Radiative Corrections to Higgs Boson Masses in the Next-to-Minimal Supersymmetric Standard Model

T. Elliott, S.F. King and P.L. White

Physics Department, University of Southampton,
Southampton SO9 5NH, UK.

Abstract

We perform a systematic study of radiative corrections to the masses of the Higgs bosons in the minimal supersymmetric standard model (MSSM) augmented by a single gauge singlet, the so-called next-to-minimal supersymmetric standard model (NMSSM). Our method is based on the one-loop effective potential and includes effects of top quark, squark, Higgs and Higgsino loops. We discuss the renormalisation group flows of Yukawa couplings and the upper bound on the lightest CP-even neutral Higgs boson mass as a function of the heavier stop mass and top mass. We then give a general discussion of Higgs boson phenomenology including radiative corrections. We survey as much of the parameter space of the Higgs sector of the NMSSM as is practicable, and analyse the full spectrum of Higgs masses and couplings in these regions of parameter space. Characteristic signatures of the NMSSM such as light charged bosons and weakly coupled neutral scalars are discussed, as are the relative sizes of the various radiative corrections. The MSSM is also discussed as a limiting case of the NMSSM for comparison.
1 Introduction

The most widely studied extension to the standard model is supersymmetry (SUSY) \[1\], in which the particles of the theory are supplemented by the inclusion of their superpartners. SUSY requires the introduction of a non-minimal Higgs sector with at least two doublets \[2\]. An important question is how heavy the lightest neutral CP-even supersymmetric Higgs boson, \(h\), can be within the framework of supersymmetric grand unified theories (SUSY GUTs). In SUSY GUTs all the Yukawa couplings are constrained to remain perturbative in the region \(M_{SUSY} \sim 1\) TeV to \(M_{GUT} \sim 10^{16}\) GeV. This constraint provides a maximum value at low energies for those Yukawa couplings which are not asymptotically-free, and is obtained from the renormalisation group (RG) equations together with the boundary conditions that the couplings become non-perturbative at \(M_{GUT}\) – the so-called “triviality limit”. In the minimal supersymmetric standard model (MSSM) \[1, 2\], the triviality limits provide a useful bound on the top quark mass \(m_t\). The upper bound on the \(h\) mass, \(m_h\), in the MSSM, including radiative corrections, has recently been the subject of much discussion \[3, 4, 5, 6, 7\].

However the MSSM is not the most general low energy manifestation of SUSY GUTs. It is possible that SUSY GUTs give rise to a low energy theory which contains an additional gauge singlet field, the so called next-to-minimal supersymmetric standard model (NMSSM) \[8, 9, 10\]. In all supersymmetric models, including those with non-minimal Higgs sectors, there is an upper bound on the lightest CP-even scalar Higgs mass, which is generally larger for non-minimal models \[11, 12\]. Although the NMSSM involves a single gauge singlet, the estimates of the bound are applicable to a model with an arbitrary number of Higgs singlets, since the singlet fields may always be redefined so that only one of them couples to the doublets and only this field contributes to the upper two-by-two mass matrix which gives the bound. This argument cannot be applied to models which contain extra non-singlet degrees of freedom (although these usually give a lower value for the bound) or to the description of the whole spectrum in any model other than the NMSSM.

There has recently been much interest in radiative corrections to Higgs boson masses in the NMSSM \[13, 14, 15, 16, 17, 18, 19, 20\]. The two approaches that have been
considered are the renormalisation group (RG) approach [16, 17, 18], and the one-loop effective potential approach [13, 14, 15, 19, 20]. In the RG approach one derives an effective low energy Higgs potential at a low energy scale $\mu$ from SUSY matching conditions at a scale $M_{SUSY}$. The squarks and other sparticles are usually assumed to be degenerate at $M_{SUSY}$, and the logarithmic radiative corrections are efficiently summed by the RG equations for the case of either one [16, 17] or two [18] light Higgs doublets. This approach, although convenient, fails to pick up non-logarithmic corrections, and in general becomes complicated when a general squark spectrum is considered [7]. The one-loop effective potential [21, 22] has been used in the MSSM to estimate the radiative corrections due to a general squark spectrum [4] and these calculations have been repeated in the NMSSM [14, 15, 19]. However in the NMSSM there are other corrections due to Higgs loops which may become large due to the presence of large couplings and trilinear soft parameters, and it is one of the purposes of the present paper to consider such contributions to the one-loop effective potential.

In this paper we shall perform a systematic study of radiative corrections to Higgs boson masses in the NMSSM, using the one-loop effective potential. We shall consider the effects of all particles which couple through relatively large (order one) couplings in the model including the effects of loops of top quarks, stop squarks, Higgs bosons, and Higgsinos. The top and stop corrections have been calculated before [14, 15, 19] and we include them in our analysis for completeness. The Higgs and Higgsino contributions to the effective potential have not been calculated before, and are dealt with here by numerical techniques since their contribution has no simple analytic form.

Having developed the techniques for dealing with radiative corrections in the NMSSM we then apply these techniques in two different ways. The first application is to the problem of the bound on the lightest CP-even Higgs boson mass, which was discussed above. More generally we give a phenomenological discussion of Higgs boson masses and couplings over as much of the parameter space of the NMSSM as it is feasible to consider. Our goal is to understand the behaviour of the Higgs boson spectrum as each of the parameters in turn is varied, and to give some examples of phenomenological signatures which would enable the NMSSM to be distinguished from the MSSM. It is important to stress that we shall not
impose GUT scale constraints on the soft SUSY-breaking parameters, so that our analysis is of a general low energy phenomenological nature.

We shall also present an RG analysis of the dimensionless Yukawa couplings of the model which will turn out to provide a useful guide to the typical values that these parameters may take at low energy.

The plan of the rest of this paper is as follows. Section 2 introduces the NMSSM and the tree-level potential and Higgs boson mass matrices. Section 3 presents an RG analysis of the dimensionless couplings between $M_{GUT}$ and $M_{SUSY}$. In section 4 we discuss in general terms the radiative corrections to the Higgs mass matrix using the one-loop effective potential. In section 5 we use numerical methods to obtain an upper bound on the lightest CP-even neutral Higgs boson mass as a function of the various parameters. This is compared to the case in the minimal model. In section 6 we discuss Higgs boson masses and couplings in some detail and discuss the effect of radiative corrections. Section 7 concludes the paper.

2 The Next-to-Minimal Supersymmetric Standard Model

The most commonly considered supersymmetric model, the MSSM, has, in addition to the usual matter and gauge particle content, a Higgs sector containing two Higgs doublet superfields $H_1$ and $H_2$. The superpotential is then of the form

$$W_{MSSM} = h_u Q u^c H_2 + h_d Q d^c H_1 + h_e L e^c H_1 - \mu H_1 H_2, \quad (2.1)$$

where generation indices are understood, $H_1 H_2 = H_1^0 H_2^0 - H_1^- H_2^+$, with $H_1^T = (H_1^0, H_1^-)$, $H_2^T = (H_2^+, H_2^0)$, and the rest of the notation is conventional. In this model the physical Higgs spectrum consists of two CP-even and one CP-odd neutral scalars, and the lightest neutral scalar $h$ has a mass which is bounded at tree level by $m_h^2 \leq m_Z^2$ [2].

In the NMSSM [8,9,10] the particle content of the MSSM is supplemented by a gauge singlet superfield, $N$. The superpotential is given by

$$W_{NMSSM} = h_u Q u^c H_2 + h_d Q d^c H_1 + h_e L e^c H_1 + \lambda N H_1 H_2 - \frac{k}{3} N^3. \quad (2.2)$$

The cubic term in $N$ is necessary to avoid a Peccei-Quinn symmetry which would force the existence of a light pseudo-Goldstone mode once the symmetry is broken. However there
still remains a $\mathbb{Z}_3$ symmetry under which all the matter and Higgs superfields $\Phi$ transform as $\Phi \to \alpha \Phi$ where $\alpha^3 = 1$ [9]. Note that we have eliminated $\mu$. This can be justified on the grounds of naturalness, and its inclusion would only complicate our analysis. The gauge singlet field acquires a vacuum expectation value (vev) which plays the role of the mass parameter $\mu$ in the MSSM.

Unlike the case in non-supersymmetric models, radiative corrections do not generate large masses of order the cut-off scale (which for SUSY GUTS is essentially the unification scale $10^{16}\text{GeV}$), although the inclusion of singlets may cause the destabilisation of the hierarchy if there are strong couplings to super-heavy particles such as Higgs colour triplets [23]. This is however strongly dependent on the structure of the model at the GUT scale, and so we shall not discuss it here. Recently it has also been noted that the inclusion of non-renormalisable operators suppressed by powers of the Planck mass may lead to the introduction of non-logarithmic divergences, which in turn can generate large mass terms for the Higgs bosons of the electroweak theory [24]. This effect can only occur in the case of a model which has a singlet, and requires that gravity violate the $\mathbb{Z}_3$ symmetry which is respected by the renormalisable operators of the theory. The coefficients of such operators have not yet been calculated and their size and importance is unclear.

In our analysis we shall drop all quark and lepton Yukawa couplings apart from that of the top quark so that the superpotential reduces to

$$W_{\text{NMSSM}} \approx h_t Q H_2 t^c + \lambda N H_1 H_2 - \frac{k}{3} N^3,$$

where the superfield $Q^T = (t_L, b_L)$ contains the left-handed top and bottom quarks, and $t^c$ contains the charge conjugate of the right-handed top quark. Adopting the usual convention of using the same symbols for both component Higgs fields and superfields, the fields $H_1$, $H_2$, and $N$ develop vevs which may be assumed to be of the form

$$< H_1 > = \begin{pmatrix} \nu_1 \\ 0 \end{pmatrix}, \quad < H_2 > = \begin{pmatrix} 0 \\ \nu_2 \end{pmatrix}, \quad < N > = x = r \nu,$$

where $\nu_1$, $\nu_2$ and $x$ are real, $\sqrt{\nu_1^2 + \nu_2^2} = \nu = 174 \text{ GeV}$, and $\tan \beta = \nu_2/\nu_1$. The low energy physical spectrum of the Higgs scalars consists of three CP-even neutral states, two CP-odd neutral states, and two charged scalars. A third CP-odd state is a Goldstone
mode which becomes the longitudinal component of the $Z^0$, while a further two charged degrees of freedom become those of the $W^\pm$.

In addition to the potential which can be derived from the superpotential in the usual manner, there is a soft supersymmetry breaking potential of the form

$$V_{\text{soft}} = h_t A_t \tilde{Q} \tilde{c} H_2 - \lambda A_N H_1 H_2 - \frac{k}{3} A_k N^3$$

$$+ m^2_{H_1} |H_1|^2 + m^2_{H_2} |H_2|^2 + m^2_N |N|^2$$

$$+ m^2_\tilde{t} |\tilde{t}|^2 + m^2_B |\tilde{b}|^2 + m^2_\tilde{Q} |\tilde{Q}|^2$$

which leads to the full low energy Higgs potential $V_0$, where

$$V_0 = \frac{i}{2} \lambda_1 (H_1^\dagger H_1)^2 + \frac{i}{2} \lambda_2 (H_2^\dagger H_2)^2 + (\lambda_3 + \lambda_4) (H_1^\dagger H_1)(H_2^\dagger H_2)$$

$$- \lambda_4 |H_2^\dagger H_1|^2 + \lambda_5 |N|^2 |H_1|^2 + \lambda_6 |N|^2 |H_2|^2$$

$$+ \lambda_7 (N^2 H_1 H_2 + N^2 H_1^* H_2^*) + \lambda_8 |N|^4$$

$$+ m_1^2 |H_1|^2 + m_2^2 |H_2|^2 + m_3^2 |N|^2$$

$$- m_4 (H_1 H_2 N + H.c.) - \frac{1}{3} m_5 (N^3 + H.c.)$$

The quartic couplings $\lambda_i$ and the mass parameters $m_i$ must satisfy the following boundary conditions at $M_{\text{SUSY}}$

$$\lambda_1 = \lambda_2 = \frac{(g_2^2 + g_1^2)}{4}, \quad \lambda_3 = \frac{(g_2^2 - g_1^2)}{4}$$

$$\lambda_4 = \lambda^2 - \frac{g_2^2}{2}, \quad \lambda_5 = \lambda_6 = \lambda^2,$$

$$\lambda_7 = -\lambda k, \quad \lambda_8 = k^2, \quad (2.7)$$

$$m_1^2 = m_{H_1}^2, \quad m_2^2 = m_{H_2}^2, \quad m_3^2 = m_N^2,$$

$$m_4 = \lambda A_\lambda, \quad m_5 = k A_k$$

where $g_1, g_2$ are the usual $U(1)$ and $SU(2)$ gauge couplings of the standard model.

