Real Time Congestion Management Using Plug in Electric Vehicles (PEV’s): A Game Theoretic Approach

FAHEEM UL HAQ, (Student Member, IEEE), PRATYASA BHUI, (Member, IEEE), AND KOTAKONDA CHAKRAVARTHI, (Student Member, IEEE)

Department of Electrical Engineering, Indian Institute of Technology Dharwad, Dharwad, Karnataka 580011, India

Corresponding author: Pratyasa Bhui (pbhui@iitdh.ac.in)

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ABSTRACT

This paper presents a game-theoretic approach for solving network congestion in the Smart Grid. A smart grid environment with high penetration of Electric Vehicles (EVs) for a bidirectional power trade provides an opportunity to use the EVs for congestion management. Optimal power injected or drawn by the EV charging stations can be obtained to mitigate the congestion problem by solving a constrained optimization problem to minimize the change in the power injected at every bus while satisfying several constraints. EV charging stations will be requested to inject or draw optimal power as obtained from the optimization problem. This paper proposes a game-theoretic approach (Greed Game) to develop a smart pricing model and design a mechanism to remove congestion in the network. A unique Nash equilibrium is achieved in the game, which optimally fulfills the line power congestion management problem and yields maximum benefit for the EVs. The algorithm was applied to 16-machine 68-bus test systems, and it has been found that the proposed mechanism maximizes the gain of the EVs and solves the congestion problem in the Grid.

INDEX TERMS

Congestion management, smart grid, generation shift factor, power system, renewable energy, game theory, electric vehicle, vehicle to grid, grid to vehicle.

I. INTRODUCTION

Increase in power demand, insufficient transmission infrastructure, and uncertainty in renewable generation cause congestion in the transmission network and overloading of transmission lines. Every transmission line is restricted by several constraints, e.g., thermal limit, stability limit, etc. The power system can be operated conservatively to avoid such congestion leading to economic loss. Alternatively, the system can be operated closer to the security limit, and any limit violation can be managed with proper corrective action. Congestion in the power network, sometimes, leads to unwanted tripping of transmission lines, curtailment of renewable energy, tripping of loads, and generation. New transmission lines can be built to solve the congestion problem, which may be expensive. Therefore, the existing power network is to be used optimally. Congestion problems are solved in several ways described below.

A. NETWORK TOPOLOGY MODIFICATION

In the last decade, there have been major changes in the generation locations (especially because of renewable energy integration) and an increase in load demand, whereas the legacy transmission infrastructure has not been updated, leading to network congestion. A Genetic Algorithm based reconfiguration Algorithm for the identification of congestion-prone areas in a power network and mitigate overload and over-voltage and the fabrication of the least loss condition in the network was discussed in [1]. Continuous evaluation of the network paths was done, and the non-congesting path with minimum transmission losses was chosen for the power transmission. Transmission switching for network topology alteration was proposed based on DC optimal power flow (DCOPF) in [2], by designing a
switching problem using binary variables. A decentralized optimization methodology for transmission line switching to locally manage the congestion of regional lines and tie lines between the regions was introduced in [3]. Congestion management was done locally by the local or regional control centers which reduced the burden on ISO (Independent System Operator) and ensured faster control actions. [4] proposed transmission topology control policies with reduced computational burden. Sensitivity information regarding line outages were utilized to restrict the topology change to few lines only. Contribution of each transaction in power market on network congestion was determined and congestion charge was calculated for each transaction in [5]. High voltage distribution network in urban power grid was partitioned into multiple smaller regions and then optimal topology was obtained for each region in [39]. It was solved as bi-level iterative line switching problem. First the operational boundary for each region was calculated, and then optimal line switching was done in each region. Topology modification procedure suffers a lot of issues in a real-time environment. For instance, the algorithms may not reach a solution because of multiple line congestion in multiple areas. Most of the topology change related works didn’t consider the effect of topology modification on the other power transactions in the neighborhood of congested zones.

B. GENERATOR POWER RESCHEDULING

The power sources near the congested lines can be rescheduled to avoid the congestion problem. Generator rescheduling-based congestion management covers several fields like optimal power flow (OPF), Economic Dispatch (ED), Market-based solutions, and unit commitment-based solutions. Optimal rescheduling of generators considering minimum deviation of generating units from scheduled levels was discussed in [6] and [7]. Optimization-based congestion management using energy storage systems (ESS) considering the state of charge (SoC) of ES, Load flexibility, and renewable penetration was discussed in [8]. A detailed study of congestion risk-aware unit commitment considering Line Transfer Margins (LTM) was done in [9]. A real-time congestion management algorithm for a competitive market considering quasi-dynamic thermal rates of transmission lines was discussed in [10]. Locational marginal price based market mechanism was designed in [11] for congestion management in distribution grids. The ISO needs to pay extra to the generators participating in congestion management, hence increases the cost [6]. Also generation rescheduling reduces generator efficiency as it deviates from the tertiary control.

C. LOAD CURTAILMENT/TRANSACTIONS

Curtailing loads in the network can avoid transmission congestion in a power network. A load control model in a virtual power plant using the concept of load reduction bidding for congestion management was discussed in [12]. A combined approach of generator rescheduling and load shedding cost minimization approach to mitigate the transmission congestion was presented in [36]. Hydro Quebec’s defense plan for extreme contingencies employing the RPTC system for detection of multiple line failures and corrective action of load shedding and resource rescheduling is detailed in [13]. With load shedding and curtailments, the DISCOMs may face reliability issues. In some instances, a DISCOM may not present a feasible load reduction bid due to critical loads. These problems make this practice of congestion management highly unreliable.

D. HARDWARE-BASED SOLUTIONS

Power electronics-based Flexible AC Transmission Systems (FACTS) can be used to change the effective transmission line reactance and control the active and reactive power through the line. A Thyristor-Controlled-Series-compensator (TCSC) was discussed in [14] which was used to improve the voltage profile of the system, transmission capability, and reduce transmission congestion. Interline Power Flow Controller (IPFC) was introduced to control the power flow of multiple transmission lines for congestion management [15]. The use of FACTS devices as control variables in a compact and reduced Security-Constrained Optimal Power Flow (SCOPF) is discussed in [16]. Concerns over the high cost and continuous operation without failure prohibit excessive deployment of the FACTS devices to solve the congestion problem. The hardware based solutions are highly localized, while the congestion can happen in any part of the network where the hardware has no corrective influence.

A real-time market-based solution for congestion management can address most of the problems faced by the conventional congestion control methods and can be adopted in the next-generation Smart Grid Environment. The line power flow, being sensitive to the bus injections [10] and [17], can be maintained within a safe limit by increasing or decreasing the bus power injections. A huge amount of distributed energy storage available in Plug-In Electric Vehicles (PEVs) provides an opportunity to control power injections at a bus. Most of the work related to Vehicle-to-Grid (V2G) has been done in the domain of the frequency regulation assistance to the grid [18]–[20]. A colored Petri-Net-based algorithm was proposed for frequency regulation support in a smart grid using EVs [20]. Frequency regulation support to the grid using EVs along with a battery energy storage system (BESS) was introduced in [18], and [40]. A decentralized control was designed in [21] to schedule the charging of EVs to fill the overnight non-PEV demand reduction of the grid. A day ahead market framework for congestion management was introduced in [22] using distribution-level market operator (DMO) and data traffic operator (DTO). In Game theory-based approaches to V2G, [23]–[25], [33], [35] and [38] used several approaches for charging EVs with certain targets. Optimal EV charging strategy without disturbing network performance was discussed in [24]. A day ahead charging schedule of the EVs was proposed in [25] considering the impacts of charging demand on spot prices.
in electricity market. A revenue maximization method using Markovian process was proposed in [35] and [38] presented a detailed survey on applications of Game-theory in vehicular technology. To the best of our knowledge, no work has been done in game-theoretic solutions to solve network congestion using EVs.

The remainder of this paper is arranged as follows. Section II describes the problem formulation. A strategic pricing model is designed in section III. A detailed overview of the game is discussed in Section IV. An analytical evaluation of the game is discussed in V. Simulation results are discussed in Section VI. Finally, a conclusion of the work is stated in section VII.

