Cosmology with gamma-ray bursts: I. The Hubble diagram through the calibrated $E_{p,i} - E_{\text{iso}}$ correlation

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ABSTRACT

Context. Gamma-ray bursts (GRBs) are the most energetic explosions in the Universe. They are detectable up to very high redshifts. They may therefore be used to study the expansion rate of the Universe and to investigate the observational properties of dark energy, provided that empirical correlations between spectral and intensity properties are appropriately calibrated.

Aims. We used the type Ia supernova (SN) luminosity distances to calibrate the correlation between the peak photon energy, $E_{p,i}$, and the isotropic equivalent radiated energy, $E_{\text{iso}}$, in GRBs. With this correlation, we tested the reliability of applying these phenomena to measure cosmological parameters and to obtain indications on the basic properties and evolution of dark energy.

Methods. Using 162 GRBs with measured redshifts and spectra as of the end of 2013, we applied a local regression technique to calibrate the $E_{p,i} - E_{\text{iso}}$ correlation against the type Ia SN data to build a calibrated GRB Hubble diagram. We tested the possible redshift dependence of the correlation and its effect on the Hubble diagram. Finally, we used the GRB Hubble diagram to investigate the dark energy EOS and implemented the Markov chain Monte Carlo (MCMC) method to efficiently sample the space of cosmological parameters.

Results. Our analysis shows once more that the $E_{p,i} - E_{\text{iso}}$ correlation has no significant redshift dependence. Therefore the high-redshift GRBs can be used as a cosmological tool to determine the basic cosmological parameters and to test different models of dark energy in the redshift regime $z > 3$, which is unexplored by the SNIa and baryonic acoustic oscillations data. Our updated calibrated Hubble diagram of GRBs provides some marginal indication (at 1σ level) of an evolving dark energy EOS. A significant enlargement of the GRB sample and improvements in the accuracy of the standardization procedure is needed to confirm or reject, in combination with forthcoming measurements of other cosmological probes, this intriguing and potentially very relevant indication.

Conclusions.

Key words. Cosmology: observations, Gamma-ray burst: general, Cosmology: dark energy, Cosmology: distance scale

1. Introduction

Starting at the end of the 1990s, observations of high-redshift supernovae of type Ia (SNIa) revealed that the Universe is now expanding at an accelerated rate (see e.g. [52], [53], [58], [60], [9], [7]). This surprising result has been independently confirmed by statistical analyses of observations of small-scale temperature anisotropies of the cosmic microwave background radiation (CMB) ([69], [54], [55]). It is usually assumed that the observed accelerated expansion is caused by the so-called dark energy, a cosmic medium with unusual properties. The pressure of dark energy $p_{de}$ is negative and is related to the positive energy density of dark energy $\rho_{de}$ by $p_{de} = \frac{w}{3} \rho_{de}$, where the proportionality coefficient, that is, the equation of state (EOS), $w$, is negative ($w < -1/3$). According to current estimates, about 70% of the matter energy in the Universe is in the form of dark energy, so that today dark energy is the dominant component in the Universe. The nature of dark energy is, however, not known. The models of dark energy proposed so far can be divided into at least three groups: a) a non-zero cosmological constant (see for instance [16]), in this case $w = -1$, b) a potential energy of some not yet discovered scalar field (see for instance [61]), or c) effects connected with the inhomogeneous distribution of matter and averaging procedures (see for instance [18]). In the last two cases, in general, $w \neq -1$ and is not constant, but depends on the redshift $z$. To test whether and how $w$ changes with redshift, it is necessary to use more distant objects. It is commonly argued that since the dark energy density term becomes sub-dominant (with respect to the dark matter) at $z \gtrsim 0.5$ ([59]), it is not important to study its EOS at earlier epochs. However, this argument is unreliable: even within the simplest model, the dark energy contributes nearly $\approx 10\%$ of the overall cosmic energy density at $z \approx 2$ and strongly alters the deceleration parameter with the cosmological constant. Moreover, for several observables the sensitivity to the dark energy equation of state increases at high redshifts. In Fig. 1 we explore this aspect following a simplified approach, considering the modulus of distance $\mu(z)$ as observable: we fixed a flat $\Lambda$CDM fiducial cosmological model, constructing the corresponding $\mu_{\text{fid}}(z, \theta)$, and plot the percentage error on the distance.
modulus with respect to different corresponding functions evaluated in the framework of a flat CPL quintessence model (17,48). We selected $\Omega_m = 0.3$ and $h = 0.7$ and varied the dark EOS parameters $w_0$ and $w_1$. It is worth noting that a higher sensitivity is reached only for $z \gtrsim 2$. Therefore, investigating the cosmic expansion of the Universe also beyond these redshifts remains a fundamental probe of dark energy. In this scenario, new possibilities opened up when gamma-ray bursts were discovered at higher redshifts. The current record is at $z = 9.4$ (70,64,19). It is worth noting that the photometric redshift on GRB 090429B is quite high, especially on the low side; GRB 090423, for which a spectroscopic redshift is available, is much better determined. GRBs span a redshift range better suited for probing dark energy than the SNIa range, as shown in Fig. 2, where we compare the distribution in redshift of our sample of 162 long GRBs/XRFs with the Union 2.1 SNIa dataset.

Gamma-ray bursts are enigmatic objects, however. First of all, the mechanism that is responsible for releasing the high amounts of energy that a typical GRB emits is not yet completely known, and only some aspects of the progenitor models are well established, in particular that long GRBs are produced by core-collapse events, see for instance (49) and (Vedrenne & Atteia (2009)). Despite these difficulties, GRBs are promising objects for studying the expansion rate of the Universe at high redshifts. One of the most important aspects of the observational property of long GRBs is that they show several correlations between spectral and intensity properties (luminosity, radiated energy). Here we consider the correlation between the observed photon energy of the peak spectral flux, $E_{p,i}$, which corresponds to the peak in the $v F_v$ spectra, and the isotropic equivalent radiated energy $E_{iso}$ (e.g., (11,2)).