Since we have three minimisation conditions, $\frac{\partial V_0}{\partial \nu_i} = 0$ and $\frac{\partial V_0}{\partial x} = 0$, we may eliminate three of the unknown parameters of the theory, which we choose to be $m_1, m_2$, and $m_3$, in favour of the three vevs. The remaining masses $m_4$ and $m_5$ are related to the parameters $A_\lambda$ and $A_k$ at $M_{\text{SUSY}}$ as above. Because we know $\nu$ (from the $W$ mass) the Higgs sector of this model is now parametrised in terms of the six parameters $\lambda$, $k$, $\tan \beta$, $r$, $m_4$, and $m_5$. 6
We shall take the parameters $\lambda$, $k$, $A_\lambda$ and $A_k$ to be real, and $\lambda$, $k$ to be positive, which is a sufficient condition for the vacuum to conserve CP and leads to a choice of vacuum in which all the vevs $x, \nu_1, \nu_2$ are real and positive [9].

The range of the parameters is restricted by the condition that the vacuum does not break QED in the Higgs sector, which is not automatic in the NMSSM, and is equivalent to the condition that $m_c^2 \geq 0$, where $m_c$ is the mass of the physical charged Higgs $H^\pm$. Another similar problem is that the vacuum may break QCD in the squark sector, but this does not occur for sufficiently small $A_t$ [9, 25]. Slepton vevs will not be discussed here since they can be avoided by an appropriate choice of soft parameters.

The full $10 \times 10$ mass squared matrix $M^2$ for the scalar fields is simple to derive by expressing all of the fields in terms of their real scalar and pseudo-scalar parts

\[
\begin{align*}
H_1^0 &= \frac{1}{\sqrt{2}} (\varphi_1 + i\varphi_4) \\
H_2^0 &= \frac{1}{\sqrt{2}} (\varphi_2 + i\varphi_5) \\
N &= \frac{1}{\sqrt{2}} (\varphi_3 + i\varphi_6) \\
H_1^- &= \frac{1}{\sqrt{2}} (\varphi_7 - i\varphi_9) \\
H_2^+ &= \frac{1}{\sqrt{2}} (\varphi_8 + i\varphi_{10})
\end{align*}
\] (2.8)

and then using

\[
M^2_{ij} = \frac{\partial^2 V_0}{\partial \varphi_i \partial \varphi_j}
\] (2.9)

When evaluated at the vevs,

\[
\begin{align*}
< \varphi_1 > &= \sqrt{2} \nu_1 \\
< \varphi_2 > &= \sqrt{2} \nu_2 \\
< \varphi_3 > &= \sqrt{2} x \\
< \varphi_i > &= 0 \quad \forall i \neq 1, 2, 3
\end{align*}
\] (2.10)

$M^2$ breaks down to consist entirely of zeros except in one $3 \times 3$ block for the CP-even, one $3 \times 3$ for the CP-odd, and two $2 \times 2$ blocks for the charged mass matrices (the CP-odd matrix and each of the charged matrices have one zero eigenvalue corresponding to a
Goldstone mode, and the two non-zero charged eigenvalues are equal). Thus the tree-level neutral CP-even (scalar) mass squared symmetric matrix, in the basis \{H_1, H_2, \nu\}, is

\[
M^2 = \begin{pmatrix}
2\lambda_1 \nu_1^2 & 2(\lambda_3 + \lambda_4) \nu_1 \nu_2 & 2\lambda_5 \nu_1 \\
2(\lambda_3 + \lambda_4) \nu_1 \nu_2 & 2\lambda_2 \nu_2^2 & 2\lambda_6 \nu_2 \\
2\lambda_5 \nu_1 & 2\lambda_6 \nu_2 & 4\lambda \nu_3^2 - m_5^2
\end{pmatrix}
\]

\[
+ \begin{pmatrix}
\tan \beta [m_4 \nu - \lambda \nu_3 x^2] & -[m_4 \nu - \lambda \nu_3 x^2] & -\frac{\nu_2}{\nu_3} [m_4 \nu - 2\lambda \nu_3 x^2]
\end{pmatrix}
\]

Similarly the tree-level neutral CP-odd (pseudoscalar) mass-squared symmetric matrix, in the basis \{H_1, H_2, N\}, is

\[
\tilde{M}^2 = \begin{pmatrix}
\tan \beta [m_4 \nu - \lambda \nu_3 x^2] & [m_4 \nu - \lambda \nu_3 x^2] & \frac{\nu_2}{\nu_3} [m_4 \nu + 2\lambda \nu_3 x^2]
\end{pmatrix}
\]

\[
\begin{pmatrix}
\frac{\nu_2}{\nu_3} [m_4 \nu + 2\lambda \nu_3 x^2] & \nu_3 [m_4 \nu + 2\lambda \nu_3 x^2] & 3\nu_3 x [m_4 \nu - 2\lambda \nu_3 x^2]
\end{pmatrix}
\]

Finally the tree-level charged mass-squared matrix, in the basis \{H_1, H_2\}, is

\[
M_c^2 = \begin{pmatrix}
\tan \beta & 1 \\
1 & \cot \beta
\end{pmatrix}
\]

\[
(m_4 \nu - \lambda \nu_3 x^2 - \lambda \nu_3 \nu_2)
\]

In the limit \(\lambda, k \to 0, x \to \infty\) with \(\lambda x\) and \(k x\) held fixed, the \(N\) components do not mix with the \(H_1, H_2\) components in the mass matrices in (2.11) and (2.12). This is just the MSSM limit of the NMSSM.

**3 Renormalisation Group Analysis**

Now let us consider which values may be taken by the dimensionless couplings \(\lambda, k\) and \(h_t\). Above some scale at which supersymmetry becomes a good symmetry, and defining \(t = \log \mu\), where \(\mu\) is the renormalisation scale, the RG equations for these couplings [25] are given by

\[
8\pi^2 \frac{\partial \lambda}{\partial t} = (2\lambda^2 + k^2 + \frac{3}{2} h_t^2 - \frac{3}{2} g_3^2 - \frac{1}{2} g_1^2) \lambda
\]

\[
8\pi^2 \frac{\partial k}{\partial t} = (3\lambda^2 + 3k^2) k
\]

\[
8\pi^2 \frac{\partial h_t}{\partial t} = \left(\frac{1}{2} \lambda^2 + 3h_t^2 - \frac{8}{3} g_3^2 - \frac{3}{2} g_2^2 - \frac{13}{18} g_1^2\right) h_t
\]

where \(g_3\) is the QCD coupling. Following the analysis of reference [26], they may be written in the suggestive form

\[
8\pi^2 \frac{\partial}{\partial t} \left(\frac{\lambda^2}{h_t^2}\right) = (3\lambda^2 + 2k^2 - 3h_t^2 + \frac{16}{3} g_3^2 + \frac{4}{9} g_1^2) \left(\frac{\lambda^2}{h_t^2}\right)
\]

\[
8\pi^2 \frac{\partial}{\partial t} \left(\frac{k^2}{h_t^2}\right) = (5\lambda^2 + 6k^2 - 6h_t^2 + \frac{16}{3} g_3^2 + \frac{3}{2} g_2^2 + \frac{13}{9} g_1^2) \left(\frac{k^2}{h_t^2}\right)
\]
These have three fixed points in the gaugeless \( g_i = 0, \forall i \) limit:

\[
\left( \frac{\lambda^2}{h_t^2} \right) = 1, \quad \left( \frac{k^2}{h_t^2} \right) = 0;
\]

\[
\left( \frac{\lambda^2}{h_t^2} \right) = 0, \quad \left( \frac{k^2}{h_t^2} \right) = 1;
\]

\[
\left( \frac{\lambda^2}{h_t^2} \right) = 3 \left( \frac{3}{4} \right), \quad \left( \frac{k^2}{h_t^2} \right) = 3 \left( \frac{3}{8} \right).
\]

Of these fixed points, only the last is infra-red stable. This is illustrated in Figure 1a, where the paths traced out by a number of equally spaced points in the \( \lambda/h_t - k/h_t \) plane with \( g_i = 0 \) and \( h_t = 10 \) at the GUT scale are plotted as the energy scale runs from the GUT scale of \( 10^{16}\text{GeV} \) to \( 10^3\text{GeV} \). A point in this plane will flow rapidly towards the central valley which passes through all three fixed points, and then more slowly along it to the stable fixed point. From this figure, it is clear that, regardless of the high energy values, the low energy values of the couplings are likely to be somewhere in this one dimensional region, but they may well be nearer an unstable than a stable fixed point.

So far, however, we have neglected the effects of the large QCD coupling in equation (3.2); this may be important as discussed in [26]. When we include the effect of gauge couplings in our numerical analysis, a different picture emerges. Although at high energy the non-zero QCD coupling is small enough not to make much difference, and the flow is much as before, at lower energies this contribution dominates, and the effect is that points flow towards and along the valley for a short distance before being forced towards the origin at lower energies. However, it is noticeable that the region of parameter space to which the couplings flow in this figure is still one dimensional. This is illustrated in Figure 1b, which again has \( h_t = 10 \) at the GUT scale.

Both of these plots have been done in the case where the couplings \( \lambda \) and \( k \) are large; if they are small \((<1 \text{ at high energy})\), then the flow towards the fixed point in the gaugeless limit is much slower, and with gauge couplings included most points flow rapidly to the origin remaining within a region bounded by the one dimensional valley described above and the \( x \)- and \( y \)-axes, as shown in Figure 1c, where \( h_t = 1 \) at the GUT scale. Finally, if \( \lambda, k \) are greater than \( h_t \) at high energies, then they flow very rapidly towards the origin and end up on the boundary of this region.
In summary, for fixed $h_t$ the renormalisation group flows give a curve in parameter space which can be attained consistent with triviality from the assumption of large couplings at the GUT scale, but any point between this curve and the origin is also possible given appropriate initial values of the couplings.

This analysis, with the assumption that the high energy behaviour of the theory is described by a GUT model, or at least that perturbative physics continues up to the unification scale of order $10^{16}$GeV, means that we can find maximum values of $\lambda$ and $k$ consistent with a given value of $h_t$. If we take $k = 0$ (in order to maximise the low energy value of $\lambda$), it is possible to solve for $\lambda_{\text{max}}$ (the value of $\lambda$ at which it just becomes non-perturbative at the unification scale) as a function of $h_t(1\text{TeV})$ (or equivalently of $m_t$ and $\tan \beta$). This has been done [11,12], and the results are reproduced in Figure 2.

