Study of Knowledge-Aided Iterative Detection and Decoding for Multiuser MIMO Systems

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Abstract—In this work, we consider the problem of reduced latency of low-density parity-check (LDPC) codes with iterative detection and decoding (IDD) receiver in multiuser multiple-antenna systems. The proposed knowledge-aided IDD (KA-IDD) system employs a minimum mean-square error detector with refined iterative processing and a reweighted belief propagation (BP) decoding algorithm. We present reweighted BP decoding algorithms, which exploit the knowledge of short cycles in the graph structure and reweighting factors derived from the expansion of hypergraphs. Simulation results show that the proposed KA-IDD scheme and algorithms outperform prior art and require a reduced number of decoding iterations.

Index Terms—iterative detection and decoding, multiuser detection, MIMO, LDPC codes.

I. INTRODUCTION

The fifth generation (5G) of wireless systems will demand higher capacity, lower latency and an improved user experience [1]. Spatially multiplexed multiuser multiple-input and multiple-output (MIMO) systems can support several independent data streams, resulting in a significant increase of the system throughput [2]. In recent years, massive MIMO [3], [4], [5], [6], [7] has been advocated as one of the key technologies to address the capacity requirements of 5G wireless communications. In this context, a great deal of effort has been made in the development of detection algorithms and their integration with channel decoding techniques [9], [10], [11], [12], [13], [14], [15], [16], [17], [18]. With the adoption of modern iteratively decodable codes such as Turbo and low-density parity-check (LDPC) codes, MIMO systems with iterative detection and decoding (IDD) have been shown to approach the performance of an interference free scenario.

A multiuser MIMO-IDD system is comprised of a soft-input soft-output (SISO) MIMO detector and an efficient SISO decoder with low delay. Specifically, the log-likelihood-ratios (LLRs) associated with the encoded bits are updated between the two components, the information exchange of detection and decoding is then repeated in an iterative manner until the maximum number of iterations is reached. However, there are many open problems for IDD schemes. These include detection/decoding delay, which depends on the number of inner and outer decoding iterations and performance degradation for codes with short block lengths [10], [11].

Capacity achieving LDPC codes [19], [20], [21], [22], [23] are a class of block code blocks with simple encoding and efficient decoding algorithms. The standard belief propagation (BP) algorithm is well-known and has been widely employed in LDPC-based IDD schemes for MIMO systems [20], [10], [13], [24], [25]. However, with the existence of cycles in the graph structure, the standard BP has a shortcoming: at low-to-moderate signal-to-noise ratios (SNR), a large number of inner iterations may be required for convergence to a codeword, which causes undesired delay and deteriorates the decoding performance. In order to address this problem, a set of reweighting factors have been introduced in [26], where the problem of finding the fixed points of the BP algorithm was shown to be equivalent to solving a variational problem. More recently, Wymeersch et al. [27] upgraded the reweighted BP algorithm from pairwise graphs to hypergraphs and reduced the set of reweighted parameters to a constant, whereas Liu and de Lamare considered the use of two possible values in [23].

In this work, we present a knowledge-aided IDD (KA-IDD) scheme and decoding algorithms for multiuser MIMO systems with reduced latency. The proposed KA-IDD scheme and BP algorithms are inspired by the reweighted BP decoding algorithms in [27], [28], which exploit the graphical distributions of the Tanner graph, iterative processing and weight optimization. The proposed KA-IDD scheme consists of a minimum mean-square error (MMSE) detector with soft interference cancelation, refined iterative processing and a reweighted BP decoding algorithm. We also present reweighted knowledge-aided BP decoding algorithms: the first one is called cycles knowledge-aided reweighted BP (CKAR-BP) algorithm, which exploits the cycle distribution of the Tanner graph, whereas the second is termed expansion knowledge-aided reweighted BP (EKAR-BP) algorithm, which expands the original graph into a number of subgraphs and locally optimizes the reweighting parameters. The proposed KA-IDD scheme and decoding algorithms can considerably improve the performance of existing schemes.

