Magnetic Activity in Thick Accretion Disks and Associated Observable Phenomena: II. Flux Storage

Sydney D’Silva and Sandip K. Chakrabarti
Tata Institute of Fundamental Research, Homi Bhabha Road, Colaba, Bombay, 400005, INDIA

November 25, 2021

Abstract

In paper I, we have studied the conditions under which flux tubes are expelled from adiabatic thick accretion disks. In the present paper, we explore a few other models of thick disks, where flux tubes could be stored. We show that flux tubes with sufficiently weak fields are not expelled out if they move adiabatically inside an isothermal disk; they continue to oscillate around mean equipotential surfaces inside the disk. If the field in the flux tube is amplified due to the shear, they are eventually expelled away. We explore a ‘toy’ model also, where the entropy increase outwards from the center of the thick disk and find a similar behavior. Flux storage in the disk, as in the case of the sun, in general, enhances the possibility of sustained magnetic activity formation of coronae in the chimney region. The existence of coronae on the disk surface may explain the short-time variability in the spectra of Blazars and the emission of energetic particles from AGNs and Quasars. It may also supply matter to the cosmic jets through magnetized winds.

Keywords: Black holes – accretion disks – magnetohydrodynamics – magnetic winds – corona – flares – BL Lac objects – AGNs – Quasars
1. INTRODUCTION

In an earlier paper (Chakrabarti & D'Silva 1993; hereafter referred to as Paper I), we have presented a detailed study of how magnetic flux tubes, either generated inside a disk or advected along the accreting flow, are expelled outside the disk. Over a large region of the parameter space, spanned by the size and the field strength, the flux tubes are seen to emerge in the chimney region of the thick accretion disk. These flux tubes emerge in buoyancy timescales, and might be advected along the jets, which are believed to be originated in the chimney of the disk. It is long known that most of the luminosity of the disk is emitted from the chimney. For the first time, we pointed out that the chimneys could also be magnetically the most active regions in the disk.

Blazars are long believed to belong to that class of active galaxies where the jets point at the observers. These objects are known to be highly variable over the entire electromagnetic spectrum. Variability time scales vary from $10^2$ s to $10^5$ s (e.g., Bregman 1988). A large number of models have been proposed to explain the cause of variability (see, Paper I for a review). If it is caused by the magnetic activity at the base of radio jets (Blandford and Königl 1979), then it is imaginable that the flux tubes are actually anchored in the chimney region of the thick disks and the activities are due to shock production by flaring events.

In the case of the sun, flux tubes are believed to be anchored beneath the convection zone where the temperature gradient is sub-adiabatic, and sunspots are believed to be formed at the regions where these flux tubes emerge from the surface (Parker 1979). This emergence takes place due to Parker’s instability (Parker 1955), which is the common cause of the formation of sunspots, coronae, prominences, flares and other magnetic regions on the solar surface. In this paper, we show that thick accretion disks could also entertain a very similar scenario where flux tubes can be anchored inside the disk and the parts of these flux tubes could emerge into the chimney, making it a region of sustained magnetic activity. So far no work has been done to show that
flux tubes could be anchored inside a thick accretion disk. In the present paper, we explore models of thick disks which render efficient anchoring. We find that isothermal disks are capable of storing flux tubes provided the tubes move adiabatically. We also explore some ‘toy’ models of thick disks, which are in hydrostatic equilibrium, and at the same time, have entropy rising outward from the center. These disks can store flux tubes provided the field strength is sufficiently weak so that even the buoyancy force is counter acted. In other words, in order to anchor a flux tube of given strength the local entropy gradient should be sufficiently sub-adiabatic. In may be noted in this context that Shibata et al. (1989), using a two dimensional MHD simulation show that the Parker instability of horizontal magnetic flux sheet could be suppressed inside a two temperature inversion medium. Such models could be physically realised in a variety of astrophysical circumstances which we elaborate. Axisymmetric flux tubes, in the absence of shear amplification and other dynamo effects, continue to oscillate around mean equipotential surfaces indefinitely, without ever being expelled out of the disk. Perturbative effects, such as deviation from axisymmetry (either of the fields, or temperature or pressure profile) can easily bring these tubes to the surface and cause magnetic activity as in the case of the sun (Moreno-Insertis, 1986; Choudhuri, 1990; D’Silva & Choudhuri, 1993). In the context of accretion disks, such perturbations could be caused by various interchange instabilities (Kaisig, Tajima & Lovelace, 1992) or non-axisymmetric shear instabilities (Balbus & Hawley, 1991, 1992; Hawley & Bal- bus, 1992). Shear amplification of the field could also enhance the field strength till it is expelled away due to Parker instability (e.g., Matsumoto et al. 1990; Shibata, Tajima, Matsumoto 1990).

The ‘toy’ models we explore have a simple entropy distribution, wherein the entire disk becomes radiative and flux tubes can be stored, or the entire disk is convective and all flux tubes get expelled out of the disk. In principle, one could try out entropy distributions where the disk can have a radiative core and a convective envelope, as in the sun. This exercise leads to the most obvious question - ‘What governs the entropy distribution in an accretion disk?’ The most serious problem is our ignorance
of viscosity and its role in controlling the dynamics of the entire disk. For instance, if accretion was surface phenomenon (Paczyński 1978) in thick disks, then, naively one would expect that the ratio of gas pressure to radiation pressure $\beta$ would be small at the surface and increase inward to the center. Since the entropy function $K$ (described in Paper I) of thick disks goes roughly as $\beta^{-4/3}$, for $\beta << 1$, this allows the entropy to rise from the center to the outer edge of the disk. As we shall show later, this is the Schwarzchild criterion for convective stability in thick disks. On the other hand, if accretion is equatorial (Paczyński and Wiita 1980; Wiita 1982, Abramowicz & Paczyński 1982, Chakrabarti, Jin & Arnett 1987), the disk can naively be expected to be convective. Convective stability also implies stability towards magnetic buoyancy, provided the field strength is low enough — the value depends on the entropy gradient.

The presence of magnetically active coronae on disk surface can not only explain the variability of Blazars, they can also supply matter to the jets through magnetic winds. The flares can accelerate particles through the first order Fermi process as in the case of the sun. Thus a large fraction of the high energy radiation from AGNs may originate at these flares. Magnetic flux tubes also preferentially filter charged particles (e.g. protons, electrons and ions) from those uncharged (e.g., neutron) and may produce neutron rich core of the tori which, in turn, can produce neutron rich elements through processes involving rapid capture of neutrons.

