Probing Unification With Precision Higgs Physics

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We propose a novel approach of probing grand unification through precise measurements on the Higgs Yukawa couplings at the LHC, which is well motivated by the appearance of effective operators not suppressed by the mass scale of unification $M_U$ in realistic models of unification with minimal Yukawa sector. These operators modify the Higgs Yukawa couplings in correlated patterns at scale $M_U$ that hold up to higher-order corrections. The coherences reveal that, the weak-scale effect on tau Yukawa coupling is the largest among the third generation, which if verified by the future LHC, can serve as a hint of unification.

I. INTRODUCTION

The Higgs Yukawa couplings to the standard model (SM) fermions such as top [1], bottom [2], and tau [3] have been verified at the LHC. With upcoming data at the future LHC, we will enter into an era of precision tests on the Higgs scalar, which may shed light on the fundamental laws underlying the electroweak symmetry breaking. While the minimal version of supersymmetry is not satisfactory such as in explaining the observed Higgs mass, it is still on the short list of frameworks that address some well-known big questions such as the hierarchy problem.

Since supersymmetry is advocated to solve the hierarchy problem, there are limited tools to probe the underlying grand unification theory (GUT), as the unification scale $M_U$ is far larger than the weak scale. Until now, proton decay is the most important tool to detect unification. For a review on unification and proton decay, see ref.[4]. According to the dependence of proton decay lifetime on $M_U$, i.e., $\tau_p \sim 1/M_U^4$, a large amount of models can evade the Super-Kamiokande limits [5, 6] by adjusting the value of $M_U$ in the mass range $10^{15–17}$ GeV. A delay of update on those experimental limits postpones our exploration along this line.

In this Letter, we propose a novel method of probing GUT through precision measurements on the Higgs Yukawa couplings. Similar to high-dimensional operators that lead to proton decay through interactions between the GUT-scale states and the SM fermions, there are analogies which modify the Higgs Yukawa couplings due to interactions between GUT-scale states $\phi$ and the Higgs doublets:

$$W_{\text{eff}} \sim \int d^2 \theta \left[ (y_u' + e_u')Q_i\bar{u}_iH_u + (y_d' + e_d')Q_i\bar{d}_iH_d + (y_e' + e_e')L_i\bar{e}_iH_d + \cdots \right],$$

where $i$ is the generation index, $y_f$ ($f = u, d, e$) refer to the tree-level Higgs couplings, with corrections denoted by $e_i$. What we have neglected in Eq.(1) are higher-order effective operators suppressed by power laws of $1/M_U$. We will show that a) $e_i$ terms are less than unity but not suppressed by $1/M_U$ for

$$e_i \sim \frac{\langle \phi \rangle}{M_U},$$

where $\langle \phi \rangle \sim M_U$ represents the vacuum expectation value (vev) of SM singlet responsible for breaking the GUT gauge group. b) All of $e_i$ terms are always correlated in specific patterns rather than independent parameters. c) The coherences lead to specific patterns of derivations in the Higgs Yukawa couplings from SM predictions, which can be verified by the future LHC. This is the subject of this Letter.

II. MINIMAL YUKAWA SECTOR

Let us briefly review the realistic models of unification with the minimal Yukawa sectors.

For the SU(5) unification, the Higgs fields in the minimal Yukawa sector are composed of a 5, a $\bar{5}$ and a $\overline{45}$. The $\overline{45}$ [7] is added to the Yukawa sector in order to adjust the lepton and down quark Yukawa couplings at the scale $M_U$. Under this Yukawa structure, the Yukawa couplings at scale $M_U$ are given by:

$$y_u = \frac{v_5}{v_4} y_{ij}^u,$$

$$y_d = \frac{v_5}{v_4} y_{ij}^d + \frac{v_8}{v_4} y_{ij}^{d45},$$

$$y_e = \frac{v_5}{v_4} y_{ij}^e - 3 \frac{v_8}{v_4} y_{ij}^{e45},$$

where $v_5$, $v_8$ and $v_4$ refers to the vevs of doublets in 5, 5 and $\overline{45}$, respectively; $v_u = v \sin \beta$, $v_d = v \cos \beta$, with $v \sim 174$ GeV; and $Y_u$, $Y_d$ and $Y_{45}$ are $3 \times 3$ matrixes in
The purpose of $V_1$ is small. For the $SO(10)$ unification, the minimal Yukawa sector [9] is composed of a 10 and an $126$. The purpose of $126$ closely follows from that of 45 in the $SU(5)$. The Yukawa couplings at scale $M_U$ read as,