The organization of this paper is as follows: Section II introduces the system model. In Section III, the proposed EKAR-BP and CKAR-BP algorithms are explained in detail. Section IV shows the simulation results along with discussions. Finally, Section V concludes the paper.

II. SYSTEM MODEL

Let us consider the uplink of a spatially multiplexing multiuser MIMO system with $K$ simultaneous single-antenna users and $N_R$ receive antennas ($N_R \geq K$) transmitting data over flat fading channels. At each time instant $i$, the $K$ users transmit $K$ symbols which are organized into a $K \times 1$ vector $s[i] = [s_1[i], s_2[i], \ldots, s_K[i], \ldots, s_K[i]]^T$ and each
entry is taken from a constellation \( A = \{a_1, a_2, \ldots, a_C\} \), where \((\cdot)^T\) denotes transpose and \(C\) denotes the number of constellation points. For a given block, the symbol vector for each user \(s_k\) is obtained by mapping it into the vector \(x_k = [x_{k,1}, \ldots, x_{k,j}, \ldots, x_{k,J}]\) with the coded bits. The received data vector \(r[i] \in \mathbb{C}^{N_R \times 1}\) at time instant \(i\) is given by

\[
\begin{align*}
    r[i] = Cx[i] + n[i] = \sum_{k=1}^{K} c_k x_k[i] + n[i],
\end{align*}
\]

where \(C \in \mathbb{C}^{N_R \times K}\) is the channel matrix with its \(k\)th column \(c_k[i] \in \mathbb{C}^{N_R \times 1}\) representing the complex channel coefficients, \(x[i] \in \mathbb{C}^{K \times 1}\) is the encoded data vector with zero mean and \(E[x[i]\!x^H[i]] = \sigma^2_1 \mathbf{I}\), where \(\sigma^2_1\) is the signal power, \(E[\cdot]\) stands for expected value, \((\cdot)^H\) denotes the Hermitian operator and \(\mathbf{I}\) is the identity matrix. The symbol \(x_k[i]\) is the encoded transmitted bit for the \(k\)th user, \(n[i] \in \mathbb{C}^{N_R \times 1}\) is complex Gaussian noise vector with \(E[n[i]\!n^H[i]] = \sigma^2_2 \mathbf{I}\) with variance \(\sigma^2_2\). The model in (1) is used to represent the transmission of data symbols that are then organized in blocks.

III. KNOWLEDGE-AIDED IDD SCHEMES

In a parallel interference cancellation (PIC) based MMSE IDD receiver, the estimates of the transmitted symbols are updated based on the a priori LLRs obtained from the channel decoder. These soft symbol estimates are retrieved from the received vector to perform interference cancellation. An MMSE filter [32], [33] is introduced to equalize the remaining noise plus interference term and the individual a posteriori LLRs of the constituent bits are obtained at the output of the filter [8]. According to this model in [8], a PIC detector cancels the interference \((q \neq k)\) with

\[
\begin{align*}
\tilde{y}_k &= y_k - \sum_{q \neq k} c_q \tilde{y}_q - \tilde{n}, \quad \forall k,
\end{align*}
\]

where the co-channel interferences are estimated according to \(\tilde{y}_q = E[y_q] = \sum_{a \in A}^P y_q = a|a\), where the vector \(c_k\) is the \(k\)th column of \(C\) and \(P[y_q = a]\) corresponds to the a priori probability of the symbol \(a\) on the constellation map. Term \(\tilde{n}\) is the noise-plus-remaining-interference vector to be equalized by a linear MMSE estimator as

\[
\tilde{y}_k = \tilde{w}_k^H \tilde{r}_k = \tilde{w}_k^H c_k x_k + \tilde{w}_k^H \tilde{n}.
\]

In Fig. 1, we set \(y_k = x_k + n_{\text{eff}}\) at the output of the detector, where \(n_{\text{eff}}\) is the effective noise factor after MMSE filtering. By assuming that the output is independent from each other [8], the approximation of the LLR of bit \(x_{k,j}\):

\[
\begin{align*}
    L_1[x_{k,j}] \approx \log \frac{P(x_{k,j} = +1|y_k)}{P(x_{k,j} = -1|y_k)} = l_1[x_{k,j}] + l_2^x[x_{k,j}],
\end{align*}
\]

where the last term represents the a priori information for the coded bits \(x_{k,j}\), which is obtained by the LDPC decoder. The first term \(l_1\) denotes the extrinsic information which is obtained by \(r[i]\) and a priori \(l_2^x\).

