Embedding based on function approximation for large scale image search

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Abstract—The objective of this paper is to design an embedding method that maps local features describing an image (e.g. SIFT) to a higher dimensional representation useful for the image retrieval problem. First, motivated by the relationship between the linear approximation of a nonlinear function in high dimensional space and the state-of-the-art feature representation used in image retrieval, i.e., VLAD, we propose a new approach for the approximation. The embedded vectors resulted by the function approximation process are then aggregated to form a single representation for image retrieval. Second, in order to make the proposed embedding method applicable to large scale problem, we further derive its fast version in which the embedded vectors can be efficiently computed, i.e., in the closed-form. We compare the proposed embedding methods with the state of the art in the context of image search under various settings: when the images are represented by medium length vectors, short vectors, or binary vectors. The experimental results show that the proposed embedding methods outperform existing the state of the art on the standard public image retrieval benchmarks.

1 INTRODUCTION

Finding a single vector representing a set of local descriptors extracted from an image is an important problem in computer vision. This single vector representation provides several important benefits. First, it contains the power of local descriptors, such as a set of SIFT descriptors [1]. Second, the representation vectors can be used in image retrieval problem (comparison using standard metrics such as Euclidean distance), or in classification problem (input to robust classifiers such as SVM). Furthermore, they can be readily used with the recent advanced indexing techniques [2], [3] for large scale image retrieval problem.

There is a wide range of methods for finding a single vector to represent a set of local descriptors proposed in the literature: bag-of-visual-words (BoW) [4], Fisher vector [5], vector of locally aggregated descriptor (VLAD) [6] and its improvements [7], super vector coding [9], vector of locally aggregated tensor (VLAD) [10], [11] which is higher order (tensor) version of VLAD, triangulation embedding (Temb) [12], sparse coding [13], local coordinate coding (LCC) [14], locality-constrained linear coding [15] which is fast version of LCC, local coordinate coding using local tangent (TLCC) [16] which is higher order version of LCC. Among these methods, VLAD [17] and VLAT [11] are well-known embedding methods used in image retrieval problem [11], [17] while TLCC [16] is one of the successful embedding methods used in image classification problem.

VLAD is designed for image retrieval problem while TLCC is designed for image classification problem. They are derived from different motivations: for VLAD, the motivation is to characterize the distribution of residual vectors over Voronoi cells learned by a quantizer; for TLCC, the motivation is to linearly approximate a nonlinear function in high dimensional space. Despite these differences, we show that VLAD is actually a simplified version of TLCC based on our original analysis. The consequence of this analysis is significant: we can depart from the idea of linear approximation of function to develop powerful embedding methods for the image retrieval problem.

In order to compute the single representation, all aforementioned methods include two main steps in the processing: embedding and aggregating. The embedding step uses a visual vocabulary (a set of anchor points) to map each local descriptor to a high dimensional vector while the aggregating step converts the set of mapped high dimensional vectors to a single vector. This paper focuses on the first step. In particular, we develop a new embedding method which can be seen as the generalization of TLCC and VLAT.

In the next sections, we first present a brief description of TLCC. Importantly, we derive the relationship between TLCC and VLAD. We then present our motivation for designing the new embedding method.

1.1 TLCC

TLCC [16] is designed for image classification problem. Its goal is to linearly approximate a smooth nonlinear function \( f(x) \), e.g. a nonlinear classification function, defined on a high dimensional feature space \( \mathbb{R}^d \). TLCC’s approach finds an embedding scheme \( \phi: \mathbb{R}^d \rightarrow \mathbb{R}^D \) mapping each \( x \in \mathbb{R}^d \) as

\[
x \mapsto \phi(x)
\]

such that \( f(x) \) can be well approximated by a linear function, namely \( w^T \phi(x) \). To solve above problem, TLCC’s authors relied on the idea of coordinate coding defined below. They showed that with a sufficient selection of coordinate coding, the function \( f(x) \) can be linearly approximated.

1. The “linear approximation” means that the nonlinear function \( f(x) \) defined on \( \mathbb{R}^d \) is approximated by a linear function \( w^T \phi(x) \) defined on \( \mathbb{R}^D \) where \( D > d \).
**Definition 1.1. Coordinate Coding [12]**

A coordinate coding of a point \( x \in \mathbb{R}^d \) is a pair \((\gamma(x), C)\)

where \( C = [v_1, \ldots, v_n] \in \mathbb{R}^{d \times n} \) is a set of \( n \) anchor points (bases), and \( \gamma \) is a map of \( x \in \mathbb{R}^d \) to \( \gamma(x) = [\gamma_{v_1}(x), \ldots, \gamma_{v_n}(x)]^T \in \mathbb{R}^n \) such that

\[
\sum_{j=1}^{n} \gamma_{v_j}(x) = 1
\]

(2)

It induces the following physical approximation of \( x \) in \( \mathbb{R}^d \):

\[
x' = \sum_{j=1}^{n} \gamma_{v_j}(x)v_j
\]

(3)

A good coordinate coding should ensure that \( x' \) closes to \( x \).

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Let \((\gamma(x), C)\) be coordinate coding of \( x \). Under assumption that \( f \) is \((\alpha, \beta, \nu)\) Lipschitz smooth, the authors showed (in lemma 2.2 [16]) that, for all \( x \in \mathbb{R}^d \)

\[
|f(x) - \sum_{j=1}^{n} \gamma_{v_j}(x) \left( f(v_j) + \frac{1}{2} \nabla f(v_j)^T (x - v_j) \right)|
\]

\[
\leq \frac{1}{2} \alpha \|x - x'\|_2 + \nu \sum_{j=1}^{n} |\gamma_{v_j}(x)| \|x - v_j\|_2
\]

(4)

To ensure a good approximation of \( f(x) \), the authors minimize the RHS of (4). Equation (4) means that the function \( f(x) \) can be linearly approximated by \( w^T \phi(x) \) where \( w = \left[ \frac{1}{2} f(v_j); \frac{1}{2} \nabla f(v_j) \right]_{j=1}^{n} \) and TLCC embedding \( \phi(x) \) defined as

\[
\phi(x) = [s \gamma_{v_1}(x); \gamma_{v_2}(x)(x - v_1); \ldots; \gamma_{v_n}(x)(x - v_n)]^T \in \mathbb{R}^{n(1 + d)}
\]

(5)

where \( s \) is a nonnegative constant.

