Z' Bosons, the NuTeV Anomaly, and the Higgs Boson Mass

Michael S. Chanowitz

Theoretical Physics Group
Lawrence Berkeley National Laboratory
University of California
Berkeley, California 94720

Abstract

Fits to the precision electroweak data that include the NuTeV measurement are considered in family universal, anomaly free $U(1)$ extensions of the Standard Model. In data sets from which the hadronic asymmetries are excluded, some of the $Z'$ models can double the predicted value of the Higgs boson mass, from $\sim 60$ to $\sim 120$ GeV, removing the tension with the LEP II lower bound, while also modestly improving the $\chi^2$ confidence level. The effect of the $Z'$ models on both $m_H$ and the $\chi^2$ confidence level is increased when the NuTeV measurement is included in the fit. Both the original NuTeV data and a revised estimate by the PDG are considered.
To Lev Okun in honor of his 80'th birthday: a kind and gentle man who does physics as he lives his life, with simple honesty and integrity. Although I have told it before, the story of my first encounter with Lev bears retelling. It was at the 1976 International Conference on High Energy Physics in Tbilisi. Andre Sakharov was not allowed to register for the conference but was told that his presence would be tolerated if he wished to attend informally. While there was doubtless considerable sympathy for Sakharov among many of the Soviet physicists attending the conference, Okun was the only one I saw who dared to associate openly on the streets of Tbilisi with Sakharov during the meeting. In another characteristic decision, in the period following the fall of the Soviet Union, Lev chose to remain in Moscow to help preserve the marvelous school of theoretical physics at ITEP, when he could have accepted offers of more comfortable positions outside of Russia.

Introduction

When the precision electroweak data began to emerge from LEP, SLC, and Fermilab, Okun and collaborators Novikov, Rozanov, and Vysotsky did a complete and independent study of the one loop radiative corrections, culminating in the LEPTOP program.[1] Using LEPTOP they then made the interesting discovery, contrary to the conventional wisdom of the time, that a fourth generation of quarks and leptons is not excluded by the precision data.[2] They also found that a fourth generation would raise the Higgs mass prediction significantly, and that it could resolve the conflict between the SM (Standard Model) prediction for $m_H$ and the 114 GeV LEP II direct lower limit on $m_H$ that arises if the inconsistent determinations of the weak interaction mixing angle $\sin^2 \theta_W^{\text{eff}}$ are attributed to underestimated systematic error in the hadronic asymmetry measurements.[3] In this paper I consider a class of $Z'$ models that would also raise the $m_H$ prediction into the LEP II allowed region. In particular I extend an earlier study[4] of the EWWG[5] (Electroweak Working Group) set of observables, to also include three low energy measurements: the NuTeV νN scattering measurement,[6] Möller scattering,[7] and atomic parity violation.[8] Principally as a result of the NuTeV measurement, the $Z'$ models have a greater effect on the fits than in the previous study: the central value of $m_H$ is raised by a factor 2 into the LEP II allowed region and the $\chi^2$ confidence level is modestly improved in some cases.

The SM Fit

The SM fit to the precision electroweak data shown in table 1 has two 3σ anomalies which together reduce the $\chi^2$ confidence level of the fit to 2%.\(^1\) One is the 3.2σ discrepancy between the two most precise measurements of the weak interaction mixing angle $\sin^2 \theta_W^{\text{eff}}$, $A_{LR}$ and $A_{FB}^\nu$, and the corresponding 3.2σ discrepancy between the three hadronic asym-

