From Copenhagen to neo-Copenhagen interpretation

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Abstract

Positive and negative features of the Copenhagen interpretation are discussed. As positive features can be mentioned its pragmatism and its awareness of the crucial role of measurement. However, the main part of the contribution is devoted to the negative features, to wit, its pragmatism (once again), its confounding of preparation and measurement, its classical account of measurement, its completeness claims, the ambiguity of its notion of correspondence, its confused notion of complementarity. It is demonstrated how confusions and paradoxes stemming from the negative features of the Copenhagen interpretation can be dealt with in an amended interpretation, to be referred to as ‘neo-Copenhagen interpretation’, in which the role of the measuring instrument is taken seriously by recognizing the quantum mechanical character of its interaction with the microscopic object. The ensuing necessity of extending the notion of a quantum mechanical observable from the Hermitian operator of the standard formalism to the positive operator-valued measure of a generalized formalism is demonstrated to yield a sound mathematical basis for a transition from the Copenhagen contextualistic-realist interpretation to the neo-Copenhagen empiricist one. Applications to the uncertainty relations and to the Bell inequalities are briefly discussed.

1 Introduction

Interpretations of physical theories are neither true nor false. Even if they are not completely internally consistent they may be thought to provide useful rules of correspondence between the mathematical formalism and the physical reality a theory purports to describe. It is well-known that the Copenhagen interpretation of quantum mechanics is no exception. Even 80 years after its conception it is impossible to say that there exists a unique and internally consistent interpretation by that name. On the contrary, this interpretation has been characterized (Feyerabend [1]) as “not a single idea but a mixed bag of interesting conjectures, dogmatic declarations, and philosophical absurdities.”

In my contribution to a ‘conference devoted to 80 years of Copenhagen Interpretation’ it therefore is not possible to present a eulogy. On the contrary, my main attention will be directed toward the many imperfections of the Copenhagen
interpretation. Among these I do not count the pragmatic attitude the founding fathers of quantum mechanics have displayed while developing the theory so as to be able to come to grips with the “strange” results their experiments confronted them with. On the contrary, at that time a pragmatic approach, not bothering too much about the physical meaning of the applied mathematics, turned out to be advantageous to rapid scientific progress. However, maintaining such a pragmatism for 80 years may have become detrimental both with respect to development of fundamental insights as well as experimental applications. As seen from table 1 the list of negative features is considerably longer than that of the Copenhagen interpretation’s positive features. Thus, we have to deal with the Copenhagen pre-

| Positive features                  | Negative features                                      |
|-----------------------------------|-------------------------------------------------------|
| +1. pragmatism                    | -1. pragmatism                                        |
| +2. crucial role of measurement   | -2. confusion of preparation and measurement          |
|                                   | -3. classical account of measurement                   |
|                                   | -4. completeness claims                                |
|                                   | -5. ambiguous notion of correspondence                 |
|                                   | -6. confused notion of complementarity                 |

Table 1: Positive and negative features of the Copenhagen interpretation

occupation with measurement, having as a consequence a confusion of preparation and measurement. Another negative feature is the classical account of measurement, overlooking a crucial property of measurements performed on microscopic objects by means of macroscopic measuring instruments. Moreover, each of the three characteristics of the Copenhagen interpretation, viz. completeness, correspondence, and complementarity, is liable to criticism and will be criticized.

Any of these issues might be sufficient to reject the Copenhagen interpretation as a useful interpretation of quantum mechanics. Yet, there is one important positive feature in the Copenhagen appreciation of quantum mechanics as a description of microscopic reality, viz. its recognition of the crucial role played by measurement, which probably outweighs its imperfections. This feature, often considered a weakness of the interpretation, is actually an asset, making quantum mechanics different from classical theory in a truly revolutionary way. It makes quantum mechanics a paradigm of the structuralist methodology (e.g. Suppe [2]) in which the Einsteinian idea of a physical theory as a ‘description of an objective reality (i.e. being independent of the observer, including his measuring instruments)’ has been relinquished.
2 The Copenhagen interpretation and measurement

