Special Relativity from the Dynamical Viewpoint

William M. Nelson

(Dated: May 19, 2014)

Abstract

Arguments are collected and extended in favor of presenting special relativity at least in part from the dynamical (a.k.a. constructive, Lorentzian, physical) point of view. The conceptual steps introducing the theory are presented in detail and contrasted with those of the customary approach. The primary dynamical mechanisms at work are catalogued and specific pedagogical demonstrations discussed for each. We advocate introducing the crucial concept of relativistic mass directly in terms of wave-packet particle models, its most natural context and the one existing in nature; also, we suggest that relativity of simultaneity should be introduced through carried clocks rather than the customary light signalling.
I. INTRODUCTION

Students major in physics because they want to learn how the world works. Unfortunately, when it comes to special relativity, they are rather shortchanged by the traditional curriculum, which places strong emphasis on what the relativistic effects are and how to calculate them, but largely ignores the underlying physical mechanisms which create these effects, and which indeed enable a Lorentz-invariant theory to exist at all.

The dynamical approach to relativity, known also as “Lorentzian pedagogy” \(^1\), “constructive relativity” \(^2\) or “physical relativity” \(^3\) aims to address this by providing the same sort of low-level understanding of relativistic effects which all students receive when studying, e.g., the dielectric properties of materials. Instead of being satisfied, for example, to see that length contraction can be derived from the principle of invariance of the speed of light, the dynamical viewpoint seeks to understand how the contraction physically arises during dynamical processes in which a real object is accelerated. Put differently, the dynamical viewpoint seeks to understand what sorts of physical processes exist (perhaps, must exist) to enable the principle of invariance of light speed to be satisfied in the first place.

It is clear that such processes occur, not only for contraction but for all relativistic phenomena; Lorentz invariance relates systems having different velocities, and velocity change happens through dynamical acceleration by forces. Nevertheless, the reality of such processes is not obvious to most students of the traditional paradigm, and indeed the opposite is often explicitly taught, leading to widespread misconceptions that relativistic effects are separate from other physical effects and have entirely non-physical causes such as “the way things are measured” or “the structure of spacetime”. The dynamical viewpoint aims to rebalance the presentation and also to provide useful, intuitive understanding of the relativistic mechanisms.

Many elements of the program pre-date relativity; indeed Fitzgerald, Lorentz, Larmor, and Thompson were all motivated by explicitly dynamical considerations in introducing the seminal notions of length contraction, time dilation, and relativistic mass.\(^4\,\text{–}\,\text{7}\) After Einstein and Minkowski the dynamical view languished, displaced by the now-standard treatment in terms of principles and spacetime. Arguably the major reasons for this were, first, the absence at the time of any workable theory of matter, and second, the highly constraining nature of Lorentz invariance, which enables it to determine many quantitative answers
without reference to underlying mechanisms.

The dynamical view was revived by J.S. Bell in his 1976 essay “How to teach special relativity” which, however, had little impact on pedagogy, possibly because it supplied a very incomplete picture and proposed rather opaque numerical computations. In the 1990’s the baton was picked up by H.R. Brown, often in collaboration with O. Pooley, with a particular focus on challenging the independent status of Minkowski spacetime in physics (these authors also introduced the term “dynamical” to describe the viewpoint). These works triggered a considerable debate, so far primarily within the philosophy of science community.

Fully constructive examples have been presented by D.J. Miller who also made the suggestion, correct in our view, that the primary aim of the dynamical approach should be to supplement the traditional view with increased qualitative understanding. The dynamical viewpoint has been presented to laypeople for, it would appear, the first time, by N. D. Mermin, albeit quite briefly. Finally, the present author has presented a complete layperson’s exposition from this viewpoint.

Section II enumerates the main benefits which, in our view, follow from this approach; section III presents one way of introducing the conceptual foundations of the theory in the dynamical paradigm, and contrasts it with the conventional; section IV argues that the mechanisms of (quantum) field theory are more central to special relativity than is usually described; section V attempts to classify all of the major mechanisms which create relativistic effects, and describes accessible models demonstrating them; section VI is devoted to the perennially vexatious phenomenon of relativity of simultaneity; and VII contains concluding remarks.

