In 1978, to explain quark confinement in hadrons, t’Hooft coined the notion of “superinsulator,” a hypothetical ground state endowed with infinite electric resistance and representing thus an extreme opposite to a superconductor. In condensed matter, the superinsulating ground state was first predicted for Josephson junction arrays (JJA) and reappeared in NbTiN films as a phase emerging at the insulating side of the superconductor-insulator transition (SIT) in superconducting films due to the duality of the phase-amplitude uncertainty principle. Superinsulators, dissipationless Bose condensates of vortices retain an infinite resistance at finite temperatures and, as such, have been gaining an intense research attention, see references therein. Here we investigate the response of a superinsulator to a dc electric field and show that small fields, $E < E_1$, are completely suppressed inside the material, as a dual, electric version of the Meissner effect. Intermediate fields $E_1 < E < E_2$ penetrate the superinsulator as electric filaments realizing Polyakov’s electric strings in compact QED as a dual, electric version of the mixed state of superconductors, and, finally, large fields, $E > E_2$, break down the superinsulation. We report transport measurements in NbTiN disordered films revealing both thresholds and their linear scaling with the system size. We demonstrate that the asymptotically free behaviour of quarks within mesons maps onto the metal-like behaviour of small films. Our findings open the route to measurements of Polyakov’s string tension as a function of the system’s parameters, enabling the exploration of strong coupling gauge theory concepts via desktop experiments.

Experimentally, superinsulating materials were revealed by transport measurements on titanium nitride (TiN) films, niobium titanium nitride (NbTiN) films, and, albeit under a different name, InO films. The mechanism of superinsulation was established in Eu films as the realization of a single-color version of quantum chromodynamics (QCD), compact QED with Cooper pairs playing the role of quarks. The infinite resistance owes to the binding of charge-anticharge pairs (i.e. fluctuation-induced thermodynamic Cooper pairs and their anti-Cooper pairs) by electric strings, resulting in a confinement potential that grows linearly with the charge separation. A superinsulator hosts a vortex Bose condensate localizing Cooper pairs. In a condensate, vortex number is not conserved and vortices can be viewed as strings connecting magnetic monopoles at their ends. In 2D systems, with $d < \xi$, vortices are pseudoscalar point particles dual to charges. Vortex number non-conservation means that they fluctuate in the condensate ground state, and events at which 2D vortices emerge and/or disappear are represented by instantons corresponding to magnetic monopoles in Euclidean space-time. In 3D systems, with $d > \xi$, the monopoles are, instead solitons at the ends of open vortices in the condensate. The superinsulating state is thus a plasma of magnetic monopoles to whom vortices delgate the property of being a superfluid Bose condensate.

Building on the approach of, we unravel a distinct electrostatic property of a superinsulator, the expulsion of static electric fields, that can lead to an electric analogue of the Meissner effect. We derive current-voltage (I-V) characteristics in superinsulators and report transport measurements in NbTiN disordered films, revealing the expected linear confinement and asymptotic freedom of Cooper pair dipoles.

The electrodynamics of the dual charge-monopole ensemble is governed by the Maxwell-Dirac equations,

$$
\partial_\mu F^{\mu \nu} = j_\nu^q, \quad \partial_\mu F^{\mu \nu} = j_\nu^m,
$$

where $\tilde{F}^{\mu \nu} = (1/2)\epsilon^{\mu \nu \alpha \beta} F_{\alpha \beta}$ is the dual electromagnetic field tensor, $j_\nu^q$ is the charge current, $j_\nu^m$ is the magnetic monopole current. Greek letters denote space-time indices, Latin letters stand for spatial indices, $\epsilon^{\mu \nu \alpha \beta}$ is the totally antisymmetric tensor, and we use natural units $c = 1, \hbar = 1$. To gain a first, qualitative insight into the electric Meissner effect, consider an infinite superinsulating slab with thickness $d$ of order of $\xi$, the superconducting coherence length, harboring an ensemble of vortices aligned along the $z$ axis, perpendicular to the slab surfaces and subject to a dc electric field $E$ parallel to, say, the $x$ axis. The electric field $E \equiv (E_x, 0, 0)$ generates a magnetic monopole current circulating around the slab.
in the $y$-direction, which, according to the dual Ampère law $\nabla \times \mathbf{E} = -j_0$, creates a shielding electric field in the opposite direction to the applied one. Since, in the condensate, monopole motion is dissipationless, the entire external applied electric field is screened, hence the electric Meissner effect.

