A highly abnormal massive star mass function in the Orion Nebula cluster and the dynamical decay of trapezium systems

J. Pflamm-Altenburg\textsuperscript{1,2} and P. Kroupa\textsuperscript{1,2}\textsuperscript{*}

\textsuperscript{1}Argelander-Institut für Astronomie\textsuperscript{†}, Auf dem Hügel 71, D-53121 Bonn, Germany
\textsuperscript{2}Rhine Stellar Dynamics Network (RSDN)

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ABSTRACT

The Orion Nebula cluster (ONC) appears to be unusual on two grounds: the observed constellation of the OB stars of the entire ONC and its Trapezium at its centre implies a time-scale problem given the age of the Trapezium, and an initial mass function (IMF) problem for the whole OB star population in the ONC. Given the estimated crossing time of the Trapezium, it ought to have totally dynamically decayed by now. Furthermore, by combining the lower limit of the ONC mass with a standard IMF it emerges that the ONC should have formed at least about 40 stars heavier than 5 M\textsubscript{\odot} while only 10 are observed. Using the N-body experiments we (i) confirm the expected instability of the Trapezium and (ii) show that beginning with a compact OB-star configuration of about 40 stars both the number of observed OB stars after 1 Myr within 1 pc radius and a compact trapezium configuration can be reproduced. These two empirical constraints thus support our estimate of 40 initial OB stars in the cluster. Interestingly, a more-evolved version of the ONC resembles the Upper Scorpius OB association.

The N-body experiments are performed with the new C-code \textit{cataena} by integrating the equations of motion using the chain-multiple-regularization method. In addition, we present a new numerical formulation of the IMF.

Key words: methods: N-body simulations – stars: kinematics – stars: luminosity function, mass function – open clusters and associations: individual: ONC.

1 INTRODUCTION

46 per cent of all O stars and 4 per cent of all B stars are runaways exceeding 30 km s\textsuperscript{-1} (Stone 1991). Furthermore, the binary fraction among runaway O stars is around 10 per cent (Gies & Bolton 1986) while it is more than 50 per cent in young star clusters (Goodwin et al. 2006). This suggests that binaries are involved in close dynamical encounters leading to stellar ejections while the binary fraction among the ejected stars is decreased. Indeed, Clarke & Pringle (1992) deduced using an analytical approach that massive stars must form in compact small-N groups. The decay of non-hierarchical three-, four- and five-body systems with equal masses as well as a mass spectrum has been investigated by Sterzik & Durisen (1998). They determined the spectrum of the remnant decay products but not the phase-space behaviour of compact few-body systems with time. Hoogerwerf, de Bruijne & de Zeeuw (2000) and Hoogerwerf et al. (2001) were able to trace back the trajectories of some runaways to nearby associations. Ramspeck, Heber & Moehler (2001) determined the age and the calculated time of flight of early-type stars at high galactic latitudes and concluded that they can have their origin in the galactic disc. In the case of the runaways, AE Aurigae and \(\mu\) Columbae, which have spatial velocities greater than 100 km s\textsuperscript{-1}, in combination with the binary \(\iota\) Orionis, Gualandris, Portegies Zwart & Eggleton (2004) have shown that the encounter of two binaries with high eccentricities 2.5 Myr ago and in the comoving vicinity of the current Orion Nebula cluster (ONC) can reproduce the spatial configuration observed today. The spatial distribution of field OB stars can thus be understood qualitatively using theoretical stellar-dynamical methods. However, to obtain a more complete picture we need to study the details and the frequency of occurrence of energetic ejections from the acceleration centres, namely the inner regions of young star clusters.

2 MOTIVATING PROBLEMS

In the case of the ONC, two main discrepancies concerning the properties of its OB stars are found as follows.

2.1 Existence of the Trapezium system

Given the total mass of all four Trapezium stars (Hillenbrand 1997) of 88.4 M\textsubscript{\odot} and their occupying space of nearly 0.05 pc in diameter
the corresponding crossing time can be estimated,
\[ t_c = \sqrt{\frac{4 \, R^3 \, M}{G \, M}} , \]
to be about 13 kyr (Table 1), if the Trapezium is assumed to be a compact virialized subsystem of the ONC. Sterzik & Durisen (1998) noted that most systems in their decay analysis decay within dozens of crossing times. So the ONC-TS is expected to have totally decayed by now, if its age is about 1 Myr (Kroupa 2004).

2.2 The number of OB stars

The virial mass of the ONC is measured to be nearly 4500 M_☉ but only about 1800 M_☉ is visible in stellar mass (Hillenbrand & Hartmann 1998) while cluster-formation models that match the ONC suggest that it may have formed with 10^4 stars plus brown dwarfs and it is expanding now resembling a Pleiades-type cluster embedded in an expanding association to a remarkable degree after 100 Myr (Kroupa, Aarseth & Hurley 2001). The observed mass of all star heavier than 5 M_☉ is 167 M_☉. If the canonical initial mass function (IMF) (Appendix B) is normalized such that 1633 M_☉ are contained in the mass interval ranging from 0.01 up to 5 M_☉ for three different physically possible upper stellar mass limits, m_{max}, of 80, 150 M_☉ or +∞ (Weidner & Kroupa 2004; Figer 2005; Oey & Clarke 2005; Koen 2006), and different IMF slopes above 1 M_☉ the corresponding maximum stellar mass m_{max} and the expected number of OB stars formed in the ONC can be calculated (Table 2). The calculations are based on the IMF by Kroupa (2001) but with a numerically more convenient description1 (Appendix A1).

