Primordial Helium Abundance from CMB: a constraint from recent observations and a forecast

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Abstract

We studied a constraint on the primordial helium abundance $Y_p$ from current and future observations of CMB. Using the currently available data from WMAP, ACBAR, CBI and BOOMERANG, we obtained the constraint as $Y_p = 0.25^{+0.10}_{-0.07}$ at 68\% C.L. We also provide a forecast for the Planck experiment using the Markov chain Monte Carlo approach. In addition to forecasting the constraint on $Y_p$, we investigate how assumptions for $Y_p$ affect constraints on the other cosmological parameters.
1 Introduction

Current cosmological observations are very precise to give us a lot of information on the evolution and present state of the universe. We usually extract the information by constraining cosmological parameters such as energy densities of baryon, dark matter and dark energy, the Hubble constant, reionization optical depth, spectral index of primordial fluctuation and so on. Among them, in this paper, we focus on the primordial helium abundance $Y_p$, which has been of great interest in cosmology. One of the reasons why the primordial helium abundance has been considered to be interesting and important is that, in the context of the standard big bang nucleosynthesis (SBBN), we can know the baryon density once $Y_p$ is determined from observations. However, it has been discussed that a significant systematic error dominates when one infers the value of $Y_p$ from measurements in low-metallicity extragalactic HII region [1–5]. Furthermore, there have been some discussions that there may be a large uncertainty in the neutron lifetime [6–8], which results in uncertainties in the predictions for the abundances of light elements. In this respect, the study of other independent measurements of the helium abundance would be interesting.

Recent precise measurements of cosmic microwave background (CMB) such as WMAP can now enable us to constrain cosmological parameters with great accuracies. However, the helium abundance has not been discussed much in the literature when one study of cosmological constraints from CMB since $Y_p$ has been considered to have little effect on CMB power spectrum. In most of analyses, $Y_p$ is fixed to be 0.24 which is probably motivated from a somewhat old value of the observed primordial helium abundance of $Y_p = 0.238$ [9]. But, in fact, there have been some works which discuss the effects of $Y_p$ on CMB and some constraints were given [10–12]. Since the helium abundance affects the recombination history, the CMB power spectrum can be affected mainly through the diffusion damping. Although the constraint is not so severe, it is important to notice that they are obtained independently from BBN, which can be used to cross-check our understanding of the helium abundance. Furthermore, the value of $Y_p$ at the time of BBN may be different from that at late time when CMB observations can probe [2]. After we studied the constraint on $Y_p$ in [12], the data from WMAP has been updated [14–18]. In addition, the data at higher multipoles where the effects of $Y_p$ become significant have been updated by ACBAR [19] and CBI [20]. Thus it is a good time to investigate the constraint on the helium abundance using these CMB data, which is one of the aims of this paper.

Furthermore, we expect a more precise measurement of CMB from the future Planck satellite [21], which can give us a much better constraint on $Y_p$. In fact, future constraint on $Y_p$ has already been studied using the Fisher matrix formalism [10, 12]. Although this method is fast and usually adopted to predict the precision of the future measurements of

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#1 Recent observations give e.g. $Y_p = 0.249 \pm 0.009$ [1, 2].
#2 For example, in a scenario to solve so-called “Lithium problem” with $Q$-balls, the decay of $Q$-balls produce extra baryon after BBN has completed [13]. In a model of this kind, the value of $Y_p$ can vary at different epochs.
cosmological parameters, it can give some inaccurate predictions in some cases, for example, when the likelihoods does not respect a Gaussian form. In addition, the Fisher matrix formalism predicts only the uncertainty for the parameter estimation since it just concerns with the derivatives with respect to parameters around the fiducial values. However, since some parameters are correlated in general, fixing the values of some parameters can bias the estimation of other parameters. Namely, priors we assume on some parameters can cause the estimated central values to deviate from the input fiducial values, but such effects cannot be quantified by the Fisher matrix approach. Thus, in this paper, we use the Markov chain Monte Carlo (MCMC) approach to extract reliable future constraints on \( Y_p \) and other cosmological parameters. In particular, when we forecast the sensitivity for other cosmological parameters, we assume some different priors on \( Y_p \) and investigate to what extent the information of \( Y_p \) is important to determine other cosmological parameters.

The organization of this paper is as follows. In the next section, the effects of \( Y_p \) on CMB power spectrum are briefly discussed. Then in section 3, we study the constraint on \( Y_p \) using CMB data currently available including ACBAR, BOOMERANG and CBI as well as WMAP5. In section 4, we investigate a constraint on \( Y_p \) and other cosmological parameters from the future Planck experiment. In addition, we also study how the prior on \( Y_p \) affects the determination of other cosmological parameters. A brief discussion on the significance of the uncertainties of the recombination theory in deriving cosmological constraint is given too. The final section is devoted to the summary of this paper.

