Intermittency analysis in diffractive DIS processes

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It is pointed out in this talk that “the colorless objects” in diffractive scattering in the small-$x_B$ region can be probed by measuring the scaled factorial moments of final-state-hadrons and the dependence of their scaling behavior upon the diffractive kinematic variables. The Monte Carlo implementation of RAPGAP and JETSET are discussed as illustrative examples. The results of these model-calculations show in particular that inclusion of the contributions from the photon gluon fusion processes can considerably enlarge the power of the scaled factorial moments. The significant $x_B$-dependence of intermittency index in small-$x_B$ region in this MC analyses shows the hadronic final states depend very much on the lifetime of the exchanged colorless object $c_0^*$. The dependence of the intermittency index upon $x_F$, $Q^2$ and $\beta$ are also discussed.

Recent years have witnessed a remarkably intense experimental and theoretical activity in research of the dynamics of diffractive deep-inelastic electron-proton (ep) scattering (DIS) in low Bjorken-$x$ kinematic range. The measurements at HERA have demonstrated the existence of a distinct class of events in which there is no hadronic energy flow in an interval of pseudo-rapidity adjacent to the proton beam direction. Our present understanding of DIS could be inadequate at low-$x_B$ because additions to the leading order QCD-based partonic picture are likely to be substantial. A natural interpretation of these so-called “large rapidity gap” events is based on the hypothesis that the deep-inelastic scattering process involves the interaction of the virtual boson probe with a colorless component of the proton. Hence there is no chromodynamic radiation in final state immediately adjacent to the direction of the scattered proton or any proton remnant. What is this colorless component originating from the nucleon? The large rapidity gap events discovered in deep inelastic scattering at HERA are usually interpreted in terms of pomeron-exchanged model. Although this seems to work reasonably well phenomenologically, there is yet no satisfactory understanding of the pomeron structure and its interactions mechanism. In this respect, it is helpful to probe the properties of such objects in the non-traditional aspects, in addition to the traditional measures such as rapidity gap distribution, structure function and the averaged cross section and so on. In the present talk we wish to point out that useful information about the exchanged colorless object in diffractive lepton-nucleon scattering process can be extracted by studying scaling behavior and fractality (intermittency) of the final state of the colorless component in proton excited by the virtual boson probe.

The manifestation of fractality (intermittency) in high energy multiparticle production process is the anomalous scaling

$$F_q(\delta x) = F_q(\Delta x) \left( \frac{\Delta x}{\delta x} \right)^{\phi_q}, \text{ as } M \to \infty, \delta x \to 0$$

of $q$-order factorial moments (FM’s) $F_q$, defined as

$$F_q(\delta x) = \frac{1}{M} \sum_{m=1}^{M} \frac{\left\langle n_m(n_m - 1) \ldots (n_m - q + 1) \right\rangle}{\left\langle n_m \right\rangle^q},$$

where $x$ is some phase space variable, e.g. (pseudo-)rapidity, the scale $\delta x = \Delta x/M$ is the bin width for a $M$-partition of the region $\Delta x$ in consideration, $n_m$ is the multiplicity in the $m$th bin. Since the factorial moments can remove the statistic noise around the probability and associated directly with the scaled moments of probabilities, the scaling exponent $\phi_q$ in Eq. (1), called intermittency index, characterizing the strength of dynamical fluctuation, is connected with the anomalous fractal dimenion $d_q$ of rank $q$ of the spatial-temporal evolution of
high energy collisions, \( d_{q} = \phi_{q}/(q - 1) \). A possible cause leading to the power-law of FM's of final particles in high-energy collisions is that the emitting source of final hadrons is self-similar fractal [8]. Furthermore, DIS experiments and the empirical analyses [9] show that the gluon-density in the nucleon in the low-\( x_{B} \) kinematical region is much higher than those for quarks/antiquarks, and it is increasing with decreasing \( x_{B} \) [10]. In this soft gluon system where the nucleons interact with each other in complicated dynamic processes, it has been proposed [8] that the gluons may 'self-organize' into the clusters in the dissipative gluon system by self-organized criticality (SOC) [11], and the colorless component in proton can be regarded as color singlet gluon clusters [11]. The spatial-temporal structure of the SOC-cluster (or BTW-cluster) is self-similar fractal [11]. So it is feasible to study the fractal structure of the colorless component of the proton by measuring the anomalous scaling behaviors of factorial moments of the final state particles originating from the scattering of virtual photon and colorless object in the diffraction lepton-nucleon scattering.