As a check, the number of stars heavier than 5 M_☉ can also be estimated by normalizing the canonical IMF to the number of stars in the mass range 1–2 M_☉ in the cluster. Using the stellar sample of Hillenbrand (1997), the number of stars heavier than 5 M_☉ can be derived from the number of stars between 1 and 2 M_☉ and noting that in this mass regime only the non-embedded sources are listed. These amount to approximately half of all stars (Hillenbrand 1997). Thus, 26 OB stars are expected to have formed in the ONC. The total mass derived from this mass regime is 1404 M_☉, 22 per cent less than the total estimated mass used above. Given this uncertainty (13–38 stars heavier than 5 M_☉), we perform computations with 10 and 40 stars. It will be apparent below that 40 OB stars are our preferred value.

Furthermore, if stars are drawn randomly from a universal IMF, the number of stars heavier than 5 M_☉ may not be the expectation value of 38. The number can be smaller. To estimate the probability that less than k of n stars have masses less than 5 M_☉ drawing stars from an IMF has to be interpreted as a Bernoulli experiment: for the mass of the ONC, M_{ONC}, the total number of stars, n_{tot}, and two different IMF slopes, α, for the mass range from 1 M_☉ to m_{max}. The mass range less than 1 M_☉ is described by a Kroupa IMF (Kroupa 2001).

\[ p = \frac{n_{≤5}}{n_{tot}} . \]
This experiment is repeated n_{tot} times. So the probability to have a star heavier than 5 M_☉ is given by the Bernoulli

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1 A utility-IMF package and CATENA including a full documentation can be downloaded from the AIfA-webpage: http://www.astro.uni-bonn.de/

Table 1. Set-up data for all three models.

| Model | Stars | M_{tot,OB}(M_☉) | σ_{3D}(km s^{-1}) | t_c(kyr) | Runs |
|-------|-------|-----------------|-------------------|----------|------|
| 4-body | 6^1 in Table 3 | 88.4 | 3.9 | 12.6 | 1000 |
| 10-body | All in Table 3 | 167.2 | 5.4 | 9.2 | 1000 |
| 40-body | 4 × Table 3 | 668.8 | 10.7 | 4.7 | 1000 |

Notes. Specification of the N-body systems. M_{tot} is the total mass in M_☉. The velocity dispersion σ is calculated from the total mass placed within the initial radius of 0.025 pc corresponding to the ONC-TS size if the virial equilibrium is assumed. t_c is the crossing time, and runs are the total number of integrated configurations.

Table 2. The number of expected OB stars and maximum stellar mass in the ONC.

| α | m_{max}(M_☉): | +∞ | 150 | 80 | Observation |
|---|---------------|----|-----|---|-------------|
| 2.35 | m_{max}(M_☉) | 76.4 | 59.5 | 46.8 | 45.7 |
| 2.35 | N_{>5} | 38.7 | 38.3 | 37.8 | 10 |
| 2.35 | M_{tot}(M_☉) | 2103 | 2076 | 2048 | 1800 |
| 2.7 | m_{max}(M_☉) | 28.6 | 27.7 | 26.1 | 45.7 |
| 2.7 | N_{>5} | 18.4 | 18.4 | 18.3 | 10 |
| 2.7 | M_{tot}(M_☉) | 1799 | 1797 | 1794 | 1800 |

Notes. Observed (Hillenbrand 1997) and expected maximum stellar mass (m_{max}), number of stars more massive than 5 M_☉ (N_{>5}), and total initial mass (M_{tot}) for the ONC in dependence of three different physically possible upper stellar mass limits, m_{max}, and two different IMF slopes, α, for the mass range from 1 M_☉ up to m_{max}. The mass range less than 1 M_☉ is described by a Kroupa IMF (Kroupa 2001).

Table 3. Identity of the stars used in the three models (Table 1).

| Name | Parenago | SpT | m(M_☉) |
|------|----------|-----|--------|
| 1A | 1865 | O9V | 18.9 |
| 1B | 1863 | B0V | 7.2 |
| 1C | 1891 | O7V | 45.7 |
| 1D | 1889 | B0Vp | 16.6 |
| 2A | 1993 | O9V | 31.2 |
| 2B | 2031 | B1V | 12.0 |
| LP Ori | 1772 | B2V | 7.2 |
| — | 1956 | B3 | 6.4 |
| NU Ori | 2074 | B1V | 16.3 |
| HD37115 | 2271 | B5V | 5.7 |

Notes. Stellar data for all OB stars over 5 M_☉ given by Hillenbrand (1997). Spectral type after van Altena et al. (1988).

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μ ≈ diameter. For the supernova progenitor, they deduced a mass of 5 M⊙ from a Kroupa IMF; the expectation value μ of the number of stars heavier than 5 M⊙ and the total number of stars ntot (equivalent to the number of repeated experiments) are calculated for two different total cluster masses and two different physically possible upper stellar mass limits. The probability to get 20, 10 or fewer OB stars for two different upper stellar mass limits is large the Poissonian limit can be applied. The probability is approximately

\[ P(k \leq 10) = \frac{\mu^k}{k!} e^{-\mu}, \]

\[ P(k \leq 20) = \frac{\mu^k}{k!} e^{-\mu}, \]

where \( \mu = n_{\text{tot}} \). So the total probability to get \( k \) or fewer OB stars is

\[ P(\leq k) = \sum_{i=0}^{k} p(i). \]

The probability to get 20, 10 or fewer OB stars for two different ONC masses and two different physically possible upper stellar mass limits is given in Table 4. It is extremely unlikely that only 10 stars have formed in the ONC if the IMF is universal.

