Measurements of $\sigma(e^+e^- \rightarrow \Upsilon(nS)\pi^+\pi^-)$ and $\sigma(e^+e^- \rightarrow b\bar{b})$ in the $\Upsilon(10860)$ and $\Upsilon(11020)$ resonance regions

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(The Belle Collaboration)

1 University of the Basque Country UPV/EHU, 48080 Bilbao
2 University of Bonn, 53115 Bonn
3 Budker Institute of Nuclear Physics SB RAS and Novosibirsk State University, Novosibirsk 630090
4 Faculty of Mathematics and Physics, Charles University, 121 16 Prague
5 University of Cincinnati, Cincinnati, Ohio 45221
6 Deutsches Elektronen-Synchrotron, 22607 Hamburg
7 Justus-Liebig-Universität Gießen, 35392 Gießen
8 The Graduate University for Advanced Studies, Hayama 240-0193
9 Gyeongsang National University, Chinju 660-701
10 Hanyang University, Seoul 133-791
11 University of Hawaii, Honolulu, Hawaii 96822
12 High Energy Accelerator Research Organization (KEK), Tsukuba 305-0801
13 IKERBASQUE, Basque Foundation for Science, 48011 Bilbao
14 Indian Institute of Technology Guwahati, Assam 781039
15 Indian Institute of Technology Madras, Chennai 600036
16 Institute of High Energy Physics, Chinese Academy of Sciences, Beijing 100049
17 Institute of High Energy Physics, Vienna 1050
18 INFN - Sezione di Torino, 10125 Torino
19 Institute for Theoretical and Experimental Physics, Moscow 117218
20 J. Stefan Institute, 1000 Ljubljana
21 Kanagawa University, Yokohama 221-8686
22 Kennesaw State University, Kennesaw GA 30144
23 Department of Physics, Faculty of Science, King Abdulaziz University, Jeddah 21589
24 Korea Institute of Science and Technology Information, Daedeon 305-806
25 Korea University, Seoul 136-713
26 Kyungpook National University, Daegu 702-701
27 École Polytechnique Fédérale de Lausanne (EPFL), Lausanne 1015
28 Faculty of Mathematics and Physics, University of Ljubljana, 1000 Ljubljana
29 Luther College, Decorah, Iowa 52101
30 University of Maribor, 2000 Maribor
31 Max-Planck-Institut für Physik, 80805 München
32 School of Physics, University of Melbourne, Victoria 3010
33 Moscow Physical Engineering Institute, Moscow 115409
34 Moscow Institute of Physics and Technology, Moscow Region 141700
35 Graduate School of Science, Nagoya University, Nagoya 464-8602
36 Kobayashi-Maskawa Institute, Nagoya University, Nagoya 464-8602
37 Nara Women’s University, Nara 630-8506
38 Department of Physics, National Taiwan University, Taipei 10617
39 H. Niewodniczanski Institute of Nuclear Physics, Krakow 31-342
40 Niigata University, Niigata 950-2181
41 Osaka City University, Osaka 558-8585
42 Pacific Northwest National Laboratory, Richland, Washington 99352
43 Peking University, Beijing 100871
44 University of Pittsburgh, Pittsburgh, Pennsylvania 15260
45 University of Science and Technology of China, Hefei 230026
46 Seoul National University, Seoul 151-742
47 Soongsil University, Seoul 156-743
48 Sungkyunkwan University, Suwon 440-746
Abstract

We report new measurements of $R_{\Upsilon \pi \pi} \equiv \frac{\sigma(e^+e^- \to \Upsilon(nS)\pi^+\pi^-)}{\sigma_{\mu\mu}^0} (n = 1, 2, 3)$ and $R_b \equiv \frac{\sigma(e^+e^- \to b\bar{b})}{\sigma_{\mu\mu}^0}$ (where $\sigma_{\mu\mu}^0$ is the muon-pair Born cross section) in the region $\sqrt{s} = 10.63$-$11.05$ GeV, based on data collected with the Belle detector. Distributions in $R_{\Upsilon \pi \pi}$ and $R_b$ are fit for the masses and widths of the $\Upsilon(10860)$ and $\Upsilon(11020)$ resonances. Unlike $R_b$, which includes a large non-resonant $b\bar{b}$ component, we find that $R_{\Upsilon \pi \pi}$ is dominated by the two resonances.

