Extraction of the Compton Form Factor $\mathcal{H}$ from DVCS measurements in the quark sector

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MENU 2010 - 06 / 02 / 2010

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2 Preliminary analysis

3 Hybrid fitting strategy

4 Results from all strategies
(Generalized) Parton Distributions.
From Deep Inelastic Scattering to Deeply Virtual Compton Scattering.

Extraction of \( \mathcal{H} \) from DVCS

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About GPDs
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Preliminary analysis
Selected data
GV formalism
Assumptions
Hybrid fitting strategy
Local fits
Global fit
Results from all strategies
\( \text{Im}\mathcal{H} \) and \( \text{Re}\mathcal{H} \)
Discussion

\( e^{-} \rightarrow e^{-} \gamma, Q^{2} \)
\( p \rightarrow X \)

Parton Distributions
nucleon

C. Weiss, AIP Conf. Proc. 1149, 150 (2009)
(Generalized) Parton Distributions.
From Deep Inelastic Scattering to Deeply Virtual Compton Scattering.

Correlation of the \textit{longitudinal momentum} and the \textit{transverse position} of the struck quark.
(Generalized) Parton Distributions.
From Deep Inelastic Scattering to Deeply Virtual Compton Scattering.

- Correlation of the **longitudinal momentum** and the **transverse position** of the struck quark.
- **3-dimensional** description of the nucleon.
- Insights on:
  - spin structure,
  - energy-momentum structure.
(Generalized) Parton Distributions.
From Deep Inelastic Scattering to Deeply Virtual Compton Scattering.

How to obtain this 3d picture from DVCS measurements?

Generalized Parton Distributions

$H(x, \xi, t)$
$E(x, \xi, t)$
$\tilde{H}(x, \xi, t)$
$\tilde{E}(x, \xi, t)$

C. Weiss,
AIP Conf.
Proc. 1149,
150 (2009)
DVCS described by 4 Compton Form Factors.
Approximations: quark sector, leading twist and leading order.

- **Example:** GPD $H$

$$
\mathcal{H} = \int_{-1}^{+1} dx \, H(x, \xi, t) \left( \frac{1}{\xi - x - i\epsilon} - \frac{1}{\xi + x - i\epsilon} \right)
$$

- Integration yields **real** and **imaginary** parts to $\mathcal{H}$:

$$
\text{Re}\mathcal{H} = \mathcal{P} \int_{-1}^{+1} dx \, H(x, \xi, t) \left( \frac{1}{\xi - x} - \frac{1}{\xi + x} \right)
$$

$$
\text{Im}\mathcal{H} = \pi \left( H(\xi, \xi, t) - H(-\xi, \xi, t) \right)
$$

- Relation between $\text{Im}\mathcal{H}$ and $\text{Re}\mathcal{H}$ **weakly constrained** by dispersion relations. However see:

  K. Kumericki and D. Müller, arXiv:0904.0458
  G. Goldstein and S. Liuti, DIS2009
Current extraction methods.
Problem: How to reduce the model dependence?

Local fits
Take each kinematic bin independently of the others.
Extraction of $Re\mathcal{H}$, $Im\mathcal{H}$, ... as independent parameters.

Global fit
Take all kinematic bins at the same time. Use a parametrization of GPDs or CFFs.

Hybrid: Local / global fit
Combine two previous methods to estimate model dependence.

Neural networks
No results yet. Work in progress.
Selected DVCS measurements (unpolarized target).
Fine kinematic binning, large kinematic coverage, several observables.

Hall A: helicity-dependent and independent cross sections

C. Muñoz Camacho et al., Phys. Rev. Lett. 97, 262002 (2006)
Restricted kinematic range, highly-precise helicity-dependent cross sections.

Hall B: Beam Spin Asymmetries

F.-X. Girod et al., Phys. Rev. Lett. 100, 162002 (2008)
Wide kinematic range, precise BSAs.

Hermes: BSAs, BCAs, TSAs

A. Airapetian et al., JHEP 0806, 017 (2008)
D. Zeiler et al., arXiv:0810.5007 [hep-ex]
Restricted kinematic range, several different observables.
Analytic $ep \rightarrow ep\gamma$ cross sections.
Interference between Bethe-Heitler and VCS processes treated exactly.

