3–Dimensional Approach to Hot Electroweak Matter for $M_{Higgs} \leq 70$ GeV

M. Gürtler $^a$, E.-M. Ilgenfritz $^{b,*}$, J. Kripfganz $^{c,*}$, H. Perlt $^a$§ and A. Schiller $^a$

$a$Institut für Theoretische Physik, Universität Leipzig, Germany

$b$Institut für Physik, Humboldt-Universität zu Berlin, Germany

$c$Institut für Theoretische Physik, Universität Heidelberg, Germany

We study the electroweak phase transition by lattice simulations of an effective 3-dimensional theory, for a Higgs mass of about 70 GeV. Exploiting a variant of the equal weight criterion of phase equilibrium, we obtain transition temperature, latent heat and surface tension and compare with $M_H \approx 35$ GeV. For the symmetric phase, bound state masses and the static force are determined and compared with results for pure SU(2) theory.

1. Introduction

There is an old belief that the electroweak standard theory possesses a first order phase transition [1]–[10] at some temperature of the order of the $W$ mass. This phase transition has become subject of intensive studies in the last years, in particular its dependence on the mass of the so far elusive Higgs boson. One motivation was the phenomenological interest in baryon asymmetry generation at the electroweak scale. The transition has to be strong enough, both in order to accomplish a sufficient rate of baryon generation during the transition and to prevent the wash–out of baryon number after it is completed. The present quantitative understanding of possible mechanisms as well as experimental lower bounds for the Higgs mass make this unlikely within the minimal standard model.

A second reason was the wish to control the behaviour of perturbative calculations of the effective action. This quantity is the appropriate tool of (non–lattice) thermal quantum field theory for dealing with symmetry breaking. Infrared problems prevent a perturbative evaluation of the free energy in the symmetric phase to higher loop order. However, the true, non–perturbative nature of the symmetric phase will be characterized by massive $W$– and Higgs bound states instead of massless $W$ gauge bosons. A selfconsistent approach to provide masses across the transition, e.g. by gap equations, can improve the ability to calculate perturbatively the symmetric phase. Gauge field condensates are another property of the symmetric phase, expected to lower its free energy density.

An independent method is needed to characterize the electroweak phase transition even within a pure SU(2) gauge–Higgs version of the theory. This model has become a testfield to control the validity of perturbative predictions over a broad range of Higgs masses. At present, one is interested to see whether the first order transition ends somewhere around a Higgs mass $M_H \approx 100$ GeV. Lattice simulations [4]–[10] are not only able to describe both phases starting from first principles but, moreover, make it possible to put both phases into coexistence near the phase equilibrium. Thus one is able to measure directly quantities like latent heat, surface tension, condensates

$^*$Talk given by E.-M. Ilgenfritz

†Supported by the DFG under grant Mu932/1-2

‡Supported by the DFG under grant We1056/2-3

§Supported by the DFG under grant Schi422/2-3
etc. quantifying the strength of the transition.

One approach to lattice calculations of the electroweak transition is based on an effective 3–dimensional Higgs model. It is attractive phenomenologically because it circumvents the problem of putting chiral fermions on the lattice. Due to dimensional reduction, fermions as well as non–static bosonic modes contribute to the effective action. In contrast to QCD, dimensional reduction should work for the electroweak theory around and above the transition temperature because $g^2$ is small. For the electroweak phase transition this approach has been pioneered by Farakos et al. (see e.g. [1,11]). This program aims at exploring the accuracy of dimensional reduction at various Higgs masses by comparing various parameters of the transition with those of 4–dimensional lattice and perturbative approaches. Perturbation theory is necessary to relate the 4–dimensional continuum theory to the parameters of the dimensionally reduced theory and, finally, to the bare coupling parameters of the lattice action. Dimensionally reduced versions retain the remnant of the temporal gauge field $A_0$ (as an adjoint Higgs field) or not (as in this work).