2 Effects of \( Y_p \) on CMB

Here we briefly discuss the effects of the primordial helium abundance \( Y_p \) on CMB power spectrum where \( Y_p = 4n_{\text{He}}/(n_H + 4n_{\text{He}}) \) with \( n_H \) and \( n_{\text{He}} \) being the number density of hydrogen and helium-4 respectively. As has been discussed in \([10, 12]\), the value of \( Y_p \) can affect the recombination history, which changes the structure of acoustic peaks. The effects of \( Y_p \) on acoustic peaks mainly come from the diffusion damping which suppresses the power on small scales and the shift of the position of acoustic peaks due to the change of the recombination epoch. Before recombination, the number density of electron \( n_e \) can be given by \( n_e = n_b(1 - Y_p) \), where \( n_b \) is the baryon number density. Thus the increase of \( Y_p \) indicates the decrease of the number of electrons. When the number of electrons is reduced, the mean free path of the Compton scattering becomes larger, which means that fluctuations on larger scales can be more affected by the diffusive mixing and rescattering. Thus the damping scale below which fluctuation of photon is exponentially suppressed becomes larger. Furthermore, due to the change of the number density of electrons, the epoch of recombination is also affected even though its effect is not so significant. This effect shifts the position of acoustic peaks slightly. In Fig. 1, we show the CMB \( TT \) spectrum with several values of \( Y_p \). For reference, we also plot the current data (left panel) and the expected data from the future Planck experiment (right panel).
As mentioned above, by increasing the value of $Y_p$, the power on small scales is damped more. In addition, it is noticeable that the position of acoustic peaks also shifts slightly.

![Figure 1: CMB $TT$ power spectra for several values of $Y_p$. In this figure, we take $Y_p = 0.1$ (blue dotted line), 0.24 (red solid line) and 0.4 (green dashed line). Other cosmological parameters are assumed to be the mean value of WMAP5 for a power-law ΛCDM model. For reference, in the left panel, data from WMAP5, ACBAR, BOOMERANG and CBI are also depicted. In the right panel, the expected data from the Planck experiment are also shown.](image)

To characterize the effects of the change in $Y_p$ and other cosmological parameters on CMB $TT$ power spectrum $C_l^{TT}$, we consider some useful quantities [22]. First of all, to see how cosmological parameters affect the position of acoustic peaks, we investigated the response of the position of the first peak by the change of the parameters, which we denote $\Delta l_1$. In addition, to see the effects of the diffusion damping and some other effects by cosmological parameters, we study the height of the first peak relative to that at $l = 10$ and the height of the second peak (and higher peaks up to 5th peak) relative to the first peak, which are denoted as $H_1$, $H_2$, $H_3$, $H_4$ and $H_5$, respectively. For clarity, we give the definitions of these quantities. The definition of $H_1$ is

$$H_1 = \left( \frac{\Delta T(l = l_1)}{\Delta T(l = 10)} \right)^2,$$

and the height of the $i$-th peak relative to the first peak is defined as

$$H_i = \left( \frac{\Delta T(l = l_i)}{\Delta T(l = l_1)} \right)^2 \quad \text{(for } i \geq 2),$$

where $(\Delta T(l))^2 = l(l+1)C_l^{TT}/2\pi$. We varied cosmological parameters including $Y_p$ around a fiducial model and obtained partial derivatives by fitting linearly around the fiducial
value. For the fiducial cosmological values, we assumed the mean values of the WMAP 5-yr result (WMAP5) for a power-law ΛCDM model. Regarding \( Y_p \), we take \( Y_p = 0.248 \), which corresponds to the value obtained in the SBBN for the WMAP5 baryon density. The resulting derivatives are:

\[
\Delta l_1 = 15.6 \frac{\Delta \omega_b}{\omega_b} - 27.0 \frac{\Delta \omega_m}{\omega_m} + 36.0 \frac{\Delta n_s}{n_s} + 0.94 \frac{\Delta Y_p}{Y_p} - 44.5 \frac{\Delta h}{h},
\]

\[
\Delta H_1 = 2.87 \frac{\Delta \omega_b}{\omega_b} - 3.13 \frac{\Delta \omega_m}{\omega_m} + 16.7 \frac{\Delta n_s}{n_s} - 2.30 \frac{\Delta h}{h},
\]

\[
\Delta H_2 = -0.290 \frac{\Delta \omega_b}{\omega_b} + 0.023 \frac{\Delta \omega_m}{\omega_m} + 0.396 \frac{\Delta n_s}{n_s} - 0.013 \frac{\Delta Y_p}{Y_p},
\]

\[
\Delta H_3 = -0.177 \frac{\Delta \omega_b}{\omega_b} + 0.206 \frac{\Delta \omega_m}{\omega_m} + 0.514 \frac{\Delta n_s}{n_s} - 0.028 \frac{\Delta Y_p}{Y_p},
\]

\[
\Delta H_4 = -0.102 \frac{\Delta \omega_b}{\omega_b} + 0.082 \frac{\Delta \omega_m}{\omega_m} + 0.317 \frac{\Delta n_s}{n_s} - 0.025 \frac{\Delta Y_p}{Y_p},
\]

\[
\Delta H_5 = -0.040 \frac{\Delta \omega_b}{\omega_b} + 0.084 \frac{\Delta \omega_m}{\omega_m} + 0.236 \frac{\Delta n_s}{n_s} - 0.023 \frac{\Delta Y_p}{Y_p},
\]

where \( \omega_b \) and \( \omega_m \) are energy densities of baryon and matter, \( n_s \) is the scalar spectral index of primordial fluctuation, \( h \) is the Hubble constant in units of \( 100 \text{ km s}^{-1} \text{Mpc}^{-1} \). In the formula for \( H_1 \), we do not show the dependence on \( \Delta Y_p/Y_p \) since its effect on \( H_1 \) is very small compared to that of the other parameters. As seen from the negative signs of \( \Delta H_i/\Delta Y_p \) for \( i = 2-5 \), the diffusion damping becomes more efficient as \( Y_p \) increases. We can also see the correlation of \( Y_p \) with other cosmological parameters, which can be useful when we interpret the results, in particular, for a Planck forecast.