Example: DVCS helicity-dependent cross section at twist 2

- BKM formalism:
\[
C_1 \sin \phi \text{Im} \left( \mathcal{H} + \frac{x_B}{2-x_B} \left( 1 + \frac{F_2}{F_1} \right) \tilde{\mathcal{H}} - \frac{t}{4M^2} \frac{F_2}{F_1} \mathcal{E} \right)
\]

A.V. Belitsky, D. Mueller and A. Kirchner
Nucl. Phys. B629, 323 (2002)

- GV formalism:
\[
C_2 \sin \phi \text{Im} \left( \mathcal{H} + c_\mathcal{E} \mathcal{E} + c_\tilde{\mathcal{H}} \tilde{\mathcal{H}} + c_\tilde{\mathcal{E}} \tilde{\mathcal{E}} \right)
\]

P.A.M. Guichon and M. Vanderhaeghen, unpublished
Analytic $ep \rightarrow ep\gamma$ cross sections.
Interference between Bethe-Heitler and VCS processes treated exactly.

Example: DVCS helicity-dependent cross section at twist 2

- BKM formalism: arXiv:1005.5209

$$C_1 \sin \phi \text{Im} \left( \mathcal{H} + \frac{x_B}{2-x_B} \left( 1 + \frac{F_2}{F_1} \right) \mathcal{H} - \frac{t}{4M^2} \frac{F_2}{F_1} \mathcal{E} \right)$$

A.V. Belitsky, D. Mueller and A. Kirchner
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- GV formalism:

$$C_2 \sin \phi \text{Im} \left( \mathcal{H} + c\mathcal{E} \mathcal{E} + c\mathcal{H} \mathcal{H} + c\tilde{\mathcal{E}} \tilde{\mathcal{E}} \right)$$

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**Example:** DVCS helicity-dependent cross section at twist 2

- **BKM formalism**: coefficients do not depend on $Q^2$
  \[
  C_1 \sin \phi \text{Im} \left( \mathcal{H} + \frac{x_B}{2-x_B} \left(1 + \frac{F_2}{F_1}\right) \tilde{\mathcal{H}} - \frac{t}{4M^2} \frac{F_2}{F_1} \mathcal{E} \right)
  \]

  A.V. Belitsky, D. Mueller and A. Kirchner
  *Nucl. Phys. B629*, 323 (2002)

- **GV formalism**: coefficients depend on $Q^2$
  \[
  C_2 \sin \phi \text{Im}\left( \mathcal{H} + \frac{c_\mathcal{E}}{20\%} \mathcal{E} + \frac{c_{\tilde{\mathcal{H}}}}{20\%} \tilde{\mathcal{H}} + \frac{c_{\tilde{\mathcal{E}}}}{30\%} \tilde{\mathcal{E}} \right)
  \]

  P.A.M. Guichon and M. Vanderhaeghen, unpublished
Main assumptions.
Expectation: extraction of $H$ with $\geq 40\%$ total uncertainty.

- Twist 2 accuracy
  - Early $Q^2$-scaling was observed in Hall A.
    
    C. Muñoz Camacho et al.
    Phys. Rev. Lett. \textbf{97}, 262002 (2006)
  - Similar recent result concerning a subset of JLab data.
    
    M. Guidal, arXiv:1003.0307
  - Small higher twist contribution in Hermes data.
    
    D. Zeiler \textit{et al.}, DIS2008

- $H$-dominance
  - Dramatically decreases the number of degrees of freedom in the fits.
  - Expectations: \textbf{systematic error between 20 and 50 \%}.
  - Systematic error $\lesssim 25\%$ from direct test of hypothesis with VGG model.
  - The most questionable assumption so far?
Local fits.
Fits on each kinematic bin to twist 2 expressions.

- Keep bins with \( \frac{|t|}{Q^2} < \frac{1}{2} \).
- Low model dependence (H-dominance, twist 2).
- But fits may still be underconstrained.
- **Estimation** of systematic errors caused by *H-dominance hypothesis* by fitting data with subdominant GPDs set to 0 or to their VGG value.
Global fit.
Fit to a parametrization from the dual model.

- DVCS cross sections depend on singlet combination $H_+$:
  \[ H_+(x, \xi, t, Q^2) = H(x, \xi, t, Q^2) - H(-x, \xi, t, Q^2) \]

- Dual model parametrization of $H_+$:
  \[ 2 \sum_{n=0}^{\infty} \sum_{l=0}^{n+1} B_{nl}(t, Q^2) \theta \left( 1 - \frac{x^2}{\xi^2} \right) \left( 1 - \frac{x^2}{\xi^2} \right) C_{2n+1}^2 \left( \frac{x}{\xi} \right) P_{2l} \left( \frac{1}{\xi} \right) \]
Global fit.
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  \]

Legendre polynomial
Global fit.
Fit to a parametrization from the dual model.

