Report on Progress

Report on progress in physics: observation of the Breit–Wheeler process and vacuum birefringence in heavy-ion collisions

James Daniel Brandenburg\(^{1}\)*, Janet Seger\(^{2}\), Zhangbu Xu\(^{3}\) and Wangmei Zha\(^{4}\)

\(^{1}\) Department of Physics, The Ohio State University, Columbus, OH 43210, United States of America
\(^{2}\) Creighton University, Omaha, NE 68178, United States of America
\(^{3}\) Brookhaven National Laboratory, Upton, NY 11973-5000, United States of America
\(^{4}\) University of Science and Technology of China, Hefei, People’s Republic of China

E-mail: brandenburg.89@osu.edu

Received 31 July 2022, revised 8 May 2023
Accepted for publication 2 June 2023
Published 26 June 2023

Corresponding editor: Dr Jean-Paul Blaizot

Abstract

This report reviews the effort over several decades to observe the linear Breit–Wheeler process \((\gamma\gamma \rightarrow e^+e^-)\) and vacuum birefringence (VB) in high-energy particle and heavy-ion collider experiment. This report, motivated by the STAR collaboration’s recent observations, attempts to summarize the key issues related to the interpretation of polarized \(\gamma\gamma \rightarrow l^+l^-\) measurements in high-energy experiments. To that end, we start by reviewing the historical context and essential theoretical developments, before focusing on the decades of progress made in high-energy collider experiments. Special attention is given to the evolution in experimental approaches in response to various challenges, to the demanding detector capabilities required to unambiguously identify the linear Breit–Wheeler process, and to the connections with VB. We close the report with a discussion, followed by a look at near-future opportunities for utilizing these discoveries and for testing quantum electrodynamics in previously unexplored regimes.

Keywords: quantum electrodynamics, Breit–Wheeler process, vacuum birefringence, vacuum dichroism, heavy-ion collisions, light-by-light scattering

(Some figures may appear in colour only in the online journal)
3. The BW process in HICs
   3.1. Validity of the photon source
      3.1.1. Theoretical description of equivalent photons from high-energy electron beams
      3.1.2. Theoretical description of equivalent photons in HICs
      3.1.3. Photon virtuality and a criterion for the BW process
   3.2. Higher order processes in QED
   3.3. Experimental identification of the BW process
      3.3.1. Measurements from electron–positron colliders
      3.3.2. Experimental progress in high-energy collisions
      3.3.3. Detectors, requirements, and limitations
      3.3.4. Early measurements (2000 ~ 2015)
      3.3.5. Observations of EM production in hadronic HICs (2016 ~ 2018)
      3.3.6. Observation of impact parameter dependence on photon kinematics
   4. Connections to VB
      4.1. Theoretical aspects of VB and polarization effects
      4.2. Connection between the STAR observation and VB
   5. Discussion
      5.1. The BW process
         5.1.1. Do the highly-Lorentz contracted fields produced in HICs provide a valid source of photons for the BW process?
         5.1.2. Are higher order effects present, and if so, are they separable or inseparable from the lowest-order BW process?
         5.1.3. Can the BW process be isolated from background and higher-order processes?
         5.1.4. Has the BW process previously been observed in high energy particle or HICs?
      5.2. VB and the STAR observation
         5.2.1. What is novel about the STAR measurement, i.e. how is it unique compared to the SLAC E-144 measurement, with respect to polarization effects?
         5.2.2. How is the azimuthal angle ($\phi$) modulation observed by STAR related to VB and/or dichroism?
   6. Related topics and future opportunities
      6.1. Opportunities as EM probes of QGP
      6.2. Probe the geometry of nuclear charge and gluon distributions
      6.3. The BW process for measuring tau g ~ 2
      6.4. Light-by-light scattering and axion searches
      6.5. Future laser facilities
   7. Conclusion
   Data availability statement
   Acknowledgments
   Appendix A. Available theoretical calculations and Monte Carlo generators
   Appendix B. Polarized photonuclear interactions and quantum interference
   References

1. Introduction

1.1. History

The birth of quantum mechanics the early part of the 20th century led to an explosion of progress in both experimental achievements and theoretical understanding. It was in the late 1920s that Dirac and Pauli undertook the work of unifying special relativity and quantum mechanics to obtain a relativistic equation of motion for the wave function of the electron. The solution, known for decades as the Dirac equation, describes all massive spin-1/2 particles. The Dirac equation produces the two expected positive energy solutions, but also predicts two negative energy states. The prediction of negative energy states was somewhat puzzling at the time, since their physical interpretation was not clear. Furthermore, available negative energy states should imply that electrons (with positive energy) could decay via a photon. In order to explain the apparent stability of the electron, Dirac proposed the hole theory, which postulates that the quantum vacuum is a quantum many-body state in which all negative-energy ‘holes’ are filled. The hole theory, along with the Pauli exclusion principle, which forbids electrons from occupying identical energy states, resolved the problem of electron decay. In the year 1930, Dirac built on his earlier work by proposing the annihilation process between positive energy states ($e^-$ and negative energy states (later known as positrons, $e^+$):

$$e^- + e^+ \rightarrow \gamma + \gamma.$$  

This process, known as Dirac annihilation, or simply as annihilation, describes the conversion of an electron and a positron into electromagnetic (EM) radiation in the form of two photons. The physical interpretation of these negative energy states was resolved with the experimental discovery of the positron in cosmic ray interactions by Anderson in 1932 [1, 2]. The discovery of the positron led to rapid progress in understanding and describing the fundamental interactions between charged particles and photons. Around the same time, Klein [3] and Sauter [4] pioneered the concept of creating electron–positron pairs from the vacuum in the presence of an EM potential above a critical value. This critical field strength corresponds to the value needed such that the energy gain of an electron accelerating over a Compton wavelength ($\lambda_C$) is the rest mass of an electron. They computed this critical value to be:

$$E_c \equiv \frac{m_e c^2}{e \lambda_C} = \frac{m_e^2 c^3}{e h} \approx 1.3 \times 10^{16} \text{ V cm}^{-1},$$

where $m_e$ is the electron mass, $c$ is the speed of light, $e$ is the charge of an electron, $h$ is the Planck constant, and $\lambda_C \equiv h/m_e c$ is the Compton wavelength of the electron. In static electric
fields of this strength the quantum vacuum becomes unstable, spontaneously breaking down into a real electron–positron pair [4]. In this class of processes, the crucial criterion is that the effective photon energy from the field strength has to be sufficiently high to overcome the pair mass. At that time, several others were also studying the creation of pairs from the vacuum via the collision of light quanta. In the year 1934, Breit and Wheeler published their theory of pair creation via the inverse of the Dirac annihilation process, namely the creation of an electron–positron pair via the collision of two light quanta (i.e. photons):

\[ \gamma + \gamma \rightarrow e^+ + e^- \tag{3} \]

This process, the time inverse of the Dirac annihilation process, has become known as the Breit–Wheeler (BW) process. While the Dirac annihilation process was quickly realized [5–7] and has since become one of the most rigorously tested processes in physics [8], experimental observation of the B process proved to be much more elusive. Indeed, Breit and Wheeler clearly recognized the difficulty of observing the proposed process of electron–positron pair creation, commenting in their 1934 paper [9] that it is ‘hopeless to try to observe the pair formation in laboratory experiments with two beams of x-rays or gamma-rays meeting each other.’ Their comment was an astute observation, considering the difficulty observing the process even after decades of technological advancements, including the invention of lasers [10]. However, Breit and Wheeler noted Fermi’s [11] method of equivalent photons and the recent work of Williams [12] and Weizsäcker [13] which indicated that ultra-relativistic beams of highly charged nuclei may provide a viable photon source:

In the considerations of Williams, however, the large nuclear electric fields lead to large densities of quanta in moving frames of reference. This, together with the large number of nuclei available in unit volume of ordinary materials, increases the effect to observable amounts.

Theoretical perspectives on the validity and viability of using Weizsäcker–Williams (WW) photons in heavy-ion collisions (HICs) to achieve the BW process will be discussed further in section 3.1.

In addition to Breit and Wheeler, several others were also investigating the concept of pair production in the same year. Unlike Breit and Wheeler, who were specifically interested in the collision of real photons, Bethe and Heitler [14] introduced the concept of pair production via a collision between one real photon and a virtual photon from the Coulomb field of a nucleus. Similarly, Landau and Lifshitz [15, 16] studied the total production cross section for electron–positron pairs from the collision of two virtual photons from high-energy particle beams.

Over the next few decades, the work of Dirac [17], Feynman [18], Schwinger [19], Baranger et al [20], Tomonaga [21], Dyson [22], and several others to describe the gauge invariant interactions of charged particles with photons grew into what we now call quantum electrodynamics (QED)—the fully covariant relativistic quantum field theory describing the interactions between light and matter. The ability of QED to precisely predict the Dirac annihilation process and many other processes has led to it being regarded as the most rigorously tested theory in physics [23, 24].

12. The BW process

Figure 1 shows the two Feynman diagrams representing the BW process at lowest-order (second-order in the EM coupling strength) in QED. Four-momentum conservation renders direct pair production from a single photon impossible, making the BW process the simplest mechanism in QED for converting photons into an electron–positron pair. The potential interaction of two photons to produce an electron–positron pair is a clear violation of the superposition principle, an essential feature of the linear theory of classical electromagnetism. Additional non-linear features of QED will be discussed in more detail in the section on vacuum polarization and vacuum birefringence (VB) (section 1.3). The central result of Breit and Wheeler’s investigation was the cross section for photon–photon collisions leading to electron–positron pair production:

\[
\sigma (\gamma \gamma \rightarrow e^+ e^-) = 4\pi \alpha^2_{\text{em}} \left[ \frac{2 + \frac{8m^2}{W^2} - \frac{16m^4}{W^4} + \ln \left( \frac{W + \sqrt{W^2 - 4m^2}}{2m} \right)}{W^2} \right] \]

\[
- \sqrt{1 - \frac{4m^2}{W^2}} \left( 1 + \frac{4m^2}{W^2} \right) \tag{4}\]

where \( \alpha_{\text{em}} \) is the EM coupling constant, \( m \) is the lepton mass (\( m = m_e \) for electron–positron production), and \( W \) is the invariant mass of the produced pair. Note that when \( W = 2m \),
\[
\ln \left( \frac{W + \sqrt{W^2 - 4m^2}}{2m} \right) = \ln(1) = 0,
\]
and
\[
\sqrt{1 - \frac{4m^2}{W^2}} = 0,
\]
illustrating a key feature of this process, namely that the pair production cross section is zero for center-of-mass energy \( s, \gamma = W^2 \) below threshold \( s = 4m^2 \). This feature is crucial to understanding the near impossibility of achieving the BW process in terrestrial laboratories, since the peak cross section is not much smaller than other more easily achievable processes (e.g. Compton scattering). Furthermore, as noted above, a minimum of two photons are needed for the center of mass energy of the system to exceed the threshold and simultaneously conserve momentum, allowing the conversion of the initial mass-less state (two photons) into a pair of massive leptons.

In addition to calculating the total cross section for \( \gamma \gamma \rightarrow e^+e^- \), Breit and Wheeler also considered the photon’s spin-1 character, a fundamental attribute of the photon’s quantum nature. Since the photon is a massless spin-1 particle, it can have spin projections of only ±1 character, a fundamental attribute of the photon’s quantum nature. However, when two sufficiently high-energy photons are not available for the linear BW process, the non-linear BW process can still be achieved through the fusion of multiple lower-energy photons:

\[
\gamma + n\gamma \rightarrow l^+l^- ,
\]  

where, in this case, one photon interacts with \( n \) additional photons to produce a lepton anti-lepton pair. This process allows photons that do not satisfy the energy threshold of the linear BW process to combine, and collectively overcome the threshold for lepton pair creation.

The E-144 experiment at the Stanford Linear Accelerator Complex (SLAC) was built with the express purpose of exploring QED in the strong field regime [28]. The E-144 experiment, illustrated in figure 2, orchestrated a collision between a low-emittance 46.6 GeV electron beam and a (green) 527 nm wavelength pulse from a terawatt Nd:glass laser. With this setup they achieved a peak laser intensity of \( \sim 1.3 \times 10^{18} \) W cm\(^{-2} \) corresponding to a value of 0.3 in the parameter \( T \), where \( T = \frac{E_{\text{rms}}}{E_c} \), \( E_{\text{rms}} \) is the root-mean-squared electric field strength of the laser in the electron rest frame, and \( E_c \) is the QED critical field strength for electron–positron production given in equation (2).

After demonstrating their sensitivity to nonlinear QED effects via measurements of nonlinear Compton back-scattering [27, 29], the E-144 experiment made a discovery observation of 106 ± 14 positrons produced in the multiphoton BW process [28]. The observed positrons resulted from a two-step process used to achieve the multiphoton BW process. In the first step of this process, an electron from the 46.6 GeV electron beam undergoes nonlinear Compton scattering by which laser photons (denoted \( \omega \)) are back-scattered, resulting in a single GeV energy photon (\( \gamma \)):

\[
e + n\omega \rightarrow e' + \gamma ,
\]

Next, as the high-energy photon (maximum energy of 29.2 GeV for Compton-backscattering from a 46.6 GeV electron beam) travels through the laser field it may interact with additional photons, overcoming the energy threshold needed to produce an electron–positron pair:

\[
\gamma + n\omega_0 \rightarrow e^+ + e^- ,
\]
where, $\omega_0$ denotes the 527 nm laser photons with an energy of $\hbar \omega_0 = 2.35$ eV, and where $\gamma$ denotes the GeV-energy photon produced in step 1 (equation (8)). Since the Compton scattering process leads to numerous electrons detected in the electron calorimeter (ECAL), the process was identified through the detection of positrons in the positron calorimeter (PCAL). The measurement was conducted for laser-on and laser-off trials to establish the background level of positron production. Comparison of the two cases demonstrates a clear excess in positron production for the laser-on case, compared to the laser-off case, as shown in figure 3. Figure 4 shows the measured positron momentum spectrum, determined from the laser-on spectrum minus the normalized laser-off spectrum. The result is in good agreement with model calculations of the multi-photon BW process, in terms of both the overall cross section and the differential distribution [30]. Since the E-144 experiment identified only the positron from the process, and was not able to identify the electron counterpart, they did not obtain full information from the process. In contrast, the recent measurement of the linear BW process by STAR [31] included precise measurement of both the produced electron and positron, allowing polarization dependent effects to be observed. Further details of the STAR measurement will be discussed in section 3.3.2. In the next subsection, the effects due to the polarization of photons will be discussed in more detail.

1.3. Vacuum birefringence

In the 1930s, Dirac [17], Heisenberg and Euler [32], and Weisskopf [33] all explored the corrections to classical electromagnetism due to virtual electron–positron pairs. The Euler–Heisenberg effective Lagrangian describes the non-linear dynamics of EM fields—a clear deviation from the classical Maxwell theory of electromagnetism, which is manifestly linear. Later, Schwinger [34] derived the effective Lagrangian in the proper-time integral within QED [19]. The work of Euler, Heisenberg, and Schwinger showed that the presence of a background field polarizes the vacuum by interacting with the short-lived virtual particles that are continuously and nearly instantaneously created and annihilated.

Toll carried out pioneering work to understand vacuum polarization in the strong-field regime of QED [35]. His work
built on the idea that the vacuum itself behaves as a polarizable and magnetizable medium, which can lead to birefringence (i.e. double refraction) of light as it passes through a region of space in the presence of an external magnetic field \([33, 36–38]\). This is known as VB, a phenomenon in which the QED vacuum gains distinct refractive indices, different from unity, for each polarization mode of the photon \([32, 39]\). The effects of VB can be concisely summarized as \([40]\):

- Maxwell’s equations are no longer linear and the superposition principle is violated;
- in vacuum light-by-light scattering can occur (see figure 5(a)) and the velocity of light is \(v_{\text{light}} < c\) in the presence of other EM fields;
- electromagnetism in vacuum is described by Maxwell’s equations in a medium.

In birefringent media, the refractive index of light depends on its polarization. The strength of the birefringence effect is often characterized by the maximum difference in the indices of refraction exhibited by the material. In materials with a single optical axis, this can be expressed as the difference in refractive indices \((\Delta n)\) for light polarized parallel \((n_{||})\) vs. perpendicular \((n_{\perp})\) to the material’s optical axis:

\[
\Delta n = n_{||} - n_{\perp}.
\]

VB is a purely quantum effect that becomes appreciable as the background field strength approaches the critical field strength, growing with the square of the electric and/or magnetic field strength according to the relation \([43]\):

\[
\begin{align*}
\{ n_{||} \} = 1 + \frac{\alpha_{\text{em}}}{\pi} & \left[ \frac{eE}{m^2} \right]^2 \sin^2 \angle(k, \vec{E}) \\
+ \left( \frac{eB}{m^2} \right)^2 \sin^2 \angle(k, \vec{B}) - \frac{2eEeB}{m^2 m^2 k^2} \cdot \hat{s},
\end{align*}
\]

where \(\hat{s}\) is the photon momentum, \(\hat{s} = \frac{\vec{E} \times \vec{E}}{\vec{E}^2}\) is the unit Poynting vector, and \(\angle(a, b)\) denotes the angle between \(a\) and \(b\). For a purely magnetic field the strength of the birefringence effect can be expressed with the simpler relation:

\[
\Delta n = k_{\text{CM}}B^2,
\]

where \(k_{\text{CM}}\) is the Cotton–Mouton coefficient \([44]\) with a value of:

\[
k_{\text{CM}} = \frac{2\alpha_{\text{em}}^2 \hbar^3}{15\mu_0 m^2 c^5} \approx 4.0 \times 10^{-24} \text{T}^{-2},
\]

where \(\mu_0\) is the vacuum magnetic permeability. This value illustrates the smallness of the effect for everyday values of the magnetic field strength and the difficulty in observing this non-linear effect of QED.

