Social computing is blossoming into a rich research area of its own, with contributions from diverse disciplines including computer science, economics, and other social sciences. The field spans everything from systems research directed at building scalable platforms for new social computing applications to HCI research directed toward user interface design, from studies of incentive alignment in online applications to behavioral experiments evaluating the performance of specific systems, and from understanding online human social behavior to demonstrating new possibilities of organized social interactions. Yet a broad mathematical foundation for social computing is yet to be established, with a plethora of under-explored opportunities for mathematical research to impact social computing.

In many fields or subfields, mathematical theories have provided major contributions toward real-world applications. These contributions often come in the form of mathematical models to address the closely-related problems of analysis—why do existing systems exhibit the outcomes they do?—and design—how can systems be engineered to produce better outcomes? In computer science, mathematical research led to the development of commonly used practical machine learning methods such as...
boosting and support vector machines, public-key cryptography including the RSA protocol, widely used data structures such as splay trees and techniques like locality-sensitive hashing, and more. Well-known examples in economics include the analysis and design of matching markets that have enabled kidney exchanges and have led to significant successes in public school admissions and residence matching for doctors and hospitals, the influence of auction theory on the design of the FCC spectrum auctions, and the design and redesign of the auctions used in online advertising markets.

As in other fields, there is great potential for mathematical work to influence and shape the future of social computing. There is a small literature using mathematical models to analyze and propose design recommendations for social computing systems including crowdsourcing markets.
prediction markets, human computation games, and user-generated content sites; see, for example, Ghosh for a survey of one facet of this work. However, we are far from having the systematic and principled understanding of the advantages, limitations, and potentials of social computing required to match the impact on applications that has occurred in other fields.

We note that social computing enjoys a close relationship with another emerging discipline, which is computational social science. But it is also distinct from that field. While human and social behavior, ability, and performance are central to both, computational social science focuses primarily on the use of modern technology, data, and algorithms to understand and describe social interactions in their “natural habitats.” In contrast, social computing (as the name suggests) has a much more deliberate focus on engineering systems that are hybrids of humans and machines, which may often entail shaping collective behavior in unfamiliar environments. Nevertheless, we anticipate a continued close relationship and even blurring of the two efforts. As an example, one should expect the vast theoretical and experimental literature on the diffusion of influence and behavior in social networks to be relevant to any effort to design a social computing system that relies on such dynamics to recruit and engage workers.

In June 2015, we brought together roughly 25 experts in related fields at a CCC-sponsored Visioning Workshop on the Theoretical Foundations of Social Computing to discuss the promise and challenges of establishing mathematical foundations for social computing. This document captures several of the key ideas discussed.

**Success Stories**

We begin by describing some examples in which mathematical research has led to innovations in social computing.

**Crowdsourced democracy.** YouTube competes with Hollywood as an entertainment channel, and also supplements Hollywood by acting as a distribution mechanism. Twitter has a similar relationship to news media, and Coursera to universities. But Washington has no such counterpart; there are no online alternatives for making democratic decisions at large scale as a society. As opposed to building consensus and compromise, public discussion boards often devolve into flame wars when dealing with contentious socio-political issues. This motivates the problem of designing systems in which crowds of hundreds, perhaps millions, of individuals collaborate together to come to consensus on difficult societal issues.

Mathematical research has recently led to new systems implementing crowdsourced democracy. This work builds upon a body of research on social choice that examines how to best take the preferences of multiple agents (human or otherwise) and obtain from them a social decision or aggregate social preference, typically accomplished through some form of voting.

Consider situations where a highly structured decision must be made. Some examples are making budgets, assigning water resources, and setting tax rates. Goel et al. made significant progress towards understanding the “right” mechanisms for such problems. One promising candidate is “Knapsack Voting.” Recall that in the knapsack problem, a subset of items with different values and weights must be placed in a knapsack to maximize the total value without exceeding the knapsack’s capacity. This captures most budgeting processes—the set of chosen budget items must fit under a spending limit, while maximizing societal value. Goel et al. prove that asking users to compare projects in terms of “value for money” or asking them to choose an entire budget results in provably better properties than using the more traditional approaches of approval or rank-choice voting. Inspired by these mathematical results, Goel et al. designed a participatory budgeting platform that is fast becoming the leader for such processes in the U.S. For example, this platform was recently used to decide how to spend $250,000 of infrastructure funds to improve Long Beach (CA) Council District 9, and how to allocate $2.4 million of Vallejo’s capital improvement budget. Looking forward, it is an interesting and open research challenge to understand if these algorithms and systems yield near-optimal aggregations of societal preferences, or decisions that are near-optimal in terms of overall societal utility.