II. ADVANTAGES OF THE DYNAMICAL VIEWPOINT

A. Qualitative Understanding

We emphasize first that the dynamical viewpoint supplements the customary viewpoint. One certainly can not dispense with the Lorentz transformation, a central symmetry principle of nature, without which virtually no quantitative results can be obtained. Nevertheless, for the vast majority of students who in later life will never need precise results of Lorentz
invariance, understanding the qualitative mechanisms which produce this invariance could well prove to be more meaningful than the elegant exact formulas. Qualitative understanding of mechanisms should also provide students with a greater feeling of belief that the various relativistic effects actually occur, something which is not necessarily trivial given the counterintuitive nature of the effects and the fact that some of the most important (length contraction, simultaneity) lack straightforward experimental verification.

B. Acknowledging Physical Phenomena

Some of the mechanisms which generate relativistic phenomena are quite obvious given what we know today about the construction of matter. The light clock demonstrates a rate-change mechanism which clearly will affect all processes within composite objects;\textsuperscript{14} likewise, the Heaviside deformation of the moving point-particle electric field shows a universal shape-deformation mechanism.\textsuperscript{15} Both of these may be traced to elementary aspects of wave behavior, with which students should be quite familiar. There is no other area of physics where interesting and instructive phenomena are routinely ignored or dismissed as “not real” in favor of axiomatic treatments.

C. Acknowledging History

It is a fact still not widely appreciated that relativistic phenomena were first inferred from dynamical considerations, starting over two decades before Einstein’s work.\textsuperscript{16} The inferences were made by outstanding scientists, for good reasons, and they were proved to be correct; students should be aware of this.

D. Understanding Why Lorentz-Invariant Theories Can Exist

Immensely powerful consequences follow quickly from the famed Principle of Relativity; this is one indication of precisely how constraining this principle is, or to put it another way, how unlikely it is \textit{a priori} that any physical theory could exist which obeys such a principle.

Many interesting principles can be imagined, but very few find any sensible mathematical expression, let alone congruence to the observed world. When students better understand the mechanisms which underly relativity they will more clearly see why its central principle actually \textit{can} be realized, and their understanding will have less of the magical-realistic flavor imparted by “it must be so” axiomatic derivations.
E. Frame Agnosticism

Lorentz invariance implies that all inertial frames are equivalent, yet the standard pedagogy tends to single out frames in which phenomena have explanations that de-emphasize the physical reality of relativistic processes. The canonical example is the Dewan-Beran scenario. In the initial rest frame, the explanation for the string breaking is clearly the Heaviside alterations to internal force fields, producing contractive forces which are frustrated by the ships to which the string is attached. However, this goes against the notion that length contraction is not “real”, and one often encounters the view that the only true explanation is found in the comoving frame of one of the ships. In this frame the distance between the ships grows, thereby stretching and eventually breaking the string.

Students must realize that Lorentz invariance implies that every reference frame has its own description for any phenomenon, and that all the descriptions are equally complete and equally valid.

F. Better Understanding of Accelerating Systems

A spaceship is accelerating, and hence contracting; what is the trajectory of its two endpoints? Does the front end contract towards the back, does the back contract towards the front, or do both ends contract towards the middle? In any given situation this question clearly has an answer, and the answer clearly relates to the process of length contraction, but it cannot be obtained by Lorentz transformation using any frame.

One can elaborate by imagining a dumbell with heavy weights on each end, accelerated lengthwise. The dumbell contracts, hence the two weights do not follow parallel trajectories and hence do not have identical accelerations. The differential acceleration of the heavy weights implies differential forces, and these are simply the internal forces which produce the length contraction in the first place.

The rotating disk is another example. No two points on the disk are moving with the same velocity, hence analysis using co-moving rest frames gets rather confusing; a much simpler and sounder intuition is gained by working in the center-of-mass frame and recognizing that the underlying dynamical mechanisms remain valid, modulo disruption by inertial forces, producing exactly the same effects on the disk as seen in other moving bodies.

G. Inclusion of Non-Lorentz-Invariant Theories

Consider a theory formed by writing down Maxwell’s equations, then writing down an-
other copy of them with a different speed constant, and coupling them together with some interaction. Such a theory clearly makes sense and has nontrivial physics, indeed may have almost-identical physics to that of our own world, if the interaction is small; nevertheless, it is not Lorentz-invariant and hence cannot be said to live in Minkowski spacetime.

Does that mean that this “two-light” theory must instead fall under the Newtonian absolute-space paradigm? Does a possible universe, simply by virtue of being non-Lorentz-invariant, automatically revert to the pre-relativistic picture? Obviously not, for this theory is no more Galilean invariant than pure electromagnetism itself.