$$
y_u^i = \frac{v_u^{10}}{v_u} Y_{10}^{ij} + \frac{v_u^{126}}{v_u} Y_{126}^{ij}, \quad y_d^i = \frac{v_d^{10}}{v_d} Y_{10}^{ij} + \frac{v_d^{126}}{v_d} Y_{126}^{ij},$$

$$
y_e^i = \frac{v_e^{10}}{v_e} Y_{10}^{ij} - 3 \frac{\nu_e^{10}}{\nu_e} Y_{126}^{ij}, \quad y_d^i = \frac{v_d}{v_u} Y_{10}^{ij} - 3 \frac{\nu_d}{\nu_u} Y_{126}^{ij},$$

where $v_u^{10}$ and $v_u^{126}$ refers to the vevs of doublets in 10 and $126$, respectively, and $y_e$ is the neutrino Yukawa coupling. Similar to $\overline{45}$, for $m_{126} \sim M_U$ the proton decay due to component fields of $126$ is small.

Fitting the values of Yukawa couplings at scale $M_U$ to their SM values of at the scale $m_Z$ in terms of the renormalization group equations (RGEs), one can fix all of the input parameters in Eq.(3) or Eq.(4).

III. UNSUPPRESSED EFFECTIVE OPERATORS

It is well known that we will obtain the effective operators [11, 12] which contribute to Eq.(1) after one integrates out heavy freedoms with characteristic mass scale $M$. One is also aware of that the ability of testing those effective operators dramatically declines as the value of $M$ increases. In the situation $M \sim M_U$, the effects on SM observables due to those operators are supposed to be tiny (e.g., $\tau_p$), unless they aren’t suppressed by power laws of $1/M$. We will show that there are indeed unsuppressed effective operators in Eq.(1) by integrating out the heavy Higgs field 45 or $\overline{126}$ in the minimal Yukawa structure as discussed in the preceding section.

We show in the left plot of Fig.1 the generation of effective operator

$$\delta W_{\text{eff}} \sim \int d^2 \theta \frac{Y_{45}^{ij}}{m_{45}} \phi(75) \bar{H}(5) \psi_i(5) \psi_j(10),$$

after integrating out the vectorlike Higgs fields 45 in the minimal Yukawa sector\(^1\). Here, $\phi$ is a 75-dimensional Higgs that spontaneous breaks the $SU(5)$ gauge group into the SM gauge group, which is often considered as the most economic solution to the doublet-triplet problem in the literature [13].

In Eq.(5), one finds the coefficients in Eq.(1)

$$\delta W_{\text{eff}} \sim \int d^2 \theta \frac{Y_{45}^{ij}}{m_{45}} \phi(75) \bar{H}(5) \psi_i(5) \psi_j(10),$$

where the overall scale $\epsilon = \langle \phi(75) \rangle / M_U$, with $M_U$ referring to the effective VL mass $m_{45}$. For simplicity, all Yukawa coefficients in the vertexes of the Feynman diagram are absorbed into $\epsilon$. The coherence $\epsilon_d^i \simeq \epsilon^i$ in Eq.(6) is a result of the GUT representation, which is independent of GUT-scale parameters such as $\epsilon$, $Y_{45}$ and the ratio of two vevs. This coherence can be a key to reveal the $SU(5)$ unification through the precision measurements on relevant Higgs Yukawa couplings.

