The Spectrum of a Binding System for a Heavy Quark with an Anti-Sbottom or for a Sbottom and Anti-Sbottom Pair

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Since long-lived light bottom squark (sbottom) and its anti-particle with a mass close to the bottom quark have not been excluded by experiments so far, we consider such a sbottom to combine with its anti-particle to form a color singlet meson-like bound state or to combine with a common anti-quark to form a fermion-like one, or accordingly their anti-particles to form an anti-particle bound system. Namely we calculate the low-lying spectrum of the systems based on QCD inspired potential model. To be as relativistic as possible, we start with the framework of Bethe-Salpeter (BS) equation even for non-relativistic binding systems. Finally, we obtain the requested spectrum by constructing general forms of the BS wave functions and solving the BS equations under instantaneous approximation.

14.40.Lb, 14.40.Nd, 13.40.Hq.

I. INTRODUCTION

The supersymmetric minimum extended standard model, i.e. Minimum Supersymmetric Standard Model (MSSM), which have consequences at or lower than weak-scale \( \langle 1 \rangle \), is arguably the most promising candidate for physics beyond the Standard Model (SM). Generally this theory predicts existence of a super partner corresponding to each particle of SM, and these 'partners', the supersymmetric particles (sparticles), should not be heavier than \( O(1) \) TeV. Thus the sparticles should be accessible at the exist and constructing colliders such as Tevatron and LHC. In recent years, great deal of effort has been made to search for SUSY. Unfortunately, no direct signal for SUSY has been observed so far and some lower mass bounds have been established for sparticles. Based on the experimental results at LEP \( \langle 2 \rangle \) and Tevatron \( \langle 3 \rangle \), squarks must be heavier than about 100 GeV. However, most of the analysis of experimental searches for the sparticles is performed with model-dependent assumptions and rely on a large missing energy cut. Hence the quoted bounds may be escaped if some of the assumptions are relaxed. In particular, a long-lived light sbottom, \( \tilde{b} \), with mass close to the bottom quark, has not been excluded by experiments. Some analyses \( \langle 4 \rangle \) showed that if the light \( \tilde{b} \) is an appropriate admixture of left-handed and right-handed bottom squarks, its coupling to \( Z \) boson can be small enough to avoid LEP-I \( Z \) decay bounds. In addition, a scenario with light gluino and long-lived light sbottom, \( \tilde{b} \), with mass close to the bottom quark, was proposed in \( \langle 5 \rangle \), with which the excess of measured \( \tilde{b} \tilde{b} \) pair production in hadron collision over QCD theoretical prediction by a factor two is explained successfully.\footnote{In this case, i.e., gluino and the sbottom \( \tilde{b} \) are both light, the loop effects of such a sbottom may cause significant effects in the Z-peak observables \( \langle 6 \rangle \), Thus more care should be paid in globally fitting the parameters of the model.}

The CLEO exclusion of a \( \tilde{b} \) with mass 3.5 to 4.5 GeV \( \langle 7 \rangle \) can also be loosened even avoided, since their analysis depends on the assumption for semi-leptonic decays of the light sbottom. Moreover, since sbottom is a scalar, based on the spin freedom counting only, its pair production rate at colliders will be smaller than the bottom quark by a factor four, so the sbottom samples must be rarer than those of bottom quark in experiments. Therefore, a light sbottom and its anti-particle, in fact, are not excluded.

In contrary, it is interesting to point out that some experiments seemingly favor such a light sbottom. The ALEPH collaboration has reported experimental hints for a light sbottom with a mass around 4 GeV and lifetime of 1 ps \( \langle 8 \rangle \). A recent reanalysis of old anomaly in the MARK-I data for cross section of \( e^+e^- \rightarrow \text{hadrons} \) shows that the existence of such a light sbottom can bring the measured cross section into agreement with the theoretical prediction \( \langle 9 \rangle \).

The phenomenology of a very light sbottom has been studied by many authors recently \( \langle 10 \rangle \). If such a light sbottom indeed exist, new meson-like bound states by a pair of the sbottoms (\( \tilde{b}\tilde{b} \)) and fermion-like ones by the sbottom plus
an ordinary quark ($Q \bar{b}$) may be formed. In this paper we study these new binding systems in the framework of BS equation, because this equation has been successfully used in the study of the heavy-heavy ($Q\bar{Q}$) bound states under some approximation.

In Sec. II we give the BS equations for the binding systems ($Q \tilde{b}$) and ($\bar{b}b$). We construct the BS kernel by calculating the lowest order Feynman diagrams and by phenomenological considerations similar to the cases of ($Q\bar{Q}$) systems, i.e. the QCD inspired kernel. Then we start with the general formulations of the BS wave functions for these bound states to establish the coupling equations for the components of the BS wave functions accordingly. In Sec. III we choose the parameters in our calculation by fitting the spectrum of $c\bar{c}$ and $b\bar{b}$ bound states. Then we numerically solve the the BS equations for the systems ($Qb$) and ($bb$). Finally we present the numerical results and some discussions.