An optical medium can, in general, permit both transmission and absorption of EM waves. Both the transmission and absorption processes can be described by generalizing the (real valued) index of refraction \((n)\) to a complex index of refraction \((\tilde{n})\):

\[
\tilde{n} = n + i \kappa,
\]

where \(n\) and \(\kappa\) are real-valued. In the complex index of refraction, the transmission process is characterized by the real part \((n)\) while the absorption process is characterized by the imaginary part \((\kappa)\). In general, a birefringent medium may have both \(\Delta n \neq 0\) and \(\Delta \kappa \neq 0\). When \(\Delta \kappa = 0\), the medium’s opacity varies with respect to the incident polarization. While the term, ‘birefringent’ entails both the transmission and absorption phenomena, historically, VB has been used to describe primarily the transmission phenomena. The polarization-dependent absorption phenomena characterized by \(\Delta \kappa \neq 0\) is known as vacuum dichroism \([45, 46]\). Photon splitting, though forbidden in vacuum, is possible in a background EM field with at least three couplings between the field and the internal fermion loop. Such a case, leading to photon splitting in the presence of a background field, can also lead to a nonzero value of \(\Delta \kappa\) and dichroism \([47–49]\). The connection between VB, vacuum dichroism, and the BW process will be discussed further in sections 4 and 5.

Vacuum polarization was confirmed with the groundbreaking discovery of the Lamb shift, a purely quantum mechanical effect that leads to the splitting of the hydrogen \(^2\text{S}_{1/2}\) and \(^2\text{P}_{1/2}\) energy levels \([50]\). However, VB has been much more difficult to confirm experimentally. Terrestrial experiments are generally limited by the difficulty in obtaining sufficiently strong EM fields over macroscopic lengths, while cosmological experiments suffer from experimental uncertainties related to the photon sources and polarizing EM fields \([40]\).

1.3.1. Earth and space based searches for VB. HICs \([51–53]\) are believed to produce some of the strongest EM fields in the Universe, with magnetic fields reaching \(10^{15} \text{T}\). However, these strong fields exist only for extremely short time scales over short femtometer distances, making their effects exceedingly difficult to observe. Some astronomical objects, such as
magnetars, a class of neutron stars, are believed to produce magnetic fields on the order of $10^{12-15}$ Gauss ($10^8-11$ T) [54] over a large spatial extent. The presence of such strong fields over macroscopic lengths would be sufficient to make quantum effects like VB visible. One of the expected signatures of VB acting on surface emission from magnetars is the apparent increase in polarization of light observed by a distant observer [55, 56]. Recent optical polarimetric measurements of the isolated neutron star RX J1856.5-3754 found an increase in the polarization of incident light which was interpreted as potential evidence for VB [57]. However, due to the low significance of the result, uncertainties in the neutron star models, and uncertainty in the direction of the neutron magnetization axis, the measurement from RX J1856.5-3754 is not able to confirm VB unambiguously [40]. In fact, more recent measurements [58] and theoretical analysis [59] cast doubt on the interpretation of this particular measurement as evidence for VB.

Discovery of VB has also been sought through terrestrial experiments. Such experiments are generally set up to search for a change in the polarization of light, since linearly polarized light may acquire an elliptical component after traveling through a birefringent medium. Specifically, one of the potentially observable effects of VB in a pure magnetic field is that it induces an ellipticity ($\psi$) to linearly polarized light (with wavelength $\lambda$) traveling through it, of the strength:

$$\psi = \pi k_{CM} \frac{L_{\theta}}{\lambda} B^2 \sin 2\theta,$$

where $L_{\theta}$ is the path length through the magnetic field and $\theta$ is the angle between the polarization and the birefringence axis (magnetic field orientation). From an experimental perspective, the $L_{\theta}B^2$ dependence of the induced ellipticity is what makes such an effect potentially observable, by utilizing laboratory strength magnetic fields over macroscopic lengths. The total ellipticity ($\Psi$) gained by linearly polarized light traversing a cavity depends on the cavity’s finesse ($F$) as $\Psi = \psi(2F/\pi)$. The PVLAS (Polarisation of Vacuum with LASer) experiment, shown schematically in figure 6, is one such experiment that has employed high-powered lasers, strong magnetic fields, and an optical Fabry–Pérot cavity to search for VB [60]. Their progress over the last 25 years and final results, which set the strongest limits on VB, are summarized in a comprehensive review [40]. Despite improvements in the measurement sensitivity over the years, the PVLAS experiment has not been able to demonstrate feasibility for performing a definitive measurement of VB, as illustrated in figure 7. On the other hand, their results have allowed exclusion of model-independent parameter space of axion-like and milli-charged particles, since the existence of such particles are expected to increase the VB effect above that predicted for standard model particles alone.

The Biréfringence Magnétique du Vide (BMV) experiment also used a very high finesse Fabry–Pérot cavity with a pulsed transverse magnetic field to search for VB [65, 69]. The latest result from BMV came in 2014 with a measured value of $k_{CM} = \Delta n/B^2 = (8.3 \pm 8.0) \times 10^{-21} \text{T}^{-2}$ [65]—several orders of magnitude away in sensitivity from the predicted QED value. The Observing Vacuum with Laser (OVAL) experiment, underway now, uses a similar apparatus as that of PVLAS and BMV, but with stronger, pulsed magnetic fields. The OVAL experiment has recently completed a calibration measurement to demonstrate the feasibility of observing VB [68] and finds value of $k_{CM} = \Delta n/B^2 = (-0.5 \pm 1.1) \times 10^{-18} \text{T}^{-2}$. However, as depicted in figure 7, none of these experiments have achieved the sensitivity required to observe VB, though advances in laser power are expected to allow more stringent tests of QED in similar terrestrial experiments in the coming decade [70–76]. To this end, significant theoretical progress has been made to identify

![Figure 6. Schematic of the PVLAS polarimeter experiment. A rotating magnetic field between the cavity mirrors generates a time dependent ellipticity. Reproduced from [40]. CC BY 4.0.](image)

![Figure 7. Historical time evolution of the measurement of vacuum birefringence normalized to $B_{CM}^2$. Error bars correspond to 1σ uncertainties. Values are taken from the following references: BFRT [61], PVLAS-LNL [62, 63], PVLAS-Test [64], BMV [65], PVLAS-FE [66, 67], and OVAL [68]. Reproduced from [40]. CC BY 4.0.](image)
the signatures of VB in various experimental laser setups [77, 78]. With all of the essential pieces in place and rapid progress being made, the next decade is sure to be an exciting time for laser-based QED experiments.

### 1.4. Context and structure of this report

The STAR collaboration recently reported observation of the linear BW process and a polarization effect related to VB. This article presents the state of affairs, as of 2023, with special emphasis given to the discovery claims made by the STAR collaboration (see section 5) and the various challenges to such claims that have been raised in past literature. While many reviews on the subject of electromagnetically produced lepton pairs in HICs exist [79–82], we find that certain key issues and their physical interpretations are discussed in conflicting ways. We list these key issues and questions in section 2 related to the BW process and VB. One of the primary goals of this report is to rectify elements in the literature that have led to confusion and contradictions relevant to the interpretation of the BW process and VB, specifically with respect to observations made in high-energy HICs. To this end, section 3 and the subsection within are devoted to presenting the relevant theoretical and experimental details needed to evaluate each of the key issues related to the BW process. Similarly, section 4 outlines the theoretical relationship between the polarized BW process and VB, which is necessary to evaluate the key issues related to the VB. We then continue with a discussion (section 5) by summarizing and directly addressing the challenges and issues raised in section 2. Finally, we close the article with perspectives for the future of the field (section 6) and with a few final conclusions (section 7).

### 2. Key issues

In this section, we summarize key issues and difficulties regarding the interpretation of photon–photon measurements from high-energy particle and HICs. We list a few important questions with respect to: (a) the BW process, and (b) VB that will be addressed throughout the remainder of this report.

#### 2.1. The BW process

Over the last many decades, high-energy particle colliders have made the creation of matter in the fusion of virtual photons a commonplace event \[\gamma \gamma \rightarrow e^+ e^-\] production. Since the photons are quantized from the extended field of a heavy ion at relativistic speed, it is also a vacuum polarization effect. However, the relationship between this newly observed effect and VB is not straightforward since the experimental measurement is not directly comparable to traditional birefringence or dichroism experimental designs. In order to clarify the connection, we seek to answer the following questions:

- Has the BW process previously been observed in high energy particle or HICs?

These key issues regarding the BW process will be addressed in section 3 through a review of the relevant theoretical and experimental advances made over the last several decades.

#### 2.2. Vacuum birefringence

STAR has recently observed the polarization dependence in \[\gamma \gamma \rightarrow e^+ e^-\] production. Since the photons are quantized from the extended field of a heavy ion at relativistic speed, it is also a vacuum polarization effect. However, the relationship between this newly observed effect and VB is not straightforward since the experimental measurement is not directly comparable to traditional birefringence or dichroism experimental designs. In order to clarify the connection, we seek to answer the following questions:

- What is novel about the STAR measurement, i.e. how does it differ from the SLAC E-144 measurement, with respect to polarization effects?
- How is the azimuthal angle ($\phi$) modulation observed by STAR related to VB and/or dichroism?

Section 4 will build upon the discussion in section 3 to address these key issues related to VB. Finally, after introducing all of the relevant theoretical progress and experimental results, section 5 will revisit each of the key issues to summarize their current status.

### 3. The BW process in HICs

This section will address the key issues regarding the observation and interpretation of the \[\gamma \gamma \rightarrow e^+ e^-\] process in high-energy particle and HICs. Each issue is addressed in a dedicated subsection.

#### 3.1. Validity of the photon source

Many publications in the three decades prior to 2021 explicitly stated that the BW process ‘is still today waiting a direct observational verification’ [86]. On the other hand, the nonlinear BW process (multiple photon version) observed in an experiment at SLAC [28] in 1997 has been widely recognized as a discovery of that related process. In the last ten years, there have been theory proposals on how to discover the BW process in photon–photon collisions using high-intensity lasers [87] and a vacuum hohlraum [88], which explicitly stated that the BW process ‘has never been observed in the laboratory’. However, creative experimental designs and increasing laser power may render the exclusive BW process achievable at laser facilities [86, 87, 89–91] in the near future. Clearly, the community considers laser photons as a valid source of photons for achieving the BW process. However, the validity of photons manifest from ultra-Lorentz contracted EM fields is less clear. Therefore, in this section we will investigate
whether or not the EM fields of HICs can be a valid source of photons for the BW process and discuss reasons for differing opinions over the decades.

It was Fermi in 1924 who first described EM fields in terms of their equivalent photon spectrum [11]. The central idea in this equivalence is that the Lorentz-contracted EM fields of a fast-moving charged particle appear as a radial electric and circular magnetic field. At a distant point, such a field configuration resembles the EM wave carried by a real photon. Based on this insight, Williams and Weizsäcker later developed the method of equivalent photons, whereby the number of photons \( n \) with a given energy \( \omega \), may be related to the Fourier transform of the time-dependent EM field [12, 13]. This concept has been employed to describe a wide range of physical phenomena in high-energy particle collisions. In this section we briefly review the application of the equivalent photon approximation (EPA) giving special attention to the question: under what conditions, if any, do WW photons from highly-Lorentz contracted Coulomb fields provide a valid and viable source for achieving the BW process?

### 3.1.1. Theoretical description of equivalent photons from high-energy electron beams

According to the method of equivalent photons, the number density of photons for the BW process can be computed as

\[
\sigma_{\gamma\gamma\rightarrow X} = \frac{\alpha_{\text{em}}}{\pi} \left[ 1 - \frac{\omega^2}{E^2} \right] \frac{d\omega}{\omega} \frac{dQ^2}{Q^2} \tag{16}
\]

This description of the equivalent photon spectrum resulting from a high energy electron beam has proven a convenient tool for many calculations over the decades even though its applicability has some known limitations [92].

In this formulation, the cross section for a process of the form \( e^+ e^- \rightarrow e^+ e^- + X \) can be computed by the convolution of the photon number density with the appropriate photoproduction cross section \( \sigma_{\gamma\gamma\rightarrow X} \), i.e. \( d\sigma_{\gamma\gamma\rightarrow X}(x) = dn_{\gamma\gamma\rightarrow X}(W^2) = \frac{d\gamma_{\gamma\gamma\rightarrow X}(W^2)}{d\gamma} d\gamma \). Where \( W \) is the invariant mass of the produced system \( X \). The total production cross section can be obtained by integrating over the \( Q^2 \) range from \( Q^2_{\min} = \frac{m_{\gamma\gamma}^2 \omega^2}{E(E - \omega)} \) to \( Q^2_{\max} \), which is determined by the details of the produced system. The resulting total cross sections for the production of various final states can be found in [93]. The minimum cutoff is especially important, as the photon flux shown in equation (16) diverges for \( Q^2 \rightarrow 0 \). For this reason, electron beams cannot provide a source of real photons which interact without significant effects due to finite virtuality. Therefore, electron beams have never been considered a viable source of photons for the BW process, even though they have provided valuable experimental information about virtual photon interactions [94].

### 3.1.2. Theoretical description of equivalent photons in HICs

The photon flux provided by the Lorentz-boosted Coulomb field of an ultra-relativistic heavy nucleus shares some similarities with the virtual photon flux of a high-energy electron beam. However, there are a number of qualitative differences between the two cases. One important difference is due to the point-like structure of the electron compared to the diffuse charge distribution within a heavy nucleus. Another practical difference arises from the significantly heavier mass of highly-charged nuclei, making them less easily deflected. The implications of these differences will be further expounded below.

In the EPA, two photon interactions are computed by factoring the calculation into a semi-classical and a quantum component. The equivalent flux of photons is dealt with in terms of an external classical field, while the quantum part of the calculation deals with the elementary cross section for the \( \gamma\gamma \rightarrow X \) cross section (where, e.g. \( X = e^+ e^- \) for the BW process).

According to the EPA, the number density spectrum of photons with energy \( \omega \) [95] manifested by the field of a single nucleus is:

\[
n(\omega) = \frac{(Ze)^2}{4\pi \omega} \int_0^\infty d^2k_\perp \left( \frac{F(\frac{\omega}{2k_\perp})}{(\frac{\omega}{2k_\perp})^2 + k_\perp^2} \right)^2 \frac{k_\perp^2}{\omega^2}, \tag{17}\]

where \( Z \) is the nuclear charge number, \( \gamma \) is the Lorentz factor, \( k_\perp \) is the photon transverse momentum, and \( F(\ldots) \) is the nuclear EM form factor. The nuclear EM form factor for a spherically symmetric nucleus can be obtained via the Fourier transform of the charge distribution as:

\[
F(k^2) = \int d^3r \rho_\lambda(r). \tag{18}\]

Generally, a Woods–Saxon distribution is assumed to describe the charge distribution of heavy nuclei [96] without any fluctuations or point-like structures:

\[
\rho_\lambda(r) = \frac{\rho_0}{1 + \exp[(r - R_{WS})/a]}. \tag{19}\]

where the radius \( R \) and skin depth \( a \) are based on fits to low energy electron scattering data such that all deformations are assumed to be higher order and are ignored [97], and \( \rho_0 \) is the normalization factor for the distribution. Since the Fourier transform of the Woods–Saxon distribution does not have an analytic form, it is commonly approximated with a hard sphere, with radius \( R_{WS} \), convolved with a Yukawa potential with range \( a \) [98]. The resulting charge distribution form factor provides a good approximation to the Woods–Saxon distribution and has an analytical form. See [99] for a direct comparison between this form factor and numerical calculations from the exact Wood–Saxon distribution.

With these, the cross section for the polarization averaged two-photon process in HICs can be computed as:

\[
\sigma_{A+A\rightarrow A+A+\gamma\gamma} = \int d\omega_1 d\omega_2 n_1(\omega_1) n_2(\omega_2) \times \sigma_{\gamma\gamma\rightarrow \gamma\gamma}(W;m), \tag{20}\]

[References]
where $W$ is the invariant mass of the produced lepton pair and $n_1(\omega_1)$ and $n_2(\omega_2)$ are the equivalent number of photons with energies $\omega_1$ and $\omega_2$ from the field of nucleus 1 and 2, respectively. In the next subsections, we review various theoretical approaches for computing the photon–photon fusion process in HICs. The following subsections review various theoretical approaches: 1) the traditional EPA approach, 2) lowest-order QED, and 3) Wigner quasi-probability distributions. Then we discuss the issue of photon virtuality and present a well-defined criterion for the domain of applicability of the BW process in HICs.

