Ageing properties of three-dimensional pure and site-diluted Ising ferromagnets

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Abstract. Ageing properties of the three-dimensional pure and diluted Ising models are investigated for case with quenching from a high temperature initial state to their critical points. The two-times autocorrelation and response functions for the systems with spin concentrations \( p = 1.0, 0.8, \) and 0.6 are computed and analysed. The fluctuation-dissipation theorem violation for considered 3D Ising models are shown with the asymptotic values of the fluctuation-dissipation ratio \( X_\infty(p) < 1/2. \) The asymptotic values of universal fluctuation-dissipation ratio \( X_\infty \) were measured, and it was shown that \( X_\infty^{\text{disorder}}(p < 1) > X_\infty^{\text{pure}}(p = 1). \)

1. Introduction
The behaviour of systems with abnormally slow dynamics stays one of the main problem in theoretical physics [1, 2]. It demonstrates a wide range of interesting phenomena such as critical slowing down, ageing, and violation of the fluctuation-dissipation theorem [3]. Originally, these features were founded in complex spin glass systems [4] but different studies of these phenomena showed that the main properties of slow dynamics can be observed in simpler systems with second-order phase transition near critical point, because the evolution of such systems is characterized by large relaxation times.

According to dynamical scaling the relaxation time diverges as \( t_{rel} \sim |T - T_c|^{-\nu} \) in the vicinity of critical point \( T_c, \) where \( z \) is the dynamic exponent, \( \nu \) is the exponent for correlation length. Therefore a system does not achieve an equilibrium state at critical point while relaxation process. During out-of-equilibrium stage of dynamics for \( t \ll t_{rel} \) ageing phenomena occur with two-time dependencies of correlation and response functions on characteristic time variables as waiting time \( t_w \) and time of observation \( t - t_w \) with \( t > t_w. \)

Another slow dynamics effect at out-of-equilibrium stage is the violation of the fluctuation-dissipation theorem (FDT) [5]. FDT suggests relation between correlation function \( C(t, t_w) \) and response function \( R(t, t_w) \) on some external field

\[
R(t, t_w) = \frac{X(t, t_w)}{T} \frac{\partial C(t, t_w)}{\partial t_w},
\]

where \( X(t, t_w) \) is so called fluctuation-dissipation ratio (FDR), and one assumes \( t > t_w. \) According to FDT \( X(t, t_w) = 1 \) in equilibrium. The asymptotic value of the FDR

\[
X_\infty = \lim_{t_w \to \infty} \lim_{t \to \infty} X(t, t_w)
\]
can be used as a new universal critical characteristic of a systems with slow dynamics with 
$X^\infty \neq 1$ as a signal of an asymptotic non-equilibrium dynamics.

Ageing effects were studied at critical point for 2D Ising model [3, 6] and 3D XY model
[7] by Monte Carlo methods. The theoretical description results of the different models was
reviewed in [5]. In the paper [8] the off-equilibrium fluctuation-dissipation relation for some
disordered systems (diluted ferromagnets, random field Ising model and four-dimensional spin
glasses) has been investigated for temperatures far from the critical point with $T < T_c$. The
value of asymptotical FDR $X^\infty = 0$ was obtained for these systems. It has been shown that
no evidences of replica symmetry breaking have been found in the diluted ferromagnetic model.
Non-equilibrium dynamics of the 2D random-bond Ising model has been discovered in [9] by
short-dynamics method

In this report we present the numerical investigation of the ageing properties and the
fluctuation-dissipation ratio in ferromagnets described by three-dimensional Ising model with
point-like defects at critical point.

2. Model and methods

The Hamiltonian of disordered ferromagnetic Ising model with local magnetic field $h_i$ is

$$H = -J \sum_{<i,j>} p_ip_jS_iS_j - \sum_i h_i S_i,$$

where $J > 0$ is the short-range exchange interaction between spins $S_i$ fixed at the lattice sites,
and assuming values of $S_i = \pm 1$. Nonmagnetic impurity atoms form empty sites. In this case,
occupation numbers $p_i$ assume the value 0 or 1, and $p_i$ equal to 1 if site contains spin and 0
otherwise. We considered the cubic lattice with periodic boundary conditions. Let us denote
$N = pL^3$ is the number of spins where $L$ is the linear size of the lattice and $p$ is the spin
concentration.

