Anti-triplet charmed baryon decays with SU(3) Flavor Symmetry

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(Dated: April 10, 2018)

Abstract

We study the decays of the anti-triplet charmed baryon state ($\Xi^0_c, \Xi^+_c, \Lambda^+_c$) based on the SU(3) flavor symmetry. In particular, after predicting $\mathcal{B}(\Xi^0_c \to \Xi^- \pi^+) = (15.7 \pm 0.7) \times 10^{-3}$ and $\mathcal{B}(\Xi^+_c \to \Xi^- \pi^+ \pi^+) = (14.7 \pm 8.4) \times 10^{-3}$, we extract that $\mathcal{B}(\Xi^0_c \to \Lambda K^- \pi^+, \Lambda K^+ K^-, \Xi^- e^+ \nu_e) = (16.8 \pm 2.3, 0.45 \pm 0.11, 48.7 \pm 17.4) \times 10^{-3}$ and $\mathcal{B}(\Xi^+_c \to p K^0_s K^0_s, \Sigma^+ K^- \pi^+, \Xi^0 \pi^+ \pi^0, \Xi^0 e^+ \nu_e) = (1.3 \pm 0.8, 13.8 \pm 8.0, 33.8 \pm 21.9, 33.8^{+21.9}_{-22.6}) \times 10^{-3}$. We also find that $\mathcal{B}(\Xi^0_c \to \Xi^0 \eta, \Xi^0 \eta') = (1.7^{+1.0}_{-1.7}, 8.6^{+11.0}_{-6.3}) \times 10^{-3}, \mathcal{B}(\Xi^0_c \to \Lambda^0 \eta, \Lambda^0 \eta') = (1.6^{+1.2}_{-0.8}, 9.4^{+11.6}_{-6.8}) \times 10^{-4}$ and $\mathcal{B}(\Xi^+_c \to \Sigma^+ \eta, \Sigma^+ \eta') = (28.4^{+8.2}_{-6.9}, 13.2^{+24.0}_{-11.9}) \times 10^{-4}$. These $\Xi_c$ decays with the branching ratios of $O(10^{-4} - 10^{-3})$ are clearly promising to be observed by the BESIII and LHCb experiments.
I. INTRODUCTION

In terms of the $SU(3)$ flavor ($SU(3)_f$) symmetry, the $\Xi_c$ decays should be in association with the $\Lambda_c^+$ ones as $\Xi_c^0$, $\Xi_c^+$ and $\Lambda_c^+$ are united as the lowest-lying anti-triplet of the charmed baryon states ($B_c$). Nonetheless, in accordance with $f_{\Xi_c^+} + f_{\Xi_c^0} + f_{\Omega_c^0} \simeq 0.136 f_{\Lambda_c^+}$ estimated in Refs. [1, 2], where $f_{B_c,\Omega_c}$ stand for the fragmentation fractions for the rates of the charmed baryon productions, the measurements of the $\Xi_c$ decays are not easy tasks compared to the $\Lambda_c^+$ ones. For example, the two-body $\Lambda_c^+ \rightarrow B_n M$ decays with $B_n (M)$ the baryon (pseudoscalar-meson) have been extensively studied by experiments. Interestingly, six decay $\Lambda_c^+$ decay modes have been recently reexamined or measured by BESIII [3, 4]. In addition, LHCb has just observed the three-body $\Lambda_c^+ \rightarrow pM M$ decays [5], together with their CP violating asymmetries [6]. However, no much progress has been made in the $\Xi_c$ decays. In particular, none of the absolute branching fractions in the $\Xi_c$ decays has been given yet. Instead, these decays are experimentally measured by relating the decays of $\Xi_c^+ \rightarrow \Xi^- \pi^+ \pi^+$ or $\Xi_c^0 \rightarrow \Xi^- \pi^+$, and can only be determined once $f_{\Xi_c^0}$ are known.

Since BESIII and LHCb are expected to search for all possible anti-triplet charmed baryon decays, one can test whether or not the studies of $\Lambda_c^+ \rightarrow B_n M$ can be applied to $\Xi_c^0 \rightarrow B_n M$. Theoretically, the factorization for the $b$ baryon decays [8–13] does not work for the charmed baryon decays, which receive corrections by taking into account the nonfactorizable effects [14–19]. On the other hand, the possible $b$ or $c$ hadron decay modes can be examined by the $SU(3)_f$ symmetry [20–31]. Furthermore, the symmetry approach has been extended to explore the doubly and triply charmed baryon decays [31], which helps to establish the spectroscopies of the doubly and triply charmed baryon states [32], such as the to-be-confirmed $\Xi_{cc}^+$ state [33–38].

