Relativistic Quantum Measurements, Unruh effect and Black Holes

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Abstract

It is shown how the technique of restricted path integrals (RPI) or quantum corridors (QC) may be applied for the analysis of relativistic measurements. Then this technique is used to clarify the physical nature of thermal effects as seen by an accelerated observer in Minkowski space-time (Unruh effect) and by a far observer in the field of a black hole (Hawking effect). The physical nature of the “thermal atmosphere” around the observer is analyzed in three cases: a) the Unruh effect, b) an eternal (Kruskal) black hole and c) a black hole forming in the process of collapse. It is shown that thermal particles are real only in the case (c). In the case (b) they cannot be distinguished from real particles but they do not carry away mass of the black hole until some of these particles are absorbed by the far observer. In the case (a) thermal particles are virtual.

1 Introduction

Nonrelativistic quantum theory of measurements is essentially based on the von Neumann’s postulate and cannot be applied for relativistic systems because of the violation of causality in the instantaneous state reduction of the measured system. This problem was considered by many authors (see for example [1]-[4]), but no consensus has been achieved about how relativistic quantum measurements may be correctly described. The general conclusion that may be drawn from this discussion is that duration of a quantum measurement in time and
dimension of the area where the measurement is arranged cannot be
neglected in the relativistic case. Relativistic quantum measurements
must be considered as continuous both in space and time.

The restricted-path-integral (RPI) approach to continuous mea-
surements has been successfully applied to relativistic as well as non-
relativistic measurement setups \[5\]-\[7\]. Particularly, this approach was
used in \[7\] to describe the measurement of the position of a relativistic
particle.

In what follows we shall elaborate this method in such a way that
it might be applied for a wide scope of quantum measurements on
elementary particles. Then some qualitative conclusions will be made
with the help of this technique for the Unruh and Hawking effects.

The RPI approach has been initiated by R.Feynman \[8\] to de-
scribe continuous (prolonged in time) non-relativistic quantum mea-
surements and was technically elaborated and extended on new areas
in \[5, 6, 9\] (see also \[10\]). An important advantage of the approach is
its being general and model-independent.

The idea of the RPI approach is that the evolution of the system
undergoing a continuous measurement must be described by the path
integral restricted on the set of paths compatible with the measure-
ment readout. Therefore an integral over a corridor of paths arises
instead of the Feynman path integral over all paths. This corridor
of paths may be called quantum corridor (QC) in analogy with the
close (but different) concept of the quantum trajectory introduced by
H.Carmichael \[11\]. QCs play an important role in the interpreta-
tion of continuous quantum measurements. A certain set of QCs deter-
mines the continuous measurement. Alternative QCs from this set
correspond to alternative measurement readouts possible in the given
measurement.

In the present paper we shall outline some features of the method
of QCs for relativistic quantum particles. Then the concept of a QC
will be used to analyze some conceptual problems in connection with
the Unruh effect for an accelerated observer and the Hawking effect
in the field of a black hole. In the course of the analysis we shall
clarify the physical nature of the “thermal atmosphere” observed by
an accelerated observer in Minkowski space-time or by an observer
moving far from a black hole. More concretely, we shall answer the
following questions:

- Is it possible, while observing thermal effects, to separate contri-
butions of different particles forming the thermal atmosphere?

- Whether the particles constituting this atmosphere are real, i.e.
  whether each of them may be observed in such a way that the
  fact of its existence be independent of the measurement?

We shall see that the answers to these questions are different not
only for the Unruh and Hawking effects, but also for the Hawking
effect in the case of the “eternal” black hole (described by the Kruskal
metric) and the black hole arising in the course of collapse. Some of
the conclusions we shall arrive at are of course known, particularly
from the important paper of W. Unruh and R. Wald [12]. However
some of them, especially the difference between eternal black holes
and those forming in collapse, seem to have never been formulated
clearly enough.

