RESEARCH

Edge resolvability of crystal cubic carbon structure

Sahil Sharma¹ · Vijay Kumar Bhat¹ · Sohan Lal¹

Received: 27 September 2022 / Accepted: 31 January 2023 / Published online: 12 February 2023
© The Author(s), under exclusive licence to Springer-Verlag GmbH Germany, part of Springer Nature 2023

Abstract
Graph theory plays an important role for modelling and designing chemical structures and complex networks. Chemical graph theory is commonly used to analyse and comprehend chemical structures and networks, as well as their features. In graph theory, a chemical structure can be represented by vertices and edges where vertices denote atoms and edges denote molecular bonds. The concept of resolvability parameters for a graph \( G = (V, E) \) is a relatively new advanced field in which the complete structure is built so that each vertex (atom) or edge (bond) represents a distinct position. It plays a very prominent role in analysing the overall symmetry and structural properties of new chemical compounds. The goal of this study is to employ chemical graph theory to determine some graph-related parameters related to chemical graphs of hypothesised carbon allotrope. In this article, we study the resolvability parameters, i.e. edge resolvability of the chemical graph of the crystal structure of cubic carbon (\( CCS(n) \)).

Keywords  Edge metric dimension · Metric dimension · Crystal cubic carbon · Cubes

Mathematics Subject Classification  13A99 · 05C12

1 Introduction
Mathematical applications have been increasingly popular in recent years. Graph theory has rapidly grown in theoretical conclusions as well as applicability to real-life issues as a handy tool for dealing with event relations. Distance is a concept that penetrates all of the graph theory, and it is utilised in isomorphism tests, graph operations, maximal and minimal connectivity problems, and diameter problems. Several characteristics linked to graph distances have piqued the interest of several scholars. One of them is the metric dimension.

The notion of metric dimension (MD) was introduced by Slater [1], Melter and Harary [2] independently. They termed the ordered subset \( K = \{a_1, a_2, a_3, \ldots, a_m\} \) of vertices of graph \( G = (V, E) \) as resolving set, if the representation \( r(u|K) = (d(u, a_1), d(u, a_2), \ldots, d(u, a_m)) \) for each \( u \in V \) is unique, where \( d(u, a_m) \) is the shortest path between \( u \) and \( a_m \). The cardinality of the minimal resolving set is called MD (locating number) of \( G \) and is denoted by \( \dim(G) \). Chartrand et al. [3] studied MD of path graph \( P_n \) and complete graph \( K_m \). Later on, Caceres et al. [4] studied about MD of the Cartesian product of graphs, hypercubes, and cycles graphs. MD of regular graphs, Jahangir graphs, prism graphs, and circulant graphs were examined by several different authors. In chemistry, MD is used to give a series of compounds a unique representation, which leads to drug discovery. Other applications of MD include navigation, combinatorial optimization, and games such as coin weighing, etc. [3, 5, 6].

A new variant of metric generator, namely edge metric dimension (EMD), in graphs was introduced by Kelenc et al. [7]. This is based on the fact that an ordered subset \( H \) of vertices of \( G = (V, E) \) is called an edge resolving set (edge metric generator), if for \( e_1 \neq e_2 \in E \), there is a vertex \( h \in H \) such that \( d(e_1, h) \neq d(e_2, h), \) where \( d(e, h) = \min\{d(a, h), d(b, h)\}, ab = e \in E \) and \( h \in H \). The cardinality of the smallest edge metric generator (edge basis) is called EMD and is denoted by \( edim(G) \). After that, Zhang and Gao [8] discussed the EMD of some convex polytopes.
EMD of path graphs, complete graphs, bipartite graphs, wheel graphs, etc., has been studied. Recently, knor et al. [9] studied graphs with $edim(G) < dim(G)$. Sharma et al. [10] computed the EMD of one heptagonal carbon nanotube. After the introduction of EMD by Kelenc et al. [7], several other authors [11–15] have studied it thoroughly.

The structure of a chemical molecule is commonly described by a group of functional groups placed on a substructure. The structure is a graph-theoretical labelled graph, with vertex and edge labels representing atom and bond types, respectively. A collection of compounds is primarily defined by the substructure common to them, which is characterised by modifying the set of functional groups and/or permuting their locations. Traditionally, these “positions” simply reflect uniquely defined atoms (vertices) of the substructure (common subgraph). These positions seldom form a minimum set $K$ for which every two distinct vertices have distinct ordered $k$-tuples forming a minimum dimensional representation of the positions. In this context, $K$ is referred to as a resolving set relative to $V(G)$.

