Wireless Secrecy Under Multivariate Correlated Nakagami-$m$ Fading

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This work was supported by the Australia Research Council under Discovery Project DP200100057 and Project DP190100559.

ABSTRACT Current wireless secrecy research in the literature has mainly been performed for one wiretapper under correlated fading. In this paper, a new wireless secrecy framework for multiple wiretappers under multivariate exponentially-correlated (exp.c.) Nakagami-$m$ fading is proposed. Using the distribution of multivariate exp.c. Nakagami-$m$ fading, new, exact, and compact expressions for the ergodic secrecy capacity, and secrecy outage probability (SOP) under multiple wiretappers are obtained for an integer fading parameter $m$. A secrecy analysis is also performed for the first time in this paper using an adaptive on/off transmission encoder under multivariate exp.c. Nakagami-$m$ fading. A secrecy analysis with three wiretappers under quadrivariate exp.c. Nakagami-$m$ fading is also given, which shows the effectiveness of the new framework. Simulation results are shown to exactly match theoretical predictions.

INDEX TERMS Physical layer security, multivariate correlated Nakagami-$m$ fading, wiretapping channels.

I. INTRODUCTION

Wireless secrecy has been one of the most popular research topics in recent years. Starting from the wiretap model proposed in [1], secrecy research has now included mathematical modelling of channel correlation, system coherence [2], and end-to-end system performance [3].

A. SURVEY OF CURRENT LITERATURE

Information security for wireless networks has become important in recent years [4]–[7]. With rapid developments of device-to-device, peer-to-peer communications, and ultra-densification [8], [9] under Fifth-Generation (5G) wireless networks, correlation between the main transmission channel, and eavesdropper channel(s) is practically likely because of their close proximity. Research on wireless secrecy has been very active with findings on several practical scenarios [10]–[24], in which (i) cooperative relays, and (ii) wireless secrecy for multi-user systems have been reported.

Wireless secrecy under spatial correlation, and imperfect channel state information (CSI) has also been studied in recent years. Specifically, the ergodic secrecy capacity (ESC), and secrecy outage probability (SOP) have been reported in the literature under correlated Rayleigh fading [2], [25], [26]. To quantitatively generalise the effects of spatial correlation, the joint signal-to-noise ratio (SNR) probability density function (pdf) of multiple correlated channels, i.e. not limited to only two correlated channels, must be obtained, which theoretically links wireless secrecy research to fundamental correlated fading research. However, it has notoriously been known that theoretical findings for correlated fading appear very scattered, and scarce, which significantly hinders its (i) progress, and (ii) applications to other branches of wireless communications such as wireless secrecy.

It is worthy to note that there are predominantly two correlation models currently available in the open literature (i) equally-correlated (equ.c.), and (ii) exponentially-correlated (exp.c.). Specifically, for pdfs of bivariate correlated fading, these two correlation models are mathematically identical, which makes the pdfs of bivariate correlated fading desirable to obtain closed-form expressions for SOP, and ESC. Therefore, wireless secrecy has commonly been examined for one wiretap channel under correlated Rayleigh fading [2], [4], [25] for simplification purposes. However, for higher pdf orders such as trivariate, quadrivariate, and multivariate, these two correlation models are mathematically different, and available findings are selectively available under specific fading environments. For example,
for the exp.c. Rician fading, the pdfs of trivariate are available in the open literature [27]–[29], suggesting that additional research is required to explore their potential. For the exp.c. Nakagami-m fading, the pdf of multivariate is available as given in [30, Eq. (3)], which makes Nakagami-m a well-equipped correlated fading environment for thorough studies on wireless secrecy under the influence of multiple wiretappers. It is also noted that the finding given in [30, Eq. (3)] is currently one of the most-advanced pdfs of multivariate exp.c. Nakagami-m fading for single-input-multiple-output (SIMO) systems, but not for multiple-input-multiple-output (MIMO) systems, suggesting that further improvement can be made.

B. RATIONALE
It is noted that (i) wireless secrecy has not been performed for more than one wiretap channel under correlated Nakagami-m fading, albeit extensive research efforts have been spent [2], [4], [17]–[23], [25]. This bottlenecks the current wireless secrecy research to only one wiretap channel under correlated fading. In addition, with the ever increasing demand for smart mobile devices over ultra-dense wireless networks, multiple correlated wiretappers targeting one Destination practically appears possible; (ii) the availability of the distribution of multivariate exp.c. Nakagami-m fading given in [30, Eq. (3)] for physical layer security (PLS),1 and (iii) Even though analyses under two correlated Rayleigh channels have been given in [2], those under multiple correlated Nakagami-m branches are not yet available in the literature. The modelling of p wiretappers, single-antenna Source, and Destination resembles the SISOME (single-input-Source-single-output-Destination-multiple-eavesdropper) outlined in [32], except that the proposed analyses are valid under multivariate exp.c. Nakagami-m fading, which is the most advanced in the existing literature, and thus generalising other findings reported in [2], [31]. The inclusion of channel correlation makes the proposed findings practical as they include the case of independent and identically distributed (i.i.d.) findings for $\rho = 0$.

C. CONTRIBUTION
Having learnt the latest correlated fading developments for wireless secrecy, to the best of the author’s knowledge, results have not been generalised for multiple wiretap channels under multivariate exp.c. Nakagami-m fading environments. This paper fills the identified knowledge gaps by (i) proposing an information theoretic framework for p wiretap channels under multivariate exp.c. Nakagami-m fading environments, and (ii) deriving exact and compact expressions for ESC and SOP under the quadrivariate exp.c. Nakagami-m fading with three wiretappers. The new expressions mathematically include the fading parameter $m$, assuming to be an integer in this paper, which can be varied to cross-verify the proposed findings against existing findings under dual correlated Rayleigh fading [2]. Findings for non-integer $m$ are currently being progressed, and will be reported in a future publication. Results under the encoder deployment are reported for multiple wiretappers, which generalise findings reported in [31] for integer $m$.

D. PAPER ORGANISATION
This paper is organised as follows. Section II briefly explains the fading severity on secrecy. A unified secrecy framework for p wiretappers with and without the encoder deployment is given in Section III, which governs the proposed findings in Section IV. By setting $p = 3$, Section IV obtains compact expressions for the ESC, and SOP under quadrivariate exp.c. Nakagami-m fading. Detailed simulation, and numerical results are given in Section V for two, three, and four correlated Nakagami-m branches, from which their matching is evident. Section VI concludes the main findings of this paper, and outlines possible future work.

Notation: The symbol $p \geq 1$ is the number of correlated wiretap channels, the number of branches is $n = p + 1$, $\lambda_1$ is the average SNR in the main channel whereas $\lambda_2, \ldots, \lambda_n$ are the average SNRs in the wiretap channels, $\frac{1}{1+w} < z_p, w = \sum_{i=1}^{n} w_i, I_\beta(.)$ is the $\beta$th-order modified Bessel function of the first kind as in [33, Eq. (2.1–120)], $m \geq \frac{1}{2}$ is a fading parameter of Nakagami-m distribution. The lower incomplete gamma function is $\Gamma(p_1, r) \triangleq \int_0^r t^{p_1-1} e^{-t} dt$, the gamma function is $\Gamma(p_1) \triangleq \int_0^{\infty} t^{p_1-1} e^{-t} dt, p_1 > 0$, the upper-incomplete gamma function is $\Gamma(p_1, r) \triangleq \int_r^{\infty} t^{p_1-1} e^{-t} dt$, and $0 \leq \rho < 1$ is the channel correlation coefficient.

