Localization of nonlocal cosmological models with quadratic potentials in the case of double roots

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Abstract
Nonlocal cosmological models with quadratic potentials are considered. We study the action with an arbitrary analytic function $F(□g)$, which has both double and simple roots. The formulae for nonlocal energy–momentum tensors, which correspond to double roots, have been obtained. The way to find particular solutions for nonlocal Einstein equations in the case when $F(□g)$ has both simple and double roots has been proposed. One and the same functions solve the initial nonlocal Einstein equations and the obtained local Einstein equations.

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1. Introduction

Recently a wide class of nonlocal cosmological models based on string field theory (SFT) (for details see reviews [1]) and the $p$-adic string theory [2] emerge and attract a lot of attention [3–19]. The SFT-inspired cosmological models are intensively considered as models for dark energy (DE). Actions of some of the cosmological models originating from SFT have terms with infinite-order derivatives, i.e. nonlocal terms.

Due to the presence of phantom excitations nonlocal models are of interest for present cosmology. The inequality $w_{DE} < -1$, where $w_{DE}$ is the DE state parameter, means the violation of the null energy condition (NEC). Field theories which violate the null energy condition are actively studied as a possible solution to the cosmological singularity problem [20–22] and as models of dark energy (see [23–32] and references therein). Generally speaking, models that violate the NEC have ghosts, and therefore are unstable and physically unacceptable. Phantom fields look harmful to the theory and a local model with a phantom scalar field is not acceptable from the general point of view. Models with higher derivative terms produce well-known problems with quantum instability [33, 34]. Several attempts to solve these problems have been recently performed [35, 36]. A physical idea that could solve
the problems is that the instabilities do not have enough time to fully develop. A mathematical one is that dangerous terms can be treated as corrections valued only at small energies below the physical cutoff. This approach implies the possibility of constructing a UV completion of the theory, and this assumption requires detailed analysis.

Note that the possibility of the existence of dark energy with $w_{\text{DE}} < -1$ is not excluded experimentally. Indeed, contemporary cosmological observational data [37] strongly support that the present universe exhibits an accelerated expansion providing thereby evidence for a dominating DE component (for a review see also [38]). Recent results of WMAP together with the data on Ia supernovae give the following bounds for the DE state parameter:

$$w_{\text{DE}} = -1.0 \pm 0.2.$$ (1)

The present cosmological observations do not exclude an evolving parameter $w_{\text{DE}}$. Moreover, the recent analysis of observation data indicates that the varying in time dark energy with the state parameter $w_{\text{DE}}$, which crosses the cosmological constant barrier, gives a better fit than a cosmological constant [39] (for details see reviews [40] and references therein).

To obtain a stable model with $w_{\text{DE}} < -1$ one should construct the effective theory with the NEC violation from the fundamental theory, which is stable and admits quantization. From this point of view, the NEC violation might be a property of a model that approximates the fundamental theory and describes some particular features of the fundamental theory. With the lack of quantum gravity, we can just trust string theory or deal with an effective theory admitting the UV completion. It can be considered as a hint towards SFT-inspired cosmological models.

Among the cosmological models with $w_{\text{DE}} < -1$, which have been constructed to be free of instability problem, we can mention the Lorentz-violating dark energy model [41], the k-essence models (see [36] and references therein) and the brane-world models [42].

For a more general discussion on string cosmology and coming out of string theory theoretical explanations of the cosmological observational data the reader is referred to [43–45]. Other models obeying nonlocality and their cosmological consequences are considered in [46]. In the flat space-time nonlocal equations are actively investigated as well [47–50]. Note that the study of differential equations of infinite order was started in the mathematical literature a long time ago [51, 52] (see [13] as a review).

The purpose of this paper is to study the nonlocal string field theory-inspired model with a quadratic potential. In this paper we consider a general form of linear nonlocal action for the scalar field keeping the main ingredient, the function $F(\square_g)$, which in fact produces the nonlocalities in question, almost unrestricted. The only strong restriction we impose is the analyticity of $F(\square_g)$. In previous papers [7, 8, 14, 15, 18] only simple roots have been considered. In this paper we consider the case of the function $F(\square_g)$ with both simple and double roots.

The possible way to find solutions of the Einstein equations with a quadratic potential of the nonlocal scalar field is to reduce them to a system of Einstein equations describing many non-interacting free local scalar fields [7, 14] (see also [18]). The masses of all local fields are roots of an algebraic or transcendental equation, which appears in the nonlocal model. Some of the obtained local scalar fields are normal and other of them are phantom ones.

