Next-to-next-to-leading order soft and virtual corrections for QCD, Higgs and SUSY processes

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Abstract

I review recent advances in the calculation of higher-order soft-gluon corrections for a variety of QCD, Higgs, and SUSY processes in hadron colliders. A unified approach and master formulas for next-to-next-to-leading order soft and virtual corrections are discussed. I present some applications of the formalism to top quark pair production at the Tevatron and the LHC, top production via anomalous couplings in flavor-changing neutral-current processes, $W$ boson hadroproduction at large transverse momentum, and charged Higgs production at the LHC.
1 Introduction

A lot of progress in calculations of higher-order QCD corrections has been achieved in the last few years. Recent theoretical advances include an array of resummations, and a few complete and several partial next-to-next-to-leading order (NNLO) calculations \[1, 2\]. These advances are welcome and needed in many cases where next-to-leading order (NLO) calculations are not precise enough, since higher-order corrections reduce the scale dependence of cross sections and increase theoretical accuracy. The search for new physics, such as supersymmetry, in hadron colliders relies on calculations of both backgrounds and new processes, and this is where accuracy becomes important.

Next-to-next-to-leading order calculations are technically difficult and have been fully completed only for the simplest, Drell-Yan type \[3, 4, 5, 6\], processes. The corrections can be separated into hard, soft, and virtual parts, corresponding to contributions from energetic, soft, and virtual gluons, respectively. The soft corrections are an important component of the total result both theoretically and numerically. In fact, in certain kinematical regions, notably threshold, they are dominant.

Recently, a new unified formalism for the calculation of soft and virtual corrections for arbitrary hard-scattering processes in hadron-hadron and lepton-hadron collisions, including QCD, electroweak/Higgs, and supersymmetric processes, was presented in Ref. \[7\]. It is based on and extends earlier work on threshold resummations \[8, 9, 10, 11\].

The calculation of cross sections in hadron-hadron or lepton-hadron collisions can be written schematically as

$$
\sigma = \sum_{f} \int \left[ \prod_{i} dx_i \phi_{f/h_i}(x_i, \mu_F) \right] \hat{\sigma}(s, t, u, \mu_F, \mu_R),
$$

(1.1)

where \( \sigma \) is the physical cross section, \( \phi_{f/h_i} \) is the distribution function for parton \( f \) carrying momentum fraction \( x_i \) of hadron \( h_i \), at a factorization scale \( \mu_F \), while \( \mu_R \) is the renormalization scale. The parton-level hard-scattering cross section is denoted by \( \hat{\sigma} \), and \( s, t, u \) are standard kinematical invariants formed from the momentum four-vectors of the particles in the partonic reaction. In a lepton-hadron collision we have one parton distribution \( i = 1 \) while in a hadron-hadron collision \( i = 1, 2 \). We note here that \( \sigma \) and \( \hat{\sigma} \) are not restricted to be total cross sections; they can represent any relevant differential cross section of interest, such as transverse momentum or rapidity distributions.

In general, the partonic cross section \( \hat{\sigma} \) includes plus distributions \( D_l(x_{th}) \) with respect to a kinematical variable \( x_{th} \) that measures distance from threshold, with \( l \leq 2n - 1 \) at \( n \)-th order in \( \alpha_s \) beyond the leading order. These are the soft corrections. The virtual corrections appear as delta functions, \( \delta(x_{th}) \). In single-particle inclusive (1PI) kinematics, \( x_{th} \) is usually called \( s_4 \) (or \( s_2 \)), \( s_4 \equiv s + t + u - \sum m^2 \), and vanishes at threshold. Then

$$
D_l(s_4) \equiv \left[ \frac{\ln^l(s_4/M^2)}{s_4} \right]_+,
$$

(1.2)

where \( M^2 \) is a hard scale relevant to the process at hand, for example the mass \( m \) of a heavy quark, the transverse momentum \( p_T \) of a jet, etc. In pair-invariant-mass (PIM) kinematics, with \( Q^2 \) the invariant mass squared of the produced pair, \( x_{th} \) is usually called \( 1 - z \) (or \( 1 - x \)),

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with $z = Q^2/s \to 1$ at threshold. Then

$$D_l(z) \equiv \left[ \frac{\ln^l(1-z)}{1-z} \right]_+.$$  

(1.3)

The logarithms raised to the power $l = 2n - 1$ are the leading logarithms (LL), those with $l = 2n - 2$ are next-to-leading (NLL), with $l = 2n - 3$ are next-to-next-to-leading (NNLL), with $l = 2n - 4$ are next-to-next-to-next-to-leading (NNNLL), etc. These logarithms can be formally resummed to all orders in perturbation theory.