II. BACKGROUND
A. FADING SEVERITY ON WIRELESS SECRECY
Typically, fading severity plays an important role in mathematically determining the secrecy analysis. Under different fading environments, the wiretapper can be correlated to the Destination, and the Source, posing challenging tasks of obtaining their joint fading distribution. In this paper, the p wiretappers are assumed to be correlated to the SD link under multivariate exp.c. Nakagami-m fading, whose joint distribution is studied under Section II-B. Depending on the joint fading distribution between the wiretappers, Source, and Destination, tractability can be accordingly obtained. Notoriously, under exp.c. Rician fading, intractability can result as explained in [34]. In addition, end-to-end analyses can mathematically lead to intractability if the joint fading distribution is more advanced than Rayleigh fading [3], because of the corresponding inverse two-dimensional Laplace transform.

The following Scenarios are studied in this paper:

Scenario 1: the Source, and Destination can partially approximate the wiretappers’ channel fading coefficients, i.e. all parties experience slow fading, hence the channel’s coherence times remain relatively constant over transmission of a codeword, under which the active wiretappers’ CSI can

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1The distribution of bivariate correlated Nakagami-m fading has been used for wireless secrecy research [31]. However, the distribution of n-variate exp.c. fading, mathematically being different from that of n-variate equ.c. fading, which explains why PLS research has been mostly limited under dual correlated fading in the literature [2], [25] for simplification purposes.
partially be estimated by the Source, and the wiretappers operate in a time-division multiple access (TDMA) wireless network. For this scenario, the ESC is employed to assess the system secrecy performance;

**Scenario 2:** The wiretappers’ channel fading coefficients are unknown, i.e. the wiretapper is passive, under which the SOP is employed to assess the system’s wireless secrecy performance. From the findings given in [31], collusion between the Source, and active users over the same wireless network can practically become important to estimate the wiretappers’ channel capacity.

The following remarks can be drawn:

**Remark 1:** It is noted that for comprehensiveness, both Scenarios will be considered in this paper. It is also assumed that the wiretappers do not have jamming capabilities so that tractability can be successfully achieved.

**Remark 2:** Collusion among the wiretappers is assumed possible with the deployment of a base wiretapper (BR). The deployment of maximal ratio combining (MRC) for the wiretappers can thus be initiated as shown in Section II-C.

**Remark 3:** In the current literature, under correlated fading environments, PLS analyses appear limited, from which the effects of multiple wiretappers have not been studied. This is partly because of intractability, and the popularity of [30, Eq. (3)] as explained in [35].

**B. THE DISTRIBUTION OF MULTIVARIATE EXP.C. NAKAGAMI-m FADING**

The joint distribution of p wiretappers, and the Destination is first studied in Lemma 1. Its corresponding infinite-summation expression is then obtained in Lemma 2, which is employed to derive the resultant SOP for the encoder deployment. It is noted that the nested infinite summations (i) fast-converge for a finite number of terms, which show their practicality, (ii) can be mathematically employed to obtain tractability, and (iii) the joint distribution of multivariate exp.c. Nakagami-m fading has recently been revised in [35], even though it was first reported in [30, Eq. (6)].

**Lemma 1:** For \( n = p + 1 \geq 2 \), the pdf of n-variate exp.c. Nakagami-m fading can be given by

\[
f_n = \frac{m^{m-1+n}(vw_{n-1})^{\frac{m-1}{2}}}{\lambda^n \Gamma(m)(1-\rho^2)^{\frac{m+1}{2}}}(vw_{n-1})^{-\frac{m+1}{2}}e^{-\frac{m}{2}(vw_{n-1})}, \tag{1}
\]

where \( g_v \equiv g_v(v, w_1, \ldots, w_{n-1}) \), \( v, w_1, \ldots, w_{n-1} \) represent the random variables (RVs) for the main-channel, and wiretapping-channel SNRs respectively.

**Proof:** Using [36, Eq. (2.1)] with the correct covariance matrix and transforming using [37, Eq. (2.3)], Lemma 1 results. It should be noted that (i) the main transmission channel SD is correlated with the \( n-1 \) wiretap channels \( SE_1, SE_2, \ldots, SE_{n-1} \) forming \( n \) correlated branches in Nakagami-m fading, and (ii) \( \rho_i > 0 \) and are finite.

**Lemma 2:** For \( \Sigma_p = \sum_{i=1}^{k_1} \ldots \sum_{i=p}^{k_p} \), and \( f_n \equiv f_n(v, w_1, w_2, \ldots, w_p) \), the infinite-summation pdf of n-variate exp.c. Nakagami-m fading can be given by

\[
f_n = A \Sigma_p B f_0, \tag{5}
\]

\[
f_0 = e^{-\frac{m}{2}(vw_{n-1})} T_w, \tag{6}
\]

\[
A = \frac{m_n^{m_n}}{\Gamma(m)n!}, \quad B = \frac{(m)_{\rho}^{2k}}{\lambda_n \Gamma(1-\rho^2)^{2k}}, \tag{7}
\]

\[
\lambda_A = \prod_{i=1}^{n} \lambda_i, \quad \lambda_B = \lambda_1^{k_1}, \lambda_2^{k_2}, \ldots, \lambda_n^{k_n} \tag{8}
\]

\[
k = \sum_{i=1}^{n-1} k_i, \quad \Gamma_n = \prod_{i=1}^{n-1} k_i! \Gamma(m + k_i), \tag{9}
\]

\[
T_w = \prod_{j=1}^{n-2} w_j^{m_j - k_j + k_{j+1}}. \tag{10}
\]

**Proof:** Using \( I_n(x) = \sum_{l=0}^{\infty} \frac{(x/2)^{n+l}}{l! \Gamma(m + l)} \), \( S_w \) can be rewritten as

\[
S_w = \sum_{l=0}^{\infty} \left( \frac{m^2 \rho^2 w_{n-2}^{m-1+l}}{\lambda_1 \lambda_2 (1-\rho^2)^2} \right) \frac{m^{m-1+l}}{l! \Gamma(m + l)} \times \frac{1}{\lambda_2 \lambda_3 (1-\rho^2)^2 \Gamma(m + l_1)} \times \frac{1}{\lambda_3 \lambda_4 (1-\rho^2)^2 \Gamma(m + l_2)} \times \cdots \times \frac{1}{\lambda_{n-2} \lambda_n (1-\rho^2)^2 \Gamma(m + l_{n-2})} \tag{11}
\]

Using (1), \( f_n \) can be rewritten as

\[
f_n = \frac{m^{m-1+n}(vw_{n-1})}{\lambda^{m} \Gamma(m)(1-\rho^2)^{\frac{m+1}{2}}}(vw_{n-1})^{-\frac{m+1}{2}}e^{-\frac{m}{2}(vw_{n-1})}, \tag{12}
\]

\[
\lambda_A = \prod_{i=1}^{n} \lambda_i, \quad \lambda_B = \lambda_1^{k_1}, \lambda_2^{k_2}, \ldots, \lambda_n^{k_n}, \tag{13}
\]

\[
k = \sum_{i=1}^{n-1} k_i, \quad \Gamma_n = \prod_{i=1}^{n-1} k_i! \Gamma(m + k_i), \tag{14}
\]

\[
\frac{1}{\lambda_2 \lambda_3 (1-\rho^2)^2 \Gamma(m + l_1)} \times \frac{1}{\lambda_3 \lambda_4 (1-\rho^2)^2 \Gamma(m + l_2)} \times \cdots \times \frac{1}{\lambda_{n-2} \lambda_n (1-\rho^2)^2 \Gamma(m + l_{n-2})} \times \frac{1}{l! \Gamma(m + l)} \tag{15}
\]

\[
\frac{1}{l! \Gamma(m + l)} \tag{16}
\]

\[
\frac{1}{l! \Gamma(m + l)} \tag{17}
\]

\[
\frac{1}{l! \Gamma(m + l)} \tag{18}
\]

\[
\frac{1}{l! \Gamma(m + l)} \tag{19}
\]

\[
\frac{1}{l! \Gamma(m + l)} \tag{20}
\]
Remark 4: Lemmas 1–2 give two alternative but equivalent expressions for the distribution of \( n \)-variate exp.c. Nakagami-\( m \) fading, which both can be employed for secrecy computation as shown in Section III. It is noted that simplified expressions can be readily obtained using Lemma 2, barring the presence of nested infinite summations, which however fast-converges for a finite number of terms, showing their practicality. Lemma 1 gives the distribution as a product of multiple \( I_{m-1}(\cdot) \), which can mathematically result in unsimplification, albeit their compact form.

