Deterministic Polynomial Solution for NP by an Abstractional Machine

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April 2, 2015

Abstract

In this paper it is argued that a particular kind of mechanical process called mechanical abstractional process offers capabilities that surpass those of a general mechanical process (e.g., Turing machines). Abstractional machines, which follow a mechanical abstractional process, are defined, and it is proved that an abstractional machine is able to solve NP-complete problems in polynomial time. This result together with the provided evidence that shows the newly defined process as a purely mechanical process supports a relevant consequence of this work, that is, P = NP.

1 Introduction

In this paper we present a particular kind of mechanical process through which machines can build their own understanding of things that they can use in performing their tasks. Arguably, this implies to surpass the capabilities of a general mechanical process (e.g., Turing machines), which can only execute precisely and unambiguously specified actions.

The particular kind of mechanical process presented is such that, based on the machine’s own criteria, builds computable abstractions from any given input to obtain corresponding representations that are meaningful for the machine and serve as the building blocks for the construction of the machine’s own understanding of things that the machine can use to perform its tasks. We refer as abstractional machines to the machines following this process that we call mechanical abstractional process. In a previous work we have introduced abstractional machines from a conceptual point of view [3].

In this paper we present the first step towards showing the validity and reach of the capabilities of abstractional machines with respect to those of a general mechanical process. To this end, we apply abstractional machines to find an efficient solution for a class of problems for which efficient solution based on a general mechanical process has not been

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found. Specifically, an abstractional machine is applied to the efficient solution of 3-SAT. It is then proved that there exists an abstractional machine able to solve NP-complete problems in polynomial time and thus, it is proved that the problems in NP can be solved efficiently by an abstractional machine. These results together with the provided evidence that shows the mechanical abstractional process as a purely mechanical process supports a relevant consequence of this work, that is, $P = NP$.

In §2 computable abstractions and the mechanical abstracting process to build them are defined. In §3 abstractional machines are presented, providing their general definition and mechanism. In §4 an abstractional machine to solve 3-SAT in polynomial time is defined. An example is provided to ease the understanding of the process of this machine, and its running time complexity is analyzed, proving it to be polynomial. In §5 two relevant consequences of this work are given and §6 provides a brief conclusion.

\section{Computable Abstractions}

\subsection{Preliminaries}

Herein, an abstraction is seen as a construct that encompasses one or many things, holding meaningful information about the things encompassed according to certain criteria. It is the meaningful information hold by the abstraction what makes possible to ‘discard’ or ‘forget about’ the things encompassed.

We say that abstractions, based on their meaningful information, can serve as the building blocks for creating one’s own understanding of things, which we see as to be built as a system of abstractions (i.e., different abstractions being related (or connected) forming a complex whole). This allows to distinguish one thing from others regardless of their level of abstraction, as well as to relate different things under different contexts and at different levels of abstraction. (For a conceptual introduction on computable abstractions, understanding, and abstractional machines see [3])

We shall call operating machine to any model operationally equivalent to a deterministic Turing machine.

We consider that an operating machine $M$ is given a sequence $S$ of objects as input, where the objects are either symbols, sets or sequences of symbols, or sets or sequences of sets or sequences.

We say that any object, except for symbols, is specified by a unique identifier (unique in $M$) and a set or sequence of objects (more precisely, a set or sequence of the identifiers of the objects). A symbol is specified only by a unique identifier, which we say it is the symbol itself. Furthermore, we shall denote an object $\varphi_a$ specified by a set of objects as $\varphi_a: \{\varphi_1, \ldots, \varphi_n\}$ and an object $\varphi_b$ specified by a sequence of objects as $\varphi_b: \langle \varphi_1, \ldots, \varphi_n \rangle$.
2.2 Computable Concepts

**Definition 1.** For any object \( x \) in some set or sequence \( \varphi \) of objects in the input of an operating machine \( M \), we call *computable concepts* to the primitive contexts that \( M \) can describe about \( x \) with respect to \( \varphi \) and its components. Computable concepts, which we will denote by \( C_i \) (\( i \) being the unique identifier of the concept, for \( M \)), can be of two types:

