Constraining axionlike particles using the distance-duality relation

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One of the fundamental results used in observational cosmology is the distance duality relation (DDR), which relates the luminosity distance, $D_L$, with angular diameter distance, $D_A$, at a given redshift $z$. We employ the observed limits of this relation to constrain the coupling of axion like particles (ALPs) with photons. With our detailed 3D ALP-photon mixing simulation in standard ΛCDM universe and latest DDR limits observed in Holand & Barros (2016) we limit the coupling constant $g_α \leq 6 \times 10^{-13} \text{GeV}^{-1} \left(\frac{\langle B_n \rangle}{10^{-4}\mu G}\right)$ for ALPs of mass $\leq 10^{-15}$ eV. The DDR observations can provide very stringent constraint on ALPs mixing in future. Also any deviation in DDR can be conventionally explained as photons decaying to axions or vice-versa.

I. INTRODUCTION

The many extensions of Standard Model (SM) predict the existence of new light particles which may weakly couple with visible matter and photon. The famous known example is axion or axion like particles (ALPs) which arise naturally as Goldstone bosons in spontaneously broken global symmetries [1–9]. The ALPs are massless if the broken symmetry was exact, however, if it was approximate, the ALPs are light pseudoscalar. These particles, if exist, will potentially show many interesting cosmological as well as astrophysical signatures. Axions are also dark matter candidate. Interestingly, the ultralight axion mixed dark matter has been proposed to resolve the missing satellites problem, the cusp-core problem and the ‘too big to fail’ problem [10]. This solely gives strong motivation to study ultra light axions. The axion coupling with photons modify almost all the cosmological observations, and many such effects have been widely studied in literature [11–33].

The coupling of ALPs with photons in presence of external magnetic field [33,45] has interesting consequences, for example the ALP can convert into a photon and vice versa i.e. the number of photons is not conserved. Also, the mixing introduces polarization in radiations [11,24,26,43]. Due to the weak coupling of axions with photons, the change in intensity and polarization is very small for light from local sources. However, it becomes significant for light coming from far cosmological sources. As a result the radiation from distant cosmological sources has been employed to constrain axion-photon mixing by several authors e.g. from Cosmic Microwave Background Radiation (CMBR) we have strong limits on coupling for a wide range of axion mass [21,33,46]. Furthermore, the axion-photon mixing has been applied to explain several physical puzzling observations, for example to explain large scale polarization alignment in distant quasars [27,47,51], polarization properties of radio galaxies [11,19,52] and the dimming of distant supernovae [16,19,21]. In addition there are various experiments looking for direct and indirect observations of axions [12,22,50,72].

In this work we employ the latest distance duality relation (DDR) limits observed by Holand & Barros [73] to constrain the axion-photon coupling [74,77]. We perform full 3D simulations for ALP-photon mixing in expanding ΛCDM universe, particularly for the galaxy clusters used in Holand & Barros [73] analysis. We compare the simulation results with observational DDR limits and constrain coupling constant $g_α$. The coupling depends on background magnetic field and so are the results. Therefore, we effectively constrain the background magnetic field times the coupling.

The axion-photon coupling framework is almost the same as in references [27,33]. The background magnetic field is simulated in 3D space on a $512^3$ cubic grid, covering the comoving space up to 3.5Gpc ($z = 1$, all sources are within this distance). The origin of background magnetic field is assumed to be primordial [78,82] and it has a power law spectrum in k-space. The background axion density is negligible (zero in simulation) as compared to photon flux emitted by source.

The paper is organized as follows. In Section II we give the estimate of axion-photon mixing on DDR relation. We briefly review the axion-photon mixing in Section III. In Section IV we provide the magnetic field generation and mixing simulation details. We present the observed limits in Section V. Finally, in Section VI we conclude and compare our results with other present limits.

II. METHOD

The reciprocity theorem for null geodesics states that the geometric properties are invariant when we exchange source and observer [83,84]. Using this fundamental argument, Etherington in 1993 [83] pointed out the distance duality relation (DDR), a relation between the lu-

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minority distance $D_L$ and the angular diameter distance $D_A$. The equation relating these two distances follows:

$$D_L = (1 + z)^2 D_A$$  \hfill (1)

The above equation (1) is quite general and true in any general Riemannian spacetime, however, requires that the source and observer are connected by null geodesics and the number of photons remains conserved. In principle the DDR relation can be tested observationally, if one can locate a source with well defined size and intrinsic luminosity. Interestingly, now the observational astronomy is becoming so efficient that by using different astronomical quantities several authors have attempted to test DDR relation. The validity of DDR relation assumes the photons conservation, however, in the presence of intergalactic magnetic field the coupling of photons with axion may potentially convert photons to axions and vice versa –this violates the photon number conversation. The mixing introduces the spread as well as deviation in DDR relation. The observed spread can be used to constrain the axion-photon mixing.

