Two-field Kähler moduli inflation in large volume moduli stabilization

Huan-Xiong Yang\textsuperscript{1,2} and Hong-Liang Ma\textsuperscript{1}

\textsuperscript{1} Interdisciplinary Center for Theoretical Study, University of Science and Technology of China, Hefei 200026, People’s Republic of China
\textsuperscript{2} Kavli Institute for Theoretical Physics China, CAS, Beijing 100190, People’s Republic of China
E-mail: hyang@ustc.edu.cn and hlma@mail.ustc.edu.cn

Received 28 April 2008
Accepted 3 August 2008
Published 20 August 2008

Abstract. In this paper we present a two-field inflation model, which is distinctive in having a non-canonical kinetic Lagrangian and comes from the large volume approach to the moduli stabilization in flux compactification of type IIB superstring on a Calabi–Yau orientifold with $h^{(1,2)} > h^{(1,1)} \geq 4$. The Kähler moduli are classified as the volume modulus, heavy moduli and two light moduli. The axion–dilaton, complex structure moduli and all heavy Kähler moduli including the volume modulus are frozen by a non-perturbatively corrected flux superpotential and the $\alpha'$-corrected Kähler potential in the large volume limit. The minimum of the scalar potential at which the heavy moduli are stabilized provides the dominant potential energy for the surviving light Kähler moduli. We consider a simplified case where the axionic components in the light Kähler moduli are further stabilized at the potential minimum and only the geometrical components are taken as scalar fields to drive an assisted-like inflation. For a certain range of moduli stabilization parameters and inflation initial conditions, we obtain a nearly flat power spectrum of the curvature perturbation, with $n_s \approx 0.96$ at Hubble exit, and an inflationary energy scale of $3 \times 10^{14}$ GeV. In our model, there is significant correlation between the curvature and isocurvature perturbations on super-Hubble scales, so at the end of inflation a great deal of the curvature power spectrum originates from this correlation.

Keywords: cosmological perturbation theory, string theory and cosmology, inflation
1. Introduction

The development of top-down cosmology has encountered a research boom in studying the superstring landscape for realizing cosmological inflation. This involves the search for metastable de Sitter-like string vacua in which the ten-dimensional spacetime behaves effectively as having only four dimensions, with six spatial extra dimensions compactified on a Calabi–Yau threefold and almost all of the compactification moduli including the Calabi–Yau volume stabilized. The advent of the KKLT mechanism [1] for moduli stabilization has opened lots of possibilities for engineering inflation in string theory, using either the light closed string moduli [2]–[5] or open string moduli [6] related to the position of a mobile D-brane in wrapped geometry as the relevant scalar fields. Different stringy realizations of cosmological inflation may not be mutually incompatible. They may arise in different regions of the landscape, among which we anticipate that there is at least one possibility that realizes the inflationary evolution of our Universe. The different inflation scenarios may also turn into one another, increasing the overall probability of inflation from a single mechanism [5].

In a reasonable inflationary scenario, the inflaton field has to be ensured to roll its potential very slowly so that there are enough e-foldings for acceleration. It is necessary that, during the inflation epoch, the potential energy curve of the scalars involved must be sufficiently flat and their effective masses have to be much smaller than the Hubble parameter, \( m^2 \ll H^2 \). Scalar fields such as the brane inter-distances in a general D-brane inflation model are conformally coupled to gravity, which enables them to acquire squared masses \( m^2 \approx \frac{1}{12} R \sim H^2 \) (the \( \eta \)-problem). To overcome this \( \eta \)-problem and have a slow-roll inflation in such a scenario, severe and sometimes unnatural fine-tunings are indispensable [6]. Alternatively, no severe \( \eta \)-problem exists in the closed string moduli inflation scenario [2]–[5] where the inflation is thought of as driven by some light closed string moduli associated with 4-cycle volumes of the compact Calabi–Yau threefolds, particularly in the large volume limit [7, 8].

The first proposed closed string moduli inflation scenarios in type IIB superstring theory were the racetrack inflation model [2] and the better racetrack inflation model [3], where the heavy moduli are stabilized at a supersymmetric anti-de Sitter \( F \)-term potential minimum and the potential is uplifted to have a de Sitter vacuum for inflation in
Two-field Kähler moduli inflation in large volume moduli stabilization

an uncontrollable manner unless some additional ingredients are added to break the supersymmetry spontaneously [9]–[12]. In the subsequent Kähler inflation model [4] and roulette inflation model [5], the scalar potential for the moduli fields is calculated in the large volume limit, where the leading $\alpha'$-correction to the Kähler potential has been taken into account so that the heavy moduli are stabilized at a supersymmetric broken anti-de Sitter $F$-term potential minimum. Because this anti-de Sitter minimum is separate from supersymmetry, it can be uplifted to a de Sitter vacuum by taking into account some $D$-term corrections to the scalar potential [13,14]. Moreover, after heavy moduli are stabilized in the large volume limit, the scalar potentials for the surviving light Kähler moduli are exponentially flat, making such approaches very promising for engineering assisted-like inflation [15].

In string theory cosmology scenario, the early evolution of our Universe is essentially driven by multiple scalar fields. When superstring is compactified on a Calabi–Yau threefold to yield a four-dimensional effective $\mathcal{N} = 1$ supergravity theory, lots of moduli fields emerge. It has been conjectured, however, that increasing the number of Kähler moduli fields probably makes the inflation easier to achieve [3]. Moreover, the presence of multiple scalar fields during inflation can lead to quite different inflationary dynamics such as the detectable non-Gaussianity in primordial density perturbations on super-Hubble scales and residual isocurvature fluctuations after inflation [16] that might appear unnatural in a single-field model.

Our aim in the present paper is to build a two-field inflation model from the large volume limit of type IIB superstring with 3-form fluxes compactified on a Calabi–Yau orientifold with $h^{(1,2)} > h^{(1,1)} \geq 4$ and discuss its implications for the COBE spectrum of the linear curvature perturbation. For that we first investigate the possibility of realizing better racetrack inflation in the large volume approach. We divide the Kähler moduli into the volume modulus, $h^{(1,1)} - 3$ heavy moduli and two light moduli. The scalar potential is dominated by its $F$-term contribution, and is corrected by a volume modulus dependent uplifting. The axion–dilaton, complex structure moduli and all heavy Kähler moduli including the volume modulus are stabilized, in the large volume limit, to a de Sitter-like minimum of the potential, whereas the two light Kähler moduli remain dynamical during the moduli stabilization process. These two light Kähler moduli are supposed to be the scalar fields driving the inflationary expansion of the four-dimensional Universe. The de Sitter minimum of the dominant potential not only stabilizes the heavy moduli by giving them masses, but also governs the evolution of the two surviving light moduli. The exponential dependence of the non-perturbatively corrected superpotential upon the Kähler moduli guarantees flatness of the scalar potential for two light Kähler moduli. If the axionic components of the light moduli are dynamical, this is a large volume version of the better racetrack model [3] in which four scalar fields are involved. In the present paper we choose to stabilize the axionic components at the potential minimum; a two-field assisted-like model [15] results instead. In some sense, this is an extension of the single-field inflationary model [4] to a two-field case, for which we calculate the linear perturbations in a strict multifield approach. By assigning some suitable model dependent parameters, we obtain the COBE normalization favored power spectrum of the linear curvature perturbation, and in particular, $n_s \approx 0.96$ at the horizon exit. Dependent upon the trajectory in phase space and thus the initial conditions that we choose, the isocurvature perturbation in this model could decay very slowly for most
Two-field Kähler moduli inflation in large volume moduli stabilization

e-folds on superhorizon scales. We have also observed that the power spectrum of curvature perturbation on the super-Hubble scales shows some remarkable departures from that of adiabatic perturbation at the end of inflation, implying that the curvature fluctuation after inflation inevitably includes significant contributions from isocurvature perturbation.

The paper is organized as follows. In section 2, the theoretical setup is introduced. We review briefly the large volume approach to the moduli stabilization developed by Quevedo and collaborators in [7,8] and establish our own two-field model. The third section is devoted to the investigation of possible inflationary dynamics in our model. We first present the equations of motion for the background fields and the linear perturbations, and then calculate the power spectra of these perturbations and their correlation by numerical integration. The curvature power spectrum obtained is nearly flat during most of the inflation and is characterized by a spectral index $n_s \approx 0.96$ at the claimed Hubble crossing. The final section gives our conclusions.

