Direct detection of dark matter in models with a light $Z'$

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**Abstract**

We discuss the direct detection signatures of dark matter interacting with nuclei via a new neutral gauge boson $Z'$, focusing on the case where both the dark matter and the $Z'$ have mass of a few GeV. Isospin violation (i.e. different couplings to protons and neutrons) arises naturally in this scenario. In particular it is possible to reconcile the preferred parameter regions inferred from the observed DAMA and CoGeNT modulations with the bounds from XENON100, which requires $f_n/f_p \simeq -0.7$. Moreover, the $Z'$ mediator can also yield a large spin-dependent cross-section which could contribute to the DAMA signal, while the spin-independent cross-section is adequate to explain the CoGeNT signal.
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1 Introduction

For many years, direct detection experiments have attempted to confirm if dark matter (DM) is indeed made of new non-baryonic relic particles [1, 2]. They have ruled out various DM candidates and severely constrained others, but have not yet been able to unambiguously observe a DM signal. Nevertheless, recent data from the DAMA [3] and CoGeNT [4, 5] experiments have been interpreted as being due to spin-independent (SI) scattering of DM particles with relatively high cross-section ($\sim 10^{-40}$ cm$^2$) and small mass ($\sim 10$ GeV) [6]. However, this explanation is in strong tension with other null results, most notably from the CDMS [7], XENON10 [8] and XENON100 [9] experiments.

The analysis of direct detection experiments usually assumes that the DM particle couples with equal strength to protons and neutrons. However, this need not be the case, e.g. if the mediator of the DM scattering couples to isospin [10]. Isospin-dependent couplings occur naturally for vector mediators, for example the photon couples only to protons, while the $Z$ boson couples predominantly to neutrons. There can even be cancellations between the scattering on protons and neutrons, reducing the sensitivity of some direct detection experiments.

It has been noted that the tension between the DAMA/CoGeNT and XENON results can be alleviated by considering such isospin-dependent couplings, in particular a ratio of neutron to proton couplings $f_n/f_p \simeq -0.7$ [10–19]. We demonstrate that this value can be obtained from a single new vector mediator. Specifically we consider the case of a light $Z'$ with a mass that is comparable to that of the DM particle, i.e. several GeV. It mediates the interaction between the SM and a new hidden sector which includes the DM particle and is uncharged under the Standard $SU(2)_L \times U(1)_Y$ Model (SM) but charged under the new $U(1)$. In this framework it is possible to get both the required value of $f_n/f_p$, as well as sufficiently high cross-sections to account for the absolute signal levels observed by DAMA and CoGeNT. Our analysis can also be applied to the composite vector particles arising in new strong dynamics models of DM.

The outline of this paper is as follows: first we discuss how to obtain the effective coupling constants $f_n$ and $f_p$ in the general case of a vector mediator which mixes with the neutral gauge bosons of the SM. We then study, using an effective Lagrangian, how the required value of $f_n/f_p$ can be realised with such a $Z'$. We discuss other experimental bounds on our model parameters and show that the required cross-section needed to explain the DAMA and CoGeNT signals, with $f_n/f_p \simeq -0.7$, remains viable. Finally we note that data from collider experiments such as BaBar, Belle, BEPC and LHCb will be able to detect or rule out such a light $Z'$, as discussed in e.g. Ref. [20].
2 DM interaction via a vector mediator $R$

In this section we discuss the general calculation of the ratio of the effective coupling constants for DM scattering on neutrons and protons $f_n/f_p$ via a vector mediator. For this purpose, we first consider an (axial)-vector mass-eigenstate $R$ interacting with the SM fermions $f$ and the fermionic DM $\chi$ via the neutral current Lagrangian

$$L_{\text{NC}}^{R} = R_{\mu} \bar{\chi} \gamma^\mu (g_{\chi}^V - g_{\chi}^A \gamma^5) \chi + R_{\mu} \bar{f} \gamma^\mu (g_{f}^V - g_{f}^A \gamma^5) f .$$

Integrating out $R$ generates the effective four-fermion interactions

$$L_{\text{eff}}^{R} = b_f^{V,A} \bar{\chi} \gamma_\mu \chi \bar{f} \gamma^\mu f + c_f^{1,2} \bar{\chi} \gamma_\mu \gamma^5 \chi \bar{f} \gamma^\mu f ,$$

where $b_f^{V,A} = g_{\chi}^{V,A} g_f^{V,A} / m_R^2$ and $c_f^{1,2} = g_{\chi}^{V,A} g_f^{V,A} / m_R^2$. In the non-relativistic limit relevant for the interaction between DM and detector nuclei, the mixing terms proportional to $c_f^{1,2}$ are suppressed, so we neglect them in our analysis.

We assume that $R$ arises from the mixing of an interaction eigenstate vector $X$ with the SM $U(1)_Y B$ field and the neutral component $W^3$ of $SU(2)_L$ weak fields

$$\begin{pmatrix} B_\mu \\ W^3_\mu \\ X_\mu \end{pmatrix} = \begin{pmatrix} N_{11} & N_{12} & N_{13} \\ N_{21} & N_{22} & N_{23} \\ N_{31} & N_{32} & N_{33} \end{pmatrix} \begin{pmatrix} A_\mu \\ Z_\mu \\ R_\mu \end{pmatrix} .$$

Here $A$, $Z$ are the physical photon and neutral massive gauge boson fields of the SM. As we discuss below, there can also be a kinetic mixing between $X$ and the SM gauge bosons in addition to the mass mixing described here. However, by redefining the fields to have standard kinetic terms, this kinetic mixing is equivalent to an additional mass mixing contribution, cf. Sec. 3.

