Finite temperature 2-color QCD for real and imaginary chemical potential

Pietro Giudice and Alessandro Papa

Dipartimento di Fisica, Università della Calabria
& Istituto Nazionale di Fisica Nucleare, Gruppo Collegato di Cosenza, Italy

In this work we study the finite temperature SU(2) gauge theory with staggered fermions for real and imaginary chemical potential. We test the method of analytical continuation of Monte Carlo results obtained for imaginary chemical potential, by comparison with those obtained directly for real chemical potential.

1. INTRODUCTION

Understanding the QCD phase diagram in the temperature–chemical potential $(T,\mu)$ plane is important for the implications in cosmology, astrophysics and heavy-ion collisions, where quark–gluon plasma could be formed. The main purpose of the most recent studies is to localize the critical lines in the phase diagram and to determine the kind of transition across them. The most powerful tool for such investigation is the formulation of the theory on a space–time lattice. Unfortunately, at nonzero chemical potential the determinant of the fermion matrix becomes complex, thus requesting nonstandard approaches to Monte Carlo simulations.

One way to circumvent this problem is based on the idea to perform numerical simulations at imaginary chemical potential $\mu = i\mu_I$ and to analytically continue the results to real $\mu$. The advantage is that for imaginary $\mu$ the fermion determinant is again real, so simulations are feasible in QCD. Moreover, this method allows to check the results obtained by other approaches. On the other side the method is applicable as long as one remains inside a region of analyticity of the partition function in the $(\mu_I)\mu_I$ plane.

2. PHASE DIAGRAM

Roberge and Weiss (RW) have shown that the partition function of the SU($N_c$) theory is periodic in the parameter $\theta = \mu_I/T$, owing to $Z(N_c)$, and have observed that there are first-order vertical lines in the $(T,\theta)$ plane, located at $\theta = 2\pi(k+1/2)/N_c$, with integer $k$. According to RW, these transitions are placed above a certain temperature $T_E$ and since the first of them appears for $\theta = \pi/N_c$, they conclude that there is not transition at $\mu = 0$. The last conclusion is contradicted in the literature where there are different arguments according to which a chiral phase transition for $\mu = 0$ does exist. Its nature depends on the number of quark, on fermion masses, and so on.

In view of this, the worst case which can occur, for the purposes of analytical continuation (AC), is that the RW critical line at $\theta = \pi/N_c$ could be prolonged downwards and bend up to reaching the $\theta = 0$ axis in correspondence of a critical temperature $T_c$. The RW periodicity and the $\mu \to -\mu$ symmetry, resulting from CP invariance, imply that knowing the expectation value of any observable for $T$ and $\theta$ inside the strip $\{0 \leq \theta \leq \pi/N_c, 0 \leq T < \infty\}$ in the $(T,\theta)$ plane is enough to fix it on the remaining part of the plane. It is more suitable for lattice simulations to introduce the inverse coupling $\beta$ and the chemical potential in lattice units, $\mu_I \equiv a\mu$. The RW transition lines in the $(\beta,\mu_I)$ plane appear at $\hat{\mu}_I = 2\pi(k+1/2)/(N_cN_{\tau})$ and we can consider only the strip $\{0 \leq \hat{\mu}_I \leq \pi/(N_cN_{\tau}); 0 \leq \beta < \infty\}$ (see Fig. 1).

3. NUMERICAL RESULT

In this work we consider SU(2) gauge theory with staggered fermions ($n_f = 8$) because in this
case numerical simulations are feasible for both real and imaginary chemical potential, thus allowing to test the method of AC by direct comparison. We performed all simulations on a $8^3 \times 4$ lattice, setting the quark bare mass to $am = 0.07$. For different values of $\beta$ we performed simulations for both real and imaginary chemical potential, varying $\tilde{\mu}$ and $\mu_R$ between 0 and $\pi/(2N_f)$ and determined expectation values of the Polyakov loop $L$ and the chiral condensate $\langle \bar{\psi}\psi \rangle$. Then, we interpolated data obtained for imaginary $\mu$ with a truncated Taylor series of the form $a + b\tilde{\mu}_I^2 + c\tilde{\mu}_I$. After “rotating” this polynomial to real chemical potential, thus leading to $a - b\mu_R^2 + c\mu_R^2$, we compared it with the determinations obtained directly for real $\mu = \mu_R$.

We selected three values of $\beta$ (1.90, 1.45, 0.90), each representative of one of the regions $\beta > \beta_E$, $\beta < \beta < \beta_E$ and $\beta < \beta_c$.

For $\beta = 1.90$ we interpolated the data obtained for $0 \leq \tilde{\mu}_I \lesssim \pi/(2N_f)$ and found (see Fig. 2) that the rotated polynomial perfectly interpolates data obtained directly for real $\mu = \mu_R$ in the same range. From this outcome we can conclude that the AC works in the region $\beta > \beta_E$ for all the $\tilde{\mu}_I$’s before the first RW critical line.

For $\beta = 1.45$ we are in the region $\beta < \beta < \beta_E$ for which we expect that by varying $\tilde{\mu}_I$ the chiral critical line is crossed, while no transitions should be met at the same $\beta$ by varying $\mu_R$. Therefore this time we interpolated data obtained for the real chemical potential $0 \leq \tilde{\mu}_I \lesssim \pi/(2N_f)$ and rotated this polynomial to its counterpart in $\tilde{\mu}_I$.

The comparison (see Fig. 3) with data obtained directly for imaginary chemical potential shows agreement for $\tilde{\mu}_I$ below $\simeq 0.18$, which can be taken as an estimate of the chiral critical value in the $(\beta, \tilde{\mu}_I)$ plane at the given value of $\beta$.

For $\beta = 0.90 < \beta_c$, we expect analyticity for all possible values of $\tilde{\mu}_I$, while the chiral critical line in the $(\beta, \mu_R)$ plane is crossed by varying $\mu_R$ at the given value of $\beta$. Therefore we interpolated data obtained for $0 \leq \tilde{\mu}_I \lesssim \pi/(2N_f)$ and rotated this polynomial to its counterpart in $\mu_R$.

The comparison (see Fig. 4) with data obtained directly for real chemical potential shows agreement for $\mu_R$ below $\simeq 0.12$, which can be taken as an estimate of the critical value in the $(\beta, \mu_R)$ plane.
Figure 3. Polyakov loop (above) and chiral condensate (below) vs $\hat{\mu}$ for $\beta = 1.45$.

Figure 4. Polyakov loop (above) and chiral condensate (below) vs $\hat{\mu}$ for $\beta = 0.90$.

4. CONCLUSION

We have presented a possible structure of the phase diagram in the $(T, \mu_I)$ plane for 2–color QCD with $n_f = 8$ and have found that the method of AC works fine within the restrictions imposed by the presence of nonanalyticities.

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