Hamiltonian analysis of Mimetic gravity with higher derivatives

Yunlong Zheng\textsuperscript{a,b}

\textsuperscript{a}CAS Key Laboratory for Researches in Galaxies and Cosmology, Department of Astronomy, University of Science and Technology of China, Hefei, Anhui 230026, China
\textsuperscript{b}School of Astronomy and Space Science, University of Science and Technology of China, Hefei, Anhui 230026, China

E-mail: zhyunl@ustc.edu.cn

Abstract. Two kinds of mimetic gravity model with higher derivatives of the mimetic field are analyzed in the Hamiltonian formalism. We first perform the Hamiltonian analysis for the mimetic gravity with a general higher derivative function and show the degrees of freedom (DOFs) is 3 which is consistent with the previous result of the Hamiltonian analysis at the perturbation level. We then perform the Hamiltonian analysis for the extended mimetic gravity with higher derivatives directly couples to the Ricci scalar in both Einstein frame and Jordan frame, and the conclusions in both frames are consistent with each other. Different from our previous research at the cosmological perturbation level where only 3 propagating DOFs show up, non-Perturbative Hamiltonian analysis shows that this generalized mimetic model has actually 4 DOFs. To clarify this issue, we find out that the DOFs is reduced to 3 when we reanalyze the model after setting the unitary gauge. Thus, we conclude the number of the propagating DOFs in the extended model is actually 3 while the extra mode does not propagate and can be eliminated by appropriate boundary conditions (specifically in the unitary gauge). What makes the system so special is the gauge dependence of the rank of the Dirac matrix, thus we give a similar but simpler example to illustrate when the rank of the Dirac matrix is gauge dependent and how gauge choice affects the number of secondary constraint and the DOFs according to Dirac.
1 Introduction

Standard cosmology based on dark energy and dark matter is very successful so far. Despite its observational success, the origins of dark matter and dark energy are still puzzles in modern cosmology and particle physics, and a number of scenarios including modifying gravity have developed.

Recently a novel interesting model dubbed mimetic dark matter has been proposed [1] as a modification of general relativity, where the physics metric is related to a scalar field and an auxiliary metric via

$$g_{\mu\nu} = (\tilde{g}_{\alpha\beta}\phi^\alpha\phi^\beta)^{\frac{1}{2}} \tilde{g}_{\mu\nu} ,$$

(1.1)

where $\phi_\alpha \equiv \nabla_\alpha \phi$ denotes the covariant derivative of the scalar field with respect to spacetime. This transformation separates the conformal mode of gravity in the covariant manner. The resulting gravitational equations by varying the usual Einstein-Hilbert action plus matter sector, which are constructed from the physical metric, contains the usual Einstein equation plus the extra contribution of the mimetic field which can mimics the cold dark matter. One can see the kinetic term of scalar field is subject to the constraint

$$g^{\mu\nu} \phi_\mu \phi_\nu = 1 .$$

(1.2)

Actually the number of degrees of freedom remains unchanged under a general invertible disformal transformation [2], one may wonder how does the new DOF arise in mimetic scenario. It has been shown that mimetic scenario can be viewed as a singular limit of general disformal transformation and therefore a new DOF $\phi$ arises in this setup [3–5].

One can view the mimetic constraint as a constraint by employing a Lagrange multiplier, that is, the action of mimetic gravity can be written as [6]

$$S = \int d^4x \left[ \frac{1}{2} R + \lambda (g^{\mu\nu} \phi_\mu \phi_\nu - 1) \right] + S_m ,$$

(1.3)
where \( S_m \) is the action for other matter in the universe and we use the most negative signature for the metric. One can see from the equation of motion that these two formalisms are equivalent, at least classically. We shall take the Lagrange multiplier formalism in this paper, as has been done in most paper of extensions of original mimetic gravity.

The model above was generalized in [7] by introducing an arbitrary potential. This generalized mimetic model has many applications in cosmology, and can provide us inflation, dark energy, bounce and so on with appropriate choice of the potential \( V(\phi) \). The mimetic constraint can also be implemented in various modified gravity models [8–19]. For astrophysical and cosmological aspects see Refs. [20–33], for recent developments in mimetic gravity see Refs. [34–40], for the Hamiltonian analysis of mimetic gravity see Refs. [41–43], and for a review see Ref. [44].

Even being offered a potential, there is no nontrival dynamics for scalar perturbation, i.e. the propagation velocity is zero \( c_s = 0 \). This may rise to caustic singularities. Besides, the notion of quantum fluctuations is lost as there is no propagating degree of freedom for the scalar perturbation. Hence, when applied to the early universe, such model fails to produce the primordial perturbations which seeds the formation of large scale structure. To remedy these issues, higher derivative terms \((\Box \phi)^2\) are introduced in [7] to promote the scalar degree of freedom to be dynamical with a non-zero sound speed. Although the equation for the scalar perturbation has the wave-like form by choosing appropriate coefficient, the analysis in the action formalism shows that the mimetic scenario with higher derivatives always suffer from ghost instability or gradient instability [45]. Actually, the mimetic model with higher derivative terms can be produced as a certain limit of the projective version of the Horava-Lifshitz gravity and such instability has already been pointed out [46]. It has been shown that simply generalizing the quadratic higher derivative terms to arbitrary function \( f(\Box \phi) \) [47] or introducing the non-minimal coupling of mimetic field to the Ricci scalar \( f(\phi) R \) [48] can not cure this pathology. To find a way out of the ghost and gradient instabilities, in [48] we show that it is possible to circumvent both the ghost and gradient instabilities by introducing the direct couplings of the higher derivatives of the mimetic field to the curvature. Similar couplings are also proposed in [49, 50]. The extended action in our previous work [48] has the form

\[
S = \int d^4x \sqrt{-g} \left[ \frac{f(\phi, \Box \phi)}{2} R + \lambda (g^{\mu\nu} \phi_\mu \phi_\nu - 1) - V(\phi) + \alpha (\Box \phi)^2 + \beta \phi^{\mu\nu} \phi_{\mu\nu} \right].
\]  

From the reduced quadratic action of the perturbations, one scalar and two tensor modes are obtained, and we showed it is indeed possible to avoid all the instabilities. It seems that we have achieved the goal to construct a healthy model without any instabilities. However, since the action (1.4) contains the direct coupling between the higher derivative terms of mimetic field and the curvature, one might be concerned whether the model has 3 DOFs exactly. Besides, the modified dispersion relation [51] (involving \( k^4 \) term) of scalar perturbation may imply the existence of extra DOF which do not show up at the perturbation level with cosmological background.

The aim of this paper is to identify the number of DOFs for an extended mimetic model (3.1) which is slightly different from (1.4). As we shall see, generally such kind of theories has 4 DOFs, of which 3 are propagating and one is non-propagating and will be eliminated in the unitary gauge.

The paper is organized as follows. In the next section, we perform the full Hamiltonian analysis for the mimetic model with a general higher derivative function and show the DOFs...
is 3 which is consistent with the previous result of the Hamiltonian analysis at perturbation level in [47]. In section 3, the full Hamiltonian analysis for the extended mimetic gravity with higher derivatives directly couples to the Ricci scalar is performed in both Einstein frame and Jordan frame, and we find 4 DOFs generally. To clarify the confusion why only 3 DOFs show up at the cosmological perturbation level, we also perform the Hamiltonian analysis in the unitary gauge where only 3 DOFs appear. Finally, we give a simple example where the rank of the Dirac matrix is gauge dependence in section 4 followed by conclusion and discussions in section 5. A special case of mimetic gravity with higher derivative terms is discussed in the Appendix.

2 Mimetic gravity with higher derivative terms

We start from the following action of mimetic theory

\[ S_1 = \int d^4x \sqrt{-g} \left[ \frac{f(\phi)}{2} R + \lambda (g^{\mu\nu} \phi_\mu \phi_\nu - 1) - V(\phi) + g(\Box \phi) \right], \tag{2.1} \]

where \( R \) is the Ricci scalar, \( \lambda \) is the Lagrange multiplier enforcing the mimetic constraint (1.2), \( g(\Box \phi) \) is the general higher derivative function and we have considered the non-minimal coupling to the curvature and \( f(\phi) \) only depends on the mimetic field. This model can be viewed as a generalization of the model in [47], and is slightly different from the model considered in [48] which includes terms \( \phi^{\mu\nu} \phi_{\mu\nu} \). Recently, the detection of the gravitational wave event GW170817 [52] has provided strict constraints on the sound speed of gravitational waves \( c_s \), which has to be equal to the light speed \( c=1 \), up to very high accuracy \( |c_s^2/c^2 - 1| \leq 5 \times 10^{-16} \). As one can see from the quadratic action of perturbation in [48], the inclusion of terms \( \phi^{\mu\nu} \phi_{\mu\nu} \) will change the sound speed of gravitational waves and leads to the deviation from the light speed, thus the \( \phi^{\mu\nu} \phi_{\mu\nu} \) terms will not be considered in this paper. The main goal of this section is to identify the number of DOFs of the theory (2.1). Introducing a new variable \( \varphi = \Box \phi \), one can rewrite the action as

\[ S_1 = \int d^4x \sqrt{-g} \left[ \frac{f(\phi)}{2} R + \lambda (g^{\mu\nu} \phi_\mu \phi_\nu - 1) - V(\phi) + g(\varphi) + \Lambda (\varphi - \Box \varphi) \right], \tag{2.2} \]

where the Lagrange multiplier \( \Lambda \) in the last term fixes \( \varphi \). To get rid of the appearance of higher derivatives of the mimetic field in the action, we simplify the action and drop the boundary term

\[ S_1 = \int d^4x \sqrt{-g} \left[ \frac{f(\phi)}{2} R + \lambda (g^{\mu\nu} \phi_\mu \phi_\nu - 1) + g^{\mu\nu} \phi_\nu \Lambda_\mu - V(\phi) + g(\varphi) + \Lambda \varphi \right]. \tag{2.3} \]

