Generation of large-scale magnetic fields from inflation in teleparallelism

Kazuharu Bamba,^a,c^ Chao-Qiang Geng^b,c,d^ and Ling-Wei Luo^b,d^

^a^Kobayashi-Maskawa Institute for the Origin of Particles and the Universe, Nagoya University, Nagoya 464-8602, Japan
^b^Department of Physics, National Tsing Hua University, Hsinchu, 300, Taiwan
^c^Physics Division, National Center for Theoretical Sciences, Hsinchu, 300, Taiwan
^d^College of Mathematics & Physics, Chongqing University of Posts & Telecommunications, Chongqing, 400065, China

E-mail: bamba@kmi.nagoya-u.ac.jp, geng@phys.nthu.edu.tw, d9622508@oz.nthu.edu.tw

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Abstract. We explore the generation of large-scale magnetic fields from inflation in teleparallelism, in which the gravitational theory is described by the torsion scalar instead of the scalar curvature in general relativity. In particular, we examine the case that the conformal invariance of the electromagnetic field during inflation is broken by a non-minimal gravitational coupling between the torsion scalar and the electromagnetic field. It is shown that for a power-law type coupling, the magnetic field on 1 Mpc scale with its strength of \( \sim 10^{-9} \) G at the present time can be generated.

Keywords: modified gravity, primordial magnetic fields, magnetic fields, inflation
1 Introduction

A number of recent cosmological observations, e.g., Type Ia Supernovae [1], baryon acoustic oscillations [2], large scale structure (LSS) [3], cosmic microwave background (CMB) radiation [4–6], and weak lensing [7], suggest the accelerated expansion of the current universe. To explain the late time cosmic acceleration, there exist two main approaches: One is the introduction of the so-called “dark energy” (for reviews on dark energy, see, e.g., [8–14]) and the other is the modification of gravity such as \( f(R) \) gravity [15–23].

Recently, “teleparallelism” [24] has attracted much attention as it can be considered as an alternative gravity theory to general relativity. Teleparallelism is formulated with the Weitzenböck connection, so that its action consists of the torsion scalar \( T \) instead of the scalar curvature \( R \) in general relativity with the Levi-Civita connection. It has been shown that by introducing a scalar with the non-minimal coupling to gravity in teleparallelism [25], the late time cosmic acceleration can be achieved. Moreover, similar to \( f(R) \) gravity, the non-linear generalization of the torsion scalar \( T \), i.e., \( f(T) \) gravity, can account for inflation [26] in the early universe as well as the cosmic acceleration in the late time [27, 28]. Various aspects on \( f(T) \) gravity have been widely investigated in the literature (see, e.g., [14] and the references therein).

On the other hand, according to astrophysical observations, it is well known that there exist magnetic fields with the strength \( \sim 10^{-6} \) G and the coherence scale 1–10 kpc. Also in clusters of galaxies, large-scale magnetic fields are observed, whose strengths are \( 10^{-7}–10^{-6} \) G and the coherence scales are estimated as 10 kpc–1 Mpc. However, the origins of these cosmic magnetic fields, in particular the large-scale magnetic fields in clusters of galaxies have not been well understood yet (for reviews on cosmic magnetic fields, see, e.g., [29]). There are several generation mechanisms of the cosmic magnetic fields, such as those from astrophysical processes based on the plasma instability [30, 31], cosmological phase transitions [32], and matter density perturbations before or at the recombination epoch [33]. Indeed, it is not so easy for these mechanisms to generate the large-scale magnetic fields observed in clusters of galaxies only with the adiabatic compression and without any secondary amplification.
mechanism as the galactic dynamo [34]. Thus, the most natural mechanism to produce the large-scale magnetic fields is considered to be electromagnetic quantum fluctuations during inflation [35], because the scale of the electromagnetic quantum fluctuations can be extended to that larger than the Hubble horizon by inflation.