We note that the same argument can be applied to a more-evolved population. In an exploration of the full stellar population of the Upper Scorpius OB association, Preibisch et al. (2002) determined a total stellar mass of 2060 M⊙ covering a volume of 35 pc in diameter. For the supernova progenitor, they deduced a mass of 50–60 M⊙. An IMF steeper than 2.3 in the regime of massive stars would not have lead to the formation of such a massive star in the young star cluster stage of the Upper Scorpius OB association 5 Myr ago for this mass of 2060 M⊙ (Weidner & Kroupa 2006). This further supports that the IMF may not be steeper than 2.3 for massive stars. Preibisch et al. (2002) listed 19 stars heavier than 5 M⊙. This is approximately half of the expected number of formed stars more massive than 5 M⊙ and constitutes the same problem as for the ONC due to similar initial cluster masses. Therefore, it can be argued that O and B stars may have been ejected from their star-forming region very early after their formation.

So two questions arise assuming the IMF is invariant: why does the Trapezium still exist and where are the missing OB stars?

### 3 INTEGRATOR

To investigate the dynamics of the OB stars in the ONC we perform direct N-body integrations. Because close encounters with high eccentricity are very frequent in compact few-body systems due to the grainy potential, a multiple regularization technique is required to reduce energy errors and speed-up the calculations. We combined in our own code (CATENA1) the very efficient CHAIN-regularization formalism developed by Mikkola & Aarseth (1990, 1993) with an embedded Runge–Kutta method of 8(9)th order using a coefficient set published by Prince & Dormand (1981), instead of the Aarseth–CHAIN–Burlisch–Stöer integrator, to integrate the regularized equations of motions.

Computer codes for studying the dynamics of few-body systems and star clusters or planetary systems are available. A very valuable review of this kind of software industry is given in Aarseth (1999, 2003). However, interestingly, there is a lack of software for calculating the dynamical decay of systems with a few \( N \leq 4 \) to several dozen stars. Our endeavour is to fill this gap by a sophisticated software tool allowing us to efficiently study the decay of hierarchical and non-hierarchical configurations of tens or hundreds stars down to the last remaining hard binaries or hierarchical higher-order multiple stars, with the long-term aim of embedding CATENA in a general purpose N-body code.

An error analysis for the present application is provided in Section 7.

### 4 INITIAL CONDITIONS

To address the questions mentioned above, we investigate three models which consist of the stars listed in Table 1.

In the first model, we study the stability of the actually observed Trapezium system consisting of \( \theta^1 \) A, \( \theta^1 \) B, \( \theta^1 \) C and \( \theta^1 \) D precisely. In the second model, it is assumed that all currently observed OB stars in the ONC (Table 3) were initially in a compact configuration as a core at the centre of the ONC, due to either mass segregation or \textit{ab initio}. In the third model, we start with an OB core coming close to the expected number of 38. To find a suitable set of stars, all presently observed OB stars are used four times giving 40 stars (four times Table 3).

The compact settings of OB stars are motivated by the outcome of the analytical investigation by Clarke & Pringle (1992) that massive stars form in compact groups. Bonnell & Davies (1998) concluded that the positions of massive stars in the Trapezium cluster in Orion cannot be due to dynamical mass segregation, but must have formed in, or near, the centre of the cluster.

For each of these three models, 1000 configurations are created where the stars from Table 1 are uniformly distributed over a sphere with the compact Trapezium radius of 0.025 pc (Hillenbrand 1997). The velocities are drawn from a Gaussian distribution with a velocity dispersion resulting from the virial theorem,

\[ \sigma = \frac{G M_{\text{tot,OB}}}{R} \]

(Table 1). After this, the velocities are re-scaled slightly to ensure initial virialization.

This simple model does not include the rest of the ONC. To estimate its effect on the core decay the ratio of the internal and external forces can be calculated. The OB-star core of radius \( r \) consists of \( n \) stars having the mean mass \( m \). The gravitational force on one star is then

\[ F_n = G \frac{nm^2}{r^2}. \]

The force exerted by the rest of the ONC on one star in the core can be estimated by the Plummer force

\[ F_p = GmM_p(r^2 + b^2)^{-3/2}. \]

assuming the cluster can be represented reasonably well by a Plummer model, which has been shown to be the case (Kroupa...
et al. 2001). The resulting force ratio is
\[ \Phi = \frac{F_s}{F_{pl}} = \frac{nm}{M_\text{cl}} \left( 1 + \frac{b^2}{r^2} \right)^{-\frac{1}{2}}. \] (9)

The mass $M_\text{cl}$ is the cluster mass minus the mass of the OB stars. The resulting force ratios can be seen in Table 5. The core dynamics is dominated by its self-gravitation.

The escape velocities for the isolated core and the total Plummer sphere can also be compared. Both are obtained from the conservation of energy theorem. The escape speed from the centre of the Plummer sphere, $v_{c,pl}$, and the escape speed from an isolated OB-core are given by
\[ v_{c,pl} = \sqrt{\frac{2Gm_{cl}}{b}}, \quad v_{c,OB} = \sqrt{\frac{2Gm_{OB}}{r_0}}. \] (10)

respectively, where $r_0 = 0.025$ pc is the initial radius of the OB-core. For the four- and 10-body model, the escape speeds for the isolated model and the true embedded situation are comparable. In the 40-body model, the escape speed from the core is dominated by the core itself.

