Toward Practical Differential Privacy in Smart Grid with Capacity-Limited Rechargeable Batteries

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ABSTRACT
The technology of differential privacy, adding a noise drawn from the Laplace distribution, successfully overcomes a difficulty of keeping both the privacy of individual data and the utility of the statistical result simultaneously. Therefore, it is prevalent to use a rechargeable battery as the noise for achieving differential privacy in the application of smart grid. Unfortunately, to the best of our knowledge, we observe that the existing privacy protection mechanisms cannot satisfy differential privacy, when considering physical restrictions of the battery in practice. In this paper, we first classify two types of challenges caused by two physical restrictions, the maximum charging/discharging rate and the capacity of battery. We then propose a stateless privacy protection scheme by exploring a boundary-changeable distribution for noise, and prove this scheme satisfies differential privacy, with regard to the first type of challenge. We further explain the difficulty to achieve differential privacy under the second type of challenge, and formalize the definition of a relaxed differential privacy. Finally, we present a stateful privacy protection scheme that satisfies the relaxed differential privacy. Experimental analysis shows that the maximum privacy leakage of our privacy protection schemes at each time point stably outperforms that of the existing work.

Categories and Subject Descriptors
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Keywords
Differential Privacy, Physical Restrictions, Battery, Smart Grid

1. INTRODUCTION

The definition of differential privacy \cite{7} indicates that it is negligible to discern two databases where only one entry is different, even the statistical result corresponding to an arbitrary query of the entire database has been acquired. Technically, a noise drawn from a $(0, \sigma)$ Laplace distribution is added to the result for the purpose of perturbing the statistical result, in order to satisfy differential privacy. In fact, achieving differential privacy brings us two benefits \cite{7}. For one thing, it is negligible to exclude any possible value of any entry of data in the database, according to the perturbed result. This preserves the privacy of individual data. For another, now that the math expectation of the noise’s distribution is zero \cite{9}, the difference between the real result and the perturbed result is negligible in the statistics \cite{8}. This assures data utility.

It is worthwhile to apply differential privacy to smart grid, because the two benefits above successfully overcome the difficulty of keeping both the privacy of individual data and the utility of the statistical result simultaneously. Intuitively, after the power generator, as a data curator, adds a noise to the statistical result of all the households’ smart meter readings, the perturbed result is still approximately accuracy, while the value of the personal data is hard to be inferred from the perturbed result simultaneously. However, the data curator itself becomes a threat to user’s privacy in recent years \cite{1,20,25,30}.

The electricity power supplier has the ability to compute their users’ activities at home, when analyzing the intermittent meter readings by some Non-intrusive Load Monitoring (NILM) algorithms \cite{5,10,21,23}. Therefore, it’s becoming more and more difficult to persuade residents to use smart meter \cite{15}. To dismiss customers’ doubts about privacy, an intuitive idea is to add some theoretical random numbers before a smart meter sends its readings. But this is impractical, because user’s bill will not be as precise as before, and no customer wants to pay more bill than one should do. An alternative idea is to install a rechargeable battery in the house. The battery is used to generate some noises by charging or discharging, so as to perturb the user’s real electricity consumption \cite{14}. Furthermore, considering the
energy saved in the battery will eventually be provided to the house electrical appliances, users will not pay more bill than they really need to do. So far, a series of battery-based privacy protection schemes have already been proposed \cite{1,2,3,4,5,6,7}. Unfortunately, to the best of our knowledge, none of these schemes satisfies the definition of differential privacy in practice.

The major obstacle to achieve differential privacy originates from the physical restrictions of data and noise in smart grid. More specifically, the user’s real consumption definitely has a lower bound which is zero, and a upper bound based on the maximum energy consumptions of all the electricity appliances in a house. So does the noise generated by the battery, because all the batteries have their own maximum charging/discharging rate and capacity. Consequently, the range of meter readings is changeable for different user’s consumptions. According to our observation, this provides an opportunity for the power supplier to exclude some possible values of user’s real consumption with respect to some certain meter readings, which leads the scheme cannot satisfy differential privacy for those values. Although Backes et al. discussed the physical limitation of battery in smart grid \cite{3}, their scheme merely proved the impact of physical limitation might be small in some cases but not tackle with the problem. Since attacks often occur at the most vulnerable segment in the whole system, it is nontrivial to face the problem. Zhao et al. tried to propose a differential privacy protection scheme in smart grid \cite{4}, but their proof was not complete when the physical limitation of noise is considered.

The contribution of this paper is summarized as follows:

- We first investigate the problems caused by the physical restrictions of data and noise, and categorize them into two classes: the stateless and the stateful. To meet the stateless challenge, we then propose a privacy protection scheme by exploring a boundary-changeable distribution for noise, and prove the scheme satisfies differential privacy theoretically.

- We further explain the difficulty to achieve differential privacy for the stateful challenge, and formalize a relaxed differential privacy definition. We also present a stateful privacy protection scheme that can adaptively adjust the noise’s distribution, and then prove the scheme satisfies the relaxed differential privacy.

- We implement the two privacy protection schemes in smart grid, and compared our work with the existing work. Experimental analysis shows that the privacy protection of our schemes outperform that of those work.

The rest of this paper is organized as below:

Section 2 briefly recalls the definition of differential privacy and Laplace distribution. Section 3 analyzes the problem and categorizes the challenge. Section 4 proposes a stateless privacy protection scheme, and proves the scheme satisfies differential privacy. Section 5 defines a relaxed differential privacy, and proposes a stateful privacy protection scheme. Besides, that scheme is also proved to satisfy the definition of the relaxed differential privacy. Section 6 compares the privacy leakage of our privacy protection schemes with that of the existing work in the experiments. Section 7 discusses the practicability of our schemes. In section 8, we reviews the related work. Section 8 draws the conclusion.

2. THE PRELIMINARIES

We first follow the formal definition of differential privacy by Dwork \cite{8}.

**Definition 1.** A randomized function $\kappa$ satisfies $(\epsilon, \delta)$ differential privacy, if for all data sets $D$ and $D'$ differing on at most one element, and all $S \subseteq Range(\kappa)$,

$$\Pr[\kappa(D) \in S] \leq \exp(\epsilon) \times \Pr[\kappa(D') \in S] + \delta$$

(1)

where $Range(\kappa)$ denotes the range of the function $\kappa$. $\epsilon$ and $\delta$ are two parameters used to quantitatively measure privacy. Here $0 \leq \delta < 1$; otherwise, achieving differential privacy becomes meaningless to preserve the privacy \cite{8,2,3}. Roughly, inequality (1) implies that the influence of any individual data is negligible to the statistical result, since $\epsilon$ often approaches to 0 in practice. Hence, the privacy of users cannot be computed by analyzing the statistical result. In addition, the more $\epsilon$ and $\delta$ approach to 0, the better privacy is protected \cite{8}.

Dwork et al. \cite{8} achieve differential privacy by adding a stochastic noise drawn from the Laplace distribution \cite{9} to the statistical result. Thus, we then briefly introduce the Laplace distribution. The probabilistic density function of a Laplace distribution is shown as below:

$$f(x) = \frac{1}{2\sigma} e^{-\frac{|x-\mu|}{\sigma}}$$

(2)

Typically, $\mu$ is often equal to 0, and $\sigma$ is determined by the sensitivity which represents the biggest impact of individual data on the result of function $f$.