2.1 Crucial role of measurement

Classical mechanics is generally presented as representing knowledge about an *objective* reality, i.e., a reality as it is independently of being observed by human observers, and, in particular, not being interfered with by their measuring instruments. The impossibility of an interpretation of quantum mechanics as an *objective* description of reality was a main reason for Einstein to disqualify quantum mechanics as not being an adequate physical theory. On the other hand, Bohr was deeply convinced that the existence of a non-vanishing quantum of action $\hbar \neq 0$ entailed a fundamental impossibility of such objective knowledge: only knowledge could be obtained about ‘microscopic reality as it is in interaction with a measuring instrument’, i.e., *contextual* knowledge (cf. section 4). The Heisenberg uncertainty relations were considered as evidence of this, the quantized interaction of an object with different measuring instruments (of incompatible observables) being held responsible for the impossibility of obtaining simultaneously sharp knowledge about e.g. position and momentum.

According to Bohr and Heisenberg quantum mechanics did not represent any objective knowledge, and it did not need to do so because such knowledge would not be verifiable. It should nevertheless be stressed here that the discussion between Bohr and Einstein was not over the question of verifiability, but over the question of whether it is possible to obtain knowledge about a microscopic object without in any way interacting with it [3], i.e. whether quantum mechanics can yield an *objective* description (Einstein), or whether we should content ourselves with a *contextual* one (Bohr). Only after many years we have gradually become convinced (e.g., by the Kochen-Specker [4] and the Bell [5] theorems) that it is impossible to attribute sharp values to quantum mechanical observables as *objective* properties possessed by a microscopic object independently of any measurement. Bohr’s contextualism, to the effect that quantum mechanical observables are well-defined only within the context of a measurement serving to measure that very observable, can be seen as an indication that physical theories do not have the absolute property of being either universally true or universally false, but that they are applicable on a certain domain of experimentation, for quantum mechanics its domain of validity being co-determined by the measurement arrangement. The Copenhagen insight that measurement plays an ineradicable role in the interpretation of quantum mechanics is a fundamental epistemological attainment, in a more general form finding application far beyond quantum mechanics and even physics.
2.2 Confusion of preparation and measurement

However, the Copenhagen preoccupation with measurement has its drawbacks. As a consequence of the emphasis put on ‘measurement’, there was a tendency to formulate all physical operations in terms of this particular one. Measurement was seen where there is none. Thus, in Heisenberg’s version of the Copenhagen interpretation a measurement result actually corresponds to a property of the microscopic object in its final state (Heisenberg [6]). Consider, for instance, the Stern-Gerlach experiment. Here, an atom with non-vanishing angular momentum, after traversing an inhomogeneous magnetic field may finally be found in one of a number of different beams (cf. figure 1). Finding the atom in a beam is interpreted as a determination of the value of a component of the atom’s angular momentum. It is important to note here that for performing such a measurement it is essential that detectors are set up in the beams. Without these detectors the experimental arrangement does not correspond to a measurement but to a conditional preparation (e.g. de Muynck [7], section 3.2.6 and 3.3.4), allowing to perform a measurement of an arbitrary observable conditional on the beam (state $|\psi_m\rangle$) the atom happens to be in. Unfortunately, following Heisenberg [6] the Stern-Gerlach arrangement is generally interpreted as a measurement also if no detectors are present (sometimes referred to as a ‘preparative measurement’), thus allowing the experiment to satisfy (be it only approximately) the von Neumann projection postulate, stating that the conditionally prepared state $|\psi_m\rangle$ equals the eigenstate $|a_m\rangle$ corresponding to the eigenvalue that allegedly would have been found if the measurement had been performed by inserting a detector. It is rather evident that if von Neumann’s postulate holds if the detector is absent, its presence will in general change the state appreciably, thus showing the absurdity of the projection postulate as a measurement principle.

As an example of the confusion of preparation and measurement should be mentioned here the EPR setup (figure 2a), presented in [3] as a measurement of a property of particle 2 (without in any way interacting with that particle). Unfortunately, Bohr accepted the EPR proposal as such, even though it is not really a measurement of particle 2 but rather a preparation of that particle. The EPR setup should be
compared with an EPR-Bell experiment (figure 2b), like the ones performed by Aspect and co-workers \cite{Aspect} to test the Bell inequalities by means of a joint measurement on both particles. Unfortunately, experiments of the EPR-Bell type are generally referred to as EPR experiments, thus veiling the fundamental difference of these experiments.