\(^1\)Radiative corrections are from ZFITTER[9], with two loop corrections to $\sin^2 \theta_W^{\text{eff}}$[10] and $m_W$.[11]
\begin{table}[h]
\centering
\begin{tabular}{lcccc}
\hline
 & Experiment & A & Pull & B & Pull \\
\hline
$A_{LR}$ & 0.1513 (21) & 0.1476 & 1.8 & 0.1494 & 0.9 \\
$A_{FB}^t$ & 0.01714 (95) & 0.01634 & 0.8 & 0.1674 & 0.4 \\
$A_{e,\tau}$ & 0.1465 (32) & 0.1476 & -0.3 & 0.1494 & -0.9 \\
$A_{FB}^b$ & 0.0992 (16) & 0.1035 & -2.7 & \\
$A_{FB}^c$ & 0.0707 (35) & 0.0739 & -0.9 & \\
$m_W$ & 80.398 (25) & 80.369 & 1.2 & 80.391 & 0.3 \\
$\Gamma_Z$ & 2495.2 (23) & 2495.7 & 0.2 & 2496.1 & -0.4 \\
$R_l$ & 20.767 (25) & 20.743 & 1.0 & 20.743 & 1.0 \\
$\sigma_h$ & 41.540 (37) & 41.477 & 1.7 & 41.479 & 1.7 \\
$R_b$ & 0.21629 (66) & 0.21586 & 0.7 & 0.21584 & 0.7 \\
$R_c$ & 0.1721 (30) & 0.1722 & -0.04 & 0.1722 & -0.04 \\
$A_b$ & 0.923 (20) & 0.935 & -0.6 & 0.935 & -0.6 \\
$A_c$ & 0.670 (27) & 0.668 & 0.07 & 0.669 & 0.04 \\
$g_L^2$ & 0.30005 (137) & 0.30396 & -2.9 & 0.30423 & -3.1 \\
$g_R^2$ & 0.03076 (11) & 0.03009 & 0.6 & 0.03004 & 0.7 \\
x_W(ee)$ & 0.23339 (140) & 0.23145 & 1.4 & 0.23122 & 1.55 \\
x_W(Cs)$ & 0.22939 (190) & 0.23145 & -1.1 & 0.23122 & -1.0 \\
m_t & 172.6 (1.4) & 172.3 & 0.2 & 172.3 & 0.2 \\
$\Delta\alpha_S(m_Z)$ & 0.02758 (35) & 0.02768 & -0.3 & 0.02754 & 0.1 \\
$\alpha_S(m_Z)$ & 0.1186 & & & 0.118 & \\
$m_H$ & & 94 & & 64 & \\
CL($m_H > 114$) & & 0.33 & & 0.07 & \\
m_H(95\%) & & 172 & & 124 & \\
$\chi^2$/dof & & 28.4/15 & & 19.0/13 & \\
CL($\chi^2$) & & 0.02 & & 0.12 & \\
\hline
\end{tabular}
\caption{SM fits with (A) and without (B) $A_{FB}^b$ and $A_{FB}^c$.}
\end{table}
metry measurements ($A_{FB}^b$, $A_{FB}^c$, $Q_{FB}$) and the three leptonic ones ($A_{LR}$, $A_{FB}^\ell$, $A_{(P)}$).[5] The second is the measurement of the weak mixing angle, $\sin^2 \theta_W$, in $\nu N$ scattering by the NuTeV experiment.[6]² Either or both could be the result of statistical fluctuations, genuine new physics, or underestimated systematic uncertainty. If the cause is systematic error in the asymmetry measurements, the hadronic asymmetries are by far the more likely candidates since they have challenging experimental and QCD-related theoretical issues in common, which have no counterparts in the leptonic asymmetry measurements that are measured by three quite independent and relatively simple techniques.³ Without prejudging the validity of any of the possibilities, in this paper we explore the consequences of the assumption that the hadronic asymmetry measurements have underestimated systematic error. We then also consider the fit without the hadronic asymmetry measurements, data set B in table 1.

As observed by Davidson et al.[13] the NuTeV anomaly is also susceptible to theoretical systematic effects from QCD.⁴ In particular, a positive asymmetry in the momentum carried by strange versus anti-strange quarks in the nucleon sea would reduce the size of the anomaly. Recently the NuTeV collaboration has found a 1.4σ indication for a positive asymmetry.[14] Taken at face value it would reduce the anomaly from 3 to 2σ, as noted by the PDG[15] (Particle Data Group), which quotes revised estimates for $g_L^2$ and $g_R^2$. In this paper we consider both the original NuTeV results and the PDG estimates.

Table 1, with the original NuTeV data, displays the SM fit with (A) and without (B) the hadronic asymmetry measurements, $A_{FB}^b$ and $A_{FB}^c$. In addition to the three low energy measurements, we include the usual EWWG data set, with two unimportant exceptions that have a negligible effect on the results: the jet charge asymmetry, the least precise of the six asymmetry measurements, which we omit because of potential flavor-dependent systematic effects that could effect the $Z'$ fits, and the $W$ boson width, which with a 2% error is not a precision measurement in the sense of the other measurements that typically have part per mil precision. The NuTeV measurement is represented by $g_L^2$ and $g_R^2$, which in the SM are given by $g_L^2 = 1/2 - \sin^2 \theta_W + 5/9 \sin^4 \theta_W$ and $g_R^2 = 5/9 \sin^4 \theta_W$. In fit A of table 1 the 3.2σ conflict between $A_{LR}$ and $A_{FB}^c$ is manifested by their pulls, +1.8 and −2.7 respectively; together with the −2.9 pull of the NuTeV measurement of $g_L^2$, they are responsible for the poor 2% confidence level of the fit. In fit B with $A_{FB}^b$ and $A_{FB}^c$ excluded, the CL increases to 12% but the central value of the Higgs mass decreases to 60 GeV with only 7% CL to be in the LEP II allowed region above 114 GeV.

The best SM fit to data set A using the PDG estimate of $g_L^2$ and $g_R^2$ has $\chi^2/N = 24.2/15$

---

²Our fits use the NuTeV measurements of $g_L^2$ and $g_R^2$, which are simple functions of $\sin^2 \theta_W$. The anomaly is manifested in $g_L^2$, which is more sensitive to $\sin^2 \theta_W$.

³See [12] for a more detailed discussion.

⁴Davidson et al. also considered an unmixed $Z'$ coupled to $B - 3L_\mu$. 
implying a still marginal 6% confidence level. The Higgs mass is 85 GeV with an acceptable 31% CL for $m_H > 114$ GeV. For data set B the confidence level of the SM fit, shown in table 5, increases to a robust 35% but the Higgs mass prediction decreases to 58 GeV, with a 6% confidence level for $m_H > 114$ GeV. This fit then also provides motivation to consider new physics that can increase the $m_H$ prediction.