For magnetic systems the conjugate response function $R(t, t_w)$ with applied external field $h$
at time $t_w$ is defined by

$$R(t, t_w) = \frac{1}{V} \int dt' \delta \left< S(x, t) \right> \frac{\delta h(x, t_w)}{\delta h(x, t_w)} \bigg|_{h=0}.$$

There are two ways to compute the FDR. The first way is connected with application of small
magnetic field after waiting time $t_w$. In this case the response function $R(t, t_w)$ can not be
calculated directly and more useful quantity is the integrated response function [11].

$$\chi(t, t_w) = T \int_{t_w}^t dt' R(t, t').$$

In the large time limit $\chi(C) = \int_0^C X(q)dq$ and the FDR can be defined as

$$X(t, t_w) = -\lim_{C\to0} \frac{\partial(T\chi(t, t_w))}{\partial C(t, t_w)}.$$

Another way to calculate the FDR is the use of heat-bath dynamics that allows to obtain the
response function directly. The derivative of the heat-bath probability transition is continuous
and this fact allows to calculate the response function without taking into account an external
magnetic field. We have applied the method suggested by Chatelain [10, 11]. Let us describe
the details.

Heat bath dynamics is one of the realization of Glauber dynamics in Monte Carlo simulations
[13]. The probability transition of single spin update $S_i \to S_i'$ reads

$$W_{sp}(S_i \to S_i') = \frac{\exp(-\beta H(S_i'))}{\sum_{S_j} \exp(-\beta H(S_j))}.$$
with \( S_j \) summed over all possible states of spin \( S_i \) before flipping, \( \beta = 1/T \) is the inverse temperature. Since there are only two possible spin’s states in Ising model \( S_j = \pm 1 \), the probability transition is

\[
W_{sp}(S_i \rightarrow S_i') = \frac{\exp(-\beta H(S_i'))}{\exp(\beta H(S_i)) + \exp(-\beta H(S_i))}. \tag{8}
\]

Let us denote by \( \varphi(\{ S \}, t) \) the probability to obtain system with spin’s configuration \( \{ S \} \) at time \( t \). The dynamic evolution in critical point is Markov process and the master equation can be written as

\[
\varphi(\{ S \}, t + \Delta t) = (1 - \Delta t)\varphi(\{ S \}, t) + \Delta t \sum_{\{ S' \}} W(\{ S' \} \rightarrow \{ S \}, t)\varphi(\{ S' \}, t). \tag{9}
\]

\( W(\{ S' \} \rightarrow \{ S \}, t) \) is the probability transition from state \( \{ S' \} \) to \( \{ S \} \) at time \( t \) with normalization condition \( \sum_{\{ S' \}} W(\{ S' \} \rightarrow \{ S \}, t) = 1 \). The conditional probability \( \varphi(\{ S \}, t|\{ S' \}, t_w) \) satisfies the master equation too

\[
\varphi(\{ S \}, t + \Delta t|\{ S' \}, t_w) = (1 - \Delta t)\varphi(\{ S \}, t|\{ S' \}, t_w) + \Delta t \sum_{\{ S'' \}} W(\{ S'' \} \rightarrow \{ S \}, t|\{ S' \}, t_w), \tag{10}
\]

The conditional probability is defined by the Bayes theorem

\[
\varphi(\{ S \}, t) = \sum_{\{ S' \}} \varphi(\{ S \}, t|\{ S' \}, t_w)\varphi(\{ S' \}, t_w). \tag{11}
\]

The probability transition from \( \{ S_i \} \) to \( \{ S_i' \} \) reads (include (7))

\[
W(\{ S_i \} \rightarrow \{ S_i' \}, t) = \frac{1}{N} \sum_{k=1}^{N} W_k(\{ S \} \rightarrow \{ S' \}) = \frac{1}{N} \sum_{k=1}^{N} \prod_{l \neq k} \delta_{S_i,S_i'} W_{sp}(S_k \rightarrow S_k'). \tag{12}
\]

The probability \( W_k \) includes only one spin-flip update \( S_k \rightarrow S_k' \). Note that in simulations time unit is Monte Carlo step on spin (mcs/s) that defines \( N = pL^3 \) discrete spin updates. The product of deltas ensures that during spin flip only the one updates.

