Complex Scalar DM in a B-L Model

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Abstract

In this work, we implement a complex scalar Dark Matter (DM) candidate in a $U(1)_{B-L}$ gauge extension of the Standard Model. The model contains three right handed neutrinos with different quantum numbers and a rich scalar sector, with extra doublets and singlets. In principle, these extra scalars can have VEVs ($V_{\Phi}$ and $V_{\phi}$ for the extra doublets and singlets, respectively) belonging to different energy scales. In the context of $\zeta \equiv \frac{V_{\Phi}}{V_{\phi}} \ll 1$, which allows to obtain naturally light active neutrino masses and mixing compatible with neutrino experiments, the DM candidate arises by imposing a $Z_2$ symmetry on a given complex singlet, $\phi_2$, in order to make it stable. After doing a study of the scalar potential and the gauge sector, we obtain all the DM dominant processes concerning the relic abundance and direct detection. Then, for a representative set of parameters, we found that a complex DM with mass around 200 GeV, for example, is compatible with the current experimental constraints without resorting to resonances. However, additional compatible solutions with heavier masses can be found in vicinities of resonances. Finally, we address the issue of having a light CP-odd scalar in the model showing that it is safe concerning the Higgs and the $Z_\mu$ boson invisible decay widths, and also the energy loss in stars astrophysical constraints.

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I. INTRODUCTION

Currently, it is well established from several observations and studies of the Universe on different scales that most of its mass is constituted of dark matter (DM) \[1\text{-}5\]. Although, the nature of DM is still a challenging question, the solution based on the existence of a new kind of neutral, stable and weakly interacting massive particles (WIMPs) is both well motivated and extensively studied. This is mainly due to two reasons. The first reason is that WIMPs appearing in a plethora of models \[6\text{-}16\] give “naturally” the observed relic abundance, \(\Omega_{\text{DM}} h^2 = 0.1199 \pm 0.0027 \[5\]. The second reason is that WIMPs may be accessible to direct detection. Currently, there is a variety of experiments involved in the search for direct signals of WIMPs which have imposed bounds on spin-independent WIMP-nucleon elastic scattering \[17\text{-}19\].

It is also well known that, although the Standard Model (SM) has been tremendously successful in describing electroweak and strong interaction phenomena, it must be extended. Physics beyond the SM has both theoretical and experimental motivations. For instance, the neutrino masses and mixing, which are required for giving a consistent explanation for the solar and atmospheric neutrino anomalies, is one of the most firm evidences to go beyond the SM. Another motivation is providing a satisfactory explanation to the nature of the DM. This last reason is the focus of our work. The preferred theoretical framework which provides a DM candidate is supersymmetry \[6\text{-}9\]. However, many other interesting scenarios have been proposed \[10\text{-}16\]. In this paper, we focus on the possibility of having a viable scalar DM candidate in a \(U(1)\) gauge extension of the SM. In particular, this model, sometimes referred as the flipped \(B-L\) model \[20, 21\] has a very rich scalar content, which allows us to obtain a complex scalar DM candidate.

The outline of this paper is the following. In Sec. II we briefly summarize the model under consideration. In Sec. III we study the vacuum structure and the scalar sector spectrum that allows us to have a viable complex scalar DM candidate in the model. In particular, we considered the scalar potential in the context of \(\zeta \equiv \frac{V_\Phi}{V_\phi} \ll 1\), where \(V_\Phi\) and \(V_\phi\) are the vacuum expectation values, VEVs, of the doublets \(\Phi_{1,2}\) and the singlets \(\phi_{1,3,X}\) respectively. In Sec. IV we present the gauge sector and choose some parameters that simplify the study of the DM candidates. In Sec. V we calculate the thermal relic density of the complex scalar DM candidate and present a set of parameters that are consistent with the current
observations. In Sec. [VI] we summarize the main features of our study. Finally, in the Appendix, we show the general minimization conditions used to calculate the scalar mass spectrum.

II. BRIEF REVIEW OF THE $B - L$ MODEL

We briefly summarize here the model in Refs. [20, 21]. It is an extension of the SM based on the gauge symmetry $SU(2)_L \otimes U(1)_{Y'} \otimes U(1)_{B-L}$ where $B$ and $L$ are the usual baryonic and leptonic numbers, respectively, and $Y'$ is a new charge different from the hypercharge $Y$ of the SM. The values of $Y'$ are chosen to obtain the hypercharge $Y$ through the relation $Y = [Y' + (B-L)]$, after the first spontaneous symmetry breaking. Assuming a generation-independent charge assignment, the non-existence of mirror fermions and restricting ourselves to integer quantum numbers for the $Y'$ charge, the anomaly cancellation constrains the number of right-handed neutrinos, $n_R \geq 3$ [20]. Considering $n_R = 3$, there is an exotic charge assignment for the $Y'$ charge where $Y'_{n_R1,n_R2} = -4$ and $Y'_{n_R3} = 5$ besides the usual one where $Y'_{n_Ri} = 1$ with $i = 1, 2, 3$. The model under consideration has that exotic $Y'$ charge assignment. The respective fermionic charge assignment of the model is shown in Table I.

| Fermion | $I_3$ | $I$ | $Q$ | $Y'$ | $B-L$ |
|---------|------|----|-----|------|------|
| $\nu_{eL}, e_L$ | 1/2, −1/2 | 1/2 | 0, −1 | 0 | −1 |
| $e_R$ | 0 | 0 | −1 | −1 | −1 |
| $u_L, d_L$ | 1/2, −1/2 | 1/2 | 2/3, −1/3 | 0 | 1/3 |
| $u_R$ | 0 | 0 | 2/3 | 1 | 1/3 |
| $d_R$ | 0 | 0 | −1/3 | −1 | 1/3 |
| $n_{1R}, n_{2R}$ | 0 | 0 | 0 | 4 | −4 |
| $n_{3R}$ | 0 | 0 | 0 | −5 | 5 |

Table I: Quantum number assignment for the fermionic fields.

In the scalar sector the model has three $SU(2)_L$ doublets, $H$, $\Phi_1$, $\Phi_2$, and four $SU(2)_L$ singlets, $\phi_1, \phi_2, \phi_3, \phi_X$. The scalar charge assignments are shown in Table II. The $H$ doublet is introduced to give mass to the lighter massive neutral vector boson $Z_{1\mu}$, the charged vector bosons $W^\pm_\mu$, and the charged fermions, as in the SM. Besides giving mass to the extra neutral
vector boson $Z_{2\mu}$, which is expected to be heavier than $Z_{1\mu}$, the other scalars are mainly motivated to generate mass for both the left and the right handed neutrinos. In order to be more specific, the other $\Phi_1$ and $\Phi_2$ doublets are introduced to give Dirac mass terms at tree level through the renormalizable Yukawa interactions $D_{im}\overline{L}_i n_{Rm}\Phi_1$ and $D_{i3}\overline{L}_i n_{R3}\Phi_2$ in the Lagrangian. The $\phi_1$, $\phi_2$ and $\phi_3$ singlets are introduced to generate Majorana mass terms at tree level ($M_{mn}(n_{Rm})^c n_{Rn}\phi_1$, $M_{33}(n_{R3})^c n_{R3}\phi_2$, $M_{m3}(n_{Rm})^c n_{R3}\phi_3$). Finally, the $\phi_X$ singlet is introduced to avoid dangerous Majorons when the symmetry is broken down as shown in Ref. [21]. These extra scalars allow the model to implement a see-saw mechanism at $O(\text{TeV})$ energy scale, and the observed mass-squared differences of the neutrino are obtained without resorting to fine-tuning the neutrino Yukawa couplings [21]. Other studies about the possibility that the model accommodates different patterns for the neutrino mass matrix using discrete symmetries ($S_3$, $A_4$) have been done [22, 23].