[Type I] such that refers to ‘appearing within some set or sequence \( \varphi \) of objects’, denoted as \( C(x): \varphi(x) \) or simply as \( C: \varphi(x) \)

[Type II] such that refers to ‘appearing within some particular set of objects’ or, for the case of sequences, ‘... in a specific position of a sequence of objects’; for instance, for a sequence \( \langle \varphi_1, \varphi_2 \rangle \), a concept \( C_j \) that refers to ‘appearing in the first position of a two-element sequence, where the second element is \( \varphi_2 \)’ can be described by the machine, being denoted as \( C_j(x): \langle x, \varphi_2 \rangle \).

A computable concept may encompass more than one object, all of which satisfy the primitive context that the concept represents. This defines an equivalence relation between the objects that satisfy the concept. Thus, for instance, if we have a concept defined as \( C_a: \varphi_a(x) \) and two objects \( a \) and \( b \) that satisfy this concept, which we denote as \( a \leftrightarrow C_a \) and \( b \leftrightarrow C_a \), then it follows that \( a \) and \( b \) are equivalent with respect to \( C_a \), i.e., \( a \sim b \).

Concepts may entail, as well, relations that are not necessarily equivalence relations. For instance, if we know that \( \varphi_a \) appears within the specification of an object \( \varphi_b \) and we define \( C_e: \varphi_b(x) \), we can say that \( \varphi_a \leftrightarrow C_e \). And, since we have \( a \leftrightarrow C_a \), with \( C_a: \varphi_a(x) \), we have then a relation between the object \( a \) and \( C_e \) through (or with respect to) \( C_a \), i.e., \( a \xrightarrow{C_a} C_e \), with which \( a \) is related to any object that satisfies \( C_e \).

An object in the input of an operating machine may satisfy in general more than one concept, through which it may get related to other objects and concepts. When the object is indeed appearing within the specification of a particular object, one specific concept gets satisfied and the extent of relations covered in general by the object gets restricted to a subset of objects and concepts that are related to the concept that is being satisfied. For instance, for \( a \) above, if we have that, in addition to \( C_a \), \( a \) satisfies \( C_f \) (i.e., \( a \leftrightarrow C_f \)) through which it is related to some objects \( c \) and \( d \). Thus, we have \( a \xrightarrow{C_a} b \xrightarrow{C_e} c, d \) and in general \( a \rightarrow b \xrightarrow{C_e} c, d \). Then, when in an execution an input being read shows \( a \) appearing in such a way as to satisfy \( C_a \) only (i.e., appearing within \( \varphi_a \)), the extent of relations of \( a \) gets restricted accordingly, that is, \( a \xrightarrow{C_a} b \xrightarrow{C_e} c, d \).

2.3 Mechanical Abstracting Process

For any object \( \alpha \) in the input sequence of an operating machine \( M \), computable abstractions of \( \alpha \) can be obtained by \( M \) considering: i) the higher level objects of \( \alpha \), by describing \( \alpha \)

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1 Equivalent objects appear underlined.
based on concepts of Type I; ii) the objects at the same level of \( \alpha \), by describing \( \alpha \) based on concepts of Type II; and, for the case of \( \alpha \) being any object specified by a set or sequence of objects (i.e., not a symbol), iii) the components of \( \alpha \) (i.e., the elements of the set or sequence that specifies \( \alpha \)), by describing \( \alpha \) based on the computable abstractions of its components.

The criteria to decide on what level(s) to consider and on what object(s) to select for the derivation of computable concepts in any of these cases are assumed to be given and are not part of the abstracting process. As long as the criteria for all cases are consistent, an object can be abstracted considering each of the three levels, with all abstractions being consistent with respect to each other.

