A PHYSICAL MODEL FOR SN 2001ay, A NORMAL, BRIGHT, EXTREMELY SLOW DECLINING TYPE Ia supernova

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ABSTRACT

We present a study of the peculiar Type Ia supernova 2001ay (SN 2001ay). The defining features of its peculiarity are high velocity, broad lines, and a fast rising light curve, combined with the slowest known rate of decline. It is one magnitude dimmer than would be predicted from its observed $\Delta m_{15}$, and shows broad spectral features. We base our analysis on detailed calculations for the explosion, light curves, and spectra. We demonstrate that consistency is key for both validating the models and probing the underlying physics. We show that this SN can be understood within the physics underlying the $\Delta m_{15}$ relation, and in the framework of pulsed delayed detonation models originating from a Chandrasekhar mass, $M_{\text{Ch}}$, white dwarf, but with a progenitor core composed of 80% carbon. We suggest a possible scenario for stellar evolution which leads to such a progenitor. We show that the unusual light curve decline can be understood with the same physics as has been used to understand the $\Delta m_{15}$ relation for normal SNe Ia. The decline relation can be explained by a combination of the temperature dependence of the opacity and excess or deficit of the peak luminosity, $\alpha$, measured relative to the instantaneous rate of radiative decay energy generation. What differentiates SN 2001ay from normal SNe Ia is a higher explosion energy which leads to a shift of the $^{56}$Ni distribution toward higher velocity and $\alpha < 1$. This result is responsible for the fast rise and slow decline. We define a class of SN 2001ay-like SNe Ia, which will show an anti-Phillips relation.

Key word: supernovae: individual: SN 2001ay

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1. INTRODUCTION

While some progress has been made in the understanding of the Type Ia supernova (SN Ia) phenomenon in recent years, there does not yet exist an agreed upon standard model of the supernova explosion that can explain normal SNe Ia (Branch et al. 2005, 2009). For cosmology, the brightness decline relation plays a key role (Phillips 1993; Phillips et al. 1999; Goldhaber et al. 2001). From theory, $\Delta m_{15}$ is well understood: the light curves (LCs) are powered by radioactive decay of $^{56}$Ni (Colgate & McKee 1969). More $^{56}$Ni increases the luminosity and causes the envelopes to be hotter. Higher temperature means higher opacity and, thus, longer diffusion timescales and slower decline rates after maximum light (Höflich et al. 1996; Nugent et al. 1997; Umeda et al. 1999; Kasen et al. 2009). The existence of a $\Delta m_{15}$ relation holds for virtually all scenarios as long as there is an excess amount of stored energy to be released (Höflich et al. 1996). Although the tightness of the relation can be understood within the framework of the single degenerate scenario and spherical models (Höflich et al. 1996, 2002, 2010), it falls apart when taking into account burning instabilities during the deflagration phase (Kasen et al. 2009). This difficulty and the observation of a set of extremely bright SNe Ia may lend support for double degenerate (DD) scenarios (Tutukov & Yungelson 1979; Iben & Tutukov 1984; Webbink 1984) with progenitors well above the Chandrasekhar mass (Howell et al. 2006; Scalzo et al. 2010; Taubenberger et al. 2011; Howell 2011). We note, however, that the inferred brightness depends on a unique relation between the $^{56}$Ni mass $M_{\text{Ni}}$, and the intrinsic color: $B-V$ at maximum light. At least in a few cases, the apparent brightness can be understood within the framework of $M_{\text{Ch}}$ mass white dwarfs (WDs) with intrinsically red color (Quimby et al. 2006).

Additionally, some progress has been made in understanding variations among the groups, with suggestions that some of the spectral diversity is due to metallicity, central density, and asymmetries (Höflich et al. 1998, 2010; Lentz et al. 2000; Maeda et al. 2010; Maund et al. 2010). The nature of the progenitor system is also controversial, with recent work on rates and the delay time distribution favoring the DD scenario (Yungelson & Livio 1998, 2000; Maoz 2008; Ruiter et al. 2009; Maoz et al. 2010, 2011; Mennekens et al. 2010; Ruiter et al. 2011). Nevertheless, theoretical work continues to favor the single degenerate scenario, with some contribution of DD scenario (Höflich & Khokhlov 1996; Saio & Nomoto 1998; Woosley & Weaver 1986; Mochkovitch & Livio 1990; Saio & Nomoto 1985; Shen et al. 2012).

Despite the uncertainties in theory, most of the known SNe Ia obey the brightness decline relation. The LCs are self-similar within $\pm 0.3$ mag (Goldhaber & Perlmutter 1998; Riess et al. 2000).
Deviations of this order can be expected from variations of the progenitor (Höflich et al. 1998; Brachwitz et al. 2000; Thielemann et al. 2003; Seitenzahl et al. 2011). The stretching method works for both the local and high-z samples (Perlmutter et al. 1999; Riess et al. 1998). The self-similarity holds even for supernovae for which super-Chandrasekhar mass progenitors have been suggested.