### 3 Current constraint on \( Y_p \)

Now we discuss the constraint on \( Y_p \) from current cosmological observations. For this purpose, we make use of the CMB data from WMAP5 [14–18], ACBAR [19], BOOMERANG [23–25] and CBI [20]. To investigate the constraint, we performed a MCMC analysis by using a modified version of cosmomc code [26]. We sampled in an 8 dimensional parameter space with \((\omega_b, \omega_c, \tau, \theta_s, n_s, A_s, Y_p, A_{SZ})\) where \( \omega_c \) is the energy density of dark matter, \( \tau \) is the optical depth of reionization, \( \theta_s \) is acoustic peak scale [27], \( A_s \) is the amplitude of primordial curvature fluctuation at the pivot scale \( k_0 = 0.05 \text{ Mpc}^{-1} \) and \( A_{SZ} \) is the amplitude of thermal Sunyaev-Zel’dovich (SZ) effect which is normalized to the \( C_l^{SZ} \) template from [28].

In this paper, we consider a flat universe and assume a cosmological constant as dark energy. We also assume no running for primordial scalar fluctuation and no tensor mode. When we report our results in the following, we also use other customarily used #3 However, the SZ effect may be so large at very high multipoles that this template may not be appropriate to adopt. Hence, we conservatively do not use the ACBAR and CBI data with \( l \geq 2100. \)

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Table 1: The prior ranges for the parameters used in the analysis. Priors shown in the 1st and 2nd columns are adopted for current and expected Planck data, respectively. Note that we adopt the prior range for $Y_p$ shown above only in the cases with $Y_p$ being treated as a free parameter whereas $Y_p$ is a derived parameter in the case where we assume the SBBN relation. For the analysis with the current data, we also vary the amplitude of the Sunyaev-Zel’dovich effect $A_{SZ}$, which is omitted in the Planck data analysis. We also include two additional parameters $F_H$ and $b_{He}$ which represent uncertainties in the theory of recombination (see section 4 for more details).

| parameters | current data | Planck |
|------------|--------------|--------|
| $\omega_b$ | $0.005 \rightarrow 0.1$ | $0.005 \rightarrow 0.1$ |
| $\omega_c$ | $0.01 \rightarrow 0.99$ | $0.01 \rightarrow 0.99$ |
| $\theta_s$ | $0.5 \rightarrow 10$ | $0.5 \rightarrow 10$ |
| $\tau$ | $0.01 \rightarrow 0.8$ | $0.01 \rightarrow 0.8$ |
| $n_s$ | $0.5 \rightarrow 1.5$ | $0.5 \rightarrow 1.5$ |
| $\ln(10^{10} A_s)$ | $2.7 \rightarrow 4$ | $2.7 \rightarrow 4$ |
| $Y_p$ | $(0 \rightarrow 1)$ | $(0 \rightarrow 1)$ |
| $A_{SZ}$ | $0 \rightarrow 2$ | — |
| $F_H$ | — | $(0 \rightarrow 2)$ |
| $b_{He}$ | — | $(0 \rightarrow 1.5)$ |

Now we discuss the constraints on $Y_p$ when WMAP5 alone is used and when the data of ACBAR, BOOMERANG and CBI are used in addition. In Fig. 2 we show one-dimensional marginalized distributions of $Y_p$ for these two cases and, in the 1st and 2nd columns of Table 2 we list the parameter estimations for $Y_p$ and other cosmological parameters. When we use WMAP5 data alone, the constraint is given as $Y_p \leq 0.44$ at 95% C.L. where we only give the upper bound since the likelihood has a sizable value at $Y_p = 0$. On the other hand, when all data are combined, the constraint is $Y_p = 0.25^{+0.10}_{-0.07} (^{+0.15}_{-0.17})$ at 68% (95%) C.L. For the analysis using WMAP5 alone, the limit we obtained is consistent with the result given in [15]. We see that by including the data from ACBAR, BOOMERANG and CBI the likelihood distribution has a well-defined peak which is close to Gaussian. It may also be interesting to notice that when the data from ACBAR and BOOMERANG and CBI are combined, the mean value becomes as $Y_p = 0.25$, which is very close to the value obtained from HII region observations although the uncertainty is still large. With the help of this data set, we may begin to see the concordance with regards the $Y_p$ measurement from CMB and that from the HII regions.
Next we discuss the effects of the prior of $Y_p$ on the determination of other cosmological parameters. For this purpose, we repeated a MCMC analysis fixing the value of the helium abundance to $Y_p = 0.24$ as in usual analyses. We use all the CMB data (i.e., WMAP5, ACBAR, BOOMERANG and CBI) here. In Table 2, in the last column, the constraints on cosmological parameters for the case with fixing $Y_p = 0.24$ are shown. When we compare the constraints for the cases with and without fixing $Y_p$, the central values as well as the errors at 68% C.L. are almost unchanged. Thus we can conclude that the usual practice of fixing of $Y_p = 0.24$ scarcely affects the constraints on other cosmological parameters with the current precision of CMB data. However, since we can expect more precise measurements of CMB in the near future, the prior on $Y_p$ may become important and can affect the constraints on other cosmological parameters. We study this issue in the next section.