Remark 5: The fading parameter \( m \) is assumed to be an integer in this paper for simplification purposes. As shown in [31], an analysis for non-integer \( m \) mathematically involves infinite summations for the incomplete gamma functions, which significantly complicates the secrecy analysis for \( p \) wiretappers. In addition, intractability, and extensive computational burden can occur, which makes the analysis for non-integer \( m \) infeasible under multivariate exp.c. Nakagami-\( m \) fading.

which completes the proof after some straightforward algebraic simplifications.

**Corollary 1:** The distribution \( f_\rho \) fast-converges for a finite number of terms with up to five-significant-digit accuracy.

**Proof:** The term \( \rho^{2k} \), \( k \to \infty \), \( \rho < 1 \) under the constant \( B \) fast-converges for about \( T = 20 \) terms. The condition on the channel correlation coefficient \( \rho \neq 1 \) ensures that the corresponding covariance matrix is positive-definite. For \( \mathcal{R}_p = \sum_{l=1}^{\infty} \), the truncation error \( E_r \) can be given by

\[
E_r = \mathcal{R}_p A B f_0 \to 0.
\] (13)

This means that

\[
E_r = \sum_{l=1}^{\infty} \frac{\left( \sum_{l=1}^{T} \frac{m^2 \rho^2 w_2 w_3}{\lambda_2 \lambda_3 (1 - \rho^2)^2} \frac{l-1}{l} \right)^{m-1} + l}{l! \Gamma(m + l)} \times \cdots \times \frac{1}{l_1! \Gamma(m + l_1)} \times \frac{1}{l_2! \Gamma(m + l_2)} \times \cdots \times \frac{1}{l_{n-2}! \Gamma(m + l_{n-2})}.
\] (14)

which approaches zero when \( T \to \infty \) because of the terms \( \rho^{m \left( \frac{m-1}{m} + T \right)} \), \( 0 \leq \rho < 1 \) as seen in (14), and hence completes the proof.

**C. SYSTEM MODEL**

Consider the system shown in Fig. 1, from which there are \( p \) correlated wiretap channels competing for transmit information in the main channel \( SD \) with channel fading coefficient \( h_s \), and average transmit SNR \( \lambda_1 \). Assuming that the wiretap channels \( w_1, \ldots, w_p \) (i) experience identical correlation coefficient \( \rho \) to the main channel, (ii) possess channel fading coefficients \( h_{w_1}, \ldots, h_{w_p} \) respectively, (iii) possess average transmit SNRs \( \lambda_2, \ldots, \lambda_n \) respectively, (iv) commit to eavesdropping via coordinated activities from the BR, (v) interference among the individual wiretappers does not occur, and (vi) communication between the BR, and the active wiretappers occurs over a time-division-multiple-access (TDMA) system.

It is noted that in the current literature under correlated fading environments for ultra-dense wireless networks, PLS research has been mostly performed for one wiretapper [3], [4], [25], [31] using the pdf of bivariate correlated Rayleigh fading. For the proposed \( p \) wiretappers under multivariate exp.c. Nakagami-\( m \) fading, it is further assumed that (i) the wiretappers collaborate to continuously eavesdrop the main channel transmission, (ii) the designated BR is equipped...
with a pth-order MRC to sum the individual wiretappers’, and its own instantaneous SNRs, giving \( w = \sum_{i=1}^{p} w_i \), (iii) the BR requires at least one wiretapper responding to its broadcast message to initiate wiretapping activities, (iv) uniform transmit power \( P_t \) for the wiretappers and the Source, (v) all channels experience quasi-static fading, (vi) fed-back CSI is not outdated for simplification purposes, (vii) active wiretappers are synchronised to the BR upon polling, (viii) data for all active wiretappers are infinitely backlogged, and (ix) the wiretappers collaborate using a broadcast protocol as follows [31], [38].

1) The BR broadcasts a message to poll for wiretapping activities from active wiretappers over the same wireless network;
2) Active wiretappers use the polling message to synchronise with the BR, and feed back their CSI via U uplink mini slots. If \( U < p \), a “first-in-first-serve” service is employed. It is noted that increasing the number of mini slots increases the overhead. In this paper, wiretappers are not contending for eavesdropping, however, their willingness to participate is important. Effects of overhead to PLS are not within the scope of this paper;
3) By responding to the broadcast message, a wiretapper is registered to wiretapping organised by the BR;
4) Upon finish polling, the BR informs the individual registered wiretappers of the identity of the (i) Source, (ii) the registered wiretappers, and (iii) Destination to avoid collusion, and interference;
5) The BR sums the individual fed-back wiretappers’ SNRs, and its own;
6) The BR reassesses eavesdropping activities at regular intervals. If after a certain amount of time, successful eavesdropping cannot be achieved, the BR calls off all activities, and starts polling again;
7) If none of the wiretappers responded to the BR, it keeps broadcasting at regular intervals until there is at least one wiretapper responding to its polling so that eavesdropping activities may recommence. For \( p \) responding wiretappers, the pdf of \( n \) exp.c. correlated Nakagami-\( m \) fading is employed for the proposed secrecy analysis;
8) If the number of responding wiretappers remains zero, the BR may decide to eavesdrop into the main transmission channel by itself. Theoretical developments for this scenario are reported in [2], [31].

Given a transmit signal \( x_s \) from the Source, the received signals at the Destination, and the wiretappers are \( y_D = \sqrt{P_t} h_{xs} + n_D, y_w_1 = \sqrt{P_t} h_{w_1 x_s} + n_{w_1}, \ldots, y_{wp} = \sqrt{P_t} h_{w_p x_s} + n_{wp} \), where \( P_t \) is the uniform transmit power, \( n_D, n_{w_1}, \ldots, n_{wp} \) are independent, zero-mean, \( \sigma^2 \)-variance complex Gaussian noise components at the Destination, wiretapper 1, …, wiretapper \( p \) respectively. The instantaneous SNRs for the main channel, and the \( 1, \ldots, p \) wiretap channels can be written as

\[
\begin{align*}
    c_v, i &= \frac{\ln(1 + v)}{\ln 2}, \\
    c_{w_i}, i &= \frac{\ln(1 + w_i)}{\ln 2}, \\ \\
    c_{wp, i} &= \frac{\ln(1 + w_p)}{\ln 2}.
\end{align*}
\]

where the subscript \( i \) indicates instantaneous quantities. At any instance, the instantaneous ESC with respect to each of the wiretappers can be given by

\[
\begin{align*}
    c_{w_1, s, i} &= \frac{1}{2} \left( \frac{\ln(1 + v) - \ln(1 + w)}{\ln 2} \right)_{w_i}, \\
    c_{wp, s, i} &= \frac{1}{2} \left( \frac{\ln(1 + v) - \ln(1 + w)}{\ln 2} \right)_{w_p}.
\end{align*}
\]

respectively. The instantaneous capacity of the main channel, and the \( 1, \ldots, p \) wiretap channels can be written as

\[
\begin{align*}
    c_v, i &= \frac{\ln(1 + v)}{\ln 2}, \\
    c_{w_i, i} &= \frac{\ln(1 + w_i)}{\ln 2}, \\ \\
    c_{wp, i} &= \frac{\ln(1 + w_p)}{\ln 2}.
\end{align*}
\]

III. SECRECY FRAMEWORK

A. WITHOUT EMPLOYING THE ENCODER

Using the distribution of \( n \)-variate exp.c. Nakagami-\( m \) fading given in Lemma 2, the ESC \( C_{ns} \) and \( E_n \) can be obtained. It is stressed that the \( C_{ns} \) and \( E_n \) are first proposed in this paper, and they can be generalised to obtain findings for any \( p \) wiretappers.