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### 1.2 TLCC as generalization of VLAD

Although TLCC is designed for classification problem and its motivation is different from VLAD, TLCC can be seen as a generalization of VLAD. Specifically, if we add the following constraint to \( \gamma(x) \)

\[
\|\gamma(x)\|_0 = 0
\]

then we have \( x \approx x' = v_s \). The RHS of (4) becomes

\[
\frac{1}{2} \alpha \|x - v_s\|_2 + \nu \|x - v_s\|_2
\]

(7)

where \( v_s \) is anchor point corresponding to the nonzero element of \( \gamma(x) \). One solution for minimizing (7) under constraints (2) and (6) is K-means algorithm. When K-means is used, we have

\[
v_s = \arg\min_{v \in C} \|x - v\|_2
\]

(8)

where \( C \) is set of anchor points learned by K-means. Now, considering (5), if we choose \( s = 0 \) and remove zero elements attached with \( s \), \( \phi(x) = [0, \ldots, 0, (x - v_s)^T, 0, \ldots, 0]^T \in \mathbb{R}^{nd} \) will become VLAD.

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**1.3 Motivation for designing new embedding method**

The relationship between TLCC and VLAD suggests that if we can find \( \phi(x) \) such that \( f(x) \) can be well linearly-approximated \((f(x) \approx w^T \phi(x))\), then \( \phi(x) \) can be a powerful feature for image retrieval problem. In TLCC’s approach, by departing from assumption that \( f \) is \((\alpha, \beta, \nu)\) Lipschitz smooth, \( f \) is approximated using only its first order approximation at the anchor points, i.e., \( f \) is approximated as sum of weighted tangents at anchor points. It is not straightforward to use the TLCC framework to have a better approximation, i.e., approximation of \( f \) using its second order or higher order approximation at anchor points.

Therefore, in this paper, we propose to use the idea of Taylor expansion for function approximation. Using our proposed framework, it is more straightforward to achieve a higher order approximation of \( f \) at the anchor points. The embedded vectors, resulted by the proposed function approximation process, are used as new image representations in our image retrieval framework. In following sections, we note our Function Approximation-based embedding method as FAemb. In order to facilitate the use of the embedded features in large scale image search problem, we further derive its fast version. The main idea is to relax the function approximation bound such that the embedded features can be efficiently computed, i.e., in an analytic form. The proposed embedding methods are evaluated in image search context under various settings: when the images are represented by medium length vectors, short vectors, or binary vectors. The experimental results show that the proposed methods give a performance boost over the state of the art on the standard public image retrieval benchmarks.

Our previous work introduced FAemb method in [18]. This paper discusses substantial extension to our previous work: We detail the computational complexity of FAemb (Section 3.2). We propose the fast version of FAemb, i.e., FAST-FAemb (Section 4). We add a number of new experiments, i.e., results on large scale datasets (Oxford105k and Flickr1M), results when the single representation is compressed to compact binary codes (see Section 5.4). We also add new experiments when Convolutional Neural Networks (CNN) features are used instead of SIFT local features to describe the images; the comparison to the recent CNN/deep learning-based image retrieval is also provided (Section 5.5).

The remaining of this paper is organized as follows. Section 2 presents related background. Section 3 presents FAemb embedding method. Section 4 presents the fast version of FAemb. Section 5 presents experimental results. Section 6 concludes the paper. The implementation of the proposed embedding methods is released at http://tinyurl.com/F-FAemb.

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### 2 Preliminaries

In this section, we review related background to prepare for detail discussion of our new embedding method in Section 3.

**Taylor’s theorem for high dimensional variables**

**Definition 2.1. Multi-index [19]:** A multi-index is a d-tuple of nonnegative integers. Multi-indices are generally denoted by \( \alpha \):

\[
\alpha = (\alpha_1, \alpha_2, \ldots, \alpha_d)
\]

where \( \alpha_j \in \{0, 1, 2, \ldots\} \). If \( \alpha \) is a multi-index, we define

\[
|\alpha| = \alpha_1 + \alpha_2 + \cdots + \alpha_d; \alpha! = \alpha_1!\alpha_2! \cdots \alpha_d!
\]
\[ x^a = x_1^{a_1} x_2^{a_2} \ldots x_d^{a_d} \]

\[ \partial^a f(x) = \frac{\partial^{a_1} f(x)}{\partial x_1^{a_1}} \partial x_2^{a_2} \ldots \partial x_d^{a_d} \]

where \( x = (x_1, x_2, \ldots, x_d)^T \in \mathbb{R}^d \)

**Theorem 2.2.** (Taylor’s theorem for high dimensional variables) Suppose \( f: \mathbb{R}^d \to \mathbb{R} \) of class of \( C^{k+1} \) on \( \mathbb{R}^d \). If \( a \in \mathbb{R}^d \) and \( a + h \in \mathbb{R}^d \), then

\[ f(a + h) = \sum_{|\alpha| \leq k} \frac{\partial^a f(a)}{\alpha!} h^\alpha + R_{a,k}(h) \]  

(9)

where \( R_{a,k}(h) \) is Lagrange remainder given by

\[ R_{a,k}(h) = \sum_{|\alpha| = k+1} \frac{\partial^a f(a + ch)}{\alpha!} h^\alpha \]  

(10)

for some \( c \in (0, 1) \).

**Corollary 2.3.** If \( f \) is of class of \( C^{k+1} \) on \( \mathbb{R}^d \) and \( |\partial^a f(x)| \leq M \) for \( x \in \mathbb{R}^d \) and \( |\alpha| = k + 1 \), then

\[ |R_{a,k}(h)| \leq \frac{M}{(k+1)!} ||h||_1^{k+1} \]  

(11)

The proof of Corollary 2.3 is given in [19].

### 3 Embedding Based on Function Approximation (FAEbm)

#### 3.1 Derivation of FAEbm

Our embedding method is inspired from the function approximation based on Taylor’s theorem. It comes from the following lemma.

**Lemma 3.1.** If \( f: \mathbb{R}^d \to \mathbb{R} \) is of class of \( C^{k+1} \) on \( \mathbb{R}^d \) and \( \nabla f(x) \) is Lipschitz continuous with constant \( M > 0 \) and \( (\gamma(x), C) \) is coordinate coding of \( x \), then

\[ |f(x) - \sum_{j=1}^n \gamma_{v_j}(x)(f(v_j) + \nabla f(v_j)^T(x - v_j))| \leq \frac{M}{(k+1)!} \sum_{j=1}^n |\gamma_{v_j}(x)||x - v_j||_1^{k+1} \]  

(12)

The proof of Lemma 3.1 is given in Appendix A.

If \( k = 1 \), then (12) becomes

\[ |f(x) - \sum_{j=1}^n \gamma_{v_j}(x)(f(v_j) + \nabla f(v_j)^T(x - v_j))| \leq \frac{M}{2} \sum_{j=1}^n |\gamma_{v_j}(x)||x - v_j||_1^2 \]  

(13)

In the case of \( k = 1 \), \( f \) is approximated as sum of its weighted tangents at anchor points.