2. The inflationary landscape in the large volume approach

Inflation in string theory is always associated with the resolvability of the moduli stabilization because the potential used to stabilize the heavy moduli might yield unacceptably large masses for the inflation drivers [6]. In type IIB superstring compactified on a Calabi–Yau orientifold, the closed string moduli consist of the complex structure moduli, the dilaton field and the Kähler moduli. All complex structure moduli including the dilaton field could be heavy moduli and be stabilized at a minimum of the effective potential if we introduce an imaginary self-dual 3-form flux into the orientifold construction [17,18]. The stabilization of the Kähler moduli requires, according to the KKLT mechanism [1], introducing the non-perturbative contributions of the Euclid D3-branes or some wrapped D7-branes to the superpotential,

$$ W = \int G_3 \wedge \Omega + \sum_{i=1}^{h^{(1,1)}} A_i e^{-a_i T_i}, $$

where $T_i = \tau_i + i \theta_i$ with $\tau_i$ the $i$th 4-cycle volume and $\theta_i$ the corresponding axions. The coefficients $A_i$ represent threshold corrections and are independent of the Kähler moduli.

The Kähler potentials arising from type IIB superstring are of no-scale or approximately no-scale form [19],

$$ K = K_{cs} - 2 \ln \left( V + \frac{\xi}{2} \right), $$

satisfying

$$ K^{ij} \partial_i K \partial_j K - 3 = \frac{3\xi}{4V - \xi} \approx 0 $$

in the large volume limit $V \to \infty$, where $\xi = -\zeta(3)\chi(M)/2(2\pi)^3 g_s^{3/2}$, $\zeta(3) \approx 1.2$ and $\chi(M) = 2(h^{(1,1)} - h^{(1,2)})$. In (2) the $\alpha'$-corrections have been included. Combining with the superpotential given in equation (1), we see that the Kähler moduli only appear
Two-field Kähler moduli inflation in large volume moduli stabilization

exponentially in the $F$-term scalar potential\(^3\),

\[
V_F = e^K (K^{ij} D_i K D_j K - 3|W|^2) = e^K K^{ij} \left[ a_i A_i A_j e^{-a_i T_i - a_j T_j} - (K_i W a_j A_j e^{-a_j T_j} + c.c.) \right],
\]

where $D_i K = \partial_i KW + \partial_i W$. The uplifting correction to the scalar potential, which has several possible sources and is necessary for fine-tuning the cosmological constant, depends on moduli only through the overall volume,

\[
V_{\text{uplift}} \approx \frac{1}{V^\rho},
\]

where $\frac{4}{3} \leq \rho \leq 3$. For concreteness, we assume $\rho = 2$ in the present paper. It is remarkable that, in the presence of several Kähler moduli, the variation of the scalar potential along each $T_i$ direction is in general uncorrelated with its magnitude. Besides, the potential is exponentially flat along some $T_i$ directions as long as these Kähler moduli are sufficient large. Therefore, the large Kähler moduli have a great chance to become the scalar fields for driving the early inflationary evolution of our Universe.

The Calabi–Yau orientifolds of Swiss-cheese type have been shown to be very useful in string inflation engineering, whose volume can be formulated in a simplified form as follows [4, 5]:

\[
V = \frac{\alpha_0}{2\sqrt{2}} \left[ (T_1 + \bar{T}_1)^{3/2} - \sum_{i=2}^{n} \lambda_i (T_i + \bar{T}_i)^{3/2} \right].
\]

Here the volume of the $i$th 4-cycle is denoted by $\tau_i = \text{Re} T_i$, among which $\tau_1$ controls the overall volume and $\tau_2, \ldots, \tau_n$ ($n = h^{(1,1)}$) are blow-ups whose only non-vanishing triple intersections are with themselves. We stabilize the dilaton and complex structure moduli with fluxes, following the plausible KKLT procedure [1]. After that, the superpotential (1) is reduced to being only Kähler moduli dependent, $W = W_0 + \sum_{i=1}^{n} A_i e^{-a_i T_i}$, where $a_i = 2\pi/N$ and for simplicity $W_0$ is assumed to be real.

We establish our formalism in the large volume limit, where $V \to \infty$, $\tau_1 \approx V^{2/3} \to \infty$, $a_i \tau_i \sim \ln V$ for $i = 2, 3, \ldots, n$ but $A_i \gg A_j$ for $i = 2, 3, \ldots, k$ and $j = k + 1, k + 2, \ldots, n$. Because the $A_i$ in superpotential (1) are directly proportional to the squared masses of the corresponding Kähler moduli, the $T_i$ are referred to as being heavy moduli when $i = 2, 3, \ldots, k$ and light moduli when $i = k + 1, k + 2, \ldots, n$, respectively. In this limit, $e^K \approx 1/V^2$, the superpotential is approximately independent of $T_1$ which we will call the volume modulus,

\[
W \approx W_0 + \sum_{i=2}^{n} A_i e^{-a_i T_i}.
\]

Up to magnitudes of order $1/V^3$, we can approximately write the scalar potential as $V \approx V_{\text{dom}} + V_{\text{corr}}$, where the dominant part $V_{\text{dom}}$ consists of the contributions of the volume modulus and the heavy moduli, and of the necessary uplifting potential,

\[
V_{\text{dom}} = \sum_{i=2}^{k} \frac{8(a_i A_i)^2 \sqrt{\omega_i}}{3V\lambda_i a_0} e^{-2a_i \tau_i} + \sum_{i=2}^{k} \frac{4a_i A_i W_0 \tau_i}{V^2} e^{-a_i \tau_i} \cos(a_i \theta_i) + \frac{3\xi W_0^2}{4V^3} + \frac{\gamma W_0^2}{V^2}.
\]

\(^3\) Throughout, we take the units $M_p = 1/\sqrt{8\pi\alpha'} = 1$. 

Journal of Cosmology and Astroparticle Physics 08 (2008) 024 (stacks.iop.org/JCAP/2008/i=08/a=024) 5
The contributions of the light Kähler moduli to the scalar potential only appear in correction term $V_{\text{corr}}$,}

$$V_{\text{corr}} = \sum_{i=k+1}^{n} \frac{8(a_i A_i)^2 \sqrt{\tau_i}}{3\sqrt{\epsilon_i} a_0} e^{-2a_i \tau_i} + \sum_{i=k+1}^{n} \frac{4a_i A_i W_0 \tau_i}{\gamma^2} e^{-a_i \tau_i} \cos(a_i \theta_i). \quad (9)$$

In equation (8), we have parameterized the uplifting potential as $V_{\text{uplift}} = \gamma W_0^2 / \sqrt{\gamma^2}$, with $\gamma$ a positive parameter. There are terms not included in equations (8) and (9), which are subleading.

