Don’t Give Up on Large Optimization Problems; POP Them!

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Abstract

Resource allocation problems in many computer systems can be formulated as mathematical optimization problems. However, finding exact solutions to these problems using off-the-shelf solvers in an online setting is often intractable for “hyper-scale” system sizes with tight SLAs, leading system designers to rely on cheap, heuristic algorithms. In this work, we explore an alternative approach that reuses the original optimization problem formulation. By splitting the original problem into smaller, more tractable problems for subsets of the system and then coalescing resulting sub-allocations into a global solution, we achieve empirically quasi-optimal (within $1.5\%$) performance for multiple domains with several orders-of-magnitude improvement in runtime. Deciding how to split a large problem into smaller sub-problems, and how to coalesce split allocations into a unified allocation, needs to be performed carefully in a domain-aware way. We show common principles for splitting problems effectively across a variety of tasks, including cluster scheduling, traffic engineering, and load balancing.

1 Introduction

As workloads become more computationally expensive and computer systems become larger, it is becoming common for systems to be shared among multiple users. As a result, deciding how resources should be shared amongst various entities while optimizing for many macro-objectives is increasingly important across a number of domains (e.g., cluster scheduling, traffic engineering, VM allocation, and load balancing).

Resource allocation problems can often be formulated as linear or integer-linear programs [4, 12, 17, 19, 20, 22, 25, 28]; the output of these programs is the allocation of resources (e.g., accelerators, servers, or network links) given to each entity (e.g., jobs, data shards, or traffic commodities). Using existing solvers for these mathematical programs can be computationally expensive at scale – the fastest solvers for linear programs are still superlinear (approximately $O(n^{2.5})$) [6, 18], where $n$ is the number of problem variables), and integer programs are even more expensive. Mathematical programs for resource allocation problems have millions of variables (e.g., one variable for every <entity, resource> pair) for even moderate-scale systems; this leads to long solution times (e.g., around 30 minutes to find allocation for a cluster with a thousand jobs [20]). Moreover, solutions often need to be recomputed at fine time granularities to keep up with changes in the system. Consequently, systems such as B4 and BwE [15, 17] for traffic engineering in Google’s software-defined WAN, the Accordion load balancer [26] for distributed databases, and the Gavel and Firmament job schedulers [12, 20], often cannot solve problems at the required time granularity for systems with thousands of entities and resource units.

Thus, the conventional wisdom in the systems community is that solving these programs directly often takes too long. Instead, systems researchers opt for heuristic-based methods that are cheaper to compute. It is common to see some version of the following statement in a paper:

“Since these algorithms take a long time, they are not practical for real-world deployments. Instead, they provide a baseline with which to compare faster approximation algorithms.” – Taft et al. [28]

The partition-placement algorithm in E-Store [28], the space-sharing-aware policy in Gandiva [31], and cluster management policies to allocate resources to containers in systems like Kubernetes [1], DRS [14], and OpenShift [3], all rely on heuristics. Heuristics, however, are rarely ideal; they can be hard to maintain as problem scale and dynamics change, as recent work [27] has shown, are far from optimal (Figures 3, 4 & 5), and often need to be completely redesigned for slightly modified objectives.

We believe the systems community is missing out on opportunities to solve their optimization problems more efficiently. In this work, we demonstrate the potential of...
optimizing these allocations more directly using the same optimization problem formulations. In particular, we present Partitioned Optimization Problems (POP for short), a technique where large allocation problems in computer systems are split into self-similar independent sub-problems. Each sub-problem has a subset of entities and resources (e.g., for cluster scheduling, each sub-problem has a subset of training jobs and workers), and can be solved in parallel. Allocations returned by each sub-problem can then be consolidated to obtain a global solution. Depending on the exact formulation, each sub-problem can be solved in parallel. Allocations returned by each sub-problem can then be consolidated to obtain a global solution. The goal of POP is to ensure that the reduced solution is still close to optimal: this occurs when each sub-problem returns a “similar” allocation to the original problem for the subset of entities it was assigned. This leads us to the following central question:

How should allocation problems for the full system be partitioned into sub-problems?

Allocations returned by the original problem formulations often consider interactions between all entities and resources, and, as we show in §4, some partitionings can lead to low-quality solutions. For example, in job scheduling for server clusters, a partitioning where a sub cluster is overloaded with the most resource-intensive jobs would result in a longer job completion time than if these large jobs were uniformly scattered across all sub-problems. We find that a simple principle can help avoid these problems: ensure sub-problems are distributionally similar to the original problem (§4.2).