$$\log \left( \frac{E_{iso}}{1 \text{ erg}} \right) = b + a \log \left[ \frac{E_{p,i}}{300 \text{ keV}} \right], \quad (1)$$

where $a$ and $b$ are constants, and $E_{p,i}$ is the spectral peak energy in the GRB cosmological rest–frame, which can be derived from the observer frame quantity, $E_p$, by $E_{p,i} = E_p (1 + z)$. This correlation not only provides constraints for the model of the prompt emission, but also naturally suggests that GRBs can be used as distance indicators. The isotropic equivalent energy $E_{iso}$ can be calculated from the bolometric fluence $S_{bolo}$ as

$$E_{iso} = 4\pi d_L^2 (z, cp) S_{bolo} (1 + z)^{-1}, \quad (2)$$

where $d_L$ is the luminosity distance and $cp$ denotes the set of parameters that specify the background cosmological model. It is clear that to be able to use GRBs as distance indicators, it is necessary to consistently calibrate this correlation. Unfortunately, owing to the lack of GRBs at very low redshifts, the calibration of GRBs is more difficult than that of SNIa, and several calibration procedures have been proposed so far (see for instance (15,26,27,56,21,47)). We here apply a local regression technique to determine the correlation parameters $a$ and $b$, using the recently updated SNIa sample and to construct a new calibrated GRB Hubble diagram that can be used for cosmological investigations. We then use this calibrated GRB Hubble diagram to investigate the cosmological parameters through the Markov chain Monte Carlo technique (MCMC), which simultaneously computes the full posterior probability density functions of all the parameters. The structure of the paper is as follows. In Sect. 2 we describe the methods used to fit the $E_{p,i} - E_{iso}$ correlation and construct the calibrated GRB Hubble diagram. In Sect. 3 we present our cosmological constraints and investigate the possible redshift dependence and Malmquist-like bias effects. In Sect. 4, as an additional self-consistency check, we apply the Bayesian method for the non–calibrated $E_{p,i} - E_{iso}$ correlation and simultaneously extract the correlation coefficients and the cosmological parameters of the model. Section 5 is devoted to the discussion of our main results and conclusions.

### 2. Standardizing GRBs and constructing the Hubble diagram

The GRBs $v F_v$ spectra are well modeled by a phenomenological smoothly broken power law, characterized by a low-energy index, $\alpha$, a high-energy index, $\beta$, and a break energy $E_0$. They show a peak corresponding to a specific (and observable) value of the photon energy $E_p = E_0 (2 + \alpha)$. For GRBs with measured spectrum and redshift it is possible to evaluate the intrinsic peak energy, $E_{p,i} = E_p (1 + z)$ and the isotropic equivalent radiated en-
energy

\[ E_{\text{iso}} = 4\pi d^2 L(z, cp)(1 + z)^{-1}\int_{1/(1+z)}^{10^{51}/(1+z)} E N(E) dE, \]

where \( N(E) \) is the Band function:

\[
N(E) = \begin{cases} 
A \left( \frac{E}{10^{52}} \right)^\alpha \exp \left( \frac{E}{\beta} \right) & \text{if } (\alpha - \beta) E_0 > 0 \\
A \left( \frac{\alpha - \beta}{10^{52}} \right)^\alpha \exp \left( \alpha - \beta \right) \left( \frac{E}{10^{52}} \right)^\beta & \text{if } (\alpha - \beta) E_0 \leq E
\end{cases}
\]

\( E_{\text{iso}} \) and \( E_p, i \) span several orders of magnitude (therefore GRBs cannot be considered standard candles), and show distributions approximated by Gaussians plus a tail at low energies. A strong correlation between these two quantities was initially discovered in a small sample of Beppo SGRAX detectors with known redshifts (1) and confirmed afterward by HETE–2 and SWIFT observations (41) (8). Several analyses of the \( E_{p,i}, E_{\text{iso}} \) plane of GRBs showed that different classes of GRBs exhibit different behaviors, and while normal long GRBs and X–ray flashes (XRF, i.e., particularly soft bursts) follow this correlation, with the exception of the two peculiar sub–energetic GRBs 980425 and 031203 (see, e.g., 8 for a discussion on possible explanations), short GRBs do not. These facts may depend on the different emission mechanisms and/or geometry involved in different classes of GRBs and makes this correlation a useful tool to distinguish between them (2, 8). Although it was the first strong correlation discovered for the GRB observables, until recent years, the \( E_{p,i} – E_{\text{iso}} \) correlation was never used for cosmology because it exhibits a significant dispersion around the best–fit power law: the residuals follow a Gaussian with a value of \( \sigma_{E_{p,i}} \approx 0.2 \). This type of additional Poissonian scatter is typically quantified by performing a maximum likelihood analysis that takes the variance and the errors on dependent and independent variables into account. By measuring \( E_{p,i} \) in keV and \( E_{\text{iso}} \) in \( 10^{52} \) erg, this method gives \( \sigma_{E_{p,i}} = 0.19 \pm 0.02 \) (4) (5). However, the recent increase in the efficiency of GRB discoveries combined with the fact that the \( E_{p,i} – E_{\text{iso}} \) correlation requires only two parameters that are directly inferred from observations (this minimizes the effects of systematics and increases the number of GRBs that can be used) makes this correlation an interesting tool for cosmology. Despite the very large number of bursts consistent with this correlation, its physical origin is still debated. Some authors claimed that it is strongly affected by instrumental selection effects (60, 11, 13, 67). However, many other studies found that such instrumental selection biases, even if they may affect the sample, cannot be responsible for the existence of the spectral–energy correlations (56, 51). Moreover, a recent time–resolved spectral analysis of GRBs that were detected by the BeppoSAX and Fermi satellite showed that \( E_{p,i} \) correlates with the luminosity (e.g., 37, 29) and radiated energy (e.g., 12) also during the temporal evolution of the bursts (and the correlation between the spectral peak energy and the evolving flux has been pointed out by, e.g., 38). Based on Konus–WIND data, the time–resolved \( E_{p,i} – \text{luminosity} \) and \( E_{p,i} – E_{\text{iso}} \) correlations within individual GRBs and that their average slope is consistent with that of correlations defined by the time–averaged spectral properties of different bursts strongly supports the physical origin of the \( E_{p,i} – E_{\text{iso}} \) correlation. It is very difficult to explain them as a selection or instrumental effect (e.g., 25, 12), and the predominant emission mechanism in GRB prompt emission produces a correlation between the spectral peak energy and intensity (which can be characterized either as luminosity, peak luminosity, or radiated energy). In addition to its existence and slope, an important property of the \( E_{p,i} – E_{\text{iso}} \) correlation is its dispersion. As shown by several works that were based on the so–called jet–breaks, in the optical afterglow light curves of some GRBs (e.g., 33, 35), 50% of the extra–Poissonian scatter of the correlation is sometimes probably a result of the distribution of jet opening angles. However, these estimates of jet opening angles are still unconfirmed and model–dependent and can be made only for a small number of GRBs. Other factors that may contribute to the dispersion of the \( E_{p,i} – E_{\text{iso}} \) and other \( E_{p,i} – \text{intensity} \) correlations include the jet structure, viewing angles, detectors sensitivity, and energy band (but see Amati et al. 2009), the diversity of shock micro–physics parameters, and the magnetization within the emitting ejecta. At the current observational and theoretical status, it is very difficult to quantify these single contributions, which seem to act randomly in scattering the data around the best fit power–law (e.g., 56, 54). Importantly, it has been shown (e.g., 5) that a small fraction (5–10%) of the scatter depends on the cosmological model and parameters assumed for computing \( E_{\text{iso}} \), which makes this correlation a potential tool for cosmology.