2 Relativistic Path Integrals

The causal propagator (transition amplitude) for a relativistic particle
can be expressed in the form of a path integral if one introduces, follow-
ing E.C. Stueckelberg [13], the fifth parameter (besides four space-time
coordinates) \( \tau \) called the proper time or historical time.

Consider for simplicity a scalar particle of the mass \( m \). Its causal
propagator is equal to the integral over the proper time

\[
K(x'', x') = \int_0^\infty d\tau \exp \left( -i(m^2 - i\epsilon)\tau \right) K_\tau(x'', x'),
\]

of a subsidiary proper-time-dependent propagator. The latter, in turn,
may be given the form of a path integral:

\[
K_\tau(x'', x') = \int_{x''\leftarrow x'} d[x]_\tau \exp \left( -\frac{i}{4} \int_0^\tau (\dot{x}, \dot{x}) d\tau \right).
\]

Here \((, )\) denotes the Lorentzian inner product and the path \([x]_\tau\) be-
tween the points \( x' \) and \( x'' \) of the Minkowski space-time is parametrized
by the interval of the proper time \([0, \tau]\).^2

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^1 We shall use in the present paper the natural units \( \hbar = c = 1 \).

^2 Notice that this proper time does not coincide with what is called proper time in
classical physics (the proper time of an observer at the given trajectory). This is why
the term ‘historical time’ seems more appropriate. However ‘proper time’ is used in this
context more often.
As a result of these definitions, the subsidiary proper-time-dependent propagator satisfies the “relativistic Schrödinger-type equation”

\[
\frac{d}{d\tau} K_\tau(x'', x') = -i \square K_\tau(x'', x')
\]

and the causal propagator \(K(x'', x')\) is a Green function of the Klein-Gordon equation:

\[
(\Box + m^2)K(x'', x') = -i\delta(x'', x').
\]

Being a Green function, the propagator \(K(x'', x')\) satisfies an important relation

\[
i \int_S \sigma^\mu K(x'', x) \overset{\leftrightarrow}{\partial_\mu} K(x, x') = K(x'', x')
\]

where \(S\) is a closed hypersurface with the point \(x'\) being inside and \(x''\) outside it, \(\sigma^\mu\) is an element of area of the hypersurface and \(\overset{\leftrightarrow}{\partial_\mu}\) is defined by

\[
f(x) \overset{\leftrightarrow}{\partial_\mu} g(x) = f(x) \frac{\partial}{\partial x^\mu} g(x) - f(x) \frac{\partial g(x)}{\partial x^\mu} \frac{\partial f(x)}{\partial x^\mu} g(x).
\]

These properties of the propagator may be generalized for the case of an arbitrary electromagnetic or gravitational field. All the derivatives must be covariant in this case and the path integral (2) should be defined covariantly [14].

For the analysis of continuous measurements on relativistic particles in the framework of the RPI approach we have to deal with path integrals of the type of Eqs. (1, 2) but restricted on the sets of paths compatible with the corresponding measurement outputs.

3 Measurements on particles

Let us shortly consider the main features of the relativistic RPI method starting with the simple case of the measurement of the particle position.

\[^{3}\text{An analogous relation but with the opposite sign in the r.h.s. is valid also for } S \text{ having } x'' \text{ inside and } x' \text{ outside it.}\]
The measurement of the position of a relativistic particle may be presented by paths going through the gate in a time slice. Alternative measurement outputs are presented by different gates in the given time slice (a). In the general situation a closed hypersurface should be taken instead of a time slice (b). Time axis is directed upward in this as well as in the following figures.

### 3.1 Measurement of the position

The measurement of the particle position at a time moment \( x^0 = t \) resulting in the measurement output \( x = a \) may be described \([7]\) by the integral over paths intersecting the space-like surface \( S = \{ x | x^0 = t = \text{const} \} \) in a narrow region around the point \( a = (t, a) \) (see Fig. 1a). We shall say that the paths go through the gate in the surface \( S \), the location of the gate corresponding to the measurement output \( a \) and the width equal to the measurement resolution \( \Delta a \).