A second issue associated with the cluster shell of low-mass stars is a two-body relaxation between an OB-star and the low-mass stars of the cluster. Energy may be transferred from the OB-star core and ejected or evaporated OB stars to the rest of the cluster. The relaxation time of the ONC is about 18 Myr (Kroupa 2005). The relaxation time for a heavy star is given by multiplying the relaxation time with the ratio of the mass of the most massive star and the mean stellar mass (Spitzer 1987) and describes the time-scale of a massive star to sink towards the cluster centre,
\[ t_{\text{relax,OB}} \approx \frac{\bar{m}}{m_{\text{OB}}} t_{\text{relax}}, \] (11)

where the average mass $\bar{m}$ of a star is 0.35 $M_\odot$ using a Kroupa IMF. The resulting energy transfer time-scale ranges from 0.14 Myr (45.7 $M_\odot$) up to 1.26 Myr (5 $M_\odot$), thus being shorter or comparable to the time spanned by the simulations and therewith probably an important issue in our context, given the age of the ONC $\approx 1$ Myr. In the case of no equipartition instability, energy transfer stops after reaching energy equipartition,
\[ \bar{m}(v^2) = m_{\text{OB}}(v_{\text{OB}}^2), \] (12)

where $(v^2)$ ($(v_{\text{OB}}^2)$) is the mean square velocity of the mean mass stars (OB stars, respectively). Using a velocity dispersion of 2 km s$^{-1}$ (Hillenbrand 1997) for the mean mass stars, the relation above and the energy theorem it can be calculated that the velocity of a 5 $M_\odot$ (45 $M_\odot$) is low enough such that the movement of the OB stars is constrained to be within a radius of 0.026 pc (0.0084 pc). So the current observed OB core has an extension consistent with energy equipartition. Following Heggie & Hut (2003), the heavy stars are so concentrated that the lighter stars have been expelled from the core and they no longer have a significant role. This is also suggested by the observed deficit of low-mass stars in the core of the ONC (Hillenbrand & Hartmann 1998).

We conclude that the effect of the two-body relaxation between the low-mass stars and the OB stars may be of minor importance and that these simulations suffice to demonstrate the time-scale problem of the ONC, and that the OB-star core-decay model may explain the OB-star number problem of the ONC. While the full-scale N-body calculations capture the entire relevant physics, our approximations allow us to compute a very large number of renditions (here 5000 in total) which is necessary given the low frequency of massive stars. Future N-body calculations of individual set-ups will be used to check our results.

### 5 FINDING TRAPEZIUM SYSTEMS

We define a trapezium system to consist of a few stars having pair-wise distances of the same order. Here, the whole system is scanned to determine the maximum number of stars in a configuration in which the pairwise distances lie between two boundaries: when studying the stability of the ONC Trapezium these boundaries are 0.01 and 0.05 pc. When studying the total OB-star distribution these boundaries are 0 and 0.05 pc.

This procedure is illustrated in Fig. 1. Consider a configuration consisting of six bodies. A table containing the pairwise distances is created. All subsets of particles having a pairwise distance between two boundaries are determined. Of all these subsets the one having the most members is the extracted trapezium system. For the explicit example above, assume that a trapezium system of dimension $a$ is searched. The pairwise distances may be allowed to deviate by 20 per cent from this dimension. Then the subsets found are: {2, 3}, {2, 4}, {2, 5}, {3, 4}, {3, 5}, {4, 5}, {2, 3, 4}, {2, 3, 5}, {2, 4, 5}, {3, 4, 5} and {2, 3, 4, 5}. So the trapezium system consists of four bodies.

If a trapezium system of the dimension $b$ is searched all subsets extracted from the distance table are: {1, 2, 6}, {1, 3, 6}, {1, 4, 6} and {1, 5, 6}. Four candidates for a trapezium system of dimension $b$ are found. However, they all have the same number of members.

Based on this algorithm a set of bodies can contain no trapezium system of a certain dimension, or a trapezium system can have two or more members. However, it is not possible to find a trapezium system consisting only of one body.

If the number of stars within a certain sphere is of interest then the lower boundary must only be set to zero.
The pairwise distances of the Trapezium stars in the ONC Θ1 lie approximately between 0.02 and 0.05 pc, whereby all of the OB stars can be found within nearly 1 pc radius around Θ1 (Hillenbrand 1997).

6 DECAY OF OB-STAR CORES

All 3000 configurations are integrated over 2 Myr, 95 per cent of all runs have a relative energy error lower than $10^{-14}$ in the case of the four-body model, $10^{-12}$ for the 10-body model and $10^{-10}$ for the 40-body model.

6.1 Four-body model

In the top panel of Fig. 2, the decay curve of a four-body trapezium consisting of the four stars of Θ1 Ori is plotted. After 1 Myr only 2.5 stars on average are found with a pairwise distance less than 0.05 pc, and one star on average is found with a pairwise distance between 0.05 and 0.01 pc, where in both cases the 3D curves lie slightly below the 2D-projection decay curve. This demonstrates the time-scale problem pointed out above for the observed trapezium which is marked by the bold x. It can be argued that these are only mean values which are gained from a number distribution, and that a compact trapezium as is observed can survive for 1 Myr with some probability. This is not the case that can be seen in Fig. 3, where the distribution of the member numbers is plotted in a histogram. In 62.2 per cent of all runs no stellar configuration is found with a pairwise distance between 0.01 and 0.05 pc, while in only 0.4 per cent of all runs a trapezium of the observed size is found after 1 Myr.