3.1.2.1. Traditional EPA calculations. The above description provides a fairly direct and straightforward method for calculating the total cross section of the BW process in HICs. However, the differential cross sections reflecting the detailed kinematical distributions of the produced electron and positron are less straightforward to compute. When the pair transverse momentum, $P_{\perp}$, is small compared to the lepton pair invariant mass $W$, the photon energies are related to the lepton pair invariant mass and rapidity $y$ as

$$\omega_{1,2} = \frac{W}{2} e^{\pm y}, \quad (21)$$

and

$$y = \frac{1}{2} \ln \frac{\omega_1}{\omega_2}. \quad (22)$$

Furthermore, the EPA calculations determine the photon transverse momentum $(k_{\perp})$ distribution using the so-called $k_{\perp}$-factorization (throughout this review we use the term "$k_{\perp}$-factorization" as defined in [100, 101]). In this approach the one-photon distribution is integrated over all transverse distances, i.e. $0 < k_{\perp} < \infty$ to obtain the transverse momentum $(k_{\perp})$ distribution of the interacting photons (equation (16) of [102]):

$$\frac{dN(\omega,k_{\perp})}{dk_{\perp}} = 2F^2(\omega/\gamma)^2 + k_{\perp}^3 \frac{\gamma^2}{2\pi^2(\omega/\gamma)^2 + k_{\perp}^2}. \quad (23)$$

An essential feature of the $k_{\perp}$-factorization approach is that the photon transverse momentum distribution is independent of the collision geometry (impact parameter). From this, the transverse momentum of the produced pair $(P_{\perp})$ is determined via a direct sum of the two colliding photon momenta. However, some experiments cannot measure the transverse momentum accurately and instead characterize the kinematics of the process via the acoplanarity ($\alpha$), defined as:

$$\alpha = 1 - \left| \phi_1 - \phi_2 \right| \pi, \quad (24)$$

where $\phi_1$, $\phi_2$ are the azimuthal angles of the first and second lepton tracks, respectively. The acoplanarity is straightforwardly related to the pair transverse momentum as $\sqrt{2}P_{\perp} \simeq \pi \omega/\gamma/2$ [103, 104], though it is less ideal compared to measurement of $P_{\perp}$ directly since it conflates the momentum and energy of the produced lepton pair.

3.1.2.2. Lowest-order QED. Pair creation via the two photon interaction at lowest order can be depicted as a process with two Feynman diagrams contributing, as shown in figure 1. There is an approximation commonly used for describing events: that of external fields generated by nuclei that are undeflected by the collision and travel along straight-line trajectories (see figure 8(c)). Following the derivation of [25, 105], the cross section for pair production of leptons is given by

$$\sigma = (Z\alpha)^4 \left( \frac{4}{\beta^2} \frac{1}{(2\pi)^6} \right) \int d^3q \frac{d^3P(\vec{q})}{d^3p_+ d^3p_-} \int d^3\vec{b} e^{i\vec{q} \cdot \vec{b}}, \quad (25)$$

where $b$ is the nucleus–nucleus impact parameter and $p_+$ and $p_-$ are the momentum (energy) of the created leptons. The full form of the differential probability $P(\vec{q})$ for QED at lowest order is given in [104]. The differential probability depends on $F(...)$, the nuclear EM form factor, $p_+$ and $p_-$, the mass of the produced leptons, and the photon momenta $q_1$ and $q_2$ where the longitudinal ($z$) component of $q_1$ is given by $q_{1z} = \frac{1}{2}(\epsilon_+ + \epsilon_-) + \beta(p_{+z} + p_{-z})$. Unlike the $k_{\perp}$-factorization method, the lepton pair kinematics computed in lowest order QED clearly depends on the nucleus–nucleus impact parameter, evident in the $\int d^3\vec{b} e^{i\vec{q} \cdot \vec{b}}$ term. In order to compute results at all impact parameters, where in general no simple analytical form is available, the multidimensional integration should be performed with numerical algorithms such as the VEGAS Monte Carlo integration routine [106].

Figure 8. Feynman diagrams representing photon–photon interactions in heavy-ion collisions. Diagram (a) represents the Landau–Lifshitz process $(\gamma^* \gamma^* \rightarrow e^+ e^-)$ which describes $\gamma^* \gamma^*$ production from all photon–photon interactions (regardless of virtuality). Diagram (b) illustrates the Bethe–Heitler process $(\gamma^* \gamma \rightarrow e^+ e^-)$ in heavy-ion collisions. In the external field approximation, the photons are manifest from the undeflected electromagnetic field (c), which puts strict limits on the photon kinematic and scattering properties, allowing the Breit–Wheeler process in special cases (see section 3.1.3). The large grey vertices indicate photon coupling to the coherent electromagnetic field of the heavy ion.
3.1.2.3. PWF formalism. The Wigner quasi-probability distribution, often shortened as the Wigner function, is a method of relating a system’s quantum wave function to a quasi-probability function in phase space [107–109]. The Wigner function is similar to a classical probability distribution, in that it describes the phase space distribution of a process. The Wigner function is a quasi-probability distribution because, unlike its classical analog, it does not satisfy all properties of a probability function. The most obvious difference is that Wigner functions can take on negative values for states of the quantum system that have no classical analogue, e.g. states characterized by quantum interference of wave functions. To our knowledge, the authors of [110] first suggested that the full spatial dependence of the $\gamma\gamma$ processes, including the impact parameter dependence, may be expressed in terms of photon Wigner distributions.

Since then, several groups have investigated the photon Wigner function (PWF) formalism for describing the $\gamma\gamma$ processes in HICs [111–113]. Following the notation of [113], the photon Wigner function can be expressed as

$$ N_{ij}(\omega, b, q) = \int \frac{d^2Q}{(2\pi)^2} \exp[-ibQ] \times E_i(\omega, q + \frac{Q}{2}) E_j(\omega, q - \frac{Q}{2}) \left( \frac{d^2s}{\exp[qs]} \times E_i(\omega, b + \frac{s}{2}) E_j(\omega, b - \frac{s}{2}) \right) $$

(26)

where $E_{i,j}$ are the electric field vectors expressed in terms of the nuclear charge form factors ($F$, see equation (18)) as

$$ E(\omega, q) = Z \sqrt{\frac{\alpha_{em}}{\pi}} \frac{qF(q^2 + q_\parallel^2)}{q^2 + q_\parallel^2} $$

(27)

where, $q_\parallel = \omega/\gamma$.

By virtue of the Wigner function definition, equation (26) is a function of both the spatial location ($b$) and transverse momentum ($q$) coordinates. The standard formula for the photon flux in either momentum space or position space may be obtained by integration over $b$ or $q$, respectively. The differential cross section for lepton pair production can then be expressed in terms of the PWFs as a convolution over the transverse momenta and transverse positions [113]

$$ \frac{d\sigma}{dbdqFP} = \int d^2b_1 d^2b_2 \delta(2)(b - b_1 + b_2) \times \int \frac{dq_1 dq_2}{\pi} \delta(2)(P - q_1 - q_2) \times \int \frac{d\omega_1 d\omega_2}{\omega_1 \omega_2} N_{ij}(\omega_1, b_1, q_1) N_{ij}(\omega_2, b_2, q_2) \times \frac{1}{2\sqrt{\lambda}} \sum_{\lambda, \lambda} M_{i\lambda} M_{j\lambda}^* d\Phi(l^+l^-) $$

(28)

where the final term (equation (28)) is a sum over the helicity amplitudes for the $\gamma\gamma \rightarrow l^+l^-$ process and $d\Phi(l^+l^-)$ is the invariant phase space for the leptons [113].

3.1.3. Photon virtuality and a criterion for the BW process. It is often considered that the transverse momentum of the photons in HICs is related to the transverse dimensions of the nuclei and the photon virtuality, according to the uncertainty principle [114]. This has been used as an argument [89, 102, 115, 116] that the $e^+e^-$ pair production from HICs is not the BW process, despite the original proposal from Breit and Wheeler in their seminal paper [9]. In this section, we follow [117] via the S-Matrix derivation to illustrate the approximation which results in the EPA and propose a well-defined criterion for the BW process in relativistic HICs to clearly separate it from the Landau–Lifshitz [16] and Bethe–Heitler [14] processes involving virtual photons (see figure 8).

Since the Coulomb field of a nucleus is a pure electric field, the Lorentz boost does not change the fact that real photons cannot be generated by the field itself. The resulting photons, quantized from the EM field, would have the form shown in equation (17), with a spacelike Lorentz vector and a ‘negative squared mass’ (imaginary mass) of $-(\omega/\gamma)^2 + k_\perp^2)$. It was argued that if one were to define the process in HICs as the BW process, the virtuality would simply have to be ignored. In fact, this is not the case. Simply ignoring the virtuality by setting this term to zero would result in the infrared divergence of the photon flux. Instead, [117] shows the approximation required for the conserved transition current ($J^{\mu\nu}$) to behave as real photon interactions in the S-Matrix. Namely, the transition current can be expressed as:

$$ u_{1\nu}u_{2\mu}J^{\mu\nu} = \gamma^2 \frac{k_1 k_2}{k_{10} k_{20}} \left( \frac{1}{\nu} \left( k_2 f_2 - k_1 f_1 \right) \right) - \frac{1}{\gamma^2 v^2} J^{33} $$

(29)

where $u_{1,2} = \gamma(1, 0, 0, \pm v)$ are the four-momenta of nucleus 1 and 2, respectively, with velocity $v$ along the beam direction. In this form one can clearly see that the second and third terms are suppressed by at least $1/\gamma^2$ compared to the first. Importantly, when $|k_\perp| \approx \omega/\gamma$ the second and third terms can be safely omitted and the vertex function of the two-photon process in relativistic HICs becomes identical to that of the real-photon interaction (for the full derivation see equations (19)–(28) of [117]). The requirement in the center-mass-frame of the HIC is that both photons satisfy the following condition:

$$ \omega/\gamma \lesssim k_\perp \ll \omega. $$

(30)

The interpretation is therefore that the single photon flux of the virtual states from the Lorentz boosted field is given by equation (17) and that the interaction behaves as photons with real-photon states characterized by energy of $\omega$ and transverse momentum of $k_\perp$, validating the implementation of the process via the PWF formalism [110–113]. In this sense, STAR’s observation of the linear BW process [31] is concisely and accurately described as: the linear BW process has only been recently observed with quasi-real photon sources [118]. The form factor (field strength) in the photon flux limits the photon transverse momentum to be $k_\perp \lesssim 1/R$. In the regime of much higher $k_\perp$, $(k_\perp \gtrsim 1/R$ and/or $\omega \gtrsim \gamma/R$) there are significant contributions from the ‘semi-coherent’ process [119] due to
photon scattering off constituent nucleons and quarks inside the target nucleus, which may invalidate EPA assumptions. This puts a further constraint on the available phase space for the photons that may participate in the BW process [120]:

$$\frac{\omega}{\gamma} \lesssim k_{\perp} \lesssim 1/R \ll \omega.$$  \hfill (31)

With decreasing beam energy ($\gamma$) in the same kinematic acceptance, the phase space for the BW process decreases, and we would expect that the photons outside this valid range ($k_{\perp} \lesssim \omega/\gamma$) to contribute substantially to the interaction cross section at low beam energy. At extremely high energy, there are constraints on the validity of the BW process as well. The criterion can be readily defined in terms of acoplanarity since it is straightforwardly related as $\sqrt{2P_{\perp}} \simeq \pi \omega \alpha/2$ [103, 104]. Therefore, the criterion of the BW process in terms of acoplanarity reads:

$$\sqrt{2} \frac{\omega}{\gamma} \lesssim \frac{\pi}{2} \alpha \lesssim \sqrt{2} \frac{\omega}{R} \ll 1.$$  \hfill (32)

For the kinematics of A Toroidal LHC ApparatuS (ATLAS) experiment at the Large Hadron Collider (LHC) [103, 121] with $\gamma = 2500$ and $\omega \gtrsim 10\text{GeV}$ and shown in figure 9, the real-photon criterion becomes $4 \lesssim k_{\perp} \lesssim 30\text{MeV}$ or, in terms of acoplanarity, 0.0004 $\lesssim \alpha \lesssim 0.003$.

To illustrate the validity of BW process under these kinematic regimes, it is useful to compare the similarities and differences among the various formalisms used to compute the $\gamma \gamma \rightarrow e^+ e^-$ process in HICs. These connections are explored most thoroughly in [112]. Figure 10 shows the STAR measurement of $e^+ e^-$ transverse momentum compared with theoretical calculations from STARLight [102], lowest order QED [104], and the classical field approach [112] (see appendix A for details). Additionally, the differential cross section with respect to $\alpha$ for the $\gamma \gamma \rightarrow \mu^+ \mu^-$ process computed via the PWF formalism is shown in figure 9 and compared to data from ATLAS. The Wigner function calculation describes the data reasonably well over the kinematic intervals listed in equations (31) and (32), reproducing the qualitative features of the data. Furthermore, the lowest order QED and Wigner function formalism render equivalent results in the region of applicability for the BW process—again demonstrating the validity of the photon source for the BW process in the specified kinematic region. Indeed, based on the comparisons shown in [112, 113], it appears that the Wigner function calculations can recover the full impact parameter dependence of the lowest order QED result except at the very small values of acoplanarity [120], where the processes may no longer be due to the pure BW process, as discussed above.

### 3.2. Higher order processes in QED

In this section we address the presents or absence of higher order QED processes in HICs and whether or not such considerations make observation of the BW process invalid in such an environment. Due to the large charge carried by a heavy ion, the effective coupling $Z r_{\text{em}}$ ($\sim 0.6$ for e.g. gold and lead nuclei) is sufficiently close to unity that higher-order terms may not be negligible. This suggests the potential for higher order effects, which signify the crossover from the perturbative to the nonperturbative regime of QED. In 1954, the pioneering studies [123, 124] of higher order QED effects were made by Bethe, Maximon and Davies in a similar process—the Bethe–Heitler process [123] (the photoproduction of electron–positron pairs in a nuclear Coulomb field at rest). Higher order effects were treated using the Sommerfeld-Maue wave functions, which are appropriate solutions of the Dirac equation...
at high energy. This approach takes higher order effects into account to all orders in $Z_0$ and can be related to the usual Feynman graph technique [125].

A sizeable negative correction (compared to the lowest-order result) was found for the Bethe–Heitler formula [126]. For the BW process in HICs, the correction might be larger, since the quasi-real photon from the projectile is also attached to the field of the source nucleus, in contrast to the Bethe–Heitler process. It was pointed out by Ivanov et al [127] that the higher-order effect (or Coulomb correction) of lepton pair production in HICs with photon energy in the range of $m_\nu \ll \omega \ll \gamma m_\nu$ (for example, $0.5 \ll \omega \ll 50$ MeV for $e^+e^-$ at RHIC) was analogous to the well-known Bethe–Heitler process on a heavy target [14]. They established the equivalence between the calculations using Sommerfeld-Maue wave function and the standard higher-order QED approach. It is, however, important to realize that most of those Coulomb corrections are only effective for photon energy of a few MeV in the rest frame of the target nucleus. Therefore, none of the conditions presented in equation (4) of [128] for such a correction are satisfied in the situation under discussion in this review. It has been argued and demonstrated that Coulomb corrections in the kinematics relevant to the BW process under discussion ($\gamma_{1,2} \gg 1$ and $\gamma/R \gg \omega_{1,2} \gg 1/R$) are vanishingly small ($< 1\%$) [80, 125, 129–131]. This result is intuitive to understand, since, in the center-of-mass frame, an $e^+e^-$ pair within these kinematics behave as a neutral object in the Coulomb field (i.e. Coulomb cancel exactly) [131]. On the other hand, it may be possible that such Coulomb corrections are indeed present at the LHC through the dimuon channel [116, 132] with photon energy of $m_\nu \ll \omega \simeq 10$ GeV $\ll \gamma m_\nu \simeq 250$ GeV, in the expected range for significant Coulomb corrections [128] but may be absent in the $e^+e^-$ channel [133]. In this context of a negligible non-perturbative QED effect, some experts [80] concluded in dismay 15 years ago: ‘In April 1990 a workshop took place in Brookhaven with the title “Can RHIC be used to test QED?”’ [134]. We think that after about 17 years the answer to this question is “no”. However, many theorists were motivated to deal with this topic.

Another type of higher order QED effect is internal photon radiation, which leads to the appearance of a tail in the transverse momentum distributions of the produced lepton pairs. The authors of [135] study the QED showering effect utilizing a Sudakov formalism, which qualitatively described the notable tail in acoplanarity distributions observed by the ATLAS collaboration [103, 111]. The QED showering process does not affect the overall cross section, however it can modify the transverse momentum (acoplanarity) distribution of the produced vacuum pair, especially at higher transverse momentum (acoplanarity). There also exists a problem in that calculations via lowest order perturbation theory may violate unitarity [79]. Unitarity can be restored by introducing the production of multiple pairs [105, 136–138], which serves as an additional source of higher order correction. However, this source of higher order corrections is exceedingly negligible in the production phase space discussed in this report.

![Figure 11](image-url) First electroproduction measurement at the VEPP-2 collider. The plot shows the distribution of electroproduction events in the angle $\Delta\phi$—showing that the $e^+$ and $e^-$ are nearly back-to-back in the plane transverse the beam. The solid curve is a theoretical prediction for the $e^+e^-\rightarrow e^+e^-+e^+e^-$ by Baier and Fadin [139]. The dashed curve illustrates the expected distribution for the same process with independent and isotropic particle distributions. Reprinted from [140], Copyright (1971), with permission from Elsevier.