Using the master equation (9) and Bayes theorem the average of spin \( \langle S_j \rangle \) at time \( t > t_w \) can be written as

\[
\langle S_j \rangle = \sum_{\{ S \}} S_j\varphi(\{ S \}, t) = \sum_{\{ S \},\{ S' \}} S_j\varphi(\{ S \}, t|\{ S' \}, t_w + \Delta t)\varphi(\{ S' \}, t_w + \Delta t) = \sum_{\{ S \},\{ S' \}} S_j\varphi(\{ S \}, t|\{ S' \}, t_w + \Delta t)\bigg[ (1 - \Delta t)\varphi(\{ S' \}, t_w) + \Delta t \sum_{\{ S'' \}} W_k(\{ S'' \} \rightarrow \{ S' \}) \bigg]. \tag{13}
\]

In order to obtain the response function one needs to calculate derivative \( \frac{\partial \langle S_j(t) \rangle}{\partial h_i} \) and takes the limit \( h_i \rightarrow 0 \). In (13) only \( W_k \) depends on \( h_i \) through Hamiltonian (3) and the derivative reads

\[
\left[ \frac{\partial \langle S_j(t) \rangle}{\partial h_i} \right]_{h_i \rightarrow 0} = \Delta t\delta_{k,i} \sum_{\{ S \},\{ S' \},\{ S'' \}} S_j\varphi(\{ S \}, t|\{ S' \}, t_w + \Delta t) \times \left[ \frac{\partial W_k}{\partial h_i} (\{ S'' \} \rightarrow \{ S' \}) \right]_{h_i \rightarrow 0}. \tag{14}
\]
The derivative defines the integrated response function when infinitesimal external field \( h_i \) is applied at time range \([t_w, t_w + \Delta t]\)

\[
\chi_{ji}(t, [t_w, t_w + \Delta t]) = T \frac{\partial \langle S_j(t) \rangle}{\partial h_i}.
\]

(15)

Taking derivative of probability transition at(14), one obtains

\[
\frac{\partial W_i}{\partial h_i}({S''} \rightarrow {S'}) = \beta W_i({S''} \rightarrow {S'}) [S'_i - S^W_i],
\]

(16)

where \( S^W_i = \tanh(\beta J \sum_{m \neq j} S_m') \).

Using master equation (9) and taking into account (16), the integrated response function reads

\[
\chi_{ji}(t, [t_w, t_w + \Delta t]) = \delta_{k,i} \sum_{\{S\}, \{S'\}} S_j \varphi(\{S\}, t | \{S'\}, t_w + \Delta t) \left[ S'_i - S^W_i \right]
\times \left[ \varphi(\{S'\}, t_w + \Delta t) - (1 - \Delta t) \varphi(\{S'\}, t_w) \right].
\]

(17)

The response function \( R_{ji} \) is connected with the integrated response function by relation

\[
\chi_{ji}(t, [t_w, t_w + \Delta t]) = T \int_{t_w}^{t_w + \Delta t} R_{ji}(t, s) ds = TR_{ji}(t, t_w) \Delta t + O(\Delta t^2).
\]

(18)

As note above, the time is a discrete quantity in Monte Carlo simulations. Assuming \( \Delta t = 1 \) at (18), we obtain

\[
\chi_{ji}(t, [t_w, t_w + \Delta t]) = \delta_{k,i} \left( S_j(t) \left[ S_i(t_w + 1) - S^W_i(t_w + 1) \right] \right).
\]

(19)

The final relation for calculation of the response function in Monte Carlo simulations reads

\[
R(t, t_w) = \frac{1}{N} \sum_{i=1}^{N} R_{ii} = \frac{1}{N} \beta \sum_{i=1}^{N} \left( S_i(t) \left[ S_i(t_w + 1) - S^W_i(t_w + 1) \right] \right),
\]

(20)

where \( R(t, t_w) \) is averaged on all spin flips during one Monte Carlo step.