**Remark 1** Actually the surface \( S \) must be closed as is shown in Fig. 1b, in accord with the relation \([7]\). In the case of null external field the integral over the past spacelike hypersurface of \( S \) as well as the integrals over the timelike side hypersurfaces are zero provided the side hypersurfaces are far enough. Therefore \( S \) may in this special case be taken to be a time slice \( \{ x | x^0 = t = \text{const} \} \). In the present paper we shall consider the general situation, hence closed surfaces with gates will play the main role.

Thus, the result of the measurement equal to \( a \in S \) may be described by a small area (gate) \( G(a) \) around the point \( a \in S \) on the surface \( S \). The corresponding amplitude \( K^{G(a)}(x'', x') \), describing the evolution of a particle undergoing the measurement under the condition that the measurement gave the result \( a \), should be defined as a path integral over the paths going from \( x' \) to \( x'' \) through the gate \( G(a) \).
The integral is the product of two integrals, one from \( x' \) to \( G(a) \) and the other from \( G(a) \) to \( x'' \). Each of these two integrals is close (though not equal) to the complete propagator between the corresponding points.

This is the reason why the amplitude \( K^{G(a)}(x'', x') \) may be defined directly through these propagators:

\[
K^{G(a)}(x'', x') = \int_{b \in G(a)} \sigma^\mu(b) K^{(b)}_\mu(x'', x')
\]

where

\[
K^{(b)}_\mu(x'', x') = i \left( K(x'', b) \hat{\partial}_\mu(b) K(b, x') \right).
\]  

(6)

This amplitude corresponds to the RPI in the corridor presented in Fig.\( \text{[1]} \). This corridor is a (closed) hypersurface with the gap. We shall consider generalizations of this quantum corridor in the following sections.

The amplitude (6) is derived for the particle which is in the space-time point \( x' \) before the measurement and in the point \( x'' \) after it. The realistic situation corresponds usually to the initial and final states given by the wave functions at the corresponding time moments \( t', t'' \) (presented by short horizontal lines in Fig.\( \text{[1]} \)). The measurement amplitude (6) must then be multiplied by the corresponding wave functions and integrated over time slices \( t' \) and \( t'' \):

\[
K^{(b)}(\psi'', \psi') = -\int \sigma^\mu(x'') \sigma^\nu(x') \bar{\psi}^m(x'') \hat{\partial}_\mu(x'') K^{(b)}_\nu(x', x') \hat{\partial}_\nu(x') \psi(x')
\]

(7)

(the bar denotes a complex conjugate).

The relation (6) (corresponding in the non-relativistic case to conservation of probabilities or unitarity of the evolution operator) may be shown to lead to the “generalized unitarity” of the measurement amplitudes provided that the dimension of \( G(a) \) is larger than the Compton wavelength \( \lambda_C = 1/m \) of the measured particle. The physical reason is that the localization of the particle in a region of the size \( \Delta a \) requires energy of the order of \( 1/\Delta a \) and may therefore lead to creation of pairs if \( \Delta a < \lambda_C \). Such a pair creation is caused by no external reason but is induced by the measurement itself. It distorts the picture of what happens and is therefore a sort of “measurement noise”. The condition \( \Delta a > \lambda_C \) guarantees that the measurement noise is negligible and the observed particle is real.

The condition \( \Delta a > \lambda_C \) makes sense only for a massive particle. However the more general condition \( \Delta a > \lambda \) may be applied for a
massless particle. Here $\lambda = 1/p$ is the “typical” wavelength of the particle in the conditions which the measurement is performed in. It is determined by the “typical” linear momentum $p$. This condition guarantees that the localization of the particle in the region of the dimension $\Delta a$ does not result in the creation of pairs of particles having momenta of the same order as the momentum of the measured particle. In this case the measurement noise is small in the interval of momenta (wavelengths) which is interesting. The particle observed in this interval of momenta may be interpreted as real.