The plan of the present paper is the following: In the next Section, we describe model equations for the thick disks which we study in this paper. The equations of the dynamics of flux tubes inside a disk have already been presented in Paper I. Here we just reproduce the equations and show how the quantities inside the flux tubes vary, when the tubes move isothermally or adiabatically. In §3, we present numerical solutions describing the behavior of flux tubes in the various thick disks. In §4, we briefly describe the astrophysical implications of our results. Finally, in §5, we summarize our paper and make concluding remarks.
2. MATHEMATICAL EQUATIONS

We consider axisymmetric flux rings, and study their dynamics under the thin flux tube approximation, both of which are described in Paper I. Here we reproduce Eqns. (10)-(12) of Paper I.

\[
\ddot{\xi} - \xi \dot{\theta}^2 + \frac{X}{(1+X)}\left[-\xi \dot{\phi}^2 \sin^2 \theta - 2\xi \omega \dot{\phi} \sin^2 \theta \right] = \frac{X}{(1+X)}\left\{\frac{M}{X}[g - \xi \omega^2 \sin^2 \theta] - T_{ens} \sin \theta - \frac{D_r}{\pi \sigma^2 \rho_e}\right\},
\]

(1)

\[
\ddot{\xi} + 2\dot{\xi} \dot{\theta} + \frac{X}{(1+X)}\left[-\xi \dot{\phi}^2 \sin \theta \cos \theta - 2\xi \omega \dot{\phi} \sin \theta \cos \theta \right] = -\frac{X}{(1+X)}\left\{\frac{M}{X} \xi \omega^2 \sin \theta \cos \theta + T_{ens} \cos \theta + \frac{D_\theta}{\pi \sigma^2 \rho_e}\right\},
\]

(2)

and

\[
\dot{\xi} \sin \theta \dot{\phi} + 2\dot{\xi} \sin \theta (\dot{\phi} + \omega) + 2\xi \cos \theta \dot{\theta} (\dot{\phi} + \omega) + \\
+ l_o (n - 2)(\xi \sin \theta)^{(n-2)} [\dot{\xi} \sin \theta + \xi \dot{\theta} \cos \theta] = 0.
\]

(3)

where \((\xi, \theta, \phi)\) describes the position of a fluid element inside the flux tube, and the quantities of drag \(D\), dimensionless angular velocity \(\omega\) and \(X\) are described in § 2.2 of Paper I. Magnetic tension is given by

\[
T_{ens} = \frac{4\pi M_o T_o}{\mu_e A (1 - M_o) \xi_o \sin \theta_o},
\]

(4)

The expansion factor \(A = (\sigma/\sigma_o)^2\) is the ratio of the cross-sectional area of the flux tube with its initial cross-sectional area, and can be derived using the mass and flux conservations as
\[ A = \frac{\rho_e(\xi, \theta)}{\rho_o(\xi, \theta)} (\xi \sin \theta_o)(1 - M_o) (1 - M), \]  

(5)

The subscripts \( e \) and \( i \) refer to the corresponding quantities outside (ambient medium) and inside the flux tube respectively. The subscript \( o \) refers to the quantities corresponding to the initial conditions.

\textbf{2.1 Isothermal thick disks}

In the case when efficient cooling mechanism for ions is not possible, ions inside the disk may remain very hot, close to the virial temperature. Such disks have been invoked in the context of two temperature tori (Lightman and Eardley, 1974) and ion pressure supported tori (Rees et al. 1982). For the purpose of the present analysis, we consider isothermal disks, as another extreme from the adiabatic case discussed in Paper I. Gas pressure obeys the equation of state, \( p_g = R \rho T_d/\mu, \) \( T_d \) being the constant temperature of the disk. The radiation pressure \( p_r = (1/3)aT_d^4 \) remains a constant. The equipotential surfaces follow the same equation as for adiabatic disks (Eqn. 2 of Paper I). The density variation of the disk obeys the distribution,

\[ \rho(\xi) = \rho(R_o) \exp\left\{ -\frac{\mu}{RT_d} \left[ \frac{c_r^2}{2} \left( \frac{1}{c_r - 1} - \frac{1}{\xi - 1} \right) + \frac{l_o^2}{2n - 2} \left[ \xi^{2n-2} - (\xi \sin \theta)^{2n-2} \right] \right] \}, \]  

(6)

The flux tubes may radiate very efficiently and remain in isothermal equilibrium with its local surroundings. On the other extreme, when the cooling is not efficient, the flux tubes can move adiabatically inside the disk without exchanging any energy. Sections 2.1.1 and 2.1.2 give the relevant equations for these two cases. Throughout the calculations we choose an initial entropic condition in which the flux tube is in thermal equilibrium with its surroundings. The condition of pressure equilibrium provides the initial buoyancy factor to be \( M_o = B^2/(8\pi p_e). \)

\textit{2.1.1 When the flux tubes move isothermally}

We now consider the case where the flux tubes are in thermal equilibrium with
its surroundings. This is possible when the exchange of energy is very efficient. The radiation pressures inside and outside the tube are equal. Using the mass and the flux conservation \((B/\rho \xi \sin \theta = \text{constant})\) and the condition of pressure equilibrium, we get a quadratic equation in \(M\), whose solution is of the form

\[
M = \frac{1}{2} [b \pm \sqrt{b^2 - 4}],
\]

where

\[
b = 2 + \left[\frac{\xi_0 \sin \theta_0}{\xi \sin \theta}\right]^2 \frac{\rho_e(\xi_0, \theta_0)}{\rho_e(\xi, \theta)} \left[\frac{1 - M_0}{M_0}\right].
\]

This equation is similar to Eqn. (15) of Paper I. The solution with a positive sign is unphysical.