The subject of this paper, SN 2001ay, shows that nature is even more diverse (Krisciunas et al. 2011). From the redshift of the host galaxy IC4423 (Freedman et al. 2001), the distance modulus $\mu = 35.55 \pm 0.1$ is well known. The reddening in our Galaxy is found to be $E(B-V) = 0.026 \pm 0.006$ mag, and interstellar absorption in Na I suggests low reddening in the host galaxy of about $E(B-V) = 0.072 \pm 0.008$ mag (Krisciunas et al. 2011). Together with the observed magnitude, the intrinsic brightness at maximum light is $M_V = -19.17$ mag with a $B-V$ color of $-0.02$ mag, which is comparable with typical core-normal supernovae.

However, the LC shape is unlike other SNe Ia: its measured decline rate of $\Delta m_B = 0.68$ is slower than any known SNe Ia, combined with a fast rise of some 16 days (Krisciunas et al. 2011). Based on its $\Delta m_{15}$, SN 2001ay should be brighter than observed by about 1 mag. Moreover, the line widths near maximum light put it solidly in the broad-line class of SNe Ia.

We show that SN 2001ay can be understood with the same physics underlying the $\Delta m_{15}$ relation, and in the framework of parameterized pulsating delayed detonation (PDD) models similar to SN 1990N (Khokhlov et al. 1992), but with an unusual progenitor star. We consistently treat the explosion, LCs, and spectra (Höflich 1995). We show that consistent models reproduce the observed LCs and spectra reasonably well. Furthermore, we show that inconsistent calculations lead to spectral features which would lead to rejection of the explosion model. Finally, we summarize our findings and discuss possible implications for the understanding of SNe Ia and cosmology.

2. MOTIVATION FOR OUR MODEL FOR SN 2001ay

For our models, an understanding of the brightness decline relation is important. The Phillips relation (Phillips 1993; Phillips et al. 1999) provides an empirical link between peak brightness and decline of the LC after maximum. Most SNe Ia fall within 0.2 mag of this relation.

As discussed in the Introduction, the $\Delta m_{15}$ relation can be understood as an opacity effect if energy in stored energy is available in excess of the instantaneous radioactive decay (Höflich et al. 1996; Nugent et al. 1997; Umeda et al. 1999; Kasen et al. 2009). The latter is a key to understand SN 2001ay. We will discuss that while the second condition is valid in most scenarios for SNe Ia, it is not guaranteed.

For our discussion, a useful quantity is the relation between instantaneous energy deposition by radioactive decay and the brightness at maximum:

$$L_{\text{bol}}(t_{\text{max}}) = \alpha S(t_{\text{max}}),$$

where $\alpha$ accounts for the fact that some of the $\gamma$-ray energy deposited prior to maximum light can be stored in the thermal energy and trapped radiation energy which cannot escape faster than a diffusion time. In a typical delayed detonation model the value of $\alpha$ is about 1.2 (Khokhlov et al. 1993; Höflich & Khokhlov 1996).

The role of the opacity condition can be seen from Arnett’s analytic solution (Arnett 1980, 1982), which also provides further insights. Constant opacity in a polytropic model with homogeneous energy input does not produce a brightness decline relation. In this model, energy balance between radioactive decay and adiabatic expansion and cooling cancel exactly in a radiation dominated regime with pure geometrical dilution. At maximum, the ratio between maximum brightness and instantaneous energy input is $\alpha = 1$. Pinto & Eastman (2000) reconsidered Arnett’s one-zone models and confirmed previous explanations. Declining opacity provide extra energy at maximum light because it accelerates the recession of the “photosphere” (Höflich et al. 1993) and results in a positive brightness decline relation in this case.

However, even an effective opacity declining with temperature may result in an anti-$\Delta m_{15}$ relation, because $\Delta m_{15}$ depends both on the opacity effect and the excess energy available. For example, the pure detonation model DET1 has an $\alpha \approx 0.73$ and $\Delta m_{15}$ of 1.37 mag. In contrast, another pure detonation model DET2 has an $\alpha \approx 1.2$ and $\Delta m_{15} \approx 1.7$ (Khokhlov et al. 1993; Höflich & Khokhlov 1996). In DET1, the high central density leads to significant electron capture in the central region producing a $^{56}$Ni free core. In contrast, lower central densities in DET2 produce $^{56}$Ni in the center, and less $^{56}$Ni in the outer region. In both models, the density structure is close to similar polytropes but the $^{56}$Ni distribution is shifted outward and inward, respectively. The differences in $\alpha$ can be understood in terms of the deviation from Arnett’s one-zone model with a flat $^{56}$Ni distribution in mass. In DET1, this distribution is shifted outward leading to an increased escape probability of $\gamma$-rays and greater expansion work which leads to reduced $\alpha$. In DET2, the shift of the $^{56}$Ni distribution inward leads to a larger value of $\alpha$.