On a first view this stands in contradiction to Allen & Poveda (1974), which is the only known work about stability of trapezium systems. Their result was that 63 per cent of all trapezia remain as trapezia after 30 crossing times ($\approx 1$ Myr). The reason is that they used a slightly different definition of a trapezium system: "let a multiple star system (of three or more stars) ... if three or more such distances are of the same order of magnitude, then the multiple system is of trapezium type.... Two distances are of the same order of magnitude, in this context, if their ratio is greater than $1/3$ but less than $3'$. A four-body system satisfying our definition of a trapezium system, i.e. all pairwise distances are of the same order of magnitude, can evolve into a system of two single stars and one close binary. Then three distances are of the same order of magnitude, i.e. the distance between the two single stars, and the distances between each single star and one component of the binary. Their definition will detect a trapezium system. Our algorithm also detects a trapezium system but consisting only of three stars and having a different size than searched for. So the criterion by Allen and Poveda takes into account neither the number of members of the trapezium system nor the size of the trapezium. For a quantification of the stability of a four-body system, the crucial point is the number of stars the trapezium consists of. The Allen-and-Poveda trapezia consist initially of six stars, and only one of all 30 configurations (3.3 per cent) retains the initial size after 1 Myr, and consists finally only of four stars. Indeed in their 30 runs binaries form consisting preferentially of the two most massive stars, therewith being quite consistent with our result.

6.2 10-body model

As in the four-body model the mean number of stars within 0.05 and 1 pc is plotted in Fig. 2. The decay of an initial 10-body OB star core can reproduce neither a four-body trapezium with a diameter of 0.05 pc nor the entire present-day ONC OB population (Fig. 3).

6.3 40-body model

If it is assumed that the ONC had an initial OB-star content of nearly 40 as expected from the canonical IMF combined with the
estimated stellar mass of the ONC of about 2200 M⊙ then the remaining number of OB stars after 1 Myr within a sphere of 1 pc radius comes close to the observed value of 10 (Fig. 2). The probability to observe a trapezium at an age of 0.5 Myr is 13.8 per cent but only 0.7 per cent at an age of 1 Myr (Fig. 3). However, when counted together with the systems containing more than four stars the probability to find a compact trapezium increases to 0.7 + 2.9 = 3.6 per cent. Note that the estimated ONC mass is comprised of the observed mass (1800 M⊙) plus the estimated mass in missing 30 OB stars (400 M⊙). The true initial ONC mass may have been about 4000 M⊙ (Kroupa et al. 2001).

In Fig. 4, the spatially cumulative and velocity distributions of the O and B stars after 1 and 2 Myr are plotted. After 1 Myr more than 75 per cent (30 of 40) of all stars have larger distances to their common centre of mass than 2 pc, and after 2 Myr 75 per cent of all stars are more than 4 pc away from their centre of mass. Therefore, only 10 of the 40 stars heavier than 5 M⊙ remain at the cluster centre as observed in the ONC. It can be seen that more O stars than B stars are at very large distances as well as more O stars than B stars have very high velocities. This comes about because initially B stars tend to evaporate by energy redistribution rather than being ejected by close encounters. Because O stars are heavier they form tighter configurations than B stars and they are then involved in close binary interactions leading to high ejection velocities. This confirms the result of Clarke & Pringle (1992) qualitatively.

Given the spatial distribution of OB stars after an evolution of 1 and 2 Myr, an extrapolation to an evolution age of 5 Myr predicts that nearly 50 per cent of all OB stars cover a volume of 25 pc in diameter which comes close to the observed properties of the Upper Scorpius OB association as pointed out in Section 2.2.

7 ERROR ANALYSIS

An important question concerning N-body simulations is whether we can trust them. This question arises from the fact that the basic process for the decay of N-body systems is close and highly eccentric encounters of stars with a consequent redistribution of energy. These kinds of encounters are the most important sources for orbit errors. Due to the exponential instability, the numerical and true solutions deviate increasingly with time. (Goodman, Heggie & Hut 1993; Heggie & Hut 2003). As there exists no general analytic solution for systems with more than two bodies the integrals of motion have to be checked for conservation to control the integration. The most sensitive quantity is the total energy. Smaller step-sizes reduce the amount of the error but prolong the duration of the integration, while step-sizes that are too small lead to the accumulation of round-off errors. So a balance between efficiency and accuracy needs to be found.

Dejonghe & Hut (1986) determined the accuracy of the integration of 14 small-N configurations using the time-reversal test compared with energy conservation, leading to the conclusion (Aarseth 2003) that using the energy error only is questionable for establishing exact integrations.

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The event probability is approximated by the mean probability 
\( p = \bar{p} = \frac{k_1 + k_2 + (k_3/3)}{n} = \frac{k}{n} \) 
\( \Delta k = np - p^2 \).

Where \( k_1 \) is the number of events in each of the three experiment series. The variance of the Bernoulli distribution is given by

\( \Delta k^2 = np - p^2 \).

Figure 5. Error analysis using the example of the four-body decay determining the maximum number of members in configurations with a pairwise distance below 0.05 pc in 3D after 1 Myr (top diagram), 2.73 × 10⁻⁶ (middle diagram) and 6.19 × 10⁻² (bottom diagram).

When using the statistical approach, high accuracy is not essential to obtain meaningful results, provided the sample is sufficiently large (Aarseth 2003). Valtosen (1974) determined that the distributions of eccentricity, terminal escape velocity and life-time in 200 3D experiments did not show any clear accuracy dependence for a relative energy error range of 5 × 10⁻⁴ to 3 × 10⁻².

