An Optimal Closed-loop Supply Chain Scheduling with Reverse Flow of Repackaging Material

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Abstract: In this paper we deal with a model for optimizing the costs of an integrated production, distribution and packaging reprocessing problem. As a contribution to integrated production and distribution model, a reverse material flow of returnable packages material (RPM) flow and a reprocessing process of the RPM are incorporated into the model and extended to a closed-loop supply chain. A linear programming mathematical model is constructed for the beverage industry, although the model could also be applied to other industries that include a RPM feedback flow. The optimization model provides a number of decision variables such as product quantity and time, RPM ratio, production line allocation, transportation routing, RPM reprocessing scheduling. The proposed model allows to introduce and change multiple parameters in each of the different parts of the closed loop supply chain. Finally, a numerical example involving a bottling industry of standardized packaging is solved utilizing the optimization software Gurobi, and a minimal cost solution is presented.

Key Words: Supply chain management, Closed loop supply chain, Production scheduling, logistics, Returnable packages material.

1. Introduction

According to the classic definition, a supply chain consists of all the elements such as corporations, companies, people, elements and resources that exist in the process of moving goods from the original raw state to the final customer. The typical supply chain has its origin in the extraction of natural resources that have to collected and gathered given its potential capabilities. It continues with the transport of those materials to the manufacturing facilities in which they will be refined and processed in order to create what we call the final product. Finally, those final products will be transported or moved towards its original objective consumers or distributors. As it is easily understood, very few companies take care of the whole process, being its different stages normally separated among different industries. Therefore, assuring that all the elements of the supply chain work correctly is a very difficult task for managers. The effectiveness of each of the processes of the supply chain depends on whether the incoming and outgoing elements are provided as necessary. Effective chain planning takes into account this risk and proposes measures in order to coordinate activities among different elements of the chain. Not only communication is provided but also containing measures in case one of the elements fails.

Until now, the supply chain has been mostly considered only from the perspective of products moving in just one direction. In the last decades however, the definition has been extended to include not only the elements that participate in this forward flow but also to take into account elements moving in a reverse flow. This means that products could move from the consumer or distributors back to the manufacturer, being this due to different purposes. For example, end of life products can be gathered in order to receive an appropriate disposal treatment and at the same time disassembled so useful components can be reused. By carrying on this activity not only the raw material costs are decreased but the waste treatment is dramatically improved. At the same time, defective products can be sent back to the manufacturers and used again after reparation.

Repackaging is as well one of the main purposes of reverse logistics, by reprocessing used containers. This activity has received little attention so far in the academic literature and it is one of the main activities to be implemented in multinational firms of all different products. In this reverse flow, a new approach is considered to deal with empty returnable packing material (RPM). Traditionally, the reutilization of containers had been already considered but at a small scale. Recently environmental concern has grown regarding the amount of annual waste that is generated by the disposal of empty RPM of all types of goods. RPM reutilization mathematical frameworks are required in order to carry on these activities in an optimum way, minimizing the total cost, reducing the environmental impact and achieving demand satisfaction at a minimum RPM purchase and maximum reutilization factor.
Supply chain managers have to deal with issues related to optimizing the activities of different elements of the chain while making sure that the other activities of the whole chain can be carried out normally. This means that not only cost minimization has to be achieved but the effects of it on one activity can affect any other activity in the chain given that some decision variables in each activities are featured in some others. This refers specially to input and output values. Therefore, an optimal solution to one problem can have effects in some other activity. The main problems faced in supply chain management (SCM) that relate to our work are the following.

The classic definition considers inventory management as a science primarily about specifying the shape and percentage of stocked goods. It is required at different locations within a facility or within many locations of a supply chain to precede the regular and planned course of production and stock of materials. As in any other kind of holding inventory amounts, it involves and associates cost and space and decisions have to be made in order to assure that the correct amount of inventory of every good is kept allowing operations to happen as planned while minimizing as much as possible the cost involved and the risk attached to unexpected instabilities and their effect on the chain. Concerns around inventory are not only of space and cost but also of availability and time. Having quick access to inventory plays a key role in the speed of service and client satisfaction.

In a supply chain, goods have to be constantly moved from one process to the next one. In almost all the industries, at some point one or more long distance movement has to be done given that manufacturing, distribution and final consumers are not usually located within the same geographic area. The decisions involved in this activity have to do with how these goods will be transported. In the first place, the optimal amounts and the optimal transportation methods have to be chosen taking into account its availability and limits. In addition, correct timing also plays a key role so all products are where they have to be while the rest of the elements of the system are not affected.

In numerous industries, the amount of products to be processed can be very high. However, sometimes the resource availability for carrying on those activities is divided among numerous elements and work forces. It becomes crucial how to utilize each of those resources in order to allow for all the planned activities to be completed within the required time frame as well as to do it in the most possible optimal way. Production scheduling problems have to do with the assignation of each task to each work element. It could be related to a geographic location context or to the processing order of products in the same production line, as it is our case.

Our objective in this work is to expand an existing mathematical framework by incorporating reverse activities and subsequently perform a linear optimization in order to minimize the total cost. The original mathematical framework considered the activities of a dairy product industry in Greece. These activities were packaging and processing of products as well as their transport to the final distribution centers in order to supply a daily demand. The time framework was a planning horizon divided in a series of time periods of equal duration. The original paper considered only a classic forward supply chain, while our model includes reverse logistic activities in the form of a closed-loop supply chain.

When it comes to the reverse flow, our model considers a case of RPM reprocessing and reutilization. Through the use of standardized packaging, we take into account a situation in which used bottles are gathered and reprocessed for further filling of same or different final products that share the same standardized bottle. This adds new complications when it comes to inventory management and production scheduling. The amount of inventories have to be expanded in order to allow for not only different bottled products but also different types of bottles, from used to filled ones. The production in each of the plants is expanded as well: Before, all production lines were used only for bottling while now new lines can be considered to be in charge of the recycling operations.

We consider a deterministic approach, given that we work with a planning horizon in mind. The main characteristic of our work is that it considers the whole supply chain. While most analysis are normally performed in order to optimize one of the different elements of the business, our cost optimization framework takes into account how inventory, production and distribution decisions affect each other considering that all elements are interrelated. By doing so, a more efficient solution can be found, calculating at the same time the optimal inventory amounts as well as production and recycling planning variables such as the order and amount of products that have to be transported at each time to each receiver. But this whole approach also means that our final objective function will feature an even bigger number of variables and it will be tightened by more constraints. Therefore our parameters are required to be chosen carefully in order to allow a feasible solution to be found. The whole industry is modeled with great flexibility allowing our framework to be easily modified and applied for optimizing similar industries.

From a mathematical perspective, the provided framework will have the structure of a linear optimization problem. Each of the different characteristics of the industry will be represented as linear constraints featuring different decisions and variables. Initially, a series of sets will be defined too featuring the main structure of each case. The final objective function is as well a linear equation set to minimize or maximize, depending on the problem. In our work we are looking to minimize the global costs. It should be noted as well that the variables and parameters on each constraint are not exclusive of that constraint. In fact, the fact that same variables will be featured in different supply chain problems is what represents how interrelated are all the different elements of the model.

The remaining of this paper is organized as follows. In sec-
tion two we review previous work which motivated our research, in section three, the structure of the proposed industry for which our framework and code can be applied will be carefully described, introducing its different sets, parameters and variables. In section four, the mathematical model will be constructed, introducing RPM flows with numerous constraints. A numerical example is provided in validating the proposed model in section five and finally in section six, we conclude our results and provide some information for possible future extension.

2. Literature review

The main influence in this paper is the work developed by Kopanos, Puigjaner and Georgiadis [1]. The authors proposed a mathematical framework representing the multiple operations of a daily product industry in Greece by incorporating the particularities of dairy products into production scheduling and performing a computational cost optimization. Our aim is to expand this proposed framework to a different industry adding at the same time closed-supply chain activities in the form of RPM reprocessing.

Chuang, Wee and Yang [2] analyzed an inventory system with both forward and reverse material flow. In their model, products belonging to the reverse flow were remanufactured and sent to the retailer so they could be sold again. As an objective function, they proposed a maximized profit between the supplier, the manufacturer and the dealers. The mathematical framework used is a multi-echelon inventory system. At the same time they compared different results using a centralized decision making system versus a decentralized one. The main elements of their proposed framework included a infinite planning horizon, the absence of product deterioration and no capacity constraints among others. In order to assure the validity of their proposal, they also performed a numerical example and a sensitivity analysis for both centralized and decentralized decision making approaches.