From the above discussion, it is obvious that a narrow $\Delta m_{15}$ relation requires similar abundance patterns and $^{56}$Ni distributions for a given $M_{\text{Ni}}$. The theoretical relation $\Delta m_{15}(M_{\text{Ni}})$ depends on the explosion scenario. Self-similarity in the LCs requires a self-similar transformation between models of differing brightness. Within the delayed-detonation scenario, $M_{\text{Ni}}$ depends mostly on the pre-expansion during the deflagration phase, since if the density is too high neutron-rich iron-group elements will be produced. However, as the nickel mass drops out of the standard 0.5–0.6 $M_\odot$ range, the $^{56}$Ni distribution shifts inward with decreasing $M_{\text{Ni}}$. This produces a $\Delta m_{15}(M_{\text{Ni}})$ relation which is in agreement with observations (Höflich et al. 2002). For core-normal SNe Ia, the temperature and, with it the opacity, remains high well after maximum light, leading to a “shallow” $\Delta m_{15}(M_{\text{Ni}})$. Below a certain $M_{\text{Ni}}$, the nickel is very centrally condensed (there is no nickel hole) and the opacity drops rapidly soon after maximum leading to a steep $\Delta m_{15}(M_{\text{Ni}})$ relation. In the case of fast decliners $^{56}$Ni is only produced during the deflagration phase.

3. SCENARIOS FOR SN 2001ay

3.1. Models with an Increased Mass

SN 2001ay was suggested to fall within the class of “Super-Chandra” mass SNe Ia. Obviously, SN 2001ay does not obey the standard $\Delta m_{15}$ relation. Within the framework of the empirical $\Delta m_{15}$ relation, SN 2001ay should be brighter than observed by roughly 1.0 mag which can be ruled out (see Figure 10 of Krisciunas et al. 2011). In fact, compared to the putative super-Chandrasekhar SNe Ia, 2007if (Scalzo et al. 2010; Yuan et al. 2010) and SN 2009dc (Yamanaka et al. 2009; Tanaka et al. 2010; Silverman et al. 2011; Taubenberger et al. 2011) estimates of...
$M_{\text{bol}}$ at maximum light differ by about 1 mag. Estimates for the brightness of SN 2001ay imply 0.5 $M_{\odot}$ (Krisciunas et al. 2011). We need an increase of the diffusion timescales by a factor of two from $\Delta \tau_{15}$ of 1.25 for typical SNe Ia (Phillips et al. 1999) to $\Delta \tau_{15} = 0.68$. The diffusion times scale as, $\tau_{\text{diff}} \propto r^2 \propto M^2$ and, for homologous expansion, $\tau \propto t^{-2}$. If we assume $M_{\text{Ch}}$ for normal SNe Ia, this would imply a progenitor mass of 2 $M_{\odot}$ which is super-Chandra and in fact typical of the mass suggested for mergers, but then one must explain why some super-Chandras are extra bright and SN 2001ay is not.

A stronger argument against a high mass progenitor comes from the rise time to decline ratio. SN 2001ay rises quickly to maximum, whereas an increased timescale would unavoidably produce a slow rise to maximum light.

3.2. The Case for an $M_{\text{Ch}}$ Mass WD with a Faster Expansion Rate

Here, we suggest another scenario outside the “classical” regime for SNe Ia, but within the delayed-detonation scenario of Chandrasekhar mass WD progenitors. To reproduce SN 2001ay, we require a fast rising LC followed by a decline slower than the slope of the instantaneous radioactive decay. We can produce models with $\alpha < 1$. This requires a model with $56\text{Ni}$ shifted to high velocities. Since eventually, the luminosity is given by the rate of instantaneous energy deposition, such a model will show a slow decline by construction.

4. RESULTS

4.1. Explosion Models and Light Curves

We describe how a change of parameters can transform the results from that given by normal LCs to that of SN 2001ay, within the same physical picture.

Even in the absence of a well agreed upon model for SNe Ia most of the basic observational properties, that is, LCs and spectra, of core-normal SNe Ia can be understood within the framework of thermonuclear explosions of Chandrasekhar mass WDs and, in particular, the delayed-detonation scenario (Khokhlov 1989) which provides a natural explanation for the wide variety and range of the $56\text{Ni}$ production. To first order, the amount of $56\text{Ni}$ produced in the explosion depends on the pre-expansion of the WD during the deflagration phase, in which spherical models can be parameterized conveniently by the density at which the transition between deflagration to detonation occurs. For a recent review of the relation between observational properties and the underlying physical model, see, for example, Höflich (2006).

We base our analysis on a spherical explosion model within the framework of the PDD scenario. Our goal is to construct a model with reduced $\alpha$ and short diffusion timescales. We seek to obtain our goals by increasing the expansion ratio and shifting of the $56\text{Ni}$ distribution to higher velocity.

4.1.1. Explosion Models

The spherical explosion model has been constructed to allow fits of optical LCs and spectra of SN 2001ay.

Within the DD scenario, the free model parameters are (1) the chemical structure of the exploding WD, (2) its central density, $\rho_c$ at the time of the explosion, (3) the description of the deflagration front, and (4) the layer at which the transition from deflagration to detonation occurs.