The $Z'$ Model

A heavy $Z'$ boson offers a natural solution to an unacceptably light prediction for $m_H$ because $Z - Z'$ mixing with a heavy $Z'$ shifts the $Z$ mass downward, corresponding to a positive contribution to the $\rho$ parameter that offsets the negative contribution from an increase in the Higgs boson mass. There is a vast literature on $Z'$ models.[16] Here we consider the highly constrained class of family-universal models that are anomaly free without addition of new fermions to the SM except the $e, \mu, \tau$ right-handed neutrinos. The Abelian generator must then act on SM matter like a linear combination of hypercharge and baryon minus lepton number,[17, 18] which we parameterize by an angle $\theta_X$ as

$$Q_X = \cos \theta_X \frac{Y'}{2} + \sin \theta_X \frac{(B - L)'}{2}. \quad (1)$$

If kinetic mixing occurs[19, 20] between the $Z$ and $Z'$ bosons we can choose a basis for the gauge fields in which it vanishes at the electroweak scale, resulting in a generator of the form of equation (1) with a shifted value of $\theta_X$ — see for instance[21]. Although $Y'$ acts like the SM hypercharge $Y$ on SM matter fields, it is a distinct $U(1)$ generator and likewise $(B - L)'$ is distinct from the $B - L$ generator of $SO(10)$. In particular, we assume the $Z'$ gets a large mass from an SM singlet Higgs field with vanishing SM hypercharge but nonvanishing $Q_X$ charge. This class of models was explored in [18]. Direct experimental bounds were extracted from LEP II data in [22] and CDF[23] has obtained even stronger bounds for couplings weaker than electroweak, $g_{Z'} \lesssim g_Z/4$. The precision electroweak constraints and Higgs boson mass predictions for these models were studied in [24] and [4] for the EWWG data set, without the NuTeV, Möller, and APV measurements.

Gauge invariance of the SM Yukawa interactions requires the SM Higgs boson to have vanishing $(B - L)'$ charge, so that $Z - Z'$ mass mixing only occurs through the $Y'$ component of $Q_X$. After diagonalizing the mass matrix assuming $m_{Z'}^2 \gg m_Z^2$ we find[4] the mass eigenstates

$$Z = \cos \theta_M Z_0 + \sin \theta_M Z_0' \quad (2)$$
$$Z' = \cos \theta_M Z_0' - \sin \theta_M Z_0. \quad (3)$$

The mixing angle is

$$\theta_M = \frac{r \cos \theta_X}{m_{Z'}^2}. \quad (4)$$
where $r$ and $\hat{m}_{Z'}$ are the $Z'$ coupling constant and mass scaled to the $Z$ coupling and mass, that is

$$r \equiv \frac{g_{Z'}}{g_Z} \tag{5}$$

and

$$\hat{m}_{Z'} \equiv \frac{m_{Z'}}{m_Z}. \tag{6}$$

The shift in the $Z$ mass generates a contribution to the oblique parameter $T_X$,[25]

$$\alpha_{T_X} = -\frac{\delta m_Z^2}{m_Z^2} = \frac{r^2 \cos^2 \theta_X}{\hat{m}_{Z'}^2}. \tag{7}$$

The effective $Zf\bar{f}$ Lagrangian can then be written as

$$\mathcal{L}_f = g_Z \left( 1 + \frac{\alpha_{T_X}}{2} \right) g'_f \bar{f} Z f \tag{8}$$

where $f$ represents a quark or lepton of chirality $L$ or $R$. Since $\hat{m}_{Z'}^2 \gg 1$ the mixing angle $\theta_M$ is very small, and to leading order in $\theta_M$ the $Zf\bar{f}$ coupling $g'_f$ is

$$g'_f = g_f + r \theta_M q_X^f \tag{9}$$

where $q_X^f$ is the $Q_X$ charge of fermion $f$,

$$q_X^f = \cos \theta_X \frac{y_f^2}{2} + \sin \theta_X \frac{b_f^f - l_f}{2} \tag{10}$$

and $y_f^f, b_f^f, l_f^f$ are respectively the weak hypercharge, baryon number, and lepton number of fermion $f$. The first term in equation (9) has the form of the SM $Zf\bar{f}$ coupling,

$$g_f = t^L_{3f} - q_f^f (\sin^2 \theta_W^{\text{eff}} + \delta_{\text{OB}} \sin^2 \theta_W) \tag{11}$$

but with the oblique correction to $\sin^2 \theta_W^{\text{eff}}$,

$$\delta_{\text{OB}} \sin^2 \theta_W = -\frac{\sin^2 \theta_W \cos^2 \theta_W}{\cos^2 \theta_W - \sin^2 \theta_W} \frac{r^2 \cos^2 \theta_X}{\hat{m}_{Z'}^2} \alpha_{T_X}. \tag{12}$$

For fixed $\theta_X$ the the effect of $Z - Z'$ mixing on the electroweak fit is determined by a single parameter which we choose to be $T_X$. The shift in the $Zf\bar{f}$ coupling from the $Z'$ admixture, the second term in equation (9), is determined by

$$\epsilon \equiv r \theta_M = \frac{\alpha_{T_X}}{\cos \theta_X}. \tag{13}$$
The $\chi^2$ fits are obtained by varying $T_X$ in addition to the four SM parameters, $m_t$, $\Delta\alpha^{(5)}_{\text{had}}(m_Z)$, $\alpha_S(m_Z)$, and $m_H$. The ratio of coupling strength to mass (the effective “Fermi constant”) is also determined by $T_X$,

$$\frac{g_{Z'}^2}{m_{Z'}^2} = \frac{\alpha T_X}{\cos^2 \theta_X} \frac{g_Z^2}{m_Z^2}$$

or in terms of the scaled coupling and mass,

$$\frac{r^2}{m_{Z'}^2} = \frac{\alpha T_X}{\cos^2 \theta_X}.$$  (15)