Lemma 3: The SOP \( E_n \) for \( p \) wiretappers under multivariate exp.c. Nakagami-\( m \) fading for Scenario 2 can be given by

\[
\begin{align*}
    E_n &= \left( 1 - \Pi(p) \right) \left[ \int_{0}^{\infty} f_{\eta} dv \right]_{ov}^{p}, \\
    \Pi(p) &= \left[ \int_{0}^{\infty} \cdots \int_{0}^{\infty} \right]_{p-fold}.
\end{align*}
\]

where \( \eta \) is a threshold rate, \( \phi = z_p(1 + w) - 1 \), \( \theta_w = d w_1 \cdots d w_p v a r \rho h_{w}, \) and \( z_p = e^{\gamma \eta \ln 2}. \)

Proof: Outage occurs when the instantaneous secrecy capacity between the main channel, and the wiretapper channels falls below a threshold capacity

\[
\begin{align*}
    E_n &= \Pr \left\{ [\ln(1 + v) - \ln(1 + w)]_{w_1} < \eta \ln 2, \ldots, \\
    & \times [\ln(1 + v) - \ln(1 + w)]_{w_p} < \eta \ln 2 \right\} \\
    &= \Pr \left\{ 1 + v < z_1 \right\} \times \cdots \times \Pr \left\{ 1 + v < z_p \right\} \\
    &= (\Pr \left\{ v < \phi \right\})^p,
\end{align*}
\]

where each realisation of the instantaneous secrecy capacity for each wiretapper is independent. It is noted that the limit
\( \phi \) on \( v \) at each wiretapper remains unchanged as the wiretappers’ total instantaneous SNRs \( w \) is employed. Using the joint SNR pdf of the wiretap channels, and \( f_n \), we arrive at

\[
\mathcal{E}_n = \left( 1 - \Pi_1(p) \int_{\phi}^{\infty} f_n dv \right)^p \tag{22}
\]

which completes the proof.

**Remark 6:** It is noted that (i) the parameter \( \phi \) under Lemma 3 limits the influence of \( v \), thus, as \( p \) is increased, so is \( w \) which makes the \( n \)-fold integration fast approaching 0, so that \( \mathcal{E}_n \to 1 \), and (ii) SOP computation does not mathematically involve the logarithmic function, but employing the upper limit on \( v \) as shown in \( \phi \). Thus, SOP computation is significantly dependent on the summation of individual wiretappers’ SNRs in \( w \). On the other hand, the conditioning of the individual wiretapper secrecy capacity mathematically involves the logarithmic function and \( f_n \) as shown in (16)–(31), increasing the secrecy capacity computational burden.

**Theorem 1:** The ESC for \( p \) wiretappers under multivariate exp.c. Nakagami-\( m \) fading for Scenario 1 can be given by

\[
C_{ns} = \min \left\{ \max[c_v - c_{w_1}, 0], \ldots, \max[c_v - c_{w_p}, 0] \right\} \tag{23}
\]

\[
c_v = \Pi_v(n) \log_2(1 + v) f_n dv, \tag{24}
\]

\[
c_{w_1} = \Pi_{w_1}(n) \log_2(1 + w_1) f_n dv, \tag{25}
\]

\[
c_{w_p} = \Pi_{w_p}(n) \log_2(1 + w_p) f_n dv, \quad \Pi_d = dv dw_1 \cdots dw_p, \tag{26}
\]

\[
\Pi_v(n) = \int_0^v \cdots \int_0^v \int_0^\infty, \quad \Pi_w(n) = \int_0^\infty \cdots \int_0^\infty \int_0^w. \tag{27}
\]

**Proof:** Given the channel fading coefficients for the wiretappers \( w_1, \ldots, w_p \) as \( h_{w_1}, \ldots, h_{w_p} \), respectively, the individual instantaneous channel capacity with respect to each wiretap channel can be rewritten as

\[
c_w|w_1 = \log_2 \left( 1 + |h_{w_1}|^2 \sigma_w^2 \right), \quad \ldots, \quad c_w|w_p = \log_2 \left( 1 + |h_{w_p}|^2 \sigma_w^2 \right).
\]

Conditioning \( c_w|w_1, \ldots, c_w|w_p \) using \( f(v, w_1, \ldots, w_p) \), and limiting the SNRs in the wiretap channels to \( v \) while unlimiting the SNR in the main channel, i.e. \( \phi \), the limits for integrals with respect to \( w_1, \ldots, w_p \) are \( [0, v] \) and those for the integral with respect to \( v \) are \( [0, \infty) \), we obtain the main channel average capacity as

\[
c_v = \Pi(p) \left[ \int_v^\infty \log_2(1 + v) f_n dv \right] q_w, \tag{28}
\]

\[
\Pi(p) = \int_v^\infty \cdots \int_v^\infty, \tag{29}
\]

where the integration order is for \( v, w_1, \ldots, w_p \), from the inner-most to the outer-most respectively, and \( v \) is given in (15). Similarly, using the limits \( [w, \infty) \) for the integral with respect to \( v \), and \( [0, \infty) \) for integrals with respect to \( w_1, \ldots, w_p \), the average capacity for the individual wiretappers \( w_1, \ldots, w_p \) defined in (15) can be respectively given by

\[
c_{w_1} = \Pi_1(p) \left[ \int_w^\infty \log_2(1 + w_1) f_n dv \right] q_w, \ldots, \tag{30}
\]

\[
c_{w_p} = \Pi_1(p) \left[ \int_w^\infty \log_2(1 + w_p) f_n dv \right] q_w, \tag{31}
\]

which completes the proof.

**Remark 7:** It is noted that (i) integration with respect to \( v \) is performed only once, whereas there are \( p \)-fold integrations with respect to \( w_1, \ldots, w_p \) for the \( p \)-th order secrecy capacity, (ii) there are \( n \)-fold integration for the ESC computation, thus, as \( p \) is increased, so is the computational burden, and (iii) finding \( f_n \) has been a long-standing problem in wireless communications because tractability is typically difficult to obtain as the number of correlated channels increases.

**Lemma 4:** Using Lemma 2, the infinite-summation secrecy capacity can be given by

\[
C_n = \frac{A \Sigma_p B}{2 \ln 2} \min \{ \max[c_v - c_{w_1}, 0], \ldots, \max[c_v - c_{w_p}, 0] \}. \tag{32}
\]

\[
\mu_i = \begin{cases} 
  m + k_{n-1}, & n = 2, i = n - 1, \\
  m + k_i + k_{i+1}, & n > 2, 1 \leq i < n - 1,
\end{cases} \tag{33}
\]

where \( c_v, c_{w_1}, \ldots, c_p \) are given in below, \( A, B \) are given in (7).