4 Radiative Corrections

There have been a number of attempts to calculate radiative corrections to the tree-level results presented in section 2 [13, 14, 15, 16, 17, 18, 19, 20]. In the MSSM, the dominant radiative corrections are those resulting from loops of top quarks and stop squarks. These corrections have been calculated using a range of diagrammatic, RG and one-loop effective potential techniques. Radiative corrections due to the top quark and stop squark for the upper $2 \times 2$ part of the mass matrix are identical to those of the MSSM if we replace $\mu$ by $\lambda x$, and these have been studied in a simple approximation [14]. The full analytic corrections to other components of the scalar and pseudo-scalar Higgs mass matrices have also been calculated [15, 19, 20].

Logarithmic effects from loops of light Higgs bosons have been studied using a full RG analysis below a hard SUSY breaking scale [16, 18]. However these analyses may be criticised on two counts. Firstly, the assumed spectrum of degenerate sparticles at some SUSY scale and Higgs bosons degenerate with the top quark at some lower scale is oversimplified. Secondly, the RG analysis fails to pick up non-logarithmic radiative corrections depending on soft trilinear couplings. These corrections are known to be significant in the squark sector, and the corresponding corrections from the Higgs sector may also be important. It is possible to complicate the RG approach to take account of some of these
effects, by systematically decoupling heavy particles below their mass thresholds, and introducing finite shifts in boundary conditions, but the elegance of the RG approach is then lost [7]. For the cases where the effects of finite diagrams are of interest, the simplest approach is to perform a one-loop effective potential calculation. In reference [19] we considered a hybrid approach in which squark corrections to all Higgs boson masses were calculated in the framework of the one-loop effective potential, and these corrections were grafted on to our previous RG analysis [18] involving light Higgs bosons and a degenerate top quark.

In this section we shall abandon the RG approach completely and perform a full one-loop effective potential analysis of radiative corrections involving the top quark, stop squarks, Higgs bosons, and Higgsinos. We shall introduce the use of the effective potential for calculating radiative corrections to mass matrices, present corrections due to the top quark and stop squarks, and discuss the calculation of Higgs and Higgsino corrections.

4.1 One-Loop Effective Potential

The full one-loop corrected scalar potential is given by

\[ V_1 = V_0 + \Delta V_1, \]  

(4.1)

where \( V_0 \) is the tree level potential, and \( \Delta V_1 \) is

\[ \Delta V_1 = \frac{1}{64\pi^2} \text{Str} \mathcal{M}^4 \left( \log \frac{\mathcal{M}^2}{\mu^2} - \frac{3}{2} \right). \]  

(4.2)

\( \mu \) is the \( \overline{\text{MS}} \) renormalisation scale, and the supertrace is a trace over all fields which couple through the mass matrix and includes a factor \((-1)^{2J} (2J + 1)\) so that a Weyl fermion acquires a factor -2, a real scalar a factor 1, and we must remember appropriate colour and flavour factors. \( \mathcal{M}^2 \) is the field dependent mass-squared matrix, in which the fields left after differentiating \( V_0 \) to obtain \( \mathcal{M}^2 \) are not replaced by their vevs.

The one-loop minimisation conditions are, of course, \( \frac{\partial V_1}{\partial \phi_i} = 0, \ i = 1, 2, 3. \) After
replacing the fields by their vevs, we have explicitly

\[
2m_1^2 \nu_1 + 2\lambda_1 \nu_1^3 + 2(\lambda_3 + \lambda_4) \nu_1^2 \nu_2 + 2\lambda_5 x^2 \nu_1 + 2\lambda_7 x^2 \nu_2 - m_4 x \nu_2 + \sqrt{2} \left. \frac{\partial \Delta V_1}{\partial \phi_1} \right|_{\text{vevs}} = 0
\]

\[
2m_2^2 \nu_2 + 2\lambda_2 \nu_2^3 + 2(\lambda_3 + \lambda_4) \nu_1^2 \nu_2 + 2\lambda_6 x^2 \nu_2^2 + 2\lambda_7 \nu_1 x^2 - m_4 \nu_1 x + \sqrt{2} \left. \frac{\partial \Delta V_1}{\partial \phi_2} \right|_{\text{vevs}} = 0
\]

\[
2m_3^2 x + 2\lambda_5 \nu_1^3 x + 2\lambda_6 x^2 \nu_2 + 4\lambda_7 \nu_1 \nu_2 x + 4\lambda_8 x^3 - 2m_4 \nu_1 \nu_2 - 2m_5 x^2 + \sqrt{2} \left. \frac{\partial \Delta V_1}{\partial \phi_3} \right|_{\text{vevs}} = 0
\]

from which we can see that

\[
m_i^2 = m_i^2|_{\text{TL}} - \frac{1}{\nu_i \sqrt{2}} \left. \frac{\partial \Delta V_1}{\partial \phi_i} \right|_{\text{vevs}} \quad (4.3)
\]

where \(m_i^2|_{\text{TL}}\) are the values of \(m_i^2\) in terms of the vevs as evaluated at tree-level. Having thus supplied the vevs, we extremise the potential by choosing the soft masses to satisfy equation (4.4). This does not guarantee that this extremum is a minimum of the potential; this can only be done by calculating the physical Higgs spectrum and ensuring that all the masses squared are positive.

The minimum so constructed need not be the global minimum of the Higgs potential. Thus we examine the potential containing these (radiatively corrected) mass parameters \(m_i^2\) to determine whether alternative minima with zero vevs for one or more of the scalar fields \(\phi_1, \phi_2,\) and \(\phi_3\) exist. In fact the case where only one of these is zero does not occur for any of our choices of parameters. We test such possible minima by comparing the value of the potential, including top and stop corrections only for simplicity, to discover whether these alternative minima are preferred; if so we discard this point in parameter space.

The radiatively corrected matrices for the CP-even, CP-odd and charged Higgs scalars may be calculated in a straightforward manner. In the basis defined by (2.8), the correction to the mass-squared matrix is

\[
\delta M^2_{ij} = \frac{\partial^2 \Delta V_1}{\partial \phi_i \partial \phi_j}.
\]

In fact, this is only approximately true due to Higgs self–energy corrections; these are expected to be small for the lightest Higgs bosons [5]. However we should recall that the formula for \(M^2\) given in equation (2.9) contains implicit dependence on the mass parameters \(m_i^2\), and so will involve dependence on the first derivative of \(\Delta V\). Thus for
example the radiative corrections to the scalar mass-squared matrix $M^2$ whose tree-level value is given in equation (2.11) are of form

$$\delta M^2_{ij} = \left( \frac{\partial^2 \Delta V_1}{\partial \phi_i \partial \phi_j} - \frac{1}{\nu_i \sqrt{2}} \frac{\partial \Delta V_1}{\partial \phi_j} \delta_{ij} \right) \left|_{\text{vevs}} \right. .$$

(4.6)

The last term represents the shift in the mass matrix due to the radiative corrections to the minimisation conditions.

4.2 Squark and Top Corrections

The corrections to the mass matrices due to top quark and stop squark loops have been calculated elsewhere [14, 15, 19, 20], so we simply give the results, in the notation of [19], without further explanation. The field dependent top quark and stop and sbottom squark mass matrices are given by

$$M^2_{\text{top}} = h_t^2 (|H_2^0|^2 + |H_2^+|^2)$$

(4.7)

$$M^2_{\text{sq}} = \begin{pmatrix}
 m_Q^2 + h_t^2 |H_2^0|^2 & h_t \lambda N H_1^0 + h_t A_t \bar{H}_2^0 & -h_t^2 \bar{H}_2^0 H_2^+ & 0 \\
 h_t \lambda N H_1^0 + h_t A_t H_2^0 & m_T^2 + h_t^2 (|H_2^0|^2 + |H_2^+|^2) & h_t \lambda N H_1^- - h_t A_t H_2^+ & 0 \\
 -h_t^2 H_2^0 \bar{H}_2^+ & h_t \lambda N H_1^- - h_t A_t \bar{H}_2^+ & m_Q^2 + h_t^2 |H_2^+|^2 & 0 \\
 0 & 0 & 0 & m_B^2
\end{pmatrix},$$

(4.8)

in the basis $\{\tilde{t}_L, \tilde{t}_R^c, \tilde{b}_L, \tilde{b}_R^c\}$, and a bar denotes complex conjugation. The sbottom squarks appear because they contribute to the charged Higgs mass. One eigenvalue of (4.8) is field independent and may be discarded. Notice that the form of this equation implies that

$$A_t + \lambda_x \cot \beta \leq \frac{|m_{\tilde{t}_2}^2 - m_{\tilde{t}_1}^2|}{2 h_t \nu_2}$$

(4.9)

Here $m_{\tilde{t}_1}$ and $m_{\tilde{t}_2}$ are the stop squark mass eigenvalues.

After considerable algebra we obtain the following corrections to the Higgs mass matrices. The correction to the CP-odd mass-squared matrix $\tilde{M}^2$ in equation (2.12) is given by $\delta \tilde{M}^2$, where

$$\delta \tilde{M}^2 = \begin{pmatrix}
 \tan \beta & 1 & \frac{\sin \beta}{r} \\
 \frac{1}{\sin \beta} & \cot \beta & \frac{\cos \beta}{r} \\
 \frac{1}{\cos \beta} & \cos \beta & \frac{\sin \beta \cos \beta}{r^2}
\end{pmatrix} \Delta^2,$$

(4.10)

with

$$\Delta^2 = \frac{3}{16 \pi^2} h_t^2 (\lambda_x) A_t f(m_{\tilde{t}_1}^2, m_{\tilde{t}_2}^2),$$

(4.11)
and the function $f$ is defined by

$$f(m_{t_1}^2, m_{t_2}^2) = \frac{1}{m_{t_2}^2 - m_{t_1}^2} \left( m_{t_1}^2 \log \left( \frac{m_{t_1}^2}{\mu^2} \right) - m_{t_2}^2 \log \left( \frac{m_{t_2}^2}{\mu^2} \right) - m_{t_1}^2 + m_{t_2}^2 \right)$$  \hspace{1cm} (4.12)

The correction to the CP-even mass-squared matrix $M^2$ in equation (2.11) is given by $\delta M^2$, where

$$\delta M^2 = \begin{pmatrix} \Delta^2_{11} & \Delta^2_{12} & \Delta^2_{13} \\ \Delta^2_{12} & \Delta^2_{22} & \Delta^2_{23} \\ \Delta^2_{13} & \Delta^2_{23} & \Delta^2_{33} \end{pmatrix} + \begin{pmatrix} \tan \beta & -1 & -\frac{\sin \beta}{r} \\ -\frac{1}{\cot \beta} & \cot \beta & -\frac{\cos \beta}{r} \\ -\frac{\sin \beta}{r} & -\frac{\cos \beta}{r} & \sin \beta \cos \beta \end{pmatrix} \Delta^2,$$  \hspace{1cm} (4.13)

where the $\Delta^2_{ij}$ are given by

$$\Delta^2_{11} = \frac{3}{8\pi^2} h_t^4 \nu_2^2 (\lambda x)^2 \left( \frac{A_t + \lambda x \cot \beta}{m_{t_2}^2 - m_{t_1}^2} \right)^2 g(m_{t_1}^2, m_{t_2}^2)$$

$$\Delta^2_{12} = \frac{3}{8\pi^2} h_t^4 \nu_2^2 (\lambda x) \left( \frac{A_t + \lambda x \cot \beta}{m_{t_2}^2 - m_{t_1}^2} \right) \left( \log \left( \frac{m_{t_2}^2}{m_{t_1}^2} \right) + \frac{A_t (A_t + \lambda x \cot \beta)}{m_{t_2}^2 - m_{t_1}^2} \right) g(m_{t_1}^2, m_{t_2}^2)$$

$$\Delta^2_{13} = \frac{3}{8\pi^2} h_t^4 \nu_2^2 (\lambda x) (\nu_1) \left( \frac{A_t + \lambda x \cot \beta}{m_{t_2}^2 - m_{t_1}^2} \right)^2 g(m_{t_1}^2, m_{t_2}^2)$$

$$\Delta^2_{22} = \frac{3}{8\pi^2} h_t^4 \nu_2^2 \left( \log \left( \frac{m_{t_1}^2}{m_{t_2}^2} \right) + \frac{2A_t (A_t + \lambda x \cot \beta)}{m_{t_2}^2 - m_{t_1}^2} \log \left( \frac{m_{t_2}^2}{m_{t_1}^2} \right) \right)$$

$$\Delta^2_{23} = \frac{\cos \beta}{r} \Delta^2_{12}$$

$$\Delta^2_{33} = \frac{3}{8\pi^2} h_t^4 \nu_2^2 (\lambda x)^2 \left( \frac{A_t + \lambda x \cot \beta}{m_{t_2}^2 - m_{t_1}^2} \right)^2 g(m_{t_1}^2, m_{t_2}^2)$$

and the function $g$ is defined by

$$g(m_{t_1}^2, m_{t_2}^2) = \frac{-1}{m_{t_2}^2 - m_{t_1}^2} \left( (m_{t_1}^2 + m_{t_2}^2) \log \left( \frac{m_{t_2}^2}{m_{t_1}^2} \right) + 2m_{t_2}^2 - 2m_{t_1}^2 \right)$$  \hspace{1cm} (4.15)

