Passive Intermodulation Interference Suppression through Sparse Discrete Fractional Fourier Transform

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Abstract. With the wide application of duplex mode antenna, passive intermodulation (PIM) interference has become a severe problem. And the traditional methods have shortcomings that cannot be overcome. In order to solve this issue, this paper proposes an algorithm to suppress passive intermodulation (PIM) interference based on digital signal processing (DSP). We choose power series model to express PIM signal, which is also called PIM interference model. In the proposed scheme, satellite transmit pilot signal during the pilot period. With the transmitted pilot signal and the received signal, the real-time parameters of PIM interference model can be obtained. Then during the data transmission period, data is transmitted in uplink and downlink separately. The PIM interference can be rebuilt through the downlink transmitted data and the PIM interference model. In this way, the PIM interference is suppressed. We chose chirp signal to be pilot signal for its wide bandwidth, which can cover all frequency in modulated system. In order to obtain the PIM interference model parameters, we use Sparse Discrete Fractional Fourier Transform (SDFrFT) through Pei method due to its low computational complexity. The simulation results show that this algorithm can reconstruct the original signal with a relatively low complexity.

1. Introduction
Augment of satellite systems’ transmit power, number of carriers and bandwidth makes PIM become a severe problem. Once satellite system is affected by PIM, the current state-of-the-art in electronic circuits impels it to turn on spare antenna and disable duplex mode. The generally way to handle this problem is promoting hardware performance in order to reduce nonlinearity, whose cost is extremely expensive.

This motivates researches on PIM suppression based on DSP. [1] presents a general framework for the adaptive feedforward PIM cancellation, but it costs extra hardware, which is impossible due to limitation in satellites resource. Works in [2] propose a method to cancel PIM signal based on DSP. However, this method needs to interrupt communication to estimate the parameters. [3] provides a novel digital PIM suppression algorithm.

In order to reduce algorithm’s computational complexity, we turn to Discrete Fractional Fourier Transform (DFrFT) for it is an efficient way to solve many challenging engineering problems. Among all DFrFT algorithms, Pei’s algorithm has the lowest complexity [4] and its complexity can be further reduced through sparse Fourier transform [5-6]. [7] gives a modified Cauchy distribution model of
high-order PIM signal. However, there is no complete model expression of PIM interference [8], we select power series model to express PIM signal.

In this paper, we propose a digital way to estimate and cancel PIM interference through power series model. We chose chirp signal to be pilot signal for its wide bandwidth, which can cover all frequency in modulated system. Because of the sparse characteristic of chirp signal in fractional Fourier domain, SDFrFT is adopted to estimate parameters of PIM signal.

The rest of this paper is organized as follows. Section 2 details PIM signal reconstruct algorithm. Simulation results are presented in section 3. Finally, conclusions are drawn in Section 4.

2. PIM signal reconstruct algorithm
As shown in figure 1, pilot signal $c_0$ and data $c$ are transmitted in $t_1$ and $t_2$ respectively. During $t_1$, received signal $d_0$ is used to estimate parameters of PIM signal and reconstruct PIM signal $\hat{p}_0$. During $t_2$, estimated signal $\hat{c}$ can be obtained by $d - \hat{p}$. Figure 2 demonstrates this process in detail.

![Figure 1. Architecture of transmitted and received signal.](image1)

![Figure 2. Architecture of overall PIM suppression algorithm.](image2)
Flow chart of the overall algorithm is showed in figure 3, where L is the time of repetitions.

2.1. PIM parameters’ estimation
The PIM signal can be expressed in power series model [8]
\[ y(t) = \sum_{i=1}^{N} a_i x(t)^i \]  
where \( a_i \) represents the amplitudes of different order \( i \in \{1,2,\cdots,N\} \), N is the number of sampling points.

Chirp signal is given by
\[ x(t) = \exp\left[j(2\pi f_0 t + \pi k t^2)\right] \]  
where \( f_0 \) is start frequency of chirp signal and \( k \) represents frequency modulation rate.

Using equation (1) and (2), PIM signal becomes
\[ y(t) = \sum_{i=1}^{N} a_i \exp\left[j(2\pi f_0 t + \pi k t^2)\right] = \sum_{i=1}^{N} y_i(t) \]  
Using Pei sampling method, we obtain
\[ y(n) = y(n\Delta t) \]
\[ X_a(m) = F(m\Delta u) \]

where \( \Delta t \) is the reverse of sampling interval, \( \alpha \) is a real number that represents rotation angle of fractional Fourier transform (FrFT) and \( \Delta u = \frac{2\pi a}{\Delta t(2M+1)} \). Besides, \( M \geq N \) should be satisfied.

