A Simple Deterministic Measurement Matrix Based on GMW Pseudorandom Sequence*

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SUMMARY Compressed sensing is an effective compression algorithm. It is widely used to measure signals in distributed sensor networks (DSNs). Considering the limited resources of DSNs, the measurement matrices used in DSNs must be simple. In this paper, we construct a deterministic measurement matrix based on Gordon-Mills-Welch (GMW) sequence. The columns' vectors of the proposed measurement matrix are generated by cyclically shifting a GMW sequence. Compared with some state-of-the-art measurement matrices, the proposed measurement matrix has relative lower computational complexity and needs less storage space. It is suitable for resource-constrained DSNs. Moreover, because the proposed measurement matrix can be realized by simple shift register, it is more practical. The simulation result shows that, in terms of recovery quality, the proposed measurement matrix performs better than some state-of-the-art measurement matrices.

key words: distributed sensor networks, compressed sensing, Gordon-Mills-Welch sequence, pseudorandom sequence

1. Introduction

Due to convenience deployment and low cost, distributed sensor networks (DSNs) are widely used in military [1], environment monitoring [2], industrial monitoring [3] and health care [4]. Moreover, DSNs are highly related to mobile crowd sensing [5] and edge computing [6], [7]. DSN faces challenges of limited bandwidth of network, limited storage space, limited computational capability and limited sensor node power. The bandwidth issue and power issue can be solved by reducing the data transmitted from sensor node to central node. Considering the limited computational capability and the limited storage space of sensor node, the data gathering algorithm used in sensor node must be simple.

Compressed sensing (CS) [8] is a novel information gathering theory proposed by Donoho et al. Compared with the Nyquist sampling theory, for a sparse signal, CS can obtain its information with much higher compression ratio. Moreover, CS transfers computational burden from sensor node to central node. Then, CS is especially suitable to measure signals in DSN. In sensor node, signal is measured by projected onto measurement matrix. In the central node, the signal is recovered by using proper recovery algorithms. Considering limited resources of sensor node, the measurement matrix used in sensor node must be simple.

The measurement matrix in CS can be divided into random measurement matrix and deterministic measurement matrix [9]. Gaussian random measurement matrix [10] is the first proposed random measurement matrix. It satisfies the Restrict Isometry Property (RIP) [10] with high probability. However, all elements of Gaussian random measurement matrix are needed to be stored. This requires large storage space. Moreover, in compressing signal, all nonzero elements of Gaussian random measurement matrix are needed to be dealt with. This brings huge computational burden. P. Indyk [11] proposed the random sparse measurement matrix. Because most elements of random sparse measurement matrix are zeros, Its computational complexity is largely reduced. However, it is hard to verify a random measurement matrix whether satisfies the RIP or not. In Recent years, deterministic measurement matrix draws researchers’ much attention [9], [12], [13]. Especially, because pseudorandom sequences have internal randomness, the deterministic measurement matrices based on pseudorandom sequences draw researchers’ much attention. Lei Yu [14] constructed the deterministic measurement matrix by using logistic chaotic pseudorandom sequence. The column vectors of the measurement matrix are generated by cyclically shifting a logistic chaotic pseudorandom sequence. H. Gan et al. [15] proposed the deterministic measurement matrix by using chebyshev chaotic pseudorandom sequence. M. Frunzete et al. [16] proposed the deterministic measurement matrix by using tent chaotic sequence. However, all these matrices are dense measurement matrices, and they have high computational complexities.

In this paper, we construct deterministic measurement matrix using a binary pseudorandom Gordon-Mills-Welch (GMW) sequence [17]. There are just 1 and 1 two elements in this measurement matrix. Compared with some widely used measurement matrices, it has lower computational complexity and needs smaller storage space. It is suitable for signal measurement in DSN. The simulation results show that, under noisy and noiseless condition, the GMW measurement matrix performs better than some state-of-the-art measurement matrices.

The remainder of this paper is organized as follows. In Sect. 2, we introduce the compressed sensing theory. In Sect. 3, we introduce the theory of GMW pseudorandom se-
sequence. In Sect. 4, we construct the measurement matrix based on GMW sequence and analyze its performance. In Sect. 5, we provide the simulation results. Conclusions are stated in Sect. 6.