Silva, Reno, Sevegnani and Truzzi [3] performed a study on the benefits and drawbacks of using returnable packaging in a closed loop supply chain comparing it with a previous real world situation that was using disposable packaging. In their research they successfully proved that using returnable packaging not only minimizes the environmental impact but is also reflected in a remarkable decrease of costs as well as an improvement in logistic activities. They also explored the benefits of using standardized packaging in a way to reduce the total volume 7 times from its original size when it is being used to its size when it is transported in the reverse flow. The methodology that they followed involved analyzing the previous literature in reutilization of RPM, interviewing with technical representatives of different companies in order to gather more information about those companies activities and policies relating to reutilization or proposing a new concept of returnable packaging. They also performed a technical analysis on the performance characteristics of this kind of RPM in order to analyze its economical benefits as well as an environmental analysis to support their conclusions. Both analysis results are deeply explained in the conclusions with both empiric and experimental results.

Given how recent is the general appearance of reverse logistics in the Supply Chain literature, some studies have been carried in order to gather the main concepts and analyze the already existing knowledge. One of those was performed by Fleischmann, Bloemhof-Ruwaard and co. [4]. In their literature review they proposed a series of frameworks for integrating the field of reverse logistics into numerical problems, reviewing previously used models and proposing further expansions. They concluded that sometimes both flows have to be treated simultaneously, something that it is shown in this paper. Finally they also addressed the importance of further improving the existing knowledge in this science. Their study is a pioneer in this field given its old publication date and can still be applied nowadays. The paper is structured addressing first the main elements of the reverse flows in the supply chain and proposing a series of questions that must be answered in order to fully comprehend this structure. The answers to those questions are obtained from analyzing the previous literature and its contributions. Their paper also focuses a lot in inventory control in those systems that have reverse flows, a matter that our paper aims to treat extensively. The authors also considered both deterministic and stochastic models for managing inventory. Finally they reviewed production planning literature in order to reuse previous parts and reassemble new products minimizing the purchase cost like in our model.

Most of the articles on reverse logistics take on a broad and extensive view of them but not so many focus on one specific field or approach. One of these is performed by Carrascogallego, Ponce-Cuetio and Dekker [5]. In their article they explored all the previous literature existing on closed-loop supply chain and then performed a classification on different types of reusable articles guessing whether they have to be reprocessed, recovered or recycled. At the same time they expanded the existing terminology with different categories of reusable articles as well as explain the peculiarities on the processing of each of them. The validity of their proposal is given by the utilization of case studies based on real industry settings. Their objective was to show how reuse closed-loop supply chain should be considered and analyzed as a different type than others closed-loop supply chains. Their new terminology separates reusable articles (RA) from returnable transportation items (RTI), returnable packaging materials (RPM) and reusable products (RP). Some of those terms were extracted from previous papers on the field. The authors claimed that reusable articles are different from the other types due to its multiple levels of disaggregation and disassembly that required more complex recovery activities. They not only focused in RA but also in the main differences between RTI, RPM and RP and compare different reusable article networks like start systems or multi-depot systems. Their numerical analysis took into account both configurations and explained the difficulties that arise in certain companies when implementing each of them.

One of the most interesting and extensive contributions to the field of reverse logistics is the one realized by Elmas and Erdogmus [6]. Given how new this field is and the scarce previous literature on it, the authors recompiled and summarized all current knowledge of the different elements and particularities of the reverse supply chain. Their focus was in how crucial and vital this processes have become and therefore how they should be taken into account and implemented in future operational research. The benefits they explored have to do with not only the environmental impact decrease or reusable value but also with the competitive advantage that it involves. They sep-
arated all the different stages of reverse logistics and also spot its main problems, allowing for further improvement. One of the most impressive elements of their publication is how different industries are separated and analyzed individually. For each of them real data is provided including the percentage of products that are returned and its main particular elements that differentiate them from the rest when it comes to closed-loop supply chains. Furthermore, the problems they suggest have to do with Balancing the supply and demand, accumulation of inventory and designing and efficient and cost-saving logistical network.

One very relevant work for the discussed topic in this paper is the one performed by Neiva de Figuereido and Mayerle [7]. Their work presented a strong analytical background in which not only coat minimizations was required but also network planning and output constraints had to be taken into account. The paper presents a analytical framework designed to represent the explained situation, an analytical model and a solving algorithm in order to determine the optimal chain design taking into account regulations concerning the minimum amount of products recycled in a period of time, an idea that it is implemented in our currently discussed paper. One of the most noticeable contributions is how easy this paper can be modified and applied to therefore different variations and its effectiveness is proved by applying it into an existing Brazil industry. The results and research can be applied to various recycling situations depending on the policy instruments being analyzed. Some of those situations are transportations in which the cost varied with routes, or in which third parties executed transport at contracted cost, with local incentives for recycling or not. The authors also proposed further research such as including strategic interaction between different producers through spatial equilibrium models. When performing their case study, the authors also discuss the accuracy of the results due to the heuristic algorithm used.

Until now we have mainly focused on the existing literature relating to reverse supply chain from multiple approaches, but in this paper we will consider more specifically reverse flows used for reprocessing and reusing previous packaging. One of the first works on this field is the one performed by Kleber, Minner and Kiesmuller [8] who analyzed the potential cost/benefits of reprocessing products coming through the reverse flows versus producing fresh new ones. Most production and inventory management models for reverse logistics are restricted to stationary demands and returns and do not address seasonal effects and product life cycles. Therefore, they considered not only stationary demands and returns but a variable one that can be affected by peaks or seasonal effects. In this situation it was important for them to decide whether some returned products should be retaken or disposed of given the 2 different demands. As a result returns could either be stored for later use for a certain demand class or being used instantly for another class. Demands had to be satisfied either from production or remanufacturing of returned products and returns not needed for recovery may be disposed of. With this situation, they determined the optimal production and remanufacturing policy for a linear cost model. The analysis points out that an optimal recovery strategy takes into account both the absolute advantage of satisfying demand from recovering returns instead of producing new items, and the time delay between return and recovery usage where inventories are subject to holding costs. Though returns are available and recovery for a demand stream is profitable compared to disposal and production of a new item, it can be more profitable to terminate recovery for a demand stream and to save returns for future recovery of a product with higher recovery advantage.

One of the main characteristic of our research is its focus on packaging recycling, remanufacturing and reusing, not only for economical reasons but also for environment protection. Relating to this one last factor, the work developed by Birgelen, Semeijn and Keicher [9] is a fundamental one when it comes to understanding customer behavior. The authors performed a study on how recycling and purchasing decisions are influenced by customer’s ecological concerns. They focused themselves in the packaging and pro-environmental German bottle industry and try to solve some questions relating to the main factors that affect customer’s decisions depending on different types of environment friendly and not friendly packaging. Their items used to measure the constructs in their framework were based on both the previous existing literature and the existing behavioral theories. The results suggested that eco-friendly purchase and disposal decisions for beverages are related to the environmental awareness of consumers and their eco-friendly attitude. Furthermore, consumers are willing to trade off almost all product attributes in favor of environmentally friendly packaging of beverages, except for taste and price. The results of testing their first hypothesis suggested that consumers’ attitudes tend to change over time: respondents are willing to trade off various product attributes in favor of environment-friendly beverage packaging, except for taste and price. In other words, consumers seem to be willing to turn toward ecological beverage RPM, as long as the taste of the beverage and the price remain largely unchanged. Finally, the findings of this study can be exploited by companies. Providing an additional purchasing motivation, such as an ecological RPM-related benefit, has the potential to influence a consumer purchase decision positively. Consequently, by emphasizing the environmental friendliness of packaging, companies may be able to create a competitive advantage. Specifically, it was shown that only the attributes of taste and price have to be fulfilled before a consumer takes ecological beverage packaging characteristics into account.

