Dimension eight operators of the Weinberg type have been shown to give important contributions to CP violating phenomena, such as the electric dipole moment of the neutron. In this note we show how operators related to these (and expected to occur on equal footing) can give rise to time-reversal violating phenomena such as atomic electric dipole moments. We also estimate the induced parity violating phenomena such as small “wrong” parity admixtures in atomic states and find that they are negligible.
Though the discovery of CP violation in the neutral Kaon system is now a revered part of history, the origins of CP violation remain mysterious. The observation of other CP violating phenomena is an important step towards an understanding of these origins. In particular, the electric dipole moment of the neutron (EDMN) has inspired much calculation, as the current experimental limit is now approaching $10^{-26}$ e cm \[1\]. Quark dipole moments are not the only source for this effect. Weinberg has introduced a dimension six operator \[2\]

$$\mathcal{W}^{(6)} = -\frac{1}{3} f^{abc} G^a_{\mu\nu} G^b_{\nu\sigma} \tilde{G}^c_{\sigma\mu}, \tag{1}$$

which can be argued to give an important contribution to the EDMN in some models. In this paper we will be concerned with the dimension eight operators

$$\mathcal{W}^{(8)}_1 = \frac{\kappa_1 \alpha_S \alpha}{M^4} G^a_{\mu\nu} G^a_{\nu\rho} F_{\sigma\rho}, \tag{2}$$

$$\mathcal{W}^{(8)}_2 = \frac{\kappa_2 \alpha_S \alpha}{M^4} \tilde{G}^a_{\mu\nu} \tilde{G}^a_{\nu\rho} F_{\sigma\rho}. \tag{3}$$

The related four gluon operators have been argued to give non-negligible contributions to CP violating phenomena, such as the EDMN \[3\]. The anomalous dimensions of these operators have been calculated \[4\], with the interesting conclusion that at least one of the dimension eight operators is not suppressed by QCD corrections. The operators $\mathcal{W}^{(8)}_{1,2}$ will occur naturally whenever the four gluon operators do. Because $\alpha_S G^a_{\mu\nu} G^a_{\nu\rho}$ and $\alpha_S \tilde{G}^a_{\mu\nu} \tilde{G}^a_{\nu\rho}$ are unrenormalized to one loop \[5\], $\mathcal{W}^{(8)}_{1,2}$ have vanishing anomalous dimension to that order \[6\].

In this note we will show that operators of this form can give rise to observable time-reversal violation in low energy processes, specifically atomic electric dipole moments. Parity violating effects are estimated and shown to be small.

Consider the operator $\mathcal{W}^{(8)}_1$. Through the diagram of Fig. 1, this operator gives rise to a parity and time-reversal violating coupling of the electron to two gluons. In order to relate this to a physical low energy process we must evaluate the gluonic part of the operator in a hadronic state. Noting that the gluonic part of $\mathcal{W}^{(8)}_1$ is equal to the leading part of the trace anomaly, we calculate its expectation value in a nucleon state in the standard way, approximating it by the value at zero momentum transfer \[7\]. We write

$$\langle N \mid tr(G_{\mu\nu}G^{\mu\nu}) \mid N \rangle = -\frac{8 \pi}{b \alpha_S} m_N \langle N \mid \overline{N}N \mid N \rangle, \tag{4}$$
Fig. 1 Diagram containing the long range contribution of $W_1^{(8)}$ to the parity violating electron-nucleon interaction.

where $b = 11 - \frac{2}{3}n_L$, $n_L$ being the number of quarks in the effective low energy theory. Thus $W_1^{(8)}$ gives rise to a parity and time-reversal violating electron nucleon coupling,

$$O_{eeNN} = \frac{32}{\pi^2} \frac{\alpha^2 \kappa_1}{b} \frac{m_N m_e}{M^4} f(t) \bar{e}_i \gamma_5 e \langle N | \gamma_j N \rangle.$$  

(5)

The function $f(t)$ has the asymptotic behaviours

$$f(t) = \frac{1}{16\pi^2} \left\{ \begin{array}{ll}
\frac{1}{4} (-t/m_e^2) \ln(-t/m_e^2), & -t/m_e^2 \to 0^+ \\
\frac{3}{2} \left[ \ln(-t/m_e^2) \right]^2, & -t/m_e^2 \to \infty
\end{array} \right\}. \quad (6)$$

In the limit $t = (p_i^f - p_i^e)^2 \to 0$, we have subtracted simple power law terms which correspond to derivatives of a delta-function interaction in position space. Such terms are cancelled by counterterms in the renormalization of the low-energy effective theory. Note that the operator (3) gives a coherent nuclear effect, which is an important enhancement for heavy nuclei. This should be contrasted with the electron-nucleus coupling induced by nucleon or electron dipole moments; such a coupling would be proportional to the spin of the nucleus (3).

