How Network Externality and Compatibility Affect the R&D Risk Choices in a Duopoly Market

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Abstract: This study develops a duopoly model describing the optimal choice of R&D risk among R&D projects with the same expected outcome in a market exhibiting network externalities. It demonstrates that, when firms’ product is incompatible, the level of R&D risk increases in the intensity of network externality. However, when product is compatible, the impact of network externality on the R&D risk choices depends on the degree of market coverage. Moreover, firms carry on higher (resp. lower) risk R&D project when their product is incompatible than compatible in a full-coverage (resp. partial-coverage) market.

Keywords: Compatibility, Hotelling model, network externality, R&D risk, uncertainty

INTRODUCTION

The uncertainty of innovation results is an important feature of technological innovation activities (Arrow, 1962). When a firm carries out a R&D project, it generally doesn’t know the final outcome. Some work has considered the optimal choice of expected R&D efforts under uncertainty (Milstein and Tishler, 2006; Cerquera, 2006; Tishler and Milstein, 2009; Xing, 2011a). Moreover, the firm has to take some risks when the outcome of R&D is uncertain. Several studies focus on the optimal choice of R&D risk (Dasgupta and Stiglitz, 1980; Choi, 1993; Cabral, 2003; Tishler, 2008; Xing, 2011b).

A market exhibits network externalities when the utility that a consumer derives from a product increases as the number of consumers of identical or compatible products increases (Katz and Shapiro, 1985; Farrel and Saloner, 1986). There exist several markets (e.g. telecommunications, consumer electronics, operating systems, etc.) that present network externalities. Katz and Shapiro (1994) suggested that “firms’ innovation incentives are altered by network considerations”. Some theoretical R&D work has taken into account network externalities (Kristiansen and Thum, 1997; Kim, 2000, 2002; Boivin and Vencatachellum, 2002; Sääskilahti, 2005; Xing et al., 2009). However, most of them focus on how the network externalities affect the optimal choice of the expenditure on R&D efforts. In this study, we investigate the effect of network externalities and product compatibility on the optimal choice of R&D risk in a duopoly market by a modified Hotelling model (Hotelling, 1929). In our framework R&D innovation is conceived in improving consumer’s reservation price.

THE MODEL

The basic setup is a variant of Hotelling duopoly model. A continuum of consumers of mass 1 is uniformly distributed (according to their preferences) over a linear market, which is denoted by [0,1]. There are two firms, denoted firm 1 and firm 2, supplying products for consumers. Each firm is located at one ultimate of the interval. Without loss of generality, we set firm 1 at 0 and firm 2 at 1. In order to improve market demand, firms perform R&D innovation activities to increase consumer’s reservation price for their product.

A consumer purchasing from firm i (i = 1, 2) obtains two parts utilities. One part is the reservation price (or say maximum intrinsic utility), denoted by \( a_i = a + \lambda_i \), where \( a \) is the reservation price before R&D, \( \lambda_i \) is the improvement of reservation price from firm i’s product innovation. The other part is network utility associated with network externalities, which is denoted by \( \alpha(d_i + kd_j) \) (i, j = 1, 2 and i ≠ j), where \( \alpha > 0 \) is the intensity of network externalities, \( \alpha \in [0,1] \) is the extent of compatibility between firms’ product and \( d_i \) and \( d_j \) are the market demands expected by consumers for firm i and firm j, respectively (Conner, 1995; Kristiansen and Thum, 1997). If a consumer located at \( x \) purchases from firm i, his/her net utility is given by:

\[
\begin{align*}
  u_i &= (a + \lambda_i + \alpha(d_i + kd_j) - p_i - tx, \quad \text{buy from 1;} \\
  u_j &= (a + \lambda_j + \alpha(d_j + kd_i) - p_j - t(1-x), \quad \text{buy from 2. (1)}
\end{align*}
\]

where, \( p_i \) is the product price of firm i, \( tx \) and \( t(1-x) \) measure the disutility caused by consuming a product not coinciding with his/her own taste and \( t > 0 \) is the differentiation parameter (let \( t = 1 \) in this study).
When the market is fully covered, all consumers derive non-negative utilities from either type of product. Assume each consumer buys exactly one unit of the good from one of the two firms. The location of consumer who is indifferent between buying from either firm is \( \bar{x} \), which is given by \( u_1 = u_2 \), i.e.,:

\[
(a + \lambda_i) + \alpha(d_i + kd_i) - p_i - \bar{x} = (a + \lambda_i) + \alpha(d_i + kd_i) - p_i - (1 - \bar{x})
\]

(2)