4. It means that all partial derivatives of \( f \) up to (and including) order \( k + 1 \) exist and continuous.
Algorithm 1 Offline learning of coordinate coding

Input:
\[ X = \{x_i\}_{i=1}^m \in \mathbb{R}^{d \times m}; \text{ training data; } T: \text{ max iteration number; } \epsilon: \text{ convergence error.} \]

Output:
\[ C: \text{ anchor points; } \Gamma: \text{ coefficient vectors of } X. \]

1: Initialize \( C^{(0)} \) using K-means
2: for \( t = 1 \rightarrow T \) do
3: \[ \Gamma^{(t)} = \emptyset \]
4: for \( t = 1 \rightarrow m \) do
5: Fix \( C^{(t-1)} \), compute \( \gamma_i \) for \( x_i \) using Newton’s method [21]
6: \[ \Gamma^{(t)} \leftarrow [\Gamma^{(t)}, \gamma_i] \]
7: end for
8: Fix \( \Gamma^{(t)} \), compute \( C^{(t)} \) using trust-region method [20].
9: if \( t > 1 \) and \( |Q_{FAemb}(\gamma^{(t)}) - Q_{FAemb}(\gamma^{(t-1)})| < \epsilon \) then
10: break;
11: end if
12: end for
13: Return \( C^{(t)} \) and \( \Gamma^{(t)} \)

vector of sample \( x_i; \Gamma = [\gamma_1, \ldots, \gamma_m] \in \mathbb{R}^{n \times m}. \) We find \( (\Gamma, C) \) which minimize the following constrained objective function
\[
Q_{FAemb}(\Gamma, C) = \frac{1}{m} \sum_{i=1}^{m} \left[ \frac{1}{2} \|x_i - C_i \gamma_i\|^2 + \frac{\mu}{2} \sum_{j=1}^{n} |\gamma_{ij}| \|x_i - \nu_j\|^2 \right]
\]
st. \( 1^T \gamma_i = 1, i = 1, \ldots, m \) (19)

3.2.2 The offline learning of coordinate coding and the online embedding

3.2.2.1 The offline learning of coordinate coding via alternating optimization

In order to minimize (19), we propose to iteratively optimize it by alternatingly optimizing with respect to \( C \) and \( \Gamma \) while holding the other fixed. For learning the anchor points \( C \), the optimization is equivalent to a regularized least squares problem. We use trust-region method [20] to solve this problem. For learning the coefficients \( \Gamma \), the optimization is equivalent to a regularized least squares problem with linear constraint. This problem can be solved by optimizing over each sample \( x_i \) individually. To find \( \gamma_i \) of each sample \( x_i \), we use Newton’s method [21].

The offline learning of coordinate coding for \( FAemb \) is summarized in Algorithm 1 In the Algorithm 1 \( C^{(t)} \), \( \Gamma^{(t)} \), \( Q_{FAemb}^{(t)} \) are values of \( C \), \( \Gamma \), \( Q_{FAemb} \) at iteration \( t \), respectively. The objective function value \( Q_{FAemb} \) after each iteration \( t \) in the Algorithm always does not increase (by the decreasing or unchanging of the objective value on both \( C \) and \( \Gamma \) steps). It can also be validated that the objective function value is lower-bounded, i.e., not smaller than 0. Those two points indicate the convergence of our algorithm. The empirical results show that the Algorithm 1 takes a few iterations to converge. Fig. 1 shows the example of the convergence of the algorithm.

3.2.2.2 The online embedding and its complexity

After learning anchor points \( C \), given a new descriptor \( x \), we achieve \( \gamma(x) \) by minimizing (18) using learned \( C \). From \( \gamma(x) \), we get the embedded vector \( \phi(x) \)-FAemb by using (17).

The computational complexity of the online embedding

The computational complexity of the online embedding depends on the computation of the coefficient \( \gamma(x) \) using Newton’s method and the computation of \( \phi(x) \) (given \( \gamma(x) \)) using (17). It is worth noting that in our experiments, the number of anchor points \( n = 8, 16, 32 \) is less than the dimension \( d = 45 \) of descriptor.

Computing \( \gamma(x) \): FAemb uses Newton’s method [21] for finding \( \gamma_i \). The main cost in the \((t + 1)^{th} \) iteration of Newton’s method lies in (i) computing the Hessian of objective function (19) and (ii) computing the Newton step \( \Delta \gamma_i \). The complexity for computing Hessian \( \nabla^2 Q_{FAemb} \) of (19) w.r.t. \( \gamma_i \) is \( O(n^2d) \). For finding the updating step \( \Delta \gamma_i \), Newton’s method solves the following equation
\[
\begin{bmatrix}

\nabla^2 Q_{FAemb}(\gamma^{(t)}) & 1 \\
1^T & 0
\end{bmatrix}
\begin{bmatrix}

\Delta \gamma_i \\
\omega
\end{bmatrix}
=
\begin{bmatrix}

-\nabla Q_{FAemb}(\gamma^{(t)}) \\
0
\end{bmatrix}
(20)
\]
where \( \omega \) is solution at the \( t^{th} \) iteration.

The size of 1st, 2nd, 3rd matrices in (20) is \((n + 1) \times (n + 1)\), \((n + 1) \times 1\) and \((n + 1) \times 1\), respectively. So, the complexity for solving (20) is \( O(n^2d) \). Overall, the complexity in one iteration of Newton’s method is \( O(n^2d) \).

For the stopping of Newton’s method, follow [21], we define a tolerance \( \epsilon \) on the objective function, i.e., the algorithm is terminated when \( Q_{FAemb}(\gamma^{(t+1)}, C) - p^* \leq \epsilon \), where \( p^* \) denotes the optimum value of objective function; \( \gamma^{(t+1)} \) is solution at the \((t + 1)^{th} \) iteration. [21] showed that this stopping criterion is equivalent to \((\delta^{(t+1)})^2 / 2 \leq \epsilon \), where \( \delta^{(t+1)} \) is Newton decrement at the \((t + 1)^{th} \) iteration and defined by the following equation which takes only \( O(n) \) in complexity.
\[
\delta^{(t+1)} = \left( -\nabla Q_{FAemb}(\gamma^{(t)}) \right)^T \Delta \gamma_i \cdot \frac{1}{2}
(21)
\]

Given a coordinate coding with 8 anchor points, a tolerance \( \epsilon = 10^{-6} \), we experiment on 100k descriptors and have the observation that \( k \approx 50 \) iterations on average for meeting the stopping criterion. Overall, the complexity of FAemb for finding \( \gamma_i \) is \( O(kn^2d) \).