II. PROBLEM STATEMENT

Line power flow through most of the lines is limited by thermal rating, whereas few major tie-lines are limited by stability limit [10], [41], [42]. The quasi dynamic thermal rating of the line can be calculated every 5-15 min in real-time, which is robust to weather conditions [10]. Line power flow limit due to stability constraint can be calculated using repeated time-domain simulations or trajectory sensitivity analysis [26].

The line power flows are sensitive to bus injections [17]. If a power change of $\Delta P_k$ at the $k^{th}$ bus in the network causes a change in line power flow of $\Delta P_{mn}$ in the line connected between bus $m$ and $n$, the bus injection sensitivity of the line $m-n$ with respect to $k^{th}$ bus represented by $\psi_k$ is defined as:

$$\psi_{mn} = \frac{\Delta P_{mn}}{\Delta P_k}$$

Plug-in electric vehicles can play a crucial role in altering the power at buses sensitive to the congested line. Both the charging and discharging require energy trading between the EVs and the grid [34]. This energy trade can be done through aggregators and charging stations with substantial incentives to the EVs to participate in congestion management. The ancillary services aggregator will have the information regarding the availability of EVs at each charging station at each bus and inform the grid operator. The grid operator will solve an optimization problem to find the optimal corrections in power injections at every bus and send the correction requests to the aggregator. The aggregator will send the incentive signal to the EV charging stations to encourage them to participate in congestion management, Fig1.

As congestion management is a type of corrective action done in real-time, it should not cause many changes in the grid operating condition, which may lead to violation of power system security and requires detailed security studies of the grid. The optimization problem in Eq.2 will be solved to get the optimal power corrections at the buses to solve the congestion problem while maintaining power balance in the network.

$$\text{Minimize} \sum_{k=1}^{n} \Delta P_k^2$$

Subject to :

$$\Psi \times \Delta P_k = \Delta P_{mn}^{des}$$

Plug-in Electric Vehicle

Central Operator/ Control Unit

Generating Station

Renewable Energy Resource

Transmission System

Electric Vehicle

FIGURE 1. Smart grid with different energy sources and EVs.
where,

\[ \Psi_I = \begin{bmatrix} (\psi_{m,n_1})_1 & (\psi_{m,n_1})_2 & \cdots & (\psi_{m,n_1})_b \\ (\psi_{m,n_2})_1 & (\psi_{m,n_2})_2 & \cdots & (\psi_{m,n_2})_b \\ \vdots & \vdots & \ddots & \vdots \\ (\psi_{m,n_g})_1 & (\psi_{m,n_g})_2 & \cdots & (\psi_{m,n_g})_b \end{bmatrix} \]

\[ \Delta P_k = \begin{bmatrix} \Delta P_1 \\ \Delta P_2 \\ \Delta P_3 \\ \vdots \\ \Delta P_b \end{bmatrix}, \quad \Delta P_{des}^{mn} = \begin{bmatrix} \Delta P_{des}^{m_1n_1} \\ \Delta P_{des}^{m_2n_2} \\ \Delta P_{des}^{m_3n_3} \\ \vdots \\ \Delta P_{des}^{m_gn_g} \end{bmatrix} \]

where \( \Psi_I \in \mathbb{R}^{g \times b} \) is defined as Generation shift factor matrix, \( \Delta P_k \in \mathbb{R}^{b \times 1} \) is defined as the bus power injection vector and \( \Delta P_{des}^{mn} \in \mathbb{R}^{g \times 1} \) is defined as the desired change in line power flow vector, \( \forall \ m, n, g \in G \), where \( G \) is the total number of buses in the network and \( b \) is the number of buses participating in congestion management, and \( g \) is the total number of congested lines in the network.

To ease the understanding of the proposed congestion management methodology, an algorithmic flowchart is presented in Fig.2, and also explained below:

1) Step 1: Synchrophasor measurements are used by the Grid operator to detect any congestion in the lines and calculate the desired change in line power flows.
2) Step 2: The Grid operator calculates the maximum and minimum power corrections possible at every bus, based on the information shared by the charging stations.
3) Step 3: Grid operator solves the optimization problem to find the optimal corrections in EV power at different buses.
4) Step 4: Grid operator requests power corrections \( \Delta P_i^{req} \) from \( i^{th} \) bus and allocates it among charging stations.
5) Step 5: EVs participate in congestion management and get rewarded for desired corrections and penalized for undesired corrections.
6) Step 6: If congestion is not cleared, repeat Step 1-5.

Pricing functions for charging as well as discharging, as an incentive mechanism, are analyzed in the next section.

### III. STRATEGIC PRICING AS REWARD/PENALIZING FUNCTION

EVs are given incentives for charging or discharging to fulfill the objective of the grid and penalized for undesired corrective actions. The pricing functions for charging, discharging, and penalizing are defined as follows:

\[ R_i^c = \frac{\Delta P_{unmet}^{i}}{\Delta P_{desired}^{i}} \times r^c + \theta sgn(\Delta P_{i}^{req} - \Delta P_{i}^{des})(\Delta P_{i}^{req} - (\Delta P_{i}^{dis} + \Delta P_{i}^{ch})) \]  

\[ R_i^d = (1 - \frac{\Delta P_{unmet}^{i}}{\Delta P_{desired}^{i}}) \times \Pi^d + \theta' exp \left[ -(\Delta P_{i}^{req} - (\Delta P_{i}^{dis} + \Delta P_{i}^{ch}))^2 \right] \]  

\[ R_i^{pe} = r^p + \theta' |\Delta P_{i}^{req}| \]

where \( r^c, \Pi^d, \theta, \theta' \) are defined as the per unit energy pricing parameters (\$/unit) at time ‘t’. \( \Delta P_{i}^{dis} \) and \( \Delta P_{i}^{ch} \) are the EV discharging and charging power at time ‘t’ and are always considered to be positive and negative respectively. \( \Delta P_{i}^{req} \) is the power correction requested by the grid to the EVs at the \( i^{th} \) bus and \( \Delta P_i = \Delta P_{i}^{dis} + \Delta P_{i}^{ch} \).


A. ANALYSIS OF CHARGING FUNCTION

The charging price has two parts: \( \alpha \) and \( (\beta \phi) \). Both \( \alpha \) and \( \phi \) are the dynamic pricing parameters, while \( \beta \) is the control parameter.

\[
R^c_i = \alpha + \beta \phi \tag{9}
\]

The EVs will be given a chance to minimize both the parts of \( R^c_i \) for their participation in congestion management. Both terms can reach zero in the best-case scenario, which means that the EVs can charge free of cost. Each term of the charging pricing function has been analyzed below:

- **Defining \( \phi \):**
  \( \phi \) is defined as \( \phi = (\Delta P^\text{req}_i - (\Delta P^\text{dis}_i + \Delta P^\text{ch}_i)) \theta \)

  **Case I:** \( (\Delta P^\text{req}_i - (\Delta P^\text{dis}_i + \Delta P^\text{ch}_i)) = 0 \)
  
  This case pertains to the condition of critical fulfillment of the power alteration request at a bus. In this condition \( \Delta P^\text{req}_i = (\Delta P^\text{dis}_i + \Delta P^\text{ch}_i) \) and \( \phi = 0 \). Hence the EVs will not be paying the \( \beta \phi \) part of the charging price.

  **Case II:** \( (\Delta P^\text{req}_i - (\Delta P^\text{dis}_i + \Delta P^\text{ch}_i)) > 0 \)
  
  In this situation \( |(\Delta P^\text{dis}_i + \Delta P^\text{ch}_i)| > |\Delta P^\text{req}_i| \). This case pertains to the condition when the EVs draw more power than requested. The overcorrection at a bus will be increasing the charging price \( R^c_i \) as \( \phi \) takes a positive value if \( \beta \) is chosen positive value.

  **Case III:** \( (\Delta P^\text{req}_i - (\Delta P^\text{dis}_i + \Delta P^\text{ch}_i)) < 0 \)
  
  This condition indicates that the EVs have partially fulfilled the requested power change at the bus and \( |(\Delta P^\text{dis}_i + \Delta P^\text{ch}_i)| < |\Delta P^\text{req}_i| \). This is also an undesired case, and therefore, the charging price should be more than case I. Therefore, if \( \beta \) chosen negative value, the EVs need to pay \(+\beta \phi\) part of the charging cost.