As a reference, we started from the delayed detonation model Sp0y2252z22.25, which has been found to be a good starting point for core-normal SNe Ia, with respect to both spectra and LCs (Domínguez et al. 2001; Höflich 2002; Quimby et al. 2006; Höflich et al. 2010). This base model originates from a star with a main-sequence mass of 5 $M_{\odot}$ and solar metallicity. Through accretion, the C+O core of the star has grown close to the Chandrasekhar limit. At the time of the explosion of the WD, its central density is $2.0 \times 10^9$ g cm$^{-3}$ and its mass is close to 1.37 $M_{\odot}$. The transition density $\rho_c$ has been identified as the main factor which determines the $56\text{Ni}$ production and, thus, the brightness of SNe Ia (Höflich 1995; Höflich et al. 1995, 2002; Iwamoto et al. 1999). The transition density $\rho_c$ from deflagration to detonation is $2.5 \times 10^8$ g cm$^{-3}$.

For SN 2001ay, we tuned the parameters. The reference model has been modified as follows. We reduced the central density, $\rho_c$ to $1 \times 10^9$ g cm$^{-3}$, which decreased the potential energy, as always, and thus, increased the radius of the WD by $\approx30\%$ to 2320 km, and we increased the C/O ratio in the former He-burning core ($M < 0.56 M_{\odot}$) to 4. Both modifications increased the explosion energy to $1.69 \times 10^{51}$ erg (1.69 foe), leading to a more rapid expanding envelope, increasing the rate of geometrical dilution, and shifting the $56\text{Ni}$ to higher velocity. These effects are responsible for both the fast rise and slower decline of SN 2001ay.

In classical DD models, the deflagration to detonation transition (DDT) occurs in a WD already bound during the deflagration phase (Khokhlov 1989; Yamaoka et al. 1992). In contrast, in pulsating delayed models, PDD, the WD remains bound after the deflagration phase, and the DDT occurs during pulsation (Khokhlov 1993; Höflich et al. 1995). Other authors have suggested variations on the PDD (Ivanova et al. 1974; Bravo & García-Senz 2006, 2009; Baron et al. 2008; Bravo et al. 2009), but here when we discuss the PDD model, we specifically refer to the model of Khokhlov (1993). Reducing the burning rate during the deflagration phase moves the model from the DD to the PDD regime in which the WD is bound at the end of the deflagration phase. We artificially reduced the Atwood number, which we approximate as a constant independent of density and composition (see the Appendix) from 0.2 to (0.14, 0.12, 0.10) for the series PDD_11a–c which all undergo weak pulsation—producing LCs with rise times between 14 and 16 days with a slow decline from maximum. The best fit to the observations is given by PDD_11b. Its details are described below. While some of the observed trends that we find with the PDD model may be obtained in a three-dimensional DD model, including the effects of rotation, exploring such models is beyond the scope of this work. We mention results from PDD_11a/c below when needed.

The density and abundance structure of PDD_11b is given in Figures 1 and 2. The pulsation leaves a shell of $\approx0.06 M_{\odot}$ of unburned C/O (with C/O $\approx1$, only the region of central helium burning has enhanced C/O), and 0.51 $M_{\odot}$ of $56\text{Ni}$. For PDD models, mixing during the pulsation may occur. In contrast to previous pulsating models (Höflich et al. 1995; Höflich & Khokhlov 1996), we did not assume full mixing during the pulsation, but instead mixing was limited to one scale height from the position of the burning front during the pulsation. The mixed region between burned matter forms the layer where the detonation is born. As a result, the density and chemical structure are similar to spherical DD models and consistent with late time line profiles and SN remnants. We abandoned full mixing in order to preserve the inner, unmixed region as required by late time line profiles and the imaging of the supernova remnant S Andromedae (Höflich et al. 2004; Motohara et al., 2012 July 10).
2006; Fesen et al. 2007; Gerardy et al. 2007; Maeda et al. 2010). Our models leave some unburned C/O. All modifications combined result in broader lines. We note that our model is highly parameterized. First, the deflagration phase depends critically on the initial condition and as discussed above, the problem of how to suppress strong mixing is still unsolved, though it may be related to high magnetic fields (Penney & Höflich 2012). Second, the amount of mixing will depend on Rayleigh–Taylor instabilities during the pulsation and possibly, rotation induced shear instabilities.

### 4.1.2. Light Curves

Let us examine the formation of the LCs and the comparison with SN 2001ay. First, consider the energy input by $\gamma$-rays, positrons, and adiabatic cooling due to expansion. We contrast PDD_11b with our reference model. Both have a similar structure with respect to the chemical layering and, thus, similar opacities. Due to the higher explosion energy, the expansion rate of the inner layers is larger by about 25%–30%. PDD_11b shows an increased energy loss due to expansion work. The opacities for $\gamma$’s depend only on the mass column height, $\tau_M$, and the electron/nucleon ratio which is close to 0.5. $\tau_M$ scales as $v \times t^{-2}$ resulting in a 50% lower optical depth due to the higher velocity. The normalized energy deposition and escape fractions of $\gamma$-rays are shown in Figure 3. The high expansion rate results in strong heating by $\gamma$-rays in the outer region and a higher escape probability for $\gamma$-rays. Between 10 and 25 days, PDD_11b has an escape fraction larger than that of the reference model by about a factor of two.

