Probing Models of Quantum Decoherence in Particle Physics and Cosmology

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Abstract. In this review we first discuss the string theoretical motivations for induced decoherence and deviations from ordinary quantum-mechanical behaviour; this leads to intrinsic CPT violation in the context of an extended class of quantum-gravity models. We then proceed to a description of precision tests of CPT symmetry and quantum mechanics using mainly neutral kaons and neutrinos. We also emphasize the possibly unique rôle of neutral meson factories in providing specific tests of models where the quantum-mechanical CPT operator is not well-defined, leading to modifications of Einstein-Podolsky-Rosen particle correlators. Finally, we discuss experimental probes of decoherence in a cosmological context, including studies of dissipative relaxation models of dark energy in the context of non-critical (non-equilibrium) string theory and the associated modifications of the Boltzmann equation for the evolution of species abundances.

1. Introduction: Decoherence, Quantum Gravity and CPT Violation
A complete theory of quantum gravity (QG) will necessarily address fundamental issues, directly related to the emergence of space-time and its structure at energies beyond the Planck energy scale \( M_P \sim 10^{19} \text{ GeV} \). From our experience with low-energy local quantum field theories on flat space-times, we are tempted to expect that a theory of QG should respect most of the fundamental symmetries that govern the standard model of electroweak and strong interactions, specifically Lorentz symmetry and CPT invariance, i.e. invariance under the combined action of Charge Conjugation (C), Parity (P) and Time Reversal Symmetry (T).

CPT invariance is guaranteed in flat space-times by a theorem applicable to any local relativistic quantum field theory of the type used to describe currently the standard phenomenology of particle physics. More precisely the CPT theorem states \([\text{I}]:\) Any quantum theory formulated on flat space-times is symmetric under the combined action of CPT transformations, provided the theory respects (i) Locality, (ii) Unitarity (i.e. conservation of probability) and (iii) Lorentz invariance.

The validity of any such theorem in the QG regime is an open and challenging issue since it is linked with our understanding of the nature of space-time at (microscopic) Planckian distances \( 10^{-35} \) m. However there are reasons to believe that the CPT theorem may not be valid (at least in its strong form) in highly curved space-times with event horizons, such as those in the vicinity of black holes, or more generally in some QG models involving quantum space-time foam backgrounds \([\text{2}].\) The latter are characterized by singular quantum fluctuations of space-time geometry, such as microscopic black holes, etc., with event horizons of microscopic Planckian
Such backgrounds result in apparent violations of unitarity in the following sense: there is some part of the initial information (quantum numbers of incoming matter) which “disappears” inside the microscopic event horizons, so that an observer at asymptotic infinity will have to trace over such “trapped” degrees of freedom. One faces therefore a situation where an initially pure state evolves in time and becomes mixed. The asymptotic states are described by density matrices, defined as
\[ \rho_{\text{out}} = \text{Tr}_M |\psi><\psi|, \]
where the trace is over trapped (unobserved) quantum states that disappeared inside the microscopic event horizons in the foam. Such a non-unitary evolution makes it impossible to define a standard quantum-mechanical scattering matrix. In ordinary local quantum field theory, the latter connects asymptotic state vectors in a scattering process
\[ |\text{out}\rangle = S |\text{in}\rangle, \quad S = e^{iH(t_f-t_i)}, \]
where \( t_f - t_i \) is the duration of the scattering (assumed to be much longer than other time scales in the problem, i.e. \( \lim t_i \to -\infty, t_f \to +\infty \)). Instead, in foamy situations, one can only define an operator that connects asymptotic density matrices
\[ \rho_{\text{out}} \equiv \text{Tr}_M |\text{out}\rangle\langle\text{out}| = S \rho_{\text{in}}, \quad S \neq SS^\dagger. \]

The lack of factorization is attributed to the apparent loss of unitarity of the effective low-energy theory, defined as the part of the theory accessible to low-energy observers performing scattering experiments. In such situations particle phenomenology has to be reformulated by viewing our low-energy world as an open quantum system and using (3). Correspondingly, the usual Hamiltonian evolution of the wave function is replaced by the Liouville equation for the density matrix
\[ \partial_t \rho = i[\rho, H] + \delta H/\rho, \]
where \( \delta H/\rho \) is a correction of the form normally found in open quantum-mechanical systems, although more general forms are to be expected in QG. This is what we denote by QG-induced decoherence, since the interaction with the quantum-gravitational environment results in quantum decoherence of the matter system, as is the case of open quantum mechanical systems in general.

The $ matrix is not invertible, and this reflects the effective unitarity loss. Since one of the requirements of CPT theorem (viz. unitarity) is violated there is no CPT invariance (in theories which have CPT invariance in the absence of trapped states). This semi-classical analysis leads to more than a mere violation of the symmetry. The CPT operator itself is not well-defined, at least from an effective field theory point of view. This is a strong form of CPT violation (CPTV) and can be summarised by: In an open (effective) quantum theory, interacting with an environment, e.g., quantum gravitational, where $ \neq SS^\dagger$, CPT invariance is violated, at least in its strong form. This form of violation introduces a fundamental arrow of time/microscopic time irreversibility, unrelated in principle to CP properties. Such decoherence-induced CPT violation (CPTV) should occur in effective field theories, since the low-energy experimenters do not have access to all the degrees of freedom of QG (e.g., back-reaction effects, etc.). Some have conjectured that full CPT invariance could be restored in the (still elusive) complete theory of QG. In such a case, however, there may be a weak form of CPT invariance, in the sense of the possible existence of decoherence-free subspaces in the space of states of a matter system. If this situation is realized, then the strong form of CPTV will not show up in any measurable quantity (that is, scattering amplitudes, probabilities etc.).

The weak form of CPT invariance may be stated as follows: Let \( \psi \in \mathcal{H}_{\text{in}}, \phi \in \mathcal{H}_{\text{out}} \) denote pure states in the respective Hilbert spaces \( \mathcal{H} \) of in and out states, assumed accessible.
to experiment. If $\theta$ denotes the (anti-unitary) CPT operator acting on pure state vectors, then weak CPT invariance implies the following equality between transition probabilities

$$P(\psi \rightarrow \phi) = P(\theta^{-1}\phi \rightarrow \theta\psi).$$

(5)

Experimentally, at least in principle, it is possible to test equations such as (5), in the sense that, if decoherence occurs, it induces (among other modifications) damping factors in the time profiles of the corresponding transition probabilities. The diverse experimental techniques for testing decoherence range from terrestrial laboratory experiments (in high-energy, atomic and nuclear physics) to astrophysical observations of light from distant extragalactic sources and high-energy cosmic neutrinos [5].

In the present article, we restrict ourselves to decoherence and CPT invariance tests within the neutral kaon system [4, 11, 12, 13, 14] and neutrinos [15, 16, 17, 18, 19, 20]. As we will argue later on, this type of (decoherence-induced) CPTV exhibits some fairly unique effects in $\phi$ factories [21], associated with a possible modification of the Einstein-Podolsky-Rosen (EPR) correlations of the entangled neutral kaon states produced after the decay of the $\phi$-meson (similar effects could be present for $B$ mesons produced in $\Upsilon$ decays).

We note for completeness two other possible mechanisms of CPTV in QG, which, however, shall not be discussed here, and which are independent from decoherence. The first is the spontaneous breaking of Lorentz symmetry (SBL) [22]; this type of CPTV does not necessarily imply (nor does it invoke) decoherence. In this case the ground state of the field theoretic system is characterized by non-trivial vacuum expectation values of certain tensorial quantities, $\langle A_\mu \rangle \neq 0$, or $\langle B_{\mu_1 \mu_2 \ldots} \rangle \neq 0$, etc. This may occur in (non-supersymmetric) ground states of string theory and other models, such as loop QG [22]. The second mechanism for CPTV is associated with deviations from locality, e.g., as advocated in [24], in an attempt to explain observed neutrino ‘anomalies’, such as the LSND result [25], pointing towards suppressed flavour oscillations in the neutrino sector as compared to the antineutrino one. Violations of locality could also be tested with high precision, by studying discrete symmetries in meson systems.

The reader should bear in mind that the important difference between the CPTV in SBL models and the CPTV due to the space-time foam is that in the former case the CPT operator is well-defined, but does not commute with the effective Hamiltonian of the matter system. In such cases one may parametrize the Lorentz and/or CPT breaking terms by local field theory operators in the effective Lagrangian, leading to a construction known as the “standard model extension” (SME) [22], which is a framework for studying precision tests of such effects.

A note on the order of magnitude of such QG CPT-Violating effects is in order. If present, such effects are expected in general to be strongly suppressed, and thus difficult to detect experimentally, due to the weakness of gravity. Naively, QG has a dimensionful constant, $G_N \sim 1/M_P^2$, where $M_P = 10^{19}$ GeV is the Planck scale. Hence, CPT violating and decohering effects may be expected to be suppressed by $E^3/M_P^2$, where $E$ is a typical energy scale of the low-energy probe. However, there could be cases where loop resummation and other effects in theoretical models result in much larger CPT-violating effects, of order $E^2/M_P$. This happens, for instance, in some loop gravity approaches to QG [23], or some non-equilibrium stringy models of space-time foam involving open string excitations [26]. Such large effects may lie within the sensitivities of current or immediate future experimental facilities (terrestrial and astrophysical), provided that enhancements due to the near-degeneracy take place, as in the neutral-kaon case. Moreover, possibly enhanced effects of QG decoherence may be encountered in theories of space time with large extra dimensions, e.g. TeV scale gravity [27]. In such cases, (high-energy) neutrinos appear the most sensitive probe to put stringent limits in various models in the foreseeable future [28].

We next notice that, when interpreting experimental results in searches for CPT violation, one should pay particular attention to disentangling ordinary-matter-induced effects, that mimic
CPTV, from genuine effects due to QG [5]. The order of magnitude of matter induced effects, especially in neutrino experiments, is often comparable to that expected in some models of QG, and one has to exercise caution, by carefully examining the dependence of the alleged “effect” on the probe energy, or on the oscillation length (in neutrino oscillation experiments). In most models, but not always, since the QG-induced CPTV is expressed as a back-reaction effect of matter onto space-time, it increases with the probe energy $E$ (and oscillation length $L$ in the appropriate situations). In contrast, ordinary matter-induced “fake” CPT-violating effects decrease with $E$.

We emphasize that the phenomenology of CPTV is complicated, and there does not seem to be a single figure of merit for it. Depending on the precise way CPT might be violated in a given model or class of models of QG, there are different ways to test the violation [5]. In this review we describe only some selected classes of such sensitive probes of CPT symmetry and quantum-mechanical evolution (unitarity, decoherence): neutral mesons and neutrinos.

The structure of the paper is as follows: in the next section we discuss the basic ideas and the underlying formalism, relevant for the study of the phenomenology of QG-induced decoherence. In Section 3 we describe tests of decoherence-induced CPTV using first (single-state) neutral kaon systems, and then entangled neutral meson states. In this respect, we discuss the novel EPR-like modifications in meson factories that may arise if the CPT operator is not well-defined, as happens in some space-time foam models of QG. We argue in favour of the unique character of such tests in providing information on the stochastic nature of quantum space-time, and we give some order-of-magnitude estimates within some string-inspired models. As we show, such models can be falsified (or severely constrained) in the next-generation (i.e. upgraded) $\phi$-meson factories, such as DAΦNE [29, 30]. The enhancement of the effect provided by the identical decay channels ($\pi^+\pi^-, \pi^+\pi^-$) is unique. In Section 4 we discuss precision tests of decoherence effects in neutrino-oscillation experiments, which, with the exception of the above-mentioned EPR-correlation tests in meson factories, constitute the most sensitive particle-physics probes of QG-decoherence to date. In section 5 we present a discussion on the rôle of decoherence in cosmology. In particular, we discuss dissipative cosmological models, which may arise in, say, string cosmology, whenever a non-equilibrium situation -due to a cosmically catastrophic event- appears. In this context, we analyse the rôle of space-time boundaries (cosmic horizons) in possibly inducing decoherence of particle physics probes propagating in such space-time backgrounds. We examine dark-energy decoherent effects in flavour oscillations, within a specific non-critical string framework [26, 31].

