The 21cm Signature of a Cosmic String Loop

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Cosmic string loops lead to nonlinear baryon overdensities at early times, even before the time which in the standard LCDM model corresponds to the time of reionization. These overdense structures lead to signals in 21cm redshift surveys at large redshifts. In this paper, we calculate the amplitude and shape of the string loop-induced 21cm brightness temperature. We find that a string loop leads to a roughly elliptical region in redshift space with extra 21cm emission. The excess brightness temperature for strings with a tension close to the current upper bound can be as high as 1\(^{\circ}\)K for string loops generated at early cosmological times (times comparable to the time of equal matter and radiation) and observed at a redshift of \(z + 1 = 30\). The angular extent of these predicted “bright spots” is about 0.1\(^{\circ}\) for a value of the string tension equal to the current upper bound. These signals should be detectable in upcoming high redshift 21cm surveys.

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I. INTRODUCTION

21cm surveys promise to become an excellent window to observationally probe the “dark ages”, the epoch in the evolution of the early universe before the main burst of star formation sets in which is the time of reionization in the standard LCDM cosmology. The reason is that the intensity of the redshifted 21cm radiation which we receive tracks the distribution of baryons and does not require the baryons to have formed stars.

From an experimental point of view the situation is promising. The LOFAR telescope array has been commissioned and has the capability of providing 21cm maps between redshift 5 and 100 at excellent angular and redshift resolution\textsuperscript{[1]}. LOFAR can in turn be viewed as a prototype for the “Square Kilometer Array” project which will have even higher resolution\textsuperscript{[2]}, and will also be able to probe up to redshifts of 30.

From the point of view of cosmology theory it is an interesting challenge to see whether the current standard paradigm of early universe cosmology, the LCDM model, will be consistent with the data. In this scenario, the primordial fluctuations form a Gaussian random field with a scale-invariant spectrum with a slight red tilt (more structure on larger scales). In this context, nonlinearities form rapidly on an extended range of scales at late times, comparable and later to the time of reionization. Observing large amplitude primordial fluctuations at high redshifts would pose a serious problem for the standard cosmological scenario.

In contrast, if our theory of matter is of a type which admits topological defects such as cosmic strings, these defects will lead to nonlinearities at very high redshifts. It was recently realized that cosmic string wakes lead to nonlinearities at very high redshifts. In this paper, we calculate the amplitude and shape of the string loop-induced 21cm brightness temperature. We find that a string loop leads to a roughly elliptical region in redshift space with extra 21cm emission. The excess brightness temperature for strings with a tension close to the current upper bound can be as high as 1\(^{\circ}\)K for string loops generated at early cosmological times (times comparable to the time of equal matter and radiation) and observed at a redshift of \(z + 1 = 30\). The angular extent of these predicted “bright spots” is about 0.1\(^{\circ}\) for a value of the string tension equal to the current upper bound. These signals should be detectable in upcoming high redshift 21cm surveys.

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evolution with an inflationary phase. There are already cosmological limits from the angular power spectrum of cosmic microwave background (CMB) anisotropy experiments which set an upper bound (see also for earlier work) on the dimensionless value $G\mu$ of $\mu$ (where $G$ is Newton’s gravitational constant) which is

$$G\mu < 1.5 \times 10^{-7}. \quad (1)$$

The scaling distribution of cosmic strings consists of a network of infinite strings with mean curvature radius comparable to the Hubble radius, plus a distribution of loops with radii $R < t$. A long string segment gives rise to a cosmic string wake whose 21cm signals were investigated in previous work. In this paper, we discuss the 21cm signature of a cosmic string loop. Cosmic string loops accrete surrounding matter in a roughly spherical pattern and lead to overdensities of matter (both dark matter and baryons) which form when the string loop is produced at very early time and then grow in size as time progresses. Thus, string loops will lead to large 21cm redshift signals at early times (long before the time of reionization in the standard LCDM scenario). The distribution of these signals is highly non-Gaussian since the string loop distribution is not Gaussian.

The outline of this paper is as follows: In the next section we discuss some preliminaries both concerning the distribution of cosmic strings and concerning 21cm cosmology. Section 3 summarizes our computation of the brightness temperature pattern induced by an individual cosmic string loop. We end with a discussion of our results.

II. PRELIMINARIES

In this paper we will calculate the 21cm signature of a single cosmic string loop. The loop distribution is determined by the cosmic string scaling solution, according to which at all times $t$ (sufficiently long after the phase transition which gives rise to the strings) there will be a network of infinite strings with mean curvature radius comparable to the Hubble radius $t$. Since the strings are relativistic objects, string segments will typically be moving with relativistic velocity. Hence, there will be frequent intersections of strings. As can be argued analytically (see for reviews), the scaling solution of the infinite strings is maintained by the process of interactions between segments of infinite strings producing loops.

