A Consideration about the Concept of Effective Thermal Conductivity in Continuous Casting

1. Introduction

In recent years, extensive mathematical modelling has been carried out to uncover various thermal phenomena encountered in continuous casting of steel. These for example include, prediction of solidified shell thickness and liquid pool profiles; numerical estimation of cast (e.g., billet or slab) surface temperature, computation of thermal fields in the CONCAST moulds, prediction of solute segregation and inclusion trajectories in solidifying billets and so on. So far, to represent these and associated thermal phenomena in terms of an appropriate mathematical framework, two fundamentally different concepts have been applied. These have accordingly resulted into two distinct groups of mathematical models, viz.,

(i) models based on the concept that convective and turbulent transport of heat within a solidifying casting can be represented adequately, if the central core of liquid metal is treated like a pseudo-solid, having a relatively large thermal conductivity (e.g., approximately seven times the molecular conductivity), and

(ii) models embodying the exact influences of liquid steel flows on convective and turbulent transport of heat via a set of appropriate conservation equations (e.g., turbulent Navier–Stokes equation together with an appropriate energy conservation equation).

Of the two modelling approaches mentioned above, the former (e.g., derived on the basis of an effective thermal conductivity concept) however, has been relatively more common. This can be anticipated since approach (i) leads to a single conduction-like p.d.e. as the governing equation of heat flow, offering considerable advantages in terms of the overall computational efforts involved. In contrast, the latter approach entails a major computational task owing to the inherent complexities of conjugate fluid heat and transfer phenomena. In this regard it needs to be emphasized here, that if the ultimate objective of the computational effort is to throw light on phenomena such as mould slag entrainment, inclusion trajectories and dispersion, solute segregation etc., and gain better insight of the overall process, an approach based on coupled heat-fluid flow becomes readily apparent.

Using the central concept of approach (i) Miziker was among the first to propose a mathematical model for continuous casting of steel slabs. Subsequently, Laist and coworkers as well as Brimacombe taking an essentially similar approach, proposed mathematical models for slab and billet casting operations. Numerical prediction has been evaluated against experimental measurements and reasonable agreement between the two has been demonstrated. Model predictions have also been extrapolated and on the basis of extensive computation, detailed account has been given on thermal requirements of continuous casting process from design and operational point of view.

Although good agreement between theory and experiments have occasionally been claimed and numerous design recommendations made, the theoretical basis underlying approach (i) has so far, not been given adequate attention. For example, necessary fundamental evidence on possible modelling of convective heat transfer effects via a unique, artificial, large thermal conductivity is lacking. Furthermore, since \( k_{\text{eff}} \) is assumed to embody the possible influences of fluid motion on heat transfer and since flow conditions vary considerably over the entire stretch of liquid pool within a solidifying casting (these are expected to vary from one casting configuration to another as well, as hydrodynamics depend on the geometry etc.), it is therefore not evident that the concept of a universal effective thermal conductivity, as it is currently applied, is physically plausible at all.

The purpose of the present work consequently, is to throw light on the issues so far raised, so that adequacy of approach (i) can be assessed from a fundamental standpoint. Towards this a rigorous analysis is presented in the following sections.

2. Theoretical Considerations

Consider therefore, continuous casting of a cylindrical shaped billet, being withdrawn at a rate of \( U_0 \). Assuming cylindrical symmetry, a steady state heat balance over a small volume element, in terms of cylindrical polar coordinates, results in the following partial differential equation:

\[
U_0 \frac{\partial}{\partial z} (\rho CT) + \frac{\partial}{\partial z} (\rho u CT) + \frac{1}{r} \frac{\partial}{\partial r} \left( \rho J \frac{\partial T}{\partial r} \right) + S = 0 \quad \text{(i)}
\]

In Eq. (1), \( u \) and \( v \) respectively represent the axial and the radial components of fluid motion, which for the present analysis have been assumed to be known from the solution of the relevant p.d.e’s. Further, \( S \) represents and appropriate volumetric thermal source (see later), which accounts for the latent heat released
during the casting process, while, $\Gamma$ can be interpreted as an effective thermal conductivity (accordingly $\Gamma/\rho C$ is the effective thermal diffusivity), which embodies the possible effects of turbulent heat-fluid interactions. In addition, $\rho$ and $C$ are respectively the density and the specific heat of the steel under consideration and for the sake of simplicity, have been assumed to be essentially invariant. The solution to Eq. (1) is the essence of thermal fields prevalent during continuous casting of steel.