II. THE RELEVANT BETHE-SALPETER (BS) EQUATIONS

A. The bound state of a quark and a sbottom

Let $|P\rangle$ (momentum $P_\mu$ and mass $M : P^2 = M^2$) denotes a bound state of a heavy quark $Q$ with a light anti-sbottom ($\tilde{b}$), $Q(x)$ denotes the heavy quark field and $\tilde{b}(x)$ the light anti-sbottom field, where the color indices are suppressed. Then the Bethe-Salpeter (BS) wave function of this bound state is defined as

$$\chi(P, x_1, x_2) = \langle 0 | T(Q(x_1) \tilde{b}(x_2)) | P \rangle,$$

The translation invariance of the system implies

$$\chi(P, x_1, x_2) = e^{ipX} \chi(P, x),$$

where the center-of-mass and relative variables, and $X$ and $x$ are coordinate of center mass system and the relative one of the two components respectively:

$$X = \lambda_1 x_1 + \lambda_2 x_2, \quad x = x_1 - x_2,$$

with $\lambda_i = m_i / (m_1 + m_2)$ ($i = 1, 2$) and $m_1, m_2$ are the masses and momentums of the quark and sbottom respectively.

We further have the BS wave function in momentum space as

$$\chi(P, q) = \int \frac{d^4 x}{(2\pi)^4} e^{-iqx} \chi(P, x),$$

where $q$ is the relative momentum of the two components of the bound state. Then in momentum space the BS equation is written as

$$\chi(P, q) = \frac{1}{p_1 - m_1 + i\epsilon} \frac{1}{p_2 - m_2 + i\epsilon} \int \frac{d^4 k}{(2\pi)^4} G(P, q, k) \chi(P, k),$$

where $G(P, q, k)$ is the BS kernel which is defined as the sum of all two-particle irreducible graphs. $m_1, p_1$ and $m_2, p_2$ are the masses and momentums of the quark and sbottom respectively, so we have

$$P = p_1 + p_2, \quad q = \lambda_2 p_1 - \lambda_1 p_2.$$

To fix the BS equation, we should decide the specific form of the BS kernel. In the present case, the kernel is chosen based on QCD spirit as a combination of a short distance part and a long distance part:

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2Here we would like to restrict ourselves to consider the systems ($Q \tilde{b}$), an ‘ordinary’ heavy quark to bind with the light sbottom, because with the restriction the systems are surely non-relativistic, thus the well-tested knowledge for heavy-heavy quark systems ($Q\bar{Q}$) can be used as solid reference for the present study. Here $Q(\bar{Q})$ denotes $b(\tilde{b})$ or $c(\bar{c})$ only.

3Note here that in Ref. [3], a similar binding system ($Q\tilde{t}$) for a light anti-top-squark and a heavy quark is considered, but it is in Chinese and some slight different approximation is taken.
\[ G(P, q, k) = iG_s(P, q, k) + iG_l(P, q, k). \]  

(7)

The short distance part of the kernel, \( G_s(P, q, k) \), reasonably is of one-gluon-exchange, corresponding to the Feynman diagram shown in Fig. 1(a). \( G_s \) is precisely written as:

\[ G_s(P, q, k) = \frac{4}{3}(4\pi\alpha_s) \frac{\not{p}_2 + \not{p}_4}{(p_2 - p_4)^2 - \alpha^2}, \]  

(8)

where \( \frac{4}{3} \) is the color factor for color singlet bound state, and \( \alpha \) in the denominator is a small constant, which is introduced here to avoid the infrared divergence in numerical calculation, and can be considered as a fictitious gluon mass. Based on QCD, here the strong coupling constant \( \alpha_s \), as heavy quarkonium, is chosen as

\[ \alpha_s = \frac{12\pi}{27} \frac{1}{ln(a + \frac{(q-k)^2}{\Lambda_{QCD}^2})}. \]  

(9)

FIG. 1. The BS kernel from one-glou-exchange: (a) for the system \((\bar{b}b)\); (b) for the system \((\bar{\tilde{b}}\tilde{b})\).

Besides Eq.(6), because of momentum conservation we also have

\[ P = p_3 + p_4, \quad k = \lambda_2 p_3 - \lambda_1 p_4. \]  

(10)

Eq.(5) is fully Lorentz invariant, but it is hard to solve even numerically. In actual investigation, as heavy quarkonium, an effective method, the so-called instantaneous approximation, is made to reduce the full equation to a three dimensional one. For this purpose, the zero components of the momentums \( p^0_i (i = 1, 2, 3, 4) \) in the kernel should be fixed. Since we know that we are considering weak binding systems, so the zero components are chosen, as an approximation, as the on-shell values accordingly:

\[ p^0_i = \sqrt{m^2_i + \not{p}_i^2}, \quad (i = 1, 2, 3, 4). \]  

(11)

Then \( G_s \) is rewritten as

\[ G_s = G_s^{(1)}(\gamma\cdot(q + k)) + G_s^{(2)}\gamma^0. \]  

(12)

with

\[ G_s^{(1)} = \frac{4}{3}(E_2 - E'_2)^2 - (q - k)^2 + \alpha^2, \]  

(13)

\[ G_s^{(2)} = G_s^{(1)}(E_2 + E'_2), \]  

(14)

where \( E_i = \sqrt{m^2_i + \not{q}^2} \) and \( E'_i = \sqrt{m^2_i + \not{k}^2} (i = 1, 2) \). As for the long distance part of the kernel, we have to construct it phenomenologically. According to the experience of heavy quarkonium, \((Q\bar{Q})\) bound state, we assume that it has Lorentz scalar property, and choose is as

\[ 4^\text{Since now on, we change the notations a little that } p^0_i = E_i (i = 1, 2) \text{ and } p^0_{3,4} = E'_{1,2}. \]  