The LEP II bounds on $Z'$ production[22] can then also be expressed in terms of $T_X$. Translated from the notation of Carena et al.[22], who extracted the $Z'$ bounds from LEP II bounds on contact interactions, the bounds for the first quadrant in $\theta_X$ are[4]

$$\alpha T_X \leq \frac{1}{(30.1 + 15.5 \tan \theta_X)^2},$$  (16)

or in terms of the scaled ratio of mass to coupling,

$$\frac{m_{Z'}}{r} > 30.1 \cos \theta_X + 15.5 \sin \theta_X.$$  (17)

Stronger bounds have been obtained by CDF[23] for $Z'$ couplings that are weaker than electroweak as discussed in some specific cases below.

For the $Z$-pole measurements the effect of $Z'$ exchange is completely negligible but for the three low energy measurements it is of the same order as the oblique corrections and as the $Z - Z'$ mixing correction to $Z$ exchange. In addition, the effect on the NuTeV measurements includes corrections to the $Z\nu\nu$ interaction which is only implicit in the notation. For instance, the quoted measurement for the hadronic coupling $g_{L}^2 = g_{uL}^2 + g_{dL}^2$ implicitly includes a factor $4g_{\nu L}^2$ which is equal to one at leading order in the SM. The result for the nonoblique corrections, including $Z'$ exchange, is

$$\frac{\delta g_{L}^2}{g_{L}^2} = \frac{1}{9}(1 + \tan \theta_X)(\sin^2 \theta_W \tan \theta_X - 9 + 18 \sin^2 \theta_W - 10 \sin^4 \theta_W)$$  (18)

$$\frac{\delta g_{R}^2}{g_{R}^2} = \frac{\sin^2 \theta_W}{9}[(1 + \tan \theta_X)(10 + \tan \theta_X - 10 \sin^2 \theta_W) - 10 - \tan \theta_X],$$  (19)

to which we add the oblique corrections,

$$\delta^{\text{OB}} g_{L}^2 = 2\alpha T_X g_{L}^2 - (1 - \frac{10}{9} \sin^2 \theta_W) \cdot \delta^{\text{OB}} \sin^2 \theta_W$$  (20)

$$\delta^{\text{OB}} g_{R}^2 = 2\alpha T_X g_{R}^2 + \frac{10}{9} \sin^2 \theta_W \cdot \delta^{\text{OB}} \sin^2 \theta_W$$  (21)
There are extensive cancellations among the various $Z'$ corrections to both Möller scattering and atomic parity violation, with a simple final result. The sum of the $Z'$ exchange correction and the $Z - Z'$ mixing correction to $Z$ exchange cancel exactly with the oblique wave function renormalization correction proportional to $\alpha T_X/2$ from the prefactor in the effective Lagrangian equation (8). The only surviving contribution is then the oblique correction to $\sin^2\theta_W^{\text{eff}}$, equation (12).

$Z'$ Model Fits

The $Z'$ fit of data set B with the best $\chi^2$ confidence level consistent with the LEP II limits on $\hat{m}_{Z'}/r$ is obtained for $\theta_X = \pi/3$. It is compared with the SM fit in table 2 and figure 1. The $\chi^2$ decreases by 2.9 units and the confidence level increases from 12% to 19%. The improvement can be attributed entirely to the improved fit of the NuTeV $g_L^2$ measurement, for which the $\chi^2$ contribution decreases by 2.7 units, from 9.3 to 6.6. More significant is the improved consistency with the 114 GeV LEP II lower limit on the Higgs boson mass. The central value is doubled, from 64 to 126 GeV, completely resolving the marginal 7% consistency level of the SM fit with the LEP II limit. The 95% upper limit is also doubled, from 124 to 223 GeV. The central value for the ratio of $Z'$ mass to coupling constant, $\hat{m}_{Z'}/r = 29$, is just consistent with the LEP II lower limit,[22] which from the inequality (16) is $\hat{m}_{Z'}/r > 28$. For very weak coupling the CDF data[23] for $\theta_X = \pi/3$ implies a stronger bound, $\hat{m}_{Z'}/r > 41$, which excludes the fit for $g_{Z'} < g_Z/5$ — see table 5 of [4].

Figure 2 and table 3 show that fits consistent with the LEP II direct bounds on $\hat{m}_{Z'}/r$ are obtained in the entire interval $0 \leq \theta_X \leq \pi/3$ while $\pi/3 < \theta_X \leq \pi/2$ is excluded. Table 4 shows that the fits to the EW data are also acceptable for $0 \leq \theta_X \leq \pi/3$, with central values for $m_H$ above 114 GeV and with $\chi^2$ confidence levels at least as good as the SM.