**Proof:** The quantities \( C_v, C_{w_{n-1}} \) are mathematically required.

**COMPUTATION OF \( C_v \)**

We have

\[
\mathcal{I}_1 = \int_0^\infty \ln(1 + v) v^{m-1+k_1} e^{-a_1 v} dv \times \int_0^v w^{m-1+k_{n-1}} e^{-a_{n-1} w_{n-1}} dw_{n-1},
\]

\[
\times \int_0^v w^{m-1+k_{n-2}} e^{-a_{n-2} w_{n-2}} dw_{n-2},
\]

\[
\cdots \times \int_0^v w^{m-1+k_2} e^{-a_2 w_2} dw_2 \tag{34}
\]

\[
\mathcal{W}_n = \prod_{i=1}^{n-1} \left( \frac{\gamma(m_{n-1}, a_{n} v)}{\mu_{n-1} a_{n}^\gamma} \right). \tag{35}
\]

which appears difficult to be given in closed form for \( n > 2 \).

**COMPUTATION OF \( C_{w_{n-1}}, n > 2 \)**

We have

\[
\left( \sum_{i=1}^{n-1} w_i \right)^l
\]

\[
= \mathcal{W}_n w_1^{l-1} w_2^{l-2} \cdots w_{n-2}^{l-2} w_{n-1}. \tag{36}
\]

\[\vdots \mathcal{I}_{n-1}\]
under Lemma 4, we have

\[ V = \sum_{i=0}^{\infty} \left( 1 \right) \times \left( 1^p \right), \quad (43) \]

\[ \Omega = m + k_{n-2} + k_p + l_p - n, \quad n_1 = m + k_1 - 1. \quad (44) \]

Proof: Letting \( z = \frac{i(1+y)}{1+y} \), we have \( \phi = (1+w)z - 1 \),

\[ \phi' = z^1 \phi(1) = l_1 w_1^{-1} w_1^{-1} l_1^{-1} l_1^{-1} \ldots l_1^{-1} l_1^{-1} w_1^{n_1}. \quad (45) \]

Using (45), we obtain \( \mathcal{I} \)

\[ \mathcal{I} = \int_{0}^{\infty} w_{n-2}^{-a_1 w_{n-2}} dw_{n-2} \times \ldots \quad (46) \]

completing the proof.

Remark 8: It should be noted that Lemma 4 can be employed for effective ESC computation using truncated infinite summations, which fast-converge for a finite number of terms with up to five-significant-digit accuracy as shown in Corollary 1.

Corollary 2: For \( m = 1, n = 2 \), Theorem 1 becomes [2, Eq. (7)].

Proof: For \( m = 1, n = 2 \), using Lemma 2 and \( c_{w_{n-1}} \) under Lemma 4, we have

\[ c_{w_1} = \sum_{i=0}^{k_1} \sum_{j=0}^{l_1} \left( \frac{i}{l_1} \right) \times \left( \frac{j}{l_1} \right), \quad (39) \]

and \( \mathcal{E}_2(\lambda) = \int_{1}^{\infty} e^{-\mu \lambda} \lambda \mu d\lambda \) is the exponential integral. After performing algebraic simplifications completes the proof.

Remark 9: Corollary 2 thoroughly validates the proposed findings by mathematically proving the equivalence of Theorem 1, and the findings reported in [2]. In addition, it is straightforward to prove the equivalence of the proposed findings, and those given in [3] for \( n = 2 \), further cementing the correctness of this paper.

Theorem 2: Using Lemma 2, the infinite-summation expression for the SOP under \( p \) wiretapper can be obtained as

\[ E_n = \left( 1 - A \right) \sum_{k=0}^{\infty} B_{k} T_{n-1} \quad \text{Eq. (26)} \]

\[ T_{n-1} = \frac{1}{\lambda_{1}^{2}} \sum_{l=0}^{k_1} \left( \frac{l}{l_1} \right) \times \left( \frac{j}{l_1} \right), \quad (46) \]

completing the proof.

Remark 10: From Corollary 1, it is clear that \( E_n \) fast-converges for about 20 terms. Other factors such as \( e^{-a_1 \lambda_1} \lambda_2^2 \times \frac{1}{(a_1 + \lambda_2 \lambda_1) e^{-a_{2} \lambda_1 \lambda_2}} \) fasten the convergence process.

Corollary 3: For \( m = 1, n = 2 \), Theorem 2 reduces to [2, Eq. (26)].

Proof: For \( m = 1, n = 2, n_1 = k_1, \mathcal{E}_2 \) is given by

\[ \mathcal{E}_2 = 1 - A_1 \sum_{k=0}^{\infty} B_{k} T_{n-1} \quad \text{Eq. (26)} \]

\[ T_{n-1} = \frac{1}{\lambda_{1}^{2}} \sum_{l=0}^{k_1} \left( \frac{l}{l_1} \right) \times \left( \frac{j}{l_1} \right), \quad (46) \]

\[ A_1 = \frac{1}{\lambda_{1} \lambda_{2} (1 - \rho^2)}, \quad B_1 = \frac{\rho^{2k}}{\lambda_{1} \lambda_{2} (1 - \rho^2) 2k}, \quad (48) \]

\[ a_1 = \frac{1}{\lambda_{1} (1 - \rho^2)}, \quad a_2 = \frac{1}{\lambda_{2} (1 - \rho^2)}, \quad (48) \]

\[ g_{e,1} = a_1 v + a_2 w_1 \quad \text{Eq. (26)} \]

\[ k = k_1, \quad \Gamma_2 = k_1 \Gamma_1 (1 + k_1), \quad T_{n-1} = 1, \quad (50) \]

which is exactly identical to [2, Eq. (26)], completing the proof.

Remark 11: Together with Corollary 2, Corollary 3 validates the proposed work by exactly re-deriving the findings reported in [2] for \( m = 1, n = 2 \).
Corollary 4: The SOP when the threshold \( z \) approaches infinity can be given by
\[
\varepsilon_{n}^{\rightarrow \infty} \approx 1, \quad \forall a_{i} > 0.
\]

Proof: For \( z \rightarrow \infty \), \( e^{\varepsilon_{n}} e^{-a_{i}z} \rightarrow 0 \), \( \varepsilon_{n}^{\rightarrow \infty} \approx 1 \). It is noted that because of the fast decaying rate of \( e^{-z} \), Corollary 4 is true for arbitrary values of \( a_{i} \).

Corollary 5: For \( a_{1} \rightarrow 0 \), \( n, z \neq \infty \), the SOP reduces to the following
\[
\varepsilon_{n,z}^{\rightarrow 0} = 1 - A \sum_{a_{i}} B I_{sa},
\]
\[
I_{sa} = n! \sum_{l_{i}=0}^{m+k_{l}} \frac{\nu_{a_{i}}^{l_{i}}(-1)^{m+k_{l}-l_{i}}}{(m+k_{l})!} \frac{\Gamma(m+k_{l}+l_{i})}{a_{n}^{|m+k_{l}+l_{i}|}} S_{n}^{-1}.
\]

Proof: For \( a_{1} \rightarrow 0 \), \( z \) finite, we have \( l = m+k_{l} \) and \( a_{1}z \approx 0 \), the index \( l \) becomes a constant, therefore we obtain Corollary 5. For \( a_{1} \rightarrow 0 \), \( z \rightarrow \infty \), \( \varepsilon_{n,z}^{\rightarrow 0} \) is given in Corollary 4.

Remark 12: The constant \( a_{i} \) is inversely proportional to the average transmit fading gains of the main channel, and wiretapping channel. As such, keep increasing the average transmit SNRs, i.e. increasing the amount of fading (AoF), can theoretically lower the SOP, which has been reported in [31], [34].

Corollary 6: The ESC and SOP for \( n \)-variate exp.c. Rayleigh fading can be obtained from Theorems 1 and 2 respectively for \( m = 1 \).

Proof: Substituting \( m = 1 \) into Theorems 1 and 2, \( e_{Rayleigh} \) \( e_{Rayleigh} \) can be readily obtained.