**Automated market makers for prediction markets.** A prediction market is a financial market designed to extract and aggregate predictions from a crowd. In a typical prediction market, traders buy and sell securities with payments that are contingent on the outcome of a future event. For example, a security may yield a payment of $1 if a Democrat wins the 2016 U.S. Presidential election and 0 otherwise. A trader who believes the true probability of a Democrat winning the election is 0.5 maximizes his expected utility by purchasing the security if it is available at a price less than 0.5 and selling the security if it is available at a price greater than 0.5. The market price of this security is thought to reflect the traders’ collective belief about the likelihood of a Democrat winning.

Prediction markets have been shown to produce forecasts at least as accurate as other alternatives in a wide variety of domains, including politics, business, disease surveillance, entertainment, and beyond, and have been widely cited by the press during recent elections. However, markets operated using traditional mechanisms like continuous double auctions (similar to the stock market) often suffer from low liquidity. Without liquidity, a market faces a chicken-and-egg problem: potential traders are dissuaded from participating due to lack of counterparties, which contributes to an even greater reduction in future trading opportunities.

Low liquidity can also lead to high price volatility and large spreads, both of which cause the market price to yield a less meaningful prediction.

To combat this problem, Hanson proposed the idea of operating mar-
Recently there has been interest in further tapping into the information efficiency of prediction markets and using them to obtain accurate predictions of more fine-grained events.
search on (computational) fair division and comes with provable mathematical guarantees. For example, the algorithm used for room assignment and rent splitting relies on the fact that there always exists an assignment of rooms and a corresponding set of prices that are envy-free: every roommate prefers the room he is assigned to any other room given the prices. Each roommate submits her own value for each of the rooms, under the constraint that the total value of all rooms matches the total rent for the apartment; viewed another way, each roommate is essentially submitting a proposed set of prices for each room such that she would be equally happy obtaining any room at the specified price. The algorithm then maximizes the minimum utility (value of room minus price) of any roommate subject to the constraint that envy-freeness is satisfied. The solution is also Pareto efficient, meaning there is no other allocation that would increase the utility of any roommate without decreasing the utility of another.

As another example, the credit assignment problem is solved using an algorithm of de Clippel et al. Each collaborator reports the relative portion of credit that he believes should be assigned to each of the other collaborators. For example, on a project with four collaborators, collaborator A might report that collaborators B and C should receive equal credit while D should receive twice as much credit. The algorithm takes these reports as input and produces a credit assignment that is impartial, meaning that an individual’s share of credit is independent of his own report, and consensual, meaning that if there is a division of credit that agrees with all collaborators’ reports then this division is chosen. While these conditions may not sound restrictive, de Clippel et al. show they are not simultaneously achievable with three collaborators. Their algorithm therefore requires at least four.

In addition to providing a useful set of tools, part of Spliddit’s mission is to “communicate to the public the beauty and value of theoretical research in computer science, mathematics, and economics, from an unusual perspective.” Indeed, the project has inspired some members of the public to take an interest in algorithms with provable fairness properties. As one example, a representative of one of the largest school districts in California approached the Spliddit team about a problem he was tasked with solving: fairly allocating unused classrooms in public schools to the district’s charter schools. This led the Spliddit team, in collaboration with the California school district, to design a practical new approach to classroom allocation that guarantees envy-freeness as well as several other desirable properties.

A Challenge Problem: The Crowdsourcing Compiler

A concrete challenge problem for future research in social computing is what might be called the “Crowdsourcing Compiler”: the development of high-level programming languages for specifying large-scale, distributed tasks whose solution requires combining traditional computational and networking resources with volunteer (or paid) human intelligence and contributions. The hypothetical compiler would translate an abstract program into a more detailed organizational plan for machines and people to jointly carry out the desired task. In the same way that today’s Java programmer is relieved of low-level, machine-specific decisions (such as which data to keep in fast registers, and which in main memory or disk), the future crowdsourcing programmer would specify the goals of their system, and leave many of the implementation details to the Crowdsourcing Compiler. Such details might include which components of the task are best carried out by machine and which by human volunteers; whether the human volunteers should be incentivized by payment, recognition, or entertainment; how their contributions should be combined to solve the overall task; and so on. While a fully general Crowdsourcing Compiler might well be unattainable, significant progress toward it would imply a much deeper scientific understanding of crowdsourcing than we currently have, which in turn should have great engineering benefits. Noteworthy research efforts which can be viewed as steps on the path to the Crowdsourcing Compiler include Emery Berger’s AutoMan Proj-

Mathematical and experimental research are complementary and both are needed to develop relevant mathematical foundations for social computing.