The two-time autocorrelation function reads

\[
C(t, t_w) = \left\{ \frac{1}{N} \sum_{i=1}^{N} S_i(t) S_i(t_w) \right\},
\]

(21)

and the autocorrelation function derivative by waiting time \( t_w \) can be calculated by

\[
\frac{\partial}{\partial t_w} C(t, t_w) = \frac{1}{N} \sum_{i=1}^{N} \left( S_i(t) \left[ S_i(t_w + 1) - S_i(t_w) \right] \right).
\]

(22)

Using definition (1) and relations (20) - (22), the fluctuation-dissipation ratio can be defined as

\[
X(t, t_w) = \frac{TR(t, t_w)}{\frac{\partial}{\partial t_w} C(t, t_w)} = \frac{\sum_{i=1}^{N} \left( S_i(t) \left[ S_i(t_w + 1) - S^W_i(t_w + 1) \right] \right)}{\sum_{i=1}^{N} \left( S_i(t) \left[ S_i(t_w + 1) - S_i(t_w) \right] \right)}.
\]

(23)
2.1. Simulations with probing magnetic field

In this part of investigations we computed time dependence of the autocorrelation function and integrated response function using the Metropolis algorithm. The simulations have been performed on cubic lattices with linear size \( L = 128 \), with periodic boundary conditions. The considered spin systems were quenched in the critical point from an infinite-temperature initial state. The averages are performed with the use of 5000 samples characterized by different independent configurations of defects. The two-time autocorrelation function was calculated via relation

\[
C(t, t_w) = \left\langle \frac{1}{pL^3} \sum_{i=1}^{pL^3} p_i S_i(t) S_i(t_w) \right\rangle,
\]

where the angle brackets stand for an average over initial configurations and realizations of the thermal noise, the square brackets are for averaging over the different impurity configurations. The two-time integrated response function \( \chi(t, t_w) \) was computed by the application of a random magnetic field with small amplitude \( h = 0.01 \) in order to avoid nonlinear effects via:

\[
\chi(t, t_w) = \left\langle \frac{1}{h^2pL^3} \sum_{i=1}^{pL^3} h_i(t_w) S_i(t) \right\rangle,
\]

where the line stands for an average over the random field realizations. The autocorrelation functions time dependencies are plotted in Figure 1 for different values of waiting time \( t_w \) and spin concentrations \( p \).

![Autocorrelation plot for different p and t_w. The error bars are smaller than the size of the symbols.](image)

We can distinguish three stages of the relaxation for each system. The first stage is observed for small time separation \( t - t_w \ll t_w \) where ageing does not exist and the dynamic evolution of the autocorrelation functions exhibits a stationary part and does not depend on waiting time. The second regime is realized for times \( t - t_w \sim t_w \gg 1 \) where the correlation functions at different waiting times do not superpose and are characterized by different slopes for each \( t_w \). It is the ageing stage of relaxation. At long time separations with \( t - t_w \gg t_w \gg 1 \) the correlation functions decay as power law

\[
C(t, t_w) \sim (t/t_w)^{-c_a}, \quad c_a = \frac{d}{z} - \theta',
\]
where exponent $c_a$ is the same which describes time dependence of the autocorrelation function in the short-time regime of non-equilibrium behaviour with well-known initial slip exponent $\theta_i$ for the magnetization $(M(t) \sim t^{\theta_i})$ [12, 15]. At this stage ageing effects are not developed as well.

At the ageing regime (dashed lines in Figure 1) we approximated the autocorrelation function time-decay as power law $C(t, tw) \sim (t - tw)^{-b}$ with the slope exponent $b$ for estimation of correlations falling. Obtained values of $b$ are presented in the table 1 which show that decay of the correlation functions is slower with increasing of waiting time $tw$. Also we point out that the presence of defects and increasing of their concentration leads to ageing gain.

