Sub-wavelength spin excitations in ultracold gases created by stimulated Raman transitions

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Abstract

Raman transitions are used in quantum simulations with ultracold atoms for cooling, spectroscopy and creation of artificial gauge fields. Spatial shaping of the Raman fields allows local control of the effective Rabi frequency, which can be mapped to the atomic spin. Evanescent Raman fields are of special interest as they can provide a new degree of control emanating from their rapidly decaying profile and for their ability to generate features below the diffraction limit. This opens the door to the formation of sub-wavelength spin textures. In this work, we present a theoretical and numerical study of Raman Rabi frequency in the presence of evanescent driving fields. We show how spin textures can be created by spatially varying driving fields and demonstrate a skyrmionium lattice—a periodic array of topological spin excitations, each of which is composed of two skyrmions with opposite topological charges. Our results pave the way to quantum simulation of spin excitation dynamics in magnetic materials, especially of itinerant spin models.

1. Introduction

The experimental science of ultracold atoms has proven to be instrumental in understanding complex many-body phenomena [1]. Furthermore, the near-perfect isolation from the environment and unrivaled controllability of neutral atoms has made them an ideal platform for quantum computation [2] and simulation [3]. In this regard, it is possible to engineer the potential landscape of neutral atoms using far-off-resonance light [4], and to simulate effects like the Lorentz force [5] and spin–orbit coupling [6] by coupling different spin states of the atom using Raman beams. Since the above-mentioned techniques utilize propagating optical fields, spatial variations are naturally limited to $\sim \lambda/2$, where $\lambda$ is the wavelength of the light. Therefore, atoms cannot be brought closer than $\sim \lambda/2$ in a controlled manner, setting a constraint on the typical interaction energy and putting many interesting collective phases beyond experimental reach in the current regime of lowest achievable temperatures.

Nanophotonics provides a promising route for generating spatial variations smaller than the diffraction limit, by applying in-plane wave-vectors larger than those available in free-space far-field propagation. In this approach, the fields decay exponentially related to the nanophotonic device, requiring that the atoms be brought very close to the surface [7, 8]. Such fields appear, for example, in guided modes propagating at the boundary of thin metallic films [9], wave-guides [10], or in total internal reflection at an interface between two dielectric materials [11].

Integrating ultracold atoms in the near-field of nanophotonic devices not only gives access to sub-wavelength spatial variations of the fields, but also to the unique behavior of evanescent electromagnetic waves. In the near-field regime, electromagnetic fields exhibit intriguing topological properties, e.g. spin-momentum locking or transverse spin [12–14], as well as topologically nontrivial patterns, the likes of optical skyrmions [15]. The latter are topologically stable field configurations [16], which have aroused great interest in the field of magnetism, as a unique and possibly applicable form of...
spin arrangement in solids [17–19]. Though previous works studied Raman coupling in the near-field regime in the context of spectroscopy and imaging [20, 21], the use of Raman transitions for coherent manipulation of the atomic spin degree of freedom in the near-field remains so far unexplored.

Here, we explore sub-wavelength spin structures in an ultracold atom gas, generated by evanescent Raman beams. We derive a generalized formalism for the Raman Rabi frequency in the presence of an arbitrary evanescent driving field and discover a new method to induce spin excitations in the quantum gas. As a proof-of-concept, we consider the excitation of a skyrmionium lattice [22]—a topological excitation that carries a trivial total topological charge ($S = 0$), but can be viewed as a combination of two concentric skyrmions with opposite topological charges of $S = ±1$. We further discuss its experimental feasibility, and how such spin textures could enable quantum simulation of magnetic materials and the study of their dynamics.

2. Atom-field interaction model

We consider an atom near a surface, interacting with an evanescent electric field. The field may be created in many ways, e.g., total internal reflection, and can be written as $\tilde{E}(\vec{r}) = \tilde{E}_1(\vec{r}) + \tilde{E}_2(\vec{r}) = \tilde{e}_1(x, y)e^{-i k_{1z}z} e^{i \omega_2 t} + \tilde{e}_2(x, y)e^{-i k_{2z}z} e^{i \omega_1 t}$, where $k_{1,2} = (1, 2)$ is a purely imaginary wave vector component and $\tilde{e}_α$ is a three dimensional vector field that depends on $(x, y)$ coordinates. The origin resides on the surface and the $z$ axis is perpendicular to the surface. The fields $\tilde{E}_α$ have components both perpendicular (out-of-plane) and parallel (in-plane) to the surface. Each of the atoms is modeled as a three-level system in a $Λ$ configuration, as depicted in figure 1 [24]. Collective effects due to close proximity of the atoms may be non-negligible, but we defer their inclusion to future works.

