Capillary-absorbed moisture transfer in a multiphase soil medium considering the load potential

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Abstract: The work presents an analytical solution to the equation of moisture permeability of structural varieties of capillary-absorbed moisture in a multiphase soil system with incomplete water saturation considering the influence of load potential. Researches were carried out and the influence of external load on the moisture profile in a multiphase system, the rate of moisture transfer of a discontinuous flow in general, and the rate of moisture propagation of the wetting boundaries have been shown. For a comparative analysis of the indicators of moisture transfer, functional relationships between the potential and pressure of the structural varieties of capillary-absorbed ground moisture are presented: capillary-gravity, proper-capillary and capillary-osmotic components of ground moisture. In the interval of incomplete water saturation, an attempt was made to create the foundations of capillary moisture transfer in the soil environment considering both the external distributed load and the own weight of the soil.

1. Introduction
Due to the economic development in the coming years of the territory of massifs of multiphase soils formation and the construction of structures on them, the task of quantitatively forecasting of the moisture content in the soil medium with incomplete water saturation becomes more and more urgent [1, 2, 3].

The main goal of this work is the creation and development of general foundations of moisture permeability considering the load potential in a multiphase soil system with incomplete water saturation [4, 5, 7, 8, 9]. The problem of dividing the full potential of pore moisture into separate terms corresponding to the acting force fields [4,5] and the development of thermodynamic approaches to the study of moisture permeability in the domestic literature on this basis is considered more promising [2,3]. At the same time, the study of each component of the term separately, as a task to achieve the goal, can be considered a solvable problem.

2. Methods and materials
Let us consider the methodology for determining the load potential: according to the works of national researchers [2,3], the load potential is found as a quotient of the elementary work produced by the multiphase soil system and the increment in the mass of absorbed liquid during moisture transfer:

$$\Omega = \frac{dA}{dm_w}$$  \hspace{1cm} (1)
The functional dependence of the moisture content \( dm_w \) on the increment in moisture \( dW \), considering the density of the skeleton of the soil system \( \rho_d \) for an elementary volume of a multiphase medium \( dxdydz \), is presented in the following form:

\[
\frac{dm_w}{W} = \rho_d \frac{dW}{dxdydz},
\]

(2)

whence, taking into account the initial coefficient of porosity of the multiphase system \( e_0 \) and the density of mineral soil particles \( \rho_s \), we can have:

\[
\frac{dm_w}{W} = \frac{s}{\rho_s} \frac{dW}{dxdydz} / (1 + e_0).
\]

(3)

In order to determine the relationship between the change in the porosity coefficient and the relative deformation of the medium, we represent the increment \( de \) as a sum:

\[
de = de_x + de_y + de_z
\]

(4)

In order to solve practical issues in construction under standard loads, it is assumed that the expansion or contraction of a multiphase soil system occurs only due to a change in pores volume, that is, mineral particles are considered incompressible. We represent the ratio of transverse and vertical changes in pore volume through nondimensional coefficients

\[
k_x = \frac{de_x}{de_z}, k_y = \frac{de_y}{de_z},
\]

(5)

with regard to the latter relations from the dependence (4) for the increment of the pores volume \( de \) we have

\[
de = (1 + k_x + k_y) de_z
\]

(6)

In this case, the increase in the linear dimensions of the elementary volume of the soil can be finally represented in the following form:

\[
\delta dx = \frac{k_x}{1 + k_x + k_y} de dx/(1 + e_0),
\]

\[
\delta dy = \frac{k_y}{1 + k_x + k_y} de dy/(1 + e_0),
\]

\[
\delta dz = de dz/(1 + k_x + k_y) (1 + e_0).
\]

(7) \hspace{1cm} (8) \hspace{1cm} (9)

The elementary work \( dA \) produced by the soil when absorbing \( dm_w \) of water will be equal to the product of the stress components by the corresponding displacements

\[
dA = \sigma_z \frac{dx}{dxdydz} + \sigma_x \frac{dx}{dxdydz} dx + \sigma_y \frac{dx}{dxdydz} dy + \sigma_x \frac{dx}{dxdydz} dz
\]

(10)

Taking into account the obtained relations (7) - (9) for linear displacements of the soil medium, the dependence (10) will be:

\[
dA = (\sigma_z + k_x \sigma_x + k_y \sigma_y) de dz / (1 + k_x + k_y) (1 + e_0).
\]

