Dispersion regions overlapping for bulk and surface polaritons in a magnetic-semiconductor superlattice

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Extraordinary dispersion features of both bulk and surface polaritons in a finely-stratified magnetic-semiconductor structure which is under an action of an external static magnetic field in the Voigt geometry are discussed in this letter. It is shown that the conditions for total overlapping dispersion regions of simultaneous existence of bulk and surface polaritons can be reached providing a conscious choice of the constitutive parameters and material fractions for both magnetic and semiconductor subsystems.

The polariton is introduced as a quasi-particle which characterizes a coupling between electromagnetic waves (photons) and a diverse variety of dipole-active elementary excitations inherent to a matter such as phonons, plasmons, excitons, etc. Although the concept of quasiparticles is related to quantum mechanics, the polariton can be considered as a macroscopic phenomena concerning interaction of electromagnetic waves with macroscopic normal modes (eigenwaves) of a matter assuming their wavelength are long enough so that the medium can be treated as a continuous one. In such a way the theory of polaritons is developed without specifying which kind of dipole excitation is coupled to electromagnetic waves, because the specific nature of this excitation is completely defined only by the dielectric function (e.g. permittivity) of a medium \[1\]. This approach implies a particular consideration of polaritons in a bulk material (bulk polariton) and on its surface (surface polariton).

Although the nature of these two particular modes is the same and is related to the medium polarizability, surface polaritons are distinguished from bulk polaritons by the fact that their amplitudes decay exponentially away from the surface in the direction normal to it \[2\]. This means that the normal component of the wavevector of a surface polariton is purely imaginary and consequently it cannot propagate away from surface. Therefore, these surface modes do not couple linearly with bulk polaritons either inside or outside the surface. In particular, this fact manifests itself in the different areas of existence of bulk and surface polaritons on their dispersion characteristics.

As already mentioned, from the macroscopic viewpoint, the dispersion conditions for bulk and surface polaritons are defined by the dielectric function of a medium, and in the case of surface modes the difference in signs of the dielectric functions of patterning materials is required (i.e. the real parts of the permittivity scalars of two patterning materials must have opposite signs). Nevertheless, if at least one of two patterning materials is anisotropic (due to its crystalline nature or as a result of an external action, such as application of the static magnetic field) the permittivity appears as a tensor quantity, and the dispersion characteristics of polaritons become to be more complicated \[3, 4\], e.g. the propagation of surface polaritons can appear to be permissible, even though the real parts of all components of the permittivity tensors of both materials are positive \[3\].

Besides the dielectric function, the properties of polaritons can also be determined by the magnetic function in a magneto-optically active medium in which electromagnetic waves can be coupled to spin waves in (anti)ferromagnetic \[5\]. Apart from flexible modulation of the polaritons properties by an external magnetic field, surface magnetic polaritons also promise nonreciprocal effect and multi-bands of propagation \[6\]. Moreover, in such systems it was demonstrated that the bulk and surface polaritons can exist in the same frequency range, but with different wavevectors \[7, 8\], as well as the possibility of obtaining very narrow frequency range where a surface polariton branch can merge into the continuum of bulk modes is found out \[10, 11\].

It is obvious that the simultaneous combining together of dielectric and magnetic functions into a single electromagnetic (or generally, gyroelectromagnetic) system can bring a number of unique dispersion features of polaritons that are unattainable in separate subsystems. In particular, in this letter our goal is to demonstrate, for the first time to our knowledge, that the dispersion regions of bulk and surface polaritons can overlap in a composite magnetic-semiconductor structure providing a proper selection of the constitutive parameters and material fractions for both magnetic and semiconductor subsystems.
Therefore, in this paper we study dispersion characteristics of both bulk and surface polaritons propagating in a semi-infinite stack of identical composite double-layered slabs arranged along the y-axis (Fig. 1). Each composite slab within the stack includes magnetic (with constitutive parameters \(\varepsilon_m, \mu_m\)) and semiconductor (with constitutive parameters \(\varepsilon_s, \mu_s\)) layers with thicknesses \(d_m\) and \(d_s\), respectively. Thus, the stack possesses a periodic structure (with period \(L = d_m + d_s\)) that fills half-space \(y < 0\) and adjoins a vacuum occupying half-space \(y > 0\).