In Section 2 I briefly review threshold resummation and present master formulas for soft and virtual corrections at next-to-next-to-leading order for processes in hadron-hadron and lepton-hadron collisions. In Section 3 results are given for top quark pair production at the Tevatron and the LHC, while Section 4 discusses FCNC top quark production via anomalous couplings at HERA and the Tevatron. Section 5 discusses $W$ boson hadroproduction at large transverse momentum at the Tevatron, and Section 6 the production of a charged Higgs with a top quark at the LHC.

2 NNLO master formula for soft and virtual corrections

Resummed cross sections have by now been studied for a variety of processes [11]. The exponentiation [8, 9, 10, 11] of soft-gluon corrections arises from the factorization [12] properties of the cross section. The cross section is separated (factorized) into a short-distance function, $H$, a soft-gluon function, $S$, and parton distribution functions. The renormalization group evolution of these functions results in resummation.

The resummation of threshold logarithms is carried out in moment space. We define moments of the partonic cross section by

$$\hat{\sigma}(N) = \int dz \, z^{N-1} \hat{\sigma}(z) \text{ (PIM)}$$

or by

$$\hat{\sigma}(N) = \int (ds_4/s) \, e^{-N s_4/s} \hat{\sigma}(s_4) \text{ (1PI)},$$

with $N$ the moment variable. Under moments the plus distributions $D_l$ give rise to $\ln^{l+1} N$. It is these logarithms of $N$ that exponentiate in moment space to all orders in the strong coupling. The resummed partonic cross section for any process in hadron-hadron and lepton-hadron collisions can be written in moment space schematically as

$$\hat{\sigma}^{res}(N) = \exp \left[ \sum_i E^{(f_i)}(N_i) + E^{(f_i)}_{\text{scale}}(\mu_F, \mu_R) \right] \exp \left[ \sum_j E'^{(f_j)}(N_j) \right]$$

$$\times \text{Tr} \left\{ H^{f_i f_j} \exp \left[ \int \frac{d\mu'}{\mu'} \Gamma_S^i \right] S^{f_i f_j} \exp \left[ \int \frac{d\mu'}{\mu'} \Gamma_S^j \right] \right\}. \quad (2.1)$$

The sums over $i$ run over incoming partons: in hadron-hadron collisions $i = 1, 2$ while in lepton-hadron collisions $i = 1$. The sum over $j$ is relevant only if we have massless partons in the final state at lowest order. Clearly the second exponent is then absent for processes such as Drell-Yan, Higgs, and top quark pair production. Detailed expressions for the exponents are given in Ref. [7].

The first exponent in Eq. (2.1) resums the $N$-dependence of incoming partons [13, 14], including the scale, while the second exponent resums the $N$-dependence of any outgoing massless partons. The trace appearing in the resummed expression is taken in the space of color
exchanges of the specific process studied. $H$ is the hard function, independent of any soft-gluon effects, a matrix in color space. The evolution of the soft function, $S$, which describes noncollinear soft gluon emission \[^{[8]}\] and is also a matrix in color space, follows from its renormalization group properties and is given in terms of the soft anomalous dimension matrix $\Gamma_S$ \[^{[8,9,11]}\]. In processes with simple color flow, $\Gamma_S$ is a trivial $1 \times 1$ matrix. For the determination of $\Gamma_S$ in processes with complex color flow, an appropriate choice of color basis has to be made. The process-dependent soft anomalous dimension matrices, evaluated through the calculation of eikonal vertex corrections \[^{[8]}\,^{[13]}\], have by now been presented at one loop for practically all partonic processes \[^{[11]}\]. They can be explicitly calculated for any process using the techniques and results in Refs. \[^{[8,11]}\]. Work is currently being done on two-loop calculations of these anomalous dimensions \[^{[13]}\], but we note that the universal component of these anomalous dimensions for quark-antiquark and gluon-gluon initiated processes are now known \[^{[7]}\].

The trace of the product of the hard and soft matrices at lowest order reproduces the Born Born cross section is of order $\alpha_s$ in the strong coupling for the soft-gluon corrections for arbitrary processes in hadron-hadron and lepton-hadron collisions. We present results in the $\overline{\text{MS}}$ scheme. For results in the DIS scheme see Ref. \[^{[7]}\].