One can switch the action of Jordan frame to the Einstein frame by weyl scaling \( g_{\mu\nu} = \Omega^2 g_{\mu\nu} \) where \( \Omega^2 = f(\phi)^{-1} \). The final action in the Einstein frame is

\[ S_E = \int d^4x \sqrt{-g} \left[ \frac{R}{2} + \frac{3f^2}{4f^2} \bar{g}^{\mu\nu} \phi_\mu \phi_\nu + \lambda \left( \bar{g}^{\mu\nu} \phi_\mu \phi_\nu - \frac{1}{f(\phi)} \right) \right. \]

\[ + \left. \frac{1}{f(\phi)} \bar{g}^{\mu\nu} \phi_\nu \Lambda_\mu + \frac{1}{f(\phi)^2} (g(\varphi) + \Lambda \varphi - V(\phi)) \right]. \tag{2.4} \]

To identify the number of DOFs in this model, We shall perform the full Hamiltonian analysis. Although the Hamiltonian analysis of this model in the case of \( f(\phi) = 1 \) has been studied at the perturbation level [47], there may be extra DOF not showing up at the perturbation level with cosmological FRW background and thereby the analysis of the general non-perturbation theory is necessary.
2.1 Hamiltonian analysis: Einstein frame

We use bar to distinguish the variables in the Einstein frame from the ones in the Jordan frame. However, all the bars over the variables have been omitted in this subsection for briefness. Under ADM decomposition the action becomes

\[
S_E = \int d^4x \sqrt{h} \left[ \frac{1}{2} (-\mathcal{R} + K_{ij}K^{ij} - K^2) + \lambda \left( \frac{\dot{\phi} - N^i\phi_i}{N^2} - h^{ij}\phi_i\phi_j - \frac{1}{f} \right) + \frac{3f_0^2}{4f^2} \left( \frac{\dot{\phi} - N^i\phi_i}{N^2} - h^{ij}\phi_i\phi_j \right) + \frac{1}{f(\phi)} \left( \frac{\dot{\Lambda} - N^i\Lambda_i(\dot{\phi} - N^j\phi_j)}{N^2} - h^{ij}\Lambda_i\phi_j \right) + \frac{1}{f(\phi)} \left( g(\phi) + \Lambda\phi - V(\phi) \right) \right],
\]

(2.5)

where \( \mathcal{R} \) denotes the 3-dimensional Ricci scalar and \( K_{ij} = (\dot{h}_{ij} - N_{ij} - N_{ij})/2N \) is the extrinsic curvature. In the following, we perform a Hamiltonian analysis of the theory (2.1). There are 14 coordinate variables \( Q_a = \{N, N^i, h_{ij}, \phi_i, \lambda, \varphi, \Lambda \} \) and for each coordinate variable \( Q_a \), define the conjugate momentum as \( \pi_a = \frac{\partial L}{\partial \dot{Q}_a} \). As the coordinates \( N, N^i, \varphi \) and \( \lambda \) have no time derivative in the action, this leads to six primary constraints

\[
\pi_N = 0, \quad \pi_i = 0, \quad \Phi_1 \equiv \pi_\lambda = 0, \quad \Phi_5 \equiv \pi_\varphi = 0.
\]

(2.6)

Other conjugate momentums are

\[
\pi^{ij} = \sqrt{h} \left( K^{ij} - h^{ij}K \right), \quad \pi_\lambda = \frac{\sqrt{h} \phi - \phi_i N^i}{f},
\]

\[
\pi_\varphi = \sqrt{h} \left[ \frac{3f_0^2}{2f^2} + 2\lambda \right] \frac{\dot{\phi} - \phi_i N^i}{N} + \frac{1}{f} \left( \dot{\Lambda} - \Lambda_i N^i \right).
\]

(2.7)

Following the standard route, we obtain the total Hamiltonian

\[
H_T = \int d^3x (N\mathcal{H} + N^i\mathcal{H}_i + N^i\pi_N + v^i\pi_i + v^\varphi\pi_\varphi + v^\lambda\pi_\lambda),
\]

(2.8)

where

\[
\mathcal{H} = \mathcal{H}_0 + \mathcal{H}_m = \sqrt{h} \left( \frac{\mathcal{R}}{2} + h^{-1} (2\pi_{ij}\pi^{ij} - \pi^2) \right) + \sqrt{h} \left[ \frac{f\pi_\varphi\pi_\lambda}{h} - (\lambda + \frac{3f_0^2}{4f^2}) \right] \frac{\pi_\lambda^2}{h} \right] + \frac{3f_0^2}{4f^2} h^{ij}\phi_i\phi_j + \lambda(h^{ij}\phi_i\phi_j + \frac{1}{f}) + \frac{1}{f} h^{ij}\Lambda_i\phi_j + \frac{1}{f^2} (V(\phi) - g(\phi) - \Lambda\phi) \right],
\]

(2.9)

and

\[
\mathcal{H}_i = \mathcal{H}_{gi} + \mathcal{H}_{mi} = -2\sqrt{h} \left( \frac{\pi_{ij}^j}{\sqrt{h}} \right)_{ij} + \pi_\phi\phi_i + \pi_\lambda\Lambda_i + \pi_\varphi\varphi_i + \pi_\lambda\lambda_i.
\]

(2.10)

Imposing the conservation of the primary constraints, enables us to determine six corresponding secondary constraints \([53, 54]\). Using Eq. (2.8) together with the primary constraints in (2.6), we find

\[
\mathcal{H} \approx 0, \quad \mathcal{H}_i \approx 0, \quad \Phi_2 \equiv -\frac{f^2\pi_\lambda}{h} + h^{ij}\phi_i\phi_j + \frac{1}{f(\phi)} \approx 0, \quad \Phi_6 \equiv g'(\varphi) + \Lambda \approx 0.
\]

(2.11)
where the weak equality sign "≈" denotes an identity up to terms that vanish on the constraint surface. By employing the constraint equation $\Phi_6$, one can express $\varphi$ in term of $\Lambda$. The conservation of constraint $\Phi_6$ determines the Lagrange multiplier $\nu^\varphi$ and so the chain of constraints for primary constraint $\Phi_5$ determinates here.

Writing the constraints in smeared form we have

$$H[N] = \int d^3x N(x) \mathcal{H}(x),$$
$$D[N^i] = \int d^3x N^i(x) \mathcal{H_i}(x).$$

(2.12)

To recognise that $\mathcal{H}_i$ is indeed the diffeomorphism constraint, we can verify the following Poisson bracket

$$\{A, D[N^i]\} = N^i A_i = \mathcal{L}_A A_i, \quad \{A_i, D[N^i]\} = A_i N^j + A_j N^i_{,i} = \mathcal{L}_N A_i,$$

$$\{\Pi, D[N^i]\} = (N \Pi)_i = \mathcal{L}_N(\sqrt{\hbar} \frac{\Pi}{\sqrt{\hbar}}), \quad \{h_{ij}, D[N^i]\} = N_{ij} + N_{ji} = \mathcal{L}_N h_{ij},$$

$$\{\pi^{ij}, D[N^i]\} = N_{ik} \pi^{jk} + N_{jk} \pi^{ik} - \sqrt{\hbar}(\frac{N_k \pi^{ij}}{\sqrt{\hbar}})_{,k} = \mathcal{L}_N(\sqrt{\hbar} \pi^{ij}).$$

(2.13)

where $A$ is a scalar quantity such as $\phi, h^{ij} \phi_i \Lambda_j$ and so on, $A_i$ is a covariant vector quantity such as $\phi_i$, and $\Pi$ can be the conjugate momentum quantities such as $\pi_\phi$ or scalar densities with weight 1 like $\sqrt{\hbar} A$. We assume that $A, A_i, \pi_a$ in the above equations only depend on $\phi, \theta, \Lambda, \lambda, h_{ij}$ and their conjugate momentums (without $N, N^i$ dependence). Therefore the Poisson bracket of any constraints $\Phi$ (without $N, N^i$ dependence) with $D[N^i]$ vanish after imposing the constraint equation, i.e. $D[N^i]$ or $\mathcal{H}_i$ is first class. This property greatly simplify the subsequent process of calculating the secondary constraints.

The following functional derivatives of $H[N]$ will be useful to derive the time evolution of variables including constraints

$$\frac{\delta H[N]}{\delta \pi_\phi} = N f \frac{\pi_\Lambda}{\sqrt{\hbar}}, \quad \frac{\delta H[N]}{\delta \pi_\Lambda} = N f \frac{\pi_\phi}{\sqrt{\hbar}} - 2f^2 \frac{\pi_\Lambda}{\sqrt{\hbar}} (\lambda + \frac{3f_\phi^2}{4f^2})$$

$$\frac{\delta H[N]}{\delta \pi^{ij}} = 2N \frac{2 \pi_{ij} - h_{ij}}{\sqrt{\hbar}}, \quad \frac{\delta H[N]}{\delta \Lambda} = -\sqrt{\hbar} \left[ \left( \frac{N \phi_{,i}}{f} \right)_{,i} + \frac{N \phi}{f^2} \right].$$

(2.14)

The time evolution of constraint $\Phi_2$ is given by

$$\dot{\Phi_2} \approx \{\Phi_2, H_T\} \approx 2N \Phi_3 \approx 0,$$

(2.15)

where the new constraint

$$\Phi_3 = \frac{\pi_\Lambda}{\sqrt{\hbar}} \left[ f_\phi (h^{ij} \phi_i \phi_j - \frac{3}{2f}) - f \phi_{,i} - \varphi(\Lambda) \right] + fh^{ij} \phi_i \left( \frac{\pi_\Lambda}{\sqrt{\hbar}} \right)_{,j} - 2\frac{\pi_\Lambda}{\sqrt{\hbar}} (\pi^{ij} \phi_i \phi_j + \frac{\pi}{2f}) \approx 0$$

(2.16)

can be derived by using Eq.(2.14). The next consistency condition generates another new constraint

$$\Phi_4 \approx \frac{1}{N} \{\Phi_3, H_T\} = \lambda \left[ \frac{1}{f} (4h^{ij} \phi_i \phi_j + \frac{3}{f}) + 2 \frac{\partial \varphi}{\partial \Lambda} (h^{ij} \phi_i \phi_j + \frac{1}{f}) \right] + J_0 (\phi, \pi_\phi, \Lambda, \pi_\Lambda, h_{ij}, \pi^{ij}, D_i).$$

(2.17)
By requiring the constraint $\Phi_4$ to be time independent, the Lagrange multiplier $v^\lambda$ is determined in terms of other variables and so the chain of constraints for the primary constraint $\Phi_1$ determinates here.