In Quantum Electrodynamics (QED) in the curved space-time, there can appear a non-minimal coupling of the scalar curvature to the electromagnetic field owing to one-loop vacuum-polarization effects [36], so that the conformal invariance of the electromagnetic field can be broken by this coupling. This can yield the quantum fluctuation of the electromagnetic field during inflation, resulting in the large-scale magnetic field at the present universe [35, 37]. Such a breaking mechanism of the conformal invariance of the electromagnetic field is necessary to generate the quantum fluctuation of the electromagnetic field, because the Maxwell theory is conformally invariant and the Friedmann-Lemaître-Robertson-Walker (FLRW) space-time is conformally flat [38]. Consequently, a lot of breaking mechanisms of the conformal invariance of the electromagnetic field have been explored (for a list of these breaking mechanisms, see, e.g., reviews in [29] and references in [44–52]).

In this paper, motivated by both astrophysical and cosmological observations, we study the generation of large-scale magnetic fields from inflation in teleparallelism. In particular, we introduce a non-minimal gravitational coupling of the torsion scalar $T$ to the electromagnetic field by analogy with such an interaction between gravity and electromagnetism in general relativity. As an illustration, we demonstrate that for the form of the coupling to be a power-law type, the magnetic field with its current strength of $\sim 10^{-9}\text{G}$ on 1Mpc scale can be generated.

It should be remarked that for example, in ref. [44] Ratra has investigated the case that the gauge kinetic term is coupled to the inflaton field. In this work, however, the observation that there can be a spectator field evolving during inflation is used essentially. This possibility has been scrutinized in a number of different works, e.g., refs. [53–55] by Giovannini. In particular, in ref. [53] a scale-invariant spectrum during the conventional inflation has been demonstrated in a specific model where the gauge coupling is not a function of the inflaton (in the latter case the flatness of the potential might be spoiled). We use units of $k_B = c = \hbar = 1$ and denote the gravitational constant $8\pi G$ by $\kappa^2 \equiv 8\pi/M_{Pl}^2$ with the Planck mass of $M_{Pl} = G^{-1/2} = 1.2 \times 10^{19}\text{GeV}$.

The paper is organized as follows. In section II, we explain the fundamental formulations in teleparallelism. In section III, in a non-minimal $I(T)$-Maxwell theory, where $I(T)$ is an arbitrary function of the torsion scalar $T$, we investigate the generation of large-scale magnetic fields in inflationary cosmology. In section IV, for a concrete model of a power-law type coupling between the torsion scalar and the Maxwell field, we estimate the current strength of the large-scale magnetic field. Finally, conclusions are presented in section V.

2 Teleparallelism

We adopt orthonormal tetrad components $e_A(x^\mu)$ in teleparallelism, where an index $A$ runs over $0, 1, 2, 3$ for the tangent space at each point $x^\mu$ of the manifold. The relations between the metric $g^{\mu\nu}$ and orthonormal tetrad components are given by $g_{\mu\nu} = \eta_{AB} e^A_\mu e^B_\nu$, where $\mu$ and $\nu$ are coordinate indices on the manifold and run over $0, 1, 2, 3$. Hence, $e^A_\mu$ form the tangent vec-

\footnote{It should be noted that this is true only for the flat FLRW space-time, but not for the FLRW background with spatial curvature, e.g., an open FLRW universe [39]. In addition, the breaking of the conformal flatness during inflation has also been studied in ref. [40]. Furthermore, there exist arguments in terms of the back reaction effect of the magnetic field generated during inflation [41–43].}
tor of the manifold. We define the torsion and contorsion tensors as \( T^\rho_{\mu\nu} \equiv e^\rho_A (\partial_\mu e^A_\nu - \partial_\nu e^A_\mu) \) and \( K^{\mu\nu\rho} \equiv -(1/2) (T^\nu_{\rho\mu} - T^\mu_{\rho\nu} - T_\rho{}^{\mu\nu}) \), respectively. Using these tensors, we construct the torsion scalar \( T \equiv S_{\rho}{}^{\mu\nu} T^\rho_{\mu\nu} \) with \( S_{\rho}{}^{\mu\nu} \equiv (1/2) (K^{\mu\nu\rho} + \delta_\rho^\mu T^\alpha_{\mu\alpha} - \delta_\rho^\nu T^\alpha_{\nu\alpha}) \). In general relativity, the Einstein-Hilbert action consists of the scalar curvature \( R \). However, in teleparallelism the torsion scalar \( T \) is used to represent the teleparallel Lagrangian density. As a result, the action in teleparallelism is described by