| Scalar | $I_3$ | $I$ | $Q$ | $Y'$ | $B - L$ |
|--------|------|-----|-----|------|--------|
| $H^{0,+}$ | $\mp 1/2$ | $1/2$ | 0, 1 | 1 | 0 |
| $\Phi_{1}^{0,-}$ | $\pm 1/2$ | $1/2$ | 0, $-1$ | $-4$ | 3 |
| $\Phi_{2}^{0,-}$ | $\pm 1/2$ | $1/2$ | 0, $-1$ | 5 | $-6$ |
| $\phi_1$ | 0 | 0 | 0 | $-8$ | 8 |
| $\phi_2$ | 0 | 0 | 0 | 10 | $-10$ |
| $\phi_3$ | 0 | 0 | 0 | 1 | $-1$ |
| $\phi_X$ | 0 | 0 | 0 | 3 | $-3$ |

Table II: Quantum number assignment for the scalar fields.

With the above matter content we can write the most general Yukawa Lagrangian respecting the gauge invariance as follows

$$- \mathcal{L}_Y = Y_{i}^{(l)} \overline{L}_i e R_i H + Y_{ij}^{(d)} \overline{Q}_{Li} d_{Rj} H + Y_{ij}^{(u)} \overline{Q}_{Li} u_{Rj} \tilde{H} + D_{im} \overline{L}_i n_{Rm} \Phi_1 + D_{i3} \overline{L}_i n_{R3} \Phi_2$$

$$+ M_{mn}(n_{Rm})^c n_{Rn} \phi_1 + M_{33}(n_{R3})^c n_{R3} \phi_2 + M_{m3}(n_{Rm})^c n_{R3} \phi_3 + \text{H.c.},$$

(1)

where $i, j = 1, 2, 3$ are lepton/quark family numbers; $m, n = 1, 2$, and $\tilde{H} = i\tau_2 H^*$ ($\tau_2$ is the Pauli matrix). Also, we have omitted summation symbols over repeated indices.

Finally, the most general renormalizable scalar potential obtained by the addition of all
these above mentioned scalar fields is given by

\[
V_{B-L} = -\mu_H^2 H^\dagger H + \lambda_H |H^\dagger H|^2 - \mu_{11}^2 \Phi_1^\dagger \Phi_1 + \lambda_{11} |\Phi_1^\dagger \Phi_1|^2 - \mu_{22}^2 \Phi_2^\dagger \Phi_2 + \lambda_{22} |\Phi_2^\dagger \Phi_2|^2
\]

\[-\mu_{sα}^2 |φ_α|^2 + \lambda_{sα} |φ_α^* φ_α|^2 + \lambda_{212} |Φ_1| |Φ_2|^2 + \lambda_{12} (Φ_1^\dagger Φ_2)(Φ_2^\dagger Φ_1) + \Lambda_{Hγ} |H|^2 |Φ_γ|^2
\]

\[+ \Lambda'_{Hγ}(H^\dagger Φ_γ)(H) + Λ_{Hsα} |H|^2 |φ_α|^2 + Λ'_{sα} |Φ_α|^2 |φ_α|^2 + Δ_{αβ}(φ_α^* φ_α)(φ_β^* φ_β)
\]

\[+ \left[ \tilde{β}_{123 φ_1 φ_2} (φ_3^* χ) + \tilde{β}_{123 φ_1 φ_3} (φ_2^* χ) + \tilde{β}_{23 φ_2 φ_3} (φ_1^* χ) - iκ_{H1X} Φ_1^\dagger τ_2 H φ_X
\]

\[−iκ_{H2X}(Φ_2^T τ_2 H)(φ_χ^* χ) + β_X (φ_X^* φ_1)(φ_2 φ_3) + β_3X (φ_X^* φ_3^2) + H.c. \right],
\]

where γ = 1, 2; α, β = 1, 2, 3, X; and α ≠ β in the Δ_{αβ}(φ_α^* φ_α)(φ_β^* φ_β) terms.

### III. THE VACUUM STRUCTURE AND THE SCALAR SECTOR SPECTRUM

In general, DM must be stable to provide a relic abundance in agreement with the one measured by WMAP and PLANCK, Ω_{DM} h^2 = 0.1199 ± 0.0027 \cite{4,5}. Although the DM stability could result from the extreme smallness of its couplings to ordinary particles, we restrict ourselves to search for a discrete, or continuous, symmetry such as Z_2, or U(1), to protect DM candidates to decay.

First, we consider the scalar potential in Eq. (2) looking for an accidental symmetry that naturally stabilizes the DM candidate. Doing so, we find that the scalar potential has just the SU(2) ⊗ U(1)_Y ⊗ U(1)_{B-L} initial symmetry. However, none of these gauge groups can generate a stable neutral scalar when they are spontaneously broken down to U(1)_Q. Therefore, we impose a discrete symmetry in the following way: Z_2(φ_2) = −φ_2 and the other scalar fields being even under this Z_2 symmetry. As a result, the β_{23 φ_1 φ_2 φ_3} (φ_1 φ_2 φ_3) and β_X (φ_X φ_1)(φ_2 φ_3) terms are prohibited from appearing in the scalar potential, Eq. (2). Actually, when these terms are eliminated from Eq. (2), the true global symmetry in the potential is SU(2) ⊗ U(1)_Y ⊗ U(1)_{B-L} ⊗ U(1)_χ, where the last one is U(1)_χ : φ_2 → exp(−iχφ_2)φ_2, where χφ_2 is the φ_2 quantum number under the U(1)_χ symmetry, and the rest of the fields being invariant. It is important to say that we have taken into account the simplicity and some phenomenological criteria to choose the Z_2 symmetry above. For example, if we impose Z_2(φ_1) = −φ_1 (and the other fields being even), the model has a massless right handed neutrino, say N_R, at tree level. That poses a tension with the experimental data of the invisible Z_μ decay width \cite{24}, since Z_μ → N_R + N_R would be allowed to exist \cite{25}. Other simple choices such as Z_2(φ_3) = −φ_3 or Z_2(φ_X, Φ_1) = −φ_X, −Φ_1 should be avoided due
to the appearance of Majorons, $J_s$, in the scalar spectra. As it is well known, the major challenges to models with Majorons come from the energy loss in stars, through the process $\gamma + e^- \rightarrow e^- + J$, and the invisible $Z_\mu$ decay width, through $Z_\mu \rightarrow RJ \rightarrow JJJ$, being $R$ a scalar [26].

For the general case of the scalar potential with the $U(1)_\chi$ symmetry, we have the minimization conditions given in the Appendix. In general, those conditions lead to different breaking symmetry patterns and to a complex vacuum structure because the scalar potential has several free parameters. In this paper, however, we restrict ourselves to find a (some) viable scalar DM candidate(s) and to study its (their) properties in a relevant subset of the parameter space.

