MULTIPLE PARTON INTERACTIONS IN HIGH-DENSITY QCD MATTER

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Abstract

Multiple interactions of quarks and gluons in high-energy heavy-ion collisions may give rise to interesting phenomena of color charges propagating in high-density QCD matter. We study the dynamics of multi-parton systems produced in nucleus-nucleus collisions at energies corresponding to the CERN SPS and the future BNL RHIC experiments. Due to the complexity of the multi-particle dynamics we choose to employ the parton cascade model in order to simulate the development of multiple parton scatterings and associated stimulated emission processes. Our results indicate a non-linear increase with nuclear mass $A$ of, e.g., parton multiplicity, energy density, strangeness, and contrasts a linear $A$-scaling as in Glauber-type approaches. If multiple interactions are suppressed and only single parton scatterings (no re-interactions) are considered, we recover such a linear behavior. It remains to be studied whether these results on the parton level can be experimentally seen in final-state observables, such as the charged particle multiplicity, the magnitude of produced transverse energy, or the number of produced strange hadrons.

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I. INTRODUCTION

The propagation of high-energy particles through matter is generally subject to enhanced interaction rates as compared to free space, and hence imply a modification of the propagators and vertices. It is therefore of fundamental interest for studying quantum field theory in a finite-density or finite-temperature environment. An enhanced probability for multiple particle interactions can of course only arise if the distribution of potential interaction partners in the system is sufficiently dense. Vice versa, the manifestation of multiple interaction effects in observable quantities should reflect the structure of dense matter environment. Hence, multiple interactions expose the dual relation between the matter as a macroscopic medium, and the particles within it and generating its dynamics on the microscopic level:

a) If systematically studied, multiple interactions may be used as a "densometer", i.e. a probe of the particle density of the medium.

a) Measuring characteristics of particles traversing dense matter by encountering multiple interactions, can be used to learn about the influence of the medium on the particle dynamics.

Multiple scattering phenomena in many-body systems has been widely studied in a variety of applications [1], for example, electron showers in cosmic rays, electron scattering in crystals or emulsion, or hadrons propagating through nuclear matter in collisions involving heavy nuclei. In many applications, however, it has been treated as sequential re-interactions of particles by means of elastic scatterings off the targets in a medium. In this case the particles essentially undergo a simple random walk in phase-space that causes both momentum and spatial broadening of the particle spectrum. The situation is very different, if one allows for inelastic interactions, in which case the particles not only get deflected, but also transfer or absorb energy-momentum to or from the medium, and moreover can multiplicite themselves through excitation and decay that is stimulated by the energy transfer.

In this paper we are concerned specifically with multiple interactions of quarks and gluons in a high-density QCD matter environment that is created in ultra-relativistic heavy-ion collisions. In these collisions short range parton interactions can play a major role for the nuclear dynamics at least during the early and most dissipative stage of the first few fm/c [2]. A copious production of quarks and gluons, liberated from the mother nuclei through (semi)hard parton-parton scatterings, is expected to initiate a complex evolution of intertwined parton cascades that consist of multiple parton (re)scatterings plus associated emission and absorption of gluons.

As we are dealing with quantum field theory, here QCD, a classical scattering picture is certainly inappropriate. This is because the mere existence of particle fluctuations, being created and annihilated, requires to account for scattering processes (elastic and inelastic) as well as emission/absorption processes (virtual and real). In particular, the interplay between collisions and stimulated emissions is non-trivially entwined, since an increased number of scatterings can initiate an enhanced particle multiplication through emissions (decays), which in turn increases the density of particles and hence potential scattering targets for further collisions, and that may stimulate further emissions. Clearly, this avalanche effect cannot grow unbridled, since unitarity requires the cross-section to be finite (the medium cannot be blacker than black), and moreover screening effects will become the more important, the larger the density. For these reasons, a ‘classical’ cascade picture is clearly not applicable. Fig. 1 illustrates schematically the differences between ‘classical’ cascades and ‘quantum’ cascades.