\[
S_{\text{Tel}} = \int d^4x |e| \left( \frac{T}{2\kappa^2} + \mathcal{L}_M \right), \tag{2.1}
\]

where \(|e| = \det (e^A_\mu) = \sqrt{-g}\) and \( \mathcal{L}_M \) is the Lagrangian of matter. The variation of the action \( S_{\text{Tel}} \) with respect to the vierbein vector fields \( e^A_\mu \) leads to the gravitational field equation \[27\], given by \( (1/e) \partial_\mu (e S^A_{\mu\nu}) - e^A_T \partial_\mu S^\rho_{\mu\nu} + (1/4) e^\nu_\lambda T = (\kappa^2/2) e^0_T T^{(M)}_{\mu}{}^\nu \), where \( T^{(M)}_{\mu}{}^\nu \) is the energy-momentum tensor of matter.

We take the flat Friedmann-Lemaître-Robertson-Walker (FLRW) universe, whose metric is given by \( ds^2 = dt^2 - a^2(t)d\tau^2 = a^2(\eta) (dt^2 + d\xi^2) \) with \( a \) the scale factor and \( \eta \) the conformal time. In this space-time, \( g_{\mu\nu} = \text{diag}(1, -a^2, -a^2, -a^2) \) and the tetrad components become \( e^A_\mu = (1, a, a, a) \). With these relations, we find that the exact value of the torsion scalar is described by \( T = -6H^2 \), where \( H \equiv \dot{a}/a \) is the Hubble parameter with the dot being the time derivative of \( \partial/\partial t \).

3 Non-minimal \( I(T) \)-Maxwell theory

In this section, we consider a non-minimal \( I(T) \)-Maxwell theory and examine the generation of large-scale magnetic fields in inflationary cosmology.

3.1 Model of the electromagnetic sector

The action describing a non-minimal \( I(T) \)-Maxwell theory is given by

\[
S = \int d^4x |e| \left( -\frac{1}{4} I(T) F_{\mu\nu} F^{\mu\nu} \right), \tag{3.1}
\]

where \( I(T) \) is an arbitrary function of the torsion scalar \( T \) and \( F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \) with \( A_\mu \) the U(1) gauge field is the electromagnetic field-strength tensor. It follows from the action in eq. (3.1) that the electromagnetic field equation is derived as

\[
- \frac{1}{\sqrt{-g}} \partial_\mu \left[ \sqrt{-g} I(T) F^{\mu\nu} \right] = 0. \tag{3.2}
\]

In the FLRW background with the Coulomb gauge \( A_0(t, x) = 0 \) and \( \partial^i A_j(t, x) = 0 \), the equation of motion for the U(1) gauge field is written as

\[
\ddot{A}_i(t, x) + \left( H + \frac{i(T)}{I(T)} \right) \dot{A}_i(t, x) - \frac{1}{a^2} \Delta^{(3)} A_i(t, x) = 0, \tag{3.3}
\]

where \( \Delta^{(3)} \) is the Laplacian in three-dimensional space.

We remark that although the electromagnetic field coupled (preferentially) to the axial vector part of the torsion tensor is more natural, we do not consider it in this study since the
resulting large-scale magnetic field is too small. In addition, we emphasize that the strength of the generated magnetic field is more related to the nature of the coupling $I$ to the gauge kinetic term than to the torsion scalar itself.

It is important to explicitly state that the key element to generate the large-scale magnetic fields in the quasi-de Sitter phase of expansion, i.e., inflation in the early universe, is the scalar coupling between $I$ and the kinetic term of the electromagnetic field, as seen in eq. (3.1). In general, the coupling term $I(\eta)$ may be a function of various scalar degrees of freedom existing in a model, e.g., the inflaton or the dilaton field or a dynamic gauge coupling. As a result, $I(\eta)$ can be a function of a spectator field evolving during the inflationary epoch. In this case, there is no connection between the evolution of $I$ and the gauge coupling. Therefore, the physical features of the various models are different. In other words, in principle $I$ can be an arbitrary function of some non-trivial background fields. On the other hand, in the case of bouncing models some of these ideas are preferentially realized, whereas other models are consistent with the standard inflationary paradigm. In this work, instead of concentrating on a specific mechanism for inflation, we execute a model-independent analysis on the generation of large-scale magnetic fields through the breaking of the conformal invariance of the electromagnetic field due to the coupling of $I$ in $\mathcal{L} = (1/4) F_{\mu\nu} F^{\mu\nu}$ in eq. (3.1).