The particular forms of $F(\square_g)$ are inspired by fermionic SFT and the most well-understood process of tachyon condensation. Namely, starting with a non-supersymmetric configuration the tachyon of the fermionic string rolls down towards the non-perturbative minimum of the tachyon potential. This process represents the non-BPS brane decay according to Sen’s conjecture (see [1] for details). From the point of view of SFT the whole picture is not yet known and only vacuum solutions are constructed. An effective field theory description
explaining the rolling tachyon in contrast is known and numeric solutions describing the
tachyon dynamics were obtained [50]. This effective field theory description does capture the
nonlocality of SFT. Linearizing the latter Lagrangian around the true vacuum one gets a model
which is of main concern in the present paper. The SFT-inspired forms of function $F(\Box_g)$,
which have the nonlocal operator $\exp(\alpha \Box_g)$, where $\alpha$ is a constant, as a key ingredient, have
been considered in [8, 14, 15]. Such functions have infinite number of simple roots and may
be one double root.

This paper is organized as follows. In section 2 we describe the nonlocal SFT-inspired
cosmological model and its generalization. In section 3 we calculate the energy–momentum
tensor for different special solutions. Using these formulae we build local actions and the
corresponding local Einstein equations. In section 4 we propose the algorithm to find particular
solutions of the nonlocal Einstein equations, solving only local ones, and prove the self-
consistence of it. Any solution for the obtained system of differential equations is a particular
solution for the initial nonlocal Einstein equations. In section 5 we summarize the obtained
results and propose directions for further investigations.

2. Model setup

The four-dimensional action with a quadratic potential, motivated by string field theory, has
been studied in [7, 8, 14, 15, 18]. Such a model appears as a linearization of the SFT-inspired
model in the neighbourhood of an extremum of the potential (see [18] for details). For linear
models, solving the nonlocal equations using the technique, proposed in [14], is completely
equivalent to solving the equations using the diffusion-like partial differential equations [15].
In [15] it has been shown that to fix the initial data for the partial differential equations one
can use the initial data of the local fields. By linearizing a nonlinear model about a particular
field value, one is able to specify initial data for nonlinear models, which can then evolve into
the full nonlinear regime using the diffusion-like equation [15].

In this paper we study nonlocal cosmological models with a quadratic potential, in other
words, a linear nonlocal model, which can be described by the following action:

$$S = \int d^4x \sqrt{-g} \alpha' \left( \frac{R}{16\pi G_N} + \frac{1}{2g_{\phi}} \phi F(\Box_g) \phi + \Lambda \right), \quad (2)$$

where $G_N$ is the Newtonian constant: $8\pi G_N = 1/M_P^2$, where $M_P$ is the Planck mass, $\alpha'$ is the
string length squared (we do not assume $\alpha' = 1/M_P^2$) and $g_{\phi}$ is the string coupling constant.
We use the signature $(-, +, +, +)$, $g_{\mu\nu}$ is the metric tensor, $R$ is the scalar curvature and $\Lambda$ is
the cosmological constant.

The function $F$ is assumed to be an analytic function, and therefore, one can represent it
by the convergent series expansion:

$$F = \sum_{n=0}^{\infty} f_n \Box_g^n. \quad (3)$$

The function $F$ may have infinitely many roots manifestly producing thereby the nonlocality
[13, 18].

This model has been studied in [7, 18] with an additional condition that all roots of the
function $F$ are simple. At the same time the obtained formulae for the nonlocal energy–
momentum tensor (formulae (4.1) in [7]) are valid in the case of multiple roots as well and we
use them in this paper.

In [8, 14] the special class of functions $F(\Box_g)$:

$$F_{sf/t}(\Box_g) = -\xi^2 \Box_g + 1 - e^{-2\Box_g}, \quad (4)$$

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where $\xi$ is a real parameter and $c$ is a positive constant has been considered. The action with $\mathcal{F}_{ST}(\Box_g)$ is interesting in context of the SFT-inspired models. In [15] the model has been generalized and a linear term has been added to the action.

The function $\mathcal{F}_{ST}(\Box_g)$ has a double root if and only if

$$c = \frac{\xi^2}{2e^{2\xi^2}}.$$  \hfill (5)

The double root $\tilde{J}_0$ is as follows:

$$\tilde{J}_0 = \frac{1}{\xi^2} - \frac{1}{2}.$$  \hfill (6)

At any $\xi$ and $c$, which satisfy (5), the function $\mathcal{F}_{ST}(J)$ has one and only one double root $\tilde{J}_0$ and $\mathcal{F}_{ST}(\tilde{J}_0) \neq 0$.

In this paper we consider in detail the case of an arbitrary analytic function $\mathcal{F}$ with both double and simple roots.