To formulate a quantitative description of the electric Meissner state we have to take into account that, in a superinsulator, the fundamental excitations are strings with the linear tension $\sigma$\textsuperscript{16,17}. The strings can be either closed pure gauge excitations (the analogues of glue-balls in QCD), or open, representing dipoles comprising bound Cooper pairs and anti-Cooper pairs (the analogue of mesons in QCD). The energy to create the former is $\Delta_0 = mv^2$, with $m$ the gauge-field mass and $v = 1/\sqrt{\epsilon \mu}$ ($\epsilon$ the dielectric permittivity and $\mu$ the magnetic permeability of the normal insulating state) is the light velocity in the medium. Open strings have the typical length $d_s = \mathcal{O}(\sqrt{v/\sigma})$ and a gap of the order $\mathcal{O}(\sqrt{v/\sigma})$.

Both $\Delta_0$ and the related string tension $\sigma$ can be expressed via the ultraviolet (UV) cutoff $\Lambda_0 = v/r_0$ as functions of the electron charge, $e^2$, the monopole fugacity, $z$, and $v$\textsuperscript{16,17}. The quantity $z(mv)^2$ plays the role of the superfluid density in superconductivity.

The formal derivation of the electric Meissner state is based, thus, on a dual analogue of the London equations\textsuperscript{10,11,21,22}, describing the electrodynamics of strings connecting charged particles. The vector current of point-like particles is replaced by the tensor current $\mathcal{J}^{\mu\nu}$ of strings. Accordingly, the vector potential $A_\mu$, related to the electromagnetic field via $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$, and coupling to the particle vector current, is replaced by the fundamental field tensor $\mathcal{F}_{\mu\nu}$, coupled directly to the string tensor current $\mathcal{J}^{\mu\nu}$\textsuperscript{10,11,12}. The dual Maxwell-London equations in vacuum become (see Methods)

$$ (\Box + m^2) \mathcal{F}^{\mu\nu} = 2\Lambda^2 \mathcal{J}^{\mu\nu}, \quad (2) $$

$$ \partial_\alpha (\partial_\mu \mathcal{F}^{\mu\nu}) - \partial_\nu (\partial_\mu \mathcal{F}^{\mu\alpha}) = 0, \quad (3) $$

where $m = \Lambda/e$, $\Lambda = \Lambda_0\sqrt{\pi}/4$. In a superinsulator with the light speed $v$, Eqs. (2,3) are generalized to (see Methods)

$$ (\Box + (mv)^2) \mathcal{F}^{\mu\nu} = 2(\Lambda v)^2 \mathcal{J}^{\mu\nu}, \quad \Box \equiv \partial_\alpha \partial_\beta - v^2 \nabla^2, \quad (4) $$

where $\mathcal{J}^{\mu\nu} = v_0 \mathcal{J}^{\mu\nu}, \mathcal{J}^{ij} = (1/v) J^{ij}$, and time derivatives and factors $\mathcal{F}^{0i}$ in [3] must be substituted by $(1/v) \partial_0$ and $(1/v) \mathcal{F}^{0i}$, respectively.

Equation (4) implies the electric Meissner effect for applied voltages below a critical value $V_{\text{cl}}$ where strings do not form in the sample (see below). In the static situation with no strings, Eq. (4) reduces to

$$ (\nabla^2 - (mv)^2) \mathcal{F}^{0i} = 0. \quad (5) $$

Let us consider a standard illustrative setup of a superinsulator occupying the $z > 0$ half-space and a uniform electric field $E_{\text{ext}}$ applied in the $x$ direction. The solution for the field within the superinsulator with the boundary condition $\mathcal{F}^{0i}(z = 0) = E_{\text{ext}}$ is

$$ \mathcal{F}^{0i}(z) = E_{\text{ext}} \exp(-z/\lambda_{\text{el}}) \quad (6) $$

where $\lambda_{\text{el}} = 1/(vm)$ is the electric dual of the London penetration length. The suppression of the electric field on scales $z \gg \lambda_{\text{el}}$ can be understood as its screening by the circular monopole currents it induces. One sees that, on spatial scales $\gg \lambda_{\text{el}}$, a superinsulator is a forbidden region for electric fields, which skim the superinsulator surface tangentially but not enter its interior. In terms of the static dielectric permittivity, $\varepsilon_{\text{si}}$, it means that, for a bulk superinsulator, $\varepsilon_{\text{si}} = 0$.