Recently [9] we have presented results obtained with the 3-dimensional lattice model on the phase transition for $M_H \approx 35$ GeV. Here we present some results of numerical work on the more realistic $M_H \approx 70$ GeV and compare them with the case of smaller Higgs mass. As expected the first order nature has become weaker but is still evident. In this talk we put our main emphasis on ways to characterize the phase equilibrium at finite lattice size in order to obtain the infinite volume limit of the transition parameters, and on non–perturbative features of the symmetric phase. The extrapolation to the continuum limit will be dealt with in a forthcoming publication [12].

2. The Model

We study the $SU(2)$–Higgs system with one complex Higgs doublet of variable modulus. The gauge field is represented by the unitary $2 \times 2$ link matrices $U_{x,\mu}$ and the Higgs fields are written as $\Phi_x = \rho_x V_x$ ($\rho_x^2 = \frac{1}{2} Tr(\Phi_x^+ \Phi_x)$) is the Higgs modulus squared, $V_x$ an element of the group $SU(2)$. The lattice action is

$$S = \beta_G \sum_p (1 - \frac{1}{2} Tr U_p) - \beta_H \sum_l \frac{1}{2} Tr(\Phi_{x,\mu}^+ U_{x,\mu} \Phi_{x,\mu}) + \sum_x (\rho_x^2 + \beta_R (\rho_x^2 - 1)^2)$$

(1)

with $\beta_G = 4/(a g_3^2)$. In three dimensions the lattice Higgs self–coupling is $\beta_R = (\lambda_3 / g_3^2) (\beta_H^2 / \beta_G)$, $g_3^2$ and $\lambda_3$ denote the 3–d continuum gauge and Higgs self couplings which are 3–d renormalization group invariants. They are related to the corresponding four dimensional couplings via $g_3^2 = T(g^2 + O(g^4))$ and $\lambda_3 = T(\lambda + O(g^4))$.

String operators like

$$E(l) = \Phi_x^+ U_{x,\mu} U_{x+\mu,\mu} \ldots U_{x+(l-1)\mu,\mu} \Phi_{x+l\mu}$$

(2)

(of extension $l$) are used to form Higgs and $W$–operators. Actually the $3 - d$ masses (inverse correlation lengths) have been obtained from the connected correlators between separated “time slice” (in $2 + 1$ dimensions l) sums of ”spatial” string operators, which project out the proper $SU(2)$ and spin content. The overlap with the lowest mass states is improved choosing an extension $l = 4$ for correlators in the $W$ and also partly in the Higgs channel (there also $\rho_x^2$ is used). A static force is (formally) defined using Wilson loops $W(R, T)$ of asymmetric extensions $2 \leq R \leq T \leq L/2$ ($L^3$ is the lattice size).

The bare lattice coupling parameters $\beta_H$ and $\beta_R$ can be translated into a physical temperature by a formula which generalizes the tree level
relation between the Higgs mass and the bare lattice parameters including perturbative corrections (too complicated to be given here). We consider here the quartic coupling $\lambda_3/g_3^2 = 0.0957$. According to the tree level based relation $\lambda_3/g_3^2 \approx \lambda/g^2 \approx M_H^2/(8 M_W^2)$ this would correspond to the case of $M_H = 70$ GeV. Corrections to this approximate relation depend on details of the dimensional reduction (loop order, the adjoint Higgs field $A_0$ integrated out or not). Taking these corrections into account (4-dimensional renormalization effects are neglected) this coupling ratio corresponds to $M_H = 71.8$ GeV for $g_2 = 4/9$.

For comparison, the lower Higgs mass had been simulated with $\lambda_3/g_3^2 = 0.0239$, corresponding to $M_H = 38.3$ GeV.

The lattice Higgs self coupling $\beta_R$ is not fixed but runs with $\beta_H$. This is important for the multihistogram technique to be used. In general, binning has to be performed in the two relevant parts of the action (corresponding to $\beta_H$ and $\beta_R$) and some other observable. Our data come from typically 100000 configurations per set of couplings (separated by one heat bath step mixed with eight Higgs field reflections). Correlation measurements were separated by 10 iterations.