To clarify the interest to consider the case of double roots let us study a trivial example with

$$\mathcal{F}(\Box_g) = (\Box_g - J_1)(\Box_g - J_2).$$  \hfill (7)

In the Minkowski space-time for $\phi$, depending only on time, we obtain the following equation of motion:

$$(\dddot{\phi} - J_1)(\dddot{\phi} - J_2)\phi(t) = 0.$$  \hfill (8)

This fourth-order differential equation can be written in the form of a system of two second-order equations:

$$(\dddot{\phi} - J_1)\xi(t) = 0, \quad (\dddot{\phi} - J_2)\phi(t) = \xi(t).$$  \hfill (9)

The first equation has the general solution

$$\xi(t) = C_1 e^{\sqrt{J_1} t} + C_2 e^{-\sqrt{J_1} t},$$  \hfill (10)

where $C_1$ and $C_2$ are constants. So, we get the following second-order equation for $\phi$:

$$(\dddot{\phi} - J_2)\phi(t) = C_1 e^{\sqrt{J_1} t} + C_2 e^{-\sqrt{J_1} t}.$$  \hfill (11)

In the non-resonance case (two simple roots $J_1$ and $J_2$) we get the following general solution:

$$\phi(t) = \tilde{C}_1 e^{\sqrt{J_1} t} + \tilde{C}_2 e^{-\sqrt{J_1} t} + \tilde{C}_3 e^{\sqrt{J_2} t} + \tilde{C}_4 e^{-\sqrt{J_2} t},$$  \hfill (12)

whereas in the resonance case (one double root $J_2 = J_1$) the general solution is

$$\phi(t) = \tilde{C}_1 e^{\sqrt{J_1} t} + \tilde{C}_2 e^{-\sqrt{J_1} t} + \tilde{C}_3 t e^{\sqrt{J_1} t} + \tilde{C}_4 t e^{-\sqrt{J_1} t},$$  \hfill (13)

where $\tilde{C}_k$ are arbitrary constants. This trivial example shows that behaviours of solutions in the cases of one double root and two simple roots are essentially different and one cannot approximate double roots by two simple roots, which are at a very small distance. Resonance phenomena are important and actively studied in various domains of physics.

3. The energy–momentum tensor

3.1. The Einstein equations and the energy–momentum tensor

From action (2) we obtain the following equations:

$$G_{\mu\nu} = \frac{8\pi G_N}{8\pi} \mathcal{T}_{\mu\nu} + 8\pi G_N \Lambda,$$  \hfill (14)
F(□_g)φ = 0,

\[ (15) \]

where \( G_{\mu\nu} \) is the Einstein tensor and \( T_{\mu\nu} \) is the energy–momentum (stress) tensor [7, 18]:

\[ T_{\mu\nu} = -\frac{2}{\sqrt{-g}} \frac{\delta S}{\delta g^{\mu\nu}} = \frac{1}{2} \sum_{n=1}^{\infty} f_n \sum_{l=0}^{n-1} \left( \partial_\mu \square_g \phi \partial_\nu \square_g \phi^{n-1-l} \phi + \partial_\nu \square_g \phi \partial_\mu \square_g \phi^{n-1-l} \phi \right. \]

\[ - \left. g_{\mu\nu} \left( g^{\rho\sigma} \partial_\rho \square_g \phi \partial_\sigma \square_g \phi^{n-1-l} \phi + \square_g \phi \square_g \phi^{n-1-l} \phi \right) \right), \]

\[ (16) \]

\[ \square_g = \frac{1}{\sqrt{-g}} \partial_\mu \sqrt{-g} g^{\mu\nu} \partial_\nu. \]

(17)

It is easy to check that the Bianchi identity is satisfied on-shell and in a simple case \( F = f_1 \square_g + f_0 \) the usual energy–momentum tensor for the massive scalar field is reproduced. Note that equation (15) is an independent equation consistent with system (14) due to the Bianchi identity.

In an arbitrary metric the energy–momentum tensor (16) can be presented in the following form:

\[ T_{\mu\nu} = E_{\mu\nu} + E_{\nu\mu} - g_{\mu\nu} \left( g^{\rho\sigma} E_{\rho\sigma} + V \right), \]

(18)

where

\[ E_{\mu\nu} = \frac{1}{2} \sum_{n=1}^{\infty} f_n \sum_{l=0}^{n-1} \partial_\mu \square_g \phi \partial_\nu \square_g \phi^{n-1-l} \phi, \]

(19)

\[ V = \frac{1}{2} \sum_{n=1}^{\infty} f_n \sum_{l=0}^{n-1} \square_g \phi \square_g \phi^{n-1-l} \phi. \]

(20)

3.2. The energy–momentum tensor for special solutions

Classical solutions to system (14)–(15) were studied and analyzed in [7, 8, 14, 18]. The main idea of finding the solutions to the equations of motion is to start with equation (15) and to solve it, assuming that the function \( \phi \) is an eigenfunction of the d’Alembertian operator \( \square_g \).

If \( \square_g \phi = J \phi \), then such a function \( \phi \) is a solution to (15) if and only if

\[ F(J) = 0. \]

(21)

The latter condition is known as the characteristic equation. Note that values of roots of \( F(J) \) do not depend on the metric.