2.1.2 When the flux tubes move adiabatically

In this case, the entropy of the flux tube remains a constant throughout its motion inside the disk. Using the mass and flux conservations along with the condition for pressure equilibrium, we get a polynomial in \(\rho_i/\rho_e\),

\[
k_1 \left(\frac{\rho_i}{\rho_e}\right)^{4/3} + k_2 \left(\frac{\rho_i}{\rho_e}\right)^2 - 1 = 0,
\]

where

\[
k_1 = \beta_e \left[\frac{(1 - \beta_i) R}{\beta_i^2 \mu^4} \right]^{1/3} \frac{3}{a} \frac{\mu_e^{1/3} \rho_e^{1/3}}{R T_e},
\]
\[ k_2 = \beta_e \frac{M_0}{(1 - M_0)^2} \left( \frac{\xi \sin \theta}{\xi_0 \sin \theta_0} \right)^2 \frac{\rho_e}{\rho_{e,o}}, \]  

(9)

\[ \beta_e = \frac{\rho_e}{\rho_e + (a \mu/3R)T_{e,o}^3}, \]  

(10)

\[ \beta_i = \frac{(1 - M_0)\rho_{e,o}}{(1 - M_0)\rho_{e,o} + (a \mu/3R)T_{e,o}^3}. \]

2.2 A toy model for the radiative thick accretion disk

Here we present a model of a thick accretion disk in which the entropy is a function of the potential \( W \) of the disk. In the adiabatic disk, entropy is independent of \( W \) and in the isothermal disk \( S \propto -\log(\rho) \). Presently, we use a simple functional form, wherein, either the entire disk remains radiative or the entire disk becomes convective, depending on whether the Schwarzschild criterion for convective stability (Landau & Lifshitz 1959) is satisfied or not. In thick accretion disks the Schwarzschild criterion for convective stability can be written in a modified form as

\[ \frac{dK}{dW} > 0. \]  

(11)

We take a linear form for the entropy function \( K \) as

\[ K(W) = K_1 + K_2 W, \]  

(12)

where \( K_1 \) and \( K_2 \) are constants,
\[ K_1 = \frac{K(W_{out})W_c - K(W_c)W_{out}}{W_c - W_{out}}, \]  
\[ K_2 = \frac{K(W_c) - K(W_{out})}{W_c - W_{out}}, \]  
(13)  
(14)  

\[ W_c = W(R_c, \theta = 90^\circ) \] is the value of \( W \) at the center of the disk and \( W_{out} = W(R_{out}, \theta = 90^\circ) \) is its value at the outer edge of the disk which is chosen to be \( R_{out} = 20r_g \). This dependence of \( K \) on \( W \) could be through \( \beta \) or through \( \mu \) or both. We assume that the composition of the disk is homogeneous. The local value of the entropy function \( K(W) \) is given by

\[ K(W) = \left[ \frac{(1 - \beta(W))}{\beta(W)^4} \right]^{1/3}, \]  
(15)  

where \( \beta(W) \) is the ratio of the gas pressure \( p_g \) to the total pressure \( p_t \) on a given equipotential surface, \( R_G \) is the gas constant, \( a \) is the radiation density constant and \( \mu \) is the mean molecular weight of the gas. According to the Schwarzschild criterion, if \( K_2 > 0 \), the disk is radiative, and if \( K_2 < 0 \), it is convective. \( K_2 = 0 \) gives the adiabatic disk. The potential \( W \) increases outwards from the center of the disk, so \( (W_c - W_{out}) < 0 \). In Eqn. (14), \( K_2 > 0 \) for \( K(W_{out}) > K(W_c) \); the disk is convectively stable (radiative disk). For a radiation pressure dominated disk (\( \beta << 1 \)) entropy goes roughly as \( \beta^{-4/3} \). By choosing \( \beta(W_c) > \beta(W_{out}) \) the disk becomes radiative (\( dK/dW > 0 \)), or by choosing \( \beta(W_c) < \beta(W_{out}) \) it can be made convective (\( dK/dW < 0 \)).

As in Paper I, we choose the polytropic equation of state \( P = K \rho^\gamma \) inside the disk, with \( K = K(W) \). We assume that the disk is a radiation pressure dominated geometrically thick disk. It is also optically thick, photon and matter act as a single fluid of polytropic index \( \gamma = 4/3 \). We also assume that the equation of state is barotropic, \( P = P(\rho) \) and the force balance equation (Eqn. 1 of Paper I) is integrable \( W(P) = \)
\(- \int (dP/\rho),\) and

\[
W - W_{out} = - \frac{1}{2(\xi - 1)} - \frac{l_o^2}{2n - 2}(\xi \sin \theta)^{2n-2},
\]  
(16)

and \(\rho\) is given by

\[
\rho = \left\{ \frac{1}{K_2} \left[ \left( \frac{K_1 + K_2 W_{out}}{K_1 + K_2 W} \right)^{1/4} - 1 \right] \right\}^3.
\]  
(17)

2.2.1. When the flux tubes move isothermally

As usual we assume that the initial entropic condition is such that the flux tube is in thermal equilibrium with its surroundings. Thus,

\[
\frac{\rho_{i,\phi}}{\rho_{e,\phi}} = 1 - M_{\phi}.
\]

If the flux tubes remain in thermal equilibrium throughout their journey in the disk, then the pressure balance condition with the mass and flux conservations give the expression of the buoyancy factor \(M\) to be the solution of a quadratic as in Eqn. (7), where the constant \(b\) is

\[
b = 2 + \left( \frac{\xi_o \sin \theta_o}{\xi \sin \theta} \right)^2 \left( \frac{\rho_e(\xi_o, \theta_o)}{\rho_e(\xi, \theta)} \right)^2 \frac{(1 - M_o)^2}{M_o} \frac{\beta_{e,\phi}}{\beta_{e}(1 - \beta_{e,\phi})} \frac{\beta_{\phi}}{(1 - \beta_{e,\phi})}.
\]

2.2.2. When the flux tubes move adiabatically

In this case, \(\beta_i(\xi, \theta) = \beta_i(\xi_o, \theta_o)\). Using the mass and flux conservations we get an equation for \(\rho_i/\rho_e\) in the similar form as Eqn. (8), where the constants \(k_1\) and \(k_2\)
are given by

\[ k_1 = \frac{1 - \beta_{e,\circ}(W)M_o K_{e,\circ}}{(1 - M_o)^{4/3} K_e} \]

\[ k_2 = \frac{\beta_e(W)^2(1 - \beta_{e,\circ})}{\beta_{e,\circ}(1 - \beta_e)} \frac{M_o}{M_o^2}(\frac{\xi \sin \theta}{\xi_\circ \sin \theta_\circ})^2(\frac{T_e}{T_{e,\circ}})^2. \]

\[ (18) \]

\[ (19) \]

2.3 Inclusion of effects of shear amplification

The field strength of the flux tubes can be amplified by convective motions and differential rotation, and the azimuthal field can be shown to grow exponentially (Galeev, Rosner & Vaiana 1979), with a growth time \( \tau \sim (\xi/v) \), where \( v \) is the convective velocity. Following Bisnovatyi-Kogan and Blinnikov (1977), we approximate convective velocities to be \( v \approx \alpha^{1/3} c_s \), where \( c_s \) is the sound speed and is roughly equal to the local infall velocity \( v \approx (\alpha/\sqrt{\xi - 1}) \); \( \alpha \) being the viscosity parameter (Shakura & Sunyaev 1973).

