A Virtual Prototyping Model on Dynamic Behaviors of Prefabricated Bridges

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Abstract. With loads on prefabricated bridges becoming more and more heavier, their dynamic behaviors should be paid more attention to. A virtual prototyping model with the object-oriented technology and the mode synthesis method is presented in MATLAB to analyze dynamic behaviors of prefabricated bridges. Using structural characteristics of the bridges properly, substructures can be looked on as objects which can be encapsulated with the mode synthesis method. The compatibility conditions on interfaces, the Lagrange’s equation and Ritz’s variational principle play the role of messages transmitting among objects. Simulation results indicate that the present model can easily study dynamic behaviors of bridges in the same model.

1. Introduction
With the development of computer technology, the concept of the virtual prototyping technique had been presented as CAE in the 1980s. Now, this technique has been widely applied to the areas of aircraft industry, aerospace engineering, building industry, medical engineering, military engineering, etc. The application of the virtual prototyping technology in the study and design of engineer products can shorten the period of development, reduce the cost of production, ameliorate the quality of design, length the life of service. Based on virtual prototype, Jia[1] researched on simulation and quantitative evaluation of performance of recoil system; Xia[2] developed a laser transmission simulation system; Meng[3] studied the design of parameters of vibration-impact rammer; Yu[4] simulated the reliability of space station expand mechanism.

Prefabricated bridges are widely used especially in military engineering. It is well-known that a vehicle moving across a bridge produces greater stresses than if the vehicle were in a static position on the structure. This increment in stress can be called the dynamic effects. With loads on bridges becoming more and more heavier, their dynamic behaviors should be attached more importance to. There are many approaches to study dynamic behaviors of structures, such as the Rayleigh-Ritz method, the subspace method, Block-Lanczos method[5] and the mode synthesis method[6,7]. A prefabricated bridge can be divided into substructures as assembly cells by using its structural characteristics properly. The mode synthesis technique which is based on FEM can be used to analyze dynamic behaviors of the bridge.

In this study, a virtual prototyping model in MATLAB is presented by using of the object-oriented technology[8] on the basis of the mode synthesis method to analyze dynamic behaviors of prefabricated bridges. Substructures can be looked on as objects. A object can be encapsulated with the mode synthesis method, and then a fixed interface substructure comes into being. With the messages of compatibility conditions on interfaces and the common messages of the Lagrange’s equation and Ritz’s variational principle, the systemic synthesis equations for the virtual prototyping model can be got.
prototyping model is programmed in MATLAB which is a high-performance language for technical computing and simulating.

2. The Virtual prototyping model

2.1. The substructure object model
Substructure objects of the prefabricated bridge usually have both left interface and right interface. The coordinates of left and right interfaces and internal coordinates can be defined as \( u_L \), \( u_R \) and \( u_I \) respectively. The dynamic equation of the substructure \( m u + ku = f \) can be of the form

\[
\begin{bmatrix}
    m_{LL} & m_{LI} & m_{LR} \\
    m_{IL} & m_{II} & m_{IR} \\
    m_{RL} & m_{RI} & m_{RR}
\end{bmatrix}
\begin{bmatrix}
    u_L \\
    u_I \\
    u_R
\end{bmatrix}
+ \begin{bmatrix}
    k_{LL} & k_{LI} & k_{LR} \\
    k_{IL} & k_{II} & k_{IR} \\
    k_{RL} & k_{RI} & k_{RR}
\end{bmatrix}
\begin{bmatrix}
    u_L \\
    u_I \\
    u_R
\end{bmatrix}
= \begin{bmatrix}
    f_L \\
    f_I \\
    f_R
\end{bmatrix}
\]  

Using the restrained mode synthesis technology, Equation(2) can be approximately written as

\[
\begin{bmatrix}
    u_L \\
    u_I \\
    u_R
\end{bmatrix}^{(a)} = \begin{bmatrix}
    \Psi_L | \Phi_I | \Psi_R
\end{bmatrix} \begin{bmatrix}
    p_L \\
    p_I \\
    p_R
\end{bmatrix}^{(a)}
\]

where \( p_L \), \( p_I \), \( p_R \) are the general coordinates of Substructure objects.