3. Localization of the phase transition

In order to characterize the order of the phase transition at Higgs mass $M_H \approx 70$ GeV and to obtain the infinite volume critical hopping parameter $\beta_{Hc}^\infty$, we have simulated lattice sizes $30^3$, $48^3$ and $64^3$. To control the approach to the continuum limit we have chosen gauge couplings $\beta_G = 12$ and $\beta_G = 16$ (not presented here). Lattice observables to monitor the phase transition are the Higgs modulus squared $\rho^2$ and the link contribution to the action $E(1)$, but other pure gauge field quantities like the average plaquette or the Polyakov line give 2-state signals, too.

We have used the Ferrenberg–Swendsen analysis and present here only results obtained for histograms of $\rho^2$. The corresponding Binder cumulants $B_{\rho^2}^L$ are shown for the lattice sizes $L = 30$, 48 and 64 in fig. 1. The minima give the respective pseudocritical $\beta_{Hc}^L$. From a linear extrapolation with $1/L^3$ we obtain the infinite volume limit $\beta_{Hc}^\infty = 0.3435433$. For comparison, the maxima of the $\rho^2$ susceptibility give $\beta_{Hc}^\infty = 0.3435430$.

In the case of a SU(2) Higgs model without top–quark this value corresponds to $T_c = 156$ GeV. For comparison, we obtain $T_c = 98.7$ GeV for $M_H \approx 35$ GeV. Finiteness and shrinking of the Binder cumulant with increasing volume are evidence for the first order nature of the transition at the larger Higgs mass, too. The dip of the Binder cumulant reaches 0.6623 in the $V \to \infty$ limit clearly different from 2/3.

![Figure 1. Binder cumulants $B_{\rho^2}^L$ for $L = 30$, 48 and 64 at $M_H \approx 70$ GeV](image-url)

Alternatively we have used and improved the equal weight criterion for the localization of the transition. In order to account for mixing of broken and symmetric phases at some floating $\beta_H$ inside the metastability range we make the superposition ansatz for the histogram:

$$P_m(\rho^2, \beta_H) = w_b P_b(\rho^2, \beta_H) + w_s P_s(\rho^2, \beta_H) + w_{int} P_{int}(\rho^2, \beta_H).$$ (3)
For finite volumes there is also a contribution of inhomogeneous states containing interfaces. All histograms as well as the sum of the appropriate thermodynamic weights \( w_i \) are normalized. The histogram \( P_m(\rho^2, \beta_H) \) is obtained by reweighting towards \( \beta_H \), merging all data by the Ferrenberg–Swendsen method. The pure phase histograms should be provided by the same technique from data taken outside the metastable range. Our data are scarce there, however.

To obtain the pure phase histograms from runs in the metastability region one could use a selection procedure removing tunneling and excursions to the ”wrong” phase from the Monte Carlo history. This method has been used to extract pure phase correlation lengths (see below) near \( T_c \). It is unsuitable for the extraction of the weights \( w_i \). We have defined pure phase histograms by Gaussian fits to the outer flanks of the full histogram \( P_m(\rho^2, \beta_H) \), which allows to define the weights as functions of \( \beta_H \) and to determine the equilibrium point where \( w_s = w_b \). More information is given by the probability of mixed phase configurations, \( w_{\text{int}} = 1 - w_b - w_s \) at \( \beta_H \), and the slopes of \( w_s \) and \( w_b \) at intersection. The infinite volume limit of the pseudocritical couplings is \( \beta_{\infty} = 0.3435428 \). We show in fig.2 the full histogram \( P_m \) for the \( L = 64 \) lattice and the contribution from inhomogeneous (mixed phase) configurations. For a lattice of this size (in contrast to the smaller \( L = 48 \) or \( L = 30 \)) the pure phase histograms do not overlap, the gap is filled exclusively by inhomogeneous states.