If the infinite strings have negligible substructure, then the loops which produce at time $t$ will be generated with a typical radius of

$$R_{\text{form}}(t) = \alpha t, \quad (2)$$

where $\alpha < 1$ is a numerical constant (whose value must be determined numerically and is at the present time still quite uncertain). Once formed, loops will oscillate, emit gravitational radiation and gradually shrink in size. Numerical simulations of the dynamics of cosmic string networks have confirmed that the loop distribution approaches a scaling solution as well, but the detailed structure of the scaling solution is still poorly understood (see e.g. for some recent progress). Looking for position space signatures of individual string loops has the advantage over approaches based on computing correlation functions that the analysis is much less dependent on the unknown parameters entering into the description of the string loop scaling solution.

We will be working in terms of the “one scale model” for the string loop distribution (see e.g. which is based on assuming that all string loops are born with the same radius and that their number density redshifts as the universe expands with the radius very slowly decreasing due to gravitational radiation. The result for the number density $n(R, t)$ of strings per unit volume for unit $R$ is hence

$$n(R, t) = \nu R^{-2} t^{-2} R > \alpha t_{\text{eq}} \gamma G\mu t < \alpha t_{\text{eq}} \quad (3)$$

where $\gamma$ is a constant determined by the strength of gravitational radiation from a string loop and which is in the range $\gamma \sim 10$ (with substantial uncertainty). The cosmic string loops which dominate the mass in strings (and hence which dominate the baryonic mass accreted onto string loops) are hence those with $R \sim \gamma G\mu$.

Since cosmic strings carry energy, they will have gravitational effects which in turn lead to cosmological signals. Most of the signatures of infinite string segments originate from the fact that space perpendicular to a long straight string segment is conical, i.e. like $R^2$ with a missing wedge, the “missing angle” being $8\pi G\mu$). This leads to line discontinuities in CMB anisotropy maps. The cosmological fluctuations induced by a network of cosmic strings are “active” rather than “passive” (as they are for example in the case of inflationary cosmology). Thus, in spite of the fact that the power spectrum of fluctuations induced by strings is scale-invariant (see e.g.), there are no characteristic oscillations in the angular power spectrum of CMB anisotropies. The fact
that such oscillations have been observed with high significance allows us to put the constraint on the cosmic string tension mentioned in (1). On the other hand, recent work [15] has shown that limits which are up to one order of magnitude stronger may be found by analyzing CMB data directly in position space making use of edge detection algorithms [14].

Long string segments moving through the primordial dark matter and baryonic gas lead to wedge-shaped overdensities in their wake. These are called “wakes” [22]. A string at time \( t \) will produce a wake which is extended (typical length scale \( t \)) in the plane of the string world sheet, and has a mean thickness of \( 4\pi G \mu t \). The initial density in the wake is twice the background density. The wakes will grow in thickness by gravitational accretion, and will lead to signatures in CMB polarization [22] and in 21cm redshift maps [3] (see also [24]).

The signature of cosmic string wakes in 21cm redshift maps is a consequence of the overdensity of baryons inside the wake. This leads to extra emission or absorption of the 21cm radiation from the direction of the wake. The signal of a specific string wake produced at time \( t_i \) is seen at a redshift \( z(t) \) corresponding to the time \( t \) when our past light cone intersects the wake. It is extended in both angular directions (with a length scale corresponding to the comoving size of the horizon at time \( t_i \)), and it is thin in redshift direction, with the characteristic thickness being [22]

\[
\frac{\delta \nu}{\nu} \sim \frac{H w}{c},
\]

where \( H \) is the expansion rate of space and \( w \) is the wake width (taking into account its increase by gravitational clustering between the time \( t_i \) and \( t \)), both evaluated at the redshift \( z \) when the past light cone intersects the wake.

21cm surveys provide a window to explore the dark ages of the universe since they are sensitive to the distribution of baryons in the universe rather than stars. Most of the baryonic matter between the time of recombination and that of reionization is in the form of neutral hydrogen. Neutral hydrogen has a 21cm hyperfine transition line. If we look back at the radiation from the direction of a baryon cloud, we will see extra emission or absorption in this 21cm line. If the gas temperature \( T_K \) is higher than that of the microwave photons at the redshift \( z \) when our past light cone intersects the cloud, the signal will be in emission, if \( T_K < T_C \) it will be in absorption [22].

A significant advantage of cosmology with the 21cm line compared to CMB cosmology is that 21cm surveys provide three dimensional maps of structure in the universe. We map out in two directions in the sky plus in redshift direction. The effects of gas clouds at different redshifts which our past light cone intersects will be seen at different frequencies, namely the 21cm frequency shifted by the respective redshifts of intersection. Applied to cosmic string models, whereas the effects of different strings which our past light cone intersects are all projected into the same two-dimensional maps, the 21cm signals will remain well separated if the strings are at different redshifts.