We finish our discussion with a brief description of non-equilibrium environmental effects on the Boltzmann equation for the evolution of the densities of cosmic relics, which may play a rôle in dark matter searches. Conclusions and outlook at presented in section 6.

2. Ideas and Methods for Quantum-Gravity Decoherence

2.1. Non-critical string-framework for decoherence

String theory is to date the most consistent theory of quantum gravity. In first-quantized string theory, de Sitter or other space-time backgrounds with cosmic horizons are not conformal on the world-sheet. The corresponding $\beta$-functions are non vanishing. Such non conformal backgrounds can be rendered conformal upon dressing the theory with the so-called Liouville mode [32]. The resulting $\sigma$-model world-sheet theory, then, contains an extra target-space coordinate. In cases where the deviation from conformality is supercritical, the Liouville mode has a time-like signature, and can be identified with the target time [26]. This formalism provides a mathematically consistent way of incorporating de Sitter and other backgrounds with cosmological horizons in string theory.

It can be shown, that the propagation of string matter in such non-conformal backgrounds is decoherent, the decoherence term being proportional to the world-sheet $\beta$-function. The following master equation for the evolution of stringy low-energy matter in a non-conformal
\( \sigma \)-model can be derived\(^{[26]} \)

\[
\partial_t \rho = i [\rho, H] + \beta^i \mathcal{G}_{ij} \left[ g^i, \rho \right] : 
\]  

(6)

where \( t \) denotes time (Liouville zero mode), the \( H \) is the effective low-energy matter Hamiltonian, \( g^i \) are the quantum background target space fields, \( \beta^i \) are the corresponding renormalization group \( \beta \) functions for scaling under Liouville dressings and \( \mathcal{G}_{ij} \) is the Zamolodchikov metric \(^{[33, 34]} \) in the moduli space of the string. To lowest order in the background field expansion the double colon symbol in \((6)\) represents the operator ordering: \( AB := [A, B] \). The index \( i \) labels the different background fields as well as space-time. Hence the summation over \( i, j \) in \((6)\) corresponds to a discrete summation as well as a covariant integration \( \int d^{D+1} y \sqrt{-g} \) where \( y \) denotes a set of \((D + 1)\)-dimensional target space-time co-ordinates and \( D \) is the space-time dimensionality of the original non-critical string.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure1}
\caption{Left: Recoil of closed string states with D-particles (space-time defects). Right: A supersymmetric brane world model of D-particle foam. In both cases the recoil of (massive) D-particle defect causes distortion of space-time, stochastic metric fluctuations are possible and the emergent post-recoil string state may differ by flavour and CP phases.}
\end{figure}

The discovery of new solitonic structures in superstring theory \(^{[35]} \) has dramatically changed the understanding of target space structure. These new non-perturbative objects are known as D-branes and their inclusion leads to a scattering picture of space-time fluctuations. As we have discussed previously, in this context one may consider superstring models of space-time foam, containing a number of point-like solitonic structures (D-particles) \(^{[36]} \). Heuristically, when low energy matter given by a closed (or open) string propagating in a \((D + 1)\)-dimensional space-time collides with a very massive D-particle embedded in this space-time, the D-particle recoils as a result. Since there are no rigid bodies in general relativity the recoil fluctuations of the brane and their effectively stochastic back-reaction on space-time cannot be neglected. On the brane there are closed and open strings propagating. Each time these strings cross with a D-particle, there is a possibility of being attached to it, as indicated in Fig. 1. The entangled state causes a back reaction onto the space-time, which can be calculated perturbatively using logarithmic
conformal field theory formalism [37]. Now for large Minkowski time $t$, the non-trivial changes from the flat metric produced from D-particle collisions are

$$ g_{0i} \simeq \mathcal{P}_i \equiv \frac{u_i}{\varepsilon} \propto \Delta p_i / M_P $$

(7)

where $u_i$ is the velocity and $\Delta p_i$ is the momentum transfer during a collision, $\varepsilon^{-2}$ is identified with $t$ and $M_P$ is the Planck mass (actually, to be more precise $M_P = M_s / g_s$, where $g_s < 1$ is the (weak) string coupling, and $M_s$ is a string mass scale); so $g_{0i}$ is constant in space-time but depends on the energy content of the low energy particle and the Ricci tensor $R_{MN} = 0$ where $M$ and $N$ are target space-time indices. Since we are interested in fluctuations of the metric the indices $i$ will correspond to the pair $M, N$. This master equation will serve as a framework for phenomenological applications.

2.2. Stochastically Fluctuating Geometries, Light Cone Fluctuations and Decoherence

If the ground state of QG consists of “fuzzy” space-time, i.e., stochastically-fluctuating metrics, then a plethora of interesting phenomena may occur, including light-cone fluctuations [38, 26] (c.f. Fig. 2). Such effects will lead to stochastic fluctuations in, say, arrival times of photons with common energy, which can be detected with high precision in astrophysical experiments [39, 38]. In addition, they may give rise to decoherence of matter, in the sense of induced time-dependent damping factors in the evolution equations of the (reduced) density matrix of matter fields [26, 7].

Such “fuzzy” space-times are formally represented by metric deviations which are fluctuating randomly about, say, flat Minkowski space-time:

$$ g_{\mu \nu} = \eta_{\mu \nu} + h_{\mu \nu} $$

with $\langle \cdot \cdot \cdot \rangle$ denoting statistical quantum averaging, and $\langle g_{\mu \nu} \rangle = \eta_{\mu \nu}$ but $\langle h_{\mu \nu}(x) h_{\lambda \sigma}(x') \rangle \neq 0$, i.e., one has only quantum (light cone) fluctuations but not mean-field effects on dispersion relations of matter probes. In such a situation Lorentz symmetry is respected on the average, but not in individual measurements.

The path of light follows null geodesics $0 = ds^2 = g_{\mu \nu} dx^\mu dx^\nu$, with non-trivial fluctuations in geodesic deviations, $\frac{D^2 n^\nu}{D \tau^2} = - R^\mu_{\alpha \nu \beta} u^\alpha n^\nu u^\beta$; in a standard general-relativistic notation, $D/D\tau$ denotes the appropriate covariant derivative operation, $R^\mu_{\alpha \nu \beta}$ the (fluctuating) Riemann curvature tensor, and $u^\mu (n^\nu)$ the tangential (normal) vector along the geodesic. Such an effect causes primarily fluctuations in the arrival time of photons at the detector ($|\phi\rangle$=state of gravitons, $|0\rangle$=vacuum state)

$$ \Delta t^2_{\text{obs}} = |\Delta t^2_{\phi} - \Delta t^2_0| = \frac{|\langle 0|\sigma^2_{\phi}|0\rangle - \langle 0|\sigma^2_0|0\rangle|}{r^2} = \frac{|\langle \sigma^2 \rangle_R|}{r} $$

where

$$ \langle \sigma^2 \rangle_R = \frac{1}{8}(\Delta r)^2 \int_{r_0}^{r_1} dr \int_{r_0}^{r_1} dr' \ n^\mu n^\nu n^\rho n^\sigma $$

$$ \langle \phi | h_{\mu \nu}(x) h_{\rho \sigma}(x') + h_{\mu \rho}(x') h_{\nu \sigma}(x) | \phi \rangle $$

Figure 2. In stochastic space-time models of QG the light cone may fluctuate, leading to decoherence and quantum fluctuations of the speed of light in “vacuo”.

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$$ \langle \phi | h_{\mu \nu}(x) h_{\rho \sigma}(x') + h_{\mu \rho}(x') h_{\nu \sigma}(x) | \phi \rangle $$
and the two-point function of graviton fluctuations can be evaluated using standard field theory techniques [38].

Apart from the stochastic metric fluctuations, however, the aforementioned effects could also induce decoherence of matter propagating in these types of backgrounds [7], a possibility of particular interest for the purposes of the present article. Through the theorem of Wald [10], this implies that the CPT operator is not well-defined, and hence one also has a breaking of CPT symmetry.

We now proceed to describe briefly the general formalism used for parametrizing such QG-induced decoherence, as far as the CPT-violating effects on matter are concerned.

2.3. Formalism for the Phenomenology of QG-induced Decoherence
In this subsection we shall be very brief, giving the reader a flavor of the formalism underlying such decoherent systems. We shall discuss first a model-independent parametrization of decoherence, applicable not only to QG media, but covering a more general situation.

If the effects of the environment are such that the modified evolution equation of the (reduced) density matrix of matter $\rho$ [9] is linear, one can write down a Lindblad evolution equation [6], provided that (i) there is (complete) positivity of $\rho$, so that negative probabilities do not arise at any stage of the evolution, (ii) the energy of the matter system is conserved on the average, and (iii) the entropy is increasing monotonically.

For $N$-level systems, the generic decohering Lindblad evolution for $\rho$ reads

$$\frac{\partial \rho}{\partial t} = \sum_{ij} h_i \rho_j f_{ij\mu} + \sum_{\nu} L_{\mu\nu} \rho_\nu \ , \ \mu, \nu = 0, \ldots, N^2 - 1, \ i, j = 1, \ldots, N^2 - 1 \ ,$$

(8)

where the $h_i$ are Hamiltonian terms, expanded in an appropriate basis, and the decoherence matrix $L$ has the form:

$$L_{00} = L_{0i} = 0 \ , \ L_{ij} = \frac{1}{4} \sum_{k,\ell,m} c_{ij} (f_{\ell m} f_{kmj} + f_{kim} f_{\ell mj}) \ ,$$

(9)

with $c_{ij}$ a positive-definite matrix and $f_{ijk\ell}$ the structure constants of the appropriate $SU(N)$ group. In this generic phenomenological description of decoherence, the elements $L_{\mu\nu}$ are free parameters, to be determined by experiment. We shall come back to this point in the next subsection, where we discuss neutral kaon decays.

A rather characteristic feature of this equation is the appearance of exponential damping, $e^{-\ldots t}$, in interference terms of the pertinent quantities (for instance, matrix elements $\rho$, or asymmetries in the case of the kaon system, see below). The exponents are proportional to (linear combinations) of the elements of the decoherence matrix [6] [4] [9].

A specific example of Lindblad evolution is the propagation of a probe in a medium with a stochastically fluctuating density [40]. The formalism can be adapted to the case of stochastic space-time foam [7] [20].

The stochasticity of the space-time foam medium is best described [20] by including in the time evolution of the neutrino density matrix a a time-reversal (CPT) breaking decoherence matrix of a double commutator form [40] [20],

$$\frac{\partial \langle \rho \rangle}{\partial t} = L[\rho] \ , \ L[\rho] = -i[H + H'_I, \langle \rho \rangle] - \Omega^2[H'_I, [H'_I, \langle \rho \rangle]]$$

(10)

where $\langle n(r)n(r') \rangle = \Omega^2 n_0^2 \delta(r - r')$ denote the stochastic (Gaussian) fluctuations of the density of the medium, and

$$H'_I = \left( \begin{array}{cc}
(a_{\nu e} - a_{\nu \mu}) \cos^2(\theta) & (a_{\nu e} - a_{\nu \mu}) \frac{\sin 2\theta}{2} \\
(a_{\nu e} - a_{\nu \mu}) \frac{\sin 2\theta}{2} & (a_{\nu e} - a_{\nu \mu}) \sin^2(\theta)
\end{array} \right)$$

(11)
is the MSW-like interaction [41] in the mass eigenstate basis, where $\theta$ is the mixing angle. This double-commutator decoherence is a specific case of Lindblad evolution [8] which guarantees complete positivity of the time evolved density matrix. For gravitationally-induced MSW effects (due to, say, black-hole foam models as in [19, 7]), one may denote the difference, between neutrino flavours, of the effective interaction strengths, $a_i$, with the environment by:

$$\Delta a_{e\mu} \equiv a_{\nu_e} - a_{\nu_\mu} \propto G_N n_0 \quad (12)$$

with $G_N = 1/M_p^2$, $M_p \sim 10^{19}$ GeV, the four-dimensional Planck scale, and in the case of the gravitational MSW-like effect [19] $n_0$ represents the density of charge black hole/anti-black hole pairs. This gravitational coupling replaces the weak interaction Fermi coupling constant $G_F$ in the conventional MSW effect [41].

