Chapter

Some Methods of Fuzzy Conditional Inference for Application to Fuzzy Control Systems

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Abstract

Zadeh proposed fuzzy logic with single membership function. Two Zadeh, Mamdani and TSK proposed fuzzy conditional inference. In many applications like fuzzy control systems, the consequent part may be derived from precedent part. Zadeh, Mamdani and TSK proposed different fuzzy conditional inferences for “if … then …” for approximate reasoning. The Zadeh and Mamdani fuzzy conditional inferences are know prior information for both precedent part and consequent part. The TSK fuzzy conditional inferences need not know prior information for consequent part but it is difficult to compute. In this chapter, fuzzy conditional inference is proposed for “if…then…” This fuzzy conditional inference need not know prior information of the consequent part. The fuzzy conditional inference is discussed using the single fuzzy membership function and twofold fuzzy membership functions. The fuzzy control system is given as an application.

Keywords: fuzzy logic, twofold fuzzy logic, fuzzy conditional inference, fuzzy control systems

1. Introduction

When information is incomplete, fuzzy logic is useful [10–26]. Many theories [1, 2] deal with incomplete information based on likelihood (probability), whereas fuzzy logic is based on belief. Zadeh defined fuzzy set with single membership function. Zadeh [3], Mamdani [4], TSK [2] and Reddy [5] are studied fuzzy conditional inferences. The fuzzy conditions are of the form “if <. Zadeh, Mamdani and TSK fuzzy conditional inference requires both precedent-part and consequent-part but 5fuzzy inferences don’t require consequent part. Precedent-part > then < consequent-part >.”

Zadeh [6] studied fuzzy logic with single membership function. The single membership function for the proposition “x is A” contains how much truth in the proposition. The fuzzy set with two membership functions will contain more information in terms of how much truth and false it has in the proposition. The fuzzy certainty factor is studied as difference on two membership functions “true” and “false” to eliminate conflict of evidence, and it becomes single membership function. The FCF is a fuzzy set with single fuzzy membership function of twofold fuzzy set.
The fuzzy control systems are considered in this chapter as application of single fuzzy membership function and twofold fuzzy set.

2. Fuzzy log with single membership function

Zadeh [6] has introduced a fuzzy set as a model to deal with imprecise, inconsistent and inexact information. The fuzzy set is a class of objects with a continuum of grades of membership.

The fuzzy set $A$ of $X$ is characterized as its membership function $A = \mu_A(x)$ and ranging values in the unit interval $[0, 1]$.

\[ \mu_A(x) : X \rightarrow [0, 1], x \in X, \]

where $X$ is the universe of discourse.

$A = \mu_A(x_1)/x_1 + \mu_A(x_2)/x_2 + \ldots + \mu_A(x_n)/x_n,$

where “+” is the union.

For instance, the fuzzy proposition “$x$ is High”

High = $0.2/x_1 + 0.6/x_2 + 0.9/x_3 + 0.6/x_4 + 0.2/x_5$

Not High = $0.8/x_1 + 0.4/x_2 + 0.1/x_3 + 0.4/x_4 + 0.8/x_5$

For instance, the fuzziness of “Temperature is high” is 0.8

The graphical representation of young and not young is shown in Figure 1.

The fuzzy logic is defined as a combination of fuzzy sets using logical operators. Some of the logical operations are given below.

For example, $A$, $B$ and $C$ are fuzzy sets. The operations on fuzzy sets are given as:

**Negation**

If $x$ is not $A$

$A' = 1 - \mu_A(x)/x$

**Conjunction**

$x$ is $A$ and $y$ is $B$ $(x, y)$ is $A \Lambda B$

$A \Lambda B = \min(\mu_A(x), \mu_B(y))(x,y)$

If $x = y$

$x$ is $A$ and $y$ is $B$ $(x, y)$ is $A \Lambda B$

$A \Lambda B = \min(\mu_A(x), \mu_B(y))/x$

For example

\[ A = 0.2/x_1 + 0.6/x_2 + 0.9/x_3 + 0.6/x_4 + 0.2/x_5 \]

\[ B = 0.4/x_1 + 0.6/x_2 + 0.9/x_3 + 0.6/x_4 + 0.1/x_5 \]

\[ A \Lambda B = 0.2/x_1 + 0.6/x_2 + 0.9/x_3 + 0.6/x_4 + 0.1/x_5 \]

The graphical representation is shown in Figures 1 and 2.

Figure 1.

Fuzzy membership function.
Disjunction
x is A and y is B⇒ (x, y) is A V B
A V B = max(μ_A(x), μ_B(y))(x, y)
If x = y
x is A and y is B⇒ (x, y) is A V B
AVB = max(μ_A(x), μ_B(y))/x
For instance,
A = 0.2/x_1 + 0.6/x_2 + 0.9/x_3 + 0.6/x_4 + 0.2/x_5
B = 0.4/x_1 + 0.6/x_2 + 0.9/x_3 + 0.6/x_4 + 0.1/x_5
AVB = 0.4/x_1 + 0.6/x_2 + 0.9/x_3 + 0.6/x_4 + 0.2/x_5
The graphical representation is shown in Figure 3.