Assuming that the shear amplification of field inside a thick disk takes place at the same rate as inside a thin disk, we choose,

\[ B = B' \exp \{t/\tau\}; \]

\[ (20) \]

where \( B' \) is the field in the previous time step and \( t \) is the time interval. It is not our intention to develop a detailed picture of the dynamo process inside the disk. Our present strategy is to get some insight about the physical processes that might take place when such an enhancement of the field is turned on.

When the field is amplified, flux inside the tube is no longer conserved, so the
Eqns. (4) and (19) change accordingly as

\[ T_{\text{ens}} = \frac{4\pi[1 - \rho_i/\rho_e]T(\xi, \theta)}{\mu_e A(\rho_i/\rho_e)\xi \sin \theta}, \]  

(21)

and

\[ k_1 = \frac{[1 - \beta_{e,o}(W)M_o]K'_e}{(1 - M_o)^{4/3}K_e}, \]

(22)

\[ k_2 = \frac{\beta_e(W)^2(1 - \beta_e')}{\beta'_e(1 - \beta_e)} \frac{[1 - (\rho_i'/\rho_e')]\xi \sin \theta}{(\rho_i'/\rho_e')^2(\xi' \sin \theta')^2} \frac{T_e}{T'_e}^2, \]  

(23)

where, a prime (') refers to the corresponding quantities at the previous integration step.

3. NUMERICAL RESULTS

We first study the isothermal disks in § 3.1 and then the radiative and convective disks in § 3.2.

3.1 Isothermal disks

Since we are interested in studying the generic behavior of the flux tubes in isothermal disks, we choose some typical parameters. We consider an isothermal thin disk described by Eqn. (8), with the boundary condition of \( \rho(R_{\text{out}}) = 10^{-15} \text{ gm cm}^{-3} \). The disk is around a black hole of mass of \( 10^6 M_\odot \), with its outer edge \( R_{\text{out}} \) at \( 20r_g \). The temperature of the disk is taken to be \( T_d = 5 \times 10^9 \text{ }^\circ \text{K} \). The angular momentum distribution \( l = l_o(\xi \sin \theta)^n \), where \( l_o \) and \( n \) are positive constants as in Paper I. In § 3.1, we study the behavior of the flux tubes when they are in isothermal equilibrium with
their surroundings and § 3.2 gives the behavior when they move adiabatically inside the disk.

3.1.1 When the flux tubes move isothermally

We take a constant angular momentum disk \((n = 0)\) with \(l_o = 2\). Flux rings of \(M_o = 0.3\) are released close to the equatorial plane \((\theta = 89^\circ)\) at \(\xi = 4, 5, 6, 7\) and 10 with the initial conditions of zero velocity, and the trajectories of the flux tubes are computed by integrating Eqns. (1)-(3) with the assumption that drag and accretion are negligibly small \((D \sim 0\) and \(u_{acc} \sim 0\)). Figure 1a shows the results; it is almost identical to the corresponding adiabatic case in Paper I, except that the flux tube released at \(\xi = 6\) also emerges into the chimney. This interesting behavior is because magnetic tension (which goes inversely to the radius of the flux ring) dominates over magnetic buoyancy. Section 3.3 of Paper I discusses the importance of magnetic tension. Simple analytic work which involves balancing magnetic tension and the buoyancy effects, shows that if \(T_d > 4 \times 10^{10}\) K, magnetic tension dominates over magnetic buoyancy and the flux tubes released at the outer edge of the disk should also fall into the chimney. Numerically, when all the effects are taken into account, this happens at somewhat lower temperature, even at \(T_d = 10^{10}\) K itself, as shown in Fig. 1b. All the conditions in Fig. 1b, except the value of \(T_d\), are the same as in Fig. 1a.

In order that the self-gravity of the disk does not become important, we chose disks of very small density, i.e., we choose a very high temperature. The black hole mass \(M_{BH} = 10^6\) M\(_\odot\) and the disk of size of \(20r_g\) with the outer boundary condition of \(\rho(R_o) = 10^{-15}\) gm cm\(^{-3}\) restrict the lower limit on the temperature of the disk to roughly \(10^9\) K. On the other hand, if \(T_d\) is increased beyond \(10^{11}\) K the disk becomes ion pressure dominated. The disk parameters of \(M_{BH}\), \(\rho(R_o)\) and \(R_o\) that we have chosen, conspire in such a way that \(T_d\) has to lie between \(10^9\) and \(10^{10}\) K in order to avoid magnetic tension from dominating the dynamics and the disk becoming self-gravitating. A smaller temperature can be chosen provided we choose a smaller disk.
size. When $T_d$ is below $10^{10}$ °K, the behavior of the flux tubes is similar to that of the tubes which move isothermally in the adiabatic disk (§ 3.1 of Paper I). However, when flux tubes move adiabatically in the isothermal disk the behaviour is remarkably different as discussed in the next subsection.