(11)

Following the dependence (1), we can obtain for the load potential:

\[
\Omega = (\sigma_z + k_x \sigma_x + k_y \sigma_y) (\delta e / \delta W) \sigma / (1 + k_x + k_y) \rho_s
\]

(12)

At the stage of preliminary calculations of the non-continuous moisture flow of a multiphase soil system, taking into account both the external uniformly distributed load and its own weight, the obtained expression (12) can be simplified for the one-dimensional case as follows:

\[
\Omega = (P + \int_0^z \frac{dx}{dxdydz})(\delta e / \delta W) \sigma / \rho_s.
\]

(13)
The obtained value of the load potential for a one-dimensional task, when the multiphase soil system operates according to the “loading-wetting” scheme, contains moisture and pressure as independent variables. On the interval of variation of compressive stresses in the foundations of structures, the partial derivative \( (\partial e/\partial W) \), with approximation sufficient for practical purposes, can be converted into the absolute \( (de/dW) \), when fluctuations in the values of partial derivatives with increasing pressure from \( p = 0 \) to \( p = 0.3 \) MPa are irrelevant. The specific statement on the compressive stresses in multiphase soil foundations for various engineering structures simplifies the obtained dependence of the load potential for the use in numerical calculations, while excluding pressure as an independent variable.

According to [2], there is a close regular relationship between the values of the porosity coefficient and the moisture content of a multiphase medium for different soils, which, after the analysis, can be approximated in the form of a logarithmic, hyperbolic, parabolic and rectilinear relationship. It is also noted that the experimental data on the regular relationship between porosity and moisture are more accurately approximated by a parabolic equation.

Considering the partial derivative \( (\partial e/\partial W) \) with sufficient accuracy for practical purposes as a function only of the moisture content of the soil system and differentiating the nonlinear parabolic equation of porosity from moisture content at a constant compressive pressure on the multiphase system, we have the following for the load potential:

\[
\Omega = (P + \int_0^z \rho W dz)(2aW + b) / \rho_v ,
\]

where \( a \) and \( b \) are experimental coefficients.

Following the works [3,4,5], the functional dependence for the total rate of moisture permeability, considering the loading potential, can be written as follows

\[
u_w = -K_w(w)\left(\alpha_{w_p}/\rho_v + \alpha_{w_0}/\rho_w + \alpha_{w_c}/\rho_w + \alpha_{w_g}/\rho_{wg}\right)\frac{\partial W}{\partial z} + K_w(w),
\]

here \( K(w) \) is the main characteristic of the soil moisture permeability.

According to our research [4,5], the equation of flow continuity in the issues of capillary-absorbed moisture transfer, considering the loading potential, is as follows:

\[
\frac{m_\rho_v}{\rho_w} \frac{dW}{dt} + \frac{d\nu_w}{dz} = 0,
\]

here \( \nu_w \) is the rate of moisture transfer through a unit of cross-sectional area of the soil system; \( t \) is time;

3. Results and discussion

For a capillary-absorbed non-continuous flow of moisture transfer under the action of a constant load on the system, according to relations (15), as well as the continuity equation from (16), the following nonlinear differential equation can be obtained

\[
\frac{m_\rho_v}{\rho_w} \frac{dW}{dt} = \frac{\partial}{\partial z} \left[ \left( \frac{K_w \alpha_{w_p}}{\rho_v} + \frac{K_w \alpha_{w_0}}{\rho_w} + \frac{K_w \alpha_{w_c}}{\rho_w} + \frac{K_w \alpha_{w_g}}{\rho_{wg}} \right) \frac{\partial W}{\partial z} - K_w(W) \right],
\]
The solution of the mixed problem for the nonlinear equation of moisture permeability requires the formulation of two types of relations describing the state of the soil system at the boundary and at the beginning of the moisture transfer process:

Initial condition

\[ W(z, 0) = W_0 \]  \quad (18)

and at the boundary - the boundary condition

\[ W(0, t) = W_x. \]  \quad (19)