Table 1. The values of the autocorrelation slope exponent for spin concentrations $p = 1.0$, $p = 0.8$ and 0.6.

| $tw$ | $b$ for $p = 1.0$ | $tw$ | $b$ for $p = 0.8$ | $tw$ | $b$ for $p = 0.6$ |
|------|------------------|------|------------------|------|------------------|
| 10   | 1.048(18)        | 50   | 0.938(34)        | 75   | 0.879(12)        |
| 25   | 1.023(14)        | 250  | 0.739(40)        | 1000 | 0.569(30)        |
| 50   | 0.934(11)        | 500  | 0.644(25)        |      |                  |
| 75   | 0.879(12)        | 1000 | 0.569(30)        |      |                  |

For stage with $t - tw \sim tw \gg 1$ theory of ageing phenomena predicts for the correlation function following two-time scaling dependence $C(t, tw) \sim tw^{-2\nu/\beta_c}F_c(t/tw)$. The scaling function $F_c(t/tw)$ should depends on the universality class of the system. We have checked the collapse of the different waiting time curves. The obtained data are plotted in Figure 2 which demonstrate a quite good collapse of the curves for different waiting times.

Figure 2. Scaling of autocorrelation function $C(t, tw)$ for different spin concentrations $p$. The error bars are smaller than the symbols.

For well-separated times with $t - tw \gg tw \gg 1$ the scaling function $F_c(t/tw)$ is characterized by power law time dependence $F_c(t/tw) \sim A_c(t/tw)^{-c_a}$ with exponent $c_a$ from (26). From our data we obtained the following values of $c_a$ for systems with different spin concentrations: $c_a(p = 1.0) = 1.333(40), c_a(p = 0.8) = 1.237(22)$, and $c_a(p = 0.6) = 0.982(30)$. We must mention that these values of $c_a$ for pure and weakly diluted Ising models with $p = 0.8$ agree.

Figure 3. $T \chi(t, tw)$ versus $C(t, tw)$ plot for $p = 0.8$ and 0.6. The error bars are smaller than the size of the symbols.
rather well with $c_a = 1.36(1)$ from paper [16] and with $c_a = 1.242(10)$ from paper [15] computed by the short-time dynamics method.

To obtain the FDR on the base of relation (6), we analysed the dependencies $T_X(t,t_w)$ from $C(t,t_w)$, found the slopes of curves for different $t_w$, and then made extrapolation $t_w \to \infty$. The results are plotted in Figure 3 for diluted systems and presented in table 2.

Table 2. The values of the FDR $X^\infty$ for spin concentrations $p = 1.0$, $p = 0.8$, and 0.6.

| $t_w$ | $p = 1.0$ | $p = 0.8$ | $p = 0.6$ |
|-------|-----------|-----------|-----------|
| 10    | 0.586(24) |           |           |
| 25    | 0.460(22) |           |           |
| 50    | 0.437(26) |           |           |
| 250   | 0.708(16) | 0.726(13) |           |
| 500   | 0.553(17) | 0.583(14) |           |
| 1000  | 0.494(17) | 0.519(29) |           |
| $\to \infty$ | 0.391(12) | 0.419(11) | 0.443(10) |

The obtained values of the fluctuation-dissipation ratio for systems with different spin concentrations will be analysed below in this paper. In next subsection the results of the calculations of the two-time response function $R(t,t_w)$ are presented.

2.2. Heat-bath dynamics without application of external magnetic field

The use of the heat-bath dynamics method allows us to compute and analyse the response function directly. In this case we have simulated the dynamics of the systems starting from the high-temperature initial state with magnetization values $m_0(p = 1) = 0.02$, $m_0(p = 0.8) = 0.01$, and $m_0(p = 0.6) = 0.005$. The response function was calculated using the relation (20). The obtained data were averaged over 90000 statistical realization in case of pure Ising model with $p = 1$ and over 6000 independent impurities configurations for site-diluted Ising models. Note, that each disordered configuration was averaged over 15 statistical realizations of initial state with $m_0 \ll 1$. The ageing properties of the response function are showed in Figure 4.