In this section we show how the \( E_{p,i} – E_{\text{iso}} \) correlation can be calibrated to standardize long GRBs and to build a GRB Hubble diagram, which we use to investigate different cosmological models at very high redshifts (see also (74, 45, 76)).

2.1. GRB data

We used a sample of 162 long GRBs/XRFs as of September 2013 taken from the updated compilation of spectral and intensity parameters of GRBs by Sawant & Amati (2016). The redshift distribution of this sample covers a broad range, \( 0.03 \leq z \leq 9.3 \), which means that it extends far beyond the SNIa range (\( z \leq 1.7 \)). These data are of long GRBs/XRFs that are characterized by firm measurements of redshifts and the rest–frame peak energy \( E_{p,i} \). The main contributions come from the joint detections by Swift/BAT and Fermi/GBM or Konus–WIND, except for the small fraction of events for which Swift/BAT can directly provide \( E_{p,i} \) when it is in the \( (15 – 150) \) keV interval. For events detected by more than one of these detectors, the uncertainties on the \( E_{p,i} \) and \( E_{\text{iso}} \) take the measurements and uncertainties provided by each individual detector into account. In Table 4 we report for each GRB the redshift, the rest–frame spectral peak energy, \( E_{p,i} \), and the isotropic–equivalent radiated energy, \( E_{\text{iso}} \). As detailed in (60), the criterion behind selecting the observations from a particular mission is based on the conditions summarized below.

1. Preferred observations with exposure times of at least two–thirds of the whole event duration. Hence Konus–WIND and Fermi/GBM were chosen whenever available. For Konus–WIND, the observations were reported in the official literature (see e.g. (72)), and also in GCN archives when needed. For Fermi/GBM, the observations were taken from Gruber at al. 2012, from several other papers (e.g., 33, 34, 28, etc.), and from GCNs.

2. The SWIFT BAT observations were chosen when no other preferred mission (Konus–WIND, Fermi/GBM) was able to provide spectral parameters and the value of \( E_{p,\text{obs}} \) was within the energy band of this instrument. In particular, the \( E_{p,i} \) values derived from BAT spectral analysis were conservatively taken from the results reported by the BAT team (62, 63). Other BAT \( E_{p,i} \) values reported in the literature were not considered because they were not confirmed by (62, 63), by a refined analysis ((see...
e.g. [14]), or because they were based on speculative methods (13).

When available, the Band model (10) was considered as the cut-off power law, which sometimes overestimates the value of \( E_{\text{p}, i} \). Finally, the error on any value was assumed to be not less than 10% to account for the instrumental capabilities and calibration uncertainties.

### 2.2. Cosmologically independent calibration: local regression of SNII

The lack of nearby GRBs creates the so-called circularity problem: GRBs can be used as cosmological tools through the \( E_{\text{p}, i} - E_{\text{iso}} \) correlation, which is based on the cosmological model assumed for the computation of \( E_{\text{iso}} \); however, in principle, this problem could be solved in several ways: it is possible, for instance, to simultaneously constrain the calibration parameters \((a, b, \sigma_{\text{int}}) \in G \) and the set of cosmological parameters \( \theta \in H \) by considering a likelihood function defined in the space \( G \otimes H \), which allows simultaneously fitting the parameters (e.g., (24)). This procedure is implemented in Sect. 5 and compared with the local regression technique. Alternatively, it has been proposed that the problem might be avoided by considering a sufficiently large number of GRBs within a small redshift bin centered on any \( z \) (35). However, this method, even if quite promising for the future, is currently unrealistic because of the limited number of GRBs with measured redshifts. In this section we implement a procedure for calibrating the \( E_{\text{p}, i} - E_{\text{iso}} \) relation in a way independent of the cosmological model by applying a local regression technique to estimate the distance modulus \( \mu(z) \) from the recently updated SNIIa sample, called Union2.1 (see also (40, 44)). Originally implemented by Cleveland and Devlin (1988), this technique combines the simplicity of linear regression with the flexibility of nonlinear regression to localized subsets of the data, and reconstructs a function describing their behavior in the neighborhood of any \( z_0 \). A low-degree polynomial is fitted to a subsample containing a neighborhood of \( z_0 \), by using weighted least-squares with a weight function that quickly decreases with the distance from \( z_0 \). The local regression procedure can be schematically sketched as follows:

1. set the redshift \( z \) where \( \mu(z) \) has to be reconstructed;
2. sort the SNIIa Union2.1 sample by increasing value of \(|z - z_i|\) and select the first \( n = \alpha N_{\text{SNIIa}} \), where \( \alpha \) is a user-selected value and \( N_{\text{SNIIa}} \) the total number of SNIIa;
3. apply the weight function
   \[
   W(u) = \begin{cases} 
   (1 - |u|^2)^2 & |u| \leq 1 \\
   0 & |u| \geq 1 
   \end{cases},
   \]  
   \( u = |z - z_i|/\Delta \) and \( \Delta \) the highest value of the \(|z - z_i|\) over the previously selected subset;
4. fit a first-order polynomial to the data previously selected and weighted, and use the zeroth-order term as the best-fit value of the modulus of distance \( \mu(z) \);
5. evaluate the error \( \sigma_p \), as the root mean square of the weighted residuals with respect to the best-fit value.

It is worth stressing that both the choice of the weight function and the order of the fitting polynomial are somewhat arbitrary. Similarly, the value of \( n \) to be used must not be too low to make up a statistically valuable sample, but also not too high to prevent the use of a low-order polynomial. To check our local regression routine, we simulated a large catalog with the same redshift and error distribution as the Union2.1 survey. We adopted a quintessence cosmological model with a constant EOS, \( w \), and fixed values of the cosmological parameters \((\Omega_M, w, h) \). For each redshift value in the Union2.1 sample, we extracted the corresponding modulus of distance from a Gaussian distribution centered on the theoretically predicted value and with the standard deviation \( \sigma = 0.15 \). To this value, we added the error, corresponding to the same relative uncertainty as the data in the Union sample. This simulated catalog was used as input to the local regression routine, and the reconstructed \( \mu(z) \) values were compared to the input ones.