For two generation neutrino models, the corresponding oscillation probability $\nu_e \leftrightarrow \nu_\mu$ obtained from (19), in the small parameter $\Omega^2 \ll 1$, which we assume here, as appropriate for the weakness of gravity fluctuations, reads to leading order:

$$P_{\nu_e \leftrightarrow \nu_\mu} =$$

$$\frac{1}{2} e^{-\Delta a^2_{e\mu} \alpha^2 t (1 + \frac{\Delta^2}{12} (\cos(4\theta) - 1))} \sin(t \sqrt{\Gamma}) \sin^2(2\theta) \Delta a_{e\mu}^2 \Omega^2 \Delta^2_{12} \left( \frac{3 \sin^2(2\theta) \Delta^2_{12}}{4 \Gamma^{5/2}} - \frac{1}{\Gamma^{3/2}} \right)$$

$$- e^{-\Delta a^2_{e\mu} \alpha^2 t (1 + \frac{\Delta^2}{12} (\cos(4\theta) - 1)) \cos(t \sqrt{\Gamma}) \sin^2(2\theta) \Delta^2_{12}}$$

$$- e^{-\Delta^2_{e\mu} \alpha^2 \Delta^2 \sin^2(2\theta)} \frac{(\Delta a_{e\mu} + \cos(2\theta) \Delta_{12})^2}{2\Gamma} \quad (13)$$

where $\Gamma = (\Delta a_{e\mu} \cos(2\theta) + \Delta_{12})^2 + \Delta a_{e\mu}^2 \sin^2(2\theta)$, $\Delta_{12} = \frac{m_2^2}{2p}$. From (13) we easily conclude that the exponents of the damping factors due to the stochastic-medium-induced decoherence, are of the generic form, for $t = L$, with $L$ the oscillation length (in units of $c = 1$):

$$\text{exponent} \sim -\Delta a^2_{e\mu} \Omega^2 t f(\theta) ; f(\theta) = 1 + \frac{\Delta^2_{12}}{4\Gamma} (\cos(4\theta) - 1) , \text{ or} \frac{\Delta^2_{12} \sin^2(2\theta)}{\Gamma} \quad (14)$$

that is proportional to the stochastic fluctuations of the density of the medium. The reader should note at this stage that, in the limit $\Delta_{12} \to 0$, which could characterise the situation in [19], where the space-time foam effects on the induced neutrino mass difference are the dominant ones, the damping factor is of the form exponent$_{\text{gravitational MSW}} \sim -\Omega^2 (\Delta a_{e\mu})^2 L$, with the precise value of the mixing angle $\theta$ not affecting the leading order of the various exponents. However, in that case, as follows from (13), the overall oscillation probability is suppressed by factors proportional to $\Delta^2_{12}$, and, hence, the stochastic gravitational MSW effect [19], although in principle capable of inducing mass differences for neutrinos, however does not suffice to produce the bulk of the oscillation probability, which is thus attributed to conventional flavour physics.

We note now that the Lindblad type evolution is not the most generic evolution for QG models. In cases of space-time foam corresponding to stochastically (random) fluctuating space-times, such as the situations causing light-cone fluctuations examined previously, there is a different kind of decoherent evolution, with damping that is quadratic in time, i.e., one has a $e^{-(...)^2}$ suppression of interference terms in the relevant observables. We now come to discuss this case.

A specific model of stochastic space-time foam is based on a particular kind of gravitational foam [26, 36, 7], consisting of “real” (as opposed to “virtual”) space-time defects in higher-dimensional space times, in accordance with the modern viewpoint of our world as a brane hyper-surface embedded in the bulk space-time [35]. This model is quite generic in some respects,
and we will use it later to estimate the order of magnitude of novel CPT violating effects in entangled states of kaons.

A model of space-time foam [36] can be based on a number (determined by target-space supersymmetry) of parallel brane worlds with three large spatial dimensions. These brane worlds move in a bulk space-time, containing a “gas” of point-like bulk branes, termed “D-particles”, which are stringy space-time solitonic defects. One of these branes is the observable Universe. For an observer on the brane the crossing D-particles will appear as twinkling space-time defects, i.e. microscopic space-time fluctuations. This will give the four-dimensional brane world a “D-foamy” structure. Following work on gravitational decoherence [26, 7], the target-space metric state, which is close to being flat, can be represented schematically as a density matrix

$$\rho_{\text{grav}} = \int d^5 r \ f (r_\mu) |g(r_\mu)\rangle \langle g(r_\mu)|. \quad (15)$$

The parameters $r_\mu$ ($\mu = 0, 1 \ldots$) pertain to appropriate space-time metric deformations and are stochastic, with a Gaussian distribution $f (r_\mu)$ characterized by the averages

$$\langle r_\mu \rangle = 0, \quad \langle r_\mu r_\nu \rangle = \Delta_\mu \delta_{\mu\nu}. \quad (15)$$

We will assume that the fluctuations of the metric felt by two entangled neutral mesons are independent, and $\Delta_\mu \sim O \left( \frac{K^2}{M_P^2} \right)$, i.e., very small. As matter moves through the space-time foam in a typical ergodic picture, the effect of time averaging is assumed to be equivalent to an ensemble average. For our present discussion we consider a semi-classical picture for the metric, and therefore $|g(r_\mu)|$ in (15) is a coherent state.

In the specific model of foam discussed in [7], there is a recoil effect of the D-particle, as a result of its scattering with stringy excitations that live on the brane world and represent low-energy ordinary matter. As the space-time defects, propagating in the bulk space-time, cross the brane hyper-surface from the bulk in random directions, they scatter with matter. The associated distortion of space-time caused by this scattering can be considered dominant only along the direction of motion of the matter probe. Random fluctuations are then considered about an average flat Minkowski space-time. The result is an effectively two-dimensional approximate fluctuating metric describing the main effects [7]

$$g^{\mu\nu} = \begin{pmatrix} - (a_1 + 1)^2 + a_2^2 & -a_3(a_1 + 1) + a_2(a_4 + 1) \\ -a_3(a_1 + 1) + a_2(a_4 + 1) & -a_3^2 + (a_4 + 1)^2 \end{pmatrix}. \quad (16)$$

The $a_i$ represent the fluctuations and are assumed to be random variables, satisfying $\langle a_i \rangle = 0$ and $\langle a_i a_j \rangle = \delta_{ij} \sigma_i$, $i,j = 1, \ldots, 4$.

Such a (microscopic) model of space-time foam is not of Lindblad type, as can be seen [7] by considering the oscillation probability for, say, two-level scalar systems describing oscillating neutral kaons, $K^0 \leftrightarrow \bar{K}^0$. In the approximation of small fluctuations one finds the following form for the oscillation probability of the two-level scalar system:

$$\langle e^{i(\omega_1 - \omega_2)t} \rangle = \frac{4d^2}{(P_1 P_2)^{1/2}} \exp \left( \frac{\chi_1}{\chi_2} \right) \exp(ibt), \quad (17)$$

where $\omega_i, \ i = 1, 2$ are the appropriate energy levels [7] of the two-level kaon system in the background of the fluctuating space-time (16), and
\[
\begin{align*}
\chi_1 &= -4(d^2 \sigma_1 + \sigma_4 k^2) \tilde{\sigma}^2 t^2 + 2i d^2 \tilde{b} \tilde{c} \tilde{k} \sigma_1 \sigma_4 t^2, \\
\chi_2 &= 4 \tilde{d}^2 - 2i \tilde{d}^2 (k^2 \tilde{c} \sigma_4 + 2b \sigma_1) t + \\
&\quad \tilde{b} \tilde{k}^2 \left( \tilde{b} \tilde{k}^2 - 2 \tilde{d}^2 \tilde{c} \right) \sigma_1 \sigma_4, \\
P_1 &= 4 \tilde{d}^2 + 2i \tilde{d}^2 \left( k^2 - \tilde{d} \right) \sigma_2 t + \tilde{b} \tilde{k}^4 \sigma_2 \sigma_1^2 t^2, \\
P_2 &= 4 \tilde{d}^2 - 2i \tilde{d}^2 \left( k^2 \tilde{c} \sigma_4 + 2b \sigma_1 \right) t + O \left( \sigma^2 \right),
\end{align*}
\]

with
\[
\begin{align*}
\tilde{b} &= \sqrt{k^2 + m_1^2} - \sqrt{k^2 + m_2^2}, \\
\tilde{c} &= m_1^2 (k^2 + m_1^2)^{-3/2} - m_2^2 (k^2 + m_2^2)^{-3/2}, \\
\tilde{d} &= \sqrt{k^2 + m_1^2} \sqrt{k^2 + m_2^2}.
\end{align*}
\]

From this expression one can see [7] that the stochastic model of space-time foam leads to a modification of oscillation behavior quite distinct from that of the Lindblad formulation. In particular, the transition probability displays a Gaussian time-dependence, decaying as \( e^{-(\ldots) t^2} \), a modification of the oscillation period, as well as additional power-law fall-off.

From this characteristic time-dependence, one can obtain bounds for the fluctuation strength of space-time foam in particle-physics systems, such as neutral mesons and neutrinos, which we restrict our attention to for the purposes of this presentation. When discussing the CPTV effects of foam on entangled states and neutrinos we make use of this specific model of stochastically fluctuating D-particle foam [36, 7], in order to demonstrate the effects explicitly and obtain definite order-of-magnitude estimates [42, 20].

3. QG Decoherence and CPTV in Neutral Kaons

3.1. Single-Kaon Beam Experiments

As mentioned in the previous subsection, QG may induce decoherence and oscillations \( K^0 \leftrightarrow \bar{K}^0 \) [4, 11], thereby implying a two-level quantum mechanical system interacting with a QG “environment”. Adopting the general assumptions of average energy conservation and monotonic entropy increase, the simplest model for parametrizing decoherence (in a rather model-independent way) is the (linear) Lindblad approach mentioned earlier. Not all entries of a general decoherence matrix are physical, and in order to isolate the physically relevant entries one must invoke specific assumptions, related to the symmetries of the particle system in question. For the neutral kaon system, such an extra assumptions is that the QG medium respects the \( \Delta S = \Delta Q \) rule. In such a case, the modified Lindblad evolution equation [4] for the respective density matrices of neutral kaon matter can be parametrized as follows [4]:

\[
\partial_t \rho = i[H, \rho] + \delta H \rho,
\]

where
\[
H_{\alpha\beta} = \begin{pmatrix}
-\Gamma & -\frac{1}{2} \delta \Gamma & -\text{Im} \Gamma_{12} & -\text{Re} \Gamma_{12} \\
-\frac{1}{2} \delta \Gamma & -\Gamma & -2 \text{Re} M_{12} & -2 \text{Im} M_{12} \\
-\text{Im} \Gamma_{12} & 2 \text{Re} M_{12} & -\Gamma & -\delta M \\
-\text{Re} \Gamma_{12} & -2 \text{Im} M_{12} & \delta M & -\Gamma
\end{pmatrix}
\]

and
\[
\delta H_{\alpha\beta} = \begin{pmatrix}
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & -2 \alpha & -2 \beta \\
0 & 0 & -2 \beta & -2 \gamma
\end{pmatrix}.
\]
Positivity of $\rho$ requires: $\alpha, \gamma > 0$, $\alpha \gamma > \beta^2$. Notice that $\alpha, \beta, \gamma$ violate both CPT, due to their decohering nature [10], and CP symmetry, as they do not commute with the CP operator $\hat{C}P$ [11]: $\hat{C}P = \sigma_3 \cos \theta + \sigma_2 \sin \theta$, $[\delta H_{\alpha \beta}, \hat{C}P] \neq 0$.

An important remark is now in order. As pointed out in [13], although the above parametrization is sufficient for a single-kaon state to have a positive definite density matrix (and hence probabilities) this is not true when one considers the evolution of entangled kaon states ($\phi$-factories). In this latter case, complete positivity is guaranteed only if the further conditions

$$\alpha = \gamma \quad \text{and} \quad \beta = 0 \quad (18)$$

are imposed. When incorporating entangled states, one should either consider possible new effects (such as the $\omega$-effect considered below) or apply the constraints (18) also to single kaon states [13]. This is not necessarily the case when other non-entangled particle states, such as neutrinos, are considered, in which case the $\alpha, \beta, \gamma$ parametrization of decoherence may be applied. Experimentally the complete positivity hypothesis can be tested explicitly. In what follows, as far as single-kaon states are concerned, we keep the $\alpha, \beta, \gamma$ parametrization, and give the available experimental bounds for them, but we always have in mind the constraint (18) when referring to entangled kaon states in a $\phi$-factory.