- **Defining \( \beta \):**

  As described in Case II and case III, \( \beta \) must be chosen +ve in case II and -ve in case III so that positive value of \(+\beta \phi\) is always added to the charging cost avoiding a negative charging price situation for the grid, as given below:

  \[
  \beta = \text{Sgn}\{\Delta P^\text{req}_i - \Delta P^g_i\} \tag{10}
  \]

  where

  \[
  \text{Sgn}(x) = \begin{cases} 
  1, & x \geq 0 \\
  -1, & x < 0 
  \end{cases}
  \]

  \( \Delta P^g_i \) represents the total power change by EVs at \( i^{th} \) bus and is equal to \( (\Delta P^\text{dis}_i + \Delta P^\text{ch}_i) \).

- **Defining \( \omega \):**

  It is defined as the overall achievement factor in managing congestion in the network. Consider a contingency where \( P^\text{line} > P^\text{rated}_\text{line} \). The desired power correction in the line is given by \( \Delta P^\text{des}_mn = P^\text{line} - P^\text{rated}_\text{line} \). In a network where multiple lines are congested, the overall desired congestion is defined as the \( L^2 \) norm of the \( \Delta P^\text{des}_mn \) defined as

  \[
  \Delta P^\text{desired} = \left\| \Delta P^\text{des}_mn \right\|_2
  = \sqrt{(\Delta P^\text{des}_{m1n1})^2 + (\Delta P^\text{des}_{m2n2})^2 + \ldots + (\Delta P^\text{des}_{mnnn})^2}
  \]

  Assume that \( \Delta P^\text{met}_mn \) is the achieved change in line power flow. Where

  \[
  \Delta P^\text{met}_mn = \begin{bmatrix}
  \Delta P^\text{met}_{m1n1} \\
  \Delta P^\text{met}_{m2n2} \\
  \vdots \\
  \Delta P^\text{met}_{mntn}
  \end{bmatrix}
  \]

  Then the unmet power correction in the congested lines is given as:

  \[
  \Delta P^\text{unmet}_mn = \left\| \Delta P^\text{unmet}_mn \right\|_2 \tag{11}
  \]

  The magnitude of the unmet congestion management is defined as the \( L^2 \) norm of \( \Delta P^\text{unmet}_mn \) and is defined as:

  \[
  \Delta P^\text{unmet} = \left\| \Delta P^\text{unmet}_mn \right\|_2 \tag{12}
  \]

  This unmet power change will be corrected in the next interval by rescheduling the EVs again using the optimization problem (2) where \( \Delta P^\text{unmet}_mn \) is used instead of \( \Delta P^\text{des}_mn \) in Equation (3). The parameter \( \alpha \) is defined as the ratio of unmet power correction to the desired power correction.

  \[
  \alpha = \frac{\Delta P^\text{unmet}}{\Delta P^\text{desired}} \tag{12}
  \]

  If the EVs at every bus have critically fulfilled the requested power correction and cleared the congestion, then \( \Delta P^\text{unmet} = 0 \). The EVs will pay zero price for the \( \alpha \) part of the normal pricing \( R^i \). If \( \Delta P^\text{unmet} \neq 0 \) i.e. the line congestion is not completely removed, \( \alpha \neq 0 \). The EVs will have to pay partially for the first part of the charging function. This will ensure that the EVs try to fulfill the grid desire completely and avoid any non-cooperation from it. Hence we define \( \alpha \) as the overall achievement factor as it strategically helps to achieve the desired grid results.

B. ANALYSIS OF DISCHARGING FUNCTION

If the central grid optimization indicates a requirement of positive change in the bus injection (discharging), EVs are given an incentive to discharge. This is done by the strategic design of \( R^d_i \) as explained below:

\[
R^d_i = \nu + \zeta \omega \tag{13}
\]

1) DEFINING \( \omega \)

\( \omega \) is per unit electricity pricing during the congestion period and is proportional to the magnitude of congestion and is defined as:

\[
\omega \propto \left\| P^\text{line} - P^\text{rated}_\text{line} \right\|_2
\]
The higher the magnitude of the congestion, the more dangers it poses to the network. In that situation, EVs need to be given more incentive to ensure their participation in congestion management. A control parameter needs to be defined, which forces the EVs to discharge close to the requested power change only.

2) DEFINING $\zeta$

$\zeta$ plays the role of the control parameter for the above discussed condition. $\zeta$ is a Gaussian function defined as:

$$\zeta = \exp \left[ -\left( \Delta P_{t}^{req} - (\Delta P_{t}^{dis} + \Delta P_{t}^{ch}) \right)^2 \right]$$

where

$$\exp[-x^2] = \begin{cases} 1, & x = 0 \\ (0, 1), & x \neq 0 \end{cases}$$

The strategy lying in $\zeta$ is that it attains maximum value at $(\Delta P_{t}^{req} - (\Delta P_{t}^{dis} + \Delta P_{t}^{ch})) = 0$ which pertains to the case where EV’s power correction is equal to the requested power change. If there is any deviation from $(\Delta P_{t}^{req} - (\Delta P_{t}^{dis} + \Delta P_{t}^{ch})) = 0$, the EVs will not be able to harvest the full value of $\omega$; as the Gaussian function attains a maximum value for $x = 0$.

3) DEFINING $\nu$

$\nu$ is the overall goal achievement factor in the discharging function as defined below:

$$\nu = \left( 1 - \frac{|\Delta P_{t}^{unmet}|}{|\Delta P_{t}^{desired}|} \right) \Pi$$

If the value of $\Delta P_{t}^{unmet} = 0$, this means the congestion has been successfully removed by the EVs, thereby making $\frac{|\Delta P_{t}^{unmet}|}{|\Delta P_{t}^{desired}|} = 0$, and making $\nu = \Pi$. Hence making the EVs harvest the full benefit of base incentive $\Pi$.

If $\Delta P_{t}^{unmet} \neq 0$, this pertains to the case where EVs couldn’t achieve the overall goal and in this case $\frac{|\Delta P_{t}^{unmet}|}{|\Delta P_{t}^{desired}|} = (0, 1)$, thereby making $0 < \nu \leq \Pi$. Fig 4 represents $R_{i}^{d}$ as a function of bus power injection for a $\Delta P_{t}^{req}$ of 0.2943.

C. DESIGN AND ANALYSIS OF PENALIZING FUNCTION

If EVs participating in congestion management at a certain bus are asked to discharge power to the grid, but they charge from the grid, they need to be penalized. A penalizing function is also designed and is defined as:

$$R_{i}^{pe} = r' + 0.5 |\Delta P_{t}^{req}|$$

where $r'$ is the fixed base price for charging at that instant and $\theta'$ is proportional to congestion i.e. $\theta' \propto \Delta P_{t}^{unmet}$.

As the congestion increases, more will be the value of $\theta'$, and difficult it will be for EVs to choose a strategy that penalizes them. It is worth mentioning that both $R_{i}^{c}$ and $R_{i}^{pe}$ are payable to the grid by the EVs while $R_{i}^{d}$ is payable to EVs by the grid. A game-theoretic approach will be used to understand the behavior of EVs. The EV to EV game frame is defined in the next section.

IV. DEFINING EV TO EV GAME FRAME

Considering the concept of the greedy games, we define the ordinal EV to EV game frame $G_i$ in strategic form as a list of quadruple as follows:

$$G_i = \{N_i, \hat{S}_i, O_i, \gamma_i\}$$

where

$N_i = 1, 2, \ldots , N$ is the set of players at $i^{th}$ bus which are EVs in this case,

$\hat{S}_i = \text{set of all the strategies of the EVs at } i^{th} \text{ bus, designated by } (c_i, c_i^l, c_i^r)$,

$c_i^l > 0, c_i^r = 0, c_i^l < 0$. If $c_i^l > 0$, it means charging rate of the EV is positive and the EV is charging from the bus, $c_i^l < 0$ means that the charging rate is negative and the EV is discharging to the bus and $c_i^l = 0$ means that EV is neither charging nor discharging.