Under a dipole approximation [25], the interaction Hamiltonian is given in the rotating frame by

$$H_{IF} = -\frac{1}{2} |g_1| \vec{d}|e⟩ \cdot \vec{E}_1^* (\vec{r}) - \frac{1}{2} |g_2| \vec{d}|e⟩ \cdot \vec{E}_2^* (\vec{r}) + \text{h.c.},$$  \hfill (1)

where $\vec{E}_α (\vec{r})$ is the electric field phasor, such that $\vec{E}_α (\vec{r}) = \text{Re} \left\{ \vec{E}_α (\vec{r}) e^{i \omega_α t} \right\}$ and $|g_α⟩⟨e|$ is the transition operator from state $|e⟩$ to state $|g_α⟩$. Moving to spherical unit-vectors [26]: $\hat{u}_{±1} = (\hat{x} ± i\hat{y})/\sqrt{2}$ and $\hat{u}_0 = \hat{z}$, each term in equation (1) becomes:

$$\langle g_α| \vec{d}|e⟩ \cdot \vec{E}_α (\vec{r}) = \frac{1}{\sqrt{2}} |g_1| \langle d | \hat{u}_{±1} | e⟩ \left[ \hat{E}_{α,±} (\vec{r}) - i \hat{E}_{α,0} (\vec{r}) \right] + \frac{1}{\sqrt{2}} |g_2| \langle d | \hat{u}_{-1} | e⟩ \left[ \hat{E}_{α,±} (\vec{r}) + i \hat{E}_{α,0} (\vec{r}) \right]$$

$$+ |g_α| \langle d | \hat{u}_0 | e⟩ \hat{E}_{α,0} (\vec{r}).$$  \hfill (2)

It is evident from this expression that the coupling strength between each of the ground states and the excited state depends on the interference pattern created by the driving fields $\vec{E}_α$. Each of the matrix elements in equation (2) can be calculated using the Wigner $3j$ [27] and $6j$ [28] symbols [26, 29].
definition of the Rabi frequency for each transition $|g_i⟩ ↔ |e⟩$ reads

$$\Omega_r(\vec{r}) = -\frac{1}{\hbar} \langle g_i | \vec{d} | e⟩ \cdot \vec{E}_n(\vec{r}),$$  \hspace{1cm} (3)$$

and is spatially dependent according to the driving field’s interference pattern. Using the definition of equation (3), the atom-field interaction of equation (1) is written as:

$$\hat{H}_{AF} = \frac{\hbar \Omega_1(\vec{r})}{2} \sigma_1 + \frac{\hbar \Omega_2(\vec{r})}{2} \sigma_2 + \text{h.c.}$$  \hspace{1cm} (4)$$

Under the Born–Oppenheimer approximation [30], we write the atomic wave-function as a series of tensor products of internal and external states: $|\psi⟩ = |\psi_1⟩|g_1⟩ + |\psi_2⟩|g_2⟩ + |\psi_3⟩|e⟩$. We substitute this expression into Schrodinger’s equation and apply adiabatic elimination [31], assuming $\hbar \Delta \gg \{\hbar |\Omega_{1,2}|, \vec{E}^2/2m\}$, thus arriving at the following equations of motion:

$$i\hbar \partial_t |\psi_1⟩ = \left[\frac{\vec{p}^2}{2m} + \hbar \Delta_1 + \frac{\hbar}{4\Delta} |\Omega_1(\vec{r})|^2 \right] |\psi_1⟩ + \hbar \Omega_1(\vec{r}) |\psi_2⟩$$

$$i\hbar \partial_t |\psi_2⟩ = \left[\frac{\vec{p}^2}{2m} + \hbar \Delta_2 + \frac{\hbar}{4\Delta} |\Omega_2(\vec{r})|^2 \right] |\psi_2⟩ + \hbar \Omega_2(\vec{r}) |\psi_3⟩,$$  \hspace{1cm} (5)$$

where $\Omega_r$ is the generalized Raman Rabi frequency defined by

$$\Omega_r(\vec{r}) = \frac{\Omega_1(\vec{r})\Omega_2(\vec{r})}{4\Delta}. $$  \hspace{1cm} (6)$$

$\Delta \overset{\text{def}}{=} \frac{\Delta_1 + \Delta_2}{2}$ is the average single photon detuning, and we assume $\Delta \gg \{|\Delta_1 - \Delta_2|, \Gamma\}$, with $\Gamma$ being the natural linewidth of the excited state. This assumption can also be viewed in terms of a Markovian condition, where we assume that timescales of spontaneous emissions are much longer than stimulated ones and that there is no coupling to the environment, e.g., coupling to the nearby surface. If this was not the case, it would be necessary to treat the system as open and account for non-Markovian effects [32–34]. The rhs of equation (5) is divided into two parts: the free evolution of each ground state, and its coupling to the other ground state due to the Raman fields. The free evolution includes three terms that account for the kinetic energy, the bare ground state energy, and the light-shift due to the Raman field. The Raman coupling term is shaped by the interference pattern of the two Raman fields, which in turn translates into the ground state populations at each point.