With $R_{\Upsilon \pi \pi}$ as the basis, the total rate at the $\Upsilon(10860)$ peak for known final states containing bottomonium(-like) resonances is estimated. We find that $\Upsilon(10860)$ is essentially saturated by such modes, raising doubts about the validity of masses measured using $R_b$. With $R_{\Upsilon \pi \pi}$, we measure $M_{10860} = (10891.1^{+0.6}_{-1.5})$ MeV/$c^2$ and $\Gamma_{10860} = (53.7^{+7.4}_{-5.6})$ MeV and report first measurements $M_{11020} = (10987.5^{+6.4}_{-2.6})$ MeV/$c^2$, $\Gamma_{11020} = (61^{+9}_{-19})$ MeV, and the relative phase $\phi_{11020} - \phi_{10860} = (-1.0^{+1.0}_{-0.1})$ rad.
The Υ(10860) is generally interpreted to be the Υ(5S); however, the Belle Collaboration has observed unexpected behavior in $e^+e^-$ events that contain bottomonia at and near this resonance. The rate for $e^+e^- \rightarrow Υ(nS)\pi^+\pi^-$ ($n = 1, 2, 3$) at its peak (center-of-mass energy $\sqrt{s} = 10.865$ GeV) is two orders of magnitude larger than that for $Υ(nS) → Υ(1S)\pi^+\pi^-$ ($n = 2, 3, 4$) [1]. Rates to $h_b(mP)\pi^+\pi^-(m = 1, 2)$ are of the same order of magnitude as to $Υ(nS)\pi^+\pi^-$, despite the $Υ(5S) → h_b(mP)\pi^+\pi^-$ process requiring a $b$-quark spin-flip[2]. A study of the substructure in the final states $Υ(nS)\pi^+\pi^-$ ($n = 1, 2, 3$) and $h_b(mP)\pi^+\pi^-(m = 1, 2)$ has yielded two new bottomonium-like resonances, $Z^0_b(10610)$ and $Z^+_b(10650)$ [3].

In light of these findings, the nature of $Υ(10860)$ cannot be said to be fully understood. For convenience, in this paper $Υ(10860)$ and $Υ(11020)$ are called $Υ(5S)$ and $Υ(6S)$, respectively. In Ref. [4], the ratio $R_{Υππ} ≡ \sigma(Υ(nS)\pi^+\pi^-)/\sigma_μμ^0$ (where $\sigma_μμ^0 = (4πα^2)/3s$ is the Born $e^+e^- → μ^+μ^-$ cross-section, with $α$ being the fine-structure constant) was measured at six energy points near the $Υ(5S)$ resonance and one near the $Υ(6S)$. A Breit-Wigner (BW) lineshape plus a constant (with complex phase) was fit to the data distribution and yielded a BW mean separated by $9±4$ MeV/c$^2$ from the corresponding value from the fit to $R_b ≡ \sigma(\bar{b}b)/\sigma_μμ^0$. To explore these anomalies, we report measurements of $R_{Υππ}$ and $R_b$ between 10.60 and 11.05 GeV, including new data in the region of the $Υ(5S)$ and $Υ(6S)$.

The data were recorded with the Belle detector [5] at the KEKB [6] $e^+e^-$ collider and consist of 121.4 fb$^{-1}$ at $\sqrt{s} = 10.865$ GeV, nominally the $Υ(5S)$ peak, approximately 1 fb$^{-1}$ at each of the six energy points above 10.80 GeV, studied in Ref. [4]; 1 fb$^{-1}$ at each of 16 additional points between 10.63 and 11.02 GeV; and 50 pb$^{-1}$ at each of 61 points taken in 5 MeV steps between 10.75 and 11.05 GeV. The non-resonant $q\bar{q}$ continuum ($q \in \{u, d, s, c\}$) background is obtained using a 1.03 fb$^{-1}$ data sample taken below the $BB$ threshold, at $\sqrt{s_{ct}} = 10.520$ GeV (where $ct$ denotes the continuum point). This “$q\bar{q}$ continuum” background is distinct from the non-resonant $b\bar{b}$ continuum signal that might be present in our data.