3.3. Experimental identification of the BW process

3.3.1. Measurements from electron–positron colliders. Significant attention was devoted to electroproduction ($e^+e^-\rightarrow e^+e^-+e^+e^-$) at the XVth International Conference on High Energy Physics held in Kiev in 1970 [141, 142] in anticipation of powerful electron–positron colliders about to come online in the Soviet Union, the United States, and Europe. The electroproduction process was first observed in 1971 at the VEPP-2, a 2000 MeV electron–positron collider [140] in Novosibirsk, Soviet Union. That observation consisted of only about 100 events found to be nearly coplanar with the colliding beams and with the peculiar feature that the produced $e^+$ and $e^-$ were nearly back to back in the plane transverse to the beam, as shown in figure 11. These characteristics distinguished the events from other known processes and supported the electroproduction hypothesis [139]; the logarithmic contribution to the back-to-back azimuthal correlation of $\gamma^-\gamma^+\rightarrow e^+e^-$ from the integration over the four-momentum transfer of the two virtual photons. Shortly after the ACO collider came online in France, the ADONE collider came online in Italy, and DORIS came online at DESY in Germany. Around the same time in the United States, SPEAR became operational at Stanford and CESR became operational at Cornell. With a host of new colliders online, the 1970s were well positioned to be a productive decade for high-energy particle physics in general and for further tests of electroproduction in particular [143, 144]. The rapid experimental progress spurred significant theoretical progress, especially in understanding electroproduction, which laid the foundation for future decades. For instance, the review article by Budnev et al [92] in 1975 outlined the fundamental calculations for...
3.3.2. Experimental progress in high-energy collisions. The first HIC experiments were envisaged as a tool for studying nuclear matter in a previously unexplored regime. Unlike high-energy $e^+e^-$ and hadron collisions, HICs produce a relatively larger and hotter system capable of melting the quarks and gluons composing protons and neutrons within nuclei—reaching temperatures of $\sim$4 trillion Kelvin [153–157]. The first manifestations of HICs were provided by accelerators with a fixed-target configuration, such as the BEVALAC at the Lawrence Berkeley National Laboratory [158]. Despite the focus on nuclear physics, HICs have been understood as an opportunistic tool for studying QED in unique regimes for nearly a century [12, 35], with significant interest in the last 40 years [129, 136, 159]. Some of the earliest measurements of EM processes in HICs were studied by the Berkeley BEVALAC [160], the Alternating Gradient Synchrotron at BNL [161], and by the CERN SPS [162].

In the year 2000, the Relativistic Heavy Ion Collider (RHIC) began operation and ushered in a new era of high-energy nuclear physics with ultra-relativistic Au beams providing collisions with a center-of-mass energy per nucleon pair ($\sqrt{s_{NN}}$) of 200GeV. The payoff from this next-generation facility was almost immediate, with all four RHIC experiments reporting observation of a new state of nuclear matter in 2004: the quark–gluon plasma (QGP) [153–156]. It should be emphasized that in an influential workshop in 1990 titled ‘Can RHIC be used to test QED?’ [134], three general subjects were addressed with the top priority as ‘to understand the validity of the best available descriptions of $e^+e^-$ pair production in peripheral heavy-ion collisions, especially for the domain where this process is known to be non-perturbative (multiple pair production’). It concluded in a positive perspective that ‘A study of EM phenomena in extremely peripheral collisions of relativistic heavy ions can become a rich and exciting field that will complement studies of central collisions.’ However, none of the three (Landau–Lifshitz, Bethe–Heitler and BW) processes were explicitly mentioned. In the next sections, the relevant detectors are described with special attention given to their unique capabilities for and limitations with respect to measuring novel QED phenomena, especially the BW process and VB.

3.3.3. Detectors, requirements, and limitations

3.3.3.1. Detectors. There were four experiments in operation shortly after RHIC began collisions in the year 2000 [163], two large and two smaller detectors. The Solenoidal Tracker at RHIC (STAR [164]) and the Pioneering High Energy Nuclear Interaction eXperiment (PHENIX [165]) were the larger general purpose detectors. In contrast, the BRAHMS [166] and PHOBOS [167] detectors were significantly smaller, more specialized detectors. Here we concentrate on the STAR and PHENIX detectors since these were used to test various aspects of QED through measurements of electromagnetically produced $e^+e^-$. In 2010, the Large Hadron Collider at the European Organization for Nuclear Research (CERN) began colliding heavy nuclei, and is to this day the highest energy collider of hadrons and heavy-ions. For the topics covered in this report, three LHC detectors are of interest: the Compact Muon Solenoid (CMS [168]), ATLAS [169] and the ALICE detector [170].

3.3.3.2. Detector subsystems and requirements. The STAR and PHENIX detectors at RHIC and the CMS [168], ATLAS [169], and ALICE [170] detectors at the LHC are each unique general purpose experimental apparatuses for studying high-energy particle and HICs. Despite their differences, they share many similarities. Here, we focus on the aspects of these experimental apparatuses that are most relevant for the measurements that will be discussed in the next sections. These detector systems are structured like onions, with each layer serving a specific purpose. See figure 12 for a schematic illustrating the typical layout of a general-purpose particle detector experiment. Note that the schematic does not directly correspond to any specific detector mentioned here but illustrates the relevant features common to those discussed here (STAR, PHENIX, CMS, ATLAS, and ALICE).

3.3.3.3. Tracker. The tracking subsystem is often the central component in any high-energy collider experiment. The primary purpose of any tracking detector is to measure the trajectory of (charged) particles produced in an event by recording their interactions with material at precise locations.

The success of the colliders in the 1970s motivated even more powerful machines with a thousand times higher reach in center of mass energy to be built in the 1980s and 1990s. At CERN in Europe, the Large Electron–Positron (LEP) collider was commissioned and brought online in 1989. Upgrades to the Stanford accelerator in the United States and to DESY in Germany provided several machines of comparable power around the world. Many more measurements of $e^+e^-$ production from two photons were performed at these machines. The increased energy and luminosity provided substantially more pairs and allowed more differential measurements.

The $e^+e^-$ pair production from the two-photon process was studied in high-energy $e^+e^-$ collisions at LEPII [94, 145], PETRA [146–148], and SLAC [149, 150] but with photons that were significantly virtual, except in a set of so-called un-tagged events. For instance, the OPAL [94] experiment used un-tagged $e^+e^-$ events to study the total cross section for producing hadronic particles from real photon collisions. We note that high-energy electrons can emit real photons in the vacuum (under a magnetic field), and a classic example is synchrotron radiation. The study of two-photon physics processes has been an active field, using both real and virtual photons to study hadron production and photon structure functions (see [84, 93, 151, 152] and references therein).
In general, they are embedded within a solenoidal magnet (STAR, CMS, ALICE) that provides an inner volume with a constant and uniform magnetic field. However, powerful large volume solenoid magnets are heavy and expensive. Therefore, some designs (e.g. PHENIX and ATLAS) utilize a combination of other magnet configurations. The purpose of the magnetic field, regardless of its exact configuration, is to allow a particle’s momentum to be determined from the measured radius of curvature ($r$) of its trajectory as it traverses the magnetic field.

For a constant uniform magnetic field, the radius of curvature is related to the magnetic field strength ($B$), the particle’s charge ($q$), and relativistic momentum $p = \gamma m|\vec{v}|$ as $r = p/(qB)$. However, in reality, a particle’s radius of curvature changes as it loses energy and/or scatters through interactions with material. For this and other cost-related reasons, tracking detectors are often placed closest to the interaction point, just outside the beam pipe. Finally, generally speaking, tracking detectors are only sensitive to charged particles and therefore do not measure the trajectories of neutrals, e.g. photons.

The STAR and ALICE experiments utilize a large volume gaseous tracker called a time projection chamber (TPC) embedded within a 0.5 Tesla magnetic solenoid. The low material gaseous TPC tracker within a relatively weak magnetic field provides optimal tracking for particles with momenta of a few hundred MeV/c. STAR and ALICE obtain optimal momentum measurement in the region $p_\perp < 100\text{ MeV/c}$ with a resolution of $dp_\perp/p_\perp \sim 1\%$. In contrast, the CMS and ATLAS detectors utilize stronger magnetic fields of 4 Tesla and 2 Tesla, respectively. Within the magnetic field volume, they employ various technologies for tracking, including silicon-based detectors which provide precise measurement of particle interaction locations with a low material budget. These detectors are optimized for measuring higher momentum particles, with a transverse momentum resolution of 200–300 MeV/c for particles with $p_\perp \lesssim 1.5 \text{ GeV/c}$ [171]. The following references provide additional details about the specific tracking subsystems (and their performance) used in STAR [164, 172], PHENIX [165, 173], CMS [174], ATLAS [175–177], and ALICE [178].

3.3.3.4. Identification of charged leptons and photons. The energy density achieved in HICs provides a sufficient energy budget for producing a plethora of subatomic particles of various species. The STAR, PHENIX, CMS, ATLAS, and ALICE apparatuses built for high-momentum particles with extreme multiplicity employ several specialized detector technologies for particle identification (PID)—i.e. the determination of a long-lived particle’s charge and mass. Here we focus on those most relevant for the identification of charged leptons (electrons, positrons, and muons) and photons. For particles observed in a tracking detector, their charge is directly determined from the sign of the measured curvature.

In general, a particle’s mass may be identified either by measuring its Lorentz factor or by measuring its energy [179] lost as it traverses a medium of known material. For particles at low momentum, measurement of its Lorentz factor is optimal, while, for particles at higher momentum, measurement of the energy is generally optimal.
Various technologies have been developed for the identification of electrons, positrons, and muons at low momentum (\(p < 1 \text{ GeV} \)). Ring Imaging Cherenkov detectors, used e.g. in PHENIX, exploit the relationship between a particle’s velocity (\(\beta \equiv v/c\)) and the emission angle (\(\theta\)) of Cherenkov light emitted as a particle passes through a medium with an index of refraction (\(n\)) of:

\[
\cos \theta = 1/(n\beta),
\]  

(33)

Time of flight (TOF) detectors perform PID by measuring a particle’s flight time (\(t\)) over a known distance (\(s\)), where \(s\) is often provided by the tracking system. The STAR, PHENIX, and ALICE experiments employ specialized TOF detectors capable of measuring particle flight times with a precision of \(\sim\)a billionth of a second. In conjunction with the momentum measured in a tracking system (\(p\)), a TOF detector allows a particle’s mass (\(m\)) to be determined through the relation:

\[
m = p\sqrt{s^2 - c^2},
\]  

(34)

where powers of \(c\), the speed of light in vacuum, are kept in this equation for clarity.

In addition to their utility as tracking detectors, the TPCs in the STAR and ALICE experiments also provide unique information for PID. As a charged particle travels through a TPC, it may ionize the gas within and lose energy as it travels. The mean ionization energy loss per unit length (\(\langle dE/dx \rangle\)) can be correlated with the measured magnetic rigidity to provide PID [164].

Identification of electrons and positrons at high momentum is generally carried out using EM calorimeters (ECALs). In high-energy physics experiments, any detector that measures the energy a particle loses as it passes through that detector is referred to as a calorimeter. They are designed in such a way as to force particles passing through them (within a certain energy range) to deposit all of their energy, therefore stopping within the calorimeter’s volume. Depending on the material composition of a calorimeter, they can be designed to measure the energy from particles that interact via different fundamental forces/interactions. The two most common types of calorimeters utilize material compositions to measure energy deposited through the EM force (ECAL) or through the strong force (hadronic calorimeters). The STAR [180], PHENIX [181], and ALICE [182] experiments all employ Pb-plastic scintillator sampling ECALs with some variations. The ATLAS experiment utilizes a Pb-liquid Argon calorimeter [169] while the CMS experiment employs a Pb-tungstate scintillating crystal-based ECAL [168]. Regardless of the specific material and approach utilized, the energy measured in the ECAL (\(E\)) along with the momentum measured by the tracker (\(p\)) can be used to identify electrons (positrons) via the ratio:

\[
\frac{E}{p} = \left[\frac{m_e^2 + p^2}{p^2}\right]^{1/2} \approx 1, \text{ for } p \gg m_e.
\]  

(35)

More information, like the shape of the EM shower, can be employed to further improve the identification of electrons with the information from ECALs.

ECALs are also essential for identifying photons. Since neutral particles undergo essentially no interaction with the materials in standard tracking detectors, their trajectory cannot be recorded. Even if a tracking detector were to be constructed capable of recording the trajectory of a neutral particle, the neutral particle will not be deflected by the magnetic field, limiting the usefulness of such a measurement. Therefore, neutral particles are primarily identified through isolated (i.e. without an associated charged track) deposits of energy observed within an ECAL. Further, identifying a neutral particle observed in an ECAL as a photon generally requires a veto on the amount of energy deposited in a hadronic calorimeter.

3.3.4. Early measurements (2000–2015). Right from the start, the two ‘large’ experiments at RHIC (STAR and PHENIX) studied the EM production of \(e^+e^-\) from the fields of the colliding ultra-relativistic Au nuclei [115, 183]. The STAR Collaboration produced the first measurement of this kind from Au + Au collisions collected in the year 2001 [115].

A minimum bias trigger was used to select events in which both gold nuclei broke up, by detecting events with one or more neutrons in zero degree calorimeters (ZDC). At that time STAR did not have a time of flight subsystem and could only identify electrons over a limited range in transverse momentum utilizing the \(\langle dE/dx \rangle\) measured in the TPC (the tracking detector). However, since this dataset was collected with a lower magnetic field (0.25 T) than STAR’s nominal setup (0.5 T), electrons could still be well identified over a relatively large range of momentum. Only 52 candidate events out of the 2001 data set, consisting of \(\sim\)800 000 minimum-bias events, survived the additional selection criteria imposed to identify electromagnetically produced \(e^+e^-\). Figure 13 shows the observed differential cross section, in terms of the pair transverse momentum. The data
exhibit reasonable agreement with the theoretical calculations based on the traditional EPA approach (solid line) and numerical lowest-order QED calculations (dotted line) shown for comparison.

While the two theoretical calculations were nearly identical in terms of the predicted total cross section, there was one crucial difference visible in the predicted pair transverse momentum distribution. The traditional EPA predicts that the cross section should continue to rise toward pair momentum of zero, while the QED calculation predicts a significant depletion of the differential cross section for pair momentum below 20 MeV/c. Even given the large statistical uncertainties of the measurement due to the small sample of only 52 pairs, the shape of the transverse momentum distribution was interpreted as evidence that the numerical lowest order QED calculation better describe the data than the traditional EPA calculation. It is at this point crucial to recognize that the theoretical developments discussed in section 3.1 came more than a decade after this early result from STAR. At the time, the discrepancy between the traditional EPA and QED calculations were interpreted as arising from the effect of photon virtuality [115]:

The main difference between this calculation and the EPA approach is that the QED calculation includes photon virtuality.

As discussed in section 3.1, this is now understood to be an incorrect statement, with the difference resulting primarily from the impact parameter dependence of the process (included in the QED calculation but not in the traditional EPA)—not from photon virtuality [104]. However, this interpretation was consistent with the representative consensus from the community at that point in time and expressed in a highly-cited review paper [184].

Over the next decade, this view became commonplace in the literature, with similar statements appearing in multiple papers over the next decade and a half [114, 185, 186]. However, as we will discuss in the next sections, experimental progress began to challenge this interpretation.

Progress measuring photoproduction processes continued over the next decade after STAR’s initial measurement published in 2004. Due to the growing interest in utilizing HICs to study EM processes, multiple experiments performed measurements of the total cross section of electromagnetically produced $\gamma^+\gamma^-$ from ultra-peripheral HICs [183, 187, 188] or from exclusive p+p [171] collisions. In 2009, the PHENIX collaboration performed measurements of photoproduced $J/\psi$ and $e^+e^-$ [183]. However, the measurement was severely limited by statistical precision, with only 28 $e^+e^-$ reconstructed in the mass range of $2.0 < M_{e^+e^-} < 6.0$ GeV/c$^2$, of which 14 were identified as continuum $e^+e^-$ with the remaining pairs resulting from $J/\psi$ decay. The observed production cross section for both the continuum $e^+e^-$ and $J/\psi$ were found to be consistent with traditional EPA predictions. Additionally, the transverse momentum spectrum for all observed $e^+e^-$ (continuum $e^+e^-$ and $J/\psi$ decay) was found to peak at low momentum ($\sim 90$ MeV/c) as expected based on traditional EPA calculations.

In 2013, ALICE made a similar but higher precision measurement of photoproduced $J/\psi$ and high mass $e^+e^-$ [187] (see figure 14). The statistical power of their measurement allowed them to study the $e^+e^-$ invariant mass spectra over the wide range of $2.2 < M_{e^+e^-} < 6.0$ GeV/c$^2$ with an order of magnitude more $e^+e^-$ compared to the PHENIX result [183]. Of the $\sim 300$ $e^+e^-$ pairs observed within that mass range, $\sim 300$ were found to be within the $J/\psi$ mass peak. The remaining $e^+e^-$ outside that mass range were identified as continuum $e^+e^-$. The production cross section of the continuum $e^+e^-$ were compared with traditional EPA prediction utilizing the BW cross section in two different invariant mass ranges, one below and one above the $J/\psi$ mass peak. The observed cross section in both regions were found to be $\sim 20\%$ above the prediction, a disagreement of only about 1.5σ including all experimental uncertainties (and with no theoretical uncertainties accounted for). This measurement did not hint at any breakdown of the traditional EPA or the assumptions about photon virtuality. The observed cross section was interpreted to be evidence for the lack of any higher-order effects taking place in such photon–photon interactions, since higher-order effects are expected to reduce the observed cross section (with respect to the lowest-order process alone, see section 3.2) by as much as $\sim 30\%$ for the ALICE experimental conditions [132, 189, 190].

In both the PHENIX and ALICE measurements, continuum $e^+e^-$ and $e^+e^-$ from $J/\psi$ decay were observed together. These two contributions were primarily separated via an invariant mass selection window, whereby any $e^+e^-$ outside the $J/\psi$ mass peak were considered to be continuum $e^+e^-$ produced via photon–photon fusion. Both measurements found the $J/\psi$ photoproduction cross section to be consistent with EPA expectations for photonic interactions, where a WW photon from one nucleus fluctuates into a quark–antiquark pair capable of directly interacting with the target nucleus. The simultaneous observation of the continuum $e^+e^-$ along with $e^+e^-$ from $J/\psi$ decay obfuscate the nature.
of the interacting photons, since two real photons (with helicity $+/-1$, zero forbidden) cannot form a vector meson (e.g. $J/\psi$). Therefore, these measurements can not experimentally demonstrate that the photons producing the observed $e^+e^-$ spectrum had only the allowed helicity states of real photons. Even if the experimental measurements had obtained significantly higher precision on the cross section, the range of theoretical predictions expected for the $\gamma + A \to J/\psi + X$ production mechanism [99, 191, 192] would have made it difficult to definitively rule out contributions from virtual photon fusion e.g. $(\gamma' + \gamma' \to J/\psi + X)$.

