Achieving bounded delay on a time-varying satellite uplink
ACHIEVING BOUNDED DELAY ON A TIME-VARYING SATELLITE UPLINK

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ABSTRACT

In this paper, we investigate the packet transmission delay in a time varying satellite uplink. Specifically, we are interested in bounding the packet transmission delay within the terminal router for uplink traffic in a future satellite network. Previous work has provided delay bounds for wireline networks with a fixed transmission rate. However, these bounds do not apply to the time-varying satellite link that we are considering here. It has been shown that the terminal router in a future SATCOM terminal can accommodate the varying uplink RF data rate by using Ethernet pause frames. We show that by encapsulating the pause frame information into two parameters: a window size and the minimum transmission time within a window, we are able to achieve a guaranteed packet transmission delay. We also quantify the tradeoff between the packet transmission delay and the throughput on the uplink. Experiments with a commercial-off-the-shelf (COTS) Juniper M120 router were conducted to verify the delay bound.

I. INTRODUCTION

The future SATCOM system will provide the space-based backbone network that supports circuit and packet communications services to the war-fighter. Its space segment consists of several interconnected satellites each equipped with an onboard packet router. Each satellite will support one or multiple uplink and downlink RF beams. Thousands of ground-based and air terminals will communicate by accessing the space backbone via the uplinks and downlinks. These terminals must be capable of providing IPv6 networking functionalities as well as providing Quality of Service (QoS), including bounds on delay, loss and jitter that are required by various applications. To reduce the development cost and complexity of a future SATCOM terminal, we are investigating the use of a Commercial Off The Shelf (COTS) router in the SATCOM terminal. However, most COTS routers are designed for wire-line network with no regard to the time-varying nature of the satellite links. Hence, it is critical for us to investigate whether a COTS router can be effectively integrated into the satellite network and provide necessary QoS. Specifically in this paper, we study how to achieve bounded packet transmission delay in a time varying satellite uplink. Here, delay is defined to be the time difference between the instant that a packet leaves the terminal router and the instant that the packet enters the terminal router.

One of the distinguishing features of the future protected SATCOM system is the dynamic assignment of resources to terminals to efficiently share the limited RF resources of time and bandwidth among the thousands of terminals. A centralized Dynamic Bandwidth Resource Allocation algorithm assigns RF resources on an epoch by epoch basis, where an epoch is about 0.5 seconds in duration. DBRA assigns a terminal its uplink transmission mode (burst rate and modulation) and a set of timeslots within an epoch in which to transmit. A terminal transmits only during its assigned timeslots, as other timeslots are assigned to other terminals. The combination of assigned mode and number of time slots yields the terminal’s average uplink data rate. Since the mode or number of time slots may change per epoch, the average uplink data rate may change on a per epoch time scale [1].

Most high data rate commercial routers are designed to operate on constant data rate links. In order for the COTS router to be integrated into the terminal, it must be capable of managing a time-varying link. Typically the egress rate of a router cannot be configured at time slot (millisecond) time scales. One mechanism for preventing packet loss due to the rate mismatch between the terminal modem and router egress data rate is to use a flow control mechanism between the COTS router and terminal modem. Our previous work investigated using Ethernet pause frames as this flow control mechanism [1]. When a pause frame is

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received at a router port, the router “pauses”, or ceases to transmit out of that port for an amount of time specified in the received pause frame. We assume the terminal router and modem are connected via Gigabit Ethernet. When the modem buffer is full, the terminal modem transmits pause frames to the terminal router to stop the transmission of packets from the router. In this way the router egress rate can be made to match the current RF link rate.

In this paper, our objective is to achieve bounded packet transmission delay while the router’s egress rate is controlled by pause frames (the pause frame distribution is further modulated by the time slots assignments of DBRA algorithm). Previous work in [2] had showed that guaranteed worst-case packet delay and throughput for different classes of traffic can be achieved in a wire-line network by using the policing and the weighted fair queueing functions in the COTS router. In our case, we show that by encapsulating the pause frame information into two parameters: a window size and the minimum transmission time within a window, we are able to achieve a guaranteed packet transmission delay together with policing and fair queueing. As we increase the delay bound, the amount of transmitted packets that satisfy the delay bound also increases. Hence, we also quantify the tradeoff between the throughput and packet transmission delay in this paper. The resulting delay bound will give us insight in configuring router’s policing rule and queue size in order to achieve bounded delay.