We can assess whether any two compounds in the collection share the same functional group at a given position using the standard pattern. When analysing whether a compound’s characteristics are responsible for its pharmacological activity, this comparison statement is important [3, 4]. Sharma et al. [16] studied the mixed metric resolvability of the chemical compound, namely polycyclic aromatic hydrocarbon networks.

The problem of computing the edge metric dimension of $CCS(n)$ and certain generalisations of these graphs are discussed in this work. One of the many chemical compounds that are useful and necessary for the survival of living organisms is carbon, oxygen, hydrogen, and nitrogen. These are helpful for the production of cells in living organisms. Carbon is an essential element for human life. It is useful in the formation of proteins, carbohydrates, and nucleic acids. It is vital for the growth of plants in the form of carbon dioxide. The carbon atoms can bond together in various ways, called allotropes of carbon. One of the forms of carbon, namely diamond, is anticipated to convert into the $C_6$ structure, which is a cubic body-centred structure having 8 elements in its unit cell, at pressures exceeding 1000 GPa (gigapascal). Its composition is comparable to cubane and is found in one of silicon’s metastable phases. Carbon sodalite was postulated as the structure of this phase in 2012. This cubic carbon phase could be useful in astronomy.

Several authors studied different topological properties of $CCS(n)$. Yang et al. [17] studied the Augmented Zagreb index, forgotten index, Balaban index, and redefined Zagreb indices of $CCS(n)$. Baig et al. [18] computed the degree-based additive topological indices mainly atom bond connectivity index, geometric index, first and second Zagreb index, etc. Further, Imran et al. [19] studied the eccentric-based bond connectivity index and eccentric-based geometric arithmetic index of $CCS(n)$. Sattar and Javaid [20] computed the connection-based Zagreb indices of $CCS(n)$. Moreover, to examine the superiority and authenticity of computed topological indices, they compared the calculated values of these indices for crystal cubic carbon structure with each other. This comparison enables us to check which connection-based Zagreb indices are more authentic and superior to predict the physical and chemical properties of crystal cubic carbon structure. Recently, Sharma et al. analysed the multiplicative topological descriptors of $CCS(n)$ in [21]. For further studies concerning the $CCS(n)$ one can refer to [22–24] and references therein.

Zhang and Naeem [25] studied the metric dimension of $CCS(n)$. They computed that for $n = 1$, $dim(CCS(n))$ is 3, and for $n ≥ 2$, $dim(CCS(n))$ is $7n^2 - 16$. To the best of our knowledge, no work has been reported regarding the edge metric basis and edge metric dimension (EMD) of $CCS(n)$. For this, we consider the baseline paper reported by (Zhang and Naeem [25]) and further extend the result with comparisons.

In this study, the EMD of $CCS(n)$ has been computed and it has been shown that MD and EMD of $CCS(n)$ are the same. In addition, the structure, origin, and applications of $CCS(n)$ have been discussed.

## 2 Crystal cubic carbon structure $CCS(n)$

The valency of carbon allows it to create a variety of allotropes. Diamond and graphite are well-known forms of carbon, while crystal cubic carbon, also known as pcb, is one of its potential allotropes. The chemical structure of $CCS(n)$ is constructed as

1. $CCS(n)$ starts from one unit cube, also called $CCS(1)$, and central cube as shown in Fig. 1.
2. At the second level all eight vertices of $CCS(1)$ are attached to different cubes through edges (bridge edges), which results in the construction of $CCS(2)$ as shown in Fig. 2.
3. Again at the next level all the $7 \times 8$ vertices of degree 3 of $CCS(2)$ are attached to different cubes through edges, which results in the construction of $CCS(3)$
4. Similarly, we construct $CCS(n)$ by attaching new cubes to the vertices of degree 3 at the preceding level, as shown in Fig. 3.

At each level, we receive a new set of cubes that are attached to the vertex of degree 3 of cubes of the previous level. These cubes are referred to as the outermost cubes. In $CCS(2)$, we can also see that there are eight outermost cubes and $7 \times 8$ vertices of degree 3. So at the third level, $7 \times 8$...
cubes are attached. Similarly, there are $7^{n-2} \times 8$ vertices of degree 3 at each subsequent level. So, this process is again repeated to get to the next level.