4 Future constraint from Planck

In this section, we forecast a constraint for the Planck experiment [21] focusing on the constraint on $Y_p$ itself and how the prior on $Y_p$ affects the constraints on other cosmological parameters. In fact, constraints from the future Planck experiment from this viewpoint have already been discussed in Refs. [10, 12] by using the Fisher matrix analysis. As mentioned in the introduction, when the likelihood of cosmological parameters can be approximated by a multivariate Gaussian function, the Fisher matrix analysis can give a reliable prediction. However, in practice, the likelihood function deviates from the Gaussian form. Furthermore, since the Fisher matrix analysis can only predict the uncertainty for a fiducial value, it cannot extract a bias effect (i.e. the estimated central value deviates from the fiducial value) which is caused by assuming priors on parameters and possible...
| parameters | WMAP5 alone | CMB all | CMB all |
|------------|-------------|---------|---------|
|            | \( (Y_p \text{ free}) \) | \( (Y_p \text{ free}) \) | \( (Y_p = 0.24) \) |
| \( \omega_b \) | 0.0228 ± 0.0006 | 0.0229 ± 0.0005 | 0.0229 ± 0.0006 |
| \( \omega_c \) | 0.109 ± 0.006 | 0.113 ± 0.007 | 0.112 ± 0.006 |
| \( \theta_s \) | 1.040 ± 0.004 | 1.043 ± 0.004 | 1.043 ± 0.003 |
| \( \tau \) | 0.088 ± 0.016 | 0.087 ± 0.018 | 0.087 ± 0.016 |
| \( n_s \) | 0.964 ± 0.016 | 0.967 ± 0.015 | 0.966 ± 0.014 |
| \( \ln(10^{10} A_s) \) | 3.06 ± 0.06 | 3.08 ± 0.04 | 3.07 ± 0.04 |
| \( Y_p \) | < 0.44(95%) | 0.25 ± 0.07 | — |
| \( A_{SZ} \) | 1.1 ± 0.9 | 1.1 ± 0.9 | 1.0 ± 1.0 |
| \( \Omega_m \) | 0.25 ± 0.03 | 0.27 ± 0.03 | 0.26 ± 0.02 |
| \( H_0 \) | 72.3 ± 2.6 | 71.7 ± 2.2 | 71.7 ± 2.3 |

Table 2: Mean values and 68% errors from current observations of CMB for the cases with WMAP5 alone and all data combined. (Regarding \( Y_p \), an upper bound at 95% C.L. is given for the case with WMAP5 alone.) In the last column, the value of \( Y_p \) is fixed as \( Y_p = 0.24 \).

correlations among parameters. Thus it may be better to make a more reliable prediction by using a MCMC method. For this purpose, we follow the approach of Ref. [29].

Here we briefly explain the method of Ref. [29]. Observed anisotropies can be expanded in spherical harmonics and their power spectra of the coefficients \( a_l^m \) are composed of signal parts \( C_l^{PP'} \) and noise parts \( N_l^{PP'} \):

\[
\langle a_l^m \rangle = \langle C_l^{PP'} \rangle + N_l^{PP'} \delta_{ll} \delta_{mm},
\]

where \( PP' \) represents three pairs of maps, \( TT, EE \) and \( TE \). The signal parts are computed from a fiducial cosmology. We assume the cosmological parameters of the WMAP5 mean values for a power-law \( \Lambda \)CDM model as a fiducial model. As for the noise power spectra, we assume a Gaussian beam and a spatially uniform Gaussian white noise. \( N_l^{PP'} \) are given as the combined effects from these and can be approximated as

\[
N_l^{PP'} = \delta_{PP'} (\theta_{FWHM} \sigma_P)^2 \exp \left[ \frac{l(l+1)}{8 \ln 2} \right],
\]

where \( \theta_{FWHM} \) is the full width at half maximum of the Gaussian beam and \( \sigma_P \) is the root mean square of the instrumental noise. For the expected data from the Planck experiment, we use three frequency channels at 100, 143 and 217 GHz. We adopt the following values for the instrumental parameters [29]: \( (\theta_{FWHM}[\text{arcmin}], \sigma_T[\mu\text{K}], \sigma_P[\mu\text{K}]) = (9.5, 6.8, 10.9), (7.1, 6.0, 11.4) \) and \( (5.0, 13.1, 26.7) \) for \( \nu = 100, 143 \) and 217 GHz, respectively. We assume other frequency channels are used to remove foregrounds and they are ideally removed.
Since the anisotropies from both signal and noise are Gaussian distributed, the likelihood function of the data \( a = \{ a_{\text{lm}}^{T,E} \} \) for a theoretical model with parameters \( \Theta = \{ \theta_i \} \) is given by

\[
\mathcal{L}(a|\Theta) \propto \frac{1}{\sqrt{C(\Theta)}} \exp \left( -\frac{1}{2} a^* [\bar{C}(\Theta)]^{-1} a \right),
\]

where \( \bar{C}(\Theta) \) is a covariance matrix of the theoretical data. Denoting a covariance matrix of mock data as \( \hat{C} \), the effective \( \chi^2 \) is given as

\[
\chi^2_{\text{eff}} = \sum_l (2l + 1) f_{\text{sky}} \left[ \ln \frac{\bar{C}_l}{\hat{C}_l} + \hat{C}_l \bar{C}_l^{-1} - 2 \right].
\]

We take \( f_{\text{sky}} = 0.65 \) as the expected sky-coverage for the Planck experiment. The factor \((2l + 1)f_{\text{sky}}\) represents the effective number of independent moments obtained from the observation. For the MCMC analysis, we include the data up to \( l = 2500 \).