We found that POP is promising on a wide range of important problems (cluster scheduling, traffic engineering, and load balancing), achieving runtime improvements for allocation computation of up to 405x. By splitting the global problem into independent distributionally-similar sub-problems, we can produce solutions within 1.5% of optimal and 1.9x better than heuristics at comparable runtime. We are hopeful that this paper will spark further discussion and research on how to more directly approximate solutions to large allocation problems.

2 Overview

In this section, we provide an overview of our technique.

2.1 Key Insight and Challenges

Optimization problems for large systems take a long time to solve because they have many variables. For example, consider an optimization problem that involves scheduling $n$ jobs on $m$ cloud VMs. Each VM has varying amounts of resources (e.g., CPU cores, GPUs, and RAM). To express the possibility of any job being assigned to any VM, an $n \times m$ matrix of variables would be needed; for $10^3$ jobs and $10^3$ VMs, the problem has $10^6$ variables. Contemporary solvers often take hours to solve problems of these sizes, although the exact runtime depends on problem properties such as sparsity [30].

We can achieve much faster solution times by decomposing the problem; for example, the problem of scheduling $10^3$ jobs on $10^3$ VMs ($10^6$ fewer variables) is much more tractable. However, the original problem can often have many global constraints and objectives (e.g., fairness) that require one to consider the interaction between entities and resources carefully. Concretely, this raises the main challenge of applying POP: how should the original problem be split into sub-problems? The split needs to be done in a way that significantly reduces end-to-end runtime and results in a feasible allocation of similar quality to the allocation from the full optimization problem.

2.2 Partitioned Optimization Problems

The first step of POP is to partition larger allocation problems into smaller allocation sub-problems. We can then re-use the map-reduce API [9, 32]: each of these sub-problems can be solved in parallel (map step), and then allocations from the sub-problems can be reconciled into a larger allocation for the entire problem (reduce step).

The partitioning step affects the runtime, the reconciliation complexity, and ultimately the quality of the final solution. One straightforward approach that we explore in this paper is to divide both entities (e.g., jobs, shards, flows) and resources (e.g., servers, links) into subsystems, as shown in the top half of Figure 2. The reduce step is cheap with this approach, as simply concatenating sub-system allocations together usually yields a feasible solution to the original problem. Other approaches of partitioning entities and resources among sub-problems are also possible, which we leave to future work.

POP has several desirable properties:
• **Simplicity:** Users can concentrate their effort on generating effective partitions, as opposed to designing new heuristics from scratch.

• **Generality across domains:** POP can be used to accelerate allocation computations for many domains.

• **Generality across objectives in a single domain:** POP makes it easy to scale up allocation computation for different objectives, e.g., fairness/makespan for cluster scheduling and total flow/link fairness for traffic engineering, by making small tweaks to the original problem formulations.

• **Tunability:** The number of sub-problems is a knob for trading off between solution quality and runtime.

### 2.3 Considerations for Good Partitions

It is easy to imagine how naive partitioning schemes can lead to poor-quality allocations. For example, in the job scheduling problem, if a single sub-cluster is assigned disproportionately many high-priority jobs, these jobs will compete for resources with each other and get a smaller fraction of cluster resources compared to solving a global optimization problem with a smaller total fraction of high-priority jobs. Consequently, end-to-end fairness metrics will suffer. This issue can also manifest itself in other problem domains; in query load balancing, data shards receiving higher load could be disproportionately assigned across sub-clusters.

Such issues can be detected and avoided, by ensuring that entity attributes like job type, shard load, and commodity paths (part of input) are not skewed across sub-systems. We avoid such skewed partitions by ensuring that the entity and resource distributions within each sub-problem are self-similar, i.e., they closely resemble their distributions in the original problem.

Formally, we can describe each entity and resource as a multi-dimensional vector, where each dimension represents some quality of the entity (job type, priority, number of resources requested) or resource (memory capacity, link bandwidth). We then partition entities and resources so that the mean and covariance matrix of the distribution of inputs in each sub-problem is close to the mean and covariance matrix in the entire problem. This would occur naturally with a cheap, random assignment when sub-problems are large (law of large numbers [2]), but could also be explicitly accounted for during the partitioning phase. We further discuss these issues in §4.