2.1 Origin of crystal cubic carbon structure CCS($n$)

Carbon, as one of the elements, has been known and used on the planet since ancient times, continues to draw attention due to its wide range of industrial and commercial applications, ranging from cutting tools to electrical and optoelectronic materials, and so on. Carbon exhibits the adaptability of $sp$, $sp^2$, and $sp^3$ hybridization states, resulting in a variety of rich allotropic carbon forms. Graphite and diamond are two prevalent allotropes of carbon, with the former being the most thermodynamically stable form of all known carbon allotropes at ambient temperatures (e.g. hexagonal diamond, amorphous carbon, carbyne, fullerenes, carbon nanotubes, crystal cubic carbon, and graphene).

Diamond is projected to convert into a body-centred cubic structure at ultra-high pressures of above 1000 GPa. This phase is significant in astrophysics and the study of the deep cores of planets such as Uranus and Neptune. In 1979, [26] a super dense and super hard material that looked like this phase was synthesised and published, having the Im3m space group and eight atoms per primitive unit cell. It was claimed that the so-called $C_8$ structure, with eight-carbon cubes comparable to cubane in the Im3m space group, had been synthesised, with eight atoms per primitive unit cell or 16 atoms per standard unit cell. The super cubane structure, the BC8 structure, a structure with clusters of four carbon atoms in tetrahedra in space group I43m with four atoms per primitive unit cell (eight per conventional unit cell), and a structure named “carbon sodalite” were all discussed in a report published in 2012, [27]. In 2017, Baig et al. [18] modified and extended the structure of carbon sodalite and named it crystal cubic.
carbon $CCS(n)$. $CCS(n)$ as a carbon allotrope is assumed to have wide a range of industrial applications. For more studies on carbon allotrope one can refer [28–30].

### 3 Edge metric dimension of $CCS(n)$

The EMD of various graphs like Mobius network, wheel graph, convex polytopes, windmill graph, etc., has been studied. We will discuss the EMD of $CCS(n)$, $n \geq 1$ in this section.

**Theorem 1** For $n = 1$, the edge metric dimension of $CCS(n) = 3$.

**Proof** First, claim that $edim(CCS(1)) \leq 3$. We have labelled all 8 vertices of the cube as $r_i$, $1 \leq i \leq 8$, and edges as $e_i$, $1 \leq i \leq 12$ as shown in Fig. 1.

Let $R_E = \{r_1, r_2, r_3\}$ be set of vertices of graph $CCS(1)$. We have to show that $R_E$ is an edge metric generator of $CCS(1)$.

The representation of each edge $e_i$, $1 \leq i \leq 12$ with respect to $R_E$ is given as

\[
\begin{align*}
    r[e_1|R_E] &= (0, 0, 1) & r[e_2|R_E] &= (0, 1, 2) \\
    r[e_3|R_E] &= (1, 0, 0) & r[e_4|R_E] &= (1, 1, 2) \\
    r[e_5|R_E] &= (1, 1, 0) & r[e_6|R_E] &= (2, 2, 1) \\
    r[e_7|R_E] &= (0, 1, 1) & r[e_8|R_E] &= (1, 0, 1) \\
    r[e_9|R_E] &= (1, 2, 1) & r[e_{10}|R_E] &= (2, 1, 0) \\
    r[e_{11}|R_E] &= (1, 2, 2) & r[e_{12}|R_E] &= (2, 1, 1) \\
\end{align*}
\]

From equation (1), the representation of each edge is unique. Therefore $edim(CCS(n)) \leq 3$.

Next, we claim that $edim(CCS(1)) \geq 3$.

Since, for any graph $G$, $edim(G) = 1$ iff $G$ is a path graph. This implies that $edim(CCS(1)) \neq 1$. Now, if $edim(CCS(1)) = 2$, then edge metric generator $R_E$ of $CCS(1)$ consists of two vertices (say $R_E = \{r_i, r_j\}$; $i \neq j$). Due to the symmetry of $CCS(1)$, we have two choices for the vertices of $R_E$.

1. Both the vertices of $R_E$ are on the same face of the cube. Further, we have two choices
   (i) $r_i$ and $r_j$ are on the face diagonal of the cube.
   (ii) $r_i$ and $r_j$ are on the same edge of the face of the cube.
2. Both the vertices of $R_E$ are on the primary diagonal of the cube.