The Belle detector is a large-solid-angle magnetic spectrometer that consists of a silicon vertex detector (SVD), a 50-layer central drift chamber (CDC), an array of aerogel threshold Cherenkov counters (ACC), a barrel-like arrangement of time-of-flight scintillation counters (TOF), and an electromagnetic calorimeter comprised of CsI(Tl) crystals (ECL), all located inside a superconducting solenoid coil that provides a 1.5 T magnetic field. An iron flux-return located outside of the coil is instrumented to detect $K^0_L$ mesons and to identify muons (KLM).

We measure $R_b$ in each of the 61 50-pb$^{-1}$ data sets and the 16 newer 1-fb$^{-1}$ sets. The center-of-mass (CM) energy at each scan point is measured with $e^+e^- → μ^+μ^-$ events. The peak position of the distribution in dimuon mass, $M_{μμ}$, is determined from a fit to the sum of three Gaussian functions. The difference between $M_{μμ}$ and $\sqrt{s}$, primarily due to initial state radiation (ISR), is determined from Monte Carlo (MC) simulations based on the KKF generator [7], taking into account dependencies on $\sqrt{s}$. An absolute calibration is made using the $Υ(1S)\pi^+\pi^-$ sample at the $Υ(5S)$ resonance, where the distribution in $E_{CM} = ∆M^2 + m_τc^2$ yields a high-resolution measurement that is referenced to the well-measured mass of the $Υ(1S)$; here, $∆M ≡ M(μμππ) − M(μμ)$ and $m_τ$ is the world-average $Υ(1S)$ mass [3]. The difference between $\sqrt{s}$ measured by $Υππ$ and $μμ$ is taken as a common offset for all other data samples. The statistical uncertainty in $\sqrt{s}$ at each scan point is 0.7 MeV (0.4 MeV) for the lower (higher) luminosity sets.

Hadronic events are selected by requiring at least five charged tracks with transverse momentum $p_T > 100$ MeV/c that satisfy track quality criteria based on their impact parameters
relative to the interaction point (IP), where the transverse and longitudinal directions are perpendicular and parallel to the $e^+$ beam, respectively. Each event must have more than one ECL cluster with energy above 100 MeV, a total energy in the ECL between 0.1 and $0.8 \times \sqrt{s}$, and an energy sum of all charged tracks and photons exceeding $0.5 \times \sqrt{s}$. We demand that the reconstructed event vertex be within 1.5 and 3.5 cm of the IP in the transverse and longitudinal dimensions, respectively. To suppress events of non-$b\bar{b}$ origin, events are further required to satisfy $R_2 < 0.2$, where $R_2$ is the ratio of the second and zeroth Fox-Wolfram moments [8].

The $b\bar{b}$ event selection efficiency $\epsilon_{b\bar{b},i}$ for the $i$th scan set is estimated via MC simulation based on EvtGen [11] and GEANT3 [11]. Efficiencies are determined for each type of open $b\bar{b}$ event found at $\sqrt{s} = 10.865$ GeV: $B^{(*)}\bar{B}^{(*)}(\pi)$ and $B^{(*)}_{\mu\tau}\bar{B}^{(*)}_{\mu\tau}$. As the relative rates of the different event types are only known at the on-resonance point, we take the average of the highest and lowest efficiencies as $\epsilon_{b\bar{b}}$ and the difference divided by $\sqrt{12}$ as its uncertainty. The value of $\epsilon_{b\bar{b}}$ increases approximately linearly from about 70% to 74% over the scan region. The value at the on-resonance point is in good agreement with $\epsilon_{b\bar{b}}$ determined with the known event mixture [8].