The latency caused by the IDD scheme is usually due to the required inner and outer iterations involving the exchange of LLRs. The proposed KA-IDD scheme aims to reduce the number of iterations and minimizing this latency of obtaining \(l_2^x[x_{k,j}]\) from the LDPC decoder.

IV. KNOWLEDGE-AIDED DECODING ALGORITHMS

The convergence behaviour of the BP algorithm is considered in the development of the proposed CKAR-BP and EKAR-BP algorithms. Both algorithms rely on the techniques of reweighting part of the hypergraph, the impact of short cycles is also considered such that the BP decoder may calculate more accurate marginal distributions. In [26], the reweighting strategy was employed in the tree-reweighted BP (TRW-BP) algorithm and the authors convert BP decoding problem to a tractable convex optimization problem, iteratively computing beliefs and factor appearance probabilities (FAPs). Later in [27], with additional constraints on FAPs, uniformly reweighted BP (URW-BP) was introduced. Compared to TRWBP and URW-BP, the proposed CKAR-BP and EKAR-BP algorithms optimize the FAPs off-line by relaxing the constraints from [26] and [27]. Furthermore, none of them impose extra complexity to on-line decoding. In what follows, we present general message passing rules for reweighted BP algorithms, then detail the proposed CKAR-BP and EKAR-BP algorithms.

A. Message Passing Rules for Knowledge-Aided Decoding

The derivation of the message passing rules of reweighted BP algorithms can be found in [27] with higher-order interactions and in [26] with pairwise interactions. Let us consider a hypergraph with \(M\) check nodes, \(N\) variable nodes and the reweighting vector \(\rho = [\rho_1, \rho_2, \ldots, \rho_M]\), the message from the \(j\)th variable node \(s_j\) to the \(i\)th check node \(c_i\) is given by

\[
\Psi_{ji} = \lambda_{in,j} + \sum_{i' \in \mathcal{N}(j) \setminus i} \rho_{i'} \Lambda_{i'j} - (1 - \rho_i) \Lambda_{ij},
\]

except \(c_i\), the neighboring set of check nodes of \(s_j\) is \(i' \in \mathcal{N}(j) \setminus i\). Because beliefs are in the form of LLRs, \(\lambda_{in,j}\) is equal to \(l_1[x_j]\) in the first decoding iteration. We use the parameter
TABLE I
PROPOSED CKAR-BP DECODING ALGORITHM

Offline Stage 1: counting of short cycles [29]
1: Counting the number of length-\(g\) cycles \(\delta_{c_i}\) passing through the check node \(c_i\), \( \forall i\);

Offline stage 2: determination of \(\rho_i\) for the hypergraph
2: Determining variable FAPs for the nodes:
   if \(\delta_{c_i} < \mu_g\) then \(\rho_i = 1\), otherwise \(\rho_i = \rho_v\) where \(\rho_v = \frac{2\alpha}{n_D}\).

Online Stage: real-time decoding
3: Iteratively updating the belief \(b(x_j)\) with reweighted message passing [5–7] with optimized \(\rho = [\rho_1, \rho_2, \ldots, \rho_M]\).

Decoding stops if \(H\hat{x}^T = 0\) or the maximum iteration is reached.

\[ \Lambda_{ij} = 2\text{tanh}^{-1}\left( \prod_{j' \in N(i) \setminus j} \text{tanh}\left(\frac{\psi_j}{2}\right) \right), \]  

where the hyperbolic tangent function is introduced to compute an LLR from \(c_i\) to \(s_j\). Finally, we have the KA-IDD updated belief \(b(x_j)\) given by

\[ b(x_j) = \lambda_{\text{ln},j} + \sum_{i \in N(j)} \rho_i \Lambda_{ij}. \]  

The proposed KA-BP algorithm employs [3, 2] to update the information for each node. Note that \(\rho_i = 1, \forall i\) corresponds to the standard BP and negligible extra complexity is required. At the end of the decoding procedure, the soft output is either used for deciding the value of \(\hat{x}_j\) or for generating the extrinsic information \(l_2[x_j]\) for the next KA-IDD iteration.