**Case 1:** Firstly, when both vertices are on the same face and along the face diagonal, then we consider $R_E = \{r_1, r_3\}$. We have $r[e_1|R_E] = (0, 1, 1) = r[e_4|R_E] = (0, 1, 2)$.

Secondly, when both vertices are on the same face and along the same edge, then we consider $R_E = \{r_2, r_3\}$. We have $r[e_1|R_E] = (1, 1, 1) = r[e_4|R_E] = (1, 2, 1)$.

**Case 2:** When both vertices are on the primary diagonal of the cube, then we consider $R_E = \{r_3, r_7\}$. We have $r[e_7|R_E] = (0, 1, 1) = r[e_{12}|R_E] = (2, 1, 1)$.

In both cases, we get a contradiction. Therefore $edim(CCS(1)) \geq 3$. Hence $edim(CCS(1)) = 3$. \[\square\]

**Theorem 2** For $n \geq 2$, the edge metric dimension of $CCS(n) = 7^{n-2} \times 16$.

**Proof** Let $R_E = \{r_1, r_2, r_3, \ldots r_k\}$ be the set of vertices of type $a_1$ and $a_2$ of outermost cubes $U_n$ as shown in Fig. 4.

The cube $U_n$ is the outermost and is attached with the chain of cubes through vertex $c$. The degree of vertex $c$ is 4, and all other vertices are of degree 3. We assume that $R_E$ is an edge metric generator and $k = 7^{n-2} \times 16$ (since there are $7^{n-2} \times 8$ cubes in the outermost layer of $CCS(n)$). The representation of any two distinct arbitrary edges of $CCS(n)$ can be compared in the following ways.

(i) When both the arbitrarily selected edges are on $U_n$ of $CCS(n)$ as shown in Fig. 4.

![Fig. 4 Cube $U_n$ (arbitrary) from the outermost cubes of $CCS(n)$](image)
We assume that \( r_1 = a_1 \) and \( r_2 = a_2 \), then 
\[ R_E = \{ a_1, a_2, r_3, r_4, ..., r_k \} \].
The representation of each edge \( e_i, 1 \leq i \leq 12 \) of cube \( U_n \) concerning \( R_E \) is given as

\[
\begin{align*}
\text{if } & e_1 | R_E = (0, 1, d(c, r_3), d(c, r_4), ..., d(c, r_k)) \\
\text{if } & e_2 | R_E = (0, 2, d(c, r_3) + 1, d(c, r_4) + 1, \ldots, d(c, r_k) + 1) \\
\text{if } & e_3 | R_E = (1, 2, d(c, r_3) + 2, d(c, r_4) + 2, \ldots, d(c, r_k) + 2) \\
\text{if } & e_4 | R_E = (1, 1, d(c, r_3) + 2, d(c, r_4) + 2, \ldots, d(c, r_k) + 2) \\
\text{if } & e_5 | R_E = (1, 0, d(c, r_3) + 1, d(c, r_4) + 1, \ldots, d(c, r_k) + 1) \\
\text{if } & e_6 | R_E = (2, 1, d(c, r_3) + 2, d(c, r_4) + 2, \ldots, d(c, r_k) + 2) \\
\text{if } & e_7 | R_E = (1, 2, d(c, r_3) + 1, d(c, r_4) + 1, \ldots, d(c, r_k) + 1) \\
\text{if } & e_8 | R_E = (2, 1, d(c, r_3) + 1, d(c, r_4) + 1, \ldots, d(c, r_k) + 1) \\
\text{if } & e_9 | R_E = (1, 1, d(c, r_3), d(c, r_4), \ldots, d(c, r_k)) \\
\text{if } & e_{10} | R_E = (1, 0, d(c, r_3), d(c, r_4), \ldots, d(c, r_k)) \\
\text{if } & e_{11} | R_E = (2, 0, d(c, r_3) + 1, d(c, r_4) + 1, \ldots, d(c, r_k) + 1) \\
\text{if } & e_{12} | R_E = (0, 1, d(c, r_3) + 1, d(c, r_4) + 1, \ldots, d(c, r_k) + 1) \\
\end{align*}
\]

From equation (2), we see that all these representations are different.

(ii) When both the arbitrarily selected edges are on the distinct cubes of chain \( CH_e \), one end of which is the cube of the outermost level as shown in Fig. 5.