Before presenting our results, here we comment on possible contributions from the thermal and kinetic SZ effect. We assume that the thermal SZ effect can be precisely estimated from the other lower frequency channels of the Planck survey than those used in our analysis, and can be removed ideally. The contribution from the kinetic SZ effect on CMB anisotropy depends on the details of the reionization process. For somewhat conventional scenario, as argued in [30], it would be only about a few percent in the range of multipoles we make use of, \( l \leq 2500 \), and also sufficiently smaller than the expected instrumental noise for the Planck survey. Ref. [31] argued that “patchy” reionization would make it significantly larger but they found that the shape of the power spectrum due to the kinetic SZ effect does not depend much on the reionization model. Then, they concluded that its effect on the determination of the cosmological parameters can be neglected by marginalizing over the amplitude of the kinetic SZ power spectrum. We thus neglect the kinetic SZ effect here.

Now we discuss a future constraint on \( Y_p \) from the Planck experiment. In Fig. 3, one-dimensional marginalized likelihood for \( Y_p \) is shown. For comparison, we also plot the constraint from current observations. We can expect that the uncertainty for \( Y_p \) at 68% C.L. becomes as \( \Delta Y_p \sim 10^{-2} \), which is 10 times smaller than that from current data (see Table 3 below). Since Planck can measure the CMB power spectrum at higher multipoles very precisely, the effects of damping due to \( Y_p \) can be well probed. It should also be noted that since likelihood functions for \( Y_p \) and other cosmological parameters have almost the Gaussian form, our results here using MCMC approach are almost the same as those obtained by Fisher matrix analysis which has already been done in Refs. [10, 12]. Thus we found that the Fisher matrix analysis can give a good estimate for Planck data for the parameter set we assumed here. The results here are consistent with those given in Ref. [29, 32] in which a forecast on \( Y_p \) is investigated using MCMC analysis too.

Next we discuss the effects of prior on \( Y_p \) on the constraints on other cosmological parameters in the Planck experiment. As mentioned in the Introduction, when one tries to constrain some cosmological parameters from CMB, the value of \( Y_p \) is fixed to be
0.24 in most of analysis. In the previous section, we showed that, when we use current cosmological data, the fixing of $Y_p = 0.24$ does not affect much the constraints on other cosmological parameters since the value of $Y_p$ itself is not constrained well. However, as just shown above, Planck can measure the value of $Y_p$ much more precisely, thus we should study the effects of the assumption of $Y_p$ when we constrain other parameters. For this purpose, we made MCMC analyses for three cases: (i) $Y_p$ is not fixed but varied freely, (ii) $Y_p$ is fixed as $Y_p = 0.24$ and (iii) $Y_p$ is regarded as a function of $\omega_b$ via the standard BBN calculation. For the case (iii), we relate the value of $Y_p$ to $\omega_b$ by the fitting formula given in [33], to which we refer as “BBN relation” in the following.

| parameters | $Y_p$ free | SBBN $Y_p(\omega_b)$ | $Y_p = 0.24$ |
|------------|------------|-----------------|---------------|
| $\omega_b$ | $0.02273^{+0.00024}_{-0.00025}$ | $0.02273^{+0.00017}_{-0.00017}$ | $0.02261^{+0.00016}_{-0.00017}$ |
| $\omega_c$ | $0.1098^{+0.0015}_{-0.0014}$ | $0.1099^{+0.0015}_{-0.0014}$ | $0.1100^{+0.0013}_{-0.0016}$ |
| $\theta_s$ | $1.04063^{+0.00057}_{-0.00061}$ | $1.04061^{+0.00057}_{-0.00056}$ | $1.04031^{+0.00034}_{-0.00038}$ |
| $\tau$ | $0.0879^{+0.0054}_{-0.0062}$ | $0.0880^{+0.0055}_{-0.0060}$ | $0.0871^{+0.0049}_{-0.0061}$ |
| $n_s$ | $0.9627^{+0.0079}_{-0.0085}$ | $0.9631^{+0.0079}_{-0.0085}$ | $0.9580^{+0.0042}_{-0.0044}$ |
| $\ln(10^{10}A_s)$ | $3.064^{+0.011}_{-0.013}$ | $3.065^{+0.010}_{-0.013}$ | $3.061^{+0.010}_{-0.012}$ |
| $Y_p$ | $0.248^{+0.011}_{-0.011}$ | $0.248586^{+0.000078}_{-0.000076}$ | — |
| $\Omega_m$ | $0.2567^{+0.0080}_{-0.0086}$ | $0.2565^{+0.0074}_{-0.0080}$ | $0.2587^{+0.0078}_{-0.0083}$ |
| $H_0$ | $71.88^{+0.80}_{-0.85}$ | $71.92^{+0.78}_{-0.66}$ | $71.61^{+0.73}_{-0.72}$ |

Table 3: Mean values and 68% errors from Planck for some assumptions on $Y_p$. 