Moreover, to test the validity of the $SU(3)_f$ symmetry in the anti-triplet charmed baryon decays, a complete numerical analysis for the decays is necessary. In fact, the decays of $\Lambda_c^+ \rightarrow B_n M$ have been explained well by the global fit in Ref. [30], together with the predictions of $B(\Xi_c^+ \rightarrow \Xi^0 \pi^+) = (8.0 \pm 4.1) \times 10^{-3}$ and $B(\Xi_c^0 \rightarrow \Lambda^0 \bar{K}^0) = (8.3 \pm 0.9) \times 10^{-3}$, in agreement with the values of $(7.2 \pm 3.5, 8.3 \pm 3.7) \times 10^{-3}$ extracted from the ratios of $B(\Xi_c^+ \rightarrow \Xi^0 \pi^+)/B(\Xi_c^+ \rightarrow \Xi^0 e^+ \nu_e)$ and $B(\Xi_c^0 \rightarrow \Lambda^0 \bar{K}^0)/B(\Xi_c^0 \rightarrow \Xi^- e^+ \nu_e)$, respectively [31].

In this report, we will systematically study the two-body weak $\Xi_c \rightarrow B_n M$ decays based on the $SU(3)_f$ symmetry and give some specific numerical results, which can be tested in
the future measurements by BESIII and LHCb. By taking the predicted \( \mathcal{B}(\Xi_c^0 \to \Xi^- \pi^+) \) as the theoretical input, we will also estimate the branching ratios of other \( \Xi_c \) decays in the PDG [7], which are related to \( \Xi_c^0 \to \Xi^- \pi^+ \).

II. FORMALISM

For the two-body anti-triplet of the lowest-lying charmed baryon decays of \( \mathbf{B}_c \to \mathbf{B}_n M \), where \( \mathbf{B}_c = (\Xi_c^0, -\Xi_c^+, \Lambda_c^+) \) and \( \mathbf{B}_n (M) \) are the baryon (pseudoscalar) octet states, the effective Hamiltonian responsible for the tree-level \( c \to su\bar{d}, c \to uq\bar{q} \) and \( c \to du\bar{s} \) transitions are given by [39]

\[
\mathcal{H}_{\text{eff}} = \sum_{i=\pm, -} \frac{G_F}{\sqrt{2}} c_i \left( V_{cs} V_{ud} O_i + V_{cd} V_{ud} O_i^\dagger + V_{cd} V_{us} O_i^\dagger \right),
\] (1)

with \( q\bar{q} = d\bar{d} \) or \( s\bar{s} \), \( G_F \) the Fermi constant, \( V_{ij} \) the CKM matrix elements, and \( c_\pm \) the scale-dependent Wilson coefficients to take into account the sub-leading-order QCD corrections. The four-quark operators \( O_{\pm}^q \) and \( O_{\pm}^{\dagger} \equiv O_{\pm}^q - O_\mp^q \) in Eq. (1) can be written as

\[
O_{\pm} = \frac{1}{2} \left[ (\bar{u}d)_{V-A}(\bar{s}c)_{V-A} \pm (\bar{s}d)_{V-A}(\bar{u}c)_{V-A} \right],
\]

\[
O_{\pm}^q = \frac{1}{2} \left[ (\bar{u}q)_{V-A}(\bar{q}c)_{V-A} \pm (\bar{q}q)_{V-A}(\bar{u}c)_{V-A} \right],
\]

\[
O_{\pm}^\dagger = \frac{1}{2} \left[ (\bar{u}s)_{V-A}(\bar{d}c)_{V-A} \pm (\bar{d}s)_{V-A}(\bar{u}c)_{V-A} \right],
\] (2)

where \( (\bar{q}_1 q_2)_{V-A} = \bar{q}_1 \gamma_\mu (1 - \gamma_5) q_2 \). By using \( (V_{cs} V_{ud}, V_{cd} V_{ud}, V_{cd} V_{us}) \simeq (1, -s_c, -s_c^2) \) in Eq. (1) with \( s_c \equiv \sin \theta_c = 0.2248 [7] \) representing the well-known Cabbibo angle \( \theta_c \), the decays with \( O_{\pm}, O_{\pm}^\dagger \) and \( O_{\mp}^q \) are the so-called Cabibbo-allowed, Cabibbo-suppressed, and doubly Cabibbo-suppressed processes, respectively. For instance of the Cabibbo-allowed decay, \( \mathcal{B}(\Lambda_c^+ \to p\bar{K}^0) = (3.16 \pm 0.16) \times 10^{-2} \) is measured to be 50 times larger than \( \mathcal{B}(\Lambda_c^+ \to \Lambda K^+) = (6.1 \pm 1.2) \times 10^{-4} \), which is the Cabibbo-suppressed case, whereas none of the doubly Cabibbo-suppressed ones has been observed [7].