Finally, the correction to the charged mass-squared matrix $M_c^2$ in equation (2.13) is $\delta M_c^2$, where

$$\delta M_c^2 = \begin{pmatrix} \tan \beta & 1 \\ 1 & \cot \beta \end{pmatrix} \Delta_c^2$$  \hspace{1cm} (4.16)
and

$$\Delta^2 = \frac{3}{16\pi^2} \sum_{i,t_1,t_2,b_1} m_i^2 \left( \log \left( \frac{m_i^2}{m_t^2} \right) - 1 \right) \frac{\partial^2 m_i^2}{\partial H_1^i \partial H_1^j} \bigg|_{\text{vevs}} \quad (4.17)$$

$$\frac{\partial^2 m_i^2}{\partial H_1^i \partial H_2^j} \bigg|_{\text{vevs}} = - \frac{h_1^4 v_2^2 (\lambda x)^2 \cot \beta}{(m_{t_1}^2 - m_{b_1}^2)(m_{t_2}^2 - m_{t_2}^2)} - \frac{h_1^2 (\lambda x) A_t}{(m_{t_1}^2 - m_{t_2}^2)} \quad (4.18)$$

$$\frac{\partial^2 m_{b_1}^2}{\partial H_1^i \partial H_2^j} \bigg|_{\text{vevs}} = - \frac{h_1^4 v_2^2 (\lambda x)^2 \cot \beta}{(m_{b_1}^2 - m_{t_1}^2)(m_{b_1}^2 - m_{b_1}^2)}$$

4.3 Higgs and Higgsino Corrections

Calculation of the corrections due to Higgs loops is much more cumbersome because the field dependent Higgs mass-squared matrix in equation (2.9), used to calculate $\Delta V_1$ in equation (4.2), is $10 \times 10$. Since it is impossible to determine the eigenvalues of general matrices larger than $4 \times 4$ analytically, resort to numerical techniques is essential. As is clear from (4.6), we are interested in the first and second derivatives of $\Delta V_1$ evaluated at the vevs, and these are given by

$$\frac{\partial \Delta V_1}{\partial \phi_i} \bigg|_{\text{vevs}} = \frac{1}{32\pi^2} m_i^2 \frac{\partial m_i^2}{\partial \phi_i} \left( \log \frac{m_i^2}{\mu^2} - 1 \right) \bigg|_{\text{vevs}}, \quad (4.19)$$

$$\frac{\partial^2 \Delta V_1}{\partial \phi_i \partial \phi_j} \bigg|_{\text{vevs}} = \frac{1}{32\pi^2} \frac{\partial m_i^2}{\partial \phi_i} \frac{\partial m_i^2}{\partial \phi_j} \log \frac{m_i^2}{\mu^2} \bigg|_{\text{vevs}} + \frac{1}{32\pi^2} m_i^2 \frac{\partial^2 m_i^2}{\partial \phi_i \partial \phi_j} \left( \log \frac{m_i^2}{\mu^2} - 1 \right) \bigg|_{\text{vevs}}, \quad (4.20)$$

where $\{m_\alpha^2\}$ is the set of the 10 eigenvalues of $M_{ij}^2$, and we implicitly sum over $\alpha$. It is straightforward numerically to obtain $m_\alpha^2 |_{\text{vevs}}$ from $M_{ij}^2 |_{\text{vevs}}$, and since the first and second derivatives of $m_\alpha^2$ evaluated at the vevs may also be obtained numerically, the problem reduces to a routine numerical task.

Similarly, since the Higgsino mass matrix is $5 \times 5$, Higgsino loop contributions must be calculated numerically as above. We have neglected the mixing with gauginos for simplicity, as in this way we can eliminate two soft mass parameters. Moreover, if we include gaugino corrections, then we should include all gauge boson corrections for consistency, particularly since there are typically cancellations between particles and their superpartners, as we shall see later. The field dependent Higgsino mass term, with Higgsinos written as 4-component
Majorana spinors, in the basis $\psi = (\tilde{H}_1^0, \tilde{H}_2^0, \tilde{N}, \tilde{H}_1^- , \tilde{H}_2^+)^T$, is given by $\frac{1}{2} \bar{\psi} M_{\tilde{H}} \psi$, with $M_{\tilde{H}}$ defined by

$$M_{\tilde{H}} = \text{Re} M' - i\gamma^5 \text{Im} M',$$  

(4.21)

where

$$M' = \begin{pmatrix}
0 & \lambda N & \lambda H_2^0 & 0 & 0 \\
\lambda N & 0 & \lambda H_2^0 & 0 & 0 \\
\lambda H_2^0 & \lambda H_1^0 & -2kN & -\lambda H_2^+ & -\lambda H_1^- \\
0 & 0 & -\lambda H_2^+ & 0 & -\lambda N \\
0 & 0 & -\lambda H_1^- & -\lambda N & 0
\end{pmatrix}. $$  

(4.22)

The contribution to $\Delta V_1$ from Higgsinos is then given by

$$-\frac{1}{128\pi^2} \text{Tr} \left\{ (M_{\tilde{H}} M_{\tilde{H}}^\dagger)^2 \left[ \log \left( \frac{M_{\tilde{H}} M_{\tilde{H}}^\dagger}{\mu^2} \right) - \frac{3}{2} \right] \right\},$$  

(4.23)

where the trace is over Dirac as well as internal indices. Note that there is a relative factor of 2 between this result and that of [22] because we are working with Majorana spinors.

The numerical calculations are of course very computationally intensive; however, numerical inaccuracies should generally be less than about 1GeV, particularly in regions of parameter space where all of the particles are light. It appears unlikely that the inclusion of two-loop effects or removing any of the approximations in the calculations would give corrections significant relative to our uncertainty in the parameters.

### 5 The Bound on the Lightest CP-even Higgs

An upper bound on the lightest neutral CP-even scalar in the NMSSM may be obtained from the real symmetric $3 \times 3$ neutral scalar mass squared matrix by using the fact that its minimum eigenvalue is less than or equal to the minimum eigenvalue of its upper $2 \times 2$ block with the result [10]

$$m_h^2 \leq M_Z^2 + (\lambda^2 v^2 - M_Z^2) \sin^2 2\beta.$$  

(5.1)

The upper bound on $m_h$ is determined by the maximum value of $\lambda(M_{\text{SUSY}})$, henceforth denoted $\lambda_{\text{max}}$. As discussed in section 3, the value of $\lambda_{\text{max}}$ is obtained by solving the SUSY RG equations for the Yukawa couplings $h_t$, $\lambda$ and $k$ in the region $M_{\text{SUSY}} = 1$.

16
TeV to $M_{GUT} = 10^{16}$ GeV [11,26]. There we found that for
$h_t(M_{SUSY}) = 0.5 - 1.0$, $\lambda_{max} = 0.87 - 0.70$ and for $h_t(M_{SUSY}) \to 1.06$, $\lambda_{max} \to 0$ (with $k = 0$ always).

We now wish to evaluate the bound including all the radiative corrections. The
parameters on which the radiatively corrected bound depends are $m_t$, $\sin \beta$, $r$, $\lambda$, $k$, $m_4$, $m_5$, $A_t$, $m_{\tilde{t}_1}$, and $m_{\tilde{t}_2}$ (sbottom masses do not affect the bound, because they only occur in the radiative corrections to the charged Higgs mass as discussed in reference [19]). We
adopt the following approach to these variables. $\lambda$ will, for given $m_t$ and $\sin \beta$, be given by its maximum value consistent with remaining perturbative, $\lambda_{max}$. Strictly speaking $k$ and $m_5$ should then be zero, but since this results in an axion and thus an unacceptable spectrum, we shall instead use the value 0.1 for $k$. In fact sensitivity of the bound (and of $\lambda_{max}$) to $k$ and $m_5$ is negligible when they are both small (0.1 is sufficiently small for $k$, while small for $m_5$ means of order tens of GeV). This should however be borne in mind when we come to discuss the spectrum, where the lightest scalar is quite sensitive to these variables. Of the remaining parameters, $m_t$ is taken as an input parameter, as is the heavier stop mass $m_{\tilde{t}_2}$.

The bound also depends on $r$ which may be saturated by requiring that the larger
eigenvalue of the upper $2 \times 2$ block is also an eigenvalue of the whole $3 \times 3$ matrix. This
can be done by performing a unitary transformation consisting of an upper $2 \times 2$ unitary
matrix, parametrised by an angle $\theta$, together with 33 element equal to unity and all other
elements equal to zero. The condition that the 13 element of the transformed matrix be
zero leads to the constraint

$$\cos \theta M_{13}^2 + \sin \theta M_{23}^2 = 0$$

where $M_{ij}^2$ are components of the mass matrix in (2.11). While this guarantees that
the bound must be saturated, it is possible that there is another lighter mass eigenstate
consisting of a mixture of singlet and doublet states. This state will certainly depend
strongly on the values of $k$ and $m_5$. Examples of this will be discussed in section 6.

We have thus succeeded in reducing our parameter space to four variables, $\sin \beta$, $m_{\tilde{t}_1}$,

\[ h_t(M_{SUSY}) \leq 1.06 \] is the triviality bound, which, together with $m_t = h_t(m_t) v \sin \beta$, where $h_t(m_t) \leq 1.12$, implies the bound $m_t \leq 195$ GeV.

17
$A_t$, and $m_4$. These variables will all be determined numerically using the Nelder-Mead simplex method [27] to maximise the bound as a function of the parameters, imposing the constraint (4.9).

At this point it is worth recalling that in the MSSM the relevant parameters may be taken to be $\tan \beta$ and $m_c$. Using the fact that in the minimal model the bound is always maximised for $\sin \beta = 1$, the bound is then equal to the 22 element of the mass matrix $M^2 + \delta M^2$ in (2.11) and (4.13), and is given by

$$m_h^2 \leq M_Z^2 + \frac{3}{4\pi^2} h_t^2 v^2 \log \left( \frac{m_{t_2}^2}{m_{\tilde{t}_1}^2} \right)$$

$$+ \frac{3}{4\pi^2} h_t^2 v^2 \left( \frac{A_t^2}{m_{t_2}^2 - m_{t_1}^2} - \frac{1}{2} \right) \log \left( \frac{m_{t_2}^2}{m_{\tilde{t}_1}^2} \right)$$

$$+ \frac{3}{8\pi^2} h_t^4 v^2 \left( \frac{A_t^4}{(m_{t_2}^2 - m_{t_1}^2)^2} \right) g(m_{\tilde{t}_1}, m_{\tilde{t}_2})$$

(5.3)

where the function $g$ is defined in (4.15). Since the coefficient for the $A_t^4$ term is always negative $^*$ the bound will be maximised for some non-zero value of $A_t^2$. The constraint (4.9) must also be implemented so that value of $A_t$ will not always reach the value which maximises the polynomial. Finally, we note that if we allow $m_{\tilde{t}_2}$ to become comparable in size to the top mass, then the coefficient of the second logarithm in equation (5.3) will become negative, and so the bound will be maximised when the two squarks are degenerate; this typically occurs only for very large top mass, even if we allow $m_{\tilde{t}_2}$ to be as low as 250GeV.