Therefore, despite the fact that the orbits are completely wrong after many crossing times, the statistical outcome from many equivalent N-body experiments is reliable.

To test what value of energy error is acceptable, we run all 1000 four-body configurations three times with different step-size parameters. The resulting mean energy errors are 5.65 × 10⁻¹² (top diagram), 2.73 × 10⁻⁶ (middle diagram) and 6.19 × 10⁻² (bottom diagram).

To compare the statistical error with the numerical error we interpret this analysis as a series of Bernoulli experiments. One experiment is the determination of a trapezium consisting of \( N_{\text{stars}} \) stars after a certain time \( T \). The outcome can be yes with probability \( p \) and no with the probability \( 1 - p \). This experiment is repeated \( n \) = 1000 times. Therefore, the probability to get \( k \) times the event yes is given by

\( P(k) = \binom{n}{k} p^k (1 - p)^{n-k} \). (13)

The event probability is approximated by the mean probability

\( p = \bar{p} = \frac{k_1 + k_2 + (k_3/3)}{n} = \frac{k}{n} \)  \( \Delta k = np - p^2 \).

Where \( k_1 \) is the number of events in each of the three experiment series. The variance of the Bernoulli distribution is given by

\( \Delta k^2 = np - p^2 \).

Table 6. 1σ errors for the four-body decay (Fig. 5) interpreted as Bernoulli experiment.

| \( \Delta E \) / \( \Delta V \) | 1 | 1 | 1 | 2 | 2 | 2 |
|---|---|---|---|---|---|---|
| \( N_{\text{stars}} \) | 2 | 3 | 4 | 2 | 3 | 4 |
| \( n \) | 1000 | 1000 | 1000 | 1000 | 1000 | 1000 |
| \( \hat{k} \) | 802.3 | 18.9 | 8.7 | 877.0 | 122.0 | 1.0 |
| \( \hat{p} \) | 0.802 | 0.189 | 0.087 | 0.877 | 0.122 | 0.010 |
| \( \Delta k^2 \) | 28.3 | 13.7 | 9.3 | 29.6 | 11.0 | 3.2 |

Notes. Resulting variances for the error histograms in Fig. 5 if the search for a trapezium consisting of \( N_{\text{stars}} \) is interpreted as a Bernoulli experiment.

The corresponding probability \( p \), number of experiments \( n \), mean number of events \( \hat{k} \), variance \( \Delta k^2 \) and 1σ errors \( \Delta \) per cent for the histograms in Fig. 5 are given in Table 6. As an example, we consider Trapezia consisting of two bodies after 1 Myr. The mean number of runs having a Trapezium of two bodies after 1 Myr is 802 out of 1000 (80.2 per cent). The 1σ variance is 2.8 per cent. So all the three experiments (81.9, 79.9 and 78.9 per cent) lie within 1σ around the mean value (80.2 ± 2.8 = 77.4–80.2 per cent). So if a numerical error arises from the different choice of the step-size parameter, then it is not larger than the statistical error. We conclude that we can trust these N-body simulations.

From the virial theorem

\[ E = \frac{V^2}{2} \]  \( \Delta E \approx \Delta V \), (17)

and with

\[ V = -G \frac{M^2}{R} \]. (18)

\[ \frac{\Delta E}{E} \approx \frac{\Delta R}{R} \]. (19)

This means that uncertainty in the pairwise distances of the order of the mean energy error. As the energy errors are smaller than 6 × 10⁻² the distance errors have only a very slight effect on the number statistics.

8 CONCLUSIONS

We have pointed out that in the case of the ONC two main problems exist: its short decay time implying the question as to why it exists, and the significant number of missing OB stars implying that either they have been lost if the IMF is invariant, or the IMF have a highly non-standard \( \alpha_3 > 2.7 \) which is unlikely because no other stellar population in a cluster with such an IMF is known to exist and the observed most massive star in the ONC is significantly larger than that expected for this non-standard IMF. It is extremely unlikely (three out of 1000 cases) that a compact trapezium system consisting of four stars can survive for more than 1 Myr. The assumption that the initial number of OB stars was about 40 increases the probability to observe a trapezium system after 1 Myr (36 out of 1000 cases). We infer that the ONC Trapezium system could be an OB-star core in its final stage of decay.

This scenario is supported by the fact that the spatial distribution of these 40 OB stars obtained from the numerical simulations...
after 5 Myr comes close to the observed spatial distribution of the OB stars in the Upper Scorpius OB association which is assumed, using the total mass, to have had the same young star cluster progenitor as the ONC.

Given its total mass the ONC indeed ought to have four times as many OB stars than those are observed, namely 40. This suggests that about 30 OB stars may have been expelled from the ONC if the IMF is invariant. Starting with an initial number of OB stars of about 40, stars are ejected due to three-body encounters, so that this model matches the observed number of OB stars in the ONC and the ONC Trapezium.

As it has been shown that the probability that only 10 OB stars have formed in the ONC is very small, the missing stars must be somewhere. After 1 (2) Myr, 90 (70) per cent of all OB stars should be found within a 10 pc radius around the ONC centre. It is straightforward to search for these missing OB stars in OB catalogues. If they cannot be found then the IMF may be steeper at the high-mass end; or OB stars form with an initially high binary fraction so that ejections occur faster and the resulting velocities are higher, placing them even further away from the ONC centre after 1 (2) Myr.