First, we impose the necessary conditions for all real neutral components of the scalar fields, except $\phi_{2R}$, to obtain nontrivial vacuum expectation values (VEVs), i.e. $\langle H^0_R \rangle = V_H$, $\langle \Phi^0_{1R} \rangle = V_{\Phi_1}$, $\langle \phi_{1R} \rangle = V_{\phi_1}$, $\langle \phi_{2R} \rangle = 0$, $\langle \phi_{3R} \rangle = V_{\phi_3}$, $\langle \phi_{XR} \rangle = V_{\phi_X}$. For the sake of simplicity, we set $V_{\Phi_1} = V_{\Phi_2} = V_{\Phi}$ and $V_{\phi_1} = V_{\phi_3} = V_{\phi_X} = V_{\phi}$. Thus, the $U(1)_\chi$ symmetry is not spontaneously broken and the model possesses two neutral stable scalars which are the real (CP-even) and the imaginary (CP-odd) parts of the $\phi_2$ field with the same mass given by

$$M_{DM}^2 = \frac{1}{2} \left[ \Lambda_{Hs2} V_{SM}^2 + (\Lambda_{12} + \Lambda_{22} - 2 \Lambda_{Hs2}) V_{\phi}^2 + (\Delta_{12} + \Delta_{23} + \Delta_{2X}) V_{\phi}^2 - 2 \mu_{s2}^2 \right] ;$$

(3)

where we have defined $V_{SM}^2 \equiv V_H^2 + V_{\Phi_1}^2 + V_{\Phi_2}^2 = V_H^2 + 2V_{\phi}^2 = (246)^2$ GeV$^2$. From here on, we work with $M_{DM}^2$ as an input parameter, thus we solve Eq. (3) for $\mu_{s2}^2$

$$\mu_{s2}^2 = \frac{1}{2} \left[ \Lambda_{Hs2} V_{SM}^2 + (\Lambda_{12} + \Lambda_{22} - 2 \Lambda_{Hs2}) V_{\phi}^2 + (\Delta_{12} + \Delta_{23} + \Delta_{2X}) V_{\phi}^2 - 2 M_{DM}^2 \right] .$$

(4)

If we allow $\langle \phi_2 \rangle \neq 0$, the real part of the $\phi_2$ field obtains mass and its imaginary part is massless and stable. In that case, the DM candidate would be the Goldstone boson related to the breakdown of the $U(1)_\chi$ symmetry. In general, such massless DM has severe constraints from the big bang nucleosynthesis [27, 28] and the bullet cluster [14, 29]. Here we do not consider this case.

Also, we work in the context of $\zeta \equiv \frac{V_{\phi}^2}{V_{\phi}^2} \ll 1$. This assumption allows us to implement a stable and natural see-saw mechanism for neutrino masses at low energies, as shown in Ref. [21]. Once $V_H^2 + 2V_{\phi}^2 = (246)^2$ GeV$^2$ and $V_H$ is the mainly responsible to give the top quark mass at tree level, we have $V_H^2 \gg V_{\phi}^2$. Choosing $V_{\phi} \sim 1$ TeV and $V_{\phi} \sim 1$ MeV, as in Ref. [21], we have that the $\zeta$ parameter is $\sim 10^{-6}$.
In general, we solve numerically the minimization conditions, and using standard pro-
cedures we construct numerically the mass-squared matrices for the charged, CP-even and
CP-odd scalar fields. We choose the parameters in the potential such that they satisfy simulta-
neously the minimization conditions, the positivity of the squared masses and the lower
boundedness of the scalar potential. In order to satisfy this last condition, we choose the
parameters such that the quartic terms in the scalar potential are positive for all directions.
Although, all those constraints are checked numerically, let us give an insight into some con-
straints coming from the minimization conditions and the positivity of the squared masses
when we do some simplifying assumption on the parameters. First, we solve the Eqs. (31)
and (37) in the limit $\zeta \rightarrow 0$. Doing so, we have
\[
\mu_H = \pm \sqrt{\lambda_H V_{SM}^2 + \frac{1}{2} (\Lambda_{H s1} + \Lambda_{H s3} + \Lambda_{H sX}) V_{\phi}^2} + \mathcal{O}(\zeta);
\]
\[
\kappa_{H1X} = \mathcal{O}(\zeta); \quad \kappa_{H2X} = \mathcal{O}(\zeta);
\]
\[
\mu_{s1} = \pm \sqrt{\frac{\Lambda_{H s1} V_{SM}^2 + (\Delta_{13} + \Delta_{1X} + 2\lambda_{s1}) V_{\phi}^2}{\sqrt{2}}} + \mathcal{O}(\zeta);
\]
\[
\mu_{s3} = \pm \sqrt{\frac{\Lambda_{H s3} V_{SM}^2 + (3\beta_{3X} + \Delta_{13} + \Delta_{3X} + 2\lambda_{s3}) V_{\phi}^2}{\sqrt{2}}} + \mathcal{O}(\zeta);
\]
\[
\mu_{sX} = \pm \sqrt{\frac{\Lambda_{H sX} V_{SM}^2 + (\beta_{3X} + \Delta_{1X} + \Delta_{3X} + 2\lambda_{sX}) V_{\phi}^2}{\sqrt{2}}} + \mathcal{O}(\zeta);
\]
From Eq. (6), we see that $\kappa_{H1X} \rightarrow 0$ and $\kappa_{H2X} \rightarrow 0$ when $\zeta \rightarrow 0$ (and keeping $V_\phi$ finite).
Thus, in our calculations we choose $\kappa_{H1X} \sim V_\Phi$ and $\kappa_{H2X} \sim V_\phi/V_\phi$.

To simplify the squared masses and obtain useful analytical expressions, let us consider
$\lambda_{11} = \lambda_{22} = \lambda_{s1} = \lambda_{s3} = \lambda_{sX}; \; \Lambda_{H1} = \Lambda_{H2} = \Lambda_{H s1} = \Lambda_{H s3} = \Lambda_{H sX} = \Lambda'_{H1} = \Lambda'_{H2};$
$\Lambda'_{11} = \Lambda'_{13} = \Lambda'_{1X} = \Lambda'_{21} = \Lambda'_{23} = \Lambda'_{2X} = \lambda_{12} = \lambda'_{12} = \Delta_{13} = \Delta_{1X} = \Delta_{3X}; \; \Lambda'_{12} = \Lambda'_{22} = \Delta_{12} = \Delta_{23} = \Delta_{2X}$ and the other parameters without restrictions. The previous constraints
have been inspired by the similitude of the respective potential terms. We have left free the
parameters that involve the DM candidates. Also, we have assumed that the $H$ scalar field
is the Higgs-like field in this model. Doing these considerations, we have, apart from the
Goldstone bosons that are eaten by the $W^\pm$ bosons, two charged scalars, $C_{1,2}^\pm$, with masses
given by
\[
m_{C_{1,2}^\pm}^2 = \frac{1}{4} \left[ 2\Lambda_{H1} V_{SM}^2 + (1 + \sqrt{2}) V_{SM} V_\phi \right]
\]
\[ V_\phi \left( \sqrt{3 - 2\sqrt{2}} V_{\phi}^2 + 4\beta_1^2 V_{\phi}^2 + 2\beta_{13} V_\phi \right) \] 

