Excited nucleon spectrum from lattice QCD with maximum entropy method

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We study excited states of the nucleon in quenched lattice QCD with the spectral analysis using the maximum entropy method. Our simulations are performed on three lattice sizes $16^3 \times 32$, $24^3 \times 32$ and $32^3 \times 32$, at $\beta = 6.0$ to address the finite volume issue. We find a significant finite volume effect on the mass of the Roper resonance for light quark masses. After removing this systematic error, its mass becomes considerably reduced toward the direction to solve the level order puzzle between the Roper resonance $N'(1440)$ and the negative-parity nucleon $N^*(1535)$.

A particular interest in the excited nucleon spectra is the level order of the positive-parity excited nucleon $N'(1440)$ (the Roper resonance) and the negative-parity nucleon $N^*(1535)$. This pattern of the level order is also seen universally in the excited states of flavor octet and decuplet baryons. The quark confining models such as the harmonic oscillator quark model or the MIT bag model have difficulties in reproducing the Roper as the first excited state of the nucleon [1].

First systematic lattice QCD calculation for the nucleon excited states in both parity channels shows that the wrong ordering between $N'$ and $N^*$ also takes place at least for heavy-quark masses [2]. After this work, many lattice calculations confirmed this puzzle, which are summarized in Ref. [3].

Recently, it was pointed out that the finite lattice volume affects the structure of the Roper resonance considerably for light quark masses [3,4]. If the “wave function” of a quark inside baryon is squeezed due to the small volume, the kinetic energy of internal quarks increases and the total energy of the bound state should be pushed up. Such effect is expected to become serious for radially excited states as compared to the ground state. In the present report, we show that the Roper resonance particularly receives large finite volume effect: its mass after removing the finite volume effect moves toward the direction to solve the level order puzzle.

In the numerical simulations, we generate quenched QCD configurations with standard single-plaquette Wilson action mainly at $\beta = 6.0$ ($a^{-1} \sim 2\text{GeV}$) for three different lattices, $16^3 \times 32$ (444 conf.), $24^3 \times 32$ (350 conf.), and $32^3 \times 32$ (200 conf.) to study finite volume effect. The quark propagators are computed using the Wilson fermion action at four values of the hopping parameter, which cover the range $M_\pi/M_\rho \simeq 0.69 - 0.92$. We adopt the conventional interpolating operator $\epsilon_{abc}(u^T_\alpha C\gamma_5 d_b)u_c$ to create the positive parity states of the nucleon.

To extract the mass of the excited nucleon, we utilize the maximum entropy method (MEM) [5] which can reconstruct the spectral function (SPF) $A(\omega)$ from given Monte Carlo data of two point hadron correlator $G(t) = \int d\omega A(\omega)\exp(-\omega t)$. Details of the MEM analysis can be found in [4,5].

First we show dimensionless SPF of the nucleon for the largest lattice size at the lightest quark mass as a typical example in Fig.1. There are two sharp peaks and two large bumps. The crosses on each peak or bump represent the statistical significance of SPF obtained by the MEM [5].

We remark that the first two peaks are physical states but two large bumps are unphysical states. This is confirmed by the observation that those bumps appear at the same dimensionless frequency $\omega a$ in simulations with different lattice
spacings. It indicates that those states become infinitely heavy and decouple from physical states in the continuum limit. The similar observation in the mesonic case has been reported in Ref. [6].

In Fig. 2 we plot the masses of the ground state (open circles) and the first excited state (filled circles), which correspond to the peak positions of first two peaks in SPF, as functions of the spatial lattice size $L$. The quoted errors are estimated by the jackknife method. Observed finite volume effect is clearly sizable for light quarks (the right panel) and its effect is larger for the excited state than the ground state. Dashed horizontal lines in Fig. 2 express the masses in the infinite volume limit and their errors guided by a formula $M_L = M_\infty + cL^{-3}$ [7].

Taking a simple linear extrapolation toward the chiral limit in Fig. 3 we find $M_{N'} = 0.73(14)$ in lattice units. Our $N'$ mass is evidently smaller than the previously published results for the $N^*$ at the same lattice spacing; $M_{N^*} = 0.85(5)$ [2] from domain wall fermions and $M_{N^*} = 0.89(2)$ [8] from clover fermions.

Next we demonstrate the importance of the large spatial volume in a different way. We calculated the quark mass dependence of the mass splitting between the ground state and radially excited state. From the phenomenological point of view, such a mass splitting is almost independent of the quark mass as pointed out in Ref. [3].

Shown in Fig. 4 is our lattice results of the $N-N'$ mass splitting for the largest size lattice $La = 3$ fm. As is evident from the figure, the mass splittings are independent of the quark mass within the statistical error bars. This result is different from those shown in Ref. [9].

In summary, we studied the excited states of
the nucleon with the use of MEM. On the basis of a systematic analysis with three different lattice volumes \((La = 1.5, 2.2\) and \(3.0\) fm), we confirmed our previous finding that the Roper resonance receives large finite volume effect for small quark masses. We found that, for enough large spatial size \((\gtrsim 3.0\text{ fm})\), the mass splitting of the ground and first excited states of the nucleon is independent of the quark mass. This is consistent with the experimental mass splittings of baryons and mesons for wide range of quark masses \([9]\). By removing the finite volume effect, the mass of the Roper resonance becomes considerably reduced and the level switching between the Roper and the negative parity resonance may take place for sufficiently small quark masses.

Needless to say, further study is required to check other systematic error stemming from a discretization artifact. Additive simulations at different lattice spacings are under way.

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