Note that $\mathcal{H} \approx 0$ and $\mathcal{H}_i \approx 0$ are expected to correspond to the Hamiltonian and momentum constraints respectively. With some manipulation the following Poisson brackets are found to be the usual ones

$$\{D[\vec{M}], D[\vec{N}]\} = D[L_{\vec{M}} \vec{N}] ,$$

$$\{D[\vec{M}], H[N]\} = H[L_{\vec{M}} \vec{N}] .$$

(2.18)

We emphasize here that $\mathcal{H}$ is not first-class, but one can construct a new Hamiltonian constraint $\tilde{\mathcal{H}}$ [55] as a linear combination of $\mathcal{H}$, $\pi_\lambda$ and $\pi_\phi$ such that (up to boundary term)

$$\int d^3x [N \mathcal{H} + v^\phi \pi_\phi + v^\lambda \pi_\lambda] = \int d^3x N \tilde{\mathcal{H}}$$

(2.19)

where the Lagrange multipliers $v^\phi$ and $v^\lambda$ are solved in terms of other variables by requiring all the above consistency conditions. One can easily see that $v^\phi$ and $v^\lambda$ are linearly dependent on lapse function $N$ or its derivative, therefore $N$ is not involved in $\tilde{\mathcal{H}}$. The new Hamiltonian constraint $\tilde{\mathcal{H}}$ is obviously first-class. The time evolution of $\tilde{\mathcal{H}}$ and $\mathcal{H}_i$ do not yield any new constraints and the chain of constraints for primary constraints $\pi_N$ and $\pi_i$ determinates here.

To sum up, The above considerations show that the system admits 14 constraints as follows:

8 first − class : $\pi_N , \pi_i , \tilde{\mathcal{H}} , \mathcal{H}_i ,$ 
6 second − class : $\Phi_1 , \Phi_2 , \Phi_3 , \Phi_4 , \Phi_5 , \Phi_6 .$$

(2.20)

These constraints reduce the dimension of phase space and the physical DOFs of the model (2.1) are

$$\frac{1}{2}(28 - 2 \times 8 - 6) = 3 .$$

(2.21)

which is consistent with the Hamiltonian analysis in [47] and [43].

Besides, there exists a very special case in the general theory (2.1). This special case can be found by requiring that in (2.17) $\Phi_4$ doesn’t contain $\lambda$ in the unitary gauge, i.e.

$$\frac{3}{2f} \partial_\phi + \frac{\partial_\phi}{\partial \lambda} = 0 ,$$

(2.22)

which gives $f(\phi) = 1$ and $g(\Box \phi) = \frac{1}{2}(\Box \phi)^2$ by taking account of the constraint equation $\Phi_6$. As the independence of $\Phi_4$ on $\lambda$ in the unitary gauge will lead to more secondary constraints than in the general gauge, thus less DOFs show up in the unitary gauge than in this special case. More discussion about this special case $f(\phi) = 1$ and $g(\Box \phi) = \frac{1}{2}(\Box \phi)^2$ can be found in the Appendix.

3 Mimetic gravity with higher derivative terms couples to the curvature

In this section, we shall consider the following extended action of mimetic theory with higher derivative terms directly couples to the curvature

$$S_2 = \int d^4x \sqrt{-g} \left[ \frac{f(\phi, \Box \phi)}{2} R + \lambda (g_{\mu \nu} \phi_\mu \phi_\nu - 1) - V(\phi) + g(\Box \phi) \right] ,$$

(3.1)
which is slightly different from the model (1.4) considered in [48]. The aim of this section is to identify the number of DOFs of the theory (3.1). Similar to the previous section, one can introduce another Lagrange multiplier $\Lambda$ which impose the constraint equation $\varphi = \Box \phi$ and rewrite the action as

$$S_2 = \int d^4x \sqrt{-g} \left[ \frac{f(\phi, \varphi)}{2} R + \lambda (g^{\mu\nu} \phi_\mu \phi_\nu - 1) - V(\phi) + g(\varphi) + \Lambda (\varphi - \Box \phi) \right]. \quad (3.2)$$

To avoid the higher derivatives in the action, we simplify the action and drop the boundary term

$$S_2 = \int d^4x \sqrt{-g} \left[ \frac{f(\phi, \varphi)}{2} R + \lambda (g^{\mu\nu} \phi_\mu \phi_\nu - 1) + g^{\mu\nu} \phi_\mu \Lambda_\nu - V(\phi) + g(\varphi) + \Lambda \varphi \right]. \quad (3.3)$$

To simplify the calculation we define $\chi = f(\phi, \varphi)$ and the inverse function $\varphi = F(\phi, \chi)$, then we have

$$S_J = \int d^4x \sqrt{-g} \left[ \frac{\chi}{2} R + \lambda (g^{\mu\nu} \phi_\mu \phi_\nu - 1) + g^{\mu\nu} \phi_\mu \Lambda_\nu - V(\phi) + g(F(\phi, \chi)) + \Lambda F(\phi, \chi) \right]. \quad (3.4)$$

One can switch the action of Jordan frame to the Einstein frame by weyl scaling $g_{\mu\nu} = \Omega^2 \bar{g}_{\mu\nu}$ where $\Omega^2 = \chi^{-1} = \exp (\frac{\chi}{\sqrt{6}} \theta)$. The final action in the Einstein frame is

$$S_E = \int d^4x \sqrt{-g} \left[ \frac{\bar{R}}{2} + \frac{1}{2} g^{\mu\nu} \theta_\mu \theta_\nu + \bar{\lambda} (\bar{g}^{\mu\nu} \phi_\mu \phi_\nu - e^{\frac{2\theta}{\sqrt{6}}} ) + e^{\frac{2\theta}{\sqrt{6}}} g^{\mu\nu} \phi_\mu \Lambda_\nu 
+ e^{\frac{4\theta}{\sqrt{6}}} (g(F(\phi, \theta)) + \Lambda F(\phi, \theta) - V(\phi)) \right]. \quad (3.5)$$

We will first perform the full Hamiltonian analysis in the Einstein frame which is simpler and then do the similar analysis in the Jordan frame. We shall see the results in both frames are consistent.

### 3.1 Hamiltonian analysis: Einstein frame

To make the notation concise, we will drop all the bars of variables in the action (3.5)

$$S_E = \int d^4x \sqrt{-g} \left[ \frac{\bar{R}}{2} + \lambda (g^{\mu\nu} \phi_\mu \phi_\nu - e^{\frac{2\theta}{\sqrt{6}}} ) + \frac{1}{2} g^{\mu\nu} \theta_\mu \theta_\nu + e^{\frac{2\theta}{\sqrt{6}}} g^{\mu\nu} \phi_\mu \Lambda_\nu 
+ e^{\frac{4\theta}{\sqrt{6}}} (g(F) + \Lambda F - V(\phi)) \right]. \quad (3.6)$$

After ADM decomposition, the action becomes

$$S_E = \int d^4x N \sqrt{\bar{h}} \left[ \frac{1}{2} \left( -R + K^{ij} K_{ij} - K^2 \right) + \lambda \left( \frac{(\dot{\phi} - N^i \dot{\phi}_i)^2}{N^2} - h^{ij} \dot{\phi}_i \dot{\phi}_j - e^{\frac{2\theta}{\sqrt{6}}} \right) 
+ \frac{1}{2} \left( \frac{(\dot{\theta} - N^i \dot{\theta}_i)^2}{N^2} - h^{ij} \dot{\theta}_i \dot{\theta}_j \right) + e^{\frac{2\theta}{\sqrt{6}}} \left( \frac{(\dot{\lambda} - N^i \dot{\lambda}_i)(\dot{\phi} - N^i \dot{\phi}_i)}{N^2} - h^{ij} \dot{\lambda}_i \dot{\phi}_j \right) 
+ e^{\frac{4\theta}{\sqrt{6}}} (g(F) + \Lambda F - V(\phi)) \right]. \quad (3.7)$$
The coordinate \( N, N^i \) and \( \lambda \) have no time derivatives in the action, which means we have five primary constraints

\[
\pi_N = 0, \quad \pi_i = 0, \quad \Phi_1 \equiv \pi_\lambda = 0.
\] (3.8)

Other non-vanishing conjugate momentums are defined as

\[
\pi^{ij} \equiv \frac{\partial \mathcal{L}}{\partial \dot{h}_{ij}} = \frac{\sqrt{h}}{2}(K^{ij} - h^{ij} K), \quad \pi_\theta \equiv \frac{\sqrt{h}}{N}(\dot{\theta} - N^i \dot{\theta}_i), \quad \pi_\phi \equiv \frac{\sqrt{h}}{N}[2\lambda (\dot{\phi} - N^i \dot{\phi}_i) + e^{\frac{2\theta}{\Lambda}}(\dot{\Lambda} - N^i \dot{\Lambda}_i)], \quad \pi_\Lambda \equiv \frac{\sqrt{h}}{N} e^{\frac{2\theta}{\Lambda}}(\dot{\phi} - N^i \dot{\phi}_i).
\] (3.9)

After some calculations we obtain the total Hamiltonian

\[
H_T = \int d^3x [N \mathcal{H} + N^i h_i + v^N \pi_N + v^i \pi_i + v^\phi \pi_\phi + v^\lambda \pi_\lambda],
\] (3.10)

where

\[
\mathcal{H} = \mathcal{H}_g + \mathcal{H}_m = \sqrt{h} \left( \frac{\mathcal{R}}{2} + h^{-1}(2\pi^{ij} \pi^{ij} - \pi^2) \right) + \sqrt{h} \left[ \left( \frac{\pi^2}{2h} + \frac{1}{2} h^{ij} \dot{\theta}_i \dot{\theta}_j \right) + h^{-1}(\pi_\phi \pi_\lambda e^{\frac{2\theta}{\Lambda}} - \lambda \pi_\phi^2 e^{-\frac{4\theta}{\Lambda}}) \right.
\]

\[
+ \lambda (h^{ij} \dot{\phi}_i \dot{\phi}_j + e^{\frac{2\theta}{\Lambda}}) + h^{ij} \Lambda_i \dot{\Lambda}_j e^{\frac{2\theta}{\Lambda}} - e^{\frac{4\theta}{\Lambda}}(g(F) + \Lambda F - V(\phi)) \bigg],
\] (3.11)

and

\[
\mathcal{H}_i = \mathcal{H}_{gi} + \mathcal{H}_{mi} = -2\sqrt{h} \left( \frac{\pi^i}{\sqrt{h}} \right)_{ij} + \pi_\theta \dot{\theta}_i + \pi_\phi \dot{\phi}_i + \pi_\Lambda \dot{\Lambda}_i.
\] (3.12)