3.2 Other relativistic effects

Many relativistic measurements (real or thoughtful experiments) may be characterized in the framework of the RPI approach by corridors of paths (quantum corridors) i.e. closed hypersurfaces, may be with gates in them. Examples are given in Fig. 2. Two alternative schemes are presented in Fig. 2 (a,c) for the observation of the pair creation and in Fig. 2 (b,d) for the observation of the causal zig-zag.

The process of measurement may result in a number of alternative measurement outputs. If the measurement is described by quantum corridors, different alternatives correspond to different corridors.
Thus, the V-type corridor in Fig. 2a is only one of many alternative corridors with different locations of the vertex. Analogously, the corridor of Fig. 2b is one of the corridors describing propagation from one time slice to another one with the trajectory observed (measured) with a finite resolution. For the realization of both these types of measurement one needs a medium consisting of objects (for example photons) weakly interacting with the measured particle and thus localizing it, with finite resolution, in space and time.

Another type of measurement corresponds to the corridors with gates in Fig. 2c–d. In this case (just as for the position measurement, Fig. 1), the closed surface is fixed and the alternative measurement results correspond to different locations of the gates. This formal scheme describes the observation arranged at the given closed surface with the width of the gates presenting the resolution of the observation. Such a measurement requires a net of objects activated in the specified time moments. We do not need to specify details of this realization because the method of quantum corridors does not depend on the concrete measurement setup but only on the kind of information supplied by the measurement.

It is essential how wide is the corridor or the gate. To make this question clear, it is reasonable to calculate restricted path integrals (RPI) in the situation when there is no fields which could cause non-trivial processes (for example pair creation or causal zig-zag). One may expect that in this situation all RPI corresponding to Fig. 2 must have negligible values. This may be shown valid if the corridor and the gate are wider than the Compton wavelength of the measured particle, \( \Delta a \gg \lambda_C \) or, more generally, if the corridor and the gate are wider than the typical wavelength of the measured particle, \( \Delta a \gg \lambda \).

If the width of the corridor or the gates is less (or of the order of) the Compton length, then the result of the RPI calculation is non-zero even for null fields. The physical reason of this fact is that the localization of a particle in the region of smaller dimension than \( \lambda_C \) requires inserting energy larger than the proper energy of the particle. This energy may lead to the creation of pairs. In this case pairs are created because of the too detailed observation of what happens. Pair creation is then the effect of the measurement itself, not of any external

\[ \text{In what follows we shall consider also such measurement schemes that the number of gates may also be different for different measurement results.} \]
reason. If $\Delta a \gg \lambda_C$, the measurement does not induce pair creation. The observed particles are then real. The weaker condition $\Delta a \gg \lambda$, where $\lambda$ is a typical wavelength of the measured particle, guarantees that the particles cannot be created with momenta of the order of one interesting for us. In this case the observed particle is real provided that we are not interested in momenta less than $1/\Delta a$.

Considering relativistic measurements in non-zero fields, we have therefore to choose wide enough corridors and gates ($\Delta a > \lambda_C$ or $\Delta a > \lambda$) to avoid too strong influence of the measurement. If on the contrary the influence of the measurement is the aim of the investigation in its own right, then the width of the corridor or the gate may be less than the wavelength.

4 Unruh effect

As it has been shown by W.Unruh [15], an accelerated observer in Minkowski space-time will see the vacuum as a thermal bath with the temperature proportional to its acceleration, $kT = w/2\pi$. This phenomenon was called Unruh effect. It is convenient to analyze the Unruh effect in the Rindler coordinates $(\eta, \xi)$, which are related to Minkowski coordinates $(x^0, x^1)$ by the transformation

$$
x^0 = \frac{1}{w} e^{w\xi} \sinh w\eta, \quad x^1 = \frac{1}{w} e^{w\xi} \cosh w\eta.
$$

(8)

The trajectory of the accelerated observer has in the Rindler coordinates the simple form $\xi = 0$ and the Rindler time $\eta$ is a proper time on this trajectory. The surfaces $x^1 = \pm x^0$ are event horizons for the accelerated observer. This means that only those events may be causally connected with him which are in the same quadrant in respect to the horizons. This quadrant is sometimes called “Rindler wedge”.