One can see that the response function has demonstrated the similar to the autocorrelation function time behaviour (see section 2.1). The early stage of critical relaxation $t - t_w \ll t_w$ is characterized by lack of the memory effects, and only at times $t - t_w \sim t_w$ the ageing is developed. On the analogy of the autocorrelation function, theoretical prediction for scaling form of the response function $R(t,t_w)$ is

$$R(t,t_w) \sim t_w^{-1-2\beta/\nu z} F_R(t/t_w).$$

(27)

To check this scaling dependency of the $R(t,t_w)$ we plotted $t^{(1+2\beta/\nu z)} R(t,t_w)$ versus $(t - t_w)/t_w$. The obtained data are presented in Figure 5. The response function also demonstrates the collapse of curves for different waiting times into single curve with universal scaling dependency. Note that systems with different spin concentrations are characterized by different curves for two-time dependencies of the response function.

The scaling function $F_R(t/t_w)$ has power law time dependency $F_R(t/t_w) \sim A_R(t/t_w)^{-c_r}$ at times $t \gg t_w \gg 1$. At this stage ageing properties has not appeared, and $c_r$ is determined by the same exponents as $c_a$ in asymptotic dependency of the scaling function $F_c(t/t_w)$. So, $c_r = d/2 - \theta'$. 


We collected the values of $c_r$ and $c_a$ from section 2.1 in table 3 and compared their with values which have been obtained by the short-time dynamics method in other works. The exponents demonstrate a good agreement with short-time dynamics results in both pure and weak diluted systems cases.

Using expression (23), we have computed the fluctuation-dissipation ratio. The obtained data are plotted in Figure 6 for different spin concentrations. The plot of the $X(t, t_w)$ evolution consists of two different stages. It is expected that at times $t - t_w \ll t_w$ ageing properties have not appeared yet, and the $X(t, t_w)$ will be close to 1. We used time interval from $t - t_w \sim t_w$ to $t - t_w \gg t_w$ to obtain the asymptotic values of $X(t_w, p)$ for different waiting times.

In order to get the asymptotic values $X^\infty(p)$ (2) we have calculated $X(t_w, p)$ from plot in Figure 6 in the limit $t_w/(t - t_w) \rightarrow 0$, i.e. $t - t_w \gg t_w$. Using the obtained values of $X(t_w, p)$, we made extrapolation $t_w \rightarrow \infty$ to gain the asymptotic FDR $X^\infty(p)$. The results are showed in table 4 and in Figure 7.

The final values of the fluctuation-dissipation ratio are $X^\infty(p = 1) = 0.380(13)$, $X^\infty(p = 0.8) = 0.413(11)$, and $X^\infty(p = 0.8) = 0.446(8)$. In the table 4 we include the FDR values obtained from analysis of the integrated response function (lower line), see section 2.1. The obtained the FDR values agree quite well with each other for both methods.

2.3. Conclusions
We have investigated the non-equilibrium behaviour of pure and disordered three dimensional ferromagnetic Ising models quenched from high-temperature initial state to critical point.
Investigation of influence of structural disorder on ageing properties and violations of the equilibrium fluctuation-dissipation theorem in these systems is carried out.

On the base of analysis of the two-time autocorrelation $C(t,t_w)$ and response $R(t,t_w)$ functions were demonstrated the ageing effects. It has been shown that the presence of defects leads to ageing gain. The scaling relations $C(t,t_w) \sim t_w^{-a} F_C(t/t_w)$ and $R(t,t_w) \sim t_w^{-1-a} F_R(t/t_w)$ with $a = (d - 2 + \eta)/z = 2\beta/\nu z$ were checked.

For regime with well-separated times $t - t_w \gg t_w \gg 1$ it was shown that the scaling functions $F_C(t/t_w)$ and $F_R(t/t_w)$ are characterized by power law time dependencies and the obtained values of the exponents $c_a$ and $c_r$ demonstrate a good agreement with relation $c_a = c_r = d/z - \theta'$ for each spin concentration $p$ and with results of the short-time dynamics method (see the table 3).