The entire $Q_X$ parameter space is spanned by the first two quadrants of $\theta_X$, which are equivalent to the third and fourth quadrants up to the sign of $g_{Z'}$. In the second quadrant, $\pi/2 \leq \theta_X \leq \pi$, there are no $Z'$ fits which improve on either the SM $m_H$ prediction or the $\chi^2$ confidence level. In particular, for the “$\chi$” boson of SO(10) which couples to $T_3R-(B-L)/2$, corresponding to $\theta_X = \arctan(-2)$, zero mixing is preferred and the $\chi^2$ is the same as in the SM fit.

Figure 1 and table 4 display “Bayesian” 95% confidence limits for $m_H$ which differ from the conventionally quoted 95% CL frequentist upper limits that are also shown. Anticipating the discovery of a Higgs boson at the LHC, today’s Bayesian fit will become the conventional frequentist fit when the Higgs boson is discovered. Since we will then not scan on $m_H$ there is one additional degree of freedom. The Bayesian 95% interval is the domain of $m_H$ with confidence level $\geq 5\%$ for fits with $m_H$ fixed and indicates the allowed range in $m_H$ after the
Figure 1: $\chi^2$ distributions as a function of $m_H$ for the SM fit (dashed lines) and the $Z'$ (solid lines) fit with $\theta_X = \pi/3$ as shown in table 2. For each fit the lower horizontal line is the symmetric 90% confidence interval and the upper horizontal line is the Bayesian 95% confidence interval defined in the text. The vertical dash-dot line indicates the LEP II direct lower limit on $m_H$. 
|                  | Experiment | SM    | Pull | Z'   | Pull |
|------------------|------------|-------|------|------|------|
| $A_{LR}$         | 0.1513 (21)| 0.1494| 0.9  | 0.1511| 0.08 |
| $A_{FB}^i$       | 0.01714 (95)| 0.1674| 0.4  | 0.0712| 0.02 |
| $A_{e,\tau}$    | 0.1465 (32)| 0.1494| -0.9 | 0.1511| -1.5 |
| $m_W$            | 80.398 (25)| 80.391| 0.3  | 80.370| 1.1  |
| $\Gamma_Z$      | 2495.2 (23)| 2496.1| -0.4 | 2495.5| -0.1 |
| $R_t$            | 20.767 (25)| 20.743| 1.0  | 20.753| 0.6  |
| $\sigma_h$      | 41.540 (37)| 41.479| 1.7  | 41.491| 1.3  |
| $R_b$            | 0.21629 (66)| 0.21584| 0.7  | 0.21582| 0.7  |
| $R_c$            | 0.1721 (30)| 0.1722| -0.04| 0.1723| -0.08 |
| $A_b$            | 0.923 (20)| 0.935| -0.6 | 0.935 | -0.6  |
| $A_c$            | 0.670 (27)| 0.669| 0.04 | 0.669 | 0.03  |
| $g_L^2$          | 0.30005 (137)| 0.30423| -3.1 | 0.30357| 2.6  |
| $g_R^2$          | 0.03076 (11)| 0.03004| 0.7  | 0.03022| 0.5  |
| $x_W^{(ee)}$     | 0.23339 (140)| 0.23122| 1.55 | 0.23147| 1.4  |
| $x_W^{(Cs)}$     | 0.22939 (190)| 0.23122| -1.0 | 0.23147| -1.1 |
| $m_t$            | 172.6 (1.4)| 172.3| 0.2  | 172.3 | 0.2   |
| $\Delta \alpha_5(m_Z)$ | 0.02758 (35)| 0.02754| 0.1  | 0.2761| -0.09 |
| $\alpha_S(m_Z)$ | 0.118      | 0.120|      |      |      |

|                  | SM    | Pull | Z'   | Pull |
|------------------|-------|------|------|------|
| $m_H$            | 64    |      | 126  |      |
| CL($m_H > 114$)  | 0.07  |      | 0.60 |      |
| $m_H(95\%)$      | 124   |      | 223  |      |
| $T_X$            |       |      | 0.037|      |
| $\hat{m}_{Z'}/r$|       |      | 29   |      |
| $\chi^2$/dof    | 19.0/13| 16.1/12|  |
| CL($\chi^2$)    | 0.12  |      | 0.19 |      |

Table 2: The SM fit of data set (B) compared to the $Z'$ fit with $\theta_X = \pi/3$. 
Figure 2: $\hat{m}_{Z'}/r$ as a function of $\theta_X$ for the $Z'$ fits (solid line) compared with the LEP II 95% lower limits given by equation (17).

| Model       | $\hat{m}_{Z'}/r$ | $\hat{m}_{Z'}/r$ [LEP II] | $m_{Z'}|_{r=1}$ (TeV) |
|-------------|------------------|---------------------------|----------------------|
| $\theta_X = 0$ | 50               | $> 30$                    | 4.6                  |
| $\theta_X = \pi/12$ | 48             | $> 33$                    | 4.4                  |
| $\theta_X = \pi/6$  | 45             | $> 34$                    | 4.1                  |
| $\theta_X = \pi/4$  | 37             | $> 32$                    | 3.4                  |
| $\theta_X = \pi/3$  | 29             | $> 28$                    | 2.6                  |

Table 3: The value of $\hat{m}_{Z'}/r$ from the electroweak fit compared with the LEP II lower bound. The last column shows the $Z'$ mass if $g_{Z'} = g_Z$, i.e., if $r = 1$. 
The Higgs boson is discovered. In contrast, the usual frequentist 95% upper limit (the maximum of the symmetric 90% confidence interval) uses the best fit with \( m_H \) scanned as a free parameter to estimate the likelihood that the Higgs boson will be discovered within the given range of values. Table 4 shows that the Bayesian limits lie above the frequentist limits and that they are also increased in the \( Z' \) models relative to the SM.