Let us denote simple roots of \( F \) as \( J_i \) and double roots of \( F \) as \( \tilde{J}_k \). We seek a particular solution of equation (15) in the following form:

\[ \phi_0 = \sum_{i=1}^{N_1} \phi_i + \sum_{k=1}^{N_2} \tilde{\phi}_k, \]

(22)

where

\[ (\square_g - J_i) \phi_i = 0, \quad (\square_g - \tilde{J}_k)^2 \tilde{\phi}_k = 0. \]

(23)

Without loss of generality we assume that for any \( i_1 \) and \( i_2 \neq i_1 \) conditions \( J_{i_1} \neq J_{i_2} \) and \( \tilde{J}_{i_1} \neq \tilde{J}_{i_2} \) are satisfied. Indeed, if, for example, sum (22) includes two summands \( \phi_{i_1} \) and...
φ_i, which correspond to one and the same Ji, then we can consider them as one summand φi ≡ φ_i1 + φ_i2, which corresponds to Ji.

In [7, 8, 14, 15, 18] only the case of simple roots has been studied. In our paper we generalize this analysis on double roots. Our first goal is to calculate the energy–momentum tensor for φ_0. To obtain the general formula; we begin from a few particular cases. Hereafter, we denote the energy–momentum tensor for the function φ(t) as Tμν(φ).

3.3. Simple roots

If we have one simple root φ_1 such that □_gφ_1 = Ji φ_1, then

$$E_{\mu\nu}(\phi_1) = \frac{1}{2} \sum_{n=1}^{\infty} f_n \sum_{l=0}^{n-1} J_1^n l_0 J_1^n \phi_1^l \phi_1 = \frac{F'(J_1)}{2} \partial_\mu \phi_1 \partial_\nu \phi_1$$

(24)

$$V(\phi_1) = \frac{1}{2} \sum_{n=1}^{\infty} f_n = \frac{J_1}{2} \sum_{n=1}^{\infty} f_n n J_1^{n-1} \phi_1^2 = \frac{J_1 F'(J_1)}{2} \phi_1^2,$$

(25)

where F' ≡ \frac{dF}{dJ}.

In the case of two simple roots φ_1 and φ_2 we have

$$E_{\mu\nu}(\phi_1 + \phi_2) = E_{\mu\nu}(\phi_1) + E_{\mu\nu}(\phi_2) + E_{\mu\nu}^{cr}(\phi_1, \phi_2),$$

(26)

where the cross term

$$E_{\mu\nu}^{cr}(\phi_1, \phi_2) = A_1 \partial_\mu \phi_1 \partial_\nu \phi_2 + A_2 \partial_\mu \phi_2 \partial_\nu \phi_1.$$  

(27)

It is easy to calculate that

$$A_1 = \frac{1}{2} \sum_{n=1}^{\infty} f_n J_1^{n-1} \sum_{l=0}^{n-1} \left( \frac{J_2}{J_1} \right)^l = \frac{F(J_1) - F(J_2)}{2(J_1 - J_2)} = 0,$$

(28)

and

$$A_2 = 0.$$  

(29)

So, the cross term E_{\mu\nu}^{cr}(\phi_1, \phi_2) = 0 and

$$E_{\mu\nu}(\phi_1 + \phi_2) = E_{\mu\nu}(\phi_1) + E_{\mu\nu}(\phi_2).$$

(30)

Similar calculations show

$$V(\phi_1 + \phi_2) = V(\phi_1) + V(\phi_2).$$

(31)

In the case of N simple roots the following formula has been obtained [18] (see also [14]):

$$T_{\mu\nu} = \sum_{k=1}^{N} F'(J_k) \left( \partial_\mu \phi_k \partial_\nu \phi_k - \frac{1}{2} g_{\mu\nu} \partial_\rho \phi_k \partial_\sigma \phi_k + J_2 \phi_k^2 \right).$$

(32)

Note that the last formula is exactly the energy–momentum tensor of many free massive scalar fields. If F(J) has simple real roots, then positive and negative values of F'(J) alternate, so we can obtain phantom fields.
3.4. One double root

Let us consider the case when all roots of $F(J)$, but one, are simple and the last root is a double root. As we mentioned above this case is interesting in context of SFT-inspired models.

Let $\hat{J}_1$ is a double root. The fourth-order differential equation

$$ (\Box_x - \hat{J}_1)(\Box_x - \hat{J}_1)\tilde{\phi}_1 = 0 $$

is equivalent to the following system of equations:

$$ (\Box_x - \hat{J}_1)\tilde{\phi}_1 = \varphi_1, \quad (\Box_x - \hat{J}_1)\varphi_1 = 0. $$

It is convenient to write $\Box_x \tilde{\phi}_1$ in terms of $\tilde{\phi}_1$ and $\varphi_1$:

$$ \Box_x \tilde{\phi}_1 = \hat{J}_1^0 \tilde{\phi}_1 + \hat{J}_1^{-1} \varphi_1. $$