The \( x_{B} \)-dependence of scaling behaviors of FM's, and in particular that of intermittency index \( \phi_{q}(Q^{2}, x_{B}; x_{F}) \) for fixed \( Q^{2} \) (and \( x_{F} \)), plays a distinguished role in such studies. This is because, when the virtual photon \( \gamma^{*} \) originating from the incident lepton is absorbed by the nucleon, its energy-momentum \( q \equiv (q^{0}, \vec{q}) \) is in fact absorbed by a virtual colorless component of the nucleon in the lepton-nucleon diffractive scattering. In a fast moving reference frame, for example the lepton-nucleon center-of-mass frame, where the nucleon's momentum \( \vec{P} \) is large in high-energy collisions, the time interval \( \tau_{\text{int}} \) in which the absorption process takes place (it is known as the lepton-nucleon interaction/collision time) can be estimated by making use of the uncertainty principle. In fact, by calculating \( 1/q^{0} \) in this reference frame we obtain [11]:

\[
\tau_{\text{int}} \sim \frac{4|\vec{P}|}{Q^{2}} \frac{x_{B}}{1 - x_{B}}. \tag{3}
\]

This means, for given \( \vec{P} \) and \( Q^{2} \equiv -q^{2} \), the interaction time \( \tau_{\text{int}} \) is directly proportional to \( x_{B} \) in the low-\( x_{B} \) (\( x_{B} \ll 1 \)) kinematic range. In other words, \( x_{B} \) is a measure of the time-interval in which the absorption of \( \gamma^{*} \) by the space-like virtual colorless object takes place. Hence, by studying the \( x_{B} \)-dependence of the intermittency index \( \phi_{q}(Q^{2}, x_{B}; x_{F}) \), we are not merely probing the anomalous scaling behaviors of the collision process between \( \gamma^{*} \) and the colorless object which we hereafter call \( c_{0}^{*} \). Since this hadronization process of the virtual colorless object \( c_{0}^{*} \) is initiated by the interaction with \( \gamma^{*} \), we are also examining whether/how the hadronization process changes with the interaction time \( \tau_{\text{int}} \). This question is of considerable interest, because a virtual photon \( \gamma^{*} \) can (logically) only be absorbed by virtual systems \( (c_{0}^{*} \gamma^{*}) \) whose lifetimes \( (\tau_{\text{int}}^{*} \gamma^{*}) \) are longer than interactive time \( \tau_{\text{int}} \) (i.e. \( \tau_{\text{int}}^{*} \leq \tau_{\text{int}} \)). That is, the average lifetime \( \langle \tau_{\gamma^{*}} \rangle \) of the \( c_{0}^{*} \gamma^{*} \) system, which can absorb a \( \gamma^{*} \) associated with interaction-time \( \tau_{\text{int}} \), is a function of \( \tau_{\text{int}} \). Hence, from the \( x_{B} \)-dependence of the scaling behavior of FM's, in particular from that of the corresponding \( \phi_{q}(x_{B}, Q^{2}; x_{F}) \)’s, we can in principle find out whether/how the dynamic fluctuation and the fractal structure in the spatial-temporal evolution originating from the exchanged colorless object \( c_{0}^{*} \) depends on its average lifetime \( \langle \tau_{\gamma^{*}} \rangle \) of the \( c_{0}^{*} \gamma^{*} \).

