Absolute Negative Conductivity in Two-Dimensional Electron Systems Associated with Acoustic Scattering Stimulated by Microwave Radiation

V. Ryzhii and V. Vyurkov

Computer Solid State Physics Laboratory, University of Aizu, Aizu-Wakamatsu 965-8580, Japan

(Dated: March 22, 2022)

We discuss the feasibility of absolute negative conductivity (ANC) in two-dimensional electron systems (2DES) stimulated by microwave radiation in transverse magnetic field. The mechanism of ANC under consideration is associated with the electron scattering on acoustic piezoelectric phonons accompanied by the absorption of microwave photons. It is demonstrated that the dissipative components of the 2DES dc conductivity can be negative ($\sigma_{xx} = \sigma_{yy} < 0$) when the microwave frequency $\Omega$ is somewhat higher than the electron cyclotron frequency $\Omega_c$ or its harmonics. The concept of ANC associated with such a scattering mechanism can be invoked to explain the nature of the occurrence of zero-resistance “dissipationless” states observed in recent experiments.

PACS numbers: PACS numbers: 73.40.-c, 78.67.-n, 73.43.-f

I. INTRODUCTION

The effect of vanishing electrical resistance and transition to “dissipationless” states in a two-dimensional electron system (2DES) subjected to a magnetic field and irradiated with microwaves has recently been observed in the experiments by Mani et al. [1], Zudov et al. [2], and Yang et al. [3]. The most popular scenario [4, 5, 6] of the occurrence of this effect is based on the concept of absolute negative conductivity (ANC) in 2DES proposed more than three decades ago by Ryzhii [7]. Later [8], a detailed theory of the ANC effect in 2DES due to the photon-assisted impurity scattering was developed. Recently, a new version of the theory of this effect was presented by Durst et al. [9]. Some nonequilibrium processes in 2DES associated with the electron-phonon interactions were studied theoretically as well [10, 11, 12, 13].

The feasibility of ANC in heterostructures with a 2DES in the magnetic field under microwave irradiation is associated with the following. The dissipative electron transport in the direction parallel to the electric field and perpendicular to the magnetic field is due to hops of the electron Larmor orbit centers caused by scattering processes. These hops result in a change in the electron potential energy $\delta \epsilon = eE\delta \xi$, where $e = |e|$ is the electron charge, $E = |E|$ is the modulus of the net in-plane electric field, which includes both the applied and the Hall components, and $\delta \xi$ is the displacement of the electron Larmor orbit center. If the electron Larmor orbit center shifts in the direction opposite to the electric field ($\delta \xi < 0$), the electron potential energy decreases ($\delta \epsilon < 0$). In equilibrium, the electron Larmor orbit center hops in this direction dominate, so the dissipative electron current flows in the direction of the electric field. However, in some cases, the displacements of the electron Larmor orbit centers in the direction of the electric field (with $\delta \epsilon > 0$) can prevail. Indeed, under sufficiently strong microwave irradiation the main contribution to the electron scattering on impurities can be associated with the processes involving the absorption of the microwave photons. If an electron absorbs such a photon and transfers to a higher Landau level (LL), a portion of the absorbed energy $\hbar \Omega N$, where $\Omega_e = eH/mc$ is the electron cyclotron frequency, $\hbar$ is the Planck constant, $m$ is the electron effective mass, $H$ is the strength of the magnetic field, $c$ is the velocity of light, and $N = 1, 2, 3, \ldots$ is the LL index, goes to an increase of the electron kinetic energy, whereas the change in the electron potential energy is $\delta \epsilon = \hbar(\Omega - \Lambda \Omega_e)$, where $\Lambda = N' - N$. If $|\Omega - \Lambda \Omega_e| > 0$, the potential energy of electrons increases with each act of their scattering. Hence, the dissipation current flows opposite to the electric filed resulting in negative (absolute, not differential) conductivity. However, the probability of the scattering with the spatial displacements of the electron Larmor orbit center $\xi$ exceeding the quantum Larmor radius $L = (\hbar c/eH)^{1/2}$ is exponentially small. Due to this, such scattering processes are effective, and the variation of the dissipative component of the current caused by microwave radiation (i.e., the photocurrent) is be substantial only in the immediate vicinities of the resonances $|\Omega - \Lambda \Omega_e| \lesssim \max\{eEL/h, \Gamma\}$, where $\Gamma$ characterizes the LL broadening [13]. Hence, at small $eEL/h$ and $\Gamma$, the ranges $\Omega - \Lambda \Omega_e$ in which ANC associated with the photon-assisted impurity scattering occurs are rather narrow. Therefore, the contributions of other scattering mechanisms should be assessed. In this paper, we calculate the dc dissipative components of the 2DES conductivity (mobility) tensor, assuming that the main scattering mechanisms are the electron scattering and the electron photon-assisted scattering on the piezoelectric acoustic phonons. This assumption is supported by high quality of the samples (with the electron mobility $\mu > 10^5$ cm$^2$/V s) used in the experiments [1, 2, 3] performed at low temperatures ($T \simeq 1$K).