In the large volume approach [7, 8], the moduli stabilization problem of the Kähler moduli is solved in the following two-step procedure [20]. First, we stabilize the axions $\theta_i (i = 2, 3, \ldots, k)$ to the potential minimum by setting $a_i \theta_i = \pi$. Second, we can find an approximate minimum of the potential by letting $V_{\text{dom}}$ be flat along the directions of $\mathcal{V}$ and the heavy moduli $\tau_i (i = 2, 3, \ldots, k)$ in moduli space. The minimum of the scalar potential turns out approximately to be

$$V_{\text{min}} \approx -\frac{3W_0^2}{2\sqrt{\gamma}} \left[ \sum_{i=2}^{k} \frac{\lambda_i a_0}{a_i^{3/2}} (\ln \mathcal{V}_i - c_i)^{3/2} - \frac{\xi}{2} \right] + \frac{\gamma W_0^2}{\sqrt{\gamma^2}}, \quad (10)$$

where $\mathcal{V}_i$ which stands for the stabilized Calabi–Yau volume is the solution of the algebraic equation

$$\mathcal{V} \approx -\frac{9\xi}{8\gamma} + \frac{9a_0}{8\gamma} \sum_{i=2}^{k} \frac{\lambda_i \sqrt{\ln \mathcal{V} - c_i}}{a_i^{3/2}} [2\ln \mathcal{V} - (2c_i + 1)], \quad (11)$$

with

$$c_i = \ln \left( \frac{3a_0 \lambda_i W_0}{4a_i A_i} \right).$$

The heavy moduli will be stabilized at $a_i \tau_{i,\text{min}} \approx \ln \mathcal{V} - c_i$. Phenomenologically, we expect $V_{\text{min}}$ to provide the dominant potential energy for the inflationary evolution of the surviving light Kähler moduli. The COBE normalization of density perturbations at the Hubble exit $\delta_H \approx 1.92 \times 10^{-5}$ then demands $(V_{\text{min}} / \epsilon_1)^{1/4} \sim 0.004 \sim 5.4 \times 10^{16}$ GeV at the Hubble exit, where $\epsilon_1$ is the first slow-roll parameter. The range of $\epsilon_1$ at the horizon exit is $\epsilon_1 \sim 10^{-12}$ for typical values of the microscopic parameters. Thus the inflationary energy scale should be rather low in such models, $V_{\text{min}} \sim 10^{-19}$ [4]. This requirement can be easily satisfied by some fine-tuning on the stringy parameters involved and, in fact, there exists a landscape in parameter space favoring phenomenology and cosmology. One possibility that we will focus on in the next section is given by an orientifold model with $h^{(1,1)} = 4$, $h^{(1,2)} \approx 100$, with geometrical parameters $a_0 = 1/\sqrt{\lambda_1}$ and $\lambda_2 = \lambda_3 = \lambda_4 = 1$ [5]. The complex structure moduli including dilatons are supposed to be fixed by imaginary self-dual 3-form fluxes in such a manner that after these moduli are stabilized we have $W_0 = 100, g_s \approx 0.132$ and consequently $\xi \approx 10$. The other microscopic parameters in the model are designated as

$$A_2 = 1, \quad A_3 = A_4 \approx 1.075 \times 10^{-4}, \quad a_2 = a_3 = a_4 = 2\pi/300, \quad \gamma \approx 1.026 \times 10^{-5}.$$
For such a model, numerical calculation tells us that $\mathcal{V}_1 \approx 2.8 \times 10^8$ and $V_{\text{min}} \approx 4.035 \times 10^{-19}$. The volume modulus $\tau_1$ and heavy modulus $\tau_2$ are found to be frozen at $\tau_{1,f} \approx 4.28 \times 10^5$ and $a_2 \tau_{2,f} \approx 13.81 \approx 0.71 \ln \mathcal{V}_1$ respectively, as expected.

The large volume approach to heavy Kähler moduli stabilization naturally provides a platform for having a stringy inflation scenario, where the surviving light Kähler moduli, say $\tau_i$ for $i = k + 1, \ldots, n \ (k \geq 2)$, play the roles of the multifield inflation drivers. The effective potential for these light moduli is

$$V \approx V_{\text{min}} + \sum_{i=k+1}^{n} \frac{8(a_i A_i)^2 \sqrt{\tau_i}}{3V_i \lambda_i \alpha_0} e^{-a_i \tau_i} + \sum_{i=k+1}^{n} \frac{4a_i A_i W_0 \tau_i}{V_i^2} e^{-a_i \tau_i} \cos(a_4 \theta_i),$$

and these scalar fields are assumed to be initially far from the minimum of the potential $V$ (please do not confuse $V_{\text{min}}$ with the minimum of $V$; the latter is expected to be zero). Because in equation (12) $V_{\text{min}}$ is one order of the fixed Calabi–Yau volume in magnitude greater than the other scalar field dependent terms, the fixed volume $\mathcal{V}_1$ and all of the stabilized heavy Kähler moduli as well as the associated axions will remain frozen during inflation. These frozen heavy moduli will not create harmful contributions to the masses of the light scalar fields.

The form of potential (12) implies that there is a stringy landscape for realizing the cosmological inflation in IIB flux compactification at the large volume limit. If we consider type IIB compactification on the Calabi–Yau orientifolds with $h^{(1,1)} = n = 3$, for example, we will get Kähler modulus inflation [4] when the axion $\theta_3$ is stabilized at the potential minimum or roulette inflation [5] when $\theta_3$ is dynamical. In this paper, we consider the IIB compactification on a Calabi–Yau orientifold with $h^{(1,1)} = n \geq 4$, where almost all Kähler moduli (including $\mathcal{V}$, $\tau_1$ and $T_i = \tau_i + i \theta_i$ for $i = 2, 3, \ldots, n - 2$) have been stabilized. The only exception is for the two light moduli $T_{n-1} = \tau_{n-1} + i \theta_{n-1}$ and $T_n = \tau_n + i \theta_n$, which correspond to four scalar fields. The dynamics of this four-field system is governed by an effective potential

$$V \approx V_{\text{min}} + \sum_{i=-1}^{n} \frac{8(a_i A_i)^2 \sqrt{\tau_i}}{3V_i \lambda_i \alpha_0} e^{-a_i \tau_i} + \sum_{i=-1}^{n} \frac{4a_i A_i W_0 \tau_i}{V_i^2} e^{-a_i \tau_i} \cos(a_4 \theta_i).$$

Recall that the effective superpotential for these fields is a sum of two exponential terms; the potential (13) might be a large volume version of the better racetrack inflation model [3].

The detailed investigation of this refined better racetrack model is scheduled to be published in the near future [22]. In the present paper, we do what Conlon and Quevedo have done, by fixing the axions to the potential minimum and ignoring the double exponentials, but regard both light Kähler moduli $\tau_3$ and $\tau_4$ as relevant scalar fields (here we take $n = 4$ for simplicity). This will yield an assisted-like extension of the single-field Kähler modulus inflation model. The potential energy for such a two-field system reads

$$V \approx V_{\text{min}} - \frac{4a_3 A_3 W_0 \tau_3}{V_i^2} e^{-a_3 \tau_3} - \frac{4a_4 A_4 W_0 \tau_4}{V_i^2} e^{-a_4 \tau_4}.$$
The kinetic Lagrangian is given by $\mathcal{L}_K = -\sum_{i,j=3}^4 K_{ij} \partial_\mu \tau_i \partial^\mu \tau_j$. Explicitly,

$$\mathcal{L}_K = -\frac{3\alpha_0 \lambda_2}{8 V_4} (\partial \tau_3)^2 - \frac{3\alpha_0 \lambda_4}{8 V_4} (\partial \tau_4)^2 - \frac{9 \alpha_0^2 \lambda_3 \lambda_4 \sqrt{V_3 \tau_4}}{4 V_4^2} (\partial_\mu \tau_3)(\partial^\mu \tau_4).$$

Let

$$\varphi_i = \sqrt{\frac{4\alpha_0 \lambda_{i+2}}{3V_4}} \tau_{i+2}^{3/4},$$

with $i = 1, 2$; we get

$$\mathcal{L}_K = -\frac{1}{2} \partial^\mu \varphi_1 \partial_\mu \varphi_1 - \frac{1}{2} \partial^\mu \varphi_2 \partial_\mu \varphi_2 - \frac{9 \varphi_1 \varphi_2}{8 \lambda_2 \lambda_4} \partial^\mu \varphi_1 \partial_\mu \varphi_2.$$  

(16)

The potential (14) can be recast as

$$V(\varphi_1, \varphi_2) = V_{\text{min}} + V_1 \psi(\varphi_1) e^{-\beta_1 \psi(\varphi_1)} + V_2 \psi(\varphi_2) e^{-\beta_2 \psi(\varphi_2)}$$

(17)

in terms of the newly defined fields. In equation (17) we have defined $\psi(\varphi_i) = \varphi_i^{4/3}$, $\beta_i = a_{i+2}(3V_4/4\alpha_0 \lambda_{i+2})^{2/3}$ and $V_i = -4\beta_i A_{i+2} W_0/V_4^2$ for convenience.

