Modeling and experiment of double-feet piezoelectric linear motor stator system

Xiangqiang Zhong \textsuperscript{1,2,3,4}, Zhimin Di\textsuperscript{1}, Shunyu Yao\textsuperscript{1}, Huajie Fang\textsuperscript{1} and Jiang Feng\textsuperscript{1}

\textsuperscript{1} School of Mechanical & Automotive Engineering, Anhui Polytechnic University, Wuhu, China;
\textsuperscript{2} Additive Manufacturing Institute, Anhui Polytechnic University, Wuhu, China;
\textsuperscript{3} State Key Laboratory of Mechanics and Control of Mechanical Structures, Nanjing University of Aeronautics and Astronautics, Nanjing, China
\textsuperscript{4} Email: xqzh@nuaa.edu.cn

Abstract. In order to study the performance of double-feet piezoelectric linear motor, the dynamic model of the stator system was established, and the prototype of double-feet piezoelectric linear motor was made. Firstly, the stator structure of double-feet piezoelectric linear motor, the two-degree-of-freedom transverse vibration system of the stator system and the experimental device of the motor velocity test were established respectively. Secondly, based on the orthogonal experiment, the speed and performance of double-feet piezoelectric linear motor were analysed with range and variance. The research result shows that the amplitude displacement of the piezoelectric motor-driven foot is about 61μm. The performance of the piezoelectric linear motor can meet the driving requirement of the motor's mover. The orthogonal experimental analysis shows that both the voltage and frequency of double-feet piezoelectric linear motor have an effect on the velocity, and the frequency has a more significant effect on the motor velocity. It is verified that the piezoelectric motor has the characteristics of large drive displacement, fast reaction rate and stable operation.

1. Introduction

The piezoelectric linear motor has the advantages of simple structure, light movement, stable operation and high accuracy. Compared with the traditional motor, the motor can work safely and effectively in the environment of low temperature, high temperature, electromagnetic interference, vacuum, and so on [1-4]. However, the traditional motor is driven by Electrothermal, Electromagnetic or mechanical mode, which cannot meet the existing working requirements due to the working principle and other reasons. Double-feet piezoelectric linear motor uses lever amplifying double-feet stator as driving element, which can drive the worktable to work and locate it accurately [5-7].

The discovery of BaTiO\textsubscript{3} in 1940s enabled the rapid development of piezoelectric actuation technology. Xu Jingjing et al designed a single-foot driving piezoelectric linear motor, and verified the correctness of the motor trajectory by experiments, but did not explore the compound magnification rate of the mechanism [8]. Chen Xifu et al carried on the experimental research on a kind of dynamic friction piezoelectric stack single-foot linear motor stator, deduced its differential equation of driving end, the experiment verified resonant characteristics of such motors, but didn't analyzed its velocity [9-11]. This paper aims at the modeling of double-feet piezoelectric linear motor stator system, and the amplitude and velocity of the driving feet are experimentally studied to verify the working performance of the motor.
In this paper, the dynamic modeling of double-feet piezoelectric linear motor stator system was completed, based on the orthogonal test, the velocity performance of the stator system was analyzed by the method of range and variance. The experimental data show that voltage and voltage frequency have a very significant effect on the velocity of piezoelectric motor, and the influence of frequency is more significant.

2. Dynamic modeling of double-feet piezoelectric linear motor stator system

2.1. Structure model and principle of double-feet piezoelectric linear motor stator

Double-feet piezoelectric linear motor stator is composed of piezoelectric stack, pretightening mechanism, pretightening force adjusting mechanism, flexure hinge, driving feet (two level lever), and so on [12-14]. The specific structure is shown in Figure 1.

![Figure 1. The stator structure of double-feet piezoelectric linear motor.](image)

Two sinusoidal voltages with phase difference of \( \pi/4 \) are used to excite four piezoelectric stacks respectively. At the same time, the stator system of the motor vibrates and forms a trajectory at the driving end. The pretightening force adjusting mechanism makes the driving foot contact with the mover, and produces the positive pressure needed for friction between the stator and the mover of the motor in order to drive the mover movement.

2.2. Vibration characteristics of double-feet piezoelectric linear motor stator system

Because the output displacement of the piezoelectric stack is relatively small, the damping of the system will be very small, it has almost no effect on the longitudinal amplitude (y direction in Figure 1), and the influence of the damping characteristics of the system is negligible. The transverse vibration of the motor driving foot is mainly considered in this paper (x direction in Figure 1) [8]. Straight circular flexure hinges and straight beam flexure hinges can be regarded as pure elastic elements because they conform to the basic assumptions of small deformation theory. According to the structure and working principle of the motor, due to symmetry, the left side of Figure 1 can be taken as the analysis object and simplify it to two degree of freedom transverse vibration system. As shown in Figure 2, it consists of motor stator, flexible hinges, one level lever and driving foot (two level lever). Among them, \( m_1 \) is the quality of the stator driving foot, \( m_2 \) is the quality of one level lever of the stator, \( k_1 \) is the stiffness of straight circular flexure hinges between one level lever and the driving foot, \( k_2 \) is the sum of stiffness of straight circular flexure hinges between the piezoelectric stack and one level lever, \( k_3 \) is the stiffness of straight beam flexure hinges between the stator and the driving foot, \( y_1(t) \) is the transverse displacement of driving foot, \( y_2(t) \) is the transverse displacement of
one level lever of the stator, $F_1(t)$ and $F_2(t)$ are the resultant force of transverse output force of piezoelectric stacks, $F_3(t)$ is the force exerted by the straight beam flexure hinges.