*Electronic address: v-ryzhii@u-aizu.ac.jp
II. GENERAL FORMULAS

The density of the in-plane dc current in a 2DES in the transverse magnetic field is given by the following equations:

\[ j_x = \sigma(E)E_x + \sigma_H E_y, \quad j_y = \sigma(E)E_y - \sigma_H E_x, \quad (1) \]

where \( \sigma(E) \) and \( \sigma_H \) are the in-plane dissipative and Hall components of the dc conductivity tensor, respectively, with \( \sigma(E) = \sigma_{xx} = \sigma_{yy} \) and, considering the classical Hall effect, \( \sigma_H = \sigma_{xy} = -sE \). Here, \( \Sigma \) is the electron sheet concentration, and the directions \( x \) and \( y \) are in the 2DES plane. Positive values of \( \sigma(E) \) correspond to the electron drift in the direction opposite to the direction of the electric field, i.e., to the usual conductivity, whereas the case \( \sigma(E) < 0 \) is referred to as ANC.

We shall assume that \( \hbar \Omega, \hbar \Omega_c > T \), where \( T \) is the temperature in the energy units. In this case, one can disregard the processes accompanied by the emission of microwave photons.

As follows from Eq. (1), the density of the dissipative current i.e., the current parallel to the net electric field \( \mathbf{E} \), can be presented as \( j_D(E) = \sigma(E)E \). The dissipative current is determined by the transitions of electrons between their quantum states with the change in the coordinate of the electron Larmor orbit center \( \xi \). The energies of the electron states in a 2DES in crossed magnetic and electric fields (neglecting the Zeeman splitting) are given by

\[ \epsilon_{N, \xi} = \left( N + \frac{1}{2} \right) e \hbar \Omega_e + e E \xi. \quad (2) \]

Adjusting the standard "orthodox" approach [15] for the calculation of the dissipative current in a 2DES (see, for example, Refs. 7, 10, 12, 16, 17), \( j_D(E) \) can be presented in the form

\[ j_D(E) = \frac{e}{\hbar} \sum_{N, N'} f_N (1 - f_{N'}) \times \int d^2 q \sqrt{|V_{q}|^2 |Q_{N, N'} (L^2 q_{\perp}^2 / 2)|^2} \]

\[ \times \left\{ N_q \delta [(N - N') \hbar \Omega_e + \hbar \omega_q + e E L^2 q_y] \right. \]

\[ + (N_q + 1) \delta [(N - N') \hbar \Omega_e - \hbar \omega_q + e E L^2 q_y] \]

\[ + I_{\Omega}(q_x, q_y) N_q \delta [\hbar \Omega + (N - N') \hbar \Omega_e + \hbar \omega_q + e E L^2 q_y] \]

\[ + I_{\Omega}(q_x, q_y) (N_q + 1) \delta [\hbar \Omega + (N - N') \hbar \Omega_e - \hbar \omega_q + e E L^2 q_y]. \quad (3) \]