Without explicitly showing the Lorentz indices, the operators in Eq. (2) behave as \( (\bar{g}_k q \bar{g}^k)c \), with \( g_i = (u, d, s) \) as the triplet of 3, which can be decomposed as the irreducible forms under the \( SU(3)_f \) symmetry, that is, \( (3 \times 3 \times 3)c = (\bar{3} + 3' + 6 + 15)c \). Accordingly, \( (O_-, O_+) \) fall into the irreducible presentations of \( (O_6, O_{15}) \), given by [25]

\[
O_6 = \frac{1}{2} (\bar{u}d\bar{s} - \bar{s}d\bar{u})c, \quad O_{15} = \frac{1}{2} (\bar{u}d\bar{s} + \bar{s}d\bar{u})c,
\] (3)
which correspond to the tensor notations of $1/2\epsilon^{i j l}H(6)_{l k}$ and $H(\overline{15})^{i j}_{k}$, respectively, with $(i, j, k)$ representing the quark indices and the non-zero entries being $H_{22}(6) = 2$ and $H_{2}^{\overline{13}}(\overline{15}) = H_{2}^{\overline{31}}(\overline{15}) = 1$. Note that $O_{\mp}^{+}$ and $O_{\mp}^{-}$ also have similar irreducible representations, resulting in the non-zero entries of $H_{23, 32}(6) = -2s_{c}$, $H_{2}^{12, 21}(\overline{15}) = -H_{3}^{13, 31}(\overline{15}) = s_{c}$, $H_{33}(6) = 2s_{c}^{2}$, and $H_{3}^{12, 21}(\overline{15}) = -s_{c}^{2}$ [25]. By using the bases of the $SU(3)_{f}$ symmetry, the effective Hamiltonian in Eq. (1) is transformed as

$$H_{e f f} = \frac{G_{F}}{\sqrt{2}} \left[ c_{-} \frac{\epsilon^{i j l}}{2} H(6)_{l k} + c_{+} H(\overline{15})^{i j}_{k} \right], \quad (4)$$

where the individual non-zero entries of $H(6)_{l k}$ and $H(\overline{15})^{i j}_{k}$ that include $O_{\mp}^{+}$, $O_{\mp}^{i}$ and $O_{\mp}^{i}$ can be presented as the matrix forms:

$$H(6) = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 2 & -2s_{c} \\ 0 & -2s_{c} & 2s_{c}^{2} \end{pmatrix}, \quad H(\overline{15}) = \begin{pmatrix} 0 & 0 & 0 \\ 0 & s_{c} & 1 \\ 0 & s_{c} & 0 \end{pmatrix}, \quad H(\overline{15}) = \begin{pmatrix} 0 & -s_{c}^{2} & -s_{c} \\ -s_{c}^{2} & 0 & 0 \\ -s_{c} & 0 & 0 \end{pmatrix}. \quad (5)$$

Correspondingly, the $B_{c}$ anti-triplet and $B_{n}$ octet states are written as

$$B_{c} = (\Xi_{c}^{0}, -\Xi_{c}^{+}, \Lambda_{c}^{+}), \quad B_{n} = \begin{pmatrix} \frac{1}{\sqrt{6}}\Lambda + \frac{1}{\sqrt{2}}\Sigma^{0} & \Sigma^{+} & p \\ \Sigma^{-} & \frac{1}{\sqrt{6}}\Lambda - \frac{1}{\sqrt{2}}\Sigma^{0} & n \\ \Xi^{-} & \Xi^{0} & -\sqrt{\frac{2}{3}}\Lambda \end{pmatrix}. \quad (6)$$

The adding of the singlet $\eta_{1}$ to the octet $(\pi, K, \eta_{8})$ leads to the nonet of the pseudoscalar meson, given by [30]

$$M = \begin{pmatrix} \frac{1}{\sqrt{2}}(\pi^{0} + c_{\phi}\eta + s_{\phi}\eta') & \pi^{-} & K^{-} \\ \pi^{+} & -\frac{1}{\sqrt{2}}(\pi^{0} - c_{\phi}\eta - s_{\phi}\eta') & \bar{K}^{0} \\ K^{+} & \bar{K}^{0} & -s_{\phi}\eta + c_{\phi}\eta' \end{pmatrix}, \quad (7)$$

where $(\eta, \eta')$ are the mixtures of $(\eta_{1}, \eta_{8})$, with the mixing angle $\phi = (39.3 \pm 1.0)^{0}$ [40] for $(c_{\phi}, s_{\phi}) = (\cos \phi, \sin \phi)$.

The amplitudes of the $B_{c} \to B_{n}M$ decays via the effective Hamiltonian in Eq. (1) appear to be $A(B_{c} \to B_{n}M) = \langle B_{n}M | H_{e f f} | B_{c} \rangle$. Since $H_{e f f}$, $B_{c(n)}$ and $M$ have been in the $SU(3)_{f}$
forms, the amplitudes of $B_c \to B_n M$ can be further derived as

$$\mathcal{A}(B_c \to B_n M) = |\langle B_n M | \mathcal{H}_{\text{eff}} | B_c \rangle| = \frac{G_F}{\sqrt{2}} T(B_c \to B_n M),$$

with $T(B_c \to B_n M)$ given by [28]

$$T(B_c \to B_n M) = T(\mathcal{O}_6) + T(\mathcal{O}_{15})$$

$$T(\mathcal{O}_6) = a_1 H_{ij}(6) T^{ik}(B_n)_k^j(M)_l^i + a_2 H_{ij}(6) T^{ik}(M)_l^i(B_n)_k^j$$

$$+ a_3 H_{ij}(6) (B_n)_k^j(M)_l^i T^{kl} + h H_{ij}(6) T^{ik}(B_n)_k^j(M)_l^i,$$

