Simulation of Steam-flooding Thermal Heavy Oil Recovery Process with Sequential Implicit Method

Zixin Ping, Mengchen Yue, Min Wang *, Xiaohong Wang

Department of Thermal Science and Energy Engineering, University of Science and Technology of China, Hefei, Anhui, 230022, P. R. China

*Corresponding author e-mail: wang159@ustc.edu.cn

Abstract. Numerical simulation of steam-flooding thermal heavy oil recovery is studied in this paper. Although the traditional full implicit scheme has high stability, it needs to calculate the huge sparse matrix and the computation cost is super expensive; so it cannot meet the needs of practical oil industry production. In this paper, sequential implicit scheme is applied to numerical computation of steam-flooding thermal heavy oil recovery. A quasi-one-dimension numerical example is tested to verify the validity and high efficiency of the proposed method against using traditional fully implicit scheme.

1. Introduction

Heavy oil recovery is hard to conduct due to high viscosity, making it super difficult for flow movement. Steam injection is one of the most efficient ways to recover heavy oil in oil production industries; the high temperature and high pressure steam is injected into the heavy oil reservoir to improve the recovery of heavy oil through several ways, including reducing the viscosity, distillation, thermal expansion, displacement and so on.

Numerical simulation, which simulate displacement process by computing nonlinear systems of partial differential equations, is one of the effective ways to study steam injection process without high experimental cost. Computation of multi-phase flow has attracted the attention of many researchers. However, in the steam thermal flooding process, due to tremendous physical quantity changes, the governing equations could be strongly nonlinear, which makes the numerical solution difficult to obtain.

Traditionally, fully implicit method is used to solve this problem for its high stability. Although time steps should also be small to guarantee stable solution, which can cause extra expenses. Moreover, the large sparse matrix has a great negative impact on the calculation; therefore, the elapsed time could be very long which makes it challenge to meet the practical requirement.

Sequential method is one of the methods for mitigating computation cost by dividing the whole problem into several sub-problems based on physical concepts. The initial work on sequential method started with IMPES scheme (implicit pressure explicit saturations). However, IMPES scheme suffers from extremely small time step in multi-phases problem [1-4]. Therefore, sequential implicit method (SI) is introduced to overcome such disadvantages. Sequential implicit scheme allows solution for pressure as the first step and then implicit treatment for getting other variables [4]. It has been extensively developed in black-oil models and compositional models [5-7] in recent years. SI, comparing to IMPES, doesn’t suffer from dramatic time step limits; and can highly reduce the...
computational cost. In addition, it lays a significant foundation of applying different approaches in each step based on each step’s properties [8]. Based on such benefits, we extend SI scheme to steam-flooding heavy oil thermal recovery models containing both mass and energy conservation equations.

In this work, we proposed sequential implicit scheme conducted on steam-flooding heavy oil thermal recovery simulation. First, we have an introduction of governing equations of this model and the SI scheme on it. Then a conceptual Quasi one-dimensional problem with one injection well and one production well is presented to prove the validity against traditional fully implicit solution. Compared to FI solution, we showed that it could get almost the same solution with less computation cost for the model above.

2. Governing equations and sequential implicit method

Since heavy oil is non-volatile, we consider that there are three phase states (oil phase, liquid water phase and gaseous water phase). In the three-phase region, when liquid water and vapor steam are both presented, pressure and temperature should meet Dalton’s law of partial pressures. In this paper, different principal variables are selected to be solved for the two-phase region and the three-phase region respectively. The selection is as follows (the other two variables can be obtained by the constraint relationship): 1) The three-phase region: \( P, S_o, S_g \); 2) Oil-water two-phase region: \( P, S_o, T \); 3) Oil and gas two-phase region: \( P, S_o, T \). [3]

2.1. Governing equations

The three main governing equations of this model (water phase mass conservation equation, oil phase mass conservation equation and energy equation) are respectively expressed as follows:

Water mass conservation equation:

\[
\frac{\partial}{\partial t} \left( \phi \rho_w S_w + \phi \rho_g S_g \right) - \nabla \cdot \left( \rho_w \bar{V}_w + \rho_g \bar{V}_g \right) = q_{w,g}
\]  

(1)

Oil mass conservation equation:

\[
\frac{\partial}{\partial t} \left( \phi \rho_o S_o \right) - \nabla \cdot \left( \rho_o \bar{V}_o \right) = Q_o
\]  

(2)

Energy conservation equation:

\[
\frac{\partial}{\partial t} \left[ \phi (\rho_o U_o S_o + \rho_w U_w S_w + \rho_g U_g S_g) + (1-\phi)U_r \right] - \nabla \cdot \left( \rho_o U_o \bar{V}_o + \rho_w U_w \bar{V}_w + \rho_g U_g \bar{V}_g \right) = \nabla \cdot (\alpha \nabla T) + q_{o,w,g}^{\text{energy}}
\]  