### 2.4 Expected Runtime Benefits

Solvers for linear programs have worst-case time complexity of \(O(f(n,m)^a)\) \((a = 2.5 [6])\) where \(f(n,m)\) is the number of variables \((n\) entities and \(m\) resources\) in the problem. If \(f(n,m) = n \cdot m\) and both entities and resources are partitioned across \(k\) sub-problems, each sub-problem will have \(k^2 \times m\) fewer variables, as illustrated in Figure 2. The runtime savings are then proportional to \(k^{2a-1}\) if each sub-problem is solved serially, and proportional to \(k^{2a}\) if each sub-problem is solved in parallel (since each sub-problem is independent), assuming a cheap **reduce** step. Some problems have an even larger potential for runtime reduction. For example, if we consider the possibility that two jobs can share a VM and the quality of an allocation depends on the specific job combination \((n^2\) possibilities), then we have \(n^2 m\) variables, and consequently a \(k^{3a}\) runtime reduction.

Mixed-integer linear programs are generally NP-Hard, so solver runtime, and consequently the runtime reduction with POP, scales exponentially. Such large reduction factors make it possible to use optimization problem formulations directly in “hyper-scale systems”.

### 3 Problem Instantiations

In this section, we describe three concrete resource allocation problems that have optimization problem formulations: job scheduling on clusters with heterogeneous resources [20], WAN traffic engineering [4], and query load balancing [7, 26, 28]. We show the full problem formulations, and then explain POP can be used to compute high-quality allocations faster.

#### 3.1 Resource Allocation for Heterogeneous Clusters

Gavel [20] is a cluster scheduler that assigns cluster resources to jobs while optimizing various multi-job objectives (e.g., fairness, makespan, cost). Gavel assumes that jobs can be time sliced onto the available heterogeneous resources, and decides what fractions of time each job should spend on each resource type by solving an optimization problem. Optimizing these objectives can be computationally expensive when scaled to 1000s of jobs, especially with “space sharing” (jobs execute concurrently on the same resource), which requires variables for every pair of runnable jobs.

Allocation problems in Gavel are expressed as optimization problems over a quantity called effective throughput: the throughput a job observes when given a resource mix according to an allocation \(X\), computed as throughput(job \(m, X\)) = \(\sum_j T_{mj} \cdot X_{mj}\) \((T_{mj}\) is the raw throughput of job \(m\) on resource type \(j\)). For example, a heterogeneity-aware fair-sharing policy [13] can be expressed as the following max-min optimization problem over all active jobs in the cluster. We assume that each job \(m\) has priority \(w_m\) and requests \(z_m\) GPUs.

\[
\text{Maximize}\sum_{m} \frac{1}{w_m} \cdot \frac{\text{throughput}(m, X)}{\text{throughput}(m, X_m^{\text{equal}})} \cdot z_m
\]

Subject to the constraints:

\[
0 \leq X_{mj} \leq 1 \quad \forall (m,j)
\]

\[
\sum_j X_{mj} \leq 1 \quad \forall m
\]

\[
\sum_m X_{mj} \cdot z_m \leq \text{num_workers}_j \quad \forall j
\]
3.2 Traffic Engineering and Link Allocation

The problem of traffic engineering for networks looks at answering how flows in a Wide Area Network (WAN) should be allocated fractions of links of different capacities to best satisfy a set of demands. One might consider several objectives, such as maximizing the total amount of satisfied flow, or minimizing the extent to which any link is loaded to reserve capacity for demand spikes.

The problem of maximizing the total flow, given a matrix of demands $D$, a pre-configured set of paths $P$, and a list of edge capacities $c_e$, can be formalized as:

$$\text{Maximize} \sum_{f \in D} f_j$$

Subject to the constraints:

$$f_j = \sum_{p \in P} f_i^p \quad \forall j \in D$$
$$f_j \leq d_j \quad \forall j \in D$$
$$\sum_{e \in E} f_i^p \leq c_e \quad \forall e \in E$$
$$f_i^p \geq 0 \quad \forall p \in P, j \in D$$

To accelerate allocation computation using POP, we can assign the entire network (all nodes and edges) to each sub-problem (but each link with a fraction of the total capacity), and distribute commodities across sub-problems. We do not partition the network itself since traffic can flow between any pair of nodes.