The steps of the mechanical abstracting process to abstract an object \( \alpha \) are:

**step 1** Assign a unique identifier to each of the objects that will be involved in the derivation of concepts (according to the type of concept). For case (i) above, it is \( \alpha \) and the higher level object in which it appears the ones that are uniquely identified. For case (ii), it is \( \alpha \) and the other elements of the corresponding set or sequence the ones that are uniquely identified. For case (iii), it is \( \alpha \) and its components the ones that are uniquely identified.

**step 2** Define the corresponding computable concept(s)

**step 3** For cases (i) and (ii), describe \( \alpha \) based on the concepts defined in previous step. For case (iii), describe the components of \( \alpha \) based on the concepts defined in previous step and then describe \( \alpha \) based on the abstractions of the components.

For instance, to abstract an object \( \varphi_1 \) in \( S \) based on the sequences \( \varphi_a: \langle \varphi_1, \varphi_2 \rangle \) and \( \varphi_b: \langle \varphi_3, \varphi_1, \varphi_4 \rangle \), \( M \) describes the concepts that \( \varphi_1 \) satisfies in \( \varphi_a \) and \( \varphi_b \). These are \( C_1: \varphi_a(x) \), \( C_2(x) : \langle x, \varphi_2 \rangle \), \( C_3: \varphi_b(x) \), and \( C_4(x) : \langle \varphi_3, x, \varphi_4 \rangle \). Finally, it obtains the abstraction of \( \varphi_1 \) as \( \varphi_1 \vdash C_1, C_2, C_3, C_4 \).

### 3 Abstractional Machines

**Definition 2.** Given a set of tasks \( \Pi \) and a related input \( I \), we define an *abstractional machine* as a machine that follows a mechanical abstractional process through which it abstracts objects in \( I \) to build its own understanding of \( I \) (built as a consistent system of computable abstractions), and perform, based on this, the tasks in \( \Pi \).

We shall use an operating machine as the ‘foundation’ from which an abstractional machine is defined, that is, the definition of an abstractional machine will be based on executable operations of an existing operating machine. Furthermore, the operating machine will be on charge of executing all operations that the abstractional machine needs to perform as part of its mechanical abstractional process.
The definition of an abstractive machine comprises the definition of its set of tasks and the specification of how its understanding or consistent system of abstractions is to be built.

3.1 System of Computable Abstractions

The system of computable abstractions $\tilde{K}$ of an abstractive machine $\tilde{M}$ is built as a consistent model where no contradiction can exist between any two abstractions in the model. To this end, we first establish an initial collection of abstractions that are considered of foundational importance and are assumed to be non-contradictory between each other. We call this the ground (or axioms) of the model. Additionally, the abstraction levels of interest for the model have to be established. This includes establishing the lowest abstraction level from which no further decomposition of abstractions is considered within the model, and a criterion of decomposition that defines the components that will be of interest for the model when considering each object. In order to build new abstractions in the model, only abstractions from its ground or that have been derived from its ground can be used. Therefore, a criteria of validity imposed over abstractions in the ground will extend to all its derived abstractions, thus making the whole model consistent under such criteria.

The behavior of an abstractive machine will depend on the current state of its system of abstractions. If we consider an abstractive machine performing tasks in which its system of abstractions do not suffer any modification, we will observe regular behavior. However, if by the abstractive process its system of abstractions gets modified, then the behavior of the machine might vary accordingly. In spite of this, since the system of abstractions is built as explained above, it is ensured that the construction will happen in a consistent manner, i.e., abstractions in different stages of the construction of the system of abstractions will never contradict each other.

3.2 Mechanical Abstractional Process

In general, the abstractional process followed by an abstractive machine comprises abstracting objects in the input according to the definition of the machine and checking if the resulting abstractions can fit consistently into the system of computable abstractions of the machine. Then, according to the definition of the abstractive machine, the resulting abstractions are included into the system of abstractions. Thus, there are three steps in this process:

step 1 Mechanical abstracting process over objects of the input, according to the definition of the abstractive machine

step 2 Search in the system of abstractions of the abstractive machine to see if any of the resulting abstractions from the abstracting process contradicts any object inside.
step 3 Include the resulting abstractions into the system of abstractions, according to the definition of the abstractive machine

step 3.1 Perform the necessary adjustments on the system of abstractions based on the newly included abstractions in order to ensure that the whole model is affected properly with regard to, for instance, the new relation(s) that the new abstraction(s) may imply (according to the definition of the abstractive machine).