### 3. Selection rules

Next, we seek the conditions under which the Raman coupling does not vanish. The coupling is proportional to $\Omega_1(\vec{r}) \sim |\langle g_1 | \vec{d} \cdot \hat{u}_{n_1} | e⟩|^2 |\langle \vec{d} \cdot \hat{u}_{n_2} | g_2⟩|^2$, where $q_{n_1} = 0, \pm 1$ is the polarization vector for the corresponding driving field components in equation (2) and $(\cdot)^\dagger$ is the Hermitian adjoint. Let us assume that $|g_1⟩$ and $|g_2⟩$ are Zeeman states with $M_{F,1} = M_{F,2} - 1$, where $M_F$ is the projection of the total angular momentum $F$ along the quantization axes. The Wigner 3j symbol implies momentum conservation which requires $q_2 = q_1 - 1$. Combining these expressions, each term of the form $|g_1 | \vec{d} \cdot \hat{u}_{n_1} | e⟩ |\langle \vec{d} \cdot \hat{u}_{n_2} | g_2⟩|^2$ can be expanded as:

$$\langle g_1 | \vec{d} \cdot \hat{u}_{n_1} | e⟩ |\langle \vec{d} \cdot \hat{u}_{n_2} | g_2⟩|^2 = q_n^2\langle n''_1L_1 || n''_2L_2⟩ \times (-1)^{1 + 2L_1 + S_n + L_n + I + I - M_{F,1}}$$

$$\times [(2F_e + 1)(2F_1 + 1)(2F_e + 1)(2F_1 + 1)] \times \left\{ \begin{array}{ccc} L_1 & I_1 & S_n \\ I_e & L_e & 1\end{array} \right\} \left\{ \begin{array}{ccc} I_1 & F_1 & I \\ F_e & I_e & 1\end{array} \right\}^2$$

$$\times \left[ \begin{array}{ccc} F_e & 1 & F_1 \\ q_1 & M_{F,e} \end{array} \right] \left[ \begin{array}{ccc} q_1 & 1 & M_{F,1} + 1 \\ M_{F,e} & F_1 \end{array} \right],$$  \hspace{1cm} (7)$$

where $L_{j=1,i}$ is the orbital angular momentum and $S_{j=1,i}$ is the spin angular momentum of the states $|g_i⟩$ and $|e⟩$ respectively, $I$ is the nuclear spin, $j_{F=1,i}$ is the total angular momentum of the electron and $F_{F=1,i}$ is the total angular momentum of the atom under hyperfine approximation. The term $\langle n''_1L_1 || n''_2L_2⟩$ is the reduced matrix element with $n''_1$ and $n''_2$ being the effective principle quantum numbers of s and p atomic orbitals [26]. The six terms element in curly (regular) brackets is the Wigner 6j (3j) symbol. Since
$q_i = 0, \pm 1$ we obtain that $\langle g_1 | \hat{d} \cdot \hat{u}_{q_i} | \rangle \langle e \rangle \left( \hat{d} \cdot \hat{u}_{q_2} \right) | g_2 \rangle \neq 0$ only for $(q_1, q_2) \in \{(+1, 0), (0, -1)\}$ [29]. The Raman Rabi frequency then becomes

$$\Omega_k(\vec{r}) = M_{q_1 q_2} \left[ E_{1,0}(\vec{r}) - i E_{2,0}(\vec{r}) \right] \vec{E}_{2,0}(\vec{r}) + M_{0,1} \left[ E_{1,0}(\vec{r}) - i E_{2,0}(\vec{r}) \right] \vec{E}_{1,0}(\vec{r}),$$

where $M_{q_1 q_2} = \frac{1}{4\sqrt{\Delta \Omega}} \langle g_1 | \hat{d} \cdot \hat{u}_{q_1} | e \rangle \langle \hat{d} \cdot \hat{u}_{q_2} | g_2 \rangle$. Equation (8) shows that Raman transitions are generated by a combination of the $(x, y)$ components of the $\vec{E}_1$ ($\vec{E}_2$) field and the $z$ component of the $\vec{E}_2$ ($\vec{E}_1$) field. Equation (8) highlights one of the advantages of using evanescent waves for Raman transitions—due to their non-transverse nature, they naturally exhibit all three electric field components at once in a phase-locked manner, whereas achieving the same feat with transverse propagating waves is considerably more difficult.