In fact Lorentz-invariant theories exist as highly special points within a vast sea of alternate theories which are, to all appearances, equally physically reasonable. All of these theories exhibit “relativity-like” effects, and through identical mechanisms, but none of them has a Minkowski space understanding. In recent times consideration of such theories has provided a rich source of ideas for precision experimental searches.20,21

H. Continuity with Prior Studies

Students learn a lot about waves in their early studies, but one thing they almost never learn is that waves on a string, water waves, sound waves, and all the other wave systems they study in the linear approximation, are mathematically identical to relativistic theories. Intuition gained by studying these systems can and should transfer directly to physical understanding of relativistic phenomena, as we discuss further in Sec. V.

I. Improved Understanding of Quantum Field Theory

This is the topic of Sec. IV but in brief, the mechanisms which enable relativity are also those central to quantum field theory. Comparatively few students ever study this subject, yet it is the foundation of current theories of matter. By taking some time to try to convey its mechanisms to students at an earlier stage of education both their understanding of relativity and their understanding of all matter will be simultaneously enhanced, and students who do continue to study quantum field theory will face a lower barrier of comprehension.

III. FOUNDATIONAL NARRATIVES

The traditional development of the foundations of special relativity is roughly as follows:

1. One starts with what is essentially an axiom, that physics should be independent
of velocity. This is the Principle of Relativity, which appears to hold in everyday experience and holds exactly in Newtonian physics.

2. Electromagnetism appears to be incompatible with the Principle due to its fundamental speed parameter.

3. In order to preserve both electromagnetism and the Principle, distances and times must be governed by the Lorentz transformation, with space and time seemingly merged into a single “spacetime”.

4. Requiring momentum conservation in different frames then leads to relativistic kinematics and \( E = mc^2 \).

These steps, while highly efficient, are notable for their formal and axiomatic character, which is rather exceptional in the physics curriculum. One derives counterintuitive and all-pervading motion-dependent effects, as well as mass/energy equivalence, without actually studying a single moving object or example of mass interchanging with energy.

The dynamical line of thought suggests a different account of the foundations of the theory, which breaks down the development into more incremental steps and attempts to build at every step on knowledge that students already have, such as electromagnetism, wave behavior, and the atomic structure of objects:

1. Note first that motion affects objects (by which we include processes as well). This can be seen through simple examples such as the light clock or the Heaviside field deformation. Such examples clearly imply motion-dependent change to all objects whose shape and/or processes are governed by electromagnetic field interactions; this includes all the matter of everyday experience.

2. If motion affects objects then it affects measuring devices, hence observers in different states of motion will measure different values for virtually every quantity. This is quite a surprising conclusion, yet it should also seem rather obvious after the previous item.

3. The situation now appears extremely complex, with observers in different states of motion being, on the face of it, completely incommensurable. Is there a possibility of method in the motion-dependent madness?
4. Despite these complex effects, we ourselves feel no adverse effects from motion, even the orbital motion of the Earth (or Sun, or galaxy...) Moreover we observe a consistent, unchanging form for Maxwell’s equations, and a constant, isotropic $c$. The latter is a concrete numerical clue: can it be that the motion-dependent effects are somehow precisely orchestrated to keep this speed constant?

5. If so then there must exist, as a bare minimum, some conceivable mathematical relation between the measurements of moving observers which is compatible with constant $c$. At this point show that the Lorentz transformation gives the unique possible relation.

6. Given this possible relation between moving measurements, are there any physical theories that can actually manifest such a law? Could our own universe be described by such a theory? Einstein’s hypothesis was that the answer is “yes” on both scores.

7. At this point the two developments begin to converge. Lorentz invariance is assumed and its consequences studied, but in the standard view there is no apparent physical cause for any of it, while in the dynamical view the causes are made evident and are postulated to have a particular organization. In the standard view, for any given effect, there is little more to say about it after proving its existence from Lorentz invariance; in the dynamical view one will incorporate specific demonstrations of key mechanisms, examples of which are catalogued in Sec. V.

8. The mass/energy connection, rather than being simply an algebraic consequence of axioms, can be traced to its field theory origins, the most important step being to understand how mass arises at all in such theories.

Thus the two pedagogical approaches arrive at the same destination, but the dynamical view takes a longer and more scenic route, providing, in the words of Bell, “more familiarity with the country”. Given that the “country” consists of the concepts of length, time, mass, and energy which underly all physical thought, a longer and more thorough tour seems well worthwhile.
IV. SPECIAL RELATIVITY AS A PHENOMENON OF QUANTUM FIELD THEORY

A philosophical argument which has been raised against the dynamical viewpoint notes that Lorentz invariance provides precise answers which are, apparently, independent of any underlying mechanisms. This suggests that these results should be viewed as being “caused” by Lorentz invariance (or Minkowski spacetime), rather than the various specific physical mechanisms. This argument has merit, however we wish to point out that there is not, in fact, a wide variety of mechanisms known to be compatible with Lorentz invariance.