3.3.5. Observations of EM production in hadronic HICs (2016～2018). In 2016, the ALICE collaboration performed measurements of the $J/\psi$ particle in hadronic HICs over a large range of impact parameters ($0 < b < 2R$) and over a large range of the decay $e^+e^-$ momentum. In peripheral collisions, they observed an anomalous excess of $J/\psi$ produced with very small transverse momentum [188]. As shown in figure 15, the yield of excess $J/\psi$ was found to be consistent with the expectation for the diffractive photonuclear process, even displaying the characteristic peak in cross section at very low transverse momentum for coherent photoproduction processes. However, the photon–photon and photonuclear interactions had conventionally been considered only in ultra-peripheral collisions (UPC), where the hadronic (strong force) interaction does not take place. As discussed in section 3.1, one ingredient in the theoretical description of coherent photoproduction is the treatment of the fields as external, with the nuclear charge distribution remaining undeflected throughout the interaction. Since the assumption of a straight-line trajectory before and after the collision is seemingly invalid in events with hadronic overlap, and the $J/\psi$ photoproduction requires photon diffraction coherently off the target nucleus as a whole, the observation of coherent photoproduction in such events was unexpected [53].

Shortly after, STAR pioneered the measurement of photon–photon production of $e^+e^-$ in the midst of hadronically interacting HICs [193], further confirming the ALICE finding that coherent photoproduction processes can occur even in events with hadronic overlap. Since high-energy hadronic interactions produce many electrons and positrons, isolating the electromagnetically produced pairs amid the numerous particles is a challenging task. STAR utilized the excellent electron and positron identification capabilities provided by its various subsystems to remove background from other types of particles. Still, a single HIC at RHIC energies can produce several $e^+e^-$ pairs per event [194] making the identification of $e^+e^-$ from photoproduction difficult. However, coherent photoproduction shows a characteristic peak in cross section at very low transverse momentum, while the cross section for $e^+e^-$ from hadronic production tends to decrease for small values of transverse momentum (the ‘Au + Au Cocktail’ in figure 16). Therefore, STAR found that the $e^+e^-$ from photoproduction could be statistically isolated in peripheral collisions for pairs over a mass range of $0.4 < M_{ee} < 2.6$ GeV/c$^2$. However, increasingly central events (smaller $b$) tend to produce more $e^+e^-$ from hadronic interactions, swamping those from photoproduction and making the measurement untenable in mid-central to central collisions.

Around the same time, the ATLAS experiment observed a significant centrality dependence in the EM production of $\mu^+\mu^-$ [103]. Unlike the STAR result, their measurements spanned an impact parameter range from $b > 0$ outward, allowing investigation of the photoproduction cross section into the most central events. The observation of $\mu^+\mu^-$ instead of $e^+e^-$ was the primary difference allowing ATLAS to measure photoproduction even in central collisions. ATLAS’s superior muon identification and the focus on muons with $p_T \gtrsim$...
4 GeV/c helped reduce the background from other types of particles to very low levels, allowing photoproduced $\mu^+\mu^-$ to be identified in head-on collisions with $b \approx 0$. Figure 17 shows an ATLAS measurement of the $\alpha$ distribution for $\mu^+\mu^-$ pairs from coherent photon–photon production in 10%-20% central Pb + Pb collisions. The total production cross section measured in central collisions is in good agreement with traditional EPA calculations (STARLight). However, the $\alpha$ distribution was found to be significantly broader than the traditional EPA prediction. At that time the broadening was interpreted as arising due to final state interaction with the produced hadronic medium [103, 135].

The ALICE [188] measurement demonstrated that coherent diffractive photoproduction interactions can occur in hadronic interactions. Then the STAR [194] and ATLAS [103] measurements conclusively demonstrated that coherent photon–photon interactions occurs not only in Us, but also in hadronic interactions. Since traditional EPA calculations have included the impact parameter dependence of the photon flux (see equation (1) of [195]) they have been able to provide predictions for the photoproduction cross section in hadronic events. However, for impact parameter ranges extending below $b_{\text{min}} \approx 2R$, a realistic nuclear charge distribution was found to make a significant impact on the expected photon flux and the resulting photoproduction cross section [122]. With these considerations taken into account, the measured cross sections from ALICE, STAR, and ATLAS were found to be consistent with traditional EPA predictions extended for hadronic events [195]. The ATLAS measurement even demonstrates that coherent photoproduction occurs in head-on collisions where the nuclei are broken almost completely apart.

While the total cross section measured by STAR and ATLAS were consistent with coherent photoproduction, both measurements showed a puzzling deviation from the traditional EPA predictions. Both STAR [193] and ATLAS [103] found a significant broadening of the lepton pair’s transverse momentum in hadronic HICs in comparison to those in UPC (see figure 12) and to traditional EPA calculations. The STAR Collaboration characterized the broadening by measuring the $P_T^2$ and the invariant mass spectra of lepton pairs in Au + Au and U + U collisions with respect to traditional EPA calculations. STAR utilized theoretical and phenomenological models to qualitatively describe the broadening by introducing the effect of a magnetic field trapped in an electrically conducting QGP [193]). The ATLAS collaboration quantified the broadening effect via the acoplanarity of lepton pairs in different centrality events, in contrast to the same measurements in UPC. Alternatively, the ATLAS collaboration proposed that the broadening effect may be due to the EM scattering of leptons in the hot and dense medium [103]. However, in each case, the broadening was measured with respect to a ‘baseline’ determined by measurements in UPC and/or from traditional EPA calculations. These approaches assumed that there is no impact-parameter dependence of the transverse momentum distribution for the lepton pair from the initial photon–photon collision, an assumption that was held as almost self-evident by the community up to that point of time.

3.3.6. Observation of impact parameter dependence on photon kinematics. According to the traditional EPA and the $k_T -$factorization approach, the photon $k_T$ distribution is primarily a result of the uncertainty principle with no dependence on $b$ (see specifically equation (19) of [185]). These two concepts go hand-in-hand, since one can not generally describe a set of conjugate variables as a function of one-another. Up to this point in time (~2018), there had been no direct evidence that the photon kinematics predicted by the traditional EPA were incorrect. In fact, the traditional EPA had successfully described experimental measurements over nearly two decades and specifically succeeded in describing the $P_T$ and $\alpha$ distributions observed in UPC by experiments at RHIC [115, 183] and the LHC [103].

While the aforementioned measurements from STAR [193] and ATLAS [103] showed transverse momentum broadening that is inconsistent with traditional EPA predictions (see figures 17 and 18), they were otherwise consistent in terms of total cross section. For this reason, the discrepancy was initially interpreted as evidence for final state effects driven by interaction with the hadronic matter (or even a QGP) [111, 135]—and not evidence that the initial photon kinematics were incorrectly described by the traditional EPA.

In order to determine the primary source of the broadening, it was necessary to determine if the observed broadening was a result of the initial photon flux or a result of final state interactions. While STAR had already measured the $e^+e^-$ transverse momentum spectrum from UPC in their 2004 paper [115], the statistical precision was insufficient to perform additional differential studies capable of investigating the impact parameter dependence. Over the next several years, various experimental techniques were developed to test the impact parameter dependence of the photon flux and kinematics of the produced dileptons.
For highly charged nuclei, \( Z_{\text{em}} \approx 0.6 (\alpha_{\text{em}} \approx 1/137 \) and \( Z_{\text{em}} = 79, Z_{\text{em}} = 82 \), meaning that the density of photons is appreciable, and therefore, the nuclei may exchange multiple photons in a single passing. In UPC, the quasi-exclusive \( \gamma\gamma \rightarrow l^+l^- \) process may be selected in collisions where additional exchanged photons lead to the excitation and subsequent dissociation of the nuclei. Mutual Coloumb excitation is the process by which at least two photons (in addition to those mediating the semi-exclusive process of interest) cause one or both nuclei to become excited [197]. The cross section is dominated by the giant dipole resonance [198] which peaks at low energy \( (E_\gamma \approx 14 \text{ MeV/c for gold and lead nuclei}) \). The giant dipole resonance excitation is responsible for several final states with one or two neutrons emitted and has been measured with high precision by various experiments [199]. All the experiments discussed herein are well-equipped to detect beam energy neutrons via ZDC.

The STAR and CMS collaborations employed a neutron tagging approach to experimentally test the impact parameter dependence of the \( \gamma\gamma \rightarrow l^+l^- \) process and to specifically investigate the photon \( k_L \) distributions in events without hadronic overlap. The STAR collaboration first demonstrated that the lepton pair momentum depends strongly on the impact parameter range of the colliding nuclei [31]—in stark contrast to the long-accepted behavior predicted by the \( k_L \)–factorization approach used in the traditional EPA models. At the same time, theoretical progress (summarized in section 3.1) via lowest order QED calculations for the \( \gamma\gamma \rightarrow l^+l^- \) process indicated that the kinematic distribution of the initial photon flux contains strong impact parameter dependence [104, 117]. Figure 18 shows measurements of the \( P_L \) distribution from the \( \gamma\gamma \rightarrow e^+e^- \) process in ultra-peripheral and peripheral collisions. The precision measurement from UPC (with two orders of magnitude more statistics than the 2004 STAR measurement) shows a significantly broader \( P_L \) spectrum compared to the traditional EPA (STARLight) prediction. Unlike the STAR 2004 measurement, the measurement shown in figure 18 utilized events with predominately one neutron in each of the east and west STAR ZDCs. The presence of neutrons biases the nucleus–nucleus impact parameter distribution to smaller values, compared to events without neutrons in the final state. Furthermore, the transverse momentum spectra from both ultra-peripheral and peripheral collisions can be well described by the same lowest-order QED calculations, suggesting that the previously observed broadening in hadronic HICs mainly results from the initial photon flux, not from final state interactions.

Figure 19 shows additional measurements by the CMS collaboration of the acoplanarity distribution of dimuons in events with various neutron emission scenarios. A significant dependence of the \( \langle \alpha_{\text{core}} \rangle \) distribution on neutron multiplicity is observed, where \( \alpha_{\text{core}} \) is the statistically isolated \( \alpha \) distribution from coherent \( \gamma\gamma \rightarrow \mu^+\mu^- \) interactions [196]. In the CMS measurement, the narrow signal \( \langle \alpha_{\text{core}} \rangle \) and broad background distributions were isolated via empirical fit functions. The observation by the CMS collaboration confirmed that made by the STAR collaboration, demonstrating that significant broadening of the lepton pair momentum results from the initial photon flux in the absence of any hadronic medium that may modify the final distributions. While these measurements do not rule out potential medium interactions proposed previously by the STAR and ATLAS collaborations, they demonstrate that the primary source of the observed broadening is the initial photon flux.

4. Connections to VB

This section explores the connections between the polarized BW process and VB. Special attention is given to the specific case relevant for the STAR collaboration measurement. The purpose of this section is to address the key issues, listed in...
section 2.2, related to the interpretation of the STAR collaboration measurements with respect to VB.

Fundamentally, the BW process and VB are connected by the optical theorem, which connects the forward scattering process to the absorption process, as illustrated in figure 5(d). The optical theorem is a direct result of energy conservation in classical electromagnetism and a consequence of the conservation of probability (unitarity) in quantum mechanics. Specifically, the number of produced pairs in the BW process can be determined from the imaginary part of the forward scattering (light-by-light) process involving electron/positron loops. In addition to the optical theorem, the Kramers–Kronig relations (commonly applied in non-linear optics [201, 202]) provide an analytic relationship between the real and imaginary parts of the amplitudes. Thus, the Kramers–Kronig relations provide the connection between amplitude level information obtained from the polarized BW process and VB. In this case we are discussing the linear BW process and its connection to linear VB. However, these same considerations apply to the non-linear case [42]. Based on this foundation, we consider the details of the connection between the STAR measurement of the polarized BW process and linear VB in the following subsections.

4.1. Theoretical aspects of VB and polarization effects

VB has received significant theoretical attention ever since it was recognized as an implication of the Euler–Heisenberg Lagrangian [32]. Several reviews exist that discuss the essential features of QED in electric and magnetic fields [203–205] and a considerable amount of work has gone into exploring methods for computing VB effects in perturbative and non-perturbative regimes [206–208]. Significant theoretical effort has also been devoted to understanding the signatures of VB in various experimental setups [70, 74, 209–214] based on approaches including laser-electron collisions [215], x-ray free electron lasers [75, 213], and utilizing high energy photon probes [73, 215].

The study of VB and dichroism has been intimately connected with the BW process since they were first predicted. Wheeler’s student, John Toll, first studied the dispersion relationships for light traveling through various combinations of background electric and magnetic fields [35]. Toll studied the behavior of the absorption and forward transmission of polarized light in various special cases. In section 4.3 of [35], Toll specifically considered the effects on polarized light undergoing Delbrück scattering [216–218] (the forward transmission process) and the BW process (the absorption process). In Toll’s calculation, the Weizsäcker–Williams [13]–Williams [12] method was applied to highly-Lorentz contracted heavy-nucleus as a photon source for the BW process. It was further noted that to investigate the polarization dependence, one must consider the photon position relative to the source nucleus, not just the total WW photon cross section, since ‘The polarization of these photons is with the electric vector directed radially with respect to the axis of nuclear motion.’ [35]. This insight and its implications were only recently rediscovered [200] and confirmed by further theoretical investigations [110, 113].

One can extend the cross-section level calculations from Breit and Wheeler to the more fundamental level of helicity amplitudes to more deeply investigate the effects of photon polarization. Such calculations were a major focus for Toll. The helicity structure [35] of the $\gamma\gamma \to t^+t^-$ process can be expressed in terms of the matrix element:

$$
\mathcal{M}_{\lambda}^{\lambda_\perp} = -\frac{i}{2} (k_{1\perp} \cdot k_{2\perp}) \left( \mathcal{M}_{\lambda_\perp}^{\lambda_\parallel} + \mathcal{M}_{\lambda_\parallel}^{\lambda_\perp} \right) 
- \frac{i}{2} |(k_{1\perp} \times k_{2\perp})| \left( \mathcal{M}_{\lambda_\perp}^{\lambda_\perp} - \mathcal{M}_{\lambda_\parallel}^{\lambda_\parallel} \right) 
+ \frac{1}{2} (k_{1\perp}^T k_{2\perp} - k_{1\perp}^T k_{2\perp}^T) \left( \mathcal{M}_{\lambda_\perp}^{\lambda_\perp} + \mathcal{M}_{\lambda_\parallel}^{\lambda_\parallel} \right) 
+ \frac{i}{2} (k_{1\perp}^T k_{2\perp}^T + k_{1\perp}^T k_{2\perp}) \left( \mathcal{M}_{\lambda_\perp}^{\lambda_\parallel} - \mathcal{M}_{\lambda_\parallel}^{\lambda_\perp} \right)
$$

(36)

where $k_{1\perp}$ and $k_{2\perp}$ are the transverse momenta of photon 1 and 2, respectively, and $\mathcal{M}_{\lambda_\perp}^{\lambda_\parallel}$ corresponds to the individual $\gamma(\pm)\gamma(\pm) \to t^+(\lambda)t^-(\bar{\lambda})$ helicity amplitudes. Since the photon is a spin-1 particle, two real photons can combine to form states with $J^z = 0^\circ, \pm 2^\circ$ which are admixtures of the individual helicity amplitudes as described by equation (36).

From the helicity level description one may calculate the differential cross section of the BW process and investigate its dependence on the photon–photon polarization angle. To this end, Toll calculated the polarization-dependent photon–photon absorption cross section as a function of the photons’ relative polarization angle $\phi$ (see equation (4.3-7) in [35]):

$$
\sigma_{\text{BW}} (\omega; \phi_{\text{lab}}) = \frac{\sigma_{\perp}(\omega) + \sigma_{\parallel}(\omega)}{2} - \frac{\sigma_{\perp}(\omega) - \sigma_{\parallel}(\omega)}{2} \cos 2\phi_{\text{lab}}.
$$

(37)

The angular dependence due to the effect of photon polarization is entirely contained in the $\cos 2\phi_{\text{lab}}$ modulation term, with a prefactor proportional to the difference in the absorption cross sections $\tau_{\gamma\gamma} \equiv \sigma_{\perp}(\omega) - \sigma_{\parallel}(\omega)$. At the cross section level, the difference in the absorption for parallel vs. perpendicular-polarized photons is directly related to vacuum dichroism and can be interpreted as the preferential absorption of a photon with respect to a given external EM field orientation [45, 46, 66, 73, 219].

The angle $\phi_{\text{lab}}$ in equation (37) is determined with respect to the photon’s polarization as measured in the laboratory coordinate system. However, in the case of heavy-ion experiments, every nucleus–nucleus collision occurs with an impact parameter vector randomly oriented in $\phi_{\text{lab}}$, naturally making measurement with respect to any laboratory angle isotropic, since it effectively averages out the photon polarization. For this reason, when calculating the total photon–photon fusion cross section, most theoretical calculations implicitly or explicitly integrated over the photon polarization and the azimuthal angle dependencies of the cross section [152].