[See http://bit.ly/20juYEX and http://bit.ly/1nIyc3P.]
effect (http://emeryberger.com/research/automata), as well as both academic and commercial efforts to automate workflow in crowdsourcing and social computing systems (see, for example, http://groups.csail.mit.edu/uid/turkit/ and http://www.crowdflower.com/).

We note the organizational schemes in most of the successful crowdsourcing examples to date share much in common. The tasks to be performed (for example, building an online encyclopedia, labeling images for their content, creating a network of website bookmark labels, finding surveillance balloons) are obviously parallelizable, and furthermore the basic unit of human contribution required is extremely small (fix some punctuation, label an image, and so on). Furthermore, there is usually very little coordination required between the contributions. The presence of these commonalities is a source of optimism for the Crowdsourcing Compiler—so far, there seems to be some shared structure to successful crowdsourcing that the compiler might codify. But are such commonalities present because they somehow delineate fundamental limitations on successful crowdsourcing—or simply because this is the “low-hanging fruit?”

Today, the Crowdsourcing Compiler is clearly a “blue sky” proposal meant more to delineate an ambitious research agenda for social computation than to serve as a guide to short-term steps. But we believe that such an agenda would both need and drive research on theoretical foundations. First steps toward developing the mathematical foundations of a Crowdsourcing Compiler include formally addressing the following questions:

- For a given set of assumptions about the volunteer force, and given the nature of the task, what is the best scheme for organizing the volunteers and their contributions? For instance, is it a “flat” scheme where all contributors are equal and their contributions are combined in some kind of majority vote fashion? Or is it more hierarchical, with proven and expert contributors given higher weight and harder subproblems? Which of these (or other) schemes should be used under what assumptions on the nature of the task and what assumptions on the volunteers?
- How can we design crowdsourced systems for solving tasks that are much more challenging and less “transactional” than what we currently see in the field—for instance, complex problems where there are strong constraints and interdependencies between the contributions of different volunteers? Behavioral research in recent years has shown that groups of humans can indeed excel on such tasks, but we are far from understanding when and why.

Finally, we note that while the comparison to traditional compilers might be a useful guide and metaphor, a crowdsourcing analogue would have to face a variety of issues that simply do not arise with standard hardware and software. In addition to the aforementioned challenges of deciding how to organize and incentivize human contributions, there may also be the potential for malicious or deceptive behavior by workers, and the need for error correction of crowd work (which is currently largely handled by redundancy and voting techniques).

Challenges to Overcome

We have argued that mathematical research has the potential to make great contributions to social computing. However, before this potential is fully realized, there are several challenges that must be addressed.

- Blending mathematical and experimental research. Mathematical and experimental research are complementary and both are needed to develop relevant mathematical foundations for social computing. The strengths of mathematical work include:
  1. Mathematical modeling and analysis can be used to cleanly formulate and answer many questions about system behavior without requiring that we build a complete system, providing us with a tool to evaluate the impact of design decisions before committing to any particular design. For example, such models can provide guidance on how to increase participation (such as, comparing a leaderboard to badges), predict whether a social computing system will achieve critical mass, and perhaps understand how the behavior of groups of users change as the system scales.
  2. Mathematical guarantees are desirable for properties like user privacy (that can be obtained, for example, using techniques from the extensive and growing literature on differential privacy), correctness of a system’s output, or the scalability of a social computing system. Theoretical work in computer science provides tools for designing and analyzing new algorithms that could lie at the heart of social computing applications, answering questions like how to aggregate noisy and unstructured estimates or information from crowds, how to optimally divide a community into subgroups, or how to bring people together in moments of spare time to achieve a common goal.

Learning from the social sciences.