We suppose that the structure is a finely-stratified one (i.e. it is a superlattice), whose characteristic dimensions \(d_m\) and \(L\) are all much smaller than the wavelength in the corresponding layer \(d_m \ll \lambda\), \(d_s \ll \lambda\), and period \(L \ll \lambda\) (the long-wavelength limit). Along the \(x\) and \(z\) directions the system is considered to be infinite. An external static magnetic field \(\vec{M}\) is directed along the \(z\)-axis, so the system is considered to be in the Voigt configuration. The structure under investigation exhibits properties of a bi-axial bigyrotropic (gyroelectromagnetic) crystal.

In accordance with the geometry of the problem (see, Fig. 1), the static magnetic field \(\vec{M}\) is directed along the \(z\)-axis, so the system is considered to be in the Voigt configuration. In such configuration the magnetic field vector in the TM mode (that has field components \((E_x, E_y, H_z)\)) is parallel to the external magnetic field \(\vec{M}\), which results in the absence of its interaction with the magnetic subsystem \([9, 10]\). Thus, further dispersion features only of the TE mode (that has field components \((H_x, H_y, E_z)\)) are of interest.

Involving a pair of the curl Maxwell’s equations for time-harmonic fields (a time factor is considered to be in the form \(\exp(-i\omega t)\)) for the wave propagating along the \(x\)-axis in a standard way \([8, 9]\) one can obtain the following relation between frequency and wavenumber \(k_x\):

\[
k_x^2 = \frac{k_0^2\varepsilon_{zz}\mu_{yy}\mu_v}{\mu_{xx}},
\]

whose inversion gives us the dispersion law of the bulk polaritons. Here \(\varepsilon_{zz}, \mu_{xx}, \mu_{yy}\), and \(\mu_v\) are all functions of frequency \(\omega\) (i.e. they are functions of \(k_0 = \omega/c\)), and \(\varepsilon_v = \varepsilon_{xx} + i\varepsilon_{yy}/\varepsilon_{yy}\) and \(\mu_v = \mu_{xx} + i\mu_{yy}/\mu_{yy}\) are introduced as the Voigt relative permittivity and relative permeability, respectively. From (2) also one can conclude that the asymptotic limits of the dispersion curves for the bulk polaritons appear to be for frequencies where \(\mu_{xx} \to 0\).

In order to find the dispersion law of the surface polaritons, corresponding boundary conditions must be imposed to match the components of the field vectors on both sides of the interface between vacuum and the composite structure (we distinguish these media with the index \(i = 1, 2\)). According to Fig. 1 the \(y\)-axis is directed normal to the interface between the media, therefore, the wavevector components, which are responsible for the wave attenuation in both positive and negative directions along the \(y\)-axis, are defined by quantities \(\kappa_i\).

Involving a pair of the divergent Maxwell’s equations and the corresponding boundary conditions the dispersion equation for the surface polaritons at the interface between vacuum and bigyrotropic semi-infinite media is obtained as

\[
\kappa_1\mu_v + \kappa_2\mu_1 + ik_x\mu_1 \frac{\mu_{xy}}{\mu_{yy}} = 0.
\]

Taking into consideration the relations with respect to the attenuation coefficients \(\kappa_1\) and \(\kappa_2\):

\[
k_x^2 - \kappa_1^2 - k_0^2\varepsilon_{1}\mu_1 = 0.
\]
Based on the typical constitutive parameters which are inherent to available materials (e.g., In$_{2−x}$Cr$_x$O$_3$, Cd$_{1−x}$Mn$_x$Te, In$_{2−x}$Cr$_x$O$_3$, FeF$_2$/TlBr) [20–22], the characteristic resonant frequencies of magnetic and semiconductor subsystems as well as the resulting structure period are chosen and fixed, and then the search for an optimal fractions balance $\delta_m$ versus $\delta_s$ ($\delta_m = d_m/L$, $\delta_s = d_s/L$, $\delta_m + \delta_s = 1$) is proceeded via altering the layers’ thicknesses within the period. As the objective function of the optimization problem the real parts of the Voigt relative permittivity $\varepsilon_v$ and relative permeability $\mu_v$ are selected since they completely characterize the properties of waves propagation through a gyrotropic magnetic medium. Three particular cases are of interest: (i) $\varepsilon_v \to 0$; $\mu_v \to 0$; (ii) $\varepsilon_v \to 1$; $\mu_v \to 1$; (iii) $\varepsilon_v \to -1$; $\mu_v \to -1$, as far as these conditions define the asymptotic limits of the polariton branches. The graphical solution of the discussed optimization problem is depicted in Fig. 2 where the resolved configurations are distinguished by the color circles. Remarkably, in all cases $L/\lambda \approx 3 \times 10^2$.