Finally, the sensitivity \cite{2} is formally defined as follows:

**Definition 2.** For $f : D \rightarrow R^d$, the sensitivity of $f$ is

$$\Delta f = \max_{D,D'}|f(D) - f(D')|$$

(3)

for all $D$ and $D'$ differing in at most one element.

To achieve differential privacy, $\Delta f / \epsilon \leq \sigma$ \cite{2,26}.

3. TECHNICAL CHALLENGES

In this section, we aim to elaborate and categorize the problems brought by the physical restrictions of battery in smart grid. Specifically, we first recall how to achieve differential privacy theoretically. Next, we discuss two new challenges caused by the physical restrictions of data and noise, when achieving differential privacy in practice. We finally classify the challenges into two groups.

3.1 Theoretical Differential Privacy

Assume $d$ is an arbitrary entry of a data set $D$, while $\kappa(D)$ represents some statistical result of $D$, according to the formal definition of differential privacy. When $n$ denotes a stochastic noise chosen from the Laplace distribution, the output $o$ of a traditional differential privacy protection scheme is often equal to $\kappa(D) + n$. If the sensitivity depends on the maximum difference of two arbitrary entries, this scheme is regarded to satisfy differential privacy \cite{8,2,3}. This is because for all the possible $d_k$ and $d_l(k, l \in \mathbb{N})$ in the two data sets $D$ and $D'$ where only one entry is different, we have
Table 1: Data set D in smart grid

| Appliance  | Wattage (kWh) |
|------------|---------------|
| Computer   | d_1 = 0.054   |
| Refrigerator| d_2 = 0.171   |
| Rice Cooker| d_3 = 0.627   |
| Dish Washer| d_4 = 0.967   |
| Hair Drier | d_5 = 1.239   |
| Oven       | d_6 = 1.438   |
| Queries    | \(\kappa(D_i)\) | 0.171 0.225 |

\[
\Pr[\omega = \kappa(D) + n] = \frac{\Pr[n = \omega - \kappa(D)]}{\Pr[n' = \omega - \kappa(D')]},
\]

\[
= \lim_\Delta x \to 0 \frac{e^{(\omega - \kappa(D) + \Delta x)/\sigma} \Phi\left(\frac{\omega - \kappa(D) + \Delta x}{\sigma}\right)}{e^{(\omega - \kappa(D') + \Delta x)/\sigma} \Phi\left(\frac{\omega - \kappa(D') + \Delta x}{\sigma}\right)} = e^{\frac{(\omega - \kappa(D) - \omega + \kappa(D'))}{\sigma}} \leq e^{\sigma |d_k - d_l|/\sigma} \leq e^{\epsilon + \delta}
\]

In the application of smart grid, assume \(i, (i \in \mathbb{N})\) stands for the \(i\)th point in the time sequence, \(d_i\) represents the electricity consumption of the \(i\)th appliance, and therefore \(d_i\) indicates the energy consumption of the \(i\)th appliance opening at time \(i\). The data set \(D\) is composed of the consumptions all the electrical appliances at all the time points for each customer. An example with six electricity appliances is shown in Table 1. Here, all the queries are composed of the meter readings. \(\kappa(D_i)\) stands for the whole energy consumptions of all the appliances at time \(i\). Since different household appliances consume different wattage, the power supplier could apply some NILM algorithms to detect what appliances are being used by analyzing the set \(\{\kappa(D_i)\}_{i \in \mathbb{N}}\). To preserve household’s privacy, a rechargeable battery is used to generate some extra noises by charging or discharging to achieve differential privacy. Here the sensitivity equals the maximum difference of the wattage for all the pairs of appliances, which is exactly \(\max |d_k - d_l|\) for all the possible \(k\) and \(l\).

### 3.2 Practical Differential Privacy

In this part, we are first to illustrate why the typical way cannot achieve differential privacy under bounded charging and discharging rates of battery in smart grid.

As we discussed before, all the \(\kappa(D_i)\) have a lower bound \(\alpha\) and a upper bound \(\beta\), because the amount of household’s consumption \(\kappa(D_i)\) at time \(i\) is limited in the area from zero to a specific positive number on account of the number of the electricity appliances in the house. Meanwhile, the discharging rate is often regarded as a negative number, and the charging rate is viewed as a positive number by convention. Therefore, all the \(n_i\) also have a lower bound \(\gamma\) and a upper bound \(\beta\) at time \(i\), according to the maximum of the discharging/charging rate of the battery.

We then demonstrate that the ranges of the outputs for two arbitrary entries may be different. For example, as shown in Figure 1, the range of output is \([\alpha + \gamma, \alpha + \eta]\), when \(\kappa(D_i) = \alpha\). But that range changes to \([\beta + \gamma, \beta + \eta]\), when \(\kappa(D_i) = \beta\). This incurs differential privacy has not been achieved any more. A simple counterexample is that all the appliances must be closed, when the output is \(\alpha + \gamma\), because the output \(o_1 > \alpha + \gamma\), so long as \(d_1 > 0\). Similarly, all the appliances are opening, if the output is \(\beta + \eta\), because the output \(o_i < \beta + \eta\), when \(d_i < \beta\). Consequently, the interval from \(\beta + \gamma\) to \(\alpha + \gamma\) is defined as the legal zone, while the interval from \(\alpha + \gamma\) to \(\beta + \eta\) and from \(\alpha + \eta\) to \(\beta + \eta\) are denoted as the restricted zone. Essentially, once the output is in the restricted zone, differential privacy cannot be achieved any more. Formally,

**Claim 1.** \((\epsilon, \delta)\) differential privacy cannot be satisfied, if there is an output in the restricted zone.

**Proof.** Assume that output \(o_i\) at time \(i\) is in the restricted zone, there must exist some \(\kappa(D_i), n_i\), such that \(o_i = \kappa(D_i) + n_i\). In the meantime, there must also exist some \(\kappa(D_i')\), such that \(o_i \neq \kappa(D_i') + n_i'\) for all the possible \(n_i' (\gamma \leq n_i' \leq \eta)\). This leads \(\Pr[o_i = \kappa(D_i) + n_i] = 0\). However, to satisfy differential privacy, we have

\[
\Pr[o_i = \kappa(D_i) + n_i] = \Pr[o_i = \kappa(D_i') + n_i'] \cdot e^\epsilon + \delta
\]

Since this equation has to be true for all the possible \(n_i (\gamma \leq n_i \leq \eta)\), \(\delta\) has to be no less than 1. This breaks Definition \(\in\).

We complete the proof. \(\square\)

Since this claim makes the requirement that output has to be generated from the legal zone become the necessary condition for achieving differential privacy in practice, our first challenge is to design a new noise distribution, such that:

1. All the possible values of the outputs are limited in the legal zone. Technically, the probability that the output is in \([\beta + \gamma, \alpha + \eta]\) is 1. Otherwise, any probability with which the output is beyond that interval should always be 0.

2. The scheme using this noise distribution should be formally proved to satisfy differential privacy, when noise is drawn from the new distribution. After all, we have not proved this requirement is the sufficient condition to achieve differential privacy yet.

Since the noise is restricted not only by the maximum charging and discharging rate of the battery, but also by the capacity of the battery in smart grid, we have to continue to take the capacity limitation of the battery into consideration. Assume that \(C\) represents the capacity of the battery, and \(c_i\) stands for the electricity stored in the battery at time \(i\). The range of the noise \(n_i\) has to be from \(-c_i\) to \(C - c_i\) at time \(i\), which is a new restriction of noise.
In fact, our first challenge will become more complicated when we consider the bounded capacity of the battery, because the range of the legal zone itself will be variable. We show more details as follows.