Considering also the topic of product recovery and reutilization, Fleischmann, Krikke, Dekker and Flapper [10] performed one of the first analysis by defining and characterizing the structure that logistic networks must have in order to allow for efficient product recovery. Their work was developed at the time when product recovery was starting to become an important matter in the academic world of supply chain analysis. The authors based their analysis on a set of recently published case studies on logistics network design in a product recovery context following the above definition. Each case study included a quantitative model and provided detailed information on the net-work considered. Bringing together these cases involving different industries appeared in itself worthwhile since literature in this area is not yet well developed. Moreover, commonalities among the cases indicated general characteristics of product recovery networks. To understand the observed differences they introduced a set of potential factors influencing logistics network design. Positioning the available case studies in this setting, they identified a number of clusters of similar network characteristics and explanatory factors and in this
way derive distinct product recovery network classes. Their work covers in details different parts of the networks such as the products, the supply chain, the resources and the different type of recovery material. The concept of recovery is analyzed on all its possible purposes like reuse, the one this paper is based on, but also on disposal or remanufacturing activities. Considering recovery situations in more detail, including product, supply chain, and resource aspects, we have seen that product recovery networks can be subdivided into a number of classes. Re-usable item networks, remanufacturing networks, and recycling networks appeared each to have their own typical characteristics. They proposed further research on the topic in order to further improve the understanding of recovery networks, if possible utilizing more different case studies and new considerations for improving decision making.

The framework we are considering in our paper takes into account a bottling plant with both forward and reverse flows in which empty bottles are reprocessed for reusing. A very similar framework was analyzed by Del Castillo and Cochran [11], who proposed a formulation for an optimal configuration of a reverse loop for reusing bottles. The validity of their model was given by the fact that it was based on a real life bottling factory in Mexico. Their research was carried utilizing two different models combined in order to cover the optimization of the whole system and provided a master plan for production and storage planning. One of the key elements of this paper is a detailed description of the economical and organizational benefits that their work had to the analyzed company showing its effectiveness as well as some observations from their work with this company. The results of their study indicated that their formulation provides a timely response in the field to key operational problems addressed by no previous approach. Included are better organizational control (through providing one-week production and distribution plans), feedback allowing modification of heuristic rules previously used in controlling the distribution of product and container reuse, and improvement in inventory behavior such as avoiding shortages.

The field of reverse logistics and reuse and reprocessing of used materials is started to be considered quite recently in the academic literature due to the absence some years ago of strong laws and regulations concerning it. The work done by Gonzalez-Torre, Adenso-Diaz and Arriba [12] performed an analysis on the reverse flows and closed-supply chain logistics policies of some bottling and packaging companies in Europe. Their paper analyzed the relationships existing between the drinks and food firms with their supplier as and their customers as well as the implantation of reverse logistics within these firms. They wanted to test the hypotheses of the existence in differences when implanting reverse logistics practices. The authors started by studying the implantation of reverse logistics in the previous literature in order to define the main characteristics of bottle and packaging recycling in the modern industry. The methodology the authors followed was based on a series of surveys send to some large bottling companies in Spain and Belgium. By doing so, they explained the main differences in bottling policies observed in both countries due to cultural factors, such as the type of drink mostly consumed in each country. They finally concluded that relating to the join implantation of environmental and reverse flows practices, few differences can be appreciated.

When it comes to recycling and reprocessing RPM, one of the key issues is whether to use standardized packaging or individual packaging fore each of the products. Each of them have their own benefits and drawbacks and the decision is not so clear. Trying to solve this issue, Dae Ko, Noh and Hwang [13] analyzed the cost benefits when using standardized glass bottles in production, an article that has a key influence on our work given that we will too considered standardized packaging. The authors considered a recycled system with two companies that use coordinate their operations through standardization of their bottles and study the associated benefits with their decision. Those benefits are in the first place a simplification of sorting and classification operations and a cost reduction though the reprocessing of recovered empty bottles. The authors developed a mathematical model in order to optimize the operations of both companies by determining an optimal recovery policy and solve a numerical example in order to prove its effectiveness. Finally, they proposed further research by expanding their domain from 2 companies to a generic situation.

Finally, it is worth mentioning one of the few works done on production planning when it comes to food and beverage industries. Kopanos, Puigjaner and Georgiadis [14] developed a Mixed-Integer Programming model to optimize the production scheduling of a real food industry. Their framework relied on a detailed modeling of all the sequencing decisions, production stages and existing constraints. Like on our work, they aimed to cover all interrelated production stages. The objective of their problem was to find the optimal production, distribution and capacity planning of the supply chain network considering the cost, responsiveness and customer service level simultaneously. For the cost, they considered the total cost of the supply chain, including the raw material cost, formulation cost, transportation cost, inventory cost, and duties cost. To find a responsive supply chain, the total flow time was optimized in the model, which is equal to the product flow multiplied by the corresponding transportation time from formulation plants to markets. Also, the total lost sales was minimized to obtain a better customer service level. They also proposed an alternative model that noticeable reduces the computational time even though it may not bring an optimal solution. They solved some numerical examples in order to prove the performance and validity of the calculated results. The e-constraint method is adapted to solve the multi-objective optimization problem, in which total cost is the only single objective to be optimized and total flow time and total lost sales are transformed into constraints. A set of Pareto-optimal solutions were obtained. To obtain a fair solution among them, the lexicographic minimax method was also applied and a new approach has been developed to transfer lexicographic mini-max problem to a minimization problem.

3. Work flow and problem describing

Adapting the original work, we consider a similar industry from a closed-loop supply chain structure. In this section we will describe the main elements of the industry as well as the mathematical sets that define them. The number and inclusion of each of the elements can be modified in order to adapt the framework to different real-case industries. In our work we will focus on a simple filling and recycling company having to distribute and collect from different distribution centers.

Our model considers a number of bottling facilities addressed
by the subindex \( s \in S \), where \( S \) is the set of plants. There are as well a number of distribution centers \( d \in D \) to be supplied from the plants where \( D \) represents set of distribution centers.

![Diagram of the proposed industry with 3 plants and 3 distribution centers](image)

Each of the distribution centers on each time period has a demand \( \zeta_{d, p} \) for each filled product \( p \). At the same time, each distribution center has a return supply of used empty bottles (containers) defined by \( \gamma_{d, \text{dn}} \).

### 3.1 Product lineup

The plants bottle different products \( p \) in bottles of different type \( t \). Each product \( p \) can be filled on one type of bottle, but each type of bottle can be used for more than one product \( p \). Which product is filled on each type of bottle is defined by the set \( P_t \). Standardized packaging allows to reduce the RPM consumption and to simplify the recycling and waste gathering operations. If the work had to be applied to a more complicated situation or in which we don’t consider standardization, by setting the number of types of bottles to be equal to the number of products, the same mathematical framework could be used. This is an aspect of the flexibility of our work.

### 3.2 Transportation

At the beginning on each time period, one truck \( l \) can transport products between one plant and one DC (distribution center) only one time. If the truck goes to a DC on one time period, it will also do the way back on the same day, transporting if necessary empty used bottles. Trucks are assigned to each distribution center according to the predefined set \( S_l \), where \( l \) represents each of the trucks. Which distribution centers can be served by each truck is contained in the set \( D_l \), given that depending on the type of truck, some of them will be able to reach some distribution centers but not others.

It has to be noticed that each of the trucks has a minimum and maximum capacity, as well as a fixed cost per day of utilization and a variable cost depending on the amount of bottles transported in both the forward and reverse flow.

### 3.3 Inventory

Each plant has 3 different inventories of products and bottles. The inventories are calculated at the end of each time period. Figure 4 graphically represents the proposed structure for inventory and production material movement.

\( I_{f, p}^n \) represents in the time period \( n \) the inventory of filled bottles of product \( p \) in plant \( s \) that passed the quality test and are ready to be sent at any moment to be delivered. After bottling each product, it is necessary to keep it to perform a quality and certification test that can take several time periods or none at all, depending on the product. The parameter \( \delta_{p} \) determines this time for each product. If the time is equal to 0, the product will be incorporated to the inventory of filled bottles ready to be sent at the end of the same time period. If it is 1 then it will be in the next time period and so it goes.

\( I_{R}^{\text{dn}} \) represents in the time period \( n \) the inventory of reprocessed bottles of type \( t \) in plant \( s \) that have been cleaned and prepared for being filled again. We consider that after retrieving, the recycled bottles are in perfect conditions and can perfectly substitute a new purchased bottle. However, since we already have them, they eliminate the need for new purchases, therefore reducing the purchase cost as well as the ecological impact. But it has to be noticed that this happens at the price of having new recycling and inventory costs, so a compromise solution will be found.