Further we consider the dispersion features of both bulk and surface polaritons which are inherent to the composite structure corresponding to three above specified configurations. In order to find the real solutions related to normal modes, absence of losses in constitutive parameters of the underlying materials is supposed. Solutions obtained from (2) are depicted in Figs. 3–4 with the red dash-dot lines, while the regions of existence of the bulk polaritons are presented by the red colored areas denoted with the abbreviation ‘BP’. There are two separated frequency regions which represent the bulk excitations. One can see that the upper area is bounded by the light lines and its lower limit is restricted by the line at which $\mu_v = 0$. At the same time, the top limit of the bottom area is at the line where $\mu_{xx} = 0$.

Still assuming that there are no losses in the constitutive materials, four solutions of the dispersion equation with respect to $k_x^2$ [3] can be expanded into the bi-quadratic equation with respect to $k_x^2$ [4]:

$$Ak_x^4 + Bk_x^2 + C = 0,$$

(6)

where $A = Y^2 + W^2$, $B = -k_0^2(2VY + \varepsilon_1\mu_1W^2)$, $C = k_0^4V^2$, $V = p - s \varepsilon_1\mu_1(\mu_v/\mu_1)^2$, $Y = u + s[(\mu_{xy}/\mu_{yy})^2 - (\mu_v/\mu_1)^2] - q(\mu_{xy}/\mu_{yy})$, $W = (\mu_v/\mu_1)[2s(\mu_{xy}/\mu_{yy}) - q]$, $p = \varepsilon_{yy}\mu_{yy}\varepsilon_v\mu_v$, $q = \varepsilon_{xy}(\mu_{yy} - \mu_{xx}) - \mu_{xy}(\varepsilon_{yy} - \varepsilon_{xx})$, $s = \varepsilon_{yy}\mu_{yy}\varepsilon_v\mu_v$, $u = \varepsilon_{xx}\mu_{xx} - \varepsilon_{xy}\mu_{xy}$, whose solution is trivial.

From four roots of (6) those must be selected which satisfy the physical conditions, namely, wave attenuation as it propagates, that imposes restrictions on the values of $\kappa_v$ whose real parts must be positive quantities. Furthermore, it is evident that the asymptotic limits of the dispersion curves for the surface polaritons appear to be for frequencies where $A \to 0$.

From the form of dispersion equations for bulk and surface polaritons it is obvious that their spectral features substantially depend on the dispersion characteristics of components of both effective permeability tensor and effective permittivity tensor of the resulting finely-stratified structure. Moreover, since the underlying components of these tensors are all functions of the frequency, external magnetic field strength, layers thicknesses, and physical properties of the materials forming the superlattice, the regions of polaritons existence are determined by the choice of the values of the corresponding quantities. Therefore, in order to bring together magnetic and semiconductor subsystems into a single structure with desired characteristics, a multiparameter optimization problem should be solved.