The screening length satisfies $\lambda_{\text{el}} \lesssim d_s \approx \sqrt{v/\sigma}$ and increases with increasing temperature, diverging at the deconfinement transition\textsuperscript{20}. For TiN $d_s \approx 60 \mu m$, while for NbTiN parameters, one can estimate $\lambda_{\text{el}} \lesssim 0.13 \text{ nm}$ (see below). In typical experiments on TiN and NbTiN films, where $d \lesssim \xi$, with $\xi$ the superconducting coherence length, the concept of circular monopole currents does not apply, since the physical size of the “bulk” monopoles exceeds the film thickness. In this case, the screening may occur via monopole instantons, as discussed above. These may be also interpreted as a dual analogue of the phase slip effect in superconductors, but a detailed microscopic theory of electric screening in thin films is beyond the scope of this work and will be discussed elsewhere.

The quantitative electromagnetic response of a superinsulator can be obtained by exploiting the mapping onto compact QED\textsuperscript{10,11} derived in [3]. The experimental results described below, on the other hand, will shed light on the fundamental strong coupling gauge theory concept of asymptotic freedom\textsuperscript{12} via desktop experiments on superinsulating systems. The interaction strength in the compact QED model of superinsulators\textsuperscript{3} is set by an effective coupling constant

$$ e^2_{\text{eff}} = 4\alpha f(\kappa)/\varepsilon g, \quad (7) $$

where $\alpha$ is the fine structure constant and the Ginzburg-Landau ratio $\kappa$, the dielectric permittivity $\varepsilon$ and the dimensionless conductance $g$ driving the SIT (occurring at $g = g_c = 1$) are the material parameters. The two relevant energy scales, the gauge-field mass $m$\textsuperscript{20} and the string tension $\sigma$, are then expressed via this dimensionless coupling as

$$ m \propto \frac{\hbar}{vr_0} e^2_{\text{eff}} \frac{1}{4e^2} \exp\left(-\frac{\pi}{4e^2}\right), \quad \sigma = \frac{e^2_{\text{eff}} v^2}{4r_0} m \propto \frac{\hbar}{r_0^2} e^2_{\text{eff}} \exp\left(-\frac{\pi}{4e^2}\right) \quad (8) $$

Pure gauge compact QED is not renormalizable since no infrared (IR) fixed point exists. Coupling the model to the dynamical matter, however, develops a non-trivial IR fixed point\textsuperscript{21,22}. The same happens in the framework of the effective field theory of the SIT. For dynamical Cooper pairs, as shown in\textsuperscript{23}, this fixed point is set by the condensation of vortices. The exact form of
Resistance ($\Omega$) vs. reversed temperature $1/T$ for fridges of various length $L$. Dashed line shows $R \propto \exp(1/T)$. Inset: Experimental setup. The Si substrates with AlN buffer layer is shown with light gray and Hall bridge of NbTiN is dark grey. The square gold contacts are given in yellow. All lateral sizes are given in millimeters. The bridges lengths (i.e. distances between measuring electrodes) shown in the legend in panel (b). (b) Same data as in (a) but replotted as conductance $G = 1/R$ vs. $T$ in log-line scale. Few curves are omitted to avoid overcrowding. Dotted lines are fits using a two dimensional Coulomb gas model \cite{ref} that generalizes the BKT formula for conductance $G \propto \exp\left[-(T/T_{\text{dec}}-1)^{1/2}\right]$ by incorporating a self-consistent solution to the effects of electrostatic screening \cite{ref}, where the screening length $\lambda$ and $T_{\text{dec}}$ enter as fitting parameters. For all bridges we find $T_{\text{dec}} \approx 400 \text{mK}$. (c) Same data as in (b) but for temperature renormalized as $(T/T_{\text{dec}}-1)^{1/2}$. Solid line corresponds to case of infinite electrostatic screening length $\lambda \to \infty$. Inset: Screening length as function of the bridge length $L$. Symbols correspond to BKT-fits of $G(T)$ (dotted lines in (b) and (c)) and dashed line is the eye guide.

the renormalization flow equations near this fixed point will be the subject of a forthcoming publication but, for present purposes, the important point is that, within the compact QED regime, this flow displays the phenomenon of asymptotic freedom\cite{ref}, since the dimensionless coupling $c_{\text{eff}}^{2}$ flows to smaller values in the ultraviolet (UV) limit. As a consequence, the string tension, the linear interaction between charges, decreases at short scales. In QCD it is impossible to probe the interior of baryons other than indirectly, via collisions, as presently done at the LHC. In the superinsulators, however, if the string length scale $d_{s}$ is large enough and the regime $\lambda_{A} < d_{s}$ is feasible, one could probe the interior of “superinsulating mesons” by experimenting on superinsulating samples with linear dimensions $L < \lambda_{\text{string}}$. In this case the “interior” interaction, the intermediate regime where $\lambda_{A} < r < d_{s}$ would be a weak screened Coulomb potential. This would lead to a strong size-dependence of the $I$-$V$ response, where superinsulating hyperactivated behaviour in sufficiently large samples changes to a metal-like behaviour for small enough samples, with $L \lesssim d_{s}$.