Concentration
μ_{very A}(x) = μ_A(x)^2
Diffusion
μ_{more or less A}(x) = μ_A(x)^{0.5}

The graphical representation of concentration and diffusion is shown in Figure 4.

Implication
Zadeh [6], Mamdani [7] and Reddy [5] fuzzy conditional inferences are considered for fuzzy control systems.

If x_1 is A_1 and x_2 is A_2 and … and x_n is A_n, then y is B
The presidency part may contain any number of “and/or”
Zadeh [6] fuzzy inference is given as:
If x_1 is A_1 and x_2 is A_2 and … and x_n is A_n, then y is B
= min(1, 1 − (A_1, A_2, …, A_n) + B)
Mamdani [4] fuzzy inference is given as:
If $x_1$ is $A_1$ and $x_2$ is $A_2$ and … and $x_n$ is $A_n$, then $y$ is $B$
$$= \min(A_1, A_2, \ldots, A_n, B)$$
Zadeh and Mamdani fuzzy inference has prior information of $A$ and $B$. The relation between $A$ and $B$ is known. Then, $B$ is derived from $A$.
Reddy [2] inference is given by:
If $x_1$ is $A_1$ and $x_2$ is $A_2$ and … and $x_n$ is $A_n$, then $y$ is $B$
$$= \min(A_1, A_2, \ldots, A_n)$$
Consider the fuzzy rule:
If $x_1$ is $A_1$ and $x_2$ is $A_2$, then $x$ is $B$
For instance,
$$A_1 = 0.2/x_1 + 0.6/x_2 + 0.9/x_3 + 0.6/x_4 + 0.2/x_5$$
$$A_2 = 0.5/x_1 + 0.7/x_2 + 0.9/x_3 + 0.7/x_4 + 0.3/x_5$$
$$B = 0.1/x_1 + 0.4/x_2 + 0.6/x_3 + 0.4/x_4 + 0.1/x_5$$
The graphical representation of $A_1$, $A_2$ and $B$ is shown in Figure 5.
The graphical representation of fuzzy inference is shown in Figure 6.

**Composition**
If some relation between $R$ and $A_1$ than $B_1$ is to infer from $R$
$$B_1 = A_1 \circ R$$
where $R = A \Rightarrow B$
Zadeh fuzzy inference is given by:
$$B_1 = A_1 \circ R = \min(\mu_A(x), \mu_R(x))$$
$$= \min(\mu_A(x), \min(1,1-\mu_{A_1}(x) + \mu_B(x)))$$
Mamdani fuzzy inference is given by:
$$= \min(\mu_{A_1}(x), \mu_A(x) + \mu_B(x))$$
If there is some relation $R$ between $A$ and $B$, then Reddy fuzzy inference is given by:
$$= \mu_{A_1}(x)$$
3. Justification of Reddy and Mamdani fuzzy conditional inference

Justification of Reddy fuzzy conditional inference may be derived in the following:

Consider Reddy fuzzy conditional inference:
If $x_1$ is $A_1$ and $x_2$ is $A_2$ and ... and $x_n$ is $A_n$, then $y$ is $B = \min\{A_1, A_2, ..., A_n\}$.

Consider TSK fuzzy conditional inference:
If $x_1$ is $A_1$ and $x_2$ is $A_2$ and ... and $x_n$ is $A_n$, then $y$ is $B = f(x_1, x_2, ..., x_n)$.

The proposed method of fuzzy conditional inference may be defined by replacing $x_1, x_2, ..., x_n$ with $A_1, A_2$ and ... and $A_n$.

If $x_1$ is $A_1$ and/or $A_2$ and/or, ... and/or $A_n$, then $y$ is $B = f(A_1, A_2, ..., A_n)$
If $x_1$ is $A_1$ or $A_2$ and $A_3$, then $y$ is $B = f(A_1, A_2, A_3) = A_1 \Lambda A_2 \Lambda A_3$
If $x_1$ is $A_1$ or $A_2$ and $A_3$, then $y$ is $B = f(A_1, A_2, A_3) = A_1 \vee A_2 \Lambda A_3$
$B = \min(\max(\mu_{A_1}(x_1), \mu_{A_2}(x_2)), \mu_{A_3}(x_3))$

The fuzzy conditional inference is given by using Mamdani fuzzy inference

If $x_1$ is $A_1$ or $A_2$ and $A_3$, then $y$ is $B = \min(A_1 \text{ or } A_2 \text{ and } A_3, B)$
If $x_1$ is $A_1$ or $A_2$ and $A_3$, then $x$ is $B = \min(\max(\mu_{A_1}(x_1), \mu_{A_2}(x_2)), \mu_{A_3}(x_3))$

Thus, the Reddy fuzzy conditional inference is satisfied.