$$T(\mathcal{O}_{15}) = a_4 H_1^H(B_n)_k^j(M)_l^i T^{kl} + a_5 (B_n)_k^j(M)_l^i H(15)_i^j k(B_c)_k$$

$$+ a_6 (B_n)_k^j(M)_l^i H(15)_i^j k(B_c)_k + a_7 (B_n)_k^j(M)_l^i H(15)_i^j k(B_c)_k$$

$$+ h' H_1^{ik}(15)(B_n)_k^j(M)_l^i T^{kl}(B_c)_j,$$  (9)

where $T_{ij} \equiv (B_c)_k^i c^{ik}$, and $(c_-, c_+)$ have been absorbed into the $SU(3)$ parameters of $(a_1, a_2, a_3, h)$ and $(a_4, a_5, a_6, a_7, h')$, respectively, and the $h^{(l)}$ terms correspond to the contributions from the singlet $\eta_1$. With the T-amps expanded in Table I we are enabled to relate all possible two-body $B_c \to B_n M$ decays with the $SU(3)_f$ parameters. To compute the branching ratios, we use the equation given by [7]

$$B(B_c \to B_n M) = \frac{|\tilde{p}_{cM}|^2 \tau_{B_c}}{8\pi m_{B_c}^2} |\mathcal{A}(B_c \to B_n M)|^2,$$  (10)

where $|\tilde{p}_{cM}| = \sqrt{[m_{B_c}^2 - (m_{B_n} + m_M)^2][m_{B_c}^2 - (m_{B_n} - m_M)^2]/2m_{B_c}}$ and $\tau_{B_c}$ is the lifetime (the inverse of the total decay width) of $B_c$. In Eq. (10), the amplitude squared is defined by

$$|\mathcal{A}(B_c \to B_n M)|^2 = \frac{(G_F V_{ij} V_{kl})^2}{2} T^i(B_c \to B_n M) T^j(B_c \to B_n M).$$  (11)

Note that, since the Lorentz indices have been neglected in the language of the $SU(3)_f$ symmetry, no contractions of the baryon spins are needed, leading to $T^i(B_c \to B_n M) = T^*(B_c \to B_n M)$.

### III. NUMERICAL RESULTS AND DISCUSSIONS

For the numerical analysis, we note that the contributions of the $SU(3)$ parameters $(a_4, a_5, a_6, a_7, h')$ from $H(15)$ would be neglected based on the following reasons. First, the contributions to the decay branching rates from $H(15)$ and $H(6)$ lead to a small ratio of
\[ \mathcal{R}(15/6) = c_+^2/c_-^2 \approx 17\% \] in terms of \((c_+, c_-) = (0.76, 1.78)\) from the QCD calculation at the scale \(\mu = 1\) GeV in the naive dimensional regularization (NDR) scheme [41, 42].
Second, it is pointed out in Ref. [19] that $O_+^{t,l}$ belong to $H(15)$ in the group structure and behave as symmetric operators in color indices, whereas the baryon wave functions are totally antisymmetric, such that the mismatch causes the disappearance of $c_+O_+^{t,l}$ in the calculation of the non-facotrizable effects, which are regarded to be significant in the charmed baryon decays. Note that even though the single ignoring of $H(15)$ is viable, a possible interference between the amplitudes with $H(6)$ and $H(15)$ may be sizable to fail this assumption, which will be tested in the fit. Hence, being from $H(6)$ the parameters $(a_1, a_2, a_3, h)$ in Eq. (9) are kept for the fit, which are in fact complex. Since an overall phase can be removed without losing generality, we set $a_1$ to be real, such that there are seven real independent parameters to be determined, given by

$$a_1, a_2 e^{i\delta_{a_2}}, a_3 e^{i\delta_{a_3}}, h e^{i\delta_h}.$$  

(12)

We use the minimum $\chi^2$ fit for the determination, given by

$$\chi^2 = \sum_i \left( \frac{B_{i_{th}} - B_{i_{ex}}}{\sigma_{i_{ex}}} \right)^2 + \sum_j \left( \frac{R_{j_{th}} - R_{j_{ex}}}{\sigma_{j_{ex}}} \right)^2,$$  

(13)

where $B_{i_{th}}$ and $R_{j_{th}}$ stand for the separated decay branching ratios and the ratios of the two-decay branching fractions from the $SU(3)$ amplitudes, while $B_{i_{ex}}$ and $R_{j_{ex}}$ are the corresponding experimental data, along with $\sigma_{i_{ex}}$ and $\sigma_{j_{ex}}$ the $1\sigma$ uncertainties, respectively. With the ten experimental data in Table 2 the global fit results in

$$(a_1, a_2, a_3, h) = (0.244 \pm 0.006, 0.115 \pm 0.014, 0.088 \pm 0.019, 0.105 \pm 0.073) \text{ GeV}^3,$$

$$(\delta_{a_2}, \delta_{a_3}, \delta_h) = (78.1 \pm 7.1, 35.1 \pm 8.7, 10.2 \pm 29.6)^\circ,$$

$$\chi^2/d.o.f = 5.32/3 = 1.77,$$  

(14)

where $d.o.f$ represents the degree of freedom. The numerical values for the parameters in Eq. (14) are the theoretical inputs, which are used to predict the two-body $B_c \to B$ decays in Table 3.