\( c_i \) is the fatal barrier to impede our steps from achieving differential privacy, because the key point to satisfy differential privacy is to keep the same range for all the outputs. Unfortunately, this cannot be hold now. For example, when \( c_i = C \) at time \( i \), the lower bound of the legal zone is \( \beta + \gamma \), and the upper bound of the legal zone has to be 0, because the battery is full. In contrast, when \( c_i' = 0 \), the lower bound has to be 0, and the upper bound is \( \alpha + \eta \), since the battery cannot provide energy at all. Hence, the data set must be \( D'_i \) if \( c_i < 0 \); otherwise, it is \( D_i \).

Since the lower and upper boundaries of output are variable regarding to \( c_i, (i \in \mathbb{N}) \), a dilemma consequently occurs. For one thing, we have to maintain the same range of all the outputs to satisfy differential privacy. For another, \( c_i \) leads the ranges of two outputs may be different, which causes the legal zone. After analyzing this dilemma, two changes essentially occur. One is there are multiple lower and upper boundaries of the output, when we consider the charging/discharging rate and the capacity of battery together. The other is the boundaries of output are not constant. Intuitively, we can easily select the intersection of all the boundaries and reduce the problem to the first challenge. More precisely, the upper bound is set to \( \min(\alpha + \eta, C - c_i) \), and the lower bound is set to \( \max(-c_i, \beta + \gamma) \). However, this way does not work, because the example in the last paragraph still be valid. Technically, when the legal zone itself is not constant, it is difficult to achieve differential privacy due to Claim \( \square \).

Since it is difficult to completely get rid of the dilemma above, our second challenge is to design a privacy protection scheme that achieves a relaxed version of differential privacy.

In sum, we classify the challenges into two groups: the stateless and the stateful, in accordance with the two type of challenges. The former is merely consider the bounded charging and discharging rate of the battery, while the latter consider both bounded rate and capacity of the battery.

4. THE PROPOSED SCHEMES

In this section, we first design a stateless privacy protection scheme that satisfies differential privacy. Later, we formalize a relaxed definition as a variant of differential privacy. We finally propose a stateful privacy protection scheme that satisfies the relaxed differential privacy.

4.1 A Stateless Privacy Protection Scheme

To meet the stateless challenge, we construct a new distribution of noise, such that the output perturbed by the noise conceals the privacy of individual data. We first determine the range of noise. Since the range of output \( o_i \) at time \( i \) should be maintained in the interval from \( \beta + \gamma \) to \( \alpha + \eta \), and \( o_i = \kappa(D_i) + n_i \), the range of noise has to be kept in the interval from \( \beta + \gamma - \kappa(D_i) \) to \( \alpha + \eta - \kappa(D_i) \).

The core idea to construct the distribution of noise is to proportionally adjust the Laplace distribution from the range of \((-\infty, +\infty)\) to the range of \([\beta + \gamma - \kappa(D_i), \alpha + \eta - \kappa(D_i)]\). The total probability \( T \) beyond this range in the \((\mu, \sigma)\) Laplace distribution is computed by Equation \( \square \).

\[
T = \int_{-\infty}^{\beta + \gamma - \kappa(D_i)} f(x) \, dx + \int_{\alpha + \eta - \kappa(D_i)}^{+\infty} f(x) \, dx
= \int_{-\infty}^{\beta + \gamma - \kappa(D_i)} \frac{1}{2\sigma} e^{-\frac{|x - \mu|}{\sigma}} \, dx + \int_{\alpha + \eta - \kappa(D_i)}^{+\infty} \frac{1}{2\sigma} e^{-\frac{|x - \mu|}{\sigma}} \, dx
= \int_{-\infty}^{\mu} \frac{1}{2\sigma} e^{-\frac{x - \mu}{\sigma}} \, dx + \int_{\mu}^{\beta + \gamma - \kappa(D_i)} \frac{1}{2\sigma} e^{-\frac{x - \mu}{\sigma}} \, dx + \int_{\mu}^{+\infty} \frac{1}{2\sigma} e^{-\frac{x - \mu}{\sigma}} \, dx
= 2 - \frac{1}{2} \frac{e^{-\frac{\beta + \gamma - \kappa(D_i) - \mu}{\sigma}} - e^{-\frac{\alpha + \eta - \kappa(D_i) - \mu}{\sigma}}}{\sigma}
\]

(4)

Since \( T \) denotes the probability of a Laplace distribution in the union of \((-\infty, \beta + \gamma - \kappa(D_i))\) and \((\alpha + \eta - \kappa(D_i), +\infty)\), we have \( 0 < T < 1 \).

Now we define the probability density function \( pdf(x) \) of the new distribution as below:

\[
pdf(x) = \begin{cases} 
\frac{1}{2\sigma} e^{-\frac{|x - \mu|}{\sigma}} + \frac{T}{\alpha + \eta - \beta - \gamma} & \beta + \gamma \leq o_i \leq \alpha + \eta \\
0 & \text{otherwise}
\end{cases}
\]

(5)

Note that the condition of \( \beta + \gamma \leq o_i \leq \alpha + \eta \) is equivalent with that of \( \beta + \gamma - \kappa(D_i) \leq n_i \leq \alpha + \eta - \kappa(D_i) \), because \( o_i = \kappa(D_i) + n_i \). We left to prove that \( pdf(x) \) is a valid probability density function in the Appendix.

Assume that the total number of appliances is \( M \) in the house. The power consumption of each appliance at time \( i \) constitutes the set \( \{d_{ij} | j \in \mathbb{N}, j \leq M \} \). The specification of the stateless privacy protection scheme is shown as below.

Alg. 1. The stateless privacy protection scheme

Input: \( \alpha, \beta, \gamma, \eta, \{d_{ij} | i \in \mathbb{N}, j \leq M \} \) and \( \epsilon \).

Output: \( n_i \) or \( \bot \).

1. If \( \alpha + \eta \leq \beta + \gamma \) return \( \bot \).
2. Return \( \bot \).
3. \( \kappa(D_i) = \sum_{j=1}^{M} d_{ij} \).
4. For all \( k, l \leq M \), \( \Delta f = \max |d_k - d_l| \).
5. \( \mu = 0 \), \( \sigma = \frac{\Delta f}{\epsilon} \).
6. \( T = 2 - \frac{1}{2} \frac{e^{-\frac{\beta + \gamma - \kappa(D_i) - \mu}{\sigma}} - e^{-\frac{\alpha + \eta - \kappa(D_i) - \mu}{\sigma}}}{\sigma} \).
7. \( pdf(x) = \frac{1}{2\sigma} e^{-\frac{|x - \mu|}{\sigma}} + \frac{T}{\alpha + \eta - \beta - \gamma} \).
8. \( n_i \leftarrow pdf(x) \).
9. Return \( n_i \).

The goal of the stateless privacy protection scheme is to generate an appropriate \( n_i \) for perturbing \( \kappa(D_i) \), so as to achieve \( (\epsilon, \delta) \) differential privacy. The crucial factor to attain our goal is to keep the range of meter readings same all the time. Hence, we first compute the lower and upper bounds for the meter readings. If the lower bound is no less than the upper bound, it is impossible to achieve differential privacy. Fortunately, this problem is simple to be avoided, when \( \alpha + \eta \leq \beta + \gamma \). After that, the remaining is to compute all the parameters of our distribution, and then pick up a random noise from that.