\( I_{\text{dn}}^{\text{t}} \) represents in the time period \( n \) the inventory of used bottles of type \( t \) in plant \( s \) that have to be reprocessed in order to be clean again. These bottles are considered to be in perfect conditions to be recycled. This inventory tends to increase with the incoming flow of used bottles on each time period but will decrease as recycling operations are performed.

### 3.4 Production

At each plant, there is a number of production lines \( j \) that can either fill bottles or recycle them. The set \( J_p \) contains the lines \( j \) that can fill bottles of product \( p \) while \( J_r^p \) contains those lines depending on the type of bottle that can be processed, given our RPM standardization. This means that if a line can fill bottles of product \( p \), it will be included as a line that can fill bottles of type \( t \) when \( p \in P_t \).

The set \( J_r^p \) includes the lines that can recycle and clean bottles of type \( t \). It should be noticed that one line can only do one task of either filling or reprocessing, therefore \( J_r^p \cap J_r^t = \emptyset \). Whether each plant can use each of the production lines is defined in \( J_p \). The flexibility of the model can also be observed here given that the model can be used even for only forward supply chain classic problems by limiting or eliminating the existence of recycling lines in some plants as necessary.

When it comes to recycling and reprocessing, each type of bottle requires a preparation time and can be processed at a different rate depending on the line and the plant. In the case of the processing lines for bottling, all products belonging to the same type of bottle are processed consecutively, each of them requiring its own preparation time and featuring a different bottling rate. Figure 5 represents an example of this, including some parameters that will be later described in the mathematical formulation.

In all lines, when more than one type of bottle is processed in the same line in the same time period, we must consider a
setup time required for adapting the machines to the new type of bottle. This setup time also involves an additional cost. Both the cost and the time are parameters depending on the order in which the setup is made. The optimal order will be obtained in the model solution as one of the decision variables.

In conclusion, the main decision variables will be:

- The amount of bottles filled and reprocessed on each time period.
- The trucks that will satisfy the DC’s demands.
- The storage amount on each of the 3 inventories.
- The production lines used on each time period for each of the processes.
- The purchased amount of new bottles.
- The order, starting times and finishing times of the filling and recycling operations for each product and type of bottle.

4. Formulation

Our problem can be modeled by using linear programming method. In the first place, there is a number of constraints that increases proportionally to the size of the sets and the complexity of the operations and will be automatically generated by the code once the data is inputted. Each of those constraints represent an element of the industry, and they are composed of both decision variables and inserted parameters. Decision variables are represented by capital Roman letters while parameters are represented by Greek non capital letters. The inclusion of the set limitations in the constraints allows its number to be reduced as much as necessary avoiding the need for generating useless variables with values equal to 0.

Some constraints will directly represent the flow of products in some activities while others will impose limitations. A third of the constraint represents boundaries of operations. The constraints have been divided into three different categories: inventory, transportation and production. However these categories are not completely separated from each other and some decision variables will be featured in more than one category.

4.1 Inventory constraints

\[ I_{xpn} = I_{xpn-1} + \sum_{j \in J, t \in T_{t}} Q_{pxjn} - \sum_{d \in D_{d}, t \in L_{dt}} U_{xpn} \]  

\[ I_{xpn} = I_{xpn} - \sum_{d \in D_{d}, t \in L_{dt}} U_{xpdn} \]  

\[ I_{xpn}^{F} = I_{xpn}^{F} + \sum_{j \in J(0,t)} Q_{pxjn} - \sum_{d \in D_{d}, t \in L_{dt}} U_{xpdn} \]  

\[ I_{xpn}^{F} = I_{xpn}^{F} - \sum_{d \in D_{d}, t \in L_{dt}} U_{xpdn} \]  

This constraints represent the variations in each time period of the inventory of filled bottles. This inventory is considered to consist entirely of final bottled products that have passed the quality test and are ready to be sent to the distribution centers when necessary. The inventory amounts are calculated at the end of each time period after all the incorporations of new products in that time period have already happened. Between the bottling process and the incorporation of this inventory there is a quality certification time that has to be fulfilled. This time is represented by \( \lambda_{p} \) and consists of an integer number bigger or equal to 0, representing the amount of time periods that the test will take. 0 means that the bottles will join the inventory at the end of the same time period that it is produced. \( Q_{psjn} \) represents the amount of bottles of product \( p \) that are filled on time \( n \) in plant \( s \) in time \( n \), while \( U_{xpdn} \) stands for the amount of transported products from each plant \( s \) to each distribution center \( d \) in each truck \( l \). This transportation happens at the beginning of each time period.

Therefore the constraint represents that the inventory of filled bottles for a product \( p \) in plant \( s \) at the end of the time period \( n \) is equal to the amount of bottles in the previous period minus the bottles that have been sent to supply demand plus the bottles that have passed the quality check and were filled \( \lambda_{p} \) time periods ago.

It is necessary to consider the case of the inventory at the end of the first time period, when \( n = 0 \). In that case a parameter \( Init_{xpn} \) has to be input representing the amount of inventory of filled bottles of product \( p \) in plant \( s \) at the very beginning of the operations.

\[ I_{xpn}^{R} = I_{xpn-1}^{R} + \sum_{j \in J, t \in T_{t}} Q_{pxjn} - Bo_{xpn}^{R} \]  

\[ I_{xpn}^{R} = Init_{xpn}^{R} + \sum_{j \in J, t \in T_{t}} Q_{pxjn} - Bo_{xpn}^{R} \]  

When considering the inventory or recycled bottles, we also count it at the end of each time period. In addition, incorporations to the inventory are considered also before counting at the end of the time period while outgoing bottles to be filled are considered to happen at the beginning. With this, the inventory of empty clean recycled bottles of type \( t \) at each time period on each plant is equal to the inventory on the previous period minus the empty bottles that will be filled on that time period \( Bo_{xpn}^{R} \) plus the amount of bottles that have been recycled in that time period \( Q_{xpn}^{R} \). As with the other inventories, an initial amount of bottles has to be input as a parameter in the first period of operations.

\[ I_{xpn}^{D} = I_{xpn-1}^{D} + \sum_{j \in J, t \in T_{t}} Q_{pxjn} - \sum_{d \in D_{d}, t \in L_{dt}} U_{xpdn}^{D} \]  

\[ I_{xpn}^{D} = Init_{xpn}^{D} + \sum_{j \in J, t \in T_{t}} Q_{pxjn} - \sum_{d \in D_{d}, t \in L_{dt}} U_{xpdn}^{D} \]
4.2 Transportation constraints

\[ I_{sln}^D = \text{Init}_{sl}^D - U_{sln}^R + \sum_{d\in D, l\in L_d} U_{slndl}^d \]  

(8)

In the case of the inventory of used bottles, there are two processes of addition and diminution of the amounts. On the one hand, used bottles leave the inventory at the beginning of each time period to be cleaned and recycled, being this amount determined by \( Q^R_{stn} \), depending on the plant and type of bottle. On the other hand, new used bottles arrive at the end of each time period from the distribution centers in the same trucks that departed at the beginning of that time period to satisfy the demand of filled products. This incoming reverse flow of used bottles is represented by \( U^d_{slndl} \), depending on the truck, type of bottle and distribution center.

As it has been constantly repeated, given that some inventory operations happen at the beginning of a time period and some others at the end, it is necessary to specify that the amounts of units leaving each inventory at the beginning of a time period cannot be bigger that the total amount of that product or bottle in the inventory at the end of the previous time period.

\[ B_{ln}^R \leq I_{ln(n-1)}^R \]  

(9)

\[ B_{ln}^R \leq \text{Init}_{ln}^R \]  

(10)

These constraints represent that the amount of recycled bottles taken at the beginning of a time period for filling them can’t be higher than the final amount of recycled bottles in the previous period.

\[ U_{ln}^R \leq I_{ln(n-1)}^D \]  

(11)

\[ U_{ln}^R \leq \text{Init}_{ln}^D \]  

(12)

Similarly, the amount of used bottles that we recycle at the beginning of a time period shall not be higher than the amount of used bottles that we have available in the previous time period.

\[ \sum_{d\in D, l\in L_d} U_{slndp}^{sp} \leq I_{ln(n-1)}^F \]  

(13)

\[ \sum_{d\in D, l\in L_d} U_{slndp}^{sp} \leq \text{Init}_{ln}^F \]  

(14)

At last, given that the sensing of new products happens at the beginning of each time period, in one period we cannot send more products that the ones available at the end of the previous time period.