Since the value of $\chi^2/d.o.f \simeq 1.8$ in Eq. (14) indicates a good fit, there exists no inconstancy by neglecting $H(15)$ in our analysis. Note that the determinations of $|a_1|$ and $|a_2|$ depend on $T(\Lambda_c^+ \to p\bar{K}^0) = -2a_1$ and $T(\Lambda_c^+ \to \Xi^0K^+) = -2a_2$ in Table 4 respectively, by ignoring $(a_5 + a_6)$ and $(a_4 + a_7)$, associated with $H(15)$. Similarly, one can extract $|a_3|$ based
TABLE 2. The data of the \( B_c \rightarrow B_n M \) decays.

| Branching ratios | Data [4, 7] | Branching ratios | Data [4, 7] |
|------------------|-------------|------------------|-------------|
| \( 10^2 \mathcal{B}(\Lambda_c^+ \rightarrow pK^0) \) | 3.16 ± 0.16 | \( 10^2 \mathcal{B}(\Lambda_c^+ \rightarrow \Sigma^+ \eta) \) | 0.70 ± 0.23 |
| \( 10^2 \mathcal{B}(\Lambda_c^+ \rightarrow \Lambda \pi^+) \) | 1.30 ± 0.07 | \( 10^4 \mathcal{B}(\Lambda_c^+ \rightarrow \Lambda K^+) \) | 6.1 ± 1.2 |
| \( 10^2 \mathcal{B}(\Lambda_c^+ \rightarrow \Sigma^+ \pi^0) \) | 1.24 ± 0.10 | \( 10^4 \mathcal{B}(\Lambda_c^+ \rightarrow \Sigma^0 K^+) \) | 5.2 ± 0.8 |
| \( 10^2 \mathcal{B}(\Lambda_c^+ \rightarrow \Sigma^0 \pi^+) \) | 1.29 ± 0.07 | \( 10^4 \mathcal{B}(\Lambda_c^+ \rightarrow p\eta) \) | 12.4 ± 3.0 |
| \( 10^2 \mathcal{B}(\Lambda_c^+ \rightarrow \Xi^0 K^+) \) | 0.50 ± 0.12 | \( \mathcal{R} = \frac{\mathcal{B}(\Xi^0 \rightarrow \Lambda K^0)}{\mathcal{B}(\Xi^0 \rightarrow \Xi^{-} \pi^+)} \) | 0.420 ± 0.056 |

TABLE 3. The numerical results of the \( B_c \rightarrow B_n M \) decays with \( \mathcal{B}_{B_n M} \equiv \mathcal{B}(B_c \rightarrow B_n M) \), where the number with the dagger (†) is the reproduction of the experimental data input, instead of the prediction.

