CMB constraints on spatial variations of the vacuum energy density

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In a recent article, a simple ‘spherical bubble’ toy model for a spatially varying vacuum energy density was introduced, and type Ia supernovae data was used to constrain it. Here we generalize the model to allow for the fact that we may not necessarily be at the centre of a region with a given set of cosmological parameters, and discuss the constraints on these models coming from Cosmic Microwave Background Radiation data. We find tight constraints on possible spatial variations of the vacuum energy density for any significant deviations from the centre of the bubble and we comment on the relevance of our results.

I. INTRODUCTION

Recent Cosmic Microwave Background (CMB) \cite{1} and type Ia supernovae \cite{2} data have provided some reasonably strong evidence for an accelerating local universe, implying that most of the ‘missing mass’ of the universe should be in a non-clustered form, such as a cosmological constant or quintessence.

Yet it is well known that these measurements are mostly local, so one should be especially careful in the way they are used. In particular, it is almost always assumed that the equation of state of the dark matter is constant for low redshifts. However, this is by no means well justified, and a redshift dependence would arise if there are space and/or non-trivial time variations of the cosmological parameters.

Two ‘natural’ ways in which this would happen are quintessence models with a non-trivial equation of state, and models where a late-time phase transition \cite{3–6} produces wall-like defects which separate regions with different values of the cosmological parameters, notably the matter and vacuum energy densities (and hence also the Hubble constant). Note that even though the motivations for the two types of models are quite different, their observational consequences can be fairly similar, and distinguishing between the two may be a non-trivial task.

In a recent paper \cite{6}, three of the present authors have introduced a very simple ‘toy model’ that aims to mimic this type of cosmological scenario with a minimal number of free parameters. It assumes that we live inside a spherical bubble with a given set of cosmological parameters, which is surrounded by a region where these parameters are different. In terms of redshift, there will be a single discrete jump on in these parameters as one goes through the wall. A perhaps more realistic, but also definitely more complicated toy model (in the sense of having more free parameters) would be one with various different ‘shells’, and hence several jumps in the cosmological parameters at various redshifts. Ultimately, one could get to the continuous limit where one has the cosmological parameters varying as continuous functions of redshift in a non-trivial way. Something along these lines is discussed in \cite{7}.

In our previous work \cite{6}, we have used the recent measurements of the luminosity versus redshift relation using Type Ia supernovae, which have now been observed out to $z \sim 1.7$ \cite{8}, to constrain the simplest models of this type.

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It was found that presently available observations are only constraining at very low redshifts $z \lesssim 0.5$ (as was to be expected), but it was also independently confirmed that the high red-shift supernovae data does prefer a relatively large positive cosmological constant.

Here we will discuss the constraints on this type of models coming from CMB data, while also allowing for a further effect. Indeed, we allow for the possibility that we are not at the centre of the inner domain or, in other words, that there is a further anisotropic component in the CMB. Obviously the observed near-isotropy of the CMB will impose quite strong constraints on our position, but it is still important to determine how much freedom is allowed if one considers that part of the observed anisotropy comes from such a displacement.

In the next section we briefly review our toy model and discuss the evolution of cosmological perturbations within its framework. In Sect. III we show, mainly as an amusing interlude, how one could design a ‘mimic model’ of this kind which would reproduce the CMB spectrum predicted by standard paradigms such as inflation. Our main results are presented and discussed in Sect. IV, and we conclude in Sect. V.

II. COSMOLOGICAL PERTURBATIONS IN A SPHERICAL BUBBLE MODEL

We shall again consider a simplified model in which the universe consists of a spherically symmetric region (domain) with a given set of cosmological parameters, which is surrounded by another region where the values of the cosmological parameters (in particular, that of the vacuum energy density) are different. We shall refer to these two values of the vacuum energy density as $V_-$ and $V_+$ respectively. Note that the cosmological parameters will in general be different in both regions, but their variations are not totally independent—they are such as to make the universe flat both inside and outside the domain (this point is discussed in detail in [6]).