The \( Q^{2} \)-dependence of the scaling behavior of the FM’s is also of considerable interest. This is because, in photon-proton scattering experiments, not only those with real \( (Q^{2} = 0) \) photons but also those with space-like \( (Q^{2} > 0) \) photons where \( Q^{2} \) is not too large \( (\leq 1 \text{GeV}^{2}/c^{2}, \text{say}) \) have very much in common with hadron-hadron collisions. Having in mind that the index of intermittency for hadron-hadron scattering is smaller than that for electron-positron annihilation processes [8], we are led to the following questions: Do we expect to see a stronger \( Q^{2} \)-dependence when we increase \( Q^{2} \) from zero to 10 or 100 GeV\(^{2}/c^{2} \), say? Is this also a way to see whether space-like photons at large \( Q^{2} \) “behave like hadrons” in such interactions?

While waiting for data to perform the above-mentioned analysis, let us now consider the following two phenomenological models as illustrative examples:

(A). If the colorless object \( (c_{0}^{*}) \) is a quark-
antiquark pair (formed by interacting gluons) which exists in the time-interval when the virtual photon $\gamma^*$ is absorbed by the object, we shall see the following: Especially when $Q^2$ is sufficiently large, the incoming $\gamma^*$ (the transverse dimension is expected to be proportional to $1/Q^2$) will hit the quark ($q$) or the antiquark ($\bar{q}$) and make them fly apart symmetrically with respect to the center of mass the $\gamma^* q\bar{q}$ system — similar to the $q\bar{q}$-pair produced in $e^+e^-$-collisions (with respect to the center of mass the $q\bar{q}$ system). That is, in this case, the final-state-hadrons of an event are fragmentation-products of the quark and/or the antiquark, and hence they are expected to show characteristic features similar as those observed in the reaction $e^+e^- \rightarrow$ hadrons at the same c.m.s. energy. It is interesting to see that the very recent inclusive measurements performed at HERA in diffractive electron-proton scattering show the following: Both the scaled longitudinal momentum ($x_F$) distribution and the transverse momentum ($p_{\perp}$) distribution are strikingly symmetric with respect to the center-of-mass of the photon and the struck colorless object; and the general features of these distributions are very much the same as those observed in electron-position collision processes. These facts strongly suggest that a more detailed comparison between these two collision processes would be useful.

For this purpose, we made use of the Monte Carlo (MC) program JETSET. We generated 50,000 MC events, and calculated the second normalized factorial moment in 3-dimensional ($\eta, p_{\perp}, \phi$) phase space at the given $q\bar{q}$-c.m.s energy $\sqrt{s}$, where the pseudorapidity $\eta$, transverse momentum $p_{\perp}$ and the azimuthal angle $\phi$ are defined with respect to the sphericity axis of the event. The usual cumulative variables $X$ translated from $x = (\eta, p_{\perp}, \phi)$, i.e. $X(x) = \int_{x_{\min}}^{x_{\max}} \rho(x) dx / \int_{x_{\min}}^{x_{\max}} \rho(x) dx$, were used to rule out the enhancement of $F_2$ from a non-uniform inclusive spectrum $\rho(x)$ of the final hadrons. The obtained results have been collected in Fig.1a in double logarithmic $F_2$ vs M plots for different $\sqrt{s}$ (or $M_X$, which is the invariant mass of the $\gamma^* c\bar{c}$ system in a corresponding diffractive lepton-nucleon scattering event). It is clear that the higher the invariant mass of $\gamma^* c\bar{c}$ is, the larger the power of FM’s, i.e. the stronger the dynamic fluctuation is. In the very low invariant mass ($\sqrt{s} = 4.5$ GeV, say), the powers of FM’s become less than 0, which can be referred to the constraint of the momentum conservation in the hadronization process.