Similarly, we can analysis the Feynman diagrams in the

\(^1\) In this minimal Yukawa sector, fine tuning is required in order to keep the Higgs doublets light.
FIG. 2. Deviations of Yukawa couplings $y_t$ (dotted blue), $y_b$ (dotted green) and $y_{\tau}$ (dotted red) relative to their SM expectation values in the case of type-II corrections $\epsilon = \epsilon_t/r = \epsilon_b = \epsilon_{\tau}$ (left) and the type-III corrections $\epsilon = -\epsilon_t/(r \cdot s) = \epsilon_b = -\epsilon_{\tau}/3$ (right), respectively. The vertical thick and dashed lines refer to LHC limits [25] with luminosity $300 \text{ fb}^{-1}$ and $3000 \text{ fb}^{-1}$, respectively, where the references of colors are the same as the points. We have chosen the SM expectation values at the scale $m_Z$ in ref.[14], $M_U = 2 \times 10^{16} \text{ GeV}$, and $\tan\beta = 10$. Deviations larger than 10% from the SM expectation values are not shown, some of which have been already disfavored by the LHC data [1–3]. See text for details.

middle and right plots of Fig.1 respective to the SO(10),

(II) : $\delta W_{\text{eff}} \sim \int d^2 \theta \frac{Y_{ij}^{126}}{m_{126}} \phi(210) H(10) \psi_i(16) \psi_j(16),$

(III) : $\delta W_{\text{eff}} \sim \int d^2 \theta \frac{Y_{ij}^{126}}{m_{126}} \phi(210) \bar{H}(126) \psi_i(16) \psi_j(16),$

where the vectorlike Higgs is 126 instead of 45, $\phi$ is 210 instead of 75, and a 16 supermultiplet represents a whole generation of SM fermions. Unlike in the SU(5), in this minimal Yukawa sector the light Higgs doublets can be dynamically obtained as in the benchmark model studied below. In Eq.(7), one obtains

(II) : $\epsilon_u^i \simeq \epsilon_\nu^i = c Y_{126}^{i} \frac{\psi_{126}}{v_u}, \quad \epsilon_d^i \simeq \epsilon_e^i \simeq c Y_{126}^{i} \frac{\psi_{126}}{v_d};$

(III) : $\epsilon_u^i \simeq \frac{\epsilon_\nu^i}{3} = c Y_{126}^{i} \frac{\psi_{126}}{v_u}, \quad \epsilon_d^i \simeq \frac{\epsilon_e^i}{3} \simeq c Y_{126}^{i} \frac{\psi_{126}}{v_d},$

where $\epsilon = \langle \phi(210) \rangle/M_U$, with $M_U$ referring to the effective VL mass $m_{126}$. Once again, one observes that the coherences for the same generation in Eq.(8) are independent of the GUT-scale parameters such as $\epsilon$ and $Y_{126}$.

IV. PRECISION MEASUREMENTS

Either in Eq.(6) or Eq.(8), the corrections to Higgs Yukawa couplings dominate the next-leading order contributions, as long as the Yukawa sector is minimal and $\epsilon$ is less than unity. Their impacts at the scale $m_Z$ rely on the magnitudes of orders of these corrections. In individual situation, there are small hierarchies among the matrix elements of $Y_{45}^{ij}$ or $Y_{126}^{ij}$ [9], which arise from the SM fermion mass hierarchies. As a result, the largest effects always occur in the third-generation Yukawa couplings $y_{\alpha}$ ($\alpha = t, b, \tau$). Since precision measurements on Yukawa couplings $y_{\alpha}$ are prior to the first two generations at the LHC, we will mainly focus on $y_{\alpha}$ as what follows.

Given an explicit $\epsilon$, the weak-scale effects on $y_{\alpha}$ can be derived as follows. First, one uses the central values [14] of SM observables at the scale $m_Z$ to determine all input parameters at the scale $M_U$ through the RGEs from $m_Z$ to $M_U$. During this process, one has to deal with various intermediate effective theories. Second, we add correlated $\epsilon$-corrections to $y_{\alpha}$ at the scale $M_U$, then perform the RGEs reversely from the scale $M_U$ to $m_Z$, which gives rise to the dependences of $\delta y_{\alpha}$ on $\epsilon$ at the scale $m_Z$. During the RGEs, there are certain uncertain-
ties are similar to those of proton decay.2

In the literature, there is a lack of “complete” fit to the SU(5) model with the minimal Yukawa sector. The “completeness” means that all SM fermion masses and mixings are addressed, with important constraints such as proton decay taken into account. Otherwise, the theoretical uncertainty is too large to invalidate the RGE analysis. For earlier studies on this model, see e.g. refs.[15, 16].