We begin at next-to-leading order. We note that at NLO the LL are $D_1(x_{th})$ and the NLL are $D_0(x_{th})$. The virtual corrections multiply $\delta(x_{th})$. The first-order soft and virtual corrections can then be written as \[^{[7]}\]

$$\delta^{(1)} = \sigma^B \alpha_s(\mu_R^2) \left\{ c_3 D_1(x_{th}) + c_2 D_0(x_{th}) + c_1 \delta(x_{th}) \right\} + \frac{\alpha_s^{d_{as}+1} (\mu_R^2)}{\pi} \left[ A^c D_0(x_{th}) + T_1^c \delta(x_{th}) \right],$$

(2.2)

where $\sigma^B$ is the Born term, and the LL coefficient is

$$c_3 = \sum_i 2C_{f_i} - \sum_j C_{f_j},$$

(2.3)

with $C_f = C_F \equiv (N_c^2 - 1)/(2N_c)$ for quarks and $C_f = C_A \equiv N_c$ for gluons, with $N_c$ the number of colors. The NLL coefficient $c_2$ is defined by

$$c_2 = -\sum_i \left[ C_{f_i} + 2C_{f_i} \delta_K \ln \left( \frac{t_i}{M^2} \right) + C_{f_i} \ln \left( \frac{\mu_F^2}{s} \right) \right] - \sum_j \left[ B_j^{(1)} + C_{f_j} + C_{f_j} \delta_K \ln \left( \frac{M^2}{s} \right) \right],$$

(2.4)

where $M$ is a relevant hard scale and $s$ is the center-of-mass energy squared of the incoming partons. Also, $\delta_K = 1$ for 1PI and 0 for PIM kinematics, $B_j^{(1)} = 3C_F/4$ and $B_j^{(1)} = \beta_0/4$, where $\beta_0 = (11C_A - 2n_f)/3$ is the lowest-order beta function, with $n_f$ the number of light quark flavors. Note that the sum over $j$ is only relevant if we have massless partons in the final state at lowest order in 1PI kinematics.

The NLL function $A^c$ is process-dependent and depends on the color structure of the hard-scattering. It is defined by

$$A^c \equiv \text{Tr} \left( H^{(0)} \Gamma^{(1)}_S S^{(0)} + H^{(0)} S^{(0)} \Gamma^{(1)}_S \right),$$

(2.5)
where $\Gamma^{(1)}_S$ is the one-loop gauge-independent part of $\Gamma_S$. Note that for processes with simple color flow, where $\Gamma_S$ is a trivial $1 \times 1$ matrix, we have $A^c = \sigma^B \alpha_s^{-d_{as} - 2} \text{Re} \Gamma^{(1)}_S$, where Re denotes the real part.

We also define scale-independent terms $T_2 \equiv c_2 + \sum_i C_{f_i} \ln (\mu_F^2/s)$ and $T_1 \equiv c_1 - c_1^\prime$, where the scale-dependent $c_1^\prime$ is given explicitly by

$$c_1^\prime = \sum_i \left[ C_{f_i} \delta K \ln \left( \frac{-t_i}{M^2} \right) - \gamma_i^{(1)} \right] \ln \left( \frac{\mu_F^2}{s} \right) + d_{\alpha s} \beta_0 \frac{\lambda}{4} \ln \left( \frac{\mu_R^2}{s} \right), \tag{2.6}$$

with $\gamma_i^{(1)} = 3C_F/4$ and $\gamma_2^{(1)} = \beta_0/4$ one-loop parton anomalous dimensions for quarks and gluons, respectively. With respect to the scale-independent virtual $\delta(x_{th})$ terms $T_1$ and $T_1^c$, we note that they have to be calculated independently for each process.