The time evolution of primary constraints generate the corresponding secondary constraints, which are the Hamiltonian constraint

\[
\mathcal{H} = \mathcal{H}_g + \mathcal{H}_m \approx 0, \quad (3.13)
\]

the diffeomorphism constraint

\[
\mathcal{H}_i = \mathcal{H}_{gi} + \mathcal{H}_{mi} \approx 0, \quad (3.14)
\]

and mimetic constraint

\[
\Phi_2 \equiv -\frac{\pi_\phi}{h} e^{-\frac{4\theta}{\Lambda}} + h^{ij} \dot{\phi}_i \dot{\phi}_j + e^{\frac{2\theta}{\Lambda}} \approx 0. \quad (3.15)
\]

Imposing the consistency condition of mimetic constraint yields

\[
\{\Phi_2, H_T\} \approx \int d^3x \left( \frac{\delta \Phi_2}{\delta h_{ij}} \frac{\delta H_g[N]}{\delta \pi^{ij}} + \frac{\delta \Phi_2}{\delta \theta} \frac{\delta H_m[N]}{\delta \pi_\theta} + \frac{\delta \Phi_2}{\delta \phi} \frac{\delta H_m[N]}{\delta \pi_\phi} - \frac{\delta \Phi_2}{\delta \Lambda} \frac{\delta H_m[N]}{\delta \pi_\Lambda} \right) \approx 0, \quad (3.16)
\]

here we have used the property of diffeomorphism constraint \( \{\Phi_2, D[N^i]\} = N^i \Phi_{2,i} \approx 0 \). To obtain \( \Phi_3 \), let us first compute the functional derivatives. The useful functional derivatives of \( \Phi_2 \) are given by

\[
\frac{\delta \Phi_2[g]}{\delta h_{ij}} = g\sqrt{h}(\frac{2\pi_\phi^2}{h} e^{-\frac{4\theta}{\Lambda}} h^{ij} - \phi^i \phi^j), \quad \frac{\delta \Phi_2[g]}{\delta \theta} = -\sqrt{h}(2g\phi^i)_{ij}, \quad \frac{\delta \Phi_2[g]}{\delta \phi} = \frac{2g\pi_\phi e^{-\frac{4\theta}{\Lambda}}}{\sqrt{h}}, \quad \frac{\delta \Phi_2[g]}{\delta \pi_\phi} = -\frac{2g\pi_\phi e^{-\frac{4\theta}{\Lambda}}}{\sqrt{h}}, \quad (3.17)
\]
and other useful functional derivatives of Hamiltonian are
\[
\frac{\delta H_o[N]}{\delta \pi_{ij}} = \frac{4N}{\sqrt{\hbar}} (\pi_{ij} - \frac{\pi}{2} h_{ij}) , \quad \frac{\delta H_m[N]}{\delta \pi_\theta} = \frac{N \pi_\theta}{\sqrt{\hbar}} , \quad \frac{\delta H_m[N]}{\delta \pi_\Lambda} = \frac{N \pi_\Lambda}{\sqrt{\hbar}} e^{-\frac{2\theta}{\sqrt{\hbar}}} , \quad \frac{\delta H_m[N]}{\delta \pi_\Lambda} = \frac{N}{\sqrt{\hbar}} \left( \pi_\Lambda e^{-\frac{2\theta}{\sqrt{\hbar}}} - 2\lambda \pi_\Lambda e^{-\frac{2\theta}{\sqrt{\hbar}}} \right) ,
\]
\[
\frac{\delta H_m[N]}{\delta \lambda} = N \sqrt{\hbar} \Phi_2 , \quad \frac{\delta H_m[N]}{\delta \lambda} = -N \sqrt{\hbar} (N \phi^i e^{\frac{2\theta}{\sqrt{\hbar}}} + N F e^{\frac{2\theta}{\sqrt{\hbar}}}) .
\] (3.18)

Plugging the above formulae in the integral we have
\[
\{ \Phi_2, H_T \} = 2N \Phi_3 \approx 0 , \quad \text{(3.19)}
\]
where the new constraint
\[
\Phi_3(\phi, \theta, \pi_\theta, \pi_\Lambda, h_{ij}, \pi_{ij}) = -\frac{2}{\sqrt{\hbar}} \left( \frac{\pi}{2} e^{\frac{2\theta}{\sqrt{\hbar}}} + \pi_{ij} \phi_i \phi_j \right) - \frac{\pi_\Lambda}{\sqrt{\hbar}} F + \frac{\pi_\theta}{\sqrt{\hbar}} \left( \frac{2 \pi_\Lambda^2}{2 \theta} - \frac{2\theta}{\sqrt{\hbar}} + e^{\frac{2\theta}{\sqrt{\hbar}}} \right) \\
- e^{-\frac{2\theta}{\sqrt{\hbar}}} \left[ \frac{\pi_\Lambda \phi^i}{\sqrt{\hbar}} - \phi^i \left( \frac{\pi_\Lambda}{\sqrt{\hbar}} \right) + \frac{4 \theta}{\sqrt{\hbar}} \phi^i \phi^j \pi_\Lambda \right] . \quad \text{(3.20)}
\]

With the constraint equations \( \mathcal{H}, \Phi_2, \Phi_3 \), one can eliminate the dependence on \( \pi_\phi, \pi_\Lambda, \pi_\theta \) in the later calculation.

It will be useful to compute the following Poisson bracket
\[
\{ \Phi_2(y), \Phi_3(z) \} = \int d^3 x \left( \frac{\delta \Phi_2(y)}{\delta y^i} \frac{\delta \Phi_3(z)}{\delta z^i} + \frac{\delta \Phi_2(y)}{\delta y^i} \frac{\delta \Phi_3(z)}{\delta \pi_\theta(x)} \right) . \quad \text{(3.21)}
\]

The functional derivatives of \( \Phi_3 \) needed are given by
\[
\frac{\delta \Phi_3(y)}{\delta \pi_{ij}(x)} = -\frac{2}{\sqrt{\hbar}} \left( \frac{h_{ij}}{2} e^{\frac{2\theta}{\sqrt{\hbar}}} + \phi_i \phi_j \right) \delta^3(y - x) , \\
\frac{\delta \Phi_3(y)}{\delta \pi_\theta(x)} = \left( \frac{2 \pi_\Lambda^2}{2 \theta} - \frac{2\theta}{\sqrt{\hbar}} + e^{\frac{2\theta}{\sqrt{\hbar}}} \right) \delta^3(y - x) . \quad \text{(3.22)}
\]

Plugging (3.17) and (3.22) into (3.21), we have
\[
\{ \Phi_2(y), \Phi_3(z) \} \approx \frac{4}{3 \sqrt{\hbar}} (\nabla \phi)^4 \delta^3(y - z) , \quad \text{(3.23)}
\]
where we have used the constraint equation \( \Phi_2 \approx 0 \). The time evolution of \( \Phi_3 \) leads to another new constraint
\[
\{ \Phi_3, H_T \} = N \Phi_4 \approx 0 \quad \text{(3.24)}
\]
where the new constraint is
\[
\Phi_4 = -\frac{4}{3} \lambda (\nabla \phi)^4 + J(\phi, \theta, \Lambda, h_{ij}, \pi_{ij}, D_i) , \quad \text{(3.25)}
\]
here the explicit expression of \( J \) function is tediously long and not important for our purpose. The key point is that direct calculation shows \( \Phi_4 \) does not depend on \( N \). Because of the dependence of \( \Phi_4 \) on \( \lambda \), the time evolution of \( \Phi_4 \) involves Lagrange multiplier \( v^\lambda \), thus the
chain of constraints for primary constraint $\Phi_1$ terminates here. Similar to the previous section, the time evolution of $\mathcal{H}, \mathcal{H}_i$ are automatically satisfied and yield nothing.

The above considerations show that five primary constraints yield seven secondary constraints, therefore the system admits 12 constraints in all: $\Phi_A = \{ \Phi_2, \Phi_3, \Phi_4, \pi_N, \mathcal{H}, \pi_i, \mathcal{H}_i \}$. The associated Dirac matrix $D_{AB} = \{ \Phi_A(x), \Phi_B(y) \}$ is given by

\[
\begin{pmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-\frac{4}{3\sqrt{h}}(\nabla \phi)^4 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\end{pmatrix}
\]

multiplied by a delta function $\delta^3(x - y)$ where "*" represents some generally non-vanishing function. Generally the rank of the $12 \times 12$ Dirac matrix is 4, i.e. there are 12 linearly independent combination of the 12 constraints $\Phi_A$, of which 8 are first class and 4 are second class. According to the usual counting degrees of freedom for constraint systems, the number of independent physical degrees of freedom in our theory (3.1) is $14 - 8 - \frac{1}{2} \times 4 = 4$.

But if the mimetic scalar field is homogeneous $\nabla \phi = 0$ (which is related to the coordinate choice), the rank of the $12 \times 12$ Dirac matrix will become 2. Then one may get the misleading result that we have 10 first class and 2 second class in our theory and the number of physical degrees of freedom is $14 - 10 - \frac{1}{2} \times 2 = 3$. This indeed implies that the number of DOFs of the effective field theory of $S_2$ is 3, just as will be shown in the subsection below. As one can always choose the gauge invariant quantities to fully describe the perturbations of the system, the linear perturbation theory should be the same between $S_2$ and its effective field theory. This leads to the conclusion that we can only see 3 degrees of freedom (1 scalar and 2 tensor modes) in the perturbation theory of our model $S_2$, and the other one scalar degree of freedom don’t appear in the cosmological background. This is consistent with our previous paper [48] which works in the Lagrangian formalism and only consider the second order action. To clarify this issue, let us perform the Hamiltonian analysis after unitary gauge fixing.