In 2019, Li et al. [200] predicted a unique spin interference effect driven by the linear photon polarization of WW photons. This spin interference effect results when the initial quantized spin of the colliding polarized photons is transferred into $e^+e^-$.
as orbital angular momentum. The predicted signature for linearly polarized photon–photon collisions is a \( \cos 4\phi \) modulation in the azimuthal angle \( \phi \), which is approximately the angle of the electron (or positron) transverse momentum with respect to the \( e^+e^- \) pair momentum. More precisely, the modulation occurs in the angle \( \phi \), defined as

\[
\cos \phi = \frac{(p_{T1}^2 + p_{T2}^2) \cdot (p_{T1} - p_{T2})}{|p_{T1} + p_{T2}| \times |p_{T1} - p_{T2}|}
\]

where \( p_{T1} \) and \( p_{T2} \) are the 2D momentum vectors of the daughter electron and positron in the plane transverse to the beam direction. Since this modulation is not measured with respect to impact parameter direction, it is not washed out over many events. For this reason, the STAR measurement [31] is conducted with respect to the angle \( \phi \), equivalently defined as the angle between \( p_{1\perp} + p_{2\perp} \) and \( p_{1\perp} - p_{2\perp} / 2 \), where \( p_{1\perp}, p_{2\perp} \) are the transverse momentum of the \( e^+ \) and \( e^- \), respectively [200].

An essential feature of the EPA is that the polarization vector \( \vec{\xi} \) and transverse momentum vector \( \vec{k}_\perp \) for photons manifest from highly-Lorentz contracted fields. The conversion of the initial photon polarization into the orbital angular momentum of the produced \( e^+ \) and \( e^- \), along with this correlation between the \( \vec{\xi} \) and \( \vec{k}_\perp \) results in a final-state correlation between the photon polarization angle \( \phi_{lab} \) and \( \phi \). However, the angle \( \phi \) computed with the \( e^+ \) and \( e^- \) momentum vectors is different in two important ways: 1) the angle \( \phi \) has a finite resolution (<100%) with respect to the true angle \( \phi_{lab} \), and 2) using the orbital angular momentum direction introduces an ambiguity in the direction of the spin. The former effect reduces only the magnitude of the modulation (compared to measurement with \( \phi_{lab} \)) while the latter results in a transformation of the \( \cos 2\phi_{lab} \) into a \( \cos 2\phi \) and a \( \cos 4\phi \) modulation. All of these effects are easily computed for the lowest order QED process, as was done in [110], allowing a direct comparison between the STAR measurement and the predictions from lowest order QED.

### 4.2. Connection between the STAR observation and VB

Ultra-relativistic nuclei produce a highly Lorentz contracted radial electric field emanating from the nucleus, with a magnetic field circling the nucleus. Both the electric and magnetic field are almost entirely Lorentz-contracted into the plane perpendicular to the direction of motion. Therefore, at any given point, the fields appear as a nearly transverse linearly polarized EM wave. It was therefore expected that the corresponding photons manifest from these fields are linearly polarized in the transverse plane with respect to the beam [35]. Despite this expectation, until recently there was no proposed technique for accessing the photon polarization information, since the primary effects are asymmetries in the azimuthal (plane perpendicular to the beam axis) distribution of the produced \( e^+e^- \).

Since each HIC occurs with a random impact parameter orientation, such azimuthal asymmetries are generally washed out over multiple events. For the kinematic ranges applicable for \( e^+e^- \) measurements in UPC with STAR, lowest-order QED calculations predict a \( \cos 4\phi \) modulation with a normalized amplitude of approximately \(-17\%\). Figure 20 shows the measurement of \( \Delta\phi = \phi_{meas} - \phi_e \) distribution from UPCs and 60%–80% central collisions for \( 0.45 < M_{ee} < 0.76 \text{ GeV} \) compared with calculations from lowest-order QED [110, 200], STARLight [102] and the publicly available SuperChic3 code [152].

![Figure 20. The \( \Delta\phi = \phi_{meas} - \phi_e \) distribution from UPCs and 60%–80% central collisions for \( 0.45 < M_{ee} < 0.76 \text{ GeV} \) compared with calculations from lowest-order QED vs. STARLight and SuperChic3 code.](image)

We examine whether a precision angular distribution in the transverse plane could have been observed and measured by other experimental collaborations from previous data sets and detectors as presented in past publications. The data from CERES [220], PHENIX [183], STAR [115], CDF [221, 222] and ALICE [187] do not have enough statistics to perform a \( \cos (4\Delta\phi) \) measurement while those measurements presented from ATLAS [114, 171] and CMS [223–225] do not have sufficient pair momentum resolution (200–300 MeV/c) to perform this measurement.

### 5. Discussion

#### 5.1. The BW process

In section 2.1 we listed a few objections, challenges, or questions to the interpretation and observation of the linear BW process in HICs. In the following discussion, we now revisit these issues in the context of the tremendous experimental and theoretical progress made in the last few decades, as covered in section 3.3.
5.1.1. Do the highly-Lorentz contracted fields produced in HICs provide a valid source of photons for the BW process? Literature from within the high-energy particle and nuclear physics community over the last three decades has reported conflicting and contradictory statements regarding the viability of WW photons for achieving the BW process [25, 102, 105, 117, 185]. The immediate source of such statements is somewhat unclear, since WW photons from highly Lorentz contracted EM fields is precisely the photon source proposed by Breit and Wheeler in their seminal work [9]: based on this, it is clear that Breit and Wheeler certainly expected the Lorentz-boosted fields of ultra-relativistic nuclei, understood theoretically through the WW approximation, to provide a viable source for achieving the BW process in laboratory. To determine if this claim, made in 1934, is still merited, one must ask if any theoretical or experimental advancement has been made that altered these conclusions? Even after QED was firmly established at the end of the 1940s, the WW method of equivalent photons was extensively used by Wheeler’s student to study theoretical aspects of the BW process in greater detail [35, 226].

Why then, in recent decades, have WW photons from Lorentz-boosted nuclear EM fields been disregarded as a viable source of photon for the BW process? We believe that one source of confusion may be due to claims about the photon virtuality resulting in the ‘special case’ believed to be important for the production of $e^+e^-$ compared to other heavier leptons [184]:

These photons are almost real, with virtuality $-q^2 < (\hbar/R_A)^2$. Except for the production of $e^+e^-$ pairs, the photons can usually be treated as real photons.

This distinction between $e^+e^-$ production (the original concept of the BW process) and the production of heavier lepton-pairs is crucial. If the above claim were correct, one would expect to observe a qualitative difference in the kinematic distribution for photoproduction of $e^+e^-$ and $\mu^+\mu^-$, since the former would be characterized by virtual photon collisions, while the latter would be characterized by real photon collisions (due to the much heavier mass of the muon). However, as discussed in section 3.3.2, recent measurements have verified that the kinematics of both the $\gamma\gamma \rightarrow \mu^+\mu^-$ process and the $\gamma\gamma \rightarrow e^+e^-$ exhibit exactly the same features, features predicted for real photons. Specifically, the broadening of the pair transverse momentum, has been proven to be a result of the EM field distribution—not a result of virtuality via the uncertainty principle [53].

Even with the above clarification, WW photons from highly-Lorentz contracted EM fields do not provide, exclusively, real photons capable of interacting via the BW process. As discussed in section 3.1, only a small fraction of the $e^+e^-$ cross section from photoproduction may be considered a result of the BW process, with the remaining overwhelming fraction resulting from processes which include virtual photons (e.g., the Bethe–Heitler and Landou–Lifshitz processes [25, 83, 117, 120]).

5.1.2. Are higher order effects present, and if so, are they separable or inseparable from the lowest-order BW process? The discovery of higher order QED effects in high energy particle and nuclear collisions would itself be an important finding. Potential signatures of Coulomb corrections in high-energy EM scattering processes have been thoroughly studied over the last few decades [129, 131, 132, 227]. While significant theoretical differences and uncertainties exist, several authors have predicted that Coulomb corrections modify the total pair photoproduction cross section by as much as 30%. However, significant experimental uncertainties prevent measurement of the photoproduction cross section to better than $\sim8\%$ [185, 228, 229]. With such a level of uncertainty, cross section measurements alone cannot unambiguously rule out the presence of Coulomb corrections.

In the specific kinematic region of the STAR measurement for which the BW process is expected to be applicable, QED calculations find that Coulomb corrections exactly cancel [131]. In this regime, the pair production process is accurately described by the lowest-order process alone. Experimentally, the unique $\phi$ modulation measured by the STAR collaboration shows that the process results from the spin carried by two real photons at lowest order [110]. The exploration of Coulomb correction at the LHC and future EIC are points of significant interest (see section 6). On the other hand, it may be possible that higher-order effects are visible through dimuon production at LHC energies, as discussed in section 3.2.

5.1.3. Can the BW process be isolated from background and higher-order processes? Even with a viable source of photons for achieving the BW process, definitive observation requires a procedure for clearly identifying the process and separating it from background processes. This experimental challenge is not unique to high-energy nuclear physics experiments. For instance, Laser-based experiments like the E-144 experiment at SLAC had to carefully estimate the number of positrons observed in the ‘Laser off’ configuration in order to identify the distribution of positrons from the multi-photon BW process.

In heavy-ion experiments, the photons capable of achieving the BW process are only found in a small region of phase-space, resulting in $e^+e^-$ pairs in which the daughters are produced nearly back-to-back. This constraint results in the characteristic peak in cross section at very low pair transverse momentum. As mentioned previously, this feature is crucial for the separation of the BW process from other sources of $e^+e^-$, since all other processes that produce an $l^+l^-$ pair have a cross section that decreases at low pair transverse momentum. This is most plainly illustrated in figure 16, where one can see that the yield of the ‘Au + Au Cocktail’ (all non BW sources of $e^+e^-$ pairs) decreases precisely in the region where the BW cross section peaks.

The separation of processes involving real vs. virtual photons via a selection in phase space has also been carried out in the well recognized observations of light-by-light scattering by the ATLAS and CMS collaborations [225].
these cases, WW photons from highly Lorentz contracted fields are used as a source for studying light-by-light scattering. However, in order to separate the light-by-light process from those involving virtual photons, only high-mass, mid-rapidity di-photon production is considered [230]. In the same way, the BW process can be isolated from background processes by considering mid-rapidity, high-mass, low transverse momentum $e^+e^-$ pairs.

5.1.4. Has the BW process previously been observed in high energy particle or HICs? The production of $e^+e^-$ pairs has been measured by different experiments in hadron and ultra-peripheral HICs at the SPS [162, 220], RHIC [115, 183], the Tevatron [221, 222] and the LHC [114, 171, 187, 223–225] facilities over the past three decades. All experiments reported cross sections for $e^+e^-$ pair production within their kinematic acceptance. All experiments compared their results with models implementing two-photon collisions. The experimental uncertainty of these measurements range from about 15% to 30% with the exception that the WA93 experiment detected only the positrons at forward angle in a fixed-target configuration. However, unambiguous identification of the BW process has not been achieved until recently since it requires experimental demonstration that the colliding photons have the energy spectrum and quantum spin states of real photons, and that any approximations do not alter the physics result of real photon collisions.

It should be noted that the photoproduction of vector mesons (e.g. $J/\psi$, $\Upsilon$) and intermediate bosons ($Z^0$) from photon–hadron interactions dominates at their respective mass over production from the two-photon process. All the previous measurements ($J/\psi$ from PHENIX [183], CDF [222] and ALICE [187], $\Upsilon$ and $Z^0$ from CDF [221], ATLAS [114, 171] and CMS [223–225]) had to exclude those mass ranges from the measurement of the two-photon cross sections. The WA93 [162], CERES [220] and previous STAR [115] studies did not have the reach in invariant mass or statistics for the vector meson exclusion measurements. None of the previous measurements had the capability of identifying the unique smooth feature in the mass range where vector mesons are known to be present, but absent from the observed production only in the case of an exclusive BW process.

Prior to [31] from STAR, no measurement made in ultra-relativistic HICs claimed to be an observation of the BW process. We also emphasize that the previous studies that went beyond cross section measurements all reached conclusions that the data indicated significant photon virtuality (and therefore were identified as a Landau–Lifshitz process) [115] and/or that there may be significant final-state interactions [103, 193] in hadronic events. Note, for instance, recent papers by the STAR [193] and ATLAS [103] collaborations with measurements of $P_{\perp}$ broadening (STAR) and increased acoplanarity, $\alpha$ (ATLAS) that were both interpreted as evidence for final state effects. With the new measurements in UPC from the LHC and RHIC in the last few years, it has been proven that these conclusions were influenced by the assumptions in the specific EPA model used [104]. On one hand, many of the publications in the literature avoided rigorous identification of the BW process and the approximation necessary for the models and experiments; on the other hand, photon–photon collisions with the BW cross section have been assumed in many model calculations and any discrepancies with experimental data would be attributed as a disproof of the BW process. In contrast, STAR performed the first measurements providing unambiguous observation of the linear BW process by demonstrating that: 1) the colliding photon energy spectrum is consistent with the collision of real photons (i.e. not determined by photon virtuality) and 2) that the quantum spin states of the colliding quanta are those of real photons. Furthermore, the observation of the $\cos 4\phi$ interference effect indicates that the process is governed by the lowest order interaction, with no significant effect from higher-order interactions. The availability of high luminosity from ultra-relativistic heavy-ion colliders and the advances in detector technology in the last decade have provided unique opportunity with multiple tools and kinematic variables for the discovery.

5.2. VB and the STAR observation

5.2.1. What is novel about the STAR measurement, i.e. how is it unique compared to the SLAC E-144 measurement, with respect to polarization effects? The SLAC E-144 measurement of the multi-photon BW process utilized a PCAL to identify events with excess positron production compared to background processes present in the ‘laser off’ case. The excess positron yield and momentum spectrum was found to be in good agreement with the expectation for the multi-photon BW process, for a mean number of photons $\langle n \rangle \approx 5$ colliding in each interaction. However, since a copious amount of electrons were scattered and produced in each event, the E-144 measurement could not identify the electron partner of each positron, and therefore could not uniquely identify the entire $e^+e^-$ pair. With limited numbers of positrons observed and the lack of full pair information, no differential measurements could be made that might be sensitive to photon polarization effects.

Unlike the multi-photon BW measurement, the measurement performed by the STAR collaboration allowed full identification of the electron and positron produced in the linear BW process. The STAR experimental setup, consisting of charged particle tracking and PID, allowed events to be selected with exactly one $e^+e^-$ pair per event. With information from both the electron and positron, reconstruction of the full kinematic information for the BW process is possible. As discussed in section 5.1, measurement of the $e^+e^-$ momentum and energy spectrum is sufficient to uniquely identify the process as the BW process. The energy and momentum spectra also provides constraints on the charge distribution producing the Lorentz contracted EM fields [53, 120].

With the statistical precision and precise trajectory measurement capabilities of the STAR tracker, additional differential measurements sensitive to the photon polarization states...
are possible. The key experimental challenge for measuring polarization effects in HICs results from the random orientation of the photon polarization vectors in each event, due to the random nucleus–nucleus impact parameter. However, as described in section 3.1, WW photons in the regime of validity for the BW process have polarization vectors in the projected transverse directions that correspond with their transverse momentum vectors. This relationship allows photon polarization effects to be studied through quantum spin-momentum correlations in the final state $e^+e^-$ pair [200], which leads to the cos$4\phi$ modulation observed by the STAR collaboration. Even though one must generally integrate over the possible combinations of photon–photon polarization to calculate the observable cross section, a net effect due to photon polarization persists into the integrated cross section because of a mismatch in the number of photons (luminosity) colliding with parallel vs. perpendicular polarization vectors.

5.2.2. How is the azimuthal angle ($\phi$) modulation observed by STAR related to VB and/or dichroism? The cos$4\phi$ modulation observed by the STAR collaboration demonstrates that the colliding photons are linearly polarized. In terms of the angle $\phi$, the differential cross section for the BW process may be expressed as [92, 110]:

$$\frac{d\sigma}{d\phi} \propto \left( \frac{\sigma_{\parallel} + \sigma_{\perp}}{2} \right) + \tau_{\gamma\gamma} \cos 2\phi \left( B + C \cos 2\phi \right),$$

and $A$, $B$, and $C$ are the amplitudes of the isotropic, cos$2\phi$, and cos$4\phi$ contributions to the cross section, respectively. In the above, we make explicit the dependence on $\tau_{\gamma\gamma}$, the modulation strength versus $p_T$.

and $\tau_{\gamma\gamma}$ is the difference in intensity for photon–photon collisions with parallel ($I_{\parallel}$) vs. perpendicular ($I_{\perp}$) polarization vectors. The STAR kinematic acceptance results in a measurement of $e^+e^-$ well above the pair mass threshold. Just above threshold $\tau_{\gamma\gamma}$ is large, but approaches zero for pairs produced at higher invariant mass. Therefore, STAR is not directly sensitive to a difference in the absorption cross section and vacuum dichroism. However, since the geometry of the EM fields in a HIC leads to different intensities $I_{\parallel}$ vs. $I_{\perp}$ with $\tau_{\gamma\gamma} \neq 0$, STAR is still sensitive to the polarization dependence ($\phi$ angular dependence) of the cross despite $\tau_{\gamma\gamma}$ being effectively zero (consistent with zero within experimental sensitivity) in this case.

