Evaluation of influence of different perturbing accelerations on accuracy of probabilistic model of asteroid 2011 MD motion

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Abstract. This paper is devoted to investigation of the influence of various perturbing factors on accuracy of probabilistic model of asteroid 2011 MD motion. The degree of influence of each perturbing acceleration has been estimated using indicator of the force model. This indicator has been determined on the basis of the algorithm in which the errors of force models are tightly connected with sizes of confidence regions and displacements of the least-square estimations defined for different models of motion of asteroids. For object 2011 MD the influence of the Sun oblateness and relativistic effects caused by the Sun has been of less importance then the influence of gravitational perturbations from the Moon and the Earth oblateness. The investigation probabilistic orbital evolution on the basis of different force models has shown that the neglect of perturbing accelerations whose indicator of the force model is greater than the threshold value can lead to divergence of orbits.

1. Introduction
Investigation of orbital evolution of the small Solar System bodies, as well as the task of probability evaluation of the object collision with the Earth, require making up a perturbing accelerations model influencing the object under research. The force model used in differential equations of asteroids motion can contain different set of perturbing accelerations depending on an object orbit type and its possible encounters with other planets. The application of a less detailed model than is needed, leads to the non-occurrence of the object under study into the confidence region of the object motion. This is highly unacceptable when dealing with task of probability evaluation of asteroid collision with the Earth. In the articles [1-3] we have proposed the way of evaluation of accuracy of asteroid motion model based on calculation indicator of displacement relative to each other confidence regions constructed for different object motion models. In the present study this way has been applied to the force model for the asteroid 2011 MD which is approaching the Earth.

2. Accuracy indicator of the force model
When investigating the probabilistic evolution of the small Solar System bodies, one can evaluate the influence of perturbing accelerations included in the force model for this object, with the help of the following indicator (1).
\[ \varepsilon = \frac{\lVert \tilde{q} - \tilde{q}' \rVert}{\lVert q - q' \rVert} = \frac{1}{r}, \quad (1) \]

where \( \tilde{q} \) is the vector of asteroid orbital parameters \( q = (q_1, q_2, ..., q_m) \), obtained by the least square method for model of perturbing accelerations \( F \); \( \tilde{q}' \) is the same, but for model \( F^* \); \( \bar{q} \) is a point lying in the parametric space along the direction \( (\tilde{q} - \tilde{q}') \) on the boundary surface of the asteroid's motion confidence region \( \Phi(q) = \Phi(q')(1 + k^2 / (n - m)) \); \( \Phi(q') \) is mean square error for the full model; the value \( k \) is estimated using the Fisher distribution statistics; \( n \) is number of object observations.

The given indicator allows to take into consideration the size of confidence regions and shifts of parameters estimation of the object orbits, determined for a complete (F) and incomplete (F*) disturbing force models. This indicator can be applied not only for confidence regions at the initial time, but also for their mapping in time. The value \( \varepsilon > 1 \) obviously denotes significant shifts of parameters estimation \( \tilde{q} \) and \( \tilde{q}' \) relatively to each other. Comparison of the received value \( \varepsilon \) with some threshold value \( \varepsilon_{th} \) allows to evaluate the degree of perturbing accelerations influence. According to our numerical evaluation, the value \( \varepsilon \) should be chosen within the interval \( 0.01 < \varepsilon < 1 \) \([4-5]\).

Other methods for the investigation of the perturbations structure are given in the works \([6-7]\).

3. Numerical experiment

A near-Earth asteroid 2011 MD has been chosen as an object of the study. This object has 1555 observations in the database of IAU Minor Planet Center (MPC) performed within the interval from June, 21st to September, 3rd, 2011. On June, 27th, 2011 the asteroid approached the Earth surface relatively close – at the distance equal to 18000 km from the centre of the Earth. The complete force model for the given object contained gravitational perturbations from all major planets, Pluto, the Moon; the Earth, the Sun and Jupiter oblateness; relativistic effects caused by the Sun. With the help of the indicator \( \varepsilon \) (1), the influence of each perturbing acceleration included in the force model has been evaluated. The value \( \varepsilon = 0.01 \) has been taken as a threshold one. Table 1 contains the \( \varepsilon \) indicator values, evaluating the influence gravitational perturbations from the Moon; relativistic effects caused by the Sun; the Earth and the Sun oblateness. Indicators have been obtained within the object’s interval of observations.

| Perturbation                                      | \( \varepsilon \)   |
|--------------------------------------------------|---------------------|
| The Moon                                         | 13.32               |
| The Earth oblateness                             | 0.461               |
| The Sun oblateness                               | \( 1 \cdot 10^{-6} \) |
| The relativistic effects of the Sun              | \( 1 \cdot 10^{-5} \) |

The table shows that the indicator \( \varepsilon \) value evaluating the influence gravitational perturbations from the Moon and the Earth oblateness exceeds the \( \varepsilon \) threshold value within the interval of the observations. This means that the given perturbing accelerations should be taken into consideration when researching motion of this object. The indicator value \( \varepsilon \) evaluating the influence of the Sun oblateness and relativistic effects caused by the Sun is considerably below the threshold value.

Figure 1 and 2 gives a clear presentation of confidence regions shift when using different force models. Black color in both figures denotes an initial confidence region, which is constructed by a
boundary surface [5] with the use of a complete model of disturbing forces. Gray color denotes an initial confidence region constructed with the use of incomplete model regardless of the Earth oblateness (figure 1) and regardless of the Sun oblateness (figure 2). Figures 1 and 2 are constructed with respect to the solution obtained on the basis of the complete motion model.

The figure depicts that in the case when we do not consider perturbing accelerations from the Earth oblateness, the regions displace relatively one another, and in the case when we do not consider perturbing accelerations from the Sun oblateness – the regions superimpose on one another. These results are in good agreement with the indicator value $\varepsilon$ given in the table 1.

The further study of probabilistic orbital evolution was conducted by numerical integration of differential equations of motion for the next 100 years. That has been done for nominal orbits received on the basis of different force models. Below, figure 3 depicts evolution of the semi-major axis $a$, the eccentricity $e$, the inclination of the orbital plane to equator $i$, the longitude of the ascending node $\Omega$, the argument of perihelion $\omega$. Figure 4 shows the closest approaches of the object to the Earth within the analyzed time interval. Here, gray color marks the results obtained on the basis of motion model regardless of the Earth oblateness, and black color marks the results obtained on the basis of a complete force model.
Figure 3. The results of the probabilistic orbital evolution investigation for asteroid 2011 MD.
As we can see from the figure, after the year 2064 two orbits have significant spatial discrepancies. Moments of encounters to the Earth and distances between the object and the planet after this year also vary (figure 4).

4. Conclusion
The indicator $\varepsilon$ allows to evaluate the influence of perturbing accelerations on accuracy of probabilistic model construction of asteroids motion. This method can be applied both in making up initial confidence regions and in displaying these regions in time. As a whole, the picture of influence of the perturbing factors on accuracy of confidence regions construction can be complicated and may depend on close approaching of asteroids to the Earth or the other planets.

Acknowledgements
This work was supported by grant of the Russian Foundation for Basic Research (project № 16-32-00191 mol_a).

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