In previous work [3] the 21cm signal of an individual string wake was studied, and in a followup paper [26] the two dimensional power spectrum at fixed redshift was computed for a scaling distribution of string wakes. In this paper, we will study the signature of a cosmic string loop.

In the case of cosmic string wakes, we [3] found that the signal in 21cm redshift surveys is a wedge in the three-dimensional redshift maps which is extended in the two angular directions, the angular extent given by the comoving scale of the Hubble radius at the time \( t_i \) that the string wake was formed, and narrow in redshift direction, the relative thickness being proportional to \( G \mu \) multiplied by the linear growth factor \( z(t_i)/z(t) \) between the redshift \( z(t_i) \) corresponding to the formation time and the redshift \( z(t) \) at which the wake is being seen (at which our past light cone intersects it). For redshifts \( z(t) \) larger than that of reionization and for values of \( G \mu \) comparable to or smaller than the current observational upper bound [11], the 21cm signal is an absorption signal with a brightness temperature amplitude of up to 400mK, the amplitude being to a first approximation independent of \( G \mu \).

Here, we find that the amplitude of the cosmic string loop signal can be even larger than the amplitude for a string wake. However, both the amplitude and the size of the string-induced feature in the 21cm sky depends on \( G \mu \). Since the accretion pattern induced by a string loop is roughly spherical, the induced 21cm signal will be an ellipsoidal region of extra 21cm emission. Thus, even though the amplitude of the signal may be larger than that of a cosmic string wake, it will be harder to observationally distinguish from foreground noise.

### III. 21CM SIGNAL OF A COSMIC STRING LOOP

#### A. Accretion of Matter onto a Cosmic String Loop

We first study the accretion of matter onto a cosmic string loop. We consider a string loop which was produced at time \( t_i \geq t_{eq} \) (and corresponding redshift \( z_i \)). As a first approximation, we will replace the string loop signal can be even larger than the amplitude for a string wake. However, both the amplitude and the size of the string-induced feature in the 21cm sky depends on \( G \mu \). Since the accretion pattern induced by a string loop is roughly spherical, the induced 21cm signal will be an ellipsoidal region of extra 21cm emission. Thus, even though the amplitude of the signal may be larger than that of a cosmic string wake, it will be harder to observationally distinguish from foreground noise. 

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3. It is an emission signal if the gas temperature \( T_K \) in the wake is higher than the temperature \( T_C \) of the CMB, and is an absorption signal if \( T_K < T_C \).

4. For a detailed review article on the cosmology with the 21cm line see [22].
then consider the motion of mass shells surrounding the point mass shell in response to the gravitational shell, making use of the Zel’dovich approximation [23].

As just mentioned, we model the gravitational effects of the cosmic string loop in terms of a point mass with associated energy density

\[ \rho_{\text{string}} = m_s \delta(x). \]  

We will assume radial symmetry. In terms of the radial coordinate \( h \) the above energy density source becomes

\[ \rho_{\text{string}} = m_s \frac{1}{4\pi h^2} \delta(h). \]  

Let us now consider a mass shell surrounding the string loop. The physical distance of the shell from the mass point is

\[ h(q, t) = a(t)[q - \psi(q, t)], \]  

where \( q \) is the comoving scale corresponding to the shell under consideration and \( \psi(q, t) \) is the comoving displacement of this shell acquired in response to the gravitational force. The cosmological scale factor \( a(t) \) is normalized to be one today: \( a(t_0) = 1 \), where \( t_0 \) stands for the present time.

In the Zel’dovich approximation the dynamics of this mass shell is described by Newtonian gravity, i.e. by

\[ \ddot{h} = -\frac{\partial \Phi}{\partial h}, \]  

where \( \Phi \) is the Newtonian gravitational potential which in turn is determined by the Poisson equation

\[ \nabla^2 \phi = 4\pi G(\rho_{bg} + \rho_{\text{string}}), \]  

where \( \rho_{bg} \) is the background energy density.