At next-to-next-to-leading order, the LL are $\mathcal{D}_3(x_{th})$, the NLL are $\mathcal{D}_2(x_{th})$, the NNLL are $\mathcal{D}_1(x_{th})$, and the NNNNLL are $\mathcal{D}_0(x_{th})$. The virtual corrections multiply $\delta(x_{th})$. The expansion of the resummed cross section, with matching to the full NLO soft-plus-virtual result, gives the NNLO soft and virtual corrections $\mathcal{R}$:

$$\hat{\sigma}^{(2)} = \sigma^B \frac{\alpha_s^2(\mu_R^2)}{\pi^2} \left\{ \frac{3}{2} c_3 \mathcal{D}_3(x_{th}) + \left[ \frac{3}{2} c_3 c_2 - \frac{\beta_0}{4} c_3 + \sum_j C_{f_j} \beta_0 \frac{\lambda}{8} \right] \mathcal{D}_2(x_{th}) \right. \left. + \left[ c_3 c_1 + c_2^2 - \frac{\beta_0}{2} T_2 + \frac{\beta_0}{4} C_3 \ln \left( \frac{\mu_F^2}{s} \right) + \sum_j C_{f_j} K \right] \mathcal{D}_1(x_{th}) \right. \left. + \left[ c_2 c_1 - \frac{\beta_0}{2} c_3 + \frac{\beta_0}{4} c_2 \ln \left( \frac{\mu_R^2}{s} \right) \right] \mathcal{D}_0(x_{th}) \right\}$$

$$+ \frac{\alpha_s^{d_{as} + 2}(\mu_R^2)}{\pi^2} \left[ \frac{3}{2} c_3 A^c \mathcal{D}_2(x_{th}) + \left[ \left( 2 c_2 - \frac{\beta_0}{2} \right) A^c + c_3 T_1^c + F^c \right] \mathcal{D}_1(x_{th}) \right. \left. + \left[ \left( c_1 - \frac{\beta_0}{2} c_3 \right) A^c + \left( c_2 - \frac{\beta_0}{2} \right) T_1^c + F^c \delta K \ln \left( \frac{M^2}{s} \right) \right] \mathcal{D}_0(x_{th}) \right\}$$

$$+ R_{\delta}^{(2)} \delta(x_{th}), \tag{2.7}$$

where $\zeta_2 = \pi^2/6$ and $\zeta_3 = 1.2020569 \cdots$, $G_{ij}^{(2)}$ and $G^c$ are process-dependent two-loop functions and $K = C_A(67/18 - \pi^2/6) - 5a_f/9$, and

$$F^c = \text{Tr} \left[ H^{(0)} \left( \Gamma^{(1)}_S \right)^\dagger S^{(0)} + H^{(0)}_S S^{(0)} \left( \Gamma^{(1)}_S \right)^2 + 2H^{(0)} \Gamma^{(1)}_S \right] \ . \tag{2.8}$$

We cannot provide a universal equation for the full virtual terms $R_{\delta}^{(2)}$, although several terms have been given in Ref. $\mathcal{R}$. However, the $\delta(x_{th})$ terms that involve the factorization and renormalization scales can be given fully and explicitly. Let $C_{f_i}^\prime \equiv \sum_i C_{f_i} \delta K \ln \left( -t_i/M^2 \right) - \gamma_i^{(1)}$. 


Then those scale-dependent terms are given by

\[
R_\delta^{(2)} = \sigma B \frac{\alpha_s^2(\mu_R^2)}{\pi^2} \left\{ \ln^2 \left( \frac{\mu_F^2}{M^2} \right) \left[ \frac{1}{2} (C_t^u)^2 - \frac{\zeta_2}{2} \left( \sum_i C_{f_i} \right)^2 - \frac{\beta_0}{8} C_t^u \right] + \ln \left( \frac{\mu_F^2}{M^2} \right) \ln \left( \frac{\mu_F^2}{M^2} \right) \frac{\beta_0}{4} (d_{\alpha_s} + 1) C_t^u + \ln^2 \left( \frac{\mu_F^2}{M^2} \right) \frac{\beta_0^2}{32} (d_{\alpha_s} + d_{\alpha_s}) + \ln \left( \frac{\mu_F^2}{M^2} \right) \left[ (C_t^u)^2 \delta_K \ln \left( \frac{M^2}{s} \right) + C_t^u \left( T_1 + d_{\alpha_s} \frac{\beta_0}{4} \delta_K \ln \left( \frac{M^2}{s} \right) \right) - \sum i \gamma_{i/i}^{(2)} \right] + \ln \left( \frac{\mu_F^2}{M^2} \right) \left[ \frac{\beta_0}{4} (d_{\alpha_s} + 1) \left( C_t^u \delta_K \ln \left( \frac{M^2}{s} \right) + d_{\alpha_s} \frac{\beta_0}{4} \delta_K \ln \left( \frac{M^2}{s} \right) + T_1 \right) + \beta_1 \frac{d_{\alpha_s}}{16} \right] + \frac{\alpha_s^{d_{\alpha_s} + 2}}{\pi^2} \left\{ (C_t^u T_1^c + \zeta_2 \sum_i C_{f_i} A_i^c) \ln \left( \frac{\mu_F^2}{M^2} \right) + (d_{\alpha_s} + 1) \frac{\beta_0}{4} \ln \left( \frac{\mu_F^2}{M^2} \right) T_1^c \right\}, \right. \]