On the contrary, there are extensive studies on the SO(10) model with the minimal Yukawa sector. We will use the latest results in ref.[9], while earlier studies can be found, e.g. in refs.[17–20]. In this benchmark model, the effective theories are composed of the SM between the scale $m_Z$ to the gaugino mass threshold of order 1 TeV, the split-supersymmetry between 1 TeV and the fermion mass threshold of order $10^2$ TeV, complete MSSM from $10^2$ TeV to the right-hand neutrino mass threshold $m_{
u R}$, and finally the MSSM with $U_{B-L}$ between the scale $m_{
u R}$ and $M_U$.

Compared to the SU(5), there are more intermediate RGE steps in the SO(10). The main reasons for this are that the SO(10) has larger structure of gauge group than the SU(5), and the fit to SM neutrino observables also plays a role in affecting the RG trajectory. In this situation, the uncertainties include the explicit values of $M_U$, $m_{
u R}$ and intermediate mass threshold corrections. Since $M_U$ and $m_{
u R}$ are restricted to narrow mass ranges $10^{16–17}$ GeV and $10^{12–13}$ GeV respectively, and $\delta y_\alpha$ are logarithm dependent on them, one expects a theoretical uncertainty to $\delta y_\alpha$ at most of order $\sim 1 – 3\%$.

The left plot in Fig.2 shows the values of $\delta y_\alpha$ in the case of type-II correction in Eq.(8), with $r = (v_u^{10}/v_u) \cdot (v_d^{10}/v_d)^{-1} = 8.73$ [9], where the one-loop RGEs of the SM [21, 22], the split supersymmetry and the MSSM [23] are used. The splitting soft mass spectrum is chosen in order to avoid the constraint from proton decay [24]. In this plot, it is clear that the parameter ranges $|\epsilon| \geq 0.2$ and $0.1 \leq |\epsilon| \leq 0.2$ can be tested through $\delta y_\nu$ and $\delta y_\tau$ by the LHC with luminosity $300$ fb$^{-1}$ and $3000$ fb$^{-1}$, respectively. Compared to $y_\nu$ and $y_\tau$, $y_s$ receives smallest correction but faces largest experimental uncertainties [25, 26].

One can perform similar analysis in the right plot in Fig.2, which shows the values of $\delta y_\alpha$ in the case of type-III correction in Eq.(8), with $s = (v_d^{10}/v_d^{12}) \cdot (v_u^{10}/v_u^{10})$ [9]. Compared to type II, $\epsilon_I$ is the largest input value due to the enhancement factor $r$, $\epsilon_\tau$ is the largest input value in the case of type III. In this case, one expects larger value of $\delta y_\tau$, which indicates that the same LHC limits can reach smaller parameter region $|\epsilon| \sim 0.01 – 0.02$, as shown in the figure. The parameter region $|\epsilon| \geq 0.02$ can be fully covered by the LHC limit with luminosity $300$ fb$^{-1}$. Whenever the corrections to $\delta y_\alpha$ are roughly of same order, the magnitude of $\delta y_\alpha$ at the scale $m_Z$ is always the largest.

While modifying $y_\alpha$, the $\epsilon$-corrections also contribute to off-diagonal elements of $y_{\alpha\beta}$ that lead to flavor violation. They appear even though $\epsilon^2$ is diagonal at the scale $M_U$ because of RGE effects [27]. In the context of type-II two Higgs doublets as we study here, the most stringent constraint in the parameter region with moderate or large tan $\beta$ arises from $\text{BR}(B_s,d \rightarrow \mu^+ \mu^-)$. A partial reason for it is that they are enhanced by the factor tan $\beta$, unlike in the other cases such as $\text{BR}(t \rightarrow u\gamma)$ ($u_i = \{u, c\}$) that are suppressed by $\cos^3(\alpha – \beta)$. Because of the feature above, $\text{BR}(B_{s,d} \rightarrow \mu^+ \mu^-)$ is actually more sensitive to parameters tan $\beta$ and the neutral Higgs boson mass rather than the deviations of a few percent level in the Yukawa couplings in Fig.2. Typically, the $\epsilon$ corrections only yield a deviation of order $\sim 0.13\%$ relative to the SM prediction $\text{BR}(B_s \rightarrow \mu^+ \mu^-)_{\text{SM}} = 3.26 \times 10^{-9}$ for tan $\beta = 10$ and $M_A = m_H = 1$ TeV, which is consistent with the LHCb limits $\text{BR}(B_{s,d} \rightarrow \mu^+ \mu^-)_{\text{exp}} = (2.8^{+0.7}_{-0.6}) \times 10^{-9}$ [28, 29].