Inserting the ansatz (17) into the basic equations (5) and (8) of the Zel’dovich approximation and linearizing in \( \psi \) yields the following equation of motion for the comoving displacement \( \psi(q, t) \)

\[ \ddot{\psi} + \frac{4}{3}t^{-1}\dot{\psi} - \frac{2}{3}t^{-2}\psi = \frac{m_s G t_0^2}{q^2 t^2}, \]  

with initial conditions that both \( \psi \) and \( \dot{\psi} \) vanish at the time \( t_i \) when the string loop is formed. This equation can be solved by means of a Born approximation. Since this is a standard calculation (see e.g. [4, 28]) we only quote the final result:

\[ \psi = \frac{3m_s G t_0^2}{2q^2} \left[ 1 - \frac{3}{5} \left( \frac{t}{t_i} \right)^2 - \frac{2}{3} \left( \frac{t}{t_i} \right)^{-1} \right]. \]  

The Zel’dovich approximation is good until the shells “turn around”, i.e. until \( h(q, t) = 0 \). This condition determines the comoving scale \( q_{nl}(t) \) which is turning around at the time \( t \). A simple calculation yields

\[ q_{nl} = \left( \frac{9}{5} m_s G t_0^2 \right)^{1/3} t_i^{2/3} \approx q_0 \left( \frac{t}{t_i} \right)^{2/9}, \]  

where we have defined a quantity \( q_0 \) which we will use in the following. Note that this growth as a power of \( t^{2/9} \) agrees with the result of linear perturbation theory that the accreted mass (which is proportional to \( q_{nl}^3 \)) scales as \( t^{2/3} \).

After turnaround, the shell will collapse and virialize at a physical radius \( R_{\text{max}} \) which is one quarter of the radius which the shell would have at the time of turnaround in the absence of gravitational accretion. Thus, the physical radius of the nonlinear structure seeded by the string loop at time \( t \) is

\[ R_{\text{max}}(t) = \frac{1}{4} q(t) q_{nl}(t) = \frac{1}{4} z(t)^{-1} q_0 \left( \frac{z_i}{z(t)} \right)^{1/3} \]  

and the total mass which is gravitationally bound is

\[ M(t) = \frac{4\pi}{3} q_{nl}(t)^3 \rho_0, \]  

in agreement with the result expected from linear perturbation theory.

The mean overdensity (in both baryons and dark matter) in the nonlinear structure induced by a string loop is

\[ \rho_{\text{av}} = 64 \rho_0 \]  

since the radius has shrunk by a factor of 4 compared to what it would be for unperturbed Hubble flow. In fact, towards the center of the structure the relative overdensity in dark matter is higher than at the edges, since the shell which virializes at a distance \( R < R_{\text{max}} \) from the center has virialized earlier when the background density was larger. The way we can compute the cold dark matter distribution is to fix \( R < R_{\text{max}} \), determine the value of \( q \) for the shell which collapses to radius \( R \), the time when that shell has collapsed, and from those data the mass \( M(R) \) within that shell. The result is \( M(R) \sim R^{3/4} \) which leads to a density scaling

\[ \rho_{\text{DM}}(R) \sim R^{-9/4}. \]  

The details of this calculation are presented in [23]. They are the same which led to the conclusion that the velocity rotation curve about a cold dark matter halo is not flat - it is more peaked at the center - and that thus a pure cosmic string model of structure formation (which is ruled out because such a model does not yield the observed oscillations in the angular power spectrum of CMB anisotropies [32]) would require hot dark matter [31]. We are interested, however, in the density of baryons. Baryons will shock heat and their density distribution is expected to be uniform, with a relative overdensity given by [16].
We close this subsection by returning to the initial assumption we made, namely taking the entire loop mass to be concentrated at the origin. A necessary condition for this approximation to be justified is that the loop radius is smaller than the physical height of the outmost virialized shell. Replacing the loop radius by the loop length \( l \) yields a slightly more conservative condition

\[
R_{\text{max}}(t) \gg l. \tag{18}
\]

For a loop formed at time \( t_i \) whose induced nonlinear structure is being considered at time \( t \gg t_i \), this condition becomes

\[
\left( \frac{l}{t_i} \right)^2 < \frac{1}{64} \left( \frac{9}{5} G\mu \right) \left( \frac{z_i}{z} \right)^4\tag{19}
\]

and is hence easier to satisfy for small loops than large ones. Based on the scaling exercise, these are the more numerous ones. Inserting numbers, if we are interested in considering string loops formed at \( \tau_{eq} \) (corresponding to a redshift \( z_{eq} \approx 3 \times 10^3 \)) which have the longest time to undergo gravitational accretion, and redshift \( z \approx 30 \), then the condition \([17]\) is satisfied for strings with tension \( G\mu = 10^{-7} \) for all \( l \ll t_i \).

**B. Gas Temperature in the Nonlinear Structure**

We assume that the temperature of the gas inside the nonlinear structure induced by the cosmic string loop is characterized by the molecular kinetic energy which the particles obtain during the infall process. We use the virialization prescription that the infalling shell ends up with a height which is half of the height at turn-around, which is also one quarter of the height the shell would have in the absence of any gravitational accretion. If we denote the velocity of particles just before the shock as \( v_{\text{shock}} \), then the gas temperature is given by

\[
\frac{3}{2} k_B T = \frac{1}{2} m v_{\text{shock}}^2, \tag{20}
\]

where \( k_B \) is the Boltzmann constant which we will usually set to one in the natural units we are using. Thus, we must now compute \( v_{\text{shock}} \).