### 3.1.1 Hamiltonian analysis : Einstein frame in the unitary gauge

If we consider our model $S_2$ in the special unitary gauge $\phi = t$ from the beginning, i.e. the effective field theory (EFT) $S_2^{(u)} = S_2 - \int d^4x \ u(\phi - t)$, and then do the similar Hamiltonian analysis as above, we can obtain the new total Hamiltonian

\[
H_T^{(u)} = H_T + \int d^3x \ u(\phi - t),
\]

which is just the former Hamiltonian plus one additional term imposing the unitary gauge condition. The primary constraints now are given by

\[
\pi_N \approx 0, \ \pi_i \approx 0, \ \tilde{\Phi}_1 \equiv \pi_\lambda \approx 0, \ \tilde{\Phi}_7 \equiv \phi - t \approx 0.
\]

Here we use tilde to distinguish the constraints in the unitary gauge. The time evolution of those constraints generate the following new constraints

\[
\mathcal{H} \approx 0, \ \mathcal{H}_i \approx 0, \ \tilde{\Phi}_2 \equiv -\pi_\lambda \sqrt{h} + e^{\frac{2\phi}{\sqrt{h}}} \approx 0, \ \tilde{\Phi}_8 \equiv N - e^{-\frac{\phi}{\sqrt{h}}} \approx 0.
\]
where these expressions have been simplified by employing the constraints equation. Requiring \( \Phi_8 \) to be time independent gives

\[
v^N + \frac{\pi_\theta}{\sqrt{\hbar}} e^{-\frac{\theta}{\sqrt{\hbar}}} \approx 0 ,
\]

which determines the Lagrange multiplier \( v^N \) and so the chain of constraints for \( \Phi_7 \) terminate here.

The time evolution of mimetic constraint \( \Phi_2 \) gives us a new constraint

\[
\Phi_3(\phi, \theta, \pi_\theta, h_{ij}, \pi^{ij}) = -\pi - \sqrt{\hbar} e^{-\frac{\theta}{\sqrt{\hbar}}} + \frac{3}{\sqrt{6}} \pi_\theta \approx 0 .
\]

(3.30)

Through a direct calculation We find out that \( \{\Phi_2, \Phi_3\} \approx 0 \) just as expected. Then the time evolution of \( \Phi_3 \) generates a new constraint

\[
\Phi_4(\phi, \theta, \Lambda, h_{ij}, \pi^{ij}, D_i) \approx \{\Phi_3, H_{T}^{(u)}\} \approx 0 ,
\]

(3.31)

which is independent of \( \lambda \). One can find out that the Poisson bracket of \( \Phi_2 \) and \( \Phi_4 \) vanish

\[
\{\Phi_2, \Phi_4\} = \{\Phi_2, \{\Phi_3, H_{T}^{(u)}\}\} = \{\{\Phi_2, \Phi_3\}, H_{T}^{(u)}\} - \{\{\Phi_2, H_{T}^{(u)}\}, \Phi_3\} \approx 0 .
\]

(3.32)

(3.33)

With the constraint equations \( \mathcal{H}, \Phi_2, \Phi_3 \), one can eliminate the dependence on \( \pi_\phi, \pi_\Lambda, \pi_\theta \) in the later calculation. The time evolution of this new constraint \( \Phi_4 \) generates another constraint

\[
\Phi_5(\phi, \theta, \Lambda, h_{ij}, \pi^{ij}, D_i) \approx \{\Phi_4, H_{T}^{(u)}\} ,
\]

(3.34)

which also has no dependence on \( \lambda \) because of Eq. (3.33). The exact expression of \( \Phi_4 \) and \( \Phi_5 \) is complicated, but fortunately for our purpose we only care which variables they depend on. As \( \lambda \) is not involved in \( \Phi_5 \), the time evolution of \( \Phi_5 \) yield another constraint \( \Phi_6 \). The time evolution of \( \Phi_6 \) involves the Lagrangian multiplier \( v^\lambda \) because of the dependence of \( \Phi_6 \) on \( \lambda \), therefore the chain of constraints for \( \Phi_1 = \pi_\lambda \approx 0 \) determinates.

Further more, as we have set the unitary gauge which breaks the first-class property of energy constraint, the time evolution of \( \mathcal{H} \) gives \( u = 0 \) while the time evolution of \( \mathcal{H}_i \) are still automatically satisfied. Therefore the chain of constraints for \( \pi_N \) and \( \pi_i \) terminate here.

Above considerations show that six primary constraints yield nine secondary constraints, therefore the system admits 15 constraints which are

\[
6 \text{ first – class : } \pi_i , \mathcal{H}_i ,
\]

\[
10 \text{ second – class : } \pi_N , \mathcal{H} , \Phi_1 , \Phi_2 , \Phi_3 , \Phi_4 , \Phi_5 , \Phi_6 , \Phi_7 , \Phi_8 .
\]

(3.35)

According to the usual counting of DOFs for constraint systems, the number of independent degrees of freedom in our theory (3.50) is \( 14 - 6 - \frac{1}{2} \times 10 = 3 \) . We emphasize here that the number of DOFs defined by Dirac is indeed different between the original theory \( S_2 \) and the EFT of \( S_2 \) which has imposed the unitary gauge [43, 55, 56]. Normally it is supposed that gauge choice should not affect the physics and the number of DOFs. What is special in our theory is that the rank of the associated Dirac matrix (3.26) happens to depends on the gauge choice.
3.2 Hamiltonian analysis: Jordan frame

Start with the action (3.4) in the Jordan frame and one can rewrite it in the ADM formalism

\[
S_J = \int d^4x N \sqrt{h} \left[ \frac{\chi}{2} \left( -\mathcal{R} + K^{ij} K_{ij} - K^2 \right) - \frac{\dot{\chi} - N^i \chi_i}{N} K + \frac{\dot{h}^{ij} \chi_i N_j}{N} \\
+ \lambda \left( \frac{(\dot{\phi} - N^i \phi_i)^2}{N^2} - h^{ij} \phi_i \phi_j - 1 \right) + \left( \frac{\dot{\Lambda} - N^i \Lambda_i}{N^2} (\dot{\phi} - N^j \phi_j) - h^{ij} \Lambda_i \phi_j \right) \\
+ g(F(\phi, \chi)) + \Lambda F(\phi, \chi) - V(\phi) \right].
\]

The aim of this subsection is to obtain the number of DOFs of the model (3.1) in the Jordan frame and compare it with the result in the Einstein frame. As the action does not include time derivatives of \(N\), \(N_i\), and \(\lambda\), we have the primary constraints

\[
\pi_N \approx 0 \quad \pi_i \approx 0 \quad \Psi_1 \equiv \pi_\lambda \approx 0,
\]

where we use \(\Psi\) to denote the constraints in Jordan frame. Other conjugate momentums are

\[
\pi^{ij} = \frac{\partial \mathcal{L}}{\partial \dot{h}^{ij}} = \frac{\sqrt{h}}{2} [\chi (K^{ij} - h^{ij} K) - \frac{\dot{\chi} - N^k \chi_k}{N} h^{ij}],
\]

\[
\pi_\phi = \sqrt{h} (2\lambda \frac{\dot{\phi} - N^i \phi_i}{N} + \frac{\dot{\Lambda} - N^i \Lambda_i}{N}) \quad \pi_\Lambda = \sqrt{h} \frac{\dot{\phi} - N^i \phi_i}{N} \quad \pi_\chi = -\sqrt{h} K.
\]

The total Hamiltonian is then given by

\[
H_T = \int d^3x (N \mathcal{H} + N^i \mathcal{H}_i + v^N \pi_N + v^i \pi_i + v^\lambda \pi_\lambda),
\]

where

\[
\mathcal{H} = \frac{1}{\sqrt{h}} \left[ \pi_\phi \pi_\Lambda - \lambda \pi_\chi^2 + \frac{\chi}{3} \pi_\chi^2 - \frac{2}{3} \pi_\pi_\chi + \frac{2}{3} (\pi^{ij} \pi^{ij} - \frac{1}{3} \pi^2) \right]
\]

\[
+ \sqrt{h} \left[ \frac{\chi}{2} \mathcal{R} + \chi |i| + \lambda (h^{ij} \phi_i \phi_j + 1) + h^{ij} \phi_i \Lambda_j - g(F) - \Lambda F + V(\phi) \right]
\]

and

\[
\mathcal{H}_i = \pi_\phi \phi_i + \pi_\Lambda \Lambda_i + \pi_\chi \chi_i + \pi_\lambda \lambda_i - 2 \sqrt{h} \frac{\pi^j}{\sqrt{h}} |ij|.
\]

With the primary constraints (3.37), the corresponding secondary constraints are found to be the Hamiltonian constraint, diffeomorphism constraint and mimetic constraint

\[
\mathcal{H} \approx 0 \quad \mathcal{H}_i \approx 0 \quad \Psi_2 = -\frac{\pi_\chi^2}{\hbar} + h^{ij} \phi_i \phi_j + 1 \approx 0.
\]

Again, one can write the constraints in smeared form as before. We will frequently use the property in the subsequent calculations that the Poisson bracket of any constraint
(without \( N, N^i \) dependence) with \( D[N^i] \) vanish, i.e. \( \{ \Psi, D[N^i] \} = \mathcal{L}_N \Psi \approx 0 \). The following functional derivatives will be useful for the subsequent calculations
\[
\frac{\delta H[N]}{\delta \pi^{ij}} = \frac{2N}{\sqrt{\hbar}} \left( \pi_{ij} - \frac{\pi h_{ij}}{3} \right), \quad \frac{\delta H[N]}{\delta \pi^\phi} = \frac{N \pi_\Lambda}{\sqrt{\hbar}}, \quad \frac{\delta H[N]}{\delta \pi_\Lambda} = \frac{N}{\sqrt{\hbar}} \left( \pi_\phi - 2 \lambda \pi_\Lambda \right), \quad \frac{\delta H[N]}{\delta \pi_i} = \frac{2N}{3 \sqrt{\hbar}} \left( \chi \pi_i - \pi \right), \quad \frac{\delta H[N]}{\delta \Lambda} = -\sqrt{\hbar} (N \phi^i)_i + NF \bigg],
\]
\[
\frac{\delta H[N]}{\delta \chi} = \frac{N}{\chi} \frac{\pi^2}{3} - \frac{2}{\chi^2} (\pi_{ij} \pi^{ij} - \pi^2 / 3) + N \sqrt{\hbar} \left[ \frac{\mathcal{R}}{2} - (g_{ij} + \Lambda) F_{ij} \right] + \sqrt{\hbar} N^i_i. \quad (3.43)
\]