In the CP-odd scalar sector, we have, besides the two Goldstone bosons which give mass to the \( Z_{1\mu} \) and \( Z_{2\mu} \) gauge bosons, the following mass eigenvalues:

\[ m_{I_3}^2 = \mathcal{O}(\zeta); \quad m_{I_4}^2 = M_{DM}^2; \quad m_{I_7}^2 = -5\beta_{3X} V_{\phi}^2 + \mathcal{O}(\zeta); \quad m_{I_5,I_6}^2 = \frac{1}{4} V_\phi \left( 1 + \sqrt{2} \right) V_{SM} - 2\beta_{13} V_\phi \mp \sqrt{4\beta_1^2 V_{\phi}^2 + \left( 3 - 2\sqrt{2} \right) V_{SM}^2} + \mathcal{O}(\zeta); \quad (11) \]

Finally, in the CP-even scalar sector we have \( m_{R_4}^2 = M_{DM}^2 \), and

\[ m_{R_5,R_6}^2 = \frac{1}{4} V_\phi \left( 1 + \sqrt{2} \right) V_{SM} - 2\beta_{13} V_\phi \mp \sqrt{4\beta_1^2 V_{\phi}^2 + \left( 3 - 2\sqrt{2} \right) V_{SM}^2} + \mathcal{O}(\zeta); \quad (12) \]

the other mass eigenvalues are not shown for shortness. As shown in the above expressions in the \( \mathcal{O}(\zeta) \) we have three degenerate mass eigenstates, i.e. \( m_{R_4}^2 = m_{I_4}^2, m_{R_5}^2 = m_{I_5}^2 \) and \( m_{R_6}^2 = m_{I_6}^2 \). Imposing that all these masses are positive, we find the following conditions:

\[ M_{DM} > 0 \land \beta_{3X} < 0 \land \left[ \left( \Lambda_{H2}' > 0 \land \beta_{13} V_\phi + \sqrt{2} V_{SM} < 2 V_{SM} \right) \lor \left( V_\phi > -2 \left( \sqrt{2} - 1 \right) \Lambda_{H2}' V_{SM} \land \beta_{13} < \frac{V_{SM} \left( \Lambda_{H2}' V_{SM} + V_\phi \right)}{V_{SM}^2 \left( 2\Lambda_{H2}' V_{SM} + (1 + \sqrt{2}) V_\phi \right)} \land \Lambda_{H2}' \leq 0 \right) \right]. \quad (14) \]

Despite the fact that the Eqs. (13\textendash}14) are only valid in the limit \( \zeta \to 0 \), these relations will be useful in our analysis, at least as a starting point.

IV. GAUGE BOSONS

In this model the gauge symmetry breaking proceeds in two stages. In the first stage, the real components of the \( \phi_1, \phi_3, \phi_X \) fields obtain VEVs, say \( V_\phi \), as discussed in the previous section. Once this happens, the gauge symmetry is broken down to \( SU(2)_L \otimes U(1)_Y \), where \( Y \) is the usual hypercharge of the SM. In the second stage, the electrically neutral components of the \( H, \Phi_{1,2} \) obtain VEVs, \( V_H \) and \( V_\Phi \), respectively, thus, breaking down the symmetry to \( U(1)_Q \).

The mass terms for the three electrically neutral \( SU(2)_L \otimes U(1)_{Y'} \otimes U(1)_{B-L} \) gauge bosons (\( W_3^3, B_{\mu}^{Y'}, \text{ and } B_{\mu}^{B-L} \)) arise from the kinetic terms for the scalar fields upon replacing
corresponding mass eigenstates, we diagonalize

$$
M_{\text{Gauge Bosons}}^2 = \begin{bmatrix}
g^2 (K + P + 2N) & -ggY' (K + N) & -gB-L (P + N) \\
-ggY' (K + N) & gY' K & gY' gB-L N \\
-gB-L (P + N) & gY' gB-L N & gB-L P
\end{bmatrix};
$$

where $g$, $gY'$ and $gB-L$ are the $SU(2)_L$, $U(1)_Y$, $U(1)_{B-L}$ coupling constants, respectively. $K$, $P$, $N$ are defined by $K \equiv \frac{1}{4} \sum_a V_a^2 Y_a^{\prime 2}$, $P \equiv \frac{1}{4} \sum_a V_a^2 (B-L)_a^2$, $N \equiv \frac{1}{4} \sum_a V_a^2 Y_a^\prime (B-L)_a$; with $Y'_a$ and $(B-L)_a$ being the quantum numbers given in the Tables[1] and [1]. Considering our aforementioned assumptions we have:

$$
K = \frac{1}{4} (V_H^2 + 41V_F^2 + 74V_0^2), \quad P = \frac{1}{4} (45V_F^2 + 74V_0^2), \quad N = \frac{1}{4} (42V_F^2 + 74V_0^2).
$$

In order to obtain the relation between the neutral gauge bosons $(W^3_\mu, B^Y'_\mu, B^{B-L}_\mu)$ and the corresponding mass eigenstates, we diagonalize $M_{\text{Gauge Bosons}}^2$. Doing so, we have:

$$
\gamma_\mu = \frac{1}{N_2} \left[ \frac{1}{g} W^3_\mu + \frac{1}{gY'} B^Y_\mu + \frac{1}{gB-L} B^{B-L}_\mu \right];
$$

$$
Z_{1\mu} = \frac{1}{N_{Z_1}} \left[ g (P g^{2 B-L}_B - N g^2_{Y'} - M_{Z_1}^2) W^3_\mu - gY' ((P + N) g^2 + P g^2_{B-L} - M_{Z_1}^2) B^Y_\mu + gB-L ((P + N) g^2 + N g^2_{Y'}) B^{B-L}_\mu \right];
$$

$$
Z_{2\mu} = \frac{1}{N_{Z_2}} \left[ g (P g^{2 B-L}_B - N g^2_{Y'} - M_{Z_2}^2) W^3_\mu - gY' ((P + N) g^2 + P g^2_{B-L} - M_{Z_2}^2) B^Y_\mu + gB-L ((P + N) g^2 + N g^2_{Y'}) B^{B-L}_\mu \right];
$$

where $N_\gamma$, $N_{Z_1}$, $N_{Z_2}$ are the corresponding normalization constants. Also, $\gamma_\mu$ corresponds to the photon, and $Z_{1\mu}$ and $Z_{2\mu}$ are the two massive neutral vector bosons of the model, and their squared masses are given by $M_\gamma^2 = 0$, and

$$
M_{Z_{1\mu},Z_{2\mu}}^2 = \frac{1}{2} R + \frac{1}{2} \left[ R^2 - 4 (KP - N^2) (g^2 (g^2_{Y'} + g^2_{B-L}) + g^2_{Y'} g^2_{B-L}) \right]^{1/2},
$$

with $R \equiv (K + P + 2N) g^2 + K g^2_{Y'} + P g^2_{B-L}$.

For future discussion, it is convenient to define the following basis

$$
Z_\mu = \cos \theta_W W^3_\mu - \sin \theta_W \sin \alpha B^Y_\mu - \sin \theta_W \cos \alpha B^{B-L}_\mu;
$$

$$
Z'_\mu = \cos \alpha B^Y_\mu - \sin \alpha B^{B-L}_\mu;
$$

and the $\gamma_\mu$ defined as in Eq. (17). The $\alpha$ angle defined as $\tan \alpha \equiv g_{B-L}/g_{Y'}$, can be understood as the parameter of a particular $SO(2)$ transformation on the two gauge bosons, $B^Y_\mu$.  