2.3 Bohr’s classical account of measurement

The stress laid by Bohr on the macroscopicity of measuring instruments, and the necessity to account for the results of measurement in classical terms (cf. section 4) is well-known and need not be discussed here. It should be emphasized, however, that by thus restricting the attention exclusively to the macroscopic part of the measurement it was neglected that a measurement performed on a microscopic object must also have a microscopic part, viz. a part that is sensitive to the microscopic information, so as to be able to realize information transfer from the microscopic object to the measuring instrument. For instance, in the Stern-Gerlach experiment the initial phase of the measurement (nowadays generally called the ‘pre-measurement’) in which the atom interacts with a magnetic field so as to slightly influence the atom’s position, is a microscopic process which should be described quantum mechanically rather than classically. In the amplification phase of the measurement the atom’s position can be thought to change classically so as to create a sufficient distance between the beams to distinguish one from the other. It therefore seems that the microscopic rather than the macroscopic part of the measurement is the interesting one. Bohr’s exclusive attention to the macroscopic part has not contributed to achieving this insight, to say the least. In particular, the development of the generalized formalism is a consequence of a rigorous application of quantum mechanics to the interaction of the microscopic object and the measuring instrument (cf. section 3.2).

It should be noted that von Neumann and Heisenberg did consider a quantum mechanical description of measurement. However, they did not challenge Bohr’s

![Figure 2: a) EPR experiment; b) EPR-Bell experiment.](image-url)
classical description; their purpose was only to prove that a quantum mechanical
description of measurement is compatible with Bohr’s classical one. In particular,
reliance on the projection postulate c.q. restriction to measurements approximately
satisfying it, was responsible for the fruitlessness of these by itself praiseworthy
undertakings.

3 (In)completeness of quantum mechanics

3.1 (In)completeness in a wider sense

Two senses of ‘(in)completeness of quantum mechanics’ should be distinguished, viz.
‘(in)completeness in a wider sense’, dealing with the question of ‘whether hidden
variables are possible’, and ‘(in)completeness in a restricted sense’, addressing the
question of ‘whether the standard formalism of quantum mechanics describes all
possible measurements within the domain of quantum mechanics’. In the past the
main interest has been attracted by the first issue. I will not go into this, but want to
restrict myself to demonstrating the relative futility of the discussion by comparing
the answer given by Copenhagen physicists A.D. 1935 (viz. “Quantum mechanics is
complete; there are no hidden variables”) to the answer most practicing physicists
would probably give A.D. 2007 (viz. “Quantum mechanics is incomplete; ‘no go’
theorems of hidden variables theories were found to be defective, possibly at the
expense of introducing nonlocality (Bohm, Bell)”).

Contrary to the Copenhagen idea that the wave function yields a complete de-
scription of an individual object, it seems nowadays to be generally recognized that
the wave function does rather describe an ensemble. This insight has been gained by
careful analysis of interference experiments at low incident particle rates in which
impacts of individual particles can be registered. From these experiments it can be
inferred that what is described by the wave function (i.e. the interference pattern)
is not generated by an individual particle but by an ensemble of such particles.
The Copenhagen idea that the wave function would be a complete description of an
individual object does seem to be obsolete by now.

Does this mean that Einstein was right when attempting in the EPR paper 3
to prove the incompleteness of quantum mechanics? Not completely so. Actually,
Einstein was bound to fail to do so because he attempted to prove the existence of
hidden variables by identifying these with ‘values of quantum mechanical observ-
ables’. The impossibility of this was rigorously demonstrated, only much later, by
Kochen and Specker’s proof 4 of the theorem named after them. However, this
does not exclude the possibility of subquantum theories involving hidden variables
of a different kind. In any case does it seem to be evident that within any theory
 aspiring at a description of ‘reality behind the quantum mechanical phenomena’ the
contextuality, noticed for the first time by Bohr and rejected by Einstein, will have to be taken into account.