Here, \( f_N \) is the filling factor of the \( N \)th Landau level given by the Fermi distribution function, \( q = (q_x, q_y, q_z) \), \( \omega_q = s q \), and \( N_q = \exp (\hbar \omega_q / T) - 1 \) -1 are the phonon wave vector, frequency, and distribution function, respectively, \( s \) is the velocity of sound, \( q = \sqrt{q_x^2 + q_y^2 + q_z^2} \), \( q_{\perp} = \sqrt{q_x^2 + q_y^2} \), \( \delta(\omega) \) is the LL form-factor which at small \( \Gamma \) can be assumed to be the Dirac delta function, \( |V_q|^2 \propto \exp (-|q_{\perp}^2 / 2) \) characterizes the piezoelectric interaction of electrons with acoustic phonons (because such an interaction is considered as most important in the 2DES under consideration at low temperatures [18]), \( L \) is the width of the electron localization in the \( z \)-direction perpendicular to the 2DES plane (\( l \ll L \), and \( |Q_{N, N'} (L^2 q_{\perp}^2 / 2)|^2 = |P_{N, N'} (L^2 q_{\perp}^2 / 2)|^2 \exp (-L^2 q_{\perp}^2 / 2) \) is the matrix element determined by the overlap of the electron wave functions before and after the hop caused by the scattering, \( |P_{N, N'} (L^2 q_{\perp}^2 / 2)|^2 \) is proportional to a Laguerre polynomial.

The quantity \( I_{\Omega}(q_x, q_y) \) is proportional to the incident microwave power. It characterizes the effect of microwave field on the in-plane electron motion. For the nonpolarized microwave radiation [19], \( I_{\Omega}(q_x, q_y) = J_{\Omega} L^2 (q_x^2 + q_y^2) \), where \( J_{\Omega} = (\xi_{\Omega} / \xi_{\Omega}^2) L^2 \), \( \xi_{\Omega} \) is the microwave electric field amplitude, which is assumed to be smaller than some characteristic microwave field \( \xi_{\Omega} \). The latter assumption implies that the Larmor orbit center oscillation amplitude in the microwave field is smaller than \( L \), and that the radiative processes with the participation of more than one microwave photon are insignificant. The inclusion of the polarization effects leads to the appearance of some anisotropy at the frequencies far from the cyclotron resonance. A more general formula can be used in line with Ref. 19 if \( J_{\Omega} \) is of the order of unity or larger. Deriving Eq. (3), we have taken into account that the displacement of the electron Larmor orbit center is \( \delta \xi = -L^2 q_y \). First two terms in the right-hand side of Eq. (3) correspond to the electron-phonon interactions whereas the third and forth terms are associated with such interactions accompanied by the absorption of a photon. Worth pointing out that, as can be seen from Eq. (3), at the resonances \( \Omega = (N' - N) \Omega_c \), the contribution of the photon-assisted processes to the dissipative conductivity turns zero (see, Refs. [7, 8]). The characteristic amplitude can be presented as [18]

\[ \xi_{\Omega} = \frac{\sqrt{2m \Omega_c^2 - \Omega^2}}{e \sqrt{\Omega_e^2 + \Omega^2}} L \quad (4) \]

In the immediate vicinity of the cyclotron resonance \( \Omega = \Omega_c \), the quantity \( \xi_{\Omega} \) is limited by the LL broadening, Eq. (4) becomes invalid, and \( \xi_{\Omega} \) can be estimated as \( \xi_{\Omega} \simeq \sqrt{2m \Omega_c^2 / e} \).

Assuming that \( e E |\xi| \ll \hbar |\Omega - \Omega_c| < \hbar \Omega_c, \hbar \Omega_e \), one can expand the expression for the dissipating current given by Eq. (3) in powers of \( (e E L^2 / hs) \) and present the dissipative dc conductivity in the following form:

\[ \sigma(E) \simeq \sigma_{dark} + \sigma_{ph}, \quad (5) \]
in which the terms of the order of \((eEL^2/\hbar s)^3\) and higher have been neglected. As a result, from Eq. (3), we obtain

\[
\sigma_{\text{dark}} = \left(\frac{e^2 L^2}{\hbar^3 s^2} \right) \sum_{N, \Lambda} \int d^3 q \ q_\|^2 |V_q|^2 \\
\times |Q_{N, N+\Lambda}(L^2 q_\|^2/2)|^2 \delta'(q - q^{(\Lambda)})
\]

\[
\times \{f_N(1 - f_{N+\Lambda})\mathcal{N}_q - f_{N+\Lambda}(1 - f_N)(\mathcal{N}_q + 1)\}, \tag{6}
\]

where \(\delta'(q) = d\delta(q)/dq\), \(q^{(\Lambda)} = \Lambda \Omega_c/s\), \(q^{(\Omega)} = (\Omega - \Lambda \Omega_c)/s\), and \(\Lambda > 0\). Substituting the integration over \(d^3 q\) for the integration over \(dq_\| dq_\perp dt\), where \(\sin \theta = q_\|/q_\perp\), Eqs. (6) and (7) can be rewritten as