Let us now focus on the interaction energy $U(r)$ between charges separated by a distance $r$, as it arises from the compact QED model of superinsulation (we henceforth restore physical units),

$$
U(r) = \frac{e^{2}r}{24r} + a \ln \left( \frac{\lambda_{A}}{r_{0}} \right) - K_{0} \left( \frac{r}{\lambda_{A}} \right),
$$

where the second term is the so-called Lüscher term\cite{ref} and the third term, containing the MacDonald function $K_{0}$, is the screened 2D Coulomb potential, reducing to $a \ln(r/r_{0})$ for $r \ll \lambda_{A}$ while decaying exponentially at $r \gg \lambda_{A}$. For $r > d_{s}$, the Lüscher term is negligible, so that $U(r_{0}) \approx 0$. Near the SIT, the strength of the Coulomb potential becomes\cite{ref}

$$
a = \frac{4e^{2}}{2\pi n_{e} d}(f(\kappa)/g) .
$$

(10)

The exact form of $f(\kappa)$ is given in\cite{ref} and is not relevant here.

Before deriving $I$-$V$ characteristics of a superinsulator, we relate the superinsulating parameters to experimentally observable quantities. First we note that a string of length $d_{s}$ has the energy $\sqrt{\sigma\epsilon}$, hence $2eV_{c1} = \sigma L$ is the energy needed to form a chain of electric strings stretching across the entire system. Accordingly, the critical electric field

$$
E_{c1} = \frac{\sigma}{2e},
$$

(11)

corresponds to the lower critical field $B_{c1}$ in superconductors and is the minimum electric field necessary for a string to penetrate all the way through the sample. For $E > E_{c1}$ the mixed state of a superinsulator forms, the Meissner state is realized for $E < E_{c1}$. A second important quantity is the deconfinement temperature $T_{\text{dec}}$, which marks the transition between linearly bound
Free charges below the deconfinement temperature $T_{\text{dec}}$ and unbound charges above it. This transition belongs in the Berezinskii-Kosterlitz-Thouless (BKT) universality class and occurs via instanton condensation.\cite{Berezinskii82} According to\cite{Kosterlitz83},

$$k_B T_{\text{dec}} = 8a = 8 \frac{f(k)}{g} \varepsilon_c$$

(12)

where $\varepsilon_c = 4e^2/2\pi \varepsilon_0 d$ is the characteristic bare Coulomb energy of the Cooper pair and $a$ is defined by Eq. (10).

A consistent theory of the string response to an applied voltage, accounting for its interaction with the environmental degrees of freedom, should be constructed in the spirit of the non-equilibrium generalization of the Feynman-Vernon influence functional\cite{Feynman63} as devised in\cite{Kosterlitz83} and will be the subject of a forthcoming publication.

Here, for the sake of transparency, we adopt a standard phenomenological approach, see, for example\cite{Kosterlitz83}, in which the current is evaluated as $I \propto 2\sigma n V$, where $V$ is the applied voltage and $n_t$ is the equilibrium density of free charges. First of all, let us remark that, for $E_{\text{ext}} < E_{c1} \equiv \sigma/2e$, the maximum of the potential lies always at a distance $L$ corresponding to the sample size and, thus, the current is simply proportional to the number of charges activated over the barrier ($\sigma - 2eE_{\text{ext}}) L$,

$$I \propto V \exp \left( \frac{-\sigma(T) L}{k_B T} \right), \quad V < V_{c1} = \frac{\sigma(T) L}{2e}$$

(13)

which, in the thermodynamic limit $L \to \infty$, implies an infinite resistance.