2. Compressed Sensing

If there are no more than \( K \) nonzero elements in the signal \( \mathbf{x} = (x_1, x_2, \ldots, x_N)^T \in \mathbb{R}^N \), \( \mathbf{x} \) is called a \( K \)-sparse signal. In CS, a \( K \)-sparse signal can be measured by projecting it onto a measurement matrix \( \mathbf{\Phi} \in \mathbb{R}^{M \times N} \) where \( M < N \). Obviously, the high-dimensional signal \( \mathbf{x} \) is reduced to a low-dimensional signal \( \mathbf{y} \). The signal measurement process can be denoted as

\[
\mathbf{y} = \mathbf{\Phi x}
\]

Because \( M < N \), the problem of recovering \( \mathbf{x} \) from \( \mathbf{y} \) is ill posed. However, CS illustrates that if \( \mathbf{x} \) is sparse, it can be perfectly recovered by solving the \( l_0 - \text{minimization} \) problem as

\[
\min ||\mathbf{x}||_0, \quad \text{s.t.} \quad \mathbf{y} = \mathbf{\Phi x}
\]

There are mainly two kinds of algorithms to solve the \( l_0 - \text{minimization} \) problem:

- Greedy Pursuit (GP) algorithms, such as Orthogonal Pursuit Match (OMP) algorithm [18], Subspace Pursuit (SP) algorithm [19] and Compressive Sampling Matching Pursuit (CoSaMP) [20].
- The convex relaxation algorithms. Chen et al. proved that the \( l_0 - \text{minimization} \) problem can be relaxed to the \( l_1 - \text{minimization} \) problem. Basic Pursuit (BP) [21] and Gradient Projection for Sparse Reconstruction (GPSR) [22] can be used to solve this problem.

In order to evaluate the performance of the measurement matrix, Candès and Tao proposed the Restricted Isometry Property (RIP) [10]. For any \( \mathbf{x} \), if the Restricted Isometry Constant (RIC) \( \delta_K \) satisfies

\[
(1 + \delta_K)||\mathbf{\Phi x}||_2^2 \leq ||\mathbf{\Phi x}||_2^2 \leq (1 - \delta_K)||\mathbf{\Phi x}||_2^2
\]

where \( ||\mathbf{x}||_0 = K \) and \( 0 < \delta_K < 1 \), then the measurement matrix \( \mathbf{\Phi} \) satisfies the RIP. When \( \delta_K \) is small enough, the problem (2) can be perfectly solved by convex relaxation algorithms or GP algorithms.

Although RIP condition is perfect, it is hard to verify a measurement matrix’s RIP property. Bourgain et al. [23] proposed the mutual coherence (MC). It plays an important role in the evaluation of deterministic measurement matrix. The mutual coherence \( \mu(\mathbf{\Phi}) \) of the measurement matrix \( \mathbf{\Phi} \) is the maximum absolute value of the normalized inner product of two arbitrary column vectors in the measurement matrix. The definition of mutual coherence is

\[
\mu(\mathbf{\Phi}) = \max_{j_1 \neq j_2} \frac{|\langle \varphi_{j_1}, \varphi_{j_2} \rangle|}{\|\varphi_{j_1}\|_2 \|\varphi_{j_2}\|_2}, \quad 1 \leq j_1 \neq j_2 \leq N
\]

Bourgain et al. [23] proved that \( \mathbf{\Phi} \) satisfies RIP of order \( K \) with \( \delta_K \leq \mu(\mathbf{\Phi})(K - 1) \), where \( K < \frac{1}{\mu(\mathbf{\Phi})} + 1 \). This means the smaller the mutual coherence of a measurement matrix is, the more information it can obtain.

3. GMW Pseudorandom Sequence

Pseudo-random sequence not only has similar correlation characteristic to white noise but also deterministic structure. GMW sequence is a kind of pseudo-random sequence which has good autocorrelation property and high linear complexity. GMW sequence is proposed by Scholtz and Welch [17]. Compared with M sequence, GMW sequence has larger linear complexities and larger sequences number.

Trace function [17] is a mapping function in finite field. It maps number from extension field to subfield. Trace function is linear. GMW sequences can be constructed by combining mapping functions. We introduce the concept of trace function.