Remark 13: The expressions given in Theorem 1, and Lemma 4 represent two alternative methods for the ESC. Using Lemma 4, it is mathematically tractable to obtain the ESC, and SOP as \( n \) is increased. The infinite summations do converge for a finite number of terms, which shows the practicality of the proposed research.

B. Employing the Encoder

To further improve secrecy performance of the proposed system, the on/off transmission encoder is employed under multivariate exp.c. Nakagami-\( m \) fading to ensure that transmission between the Source, and Destination not only secure but successful. To achieve this goal, the Destination regularly feeds back its CSI to the Source. An SNR threshold \( \mu \) is then numerically set by the encoder to be as close to the Destination’s channel capacity to practically avoid transmission errors. Since the secrecy rate is first set by the Source, the availability of \( \mu \) (i) allows the Source to assess its state, (ii) chooses an appropriate value for the secrecy rate the higher the secrecy rate, the more insecure the transmission channel (iii) hence helps strengthen the system’s security [31]. To the best of the author’s knowledge, the proposed analysis is novel, and has not yet been reported in the literature.

Lemma 5: The SOP under \( n \)-variate exp.c. Nakagami-\( m \) fading employing the encoder can be given by
\[
P_{o} = \Sigma_{p} \frac{\gamma_{1}}{\gamma_{2}},
\]
\[
\gamma_{1} = \Gamma \left( \frac{m+k_{n-1}+a_{i}(\frac{m+1}{\chi}-1)}{a_{n}^{m+k_{n-1}}} \right) \prod_{i=1}^{n-2} \Gamma \left( \frac{m+k_{i}+a_{i}(\frac{m+1}{\chi}-1)}{a_{i+1}^{m+k_{i}+k_{i+1}}} \right),
\]
(54)
\[
\gamma_{2} = \Gamma \left( \frac{m+k_{n-1}+a_{i}(\frac{m+1}{\chi}-1)}{a_{i+1}^{m+k_{i}+k_{i+1}}} \right),
\]
(55)
where \( a_{i}, \Sigma_{p} \) are given in (2), and Lemma 2 respectively.

Proof: Under multivariate exp.c. Nakagami-\( m \) fading, the findings in [40] can be modified
\[
\gamma = \frac{Pr\{\mu < v < 2^{R_{c}(1+w)-1}\}}{Pr\{v > \mu, 0 \leq w < \infty\}} = \frac{\gamma_{1}}{\gamma_{2}},
\]
(56)
\[
\gamma_{1} = \int_{\mu}^{\infty} \cdots \int_{\mu}^{\infty} \int_{\mu}^{\infty} f_{n} dv \phi_{1},
\]
(57)
\[
\gamma_{2} = \Pi_{1} \int_{\mu}^{\infty} f_{n} dv \phi_{2},
\]
(58)
where \( \xi = 2^{R_{c}}, R_{s} \) is the secrecy rate, \( f_{n} \) is given in (1), and the term \( \gamma_{2} \) is to ensure that transmission only occurs when the instantaneous SNR in the main transmission channel \( v \) is larger than a SNR threshold \( \mu \) for any value of \( 0 \leq w < \infty \).

Using \( \Sigma_{p} = \sum_{k_{1}, \ldots, k_{p}=0}^{m} \sum_{i} \phi_{1}, \phi_{2} \) can be given by
\[
\Phi_{1} = \Sigma_{p} \cdot A \left( \int_{0}^{\infty} f_{0} dv - \int_{0}^{\xi \mu} f_{0} dv - \int_{\xi \mu}^{\infty} f_{0} dv \right),
\]
(59)
\[
\Phi_{2} = \Sigma_{p} \cdot A \left( I_{3} - I_{4} \right),
\]
(60)
\[
I_{3} = g_{w} \Gamma \left( \frac{m+k_{1}}{a_{1}^{m+k_{1}}} \right), \quad I_{4} = g_{w} \frac{\gamma(m+k_{1}, a_{1} \mu)}{a_{1}^{m+k_{1}}},
\]
(61)
\[
I_{5} = g_{w} \Gamma \left( \frac{m+k_{1}+a_{1}(\chi(1+w)-1)}{a_{1}^{m+k_{1}}} \right),
\]
(62)
\[
g_{w} = e^{-g_{w}} T_{w} w_{m-1+k_{n-1}},
\]
(63)
which completes the proof after algebraic manipulations.

Remark 14: Following the procedures shown in Lemma 4, the infinite-summation expression for \( P_{o} \) can be similarly obtained. The encoder deployment relies on the feedback link between the Source, and Destination, which can be practically achieved without extensive overhead. In addition, the feedback link deployment is to achieve possible collusion.
between the Source, and other active users over the same wireless network under Scenario 2.

**Remark 15:** The probability $\gamma_1$ ensures that the system's security by imposing that $\nu > \mu$, and simultaneously $\nu < 2^{\frac{K}{3}}(1 + \omega) - 1$, whereas the probability $\gamma_2$ represents the probability for successful transmission delivery by imposing $\nu > \mu$ for any $\omega$. By forming their ratio $\mathcal{P}_o = \frac{\gamma_1}{\gamma_2}$, it is theoretically possible to achieve both transmission security, and transmission success. The numerical settings for $R_s$ have been also recently proposed in [31], which gives a useful guidance to achieve this goal.

## IV. Secrecy Analysis for Four Correlated Channels, $P = 3$

### 1) SNR PDF of Quadrivariate Exp.c. Nakagami-m Fading

To compute $\mathcal{E}_4$, and ESC, one must obtain the pdf of quadrivariate exp.c. Nakagami-m fading. Using [37, Eq. (2.3)] to convert the pdf into its equivalent SNR version, we obtain the joint SNR pdf of quadrivariate exp.c. Nakagami-m fading as given in (64), after lengthy algebra, $f_4 \equiv f(\nu, w_1, w_2, w_3)$.

**Corollary 7:** The pdf of quadrivariate exp.c. Nakagami-m fading can be given by

$$f_4 = \frac{m^{3m_1}e^{-\frac{m_1}{\nu w_3}}}{\Gamma(m)(1 - \nu^2)^2} \left( \frac{m_2}{\nu^2 w_1} \right) I_{m-1} \left( \frac{m_2}{\nu^2 w_1} \right) \left( \frac{m_3}{\nu^2 w_2} \right) I_{m-1} \left( \frac{m_3}{\nu^2 w_2} \right) \left( \frac{m_4}{\nu^2 w_3} \right) I_{m-1} \left( \frac{m_4}{\nu^2 w_3} \right)$$

### 2) $C_{45}$ AND $\mathcal{E}_4$ Computation

**Corollary 9:** The ESC with three wiretappers under quadrivariate exp.c. Nakagami-m fading can be given by

$$C_4 = \frac{A}{2\ln 2} \sum_{k=1}^{m} \ln \left[ \frac{1}{\Gamma(m)} \right]$$

### Remark 17:** It is noted that secrecy analyses for one wiretapper can be found in (i) [31] under dual correlated Nakagami-m fading, and (ii) under dual correlated Rayleigh fading, from which (i) the findings reported in [31] generalise those given in [2] for $m = 1$, and (ii) the proposed findings generalise those in [2], [31] for $n = 2$.