If $x_1$ is $A_1$ and $x_2$ is $A_2$ and ... and $x_n$ is $A_n$, then $y$ is $B = \min(A_1, A_2, ..., A_n)$.

Justification of Mamdani fuzzy conditional inference may be derived in the following:

If some relation $R$ between $A$ and $B$ is known, then Mamdani fuzzy conditional inference is given by:
If $x$ is $A$, then $y$ is $B = A \times B$

Zadeh fuzzy conditional inference for “if ... then ... else ...” is given by:
If $x$ is $A$, then $y$ is $B$ else $y$ is $C = A \times B \vee A' \times C$
If $x$ is $A$, then $y$ is $B$ else $y$ is $C = A \times B$ if $x$ is $A'$ then $y$ is $C = A \times B$
$v A' \times C$

It is logically divided into:
If $x$ is $A$, then $y$ is $B = A \times B$
If $x$ is $A'$, then $y$ is $C = A' \times C$

Thus, the Mamdani fuzzy conditional inference is satisfied.

If $x$ is $A$, then $y$ is $B = A \times B$. 

Figure 6.
Fuzzy conditional inference.
4. Fuzzy control systems using single fuzzy membership function

Zadeh introduced fuzzy algorithms. The fuzzy algorithm is a set of fuzzy statements. The fuzzy conditional statement is defined as fuzzy algorithm:
If \( x_i \) is \( A_{1i} \) and \( x_i \) is \( A_{2i} \) and... and \( x_i \) is \( A_n \), then \( y \) is \( B_i \).
The consequent part may not be known in control systems.
The fuzziness may be given for Reddy fuzzy inference as
If \( B_Z \) is low (0.6) and \( B_E \) is normal (0.7) then reduce fan speed
\[ = \min(0.6, 0.7) \]
\[ = 0.6 \]

The fuzzy set type-2 is a type of fuzzy set in which some additional degree of information is provided.

**Definition:** Given some universe of discourse \( X \), a fuzzy set type-2 \( A \) of \( X \) is defined by its membership function \( \mu_A(x) \) taking values on the unit interval \([0,1]\), i.e., \( \mu_A(x) \rightarrow [0,1] \)
Suppose \( X \) is a finite set. The fuzzy set \( A \) of \( X \) may be represented as
\[ A = \mu_{A_1}(x_1)/A_1 + \mu_{A_2}(x_2)/A_2 + ... + \mu_{A_n}(x_n)/A_n \]
Temperature = \{0.4/low, 0.6/medium, 0.9/high\}
John has “mild headache” with fuzziness 0.4
The fuzzy control system for boiler consists of a set of fuzzy rules [4].
If a set of conditions is satisfied, then the set of consequences is fired.
The fuzzy control system is shown in **Figure 7**.
The fuzzy control system containing fuzzy variables are represented in decision

**Table 1.**
The fuzzy control system of boiler is given in **Table 2**.
For instance,
If \( B_Z \) is low and \( B_E \) is normal then reduce fan speed.
For instance, consider the fuzzy control system (**Table 3**).
The computation of proposed method (3.4) is given in **Table 4**.

**Defuzzification**
The centroid technique is used for defuzzification. It finds value representing the centre of gravity (COG) aggregated fuzzy generalized fuzzy set:

![Fuzzy control system](image-url)
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\[ \text{COG} = \sum C_i \mu_{A_i}(x) / \sum C_i \]

For instance,
\[ \text{Speed} = \{0.1/20 + 0.3/40 + 0.5/60 + 0.7/80 + 0.9/100\} \]
\[ \text{COG} = (0.1*20 + 0.3*40 + 0.5*60 + 0.7*80 + 0.9*100) / (0.1 + 0.3 + 0.5 + 0.7 + 0.9) = 73.6 \]

Table 1.
Fuzzy rules.

| Condition | Burning zone (BZ) temperature | Back-end (BE) temperature | Action |
|-----------|-------------------------------|---------------------------|--------|
| AND       | Drastically low               | Low                       | Reduce Klin speed |
| AND       | Drastically low               | Low                       | Reduce fuel    |
| AND       | Slightly low                  | Low                       | Increase fan speed |
| AND       | Low                           | High                      | Reduce fuel    |
| AND       | Low                           | Normal                    | Reduce fan speed |

Table 2.
Boiler controller.

| Condition | Burning zone (BZ) temperature | Back-end (BE) temperature | Action |
|-----------|-------------------------------|---------------------------|--------|
| AND       | Drastically low (0.7)         | Low (0.6)                 | Reduce Klin speed |
| AND       | Drastically low (0.7)         | Low (0.8)                 | Reduce fuel    |
| AND       | Slightly low (.8)             | Low (.9)                  | Increase fan speed |
| AND       | Low (0.7)                     | High (0.65)               | Reduce fuel    |
| AND       | Low (0.6)                     | Normal (0.7)              | Reduce fan speed |

Table 3.
Boiler fuzzy controller.