where \( \beta_1 = 34C_A^2/3 - 2n_f(C_F + 5C_A/3) \) is the NLO beta function, and

\[
\gamma_{q/q}^{(2)} = C_F^2 \left( \frac{3}{32} \delta \zeta_2 \left( \frac{3}{4} \zeta_3 + \frac{3}{12} \zeta_5 + \frac{17}{96} \right) - n_f C_F \left( -\frac{\zeta_2}{6} - \frac{1}{48} \right) \right), \quad \text{(2.10)}
\]

and

\[
\gamma_{g/g}^{(2)} = C_A^2 \left( \frac{2}{3} + \frac{3}{4} \zeta_3 \right) - n_f \left( \frac{C_F}{8} + \frac{C_A}{6} \right) \quad \text{(2.11)}
\]

are two-loop anomalous dimensions for quarks and gluons [7].

As shown in Ref. [7], further virtual corrections can also be deduced, including \( \zeta \) terms that arise from the inversion from moment to momentum space. For further details we refer the reader to Ref. [2]. Eqs. (2.7) and (2.9) constitute master formulas for the calculation of NNLO soft-gluon corrections and virtual corrections involving the scale for arbitrary processes in hadron-hadron and lepton-hadron colliders. They will be applied in the following sections to various processes of phenomenological interest.

### 3 Top quark production

As the first application of the unified formalism we consider top quark production, recently studied in Ref. [17]. The top quark is currently being re-observed at Run II of the Tevatron [18, 19, 20], and it is expected that improvements in the measurements of the values of the top mass and production cross section will be achieved.

The cross section for \( t\bar{t} \) production can be calculated in both 1PI and PIM kinematics. When not all terms are known in the theoretical expression for the cross section at NNLO, there is some difference between single-particle-inclusive (1PI) and pair-invariant-mass (PIM) kinematics. When NNNLL terms are included [17], the kinematics dependence of the cross section vanishes near threshold and is reduced away from it relative to NNLL accuracy [21, 22]. The factorization and renormalization scale dependence of the cross section is also greatly reduced. Thus top production is brought under theoretical control.
We study the partonic processes $ij \to t\bar{t}$ with $ij = q\bar{q}$ and $gg$. In 1PI kinematics, a single top quark $t$ is identified, $i(p_a) + j(p_b) \to t(p_1) + X[p_2]$, where $X[p]$ is the remaining final state that contains the $\bar{t}$. We define the kinematical invariants $s = (p_a + p_b)^2$, $t_1 = (p_b - p_1)^2 - m^2$, $u_1 = (p_a - p_1)^2 - m^2$, with $m$ the top quark mass, and $s_4 = s + t_1 + u_1$. At threshold, $s_4 \to 0$, and the soft corrections appear as $[\ln^l(s_4/m^2)/s_4]^+$. In PIM kinematics, we have instead $i(p_a) + j(p_b) \to t\bar{t}(p) + X(k)$. At partonic threshold, $s = M^2$, with $M^2$ the pair-mass squared. The soft corrections appear as $[\ln^l(1-z)/(1-z)]^+$, with $z = M^2/s \to 1$ at threshold. In either kinematics we calculate the NNLO soft corrections to the double differential cross section and add them to the exact NLO result.

The partonic scaling functions for top production were studied in detail in Ref. [17]. Note that to NNLL, the two kinematics choices give rather different results, even near threshold. When the NNNLL contribution is added, both the 1PI and PIM results are reduced relative to NNLL and the agreement between the two kinematics is much improved. Adding virtual $\zeta$ terms resulting from inversion of the resummed cross section from moment to momentum space improves the agreement between the 1PI and PIM kinematics further away from threshold. We note here that the effect of the virtual $\zeta$ terms is numerically small, as is also the case for the hadronic results for both channels. This small effect is in agreement with the arguments in Ref. [21] concerning resummation prescriptions, where it was shown that when subleading terms from inversion are calculated exactly they do not have an unwarranted large effect on the numerical results.