Using the restrained mode synthesis technology, Equation(2) can be approximately written as

\[
\begin{bmatrix}
    p_L \\
    p_I \\
    p_R
\end{bmatrix}^{(a)} = H p
\]

where \( H = [ \Psi_L | \Phi_I | \Psi_R ] \) is the synthesized mode base; \( \Psi_L = [I_{LL} | C_L^* | O]^T \) is the left restrained mode matrix with \( C_L = -k_L^T k_L \); \( \Psi_R = [O | C_R^* | I_{RR}]^T \) is the right restrained mode matrix with \( C_R = -k_R^T k_R \); \( \Phi_I \) is a matrix composed by the former \( k \) mode column vectors of \( \Phi_i \) (\( k < I \)); \( \Phi_R \) is the elastic mode matrix with dimension \( I \times I \) on the conditions of both \( u_L \) and \( u_R \) restrained; \( I_{LL} \) and \( I_{RR} \) are a \( L \) identity matrix and a \( R \) identity one, and \( O \) is a zero matrix.

From Equation(1) we obtain the dynamic equation of the encapsulated object, which is

\[
\begin{bmatrix}
    u_L \\
    u_I \\
    u_R
\end{bmatrix}
\]

\[
\begin{bmatrix}
    u_L \\
    u_I \\
    u_R
\end{bmatrix}
\begin{bmatrix}
    g_L \\
    O \\
    g_R
\end{bmatrix}
\]

2.2. The encapsulated object model
Dividing the bridges into \( m \) substructures, the physical coordinates of the \( \alpha \)-th object can be expressed as

\[
\begin{bmatrix}
    u_L \\
    u_I \\
    u_R
\end{bmatrix}^{(a)} = \begin{bmatrix}
    \Psi_L | \Phi_I | \Psi_R
\end{bmatrix} \begin{bmatrix}
    p_L \\
    p_I \\
    p_R
\end{bmatrix}^{(a)}
\]

where \( p_L \), \( p_I \), \( p_R \) are the general coordinates of Substructure objects.

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\[
\begin{bmatrix}
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    p_I \\
    p_R
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where \( H = [ \Psi_L | \Phi_I | \Psi_R ] \) is the synthesized mode base; \( \Psi_L = [I_{LL} | C_L^* | O]^T \) is the left restrained mode matrix with \( C_L = -k_L^T k_L \); \( \Psi_R = [O | C_R^* | I_{RR}]^T \) is the right restrained mode matrix with \( C_R = -k_R^T k_R \); \( \Phi_I \) is a matrix composed by the former \( k \) mode column vectors of \( \Phi_i \) (\( k < I \)); \( \Phi_R \) is the elastic mode matrix with dimension \( I \times I \) on the conditions of both \( u_L \) and \( u_R \) restrained; \( I_{LL} \) and \( I_{RR} \) are a \( L \) identity matrix and a \( R \) identity one, and \( O \) is a zero matrix.

From Equation(1) we obtain the dynamic equation of the encapsulated object, which is

\[
\begin{bmatrix}
    u_L \\
    u_I \\
    u_R
\end{bmatrix}
\begin{bmatrix}
    g_L \\
    O \\
    g_R
\end{bmatrix}
\]

2.3. Messages transmitted among objects
The compatibility conditions on interfaces play the role of messages (shown in Fig.1) transmitted among objects. The message is a vector.
There are two types of messages. One type is compatibility condition of displacement
\[ (\beta \mathbf{u}_\alpha, \beta = 1, 2, \ldots, m-1) \]  

The other type is equilibrium of forces
\[ (\beta \mathbf{f}_s, \beta = 1, 2, \ldots, m-1) \]  