As already mentioned, when testing CPT symmetry with neutral kaons one should be careful to distinguish two types of CPTV: (i) CPTV within Quantum Mechanics [14], leading to possible differences between particle-antiparticle masses and widths: $\delta m = m_{K^0} - m_{\bar{K}^0}$, $\delta \Gamma = \Gamma_{K^0} - \Gamma_{\bar{K}^0}$. This type of CPTV could be, for instance, due to (spontaneous) Lorentz violation [22]. In that case the CPT operator is well-defined as a quantum mechanical operator, but does not commute with the Hamiltonian of the system. This, in turn, may lead to mass and width differences between particles and antiparticles, among other effects. (ii) CPTV through decoherence [4, 5] via the parameters $\alpha, \beta, \gamma$ (entanglement with the QG “environment”, leading to modified evolution for $\rho$ and $S \neq S^\dagger$). In the latter case the CPT operator may not be well-defined, which implies novel effects when one uses entangled states of kaons, as we shall discuss in the next subsection.

Table 1. Qualitative comparison of predictions for various observables in CPT-violating theories beyond (QMV) and within (QM) quantum mechanics. Predictions either differ ($\neq$) or agree ($=$) with the results obtained in conventional quantum-mechanical CP violation. Note that these frameworks can be qualitatively distinguished via their predictions for $A_T$, $A_{CPT}$, $A_{\Delta m}$, and $\zeta$.

| Process | QMV | QM |
|---------|-----|----|
| $A_{2\pi}$ | $\neq$ | $\neq$ |
| $A_{3\pi}$ | $\neq$ | $\neq$ |
| $A_T$ | $\neq$ | $=$ |
| $A_{CPT}$ | $=$ | $\neq$ |
| $A_{\Delta m}$ | $\neq$ | $=$ |
| $\zeta$ | $\neq$ | $=$ |

The important point to notice is that the two types of CPTV can be disentangled experimentally [11]. The relevant observables are defined as $\langle O_i \rangle = \text{Tr} [O_i \rho]$. For neutral kaons, one looks at decay asymmetries for $K^0, \bar{K}^0$, defined as:

$$A(t) = \frac{R(\bar{K}^0_{t=0} \rightarrow \bar{f}) - R(K^0_{t=0} \rightarrow f)}{R(\bar{K}^0_{t=0} \rightarrow \bar{f}) + R(K^0_{t=0} \rightarrow f)},$$
where $R(K^0 \to f) \equiv \text{Tr} [O_f \rho(t)]$ denotes the decay rate into the final state $f$ (starting from a pure $K^0$ state at $t=0$).

In the case of neutral kaons, one may consider the following set of asymmetries: (i) identical final states: $f = \bar{f} = 2\pi$: $A_{2\pi}$, $A_{3\pi}$, (ii) semileptonic: $A_T$ (final states $f = \pi^+ l^- \bar{\nu} \neq \bar{f} = \pi^- l^+ \nu$), $A_{CPT}$ ($\bar{f} = \pi^+ l^- \bar{\nu}$, $f = \pi^- l^+ \nu$), $A_{\Delta m}$. Typically, for instance when final states are $2\pi$, one has a time evolution of the decay rate $R_{2\pi}$: $R_{2\pi}(t) = c_S e^{-\Gamma_S t} + c_L e^{-\Gamma_L t} + 2c_I e^{-\Gamma_I t} \cos(\Delta m t - \phi)$, where $S =$ short-lived, $L =$ long-lived, $I =$ interference term, $\Delta m = m_L - m_S$, $\Delta \Gamma = \Gamma_S - \Gamma_L$, $\Gamma = \frac{1}{2} (\Gamma_S + \Gamma_L)$. One may define the decoherence parameter $\zeta = 1 - \frac{c_I}{\sqrt{c_S c_L}}$, as a (phenomenological) measure of quantum decoherence induced in the system [14]. For larger sensitivities one can look at this parameter in the presence of a regenerator [11]. In our decoherence scenario, $\zeta$ corresponds to a particular combination of the decoherence parameters [11]:

$$\zeta \to \frac{\hat{\gamma}}{2|\epsilon^2|} - 2 \frac{\hat{\beta}}{|\epsilon|} \sin \phi,$$

with the notation $\hat{\gamma} = \gamma / \Delta \Gamma$, etc. Hence, ignoring the constraint [18], the best bounds on $\beta$, or - turning the logic around- the most sensitive tests of complete positivity in kaons, can be placed by implementing a regenerator [11].

The experimental tests (decay asymmetries) that can be performed in order to disentangle decoherence from quantum-mechanical CPT violating effects are summarized in Table 1. In Figure 3 we give a typical profile of a decay asymmetries [11], from where bounds on QG decohering parameters can be extracted. At present there are experimental bounds available from CPLEAR measurements [43] $\alpha < 4.0 \times 10^{-17}$ GeV , $|\beta| < 2.3 \times 10^{-19}$ GeV , $\gamma < 3.7 \times 10^{-21}$ GeV, which are not much different from theoretically expected values in some optimistic scenarios [11] $\alpha , \beta , \gamma = O(\xi^{2/3})$.

Recently, the experiment KLOE at DAΦNE updated these limits by measuring for the

Figure 3. Neutral kaon decay asymmetries $A_{2\pi}$ [11], as a typical example indicating the effects of QG-induced decoherence.
first time the $\gamma$ decoherence parameter for entangled kaon states \cite{30}, as well as the (naive) decoherence parameter $\zeta$ (to be specific, the KLOE Collaboration has presented measurements for two $\zeta$ parameters, one, $\zeta_{LS}$, pertaining to an expansion in terms of $K_L, K_S$ states, and the other, $\zeta_{00}$, for an expansion in terms of $K^0, \bar{K}^0$ states). We remind the reader once more that, under the assumption of complete positivity for entangled meson states \cite{13}, theoretically there is only one parameter to parametrize Lindblad decoherence, since $\alpha = \gamma$, $\beta = 0$. In fact, the KLOE experiment has the greatest sensitivity to this parameter $\gamma$. The latest KLOE measurement \cite{30} for $\gamma$ yields $\gamma_{\text{KLOE}} = (1.1^{+2.9}_{-2.4} \pm 0.4) \times 10^{-21} \text{GeV}$, i.e. $\gamma < 6.4 \times 10^{-21} \text{GeV}$, competitive with the corresponding CPLEAR bound \cite{43} discussed above. It is expected that this bound could be improved by an order of magnitude in upgraded facilities, such as KLOE-2 at DAΦNE-2 \cite{30}, where one expects $\gamma_{\text{upgrade}} \rightarrow \pm 0.2 \times 10^{-21} \text{GeV}$.

The reader should also bear in mind that the Lindblad linear decoherence is not the only possibility for a parametrization of QG effects, see for instance the stochastically fluctuating space-time metric approach discussed in Section 3.1 above. Thus, direct tests of the complete positivity hypothesis in entangled states, and hence the theoretical framework per se, should be performed by independent measurements of all the three decoherence parameters $\alpha, \beta, \gamma$; as far as we understand, such data are currently available in kaon factories, but not yet analyzed in detail \cite{30}.

3.2. CPTV and Modified EPR Correlations of Entangled Neutral Kaon States

3.2.1. The $\omega$-Effect

We now come to a description of an entirely novel effect \cite{21} of CPTV due to the ill-defined nature of the CPT operator, which is exclusive to neutral-meson factories, for reasons explained below. The effects are qualitatively similar for kaon and $B$-meson factories \cite{44}, with the important observation that in kaon factories there is a particularly good channel, that of both correlated kaons decaying to $\pi^+\pi^-$. In that channel the sensitivity of the effect increases because the complex parameter $\omega$, parametrizing the relevant EPR modifications \cite{21}, appears in the particular combination $|\omega|/|\eta_{+-}|$, with $|\eta_{+-}| \sim 10^{-3}$. In the case of $B$-meson factories one should focus instead on the “same-sign” di-lepton channel \cite{44}, where high statistics is expected.

In this article we restrict ourselves to the case of $\phi$-factories, referring the interested reader to the literature \cite{44} for the $B$-meson applications. We commence our discussion by briefly reminding the reader of EPR particle correlations.

The EPR effect was originally proposed as a paradox, testing the foundations of Quantum Theory. There was the question whether quantum correlations between spatially separated events implied instant transport of information that would contradict special relativity. It was eventually realized that no super-luminal propagation was actually involved in the EPR phenomenon, and thus there was no conflict with relativity.

The EPR effect has been confirmed experimentally, e.g., in meson factories: (i) a pair of particles can be created in a definite quantum state, (ii) move apart and, (iii) eventually decay when they are widely (spatially) separated (see Fig. 4 for a schematic representation of an EPR effect in a meson factory). Upon making a measurement on one side of the detector and identifying the decay products, we infer the type of products appearing on the other side; this is essentially the EPR correlation phenomenon. It does not involve any simultaneous measurement on both sides, and hence there is no contradiction with special relativity. As emphasized by
Lipkin [45], the EPR correlations between different decay modes should be taken into account when interpreting any experiment.

In the case of φ factories it was claimed [46] that due to EPR correlations, irrespective of CP, and CPT violation, the final state in φ decays: \( e^+e^- \Rightarrow φ \Rightarrow K_SK_L \) always contains \( K_LK_S \) products. This is a direct consequence of imposing the requirement of Bose statistics on the state \( K^0\Kbar^0 \) (to which the φ decays); this, in turn, implies that the physical neutral meson-antimeson state must be symmetric under CP, with C the charge conjugation and \( P \) the operator that permutes the spatial coordinates. Assuming conservation of angular momentum, and a proper existence of the antiparticle state (denoted by a bar), one observes that: for \( K^0\Kbar^0 \) states which are C-conjugates with \( C= (−1)\ell \) (with \( \ell \) the angular momentum quantum number), the system has to be an eigenstate of the permutation operator \( P \) with eigenvalue \((-1)^\ell \). Thus, for \( \ell = 1 \):

\[ C= − \rightarrow P = − . \]

Bose statistics ensures that for \( \ell = 1 \) the state of two identical bosons is forbidden. Hence, the initial entangled state:

\[
|i> = \frac{1}{\sqrt{2}} \left( (|K^0(\vec{k}), \Kbar^0(−\vec{k}) > -|\Kbar^0(\vec{k}), K^0(−\vec{k}) > \right) \nonumber \\
= \mathcal{N} (|K_S(\vec{k}), K_L(−\vec{k}) > -|K_L(\vec{k}), K_S(−\vec{k}) > \right) \nonumber
\]

with the normalization factor \( \mathcal{N} = \frac{\sqrt{1+|\epsilon_1|^2(1+|2\epsilon_2|)}}{\sqrt{2(1-\epsilon_1\epsilon_2)}} \), and \( K_S = \frac{1}{\sqrt{1+|\epsilon_1|^2}}(|K_+ > +\epsilon_1|K_− > \), \( K_L = \frac{1}{\sqrt{1+|\epsilon_2|^2}}(|K_− > +\epsilon_2|K_+ > \), where \( \epsilon_1, \epsilon_2 \) are complex parameters, such that \( \epsilon \equiv \epsilon_1 + \epsilon_2 \) denotes the CP- & T-violating parameter, whilst \( \delta \equiv \epsilon_1 - \epsilon_2 \) parametrizes the CPT & CP violation within quantum mechanics [14], as discussed previously. The \( K^0 \leftrightarrow \Kbar^0 \) or \( K_S \leftrightarrow K_L \) correlations are apparent after evolution, at any time \( t > 0 \) (with \( t = 0 \) taken as the moment of the φ decay).

In the above considerations there is an implicit assumption, which was noted in [21]. The above arguments are valid independently of CPTV, provided such violation occurs within quantum mechanics, e.g., due to spontaneous Lorentz violation, where the CPT operator is well defined.

If, however, CPT is intrinsically violated, due, for instance, to decoherence scenarios in space-time foam, then the factorizability property of the super-scattering matrix $S$ breaks down, $S \neq SS^\dagger$, and the generator of CPT is not well defined [10]. Thus, the concept of an “antiparticle” may be modified perturbatively! The perturbative modification of the properties of the antiparticle is important, since the antiparticle state is a physical state which exists, despite the ill-definition of the CPT operator. However, the antiparticle Hilbert space will have components that are independent of the particle Hilbert space.