We tested POP on several large networks from the Topology Zoo repository [16], with similar results. Figure 4 shows the trade-off between runtime and allocated flow on the Kentucky Data Link network, which has 754 nodes and 1790 edges spanning the Eastern half of continental USA. We instantiated over $5 \times 10^5$ demands to up to 4 paths in the network. The flow allocated by POP is within 1.5% of optimal when using 64 sub-problems, yet 100× faster than the original problem. We also compare favourably to the Constrained Shortest Path First (CSPF) heuristic [10].

3.3 Query Load Balancing

Systems like Accordion [26], E-Store [28], and Kairos [7] need to determine how to place data items in a distributed store to spread load across available servers.

We consider the problem of load balancing data shards (collections of data items). This is similar to the single-tier load balancer in E-Store, but acting on collections of data items instead of individual tuples. The objective is to minimize shard movement across servers as load changes, while constraining the load on each server to be within a tolerance $\epsilon$ of average system load $L$. Each shard $i$ has load $l_i$ and requires $m_i$ memory. Each server $j$ has memory constraint $C_j$ that restricts the number of shards it can host. The initial placement of shards is given by a matrix $r$, where $r_{ij} = 1$ if partition $i$ is on server $j$. $r$ is a shard-to-server map, where $r_{ij}$ is the fraction of queries on partition $i$ served by $j$, and $r'_{ij} = 1$ if $r_{ij} > 0$, 0 otherwise. Finding the balanced shard-to-server map that minimizes data movement can then be formulated as the following mixed-integer linear program:

$$\text{Minimize} \sum_{i} \sum_{f} (1 - t_{ij}) r'_{ij} m_i$$

Subject to the constraints:

$$L - \epsilon \leq \sum_r r_{ij} l_i \leq L + \epsilon \quad \forall j$$
$$\sum_i r_{ij} = 1 \quad \forall i$$
$$\sum_i r'_{ij} m_i \leq C_j \quad \forall j$$
$$r_{ij} < r'_{ij} \leq r_{ij} + 1 \quad \forall (i, j)$$

The load balancing problem can be accelerated using POP by dividing the shard set and server cluster into shard subsets and server sub-clusters, while ensuring that each shard subset has the same total load. Figure 5 shows
the performance of POP with various numbers of sub-problems, compared to the original optimization problem, and a greedy heuristic algorithm from E-Store [28].

4 Discussion

In this section, we discuss some directions for future work that would make POP more easily deployable.

4.1 Theoretical Bounds for Optimality Gap

Our empirical results thus far show that POP in practice gives strong runtime improvements with little drop in allocation quality. Proving theoretical guarantees on the quality is difficult; one could imagine pathological examples of problems with brittle optimal allocations, where small perturbations cause large changes in quality. It may be easier to prove guarantees on more well-behaved problems with self-similar partitions.

4.2 Automating Good Splits

Our experiments thus far have shown that dividing systems into equal-capacity sub-systems and randomly assigning entities to sub-systems usually works well. However, this is not always the case. Figure 6 shows the degraded performance of a skewed split for the traffic engineering experiment described in §3.2, where all commodities originating at a particular node are assigned to the same sub-problem. Since each sub-problem is only assigned a fraction of each network link’s true capacity, the flow assigned to these commodities decreases.

One interesting problem here is to devise an automatic method for generating self-similar sub-problems. For inputs that have a continuous distribution across all dimensions, such as data shard load and memory in the query load balancing problem (§3.3), stratified sampling can be used to split the inputs into per-dimension strata. Inputs can also be clustered by key properties such as job type. Sub-problems can then be assigned inputs by sampling evenly from each strata or cluster. Automating this would make POP more straightforward to apply.

4.3 Ensuring Feasibility

In some cases, it might not be possible to return a feasible solution to an allocation problem by merely assigning each entity and a fraction of available resources to sub-problems. Skewed workloads with heavy tails are com-

Figure 5: Scatterplot showing runtimes and number of shard movements for the original formulation and its POP variants, and E-Store’s greedy algorithm [28]. POP-κ uses κ partitions.

