Longitudinal mechanical response of tunnels under reverse faulting and its analytical solution

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Abstract. The number of tunnels crossing fault zones in Western China has inevitably increased. These tunnels would undergo serious damage under the dislocation, especially for the reverse fault condition. Therefore, this study aims to reveal the longitudinal mechanical responses of tunnels under the dislocation of reverse faults by conducting a comprehensive review of available experimental data. The longitudinal deformation modes and longitudinal strain distributions are collected and analyzed. The bending and axial strains along the longitudinal direction are computed and used to highlight the contributions of the bending and axial compression mechanisms. Moreover, the longitudinal rock pressure distributions at the vault and inverted arch are analyzed to support such response mechanisms of tunnels. Finally, a novel analytical solution to the longitudinal mechanical response of tunnels is proposed on the basis of the Pasternak beam model with consideration of the bending and axial compression mechanisms. The key parameters of the solution are discussed, and the solution itself is validated by comparing the analytical results with the experimental data. The outcomes of this work would provide technical support for the design of tunnels crossing active reverse faults.

1. Introduction

Many tunnels inevitably cross faults because of layout requirements. Hence, significant challenges emerge in the design, construction, and operation of tunnels, especially when faults are active. For example, several tunnels collapsed during the Chi-Chi earthquake [1], and a clear dislocation was recorded in the Longxi Tunnel because of faulting during the Wenchuan earthquake [2]. Therefore, extensive research has aided the understanding of the behavior of tunnels across faults. A particularly hot topic is tunnel response under dislocation.

Model tests have been widely used to investigate tunnel deformation modes and damage patterns, and they include the centrifuge test [3] and 1-g condition model test [4-8]. The centrifuge test can
simulate real stress conditions, but its application is costly, hence the limited data available in the literature. By contrast, the 1-g condition model test is relatively convenient and inexpensive; thus, it has been widely performed and reported. Many factors, including dislocation, fault dip angle, fault type, burial depth of tunnel, and tunnel type, have been investigated using model tests, and their effects have been clearly highlighted. However, some unconformities exist in the observations derived from different model tests. These gaps may be related to the differences in model scale, model material, and boundary condition.

In general, test observations are subjected to analytical analysis or numerical simulation for clear understanding. Analytical analysis, which involves adopting clear concepts and assumptions, is convenient for engineering applications. For example, a comprehensive method was built for the analysis of pipeline crossing faults [9-10]. Our team extended this method to analyze the behavior of a tunnel while neglecting axis force [11]. This shortcoming was solved in the current work by introducing a distribution model of axis force. In terms of numerical simulation, 2D and 3D analyses have been reported, and many key factors have been investigated [12-14]. In numerical simulation, the interactions between soil/rocks and tunnels, and the internal force of tunnel linings can be obtained. However, the calibration of model parameters and the simulation technology are key issues that need to be studied further. These topics are beyond the scope of this work.

This study aims to summarize the key mechanical responses of tunnels under reverse faulting and to improve the analytical model in [11] by introducing the axis force effect. In the following sections, four available model tests are identified and analyzed to highlight the generalized features of tunnel behavior under reverse faulting. Then, the analytical model is presented and verified on the basis of the experimental data from one model test.

2. Overview of available experimental data

Four model tests under the 1-g condition were collected in this section, and the observations are analyzed in terms of longitudinal strain, bending strain, axial strain, and rock pressure.

2.1. Overview of model test and analysis methodology

Table 1 summarizes all details of the four tests. All these tests were performed using the split box approach. Different geometries, rock properties, and tunnel properties were considered in these tests. A normalization method is adopted to eliminate the deficiencies induced by the variations. Strain and pressure are normalized by dividing their maximum values; geometry is scaled according to the tunnel diameter. Different dislocations were reported in these tests, but only the results at the dislocation at 1/10 of the diameter are discussed and compared in this section.

| Model size /m | Rock | Fault | Tunnel |
|---------------|------|-------|--------|
|               | Young’s modulus/MPa | Friction angle | Cohesion /kPa | Dip angle | Material | Young’s modulus/GPa | Diameter /cm |
| /             | 33.3 | 23    | 2.8    | 70        | Gypsum   | /        | 33.7       | [5]         |
| 2.0×0.8×      | 39   | 26.6  | 1.5    | 75        | Gypsum   | 1.0      | 25.0       | [6]         |
The longitudinal strain and rock pressure at the vault and inverted arch are often measured to directly evaluate the safety status of the tunnel lining structure. The obtained values are induced by different mechanisms under faulting, such as bending, shearing, and compression/tension. To distinguish the contributions of these mechanisms, this study presents the longitudinal strain in terms of bending and axial strains. Bending strain is used to represent the contribution of the bending mechanism while axial strain relates to compression or tension. The shearing effect is not considered herein because its influence zone is smaller than those of the other two mechanisms. The bending and axial strains are defined as