$O_i = \text{set of payoffs corresponding to the set of strategies } \hat{S}_i \text{ at } i^{th} \text{ bus.}$

$\gamma_i: \hat{S}_i \rightarrow O_i$ is the function that maps elements of the strategy profile set to the elements of payoff set. In our case the strategic pricing functions $R_{i}^{c}$ and $R_{i}^{d}$ and $R_{i}^{pe}$ play the role of $\gamma_i$.

A. CHARACTERIZING EV TO EV GAME

EVs at a bus are either expected to charge or discharge to make the desired power correction. This expected action of
the EVs at a bus is defined as “Desired Strategy” allocated to the bus and is represented as $S_{i}^{\text{per}}$. Based on the desired strategy assigned to the buses, the charging and discharging prices will be accordingly allotted to the buses as explained in the following sub-sections.

### B. Desired Strategy Distribution

The corrective action at the buses are to be done by the EVs which have an arbitrary set of strategies completely beyond the control of the Grid, ranging from charging, discharging to layoff as defined by $\hat{S}_{i}$ in section IV. To gain a hold on the EV to EV game from the grid side, a certain expected action called Desired strategy $S_{i}^{\text{per}}$ is allocated to the buses such that $S_{i}^{\text{per}} \in \hat{S}_{i}$.

It is worth mentioning that the allocated strategies do not control the strategy of the EVs, though it controls the payoffs given in different strategic scenarios of the EVs. The discussion on strategic payoff settlement is detailed in section IV.C. The solution $\Delta P_{i}^{\text{bus}}$ of the optimization problem [2] can be either positive, negative, or zero for different buses participating in the congestion management. If the optimized power correction at a bus is positive, i.e., $\Delta P_{i}^{\text{bus}} > 0$, the grid would expect the EVs to discharge. Similarly, if the optimized power correction is negative, i.e., $\Delta P_{i}^{\text{bus}} < 0$, then the bus is expected to reduce the power, which can be interpreted as EVs being charged. This expected action of the bus is defined as the “Desired Strategy” allocated to the bus. The buses with optimized power correction $\Delta P_{i}^{\text{bus}} = 0$ will not be assigned any desired strategy. The desired strategy distribution is given in Table 2 which provides a clear idea of how a centralized control assigns desired strategies to different buses in different situations.

### C. Strategic Payoff Settlement

If the strategy chosen by the EV matches the allocated desired strategy at a bus, then the EV will be rewarded, and if it opposes the desired strategy, then the EV will be penalized. Irrespective of the allocated permissible strategy, in layoff condition, i.e., $c_{i}^{\text{ch}} = 0$, an EV will neither be penalized nor rewarded. The reward is the pricing functions defined in (6) and (7) while the function in (8) is a penalty. The settlement of the payoffs is given in Table 3.

### V. Analytical Evaluation of EV to EV Game

#### A. Best Strategy Response

The best response of an EV to a fixed desired strategy and to the strategies of all other EVs is a set of all the strategies that follow either of the two below equations:

$$s_{n}^{\text{best}}(S_{i}^{\text{per}}, S_{-n}) = \arg \max_{S_{n}} (R_{i}^{d})$$  \hspace{1cm} \text{(13)}

$$s_{n}^{\text{best}}(S_{i}^{\text{per}}, S_{-n}) = \arg \min_{S_{n}} (R_{i}^{e})$$  \hspace{1cm} \text{(14)}

where $S_{n}^{\text{best}}$, $S_{i}^{\text{per}}$, $S_{-n}$ are defined as the best strategy response of $n^{th}$ EV at the $i^{th}$ bus, permissible strategy at the $i^{th}$ bus and strategy of all other EVs other than $n^{th}$ EV respectively. $s_{n}^{\text{best}}$, $S_{i}^{\text{per}}$, $S_{-n}$ are the cardinalities of the strategy set defined in the ordinal game frame $G_{i}$, i.e., $s_{n}^{\text{best}}, S_{i}^{\text{per}}, S_{-n} \subseteq \hat{S}_{n}$.

Equations (13) and (14) signifies the fact that the best strategy of the EV should be able to either maximize the discharging revenue or minimize the charging expenditure of the EV.

#### B. Nash Equilibrium of EV to EV Game

**Claim 1:** We claim that the EV to EV game for congestion management attains a solution at each bus which is a function of the EV strategies and is known as Nash equilibrium. The Nash equilibrium is defined as follows.

**Definition 1:** Nash equilibrium of a game is defined as the set of strategies of all the players, such that there is no incentive for unilateral deviation from that set of strategies for any player of the game. Alternatively, a Nash equilibrium is also defined as the intersection of the best response strategies of the players in a game. The Nash equilibrium of the EV to EV game is a strategy vector $S_{i} = [S_{n}^{*}, \forall n \in N_{i}], (S_{n}^{*} \in \hat{S}_{n})$ for which an incentive of an EV at the Nash Equilibrium is more than an incentive to an EV at any other strategy.

$$O_{i}(S_{n}^{*}, S_{-n}^{*}, S_{i}^{\text{per}}) \geq O_{i}(S_{n}, S_{-n}, S_{i}^{\text{per}})$$  \hspace{1cm} \text{(15)}

To validate the above statement, we consider bus I and bus II, where bus I is allotted the desired strategy of charging, i.e., $c_{i}^{\text{ch}} > 0$, and bus II is assigned the desired strategy of discharging, i.e., $c_{i}^{\text{dis}} < 0$ to clear certain congestion in the network. Consider the grid has sent the requests as $P_{i}^{\text{req}}$ and $P_{i}^{\text{req}}$ to buses I and II, respectively. There may be two cases in EV to EV game.

1) **CASE I:** $\Delta P_{i}^{\text{dis}} + \Delta P_{i}^{\text{ch}} = \Delta P_{i}^{\text{req}}$

Considering the co-operation of the EV’s at two buses to make $\Delta P_{i}^{\text{net}} = 0$. Then for both the buses, the Nash equilibrium is achieved on all those sets of strategies where:

$$\Delta P_{i}^{\text{dis}} + \Delta P_{i}^{\text{ch}} = \Delta P_{i}^{\text{req}}$$  \hspace{1cm} \text{(16)}
Hence for bus I, the Nash equilibrium will occur at all those strategies where \((\Delta P_{I}^{\text{dis}} + \Delta P_{I}^{\text{ch}}) = \Delta P_{I}^{\text{req}}\). Similarly for bus II, the Nash equilibrium will occur at all the sets of strategies that follow \(\Delta P_{II}^{\text{dis}} = (\Delta P_{I}^{\text{dis}} + \Delta P_{II}^{\text{ch}})\). Hence we can say \((\Delta P_{I}^{\text{dis}} + \Delta P_{II}^{\text{ch}})^{*}\) is the Nash Equilibrium of the EV to EV game for case I if and only if at each bus. \((\Delta P_{I}^{\text{dis}} + \Delta P_{II}^{\text{ch}})^{*} = \Delta P_{I}^{\text{req}}\). There can be infinite possibilities of \((\Delta P_{I}^{\text{dis}} + \Delta P_{II}^{\text{ch}})^{*}\) that can add up and equalize to \(\Delta P_{I}^{\text{req}}\).

2) CASE II \(\Delta P_{I}^{\text{dis}} + \Delta P_{II}^{\text{ch}} \neq \Delta P_{I}^{\text{req}}\)

This case pertains to the non-cooperation of the EVs at the two buses. Non-cooperation of EVs is defined as the condition when EVs fail to fulfill the grid request either fully or partially, for which there is a possibility of \(\Delta P_{\text{unmet}} \neq 0\). The number of EVs available for charging/discharging in a charging station keeps on changing, and the SoC of some EVs may reach the maximum or minimum limits. Therefore, power corrections by the EVs may not be exactly equal to the power requested by the grid operator. In this case, the Nash equilibrium is achieved at

\[
(\Delta P_{I}^{\text{dis}} + \Delta P_{II}^{\text{ch}})^{*} = \min \left( \left| \Delta P_{I}^{\text{req}} \right| , \left| \Delta P_{I}^{\text{dis}} + \Delta P_{II}^{\text{ch}} \right| \right)
\]

For the Nash equilibrium defined in case II, there are again infinite combinations of \(\Delta P_{I}^{\text{dis}}\) and \(\Delta P_{II}^{\text{ch}}\) for which the term \((\Delta P_{I}^{\text{dis}} + \Delta P_{II}^{\text{ch}})^{*}\) attains the same value.