Definition 1: Let \( P \) and \( J \) be two positive integers where \( P \) is a divisor of \( J \). The trace function \( tr_J^P(s) \) maps each element of finite field \( GF(2^P) \) to finite field \( GF(2^L) \).

\[
tr_J^P(s) = \sum_{i=0}^{(P/J-1)} s^{2^i}
\]

The binary GMW sequence \( b(b_0, b_1, \cdots, b_L) \) can be defined as

\[
b_j = tr_J^P([tr_J^P(a^\tau)])^\tau, \quad j = 1, 2, \cdots, 2^P - 1
\]

where \( a \) is a primitive element of the finite field \( GF(2^M) \), \( \gamma \) and \( 2^j - 1 \) are arbitrary relatively prime integer where \( 1 < \gamma < 2^j - 1 \).

Binary GMW sequence has following characters:

- The period of GMW sequence is \( L = 2^P - 1 \).
- The autocorrelation function of GMW sequence is

\[
\varphi(\tau) = \sum_{j=1}^{L-1} b_j b_{j+\tau} = \begin{cases} L, & \tau = 0; \\ -1, & 1 \leq \tau \leq L \end{cases}
\]

- The number of zeros in each period of binary GMW sequence is \( 2^{P-1} - 1 \), the number of 1s in each period of the binary GMW sequence is \( 2^{P-1} \). This means, in each period, the number of zeros is one less than the number of 1s.
- Easy hardware realization: the GMW sequence can be constructed by crossing and shifting \( m \) sequence. Because \( m \) sequence realization: the GMW sequence can be realized by using simple shift register, GMW sequence is also easily realized.

4. The Measurement Matrix Based on GMW Pseudorandom Sequence

4.1 The Construction of GMW Measurement Matrix

The construction of measurement matrix based on GMW sequence \( b \) can be divided into three steps:
1. Construct a temporary matrix, the first column is the GMW sequence $b$ and the $t-th, (2 \leq t \leq L)$ column is constructed by cycle-left-shifting $b$ $t - 1$ times.

2. The measurement matrix $\Phi$ can be constructed by randomly choosing $T, (1 \leq T \leq L)$ rows from the temporary matrix.

3. Replace all 0s in $\Phi$ with $-1$s.

4.2 Performance Analysis

Form Sect. 2, we know that the smaller the mutual coherence of a measurement matrix, the stronger its ability of obtaining information. In this part, we analyze the mutual coherences of the following measurement matrices.

- The GMW measurement matrix proposed in this paper.
- The Gaussian random measurement matrix [10] whose elements following standard normal distribution.
- The random sparse measurement matrix [11], each column of which has four 1s and other elements are zeros. And, the positions of 1s are randomly generated.
- The deterministic measurement matrix constructed from logistic chaotic sequence [14].

Figure 1 shows the mutual coherences of above measurement matrices with the size of $M \times N$ where $N = 63$. We can see that the mutual coherences of all measurement matrices decline with the increase of measurement number $M$. This demonstrates that the larger the measurement number, the stronger the measurement matrix’s ability of obtaining information. The random sparse measurement matrix has the largest mutual coherence. The mutual coherence of Gaussian random measurement matrix is similar to that of chaotic measurement matrix. The GMW measurement matrix has the smallest mutual coherence. Then, we forecast that the GMW measurement matrix has the strongest ability of obtaining information.

Table 1 shows the computation complexities and the storage spaces needed of different measurement matrices. For the storage space, we assume each element of random measurement matrix needs one storage space. All elements of Gaussian random measurement matrix are needed be stored. Then it needs the maximum storage space. Chaotic measurement matrix and GMW measurement matrix are deterministic measurement matrix. They just need a little initialization parameters. Then, they need the minimum storage space, approximately to zero.

For the computational complexity, all elements of Gaussian random measurement matrix and chaotic measurement matrix are needed be multiplication calculated and addition calculated. They need the maximum computational complexity. The computational complexity of GMW measurement matrix is higher than that of random sparse measurement matrix, but lower than that of Gaussian random measurement matrix and chaotic measurement matrix.