![Figure 1](image.png)

Figure 1: The $t\bar{t}$ total cross section in $p\bar{p}$ collisions at $\sqrt{S} = 1.96$ TeV is shown as a function of $m$ for $\mu = m$ (left frame), and $\mu/m$ for $m = 175$ GeV (right frame). The LO (dot-dot-dot-dashed), NLO (solid), and NNLO-NNNLL+$\zeta$ 1PI (dashed), PIM (dot-dashed) and average (dotted) results are plotted.

In Fig. 1 we present the NLO and soft NNLO $t\bar{t}$ cross sections in $p\bar{p}$ collisions at the Tevatron with $\sqrt{S} = 1.96$ TeV as functions of mass and scale [17]. We use the MRST2002 NNLO parton densities [25] (results with the CTEQ6M densities [26] are similar). The NNLO results include the soft NNNLL and virtual $\zeta$ terms, and thus denoted NNLO-NNNLL+$\zeta$, in 1PI and PIM kinematics. We also show the average of the two kinematics results which may perhaps be closer to the full NNLO result. In the left frame we plot the cross section as a...
function of top quark mass for $\mu \equiv \mu_F = \mu_R = m$. In the right frame we plot the scale dependence of the cross section over two orders of magnitude in $\mu/m$ with $m = 175$ GeV. The NLO cross section has a milder dependence on scale than the leading order (LO) result. The NNLO cross section exhibits even less dependence on $\mu/m$, approaching the independence of scale corresponding to a truly physical cross section. The change in the NNLO cross section in the range $m/2 < \mu < 2m$, normally displayed as a measure of uncertainty from scale variation, is less than 3%.

Figure 2: $t\bar{t}$ production in $pp$ collisions at the LHC with $\sqrt{S} = 14$ TeV. Left: The mass dependence of the total cross section. Right: The top quark transverse momentum distribution with $m = 175$ GeV. The NLO (solid), and NNLO-NNNLL+$\zeta$ 1PI (dot-dashed) results are shown for $\mu = m$. The NLO results are also shown for $\mu = m/2$ (upper dashed) and $2m$ (lower dashed).

In Fig. 2 we present results in 1PI kinematics for top production in $pp$ collisions at the LHC with $\sqrt{S} = 14$ TeV. We note that PIM results are unreliable for processes where the gluon contribution is dominant, such as top production at the LHC [17] and bottom and charm production at fixed-target experiments [27]. The left frame shows the total cross section versus the top mass. In the right frame the top quark transverse momentum distribution is plotted. At NNLO we observe an enhancement of the NLO $p_T$ distribution with no significant change in shape.

4 FCNC top quark production via anomalous $tqV$ couplings

Recently NNLO threshold corrections were calculated for top quark production via anomalous $tqV$ couplings in flavor-changing neutral-current (FCNC) processes at the Tevatron and HERA colliders [28, 29].
The study of anomalous couplings involving the top quark is well motivated in various models involving new physics. The effective Lagrangian involving such couplings of a $t, q$ pair to massless bosons is the following:

$$\Delta L^{\text{eff}} = \frac{1}{\Lambda} \kappa_{tqV} \bar{t} \sigma_{\mu\nu} q F_{\mu\nu}^{V} + h.c., \quad (4.1)$$

where $\kappa_{tqV}$ is the anomalous FCNC coupling, with $q$ a $u$- or $c$-quark and $V$ a photon or $Z$-boson; $F_{\mu\nu}^{V}$ are the usual photon/Z-boson field tensors; $\sigma_{\mu\nu} = (i/2)(\gamma_{\mu}\gamma_{\nu} - \gamma_{\nu}\gamma_{\mu})$ with $\gamma_{\mu}$ the Dirac matrices; $e$ is the electron charge; and $\Lambda$ is an effective scale which we will take to be equal to the top quark mass, denoted by $m$.

Current colliders such as the Tevatron and HERA have a good potential to search for FCNC interactions in the top-quark sector. In order to establish accurate limits on anomalous couplings, experiments need accurate predictions of cross sections for FCNC processes. It was shown in Refs. [28, 29] that at HERA uncertainties for the tree-level FCNC cross section $ep \to et$ can be very big, since the cross section can vary by a factor of two due to the choice of the factorization and renormalization scale. Similar conclusions were drawn in Ref. [29] for the Born-level cross sections for the FCNC processes $p\bar{p} \to tZ$, $p\bar{p} \to t\gamma$, and $p\bar{p} \to tt$ at the Tevatron. This fact unavoidably requires the calculation of higher-order corrections for the stabilization of the FCNC cross section.