The time evolution of mimetic constraint is
\[
\{ \Psi_2, H_T \} = \int d^3 x \left( \frac{\delta \Psi_2 \delta H[N]}{\delta h_{ij}} + \frac{\delta \Psi_2 \delta H[N]}{\delta \phi} - \frac{\delta \Psi_2 \delta H[N]}{\delta \pi_i} \right) = 2N \Psi_3 \approx 0, \quad (3.44)
\]

here we have used \( \{ \Psi_2, D[N^i] \} \approx 0 \). The explicit expression of the new constraint is
\[
\Psi_3 = -\frac{\pi_\Lambda}{\sqrt{\hbar}} (\phi^i_i + F) + h^{ij} \phi_i (\frac{\pi_\Lambda}{\sqrt{\hbar}})_{ij} = \frac{\pi^2_\Lambda}{\hbar} - \frac{2}{\chi} \frac{\pi^2}{\sqrt{\hbar}} - \frac{\pi h_{ij}}{\sqrt{\hbar}} (\pi_{ij} - \frac{\pi}{3} h_{ij}) \phi_i \phi_j + \frac{\pi^2_\Lambda}{3 \sqrt{\hbar}} h^{ij} \phi_i \phi_j. \quad (3.45)
\]

With the constraint equations \( \mathcal{H}, \Psi_2, \Psi_3 \), one can eliminate the dependence on \( \pi_\phi, \pi_\Lambda, \pi_\theta \) in the later calculation.

The time evolution of \( \Psi_3 \) leads to another constraint
\[
\{ \Psi_3, H_T \} \approx N \Psi_4 \approx 0. \quad (3.46)
\]

Using the result of the following Poisson bracket
\[
\{ \Psi_2(y), \Psi_3(z) \} = \frac{4}{3 \sqrt{\hbar} \chi} (\nabla \phi)^4 \delta^3(y - z), \quad (3.47)
\]

the new constraint is obtained to be
\[
\Psi_4 = -\frac{4\lambda}{3 \chi} (\nabla \phi)^4 + J_2 (\phi, \chi, \Lambda, h_{ij}, \pi_{ij}, D_i), \quad (3.48)
\]

here the explicit expression of \( J_2 \) is tedious and not important for us. The key point is that through direct calculation we find out all the terms involving \( N \) cancel exactly, i.e. \( \Psi_4 \) does not depend on \( N \). Requiring this constraint to be time independent, determines the Lagrangian multiplier \( \nu^A \) in terms of phase space variables and the chain of constraints for primary constraint \( \Psi_1 = \pi_\Lambda \approx 0 \) determinates. Besides, the time evolution of \( \mathcal{H}_t \) are automatically satisfied and yield no extra constraint.

Therefore the system admits 12 constraints \( \Psi_A = \{ \Psi_2, \Psi_3, \Psi_1, \Psi_4, N, \mathcal{H}, \pi_i, \mathcal{H}_t \} \). The associated Dirac matrix \( D_{AB} = \{ \Psi_A(x), \Psi_B(y) \} \) is given by
\[
\begin{pmatrix}
0 & 0 & \frac{4}{3 \sqrt{\hbar} \chi} (\nabla \phi)^4 & 0 & \ast & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & \frac{4}{3 \sqrt{\hbar} \chi} (\nabla \phi)^4 & 0 & 0 & 0 & \ast & 0 \\
\ast & \ast & -\frac{4}{3 \chi} (\nabla \phi)^4 & 0 & 0 & \ast & 0 & 0 & 0 \\
0 & \ast & 0 & 0 & 0 & \ast & 0 & 0 & 0 \\
0 & 0 & 0 & \ast & 0 & 0 & \ast & 0 & 0 \\
0 & 0 & 0 & 0 & \ast & 0 & 0 & \ast & 0 \\
0 & 0 & 0 & 0 & 0 & \ast & 0 & 0 & \ast \\
0 & 0 & 0 & 0 & 0 & 0 & \ast & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & \ast & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \ast
\end{pmatrix}. \quad (3.49)
\]
multiplied by a delta function $\delta^3(x - y)$. Generally the rank of the $12 \times 12$ Dirac matrix is 4, i.e. there are 12 linearly independent combination of the 12 constraints $\Psi_A$, of which 8 are first class and 4 are second class. Thus, the number of independent physical degrees of freedom in the model (3.1) is $14 - 8 - \frac{1}{2} \times 4 = 4$ which is consistent with the analysis in the Einstein frame. Note $H$ is not first-class, but a particular linear combination of $H$ and $\pi_\lambda$ is. One can see that if the mimetic field is homogeneous $\nabla \phi = 0$, the rank of this Dirac matrix will be reduced to 2. Now let’s work out the Hamiltonian analysis in the special gauge, i.e. the unitary gauge, and see whether the degrees of freedom change.

3.2.1 Hamiltonian analysis: Jordan frame in the unitary gauge

Consider the action $S_J$ in the special unitary gauge $\phi = t$, and then one obtain the new total Hamiltonian

$$ H^{(u)}_T = H_T + \int d^3x \, u(\phi - t) , \tag{3.50} $$

which is the former Hamiltonian plus one additional term imposing the unitary gauge condition. The primary constraints now are given by

$$ \pi_N \approx 0 \, , \, \pi_i \approx 0 \, , \, \tilde{\Psi}_1 \equiv \pi_\lambda \approx 0 \, , \, \tilde{\Psi}_7 \equiv \phi - t \approx 0 . \tag{3.51} $$

The time evolution of those constraints generate the following new constraints

$$ H \approx 0 \, , \, H_i \approx 0 \, , \, \tilde{\Psi}_2 \equiv -\frac{\pi_\lambda}{\hbar} + 1 \approx 0 \, , \, \tilde{\Psi}_8 \equiv N - 1 \approx 0 . \tag{3.52} $$

where these expressions have been simplified by using the constraints equation. Requiring $\tilde{\Psi}_8$ to be time independent gives

$$ v^N + \frac{\pi_\theta}{\sqrt{\hbar}} e^{-\frac{\theta}{\sqrt{\hbar}}} \approx 0 , \tag{3.53} $$

which determines the Lagrange multiplier $v^N$ and so the chain of constraints for unitary gauge $\tilde{\Psi}_7$ terminate here.

The time evolution of mimetic constraint $\tilde{\Psi}_2$ gives us a new constraint

$$ \tilde{\Psi}_3(\phi, \theta, \pi_\theta, h_{ij}, \pi_{ij}) = - (\pi_\lambda F + \pi_\chi) \approx 0 . \tag{3.54} $$

One can easily see $\{\tilde{\Psi}_2, \tilde{\Psi}_3\} \approx 0$ just as expected. The time evolution of $\tilde{\Psi}_3$ generates a new constraint

$$ \tilde{\Psi}_4(\phi, \chi, \Lambda, h_{ij}, \pi_{ij}, D_i) \approx \{\tilde{\Psi}_3, H^{(u)}_T\} \approx 0 . \tag{3.55} $$

One can verify that the Poisson bracket of $\tilde{\Psi}_2$ and $\tilde{\Psi}_4$ vanish just as the case in the Einstein frame. The time evolution of this new constraint $\tilde{\Psi}_4$ also gives another constraint

$$ \tilde{\Psi}_5(\phi, \chi, \Lambda, h_{ij}, \pi_{ij}, D_i) = \{\tilde{\Psi}_4, H^{(u)}_T\} . \tag{3.56} $$

which has no dependence on $\lambda$. Although the exact expression of $\tilde{\Psi}_4$ and $\tilde{\Psi}_5$ is complicated, the key point is $\tilde{\Psi}_4$ and $\tilde{\Psi}_5$ do not include $\lambda$ and the time evolution of $\tilde{\Psi}_5$ yield another constraint $\tilde{\Psi}_6$ involving $\lambda$. Therefore $\tilde{\Psi}_6 \approx 0$ involves the Lagrangian multiplier $v^\lambda$ and so the chain of constraints for $\tilde{\Psi}_4 = \pi_\lambda \approx 0$ determinates here.

Further more, as we have set the unitary gauge which satisfy $\{\phi - t, H[N]\} \neq 0$, the time evolution of $H$ determines $u = 0$ and so the chain of constraints for $\pi_N \approx 0$ determinates
here. The first class property of spatial diffeomorphism is unspoiled in the unitary gauge, and the time evolution of \( H_i \) is automatically preserved, so the chain of constraints for \( \pi_i \) determinates here.

Above considerations show that six primary constraints yield nine secondary constraints, therefore the system admits 16 constraints \( \Psi_A = \{ \Psi_1, \Psi_2, \Psi_3, \Psi_4, \Psi_5, \Psi_6, \Psi_7, \Psi_8, \pi_N, H, \pi_i, H_i \} \). As one can see, all the constraints are second-class except \( \pi_i, H_i \), i.e. of the 16 constraints, 6 are first class and 10 are second class. Therefore, the number of independent degrees of freedom in the theory (3.50) is \( 14 - 6 - \frac{1}{2} \times 10 = 3 \), which is different from the theory without gauge fixing.

We see that the Hamiltonian analysis in the Jordan frame is consistent with the analysis in the Einstein frame. Thus we have shown that our conclusion is independent of the frame: the number of DOFs according to Dirac in the general theory (3.1) is 4, and it is reduced to 3 in the unitary gauge. We deduce the extra DOF is non-propagating and will be eliminated in the unitary gauge.