Let us now proceed to the discussion of our results for the bound in the NMSSM (and MSSM for comparison) with all radiative corrections included. Figure 3 shows the values of $\sin \beta$, $h_t$, and $\lambda$ for which the bound is saturated as a function of the top mass, derived as described above. Although this figure is for $m_{\tilde{t}_2} = 500\text{GeV}$, the results which can be derived for other values of $m_{\tilde{t}_2}$ are virtually identical. This figure is very similar to Figure 1 of reference [18], which was, however, derived without the inclusion of squark effects, as discussed there.

$^*$ Note that $g \leq 0$ and $g/(m_{\tilde{t}_2}^2 - m_{\tilde{t}_1}^2)^2$ remains finite as $m_{\tilde{t}_1}$ tends to $m_{\tilde{t}_2}$. 

18
Values of the bound as a function of the top mass with the heavier stop mass $m_{\tilde{t}_2}$ taken to be 500 and 1000 GeV are shown in Figure 4. It is clear from this graph that for low top mass the dependence on the stop mass is very slight (as should be obvious, as here $h_t$ is small), while for larger top masses this dependence is far greater, as the radiative corrections are very large. At tree level the decreasing value of $\lambda$ means that the bound is a monotonically decreasing function of $m_t$, but for large enough $m_{\tilde{t}_2}$ the bound actually starts increasing again with increasing top mass (both the top corrections which have a $\log(m_{\tilde{t}_2}/m_t)$ dependence and the stop corrections can become large only if $m_{\tilde{t}_2}$ is large). Numerically the bound is around 150 to 155 GeV, and is maximised for small top mass; but for top masses near the triviality bound and stop masses up to 1 TeV this same value is also approached. The radiative corrections to the bound from Higgs and Higgsino loops are relatively small, of order a few GeV, and since they are of opposite sign (including Higgs loops decreases the bound, while including Higgsinos increases it) the total effect is typically only one or two GeV. In general the optimum value of $A_t$ is of order 1.6 TeV to 2 TeV for $m_{\tilde{t}_2} = 1$ TeV and of order 600 GeV to 800 GeV for $m_{\tilde{t}_2} = 500$ GeV. The dependence of this optimum value on the top mass is very small, but generally it decreases as $m_t$ increases. Typically the value of $m_{\tilde{t}_1}$ which maximises the bound is around a third to three quarters the value of $m_{\tilde{t}_2}$.

The MSSM bound for the same values of the stop masses is also shown in Figure 4 for comparison. In this figure Higgs and Higgsino radiative corrections are included in addition to the top quark and stop squark corrections in (5.3). Note that, unlike the case in the NMSSM, the MSSM bound is a monotonically increasing function of top mass, and that it approaches the NMSSM bound for sufficiently large $m_t$.

The bound is also presented in Figures 5 but here as a function of $m_{\tilde{t}_1}$, the lighter stop mass, for $m_t = 150$ GeV and $m_{\tilde{t}_2} = 500$ GeV and 1000 GeV respectively. As can be seen from these graphs, there is a gradual rise in the bound until it reaches the maximum before falling off more rapidly. The very rapid drop in the bound as $m_{\tilde{t}_1}$ approaches $m_{\tilde{t}_2}$ is caused by the constraint (4.9), which forces $A_t + \lambda x \cot \beta$ to zero in this region. The most interesting feature of these figures is that they suggest that an arbitrarily chosen value of the lighter stop mass is not unlikely to give a value of the bound close to the maximum
possible, since the variation of the bound with \(m_{\tilde{t}_1}\) is relatively small over much of its range.

6 Higgs Boson Phenomenology

6.1 Preliminary Remarks

In this section we shall explore some of the phenomenological implications of our results. The qualitative features of certain aspects of our discussion are well known at the tree-level [9]; however we are now in a position to re-examine some of these issues in the light of our treatment of radiative corrections.

We have already emphasised that the lightest CP-even neutral scalar \(h\) (the analogue of the Higgs boson of the standard model) may be heavier in the NMSSM than in the MSSM, although as \(m_t \rightarrow 190\text{GeV}\) the two bounds coalesce as shown in Figure 4. However from a pragmatic standpoint the key question is whether \(m_h\) can exceed \(M_Z\) and so evade discovery at LEPII. This can happen in both the MSSM and the NMSSM due to radiative corrections and thus failure to observe \(h\) at LEPII cannot be interpreted as evidence for favouring the NMSSM over the MSSM. Also we observe that the NMSSM bound \(m_h < 150\text{GeV}\) means that if \(h\) is not found at LEPII it must necessarily be in the intermediate mass region \(M_Z - 2M_W\) (i.e., too heavy to be produced at LEPII but too light to decay into pairs of Ws or Zs) in both models. Thus the upper bound on \(m_h\) may not be particularly helpful in enabling us to distinguish between the MSSM and the NMSSM.

We can turn the argument of the preceding paragraph around and ask whether the observation of \(h\) would enable the MSSM and NMSSM to be distinguished. This question is analogous to that of distinguishing between the minimal standard model (MSM) and the MSSM from an observation of the Higgs boson, and has been widely considered [2,28]. The answer depends in part on measuring the \(ZZh\) coupling which contains a factor of \(R_{ZZh} = \sin(\beta - \alpha)\) relative to the usual standard model coupling, where \(\alpha\) is a mixing angle which results from diagonalising the scalar \(2 \times 2\) matrix. Although \(R_{ZZh}\) may be small, the \(ZAh\) coupling contains a factor of \(R_{ZAh} = \cos(\beta - \alpha)\) so that both couplings
cannot simultaneously be small. In the NMSSM the situation is not so simple since the $ZZh$ coupling is derived from diagonalising the scalar $3 \times 3$ matrix and so $R_{ZZh}$ is more complicated [9]. Similarly the $ZAh$ coupling, where $A$ is the lightest pseudoscalar, in the NMSSM will contain a more complicated factor $R_{ZAh}$ [9].

Given that in the NMSSM physical Higgs boson spectrum there are three CP-even scalars and two CP-odd pseudoscalars, one more in each case than in the MSSM, it may seem at first sight that the Higgses of the NMSSM, being more numerous, are therefore easier to discover. Unfortunately this is not so. The only reason why there are more neutral states is due to the complex singlet $N$, whose scalar and pseudoscalar components mix with the neutral components of $H_1$ and $H_2$. Since $N$ has no gauge couplings, this has the effect of diluting the couplings of the neutral Higgs particles: there are more of them but they all couple more weakly, making them harder to produce. However the dilution of the neutral Higgs couplings may be the key to distinguishing between the NMSSM and the MSSM, since in the NMSSM there is the possibility of a very light Higgs boson waiting to be discovered with higher statistics data from LEPI. This is an intriguing possibility, unique to the NMSSM, because although it is possible to have $R_{ZZh} = \sin(\beta - \alpha) \approx 0$ in the MSSM, this is always accompanied by $R_{ZAh} = \cos(\beta - \alpha) \approx 1$ and since small $m_h$ is associated with small $m_A$ the light decoupled Higgs scenario in the MSSM is ruled out. Thus the discovery of a light weakly coupled Higgs boson would be exactly the sort of signature which would suggest the NMSSM.

The charged scalars remain unaffected by the presence of the additional singlet, and their gauge and matter couplings in the NMSSM are identical to those of the MSSM. However in contrast to the MSSM bound $m_c > M_W$ (in the absence of radiative corrections), in the NMSSM we may have $m_c < M_W$. There will however be a constraint on how light the charged scalars can be since they will give a contribution to $b \rightarrow s\gamma$ via a penguin diagram involving an intermediate top quark line [29]. Such contributions may be partially or completely cancelled by other SUSY diagrams [30]. It would therefore be prudent to keep an open mind on the existence of light charged scalars in the range $m_c = 45 GeV - M_W$ which would be accessible to LEPII. We emphasise that this range is not consistent with the MSSM and so the discovery of charged Higgs bosons at LEPII would be, if not a smoking
gun of the NMSSM, then at least one which is loaded and cocked.

The Higgs boson mass spectrum of the MSSM at tree level may be expressed in terms of the parameter set \( \{m_A, \tan \beta\} \), so that at tree-level its properties may be expressed as contour plots in the \( m_A, \tan \beta \) plane [28]. In the NMSSM, the Higgs boson mass spectrum may be expressed in terms of the parameter set \( \{\lambda, k, m_c, A_k, \tan \beta, r\} \). These masses are conventionally plotted as a function of \( m_c \) for a particular choice of \( \lambda, k, A_k, \tan \beta, r \). There are thus four additional parameters in the NMSSM which may be taken to be \( \lambda, k, A_k \) and \( r \) which are simply not present in the MSSM. For these four additional parameters, reference [9] uses \( \lambda = 0.87, k = 0.63 \) corresponding to an approximate fixed point, and varies \( A_k \) over allowed ranges to produce bands of Higgs boson masses, for three different \( r \) values of 0.1, 1, 10. Although it is clear from Figure 1a that there is indeed a fixed point in the gaugeless limit, it is equally clear from Figures 1b and 1c that with the QCD and other gauge couplings switched on this fixed point is washed away. Instead there seems to be a fringe of preferred values of \( \lambda, k \) roughly corresponding to a quarter-circle in the \( \lambda-k \) plane.

If one is tempted to despair at the six dimensional parameter space of the NMSSM compared to the two dimensional parameter space of the MSSM, it is worth remembering that when radiative corrections are taken into consideration things get much worse. In addition to \( m_A \) and \( \tan \beta \), the MSSM Higgs sector depends on five additional parameters \( m_t, \mu, m_{\tilde{t}_1}, m_{\tilde{t}_2} \) and \( A_t \), even neglecting the bottom quark Yukawa coupling. Actually the NMSSM fares slightly better since with radiative corrections due to top quarks and squarks included there are only four additional parameters \( m_t, m_{\tilde{t}_1}, m_{\tilde{t}_2} \) and \( A_t \) since there is no \( \mu \) parameter. The Higgs and Higgsino radiative corrections do not introduce any further parameters since their tree-level masses, which are used to generate the one-loop effective potential, are determined in terms of the tree-level parameters. This would of course not be the case if we had included the full effects of Higgsino-gaugino mixing, introducing extra gaugino mass parameters. One way to reduce the number of parameters is to assume universal soft parameters at the GUT scale and this has been done in the NMSSM [9, 31]. Here we shall not restrict the soft parameters in this way, and instead attempt a crude exploration of the full parameter space without any constraints from the GUT scale apart from the usual SUSY desert assumption that no couplings blow up below \( M_{GUT} \).
We shall plot Higgs boson masses as a function of $m_c$ over the regions of $m_c$ which correspond to all Higgs boson masses being greater than zero at tree-level. This turns out to be a non-trivial restriction, usually (but not always) due to the lightest scalar mass $m_h$ diving to zero at the edges of the region. Although radiative corrections tend to expand this region, since typically they serve to increase Higgs boson masses, we use the tree-level restriction since it is the tree-level mass matrices which are used in the calculation of the one-loop effective potential. Charge breaking in the Higgs sector is trivial to check for since it just corresponds to $m_c^2$ becoming negative. The possibility of squark and slepton vevs as discussed earlier will not concern us here. However we search for alternative vacua in which one or more of $v_1, v_2, x$ take zero values corresponding to a deeper minimum of the (radiatively corrected) potential, as discussed in section 4. This can be an important constraint on the allowed range of $m_c$.