**Corollary 8:** The infinite-summation pdf of quadrivariate exp.c. Nakagami-m fading can be given by

$$f_4 = \frac{\alpha_1 \beta_1 \gamma_1}{\Gamma(m)(1 - \nu^2)^2} \left( \frac{m_2}{\nu^2 w_1} \right) I_{m-1} \left( \frac{m_2}{\nu^2 w_1} \right) \left( \frac{m_3}{\nu^2 w_2} \right) I_{m-1} \left( \frac{m_3}{\nu^2 w_2} \right) \left( \frac{m_4}{\nu^2 w_3} \right) I_{m-1} \left( \frac{m_4}{\nu^2 w_3} \right)$$

### Remark 18:** Under i.i.d. Nakagami-m fading, the constants $A_4, B_4$ can be rewritten as

$$A_{4, \rho=0} = \frac{m^{4m}}{\Gamma(m)(1 - \nu^2)^2}, \quad B_{4, \rho=0} = \frac{1}{\Gamma(m)^2}$$

which can be employed for further secrecy computation.

Let $\mathcal{K} = \sum_{l=0}^{m+k-1} \sum_{l=0}^{\nu} \left( \frac{l}{l_1} \right) \frac{(m+k-l)}{(a_1 \nu^2)}$, we have

$$c_{w_1} = KG(1, m - 1 + k_1 + k_2 + l - l_1, a_1 + a_2) \cdot \Gamma(m + k_2 + k_3 + l - l_1, a_1 + a_3) \cdot \left( a_1 + a_3 \right)^{m+k+1}$$

**Proof:** Substituting $p = 3$, or $n = 4$ into Lemma 1, after simplification, completes results.

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\[ c_{w_2} = \frac{K}{(a_1 + a_2)^{m+k_1+k_2+l-i}} \]
\[ \times G(1, m-1+k_1+k_2+l-i, l_1) \]
\[ \times \frac{(a_1 + a_2)^{m+k_1+k_2+l-i}}{(a_1 + a_2)^{m+k_1+k_2+l-i}} \]
\[ \times \frac{\Gamma(m+k_1+k_2+l-i)}{\Gamma(m+2+k_1+k_2+l-i)}, \] (74)

\[ c_{w_3} = \frac{K}{(a_1 + a_2)^{m+k_1+k_2+l-i}} \]
\[ \times G(1, m-1+k_1+k_2+l-i, l_1) \]
\[ \times \frac{(a_1 + a_2)^{m+k_1+k_2+l-i}}{(a_1 + a_2)^{m+k_1+k_2+l-i}} \]
\[ \times \frac{\Gamma(m+k_1+k_2+l-i)}{\Gamma(m+2+k_1+k_2+l-i)}, \] (75)

**Proof:** Using \( f_3 \) into Theorem 1, after algebraic simplification completes the proof. \( \blacksquare \)

**Corollary 10:** The SOP with three wiretappers under quadrivariate exp.c. Nakagami-\( m \) fading can be given by

\[ E_4 = n! \sum_{n=0}^{m} \frac{V_4}{a_1^{m+k_1+l-i}} \]
\[ \times \frac{\Gamma(m+k_1+l-i)\Gamma(m+k_2+l-i)\Gamma(m+k_3+l-i)}{(a_1+1)^{m+k_1+k_2+k_3+l-i}} \]
\[ \times \frac{\Gamma(m+k_1+k_2+l-i)}{\Gamma(m+2+k_1+k_2+l-i)}, \] (76)

**Proof:** Letting \( \frac{1+v}{1+w} < z, w = w_1 + w_2 + w_3 \), thus

\[ \phi = z^{l} V_4 \frac{(-1)}{a_1} \]
\[ w_1^{l} w_2^{l} w_3^{l} \] (77)

Using (77), we obtain \( Z_5 \)

\[ I_5 = \int_0^{\infty} w_3^{m-1+k_3} e^{-a_3 w_3} dw_3 \]
\[ \times \int_0^{\infty} w_2^{m-1+k_2} e^{-a_2 w_2} dw_2 \]
\[ \times \int_0^{\infty} w_1^{m-1+k_1} e^{-a_2 w_1} dw_1 \]
\[ \times \int_0^{\infty} v^{m-1+k_1} e^{-av} dv, \] (78)

completing the proof. \( \blacksquare \)

**Corollary 11:** The SOP for large threshold \( z \) can be given by

\[ E_{4z \rightarrow z} \approx 1, \quad \forall a_2, a_3, a_4 > 0. \]

**Proof:** For \( z \rightarrow \infty, \phi = e^{-az} \rightarrow 0 \); \( E_{4z \rightarrow z} \approx 1 \).

It is noted that because of the fast decaying rate of \( e^{-az} \), Corollary 11 is true for arbitrary values of \( a_1, a_2, a_3, a_4 \). \( \blacksquare \)

**Corollary 12:** The SOP for \( a_1 \rightarrow 0 \) can be given by

\[ E_{4z \rightarrow z} = (1 - \delta A) z^{l} (B_5 Z_6)^3, \] (79)

\[ I_6 = \frac{(l_2-1)! (l_3-1)! (l_4-1)!}{l_2! l_3! l_4!} \sum_{i=0}^{l_2} \frac{V_4}{a_1^{m+k_1+k_2+l-i}} \]
\[ \times \frac{\Gamma(l+k_2+l_2-l_3)}{\Gamma(l+k_2+l_3)}, \quad l_2 = m+k_1, \] (80)

\[ \delta A = \frac{m^m}{\Gamma(m)(\lambda_2 \lambda_3 \lambda_4)^m (1 - \rho^2) \lambda_5}, \] (81)

\[ \delta B \] (82)

**Corollary 13:** The ESC under quadrivariate i.i.d. Nakagami-\( m \) fading can be given by

\[ c_{\rho=0} = \frac{A_4, \rho=0}{\sum_{l=0}^{m} \sum_{l=0}^{l} \sum_{l=0}^{l} (\frac{l_1}{l_2}) (\frac{l_1}{l_2}) (\frac{l_1}{l_2}) (\frac{l_1}{l_2}) \]

\[ \times \frac{\Gamma(l+k_1+l_2-l_1)}{\Gamma(l+k_2+l_3)}, \quad l_2 = m+k_1, \] (83)

\[ c_{\rho=0} = \frac{A_4, \rho=0}{\sum_{l=0}^{m} \sum_{l=0}^{l} \sum_{l=0}^{l} (\frac{l_1}{l_2}) (\frac{l_1}{l_2}) (\frac{l_1}{l_2}) (\frac{l_1}{l_2}) \]

\[ \times \frac{\Gamma(l+k_1+l_2-l_1)}{\Gamma(l+k_2+l_3)}, \quad l_2 = m+k_1, \] (84)

\[ c_{\rho=0} = \frac{A_4, \rho=0}{\sum_{l=0}^{m} \sum_{l=0}^{l} \sum_{l=0}^{l} (\frac{l_1}{l_2}) (\frac{l_1}{l_2}) (\frac{l_1}{l_2}) (\frac{l_1}{l_2}) \]

\[ \times \frac{\Gamma(l+k_1+l_2-l_1)}{\Gamma(l+k_2+l_3)}, \quad l_2 = m+k_1, \] (85)

\[ c_{\rho=0} = \frac{A_4, \rho=0}{\sum_{l=0}^{m} \sum_{l=0}^{l} \sum_{l=0}^{l} (\frac{l_1}{l_2}) (\frac{l_1}{l_2}) (\frac{l_1}{l_2}) (\frac{l_1}{l_2}) \]

\[ \times \frac{\Gamma(l+k_1+l_2-l_1)}{\Gamma(l+k_2+l_3)}, \quad l_2 = m+k_1, \] (86)

\[ A = \frac{1}{2 \ln 2} \sum_{l=0}^{m} \sum_{l=0}^{l} \sum_{l=0}^{l} (\frac{l_1}{l_2}) (\frac{l_1}{l_2}) (\frac{l_1}{l_2}) (\frac{l_1}{l_2}) \]

\[ \times \frac{\Gamma(l+k_1+l_2-l_1)}{\Gamma(l+k_2+l_3)}, \quad l_2 = m+k_1, \] (87)

where \( b_1, b_2, b_3, b_4 \) are given in (66), \( \kappa_1 = \mu_1, \rho=0, \kappa_2 = \mu_2, \rho=0, \mu_1 = m + k_1 + k_2, \mu_2 = m + k_2 + k_3, \nu = m + k_3. \)

**Proof:** Assigning \( \rho = 0 \) forces \( k_1 = k_2 = k_3 = 0 \), which completes the proof. \( \blacksquare \)

**V. DISCUSSION**

Using the generalised results in this paper, statistics of two, three, and four correlated Nakagami-\( m \) branches are obtained. From that, their corresponding SOP, and ESC are derived, and plotted in Figs. 2–8, from which numerical and simulation results are clearly matched, validating the proposed theoretical predictions.