3
\[ G_t(P, q, k) = (2\pi)^3 (2m_2) \frac{\lambda \delta^3(q - k) - \frac{\lambda}{\pi^2 ((q - k)^2 + \alpha^2)^2}}{2E_2 (M - E_1 - E_2)} \]  

(15)

where \( \lambda \) is the string tension. In the non-relativistic limit, such a \( G_t \) corresponds to a potential in space configuration, which has a form as follows

\[ \frac{\lambda}{\alpha} (1 - e^{-\alpha r}). \]  

(16)

Since the ‘on-shell’ assumption Eq.(11), the kernel \( G(P,q,k) \) in Eq.(5) is \( q_0 \) and \( k_0 \) independent. We further define the three dimensional (instantaneous) wave function as

\[ \phi(P, q) = \int dq_0 \chi(P, q) . \]  

(17)

For convenience, we will adopt the center-of-mass frame in following treatment. In this frame, performing \( dq_0 \) integration on both sides of Eq.(5), we obtain the reduced BS equation

\[ \phi(q) = (\frac{\Lambda^+(P_1)\gamma_0}{2E_2 (M - E_1 - E_2)} + \frac{\Lambda^-(P_1)\gamma_0}{2E_2 (M + E_1 + E_2)}) \int d^3 k (2\pi)^3 (-i)G(P, q, k)\phi(P, k), \]  

(18)

where \( \Lambda^\pm(P_1) \) are project operators defined as

\[ \Lambda^\pm(P_1) = \frac{E_1 \pm \gamma_0 (\gamma \cdot P_1 + m_1)}{2E_1}. \]  

(19)

To solve the equation, we should construct the general form of the wave function with the given quantum numbers. The possible quantum numbers of \( (Q\hat{0}) \) bound states are \((\frac{1}{2})^+, \(\frac{1}{2})^-, (\frac{3}{2})^+, (\frac{3}{2})^+, \) etc.

For \((\frac{1}{2})^+ \) state, the most general instantaneous wave function in three dimensions has the form

\[ \phi(q) = (\phi_1 + \phi_2 \gamma \cdot q)u(P), \]  

(20)

where \( \phi_1 \) and \( \phi_2 \) are scalar functions of \( q^2 \), and \( u(P) \) is the Dirac spinor which satisfies the Dirac equation

\[ (P - M)u(P) = 0. \]  

(21)

However, for convenience of the following treatment, we would like to rewrite the wave function in another form

\[ \phi(q) = (\Lambda^+(q)f_1(\frac{1}{2}^+) + \Lambda^-(q)f_2(\frac{1}{2}^+))u(P), \]  

(22)

where \( f_1 \) and \( f_2 \) are also scalar functions of \( q^2 \), which are related to \( \phi_1 \) and \( \phi_2 \) through

\[ \phi_1 = \frac{E_1 + m_1}{2E_1} f_1 + \frac{E_1 - m_1}{2E_1} f_2, \]  

(23)

\[ \phi_2 = \frac{1}{2E_1} (f_2 - f_1). \]  

(24)

For \((\frac{1}{2})^- \) state, the three dimensional wave function is

\[ \phi(q) = (\Lambda^+(q)f_1(\frac{1}{2}^-) + \Lambda^-(q)f_2(\frac{1}{2}^-))\gamma_5 u(P). \]  

(25)

For \((\frac{3}{2})^+ \) state, the three dimensional wave function is

\[ \phi(q) = (\Lambda^+(q)f_1(\frac{3}{2}^+) + \Lambda^-(q)f_2(\frac{3}{2}^+))\gamma_5 q \cdot u(P), \]  

(26)

where \( u(P) \) is a vector spinor which satisfy the following equations

\[ (P - M)u(P) = 0; \quad \gamma \cdot u(P) = 0. \]  

(27)

For \((\frac{3}{2})^- \) state
\[ \phi(q) = (\Lambda^+(q)f_1^{(2^-)} + \Lambda^-(q)f_2^{(2^-)})q \cdot u(P). \]  

Inserting the wave function Eq.(22) into Eq.(18), we obtain two coupled equations:

\[
(M - E_1 - E_2)\Lambda^+(q)f_1^{(2^+)}(q)u = \frac{\Lambda^+(q)\gamma_0}{2E_2} \int \frac{d^3k}{(2\pi)^3} (-i)G(P, q, k) \\
\times \left[ \Lambda^+(k)f_1^{(2^+)}(k) + \Lambda^-(k)f_2^{(2^+)}(k) \right] u, \quad (29)
\]

\[
(M + E_1 + E_2)\Lambda^-(q)f_2^{(2^+)}(q)u = \frac{\Lambda^-(q)\gamma_0}{2E_2} \int \frac{d^3k}{(2\pi)^3} (-i)G(P, q, k) \\
\times \left[ \Lambda^+(k)f_1^{(2^+)}(k) + \Lambda^-(k)f_2^{(2^+)}(k) \right] u. \quad (30)
\]