Indeed the only physically significant realizations of special relativity appear to be found in quantized field theories. A plausible chain of reasoning shows why this is so: Relativistic interactions must be mediated by fields in order to respect causality (no “action at a distance”), and fields, in turn, must be quantized to remove their instabilities (e.g., the “ultraviolet catastrophe” which prompted Planck’s original work on quantum mechanics).

Systems consisting of classical particles alone can only have contact interactions, hence effectively no interactions. Systems consisting of only classical fields are mathematically interesting but they do not support stable objects and hence do not meaningfully exhibit, for example, length contraction. Hybrid field+particle systems such as classical relativistic particles interacting with an electromagnetic field are, at best, extremely difficult to consistently define due to the singular nature of the point particles, and are also disastrously unstable (failure to build stable atoms in this framework led to Bohr’s quantization hypotheses).

String theory could be taken as a counterexample showing Lorentz invariance emerging from something other than a quantum field theory; however, we note that string theory, like any gravitational theory, is only Lorentz-invariant in the limit of small curvature, and this is also the limit in which it can be approximated by a quantum field theory. It is, therefore, not clear whether any really new physical mechanisms underly the Lorentz invariance of string theory.

Hence it seems not unreasonable to assert that the only physical framework capable of supporting Lorentz invariance in a physically meaningful way is quantum field theory (which we henceforth abbreviate to “field theory”). It follows that Lorentz invariance, rather than being independent of or prior to mechanism, is closely tied to the particular set
of mechanisms found in field theories; further, one can make a case for conceptual priority of the mechanisms, since they can exist without Lorentz invariance (in a non-Lorentz-invariant field theory) but Lorentz invariance cannot exist without the mechanisms.

This philosophical debate certainly will not be resolved here, but to us the considerations above strongly suggest that the mechanisms, the symmetry, and the resulting phenomena should be viewed as part of a single unified package, which students should understand from all angles, just as biologists study both chickens and eggs.

V. MECHANISMS

Regardless of one’s position on philosophical questions, it seems indisputable that students would better and more accurately understand the world around them by understanding as early as possible the mechanisms within field theory which make special relativity possible in the world that we actually inhabit. Moreover, the number of distinct mechanisms is not really so large, and they all can be understood reasonably well in terms of elementary model systems. Here we briefly describe the main mechanisms and associated models, following a more mathematically-oriented treatment which has been presented elsewhere.

A. Wave propagation and rigidity

In a field theory no object influences any other object except through disturbances which propagate as waves. Wave propagation, moreover, inherently occurs at finite speed. A composite object whose components can only interact with each other through disturbances which travel at finite speed cannot be rigid. Force applied to one side of the object will set that side into motion before the other side even feels any change, resulting in shape deformation which can’t be removed by any design of the material. Whether the deformation persists after the acceleration stops requires additional consideration (see E,F below) but Lorentz contraction certainly could not happen without this type of material-independent failure of rigidity.

B. Wave propagation and time dilation

Not only does wave propagation inherently occur at finite speed, but the wave speed is also generically independent of the speed of the source. The speed of sound waves from a jet doesn’t depend on the speed of the jet; the speed of water waves in a wake doesn’t
depend on the speed of the boat that created them. We stress that we are talking about *source-independence* here, not *observer-independence*, the vastly stronger assertion which implies Lorentz invariance.

Source-independence of wave speeds causes changes to the tick rates of moving clocks, as illustrated vividly in the light clock. Any clock consisting of separated subcomponents will be affected by this to some degree, because its components can only interact through wave transmission, the only form of interaction existing in a field theory. This mechanism is so simple that it is almost difficult to accept that it relates to something as apparently recondite as the flow of time; yet, the connection is evident.

*C. Wave packets, group velocity, and relativistic mass*

Particles in a field theory, and hence (apparently) in the actual world, are really wave packets, hence it should not be surprising that their behavior differs considerably from expectations derived from classical particles. Students can find close analogies to their behavior in simple studies of waves moving on a string (a one dimensional model field theory). A freely vibrating string analogizes massless particles, while massive particles can be modelled using a string with springs attached along its length, giving an extra restoring force at each point. “Rest mass” in a field theory is exactly this restoring force.