The \( Z' \) fits to data set A, which includes the hadronic asymmetries \( A_{FB}^b \) and \( A_{FB}^c \), are not shown, because they are little changed with respect to the SM. For most of the \( \theta_X \) domain they are not changed at all. In the most favorable fit \( \chi^2 \) only decreases from 28.4 to 28.0, with a confidence level of 0.014, less than the 0.019 confidence level of the SM fit, and with only a modest increase in the Higgs boson mass, from 94 to 130 GeV.

As observed in [13] the results presented by NuTeV [6] for \( \sin^2 \theta_W \) and \( g_{L,R}^2 \) are sensitive to the assumption that the nucleon strange quark sea is symmetric between \( s \) and \( \bar{s} \) quarks. The NuTeV collaboration has since found a 1.4\( \sigma \) indication of an asymmetry in the momentum carried by strange quarks, \( S^- = +0.0020(14) \).[14] The collaboration has not revised their value for \( \sin^2 \theta_W \) based on this result, but they do state that an asymmetry of \(+0.007\) would make their measurement of \( \sin^2 \theta_W \) consistent with SM prediction. Using this statement and taking the new \( S^- \) measurement at face value, the PDG[15] has estimated revised values for \( \sin^2 \theta_W \), \( g_{L}^2 \), and \( g_{R}^2 \), which we use in the alternative fits shown in figure 3 and tables 5 and 6. With these estimates the disagreement with the SM is reduced to a 2\( \sigma \) effect.

As can be seen by comparing the SM fits of tables 1 and 5, the effect of the revised estimate of the NuTeV results on the global fit to data set B is to raise the confidence level from 0.12 to 0.35, while the inconsistency with the LEP II constraint on \( m_H \) remains: the central value is \( m_H = 58 \) GeV and the confidence level for \( m_H \) above 114 GeV is 6\%. Again the \( Z' \) models can resolve the inconsistency with the LEP II lower bound on \( m_H \) while maintaining the \( \chi^2 \) confidence level of the fit. In particular, for \( \theta_X = \pi/3 \), shown in figure 3 and table 5, the central value doubles, to 109 GeV, and the CL for \( m_H > 114 \) GeV increases to 45\%. All \( Z' \) models in the domain \( 0 \leq \theta_X \leq \pi/3 \) increase the \( m_H \) prediction into the LEP II allowed region, with acceptable fits to the precision data and with values of \( m_{Z'}/g_{Z'} \) consistent with the LEP II lower bounds as shown in tables 6 and 7.

For data set A with the PDG estimates of \( g_{L}^2 \) and \( g_{R}^2 \), the SM fit yields \( \chi^2/N = 24.2/15 \) with a still marginal confidence level of 6\%. The Higgs mass prediction is acceptable, at \( m_H = 85 \) GeV, with 30\% likelihood for \( m_H > 114 \) GeV. In most cases the \( Z' \) fits prefer zero mixing, and in all cases the effect on \( \chi^2 \) and \( m_H \) is negligible.

---

\footnote{The anomaly in \( \sin^2 \theta_W \) is 0.2277 - 0.2227 = 0.0050 so the shift from \( S^- = +0.0020(14) \) is 2/7 of the anomaly or \(-0.0014(10)\). Combining errors in quadrature then gives the values quoted by the PDG.}
Table 4: Best fit value of $T_X$, $\chi^2$ confidence level, and $m_H$ predictions for SM and $Z'$ fits to data set B.

| Model  | $T_X$ | $\chi^2$ | CL($\chi^2$) | $m_H$ | CL($m_H > 114$) | $m_H^{95\%}$ (Freq.) | $m_H^{95\%}$ (Bayes) |
|--------|-------|----------|---------------|-------|-----------------|------------------------|------------------------|
| SM     |       | 19.0     | 0.12          | 64    | 0.07            | 124                    | 149                    |
| $\theta_X = 0$ | 0.052 | 17.9     | 0.12          | 120   | 0.56            | 215                    | 249                    |
| $\theta_X = \pi/12$ | 0.052 | 17.4     | 0.14          | 126   | 0.58            | 224                    | 266                    |
| $\theta_X = \pi/6$  | 0.048 | 16.9     | 0.15          | 126   | 0.59            | 223                    | 261                    |
| $\theta_X = \pi/4$  | 0.046 | 16.5     | 0.17          | 126   | 0.60            | 230                    | 300                    |
| $\theta_X = \pi/3$  | 0.037 | 16.1     | 0.19          | 126   | 0.60            | 223                    | 288                    |

Figure 3: $\chi^2$ distributions as a function of $m_H$ for the SM (dashed lines) and $Z'$ (solid lines) fits shown in table 5, with PDG values for $g_L^2$ and $g_R^2$. For each fit the lower horizontal line is the symmetric 90% confidence interval and the upper horizontal line is the Bayesian 95% confidence interval defined in the text. The vertical dash-dot line indicates the LEP II direct lower limit on $m_H$. 