| Condition | Burning zone (BZ) temperature | Back-end (BE) temperature | Action |
|-----------|-------------------------------|---------------------------|--------|
| AND       | Drastically low (0.7)         | Low (0.6)                 | Reduce Klin speed (0.6) |
| AND       | Drastically low (0.7)         | Low (0.8)                 | Reduce fuel (0.7) |
| AND       | Slightly low (.8)             | Low (.9)                  | Increase fan speed (0.8) |
| AND       | Low (0.7)                     | High (0.65)               | Reduce fuel (0.65) |
| AND       | Low (0.6)                     | Normal (0.7)              | Reduce fan speed (0.6) |

Table 4.
Fuzzy inference.
5. Fuzzy logic with twofold fuzzy sets

Generalized fuzzy logic is studied for incomplete information [8, 9].

Given some universe of discourse $X$, the proposition “$x$ is $A$” is defined as its twofold fuzzy set with membership function as

$\mu_A(x) = \{\mu_A^{\text{True}}(x), \mu_A^{\text{False}}(x)\}$

or

$A = \{\mu_A^{\text{True}}(x), \mu_A^{\text{False}}(x)\}$

where $A$ is the generalized fuzzy set and $x \in X$,

$0 < \mu_A^{\text{True}}(x) < 1$ and, $0 < \mu_A^{\text{False}}(x) < 1$

$A = \{\mu_A^{\text{True}}(x_1)/x_1 + \ldots + \mu_A^{\text{True}}(x_n)/x_n, \mu_A^{\text{False}}(x_1)/x_1 + \ldots + \mu_A^{\text{False}}(x_n)/x_n, x_i \in X$,

$\mu_A^{\text{True}}(x) + \mu_A^{\text{False}}(x) < 1,$

$\mu_A^{\text{True}}(x) + \mu_A^{\text{False}}(x) > 1$ and

$\mu_A^{\text{True}}(x) + \mu_A^{\text{False}}(x) = 1$

The conditions are interpreted as redundant, insufficient and sufficient, respectively.

For instance,

$A = \{0.5/x_1 + 0.7/x_2 + 0.9/x_3 + 0.7/x_4 + 0.5/x_5,$

$0.1/x_1 + 0.2/x_2 + 0.3/x_3 + 0.2/x_4 + 0.1/x_3\}$

The graphical representation is shown in Figure 8.

The fuzzy logic is defined as a combination of fuzzy sets using logical operators. Some of the logical operations are given below.

Let $A$, $B$ and $C$ be the fuzzy sets. The operations on fuzzy sets are given below for twofold fuzzy sets.

Negation

$A' = \{1 - \mu_A^{\text{True}}(x), 1 - \mu_A^{\text{False}}(x)\}/x$

Disjunction

$AVB = \{\max(\mu_A^{\text{True}}(x), \mu_A^{\text{True}}(y)), \max(\mu_B^{\text{False}}(x), \mu_B^{\text{False}}(y))\}/(x,y)$

Conjunction

$A\wedge B = \{\min(\mu_A^{\text{True}}(x), \mu_A^{\text{True}}(y)), \min(\mu_B^{\text{False}}(x), \mu_B^{\text{False}}(y))\}/(x,y)$

Composition

$A \circ R = \{\min_x (\mu_A^{\text{True}}(x), \mu_R^{\text{True}}(x)), \min_x (\mu_R^{\text{False}}(x), \mu_R^{\text{False}}(x))\}/y$

The fuzzy propositions may contain quantifiers like “very”, “more or less”.

These fuzzy quantifiers may be eliminated as follows:

Concentration

“$x$ is very $A$”

| Condition | Burning zone (BZ) temperature | Back-end (BE) temperature | Action |
|-----------|-------------------------------|---------------------------|--------|
| AND/OR    | Drastically low (0.7,0.1)     | Low (0.8,0.1)             | Reduce Klin speed (0.6,0.2) |
| AND/OR    | Drastically low (0.8,0.1)     | Low (0.9,0.1)             | Reduce fuel (0.7,0.2)       |
| AND/OR    | Slightly low (1.0,0.2)        | Low (1.0,0.1)             | Increase fan speed (0.9,0.2) |
| AND/OR    | Low (0.8,0.1)                 | High (0.9,0.2)            | Reduce fuel (0.6,0.1)       |
| AND/OR    | Low (0.7,0.1)                 | Normal (0.8,0.2)          | Reduce fan speed (0.5,0.1)  |

Table 5.