Events passing the above criteria include direct $b\bar{b}$, $q\bar{q}$ continuum, and bottomonia produced via ISR: $e^+e^- \rightarrow \gamma \Upsilon(nS)\ (n=1, 2, 3)$. The number of selected events is

$$N_i = \mathcal{L}_i \times \left[ \sigma_{b\bar{b},i}\epsilon_{b\bar{b},i} + \sigma_{q\bar{q},i}\epsilon_{q\bar{q},i} + \sum \sigma_{ISR,i}\epsilon_{ISR,i} \right]$$

(1)

where $i$ denotes the data set, $\mathcal{L}_i$ is its integrated luminosity, and the sum is over the three $\Upsilon$ states produced via ISR. Ref. [12] is used to calculate $\sigma_{ISR}$ with measured electronic widths of $\Upsilon(nS)$. The contribution from $\sigma(q\bar{q})$, which scales as $1/s$, is estimated from the data taken at $\sqrt{s_{ct}}$, where $\sigma_{b\bar{b}} = 0$, and is corrected for luminosity and energy differences. The subtracted quantity

$$\tilde{R}_{b,i} = \frac{1}{\epsilon_{b\bar{b}}} \left( \frac{N_i}{\mathcal{L}_i \sigma_{\mu\mu,i}} - \frac{N_{ct}}{\mathcal{L}_{ct} \sigma_{\mu\mu,ct}^{0}} \epsilon_{q\bar{q},ct} \right)$$

(2)

includes a residual contribution from ISR, which differs from $q\bar{q}$ continuum in its $s$-dependence. For comparison with a previous measurement by BABAR [13], we define $R_b$ to include the ISR events and correct the result based on theoretical calculations [12] and MC simulation, as described above. For comparison with $R_{\Upsilon\pi\pi}$, the ISR portion is excluded, and we define $R_{b,i} \equiv \tilde{R}_{b,i} - \sum \sigma_{ISR,i} / \sigma_{\mu^+\mu^-;i}$. These measurements are of the visible cross-sections and include neither corrections due to ISR nor the vacuum polarization necessary to obtain the Born cross-section [14].

The function

$$\mathcal{F} = |A_{mS}|^2 + |A_{t} + A_{6S}e^{i\delta_{6S}}f_{6S}|^2$$

(3)

is fit to the $\{R_{b,i}\}$ and $\{R_{b,i}'\}$ distributions, where $f_{nS} = M_{nS}\Gamma_{nS}/[(s - M_{nS}^2) + iM_{nS}\Gamma_{nS}]$. The parameters $A_t$ and $A_{mS}$ are coherent and incoherent continuum terms, respectively. The fitting range is restricted to 10.82-11.05 GeV to avoid complicated threshold effects below 10.8 GeV [13]. The $R_b$ data and fit are shown in Fig.1. The results from both fits are shown in Table1. The masses, widths, and phase difference for $R_b$ are consistent with those from earlier measurements by Belle [4] and BABAR [13].

Candidate $\Upsilon(nS)[\rightarrow \mu^+\mu^-]\pi^+\pi^-$ events are required to have exactly four charged tracks satisfying track quality criteria, with distances of closest approach to the IP of less than 1 cm and 5 cm in the transverse and longitudinal directions, respectively, and with $p_T > 100$
TABLE I. $\Upsilon(5S)$ and $\Upsilon(6S)$ masses, widths, and phase difference, extracted from fits to data. The errors are statistical and systematic. The 1 MeV uncertainty on the masses due to the systematic uncertainty in $\sqrt{s}$ is not included. Fit C is our nominal result.