An abstractional machine reaches its accepting state when, according to the definition of the machine, a consistent understanding is built from the given input. The rejecting state is reached when no understanding can be built by the machine from the given input.

Lemma 1. The process followed by any abstractional machine is a purely mechanical process.

4 Abstractional Machine for 3-SAT

4.1 Definition of the machine

We define an abstractional machine \( \tilde{M}_S \) with system of abstractions \( \tilde{K}_S \) to solve 3-SAT in polynomial time. The set of tasks of \( \tilde{M}_S \) comprises one procedure called Traverse described in \([L.1.2]\). Thus, given a 3CNF formula \( \phi \) as input, \( \tilde{M}_S \) will execute Traverse to build, if possible, a consistent understanding of \( \phi \) in \( \tilde{K}_S \), according to the definition of the problem. If \( \phi \) is satisfiable, then \( \tilde{K}_S \) will contain the satisfiability assignment for \( \phi \) which \( \tilde{M}_S \) will output together with \( \phi \text{ SAT} \). In case no satisfiability assignment exists for \( \phi \), \( \tilde{M}_S \) will be unable to build a consistent understanding of it and \( \phi \text{ UNSAT} \) will be the output.

4.1.1 Ground of \( \tilde{K}_S \)

The ground of \( \tilde{K}_S \) includes axioms related to the general definition of the problem, and abstractions that define the logical operations involved, that is, disjunction and negation (conjunction is not necessary as we start accepting that all clauses in the 3CNF formula must be related to \( T \)).

General

i) The abstract levels of interest for \( \tilde{M}_S \) comprise clauses and literals, which form a 3CNF formula \( \phi \) given as input

ii) Any literal \( x \) can be related to either \( T \) (i.e., \( x \rightarrow T \)), \( F \) (i.e., \( x \rightarrow F \)), or be free (i.e., potentially related to any of \( T \) and \( F \), it being denoted as \( x \rightarrow [T, F] \) or by the literal alone)
iii) clause \(i \rightarrow T\), for \(i \in [1, m]\), where \(m\) is the number of clauses in \(\phi\)

Disjunction

For any literal \(x\) in a disjunction of three distinct literals that is related to \(T\), the computable concepts that can be encountered are:

\[
\begin{align*}
C_a(x) & : \{x, y, z\} \rightarrow T, \\
C_b(x) & : \{x, y \rightarrow T, z \rightarrow T\} \rightarrow T, \\
C_c(x) & : \{x, y \rightarrow F, z \rightarrow F\} \rightarrow T, \\
C_d(x) & : \{x, y \rightarrow F, z \rightarrow T\} \rightarrow T, \\
C_e(x) & : \{x, y \rightarrow F, z \rightarrow F\} \rightarrow T
\end{align*}
\]

Then, based on the definition of the operation of disjunction we have that for any literal \(a\):

\[
a\mid C_a \rightarrow T, a\mid C_e \rightarrow T, a\mid C_f \rightarrow T, a\mid C_b \rightarrow |T,F|, a\mid C_c \rightarrow |T,F|, a\mid C_d \rightarrow |T,F|.
\]

Negation

For any literal \(x\) we write the specification of its negation as \(\bar{x} : \{\neg, x\}\). Based on the definition of disjunction above, the only concern will be on literals restricted to be related to \(T\). Thus, if a literal \(a\) is restricted to \(T\) by a concept due to a clause where it appears, it follows that its negation satisfies the concept

\[
C_{\neg}(x) : \{\neg, x\} \rightarrow T
\]

with which (according to the definition of the operation of negation) we have \(a\mid C_{\neg} \rightarrow F\).