4.2 Transportation constraints

\[ \overline{U}_{slln}^d = \sum_U U_{slndp}^s \]  

(15)

The total transported amount in the forward flow by each truck on each time period between a plant and a distribution center \( \overline{U}_{slln}^d \) has to be equal to the sum of the amounts of all the products transported in both the forward flow in that time period.

\[ \overline{U}_{slln}^d = \sum_U U_{slndp}^s \]  

(16)

Similarly, the total amount of empty bottles transported by each truck in the reverse flow \( \overline{U}_{slln}^d \) is equal to the sum of all the different types of bottles transported by that truck from a determined distribution center.

\[ \epsilon_l^{\min} z_{slln} \leq \overline{U}_{slln}^d \leq \epsilon_l^{\max} z_{slln} \]  

(17)

\[ \epsilon_l^{\min} z_{slln} \leq \overline{U}_{slln}^d \leq \epsilon_l^{\max} z_{slln} \]  

(18)

As it can be expected, in both the forward and the reverse flow, each of the trucks has a limitation in the amount of bottles, either filled or empty that can transport. Therefore it is necessary to introduce a parameter that determines that maximum (and or minimum) amount in the model. This constraints also feature for the first time the binary decision variable \( Z_{slln} \). This variable is equal to 1 when truck \( l \) is being used in time period \( n \) to transport goods between factory \( s \) and the distribution center \( d \), and it is equal 0 otherwise. The solution of the optimization will determine its value.

\[ \sum_{s \in S, d \in (D \setminus D_s)} Z_{slln} \leq 1 \]  

(19)

It has been previously mentioned that one truck can only do trip forward and back between one plant and one distribution center on each time period. This constraint represents that limitation for each truck on each time period utilizing the previously introduced binary variable \( Z \).

\[ \sum_{s \in S, l \in L_d} U_{slndp}^s = \xi_{slpn} \]  

(20)

\[ \sum_{s \in S, l \in L_d} U_{slndp}^d = \zeta_{slpn} \]  

(21)

Our model is designed in order to optimize the operations of an industry that has to satisfy the demand of certain products and collect and reprocess some other empty RPM for reprocessing. The last transportation constraints represent that the total amount of a product brought by all trucks in one distribution center in one time period has to be equal to the demand of that product on that distribution center on that time period \( \xi_{slpn} \). At the same time the sum of all the empty RPM or bottles of the same type leaving each distribution center on each period is equal to the total returned amount of that type in that distribution center \( \zeta_{slpn} \).

4.3 Production scheduling constraints

\[ \sum_{p \in F} \sum_{j \in (J \setminus J_F)} Q_{p,jn} = \text{Bo}_{ln}^N + \text{Bo}_{ln}^E \]  

(22)
When coming to the filling operation, new products can be filled in previously used and recycled bottles or in new purchased bottles, which carry an additional cost. The total amount of bottles of the same type filled in one plant in one time period has to be equal to the sum of all empty recycled bottles \( B_{0\text{s}n} \) utilized plus new empty bottles purchased \( B_{0\text{p}n} \) in that plant in that time period. That total amount is calculated as the sum of all products filled that share the same type of bottle \( t \).

\[
\pi_{\text{ps}jn} \sum_{p \in P_t} Y_{\text{ps}jn} \leq Q_{\text{ps}jn} \leq \pi_{\text{ps}jn} \sum_{p \in P_t} Y_{\text{ps}jn}
\]

(23)

\[
\pi_{\text{ts}jn} Y_{\text{ts}jn} \leq Q_{\text{ts}jn} \leq \pi_{\text{ts}jn} Y_{\text{ts}jn}
\]

(24)

Each production line on each plant has a limited amount of bottles that can fill and bottles that it can recycle each time period. In the case of the filling operations, that amount depends on the product that is being bottled on each line. The maximum and minimum limits are represented by \( \pi_{\text{ps}jn} \) and \( \pi_{\text{ts}jn} \) for the filling lines and \( \pi_{\text{ts}jn}^{\text{min}} \) and \( \pi_{\text{ts}jn}^{\text{max}} \) for the recycling lines. At the same time, new binary variables are introduced. \( Y_{\text{ps}jn} \) is a binary decision variables equal to 1 if line \( j \) is being used in time period \( n \) on product \( p \) for bottling product \( p \), and equal to 0 otherwise. \( Y_{\text{ts}jn} \) is a binary variable equal to 1 if bottles of type \( t \) are being processed in production line \( j \) for being filled or being recycled. Noticed that since \( Y_{\text{ts}jn} \) is used for both filling and recycling operations, it is not directly analogous to \( Y_{\text{ps}jn} \).

\[
T_{\text{ts}jn} = \sum_{p \in P_t} \left( \frac{Q_{\text{ts}jn}}{p_{tsj}} + \delta_{psj} Y_{\text{ps}jn} \right)
\]

(25)

As it has been already described, the proposed model gives an optimal solution in terms of transportation, inventory and production scheduling. Relating to the former, one of the key decision variables is the processing time for each type of bottle. This variable represents the total amount of time, measured in hours, throughout which both the bottle system is processed in a production line of either recycling or bottling, for one plant on one time period. In the case of bottling operations, that time is composed of the sum of the different bottling times of each product of the same type of bottle, plus the preparation time \( \delta_{psj} \) that is required for each of those products, depending on whether they will be processed or not in that time period, represented by the variable \( Y_{\text{ps}jn} \). The processing time of each product is calculated as the amount of products bottled on that line on that time period \( Q_{\text{ps}jn} \) divided by the parameter \( p_{psj} \) that represents the bottling rate of that product in that line, measured in bottles per hour.

\[
T_{\text{ts}jn} = \frac{Q_{\text{ts}jn}}{\rho_{tsj}} + \delta_{tsj} Y_{\text{ts}jn}
\]

(26)

In the case of the recycling lines, since there is no separation per product on each type of bottle (thanks to the standardization), there is no need to consider the preparation times for each product of the same type, and only one preparation time per type of bottle is required. The processing time is calculated then as the time taken in the line, amount divided by rate, plus the preparation time of that type of bottle \( \delta_{tsj} Y_{\text{ts}jn} \).

\[
\overline{Y}_{\text{ps}jn} \leq Y_{\text{ts}jn}
\]

(27)

\[
Y_{\text{ts}jn} \leq \sum_{p \in P_t} \overline{Y}_{\text{ps}jn}
\]

(28)

We have to include some logical constraints relating to the bottle type allocation variables. They specify that if a product that uses a type of bottle is being processed in a line at some moment, that type of bottle must be considered to be processed on that line. This only applies for the bottling lines.

\[
\sum_{r \neq t \in T_j} X_{\text{ts}rn} \leq Y_{\text{ts}jn}
\]

(29)

\[
Y_{\text{ts}jn} \leq \sum_{r \in T_j} \overline{Y}_{\text{ps}jn}
\]

(30)

One of the decision variables of our model represents in which order should we process the different types of bottles in order to minimize the associated setup cost in both the recycling and bottling lines. This setup also takes some time that must be taken into account. Therefore, we propose \( X_{\text{ts}rn} \), a binary variable equal to 1 if the bottle \( r \) is processed before \( t \). The proposed constraints limit the type of bottles that can be processed before and after each other type of bottle to a maximum of 1, acting then as logical constraints. It must be noticed that all this constraints refer to each type of bottle being processed and not to the products themselves in the production lines.

\[
\sum_{r \neq t \in T_j} X_{\text{ts}rn} + V_{\text{ts}jn} = \sum_{r \in T_j} Y_{\text{ts}rn}
\]

(31)

\[
V_{\text{ts}jn} \geq Y_{\text{ts}jn}
\]

(32)

A new variable is introduced to define whether a production line is being utilized at all or not in each plant in each time period. This variable is \( V_{\text{ts}jn} \) and is equal to 1 when the line is being used and 0 otherwise. Two logical constraints are added to include this new variable, representing the first one that there is at least one type changeover and the line is being used, the number of types of bottles processed is equal to the number of changeovers plus 1. The second constraint specifies that if a line is being used, at least one type of bottle has to be processed in that line.