The massive model is the really useful one, since massless phenomena are already familiar from studies of electromagnetism. The massive model, although simple, may require a slightly more in-depth discussion of concepts such as group velocity and wave momentum than students will have received previously. If time can be found for these topics then students have a great opportunity to understand two related aspects of special relativity which are persistently opaque in the traditional treatment.

The first is the “cosmic speed limit”, i.e., the fact that no massive particle can exceed (or reach) the speed of $c$. This is transparent in the massive model because $c$ is just (analogous to) the underlying wave speed without springs, and the wave propagation is then hindered by the attached springs. The maximal group velocity is easily calculated as $c$.

The relativistic mass is likewise understandable because the “particle” now contains within itself a natural way to store energy, namely through faster oscillations. As one attempts to accelerate a wave packet up to the maximal velocity, a larger and larger fraction of the energy is diverted into faster vibrations, hence the particle resists acceleration and its
inertia grows to infinity.

D. Relativistic mass in time dilation

Once relativistic mass is familiar, students can understand the other main mechanism of time dilation (the first being item B above). As an object accelerates in one direction, the relativistic mass increase of its subcomponents results in a slowing of any periodic movements taking place inside the object.

It is useful to understand this directly in terms of wave packets. The velocity of a wave packet is its group velocity, which depends on the overall frequency, and the frequency is changed by applied forces. Acceleration of the wave packet in one direction increases its frequency, and this reduces the group velocity of the packet in transverse directions. The different components of velocity in a wave packet are thus interrelated in a way which has no analog for particles. (To model this the one-dimensional string must be extended at least to a two-dimensional vibrating sheet.)

Students can see an example of the slowing of periodic motions by studying an orbiting particle (wave packet), accelerated slowly transverse to the orbit. (Other orbit orientations become quickly intractable, hence Bell’s original suggestion to study them through numerical simulation.)

E. Doppler effect and deformation of potential fields

Length contraction (more correctly, shape deformation) first occurred to Fitzgerald upon seeing Heaviside’s solution showing the deformed electric field about a moving charged particle. This remains one of the most striking demonstrations of the inevitability of motion-dependent effects, but direct calculation of the deformed field is not altogether enlightening, and for qualitative purposes it is helpful to understand it in even simpler terms, namely as a manifestation of the doppler effect.

The field of the moving particle establishes itself through the emission of electromagnetic waves during acceleration, and these waves are doppler-shifted like any other, leading to an asymmetrical final field, which can be thought of as a “zero frequency” doppler shift. This gives at least some idea of the inevitability of moving-field deformations in any field theory, in terms of a very generic phenomenon familiar to all students.
F. Quantum numbers and contraction

The spacetime viewpoint can leave the somewhat incorrect impression that all things must contract when accelerated. Of course the Lorentz symmetry does not really guarantee this; obvious counterexamples are malleable systems such a cloud of gas. These systems have many nearly-degenerate energy states and it is impossible to accelerate them without inducing transitions between states. Another example is the wave packets of the elementary particles themselves, which generally do not contract when falling through a potential field. Nevertheless, solid objects do contract when accelerated, matching the predictions of Lorentz invariance; why is this? The reason is the same as that which makes objects stable in the first place, namely quantization. Quantum bound states are measured by discrete integers (“quantum numbers”) which cannot change under any smooth change of conditions such as a slow acceleration (the technical term for such quantities is “adiabatic invariants”; quantum numbers are closely related to classical adiabatic invariants).

The student who has not studied quantum mechanics can still appreciate this concept by thinking about the nodes of a standing wave, whose number is a discrete integer that must remain the same if the conditions of the wave medium are changed in a smooth way.

G. Back-reaction and mass

The mass/energy equivalence implies that the electric field within a capacitor adds to its inertia, i.e., makes the capacitor harder to accelerate, but how does this come about? Likewise when an atom emits a photon and one electron drops to a lower energy level, the atom must become lighter and hence easier to accelerate; part of this change is due to the reduced electrical field energy inside the atom - but how does this changed field translate into reduced difficulty accelerating the atom?

The answer is back-reaction, the process by which a field acts back on its source to (generically) resist the acceleration of the source. For example, for a capacitor accelerated parallel to its plates, self-inductance generates an electric field which acts oppositely on the charges. Almost all of the mass of everyday matter arises, in fact, through this mechanism (with the strong nuclear field rather than the electromagnetic).