### 3. The two-field inflation model with the non-canonical Lagrangian

In this section, we intend to study the probable implications of the above two-field model in cosmology. The model emerges from the large volume approach to the moduli stabilization of type IIB superstring flux compactification and is described by the effective action

$$S = \int d^4x \sqrt{-G} \left[ \frac{1}{2} R - \frac{1}{2} \partial^\mu \varphi_1 \partial_\mu \varphi_1 - \frac{1}{2} \partial^\mu \varphi_2 \partial_\mu \varphi_2 - \frac{9}{8} \varphi_1 \varphi_2 \partial^\mu \varphi_1 \partial_\mu \varphi_2 - V(\varphi_1, \varphi_2) \right],$$

(18)

with $V(\varphi_1, \varphi_2)$ given in equation (17). Like in the assisted inflation model [15], the potential $V(\varphi_1, \varphi_2)$ consists of the sum of two exponentials of scalar fields. The model can be thought of as a simplified version of the large volume better racetrack model in which two axionic fields have been stabilized at the potential minimum. In action (18), $G$ stands for the determinant of the four-dimensional metric tensor $G_{\mu \nu}$, whose Ricci scalar curvature is denoted by $R$. From the assigned stringy parameters for the model, the phenomenological parameters involved take values of $V_{\text{min}} \approx 4.035 \times 10^{-19}$, $V_1 = V_2 \approx -2.212 \times 10^{-14}$ and $\beta_1 = \beta_2 \approx 40 \, 337 \, 578$ respectively. The potential has a Minkowski minimum, $V(\varphi_1, \varphi_2) \approx 0$, at $\varphi_1 = \varphi_2 \approx 3.51 \times 10^{-14}$; however, it is very flat far away from this minimum.

The crossing term in the kinetic Lagrangian brings in much unnecessary inconvenience. To overcome this, we redefine the independent scalar fields by

$$\varphi_1 = (\varphi + \chi)/\sqrt{2}, \quad \varphi_2 = (\varphi - \chi)/\sqrt{2}.$$  

(19)

The action becomes

$$S = \int d^4x \sqrt{-G} \left[ \frac{1}{2} R - \cos^2 \alpha \partial^\mu \varphi \partial_\mu \varphi - \sin^2 \alpha \partial^\mu \chi \partial_\mu \chi - V(\varphi, \chi) \right],$$

(20)

7 Correspondingly, the light Kähler moduli at this Minkowski minimum take values $\tau_3 = \tau_4 \approx 47.75$. 

Journal of Cosmology and Astroparticle Physics 08 (2008) 024 (stacks.iop.org/JCAP/2008/i=08/a=024)
Two-field Kähler moduli inflation in large volume moduli stabilization

where

\[ V(\varphi, \chi) = V_{\text{min}} + V_1 \psi((\varphi + \chi)/\sqrt{2}) e^{-\beta \psi((\varphi + \chi)/\sqrt{2})} + V_2 \psi((\varphi - \chi)/\sqrt{2}) e^{-\beta \psi((\varphi - \chi)/\sqrt{2})} \]

and \( \cos 2\alpha = \frac{9}{16} (\varphi^2 - \chi^2) \). Because in the large volume limit the magnitudes of both \( \varphi \) and \( \chi \) could be much smaller than 1, the introduction of the field dependent auxiliary quantity \( \alpha(\varphi, \chi) \) through its cosine value is reasonable. The appearance of \( \alpha(\varphi, \chi) \) in the kinetic Lagrangian implies that we get a non-trivial diagonal metric in field space, which does not coincide with the known non-canonical two-field kinetic Lagrangian studied in [23], and is expected to bring in some new features in cosmological application. The potential surface is plotted in figure 1. In terms of newly defined scalars, the Minkowski minimum of the potential (21) occurs at \( \varphi \approx 4.97 \times 10^{-4} \) and \( \chi = 0 \). The departure from this minimum is a plateau mainly along the \( \varphi \) direction which might be adequate for inflation.

We begin with the equations of motion of the homogeneous background fields. Consider a spatially flat Robertson–Walker spacetime

\[ ds^2 = -dt^2 + a(t)^2 d\mathbf{x}^2. \]  

(22)

Here \( t \) is the cosmic time. It is highly convenient to formulate the equations of motion of the scalar fields in terms of the e-fold time \( n = \ln(a(t)/a_{\text{ini}}) \), through which the equations of motion of the scalar fields are decoupled from the metric evolution. For the sake of convenience, we define a homogeneous inflaton field \( \sigma(t) \) through its velocity,

\[ \dot{\sigma} = \sqrt{2 \cos^2 \alpha \dot{\varphi}^2 + 2 \sin^2 \alpha \dot{\chi}^2}. \]  

(23)

In equation (23) and hereafter, a dot stands for a derivative with respect to the e-fold time \( n \). The equations of motion of the scale factor in metric (22) and the homogeneous background scalar fields are found to be

\[ H^2 = \frac{V}{3 - \dot{\sigma}^2/2}, \]  

(24)

\[ \frac{\dot{H}}{H} = -\frac{1}{2} \dot{\sigma}^2, \]  

(25)

\[ \ddot{\varphi} + \tan \alpha \partial_\varphi \alpha (\dot{\varphi}^2 + \dot{\chi}^2) - 2 \tan \alpha \partial_\chi \alpha \dot{\varphi} \dot{\chi} + \varphi = -\frac{\sec^2 \alpha \partial_\varphi V}{2V}, \]  

(26)

\[ \ddot{\chi} + \cot \alpha \partial_\chi \alpha (\dot{\varphi}^2 + \dot{\chi}^2) + 2 \cot \alpha \partial_\varphi \alpha \dot{\varphi} \dot{\chi} + \chi = -\frac{\csc^2 \alpha \partial_\chi V}{2V}, \]  

(27)

where \( V \) is the abbreviation for the potential \( V(\varphi, \chi) \), \( H = da/adt \) is the Hubble parameter, but \( \dot{H} = dH/dn \). Inflation occurs for \( d^2a/dt^2 > 0 \). Hence, having an inflation driven by the above two scalar fields requires

\[ \epsilon_1 = -\frac{\dot{H}}{H} = \dot{\sigma}^2/2 < 1. \]  

(28)

\( \epsilon_1 \) is the so-called first slow-roll parameter\(^8\). Is there an inflationary epoch with \( \epsilon_1 < 1 \) for our two-field model? To have a definite answer to this question, we have to numerically

\(^8\) The slow-roll approximation in general figures in terms of numerous Hubble flow functions [24], among which the first two are \( \epsilon_1 = -\dot{H}/H \) and \( \epsilon_2 = \dot{\epsilon}/\epsilon_1. \)
Two-field Kähler moduli inflation in large volume moduli stabilization

Figure 1. The potential surface of our two-field model defined in equations (20) and (21) where the values $V_{\text{min}} \approx 4.035 \times 10^{-19}$, $V_1 = V_2 \approx -2.212 \times 10^{-14}$ and $\beta_1 = \beta_2 \approx 40337.578$ have been assigned to the parameters. The potential has a Minkowski minimum at $\varphi \approx 4.97 \times 10^{-4}$ and $\chi = 0$ and is very flat along the $\varphi$ direction far away from this minimum.

solve equations (24)–(27) under appropriate initial conditions. In fact, it is sufficient to integrate equations (26) and (27) only, since these scalar field equations are decoupled from the metric evolution. From the Cauchy theorem, the solution to equations (26) and (27) is unique provided the initial fields and initial field velocities are given at some initial instant $n = n_{\text{ini}}$. However, just as pointed out by Ringeval [24], the attractor behavior induced by the friction terms erases any effect associated with the initial field velocities after a few e-folds. The integration of equations (26) and (27) depends essentially upon the initial field values only. The attractor behavior also ensures the stability of forward numerical integration schemes, so we use a Runge–Kutta integration method of order 4. Numerically, we take $n_{\text{ini}} \approx -4.60516$ and $\varphi_{\text{ini}} \approx 5.67 \times 10^{-3}$. It follows from equation (19) that, if $\chi_{\text{ini}}$ vanished, $\varphi_{1,\text{ini}} = \varphi_{2,\text{ini}}$, the evolutions of $\varphi_1$ and $\varphi_2$ would be exactly the same; our multifield model would effectively reduce to a single-field model. To have a small but significant deviation from such an uninteresting situation, we take $\chi_{\text{ini}} \approx -1.13 \times 10^{-5}$. Such a choice for initial fields ensures that they are far away from their possible values at the Minkowski minimum, and correspondingly the light Kähler moduli are initially set to be $\tau_3 \approx 1224.14$ and $\tau_4 \approx 1230.67$, respectively. The initial field