|     | $M_{5S}$ (MeV/$c^2$) | $\Gamma_{5S}$ (MeV) | $M_{6S}$ (MeV/$c^2$) | $\Gamma_{6S}$ (MeV) | $\phi_{6S}-\phi_{5S}$ (rad) | $\chi^2$/dof |
|-----|---------------------|---------------------|---------------------|---------------------|-----------------------------|--------------|
| $R_b$ | 10881.9 ± 1.0 ± 1.2 | 49.8 ± 1.9 ± 2.2 | 11002.9 ± 1.1 ± 0.9 | 38.5 ± 1.7 ± 1.3 | -1.86 ± 0.24 ± 0.10 | 55/50 |
| $\Gamma_b$ | 10881.8 ± 1.1 ± 1.2 | 48.5 ± 1.8 ± 2.2 | 11003.0 ± 1.1 ± 0.9 | 39.3 ± 1.6 ± 2.4 | -1.87 ± 0.51 ± 0.16 | 56/50 |
| $R_{\Upsilon \pi \pi}$ (fit A) | 10883.2 ± 1.4 ± 1.0 | 51.6 ± 3.1 | 11003.0 | 39.3 | -1.87 | 63/62 |
| $R_{\Upsilon \pi \pi}$ (fit B) | 10890.4 ± 3.5 ± 0.6 | 54.7 ± 7.4 ± 0.5 | 10988.4 ± 6.2 ± 2.2 | 58.1 ± 4.1 ± 0.5 | -0.7 ± 0.5 ± 0.0 | 52/59 |
| $R_{\Upsilon \pi \pi}$ (fit C) | 10891.1 ± 3.2 ± 0.6 | 53.7 ± 5.6 ± 0.4 | 10987.5 ± 2.5 ± 2.1 | 58.1 ± 4.1 ± 0.5 | -1.0 ± 0.4 ± 0.1 | 51/56 |
| $R_{\Upsilon \pi \pi}$ (fit D) | 10891 ± 3 ± 2 | 54.7 ± 4.5 | 10988 ± 3.7 | 60.0 ± 2.0 | 51/57 |

MeV/$c^2$, including two oppositely-charged tracks with an invariant mass above 8 GeV/$c^2$, each consistent with the muon and inconsistent with the kaon hypothesis and two oppositely charged tracks, each consistent with the pion and inconsistent with the electron hypothesis. Radiative muon pair events, $e^+e^- \rightarrow \gamma \mu^+\mu^- [\gamma \rightarrow e^+e^-]$, are suppressed by requiring the $\mu^+\mu^-$ and $\pi^+\pi^-$-candidate vertices be separated in the plane transverse to the $e^+$ beam by less than 3 (4.5) mm for $\Upsilon(1S, 2S)$ ($\Upsilon(3S)$). We require $|M(\mu^+\mu^- - \sqrt{s}/c^2)| < 200$ MeV/$c^2$, where the resolution is $\approx 60$ MeV/$c^2$. Signal candidates are selected by requiring $\delta M \equiv |D_\pi - (\sqrt{s}/c^2 - m_\pi)| < 25$ MeV/$c^2$, where the $\delta M$ resolution is $\approx 7$ MeV/$c^2$. We select sideband events in the range 50 MeV/$c^2 < |\delta M| < 100$ MeV/$c^2$ to estimate background.

Reconstruction efficiencies are estimated via MC simulation. Because the relative contributions of intermediate resonances such as the $Z_0^\pm$ may vary with $\sqrt{s}$, the efficiency is modeled analytically as a function of $s_1 \equiv M^2(\Upsilon \pi^\pm)$, $s_2 \equiv M^2(\Upsilon \pi^-)$, and $E_{CM}$ using MC datasets generated at six values of $\sqrt{s}$, with the $\sqrt{s}$-dependence of the efficiency parameters modeled by second-order polynomials. The efficiencies are 42.5-44.5%, 31-41%, and 15-35% over the range of $\sqrt{s}$ for $\Upsilon(1S)$, $\Upsilon(2S)$, and $\Upsilon(3S)$, respectively.