Transmitting the above two types of message among substructure objects and introducing the common messages of the Lagrange’s equation and Ritz’s variational principle, we can get the general virtual prototyping model equation of the whole bridge as
\[ \mathbf{M} \dot{\mathbf{q}} + \mathbf{K} \mathbf{q} = \mathbf{0} \]  

where \( \mathbf{M} = \sum \mathbf{B}_\mu \) is the general mass matrix; \( \mathbf{K} = \sum \mathbf{B}_\eta \) is the general rigidity matrix; \( \mathbf{q} = (\mathbf{q}^T, \mathbf{q}_1^T, \mathbf{q}_2^T, \mathbf{q}_3^T) \) is the general coordinates (where \( \alpha = 1, 2, \ldots, m \), where \( m \) is the number of substructures of the bridge); \( \mathbf{B} \) is the matrix taking mode matrices’ seat according to the whole FEM model.

2.4. Simulation in MATLAB
Developing over a period years, MATLAB has been the standard instructional tool for introductory and advanced courses in mathematics, engineering, and science. It is an interactive system whose basic data element is an array that does not require dimensioning. This allows you to solve problems with matrix and vector formulations. In the present study, the model is programmed in M-file.

The geometric model of the substructure object is displayed with high-level functions for two-dimensional and three-dimensional data visualization. Then the whole geometric model of a bridge is obtained with arraying substructures. The model equation can be solved directly with matrix and vector formulations.

3. Examples
There is a type of prefabricated truss bridge composed of two types of substructures: the end substructure shown in Fig.2 and the inner one shown in Fig.3. In this study, bridges with four and six substructures (shown in Fig.4 and Fig.5) are investigated. Table.1 and Table.2 list the former three natural frequencies of vibration with vertical bending and torsion respectively for both bridges. Fig.6 to Fig.17 display modes.
Fig. 3  The inner substructure

Fig. 4  The geometric model of the bridge with four substructures

Fig. 5  The geometric model of the bridge with six substructures

TABLE I. The natural frequencies of vibration with vertical bending

|                  | A bridge with four substructures | A bridge with six substructures |
|------------------|----------------------------------|---------------------------------|
| Frequency        | 3.27 Hz                          | 2.28 Hz                         |
|                  | 10.33 Hz                         | 9.05 Hz                         |
|                  | 14.65 Hz                         | 12.95 Hz                        |

TABLE II. THE NATURAL FREQUENCIES OF VIBRATION WITH TORSION

|                  | A bridge with four substructures | A bridge with six substructures |
|------------------|----------------------------------|---------------------------------|
| Frequency        | 5.12 Hz                          | 4.61 Hz                         |
|                  | 13.36 Hz                         | 11.61 Hz                        |
|                  | 20.87 Hz                         | 18.30 Hz                        |

Fig. 6  The first vertical bending mode of the bridge with four substructures
Fig. 7  The second vertical bending mode of the bridge with four substructures

Fig. 8  The third vertical bending mode of the bridge with four substructures

Fig. 9  The first vertical bending mode of the bridge with six substructures

Fig. 10  The second vertical bending mode of the bridge with six substructures

Fig. 11  The third vertical bending mode of the bridge with six substructures

Fig. 12  The first torsion mode of the bridge with four substructures
4. Conclusion

The virtual prototype of prefabricated bridges is built by the use of the object-oriented technology and the mode synthesis method to analyze their dynamic behaviors. Simulations of two truss bridges with the same type are carried out to obtain their dynamic behaviors. Results indicate that the present model can easily study bridges in the same model.

With the wide application of fabricated structures such as fabricated bridges, the virtual prototype model based on the combination of the object-oriented technology and the mode synthesis method will be more and more widely used. If a digital structural assembly unit model library is established in the design process, the virtual prototype model established in this study can not only be used to carry out
structural optimization design quickly and effectively, but also provide technical support for subsequent structural health monitoring and reinforcement.

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