\[
C_{\text{ts}jn} + \gamma_{\text{ts}jn} \leq C_{\text{ts}jn} - T_{\text{ts}jn} + (\alpha_{\text{ts}jn} - \beta_{\text{ts}jn})(1 - X_{\text{ts}jn})
\]

\forall s, t, t' \neq t, j \in ((J^p_t \cup J^p_{t'}) \cup (J^s_t \cup J^s_{t'})) \cap J_s, n

(33)

\[
C_{\text{ts}jn} - T_{\text{ts}jn} \geq \alpha_{\text{ts}jn} Y_{\text{ts}jn} + \sum_{r \neq t \in T_j} \gamma_{\text{t}r} X_{\text{ts}rn}
\]

\forall s, t, j \in ((J^p_t \cup J^p_{t'}) \cap J_s), n

(34)
\[ C_{t_j,n} \leq (\omega_{t_j,n} - \beta_{t_j,n}) Y_{t_j,n} \]  

(35)

The production in both the bottling line and the recycling one has to be constraint by some time limits. A new key decision variable \( C_{t_j,n} \) is introduced, representing the completion time in which the processing of a type of bottle in a production line finishes. This time is counted in hours from the beginning of operations in each line. The first of the constraints sets a limit for this completion time of a type of bottle \( t \). It establishes that in case a type of bottle \( t' \) is processed after type \( t \) \((X_{t',t_j,n} = 1)\), the starting processing time of type \( t' \) (its completion time minus its processing time) has to be bigger than the completion time of type \( t \) plus the changeover time between both types of bottles. This applies for bottling and recycling lines.

The second constraints determines that the starting processing time of a type of bottle \( t \) has to be at least bigger than the opening time of a processing line plus all the previous changeovers regarding the existing previous types of bottles being processed in that same line on that time period. The last constraint establishes that the processing finishing time of a type of bottle can’t be later than the total available time minus the required closing time on a line on a time period.

4.4 Costs definition

Before we propose the objective function, we are going to define all the different costs that are involved in the operations of the analyzed industry.

The name of costs used below are listed below.

- **PIC**: production inventory cost
- **RIC**: recycled inventory cost
- **FIC**: final product inventory cost
- **FOC**: filling operation cost
- **ROC**: recycling operation cost
- **UC**: utilization cost
- **SC**: setup cost
- **TC**: transportation cost
- **PC**: purchasing cost

\[ PIC = \sum_s \sum_p \sum_n \xi_{s,n}^D f_{s,n}^D \]  

(36)

When we keep in storage one finished product during one time period there is a variable cost per unit stored that accounts for space utilization, refrigeration and maintenance operations. That cost is represented by \( \xi_{s,n}^D \) and depends on the product and the plant on each time period. Filled bottle costs are calculated per bottle stored at the end of a time period and does not consider the amount of bottles throughout a period and neither the amount of bottles that are going through quality inspection and haven’t been yet incorporated to this inventory.

\[ RIC = \sum_s \sum_t \sum_n \xi_{s,n}^R f_{s,n}^R \]  

(37)

In the case of the inventory for recycled bottles, the cost \( \xi_{s,n}^R \) is very similar to the one for filled bottles but with some small difference. In this case the amount of parameters to be defined is lower given that the cost has to be defined per type of bottle in each plant in each time period rather for each different product.

\[ FIC = \sum_t \sum_n \sum_p \xi_{s,n}^D f_{s,n}^D \]  

(38)

The last inventory cost \( \xi_{s,n}^D \) is related to the amount of used bottles that have to be stored waiting for them to be reprocessed. This inventory is increasing at the end of each time period as new bottles are arriving from each of the distribution centers.

\[ FOC = \sum_t \sum_n \sum_{j \in \mathcal{J}_p} \sum_{s \in \mathcal{S}_p} \theta_{t_p,n} Q_{p,s,j} \]  

(39)

\[ ROC = \sum_t \sum_n \sum_{j \in \mathcal{J}_p} \sum_{s \in \mathcal{S}_p} \theta_{t_p,n} Q_{p,s,j}^R \]  

(40)

There is a cost involved in carrying on all the filling operations in our plants. This cost accounts for the use of machines, human capital, maintenance and raw materials and it is proportional to the time throughout which the filling operations take place. Therefore the total cost is calculated multiplying the cost per hour \( \theta_{t_p,n} \) for the total amount of hours used, which it’s calculated dividing the total amount of bottles filled by the filling rate for that product in that line.

\[ UC = \sum_t \sum_n \sum_{j \in \mathcal{J}_p} \sum_{s \in \mathcal{S}_p} \theta_{t_p,n} Q_{p,s,j} \]  

(41)

Utilizing each line on each plant on a time period is not cost-free. Whenever a line is used there is a fixed cost involved to the operations of opening and closing it, no matter for how long it has been working or the amount of products processed. This means that an optimized solution will try to minimize this cost by assignment as many products as possible to the same line. The total line utilization cost is calculated multiplying the unitary cost per line open \( \psi_{s,j,n} \) for the decision variable \( V_{s,j,n} \) that determines if that line is being used on that time period in that plant.

\[ SC = \sum_s \sum_t \sum_{j \in \mathcal{J}_p} \sum_{s \in \mathcal{S}_p} \sum_{n \in \mathcal{N}_p} \phi_{t_p,n} X_{t_p,j} \]  

(42)

As it has been mentioned above, whenever two different type of bottles are being processed in the same line for either filling or recycling them, the order in which they are processed matters. This happens because the setup of the machines to adapt them to each type involves a setup time that depends on the order. And, on the other hand, that setup involves a cost that depends on the order in which bottles are processed. The cost per setup from type of bottle \( t \) to \( t' \) in line \( j \) in plant \( s \) on time period \( n \) is defined by the parameter \( \phi_{t_p,n} \). Therefore, the total setup cost in all the operations will be the sum of all the
setup costs, calculated multiplying the previous parameter for
the binary variable that determines if an exact setup happened
or not.

\[
TC = \sum_{s} \sum_{d} \sum_{n} \sum_{l} \sum_{a} (\psi_{sl} Z_{adln} + \nu_{sdl} (U_{adln} + V_{adln}))
\]

When calculating the total transportation costs we have to
consider both fixed and variable costs. The fixed cost associated
to utilizing a truck on a route between plant \(s\) and distribution
center \(d\) on a time period \(n\) is represented by \(\psi_{sl}\). Therefore, it
has to be multiplied for the binary variable \(Z\) that represents if
that route takes place.

On the other hand there is a unitary variable cost \(\nu_{sdl}\) per each
unit transported in the forward and reverse flow depending on
the route and truck. It accounts the same value for both filled
and empty bottles for the sake of simplicity in the calculations.

\[
PC = \sum_{s} \sum_{n} \sum_{l} \sum_{a} R_{sln}^{p} \theta_{sln}^{p}
\]

The last of the costs involved in the operations is related to
the purchase of new empty bottles for filling them. This cost
is minimized by the recycling operations. The unitary cost for
each new bottle is represented by \(\theta_{sln}^{p}\) and it depends on the
plant \(s\) and in the type of bottle \(t\) that is purchased.

4.5 Objective function

\[
\min PIC + RIC + FIC + FOC + ROC + UC + SC + TC + PC
\]

The objective function represents the total costs of all op-
erations, being the sum of all the previously described costs.
Summarizing, in the first place we have the cost for all 3 in-
ventories of filled, cleaned and used bottles multiplied by their
unitary costs per time period \((\xi_{s,pp}^{f}, \xi_{s,pp}^{c}, \xi_{s,pp}^{u})\). We consider also
the cost per time unit of filling \((\theta_{spp}^{p})\) and cleaning \((\theta_{spp}^{c})\) op-
erations and multiply it for the amount processed divided by the
processing rate. In addition, there are the additional fixed costs
\((\nu_{s,sl})\) depending on whether a line is used or not every time pe-
riod and a cost \((\phi_{p,sl})\) involved in the bottle setup operations
within the same line. At last, we consider too the fixed \((\phi_{s,sl})\) and
variable cost \((\nu_{s,sl})\) of transportation items in both the forward
and reverse flow as well as the purchase unitary cost \((\theta_{sln}^{p})\) of
new bottles multiplied by the amount of bottles purchased on
each time period.