As in Katz and Shapiro (1985), we assume consumers’ expectations on market demand are fulfilled, i.e., \( d_i \) (i.e. \( E(d_i) \)) is the same as firm i’s actual demand. All consumers to the left (resp. right) of the marginal consumer will purchase from firm 1 (resp. firm 2). Therefore, \( d_1 \) and \( d_2 \) satisfy equations:

\[
d_1 = \int_0^x dx = \bar{x} \quad \text{and} \quad d_2 = \int_{\bar{x}}^1 dx = 1 - \bar{x}
\]

By solving (2), we obtain:

\[
\bar{x} = \frac{1 - (1 - k)\alpha + \lambda_i - \lambda_j + p_j - p_i}{2[1 - (1 - k)\alpha]}
\]

(3)

Therefore, the demand functions are:

\[
d_i = \frac{1 - (1 - k)\alpha + \lambda_i - \lambda_j + p_j - p_i}{2[1 - (1 - k)\alpha]} \quad \text{(4)}
\]

\[
d_2 = 1 - \frac{1 - (1 - k)\alpha + \lambda_i - \lambda_j + p_j - p_i}{2[1 - (1 - k)\alpha]} \quad \text{(5)}
\]

When the market is not fully covered, there exist consumers not buying from any firms. The location of consumer who is indifferent between buying from firm i and not buying is \( \bar{x}_i \), which is given by \( u_i = 0 \), i.e.,:

\[
(a + \lambda_i) + \alpha(d_i + kd_i) - p_i - \bar{x}_i = 0
\]

(6)

\[
(a + \lambda_i) + \alpha(d_i + kd_i) - p_i - (1 - \bar{x}_i) = 0
\]

(7)

According to fulfilled consumers’ expectations on market demand, we obtain:

\[
d_1 = \int_0^{\bar{x}_1} dx = \bar{x}_1 \quad \text{and} \quad d_2 = \int_{\bar{x}_1}^1 dx = 1 - \bar{x}_2
\]

\( \bar{x}_1 \) and \( \bar{x}_2 \) meet \( \bar{x}_1 < \bar{x}_2 \) because the market isn’t full coverage. Therefore, the demand functions are:

\[
d_i = \frac{(1 - \alpha)(a + \lambda_j - p_i) + \alpha k(a + \lambda_j - p_i)}{(1 - \alpha) - \alpha k^2} \quad \text{(8)}
\]

\[
d_2 = \frac{(1 - \alpha)(a + \lambda_j - p_i) + \alpha k(a + \lambda_j - p_i)}{(1 - \alpha) - \alpha k^2} \quad \text{(9)}
\]

Assume the marginal cost is equal to zero and the fixed cost is only caused by the R&D investment for both firms. Thus, the profit function for firm i is given by:

\[
\pi_i = p_i d_i - I(\mu_i, \sigma_i), \quad i = 1, 2
\]

(10)

where, \( I(\mu_i, \sigma_i) \) stands for the investment cost of firm i. We suppose the R&D outcome for firm i (i.e. \( \lambda_i \)) is uncertain when it performs innovation activities, whose probability distribution is \( \lambda_i \sim [\mu_i, \sigma_i] \), where, \( \mu_i \in [0, \infty) \) and \( \sigma_i \in [0, \infty) \) represent the expected value and variance of \( \lambda_i \), respectively (i.e. \( E(\lambda_i) = \mu_i \) and \( V(\lambda_i) = \sigma_i \)). Note that the variance of R&D outcome represents the risk of the R&D program and both firms are risk neutral in this study. Provide the R&D cost function has the following structure:

\[
I(\mu_i, \sigma_i) = f(\mu_i) + g(\sigma_i), \quad i = 1, 2
\]

(11)

We further assume that \( f' \geq 0, g' \geq 0, g'(0) = 0 \) and that \( g'' > 0 \), which guarantees that the second-order conditions for R&D are satisfied and that the unique equilibrium risk level is interior.

In this study, our setup is a two-stage game model. In the first stage, each firm chooses its R&D program (the optimal risk). In the second stage, each firm chooses the price of its product.

**EQUILIBRIUM**

As usual, we solve the equilibrium by backwards induction.

**When market is fully covered:**

**Stage 2: Each firm chooses price:** In this stage, each firm chooses product price in order to maximize its profit function, taking its rival’s product price and the outcomes of R&D program (which were completed in the first stage and are certain in the second stage) as given. The first-order conditions yield the following equilibrium prices:

\[
p_i' = \frac{3[1 - (1 - k)\alpha] + (\lambda_i - \lambda_j)}{3}
\]

(12)

\[
p_2' = \frac{3[1 - (1 - k)\alpha] - (\lambda_i - \lambda_j)}{3}
\]

(13)

Note that the second-order conditions \( \frac{\partial^2 \pi_i}{\partial p_i^2} = -1/[1 - (1 - k)\alpha] < 0 \) for all \( i = 1, 2 \) are met when \( (1 - k)\alpha < 1 \). Provide this condition is satisfied in this study.