3.2 (In)completeness in a restricted sense

Whereas at this moment the question of '(in)completeness in a wider sense' is not experimentally relevant because we do not have any indication which kind of experiments should be performed so as to transcend the quantum mechanical description in an observationally relevant way, the situation is different as regards '(in)completeness in a restricted sense', at least if we restrict ourselves to the standard (textbook) formalism of that theory (in which measurement probabilities of quantum mechanical observables are represented by expectation values of the projection operators of the spectral representations of Hermitian operators). The standard formalism is easily seen to be 'incomplete in a restricted sense'. For instance, consider the 'which-way polarization measurement of a photon' depicted in figure 3. Here a photon has probability $\gamma$ to be transmitted by a nonpolarizing semi-transparent mirror toward a polarization measurement setup in direction $\theta$, and probability $1 - \gamma$ to be reflected toward a polarization measurement setup in direction $\theta'$. Since the detection probabilities of detectors $D$ and $D'$ are given by $p_D = \gamma \langle E^\theta_+ \rangle$ and $p_{D'} = (1 - \gamma) \langle E^{\theta'}_+ \rangle$, respectively, $E^\theta_+$ and $E^{\theta'}_+$ being projection operators, it is evident that the detection probabilities are not described by the standard formalism (e.g. $\gamma E^\theta_+ \neq (\gamma E^\theta_+)^2$). From the whole range $0 \leq \gamma \leq 1$ of possibilities of the present experiment only the 'set of measure zero $\gamma = (0, 1)$' satisfies the standard formalism.

This example can be supplemented by many other ones; actually, there are very few realistic experiments satisfying the standard formalism (see e.g. de Muynck [7]). If it is taken into account that the interaction of object and measuring instrument is a quantum mechanical process (cf. section 2.3), detection probabilities are seen to be expectation values of the operators $M_m$ of a positive operator-valued measure (POVM) corresponding to the resolution of the identity $\{M_m\}$, $M_m \geq O$, $\sum_m M_m = I$, rather than a projection-valued measure (PVM) corresponding to
the orthogonal decomposition \( \{E_m\} \), \( E_m^2 = E_m \), \( \sum_m E_m = I \) of the standard formalism. Indeed, denoting by \( \rho_o \) and \( \rho_a \) the initial density operators of microscopic object and measuring instrument, respectively, the final density operator is given by \( \rho_{oaf} = U \rho_o \rho_a U^\dagger \), \( U = e^{-i\hbar HT} \), \( T \) the interaction time. Then detection probabilities of the pointer positions (represented by a pointer observable having spectral representation \( \{E_{am}\} \)) are given by

\[
p_m = Tr_o \rho_{oaf} E_{am} = Tr \rho_o M_m, \quad M_m = Tr \rho_a U^\dagger E_{am} U.
\]

From the physical properties of the distribution \( \{p_m\} \) it follows that the operators \( M_m \) satisfy all properties of the elements of a POVM defined above. It should be noted that there is no single reason to require idempotency of these operators.

### 4 The correspondence principle (mature form)

Disregarding here Bohr’s early use of the correspondence principle (in the sense of requiring a classical limit for defining the notion of a quantum mechanical observable), I want to restrict myself here to his later characterization, according to which the following requirements should be met:

1. A quantum mechanical observable is exclusively defined within the context of the measurement serving to measure that observable.

2. Experimental arrangement and measurement results have to be described in classical terms.

Usually these requirements are considered part of the complementarity principle, while restricting the notion of ‘correspondence’ to the requirement of a classical limit. It, however, seems more appropriate to stick to Bohr’s later use, and restrict ‘complementarity’ to considerations on pairs of ‘complementary observables’ only (cf. section 5).

The importance of the first item has been stressed already in section 2.1, whereas the second item was criticized in section 2.3. It is my intention in the present section to demonstrate that by not consistently sticking to the first item Bohr introduced an ambiguity in his notion of ‘correspondence’, thus being responsible for much confusion. The cause of this ambiguity can be traced back to not distinguishing between two different possibilities of what is meant by the word ‘phenomenon’, viz. either ‘a macroscopically observable property of a microscopic object’, or rather ‘a property of a macroscopic measuring instrument obtained by letting the instrument interact with the object (e.g. a pointer triggered by the microscopic object to take a certain position on a measurement scale)’. From the way Bohr reacted to the
EPR proposal \([3]\) it is evident that he had in mind the first possibility. Indeed, since no measuring instrument is present for particle 2 (cf. figure 2a) Bohr could accept Einstein’s proposal to view the EPR measurement setup as a *measurement* on particle 2 only if the measurement result would be taken as a property of that particle.