For $E_{c1} < E_{\text{ext}}$, the sign of the linear barrier is changed by the sufficiently strong applied electric field. The result is that a maximum at the distance $r^*$ determined by the equation $K_1'(r^*/\lambda_d) = (2eE_{\text{ext}} - \sigma) \lambda_d/a$. Therefore, two regimes are possible at $E_{c1} < E_{\text{ext}}$. The first is realized for small samples such that

$$K_1'(L/\lambda_d) > \frac{4e \Delta V}{d_s V_{c1}}$$

(14)

where $\Delta V = V - V_{c1}$ and we have used the estimate $d_s = \hbar v/k_B T_{\text{dec}}$ for the string scale. In this case, the equation for the maximum has no solution and the potential for $\lambda_d < r < L$ is essentially flat. This is the asymptotic free region, where charges effectively do not interact, and we expect thus a metallic saturation at the lowest temperatures. Given that the ratio $d_s/\lambda_d > 1$ but not typically extremely large\cite{Kosterlitz83} and that the function $K_1(x) \approx \exp(-x)/\sqrt{x}$ at $x \gg 1$, the typical sample size for which this metallic behaviour may emerge is $O(d_s)$, although it can become quite larger if measurements are taken just above $V_{c1}$.

In the opposite regime, the total energy $U_{\text{ext}}$ of the charge-anticharge pair following from\cite{Kosterlitz83} is

$$U_{\text{ext}} = a \ln(r/r_0) - Fr,$$

(15)

where $F = 2eE_{\text{ext}} - \sigma$ is the effective force pulling the charge-anticharge pair apart. The saddle point $r^*$ of this potential, controlling the activated current, is $r^* = a/F$, so that the energy barrier is $U^* = U_{\text{ext}}^* (r^*) = a \ln(a/Fr_0) - 1$. In equilibrium, the ionization rate $R \propto \exp(-U^*/k_BT)$ and the recombination rate, $R_r$, of the $\pm$ charges, are equal. Since $R_r \propto n_+ n_- = n_t^2$, then with logarithmic accuracy $n_t = \sqrt{R} \propto \exp(-U^*/2k_BT) \propto (Fr_0/a)^{2/(2k_BT)}$, and Eq. (12) yields

$$I \propto (V - \sigma L/2e)^{1+T_{\text{dec}}/16T}, \quad V_{c1} < V < V_{c2} \equiv L \left[ \frac{\sigma(T)}{2e} + \frac{k_B T_{\text{dec}}}{2.718 r_0} \right]$$

(16)

where the upper critical voltage $V = V_{c2} \equiv V_{\text{ib}}$ is the voltage where the energy barrier $U^*$ vanishes, the $I$-$V$ curve experiences a jump and the system switches from superinsulation into a normal insulating state. In the field interval such that $V_{c1} < V < V_{c2}$, the superinsulator is in the mixed state, where an ensemble of electric strings penetrates the system. This is the analogue to the mixed, or Abrikosov state in superconductors.

Measurements are taken on a polycrystalline 10 nm thin NbTiN film obtained by atomic layer deposition (ALD)\cite{Hebard} at temperature 350 °C. The temperature dependencies of the resistances are measured on a sample patterned by photolithography into Hall bar with the width 40 μm, see inset in Fig. [4]. The chosen geometry enables measurements on bridges of different lengths. The experiment is carried out in a $^3\text{He}/^4\text{He}$ dilution refrigerator. The resistance is measured by two-terminal circuit under the low-frequency, $f \sim 1$ Hz, ac voltage, $V \sim 100 \mu$V, in the linear regime as verified by the direct measurements of the films’ I-$V$s.

The Arrhenius plots of the resistances per square $R_0(T)$ versus inverse temperature $1/T$ for bridges of different lengths of the same NbTiN film are shown in Fig. [4]. At $T \gtrsim 0.8$ K, $R_0(T)$ all curves collapse on top of each other. For temperatures below $T_{\text{dec}} = 400$ mK the long bridges show the hyperactivated behaviours typical of superinsulators.\cite{Kosterlitz83, Kosterlitz84} The shortest bridge of size 0.2 mm, however, clearly shows a metallic-like saturation at very low temperatures. Using the measured critical temperature of 400 mK and the known dielectric constant $\varepsilon = 800$ and coherence length $r_0 = \xi = 10$ nm of NbTiN, one can estimate the string size as $d_s \approx \hbar v/k_BT_{\text{dec}}$, obtaining $d_s \approx 0.13$ mm in remarkable quantitative agreement with the experimental result in Fig. [4].