Based on the shell dynamics studied in the previous subsection, the physical velocity of the shell labelled by \( q \) at late times \( t \) (times when we can neglect the decaying mode of \( \psi(t) \)) is

\[
h = -\frac{3}{2} \frac{m_G}{q^2} \left( \frac{1}{5} \left( \frac{t_i}{t_0 t_1} \right)^{2/3} q \right)^{1/3}. \tag{21}
\]

The shock time \( t_{\text{shock}} \) is given by

\[
h(q, t_{\text{shock}}) = \frac{1}{2} h_{\text{max}}(q, t_{\text{nl}}), \tag{22}
\]

where \( h_{\text{max}} \) is \( h_{\text{max}} = 1/2a(t_{\text{nl}})q \). We therefore have

\[
h(q, t_{\text{shock}}) = \frac{1}{4} a(t_{\text{nl}})q. \tag{23}
\]

Based on our expressions for \( h(q, t) \) it is straightforward to compute the time \( t_s \) when the shell which turns around at time \( t_{\text{nl}}(q) \) hits the shock. The result is that to a good approximation

\[
t_s^{2/3} = t_{\text{nl}}^{2/3} \left( \frac{1 + \sqrt{2}}{\sqrt{2}} \right). \tag{24}
\]

Hence, the velocity at the shock is given by

\[
h = \left( \frac{1 + \sqrt{2}}{\sqrt{2}} \right)^{1/2} \frac{1}{5} \left( \frac{t_i}{t_0 t_1} \right)^{1/3} t_{\text{nl}}^{2/3}. \tag{25}
\]

Making use of \([20]\), this leads to a gas temperature of

\[
T_K(t) = \left( \frac{3}{2} \right)^{1/2} \frac{m_{HI}}{G\mu} \left( \frac{1}{2} \right)^{4/3} \left( \frac{m_G}{G\mu} \right)^{1/3} \left( \frac{t_i}{t_0 t_1} \right)^{1/3} t_{\text{nl}}^{-2/3}. \tag{26}
\]

or, multiplying out the factors of order unity,

\[
T_K(t) = \left[ 0.4 m_{HI} (m_G)^{2/3} t_i^{-4/9} t_{\text{nl}}^{-2/9} \right]. \tag{27}
\]

The gas temperature \( T_K \) needs to be computed with the temperature \( T_{\gamma} \) of the background CMB photons to determine whether the 21cm signal from the string loop is in emission or in absorption. As a first step, we must insert the string loop mass \( m_s \) into \([27]\). We express the string loop mass as a function of the ratio of the string length \( l \) to the formation time \( t_i \):

\[
m_s G = \frac{l}{t_1} G\mu t_1, \tag{28}
\]

and thus

\[
\frac{T_K(t)}{T_{\gamma}(t)} = \frac{0.4 \times m_{HI} (G\mu)^{2/3} \left( \frac{l}{t_i} \right)^{2/3} z(t)^{1/3} z_i^{-1/3}}{3 \times 10^{-13} \text{GeV}(t)}. \tag{29}
\]

The background photon temperature is given by

\[
T_{\gamma}(t) \approx 3 \times 10^{-13} \text{GeV}(t), \tag{30}
\]

and hence the ratio of temperatures is

\[
\frac{T_K(t)}{T_{\gamma}(t)} \approx \frac{0.4 \times m_{HI} (G\mu)^{2/3} \left( \frac{l}{t_i} \right)^{2/3} z(t)^{1/3} z_i^{-1/3}}{3 \times 10^{-13} \text{GeV}(G\mu)^0} \[z(t)]^{-2/3} z_i^{-1/3}. \tag{31}
\]

(Where \( (G\mu)_0 \) indicates the value of \( G\mu \) in units of \( 10^{-6} \)). Evaluating this result for string loops formed at \( z_i = 10^3 \)
and observed at \( z \approx 30 \) we obtain

\[
\frac{T_K(t)}{T_{\gamma}(t)} \approx 3 \times 10^6 \alpha^{2/3} (G\mu)_0^{2/3} \tag{32}
\]
for the largest loops present at time $t_i$ and

$$\frac{T_K(t)}{T_\gamma(t)} \simeq 3 \times 10^2 \gamma^{2/3} (G\mu)^{4/3}$$

(33)

for loops which dominate the distribution at the time $t_i$ which have radius $R = \gamma G\mu t_i$ at that time $^5$

The main message from this subsection is that the gas temperature in the nonlinear structure formed by a string loop is typically larger than that of the CMB photons. Hence, the 21cm signal is an emission signal. This is a difference compared to the string wake case $^3$, where for interesting values of $G\mu$ the 21cm signal will be in absorption. The difference is due to the fact that string loops exert a stronger gravitational attractive force than that induced by string wakes.