9
and $B_{\mu}^{B-L}$, that rotates the $U(1)_Y \otimes U(1)_{B-L}$ gauge group into the $U(1)_Y \otimes U(1)_Z$ gauge group. In the last expression $U(1)_Y$ is the usual hypercharge gauge group. Also, we have that $g^2 \sin^2 \theta_W = e^2 = (1/g^2 + 1/g_{Y'}^2 + 1/g_{B-L}^2)^{-1}$. The $U(1)_Z$ can be understood as the gauge group with the coupling $g_Z^2 = g_{Y'}^2 + g_{B-L}^2$. Using Eqs. (21) and (22), we can write the two massive gauge bosons $Z_{1\mu}$ and $Z_{2\mu}$ in terms of $Z_{\mu}$ and $Z_{\mu}'$ as follows:

$$Z_{1\mu} = \cos \beta Z_{\mu} + \sin \beta Z_{\mu}',$$  \hspace{1cm} (23)

$$Z_{2\mu} = -\sin \beta Z_{\mu} + \cos \beta Z_{\mu}';$$  \hspace{1cm} (24)

where

$$\tan \beta = \sqrt{\frac{g^2 (g_{Y'}^2 + g_{B-L}^2) + g_{Y'}^2 g_{B-L}^2 (g_{B-L}^2 P - g_{Y'}^2 N - M_{Z_1}^2)}{g^2 (g_{Y'}^2 + g_{B-L}^2) (P + N) + g_{Y'}^2 (g_{B-L}^2 (P + N) - M_{Z_2}^2)}}. \hspace{1cm} (25)$$

From Eqs. (20), (23) and (24), we can see that $\tan \beta = 0$ when $V_\phi \to \infty$ or $V_H^2 = (g_{Y'}^2 + 3g_{B-L}^2) V_\phi^2 / g_{Y'}^2$. However, this last solution is not allowed since in our case we have $V_H \gg V_\phi$ and $O(g_{Y'}) \sim O(g_{B-L})$.

In this work, we use the following gauge couplings, $g \simeq 0.65$, $g_{Y'} = g_{B-L} \simeq 0.505$, such that $\tan \beta \simeq 4 \times 10^{-4}$. Doing so, we have $Z_{1\mu} \simeq Z_{\mu}$ and $Z_{2\mu} \simeq Z_{\mu}'$. In general, the $\beta$ angle must be quite small, $\beta \lesssim 10^{-3}$, to be in agreement with precision electroweak studies [30–32] since a new neutral boson $Z_{2\mu}$ which mixes with the SM $Z_{\mu}$ distorts its properties, such as couplings to fermions and mass relative to electroweak inputs. Using those parameters for the gauge couplings and the VEVs discussed in the previous section, we obtain $M_{Z'} \simeq 3.1$ TeV besides the already known masses for the SM gauge bosons. In general, a new neutral vector boson must have a mass in the order of few TeV, or be very weakly coupled to the known matter to maintain consistency with the present phenomenology [30–32]. Doing a phenomenological study of the bounds on the parameter space imposed by data coming from LEP II, Tevatron and LHC in the present model is out of the scope of this work. However, we see that the $M_{Z'}$ value above is consistent with the relation $M_{Z'}/g_{B-L} \simeq 6.13 \gtrsim 6$ TeV [33, 34].

Finally, the charged gauge bosons $W_{\mu}^{\pm}$ are not affected by the presence of one additional neutral gauge boson $Z_{2\mu}$. These have the same form as in the SM, $W_{\mu}^{\pm} = \frac{1}{\sqrt{2}} (W_{\mu}^1 \mp iW_{\mu}^2)$, with masses given by $M_{W^{\pm}}^2 = \frac{1}{4}g^2 V_{SM}^2 = \frac{1}{4}g^2 (V_H^2 + 2V_\phi^2).$
V. DARK MATTER

A. Thermal Relic Density

In order to calculate the present-day DM mass density, \( \Omega_{\text{DM}} h^2 \), arising from \( R_{\text{DM}} \) and \( I_{\text{DM}} \) scalars freezing out from thermal equilibrium, we follow the standard procedure in Refs. [36, 37]. Thus, we should find the solution to the Boltzmann equations for the \( Y_{R_{\text{DM}}} \) and \( Y_{I_{\text{DM}}} \), which are defined as the ratio of the number of particles (\( n_{R_{\text{DM}}} \) and \( n_{I_{\text{DM}}} \)) to the entropy, \( Y_i \equiv n_i / s \) (\( i = R_{\text{DM}}, I_{\text{DM}} \)), with \( s \) being the total entropy density of the Universe. Usually, \( s \) is written in terms of the effective degrees of freedom \( h_{\text{eff}}(T) \) as follows:

\[
s = \frac{2\pi^2}{45} h_{\text{eff}}(T) T^3;
\]

where \( T \) is the photon temperature and \( h_{\text{eff}} \) is calculated as in the Ref. [36]. Actually, in our case, due to the \( U(1)_\chi \) symmetry introduced in Sec. III, \( M_{I_{\text{DM}}} = M_{R_{\text{DM}}} = M_{\text{DM}} \), \( Y_{R_{\text{DM}}} = Y_{I_{\text{DM}}} \equiv Y \), and, \( \Omega_{\text{DM}} h^2 = \Omega_{R_{\text{DM}}} h^2 + \Omega_{I_{\text{DM}}} h^2 = 2\Omega_{R_{\text{DM}}} h^2 = 2\Omega_{I_{\text{DM}}} h^2 \). Therefore, the Boltzmann equation that we have to solve is

\[
\frac{dY}{dx} = - \left( \frac{45}{\pi} G \right)^{-1/2} \frac{g_1^{1/2} M_{\text{DM}}}{x^2} \langle \sigma v_{\text{Moller}} \rangle_{\text{ann}} \left[ Y^2 - Y_{\text{eq}}^2 \right];
\]

where \( x = M_{\text{DM}} / T \), \( G \) is the gravitational constant, and \( Y_{\text{eq}} = n_{\text{eq}} / s \). \( n_{\text{eq}} \) is the thermal equilibrium number density and when \( M_{\text{DM}} / T \gg 1 \), it is \( n_{\text{eq}} = g_i \left( \frac{M_{\text{DM}} T}{2\pi} \right)^{3/2} \exp \left[ - \frac{M_{\text{DM}}}{T} \right] \); where \( g_i = 1 \) is the internal degree of freedom for the scalar dark matter. The \( g_* \) parameter in the Eq. (26) is calculated as in the Ref. [36].

Also, we have that the thermal-average of the annihilation cross section times the Moller velocity, \( \langle \sigma v_{\text{Moller}} \rangle_{\text{ann}} \), has the following form

\[
\langle \sigma v_{\text{Moller}} \rangle_{\text{ann}} = \frac{1}{8 M_{\text{DM}}^2 T K_2^2 (M_{\text{DM}} / T)} \int_{4M_{\text{DM}}^2}^\infty \sigma_{\text{ann}} (s - 4M_{\text{DM}}^2) \sqrt{s} K_1 \left( \sqrt{s}/T \right) ds,
\]

where \( K_i \) are the modified Bessel functions of order \( i \). The variable \( s \), in the integral above, is the Mandelstam variable. Finally, once the \( Y \) is numerically calculated for the present time, \( Y_0 \), we can obtain \( \Omega_{\text{DM}} h^2 = 2.82 \times 10^8 \times (2 \times Y_0) \times \frac{M_{\text{DM}}}{\text{GeV}} \).