| \( \Xi^0 \) | our results | Ref. [43] | \( \Xi^+ \) | our results | Ref. [43] | \( \Lambda_c^+ \) | our results | Ref. [43] |
|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|
| \( 10^3 \mathcal{B}_{\Sigma^+ K^-} \) | 3.5 ± 0.9 | 3.1 ± 0.9 | \( 10^3 \mathcal{B}_{\Xi^+ K^-} \) | 8.0 ± 3.9 | 0.1 - 102.2 | \( 10^3 \mathcal{B}_{\Sigma^0 p} \) | (1.3 ± 0.2)† | (1.27 ± 0.17)† |
| \( 10^3 \mathcal{B}_{\Sigma^0 K^0} \) | 4.7 ± 1.2 | 4.6 ± 1.4 | \( 10^4 \mathcal{B}_{\Xi^0 p} \) | 8.1 ± 4.0 | 1.2 - 96.8 | \( 10^3 \mathcal{B}_{\Sigma^0 p} \) | (1.3 ± 0.2)† | (1.27 ± 0.17)† |
| \( 10^3 \mathcal{B}_{\Xi^0 K^0} \) | 4.3 ± 0.9 | 0.7 - 18.1 | \( 10^4 \mathcal{B}_{\Xi^0 \eta} \) | 1.7 - 1.0 | 1.0 - 6.0 | \( 10^4 \mathcal{B}_{\Sigma^0 \eta} \) | (0.7 ± 0.3)† | (0.7 ± 0.3)† |
| \( 10^4 \mathcal{B}_{\Xi^0 K^0} \) | 8.6 ± 11.0 | 6.3 | \( 10^4 \mathcal{B}_{\Xi^0 \eta} \) | 0.5 ± 0.1 | (0.5 ± 0.1)† | \( 10^4 \mathcal{B}_{\Xi^0 \eta} \) | (0.5 ± 0.1)† | (0.5 ± 0.1)† |
| \( 10^4 \mathcal{B}_{\Xi^0 \eta} \) | 15.7 ± 0.7 | 22.4 ± 3.4 | \( 10^5 \mathcal{B}_{\Xi^0 K^0} \) | 8.3 ± 0.9 | 9.4 ± 1.6 | \( 10^5 \mathcal{B}_{\Xi^0 K^0} \) | (3.3 ± 0.2)† | (2.72 ± 3.60)† |
| \( 10^5 \mathcal{B}_{\Xi^0 K^0} \) | 2.0 ± 0.5 | | \( 10^5 \mathcal{B}_{\Xi^0 K^0} \) | 18.5 ± 2.2 | | \( 10^5 \mathcal{B}_{\Xi^0 K^0} \) | (4.0 ± 0.8)† | (4.0 ± 0.8)† |
| \( 10^5 \mathcal{B}_{\Xi^0 K^0} \) | 9.0 ± 0.4 | | \( 10^5 \mathcal{B}_{\Xi^0 K^0} \) | 28.4 ± 8.2 | | \( 10^5 \mathcal{B}_{\Xi^0 K^0} \) | 5.7 ± 1.5 | 5.7 ± 1.5 |
| \( 10^5 \mathcal{B}_{\Xi^0 K^0} \) | 3.2 ± 0.3 | | \( 10^5 \mathcal{B}_{\Xi^0 K^0} \) | 13.2 ± 24.0 | | \( 10^5 \mathcal{B}_{\Xi^0 K^0} \) | (12.5 ± 3.8)† | (12.5 ± 3.8)† |
| \( 10^5 \mathcal{B}_{\Xi^0 K^0} \) | 3.6 ± 0.9 | | \( 10^5 \mathcal{B}_{\Xi^0 K^0} \) | 18.0 ± 4.7 | | \( 10^5 \mathcal{B}_{\Xi^0 K^0} \) | 12.2 ± 14.2 | 12.2 ± 14.2 |
| \( 10^5 \mathcal{B}_{\Xi^0 K^0} \) | 1.7 ± 0.3 | | \( 10^5 \mathcal{B}_{\Xi^0 K^0} \) | 20.3 ± 4.2 | | \( 10^5 \mathcal{B}_{\Xi^0 K^0} \) | 11.3 ± 2.9 | 11.3 ± 2.9 |
| \( 10^5 \mathcal{B}_{\Xi^0 K^0} \) | 7.6 ± 0.4 | | \( 10^4 \mathcal{B}_{\Lambda^0 K^0} \) | 1.6 ± 1.2 | | \( 10^4 \mathcal{B}_{\Lambda^0 K^0} \) | 4.6 ± 0.9† | 4.6 ± 0.9† |
| \( 10^5 \mathcal{B}_{\Xi^0 K^0} \) | 6.3 ± 1.2 | | \( 10^5 \mathcal{B}_{\Xi^0 K^0} \) | 8.8 ± 0.4 | | \( 10^5 \mathcal{B}_{\Xi^0 K^0} \) | 12.2 ± 6.0 | 12.2 ± 6.0 |
| \( 10^5 \mathcal{B}_{\Xi^0 K^0} \) | 2.1 ± 0.5 | | \( 10^5 \mathcal{B}_{\Xi^0 K^0} \) | 17.6 ± 0.8 | | \( 10^5 \mathcal{B}_{\Xi^0 K^0} \) | 12.2 ± 6.0 | 12.2 ± 6.0 |
| \( 10^5 \mathcal{B}_{\Xi^0 K^0} \) | 7.9 ± 1.4 | | \( 10^5 \mathcal{B}_{\Xi^0 K^0} \) | 23.8 ± 6.1 | | \( 10^5 \mathcal{B}_{\Xi^0 K^0} \) | 23.8 ± 6.1 | 23.8 ± 6.1 |
| \( 10^5 \mathcal{B}_{\Xi^0 K^0} \) | 0.2 ± 0.2 | | \( 10^5 \mathcal{B}_{\Xi^0 K^0} \) | 10.5 ± 4.5 | | \( 10^5 \mathcal{B}_{\Xi^0 K^0} \) | 10.5 ± 4.5 | 10.5 ± 4.5 |
| \( 10^5 \mathcal{B}_{\Xi^0 K^0} \) | 1.2 ± 0.8 | | \( 10^5 \mathcal{B}_{\Xi^0 K^0} \) | 12.1 ± 6.7 | | \( 10^5 \mathcal{B}_{\Xi^0 K^0} \) | 12.1 ± 6.7 | 12.1 ± 6.7 |
| \( 10^5 \mathcal{B}_{\Xi^0 K^0} \) | 1.4 ± 1.6 | | \( 10^5 \mathcal{B}_{\Xi^0 K^0} \) | 17.3 ± 24.4 | | \( 10^5 \mathcal{B}_{\Xi^0 K^0} \) | 17.3 ± 24.4 | 17.3 ± 24.4 |
| \( 10^5 \mathcal{B}_{\Xi^0 K^0} \) | 4.4 ± 3.7 | | \( 10^5 \mathcal{B}_{\Xi^0 K^0} \) | 56.8 ± 14.5 | | \( 10^5 \mathcal{B}_{\Xi^0 K^0} \) | 56.8 ± 14.5 | 56.8 ± 14.5 |
on $T(\Xi_c^+ \to \Xi^0\pi^+) = 2a_3 + (a_4 + a_6) \simeq 2a_3$. Consequently, we get