Figure 3: One-dimensional marginalized distributions of $Y_p$. Shown are the distributions from the Planck experiment for the cases with no priors on $Y_p$ (red solid line) and assuming the BBN relation (green dashed line). For reference, the distribution for the case with no priors on $Y_p$ using current CMB data is also shown (dash-dotted magenta line).
Figure 4: One-dimensional marginalized distributions of $\omega_b$, $\omega_c$, $n_s$, $A_s$, $\tau$, $H_0$, using same data as in Fig. 3. Additionally, the distributions from Planck for the case with fixing $Y_p = 0.24$ are also shown (dotted blue line).

Now we show one-dimensional marginalized likelihood for $\omega_b$, $\omega_c$, $n_s$, $A_s$, $\tau$, $H_0$ in Fig. 4. In the figure, three cases (i), (ii) and (iii) are depicted. In Table 3, the mean values and errors at 68% C.L. are shown for representative parameters. By looking at Fig. 4 some features can be noticed. For $\omega_c$, $A_s$ and $\tau$, the effects of the prior on $Y_p$ are very small even with the precision of Planck. However, for $\omega_b$, $n_s$ and $H_0$, marginalized distributions are changed depending on the prior on $Y_p$. This tendency can also be seen by reading the errors at 68% C.L. from Table 3. For $\omega_b$, $n_s$ and $H_0$, when we assume the BBN relation or fix the value of the helium abundance as $Y_p = 0.24$, the errors are reduced to some extent, which clearly indicates that the assumption of $Y_p$ can affect the determination of other cosmological parameters. Furthermore, for these parameters, when we fix $Y_p = 0.24$, the central values differ from the fiducial values by about the uncertainties at 68% C.L. Therefore, in the Planck era, we advocate varying the value of $Y_p$ freely in the cosmological parameter estimation for a conservative constraint, or, if we would like to do the cosmological parameter estimation in the framework of the standard cosmology, we should impose the BBN relation.

To see how these parameters are correlated with $Y_p$, 2D marginalized contours may be useful, which are shown in Fig. 5. From this figure, we can see that $Y_p$ and these

#4 A similar analysis has been done in Ref. [32] recently and their results are consistent with ours.
parameters $\omega_b, n_s$ and $H_0$ are positively correlated. Positive correlations of $Y_p$ with $n_s$ mainly come from the degeneracy at higher multipoles where the effect of the diffusion damping is significant. From Eqs. (5), (6), (7) and (8), the response of $H_i$ for $i \geq 2$ to the change of $Y_p$ and $n_s$ are opposite sign, which indicates that the correlation between these are positive. Correspondingly, $\omega_b$ and $Y_p$ becomes positively correlated because of the positive correlation between $\omega_b$ and $n_s$ which can be read off, in particular from Eq. (4). For the correlation of $Y_p$ with $H_0$, it should be noticed that the position of the first peak can be significantly affected by changing $n_s$ and $H_0$. By increasing the value of $Y_p$, the diffusion damping suppresses the power on small scales. To compensate this effect to fit the data well, increasing $n_s$ can enhance the power. Due to the change of $n_s$, the first peak position is in turn also shifted. However, as is already well determined by WMAP, the position of the first peak should be tuned to the right position to fit the data well. This can be done by changing $H_0$ as can be seen from Eq. (3). In fact, the change of $Y_p$ itself can also shift the peak position. However the direct effect of $Y_p$ on $l_1$ is very small compared to other quantities.

Figure 5: Two-dimensional marginalized constraints in several combinations of cosmological parameters. Shown are a Planck forecast on the constraints for the cases with no prior on $Y_p$ (orange and yellow shaded region), assuming the BBN relation (solid red line) and fixing $Y_p = 0.24$ (blue dotted line). For reference, the constraint for the case with no prior on $Y_p$ using current CMB data is also shown (dash-dotted magenta line).

Finally, we would briefly discuss how our arguments above are affected by considering the uncertainties arising from the recombination process. Several authors have claimed
that the uncertainties in the theory of recombination make some effects on the CMB power spectra and the determination of the cosmological parameters from CMB [34,35]. Although some detailed studies of the recombination modeling have been done [36–41], more developments of the recombination modeling may be needed [35]. Since the primordial helium abundance, which we are studying in this paper, directly affects the recombination history, it may be interesting to check how the uncertainties in the recombination modeling affect the determination of $Y_p$ and other cosmological parameters in the future Planck experiment. For this purpose, we treat two numerical parameters, the so-called fudge factors $F_H$ and $b_{He}$ used in recfast [35,42,43] as free parameters to represent the uncertainties of the recombination modeling. $F_H$ is introduced to fit the hydrogen recombination rate in the three-level approximation to the result from multi-level calculations. $b_{He}$ is a fitting parameter for the recombination rate of HeI. We repeated a MCMC analysis including $F_H$ and $b_{He}$ and marginalized over these parameters with the top hat priors given in Table 1 which are very conservative ones. In Table 4 we summarize the resultant constraints on the cosmological parameters from expected Planck data. We also show the probability distributions for several cosmological parameters in Fig. 6. Table 4 shows that, even if we adopt a very conservative prior on the fudge factors, the uncertainties of the recombination modeling which is represented by $F_H$ and $b_{He}$ do not significantly affect the determination of cosmological parameters in the Planck era. (Errors for some parameters are changed at most by 10%.) Therefore, we can say at least that the theoretical uncertainties of the recombination process which are discussed recently do not affect our previous discussions. However, since our analysis is limited, we need more understanding of the recombination process and more detailed analysis in order to reduce systematic errors in the helium estimation from future CMB data.