\[
\sigma_{\text{dark}} \propto \sum_{N, \Lambda} \int_0^\infty dq dq_\| q_\|^2 \ \exp \left[ -\frac{t^2 (q^2 - q_\|^2)}{2} \right] \\
\times |Q_{N, N+\Lambda}(L^2 q_\|^2/2)|^2 \delta'(q - q^{(\Lambda)})
\]

\[
\times \{f_N(1 - f_{N+\Lambda})\mathcal{N}_q - f_{N+\Lambda}(1 - f_N)(\mathcal{N}_q + 1)\}, \tag{8}
\]

and reduced to

\[
\sigma_{\text{dark}} \propto \sum_{N, \Lambda} f_N(1 - f_{N+\Lambda}) \int_0^\infty dq \ \exp \left[ -\frac{t^2 q^2}{2} \right] G_N^{(\Lambda)}(q)
\]

\[
\times [\mathcal{N}_q - \exp(-\Lambda \Omega_c/T)(\mathcal{N}_q + 1)]\delta'(q - q^{(\Lambda)}), \tag{10}
\]

\[
\sigma_{\text{ph}} \propto J_\Omega \sum_{N, \Lambda} f_N(1 - f_{N+\Lambda}) \int_0^\infty dq \ \exp \left[ -\frac{t^2 q^2}{2} \right] H_N^{(\Lambda)}(q)
\]

\[
\times \left[ \mathcal{N}_q\delta'(q + q^{(\Lambda)}) - (\mathcal{N}_q + 1)\delta'(q - q^{(\Lambda)}) \right], \tag{11}
\]

where

\[
G_N^{(\Lambda)}(q) = \int_0^{Lq} dt^3 \ \exp \left[ \frac{(t^2 - L^2)^2}{2L^2} \right] |P_N^{(\Lambda)}(t^2/2)|^2, \tag{12}
\]

\[
H_N^{(\Lambda)}(q) = \int_0^{Lq} dt^{5} \ \exp \left[ \frac{(t^2 - L^2)^2}{2L^2} \right] |P_N^{(\Lambda)}(t^2/2)|^2. \tag{13}
\]

At \(N \gg 1\), using the asymptotic expressions for the Laguerre polynomials\(^\text{20}\), one can obtain \(|P_N^{(\Lambda)}(t^2/2)|^2 \approx J_\Lambda^2(\sqrt{2N} t L)/J_\Lambda(t)^2\), where \(J_\Lambda(t)\) is the Bessel function, so that \(|P_N^{(\Lambda)}(t^2/2)|^2 \approx \cos^2(\sqrt{2N} t - (2\Lambda + 1)\pi/4)/\pi N t^2\) if \(t \gg 1/\sqrt{2N}\), and \(|P_N^{(\Lambda)}(t^2/2)|^2 \approx t^2\Lambda^2\) when \(t < 1/\sqrt{2N} \ll 1\). Therefore, \(G_N^{(\Lambda)}(q)\) and \(H_N^{(\Lambda)}(q)\) become

\[
G_N^{(\Lambda)}(q) \approx \frac{1}{\pi N Lq} \int_0^\infty dt \ t^2 \ \exp \left[ -\frac{(t^2 - L^2)^2}{2L^2} \right] \cos^2 \left( \sqrt{2N} t - \frac{(2\Lambda + 1)\pi}{4} \right),
\]

\[
H_N^{(\Lambda)}(q) \approx \frac{1}{\pi N Lq} \int_0^\infty dt \ t^4 \ \exp \left[ -\frac{(t^2 - L^2)^2}{2L^2} \right] \cos^2 \left( \sqrt{2N} t - \frac{(2\Lambda + 1)\pi}{4} \right)
\]

at \(Lq \gg 1\), and

\[
G_N^{(\Lambda)}(q) \approx \frac{1}{\pi N} \int_0^{Lq} dt \ \frac{t^2}{\sqrt{L^2 q^2 - t^2}} \cos^2 \left( \sqrt{2N} t - (2\Lambda + 1)\pi \right),
\]