In such a case, the neutral mesons \( K^0 \) and \( \Kbar^0 \) should no longer be treated as indistinguishable particles. As a consequence [21], the initial entangled state in φ factories \( |i> \), after the φ-meson decay, will acquire a component with opposite permutation (\( P \)) symmetry:

\[
|i> = \frac{1}{\sqrt{2}} \left( (|K_0(\vec{k}), \Kbar_0(−\vec{k}) > -|\Kbar_0(\vec{k}), K_0(−\vec{k}) > \right) \\
+ \frac{\omega}{2} \left( (|K_0(\vec{k}), \Kbar_0(−\vec{k}) > +|\Kbar_0(\vec{k}), K_0(−\vec{k}) > \right) \nonumber \\
= \mathcal{N} (|K_S(\vec{k}), K_L(−\vec{k}) > -|K_L(\vec{k}), K_S(−\vec{k}) > \right) \\
+ \omega \left( (|K_S(\vec{k}), K_S(−\vec{k}) > -|K_L(\vec{k}), K_L(−\vec{k}) > \right) ,
\]

where \( \mathcal{N} \) is an appropriate normalization factor, and \( \omega = |\omega|\epsilon^\Omega \) is a complex parameter, parametrizing the intrinsic CPTV modifications of the EPR correlations. Notice that, as a
result of the \( \omega \)-terms, there exist, in the two-kaon state, \( K_S K_S \) or \( K_L K_L \) combinations, which entail important effects to the various decay channels. Due to this effect, termed the \( \omega \)-effect by the authors of [21], there is contamination of \( P(\text{odd}) \) state with \( P(\text{even}) \) terms. The \( \omega \)-parameter controls the amount of contamination of the final \( P(\text{odd}) \) state by the “wrong” \( (P(\text{even})) \) symmetry state.

Later in this section we will present a microscopic model where such a situation is realized explicitly. Specifically, an \( \omega \)-like effect appears due to the evolution in the space-time foam, and the corresponding parameter turns out to be purely imaginary and time-dependent [42].

3.2.2. \( \omega \)-Effect Observables

To construct the appropriate observable for the possible detection of the \( \omega \)-effect, we consider the \( \phi \)-decay amplitude depicted in Fig. 4, where one of the kaon products decays to the final state \( X \) at \( t_1 \) and the other to the final state \( Y \) at time \( t_2 \). We take \( t = 0 \) as the moment of the \( \phi \)-meson decay.

The relevant amplitudes read:

\[
A(X, Y) = \langle X | K_S \rangle \langle Y | K_S \rangle N \left( A_1 + A_2 \right),
\]

with

\[
A_1 = e^{-i(\lambda_L + \lambda_S)t/2} \left[ \eta_X e^{-i\Delta t/2} - \eta_Y e^{i\Delta t/2} \right],
\]

\[
A_2 = \omega \left[ e^{-i\lambda_L t} - \eta_X \eta_Y e^{-i\lambda_L t} \right]
\]

denoting the CPT-allowed and CPT-violating parameters respectively, and \( \eta_X = \langle X | K_L \rangle / \langle X | K_S \rangle \) and \( \eta_Y = \langle Y | K_L \rangle / \langle Y | K_S \rangle \). In the above formulae, \( t \) is the sum of the decay times \( t_1, t_2 \) and \( \Delta t \) is their difference (assumed positive).

![Figure 5](image)

**Figure 5.** A characteristic case of the intensity \( I(\Delta t) \), with \( |\omega| = 0 \) (solid line) vs \( I(\Delta t) \) (dashed line) with \( |\omega| = |\eta_{+-}|, \Omega = \phi_{+-} - 0.16\pi \), for definiteness [21].

The “intensity” \( I(\Delta t) \) is the desired observable for a detection of the \( \omega \)-effect,

\[
I(\Delta t) = \frac{1}{2} \int_{\Delta t}^{\infty} dt |A(X, Y)|^2.
\]

depending only on \( \Delta t \).

Its time profile reads [21]:

\[
I(\Delta t) = \frac{1}{2} \int_{|\Delta t|}^{\infty} dt |A(\pi^+\pi^-, \pi^+\pi^-)|^2 =
|\langle \pi^+\pi^- | K_S \rangle|^4 |N|^2 |\eta_{+-}|^2 \left[ I_1 + I_2 + I_{12} \right],
\]

where
where the assumptions for the parameters matter state, are:

\[ I_1(\Delta t) = \frac{e^{-r_5 \Delta t} + e^{-r_4 \Delta t} - 2e^{-r(\Gamma_S + \Gamma_L)} \Delta t/2 \cos(\Delta m \Delta t)}{\Gamma_L + \Gamma_S} \]

\[ I_2(\Delta t) = \frac{|\omega|^2}{|\eta_+|^2} \frac{e^{-r_5 \Delta t}}{2\Gamma_S} \]

\[ I_{12}(\Delta t) = \frac{4}{4(\Delta m)^2 + (3\Gamma_S + \Gamma_L)^2 |\eta_-|} \times \]

\[ 2\Delta m \left( e^{-r_5 \Delta t} \sin(\phi_+ - \Omega) - e^{-r(\Gamma_S + \Gamma_L) \Delta t/2 \sin(\phi_+ - \Omega + \Delta m \Delta t)} \right) \]

\[ -3\Gamma_S \left( e^{-r_5 \Delta t} \cos(\phi_+ - \Omega) - e^{-r(\Gamma_S + \Gamma_L) \Delta t/2 \cos(\phi_+ - \Omega + \Delta m \Delta t)} \right) \],

with \( \Delta m = m_S - m_L \) and \( \eta_{+-} = |\eta_+|e^{i\phi_+} \) in the usual notation [14].

A typical case for the relevant intensities, indicating clearly the novel CPTV \( \omega \)-effects, is depicted in Fig. [5].

As announced, the novel \( \omega \)-effect appears in the combination \( |\omega|/|\eta_+| \), thereby implying that the decay channel to \( \pi^+\pi^- \) is particularly sensitive to the \( \omega \) effect [21], due to the enhancement by \( 1/|\eta_+| \sim 10^3 \), implying sensitivities up to \( |\omega| \sim 10^{-6} \) in \( \phi \) factories. The physical reason for this enhancement is that \( \omega \) enters through \( K_S K_S \) as opposed to \( K_L K_S \) terms, and the \( K_L \to \pi^+\pi^- \) decay is CP-violating.

3.2.3. Microscopic Models for the \( \omega \)-Effect and Order-of-Magnitude Estimates

For future experimental searches for the \( \omega \)-effect it is important to estimate its expected order of magnitude, at least in some models of foam.

A specific model is that of the D-particle foam [36, 7, 42], discussed already in connection with the stochastic metric-fluctuation approach to decoherence. An important feature for the appearance of an \( \omega \)-like effect is that, during each scattering with a D-particle defect, there is (momentary) capture of the string state (representing matter) by the defect, and a possible change in phase and/or flavour for the particle state emerging from such a capture (see Fig. [1]).

The induced metric distortions, including such flavour changes for the emergent post-recoil matter state, are:

\[ g^{00} = (-1 + r_4) 1, \quad g^{01} = g^{10} = r_0 1 + r_1 \sigma_1 + r_2 \sigma_2 + r_3 \sigma_3, \quad g^{11} = (1 + r_3) 1 \]  

where the \( \sigma_i \) are Pauli matrices.

The target-space metric state is the density matrix \( \rho_{\text{grav}} \) defined at [15, 42], with the same assumptions for the parameters \( r_\mu \) stated there. The order of magnitude of the metric elements \( g_{0\mu} \approx \bar{\nu}_{\mu,\text{rec}} \propto g_s \Delta p_i \), where \( \Delta p_i \sim \xi p_i \) is the momentum transfer during the scattering of the particle probe (kaon) with the D-particle defect, \( g_s < 1 \) is the string coupling, assumed weak, and \( M_\ast \) is the string scale, which in the modern approach to string/brane theory is not necessarily identified with the four-dimensional Planck scale, and is left as a phenomenological parameter to be constrained by experiment.

To estimate the order of magnitude of the \( \omega \)-effect we construct the gravitationally-dressed initial entangled state using stationary perturbation theory for degenerate states [21], the degeneracy being provided by the CP-violating effects. As Hamiltonian function we use
\[ \tilde{H} = g^{01} (g^{00})^{-1} \tilde{k} - (g^{00})^{-1} \sqrt{(g^{00})^{2} k^{2} - g^{00}(g^{11}k^{2} + m^{2})} \]

describing propagation in the above-described stochastically-fluctuating space-time. To leading order in the variables \( r \) the interaction Hamiltonian reads:

\[ \tilde{H}_{I} = - (r_{1}\sigma_{1} + r_{2}\sigma_{2}) \tilde{k} \]  

with the notation \( |K_{L}\rangle = |\uparrow\rangle, \quad |K_{S}\rangle = |\downarrow\rangle \). The gravitationally-dressed initial states then can be constructed using stationary perturbation theory:

\[ |k^{(\alpha)}_{\downarrow}\rangle^{(i)}_{QG} = |k^{(\alpha)}_{\downarrow}\rangle^{(i)} + |k^{(\alpha)}_{\uparrow}\rangle^{(i)} \alpha^{(i)} , \]

where \( \alpha^{(i)} = \frac{(i,k^{(\alpha)}|\tilde{H}_{I}|k^{(\beta)}_{\downarrow})_{E_{2}-E_{1}}}{E_{2}-E_{1}} \). For \( |k^{(\alpha)}_{\downarrow}\rangle^{(i)} \) the dressed state is obtained by \( |\downarrow\rangle \leftrightarrow |\uparrow\rangle \) and \( \alpha \rightarrow \beta \) where \( \beta^{(i)} = \frac{(i,k^{(\beta)}|\tilde{H}_{I}|k^{(\alpha)}_{\downarrow})}{E_{2}-E_{1}} \).

The totally antisymmetric “gravitationally-dressed” state of two mesons (kaons) is then:

\[
|k^{(\alpha)}\rangle^{(i)}_{QG} \rightarrow |k^{(\alpha)}\rangle^{(i)}_{QG} - |k^{(\beta)}\rangle^{(i)}_{QG} - |k^{(\alpha)}\rangle^{(i)}_{QG} - |k^{(\beta)}\rangle^{(i)}_{QG} = \\
\xi = |k,k^{(\alpha)}\rangle^{(i)}_{QG} - |k^{(\alpha)}\rangle^{(i)}_{QG} - |k^{(\beta)}\rangle^{(i)}_{QG} + |k^{(\beta)}\rangle^{(i)}_{QG} + \xi |k,k^{(\alpha)}\rangle^{(i)}_{QG} - |k^{(\alpha)}\rangle^{(i)}_{QG} - |k^{(\beta)}\rangle^{(i)}_{QG} - |k^{(\beta)}\rangle^{(i)}_{QG} ,
\]

Notice here that, for our order-of-magnitude estimates, it suffices to assume that the initial entangled state of kaons is a pure state. In practice, due to the omnipresence of foam, this may not be entirely true, but this should not affect our order-of-magnitude estimates based on such an assumption.

With these remarks in mind we then write for the initial state of two kaons after the \( \phi \) decay:

\[ |\psi\rangle = |k,\uparrow\rangle^{(1)}_{QG} - |k,\downarrow\rangle^{(2)}_{QG} - |k^{(\alpha)}\rangle^{(1)}_{QG} - |k^{(\beta)}\rangle^{(2)}_{QG} + \xi |k,\downarrow\rangle^{(1)}_{QG} - |k^{(\alpha)}\rangle^{(2)}_{QG} + \xi |k,\uparrow\rangle^{(1)}_{QG} - |k^{(\beta)}\rangle^{(1)}_{QG} - |k^{(\beta)}\rangle^{(2)}_{QG} , \]

where for \( r_{i} \propto \delta_{i} \) we have \( \xi = \xi' \), that is strangeness violation, whilst for \( r_{i} \propto \delta_{i} \rightarrow \xi = -\xi' \) (since \( \alpha^{(i)} = \beta^{(i)} \)) we obtain a strangeness conserving \( \omega \)-effect.