It is evident that an exact treatment of multiple QCD interactions in a high-density environment is a problem of (at present) intractable complexity, both on the perturbative and the non-perturbative level. It requires a detailed understanding of, e.g., the non-abelian interference of interaction amplitudes, the propagation and screening of color charges due to these interactions, the infra-red regime of very low energy/momentum transfer, the confinement dynamics and hadron formation in dense matter. Even though recent remarkable progress within the isolated issue of parton propagation in finite-density matter has been made on the basis of QCD’s first principles [3–5], the results, being based on highly idealized assumptions, are at present not useful for experimental study and verification. Therefore, in order to take pragmatical steps, we need to rely on a model description that serves as a baby-version of the (impossible) exact QCD treatment.
Specifically, we employ here the Parton Cascade Model (PCM) [6], to quantify the above intuitive picture of multiple interaction physics in a phenomenological manner that makes contact with the experiments at the CERN SPS and the BNL RHIC. The PCM assumes that the early stage of nuclear collisions at sufficiently high energies can well be described in terms of the space-time evolution of interwoven parton cascades [2], based on renormalization-group improved perturbative QCD in conjunction with quantum kinetic theory [7], whereas the later non-perturbative development of the transformation of partons to final-state hadronic states must be treated within a phenomenological hadronization scheme. Hence, within this framework, the early stage of parton evolution rests on the robust principles of perturbative QCD, whereas the modeling of the later development of parton-hadron conversion has no such firm theoretical basis, and moreover is not unique in view of lack of knowledge of the underlying confinement mechanism. Therefore we focus our analysis on the partonic sector, which is free of the ambiguities of a particular hadronization model. However, we plan to investigate to which degree multiple parton interactions may reflect themselves in final-state hadron yields. This latter aspect requires a parton-hadron conversion model, for instance the hadronization scheme of Ref. [8] in which partons coalesce to colorless clusters as controlled by the local density of color charges and the subsequent decay of these prehadronic clusters into hadron states. In the PCM, the colliding nuclei are viewed as two coherent clouds of valence quarks plus virtual gluon- and seaquark-fluctuations that materialize into “real” excitations due to primary parton-parton scatterings. For large nuclei, this primary parton production can result in a large initial particle and energy density in the central collision region, which increases further by subsequent intense gluon bremsstrahlung, secondary scatterings and rescatterings. Detailed balance is assured by including absorption or fusion of partons when the local density becomes so large that the quanta begin to overlap in phase-space, so that the unitarity principle is obeyed.

In the remainder of the paper, we report and discuss results obtained with the Monte Carlo program VNI [9] which allows one to simulate heavy-ion collisions on the basis of the aforementioned PCM concepts. Within this model framework, we address effects of multiple parton scattering and associated multi-gluon radiation during nucleus-nucleus collisions at the highest beam energies, corresponding to $\sqrt{s} \simeq 20$ A GeV at the CERN SPS accelerator and $\sqrt{s} = 200$ A GeV at the BNL RHIC setup. From a phenomenological point of view it is important to investigate whether the final-state particles carry some information about their partonic interaction history into the detectors, and whether it is possible to extract observable manifestations of multiple parton interactions in the final-state spectra. Although we do not address these connections to the final hadronic yields and spectra (which we postpone to a sequel to this work) here, we think that the partonic dynamics serves as an estimate of the type and magnitude of observable effects.
Specifically, we focus on three selected topics, which, we believe, are of foremost interest:

- **Development of initial conditions**: What is the characteristic development of the initial partonic system during the first fraction of a fm/c? How fast does the materialization of partons through collisions and emissions occur? How large is the resulting particle and energy density of the partonic system that determines the initial conditions for the subsequent expansion, hadronization and freeze-out?

- **Multiple scattering as response to dense matter**: What are the significances and potential experimentally measurable implications of multiple parton interactions as a response to the high-density matter? What is the growth pattern of the total partonic entropy and transverse energy when the size of the nuclei is increased? Do multiple scatterings result in a visible non-linear growth with nuclear mass number $A$, in contrast to a linear scaling as in Glauber models?

- **Interplay between multiple scatterings and parton emissions**: How large is the effect (e.g., on rapidity density and transverse energy) of multiple scatterings compared to single scatterings of partons? What is the relative importance of multiple parton collisions and associated parton emissions? How different are the parton spectra with/without multiple interactions and with/without stimulated emissions?