The proof of claim 1 is given in the appendix.

C. UNIQUENESS OF NASH EQUILIBRIUM

Claim 2: Though there being infinite combinations of \(\Delta P_{I}^{\text{dis}}\) and \(\Delta P_{II}^{\text{ch}}\) that sum up to achieve the Nash equilibrium, we claim that there is a unique combination of \(\Delta P_{I}^{\text{dis}}\) and \(\Delta P_{II}^{\text{ch}}\) that achieve the Nash Equilibrium for EV to EV game for both the cases discussed above. The unique combination of \(\Delta P_{I}^{\text{dis}}\) and \(\Delta P_{II}^{\text{ch}}\) are given as follows:

- **Uniqueness for case I**

For all the buses satisfying to be the cardinalities of the below mentioned Set

\[ i : i \in b \quad \Delta P_{i}^{*} < 0 \]

The unique Nash Equilibrium is defined as:

\[
\Delta P_{I}^{\text{ch}} = \Delta P_{I}^{\text{req}}, \quad \Delta P_{I}^{\text{dis}} = 0
\]

This is a unique combination of \(\Delta P_{I}^{\text{ch}}\) and \(\Delta P_{I}^{\text{dis}}\) for which \((\Delta P_{I}^{\text{dis}} + \Delta P_{II}^{\text{ch}})^{*} = \Delta P_{I}^{\text{req}}\).

Similarly, for the buses qualifying for the cardinalities of the below-given set.

\[ i : i \in b \quad \Delta P_{i}^{*} > 0 \]

The unique Nash equilibrium is given as:

\[
\Delta P_{I}^{\text{ch}} = 0, \quad \Delta P_{I}^{\text{dis}} = \Delta P_{I}^{\text{req}}
\]

This unique combination of \(\Delta P_{I}^{\text{dis}}\) and \(\Delta P_{II}^{\text{ch}}\) achieves the optimal solution for both grid and the EVs.

- **Uniqueness for Case II**

As discussed earlier there can be several combinations of \(\Delta P_{I}^{\text{dis}}\) and \(\Delta P_{II}^{\text{ch}}\) that yield the Nash Equilibrium for case II of EV to EV game. But as per the claim 2 we define a unique combination of \(\Delta P_{I}^{\text{ch}}\) and \(\Delta P_{I}^{\text{dis}}\) that yields the Nash Equilibrium for case II. The buses qualified for the cardinalities of the below given set.

\[ i : i \in b \quad \Delta P_{i}^{*} < 0 \]

The unique Nash Equilibrium for case II is defined as:

\[
\Delta P_{i}^{\text{ch}} = \min \left( \left| \Delta P_{i}^{\text{ch}} \right| , \left| \Delta P_{i}^{\text{req}} \right| \right), \quad \Delta P_{i}^{\text{dis}} = 0
\]

Similarly for the cardinalities of the below set

\[ i : i \in b \quad \Delta P_{i}^{*} > 0 \]

The Nash Equilibrium is defined as:

\[
\Delta P_{i}^{\text{dis}} = \min \left( \left| \Delta P_{i}^{\text{dis}} \right| , \left| \Delta P_{i}^{\text{req}} \right| \right), \quad \Delta P_{i}^{\text{ch}} = 0
\]

This defines the uniqueness of the Nash Equilibrium of EV to EV game. The proof for uniqueness of the of the Nash Equilibrium is given in the Appendix.

VI. SIMULATION RESULTS AND DISCUSSION

A. CONGESTION MANAGEMENT IN TRADITIONAL POWER SYSTEM

The proposed method is tested on IEEE 16 machine 68 Bus test bed [27]. The lines between buses 27-53 and 53-54 are considered congested, and a congestion clearance of −0.5 p.u. each is considered for this case. 16 load buses with significant EV load are considered for congestion control; 8 from each area 1 and 2. Considering the growth of EVs in the market, it can be estimated that EVs will be the significant load share in the near future [28]. For our case, we have considered 40% of the EV load at these buses, which gives us the flexibility of altering up to 40% load at the buses(either charging or discharging). Based on solutions obtained from the optimization problem we have,

\[ i \in (25, 26, 27, 28, 64) : \Delta P_{i}^{*} < 0 \]

And

\[ i \in (17, 33, 36, 40, 47, 48, 53, 56, 59, 60, 61) : \Delta P_{i}^{*} > 0 \]

The time instant of the EVs for participating in congestion control has been taken as random, in between 0 and 200s. Charging stations at a particular bus will start participating in congestion control at different instants. However, for simplicity, it has been assumed that they participate simultaneously for ease of analysis. Considering one bus for charging analysis and one bus for discharging analysis will represent all other buses with similar desired strategy. In our case, we have considered bus 25 for charging analysis and bus 60 for discharging analysis. Charging starts at bus 25 around 20 sec, and discharging starts at bus 60 around 150 sec of the congestion interval. The analysis is done for two cases.

case 1: Exact bus power injections.

case 2: Random bus power injections.
1) EXACT BUS POWER CORRECTION
In this case, we consider a highly optimistic situation rather than a realistic one where we consider that every bus draws or injects exactly the same amount as requested. The line congestion in lines 27-53 and 53-54 is completely cleared, as shown in Fig(5). Considering the charging price at bus 25 in this case, it is observed that the charging price becomes almost equal to zero, Fig 6; because the bus is correcting exactly by $-0.4931$ p.u. as requested to it. Similarly, bus 60 also corrects exactly by the requested amount of power correction i.e., $+0.2943$ p.u., so it can be observed that the discharging price settles almost to ₹ 20/kWh Fig(7).

2) RANDOM BUS POWER INJECTION
This case is a realistic approach towards congestion management and considers the uncertainties in fulfilling the grid request by the EVs. The grid operator sends the power requests to charging stations considering the EV availability at the start of the interval. However, the number of EVs available for charging and discharging changes continuously, and also SoC of some EVs may reach maximum or minimum limits. This may cause the EV power to vary from the requested power. However, the amount of deviation from the requested power is uncertain.

A statistical analysis of this case shows that if the EVs correction injections are between $-50\%$ to $+50\%$ of what is requested, the line power is corrected within 90\% to 110\% of the desired value with a 60.012\% probability. Further, if the uncertainty of EVs is restricted to $+25\%$ to $-25\%$ of the requested power, then the probability that the line powers will remain within 90\% to 110\% of the desired line increases from 60.012\% to 90.011\%, as shown in Fig (8). Suppose Bus 25 provides a correction of $-0.2693$ p.u instead of desired correction of $-0.4931$ p.u. Other buses also inject random power between 50\% and 150\% of desired power. There is 1.25\% and 2.59\% unmet of the desired power correction in the lines 27-53 and 53-54, respectively. Since the unmet correction is small so they will be able to extract a large percentage of the benefit of the first part of the charging cost, but due to an ample amount of the deviation from
the requested correction, EVs at the Bus 25 will not be able to enjoy the second part of the charging cost thereby making the charging price settle at a non-zero value lesser than the nominal charging price as shown in Fig (10).

In the case of the discharging price analysis, we consider bus 60 having a request of +0.2943 p.u. A random but optimistic discharging correction of +0.2912 p.u. was done on bus 60. Due to the small percentage of unmet desired correction in the lines and the requested power correction deviation, they can extract a large percentage of benefit from both the parts of the discharging price, thereby settling the discharging price at ₹18.75/kWh. This can be seen from the Fig (11).

The unmet line power correction can be further reduced by sending a re-dispatch request to the EVs by the Grid considering unmet power correction as the new congestion.

The simulation results are in coherence with the fact that the best case for the Grid and the EVs is that every bus tries to achieve a Nash Equilibrium by sticking as close as possible to the requested bus power correction.