The causal propagator of the (massless) particle in the Minkowski space-time between two points on the trajectory of the accelerated observer is equal (up to the number factor) to [16]

$$
\frac{w^2/4}{\sinh^2 \frac{1}{2} w(\eta' - \eta'')} = \sum_{n=\pm\infty} \frac{1}{[(\eta' - \eta'') + i\beta n]^2}
$$

(9)

where $\beta = (kT)^{-1}$. In the energy-momentum representation

$$
\frac{w^2/4}{\sinh^2 \frac{1}{2} w(\eta' - \eta'')} = -\frac{1}{(2\pi)^2} \int dE \, dp \, e^{iE(\eta' - \eta'')} D_\beta(E, p)
$$

(10)
Figure 3: Paths with different winding numbers $n$ in respect to the origin of the Rindler plane. Topologically nontrivial paths $n \neq 0$ are responsible for the Unruh effect. The trajectory of the accelerated observer (thick line) and his event horizons (thin direct lines) are also drawn in the figure.

The propagator has the form

$$D_\beta(E,p) = \frac{i}{E^2 - p^2 + i\epsilon} + \frac{2\pi\delta(E^2 - p^2)}{e^{\beta|E|} - 1}$$

(11)

with the first term corresponding to $n = 0$ in (8).

This means that the usual propagator in the Minkowski space-time has the form of the thermal propagator in respect to the Rindler time (a proper time of the accelerated observer). Formally this leads to the conclusion that the accelerated observer will see the thermal bath instead of the vacuum. In the expansion (9) the terms with $n \neq 0$ are responsible for the thermal effects.

As it is shown by W. Troost and H. Van Dam [16], in the path-integral representation of the propagator the term with the given $n$ is presented by the paths having the winding number $n$ in respect to the origin of the plane $(x^0, x^1)$. This means that only those paths which go around the origin precisely $n$ times contribute to the $n$th term (see Fig. 3). Thermal effects are therefore presented by topologically non-trivial paths having $n \neq 0$.

Let us apply now the RPI approach to analyze the Unruh effect. Consider first the measurement setup which does not induce pair creation. According to what has been said at the end of Sect. 3.2, if we want to arrange the observation in such a way that the measure-
Figure 4: Observation of the Unruh effect: (a) the measurement includes the whole “thermal effect” as the result of vacuum fluctuations (wide corridor); (b) the observation distinguishes single thermal particles, however their creation under influence of the measurement cannot be excluded (narrow corridor).

ment itself does not induce the pair creation, then we have to choose the quantum corridors (describing this measurement) wider than the wavelengths of particles in the given thermal bath. The typical energy for these particles is $kT$, so that the wavelength is of the order of $\lambda = 1/kT$. It can be shown that any point in the trajectory of the accelerated observer is separated by the distance of the order of $\lambda$ from the corresponding point at the “trajectory of the antiobserver” obtained by the reflection through the origin ($x^0 \rightarrow -x^0, x^1 \rightarrow -x^1$).

Therefore among all alternative wide corridors we have to consider those which include, together with any part of the observer trajectory, also the corresponding part of the trajectory of the “antiobserver” and the whole region between these lines. All topologically nontrivial paths responsible for thermal effects will be included in this corridor (Fig. 4a). Individual particles from the “thermal atmosphere” of the accelerated observer cannot be separated with the help of the measurement of this type. Thermal terms are interpreted in this case as “vacuum fluctuations in the Minkowski vacuum”.