3.1.2 When the flux tubes move adiabatically

We repeat the calculations with parameters as in Fig. 1a, except that the flux tubes move adiabatically inside the disk. This is done by calculating the buoyancy factor $M$ using Eqn. \[8\]. To start with, the flux tubes are buoyant because of the particular initial condition chosen such that they are in thermal equilibrium with their local surroundings, but not in hydrostatic equilibrium. The flux tubes move out in the direction of the pressure gradient force and in doing so they expand in order to be in pressure equilibrium with its immediate surroundings. The reduction in density due to the expansion, however, is not sufficient to keep its density smaller than its surroundings, and the tube becomes less buoyant as it rises, till it reaches a point where the tube is no longer buoyant ($\rho_e = \rho_i$). The rising flux tube overshoots this point and enters into a region where it becomes heavier than its surroundings and begins to fall back causing it to oscillate back and forth. Figure 2a shows the results. Flux tubes are seen to oscillate about the equipotential surfaces and in addition they move along it. The movement along the equipotential surfaces is due to the tension which pulls the flux tube toward the rotation axis. If $T_d$ is sufficiently low, where tension can be totally neglected, the flux tube will not move along the equipotential surface, but just oscillate both radially and vertically. Figure 2b shows the results for a $n = 0.1$ disk. In addition to the tension which acts toward the rotation axis, there is now a Coriolis force which acts away from it; these two competing forces determine the fate of the flux tube. For example, $M_0 = 0.3$ flux tubes released at $\xi = 10$ and 14 revert back as soon as they cross the dot-dashed curve. This marks the surface where the component of effective gravity perpendicular to the rotation axis becomes zero. This is also the surface where the component of buoyancy perpendicular to the rotation axis changes sign from being
away from it outside this surface to being toward it inside this surface. If the field is sufficiently large as for the $M = 0.7$ case, then tension makes the flux tube overshoot so much that buoyancy makes it oscillate about the equatorial plane.

### 3.2 Radiative and convective disks

Hereafter, we confine ourselves to the study of flux tubes which move adiabatically. We do not deal with the isothermal case because the generic behaviour of the flux tubes is similar to that in adiabatic or isothermal disks. We choose a radiative thick accretion disk (convectively stable) by choosing $\beta(W_c) = 0.0002$ and $\beta(W_{out}) = 0.0001$ (See § 2.2). Equation (17) gives the distribution of $\rho$ in such a disk. Flux rings of $M_o = 0.3$ are released from $\xi = 5, 7, 10$ and $15$ and $\theta = 89^\circ$ in a $n = 0$ disk with zero initial velocity. The trajectories are computed by integrating Eqns. (1)-(3) assuming that drag and accretion are negligible ($D = 0$ and $u_{acc} = 0$) and the flux tubes move adiabatically in the disk (this is ensured by using Eqn. 19). Except for replacing the isothermal disk with the radiative disk, the conditions are same as in Fig. 2a. The results are shown in Fig. 3a. The flux tubes oscillate about an equipotential surface as in an isothermal disk. For any given entropy gradient there is a maximum $M_o$ which can be confined in the disk at a given point. Figure 3a shows that the flux tube released at $\xi_o = 15$ emerges because the oscillation amplitude increases with $\xi_o$ (Eqn. 26 given below) and the amplitude is so large that it emerges before it can turn back. Here we present a simple derivation to show how the oscillation amplitude depends on $M_o$ and $\xi_o$.

Section 3.1.2 described, in detail, how and why the flux tubes oscillate around an equipotential surface, where its density is equal to the external density. (On the equipotential surface, densities will be exactly equal if $M \sim 0$.) The variation of density with height is described by Eqn. (19). If a small displacement of the flux tube equalises the external and internal densities, $(\rho_i/\rho_e) = 1$, we can substitute the constants $k_1$ and $k_2$ from Eqn. (19) and approximate Eqn. (19) in the limit when $\beta_e \ll 1$, as
Expanding $K_e$ by a Taylor series about the point where the flux tube was released and retaining only the first term, we have

$$K_{e,0} + \frac{\partial K}{\partial W} \frac{\partial W}{\partial x} \bigg|_0 dx \sim \frac{K_{e,0}}{1 - M_0},$$

which on simplification gives the amplitude of oscillation as

$$dx = \frac{K_e M_0}{(1 - M_0) K_2 (\partial W / \partial x)}.$$

For $M_0 << 1$, the amplitude of oscillations increases with $M_0$ which measures the initial departure from hydrostatic equilibrium. Beyond the dot-dashed curve (cf. Fig. 2b), $(\partial W / \partial x) > 0$ indicating that the initial displacement of the flux tube is outward in the direction of the pressure gradient force. Similarly, between the inner edge of the disk and the center $(\partial W / \partial x) < 0$, hence the initial displacement of the flux tube is toward the rotation axis along the pressure gradient force. The constant $K_2 = \partial K / \partial W$ indicates whether the disk is adiabatic, convective or radiative. If $K_2 = 0$, the disk is adiabatic, and the initial displacement is infinite; in other words, the flux tube emerges out in the direction of the pressure gradient force (Fig. 8 of Paper I). If $K_2 > 0$ the disk is radiative and the displacement, or oscillation amplitude, goes inversely to it. The derivation is not valid for $K_2 < 0$. When $\beta(W_c)$ is made even slightly greater than $\beta(W_{out})$ flux tubes are expelled out as in the convective disks.

3.3 Inclusion of the effects of shear amplification

We study the behavior of the flux tubes that are anchored in a radiative disk
under shear amplification. This is done by amplifying the field through Eqn. (20) and substituting Eqn. (21) for the tension and Eqn. (23) for the buoyancy factor, while numerically integrating Eqns. (1)-(3). Figure 4 shows the results under similar conditions as Fig. 3a; the amplitude of the oscillations increases and also the mean surface about which the oscillations take place keeps shifting outward till the flux tube emerges. The process takes several dynamical timescales before the flux tubes emerge.

4. IMPLICATIONS OF MAGNETIC ACTIVITY INSIDE THICK ACCRETION DISKS

The magnetic field brought by a flux tube depends upon the point of release, the degree to which the field is saturated in comparison to the equipartition value, as well as the location where it surfaces. Because of the uncertainties, we have refrained from providing any quantitative description of the field itself. In general, for typical values of the parameters, the field is found to range between a few thousand Gauss at the inner edge of the disk to a few Gauss at the outer edge. This is quite strong and a large number of important observational effects are expected if the chimneys of the accretion disks do harbour such fields in the form of loops, coronae and prominences. In the case of Blazars (i.e., the optically violent variables and the BL Lac objects), the effects would be more prominent since the jets are supposed to be beamed directly towards the observers. Indeed, most of the multifrequency observations of these objects do invoke the presence of very strong magnetic flares for a satisfactory explanation of the magnitude and the time scale of variabilities as well as the energies associated with them. Recent observation of TeV gamma rays from a highly variable source MK421 may also be explained through repeated accelerations in magnetic loops. In the case of other radio sources, which are not beamed towards the observer, the presence of magnetic fields in the chimney region can only be inferred through indirect means, by comparing the kinematic luminosity of cosmic jets with the mass loss through magnetic winds from the chimney surface, for instance. Similarly, if significant magnetic flux tubes are present inside the disks, they would play an important role in influencing
nuclear compositions both inside the disk, as well as outside. Below, we discuss some of these suggestive effects.