Twofold fuzziness.
\[ \mu_{\text{very} \ A}(x) = \{\mu_{A\ True}(x)^2, \mu_{A\ False}(x)\mu_A(x)^2\} \]

**Diffusion**

“x is more or less A”

\[ \mu_{\text{more or less} \ A}(x) = (\mu_{A\ True}(x)^{1/2}, \mu_{A\ False}(x)\mu_A(x)^{0.5}) \]

\[ A = \{0.5/x_1 + 0.7/x_2 + 0.9/x_3 + 0.7/x_4 + 0.5/x_5, 0.1/x_1 + 0.2/x_2 + 0.3/x_3 + 0.2/x_4 + 0.1/x_5\} \]

\[ B = \{0.4/x_1 + 0.6/x_2 + 0.8/x_3 + 0.6/x_4 + 0.4/x_5, 0.1/x_1 + 0.2/x_2 + 0.3/x_3 + 0.2/x_4 + 0.1/x_5\} \]

\[ A' = \{0.5/x_1 + 0.3/x_2 + 0.1/x_3 + 0.3/x_4 + 0.5/x_5, 0.9/x_1 + 0.8/x_2 + 0.7/x_3 + 0.8/x_4 + 0.9/x_5\} \]

\[ A \lor B = \{0.5/x_1 + 0.7/x_2 + 0.9/x_3 + 0.7/x_4 + 0.5/x_5, 0.1/x_1 + 0.2/x_2 + 0.3/x_3 + 0.2/x_4 + 0.1/x_5\} \]

\[ A \land B = \{0.4/x_1 + 0.6/x_2 + 0.8/x_3 + 0.6/x_4 + 0.4/x_5, 0.1/x_1 + 0.2/x_2 + 0.3/x_3 + 0.2/x_4 + 0.1/x_5\} \]

\[ \text{Very} \ A = \{0.25/x_1 + 0.49/x_2 + 0.81/x_3 + 0.49/x_4 + 0.25/x_5, 0.01/x_1 + 0.04/x_2 + 0.09/x_3 + 0.04/x_4 + 0.01/x_5\} \]

More or less \ A = \{0.70/x_1 + 0.83/x_2 + 0.94/x_3 + 0.83/x_4 + 0.70/x_5, 0.31/x_1 + 0.44/x_2 + 0.54/x_3 + 0.44/x_4 + 0.31/x_5\} \]

\[ A \rightarrow B = \{1/x_1 + 0.8/x_2 + 0.9/x_3 + 0.9/x_4 + 1/x_5, 1/x_1 + 1/x_2 + 1/x_3 + 0.8/x_4 + 1/x_5\} \]

\[ A \bigcirc B = \{0.8/x_1 + 0.7/x_2 + 0.7/x_3 + 0.5/x_4 + 0.5/x_5, 0.4/x_1 + 0.3/x_2 + 0.4/x_3 + 0.5/x_4 + 0.6/x_5\} \]

**Implication**

Consider the fuzzy condition “if x is A1 and x is A2, then x is B.”

The presidency part may contain any number of “and”/“or.”

Zadeh fuzzy conditional inference given as

\[ = \{\min (1, 1 - \min(\mu_{A1\ True}(x), \mu_{A2\ True}(x), \ldots, \mu_{An\ True}(x)) + \mu_{B\ True}(y)), \min (1, 1 - \min(\mu_{A1\ False}(x), \mu_{A2\ False}(x), \ldots, \mu_{An\ False}(x)) + \mu_{B\ False}(y))\}(x,y) \]

Mamdani fuzzy conditional inference given as

\[ = \{\min(\mu_{A1\ True}(x), \mu_{A2\ True}(x), \ldots, \mu_{An\ True}(x)), \mu_{B\ True}(y)), \min(\mu_{A1\ False}(x), \mu_{A2\ False}(x), \ldots, \mu_{An\ False}(x))\}(x,y) \]

Reddy [5] fuzzy conditional inference given by

\[ = \{\min(\mu_{A1\ True}(x), \mu_{A2\ True}(x), \ldots, \mu_{An\ True}(x)), \min(\mu_{A1\ False}(x), \mu_{A2\ False}(x), \ldots, \mu_{An\ False}(x))\}(x,y) \]

Consider the fuzzy condition “if x is A1 and x is A2, then x is B”

The presidency part may contain any number of “and”/“or.”
For instance, A1 = \{0.5/x_1 + 0.7/x_2 + 0.9/x_3 + 0.7/x_4 + 0.5/x_5, 0.1/x_1 + 0.2/x_2 + 0.3/x_3 + 0.2/x_4 + 0.1/x_5\} \\
A2 = \{0.4/x_1 + 0.6/x_2 + 0.8/x_3 + 0.6/x_4 + 0.4/x_5, 0.1/x_1 + 0.2/x_2 + 0.3/x_3 + 0.2/x_4 + 0.1/x_5\} \\
B = \{0.5/x_1 + 0.7/x_2 + 1/x_3 + 0.7/x_4 + 0.5/x_5, 0.4/x_1 + 0.5/x_2 + 0.6/x_3 + 0.5/x_4 + 0.4/x_5\}

Zadeh fuzzy conditional inference given as
\[
\begin{align*}
&= \{\min (1, 1 - \min(\mu_{A1}^{\text{True}}(x), \mu_{A2}^{\text{True}}(x))) + \mu_{B}^{\text{True}}(x)), \min (1, 1 - \min(\mu_{A1}^{\text{False}}(x), \mu_{A2}^{\text{TrueFalse}}(x))) + \mu_{B}^{\text{False}}(x)) \}
\end{align*}
\]