The observed cos$4\phi$ modulation for the BW process results from the behavior of the helicity amplitudes relevant for polarized photons [113, 152]. While this particular observation is not sensitive to $\tau_{\gamma\gamma}$, it demonstrates the physical splitting of the wavefunction governing the momentum distribution of the $e^+e^-$ for parallel vs. perpendicular photon polarizations. For two real photons, the absorption process (the BW process) and the forward scattering process (light-by-light scattering) are related by the optical theorem [231], with exact analytical expressions for the dispersive and absorptive parts of the amplitude. Additional theoretical work is needed to relate the observed modulation directly to birefringence in the forward scattering process. As an example, according to the SuperChic3 [152] code, the light-by-light scattering process exhibits a cos$2\phi$ modulation when restricted to approximately the same kinematics as the STAR BW measurement (mid-rapidity with large invariant mass). These effects result from the structure of the helicity amplitudes for the BW process and light-by-light scattering, taking into account the photon polarization determined by the electric field lines of the source from the Lorentz contracted Coulomb field. The angular modulation effect is similar to proposed laser experiments for probing VB and dichroism [73]. As a final note, we recognize that these observed effects operate in a purely perturbative regime of QED as depicted by the diagrams shown in figure 5 and do not address the strong field regime of QED [232], at least for HICs in the ultra-relativistic energies with the specific kinematics discussed currently.

6. Related topics and future opportunities

Significant progress through multiple discoveries has been made in the physics of photon interactions. Experimental measurements and theoretical descriptions have been progressing from the initial discoveries toward quantitative and precise comparisons. At this point in time, polarized photons have been used and proposed as a tool to test and define the PWF [104, 111–113, 131, 196, 227], to probe the properties of the QGP [53, 100, 103, 135, 193, 233, 234], to measure nuclear charge and mass radii, to study gluon structure inside nuclei [237–239] and to investigate new quantum effects [235, 238, 240–242]. In this section, we provide some examples of these future opportunities.

6.1. Opportunities as EM probes of QGP

As discussed in [53], the observation of these $\gamma\gamma$ processes in events with hadronic overlap potentially allow the pure QED processes to be used as a probe of the produced nuclear medium. Both RHIC and the LHC plan future data collection that will allow high precision multi-differential analysis of these $\gamma\gamma$ processes [243, 244]. Future measurements at STAR are expected to provide significantly higher precision measurements of the $e^+e^-$ transverse momentum spectra and the cos$4\phi$ modulation. Additionally, multi-differential measurements, such as the cos$4\phi$ modulation strength versus pair $p_T$, will be possible. The increased precision on the pair $p_T$ will provide additional constraining power to investigate the proposed final-state broadening effects. In addition to
their effect on the $p_T$ spectra, final state interactions would wash out the $\cos 4\phi$ modulation strength that results from the initial colliding photon polarization. Figure 21 shows the predicted future precision that will be achieved for the $p_T$ (a) and $\cos 4\phi$ modulation measurements (b) in future STAR analyses. The added precision in the $\cos 4\phi$ modulation measurement is expected to allow experimental verification of impact parameter dependence predicted by the lowest order QED calculations (and therefore further exclude the $k_{L}$ factorization plus transverse-momentum dependent treatment of the photon polarization [152, 200]). The future data taking campaigns planned for the LHC experiments will also allow improved measurements from ALICE of the $\gamma\gamma \rightarrow e^+e^-$ process (figure 14) in a similar region of phase space as measured by STAR, but in collisions with a much larger Lorentz-boost factor. Such measurements will provide further constraints on the treatment of the photon kinematic distributions over a range of photon energies. Similarly, future data taking and analyses by CMS and ATLAS [121, 244] will allow additional precision measurements of the $\gamma\gamma \rightarrow \mu^+\mu^-$ process in events with hadronic overlap, possibly shedding light on the presence (or lack) of medium induced modifications via differential measurements of the produced dilepton kinematics.

6.2. Probe the geometry of nuclear charge and gluon distributions

As discussed in the previous sections, the BW process is sensitive to the nuclear charge distribution. Unlike in the case for an $e^+e^-$ collider, the photon flux does not diverge in UPC because the low-energy photon flux is regulated by the finite Lorentz factor of the ions and the high-energy photon flux is naturally cut off by the finite field strength due to the finite size of the ion’s charge distribution in the form factor. However, there is an additional important factor which makes the BW process sensitive to the nuclear geometry. The WW photons are linearly polarized, and the two Feynman diagrams [25] (see figure 1) taken into account in equation (25) cancel at low $k_{L}$. The phase modulation is of the form $\exp(-i\vec{b}\cdot\vec{k}_{\perp})$, and depends on the impact parameter, which is related to the nuclear geometry. This is also what results in an impact-parameter dependence of the BW process. Therefore, in high-energy ultra-peripheral HICs, the low-$k_{L}$ is modulated by $\exp(-i\vec{b}\cdot\vec{k}_{\perp})$ and high-$k_{L}$ by the form factor. Both of these factors are a function of the nuclear geometry [120]. The first attempt of extracting the nuclear charge radius was performed using the available Au + Au collisions at 200 GeV, and shows comparable result with that from low-energy electron scattering [120]. Future theoretical and experimental progress could yield quite precise charge radius measurements.

When two relativistic heavy nuclei pass one another at a distance of a few nuclear radii, the photon from one nucleus may interact through a virtual quark-antiquark pair with gluons from the other nucleus forming a short-lived vector meson (e.g. $p^0$) as described in appendix B. The polarization was utilized in diffractive photoproduction to observe a unique spin interference pattern in the angular distribution of $p^0 \rightarrow \pi^+\pi^-$ decays [235]. The observed interference is a result of an overlap of two wave functions at a distance an order of magnitude larger than the $p^0$ travel distance within its lifetime. The observable is an example of the quantum interference of non-identical particles. Independent of theoretical models, the interference patterns observed in Au + Au and U + U measurements can be quantified by studying the polarization dependence of the $|t|$ distribution in two dimensions. In this way, the effects of photon transverse momentum and two-source interference can be removed to extract the nuclear radius of gold and uranium. The strong-interaction nuclear radii were extracted from these diffractive interactions, and found to be $6.53\pm0.06$ fm ($^{197}$Au) and $7.29\pm0.08$ fm ($^{238}$U), larger than the nuclear charge radii [235]. These could be used to extract neutron skins [235]. The new measurement of neutron skin ($S$) for $^{197}$Au of $0.17\pm0.03$ (stat.) $\pm0.08$ (syst.) fm seems to follow the trend of world measurements at low energies [245] while that of $^{238}$U of $0.44\pm0.05$ (stat.) $\pm0.08$ (syst.) fm significantly non-zero and indicates a value larger than that expected for neutron skins of similar nuclei [246] (in terms of the fraction of neutron excess (N-Z)/A) [245]. It further demonstrates that this spin-induced orbital angular momentum interferometry offers a new avenue for studying nuclear geometry and gluon distribution within large nuclei at a quantitative level.

The photon-nuclear interactions can also be viewed as a photon-Pomeron fusion process, and are therefore sensitive to the gluon distribution in nuclei, especially for heavy quarkonia photoproduction, in which the process can be treated with perturbative QCD. The momentum transfer of this process is determined by the local density, which enables probing the spatial gluon distribution within nuclei. Furthermore, the linear polarization feature of photoproduction in relativistic HIC makes it possible to align the initial collision geometry, which allows 2D tomography of the gluon distribution [235, 238, 241]. Currently, various measurements of $J/\psi$ photoproduction have been made in UPC [183, 247–251], however most of them are only performed in one dimension (magnitude of transverse momentum). The photoproduction
processes could be accompanied by violent hadronic interaction when achieved in events with nuclear overlap [188, 251]. In these events, the photon produced products could serve as a novel probe to detect the properties of the hot medium created in the overlap region. However, the existing experimental measurements lack sufficient statistics to infer that. Both RHIC and LHC plan future data collection that will allow high precision multi-differential analysis of these photoproduction processes [243, 252] and provide significantly higher precision of the two-dimensional transverse momentum spectra of photon produced vector meson. Future measurements at STAR with iTPC upgrade are expected to make the first observation of $\phi$ photoproduction in HICs which may provide novel information about gluon dynamics within large nuclei [253]. Such future data taking campaigns planned at RHIC and LHC will significantly improve our understanding of the little known gluon distribution in nuclei and provide powerful tools to quantitatively extract the properties of matter governed by strong interaction.

6.3. The BW process for measuring tau $\gamma - 2$

The BW process, realized as the photoproduction of a pair of tau leptons ($\gamma \gamma \rightarrow \tau^+\tau^-$), provides a unique opportunity to measure the anomalous magnetic moment of the tau lepton: $\alpha_\tau = (g_\tau - 2)/2$ [254]. This is possible because, in the Lagrangian formulation of the interaction of photons in creation of lepton pairs, the production cross section is directly related to the magnetic moment of the lepton. Due to the heavy mass of the tau, measurement of the anomalous magnetic moment provides stringent tests of fundamental predictions from QED and potentially probes physics beyond the standard model [255] with precision $m_\tau^2/m_\mu^2 \sim 280$ times more than for measurements of the muon. Currently, the highest precision experimental measurement of $\alpha_\tau$ results from a 2004 measurement by the DELPHI collaboration [256] yielding $\alpha_\tau = -0.018(17)$. The central value of this measurement is surprisingly an order of magnitude larger than the QED prediction $\alpha_\tau^{QED} = 0.00117721(5)$, allowing significant room for physics beyond the standard model.

Past measurements of the electron and muon have yielded $\alpha_e$ to a precision of $\sim 0.28$ ppt [257] and $\alpha_\mu$ to a precision of $\sim 0.7$ ppb [258]. Considering that the value of $\alpha_\mu$ shows a $3 - 4\sigma$ tension with the standard model predictions [24, 259-261], measurement of $\alpha_\tau$ may prove an extremely important tool for uncovering new physics beyond the standard model. Measurement of $\alpha_\tau$ via the BW process in ultra-peripheral HICs provides multiple benefits: the $Z^2$ enhancement of the coherent photon field leads to large cross sections, clean events allow individual tracks from tau decays to be identified, and their unique event characteristics that allow efficient triggering and selection. By employing a recently proposed analysis, experiments at the LHC may be able to improve upon the DELPHI measurement, increasing the precision on $\alpha_\tau$ by a factor of $\sim 3$ with existing datasets [255] as illustrated in figure 22. Future datasets to be collected at the High Luminosity LHC will further allow constraints an order of magnitude more stringent than the DELPHI result.

6.4. Light-by-light scattering and axion searches

Light-by-light scattering is a purely quantum mechanical process in electrodynamics. Within the standard model, it proceeds at lowest order via virtual one-loop diagrams that involve charged fermions or W bosons. Light-by-light scattering is challenging to observe experimentally because it occurs at order $\alpha_E^4 \times 3 \times 10^{-9}$. However, the large photon fluxes produced in ultra-peripheral heavy ion collisions at the LHC finally made this observation possible. In 2016, the ATLAS collaboration presented the first evidence [114], for the direct observation of this phenomenon in $\text{Pb} + \text{Pb}$ collisions at $\sqrt{s_{NN}} = 5.02$ TeV, $\text{Pb} + \text{Pb} \rightarrow \text{Pb}(\ast) + \text{Pb}(\ast) + \gamma + \gamma$. The two lead nuclei continue down the beam pipe, although they may be left in an excited state (represented by the $\ast$) and subsequently emit neutrinos. A Feynman diagram for the Light-by-L processes is shown in figure 5(a).

The observed signal is two exclusive photons in the central detector, with possibly some neutrons deposited in the ZDCs from the decay of the excited lead nuclei. The primary background is from electrons misidentified as photons. This can happen if, for example, the electron track is not reconstructed and the electron is only detected through energy deposited in the ECALs. The dominant source of electrons is the process $\gamma\gamma \rightarrow e^+e^-$. The exclusive two-photon final state can also be produced via the strong interaction through the central exclusive process $g g \rightarrow \gamma\gamma$. The backgrounds are effectively suppressed by selecting events with low di-photon transverse momentum, where the acoplanarity of the two photons is very small ($\alpha < 0.01$). The ATLAS dataset collected in 2015, with a luminosity of 0.480 nb$^{-1}$, gave a signal with a significance of 4.4 standard deviations. The observation was confirmed by the CMS collaboration [225]—a signal with a significance of 4.1 standard deviation was observed in the 2015 CMS dataset with a luminosity of 0.39 nb$^{-1}$. A subsequent measurement by the ATLAS collaboration with data collected in 2018, with a luminosity of 1.73 nb$^{-1}$, brought the significance of
the observation to 8.2 standard deviations [186]. The measured two-photon production cross sections are consistent with standard model predictions for light-by-light scattering.

Photon–photon fusion processes can also be used to search for the existence of Axion-like particles (ALPs, a) with lepton-flavor-violating couplings that could potentially lead to an anomalous magnetic moment of the leptons. In the standard model, the leading order light-by-light process proceeds through box diagrams of virtual charged particles. However, measurement of the process has also been proposed as an avenue for testing physics beyond the standard model, since the process may proceed through predicted ALPs (a through γγ → a → γγ) [262]. Existing measurements from the ATLAS collaboration have utilized the γγ → μ⁺μ⁻ to set the most stringent limits on the existence of ALPs with an invariant mass between 6 and 100 GeV [263, 264] as depicted in figure 23. Event more stringent limits are expected with the planned 10 nb⁻¹ data to be collected in the LHC Run 3 and Run 4. Finally, the strong EM fields of ultra-relativistic heavy nuclei may provide sufficient energy density to manifest as dark photons (A) [265], a massive gauge boson that couples to the standard model through kinetic mixing. The existence of dark photons may lead to an anomalous signal in the yield of l⁺l⁻ within the kinematic region dominated by the BW process due to production via one or two dark photons (γA' → l⁺l⁻ or A'A' → l⁺l⁻) [53].

6.5. Future laser facilities

In 1996 the NOVA Petawatt laser [266] at Lawrence Livermore National Laboratory achieved the first petawatt (PW) laser pulse via chirped pulse amplification [10]. Since then the high-field laser community has been making rapid progress [267] with multiple PW grade facilities such as the Extreme Light Infrastructure [268] in Europe, the Shanghai Superintense Ultrafast Laser Facility [269] in China, and the Laboratory for Laser Energetics (LLE) at the University of Rochester, in New York, USA [270, 271]. Even higher powered laser facilities are expected to come online in the next decade, such as the 100-PW laser at the Station of Extreme Light (SEL) in China, the proposed 180-PW laser at the Exawatt Center for Extreme Light Studies in Russia, or the Optical Parametric Amplifier Line (OPAL), a 75-PW laser upgrade proposed at the LLE [272]. Though, up till now, the highest intensities achieved by laser are still sub-critical (2 × 10²² W cm⁻²). This rapid progress in power toward that needed to reach the critical field strength makes it feasible to test effects of strong-field QED with these laser experiments in the near future. In light of these recent advances in laser power, experiments have even been proposed for the express purpose of testing QED in the strong field regime, such as the LUXE experiment at the European x-ray Free-Electron Laser Facility [273].

7. Conclusion

Motivated by the recent discoveries in polarized photon collisions by the STAR collaboration [31], we have reviewed the evolution of photon–photon collisions from the time of Breit and Wheeler’s proposal to the current time. Efforts in particle accelerators from e⁺e⁻ to HICs at RHIC and the LHC have been reviewed. The milestones in experimental measurements of the cross section, transverse momentum, acoplanarity, angular distributions, and other observables have been summarized. Comparisons to various theoretical calculations have been made to emphasize the progress made over the last decades. We have sought to clarify confusions about the role of photon virtuality and to define a criterion for the specific domain of validity for the BW process in relativistic HICs. The unique features of polarized photon interactions have been discussed with connection to the pioneering work by Toll. The importance of the experimental demonstration of polarized photons from highly Lorentz-contracted Coulomb fields has been highlighted and some examples of future opportunities of their utilization have been given. It is evident from the tremendous progress made in the last half decade with multiple discoveries and measurements related to photon induced processes at both RHIC and the LHC that indeed relativistic HICs can be used to test QED and beyond.

Data availability statement

No new data were created or analysed in this study.

Acknowledgments

The authors would like to thank Dr Spencer Klein, Prof. Gorden Baym, Prof. Jian Zhou, Prof. Jinfeng Liao, Prof. Shi Pu, Prof. Qun Wang, Prof. Uli Heinz, Prof. Yuri Kovchegov, Prof. Bowen Xiao, Dr Feng Yuan and many members of STAR, ATLAS, ALICE, CMS, and PHENIX Collaborations. This work was funded in part by the U.S. DOE. 

Figure 23. Compilation of exclusion limits at 95% CL in the ALP-photon coupling (1/λₐᵥ) versus ALP mass (mₐ) plane obtained by different experiments. All measurements assume a 100% ALP decay branching fraction into photons. Reproduced from [264].
Appendix A. Available theoretical calculations and Monte Carlo generators

We list below the most commonly used theoretical calculations and Monte Carlo generators used for photon–photon processes in HICs, along with notes on the methodology each implements.

• **STARLight** [102]: Monte Carlo generator which exemplifies the traditional EPA approach. Does not include any photon polarization effects. Another essential feature is the treatment of the photon $k_\perp$ independently of the nucleus–nucleus impact parameter, and the cross section and transverse momentum distribution are calculated separately. By default, STARLight does not include pair production within the geometrical radius of the nuclei. Calculations with STARLight have been carried out taking into account production within the nuclei, e.g. important for peripheral collisions [195].

• **SuperChic3** [152]: Monte Carlo generator which implements photon–photon processes based on [92, 144]. SuperChic3 implements the EPA at the level of helicity amplitudes such that the photon polarization is taken into account. The code only provides calculations of the exclusive photon–photon process. Therefore, it cannot provide impact parameter dependent calculations for events with hadronic overlap or neutron tagging.

• **Lowest-Order QED** [104, 110, 200]: implementation of the lowest-order QED calculations following the prescription developed in [25, 105] and via low-x formalism in [200]. These calculations perform numerical computation of the process at all impact parameters. [104] primarily investigates the impact parameter dependent photon $k_\perp$ distribution, while [200] primarily investigates polarization driven effects in photon–photon interactions.