Due to the absence of primordial binaries, ours is a conservative result because binaries enhance the ejection rates. Therefore the influence of primordial binaries must be investigated in further experiments, which will also need to consider the gas expulsion from the embedded cluster (Kroupa et al. 2001; Vine & Bonnell 2003). Vine & Bonnell (2003) investigated the evolution of cores of young star clusters and their massive stars but used a smoothed potential and the energy loss by the two-body relaxation may return potential and the energy loss by the two-body relaxation may return.

The influence of the cluster potential and especially of the two-body relaxation with low-mass stars in the cluster shell may also be investigated in further simulations. The nature of this influence is still somewhat unclear. Stronger constraining forces by the cluster potential and the energy loss by the two-body relaxation may return some of the OB stars evaporated with low velocities from the core. This may stabilize the core on the one hand, but may also lead to a faster decay by shifting the velocity spectrum of the ejected stars to higher velocities because the stellar density at the centre would be higher. This would increase the probability of close encounters and high-velocity ejections. If relaxation indeed stabilizes the core then more OB stars are expected to remain in the ONC worsening the discrepancy of OB stars. Ours is therefore a conservative result.

As a final note, Kroupa (2001) have presented star cluster formation calculations that reproduce the ONC at an age of 1 Myr and the Pleiades at an age of 100 Myr. These models, however, are about 1.8 times as massive as the ONC mass used here (2200 M⊙) implying that if the IMF was canonical then the ONC may have had 1.8 × 40 = 72 stars more massive than 5 M⊙. This would pose an increased challenge, because as is evident from Fig. 2, 40 OB stars already lead to an acceptable match with the data, so 72 would increase the probability of finding a trapezium configuration at an age of 1 Myr, but would lead to too many OB stars within the cluster (as can be deduced from the lower panel in Fig. 2).

Clearly, such models with a high initial multiplicity fraction need to be constructed for further studies of the intricate interrelation of the IMF with stellar dynamics in young clusters.

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APPENDIX A: A PRACTICAL NUMERICAL FORMULATION OF THE IMF

A1 The general IMF

When doing research using the IMF a multipower law,

\[ \xi(m) = k \begin{cases} \frac{m}{m_i}^{-\alpha_1} & , \quad m_{\text{low}} \leq m \leq m_i, \\ \frac{m}{m_i}^{-\alpha_2} \left( \frac{m}{m_0} \right)^{-\alpha_3} & , \quad m_i \leq m \leq m_0, \\ \frac{m}{m_i}^{-\alpha_0} \left( \frac{m}{m_0} \right)^{-\alpha_1} \left( \frac{m}{m_{\text{max}}} \right)^{-\alpha_2} & , \quad m_0 \leq m \leq m_1, \\ \frac{m}{m_i}^{-\alpha_0} \left( \frac{m}{m_{\text{max}}} \right)^{-\alpha_3} & , \quad m_1 \leq m \leq m_{\text{max}}. \end{cases} \]  

(A1)

with exponents for its canonical form,

\[ \alpha_0 = +0.30, \quad 0.01 \leq m/M_\odot \leq 0.08, \]
\[ \alpha_1 = +1.30, \quad 0.08 \leq m/M_\odot \leq 0.50, \]
\[ \alpha_2 = +2.30, \quad 0.50 \leq m/M_\odot \leq 1.00, \]
\[ \alpha_3 = +2.35, \quad 1.00 \leq m/M_\odot \leq +\infty, \]  

(Kroupa et al. 1993; Kroupa 2001; Reid et al. 2002; Weidner & Kroupa 2004) requires many if-statements in code implementations. Note, however, that the canonical IMF parametrization constitutes a two-part power-law IMF in the stellar regime: \( \alpha_1 = 1.3 \) for \( m \leq 0.5 M_\odot \) and \( \alpha_{2,3} = 2.3 \) for \( m > 0.5 M_\odot \). Here, we present a general formulation for an IMF that avoids these difficulties with if-statements and allows any number of mass segments and types of interpolating functions. Complicated code constructions containing if-statements are replaced by two easy loops. The described method is realized by some very handy routines implemented in a shared C-library, and is available on request or at the AHA homepage.\(^2\)

The formulation described below can be applied to any arbitrary distribution functions for any purpose.

Due to historical reasons multipower-law IMFs start indexing intervals and slopes at zero instead of one. For simplicity, we here index \( n \) intervals from one up to \( n \).

Consider an arbitrary IMF with \( n \) intervals fixed by the mass array \( [m_0, \ldots, m_n] \) and the array of functions \( f_1, \ldots, f_n \). So on the \( i \)th interval \( [m_{i-1}, m_i] \) the IMF is described by the function \( f_i \). For the case of a multipower law the functions may be chosen to be

\[ f_i(m) = m^{-\alpha_i}. \]  

(A3)

To make this power-law description corresponds to the multipower law given above, the IMF-slope indices above need to be shifted by one.

The following general IMF description does not require power laws for the functions \( f_i \), but also any kind of function is allowed. This includes lognormal IMFs (Miller & Scalo 1979; Chabrier 2003).