Upon averaging the density matrix over \( r_{i} \), only the \( |\omega|^{2} \) terms survive:

\[ |\omega|^{2} = \mathcal{O} \left( \frac{1}{(E_{1} - E_{2})^{2}} |(\uparrow, k|H|k, \uparrow\rangle)^{2} \right) \sim \frac{\Delta_{2}k^{2}}{(m_{1} - m_{2})^{2}} \]

for momenta of order of the rest energies, as is the case of a \( \phi \) factory.

Recalling that in the recoil D-particle model under consideration we have \( \Delta_{2} = \frac{\tilde{\xi}^{2}k^{2}}{M_{D}^{2}} \), we obtain the following order of magnitude estimate of the \( \omega \) effect:

\[ |\omega|^{2} \sim \frac{\tilde{\xi}^{2}k^{4}}{M_{D}^{2}(m_{1} - m_{2})^{2}} . \]  

For neutral kaons with momenta of the order of the rest energies \( |\omega| \sim 10^{-4}|\tilde{\xi}| \). For \( 1 > \tilde{\xi} \geq 10^{-2} \) this not far below the sensitivity of current facilities, such as KLOE at DAΦNE. In fact, the KLOE experiment has just released the first measurement of the \( \omega \) parameter \( [30] \):

\[ \text{Re}(\omega) = (1.1^{+8.7}_{-5.3} \pm 0.9) \times 10^{-4}, \quad \text{Im}(\omega) = (3.4^{+4.8}_{-5.0} \pm 0.6) \times 10^{-4} . \]
One can constrain the $\omega$ parameter (or, in the context of the above specific model, the momentum-transfer parameter $\xi$) significantly in upgraded facilities. For instance, there are the following perspectives for KLOE-2 at (the upgraded) DAΦNE-2 [30]: Re($\omega$), Im($\omega$) $\rightarrow 2 \times 10^{-5}$.

Let us now mention that $\omega$-like effects can also be generated by the Hamiltonian evolution of the system as a result of gravitational medium interactions. To this end, let us consider the momentum-transfer parameter $\xi$. Hamiltonian evolution in our stochastically-fluctuating D-particle-recoil distorted space-times, $|\psi(t)\rangle = \exp[-i(\hat{H}^{(1)} + \hat{H}^{(2)}t)]|\psi\rangle$.

Assuming for simplicity $\xi = \xi' = 0$, it is easy to see [12] that the time-evolved state of two kaons contains strangeness-conserving $\omega$-terms:

$$|\psi(t)\rangle \sim e^{-i\left(\lambda_0^{(1)} + \lambda_0^{(2)}\right)\frac{\omega}{t}} \varpi(t) \times \left\{|k, \uparrow\rangle^{(1)}|\uparrow\rangle^{(2)} - |k, \downarrow\rangle^{(1)}|\downarrow\rangle^{(2)}\right\},$$

The quantity $\varpi(t)$ obtained within this specific model is purely imaginary,

$$\varpi(t) = \frac{2\Delta_1^{1/2}k}{(k^2 + m_1^2)^{1/2} - (k^2 + m_2^2)^{1/2}} \times \cos\left(|\lambda\rangle^{(1)}|\uparrow\rangle\right) \sin\left(|\lambda\rangle^{(1)}|\uparrow\rangle\right) = \varpi_0 \sin\left(2|\lambda\rangle^{(1)}|\uparrow\rangle\right),$$

with $\Delta_1^{1/2} \sim \frac{m_1}{M_{\pi}}, \quad \varpi_0 \equiv \frac{\Delta_1^{1/2}k}{(k^2 + m_1^2)^{1/2} - (k^2 + m_2^2)^{1/2}}, \quad |\lambda\rangle \sim \left(1 + \Delta_4^{1/2}\right)\sqrt{\frac{\lambda_1^2 + \lambda_3^2}{\chi_1^2 + \chi_3^2}}.$

It is important to notice the time dependence of the medium-generated effect. It is also interesting to observe that, if in the initial state we have a strangeness-conserving (-violating) combination, $\xi = -\xi'$ ($\xi = \xi'$), then the time evolution generates time-dependent strangeness-violating (-conserving $\omega$) imaginary effects.

The above description of medium effects using Hamiltonian evolution is approximate, but suffices for the purposes of obtaining order-of-magnitude estimates for the relevant parameters. In the complete description of the above model there is of course decoherence [42, 26], which affects the evolution and induces mixed states for kaons. A complete analysis of both effects, $\omega$-like and decoherence in entangled neutral kaons of a $\phi$-factory, has already been carried out [21], with the upshot that the various effects can be disentangled experimentally, at least in principle.

Finally, as the analysis of [42] demonstrates, no $\omega$-like effects are generated by thermal bath-like (rotationally-invariant, isotropic) space-time foam situations, argued to simulate the QG environment in some models [47]. In this way, the potential observation of an $\omega$-like effect in EPR-correlated meson states would in principle distinguish various types of space-time foam.

3.2.4. Disentangling the $\omega$ Effect from the C(even) Background and Decoherent Evolution Effects When interpreting experimental results on delicate violations of CPT symmetry, it is important to disentangle (possible) genuine effects from those due to ordinary physics. Such a situation arises in connection with the $\omega$-effect, that must be disentangled from the C(even) background characterizing the decay products in a $\phi$-factory [46]. The C(even) background $\gamma^+\gamma^- \Rightarrow 2\gamma \Rightarrow K^0\bar{K}^0$ leads to states of the form

$$|b\rangle = |K^0\bar{K}^0(C\text{(even)})\rangle = \frac{1}{\sqrt{2}} \left(K^0(\bar{k})\bar{K}^0(-\bar{k}) + \bar{K}^0(\bar{k})K^0(-\bar{k})\right),$$

which at first sight mimic the $\omega$-effect, as such states would also produce contamination by terms $K_SK_S, K_LK_L$.

Closer inspection reveals, however, that the two types of effects can be clearly disentangled experimentally [21, 3] on two accounts: (i) the expected magnitude of the two effects, in view
of the above-described estimates in QG models and the unitarity bounds that suppress the ‘fake’ (C(even)-background) effect [48], and (ii) the different way the genuine QG-induced ω-effect interferes with the the C(odd) background [21].

Finally, we remark that it is also possible to disentangle [21] the ω-effect from decoherent evolution effects [21], due to their different structure. For instance, the experimental disentanglement of ω from the decoherence parameter γ in completely-positive QG-Lindblad models of entangled neutral mesons [4, 11, 13] is possible as a result of different symmetry properties and different structures generated by the time evolution of the pertinent terms.

4. Neutrino Tests of QG Decoherence

Neutrino oscillations is one of the most sensitive particle-physics probes of QG-decoherence to date. The presence of QG decoherence effects would affect the profiles of the oscillation probabilities among the various neutrino flavours, not only by the presence of appropriate damping factors in front of the interference oscillatory terms, but also by appropriate modifications of the oscillation period and phase shifts of the oscillation arguments [15, 16, 17, 5].

In the context of our present talk, we shall restrict ourselves to a brief discussion of a three-generation fit, including QG-decoherence effects, to all presently available data, including LSND results [25]. For more details we refer the reader to the literature [20].

We assume for simplicity a Lindblad environment, for which the evolution [8] applies. However, as we have discussed in section 2, a combination of both Lindblad environment and stochastically fluctuating space-time backgrounds [10], affecting also the Hamiltonian parts of the matter-density-matrix evolution [5], could also be in place. Without a detailed microscopic model, in the three generation case, the precise physical significance of the decoherence matrix cannot be fully understood. In [20] we considered the simplified case in which the matrix $C \equiv (c_{kl})$ [9] is assumed to be of the form

$$C = \begin{pmatrix}
  c_{11} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
  0 & c_{22} & 0 & 0 & 0 & 0 & 0 & 0 \\
  0 & 0 & c_{33} & 0 & 0 & 0 & 0 & c_{38} \\
  0 & 0 & 0 & c_{44} & 0 & 0 & 0 & 0 \\
  0 & 0 & 0 & 0 & c_{55} & 0 & 0 & 0 \\
  0 & 0 & 0 & 0 & 0 & c_{66} & 0 & 0 \\
  0 & 0 & 0 & 0 & 0 & 0 & c_{77} & 0 \\
  0 & 0 & c_{38} & 0 & 0 & 0 & 0 & c_{88}
\end{pmatrix}$$

(22)

Positivity of the evolution can be guaranteed if and only if the matrix $C$ is positive and hence has non-negative eigenvalues. As discussed in detail in [7, 20], such special choices can be realised for models of the propagation of neutrinos in models of stochastically fluctuating environments [10], where the decoherence term corresponds to an appropriate double commutator involving operators that entangle with the environment. The quantum-gravity space-time foam may in principle behave as one such stochastic environment [3, 7, 11], and it is this point of view that we critically examined in [20] and review here, in the context of the entirety of the presently available neutrino data.
The simplified form of the $c_{ij}$ matrix given in (22) implies a matrix $L_{ij}$ in (8) of the form:

$$L = \begin{pmatrix}
D_{11} & -\Delta_{12} & 0 & 0 & 0 & 0 \\
\Delta_{12} & D_{22} & 0 & 0 & 0 & 0 \\
0 & 0 & D_{33} & 0 & 0 & 0 \\
0 & 0 & 0 & D_{44} & -\Delta_{13} & 0 \\
0 & 0 & 0 & \Delta_{13} & D_{55} & 0 \\
0 & 0 & 0 & 0 & D_{66} & -\Delta_{23} \\
0 & 0 & 0 & 0 & 0 & D_{77} \\
0 & 0 & D_{83} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & D_{88}
\end{pmatrix}$$

(23)

where we have used the notation $\Delta_{ij} = \frac{m_i^2 - m_j^2}{2p}$, and the matrix $D_{ij}$ is expressed in terms of the elements of the $C$-matrix: $D_{ij} = \sum_{k,l,m} \frac{1}{2} \left( -f_{ikm} f_{jlm} + f_{kjm} f_{lim} \right)$.

The probability of a neutrino of flavor $\nu_\alpha$, created at time $t = 0$, being converted to a flavor $\nu_\beta$ at a later time $t$, is calculated in the Lindblad framework [6] to be

$$P_{\nu_\alpha \rightarrow \nu_\beta}(t) = \text{Tr} \left[ \rho^{(t)} \rho_\beta \right] = \frac{1}{3} + \frac{1}{2} \sum_{i,j,k} e^{\lambda_k t} D_{ik} D_{kj}^{-1} \rho_i^{(t)} \rho_\beta.$$

(24)

where $\lambda_k$ denote the eigenvalues of the Lindblad-decoherence matrix $L_{ij}$.

A detailed analysis [20] gives the the final expression for the probability as:

$$P_{\nu_\alpha \rightarrow \nu_\beta}(t) = \frac{1}{3} + \frac{1}{2} \left[ \rho_1^\alpha \rho_1^\beta \cos \left( \frac{\Omega_{12} t}{2} \right) + \frac{\Delta D_{21} \rho_1^\alpha \rho_1^\beta}{\Omega_{12}} \sin \left( \frac{\Omega_{12} t}{2} \right) \right] e^{(D_{11} + D_{22}) t} +$$

$$+ \left[ \rho_2^\alpha \rho_2^\beta \cos \left( \frac{\Omega_{13} t}{2} \right) + \frac{\Delta D_{54} \rho_2^\alpha \rho_2^\beta}{\Omega_{13}} \sin \left( \frac{\Omega_{13} t}{2} \right) \right] e^{(D_{44} + D_{55}) t} +$$

$$+ \left[ \rho_3^\alpha \rho_3^\beta \cos \left( \frac{\Omega_{23} t}{2} \right) + \frac{\Delta D_{76} \rho_3^\alpha \rho_3^\beta}{\Omega_{23}} \sin \left( \frac{\Omega_{23} t}{2} \right) \right] e^{(D_{66} + D_{77}) t} +$$

$$+ \left[ \rho_4^\alpha \rho_4^\beta + \rho_5^\alpha \rho_5^\beta \right] \cosh \left( \frac{\Omega_{34} t}{2} \right) +$$

$$+ \frac{2D_{38} (\rho_3^\alpha \rho_5^\beta - \rho_5^\alpha \rho_3^\beta) + \Delta D_{83} (\rho_3^\alpha \rho_3^\beta - \rho_5^\alpha \rho_5^\beta)}{\Omega_{38}} \sinh \left( \frac{\Omega_{38} t}{2} \right) \right] e^{(D_{33} + D_{38}) t} .$$

(25)

Above we have used the notation that $\Delta D_{ij} = D_{ij} - D_{jj}$. We have assumed that $2|\Delta_{ij}| < |\Delta D_{ij}|$ with the consequence that $\Omega_{ij}, \ ij = 12, 13, 23$ is imaginary. However, $\Omega_{38} = \sqrt{(D_{33} - D_{88})^2 + 4D_{38}^2}$ will be real. Taking into account mixing, with the up-to-date values of neutrino mass differences and mixing angles, we can proceed into a fit of (25) to all the presently available data, including the results of the neutrino sector of the LSND experiment [25], and KamLand spectral distortion data [49].