(B). Based on pomeron-exchanged model, several Monte Carlo generators, such as...
POMPYT, RAPGAP have been set up. These models have been used to describe quite well the HERA data in the global and averaged features, such as the rapidity gap distribution, the diffraction and total cross section and so on\[9]. The theoretic models are usually confronted with incorrigible discrepancy, when the data about the locally nonstatistic fluctuation in small phase space are involved in the comparison with them\[4]. No evidence has shown that this kind of locally dynamic fluctuation could be certainly referred only to the hadronization process but have nothing to do with the initial stage of high energy collisions. In this respect, it is also relevant to see whether the features of scale invariance and fractality in small phase space of the lepton-nucleon diffractive processes, specially their dependence upon the diffractive trajectory is given by Berger et al. and Streng\[2], and Schlein\[2]. Here the kinematic variables \(c \) usually hard' gluons as suggested by Ingelman and when the pomeron is made of 2 'unrealistic (Fig.2b). In both cases a color octet remnant is left at low \(s\) assumption (Fig.2b). It is understandable since the branch number of the parton cascading process is just same as shown in Fig. 2a, the difference between cases (A) and (B) is that the higher-order photon-parton interaction has been taken into account in RAPGAP (see Fig. 2b). It is understandable since the branch number of the parton cascading process in parton shower level is larger when the photon gluon fusion is involved in Fig. 2b, while it is believed generally that\[2] power-law behaviors.

We generated 500,000 RAPGAP events, and divided the whole sample into 10 subsamples according to the invariable mass \(M_X\) of \(\gamma^*c_0\) in the MC event. The scaling behaviors of FM’s for different \(M_X\) intervals are shown in Fig. 1b. The dependence of scaling behaviors of FM’s upon \(M_X\) is similar with the result of JETSET in Fig.1a, i.e. the powers increase for increasing scattering energy of \(\gamma^*c_0\). But it is noticeable that the intermittency index \(\phi_2\) for a given \(M_X\) interval is much larger in RAPGAP than that in JETSET. Having in mind that in Fig. 1a the colorless object is considered as a quark-antiquark pair formed by gluons and the Feymann graph of the diffractive process in this aspect is just same as shown in Fig. 2a, the difference between cases (A) and (B) is that the higher-order photon-parton interaction has been taken into account in RAPGAP (see Fig. 2b). It is understandable since the branch number of the parton cascading process in parton shower level is larger when the photon gluon fusion is involved in Fig. 2b, while it is believed generally that\[2] power-law behaviors.

\[
f_{PT}(t, x_F) = \frac{\beta G(t)}{16\pi}, \tag{4}
\]

given by Berger et al. and Streng\[2], and

\[
\beta G(\beta) = 6\beta(1 - \beta), \tag{5}
\]

when the pomeron is made of 2 'unrealistically hard’ gluons as suggested by Ingelman and Schlein\[3]. Here the kinematic variables \(t, x_F\) and \(\beta\) are defined as \(t = (P - P')^2, x_F = q \cdot (P - P')/(q \cdot P), \) and \(\beta = -q^2/[2q \cdot (P - P')]\), where \(P'\) is the 4-momentum of the final state colorless remnant of nucleon. The pomeron Regge trajectory is given by \(\alpha_P(t) = 1+\epsilon+\alpha't, \epsilon \approx 0.085\) and the slope \(\alpha' = 0.25\) obtained from a fit to data\[9].
in the color-string type of models are referred in large part to the randomly cascading process of parton energy in perturbative phase, so the fractal in the evolution processes with longer cascade branch would be stronger.

In order to see the dependence of dynamic fluctuation upon the other diffractive variables as we argued above, we divided the RAPGAP MC sample into 10 subsamples according to $x_B$ and $Q^2$. The results of scaling behaviors of FM’s are shown in Figs. 3a and b respectively. This example explicitly shows how the proposed method can be used to analyze the feature of the evolution process of the colorless object in diffractive lepton-nucleon scattering processes. The significant $x_B$-dependence of the intermittency index observed in this example not only shows that the hadronic final states depend very much on the average life-time of the exchanged colorless object $c_0^*$ if $c_0^*$ can be indeed considered as a pomeron as simulated in RAPGAP, but also the following: The longer the average lifetime of $c_0^*$ is, the stronger the fractal structure of the space-time evolution processes originating from the $\gamma^* c_0^*$’s interaction will be. The smaller the $Q^2$ becomes, i.e. the larger transverse dimension of virtual $\gamma^*$ is, the more $\gamma^*$ “behaves like a hadron”.