Candidate weights are summed event-by-event after correcting for reconstruction efficiency for each of the signal and sideband samples. The net signal $N_{\Upsilon \pi \pi,i}$ is equal to the signal sum minus half the sideband sum. The resulting distribution of $R_{\Upsilon \pi \pi,i} = N_{\Upsilon \pi \pi,i}/(L_0 B(\Upsilon(nS) \rightarrow \mu^+\mu^-) e^{0/\mu_{\mu}^0(e^{0^3}s)})$, shown in Fig. 2, is tested for consistency with $R_b$ by fitting to $F \cdot \Phi_n(\sqrt{s})$. Here, the phase-space factor $\Phi_n$ is the ratio of phase-space volumes of $e^+e^- \rightarrow \Upsilon(nS)\pi\pi$ and $e^+e^- \rightarrow \Upsilon\gamma\gamma$ and accounts for the behavior near the mass threshold of the $\Upsilon(nS)\pi\pi$ final state. All masses, widths, and phases are fixed to the values obtained from the fit to $R_b$. The fit yields a $\chi^2$ per degree of freedom (dof) of 70/64, with insignificant contributions of $A_r$ and $A_{nr}$ in all $\Upsilon(nS)$ channels. We fix $A_r$ and $A_{nr}$ to zero in all subsequent fits. As there are few events in the $\Upsilon(6S)$ region, we perform two additional fits to $F \cdot \Phi_n(\sqrt{s})$: one with $M_{6S}$ and $\Gamma_{6S}$ allowed to float and common across the three $\Upsilon(nS)$ modes (fit "A") and the other with both $\Upsilon(5S)$ and $\Upsilon(6S)$ masses, widths, and phases allowed to float and common across the three channels (fit "B"). Implicit in $F$ is the assumption that the resonance substructures of $\Upsilon(5S)$ and $\Upsilon(6S)$ are identical, such that the normalized amplitude distributions over the Dalitz plot in $(s_1, s_2)$ at a given $\sqrt{s}$, $D_{5S,n}(s_1, s_2)$ and $D_{6S,n}(s_1, s_2)$, satisfy

$$\int D_{5S,n}(s_1, s_2) D_{6S,n}(s_1, s_2) ds_1 ds_2 \equiv k_n e^{i\delta},$$

where $k_n$, the decoherence coefficient for the $\Upsilon(nS)\pi\pi$ final state, is unity and $\delta \equiv \phi_{6S} - \phi_{5S}$. Given the rich structure found at $\sqrt{s} = 10.865$ GeV [3], some deviation of both $k_n$ and $\delta$
from this assumption are likely. We allow for this relaxation in fit “C” using the fitting function

\[ F_n = |A_{5S,n} f_{5S}|^2 + |A_{6S,n} f_{6S}|^2 + 2k_n A_{5S,n} A_{6S,n} \Re(e^{i\delta_n} f_{5S} f_{6S}^*) , \] (5)

wherein \( k_n \) and \( \delta_n \) are allowed to float but the three \( \delta_n \) are constrained to a common value. We find \( k_1 = 1.04 \pm 0.19, k_2 = 0.87 \pm 0.17, k_3 = 1.07 \pm 0.23, \) and \( \delta_n = -1.0 \pm 0.4 \). Finally, in fit “D,” we fix \( k_n \) to unity, and allow the three \( \delta_n \) to float independently. We find \( \delta_1 = -0.5 \pm 1.9, \delta_2 = -1.1 \pm 0.5, \) and \( \delta_3 = 1.0^{+0.8}_{-0.5} \). The masses and widths found in fits C and D are not significantly different from those found in fits A and B, as can be seen in Table I. The results from fit C are taken as the nominal values and shown in Fig. 2. The difference in \( M_{5S} \) between fit C and the fit to \( R'_b \) is 9.2 ± 3.4 ± 1.9 MeV.

As can be seen from Eq. (3), the distributions in \( R \) are described by the absolute square of the sum of two or more amplitudes. The expanded sum includes absolute squares of amplitudes for individual processes and interference terms. In principle, the term proportional to the absolute square of the \( \Upsilon(5S) \) amplitude in \( R_{\Upsilon\pi\pi} \), summed with corresponding terms for all other event types, is expected to result in the corresponding term for \( R'_b \). We calculate \( P_n = |A_{5S}(nS) f_{5S}|^2 \times \Phi_n \) (for \( n = 1, 2, 3 \)) and \( P_b = |A_{5S}(0) f_{5S}|^2 \) at the on-resonance energy point (\( \sqrt{s} = 10.865 \) GeV) using the results from fit A and the fit to \( R'_b \), respectively. We determine the “branching fraction” \( \mathcal{P} = \sum_n P_n/P_b = 0.170 \pm 0.009 \). It is worthwhile to expand this definition of \( \mathcal{P} \) to include several known final states related to \( \Upsilon(nS)\pi^+\pi^- \), which may also be expected to contain very little continuum. The \( \Upsilon(nS)\pi^0\pi^0 \) is related through isospin, and the observed rate is consistent with being half of the \( \Upsilon(nS)\pi^+\pi^- \) rate, as expected [16]. As \( \Upsilon(nS)\pi^+\pi^- \) (\( \Upsilon(nS)\pi^0\pi^0 \)) includes a substantial fraction of \( Z_b^{\pm} \pi^\mp (Z_b^0\pi^0) \), we can conclude that other final states with \( Z_b^{0/\pm} \pi^0/\mp \) behave similarly, \( \text{i.e.} \), with little or no \( b\bar{b} \) continuum. These include \( h_b(mP)\pi^+\pi^- \) (\( m = 1, 2 \)), which is found to be saturated by \( \Upsilon(5S) \rightarrow Z_b^{\pm} \pi^\mp \) [2, 3], and \( h_b(mP)\pi^0\pi^0 \), which we assume contributes at half the rate.
FIG. 2. $R_{\Upsilon \pi \pi}$ data for $\Upsilon(1S)$ (top), $\Upsilon(2S)$ (center), and $\Upsilon(3S)$ (bottom), with results of fit C. Error bars are statistical only.