12
|                                | Experiment | SM            | Pull  | Z'    | Pull  |
|--------------------------------|------------|---------------|-------|-------|-------|
| $A_{LR}$                       | 0.1513 (21)| 0.1498 0.7    | 0.1510| 0.1   |
| $A_{FB}^{i}$                   | 0.01714 (95)| 0.1682 0.3   | 0.0710| 0.05  |
| $A_{e,\tau}$                  | 0.1465 (32) | 0.1498 -1.0  | 0.1510| -1.4  |
| $m_W$                          | 80.398 (25) | 80.396 0.08  | 80.376| 0.9   |
| $\Gamma_Z$                    | 2495.2 (23) | 2496.2 -0.4  | 2495.5| -0.1  |
| $R_l$                          | 20.767 (25) | 20.744 0.9   | 20.750| 0.7   |
| $\sigma_h$                    | 41.540 (37) | 41.479 1.7   | 41.491| 1.3   |
| $R_b$                          | 0.21629 (66)| 0.21584 0.7  | 0.21581| 0.7   |
| $R_c$                          | 0.1721 (30) | 0.1722 -0.05| 0.1723| -0.07 |
| $A_b$                          | 0.923 (20)  | 0.935 -0.6   | 0.935 | -0.6  |
| $A_c$                          | 0.670 (27)  | 0.669 0.04   | 0.669 | 0.03  |
| $g_L^2$                        | 0.3010 (15) | 0.3043 -2.2  | 0.3037| 1.8   |
| $g_R^2$                        | 0.03080 (11)| 0.03003 0.7  | 0.03020| 0.5   |
| $x_W(ee)$                      | 0.23339 (140)| 0.23117 1.6 | 0.23144| 1.4   |
| $x_W(Cs)$                      | 0.22939 (190)| 0.23117 -0.9| 0.23144| -1.1  |
| $m_t$                          | 172.6 (1.4) | 172.3 0.2   | 172.3 | 0.2   |
| $\Delta\alpha_5(m_Z)$         | 0.02758 (35)| 0.02754 0.1 | 0.2768| -0.3  |
| $\alpha_S(m_Z)$               | 0.118       | 0.120        |       |       |

|                                |            |               |       |       |       |
|--------------------------------|------------|---------------|-------|-------|-------|
| $m_H$                          |            | 58            | 109   |       |       |
| CL($m_H > 114$)                |            | 0.06          | 0.45  |       |       |
| $m_H(95\%)$                    |            | 118           | 201   |       |       |
| $T_X$                          |            |               |       |       | 0.033 |
| $\hat{m}_{Z'}/r$               |            |               |       |       | 31    |
| $\chi^2$/dof                  |            | 14.4/13       | 12.4/12|       |       |
| CL($\chi^2$)                  |            | 0.35          | 0.42  |       |       |

Table 5: SM and $Z'$ ($\theta_X = \pi/3$) fits to data set B with PDG values for $g_L^2$ and $g_R^2$. 
\begin{table}[h]
\centering
\begin{tabular}{|c|c|c|c|c|c|c|c|}
\hline
Model & $T_X$ & $\chi^2$ & CL($\chi^2$) & $m_H$ & CL($m_H > 114$) & $m_H^{95\%}\text{(Freq.)}$ & $m_H^{95\%}\text{(Bayes)}$ \\
\hline
SM & & & & & & & \\
$\theta = 0$ & 0.043 & 14.3 & 0.35 & 58 & 0.06 & 118 & 192 \\
$\theta = \pi/12$ & 0.044 & 13.3 & 0.35 & 104 & 0.42 & 189 & 285 \\
$\theta = \pi/6$ & 0.043 & 13.0 & 0.37 & 114 & 0.50 & 197 & 300 \\
$\theta = \pi/4$ & 0.039 & 12.7 & 0.39 & 114 & 0.50 & 203 & 312 \\
$\theta = \pi/3$ & 0.033 & 12.4 & 0.42 & 109 & 0.45 & 202 & 317 \\
\hline
\end{tabular}
\caption{Best fit value of $T_X$, $\chi^2$ confidence level, and $m_H$ predictions from SM and $Z'$ fits to data set B with the PDG estimates for $g_{L,R}^2$.}
\end{table}

\begin{table}[h]
\centering
\begin{tabular}{|c|c|c|c|c|}
\hline
Model & $\hat{m}_{Z'}/r$ & $\hat{m}_{Z'}/r[\text{LEP II}]$ & $m_{Z'}|_{r=1}$ (TeV) \\
\hline
$\theta = 0$ & 55 & > 30 & 5.0 \\
$\theta = \pi/12$ & 52 & > 33 & 4.7 \\
$\theta = \pi/6$ & 47 & > 34 & 4.3 \\
$\theta = \pi/4$ & 40 & > 32 & 3.6 \\
$\theta = \pi/3$ & 31 & > 28 & 2.8 \\
\hline
\end{tabular}
\caption{The value of $\hat{m}_{Z'}/r$ from the fit with PDG estimates for $g_{L,R}^2$ compared with the LEP II lower bound. The last column shows the $Z'$ mass if $g_{Z'} = g_Z$, i.e., if $r = 1$.}
\end{table}
Conclusion