4 Gauge dependence of the rank of the Dirac matrix: a simple example

To better understand the results above, consider a simple constrained system with countable degrees of freedom in which the rank of the Dirac matrix is gauge dependent. The ideal is to construct a Hamiltonian system with one first-class \( f_1 \) (gauge system) and two second-class constraints \( f_2 \) and \( f_3 \), and the rank of the associated Dirac matrix is related to the gauge choice, i.e.

\[
\begin{align*}
    f_1 \text{ is first} - \text{class} : & \quad \{f_1, f_2\} \approx 0, \quad \{f_1, f_3\} \approx 0, \\
    f_2 \text{ and } f_3 \text{ are second} - \text{class} : & \quad D_{23} = \{f_2, f_3\} \neq 0, \\
    \text{gauge dependence of the rank} : & \quad \{f_1, D_{23}\} = \{f_1, \{f_2, f_3\}\} \neq 0, \\
\end{align*}
\]

(4.1)

where the sign \( \neq \) denotes an inequality up to terms that vanish on the constraint surface.

To realize the ideal, we can first assume the total Hamiltonian of the system is given by

\[
H_T = H(q^a, p_a) + uf_1 + vf_2, \quad a = 1, \ldots, N
\]

(4.2)

where \( f_1, f_2 \) are two primary constraints and \( u, v \) are the corresponding Lagrange multiplier. As \( f_1 \) is supposed to be first-class, we require

\[
\{f_1, f_2\} \approx 0, \quad \{f_1, H\} \approx 0.
\]

(4.3)

The evolution of the constraint \( f_1 \) is automatically satisfied and yields no new constraint. For the time evolution of the constraint \( f_2 \), we obtain a new constraint \( f_3 = \{f_2, H\} \). Requiring \( f_3 \) to be time independent gives

\[
\{f_3, H\} + v\{f_3, f_2\} = 0.
\]

(4.4)

As \( f_2, f_3 \) are second-class, \( D_{23} = \{f_3, f_2\} \) is generally not vanishing on the constraint surface and the above equation determines the Lagrange multiplier \( v \). Therefore, the number of DOFs is \( (2N - 2 \times 1 - 2)/2 = N - 2 \).

We assume that the rank of the associated Dirac matrix is gauge dependent, i.e. \( \{f_1, D_{23}\} \neq 0 \). Then in the special gauge

\[
D_{23} = 0,
\]

(4.5)
the new total Hamiltonian becomes

\[ H_{T}^{\text{gauge}} = H_T + wD_{23} = H(q^a, p_a) + uf_1 + v f_2 + wD_{23} \quad (4.6) \]

where the Lagrange multiplier \( w \) enforces the gauge fixing (4.5). We have 3 primary constraints

\[ f_1 \approx 0, \quad f_2 \approx 0, \quad D_{23} \approx 0. \quad (4.7) \]

The time evolutions of \( f_1 \) determine \( w = 0 \), the time evolutions of \( D_{23} \) involves Lagrange multiplier \( u \), while the time evolutions of \( f_2 \) yield the secondary constraint \( f_3 \approx 0 \). The consistency relation of \( f_3 \) generate a new constraint

\[ f_4 \equiv \{ f_3, H \} \approx 0 \quad (4.8) \]

after using the gauge condition \( D_{23} \approx 0 \).

Thus in the gauge fixing (4.5), we have at least 5 constraints \{ \( f_1, f_2, f_3, f_4, D_{12} \) \} while only 4 constraints exist in the general gauge. Such a simple example is a good demonstration that there exists some special systems where some DOF may not like the usual DOF and will be eliminated by appropriate gauge fixing. We shall further analyze this issue in the future work.

5 Conclusion and discussions

Recently, there is an increasing investigation in exploring the instability issue [45] of mimetic model with higher derivative terms. In the previous work [48] we pointed out that it is possible to overcome this pathology by introducing the direct coupling of the higher derivatives of the mimetic field to the Ricci scalar of the spacetime. Although it seems that our setup have one scalar mode and two tensor modes by analyzing the quadratic actions of perturbation, the modified dispersion relation of scalar perturbation may imply the existence of extra DOF which do not show up at the cosmological perturbation level. In this paper we first confirmed that the mimetic gravity with a general higher derivative function of the mimetic field (2.1) has 3 DOFs which is consistent with the previous result of the Hamiltonian analysis in [47] and [43]. Then we perform a detailed Hamiltonian constraint analysis for the extended mimetic model (3.1) (which is slightly different from the model considered in [48]) in both Einstein frame and Jordan frame. The conclusion is consistent with each other in both frames: generally such kind of theories has 4 DOFs while only 3 propagating DOFs show up at the cosmological perturbation level [48]. To clarify why the number of DOFs is not the same, we reanalyze the model after fixing the unitary gauge and interestingly, the DOFs is reduced to 3. Therefore, we can conclude that the number of propagating DOFs in the model (3.1) is actually 3, and the extra DOF is not propagating and can be eliminated in the unitary gauge condition.

Thus, we conclude that there exist some kind of special theories in which the DOFs of the space-time covariant version may not always be equivalent to the DOFs of its effective spatially covariant version, and some DOF may not show up on the FRW cosmological background. Actually this situation has already been studied in [57–59], and it was argued that this apparently dangerous mode is non-propagating and can be eliminated by choosing appropriate boundary conditions. For our case, the unitary gauge leads to the elimination of this extra mode. As the existence of the extra DOF which is non-propagating, we may need to extend the definition of DOFs by Dirac, and make a clear distinction among different kinds of DOFs to avoid confusion.
This also gives us another important hint: the reason why the XG3 theory [60] is larger than the DHOST theory [56, 61] may be because there exist some special theory which belong to the XG3 theory but not DHOST theory and the spacetime covariant version of those theories have actually 4 DOFs just like the model (3.1) studied in this paper.

Another comment is that even the number of DOFs according to Dirac in the spacetime covariant version and the spatially covariant version are not equal in some cases, the perturbative theory in FRW universe is always the same. Furthermore, we should point out the appearance of higher power of \( w \) and \( k \) than two in the dispersion relation (such as in the case of the XG3 theory and Horava gravity [62]) suggests the existence of extra non-propagating DOF. The relation between the modification of dispersion relation and the existence for extra DOF deserves detailed investigations in the future.

**Acknowledgments**

I would like to thank Mingzhe Li, Liuyuan Shen, Haoming Rao, Dehao Zhao for useful discussions, and I am also grateful to Xian Gao, Karim Noui and Yifu Cai for helpful correspondences. This work is funded by the CAS Key Laboratory for Research in Galaxies and Cosmology, Chinese Academy of Science (No. 18010203), and supported in part by NSFC under Grant No. 11422543 and No. 11653002.

**A a special case**

Here we consider the case of \( f(\phi) = 1 \) and \( g(\Box \phi) = \alpha (\Box \phi)^2 \), the action \( S_1 \) reduces to

\[
S_1 = \int d^4x \sqrt{-g} \left[ \frac{1}{2} R + \lambda (g^\mu\nu \phi_\mu \phi_\nu - 1) - V(\phi) + \alpha (\Box \phi)^2 \right].
\] (A.1)

In the special case \( \alpha = \frac{1}{3} \), one has the Poisson bracket \( \{ \Psi_2, \Psi_3 \} \propto (\nabla \phi)^2 \), and \( \Psi_4 = \lambda (\nabla \phi)^2 + J_0 \). As \( \Psi_4 \) involves \( \lambda \), the time evolution will fix the Lagrangian multiplier \( \nu^\lambda \) and the chain of constraints for \( \pi^\lambda \) determinates here. The number of DOFs will be 3 according previous analysis. But if we set the unitary gauge from the beginning, the situation will change. the evolution of \( \Psi_4 \) will generate two secondary constraints \( \Psi_5 \) and \( \Psi_6 \). This will reduce the DOFs to be 2 which means there are only two tensor modes at the corresponding perturbation theory.

The quadratic action for scalar perturbation in the model (A.1) is [45]

\[
S^{(2)}_\zeta = \int d^4x a^3 \left[ - \frac{1 - 3\alpha}{\alpha} \zeta^2 + \frac{(\nabla \zeta)^2}{a^2} \right].
\] (A.2)

where the coefficient of time derivative term happens to be vanishing in the special case \( \alpha = \frac{1}{3} \). The EOM gives \( \zeta = 0 \). Therefore indeed only two tensor perturbations contribute to the DOFs. However, it is strange that the background equation in this special case becomes [45]

\[
0 = V(t),
\] (A.3)

which will be not self-consistent unless the model have no potential term. But if the potential is vanishing, the background equation (A.3) will be automatically satisfied and gives us nothing, i.e. we don’t have the evolution equation for the scale factor at all!
References

[1] A. H. Chamseddine and V. Mukhanov, JHEP 1311 (2013) 135 doi:10.1007/JHEP11(2013)135 [arXiv:1308.5410 [astro-ph.CO]].

[2] J. D. Bekenstein, Phys. Rev. D 48 (1993) 3641 doi:10.1103/PhysRevD.48.3641 [gr-qc/9211017].

[3] N. Deruelle and J. Rua, JCAP 1409 (2014) 002 doi:10.1088/1475-7516/2014/09/002 [arXiv:1407.0825 [gr-qc]].

[4] F. Arroja, N. Bartolo, P. Karmakar and S. Matarrese, JCAP 1509 (2015) 051 doi:10.1088/1475-7516/2015/09/051 [arXiv:1506.08575 [gr-qc]].

[5] G. Domenech, S. Mukohyama, R. Namba, A. Naruko, R. Saitou and Y. Watanabe, Phys. Rev. D 92 (2015) no.8, 084027 doi:10.1103/PhysRevD.92.084027 [arXiv:1507.05390 [hep-th]].

[6] A. Golovnev, Phys. Lett. B 728 (2014) 39 doi:10.1016/j.physletb.2013.11.026 [arXiv:1310.2790 [gr-qc]].

[7] A. H. Chamseddine, V. Mukhanov and A. Vikman, JCAP 1406 (2014) 017 doi:10.1088/1475-7516/2014/06/017 [arXiv:1403.3961 [astro-ph.CO]].

[8] S. Nojiri and S. D. Odintsov, Mod. Phys. Lett. A 29 (2014) no.40, 1450211 doi:10.1142/S0217732314502113 [arXiv:1408.3561 [hep-th]].

[9] G. Leon and E. N. Saridakis, JCAP 1504 (2015) no.04, 031 doi:10.1088/1475-7516/2015/04/031 [arXiv:1501.00488 [gr-qc]].