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$$2 \sum_{n=0}^{\infty} \sum_{l=0}^{n+1} B_{nl}(t, Q^2) \theta \left(1 - \frac{x^2}{\xi^2}\right) \left(1 - \frac{x^2}{\xi^2}\right) C_{2n+1}^2 \left(\frac{x}{\xi}\right) P_{2l} \left(\frac{1}{\xi}\right)$$

Gegenbauer polynomial
Global fit.
Fit to a parametrization from the dual model.

- DVCS cross sections depend on singlet combination $H_+$:
  \[ H_+(x, \xi, t, Q^2) = H(x, \xi, t, Q^2) - H(-x, \xi, t, Q^2) \]

- Dual model parametrization of $H_+$:
  \[
  2 \sum_{n=0}^{\infty} \sum_{l=0}^{n+1} B_{nl}(t, Q^2) \theta \left( 1 - \frac{x^2}{\xi^2} \right) \left( 1 - \frac{x^2}{\xi^2} \right) C_{2n+1}^3 \left( \frac{x}{\xi} \right) P_{2l} \left( \frac{1}{\xi} \right)
  \]

Support: Smeared
Global fit.
Fit to a parametrization from the dual model.

- DVCS cross sections depend on singlet combination $H_+$:

$$H_+(x, \xi, t, Q^2) = H(x, \xi, t, Q^2) - H(-x, \xi, t, Q^2)$$

- Dual model parametrization of $H_+$:

$$2 \sum_{n=0}^{\infty} \sum_{l=0}^{n+1} B_{nl}(t, Q^2) \theta \left(1 - \frac{x^2}{\xi^2}\right) \left(1 - \frac{x^2}{\xi^2}\right) C_{2n+1}^{3/2} \left(\frac{x}{\xi}\right) P_{2l} \left(\frac{1}{\xi}\right)$$

Model $t$-dep.

with $B_{nl}(t, Q^2) = \left(\ln \frac{Q^2_0}{\Lambda^2} / \ln \frac{Q^2}{\Lambda^2}\right)^{\frac{\gamma_p}{\beta_0}} B_{nl}(t, Q^2_0)$. 

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**Preliminary analysis**
Selected data
GV formalism
Assumptions

**Hybrid fitting strategy**
Local fits
Global fit

**Results from all strategies**
$\text{Im} H$ and $\text{Re} H$

**Discussion**

**Conclusions**
Global fit.
Fit to a parametrization from the dual model.

- DVCS cross sections depend on singlet combination $H_+$:
  \[
  H_+(x, \xi, t, Q^2) = H(x, \xi, t, Q^2) - H(-x, \xi, t, Q^2)
  \]

- Dual model parametrization of $H_+$:
  \[
  2 \sum_{n=0}^{N} \sum_{l=0}^{n+1} B_{nl}(t, Q^2) \theta \left(1 - \frac{x^2}{\xi^2}\right) \left(1 - \frac{x^2}{\xi^2}\right) C_{2n+1}^2 \left(\frac{x}{\xi}\right) P_{2l} \left(\frac{1}{\xi}\right)
  \]

  Model t-dep.

  with $B_{nl}(t, Q^2) = \left(\ln \frac{Q_0^2}{\Lambda^2} / \ln \frac{Q^2}{\Lambda^2}\right)^{\gamma_p/\beta_0} \frac{a_{nl}}{1+b_{nl}(t-t_0)^2}$.

- Non-trivial correlation between $x$ and $t$.
- $a_{nl}$ and $b_{nl}$ are fitted. $t_0$ is chosen prior to the fits.
Global fit.
Iterative fitting procedure and systematic uncertainties.

- Keep bins with $\frac{|t|}{Q^2} < \frac{1}{2}$ (1001 $\phi$-bins fitted).

- $\frac{N(N+3)}{2}$ fitted coefficients for a given truncation $N$.
  - 10, 18 and 28-parameter fits for $N = 2, 3$ and 4.
  - **Estimation** of the truncation error by comparison of the results of these 3 fits.

- Iterative fitting procedure to handle large number of parameters.

- **Estimation** of systematic errors caused by $H$-dominance hypothesis by fitting data with subdominant GPDs set to 0 or to their VGG value.