Finally, we prove our stateless privacy protection scheme achieves the definition of differential privacy.

Theorem 1. The stateless privacy protection scheme achieves \( (\epsilon, \delta) \) differential privacy.
Proof. Assume that \( D \) and \( D' \) are two arbitrary data sets where only one entry is different. Since \( o_i = \kappa(D_i) + n_i \), for any value of \( o_i \), we have

\[
\frac{\Pr[o_i = \kappa(D_i) + n_i]}{\Pr[o_i = \kappa(D_i') + n_i']} = \frac{\Pr[n_i = o_i - \kappa(D_i)]}{\Pr[n_i' = o_i - \kappa(D_i)']} \leq \frac{\int_{\frac{o_i - \kappa(D_i) + \Delta x}{\sigma}}^{\frac{o_i - \kappa(D_i) + \Delta x}{\sigma}} \left( e^{\frac{-|x|}{\sigma}} + \frac{2}{\alpha + \eta - \beta - \gamma} \right) dx}{\int_{\frac{o_i - \kappa(D_i) + \Delta x}{\sigma}}^{\frac{o_i - \kappa(D_i) + \Delta x}{\sigma}} \left( e^{\frac{-|x|}{\sigma}} + \frac{2}{\alpha + \eta - \beta - \gamma} \right) dx},
\]

where

\[
T_1 = 2 - \frac{1}{2} e^{\frac{-\beta + \gamma - \kappa(D_i) - \mu}{\sigma}} - \frac{1}{2} e^{\frac{-\alpha + \eta - \kappa(D_i) - \mu}{\sigma}}
\]

\[
T_2 = 2 - \frac{1}{2} e^{\frac{-\beta + \gamma - \kappa(D_i') - \mu}{\sigma}} - \frac{1}{2} e^{\frac{-\alpha + \eta - \kappa(D_i') - \mu}{\sigma}}
\]

and

\[
\sigma = \Delta f/\epsilon, \quad T_1, T_2 > 0,
\]

where \( \sigma = \Delta f/\epsilon \). When \( o_i \in [\beta + \gamma, \alpha + \eta] \), \( \mu = 0 \), \( \Delta f = \max|d_k - d_l| \), and \( T_1, T_2 > 0 \), we have

\[
\lim_{\Delta x \to 0} \frac{\int_{\frac{o_i - \kappa(D_i) + \Delta x}{\sigma}}^{\frac{o_i - \kappa(D_i) + \Delta x}{\sigma}} \left( e^{\frac{-|x|}{\sigma}} + \frac{T_1}{\alpha + \eta - \beta - \gamma} \right) dx}{\int_{\frac{o_i - \kappa(D_i) + \Delta x}{\sigma}}^{\frac{o_i - \kappa(D_i) + \Delta x}{\sigma}} \left( e^{\frac{-|x|}{\sigma}} + \frac{T_2}{\alpha + \eta - \beta - \gamma} \right) dx} = \frac{1}{2} e^{\frac{-\beta + \gamma - \kappa(D_i) - \mu}{\sigma}} + \frac{T_1}{\alpha + \eta - \beta - \gamma} + \frac{T_2}{\alpha + \eta - \beta - \gamma} \leq e^\epsilon + \frac{T_1}{T_2} \leq e^\epsilon + \frac{T_1}{T_2}
\]

**Definition 3.** Assume \( 0 \leq \lambda \leq 1 \). A privacy protection scheme satisfies \((\lambda, \epsilon, \delta)\) differential privacy, if this scheme satisfies \((\epsilon, \delta)\) differential privacy with the probability \( \lambda \).

Here \( \lambda \) is used to show the probability that a privacy protection scheme achieves differential privacy. In other words, this definition permits a privacy protection scheme to break \((\epsilon, \delta)\) differential privacy with the probability \( 1 - \lambda \). The more \( \lambda \) approaches to 1, the better the scheme achieves differential privacy. Theoretically, when \( \lambda = 1 \), this definition is totally same as that of \((\epsilon, \delta)\) differential privacy.

Finally, we aim to propose a stateful privacy protection scheme, and prove it satisfies \((\lambda, \epsilon, \delta)\) differential privacy.

Recall that the range of the legal zone should always be the same, in order to achieve differential privacy. However, breaking this rule is inevitable under the stateful challenge in practice as we discussed before. Thus, our fundamental strategy is to reduce the probability it may occurs. Specifically, we adjust our distribution of noise to increase the probability of charging the battery when \( c_i \) approaches 0; meanwhile, we adjust the distribution to increase the probability of discharging the battery when \( c_i \) approaches \( C \).

We implement this strategy by changing the value of the parameter \( \mu \) in our noise distribution adaptively. For concreteness, assume that \( \mu_0 \) denotes the lower bound of \( \mu \), and \( \mu_u \) represents the upper bound of \( \mu \). The \( \mu \) at time \( i \) is determined as follows:

\[
\mu = \frac{c_i - 1}{C}(\mu_i - \mu_u) + \mu_u
\]

(6)

We can see that \( \mu = \mu_u \) when \( c_i = 0 \), and \( \mu = \mu_i \) when \( c_i = C \). Moreover, the more \( c_i \) approaches 0, the bigger the probability of charging the battery has, and vice versa. In addition, the charging and discharging rate is adaptive according to \( c_i \).

We propose our stateful privacy protection scheme as below:

**Alg. 2.** The stateful privacy protection scheme

**Input:** \( \alpha, \beta, \gamma, \eta, C, c_i, \mu_i, \mu_u, \{d_j^t\in \mathbb{N}, j \leq M\} \) and \( \epsilon \).

**Output:** \( n_j \) or \( \perp \).

1. If \( \alpha + \eta \leq \beta + \gamma \)
2. Return \( \perp \)
3. \( \kappa(D_i) = \sum_{j=1}^{M} d_j^t \)
4. For all \( k, l \leq M \), \( \Delta f = \max|d_k - d_l| \)
5. \( \mu = \frac{c_i - 1}{C} (\mu_i - \mu_u) + \mu_u \), \( \sigma = \Delta f/\epsilon \)
6. \( T = 2 - \frac{2}{\epsilon} - \frac{\epsilon}{\alpha + \eta - \beta - \gamma} \)
7. pdf \((x) = \frac{1}{2} e^{-\frac{\epsilon x}{\alpha + \eta - \beta - \gamma}} + \frac{T}{\alpha + \eta - \beta - \gamma} \)
8. Do
9. \( n_i \leftarrow \text{pdf}(x) \)
10. While \( (\kappa(D_i) + n_i - c_{i-1} > C|\kappa(D_i) + n_i - c_{i-1} < 0) \)
11. \( c_i = c_{i-1} + n_i \)
12. Return \( n_i \)

Finally, we prove our stateful privacy protection scheme satisfies the relaxed differential privacy.

**Theorem 2.** The stateful privacy protection scheme achieves \((\lambda, \epsilon, \delta)\) differential privacy.

**Proof.** The proof consists of two steps. The first step is to prove our stateful privacy protection scheme satisfies \((\epsilon, \delta)\) differential privacy, when \( (\beta + \gamma) \leq c_{i-1} \leq (\alpha + \eta) \).