Let us assume that two selected edges \( e_1 \) and \( e_2 \) are on two distinct cubes, namely \( L \)-cube and \( M \)-cube, respectively, on the chain of cubes. Assume that the chain of cubes \( CH_e \) in \( CCS(n) \) has one end as its central cube and the other end as the outermost cube which has a pair of distinct arbitrary edge resolving vertices (say \( r_1 \) and \( r_2 \)). Since \( e_1 \) is one of the edges of the \( L \)-cube and \( e_2 \) is edge of \( M \)-cube, then it is clear that \( d(e_1, R_E) < d(e_2, R_E) \). Hence, representation of both edges concerning \( R_E \) is distinct.

(iii) When both the arbitrarily selected edges are bridge edges on the chain of cubes \( CH_e \), one end of which is the cube of the outermost level as shown in Fig. 5.

Let us assume that two selected edges \( e_{b_1} \) and \( e_{b_2} \) are on the chain of cubes. Assume that the chain of cubes \( CH_e \) in \( CCS(n) \) has one end as its central cube and the other end as the outermost cube which has a pair of distinct arbitrary edge resolving vertices (say \( r_1 \) and \( r_2 \)). Then, it clear that \( d(e_{b_1}, R_E) < d(e_{b_2}, R_E) \). Hence, representation of both edges concerning \( R_E \) is distinct.

(iv) When both the arbitrarily selected edges are on distinct chains of cubes (say \( CH_{e_1} \) and \( CH_{e_2} \)) and these two chains are connected to a common cube called a branching cube.

Let us assume that the two selected edges \( e_M \) and \( e_N \) are on two distinct cubes, namely \( M \)-cube and \( N \)-cube, respectively. Further, these cubes are on distinct chains of cubes (say \( CH_{e_1} \) and \( CH_{e_2} \)) connected at the branching \( C \) cube as shown in Fig. 6.

Assume that the chain of cubes \( CH_{e_1} \) and \( CH_{e_2} \) in \( CCS(n) \) has one end as a branching curve and the other end as outermost curves, which has a pair of distinct arbitrary edge resolving vertices \( r_i \) and \( r_j \), and \( r_j = r_3, r_{j+1} \). Also, the length of the shortest path between vertex \( r_1 \) and edge \( e_M \) on the \( M \)-cube is greater than the length of the shortest path between vertex \( r_3 \) and edge \( e_N \) on the \( N \)-cube. This implies \( d(e_N, r_1) \neq d(e_M, r_1) \), and hence \( r(e_N, R_E) \neq r(e_M, R_E) \).
(v) When both the arbitrarily selected bridge edges are on distinct chains of cubes (say $CH_{c_1}$ and $CH_{c_2}$) and these two chains are connected to a common cube called a branching cube.

Let us assume that the two selected edges $e_{bi}$ and $e_{bj}$ are on two distinct chains of cubes, namely $CH_{c_1}$ and $CH_{c_2}$, respectively. Further, these distinct chains are connected at the branching $B$ cube as shown in Fig. 6.

Assume that the chain of cubes $CH_{c_1}$ and $CH_{c_2}$ in $CCS(n)$ has one end as branching cube and the other end as outermost cubes, which has a pair of distinct arbitrary edge resolving vertices $r_i = r_1, r_{i+1}$, and $r_j = r_3, r_{j+1}$. Also, the length of the shortest path between vertex $r_i$ and edge $e_{bi}$ is greater than the length of the shortest path between vertex $r_i$ and edge $e_{bj}$. This implies $d(e_{bi}, r_i) \neq d(e_{bj}, r_i)$, and hence $r(e_{bi}, R_E) \neq r(e_{bj}, R_E)$.

(vi) When both the arbitrarily selected edges are on the central cube as shown in Fig. 7.

Assume that the two arbitrarily selected edges are on the central cube. We have labelled all 8 vertices as $a_1, a_2, ..., a_8$ and edges as $e_i$, $i = 1, 2, 3, ..., 12$. Without loss of generality, we assume that vertices $r_{2i-1}, r_{2i}$, $i = 1, 2, 3, ..., 8$, are on the outermost layer cubes in $CCS(n)$, and these outermost cubes having vertices.