Computing \( \phi(x) \) (using (17), given \( \gamma(x) \)): From (17), the complexity mainly depends on the computing the tensor between

5. The step-size \( \alpha \) for updating \( \gamma_i \), i.e., \( \gamma^{(t+1)} = \gamma^{(t)} + \alpha \Delta \gamma_i \) at each iteration is selected by empirical experiments and equals to 0.1 in our experiments.
x and v_j which takes O(d^2). So, the computational complexity for computing \( \phi'(x) \) is \( O(nd^2) \).

From above analysis, we find that the computational complexity of the whole embedding process of FAemb is dominated by the computing of \( \gamma(x) \).

3.3 Relationship to other methods

The most related embedding methods to FAemb are TLCC [16] and VLAT [10].

Compare to TLCC [16], our assumption on \( f \) in lemma 3.1 is different from the assumption of TLCC (lemma 2.2 [16]), i.e., our assumption only needs that \( \nabla^2 f(x) \) is Lipschitz continuous while TLCC assumes that all \( \nabla^2 f(x) \) are Lipschitz continuous, \( j = 1, \ldots, k \). Our objective function (18) is different from TLCC [4], i.e., we rely on \( l_1 \) norm of \( x - v_j \) in the second term while TLCC uses \( l_2 \) norm; we solve the constraint on the coefficient \( \gamma \) in our optimization process while TLCC does not. FAemb approximates \( f \) using \( F - FAemb \) up to its second order derivatives while TLCC approximates \( f \) only using its first order derivatives.

FAemb can also be seen as the generalization of VLAT [10]. Similar to the relationship of TLCC and VLAD presented in Section 3.2, if we add the constraint (6) to \( \gamma(x) \), the objective function (19) will become

\[
Q_1(\Gamma, C) = \frac{1}{m} \sum_{i=1}^{m} \left[ \frac{1}{2} \|x_i - v_*\|^2_2 + \frac{\mu}{2} \|x_i - v_+\|_1^2 \right] \quad \text{st. } 1^T \gamma_i = 1, i = 1, \ldots, m
\]

\[
\|v_i\|_0 = 1, i = 1, \ldots, m \quad (22)
\]

where \( v_* \) is anchor point corresponding to the nonzero element of \( \gamma \).

If we relax \( l_1 \) norm in the second term of \( Q_1(\Gamma, C) \) into \( l_2 \) norm, we can use K-means algorithm for minimizing (22). After learning \( C \) by using K-means, given an input descriptor \( x \), we have

\[
x \approx v_* = \arg\min_{v \in C} \|x - v\|_2 \quad (23)
\]

Now, consider (17), if we choose \( s_1 = 0, s_2 = 0 \) and we remove zero elements attached with them, \( \phi(x) = [0, \ldots, 0, (V ((x - v_*)(x - v_+)^T))^T, 0, \ldots, 0]^T \in \mathbb{R}^{nd^2} \) will become VLAT.

In practice, to make a fair comparison between FAemb and VLAT, we assign \( s_1, s_2 \) in (17) to 0. This makes the embedded vectors produced by two methods have same dimension. It is worth noting that in (17), as matrix \((x - v_*)(x - v_+)^T\) is symmetric, only the diagonal and upper part are kept while flattening it into vector. The size of VLAT and FAemb is then \( \frac{nd(d+1)}{2} \).

4 Fast embedding based on function approximation (F-FAemb)

FAemb needs an iterative optimization at the online embedding step. While FAemb is applicable for small/medium-size datasets, it may not be suitable for large scale datasets. In this section, we develop the fast version of FAemb. The main idea is to find reasonable relaxation for the function approximation bound of FAemb (i.e., the RHS of (14)) such that the coefficient vector \( \gamma(x) \) can be efficiently computed, i.e., it can be computed in a closed-form.

4.1 Derivation of F-FAemb

The relaxed bound is based on the following observation

\[
1 = \sum_{j=1}^{n} \gamma_v(x) \leq \|\gamma(x)\|_1 \leq n \|\gamma(x)\|_2^2 \quad (24)
\]

Thus,

\[
B_{FAemb} \leq \frac{nM}{6} \|\gamma(x)\|_2^2 \sum_{j=1}^{n} \|x - v_j\|_1^3 \quad (25)
\]

We define the relaxed bound \( B_{F-FAemb} \) as

\[
B_{F-FAemb} = \frac{nM}{6} \|\gamma(x)\|_2^2 \sum_{j=1}^{n} \|x - v_j\|_1^3 \quad (26)
\]

means that the relaxed bound \( B_{F-FAemb} \) is still upper bound of the function approximation, i.e., the LHS of (14). This relaxed bound allows analytic solution for the embedding as shown in Section 4.2.

Similar to FAemb, in order to ensure a good reconstruction error (which is necessary condition for a good coordinate coding) and a good function approximation, we jointly minimize over the reconstruction error and the bound \( B_{F-FAemb} \). Specifically, the coordinate coding is learned by minimizing the following constrained objective function

\[
Q_{F-FAemb}(\Gamma, C) = \min_{C, \Gamma} \frac{1}{m} \sum_{i=1}^{m} \left[ \frac{1}{2} \|x_i - C\gamma_i\|^2_2 + \frac{\mu}{2} \|\gamma_i\|_2^2 \sum_{j=1}^{n} \|x_i - v_j\|_1^2 \right] \quad \text{st. } 1^T \gamma_i = 1, i = 1, \ldots, m \quad (27)
\]

4.2 The offline learning of coordinate coding and the online embedding

4.2.1 The offline learning of coordinate coding via alternating optimization

Similar to FAemb, in order to optimize (27), we alternatingly optimize with respect to \( C \) and \( \Gamma \) while holding the other fixed.

For learning the anchor points \( C \), the optimization problem is unconstrained regularized least squares. We use trust-region method [20] for solving.

For computing the coefficients \( \Gamma \), we can solve over each sample \( x_i \) individually. The optimization problem is equivalent to a \( l_2 \) regularized least squares problem with linear constraint. Thus, we achieve the closed-form for the solution.

Let \( a = [(\|x_i - v_1\|_1^2, \ldots, \|x_i - v_n\|_1^2]^T : a = 1^T a; B = (C^T C + a\mu I)^{-1} \). Let

\[
\lambda = \frac{1^T B C^T x_i - 1}{1^T B 1} \quad (28)
\]

We have the closed-form for the coefficient vector as

\[
\gamma_{F-FAemb} = B(C^T x_i - \lambda I) \quad (29)
\]

where \( I \) is identity matrix having size of \( n \times n \); \( 1 \) is column vector having \( n \) elements equaling to 1.

It is worth noting that although the function bound of F-FAemb is the relaxed version of the function bound of FAemb, F-FAemb provides an optimum solution on the coefficient vector while FAemb does not. This explains for the results that the performance F-FAemb is competitive to FAemb in our experiments.
4.2.2 The online embedding and its complexity

After learning anchor points $C$, given a new input $x$, we use \( (29) \) for computing the coefficient vector $\gamma$. After getting the coefficient vector, the embedded vector $\phi(x)$ is achieved by using \( (17) \). Similar to FAemb, the values of $s_1$, $s_2$ in \( (17) \) are assigned to 0.