Then from these two equations we may abstract two coupled and independent equations about the two scalar functions \( f_1^{(2^+)} \) and \( f_2^{(2^+)} \):

\[
(M - E_1 - E_2)f_1^{(2^+)} = \frac{1}{2E_2} \int \frac{d^3k}{2E_1(2\pi)^3} \left\{ G_s^{(1)}[\mathbf{k} \cdot (q + k) + \frac{E_1 + m_1}{E_1 + m_1} q \cdot (q + k)] \\
+ G_s^{(2)}[(E'_1 + m_1) + \frac{q \cdot k}{E_1 + m_1}] - G_s[(E'_1 + m_1) - \frac{q \cdot k}{E_1 + m_1}] \right\} f_1^{(2^+)} \\
+ \frac{1}{2E_2} \int \frac{d^3k}{2E_1(2\pi)^3} \left\{ G_s^{(1)}[-\mathbf{k} \cdot (q + k) + \frac{E_1 + m_1}{E_1 + m_1} q \cdot (q + k)] \\
+ G_s^{(2)}[(E'_1 - m_1) - \frac{q \cdot k}{E_1 + m_1}] - G_s[(E'_1 - m_1) + \frac{q \cdot k}{E_1 + m_1}] \right\} f_2^{(2^+)}, \quad (31)
\]

\[
(M + E_1 + E_2)f_2^{(2^+)} = \frac{1}{2E_2} \int \frac{d^3k}{2E_1(2\pi)^3} \left\{ G_s^{(1)}[\mathbf{k} \cdot (q + k) - \frac{E_1 + m_1}{E_1 - m_1} q \cdot (q + k)] \\
+ G_s^{(2)}[(E'_1 + m_1) - \frac{q \cdot k}{E_1 - m_1}] - G_s[(E'_1 + m_1) + \frac{q \cdot k}{E_1 - m_1}] \right\} f_1^{(2^+)} \\
+ \frac{1}{2E_2} \int \frac{d^3k}{2E_1(2\pi)^3} \left\{ G_s^{(1)}[-\mathbf{k} \cdot (q + k) - \frac{E_1 - m_1}{E_1 - m_1} q \cdot (q + k)] \\
+ G_s^{(2)}[(E'_1 - m_1) + \frac{q \cdot k}{E_1 - m_1}] - G_s[(E'_1 - m_1) - \frac{q \cdot k}{E_1 - m_1}] \right\} f_2^{(2^+)}. \quad (32)
\]

In a similar way, we can obtain coupled equations for the bound state with the other quantum numbers, which are shown in the appendix.

**B. The bound states of a pair of sbottoms**

The BS wave function for a \( \tilde{b}\tilde{b} \) bound state is defined as

\[ \chi(P, x_1, x_2) = \langle 0|T(\tilde{b}(x_1)\tilde{b}(x_2))|P \rangle, \]  

(33)

where \( |P \rangle \) is now a bound state of a pair of scalar bottom quarks. The wave function in momentum space can be defined similarly as in Eq.(4), then in momentum space the equation for such a bound state is written as

\[ \chi(P, q) = \frac{1}{p_1^2 - m^2 + i\epsilon} \frac{1}{p_2^2 - m^2 + i\epsilon} \int \frac{d^4k}{(2\pi)^4} \bar{G}(P, q, k)\chi(P, k), \]  

(34)

where \( p_1 \) and \( p_2 \) are momentums of \( \tilde{b} \) and \( \tilde{b} \) with mass \( m \). In the case of the \( \tilde{b}\tilde{b} \) bound state, \( \lambda_1, \lambda_2 \) as in Eqs.(3,6,10) now have \( \lambda_1 = \lambda_2 = \frac{1}{2} \). The kernel \( \bar{G} \) in the above equation is also assumed to be a combination of two parts

\[ \bar{G} = \bar{G}_s + \bar{G}_t. \]  

(35)

The short distance part can be obtained by calculating one-gluon-exchanged Feynman diagram shown in Fig. 1(b).
\[ G_s(P, q, k) = \frac{4}{3} (4\pi \alpha_s) \frac{(p_1 + p_3) \cdot (p_2 + p_4)}{(p_2 - p_4)^2 - \alpha^2}. \]  

The zero components of momentums are again fixed at their on-shell values. The long distance part of the kernel is chosen as

\[ G_l(P, q, k) = i(2\pi)^3 (4m_b^2) \{ \frac{1}{2} \partial^3 (q - k) \partial^2 \} - \frac{1}{2} \delta^3 (q - k) \partial^2 \} \].

The three dimensional equal time wave function is also defined as in Eq.(17), and similarly we obtain the reduced instantaneous equation for the bound state of a pair of scalar bottoms,

\[ (M - 2E)\phi(q) = \frac{1}{E(M + 2E)} \int \frac{d^3k}{(2\pi)^3} (-i)G(P, q, k)\phi(P, k), \]

where \( E = \sqrt{m^2 + q^2} \).