Alternatively, incorporating the central concept of approach (i) into the previous equation (e.g., representing $\frac{\partial}{\partial z} \left( \Gamma \frac{\partial T}{\partial z} - \rho u C T \right)$ via $\frac{\partial}{\partial z} \left( K_{ef} \frac{\partial T}{\partial z} \right)$ etc.), Eq. (1) can also be expressed as:

$$U_0 \frac{\partial}{\partial z} (\rho C T) = \frac{\partial}{\partial z} \left( K_{ef} \frac{\partial T}{\partial z} \right) + \frac{1}{r} \frac{\partial}{\partial r} \left( r K_{ef} \frac{\partial T}{\partial r} \right) + S$$

...(2)

in which, the differentials on the righthand side have been assumed to embody the influences of fluid motion and turbulence on heat transfer. $K_{ef}$ in Eq. (2), thus represents and artificial effective thermal conductivity which takes a value of approximately $\frac{7K c^{2}}{r}$ in the liquid pool of solidifying metal (e.g., in regions where local temperature is in excess of the liquidus temperature). Elsewhere, $K_{ef}$ is equivalent to the molecular conductivity, $K$. If the assumption made by Lait and coworkers (e.g., under practical casting situations, $U_0 \frac{\partial}{\partial z} (\rho C T) \geq \frac{\partial}{\partial z} \left( K_{ef} \frac{\partial T}{\partial z} \right)$) is further induced into Eq. (2), the following final equation results:

$$U_0 \frac{\partial}{\partial z} (\rho C T) = \frac{1}{r} \frac{\partial}{\partial r} \left( r K_{ef} \frac{\partial T}{\partial r} \right) + S$$

...(3)

Eq. (3) within the framework of assumptions mentioned already, represents an expression of heat flow via approach (i) during continuous casting of a cylindrical shaped billet. The boundary conditions considered applicable to Eq. (3) can be conveniently expressed in terms of pertinent casting variables such as, pouring temperature, average mould dwell time etc., which are specific to the particular casting configuration chosen.

Since models based on approaches (i) and (ii) have already been shown to yield results which correspond reasonably well to the experimental observations, it is therefore, reasonable to expect Eqs. (1) and (3) to be essentially equivalent, representing one and the same physical phenomenon. To assess this, a control volume based computational procedure embodying Eq. (3) and associated boundary conditions has been developed. In that, the latent heat release effect has been estimated assuming equilibrium freezing of the alloy system and represented according to:

$$S = \rho dH T U_0 \frac{\partial T_s}{\partial z}$$

...(4)

Since solid fractions at axial stations $z$ and $z + \Delta z$ are required for the estimation of $S$, and further, since the solid fraction at $z + \Delta z$ is not known a priori, an iterative calculation procedure has been adopted to seek a numerical solution to Eq. (3). The governing equation together with the associated boundary conditions have been integrated numerically over a large number of non overlapping control volumes to result in a set of algebraic equations. These were subsequently solved via a digital computer incorporating the popular Tri-diagonal Matrix Algorithm. The initial and the boundary conditions applied were:

(i) $z = 0, \quad 0 \leq r \leq R, \quad T = T_i$ ............(5)

(ii) $z > 0, \quad r = 0, \quad -K \frac{dT}{dr} = 0$ .......(6)

and (iii) $z > 0, \quad r = R, \quad -K \frac{dT}{dr} = q_s$ .......(7)

Further, in Eq. (7), $q_s$ represents the surface heat flux across the casting face, which in the mould and the submould regions has been prescribed as

$$q_s = 2.67 - 0.33 \sqrt{r} \text{ MWm}^{-2} \text{ for } z < Z_M$$

and $$q_s = h(T_r - T_w) \text{ for } z > Z_M$$

Numerical values of relevant parameters incorporated in the present calculation procedure are summarised in Table 1.