4.2 A Stateful Privacy Protection Scheme

In this section, we define a relaxed differential privacy. We then propose a stateful privacy protection scheme, and prove our scheme satisfies the relaxed differential privacy.

We define a new relaxed differential privacy called \((\lambda, \epsilon, \delta)\) differential privacy as below.

We complete the proof. \( \square \)

According to Equation 3 and Theorem 1, our stateless privacy protection scheme satisfies the two requirements of our first challenge introduced in Section 3.2.

### 4.2 A Stateful Privacy Protection Scheme

In this section, we define a relaxed differential privacy. We then propose a stateful privacy protection scheme, and prove our scheme satisfies the relaxed differential privacy.

We define a new relaxed differential privacy called \((\lambda, \epsilon, \delta)\) differential privacy as below.
The second step is to prove the probability that $c_j < (\beta + \gamma)$ or $c_j > (\alpha + \eta)$ is no more than $1 - \lambda$, for all $j \geq 1$.

Step 1. For $(\beta + \gamma) \leq c_{j-1} \leq (\alpha + \eta)$:

Assume that $D_i$ and $D'_i$ are two arbitrary data sets where only one entry is different. Since $\alpha_i = \kappa(D_i) + \eta_i$, for any value of $\alpha_i$, we have

$$
\Pr[\alpha_i = \kappa(D_i) + \eta_i] = \Pr[n_i = \alpha_i - \kappa(D_i)]
$$

$$
\Pr[\alpha_i = \kappa(D'_i) + \eta_i] = \Pr[n'_i = \alpha_i - \kappa(D'_i)]
$$

$$(\beta + \gamma) = \int_{\kappa(D_i) - \sigma}^{\kappa(D'_i) + \sigma} \frac{1}{2\sigma} e^{-\frac{(x-\mu)}{\sigma^2}} dx + \frac{T_1}{T_2}, \quad \eta_i \in [\beta + \gamma, \alpha + \eta], \quad \eta_i \notin [\beta + \gamma, \alpha + \eta]
$$

where

$$
T_1 = \frac{1}{\kappa(D_i) - \sigma} + \frac{T_2}{\kappa(D'_i) + \sigma}
$$

$$
T_2 = \frac{1}{\kappa(D'_i) + \sigma} + \frac{T_2}{\kappa(D_i) - \sigma}
$$

(1) When $\sigma \in [\beta + \gamma, \alpha + \eta]$. In addition, $\mu_1 \leq \mu_2 \leq \mu_0$

$$
\Delta f = \max\{d_i - d_i, |\sigma| = \Delta f / \epsilon, \text{and} T_1, T_2 > 0
$$

Assume that $\Delta \mu = \max\{\mu_i - \mu_2\}$, we have

$$
\Pr[\alpha_i = \kappa(D_i) + \mu_2 - \mu_1 - \mu_2] = \Pr[n_i = \alpha_i - \kappa(D_i) + \mu_2 - \mu_1] + \frac{T_1}{T_2}
$$

$$
\leq \epsilon^c \Delta \mu + \frac{T_1}{T_2} \leq \epsilon \Delta \mu + \frac{2 - \frac{1}{\kappa(D'_i) + \sigma}}{2 - \frac{1}{\kappa(D_i) - \sigma}} \epsilon \Delta \mu - \frac{1}{2} \epsilon \Delta \mu
$$

$$
\leq \epsilon \Delta \mu + \frac{1}{2} \epsilon \Delta \mu - \frac{1}{2} \epsilon \Delta \mu
$$

Assume $\delta = (\theta_1 + \theta_2) \Pr[\alpha_i = \kappa(D'_i) + n'_i]$, where

$$
\theta_1 = \epsilon \Delta \mu + 1
$$

$$
\theta_2 = \frac{2 - \frac{1}{\kappa(D'_i) + \sigma}}{2 - \frac{1}{\kappa(D_i) - \sigma}} \epsilon \Delta \mu - \frac{1}{2} \epsilon \Delta \mu
$$

We have $\Pr[\alpha_i = \kappa(D_i) + \eta_i] = \epsilon \Pr[\alpha_i = \kappa(D'_i) + n'_i] + \delta$.

Considering that $\Pr[\alpha_i = \kappa(D_i) + n'_i]$ and $\theta_1$ approaches to 0 when $\Delta \mu$ is small. In addition, $\theta_2$ is always a fixed value. Therefore, $\delta$ approaches to 0. Finally, $(\epsilon, \delta)$ differential privacy is satisfied in this case.

(2) When $\sigma \notin [\beta + \gamma, \alpha + \eta]$, $\Pr[\alpha_i = \kappa(D_i) + \eta_i] = \Pr[n_i = \kappa(D'_i) + n'_i] = 0$. This is conventionally viewed as 1 [9], and satisfies (0,0) differential privacy.

Step 2. For all $j \geq 1$, we show the probability of $c_j < (\beta + \gamma)$ or $c_j > (\alpha + \eta)$ is no more than $1 - \lambda$.

(1) For the probability of $c_j < (\beta + \gamma)$: Since $n_j = c_j - c_{j-1}$, $\mu_j \leq \mu \leq \mu_0$, $0 \leq c_{j-1} \leq C$, we have

$$
\Pr[c_j < (\beta + \gamma)] = \Pr[n_j + c_{j-1} < (\beta + \gamma)]
$$

$$
= \Pr[n_j + c_{j-1} + (\beta + \gamma)] \leq \Pr[n_j + (\beta + \gamma)]
$$

$$
= \Pr[n_j - (\beta + \gamma) < (\beta + \gamma) - (\beta + \gamma)]
$$

Based on the Chebyshev’s inequality, we have

$$
\Pr[c_j < (\beta + \gamma)] \leq \Pr[n_j - (\beta + \gamma) < (\beta + \gamma) - (\beta + \gamma)]
$$

$$
E(n_j) \leq (\beta + \gamma - E(n_j))^2
$$

where $n_j$ is a random variable with expected value $E(n_j)$ and finite variance $D(n_j)$.

(2) For the probability of $c_j > (\alpha + \eta)$: Since $n_j = c_j - c_{j-1}$, $\mu_j \leq \mu \leq \mu_0$, $0 \leq c_{j-1} \leq C$, we have

$$
\Pr[c_j > (\alpha + \eta)] = \Pr[n_j + c_{j-1} > (\alpha + \eta)]
$$

$$
= \Pr[n_j + c_{j-1} + (\alpha + \eta)] \leq \Pr[n_j + C + (\alpha + \eta)]
$$

$$
= \Pr[n_j - (\alpha + \eta) > C + (\alpha + \eta) - (\alpha + \eta)]
$$

Similarly, based on the Chebyshev’s inequality, we have

$$
\Pr[c_j > (\alpha + \eta)] \leq \Pr[n_j - (\alpha + \eta) > C + (\alpha + \eta) - (\alpha + \eta)]
$$

$$
E(n_j) \leq (\alpha + \eta - E(n_j))^2
$$

where $n_j$ is a random variable with expected value $E(n_j)$ and finite variance $D(n_j)$.

We set

$$
\lambda = 1 - \frac{D(n_j)}{(\beta + \gamma - E(n_j))^2} - \frac{D(n_j)}{(\alpha + \eta + E(n_j))^2}
$$

(7)

In the Appendix, we have shown that $E(n_j)$ and $D(n_j)$ are two fixed numbers, given $\sigma, \mu, \alpha, \beta, \gamma, \eta$ and $\kappa(D_i)$. As a result, the probability that $c_j < (\beta + \gamma)$ or $c_j > (\alpha + \eta)$ is no bigger than a constant value $1 - \lambda$.