---

9 See equation (47) below for an explanation.
velocities are chosen on the attractor by setting
\[ \dot{\phi}_{\text{ini}} = -\partial_{\phi} \ln V |_{\phi_{\text{ini}}, \chi_{\text{ini}}}, \quad \dot{\chi}_{\text{ini}} = -\partial_{\chi} \ln V |_{\phi_{\text{ini}}, \chi_{\text{ini}}}. \] (29)

Figure 2 gives our numerical solution to the classical trajectory for the scalar fields for $-4.605 \leq n \leq 56.83847$. During most of the e-folds, $-4.605 \leq n \leq 55.85$, the scalars roll very slowly and the trajectory is almost in the $(\varphi - \chi)$ direction. As this is a two-field model, the trajectory is not a straight line and in fact a sharp turn occurs in the trajectory, at roughly $n \approx 55.85$. Within the last e-fold time interval the trajectory becomes another straight line along the $(\varphi + \chi)$ direction. Plugging this solution for $\varphi(n)$ and $\chi(n)$ into equations (24) and (25) determines the evolution of the Hubble parameter and the first slow-roll parameter $\epsilon_1$. Our choice of the initial conditions ensures that both $\epsilon_1$ and $\epsilon_2$ are much smaller than 1 for most of the e-folds (see figure 3 for the evolution of $\epsilon_1$). In particular, $\epsilon_1 \approx 4.82 \times 10^{-12}$ and $\epsilon_2 \approx 0.036$ at the Hubble exit ($n \approx 2$). There is indeed an inflationary epoch in our model during which the Hubble parameter hardly changes. The potential $V$ is very flat along the classical trajectory and, just as in the single-field case [4], it is almost a constant, $V \approx 4.035 \times 10^{-19}$, for most of inflation. Only within the final fractions of the last e-fold does a steep decay of the potential take place. When $n$ comes near 56.83847, the potential sharply drops off from the preceding constant value to a metastable minimum with $V \approx 2.725 \times 10^{-19}$, where $\epsilon_1 \approx 0.97$ and the inflation is close to its end.

The incontrovertible deviation of trajectory from an entirely straight line shows that what we consider is a genuine multifield model. Provided that the isocurvature perturbation decays fast enough on the super-Hubble scales, it will not influence the super-Hubble curvature perturbation heavily even if the trajectory in field space deviates greatly from the straight line one of a single-field model. The steep decay of the isocurvature perturbations takes place in many supergravity or string theory inspired multifield models, and in particular in the string theory inspired roulette model [5]. The inflation in these models is effectively driven by a single scalar field where one can use the single-field approximation to account for the superhorizon power spectrum of the curvature perturbation. However, the same is not true for the present situation. In our two-field model, the decay of the isocurvature perturbation on super-Hubble scales is not fast and the spectrum of the curvature perturbation in the vicinity of the Hubble crossing is remarkably deviated by that at the end of inflation, where the correlation between the curvature and isocurvature modes becomes sufficiently strong that the final curvature perturbation substantially results from the isocurvature perturbation.

We are going to confirm the above conclusion for the two-field model under consideration through numerical calculations of the power spectra of the linear perturbations and their correlation. Due to the fact that at the Hubble exit $\epsilon_1 \approx 4.82 \times 10^{-12}$, there are no important tensor fluctuations in our model. This might be a common feature of the stringy inflation models where the inflation is driven by some closed string moduli [2]–[5]. So we consider only the scalar perturbations. Because the matter is composed of two scalar fields, the stress tensor is diagonal and the perturbed metric is of the form

\[ ds^2_4 = -[1 + 2\Psi(t, \mathbf{x})] dt^2 + a(t)^2 [1 - 2\Psi(t, \mathbf{x})] dx^2 \] (30)
Classical inflationary trajectory of the two-field model under consideration, for e-folds $-4.60516 \leq n \leq 56.83847$. The inflationary epoch starts at $n \approx -4.60516$ where we set the initial conditions $\varphi_{\text{ini}} \approx 5.67 \times 10^{-3}$ and $\chi_{\text{ini}} \approx -1.13 \times 10^{-5}$. In the first 35 e-folds the trajectory is approximately along the $(\varphi - \chi)$ direction (red curve). When $n \gtrsim 30$, the fields $(\varphi + \chi)/\sqrt{2}$ begin to lose their synchrony in evolution. A steep turn in field space takes place roughly at $n \approx 55.85$ where $\varphi \approx 5.04 \times 10^{-3}$, $\chi \approx -2.57 \times 10^{-4}$, and after that $(\varphi - \chi)/\sqrt{2}$ no longer evolves but $(\varphi + \chi)/\sqrt{2}$ continues to decrease. During the last e-fold, $55.85 \leq n \leq 56.83847$, the trajectory is along the $(\varphi + \chi)$ direction (magenta curve). The inflation approaches to its end at $n \approx 56.83847$, where $\varphi \approx 3.34 \times 10^{-3}$, $\chi \approx -1.94 \times 10^{-3}$ and $\epsilon_1 \approx 0.97$. 
in the longitudinal gauge. The scalar fields are decomposed into their homogeneous backgrounds plus anisotropic perturbations:

\[ \varphi(t, x) = \varphi(t) + \delta \varphi(t, x), \quad \chi(t, x) = \chi(t) + \delta \chi(t, x). \]  

(31)

To facilitate solving the evolution equations of these cosmological perturbations, we can alternatively decompose them into the instantaneous curvature and isocurvature components, relying on the fact that the isocurvature modes only generate the curvature perturbations approximately [25, 26]. The curvature and isocurvature perturbations are parallel with and orthogonal to the trajectory of the homogeneous inflaton field \( \sigma(t) \), respectively. To define them, we first introduce a time dependent angle \( \theta \),

\[ \cos \theta = \sqrt{2} \cos \alpha \frac{\dot{\varphi}}{\dot{\sigma}}, \quad \sin \theta = \sqrt{2} \sin \alpha \frac{\dot{\chi}}{\dot{\sigma}}, \]  

(32)

using which we can recast the background equations (26) and (27) as

\[ \ddot{\sigma} + (3 - \dot{\sigma}^2/2)\dot{\sigma} + \frac{V_{\sigma}}{H^2} = 0, \]  

(33)

\[ \dot{\theta} + \frac{V_{\chi}}{H^2 \dot{\sigma}} + \frac{\dot{\sigma}}{\sqrt{2}}(\cos \theta \sec \alpha \alpha_\chi + \sin \theta \csc \alpha \alpha_\varphi) = 0. \]  

(34)

The curvature perturbation \( \delta \sigma(t, x) \) and isocurvature perturbation \( \delta s(t, x) \) are defined through a linear transformation in field space:

\[ \begin{bmatrix} \delta \sigma(t, x) \\ \delta s(t, x) \end{bmatrix} = \sqrt{2} \begin{bmatrix} \cos \alpha \cos \theta & \sin \alpha \sin \theta \\ -\cos \alpha \sin \theta & \sin \alpha \cos \theta \end{bmatrix} \begin{bmatrix} \delta \varphi(t, x) \\ \delta \chi(t, x) \end{bmatrix}. \]  

(35)
From equation (35) we have $\delta s = \sin 2\alpha (\dot{\varphi} \delta \chi - \dot{\chi} \delta \varphi) / \dot{\sigma}$. The isocurvature perturbations turn out to be the entropy perturbations.