Numerically predicted solidified shell thickness (deduced on the basis of the solidus temperature of the steel) corresponding to a specific casting configuration (viz., $\{z, r\}$, Table 1) have been compared in Fig. 1 with an earlier prediction of Asai and Szekely. This latter prediction as one would note, was derived through numerical solution of turbulent Navier-Stokes equation together with an appropriate energy conservation equation (e.g., similar in essence to Eq. (1)). The comparison illustrated in Fig. 1 clearly indicates large differences between the two sets of predictions and the respective estimates of shell thickness are seen to differ approximately by a factor of 3. Fig. 1, for the sake of further elucidation, also includes experimental estimates of shell thickness, as reported by Ushijima. There discrepancy between

| Table 1. Numerical values of parameters used in the computation of Figs. 1 and 2. |
|-----------------------------------------------|
| Parameter | Value |
| Casting size (m) | 0.115d |
| Melt carbon (%) | 0.11 |
| Melt superheat (°C) | 21 |
| Solidus temperature (°C) | 1499 |
| Liquidus temperature (°C) | 1529 |
| Mould length (m) | 0.30 |
| Casting speed (m/s) | 0.0817 |
| Primary cooling flux (MWm⁻²) | 2.67 - 0.33 \sqrt{r} |
| Spray heat transfer coefficient (W/(m²·°C)) | 1079.45 |
| Density of steel (kg/m³) | 7400 |
| Specific heat of steel (J/(kg·°C)) | 581.97 |
| Thermal conductivity of steel (W/(m·°C)) | 34.60 × 15.89 |
| Latent heat of solidification (J/kg) | 271954 |
present estimates and reported measurements is readily apparent.

Similarly in Fig. 2 numerically predicted temperature isotherms in the solidifying billet via approach (i) has been compared directly with equivalent prediction of Asai and Szekely. Significant differences between the two set of predictions are again readily evident. It is, however, important to note here that the qualitative nature of the thermal profiles in the solid and the liquid regions (e.g., \( T > 1529^\circ\text{C} \)) deduced via approach (i), while are essentially similar, approach (ii) in contrast, show marked differences between the isotherm contours in these two regions of the solidifying casting. These latter isotherms obviously demonstrate the influences that fluid motion is likely to exert on heat transfer. It is therefore, reasonable to anticipate that the complex nature of fluid motion and its associated influence on heat flow is perhaps not been adequately accounted for, through prescription of a unique, artificial, large thermal conductivity in the liquid pool of the solidifying casting.

The origin of these discrepancies between the two set of theoretical predictions can be traced, if the principal assumption made in deriving Eq. (3) from the exact transport equation (viz., Eq. (1)) is critically analysed. To demonstrate this in terms of relevant fundamental principles, consider further a steady, uni-dimensional physical situation of combined convection and diffusion of a scalar variable \( \phi \), governed by the following equation:

\[
\frac{d}{dz} (\rho u \phi) = \frac{d}{dz} \left( \Gamma_0 \frac{d \phi}{dz} \right) \quad \text{.........(10)}
\]

In Eq. (10), \( \Gamma_0 \) is an appropriate exchange coefficient and is specific to a particular \( \phi \) chosen. Thus, if \( \phi \) represents enthalpy, \( h \), equivalent \( \Gamma_0 \) can be readily seen to be \( K/C \). The boundary conditions considered applicable to Eq. (10) are, (i) \( z = 0, \phi = \phi_0 \) and (ii) \( z = L, \phi = \phi_L \), respectively.