## II. MULTIPLE PARTON INTERACTIONS DURING THE EARLY STAGE OF HEAVY-ION COLLISIONS AT CERN SPS AND RHIC ENERGIES

Since the notion of multiple interactions is *defined* as successions of similar processes that change the direction of momentum (and possibly the invariant mass) and which are statistically independent (at least approximately), it is clear that truly quantum-mechanical interference between the interaction amplitudes cannot be accounted for — except when it is possible to describe them effectively in a probabilistic manner. Moreover, in view of lack of knowledge of the details of new phenomena associated with “dense-matter QCD”, we do not attempt to model unknown medium effects, but rather restrict ourselves to those effects that can arise from statistical many-body kinetics. The main assumptions and approximations of the probabilistic parton cascade picture that we employ, are:

(i) **Factorization of short distance interactions of cascading partons with the nuclear medium from long range non-perturbative forces** is assumed. This means, even in the presence of dense medium where a parton can encounter multiple successive interactions, a probabilistic description of local, non-interfering interactions applies at sufficiently high energies.

(ii) **Non-trivial interference effects are generally neglected**, except: destructive interference of interaction amplitudes in both, coherent successive small angle emissions (‘angular ordered emission’), and multiple sequential scatterings (‘Landau-Pomerantchuk-Migdal effect’).

(iii) **Nuclear effects due to long range correlations**, e.g. nuclear shadowing (anti-shadowing) which are not associated with truly perturbative parton interactions are ignored. Similarly, possible mean field effects arising from a collective motion of the particle system are beyond our concern.

In the following we first study the evolution of parton distributions in central collision of two gold nuclei, at $\sqrt{s} = 20$ A GeV and 200 A GeV. We first address the early stage of the parton cascade development. Then we turn to our results for the energy densities achieved in collisions involving several identical nuclei across the periodic table. Finally we discuss characteristic effects of multiple parton interactions as a response to the high-density matter.

In our analysis of parton spectra and density profiles, we have considered - aside from the initially present valence quarks - only *materialized* quarks and gluons that have suffered *at least one interaction*, i.e., any parton that was either knocked out from the nuclear wavefunction by a collision with another initial-state parton (primary scattering) or with an already interacted parton (rescattering), or being created through a pair production, or emitted by a gluon bremsstrahlung process. Moreover, we have considered only (semi)hard parton-parton collisions with momentum transfer $q^2 > p_0^2$, where $p_0 = 1.1$ GeV/c for $\sqrt{s} = 20$ A GeV and $p_0 = 2.1$ GeV/c for $\sqrt{s} = 200$ A GeV. These values for $p_0$ correspond to the default values of VNI, which were found to agree well with measured particle spectra from $pp$ collisions at ISR to Tevatron energies, as well as from $AB$ collisions at CERN SPS energy. In principle one could extend to include parton scatterings with $q^2 < p_0^2$, as discussed in Refs., by allowing for the whole range $0 \leq q^2 \leq (p_1 + p_2)^2$. In this case one would divide (semi)hard ($q^2 \geq p_0^2$) and soft ($q^2 < p_0^2$) parton collisions and
matching them at $p_0^2$, the former treated perturbatively and the latter described by a reasonable phenomenological ansatz. However, in order to avoid the ambiguities and model-dependence associated with non-perturbative soft scatterings, we choose to restrict ourselves to include exclusively (semi)hard scatterings for which we have a confident, perturbative description available.

A. Development of initial conditions

We start by comparing the parton dynamics at $\sqrt{s} = 20$ A GeV and $\sqrt{s} = 200$ A GeV during the first 0.5 $fm/c$ after nuclear overlap, the stage that determines the initial conditions for the subsequent expansion. Figs. 2 - 5 sum up our findings regarding the early time evolution of the parton distributions, starting at $t = 0$ when the nuclei overlap.

![Graph](image)

**FIG. 2.** Distribution of materialized partons per unit length in the centre of mass frame of the colliding nuclei at different times along the longitudinal $z$-axis at 20 A GeV (upper panel) and 200 A GeV (lower panel). The zero of time is taken as the instant of complete overlap.