In sum, our stateful privacy protection scheme satisfies $(\lambda, \epsilon, \delta)$ differential privacy.

We complete the proof. □

According to Theorem 2 our stateless privacy protection scheme satisfies a relaxed differential privacy, which solves our second challenge introduced in Section 3.2.

We can adjust $\mu, \alpha, \beta, \gamma, \sigma$ and $C$ to maximize $\lambda$. From the point of preserving privacy, the difference of $\mu_i$ and $\mu_0$ should be as big as possible, because this could stimulate $n_i$ to charge when $c_{i-1}$ approaches to 0, or to discharge when $c_{i-1}$ approaches $C$. This increases the value of $\lambda$. So $\mu = \beta + \gamma - \kappa(D_i)$ and $\mu_0 = \alpha + \eta - \kappa(D_i)$ are the best in this situation. This implies that the bigger $\eta - \gamma$ is, the better the privacy is, if $\alpha$ and $\beta$ are determined. Furthermore, $C^2$ is proportionate with $\lambda$ based on equation 7. So the larger $C$ is, the bigger $\lambda$ is. In sum, to preserve the privacy, $C$ and $\eta - \gamma$ should be as large as possible. This principle could guide us to select an appropriate battery in practice. More quantitative details about the relationship among $\lambda, C, \eta - \gamma$ will be analyzed in the next section.

5. PRIVACY ANALYSIS

From the point of privacy protection, users’ privacy at each time point has been proved to implement about the
Table 2: Default value of the parameters

| Parameter | Value |
|-----------|-------|
| $\Delta t$ | 0.25 H |
| $\Delta f$ | $\frac{4.662}{2.246}/4.008$ kW |
| $\epsilon$ | 0.1 |
| $\alpha$ | 0 kWh |
| $\eta$ | 1 kW |
| $\beta$ | 6.081 kWh |
| $\gamma$ | -7.081 kW |
| $c(0)$ | 50 kWh |
| $\mu$ | -1 kW |
| $\nu$ | 1 kW |

same privacy assurance level theoretically, since the stateless and stateful privacy protection schemes satisfy differential privacy. Therefore, we just need to statistically measure the absolute value of privacy leakage at each time point. In this section, we compute the maximum privacy leakage at each time point by leveraging the metric of mutual information, compared our stateless and stateful privacy protection schemes with the existing BE, NILL, LS\textsuperscript{1}, PRIVATUS, MBE, OPPEM and DPSM schemes [3,10,12,32,34]. We then clarify how many times the event that $(\epsilon, \delta)$ differential privacy cannot be satisfied for the stateful privacy protection scheme, MBE scheme and DPSM scheme occurs. We also show the relationship between $\epsilon$ and $\delta$ in the stateless and stateful privacy protection schemes.

Following the same strategy in MBE [34], we pick up three houses where the electricity consumes at most from five houses in the REDD [13] as our data sets. In the experiments, the time interval $\Delta t$ is set to 15 minutes, $\epsilon$ is 0.1, and $\Delta f$ is 4.662 kW, 2.246kW and 4.008kW respectively. The lower bound of $\kappa(D_i)$ is 0, when all the appliances are closed. The upper bound of $\kappa(D_i)$ is the highest consumption among the three houses, when all the energy appliances are opened. The maximum values of the discharging and charging rates are -7.081 kW and 1 kW respectively. The initial energy $c(0)$ in the battery is a half of $C$, where $C$ is 100 kWh. The lower bound and upper bound of $\mu$ are -1 kW and 1 kW respectively. The default values of parameters are listed in Table 2.

Since the difference of meter readings at two neighboring time points and the meter readings themselves are regarded as both sensitive information [34], we define mutual information [10,29] for both as $MI_0$ and $MI_1$, respectively. Since the privacy loss is proportionate with the value of mutual information, and the most probable attack intuitively appears in the weakest segment of the entire system, we eventually appraise the privacy loss of a privacy protection scheme $MI$ at the bigger value of $MI_0$ and $MI_1$. This appraisal of privacy loss is a little different with that in the previous work where the sum of the mutual information at each time point is used as the privacy leakage. After all, the biggest privacy loss at all the time points is more nature to represent the final privacy leakage for a privacy protection scheme than the total amount of that at all the time points, just as the worse time complexity is frequently used to evaluate the efficiency of an algorithm but not the average time complexity. In formal, assume that $\kappa'(D_i) = \kappa(D_i) - \kappa(D_{i-1})$, and $\alpha_i = \alpha_i - \alpha_{i-1}$, for all $\kappa(D_i)$, $\kappa(D_{i-1})$, $\alpha_i$ and $\alpha_{i-1}$, we have

$$MI_0 = \max_i \left( \log \frac{Pr(\kappa'(D_i), \alpha'_i)}{Pr(\kappa'(D_i)) \cdot Pr(\alpha'_i)} \right)$$

$$MI_1 = \max_i \left( \log \frac{Pr(\kappa(D_i), \alpha_i)}{Pr(\kappa(D_i)) \cdot Pr(\alpha_i)} \right)$$

$^3$The unit is $0.001$ kW.

$^4$Note that the condition of MBE scheme does not always work. In fact, the parameter $q(t)$ in (33) may be less than 0, because $\frac{2(d(t)+k(t))}{b(t)}$ is not always bigger than 0. This leads the binomial distribution cannot be constructed sometimes. We force $q(t) > 0$, if it occurs.

$^5$Note that the value of $\lambda$ is always 1 for the stateless privacy protection scheme, based on Theorem 1.

We first shows the privacy leakage for all the schemes in three houses, when using the default parameters. The lower $MI$ is, the less the privacy leakage. As shown in Figure 2, the privacy leakage of the stateless and stateful privacy protection schemes are less than that of the other schemes.

We further change the capacity of the battery to 10 different values for observing how big capacity will impact the privacy loss in the house 1. Specifically, we compute the expected value and 90% confidence interval of $MI$ after running 50 times for all the schemes during in the experiments. As shown in Figure 3, we can see that the privacy leakage of the stateless and stateful privacy protection schemes stably outperforms that of the other schemes for all the different capacity. Likewise, we continue to change the maximum charging and discharging rates of the battery to 10 different values which is from the start of 100 kWh and to increment by 10 kWh. We then watch the change of the privacy leakage. As shown in Figure 4, both of our schemes are also stably better than the other schemes for all the different charging and discharging rates.

Next, as an extension of how to choose an appropriate battery which has mentioned at the end of the last previous section, we show the relationship among $\lambda C$ and $\eta - \gamma$. As shown in Figure 5 and Figure 6, the value of $\lambda$ in the stateful privacy protection scheme is proportionate with $C$ and disproportionate with $\eta - \gamma$. In addition, $\lambda$ in our stateful privacy protection scheme is always higher than that in MBE and DPSM schemes in all the three houses $H1$, $H2$ and $H3$. Finally, we demonstrate the relationship between $\epsilon$ and $\delta$ in the stateless and stateful privacy protection schemes. Specifically, we observe the value of $\delta$ for $MI_0$ and $MI_1$, after we change the value of $\epsilon$ from 0.1 to 0.5. As shown in Figure 7, all the values of $\delta$ approach to 0.
6. DISCUSSION

Utility: We consider the data utility here. In fact, the individual's output is essentially the real energy consumption, when the battery is regarded as a special appliance. Furthermore, since the noise saved in the battery will eventually be used for the other appliances in the house, it is trivial to distinguish whether the energy usage is used for a battery or the other appliances, from the point view of electricity company. Therefore, the statistical result of each customer's energy usage is accurate in the application of smart grid.