Considering \( \rho u \) to be constant (e.g., from equivalent continuity consideration), Eq. (10) can also be expressed in the following dimensionless form:

\[
\frac{d \theta}{d\eta} = \frac{d}{d\eta} \left( \frac{1}{P} \frac{d \theta}{d\eta} \right) \quad \text{.........(11)}
\]

with corresponding boundary conditions (i) \( \eta = 0, \theta = 0 \) and (ii) \( \eta = 1, \theta = 1 \). \( P \), in Eq. (11) is a local pecllet number \( \left( = \rho u L / \Gamma_0 \right) \), which can be seen to be equivalent to a ratio of strengths of convection to diffusion (of thermal and/or mass) processes.

Now, if the concept of effective thermal conductivity discussed in the preceding section, is applied to Eq. (11), it is to be expected that the following equations, viz.,

\[
\frac{d}{d\eta} \left( \beta \frac{d \theta}{d\eta} \right) = 0 \quad \text{.........(12)}
\]

where, \( \beta \) is any constant, and

\[
\frac{d}{d\eta} \left( \frac{1}{P} \frac{d \theta}{d\eta} - \theta \right) = 0 \quad \text{.........(13)}
\]

would result in equivalent estimates of \( \theta \). This can, however, be assessed readily if \( \Gamma_0 \), the exchange coefficient (or \( P \), the Pecllet number) is taken to be a constant. Eqs. (12) and (13) therefore, under the applied boundary conditions can be solved analytically to yield the respective estimates of \( \theta \) in the domain of interest. Thus on solution, the convection diffusion equation (viz., Eq. (13)) provides,

\[
\theta = \frac{\exp \left( Q_0 \right) - 1}{\exp (Q) - 1} \quad \text{.........(14)}
\]

while as shown below, a linear distribution of \( \theta \) follows from Eq. (12), viz.,

\[
\theta = \eta \quad \text{.........(15)}
\]

These analytical results clearly demonstrate that Eq. (12), while would suggest a linear \( \theta \sim \eta \) distribution, Eq. (13) embodying the conjugate influences of convection and diffusion phenomena, in contrast, would indicate an exponential variation of \( \theta \) in the flow domain. Indeed referring to above, it can be at once recognised that estimates of \( \theta \) via Eqs. (12) and (13) are expected to be equivalent only in the lower limit.
of the Peclet number, suggesting essentially a no flow situation \((e.g., \; u \to 0 \; \text{and} \; \beta \to 1)\).

Proceeding with an integral balance over \(\theta\) and further, equating net transport of \(\theta\) via Eqs. (12) and (13) it can also be shown that if \(\beta\), the constant in Eq. (12), is assigned to a value of \(P/(\exp(P)-1)\), the flux of \(\theta\) at any location estimated from either equations will be essentially identical. This evidently indicates that even though the net flux of \(\theta\) at equivalent locations can be made identical via an appropriately selected constant value of \(\beta\) in Eq. (12), nevertheless, the absolute values of \(\theta\) deduced from Eqs. (12) and (13) will not be the same \((e.g., \; \text{linear vs. exponential})\). This follows from the fact that diffusion flux has relevance to the difference \((\text{say}, \; \theta_2-\theta_1)\), rather than to the absolute values \((e.g., \; \theta_1 \; \text{and} \; \theta_2)\), and thus, a correct expression for flux may result, provided this difference is meaningful, though the absolute values are not.

In an equivalent multidimensional physical situation involving convection and diffusion of thermal energy, the actual transport phenomena involved are relatively more complex than is considered in the preceding analysis. Release of latent heat, variation of thermal properties with temperature \(\text{etc.}\) although increase the complexity further, these as one would normally anticipate, cannot possibly affect the fundamental nature of the transport processes. Consequently, the inferences drawn so far appear to be equally applicable to even more complex multi-dimensional physical situations. In this regard, however, situations with a spatially dependent exchange coefficient might as well be given some considerations.