### 4. Sub-wavelength spin excitations

A major outcome of the position dependent Raman coupling in the near-field, given by equation (8), is the possible generation of complex spin excitations. We assume that the atoms are held in a two-dimensional trap at some distance $z_0$ from the surface, and that their initial state is $|\varphi\rangle = |g_1\rangle$. We also assume that the Raman fields are pulsed for a duration $T$ much shorter than the dynamical timescales of the free evolution. We can therefore focus only on the coupling term in equation (5), and leave the analysis of itinerant spins to future works. Each of the atoms is an effective two-level system that can be mapped to a spinor:

$$|\varphi\rangle = e^{-i\delta} \cos \left( \frac{\theta}{2} \right) |\uparrow\rangle + \sin \left( \frac{\theta}{2} \right) |\downarrow\rangle,$$

with $|\downarrow\rangle \overset{\text{def}}{=} |g_1\rangle$ and $|\uparrow\rangle \overset{\text{def}}{=} |g_2\rangle$ and the angles given by

$$\theta = 2 \arccos \left( \frac{T}{\pi} |\Omega_k(\vec{r})| \right) \quad \phi = \angle \Omega_k(\vec{r}).$$

We take the duration $T = \pi / \max_{\vec{r}} (|\Omega_k(\vec{r})|)$ such that spinors at the points with maximal Raman coupling are brought to the south pole of the Bloch sphere while atoms at points with vanishing $\Omega_k(\vec{r})$ will stay at the north pole. For this choice, equations (9) and (10) map the spatially-varying Raman Rabi frequency onto a Bloch sphere, similarly to a spin-1/2 particle. In reality, the atoms are not necessarily spin-1/2 particles, but at high magnetic field the second-order Zeeman splitting induced between the other states is sufficiently far-detuned from the Raman resonance to neglect them. Thus, we can treat a multi-level atom as an effective spin-1/2 particle, as long as we isolate the Raman coupling to only two states.

We are now ready to calculate the resulting sub-wavelength spin excitation for several field configurations. In these calculations we assume 40K atoms are suspended inside a vacuum chamber in a two-dimensional trap, which is formed at a distance of $z_0 \sim 100$ nm from a single surface of a dielectric material. We envision coupling the Raman beams into the vacuum chamber through an optical viewport made of this transparent material. Such windows can be manufactured with surface roughness much smaller than $\lambda_0$ value. Current state-of-the-art manufacturing facilities may provide surface roughness as low as 0.5 nm—much smaller than the typical distance between the atoms and the surface. Another issue to consider is decoherence and heating of the atoms induced by the close-by surface. However, a previous study found that for atoms at close proximity to a dielectric surface, if such processes exist, their timescale is larger than a millisecond [35].

The surface is illuminated by two groups of $n$ beams undergoing a total internal reflection as they impinge on it. These beams are the source for the evanescent fields that drive the Raman transitions at $z > 0$. The phasor of a transverse-magnetic driving field $\vec{E}_{\alpha}$, for a single group, is given by:

$$\vec{E}_{\alpha}(\vec{r}) = E_{0,\alpha} e^{-i k_{\alpha,\perp} x} \sum_{i=1}^{n} \left( \frac{| k_{\alpha,\parallel} | A_{\alpha,i}}{| k_{\alpha,\parallel} |} \frac{| k_{\alpha,\perp} | B_{\alpha,i}}{| k_{\alpha,\perp} |} 1 \right) \times e^{i \delta_{\alpha,i}} e^{-i k_{\alpha,\parallel} \left( \delta_{\alpha,i} + \delta_{\alpha,\perp} \right)},$$

where $E_{0,\alpha}$ is the driving field’s amplitude (assumed to be real), $k_{\alpha,\parallel} = \sqrt{k_{\alpha,\perp}^2 + k_{\alpha,\parallel}^2}$ is the in-plane wave-vector, $k_{\alpha,\parallel}^2 = k_{\alpha,\perp}^2 + k_{\alpha,\parallel}^2$, $\delta_{\alpha,i}$ is a constant initial phase of
At this point, we examine how relative phases between the beams generate different skyrmionium lattices in wave-vectors \( \mathbf{k} \) driving fields. The length scale on which the spins are rotated in the Bloch sphere is defined by the in-plane section. As can be seen, the symmetry of the spin excitation lattice is directly related to the symmetry of the configurations with no relative phase between the beams (relative phases will be discussed in the next section).