The redshift dependent deviation of DDR relation is expressed as,

$$\frac{D_L}{(1 + z)^2 D_A} = \eta(z),$$  \hfill (2)

we have $\eta(z) = 1$ for strict DDR relation. Holanda et al. [85] parametrized the redshift dependence of $\eta(z)$ in two distinct forms, $\eta(z) = 1 + \eta_0 z$ (P1) and $\eta(z) = 1 + \eta_0 (1 + z)$ (P2) and investigated the $\eta_0$ parameter by employing the luminosity distance $D_L$ measurements from Type Ia supernovae (SNe Ia) and diameter distance $D_A$ from galaxy clusters [86, 87]. Several other authors have also tested the DDR relation using different observations: SNe Ia plus cosmic microwave background (CMB) and barion acoustic oscillations (BAO) [88], SNe Ia plus H(z) data [77, 89, 91], gas mass fraction of galaxy clusters and SNe Ia [92, 93], CMB spectrum [94], gamma-ray burst (GRB) plus H(z) [95], SNe Ia plus BAO [96], gas mass fraction plus H(z) [97], gravitational lensing plus SNe Ia [98], SNe Ia and radio galaxy plus CMB [99]. Most of the above authors obtain no significant deviation in DDR relation, although, roughly the scatter in $\eta_0$ parameter is observed as ±0.1 to ±0.3. Recently, Holand & Barros [73] test the DDR relation with $D_A$ measurements from galaxy clusters [80] plus $D_L$ measurements from SNe Ia. They report $\eta_0 = 0.039 \pm 0.106$ (with parametric form P1) and $\eta_0 = 0.097 \pm 0.152$ (P2) with their method I and $\eta_0 = -0.0 \pm 0.153$ (P1) and $\eta_0 = -0.03 \pm 0.20$ (P2) with their method II. Several other measurements of $\eta_0$ for the form P1, and P2 are also available in literature viz. Li et al. [100] found $\eta_0 = -0.07 \pm 0.19$ (P1) and $\eta_0 = -0.11 \pm 0.26$ (P2), Meng et al. [101] report $\eta_0 = -0.047 \pm 0.178$ (P1) and $\eta_0 = -0.083 \pm 0.246$ (P2).

In axion-photon coupling scenario the photons can convert to axions (and vice-versa) and the number of photons remains no more conserved. The observed flux and the luminosity relation is expressed as:

$$F = \frac{L}{4\pi D_L^2} \Rightarrow D_L \propto \frac{1}{\sqrt{F}}.$$  \hfill (3)

Assuming that the DDR is exact and the scatter is all due to axion-photon mixing, we can write $\eta(z)$ as,

$$\eta(z) = \frac{D_{\text{obs}}}{D_L} = \sqrt{\frac{F}{F_{\text{obs}}}},$$  \hfill (4)

here $D_{\text{obs}}^2$ and $F_{\text{obs}}$ are the observed luminosity distance and flux respectively. Equation (4) for $\eta(z) \to 1$ can be approximated as,

$$\eta(z) - 1 = \frac{D_{\text{obs}} - D_L}{D_L} \approx \frac{1}{2} \frac{F - F_{\text{obs}}}{F_{\text{obs}}}. $$  \hfill (5)

If the redshift $z$ is known precisely then the scatter in $\eta_y$ (times $z$ for P1 and $z/(1 + z)$ for P2) is simply the scatter in $\sqrt{\frac{F - F_{\text{obs}}}{F_{\text{obs}}}}$. Alternatively, the observed $\eta_y$ scatter constrains the flux scatter due to photon-axion mixing or in other words the coupling of axions with photons.