**Computing $\gamma(x)$:** From \( (29) \), the computational complexity for computing $B$, $\lambda$ and $\gamma$ is $O(n^2d)$, $O(nd)$ and $O(nd)$, respectively. So the overall computational complexity for computing $\gamma(x)$ is $O(nd)$.

**Computing $\phi(x)$:** As presented in Section 3.2.2, the computational complexity for computing $\phi(x)$ is $O(nd^2)$.

As in our experiments, the number of anchor points $n$ (= 8, 16, 32) is less than the dimension $d$ (= 45) of descriptor, the complexity of the whole embedding process of FAemb is dominated by the computing of $\phi(x)$.

4.3 The computational complexity comparison between FAemb/F-FAemb and other methods

In this section, we compare the computational complexity to embed a local descriptor of FAemb/F-FAemb and other methods which also use high order (i.e., second order) information for embedding such as VLAT \( [10] \), [11], Fisher \( [17] \), \( [22] \).

The fifth row of Table \( [1] \) presents the asymptotic complexity (hence the constant of the complexity is different for each method). Note that, although the dimension of local descriptors ($d$) and the number of anchor points ($n$) of methods are different, the dimension of the embedded vector produced by methods are comparable. It is worth noting that for Fisher \( [17] \), \( [22] \), although the complexity is $O(nd)$, it has a large constant, i.e., by computing posterior probabilities, the gradient with respect to both the mean and the standard deviation.

The sixth row of Table \( [1] \) presents the timing to embed a local descriptor. For Fisher, we use the implementation provided by VLFeat \( [23] \) where the implementation is optimized with mex files. For standard VLAT \( [10] \), we re-implement it as there is no Matlab implementation available. The experiments are run on a processor core (2.60 GHz Intel CPU). We report the CPU times which is larger than elapsed ones because CPU time accumulates all active threads. From \( 6^{th} \) row of Table \( [1] \) F-FAemb is $\sim$ 11 times faster than FAemb. F-FAemb is slower than VLAT while it is faster than Fisher. In practical, F-FAemb takes less than 1s to embed an image having 1,000 local descriptors. This efficient computation allows F-FAemb to be used in large scale problems and in applications requiring fast retrieval. It is worth noting that the embedding can be further speeded up by optimizing the implementation, i.e., using mex files. In our experiments, when using mex file implementation for computing $\phi(x)$ \( [17] \), F-FAemb takes only 0.20 ms to embed a local descriptor.

5 Experiments

This section presents results of the proposed FAemb and F-FAemb embedding methods and compare them to the state of the art. Specifically, in Section 5.3, we compare FAemb, F-FAemb to other three methods: VLAD \( [17] \), Temb \( [12] \) and VLAT \( [10] \) when the same test bed are used. In Section 5.4, we compare FAemb, F-FAemb to the state of the art under various setting, i.e., when the images are represented by mid-size vectors, short vectors, or binary vectors. Furthermore, in Section 5.5 we present the results when Convolutional Neural Network features are used as the local features to describe the images. The comparison to the recent CNN/deep learning-based image retrieval are also provided.

5.1 Dataset and evaluation metric

INRIA holidays \( [24] \) consists of 1,491 images of different locations and objects, 500 of them being used as queries. The search quality is measured by mean average precision (mAP), with the query removed from the ranked list. In order to evaluate the search quality on large scale, we merge Holidays dataset with 1M negative images downloaded from Flickr \( [25] \), forming the Holidays+Flickr1M dataset. For this large scale dataset, following common practice \( [12] \), we evaluate search quality on the short representations of the aggregated vectors. For all learning stages, we use a subset from the independent dataset Flickr60k provided with Holidays.

Oxford buildings \( [26] \) consists of 5,063 images of buildings and 55 query images corresponding to 11 distinct buildings in Oxford. Each query image contains a bounding box indicating the region of interest. When local SIFT features are used, we follow the standard protocol \( [7] \), \( [8] \), \( [12] \): the bounding boxes are cropped and then used as the queries. This dataset is often referred to as Oxford5k. The search quality is measured by mAP computed over the 55 queries. Images are annotated as either relevant, not relevant, or junk, which indicates that it is unclear whether a user would consider the image as relevant or not. Following the recommended configuration \( [6] \), \( [7] \), \( [12] \), the junk images are removed from the ranking before computing the mAP. In order to evaluate the search quality on large scale, Oxford5k is extended with 100k negative images \( [26] \), forming the Oxford105k dataset. For all learning stages, we use the Paris6k dataset \( [27] \).

5.2 Implementation details

5.2.1 Local descriptors

Local descriptors are detected by the Hessian-affine detector \( [28] \) and described by the SIFT local descriptor \( [1] \). RootSIFT variant \( [29] \) is used in all our experiments. For VLAT, FAemb, F-FAemb, at beginning, the SIFT descriptors are reduced from 128 to 45 dimensions using PCA. This makes the dimension of VLAT,
FAemb, and F-FAemb comparable to dimension of compared embedding methods.

5.2.2 Whitenning and aggregating the embedded vectors

Whitenning Successful instance embedding methods consist of several feature post-processing steps. In [12], authors showed that by applying the whitening processing, the discriminative power of embedded vectors can be improved, hence improving the retrieval results. In particular, given \( \phi(x) \in \mathbb{R}^D \), we achieve whitened embedded vectors \( \phi_w(x) \) by

\[
\phi_w(x) = \text{diag} \left( \lambda_1^{-\frac{1}{2}}, \ldots, \lambda_D^{-\frac{1}{2}} \right) P^T \phi(x)
\]

(30)

where \( \lambda_i \) is the \( i \)th largest eigenvalue. \( P \in \mathbb{R}^{D \times D} \) is matrix formed by the largest eigenvectors associated with the largest eigenvalues of the covariance matrix computed from learning embedded vectors \( \phi(x) \).

In [12], the authors further indicated that by discarding some first components associated with the largest eigenvalues of \( \phi_w(x) \), the localization of whitened embedded vectors will be improved. We apply this truncation operation in our experiments. The setting of this truncation operation is detailed in Section 5.3.