Substituting (12) and (13) into (4) and (5), we obtain the demand functions:

\[
d_i = \frac{3[1 - (1 - k)\alpha] + (\lambda_i - \lambda_j)}{6[1 - (1 - k)\alpha]}
\]

(14)

\[
d_2 = \frac{3[1 - (1 - k)\alpha] - (\lambda_i - \lambda_j)}{6[1 - (1 - k)\alpha]}
\]

(15)

The resulting profits of the firms are:

\[
\pi_i = \frac{3[1 - (1 - k)\alpha] + (\lambda_i - \lambda_j)^2}{18[1 - (1 - k)\alpha]} - I(\mu_i, \sigma_i)
\]

(16)
\[
\pi_i = \frac{[1-(1-k)x]-(\lambda_i-\lambda_j)^2}{18(1-(1-k)x)}=I(\mu_i, \sigma_i)
\]  
(17)

**Stage 1: Each firm chooses R&D risk:** In this stage, each firm determines the risk of its R&D project taking the rival’s as given. Since the R&D efforts are uncertain in the first stage, both firms choose their optimal decisions by maximizing their expected profit functions. The expectation for (16) and (17) are respectively given by:

\[
E(\pi_i) = \frac{1}{18(1-(1-k)x)}[[3(1-(1-k)x)+(\mu_i-\mu_j)]^2
+ \sigma_i + \sigma_j - 2\text{cov}(\lambda_i, \lambda_j)]-I(\mu_i, \sigma_i)
\]  
(18)

\[
E(\pi_j) = \frac{1}{18(1-(1-k)x)}[[3(1-(1-k)x)-(\mu_i-\mu_j)]^2
+ \sigma_i + \sigma_j - 2\text{cov}(\lambda_i, \lambda_j)]-I(\mu_i, \sigma_i)
\]  
(19)

where, \(\text{cov}(\lambda_1, \lambda_2)\) represents the covariance of \(\lambda_1\) and \(\lambda_2\) and is assumed to equal a constant.

Firms need to evaluate the risk when they carry out R&D projects because uncertainty exists. As in Tishler (2008), we focus on the choices of the optimal R&D risk by comparing R&D projects with identical expected efforts (i.e., \(\mu_1 = \mu_2\)). The first-order conditions of (18) and (19) are:

\[
\frac{1}{18(1-(1-k)x)} \frac{\partial E(\pi_i)}{\partial \sigma_i} = 0
\]  
(20)

\[
\frac{1}{18(1-(1-k)x)} \frac{\partial E(\pi_j)}{\partial \sigma_{i}} = 0
\]  
(21)

Substituting (11) into (20) and (21), we have:

\[
\frac{1}{18(1-(1-k)x)} g(\sigma')_i = 0, \quad i = 1, 2
\]  
(22)

where, \(\sigma'^{E}_{i}\) is the equilibrium R&D risk level for firm i. Obviously, \(\sigma'^{E}_{1} = \sigma'^{E}_{2}\). Let \(\sigma'^{E}_{i} = \sigma'^{E}_{j} = \sigma'^{E}_{i}\), \(i = 1, 2\). We set \(\eta'^{E}_{i} = 1/[18(1-k)x]\) and can prove \(\frac{\partial \sigma'^{E}_{i}}{\partial k} > 0\) when \(k = 0\), \(\frac{\partial \sigma'^{E}_{i}}{\partial a} = 0\) when \(k = 1\) and \(\frac{\partial \sigma'^{E}_{i}}{\partial k} < 0\). Using (22), we have the following result.

**Proposition 1:**

- When \(k = 0\), \(\sigma'^{E}_{i}\) increases in \(a\); when \(k = 1\), \(\sigma'^{E}_{i}\) doesn’t depend on \(\sigma'^{E}_{i}\) when \(k = 0\) than when \(k = 1\).

The result suggests that, in a full-coverage market, when firms’ product is incompatible, they execute higher risk R&D project if the network externality is more important. However, when firms’ product is compatible, their optimal R&D risk choices don’t depend on the network externality. Moreover, the extent of product compatibility affects the optimal R&D risk choices. Further, firms will carry on higher risk R&D project when their product is incompatible than compatible.