![Picture](image)

Figure 3: The Born, NLO-NLL, and NNLO-NLL cross sections for $gu \to tZ$ in $p\bar{p}$ collisions with $\sqrt{S} = 1.96$ TeV and $\kappa_{tuZ} = 0.1$. Left: cross section versus top-quark mass at scale $\mu = m$. Right: cross section versus $\mu/m$ with $m = 175$ GeV.

As was shown in Refs. [28, 29] the inclusion of NLO and NNLO threshold corrections indeed greatly stabilizes the cross sections for the aforementioned FCNC processes. For $ep \to et$ the uncertainty due to scale variation drops to around 10 percent. This is of particular importance for HERA studies of FCNC processes [30, 31, 32, 33, 34].

Similarly, for the several possible FCNC processes at the Tevatron the theoretical predictions are stabilized with inclusion of threshold corrections. As an example, we consider the FCNC
process $p\bar{p} \rightarrow t\bar{Z}$ [35, 36] at the Tevatron, which at the partonic level proceeds mainly through $gu \rightarrow t\bar{Z}$ via an anomalous $\kappa_{tu\bar{Z}}$ coupling (the contribution from $gc \rightarrow t\bar{Z}$ is much smaller). The NLO and NNLO threshold corrections at NLL accuracy were calculated in Ref. [29]. The left frame of Fig. 3 shows the total cross section versus top quark mass for scale $\mu = m$. We see that the NLO-NLL threshold corrections provide an important enhancement of the Born result. The NNLO-NLL corrections provide an additional increase of the cross section. The right frame plots the scale dependence of the cross section over two orders of magnitude: $0.1 < \mu/m < 10$. We note the significant stabilization of the cross section when the NLO and NNLO threshold corrections are included.

5 \ W-boson hadroproduction at large transverse momentum

The production of $W$ bosons in hadron colliders is a process of relevance in testing the Standard Model, calculating backgrounds to new physics such as associated Higgs boson production, and luminosity monitoring.

The calculation of the complete NLO cross section for $W$ hadroproduction at large transverse momentum was presented in Refs. [37, 38]. The NLO corrections enhance the differential distributions in transverse momentum $Q_T$ of the $W$ boson. The $Q_T$ distribution falls rapidly with increasing $Q_T$, spanning six orders of magnitude in the $10 \text{ GeV} < Q_T < 190 \text{ GeV}$ region at the Tevatron.

$W$-boson production at high transverse momentum receives important corrections from the emission of soft gluons from the partons in the process. Soft-gluon resummation and NNLO-NNLL corrections for this process were studied in Ref. [39]. More recently the NNLO-NNNLL corrections were studied in Ref. [40]. These threshold corrections further enhance the cross section and reduce the scale dependence [40].

For the hadronic production of a high-$Q_T$ $W$ boson, with mass $m_W$, the lowest-order partonic subprocesses are $q(p_a) + g(p_b) \rightarrow W(Q) + q(p_c)$ and $q(p_a) + \bar{q}(p_b) \rightarrow W(Q) + g(p_c)$. The partonic kinematical invariants in the process are $s = (p_a + p_b)^2$, $t = (p_a - Q)^2$, $u = (p_b - Q)^2$, which satisfy $s_2 \equiv s + t + u - Q^2 = 0$ at partonic threshold. Note that $s_2 = (p_a + p_b - Q)^2$ is the invariant mass of the system recoiling against the $W$ boson. The partonic cross section $\hat{\sigma}$ includes distributions with respect to $s_2$ of the type $[\ln^l(s_2/Q_T^2)]/s_2^+$. In Fig. 4 we plot the transverse momentum distribution, $d\sigma/dQ_T^2$, for $W$ hadroproduction at the Tevatron Run II with $\sqrt{S} = 1.96 \text{ TeV}$. We use the MRST2002 NNLO parton densities [25]. In the left frame, we set the factorization scale $\mu_F$ and the renormalization scale $\mu_R$ equal to $Q_T$. We focus on the large $Q_T$ region where the soft-gluon approximation holds well and the corrections are important. We see that the NLO corrections provide a significant enhancement of the Born cross section. The NNLO-NNLL corrections provide a further modest enhancement of the $Q_T$ distribution. If we increase the accuracy by including the NNLL contributions, which are negative, then we find that the NNLO-NNNLL cross section lies between the NLO and NNLO-NNLL results. In the right frame, we vary the scale by a factor of two. We see that the NNLO-NNNLL distribution is more stable under scale variation than the NLO result, which in turn is much more stable than LO. In fact the two NNLO-NNNLL curves and the $\mu = Q_T/2$ Born and NLO curves all lie on top of each other.