(a) *X-ray flickering and optical microvariabilities in AGN:*

Emission from Blazars exhibit high polarization and variability at all wavelengths (e.g., Brown et al. 1989) on time scales varying from roughly months (quiescent emission component) at wavelengths longer than 1 cm to weeks at submillimeter wavelengths to roughly days (flaring component) in UV and smaller wavelengths. X-rays can also vary between days to months. It is believed that the emission originates from a single compact region (Gear et al. 1986) of size $r \sim 10^{15}$ to $10^{17}$ cm and the magnetic field, which is largely turbulent (Jones et al. 1985) is $B \sim 0.1 - 1$ G (Gear et al. 1985, Brown et al. 1989). A large number of these objects such as, OJ287, 3C 345, BL Lac, and 3C 454.3 etc. show clear evidence of a net magnetic field aligned orthogonal to the direction of the jet axis. The spectral index $\alpha$ shows a break at a frequency $\nu_b$ which is known to evolve with time as the emission mechanism changes. At low frequencies, where the loss during the acceleration is not important, $\alpha \sim -0.75$ but at a break frequency, $\nu_b$, when the electrons experience significant radiative loss during acceleration, the spectrum steepens to have $\alpha \sim -1.25$. But if the electrons experience a burst of injection and then the radiative loss, the spectrum should have a slope of $\alpha \sim -2$ (Khardashev 1962). This variation is indeed observed. For an efficient radiative loss which produces steepening at infrared and which is clearly observed, one requires the field be very high. All these points to the evidence that a strong magnetic activity, very similar to what happens of the solar surface, must be taking place at the chimney wall as well. Blandford and Königl (1979) suggest that if a succession of mild shocks continuously reaccelerate the electrons, the flow should behave as roughly isothermal, and the field perpendicular to the flow axis at the shock should vary as $B \propto R^{-1}$. This condition produces a very high field of around 1G at the flaring region. In a highly variable source, such as MK421 ($t_{var} \sim 10000s$), (Brodie, Bowyer & Tennant 1987) field varies as $B = B_0 r^{-1.4}$ with $B \sim 0.2$G. In PKS2155-304, field varies as $B = B_0 r^{-1.3}$ with
\(B_0 \sim 100\text{G}\) close to the core to 70 \(\mu\text{G}\) at 1pc. The trends of these observational results agrees with our calculated field values near the chimney.

In the Introduction of Paper I, we mentioned some models to account for intraday variability of the blazars. We propose here that magnetic flares in the chimney can be considered to be a major contributor to this variability. Though the non-linear physics of the flares on the solar surface is far from known, it is very reasonable to assume that the reconnection of the abundant flux tubes in the chimney should produce flares in time-scales \((t_f)\) of the order of a few times the infall time-scale \(t_i, t_i \sim \alpha^{-1}r_c/v_r\), since the flux tubes typically saturate to equipartition value in about \(t_i\). Here, \(\alpha\) is the viscosity parameter, \(r_c\) is the center of the disk and \(v_r \sim (2GM/r)^{1/2}\). For a disk around a black hole of mass \(10^8M_\odot\), \(r_c \sim 10r_g\) and \(\alpha \sim 0.01\), \(t_f \sim 1-3\text{d}\). The energy deposited at each flare is about \(L^3B_f^2/4\pi\). Here, \(L\) is the length scale of a typical flare and \(B_f\) is the average field on the chimney surface. The energy is released in time \(L/V_A\), where, \(V_A\) is the Alfven velocity \((B_f^2/4\pi\rho_f)^{1/2}\), \(\rho_f\) being the density of matter inside the chimney. With reasonable estimates of \(L \sim r_g, B \sim 100\text{G}, \text{ and } \rho \sim 10^{-14}\text{gm cm}^{-3}\), one obtains the energy release, time-scale and the rate of release to be given by \(2 \times 10^{37}\) ergs, 0.3d and \(2 \times 10^{34}\) erg s\(^{-1}\), respectively. Depending upon the size of the flares their occurrence rate will change. Thus a large number of frequent micro- and miniflares are expected inside the chimney. Toward the outer edge of the disk, flares will be very less energetic if frequent, or highly energetic but very less frequent. Away from the hole, the flare timescale becomes longer because both the amplification as well as the buoyancy time scales should grow as \(r^{3/2}\) and naively the flare time scales should also grow as \(r^{3/2}\). (The unknown non-linear physical processes which govern flarings may change this dependence significantly.) This property explains the rapid flickering in X-rays and slower variations in optical and radio emissions.

(b) \textit{High energy gamma rays}:

Recently 0.5TeV gamma rays have been detected from a highly active and variable
source Mk421 (Punch et al. 1992). It is possible to produce such energetic gamma rays, if one assumes that they are originated from first order Fermi accelerated protons radiating at the ‘thick targets’ close to the base of the magnetic field loops, similar to what is believed to be the source of hard X-rays and gamma rays on the surface of the sun. However, in the chimney region where crowded coronae can be present, the particles may be energized in multiple steps, and not just only in two steps as in the solar corona (Bai et al. 1983). In this process, moderately energetic protons released in a flux tube, traverse back and forth and are repeatedly accelerated at each encounter with shock fronts moving parallel to the field lines from each footpoint.