5. Numerical example

In this section we are going to proved several numerical exam-
ple validating the proposed model. Firstly, we explain the sets
that determine the the problem. Then, all the parameters rela-
ting the operations in the industry are established, and finally,
the costs on each operation are provided. After parameter set-
ing the problem is solved by using optimization software.

5.1 Example industry structure (Sets)

We consider a planning horizon of 4 different time periods, each
of them representing one day. Those time periods are numbered
starting from 0 to 3. Therefore set \(N = 0, 1, 2, 3\). Our proposed
business consists of 2 different manufacturing plants and 2 dis-
tribution centers. Sets \(S\) and \(D\) will include two values 0 and

I respectively. The plants are labeled as \(s_0\) and \(s_1\) while the
distribution centers will be \(d_0\) and \(d_1\). Our industry transports
forward 4 different products between the plants and the distri-
bution centers, and brings 2 different types of empty bottles in
the reverse flow. The 4 products are labeled \(p_0, p_1, p_2\) and \(p_3\)
while the types of bottles are \(t_0\) and \(t_1\). Both \(p_0\) and \(p_1\) are bot-
tles using empty bottles of type \(t_0\) while products \(p_2\) and \(p_3\) are
bottled in bottles of type \(t_1\).

For transporting products between the plants and the distri-
bution centers there are 4 different trucks \((L = 0, 1, 2, 3)\). The
trucks are labeled \(l_0, l_1, l_2\) and \(l_3\). Trucks \(l_0\) and \(l_1\) can trans-
port any product back and forth from plant \(s_0\) to any of the two
distribution centers. Trucks \(l_2\) and \(l_3\) would do the same thing
between plant \(s_1\) and any of the \(DC\) as well.

Inside each of the production plants there are three different
production lines \((J = 3)\) which can work in any of the different
time periods. The three production lines are labeled \(j_0, j_1\) and
\(j_2\). Lines \(j_0\) and \(j_1\) can only take charge of filling operations
while \(j_2\) is the only one and exclusively in charge of the clean-
ing and recycling operations. All the production lines are com-
patible with all the products and the types of bottles.

5.2 Example industry parameters

As it has been previously introduced, each of the distribution
centers will have a daily demand of products and at the same
time a return amount. Those amounts are reflected in figures 7
and 8.

\[
\begin{align*}
\text{Product Demand on each Distribution Center} \\
\text{N0} & \text{ N1} & \text{ N2} & \text{ N3} \\
\text{DC d0} & \text{DC d1} & \text{DC d2} & \text{DC d3} \\
\end{align*}
\]

The initial inventory amounts in each plant for each of the
three proposed inventories on each plant is of 3000 units of
each finished product and of 2000 empty bottles of each type
in both the recycled and the used bottles inventory.

When it comes to transportation, all trucks have a maximum
capacity of 7000 bottles (either filled or empty) and a minimum
of 1000.

Inside the plants, parameters vary depending on each of the
production lines. In the first case we consider different produc-
tion rates. In the case of the bottling lines, \(j_0\) has a processing
rate $\rho$ of 700 bottles per hour and $j_1$, a rate of 1000 bottles per hour. $j_2$ is the one in charge of the recycling operations with a processing rate $\rho^R$ of 600 bottles per hour. The preparation times $\delta$ have the same value for all products are types of bottles, being it of 0.5 hours for each. In each of the plants on each time period the total physical available time $\omega$ is 10 hours for all lines. The opening time $\alpha$ has a value of 1 hour for all lines, but the closing time $\beta$ depends on the line, whether it is a filling or a recycling one. In the case of the bottling ones 1 hour is required and for the recycling line this parameter takes a value of 2 hours.

The last group parameters that have to be defined include the changeover time between the two different types of bottles in each of the production lines. For the recycling line the changeover time is the same no matter the order of the change being this of 0.5 hours. For the bottling lines this parameter takes a value of 1 hour. The last parameters are the daily production limits on the lines. Each filling line can produce 4000 units of each product each day while the recycling lines can clean 3000 units of each type of bottle each day.

### 5.3 Example industry costs

When it comes to the costs, the unitary daily inventory cost is 0.1 dollars for filled bottles and 0.05 for recycled and used ones. The processing cost for both filling and cleaning operations is 10 dollars per hour, while the fixed production line utilization cost is equal to 100. The change cost between types of bottles is 20 dollars for the filling lines and 10 dollars for the recycling lines. The fixed cost per day of utilization of the trucks is 100 dollars for trucks $l_0$ and $l_2$ and 130 dollars for trucks $l_1$ and $l_3$. The additional variable cost per bottle transported is 0.01 dollars for trucks $l_0$ and $l_2$ and 0.005 dollars for trucks $l_1$ and $l_3$. Finally, the empty bottle purchase cost is 0.4 dollars per bottle, a cost that will be minimized with the recycling operations.

### 5.4 Example solving and results

Once the whole example is defined with all its different elements, we perform a computing linear optimization using Gurobi. In order to do this, our whole mathematical framework is written using Python, allowing for the different sets and parameters to be modified very easily. After that, we solved the problem using a CPU Intel i7-3770 with 8GB of RAM with Gurobi Optimizer 5.5. The final optimization problem featured 942 constraints and 652 variables, taking a total computational time of 98.26 seconds.

The output of our problem includes all detailed transportation operations on each time period as well as filling, recycling and inventory amounts.

Tables 1 and 2 represent the optimal inventory amounts at the end of each time period for recycled and used bottles $I^R, I^D$ and filled products $I^F$ as well as the amount of products and bottles filled and recycled during that time period.

In addition, figure 9 represents this same optimal evolution of the inventory represented in the previously presented graphical structure. In this example representation we have chosen to show the evolution for products $p_0$ and $p_1$ in plant $s_0$ in time period 0. As a reminder, these products are filled in empty bottles of type 0.

Table 1 | Inventory amounts of filled bottles and amount of filled products on each period

| Plant $s$ | Product $p$ | Time period $n$ | $F^p$ | Filled products |
|-----------|-------------|----------------|------|----------------|
| 0         | 0           | 0              | 2000 | 1300           |
| 0         | 1           | 0              | 2400 | 900            |
| 0         | 2           | 0              | 810  | 0              |
| 0         | 3           | 0              | 1790 | 510            |
| 1         | 0           | 0              | 1300 | 0              |
| 1         | 1           | 0              | 2500 | 0              |
| 1         | 2           | 0              | 1690 | 0              |
| 1         | 3           | 0              | 2210 | 2000           |
| 0         | 0           | 1              | 1000 | 1000           |
| 0         | 1           | 1              | 2370 | 2370           |
| 0         | 2           | 1              | 1020 | 1020           |
| 0         | 3           | 1              | 3980 | 3980           |
| 1         | 0           | 1              | 2130 | 2130           |
| 1         | 1           | 1              | 2430 | 2430           |
| 1         | 2           | 1              | 1550 | 1550           |
| 1         | 3           | 1              | 1020 | 1020           |
| 0         | 0           | 2              | 4000 | 4000           |
| 0         | 1           | 2              | 1000 | 1000           |
| 0         | 2           | 2              | 1050 | 1050           |
| 0         | 3           | 2              | 2450 | 2450           |
| 1         | 0           | 2              | 1720 | 1590           |
| 1         | 1           | 2              | 1430 | 1430           |
| 1         | 2           | 2              | 1480 | 1480           |
| 1         | 3           | 2              | 0    | 0              |
| 0         | 0           | 3              | 0    | 0              |
| 0         | 1           | 3              | 0    | 0              |
| 0         | 2           | 3              | 0    | 0              |
| 0         | 3           | 3              | 0    | 0              |
| 1         | 0           | 3              | 0    | 0              |
| 1         | 1           | 3              | 0    | 0              |
| 1         | 2           | 3              | 0    | 0              |
| 1         | 3           | 3              | 0    | 0              |

Fig. 9 | Evolution of the inventory of products $p_0$ and $p_1$ in time period 0 in plant $s_0$ according to the optimal solution of the numerical example

Table 3 includes the optimal transportation amounts results in both the forward $U^F$ and the reverse $U^D$ flow, as well as the
was calculated to be 16026 dollars, with the following cost
minimize the total cost is presented in table 8.

Tables 6 and 7 represent all the time needed for either filling or recycling the
optimal choices of trucks for each time period.