In Fig. 2a we plot the number distribution of partons along the collision axis at several times $0 < t \leq 0.5$ $fm/c$ (with respect to the laboratory frame) for $\sqrt{s} = 20$ A GeV, while the corresponding results for 200 A GeV are given in Fig. 2b. We see that at 20 A GeV the number density of partons continues to increase and peaks at $z = 0$ up to a lapse of $t = 0.5$ $fm/c$. Among other things this is indicative of the fact that until then the crossing over of the Lorentz contracted ($\gamma \approx 10$) nuclei is not complete. On the other hand the number of partons per unit length reaches
its maximum around $t = 0.3 \, fm/c$ at RHIC energy of 200 A GeV. By this time the Lorentz contracted ($\gamma \approx 100$) nuclei have crossed each other and in fact the bulk of the matter is piled away from $z = 0$ on either side.

Figs. 3a and 3b further support this view through the shape of the $p_T$ distribution of the partons at different times. We see that the number of partons having a given $p_T$ increases rapidly with time at 20 A GeV while at 200 A GeV it reaches a saturation value by the $t = 0.3 \, fm/c$. We also see a noticeable bulge in their number around $p_T \approx p_0$ for the two cases, where $p_0$ is equal to 1.1 and 2.1 GeV/$c$, respectively.

It is quite instructive to see the time evolution of the number and the energy density profile of the partons, shown for the two cases in Figs. 4a and 4b for $z = 0$. They attain their maximum values around the common time of $\tau = 0.3 \, fm/c$ for both energies. We also find that the number and energy density of the partons can get rather large and at RHIC energies it can get up to five times as large as at SPS energies, around the same point of time.

Finally in Figs. 5a and 5b we give our predictions for the energy density profiles at $z = 0$ and $\tau = 0.3 \, fm/c$ for the two energies, for central collisions of several identical nuclei. One interesting aspect of these results is that even in collisions involving sulfur nuclei at SPS energies, an energy density $\varepsilon$ above a critical value $\varepsilon_{crit} \approx 2 \text{ GeV/fm}^3$ may be attained for a short time. Of course, as expected, we get larger energy densities extending up to larger radii as we collide heavier nuclei.

FIG. 3. The transverse momentum distribution of materialized partons of the colliding nuclei at different times for 20 A GeV (upper panel) and 200 A GeV (lower panel). Only hard scatterings along with radiations from the partons have been included in the development of the cascade.
FIG. 4. The time evolution of the number density (left half) and the energy density (right half) of the materialized partons in the central region ($|z| \leq 0.5$ fm) of the colliding nuclei at 20 A GeV (upper panel) and 200 A GeV (lower panel). Only hard scatterings along with radiations from the partons have been included in the development of the cascade.
B. Multiple scattering as response to dense matter

Does the creation of a partonic matter which may attain a rather high energy density also imply enhanced multiple scatterings? Not necessarily, as a consideration of free-streaming matter readily shows. It is quite clear that multiple scatterings distinguish a freely streaming matter from a matter which expands and cools. Thus multiple scatterings are essential to obtain and maintain a thermal equilibrium, and to associate a temperature to the system under consideration. They also facilitate chemical equilibration.

What are the measures of multiple scatterings? Clearly, if multiple scatterings succeed in maintaining a thermal equilibrium in an expanding system, then we shall witness a decreasing temperature with the passage of time. Moreover, multiple scatterings should increase the total entropy production, which would result from an enhanced production of particles. As we have elucidated in the previous subsection, the multiplicity of produced partons reaches a maximum within a time interval of 0.5 fm, subsequent to which rescatterings may drive the system towards equilibration. Therefore one would expect that multiple parton scatterings manifest themselves in an enhanced production of gluons, light quarks, and even strange quarks, when compared to a dynamics with only single, primary parton scatterings as in hadronic collisions. In the latter case the multiplicity would scale linearly with the mass number $A$, whereas in the case of multiple scatterings, one should see a non-linear increase of the multiplicity. In effect, final-state particles that emerge from this early stage, such as photons, dileptons and strange hadrons, should exhibit...
such a non-linear behaviour as one increases the size of the nuclei by going to heavier collision systems. An indication of the emergence of this aspect was noted in our recent work [11] on the production of single photons at SPS and RHIC energies.