In the first order Fermi acceleration, the energy gain is proportional to the total energy of the particle. Thus protons are more efficiently accelerated than the electrons for a given velocity. The energy gain per encounter can be written as (Jokipii 1966):

$$\Delta W = 2\beta_s \gamma_s^2 W (\beta_p \mu_p + \beta_s)$$

where $\beta_s$ is the shock velocity component parallel to the field lines and $\beta_p$ is the speed of the particle in units of velocity of light, $W$ is the total energy of the particle, $\mu_p$ is the cosine of the pitch angle and $\gamma_s$ is the Lorentz factor of the shock motion. The average time interval between two successive collisions with the approaching shock fronts is given by (Bai et al. 1983),

$$\Delta t = \frac{d}{c\beta_p < \mu_p >}$$

where $d$ is the half length of the flux tube. Assuming shocks move non-relativistically, i.e., $\beta_s << \beta_p$, and using the relativistic relation $m = m_p/(1 - \beta_p)^{1/2}$, where, $m_p$ is the rest mass of the proton, one obtains the time variation of total energy during $i$-th step as,

$$W_i^2(t) - W_p^2 = (W_{i-1}(t_i)^2 - W_p^2) \exp[4 < \mu_p^2 > (v_s/d)(t - t_i)]$$

where, $W_p = m_pc^2$. Here, $W_{i-1}(t_i)$ is the injection energy of the proton at the beginning of the $i$-th step at time $t_i$ (i.e., the final energy of the proton at the end of $(i-1)$-th step). It is easy to see that in the non-relativistic limit, assuming the duration of acceleration to be equal to the shock traversal time of the tube $d/v_s$ and $< \mu_p^2 > = 0.75$, the energy
goes up by a factor of $\exp(3) \sim 20$ as in the solar flux tubes (Bai et al. 1983). However, as the total energy goes up, this efficiency factor decreases to about $\exp(1.5) \sim 4.5$ when the ultra-relativistic equation (given above) is used. Assuming initial injection energy $E_{ini}$ to be about 1MeV (which is just above the energy where the gain rate is higher than the Coulomb energy loss rate), a final energy $W_n$ of $1 - 2$TeV is obtained in $n$-steps where,

$$W_n^2 \sim 2W_pE_{ini}\exp(3n)$$

which yields, $n \sim 7$. Because a large number of steps are required, it may seem to be an inefficient process. But the chimney is more confined than an almost flat solar surface, and such multiple steps could be possible. Thus, in general, production of high energy cosmic rays in the chimney region cannot be ruled out.

One way to verify if this is indeed what takes place in Mk421, TeV gamma rays can be observed after a delay of several days or even weeks after the detection of $1 - 10$MeV gamma rays, as a considerable time is spent during the acceleration process. (Indeed, at each step, $d/v_s \sim 2GM/v_sc^2 \sim 3d$ is required, where, $d \sim r_g$ and $v_s \sim 10^8$cm s$^{-1}$ are used.) Furthermore, intermediate energy photons should also be observed. Note that the acceleration within shock traversal time does not put any constraint on the size of the magnetic flux tube. But a delay between different bands, if observed, would constrain the sizes involved.

(c) Correlations between radio and optical variabilities

In the case of BL Lac 0716+714, it has been observed that variabilities in radio and optical are well correlated (Wagner et al. 1990; Quirrenbach et al. 1991) and some kind of quasi periodicities are observed in time scales of $1.2$d in the first week and of $\sim 7$d in later weeks of observations. This behavior could be understood using our observation of the fact that strength of magnetic fields on the disk surface is well correlated with the pressure inside the disk where the flux tube is originated. If, for example, the optical variabilities are caused by the formation and disappearance of
the hot-spots (non-axisymmetric density perturbations) inside the disk (Chakrabarti & Wiita 1993), then the radio flares caused by magnetic fields anchored with these density waves should also follow a similar variability. A possible delay, corresponding to the buoyancy timescale of a $\leq$ day (inner disk) to several days (outer disk) is expected.

(d) *Matter supply to cosmic jets via magnetic winds*

Magnetic winds carry almost $10^{-14} M_\odot$ yr$^{-1}$ from the surface of the Sun. The active surface of the chimney of a thick disk, which may have a typical area ($\sim 2\pi r_c h$, $h$ being the maximum height of the disk) of $A_f \sim 1000 r_g^2$ will release roughly $\dot{M}_w \sim A_f \rho_f V_E$ amount of matter per second, where $V_E$ is the escape velocity of matter. At an average distance of $r \sim 10 r_g$, and with $\rho_f = 10^{-14} \text{gm cm}^{-3}$, the rate will be $\sim M_\odot$ yr$^{-1}$. This is comparable to the typical kinematic flux observed in a jet. Thus, the magnetic winds from the chimney can be responsible for supplying necessary matter in the jet.

(e) *Formation of neutron disks and neutron rich isotopes*

In the presence of a large number of flux tubes, which are originated deep inside, the nuclear composition of the disk could change in buoyancy time scale. In the case of ion pressure supported thick disks (Rees et al 1982), the central temperature of the disk could be high enough ($\geq 10^{10}$K) that the accreting matter would be photo-dissociated completely (Chakrabarti, Jin and Arnett 1987; Hogan and Applegate 1987). However, the buoyant flux tubes will carry with them only the protons and electrons. Neutrons left behind can produce a neutron rich disk close to the center. Newly accreting matter can capture these neutrons and produce neutron-rich nuclear isotopes which eventually escape from the disk by the rising and flaring flux tubes.

5. CONCLUSIONS

In the present paper, we have been able to show that it is possible to construct
physical models of thick accretion disks around black holes which can act as a store house of magnetic fields. In isothermal disks or in radiative disks, weaker flux tubes are not expelled away. Instead, they continue to oscillate around equipotential surfaces, dictated by the forces of gravity and the centrifugal motion, till they are amplified by shear and become more buoyant. It is well known that in the case of the sun, the flux tubes are stored in the regions near the boundary between the radiative and the convective zones, and appear on the surface due to instabilities. Similarly, the anchored flux tubes in the disk are also expected to appear close to the chimney due to perturbative effects and cause magnetic activities. We have discussed briefly that a large number of astrophysical processes, such as supply of matter through magnetic winds, variability of Blazars through flaring events, production of highly energetic particles in the coronae through Fermi acceleration processes, etc. could be related to the interesting magnetic flux tube behavior that we discover.

In our work, in both Paper I and Paper II, we ignored the effects of reconnection of flux tubes within the disk. Reconnection takes place in diffusion timescale, and is slower compared to the shear amplification timescale. Therefore, it is not expected that it would have any important effect on the dynamics of flux tubes inside the disk. However, outside the disk, in the magnetosphere, the Alfven velocity is very high (since the matter density is low) and thus reconnection is important. Indeed, the flares on the solar surface are believed to be due to this reconnection process.