Aggregating Let \( \mathcal{X} = \{ x \} \) be set of local descriptors describing the image. Sum-pooling [30] and max-pooling [31, 32] are two common methods for aggregating set of whitened embedded vectors \( \phi_w(x) \) of the image to a single vector. Sum-pooling lacks discriminability because the aggregated vector is more influenced by frequently-occurring uninformative descriptors than rarely-occurring informative ones. Max-pooling equalizes the influence of frequent and rare descriptors. However, classical max-pooling approaches can only be applied to BoW or sparse coding features. Recently, in [12], the authors introduced a new aggregating method named democratic aggregation applied to image retrieval problem. This method bears similarity to generalized max-pooling [33] applied to image classification problem. Democratic aggregation can be applied to general features such as VLAD, Fisher vector [17], Temb [12]. The authors [12] showed that democratic aggregation achieves better performance than sum-pooling. The main idea of democratic aggregation is to find a weight for each \( \phi_w(x) \) such that \( \forall x_i \in \mathcal{X} \)

\[
\lambda_i \left( \phi_w(x_i) \right)^T \sum_{x_j \in \mathcal{X}} \lambda_j \phi_w(x_j) = 1
\]

(31)

In summary, the process for producing the single vector from the set of local descriptors describing the image is as follows. First, we map each \( x \in \mathcal{X} \rightarrow \phi(x) \) and whiten \( \phi(x) \), producing \( \phi_w(x) \). We then use democratic aggregation to aggregate vectors \( \phi_w(x) \) to the single vector \( \psi \) by

\[
\psi(\mathcal{X}) = \sum_{x_i \in \mathcal{X}} \lambda_i \phi_w(x_i)
\]

(32)

5.2.3 Power-law normalization

The burstiness visual elements [34], i.e., numerous descriptors almost similar within the same image, strongly affects the measure of similarity between two images. In order to reduce the effect of burstiness, we follow the common practical setting [6, 12]: applying power-law normalization [35] to the final image representation \( \psi \) and subsequently \( L_2 \) normalize it. The power-law normalization is applied to each component \( a \) of \( \psi \) by \( a := |a|^{\alpha} \text{sign}(a) \), where \( 0 \leq \alpha \leq 1 \) is a constant. We standardly set \( \alpha = 0.5 \) in our experiments.

5.3 Impact of parameters and comparison between embedding methods

In this section, we compare FAemb, F-FAemb to other three methods including VLAD [17], Temb [12] and VLAT [10] under same test bed, i.e., the whitening, the democratic aggregation, and the power-law normalization are applied for all five embedding methods. We reimplement VLAD and VLAT in our framework. For Temb [12], we use the implementation provided by the authors.

Follow the suggestion in [12], for Temb and VLAD methods, we discard first \( d \) components of \( \phi_w(x) \). The final dimension of \( \phi_w(x) \) is therefore \( D = (n-1) \times d \). For VLAT, FAemb and F-FAemb methods, we discard first \( \frac{d \times (d+1)}{2} \) components of \( \phi_w(x) \). The final dimension of \( \phi_w(x) \) is therefore \( D = \frac{(n-1) \times (d+1)}{2} \). The value of \( \mu \) in [19] and [17] is selected by empirical experiments and is fixed to \( 10^{-2} \) for all FAemb, F-FAemb results reported bellow.

The comparison between the implementation of VLAD and VLAT in this paper and their improved versions [11, 17] on Holidays dataset is presented in Table 2. It is worth noting that even with a lower dimension, the implementation of VLAD and VLAT in our framework (RootSIFT descriptors, VLAD/VLAT embedding, whitening, democratic aggregation and power-law normalization) achieves better retrieval results than their improved versions reported by the authors [11, 17].

5.3.1 Impact of parameters

The main parameter here is the number of anchor points \( n \). The analysis for this parameter is shown in Fig. 2 and Fig. 3 for Holidays and Oxford5k datasets, respectively. We can see that the mAP increases with the increasing of \( n \) for all four methods. For all methods, the improvement tends to be smaller for larger \( n \). This phenomenon has been discussed in [12]. For larger vocabularies, the interaction between descriptors is less important than for small ones. For VLAT, FAemb and F-FAemb, we do not report the results for \( n > 32 \) as with this setting, the democratic aggregation is very time consuming. It has been indicated in [12] that when
dimension of the embedded vector is high, e.g., $>32,000$, the benefit of democratic aggregation is not worth the computational overhead.

5.3.2 Comparison between embedding methods

We find the following observations are consistent on both Holidays and Oxford5k datasets.

The mAP of F-AEmb is slightly better than the mAP of F-AEmb at small $n$, i.e., $n = 8$. When $n$ is large, i.e. $n = 32$, F-AEmb and F-AEmb achieve very competitive results.

At same $n$, F-AEmb, F-AEmb, and VLAT have same dimension. However, F-AEmb and F-AEmb improve the mAP over VLAT by a fair margin. For examples, at $n = 8$, the improvement of F-AEmb over VLAT is +1.8% and +3.9% on Holidays and Oxford5k, respectively. At $n = 16, 32$, the improvement is about +3% on both datasets.

At comparable dimensions, F-AEmb and F-AEmb significantly improve the mAP over VLAD and Temb. For examples, comparing F-AEmb at ($n = 16, D = 15,525$) with VLAD/Temb at ($n = 128, D = 16,256$), the gain of F-AEmb over VLAD/Temb is +7.5%/+2% on Holidays and +8.1%/+5% on Oxford5k.

5.4 Comparison with the state of the art

In this section, we compare our framework with benchmarks having similar representation, i.e., they represent an image by a single vector. Due to the efficient computation of F-AEmb, it not only allows F-AEmb to use more anchor points for the function approximation but also allows F-AEmb to work on large scale datasets. Thus, we put more interest on F-AEmb when comparing to the state of the art. The main differences between the compared embedding methods are shown in Table 3

In VLATimproved [11], VLAT [7] and CVLAD [37], PCA and sum pooling are applied on Voronoi cells separately. Then, pooled vectors are concatenated to produce the single representation. In addition to methods listed in Table 3 we also compare with the recent embedding method VLAD_{LCS}+Exemplar SVM (VLAD_{LCS}+ESVM) [38] and Convolutional Neural Network features. We consider the recent work [39] as the baseline for CNN-based image retrieval. In [38], the authors use the exemplar SVMs (linear SVMs trained with one positive example only and a vast collection of negative examples) as encoders. For each image, its VLAD_{LCS} representation is used as positive example for training an exemplar SVM. The weight vector (hyperplane) of the trained exemplar SVM is used as new representation. In [39], the authors use the deep Convolutional Neural Network (CNN) model proposed in [40] for extracting image presentation. The network is first trained on ImageNet dataset [41]. It is then retrained on the Landmarks dataset [39] containing ~213,000 images which are more relevant to the Holidays and the Oxford5k datasets. The activation values invoked by an image within top layers of the network are used as the image representation. It is worth noting that training CNN [39] is a supervised training task coming with challenges including: (i) the requirement for large amounts of labeled training data. According to [39], the collecting of the labeled Landmarks images is a nontrivial task; (ii) the high computational cost and the requiring of GPUs. Contrary to CNN [39], the training for our embedding is totally unsupervised, requiring of only several ten thousands of unlabeled images and without requiring of GPUs.