Figs. 6 and 7 exhibit this non-linear behavior due to multiple parton scatterings in \( A + A \) collisions at 20 A GeV and 200 A GeV, respectively, in comparison to the same collisions accounting only for first scatterings of partons. In both Figures, the top part shows the increase with \( A + A \) of the total number of produced gluons, light quarks and strange quarks, while the bottom part shows the partonic rapidity density \( dN^{q+g}/dy|y=0 \), the transverse energy density \( dE_{\perp}^{q+g}/dy|y=0 \), and the accumulated number \( N_{\text{hard coll}} \) of (semi)hard parton-parton collisions. The symbols are the results of our simulations for \( A + A \) from Helium all the way up to Uranium, whereas the lines serve to guide the eye.

![Graph showing materialized partons at \( \tau = 3 \) fm/c](image1)

![Graph showing effect of multiple scatterings](image2)

**FIG. 6.** Effect of multiple scatterings on the variation of number of materialized partons at \( \tau = 3 \) fm/c as a function of number of participating nucleons in central collision of identical nuclei at \( \sqrt{s} = 20 \) A GeV (uppermost panel). The three lower panels show the effect of multiple scatterings on the total number of hard collisions, and the rapidity density of partons and their transverse energy.

The common feature of all the plots is the non-linear departure of the multiple scattering picture (solid lines) from a linear scaling with \( A \) in the case of only primary scatterings (dashed lines), which sets in for systems \( A + A \) above...
220, corresponding to collisions of Silver nuclei. Also remarkable is that the qualitative behavior of this non-linearity is the same for $\sqrt{s} = 20$ A GeV (Fig. 6) and for $\sqrt{s} = 200$ A GeV (Fig. 7), however, the absolute magnitude of the various quantities shown is of course drastically different due to the factor 10 difference in the total energy. For example, the multiplicities of materialized gluons, light and strange quarks at 200 A GeV are about a factor of six larger for the heaviest system, than the corresponding numbers for 20 A GeV. Most remarkable, however, is the substantial difference between the multiple and the single scattering scenarios: at $\sqrt{s} = 20$ A GeV, the multiple scatterings enhance the materialized parton numbers up to 25-30 %, whereas at $\sqrt{s} = 200$ A GeV, the amplification can reach up to 40 %.

FIG. 7. Effect of multiple scatterings on the variation of number of materialized partons at $\tau = 3$ fm/c as a function of number of participating nucleons in central collision of identical nuclei at $\sqrt{s} = 200$ A GeV (uppermost panel). The three lower panels show the effect of multiple scatterings on the total number of hard collisions, and the rapidity density of partons and their transverse energy.
C. Interplay between multiple scatterings and parton emissions

So far we have discussed the role of multiple scatterings alone, and have not addressed the relevance of gluon and quark-antiquark production through multiple emission processes. As we have mentioned in the Introduction, one cannot really disentangle the parton emission processes from the scatterings, because the latter are the source of the former, but the former in effect influence the latter: Each (semi)hard parton-parton scattering is a potential trigger for a bremsstrahlung cascade of emissions, if the scattering was sufficiently inelastic. Therefore the emission of partons stimulated by scatterings grows naturally when the number of parton collisions increases. On the other hand, an enhanced emission of quanta feeds back on the dynamics by increasing the local parton density and hence the rescattering probability as well as the probability for further emissions.

What limits this avalanche effect and prevents a parton population catastrophe? There are three mechanisms: The first one is the multi-particle dynamics in heavy-ion collisions itself, which eventually causes the system to expand and diffuse rapidly, and hence the interactions cease. The second is due to energy conservation, which limits the maximum amount of energy that can be harnessed in the particle interactions, and results in lower-energetic interactions with decreasingly smaller cross-sections as time goes on. The third mechanism is the Landau-Pomeranchuk-Migdal effect [12,13], which is mimicked in the PCM by assigning each parton that emerges from a collision or an emission, a specific formation time \( \tau = E/q_{\perp}^2 \) during which the parton cannot interact. As a consequence, the production and the re-interaction of particles is the more delayed the smaller the momentum transfer \( q_{\perp} \) of the interaction is, which causes a limitation on the parton density increase, because it is mainly due to the production of low-energy gluons with the largest re-interaction cross-sections. The elongated formation time for these soft gluons makes time for the dense region to expand and dilute.