The results are summarised in Fig. 3 which demonstrates the agreement (left) of our model with the KamLand spectral distortion data [49], and our best fit (right) for the Lindblad decoherence model used in ref. [20], and in Table 2 where we present the $\chi^2$ comparison for the model in question and the standard scenario.

The best fit has the feature that only some of the oscillation terms in the three generation probability formula have non trivial damping factors, with their exponents being independent.
Figure 6. Left: Ratio of the observed $\nu_e$ spectrum to the expectation versus $L_0/E$ for our decoherence model. The dots correspond to KamLAND data. Right: Decoherence fit. The dots correspond to SK data.

|                  | decoherence | standard scenario |
|------------------|-------------|-------------------|
| SK sub-GeV       | 38.0        | 38.2              |
| SK Multi-GeV     | 11.7        | 11.2              |
| Chooz            | 4.5         | 4.5               |
| KamLAND          | 16.7        | 16.6              |
| LSND             | 0.0         | 6.8               |
| TOTAL            | 70.9        | 77.3              |

Table 2. $\chi^2$ obtained for our model and the one obtained in the standard scenario for the different experiments calculated with the same program.

of the oscillation length, specifically [20]. If we denote those non trivial exponents as $\mathcal{D} \cdot L$, we obtain from the best fit of [20]:

$$\mathcal{D} = -\frac{1.3 \cdot 10^{-2}}{L},$$

in units of 1/km with $L = t$ the oscillation length. The $1/L$-behaviour of $\mathcal{D}_{11}$, implies, as we mentioned, oscillation-length independent Lindblad exponents.

In [20] an analysis of the two types of the theoretical models of space-time foam, discussed in section 2, has been performed in the light of the result of the fit [20]. The conclusion was that the model of the stochastically fluctuating media [16,17] cannot provide the full explanation for the fit, for the following reason: if the decoherent result of the fit [20] was exclusively due to this model, then the pertinent decoherent coefficient in that case, for, say, the KamLand experiment with an $L \sim 180$ Km, would be $|\mathcal{D}| = \Omega^2 G_N^2 n_0^2 \sim 2.84 \cdot 10^{-21}$ GeV (note that the mixing angle part does not affect the order of the exponent). Smaller values are found for longer $L$, such as in atmospheric neutrino experiments or, in future, for high-energy cosmic neutrinos [17]. The independence of the relevant damping exponent from the oscillation length, then, as required by [20] may be understood as follows in this context: In the spirit of [19], the quantity $G_N n_0 = \xi \frac{\Delta m^2}{E}$, where $\xi \ll 1$ parametrises the contributions of the foam to the induced neutrino mass differences, according to our discussion above. Hence, the
damping exponent becomes in this case $\xi^2 \Omega^2 (\Delta m^2)^2 \cdot L/E^2$. Thus, for oscillation lengths $L$ we have $L^{-1} \sim \Delta m^2/E$, and one is left with the following estimate for the dimensionless quantity $\xi^2 \Delta m^2 \Omega^2 / E \sim 1.3 \cdot 10^{-2}$. This implies that the quantity $\Omega^2$ is proportional to the probe energy $E$. In principle, this is not an unreasonable result, and it is in the spirit of $[19]$, since back reaction effects onto space time, which affect the stochastic fluctuations $\Omega^2$, are expected to increase with the probe energy $E$. However, due to the smallness of the quantity $\Delta m^2 / E$, for energies of the order of GeV, and $\Delta m^2 \sim 10^{-3}$ eV$^2$, we conclude (taking into account that $\xi \ll 1$) that $\Omega^2$ in this case would be unrealistically large for a quantum-gravity effect in the model.

We remark at this point that, in such a model, we can in principle bound independently the $\Omega$ and $n_0$ parameters by looking at the modifications induced by the medium in the arguments of the oscillatory functions of the probability $[17]$, that is the period of oscillation. Unfortunately, this is too small to be detected in the above example, for which $\Delta a_{\text{eff}} \ll \Delta a_{12}$.

The second model $[16],[17]$ of stochastic space time can also be confronted with the data, since in that case $[20]$ would imply for the pertinent damping exponent

$$
\left(\frac{(m_1^2 - m_2^2)^2}{2k^2}\right)^{2} (9\sigma_1 + \sigma_2 + \sigma_3 + \sigma_4) + \frac{2V \cos 2\theta (m_1^2 - m_2^2)}{k} (12\sigma_1 + 2\sigma_2 - 2\sigma_3) t^2 \\
\sim 1.3 \cdot 10^{-2}.
$$

Ignoring subleading MSW effects $V$, for simplicity, and considering oscillation lengths $t = L \sim 2k / (m_1^2 - m_2^2)$, we then observe that the independence of the length $L$ result of the experimental fit, found above, may be interpreted, in this case, as bounding the stochastic fluctuations of the metric to $9\sigma_1 + \sigma_2 + \sigma_3 + \sigma_4 \sim 1.3 \cdot 10^{-2}$. Again, this is too large to be a quantum gravity effect, which means that the $L^2$ contributions to the damping due to stochastic fluctuations of the metric, as in the model of $[7]$ above $[16]$, cannot be the exclusive explanation of the fit.

The analysis of $[20]$ also demonstrated that, at least as far as an order of magnitude of the effect is concerned, a reasonable explanation of the order of the damping exponent $[20]$, is provided by Gaussian-type energy fluctuations, due to ordinary physics effects, leading to decoherence-like damping of oscillation probabilities $[50]$. The order of these fluctuations, consistent with the independence of the damping exponent on $L$ (irrespective of the power of $L$), is

$$
\frac{\Delta E}{E} \sim 1.6 \cdot 10^{-1}
$$

if one assumes that this is the principal reason for the result of the fit. However, the selective damping implied by the result of the fit $[20]$, implies that this cannot be the explanation. The fact that the best fit model includes terms which are not suppressed at all calls for a more radical explanation of the fit result, and the issue is still wide open.

It is interesting, however, that the current neutrino data can already impose stringent constraints on quantum gravity models, and exclude some of them from being the exclusive source of decoherence, as we have discussed above. Of course, this is not a definite conclusion because one cannot exclude the possibility of other classes of theoretical models of quantum gravity, which could escape these constraints. At present, however, we are not aware of any such theory.

Before closing this section we would like to remark that the above analysis took into account the neutrino sector results of the LSND experiment $[25]$. In the antineutrino sector, the original indication from that experiment was that there was evidence for much more profound oscillations as compared to the neutrino sector, which, if confirmed, should be interpreted as indicating a
direct CPT Violation. There were attempts to interpret such a result as being due to either CPT violating mass differences between neutrinos and antineutrinos, characterising, for instance, non-local theories [24], or CPT-Violating differences between the strengths of the QG-environmental interactions of neutrinos as compared to those of antineutrinos [18], without the need for CPT violating mass differences. In either case, the order of magnitude of the effects indicated by experiment was too large for the effect to correspond to realistic QG models, probably pointing towards the need of confirming first the LSND results in the antineutrino sector by future experiments before embarking on radical explanations.

5. Decoherence in Cosmology: some remarks

5.1. Cosmic Horizons and Decoherence

Recent astrophysical observations, using different experiments and diverse techniques, seem to indicate that 70% of the Universe energy budget is occupied by “vacuum” energy density of unknown origin, termed Dark Energy [51, 52]. Best fit models give the positive cosmological constant Einstein-Friedman Universe as a good candidate to explain these observations, although models with a vacuum energy relaxing to zero (quintessential, i.e. involving a scalar field which has not yet reached the minimum of its potential) are compatible with the current data.

From a theoretical point of view the two categories of Dark Energy models are quite different. If there is a relaxing cosmological vacuum energy, depending on the details of the relaxation rate, it is possible in general to define asymptotic states and hence a proper scattering matrix (S-matrix) for the theory, which can thus be quantised canonically. On the other hand, Universes with a cosmological constant Λ > 0 (de Sitter) admit no asymptotic states, as a result of the Hubble horizon which characterises these models, and hampers the definition of proper asymptotic state vectors, and hence, a proper S-matrix. Indeed, de Sitter Universes will expand for ever, and eventually their constant vacuum energy density component will dominate over matter in such a way that the Universe will enter again an exponential (inflationary) phase of (eternal) accelerated expansion, with a Hubble horizon of radius \( \delta H \propto 1/\sqrt{\Lambda} \). It seems that the recent astrophysical observations [51, 52] seem to indicate that the current era of the universe is the beginning of such an accelerated expansion.

Canonical quantisation of field theories in de Sitter space times is still an elusive subject, mostly due to the above mentioned problem of defining a proper S-matrix. One suggestion towards the quantisation of such systems could be through analogies with open systems in quantum mechanics, interacting with an environment. The environment in models with a cosmological constant would consist of field modes whose wavelength is longer than the Hubble horizon radius. This splitting was originally suggested by Starobinski [53], in the context of his stochastic inflationary model, and later on was adopted by several groups [54]. Crossing the horizon in either direction would constitute interactions with the environment. An initially pure quantum state in such Universes/open-systems would therefore become eventually mixed, as a result of interactions with the environmental modes, whose strength will be controlled by the size of the Hubble horizon, and hence the cosmological constant. Such decoherent evolution could explain the classicality of the early Universe phase transitions [55] (or late in the case of a cosmological constant). The approach is still far from being complete, not only due to the technical complications, which force the researchers to adopt severe, and often unphysical approximations, but also due to conceptual issues, most of which are associated with the back reaction of matter onto space-time, an issue often ignored in such a context. It is our opinion that the latter topic plays an important rôle in the evolution of a quantum Universe, especially one with a cosmological constant, and is associated with issues of quantum gravity. The very origin of the cosmological constant, or in general the dark energy of the vacuum, is certainly a property of quantum gravity.

This link between quantum decoherence and a cosmological constant may have far reaching
consequences for the phenomenology of elementary particles, especially neutrinos. In [19, 7] we proposed a scenario according to which the mass differences of neutrinos may have (part of) their origin in the quantum gravity decoherence medium of space time foam. The induced decoherence, then, will affect their oscillation, a notable consequence being the appearance of intrinsic CPT violating damping terms in front of the oscillation amplitudes. This fundamental (and local) form of CPT violation has its origin in the ill-defined nature of the corresponding CPT operator in such decoherent quantum theories, due to the mathematical theorem by Wald [10], mentioned previously. This local form of CPT violation, as a result of the interaction of the elementary particle with a decoherent medium is linked to a cosmological (global) violation of CPT symmetry of the type proposed in [56] by means of a generation of a cosmological constant as a result of neutrino mixing and non-trivial mass differences due to the quantum gravity vacuum. The framework in which such a cosmological constant may be generated by the neutrinos is the approach of [57], according to which the problem of mixing in a quantum field theory is treated by means of a canonical Fock-space quantization.