Mamdani fuzzy conditional inference given as
\[
\begin{align*}
&= \{\min(\mu_{A1}^{\text{True}}(x), \mu_{A2}^{\text{True}}(x), \ldots, \mu_{An}^{\text{True}}(x), \mu_{B}^{\text{True}}(x)), \min(\mu_{A1}^{\text{False}}(x), \mu_{A2}^{\text{TrueFalse}}(x), \ldots, \mu_{An}^{\text{False}}(x), \mu_{B}^{\text{False}}(x)) \}
\end{align*}
\]

Reddy fuzzy conditional inference given as
\[
\begin{align*}
&= \{\min(\mu_{A1}^{\text{True}}(x), \mu_{A2}^{\text{True}}(x)), \min(\mu_{A1}^{\text{False}}(x), \mu_{A2}^{\text{TrueFalse}}(x)) \}
\end{align*}
\]

Composition
If some relation \( R \) between A and B is known and some value \( A_1 \) than \( B_1 \) is inferred from \( R \),
\[
B_1 = A_1 \circ R,
\]
where \( R = A \rightarrow B \). Zadeh fuzzy inference is given by
\[
B_1 = A_1 \circ R = A_1 \circ \{\min (1, 1 - \mu_{A}^{\text{True}}(x)) + \mu_{B}^{\text{True}}(x)), \min (1, 1 - \mu_{A}^{\text{False}}(x)) + \mu_{B}^{\text{False}}(x)) \}
\]

Mamdani fuzzy inference is given by
\[
B_1 = A_1 \circ \{\min(\mu_{A1}^{\text{True}}(x), \mu_{B}^{\text{True}}(x)), \min(\mu_{A1}^{\text{False}}(x), \mu_{B}^{\text{False}}(x)) \}
\]

Reddy fuzzy inference is given by
\[
B_1 = \{\min(\mu_{A1}^{\text{True}}(x), \mu_{A2}^{\text{True}}(x)), \min(\mu_{A1}^{\text{False}}(x), \mu_{A2}^{\text{TrueFalse}}(x)) \}
\]

The fuzzy set \( A \) of \( X \) is characterized as its membership function \( A = \mu_{A}(x) \) and ranging values in the unit interval \([0, 1]\)
\[
\mu_{A}(x): X \rightarrow [0, 1], x \in X, \text{ where } X \text{ is universe of discourse.}
\]
A = \( \mu_{A}(x) = \mu_{A}(x_1)/x_1 + \mu_{A}(x_2)/x_2 + \ldots + \mu_{A}(x_n)/x_n \) ”is union

The generalized fuzzy certainty factor (GFCF) is defined as
\[
\mu_{A}^{\text{GFCF}}(x) = \mu_{A}^{\text{True}}(x) - \mu_{A}^{\text{False}}(x)
\]
The generalized fuzzy certainty factor becomes single fuzzy membership function.
\[
\mu_{A}^{\text{GFCF}}(x): X \rightarrow [0, 1], x \in X, \text{ where } X \text{ is universe of discourse.}
\]
The generalized fuzzy certainty factor (GFCF) will compute the conflict of evidence in the uncertain information.

For example,
\[
A = \{0.5/x_1 + 0.7/x_2 + 0.9/x_3 + 0.7/x_4 + 0.5/x_5, 0.1/x_1 + 0.2/x_2 + 0.3/x_3 + 0.2/x_4 + 0.1/x_5\}
\]
\[
\mu_{A}^{\text{GFCF}}(x) = \{0.5/x_1 + 0.7/x_2 + 0.9/x_3 + 0.7/x_4 + 0.5/x_5 - 0.1/x_1 + 0.2/x_2 + 0.3/x_3 + 0.2/x_4 + 0.1/x_5 \}
\]
\[
= 0.4/x_1 + 0.5/x_2 + 0.6/x_3 + 0.5/x_4 + 0.4/x_5
\]
For instance, “x is high temperature” with fuzziness \([0.8,0.2]\)
The GFCF is 0.6
The graphical representation of GFCF is shown in Figure 9.
For example, A and B are generalized fuzzy sets.

\[
A = \{0.5/x_1 + 0.7/x_2 + 0.9/x_3 + 0.7/x_4 + 0.5/x_5 - 0.1/x_1 + 0.2/x_2 + 0.3/x_3 + 0.2/x_4 + 0.1/x_5\}
\]

\[
B = \{0.4/x_1 + 0.6/x_2 + 0.8/x_3 + 0.6/x_4 + 0.4/x_5 - 0.1/x_1 + 0.2/x_2 + 0.3/x_3 + 0.2/x_4 + 0.1/x_5\}
\]

The operations on GFCF are given as follows:

**Negation**

\[
A' = 1 - \mu_A^{GFCF}(x)/x
\]

\[
A' = 0.6/x_1 + 0.5/x_2 + 0.4/x_3 + 0.5x_4 + 0.6/x_5
\]

The graphical representation is shown in Figure 10.