The rest of the paper is organized as follows: in Section II, we translate an actual pause frame pattern into a simple description of the pause frames from which the delay bound can be easily obtained. Section III explores the tradeoff between the transmission delay and the delay bounded throughput. This further helps us providing differentiated service to different classes of traffic with different delay requirements. In Section IV, experiments with a Juniper M120 router were conducted to verify the delay bound. Section V concludes the paper.

II. PAUSE FRAME CHARACTERIZATION

Adapting to the change in the satellite link capacity, the modem will send pause frames to the terminal router’s egress port. From a router’s perspective, it sees transmission window which can be described by the following process in Figure 1.

The terminal router can transmit packets to the modem only during the interval which f(t) is equal to one. The exact shape of f(t) depends on the DBRA assignment which further depends on the channel link capacity and users’ demand. Suppose a user wants to transmit a file at time 0 and wishes to know the worst delay that a packet can possibly experience. If we do not know anything about the transmission process f(t), we cannot determine the worst delay since there is a possibility that f(t) is equal to 0 for all time t (i.e., the channel is down all the time). If we know f(t) for all t at time 0, the worst case delay can be trivially computed. However, an appropriate question to raise now is whether it is possible to know f(t) for all t at time 0. Given the random nature of the link and traffic demand, it is impossible to know f(t) for all t exactly. But we do need to characterize f(t) to certain extent in order to get a bound on packet delay. Here, we first propose a simple description of f(t) which involves only two parameters: a window size W and a transmission time \( \alpha(W) \) which denotes the total time that the router is allowed to transmit during the window of size W. A more comprehensive description of f(t) can be obtained by giving a list of window sizes \( W_1, W_2, ..., W_n \) and the associated \( \alpha(W_1), \alpha(W_2), ..., \alpha(W_n) \). We will focus on the simple description first. Given a window size W, we define the function \( F_W(t) \) as follows:

\[
F_W(t) = \int_{t}^{t+W} f(s)ds
\]

\( \alpha(W) \) is then defined as follows:

\[
\alpha(W) = \min \{ F_W(t) \}
\]

Note that in [3], the authors define a measure of guaranteed availability as \( F_W(t)/W \). Here, we are more interested in the actual minimum transmission time available during the window W (i.e., \( \alpha(W) \)).
In Fig. 2, \( f_1(t) \) represents an actual transmission process that a router may see, and each rectangle represents a transmission opportunity for the router, which has a constant output rate of \( r \). Given a window size \( W \) and the actual transmission process \( f_1(t) \), we can calculate the transmission time \( \alpha(W) \). We can further construct a periodic transmission process \( f_2(t) \) by placing the transmission opportunity with duration \( \alpha(W) \) at the end of each period of duration \( W \). The periodic transmission process \( f_2(t) \) offers a more structured look at the transmission process \( f_1(t) \). Given the window size \( W \), both \( f_1(t) \) and \( f_2(t) \) have the same \( \alpha(W) \)—the minimum amount of transmission opportunity within any window of size \( W \). Since there is \( \alpha(W) \) transmission opportunity within a window of duration \( W \) in \( f_2(t) \), it is easy to see that the packet transmission delay will be less than \( W \) if the number of packets arrived in a period of duration \( W \) is less than \( r \alpha(W) \). So far, we translated an arbitrary transmission process \( f_1(t) \) into a much simpler transmission process \( f_2(t) \), from which the transmission delay for \( f_1(t) \) can now be easily characterized.

III. DELAY BOUNDED THROUGHPUT

Before translating an arbitrary transmission process \( f_1(t) \) into a simpler transmission process \( f_2(t) \), we need to have a window size \( W \). The window size \( W \) determines the delay of packet transmission. Here, given a transmission process \( f(t) \), we will explore the relationship between the window size and the number of packets that can be transmitted during a window. This further relates to the throughput with bounded delay which we will explain later.

First, given a window size \( W \) and a transmission process \( f(t) \), we let \( S(W) \) denote the minimum amount of packets that can be transmitted on any arbitrary window of size \( W \). Let \( r \) denote the router’s output rate. We then have the following:

\[
S(W) = \alpha(W)r
\]

The following theorems characterize the function \( S(W) \).

**Theorem 1:** \( S(W) \) is a continuous, non-decreasing function of \( W \).

*Proof:* See Appendix.