Defining the percentage deviation \( \varepsilon = \frac{\mu_p(z) - \mu_{\text{rec}}(z)}{\mu_{\text{rec}}} \) with \( \mu_{\text{rec}}, \) and \( \mu_p \) the local regression estimate and the input values, respectively, and averaging over 500 simulations, we found that the choice \( \alpha = 0.02 \) gives \((\delta\mu/\mu)_{\text{rms}} \approx 0.3\% \) with \( |\varepsilon| \leq 1\% \) independently on the redshift \( z \). This result implies that the local regression method allows correctly reconstructing the underlying distance modulus regardless of redshift \( z \leq 1.4 \) from the Union SNIIa sample. We also tested this results in different cosmological backgrounds by adopting an evolving EOS and averaging over five realizations of the mock catalog. With this efficient way of estimating \( \mu(z) \) at redshift \( z \) in a model-independent way, we can now fit the \( E_{\text{p}, i} - E_{\text{iso}} \) correlation relation, using the reconstructed \( \mu(z) \). We considered only GRBs with \( z \leq 1.414 \) to cover the same redshift range as is spanned by the SNIIa data. To standardize the \( E_{\text{p}, i} - E_{\text{iso}} \) relation as expressed by the Eq. (1), we need to fit a data array \( \{x_i, y_i\} \) with uncertainties \( \{\sigma_{x,i}, \sigma_{y,i}\} \) to a straight line

\[
y = b + ax,
\]

and determine the parameters \( (a, b) \). We expect a certain amount of intrinsic additional Poissonian scatter, \( \sigma_{\text{int}} \), around the best-fit line that has to be taken into account and determined together with \( (a, b) \) by the fitting procedure. We used a likelihood, implemented by Reichart (47), that solved this problem of fitting data that are affected by extrinsic scatter in addition to the intrinsic uncertainties along both axes:

\[
L_{\text{Reichart}}(a, b, \sigma_{\text{int}}) = \frac{1}{\sqrt{2\pi}} \sum \log \left( \frac{\sigma_{\text{int}}^2 + \sigma_i^2 + a^2 \sigma_{\text{int}}^2}{\log(1 + a^2)} \right)
\]

\[
+ \frac{1}{2} \sum \frac{(y_i - ax_i - b)^2}{\sigma_{\text{int}}^2 + \sigma_i^2 + a^2 \sigma_{\text{int}}^2},
\]

where the sum is over the \( N \) objects in the sample. We note that this maximization was performed in the two-parameter space \( (a, \sigma_{\text{int}}) \) since \( b \) may be calculated analytically by solving the equation

\[
\frac{\partial}{\partial b} L(a, b, \sigma_{\text{int}}) = 0,
\]

obtained

\[
b = \left[ \sum \frac{y_i - ax_i}{\sigma_{\text{int}}^2 + \sigma_i^2 + a^2 \sigma_{\text{int}}^2} \left[ \sum \frac{1}{\sigma_{\text{int}}^2 + \sigma_i^2 + a^2 \sigma_{\text{int}}^2} \right]^{-1} \right]^{-1}.
\]

To quantitatively estimate the goodness of this fit, we used the median and root mean square of the best-fit residuals, defined as \( \delta = \sqrt{\delta_{\text{tot}}^2 / \sigma_{\text{fit}}^2} \). To quantify the uncertainties of some fit parameter \( p_i \), we evaluated the marginalized likelihood \( L_\theta(p_i) \) by...
integrating over the other parameters. The median value for the parameter \( p_i \) was then found by solving

\[
\int_{p_{i,\text{min}}}^{p_{i,\text{med}}} L_i(p_i) d p_i = \frac{1}{2} \int_{p_{i,\text{min}}}^{p_{i,\text{max}}} L_i(p_i) d p_i.
\]

The 68% (95%) confidence range \((p_{i,\text{med}}, p_{i,\text{h}})\) was then found by solving (e.g., [20])

\[
\int_{p_{i,\text{med}}}^{p_{i,\text{med}}} L_i(p_i) d p_i = 1 - \varepsilon \quad \text{and} \quad \int_{p_{i,\text{h}}}^{p_{i,\text{max}}} L_i(p_i) d p_i = 1 - \varepsilon,
\]

with \( \varepsilon = 0.68 \) and \( \varepsilon = 0.95 \) for the 68% and 95% confidence level. The maximum likelihood values of \( p_i \) and \( \sigma_{\text{iso}} \) are \( a = 1.75^{+0.10}_{-0.10} \) and \( \sigma_{\text{iso}} = 0.37^{+0.07}_{-0.07} \). In Fig. 3 we show the correlation \( \log E_{\text{iso}} - \log \hat{E}_{\text{iso}} \). The solid line is the best fit obtained using the Reichart likelihood, and the dashed line is the best fit obtained by the weighted \( \chi^2 \) method.

The marginalized likelihood functions are shown in Fig. 4.

As noted above, \( b \) can be evaluated analytically. We obtained \( b = 52.53 \pm 0.02 \). It is worth noting that in the literature results for the inverse correlation are commonly reported, that is, the correlation \( E_{\text{iso}} - E_{p,1} \); using the local regression technique and the Reichart likelihood, we also obtained this inverse calibration. We obtained \( \sigma_{\text{iso}}^{\text{inverse}} = 0.58^{+0.07}_{-0.05} \) and \( \sigma_{\text{iso}}^{\text{inverse}} = 0.24^{+0.04}_{-0.03} \). In Fig. 5 we plot the best-fit curves for the \( E_{\text{iso}} - E_{p,1} \) correlation relation superimposed on the data.

2.2.1. Constructing the Hubble diagram

After fitting the correlation and estimating its parameters, we used them to construct the GRB Hubble diagram. We recall that the luminosity distance of a GRB with the redshift \( z \) may be computed as

\[
d_L(z) = \left( \frac{E_{\text{iso}}(1+z)}{4\pi S_{\text{bolo}}} \right)^{1/2}.
\]

The uncertainty of \( d_L(z) \) was then estimated through the propagation of the measurement errors on the involved quantities. In particular, recalling that our correlation relation can be written as a linear relation, as in Eq. (4), where \( y = E_{\text{iso}} \) is the distance dependent quantity, while \( x \) is not, the error on the distance dependent quantity \( y \) was estimated as

\[
\sigma(y) = \sqrt{\sigma_x^2 + \sigma_y^2 + \sigma_{\text{ISO}}^2}.
\]

where \( \sigma_x \) is properly evaluated through the Eq. (26), which implicitly defines \( b \) as a function of \( a \) and \( \sigma_{\text{ISO}} \), and is then added in quadrature to the uncertainties of the other terms entering Eq. (9)
to obtain the total uncertainty. The distance modulus \( \mu(z) \) is easily obtained from its standard definition

\[
\mu(z) = 25 + 5 \log d_L(z),
\]

with its uncertainty obtained again by error propagation:

\[
\sigma_{\mu}^2 = \left( \frac{5}{2} \sigma^2(y) \right)^2 + \left( \frac{5}{2 \ln 10} \frac{\sigma_{\text{bolo}}}{\text{S} \text{bolo}} \right)^2.
\]