### C. Brightness Temperature

Consider the CMB radiation at frequency $\nu$ in direction of a gas cloud. The brightness temperature in this direction due to 21cm emission for a frequency $\nu$ is denoted by $T_b(\nu)$ and is given by (see $^{[23]}$)

$$T_b(\nu) = T_S(1 - e^{-\tau_\nu}) + T_\gamma(\nu)e^{-\tau_\nu}$$

(34)

where $T_\gamma$ is the background CMB temperature, $\tau_\nu$ is the optical depth and $T_S$ the spin temperature of the gas. The first term describes the extra emission as a consequence of the hot gas, the second term represents the absorption of the background CMB radiation by the gas cloud.

Because UV scattering is insignificant in our case, the spin temperature $T_S$ depends only on the gas temperature $T_K$ of the wake and the collision coefficient $x_c$:

$$T_S = \frac{1 + x_c}{1 + x_c} T_\gamma$$

(35)

The collision coefficient $x_c$ describes the rate at which hydrogen atoms and electrons are scattered. We will use values of $x_c$ for which are of observational interest. The spin temperature $T_S$ itself describes the relative number density

$$\frac{n_1}{n_0} = 3 \exp(-T_\gamma/T_S)$$

(36)

of atoms in the two hyperfine states that when excited produce the 21cm radiation. The quantity $T_\gamma = 0.068 K$ is the temperature that corresponds to the energy splitting between these two states. Furthermore, $n_1$ and $n_0$ are the individual number densities of atoms in the hyperfine states. As before, $\tau_\nu$ is the optical depth which can be determined by computing the absorption coefficient of the light way along the gas cloud. The frequency $\nu$ is the blue shifted frequency at the position of the cloud corresponding to the observed frequency $\nu_0$. The term proportional to $T_\gamma$ is due to absorption and stimulated emission.

As we have mentioned, what is of interest to us is the comparison of the temperature observed today in direction of the hydrogen gas cloud of the defect as compared to the background temperature. This is given by

$$\delta T_b(\nu) = \frac{T_b(\nu) - T_\gamma(\nu)}{1 + z} \simeq \frac{T_S - T_\gamma}{1 + z} \tau_\nu,$$

(37)

where we have assumed in the last step that the optical depth is smaller than 1 and that one thus Taylor expand the exponential factor to linear order in $\tau_\nu$. $^6$ The factor $1 + z$ comes from the redshifting of temperatures between the time of emission and the present time. We can now express $T_\gamma$ in terms of $T_S$ to obtain

$$\delta T_b(\nu) \simeq T_S \frac{x_c}{1 + x_c} \frac{T_\gamma}{1 + z} (1 - T_\gamma/T_K).$$

(38)

The optical depth for a general cloud of hydrogen is (in natural units) $^{[23]}$

$$\tau_\nu = \frac{3 A_{10} N_{H I} \nu_0}{4 \nu^2} \frac{T_S}{\phi(\nu)}$$

(39)

where $A_{10}$ is the spontaneous emission coefficient of the 21cm transition, $N_{H I}$ is the column density of hydrogen within the gas cloud which the photons reaching us pass through, and $\phi(\nu)$ is the line profile. The hydrogen column density is given by the number density $n_{HI}$ of hydrogen atoms in the cloud multiplied by the physical diameter $2R$ of the cloud, where $R$ is the height computed in the first subsection $^7$.

$$N_{H I} = 2 n_{HI} R.$$ 

(40)

The line profile describes the broadening of the emission lines as a consequence of the spatial extent of the gas cloud and the resulting redshift difference $\delta \nu$ in the frequency

$$\frac{\delta \nu}{\nu} = 2 H R,$$

(41)

where $H$ is the Hubble expansion constant. Because $\phi(\nu)$ is normalized to unity we find that

$$\phi(\nu) = \frac{1}{\delta \nu}$$

(42)

$^5$ Loops with radius $R < c t_i$ at time $t_i$ were formed earlier and so started accreting earlier if $t_i > t_{eq}$. For such loops $^{[23]}$ is not the correct formula to use. Instead, one should use the analog of $^{[23]}$ for the earlier initial time. However, since we are taking $z_i \sim z_{eq}$, and small loops present at that time only start the gravitational accretion process at the time of equal matter and radiation, then $^{[23]}$ is the equation which is applicable.

$^6$ However in practice this result is skewed by the intergalactic medium which can also be considered a hydrogen gas cloud.

