BOUNDS ON GENERIC HIGH-ENERGY PHYSICS MODIFICATIONS TO
THE PRIMORDIAL POWER SPECTRUM FROM BACK-REACTION ON
THE METRIC

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Abstract

Modifications to the primordial power spectrum of inflationary density perturbations have been studied recently using a boundary effective field theory approach. In the approximation of a fluctuating quantum field on a fixed background, the generic effect of new physics is encoded in parameters of order $H/M$. Here, we point out that the back-reaction on the metric can be neglected only when these parameters obey certain bounds that may put them beyond the reach of observation.

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1 Introduction

The possibility of observing very high-energy, “trans-planckian” physics in the cosmic microwave background radiation, thanks to the enormous stretch in proper distance due to inflation is one of the most exciting possibility for probing string theory, or any other model for quantum gravity. As such, it has received enormous attention, once the possibility was raised that these effects can be as large as $H/M$, with $H$ the Hubble parameter during inflation, and $M$ the scale of new physics (e.g. the string scale). A partial, derivative list of references is [1].

Due to our ignorance of the ultimate theory governing high-energy physics, the most natural, model-independent approach to study any modifications to the primordial power spectrum is effective field theory [2, 3]. Using an effective field theory approach, the authors of [2] concluded that the signature of any trans-planckian modification of the standard inflationary power spectrum is $O(H^2/M^2)$, well beyond the reach of observation even in the most favorable scenario ($H \sim 10^{14}\,\text{GeV}, M \sim 10^{16}\,\text{Gev}$).

This conclusion has been recently criticized in [4]. There, it was pointed out that the effect of high-energy physics manifests in two ways. First, through the appearance of irrelevant operators in the four-dimensional local field theory describing physics below the cutoff. The most relevant of these effects, studied in [2], are parametrized by local operators of dimension six, so, they are naturally of order $H^2/M^2$. Second, through a change in the initial conditions at some (arbitrary) early time $t_0$. This change of the initial quantum state can also be described in the effective field theory language, by adding appropriate boundary terms at the initial-time hypersurface. In [4], it was shown that the most relevant change in initial conditions is described in the three-dimensional boundary Lagrangian by operators of dimension four. They do generate effects of order $H/M$, generically.

Reference [4] does not take into account all effects due to the back-reaction of the modified stress-energy tensor on the metric. Specifically, any change in the boundary conditions of the effective field theory generates finite, regularization-independent modifications to expectation value of the matter stress-energy tensor. These modifications can become large near the (space-like) boundary hypersurface. By requiring that the back-reaction remain under control, we shall get new bounds on the size of the parameters encoding new physics. These bounds depend on some mild assumptions on the inflationary potential. Ranging from the weakest, most generic bound to the strongest, least generic one, we get:

$$\beta < \frac{4\pi}{GM^2} \left(\frac{H}{M}\right)^2,$$

$$\beta < \frac{8\pi}{5GM^2} \left(\frac{H}{M}\right)^3,$$
\[ \beta < \epsilon \eta \frac{8 \pi}{15GM^2} \left( \frac{H}{M} \right)^4, \]  

Here \( \beta \) is the dimensionless quantity parametrizing the effects of high-energy physics, while \( \epsilon \) and \( \eta \) are standard quantities parametrizing slow-roll inflation. They will all be defined in Section 3. \(^1\)

In this note, we re-derive for completeness the result of [4], in a formalism appropriate to our purpose, and then we proceed to derive the bounds in Eq. (3). Back-reaction in inflation was also considered within a different approach in [6].

2 Boundary Interactions and Their Effect

We follow the formalism developed in ref [4], where changes in the inflationary vacuum were parametrized by higher-dimension operators in the boundary conditions imposed on the inflaton (or any other fluctuating field) at an arbitrary “initial time” hypersurface.

We use the metric
\[ ds^2 = -dt^2 + a^2(t)dx \cdot dx. \]  

As usual, we denote the time derivative with an overdot, and we define the Hubble parameter as \( H = \dot{a}/a \). By decomposing a minimally-coupled, free scalar field \( \phi \) of mass \( m \) into plane waves of co-moving momentum \( k \), we find the equation of motion
\[ \left[ \frac{d}{dt} a^3(t) \frac{d}{dt} + a(t) k^2 + a^3(t) m^2 \right] \phi(t, k) = 0. \]  

The Feynman Green’s function obeys the equation
\[ \left[ \frac{d}{dt} a^3(t) \frac{d}{dt} + a(t) k^2 + a^3(t) m^2 \right] G_F(t, t', k) = i\delta(t - t'). \]  

This function can be written in terms of two independent solutions to the homogeneous wave equation, \( \phi^+(t, k) \) and \( \phi^-(t, k) \), as:
\[ G_F(t, t', k) = \phi^+(t, k)\phi^-(t', k)\theta(t - t') + \phi^-(t, k)\phi^+(t', k)\theta(t' - t). \]  

Eq. (5) gives us the normalization of \( \phi^\pm(t, k) \)
\[ \phi^-(t, k)\dot{\phi}^+(t, k) - \phi^+(t, k)\dot{\phi}^-(t, k) = \frac{i}{a^3(t)}. \]  