As a consequence of this Bohr applied his correspondence principle to EPR in an inconsistent way. As a matter of fact, Bohr did not recognize the correlation of two quantum mechanical observables \(A_1\) and \(A_2\), proposed in the EPR paper as a *classical* correlation (\(A_i\) either position or momentum of particle \(i\), \(i = 1, 2\)), as an ordinary *quantum mechanical* observable. He granted Einstein well-definedness of correlation observable \(A_1A_2\) although only \(A_1\) is measured in the EPR experiment. But, according to his correspondence principle the correlation observable \(A_1A_2\) is well-defined only in an EPR-Bell experiment. Hence, Bohr’s conclusion that Einstein’s ‘element of physical reality’ is ambiguous (due to its dependence on the context of the measurement of either \(P_1\) or \(Q_1\)) must be completed by the observation that Bohr’s conclusion is itself a consequence of an ambiguity as regards the meaning of a quantum mechanical observable. Actually, like Einstein, also Bohr never considered the possibility of looking upon a quantum mechanical measurement result as a pointer position rather than as a property of the microscopic object. Only by noting the difference between EPR experiments and experiments of the EPR-Bell type (cf. figure 2) the inconsistency of Bohr’s reaction to EPR could become obvious.

Unfortunately, negligence of the difference between EPR and EPR-Bell experiments has been perpetuated until the present day. As a consequence it is still widely thought that properties of particle 2 may be nonlocally influenced by the measurement performed on particle 1. It will be seen in section 6 that there is no reason for such a conclusion if quantum mechanical measurement results are assumed to correspond to pointer positions of a measuring instrument.

In order to evade confusions like the one discussed here it is necessary to be as precise as possible with respect to the correspondence between the mathematical formalism of quantum mechanics and physical reality. For this reason two different notions of ‘correspondence’ should be distinguished here, viz. (restricting ourselves to standard observables)

1. **Realist correspondence:**
   Quantum mechanical observable \(A\), as well as its values \(a_m\) refer to properties of the microscopic object.

2. **Empiricist correspondence:**
   Quantum mechanical observable \(A\) is a label of a measurement procedure, \(a_m\) is a label of a pointer position.
More generally, we should distinguish two different interpretations of the mathematical formalism of quantum mechanics, viz. a realist interpretation (either objectivistic or contextualistic) interpretation (cf. figure 4(a)) in which quantum mechanics is thought to describe microscopic reality most in the same way classical mechanics is generally thought to describe microscopic reality, and an empiricist interpretation (cf. figure 4(b)) in which state vector and density operator are thought to correspond to preparation procedures, and quantum mechanical observables (either standard or generalized) correspond to measurement procedures and the phenomena induced by a microscopic object in the macroscopically observable pointer of a measuring instrument.

The empiricist interpretation is particularly suited to encompass the generalization from the standard notion of a quantum mechanical observable to the generalized one referred to in section 3.2. An important application of the empiricist interpretation to the generalized formalism is that POVMs can often be interpreted as labeling nonideal measurement procedures, not registering ‘reality as it objectively is’, but taking into account possible disturbing influences introduced by the measurement itself. Thus, of two measurement procedures labeled by POVMs \( \{ M_m \} \) and \( \{ N_n \} \), respectively, such that

\[
M_m = \sum_n \lambda_{mn} N_n, \quad \lambda_{mn} \geq 0, \quad \sum_m \lambda_{mn} = 1, \tag{1}
\]

the first is a nonideal version of the second, the nonideality matrix \( (\lambda_{mn}) \) representing the conditional probability that a measurement of POVM \( \{ M_m \} \) yields result \( m \) if a measurement of POVM \( \{ N_n \} \) would have given result \( n \). In the following sections applications of this will be discussed. Restricting ourselves to discrete spectra it is possible to quantify the nonideality by means of certain properties of the nonideality.
matrix. A convenient quantity is the average row entropy

\[ J(\lambda) = -\frac{1}{N} \sum_{mn} \lambda_{mn} \ln \frac{\lambda_{mn}}{\sum_{n'} \lambda_{mn'}}. \]  

(2)

5 The complementarity principle

The complementarity principle must be seen as a ‘no go’ theorem as regards general applicability of ‘correspondence as obtains in classical physics’: it was soon established that the notion of ‘correspondence’ as defined in section 4 cannot be applied simultaneously to quantum mechanical position and momentum observables, or, more generally, to incompatible standard observables (corresponding to non-commuting Hermitian operators). From discussions of the so-called ‘thought experiments’ it was seen that measurements of incompatible standard observables had mutually exclusive measurement arrangements, in such a way that measurement of one observable would disturb incompatible ones. The mutual disturbance of measurement results of standard observables, when measured simultaneously, was thought to be described by the Heisenberg uncertainty relation

\[ \Delta A \Delta B \geq \frac{1}{2} |\langle \psi | [A, B] | \psi \rangle| \]  

(3)
derived from the standard formalism.