As in our previous work [6], we assume that the red-shift of the domain wall, as measured by an observer at the centre of the inner spherical domain, is $z_*$. However, we now allow for the possibility that we are not at the centre of the domain. Specifically, we assume that we live a red-shift $z_{\Delta} \leq z_*$ away from this ‘central’ observer.

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Again, we are assuming that the thin region separating the two domains considered (domain wall) does not generate relevant CMB fluctuations. This happens if the potential of the field is small enough at the origin [6]. We shall also assume that the domain walls are frozen in comoving coordinates, which again will be a good approximation provided that friction is important [6].

We shall parametrize the vacuum energy density by

$$\Omega_\Lambda \equiv \frac{V_-}{\rho_c},$$

where $\rho_c$ is the critical density, and we define $\Delta \Omega_\Lambda$ as

$$\Delta \Omega_\Lambda(r) = \frac{\delta \rho_\Lambda(r)}{\rho_c} = \frac{\rho_\Lambda(r) - V_-}{\rho_c},$$

where $\rho_\Lambda(r)$ is the vacuum energy density at the point in question. Hence, this can have two possible values: 0 if we are inside the inner region, and $(V_+ - V_-)/\rho_c$ in the outer domain.

In the conformal-Newtonian gauge, the line-element for a flat Friedmann-Robertson-Walker background and scalar metric perturbations can be written as

$$ds^2 = a^2(\eta) \left[ (1 + 2\Phi)c^2 d\eta^2 - (1 - 2\Phi)(dr^2 + r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2) \right],$$

assuming that the anisotropic stresses are small. Here, $\Phi$ is the metric perturbation, $c$ is the speed of light in vacuum, $a$ is the scale factor, $\eta$ is the conformal time, and $r$, $\theta$ and $\phi$ are spatial coordinates.

Given that the vacuum energy becomes dominant only for recent epochs we shall be concerned with the evolution of perturbations only in the matter-dominated era, thus neglecting the contribution of the radiation component. The evolution of the scale factor $a$ is governed by the Friedmann equation[6], which can be written as

$$\mathcal{H}^2 = \Omega_m^0 a^{-1} + \Omega_\Lambda^0 a^2.$$

\[1\] A dot denotes a derivative with respect to conformal time, $\dot{\mathcal{H}} = \dot{a}/a$, the index ‘0’ means that the quantities are to be evaluated at the present time and we have taken $a_0 = 1$ and $\mathcal{H}_0 = 1$ (so that the conformal time is measured in units of $\mathcal{H}_0^{-1}$).
Note that the background matter and vacuum energy densities at an arbitrary epoch can be written as

$$\Omega_m = \frac{\Omega^0_m}{\Omega^0_m + \Omega^0_\Lambda a^3}$$

and $$\Omega_\Lambda = 1 - \Omega_m$$.

The linear evolution equation of the scalar perturbations is given by (see for example [10])

$$\ddot{\Phi} + 3H \dot{\Phi} + (2\dot{H} + H^2)\Phi = 4\pi G a^2 \delta p = \frac{3}{2} a^2 \Delta \Omega^0_\Lambda,$$

where $$\delta p$$ is the pressure perturbation and

$$2\dot{H} = -\Omega^0_m a^{-1} + 2\Omega^0_\Lambda a^2.$$  

Given that the source term in the outer domain ($\propto a^2 \Delta \Omega^0_\Lambda$) is only important near the present time, we assume the following initial conditions for eq. (6):

$$\Phi(0) = 0, \quad \dot{\Phi}(0) = 0,$$

both in the inner and the outer regions. We note that since the source term is absent in the inner region, the metric perturbation is always zero there.

