Generation of 2D Vector Magnetic Field by Mangle-Type Magnetic Field Source Using Permanent Magnets

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Generating a magnetic field with a desired magnitude and direction in a plane by using a mangle-type magnetic field source with four permanent magnets is discussed. Two rotation patterns were compared by using a 2D finite element method in terms of the controllability of the magnetic field and the uniformity of the generated magnetic field. In both patterns, the arrangement of magnets that generates the maximum magnetic field corresponds to a Halbach cylinder. It was found that one of the rotation patterns is superior for controlling the magnitude and direction of the magnetic field independently, while it is inferior in terms of uniformity. Finally, the above findings were demonstrated with a prototype, although a slight deviation was seen between the simulation and the demonstration.

Keywords: magnetic field source, permanent magnet, magnetic field vector, finite element method, Halbach cylinder

1. Introduction

A magnetic field source using permanent magnets is an energy-saving device since it does not require electric power to generate a magnetic field; consequently, cooling is also unnecessary. A mangle-type magnetic field source is typically equipped with four or six cylindrical magnets magnetized in radial directions. It generates a magnetic field with an arbitrary magnitude by rotating the magnets. It is superior to the Halbach cylinder in two ways. First, it enables lights parallel to the magnetic field to be introduced for taking measurements such as of magneto-optical effects because each magnet is separated. Second, it does not use special wedge-shaped magnets but uses common cylindrical magnets.

In our previous papers, the design of the mangle-type magnetic field source was discussed for generating a magnetic field in one direction. Such a magnetic field is

![Fig. 1 Dimensions and definitions of magnets M1–M4, \( \theta \) and \( \phi \) for expressing angle of each magnet (see Table 1), and magnitude and direction of magnetic flux density \( B_{\text{mag}} \) and \( B_{\text{direc}} \).](image)

![Fig. 2 Rotation patterns of magnets.](image)
used for taking various measurements including Hall and magneto-optical effects. The capability of controlling the direction of the magnetic field in addition to the magnitude is necessary for taking measurements such as of magnetostriction and magnetic torque. However, to the author’s knowledge, there is no literature on controlling the direction of a magnetic field with a mangle-type magnetic field source. In this paper, the control of the magnetic field direction in a plane, in other words, the generation of a 2D vector magnetic field, is discussed.

2. Simulation conditions

A mangle-type magnetic field source equipped with four permanent magnets was studied. Figure 1 shows the definitions of magnets M1–M4, \( \theta \) and \( \phi \) for expressing the angle of each magnet, and the magnitude and direction of magnetic flux density \( B_{\text{mag}} \) and \( B_{\text{line}} \). The four magnets were positioned on the corners of a square with sides 60 mm long. The diameter of the magnets was set at 20 mm. These dimensions were designed for a magnetic field source that is capable of generating a magnetic flux density above 0.1 T and into which a probe microscope can be placed. The magnetic flux density at the origin in Fig. 1 was calculated by using a 2D finite element method (Field Precision, TriComp). The mesh size was set at 1 mm, and the target of relative error was set at 10^{-6}. The magnetic flux density of the magnets was set at 1.29 T, which is a typical value for the N40-grade neodymium magnets used in the prototype. The demagnetization curve was set at the ideal straight line with a slope of −1.

3. Control of magnetic field vector

3.1 Rotation patterns of magnets

Figure 2 shows two rotation patterns of magnets based on Ref. 5, in which the angle of each magnet is expressed with two parameters \( \theta \) and \( \phi \) as listed in Table 1. The angles that generate the highest \( B_{\text{mag}} \) are, for example, \( \theta = 90^\circ \), \( \phi = 0^\circ \) (patterns 1 and 2 are identical for \( \theta \) and \( \phi \)). If \( B_{\text{mag}} \) is varied while \( B_{\text{line}} \) is kept in the horizontal direction, \( \theta \) should be varied. \( B_{\text{line}} \) is changed by simply

\[
\begin{array}{ccc}
\text{Pattern} & M1 & M2 \\
1 & \theta - \phi & -\theta - \phi \\
2 & \theta - \phi & 180^\circ + \theta - \phi \\
\end{array}
\]

\[
\begin{array}{ccc}
\text{Pattern} & M3 & M4 \\
1 & \theta - \phi & -\theta - \phi \\
2 & \theta - \phi & 180^\circ - \theta - \phi \\
\end{array}
\]

Fig. 3 (a),(b) \( B_{\text{mag}} \), (d),(e) \( B_{\text{line}} \), and (g),(h) \( B_{\text{ld}} \) maps for patterns 1 and 2. (c),(f),(i) are representative \( \phi = 45^\circ \) curves as functions of \( \theta \).
rotating all magnets in the same direction. The rotation angle is $-\phi$. The minus sign means that the change in $B_{\text{disc}}$ is opposite $\phi$.

### 3.2 Simulation results

Figure 3(a) and (b) shows $B_{\text{mag}}$ maps for patterns 1 and 2, respectively. Representative $B_{\text{mag}}$ curves ($\phi = 45^\circ$) as functions of $\theta$ are shown in Fig. 3(c). $B_{\text{mag}}$ monotonically increased with $\theta$ for both patterns. For pattern 1, however, it slightly depended on $\phi$. As shown in Fig. 3(d)–(f), $B_{\text{disc}}$ monotonically increased with $\phi$ for both patterns, except $\theta$ around zero. $B_{\text{disc}}$ diverged at $\theta = 0$ for both patterns, although it was confined to a narrow range of $\theta$ for pattern 2. The divergence of $B_{\text{disc}}$ is ascribable to two facts. First, $B_{\text{disc}}$ is not definitive for $B_{\text{mag}} = 0$. Second, the magnetic flux around the origin is complex because those from the four magnets are incompatible. Figure 3(g)–(i) shows the standard deviation of $B_{\text{mag}}$, $B_{\text{disc}}$ for a region of $10 \times 10 \text{ mm}^2$ around the origin in Fig. 1. The maximum $B_{\text{disc}}$ for pattern 2 was about three times larger than that for pattern 1. Thus, pattern 2 is advantageous for the independent control of $B_{\text{mag}}$ and $B_{\text{disc}}$ with $\theta$ and $\phi$, while it is inferior to pattern 1 in terms of the uniformity of the magnetic field. It should be noted that all magnets are rotated evenly (with $\theta$) to change $B_{\text{mag}}$ for both patterns. Such movement is not optimal for producing a uniform magnetic field, and this is a challenge for the future.