It is important here to remind a criticism by Ballentine [10], to the effect that the Heisenberg uncertainty relation (3) does not refer to ‘joint measurement’ because it can be tested by means of separate ideal measurements of observables \( A \) and \( B \). Hence, it is impossible that this inequality is an expression of mutual disturbance as found in the ‘thought experiments’; it should be seen as a restriction of our preparing abilities rather than of our measuring ones. Probably Einstein was closer to the real meaning of the Heisenberg uncertainty relation (viz., as a restriction on the values of standard deviations in an ensemble) than were Bohr and Heisenberg, who seem to have been jumping to conclusions by associating the results of their considerations on the ‘thought experiments’ with equation (3).

Perhaps Bohr and Heisenberg should not be blamed too much for this error because they did not have at their disposal the generalized formalism referred to in section 3.2, which turns out to be necessary to clarify the situation. It, indeed, is possible by means of the POVMs of the generalized formalism to prove that in trying to jointly measure incompatible standard observables mutual disturbance necessarily occurs (Martens, de Muynck [11]; also de Muynck [7], section 7.10), thus corroborating the results obtained by Bohr and Heisenberg by studying ‘thought experiments’.
In order to do so a bivariate POVM \( \{ R_{mn} \} \) was considered, the expectation values of which describing the joint probability distributions of the two pointers intended to yield the measurement results of the incompatible observables \( A = \sum_m a_mE_m \) and \( B = \sum_n b_nF_n \) to be measured jointly. Then these latter probability distributions are given by the expectation values of the marginals \( \{ \sum_n R_{mn} \} \) and \( \{ \sum_m R_{mn} \} \), respectively. We will call a measurement procedure a joint measurement of \( A \) and \( B \) if \( \sum_n R_{mn} = E_m \) and \( \sum_m R_{mn} = F_n \). It is not difficult to prove that this is possible only if \( A \) and \( B \) are compatible. However, if \( A \) and \( B \) are incompatible it is possible that the marginals of \( \{ R_{mn} \} \) correspond to nonideal measurements (cf. (1)) of \( A \) and \( B \), respectively. Thus, the POVM \( \{ R_{mn} \} \) describes a joint nonideal measurement of standard observables \( A \) and \( B \) if nonideality matrices \( (\lambda_{mm'}) \) and \( (\mu_{nn'}) \) exist such that

\[
\begin{align*}
\sum_n R_{mn} &= \sum_{m'} \lambda_{mm'} E_{m'}, \quad \lambda_{mm'} \geq 0, \quad \sum_m \lambda_{mm'} = 1, \\
\sum_m R_{mn} &= \sum_{n'} \mu_{nn'} F_{n'}, \quad \mu_{nn'} \geq 0, \quad \sum_n \mu_{nn'} = 1.
\end{align*}
\]

(4)

It is easily seen that measurement procedures exist satisfying (4). Thus, the measurement arrangement depicted in figure 3 is a joint nonideal measurement of incompatible standard polarization observables in directions \( \theta \) and \( \theta' \), respectively. Indicating a detection event by \(+\), and non-detection by \( -\), we find

\[
(R_{mn}) = \begin{pmatrix}
O & \gamma E_{+}^\theta \\
(1 - \gamma)E_{+}^{\theta'} & I - \gamma E_{+}^\theta - (1 - \gamma)E_{+}^{\theta'}
\end{pmatrix},
\]

(5)

event ‘ + ‘ having probability zero, and \( I - \gamma E_{+}^\theta - (1 - \gamma)E_{+}^{\theta'} \) corresponding to the possibility that the photon is not detected by either one of the detectors \( D \) or \( D' \) but is absorbed in one of the analyzers. It is straightforward to calculate the nonideality matrices for this experiment as

\[
(\lambda_{mm'}) = \begin{pmatrix}
\gamma & 0 \\
1 - \gamma & 1
\end{pmatrix}, \quad (\mu_{nn'}) = \begin{pmatrix}
1 - \gamma & 0 \\
\gamma & 1
\end{pmatrix}.
\]