In Figure 4, we give a comparison between LCs of PDD_11b and the reference model. PDD_11b rises faster and is brighter at early times. The higher escape of $\gamma$-rays and the increased expansion work leads to $\alpha < 1$ and, therefore, a lower peak relative to the rate of instantaneous $\gamma$-ray input. The second ingredient reducing the value of $\alpha$ is the opacity difference between optical photons and $\gamma$-rays. They are 0.1–0.2 and 1/35 cm$^2$ g$^{-1}$, respectively. $\gamma$-rays leak into the center where the energy is trapped without contributing to the LC at maximum (Figure 3).

We therefore have a deficit in luminosity (with respect to the instantaneous $\gamma$-ray deposition rate), and the optical LCs approaches $\dot{E}_\gamma \, \text{from below}.$ The same opacity mechanism responsible for the regular brightness decline relation results in an “anti-Phillips” relation. More $^{56}$Ni and with it, larger opacities will lead to steeper declines.
uniformly shifted by +0.05 dex to better illustrate the variation of the gradients.

(An color version of this figure is available in the online journal.)

Figure 5. \( B \) and \( V \) LCs of SN 2001ay (Krisciunas et al. 2011) in comparison with theory. The comparison between SN 2001ay and PDD_11b in \( B \) (upper) and \( V \) (lower) as a function of time since maximum light in the \( V \) band. The model light curves have been corrected for reddening and redshift. We assume a distance modulus \( m - M \) of 35.55 mag, and following Krisciunas et al. (2011), we take \( A_V = 0.253 \) and \( A_B = 0.353 \). The data include the photometric errors.

(An color version of this figure is available in the online journal.)

4.1.3. Comparison to SN 2001ay

In Figure 5, the theoretical LCs in \( B \) and \( V \) are compared with the observations. The agreement is reasonable, and they meet the brightness limit imposed by the early non-detection in \( R \). The theoretical \( B \) and \( V \) have been corrected for the redshift \( z \) of the host galaxy. As discussed in the Introduction, the distance modulus of the host galaxy is 35.5 \( \pm 0.1 \) mag, and the galactic foreground extinction \( E(B - V) = 0.026 \) mag. Krisciunas et al. (2011) use a reddening of the host galaxy of \( E(B - V) = 0.072 \) mag with an \( R_V \) of 3.1 and 2.4 for the Galaxy and host galaxy, respectively, giving \( A_V = 0.253 \) mag. Using our theoretical color, \( (B - V)_{\text{max}} = 0.0 \) mag, and from an optimized fit, we find the host galaxy reddening \( E(B - V) = 0.02 \) mag with a global \( R_V = 3.1 \) for a total extinction \( A_V = 0.144 \) mag.

4.2. Spectral Analysis

Based on the explosion models, \( \gamma \)-ray transport, and LCs, we analyzed the spectrum of SN 2001ay at maximum light using the NLTE-code PHOENIX (see the Appendix).

We use the density, abundance structure, and \( \gamma \)-ray and positron deposition given by the explosion models and LC calculations. For the reddening and distance modulus, we use the values found from our LC fit. For the redshift, we use \( z = 0.030244 \). Converged models required 256 optical depth points.

For a consistent absolute \( M_V \) magnitude of \(-19.1 \) mag, the resulting synthetic spectrum is compared to the spectrum observed at maximum light in Figure 6. The continuum colors are well reproduced. The synthetic value of \( B - V = -0.06 \) mag, roughly consistent with 0 mag of SN 2001ay. In order to avoid the complications of two dust populations that was assumed by Krisciunas et al. (2011), we have taken \( E(B - V) = 0.096 \) and \( R_V = 3.1 \), so our reddening is a bit “bluer” that was assumed in the LC comparison, but it is well within the 0.1 mag error of the photometry.

The spectrum is dominated by single ionized lines of \( \text{Si} \), \( \text{Si} \), \( \text{Ca} \), \( \text{Fe} \), \( \text{Co} \) as well as blends of doubly ionized species in the blue. The synthetic and observed spectra show good agreement. The Doppler shifts of lines from elements undergoing incomplete oxygen burning include \( \text{Si} \lambda 6355 \), \( \text{Si} \lambda 5972 \) and \( \text{Si} \lambda 5468 \lambda 5654 \), and the \( \text{Ca} \) \( \text{H} + \text{K} \) and the IR triplet are reasonably well reproduced within 10, 20, and 30 \( \AA \), respectively, which corresponds to a velocity shift of 500–1000 km s\(^{-1} \) at the measured velocity of 14.400 km s\(^{-1} \). The strength of the absorption components agrees well. A large number of weak lines form the quasi-continuum in the near IR. The feature due to blends of \( \text{Fe} \text{Co}/ \text{Ni} \) which could possibly be misidentified as due to \( \text{C} \) \( 6580 \) is reasonably strong, but the feature does not agree with the absorption notch identified as \( \text{C} \) by Krisciunas et al. (2011). However, the evidence for \( \text{C} \) claimed in Krisciunas et al. (2011) is stronger than warranted based on the analysis that were done for that paper (R. Thomas 2012, private communication). Some disagreement is evident. The \( \text{Si} \) at about 5000 \( \AA \), blended with other lines, and the \( \text{Si} \lambda 5970 \) line are too weak. In the red, the \( \text{Ca} \) IR triplet is too weak, as is the \( \text{O} \lambda 7773 \) line, which is, however, severely contaminated by Telluric absorption. The lack of \( \text{O} \) is an indication that the outer part of our model is not correct, either in density structure or due to temperature effects.
To probe the importance of consistency and the sensitivity of the spectra, we calculated a series of maximum light spectra which are underluminous and overluminous, with $M_V = (-18.92, -19.23, -19.35, -19.45)$ mag.