Computer scientists cannot develop the mathematical foundations of social computing in isolation. Social computing systems are fundamentally social. These systems cannot be properly modeled or analyzed without accounting for the behavior of their human components. Much of the literature thus far uses standard models of economic agents and corresponding assumptions about agent preferences, but a growing literature based on experimental work on online platforms suggests that human behavior in several online settings might deviate from these models, and these deviations can have significant consequences for how to optimally design social computing systems.

In order for mathematical foundations to provide useful practical results, it is necessary to base it on models that better reflect human behavior. This is most effectively achieved via a dialog between theoretical and experimental and empirical research, with studies of human behavior informing mathematical...
such as discrimination and fairness. Examining and avoiding the unintended consequences of opaque decisions made by algorithms is a topic that has been gaining interest in the machine learning and big data communities. Such concerns will undoubtedly need to be addressed in the context of social computing as well.

Acknowledgments. We thank the participants of the Visioning Workshop on Theoretical Foundations for Social Computing for their contributions. We also thank Ashish Goel, Vince Conitzer, David McDonald, David Parkes, and Ariel Procaccia for their feedback.

References

1. Abendeth, J., Chen, Y. and Vaughan, J.W. Efficient market making via convex optimization, and a connection to online learning. ACM Trans. Economics and Computation 1, 2 (2013), Article 12.
2. Abendeth, J., Frongillo, R., Li, X. and Vaughan, J.W. A general volume-parameterized market-making framework. In Proceedings of the 15th ACM Conference on Economics and Computation, 2014.
3. Barowy, D., Curtlinger, C., Berger, E.D. and Mcgregor, A. Automaton: A platform for integrating human-based and digital computation. In Proceedings of the Object-Oriented Programming, Systems, Languages, and Applications, 2012.
4. Brandt, F., Conitzer, V., Endriss, U., Lang, J. and Procaccia, A. Complexity of combinatorial market makers. In Proceedings of the 8th ACM Conference on Electronic Commerce, 2008.
5. Cole, R., Introduction to Mathematical Sociology. Free Press of Glencoe, 1964.
6. Dasgupta, A. and Ghosh, A. Crowdsourced judgment elicitation with endogenous proficiency. In Proceedings of the 22nd International World Wide Web Conference, 2013.
7. de Clippel, G., Moulin, H. and Tideman, N. Impartial division in combinatorial market maker using constraint generation. In Proceedings of the 4th ACM Conference on Electronic Commerce, 2012.
8. Doudk, M., Lahaye, S. and Pennock, D.M. A tractable combinatorial market maker using constraint generation. In Proceedings of the 13th ACM Conference on Electronic Commerce, 2012.
9. Doudk, M., Lahaye, S. and Pennock, D.M and Rothschild, D. A combinatorial prediction market for the U.S. elections. In Proceedings of the 14th ACM Conference on Electronic Commerce, 2013.
10. Dwork, C. and Roth, A. The algorithmic foundations of differential privacy. Foundations and Trends in Theoretical Computer Science 9, 3–4 (2014), 211–407.
11. Easley, D. and Ghosh, A. Incentives, gamification, and game theory. In Proceedings of the 12th ACM Conference on Web Information and Knowledge Management, 2013.
12. Easley, D. and Kleinberg, J. Behavioral mechanism design: Optimal crowdsourcing contracts and prospect theory. In Proceedings of the 18th ACM Conference on Economics and Computation, 2015.
13. Ghosh, A. Exper theory and incentives in human computation. Handbook of Human Computation. P. Michelucci, ed. Springer, 2014.
14. Ghosh, A. and Mcnair, P. A game-theoretic analysis of rank-order mechanisms for user-generated content. In Proceedings of the 12th ACM Conference on Electronic Commerce, 2013.
15. Ghosh, A. and Reingber, R. Behavioral mechanism design: Optimal contests for simple agents. In Proceedings of the 19th ACM Conference on Economics and Computation, 2014.
16. Ghosh, A. Online learning and big data communities. In Proceedings of the 20th International World Wide Web Conference, 2011.