\[ R_0 B(\Lambda_c^+ \to p\bar{K}^0) = B(\Xi_c^0 \to \Xi^-\pi^+) = (15.7 \pm 0.7) \times 10^{-3}, \]
\[ R_0 B(\Lambda_c^+ \to \Xi^0 K^+) = B(\Xi_c^0 \to \Sigma^+ K^-) = (0.4 \pm 0.1) \times 10^{-2}, \]
\[ B(\Xi_c^+ \to \Sigma^+\bar{K}^0) = B(\Xi_c^+ \to \Xi^0\pi^+) = (8.1 \pm 4.0) \times 10^{-3}, \]

without the contributions from $H(15)$, where $R_0 = \tau_{\Xi_c^0}/\tau_{\Lambda_c^+} = 0.56 \pm 0.07$. To check if the $H(15)$ terms are indeed negligible, we may use the relations from Table 1 given by

\[ T(\Lambda_c^+ \to p\bar{K}^0) + T(\Xi_c^0 \to \Xi^-\pi^+) = 2(a_5 + a_6), \]
\[ T(\Lambda_c^+ \to \Xi^0 K^+) + T(\Xi_c^0 \to \Sigma^+ K^-) = 2(a_4 + a_7), \]
\[ T(\Xi_c^+ \to \Xi^0\pi^+) + T(\Xi_c^+ \to \Sigma^+\bar{K}^0) = 2(a_4 + a_6). \]

Clearly, if the results in Eq. (15) do not agree with the future measurements, the contributions from $H(15)$ should be reconsidered as seen in Eq. (16).

According to the PDG [7], the branching fractions in the $\Xi_c^0$ decays are observed to be relative to $B_{\Xi^-\pi^+} \equiv B(\Xi_c^0 \to \Xi^-\pi^+)$, predicted in Table 3. Hence, by using the partial observations of $B(\Xi_c^0 \to \Lambda K^-\pi^+) = (1.07 \pm 0.14)B_{\Xi^-\pi^+}$, $B(\Xi_c^0 \to \Lambda K^+ K^-) = (0.029 \pm 0.007)B_{\Xi^-\pi^+}$, and $B(\Xi_c^0 \to \Xi^- e^+ \nu_e) = (3.1 \pm 1.1)B_{\Xi^-\pi^+}$, we obtain

\[ B(\Xi_c^0 \to \Lambda K^-\pi^+) = (16.8 \pm 2.3) \times 10^{-3}, \]
\[ B(\Xi_c^0 \to \Lambda K^+ K^-) = (4.5 \pm 1.1) \times 10^{-4}, \]
\[ B(\Xi_c^0 \to \Xi^- e^+ \nu_e) = (48.7 \pm 17.4) \times 10^{-3}. \]

Similarly, the branching fractions in the $\Xi_c^+$ decays are measured to be relative to $B(\Xi_c^+ \to \Xi^-\pi^+\pi^+)$, which has not been theoretically and experimentally studied yet. With $B(\Xi_c^+ \to \Xi^0\pi^+)/B(\Xi_c^+ \to \Xi^-\pi^+\pi^+) = 0.55 \pm 0.16$ [7] and $B(\Xi_c^+ \to \Xi^0\pi^0)$ in Table 3, we find

\[ B_{\Xi^-2\pi^+} \equiv B(\Xi_c^+ \to \Xi^-\pi^+\pi^+) = (14.7 \pm 8.4) \times 10^{-3}. \]

Subsequently, the relative branching fractions of $B(\Xi_c^+ \to pK_s^0 K_s^0) = (0.087 \pm 0.021)B_{\Xi^-2\pi^+}$, $B(\Xi_c^+ \to \Sigma^+ K^-\pi^+) = (0.94 \pm 0.10)B_{\Xi^-2\pi^+}$, $B(\Xi_c^+ \to \Xi^0\pi^+\pi^0) = (2.3 \pm 0.7)B_{\Xi^-2\pi^+}$ and $B(\Xi_c^+ \to \Xi^0 e^+ \nu_e) = (2.3^{+0.7}_{-0.5})B_{\Xi^-2\pi^+}$ lead to

\[ B(\Xi_c^+ \to pK_s^0 K_s^0) = (1.3 \pm 0.8) \times 10^{-3}, \]
\[ B(\Xi_c^+ \to \Sigma^+ K^-\pi^+) = (13.8 \pm 8.0) \times 10^{-3}, \]
\[ B(\Xi_c^+ \to \Xi^0\pi^+\pi^0) = (33.8 \pm 21.9) \times 10^{-3}, \]
\[ B(\Xi_c^+ \to \Xi^0 e^+ \nu_e) = (33.8^{+21.9}_{-22.6}) \times 10^{-3}. \]
By adding the $h^{(0)}$ terms, we are able to include the contributions from the singlet $\eta_1$ in the $SU(3)_f$ amplitudes, which have been used to explain the observations of $B(\Lambda_c^+ \to \Sigma^+\eta)$ and $B(\Lambda_c^+ \to p\eta)$. Nonetheless, the estimations of $B(\Lambda_c^+ \to \Sigma^+(p)\eta') \approx B(\Lambda_c^+ \to \Sigma^+(p)\eta)$ show no inequality as $B(B \to K\eta') \gg B(B \to K\eta)$ or $B(B \to K^*\eta) \gg B(B \to K^*\eta')$. On the other hand, it is interesting to note that, despite of the large uncertainties, the $\Xi_c \to B_n\eta^{(0)}$ decays contain the similar inequalities between the $\eta$ and $\eta'$ modes, given by