B. Cycles Knowledge-Aided Reweighted BP (CKAR-BP)

The distribution of short cycles in the graph has an impact on statistical dependency among the incoming messages being exchanged by nodes, leading to low reliability. With the knowledge of the cycle distribution, the proposed CKAR-BP algorithm updates the reweighting parameters in order to mitigate the effect of short cycles. For counting short cycles, a matrix multiplication technique [29] which can calculate the number of cycles with girth of \(g\), \(g+2\) and \(g+4\), explicitly.

In the offline stage shown in Table I the parameter \(\delta_{c_i}\) denotes the number of cycles passing through check node \(c_i\) which affects the convergence behaviour of the LDPC decoding, is determined. The average number of of length-\(g\) cycles passing a check node denoted by \(\mu_g\), can be used to compute the reweighting parameters \(\rho_i (i = 0, 1, \ldots, M-1)\), we adopt a simple criterion:

\[ \text{if } \delta_{c_i} < \mu_g \text{ then } \rho_i = 1, \text{ otherwise } \rho_i = \rho_v, \]  

where \(\rho_v = \frac{2\alpha}{n_D}\), \(0 < \alpha < 1\) and \(n_D\) denotes the average connectivity for \(N\) variable nodes given by

\[ n_D = \frac{1}{\int_0^1 v(x)dx} = \frac{M}{N \int_0^1 \nu(x)dx}, \]  

where \(v(x)\) and \(\nu(x)\) represent the distributions of the variable nodes and the check nodes, respectively. As an improvement of URW-BP [27], cycle counting [29] is required and CKAR-BP needs some extra complexity. It is important to note that when decoding LDPC codes, the proposed CKAR-BP algorithm can improve the performance of BP with either uniform structures (regular codes) or non-uniform structures (irregular codes).

C. Expansion Knowledge-Aided Reweighted BP (EKAR-BP)

The proposed EKAR-BP algorithm first transforms the original hypergraph \(G\) into a set of subgraphs and then locally optimizes the reweighting parameter vector \(\rho_t, t = 1, 2, \ldots, T\) for each subgraph. The dimension of \(\rho_t\) is determined by the size of the subgraph. The TRW-BP algorithm [26] (corresponds to \(T = 1\)) has a very slow convergence for large graphs and a computational complexity of \(O(M^2N)\). Nevertheless, the optimization of \(\rho\) could be significantly simpler when more subgraphs are considered. Thus, there is need for a flexible method to transform the original hypergraph into many subgraphs. In general, the number of subgraphs \(T\) depends on a pre-defined maximum expansion level \(d_{\text{max}}\), larger \(d_{\text{max}}\) usually results in a smaller \(T\) but a higher probability of short cycles within subgraphs. Inspired by [30], a modified progressive-edge growth (PEG) approach is applied to achieve the hypergraph expansion. Compared to the greedy version of PEG [30], the proposed PEG expansion has two main updates:

(i) the expansion stops as soon as every member of the set of nodes \(V_t\) has been visited;
(ii) the number of edges incident to node \(s_j\) might be less than its degree since some short cycles are excluded in subgraphs to guarantee that the local girth of each subgraph \(g_t\) is larger than the global girth of the original graph \(g\).