With the two \( \Theta \)-mappings

\[ \Theta_1(x) = \begin{cases} 1 & , \quad x \geq 0 \\ 0 & , \quad x < 0 \end{cases} \]  

(A4)

and

\[ \Theta_1(x) = \begin{cases} 1 & , \quad x > 0 \\ 0 & , \quad x \leq 0 \end{cases} \]  

(A5)

the function

\[ \Gamma_{ij}(m) = \Theta_1(m - m_{i-1}) \Theta_1(m_i - m) \]  

(A6)

can be defined. It is unity on the interval \( [m_{i-1}, m_i] \) and zero otherwise. The complete IMF can be conveniently formulated by

\[ \xi(m) = k \prod_{j=1}^{n-1} \Delta(m - m_j) \sum_{i=1}^{n} \Gamma_{ij}(m) \Psi_i f_i(m), \]  

(A7)

where \( k \) is a normalization constant and the array \( (\Psi_1, \ldots, \Psi_n) \) is to ensure continuity at the interval boundaries. They are defined recursively by

\[ \Psi_1 = 1, \quad \Psi_i = \Psi_{i-1} \frac{f_{i-1}(m_{i-1})}{f_i(m_{i-1})}. \]  

(A8)

For a given mass \( m \) the \( \Gamma_{ij} \) makes all summands zero except the one in which \( m \) lies. Only on the inner interval boundaries do both adjoined intervals give the same contribution to the total value. The product over

\[ \Delta(x) = \begin{cases} 0.5 & , \quad x = 0 \\ 1 & , \quad x \neq 0 \end{cases} \]  

(A9)

halves the value due to this double counting at the interval-boundaries. In the case of \( n \) equals one (one single power law), the empty product has, by convention, the value of unity.

An arbitrary integral over the IMF is evaluated by

\[ \int_{m_0}^{m_1} \xi(m) \, dm = \int_{m_0}^{m_1} \xi(m) \, dm - \int_{m_0}^{m_1} \xi(m) \, dm, \]  

(A10)

where the primitive of the IMF is given by

\[ \int_{m_0}^{m_1} \xi(m) \, dm = k \sum_{i=1}^{n} \Theta_1(a - m_i) \Psi_i \int_{m_{i-1}}^{m_i} f_i(m) \, dm + k \sum_{i=1}^{n} \Gamma_{ij}(a) \Psi_i \int_{m_{i-1}}^{m_i} f_i(m) \, dm. \]  

(A11)

The expressions for the mass content, i.e. \( m \xi(m) \), and its primitive are easily obtained by multiplying the above expressions in the integrals by \( m \).

A2 The individual cluster IMF

A2.1 Normalizing the IMF

The IMF denotes the number of stars per mass interval. Therefore, the normalization depends on the cluster mass. Here, we follow the normalization strategy by Weidner & Kroupa (2004). This method requires two further masses. \( m_{\text{max}} \) is the maximum physically possible stellar mass and \( m_{\text{max}}^\ast \) is the expected maximum stellar mass in a given cluster of mass \( M_\Delta \). With \( \xi = k \xi_i(m) \) two equations defining \( m_{\text{max}} \) and \( m_{\text{max}}^\ast \) result:

\[ M_\Delta = k \int_{m_0}^{m_{\text{max}}} m \xi_i(m) \, dm, \]  

(A12)

\[ 1 = k \int_{m_{\text{max}}}^{m_{\text{max}}^\ast} \xi_i(m) \, dm. \]  

(A13)

To solve these two equations for \( k \) and \( m_{\text{max}}^\ast \) they can be divided by each other leading to an expression for the cluster mass as a function of \( m_{\text{max}} \)

\[ M_\Delta = \int_{m_0}^{m_{\text{max}}} m \xi_i(m) \, dm / \int_{m_{\text{max}}}^{m_{\text{max}}^\ast} \xi_i(m) \, dm. \]  

(A14)

As a function of \( m_{\text{max}} \) it is continuous and strictly monotonically increasing and its image is \( R_{\geq 0} \). Therefore the existence of a solution

\[ \int_{m_0}^{m_{\text{max}}} \xi_i(m) \, dm \leq \int_{m_{\text{max}}}^{m_{\text{max}}^\ast} \xi_i(m) \, dm. \]  

(A15)
$m_{\text{max}}$ is concluded by a connectivity argument and the uniqueness follows from the strict monotony. This solution can be gained using any equation-solving method.

Above $m_{\text{max}}$, no stars can be found and the IMF in a star cluster can be expressed by

$$\xi_{\text{cl}}(m) = \Theta_1 (m_{\text{max}} - m) \xi(m). \quad (A15)$$

**APPENDIX B: FINDING THE NUMBER OF EXPECTED OB STARS IN THE ONC**

The observed total stellar mass of the ONC may be less than the initial one if OB stars have been ejected. The total stellar mass $M_\text{cl}$ is related to the IMF by

$$M_\text{cl} = M_\text{<5} + \int_5^{m_{\text{max}}} m \xi(m) \, dm, \quad (B1)$$

where $M_\text{<5}$ is the observed mass in stars less massive than 5 $M_\odot$. For a total mass for the ONC of 1800–3300 $M_\odot$ the expected maximum mass $m_{\text{max}}$ lies in the range 50–63 $M_\odot$ (Weidner & Kroupa 2006). If $5 M_\odot < m_{\text{max}}$, which is the case, the IMF can be normalized directly by

$$M_\text{<5} = k \int_{m_0}^5 m \xi(m) \, dm, \quad (B2)$$

where $m_0 = 0.01 M_\odot$ is the opacity-limited minimum fragmentation mass. The maximum stellar mass is then determined in the second step solving

$$1 = \int_{m_{\text{max}}}^{m_{\text{max}}^\ast} m \xi(m) \, dm, \quad (B3)$$

which means there exists one most massive star in the cluster (Weidner & Kroupa 2004). The expected number of OB stars is then given by

$$N_{\text{OB}} = \int_5^{m_{\text{max}}^\ast} \xi_{\text{cl}} \, dm. \quad (B4)$$

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