**Theorem 2:** \( S(W) \) has a constant slope of \( r \) for strictly increasing intervals, where \( r \) is the router’s output rate.

*Proof:* See Appendix.

Based on the previous two theorems, we are able to construct the general shape of \( S(W) \). Figure 3 shows a sample of \( S(W) \) which were drew in the bold line. An arbitrary plot of \( S(W) \) will consist of strictly increasing segments with slope \( r \) and horizontal line segments. Fixing a window size \( W \), and connecting the point \((W_i, S(W_i))\) with the origin, we have the delay bounded throughput as the slope of the line, \( S(W_i)/ W_i \). The delay bounded throughput is the maximum input rate which guarantees that the packet transmission delay is no larger than \( W_i \). In Figure 3, point \( a \) represents the end of the first strictly segment of \( S(W) \). More precisely, we define \( W_a \) as follows:

\[
W_a = \min\{W > 0 | \exists \varepsilon > 0 \text{ s.t. } \forall \delta < \varepsilon \quad S(W + \delta) = S(W) \}
\]

**Corollary 1:** Given \((W_a, S(W_a))\), for any \( W > 0 \) there exists a \( W > \bar{W} \) such that \( S(W)/W \geq S(W_a)/W_a \).
Proof: If we fix the window size to be $W_a$, we know that we can send $S(W_a)$ amount of packets with delay less than $W_a$. Similarly, if we fix the window size to be $2W_a$, we can send $2S(W_a)$ amount of packets with delay less than $2W_a$. This is also true when we set the window size to $3W_a$, $4W_a$, ..., $nW_a$. For any $W > 0$, we can find a $k$ such that $kW_a > W$. Thus, we have

$$S(kW_a)/(kW_a) \geq S(W_a)/W_a.$$  

Corollary 1 shows that there will be points on $S(W)$ lie above or on line $A$ shown in Figure 3. The following corollary states that for $W > W_a$ $S(W)$ will always lie below line $B$ in Figure 3.

**Corollary 2**: Given $(W_a, S(W_a))$, for all $W > W_a$, we have $S(W) \leq r \cdot W - r \cdot W_a + S(W_a)$.

Proof: The line $r \cdot W - r \cdot W_a + S(W_a)$ is generated by assuming the transmission process has only a single off interval of duration $W_a - S(W_a)/r$. It has much more transmission opportunities than the original transmission process. Hence, we have $S(W) \leq r \cdot W - r \cdot W_a + S(W_a)$.

The previous two corollaries essentially stated that $S(W)$ will lie in the region between line $A$ and line $B$ shown in Figure 3 given the point $(W_a, S(W_a))$. This tells us that there exists $W > W_a$ such that $S(W)/W \geq S(W_a)/W_a$.

That is, it is possible for us to obtain a larger delay bounded throughput when we increase the delay allowed for packet transmission. Given different slopes of lines connected by points on $S(W)$ and the origin (i.e., different delay bounded throughput), we can differentiate the quality of service by allowing different classes of packets to experience different delay. This is best illustrated by the following example.

Suppose from an arbitrary transmission process $f(t)$ we obtain $S(W)$ in Figure 4. Also assume that there are three classes of traffic with different priority levels and delay constraints as denoted in Table 2.

| Priority | Maximum Delay (ms) | Input Rate (kbps) |
|----------|--------------------|-------------------|
| High     | 6                  | 333.3             |
| Medium   | 9                  | 111.1             |
| Low      | 13                 | 94.1              |

*Table 1: Supportable delay and corresponding input rates.*

The graph of $S(W)$ can be used to determine the maximum throughput that can be supported for each class of traffic while upholding their maximum delay constraints. Specifically, high-priority traffic has a maximum delay of 6ms. To determine the maximum input rate for high-priority traffic such that the delay does not exceed 6ms, we look at the point 6ms on $S(W)$ and see that it corresponds to 2kb. Therefore, an input rate of 2kb/6ms = 333kbps can be supported while the packet transmission delay is less than 6ms. For medium-priority traffic, the maximum delay is 9ms, which corresponds to 4kb on the graph of $S(W)$. This gives a total delay bounded throughput of 4kb/9ms = 444kbps. However, since high-priority traffic already has an input rate of 333kbps, the input rate for medium-priority traffic is 444kbps - 333kbps = 111kbps. The same process is repeated again to provide low-priority traffic an input rate of 94.1kbps.