Tesla’s Battery: Tesla released a new battery product named the power packs for both home and utility use recently [17]. The capacity of the whole power pack is exactly 100 kWh. Based on Figure 5, $\lambda$ can reach about 0.92 in house 1, 0.62 in house 2 and 0.91 in house 3 for the stateful privacy protection scheme, when this power pack is used as the battery. Therefore, our stateful privacy protection scheme satisfies differential privacy in 92% of the time in house 1, 62% in house 2 and 91% in house 3. All of these values are higher than those in the MBE scheme (about 48%, 47%, 43%) and in the DPSM scheme (about 6%, 5%, 5%). This demonstrates that our stateful privacy protection scheme is practical.

Battery Cost: Some customers may worry about the cost of the battery. After all, the price of a tesla power pack is approximately $30000 now. However, the cost of battery is drastically decreasing now. It is estimated that the price may fall down to $100 per kWh in 2030 [3]. Assuming that a battery could be used in 10 years, the total cost of preserving privacy is about $1000 per year. Since differential privacy can also maintain data utility which is useful to the electricity company, part of the cost may be paid by the company. Besides, our scheme could also have opportunity to earn money, when combined with some studies about saving the cost by the battery. Therefore, cost will be not a big problem in the future.

7. RELATED WORK

Since smart meter has been widely used in our daily life [11, 12, 13, 27, 31], researchers gradually focus on the customer's privacy in smart grid. Consequently, the cryptography-based and the battery-based solutions become two main kinds of privacy protection schemes. The former tries to design some cryptographic primitives or protocols, such as a commitment scheme [11] or an anonymous authentication protocol [30], in order to assure either the metering reading is encrypted or it will not expose the identity of household to the utility company.
suppliers. Unfortunately, these algorithms cannot essentially prevent the privacy loss, because utility providers could still apply some NILM algorithms to analyze the customers’ behavior. After all, smart readings always faithfully report the total amount of the electricity load in these solutions.

By contrast, a series of battery-based privacy preserving scheme are also proposed in smart grid. Kalogridis et al. [14] was first to propose a best effort (BE) scheme. This scheme tries to flatten meter reading in a constant value, regardless of what the users’ real electric usage is at time $t$. A battery is used to counteract the inadequate or excessive electricity consumption by keeping charging or discharging. Unfortunately, BE scheme is difficult to be implemented for accommodating a variety of users, since all the batteries in practical have a bounded capacity, a maximum discharging rate, and a maximum charging rate. When the energy left in the battery is too low or too high to maintain the difference between the expected value of the meter reading and a user’s real energy usage, or that difference is too large to be covered, BE will have to expose users’ privacy [22].

To resolve this problem, McLaughlin et al. [22] developed a non-intrusive load leveling (NILL) scheme. They set the charging/discharging rate, due to the current state of the battery. They change the expected value of the meter reading to a new one, when the current value is out of bound. More precisely, the battery will try to be discharged in a constant rate, if energy saved in the battery is too high in the battery, and vice versa. However, privacy is still leaked in these two cases [33]. Yang et al. [33] introduced three lazy stepping (LS) schemes. The meter reading is not always a constant value too. In the LS scheme, the expected value will be adjusted to a new value if the battery cannot afford the current value. Nonetheless, privacy is exposed at the time when the expected value changes [34]. In general, privacy is hard to be protected by maintaining one or several constant meter readings under the physical limitation of battery.

Recently, Zhao et al. [34] proposed a Multitasking-BLH-Exp3 (MBE) scheme. This scheme tries to avoid leaving too much or too little energy in the battery, by maintaining the energy left in the battery at half capacity as much as possible. This is done by the notion of Multi-Armed Bandit problem [24, 28]. In addition, MBE scheme attempts to analyze privacy protection by leveraging the definition of differential privacy. However, their proof is not complete, when considering the constraints of the battery.

Yang et al. [32] designed an optimal privacy preserving energy management (OPPEM) scheme. This scheme minimizes the variance of the meter readings by using Lyapunov optimization technology. Reducing the bill under dynamic pricing policy is also considered in this scheme. However, users’ privacy is still in danger, because their scheme is not a randomize algorithm. Once the adversaries obtain the battery state at a time point, they may infer the noise in the future. After all, the same noise will be generated in the same circumstance for a deterministic algorithm. Koo et al. [10] proposed a wallet friendly privacy protection (PRIVATUS) scheme. This scheme uses dynamic programming to lower electricity bill. However, the privacy is also leaked, because the battery will definitely charge at the lower prize zone and discharge at the higher price zone. For example, if the meter reading approaches to 0 at the lower prize zone, all the appliances whose consumption is higher than that reading will not be opened.

Backes and Meiser [3] proposed a differentially private smart metering with battery recharging (DPSM) scheme. Their scheme quantitatively measured the distance between an ideal noise and a practical noise generated by the physical limited battery, and proved that the distance was small, if a proper battery is chosen. Thus, their scheme also satisfies $(\epsilon, \delta, \lambda)$ differential privacy. However, the value of $\lambda$ in our scheme is smaller than that in DPSM scheme, which means that our scheme has more opportunity to satisfy differential privacy than DPSM does. Meanwhile, the privacy loss also less than that in DPSM scheme as well. Besides, a secondary kind of energy like solar has to be used for keeping the energy left in the battery to $C/2$ at every $n$ steps [3].

8. CONCLUSIONS AND FUTURE WORK

In this paper, we design two practical differential privacy protection schemes under the physical limitation of the battery. To the best of our knowledge, our stateless privacy protection scheme is first to satisfy differential privacy considering the restricts of data and noise. Furthermore, we propose the definition of a new relaxed differential privacy, and prove our stateful privacy protection scheme satisfies that definition according to the theoretical analysis. Experimental analysis shows that the privacy protection of our schemes are stably better than that of BE, NILL, LS, MBE, PRIVATUS, OPPEM and DPSM schemes in practical.

Our future work will mainly focus on two aspects. On one hand, create a proper loss model for the battery. On the other hand, consider cost saving problem besides the privacy in smart grid.

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APPENDIX

We first prove \(\text{pdf}(x)\) is a valid probability density function, and then we compute \(\mathbb{E}(x)\) and \(\mathbb{D}(x)\), given \(\sigma,\mu,\alpha,\beta,\gamma,\eta \) and \(\kappa(D_i)\).