We finally estimated the distance modulus for each \( i \)-th GRB in the sample at redshift \( z_i \) to build the Hubble diagram plotted in Fig. 6. We refer to this data set as the calibrated GRB Hubble diagram below since to compute the distances, the Hubble diagram we relied on the calibration was based on the SNIa Hubble diagram. The derived distance moduli are divided into two subsets, listed in Table 3 and to Table 4. Table 4 lists GRBs with \( z \leq 1.46 \), the same redshift range as for known SNIa, and Table 5 lists GRBs with \( z \geq 1.47 \). In Fig. 7 we finally compare the GRB Hubble diagram (black points) with the SNIa Hubble diagram (blue points) and with BAO data (red points).

3. Cosmological constraints derived from the calibrated GRB Hubble diagram

Here we illustrate the possibilities of using the GRB Hubble diagram to constrain the cosmological models and investigate the dark energy EOS. Within the standard homogeneous and isotropic cosmology, the dark energy appears in the Friedman equations:

\[
\ddot{a} = -\frac{4\pi G}{3}(\rho_m + \rho_{de} + 3\rho_{de}),
\]

\[
H^2 + \frac{k}{a^2} = \frac{8\pi G}{3}(\rho_m + \rho_{de}),
\]

where \( a \) is the scale factor, \( H = \dot{a}/a \) the Hubble parameter, \( \rho_m \) the density of matter, \( \rho_{de} \) the density of dark energy, \( \rho_{de} \) its pressure, and the dot denotes the time derivative. The continuity equation for any component of the cosmological fluid is

\[
\frac{\dot{\rho}_i}{\rho_i} = -3H \left( 1 + \frac{\rho_{de}}{\rho_i} \right) = -3H \left[ 1 + w_i(t) \right],
\]

where the energy density is \( \rho_i \), the pressure \( p_i \), and the EOS of the \( i \)-th component is defined by \( w_i = \frac{p_i}{\rho_i} \). The standard non-relativistic matter has \( w = 0 \), and the cosmological constant has \( w = -1 \). The dark energy EOS and other constituents of the Universe determine the Hubble function \( H(z) \) and any derivations of it that are needed to obtain the observable quantities. When only matter and dark energy are present, the Hubble function is given by

\[
H(z, \theta) = H_0 \sqrt{(1 - \Omega_m)g(z, \theta) + \Omega_m(z + 1)^3 + \Omega_k(z + 1)^2},
\]

where \( g(z) = \exp(3 \int_0^z w(x, \theta) \frac{dx}{x + 1}) \), and \( w(z, \theta) \) describes the dark energy EOS, characterized by \( n \) parameters \( \theta = (\theta_1, \theta_2, \ldots, \theta_n) \). We limit our analysis below to the CPL parametrization,

\[
w(z) = w_0 + w_1(z + 1)^{-1},
\]

where \( w_0 \) and \( w_1 \) are constant parameters and they represent the \( w(z) \) present value and its overall time evolution, respectively [17, 48]. If we introduce the dimensionless Hubble parameter \( E(z, \theta) \):

\[
E(z, \theta) = \sqrt{(z + 1)^3\Omega_k + (z + 1)^3\Omega_m + \Omega_\Lambda e^{-\frac{9H_0^2}{4\pi G}(z + 1)^3w_0 + w_1^1}},
\]

we can define the luminosity distance and the modulus of distance. Actually the luminosity distance \( d_L \) given by:

\[
d_L(z, \theta) = d_H (1 + z) d_M(z, \theta),
\]

where \( d_H = \frac{c}{H_0} \), \( d_M(z, \theta) \) is the transverse co-moving distance and it is defined as

\[
d_M(z, \theta) = \left\{ \begin{array}{ll}
\frac{dS}{\sqrt{\Omega_k}} \sin \frac{dc(z, \theta)}{dM} & \Omega_k > 0, \\
\frac{dS}{\sqrt{|\Omega_k|}} \pi \Omega_k & \Omega_k < 0, \\
d_c(z, \theta) & \Omega_k = 0,
\end{array} \right.
\]

being \( dc(z, \theta) \) the co-moving distance:

\[
d_c(z, \theta) = d_H \int_0^z \frac{d\zeta}{E(\zeta, \theta)}. \]

Therefore we can define the modulus of distance \( \mu_{tb}(z, \theta) \):

\[
\mu_{tb}(z, \theta) = 25 + 5 \log d_L(z, \theta).
\]

The standard \( \Lambda \)CDM model corresponds to \( w_0 = -1, w_1 = 0 \).

To constrain the parameters specifying different cosmological models, we maximized the likelihood function \( \mathcal{L}(\theta) \propto \exp[-\chi^2(\theta)/2] \), where \( \theta \) indicates the set of cosmological parameters and the \( \chi^2(\theta) \) was defined as usual by

\[
\chi^2(\theta) = \sum_{i=1}^{N_{\text{GRB}}} \left[ \frac{\mu_{tb}(z_i) - \mu_{tb}(z_i, \theta)}{\sigma_i} \right]^2.
\]

Here, \( \mu_{tb} \) is the observed and theoretically predicted values of the distance modulus. The parameter space is efficiently sampled by using the MCMC method, thus running five parallel chains and using the Gelman-Rubin test to check the convergence [31]. The confidence levels are estimated from the likelihood quantiles. We recall that we performed the analysis assuming a non-zero spatial curvature (not flat \( \Lambda \)CDM), and only
in this case did we take \( w = -1 \). To alleviate the strong degeneracy of the curvature parameters and any EOS parameters, we added a Gaussian prior on \( \Omega_m \), centered on the value provided by the Planck collaboration \((54)\), \( 100\Omega_m^{\text{Planck}} = -4.2^{+1.3}_{-4.8} \), and with a dispersion ten times of \( \sigma_{\Omega_m}^{\text{Planck}} \), where \( 100\sigma_{\Omega_m}^{\text{Planck}} = 4.5 \). We investigated the CPL parameters assuming a flat universe. In a forthcoming paper we will present a full cosmological analysis using the high-redshift GRB Hubble diagram to test different cosmological models, where several parameterizations of the dark energy EOS will be used and also different dark energy scenarios, for instance the scalar field quintessence. In Table 1 we summarize the results of our analysis. There are indications that the dark energy EOS is evolving.