In summary, a guaranteed delay can be achieved when the transmission process $f(t)$ is available and from which $S(W)$ can be constructed. However, sometimes $f(t)$ may not be completely known. In our case, $f(t)$ is determined by the queue state at the modem: if the modem queue is full, the router is not allowed to transmit; if the modem queue is empty, the router is allowed to transmit. To predict the exact value of $f(t)$ in the future may be difficult due to the time-varying satellite link; however, it is possible to obtain $a(W)$. To get $a(W)$, we only need to know a lower bound on the total transmission opportunities with in a window of size $W$. For example, a terminal will be assigned in its Service Level Agreement (SLA) a certain Committed Information Rate (CIR), which specifies an assigned rate that a terminal can expect to receive with a very high level or probability of availability. Hence, we know that the total transmission opportunity within an epoch is at least its CIR (Committed Information Rate) in a satellite uplink. A small window size $W$ may increase the difficulty in estimating $a(W)$, but this difficulty can always be alleviated by using a larger window size. The DBRA agent can estimate $a(W)$ by examining the past transmission window process.
In the previous example, we demonstrated how throughput of different traffic class with different delay requirement can be derived from $S(W)$. As we mentioned previously, we have to estimate $a(W)$, which may not be possible for all window sizes. Nevertheless, we can still provide differentiated service for different traffic classes. Only a few points on the curve of $S(W)$ is necessary, provided that the lines constructed by connecting point on $S(W)$ to the origin have different slopes.

**IV. EXPERIMENTAL VALIDATION**

With a better understanding of the delay bounded throughput, we now verify that delay bounded throughput can indeed be experimentally achieved on a COTS router. In this experiment, a constant stream of traffic will be routed through the Juniper M120 router. The router’s egress rate is configured to a fixed value, but this value is further throttled by a stream of pause frames generated by an Ixia traffic analyzer. The pause frame pattern, or the transmission process as we called earlier, is shown as $f(t)$ in Figure 5(a). The On periods, intervals during which the router can transmit packets, have durations of 2.2 msec and 10.2 msec; the Off periods, the intervals during which the router cannot transmit packets, have duration of 20 msec and 10 msec. We consider to window sizes of 22.2 msec and 111 msec. Relating to our earlier discussion on delay bound, the minimum duration of the transmission opportunities, $a(W)$, is 2.2 msec if we choose the window size $W = 22.2$ msec. Likewise, $a(W)$ equals 19 msec if we choose the window size $W = 111$ msec.

With the egress rate fixed at 1Gbps (prior to throttling of rate by pause frames), the maximum delay is measured for different traffic input rates. The results of these experiments for window sizes of 22.2 ms and 111 ms are shown in Tables 2(a) and 2(b), respectively. From the simple transmission process $f(t)$, we see that as long as the input rate is less than $(2.2/22.2)\times 1$Gbps = 99 Mbps, the maximum delay should not exceed $W = 22.2$ ms. In Table 2(a), the input rates considered are all less than 99 Mbps, and the maximum delays are measured to be less than our theoretical bound of 22.2 ms. Similarly, using a window size of 111 ms results in a delay bounded throughput of $(19/111)\times 1$Gbps = 171 Mbps and a maximum delay of $W = 111$ ms as depicted in Figure 1(c). Thus, whenever the input rate is less than 171 Mbps, the maximum delay should not exceed 111 ms. For the first three experiments in Table 2(b), the input rate is less than 171 Mbps, and the delay is less than our theoretical bound of 111 ms. However, no assumptions about the maximum delay can be made when the input rate exceeds 171 Mbps as it does in the fourth experiment. Thus, we see that these experimental results are consistent our delay bound.

| Input Rate (Mbps) | 48.8 | 73.2 | 97.6 |
|-------------------|------|------|------|
| Maximum Delay (ms)| 20   | 20   | 22   |
| **Table 2(a): Experimental results for $W = 22.2$ ms** |

| Input Rate (Mbps) | 122  | 146  | 171  | 195  |
|-------------------|------|------|------|------|
| Maximum Delay (ms)| 36   | 44   | 50   | 837  |
| **Table 2(b): Experimental results for $W = 111$ ms** |

**V. CONCLUSION**

We provided a packet transmission delay bound for the time-varying uplink of the future satellite network. Our proposed delay bound is achieved when certain information about the transmission process $f(t)$, such as the window size and the minimum transmission time within the window, are known to us. Given a sequence of window
sizes $W_1, W_2, \ldots W_n$ and the associated transmission opportunities $a(W_1), a(W_2), \ldots, a(W_n)$ of the transmission process $f(t)$, we can predict the throughput of different traffic class with different delay requirement. Our delay bound is consistent with the maximum delay measured from a Juniper M120 router.