Consider now another type of measurements, for which all alternative corridors are restricted by the event horizons of the accelerated observer (are enclosed in the “Rindler wedge”). Then these corridors are narrow (as compared with the wavelength). Effects of the measurement cannot be excluded in such a measurement. Let us analyze this type of measurements.

It is possible to choose narrow corridors to characterize thermal
effects in more detail. For example the quantum corridor presented in Fig. 4b gives an amplitude for the propagation of the particle with the creation of not less than two thermal particles \((n \geq 2)\), one of which travels freely through the area of the measurement. It is seen from Fig. 4b that the thermal effect will be interpreted by the corresponding observer as the effect of particles coming from the past horizon and going to the future horizon. Amplitudes corresponding to the corridors of this type may be calculated and in principle they may be compare with experimental data. However the corridor in this case will be narrow (as compared with the wavelength of thermal particles). This means that the influence of the measuring setup is not negligible in the corresponding experiments. The observed particles cannot be interpreted as real particles existing independently of the measurement.

We can consider path integrals describing the absorption of a thermal particle by the accelerated observer (Fig. 5). The absorption of a “Rindler particle” is accompanied in this case by the creation of one more particle which can be absorbed by an inertial observer. A wide corridor (with a wide gate) exists in this case (Fig. 5a). It includes all thermal terms (all winding numbers \(n\)). The absorption of a ‘Rindler particle’ and accompanying radiation of a ‘Minkowski particle’ is a real (not virtual) process, but the contributions of different \(n\) to this process cannot be separated experimentally. A more detailed observation separating these contributions is described by narrow quantum corridors (Fig. 5b). However the influence of the measuring setup is not negligible in this case. When observing thermal particles, the observer cannot interpret them as real ones existing independently of the observation.

Conclusions for the Unruh effect:

- The “thermal atmosphere” of an accelerated observer consists of virtual rather than real particles which are parts of a long loop presenting a vacuum fluctuation (Fig. 4a).
- The observation performed in a narrow region (as compared with the wavelengths of thermal particles) may lead to “discovery” of single thermal particles, but the influence of the measurement onto creation of these particles cannot be excluded (Fig. 4b).
- If a thermal particle is absorbed by an accelerated observer, then the loop is broken and the counterpart antiparticle becomes real.
Figure 5: Absorption of a particle by an accelerated observer: (a) Observation in a wide corridor of two real processes, absorption of a ‘Rindler particle’ and creation of the corresponding ‘Minkowski particle’. Contributions of single thermal particles cannot be separated. (b) Observation in a narrow corridor separates contributions of single thermal particles, but creation of these particles under the influence of the measurement cannot be excluded.

and may be observed as a real particle (Fig. 5a).

5 Black Holes

Theoretically two qualitatively different types of black holes (BH) may exist (see Fig. 6): an eternal BH and a BH forming in the process of collapse of usual matter (for example a star). An eternal BH (if it has null angular momentum and charge) is described by the Kruskal metric and have two event horizons (the future and past horizons) and two singularities (the future and past singularities), see Fig. 6a. A BH forming in collapse have only one (future) horizon and only one (future) singularity. For both types of BH a trajectory of an observer moving at a constant distance from the BH is drawn in Fig. 6.

It has been shown by S. Hawking [18] that an observer moving far from the BH will see a thermal bath having the temperature inversely proportional to the BH mass: \( kT = \frac{1}{8\pi GM} \) where G is the gravitational constant. However the nature of thermal effects is not quite clear up to now [19, 20]. We shall apply the RPI approach to analyze this question.

\(^5\)We suppose that the reader is familiar with basic features of BH which may be found for example in [17]
Figure 6: Two types of black holes: (a) an eternal (Kruskal) BH has future and past event horizons (thin direct lines) and future and past singularities (thick lines on the top and bottom of the diagram); (b) a BH forming by collapse has only future singularity and only future horizon starting at the surface of the collapsing body. The trajectory of a far observer is presented in both cases (thick line on the right).