### B. CONGESTION CONTROL WITH HIGH RENEWABLE ENERGY RESOURCE (RER) PENETRATION

High Penetration of the Renewable Energy Resources in the Grid may cause transmission Congestion and highly uncertain ambient conditions of renewable energy resources are responsible for curtailment in the renewable generation [29], [30], [31], [37]. Nine doubly-fed induction generators (DFIGs) were connected into the original 16 machine 68 test system at the buses indexed as 21, 28, 33, 37, 40, 42, 52, 56 and 65 with an initial power of 32.86 p.u. and an equal amount of generation was reduced from conventional generators. Wind speed was generated using Auto Regressive Moving Average model (ARMA) [32], [43]. The tie-line power between bus 60 and 61 carrying power from area 2 to area 1 varies between −2.529pu to −4.6077pu due to variable wind power as shown in (Fig. 13) and Fig(12). Power from area 1 to area 2 is taken positive while from area 2 to area 1 is taken negative. The proposed method was used, and the EVs were dispatched every minute. It was observed that the RMS of the line power deviation was reduced by about 89%, and the max swing in line power deviation was reduced by 77%, Fig(13).

The line power deviation can be further reduced by re-dispatching the EVs faster e.g., once every 30 seconds, Fig(13). The analysis of the observations is given in Table 4.

| Line Power Deviation | No Control | 1 Min Control | 30 Sec Control |
|----------------------|------------|---------------|----------------|
| Line Power Deviation (KVR) | -2.529pu | -0.4789pu | -0.2466pu |
| Max Deviation | -2.529pu | -0.4789pu | -0.2466pu |
VII. CONCLUSION AND FUTURE WORK

This work considers EVs as Mobile Energy Storage System (MESS), which can provide a number of ancillary services in a smart grid environment to maintain the security of the power system. We provide a novel method of congestion control in a smart grid environment with EVs using the concept of greed games. The conclusions of this paper are summarized as follows:

1) A constrained optimization problem was formulated to find optimal injections at sensitive buses to correct line power deviations and solve the congestion problem.
2) The aggregator can successfully allocate the optimal injection request among different charging stations at a bus. The charging stations give EVs incentives to participate in congestion control.
3) It has been proven that the EVs can reduce their charging price below the nominal price by participating in congestion management. Under an ideal scenario, the charging price can reach zero. Similarly, the EVs can discharge to the grid and earn significantly higher than the nominal charging price while solving the grid congestion problem.
4) There exists a Nash equilibrium, and there is no unilateral benefit for deviating from this Nash equilibrium.
5) The method has been successfully applied to IEEE 16 Machine 68 Bus Test Systems with traditional generators and high renewable penetration. The method can control the line power flow and maintain the desired value and successfully solve the congestion problem.

Even after considering uncertainties in EV participation, there is a significant improvement in congestion. Therefore, power correction by the EVs is allowed to vary from the requested power depending on the EV priorities. The charging stations can design a localized optimization problem that effectively accounts for the priorities of the EVs and can accordingly schedule the power corrections to the grid request. However, this is out of the scope of the present work, and we plan to include this in the future extension of this work.

APPENDIX

A. PROOF OF CLAIM I

A two-bus case generalizes the problem by considering charging at one bus and discharging at another bus. Discussing the charging case of one bus will be valid for all the buses where charging is desired and discussion of one discharging bus is valid for all other buses with discharging as the desired strategy.

For a congestion vector \( \Delta P_{\text{des}} \) in a network and \( \Delta P_{\text{desired}} \) being expande of the congestion vector. Assuming for bus I, \( S_{\text{per}}^{\text{I}} = c_i > 0 \) and the charging request of \( \Delta P_{\text{req}}^{\text{I}} \). For bus II, \( S_{\text{per}}^{\text{II}} = c_{\text{II}} > 0 \) and a discharging request of \( \Delta P_{\text{req}}^{\text{II}} \) in response to \( \Delta P_{\text{des}} \). The pricing signal at buses I and II will be \( R_i^{\text{I}} \) and \( R_i^{\text{II}} \), respectively for desired strategies. Incentives to EVs are maximum at the Nash Equilibrium as explained below.

1) CASE I: CRITICAL FULFILMENT OF POWER CORRECTION REQUESTS AT EACH BUS

If \( \Delta P_{\text{dis}}^{\text{I}} + \Delta P_{\text{ch}}^{\text{I}} = \Delta P_{\text{req}}^{\text{I}} \) and \( \Delta P_{\text{dis}}^{\text{II}} + \Delta P_{\text{ch}}^{\text{II}} = \Delta P_{\text{req}}^{\text{II}} \).

Since power correction requests are the solution of the optimization problem (2) and the solution satisfies the constraint (3) i.e.

\[
\Psi_I \times \left[ \Delta P_{\text{dis}}^{\text{I}} + \Delta P_{\text{ch}}^{\text{I}} = \Delta P_{\text{req}}^{\text{I}} \right] = \Delta P_{\text{met}} = \Delta P_{\text{des}}
\]

As Equation 18 holds true, the unmet congestion vector is a null vector given as.

\[
\Delta P_{\text{unmet}} = \Delta P_{\text{des}} - \Delta P_{\text{met}} = 0
\]

This means that \( \Delta P_{\text{unmet}} = 0 \) and

\[
\frac{\Delta P_{\text{unmet}}}{\Delta P_{\text{desired}}} = 0
\]

At Bus I

\[
\theta \text{sgn}(\Delta P_{\text{I}}^{\text{req}} - \Delta P_I)(\Delta P_{\text{I}}^{\text{req}} - [\Delta P_{\text{I}}^{\text{dis}} + \Delta P_{\text{I}}^{\text{ch}}]) = 0 \quad \forall \quad \Delta P_{\text{I}}^{\text{dis}} + \Delta P_{\text{I}}^{\text{ch}} = \Delta P_{\text{I}}^{\text{req}}
\]

Combining Equations 19 and 20

\[
R_I = 0
\]

Since

\[
\min(R_I) = 0
\]

As evident from Equations 22 and 23, the strategy profile \( \Delta P_{\text{I}}^{\text{dis}} + \Delta P_{\text{I}}^{\text{ch}} = \Delta P_{\text{I}}^{\text{req}} \) minimizes the charging price according to Equation 14. Hence can be defined as the best strategy response of the EVs at a bus with charging as the desired strategy.

At Bus II

\[
\exp \left[ -\left( \Delta P_{\text{II}}^{\text{req}} - \left( \Delta P_{\text{II}}^{\text{dis}} + \Delta P_{\text{II}}^{\text{ch}} \right) \right)^2 \right] = 1 \quad \forall \quad \Delta P_{\text{II}}^{\text{dis}} + \Delta P_{\text{II}}^{\text{ch}} = \Delta P_{\text{II}}^{\text{req}}
\]

Since for bus II, the incentive is.

\[
R_{\text{II}}^{\text{I}} = \left( 1 - \frac{\Delta P_{\text{unmet}}}{\Delta P_{\text{desired}}} \right) \Pi' + \theta' \exp \left[ -\left( \Delta P_{\text{II}}^{\text{req}} - \left( \Delta P_{\text{II}}^{\text{dis}} + \Delta P_{\text{II}}^{\text{ch}} \right) \right)^2 \right]
\]

\[
R_{\text{II}}^{\text{I}} = (1 - 0) \quad \Pi' + \theta' = \Pi' + \theta'
\]
Since.
\[
\max(R_P^i) = \Pi^I + \theta^r
\]  
(27)
Evident from Equations 26 and 27, the strategy profile \(\Delta P_{i}^{dis} + \Delta P_{i}^{ch} = \Delta P_{i}^{req}\) maximizes the discharging price in accordance with Equation 13. Hence can be defined as the best strategy response of the EVs at a bus with discharging as the desired strategy. Based on Definition 1, the Nash Equilibrium is the intersection of the best strategy responses of players in a game.