5.4.1 Evaluation on Holidays and Oxford5k datasets

Table 4 shows the results of F-AEmb, F-AEmb, and the compared methods on Holidays and Oxford5k datasets.

Without RN post-processing, F-AEmb outperforms or is competitive to most compared methods. CNN features [39] achieve best performance on the Holidays dataset; its mAP is higher than F-AEmb ($n = 32$) +2.3%. However, on the Oxford5k dataset,
F-FAemb outperforms CNN features \([39]\) by a fair margin, i.e., +16.2%.

When RN is used, it boosts performance for all Temb, FAemb and F-FAemb. The performance of F-FAemb +RN at \(D = 7,245\) is slightly lower than Temb+RN at \(D = 8,064\). However, at higher dimension, i.e., \(D = 15,525\), the performance of F-FAemb + RN outperforms all performances of Temb+RN a fair margin.

The efficient computation of F-FAemb allows it to use high number of anchor points, i.e., \(n = 32\); and at this setting, F-FAemb +RN outperforms all compared methods on both datasets. The gain is more significant on the Oxford5k dataset, i.e., F-FAemb +RN outperforms the recent embedding method VLAD\(_{LC}\) + ESVM \([38]\) +16.7% and outperforms the CNN features \([39]\) +19.7%. It is worth noting that the dimension of the CNN features is lower than ones of F-FAemb +RN. We evaluate the performance of F-FAemb +RN in case of short representation in Section 5.4.3.

### 5.4.2 Evaluation on large scale dataset: Oxford105k

The Oxford105k dataset is used for large scale testing in a few benchmarks \([7, 12, 39]\). The comparative mAP between methods is shown in Table 5. The results show that even with a lower dimension, the proposed F-FAemb (at \(D = 7,245\)) outperforms the compared methods (VLAD\(_{LC}\), Temb+RN) by a large margin. The best result of F-FAemb (i.e. at \(D = 15,525\)) sets up state-of-the-art performance on this large scale dataset. It outperforms the current state of the art, i.e., Temb+RN \([12]\) +7%.

### 5.4.3 Evaluation on short representation

As the F-FAemb features are high-dimensional, a question of their performance on short representations arises. In this section, we evaluate the performance of F-FAemb at short representations achieved by keeping only first components after the rotation normalization of aggregated vectors. Table 6 reports comparative mAP for varying dimensionality.

Compare to Temb+RN at the same dimension, Temb+RN is slightly better than our method on Holidays dataset while our method is better than Temb+RN on other three datasets. Especially on large scale datasets Oxford105k and Holidays+Flickr1M, our method significantly improves the mAP over Temb+RN. On Oxford105k, the gains are +3% and +4.8% at 1,024 and 512 dimensions, respectively. On Holidays+Flickr1M, the gains are +19.1% and +18.4% at 1,024 and 512 dimensions, respectively.

Compare to CNN features having 4,096 dimension, the performance of CNN features is higher than our method +5.2% on Holidays dataset, but we see much larger variances on Oxford5k and Oxford105k datasets. The gains of our method over CNN features on Oxford5k and Oxford105k are +9.2% and +11%, respectively.

### 5.4.4 Evaluation on binary representation

Two main problems which need to be considered in large scale image search are fast searching and efficient storage. An attractive approach for handling those problems is to represent each image by a very compact code, i.e., binary code. In this section, we further evaluate the performance of the proposed F-FAemb when the single representation is compressed to compact binary codes. In order to achieve compact codes for the single representation, we use the state-of-the-art hashing method Iterative Quantization (ITQ) \([42]\). The ITQ has two main steps: the first step is to apply PCA for dimensionality reduction; the second step is to seek an optimal rotation matrix which rotates the projected data to binary codes.

We compare our F-FAemb to the recent embedding method Temb \([12]\) when both of them are compressed to binary codes. The comparative results are presented in Table 7 on Holidays dataset,
F-FAemb achieves better results than Temb for all code lengths; the improvement increases with the increasing of the code length. On Oxford5k and Oxford105k datasets, Temb is better than F-FAemb at low code length, i.e., 128-bit codes. However, F-FAemb outperforms Temb when the number of bits is increase, i.e., > 128; the improvement is more clear at high code lengths.

### 5.5 Results when CNN are used as local features

In this section we further evaluate the proposed F-FAemb when the image is described by a set of CNN features which are state-of-the-art image representation for various computer vision tasks [43]. Specifically, instead of using set of local SIFT features to describe the image as previous experiments, we extract CNN activations for local patches at multiscale levels. We then take the union of all the patches from the image, regardless of scale. This union set can be considered as local features describing the image. The same processing (i.e., F-FAemb embedding, whitening, democratic aggregating, rotation normalization, power normalization) is applied on the set of CNN features to produce the single representation. We use the output of the last fully connected layer of the pretrained AlexNet model [44] as CNN features representing for patches. We extract CNN activations at 3 levels. For the first level, we simply take 4096-dimensional CNN activations for the whole image. For the second and the third levels, we extract CNN activations for all $128 \times 128$, $64 \times 64$ patches sampled with a stride of 32 pixels. In order to make the computation of the embedding more efficient, we use PCA to reduce 4096-dimensional features to 45-dimensional features.

We compare our CNN features-based F-FAemb with the state of the art which use Convolutional Neural Network, deep learning techniques for image retrieval [43, 45, 46, 47]. Note that in this experiment, we follow [43] which uses the full query images without cropping when evaluating the Oxford5k dataset.

In [45], the authors propose a new family of convolutional descriptors for patch presentations based on the Convolutional Kernel Networks (CKN) [48] in which the learning is performed in a unsupervised manner, i.e., instead of learning filters by optimizing a loss function as Convolutional Neural Networks, CKN is trained to approximate a particular nonlinear match-kernel, and therefore no labeled data is required. The network is trained on 1M patches from RomePatches dataset [45]. In order to achieve the single representation for an image, they first extract Hessian-Affine regions [28] in the image, feed them to the trained CKN, producing a high dimensional vector (i.e., 41K dimensions) for each patch. The PCA-whitening is applied to reduce these vectors to 1024-dimensional vectors. After that they use VLAD encoding [17] with 256 centroids to encode the reduced vectors, resulting a very high dimensional (i.e., 262K dimensions) single VLAD representation. Finally, they apply PCA-whitening to reduce VLAD vector to 4,096 dimensional vector, used for retrieval.