Using (35) we obtain

$$ E_{\mu\nu}(\tilde{\phi}_1) = B_1 \partial_\mu \tilde{\phi}_1 \partial_\nu \varphi_1 + B_2 \partial_\mu \tilde{\phi}_1 \partial_\nu \varphi_1 + B_3 \partial_\mu \tilde{\phi}_1 \partial_\nu \varphi_1 + B_4 \partial_\mu \varphi_1 \partial_\nu \varphi_1, $$

where

$$ B_1 = \frac{1}{2} \sum_{n=1}^{\infty} \sum_{l=0}^{n-1} f_n \sum_{l=0}^{n-1} \tilde{j}_1^{n-1} \frac{F'(\hat{J}_1)}{2} = 0. $$

$$ B_2 = \frac{1}{2} \sum_{n=1}^{\infty} \sum_{l=0}^{n-1} f_n \sum_{l=0}^{n-1} \tilde{j}_1^{n-2} (n - l - 1) = \frac{1}{4} \sum_{n=1}^{\infty} f_n n(n - 1) \tilde{j}_1^{n-2} = \frac{F''(\hat{J}_1)}{4}. $$

$$ B_3 = \frac{1}{2} \sum_{n=1}^{\infty} \sum_{l=0}^{n-1} f_n \sum_{l=0}^{n-1} l \tilde{j}_1^{n-2} = \frac{F''(\hat{J}_1)}{4} = B_2. $$

$$ B_4 = \frac{1}{2} \sum_{n=1}^{\infty} \sum_{l=0}^{n-1} f_n \sum_{l=0}^{n-1} \tilde{j}_1^{n-3} (n - l - 1)l = \frac{F'''(\hat{J}_1)}{12}. $$

We have used the well-known formulae:

$$ \sum_{l=0}^{n-1} l = \frac{n(n - 1)}{2} \quad \text{and} \quad \sum_{l=0}^{n-1} l^2 = \frac{n(n - 1)(2n - 1)}{6}. $$

$$ V(\tilde{\phi}_1) = C_1 \tilde{\phi}_1^2 + C_2 \tilde{\phi}_1 \varphi_1 + C_3 \varphi_1^2. $$

where

$$ C_1 = \frac{1}{2} \sum_{n=1}^{\infty} \sum_{l=0}^{n-1} J_1^l = \frac{\hat{J}_1}{2} \sum_{n=1}^{\infty} f_n n J_1^{n-1} = \frac{\hat{J}_1 F'(\hat{J}_1)}{2} = 0, $$

$$ C_2 = \frac{1}{2} \sum_{n=1}^{\infty} \sum_{l=0}^{n-1} J_1^{n-1} = \frac{\hat{J}_1 F''(\hat{J}_1)}{2} + \frac{F'(\hat{J}_1)}{2} = \frac{\hat{J}_1 F''(\hat{J}_1)}{2}. $$

$$ C_3 = \frac{1}{2} \sum_{n=1}^{\infty} \sum_{l=0}^{n-1} l(n - l) J_1^{n-2} = \frac{\hat{J}_1 F'''(\hat{J}_1)}{12} + \frac{F''(\hat{J}_1)}{4}. $$
Thus, for one double root we obtain the following result:

\[ E_{\mu\nu}(\phi_1) = \frac{\mathcal{F}^r(J_1)}{2} \partial_\mu \partial_\nu \phi_1 + \frac{\mathcal{F}''(J_1)}{12} \partial_\mu \partial_\nu \phi_1, \quad (46) \]

\[ V(\phi_1) = \frac{J_1 \mathcal{F}''(J_1)}{2 \mathcal{F}''(J_1)} \phi_1 \phi_1 + \frac{\mathcal{F}''(J_1)}{4} \phi_1^2. \quad (47) \]

For one simple root \( J_2 \) (the function \( \phi_2 \) satisfies the equation \( \Box_g \phi_2 = J_2 \phi_2 \)) and one double root \( J_1 \) we obtain

\[ E_{\mu\nu}(\phi_1 + \phi_2) = E_{\mu\nu}(\phi_1) + E_{\mu\nu}(\phi_2) + E_{\mu\nu}^{cr}(\phi_1, \phi_2), \quad (48) \]

where

\[ E_{\mu\nu}^{cr}(\phi_1, \phi_2) = B_3 \partial_\mu \phi_1 \partial_\nu \phi_2 + B_6 \partial_\mu \phi_1 \partial_\nu \phi_2 + B_7 \partial_\mu \psi_1 \partial_\nu \phi_2 + B_8 \partial_\mu \phi_1 \partial_\nu \phi_2. \quad (49) \]

It is easy to calculate

\[ B_3 = \sum_{n=1}^{\infty} f_n J_2^{n-1} \sum_{l=0}^{n-1} \left( \frac{J_1}{J_2} \right)^l = \frac{\mathcal{F}(J_2) - \mathcal{F}(J_1)}{2(J_2 - J_1)} = 0, \]

\[ B_6 = 0. \quad (50) \]