(6)

From (6) it is easily seen how the mutual disturbance of the measurement results of the two standard observables changes if the parameter \( \gamma \) is changed. In particular, complementary behaviour is evident, deviation of one nonideality matrix from the unit matrix being maximal whenever the other’s is minimal. This can be expressed by means of the nonideality measure (2). For measurements of the type described by (1) it is possible (Martens, de Muynck [11]) to derive the following general inequality,

\[
J_{(\lambda)} + J_{(\mu)} \geq -\ln \{ \max_{mn} \text{Tr} E_m F_n \},
\]

(7)

to be referred to as the Martens inequality. It is important to note that, contrary to the Heisenberg inequality (3), the Martens inequality does not depend on the state vector or density operator but in an unambiguous way is a property of the
measurement procedure only. From an application of (7) to the joint nonideal measurement of polarization observables depicted in figure 5 it is seen that the Martens inequality yields a perfect illustration of the results obtained by Bohr and Heisenberg when studying the ‘thought experiments’. Indeed, their physical observations based on these experiments were completely correct; only their reluctance to exert a full-blown quantum mechanical analysis of the measurement process prevented them from a sufficiently complete analysis.

It, incidentally, should be noticed that the mutual disturbance considered here does refer to the final pointer positions of the measuring instruments. This should be distinguished from ‘mutual disturbance in a preparative sense’ as discussed in the Copenhagen literature, referring to the final state of the microscopic object rather than to the final state of the measuring instrument. Actually, complementarity can manifest itself in two different ways, viz., as a property of ‘preparation’ described by the Heisenberg inequality, and as a property of ‘joint measurement’ described by the Martens inequality. To a large extent the Copenhagen confusion with respect to ‘complementarity’ has its origin in the confusion with respect to preparation and measurement noted in section 2.2.

6 The Bell inequalities and nonlocality

As is well-known the Bell inequalities are satisfied if all standard observables that are involved are mutually compatible. This entails the conclusion that incompatibility is a necessary condition for the Bell inequalities to be violated. But, due to the ‘postulate of local commutativity’ (stating that observables measured in causally
disconnected regions of space-time must be compatible) incompatibility of observables is a local affair. Hence, violation of the Bell inequalities must have a local origin. It is the intention of the present section to demonstrate that, indeed, violation of the Bell inequalities may be caused by local mutual disturbance in joint nonideal measurements of incompatible standard observables rather than by nonlocal influences.

In order to do so we consider the generalized EPR-Bell experiment depicted in figure 6, the difference with the Aspect experiments being that the mirrors indicated by $\gamma_1$ and $\gamma_2$ are ‘semi-transparant, and stationary’ rather than ‘completely reflecting, and either stationary or switching’. We consider here two measurements of the type depicted in figure 3, simultaneously performed on particles 1 and 2. The Aspect experiments are special cases of the present experiment, satisfying $(\gamma_1, \gamma_2) = (1, 1), (1, 0), (0, 1)$ and $(0, 0)$, respectively. For general $(\gamma_1, \gamma_2)$ the (quadrivariate) POVM follows directly from the bivariate POVM as

$$R_{m_1n_1m_2n_2}(\gamma_1, \gamma_2) = R_{m_1n_1}(\gamma_1) R_{m_2n_2}(\gamma_2),$$

in which the parameters $\gamma_1$ and $\gamma_2$ are added to distinguish the two arms of the interferometer. The expectation value of POVM yields the quadrivariate probability distribution of a joint nonideal measurement of four standard polarization observables in directions $\theta_1, \theta'_1, \theta_2,$ and $\theta'_2$, respectively. There is mutual disturbance of the type discussed in section 5 separately in each of the arms of the interferometer. As is to be expected from the postulate of local commutativity there is no mutual disturbance between different arms of the interferometer.