Figure 6: Performance comparison of skewed (●) and self-similar (○) sub-problems for traffic engineering.

mon in practice [29]. As an example, consider the query load balancing problem from before; it is common for single shards to be hot: for example, Taylor Swift’s Twitter account receives much more activity (and hence request traffic) compared to the average Twitter user. In light of these hot shards, it might not be possible to assign shards to individual sub-problems and obtain self-similar sub-problem input distributions. To route around this problem, we could replicate variables for resource-hungry entities across several sub-problems (i.e., an entity would belong to multiple sub-problems), giving them access to more resources. Each of the sub-allocations assigned to an entity could be added to obtain the final allocation.

5 Related Work

The optimization community has developed various methods for scaling optimization solvers to handle “hyper-scale” problems. Fundamentally, these approaches rely strictly on identifying and then exploiting certain mathematical structures (if they exist) within the problem to extract parallelism; they make no domain-aware assumptions about the underlying problem. For example, Benders’ decomposition [11, 23] only applies to problems that exhibit a block-diagonal structure; ADMM [5, 21] has been applied to select classes of convex problems, and Dantzig-Wolfe decomposition [8], while more broadly applicable, offers no speedup guarantee. This poses a significant limitation when applying these methods to real-world systems, which often do not meet their criteria. Off-the-shelf solvers, such as Gurobi, Mosek, and SCS, do not use these techniques [24].

6 Conclusion

Using solvers to compute resource allocations can be extremely expensive for large systems. In this paper, we proposed partitioning large optimization problems consisting of entities and resources into more tractable sub-problems. Our technique, Partitioned Optimization Problems, shows promising results across a variety of tasks, including cluster scheduling, traffic engineering, and load balancing. We hope this work prompts more discussion around not resorting to heuristics in computer systems for performance reasons. Don’t give up on large optimization problems; POP them!
References

[1] Kubernetes. https://github.com/kubernetes/kubernetes.

[2] Law of Large Numbers. https://en.wikipedia.org/wiki/Law_of_large_numbers.

[3] OpenShift. https://openshift.com.

[4] F. Abuzaid, S. Kandula, B. Arzani, I. Menache, M. Zaharia, and P. Bailis. Contracting Wide-area Network Topologies to Solve Flow Problems Quickly. In 18th USENIX Symposium on Networked Systems Design and Implementation (NSDI 21), 2021.

[5] S. Boyd, N. Parikh, and E. Chu. Distributed Optimization and Statistical Learning via the Alternating Direction Method of Multipliers. Foundations and Trends in Machine Learning, pages 1–122, 2011.

[6] G. B. Dantzig and P. Wolfe. Decomposition Principle for Linear Programs. Operations Research, 8(1):101–111, 1960.

[7] J. Dean and S. Ghemawat. MapReduce: Simplified Data Processing on Large Clusters. Communications of the ACM, 51(1):107–113, 2008.

[8] B. Fortz, J. Rexford, and M. Thorup. Traffic Engineering with Traditional IP Routing Protocols. IEEE Communications Magazine, 40(10):118–124, 2002.

[9] A. M. Geoffrion. Generalized Bender’s Decomposition. Journal of optimization theory and applications, 10(4):237–260, 1972.

[10] I. Gog, M. Schwarzkopf, A. Gleave, R. N. Watson, and S. Hand. Firmament: Fast, Centralized Cluster Scheduling at Scale. In 12th USENIX Symposium on Operating Systems Design and Implementation (OSDI 16), pages 99–115, 2016.

[11] J. Gu, M. Chowdhury, K. G. Shin, Y. Zhu, M. Jeon, J. Qian, H. Liu, and C. Guo. Tiresias: A GPU Cluster Manager for Distributed Deep Learning. In 16th USENIX Symposium on Networked Systems Design and Implementation (NSDI 19), pages 485–500, 2019.

[12] A. Gulati, A. Holler, M. Ji, G. Shanmuganathan, C. Waldspurger, and X. Zhu. VMware Distributed Resource Management: Design, Implementation, and Lessons Learned. VMware Technical Journal, 1(1):45–64, 2012.

[13] C.-Y. Hong, S. Mandal, M. Al-Fares, M. Zhu, R. Alimi, C. Bhagat, S. Jain, J. Kaimal, S. Liang, K. Mendelev, et al. B4 and After: Managing Hierarchy, Partitioning, and Asymmetry for Availability and Scale in Google’s Software-Defined WAN. In Proceedings of the 2018 Conference of the ACM Special Interest Group on Data Communication, pages 74–87, 2018.