The joint probability for different sets of parameters that characterize the CPL EOS is shown in Fig. 8.

4. \( E_{\text{iso}} \) and \( E_{\text{p,i}} \) correlation

Since we here used the calibrated GRB Hubble diagram to perform the cosmological investigation described above, we discuss some arguments about the reliability of using the \( E_{\text{iso}} = E_{\text{p},i} \) relation for cosmological tasks. For instance, the calibration method we implemented so far relies on the underlying assumption that the calibration parameters do not evolve with redshift. It is worth noting that the problem of the redshift dependence of the GRBs correlations is widely debated in the literature, with different conclusions. This problem is intimately connected to the problem of the influence of possible selection effects or biases on the observed correlations, see, for instance, \((22)\). Any answer to these fundamental questions is far from being settled until more data with known redshifts are available.

In this section, however, we revisit this question from an observational point of view: we test the validity of this commonly adopted working hypothesis and search for any evidence of such a redshift dependence. We also investigate possible effects on the GRB Hubble diagram. As a first simplified approach we considered two subsamples with a comparable number of bursts, divided according to redshift: a lower redshift sample of 97 bursts with \( z \leq 2 \), and a higher redshift sample of 67 burst with \( z > 2 \). We estimated the cosmological parameters for flat \( \Lambda \text{CDM} \) and CPL dark energy EOS cosmological models, considering the two subsamples separately, and compared the results. Even when the bursts belonging to these two samples experience different environment conditions, we did not find significant indication that a spurious \( z \)-evolution of the slope affects the cosmological fit: the values of the cosmological parameters derived from these two samples are statistically consistent within 1\( \sigma \), as shown in Table 2. This result indirectly shows that redshift evolution dependence, even if it exists, does not undermine the reliability of the GRBs as probes of the cosmological expansion. Moreover, to investigate this redshift dependence in more detail, that is, how it affects \( E_{\text{iso}} \) and/or \( E_{\text{p},i} \) (which we generically denote by \( y \)), we used two different approaches. First, we evaluated the Spearman rank correlation coefficient, \( C(z,y) \), taking into account that since our sample is not too large, a few points could dominate the final value of the rank correlation, which would introduce a bias. We applied a jackknife re-sampling method by evaluating \( C(z,y) \) for \( N-1 \) samples obtained by excluding one GRB at a time, and we adopted the mean value and the standard deviation to estimate \( C(z,y) \). We found \( C(z,E_{\text{iso}}) = 0.299 \pm 0.004 \), and \( C(z,E_{\text{p},i}) = 0.278 \pm 0.004 \), which indicates a moderate evolution of the correlation. The Spearman rank correlation coefficient, however, does not include the errors of \( E_{\text{iso}} \) and \( E_{\text{p},i} \), so that we also implemented an alternative, and completely different, approach to determine whether and how strongly these variables are correlated with redshift. We assumed that the evolutionary functions can be parametrized by a simple power-law functions: \( g_{\text{iso}}(z) = (1+z)^{k_{\text{iso}}} \) and \( g_{\text{p},i}(z) = (1+z)^{k_{\text{p},i}} \) (see for instance \((22)\)), so that \( E_{\text{iso}}' = E_{\text{iso}}(z_{\text{iso}}) \) and \( E_{\text{p},i}' = E_{\text{p},i}(z_{\text{p},i}) \) are the de-evolved quantities. In this case, the effective \( E_{\text{iso}} - E_{\text{p},i} \) correlation can be written as a 3D correlation:

\[
\log \left[ \frac{E_{\text{iso}}}{1 \text{ erg}} \right] = b + a \log \left[ \frac{E_{\text{p},i}}{300 \text{ keV}} \right] + (k_{\text{iso}} - ak_{\text{p},i})(1+z).
\]

(24)

Calibrating this 3D relation means determining the coefficients \( (a, b, k_{\text{iso}}, \text{ and } k_{\text{p},i}) \) plus the intrinsic scatter \( \sigma_{\text{int}} \). Low values for \( k_{\text{iso}} \) and \( k_{\text{p},i} \) would indicate a lack of evolution, or at least negligible evolutionary effects. To fit these coefficients, we constructed a 3D Reichart likelihood, but we consider no error on the redshift of each GRB:

\[
L_{\text{Reichart}}(a,k_{\text{iso}},k_{\text{p},i},b,\sigma_{\text{int}}) = \frac{1}{2} \sum_{i=1}^{N} \log \left( \sigma_{\text{int}}^2 + \sigma_{\gamma}^2 + a^2 \sigma_{z}^2 \right) \log (1+a^2) + \frac{1}{2} \sum_{i=1}^{N} \left( y_i - a x_i - (k_{\text{iso}} - \alpha) z_i - b \right)^2 \sigma_{\gamma}^2 \sigma_{\text{int}}^2 + a^2 \sigma_{z}^2 \sigma_{\text{int}}^2 \sigma_{\gamma}^2 \sigma_{\text{int}}^2 + a^2 \sigma_{z}^2 \sigma_{\gamma}^2 \sigma_{\text{int}}^2.
\]

(25)

where \( \alpha = ak_{\text{p},i}. \) As in the 2D case, we maximized our likelihood in the space \((a,k_{\text{iso}}, \text{ and } \alpha) \) since \( b \) was evaluated analytically by solving the equation

\[
\frac{\partial}{\partial b} L_{\text{Reichart}}(a,k_{\text{iso}},k_{\text{p},i},b,\sigma_{\text{int}}) = 0,
\]

we obtain

\[
b = \left[ \frac{\sum_{i=1}^{N} y_i - a x_i - (k_{\text{iso}} - \alpha) z_i}{\sigma_{\text{int}}^2 + \sigma_{\gamma}^2 + a^2 \sigma_{z}^2} \right]^{-1} \left[ \sum_{i=1}^{N} \frac{1}{\sigma_{\text{int}}^2 + \sigma_{\gamma}^2 + a^2 \sigma_{z}^2} \right]^{-1}.
\]

(26)
Table 1. Constraints on the cosmological parameters. Columns report the mean $\langle x \rangle$ and median $\tilde{x}$ values and the 68% and 95% confidence limits. The investigation of the CPL parametrization for the dark energy was performed assuming a flat universe (upper side). The analysis performed by assuming a non-zero spatial curvature is limited to the case $w = -1$ (non-flat ΛCDM). The GRB Hubble diagram alone provides $\Omega_k^{\text{median}} = -0.00046$, in agreement with the CMBR results.