$$B(\Xi_c^0 \to \Xi^0\eta, \Xi^0\eta') = (1.7^{+1.0}_{-1.7}, 8.6^{+11.0}_{-6.3}) \times 10^{-3},$$
$$B(\Xi_c^0 \to \Lambda^0\eta, \Lambda^0\eta') = (1.6^{+1.2}_{-0.8}, 9.4^{+11.6}_{-6.8}) \times 10^{-4},$$
$$B(\Xi_c^+ \to \Sigma^+\eta, \Sigma^+\eta') = (28.4^{+8.2}_{-6.9}, 13.2^{+24.0}_{-11.9}) \times 10^{-4}. \quad (20)$$

We remark that as shown in Table 3, our numerical results for the Cabibbo-allowed processes are consistent with those in Ref. [43], where $B(B_c \to B_n\bar{K}^0)$ are taken from $B(B_c \to B_nK^0_S)$. Finally, we emphasize that there is a discrepancy between the theory and data for $B(\Lambda_c^+ \to p\pi^0)$. In Table 3, $B(\Lambda_c^+ \to p\pi^0)$ is predicted to be $(5.7 \pm 1.5) \times 10^{-4}$, whereas it is measured to be less than $3 \times 10^{-4}$ [4]. Nonetheless, the estimation in the factorization approach also gives $B(\Lambda_c^+ \to p\pi^0) = f_c^2/(2f_K^2)s_c^2B(\Lambda_c^+ \to p\bar{K}^0) = (5.5 \pm 0.3) \times 10^{-4}$ to be as large as our $SU(3)_f$ prediction in Table 3, with the experimental input of $B(\Lambda_c^+ \to p\bar{K}^0) = (3.16 \pm 0.16) \times 10^{-2}$ [7].

Clearly, to resolve this inconsistency, it is necessary to re-measure the decay of $\Lambda_c^+ \to p\pi^0$ in the future experiment.

IV. CONCLUSION

With the $SU(3)_f$ symmetry, we have studied the two-body anti-triplet charmed baryon weak decays. We have predicted that $B(\Xi_c^0 \to \Xi^-\pi^+) = (15.7 \pm 0.7) \times 10^{-3}$ and $B(\Xi_c^0 \to \Xi^-\pi^+\pi^+) = (14.7 \pm 8.4) \times 10^{-3}$, while the branching ratios of the $\Xi_c^0$ and $\Xi_c^+$ decays are measured to be relative to $B(\Xi_c^0 \to \Xi^-\pi^+)$ and $B(\Xi_c^+ \to \Xi^-\pi^+\pi^+)$, respectively. Hence, we have extracted that $B(\Xi_c^0 \to \Lambda K^+\pi^+, \Lambda K^+K^-, \Xi^-e^+\nu_e) = (16.8 \pm 2.3, 0.45 \pm 0.11, 48.7 \pm 17.4) \times 10^{-3}$ and $B(\Xi_c^+ \to pK^0\bar{K}_S^0, \Sigma^+K^-\pi^+, \Xi^0\pi^+\pi^0, \Xi^0e^+\nu_e) = (1.3 \pm 0.8, 13.8 \pm 8.0, 33.8 \pm 21.9, 33.8^{+21.9}_{-22.6}) \times 10^{-3}$. In addition, we have shown that $B(\Xi_c^0 \to \Xi^0\eta, \Xi^0\eta') = (1.7^{+1.0}_{-1.7}, 8.6^{+11.0}_{-6.3}) \times 10^{-3}$, $B(\Xi_c^0 \to \Lambda^0\eta, \Lambda^0\eta') = (1.6^{+1.2}_{-0.8}, 9.4^{+11.6}_{-6.8}) \times 10^{-4}$ and $B(\Xi_c^+ \to \Sigma^+\eta, \Sigma^+\eta') = (28.4^{+8.2}_{-6.9}, 13.2^{+24.0}_{-11.9}) \times 10^{-4}$, representing the inequalities, similar to those of $B(B \to K\eta') \gg B(B \to K\eta)$ or $B(B \to K^*\eta) \gg B(B \to K^*\eta')$ in the mesonic $B$ decays.
involving $\eta^\prime$. According to our predictions, the branching ratios of two and three-body $\Xi_c$ decays are accessible to the experiments at BESIII and LHCb.

\section*{ACKNOWLEDGMENTS}

This work was supported in part by National Center for Theoretical Sciences, MoST (MoST-104-2112-M-007-003-MY3), and National Science Foundation of China (11675030).

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