6.2 Higgs Boson Masses and Couplings

With the preliminaries over we now embark on our exploration of the ten dimensional parameter space which characterises the Higgs boson spectrum of the NMSSM. Clearly a full survey of Higgs boson masses over all of parameter space is not feasible. The best one can do is to define a set of baseline parameters and explore the effect of varying each of the parameters in turn. Our baseline parameters are

$$\lambda = 0.65 \quad k = 0.1 \quad A_k = 0 \quad \tan \beta = 1.7 \quad r = 1 \quad m_t = 150\text{GeV} \quad m_{\tilde{t}_1} = 150\text{GeV} \quad m_{\tilde{t}_2} = 500\text{GeV} \quad A_t = 0 \quad (6.1)$$

We choose these particular values of $\lambda, k, \tan \beta$ because, for a top mass of 150GeV, they are approximately those which saturate the bound on $m_h$ as discussed in section 5, and are therefore associated with heavier values of the lightest CP-even scalar. Of the remaining parameters, $A_k = A_t = 0, r = 1$, are chosen for simplicity, and $m_t = 150\text{GeV}$ is selected as a typical value. The stop masses (with $m_{\tilde{t}_2}$ chosen by convention to be the heavier) are again familiar from our discussion of the bound, and ensure that the condition on the stop eigenvalues in equation (4.9) is always satisfied over our ranges of parameters, with the top quark mass equal in mass to the lightest stop. In all our plots we shall take $A_k = 0$, in accordance with the baseline parameter set defined above. In fact, there is often very little
freedom of choice of $A_k$ since it is clear from the tree-level mass matrices (2.11) and (2.12), that allowing a large positive value of $m_5$ (recall that $m_5 = kA_k$) forces the scalar mass matrix to have a negative eigenvalue, while allowing a large negative eigenvalue forces the pseudo-scalar mass matrix to have a negative value. This is further compounded by the structure of the potential, which means that for large positive values of $m_5$ the coefficient of the $x^3$ term is large and negative, and so the alternate minimum with $x \neq 0$ and $\nu_1 = \nu_2 = 0$ will be preferred.

Figure 6a shows the Higgs boson masses for the set of parameters defined in equation (6.1), including radiative corrections due to loops of top quarks, squarks, Higgs bosons and Higgsinos. Over the charged Higgs mass range $m_c = 200 - 250 GeV$ the lightest scalar mass (lowest solid) varies between $m_h \approx 50 - 90 GeV$ while the lightest pseudoscalar mass (lower dashed) is approximately 80 GeV. For smaller values of $m_c$ than plotted the lightest scalar mass dives down to zero, while for larger $m_c$ than plotted vacuum with $v_i, x = 0$ is preferred, and these two constraints thus define the allowed region. In Figure 6b we display the amplitude of $N$ contained in the lightest scalar (dots), the next-to-lightest scalar (dot-dash) and the lightest pseudoscalar (double-dot-dash). It is clear that the lightest scalar is predominantly $N$, especially near $m_c = 235 GeV$, while the next-to-lightest scalar shows the opposite behaviour, with a very small $N$ component, especially where the lightest scalar is predominantly $N$. The lightest pseudoscalar is nearly entirely singlet. Also shown in Figure 6b are the couplings $R_{ZZh}$ (solid) and $R_{ZhA}$ (dashed) which confirm that for $m_c \approx 235 GeV$ the lightest scalar $h$ and pseudoscalar $A$ effectively decouple. This behaviour was also noted in reference [18] where it was pointed out that the next-to-lightest scalar must and does respect the bound at the point where the lightest scalar decouples. However since its mass exceeds 100 GeV over the whole region, the next-to-lightest scalar will not be visible at LEPII. The lightest scalar may be accessible to LEPI near the left-hand end of the region, however.

As an example of a region of parameter space which is excluded by LEPI, in Figure 7a we show the Higgs boson mass spectrum for a value of $r = 0.1$, with all the other parameters set equal to their baseline values as in Figure 6. Smaller $r$ is associated with smaller charged Higgs masses, so that the LEPI bound $m_c > 45 GeV$ is sufficient to exclude the
left-hand half of the region immediately. The constraint from $b \to s\gamma$ may exclude the whole of the region as discussed above, but even without this constraint the right hand half of the region is probably excluded from LEPI Higgs searches, as we now discuss. The lightest scalar has a mass of around 50 GeV or less, and the lightest pseudoscalar has a mass of about 20 GeV, and both these particles have very little singlet component, as shown in Figure 7b. The heaviest scalar and the other pseudo-scalar primarily consist of singlet, and are approximately degenerate at around half a TeV, playing no important phenomenological role at LEP. However the LEP complementary searches for $Z \to Z^*h$ and $Z \to hA$ probably eliminate the entire allowed region of masses, since both $R_{ZZh}$ (solid) and $R_{ZhA}$ (dash) in Figure 7b are quite sizeable on the right-hand part of the region. It therefore seems likely that the whole of the region is either excluded or on the point of being excluded by LEPI.

In Figure 8a we show the spectrum of Higgs bosons for $r = 5$ and the other parameters set equal to their baseline values. In this example the lightest scalar $h$ contains very little $N$ component particularly in the middle part of the range, according to Figure 8b, and couples to the $Z$ almost identically to the Higgs boson of the MSM, according to the solid line in the figure. The lightest scalar mass exceeds 100 GeV over much of the region, and so is inaccessible to LEPII except at the extreme ends of the range. The lightest pseudoscalar and second lightest scalar are both predominantly $N$. The charged scalar mass, and the heaviest scalar and pseudoscalar masses exceed 1 TeV over the entire range for this value of $r$. Thus this large $r$ parameter set produces a Higgs spectrum which will not be easy to study even at LHC/SSC.

All the above plots are for $\tan\beta = 1.7$. In Figure 9a we show the Higgs masses for $\tan\beta = 10$ with all the other parameters set equal to their baseline values. Comparing Figure 9a to Figure 6a we see that one of the main effects of choosing a larger value of $\tan\beta$ is substantially to increase the values of $m_c$ over which an allowed solution exits, as we would expect from the tree-level mass matrix (2.13). Thus we see that larger values of $m_c$ are associated with larger values of $\tan\beta$, as well as with larger values of $r$ as previously noted. With $m_c$ now in the TeV range the lightest scalar mass is pulled down to 30-50 GeV although since it consists almost completely of $N$ and couples weakly to
the Z (see Figure 9b), it is not ruled out by current Higgs searches at LEPI. This is a good example of a light weakly coupled Higgs boson, characteristic of the NMSSM. High statistics LEP data would be required to discover this weakly coupled Higgs boson. Similarly the lightest pseudoscalar in Figure 9a, although lighter than its counterpart in Figure 6a, remains decoupled and undetectable according to Figure 9b. The physically relevant next-to-lightest scalar in Figure 9a remains above 100 GeV, out of range of LEPII.

Next we turn to the question of the effect of varying $k$ and $\lambda$. For the above plots we selected $k = 0.1$ and $\lambda = 0.65$ because these values served to maximise the lightest scalar mass in our analysis of the bound in section 5. Now we shall explore the effect of choosing different values of $k$ and $\lambda$, and in this discussion we shall be guided by our study of RG flows in section 3. By comparing Figures 1a and 1b it is clear that the fixed point $\lambda = 0.87 h_t$ and $k = 0.63 h_t$ is blown away by gauge (in particular QCD) effects. Instead we are left with a preferred fringe in the $(\lambda/h_t)$ and $(k/h_t)$ plane of Figure 1b roughly corresponding to a circle of radius $\approx 0.6$. This plot is for large $h_t$ at the GUT scale, corresponding to $h_t \approx 1.1$ at low energy; however Figure 1c shows similar behaviour for $h_t = 1$ at the GUT scale. Our baseline parameters have $m_t = 150 GeV$ and $\tan \beta = 1.7$ corresponding to $h_t = 0.92$ at a scale of 1TeV; this is the value by which the points on the fringe of Figure 1b must be multiplied to obtain the values of $\lambda$ and $k$.

Following the discussion of the preceding paragraph, a central point on the fringe of $(\lambda/h_t) = (k/h_t) = 0.44$ corresponds to $\lambda = k = 0.4$ and the spectrum with these values for the couplings and all other parameters set equal to their baseline values is plotted in Figure 10a. Compared to the baseline values of $\lambda = 0.65$, $k = 0.1$ in Figure 6a, Figure 10a shows $m_c$ values of 120-200 GeV, smaller by about 100GeV, with the left hand end of the range determined by the lightest pseudoscalar mass dropping to zero, and the right hand end by the lightest scalar mass dropping to zero. The latter has a mass of up to 80 GeV, and Figure 10b shows that the lightest scalar (dots) contains little singlet resulting in a SM-type ZZ$h$ coupling (solid) in contrast to Figure 6b. This puts the lightest scalar within the LEP range, with an excluded region corresponding to the LEP limit $m_h < 60$GeV advancing to the left. As before, the lightest pseudoscalar is mainly singlet and so has small physical couplings (dashes in Figure 10b). The heavier Higgses are somewhat lighter.
than before but out of range of LEP.

Figure 11a shows the Higgs mass spectrum for $\lambda = 0.1$ and $k = 0.6$, at the other end of the fringe to the baseline values of Figure 6a. Note that Figure 11a is similar to Figure 10a, but the lightest scalar is lighter at around 30 GeV, and since it contains very little $N$ and couples in a standard Higgs like way to the Z (Figure 11b) it is excluded by present Higgs searches. We conclude that the small $\lambda$, large $k$ region is not preferred phenomenologically.

Having explored all the regions of the fringe of preferred values of $\lambda$ and $k$ in Figure 1b, we shall now consider taking these parameters both to be very small. The RG equations in section 3 show that $\lambda = k = 0$ is an acceptable choice, since if $\lambda$ and $k$ are small at the GUT scale, they may still be small at low energies. Furthermore, it is known that in the limit $\lambda \to 0$, $k \to 0$, $r \to \infty$, with $\lambda r$ and $kr$ held fixed, the NMSSM Higgs sector reduces to that of the MSSM, so this region is of intrinsic interest for comparisons to the minimal model. In Figure 12a we show the Higgs spectrum of the NMSSM for $\lambda = 0.1$, $k = 0.1$, $r = 5$ with all other parameters set equal to their baseline values. This spectrum should and does resemble that of the MSSM. For one thing the charged mass exceeds $M_W$, as it should. Also it is clear from Figure 12b that the lightest pseudoscalar (double-dot-dash) and the second lightest scalar (single-dot-dash) can be identified with $N$ to a good approximation over most of the region. The remaining states of Figure 12a then correspond to the MSSM states. The lightest scalar of mass 50 GeV couples in the SM way (solid curve in Figure 12b) and so is already excluded by LEPI. Additional radiative corrections due to non-zero $A_t$ values and/or larger top and stop masses could increase the lightest scalar mass beyond current limits. Such additional radiative corrections are discussed in section 6.3.

We can take the MSSM limit of our results in a much more direct way by removing the $N$ components of the Higgs mass matrices by hand, and numerically taking $\lambda$ and $k$ to be very small, and $r$ to be very large, with $\mu$ selected arbitrarily and appearing in the radiative corrections to the $2 \times 2$ mass matrices instead of $\lambda x$. The resulting spectrum in Figure 13a, for $\mu = 0$ and $\tan \beta = 1.7$ is quite similar to that in Figure 12a, once the lightest pseudoscalar and second lightest scalar of that plot have been removed. The range
of $m_c$ is extended with $m_c > 80$ GeV as expected. The lightest scalar again has a mass $m_h \approx 50$ GeV with $R_{ZZh} = \sin(\beta - \alpha) \approx 1$.