| $\Omega_m$ | 0.24 | 0.19 | (0.12, 0.37) | (0.10, 0.58) |
| $w_0$ | -0.29 | -0.26 | (-0.5, -0.1) | (-1.01, 0.1) |
| $w_1$ | -0.12 | -0.13 | (-0.43, 0.19) | (-0.88, 0.6) |
| $h$ | 0.74 | 0.74 | (0.69, 0.78) | (0.65, 0.8) |

| $\Omega_m$ | 0.33 | 0.32 | (0.19, 0.49) | (0.12, 0.59) |
| $\Omega_\Lambda$ | 0.66 | 0.677 | (0.51, 0.8) | (0.42, 0.87) |
| $h$ | 0.74 | 0.74 | (0.70, 0.77) | (0.66, 0.79) |
| $\Omega_k$ | -0.00049 | -0.00046 | (-0.007, 0.0064) | (-0.014, 0.013) |

We also used the MCMC method and ran five parallel chains and the Gelman-Rubin convergence test, as previously explained. We finally studied the median and 68% confidence range of $k_{iso}$ and $\alpha$ to test whether the correlation evolves and noted that a null value for these parameters is strong evidence for a lack of any evolution. We found that $a = 1.86^{+0.07}_{-0.09}$, $k_{iso} = -0.04 \pm 0.1$, $\alpha = -0.02 \pm 0.2$, $\sigma_{int} = 0.35 \pm 0.03$, so that $b = 52.40^{+0.03}_{-0.06}$. We can safely conclude that the $E_{iso}$ and $E_{p,i}$ correlation shows, at this stage, weak indications of evolution. In Fig. 9 we plot both the de-evolved and evolved/original correlation, and the de-evolved and evolved/original Hubble diagram: we do not see any signs of evolution.

5. Fully Bayesian analysis

In this section we simultaneously constrain the calibration parameters $(a, b, \sigma_{int})$ and the set of cosmological parameters by considering the same likelihood function as in Eq. (6). Our task is to determine the multidimensional probability distribution function (PDF) of the parameters $(a, b, \sigma_{int}, p)$, where $p$ is the $N$-dimensional vector of the cosmological parameters. The Amati correlation can be written in the form

$$\log_{10} S_{\text{bol}} = a + b \log_{10} E_{p,i} - \log_{10}[4\pi d_L^2(z, p)].$$

We note that the best-fit zero point $b$ can be analytically expressed as a function of $(a, \sigma_{int})$ when the cosmological parameter $p$ is specified. A more computationally efficient strategy is to let $b$ free and add it to the list of quantities to be determined, which thus sums up to $N + 3$. To efficiently sample the $N + 3$ dimensional parameter space, we used the MCMC method and ran five parallel chains and used the Gelman-Rubin convergence test, as described in the previous section. Since we are mainly interested in the calibration problem and not in constraining the cosmological parameters $(\Omega_M, \Omega_\Lambda, h)$, we considered here only the particular case of a flat ΛCDM model, as already done in the previous analysis. The analysis determines, at the same time, the cosmological parameters and the correlation coefficients, which
Fig. 10. Regions of confidence for the marginalized likelihood function $L(a, \sigma)$, obtained by marginalizing over $b$ and the cosmological parameters. The gray regions indicate the 1σ (full zone) and 2σ (empty zone) regions of confidence. On the axes we also plot the box-and-whisker diagrams for the $a$ and $\sigma_{\text{int}}$ parameters: the bottom and top of the diagrams are the 25th and 75th percentile (the lower and upper quartiles, respectively), and the band near the middle of the box is the 50th percentile (the median).

Fig. 11. Fiducial GRB Hubble diagram, superimposed on the calibrated Hubble diagram.

are listed in Table 3. Although the calibration procedure is different (since we now fit for the cosmological parameters as well), it is nevertheless worth comparing this determination of $(a, b, \sigma_{\text{int}})$ with the one obtained in the previous analysis based on the SNIa sample. It is evident that although the median values change, the 95% confidence levels are in full agreement so that we cannot find any statistically significant difference. Figure 10 shows the marginalized probability distribution function of the correlation coefficients of the relation $E_{\text{iso}} - E_{\text{p}}$.

As far as the cosmological parameters are concerned, it turns out that $\Omega_{m, \text{median}} = 0.25$, the range of confidence at 1σ is (0.13, 0.54), and $\Omega_{\Lambda, \text{median}} = 0.75$, the range of confidence at 1σ is (0.50, 0.87). This result implies that $\Omega_{m, \text{median}} = -0.0006$, and that the range of confidence is $(-0.00730, 0.0065)$, in good agreement with results derived by using the calibrated GRBs Hubble diagram.

Also in this case we finally estimate the distance modulus for each $i$-th GRB in our sample at redshift $z_i$, to build the fiducial Hubble diagram, by using Eqs. (1) and (9). It turns out that the fiducial and calibrated Hubble diagrams are fully statistically consistent, as shown in Fig. 11.

6. Discussion and conclusions

The $E_{\text{p}} - E_{\text{iso}}$ correlation is one of the most intriguing properties of the long GRBs, with significant implications for the use of GRBs as cosmological probes. Here we explored the Amati relation in a way independent of the cosmological model. Using the recently updated data set of 162 high-redshift GRBs, we applied a local regression technique to estimate the distance modulus using the recent Union SNIa sample (Union2.1). The derived calibration parameters are statistically fully consistent with the results of our previous work (20). Moreover, we tested the validity of the commonly adopted working hypothesis that the GRB Hubble diagram is slightly affected by redshift dependence of the $E_{\text{p}} - E_{\text{iso}}$ correlation. As a first qualitative and simplified approach we considered a lower redshift sample of 97 bursts with $z < 2$, and a higher redshift sample of 67 burst with $z > 2$. We estimated the cosmological parameters for flat $\Lambda CDM$ and CPL dark energy EOS cosmological models, considering the two subsamples separately, and compared the results. Even when the bursts belonging to these two samples had different environment conditions, we found no significant indications that a spurious $z$-evolution of the slope affected the cosmological fit. Moreover, to quantify this redshift dependence, we used two different approaches. First, we evaluated the Spearman rank correlation coefficient, $C(z, y)$ by applying a jacknife re–sampling method by evaluating $C(z, y)$ for $N - 1$ samples obtained by excluding one GRB at a time, and we adopted the mean value and the standard deviation to estimate $C(z, y)$, $C(z, E_{\text{p}}) = 0.299 \pm 0.004$, and $C(z, E_{\text{iso}}) = 0.278 \pm 0.004$, which indicates a negligible evolution of the correlation. Moreover, we also implemented an-