In order to calculate \( \sigma_{\text{ann}} \), we have taken into account all dominant annihilation processes which are shown in Fig. (1). In our case, the dominant annihilation contributions come from the scalar exchange. This is due to the fact that our DM candidates, \( R_{\text{DM}} \) and \( I_{\text{DM}} \), couple neither to \( Z_\mu \) nor to \( W^\pm_\mu \) gauge bosons at tree level, since they are SM singlets. Also, we have found that contributions coming from \( Z'_\mu \) exchange are negligible for the parameter set considered here.
Taking into account all previously mentioned, we have solved numerically Eq. (26) for a representative set of parameters. Although the scalar potential in this model has many free parameters, we have found that the most relevant parameters in determining the correct DM relic density and in satisfying the currently direct experimental limits are $\Lambda_{Hs_2}, \Delta_{\alpha^2}$ (with $\alpha = 1, 3, X$) and $\Lambda'_{\gamma^2}$ (with $\gamma = 1, 2$). The $\Lambda_{Hs_2}$ coupling strongly controls the direct detection signal, since in our case both the Higgs-like scalar is almost totally the neutral CP-even component of the $H$ field and as discussed below, the direct detection is mainly mediated by the $t-$channel Higgs exchange. In order to obtain the correct direct detection limits without resorting to resonances, we found that $\Lambda_{Hs_2} \sim 10^{-4}$. The $\Delta_{\alpha^2}$ and $\Lambda'_{\gamma^2}$ parameters are also crucial in obtaining the correct $\Omega_{DM}h^2$ because they mostly control the $DM - DM - R_i (I_i) - R_i (I_i)$ and $DM - DM - R_i$ couplings and, therefore, the $\sigma_{\text{ann}}$. The latter is not allowed to vary in a wide range since, roughly, $\Omega_{DM}h^2 \sim 1/\langle \sigma v_{\text{Moller}} \rangle_{\text{ann}}$ and we aim to obtain values close to $\Omega_{DM}h^2 \sim 0.11$. In other words, the larger $\Delta_{\alpha^2}$ and $\Lambda'_{\gamma^2}$ parameters are, the smaller the $\Omega_{DM}h^2$ is. In (2), we have used $\Lambda'_{\gamma^2} \simeq 10^{-2}$ and $\Delta_{\alpha^2} \simeq 9 \times 10^{-2}$. It is also important here to mention that the dominant process is the $DM + DM \rightarrow I_3 + I_3$ annihilation, where $I_3$ refers to the lightest.
CP-odd scalars, as in Sec (III). Although the other parameters in the scalar potential are not as critical in determining the $\Omega_{DM}h^2$, they give the other quantitative characteristics appearing in Fig. (2). In order to be more specific, we have choose the other parameters such that the mass scalar spectrum is given by: 1437.6, 1016.9, 631.7, 544.9, 379.6, 125 GeV and 707.1, 544.9, 379.6, 2.3 $\times$ $10^{-6}$ GeV for the CP-even and CP-odd scalars, respectively. The CP-even scalars with masses 1437.6, 1016.9, 631.7 GeV have components only in the singlets $\phi_{1,3,X}$ and the CP-even scalars with 544.9, 379.6 GeV have components only in the scalar doublets $\Phi_{1,2}$. The CP-even scalar with 125 GeV has component in the $H$ doublet and it is the Higgs-like scalar in our model. In Fig. (2), we can also observe three resonances in $\approx$ 315.8, 508.5, 718.8 GeV corresponding to the $s-$channel exchange of CP-even scalars with components in the singlets. Let us also mention that the processes via the $s-$channel due to the exchange of the CP-even scalars with masses 125, 379.6, 544.9 GeV are highly suppressed because of the smallness of their couplings. Thus, their resonances do not appear in Fig. (2).

![Relic Density vs. DM Mass](image)

Figure 2: The total thermal relic density of the $I_{DM}$ and $R_{DM}$ as a function of $M_{DM}$. We have used three different parameters for $\Lambda_{Hs2} = 0.3 \times 10^{-4}, 1 \times 10^{-4}, 5 \times 10^{-4}$. 

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B. Direct Detection

Despite weakly coupled to baryons, WIMPs can scatter elastically with atomic nuclei, providing the opportunity for direct detection. Currently, there are several experiments which aim to directly observe WIMP dark matter \[17–19\]. The signal in these experiments is the kinetic energy transferred to a nucleus after it scatters off a DM particle. The energies involved are less or of the order of 10 keV. At these energies the WIMP sees the entire nucleus as a single unit, with a net mass, charge and spin. In general, the WIMP-nucleus interactions can be classified as either spin-independent or spin-dependent. In our case, these interactions are spin-independent because the two DM candidates are scalars. The relevant WIMP-nucleus scattering process for direct detection in the case considered here takes place mainly through the $t$–channel elastic scattering due to Higgs exchange: \((I_{\text{DM}}, R_{\text{DM}}) + N \rightarrow (I_{\text{DM}}, R_{\text{DM}}) + N\) \((N\) refers to the atomic nucleus). The spin-independent cross section is given by

$$
\sigma_{\chi N}^{SI} = \frac{4}{\pi} \frac{M_{\text{DM}}^2 m_N^2}{(M_{\text{DM}} + m_N)^2} [Z f_p + (A - Z) f_n]^2 ; \tag{28}
$$

where the effective couplings to protons and neutrons, \(f_{p,n}\), are

$$
f_{p,n} = \sum_{q=u,d,s} \frac{G_{\text{eff},q} f_{Tq}^{(p,n)} m_{p,n}}{\sqrt{2} m_q} + \frac{2}{27} f_{TG}^{(p,n)} \sum_{q=c,b,t} \frac{G_{\text{eff},q} m_{p,n}}{\sqrt{2} m_q}. \tag{29}
$$

By using \(f_{Tq}^{(p,n)}\) and \(f_{TG}^{(p,n)}\) given in Ref. \[38\] and the fact that, in our case, \(G_{\text{eff},q} = G_0 \times m_q \equiv \frac{C_{\text{DM}^2 H}}{C_{\text{Higgs}}^{\text{DM}} m_q}\) \((\text{with } C_{\text{DM}^2 H} \text{ being the coupling } DM - DM - \text{Higgs which depends on the parameters of the model})\), we arrive at the cross section per nucleon of

$$
\sigma_{\chi p}^{SI} \approx 2.7 \times 10^7 \times \frac{M_{\text{DM}}^2 m_N^2}{(M_{\text{DM}} + m_N)^2} \times G_0^2 \text{ in pbarn.} \tag{30}
$$

Recently, the Large Underground Xenon (LUX) experiment \[19\] has reported its first results, setting limits on spin-independent WIMP-nucleon elastic scattering with a minimum upper limit on the cross section of \(7.6 \times 10^{-10}\) pbarn at a WIMP mass of 33 GeV/c\(^2\). We have found that choosing \(\Lambda_{Hs2} \sim 10^{-4}\) we obtain the LUX bound without resorting to resonances. It is clear that values of \(\Lambda_{Hs2}\) larger can be considered. However, we have chosen this conservative value for \(\Lambda_{Hs2}\). Our results are shown in Fig. \[3\]. The parameters are the same as in Fig. \[2\].
Figure 3: The spin-independent elastic scattering cross section, $\sigma_{\chi,p}^{SI}$, off a proton $p$ as a function of $M_{DM}$ for the same parameters as in Fig. 2, appropriately scaled to relic density. We also show the XENON100 and LUX exclusion limits [17, 19].