The total of the above is found to be $\mathcal{P} = 0.42 \pm 0.04$. Preliminary evidence for $Z_b$ via $\Upsilon(5S) \rightarrow Z_b^\pm [\rightarrow B^*B^{(*)}\pi^\mp]$ [17] indicates that $[B^*B^{(*)}]^{\pm} \pi^\mp$ is consistent with being exclusively $Z_b^\pm \pi^\mp$, and we assume again that $[B^*B^{(*)}]^0 \pi^0$ contributes at half the rate. The total, including $[B^*B^{(*)}]\pi$, is $\mathcal{P} = 1.09 \pm 0.15$.

We have considered the following sources of systematic uncertainty: integrated luminosity, event selection efficiency, energy calibration, reconstruction efficiency, secondary branching fractions, and fitting procedure. The effects of the uncertainties in $R_b$ and $R_{\Upsilon \pi \pi}$ on $M_{5S}$, $\Gamma_{5S}$, and $\mathcal{P}$ depend on whether they are correlated or not over the data sets at different energy points. The overall uncertainty in the integrated luminosity is 1.3%, while the uncorrelated variation is 0.1%-0.2%. The overall uncertainty in $\sqrt{s}$ is 1 MeV. The uncertainty in the $R_b$ event selection efficiency, $\epsilon_{b\bar{b}}$, stems from uncertainties in the mix of event types, containing $B_d$, $B_s$, bottomonia, tau pairs, two-photon events, and $q\bar{q}$ continuum, and is estimated to be 1.1%. The systematic effects in fitting due to uncertainties in the measurements of $\sqrt{s}$, fixed parameters, and fitting range are determined by varying each source by the value of the uncertainty and refitting, noting the shifts in $M_{5S}[R_{\Upsilon \pi \pi}]$, $M_{5S}[R'_b]$, $\Gamma_{5S}$, and $\mathcal{P}$. The uncertainty on the rate of $R_{\Upsilon \pi \pi}$ for each $\Upsilon(nS)$ is dominated by those of the branching fractions, $B(\Upsilon(nS) \rightarrow \mu^+\mu^-)$ [8]: ±2%, ±10%, and ±10% for $n = 1, 2$, and 3, respectively. The uncertainties from possible non-zero $A_r$ and/or $A_{ur}$ in $R_{\Upsilon \pi \pi}$ are obtained by allowing them to float in the fit and taking the variation of the fitted values of the other parameters with respect to default results. The event-by-event efficiency correction to obtain $R_{\Upsilon \pi \pi}$ is insensitive, but not immune, to intermediate states in the three-body decay. MC studies of
To the visible cross-section measurements are made, as in Ref. [14], and the fits are repeated. The end of the fit range is varied between 10.63 and 10.82 GeV. Approximate radiative corrections with \( R_{Z} \) include both twice the nominal contribution from Table I.