It is clear that the interplay between collisions and stimulated emission is a complex net of interactions which is difficult to disentangle. Nevertheless, by switching on/off multiple scattering and QCD radiation, one may get at least a qualitative impression of the different parton cascade aspects. In Fig. 8 we plot the results for three different scenarios: The first scenario (dashed-dotted lines) includes solely elastic scatterings with all parton emission switched off, and moreover considers only single collisions of initial-state partons, i.e., the first scatterings of two primary partons with no further collision of those. The second scenario (dashed lines) includes now inelastic collisions with radiative emission of partons, but still allows only for single collisions of initial-state partons. Finally, the third option (solid lines) is the full parton-cascade development with multiple parton scatterings plus associated parton decays.

Fig. 8a (top) shows for \( Au + Au \) collisions at \( \sqrt{s} = 20 \) A GeV the rapidity spectra \( dN/dy \) of materialized quarks and gluons (left panel) and the corresponding transverse energy distributions \( dE_{\perp}/dy \) at \( t = 3 \) fm/c. Fig. 8b (bottom) displays the same quantities for \( \sqrt{s} = 200 \) A GeV at \( t = 3 \) fm/c.

The qualitative features of the \( dN/dy \) spectra, common to both energies, are a substantial contribution to particle production from radiative emissions. For example, even if only primary collisions are considered, the associated parton emission increases \( dN/dy \) at \( y = 0 \) by about a factor of 1.5 at 20 A GeV and by more than a factor of 2 at 200 A GeV, as compared to primary collisions with radiative emission switched off. Allowing then also for multiple scatterings, one observes a further particle enhancement at mid-rapidity, however much smaller than the difference due to parton emission. These results are immediately plausible, because primary scatterings involve, on the average, the most energetic partons with their full initial energy, and therefore most primary scatterings are inelastic, accompanied by the emission of at least one gluon. On the other hand, secondary scatterings involve mainly partons that have lost already energy in previous interactions, so that the majority of them is of elastic nature without gluon emission. Therefore primary collisions provide a substantial parton production from emission processes, whereas secondary rescatterings do not drastically change the multiplicity further, but mainly redistribute energy and momentum among the particles. Turning to the \( dE_{\perp}/dy \) spectra, the growth of the transverse energy at \( y = 0 \) from primary collisions without and with radiation to multiple scatterings with radiation, follows the rapidity distributions just discussed. However, the differences between the three curves are much less pronounced. This is due to the fact that a large part of the multiplicity arises from low-energy radiative gluons, which leave a prominent mark in the multiplicity spectra, but add little to the total transverse energy.
FIG. 8. Consequences of radiations and multiple scatterings on rapidity density of number of partons (left half) and their transverse energy (right half) at $\sqrt{s} = 20$ A GeV (upper panel) and 200 A GeV (lower panel). Only hard scatterings are included and the parton cascade is evolved till $t = 3$ fm/c.

In Fig. 9, the $p_T$ distribution the partons in the central rapidity region is seen to also reflect many of these developments in an interesting manner. It is evident that the simulations with only primary - primary collisions lead to a distribution of partons with a noticeable peak around $p_T \approx p_0$ (where $p_0 = 1.1$ (2.1) GeV/c for $\sqrt{s} = 20$ (200) A GeV). The radiations of gluons then reallocate the high $p_T$ partons to low $p_T$ region, and their number also goes up considerably. The multiple scatterings, however, affect the $p_T$ spectra differently at 20 A GeV and 200 A GeV.

The most interesting aspect is that the multiple scatterings enhance the number of partons having large $p_T$ at 20
A GeV, while they decrease their number at 200 A GeV. The origin lies in the fact that once the scatterings lead to partons which do not have large virtualities, they would not radiate, and thus the scatterings will lead to increase in $p_T$. This is apparently the case at 20 A GeV, whereas at 200 A GeV the virtualities of the scattered partons could still be large and lead to radiation of gluons having low $p_T$, and thus the multiple scatterings do not enhance the $p_T$.

