Modal Parameters Identification Based on Equalization Principal Component Analysis

Xuan Zhang¹, Jingjing Zhang²*, Zequan Xu² and Jinglin Luo³

¹Tianjin Aerospace Reliability Technology Co., Ltd, Zhongbei Third Street, TeDA, Tianjin, China
²School of Mechatronics Engineering, Foshan University, 33 Guang-yun-lu, Shishan, Nanhai, Foshan, Guangdong, China
³School of Electromechanical Engineering, Guangdong University of Technology, Guangzhou, Guangdong, China
Email: zjj2_012@163.com

Abstract. Aiming at the magnitude influence of different discrete points in modal parameters identifications with traditional principal component analysis (PCA), a method of modal parameters identifications by equalization PCA is proposed in this paper. The basic idea is using the mean of time domain response data before identifying modal parameters with PCA. On the premise of not losing the main information of the original data, the influence of magnitude on the data is weakened. Taking a doubly-clamped beam as an example, modal parameters of the beam are identified by traditional PCA and equalization PCA with the same constraints of principal component cumulative contribution rate. The results show that compared with the traditional PCA, equalization PCA can effectively identify more orders of modal parameters. The errors of vibration modal shapes and natural frequencies identified by equalization PCA are allowable.

1. Introduction
Modal parameters are the fundamental theory bases for structural vibration characteristic analysis [1]. Traditional method of identifying modal parameters is test modal analysis method, which applies specific excitation to the structure and uses modal analysis theory to fit the frequency characteristic function curves to identify the modal parameters. However, due to the limitation of test equipment and technology, test modal analysis method can only obtain the time-domain vibration response data of the structure under certain working conditions, and cannot directly obtain the modal parameters of these systems. Therefore, modal parameters can be identified directly from the time-domain response data with the idea of “inverse problem” [2].

At present, some methods have been developed to identify structural modal parameters from response data directly, which are mainly divided into time domain method and frequency domain method [3]. The frequency domain method has clear physical meaning. But these methods usually have problems of truncation error caused by Fourier transform [4], which may occur the phenomenon of frequency aliasing in the identification process. The time-domain method can effectively avoid the shortcomings of frequency-domain method as they use response data directly for modal parameters identification. Poncelet et al. [5] proposed a method of identifying modal parameters by blind source separation. Bai et al. [6] proposed a method of identifying modal parameters by local linear embedding method (LLE). However, the above methods may have the problems of identifying
missing and false modes. Wang et al. [7] proposed a method of identifying modal parameters based on principal component analysis (PCA). The essentiality of this method find was to find the relationship between linear compound matrixes and modal shapes and the relationship between principal components and modal responses. Identifying modal parameters was then changed into decomposing principal components. But the magnitude of different discrete points’ response data made this method influence the orders of identifying modal parameters.

In this paper, a method of modal parameter identification based on equalization PCA is proposed. A numerical example will be taken to verify the algorithm.

2. Theory of Modal Parameter Identification

The vibration differential equation of a freedom linear system with \( n \) degrees is shown in equation (1) in a physical coordinate system [8].

\[
M \ddot{X}(t) + C \dot{X}(t) + K X(t) = F(t)
\]  

\( M, C \) and \( K \) are the mass matrix, damping matrix and stiffness matrix \((n \times n)\). \( F(t) \) is the external excitation to the system \((n \times 1)\). \( X(t), \dot{X}(t) \) and \( \ddot{X}(t) \) are responses of displacement, velocity and acceleration \((n \times 1)\).

For general engineering structures with small damping system, the displacement response can be expressed as equation (2).

\[
X(t) = \Phi Q(t) = \sum_{j=1}^{m} \phi_j q_j(t)
\]

where \( \Phi \) is the modal matrix which is composed of the modal vectors \( \phi_j \). \( Q(t) \) is composed of the modal responses \( q_j(t) \).

3. Modal Parameters Identification based on Equalization PCA

3.1. Modal Parameters Identification Based on Traditional PCA

Contrasting the theory of modal parameter identification and the mathematical model of PCA, we find that the linear transformation matrix \( W \) in PCA can be corresponded to the modal matrix \( \Phi \) in the modal parameter identification theory. The principal component \( Y \) in PCA can be corresponded to the modal response \( Q(t) \) in modal parameter identification theory. Wang et al. [4] has proved the certainty, existence and uniqueness of modal parameters identified by traditional PCA. Figure 1 shows the flow chart of modal parameter identification algorithm based on PCA.