Shown in Fig. [4] are the same resistive curves plotted as $G = 1/R_0$ vs. temperature. Fitting them by the standard BKT critical formula $G = G_0 \exp(-\text{const}/\sqrt{T/T_{\text{dec}} - 1})$, yields indeed $T_{\text{dec}} \approx 400$ mK. The deviations from the BKT criticality at lowest temperatures in Fig. [4] indicate the point when the typical size of the dissipating charge-anticharge dipole matches the size of the finite electrostatic screening length $\lambda$ which appears in the normal insulator due to screening effect mediated by the free charges (at the BKT transition $\lambda \to \infty$, and as a practical matter, the perfect criticality would have been observed for $\lambda > 10^{15}$).
Following the treatment of\(^\dagger\) and recalling that \(G \propto n_f\), where \(n_f\) is the density of free charge carriers, one can obtain \(\Lambda\) using it as an adjusting parameter to fit the curves in Fig. 1. When doing this, one has to take into account the image forces that appear in the Poisson equation leading to \(\lambda = \beta_1 L\), where \(\beta_1\) is a numerical coefficient that serves as an adjusting parameter and \(L\) is the length of the bridge. The result is shown in the inset in Fig. 1, perfectly confirming that \(\lambda \propto L\).

Figure 2 presents the threshold behaviors of the \(I-V\) curves corresponding to bridges of different length for two samples having the resistances \(R_1 = 0.2\) and \(2 \Omega\), respectively at \(T = 2\) K. The current jumps span a range of a few orders of magnitude and become less sharp upon shortening the distance between electrodes. The threshold voltage \(V_{th}\) exhibits a linear dependence upon \(L\), see insets in Fig. 2, implying that the threshold electric field \(E_{th} = V_{th}/L\) is independent on the distance between electrodes in full accordance with (19). For the low-resistance sample, shown in figure 2a, the string tension can be estimated from \(d_s \simeq 0.13\)mm as \(\sigma \simeq 4.15 \times 10^{-21}\)J/m = 26 meV/m, which leads to a contribution \(\sigma/2e = 1.3 \times 10^{-5}\)V/mm to the slope. This is negligible with respect to the Coulomb contribution (second term in (16)), which amounts to a predicted slope of 0.63 V/mm, again in remarkable quantitative agreement with the measured slope 0.78 V/mm. The IV characteristics below the threshold, measured at 50 mK, are linear. This has to be compared with the theoretically predicted power-law with exponent \(T_{dec}/16T = 400\)mK/16 \(\times 50\)mK = 1.5, in fair agreement.

The high-resistance sample displays a richer structure in its IV characteristics, shown in figure 2b. The two smallest bridges are clearly too small for the lower threshold \(V_{c1}\) to be identifiable in the available experimental voltage range. This is not so, however, for the larger bridges, which clearly show both thresholds \(V_{c1}\) and \(V_{c2} = V_{th}\). This detection of \(V_{c1}\), eq. (11), can be considered as the first ever direct measurement of a string tension. While the upper threshold clearly scales linearly with the bridge length, it is harder to argue conclusively in favor of a linear behaviour for the lower threshold \(V_{c1}\), given that only three points are available, see lower inset in figure 2b. If we do so, however, we can subtract the string contribution 1 of \(V_{c1}\) from the total slope 2.1 to obtain the purely Coulomb contribution 1.1. From (10) we can then estimate the deconfinement temperature 692 mK for this sample, which implies an exponent \(T_{dec}/16T = 692\)mK/16 \(\times 50\)mK = 1.86 for the IV characteristics in the mixed state \(V_{c1} < V < V_{c2}\). The measured exponents for the three largest bridges are 1.6, 1.8 and 2, respectively, yet again a remarkable quantitative agreement.

To conclude, let us mention two other effects we anticipate for superinsulators. First, as mentioned above, for applied electric fields above the critical value \(E_{c1} = \sigma/2e\), we expect the superinsulator to be in its mixed state, with penetration of electric flux tubes along an entire dimension of the sample. The interaction of such electric flux tubes is given by the massive Poisson kernel \(1/((mv)^2 - \Delta)^5\). If we apply an in-plane electric field along the x-axis to a film of thickness \(d \approx \lambda_d\) this reduces to a repulsive potential \(U(y) = \exp(-y/\lambda_d)\) in the transverse direction between tubes carrying flux of the same sign. The electric flux tubes will thus arrange themselves in a 1D lattice of spacing \(\lambda_d\), with the total number of penetrating tubes in a sample of width \(W\) being the nearest integer function \([W/\lambda_d]\). When the electric field is applied in the perpendicular direction to a film of thickness \(d > 1/\sqrt{\sigma}\) we expect, instead, an arrangement of electric flux tubes in an Abrikosov lattice. The full quantitative description of the expected periodic array of strings and the calculation of the corresponding dielectric permittivity will be the subject of a forthcoming publication. Finally, since bulk superinsulators expel electric fields, we expect a stable electric levitation phenomenon when a thick superinsulating slab is placed above a sufficiently charged capacitor plate.