At $+0.1$ mag (Figure 7), the $B - V$ color becomes 0.03 mag. The Si $\lambda 6355$ line is well fit and the Ca $\lambda$ IR triplet is relatively well reproduced. In the blue, Co $\lambda$ lines dominate and a Co $\lambda$ line at 7400 Å appears. The feature at 6500 Å is weaker. The Doppler shift of the Si $\lambda$ and S $\lambda$ remains in good agreement because they are formed within the region of incomplete oxygen burning which produced nearly constant abundances due to burning in quasi-statistical equilibrium (QSE) conditions (Figure 2).

At $-0.15$ mag (Figure 8), $B - V$ equals $-0.11$ mag, but is clearly a little bit too blue as can be seen in the red part of the spectrum. However, the Si $\lambda 5970$ feature is too weak and the S $\lambda$ lines are still significantly too weak. The Ca $\lambda$ IR triplet is much too weak. In the blue, Fe $\lambda$ lines are stronger and the fit is somewhat better. This could imply that the model should have had an initial metallicity somewhat higher, which would create more Fe $\lambda$ at the expense of Co $\lambda$ (from radioactive nickel).

At $-0.25$ mag (Figure 9), the spectrum becomes too blue, $B - V = -0.08$ mag, and the UV flux becomes too large. The Si $\lambda$ and S $\lambda$ lines are still significantly too weak and the IR flux is too low. The Ca H+K line is now too weak.

At $-0.35$ mag (Figure 10), the spectrum is much too blue, $B - V = -0.07$ mag and the Ca $\lambda$ features are completely absent. The Si $\lambda$ and S $\lambda$ features are now much too weak.

Figure 11 shows the model fit to the observed Hubble Space Telescope (HST)+Keck spectrum of April 29. The Si $\lambda$ line is very nicely fit, the S $\lambda$ “W” is well fit, although the blue line is a bit stronger in the model than in the observed spectrum. The rest of the blue is reasonably well fit, although the flux is about a factor of two too high in the UV, which may be an indication of the need for a higher metallicity, which would produce more line blanketing and reduce the UV flux. Both the Ca H+K and IR triplet are reasonably well fit, and the Co $\lambda$ + Fe $\lambda$ + Mn $\lambda$ feature at about 7350 Å is a bit too strong. The O $\lambda 7773$ line is much too weak. Figure 12 shows the model fit for a somewhat brighter model $M_V = -19.22$ and $B - V = 0.22$, still a bit too red, but the line features fit similarly well to the redder case.
Figure 12. Same as Figure 11, but \( M_V = -19.22 \) mag and \( B - V = 0.22 \).
(A color version of this figure is available in the online journal.)

Figure 13. Same as Figure 11, but \( M_V = -19.4 \) mag and \( B - V = 0.10 \).
(A color version of this figure is available in the online journal.)

Figure 14. \( \text{Si}^{2+} \lambda 6355 \) feature for 23 days after explosion compared with the observed spectrum obtained on 2001 April 29 (black) with the model shown in Figure 13 (green) and a somewhat hotter model (blue). The blueward shift in the absorption is clearly a temperature effect and is also due to blending.
(A color version of this figure is available in the online journal.)

Spectral analysis requires underlying models which are consistent including the luminosity. Then, however, they allow the study of secondary effects of individual spectral features.

5. DISCUSSION AND CONCLUSION

The peculiar Type Ia SN 2001ay is an important milestone for our understanding of the explosion, LCs, and spectra. Even with its extremely unusual LC shape and spectral features, we showed that it still can be accommodated within the framework of the explosion of a Chandrasekhar mass WD.

Although it does not follow the \( \Delta m_{15} \) relation, the LC can be reproduced within the physics of normal SNe Ia, and in the framework of PDD models. In our models, the key difference is a high, 80% carbon mass fraction, rather than the 15%–20% carbon mass fraction usual for stellar central He burning. The size of the carbon rich core is 0.56 \( M_{\odot} \). The excess of carbon coupled with a lower central density of the initial WD results in an increase of the final kinetic energy by about 40% and, in turn, shifts the distribution of \( ^{56}\text{Ni} \) to larger velocities. These modifications are responsible for both the LC characteristics and the broad spectral features.

The peculiar LC shape is a consequence of the shift of the \( ^{56}\text{Ni} \) distribution in velocity space and higher rate of central expansion. Transport effects of \( \gamma \)-rays become more important and the escape probability is increased. This leads to a fast, early rise and a value of \( \alpha < 1 \), where \( \alpha \) is the ratio between the luminosity and the instantaneous energy generation rate by radioactive decays at maximum. In normal SNe Ia, the \( \Delta m_{15} \) relation can be understood as a consequence of the temperature dependence of the opacity in combination with \( \alpha > 1 \). As discussed in Section 2, more \( ^{56}\text{Ni} \) causes higher luminosities and temperatures. Higher temperatures lead to larger opacities and therefore, a slower drop of the luminosity from the typical value of \( \approx 1.2 \times \) the instantaneous energy input rate at maximum, to the instantaneous energy input rate at later times. Accordingly, for SN 2001ay with \( \alpha \approx 0.55 \), the LC approaches the instantaneous energy input rate from below which explains the unusually slow decline. Within our models and as a corollary, we expect that there exists a sub-class of supernovae which obeys an anti-Phillips relation. As discussed below, this sub-class should be rare.