17. Ghosh, A. and Mcnair, P. Crowdsourcing with endogenous entry. In Proceedings of the 21st International World Wide Web Conference, 2012.
18. Ghosh, A. Computational social science: Making the links. Nature 488, 7412 (2012).
19. Ghosh, A. and McAfee, P. Incentivizing high-quality user generated content. In Proceedings of the 26th ACM Conference on Human Computer Interaction, 2013.
20. Gneezy, T. and Raffety, A.E. Strictly proper scoring rules, prediction, and estimation. J. American Statistical Association 102, 477 (2007).
21. Goel, A., Krishnamoorthy, A., Sahsoungw, S. and Alammaruto, T. Knapsack voting. Collective Intelligenz, 2015.
22. Goldman, J. and Procaccia, A.D. Spliddit: Unleashing fair division algorithms. SIGecom Exchanges 13, 2 (2014), 41–48.
23. Hansen, R. Combinatorial information market design. Information Systems Frontiers 1, 3 (2003), 105–119.
24. Ho, C.J. and Vaughan, J.W. Online task assignment in crowdsourcing markets. In Proceedings of the 26th AAAI Conference on Artificial Intelligence, 2012.
25. Ho, C.J., Jabbir, S. and Vaughan, J.W. Adaptive task assignment for crowdsourced classification. In Proceedings of the 28th International Conference on Machine Learning, 2013.
26. Ho, C.J., Shivas, A., Suri, S. and Vaughan, J.W. Incentivizing high-quality crowdwork. In Proceedings of the 29th International World Wide Web Conference, 2015.
27. Immorlica, N., Sodderland, P. and Syrgkanis, V. Social status and badge design. In Proceedings of the 24th International World Wide Web Conference, 2015.
28. Jain, S. and Parkes, D.C. A game-theoretic analysis of the ESP Game. ACM Trans. Economics and Computation 1, 1 (2013), 1–39.
29. Jain, S., Chen, Y. and Parkes, D.C. Designing incentives for online question-and-answer forums. Games and Economic Behavior 80 (2014), 458–474.
30. Karger, D., S. and Shah, D. Incentivizing learning for reliable crowdsourcing systems. Advances in Neural Information Processing Systems, 2011.
31. Kearns, M. Experiments in social computation. Commun. ACM 55, 10 (Oct. 2012).
32. Kurokawa, D., Procaccia, A.D. and Shah, N. Lesimmin allocations in the real world. In Proceedings of the 10th ACM Conference on Economics and Computation, 2015.
33. Lambert, N.S., Langford, J., Vaughan, J.W., Chen, Y., Reeves, D., Shoham, Y. and Pennock, D.M. An axiomatic characterization of wagering mechanisms. J. Economic Theory 156 (2015), 389–418.
34. Lazer, D. et al. Computational social science. Science 323, 5935 (2009), 721–723.
35. Mason, W. and Watts, D. Financial incentives and the “performance of crowds.” In Proceedings of the First Human Computation Workshop, 2009.
36. Procaccia, A.D. Cake cutting not just child’s play. Commun. ACM 56, 7 (July 2013), 78–87.
37. Sliksins, A. and Vaughan, J.W. Online decision making in crowdsourcing markets: Theoretical challenges. ACM Trans. Economics and Computation, 2015.
38. Ungar, L., Mellors, B., Satopäälä, V., Baron, J., Tieto, P., Ramnio, J. and Swift, S. The good judgment project: A large-scale test of different methods of combining expert predictions. AAAI Technical Report FS-12-06, 2012.
39. Waggoner, B. and Chen, Y. Outpute agreement mechanisms and common knowledge. In Proceedings of the 24th AAAI Conference on Human Computer Interaction and Crowdsourcing, 2014.
40. Yin, M., Chen, Y. and Sun, Y.-A. The effects of performance-contingent financial incentives in online labor markets. In Proceedings of the 27th AAAI Conference on Artificial Intelligence, 2013.

Yiling Chen (yl@eecs.harvard.edu) is Gordon McKay Professor of Computer Science at Harvard University, Cambridge, MA.

Arpita Ghosh (arpithaghosh@cornell.edu) is an associate professor of information science at Cornell University, Ithaca, NY.

Michael Kearns (mkearns@cisi.upenn.edu) is a professor and National Center Chair of Computer and Information Science at the University of Pennsylvania, Philadelphia, PA.

Tim Roughgarden (tim@cs.stanford.edu) is an associate professor of CS at Stanford University, Stanford, CA.

Jennifer Wortman Vaughan (jenny@microsoft.com) is a senior researcher at Microsoft Research, New York, NY.

Copyright held by owners/authors. Publication rights licensed to ACM. $15.00.