Next, we have to find the appropriate basis for \( \phi^\pm(t, k) \). In the presence of a space-like boundary at some fixed time \( t = t_0 \), the action for the scalar field is
\[ S_{\text{bulk}} + S_{\text{boundary}} = \frac{1}{2} \int d^4x \sqrt{-g} \left[ g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi + m^2 \phi^2 \right] + \frac{1}{2} \int_{t=t_0} d^3x d^3y \sqrt{h(x)} \sqrt{h(y)} \phi(x) \kappa(x, y) \phi(y). \]  

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\(^1\)Bounds on \( \beta \) were also derived on somewhat different physical grounds in [5].
Here, $h_{ij}$ is the induced metric on the boundary, and $\kappa(x,y)$ encodes the initial quantum state of the scalar $\phi$. The Bunch-Davies vacuum corresponds to choosing a specific boundary action, which, among other properties, is invariant under space translations: $\kappa(x,y) = \tilde{\kappa}_{BD}(x - y)$. By performing a Fourier transform, it can be written somewhat formally as

$$S_{\text{boundary}} = \frac{1}{2} a^3(t_0) \int \frac{d^3k}{(2\pi)^3} \left[ |\kappa_{BD}(k)|\phi^+(t_0, k)|^2 - |\kappa_{BD}(k)|\phi^-(t_0, k)|^2 \right]. \quad (10)$$

The precise form of $\kappa_{BD}(k)$ is immaterial to our computation. What matters is that two modes $\phi^\pm(t,k)$ obey

$$\left[ -\frac{d}{dt} \pm \kappa_{BD} \right] \phi^\pm(t,k) \bigg|_{t=t_0} = 0. \quad (11)$$

Actually, Eq. (11) is valid more generally. By considering a generic translation-invariant boundary term $\kappa(x,y) = \tilde{\kappa}(x - y)$, and extremizing with respect to all variations $\delta \phi$ (including those that do not vanish at the boundary) we find the equation

$$\left[ -\frac{d}{dt} \pm \kappa \right] \phi^\pm(t,k) \bigg|_{t=t_0} = 0, \quad (12)$$

where $\kappa(k)$ is of course the Fourier transform of $\tilde{\kappa}(x)$.

Suppose now that the boundary conditions are changed. To take into account the possibility that the change is different for the two modes, we define $\kappa^\pm \equiv \kappa_{BD} + \delta \kappa^\pm$, $\kappa^- \equiv \kappa_{BD} + \delta \kappa^-$. To first order in the change, we have

$$\left[ -\frac{d}{dt} \pm \kappa_{BD} \right] \delta \phi^\pm(t,k) \bigg|_{t=t_0} = 0. \quad (13)$$

Now, expand the modes $\delta \phi^\pm(t,k)$ to first order in the perturbation as

$$\delta \phi^+(t,k) = \delta b^+ \phi^-(t,k), \quad \delta \phi^-(t,k) = \delta b^- \phi^+(t,k). \quad (14)$$

Substituting this expansion into Eq. (13), we find

$$\delta b^+ = \frac{1}{2} \delta \kappa^+ \frac{\phi^+(t_0,k)}{\phi^-(t_0,k)}, \quad \delta b^- = -\frac{1}{2} \delta \kappa^- \frac{\phi^-}{\phi^+(t_0,k)}. \quad (15)$$

Define next the symmetric Green’s function $G^{(1)}(t,t') = (1/2)\langle 0|\phi(t)\phi(t') + \phi(t')\phi(t)|0\rangle$. After using the normalization condition Eq. (5), and the identity $\phi^-(t_0,k)\phi^+(t_0,k) = -\dot{\phi}^-(t_0,k)\phi^+(t_0,k)$, we can express the change in $G^{(1)}(t,t')$ as

$$\delta G^{(1)}(t,t',k) = ia^3(t_0) \left[ \delta \kappa^+ \phi^{+2}(t_0,k)\phi^-(t,k)\phi^-(t',k) + \delta \kappa^- \phi^{-2}(t_0,k)\phi^+(t,k)\phi^+(t',k) \right]. \quad (16)$$

$G^{(1)}$ is particularly useful when computing expectation values of composite operators by point-splitting. In the next section we use it to compute one such operator: the stress-energy tensor.
3 Back-Reaction and Bounds

We will consider just one example of IR irrelevant boundary perturbation encoding the effects of high-energy physics. It is the dimension-four boundary term

$$\delta S_{\text{boundary}} = a(t_0) \frac{\beta}{2M} \int d^3 x (\nabla \phi)^2. \tag{17}$$

It induces a change \( \delta \kappa^\pm(k) = \pm \beta k^2 / a^2(t_0) M \). From now on, we shall choose for convenience \( a(t_0) = 1 \).