Analogous to the concept of effective thermal conductivity outlined earlier, implicit in Eq. (1) is a similar assumption \(\text{via} \; \text{which turbulent heat-fluid vector has been approximated (this is necessary to represent the governing equations of turbulent transport in a closed form), i.e.,} \)

\[
\rho C_a \overline{v T'} = -\Gamma, \text{ grad } T \quad \quad \quad \quad \quad \text{(16)}
\]

This gradient formulation of turbulent transport of heat, however, unlike the effective thermal conductivity concept \((e.g., \; K_{\text{eff}}=7 \; K)\), incorporates a spatially dependent turbulent exchange coefficient, \(\Gamma\). This latter parameter is essentially considered to be a function of local hydrodynamic properties and therefore, is expected to vary considerably over a flow domain and from one flow situation to another. It is however, important to note here further, that despite considering a spatially dependent \(\Gamma\), the gradient assumption \(\text{(viz., Eq. (16))} \) is not obeyed always and breaks down for certain specific kind of flows, which for example include, buoyant flows, axisymmetric jet \(\text{etc.} \)\) Indeed, this gradient hypothesis, central to the formulation of turbulent transport of heat, mass and momentum has often been recognised as a premise of major weakness in turbulence modelling, as it is currently practised.\(^{11}\)

As a final point, typical flow regimes in the liquid pool of a solidifying continuously cast billet has been illustrated schematically \(\text{via} \; \text{Fig. 3.} \) This shows that while a strong convection current prevails in the immediate vicinity of the inlet metal stream \((e.g., \; \text{essentially restricted to the mould region}), \) the strength of convection decays appreciably in the submould region. Further down the liquid pool, there is practically no or little flow, resulting essentially from free convection phenomena \((e.g., \; \text{thermal and/or concentration gradients})\). In such a situation, where hydrodynamic conditions are expected to vary to these extents, it is difficult to conceptualise the proposition for a universal effective thermal conductivity \((e.g., \; K_{\text{eff}}=7 \; K)\), since \(K_{\text{eff}}\) is assumed to incorporate possible influences of fluid motion on thermal energy transport.

The analysis presented so far, therefore appears to demonstrate that the effective thermal conductivity based models \((e.g., \; \text{approach (i)}\) embodying the concept of a unique effective thermal conductivity is a crude, overidealised description of the actual thermal phenomena encountered in continuous casting of steel. Indeed, the preceding analysis has clearly shown that neglect of convection, prescription of a large thermal conductivity in the liquid pool \(\text{etc.}\) are not helpful, rather misleading. This as a result, makes the alternative approach of modelling \(\text{(viz., approach (ii))} \) almost evident for any successful theoretical analysis of continuous casting process.

### 3. Concluding Remarks

The concept of effective thermal conductivity as applied to the mathematical modelling of continuous casting operations has been analysed critically. Through considerations of relevant fundamental principles, the concept has been shown to be rather arbitrary, having little or not theoretical basis. Moreover, hydrodynamic conditions prevalent in the liquid pool of a solidifying casting has been given some attention and need for an alternative approach of modelling highlighted.

### Nomenclature

- \(C\): Specific heat \((\text{J/(kg·°C)}))
- \(K\): Thermal conductivity \((\text{W/(m·°C)}))
- \(K_{\text{eff}}\): Effective thermal conductivity \((\text{W/(m·°C)}))
- \(L\): Length \((\text{m})\)
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P: Peclet number \( (= \rho u_L/\Gamma_0) \)

r: Radial coordinate (m)

S: Volumetric heat source (W/m³)

T: Temperature (°C)

U₀: Casting speed (m/s)

z: Axial coordinate (m)

\( \Gamma' \): Effective thermal conductivity (molecular+ turbulent) (W/(m·°C))

\( \Gamma_0 \): Any general exchange (of mass/heat) coefficient

\( \eta \): Dimensionless distance, \( z/L \)

\( \theta \): A dimensionless scalar, \( (\phi - \phi_0)/(\phi_L - \phi_0) \)

\( \rho \): Density (kg/m³)

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