In addition to any direct couplings of $X$ to SM fermions, denoted by $f^{V,A}$, the mixing will introduce additional terms, so that the overall couplings of $R$ to light quarks will be given in terms of the mass mixing matrix, as

$$g^V_u = -\frac{1}{12} (5 g' N_{13} + 3 g N_{23}) - f_u^V N_{33} , \quad g^A_u = \frac{1}{4} (g' N_{13} - g N_{23}) - f_u^A N_{33} ,$$

$$g^V_d = \frac{1}{12} (g' N_{13} + 3 g N_{23}) - f_d^V N_{33} , \quad g^A_d = -\frac{1}{4} (g' N_{13} - g N_{23}) - f_d^A N_{33} ,$$

where the numerical coefficients are determined from the hypercharge and weak quantum numbers of the quarks. Similarly, the effective vector and axial couplings of $R$ to the DM particle $\chi$ are given by

$$g_\chi^V = f_\chi^V N_{33} , \quad g_\chi^A = f_\chi^A N_{33} .$$
In the case where the direct couplings between fermions and $X$ arise from a charge under some new gauge group, the coupling constants $f^{V,A}$ will be given by the product of the corresponding gauge coupling $g_X$ and the respective charge.

A mass mixing as in Eq. (3) not only induces couplings of $R$ to the SM fermions, but also couplings of $Z$ (and in some cases $A$) to the DM particle. The corresponding coupling constants can be calculated analogously, by examining the corresponding column of the mixing matrix. The effective coupling constants for $A$ are obtained from the first column, i.e. replacing $N_{i3}$ by $N_{i1}$, and the effective coupling constants for $Z$ are obtained from the second column. Note that if $N_{31} = 0$, there is no coupling of the physical photon to the DM state $\chi$ and so there are no DM millicharges. In this case, the couplings between $\chi$ and the SM fermions are given by

$$b^{A,V}_f = b^{A,V}_{fR} + b^{A,V}_{fZ} = \frac{g^{A,V}_{\chi R} g^{A,V}_{fR}}{m^2_R} + \frac{g^{A,V}_{\chi Z} g^{A,V}_{fZ}}{m^2_Z}. \tag{6}$$

Let us for now focus on the induced effective vector-vector interaction between the DM particle and nucleons $(p,n)$

$$\mathcal{L}^V = f_p \bar{\chi} \gamma_\mu \chi \bar{p} \gamma^\mu p + f_n \bar{\chi} \gamma_\mu \chi \bar{n} \gamma^\mu n, \tag{7}$$

with coefficients $f_{p,n}$ given by

$$f_p = 2b^V_u + b^V_d, \quad f_n = 2b^V_d + b^V_u. \tag{8}$$

Note that because of the conservation of the vector current, there is no contribution of sea quarks or gluons to the effective couplings.

Ultimately, we are interested in $\chi$ scattering off nuclei $N$ with charge $Z$ and mass number $A$. In the limit of zero momentum transfer, the DM particle will interact coherently with the entire nucleus $N$, resulting in the DM-nucleus cross-section

$$\sigma_N = \frac{\mu_{\chi N}^2}{\mu_{\chi n}^2} \left( \frac{Z f_p}{f_n} - \frac{A - Z}{\sigma_n} \right)^2 \sigma_n, \tag{9}$$

with $\sigma_n = \mu_{\chi n}^2 f_n^2 / \pi$ the DM-neutron cross section and $\mu_{XY}$ the reduced mass of the $(X,Y)$ system. If the DM scattering satisfies $f_n/f_p = Z/(Z - A)$ for a given nuclear isotope, the corresponding cross-section is zero and this isotope will then not contribute to the constraint on DM-nucleon scattering. Xenon, which typically gives the strongest constraint on the DM-nucleon cross-section in the case of $f_n/f_p = 1$ has $Z = 54$, while $A$ varies between 74 and 80. Consequently, if $-0.72 < f_n/f_p < -0.68$ (corresponding to
$-1.14 < b_u^V/b_d^V < -1.11$), the scattering cross-section for DM particles on xenon nuclei will be significantly suppressed. In Fig. 1, we show the suppression of the signal for different targets as a function of $f_n/f_p$ and $b_u^V/b_d^V$.

At this point, it is instructive to look at a few familiar cases

1. For a mediator coupling to the baryonic current we have $b_u^V/b_d^V = f_n/f_p = 1$.
2. For a mediator coupling to the weak isospin current we have $b_u^V/b_d^V = f_n/f_p = -1$.
3. For a coupling to the EM current we have $b_u^V/b_d^V = -2$ and thus $f_n/f_p = 0$.
4. Finally, a coupling to the vectorial part of the SM $Z$ current gives $b_u^V/b_d^V \sim -1/2$ and thus $f_n/f_p \gg 1$, corresponding to a coupling dominantly to neutrons.