From Figs. (2) and (3), we see that for a DM candidate with mass around 200 GeV and $\Lambda_{Hs2} = 0.3 \times 10^{-4}$, $1 \times 10^{-4}$, the two conditions, $\Omega_{DM}h^2$ and the direct detection, are satisfied outside the resonance regions. We also have verified that this is a general characteristic of this model. Due to the existence of the light $I_3$ scalar the annihilation process $DM + DM \rightarrow I_3 + I_3$ Fig. (1a) is the dominant one so that we do not have to appeal to resonances to get compatibility with experiments. Other $M_{DM}$ values which satisfy the experimental bounds are shown in Figs (2) and (3). Specifically, $M_{DM} \approx 319, 410, 511, 590, 737$ GeV are also possible solutions. However, these are within regions with resonances.

Let us now do some important remarks about the impact of the existence of $I_3$ in this model. First of all, we have a tree level contribution to the Higgs invisible decay, $\Gamma_h^{inv}$, due to the coupling of the Higgs field with the light pseudo-scalar field, $c_{hI_3I_3}$, which comes from the Lagrangian terms of the form $|H|^2|\phi_{1,2,x}|^2$, and gives $\Gamma_{hI_3I_3}^{inv} = c_{hI_3I_3}^2/32\pi m_h$ for $m_{I_3} \ll m_h$. Actually, when $2M_{DM} < m_h$ the $h \rightarrow I_{DM} I_{DM}$ and $h \rightarrow R_{DM} R_{DM}$ decays are also allowed, thus, further contributing to $\Gamma_h^{inv}$ according to $\Gamma_{hDMDM}^{inv} = \Gamma_{hI_{DM}I_{DM}}^{inv}$.
\[ \Gamma_{h_{\text{DM}}R_{\text{DM}}} = 2 \times c_{h_{\text{DM}}R_{\text{DM}}}^2 / (32 \pi m_h) \times \sqrt{1 - 4M_{\text{DM}}^2 / m_h^2} \text{ with } c_{h_{\text{DM}}R_{\text{DM}}} \approx \Lambda_{Hs} \sqrt{V_H}. \]

The current limit on the branching ratio into invisible particles of the Higgs, \( BR_{h_{\text{Inv}}} \), is around 10\%–15\% \[39, 40\]. A stronger bound of \( BR_{h_{\text{Inv}}} < 5\% \) at 14 TeV LHC has been claimed \[41\].

From the set of parameters used to obtain Fig. (2) and Fig. (3) we have that \( BR_{h_{\text{Inv}}} = (\Gamma_{h_{I_3I_3}} + \Gamma_{h_{\text{DM}}}) / (\Gamma_{h_{\text{Vis}}} + \Gamma_{h_{I_3I_3}} + \Gamma_{h_{\text{DM}}}) \approx 3.78\% \) for \( M_{\text{DM}} = 50 \text{ GeV} \). For different \( M_{\text{DM}} \) values we have found \( BR_{h_{\text{Inv}}} < 5\% \). Also, we have used \( \Gamma_{h_{\text{Vis}}} = 4.07 \text{ MeV} \) for \( m_H = 125 \text{ GeV} \).

The model is also safe regarding the severe existing constraints on the invisible decay width of \( Z \mu \) boson since there is no a process like \( Z \mu \rightarrow RI_3 \rightarrow I_3I_3I_3 \) \[25\] due to the fact that \( I_3 \) has only components in the SM singlets. (It would be kinetically forbidden anyway once all real scalar fields of the model are heavier than the \( Z \mu \) boson.) For the same reason, there is no issue with the energy loss in stars astrophysical constraint since there is no tree level coupling inducing the \( \gamma + e^- \rightarrow e^- + I_3 \) \[26\]. Finally, some last comments are necessary. In general, the \( I_3 \) could also contribute to the \( \Omega_{\text{DM}} h^2 \) because it is massive. However, the \( I_3 \) pseudo-scalar is not stable. It decays mainly in active neutrinos, \( \nu' \)'s, with \( \Gamma_{I_3 \rightarrow \nu\nu} \approx m_{I_3} / 16\pi \sum_i m_{\nu_i}^2 V_{\nu_i}^{2} \) \[42\]. For the parameter set used here, we have \( \tau_{I_3} \approx 1 / \Gamma_{I_3 \rightarrow \nu\nu} \approx 10^9 \text{ s} \), where we have used \( \sum_i m_{\nu_i}^2 \lesssim 0.01 \text{ eV}^2 \). With \( \tau_{I_3} \) given here and \( t_U \approx 4.3 \times 10^{17} \text{ s} \) (age of the Universe), the \( \Omega_{I_3} h^2 \approx \frac{m_{I_3}}{1.25 \text{ keV}} \exp (-t_U / \tau_{I_3}) \approx 0 \). In the last expression for \( \Omega_{I_3} h^2 \) we have considered that the \( T_{DI_3} > 175 \text{ GeV} \) (where \( T_{DI_3} \) is the decoupling temperature of the \( I_3 \)). There is also a constraint comes from the observed large scale structure of the Universe \[43, 44\]. Roughly speaking, this last condition impose \( r_{I_3} \approx g_{\text{eff}}(T_0) / g_{\text{eff}}(T_{DI_3}) \approx 1 / 25 \), being \( g_{\text{eff}} \) the effective number of the relativistic degrees of freedom. With our parameter set this condition is satisfied.

VI. CONCLUSIONS

We have discussed in this work a scenario where a complex DM candidate is possible. In particularly, the model studied here is a gauge extension of the SM based on a \( SU(2)_L \otimes U(1)_{Y'} \otimes U(1)_{B-L} \) symmetry group. This model contains three right handed neutrinos and some extra scalars, doublets and singlets, with different quantum numbers. In principle, those scalars are introduced to generate Majorana and Dirac mass terms at the tree level and to allow the implementation of a see-saw mechanism at the TeV scale as shown in Ref. \[21\]. The non-standard doublets and singlets introduce two new energy...
scales, besides the electroweak one given $V_H = 246$ GeV: $V_\Phi$ (the VEVs of the extra doublet neutral scalars) and $V_\phi$ (the VEVs of the extra singlet neutral scalars). If $\zeta \equiv V_\Phi/V_\phi \ll 1$ the see-saw mechanism becomes natural \[21\]. In this context, we have studied the scalar spectrum and imposed a $Z_2$ symmetry on the $\phi_2$ singlet scalar (which accidentally became a $U(1)_\chi$ symmetry: $\phi_2 \rightarrow \exp(-i\chi\phi_2)\phi_2$) in order to allow a complex DM candidate. Before studying the constraints coming from the thermal relic density ($\Omega_{DM}h^2$) and direct detection experiments on this DM candidate, we have done a brief analysis of the gauge sector concerning the $Z_\mu, Z'_\mu$ mixing angle ($\tan\beta \simeq 4 \times 10^{-4}$) which satisfies the $\beta \lesssim 10^{-3}$ electroweak precision constraint, and we have verified that the $Z'_\mu$ mass emerging from the model is consistent with the relation $M_{Z'}/g_{B-L} \simeq 6.13 \gtrsim 6$ TeV. Then, we have chosen some parameters that simultaneously allow us to have a compatible $\Omega_{DM}h^2$ and satisfy the direct detection experiments. Although the scalar potential has many parameters, we have found that the $\Lambda_{H_{s2}}, \Delta_{\alpha^2}$ (with $\alpha = 1, 3, X$) and $\Lambda'_{\gamma^2}$ (with $\gamma = 1, 2$) parameters mostly control these two constraints. The $\Lambda_{H_{s2}}$ parameter is fundamental in satisfying the limits coming from direct detection, since in our case it takes place through the $t$–channel elastic scattering due to the Higgs exchange. Choosing $\Lambda_{H_{s2}} \sim 10^{-4}$ roughly satisfies the bounds from the LUX experiment and allows a $\Omega_{DM}h^2$ in agreement with the WMAP and PLANCK experiments.