**Conjunction**

\[
A \land B = \min(\mu_A(x), \mu_B(x))/x
\]

\[
A \land B = 0.3/x_1 + 0.4/x_2 + 0.5/x_3 + 0.4/x_4 + 0.3/x_5
\]

The graphical representation is shown in Figure 11.

**Disjunction**

\[
A \lor B = \max(\mu_A(x), \mu_B(y))/x
\]

\[
A \lor B = .4/x_1 + 0.6/x_2 + 0.9/x_3 + 0.6/x_4 + 0.2/x_5
\]

The graphical representation is shown in Figure 12.

**Concentration**

\[
\mu_{\text{con}} A^{GFCF}(x) = \mu_A^{GFCF}(x)^2
\]
\[ 0.16/x_1 + 0.25/x_2 + 0.36/x_3 + 0.25/x_4 + 0.16/x_5 \]

**Diffusion**

\[ \mu_{\text{more or less } A}^{\text{GFCF}}(x) = \mu_A^{\text{GFCF}}(x)^{0.5} \]

\[ = 0.63/x_1 + 0.71/x_2 + 0.77/x_3 + 0.71/x_4 + 0.63/x_5 \]

The graphical representation of concentration and diffusion are shown in **Figure 13**.

**Implication**

Zadeh [9], Mamdani [7] and Reddy [5] fuzzy conditional inferences are considered keeping in view of fuzzy control systems.
Zadeh fuzzy inference is given as follows:
If \( x_1 \) is \( A_1 \) and \( x_2 \) is \( A_2 \) and \( \ldots \) and \( x_n \) is \( A_n \), then \( y \) is \( B \)

\[ = \min \left( 1, \frac{1}{\text{C}_0(A_1, A_2, \ldots, A_n) + B} \right) \]

Mamdani fuzzy inference is given as follows:
If \( x_1 \) is \( A_1 \) and \( x_2 \) is \( A_2 \) and \( \ldots \) and \( x_n \) is \( A_n \), then \( y \) is \( B \)

\[ = \min(\min(A_1, A_2, \ldots, A_n), B) \]

Reddy inference is given as follows:
If \( x_1 \) is \( A_1 \) and \( x_2 \) is \( A_2 \) and \( \ldots \) and \( x_n \) is \( A_n \), then \( y \) is \( B \)

\[ = \min(\min(A_1, A_2, \ldots, A_n)) \]

Consider the fuzzy rule:
If \( x_1 \) is \( A_1 \) and \( x_2 \) is \( A_2 \), then \( x \) is \( B \)

For instance,

\[ A_1 = \{0.5/x_1 + 0.7/x_2 + 0.9/x_3 + 0.7/x_4 + 0.5/x_5, 0.1/x_1 + 0.2/x_2 + 0.3/x_3 + 0.2/x_4 + 0.1/x_5\} \]

\[ = 0.4/x_1 + 0.5/x_2 + 0.6/x_3 + 0.5/x_4 + 0.4/x_5 \]

\[ A_2 = \{0.4/x_1 + 0.6/x_2 + 0.8/x_3 + 0.6/x_4 + 0.4/x_5, 0.1/x_1 + 0.2/x_2 + 0.3/x_3 + 0.2/x_4 + 0.1/x_5\} \]

\[ = 0.3/x_1 + 0.4/x_2 + 0.5/x_3 + 0.4/x_4 + 0.3/x_5 \]

\[ B = \{0.5/x_1 + 0.7/x_2 + 1/x_3 + 0.7/x_4 + 0.5/x_5, 0.4/x_1 + 0.5/x_2 + 0.6/x_3 + 0.5/x_4 + 0.4/x_5\} \]

\[ = 0.1/x_1 + 0.2/x_2 + 0.4/x_3 + 0.2/x_4 + 0.1/x_5 \]

The graphical representation of \( A_1 \), \( A_2 \) and \( B \) is shown in Figure 14.

Zadeh fuzzy inference is given as

\[ = \min(1, \frac{1}{\text{C}_0(A_1, A_2) + B}) \]

\[ = 0.8/x_1 + 0.8/x_2 + 0.9/x_3 + 0.8/x_4 + 0.8/x_5 \]

Mamdani fuzzy inference is given as

\[ = \min(A_1, A_2, \ldots, A_n, B) \]

\[ = 0.1/x_1 + 0.2/x_2 + 0.4/x_3 + 0.2/x_4 + 0.1/x_5 \]

Reddy fuzzy inference is given as

\[ = \min(A_1, A_2, \ldots, A_n) \]

\[ = 0.2/x_1 + 0.4/x_2 + 0.5/x_3 + 0.4/x_4 + 0.3/x_5 \]

The graphical representation of fuzzy inference is shown in Figure 15.