| parameters | $Y_p$ free | SBBN $Y_p(\omega_b)$ |
|------------|------------|----------------------|
| $F_H$, $b_{He}$ fixed | $F_H$, $b_{He}$ free | $F_H$, $b_{He}$ fixed |
| $\omega_b$ | $0.02273^{+0.00124}_{-0.00025}$ | $0.2273^{+0.00024}_{-0.00024}$ | $0.02273^{+0.00017}_{-0.00017}$ | $0.0002273^{+0.000017}_{-0.000017}$ |
| $\omega_c$ | $0.1098^{+0.0015}_{-0.0014}$ | $0.1098^{+0.0014}_{-0.0016}$ | $0.1099^{+0.0015}_{-0.0014}$ | $0.1098^{+0.0014}_{-0.0016}$ |
| $\theta_s$ | $1.0406^{+0.00057}_{-0.00061}$ | $1.0406^{+0.00060}_{-0.00061}$ | $1.0406^{+0.00037}_{-0.00036}$ | $1.0406^{+0.00036}_{-0.00036}$ |
| $\tau$ | $0.0879^{+0.00054}_{-0.00054}$ | $0.0882^{+0.00052}_{-0.00052}$ | $0.0880^{+0.00065}_{-0.00060}$ | $0.0880^{+0.00056}_{-0.00064}$ |
| $n_s$ | $0.9627^{+0.0005}_{-0.0005}$ | $0.9628^{+0.0005}_{-0.0005}$ | $0.9631^{+0.0005}_{-0.0004}$ | $0.9626^{+0.0005}_{-0.0005}$ |
| $\ln(10^{10} A_s)$ | $3.064^{+0.014}_{-0.013}$ | $3.064^{+0.013}_{-0.014}$ | $3.065^{+0.013}_{-0.013}$ | $3.064^{+0.012}_{-0.012}$ |
| $Y_p$ | $0.248^{+0.014}_{-0.011}$ | $0.249^{+0.013}_{-0.012}$ | $0.2485^{+0.000078}_{-0.000076}$ | $0.24858^{+0.000078}_{-0.000076}$ |
| $\Omega_m$ | $0.2567^{+0.0080}_{-0.0086}$ | $0.2564^{+0.0081}_{-0.0088}$ | $0.2565^{+0.0080}_{-0.0080}$ | $0.2565^{+0.0080}_{-0.0080}$ |
| $H_0$ | $71.88^{+0.80}_{-0.85}$ | $71.91^{+0.86}_{-0.82}$ | $71.92^{+0.78}_{-0.66}$ | $71.89^{+0.75}_{-0.75}$ |

Table 4: Comparison for the cases with and without uncertainties from the recombination process being considered. In the first two columns $Y_p$ is treated as a free parameter and in the latter two columns the SBBN relation is assumed.
We studied the constraint on $Y_p$ and the effects of the priors for $Y_p$ on constraining other cosmological parameters using current CMB data from WMAP5, ACBAR, BOOMERANG and CBI, and also from the future Planck experiment. After briefly reviewing the effects of $Y_p$ on CMB, we studied current constraints on the primordial helium abundance. We obtained the current limit on $Y_p$ from WMAP5 alone as $Y_p \leq 0.44$ at 95% C.L., which is improved to be $Y_p = 0.25^{+0.10}_{-0.07}(+0.15)$ at 68% (95%) C.L. by adding the data of ACBAR, BOOMERANG and CBI around the damping tail. We have also considered how the prior of $Y_p$ can affects the constraints on other cosmological parameters using currently available data. We found that, at the present precision level of CMB measurements, the prior on $Y_p$ has little effect for determinations of other cosmological parameters.

We have also investigated the future constraint from the Planck experiment. By performing a MCMC analysis, we derived an expected error for the helium abundance from the future Planck experiment and found that it will be well measured with the accuracy of $\Delta Y_p \sim 10^{-2}$ (68% C.L.) in the Planck experiment, which is 10 times smaller values
compared with current data. Furthermore, it may be interesting to notice that this precision is comparable to that obtained by HII region observations. As for the effects of the prior on $Y_p$ on the determination of other cosmological parameters, we found that, with the precision of Planck, the assumption on $Y_p$ can affect the constraints on other cosmological parameters such as $\omega_b$ and $n_s$. In this respect, the prior on $Y_p$ can be important for determining the other parameters. In addition, the constraint on $Y_p$ from CMB itself can be an independent test from other methods such as using HII region observations.

In the near future, we can have more precise measurements of CMB. Such upcoming data would give us more precise information of the primordial helium abundance. At the same time, it is necessary to study the effects of the helium abundance more rigorously in order to extract information of other cosmological parameters.

Note added: While we were finishing the present work, Ref. [32] appeared on the arXiv, which has some overlap with our analysis on the constraints from the future Planck experiment.