In Figure 13b we show the corresponding MSSM plots with $\tan\beta = 10$, and all other parameters as before. The limit of $\tan\beta \to \infty$ is of course where the MSSM bound is saturated. In this limit the $2 \times 2$ scalar mass-squared matrix is diagonal with elements $m_A^2$ and $M_Z^2$ (plus radiative corrections). Furthermore, $m_c^2 = m_A^2 + M_W^2$ at tree-level. This explains why the spectrum in Figure 13b consists of a flat scalar mass plus a rising scalar mass degenerate with the pseudoscalar. In this case, since $\beta \approx \pi/2$ and $\alpha \approx 0$ (as the scalar matrix is diagonal), it is clear that $R_{ZZh} = \sin(\beta - \alpha) \approx 1$ and $R_{ZAh} = \cos(\beta - \alpha) \approx 0$, where $h$ is defined to be the flat curve in Figure 13b (id est, for values of $m_c$ less than the point at which the two scalar curves bounce off each other, the lighter of the two scalars is decoupled, while for larger values of $m_c$ the converse is true.)

6.3 The Effect of Radiative Corrections

All the spectra plotted in section 6.2 include top quark and squark radiative corrections as well as Higgs and Higgsino radiative corrections. However in all these plots $A_t = 0$, with $m_t = m_{\tilde{t}_1} = 150$ GeV, $m_{\tilde{t}_2} = 500$ GeV. We shall now examine the effect of stop squark spectra with $A_t \neq 0$. To understand how important the effect of non-zero $A_t$ may be, we shall begin by considering the effects of each of the other radiative corrections in turn. Since these effects are far harder to compare accurately on a logarithmic scale, we shall restrict our discussion to the lightest Higgs boson masses which we shall plot on a linear scale. We shall use the baseline parameters defined in (6.1), and for comparison we shall in each case plot the tree-level masses as dotted lines, so that the effect of the particular radiative correction is easily seen.

Figure 14 shows the lightest Higgs boson masses corresponding to the baseline parameters above, so that the Higgs masses are identical to those in Figure 6a. The corresponding tree-level masses without top, stop, Higgs or Higgsino radiative corrections are indicated by dotted lines. The lightest scalar mass is unaffected by radiative corrections at its peak, where it decouples, but at this point the corrections are largest for the next-to-lightest scalar. Away from this point the corrections are shared between the two scalars, with
the pseudoscalar – which is only weakly coupled across the whole range – receiving small corrections. Radiative corrections in this case increase the second lightest CP-even scalar mass by as much as 7 GeV.

Figure 15 shows the effect of radiative corrections due to top quark loops alone, corresponding to the parameters and notation of Figure 14, but with the Higgs and Higgsino radiative corrections subtracted. It is clear by comparing Figure 15 to Figure 14 that the Higgs and Higgsino corrections have relatively little effect compared to top quark loops, the most noticeable change being felt by the pseudoscalar mass. As we shall see, the Higgs and Higgsino radiative corrections are individually quite large but tend to cancel between themselves (the cancellation being exact in the limit of exact supersymmetry). Top quark loops are therefore responsible for the bulk of the radiative corrections for the baseline parameters.

Figure 16 shows the effect of radiative corrections due to Higgs bosons alone, corresponding to the parameters and notation of Figure 14, but this time with only the Higgs boson radiative corrections included. Compared to the tree-level results (dots), the Higgs radiative corrections can be quite substantial and tend to depress the scalar masses and enhance the pseudoscalar mass. Figure 17 shows the effect of radiative corrections due to Higgsinos alone for the usual parameters and notation. In Figure 17 the comparison to the tree-level dotted lines shows that Higgsinos tend to enhance scalar masses in such a way as to partially cancel the effects of Higgs corrections in Figure 16. The pseudoscalar mass is also enhanced in this case, however.

We are now ready to examine the radiative corrections due to squark spectra with $A_t \neq 0$. Again we use the baseline parameters in (6.1), apart from the following two sets of stop parameters which were encountered previously in our discussion of the bound in section 5:

(a) $m_{\tilde{t}_1} = 150\text{GeV}, m_{\tilde{t}_2} = 500\text{GeV}, A_t = 700\text{GeV}$

(b) $m_{\tilde{t}_1} = 600\text{GeV}, m_{\tilde{t}_2} = 1\text{TeV}, A_t = 1.8\text{TeV}$

Parameter set (a) uses the same stop masses as in the baseline parameters but now involves a non-zero $A_t$ value chosen to maximise the effect of radiative corrections. Parameter set (b) involves heavier stop masses and larger $A_t$ value, again chosen to maximise
the effects of radiative corrections. In Figure 18a we show the Higgs spectrum for parameter set (a), including the radiative corrections due to the squark spectrum as well as top, Higgs and Higgsino loops, with the corresponding tree-level spectrum again represented by dotted lines. We see that the effect of the $A_t = 700$ GeV value in Figure 18a is dramatic for the physical second lightest scalar with radiative corrections to its mass of up to 18 GeV as compared to 7 GeV with $A_t = 0$ in Figure 14 (with all other parameters the same in the two cases).

In Figure 18b we show the spectrum for the heavier squark spectrum set (b) above, again including all other radiative corrections in this plot and comparing it to the baseline tree-level masses represented by dots. The physical second lightest scalar in Figure 18b has radiative corrections to its mass of up to 25 GeV as compared to 18 GeV in Figure 18a. We conclude that the effect of choosing larger stop masses and large non-zero $A_t$ values may significantly raise the mass of the lightest physical Higgs bosons from their values shown in the plots of section 6.2.

7 Conclusion

We have performed a comprehensive analysis of the radiative corrections to the Higgs boson mass spectrum in the NMSSM using the one-loop effective potential. Analytic results for top quark and squark corrections are reviewed, and our numerical procedure for including Higgs and Higgsino corrections is described. The bound on the lightest CP-even scalar, including the radiative corrections mentioned above, was then discussed. In order to find the absolute bound as a function of $m_t$ including all radiative corrections, we maximised over the parameter space using analytic and numerical techniques. Our final results are summarised in Figure 4.

We have presented a numerical analysis of the RG flows for the dimensionless couplings, relying on the assumption of perturbative behaviour up to $M_{GUT}$. We emphasise the infra-red fixed points of the gaugeless limit are not the relevant points for the low energy behaviour because they are washed away by the large QCD corrections, and instead the parameter space is limited to a region of the $(\lambda/h_t) - (k/h_t)$ plane bounded by the axes and an approximate quarter circle, as shown in Figure 1b.
A general discussion of Higgs boson phenomenology including radiative corrections was then given. Since the parameter space of the Higgs sector of the NMSSM is multi-dimensional, we selected a baseline set of parameters given in (6.1), and discussed the effect of varying each of the parameters in turn. Our selection of $k$ and $\lambda$ was based on our study of RG flows described above. We found that larger $m_c$ values are associated with larger values of $\tan \beta$ and $r$. The NMSSM with small $k$ and $\lambda$ and large $r$ was also compared to the MSSM (see Figures 12 and 13). Possible characteristic signatures of the NMSSM include light charged scalars, and weakly coupled light neutral scalars, and there are regions of parameter space where either or both signals is possible (see Figures 7 and 9). Equally there are other regions of parameter space where neither of these characteristic signals is present (see Figures 8 and 10).

Finally Figures 14-18 show the relative sizes of the various radiative corrections. These show that the dominant effect is usually that of the top quark in conjunction with the stop sector. Figure 18 emphasises the huge radiative corrections which are possible for large $A_t$ and non-degenerate stop squarks. Higgs and Higgsino corrections are typically rather smaller, and tend to cancel due to the (softly broken) supersymmetry, as shown in Figures 16 and 17. This cancellation also occurs in the top-stop sector, but is less noticeable due to the large soft squark masses which are assumed.

If Higgs bosons are discovered at LEP or LHC/SSC, then an important question will be whether they are associated with the MSM, the MSSM, or some other model. If they are not associated with the MSSM, then it is possible that they arise from the NMSSM, which has a more general structure. The discussion of radiative corrections to Higgs boson masses in the NMSSM and the phenomenological discussion presented in this paper will help to decide this question.

Acknowledgements

We are grateful to the SERC for financial support. PLW would like to thank Steve Kelley for some very helpful suggestions about effective potential techniques.
References

[1] H.P. Nilles, *Phys. Rep.* **110** (1984) 1;
    H.E. Haber and G.L. Kane, *Phys. Rep.* **117** (1985) 75.
[2] J.F. Gunion, H.E. Haber, G.L. Kane and S. Dawson, “The Higgs Hunter’s Guide” (Addison-Wesley, Reading, MA, 1990).
[3] H. Haber and R. Hempfling, *Phys. Rev. Lett.* **66** (1991) 1815;
    Y. Okada, M. Yamaguchi and T. Yanagida, *Prog. Theor. Phys.* **85** (1991) 1;
    *Phys. Lett. B262* (1991) 54.
[4] J. Ellis, G. Ridolfi and F. Zwirner, *Phys. Lett.* **B257** (1991) 83;
    *Phys. Lett. B262* (1991) 477;
    A. Brignole, J. Ellis, G. Ridolfi and F. Zwirner, *Phys. Lett. B271* (1991) 123.
[5] A. Brignole, *Phys. Lett. B277* (1992) 313;
    *Phys. Lett. B281* (1992) 284.
[6] J.L. Lopez and D.V. Nanopoulos, *Phys. Lett.* **B266** (1991) 397;
    K. Sasaki, M. Carena and C.E.M. Wagner, *Nucl. Phys. B381* (1992) 66.
[7] H.E. Haber and R. Hempfling, preprint number SCIPP 91/33.
[8] P. Fayet, *Nucl. Phys. B90* (1975) 104.
[9] J. Ellis, J.F. Gunion, H.E. Haber, L. Roszkowski and F. Zwirner, *Phys. Rev. D39* (1989) 844.
[10] L. Durand and J.L. Lopez, *Phys. Lett. B217* (1989) 463;
    L. Drees, *Int. J. Mod. Phys. A4* (1989) 3635.
[11] J.R. Espinosa and M. Quiros, *Phys. Lett. B279* (1992) 92.
[12] G. Kane, C. Kolda and J. Wells, *Phys. Rev. Lett.* **70** (1993) 2686.
[13] U. Ellwanger and M. Rausch de Traubenberg, *Z. Phys. C53* (1992) 521.
[14] U. Ellwanger and M. Lindner, *Phys. Lett. B301* (1993) 365.
[15] U. Ellwanger, *Phys. Lett. B303* (1993) 271.
[16] W. ter Veldhuis, Purdue preprint PURD-TH-92-11 and [hep-ph/9211281].
[17] J.R. Espinosa and M. Quiros, *Phys. Lett. B302* (1993) 51.
[18] T. Elliott, S.F. King and P.L. White, *Phys. Lett. B305* (1993) 71.
[19] T. Elliott, S.F. King and P.L. White, Southampton preprint SHEP 92/93-18, *Phys. Lett.* to be published.

[20] P.N. Pandita, Northeastern Hill University preprint PRINT-93-0465, *Z.Phys.* to be published.

[21] S. Coleman and E. Weinberg, *Phys. Rev.* **D7** (1973) 1888;
    S. Weinberg, *Phys. Lett.* **D7** (1973) 2887.

[22] M. Sher, *Phys. Rep.* 179 (1989) 273.