![Figure 1](image_url)

Figure 1: The factor by which the cross-section for dark matter scattering on various isotopes is suppressed (compared to the standard case where $f_n = f_p$), as a function of the ratio $f_n/f_p$ (left) and the corresponding quark couplings $b_u^V/b_d^V$ (right). In all cases, the natural isotopic abundance ratios have been assumed for the detector material.

In isolation, none of the possibilities above yields the ratio of $f_n/f_p$ required to adequately suppress the DM scattering signal from xenon. However as we will show in the following section, a single light vector mediator which mixes with the neutral SM gauge bosons can generate $f_n/f_p \sim -0.7$ with sufficiently large cross-sections to explain both the DAMA and CoGeNT signals, and be in agreement with all other experimental constraints. Alternatively there may be several independent mediators which interfere to give the desired value for $f_n/f_p$ \cite{15,21}.
3 Effective Lagrangian for a light $Z'$

From now on we will take the interaction eigenstate $X$ to be the new gauge boson corresponding to an additional $U(1)_X$ symmetry \[22\]–\[25\]. The corresponding mass eigenstate $Z'$ is then a specific realisation of the vector mediator $R$. The general possibility of DM (in particular light DM) coupling to the SM via a $Z'$ has been considered earlier \[26\]–\[32\].

We consider the following effective Lagrangian, which includes kinetic mixing and mass mixing \[23\]–\[31\]:

$$\mathcal{L} = \mathcal{L}_{SM} - \frac{1}{4} \hat{X}^{\mu\nu} \hat{X}_{\mu\nu} + \frac{1}{2} m_X^2 \hat{X}_\mu \hat{X}^\mu - m_\chi \bar{\chi} \chi - \frac{1}{2} \sin \epsilon \hat{B}_{\mu\nu} \hat{X}^{\mu\nu} + \delta m^2 \hat{Z}_\mu \hat{X}^\mu - \sum f f V_f \hat{X}_\mu \bar{f} \gamma^\mu f - f^\dagger \hat{X}_\mu \bar{\chi} \gamma^\mu \chi, \quad (10)$$

where the $U(1)_X$ is assumed to be broken and the corresponding vector boson mass is $m_\hat{X}$. We denote fields in the original basis with hats ($\hat{B}, \hat{W}^3, \hat{X}$) and define $\hat{Z} \equiv \hat{c}_W \hat{W}^3 - \hat{s}_W \hat{B}$, where $\hat{s}_W$ ($\hat{c}_W$) is the sine (cosine) of the Weinberg angle and $\hat{g}'$, $\hat{g}$ are the corresponding gauge couplings. Canonically normalised interaction eigenstates after kinetic diagonalisation and normalisation are denoted without hats, while the mass eigenstates after mass diagonalisation are denoted by $(A, Z, Z')$. In the following we will abbreviate $\sin \theta \equiv s_\theta, \cos \theta \equiv c_\theta, \tan \theta \equiv t_\theta$.

The diagonalisation of the above Lagrangian is discussed in detail in \textit{e.g.} Ref. \[23\]. The field strengths are diagonalised and canonically normalised via the following two consecutive transformations

$$\begin{pmatrix} \hat{B}_\mu \\ \hat{W}^3_\mu \\ \hat{X}_\mu \end{pmatrix} = \begin{pmatrix} 1 & 0 & -t_\epsilon \\ 0 & 1 & 0 \\ 0 & 0 & 1/c_\epsilon \end{pmatrix} \begin{pmatrix} B_\mu \\ W^3_\mu \\ X_\mu \end{pmatrix}, \quad (11)$$

$$\begin{pmatrix} B_\mu \\ W^3_\mu \\ X_\mu \end{pmatrix} = \begin{pmatrix} \hat{c}_W & -\hat{s}_W c_\xi & \hat{s}_W s_\xi \\ \hat{s}_W & \hat{c}_W c_\xi & -\hat{c}_W s_\xi \\ 0 & s_\xi & c_\xi \end{pmatrix} \begin{pmatrix} A_\mu \\ Z_\mu \\ Z'_\mu \end{pmatrix}, \quad (12)$$

where

$$t_{2\xi} = \frac{-2c_\xi (\delta m^2 + m_X^2 \hat{s}_W s_\xi)}{m_X^2 - m_Z^2 c_\epsilon^2 + m_Z^2 \hat{s}_W^2 s_\xi^2 + 2 \delta m^2 \hat{s}_W s_\xi}. \quad (13)$$

\[1\]There could also be a mass mixing term between $Z'$ and $B$, which arises \textit{e.g.} in Stückelberg models. However, we consider here only such mass mixing as can be induced by a Higgs vev, which mixes only $Z$ and $Z'$. 
Multiplying the two matrices, we obtain

\[ N_{13} = \hat{s}_W s_\xi - c_\xi t_\epsilon, \quad N_{23} = -\hat{c}_W s_\xi, \quad N_{33} = c_\xi/c_\epsilon. \] (14)