\[
\varepsilon_{\text{bending}} = \frac{\varepsilon_{\text{vault}} - \varepsilon_{\text{inverted arch}}}{2} \quad (1a)
\]

\[
\varepsilon_{\text{axial}} = \frac{\varepsilon_{\text{vault}} + \varepsilon_{\text{inverted arch}}}{2} \quad (1b)
\]

where \( \varepsilon_{\text{vault}} \) and \( \varepsilon_{\text{inverted arch}} \) are the strains at the vault and inverted arch, respectively. \( \varepsilon_{\text{bending}} \) and \( \varepsilon_{\text{axial}} \) are the bending and axial strains, respectively.

### 2.2. Longitudinal strain

Figure 1 summarizes the obtained strains at the vault and inverted arch during the test. Tension strain (positive) occurs at the vault in the hanging wall and at the inverted arch in the footwall. The compression strain is noted at the inverted arch in the hanging wall, and its value and distribution zone are greater than those of the tension strain at the vault. The same trend is observed for the strain at the inverted arch. Therefore, under reverse faulting, the tunnel failure may be dominated by compression at the vault in the footwall or at the inverted arch in the hanging wall.
2.3. Bending and axial strains

With Eq. (1), the original data are calculated and represented in terms of bending strain (Figure 2a) and axial strain (Figure 2b). The bending strain is concave in the footwall and convex in the hanging wall and is shaped like an inverted “S.” The value of the bending strain in the hanging wall is greater than that in the footwall. The effect zone in the hanging wall is almost 2D and is slightly smaller than that in the footwall. All these observations may be related to the disturbance induced by movement during faulting.

The axial strain distribution is mainly concentrated around the offset plane, and a remarkable compressive strain is obtained. Such results indicate that the tunnel suffers large compression under reverse faulting because of the relative motion between the hanging wall and the footwall. This result also means that the compression/tension effect should not be neglected in the analytical analysis. The main distribution zone of the axial strain is 2D relative to the offset plane in the hanging wall and is comparatively large in the footwall.

![Figure 2a](image.png)

![Figure 2b](image.png)
2.4. Rock pressure

Figure 3 summarizes the measured rock pressure at the vault and inverted arch of the tunnel. Positive and negative values mean that the pressure follows downward and upward directions, respectively. To compare the relative magnitudes of pressure at the vault and inverted arch, this study normalizes the pressure by dividing the common peak pressures. According to the data in [6] and [8], the pressure at the inverted arch is positive; hence, the values may not be credible. Herein, these two sets of data are thus neglected in the analysis. In the hanging wall, the pressure at the inverted arch is much greater than that at the vault. Meanwhile, a different trend is obtained in the footwall. This distribution means that a compression zone is generated at the inverted arch in the hanging wall and at the vault in the footwall during dislocation. These two compression zones result in bending in the longitudinal direction and shear around the offset plan. Owing to the limited observation zone, the effect zone of rock pressure cannot be clearly given.

3. Analytical solution to the longitudinal mechanical response of tunnels under dislocation

According to the discussion, the compression/tension effect cannot be neglected when analyzing tunnel response under reverse faulting. Therefore, a novel analytical solution is developed herein on the basis of that in [11] with consideration of compression/tension.

3.1. Main assumptions

To simplify the derivation, this study adopts the following assumptions:

(a) The tunnel is continuous, uniform, and isotropic along the longitudinal direction;
(b) The interaction between rocks and the tunnel satisfies the Pasternak model;
(c) The rate of dislocation is very small, and the inertial effect can be neglected;
(d) The bending and compression/tension mechanisms are independent;
(e) The initial internal force of the tunnel is not considered, and only the increments induced by dislocation is taken into account;
(f) The frictional coefficient between the tunnel and rocks is assumed to be constant during the dislocation.

**Figure 3.** Rock pressure distribution along the tunnel under reverse faulting.

### 3.2. Bending mechanism

The bending mechanism is described using the Pasternak model. The main differential equation is as follows:

\[
EI \frac{d^4 y}{dx^4} + kby - Gb \frac{d^2 y}{dx^2} = q(x)
\]

where \( EI \) is the bending stiffness, \( k \) is the elastic foundation coefficient, \( b \) is the width of the beam, \( G \) is the vertical shear stiffness of the rocks, \( q(x) \) is the external load, \( y \) is the deflection, and \( x \) is the position. The detailed solution was reported in [11].