We also describe the realization of inflation. In this work, we suppose that the generic slow-roll inflation is realized without identifying the specific mechanism to lead to inflation. There are various possibilities to realize inflation. For example, one can introduce the inflaton coupling to the electromagnetic field [44, 48], which can be considered to be the dilaton [46], or both inflaton and dilaton fields [47]. Namely, if the dilaton field is not responsible for inflation, the coupling $I$ can be a function of the inflaton field. Furthermore, in these cases forms of the inflaton potential are assumed to be flat enough to realize the slow-roll inflation, namely, the quasi exponential expansion of the universe. Concrete demonstrations have been investigated in refs. [53–55].

### 3.2 Quantization

We now execute the quantization of $A_{\mu}(t, \mathbf{x})$. From the action of the electromagnetic fields in eq. (3.1), we find that the canonical momenta conjugate to $A_{\mu}(t, \mathbf{x})$ become $p_0 = 0$, $p_i = ia(t) A_i(t, \mathbf{x})$. The canonical commutation relation between $A_i(t, \mathbf{x})$ and $\pi_j(t, \mathbf{y})$ is given by $[A_i(t, \mathbf{x}), \pi_j(t, \mathbf{y})] = i \int d^3k (2\pi)^{-3} e^{ik \mathbf{x} - \omega} (\delta_{ij} - k_i k_j/k^2)$ with $k$ being the comoving wave number and $k = |\mathbf{k}|$. By imposing this relation, $A_i(t, \mathbf{x})$ is described as $A_i(t, \mathbf{x}) = \int d^3k (2\pi)^{-3/2} \sum_{\sigma=1,2} \left[ \hat{b}(\mathbf{k}, \sigma) \epsilon_i(\mathbf{k}, \sigma) A(t, k)e^{ik \mathbf{x}} + \hat{b}^\dagger(\mathbf{k}, \sigma) \epsilon^*_i(\mathbf{k}, \sigma) A^*(t, k)e^{-ik \mathbf{x}} \right]$, where $\epsilon_i(\mathbf{k}, \sigma) (\sigma = 1, 2)$ stand for the two orthonormal transverse polarization vectors, and $\hat{b}^\dagger(\mathbf{k}, \sigma)$ is the annihilation (creation) operator, satisfying the relations $[\hat{b}(\mathbf{k}, \sigma), \hat{b}^\dagger(\mathbf{k}', \sigma')] = \delta_{\sigma, \sigma'} \delta^3(\mathbf{k} - \mathbf{k}')$ and $[\hat{b}(\mathbf{k}, \sigma), \hat{b}^\dagger(\mathbf{k}', \sigma')] = [\hat{b}^\dagger(\mathbf{k}, \sigma), \hat{b}(\mathbf{k}', \sigma')] = 0$. It follows from eq. (3.3) that the Fourier mode $A(k, t)$ obeys $\dot{A}(k, t) + \left( H + \dot{I}/I \right) A(k, t) + (k^2/a^2) A(k, t) = 0$ together with the normalization condition, $A(k, t) A^*(k, t) - \dot{A}(k, t) A^*(k, t) = i/\langle Ia \rangle$. If we use the conformal time $\eta$, this equation is rewritten as $A''(k, \eta) + (\dot{I}/I) A'(k, \eta) + k^2 A(k, \eta) = 0$, where the prime denotes the derivative in terms of $\eta$ as $\partial/\partial \eta$.

### 3.3 Procedure to obtain analytic solutions

With the WKB approximation on subhorizon scales and the long wavelength approximation on superhorizon scales and matching these solutions at the horizon crossing, it is possible to
acquire an analytic solution for this equation [49] approximately. In this case of the exact de Sitter background, we find $a = 1/(-H\eta)$ with $H$ being the Hubble parameter during the de Sitter expansion, and $-\kappa n = 1$ at the horizon-crossing when $H = k/a$. For subhorizon (superhorizon) scales, we have $k|\eta| \gg 1$ ($k|\eta| \ll 1$). This is considered to be sufficiently well defined also for the general slow-roll inflation, i.e., nearly exponential inflation.