The resulting coupling structure of \( Z' \) to the DM \( \chi \) and SM fermions can be written as

\[ \mathcal{L}_{Z'} = -\frac{e^2}{2\hat{c}_W \hat{s}_W} c_\xi Z'_\mu \bar{f}_\gamma^\mu \left\{ T^f_3(\hat{s}_W t_\epsilon - t_\xi) + Q^f(\hat{s}_W^2 t_\xi - \hat{s}_W t_\epsilon) \right\} f \]

\[ - \sum_f \hat{f}^V_c Z'_\mu \bar{f}_\gamma^\mu f - \hat{f}_\chi^V c_\xi Z'_\mu \bar{\chi}_\gamma^\mu \chi, \] (15)

where the first term is due to mixing, with \( T^f_3 \) and \( Q^f \) being the weak isospin and electromagnetic charge of the fermion \( f \), and the second and third terms correspond to the rescaled direct couplings. Thus, the effective couplings between \( Z' \) and quarks are

\[ g_U^V = \frac{5e c_\xi t_\epsilon}{12 \hat{c}_W} + \frac{1}{4 \hat{c}_W \hat{s}_W} - \frac{2 e \hat{s}_W s_\xi}{3 \hat{c}_W} - \frac{c_\xi}{c_\epsilon} f_U^V, \]

\[ g_D^V = -\frac{1}{12 \hat{c}_W} - \frac{1}{4 \hat{c}_W \hat{s}_W} + \frac{2 e \hat{s}_W s_\xi}{3 \hat{c}_W} - \frac{c_\xi}{c_\epsilon} f_D^V. \] (16)

Note also that in the case we consider, \( N_{31} = 0 \), which implies that there is no DM-photon coupling. This result is due to the particular choice of mass mixing of \( \hat{X} \) with only the \( \hat{Z} \) interaction eigenstate. If the mass mixing were to include a component with the interaction eigenstate photon \( A \), then millicharges for the DM would result as discussed e.g. in Ref. [33]. In our case, an important consequence is that even if we were to charge the SM fermions under \( X \) as in e.g. Ref. [23], the electric charge remains unchanged, such that \( \hat{c} = e \). The physical \( Z \) and \( Z' \) masses after diagonalisation are given by

\[ m^2_Z = m^2_Z(1 + \hat{s}_W t_\xi + \delta m^2 c_\epsilon t_\xi), \] (17)

\[ m^2_{Z'} = \frac{m^2_X + \delta m^2(\hat{s}_W s_\epsilon - c_\xi t_\epsilon)}{c^2_\epsilon(1 + \hat{s}_W t_\xi t_\epsilon)}. \] (18)

For the “physical” weak angle, we adopt the definition

\[ s^2_W c^2_W = \frac{\pi \alpha(m_Z)}{\sqrt{2} G_F m^2_Z}. \] (19)

Eq. (19) is also true with the replacements \( s_W \rightarrow \hat{s}_W, \quad c_W \rightarrow \hat{c}_W \) and \( m_Z \rightarrow \hat{m}_Z \), leading to the identity \( s_W c_W m_Z = \hat{s}_W \hat{c}_W \hat{m}_Z \). From these equations we fix \( \hat{s}_W \) and \( \hat{m}_Z \).
such that the experimentally well-measured quantities $G_F$ (or alternatively $s_W$) and $m_Z$ come out correctly. From the mixing matrix $N_{ij}$, the nucleon-DM couplings can be calculated using the formulae from Sec. 2.

$$f_p = \frac{\hat{g} f^V_X c^2_\xi t_\xi}{4 \hat{c}_W c_e} \left[ (1 - 4 s^2_W) \left( \frac{1}{m^2_Z} - \frac{1}{m^2_{Z'}} \right) - 3 s_W t_\xi \left( \frac{t^2_\xi}{m^2_Z} + \frac{1}{m^2_{Z'}} \right) \right] - \frac{c^2_\xi f^V_X}{c^2_e} \frac{2 f^V_u + f^V_d}{m^2_{Z'}},$$

$$f_n = - 4 \frac{\hat{g} f^V_X c^2_\xi t_\xi}{4 \hat{c}_W c_e} \left[ \left( \frac{1}{m^2_Z} - \frac{1}{m^2_{Z'}} \right) + s_W t_\xi \left( \frac{t^2_\xi}{m^2_Z} + \frac{1}{m^2_{Z'}} \right) \right] - \frac{c^2_\xi f^V_X}{c^2_e} \frac{f^V_u + 2 f^V_d}{m^2_{Z'}}. \quad (20)$$

Now that we have established the formalism, let us briefly come back to the low energy Lagrangian from Eq. (10). There are three terms which are of particular importance for the phenomenology

1. the direct fermion couplings $\sum_f f^V_f \hat{X}^\mu \bar{f} \gamma_\mu f$,
2. the kinetic mixing term $\frac{1}{2} s_e \hat{B}_{\mu\nu} \hat{X}^{\mu\nu}$, and
3. the mass mixing term $\delta m^2 \hat{Z}_\mu \hat{X}^\mu$.

In the following we will only consider tree-level couplings to quarks, since couplings to leptons have to be strongly suppressed for the $Z'$ mass range we are interested in here. In order to allow for the standard Yukawa couplings, the Higgs field has to be uncharged as well. This leaves us with two possible scenarios for the $Z'$. If the $Z'$ couples to SM states, it has to couple to the baryon current and hence corresponds to a gauged version of the $U(1)_B$ (a baryonic $Z'$). The other possibility is that the SM is completely uncharged under the $U(1)_X$ (a dark $Z'$). In both cases there has to be an additional Higgs field $h'$ which gives mass to the $Z'$. We discuss both cases below.