In this paper, we are able to provide guaranteed delay bounds when the window size and the minimum transmission opportunities within a window about the transmission process $f(t)$ is known to the router. It is hoped that in an operational system this information can be obtained from the terminal’s Committed Information Rate specified in its Service Level Agreement. Unfortunately, with a time-varying channel this information may not always be accurate. However, if a probabilistic description of the transmission process is available, we are able to provide a delay bound with a certain probability. In future work, we will provide two probabilistic descriptions of $f(t)$ and present probabilistic delay bounds.

**APPENDIX**

Proof of Theorem 1:

A. $S(w)$ is a continuous function of $w \in [0, \infty)$.

Let $c \in [0, \infty)$. Given any $\varepsilon > 0$, $\exists \delta > 0$ such that $\forall x \in [0, \infty)$, $|x - c| < \delta$ implies that $|S(x) - S(c)| < \varepsilon$. Let $r\delta = \varepsilon$.

$$|S(x) - S(c)| = \left| \min_{t \leq x} \int_{t}^{x} f(y)dy - \min_{t \leq c} \int_{t}^{c} f(y)dy \right|$$

$$= r \left| \min_{t \leq x} \int_{t}^{x} f(y)dy + \int_{t}^{c} f(y)dy - \min_{t \leq c} \int_{t}^{c} f(y)dy \right|$$

$$\leq r \min_{t \leq x} \int_{t}^{x} f(y)dy + \int_{t}^{c} f(y)dy - \min_{t \leq c} \int_{t}^{c} f(y)dy$$

$$= r \int_{t}^{c} f(y)dy$$

$$\leq r \int_{t}^{c} f(y)dy$$

$$= r \int_{t}^{x} f(y)dy - \int_{t}^{x} f(y)dy$$

$$= r \left| t + x - \hat{t} + (t + c) \right|$$

$$= r \left| x - c \right|$$

$$< r\delta = \varepsilon$$

where $t = \arg \min_{t \leq x} \int_{t}^{x} f(y)dy$. Thus, $|x - c| < \delta$ implies that $|S(x) - S(c)| < \varepsilon$. Therefore, $S(w)$ is continuous for all $w \in [0, \infty)$.

B. $S(w)$ is a non-decreasing function of $w \in [0, \infty)$. Let $a \in [0, \infty)$ and $b \in [0, \infty)$. WLOG, assume $b > a$. This implies

$$r \int_{i}^{b} f(y)dy \geq r \int_{i}^{a} f(y)dy$$

since $f(y)$ is a non-negative function of $y$. Therefore,

$$S(b) = \min_{t \leq b} \int_{t}^{b} f(y)dy \geq \min_{t \leq a} \int_{t}^{a} f(y)dy = S(a)$$

which implies that $S(b) \geq S(a)$. Therefore, $S(w)$ is a non-decreasing function of $w \in [0, \infty)$.

C. $S(w)$ has the same constant slope for strictly increasing intervals.

Let $A_i$ and $B_i$ denote the time that the $i^{th}$ strictly increasing interval begins and ends, respectively for $S(w)$. Thus, $(A_i, B_i), 1 \leq i \leq N$ completely characterizes $S(w)$ where $N$ is the number of strictly increasing intervals. We need to prove that all strictly increasing intervals have the same constant slope. Thus, we need to prove that

$$\frac{S(B_i) - S(A_i)}{B_i - A_i} = c, \ 1 \leq i \leq N$$

where $c$ is a constant.

$$S(B_i) - S(A_i) = r \left| \min_{t \leq b} \int_{t}^{b} f(y)dy - \min_{t \leq a} \int_{t}^{a} f(y)dy \right|$$

$$= r \left| \min_{t \leq b} \int_{t}^{b} f(y)dy + \int_{t}^{b} f(y)dy - \min_{t \leq a} \int_{t}^{a} f(y)dy \right|$$

$$\geq r \min_{t \leq b} \int_{t}^{b} f(y)dy + \int_{t}^{b} f(y)dy - \min_{t \leq a} \int_{t}^{a} f(y)dy$$