\[
H_N^{(\Lambda)}(q) \approx \frac{1}{\pi N} \int_0^{Lq} dt \ \frac{t^4}{\sqrt{L^2 q^2 - t^2}} \cos^2 \left( \sqrt{2N} t - (2\Lambda + 1)\pi \right)
\]

at \(1/\sqrt{2N} < Lq < 1\). After the integration involving averaging over fast oscillations, one can obtain

\[
G_N^{(\Lambda)}(q) \approx |2\sqrt{2N}(1 - L^2/2)^{3/2} NLq|^{-1} \approx (2\sqrt{2N Lq})^{-1}
\]
at $Lq \gg 1$, and $G_N^{(A)}(q) \simeq (Lq)^2/8N$ in the range $1/\sqrt{2N} < Lq < 1$. Analogously, for $H_N^{(A)}(q)$ in these ranges one obtains, respectively, $H_N^{(A)}(q) \simeq 3[2\sqrt{2}\pi (1 - l^2/L^2)^{1/2}NLq]^{-1} \simeq 3(2\sqrt{2}\pi NLq)^{-1}$ and $H_N^{(A)}(q) \simeq 3(Lq)^4/32N$. The derivative of function $H_N^{(A)}(q)$ is proportional to the average displacement of the electron Larmor orbit center in the direction of the electric field associated with the electron transition between the $N$th and $(N + 1)$th LL's involving a microwave photon and an acoustic phonon with the energies $\hbar \Omega$ and $\hbar s q$, respectively. As an example, function $H_{30}^{(1)}(q)$ calculated using Eq. (13), in which the asymptotic $|F_N(t^2/2)|^2 \simeq J_\lambda^2(\sqrt{2N}mt)$ was used, is shown in Fig. 1.

III. DARK CONDUCTIVITY

Considering the explicit formula for the phonon distribution (Planck’s function), Eq. (10) can be presented in the following form:

$$\sigma_{\text{dark}} \propto \sum_{N,A>0} f_N(1 - f_{N+\Lambda}) \int_0^\infty dq \exp\left(-\frac{l^2q^2}{2}\right) G_N^{(A)}(Lq)$$

$$\times \frac{1 - \exp[h(sq - \Lambda \Omega_c)/T]}{\exp(hsq/T) - 1} \vartheta\left(q - \frac{\Lambda \Omega_c}{s}\right),$$

Neglecting the terms containing higher powers of $\exp(-h\Omega_c/k_BT)$ and denoting

$$G(q) = f_{N_m}(1 - f_{N_m+1})G_{N_m}^{(1)}$$

and

$$F^{(A)}(q) = 1 - \exp\left[\frac{h(\Lambda \Omega_c - sq)}{T}\right],$$

where $N_m$ in the index of the LL immediately below the Fermi level, we present Eq. (14) in the following form:

$$\sigma_{\text{dark}} \propto \exp\left(-\frac{h\Omega_c}{T}\right) \frac{d}{dq} \left[\exp\left(-\frac{l^2q^2}{2}\right) G(q) F(q)\right]_{q = q(1)}.$$

$$= \overline{\mathcal{C}} \left(\frac{h\nu}{T}L\right) \left(\frac{s}{L\Omega_c}\right) \exp\left(-\frac{h\Omega_c}{T}\right) \exp\left(-\frac{l^2\Omega_c^2}{2s^2}\right).$$

We have taken into account that $Lq^{(1)} = L\Omega_c/s \gg 1$. For example, if $H = 2 \text{kG}$, one obtains $Lq \approx 10$. Hence, for $q \approx q^{(1)}$ one can set $G(q) \simeq \overline{\mathcal{C}}/Lq$, where $\overline{\mathcal{C}} \simeq 1/2\sqrt{2\pi}N$. Similar formula for $\sigma_{\text{dark}}$ can be derived also for the deformation mechanism of the electron-phonon interaction.

One can see from Eq. (15) that the expression for $\sigma_{\text{dark}}$ contains a small exponential factor $\exp(-h\Omega_c/T)$. This factor arises because the dark conductivity is associated with the absorption of acoustic phonons with the energy $\hbar s q$ close to the LL spacing by electrons transferring from almost a fully filled LL (just below the Fermi level, $N = N_m$), to the next one which is nearly empty. However, the number of such photons at $T \ll \hbar\Omega_c$ is small. The rate of the processes with the emission of acoustic phonons accompanying the electron transitions from an upper LL is also exponentially small due to a low occupancy of this level and the Pauli exclusion principle.