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References

[1] K. A. Olive and E. D. Skillman, Astrophys. J. 617, 29 (2004) [arXiv:astro-ph/0405588];
[2] R. H. Cyburt, B. D. Fields, K. A. Olive and E. Skillman, Astropart. Phys. 23, 313 (2005) [arXiv:astro-ph/0408033].
[3] M. Fukugita and M. Kawasaki, Astrophys. J. 646, 691 (2006).
[4] M. Peimbert, V. Luridiana and A. Peimbert, arXiv:astro-ph/0701580.
[5] Y. I. Izotov, T. X. Thuan and G. Stasinska, Astrophys. J. 662, 15 (2007) [arXiv:astro-ph/0702072].
[6] A. Serebrov et al., Phys. Lett. B 605, 72 (2005) arXiv:nucl-ex/0408009.
[7] G. J. Mathews, T. Kajino and T. Shima, Phys. Rev. D 71, 021302 (2005) arXiv:astro-ph/0408523.
[8] A. P. Serebrov, Phys. Lett. B 650, 321 (2007) arXiv:nucl-ex/0611038.
[9] B. D. Fields and K. A. Olive, Astrophys. J. 506, 177 (1998) arXiv:astro-ph/9803297.

[10] R. Trotta and S. H. Hansen, Phys. Rev. D 69, 023509 (2004) arXiv:astro-ph/0306588.

[11] G. Huey, R. H. Cyburt and B. D. Wandelt, Phys. Rev. D 69, 103503 (2004) arXiv:astro-ph/0307080.

[12] K. Ichikawa and T. Takahashi, Phys. Rev. D 73, 063528 (2006) arXiv:astro-ph/0601099.

[13] K. Ichikawa, M. Kawasaki and F. Takahashi, Phys. Lett. B 597, 1 (2004) arXiv:astro-ph/0402522.

[14] E. Komatsu et al. [WMAP Collaboration], arXiv:0803.0547 [astro-ph].

[15] J. Dunkley et al. [WMAP Collaboration], arXiv:0803.0586 [astro-ph].

[16] G. Hinshaw et al. [WMAP Collaboration], arXiv:0803.0732 [astro-ph].

[17] R. S. Hill et al. [WMAP Collaboration], arXiv:0803.0570 [astro-ph].

[18] M. R. Nolta et al. [WMAP Collaboration], arXiv:0803.0593 [astro-ph].

[19] C. L. Reichardt et al., arXiv:0801.1491 [astro-ph].

[20] J. L. Sievers et al., arXiv:astro-ph/0509203.

[21] [Planck Collaboration], arXiv:astro-ph/0604069.

[22] W. Hu, M. Fukugita, M. Zaldarriaga and M. Tegmark, Astrophys. J. 549, 669 (2001) arXiv:astro-ph/0006436.

[23] W. C. Jones et al., Astrophys. J. 647, 823 (2006) arXiv:astro-ph/0507494.

[24] F. Piacentini et al., Astrophys. J. 647, 833 (2006) arXiv:astro-ph/0507507.

[25] T. E. Montroy et al., Astrophys. J. 647, 813 (2006) arXiv:astro-ph/0507514.

[26] A. Lewis and S. Bridle, Phys. Rev. D 66, 103511 (2002) arXiv:astro-ph/0205436.

[27] A. Kosowsky, M. Milosavljevic and R. Jimenez, Phys. Rev. D 66, 063007 (2002) arXiv:astro-ph/0206014.

[28] E. Komatsu and U. Seljak, Mon. Not. Roy. Astron. Soc. 336, 1256 (2002) arXiv:astro-ph/0205468.
[29] L. Perotto, J. Lesgourgues, S. Hannestad, H. Tu and Y. Y. Y. Wong, JCAP **0610**, 013 (2006) [arXiv:astro-ph/0606227].

[30] P. J. Zhang, U. L. Pen and H. Trac, Mon. Not. Roy. Astron. Soc. **347**, 1224 (2004) [arXiv:astro-ph/0304534].

[31] M. G. Santos, A. Cooray, Z. Haiman, L. Knox and C. P. Ma, Astrophys. J. **598**, 756 (2003) [arXiv:astro-ph/0305471].

[32] J. Hamann, J. Lesgourgues and G. Mangano, JCAP **0803**, 004 (2008) [arXiv:0712.2826 [astro-ph]].

[33] S. Burles, K. M. Nollett and M. S. Turner, Astrophys. J. **552**, L1 (2001) [arXiv:astro-ph/0010171].

[34] A. Lewis, J. Weller and R. Battye, Mon. Not. Roy. Astron. Soc. **373**, 561 (2006) [arXiv:astro-ph/0606552].

[35] W. Y. Wong, A. Moss and D. Scott, [arXiv:0711.1357 [astro-ph]].

[36] J. Chluba and R. A. Sunyaev, Astron. Astrophys. **446**, 39 (2006) [arXiv:astro-ph/0508144].

[37] J. Chluba, J. A. Rubino-Martin and R. A. Sunyaev, Mon. Not. Roy. Astron. Soc. **374**, 1310 (2007) [arXiv:astro-ph/0608242].

[38] J. Chluba and R. A. Sunyaev, [arXiv:astro-ph/0702531].

[39] E. R. Switzer and C. M. Hirata, Phys. Rev. D **77**, 083006 (2008) [arXiv:astro-ph/0702143].

[40] C. M. Hirata and E. R. Switzer, Phys. Rev. D **77**, 083007 (2008) [arXiv:astro-ph/0702144].

[41] E. R. Switzer and C. M. Hirata, [arXiv:astro-ph/0702145].

[42] S. Seager, D. D. Sasselov and D. Scott, Astrophys. J. Suppl. **128**, 407 (2000) [arXiv:astro-ph/9912182].

[43] S. Seager, D. D. Sasselov and D. Scott, Astrophys. J. Lett. **523**, L1 (1999) [arXiv:astro-ph/9909275].