![Figure 1. The flow chart of modal parameter identification based on PCA.](image-url)
3.2. Modal Parameters Identification Based on Equalization PCA

The original response data $X(t)$ generally contains two parts of information: (1) The information reflecting the differences of different discrete points, which is reflected by the variance of response data of all discrete points at the same time; (2) The information reflecting the mutual influence in different times, which is reflected by the correlation matrix. In the process of identifying modal parameters based on traditional PCA, there may exist differences in the magnitude of the response data value at different discrete points. The differences will affect the contribution rate of different principal components. In order to reduce the influence of magnitude differences on the identification process, an improved PCA by equalizing the original data in each time sample to standardize the data is proposed.

$$x_{ij}^* = \frac{x_{ij}}{E(x_j)} \quad (i = 1,2, ..., m \quad j = 1,2, ..., n) \quad (3)$$

where the subscript $i$ means different time and subscript $j$ means different discrete point. The response data values have different positive and negative signs and the different signs will affect the mean value $E(x_j)$. Therefore, the data in this paper are pre-processed in formula (2) before equalizing the original data.

$$x_{ij}' = x_{ij} + |\min (x_j)| \quad (i = 1,2, ..., m \quad j = 1,2, ..., n) \quad (4)$$

Substitute equation (4) into equation (3),

$$x_{ij}'' = \frac{x_{ij}'}{E(x_j')} \quad (i = 1,2, ..., m \quad j = 1,2, ..., n) \quad (5)$$

This method can reduce the influence of magnitude difference on modal parameter identification. Since equation (4) processed the original data to eliminate the influence of different signs on equalizing the original data, $x_{ij}'$ should be processed as follows after equation (5).

$$x_{ij}^* = x_{ij}'' - |\min (x_j)| \quad (i = 1,2, ..., m \quad j = 1,2, ..., n) \quad (6)$$

Figure 2 shows the flowchart of modal parameter identification algorithm by equalization PCA.

![Figure 2. The flowchart of modal parameter identification by equalization PCA.](image-url)
4. Simulation
To verify the method of identifying modal parameters by equalization PCA, a doubly-clamped beam is taken as an example. The length of the beam is 1m and the beam is divided into 1000 parts. Apply a series of gaussian white noise loads at the middle of the beam and collect the response data of every discrete points. The sampling time is 1s and the sampling frequency is 4096Hz. The modal parameters of the doubly-clamped beam will be identified from the response data directly by traditional PCA and equalization PCA with the same constraints of principal component cumulative contribution rate P. The principal component P is set to 0.99. Contrast the results identified by these two methods and use modal assurance criterion (MAC) to evaluate the accuracy of identified modal shapes. If the MAC value is closer to 1, the identified modal shape has a higher accuracy.

Figure 3 shows the contribution of every principal component identified by the two methods. The curve is the values of P. The results in figure 3 indicate that the contribution rate of the first principal component identified by traditional PCA has reached 99.62%. Subjected to the constraint of values of P, the traditional PCA method can only identify the first-order mode. The equalization PCA method can identify the first three modes as the contribution rate $P_1$ of the first principal component identified by equalization PCA is 97.12%, the contribution rate $P_2$ of the second principal component identified by equalization PCA is 1.56% and the contribution rate $P_3$ of the third principal component identified by equalization PCA is 0.42%.

![Figure 3. Contribution rate.](image)

The contrast of the beam’s modal shapes identified by the two methods and finite element model of each order are shown in figure 4 and table 1. The natural frequencies identified are shown in table 2.

![Figure 4. Comparison diagrams of modal shapes of each order.](image)
Table 1. Comparison of MAC values.

| Modes orders of finite element model | Modes orders identified by traditional PCA | MAC1        | Modes orders identified by equalization PCA | MAC2   |
|--------------------------------------|--------------------------------------------|-------------|---------------------------------------------|--------|
| 1                                    | 1                                          | 1.0000      | 1                                           | 0.9999 |
| 2                                    | ---                                        | ---         | 2                                           | 0.9815 |
| 3                                    | ---                                        | ---         | 3                                           | 0.9772 |

Table 2. Comparison diagrams of modal shapes of each order.

| Natural frequencies obtained by FEM (Hz) | Frequencies identified by traditional PCA (Hz) | Relative errors1 (%) | Frequencies identified by equalization PCA (Hz) | Relative errors2 (%) |
|-----------------------------------------|------------------------------------------------|-----------------------|-------------------------------------------------|-----------------------|
| 91.71                                   | 92.30                                          | 0.64                  | 92.31                                           | 0.65                  |
| 252.3                                   | ---                                            | ---                   | 256.41                                          | 1.63                  |
| 493.4                                   | ---                                            | ---                   | 482.05                                          | 2.30                  |

Figures 3 and 4 show us the qualitative comparisons of modal shapes obtained by FEM and two PCA methods. Tables 1 and 2 show us the quantitative comparisons of modal shapes and natural frequencies obtained by FEM and two PCA methods. The results show that equalization PCA can identify more orders of modal parameters with the same conditions and constraints. The relative errors of vibration modal shapes and natural frequencies identified by equalization PCA are allowable.

5. Conclusions

In the process of identifying modal parameters based on traditional PCA, there may exist differences in the magnitude of the response data value at different discrete points. The differences will affect the contribution rate of different principal components. To reduce the influence of magnitude differences on the identification process, this paper improved PCA by equalizing the original data in each time sample to standardize the data. The main information of the original data still has been saved. To verify the method of modal parameters identification based on equalization PCA, a doubly-clamped beam is taken as an example. The results show that the equalization PCA method can identify more orders of modal parameters with the same conditions and constraints. The relative errors of vibration modal shapes and natural frequencies identified by equalization PCA are allowable.

At the same time, how to improve the identification accuracy of parameters is the direction of further research during the process of identifying structural modal parameters by equalization PCA.

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