5.2. Non-critical string-framework for Cosmological decoherence
Let us return to the master equation in (6) for non-critical strings. Recent astrophysical observations from different experiments all seem to indicate that 73% of the energy of the Universe is in the form of dark energy. Best fit models give the positive cosmological constant Einstein-Friedman Universe as a good candidate to explain these observations. For such de Sitter backgrounds $R_{MN} \propto \Omega g_{MN}$ with $\Omega > 0$ a cosmological constant. Also in a perturbative derivative expansion (in powers of $\alpha'$ where $\alpha' = l_s^2$ is the Regge slope of the string and $l_s$ is the fundamental string length) in leading order

$$\beta_{\mu\nu} = \alpha' R_{\mu\nu} = \alpha' \Omega g_{\mu\nu}$$

and

$$G_{ij} = \delta_{ij}.$$  

This leads to

$$\partial_t \rho = i \{ \rho, H \} + \alpha' \Omega : g_{MN} \left[ g^{MN}, \rho \right]:$$

For a weak-graviton expansion about flat space-time, $g_{MN} = \eta_{MN} + h_{MN}$, and

$$h_{0i} \propto \frac{\Delta p_i}{M_P}.$$  

If an antisymmetric ordering prescription is used, then the master equation for low energy string matter assumes the form

$$\partial_t \rho_{\text{Matter}} = i \{ \rho_{\text{Matter}}, H \} - \Omega \left[ h_{0j}, \left[ h^{0j}, \rho_{\text{Matter}} \right] \right]$$

( when $\alpha'$ is absorbed into $\Omega$). In view of the previous discussion this can be rewritten as

$$\partial_t \rho_{\text{Matter}} = i \{ \rho_{\text{Matter}}, H \} - \Omega \left[ \pi_j, \left[ \pi^j, \rho_{\text{Matter}} \right] \right],$$

thereby giving the master equation for Liouville decoherence in the model of a D-particle foam with a cosmological constant.

The above D-particle inspired approach deals with possible non-perturbative quantum effects of gravitational degrees of freedom. The analysis is distinct from the phenomenology of dynamical semigroups which does not embody specific properties of gravity. Indeed the phenomenology is sufficiently generic that other mechanisms of decoherence such as the MSW
effect [41] can be incorporated within the same framework. Consequently an analysis which is less
generic and is related to the specific decoherence implied by non-critical strings is necessary. It is
sufficient to study a massive non-relativistic particle propagating in one dimension to establish
qualitative features of D-particle decoherence. The environment will be taken to consist of both
gravitational and non-gravitational degrees of freedom; hence we will consider a generalisation of
quantum Brownian motion for a particle which has additional interactions with D-particles.
This will allow us to compare qualitatively the decoherence due to different environments. The
non-gravitational degrees of freedom in the environment (in a thermal state) are conventionally
modelled by a collection of harmonic oscillators with masses $m_n$, frequency $\omega_n$ and co-ordinate
operator $\hat{q}_n$ coupled to the particle co-ordinate $\hat{x}$ by an interaction of the form $\sum_n g_n \hat{x} \hat{q}_n$.
The master equation which is derived can have time dependent coefficients due to the competing
timescales, e.g. relaxation rate due to coupling to the thermal bath, the ratio of the time scale
of the harmonic oscillator to the thermal time scale etc. However an ab initio calculation of
the time-dependence is difficult to do in a rigorous manner [58]. The dissipative term in (34) involves the momentum
transfer operator due to recoil of the particle from collisions with D-particles (7). This transfer process will be modelled by a classical
Gaussian random variable $r$ which multiplies the momentum operator $\hat{p}$ for the particle:

$$\text{d}_t \rightarrow \frac{r}{M \bar{p}}$$

Moreover the mean and variance of $r$ are given by

$$\langle r \rangle = 0, \quad \text{and} \quad \langle r^2 \rangle = \sigma^2.$$

On amalgamating the effects of the thermal and D-particle environments, we have for the reduced
master equation [31] for the matter (particle) density matrix $\rho$ (on dropping the Matter index)

$$i \frac{\partial}{\partial t} \rho = \frac{1}{2m} \left[ \hat{p}^2, \rho \right] - i \Lambda [\hat{x}, [\hat{x}, \rho]] + \frac{\gamma}{2} \left[ \hat{x}, \{\hat{p}, \rho\} \right] - i \Omega r^2 [\hat{p}, [\hat{p}, \rho]]$$

where $\Lambda, \gamma$ and $\Omega$ are real time-dependent coefficients. As discussed in [31] a possible model for
$\Omega (t)$ is

$$\Omega (t) = \Omega_0 + \frac{\tilde{\gamma}}{a + t} + \frac{\tilde{T}}{1 + bt^2}$$

where $\omega_0, \tilde{\gamma}, a, \tilde{T}$ and $b$ are positive constants. The quantity $\tilde{\gamma} < 1$ contains information on the
density of D-particle defects on a four-dimensional world. The time dependence of $\gamma$ and $\Lambda$ can be calculated in the weak coupling limit for general $n$ (i.e. ohmic, $n = 1$ and non-ohmic $n \neq 1$
environments) where

$$I (\omega) = \frac{2}{\pi} m \gamma_0 \omega \left[ \frac{\omega}{\varpi} \right]^{n-1} e^{-\omega^2/\varpi^2}$$

and $\varpi$ is a cut-off frequency. The precise time dependence is governed by $\Lambda (t) = \int_0^t ds \nu (s)$ and
$\gamma (t) = \int_0^t ds \nu (s) s$ where $\nu (s) = \int_0^\infty d\omega I (\omega) \coth (\beta \hbar \omega / 2) \cos (\omega s)$. For the ohmic case, in the
limit $\hbar \varpi \ll k_B T$ followed by $\varpi \rightarrow \infty$, $\Lambda$ and $\gamma$ are given by $m \gamma_0 k_B T$ and $\gamma_0$ respectively after a rapid
initial transient. For high temperatures $\Lambda$ and $\gamma$ have a powerlaw increase with $t$ for the
subohmic case whereas there is a rapid decrease in the supraohmic case.
This formalism can be applied to study flavour oscillations in the presence of such dark-energy induced decoherence [31]. With the general \(\Omega(t)\) of [38] the probability for flavour oscillation, \(P_{1\rightarrow 2}\), is found proportional to 

\[
P_{1\rightarrow 2} \propto (\sin 2\theta)^2 \left(1 - \exp\left[-4\sigma^2 p^2 \mathcal{I}(t)\right]\right) \times \text{[conventional oscillatory terms]} .
\]  

(40)

with \(\mathcal{I}(t) \equiv \int_0^t \Omega(t')dt' = \Omega_0 t + \tilde{\gamma}\ln(1 + t/a) + \frac{\tilde{\delta}}{\sqrt{6}} \tan^{-1}(\sqrt{6}t)\).

From (40) it is evident that decoherence affects this probability with an exponential damping only if the cosmological term \(\Omega\) is the constant \(\Omega_0\). In particular, in the absence of the \(\Omega_0\) term, and in the limit \(\tilde{\Gamma} = 0\), the decoherence due to the D-particle foam results in power damping for large times \(t \rightarrow \infty\), the terms \(\tilde{\rho}_i \sim t^{-\delta_i}\), \(i = 0,3\), while \(\tilde{\rho}_i \sim t^{-p^2\delta_i}\), \(i = 1,2\), i.e. the scaling power depends on the probe’s momentum (with \(\delta_i\) appropriate constants depending on the term we look at).

Although tiny, for laboratory scales, such cosmological decoherence effects may play an important rôle in the propagation of sensitive particle physics probes, such as high energy neutrinos, over cosmological distances, especially in theories where the gravity scale may be at TeV [28].

5.3. Non-Critical String effects on relic abundances and dark matter constraints

Before closing this section we would like to make a final remark on contributions to the Boltzmann equation (determining cosmic relic abundances) from non-critical string terms, proportional to the \(\beta\)-functions \(\beta^i\). Such quantities are important for constraining dark matter (in particular supersymmetric) candidates, from astrophysical data [52, 59].

In effective four-dimensional space-times (after appropriate string compactification of the extra dimensions) the relic density of a species \(\tilde{\chi}\) of mass \(m_{\tilde{\chi}}\), assumed to be the dominant dark matter candidate, is given generically by [60]:

\[
\Omega_{\tilde{\chi}} h_0^2 = \left(\Omega_{\tilde{\chi}} h_0^2\right)_{\text{no-source}} \times \left(\frac{g_*}{g_*}\right)^{1/2} \exp\left(\int_{x_0}^{x_f} \frac{\Gamma H^{-1}}{x} dx\right) .
\]

(41)

In the above formula, \(H\) is the Hubble parameter of the Universe, \(x \equiv T/m_{\tilde{\chi}}\), with \(T\) the temperature, and \(\left(\Omega_{\tilde{\chi}} h_0^2\right)_{\text{no-source}} = 0.066 \times 10^9 \text{GeV}^{-1}\), \(J \equiv \int_{x_0}^{x_f} \langle \sigma v \rangle dx\), is the standard-cosmology result without sources [61]. The source \(\Gamma\) contains also, in our approach, both time-dependent dilaton and non-critical-string terms: \(\Gamma(t) \equiv \Phi - \frac{1}{2} \mathcal{J}_\mu \mathcal{J}_\mu \), where the suffix “Grav” denotes graviton background \(\beta\)-functions. The time-dependent dilaton sources \(\Phi(t)\) could play the rôle of quintessence fields in string cosmology [62] and be responsible for the Universe’s current acceleration (through dilatonic dark energy which relaxes asymptotically to zero).

The freeze out point \(x_f\) (defined to be the temperature below which all (local) particle interactions are frozen due to the expansion of the Universe) contains source-term modifications:

\[
x_f^{-1} = \ln \left[ 0.03824 \frac{g_* M_{\text{Planck}} m_{\tilde{\chi}}^{1/2} \langle \sigma v \rangle}{\sqrt{g_*}} x_f^{1/2} \langle \sigma v \rangle f \right] + \frac{1}{2} \ln \left(\frac{g_*}{g_*}\right) + \int_{x_f}^{x_{\infty}} \frac{\Gamma H^{-1}}{x} dx .
\]

(42)

whereby \(g_{cf} = g_{cf} + \frac{30}{x_f^2} T^{-4} \Delta \rho\), with \(g_{cf}\) the ordinary matter degrees of freedom, assumed thermal.

The merit of casting the relic density in such a form is that it clearly exhibits the effect of the presence of the source. It is immediately seen from (41) that, depending on the sign of the source \(\Gamma\), one may have increase or reduction of the relic density as compared with the corresponding
value in the absence of the source. For instance, for simple de-Sitter backgrounds, with positive cosmological constant $\Omega_0 > 0$, and constant dilaton, $\Phi = \phi_0$, the source term $\Gamma$ in (41) is simply $\Gamma \sim -2e^{\phi_0}\Omega_0$, in (effective) four space-time dimensions, to lowest order in the Regge slope $\alpha'$ (c.f. 29), and $H^{-1}\Gamma \propto -e^{-\phi_0}$. However, in realistic string cosmologies with matter, there may be time-dependent dilatons and relaxation dark-energy components present [62], and thus the situation is more complicated [60]. It becomes evident from the above that the constraints imposed on physically appealing particle physics models, such as supersymmetric extensions of the Standard Model, from astrophysical bounds [52] on, say, cold dark matter relic densities [59], need to be revisited in the non-critical (decoherent) string context [60].

6. Conclusions and Outlook

In this review we have outlined several aspects of decoherence-induced CPTV and the corresponding experiments. We described the interesting and challenging precision tests that can be performed using kaon systems, especially $\phi$-meson factories, where some unique (\omega) effects on EPR-correlation modifications, associated with the ill-defined nature of CPT operator in decoherent QG, may be in place.

We have presented sufficient theoretical motivation and estimates of the associated effects to support the case that testing QG experimentally at present facilities may turn out to be a worthwhile endeavour. In fact, as we have argued, CPTV may be a real feature of QG, that can be tested and observed, if true, in the foreseeable future. Indeed, as we have seen, some theoretical (string-inspired) models of space-time foam predict $\omega$-like effects of an order of magnitude that is already well within the reach of the next upgrade of $\phi$-factories, such as DA\PhiNE-2. Neutrino systems is another extremely sensitive probe of particle physics models which can already falsify several models or place stringent bounds in others. Finally, QG-decoherence effects in Cosmology may also play a rôle in our understanding of the Universe evolution, or even modify the current astrophysical constraints on models of particle physics, where such possible decoherent effects are ignored.

Therefore, the current experimental situation for QG signals appears exciting, and several experiments are reaching interesting regimes, where many theoretical models can be falsified. More precision experiments are becoming available, and many others are being designed for the immediate future. Searching for tiny effects of this elusive theory may at the end be very rewarding. Surprises may be around the corner, so it is worth investing time and effort.

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