4.1.2 Abstractional process

1: procedure Traverse
2: \hspace{1em} for \(i \leftarrow 1, m\) number of clauses in \(\phi\) do
3: \hspace{2em} if \(\bar{M}_S\) abstracts clause \(i\) into \(\bar{K}_S\) then
4: \hspace{3em} continue
5: \hspace{2em} else if AdaptTo(clause \(i\)) then
6: \hspace{3em} continue
7: \hspace{2em} else
8: \hspace{3em} return \(\phi\) UNSAT
9: \hspace{2em} end if
10: \hspace{1em} end for
11: return \(\phi\) SAT and relation between literals and truth values in \(\bar{K}_S\)
12: end procedure

where

1: procedure AdaptTo(clause)
2: \hspace{1em} for each literal \(x_i\) in clause do
for all clauses dependent on the negation of $x_i$ do
  if $\text{CHANGE}(\text{free literals})$ then
    return True
  else if $\text{CHANGE}(\text{literals restricted to } T)$ then
    return True
  end if
end for
end for
return False
end procedure

with $\text{CHANGE}$ being such that $\tilde{M}_S$ abstracts (tentatively) each of the literals considered as to be restricted to $T$ (with respect to its corresponding clause), checking if the result is consistent with $\tilde{K}_S$, adding the result into $\tilde{K}_S$ in such case, and returning False otherwise.

$\tilde{M}_S$ abstracts clause$_i$ into $\tilde{K}_S$ (line 3 in TRAVERSE) by obtaining for each of the literals in clause$_i$ (first literals already restricted to $T$ in $\tilde{K}_S$, non-negated and left to right\footnote{The decision on the processing is arbitrary}) a computable concept first with respect to clause$_i$ (Type I) and then with respect to the other literals in clause$_i$ (Type II). The concept of Type II is (temporarily) defined and matched to one in (1) with which the literal will be described with respect to the concept of Type I, obtaining thus its computable abstraction for $\tilde{K}_S$. This subprocess being successful has as consequence to relate clause$_i$ to $T$ based on the literal that has been restricted to $T$, which after being checked against the ground of $\tilde{K}_S$ (specifically, \textbf{III} in General) it returns True; in any other case it returns False. For instance, for a clause defined as $\varphi_1: \{\bar{x}_1, x_2, x_3\}$, $\tilde{M}_S$ would obtain the abstractions $\tilde{c}_{\varphi_1} : \varphi_1(x), (x_2|\tilde{c}_{\varphi_1})|\tilde{c}_u \rightarrow T, (x_3|\tilde{c}_{\varphi_1})|\tilde{c}_b \rightarrow |T, F|$, and $(\bar{x}_1|\tilde{c}_{\varphi_1})|\tilde{c}_b \rightarrow |T, F|$, with the immediate consequence being $\varphi_1 \xrightarrow{\bar{x}_2} T$, and thus the process returns True.

With regard to the general definition of abstractional machines, when $\tilde{M}_S$ abstracts a literal and includes the resulting abstractions into $\tilde{K}_S$, $\tilde{M}_S$ performs the necessary adjustments in $\tilde{K}_S$ according to its definition, affecting $\tilde{K}_S$ properly if anything is modified, e.g., based on a new clause that has just been abstracted, a literal may go from being free to being related to $T$, or vice versa. Thus, if in the above example $\bar{x}_1$ had been restricted to $T$, $x_1$ would have been abstracted accordingly, that is, if $\bar{x}_1 \rightarrow T$, then $x_1|\tilde{c}_u \rightarrow F$.