To calculate

\[ B_7 = \sum_{n=1}^{\infty} f_n J_2^n \sum_{l=0}^{n-1} l \left( \frac{J_1}{J_2} \right)^l, \quad (52) \]

we use

\[
\sum_{l=0}^{n-1} l y^{l-1} = \frac{d}{dy} \sum_{l=0}^{n-1} y^l = \frac{d}{dy} \left( \frac{1 - y^n}{1 - y} \right) = \frac{(n - 1)y^n - ny^{n-1} + 1}{(1 - y)^2}
\]

and obtain

\[ B_7 = \frac{J_2^2}{2(J_2 - J_1)} \mathcal{F}(J_1) + \frac{J_2^2}{2(J_2 - J_1)^2} (\mathcal{F}(J_1) + \mathcal{F}(J_2)) = 0. \quad (54) \]

Similar calculations give \( B_8 = 0. \)

It is easy to obtain that

\[ V(\phi_1 + \phi_2) = \sum_{n=1}^{\infty} f_n \sum_{l=0}^{n-1} \Box_g (\phi_1 + \phi_2) \Box_g^{l-1} (\phi_1 + \phi_2) = V(\phi_1) + V(\phi_2). \quad (55) \]

The calculations are straightforwardly generalized on the case of one double root and an arbitrary number of simple roots. Therefore, we obtain the following formula:

\[ T_{\mu\nu} \left( \phi_1 + \sum_{k=1}^{N} \phi_k \right) = T_{\mu\nu}(\phi_1) + T_{\mu\nu} \left( \sum_{k=1}^{N} \phi_k \right), \quad (56) \]

where

\[ T_{\mu\nu}(\phi_1) = E_{\mu\nu}(\phi_1) + E_{\nu\mu}(\phi_1) - g_{\mu\nu} \left( g^{\alpha\sigma} E_{\alpha\sigma}(\phi_1) + V(\phi_1) \right). \quad (57) \]

So, we conclude that in the case of one double root the energy–momentum tensor can be separated into energy–momentum tensors for different modes of the nonlocal scalar field, which correspond to different roots of \( \mathcal{F} \).
3.5. The General Formulae

Let us consider the case of two double roots \( \tilde{J}_1 \) and \( \tilde{J}_2 \). We can write

\[
E_{\mu \nu}(\tilde{\phi}_1 + \tilde{\phi}_2) = E_{\mu \nu}(\tilde{\phi}_1) + E_{\mu \nu}(\tilde{\phi}_2) + E_{\mu \nu}^{(\prime)}(\tilde{\phi}_1, \tilde{\phi}_2),
\]

(58)

where

\[
E_{\mu \nu}^{(\prime)}(\tilde{\phi}_1, \tilde{\phi}_2) = B_{10} \partial_\mu \tilde{\phi}_1 \partial_\nu \tilde{\phi}_2 + B_{11} \partial_\gamma \tilde{\phi}_1 \partial_\mu \tilde{\phi}_2 + B_{12} \partial_\mu \tilde{\phi}_1 \partial_\nu \tilde{\phi}_2 + B_{13} \partial_\gamma \tilde{\phi}_1 \partial_\nu \tilde{\phi}_2 + B_{14} \partial_\mu \tilde{\phi}_1 \partial_\gamma \tilde{\phi}_2 + B_{15} \partial_\nu \tilde{\phi}_1 \partial_\gamma \tilde{\phi}_2.
\]

(59)

Using computations, which are similar to computations of \( B_5 \) and \( B_7 \), it is easy to see that

\[
B_{10} = B_{11} = B_{12} = B_{13} = B_{14} = B_{15} = 0.
\]

(60)

It is suitable to present

\[
B_{16} = \frac{1}{2} \sum_{n=1}^{\infty} f_n \sum_{l=0}^{n-1} \tilde{J}_1^{l-1} (n-l-1) \tilde{J}_2^{l-2}
\]

(61)

in the following form:

\[
B_{16} = \frac{1}{2} \sum_{n=1}^{\infty} f_n \sum_{l=0}^{n-1} (n-l-1) \sigma^{l-1},
\]

(62)

where \( \sigma = \tilde{J}_1 / \tilde{J}_2 \). Using

\[
\sum_{l=0}^{n-1} (n-l-1) \sigma^{l-1} = n \frac{d}{d\sigma} \left( \sum_{l=0}^{n-1} \sigma^l \right) - \sigma \frac{d^2}{d\sigma^2} \left( \sum_{l=0}^{n-1} \sigma^l \right)
\]

(63)

and

\[
\sum_{l=0}^{n-1} \sigma^l = \frac{1 - \sigma^n}{1 - \sigma},
\]