The change in boundary conditions induces a change in \( G^{(1)}(t,t') \). This change is finite [see Eq. (16)] and regularization independent away from the boundary, i.e. as long as \(|t - t_0| > 1/M, |t' - t_0| > 1/M\)\(^2\). Now, when we compute the expectation value of the stress-energy tensor \( T_{\mu\nu} = \partial_{\mu} \phi \partial_{\nu} \phi - (1/2) g_{\mu\nu} (\partial_\lambda \phi \partial_\lambda \phi + m^2 \phi^2) \) by using \( G^{(1)}(t,t') \), we get a new contribution to first order in \( \delta \kappa^\pm \). Notice that in our formalism one does not have to independently specify the vacuum state: that information is already contained in \( G^{(1)}(t,t') \).

Again, the first-order contribution is unambiguous and finite as long as \(|t - t_0| > 1/M\). The effect of the perturbation in, say, \( \langle 0 | T^\mu_{\mu}(t,x) | 0 \rangle \), is easy to estimate for times \( 1/M < |t - t_0| < 1/M_0 \), where \( M_0 \equiv \max(H,m) \). In this case, the VEV is determined by an integration over momenta \( M > k \gg M_0 \). For these momenta, much larger than either \( H \) or \( m \), we have \( (\Delta t \equiv |t - t_0|) \):

$$\phi^+(t,k) \phi^-(t_0,k) \approx \frac{1}{2|k|} e^{i|k|\Delta t}, \quad \phi^-(t,k) \phi^+(t_0,k) \approx \frac{1}{2|k|} e^{-i|k|\Delta t}, \quad 1/M < \Delta t \ll 1/M_0. \tag{18}$$

By using this estimate, we find

$$\delta \langle 0 | T^\mu_{\mu}(t,x) | 0 \rangle \approx \frac{\beta}{M} \int_{|k| > M_0} \frac{d^3 k}{(2\pi)^3} |k|^2 \sin(2|k|\Delta t) \approx \frac{3}{16\pi^2 M} \left( \frac{\beta}{M} \right)^2 (\Delta t)^{-5}, \quad 1/M < \Delta t \ll 1/M_0. \tag{19}$$

Equation (19) is our main estimate. Now it is clear what to do next: we substitute Eq. (19) into Einstein’s equations for the background geometry and we ask ourselves which bounds must be satisfied by the “new-physics” parameter \( \beta \) so that the back-reaction is negligible at all times \( \Delta t > 1/M \)\(^3\).

The first bound, Eq. (11), is the most general. It follows from demanding that the change in the vacuum energy, proportional to \( \delta \langle 0 | T^\mu_{\mu}(t,x) | 0 \rangle \), is much smaller than the unperturbed vacuum energy \( V = 3H^2 / 8\pi G \). Setting \( \Delta t \sim 1/M \) we find

$$\delta \langle 0 | T^\mu_{\mu}(t_0 + 1/M,x) | 0 \rangle = \frac{3}{16\pi^2} \beta M^4 < \frac{3H^2}{4\pi G}, \tag{20}$$

whence Eq. (11). The bound is parametrically \( H^2/M^2 \), but it is multiplied by a coefficient \( 2\pi/GM^2 \), which can be easily as large as \( 10^6 \) if \( M \sim 10^{16} \) GeV.

\(^2\)To play safe, one should take an \( M' \) somewhat smaller than \( M \). This will not affect significantly our estimates.

\(^3\)Clearly, at \( \Delta t < 1/M \) our estimate becomes ambiguous, i.e. regularization-dependent.
The second bound comes from demanding that an inflationary period exists for some time after $t_0$. This is reasonable whenever the scale of inflation, $H$, is smaller than the scale of new physics, $M$. It requires the Hubble parameter to remain almost constant over a Hubble time, i.e. $\dot{H} \ll H^2$. Since Eq. (19) introduces a time dependence into $H$, we must set

$$|\delta \dot{H}| = \frac{5}{2H} \frac{8\pi G}{3} \frac{3}{16\pi^2} \beta M^5 < |\dot{H}|$$ (21)

By introducing the slow-roll parameter $\epsilon = |\dot{H}/H^2|$, we get Eq. (2). For the choice of parameters $H = 10^{14}$ GeV, $M = 10^{16}$ GeV, we can allow for a coefficient $\beta \sim \epsilon$, which may still fall within the range of future detectability.

The third bound requires a further assumption, namely that the quantum-corrected potential, inclusive of $\delta \langle 0 | T_{\mu\nu} | 0 \rangle$, can be written as a function of a single scalar $\phi$ (the inflaton e.g.). If this is the case, then it is easy to show that, by introducing a second slow-roll parameter, $\eta = |\ddot{\phi}/H\dot{\phi}|$, we have $2\epsilon \eta = |\ddot{H}/H^3|$. So, from

$$|\delta \ddot{H}| < |\ddot{H}| = 2\epsilon \eta H^3,$$ (22)

we arrive at the last bound, Eq. (3). This is the most stringent bound: by requiring only that both $\eta$ and $\epsilon$ have “standard,” $O(10^{-1})$ values, we find $\beta \sim \epsilon \eta 10^{-2} \sim 10^{-4}$, easily below any chance of detection.

Acknowledgments

We would like to thank G. Gabadadze and L. Randall for comments. M.P. is supported in part by NSF grants PHY-0245068 and PHY-0070787.

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