FIG. 2. (Color online) Two surfaces depict behaviors of the real parts of the Voigt relative permittivity (yellow surface) and relative permeability (blue surface) versus wavenumber in free space and the ratio of the layers’ thicknesses. The green, red and blue circles plotted at the figure bottom are projections of the conditions (i) $\varepsilon_v = \mu_v = 0$, (ii) $\varepsilon_v = \mu_v = +1$, and (iii) $\varepsilon_v = \mu_v = -1$, respectively.

$$k_x^2(\varepsilon_{xx}\mu_{xx} - \varepsilon_{xy}\mu_{xy}) - \kappa_v^2(\varepsilon_{yy}\mu_{yy} - \varepsilon_{xy}\mu_{xy}) - k_0^2\varepsilon_{yy}\mu_{yy}\varepsilon_v\mu_v + ik_x\kappa_v(\mu_{xy}(\varepsilon_{yy} - \varepsilon_{xx}) - \varepsilon_{xy}(\mu_{yy} - \mu_{xx})) = 0,$$

(5)
system $Y$ is a real number, while $W$ is an imaginary number, therefore, $A$ is always a real number, too). As a result the surface polariton branches are restricted by two asymptotic conditions: $Y-iW=0$ and $Y+iW=0$ for branches SP2/SP4 and SP1/SP3, respectively. At these particular conditions $\text{Re}(\kappa_1) \to \infty$ resulting in that the corresponding penetration depths $\tau_i = 1/\text{Re}(\kappa_i)$ become to be zero. Also it should be noted that the branches SP1/SP3 are restricted by the light line $k_0 = -\omega/c$ while their lower limit is restricted by the line at which $\text{Re}(\kappa_1) = 0$ at $\omega_3 = k_3c$ and $\omega_4 = k_4c$, respectively. At the same time, the lower limit of the branches SP2/SP4 are restricted by the light line at which $\text{Re}(\kappa_2) = 0$. At these lower limits the penetration depth of the surface polariton into the appropriate medium becomes to be infinite.

Fig. 3 (a,b) depicts the dependence of the polariton dispersion characteristics on the choice of the structural parameters of the superlattice for both positive and negative $k_x$ directions. Firstly, we consider the superlattice configurations which appear at the reached condition $\varepsilon_v = \mu_v = 0$. The first found filling factors balance is: $\delta_n = 0.081$ and $\delta_s = 0.919$, which is presented in Fig. 3 (a). It is clear that the surface polariton branches possess nonreciprocal nature and exist in different spectral ranges. In the direction of negative $k_x$ the branch SP1 of the surface polariton appears to be in the same range where the bulk polariton exists. Besides, in the direction of positive $k_x$ the regions of existence of the bulk and surface polaritons (branch SP2) do not overlap at all. Note, the branches SP1 and SP2 have different cut-off frequencies. Fig. 3 (b) presents the dispersion curves of the bulk and surface modes for the second found filling factors balance: $\delta_n = 0.396$ and $\delta_s = 0.604$. As in the previous case, the spectrum of the surface polaritons is nonreciprocal, but the frequency ranges of branches of the surface polaritons in positive and negative $k_x$ directions coincide. Moreover, the branches SP3 and SP4 reemerge from the bulk continuum at some finite values of $k_x$ and tend to the asymptotic limits discussed above.

The regions of existence of the surface polaritons drastically change when the superlattice configurations correspond to the reached conditions $\varepsilon_v = \mu_v = \pm 1$, as it is depicted in Fig. 4 (b). The obtained results show that for negative quantities of $\varepsilon_v$, $\mu_v$ the complete coincidence of the regions of existence of the bulk polaritons and the branches of the surface polaritons is found to be in the negative $k_x$ direction (Fig. 4 (a)). Besides, for positive quantities of $\varepsilon_v$, $\mu_v$ the complete coincidence of the mentioned characteristics appear to be in the positive $k_x$ direction (Fig. 4 (c)). Moreover, there are two extreme cases when asymptotic limits of lower BP-branch and SP-branch coincide. These conditions are $Y+iW = \mu_{xx} = 0$ in the negative $k_x$ direction and $Y-iW = \mu_{xx} = 0$ in the positive $k_x$ direction.

To conclude, we have examined dispersion characteristics of the bulk and surface polaritons in the magnetic-semiconductor superlattice which is under an action of
an external static magnetic field in the Voigt geometry. It is observed that in the case when specific conditions related to the superlattice’s constitutive parameters and filling factor are satisfied, the regions of existence of the bulk and surface polaritons overlap, moreover, they can have the same asymptotic limits. We believe that such peculiarities can give great advantages when providing excitation of the surface polaritons via nonlinear coupling.

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