Table 2  Inventory amounts of recycled and used bottles and amount at
the end of each period as well as amount of recycled products on
each time period

| Plant s | Bottle t | Time period n | $I^p$ | $I^d$ | Recycled bottles |
|---------|----------|---------------|-------|-------|------------------|
| 0       | 0        | 0             | 1300  | 1700  | 1300             |
| 0       | 1        | 0             | 3490  | 2500  | 2000             |
| 1       | 0        | 0             | 4000  | 2000  | 2000             |
| 1       | 1        | 0             | 1300  | 1700  | 1300             |
| 0       | 0        | 1             | 800   | 1400  | 800              |
| 0       | 1        | 1             | 2500  | 1000  | 2500             |
| 1       | 0        | 1             | 1820  | 2680  | 1820             |
| 1       | 1        | 1             | 1480  | 1220  | 1480             |
| 0       | 0        | 2             | 3900  |       | 3900             |
| 0       | 1        | 2             | 3000  |       | 3000             |
| 1       | 0        | 2             | 2680  |       | 2680             |
| 1       | 1        | 2             | 2420  |       | 2420             |
| 0       | 0        | 3             | 6900  |       | 6900             |
| 0       | 1        | 3             | 4000  |       | 4000             |
| 1       | 0        | 3             | 3180  |       | 3180             |
| 1       | 1        | 3             | 3420  |       | 3420             |

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When it comes to the production scheduling problem, one of
the key decision variables represents in which order will be pro-
cessed the different types of bottles in the same production line,
in order to minimize the cost involved in the operation while
maintaining it within the valid operation time limits. That de-
cision variable is equal to 1 in the solution when a determinate
pattern is followed and 0 otherwise. Tables 4 and 5 represent
the value of that variable in our problem. If it is equal to 1, it
means that in that same line, types will be processed in the or-
der established in that row (of the solution). If it is equal to 0,
that order won’t be followed.

The second part of our production scheduling results is the
completion time and the finishing time of each type of bottle in
each of the processing lines. The completion time is measured
in hours since the opening of the plants while the processing
time represents all the time needed for either filling or recy-
cling the bottles of that type, including the preparation times
and not including type changeover types. Tables 6 and 7 rep-}
represent both times for the filling and recycling operations respec-
tively, only in the cases in which those operations are carried.

The optimal amount of new bottles purchased in order to
minimize the total cost is presented in table 8.

Under the given parameter setting, the minimum total cost
was calculated to be 16026 dollars, with the following cost struc-

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Inventory Filled Cost: 4333.0 → 27.04%
Inventory Recycled Cost: 834.5 → 5.21%
Inventory used Cost: 2185.0 → 13.63%
Filling Processing Cost: 372.56 → 2.32%
Recycling Processing Cost: 220.0 → 1.37%
Line Operation Cost: 1300.0 → 8.11%
Table 6  Completion and processing times for each type of bottle in the filling operations

| Time period n | Plant s | Line j | Bottle type t | Completion time | Processing time (hours) |
|---------------|---------|--------|---------------|-----------------|------------------------|
| 0             | 0       | 1      | 0             | 5.49            | 3.20                   |
| 0             | 0       | 1      | 1             | 8.00            | 1.51                   |
| 0             | 1       | 1      | 1             | 7.00            | 3.00                   |
| 1             | 0       | 0      | 0             | 7.00            | 5.81                   |
| 1             | 0       | 1      | 1             | 7.00            | 6.00                   |
| 1             | 1       | 0      | 1             | 7.00            | 4.67                   |
| 1             | 1       | 1      | 0             | 7.00            | 5.56                   |
| 2             | 0       | 0      | 1             | 7.00            | 6.00                   |
| 2             | 0       | 1      | 0             | 7.00            | 6.00                   |
| 2             | 1       | 1      | 0             | 5.02            | 4.02                   |
| 2             | 1       | 1      | 1             | 8.00            | 1.98                   |

Table 7  Completion and processing times for each type of bottle in the recycling operations

| Time period n | Plant s | Line j | Bottle type t | Completion time | Processing time (hours) |
|---------------|---------|--------|---------------|-----------------|------------------------|
| 0             | 0       | 2      | 0             | 8.00            | 2.67                   |
| 0             | 0       | 2      | 1             | 4.83            | 3.83                   |
| 0             | 1       | 2      | 0             | 4.83            | 3.83                   |
| 0             | 1       | 2      | 1             | 8.00            | 2.67                   |
| 1             | 0       | 2      | 0             | 2.83            | 1.83                   |
| 1             | 0       | 2      | 1             | 8.00            | 4.67                   |
| 1             | 1       | 2      | 0             | 8.00            | 3.53                   |
| 1             | 1       | 2      | 1             | 3.97            | 2.97                   |

Table 8  Purchased amount of new bottles

| Plant s | Bottle type t | Time period n | Purchased Bottles |
|---------|---------------|---------------|-------------------|
| 0       | 0             | 0             | 200               |
| 0       | 1             | 0             | 0                 |
| 1       | 0             | 0             | 0                 |
| 1       | 1             | 0             | 0                 |
| 0       | 0             | 1             | 2070              |
| 0       | 1             | 1             | 1510              |
| 1       | 0             | 1             | 560               |
| 1       | 1             | 1             | 1270              |
| 0       | 0             | 2             | 4200              |
| 0       | 1             | 2             | 1000              |
| 1       | 0             | 2             | 1200              |
| 1       | 1             | 2             | 0                 |
| 0       | 0             | 3             | 0                 |
| 0       | 1             | 3             | 0                 |
| 1       | 0             | 3             | 0                 |
| 1       | 1             | 3             | 0                 |

Type Changeover Cost: 80.0 → 0.5%
Transportation Cost: 1897.45 → 11.84%
Bottle purchase Cost: 4804.0 → 29.98%

5.5  Comparison and model success evidence

In order to verify the success of the model, we decided to run the same code but altering some parameters. In this new example, all reverse operations are eliminated in order to prove the benefits of our proposal. This can be done very quickly by setting the return amounts from each distribution center to be 0, the same amount that the initial inventories of empty and recycled bottles. The last small modification is to set the minimum truck transportation amount to 0, allowing the reverse flow to be null. Finally, we modify the set of recycling lines, making them useless. With this small modifications we run again the same example with the same parameters, obtaining the following results and a final total cost of 20650 dollars:

Inventory Filled Cost: 4335.0 → 20.99%
Inventory Recycled Cost: 0 → 0%
Inventory used Cost: 0 → 0%
Filling Processing Cost: 374.06 → 1.81%
Recycling Processing Cost: 0 → 0%
Line Operation Cost: 900 → 4.36%
Type Changeover Cost: 0 → 0%
Transportation Cost: 1757.75 → 8.51%
Bottle purchase Cost: 13284 → 64.33%

As can be seen from above results, when eliminating the reverse flow, the transportation cost is slightly decreased due to the absence of variable reverse transportation costs and all the recycling and inventory costs of empty and used bottles are eliminated. However, all new filled products require a new bottle, greatly increasing as much as 3 times the purchase cost.

In conclusion, our model implementing the repackaging reverse flow has reduced the total cost of operations by around 20%. Surely the benefits of the reverse flow of RPM is depends on the purchasing cost of new RPM. In most cases, the RPM itself is introduced from the cost benefits without considering any benefit which comes from optimization of production schedule, we can say that the purchasing cost of new RPM will not be a problem which may refuse our model.

6. Conclusion

This paper we extended a previous mathematical framework proposed by Kopanos et. al.[1] in order to implement reverse flows and recycling activities in a linear-programming cost optimization problem. A business of bottling and recycling plants has been introduced taking into account that it must satisfy a series of demands and at the same time recycle and reuse empty incoming bottles in order to reduce the ecological impact and the RPM purchase cost.

In order to prove the model’s efficiency, a numerical example has been solved taking into account all the describe equa-
tions and the full objective function. The output of this example has given detailed information on inventory management, transportation direction, production scheduling, material acquisition and operation placement. The combined implementation of reverse repackaging logistics with a whole industry optimization constitutes an innovative approach in the closed-loop supply chain field.

The output of the numerical example and its comparison with the same numerical problem using the original framework that our work has proven that the contribution allows to successfully decrease the total cost of operations and reduce the ecological damage, favoring the environment preservation and the company social responsibility policy.

Further improvement of the model could be focused on considering alternative remanufacturing processes as well as separate activities and disposal activities. A multi-optimization approach focusing on demand peaks could also be one interesting future extension.

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