In [46], the authors combine Fisher Vector encoding [5, 35] and Deep Neural Network into a single model (FV-DNN) for producing the single representation. Specifically, in their hybrid architecture, the image patches are described by PCA-projected SIFT and color descriptors [49]. The projected SIFT and color descriptors are then separately encoded using Fisher encoding. These two Fisher vectors are concatenated; the concatenated vector is subsequently normalized by square-rooting and $l_2$ normalization. The $l_2$ normalized vector is PCA-projected and $l_2$ normalized again. The resulted vector is fed into supervised layers which is a deep neural network comprising by a sequence of 3 fully connected (hidden) layers, ending by a softmax layer. The model is trained using the ImageNet dataset [41]. The activations of the first supervised hidden layer are used as the image feature in their image retrieval task. In order to boost the performance, they train 8 different models using different random seed initializations. They concatenates the representations produced by 8 models, and reduce the dimension of the concatenated representation with PCA-whitening.

In [47], the authors propose a CNN-based embedding method named multiscale orderless pooling (MOP-CNN). This scheme first extracts CNN activations for local patches at multiple scales. To extract CNN activations for patches, they use the AlexNet model [44] pretrained on ImageNet dataset [41]. They extract CNN activations at three levels. For the first level, they take the 4096-dimensional CNN activations for the whole image. For the remaining two levels, they extract activations for all $128 \times 128$ and $64 \times 64$ patches with a stride of 32 pixels, resulting 4096-dimensional activation vectors. In order to make the computation more efficient, they use PCA to reduce CNN activation vectors on the second and the third levels to 500-dimensional vectors. After that they use soft assignment VLAD encoding [50] to encode the reduced vectors of the second level and the third level separately, resulting 50,000-dimensional VLAD vector for each level. They further perform PCA on the VLAD encoded vectors and reduce them to 4096 dimensions. They concatenate the 4096-dimensional CNN feature at the first level and two reduced VLAD encoded vectors at the second and the third level to form a single image representation, and reduce the dimension of the concatenated representation with PCA-whitening. Note that the way we process CNN features differs from [47]. Instead encoding the CNN features of levels separately and then concatenating encoded features which causes the dimension of the concatenated vector rapidly increasing, we first make union CNN features of all levels and then perform the encoding only one time. This make the dimension of the final representation independent to the number of levels.

In [43], the authors use the OverFeat model [51] pretrained on the ImageNet [41] to extract CNN features. The output 4096-dimensional vector of the first fully connected layer of the network is further post-processing, i.e., $l_2$ normalization, whitening, $l_2$ renormalization, power normalization, and is used as the feature representation for the retrieval. At the retrieval stage, they use the spatial search as follows. For each image they extract multiple patches of different sizes at different scales. For each extracted patch they compute its CNN representation. The distance between

### TABLE 7

mAP comparison between F-FAemb and Temb in binary representation with varying code lengths on three datasets (Holidays, Oxford5k and Oxford105k). The original dimension of F-FAemb and Temb are 7, 245 and 8, 004, respectively.

| Dataset  | Method | Code length (bits) |
|----------|--------|--------------------|
|          |        | 128 256 512 1024   |
| Holidays | Temb [12] | 39.2 46.5 53.0 57.3 |
|          | F-FAemb [14] | 40.1 47.9 54.5 59.7 |
| Ox5k     | Temb [12] | 27.1 33.1 38.5 43.4 |
|          | F-FAemb [14] | 26.4 32.8 40.7 45.9 |
| Ox105k   | Temb [12] | 25.9 31.6 37.7 42.9 |
|          | F-FAemb [14] | 24.2 32.0 38.5 44.7 |
a query patch and a reference image is defined as the minimum $l_2$ distance between the query patch and respective reference patches. The distance between the query and the reference image is set to the average distance of each query patch to the reference image.

As presented, similar to F-FAemb, all approaches in [43], [45], [46], [47] apply an whitening on the single representation. Thus, for clear presentation, we ignore the +RN notation when presenting results of F-FAemb. The mAP of F-FAemb and compared methods is shown in Table 8. The short vector representations of F-FAemb (i.e., when $D < 7, 245$) is achieved by keeping only first components after the rotation normalization (whitening) step.

On the Holidays dataset, at the same dimension, F-FAemb slightly improves over FV-DNN [46], CNN [43] and considerably improves over MOP-CNN [47]. On the Oxford5k dataset, at the same dimension, F-FAemb is on par to KNN [45] while it is lower than CNN [43]. However it is worth noting that the spatial search used in [43] is a costly searching as it has two drawbacks. Firstly, all the patch vectors of the image have to be stored. This increases the memory requirements by a factor of $P$ where $P$ is number of extracted patches per image. Secondly, the complexity for computing the distance between two images is increased by a factor of $P^2$. Contrary to [43], we only store a single representation per image and compute only one Euclidean distance when comparing two images.

### 6 Conclusion

Embedding local features to high dimensional space is a crucial step for producing the single powerful image representation in many state-of-the-art large scale image search systems. In this paper, by departing from the goal of linear approximation of a nonlinear function in high dimensional space, we first propose a novel embedding method. The proposed embedding method, FAemb, can be seen as the generalization of several well-known embedding methods such as VLAD, TLCC, VLAT. In order to speed up the embedding process, we then derive the fast version of FAemb, in which the embedded vector can be efficiently computed, i.e., in the closed-form. The proposed embedding methods are evaluated with different state-of-the-art local features such as SIFT, CNN, in image search context under various settings. The experimental results show that the proposed embedding methods give a performance boost over the state of the art on several standard public image retrieval benchmarks.

6. The results of CKN [45] on Oxford5k is with cropped queries. According to [45], the cropped queries achieve better performance than full queries, i.e., their best mAP with full queries is 55.4.

### APPENDIX A

#### PROOF OF LEMMA 3.1

By assumption, we have (i) $f$ is of class $C^{k+1}$. Because $\nabla^k f(x)$ is Lipschitz continuous with constant $M > 0$, we have $\|\nabla^{k+1} f(x)\|_2 \leq M$. So for $|\alpha| = k + 1$, we have (ii) $|\alpha| \leq \|\nabla^{k+1} f(x)\|_2 \leq M$. (i) and (ii) make the condition of the Corollary 2.3 is held.

We have

$$f(x) - \sum_{j=1}^{n} \gamma_{v_j}(x) \sum_{|\alpha| \leq k} \frac{\partial^\alpha f(v_j)}{\alpha!} (x - v_j)^\alpha = \sum_{j=1}^{n} \gamma_{v_j}(x) \left(f(x) - \sum_{|\alpha| \leq k} \frac{\partial^\alpha f(v_j)}{\alpha!} (x - v_j)^\alpha\right) \leq \sum_{j=1}^{n} \gamma_{v_j}(x) \left| x - v_j \right|^{k+1} = \frac{M}{(k+1)!} \sum_{j=1}^{n} \gamma_{v_j}(x) \left| x - v_j \right|^{k+1}.$$

where the last inequation comes from the Corollary 2.3.

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