Provided that in the short-wavelength limit of $k/(aH) \gg 1$ the vacuum asymptotically approaches the Minkowski vacuum, the WKB subhorizon solution reads $A_{in}(k, \eta) = \left(1/\sqrt{2k}\right) I^{-1/2} e^{-ik\eta}$.

While, with the long-wavelength expansion in terms of $k^2$, we can have the solution on superhorizon scales $A_{out}(k, \eta)$. By matching this solution with the above WKB subhorizon solution at the horizon crossing time $\eta = \eta_k \approx 1/k$, we obtain the lowest order approximate solution of $A_{out}(k, \eta)$ [49] as

$$A_{out}(k, \eta) = A_1(k) + A_2(k) \int_{\eta}^{\eta_k} \frac{1}{I(\eta)} d\eta,$$

$$A_1(k) \equiv \frac{1}{\sqrt{2k}} I^{-1/2} \left[ 1 - \left( \frac{i}{2} I' + ikI \right) \right] \left( 1 - \left( \frac{i}{2} I' + ikI \right) \right) I(\eta) d\eta \bigg|_{\eta = \eta_k},$$

$$A_2(k) \equiv \frac{1}{\sqrt{2k}} I^{-1/2} \left( \frac{1}{2} I' + ikI \right) e^{-ik\eta} \bigg|_{\eta = \eta_k}. \quad (3.6)$$

We neglect the decaying mode solution, which is the second term of the right-hand side of eq. (3.4). Equations (3.5) and (3.6) lead to $|A(k, \eta)|^2$ at the late times, given by

$$|A(k, \eta)|^2 = |A_1(k)|^2 = \frac{1}{2k I(\eta_k)} \left| 1 - \left( \frac{i}{2} I' + ikI \right) \right| k \int_{\eta_k}^{\eta_R} \frac{I(\eta)}{I(\eta_k)} d\eta \bigg|_{\eta = \eta_k}^2. \quad (3.7)$$

Here, we have supposed the instantaneous reheating after inflation and therefore, $\eta_R$ is considered to be the conformal time at the reheating stage. By using the comoving magnetic field $B_i(t, \mathbf{x})$, the proper magnetic field is expressed as $B_{i, \text{proper}}(t, \mathbf{x}) = a^{-1} B_i(t, \mathbf{x}) = a^{-2} \epsilon_{ijk} \partial_j A_k(t, \mathbf{x})$, where $\epsilon_{ijk}$ is the totally antisymmetric tensor with $\epsilon_{123} = 1$. Accordingly, we find that the spectrum of the magnetic field is described as $|B_{\text{proper}}(k, \eta)|^2 = 2 \left( k^2/a^4 \right) |A(k, \eta)|^2 = 2 \left( k^2/a^4 \right) |A_1(k)|^2$, where we have taken into account the factor 2 originating from the two degrees of freedom for the polarization. In the Fourier space, the energy density of the magnetic field becomes $\rho_B(k, \eta) = \left(1/2\right) |B_{\text{proper}}(k, \eta)|^2 I(\eta)$. With multiplying this by the phase-space density $4\pi k^3/(2\pi)^3$, we derive the energy density of the generated magnetic field per unit logarithmic interval of $k$ as

$$\rho_B(k, \eta) \equiv \frac{1}{2} 4\pi k^3 |B_{\text{proper}}(k, \eta)|^2 I(\eta) = \frac{k |A_1(k)|^2 k^4}{2\pi^2 a^4} I(\eta). \quad (3.8)$$