As shown in Table II with the obtained \(T\) subgraphs, we introduce the vector \(L = [L_1, L_2, \ldots, L_T]\) where \(L_t\) is
the number of check nodes in the $t$th subgraph. Due to the expansion, we have $\sum_{t} L_t > M$ due to duplicated nodes. Similar to TRW-BP [20], in the $t$th subgraph, the associated FAPs $\rho_i = [\rho_{i,1}, \rho_{i,2}, \ldots, \rho_{i,L_t}]$ are optimized recursively, but with higher-order interactions and related message passing [5–7]. The optimization problem is recursively solved:

i) the message passing rules [5–7] are used to compute the mutual information $I_t = [I_{t,1}, I_{t,2}, \ldots, I_{t,L_t}]$ and the beliefs of $b(x_i)$ for all $T$ parallel subgraphs and fixed $\rho_i^{(r)}$.

ii) given $\{I_t\}_{t=1}^T$, we use the conditional gradient method to update $\rho_i^{(r)}$ for all $T$ subgraphs in parallel, then go back to step 1). The objective function used by the conditional gradient method is given by

$$\text{minimize} \quad -\rho_i^\dagger I_t$$

$$\text{s.t.} \quad \rho_i \in \mathbb{T}(\mathcal{G}_t),$$

where $(\cdot)^\dagger$ denotes transpose, $\mathbb{T}(\mathcal{G}_t)$ is the set of all valid FAPs over the subgraph $\mathcal{G}_t$, and $I_{t,\ell}$ is a mutual information term depending on $\rho_{t,\ell}^{(r)}$, the previous value of $\rho_t$, representing the objective function by $f(\rho_t) = \rho_t^\dagger I_t$, we first linearize the objective around the current value $\rho_t^{(r)}$:

$$f_{\text{lin}}(\rho_t) = f(\rho_t^{(r)}) + \nabla_{\rho_t} f(\rho_t^{(r)})(\rho_t - \rho_t^{(r)}), \quad (10)$$

where $\nabla_{\rho_t} f(\rho_t^{(r)}) = -I_t$. Then, the term $f_{\text{lin}}(\rho_t)$ is minimized with respect to $\rho_t$, denoting the minimizer by $\rho_t^*$ and $z_t^{(r+1)} = \max(f_{\text{lin}}(\rho_t^*), z_t^{(r)})$, where $z_t^{(0)} = -\infty$. Finally, $\rho_t^{(r)}$ is updated as:

$$\rho_t^{(r+1)} = \rho_t^{(r)} + \alpha[\rho_t^* - \rho_t^{(r)}], \quad (11)$$

and $\alpha$ is obtained as:

$$\arg \min_{\alpha \in [0,1]} f(\rho_t^{(r)} + \alpha[\rho_t^* - \rho_t^{(r)}]). \quad (12)$$

In each recursion, $f(\rho_t^{(r)} + \alpha[\rho_t^* - \rho_t^{(r)}])$ is an upper bound on the optimized objective, while $z_t^{(r+1)}$ is a lower bound. Note that the proposed EKAR-BP algorithm can be straightforward applied if LDPC codes have been designed by the PEG principle and its variations [31], but is not limited to such designs.

V. SIMULATION RESULTS

In this section, we present the proposed KA-IDD scheme with CKAR-BP and EKAR-BP using an LDPC-coded uplink multiuser MIMO system with single-antenna users. The LDPC code adopted is a regular code designed by the PEG algorithm [30] with block length $N = 1000$, rate $R = 0.5$, girth $g = 6$, and the degree distributions are $3(\nu(x) = x^4)$ and $5(\nu(x) = x^5)$, respectively. For CKAR-BP, we employ $\alpha = 0.85$. For EKAR-BP, $T = 20$ subgraphs are generated where the check nodes are allowed to be re-visited. EKAR-BP requires around 600 recursions to converge for this code.

The iterative processing principle provides substantial gains in each iteration. Here, we employ the extrinsic information transfer (EXIT) chart to analyze the behavior of the constituent components of KA-IDD scheme. Using an uncorrelated Rayleigh flat fading channel, an EXIT chart for different decoding algorithms with the standard BP and URW-BP algorithms are given in Fig. 2. Even if the curve of the PIC detector does not reach the top-right $(1, 1)$ corner at the given SNR, it is obvious that the combination of PIC detector and the proposed EKAR-BP decoding algorithm creates the widest detection and decoding tunnel. Additionally, only the tunnel between the PIC detector and standard BP decoding algorithm is closed at an early stage, which indicates that performance gain from the IDD process could be significantly diminished in this case. To verify the result of the EXIT chart, we examine the performance in terms of average bit-error ratio (BER).