**Claim 2.** pdf \((x)\) is a valid probability density function.

**Proof.** The validity of pdf \((x)\) as a probability density function is guaranteed, if

\[
\begin{align*}
\text{pdf}(x) & \geq 0 \\
\int_{-\infty}^{+\infty} \text{pdf}(x)dx & = 1
\end{align*}
\]

As we discussed before, \(\alpha + \eta > \beta + \gamma\) is hold, when an appropriate battery is chosen. Furthermore, \(0 < T < 1\), because it represents the probability of two subsets that consist of variables from \((\mu, \sigma)\) Laplace distribution. Since

\[
\sigma, e^{-\frac{|x-\mu|}{\sigma}}, T > 0, \text{ and } \alpha + \eta > \beta + \gamma,
\]

we have

\[
\frac{1}{2\sigma} e^{-\frac{|x-\mu|}{\sigma}} + \frac{T}{\alpha + \eta - \beta - \gamma} \geq 0.
\]

In addition,

\[
F(x) = \int_{-\infty}^{+\infty} f(x)dx = \int_{\beta+\gamma-\kappa(D_i)}^{\alpha+\eta-\kappa(D_i)} f(x)dx = \int_{\beta+\gamma-\kappa(D_i)}^{\alpha+\eta-\kappa(D_i)} \left(\frac{1}{2\sigma} e^{-\frac{|x-\mu|}{\sigma}} + \frac{T}{\alpha + \eta - \beta - \gamma}\right) dx = 1 - \frac{1}{2\sigma} \left(\int_{-\infty}^{\beta+\gamma-\kappa(D_i)} e^{-\frac{|x-\mu|}{\sigma}} dx + \int_{\alpha+\eta-\kappa(D_i)}^{+\infty} e^{-\frac{|x-\mu|}{\sigma}} dx\right) + T = 1
\]

We complete the proof. \(\Box\)

Now we compute the expected value \(\mathbb{E}(x)\) of our noise distribution, given \(\sigma,\mu,\alpha,\beta,\gamma,\eta\) and \(\kappa(D_i)\).

\[
\mathbb{E}(x) = \int_{-\infty}^{+\infty} xf(x)dx = \int_{\beta+\gamma-\kappa(D_i)}^{\alpha+\eta-\kappa(D_i)} xf(x)dx = \frac{1}{2\sigma} \int_{\beta+\gamma-\kappa(D_i)}^{\alpha+\eta-\kappa(D_i)} e^{-\frac{|x-\mu|}{\sigma}} \frac{T}{\alpha + \eta - \beta - \gamma} dx
\]

\[
\int_{\beta+\gamma-\kappa(D_i)}^{\alpha+\eta-\kappa(D_i)} \frac{T}{\alpha + \eta - \beta - \gamma} dx = \frac{\sigma e^{-\frac{|\mu-\kappa(D_i)|}{\sigma}}}{2} (x-1) e^{x} \left|_{\beta+\gamma-\kappa(D_i)}^{\alpha+\eta-\kappa(D_i)}\right| + \frac{\sigma e^{-\frac{|\mu-\kappa(D_i)|}{\sigma}}}{2} (x-1) e^{x} \left|_{\beta+\gamma-\kappa(D_i)}^{\alpha+\eta-\kappa(D_i)}\right|
\]

where

\[
T = 2 - \frac{1}{2} e^{-\frac{\beta+\gamma-\kappa(D_i)-\mu}{\sigma}} - \frac{1}{2} e^{-\frac{\alpha+\eta-\kappa(D_i)-\mu}{\sigma}}
\]

This equation illustrates that \(\mathbb{E}(x)\) is fixed, given \(\sigma,\mu,\alpha,\beta,\gamma,\eta\) and \(\kappa(D_i)\).

Finally, given \(\sigma,\mu,\alpha,\beta,\gamma,\eta\) and \(\kappa(D_i)\), the variance \(\mathbb{D}(x)\) of noise distribution is computed as follows:

\[
\mathbb{D}(x) = \int_{-\infty}^{+\infty} (x - \mathbb{E}(x))^2 f(x)dx = \int_{\beta+\gamma-\kappa(D_i)}^{\alpha+\eta-\kappa(D_i)} (x - \mathbb{E}(x))^2 f(x)dx = \int_{\beta+\gamma-\kappa(D_i)}^{\alpha+\eta-\kappa(D_i)} \left(\frac{1}{2\sigma} e^{-\frac{|x-\mu|}{\sigma}} + \frac{T}{\alpha + \eta - \beta - \gamma}\right) dx
\]

\[
\int_{\beta+\gamma-\kappa(D_i)}^{\alpha+\eta-\kappa(D_i)} (x^2 - 2\mathbb{E}(x)x + \mathbb{E}(x)^2) \frac{1}{2\sigma} e^{-\frac{|x-\mu|}{\sigma}} dx + \int_{\beta+\gamma-\kappa(D_i)}^{\alpha+\eta-\kappa(D_i)} T(x - \mathbb{E}(x))^2 \frac{1}{2\sigma} e^{-\frac{|x-\mu|}{\sigma}} dx
\]

\[
\int_{\beta+\gamma-\kappa(D_i)}^{\alpha+\eta-\kappa(D_i)} \frac{x^2}{2\sigma} e^{-\frac{|x-\mu|}{\sigma}} dx - 2\mathbb{E}(x) \int_{\beta+\gamma-\kappa(D_i)}^{\alpha+\eta-\kappa(D_i)} \frac{x}{2\sigma} e^{-\frac{|x-\mu|}{\sigma}} dx + \frac{\mathbb{E}(x)^2}{2\sigma} \int_{\beta+\gamma-\kappa(D_i)}^{\alpha+\eta-\kappa(D_i)} e^{-\frac{|x-\mu|}{\sigma}} dx
\]

\[
\int_{\beta+\gamma-\kappa(D_i)}^{\alpha+\eta-\kappa(D_i)} \frac{x}{2\sigma} e^{-\frac{|x-\mu|}{\sigma}} dx + \int_{\beta+\gamma-\kappa(D_i)}^{\alpha+\eta-\kappa(D_i)} \frac{\mu}{2\sigma} e^{-\frac{|x-\mu|}{\sigma}} dx
\]

\[
\mathbb{D}(x) = \frac{\sigma_\mu}{2} x^2 (x-1) e^{x} \left|_{\beta+\gamma-\kappa(D_i)}^{\alpha+\eta-\kappa(D_i)}\right| - \mathbb{E}(x) \mathbb{E}(x) e^{-\frac{|\mu-\kappa(D_i)|}{\sigma}} - \frac{\mathbb{E}(x)^2}{2} e^{x} \left|_{\beta+\gamma-\kappa(D_i)}^{\alpha+\eta-\kappa(D_i)}\right|
\]

\[
T(x - \mathbb{E}(x))^2 \left|_{\beta+\gamma-\kappa(D_i)}^{\alpha+\eta-\kappa(D_i)}\right|
\]

where

\[
T = 2 - \frac{1}{2} e^{-\frac{\beta+\gamma-\kappa(D_i)-\mu}{\sigma}} - \frac{1}{2} e^{-\frac{\alpha+\eta-\kappa(D_i)-\mu}{\sigma}}
\]

\[
\mathbb{E}(x) = \frac{\sigma e^{-\frac{|\mu-\kappa(D_i)|}{\sigma}}}{2} (x-1) e^{x} \left|_{\beta+\gamma-\kappa(D_i)}^{\alpha+\eta-\kappa(D_i)}\right| + \frac{\sigma e^{-\frac{|\mu-\kappa(D_i)|}{\sigma}}}{2} (x-1) e^{x} \left|_{\beta+\gamma-\kappa(D_i)}^{\alpha+\eta-\kappa(D_i)}\right|
\]

\[
\int_{\beta+\gamma-\kappa(D_i)}^{\alpha+\eta-\kappa(D_i)} x \frac{T}{\alpha + \eta - \beta - \gamma} dx
\]

This equation illustrates that \(\mathbb{D}(x)\) is also fixed, given \(\sigma,\mu,\alpha,\beta,\gamma,\eta\) and \(\kappa(D_i)\).