5. Skyrmionium and antiskyrmionium lattices

We focus on figure 2(a) and note that the spinor undergoes a continuous rotation from \(|g_1\rangle\) to \(|g_2\rangle\) and then back to \(|g_1\rangle\) as we traverse over a straight line from the center of a unit cell to the boundary. To characterize the topology of the spin structure, we consider the topological charge density defined as

\[
S = \frac{1}{4\pi} \int_A s \, dA,
\]

where \( s \) is the two-dimensional manifold. To investigate how the Pontryagin number changes as we expand the domain of integration \( A \) from the center of a unit cell to its boundary, we define the domain

\[
A = \{ (x,y) \mid x^2 + y^2 \leq R \} \cap B
\]

where \( B \) is the unit cell boundary and \( R \) is some radius of choice. This definition essentially takes the intersection of ever increasing circles and the boundary of a unit cell [29]. At this point, we examine how relative phases between the beams generate different skyrmionium lattices in the quantum gas. Skyrmionium is a spatial spin excitation with a total topological charge of \( S = 0 \) which is a combination of two concentric skyrmions with opposite topological charges of \( S = \pm 1 \) [38]. Such spin excitations were only recently observed in solid-state systems [22, 39–42] and were suggested as a platform for magnetic logic that is immune to parasitic Hall effects [43]. We focus on the configuration in

![Figure 2](image_url)

**Figure 2.** Spin textures generated by near-field Raman transitions. **Top row:** Field configuration and interference patterns. Arrows (left) represent the in-plane propagation direction, each arrow corresponds to both \( E_1 \) and \( E_2 \). Amplitude of the in-plane (upper right) and the out-of-plane (lower right) field components are shown next to the arrows. Bright (dark) yellow (blue) denotes the maximal (minimal) value. Each configuration consists of two groups of in-plane propagating beams, organized as (a) three beams, (b) four beams and (c) six beams. **Middle row:** Spatially dependent Raman Rabi Frequency. Amplitude (left) and phase (right) are shown for each configuration in the top row, resulting in a hexagonal (a), (c) or square (b) lattice. Bright (dark) yellow (blue) denote the maximal (minimal) value. **Bottom row:** Spin excitation in an atom cloud after Raman fields interact with the atoms for a duration \( T = \pi/\max (|g_1|, |g_2|) \) (“\( \pi \)-pulse”). Spatial changes in coupling frequency affect the spinor state according to the mapping in equation (10), such that the resulting spin texture follows the Raman Rabi frequency amplitude closely. Colorbar shows the state of the spinor as a function of position within the cold atom cloud. For these simulations we used typical numbers of \( ^{87}\text{Rb} \) atoms. The Raman fields were taken to be red-detuned, \( \Delta \approx -10 \text{ THz} \), from the \( D_2 \) transition \( \left( ^2S_1/2 \rightarrow ^2P_{3/2} \right) \). Spinor states are defined as \(|g_1\rangle = |M_F = -9/2\rangle \) and \(|g_2\rangle = |M_F = -7/2\rangle \) on the \( F = 9/2 \) hyperfine manifold. The width of the insets is 1.8\( \lambda_0 \), where \( \lambda_0 \approx 766.7 \text{ nm} \) is the \( D_2 \) transition wavelength.
**Figure 3.** Calculation of the integrated topological charge as a function of the manifold $A$ from equation (12) and the $z$-component of the Bloch vector $\vec{m}$. **Top row:** Unit cells corresponding to anti-skyrmionium, Bloch-type skyrmionium and Neel-type skyrmionium lattices (from left to right). Horizontal width of a unit cell is $0.7\lambda_0$. **Bottom row:** (Left) Calculation of the topological charge as a function of the manifold $A$ from equation (12) and the $z$-component of the Bloch vector $\vec{m}$. Solid blue curve labeled $S_a$ corresponds to the topological charge for the case of anti-skyrmionium and solid red curve labeled $S_b$ corresponds to the topological charge for the Neel and Bloch-type skyrmionium. Dashed black curve is the $z$-component of the Bloch vector $\vec{m}$ taken over horizontal width of a unit cell. As the topological charge reaches $-1 (+1)$, the Bloch vector completes a rotation of $-\pi$. The topological charge vanishes when the integration domain is taken over the entire unit cell. (Right) Topological charge densities over which integration is performed. The topological charge densities for the anti-skyrmionium (lower inset) and Bloch and Neel-type skyrmionium (upper inset) are of opposite sign and the same magnitude. Dashed white circle corresponds to the $R_0 = 0.28$ where the topological charge vanishes for the first time.