Much remains to be studied, both theoretically and experimentally. Our present results, however, show that, already at the level of a first crude model one can obtain remarkable quantitative agreement between experimental data and theoretical predictions.

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**APPENDIX**

**A. Electric London equations**

The tensor current describing the motion of strings replaces the vector current encoding the motion of point particles. Correspondingly, the vector potential \(A_\mu\) related to the original electromagnetic field strength via \(F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu\) is replaced by a fundamental tensor field \(\mathcal{F}_{\mu\nu}\) coupled to the tensor current \(J^{\mu\nu}\). The associated field strength is given by the three-tensor \(H_{\mu\nu\rho} = \partial_\mu \mathcal{F}_{\nu\rho} + \partial_\nu \mathcal{F}_{\rho\mu} + \partial_\rho \mathcal{F}_{\mu\nu}\). The Euclidean action for electric and magnetic fields in the superinsulating vortex
Threshold voltage of bridges of different length. The I-V curves of two different NbTiN samples with (a) \( R_c(T=2\,\text{K}) = 0.2\,\text{M\Omega} \) and (b) \( R_c(T=2\,\text{K}) = 2\,\text{M\Omega} \) taken at the same temperature \( T = 50\,\text{mK} \). Different colors correspond to different bridge’s length \( L \) (distance between electrodes). Upper inset: Dependence of the threshold voltage on the bridge’s length. Dashed line shows \( V_{1b} = 0.78 \cdot L \) in (a) and \( V_{1b} = 2.1 \cdot L \) in (b). Bottom inset: The dependence of the kink voltage \( V_k \) which we associate with the critical voltage \( V_{c1} \) upon the bridge length.

Condensate acquires the form

\[
S = \int d^4x \left[ \frac{1}{12\Lambda^2} \mathcal{H}_{\mu\nu\rho} \mathcal{H}^{\mu\nu\rho} + \frac{1}{4\epsilon_0^2} F_{\mu\nu} F_{\mu\nu} + \imath \frac{1}{2} F_{\mu\nu} \mathcal{J}_{\mu\nu} \right],
\]

where \( \Lambda = \Lambda_0 \sqrt{\epsilon} / 4 \), with \( \Lambda_0 = 1/\kappa_0 \) the UV cutoff and \( \kappa_0 \) the magnetic monopole quantum fugacity. Varying with respect \( F_{\mu\nu} \) yields the equations of motion

\[
\partial_\mu \mathcal{H}^{\mu\nu\alpha\beta} + m^2 \mathcal{F}^{\alpha\beta} = 2\Lambda^2 \mathcal{J}^{\alpha\beta},
\]

which, in turn reduce to Eqs. 23 of the main text. To make these equations plausible we now show how they reduce to the usual Maxwell equations in the limit of vanishing monopole fugacity, \( \kappa \to 0 \), when the vortex condensate disappears. In this limit we have \( \Lambda \to 0 \) and finiteness of the partition function \( Z = \int \mathcal{D}F_{\mu\nu} \exp(-S) \) requires that the 3-tensor field strength vanishes, \( \mathcal{H}_{\mu\nu\rho} = 0 \). This is generically the case for “pure gauge configurations” \( F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu = F_{\mu\nu} \) so that, in this limit, the partition function of classical Maxwell electrodynamics is recovered,

\[
Z \to \int \mathcal{D}A_\mu \exp \left( -\int d^4x \left[ \frac{1}{4\epsilon_0^2} F_{\mu\nu} F_{\mu\nu} + \imath A_\mu j_\mu \right] \right),
\]

with the current \( j^\mu = \partial_\nu \mathcal{J}^{\mu\nu} \) describing the now free charges at the end of the original strings. This shows what is the relation between electrodynamics within the vortex condensate and the familiar Maxwell theory. The latter is recovered as a “pure gauge” version of the former.