Students can easily calculate these effects in simple cases such as the capacitor, but there is a catch: the answers are not quantitatively correct because extra “hidden momentum” is contained inside the material of the objects, in the motion of their electrons. This in itself
is a source of interesting exercises for students.

H. Mass-energy equivalence

In the traditional pedagogy of relativity, equivalence of mass and energy is rather opaque because objects are black boxes whose energy and mass are not further analyzed. Within field theory, however, the equivalence is much easier to understand, at least qualitatively.

For starters, every object in the theory is a field configuration (perhaps organized into a wave packet, perhaps not). It is not surprising that energy can convert freely between different kinds of field configurations; the whole theory is, after all, just wave fields exciting other wave fields through processes not much different from shaking a string.

One can consider then a very massive particle sitting at rest. In the string model, high mass means very stiff attached springs whose vibrations store a large amount of energy. If we now connect that string to a very long massless string, modelling an interaction, then the vibrational energy on the massive string will leak out onto the massless one, until the “massive particle” is gone and all that remains is “energy”, i.e., waves propagating on the massless string. Thus mass is converted to energy, and the greater the mass, the more the resulting energy.

The two other sources of mass described above also have intuitive mechanisms for conversion to other forms of energy. Relativistic mass arises from the same internal wave-packet oscillations as rest mass, and converts to other forms of energy in the same way just shown. Back-reaction is more involved since the internal field configurations which cause it can be very complicated (e.g., the strong nuclear field inside a nucleon); however, it is qualitatively clear that a greater energy of internal fields implies greater back-reaction, showing the mass-energy relation.

I. Time dilation of particle decays

Particle decays are the most commonly observed manifestations of time dilation, yet they are quite inscrutable in traditional relativity courses. This is not necessary, for simple wave-on-string models capture the essence of the mechanism, including the correct dilation factor. Indeed, this factor is visible already in a driven harmonic oscillator; the rate of amplitude gain (for resonant driving) is inversely proportional to the frequency, and that frequency translates directly into the dilation factor for a moving wave packet.
VI. SIMULTANEITY

Relativity of simultaneity is persistently one of the most confusing pieces of the relativity puzzle, owing, perhaps, to its “nonlocal” nature, connecting observations made at separated locations. It is a very subtle and beautiful physical effect, one which students should truly enjoy coming to understand. From the dynamical perspective the crucial point can be expressed in a simple mnemonic: motion affects clocks, and synchronization of separated clocks requires additional motion, resulting in an additional effect.

More concretely, imagine that we are watching a scientist on board a moving spaceship, and that scientist is in the process of synchronizing clocks at two different places in her lab. How is she doing it? Simple: she sets both clocks together in one location, and then carries one clock to the new location. It seems very reasonable to expect that if there is any good way at all to synchronize two clocks in a lab, then this must be one good way.

But as we watch this scientist carry out the simple procedure, something goes wrong. For starters, since she is already moving, along with her whole lab, all her clocks are running slow, due the physical effects described in Sec.VI (B,D). Moreover, and crucially, as she carries the clock within her lab she is moving at a slightly different speed from her ship, hence the carried clock is running at a slightly different rate from the clock left behind. This means that when she arrives at the new location her synchronized clocks will, in fact, no longer be synchronized, at least according to us.

One’s first thought is that this discrepancy should go away if the scientist is careful to carry the clock extremely slowly; however, it does not. Slower carrying implies a longer carrying time, meaning that the rate difference has a longer time to accumulate, and one finds, amazingly, an irremovable discrepancy in synchronization which depends only on the distance between the two clocks being synchronized.

Students can see a concrete and geometric demonstration of this effect by studying the ever-helpful light clock. Grasping this phenomenon should, in our view, be one of the very high points of a student’s physics education.

Having understood this, the student is at last ready to see how the pieces of the relativity puzzle fit together to make it physically possible to realize Einstein’s bizarre assertion that a particular speed should look the same to everyone. The student is ready to understand the true content of the Lorentz transformation, and to see how paradoxes of the “each person
sees the other contracted" variety are resolvable.

This method of clock synchronization is known as “slow carrying” and has been studied at least the work of Eddington in the 1920’s. It contrasts with the “light signalling” method originally employed by Einstein, and in our opinion is conceptually preferable in several ways:

- It exposes the underlying physical mechanism, i.e., effects accruing from additional motion.
- It builds on students’ prior understanding of the dynamical effects in clocks.
- It is operationally prior to signalling, because one would not employ synchronization by signalling without first verifying that it agrees with the result of clock-carrying.
- It applies also in theories which have no propagating massless fields to use for signalling.