figure 2(a). The driving field consists of two groups of $n = 3$ beams where the relative phases of the beams are $\delta_{\alpha,i}$ as defined in equation (11). Setting these relative phases to $\delta_1 = (0, \frac{2\pi}{3}, \frac{4\pi}{3})$ and $\delta_2 = (-\frac{2\pi}{3}, \frac{4\pi}{3}, 0)$ yields a Bloch-type skyrmionium lattice. Changing the relative phase of the first group to $\delta_1 = (\frac{4\pi}{3}, 0, \frac{2\pi}{3})$ yields a Neel-type skyrmionium lattice. If all relative phases are set to 0, as in figure 2(a), the result is an anti-skyrmionium lattice. We present the unit cell of these three lattices in figure 3 and plot the integrated topological charge along with the topological charge density. The main difference between Neel and Bloch-type skyrmioniums is in the direction of the gradual reorientation of individual spinors on the Bloch sphere across a finite distance with respect to the $z$ axis of the sphere. In Neel-type, the direction of the spinor rotation is parallel to $\hat{r}$, the radial unit vector pointing away from the center of the unit cell, while in Bloch-type the rotation is perpendicular to $\hat{r}$. In our spin texture there are two such spinor re-orientations, one between $m_z = -1$ and $m_z = 1$ and another between $m_z = 1$ and $m_z = -1$. While the total topological charge is identical for skyrmionium and anti-skyrmionium, the integrated topological charge reveals the difference between them. Figure 3 shows that anti-skyrmionium and Bloch or Neel-type skyrmionia have the opposite topological charge within the unit cell. That is, these structures are topologically distinct for all but three domains within the unit cell.

6. Discussion and outlook

Our analysis shows that when atoms are driven by evanescent fields, a Raman process can be used to generate a sub-wavelength lattice of spin excitations, with the skyrmionium lattices presented above as a specific example. This opens the door to the study of spin excitation dynamics in magnetic materials using ultracold gases, though there are several challenges in this route. First, it is required to efficiently load the ultracold gas to a trap very close to a surface, which is at a much higher temperature than the atoms—a process usually achieved only with low-loss wave-guides [44]. Another challenge is to find a method to characterize the state of the gas with an underlying structure at a lengthscale smaller than the diffraction
limit. One possible route is by using super-resolution imaging methods for atoms [45]. Another possible route to probing the atoms near the surface is by using heterodyne fluorescence spectroscopy [46]. Finally, if the average distance between the atoms is considerably shorter than the wavelength, collective effects such as sub- and super-radiance may occur as well. These effects can also be potentially useful in probing and characterizing the atomic spin structure [47].

The use of ultracold atoms allows studying interesting models of magnetism which are hard to implement in solid systems. For example, the spin textures presented in figure 3 represent different types of excitations in magnetic materials [19]; hence their implementation in an ultracold atomic system is of great interest in the context of the quantum simulation of magnetic materials. Furthermore, by trapping the atoms in an optical lattice (which may also be sub-wavelength in the near-field regime), one can explore a model of itinerant spins and the interplay between effective magnetic interactions and diffusion. In addition, Feshbach resonances enable tuning the interaction strength between the two spin states. We believe that the most interesting regime will be around unitarity, where the spin diffusivity achieves its minimum [48].

Another interesting direction is to study the dynamics of a skyrmionium lattice in the repulsive branch of the Feshbach resonance. The Stoner model predicts itinerant fermions with repulsive interactions will develop ferromagnetic ordering [49], but rapid decay from the excited repulsively-interacting state prevented the experimental observation of this phenomenon thus far [50]. The rapid decay was due to the formation of bound pairs of particles with opposite spins. However, in a skyrmionium, the spatial spin rotation happens at a low wave-vector, hence adjacent spins are pointing almost to the same direction and Pauli repulsion is expected to suppress recombination processes. Therefore, it is plausible that in the repulsive branch a spin excitation will actually be more stable than in a fully mixed spin state.

It is worth noting that our results bear some resemblance to the study of optical flux lattices in ultracold atomic gases, where spin textures consisting of both zero [51] and non-zero [52–54] topological charges were envisioned. In these works, spin textures were obtained by either minimization of the Gross–Pitaevskii energy functional [51, 52] or by considering the eigenstates of the two-level system [53, 54]. Another possible route for spin texture generation in ultracold atoms is the onset of a Rashba Hamiltonian in the interaction of atoms with an optical lattice [55]. The derivation performed in this paper, however, offers a different treatment to this problem, which may be implemented using extension of existing techniques.