- Purpose: smooth parametrization of data. **No extrapolation** outside the domain of the fit.
Effect of the truncation of the series.
Hall B data (hybrid fitting strategy).

- 3 global fits qualitatively similar:
  \[ N \quad \chi^2/d.o.f. \]
  \[
  \begin{array}{ccc}
  2 & 1.73 \\
  3 & 1.61 \\
  4 & 1.78 \\
  \end{array}
  \]

- No differences on Hall A data (next slide).
- \( N=2 \) fails to reproduce BSAs at small \( \xi \).
- \( N=3 \) always good and close to local fits.
- \( N=4 \) is uncontrolled at large \( \xi \).
$ImH$ on Hall B kinematics (hybrid fitting strategy).

$Q^2$-dependence.

- Compatible results of local and global fits: **strong** consistency check.
- Realistic estimation of systematic uncertainties:
  - Comparable accuracy from local and global fits.
  - Accuracy in agreement with expectations.
- Restricted kinematic region suitable for GPD-analysis.
$ReH$ on Hall B kinematics (hybrid fitting strategy).
$Q^2$-dependence.

- Large fluctuations in $ReH$ from local fits. Global fit is smoother.
- Unreliable extraction of $ImH$ or $ReH$ at large $\xi$.
- $ReH$ weakly constrained.
- Noticeable deviations if
  \[ \xi = x_B \frac{1 + \frac{t}{2Q^2}}{2 - x_B + \frac{x_B t}{Q^2}} \rightarrow \frac{x_B}{2 - x_B} \]

- Call for a twist 3 analysis!
Hermes data (local fits).
Data show a small higher-twist contribution.
Hermes data (local fits).
Data show a small higher-twist contribution.

BCA (5 harmonics)
Hermes data (local fits).
Data show a small higher-twist contribution.
Hermes data (local fits).
Data show a small higher-twist contribution.

BSA (8 harmonics)
Hermes data (local fits).
Data show a small higher-twist contribution.

$A_{DLU}^{DVCS} = 0$
at twist 2
Hermes data (local fits).
Data show a small higher-twist contribution.

Original point
Fitted point
Fit
--- VGG

Small higher twist effect.
All observables are fitted at the same time.
Comparison with other studies (Hall A data).
Several approaches: BKM, BKM + "hot fix", GV, VGG.

- First extraction: BKM formalism without "hot fix".
  
  C. Muñoz Camacho et al.
  Phys. Rev. Lett. **97**, 262002 (2006)

- Model-dependent prediction. Fit in progress.
  
  S. Ahmad et al., arXiv:0708.0268

- VGG fitter code.
  
  M. Guidal, Eur. Phys. J. **A37**, 319 (2008)
  M. Guidal, Phys. Lett. **B689**, 156 (2010)

- "Hot fix" for power suppressed contributions in BKM.
  
  A. Belitsky and D. Müller, Phys. Rev. **D79**, 014017 (2009)

- Global fit for all unpolarized proton target with BKM + "hot fix".
  
  K. Kumericki and D. Müller, arXiv:0904.0458
Comparison with previous studies (Hall A data).
Where are we today?

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- Conclusions

Comparison with previous studies (Hall A data).
Where are we today?

- EPJA 39, 5 (2009)
- PRL 97, 262002 (2006)

H. Moutarde (Irfu/SPhN, CEA-Saclay)
Hybrid fitting strategy results compared to VGG. Similar $x_B$-dependence but loss of information during the extraction.
Conclusions.
DVCS measurements are still a challenge to phenomenology.

- \( \text{Im} \mathcal{H} \) extracted with 20 to 50 % accuracy on a wide kinematic range.

- \( \text{Re} \mathcal{H} \) still poorly known.

- Realistic first estimation of systematic errors.

- Plausible early \( Q^2 \)-scaling but twist 3 study necessary.

- Working without \( H \)-dominance hypothesis ? In progress.

- More generally, a global fitting strategy to obtain a ”experimental” 3d nucleon picture is still missing.
Acknowledgments.

I am indebted to :

- the CLAS group at Saclay :
  * J. Ball
  * M. Garçon
  * P. Konczykowski

- and also :
  * M. El Yakoubi
  * F.-X. Girod
  * M. Guidal

  * B. Moreno
  * S. Procureur
  * F. Sabatié
  * C. Muñoz Camacho
  * P. Guichon
  * M. Vanderhaeghen