Consequently, the density parameter of the magnetic field per unit logarithmic interval of $k$ and its spectral index are given by [49]

$$\Omega_B(k, \eta) = \frac{\rho_B(k, \eta_R)}{\rho_{\gamma}(\eta_R)} \frac{I(\eta)}{I(\eta_R)} = \frac{k^4}{T_\text{R}^4 a_R^4} \frac{15 k |A_1(k)|^2}{N_{\text{eff}} \pi^4} I(\eta),$$

$$n_B \equiv \frac{d \Omega_B(k)}{d \ln k} = 4 + \frac{d \ln |A_1(k)|^2}{d \ln k}, \quad (3.10)$$
respectively, where, \( \rho_r(\eta_R) = N_{\text{eff}} (\pi^2/30) T_R^4 \) \[56\] is the energy density of radiation at the reheating stage with the reheating temperature \( T_R \), \( a_R \) is the scale factor at \( \eta = \eta_R \) and \( N_{\text{eff}} \) is the effective massless degrees of freedom (e.g., for photons, 2) thermalized at the reheating stage.

4 Large-scale magnetic field generated in teleparallelism

4.1 Current strength of the magnetic field

For the purpose of analyzing the strength of the magnetic field quantitatively, we examine the case of the specific form of \( I(\eta) \), given by \[49\]

\[ I(\eta) = I_*(\frac{\eta}{\eta_*})^{-\beta}, \] (4.1)

where \( \eta_* \) is some fiducial time at the inflationary stage, \( I_* \) is the value of \( I \) at \( \eta = \eta_* \), and \( \beta (> 0) \) is a positive constant, whose positivity makes \( I \) increase monotonically during inflation. For this form of \( I \), \( |A| \) in eq. (3.7) reads \( k|A|^2 = [1/(2I(\eta_k))][1-(\beta+2i)/2(\beta+1)]^2 \equiv A/(2I(\eta_k)) \), where \( A(=O(1)) \) is a constant of the order of unity. By plugging this relation into eqs. (3.9) and (3.10), the density parameter of the magnetic field at the present time \( \eta_0 \) is expressed as \( \Omega_B(k, \eta_0) = [k^4/(T_R^4a_R^4)] [15A/(2N_{\text{eff}}\pi^4I_*)] (\eta_0/\eta_*)^\beta \) with \( n_B = 4 - \beta \), where we have used \( I(\eta_k) \propto k^\beta \) and \( I(\eta_0) = 1 \). If \( I_* \) is very small and the spectrum is nearly scale-invariant, i.e., \( \beta \sim 4 \), the resultant amplitude of the large-scale magnetic field becomes large. From eq. (3.9), we find that the current density parameter of the magnetic field is described by \[49\]

\[ \Omega_B(k, \eta_0) = A \frac{N_{\text{eff}}}{1080} \left( \frac{T_R}{M_{\text{Pl}}} \right)^4 (-k\eta_R)^{4-\beta} \frac{1}{T(\eta_R)} . \] (4.2)

Here, we have used the relation \( a_R^2\eta_R^2 \approx H_R^{-2} \) with \( a_R \) and \( H_R \) being the scale factor and the Hubble parameter at the reheating stage, respectively, and the Friedmann equation \( 3H_R^2 = \rho_r(\eta_R)/M_{\text{Pl}}^2 \) at the reheating stage, where \( M_{\text{Pl}} = M_{\text{Pl}}/\sqrt{8\pi} = 1/\kappa \). We can further rewrite the term \( (-k\eta_R) \) as \[50\]

\[ -k\eta_R = \frac{k}{a_R H_R} \approx \left( \frac{1.88}{h} \right) 10^4 \left( \frac{L}{[\text{Mpc}]} \right) \left( \frac{T_R}{T_0} \right) \left( \frac{H_0}{H_R} \right) \]

\[ = 5.1 \times 10^{-25} N_{\text{eff}}^{-1/2} \left( \frac{M_{\text{Pl}}}{T_R} \right) \left( \frac{L}{[\text{Mpc}]} \right)^{-1} . \] (4.4)