![Histograms \( P_m \) and \( P_{\text{int}} \) at \( \beta_H \) for \( L = 64 \) at \( M_H \approx 70 \) GeV](image)

Figure 2. Histograms \( P_m \) and \( P_{\text{int}} \) at \( \beta_H \) for \( L = 64 \) at \( M_H \approx 70 \) GeV

The phase separated histograms with respect to \( \rho^2 \) at \( \beta_H \) give \( \Delta(\rho^2) \) slightly decreasing with growing lattice size. The infinite volume extrapolation with \( 1/L^2 \) gives \( \Delta \rho^2_{\infty} = 0.499(10) \), which amounts to \( \Delta \epsilon/T_c^4 = 0.024(2) \). For comparison, we have obtained \( \Delta \epsilon/T_c^4 = 0.20(1) \) for the case \( M_H = 38.3 \) GeV (at \( \beta_G = 12 \), too).
The equal weight method makes it possible to reconstruct the free energy densities of the pure phases in the vicinity of the phase equilibrium. The latent heat can then be expressed as the jump \( \Delta \epsilon \) of the energy density by

\[
\frac{\Delta \epsilon}{T_c^4} = \left( \frac{\beta G g^2}{4} \right)^3 T \frac{d \beta H}{dT} \times \\
\frac{d}{d \beta_H} \left( \frac{\log w_s}{L^3} - \frac{\log w_b}{L^3} \right)
\]

(6)

with derivatives taken at \( \beta_{Hc} \). Thus we obtain \( \Delta \epsilon / T_c^4 = 0.0238 \) for \( L = 64 \). Both methods give compatible results for \( \Delta \epsilon \) in the infinite volume limit within a \( 1/L^2 \) extrapolation.

The jump in \( \langle \rho^2 \rangle \) at the critical temperature (at \( \beta_{Hc} \)) may be translated into continuum units

\[
\frac{\Delta \langle \phi^2 \rangle}{(gT_c)^2} = \frac{\beta_{Hc} \beta G}{4} \Delta \langle \rho^2 \rangle.
\]

(7)

This jump is independent of the 3-dimensional renormalization scale \( \mu_3 \), and is therefore more appropriate to consider than the Higgs condensate itself. Extrapolating to infinite volume we find \( \sqrt{\Delta \phi^2} = 0.706 g T_c \) (at \( \beta_G = 12 \)). This value may be compared with predictions from 2–loop perturbation theory (in Landau gauge) \[13,14\] \( \sqrt{\Delta \phi^2}_{\text{pert}} = 0.765 g T_c \). One has to realize that \( \sqrt{\Delta \phi^2} \) is not identical with the condensate value \( v(T_c) \). There is a relation \( \sqrt{\Delta \phi^2} = 0.904 v(T_c) \) at \( \lambda_3 / g_3^2 = 0.0957 \) and with a renormalization scale \( \mu_3 = v(T_c) \).

We conclude that the phase transition at \( M_H \approx 70 \) GeV shows a clear 2–state signal, and quantities like the latent heat, the surface tension and the Higgs condensate (all scaled by an appropriate power of \( T_c \)) have a clearly non–vanishing infinite volume limit. For the latent heat and the surface tension it must be stressed that, contrary to the cases of very small Higgs mass, the transition appears to be weaker than predicted by perturbation theory.

5. The strongly coupled symmetric phase

This phase is characterized by a mass scale \( g^2 T \). It manifests itself e.g. in the string tension of the dimensionally reduced variant of the theory. As in the 4–dimensional theory a force can be defined formally between static non–abelian charges through Creutz ratios of Wilson loops. In a previous work \[2\] we have calculated the force in the case of \( M_H \approx 35 \) GeV in the symmetric as well as in the broken phase. We have found a string tension \( \sigma = 0.11 (g^2 T)^2 \) in the symmetric phase, whereas it vanishes in the broken phase. This value has to be compared to \( \sigma = 0.13 (g^2 T)^2 \) for the 3–dimensional pure gauge theory \[15\].