(64)

we obtain

\[
\sum_{l=0}^{n-1} (n-l-1) \sigma^{l-1} = n \frac{1 + \sigma^{n-1} (2\sigma - 1)}{(\sigma - 1)^2} + 2\sigma \frac{1 - \sigma^n}{(\sigma - 1)^3}.
\]

(65)

Thus, we get

\[
B_{16} = \frac{\tilde{J}_2 (F(\tilde{J}_2) - F(\tilde{J}_1))}{(J_2 - \tilde{J}_1)^3} + \frac{\tilde{J}_2 (2\tilde{J}_1 - \tilde{J}_2) F(\tilde{J}_1) + \tilde{J}_2^2 F(\tilde{J}_2)}{2(J_1 - \tilde{J}_2)^2}.
\]

(66)

So, \( B_{16} = 0 \). The similar calculations prove that \( B_{17} = 0 \) and we come to the following result:

\[
E_{\mu \nu}(\tilde{\phi}_1 + \tilde{\phi}_2) = E_{\mu \nu}(\tilde{\phi}_1) + E_{\mu \nu}(\tilde{\phi}_2).
\]

(67)

We also obtain

\[
V(\tilde{\phi}_1 + \tilde{\phi}_2) = V(\tilde{\phi}_1) + V(\tilde{\phi}_2).
\]

(68)

The results, obtained for two summands, can be straightforwardly generalized on an arbitrary number of summands. So, we obtain that for any analytical function \( F \), which has simple roots \( J_i \) and double roots \( \tilde{J}_k \), and any \( \phi_0 \) given by (22) the energy–momentum tensor

\[
T_{\mu \nu}(\phi_0) = T_{\mu \nu} \left( \sum_{i=1}^{N_1} \phi_i + \sum_{k=1}^{N_2} \hat{\phi}_k \right) = \sum_{i=1}^{N_1} T_{\mu \nu}(\phi_i) + \sum_{k=1}^{N_2} T_{\mu \nu}(\hat{\phi}_k).
\]

(69)

The result has been obtained for an arbitrary metric \( g_{\mu \nu} \).
Considering the following local action:

\[ S_{\text{loc}} = \int d^4x \sqrt{-g} \left( \frac{R}{16\pi G_N} + \Lambda - \frac{1}{2g_0^2} \sum_{i=1}^{N_1} \mathcal{F}'(J_i)(g^{\mu\nu}\partial_\mu\phi_i\partial_\nu\phi_i + J_i\phi_i^2) \right. \]

\[ - \sum_{k=1}^{N_2} \left( \frac{g^{\mu\nu}}{2} \partial_\mu\tilde{\phi}_k\partial_\nu\varphi_k + \frac{\mathcal{F}'''(J_k)}{6}\partial_\mu\varphi_k\partial_\nu\varphi_k \right) \]

\[ + \left( \frac{J_k\mathcal{F}''(J_k)}{12} + \frac{\mathcal{F}'(J_k)}{4} \right) \varphi_k^2 \left) \right. \]

we can see that solutions of the system of the Einstein equations and equations in \( \phi_k \), \( \tilde{\phi}_k \) and \( \varphi_k \), obtained from this action, solve the initial system of nonlocal equations (14) and (15). Thus, we obtained special solutions of nonlocal equations by solving a system of local (differential) equations.

To clarify physical interpretation we diagonalize the kinetic terms of scalar fields \( \tilde{\phi}_k \) and \( \varphi_k \) in action (70). It is convenient to present (70) as follows:

\[ S_{\text{loc}} = \int d^4x \sqrt{-g} \left( \frac{R}{16\pi G_N} + \Lambda + \sum_{i=1}^{N_1} S_i + \sum_{k=1}^{N_2} \tilde{S}_k \right), \]  

where

\[ S_i = -\frac{1}{2g_0^2} \int d^4x \sqrt{-g} \mathcal{F}'(J_i)(g^{\mu\nu}\partial_\mu\phi_i\partial_\nu\phi_i + J_i\phi_i^2), \]

\[ \tilde{S}_k = -\frac{1}{2g_0^2} \int d^4x \sqrt{-g} \left( \frac{g^{\mu\nu}}{4} \partial_\mu\tilde{\phi}_k\partial_\nu\varphi_k + \frac{\mathcal{F}'''(J_k)}{6}\partial_\mu\varphi_k\partial_\nu\varphi_k \right) \]

\[ + \frac{J_k\mathcal{F}''(J_k)}{12} \tilde{\phi}_k\varphi_k + \left( \frac{J_k\mathcal{F}'(J_k)}{4} \right) \varphi_k^2 \].