B. Electric field equations in materials with the light velocity \( v = c/\sqrt{\frac{\varepsilon}{\mu}} \)

The above “electric London equations” hold if the speed of light is \( c \) (1 in our natural units), which is reflected in the Lorentz invariance of the action (17) and the Lorentz covariance of the equations of motion (2). For real materials, however, this is surely not the case. The (Euclidean) action for electromagnetic fields in a linear material is given by

\[
S_{\text{linear}} = \frac{1}{2\epsilon_0^2} \int dt \, d^3x \left( \frac{1}{v} E^2 + v B^2 \right),
\]

where \( v = 1/\sqrt{\frac{\varepsilon}{\mu}} \) is the light velocity in the material, expressed in terms of the dielectric permittivity \( \varepsilon \) and the magnetic permeability \( \mu \). Due to \( v < 1 \), the symmetry of this action is now restricted to non-relativistic Galilean invariance. Correspondingly, the Galilean-invariant action for a superinsulating thin film of this material when vortices condense is

\[
S = \int dt \, d^3x \left[ \frac{1}{12} \frac{6}{v^2} \sqrt{\mathcal{H}^{ij} \mathcal{H}_{ij} + \frac{1}{v} \mathcal{H}_{0ij} \mathcal{H}_{0ij}} + \frac{1}{4\epsilon_0^2} \left( v \mathcal{F}_{ij} \mathcal{F}_{ij} + 2 \frac{1}{v} \mathcal{F}_{0ij} \mathcal{F}_{0ij} \right) + \frac{1}{2} \mathcal{F}_{\mu\nu} \mathcal{J}_{\mu\nu} \right]
\]

where \( v \) is the light velocity expressed in terms of material parameters of the normal insulating state. The equations of motion (2) and (3) are modified to

\[
\Box + (mv^2)^2 \mathcal{F}_{\mu\nu} = 2 \left( \Lambda \nu^2 \right) \mathcal{J}_{\mu\nu}
\]

\[
\Box = \partial_\mu \partial_\mu - v^2 \nabla^2.
\]
where $\Delta_{\Sigma 1} = \frac{mv^2}{a}$ represents the energy gap of the superinsulator, $\mathcal{F}^{\alpha i} = v \mathcal{F}^{i \alpha}$, $\mathcal{F}^{ij} = (1/v) \mathcal{F}^{ji}$, and every time derivative and factor $\mathcal{F}^{\alpha i}$ in the gauge condition \[ must be substituted by $(1/v) \partial_0$ and $(1/v) \mathcal{F}^{\alpha i}$, respectively.

C. Sample preparation

The sample growths were carried out in a custom-made viscous flow ALD reactor in the self limiting regime. A constant flow of ultrahigh-purity nitrogen (UHP, 99.999%, Airgas) at $\sim 350$ sccm with a pressure of $\sim 1.1$ Torr was maintained by mass flow controllers. An inert gas purifier (Entegris GateKeeper) was used to further purify the N$_2$ gas by reducing the contamination level of H$_2$, CO, and CO$_2$ to less than 1 ppb and O$_2$ and H$_2$O to less than 100 ppt. The thermal ALD growth of the AlN/NbTiN multilayer was performed using alternating exposures to the following gaseous reactants with the corresponding total ALD cycles $240$ to produce the NbTiN film. The growth rate of the AlN/NbTiN multilayer was performed using alternate exposures to the following gaseous reactants with the corresponding total ALD cycles $240$ to produce the NbTiN film. The sample parameters are as follows: the diffusion constant $D \sim 0.25 cm^2 s^{-1}$, the superconducting coherence length $\xi(0) \sim 5$ nm, the density of states as $n \sim 4 \cdot 10^{21} cm^{-3}$, NbTiN film was patterned using photolithography and plasma etching into resistivity bar 50 $\mu$m wide and with 100, 250, and 100 $\mu$m separation between additional perpendicular bars. On this additional bars were making gold contacts. The chosen design allows for two-probe resistivity measurements of regions with different length (2.4, 1.34, 0.55, 0.44, 0.32, 0.2 $\mu$m). Measurements of the temperature dependencies of the resistance were carried out in $^3$He/$^4$He dilution refrigerator Triton400. Two-probe technique was used with the 100 $\mu$V and 1 Hz. The chosen values of applied voltage guaranteed that the resistance was obtained in the linear response regime by the direct measurements of current-voltage characteristics. AC measurements were performed using SR830 lock-in amplifier, dc measurements were performed using nanovoltmeter Agilent 34420, current was transformed into voltage with Agilent 34420, current was transformed into voltage with SR570 low-noise current preamplifier with filtration system.

D. Samples parameters and measurements

Data sharing not applicable to this article as no datasets were generated or analyzed during the current study.

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