V. CONCLUSIONS

Unification is an important theoretical idea of new physics beyond SM. Yet, there are limited ways in testing it except proton decay experiment in the last a few decades. Unfortunately, the advance along this line is delayed due to the experimental status. In this Letter, we proposed the novel approach of probing unification through precision measurements on the Higgs Yukawa couplings especially of the third generation. Our analysis shows that a large deviation in $y_\tau$ but small in $y_t$ and $y_\nu$, if favored by the future LHC data, can serve as a smoking gun of any realistic model of unification with the minimal Yukawa sector.

Our results are supported by three observations. The first observation is the appearance of unsuppressed effective operators through integrating out the heavy Higgs freedom $\Phi$ or $\Phi^*$ in any realistic model with the minimal Yukawa sector. These operators dominate the next-leading-order corrections to Yukawa couplings. The second observation is that the corrections to $y_\alpha$ at the scale $M_U$ are in three specific patterns, which are results of GUT representations. Lastly, the deviations to $y_\alpha$ at the scale $m_Z$ can be verified by the future LHC limits (see Fig.2), although there are subject to certain uncertain-

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2 Compared to proton decay, the theoretical uncertainties in our approach are improved in the sense that $\delta y_\alpha$ are only logarithm rather than power-law dependent on mass scales such as $M_U$.\[...\]
ties in an explicit RG trajectory between the scales $m_Z$ and $M_U$.