Let us finally see where the violation of the Bell inequalities in the Aspect experiments comes from (see de Muynck, chapter 9). In order to do so it is important to note that these experiments can be demonstrated to violate the Bell inequalities only by combining results obtained within four different arrangements for measuring bivariate probabilities. By contrast, the bivariate probabilities derived from the generalized experiment for fixed $(\gamma_1, \gamma_2)$ from the experimentally obtained

Figure 6: Generalized EPR-Bell experiment.
quadrivariate one do satisfy the Bell inequalities because they are obtained within one single measurement setup. This can be traced back to the fact that each individual particle pair yields a quadruple \((m_1, n_1, m_2, n_2)\) of measurement results (pointer readings); this is a sufficient condition for the statistics to have a Kolmogorovian character. This evidently holds true for each of the Aspect experiments separately, but not for measurement results of the four experiments pasted together.

Violation of the Bell inequalities is evidence of the non-Kolmogorovian character of the statistics. Indeed, by changing the measurement arrangement in the experiments performed by Aspect the quadruples are changed in such a way that an octuple of measurement results of the four measurements corresponding to \((\gamma_1, \gamma_2) = (1, 1), (1, 0), (0, 1),\) and \((0, 0),\) cannot be accommodated into one single quadruple. The cause of this is seen to be a changing of the mutual disturbance in one arm of the interferometer by changing the measurement arrangement in that arm (and analogously in the other arm). These changes are completely accounted for by the the idea of complementarity in a joint measurement of incompatible standard observables as discussed in section 5. Nonlocal influences between different arms are not necessary to explain the ensuing violation of the Bell inequalities.

It should finally be stressed that the latter conclusion, although derived from (generalized) quantum mechanics, is actually a conclusion about the physical processes obtaining in experiments of the kind discussed here. Therefore, the often-heard assertion that quantum mechanical locality might be compatible with subquantum nonlocality (the latter, allegedly, being necessary to cope with derivations of the Bell inequalities from local hidden variables theories), is not applicable: it is utterly unreasonable to suppose that one and the same phenomenon would have completely different physical explanations within different theories. If violation of the Bell inequalities has a local explanation within a quantum mechanical description of certain phenomena, it should have a local explanation within any theory describing the reality behind these phenomena. Rather than acquiescing in an unfortunate marriage of ‘observable locality’ with ‘unobservable nonlocality’ by relying on such fancy expressions like ‘entanglement’, ‘inseparability’ or ‘passion at a distance’, it seems more appropriate to perform as precise as possible analyses of measurement processes, and to study the relations between its measurement results and the properties of microscopic objects as described in different theories.

7 From Copenhagen to neo-Copenhagen interpretation (summary)

Due to Bohr’s reference to ‘phenomena’ the Copenhagen interpretation has an empiricist reputation (e.g. Reichenbach [12]). However, on closer scrutiny Bohr’s ‘phenomenon’ is probably too much inspired by the rather primitive detection methods
(like flashes on a scintillation screen) of that time. It does not take into account more modern measurement procedures in which the final observational data are processed so as to be only vaguely reminiscent of their microscopic origin: what is seen by the observer is often just a graph coming out of his printer. Contrary to the Copenhagen interpretation (which, for the main part, is a realist interpretation, be it of a contextualistic kind) the neo-Copenhagen interpretation is an empiricist one, adapted to the generalized formalism of quantum mechanics necessary to also describe more modern experiments (cf. de Muynck [7], chapter 8).

Comparing, in summary, the neo-Copenhagen interpretation with the Copenhagen one with respect to the notions of completeness, correspondence and complementarity, we arrive at the following results:

| Interpretation | Copenhagen | Neo-Copenhagen |
|----------------|------------|----------------|
| Completeness:  |            |                |
| In a wider sense: | yes (claimed) | no             |
| In a restricted sense (standard QM): | yes (claimed) | no             |
| In a restricted sense (generalized QM): | yes |                |
| Correspondence: |            |                |
| Contextualistic-realist | empiricist | |
| Partly classical | completely QM | |
| Complementarity: |            |                |
| Preparation: | Heisenberg inequality | Heisenberg inequality |
| Joint Measurement: | Heisenberg inequality | Martens inequality |

Notwithstanding the considerable differences does it nevertheless seem appropriate to pay a tribute to Copenhagen by referring to that name in the new interpretation, because in the latter the important development away from the classical idea of ‘physical theory as an objective description of reality’, which was started by the Copenhagen interpretation, has been continued, and has been pushed to a logical completion.

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