**Figure 2.** The longitudinal vibration model of the stator.

According to Figure 2, the transverse vibration differential equation of the stator is as follows.

$$
\begin{bmatrix}
m_1 & 0 & y_1(t) \\
0 & m_2 & y_2(t)
\end{bmatrix} + \begin{bmatrix}
k_1 + k_3 & -k_1 & -k_3 \\
-k_1 & k_1 + k_2 & 0
\end{bmatrix} \begin{bmatrix}
y_1(t) \\
y_2(t)
\end{bmatrix} = \begin{bmatrix}
F_1(t) + F_2(t) \\
F_3(t)
\end{bmatrix}
$$

(1)

Where, $\begin{bmatrix} F_1(t) + F_2(t) \\ F_3(t) \end{bmatrix} = \sqrt{2} F_{\text{max}} \left[ \sqrt{2 + \sin(\pi t + \frac{\pi}{4})} \right]$, $F_{\text{max}}$ represents the maximum transverse output force of the single stack of the stator, it is 1000 N. The solution is as follows.

$$
\begin{bmatrix}
y_1(t) \\
y_2(t)
\end{bmatrix} = \frac{\sqrt{2} F_{\text{max}}}{(k_1 + k_3 - \omega^2 m_1)(k_1 + k_3 - \omega^2 m_2)(k_1 + k_3 + k_4)\omega^2 m_1} \begin{bmatrix}
k_2 - \omega^2 m_2 \\
-\omega^2 m_1
\end{bmatrix} \sin(\omega \cdot t + \frac{\pi}{4}) + \frac{F_{\text{max}}}{k_1 + k_3} \left[ 2 \right]
$$

(2)

The transverse vibration equation of the driving end is as follows.

$$
y_1(t) = \frac{\sqrt{2} F_{\text{max}}}{(k_1 + k_3 - \omega^2 m_1)(k_1 + k_3 - \omega^2 m_2)(k_1 + k_3)\omega^2 m_1} (k_2 - \omega^2 m_2) \sin(\omega \cdot t + \frac{\pi}{4}) + \frac{2 F_{\text{max}}}{k_1 + k_3}
$$

(3)

The amplitude $A$ of the driving end is as follows.

$$
A = \frac{\sqrt{2} F_{\text{max}}}{(k_1 + k_3 - \omega^2 m_1)(k_1 + k_3 - \omega^2 m_2)(k_1 + k_3)\omega^2 m_1} (k_2 - \omega^2 m_2)
$$

(4)

According to two level lever model of the stator system in the piezoelectric linear motor, the volumes are measured and calculated, the volume of one level lever is $2280 \, mm^3$, the volume of two level lever is $4018.1 \, mm^3$. From the formula, $m = \rho v$, the quality of one level lever is 0.0179 kg, and the quality of two level lever is 0.0315 kg. The equivalent structural parameters of the stator transverse vibration displacement model are determined by the main structural parameters of the piezoelectric linear motor model, it is as shown in Table 1.

The values in Table 1 are substituted in formula (4), the amplitude of the driving foot is $61 \, \mu m$. The amplitude test device for the driving foot of the double-feet piezoelectric linear motor stator is as shown in Figure 3, 1 is the stator, 2 is the fixture with fixed stator 1, 3 is the driving foot, 4 is laser displacement sensor. The $100 \, Hz$ triangular wave signal is generated by the signal generator in Figure 4, and the voltage is amplified to 120 V by the signal amplifier, and causes the driving foot to move. The displacement of driving foot is measured by laser displacement sensor and uploads the data to the
computer. The experimental results are shown in Figure 5, the amplitude of the driving foot is about 61 $\mu m$, which is consistent with the theoretical calculation.

**Table 1.** Equivalent structural parameters of double-feet piezoelectric linear motor stator system.

| Structural parameters | $m_1 / kg$ | $m_2 / kg$ | $k_1 / (N\cdot m^{-1})$ | $k_2 / (N\cdot m^{-1})$ | $k_3 / (N\cdot m^{-1})$ |
|-----------------------|------------|------------|-------------------------|-------------------------|-------------------------|
| Value                 | 0.0179     | 0.0315     | $1\times10^5$           | $1\times10^3$           | $5\times10^4$           |

**Figure 3.** Amplitude measuring device of the driving foot.

**Figure 4.** Velocity experiment device of piezoelectric motor.
3. Velocity performance analysis of double-feet piezoelectric linear motor based on orthogonal test

3.1. Test analysis
The above theoretical model and amplitude test can confirm the design correctness of the double-feet piezoelectric linear motor, but the influence of voltage and frequency on the motor velocity cannot be further obtained. Therefore, the test data of velocity are sorted out, the factors affecting the velocity of piezoelectric linear motor are found out by orthogonal experiment design method and the corresponding relations are obtained. The velocity test device of piezoelectric motor is as shown in Figure 4. 1 is the piezoelectric motor stator, 2 is 4-way signal generator, 3 is the oscilloscope, and 4 is the power amplifier.