Taking linear superposition of FrFT into consideration, we process one argument of \( y_0(n) \) at a time. Assuming the sampling rate is \( f_s \), which means \( \Delta t = 1/f_s \), then \( X_{0a}(m) \) and \( x(n) \) become
\[ x(n) = \exp[j(2\pi f_0/n + k\pi (n/f_s)^2)] \]
\[ X_a(m) = \left(\frac{\sin\alpha - j \cos\alpha}{2M+1}\right)^{1/2} \exp\left(\frac{1}{2} \cot\alpha^2 \Delta u^2\right) \sum_{n=-N}^{N} \exp\left[\frac{1}{2} \cot\alpha^2 (n/f_s)^2 - j \frac{2\pi n \alpha}{2M+1}\right] y_i(n) \]  
\[ = a_i A_{\alpha} \exp\left(\frac{1}{2} \cot\alpha^2 \Delta u^2\right) \sum_{n=-N}^{N} \exp\left(\frac{1}{2} \cot\alpha^2 (n/f_s)^2 - j \frac{2\pi \alpha n \pi}{2M+1}\right) \exp[j(2\pi f_0/n + k\pi (n/f_s)^2)] \]
It is obvious that the maximum value of $|X_\alpha(m)|$ can be achieved when

$$\left( \frac{1}{2} \cot \alpha + \pi k \right) (n/f_s)^2 = 0$$

and

$$\max |X_\alpha(m)| = (2N + 1) |a_i A_\alpha|$$

Then $i$ and $a_i$ is

$$i = -\frac{\cot \alpha}{2\pi k}$$

and

$$a_i = \frac{\max |X_\alpha(m)|}{(2N+1) |A_\alpha|}$$

2.2. Normalization

Sampling time is $T = N/f_s$, and range of time and frequency domain is $[-T/2, T/2] \times [-f_s/2, f_s/2]$ [10]. Adopt time dimension factor $S = (T/f_s)^{1/2}$. New coordinates are defined as $t \rightarrow t/S$ and $f \rightarrow f \times S$, then range of time and frequency domain become $[-(Tf_s)^{1/2}/2, (Tf_s)^{1/2}/2] \times [-{(Tf_s)^{1/2}}/2, (Tf_s)^{1/2}/2]$. Let $\Delta x = (Tf_s)^{1/2}$, then time-frequency distribution of signal after normalization is a circle centered on origin with a radius of $\Delta x$. Therefore, the sampling interval becomes $1/\Delta x$, and we have $N = (\Delta x)^2$.

After normalization, sampling data changes from $x \left( \frac{n}{f_s} \right)$ into $x \left( \frac{n}{\Delta x} \right)$. The DFrFT of sampling data becomes

$$X_\alpha(m) = \left( \frac{\sin \alpha - \cos \alpha}{2M+1} \right)^{1/2} \exp \left( \frac{1}{2} \cot \alpha \Delta u^2 \right) \sum_{n=-N}^{N} \exp \left[ \frac{\cot \alpha n^2}{2(\Delta x)^2} - \frac{2\pi m n}{2M+1} \right] y_i(n)$$

Equation (9) and (11) turn into

$$\cot \alpha + \pi k (n/f_s)^2 = 0$$

$$i = -\frac{\cot \alpha}{2\pi k (\Delta x/f_s)^2}$$

2.3. SDFrFT

Steps of SDFrFT with complexity $O((nklogn)^{1/2} log n)$ [5] detail in figure 4.

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**Figure 4.** Flow chart of SDFrFT algorithm.

The first step, permutation, enables the algorithm to distinguish signals whose spectrums are close to each other. And window function reduces the spectrum leakage. After subsampling and fast Fourier transform (FFT), error estimation takes part through Hash function and penalty function. After L
repetitions, save locations of points that occur more than half of the internal circulation and estimate the median. Multiple the result by chirp signal to get its FrFT.

2.4. Multi-component signal process and PIM signal reconstruction
Setting chirp signal as pilot signal makes PIM signal feature several peaks in fractional domain. Set a threshold value $P_{\text{threshold}}$. In order to get all $|\tilde{F}_a(m)|$ and $\alpha$, we first search the maximum magnitude value $\max|\tilde{F}_a(m)|$ in two-dimension fractional domain. Next, obtain $i$ and $\alpha_i$ from (15) and (12) respectively if $\max|\tilde{F}_a(m)| > P_{\text{threshold}}$. Then, we apply a narrow band filter $G(m)$ to weaken the maximum peak, where $F^\prime_a(m) = \tilde{F}_a(m)G(m)$, and find the next maximum value by steps above until $\max|\tilde{F}_a(m)| < P_{\text{threshold}}$.

3. Simulation
Set $f_e$ as 112MHz/s, $N$ as 8192, $f_0$ as 0.5MHz and $k$ as 17.2GHz. The order of FrFT ranges from 0 to 2 with step of 0.001. The number of sparse spectrum line is set to 1, and the number of loop is 9. The order of two components are 7.3 and 8.9 respectively.

Figure 5 shows the original signal, the signal with PIM interference and the signal after cancellation in time domain. As can be seen from figure 5, although the power of the PIM interference is larger than that of the signal, the signal after cancellation and the original signal are almost superposition.

Figure 6 shows the power spectral density (PSD) of original signal, PIM interference, signal with interference and signal after cancellation separately. Figure 6(c) shows that the original signal is submerged in PIM interference. However, as can be seen from figure 6(d), although there is several sub-peak in PSD of the signal after cancellation, the PSD of the original signal can be identified easily.

![Figure 5. Cancellation results in time domain.](image_url)
Figure 6. PSD of original signal, PIM interference, signal with interference and signal after cancellation.

4. Conclusion
In this paper, we propose a PIM suppression algorithm with low computational complexity. We use SDFrFT to reconstruct PIM signal in power series model. Simulation results show that this algorithm performs well in PIM cancellation.

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