• **All-order QED** [227]: investigation of the potential influence of higher-order QED effects for photon–photon interactions in HICs. Only calculations of the total cross section (within experimental acceptance where applicable) are currently supplied. Differential cross sections and polarization dependent effects have not yet been explored.

• **Photon Wigner Formalism** [110, 111, 113, 135]: a complete formalism for the calculation of differential distributions of photon–photon fusion to dileptons via the Wigner function formalism [107]. Provides invariant mass, dilepton transverse momentum, and dilepton acoplanarity for arbitrary nucleus–nucleus impact parameter ranges.

• **Classical Field Approximation** [112]: Derivation of a general form of the cross section in terms of photon distributions, which depend on the transverse momentum and coordinates of the wave packet form of nuclear wave functions. Connections to the EPA and corrections in the Born approximation are clearly indicated. Connections to the PWF approach and those utilizing transverse momentum dependent photon distributions [200] are also explored.

Appendix B. Polarized photonuclear interactions and quantum interference

In addition to photon–photon interactions, the quasi-real photons generated by the projectile nucleus can also interact directly with the target nucleus in so-called photonuclear interactions. While not directly related to the BW process or VB, we briefly mention the theoretical formulation of photonuclear interactions since they provide another avenue for utilizing and investigating the wavefunction of the incident linearly polarized photon.

The photonuclear production process can be explained by the photon first fluctuating into a $q\bar{q}$ pair, which may then interact with the target via the strong nuclear force. This provides an effective tool to further validate the photon kinematics and polarization discussed in the previous sections in addition to the measurements of nuclear geometries. In this subsection, we discuss how the newly discovered photon polarization in HICs could be used to enable precise nuclear tomography [235]. The wave function of a quasi-real photon can be written as a Fock decomposition [274]:

$$|γ⟩ = C_{bare}|γ_{bare}⟩ + C_{V}|V⟩ + ⋯ + C_{q}|q\bar{q}⟩.$$  

Here $C_{bare} ≈ 1$ and $C_{V} ∼ \sqrt{α}(V = ρ, ω, φ,J/ψ,…)$. The coefficient $C_{V}$ is determined by the photon-vector meson coupling, $f_{V}$, through $C_{V} = √{4πα/f_{V}}$, where the coupling $f_{V}$ can be extracted from the vector meson leptonic decay width $Γ(V → e^+e^-)$. The photon has quantum numbers $J^{PC} = 1^{−}$, which makes it preferentially fluctuate to a vector meson. According to the vector meson dominance model (VMD), the scattering amplitude for the process $γ + A → B$ is the sum over the corresponding vector meson scattering amplitudes [275]:

$$A_{γ + A → B}(s,t) = \sum_{V} C_{V}A_{V + A → B}(s,t).$$

For coherent photoproduction, $γ + A → V + A$, the off-diagonal terms ($V + A → V + A$) can be neglected. The elastic vector meson scattering on nuclei ($A_{V + A → V + A}(s,t)$) can be related to the vector meson–nucleon cross section, $σ_{VN}$, via quantum Glauber [276] and the optical theorem relation:

$$A_{γ + A → B}(s,t) = \frac{2}{h} \int e^{i\mathbf{q}_L \cdot \mathbf{x}_L} \left[ 1 - \exp \left( -\frac{σ_{VN}}{2} T'(\mathbf{x}_L) \right) \right] d^2\mathbf{x}_L.$$  

$T'(\mathbf{x}_L)$ is the modified thickness function accounting for the coherent length effect:

$$T'(\mathbf{x}_L) = \int_{-∞}^{+∞} ρ(x) \sqrt{\mathbf{x}_L^2 + z^2} e^{i\mathbf{q}_L \cdot z} dz,$$

$$q_L = \frac{M_{\gamma} e^{iφ}}{2\gamma_c}.$$
where \( q_{\bot} \) is the longitudinal momentum transfer required to produce a real vector meson. The vector meson–nucleon cross section \( \sigma_{\gamma NN} \) can be determined from the measurements of the forward-scattering cross section \( \sigma_{\gamma NN}^{PB} \) through VMD relation, which is well parameterized in [99]. In the coherent photoproduction process, the produced vector meson inherits the quantum state of the photon, which is fully linearly polarized. This leads to the preferential orientation of the decay angle along the direction of polarization. Here, we take the process of \( \gamma + A \rightarrow \rho^0 + A \rightarrow \pi^+ + \pi^- + A \) as an example. Following the derivation in [277], the decay angular distribution of vector meson to two spinless daughters \( (\rho^0 \rightarrow \pi^+ + \pi^-) \) is

\[
\frac{d^2N}{d\cos\theta d\phi} = \frac{3}{8\pi} \sin^2\theta [1 + \cos(2\phi)],
\]

where the decay angles \( \theta \) and \( \phi \) are the polar and azimuthal angles, respectively, which denote the direction of one of the decay daughters in the vector meson rest frame.

In relativistic HICs, the coherent vector meson photoproduction consists of two indistinguishable processes: either nucleus 1 emits a photon and nucleus 2 acts as a target, or vice versa. The two processes interfere with each other, forming a Young’s double-slit experiment at the Fermi scale. Destructive interference of the cross section was first proposed in [278] and confirmed by the STAR collaboration [247]. Due to the linear polarization of the interacting photon, the destructive interference can also reveal itself in polarization space, which leads to a periodic oscillation pattern for the asymmetries of the decay angular distribution [235, 241]. Alternatively, the diffractive process can be implemented in terms of polarized photon–gluon interactions [238], employing the gluon saturation mechanism relevant at small-\( x \). The \( \rho^0 \) wave function cannot be implemented in perturbative QCD, but has been measured experimentally with good precision and been adapted [235, 238, 279]. Regardless of the exact theoretical prescription used, this effect is remarkable since it demonstrates quantum interference between distinguishable particles.

**ORCID iDs**

James Daniel Brandenburg  
https://orcid.org/0000-0002-6327-5947  
Janet Seger  
https://orcid.org/0000-0003-1423-6973  
Zhangbu Xu  
https://orcid.org/0000-0001-8853-0409  
Wangmei Zha  
https://orcid.org/0000-0001-6094-9574

**References**

[1] Anderson C D 1932 *Science* 76 238  
[2] Anderson C D 1933 *Phys. Rev.* 43 491  
[3] Klein O 1929 *Z. Phys.* 53 157  
[4] Sauter F 1931 *Z. Phys.* 69 742  
[5] Chao C Y 1939 *Phys. Rev.* 56 1519  
[6] Klempner O 1934 *Math. Proc. Camb. Phil. Soc.* 30 347  
[7] Goworek T 2014 *Acta Phys. Pol. A* 125 685–7  
[8] Feynman R P 1998 *Quantum Electrodynamics* (Emeryville, CA: Avalon Publishing)  
[9] Breit G and Wheeler J A 1934 *Phys. Rev.* 46 1087  
[10] Strickland D and Mourou G 1985 *Opt. Commun.* 56 219  
[11] Fermi E 1924 *Z. Phys.* 29 315  
[12] Williams E J 1934 *Phys. Rev.* 45 729  
[13] Weizsäcker C F V 1934 *Z. Phys.* 88 612  
[14] Bethe H and Heitler W 1934 *Proc. R. Soc. A* 146 83  
[15] Landau L 1965 *Collected Papers of L.D. Landau* ed D Ter Haar (Oxford: Pergamon) pp 84–95  
[16] Landau L D and Lifshitz E M 1934 *Phys. Z.* 6 244  
[17] Dirac P A M 1928 *Proc. R. Soc. A* 117 610  
[18] Feynman R P 1949 *Phys. Rev.* 76 749  
[19] Schwinger J 1949 *Phys. Rev.* 76 790  
[20] Baranger M, Bethe H and Feynman R 1953 *Phys. Rev.* 92 482–501  
[21] Tomonaga S-i 1946 *Prog. Theor. Phys.* 1 27  
[22] Dyson F J 1949 *Phys. Rev.* 75 486  
[23] Sailer T et al 2022 *Nature* 606 479  
[24] Jegerlehner F 2018 *Acta Phys. Pol. B* 49 1157  
[25] Hencken K, Trautmann D and Baur G 1995 *Phys. Rev.* A 51 1874  
[26] Baym G 1969 *Lectures on Quantum Mechanics* (San Francisco, CA: The Benjamin/Cummings Publishing Company)  
[27] Bamber C et al 1999 *Phys. Rev.* D 60 092004  
[28] Burke D L et al (E-144 Experiment) 1997 *Phys. Rev. Lett.* 79 1626  
[29] Bula C, McDonald K and Prebys E 1996 Preliminary observation of nonlinear effects in Compton scattering *Report* (Stanford Linear Accelerator Center)  
[30] Hu H, Müllner C and Keitel C H 2010 *Phys. Rev. Lett.* 105 080401  
[31] Adam J et al (STAR Collaboration) 2021 *Phys. Rev. Lett.* 127 052302  
[32] Heisenberg W and Euler H 1936 *Z. Phys.* 98 714  
[33] Weisskopf V 1936 *Kong. Dan. Vid. Sel. Mat. Fys. Med.* 14 1  
[34] Schwinger J 1951 *Phys. Rev.* 82 664  
[35] Toll J S 1952 *The dispersion relation for light and its application to problems involving electron pairs* PhD Thesis Princeton University, Princeton, NJ  
[36] Serber R 1935 *Phys. Rev.* 48 49  
[37] Pauli W and Rose M E 1936 *Phys. Rev.* 49 462  
[38] Uhling E A 1935 *Phys. Rev.* 48 55  
[39] Baier R and Breitenlohner P 1967 *Nuovo Cimento* B 47 117  
[40] Ejlli A, Della Valle F, Castaldi U, Messineo G, Pengo R, Ruoso G and Zavattini G 2020 *Phys. Rep.* 871 1  
[41] Jackson J D 1975 *Classical Electrodynamics* (New York: Wiley)  
[42] Borysoy O, Heinemann B, Ilderton A, King B and Potylitsyn A 2022 *Phys. Rev. D* 106 116015  
[43] Kärkäinen F 2017 *Proc. Helmholtz Int. Summer School 2016 (HQ 2016) (DESY-PROC)* p 44  
[44] Rizzo C, Rizzo A and Bishop D M 1997 *Int. Rev. Phys. Chem.* 16 81  
[45] Klein J J and Nigam B P 1964 *Phys. Rev.* 136 B 1540  
[46] Heyl J S and Hernquist L 1997 *J. Phys. A: Math. Gen.* 30 6485  
[47] Adler S L, Bahcall J N, Callan C G and Rosenbluth M N 1970 *Phys. Rev. Lett.* 25 1061  
[48] Bialynicka-Birula Z and Bialynicki-Birula I 1970 *Phys. Rev. Lett.* 20 2341  
[49] Adler S L 1971 *Ann. Phys., NY* 67 599  
[50] Lamb W E and Retherford R C 1947 *Phys. Rev.* 72 241  
[51] Karzeev D E, Liavn I, Voloshin S A and Wang G 2016 *Prog. Part. Nucl. Phys.* 88 1  
[52] Karzeev D E, McLerran L D and Warringa H J 2008 *Nucl. Phys. A* 803 227  
[53] Brandenburg J D, Zha W and Xu Z 2021 *Eur. Phys. J. A* 57 299
134826
Hatta Y, Xiao B-W, Yuan F and Zhou J 2021 
Abelev B I
ATLAS Collaboration 2018 Prospects for measurements of 
Battesti R
137
160
332
Gales S
Schilling K, Seyboth P and Wolf G 1970 
L041901
JHEP12(2017)044 
Sakurai J J 1960 
Budker D
033007
Borysov O 2021 The strong-field QED experiment LUXE at 
Adamo T, Ilderton A and MacLeod A J 2021 
1017
417
117
471
Aoyama T, Hayakawa M, Kinoshita T and Nio M 2012 
[hep-ph]
Hattori K, Taya H and Yoshida S 2021 
256
122502
STAR Collaboration 2021 
King B and Elkina N 2016 
1
Cartlidge E 2018 Physicists are planning to build lasers so 
Lutzky M and Toll J S 1959 
Filippi A and De Napoli M 2020 
JHEP03(2021)243 
Milstein A I and Schumacher M 1994 
Rep. Phys. 243 183 
Hattori K, Taya H and Yoshida S 2021 J. High Energy Phys. JHEP01(2021)093 
[245] Centelles M, Roca-Maza X, Vinás X and Warda M 2009 Phys. Rev. Lett. 102 122502 
[246] Adhikari D et al (PREX Collaboration) 2021 Phys. Rev. Lett. 126 172502 
[247] Abelev B I et al (STAR Collaboration) 2009 Phys. Rev. Lett. 102 112301 
[248] Adam J et al (ALICE collaboration) 2015 J. High Energy Phys. JHEP09(2015)095 
[249] Acharya S et al (ALICE collaboration) 2021 Phys. Lett. B 820 136481 
[250] Adamczyk L et al (STAR Collaboration) 2017 Phys. Rev. C 96 054904 
[251] Adam J et al (STAR Collaboration) 2019 Phys. Rev. Lett. 123 132302 
[252] ATLAS Collaboration 2018 Prospects for measurements of photon-induced processes in ultra-peripheral collisions of heavy ions with the ATLAS detector in the LHC runs 3 and 4 Technical Report (available at: https://cds.cern.ch/record/2641655/files/ATL-PHYS-PUB-2018-018.pdf) 
[253] Toll T and Ulirich T 2013 Phys. Rev. C 87 024913 
[254] del Aguila F, Cornet F and Illana J I 1991 
397 
Keshavarzi A, Nomura D and Teubner T 2018 
Chatrchyan S 
F143 
Boehly T R 
Adhikari D 
85 
1649 
Hanneke D, Fogwell S and Gabrielse G 2008 
061901 
Toll T and Ullrich T 2013 
77 
[physics]) 
Zha W, Ruan L, Tang Z, Xu Z and Yang S 2019 
Bennett G W 
Baltz A J, Chasman C and White S N 1998 
171801 
112001 
103 
023010 
Adler S L 2007 
JHEP09(2015)095 
Karbstein F, Gies H, Reuter M and Zepf M 2015 Phys. Rev. D 92 012302 
Schuler G A and Sjostrand T 1993 
116013 
Xing H, Zhang C, Zhou J and Zhou Y-J 2020 
242001 
094301 
114801 
Bauer R 
Klein S R and Nystrand J 2000 
Adam J 
B 
Phys. Rev. Lett. 
of Heavy Ions with the ATLAS Detector in the LHC Runs 3 
Photon-Induced Processes in Ultra-Peripheral Collisions 
105 
012302 
Meitner L and Kösters H 1933 
[240] Schlenvoigt H-P, Heinzl T, Schramm U, Cowan T E and 
Sauerbray R 2016 Phys. Scr. 91 023010 
[237] 
[236] 
[235] 
[234] 
[233] 
[232] 
[231] 
[230] 
[229] 
[228] 
[227] 
[226] 
[225] 
[224] 
[223] 
[222] 
[221] 
[220] 
[219] 
[218] 
[217] 
[216] 
[215] 
[214] 
[213] 
[212] 
[211] 
[210] 
[209] 
[208] 
[207] 
[206] 
[205] 
[204] 
[203] 
[202] 
[201] 
[200] 
[199] 
[198] 
[197] 
[196] 
[195] 
[194] 
[193] 
[192] 
[191] 
[190] 
[189] 
[188] 
[187] 
[186] 
[185] 
[184] 
[183] 
[182] 
[181] 
[180] 
[179] 
[178] 
[177] 
[176] 
[175] 
[174] 
[173] 
[172] 
[171] 
[170] 
[169] 
[168] 
[167] 
[166] 
[165] 
[164] 
[163] 
[162] 
[161] 
[160] 
[159] 
[158] 
[157] 
[156] 
[155] 
[154] 
[153] 
[152] 
[151] 
[150] 
[149] 
[148] 
[147] 
[146] 
[145] 
[144] 
[143] 
[142] 
[141] 
[140] 
[139] 
[138] 
[137] 
[136] 
[135] 
[134] 
[133] 
[132] 
[131] 
[130] 
[129] 
[128] 
[127] 
[126] 
[125] 
[124] 
[123] 
[122] 
[121] 
[120] 
[119] 
[118] 
[117] 
[116] 
[115] 
[114] 
[113] 
[112] 
[111] 
[110] 
[109] 
[108] 
[107] 
[106] 
[105] 
[104] 
[103] 
[102] 
[101] 
[100] 
[99] 
[98] 
[97] 
[96] 
[95] 
[94] 
[93] 
[92] 
[91] 
[90] 
[89] 
[88] 
[87] 
[86] 
[85] 
[84] 
[83] 
[82] 
[81] 
[80] 
[79] 
[78] 
[77] 
[76] 
[75] 
[74] 
[73] 
[72] 
[71] 
[70] 
[69] 
[68] 
[67] 
[66] 
[65] 
[64] 
[63] 
[62] 
[61] 
[60] 
[59] 
[58] 
[57] 
[56] 
[55] 
[54] 
[53] 
[52] 
[51] 
[50] 
[49] 
[48] 
[47] 
[46] 
[45] 
[44] 
[43] 
[42] 
[41] 
[40] 
[39] 
[38] 
[37] 
[36] 
[35] 
[34] 
[33] 
[32] 
[31] 
[30] 
[29] 
[28] 
[27] 
[26] 
[25] 
[24] 
[23] 
[22] 
[21] 
[20] 
[19] 
[18] 
[17] 
[16] 
[15] 
[14] 
[13] 
[12] 
[11] 
[10] 
[9] 
[8] 
[7] 
[6] 
[5] 
[4] 
[3] 
[2] 
[1]