Hence for charging and discharging buses the Nash Equilibrium is defined as
\[
(\Delta P_{i}^{dis} + \Delta P_{i}^{ch}) = \Delta P_{i}^{req}
\]

2) CASE II: NON CRITICAL FULFILMENT OF POWER REQUEST AT THE BUS

For such a case there arise two sub cases i.e., \(|\Delta P_{i}^{avd}|<|\Delta P_{i}^{req}| \) and \(|\Delta P_{i}^{avd}| >|\Delta P_{i}^{req}| \)

Where \(\Delta P_{i}^{avd}\) is the power available with the EVs for bus power correction

a: SUB-CASE I: DEFICIENT IN POWER AVAILABILITY

\(|\Delta P_{i}^{avd}|<|\Delta P_{i}^{req}|\)

In such as case, the EVs at a bus cannot provide the desired power correction. For such a situation, the EVs have two options available either they can do the power correction available to them or less than that. Considering that the EVs do all the power correction available to them i.e.,
\[
\Delta P_{i}^{dis} + \Delta P_{i}^{ch} = \Delta P_{i}^{avd} \text{ and } |\Delta P_{i}^{avd}| < |\Delta P_{i}^{req}|.
\]
Considering bus I with \(S_{i}^{per} = c_{i} > 0\), the payoff of EVs will be.

\[
R_{i}^{c} = \left| \frac{\Delta \text{pannet}}{\Delta \text{desired}} \right| r'(D') + \theta Sgn(\Delta P_{i}^{req} - \Delta P_{i}^{ch})(\Delta P_{i}^{req} - |\Delta P_{i}^{dis} + \Delta P_{i}^{ch}|)
\]
Since \(\Delta P_{i}^{dis} + \Delta P_{i}^{ch} = \Delta P_{i}^{avd}\)
\[
R_{i}^{c} = \left| \frac{\Delta \text{pannet}}{\Delta \text{desired}} \right| r'(D') + \theta Sgn(\Delta P_{i}^{req} - \Delta P_{i}^{ch})(\Delta P_{i}^{req} - \Delta P_{i}^{avd})
\]
Now consider the case where EVs will do lesser power correction than their available power correction capability. Assuming the power correction done by the EVs in this case is
\[
(\Delta P_{i}^{dis} + \Delta P_{i}^{ch}) = \Delta P_{i}^{req}
\]
where \(|\Delta P_{i}^{dis}| < |\Delta P_{i}^{req}|\).

The payoff for EVs is
\[
R_{i}^{c} = \left| \frac{\Delta \text{pannet}}{\Delta \text{desired}} \right| r'(D') + \theta Sgn(\Delta P_{i}^{req} - \Delta P_{i}^{ch})(\Delta P_{i}^{req} - \Delta P_{i}^{avd})
\]
Since the designing of \(R_{i}^{c}\) is done in such a way that every value closer to the requested power yields better results for the EVs than the values farther from the requested power \(\Delta P_{i}^{req}\). Hence both \(|\Delta P_{i}^{avd}|<|\Delta P_{i}^{req}|\) but \(|\Delta P_{i}^{avd}|>|\Delta P_{i}^{req}|\) which yields the conclusion that for a certain value of \(|\Delta P_{i}^{avd}|\)
\[
\frac{\Delta \text{pannet}}{\Delta \text{desired}} > r'(D')
\]
This is because for a certain value of \(|\Delta P_{i}^{avd}|\), the following three strategies.

I. \(\Delta P_{i}^{dis} + \Delta P_{i}^{ch} < \Delta P_{i}^{req}\)

II. \(\Delta P_{i}^{dis} + \Delta P_{i}^{ch} = \Delta P_{i}^{req}\)

III. \(\Delta P_{i}^{dis} + \Delta P_{i}^{ch} > \Delta P_{i}^{req}\)

For Bus I.
\[
R_{i}^{c}(\Delta P_{i}^{dis} + \Delta P_{i}^{ch} < \Delta P_{i}^{req}) > R_{i}^{c}(\Delta P_{i}^{dis} + \Delta P_{i}^{ch} = \Delta P_{i}^{req})
\]
This is because for a certain value of \(|\Delta P_{i}^{avd}|\)
\[
\frac{\Delta \text{pannet}}{\Delta \text{desired}} > r'(D')
\]
\[
\theta Sgn(\Delta P_{i}^{req} - \Delta P_{i}^{ch})(\Delta P_{i}^{req} - |\Delta P_{i}^{dis} + \Delta P_{i}^{ch}|)
\]
while
\[
\theta Sgn(\Delta P_{i}^{req} - \Delta P_{i}^{ch})(\Delta P_{i}^{req} - |\Delta P_{i}^{dis} + \Delta P_{i}^{ch}|)
\]
this pertains to the fact
\[
R_{i}^{c}(\Delta P_{i}^{dis} + \Delta P_{i}^{ch} = \Delta P_{i}^{req}) > R_{i}^{c}(\Delta P_{i}^{dis} + \Delta P_{i}^{ch} < \Delta P_{i}^{req})
\]
In a similar manner.
\[
\theta Sgn(\Delta P_{i}^{req} - \Delta P_{i}^{ch})(\Delta P_{i}^{req} - \Delta P_{i}^{ch})
\]
\[
> 0 \quad \forall \quad (\Delta P_{i}^{dis} + \Delta P_{i}^{ch} < \Delta P_{i}^{req})
\]
while
\[ \theta Sgn(\Delta P_{i}^{req} - \Delta P_{i})(\Delta P_{i}^{req} - (\Delta P_{i}^{dis} + \Delta P_{i}^{ch})) = 0 \quad \forall (\Delta P_{i}^{dis} + \Delta P_{i}^{ch} = \Delta P_{i}^{req}) \]

this pertains to the fact
\[ R_{i}(\Delta P_{i}^{dis} + \Delta P_{i}^{ch} = \Delta P_{i}^{req}) > R_{i}(\Delta P_{i}^{dis} + \Delta P_{i}^{ch} > \Delta P_{i}^{req}) \]

Since both the payoffs i.e. \( R_{i} \) and \( R_{i}^{d} \) are designed for the best value to be attained at \( \Delta P_{i}^{dis} + \Delta P_{i}^{ch} = \Delta P_{i}^{req} \). So the best value will be yielded at II instead of I and III. This can be proven as follows.

For charging buses the payoff is
\[ R_{i}^{c} = \frac{\Delta P_{i}^{dis}}{\Delta P_{i}^{dis} - \Delta P_{i}^{ch}} + \theta Sgn(\Delta P_{i}^{req} - \Delta P_{i})(\Delta P_{i}^{req} - [\Delta P_{i}^{dis} + \Delta P_{i}^{ch}]) \]

For a certain value of \( \frac{\Delta P_{i}^{dis}}{\Delta P_{i}^{dis} - \Delta P_{i}^{ch}} \).
\[ \min[\theta Sgn(\Delta P_{i}^{req} - \Delta P_{i})(\Delta P_{i}^{req} - [\Delta P_{i}^{dis} + \Delta P_{i}^{ch}])] = 0 \quad \forall (\Delta P_{i}^{dis} + \Delta P_{i}^{ch} = \Delta P_{i}^{req}) \]

And
\[ \min[\theta Sgn(\Delta P_{i}^{req} - \Delta P_{i})(\Delta P_{i}^{req} - [\Delta P_{i}^{dis} + \Delta P_{i}^{ch}])] = 0 \quad \forall (\Delta P_{i}^{dis} + \Delta P_{i}^{ch} = \Delta P_{i}^{req}) \]

This means the charging price is minimum when the EVs use only the requested capacity of their total available capacity.

For buses with discharging as desired strategy, the payoff is
\[ R_{i}^{d} = (1 - \frac{\Delta P_{i}^{dis}}{\Delta P_{i}^{dis} - \Delta P_{i}^{ch}})P_{i}^{t} + \theta \exp[-(\Delta P_{i}^{req} - (\Delta P_{i}^{dis} + \Delta P_{i}^{ch}))^{2}] \]

The three cases defined in I, II and III are also valid for the discharging buses. For a certain value of \( \frac{\Delta P_{i}^{dis}}{\Delta P_{i}^{dis} - \Delta P_{i}^{ch}} \).
\[ \max[\exp[-(\Delta P_{i}^{req} - (\Delta P_{i}^{dis} + \Delta P_{i}^{ch}))^{2})] = 1 \quad \forall (\Delta P_{i}^{req} - (\Delta P_{i}^{dis} + \Delta P_{i}^{ch}))^{2} = 0 \]

Which is achievable for \( \Delta P_{i}^{dis} + \Delta P_{i}^{ch} = \Delta P_{i}^{req} \). All other cases defined in I and III will yield \( \exp[-(\Delta P_{i}^{req} - (\Delta P_{i}^{dis} + \Delta P_{i}^{ch}))^{2}) < 1 \) and cannot be considered as the best strategies for discharging EVs as the discharging price in I and III is lesser than that of II.