There is a considerable literature on different possible synchronization conventions, but we feel that the question should not be central to pedagogy. Rather, the primary challenge for the special relativity curriculum when it comes to simultaneity is simply to persuade students that this very subtle effect actually exists at all, something which, in our opinion, is best done by exhibiting a physical mechanism which the student can explore based on prior knowledge.

VII. CONCLUSION

We have tried to provide convincing arguments in favor of adding the dynamical viewpoint to the standard pedagogy of special relativity, as well as specific explanations and demonstrations usable for that purpose. Teaching time is obviously limited, and one may question whether it is productive to “muddy the waters” by discussing modes of understanding which are primarily qualitative (although students will be amazed to find fully relativistic formulas materialize from very simple model systems). We feel, nevertheless, that it should be seriously considered, because of the absolute centrality of the concepts involved; this is, after all, the only part of the curriculum that deals solely with the core concepts of distance, time, energy, mass, and measurement that permeate all physical thinking.
The surprising ease of deriving the Lorentz transformation belies its great conceptual difficulty, which requires students to re-think virtually everything they previously believed about how the world works. Overly abstract explanations based on constancy of the speed of light hide the true complexity of the underlying processes and do not provide a solid foundation for reasoning about the fundamental concepts involved, nor indeed a really compelling reason to believe that the many counterintuitive phenomena are true to begin with. Many students are left with lingering sensations of circularity in the derivations as well as serious uncertainties about what the theory covers, what the alternatives to it are (generally none are discussed, aside from Newtonian physics), whether it is an empirical theory or holds by philosophical necessity, and generally how to reason about any question which goes beyond textbook problems. In our view this is not necessary, but results from the unbalanced presentation which has become standard.

VIII. ACKNOWLEDGMENTS

The author gratefully acknowledges helpful discussions with and suggestions from Dean Welch, Larry Hoffman, and Kirk McDonald.

*wmn@cox.net

1 H.R. Brown, O. Pooley, “The origins of the spacetime metric: Bell’s ‘Lorentzian pedagogy’ and its significance in general relativity,” in *Physics Meets Philosophy at the Planck Scale*, edited by C. Callender and N. Huggett (Cambridge University Press, Cambridge, 2001), p.256. (Arxiv:gr-qc:9908048)

2 D.J. Miller, “A constructive approach to the special theory of relativity,” Am. J. Phys. 78, 633-638 (2010).

3 H.R. Brown, *Physical Relativity*, 1st ed. (Oxford University Press, Oxford, 2005).

4 H.R. Brown, “Michelson, FitzGerald and Lorentz: the origins of special relativity revisited,” e-print PITT-PHIL-SCI 00000987; http://philsci-archive.pitt.edu/987/1/Michelson.pdf.

5 H.R. Brown, “The origins of length contraction: I. The FitzGerald-Lorentz deformation hypothesis,” Am. J. Phys. 69, 1044-1054 (2001).
6 J. Larmor, “On a dynamical theory of the electric and luminiferous medium,” Philosophical Transactions of the Royal Society 190, 205 (1897).
7 J.J. Thompson, “On the electric and magnetic effects produced by the motion of electrified bodies,” Philosophical Magazine 11, 229-249 (1881).
8 J.S. Bell, reprinted in Speakable and Unspeakable in Quantum Mechanics, 2nd ed. (Cambridge University Press, Cambridge, 2004), pp. 67-80.
9 H.R. Brown, O. Pooley, “Minkowski space-time: a glorious non-entity,” in The Ontology of Spacetime, edited by Dennis Dieks (Elsevier Science, Amsterdam, 2006), p.67. (Arxiv:physics:0403088)
10 see, e.g., M. Janssen, “Drawing the line between kinematics and dynamics in special relativity,” Studies in History and Philosophy of Modern Physics 40 26-52 (2008).
11 M. Frisch, “Principle or constructive relativity?,” Studies in History and Philosophy of Modern Physics 42 176-183 (2011).
12 N.D. Mermin, It’s About Time (Princeton University Press, Princeton, 2005).
13 W.M. Nelson, Relativity Made Real, 2nd ed. (CreateSpace Publishing, 2013).
14 The light-clock example is described in many places, e.g. refs. 12,13,26.
15 This calculation is described in standard texts such as J.D. Jackson, Classical Electrodynamics, 2nd ed. (John Wiley & Sons, New York, 1975). A more qualitative understanding is presented in ref. 26. The history surrounding Heaviside’s calculation is well described in B. Hunt, The Maxwellians (Cornell University Press, Ithaca, 1994).
16 E.g., in the 1881 work of Thompson (ref. 7). The book of Brown (ref. 3) contains very extensive historical references.
17 E. Dewan and M. Beran, “Note on stress effects due to relativistic contraction,” Am. J. Phys. 27, 517 (1959).
18 For our purposes it is mainly a question of principle, however it has been studied quantitatively by H. Nikolic, “Relativistic contraction of an accelerated rod,” Am. J. Phys. 67, 1007 (1999).
19 e.g. J. Stachel, “Einstein and the rigidly rotating disk,” in “Einstein and the History of General Relativity”, edited by J. Stachel and D. Howard (Birkhauser, 1986).
20 A. Kostelecky, “The Search for relativity violations,” Sci. Am., September 2004, p. 92.
21 They can be written formally in terms of Minkowski-space theories plus external, Lorentz-breaking background fields; however, the structure of Minkowski space has no real relevance in
this formalism beyond the stipulation that the field equations should be hyperbolic.