$$= r \int_{t}^{b} f(y)dy - \int_{t}^{a} f(y)dy$$

where $t = \arg \min_{t \leq b} \int_{t}^{b} f(y)dy$. Also,

$$S(B_i) - S(A_i) = r \left| \min_{t \leq b} \int_{t}^{b} f(y)dy - \min_{t \leq a} \int_{t}^{a} f(y)dy \right|$$

$$= r \left| \min_{t \leq b} \int_{t}^{b} f(y)dy + \int_{t}^{b} f(y)dy - \min_{t \leq a} \int_{t}^{a} f(y)dy \right|$$

$$\leq r \min_{t \leq b} \int_{t}^{b} f(y)dy + \int_{t}^{b} f(y)dy - \min_{t \leq a} \int_{t}^{a} f(y)dy$$

$$= r \int_{t}^{b} f(y)dy - \int_{t}^{a} f(y)dy$$

where $t = \arg \min_{t \leq b} \int_{t}^{b} f(y)dy$. Therefore,

$$r \int_{i}^{b} f(y)dy = \int_{i}^{a} f(y)dy$$

To prove that $S(w)$ has the same constant slope, we need the following Lemma.

**Lemma 1**:

$$\tilde{t} = \arg \min_{t \leq a} \int_{t}^{b} f(y)dy = \min_{t \leq a} \int_{t}^{b} f(y)dy$$
Proof of Lemma 1:

Suppose \( t = t \).

Suppose \( t \neq t \). We know that \( \int_i^t f(y)dy \geq \int_i^t f(y)dy \), and we can find a \( \beta \in [0,B_i] \) such that \( \int_i^\beta f(y)dy = \int_i^\beta f(y)dy \). By the definition of \( t \), we must have \( \beta \geq A_i \). We must consider the following two cases.

**Case 1:** Assume \( \beta = A_i \).

This implies \( \int_i^\beta f(y)dy = \min \int_i^t f(y)dy \), which further implies that \( t \) also achieves the minimum even though \( t \neq t \), because \( t = \arg\min \int_i^t f(y)dy \) has multiple solutions.

**Case 2:** Assume \( \beta > A_i \).

We know that \( \int_i^\beta f(y)dy = \int_i^\beta f(y)dy \), and we can find a \( \delta > 0 \) such that \( \beta - \delta > A_i \) and \( \int_i^{\beta - \delta} f(y)dy \leq \int_i^{\beta - \delta} f(y)dy \). This implies \( \min \int_i^{\beta - \delta} f(y)dy \leq \int_i^{\beta - \delta} f(y)dy \), which implies \( S(\beta - \delta) \leq S(A_i) \text{ and } \beta - \delta > A_i \). This is a contradiction because \((A_i, B_i)\) is a strictly increasing interval of \( S(w) \). Thus, \( \beta = A_i \) and \( t = \arg\min \int_i^t f(y)dy \).

Since \( t = t \), \( \frac{r \int_i^t f(y)dy}{B_i - A_i} = \frac{r \int_i^t f(y)dy}{B_i - A_i} \). We now prove that \( r \int_i^t f(y)dy = c(B_i - A_i) \) where \( c \in \mathbb{R} \). To do this, we need the following Lemma.

**Lemma 2:** \( f(y) \) is exactly 1 in the interval \((t + A_i, t + B_i)\).

Proof of Lemma 2:

We know that the interval \((A_i, B_i)\) is a strictly increasing interval of \( S(w) \), and that \( f(y) \) takes on values, 0 and 1. Assume \( f(y) = 0 \) for \( y \in (t + A_i, t + A_i + \varepsilon) \) where \( \varepsilon > 0 \) and \( A_i + \varepsilon < B_i \). Then \( \frac{S(A_i) - S(A_i + \varepsilon)}{A_i + \varepsilon - A_i} = \frac{r \int_i^{A_i + \varepsilon} f(y)dy}{A_i + \varepsilon - A_i} \), which means that \( S(A_i) - S(A_i + \varepsilon) = 0 \). This is a contradiction since \((A_i, B_i)\) is a strictly increasing interval of \( S(w) \). Therefore \( f(y) = 1 \) for all \( y \in (t + A_i, t + B_i) \).

Therefore, \( S(B_i) - S(A_i) = \frac{r \int_i^t f(y)dy}{B_i - A_i} = \frac{r \int_i^{A_i} f(y)dy}{B_i - A_i} = r(B_i - A_i) = r = c \).

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