4.2 Example

To ease the understanding of the abstractional process above, we outline next its application to a simple 3CNF formula presented in \[5\] as a shortest interesting formula for 3-SAT:

$$\phi = (x_2 \lor x_3 \lor \bar{x}_4) \land (x_1 \lor x_3 \lor x_4) \land (\bar{x}_1 \lor x_2 \lor \bar{x}_4) \land (\bar{x}_1 \lor \bar{x}_2 \lor x_4) \land (\bar{x}_2 \lor \bar{x}_3 \lor x_4) \land (\bar{x}_1 \lor \bar{x}_3 \lor x_4) \land (x_1 \lor \bar{x}_2 \lor \bar{x}_4) \land (x_1 \lor x_2 \lor \bar{x}_3) \ (3)$$
For the sake of shortening the exposition of the example, let us define (beyond what has been defined in §2.2) concepts that encompass any of a set of concepts. Thus, we define a concept $C_T$ encompassing the concepts in (1) that restrict a literal to $T$ and a concept $C_L$ encompassing the concepts that restrict a literal to $[T, F]$. That is,

\[
C_T : |c_a, c_e, c_f| \\
C_L : |c_b, c_c, c_d|
\]

As it was defined in the abstractional process in §4.1.2, a literal will be first restricted by a concept of Type I with respect to the clause where it appears and this in turn will make it being restricted by any of the concepts in (1). In this example the second restriction is made by concepts in (4). Thus, we would have for any literal $x$, depending on the case, either $(x | c_{\text{clause}, i}) | c_T \rightarrow T$ or $(x | c_{\text{clause}, i}) | c_L \rightarrow T$.

Finally, we further shorten the notation with

\[
C_i \triangleq |c_i | c_T \rightarrow T \\
C^*_i \triangleq |c_i | c_T \rightarrow [T, F]
\]

where $i$ is the identifier of the clause.

Thus, if a literal $a$ is restricted to $T$ by a clause $\varphi_1$ we write $a \upharpoonright C_1$, and if it is restricted to $[T, F]$ we write $a \upharpoonright C^*_1$.

Next, the process followed by $\tilde{M}_S$ in solving $\phi$ in (3) is presented, with each circled number referring to the particular clause being processed by $\tilde{M}_S$ on each case. Literals that are restricted to $F$ are not mentioned. Those abstractions that change a literal from being restricted to $T$ to becoming free or vice versa appear inside a box and are followed (in the next line, indented) by the immediate consequences.

1. $x_2 \upharpoonright C_1$, $x_3 \upharpoonright C^*_1$, $\bar{x}_4 \upharpoonright C^*_1$
2. $x_1 \upharpoonright C_2$, $x_3 \upharpoonright C^*_2$, $x_4 \upharpoonright C^*_2$
3. $x_2 \upharpoonright C_3$, $x_4 \upharpoonright C^*_3$
4. $\underline{x_3 \upharpoonright C_4}$
   1. $x_2 \upharpoonright C^*_1$, $x_3 \upharpoonright C^*_1$, $\bar{x}_4 \upharpoonright C^*_1$
   2. $\underline{x_1 \upharpoonright C^*_2}$, $x_3 \upharpoonright C_2$, $x_4 \upharpoonright C^*_2$
      $\bar{x}_1 \upharpoonright C^*_3$, $C^*_4$
5. $\underline{x_4 \upharpoonright C_5}$
   2. $x_3 \upharpoonright C^*_2$, $x_4 \upharpoonright C^*_2$, $x_1 \upharpoonright C^*_2$
   3. $x_4 \upharpoonright C_3$, $\underline{x_2 \upharpoonright C^*_3}$, $\bar{x}_1 \upharpoonright C^*_3$
\[ \begin{align*}
1 & \ x_3 \mid C_1, \ x_2 \mid C_1^* \\
2 & \bar{x}_1 \mid C_6 \\
3 & \ x_1 \mid C_3^* \mid C_4^* \mid C_2^* \\
4 & \ x_1 \mid C_4^* \mid C_3^* \mid C_2^* \\
5 & \bar{x}_2 \mid C_7 \\
6 & \ x_2 \mid C_5 \mid C_4 \mid C_3^* \\
7 & \bar{x}_2 \mid C_5 \mid C_4 \mid C_3^* \\
8 & \bar{x}_1 \mid C_3 \mid C_4 \mid C_3^* \\
\end{align*}\]

As this stage (i.e., \(\bar{x}_1 \mid C_3, C_6, C_1^*; \bar{x}_2 \mid C_5, C_7, C_4^*; x_3 \mid C_1, C_2, C_4^*\)), there is no possible abstraction which based on this clause can be consistent with \(\bar{K}_S\). Then, execute \textsc{AdaptTo}.