**When market isn’t fully covered:**

**Stage 2:** Each firm chooses price: The first-order conditions yield the following equilibrium prices:

\[
p^e_1 = \frac{[2(1-a)^{-\alpha} - \alpha^{-\alpha}k^2] \lambda_i + (1-a)\lambda k \mu_i + \zeta}{4(1-a)^{-\alpha} - \alpha^{-\alpha}k^2} + \zeta
\]  
(23)

\[
p^e_2 = \frac{[2(1-a)^{-\alpha} - \alpha^{-\alpha}k^2] \lambda_j + (1-a)\lambda k \mu_j + \zeta}{4(1-a)^{-\alpha} - \alpha^{-\alpha}k^2} + \zeta
\]  
(24)

where, \(\zeta = [2(1-a)-\alpha k][1-(1-a)+\alpha k]a\). Note that the second-order conditions meet when \((1-a)^{-\alpha} - \alpha^{-\alpha}k^2 > 0\). Provided this condition is satisfied in this study.

Substituting (23) and (24) into (8) and (9), we obtain:

\[
d_1 = \frac{[2(1-a)^{-\alpha} - \alpha^{-\alpha}k^2] \lambda_i + (1-a)\lambda k \mu_i + \zeta}{[1-(1-a)^{-\alpha} - \alpha^{-\alpha}k^2][4(1-a)^{-\alpha} - \alpha^{-\alpha}k^2]} + \sigma
\]  
(25)

\[
d_2 = \frac{[2(1-a)^{-\alpha} - \alpha^{-\alpha}k^2] \lambda_j + (1-a)\lambda k \mu_j + \zeta}{[1-(1-a)^{-\alpha} - \alpha^{-\alpha}k^2][4(1-a)^{-\alpha} - \alpha^{-\alpha}k^2]} + \sigma
\]  
(26)

where, \(\sigma = \frac{(1-a)\alpha}{(1-a)\alpha + 2(1-a)\alpha k}\). The resulting profit functions are:

\[
\pi_i = \left[\frac{[2(1-a)^{-\alpha} - \alpha^{-\alpha}k^2] \lambda_i + (1-a)\lambda k \mu_i + \zeta}{4(1-a)^{-\alpha} - \alpha^{-\alpha}k^2}\right] \times \left[\frac{[2(1-a)^{-\alpha} - \alpha^{-\alpha}k^2] \lambda_i + (1-a)\lambda k \mu_i + \zeta}{[1-(1-a)^{-\alpha} - \alpha^{-\alpha}k^2][4(1-a)^{-\alpha} - \alpha^{-\alpha}k^2]} + \sigma\right] - I(\mu_i, \sigma_i)
\]  
(27)

\[
\pi_j = \left[\frac{[2(1-a)^{-\alpha} - \alpha^{-\alpha}k^2] \lambda_j + (1-a)\lambda k \mu_j + \zeta}{4(1-a)^{-\alpha} - \alpha^{-\alpha}k^2}\right] \times \left[\frac{[2(1-a)^{-\alpha} - \alpha^{-\alpha}k^2] \lambda_j + (1-a)\lambda k \mu_j + \zeta}{[1-(1-a)^{-\alpha} - \alpha^{-\alpha}k^2][4(1-a)^{-\alpha} - \alpha^{-\alpha}k^2]} + \sigma\right] - I(\mu_j, \sigma_j)
\]  
(28)

**Stage 1:** Each firm chooses R&D risk: The expectation for (27) and (28) are given by:

\[
E(\pi_i) = \left[\frac{[2(1-a)^{-\alpha} - \alpha^{-\alpha}k^2] \lambda_i + (1-a)\lambda k \mu_i + \zeta}{4(1-a)^{-\alpha} - \alpha^{-\alpha}k^2}\right] \times \left[\frac{[2(1-a)^{-\alpha} - \alpha^{-\alpha}k^2] \lambda_i + (1-a)\lambda k \mu_i + \zeta}{[1-(1-a)^{-\alpha} - \alpha^{-\alpha}k^2][4(1-a)^{-\alpha} - \alpha^{-\alpha}k^2]} + \sigma\right] - I(\mu_i, \sigma_i)
\]  
(29)
This study investigates the impact of network externality and compatibility on the optimal choices of R&D risk in a duopoly market. Firms carry out R&D programs with identical expected outcome. We find that, when firms’ product is incompatible, the optimal level of R&D risk increases in the intensity of network externality. However, when their product is compatible, how network externality affects the optimal R&D risk choices depends on the degree of market coverage: if the market is fully covered, the optimal level of R&D risk doesn’t depend on network externality; while if the market is partially covered, it increases in the intensity of network externality. Moreover, firms will carry on higher (resp. lower) risk R&D project when their product is incompatible than compatible in a full-coverage (resp. partial-coverage) market. That is how compatibility affects the optimal R&D risk choices depends on the degree of market coverage. Finally, when firms’ product is incompatible or compatible, they both execute higher risk R&D project if the market is partially covered than fully covered.

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