### III. AXION-PHOTON MIXING

#### A. Mixing model

The coupling of electromagnetic radiation with axions or ALPs in presence of external magnetic field in flat expanding universe can be written as [33, 102, 103],

$$S = \int d^4x \sqrt{-g} \left[ -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \frac{1}{4} g_{\phi} F_{\mu\nu} F^{\mu\nu} + \frac{1}{2} (\omega^2 a^{-3}) A_{\mu} A^\mu + \frac{1}{2} a^{-3} \Phi_{\mu\nu} \Phi^{\mu\nu} - \frac{1}{2 m_p^2} \phi^2 \right],$$  \hfill (6)

where $F_{\mu\nu}$ and $\tilde{F}_{\mu\nu}$ are electromagnetic field tensor and dual tensor respectively, $\phi$ represents the pseudoscalar axion field, $g_{\phi}$ the coupling between $\phi$ to electromagnetic field, $\omega_p = \frac{4\pi e^2}{m_e}$ is the plasma frequency (while $n_e$ and $m_e$ are the number density and mass respectively), ‘$a$’ the cosmological scale factor, $A^\mu$ the electromagnetic four-potential and $\Phi_{\mu\nu}$ is the axion mass. The Maxwell’s equations following this action are given in [27]. Assuming the $z$-axis as the photon propagation direction, the mixing equation in expanding Universe is written as,

$$\left( \omega^2 + \partial^2 \right) \left( \frac{A_{||}}{\chi} \right) - M \left( \frac{A_{\perp}}{\chi} \right) = 0,$$  \hfill (7)

where $\omega$ is the radiation frequency and we have replaced $\phi$ by $\chi$. $A_{||} (\pm \frac{\sqrt{3}}{2} E_\perp)$ refers to the component parallel to transverse magnetic field (B_L) and the mixing matrix, $M$, in above equation (7) has the following form,

$$M = \left( \begin{array}{cc} \frac{\omega^2}{\sqrt{3} (a^2 B_L)} \omega & -\frac{2g_{\phi}}{\sqrt{3}} (a^2 B_L) \omega \\ -\frac{2g_{\phi}}{\sqrt{3}} (a^2 B_L) \omega & \frac{2g_{\phi}}{\sqrt{3}} \frac{\omega^2}{m_\phi^2} a^2 \end{array} \right).$$  \hfill (8)
The solution to mixed field equation \[ (17) \] is described in Ref. \[27, 43, 46\], we briefly review the same. The mixing matrix \( M \) is diagonalized by an orthogonal transformation \( OMO^T = M_D \), where

\[
O = \begin{pmatrix}
\cos \theta & -\sin \theta \\
\sin \theta & \cos \theta
\end{pmatrix},
\]

and \( \theta \) is such as \( \tan 2\theta = 2g_\phi \omega a^{-2}(a^2 B_\perp)/\left(\frac{\omega^2}{a} - m_\phi^2 a^2\right) \).

The first two row diagonal term of density matrix are defined as,

\[
\gamma \rightarrow \theta = \frac{\omega^2}{a} - m_\phi^2 a^2.
\]

The eigenvalues, \( \mu_\pm \), of matrix \( M \) are given as,

\[
\mu_\pm = \frac{\omega^2}{a} + m_\phi^2 a^2 \pm \frac{1}{2} \sqrt{\left(\frac{\omega^2}{a} + m_\phi^2 a^2\right)^2 + (2g_\phi \omega B_\perp)^2}.
\]

\[
P(z) = e^{i(\omega + \Delta \lambda)z},
\]

\[
\begin{pmatrix}
1 - \gamma \sin^2 \theta & 0 \\
0 & \gamma \cos \theta \sin \theta \\
\gamma \cos \theta \sin \theta & 0 \\
0 & 1 - \gamma \cos^2 \theta
\end{pmatrix}.
\]

IV. SIMULATION DETAILS

We simulate the axion-photon coupling for sources used in DDR validity tests by Holanda & Barros \[73\] in presence of external magnetic field. De Filippis et al. \[80\] provide the measurements of angular diameter distance (DA) for 25 clusters as a function of redshift (see figure \[1\]). Holanda & Barros \[73\] employ the same for DDR validity test. We fix the initial source position as according to De Filippis et al. sample. To simulate axion-photon mixing we divide 3D space in 512\(^3\) cubic grids, covering the comoving space up to redshift 1, all the sources in De Filippis et al. sample are within this distance. The grid size in our simulation is \( \sim 13 \) Mpc.