Coming to the kinetic and mass mixing terms, one might wonder how they are generated in these setups. It is well known that kinetic mixing will be zero at tree level if both $U(1)$’s arise from the breaking of a simple group. However, if there is matter which is charged under both $U(1)$’s, kinetic mixing will in general be induced at 1-loop. If there are fields which are charged under both $U(1)$’s, the mass mixing term can also be generated, e.g. via the operator $\frac{1}{\lambda^2} h^i D^\mu h''^i D^n h' \rightarrow \frac{v^2 m^2}{\lambda^2} Z Z'$. Note however that while the kinetic mixing term is renormalisable, we have to invoke higher dimensional operators to generate a mass mixing term, unless one of the Higgs fields is charged under both the SM gauge group and the new $U(1)_X$. 

8
3.1 Dark $Z'$

Let us first consider the case of a dark $Z'$, where the SM fields are uncharged under the new gauge group. If we also set $\delta m^2 = 0$, i.e. consider kinetic mixing alone, the mass eigenstate $Z'$ has photon-like couplings to quarks, with $|f_n/f_p| \ll 1$. On the other hand, for $\epsilon = 0$, i.e. considering only mass mixing, the resulting $Z'$ has $Z$-like coupling to quarks which leads to $|f_n/f_p| \gg 1$. If both parameters are non-zero, we can achieve $f_n/f_p \sim -0.7$ as required to evade the XENON constraints. In Fig. 2 we plot the contour levels of $f_n/f_p$ with $m_{Z'} = 4$ GeV. The figure shows that the required ratio of proton to neutron coupling can be achieved by adjusting the two parameters appropriately.

![Figure 2: Contours of $f_n/f_p$ in the plane of kinetic and mass mixing parameters $\epsilon$ and $\delta m$, with the dark green trough corresponding to $f_n/f_p \simeq -0.7$.](image)

Interestingly the required value of $f_n/f_p \simeq -0.7$ is achieved for $\epsilon \simeq \delta m^2/m_{Z'}^2$, as can be seen in Fig. 2. In this limit, provided $\epsilon \ll 1$, we have $\xi \approx \epsilon (1 + s_W)$ leading to

$$f_p \approx -\frac{g_Y}{4c_W} \frac{c_T}{\xi} \frac{1}{m_{Z'}^2} \frac{3\hat{s}_W}{1 + \hat{s}_W}, \quad f_n \approx \frac{g_Y}{4c_W} \frac{c_T}{\xi} \frac{1}{m_{Z'}^2} \left( 1 - \frac{\hat{s}_W}{1 + \hat{s}_W} \right). \quad (21)$$

The corresponding ratio of the couplings is $f_n/f_p \approx -1/3\hat{s}_W \approx -0.7$. The value of $f_n/f_p$ is linearly sensitive to a rescaling $\epsilon \rightarrow \alpha \delta m^2/m_{Z'}^2$ with $\alpha$ of $O(1)$. Of course the underlying DM model needs to satisfy this relation in some natural way.
3.2 Baryonic $Z'$

Next we consider the case where the SM is charged under the new $U(1)_X$ gauge group. As discussed above, when we constrain the leptons to be uncharged under $U(1)_X$, the unique possibility is a baryonic $Z'$ with $U(1)_X \equiv U(1)_B$. Anomaly-free models that have a light $Z'$ coupling directly to baryon number such that $f_u^V = f_d^V$ have been considered e.g. in Refs. [35, 36]. In the framework of a baryonic $Z'$ we can also have kinetic mixing, but there is no $Z-Z'$ mixing induced by the SM Higgs at tree-level as it does not carry baryon number [36]. However, mass mixing can still be induced at the non-renormalizable level via e.g. the dim-6 operator $h_1^T D_\mu h_1 h'^T D_\mu h'$ discussed above.

Since up and down quarks have equal charges under the baryonic $U(1)_X$, large direct couplings will lead to $f_n/f_p \simeq 1$ for $f_q^V \gtrsim \epsilon$. In fact, $f_q^V$ must be over an order of magnitude smaller than $\epsilon$ to get $f_n/f_p \simeq -0.7$, as shown in the left panel of Fig. 3. As discussed in the next section, the coupling $\epsilon$ is constrained to be of order $10^{-2}$ or smaller, such that $f_q^V \lesssim 10^{-3}$.

![Figure 3](image)

Figure 3: Left: The ratio $f_n/f_p$ as a function of $f_q$ and $\epsilon$ for $\delta m^2 = 0$. Right: $f_n/f_p$ as a function of $\delta m^2$ and $\epsilon$ for $f_q^V = 10^{-5}$.

Since the direct quark couplings must be very small in order to obtain the desired ratio $f_n/f_p$, we can in fact relax the restriction that the additional $U(1)$ must be baryonic. Allowing for couplings to leptons would facilitate the construction of an anomaly free model.