FIG. 9. Interplay of radiations and multiple scatterings on the transverse momentum distribution of quarks and gluons in the central rapidity region at $\tau = 3 \text{ fm}/c$ at $\sqrt{s} = 20$ A GeV (left panels) and 200 A GeV (right panels). Only hard scatterings leading to $p_T > p_T^0$, where $p_T^0 \approx 1.1$ GeV at 20 A GeV and $\approx 2.1$ GeV at 200 A GeV, are included.

**III. SUMMARY**

Within the PCM framework and using the computer simulation program VNI, we have studied the development of dense partonic systems in central collisions of several nuclei at 20 A GeV and 200 A GeV, in order to bring out the role of quark-gluon cascades due to multiple scatterings and radiative emission of partons during the early stage of these collisions. Since we restricted our analysis to the perturbative QCD sector and avoided confronting model-dependencies associated with a hadronization prescription, we are confident that our results draw a fairly realistic picture of the short-range QCD dynamics at high density. Our main findings may be summarized as follows:

(i) The initial materialization of partons through scatterings or radiative emission reaches its maximum around 0.5 fm/c (at 20 A GeV) and 0.3 fm/c (at 200 A GeV). After that lapse, the liberation of initial-state partons damps out strongly, however, radiative parton emissions and rescatterings between these produced partons continue.

(ii) The total number of produced partons around $z = 0$ achieves its maximum of about 600 for 20 A GeV and close to 4000 at 200 A GeV for central collision of gold nuclei. This increase of a factor 6.5 reflects itself in the corresponding number densities which increase by a factor of 5 from 20 A GeV to 200 A GeV, and an even
stronger increase in the energy density at $z = 0$ from $\sim 8$ GeV/fm$^3$ to to $\sim 50$ GeV/fm$^3$.

(iii) Multiple scatterings along with the associated parton emission lead to a non-linear increase with nuclear mass number $A$ of parton multiplicity and number of parton collisions, as compared to single, primary scatterings only, in which case the growth pattern scales linearly with $A$.

(iv) The emission of partons in radiative processes associated parton collisions contributes substantially to the multiplicity, making out up to 30 % at 20 A GeV and about 50 % at 200 A GeV. The radiative emissions are mainly due to inelastic, primary scatterings of initial-state partons, whereas secondary scatterings are to large extent of elastic nature without additional emission, but do redistribute energy and momentum among the particles.

What should be done in the near future, is to establish a connection between the above partonic properties and the final-state hadronic and electromagnetic particles. With a suitable parton-hadron conversion model, one must attempt to identify the high-density aspects of parton cascades in measurable signatures that one can extract from data. For instance:

- Does the the final-state transverse energy spectrum $dE_\perp/dy$ reflect the non-linear behaviour of the corresponding partonic quantity? How does, e.g., the ratio $dE_\perp/(200 A GeV)/dE_\perp/(20 A GeV)$ scale? How does the event distribution distribution $dN_{\text{event}}/dE_\perp$ of the total transverse energy per event (which has the famous ‘horseback’ shape) stretch out to larger $E_\perp$ as the collision system or the collision energy is increased?

- Can one actually ‘see’ some remnants of minijets and parton cascades in final-state observables? E.g., how does the enhancement of the angular correlation (between pairs of partons) at small angles due to evolving cascades show up in the hadron pair-correlations? In the absence of minijets, one would expect a flat correlator when plotted versus the relative angle, while for prominent and energetic cascading one would see a pronounced peak at very small angles.

This incomplete list gives examples of experimental quantities, which are presumably difficult to trace back to the partonic history due to insufficient statistics, but that can already be analysed at the end of day one at BNL RHIC when it starts operation.

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Note that due to the different Lorentz contraction and the different times (0.5 $fm/c$ and 0.3 $fm/c$) at which the maximum parton multiplicity is achieved, the associated volume of the collision system is not the same in the two cases.
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