(3)

Where, \( \phi, \lambda, \rho_o, S_o, \bar{V}_o, U_o, H_o, \alpha = o, w, h \) are rock porosity, heat conductivity coefficient, density, saturation, flow velocity, internal energy and enthalpy of gaseous water respectively. \( q_o, q_{w,g} \) Present the mass source terms of oil and water. \( q_{o,w,g}^{\text{energy}} \) Presents the energy source terms. According to Darcy’s law, flow velocity is calculated as follows:

\[
\bar{V}_a = -K \lambda_a (\nabla P - \rho_a M_a \bar{g}), \quad \alpha = o, w, h
\]  

(4)

When liquid water and vapor steam are both presented, it should meet:

\[
T = Tsap(P)
\]  

(5)

With the constraint relationship between saturations:

\[
S_o + S_w + S_g = 1
\]  

(6)

Along with these two relations in addition to three main governing equations mentioned above, this is a closed mathematical model.
2.2. Sequential implicit scheme
Different from the traditional fully implicit scheme, the sequential implicit scheme is divided the whole steam-flooding oil recovery problem into three steps to solve for each variable. Then, it is allowed to apply different approaches for different equations in each step based on their characteristics. Specifically, the procedure is shown as follows: 1) Establish pressure equations; 2) Solve oil mass conservation equations; 3) Solve energy conservation equations.

The pressure equations are a nonlinear system of equations, which can be solved by Newton-Raphson iteration. After the pressure is obtained, we update the coefficients related to pressure and total velocity. Next, oil mass conservation equations are solved implicitly for oil saturation. Last, energy conservation equations are solved for the remaining variables. It may require several iterations in the outer loop sometimes.

3. Numerical Tests
Here, we use a Quasi one-dimension example to verify the proposed method which conducted on steam-flooding thermal model is valid. The solution of using sequential implicit scheme is compared with that of fully implicit scheme. In this example, there are 128 grid cells. Permeability is 1D. There is an injection well on the left with continuously injecting 98% wet steam at 4MPa; and a production well on the right which continues to be produced at 2.5MPa. Both the injection and production wells have a radius of 0.15 m. The initial temperature is 400.15K. The initial oil phase saturation was 0.7 and the water phase saturation was 0.3. We calculate for 200 days.

Figure 1. Pressure (MPa).

Figure 2. Temperature (K).
Here, we give sequential implicit scheme about 2-3 times iteration for this example. Still, using sequential implicit scheme highly reduces computation cost. In this example, which is shown in Table 1, the total running time is reduced by more than 1 time against using fully implicit scheme.

**Table 1.** Elapsed time comparing.

|                      | Running time |
|----------------------|--------------|
| Fully implicit       | 35s          |
| Sequential implicit  | 15s          |

4. **Conclusion**
In this work, we propose to apply sequential implicit scheme to the simulation of thermal heavy oil recovery model with steam injection. The whole problem is divided into three sub-problems; and we select the appropriate method for each sub-problem according to their characteristics. In this way, we
avoid solving huge sparse matrix when using traditional fully implicit scheme and improve the computational efficiency. We use a quasi-one-dimensional conceptual model to verify the validation of the proposed method. Moreover, by comparing the running time, we verify that SI can highly reduce the computation cost with the accuracy of which basically consistent with that of FI.

Acknowledgments
This work was supported by the National Science and Technology Major Project of China (No. 2017ZX05072-005).

References
[1] Aziz, K. and Settari, A. (1979) Petroleum Reservoir Simulation. Applied Science Publishers, London, 135-139.
[2] Coats, & K., H. . (1976). Simulation of steamflooding with distillation and solution gas. Society of Petroleum Engineers Journal, 16(05), 235-247.
[3] Coats, K. H. . (1978). A highly implicit steamflood model. Society of Petroleum Engineers Journal, 18(5), 369-383.
[4] Watts, J. W. . (1986). A compositional formulation of the pressure and saturation equations. SPE Reservoir Engineering, 1(3).
[5] Møyner, Olav, & Tchelepi, H. A. . (2018). A mass-conservative sequential implicit multiscale method for isothermal equation of state compositional problems. SPE Journal, 23.
[6] Wong, Z. Y. , Kwok, F. , Horne, R. N. , & Tchelepi, H. A. . (2019). Sequential-implicit newton method for multiphysics simulation. Journal of Computational Physics.
[7] Qiao, C. , Wu, S. , Xu, J. , & Zhang, C. S. . (2017). Analytical decoupling techniques for fully implicit reservoir simulation. Journal of Computational Physics, 336, 664-681.
[8] Natvig, J. R. , & Lie, K. A. . (2008). Fast computation of multiphase flow in porous media by implicit discontinuous galerkin schemes with optimal ordering of elements. Journal of Computational Physics, 227(24), 10108-10124.