To generate background magnetic field we assume it to be primordial \[78, 82, 104\]. The k-space correlation of homogeneous and isotropic magnetic field is expressed as,

\[
\langle b_i(k)b_j^*(q) \rangle = \delta_{k,q} P_{ij}(k) M(k)
\]

where \( b_{i,j}(k) \) are the \( i,j \)th component of the magnetic field in k-space, \( P_{ij}(k) = (\delta_{ij} - k_i k_j/k^2) \) is the projection operator and \( M(k) = Ak^{\mu\nu} \) contains the power-law nature of spectrum while \( n_n \) is power spectral index

\[1\] The real space and k-space field transformations are as following:

\[
B_j(r) = \frac{1}{V(2\pi)^3} \sum b_j(k) e^{ik\cdot r},
\]

\[
b_j(k) = \frac{1}{V} \sum B_j(r) e^{-ik\cdot r}.
\]
and constant $A$ is normalization. We fix spectral index $n_B = -2.37$ in our simulation. The results slightly depends on spectral index, however, the final effect on photon flux and polarization is expected to be small [33]. The normalization factor $A$ is set to fix real space magnetic field $B_i(r)$, such as $\sum_k <B_i(r)B_i(r) >= B_0^2$, where $B_0$ is the strength of the magnetic field in real space averaged over a distance $r$, we set $B_0 = 1$ nG [80, 105] for a comoving scale of 1 Mpc. There is no k-space cutoff on correlation in our formulation and so in real space the correlation have no distance cutoffs. We first generate the k-space magnetic field in each grid according to spectral distribution as in equation (17). In k-space the grids are uncorrelated, for any given $k$, $b_k = 0$ and $b_\theta$, $b_\phi$ are generate independently from a Gaussian distribution [27, 33, 51] as,

$$f(b_\theta(k), b_\phi(k)) = N \exp \left[ -\frac{b_\theta^2(k) + b_\phi^2(k)}{2\theta M(k)} \right] , \quad (20)$$

where $N$ is a normalization factor. We Fourier transform the k-space field and obtain the real space magnetic field in each grid.

Having generated the background magnetic field in each grid, we fix the sources as in De Filippis et al. sample and propagate to us, the observer at central grid. The initial axion density is set to zero, the plasma density $n_e$ is fixed to $10^{-8} a^{-3} \text{cm}^{-3}$ and the radiation wavelength is set to visible (2 eV). We have a fix coordinate (comoving distances) system in our simulation. In order to propagate through each grid, we first rotate the coordinate so that the transverse component of the magnetic field aligns along one of the coordinate axes. We then use equation (12) to calculate the modification due to mixing after propagating the particular grid. We then rotate back to resume our fixed coordinate system. This procedure is repeated for all grids along the line of sight of a source. We have presented a few realizations from our simulation in figure [2]. The fluctuation of photon flux due to ALP-photon mixing is evident in figure.

We average over magnetic field seed by generating 400 random angular positions for each individual source while keeping its radial distance (redshift) fix. Each individual line of sight can be thought as a magnetic field configuration. Averaging over these 400 random angular positions (line of sights) yields an average over magnetic field configurations. In total we simulate 10,000 (25 clusters x400) intensity deviations $(\frac{1}{2} F - F_{\text{obs}})$.

V. RESULTS

We obtain $(\eta(z) - 1)$ from the fractional change in initial density to observed intensity (see equation (5)). The critical scatter observed due to axion-photon mixing for polynomial form P1 and P2 is shown in figure [3] and [4] respectively. We have tuned the coupling constant $g_a = 6 \times 10^{-13} \text{GeV}^{-1}$ with axion mass $10^{-15} \text{GeV}$ to obtain root mean square (RMS) scatter in $(\eta(z) - 1)$ within DDR observed limits as obtained by Holanda & Barros [73]. This is done by performing simulations for discrete values of $g_a$ in intervals of $10^{-15} \text{GeV}^{-1}$, this interval is small, we expect larger uncertainties in $g_a$ due to assumed background magnetic field and plasma density. The mean value of $\eta_a$ is not crucial as various DDR tests in literature give $\eta_a (\propto \eta(z) - 1)$ consistent with zero, namely Li, Wu & Yu [100] find $\eta_a = -0.07 \pm 0.19$ and $\eta_a = -0.11 \pm 0.26$ respectively for P1 and P2.
The principle of our method is the fact that photon conservation mixing from DDR validity observations. The basic principle of ALP-photon mixing and constrained the axion-photon coupling observations. The results obtained in our analysis to zero initial axion density, however, there are processes which can generate some axion flux at the source and those axions can convert to photons during propagation form source to observer. For the time being, with available DDR validity limits the RMS scatter of \( \eta \) parameter constrains the axion-photon coupling \( g_\phi \) limits as a function of axion mass less than \( 10^{-15} \)GeV. This is assuming that there is no systematic and statistical error in Holanda & Barros DDR validity limits, including the scatter from these procedural errors the coupling \( g_\phi \) limits will go even lower.