Expressing \( \tilde{\phi}_k \) and \( \varphi_k \) in terms of new fields \( \xi_k \) and \( \chi_k \):

\[ \tilde{\phi}_k = \frac{1}{2\mathcal{F}'(J_k)} \left( (\mathcal{F}'(J_k) - \frac{2}{3}\mathcal{F}''(J_k))\xi_k - (\mathcal{F}'''(J_k) + \frac{2}{3}\mathcal{F}''(J_k))\chi_k \right), \]

\[ \varphi_k = \xi_k + \chi_k, \]

we obtain the corresponding \( \tilde{S}_k \) in the following form:

\[ \tilde{S}_k = -\frac{1}{2g_0^2} \int d^4x \sqrt{-g} \left( g^{\mu\nu} \frac{\mathcal{F}''(J_k)}{4} (\partial_\mu\xi_k\partial_\nu\xi_k - \partial_\mu\chi_k\partial_\nu\chi_k) \right. \]

\[ + \frac{J_k}{4} \left( (\mathcal{F}''(J_k) - \frac{2}{3}\mathcal{F}''(J_k))\xi_k - (\mathcal{F}'''(J_k) + \frac{2}{3}\mathcal{F}''(J_k))\chi_k \right) \xi_k + \chi_k \]

\[ \left. + \left( \frac{J_k\mathcal{F}''(J_k)}{12} + \frac{\mathcal{F}'(J_k)}{4} \right) (\xi_k + \chi_k)^2 \right). \]

It is easy to see that each \( \tilde{S}_k \) includes one phantom scalar field and one standard scalar field. So, in the case of one double root we obtain a quintom model. In the Minkowski space appearance of phantom fields in models, when \( \mathcal{F}(\Box) \) has a double root, has been obtained in [47].
4. The algorithm of localization

The obtained formulae allow us to seek particular solutions for nonlocal gravitational models with quadratic potentials, which are described by action (2), in the following way.

• Find roots of the function $F(J)$ and calculate their orders.
• Select a finite number of simple and double roots.
• Construct the corresponding local action by formula (70).
• Obtain a system of the Einstein equations and equations of motion. The obtained system is a finite-order system of differential equations; in other words we get a local system.
• Seek solutions of the obtained local system.

Remark 1. If $F(J)$ has an infinity number of roots then one nonlocal model corresponds to an infinite number of different local models. In this case the initial nonlocal action (2) generates an infinite number of local actions (70).

Remark 2. We should prove that our algorithm is self-consistent. To construct local action (70) we assume that equations (23) are satisfied. Therefore, our algorithm is correct only if these equations can be obtained from the local action (70). The straightforward calculations show that

$$\frac{\delta S_{\text{loc}}}{\delta \phi_i} = 0 \iff \Box_g \phi_i = J_i \phi_i,$$

(76)

$$\frac{\delta S_{\text{loc}}}{\delta \phi_k} = 0 \iff \Box_g \phi_k = J_k \phi_k.$$

(77)

Using (77) we obtain

$$\frac{\delta S_{\text{loc}}}{\delta \tilde{\phi}_k} = 0 \iff \Box_g \tilde{\phi}_k = \tilde{J}_k \tilde{\phi}_k + \psi_k.$$

(78)

So, our way of localization is self-consistent in the case of $F(J)$ with simple and double roots. The self-consistence of similar approach for $F(J)$ with only simple roots has been proven in [14, 18].

In spite of the above-mentioned equations we obtain from $S_{\text{loc}}$ the Einstein equations

$$G_{\mu\nu} = \frac{8\pi G_N}{s_0} T_{\mu\nu}(\phi_0) + 8\pi G_N \Lambda,$$

(79)

where $\phi_0$ is given by (22) and $T_{\mu\nu}(\phi_0)$ can be calculated by (69).

So, we obtained such systems of differential equations so that any solutions of these systems are particular solutions of the initial nonlocal equations (14) and (15).

5. Conclusion

The main result of this paper is the explicit proof that a nonlocal cosmological model can be localized not only in the case when $F(\Box_g)$ has only simple roots. We have found the way to find particular solutions of the nonlocal Einstein equations in the case when an analytic function $F(\Box_g)$ has both simple and double roots. We prove that the same functions solve the initial nonlocal Einstein equations and the obtained local Einstein equations. We have found the corresponding local actions and proved the self-consistence of our approach. The result has been obtained for an arbitrary metric, so it can be used not only to find solutions
in the Friedmann–Robertson–Walker metric, but also to find other interesting solutions, for example, black hole solutions. In the case of simple roots some exact solutions in the Friedmann–Robertson–Walker metric have been found in [14] (the stability of these solutions is considered in [53]).

Looking a step further it is interesting to consider nonlocal models with an arbitrary analytic \( \mathcal{F}(\Box_g) \), without any restrictions on order of roots. The consideration of simple roots in papers [14, 18] and double roots in this paper allows us to make the conjecture that the existence of local actions, which correspond to a nonlocal action, does not depend on the order of \( \mathcal{F}(\Box_g) \) roots and the method of finding particular solutions of the nonlocal Einstein equations can be generalized on a nonlocal action with an arbitrary analytic \( \mathcal{F}(\Box_g) \).

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