### 3.3. Compression/tension mechanism

Compression/tension only occurs around the offset plane, and it reaches its maximum value at the fault. Thus, the distribution of axial compression/tension force is assumed to satisfy the pattern illustrated in Figure 4. The maximum force \( N_{\text{max}} \) is at the offset plane, and the force decreases linearly with the increase of distance to the offset plane. The effect zone is assumed to be \( L \) (Figure 4a). At the cross section, the maximum value occurs at the vault and inverted arch (Figure 4b). The axial force \( N(x) \) can be calculated as

\[
N(x) = \begin{cases} 
N_{\text{max}} + \frac{2N_{\text{max}}}{L}x \\
N_{\text{max}} - \frac{2N_{\text{max}}}{L}x 
\end{cases}
\]

The frictional force along the tunnel can be computed as follows:

\[
f(x) = \mu \int_0^{2\pi} P(\theta, x) d\theta
\]
Given the balance between the axial force and the frictional force,

\[ \int_{\frac{L}{2}}^{L} N(x)dx = \int_{\frac{L}{2}}^{L} f(x)dx \]  

(5)

By combining Eqs. (3–5), the maximum force \( N_{\text{max}} \) can be solved as follows:

\[ N_{\text{max}} = \frac{4 \cdot m \cdot \mu D}{L} \int_{\frac{L}{2}}^{L} P(x)dx \]  

(6)

Figure 4. (a) Distribution of axial compression/tension force in the longitudinal direction and (b) compression pressure at the cross section.

3.4. Validation

With the distribution of moment \( M(x) \) and axial force \( N(x) \) along the longitudinal direction, the strains at the vault and inverted arch can be calculated as follows:

\[ \varepsilon_{\text{vault}}(x) = \frac{N(x)}{EA} + \frac{M(x)}{EI} \cdot \frac{D}{2} \]

\[ \varepsilon_{\text{inverted}}(x) = \frac{N(x)}{EA} \cdot \frac{M(x)}{EI} \cdot \frac{D}{2} \]  

(7)

where \( A \) is the area of the tunnel section.

The model test data in [7] are used to validate the proposed analytical solution. According to the model information in [7], the model parameters can be derived, and they are summarized in Table 2. The analytical results are presented and compared with the test data in Figure 5.

Table 2. Model parameters used in the analytical solution.

| \( E/\text{MPa} \) | \( D/\text{m} \) | \( t/\text{m} \) | \( E_R/\text{MPa} \) | \( v \) | \( d/\text{m} \) | Dip angle ° | \( w/\text{m} \) | \( H/\text{m} \) |
|-----------------|-----------|-----------|-----------------|----|-----------|------------|--------|--------|
| 1130            | 0.24      | 0.015     | 6               | 0.4| 0.01-0.04 | 75         | 0.8    | 0.52   |

\( t \) is the thickness of the lining, \( E_R \) is the rock’s Young’s modulus, \( v \) is the Poisson’s ratio, \( d \) is the dislocation, \( w \) is the width of the fault zone, and \( H \) is the buried depth of the tunnel.

For the vault strain, the values in the footwall are much greater than those in the hanging wall; and the maximum value appears at 0.6 m relative to the offset plan (Figure 5a). For the strain at the
inverted arch, the strain in the hanging wall is relatively high (Figure 5b). As the dislocation increases, the strain at the vault and that at the inverted arch increase while the location of the peak strain remains almost constant (Figures 5a and 5b). This result indicates that the dislocation only affects the strain values and does not influence the pattern of the strain distribution along the longitudinal direction.

Figures 5c and 5b respectively compare the evolution of the peak strains at the vault and inverted arch with the dislocation. The analytical solution can reproduce the increasing trend of the peak strain with the dislocation, and the analytical values are comparable to the experimental data. However, a linear trend is obtained in the analytical solution while nonlinear characteristics are derived from the test. This difference may be related to the uncertainty and errors during the test. Overall, the analytical solution seems to have the capacity to reproduce the tunnel responses under reverse faulting.

4. Conclusion

Four available model tests on tunnel response under reverse faulting are analyzed in this work. Bending and axial strains were used to re-analyze the experimental data and represent the contribution of the bending and compression/tension mechanisms. The results reveal that the longitudinal mechanical response of the tunnel under reverse faulting is a combined result of bending and compression and that the compression effect could not be neglected. To consider this compression contribution, this study develops a new analytical solution by extending an existing model. The proposed model can effectively reproduce the test results and can thus be used to calculate generalized tunnel responses under faulting.
Figure 5. Analytical results in terms of (a) vault strain; (b) inverted arch strain; (c) comparison of peak values at the vault; and (d) comparison of peak values at the inverted arch.

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