We consider next the proposed KA-IDD scheme and decoding algorithms in two scenarios. In the first scenario, we consider independent and identically distributed (i.i.d) fading channel models whose coefficients are complex Gaussian random variables with zero mean and unit variance. In the second scenario, we consider a channel described by

$$c_k = \alpha_k \beta_k h_k; \quad k = 1, \ldots, K, \quad (13)$$

where $c_k$ represents the distance based path-loss between the $k$th transmitter and the receiver, and $\beta_k$ is a log-normal variable, representing the shadowing between the transmitter and the receiver. The parameters $\alpha_k$ and $\beta_k$ are calculated by $\alpha_k = \sqrt{L_p^{(k)}}$, and $\beta_k = 10 \log_{10} \frac{\sigma_k^2}{\beta_0}$, respectively, where $L_p^{(k)}$ is the base power path loss, $N_p^{(k)}(0,1)$ denotes a Gaussian distribution with zero mean and unit variance and $\sigma_k$ is the shadowing spread in dB. The vector $c_k$ in (13) is modeled as the Kronecker channel model expressed by

$$c_k = \mathbf{R}^{1/2}_{\tau_x} \mathbf{h}_{0_k}, \quad (14)$$

where $\mathbf{h}_{0_k}$ is the channel vector for the first scenario and $\mathbf{R}_{\tau_x}$
denotes the receive correlation matrix given by
\[
R_x = \begin{pmatrix}
1 & \rho & \cdots & \rho^{(N_R-1)^2} \\
\rho & 1 & \cdots & \vdots \\
\vdots & \rho & \cdots & \rho \\
\rho^{(N_R-1)^2} & \cdots & \rho & 1
\end{pmatrix}.
\] (15)

Assuming \(L_p^{(k)}\), \(\sigma_k\), no correlation for the \(K\) transmitters with a single antenna and the correlation coefficient \(\rho = 0.8\) for all the receiver, the SNR is defined as \(10\log_{10} \frac{\sigma_n^2}{\sigma_x^2}\), where \(\sigma_x^2\) is the variance of the received symbols and \(\sigma_n^2\) is the noise variance. The LDPC coded bits are modulated to QPSK symbols with anti-gray coding. We used 3 outer detection and decoding iterations. The performance curves after 2 outer iterations are denoted by solid lines while the curves after 3 outer iterations are denoted by dashed lines.

In the first propagation scenario shown in Fig. 3, we employed 30 inner decoding iterations and both CKAR-BP and EKAR-BP decoders outperform the standard BP and URW-BP decoder in the first detection and decoding iteration. In the third outer iteration, two proposed decoders are still able to generate relatively good performance when considering the low SNR range and the block length of code.

In the second scenario, we have \(L_p^{(k)}\) taken from a uniform random variable between 0.7 and 1, \(\tau_k = 2\) as the path loss exponent, and the shadowing spread is \(\sigma_k = 3\) dB. We employed 20 inner iterations and 3 outer iterations. Fig. 4 a) depicts a multiuser MIMO scenario with \(N_R = 8\) receive antennas and \(K = 8\) single-antenna users. Fig. 4 b) demonstrate a massive multiuser MIMO case with \(N_R = 32\) receiving antennas at the base station and \(K = 8\) simultaneous users. The results indicate that with a higher number of users, the proposed algorithms also outperform the standard BP even with a small number of outer iterations.

**VI. Conclusions**

We have proposed a KA-IDD scheme for multiuser MIMO systems and two novel KA-BP decoders, which employ reweighting strategies for decoding regular or irregular LDPC codes. The proposed CKAR-BP and EKAR-BP algorithms have different computational complexities in the optimization phase and can reduce the latency caused by iterations. The results show that the proposed KA-IDD scheme has improved performance while using a lower number of iterations.

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