The $\Delta_{\alpha^2}$ and $\Lambda'_{\gamma^2}$ parameters control $\sigma_{\text{ann}}$ mostly and, therefore $\Omega_{DM}h^2$. As an example, we have shown $\Omega_{DM}h^2$ and $\sigma_{\chi,\mu}^{SI}$, for $\Lambda'_{\gamma^2} \simeq 10^{-2}$ and $\Delta_{\alpha^2} \simeq 9 \times 10^{-2}$, in Figs. \[2\] and \[3\]. It is interesting to note that this model, for the same set of parameters fixed, except $M_{DM}$'s, has several $M_{DM}$ values satisfying the experimental bounds. In other words, we have found solutions in the region outside and inside of the resonances for the same parameters, varying $M_{DM}$ only. As previously mentioned, the presence of a light scalar, $I_3$, in this model makes the process $DM + DM \rightarrow I_3 + I_3$ to be dominant for $\Omega_{DM}h^2$. However, $I_3$ may bring some potential problems, so that we have discussed some constraints imposed on $I_3$ coming from the Higgs and the $Z_\mu$ invisible decay widths, the energy loss in stars and the observed large scale structure of the Universe. We have found that in our context all of these constraints are satisfied. Finally, we would like to point out the recent work studying the possibility of having a Majoron DM candidate \[45\].
Acknowledgments

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APPENDIX: THE MINIMIZATION CONDITIONS

The general minimization conditions coming from $\partial V_i/\partial R_i = 0$, where $V_i$ is the scalar potential with $U(1)_X$ symmetry and $R_i = \{H_R^0, \Phi_{1R}^0, \Phi_{2R}^0, \phi_{1R}, \phi_{2R}, \phi_{3R} \phi_{XR}\}$ are the neutral real components of the scalar fields, can be written as:

\begin{align*}
0 &= V_H \left( 2\lambda_H V_H^2 + \Lambda_{H1} V_{\phi_1}^2 + \Lambda_{H2} V_{\phi_2}^2 + \Lambda_{Hs1} V_{\phi_1}^2 + \Lambda_{Hs2} V_{\phi_2}^2 + \Lambda_{Hs3} V_{\phi_3}^2 + \Lambda_{HsX} V_{\phi_X}^2 - 2\mu_H^2 \right) \\
& \quad - \sqrt{2} \kappa_{H1X} V_{\phi_1} V_{\phi_X} - \kappa_{H2X} V_{\phi_2} V_{\phi_X}^2; \quad (31)
0 &= V_{\phi_1} \left( \Lambda_{H1} V_H^2 + 2\lambda_{11}^1 V_{\phi_1}^2 + (\lambda'_{11} + \lambda_{12})^1 V_{\phi_2}^2 + \Lambda'_{11} V_{\phi_1}^2 + \Lambda'_{12} V_{\phi_2}^2 + \Lambda'_{13} V_{\phi_3}^2 + \Lambda'_{1X} V_{\phi_X}^2 - 2\mu_{11}^2 \right) \\
& \quad - \sqrt{2} \kappa_{H1X} V_H V_{\phi_X} + \beta_{13} V_{\phi_2} V_{\phi_1} V_{\phi_3}; \quad (32)
0 &= V_{\phi_2} \left( \Lambda_{H2} V_H^2 + (\lambda_{12} + \lambda'_{12})^2 V_{\phi_1}^2 + 2\lambda_{22}^2 V_{\phi_2}^2 + \Lambda'_{21} V_{\phi_1}^2 + \Lambda'_{22} V_{\phi_2}^2 + \Lambda'_{23} V_{\phi_3}^2 + \Lambda'_{2X} V_{\phi_X}^2 - 2\mu_{22}^2 \right) \\
& \quad - \kappa_{H2X} V_H V_{\phi_X}^2 + \beta_{13} V_{\phi_1} V_{\phi_2} V_{\phi_3}; \quad (33)
0 &= V_{\phi_3} \left( \Lambda_{Hs1} V_H^2 + \Lambda'_{s1} V_{\phi_1}^2 + \Lambda'_{s2} V_{\phi_2}^2 + 2\lambda_{s1} V_{\phi_1}^2 + \Delta_{12} V_{\phi_2}^2 + \Delta_{13} V_{\phi_3}^2 + \Delta_{1X} V_{\phi_X}^2 - 2\mu_{s1}^2 \right) \\
& \quad + \beta_{13} V_{\phi_1} V_{\phi_2} V_{\phi_3}; \quad (34)
0 &= V_{\phi_2} \left( \Lambda_{Hs2} V_H^2 + \Lambda'_{s2} V_{\phi_1}^2 + \Lambda'_{s2} V_{\phi_2}^2 + \Delta_{12} V_{\phi_2}^2 + 2\lambda_{s2} V_{\phi_2}^2 + \Delta_{23} V_{\phi_3}^2 + \Delta_{2X} V_{\phi_X}^2 - 2\mu_{s2}^2 \right); \quad (35)
0 &= V_{\phi_3} \left( \Lambda_{Hs3} V_H^2 + \Lambda'_{s3} V_{\phi_1}^2 + \Lambda'_{s3} V_{\phi_2}^2 + \Delta_{13} V_{\phi_2}^2 + \Delta_{23} V_{\phi_3}^2 + 2\lambda_{s3} V_{\phi_3}^2 + \Delta_{3X} V_{\phi_X}^2 + 3\beta_{3X} V_{\phi_3} V_{\phi_X} \\
& \quad - 2\mu_{s3}^2 \right) + \beta_{13} V_{\phi_1} V_{\phi_2} V_{\phi_3}; \quad (36)
0 &= V_{\phi_X} \left( \Lambda_{HsX} V_H^2 + \Lambda'_{sX} V_{\phi_1}^2 + \Lambda'_{sX} V_{\phi_2}^2 + \Delta_{1X} V_{\phi_1}^2 + \Delta_{2X} V_{\phi_2}^2 + 2\lambda_{sx} V_{\phi_X}^2 - 2\kappa_{H2X} V_H V_{\phi_2} - 2\mu_{sx}^2 \right) \\
& \quad - \sqrt{2} \kappa_{H1X} V_H V_{\phi_1} + \beta_{3X} V_{\phi_3}^3 + \Delta_{3X} V_{\phi_3} V_{\phi_X}; \quad (37)
\end{align*}

In the Eqs. (31)-(37) above, $V_H, V_{\phi_1}, V_{\phi_2}, V_{\phi_3}, V_{\phi_X}$ are the VEVs of $H_R^0, \Phi_{1R}^0, \Phi_{2R}^0, \phi_{1R}, \phi_{2R}, \phi_{3R}, \phi_{XR}$, respectively.

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