Within \textsc{AdaptTo}, for \(x_1\) the process attempts \(x_1 \mid C_8\) and goes to the clauses that are dependent on the negation of \(x_1\) (i.e., \(\bar{x}_1\)), that is, \(\text{clause}_3\) and \(\text{clause}_6\), to check if they can depend on a different literal. In trying free literals in \(\text{clause}_3\) and \(\text{clause}_6\) a contradiction is found, namely, \(\text{clause}_3 \xrightarrow{x_4} T \& \text{clause}_6 \xrightarrow{\bar{x}_4} T\). Then, in trying literals related to \(F\) another contradiction is found, as \(\text{clause}_3\) and \(\text{clause}_6\) would be related to \(T\) due to \(x_2\) and \(\bar{x}_3\), respectively, while \(\text{clause}_4\) would remain dependent on either \(x_3\) or \(\bar{x}_2\). Therefore, the process will continue with the other literals in \(\text{clause}_8\). Here we skip going through the process for \(x_2\) and \(\bar{x}_3\), since it has similar results as in the case of \(x_1\). Finally, \(\bar{M}_S\) returns \(\phi\) UNSAT.

4.3 Complexity

\textbf{Lemma 2.} There exists an abstractive machine able to solve 3-SAT in polynomial time

\textit{Proof.} The abstractive process \textsc{ Traverse} described in §4.1.2 can be seen as to be divided into three major subprocesses. Traversing the formula, abstracting each of the clauses in \(\phi\) requires \(cm\) steps, where the constant \(c\) accounts for abstracting process steps and \(m\) for the number of clauses in the formula. The process by which \(\bar{M}_S\) adjusts \(\bar{K}_S\) to the new abstractions being built as the formula is traversed has a similar worst case running time to the process through which \(\bar{M}_S\) checks if \(\bar{K}_S\) can be adapted to a particular required change (\textsc{AdaptTo}). In both cases would be required, at each step forward, to iterate back from the current clause to the part of the formula that has been already abstracted into \(\bar{K}_S\). Thus, the upper bound for the running time of any of these two subprocesses can be described by a quadratic function of \(m\). Therefore, the time complexity of \textsc{Traverse} is polynomial (specifically, \(O(n^2)\)).
5 Relevant Consequences

Theorem 1 (\[\textbf{1}\]). 3-SAT is NP-complete

Corollary 1.1. \( P = NP \iff 3\text{-SAT} \in P \)

Theorem A. There exists an abstractional machine able to solve the problems in NP in polynomial time

Proof. This follows from Lemma 2, Theorem 1, and the definition of NP-completeness (\[\textbf{1}\],\[\textbf{4}\], and \[\textbf{6}\])

Theorem 2. \( P = NP \)

Proof. We take the result of Lemma 2 and use Lemma 1 to make it valid for the case of a general mechanical process. Based on this and Corollary 1.1 we conclude \( P = NP \)

6 Conclusion

In this paper we have presented the first step towards showing the validity and reach of the capabilities of abstractional machines with respect to a general mechanical process. In a strict sense, this endeavor would arguably require to present a proof over an infinitary case with regard to, for instance, (un)computability or unsolvability as defined by A.M. Turing (\[\textbf{7}\], \[\textbf{8}\], respectively) or undecidability (or incompleteness) as defined by K.F. Gödel (\[\textbf{2}\]). However, that is left for future work, taken now first a finitary case, that is, the polynomial solution of the problems in NP, which already allows to exhibit the potential of abstractional machines.

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