$^7$ Note that $n_{HI}$ is the hydrogen number density at the time $t$. 
for \( \nu \in [\nu_{10} - \delta \nu/2, \nu_{10} + \delta \nu/2] \) and \( \phi(\nu) = 0 \) otherwise. This frequency shift plays an integral role in the observability of the 21cm signal since it determines the width of the string signal in the redshift direction.

Note that the dependence on \( G \mu \) cancels out between the column height and the line profile, as it did in the case of cosmic string wakes. Thus, we obtain

\[
\tau_\nu = \frac{3}{16 \pi^2} \frac{A_{10}}{T_\gamma} \frac{n_{HI}}{H_0 \Omega_m^{1/2} (1 + z)^{3/2}},
\]

where we have used \( H(z) = H_0 \Omega_m^{1/2} (1 + z)^{3/2} \) in which \( \Omega_m \) is the ratio between the matter energy density and the critical density (the density for a spatially flat universe). We can insert this into the expression for \( \delta T_b(\nu) \) to obtain the signal strength

\[
\delta T_b(\nu) = \frac{x_c}{1 + x_c} \left( 1 - \frac{T_\gamma}{T_K} \right) \frac{3 A_{10}}{16 \pi^2} \frac{n_{HI}^{string}}{H_0 \Omega_m^{1/2}} (1 + z)^{-5/2},
\]

We can remove the dependence on the hydrogen number density \( n_{HI} \) by introducing the background density \( n_{bg} \) and noting that the overdensity of hydrogen \( n_{HI}^{string} = 64 \) times that of the background. This yields

\[
\delta T_b(\nu) = \frac{x_c}{1 + x_c} \left( 1 - \frac{T_\gamma}{T_K} \right) \frac{3 A_{10}}{16 \pi^2} \frac{n_{HI}^{string}}{n_{bg}} \frac{n_{bg}}{H_0 \Omega_m^{1/2}} (1 + z)^{1/2},
\]

where we have rescaled the background number density to the present time.

In the expression \( 14 \) for the brightness temperature, the string tension \( G \mu \) enters only in two minor ways. Firstly, it determines the gas temperature \( T_\gamma \), but since \( T_K \gg T_\gamma \), for the parameter values we are interested in, this dependence is negligible. The second place where \( G \mu \) enters is in the collision coefficient \( x_c \), which depends on the gas temperature and hence on \( G \mu \). However, for our parameter values \( x_c \gg 1 \) and hence this second dependence is negligible as well.

Inserting the values \( H_0 = 73 \text{ km s}^{-1} \text{ Mpc}^{-1} \), \( A_{10} = 2.85 \times 10^{-15} \text{s}^{-1} \), \( v_{10} = 1420 \text{ MHz} \), \( \Omega_m = 0.26 \), \( T_\gamma = 0.068 \text{ K} \), and \( n_{bg} = 1.9 \times 10^{-7} \text{cm}^{-3} (1 + z)^3 \) we obtain

\[
\delta T_b(\nu) \simeq [1.1] \frac{x_c}{1 + x_c} (1 + z)^{1/2}
\]

where the collision coefficient \( x_c \) is given by

\[
x_c = \frac{n_{HI}^{string} T_\gamma}{A_{10} T_\gamma}
\]

and \( n_{10}^{HH} \) is the de-excitation cross section whose value is \( 2.7 \times 10^{-9} \text{ m}^3 \text{s}^{-1} \) at high temperatures corresponding to \( G \mu \sim 0.3 \times 10^{-6} \) at a redshift of \( (1 + z) \sim 30 \) and \( (1+z_1) \sim 10^5 \). These values lead to the collision coefficient of \( x_c \gg 1 \). We have also made use of the fact that the overdensity of the hydrogen gas is 64 times that of the background. The ensuing signal of the cosmic string loop is then

\[
\delta T_b(\nu) \sim 6 K,
\]

which is an emission signal in the 21cm map. This temperature is even larger than that obtained for cosmic string wakes, the reason being that the overdensity inside of a string loop-induced structure is larger than that in a string wake by a factor of 16.