[23] S. Ferrara, D.V. Nanopoulos and C.A. Savoy, *Phys. Lett.* **B123** (1983) 214;
    J. Polchinski and L. Susskind, *Phys. Rev.* **26** (1982) 3661;
    H.P. Nilles, M. Srednicki and D. Wyler, *Phys. Lett.* **B124** (1983) 337;
    A.B. Lahanes, *Phys. Lett.* **B124** (1983) 341;
    L. Alvarez-Gaume, J. Polchinski and M.B. Wise, *Nucl. Phys.* **B221** (1983) 495.

[24] J. Bagger and E. Poppitz, John Hopkins preprint number JHU-TIPAC-93018 and hep-ph/9307317.

[25] J.-P. Derendinger and C.A. Savoy, *Nucl. Phys.* **B237** (1984) 307.

[26] P. Binetruy and C.A. Savoy, *Phys. Lett.* **B277** (1992) 453.

[27] S. Jacoby, J. Kowalik and J. Pizzo, “Iterative Methods for Non-Linear Optimization Problems” (Prentiss-Hall, 1972);
    W.H. Press, B.P. Flannery, S.A. Teukolsky and W. T.Vetterling, “Numerical Recipes” (Cambridge University Press, 1986).

[28] V. Barger, K. Cheung, R.J.N. Phillips and A.L. Stange, *Phys. Rev.* **D46** (1992) 4914;
    H. Baer *et al.*, *Phys. Rev.* **D46** (1992) 1067;
    J. Gunion *et al.*, *Phys. Rev.* **D46** (1992) 2040, 2052;
    J. Gunion *et al.*, *Phys. Rev.* **D47** (1993) 1030;
    Z. Kunszt and F. Zwirner, *Nucl. Phys.* **B285** (1992) 3.

[29] J.L. Hewett, *Phys. Rev. Lett.* 70 (1993) 1045;
    V. Barger, M. Berger and R.J.N. Phillips, *Phys. Rev. Lett.* 70 (1993) 1368.

[30] R. Barbieri and G.F. Giudice, CERN preprint CERN-TH.6830/93.

[31] U. Ellwanger, M. Rausch de Traubenberg and C.A. Savoy, Heidelberg preprint HD-THEP-93-95 and hep-ph/9307322.
Figure Captions

**Figure 1a:** Renormalisation group flow of a set of points in the $(\lambda/h_t)-(k/h_t)$ plane as the energy scale is varied from $10^{16}$GeV to 1TeV. $h_t(10^{16}$GeV$) = 10$ and gauge couplings have been set to zero.

**Figure 1b:** Renormalisation group flow of a set of points in the $(\lambda/h_t)-(k/h_t)$ plane as the energy scale is varied from $10^{16}$GeV to 1TeV. $h_t(10^{16}$GeV$) = 10$ and gauge couplings have been included.

**Figure 1c:** Renormalisation group flow of a set of points in the $(\lambda/h_t)-(k/h_t)$ plane as the energy scale is varied from $10^{16}$GeV to 1TeV. $h_t(10^{16}$GeV$) = 1$ and gauge couplings have been included.

**Figure 2:** $\lambda_{\text{max}}$ against $h_t(1\text{TeV})$

**Figure 3:** The values of $\sin \beta$ (long dashes), $\lambda$ (short dashes), and $h_t$ (solid line) for which the bound is maximised, for the value $m_{t_2} = 1$TeV. Results for other values of $m_{t_2}$ are very similar.

**Figure 4:** The bound on the lightest neutral CP-even scalar against $m_t$. The two solid lines are for the NMSSM; the dotted lines for the MSSM. In each case the upper (lower) line is for $m_{t_2} =1$TeV (500GeV).

**Figure 5:** The bound on the lightest scalar against $m_{t_1}$ expressed in units of $m_{t_2}$. The upper (lower) line represents the case where $m_{t_2} =1$TeV (500GeV).

**Figure 6a:** Higgs boson masses against $m_c$ in the NMSSM. The parameters are $r = 1.0$, $\tan \beta = 1.7$, $\lambda = 0.65$, $k = 0.1$, $A_k = 0$, $m_t = 150$GeV, $m_{t_1} = 150$ GeV, $m_{t_2} = 500$ GeV, and $A_t = 0$. Solid lines represent CP-even, and dashed lines CP-odd mass eigenstates.

**Figure 6b:** Higgs boson couplings and N-amplitudes corresponding to Fig.6a. The solid line represents the $R_{ZZh}$ coupling, the dashed line the $R_{ZhA}$ coupling. The amplitude of
singlet field $N$ contained in each of the mass eigenstates are indicated by a dotted line for the lightest scalar $h$, a dot-dashed line for the next-to-lightest scalar, and a dot-dot-dashed line for the lightest pseudo-scalar $A$.

**Figure 7a:** Higgs boson masses against $m_c$ in the NMSSM. The parameters are as in Fig.6a except that $r = 0.1$. Solid lines represent CP-even, and dashed lines CP-odd mass eigenstates.

**Figure 7b:** Higgs boson couplings and N-amplitudes corresponding to Fig.7a. The solid line represents the $R_{ZZh}$ coupling, the dashed line the $R_{ZhA}$ coupling. The amplitude of singlet field $N$ contained in each of the mass eigenstates are indicated by a dotted line for the lightest scalar $h$, a dot-dashed line for the next-to-lightest scalar, and a dot-dot-dashed line for the lightest pseudo-scalar $A$.

**Figure 8a:** Higgs boson masses against $m_c$ in the NMSSM. The parameters are as in Fig.6a except that $r = 5.0$. Solid lines represent CP-even, and dashed lines CP-odd mass eigenstates.

**Figure 8b:** Higgs boson couplings and N-amplitudes corresponding to Fig.8a. The solid line represents the $R_{ZZh}$ coupling, the dashed line the $R_{ZhA}$ coupling. The amplitude of singlet field $N$ contained in each of the mass eigenstates are indicated by a dotted line for the lightest scalar $h$, a dot-dashed line for the next-to-lightest scalar, and a dot-dot-dashed line for the lightest pseudo-scalar $A$.

**Figure 9a:** Higgs boson masses against $m_c$ in the NMSSM. The parameters are as in Fig.6a except that $\tan \beta = 10$. Solid lines represent CP-even, and dashed lines CP-odd mass eigenstates.

**Figure 9b:** Higgs boson couplings and N-amplitudes corresponding to Fig.9a. The solid line represents the $R_{ZZh}$ coupling, the dashed line the $R_{ZhA}$ coupling. The amplitude of singlet field $N$ contained in each of the mass eigenstates are indicated by a dotted line for the lightest scalar $h$, a dot-dashed line for the next-to-lightest scalar, and a dot-dot-dashed line for the lightest pseudo-scalar $A$.

35
**Figure 10a:** Higgs boson masses against $m_c$ in the NMSSM. The parameters are as in Fig.6a except that $\lambda = 0.4$, $k = 0.4$. Solid lines represent CP-even, and dashed lines CP-odd mass eigenstates.

**Figure 10b:** Higgs boson couplings and N-amplitudes corresponding to Fig.10a. The solid line represents the $R_{ZZh}$ coupling, the dashed line the $R_{ZhA}$ coupling. The amplitude of singlet field $N$ contained in each of the mass eigenstates are indicated by a dotted line for the lightest scalar $h$, a dot-dashed line for the next-to-lightest scalar, and a dot-dot-dashed line for the lightest pseudo-scalar $A$.

**Figure 11a:** Higgs boson masses against $m_c$ in the NMSSM. The parameters are as in Fig.6a except that $\lambda = 0.1$, $k = 0.6$. Solid lines represent CP-even, and dashed lines CP-odd mass eigenstates.

**Figure 11b:** Higgs boson couplings and N-amplitudes corresponding to Fig.11a. The solid line represents the $R_{ZZh}$ coupling, the dashed line the $R_{ZhA}$ coupling. The amplitude of singlet field $N$ contained in each of the mass eigenstates are indicated by a dotted line for the lightest scalar $h$, a dot-dashed line for the next-to-lightest scalar, and a dot-dot-dashed line for the lightest pseudo-scalar $A$.

**Figure 12a:** Higgs boson masses against $m_c$ in the NMSSM. The parameters are as in Fig.6a except that $\lambda = 0.1$, $k = 0.1$. Solid lines represent CP-even, and dashed lines CP-odd mass eigenstates.

**Figure 12b:** Higgs boson couplings and N-amplitudes corresponding to Fig.12a. The solid line represents the $R_{ZZh}$ coupling, the dashed line the $R_{ZhA}$ coupling. The amplitude of singlet field $N$ contained in each of the mass eigenstates are indicated by a dotted line for the lightest scalar $h$, a dot-dashed line for the next-to-lightest scalar, and a dot-dot-dashed line for the lightest pseudo-scalar $A$.

**Figure 13a:** Higgs boson masses against $m_c$ for the MSSM. The parameters are $\tan \beta = 1.7$, $m_t = 150 \text{ GeV}$, $m_{\tilde{t}_1} = 150 \text{ GeV}$, $m_{\tilde{t}_2} = 500 \text{ GeV}$, $\mu = 0$ and $A_t = 0$. CP-even and CP-odd mass eigenstates are represented by solid and dashed lines respectively.
Figure 13b: Higgs boson masses against $m_c$ for the MSSM. The parameters are as in Fig.13a except that $\tan \beta = 10$. CP-even and CP-odd mass eigenstates are represented by solid and dashed lines respectively.

Figure 14: Higgs boson masses against $m_c$ in the NMSSM. The parameters are $r = 1.0$, $\tan \beta = 1.7$, $\lambda = 0.65$, $k = 0.1$, $A_k = 0$, $m_t = 150 \text{GeV}$, $m_{\tilde{t}_1} = 150 \text{ GeV}$, $m_{\tilde{t}_2} = 500 \text{ GeV}$, and $A_t = 0$. Solid lines represent CP-even, and dashed lines CP-odd mass eigenstates and include radiative corrections due to loops of top quarks, squarks, Higgs bosons and Higgsinos. This plot is as in Fig.6a except that it is rescaled to show the lighter mass eigenstates in greater detail. The dotted lines indicate the tree-level spectrum for the same parameters.

Figure 15: Higgs boson masses against $m_c$ in the NMSSM. The parameters are as in Fig.14. Solid lines represent CP-even, and dashed lines CP-odd mass eigenstates and include radiative corrections due to loops of top quarks and squarks only. The dotted lines indicate the tree-level spectrum for the same parameters.

Figure 16: Higgs boson masses against $m_c$ in the NMSSM. The parameters are as in Fig.14. Solid lines represent CP-even, and dashed lines CP-odd mass eigenstates and include radiative corrections due to loops of Higgs bosons only. The dotted lines indicate the tree-level spectrum for the same parameters.

Figure 17: Higgs boson masses against $m_c$ in the NMSSM. The parameters are as in Fig.14. Solid lines represent CP-even, and dashed lines CP-odd mass eigenstates and include radiative corrections due to loops of Higgsinos only. The dotted lines indicate the tree-level spectrum for the same parameters.

Figure 18a: Higgs boson masses against $m_c$ in the NMSSM. The parameters are as in Fig.14 except that $A_t = 700 \text{GeV}$. Solid lines represent CP-even, and dashed lines CP-odd mass eigenstates and include radiative corrections due to loops of top quarks, squarks, Higgs bosons and Higgsinos. The dotted lines indicate the tree-level spectrum for the same parameters.
**Figure 18b:** Higgs boson masses against $m_c$ in the NMSSM. The parameters are as in Fig.14 except that $m_{\tilde{t}_1} = 600$ GeV, $m_{\tilde{t}_2} = 1.0$ TeV, $A_t = 1.8$ TeV. Solid lines represent CP-even, and dashed lines CP-odd mass eigenstates and include radiative corrections due to loops of top quarks, squarks, Higgs bosons and Higgsinos. The dotted lines indicate the tree-level spectrum for the same parameters.