5.1 An eternal black hole

As was demonstrated by W.Troost and H.Van Dam [16], thermal effects in the field of the eternal BH are (in complete analogy with the Unruh effect) described by the paths which are topologically non-trivial in respect to the origin of the Kruskal coordinates (the point where the horizons cross each other). Just as in the Rindler plane, the winding number in respect to the origin of the Kruskal coordinates coincides with the number of thermal particles. We shall analyze these paths with the help of different quantum corridors (see Fig. 7).

Despite of the deep analogies, one feature essentially distinguishes the Hawking effect from the Unruh effect. In the Unruh effect the temperature tends to zero when the acceleration $a$ decreases (i.e. for the observer far from the origin). In the Hawking effect the temperature also decreases with the distance from the BH increasing. However it tends to a constant value $\frac{1}{8\pi GM}$ for an infinitely far observer. The temperature stays finite (and close to this constant) in infinite interval of distances. Therefore, in the case of a BH wide corridors around the observer trajectory do not include the origin of the Kruskal plane. We shall see that the existence of such corridors make possible the measurements separating the contributions of single thermal particles.

Let us consider the measurement corresponding to the corridor of Fig. 7a having the width larger than the typical wavelength of the thermal particles $\lambda = 1/kT$. Alternative measurement results are
Figure 7: Observations in the field of an eternal BH: (a) The observation of the propagation arranged in a wide region simultaneously discovers singular thermal particles having all properties of real ones. No energy is taken from the BH. (b) The absorption of a particle observed in a wide region simultaneously discovers single thermal particles with the properties of real particles. The counterpart antiparticle is absorbed by the BH, extracting energy from it.

described in this case by the number and location of gates which are also wide enough. The number of gates (divided by 2) determines the number of thermal particles observed in the given measurement result. Since both the corridor and the gates are wide, all observed particles are real (not originated by the too narrow localization during the measurement). However, no particle is absorbed or issued in these processes by the BH, therefore the BH mass cannot be changed in this way. This of course could be expected for the eternal BH.

The question naturally arises: if the observed thermal particle cannot be distinguished from a real one, then it should carry an energy and contribute to the general mass of the BH and its environment as it is seen by a far observer. The answer is yes, each of these particles contributes to the general energy, but the sum of all these contributions is zero. This is connected with the special properties of time of the far observer. A surface of constant time for such an observer is presented at the Kruskal diagram (as in Fig. 7) by the direct line passing through the origin. If the right end of such a line goes upward (positive direction of time), its left end goes downward.

Because of the loop-like structure of paths of thermal particles, they are divided in pairs consisting of a particle and an antiparticle, the particle in the ‘causal wedge of the observer’ and the antiparticle in the opposite wedge. In respect to the observer’s time, the particle in each pair propagates in the positive direction of time, while the cor-
responding antiparticle does in the negative time direction. Therefore, if the particle have positive energy, the antiparticle has negative energy. Because of the complete symmetry of the set of all paths, these energies compensate each other so that the complete contribution to the mass observed by the far observer is null.

The symmetry however breaks down if one of the particles is absorbed by the observer (the corridor of Fig. 7b). The breakdown occurs at the moment of the absorption (in the observer’s time). Beginning from this time moment the number of antiparticles is larger by unit than the number of particles. The absorption of a thermal particle is accompanied by another process: one of the antiparticles becomes real and falls onto the future singularity of the BH. The negative energy of this antiparticle contributes now to the general mass of the BH as it is seen by the observer. The observer when measuring gravitational field will see that this field corresponds now to smaller general mass. This may be interpreted as diminishing of the mass of the BH. From another point of view the origin of this mass is not the BH, but a particle moving in its vicinity.

Conclusions for the eternal (Kruskal) BH:

- Particles forming the “thermal atmosphere” of the far observer cannot be distinguished from real ones, but they do not change the mass of the BH if they are not absorbed. Together with their counterpart antiparticles they form a loop, and their energies (in respect to the time of the far observer) compensate each other.