It is worthwhile to stress that, among the linear scalar perturbations $\Psi(t, x), \delta \sigma(t, x)$ and $\delta s(t, x)$, only two of them are independent. The independent scalar perturbations can also be chosen as the gauge invariant Mukhanov–Sasaki variable

$$Q_\sigma(t, x) = \delta \sigma(t, x) + \dot{\sigma} \Psi(t, x)$$

and $\delta s(t, x)$. In terms of these gauge invariant perturbations, the perturbed Klein–Gordon equations and perturbed Einstein equations [24] can be unified into two coupled second-order differential equations:

$$\ddot{Q}_\sigma + (3 - \dot{\sigma}^2/2) \dot{Q}_\sigma + \frac{2V_\sigma}{H^2 \dot{\sigma}} \dot{\delta} s - \frac{1}{H^2 a^2} \nabla^2 Q_\sigma + \frac{C_{\sigma \sigma}}{H^2} Q_\sigma + \frac{C_{\sigma s}}{H^2} \dot{\delta} s = 0,$$

$$\ddot{\delta} s + (3 - \dot{\sigma}^2/2) \dot{\delta} s - \frac{2V_\sigma}{H^2 \dot{\sigma}} \dot{Q}_\sigma - \frac{1}{H^2 a^2} \nabla^2 \delta s + \frac{C_{\sigma \sigma}}{H^2} Q_\sigma + \frac{C_{ss}}{H^2} \dot{\delta} s = 0,$$

where

$$C_{\sigma \sigma} = V_{\sigma \sigma} + H^2 (3 - \dot{\sigma}^2/2) \dot{\sigma}^2 - \frac{1}{H^2} \left( \frac{V_\sigma}{\dot{\sigma}} \right)^2 + 2 \dot{\sigma} V_\sigma$$

$$+ \frac{\sqrt{2} V_\sigma}{\sin^2 2\alpha} (\cos 2\theta - \cos 2\alpha) (\cos \theta \sin \alpha \alpha_\varphi + \sin \theta \cos \alpha \alpha_\chi),$$

$$+ \frac{\sqrt{2} V_\sigma}{\sin^2 2\alpha} \left[ \cos \theta \cos \alpha (\cos 2\theta + \cos 2\alpha - 2) \alpha_\chi - \sin \theta \sin \alpha (\cos 2\theta + \cos 2\alpha + 2) \alpha_\varphi \right],$$

$$C_{\sigma s} = (6 + \dot{\sigma}^2) \frac{V_\sigma}{\dot{\sigma}} + \frac{2V_\sigma V_\sigma}{H^2 \dot{\sigma}^2} + 2V_\sigma$$

$$+ \frac{2 \sqrt{2} V_\sigma}{\sin^2 2\alpha} (\cos 2\theta - \cos 2\alpha) (\cos \theta \cos \alpha \alpha_\chi - \sin \theta \sin \alpha \alpha_\varphi),$$

$$- \frac{2 \sqrt{2} V_\sigma}{\sin^2 2\alpha} (\cos 2\theta + \cos 2\alpha) (\cos \theta \sin \alpha \alpha_\varphi + \sin \theta \cos \alpha \alpha_\chi),$$

$$C_{ss} = -6 \frac{V_\sigma}{\dot{\sigma}} - \frac{2V_\sigma V_\sigma}{H^2 \dot{\sigma}^2} + V_\sigma \dot{\sigma},$$

$$C_{ss} = V_{ss} - \frac{1}{H^2} \left( \frac{V_\varphi}{\dot{\sigma}} \right)^2 - \frac{\dot{\sigma}^2}{\sin 2\alpha} (\alpha_{\varphi \varphi} - \alpha_{\chi \chi})$$

$$- \frac{\sqrt{2} V_\varphi}{\sin^2 2\alpha} (\cos 2\theta - \cos 2\alpha) (\cos \theta \sin \alpha \alpha_\chi - \sin \theta \cos \alpha \alpha_\varphi),$$

$$+ \frac{\sqrt{2} V_\varphi}{\sin^2 2\alpha} \left[ \cos \theta \sin \alpha (2 + \cos 2\theta + 2 \cos 2\alpha) \alpha_\varphi - \sin \theta \cos \alpha (2 + \cos 2\theta - \cos 2\alpha) \alpha_\chi \right].$$

As is usually done, we will work with the Fourier components of the perturbations, $Q_k(n)$ and $\delta s_k(n)$, with $k$ a given comoving wavenumber. For the sake of convenience, in equations (37)–(42) we have defined various derivatives of the potential with respect to
the curvature and isocurvature directions, which are associated with the derivatives of the potential with respect to the original scalars $\varphi$ and $\chi$ through the linear transformations
\[
\begin{bmatrix}
V_{\sigma} \\
V_s
\end{bmatrix} = T
\begin{bmatrix}
V_{\varphi} \\
V_{\chi}
\end{bmatrix}
\] (43)
and
\[
\begin{bmatrix}
V_{\sigma\sigma} & V_{\sigma s} \\
V_{s\sigma} & V_{ss}
\end{bmatrix} = T
\begin{bmatrix}
V_{\varphi\varphi} & V_{\varphi\chi} \\
V_{\varphi\chi} & V_{\chi\chi}
\end{bmatrix} T^T.
\] (44)
The transformation matrix $T$ reads
\[
T = \frac{1}{\sqrt{2}}
\begin{bmatrix}
\cos \theta \sec \alpha & \sin \theta \csc \alpha \\
-\sin \theta \sec \alpha & \cos \theta \csc \alpha
\end{bmatrix}.
\] (45)

To study the generation of perturbations from the vacuum fluctuations, we have to solve the closed set of equations (37) and (38) under appropriate initial conditions. Though the curvature and isocurvature fluctuations are finally coupled to each other beyond the Hubble crossing, they might be statistically independent deep inside the Hubble radius. The initial e-fold time $n_{ini}$ at which the perturbations are thought of as mutually independent must be much less than the Hubble crossing instant $n_*$ for Fourier mode perturbations of wavenumber $k$, $k \lesssim a(n_*)H(n_*)$. The value of $n_*$ depends on the e-fold number during which the Universe reheated before the start of the radiation era [24] and it is known [27] that typically $40 \lesssim N_* \lesssim 60$ for $N_* = n_{end} - n_*$. Without knowledge of details of the reheating mechanism we cannot be more precise in saying what $n_*$ should be. However, if $n_*$ is known somehow, the e-folding number $N_*$ of inflation after horizon crossing can be estimated from the assumption that the reheat temperature is of the same order as but slightly lower than the energy scale of inflation, $T_{reh} \approx 3 \times 10^{14}$ GeV, through the formula [3]
\[
N_* \approx 53 + \ln \left( \frac{T_{reh}}{10^{14} \text{ GeV}} \right) \lesssim 57.
\] (46)

We will simply set $n_* = 2$. Then $N_* \approx 54.83847$, compatible with the constraint from cosmological observation. The pivot wavenumber of the Fourier mode perturbation is taken as $k = a(0)H(0)$ for which the initial e-fold time $n_{ini}$ can be defined by a cutoff equation [28]
\[
\frac{k}{a(n_{ini})H(n_{ini})} = \frac{H(0)}{e^{n_{ini}}H(n_{ini})} = C_q.
\] (47)
The constant $C_q$ should be sufficiently large in order that the probable interferences between the initial curvature and isocurvature modes remain negligible [29]. We take $C_q \approx 100$ for concreteness, which leads to $n_{ini} = -4.60516$. To take into account the statistical independence of the initial perturbations in our numerical procedure, we integrate equations (37) and (38) twice: first with the initial conditions,
\[
\begin{align*}
Q_{\sigma}(n_{ini}) & \approx Q_{ini}, \\
\dot{Q}_{\sigma}(n_{ini}) & \approx -(1 + iC_q)Q_{ini}, \\
\delta s(n_{ini}) & \approx \delta s(n_{ini}) \approx 0,
\end{align*}
\] (48)
and second with the initial conditions,
\[
Q_\sigma(n_{ini}) \approx \dot{Q}_\sigma(n_{ini}) \approx 0, \\
\delta s(n_{ini}) \approx \delta s_{ini}, \\
\dot{s}(n_{ini}) \approx -(1 + iC_q)\delta s_{ini}, \\
\] (49)
with \(Q_{ini} = \delta s_{ini} = 1/a(n_{ini})\sqrt{2k}\). The curvature and isocurvature perturbations are customarily described by \(R = Q_\sigma/\dot{\sigma}\) and \(S = \delta s/\dot{\sigma}\), for which we get \(R_i\) and \(S_i\) \((i = 1, 2)\) after finishing two individual integrations of equations (37) and (38). \(R_1\) \((S_2)\) can be thought of as the curvature (isocurvature) perturbation without taking into account the coupling to the isocurvature (curvature) perturbation, and \(R_2\) \((S_1)\) just comes from such a coupling. The final power spectra and their correlation are calculated as follows [30]:
\[
\mathcal{P}_R = \frac{k^3}{2\pi^2} (|R_1|^2 + |R_2|^2), \\
\mathcal{P}_S = \frac{k^3}{2\pi^2} (|S_1|^2 + |S_2|^2), \\
\mathcal{C}_{RS} = \frac{k^3}{2\pi^2} \left( R_1^\dagger S_1 + R_2^\dagger S_2 \right). \\
\] (50)
In fact, the correlation between the curvature and isocurvature perturbations can be simply described using the so-called relative correlation coefficient \(\mathcal{C} = |\mathcal{C}_{RS}|/\sqrt{\mathcal{P}_R \mathcal{P}_S}\). The value of \(\mathcal{C}\) lies between 0 and 1, which measures to what extent the final curvature perturbation results from the interactions with the isocurvature perturbation.