![Fig. 6 The distinct chains of cubes through branching cube](image)

![Fig. 7 Central Cube of $CCS(n)$](image)
$r_{2i-1}, r_{2i}$ are connected to the central cube at vertex $a_i$, $1 \leq i \leq 8$, through a chain of cubes. Since each vertex is joined by three edges of the central cube ($a_1$ is joined by edge $e_1, e_4, e_8$) so the shortest distance of these edges from vertex $r_1$ and $r_2$ is the same but simultaneously edge $e_i$ is also joined by vertex $a_2$ but edges $e_4$ and $e_8$ are not joined by $a_2$. This further implies that the shortest path between edge $e_1$ and vertex $r_3$ is not the same as edges $e_4$ and $e_8$, and so on for other edges. Hence, the representation is distinct for each edge concerning $R_E$.

(vii) When both the arbitrarily selected edges are on the middle cube, which is neither the outer cube nor the central cube.

Assume that the middle cube contains two arbitrarily selected edges as shown in Fig. 8. We have labelled all 8 vertices as $c, a_1, a_2, \ldots, a_7$ and edges as $e_i, i = 1, 2, 3\ldots, 12$. We can assume that the vertices $r_1$ and $r_2$ lie on the cube (say $C_j$) of the outermost layer of $CCS(n)$ and that $C_j$ is joined to the middle cube $C_m$ at vertex $a_1$ by a chain of cubes without losing generality. Similarly, we can suppose that $r_{2i-1}, r_{2i}$ lie on the cubes of the outermost layer in $CCS(n)$, and these cubes are connected to middle cube $C_m$ at vertices $a_i, 2 \leq i \leq 7$, through the chain of cubes. As in case (vi), we find that each vertex is joined by three edges of cube $C_m$. Similar to the above case the representation is distinct for each edge concerning $R_E$.

(viii) When one of the arbitrarily selected edges is on one of the cube, and the other is the bridge edge between cubes.

Clearly, by the symmetry of the structure, the representation is distinct for each edge concerning $R_E$.

All these cases prove that $R_E = \{r_1, r_2, r_3\ldots r_k\}$ is resolving set. Hence $edim(CCS(n)) \leq 7^{n-2} \times 16$.

Next, we claim that $edim(CCS(n)) \geq 7^{n-2} \times 16$. Let $C_n$ be an arbitrary cube on the outermost layer of $CCS(n)$ as shown in Fig. 9. We have labelled all the vertices of $C_n$ as $a_i, i = 1, 2, 3\ldots, 8$, and edges as $e_i, i = 1, 2, 3\ldots, 12$. The degree of each vertex is 3, except the vertex $a_1$, which is attached to the chain of the cube through an edge. Let $R_E = \{r_1, r_2, \ldots, r_k\}$ be an edge metric generator of $CCS(n)$. In order to complete the proof, we have to show that the edge metric generator $R_E$ has a minimum of two vertices from $C_n$. To begin, we assume $v \in R_E$ such that $v \notin C_n$, then $r_{[e_1]}R_E = (d(a_1, r_1), d(a_1, r_2), d(a_1, r_3), \ldots, d(a_1, r_k)) = r_{[e_1]}R_E$ which is a contradiction.

Secondly, we suppose that only one vertex of $R_E$ belongs to $C_n$. Without loss of generality, we assume that this common vertex is $r_1$.
The edge resolvability of various well-known graphs has recently been computed. For instance, convex polytopes, web graphs, circulant graphs, certain chemical structures, and so on. In chemistry, one of the most important problems is to represent a series of chemical compounds mathematically so that each component has its representation. Graph invariants play an important role to analyse the abstract structures of chemical graphs. But there are still some chemical graphs for which the vertex and edge resolvability have not yet been studied. Many researchers have conducted studies on the edge resolvability of chemical graphs; however, studies on the edge resolvability of some chemical graphs have not been performed. Therefore, the aim of this work is to compute the edge resolvability of some well-known chemical graphs.

Theoretical Chemistry Accounts (2023) 142:24

4 Conclusion and future work

In this article, we analysed the chemical structure of crystal cubic carbon, which is one of hypothesised carbon allotrope. Since it is not yet synthesised, there is a little amount of reported data to support their usage in different fields. In order to gain a deeper knowledge about the structure, bonding, arrangement and resolvability of atoms, the edge metric dimension of $CCS(n)$ has been computed. It has been proved that for $n=1$, $edim(CCS(n)) = 3$, and for $n \geq 2$, $edim(CCS(n)) = 7^{n-2} \times 16$, which is similar to metric dimension of $CCS(n)$. These results may be useful in studying the structural properties and finding the minimum number of locations on the substructure of crystal carbon. Also towards linking distinct functional groups at the locations for increasing the efficiency of $CCS(n)$ in different fields of electronics. In relation to this study, our upcoming research may include:

- Fault-tolerant metric dimension of $CCS(n)$.
- Fractional metric dimension of $CCS(n)$.
- Topological indices of $CCS(n)$ to analyse its chemical properties.