- The absorption of a particle by the far observer is accompanied by falling an antiparticle onto the singularity resulting in the change of the BH mass as it is seen by the far observer. Instead, the absorbed particle may be issued from the past singularity.

5.2 A black hole forming in collapse

Consider now a BH forming in real collapse (Fig. 8). There is one essential new feature of such a BH as compared with the eternal BH. In the space-time point coinciding with the origin of the horizon, a virtual pair may be “torn off” with forming a real particle escaping to infinity and a real antiparticle falling into the BH (Fig. 8a). The particle

\[6\] Instead of the future singularity, the free end of the torn loop may begin at the past singularity. Then the absorbed particle is issued by the BH.
of a virtual pair may go through the collapsing body (which has at this stage the size of the order of the wavelength of the considered particle), exit from the other side of it and escape to infinity. The corresponding antiparticle bypasses the body and falls into the BH.

As it is proved by S.Hawking [18], the energy spectrum of particles escaping to infinity due to this mechanism is thermal with the temperature $kT = \frac{1}{8\pi GM}$. This seems similar to what takes place around the eternal BH (though due to another mechanism). However in the case of the BH forming in collapse the “thermal atmosphere” consists of real rather than virtual particles. Creation of each particle is accompanied by the creation of an antiparticle carrying negative energy into the BH. The mass of the BH decreases in the result of such a process. Each of the thermal particles formed in this way may be absorbed by the far observer. The origin of the observed particle may be in principle traced back to the moment of its formation near the horizon origin (Fig. 8b).

Conclusions for the BH resulting in the process of collapse:

- Near the origin of the horizon a virtual pair may be converted to a real one, with the antiparticle falling into the BH diminishing its mass and the particle escaping to infinity along the horizon.
- The real particles escaping to infinity may be seen (and absorbed) by a far observer. However independently of their absorption these particles are carrying mass of the BH away.
- The trajectory of an absorbed (or only observed) particle may
in principle be traced back to the point near the origin of the horizon.

6 Concluding remarks

The technique of relativistic restricted path integrals (RPI) and quantum corridors (QC) has been here only outlined. Some important procedures characteristic for this technique were not properly discussed, for example summing up over all alternative measurement results. Besides, no RPI has been really calculated in the present paper. Nevertheless, the estimate of the width of a QC in different physical situations led us to some new conclusions or at least made more clear some points of view on the Unruh and Hawking effects.

The main of these conclusions is a subtle distinction between the cases of a) the Unruh effect, b) an eternal black hole (BH) and c) the BH resulting in the process of collapse. The nature of the “thermal atmosphere” of the observer is different in these three cases. This atmosphere consists of virtual particles in the case (a) and of real particles in the case (c). In the intermediate case (b) thermal particles may be observed in a wide enough region so that they have all properties of real particles. If some of them are absorbed, the mass of the BH decreases by the corresponding amount. However until being absorbed these particles do not carry away mass of the BH so that this mass is constant. In fact, thermal particles attain more features of real particles with each step of advancing along the chain (a)→(b)→(c).

The difference between the eternal BH and the BH forming in collapse may have consequences for astrophysical observations. If some BH is observed, it is not necessary to expect that it will finally evaporate. This depends on its prehistory. If the BH has been formed by collapse, it will finally evaporate, but if it was existing at any time in the past, it will be existing infinitely also in the future. Such a BH is actually “eternal”. At least this is the case if the environment of the BH is not too dense, because otherwise absorption of particles from the “thermal atmosphere” by the environment will lead to falling their antiparticle counterparts into the BH and resulting decrement of the

\footnote{The distinction between an eternal BH and one forming in collapse was discussed in \cite{21}. The conclusion was that the vacuum must be stable in the field of the eternal BH but not in the case of a collapsing body.}
observed BH mass.

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