The unusually broad spectral features can be understood by the overall shift of the overall element pattern to higher
velocities. However, this pattern remains similar to that found in normal SNe Ia (Figure 2). We demonstrated the power and sensitivity of spectral analysis. High sensitivity demands a consistent treatment with the explosion and LC models. Otherwise, even valid models may be rejected because of poor fits. However, since a good overall fit is achieved with a consistent treatment, individual features are a powerful tool to study details of the explosion physics.

We have shown that limiting the mixing during the pulsation to small scales produces a very similar abundance pattern to the one produced in standard DD models which have been shown to reproduce the observables for the majority of SNe Ia. PDD may be a promising mechanism for the DDT.

While our model fits the basic observed features of SN 2001ay, we do not mean to imply that other models are not possible. In particular, a DD model including three-dimensional effects as discussed above with a high central C/O ratio may also be a viable model and there may exist other models that fit the observed trends. Pursuing other explanations is beyond the scope of the present work.

Our model agrees reasonably well with the observations (Figure 5) but the high carbon abundance in the progenitor poses a challenge. The central region of the progenitor originates from central He burning in stars with less than 7–8 $M_\odot$ (Becker & Iben 1980). During the early stages of central He burning, high carbon abundances are produced by $^4$He($\alpha$, $\gamma$)$^{12}$C burning. With time, the helium abundance is reduced in the core. Then, $^{12}$C($\alpha$, $\gamma$)O depletes $^{12}$C to a typical value of 10%–25% (Umeda et al. 1999; Domínguez et al. 2001). The final amount of $^{12}$C/O depends on the mass of the progenitor, and the $^{12}$C($\alpha$, $\gamma$)O rate (Buchmann 1997), and the amount chemical mixing assumed (Castellani et al. 1985; Caputo et al. 1989; Renzini & Fusi Pecci 1988; Domínguez et al. 2001). Increased chemical overshooting (or semiconvection) prolongs the phase of burning under helium depleted conditions, leading to a lower value of C/O. Although chemical overshooting prescriptions vary widely between various groups studying stellar evolution, the final outcome is a C-poor mixture. To reach high central carbon abundances, burning under He-depleted conditions must be avoided (see Straniero et al. 2003 and references therein). Straniero et al. (2003) found that they could increase the central carbon abundance somewhat by increasing mechanical overshooting. As a possible solution, we propose a common envelope phase with very strong mixing induced by a compact secondary such as a brown dwarf or planet. Common envelope evolution is generally assumed to be responsible for the formation of close binaries leading to cataclysmic variables, X-ray binaries, and supersoft-X-ray sources (Livio & Soker 1988; Ricker & Taam 2008). Moreover, close binary systems with planet or brown dwarfs have recently been detected (Neuhäuser & Posselt 2007; Hessman et al. 2011). Unfortunately, there are no systematic studies that allow one to estimate the amount of mixing. Detailed, numerical studies are few, and analytic models are insufficient to quantify the amount of chemical mixing (Meyer & Meyer-Hofmeister 1979). One observational clue may be SN 1987A, which is believed to be the result of a common envelope evolution (Podsiadlowski et al. 1990). Blue progenitors may also be a result of low metallicity (Brunish & Truran 1982; Chieffi et al. 2003), and very few SN 1987A-like events have been found (Pastorello et al. 2012; Taddia et al. 2012). Pastorello et al. (2012) estimate that SN 1987A-like events are seen about 1% of all SNe II; however, this is likely an extreme upper limit to our scenario, since the preponderance of these objects may be just from higher mass compact stars, rather than from systems that have undergone common envelope evolution. Similarly, one can use observations of WDs to provide another estimate. The number of WDs with sub-stellar companions is $\lesssim 0.5\%$ (Farhi et al. 2005), and the number of WDs with debris disks from tidally disrupting minor planets is 1%–3% (Farhi et al. 2009). Therefore, our scenario should be quite rare, representing 0.05%–0.5% of all SNe Ia.

Finally, we want to mention the long list of limitations of our studies. We have suggested the existence of a rare subclass of SNe Ia which should obey an anti-Phillips relation. It is up to future systematic surveys, such as LSST, to find a sufficiently large statistical sample. Note that this sub-class may be hard to separate at the bright end of SNe Ia because the decline rates for both normal and “SN 2001ay-like” supernovae are low and similar. Chemical mixing during the common envelope phase needs to be studied in detail. Our model for SN 2001ay requires a shift of the $^{56}$Ni distribution and a central hole in $^{56}$Ni. Although verified for a number of normal SNe Ia, late time spectra for an SN 2001ay-like are required to confirm our assumptions of little mixing. Detailed time-series of early time spectra may help to probe whether PDDs provide a common mechanism for the transition from deflagration to detonation. Detailed three-dimensional models of the pulsation phase are required to test and quantify possible mixing.