Now let us construct the ‘general form’ of the instantaneous wave function of \( \bar{b}b \) bond state. The possible quantum numbers are \( 0^{++}, 1^{--}, 2^{++}, \) etc. For the state with quantum number \( 0^{++} \), the three dimensional wave function is a scalar function

\[ \phi(q) = f_0(q). \]

For the \( 1^{--} \) state

\[ \phi(q) = f_1(q)q \cdot e, \]

where \( e \) is the polarization vector of the bound state, and for \( 2^{++} \) state the wave function reads

\[ \phi(q) = f_2(q)q_i q_j \eta_{ij}, \]

where \( \eta_{ij} \) is the polarization tensor of the bound state. Substituting the wave function Eq.(39) into Eq.(38), we obtain the equation for \( 0^{++} \) state. The equations for the states \( 1^{--}, 2^{++} \) and \( 3^{--} \) may be obtained obtained similarly. These equations are given explicitly in the appendix.

III. THE NUMERICAL RESULTS AND DISCUSSIONS

Since the integration kernel (potential) is too complicated for analytically solving, we solve the instantaneous equations obtained above numerically.

We adopt the method to achieve the low-lying eigenvalues and states of the equations, first by making the integral equations into discrete ones, namely to turn the equations into algebra equations, then to make the matrix for the discrete algebra ones being diagonal.

Besides the masses of quark and scalar quark, there are four parameters in our investigation: \( a \) and \( \Lambda_{QCD} \) appearing in the running strong coupling constant, the constant \( \alpha \) and the string tension \( \lambda \). These parameters are fixed by fitting the spectrum of \( (QQ) \) bound states in the same framework of BS equation. Namely to test the reliability of our numerical method and to choose suitable parameters appearing in the equations, we calculate the spectrum of the ordinary heavy quarkonia \( (QQ) \) and request the calculated values for the heavy quarkonia are in good agreement with their corresponding experimental ones. The values for the parameters are fixed as the below values:

\[ \Lambda_{QCD} = 0.162GeV, a = 2.713, \lambda = 0.23(GeV)^2, \alpha = 0.06GeV, m_b = 4.83GeV, m_c = 1.55GeV. \]

To see the fit of these parameters we have adopted, the calculated spectrum and the experimental one for heavy quarkonia are put in Table 1. From Table 1 one may conclude how well the chosen parameters in fitting heavy quarkonium data.

In our investigation, the sbottom mass, \( m_{\tilde{b}} \), is unknown, so one should treat it free. In the present paper, we constrain ourselves mainly to interest light sbottom with a mass close to \( m_b (~5GeV) \), partly because of the ALEPH indication \[ [7] \] and partly because of interesting scenario to explain the excess of \( bb \) pair production in hadron collisions than theoretical prediction by a factor two. However, in order to have a full knowledge about the spectrum, we also give the results for a heavier sbottom. The numerical results about \( (\bar{Q}\tilde{b}),(Q = b, c) \) states are given in Table 2 and Table 3. In Table 4 we show the results about \( (\tilde{b}\tilde{b}) \).
Because the sbottom is a spin zero particle, the spectrum of the corresponding bound states is simpler than that of \((Q\bar{Q})\) states. We can see from these tables that when \(m_b\) is close to the mass of bottom quark the masses of the lowest state of \((b\bar{b})\), \((\bar{b}b)\) and \((\tilde{b}\bar{b})\) are very close to each other. This is reasonable because the strong interaction for these states is similar. We also see that the spin-orbit interaction for \((Q\bar{b})\) system is rather weak: the mass difference between \((\frac{1}{2}^-)\) and \((\frac{3}{2}^-)\) is quite small. The reason is that the quark and scalar quark, being considered here, both are heavy.

The parameters are obtained by fitting the spectrum for heavy quarkonia \((Q\bar{Q})\). Of the parameters, we find that the masses of the bound states are not sensitive to the parameters \(a, \Lambda_{QCD}\) and \(\lambda\). As examples, We show \(\Lambda_{QCD}\) and \(\lambda\) dependence of the masses of the lowest \((Q\bar{b})\) and \((b\bar{b})\) bound states in Table 5 and Table 6.

It should be pointed out that the study of the production and decay properties of such bound states beyond the scope of this paper and we will be presented elsewhere soon [3].

ACKNOWLEDGMENTS

One of the authors (J. Y. Cui) acknowledges the hospitality from Institute of Theoretical Physics, Chinese Academy of Sciences, when he visited there. This work was supported in part by Nature Science Foundation of China (NSFC), and in part by a grant of Chinese Academy of Science for Outstanding Young Scholars.

APPENDIX

A. The equations of \((Q\bar{b})\) states

For \((\frac{1}{2}^-)\) state the coupled equations are

\[
(M - E_1 - E_2) f_1^{(\frac{1}{2}^-)} = \frac{1}{2E_2} \int \frac{d^3k}{2E_1(2\pi)^3} \left\{ G_2^{(1)}[k \cdot (q + k) + \frac{E_1 - m_1}{E_1 - m_1} q \cdot (q + k)] + G_2^{(2)}[(E'_1 - m_1) + \frac{q \cdot k}{E_1 - m_1}] + G_1[(E'_1 - m_1) - \frac{q \cdot k}{E_1 - m_1}] \right\} f_1^{(\frac{1}{2}^-)},
\]

\[
(M + E_1 + E_2) f_2^{(\frac{1}{2}^-)} = \frac{1}{2E_2} \int \frac{d^3k}{2E_1(2\pi)^3} \left\{ G_2^{(1)}[k \cdot (q + k) - \frac{E_1 - m_1}{E_1 + m_1} q \cdot (q + k)] + G_2^{(2)}[(E'_1 - m_1) - \frac{q \cdot k}{E_1 + m_1}] + G_1[(E'_1 + m_1) + \frac{q \cdot k}{E_1 + m_1}] \right\} f_2^{(\frac{1}{2}^-)},
\]