In deriving eq. (4.3), we have used \( H_0^{-1} = 3.0 \times 10^3 h^{-1} \text{Mpc} \) and \( T \propto a^{-1} \), which leads to \( (a_0/a_R) = (T_R/T_0) \). Moreover, in analyzing eq. (4.4), we have adopted the Friedmann equation \( 3H^2 = \rho_r(\eta_R)/M_{\text{Pl}}^2 \) with \( \rho_r(\eta_R) = N_{\text{eff}} (\pi^2/30) T_R^4 \), \( T_0 = 2.73 \text{K} \) and \( H_0 = 2.47h \times 10^{-21} \text{K} \) \[56\] with \( h = 0.7 \) \[6, 57, 58\]. Since the current amplitude of the magnetic field is given by \( |B(\eta_0)|^2 = 2\rho_B(\eta_0) = 2\Omega_B(\eta_0, k) \rho_r(\eta_0) \), with \( \rho_r(\eta_0) \approx 2 \times 10^{-51} \text{GeV}^4 \) and \( 1 \text{G} = 1.95 \times 10^{-20} \text{GeV}^2 \) we find \[50\]

\[ |B(\eta_0, L)| = 2.7 \left[ \frac{7.2}{(5.1)^4} \right]^{\beta/8} \times 10^{-56+5\beta/4} N_{\text{eff}}^{(\beta-4)/8} \sqrt{A I(\eta_0)/I(\eta_R)} \left( \frac{H_R}{M_{\text{Pl}}} \right)^{\beta/4} \left( \frac{L}{[\text{Mpc}]} \right)^{\beta/2-2} \text{G} . \] (4.5)
We note that the reheating temperature $T_R$ is described by using the Hubble parameter at the end of inflation, namely, instantaneous reheating stage, $H_R$ as

$$T_R = \left[ \frac{90}{(8\pi^3 N_{\text{eff}})} \right]^{1/4} \sqrt{M_{\text{Pl}}H_R}.$$ 

Furthermore, there exists the upper limit of $H_R$ from tensor perturbations. With the Wilkinson Microwave Anisotropy Probe (WMAP) five year data in terms of the anisotropy of the CMB radiation [5], we have $H_R < 6.0 \times 10^{14} \text{GeV}$ [59].

### 4.2 Estimation of the current strength of the large-scale magnetic field

We suppose that power-law inflation occurs, in which the scale factor is given by

$$a = a_0 \left( \frac{t}{t_0} \right)^p,$$  \hspace{1cm} (4.6)

with $p \gg 1$, where $a_0$ and $t_0$ are constants. The larger the value of $p$ is, the closer power-law inflation goes to exponential inflation. In this case, with the relation $\eta = \int \left( \frac{1}{a} \right) dt$, we get

$$\frac{t}{t_0} = [a_0 t_0 (p - 1) (-\eta)]^{-1/(p-1)}.$$  \hspace{1cm} (4.7)

We examine the case of a power-law type coupling as

$$I(T) = \left( \frac{T}{T_0} \right)^n,$$  \hspace{1cm} (4.8)

where $T_0$ is a current value of $T$ and $n(\neq 0)$ is a non-zero constant. In this case, by using $T = -6H^2$, $H = p/t$ and eq. (4.7), we obtain

$$I(\eta) = \left( -\frac{6}{T_0} \right)^n \left( \frac{p}{t_0} \right)^{2n} [a_0 t_0 (p - 1)]^{2n/(p-1)} (-\eta)^{2n/(p-1)}.$$  \hspace{1cm} (4.9)

By comparing this equation with eq. (4.1), we acquire

$$\beta = \frac{-2n}{p - 1}.$$  \hspace{1cm} (4.10)

For $N_{\text{eff}} = 100$, $H_R = 1.0 \times 10^{14} \text{GeV}$ ($T_R = 8.6 \times 10^{15} \text{GeV}$), $L = 1 \text{Mpc}$, $A = 1$, $I(\eta_R) = I(\eta_0)$, and $\beta = 4.2$, which can be realized for $p = 10$ and $n = -18.9$, we have

$$|B(\eta_0, L = 1 \text{Mpc})| = 2.5 \times 10^{-9} \text{G}.$$  \hspace{1cm} (4.11)

Similarly, for the above values except $H_R = 1.0 \times 10^{10} \text{GeV}$ ($T_R = 8.6 \times 10^{13} \text{GeV}$) and $\beta = 4.6$, met for $p = 10$ and $n = -19.7$, we obtain

$$|B(\eta_0, L = 1 \text{Mpc})| = 2.3 \times 10^{-9} \text{G}.$$  \hspace{1cm} (4.12)

Here, it should be mentioned that in order to demonstrate the estimation of the generated magnetic field strength at the present time, we have considered the case in which the non-minimal gravitational coupling of the electromagnetic field $I(T)$ changes in time only during inflation, whereas it does not evolve any more, i.e., $I(\eta_R) = I(\eta_0)$, after the instantaneous reheating stage following inflation.