The $K$-factors were studied in Ref. [40] where it was shown that they are moderate and
nearly constant over the high-\(Q_T\) range even though the distributions themselves span a few orders of magnitude. It was also shown that in the high-\(Q_T\) region the soft-gluon approximation holds very well, as the NLO-NLL cross section is almost identical to the full NLO result. The scale dependence of the differential cross section was also studied over two orders of magnitude, \(0.1 < \mu/Q_T < 10\), at fixed \(Q_T\). There is good stabilization of the cross section over the entire range in \(\mu/Q_T\) when the NLO corrections are included, and further improvement when the NNLO-NNNLL corrections are added.

Finally, we note that there are related NNLO threshold results for direct photon production \[41\], a process for which the partonic subprocesses are essentially the same as for \(W\) hadroproduction.

6 Charged Higgs production via the process \(bg \rightarrow tH^-\)

The discovery of the Higgs boson(s) is one of the main aims of the current run at the Tevatron and the future program at the LHC. The Minimal Supersymmetric Standard Model (MSSM) introduces charged Higgs bosons in addition to the Standard Model neutral Higgs boson. The charged Higgs would be an important signal of new physics beyond the Standard Model.

An important partonic channel for charged Higgs discovery at the LHC is associated production with a top quark via bottom-gluon fusion, \(b(p_b) + g(p_g) \rightarrow t(p_t) + H^-(p_H)\), for which we define the kinematical invariants \(s = (p_b + p_g)^2\), \(t = (p_b - p_t)^2\), \(u = (p_g - p_t)^2\), and \(s_4 = s + t + u - m_t^2 - m_H^2\), where \(m_H\) is the charged Higgs mass, \(m_t\) is the top quark mass, and we ignore the bottom quark mass \(m_b\). Threshold corrections appear as \([\ln(s_4/m_H^2)/s_4]_+\). The next-to-leading order (NLO) corrections to this process were calculated in Refs. \[42\,43\].
Here, I discuss soft-gluon corrections to charged Higgs production, which are expected to be important near threshold, the kinematical region where the charged Higgs may be discovered at the LHC.

In Fig. 5 we show results for charged Higgs production in $pp$ collisions at the LHC. We set $\mu_F = \mu_R = m_H$ and use the MRST2002 NNLO [25] parton densities. We keep $m_b$ non-zero only in the coupling. In the left frame of Fig. 5 we plot the cross section for charged Higgs production with $\sqrt{s} = 14$ TeV versus the charged Higgs mass. We use $m_t = 175$ GeV, $m_b = 4.5$ GeV, and $\tan \beta = 30$, where $\tan \beta = v_2/v_1$ is the ratio of the vacuum expectation values of the two Higgs doublets in the MSSM. The Born, NLO-NLL, and NNLO-NLL results are shown. Both the NLO and the NNLO threshold corrections are important as can be more clearly seen in the right frame of Fig. 1 where we plot the $K$-factors, i.e. the ratios of the NLO-NLL over Born and the NNLO-NLL over Born cross sections. As expected, the corrections increase for larger charged Higgs mass at fixed $\sqrt{s}$, since then we get closer to the threshold region. Finally, we note that the cross section for $bg \rightarrow \bar{t}H^+$ is the same as for $bg \rightarrow tH^-$.

7 Conclusions

I have reviewed recent calculations of soft-gluon corrections for processes in hadron-hadron and lepton-hadron collisions. A unified approach and master formulas that allow the explicit calculation of next-to-next-to-leading order soft and virtual corrections for arbitrary processes have been presented. I have also presented phenomenological applications of the unified formalism to top quark pair production, FCNC top production via anomalous $tqV$ couplings, $W$-boson hadroproduction at large transverse momentum, and charged Higgs production. In all cases the soft corrections are significant and important, and they greatly decrease the dependence of the cross sections on factorization and renormalization scales.
Acknowledgments

I would like to thank Sasha Belyaev, Jeff Owens, Agustin Sabio Vera, and Ramona Vogt for fruitful collaborations. The author’s research has been supported by a Marie Curie Fellowship of the European Community programme “Improving Human Research Potential” under contract number HPMF-CT-2001-01221.

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