Summing up the entire discussion of case II of claim 1 into a mathematical statement, the Nash equilibrium for case II of claim 1 is defined as.
\[ \Delta P_{i}^{dis} + \Delta P_{i}^{ch} = \min \left[ \left( \frac{\Delta P_{i}^{req}}{\Delta P_{i}^{dis} + \Delta P_{i}^{ch}} \right), (\Delta P_{i}^{dis} + \Delta P_{i}^{ch}) \right] \]

where \( (\Delta P_{i}^{dis} + \Delta P_{i}^{ch}) = \Delta P_{i}^{req} \) is the total available power correction capability of EVs at th bus. Hence we prove that EV to EV game for congestion management attains a Nash Equilibrium at each bus as reflected in claim 1.

**B. PROOF OF CLAIM II**

To justify the statement in claim 2 we hold that for power exchange between the bus and the charging station, both charging and discharging will not happen simultaneously. This can be proven by taking certain considerations into account.

To prove that discharging and charging will not happen at the same time at a bus we have to prove the following two situations.

- No EV at the buses with the desired strategy as charging will choose to discharge.
- No EV at the buses with the desired strategy as discharging will choose to charge.

We will first prove that the EVs at charging buses will not choose to discharge. Considering a bus with the desired strategy as charging and let us take an EV that chooses to charge. We will show that the EV at this bus will not choose to discharge or switch to discharge. This can be proved by considering the payoffs of the EV in case of charging and switching from the charging strategy.

For an EV at the charging bus, the payoff for charging will be
\[ \gamma_{i}(s_{i} = c_{i} > 0, s_{-i} \in \hat{S}) = R_{i}^{c} \]

The above payoff for the charging is a reward as it minimizes the charging price.

The payoff for discharging of an EV at a charging bus is
\[ \gamma_{i}(s_{i} = c_{i} < 0, s_{-i} \in \hat{S}) = R_{i}^{pe} \]

Based on the interests of the EVs
\[ \gamma_{i}(s_{i} = c_{i} > 0, s_{-i} \in \hat{S}) > \gamma_{i}(s_{i} = c_{i} < 0, s_{-i} \in \hat{S}) \]

This accounts for the fact that the EV with a charging strategy will be paying less than the regular price for charging, and the discharging EV will be paying towards the grid for selling the energy. Paying for selling its own power is the worst case for an EV rather than paying less than the regular price for charging. Hence an EV at a bus with charging as the desired strategy defined by \( [i : i \in b \quad \forall \Delta P_{i}^{c} < 0] \) will not choose to discharge or switch to discharge.

Now we will prove that the EVs at the buses with the desired strategy as discharging will not choose or switch to charging.

For a bus with discharging as the desired strategy, the payoff for the discharging EV is
\[ \gamma_{i}(s_{i} = c_{i} < 0, s_{-i} \in \hat{S}) = R_{i}^{d} \]

And the payoff for the charging EVs is
\[ \gamma_{i}(s_{i} = c_{i} > 0, s_{-i} \in \hat{S}) = R_{i}^{pe} \]

Since \( R_{i}^{d} \) is a reward as it enables the EVs to maximize their selling price of the discharging power, while \( R_{i}^{pe} \) is a penalization function and is a function of the magnitude of the congestion. As the EV at a discharging bus starts to charge,
the magnitude of congestion increases, and the EV has to pay a much higher price for charging than it pays in normal times. Hence this penalization function makes sure that the EVs do not charge at the discharging buses. For the buses defined by

\[ \{i : i \in b \, \forall \Delta P^s_i > 0\} \]

we can say that

\[ \gamma_i(s_i = c_i^l < 0, s_{-i} \in \delta) > \gamma_i(s_i = c_i^l > 0, s_{-i} \in \delta). \]

Since the payoff for discharging is better than the charging, they will choose to discharge, which assures that they will not switch to charging.

Based on the above discussion, we claim that for every bus with an allotted desired strategy, the following equation is valid with complementary slackness:

\[ \Delta P_{\text{dis}}^i \times \Delta P_{\text{ch}}^i = 0 \]

If that is the case then all the buses with desired strategy \( c_i^l > 0 \) will have \( \Delta P_{\text{dis}}^i = 0 \). Hence for the Nash equilibrium discussed in claim 1 i.e. \( (\Delta P_{\text{dis}}^i + \Delta P_{\text{ch}}^i) = \Delta P^s_i \) reduces to \( \Delta P_{\text{dis}}^i = \Delta P_{\text{ch}}^i \). While, for any non-zero value of \( \Delta P_{\text{dis}}^i \) will yield a worse outcome for the EV’s than for a zero value of \( \Delta P_{\text{dis}}^i \), therefore We define \( \Delta P_{\text{dis}}^i = 0 \). Hence for all the buses which are cardinalities of the set \( \{i : i \in b \, \forall \Delta P_i^s < 0\} \) we defined Nash Equilibrium as

\[ \Delta P_{\text{dis}}^i = \Delta P_{\text{req}}^i, \Delta P_{\text{dis}}^i = 0. \]

Similarly for the elements of the set

\[ \{i : i \in b \, \forall \Delta P_i^s > 0\} \]

we define Nash Equilibrium at the point \( \Delta P_{\text{dis}}^i = \Delta P_{\text{req}}^i, \Delta P_{\text{dis}}^i = 0. \)

For case II, we already know that the buses where charging is desired will have \( \Delta P_{\text{dis}}^i = 0 \) and for the buses where discharging is desired will have \( \Delta P_{\text{ch}}^i = 0 \). So the unique Nash Equilibrium for the set \( \{i : i \in b \, \forall \Delta P_i^s < 0\} \) is defined as:

\[ \Delta P_{\text{dis}}^i = \min \left( \Delta P_{\text{req}}^i, \left( \Delta P_{\text{dis}}^i + \Delta P_{\text{ch}}^i \right) \right), \Delta P_{\text{dis}}^i = 0. \]

And for the buses satisfying \( \{i : i \in b \, \forall \Delta P_i^s > 0\} \) the Nash Equilibrium is defined at

\[ \Delta P_{\text{dis}}^i = \min \left( \Delta P_{\text{req}}^i, \left( \Delta P_{\text{dis}}^i + \Delta P_{\text{ch}}^i \right) \right), \Delta P_{\text{dis}}^i = 0. \]

This proves our claim for the uniqueness of the EV to EV game.

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FAHEEM UL HAQ (Student Member, IEEE) received the bachelor’s degree in electrical and renewable energy resources engineering from BGSB University, Jammu and Kashmir, India. He is currently pursuing the master’s degree in power system engineering with the Indian Institute of Technology Dharwad, Karnataka, India. His current research interests include electricity market, game theory applications in smart grid, and power system stability.

PRATYASA BHUI (Member, IEEE) received the master’s degree in power and energy systems from IIT Kharagpur, in 2013, and the Ph.D. degree in electrical engineering from IIT Delhi, in 2017. He also worked as a Postdoctoral Fellow with Texas A&M University, College Station, USA, from 2017 to 2018. He is currently working with IIT Dharwad as an Assistant Professor. His research interests include power system dynamics, wide area measurement systems, and ancillary services.

KOTAKONDA CHAKRAVARTHI (Student Member, IEEE) received the master’s degree in machine drives and power electronics from IIT Kharagpur, in 2013. He is currently pursuing the Ph.D. degree with the Department of Electrical Engineering, IIT Dharwad. His research interests include power system dynamics, renewable energy, and power system control.