Finally, we compare our results with the analysis executed in ref. [55]. The parameter of $\beta$ in our study corresponds to that of $\nu$ in ref. [55]. In particular, $\beta = 4.2$ in the present work
correlate with $\nu = 2.6$. Note that the scale-invariant spectrum of the magnetic fields is obtained for $\nu = 5/2$ in ref. [55]. In comparison with the analysis in ref. [55], for $\nu = 2.6$ the resultant strength of the magnetic field is estimated as $5.4 \times 10^{-9} \text{G}$, whereas in the scale-invariant limit the magnetic field would be $1.4410 \times 10^{-11} \text{G}$. These figures may change depending on the assumptions on the reheating stage. Hence, we suppose the sudden (i.e., spontaneous) reheating where all the energy density of the inflaton can safely be assumed to be released into the energy density of the radiation. In this sense, for the case where $H_R = 10^{10} \text{GeV}$ ($T_R = 8.6 \times 10^{13} \text{GeV}$) with the above values such as $\beta = 4.2$ (i.e., $p = 10$ and $n = -18.9$), we find

$$|B(\eta_0, L = 1 \text{Mpc})| = 1.6 \times 10^{-13} \text{G}.$$  (4.13)

Clearly, this strength can satisfy the scale-invariant limit, namely, less than $1.4410 \times 10^{-11} \text{G}$.

5 Conclusions

We have investigated the generation of large-scale magnetic fields in inflationary cosmology in the context of teleparallelism. We have examined a non-minimal gravitational coupling of the torsion scalar to the electromagnetic field, which breaks its conformal invariance and hence, the quantum fluctuations of the electromagnetic field can be produced during inflation. It has explicitly illustrated that if the form of the coupling is a power-law type, the magnetic field with its strength of $\sim 10^{-9} \text{G}$ and the coherence scale of $1 \text{Mpc}$ at the present time can be generated. This field strength is enough to account for the large-scale magnetic fields observed in clusters of galaxies only through the adiabatic compression during the construction of the large scale structure of the universe without the dynamo amplification mechanism.

Finally, we remark that the resultant field strength of $\sim 10^{-9} \text{G}$ on $1 \text{Mpc}$ scale is compatible with the upper limit of $\sim 2-6 \times 10^{-9} \text{G}$ obtained from the observation of CMB radiation [60, 61] as well as that of being smaller than $4.8 \times 10^{-9} \text{G}$ from CMB radiation on the present strength with scales larger than the present horizon [62]. There also exist constraints on the strength of the large-scale magnetic fields from the matter density fluctuation parameter $\sigma_8$ [63], the fifth science (S5) run of laser interferometer gravitational-wave observatory (LIGO) [64], Chandra X-ray galaxy cluster survey and Sunyaev-Zel’dovich (S-Z) survey [65], which are compatible with or weaker than those from CMB. Incidentally, generic features of the spectrum of the large-scale magnetic fields generated at the inflationary stage have been investigated in ref. [66]. Moreover, it is also known that from the Big Bang Nucleosynthesis (BBN), there are limits on the primordial magnetic fields The constraint on the current strength of the magnetic fields on the BBN horizon scale $\sim 9.8 \times 10^{-5} h^{-1} \text{Mpc}$, where $h = 0.7$ [57], is smaller than $10^{-6} \text{G}$ [67]. Furthermore, it is meaningful to note that the large-scale magnetic fields with the strength $\sim 4 \times 10^{-11} - 10^{-10} \text{G}$ at the present time can be observed [68] by various future polarization experiments on CMB radiation, e.g., PLANCK [69, 70], QUIET [71, 72], B-Pol [73] and LiteBIRD [74]. If such large-scale magnetic fields in void regions and/or inter galactic medium are detected, the possibility that those origin is the quantum fluctuations of the electromagnetic field generated at the inflationary stage would become higher. Thus, physics in the early universe including inflation may be understood through the future detection of the large-scale magnetic fields.

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