Our result from numerical integration for linear perturbations is displayed in figure 4, where the power spectra of the curvature and isocurvature perturbations as well as the correlation between them are plotted as functions of the number \(n\) of e-folds. On super-Hubble scales the relative correlation coefficient continues to increase; this is displayed in figure 5.
Figure 5. Relative correlation coefficient $C$ in the present model. It vanishes initially, as expected. At $n = -1, 0, 1$ and 2, $C \approx 5.18 \times 10^{-4}, 1.35 \times 10^{-3}, 1.69 \times 10^{-3}$ and $3.6 \times 10^{-3}$, respectively. The coefficient increases monotonically on super-Hubble scales. When inflation approaches its end, $n \approx 56.83847$, $C \approx 0.66$.

The spectra drop off sharply on subhorizon scales until the supposed Hubble exit for the Fourier mode perturbation is achieved. On superhorizon scales, however, the spectra become sufficiently flat for $2 \lesssim n \lesssim 30$, with $\mathcal{P}_R \approx \mathcal{P}_S \approx 3.96 \times 10^{-10}$, and correlator $\mathcal{C}_{RS}$ increases steadily from $1.46 \times 10^{-12}$ to $5.041 \times 10^{-11}$ ($3.6 \times 10^{-3} \lesssim C \lesssim 0.132$). During the period of $n \gtrsim 30$, though $\mathcal{C}_{RS}$ increases at first and then drops off steeply, $C$ increases monotonically. For $n \gtrsim 30$, the spectrum of curvature perturbation begins to increase while the spectrum of isocurvature begins to decrease once more. The persistent increase of the curvature perturbation spectrum for $n \gtrsim 30$ reflects the fact that the correlation between the curvature and isocurvature perturbations during this time interval has become sufficiently strong that it can no longer be neglected. The decay of the isocurvature perturbation spectrum accelerates in the final stage of inflation, and becomes unbelievably steep within the last e-fold, resulting in an unbelievable steep decay of correlator $\mathcal{C}_{RS}$. For $n \gtrsim 56.6$, in particular, both $\mathcal{C}_{RS}$ and $\mathcal{P}_S$ become negligible, and $\mathcal{P}_R$ stops varying. Therefore, in the present two-field model, the power spectrum of curvature perturbation on superhorizon scales is changeable with respect to time. This is in contrast with the single-field inflation case where the curvature power spectrum remains approximately a constant after Hubble crossing. Figure 6 gives the details of our numerical result for the power spectrum of the curvature perturbation, where the three curves stand for the complete curvature spectrum $\mathcal{P}_R = (k^3/2\pi^2)(|\mathcal{R}_1|^2 + |\mathcal{R}_2|^2)$ and its two terms

\[ \alpha = \frac{\mathcal{P}_S}{\mathcal{P}_R + \mathcal{P}_S} \]

to measure the relative contribution of the isocurvature fluctuations. $\alpha|_{n=56.83847} \approx 3.7 \times 10^{-10}$. Hence the constraint on isocurvature perturbations from observations [31] is trivially matched and there is actually no observable isocurvature perturbation spectrum after inflation in the present two-field model.
The power spectrum of curvature perturbation in the e-fold time interval \(-4.60516 \lesssim n \lesssim 56.83847\). The red curve shows the evolution of the complete curvature spectrum. Pink and magenta curves stand for the partial curvature spectra from fluctuations along the inflationary trajectory and from the interactions with the isocurvature modes, respectively. The curvature spectrum at the end of inflation is \(P_R|_{n \approx 56.83847} \approx 6.79 \times 10^{-10}\), which is different from the curvature spectrum \(P_R|_{n \approx 2} \approx 3.96 \times 10^{-10}\) at Hubble exit.

\[(k^3/2\pi^2)|R_1|^2\) and \((k^3/2\pi^2)|R_2|^2\). We see that for \(2 \lesssim n \lesssim 30\) on superhorizon scales the spectrum of curvature perturbation originates almost exclusively from the fluctuations of the initial curvature perturbation along the inflationary trajectory. While for \(n \gtrsim 30\), the curvature–isocurvature correlation becomes important, the spectrum at the final stage of inflation contains considerable contributions from the interactions with the isocurvature modes. As a result, the curvature perturbation spectrum after inflation is slightly higher than the curvature perturbation spectrum at Hubble crossing.

The power spectrum obtained for the curvature perturbation in the present two-field model is in good agreement with the COBE normalization of the power spectrum \((51)\), for \(2 \lesssim n \lesssim 30\). \(n \approx 2\) is at an instant approximately 55 e-foldings before the end of inflation; it is near the COBE normalization point. So \(n \approx 2\) is a reasonable e-fold time corresponding to Hubble crossing. The scalar spectral index of the curvature power spectrum

\[n_s = 1 + \frac{d \ln P_R}{d \ln k}\]

on superhorizon scales is also time dependent; this has been calculated numerically with partial results given in figure 7. At the claimed Hubble crossing, \(n_s|_{n \approx 2} \approx 0.96\), which is compatible with the best observational constraint available at present \((33)\), \(n_s = 0.98 \pm 0.02\).

It follows from the above numerical investigation that, in the two-field model under consideration, strong interactions exist between the curvature and isocurvature...
Figure 7. Spectral index $n_s$ of the curvature power spectrum in the vicinity of Hubble exit. The instant for COBE-like normalization corresponds roughly to $n = 2$, where $n_s \approx 0.96$. However, the index running is sufficiently small that for $1.8 \lesssim n \lesssim 50$, almost all values of $n_s$ are compatible with the observational constraint.

perturbations during the final stage of inflation. Because the relative correlation coefficient increases steadily from the assigned tiny value deep inside the Hubble radius to values very close to 1 at the end of inflation, a great deal of curvature perturbation after inflation originates from interactions between the curvature and isocurvature perturbations on super-Hubble scales, not only from the quantum fluctuations of the initial curvature perturbation along the classical trajectory. At this point, our model is somewhat similar to the double inflation with non-canonical kinetic terms and roulette inflation studied in [23]. What distinguishes our model from the double inflation with non-canonical kinetic terms and roulette inflation is that in the former the power spectrum of isocurvature perturbation remains almost a constant during most of the inflationary epoch while in the latter it decays rapidly after the Hubble radius crossing. This implies that in our model the isocurvature perturbation continues to interact with the curvature perturbation on superhorizon scales, so there is no effective single-field treatment available for it [5].

4. Discussion

We have established a multifield inflationary model on the large volume flux compactification scheme in type IIB superstring orientifolds with $h^{(1,2)} > h^{(1,1)}$. The Kähler moduli are categorized into the volume modulus, heavy moduli and light moduli. Almost all of the closed string moduli emerging from the flux compactification are frozen at the potential minimum in the spirit of the KKLT mechanism in the large volume limit. However, this is not the case for light Kähler moduli. The light Kähler moduli have very small masses, they are not fixed by either the 3-form fluxes or non-perturbative effects in the effective superpotential, and they could be the driving force for the potential
Inflationary evolution of the Universe in its early history. To have a multifield inflation model, the superpotential is expected to contain several exponent terms for the light Kähler moduli. Hence we have considered the orientifolds with $h^{(1,1)} \geq 4$ and supposed that in our constructions the number of light Kähler moduli is equal to or greater than 2. In some sense, in this paper we have proposed a plausible generalization of the better racetrack model [3] in the large volume approach [7,8].