### 4. Comparison with Halbach cylinder

Figure 4(a) shows a schematic of a Halbach cylinder. The cylinder consists of a permanent magnet with magnetization rotating $360^\circ$ in a half lap. The upper and lower magnetic flux loops generate a strong and uniform magnetic field in the interior of the cylinder. It is known that the magnetic flux density generated in a Halbach cylinder is given by the following expression:\(^{10}\)

\[
B = B_i \ln \frac{r_2}{r_1}
\]

where $B_i$ is the (residual) magnetic flux of a magnet, and $r_1$ and $r_2$ are the inner and outer radii. Theoretically, $B$ approaches 10 T under the assumption of a neodymium magnet with a very large $r_2/r_1$.

A mangle-type magnetic field source with four magnets is a simplified Halbach cylinder having the magnetization components of $\alpha = 45^\circ$, $135^\circ$, $225^\circ$, $315^\circ$, on the condition that $\theta = 90^\circ$. The magnetic field is weakened by the reduced volume of the magnets. Figure 4(b) shows a Halbach cylinder rotated by angle $\phi$. Consequently, the magnetic field is directed to angle $\phi$. This corresponds with the mangle-type magnetic field source with $\theta = 90^\circ$, $\phi = \phi$ as shown in Fig. 4(c). Note that

![Fig. 4](image_url)

**Fig. 4** (a) Halbach cylinder. (b) Rotation of whole cylinder by angle $\phi$. Magnetic field is also rotated by angle $\phi$. (c) Mangle-type magnetic field source. Each magnet is rotated by angle $-\phi (\theta = 90^\circ)$

![Fig. 5](image_url)

**Fig. 5** Comparison between Halbach cylinder (12 segments, $r_1 = 40 \text{ mm}$, $r_2 = 80 \text{ mm}$) and mangle-type magnetic field source ($\theta = 90^\circ$). (a) $B_{\text{mag}}$ and (b) $B_{\text{disc}}$. 
the magnetization direction on the dotted lines in Fig. 4(b) is identical to that in Fig. 4(c). Thus, the mangle-type magnetic field source with a magnetic field at the maximum magnitude corresponds to a Halbach cylinder at any rotation angle. This applies to any mangle-type magnetic field source with six or more magnets.

This was confirmed by simulation. Figure 5 shows $B_{\text{mag}}$ and $B_{\text{direct}}$ as functions of $\phi$. The Halbach cylinder was modeled with 12 segments of magnets. For both magnetic field sources, $B_{\text{mag}}$ was perfectly constant with the change in $\phi$, and $B_{\text{direct}}$ perfectly agreed with $\phi$ despite the four-fold symmetry of the mangle-type magnetic field source.

5. Demonstration

A demonstration was performed with a prototype of the magnetic field source (Fig. 6). The permanent magnets used in the prototype were N40-grade neodymium magnets 20 mm in diameter and 50 mm in length. The separation between magnets was 60 mm (center to center) as with the above simulation. Figure 6 shows a photograph of the prototype set on a probe microscope (7). The magnets were rotated according to pattern 2. Two Hall sensors were set on the position corresponding to the origin in Fig. 1 to monitor the x and y components of the magnetic flux density.

As shown in Fig. 7(a) and (b), $B_{\text{mag}}$ continuously increased with $\theta$, and no dependence on $\phi$ was seen. This agrees with the simulation, except that the maximum $B_{\text{mag}}$ was 0.106 T, which was about 25% smaller than the simulation. The degradation was mainly due to the overestimation of the simulation, for which an infinite length of magnets was assumed; the magnetic flux diverged in the depth dimension when magnets with a length of 50 mm were used. A 3D simulation and measurement in the depth dimension may be required for quantitative agreement.

$B_{\text{direct}}$ shown in Fig. 7(c) and (d) approximately agreed with the simulation. However, a small discrepancy was found between $B_{\text{direct}}$ and $\phi$, and the divergence of $B_{\text{direct}}$ was seen in a wider range of $\theta$ than the simulation. This was most likely caused by the deviation from the intended angle of the magnets (no marks indicating N/S poles were placed on the magnets, and we checked them by measuring the surface magnetic field).

6. Summary

Controlling a magnetic field in a plane by using a mangle-type magnetic field source with four permanent magnets was discussed. The magnitude and direction of the field were independently controlled by introducing two parameters ($\theta$ and $\phi$) and one particular rotation pattern of the magnets (pattern 2). The arrangement of the magnets that generates the maximum magnetic field corresponds to a Halbach cylinder. The above findings

Fig. 6 Prototype of magnetic field source set on probe microscope.

Fig. 7 Measured (a) $B_{\text{mag}}$ and (c) $B_{\text{direct}}$ maps for prototype magnetic field source with rotation pattern 2. (b) and (d) are representative ($\phi = 45^\circ$) curves as functions of $\theta$ with simulation results.
were demonstrated with a prototype, although a slight deviation was seen.

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