Author contributions Concept and final draft by SS and VKB; figures and graphs were prepared by SS and SL; supervision and editing by VKB, and all authors reviewed the final draft.

Funding The work of Sahil Sharma is supported by Shri Mata Vaishno Devi University (ref no SMVVDU / R &D / 19 / 2661-65).

Availability of data and material This article does not qualify for data sharing since no data sets were generated or analysed during this research.

Declarations

Conflict of interest The authors disclose that they have no competing interests.

Ethics approval and consent to participate Not applicable.

Novelty statement The edge resolvability of various well-known graphs has recently been computed. For instance, convex polytopes, web graphs, circulant graphs, certain chemical structures, and so on. In chemistry, one of the most important problems is to represent a series of chemical compounds mathematically so that each component has its representation. Graph invariants play an important role to analyse the abstract structures of chemical graphs. But there are still some chemical graphs for which the vertex and edge resolvability have not yet been studied. Many researchers have conducted studies on the edge resolvability of chemical graphs; however, studies on the edge resolvability of some chemical graphs have not been performed. Therefore, the aim of this work is to compute the edge resolvability of some well-known chemical graphs.\n
Our claim was backed by inconsistency in all of the circumstances (equation (3)). As a result, the edge metric generator $R_e$ of $CCS(n)$ has a minimum of two vertices from the outermost cube $C_n$. The cubes in $CCS(n)$ are increased by a number equal to 7 multiplied by the number of cubes in the outermost layer of the preceding level at each step or level, as can be seen from the construction of $CCS(n)$. This implies that $edim(CCS(n)) \geq 7^{n-2} \times 16$. Hence $edim(CCS(n)) = 7^{n-2} \times 16$. □
been found yet, one such compound is crystal cubic carbon structure $CCS(n)$. Therefore, in this article, we compute the edge resolvability of $CCS(n)$, for $n \geq 1$ and see that $\text{edim}(CCS(n)) = \text{dim}(CCS(n))$.