Charging the SM fermions under the new gauge group induces kinetic mixing via loops [36]. Below the electroweak symmetry breaking scale the induced mixing of the $Z'$ with the $Z$ ($\epsilon_Z$) and the photon ($\epsilon_A$) will be different. At low energy one can then perform
a transformation as in Eq. (11) and recast the mixings $\epsilon_{A,Z}$ into a kinetic mixing $\epsilon$ and an additional $Z'-Z$ mass mixing term $\delta m^2$ as in Eq. (10). The resulting mass mixing term will be of the same order as $\epsilon m^2_Z$, which was needed to achieve $f_n/f_p \approx -0.7$ in the absence of direct couplings to quarks. However, both these terms will be small compared to the direct coupling of $Z'$ to quarks, $f_q^{V'}$, so that one actually obtains $f_p \approx f_n$ in the absence of additional contributions. One possible way of avoiding the direct coupling contribution to $f_{n,p}$ is to couple the $Z'$ only to the second and third generation, so that the direct couplings do not contribute to $f_n/f_p$, but do induce kinetic mixings via quark loops. In this case, the required difference between $\epsilon_Z$ and $\epsilon_A$ can be achieved by imposing the initial condition $\epsilon_Z = \epsilon_A = 0$ at the TeV scale.

4 Limits on the mixing parameters

In this section, we discuss various constraints on a light $Z'$ coupled to DM. We do not require the $Z'$ interactions with the SM to yield the correct thermal relic density for $\chi$ as the DM relic density may well be of asymmetric origin. In particular the mass range required to fit DAMA and CoGeNT is natural in models of asymmetric DM (ADM), where the observed cosmological DM energy density is realised for $m_\chi \sim 1 - 10$ GeV [37, 38]. Moreover the ADM scenario avoids the otherwise significant constraints from annihilation of light DM, such as from the Sun [39, 40].

One class of relevant constraints arise from measurements of the $Z$-pole — these constrain directly the magnitude of the mixing between $Z$ and $Z'$, while measurements of low energy observables are sensitive to both the $Z'$ couplings and mass. Collider constraints from mono-jet [41–43] and di-jet searches [44] also constrain the couplings to quarks; however if the mediator is light, these constraints are fairly weak.

4.1 Electroweak precision tests

The constraints from electroweak precision measurements are encoded in the $S$ and $T$ parameters. From the effective Lagrangian formulation of the interaction between $Z$ and the SM fermions [23, 45]

$$\mathcal{L}_Z = -\frac{e}{2 s_W c_W} \left( 1 + \frac{\alpha T}{2} \right) \bar{f} \gamma^\mu f$$

$$\times \left[ \left( T_3^{f'} - 2Q^f \left( s_W^2 + \frac{\alpha S - 4c_W^2s_W^2\alpha T}{4(c_W^2 - s_W^2)} \right) \right) - T_3^{f} \gamma^5 \right] f Z_{\mu} ,$$

(22)
we can determine $S$ and $T$ to quadratic order in $\xi$ and $\epsilon^2$

\[
\alpha S = 4c_W^2 s_W \xi (\epsilon - s_W \xi) ,
\]

\[
\alpha T = \xi^2 \left( \frac{m_{Z'}^2}{m_Z^2} - 2 \right) + 2 s_W \xi \epsilon ,
\]

(23)

(24)

where $\alpha = \epsilon^2/4\pi$. In the framework that we consider, one typically has $\epsilon > s_W \xi$ so that the $S$ parameter is slightly positive. On the other hand, we require $m_{Z'} < m_Z$ so that the $T$ parameter will generally be negative. In this direction, $S$ and $T$ are tightly constrained; however, given that in our preferred parameter region we have $\xi \sim \epsilon \sim 0.01$ (see Sec. 5), we get $S \simeq 0.01$, $T \simeq -0.015$ which is adequately within the current constraints [46].

The bound on $\xi$ and $\epsilon$ also implies that the $\rho$ parameter, given by

\[
\rho = \frac{m_W^2}{m_{Z'}^2} = \frac{s_W^2}{c_W^2} ,
\]

(25)

is within experimental uncertainties. To quadratic order in $\xi$ and $\epsilon$ we obtain

\[
\rho - 1 = \frac{c_W^2 s_W^2}{c_W^2 - s_W^2} \left( \frac{m_{Z'}^2}{m_Z^2} - 1 \right) ,
\]

(26)

which gives $\rho - 1 = -3 \times 10^{-4}$ for $\xi = 0.015$.

4.2 $Z$ decay width

The induced coupling of the $Z$ to the DM particle increases the invisible $Z$ decay width. The contribution is approximately

\[
\Gamma(Z \to \bar{\chi}\chi) = \frac{G_F m_Z^3}{24\pi \sqrt{2}} \left( |g^V_\chi|^2 + |g^A_\chi|^2 \right) = \frac{G_F m_Z^3}{24\pi \sqrt{2}} s_\xi^2 \left( |f^V_\chi|^2 + |f^A_\chi|^2 \right) .
\]

(27)

Consequently, as long as $s_\xi < 0.015$, we satisfy the experimental limit $\Gamma(Z \to \bar{\chi}\chi) < 1.5$ MeV even if we saturate the perturbative bound.