As a measure of \( M_{5S} \), \( R_{b} \) has the advantages of large statistics. However, a uniform \( b\bar{b} \) continuum may be overly simplistic, as the region above and below the \( \Upsilon(5S) \) includes mass thresholds for many \( \{b\bar{b}\} \) final states, not all of which are known. A measurement with \( R_{\Upsilon\pi\pi} \) is cleaner, as it has a negligible \( b\bar{b} \) continuum contribution. Furthermore, upon inspecting rates, we reach the somewhat surprising conclusion that states including bottomonia and bottomonium-like resonances dominate the \( \Upsilon(10860) \) resonance, leaving little room for \( B_{(s)}^{(s)} \). Paradoxically, if the \( \Upsilon(10860) \) is dominated by the bottomonium and \( Z_{b} \)-related final states, which are not produced in the \( b\bar{b} \) continuum, then \( R_{b} \) would be expected to exhibit little interference between \( f_{5S} \) and continuum, yet this is clearly not the case, as can be seen in Fig. 1. It is apparent that the behavior of the \( b\bar{b} \) cross section in this region is poorly modeled by Eq.(3) [15], rendering questionable its use with \( R_{b} \) to extract masses and widths. We suggest that modes with low continuum content, such as \( \Upsilon\pi\pi \), are more reliable.

To summarize, we have measured the cross sections for \( e^{+}e^{-} \rightarrow b\bar{b} \) and \( e^{+}e^{-} \rightarrow \Upsilon(nS)\pi^{+}\pi^{-} \) \( (n = 1, 2, 3) \) in the region \( \sqrt{s} = 10.8\text{-}11.05 \text{ GeV} \) to determine masses and widths for \( \Upsilon(10860) \) and \( \Upsilon(11020) \). From \( R_{\Upsilon\pi\pi} \) \( R_{b} \), we find \( M_{10860} = (10891.1^{+3.2}_{-3.1}) \text{ MeV/c}^{2} \) \((10881.8^{+1.0}_{-1.1} \pm 1.2) \text{ MeV/c}^{2} \), \( \Gamma_{10860} = (53.7^{+7.1}_{-7.0} +0.9) \text{ MeV/c}^{2} \) \((48.5^{+1.9}_{-2.1} +2.0) \text{ MeV/c}^{2} \), \( M_{11020} = (10987.5^{+6.4}_{-6.5} -2.1) \text{ MeV/c}^{2} \) \((11003.0 \pm 1.2^{+0.9}_{-1.0}) \text{ MeV/c}^{2} \), \( \Gamma_{11020} = (61^{+9}_{-19} +2.0) \text{ MeV/c}^{2} \) \((39.3^{+7.1}_{-6.4} +2.3) \text{ MeV/c}^{2} \), and \( \phi_{11020} - \phi_{10860} = (-1.0 \pm 0.4^{+1.0}_{-0.5}) \text{ rad} \) \((-1.87^{+0.32}_{-0.51} \pm 0.16) \text{ rad} \). We find that \( R_{\Upsilon\pi\pi} \) is dominated by the two resonances, with \( b\bar{b} \) continuum consistent with zero. The resonance masses and widths obtained from \( R_{b} \) and \( R_{\Upsilon\pi\pi} \) are mutually consistent. We do not see the peaking structure at 10.9 GeV in the \( R_{b} \) distribution that was suggested by M. Ali [18] based on the BABAR measurement of \( R_{b} \) [13]. We set an upper limit on \( \Gamma_{ee} \) for the proposed structure of 9 eV with a 90% confidence level. The amplitude for the \( \Upsilon(10860) \) component of \( R_{b} \) appears to be saturated by \( \Upsilon(nS)\pi^{+}\pi^{-} \) and several other non-\( B_{(s)}^{(s)} \)-pair events. Paradoxically, this leaves little room for \( B_{(s)}^{(s)} \)-pairs and is inconsistent with the large interference between \( \Upsilon(10860) \) and \( b\bar{b} \) continuum implied from the fit to \( R_{b} \). It is probable that, exclusive of the two resonances, \( R_{b} \) is not well modeled as a flat \( b\bar{b} \) continuum. As interference with other states distorts the shape, this uncertainty adds unknown systematic errors to the resonance masses and widths. We thus suggest that \( R_{\Upsilon\pi\pi} \) is preferable for measuring masses and widths.

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