Equation (15) differs from that calculated previously by Erukhimov [17] for the dark conductivity associated with the acoustic phonon scattering by the last exponential factor which, according to Eq. (15), depends on the width of the electron localization $l$. This dependence is attributed to a significant contribution of the electron scattering processes with $q_z \neq 0$ which were neglected in Ref. 17. As can be seen from comparison of Eq. (15) and that obtained in Ref. 17, the inclusion of such scattering processes results in the replacement of factor $\exp(-L^2\Omega_c^2/2s^2) = \exp(-h\Omega_c^2/2ms^2)$ (as in ref. 17) by much larger factor $\exp(-l^2\Omega_c^2/2s^2)$ (as in Eq. (15)). This yields a substantially higher (exponentially) value of $\sigma_{\text{dark}}$ than if the transitions with $q_z \neq 0$ were neglected. However, the temperature dependences of the dark conductivity obtained in Ref. 17 and in the present paper coincide.

IV. PHOTOCONDUCTIVITY

Using Eq. (9), we arrive at

$$\sigma_{\text{ph}} = \sum_\Lambda \sigma_{\text{ph}}^{(A)};$$

where

$$\sigma_{\text{ph}}^{(A)} \propto J_\lambda \sum_{N} f_N(1 - f_{N+\Lambda}) \int_0^\infty dq \exp\left(-\frac{l^2q^2}{2}\right) H_N^{(A)}(q)$$
\[ \times \{ N_q \delta'(q + q^{(A)}_q) - (N_q + 1) \delta'(q - q^{(A)}_q) \} \]

\[ \simeq \mathcal{J}_\Omega \int_0^\infty dq \exp \left( - \frac{l^2 q^2}{2} \right) H^{(A)}(q) \]

\[ \times \{ N_q \delta'(q + q^{(A)}_q) - (N_q + 1) \delta'(q - q^{(A)}_q) \}. \] \hspace{1cm} (17)

Integrating in the right-hand side of Eq. (17), we obtain

\[ \sigma^{(A)}_{ph} \propto - \mathcal{J}_\Omega \frac{d}{dq} \left[ \exp \left( - \frac{l^2 q^2}{2} \right) H^{(A)}(q) N_q \right]_{q = -q^{(A)}_q} \]

at \( \Omega - \Lambda \Omega_c < 0 \), and

\[ \sigma^{(A)}_{ph} \propto \mathcal{J}_\Omega \frac{d}{dq} \left[ \exp \left( - \frac{l^2 q^2}{2} \right) H^{(A)}(q) (N_q + 1) \right]_{q = q^{(A)}_q} \]

at \( \Omega - \Lambda \Omega_c > 0 \). Here,

\[ H^{(A)}(q) \simeq \sum_{N = N - \Lambda + 1}^N f_N(1 - f_{N + \Lambda}) H^{(A)}_N(q). \] \hspace{1cm} (19)

The photoconductivity given by Eqs. (16) - (19) as a function of the microwave frequency exhibits pronounced oscillations in which the photoconductivity sign alternates. At the resonances \( \Omega = \Lambda \Omega_c \), the photoconductivity \( \sigma_{ph} \) is zero. Near the resonances \( (s/\sqrt{2N_m L}), (T/\sqrt{2N_m}) < h[\Omega - \Lambda \Omega_c] < (s/L), (T/h) \), the value of the photoconductivity varies as a power of \( (\Omega - \Lambda \Omega_c) \):

\[ \sigma^{(A)}_{ph} \propto - \mathcal{J}_\Omega \frac{TL}{l} \frac{L^2(\Omega - \Lambda \Omega_c)^2}{s^2} < 0 \] \hspace{1cm} (20)

at \( \Omega - \Lambda \Omega_c < 0 \), and

\[ \sigma^{(A)}_{ph} \propto \mathcal{J}_\Omega \frac{TL}{l} \frac{L^2(\Omega - \Lambda \Omega_c)^2}{s^2} > 0 \] \hspace{1cm} (21)

when \( \Omega - \Lambda \Omega_c > 0 \).