For \((\frac{3}{2}^-)\) state, the coupled equations are (in the following \(\theta\) represents the angle between the vectors \(q\) and \(k\)):

\[
(M - E_1 - E_2) f_1^{(\frac{3}{2}^-)} = \frac{1}{2E_2} \int \frac{d^3k}{2E_1(2\pi)^3} \left\{ G_2^{(1)}[k \cdot (q + k) + \frac{E_1 + m_1}{E_1 + m_1} q \cdot (q + k)] + G_2^{(2)}[(E'_1 + m_1) + \frac{q \cdot k}{E_1 + m_1}] - G_1[(E'_1 + m_1) - \frac{q \cdot k}{E_1 + m_1}] \right\} \cos \theta f_1^{(\frac{1}{2}^-)},
\]

\[
(M + E_1 + E_2) f_2^{(\frac{3}{2}^-)} = \frac{1}{2E_2} \int \frac{d^3k}{2E_1(2\pi)^3} \left\{ G_2^{(1)}[-k \cdot (q + k) - \frac{E_1 - m_1}{E_1 + m_1} q \cdot (q + k)] + G_2^{(2)}[(E'_1 - m_1) - \frac{q \cdot k}{E_1 + m_1}] + G_1[(E'_1 - m_1) + \frac{q \cdot k}{E_1 + m_1}] \right\} \cos \theta f_2^{(\frac{1}{2}^-)},
\]
\[
(M + E_1 + E_2)f_2^{(\frac{3}{2}^-)} = \frac{1}{2E_2} \int \frac{d^3k}{2E'_1(2\pi)^3} \left\{ G_s^{(1)}[k \cdot (q + k) - \frac{E'_1 + m_1}{E_1 - m_1} q \cdot (q + k)] \\
+ G_s^{(2)}[(E'_1 + m_1) - \frac{q \cdot k}{E_1 - m_1}] - G_i[(E'_1 + m_1) + \frac{q \cdot k}{E_1 - m_1}] \right\} \cos \theta f_1^{(\frac{3}{2}^-)} \\
+ \frac{1}{2E_2} \int \frac{d^3k}{2E'_1(2\pi)^3} \left\{ G_s^{(1)}[-k \cdot (q + k) - \frac{E'_1 - m_1}{E_1 - m_1} q \cdot (q + k)] \\
+ G_s^{(2)}[(E'_1 - m_1) + \frac{q \cdot k}{E_1 - m_1}] - G_i[(E'_1 - m_1) - \frac{q \cdot k}{E_1 - m_1}] \right\} \cos \theta f_2^{(\frac{3}{2}^-)}. \tag{45}
\]

For \((\frac{3}{2}^+)^\star\) state the coupled equations are

\[
(M - E_1 - E_2)f_1^{(\frac{3}{2}^+)} = \frac{1}{2E_2} \int \frac{d^3k}{2E'_1(2\pi)^3} \left\{ G_s^{(1)}[k \cdot (q + k) + \frac{E'_1 - m_1}{E_1 - m_1} q \cdot (q + k)] \\
+ G_s^{(2)}[(E'_1 - m_1) + \frac{q \cdot k}{E_1 - m_1}] + G_i[(E'_1 - m_1) + \frac{q \cdot k}{E_1 - m_1}] \right\} \cos \theta f_1^{(\frac{3}{2}^+)} \\
+ \frac{1}{2E_2} \int \frac{d^3k}{2E'_1(2\pi)^3} \left\{ G_s^{(1)}[-k \cdot (q + k) + \frac{E'_1 + m_1}{E_1 - m_1} q \cdot (q + k)] \\
+ G_s^{(2)}[(E'_1 + m_1) - \frac{q \cdot k}{E_1 - m_1}] + G_i[(E'_1 + m_1) - \frac{q \cdot k}{E_1 - m_1}] \right\} \cos \theta f_2^{(\frac{3}{2}^+)}, \tag{46}
\]

\[
(M + E_1 + E_2)f_2^{(\frac{3}{2}^+)} = \frac{1}{2E_2} \int \frac{d^3k}{2E'_1(2\pi)^3} \left\{ G_s^{(1)}[k \cdot (q + k) - \frac{E'_1 - m_1}{E_1 + m_1} q \cdot (q + k)] \\
+ G_s^{(2)}[(E'_1 - m_1) - \frac{q \cdot k}{E_1 + m_1}] + G_i[(E'_1 - m_1) + \frac{q \cdot k}{E_1 + m_1}] \right\} \cos \theta f_1^{(\frac{3}{2}^+)} \\
+ \frac{1}{2E_2} \int \frac{d^3k}{2E'_1(2\pi)^3} \left\{ G_s^{(1)}[-k \cdot (q + k) - \frac{E'_1 + m_1}{E_1 + m_1} q \cdot (q + k)] \\
+ G_s^{(2)}[(E'_1 + m_1) + \frac{q \cdot k}{E_1 + m_1}] + G_i[(E'_1 + m_1) - \frac{q \cdot k}{E_1 + m_1}] \right\} \cos \theta f_2^{(\frac{3}{2}^+)} \tag{47}
\]