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alternative method, assuming that the redshift evolution can be parametrized by simple power-law functions: $g_{\text{iso}}(z) = (1 + z)^{k_{\text{iso}}}$ and $g_{p}(z) = (1 + z)^{k_{p}}$ and that the correlation holds for the de-evolved quantities $E_{\text{iso}}' = \frac{E_{\text{iso}}}{g_{\text{iso}}(z)}$ and $E_{p,1}' = \frac{E_{p,1}}{g_{p}(z)}$. In this case, we rewrote an effective 3D $E_{\text{iso}} - E_{p,1}$ correlation, with included the evolutionary terms. Since we were interested in the implications of possible evolutionary effects of the GRB Hubble diagram and to simplify the fit, we introduced an auxiliary variable $\alpha = ak_{p}$. A null value for $k_{\text{iso}}$ and $\alpha$ is strong evidence for a lack of any evolution. To fit the coefficient of our 3D correlation we constructed a 3D Reichter likelihood and used a MCMC method running five parallel chains and using the Gelman-Rubin convergence test. The $E_{\text{iso}}$ and $E_{p,1}$ correlation shows at this stage only weak indications of evolution. The derived calibration parameters were used to construct an updated calibrated GRB Hubble diagram, which we adopted as a tool to constrain the cosmological models and to investigate the dark energy EOS. In particular, we searched for any indications that $w(z) \neq -1$, which reflects the possibility of a deviation from the $\Lambda$CDM cosmological model. To accomplish this task, we focused on the CPL parametrization as an example. To efficiently sample the cosmological parameter space, we again used a MCMC method. At $1\sigma$ level we found indications for a time evolution of the EOS in the considered parametrization; we conclude that the $\Lambda$CDM model is not favored even though it is not ruled out by these observations so far. Moreover, for $w = -1$ we also performed our analysis assuming a non-zero spatial curvature, adding a Gaussian prior on $\Omega_{k}$ centered on the value provided by the Planck data [54]. The GRB Hubble diagram alone provides $\Omega_{k,\text{median}} = -0.00046$ for the range of confidence at $1\sigma$. Finally, to investigate the reliability of the $E_{p,1} - E_{\text{iso}}$ correlation in greater detail, we also used a different method to simultaneously extract the correlation coefficients and the cosmological parameters of the model from the observed quantities. To illustrate this method we assumed here, by way of an example, only the particular case of a non-flat $\Lambda$CDM model, as already done in the previous analysis. This analysis simulta-

| Table 2. Constraints on the cosmological parameters. Columns report the mean $\langle x \rangle$ and median $\bar{x}$ values and the 68% and 95% confidence limits. The analysis is related to a spatially flat model with the dark energy parametrized through the CPL ansatz. |
|---|
| **Lower redshift sample** | **Higher redshift sample** |
| $\Omega_{m}$ | 0.28 | 0.27 | (0.15, 0.4) | 0.22 | 0.24 | (0.14, 0.2) | (0.10, 0.35) |
| $h$ | 0.74 | 0.74 | (0.70, 0.77) | 0.75 | 0.75 | (0.72, 0.77) | (0.69, 0.8) |
| $\Omega_{\Lambda}$ | 0.74 | 0.74 | (0.66, 0.8) | 0.74 | 0.74 | (0.70, 0.77) | (0.67, 0.8) |
| $w_{0}$ | -0.65 | -0.61 | (-0.77, -0.57) | -0.64 | -0.63 | (-0.75, -0.53) | (-0.86, -0.51) |
| $w_{1}$ | -0.11 | -0.11 | (-0.3, 0.15) | -0.56 | -0.57 | (-0.63, -0.37) | (-0.69, 0.42) |

| Table 3. Constraints on the calibration parameters. Columns report the mean $\langle x \rangle$ and median $\bar{x}$ values and the 68% and 95% confidence limits. The calibration procedure based on the local regression technique does not directly provide the zero-point parameter $b$, which can be analytically evaluated as a function of $(a, \sigma_{\text{int}})$ by the Eq. (26). The fully Bayesian analysis, moreover, is related to a spatially flat model with dark energy parametrized through the CPL ansatz. |
|---|
| **Fully Bayesian Analysis** | **Local Regression with SNIa** |
| $a$ | 1.69 | 1.71 | (1.64, 1.76) | (1.59, 1.82) | 1.74 | 1.74 | (1.59, 1.93) | (1.45, 2.16) |
| $b$ | 52.5 | 52.5 | (52.48, 52.55) | (52.44, 52.60) | – | – | – | – |
| $\sigma_{\text{int}}$ | 0.23 | 0.23 | (0.22, 0.25) | (0.20, 0.26) | 0.36 | 0.37 | (0.33, 0.49) | (0.31, 0.5) |
neously determines the cosmological parameters and the correlation coefficients. Although the calibration procedure is different (since we now fit for the cosmological parameters as well), it is nevertheless worth comparing this determination of \((a, b, \sigma)\) with the one obtained in the previous analysis based on the SNIa sample. It is evident that although the median values change, the 95% confidence levels are in full agreement so that we cannot find any statistically significant difference. This means that the high-redshift GRBs can be used as cosmological probes, mainly in a redshift region, \(z > 3\), which is unexplored by SNIa and BAO samples, and that both the calibration technique based on a local regression with SNIa and a fully Bayesian approach are reliable. As a final remark we note that our results for the cosmological parameters are statistically consistent with those previously obtained by other teams, as shown in Table 6, where we display some of the most recent measurements of cosmological parameters obtained with GRBs, even if following a slightly different approach. The main peculiarity in our analysis consists of the procedure used to build up the dataset, consisting of 162 objects, as discussed in Sect. 2.1 (see also [65, 66]), even more than the specific statistical analysis, which follows a Bayesian approach through the implementation of the MCMC. Because the reliability of GRBs as distance indicators has not yet been clearly proved and because it constitutes an independent topic in the field of GRB research, it is not meaningless to investigate the reliability of the \(E_p - t_{160}\) correlation whenever new and improved datasets are available. In the near future we intend to enhance the cosmological analysis by using the high-redshift GRB Hubble diagram to test different cosmological models, where several parameterizations of the dark energy EOS are used, but a cosmographic approach will also be implemented, which will update the analysis performed in [27] to check whether with this new updated dataset the estimates of the jerk and the dark energy parameters will confirm deviations from ΛCDM cosmological model, as has been indicated in our previous analysis.

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Table 6. Some of the most recent constraints of cosmological parameters by GRBs.