Outside the resonances, i.e., in the ranges \( (s/L), (T/h) < |\Omega - \Lambda \Omega_c| < \Omega_c \), Eqs. (18) and (19) lead to

\[ \sigma^{(A)}_{ph} \propto \mathcal{J}_\Omega \exp \left[ \frac{h(\Omega - \Lambda \Omega_c)}{T} \right] \exp \left[ - \frac{l^2(\Omega - \Lambda \Omega_c)^2}{2s^2} \right] > 0 \] \hspace{1cm} (22)

at \( \Omega - \Lambda \Omega_c < 0 \), and

\[ \sigma^{(A)}_{ph} \propto - \exp \left[ - \frac{l^2(\Omega - \Lambda \Omega_c)^2}{2s^2} \right] < 0 \] \hspace{1cm} (23)

at \( \Omega - \Lambda \Omega_c > 0 \). In this frequency range, the photoconductivity is determined by two components: \( \sigma^{(A)}_{ph} \) (which is negative) and \( \sigma^{(A+1)}_{ph} \) (whose contribution is positive).

As a result, when \( \Lambda \Omega_c < \Omega < (\Lambda + 1) \Omega_c \), we obtain

\[ \sigma_{ph} \simeq \sigma^{(A)}_{ph} + \sigma^{(A+1)}_{ph}, \]

so that

\[ \sigma_{ph} \propto \mathcal{J}_\Omega \left\{ - \exp \left[ \frac{l^2(\Omega - \Lambda \Omega_c)^2}{2s^2} \right] \right\} \]

\[ + \exp \left[ \frac{h(\Omega - \Lambda \Omega_c - \Omega_c)}{T} \right] \exp \left[ - \frac{l^2(\Omega - \Lambda \Omega_c - \Omega_c)^2}{2s^2} \right] \} \]. \hspace{1cm} (25)

V. DISCUSSION

The negativity of \( \sigma_{ph} \) associated with photon-assisted acoustic scattering near the resonances at \( \Omega \lesssim \Lambda \Omega_c \) and in the ranges \( \Lambda \Omega_c < \Omega (\Lambda + 1) \Omega_c \) is attributed to the following. When \( \Omega \lesssim \Lambda \Omega_c \), the electron transitions between LL’s contributing to the absorption of acoustic phonons with the energies \( h \omega_q \simeq \Lambda \Omega_c - \Omega \) which are rather small. In this situation, the probability of the photon-assisted phonon absorption decreases with decreasing phonon energy \( h \omega_q \propto q \) because of a decrease in the matrix elements of the scattering in question with decreasing \( q \) (see Fig. 1). Since the energies of phonons absorbed near the resonances are small, the distinction between \( N_q \) with sufficiently small and very close values of \( q \) is insignificant. Due to this, the rate of the absorption of acoustic phonons with \( h \omega_q \) slightly higher than \( h(\Lambda \Omega_c - \Omega) \) exceeds that of acoustic phonons with \( h \omega_q \lesssim h(\Lambda \Omega_c - \Omega) \). In the first case, the electron Larmor orbit center displacement \( \delta \xi = h(\Omega - \Lambda \Omega_c + \omega_q)/eE > 0 \) and therefore the change in the electron potential energy \( \delta \epsilon > 0 \) so that such an act of the electron scattering provides a negative contribution to the dissipative current, i.e., to ANC. In contrast, near the resonances but at \( \Omega \) slightly larger than \( \Lambda \Omega_c \), the scattering acts with \( \delta \xi < 0 \) dominate.

Sufficiently far from the resonances the energies of the emitted (absorbed) phonons are relatively large. The matrix elements of the scattering involving such phonons and the phonon number steeply decrease with increasing phonon energy (momentum) as shown in Fig. 1. In this situation, the electron Larmor orbit center displacements \( \delta \xi = h(\Omega - \Lambda \Omega_c - \omega_q)/eE > 0 \) corresponding to the emission of less energetic phonons prevail, that results in \( \sigma_{ph} < 0 \) if \( \Omega > \Lambda \Omega_c \) but \( \Omega \) is still not too close to \( (\Lambda + 1) \Omega_c \). When \( \Omega \) increases approaching to and passing the next resonance, the situation repeats leading to an oscillative dependence of \( \sigma_{ph} \) on \( \Omega \) if \( \Omega_c \) is kept constant or on \( \Omega_c \) at fixed \( \Omega \). The dependence of the photoconductivity on the frequency of microwave radiation calculated using Eqs. (16) - (19) for \( L \Omega_c/s = 10 \), \( l/L = 0.1 \), and different values of parameter \( b = (hs/TL) \) is shown in Fig. 1.
As can be drawn from the above formulas at low temperatures \( b \gg 1 \), when \( \Omega \) increases in the range \( \Delta \Omega_c \leq \Omega \leq (\Lambda + 1) \Omega_c \), \( \sigma_{ph} \) sequentially turns to zero at \( \Omega = \Lambda \Omega_c \), \( \Omega = \Delta \Omega_c + \delta_0 \), and \( \Omega = \Lambda \Omega_c + \Delta_0 \). The photoconductivity becomes either small in the range \( \Delta \Omega_c + \Delta_0 < \Omega \leq (\Lambda + 1) \Omega_c \). The quantity \( \delta_0 \) can be estimated as \( \delta_0 \approx \frac{2s}{L} \). As follows from Eq. (25), \( \Delta_0 = \Omega_c \left[ 1 - \left( \frac{1}{2(1 + \gamma)} \right) \right] \), (27)