The novel feature of this type of model is that in the presence of the scalar metric perturbations defined by eqn. (6) there is a shift in the CMB temperature given by

$$\Delta T = -2\Phi_+.$$

We find it useful to parametrize our position within the bubble by

$$\epsilon \equiv \frac{d(z_\Delta)}{d(z_\ast)},$$

that is, the ratio of our comoving distance to the bubble centre and the comoving bubble radius. On the other hand, the comoving distance to the domain wall as seen by us as a function of the angle $$\theta$$ is given by

$$D(\theta) = \sqrt{(d(z_\ast) \cos \theta - d(z_\Delta))^2 + (d(z_\ast) \sin \theta)^2},$$

and the conformal time when the CMB radiation that reaches us today crossed the domain wall at that particular direction is then simply given by

$$\eta(\theta) = \eta_0 - \frac{D(\theta)}{c}.$$  

The value of $$\Phi_+(\theta) \equiv \Phi(\eta)$$ can now be calculated using eqn. (6).

Before moving on to discuss the results of this analysis in Sect. IV, we make a brief detour to discuss an interesting property of this type of models.
III. A MIMIC MODEL

Let us assume for the sake of simplicity that $\Omega_m = 1$ and $\Omega_\Lambda = 0$ such that eq. (6) becomes

$$\ddot{\Phi} + \frac{6}{\eta} \dot{\Phi} = \eta^{-6} \frac{\partial (\eta^6 \dot{\Phi})}{\partial \eta} = -\frac{3}{8} \eta^2 \Delta \Omega_\Lambda^0,$$

(14)

where we have made use of the fact that with our conventions one has $a = \eta^2 / \eta_0^2$ and $\eta_0 = 2H_0^{-1} = 2$. Solving this equation with the boundary conditions

$$\Phi(0) = 0, \quad \dot{\Phi}(0) = 0,$$

(15)

in the outer region we obtain

$$\Phi(\eta) = -\frac{1}{96} \eta^4 \Delta \Omega_\Lambda^0 = -\frac{1}{6} (1 + z)^{-2} \Delta \Omega_\Lambda^0.$$  

(16)

Let us consider a domain wall which is at a red-shift $z$ from us. Consider small spatial fluctuations in the red-shift of the domain wall parametrized by $\Delta z(\theta, \phi)$ where $\theta$ and $\phi$ are angular coordinates. These fluctuations will induce CMB perturbations with

$$\Delta T = -2 \Delta \Phi_+ = -\frac{2}{3} \Delta z(1 + z)^{-3} \Delta \Omega_\Lambda^0.$$  

(17)

where we have subtracted the multipole of order 0 of the CMB fluctuations. This has been done for a flat matter dominated background but it is obvious to see that it can be done more generally.

This demonstrates that it would be possible in general to generate any spectrum of CMB fluctuations by an appropriate design of the domain wall. It would, of course, be extremely unlikely that a domain wall would have the precise shape necessary to produce the CMB spectrum predicted in popular models for the origin of CMB and large scale structure (e.g. inflation). However, it would not be so implausible that this model could have some kind of impact on the CMB fluctuations.

Irrespective of the likelihood of this model, we think it serves to highlight an important point, namely that a good test to the predictions of current models for structure formation consists in studying to what extent these could be produced by other models (even if they might seem unlikely in the light of our current theoretical prejudices).

IV. RESULTS AND DISCUSSION

Our results can be conveniently summarized by figs. 1 and 2. We have assumed that the present local universe is characterized by cosmological densities $\Omega_m^0 = 0.3$ and $\Omega_\Lambda^0 = 0.7$, and calculated the maximum value allowed by currently existing observations for $\Delta \Omega_\Lambda^0$ in the outer domain ($\Delta \Omega_\Lambda^{max}$) as a function of the parameters $z_*$ and $\epsilon$. Figs. 1 and 2 show the result of this calculation for the dipole and quadrupole measured by COBE [11].

As expected, there are no direct CMB constraints, to first order, if we happen to be at the centre of the domain, and they become progressively tighter as we move away from this position. We note that if the observer is right at the centre of the spherical domain, a further (second-order) contribution would come from the integrated Sachs-Wolfe effect. Being second order, this would be subdominant everywhere except extremely close to the centre ($\epsilon \sim 0$), and in this limit the supernovae constraints discussed in [6] are currently much stronger. For this reason we have ignored this effect in the present analysis, although it should be considered in the future when more precise data becomes available.