22 Classical models do support solitons, but the author is not aware of a theory in which composite material objects can be built from these. In any case, the relativistic mechanisms in a classical field theory are a subset of those found in quantum field theories.

23 The argument is a variant of conventional “lore” stating that a relativistic quantum theory has to be a quantum field theory; cf. A. Hobson, “There are no particles, there are only fields,” Am. J. Phys. 81, 211 (2013), or T. Banks, Modern Quantum Field Theory (Cambridge University Press, Cambridge, 2008), ch. 1.

24 see, e.g., the chapters by P. Pearl and S. Coleman in Electromagnetism: Paths to Research, edited by D. Teplitz (Plenum, New York, 1982)

25 cf. J. Polchinski, String Theory (Cambridge University Press, Cambridge, 1998), Vol. I, ch. III.

26 W.M. Nelson, “A wave-centric view of special relativity,” Arxiv:1305.3022.

27 Moreover, source-independence is generically true even if an underlying distinguished frame or aether exists; the only thing that matters is that the phenomena which we can observe are governed by propagation of waves. This point has been emphasized by M. Shupe, “The Lorentz-Invariant Vacuum Medium,” Am. J. Phys. 53, 122 (1985).

28 Indeed the massless model does not support “resting” oscillations; all disturbances propagate away. With springs attached, however, a disturbance may oscillate in place, because the springs provide a restoring force which is not connected to neighboring parts of the string.

29 We note that the “restoring force” does not have to be entirely fundamental but can instead be created by the interaction with yet another field, e.g. the Higgs or something similar. It would be nice to incorporate a higgs-like interaction into the string+spring model in a natural way; cf. notes of K.T. McDonald, http://puhep1.princeton.edu/~mcdonald/examples/higgs.pdf

Additional interesting intuitive pictures of the origin of mass in field theories can be found in B. Hoeneisen, “Trying to understand mass,” Arxiv:hep-th/0609080.

30 The exception is gravitational potentials, a reflection of the equivalence principle (ref. 26)

31 H. Goldstein, Classical Mechanics, 2nd ed. (Addison Wesley, 1980), ch. 11-7.

32 The other part of the mass change is actually an increase, because the electron in a lower energy state moves faster and hence has a larger relativistic mass. This increase is more than offset by the decrease in field energy.

33 F. Wilczek, “Happy birthday, electron,” Sci. Am. June 2012, 24.
34 D.J. Griffiths, “Resource letter EM-1: electromagnetic momentum,” Am. J. Phys 80 7-18 (2012).
35 W.M. Nelson, “On back-reaction in special relativity,” Am. J. Phys. 81, 492 (2013).
36 see also the pedagogical notes of K. T. McDonald, http://puhep1.princeton.edu/~mcdonald/examples/cap_
37 Internal electron motion isn’t the whole resolution of hidden momentum; ultimately it comes
about because the momentum (and energy) of interacting fields (or fields and particles) has an
interaction component which cannot be canonically divided between the subsystems.
38 This is discussed in detail in ref. 26; also cf. the $\frac{1}{\omega}$ factor in the green function, eq. (34) of
http://www.physics.ohio-state.edu/~mathur/greensfunctions.pdf
39 A. Eddington, The Mathematical Theory of Relativity (Cambridge University Press, 1923).
40 R. Anderson, I. Vetharaniam, and G.E. Stedman, “Conventionality of synchronization, gauge
dependence and test theories of relativity,” Physics Reports 295, 93-180 (1998).