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[1] A. M. Sirunyan et al. [CMS Collaboration], “Observation of $t\bar{t}H$ production,” Phys. Rev. Lett. 120, no. 23, 231801 (2018), [arXiv:1804.02610 [hep-ex]].
[2] A. M. Sirunyan et al. [CMS Collaboration], “Evidence for the Higgs boson decay to a bottom quark-antiquark pair,” Phys. Lett. B 780, 501 (2018), [arXiv:1709.07497 [hep-ex]].
[3] G. Aad et al. [ATLAS Collaboration], “Evidence for the Higgs boson Yukawa coupling to tau leptons with the ATLAS detector,” JHEP 1504, 117 (2015), [arXiv:1501.04943 [hep-ex]].
[4] P. Langacker, “Grand Unified Theories and Proton Decay,” Phys. Rept. 72, 185 (1981).
[5] H. Nishino et al. [Super-Kamiokande Collaboration], “Search for Proton Decay via $p \rightarrow \bar{\nu} e^+ \pi^0$ and $p \rightarrow \bar{\nu} \mu^+ \pi^0$ in a Large Water Cherenkov Detector,” Phys. Rev. Lett. 102, 141801 (2009), [arXiv:0903.0676 [hep-ex]].
[6] Y. Hayato et al. [Super-Kamiokande Collaboration], “Search for proton decay through $p \rightarrow \bar{\nu} \pi^+$ anti-neutrino K+ in a large water Cherenkov detector,” Phys. Rev. Lett. 83, 1529 (1999), [hep-ex/9904020].
[7] H. Georgi and C. Jarlskog, “A New Lepton - Quark Mass Relation in a Unified Theory,” Phys. Lett. 86B, 297 (1979).
[8] S. Zheng, “Towards realistic SUSY grand unification for extended MSSM models,” Phys. Rev. D 99, no. 7, 075033 (2019), [arXiv:1809.08724 [hep-ph]].
[9] K. S. Babu, B. Bajc and S. Saad, “Resurrecting Minimal Yukawa Sector of SUSY SO(10),” JHEP 1810, 135 (2018), [arXiv:1805.10631 [hep-ph]].
[10] K. S. Babu and R. N. Mohapatra, “Predictive neutrino spectrum in minimal SO(10) grand unification,” Phys. Rev. Lett. 70, 2845 (1993).
[11] A. Brignole, J. A. Casas, J. R. Espinosa and I. Navarro, “Low scale supersymmetry breaking: Effective description, electroweak breaking and phenomenology,” Nucl. Phys. B 666, 105 (2003), [hep-ph/0301121].
[12] M. Dine, N. Seiberg and S. Thomas, “Higgs physics as a window beyond the MSSM (BMSSM),” Phys. Rev. D 76, 095004 (2007), [arXiv:0707.0005 [hep-ph]].
[13] G. Altarelli, F. Feruglio and I. Masina, “From minimal to realistic supersymmetric SU(5) grand unification,” JHEP 0011, 040 (2000), [hep-ph/0007254].
[14] S. Antusch and V. Maurer, “Running quark and lepton parameters at various scales,” JHEP 1311, 115 (2013), [arXiv:1306.6879 [hep-ph]].
[15] M. Fukugita, M. Tanimoto and T. Yanagida, “Embedding phenomenological quark - lepton mass matrices into SU(5) gauge models,” Phys. Rev. D 59, 113016 (1999), [hep-ph/9809554].
[16] P. Fileviez Perez, Phys. Lett. B 654, 189 (2007), [hep-ph/0702287].
[17] B. Bajc, G. Senjanovic and F. Vissani, “b - tau unification and large atmospheric mixing: A Case for non-canonical seesaw,” Phys. Rev. Lett. 90, 051802 (2003).
[18] T. Fukuyama, A. Ilakovac, T. Kikuchi, S. Meljanac and N. Okada, “SO(10) group theory for the unified model building,” J. Math. Phys. 46, 033505 (2005), [hep-ph/0405300].
[19] A. S. Joshipura and K. M. Patel, “Fermon Masses in SO(10) Models,” Phys. Rev. D 83, 095002 (2011), [arXiv:1102.5148 [hep-ph]].
[20] A. Dueck and W. Rodejohann, “Fits to SO(10) Grand Unified Models,” JHEP 1309, 024 (2013).
[21] M. E. Machacek and M. T. Vaughn, “Two Loop Renormalization Group Equations in a General Quantum Field Theory. 1. Wave Function Renormalization,” Nucl. Phys. B 222, 83 (1983).
[22] M. E. Machacek and M. T. Vaughn, “Two Loop Renormalization Group Equations in a General Quantum Field Theory. 2. Yukawa Couplings,” Nucl. Phys. B 236, 221 (1984).
[23] V. D. Barger, M. S. Berger and P. Ohmann, “Super-symmetric grand unified theories: Two loop evolution of gauge and Yukawa couplings,” Phys. Rev. D 47, 1093 (1993), [hep-ph/9209232].
[24] H. Murayama and A. Pierce, “Not even decoupling can save minimal supersymmetric SU(5),” Phys. Rev. D 65, 055009 (2002).
[25] S. Dawson et al., “Working Group Report: Higgs Boson,” [arXiv:1310.8361 [hep-ex]].
[26] ATLAS and CMS Collaborations [ATLAS and CMS Collaborations], “Report on the Physics at the HL-LHC and Perspectives for the HE-LHC,” [arXiv:1902.10229 [hep-ex]].
[27] S. Gori, H. E. Haber and E. Santos, “High scale flavor alignment in two-Higgs doublet models and its phenomenology,” JHEP 1706, 110 (2017), [arXiv:1703.05873 [hep-ph]].
[28] V. Khachatryan et al. [CMS and LHCb Collaborations], “Observation of the rare $B^0_s \rightarrow \mu^+ \mu^-$ decay from the combined analysis of CMS and LHCb data,” Nature 522, 68 (2015), [arXiv:1411.4413 [hep-ex]].
[29] R. Aaij et al. [LHCb Collaboration], “Measurement of the $B^0_s \rightarrow \mu^+ \mu^-$ branching fraction and effective lifetime and search for $B^0 \rightarrow \mu^+ \mu^-$ decays,” Phys. Rev. Lett. 118, no. 19, 191801 (2017), [arXiv:1703.05747 [hep-ex]].