[10] A. V. Astashenok, S. D. Odintsov and V. K. Oikonomou, Class. Quant. Grav. 32 (2015) no.18, 185007 doi:10.1088/0264-9381/32/18/185007 [arXiv:1504.04861 [gr-qc]].

[11] R. Myrzakulov, L. Sebastiani and S. Vagnozzi, Eur. Phys. J. C 75 (2015) 444 doi:10.1140/epjc/s10052-015-3672-6 [arXiv:1504.07984 [gr-qc]].

[12] Y. Rabochaya and S. Zerbini, Eur. Phys. J. C 76 (2016) no.2, 85 doi:10.1140/epjc/s10052-016-3926-y [arXiv:1509.03720 [gr-qc]].

[13] F. Arroja, N. Bartolo, P. Karmakar and S. Matarrese, JCAP 1604 (2016) no.04, 042 doi:10.1088/1475-7516/2016/04/042 [arXiv:1512.09374 [gr-qc]].

[14] G. Cognola, R. Myrzakulov, L. Sebastiani, S. Vagnozzi and S. Zerbini, Class. Quant. Grav. 33 (2016) no.22, 225014 doi:10.1088/0264-9381/33/22/225014 [arXiv:1601.00102 [gr-qc]].

[15] S. Nojiri, S. D. Odintsov and V. K. Oikonomou, Class. Quant. Grav. 33 (2016) no.12, 125017 doi:10.1088/0264-9381/33/12/125017 [arXiv:1601.07057 [gr-qc]].

[16] D. Momeni, A. Altaibayeva and R. Myrzakulov, Int. J. Geom. Meth. Mod. Phys. 11 (2014) 1450091 doi:10.1142/S0219887814500911 [arXiv:1407.5662 [gr-qc]].

[17] A. H. Chamseddine and V. Mukhanov, JHEP 1806 (2018) 060 doi:10.1007/JHEP06(2018)060 [arXiv:1805.06283 [hep-th]].

[18] A. H. Chamseddine and V. Mukhanov, JHEP 1806 (2018) 062 doi:10.1007/JHEP06(2018)062 [arXiv:1805.06598 [hep-th]].

[19] E. Alvarez, J. Anero, G. Milans Del Bosch and R. Santos-Garcia, arXiv:1806.10507 [hep-th].

[20] H. Saadi, Eur. Phys. J. C 76 (2016) no.1, 14 doi:10.1140/epjc/s10052-015-3856-0 [arXiv:1411.4531 [gr-qc]].

[21] L. Mirzagholi and A. Vikman, JCAP 1506 (2015) 028 doi:10.1088/1475-7516/2015/06/028 [arXiv:1412.7136 [gr-qc]].

[22] J. Matsumoto, S. D. Odintsov and S. V. Sushkov, Phys. Rev. D 91 (2015) no.6, 064062 doi:10.1103/PhysRevD.91.064062 [arXiv:1501.02149 [gr-qc]].
[23] S. Ramazanov, JCAP 1512 (2015) 007 doi:10.1088/1475-7516/2015/12/007 [arXiv:1507.00291 [gr-qc]].

[24] R. Myrzakulov, L. Sebastiani, S. Vagnozzi and S. Zerbini, Class. Quant. Grav. 33 (2016) no.12, 125005 doi:10.1088/0264-9381/33/12/125005 [arXiv:1510.02284 [gr-qc]].

[25] A. V. Astashenok and S. D. Odintsov, Phys. Rev. D 94 (2016) no.6, 063008 doi:10.1103/PhysRevD.94.063008 [arXiv:1512.07279 [gr-qc]].

[26] A. H. Chamseddine and V. Mukhanov, JCAP 1602 (2016) no.02, 040 doi:10.1088/1475-7516/2016/02/040 [arXiv:1601.04941 [astro-ph.CO]].

[27] S. Nojiri, S. D. Odintsov and V. K. Oikonomou, Phys. Rev. D 94 (2016) no.10, 104050 doi:10.1103/PhysRevD.94.104050 [arXiv:1608.07806 [gr-qc]].

[28] E. Babichev and S. Ramazanov, Phys. Rev. D 95 (2017) no.2, 024025 doi:10.1103/PhysRevD.95.024025 [arXiv:1609.08580 [gr-qc]].

[29] A. H. Chamseddine and V. Mukhanov, Eur. Phys. J. C 77 (2017) no.3, 183 doi:10.1140/epjc/s10052-017-4759-z [arXiv:1612.05861 [gr-qc]].

[30] N. Sadeghnejad and K. Nozari, arXiv:1703.06269 [gr-qc].

[31] L. Shen, Y. Mou, Y. Zheng and M. Li, Chin. Phys. C 42 (2018) no.1, 015101 doi:10.1088/1674-1137/42/1/015101 [arXiv:1710.03945 [gr-qc]].

[32] M. Rinaldi, L. Sebastiani, A. Casalino and S. Vagnozzi, arXiv:1803.02620 [gr-qc].

[33] K. Hammer and A. Vikman, arXiv:1512.09118 [gr-qc].

[34] A. Golovnev, Phys. Lett. B 779 (2018) 441 doi:10.1016/j.physletb.2018.02.044 [arXiv:1801.07958 [gr-qc]].

[35] S. D. Odintsov and V. K. Oikonomou, Nucl. Phys. B 929 (2018) 79 doi:10.1016/j.nuclphysb.2018.01.027 [arXiv:1801.10529 [gr-qc]].

[36] D. Langlois, M. Mancarella, K. Noui and F. Vernizzi, arXiv:1802.03394 [gr-qc].

[37] H. Firouzjahi, M. A. Gorji, S. A. Hosseini Mansoori, A. Karami and T. Rostami, arXiv:1806.11472 [gr-qc].

[38] M. A. Gorji, S. Mukohyama, H. Firouzjahi and S. A. Hosseini Mansoori, arXiv:1807.06335 [hep-th].

[39] S. A. Paston and A. A. Sheykin, arXiv:1806.10902 [gr-qc].

[40] O. Malaeb, Phys. Rev. D 91 (2015) no.10, 103526 doi:10.1103/PhysRevD.91.103526 [arXiv:1404.4195 [gr-qc]].

[41] M. Chaichian, J. Kluson, M. Oksanen and A. Tureanu, JHEP 1412 (2014) 102 doi:10.1007/JHEP12(2014)102 [arXiv:1404.4008 [hep-th]].

[42] K. Takahashi and T. Kobayashi, JCAP 1711 (2017) no.11, 038 doi:10.1088/1475-7516/2017/11/038 [arXiv:1708.02951 [gr-qc]].

[43] L. Sebastiani, S. Vagnozzi and R. Myrzakulov, Adv. High Energy Phys. 2017 (2017) 3156915 doi:10.1155/2017/3156915 [arXiv:1612.08661 [gr-qc]].

[44] A. Ijjas, J. Ripley and P. J. Steinhardt, Phys. Lett. B 760 (2016) 132 doi:10.1016/j.physletb.2016.06.052 [arXiv:1604.08586 [gr-qc]].

[45] S. Ramazanov, F. Arroja, M. Celoria, S. Matarrese and L. Pilo, JHEP 1606 (2016) 020 doi:10.1007/JHEP06(2016)020 [arXiv:1601.05405 [hep-th]].
[47] H. Firouzjahi, M. A. Gorji and A. Hosseini Mansoori, arXiv:1703.02923 [hep-th].
[48] Y. Zheng, L. Shen, Y. Mou and M. Li, JCAP 1708 (2017) no.08, 040
doi:10.1088/1475-7516/2017/08/040 [arXiv:1704.06834 [gr-qc]].
[49] M. A. Gorji, S. A. Hosseini Mansoori and H. Firouzjahi, JCAP 1801 (2018) no.01, 020
doi:10.1088/1475-7516/2018/01/020 [arXiv:1709.09988 [astro-ph.CO]].
[50] S. Hirano, S. Nishi and T. Kobayashi, arXiv:1704.06031 [gr-qc].
[51] Y. F. Cai and X. Zhang, Phys. Rev. D 80 (2009) 043520
doi:10.1103/PhysRevD.80.043520 [arXiv:0906.3341 [astro-ph.CO]].
[52] B. P. Abbott et al. [LIGO Scientific and Virgo Collaborations], Phys. Rev. Lett. 119 (2017)
no.16, 161101 doi:10.1103/PhysRevLett.119.161101 [arXiv:1710.05832 [gr-qc]].
[53] Y. F. Cai, F. Duplessis and E. N. Saridakis, Phys. Rev. D 90 (2014) no.6, 064051
doi:10.1103/PhysRevD.90.064051 [arXiv:1307.7150 [hep-th]].
[54] Y. F. Cai and E. N. Saridakis, Phys. Rev. D 90, no. 6, 063528 (2014)
doi:10.1103/PhysRevD.90.063528 [arXiv:1401.4418 [astro-ph.CO]].
[55] D. Langlois and K. Noui, JCAP 1607 (2016) no.07, 016
doi:10.1088/1475-7516/2016/07/016 [arXiv:1512.06820 [gr-qc]].
[56] D. Langlois and K. Noui, JCAP 1602 (2016) no.02, 034
doi:10.1088/1475-7516/2016/02/034 [arXiv:1510.06930 [gr-qc]].
[57] A. De Felice, D. Langlois, S. Mukohyama, K. Noui and A. Wang, arXiv:1803.06241 [hep-th].
[58] D. Blas, O. Pujolas and S. Sibiryakov, JHEP 0910 (2009) 029
doi:10.1088/1126-6708/2009/10/029 [arXiv:0906.3046 [hep-th]].
[59] X. Gao and Z. b. Yao, arXiv:1806.02811 [gr-qc].
[60] X. Gao, Phys. Rev. D 90 (2014) 081501
doi:10.1103/PhysRevD.90.081501 [arXiv:1406.0822 [gr-qc]].
[61] J. Ben Achour, M. Crisostomi, K. Koyama, D. Langlois, K. Noui and G. Tasinato, JHEP 1612
(2016) 100 doi:10.1007/JHEP12(2016)100 [arXiv:1608.08135 [hep-th]].
[62] P. Horava, Phys. Rev. D 79 (2009) 084008
doi:10.1103/PhysRevD.79.084008 [arXiv:0901.3775 [hep-th]].