Since $x_F$ may be interpreted as the fraction of the 4-momentum of the proton carried by the exchanged colorless objects, and $\beta$ as the fraction of the 4-momenta of the exchanged objects carried by the parton interacting with the virtual boson, it meaningful to calculate the dependence of scaling behavior upon $x_F$ and $\beta$ (Fig. 3c and d respectively). It seems from RAPGAP that only when the momentum of colorless object is large enough, can scaling behavior of the final state originating from $\gamma^* c_0^*$ be significant; and it is also clear from Fig. 3d that the dynamic fluctuation in the diffractive scattering don’t increase monotonously for increasing the fraction of energy of stricken parton in the pomeron.

In conclusion, we have shown in this note that the colorless object $c_0^*$ in diffractive lepton-nucleon scattering can be probed in a model-independent way, i.e. by performing the scaled factorial moment analyses for the hadronic final states originating from the scattering of virtual boson $\gamma^*$ and the $c_0^*$ on an event-by-event bases. The Monte Carlo implementation of RAPGAP and JETSET have been discussed as illustrative examples. The results of these model-calculations show in particular that inclusion of the contributions from the photon gluon fusion processes can considerably enlarge the power of the scaled factorial moments. The significant $x_B$-dependence of intermittency index in small-$x_B$ region in this MC analyses shows the hadronic final states depend very much on the lifetime of the exchanged colorless object $c_0^*$, i.e. the longer the lifetime of $c_0^*$ is, the stronger the fractal structure of the space-time evolution processes originating from the $\gamma^* c_0^*$’s interaction will be.
Acknowledgments

Author thanks C. Boros, M. Derrick, E. De Wolf, D. Gross, Z. Liang, T. Meng, R. Rittel, K. Schotte and K. Tabelow for helpful discussions, H. Jung for the patient helps in RAPGAP Monte Carlo generator. He also wants to thank Alexander von Humboldt Stiftung for the financial support.

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11. The lifetime $\tau_{\text{int}}$ of virtual photon in the lepton-nucleon center-of-mass frame in DIS can be easily formed in the following simple kinematic exercise: $k^2 \equiv m_e^2 = (k - q)^2 = m_e^2 + q^2 - 2k \cdot q$, where $k \equiv (k^0, \vec{k})$, $k' \equiv (k^0', \vec{k}')$, $q \equiv (q^0, \vec{q})$ and $P \equiv (P^0, \vec{P})$ are the 4-momentum of incident lepton, outgoing lepton, exchanged virtual photon and incident nucleon in lepton-nucleon center-of-mass frame respectively, $m_e$ and $m_p$ are the invariance mass of lepton and nucleon. Taking into account $\vec{k} = -\vec{P}$ and $|\vec{k}| \gg m_e, |\vec{P}| \gg m_p$, then $m_e^2 \approx m_e^2 + q^2 - 2|\vec{P}|q^0 - 2\vec{P} \cdot \vec{q} = m_e^2 + q^2 - 2|\vec{P}|q^0 - 2q^0 \sqrt{P^2 + m_p^2 + 2P \cdot q}$ i.e. $q^0 = \frac{2P \cdot q + q^2}{2\sqrt{|\vec{P}|^2 + m_p^2 + 2|\vec{P}|}} \approx \frac{2P \cdot q - Q^2}{4|\vec{P}|}$. So the lifetime of virtual photon is $\tau_{\text{int}} \sim \frac{1}{q^0} \sim \frac{x_B}{1-x_B} \frac{4|\vec{P}|}{Q^2}$.

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