B. The equations of \((b\bar{b})\) states

For \(0^{++}\) state

\[
(M - 2E)f_0 = \frac{1}{E(M + 2E)} \int \frac{d^3k}{(2\pi)^3} (-i) \tilde{G}(P, q, k)f_0. \tag{48}
\]

For \(1^{--}\) state

\[
(M - 2E)f_1 = \frac{1}{E(M + 2E)} \int \frac{d^3k}{(2\pi)^3} (-i) \tilde{G}(P, q, k) \cos \theta f_1. \tag{49}
\]

For \(2^{++}\) state

\[
(M - 2E)f_2 = \frac{1}{E(M + 2E)} \int \frac{d^3k}{(2\pi)^3} (-i) \tilde{G}(P, q, k) \frac{3 \cos^2 \theta - 1}{2} f_2. \tag{50}
\]

For \(3^{--}\) state

\[
(M - 2E)f_3 = \frac{1}{E(M + 2E)} \int \frac{d^3k}{(2\pi)^3} (-i) \tilde{G}(P, q, k) \frac{5 \cos^3 \theta - 3 \cos \theta}{2} f_3. \tag{51}
\]
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TABLE I. The calculated mass spectrum for heavy quarkonia (in GeV).

|              | Calculated | Observed[14]        |
|--------------|------------|---------------------|
| $1^+ S_0(\bar{c}c)$ | 2.960      | 2.978$\pm$ 0.0019   |
| $1^3 S_1(\bar{c}c)$ | 3.100      | 3.0968$\pm$ 0.00004 |
| $2^1 S_0(\bar{c}c)$ | 3.616      |                     |
| $2^3 S_1(\bar{c}c)$ | 3.666      | 3.680$\pm$ 0.00010  |
| $1^+ S_0(\bar{b}b)$ | 9.421      |                     |
| $1^3 S_1(\bar{b}b)$ | 9.463      | 9.4603$\pm$ 0.00021 |
| $2^1 S_0(\bar{b}b)$ | 9.980      |                     |
| $2^3 S_1(\bar{b}b)$ | 9.996      | 10.0233$\pm$ 0.00031|
| $3^1 S_0(\bar{b}b)$ | 10.331     |                     |
| $3^3 S_1(\bar{b}b)$ | 10.340     | 10.355$\pm$ 0.0005  |
| $4^1 S_0(\bar{b}b)$ | 10.601     |                     |
| $4^3 S_1(\bar{b}b)$ | 10.609     | 10.580$\pm$ 0.0035  |

TABLE II. The mass spectrum (in GeV) for $(\bar{b}b)$ with various $m_{\bar{b}}$ (in GeV).

| $m_{\bar{b}}$ | n=1 | n=2 | n=3 | n=4 |
|---------------|-----|-----|-----|-----|
| 3.0           |     |     |     |     |
| n=1           | 7.659 | 8.054 | 8.067 | 8.121 |
| n=2           | 8.218 | 8.444 | 8.453 | 8.424 |
| n=3           | 8.575 | 8.741 | 8.747 | 8.800 |
| 3.5           |     |     |     |     |
| n=1           | 8.145 | 8.538 | 8.551 | 8.604 |
| n=2           | 8.700 | 8.925 | 8.932 | 9.003 |
| n=3           | 9.054 | 9.219 | 9.224 | 9.278 |
| 4.0           |     |     |     |     |
| n=1           | 8.633 | 9.024 | 9.036 | 9.089 |
| n=2           | 9.184 | 9.407 | 9.415 | 9.485 |
| n=3           | 9.535 | 9.699 | 9.704 | 9.758 |
| 4.5           |     |     |     |     |
| n=1           | 9.122 | 9.512 | 9.523 | 9.576 |
| n=2           | 9.670 | 9.892 | 9.899 | 9.969 |
| n=3           | 10.019 | 10.182 | 10.187 | 10.240 |
| 5.0           |     |     |     |     |
| n=1           | 9.613 | 10.001 | 10.012 | 10.064 |
| n=2           | 10.158 | 10.379 | 10.376 | 10.455 |
| n=3           | 10.505 | 10.667 | 10.672 | 10.725 |
| 5.5           |     |     |     |     |
| n=1           | 10.104 | 10.491 | 10.502 | 10.554 |
| n=2           | 10.648 | 10.867 | 10.874 | 10.943 |
| n=3           | 10.992 | 11.154 | 11.158 | 11.211 |
| 6.0           |     |     |     |     |
| n=1           | 10.596 | 10.983 | 10.993 | 11.045 |
| n=2           | 11.138 | 11.357 | 11.363 | 11.432 |
| n=3           | 11.481 | 11.642 | 11.646 | 11.699 |
| 6.5           |     |     |     |     |
| n=1           | 12.573 | 12.956 | 12.965 | 13.015 |
| n=2           | 13.109 | 13.325 | 13.330 | 13.400 |
| n=3           | 13.447 | 13.605 | 13.608 | 13.661 |
| 7.0           |     |     |     |     |
| n=1           | 14.557 | 14.938 | 14.946 | 14.996 |
| n=2           | 15.090 | 15.302 | 15.307 | 15.374 |
| n=3           | 15.424 | 15.580 | 15.583 | 15.634 |
| 8.0           |     |     |     |     |
| n=1           | 24.519 | 24.895 | 24.902 | 24.950 |
| n=2           | 25.045 | 25.251 | 25.255 | 25.320 |
| n=3           | 25.371 | 25.521 | 25.523 | 25.573 |
| 10.0          |     |     |     |     |
| n=1           | 44.497 | 44.871 | 44.878 | 44.922 |
| n=2           | 45.020 | 45.222 | 45.230 | 45.288 |
| n=3           | 45.339 | 45.487 | 45.496 | 45.554 |
| 20.0          |     |     |     |     |
| n=1           | 64.492 | 64.865 | 64.869 | 64.914 |
| n=2           | 65.015 | 65.218 | 65.219 | 65.378 |
| n=3           | 65.336 | 65.483 | 65.484 | 65.524 |
TABLE III. The mass spectrum (in GeV) for $(c\bar{b})$ with various $m_{\tilde{b}}$ (in GeV).