The asymptotic values of the universal fluctuation-dissipation ratio were computed for these systems, and it has been proved the violation of the fluctuation-dissipation theorem for critical non-equilibrium behaviour of the three-dimensional diluted Ising models with $X^\infty(p) < 1/2$. So, the obtained values of $X^\infty(p)$ for pure and diluted systems equal $X^\infty(p = 1) = 0.380(13), X^\infty(p = 0.8) = 0.413(11)$, and $X^\infty(p = 0.8) = 0.446(8)$. From

\begin{table}[h]
\centering
\begin{tabular}{ccc}
$t_w$ & $p = 1.0$ & $p = 0.8$ & $p = 0.6$ \\
\hline
10 & 0.361(2) & & \\
15 & 0.371(4) & & \\
20 & 0.365(10) & 0.373(9) & \\
25 & 0.369(9) & & \\
30 & 0.374(14) & 0.384(5) & 0.382(1) \\
50 & 0.379(10) & 0.397(4) & 0.407(3) \\
100 & 0.406(6) & 0.427(6) & \\
150 & 0.412(9) & 0.437(9) & \\
$\rightarrow \infty$ & 0.380(13) & 0.413(11) & 0.446(8) \\
$\rightarrow \infty$, $\chi(t,t_w) vs C(t,t_w)$ & 0.391(12) & 0.419(11) & 0.443(10) \\
\end{tabular}
\caption{Values of the FDR $X^\infty$ for spin concentrations $p = 1.0, p = 0.8,$ and 0.6.}
\end{table}
these results of our numerical investigations we can conclude that the non-equilibrium critical dynamics of the three-dimensional diluted Ising model is characterized by new FDR with $X_{\infty}^{\text{disorder}} (p < 1) > X_{\infty}^{\text{pure}} (p = 1)$, and values of the FDR are increased with growth of defect concentrations.

Renormalization-group investigations of non-equilibrium critical behaviour in pure Ising model and weakly dilute Random Ising Model (RIM) for purely relaxational dynamics were carried out in papers [17] and [18]. The asymptotic values of the FDR $X_{\infty}^{\text{pure}} = 0.429(6)$ and $X_{\infty}^{\text{weak--diluted}} \simeq 0.416$ were calculated for these models with the use of $\varepsilon$-expansion method. In the article [18] the authors has written that it is too complicated to define the influence of impurities based only on results $\varepsilon$-expansion in nearest orders of theory. They believe that calculations in higher orders of theory can help to clarify this complex question. The results of our study gave the answer on this question.

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References
[1] Henkel M and Pleimling M 2010 (Springer, Heidelberg) vol 2
[2] Afzal N and Pleimling M 2013 Phys. Rev. E 87 012114
[3] Calabrese P, Gambassi A, and Krzakala F 2006 J. Stat. Mech. , 6 2
[4] Berthier L and Kurchan J 2013 Nature Phys. 9 310
[5] Calabrese P and Gambassi A 2005 J. Phys. A 38 R133
[6] Krzakala F and Ricci-Tersenghi F 2006 J. Phys.: Conf. Ser. 40 42-49
[7] Abriet S and Karevski D 2004 Eur. Phys. J. B 41 79
[8] Parisi G, Ricci-Tersenghi F and Ruiz-Lorenzo J J 1999 Eur. Phys. J. B 11 317
[9] Luo H J, Schülke L and Zheng B 2001 Phys. Rev. E 64 036123
[10] Chatelain C 2003 J. Phys. A 36 10739
[11] Ricci-Tersenghi F 2003 Phys. Rev. E, 68 065104
[12] Janssen H K, Schaub B, and Schmittmann B 1989 Z. Phys. B 73 539
[13] Janke W 2008 (Springer Berlin Heidelberg) vol 739 pp 79-140
[14] Prudnikov V, Prudnikov P, Vakilov A and Krinitsyn A 2007 JETP 105 371
[15] Prudnikov V, Prudnikov P, Krinitsyn A, Vakilov A, Pospelov E and Rychkov M 2010 Phys. Rev. E 81 011130
[16] Jaster A, Mainville J, Scheulke L and Zheng B 1999 J. Phys. A 32 1395
[17] Calabrese P and Gambassi A 2002 Phys. Rev. E 66 066101
[18] Calabrese P and Gambassi A 2002 Phys. Rev. B 66 212407