### D. Geometry of the Signal

Projected onto the two angular directions, the 21cm signal of a cosmic string loop looks like a filled circular region of extra 21cm emission. The angular scale \( \theta \) of this region is determined by the ratio of the comoving distance \( \ell_{com} \) corresponding to \( R_{max} \) and the distance of the observer’s past light cone. We can express this as

\[
\frac{\theta}{180^\circ} = \frac{\ell_{com}}{l_0} = \frac{(1 + z) R_{max}(z)}{l_0}.
\]

The comoving length is related to the physical distance by \( a(t) \):

\[
\ell_{com} = a^{-1} \ell_{phys} = (1 + z) R_{max},
\]

and thus the angular size of the defect is determined by the maximum radius of the nonlinear region which has collapsed onto the cosmic string loop. Thus,

\[
\theta \sim 180^\circ \times \frac{1}{4} \left( \frac{G \mu}{G_M} \right)^{1/3} \left( \frac{L}{l_i} \right)^{1/3} t_0 z_i^{-1/6} z^{-4/3}
\]

The angular size therefore becomes

\[
\theta \sim 2^\circ \times 10^{-1} (G \mu)_b^{1/3} \alpha^{-1/3} z_i^{1/3} z^{-1/3}.
\]

Smaller loops at \( t_i \) give a correspondingly smaller angular scale.

The redshift extent of the blob of extra 21cm emission coming from the nonlinear structure seeded by a string loop was derived previously. It is

\[
\frac{\delta \nu}{\nu} = 2 H R_{max}(z).
\]

Inserting \( 52 \) and using the same parameters as above we obtain

\[
\frac{\delta \nu}{\nu} \sim 2 \times 10^{-3} (G \mu)_b^{1/3} \alpha^{-1/3},
\]

which is a larger value than the thickness for the 21cm signal of a cosmic string wake. The reason for the larger value is that in the case of a string wake (planar accretion) the height of the nonlinear structure scales linearly in \( G \mu \) whereas for a string loop (spherical accretion) it scales as \( (G \mu)^{1/3} \).
In this paper we have discussed the 21cm signature of cosmic strings loops. We found that for values of the string tension close to the current upper bounds, the signature is an ellipsoidal region of extra 21cm emission. The brightness temperature of this signal is several degrees Kelvin for string loops produced at a redshift of \( z = 10^3 \) and observed at a redshift of \( z = 30 \). The brightness temperature is to a first approximation independent of the string tension, as in the case of the signature of a cosmic string wake.

Projected onto the celestial sphere, the angular extent of the region of extra 21cm emission is (for the same redshifts \( z_i \) and \( z \) mentioned in the previous paragraph) \( 0.2^\circ \) for a value of \( G\mu = 10^{-6} \) and for a string loop with width \( l_s = r_s \). This angle scale as \((G\mu)^{1/3}\) and also as \( l_s^{1/3} \). In redshift direction the relative thickness of the region of extra 21m emission is \( \Delta z \sim 2 \times 10^{-3} \) (for the same values of \( G\mu \) and \( z \) as above), and this thickness has the same scaling in both \( G\mu \) and \( l_s \) as the angular scale.

In terms of amplitude and both angular and redshift extent the cosmic string signal should be easily detectable by the SKA experiment, and possibly even with LOFAR. However, the signal might be hard to disentangle from noise and foregrounds, in contrast to the signal of a string wake which forms a very specific geometrical pattern.

A scaling distribution of string loops will lead to a superposition of many ellipsoidal regions of extra 21cm emission. However, since 21cm redshift surveys will produce three dimensional maps, the signatures of individual string loops will be spaced further apart as the corresponding signals in CMB anisotropy maps would be. Hence, we do not expect the string loop signal to become Gaussian as a consequence of the central limit theorem.

Since the string loop distribution can be viewed as statistically independent from the distribution of string wakes, the 21cm signature of a scaling string model will be the linear superposition of what we found in the current paper with what was found in \( \text{[2]} \). It would be interesting to compute the total angular power spectrum of the 21cm signal, following what was done in \( \text{[24]} \). However, in computing two dimensional correlation function, and in doing this for a projected map, a lot of discriminatory power is lost. An improvement could be obtained by computing a three-dimensional power spectrum. Even more promising, however, would be to find position space algorithms to search for string signatures.

Our analysis is applicable with very few changes to a second type of topological defect, namely to global monopoles, pointlike defects arising in a theory in which a global symmetry is broken in a way that the second homotopy group of the vacuum manifold is non-trivial \( \text{[32]} \). Note, however, that theories with broken global symmetries have potential problems and are hence not considered as interesting as those with broken local symmetries \( \text{[23]} \). Global monopoles also give rise to a spherical accretion pattern and to nonlinear density perturbations at early times. The density distribution of a monopole is different from that of a cosmic string loop since field gradient energy extends arbitrarily far from the center of the monopole \( \text{[8]} \). As a consequence, the dark matter distribution has a different radial profile compared to that of the structure induced by a cosmic string loop. However, the baryon distribution inside the region which has undergone shock heating will be identical for string loops and global monopoles, and hence the 21cm signal of a single global monopole will not be distinguishable from that of a cosmic string loop (see \( \text{[29]} \) for details). On the other hand, the scaling distribution of monopoles will differ from that of string loops, and hence the overall 21cm maps can in principle be distinguished.

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