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APPENDIX

BRIEF DESCRIPTION OF NUMERICAL METHODS

A.1. Explosion

We have calculated explosion models and LCs using the one-dimensional radiation-hydro code HYDRA using computational modules for spherical geometry (Höflich 1995, 2003a, 2003b, 2009). We solve the hydrodynamical equations explicitly by the piecewise parabolic method on 910 depth points (Colella & Woodward 1984). Because a simple $\alpha$ network is insufficient
Here, \( \alpha_T \) is the Atwood number, \( L_f \) is the characteristic length scale, and \( \rho^+ \) and \( \rho^- \) are the densities in front of and behind the burning front, respectively. The quantities \( \alpha \) and \( L_f \) are directly taken from the hydrodynamical model at the location of the burning front and we take \( L_f = r_{\text{burn}}(t) \). The transition density is treated as a free parameter. Table 1 gives the yields of the stable elements as well as the total amount of \(^{56}\text{Ni} \) produced in the explosion.

### A.3. Spectra

The calculations were performed using the multi-purpose stellar atmospheres program PHOENIX/1D version 16 (Hauschildt & Baron 1999; Baron & Hauschildt 1998; Hauschildt et al. 1997a, 1997b, 1996). Version 16 incorporates many changes over previous versions used for supernova modeling (Baron et al. 2007, 2006) including many more species in the equation of state (83 versus 40), twice as many atomic lines, and many more species treated in full non-LTE (NLTE) and an improved equation of state. PHOENIX/1D solves the radiative transfer equation along characteristic rays in spherical symmetry including all special relativistic effects. The NLTE rate equations for many ionization states are solved including the effects of ionization due to non-thermal electrons from the \( \gamma \)-rays produced by the radiative decay of \(^{56}\text{Ni} \), which is produced in the supernova explosion. The atoms and ions calculated in NLTE are \( \text{He}^{+}, \text{C}^{+}, \text{O}^{+}, \text{Mg}^{+}, \text{Si}^{+} \), \( \text{O}^{+} \), \( \text{Ne}^{+} \), \( \text{Na}^{+} \), \( \text{Mg}^{++} \), \( \text{Ni}^{++} \), \( \text{Fe}^{++} \), \( \text{Co}^{++} \), and \( \text{Ni}^{++} \). These are all the elements whose features make important contributions to the observed spectral features in SNe Ia. \( \gamma \)-ray deposition was that calculated by the LC.

Each model atom includes primary NLTE transitions, which are used to calculate the level populations and opacity, and weaker secondary LTE transitions which are included in the opacity and implicitly affect the rate equations via their effect on the solution to the transport equation (Hauschildt & Baron 1999). In addition to the NLTE transitions, all other LTE line opacities for atomic species not treated in NLTE are treated with the equivalent two-level atom source function, using a thermalization parameter, \( \alpha = 0.10 \) (Baron et al. 1996). The atmospheres are treated to energy balance in the comoving frame; while we neglect the explicit effects of time dependence in the radiation transport equation, we do implicitly include these effects, via explicitly including \( p \, dV \) work and the rate of gamma-ray deposition in the generalized equation of radiative equilibrium and in the rate equations for the NLTE populations.

The outer boundary condition is the total bolometric luminosity in the observer’s frame. The inner boundary condition is that the flux at the innermost zone (\( v = 700 \text{ km s}^{-1} \)) is given by the diffusion equation. Converged models required 256 optical depth points to correctly obtain the \( \text{Si} \) ii \( \lambda 6355 \) profile.

PHOENIX and HYDRA have been well tested and compared on SNe Ia (Nugent et al. 1995a, 1995b, 1997; Lentz et al. 2001; Baron et al. 2006; Höflich 2002, 2005; Höflich et al. 1998) and, in particular, compared with observed LCs and spectra of SN 1994D and SN 2001ay.

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| Element | Yield \((M_\odot)\) |
|---------|------------------|
| He      | 0.01             |
| C       | 0.021            |
| O       | 0.043            |
| Ne      | \(3 \times 10^{-3}\) |
| Na      | \(7.8 \times 10^{-5}\) |
| Mg      | \(1 \times 10^{-5}\) |
| Si      | 0.182            |
| P       | \(9 \times 10^{-5}\) |
| S       | 0.104            |
| Cl      | \(4.2 \times 10^{-5}\) |
| Ar      | 0.022            |
| K       | \(4.8 \times 10^{-5}\) |
| Ca      | 0.022            |
| Sc      | \(3.1 \times 10^{-7}\) |
| Ti      | \(1.5 \times 10^{-3}\) |
| V       | 0.022            |
| Cr      | 0.106            |
| Mn      | 0.024            |
| Fe      | 0.661            |
| Co      | \(1.1 \times 10^{-3}\) |
| Ni      | 0.146            |
| \(^{56}\text{Ni}\) | 0.515 |

Notes. The yield of the stable elements at time infinity along with the total mass of \(^{56}\text{Ni} \) produced in the explosion.