| $m_{\tilde{b}}$ | n=1    | n=2    | n=3    | n=4    |
|-----------------|--------|--------|--------|--------|
| 3.0             | 4.530  | 4.906  | 4.931  | 4.994  |
| 3.5             | 5.029  | 5.402  | 5.425  | 5.488  |
| 4.0             | 5.527  | 5.899  | 5.919  | 5.983  |
| 4.5             | 6.025  | 6.395  | 6.414  | 6.478  |
| 5.0             | 6.523  | 6.892  | 6.910  | 6.974  |
| 5.5             | 7.021  | 7.389  | 7.406  | 7.469  |
| 6.0             | 7.519  | 7.887  | 7.902  | 7.966  |
| 8.0             | 9.513  | 9.878  | 9.891  | 9.955  |
| 10.0            | 11.509 | 11.872 | 11.884 | 11.947 |
| 20.0            | 21.500 | 21.859 | 21.867 | 21.929 |
| 40.0            | 41.494 | 41.851 | 41.857 | 41.919 |
| 60.0            | 61.490 | 61.848 | 61.854 | 61.914 |
### TABLE IV. The mass spectrum (in GeV) for $\tilde{b}\bar{\tilde{b}}$ with various $m_{\tilde{b}}$ (in GeV).

| $m_{\tilde{b}}$ | n=1 | n=2 | n=3 | n=4 |
|-----------------|-----|-----|-----|-----|
| 3.0             | 5.969 | 6.313 | 6.553 | 6.742 |
| 3.5             | 6.930 | 7.280 | 7.520 | 7.709 |
| 4.0             | 7.895 | 8.250 | 8.489 | 8.678 |
| 4.5             | 8.863 | 9.221 | 9.461 | 9.649 |
| 5.0             | 9.833 | 10.195 | 10.434 | 10.621 |
| 5.5             | 10.805 | 11.170 | 11.409 | 11.595 |
| 6.0             | 11.779 | 12.147 | 12.385 | 12.571 |
| 8.0             | 15.689 | 16.069 | 16.305 | 16.496 |
| 10.0            | 19.615 | 20.005 | 20.240 | 20.420 |
| 20.0            | 39.374 | 39.801 | 40.037 | 40.208 |
| 40.0            | 79.146 | 79.602 | 79.842 | 80.010 |
| 60.0            | 119.035 | 119.500 | 119.742 | 119.909 |

### TABLE V. The mass spectrum dependence of $\Lambda_{QCD}$.

| $\Lambda_{QCD}$ | $\langle \bar{b}b \rangle (\frac{1}{2})^+$ (GeV) | $\langle \bar{b}b \rangle (0^{++})$ (GeV) |
|-----------------|---------------------------------|---------------------------------|
| 0.11            | 9.745                           | 9.943                           |
| 0.13            | 9.692                           | 9.898                           |
| 0.15            | 9.641                           | 9.85                            |
| 0.17            | 9.594                           | 9.817                           |
| 0.19            | 9.577                           | 9.780                           |
| 0.21            | 9.504                           | 9.744                           |
| 0.23            | 9.461                           | 9.711                           |
| 0.25            | 9.420                           | 9.679                           |
### TABLE VI. The mass spectrum dependence of $\lambda(\text{GeV}^2)$.

| $\lambda$ | $(\bar{b}b)(\frac{1}{2})^+(\text{GeV})$ | $(\bar{b}b)(0^{++})(\text{GeV})$ |
|-----------|--------------------------------|---------------------------------|
| $\lambda = 0.15$ | 9.541 | 9.764 |
| $\lambda = 0.16$ | 9.550 | 9.773 |
| $\lambda = 0.17$ | 9.559 | 9.782 |
| $\lambda = 0.18$ | 9.568 | 9.790 |
| $\lambda = 0.19$ | 9.578 | 9.799 |
| $\lambda = 0.20$ | 9.586 | 9.807 |
| $\lambda = 0.21$ | 9.595 | 9.816 |
| $\lambda = 0.22$ | 9.604 | 9.824 |
| $\lambda = 0.23$ | 9.612 | 9.832 |
| $\lambda = 0.24$ | 9.621 | 9.840 |
| $\lambda = 0.25$ | 9.630 | 9.848 |