The quantitative analysis of the inflationary property in the resulting multifield inflationary model has been done in a simplified two-field model where there are only two light Kähler moduli with the axionic components being stabilized at the potential minimum also. This assisted-like model is an extension of the Kähler moduli inflation model [4] to two-field inflation, which is distinctive in having a kinetic Lagrangian that is of neither the canonical nor the non-standard type studied in [23]. Our investigation consists of two steps. First, by some inevitable fine-tuning in the initial conditions of the scalar fields, we integrate the background equations numerically and obtain the inflationary trajectory of the scalar fields. As a remarkable characteristic of the multifield model, the trajectories in field space in our model are strongly bent roughly 50 e-folds after the Hubble crossing. The first slow-roll parameter is numerically calculated; it turns out to almost vanish for the first 60 e-folds, implying that the scalar potential is very flat along the trajectory. Second, we derive the evolution equations of the linear scalar perturbations. These equations are numerically integrated; for them we have calculated the curvature, isocurvature spectra, and the correlation between them. Our numerical estimation for the power spectrum of the curvature perturbation and the corresponding spectral index is in good agreement with the COBE normalization and the WMAP observation data set [32,34]. In our model, the correlation between the curvature and isocurvature perturbations becomes gradually stronger on super-Hubble scales, so the curvature spectrum approximately thirty e-folds after the Hubble exit is remarkably different from the curvature perturbation spectrum at the Hubble exit, and there might be significant non-Gaussianity in the bispectrum and trispectrum of the primordial perturbations. The investigation of the deviations from Gaussianity in power spectra in our two-field inflation model is in progress.

Acknowledgments

We would like to thank M Li, J X Lu and Y Wang for stimulating discussions. This work was supported in part by CNSF-10375052, the Startup Foundation of the University of Science and Technology of China and the Project of Knowledge Innovation Program (PKIP) of the Chinese Academy of Sciences.

References

[1] Kachru S, Kallosh R, Linde A and Trivedi S, de Sitter vacua in String theory, 2003 Phys. Rev. D 68 046005 [SPIRES]
[2] Blanco-Pillado J, Burgess C, Cline J, Escoda C, Gomez-Reino M, Kallosh R, Linde A and Quevedo F, Racetrack inflation, 2004 J. High Energy Phys. JHEP11(2004)063 [SPIRES]
[3] Blanco-Pillado J, Burgess C, Cline J, Escoda C, Gomez-Reino M, Kallosh R, Linde A and Quevedo F, Inflating in a better racetrack, 2006 J. High Energy Phys. JHEP09(2006)002 [SPIRES]
[4] Conlon J and Quevedo F, Kähler moduli inflation, 2006 J. High Energy Phys. JHEP01(2006)146 [SPIRES]
[5] Bond J, Kofman L, Prokhorov S and Vanderven P, Roulette inflation with Kähler moduli and their axions, 2006 Preprint hep-th/0612197
Two-field Kähler moduli inflation in large volume moduli stabilization

[6] Kachru S, Kallosh R, Linde A, Maldacena J, McAllister L and Trivedi S, Towards Inflation in String Theory, 2003 J. Cosmol. Astropart. Phys. JCAP10(2003)013 [SPIRES]

[7] Balasubramanian V, Berglund P, Conlon J and Quevedo F, Systematics of moduli stabilization in Calabi–Yau flux compactifications, 2005 J. High Energy Phys. JHEP03(2005)007 [SPIRES]

[8] Conlon J P, Quevedo F and Suruliz K, Large volume flux compactifications: moduli spectrum and D3/D7 soft supersymmetry breaking, 2005 J. High Energy Phys. JHEP08(2005)007 [SPIRES]

[9] Achucarro A, de Carlos B, Casas J A and Doplicher L, de Sitter vacua from uplifting D-terms in effective supergravities from realistic strings, 2006 J. High Energy Phys. JHEP06(2006)014 [SPIRES]

[10] Cremades D, del Moral M P G, Quevedo F and Suruliz K, Moduli stabilisation and de Sitter string vacua in a refined racetrack

[11] Wen W, Multiple field inflation

[12] Misra A and Shukla P, Large volume axionic Swiss-cheese inflation

[13] Choi K, Falkowski A, Nilles H P and Olechowski M, Soft supersymmetry breaking in KKLT flux assisted inflation

[14] Liddle A R, Mazumdar A and Schunck F B, Effective potentials for light moduli

[15] de Alwis S, How long before the end of inflation were observable perturbations produced?

[16] Wands D, The quest for non-gaussianity

[17] Dasgupta K, Rajesh G and Sethi S, M theory, orientifolds and G-flux

[18] Giddings S, Kachru S and Polchinski J, Multiple field inflation

[19] Grimm T and Louis J, The effective action of Adiabatic and entropy perturbations from inflation

[20] Gordon C, Wands D, Bassett B A and Maartens R, Minimal modifications of the primordial power spectrum from an adiabatic short distance cutoff, 2002 Phys. Rev. D 66 083510 [SPIRES]

[21] Ringeval C, The exact numerical treatment of inflationary models, 2008 Lect. Notes Phys. 738 243–73

[22] Lalak Z, Langlois D, Pokorski S and Turzynski K, Better racetrack inflation in large volume approach

[23] Gordon C, Wands D, Bassett B A and Maartens R, Correlation-consistency cartography of the double inflation landscape, 2003 Phys. Rev. D 67 083516 [SPIRES]

[24] Liddle A R, Mazumdar A and Schunck F B, Effective potentials for light moduli

[25] Marco F D and Finelli F, Curvature and isocurvature perturbations in two-field inflation

[26] Gordon C, Wands D, Bassett B A and Maartens R, Minimal modifications of the primordial power spectrum from an adiabatic short distance cutoff, 2002 Phys. Rev. D 66 083510 [SPIRES]

[27] Liddle A and Leach S M, Slow-roll inflation for generalized two-field lagrangians

[28] Langlois D, Pokorski S and Turzynski K, Better racetrack inflation in large volume approach

[29] Wen W, Minimal modifications of the primordial power spectrum from an adiabatic short distance cutoff, 2002 Phys. Rev. D 66 083510 [SPIRES]

[30] Niemeyer J, Parentani R and Campo D, Designing density fluctuation spectra in inflation, 1989 Phys. Rev. D 40 1753 [SPIRES]

[31] Holman R and Hutasoit J A, Systematics of moduli stabilization, inflationary dynamics and power spectrum, 2006 J. High Energy Phys. JHEP08(2006)053 [SPIRES]

[32] Achucarro A, de Carlos B, Casas J A and Doplicher L, de Sitter vacua from uplifting D-terms in effective supergravities from realistic strings

[33] Seljak U et al, Designing density fluctuation spectra in inflation, 1989 Phys. Rev. D 40 1753 [SPIRES]

[34] Seljak U et al, Cosmological parameter analysis including SDSS Ly-alpha forest and galaxy bias: constraints on the primordial spectrum of fluctuations, neutrino mass, and dark energy, 2005 Phys. Rev. D 71 103515 [SPIRES]

[35] Seljak U et al, Wilkinson Microwave Anisotropy Probe (WMAP) Three Year Results: Implications for Cosmology, 2007 Astrophys. J. Suppl. 170 377

[36] Seljak U et al, Cosmological parameter analysis including SDSS Ly-alpha forest and galaxy bias: constraints on the primordial spectrum of fluctuations, neutrino mass, and dark energy, 2005 Phys. Rev. D 71 103515 [SPIRES]

[37] Seljak U et al, Wilkinson Microwave Anisotropy Probe (WMAP) Three Year Results: Implications for Cosmology, 2007 Astrophys. J. Suppl. 170 377

[38] Seljak U et al, Wilkinson Microwave Anisotropy Probe (WMAP) Three Year Results: Implications for Cosmology, 2007 Astrophys. J. Suppl. 170 377