Composition

The GFCF is a single fuzzy membership function
If some relation \( R \) between \( A_1 \), then \( B_1 \) is to infer from \( R \):

\[
B_1 = A_1 \circ R = \min\{\mu_{A_1}^{GFCF}(x), \mu_{R}^{GFCF}(x)\}/x
\]

Zadeh fuzzy inference is given by

\[
B_1 = A_1 \circ R = \min\{\mu_{A_1}^{GFCF}(x), \mu_{R}^{GFCF}(x)\}
\]

Mamdani fuzzy inference is given by

\[
= \min\{\mu_{A_1}^{GFCF}(x), \min(1, 1- \mu_{A_1}^{GFCF}(x) + \mu_{B}^{GFCF}(x))\}
\]

Reddy fuzzy inference is given by

\[
\mu_{A_1}^{GFCF}(x)
\]

where \( A, B, A_1, \) and \( B_1 \) are the GFCF.

\[
A = \{0.5/x_1 + 0.7/x_2 + 0.9/x_3 + 0.7/x_4 + 0.5/x_5 - 0.1/x_1 + 0.2/x_2 + 0.3/x_3 + 0.2/x_4 + 0.1/x_5\}
\]

\[
= 0.4/x_1 + 0.5/x_2 + 0.6/x_3 + 0.5/x_4 + 0.4/x_5
\]

\[
B = \{0.5/x_1 + 0.7/x_2 + 1/x_3 + 0.7/x_4 + 0.5/x_5, 0.4/x_1 + 0.5/x_2 + 0.6/x_3 + 0.5/x_4 + 0.4/x_5\}
\]

\[
= 0.1/x_1 + 0.2/x_2 + 0.4/x_3 + 0.2/x_4 + 0.1/x_5
\]

\[
A_1 = \text{more or less } A
\]

\[
= = 0.55/x_1 + 0.63/x_2 + 0.71/x_3 + 0.63/x_4 + 0.55/x_5
\]

The composition of Zadeh, Mamdani and Reddy fuzzy inference is shown in Figure 16.
6. **Fuzzy control systems using two fuzzy membership functions**

Zadeh [6] introduced fuzzy algorithms. The fuzzy algorithm is a set of fuzzy statements. The fuzzy conditional statement is defined as follows:

If \( x_i \) is \( A_1 \) and \( x_i \) is \( A_2 \) and ... and \( x_i \) is \( A_n \), then \( y_i \) is \( B_i \)

The precedence part may contain and/or/not.

The fuzzy control system consist of a set of fuzzy rules.

If a set of conditions is satisfied, then a set of consequences is inferred.

The fuzzy set with twofold membership function will give more information than the single membership function.

The generalized fuzzy certainty factor (GFCF) is given as

\[
\mu_{GFCF}^A(x) = \{\mu_A^{True}(x) - \mu_A^{False}(x)\}
\]

For instance, “x has fever”

The GFCF for fever given as

\[
\mu_{Low}^{GFCF}(x) = \{\mu_{Low}^{True}(x) - \mu_{Low}^{False}(x)\}
\]

Consider the rule in fuzzy control system

If BZ is low

and BE is normal

then reduce fan speed

For instance, fuzziness may be given as follows:

If BZ is low \((0.9,0.2)\)

and BE is normal \((0.8,0.2)\)

then reduce fan speed \((0.6,0.3)\)

Fuzziness of GFCF may be given as follows:

If BZ is low \((0.7)\)

and BE is normal \((0.6)\)

then reduce fan speed \((0.3)\)

For instance, consider the twofold fuzzy relational model of fuzzy control system.

The graphical representation of twofold fuzzy relational model is shown in Figure 17.

![GFCF](image)

**Figure 17.**

GFCF for Table 5.
Defuzzification

Usually, centroid technique is used for defuzzification. It finds value representing the centre of gravity (COG) aggregated fuzzy generalized fuzzy set.

\[ \text{COG} = \frac{\sum C_i \mu_{A_i}^{GFCF}(x)}{\sum C_i} \]

For instance,

\[ \text{Speed} = \{0.1/20 + 0.3/40 + 0.5/60 + 0.7/80 + 0.9/100\} \]

\[ \text{COG} = \frac{(0.1 \times 20 + 0.3 \times 40 + 0.5 \times 60 + 0.7 \times 80 + 0.9 \times 100)}{(0.1 + 0.3 + 0.5 + 0.7 + 0.9)} = 73.6 \]

The defuzzification is shown in Figure 20.

7. Conclusion

The fuzzy set of two membership function will give more information than single fuzzy membership function for incomplete information. The fuzzy logic and
fuzzy conditional inference based on single membership function and twofold fuzzy set are studied. The FCF is studied as difference between “True” and “False” membership functions to eliminate conflict of evidence and to make as single fuzzy membership function. FCF = [True-False] will correct truthiness of single membership function. The methods of Zadeh, Mamdani and Reddy fuzzy conditional inference studied for fuzzy control systems are given as application.

Conflict of interest

The author states that he has no conflict of interest and that he has permission to use parts of his previously published work from the original publisher.

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