For the coefficients of permeability \( K(W) \), diffusion \( D(W) \) and intensity of force fields \( \alpha_w \) of a multiphase system, according to our research [4,5], we can recommend the well-known relations showing the efficiency of a non-continuous flow and omitting the intermediate results [4,5,6], and present the final solution for determining moisture depending on the coordinates of moisture movement and the current time:

\[ W = W_0 + (n - W_0) H_{\lambda} \left[ 1 - \exp \left( \frac{K_0 (k - 1)}{D_0 \Sigma} (z - \lambda t) \right) \right]^{1/2} \]  \quad (20)

where

\[ H_{\lambda} = \frac{1}{\lambda} \frac{\alpha_w (W_i - W_0) (n - W_0) \rho_\lambda}{K_0 \rho_w} \left( \frac{W_i - W_0}{n - W_0} \right)^{k - 1} \]  \quad (21)

here, \( \rho_w, W_i, \alpha_w \) are the density, humidity and tension coefficients respectively, for the studied intervals of moisture change.

At that, the dependence for determining the rate of advancement of the wetting boundaries in a multiphase soil system is written as:

\[ \lambda = \frac{1}{\lambda} \frac{\alpha_w (W_i - W_0) (n - W_0) \rho_\lambda}{K_0 \rho_w} \left( \frac{W_i - W_0}{n - W_0} \right)^{k - 1}. \]  \quad (22)
Figure 1. Influence of the load potential on moisture redistribution in a multiphase medium (time - 1 day). 1st curve at a pressure of $P = 0.0$ MPa; 2nd curve at a pressure of $P = 0.025$ MPa; 3rd curve at a pressure of $P = 0.05$ MPa; 4th curve at a pressure of $P = 0.1$ MPa; 5th curve at a pressure of $P = 0.2$ MPa; 6th curve at a pressure of $P = 0.4$ MPa.

Figure 2. Formation of a complete moisture profile in a multiphase system depending on coordinates and time (1st curve for the term of 1 day, 2nd curve – for 3 days, 3rd curve – for 6 days, 4th curve – for 12 days, 5th curve – for 24 days).
I-interval of bound moisture $W \leq \bar{W}$;  
II-interval of polymolecular and capillary moisture $WA < W \leq WC$;  
III-interval of gravity water $WC < W \leq WS$.

The presented theoretical foundations of the moisture permeability of capillary-absorbed ground moisture, considering the potential of both the external load and the own weight of the multiphase system, and the presented methodology, allowed solving the problems of modeling moisture transfer in space and time.

The following boundary and initial conditions for describing the state of the soil system have been accepted: the multiphase soil system is moistened from the strip $W(z,0)=W_3$; the initial moisture content is $W(z,0)=W_0=0.03$.

Numerical results have been obtained for the calculated values of the parameters of the multiphase system: $K_0 = 3.5 \cdot 10^{-5}$ cm/s, $n=0.4$, $m=0.6$, $\rho_s = 2.7$ g/cm$^3$, $\rho_w = 0.997$ g/cm$^3$, $k=3.56$, $\alpha_w = 0.006$ MPa, $\alpha_{wp} = 0.106$ MPa.

The studies on the redistribution of moisture in a multiphase medium depending on the uniformly distributed pressure are shown in Fig. 1. The results of the complete moisture profile formation in a multiphase system depending on coordinates and time are shown in Fig. 2.

4. Summary
The presented theoretical basis of moisture transfer of capillary-absorbed pore water considering the load potential and the results of solving the model problem allow drawing the following conclusions:

- The solution of the nonlinear equation of moisture permeability of structural varieties of capillary-absorbed moisture in soils with incomplete water saturation taking into account the load potential, is considered as both external distributed load and own weight of a multiphase soil medium.
- The tasks of comparative assessment of both the rate of advancement of the wetting boundaries and the rate of moisture transfer of capillary-absorbed pore moisture are easily solved with the presented solution of the moisture permeability equation.
- Comparative analysis of the results of solving the model problem showed that the main characteristics of moisture transfer - the moisture profile in a multiphase medium, the rate of moisture transfer and the rate of advancement of the wetting boundaries significantly depend on the external uniformly distributed load.
- The rate of advancement of the wetting boundaries in soil massifs is variable: at the initial moment of wetting of multiphase massifs, it reaches the maximum value, and then it decreases significantly.
- The analytical solution of the moisture permeability equation has an obvious advantage in identifying the regularities of moisture permeability of structural varieties of capillary-absorbed pore moisture, taking into account both the external uniformly distributed load and the own weight of the multiphase soil medium at the considered interval of incomplete water saturation.

5. References
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