Though the toy model of the disk we had used is sufficiently realistic under the circumstances mentioned in the text, it is desirable to construct a disk model which is fully self-consistent, namely, in which the structure of the disk is determined from a minimum number of parameters. For example, we have ignored the effects of nuclear reaction and associated energy generation inside a hot disk. In Paper I, we have already shown how various parameters that we use are closely linked with the ‘unknown’ viscous mechanism operating inside a disk. There is also a large uncertainty in the size and field distribution of the flux tubes of the accreted matter. Though, once they are inside
the disk they are amplified to the local equipartition value in a dynamical timescale. It would therefore appear that a more complete knowledge of the behavior of the flux tubes and their influence on the observable phenomena must await an improvement of our understanding of the uncertainties mentioned above. On the other hand, since we have chosen very reasonable sets of parameters, we believe that the major conclusions drawn in our papers (I & II) will remain intact.
REFERENCES

Bai, T. et al. 1983, ApJ, 267, 433
Balbus, S.A. & Hawley, J.F., 1991, ApJ, 376, 214
Balbus, S.A. & Hawley, J.F., 1992, ApJ, 400, 610
Balbus, S.A. & Hawley, J.F., 1992, ApJ, 400, 620
Bisnovatyi-Kogan, G.S. and Blinnikov, 1977, A & A, 59, 111
Blandford, R.D. & Königl, A. 1979, ApJ, 232, 34
Bregman, J.N., 1988, in Supermassive Black Holes, ed. M. Kafatos (Cambridge University Press), p. 43
Brodie J., Bowyer S. and Tennant A.: 1987, ApJ, 318, 175
Brown et al. 1989, ApJ, 340, 129
Brown, L.M., Robson, E.I., Gear, W.K. & Smith, M.G. 1989, ApJ, 340, 150
Chakrabarti, S.K. & D'Silva, S. 1994, ApJ, (to appear)
Chakrabarti, S.K., Jin, L. & Arnett, W.D. 1987, ApJ, 336, 572
Chakrabarti, S.K. & Wiita, P.J. 1992, ApJ (submitted)
Choudhuri, A. R. 1990, ApJ, 355, 733
D'Silva, S. & Choudhuri, A.R., 1993, A&A, 272, 621
Galeev, A. A., Rosner, R. & Vaiana, G. S., 1979, ApJ, 229, 318
Gear, W.K. et al. 1985, ApJ, 291, 511
Gear, W.K. et al. 1986, ApJ, 304, 295
Hawley, J. F. & Balbus, S. A. 1991, 376, 223
Hogan, C. & Applegate, J. 1987, Nat, 330, 236
Jokipii, J.R., 1966, ApJ, 143, 961
Jones, T.W. et al. 1985, ApJ, 290, 627
Khardashev, N.S. 1962, Sov. Astr.– AJ, 6, 317
Kaisig, M., Tajima, T. & Lovelace, R.V.E., 1992, ApJ, 386, 83
Lawrence, A., Watson, M.G., Pounds, K.A., & Elvis, M. 1987, Nat, 325, 694
Landau, L.D. and Lifshitz, F.D. 1959, Fluid Mechanics, Pergamon Press (New York)
Lightman, A.P. & Eardley, D.M. 1974, ApJ, 187, L1
Matsumoto, R., Horiuchi, T., Hanawa, T. & Shibata, K., 1990, ApJ, 356, 259
Moreno-Insertis, F., 1986, A&A, 166, 291
Paczyński B. 1978, Acta. Astron. 28, 91
Paczyński, B. & Abramowicz, M. A. 1982, ApJ, 253, 897
Paczyński B. & Wiita, P. 1980, Astron. Ap, 88, 23
Parker, E. N., 1955, ApJ, 121, 491
Parker, E. N., 1979, Cosmical Magnetic Fields, Oxford University Press
Punch, M. et al. 1992, Nat, 358, 477.
Quirrenbach, A. et al. 1991, ApJ, 372, L71
Rees, M.J., Begelman, M.C., Blandford, R.D, & Phinney, E.S, 1982, Nat, 295, 17
Shakura, N. I. & Sunyaev, R. A., 1973, A&A, 24, 337
Shibata, K, Tajima, T. & Matsumoto, R., 1990, ApJ, 350, 295
Wagner, S., Sanchez-Pons, F., Quirrenbach, A., & Witzel, A. 1990, A&A, 235, L1
Wiita, P.J. 1982, ApJ, 256, 666
Fig. 1 — Trajectories of flux tubes in the $R = \xi \sin \theta - z = \xi \cos \theta$ plane, released at $\xi = 4$, 5, 6, 7, 10 and $\theta = 89^\circ$, with zero initial velocity and no drag, under isothermal conditions. The dotted line shows the outer boundary of the disk. The flux tubes have $M_0 = 0.1$, the disk has $n = 0$, and the markers are at time steps of $100r_g/c$. (a) The disk temperature $T_d = 5 \times 10^9$ °K. (b) At $T_d = 10^{10}$ °K where the tension is the dominant force.

Fig. 2 — Trajectories as in Fig. 1, except that the flux tubes move adiabatically and they have $M_0 = 0.3$ (to highlight the oscillations). (a) Here, $n = 0$ i.e., flux tubes do not feel any Coriolis force, (b) $n = 0.1$ and flux tubes feel Coriolis force.

Fig. 3 — Trajectories as in Fig. 2, except that the flux tubes ($M_0 = 0.1$) move adiabatically inside a radiative disk; (a) $n = 0$, (b) $n = 0.1$.

Fig. 4 — Trajectories of flux tubes released at $\xi = 5$ and 7, when the field is shear amplified exponentially. Other parameters are same as in Fig. 3.
This figure "fig1-1.png" is available in "png" format from:

http://arxiv.org/ps/astro-ph/9311009v1
This figure "fig1-2.png" is available in "png" format from:

http://arxiv.org/ps/astro-ph/9311009v1
This figure "fig1-3.png" is available in "png" format from:

http://arxiv.org/ps/astro-ph/9311009v1
This figure "fig1-4.png" is available in "png" format from:

http://arxiv.org/ps/astro-ph/9311009v1