Also as expected, there are no constraints in the limits of an infinitely small ($z_* \to 0$) or infinitely large ($z_* \to \infty$) inner region. This means that for any given $\epsilon$ there will be some intermediate value of the redshift for which the the allowed variation in the vacuum energy density is minimal. In fact it turns out that, at least to a first approximation, this particular redshift is independent of our position within the bubble (which is measured by $\epsilon$). For the currently favoured cosmological model, with $\Omega_m^0 = 0.3$ and $\Omega_\Lambda^0 = 0.7$, this redshift is found to be $z \sim 2.2$. We also point out that, the quadrupole constraints (Fig 2) are, in general, much stronger than the dipole constraints (Fig 1). This is due to the much larger amplitude of the observed dipole CMB anisotropy compared with the quadrupole.

Several models attempting to reconstruct the matter distribution from the observed velocity field have been proposed. However, even the best ones are left with an unexplained relatively high residual velocity of $\sim 200 \text{ km} \text{ s}^{-1}$, too large to be an unknown motion of our galaxy with respect to the Local Group reference frame (see for example
FIG. 1. The maximum variation in the vacuum energy density with respect to the present local value (parametrized by $\Delta \Omega_A^{\text{max}}$) allowed by the COBE dipole, as a function of our position within the spherical bubble. This position is parametrized by the redshift, $z_*$, of the spherical wall as measured by an observer at its centre, and by the ratio, $\epsilon$, of our comoving distance to the bubble centre and the comoving bubble radius.

FIG. 2. The maximum variation in the vacuum energy density with respect to the present local value (parametrized by $\Delta \Omega_A^{\text{max}}$) allowed by the COBE quadrupole, as a function of our position within the spherical bubble. This position is parametrized by the redshift, $z_*$, of the spherical wall as measured by an observer at its centre, and by the ratio, $\epsilon$, of our comoving distance to the bubble centre and the comoving bubble radius.
The possibility that this discrepancy might be explained by a model such as the ‘bubble’ model presented in this paper is, however, unfavoured by our results. The relatively large values of $\Delta \Omega_{\Lambda}^{\text{max}}$ allowed by the COBE dipole make this possibility rather unlikely.

In any case, our analysis confirms the intuitive expectation that the CMB constraints on anisotropies of this kind are extremely tight, or in other words that if such a model is a realistic approximation to the evolution of the universe, then we must be very close to the centre of the domain. Still another way of saying it is that, if non-trivial variations of the cosmological parameters as a function of the redshift did occur in the past, they are more likely to result from scenarios where the redshift variations are the result of time variations which maintain spatial homogeneity or isotropy (such as quintessence models, for example), since in scenarios where there are also spatial variations we are forced by current observations to occupy a fairly privileged (and arguably unlikely) position.

V. CONCLUSIONS

In an earlier work we have introduced a very simple ‘spherical bubble’ toy model for a spatially varying vacuum energy density, and type Ia supernovae data was used to constrain it. Here we have generalized this model, in order to account for the possibility that we may not be at the centre of a region with a given set of cosmological parameters. We have then discussed the constraints on this model coming from CMBR data. As expected, any significant deviations from isotropy are tightly constrained.

Ultimately, our toy model aims to be a simple mimic for non-trivial variations of cosmological parameters. As we have pointed out before, these variations as one probes higher redshifts could result either from genuine time-variations (an example of which would be quintessence models with a non-trivial equation of state) or from spatial variations (an example of which would be models where late-time phase transitions produce a domain-like structure).

The present work, together with [3] quantitatively confirms the intuitive expectation that models with spatial variations are much more constrained, in the sense that the near-isotropy of the CMBR imposes quite strong restrictions on our position within such a domain, whereas no such constraints exist in the case of genuine time variations. Obviously, our toy model is rather simplified, and a more detailed analysis is required to constrain specific models. We shall return to this interesting topic in a future publication.

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