\footnote{Note that due to the requirement $f_n/f_p \simeq -0.7$, $\epsilon$ must be of the same order of magnitude as $\xi$. We therefore have a -2 appearing in the expression for $T$ as opposed to a -1 in [23].}
4.3 Muon $g - 2$

In the presence of a new vector mediator the anomalous magnetic moment of the muon will generally change. In the case of a dark $Z'$ with $m_{Z'} \ll m_Z$, the contribution to $a_\mu = (g_\mu - 2)/2$ can be estimated to be a factor of order unity.

\[ \delta a_\mu \approx \frac{\alpha \xi^2}{3 \pi c_w^2 s_w^2} \frac{m_\mu^2}{m_{Z'}^2} \]

(28)

up to a factor of order unity. The requirement $\delta a_\mu \approx 4 \times 10^{-9}$ then implies an approximate limit of $m_{Z'} > 1 \text{ GeV}$ for $\xi \approx 10^{-2}$.

4.4 Atomic parity violation

The contribution of the $Z'$ to atomic parity violation (APV) is proportional to the product of the axial coupling of the $Z'$ to electrons, $g^A_{eZ'}$, and the vector coupling to quarks $g^V_{qZ'}$. In practice, however, one can only measure the vector coupling to an entire nucleus, which is proportional to $Z f_p + (A - Z) f_n$. Because $f_p$ and $f_n$ have different signs in all the cases that we consider, the two contributions nearly cancel, so that no relevant constraint arises from APV. Note that in fact, the strongest bounds on APV come from measurements of cesium, which has almost the same ratio of protons to neutrons as xenon (see Fig. 1). Consequently, if indeed $f_n/f_p \approx -0.7$, the $Z'$ will give almost no contribution to APV in cesium.

4.5 Hadronic decays

Measurements of the decays of $\psi$ and $\Upsilon$ strongly constrain the axial coupling of the $Z'$ to $c$ and $b$ quarks. According to Ref. [47], these limits are

\[ |g^A_c| \lesssim 1.5 \times 10^{-3} \frac{m_{Z'}}{\text{GeV}}, \]

(29)

\[ |g^A_b| \lesssim 0.8 \times 10^{-3} \frac{m_{Z'}}{\text{GeV}}. \]

(30)

In our model, there is no direct axial coupling of the $Z'$ to quarks, so that such a coupling can only arise from $Z - Z'$ mixing. The resulting coupling constants are proportional to $\xi$, i.e. small enough, given the previous constraints on $\xi$, if $m_{Z'} > 1 \text{ GeV}$. The constraint for vector couplings is significantly weaker (especially, if the decay of the $\Upsilon$ into two dark

---

3Here we assume that there are no cancellations between the axial and the vectorial part
matter particles is not possible). In fact, according to Ref. [48] it is easily possible to have $|g_Y| \sim 0.1$. Ongoing and forthcoming searches for a light $Z'$ at collider experiments such as BaBar, Belle, BEPC and LHCb will be able to constrain these parameters more tightly [20].

5 Dark matter direct detection

5.1 Spin-independent interactions

We have previously determined best-fit parameters for a DM explanation of the observed DAMA and CoGeNT modulations with SI scattering, assuming that the relation $f_n/f_p = -0.7$ holds [14]. The results were $m_\chi \sim 10$ GeV and $\sigma_{SI} \sim 10^{-38} - 10^{-37}$ cm$^2$ (depending on whether a small inelasticity is included). We will now demonstrate that such a large cross-section can be realized while evading all the limits discussed above.

First, we consider the case of a dark $Z'$ with no direct couplings to SM quarks. In this case, there are 4 free parameters in our model: $m_{Z'}$, $\delta m$, $m_\chi$ and $f_V^{\chi}$. Note that in the mass range $5 < m_\chi < 15$ GeV, the DM-neutron cross-section depends only very weakly on the DM mass. As an example we take $m_{Z'} = 4$ GeV, $\delta m = 8$ GeV and $m_\chi = 8$ GeV, leading to $\epsilon = 0.007$ and $\xi = 0.011$. The resulting cross-section is

$$\sigma_n \simeq 8 \cdot 10^{-37} (f_V^{\chi})^2 \text{ cm}^2.$$ (31)

Thus, a sufficiently large cross section can be achieved for $f_V^{\chi} \sim 0.1$. Note that there is no reason why $f_V^{\chi}$ cannot be as large as its perturbative bound $f_V^{\chi} \lesssim \sqrt{4\pi}$. In fact, if the DM sector is strongly interacting, $f_V^{\chi}$ could be even larger. By making $f_V^{\chi}$ larger, one could still obtain cross sections of the correct magnitude even if the mixing parameters were smaller or $m_{Z'}$ larger than assumed above. Note however, that since $\sigma_n \propto m_{Z'}^{-4}$, the cross-section quickly becomes too small if $m_{Z'}$ is heavier than $\sim 15$ GeV.

Baryonic $Z'$

We now turn to the case of a baryonic $Z'$. As we have seen before, direct couplings greater than $10^{-3}$ generally spoil the ratio $f_n/f_p$. Thus, we must assume that the $Z'$ is extremely weakly coupled, meaning that for charges of $O(1)$ the gauge coupling $g_X \ll 1$. Setting $\delta m^2 = 0$ and $f_q^V = f_\chi^V = 10^{-3}$ yields $f_n/f_p = -0.7$ if $\epsilon = 0.0265$ and $\xi = 0.0128$, which is still consistent with all constraints. However, the resulting cross-section is

\footnote{The kinetic mixing $\epsilon$ is fixed by the requirement $f_n/f_p \simeq -0.7$, while $\xi$ is fixed as soon as $\delta m$ and $m_{Z'}$ are chosen.}
too small: $\sigma_n < 10^{-41}\text{cm}^2$. Allowing a non-zero $\delta m^2$ does not improve the situation significantly. Consequently, if we wish to achieve a sufficiently high cross-section, the coupling of the $Z'$ to the DM particle has to be significantly larger than to quarks, $f^V_X \gg f^V_q$. While this might seem unnatural in a ‘standard’ $Z'$ model, it may well be possible in a framework that derives from new strong dynamics, such as in Ref. [49]. We are investigating this possibility.