References

1. Slater PJ (1975) Leaves of trees. Congress Numer 14:549–559
2. Harary F, Melter RA (1966) On the metric dimension of a graph. Ars Comb 2:191–195
3. Chartrand G, Eroh L, Johnson MA, Oellermann OR (2000) Resolvability in graphs and metric dimension of a graph. Discrete Appl Math. https://doi.org/10.1016/S0166-218X(00)00198-0
4. Caceres J, Hernando C, Mora CM, Pelayo I, Puertas M, Seara C, Wood D (2007) On the metric dimension of Cartesian products of graphs. SIAM J Discrete Math. https://doi.org/10.1137/050641867
5. Junaidi A, Aldino AA, Munandar A (2022) The Alternative of sensor placement in multi-story buildings (representation of generalized Petersen graphs with certain operation) through the metric dimension approach. ComTech: Comput Math Eng Appl. https://doi.org/10.21512/comtech.v13i2.7268
6. Parera CO, Salmin M, Bustan AW, Mahmud R (2022) Application of metric dimensions to minimize the installation of fire sensors on the rectorate building of Pasifik Morotai University. MATEC Web Conf. https://doi.org/10.1051/mateconf/202237204005
7. Kelenc A, Tratnik N, Yero IG (2018) Uniquely identifying the edges of a graph: the edge metric dimension. Discrete Appl Math. https://doi.org/10.1016/j.dam.2018.05.052
8. Zhang Y, Gao S (2020) On the edge metric dimension of convex polytopes and its related graphs. J Comb Optim. https://doi.org/10.1007/s10878-019-00472-4
9. Knor M, Majstorovic S, Toshi ATM, Skrekovski R, Yero IG (2019) Vertex metric dimension of windmill graphs. AIMS Math. https://doi.org/10.3934/math.2021531
10. Sharma K, Bhat VK, Sharma SK (2022) Edge metric dimension and edge basis of one-heptagonal nanocarbon networks. IEEE Access. https://doi.org/10.1109/ACCESS.2022.3158982
11. Deng B, Nadeem MF, Azem M (2021) On the edge metric dimension of different families of Möbius networks. Math Probl Eng. https://doi.org/10.1155/2021/6623208
12. Sedlar J, Skrekovski R (2021) Bounds on metric dimensions of graphs with edge disjoint cycles. Appl Math Comput. https://doi.org/10.1016/j.amc.2020.125908
13. Sharma S, Bhat VK (2022) Fault-tolerant metric dimension of zero-divisor graphs of commutative rings. AKCE Int J Graphs Comb. https://doi.org/10.1080/09728600.2021.2009746
14. Singh P, Sharma S, Sharma SK, Bhat VK (2021) Dimension and edge metric dimension of windmill graphs. AIMS Math. https://doi.org/10.3934/math.2021531
15. Wei C, Salman M, Shahzab S, Rehman MU, Fang J (2021) Classes of planar graphs with constant edge metric dimension. Complexity. https://doi.org/10.1155/2021/5599274
16. Sharma SK, Bhat VK, Raza H, Sharma S (2022) On mixed metric dimension of polycyclic aromatic hydrocarbon networks. Chem Pap. https://doi.org/10.1007/s11696-022-02151-x
17. Yang H, Naem M, Siddiqui MK (2020) Molecular properties of carbon crystal cubic structures. Open Chem. https://doi.org/10.1515/chem-2020-0035
18. Baiq AQ, Imran M, Khalid W, Naem M (2017) Molecular description of carbon graphite and crystal cubic carbon structures. Can J Chem. https://doi.org/10.1139/cjc-2017-0083
19. Imran M, Naem M, Baiq AQ, Siddiqui MK, Zahid MA, Gao W (2019) Modified eccentric descriptors of crystal cubic carbon. J Discrete Math Sci Cryptogr. https://doi.org/10.1080/09720529.2019.1700922
20. Sattar A, Javaid M (2022) Topological characterizations of crystal cubic carbon structures. Can J Chem. https://doi.org/10.1139/cjc-2022-0087
21. Sharma S, Bhat VK, Lal S (2023) Multiplicative topological indices of the crystal cubic carbon structure. Cryst Res Technol. https://doi.org/10.1002/crat.202200222
22. Yang H, Naem M, Baiq AQ, Shaker H, Siddiqui MK (2019) Vertex Szeged index of crystal cubic carbon structure. J Discrete Math Sci Cryptogr. https://doi.org/10.1080/09720529.2019.1688966
23. Zahid MA, Naem M, Baiq AQ, Gao W (2018) General fifth M-zagreb indices and fifth M-zagreb polynomials of crystal cubic carbon. Util Math 109:263–270
24. Javaid M, Sattar A, Bonyah E (2022) Topological aspects of molecular networks: crystal cubic carbons. Complexity. https://doi.org/10.1155/2022/3458094
25. Zhang X, Naem M (2021) Metric dimension of crystal cubic carbon structure. J Math. https://doi.org/10.1155/2021/3438611
26. Matyushenko NN, Strel’Nitskii VE, Gusev VA (1979) A dense new version of crystalline carbon $C_8$. JETP Lett 30:199–202
27. Pokropivny A, Volz S (2012) ‘C8 phase’: Supercubane, tetrahe- dral, BC-8 or carbon sodalite? Phys Status Solidi B. https://doi.org/10.1002/pssb.201248185
28. Hatala M, Gmeiner P, Lorencova L, Mikula M, Hvojnik M, Pavličková M, Ház A, Kosnáč D, Bertók T, Tkač J (2021) Screen-printed conductive carbon layers for dye-sensitized solar cells and electrochemical detection of dopamine. Chem Pap. https://doi.org/10.1007/s11696-021-01601-2
29. Kharissova OV, Rodríguez J, Kharisov BI (2019) Non-standard ROS-generating combination “theraphthal-ascorbic acid” in low-temperature transformations of carbon allotropes. Chem Pap. https://doi.org/10.1007/s11696-018-0571-y
30. Majidi R (2017) Density functional theory study on structural and mechanical properties of graphene, T-graphene, and R-graphyne. Theor Chem Acc. https://doi.org/10.1007/s00214-017-1248-1

Publisher's Note Springer Nature remains neutral with regard to jurisdictional claims in published maps and institutional affiliations.