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References

[1] Bloch I, Dalibard J and Zwerger W 2008 Many-body physics with ultracold gases Rev. Mod. Phys. 80 885–964
[2] Bloch I, Dalibard J and Nascimbène S 2012 Quantum simulations with ultracold quantum gases Nat. Phys. 8 267–76
[3] Georgescu I M, Ashhab S and Nori F 2014 Quantum simulation Rev. Mod. Phys. 86 153–85
[4] Grimm R, Weidemuller M and Ovchinnikov Y B 2000 Optical dipole traps for neutral atoms Advances in Atomic, Molecular, and Optical Physics vol 42 (New York: Academic) pp 95–170
[5] Lin Y-J, Compton R L, Jimenez-Garcia K, Porto J V and Spielman I B 2009 Synthetic magnetic fields for ultracold neutral atoms Nature 462 628–32
[6] Lin Y-J, Jimenez-Garcia K and Spielman I B 2011 Spin–orbit-coupled Bose–Einstein condensates Nature 471 83–6
[7] González-Tudela A, Hung C-L, Chang D E, Cirac J I and Kimble H J 2015 Subwavelength vacuum lattices and atom–atom interactions in two-dimensional photonic crystals Nat. Photon. 9 320–5
[8] Chang D E, Douglas J S, González-Tudela A, Hung C L and Colloquium H J K 2018 Quantum matter built from nanoscopic lattices of atoms and photons Rev. Mod. Phys. 90 31002
[9] Economou E N 1969 Surface plasmons in thin films Phys. Rev. 182 539–54
[10] Midwinter J 1970 Evanescent field coupling into a thin-film waveguide IEEE J. Quant. Electron. 6 583–90
[11] Axelrod D, Thompson N L, Burghardt T P 1984 Ann. Rev. Biophys. Bioeng. 13 247–68
[12] Petersen J, Volz J and Rauschenbeutel A 2014 Chiral nanophotonic waveguide interface based on spin–orbit interaction of light Science 346 67–71
[13] Van Mechelen T and Jacob Z 2016 Universal spin-momentum locking of evanescent waves Conf. on Lasers and Electro-Optics, (CLEO 2016) vol 3
[14] Kalhori F, Thundat T and Jacob Z 2016 Universal spin-momentum locked optical forces Appl. Phys. Lett. 108 1–6
[15] Tseses S, Ostrovsky E, Cohen K, Gjonaj B, Lindner N H and Bartal G 2018 Optical skyrmion lattice in evanescent electromagnetic fields Science 361 993–6
[16] Skyrme T H R 1962 A unified field theory of mesons and baryons Nucl. Phys. 31 556–69
[17] Mühlbauer S, Binz B, Jonietz F, Pfleiderer C, Rosch A, Neubauer A, Georgii R and Bönß P 2009 Skyrmion lattice in a chiral magnet Science 323 915–9
[18] Yu X Z, Onose Y, Kanazawa N, Park J H, Han J H, Matsui Y, Nagaosa N and Tokura Y 2010 Real-space observation of a two-dimensional skyrmion crystal Nature 465 901–4
[19] Nagaosa N and Tokura Y 2013 Topological properties and dynamics of magnetic skyrmions Nat. Nanotechnol. 8 899–911
[20] Hartusch A, Sánchez E J, Xie X S and Novotny L 2003 High-resolution near-field Raman microscopy of single-walled carbon nanotubes Phys. Rev. Lett. 90 4
[21] Kawata S, Inouye Y and Verma P 2009 Plasmonics for near-field nano-imaging and superlensing Nat. Photon. 3 388–94
[22] Zhang X, Xia J, Zhou Y, Wang D, Liu X, Zhao W and Ezawa M 2016 Control and manipulation of a magnetic skyrmionium in nanostructures Phys. Rev. B 94 094420
[23] Ovchinnikov Y B, Shul’ga S V and Balykin V I 1991 An atomic trap based on evanescent light waves J. Phys. B: At. Mol. Opt. Phys. 24 3173–8
[24] Wu Y 1996 Effective Raman theory for a three-level atom in the Λ configuration Phys. Rev. A 54 1586–92
[25] Cohen-Tannoudji C Atom-Photon Interactions (Basic Processes and Applications) (New York: Wiley)
[26] Metcalf H J 1999 Laser cooling and trapping Graduate Texts in Contemporary Physics (Berlin: Springer) 1st edn 19 edition
[27] Aquilanti V, Haggard H M, Littlejohn R G and Yu L 2007 semiclassical analysis of Wigner Sj-symbol J. Phys. A: Math. Theor. 40 5637–74
[28] Aquilanti V, Haggard H M, Hedelman A, Jeveanjee N, Littlejohn R G and Yu L 2012 Semiclassical mechanics of the Wigner Sj-symbol J. Phys. A: Math. Theor. 45 065209