3.2. Determination of test factors, levels and indicators
The test mainly analyzes the influence of voltage and voltage frequency on velocity, and expresses in $X_1$ and $X_2$ respectively. The initial data are selected as the basic reference data, and the other data fluctuates up and down as the standard, the variable amplitude value is 10 V and 10 Hz. The level of each factor is 6, the values of corresponding voltage $X_1$ and voltage frequency $X_2$ are shown in Table 2.

| Level | $X_1$ (V) | $X_2$ (Hz) |
|-------|-----------|------------|
| 1     | 70        | 50         |
| 2     | 80        | 60         |
| 3     | 90        | 70         |
| 4     | 100       | 80         |
| 5     | 110       | 90         |
| 6     | 120       | 100        |

Figure 5. Amplitude of driving foot 1.
3.3. Test result
The effect of voltage and voltage frequency on velocity is as shown in Figures 6 and 7. It can be seen from Figures 6 and 7 that the motor velocity increases with the increase of voltage and frequency, but it is impossible to get the exact relationship of the influence of voltage and voltage frequency on motor velocity. In order to get the exact relationship, it is necessary to use the range and variance analysis method to make further comparison and analysis of the experimental data.

![Figure 6. Velocity curves under different voltage frequencies.](image)

![Figure 7. Velocity curves under different voltages.](image)

3.4. Range analysis
The range analysis table of the double-feet piezoelectric linear motor is obtained from the range analysis of the preceding experimental data. K is level factor, R is the range, and it is shown in Table 3.
Table 3. Range analysis table.

| Level | Voltage $X_1$ | Voltage frequency $X_2$ |
|-------|---------------|-------------------------|
| K1    | 25.60         | 17.7                    |
| K2    | 30.14         | 21.78                   |
| K3    | 29.52         | 27.83                   |
| K4    | 34.96         | 37.38                   |
| K5    | 40.56         | 42.94                   |
| K6    | 44.7          | 56.27                   |
| R     | 19.1          | 38.57                   |

As can be seen from Table 3, the velocity of the double-feet piezoelectric linear motor is affected by both voltage frequency and voltage of the motor, and the range of voltage frequency $X_2$ is greater than that of voltage $X_1$, that is, $R_{X_2} > R_{X_1}$.

3.5. Variance analysis

The variance value of the velocity of double-feet piezoelectric linear motor is obtained by the variance method, $e$ is the error sequence. The analysis results are shown in Tables 4 and 5.

Table 4. Calculation parameters of velocity variance.

| Level | Voltage | Voltage frequency | $T$     | $Q$       | $P$       |
|-------|---------|-------------------|---------|-----------|-----------|
| K1    | 25.60   | 17.7              | 205.48  | 1175.49   | 1172.83   |
| K2    | 30.14   | 21.78             |         |           |           |
| K3    | 29.52   | 27.83             |         |           |           |
| K4    | 34.96   | 37.38             |         |           |           |
| K5    | 40.56   | 42.94             |         |           |           |
| K6    | 44.7    | 56.27             |         |           |           |
| R     | 19.1    | 38.57             |         |           |           |
| SST   | C       | SS$_i$            | -1172.83| 1175.49   | 1.10      |

Table 5. Variance analysis of velocity test results.

| Difference source | Squared deviation and Freedom $d_f$ | Mean square | F value |
|-------------------|-------------------------------------|-------------|---------|
| Voltage           | 1.10                                | 0.18        | 35.76   |
| Voltage frequency | 2.19                                | 0.37        | 149.59  |
| Error e           | ---                                 | -39.18      |         |
| SUM               | 3.30                                |             |         |

From the experimental data and analysis results, it can be seen that F value of voltage and voltage frequency of double-feet piezoelectric linear motor is larger than the critical value of variance $F=8.47$, which indicates that both voltage and voltage frequency have an effect on velocity, and F value of voltage frequency is much larger than that of voltage F value, which indicates that the influence of voltage frequency on the velocity of double-feet piezoelectric linear motor is more significant.
4. Conclusion
The dynamic modeling method of the stator system of double-feet piezoelectric linear motor is studied, the dynamic model of the stator of double-feet piezoelectric linear motor is established, the amplitude of the driving foot is 61 µm by theoretical calculation, the experimental test results agree with the theoretical calculation.

Based on the theoretical calculation results, the structure design of the motor stator, flexure hinge, one level lever and driving foot is carried out, and the amplitude and velocity of the driving foot of the motor model are experimentally analyzed. The experimental results show that velocity is affected by voltage frequency and voltage, and the effect of voltage frequency on velocity is more significant. The analysis results verify that the piezoelectric motor meets the theoretical design requirements, and the motor has simple structure and stable operation, which provides a theoretical basis for the follow-up study of piezoelectric motor.

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