where \( \gamma = \left( \hbar s / T L \right) \left( s / \Omega \right) \). One can see from Eq. (27) that \( \delta_0 / \Omega_c > 1/2 \). The value \( \Delta_0 \) increases with decreasing temperature. At sufficiently low temperature \( T \ll \hbar s / L \), one obtains \( \delta_0 / \Omega_c \ll 1 \). For \( s = 3 \times 10^5 \text{ cm/s} \), \( l = (5 - 10) \times 10^{-7} \text{ cm} \), \( H = 2 \text{ kG} \), and \( T = 1 \text{ K} \), Eqs. (26) and (27) yield \( \delta_0 / \Omega_c \approx 1/5 \) and \( \Delta_0 / \Omega_c \approx 3/5 - 3/4 \). Worth noting the ranges where the calculated photoconductivity becomes negative approximately correspond to those with zero-resistance [1, 2]. One needs to point out that the behavior of the photoconductivity near the resonances can be markedly affected by the photon-assisted impurity scattering mechanisms and those associated with other types of the 2DES disorder [2, 3, 4]. The combined contribution of the photon-assisted impurity and acoustic phonon scattering mechanisms can result in the formation of the ANC ranges from \( \Omega = \Lambda \Omega_c \) to \( \Omega = \Delta \Omega_c + \Delta_0 \).

The net dissipative conductivity and its sign are determined by both \( \sigma_{dark} \) and \( \sigma_{ph} \). Different scattering mechanisms can markedly contribute to the dark component. However, the expression for \( \sigma_{dark} \) given by Eq. (15) comprises a small exponential factor \( \exp(-\hbar \Omega_c / T) \). Due to this factor, the photoconductivity associated with the electron-phonon scattering accompanied by the absorption of microwave photons can dominate at rather small values of \( J_\Omega \). As follows from Eq. (4), the photocurrent becomes more sensitive to microwave radiation near the first (cyclotron) resonance owing to a resonant increase in \( J_\Omega \).

Both the dark conductivity and photoconductivity depend to some extent on \( |V_{q l}|^2 \). In our calculation we assumed that for the piezoelectric acoustic scattering \( |V_{q l}|^2 \propto q^{-1} \exp(-l^2 q_z^2 / 2) \). One can also use another approximation \( |V_{q l}|^2 \propto q^{-1}(1 + l^2 q_z^2)^{-2} \) which corresponds to the wave functions of 2D electrons proportional to \( \exp(-|z|/l) \). However, such a change in \( |V_{q l}|^2 \) does not significantly affect the obtained results.

VI. CONCLUSION

We have calculated the dissipative component of the dc conductivity tensor of a 2DES in the transverse magnetic field and irradiated with microwaves. We have demonstrated that the electron transitions between the Landau levels stimulated by the absorption of microwave photons accompanied by the emission of acoustic (piezoelectric) phonons can result in the absolute negative conductivity in rather wide ranges of the resonance detuning \( \Omega - \Lambda \Omega_c \). Thus, the "acoustic" mechanism of the absolute negative conductivity can contribute to the formation of the zero-resistance states.

Acknowledgments

One of the authors (V.R.) is grateful to V. Volkov for bringing Refs. 1 and 2 to his attention and valuable discussions, and R. Suris and I. Aleiner for stimulating comments. The authors thank A. Satou for numerical calculations.

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