5. Simulation Result and Analysis

In this part, GMW measurement matrix is compared with some state-of-the-art measurement matrices, including Gaussian random measurement matrix [10], random sparse measurement matrix [11] and Logistic chaotic measurement matrix [14]. Orthogonal match pursuit (OMP) algorithm [18] is used to recovery signals. The experimental design is as follows:

- $K - sparse$ vector is used as test signal with length $N = 511$.
- All simulation results are averaged over independent trials. In each trial, the measurement matrix $\Phi$ and signal $x$ are generated independently. Each signal has $K$ nonzero elements and $N - K$ zero elements. The positions of nonzero elements are randomly chosen from $I = \{1, 2, \cdots, N\}$. The values of signal’s nonzero elements are randomly drawn from standard normal distribution.
- We use perfect recovery percentages to evaluate algorithms’ performances. For the $i-th$ trial, $\hat{x}^{(i)}$ and $x^{(i)}$ respectively denote the recovered signal and the original signal. If $\|x^{(i)} - \hat{x}^{(i)}\|_2 < 10^{-5}\|x^{(i)}\|_2$, we announce the $i-th$ trial is perfect. Suppose $SE$ recoveries are perfect in 500 trials, the perfect recovery percentage is $SE/500$.
- We perform all simulations in Matlab R2015b running on a computer with 2.5 GHz, Intel Celeron G540 processor, 4.0GB of RAM, and Windows 10 system.

5.1 Recovery from Noiseless Measurement

Figure 2 shows the perfect recovery percentages of different measurement matrices against the change of measurement number in noiseless situation. The measurement number changes from 20 to 120 with step 5, other parameters are
fixed as \( N = 511 \) and \( K = 15 \). The perfect recovery percentages of all measurement matrices increase with measurement number. GMW measurement matrix performs always better than others. GMW measurement matrix just needs 95 measurement numbers to reach 100% recovery. Gaussian random measurement matrix needs 120 measurement numbers to reach 100% recovery. Random sparse measurement matrix and chaotic measurement matrix need more measurement numbers.

Figure 3 shows the perfect recovery percentages of different measurement matrices against the change of sparsity in the noiseless situation. The signal sparsity changes from 5 to 40 with the step 5. Other parameters are fixed as \( N = 511 \) and \( M = 100 \). We can see that with the increase of sparsity, the perfect recovery percentages of all measurement matrices decline. GMW measurement matrix performs always better than other measurement matrices. When signal sparsity \( K = 15 \), the perfect recovery percentage of GMW measurement matrix is still 100%, however, the perfect recovery percentages of other measurement matrices have fallen below 100%.

The above experiments demonstrate that the proposed measurement matrix has stronger ability of obtaining information than other measurement matrices.

5.2 Recovery from Noisy Measurement

In this part, we test measurement matrices’ performance against noise. In noisy case, signal compression process can be denoted as

\[ y = \Phi x + n \]  

where \( n = (n_1, n_2, \ldots, n_M)^T \in \mathbb{R}^M \) is Gaussian white noise. Relative error is used to evaluate the measurement matrices’ performances. Its definition is

\[ \text{Relative error} = \frac{500}{\sum_{i=1}^{500} \left( \frac{||x^{(i)} - \hat{x}^{(i)}||_2}{||x^{(i)}||_2} \right)^2} \]  

Figure 4 shows the performances of different measurement matrices against the change of measurement number. Other parameters are fixed as \( N = 511, K = 15 \) and \( SNR = 20db \). With the increase of measurement number, relative errors of all measurement matrices decline. The relative error of GMW measurement matrix is always lower than those of other measurement matrices.

Figure 5 shows the performances of different measurement matrices against the change of sparsity. Other parameters are fixed as \( N = 511, M = 100 \) and \( SNR = 20db \). With the increase of sparsity, relative errors of all measurement matrices decline. The relative error of GMW measurement matrix is always lower than those of other measurement matrices.

Figure 6 shows the performances of different measurement matrices against the change of SNR. Other parameters
are fixed as $N = 511$, $M = 100$ and $K = 50$. We can see that the strength of noise affects the performances of all measurement matrices significantly. With the increase of SNR, relative errors of all measurement matrices decline. The relative error of GMW measurement matrix is always lower than those of others when SNR $> 10$db. This demonstrates that GMW measurement matrix is more robust against noise than other measurement matrices.

6. Conclusions

In this paper, deterministic measurement matrix based on GMW pseudorandom sequence is proposed. The column vectors of the proposed measurement matrix are generated by cyclically shifting a GMW sequence. Compared with some state-of-the-art measurement matrices, the proposed measurement matrix needs less storage space and has relative lower computational complexity. It is suitable for the resource-constrained DSNs. The simulation result shows that the proposed measurement matrix performs better than some state-of-the-art measurement matrices. Moreover, the proposed measurement matrix can be realized by using simple shift register, it’s more practical.

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