5.2 Spin-dependent interactions

If the $Z'$ couples to the axial DM current, there is an effective axial-axial coupling between the DM particle and quarks given by

$$ b^A_q = \frac{g^A_X g^A_q}{m^2_{Z'}} , \quad (32) $$

neglecting the contribution of the $Z$, which gives a correction of order 1%.

The quark-level couplings $b^A_q$ induce the effective nucleon couplings $a_{p,n}$ according to

$$ a_{p,n} = \sum_{q=u,d,s} \Delta q^{(p,n)} b^A_q , \quad (33) $$

where [40]

$$ \Delta u^{(p)} = \Delta d^{(n)} = 0.84 \pm 0.03 $$
$$ \Delta d^{(p)} = \Delta u^{(n)} = -0.43 \pm 0.03 $$
$$ \Delta s^{(p)} = \Delta s^{(n)} = -0.09 \pm 0.03 . \quad (34) $$

We assume that there is no direct axial coupling, i.e. $f^A_u = f^A_d = 0$. In that case, according to Eq. (4), $b^A_d = b^A_s = -b^A_u$. Consequently,

$$ a_p = -1.36b^A_d , $$
$$ a_n = 1.18b^A_d , \quad (35) $$

meaning that the DM particle couples with roughly the same strength but opposite sign to protons and neutrons.

For zero momentum transfer, the spin-dependent (SD) DM-nucleon cross section $\sigma_{SD}^{p,n}$ is simply given by

$$ \sigma_{SD}^{p,n} = \frac{3}{\pi} \mu^2_{X(p,n)} a^2_{p,n} . \quad (36) $$
Using the results from Sec. 3, we get

\[ \sigma_{p,n}^{SD} \simeq 0.1 \frac{\mu_{\chi}^2}{m_{Z'}} \tilde{g}^2 (f_A^A)^2 \left( \frac{c_{\xi}}{c_e} \right)^2 \left( s_W s_\xi - c_{\xi} t_e + \frac{c_W}{s_W} s_\xi \right)^2 \]

\[ \simeq 3 \cdot 10^{-36} \text{cm}^2 (f_A^A)^2, \]  

(37)

where in the last line we have substituted the same benchmark parameters as above. We observe that the SD cross-section is significantly larger than the SI one. It was shown \(^5\) (see also Ref. \(^\text{[16]}\)) that a SD DM-proton cross section of \(\sim 10^{-36} \text{cm}^2\) is sufficient to explain the DAMA annual modulation. Such a cross section can easily be obtained from a light \(Z'\) mediator.

Consequently, a dark \(Z'\) generically gives rise to both SI and SD interactions of dark matter with nuclei. In the particular case that we consider, SI interactions do not benefit from an enhancement proportional to \(A^2\), so that for sufficiently large axial couplings both interactions should give similar signals in direct detection experiments. This allows for the very interesting possibility that the CoGeNT signal arises from SI interactions, while the DAMA signal arises partly from SD interactions.

6 Conclusion

The apparent conflict between the dark matter interpretation of the DAMA and CoGeNT signals and the null results from other experiments is a challenge for our understanding of dark matter. It has highlighted the fact that various assumptions, which are made in analysing the experimental data in order to derive the DM scattering cross-section (or upper limits thereon), may be inappropriate. In particular, the DM particle need not couple equally to protons and neutrons. Specifically a ratio of neutron to proton coupling \(f_n/f_p \simeq -0.7\) can reduce the tension between experiments using different target materials.

In this paper, we have demonstrated that negative values of \(f_n/f_p\) occur naturally if dark matter interactions with the experimental targets are mediated by a GeV scale \(Z'\) arising from a new \(U(1)\) gauge group extension of the SM. There is a viable parameter region in this model that leads to the desired value for \(f_n/f_p\) and at the same time gives sufficiently high cross sections to explain the recently observed signals.

An interesting feature of light \(Z'\) mediators is that they can also yield a spin-dependent cross-section which is sufficiently large to account for the absolute signal level observed by DAMA. Forthcoming measurements of the \(\Upsilon\) branching ratios will constrain the mass and couplings of the \(Z'\). Moreover, if the dark matter is asymmetric,
such a large cross-section will affect heat transport in the Sun (which has been sweeping up dark matter for several billion years) and measurably alter the fluxes of low energy Solar neutrinos [52–54], thus providing another diagnostic.

Acknowledgements

We thank John March-Russell, Chris McCabe, Graham Ross and Stephen West for useful discussions and the European Research and Training Network “Unification in the LHC era” (PITN-GA-2009-237920) for partial support. FK is supported by the DAAD and KSH acknowledges support from ERC Advanced Grant BSMOXFORD 228169. This work was finalised during the CERN TH-Institute DMUH’11 and we thank all participants for stimulating discussions.

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