We have extended a previous study of a simple class of $Z'$ models to include three low energy measurements in addition to the EWWG set of $Z$-pole observables. In the earlier study we found that the $Z'$ models had little effect on the $\chi^2$ or on the central value of $m_H$ but that they did significantly extend the upper limit on $m_H$ for the data set without $A^b_{FB}$ and $A^c_{FB}$, increasing $CL(m_H > 114\text{GeV})$ from 2% in the SM to as much as 30% in the $Z'$ models. With the inclusion of the low energy measurements and, in particular, the NuTeV data, the effect of the $Z'$ models is more pronounced. Both the central value and the upper limit of the Higgs boson mass are increased significantly, by as much as a factor two, and the $\chi^2$ confidence levels are modestly improved. If underestimated systematic error proves to be the explanation of the $3.2\sigma$ difference in the SM determination of $\sin^2\theta_W^{\ell\text{ eff}}$ from the leptonic and hadronic asymmetry measurements, then the $Z'$ models discussed here can resolve the resulting problematic prediction for the Higgs boson mass, providing completely satisfactory fits to the precision EW data. Though it remains to be confirmed by a careful study, we expect that the $Z'$ models favored by the fits can be excluded or confirmed at the LHC.

This work was supported in part by the Director, Office of Science, Office of High Energy and Nuclear Physics, Division of High Energy Physics, of the U.S. Department of Energy under Contract DE-AC02-05CH11231

References

[1] V. Novikov, L. Okun, Alexandre N. Rozanov, M. Vysotsky. CPPM-95-1, ITEP-19-95, Mar 1995. 38pp. e-Print: hep-ph/9503308

[2] V.A. Novikov, L.B. Okun, Alexandre N. Rozanov, Phys.Lett.B529:111-116,2002, e-Print: hep-ph/0111028; JETP Lett.76:127-130,2002, Pisma Zh.Eksp.Teor.Fiz.76:158-161,2002, e-Print: hep-ph/0203132.

[3] M. Chanowitz, Phys.Rev.Lett.87:231802,2001, hep-ph/0104024; M. Chanowitz, Phys.Rev.D66:073002,2002, hep-ph/0207123; M. Chanowitz, Presented at Mini-Workshop on Electroweak Precision Data and the Higgs Mass, Zeuthen, Germany, 28 Feb - 1 Mar 2003. Published in *Zeuthen 2003, Electroweak precision data and the Higgs mass* 15-24 e-Print: hep-ph/0304199.

[4] M. Chanowitz, arXiv:0806.0890v2 [hep-ph] (2008).
[5] The ALEPH, DELPHI, L3, OPAL, SLD Collaborations, the LEP Electroweak Working Group, the SLD Electroweak and Heavy Flavour Groups, Physics Reports: Volume 427 Nos. 5-6 (May 2006) 257-454, hep-ex/0509008. See http://lepewwg.web.cern.ch/LEPEWWG/ for the most recent data.

[6] G.P. Zeller et al. (NuTV Collaboration), Phys. Rev. Lett. 88, 091802 (2002); (190, 239902(E) (2003).

[7] P.L. Anthony et al. (SLAC E158 Collaboration), Phys. Rev. Lett. 95, 081601 (2005).

[8] S.C. Bennett and C.E. Wieman, Phys. Rev. Lett. 82, 2484 (1999).

[9] A.B. Arbuzov et al., Comput.Phys.Commun.174:728-758,2006. e-Print: hep-ph/0507146.

[10] M. Awramik, M. Czakon, A. Freitas, JHEP 0611:048,2006. e-Print: hep-ph/0608099.

[11] M. Awramik, M. Czakon, A. Freitas, G. Weiglein, Phys.Rev.D69:053006,2004. e-Print: hep-ph/0311148.

[12] Talk posted at http://theory.fnal.gov/jetp/talks/chanowitz.pdf, Feb. 2007.

[13] S. Davidson et al., JHEP 0202:037,2002. e-Print: hep-ph/0112302.

[14] D. Mason (NuTV Collaboration), Phys. Rev. Lett. 99, 192001 (2007).

[15] C. Amsler et al. (Particle Data Group), Physics Letters B667, 1 (2008).

[16] P. Langacker, e-Print: arXiv:0801.1345 [hep-ph]

[17] Michael S. Chanowitz, John R. Ellis, Mary K. Gaillard, Nucl.Phys.B128:506 (1977).

[18] Thomas Appelquist, Bogdan A. Dobrescu, Adam R. Hopper, Phys.Rev.D68:035012,2003. e-Print: hep-ph/0212073

[19] B. Holdom, Phys.Lett.B166:196 (1986).

[20] F. del Aguila, M. Masip, M. Perez-Victoria, Nucl.Phys.B456:531-549 (1995). e-Print: hep-ph/9507455

[21] P. Galison and A. Manohar, Phys.Lett.B136:279 (1984).

[22] Marcela S. Carena, Alejandro Daleo, Bogdan A. Dobrescu, Tim M.P. Tait, Phys.Rev.D70:093009,2004. e-Print: hep-ph/0408098.
[23] A. Abulencia et al. (CDF Collaboration), Phys.Rev.Lett.96:211801 (2006). e-Print: hep-ex/0602045.

[24] A. Ferroglia, A. Lorca, J.J. van der Bij, Annalen Phys.16:563-578,2007, e-Print: hep-ph/0611174.

[25] B. Holdom, Phys. Lett. B 259:329 (1991).