Finally, we extrapolate the \( g_\phi \) limits as a function of axion mass \( m_\phi \) assuming the cosmological scale factor \( a = 1 \) (static universe). The results are presented in figure 5, our simulation in expanding Universe is shown as a red dot in figure. The extrapolated value of \( g_\phi \) for different axion mass \( m_\phi \) may differ by a factor of 2 or 3 if compared with exact simulation in expanding universe.

VI. CONCLUSION AND DISCUSSION

In this work, we have presented the detailed simulation of ALP-photon mixing and constrained the axion-photon mixing from DDR validity observations. The basic principle of our method is the fact that photon conservation is violated in axion-photon mixing scenario, which is crucial to DDR validity [74][77]. We employ the latest observation of DDR tests and present the limits from the observed scatter in relation. With present DDR validity observations, the mixing limit obtained are competitive and can resolve major problems in galaxy formation [10].

The limits obtained in this work from DDR validity tests are much improved if compared with direct experiments i.e. using the CERN Axion Solar Tele-

Figure 3. \( \eta_0 \) scatter following P1 distribution. The axion mass is set to \( 10^{-15} \)GeV and axion-photon coupling \( g_\phi \) is set to \( 6 \times 10^{-13} \text{GeV}^{-1} \). The RMS scatter is 0.1167 which is almost the Holanda & Barros [73] observed \( \eta_0 \) scatter. We have averaged over magnetic field seed by randomly generating the 400 angular positions for each individual source (25 clusters × 400).

Figure 4. \( \eta_0 \) scatter following P2 distribution. The axion mass and axion-photon coupling values are same as in figure 3. Note that the RMS scatter is 0.1535 which is again almost the Holanda & Barros [73] observed \( \eta_0 \) scatter for P2.

Figure 5. The extrapolated axion-photon mixing limits as a function of axion mass \( m_\phi \) in static Universe (\( a = 1 \)). The strong dip between axion mass \( 10^{-15} - 10^{-14} \) eV is mixing resonance when axion mass becomes comparable to plasma frequency \( \omega_p \). The simulation result in expanding Universe obtained for mass \( 10^{-15} \)GeV is shown as red dot.
scope (CAST) Andriamonje et al. (2007) find $g_\phi < 8.8 \times 10^{-11}\text{GeV}^{-1}$ at 95% CL for $m_\phi \leq 0.02$ eV. The constraint on $g_\phi$ from Galactic Globular Clusters is $g_\phi < 0.66 \times 10^{-10}\text{GeV}^{-1}$ at 95% CL which is two orders of magnitude higher than this work. From X-ray observation the coupling $g_\phi \leq 8.3 \times 10^{-12}\text{GeV}^{-1}$ for ALPs below mass $7 \times 10^{-12}$ eV.

The polarization measurement of ultraviolet photons from active galactic nuclei yields a constraint $g_\phi B \leq 10^{-11}\text{GeV}^{-1}\mu\text{G}$ for ALPs with mass $\lesssim 10^{-15}\text{eV}$ [61]. The polarization measurements from quasars give better constrain on $g_\phi$, Payez et al. [110] found $g_\phi \lesssim 2.5 \times 10^{-13}\text{GeV}^{-1}$ for ultra light ALPs. The CMBR polarization provide even more stringent constrain on coupling and P. Tiwari [33] found $g_\phi \leq 1.6 \times 10^{-13}$ and $3.4 \times 10^{-15}\text{GeV}^{-1}$ $(\frac{mG}{B(Mpc)})$ for ALPs of mass $10^{-10}$ and $10^{-15}$ eV respectively. We conclude that the present DDR validity tests provide a good measure of axion-photon coupling limits. With time the DDR test will immensely improve and will limit the axion photon mixing drastically. Alternatively, the deviation observed in DDR validity test can be conventionally explained as axion-photon mixing, unless we have strong limits on coupling of ALPs with photon from some other observation or rule out the existence of ALPs somehow.

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