![Figure 3. Static force F vs. distance R for M_H ≈ 35 GeV and M_H ≈ 70 GeV](image)

In fig. 3 we show the force in the symmetric phase for the cases \( M_H \approx 35 \) GeV and \( M_H \approx 70 \) GeV, both at \( \beta_G = 12 \). From these data for \( M_H \approx 35 \) GeV the above string tension had to be fitted. The errors are large but there is no evidence for the expected breakdown of confinement beyond some screening length. For \( M_H \approx 70 \)
GeV (due to the smaller $W$ mass in the symmetric phase compared to the lighter Higgs case) the perturbative contribution extends further in distance. Thus we can obtain only an upper bound for the string tension, $\sigma \leq 0.1 (g^2 T)^2$.

There is another possibility to calculate the string tension in the vicinity of the phase transition. We consider the ratio of Wilson loops from both sides of the phase transition which amounts to a subtraction of the perturbative part. This allows to anticipate the string tension already at much smaller distance. By this method we make more precise for $M_H \approx 35$ GeV the string tension $\sigma = 0.124 (g^2 T)^2$ and predict $\sigma = 0.078 (g^2 T)^2$ for $M_H \approx 70$ GeV.

Figure 4. Correlation lengths in Higgs and $W$ channel near the transition for $M_H \approx 70$ GeV

In order to characterize immediately the symmetric phase we have studied masses in various channels. The behaviour of two correlation lengths at $M_H \approx 70$ GeV with Higgs and gauge boson quantum numbers is presented in fig. 4 in the vicinity of the phase transition. A discussion of these results together with the question of gauge condensates are postponed to Ref. [12].

Acknowledgements: The calculations have been mainly performed on the DFG–Quadrics QH2 parallel computer in Bielefeld. We wish to thank the system manager M. Plagge for his help. Additionally, we thank the council of HLRZ Jülich for providing CRAY-YMP resources.

REFERENCES

1. D.A. Kirzhnits and A.D. Linde, Phys. Lett. 72B, 471
2. W. Buchmüller, Z. Fodor, T. Helbig and D. Walliser, Ann. Phys. 234 (1994) 260
3. K. Farakos, K. Kajantie, K. Rummukainen and M. Shaposhnikov, Nucl. Phys. B425 (1994) 67
4. B. Bunk, E.-M. Ilgenfritz, J. Kripfganz and A. Schiller, Phys. Lett. B284 (1992) 371; Nucl. Phys. B403 (1993) 453
5. Z. Fodor, J. Hein, K. Jansen, A. Jaster, I. Montvay, Nucl. Phys. B439 (1995) 147
6. E.-M. Ilgenfritz and A. Schiller, Nucl. Phys. B (Proc. Suppl.) 42 (1995) 578
7. K. Farakos, K. Kajantie, K. Rummukainen and M. Shaposhnikov, Nucl. Phys. B407 (1993) 356
8. K. Farakos, K. Kajantie, K. Rummukainen, M. Shaposhnikov, Phys. Lett. B336 (1994) 494
9. E.-M. Ilgenfritz, J. Kripfganz, H. Perl and A. Schiller, Phys. Lett. B356 (1995) 561
10. K. Kajantie, M. Laine, K. Rummukainen and M. Shaposhnikov, CERN-TH/95-263 (1995)
11. K. Kajantie, M. Laine, K. Rummukainen and M. Shaposhnikov, CERN-TH/95-226 (1995)
12. M. Gürtler, E.-M. Ilgenfritz, J. Kripfganz, H. Perl and A. Schiller, in preparation
13. J. Kripfganz, A. Laser and M.G. Schmidt, Nucl. Phys. B433 (1995) 467
14. J. Kripfganz, A. Laser and M.G. Schmidt, Phys. Lett. B 351 (1995) 266
15. M. Teper, Phys. Lett. B289 (1992)115, Phys. Lett. B311 (1993) 223