The operator $W_2^{(8)}$ involves a nucleon spin-flip in the non-relativistic limit, which is thus suppressed by $m_e/m_N$ relative to the operator (3). It also gives rise to a $\pi^0\gamma\gamma$
vertex which will induce an electron-nucleon coupling through pion exchange. However, this operator is suppressed by a factor of \( f_\pi / m_N \) relative to the operator (5). Thus we consider only the effect of the operator (5).

In the presence of a static nucleon, the operator of eqn. (5) gives rise to an effective perturbation of the electron Dirac equation of the form

\[
H' = \frac{1}{2m_e} ic V(r) \vec{\sigma} \cdot \vec{p},
\]

where \( c \) is the operator coefficient and \( V(r) \) is determined from \( f(t) \),

\[
V(r) = \frac{3}{16\pi^3} \frac{1}{r^3} \left\{ \begin{array}{ll}
(4rm_e)^{-2}, & rm_e \gtrsim 1 \\
\ln(rm_e), & rm_e \lesssim 1
\end{array} \right\}.
\]

This perturbation will create a mixing between \( s \) and \( p \) atomic levels, similar to the mixing induced by \( P \) violating \( Z^0 \) exchange. In the case of \( Z^0 \) exchange, the structure of the wave function at the nucleus is very important due to the locality of the potential, and the calculation of parity-violating mixing effects is difficult. However, for our long-range potential there is no great subtlety. Of course, treated literally, the \( \frac{1}{r^3} \) perturbation gives a divergent first Born term, but the potential is actually mollified by the nuclear size, \( r_N \approx 1.2 A^{1/3} \) fm.

In order to estimate the effect of this perturbation on atomic states, we calculate the induced atomic electric dipole moment for \(^{55}\text{Cs}\). The perturbation (5) gives rise to a mixing of \( \Psi(6s) \) and \( \Psi(6p) \) states, thus inducing an electric dipole moment. For the induced dipole moment we find

\[
|d_{Cs}| = d_{6s,6p} \alpha^2 \frac{1}{b} \frac{m_e m_N}{2m_e M^4 \Delta E_{6s-6p}} I_O
\]

\[
= (2.9 \times 10^{-25} \text{ e cm}) \kappa_1 \left[ \frac{52 \text{ GeV}}{M} \right]^4,
\]

where \( d_{6s,6p} = 2.9 \times 10^{-8} \text{ e cm} \) is the measured dipole moment, \( \langle 6s | e\vec{r} | 6p \rangle \), and \( I_O \) is the matrix element of \( V(r) \vec{\sigma} \cdot \vec{p} \). We have used \( b = 11 \) and \( \Delta E_{6s-6p} = 11000 \text{ cm}^{-1} \). We find that the matrix element is dominated by the contribution from \( r \simeq r_N \), giving \( I_O \approx 12\sqrt{35}/\pi (Z\alpha m_e/6)^4 [\ln(r_N m_e)]^2 \). The current limits on such a dipole moment are \( d_{Cs} < 7.2 \times 10^{-24} \text{ e cm} \). The implied limit on the operator coefficient is \( \kappa_1^{1/4} \left( \frac{42 \text{ GeV}}{M} \right) < 1 \). This limit is even stronger than it first appears since one typically has \( M^4 \approx \Lambda^2 m_Q^2 \), where \( \Lambda \) is the scale of the new physics, typically a mass in the Higgs sector, and \( m_Q \) is the
mass of the heavy quark which is integrated out in the evaluation of the effective dimension eight operator. Using $m_Q = m_b$ and $\kappa_1^{1/4} = 1$, this translates into a bound $\Lambda > 350$ GeV, which is quite restrictive.

We have also estimated the effect of this perturbation on atomic parity violating phenomena. Note that it is unclear how a practical measurement of this effect might be accomplished, since it cannot be measured in polarization experiments in the way that the $Z$ exchange effect is measured [10]. In any case, the effect is quite small. We choose to calculate the $2s_{1/2} - 2p_{1/2}$ mixing in hydrogen, and we find $|\delta| \simeq 10^{-11}\kappa_1 \left[\frac{1\text{ GeV}}{M}\right]^4$. An estimate of the mixing induced by $Z$-exchange gives $|\delta_Z| \simeq 10^{-11}$, and we see that any observable effect is already ruled out by our previous bound.

While completing this work, we became aware of ref. [11], whose authors, calculating in the context of the minimal supersymmetric standard model, consider the contribution of the operator $W_1^{(8)}$ to atomic parity violation. They use “naive dimensional analysis” to relate $W_1^{(8)}$ to the local operator $\bar{e}i\gamma_5 e \bar{N}N$. As the authors note, this is may not be a good approximation, particularly for large $Z$. Our calculation yields an effect which is larger than argued in ref. [11].

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