[29] See supplementary material
[30] Combes J M, Duclos P and Seiler R 1981 The Born–Oppenheimer approximation Rigorous Atomic and Molecular Physics (Berlin: Springer) pp 185–213
[31] Brion E, Pedersen L H and Mølmer K 2007 Adiabatic elimination in a lambda system J. Phys. B: At. Mol. Opt. Phys. 40 1–10
[32] Breuer H-P, Petruccione F 2002 The Theory of Open Quantum Systems (Oxford: Oxford University Press)
[33] Navarrete-Benlloch C, Vega I, Porras D and Ignacio Cirac J 2011 Simulating quantum-optical phenomena with cold atoms in optical lattices New J. Phys. 13 023024
[34] Shen H Z, Qin M and Yi X X 2013 Single-photon storing in coupled non-Markovian atom-cavity system Phys. Rev. A 88 033835
[35] Reitz D, Sayrin C, Mitsch R, Schneeweiss P and Rauschenbeutel A 2013 coherence properties of nanofiber-trapped cesium atoms Phys. Rev. Lett. 110 243603
[36] Tseses S, Cohen K, Ostrovsky E, Gjonaj B and Bartal G 2019 Spin–orbit interaction of light in plasmonic lattices Nano Lett. 19 4010–6
[37] Ping Liu J, Zhang Z and Skrymions G Z 2016 Skyrmions (Topological Structures, Properties, and Applications) (Boca Raton, FL: CRC Press)
[38] Bogdanov A and Hubert A 1999 The stability of vortex-like structures in uniaxial ferromagnets J. Magn. Magn. Mater. 195 182–92
[39] Finazzi M, Savoini M, Khorsand A R, Tsukamoto A, Itoh A, Duò L, Kirilyuk A, Rasing T and Ezawa M 2013 Laser-induced magnetic nanostructures with tunable topological properties Phys. Rev. Lett. 110 1–5
[40] Komineas S and Papanicolaou N 2015 Skyrmion dynamics in chiral ferromagnets under spin-transfer torque Phys. Rev. B 92 1–7
[41] Komineas S and Papanicolaou N 2015 Skyrmion dynamics in chiral ferromagnets Phys. Rev. B 92 1–10
[42] Zhang S, Kronast F, van der Laan G and Hesjedal T 2018 Real-space observation of skyrmionium in a ferromagnet–magnetic topological insulator heterostructure Nano Lett. 18 1057–63
[43] Kolesnikov A G, Stebliy M E, Samardak A S and Ognev A V 2018 Skyrmionium high velocity without the skyrmion hall effect Phys. Rev. Lett. 122 160601
[44] Vetsch E, Reitz D, Sagué G, Schmidt R, Dawkins S T and Rauschenbeutel A 2010 Optical interface created by laser-cooled atoms trapped in the evanescent field surrounding an optical nanofiber Phys. Rev. Lett. 104 203603
[45] McDonald M, Trindadi J, Yao K–X and Chin C 2019 Supersolution microscopy of cold atoms in an optical lattice Phys. Rev. X 9 021001
[46] Hümmel D, Romero-Isart O, Rauschenbeutel A and Schneeweiss P 2020 Probing surface-bound atoms with quantum nanophotonics (arXiv:2006.12853)
[47] Rui J, Wei D, Rubio-Abadal A, Hollerith S, Zeiher J, Stamper-Kurn D M, Gross C and Bloch I 2020 A subradiant optical mirror formed by a single structured atomic layer Nature 583 369–74
[48] Sommer A, Ku M, Roati G and Zwierlein M W 2011 Universal spin transport in a strongly interacting Fermi gas Nature 472 201–4
[49] Snoke D and Snoke D W 2009 Solid State Physics: Essential Concepts (Reading, MA: Addison-Wesley) 01
[50] Sanner C, Su E J, Huang W, Keshet A, Gillen J and Ketterle W 2012 Correlations and pair formation in a repulsively interacting fermi gas Phys. Rev. Lett. 108 240404
[51] Price H M and Cooper N R 2011 Skyrmion–antiskyrmion pairs in ultracold atomic gases Phys. Rev. A: At., Mol., Opt. Phys. 83 3–6
[52] Kasamatsu K, Tsubota M and Ueda M 2004 Vortex molecules in coherently coupled two-component Bose–Einstein condensates Phys. Rev. Lett. 93 1–4
[53] Cooper N R 2011 Optical flux lattices for ultracold atomic gases Phys. Rev. Lett. 106 1–4
[54] Cooper N R and Dalibard J 2011 Optical flux lattices for two-photon dressed states Europhys. Lett. 95 60004
[55] Dudarev A M, Diener R B, Carusotto I and Niu Q 2004 Spin–orbit coupling and berry phase with ultracold atoms in 2D optical lattices Phys. Rev. Lett. 92 1–4