**A. SOP**

1) **VERSUS \( \lambda_1, \lambda_2, \lambda_3, \lambda_4 \)**

Increasing average SNRs generally improves performance. Under two, three, and four correlated Nakagami-\( m \) branches, their corresponding SOP (i) increases as the wiretap average SNR \( \lambda_2, \lambda_3, \lambda_4 \) are increased, and (ii) decreases as the main transmission average SNR \( \lambda_1 \) is increased. Simulation results show that \( \lambda_1 \) is the most dominant factor that can be effectively used to lower the SOP, hence making main channel transmission more secure. It is noted that (i) keep increasing \( \lambda_1 \) lowers the SOP but also saturates it, which suggests that this approach, even though can be effective, may not be very efficient, (ii) the effectiveness of increasing
$\lambda_1$ can be lessened if the channel correlation is increased beyond $\rho = 0.5$ as seen from Fig. 2. (iii) with respect to $\lambda_1$, $E_4$ is significantly larger than $E_3$, which shows the severity of 3 wiretappers over 2 wiretappers. This interesting fact cannot clearly be seen if only one wiretapper is employed for PLS research. As the number of wiretappers $p$ is increased,
it appears that $E_n$ saturates to its peak, and keep increasing $\lambda_1$ does not help improve main channel transmission security. This shows the severity of coordinated eavesdropping, which deserves extra research efforts to avoid, prevent, and rectify for ultra-dense wireless networks.

2) VERSUS $z$

Lifting the threshold $z$, i.e. magnifying the total wiretap SNR so that it becomes much larger than the main transmission SNR, appears to be more severe than increasing the individual wiretappers SNRs as seen in Figs. 3, and 4 with the sharp increase in the SOP curves’ slope. Because the threshold $z$ is directly proportional to the total wiretappers SNRs, this means that the more individual wiretappers joining the eavesdropping activities, the less secure the main channel transmission. It is reminded that the BR is assumed to accept any eavesdropping registration from active wiretappers without imposing an SNR threshold on them. It is thus straightforward that coordinated eavesdropping can become very severe, and the vulnerability of wireless transmission is thus one of the key network design criteria for wireless networks.

3) VERSUS $\rho$

It clearly appears that the SOP is directly proportional to the number of wiretap channels. From Fig. 5, $E_4$ is not as significantly larger than $E_3$ compared to Fig. 2 for plotting the SOP against $\lambda_1$. It is also noted that $E_2$ is much smaller than $E_3, E_4$ but the sharp rise in the slope of $E_2$ curves as $\rho$ is increased means that, under the influence of channel correlation, (i) $E_3, E_4$ are more stable than $E_2$, (ii) wiretapper diversity can be employed to combat channel correlation, (iii) it can be suggested that $E_2$ saturates toward 1, and this stability appears to be strengthened as $p, \rho$ are increased. For $\rho = 0.8$, SOP approaches unity, which suggests that outage occurs very frequently if there are three or more wiretappers in the proximity of the main transmission channel. This phenomenon cannot be clearly seen when there is only one wiretapper [4], [25], which shows the necessity of the proposed work. It can be further suggested that the quadruple-wiretap scenario approaches the asymptotic SOP, which also means that the SOP fast-approaches its maximum as $p \geq 3$.

B. ESC

The ESC appears to be inversely proportional to the number of wiretap channels [39]. As such, there appears to be a trade-off between SOP, and ESC, i.e. increasing $\rho$ increases $E_n$, but lowers the ESC $C_{ns}$.

1) VERSUS $\lambda_1, \lambda_2, \lambda_3, \lambda_4$

From Fig. 6, increasing $\lambda_1$ decreases SOP, i.e. improves main channel’s secrecy, and decreases ESC. The sharp fall in $C_{4s}$ is in contrast to the gradual rise in $C_{2s}, C_{3s}$, which clearly verifies the trade-off between $E_n$, and $C_{ns}$. Keep increasing $\lambda_1$ does decrease the SOP but this process suffers from SOP saturation, and the sharp ESC decrease for $\rho \geq 3$. If $p < 3$, simulation results show that increasing $\lambda_1$ can be effective in combating eavesdropping by decreasing the SOP, and simultaneously increasing the corresponding ESC. The proposed analysis under multiple wiretappers is thus necessary to thoroughly deduce the trade-off between $E_n$, and $C_{ns}$.

From Fig. 7, increasing the individual wiretappers’ SNRs increases the ESC but simultaneously saturates it, i.e. wiretappers can freely increase their average SNRs but this process appears inefficient if the average SNR is beyond 10dB. It also appears clear that $C_{4s}$ is lower than $C_{3s}$, as the individual wiretappers’ SNRs are increased, which verifies the trade-off in ESC explained earlier. Under multiple wiretappers, increasing the channel correlation coefficient reduces the ESC, and increases the SOP. However, channel correlation becomes less severe as the number of wiretappers is increased.

2) VERSUS $\rho$

The ESC appears unstable for $\rho \geq 0.5$ mainly because of the unavoidable mathematical singularity at $\rho = 1$. It can be suggested that increasing $\rho$ decreases ESC as reported in [2], and as evidenced in the proposed findings in this paper. However, it is noted that increasing $\rho$ does not always degrade secrecy performance as noted in [4]. As $p$ is increased, increasing $\rho$ decreases the ESC. For $p \geq 3$, there appears to be a trade-off between the fading parameter $m$, and $\rho$ as evidenced in the cross-over point from Fig. 8, which is mainly caused by the change in the system’s amount of fading as explained in [3], [31], [34]. Optimisation for the ESC with respect to $\rho$ appears to be very difficult, i.e. tractability may not be obtained, and thus is out of the scope of this paper.

VI. CONCLUSION

A new wireless secrecy framework for multiple coordinated wiretappers under multivariate exp.c. Nakagami-\textit{m} fading has been proposed, and analysed for the first time in this paper. Active wiretappers have been coordinated by the base wiretapper using the proposed broadcast protocol to
avoid interference. Using the new framework, and the distribution of multivariate exp.c. Nakagami-m fading, ESC, and SOP for three and four correlated Nakagami-m branches have been derived as special cases in this paper for integer fading parameter m. For one wiretapper, the new findings exactly reduce to existing results under dual correlated Rayleigh fading, validating the proposed work. Matching of simulation, and numerical results have been shown, which verifies the proposed findings. The new findings can be useful for device-to-device wireless security studies under multiple correlated Nakagami-m fading without a line-of-sight path, which is a practical scenario for Fifth-Generation wireless networks. Further work on wireless security for (i) wiretapper interference, and (ii) correlated Nakagami-m fading employing amplify-and-forward relays will be reported in a future publication. Findings for non-integer m are currently being progressed, and will also be reported in a future publication.

ACKNOWLEDGMENT

The authors are grateful to the Associate Editor, and the anonymous reviewers whose comments improved the earlier versions of this paper.

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