[14] S. Knight, H. X. Nguyen, N. Falkner, R. Bowden, and M. Roughan. The internet topology zoo. IEEE Journal on Selected Areas in Communications, 29(9):1765–1775, 2011.

[15] A. Kumar, S. Jain, U. Naik, A. Raghuraman, N. Kasinadhuni, E. C. Zermeno, C. S. Gunn, J. Ai, B. Carlin, M. Amarandei-Stavila, et al. BwE: Flexible, Hierarchical Bandwidth Allocation for WAN Distributed Computing. In Proceedings of the 2015 ACM Conference on Special Interest Group on Data Communication, pages 1–14, 2015.

[16] Y. T. Lee and A. Sidford. Efficient inverse maintenance and faster algorithms for linear programming. In 2015 IEEE 56th Annual Symposium on Foundations of Computer Science, pages 230–249. IEEE, 2015.

[17] X. Li and K. L. Yeung. Traffic Engineering in Segment Routing Networks Using MILP. IEEE Transactions on Network and Service Management, 17(3):1941–1953, 2020.

[18] D. Narayan, K. Sankaranam, F. Kazhamiaka, A. Phanishayee, and M. Zaharia. Heterogeneity-Aware Cluster Scheduling Policies for Deep Learning Workloads. In 14th USENIX Symposium on Operating Systems Design and Implementation (OSDI 20), pages 481–498, 2020.

[19] B. O’Donoghue, G. Stathopoulos, and S. Boyd. A Splitting Method for Optimal Control. IEEE Transactions on Control Systems Technology, 21(6):2432–2442, 2013.

[20] Q. Pu, G. Ananthanarayanan, P. Bodik, S. Kandula, A. Akella, P. Bahl, and I. Stoica. Low Latency Geo-Distributed Data Analytics. ACM SIGCOMM Computer Communication Review, 45(4):421–434, 2015.

[21] R. Rahmanian, T. G. Crainic, M. Gendreau, and W. Rei. The Benders Decomposition Algorithm: A Literature Review. European Journal of Operational Research, 259(3):801–817, 2017.

[22] J. Rios and K. Ross. Massively Parallel Dantzig-Wolfe Decomposition Applied to Traffic Flow Scheduling. Journal of Aerospace Computing, Information, and Communication, 7(1):32–45, 2010.

[23] S. Rizvi, X. Li, B. Wong, F. Kazhamiaka, and B. Cassell. Mayflower: Improving Distributed Filesystem Performance through SDN / Filesystem Co-Design. In 2016 IEEE 36th International Conference on Distributed Computing Systems (ICDCS), pages 384–394. IEEE, 2016.

[24] M. Serafini, E. Mansour, A. Aboulnaga, K. Salem, T. Rafiq, and U. F. Minhas. Accordion: Elastic Scalability for Database Systems Supporting Distributed Transactions. Proceedings of the VLDB Endowment, 7(12):1035–1046, 2014.

[25] L. Suresh, J. Loff, F. Kalim, S. Jyothi, N. Narodytska, L. Ryzhyk, S. Gamage, B. Oki, P. Jain, and M. Gasch. Building Scalable and Flexible Cluster Managers Using Declarative Programming. In 14th USENIX Symposium on Operating Systems Design and Implementation (OSDI 20), pages 827–844, 2020.
Fine-Grained Elastic Partitioning for Distributed Transaction Processing Systems. *Proceedings of the VLDB Endowment*, 8(3):245–256, 2014.

[29] M. Tirmazi, A. Barker, N. Deng, M. E. Haque, Z. G. Qin, S. Hand, M. Harchol-Balter, and J. Wilkes. Borg: The Next Generation. In *Proceedings of the Fifteenth European Conference on Computer Systems*, pages 1–14, 2020.

[30] R. J. Vanderbei et al. *Linear Programming*, volume 3. Springer, 2015.

[31] W. Xiao, R. Bhardwaj, R. Ramjee, M. Sivathanu, N. Kwatra, Z. Han, P. Patel, X. Peng, H. Zhao, Q. Zhang, et al. Gandiva: Introspective Cluster Scheduling for Deep Learning. In *13th USENIX Symposium on Operating Systems Design and Implementation (OSDI 18)*, pages 595–610, 2018.

[32] M. Zaharia, M. Chowdhury, T. Das, A. Dave, J. Ma, M. McCauly, M. J. Franklin, S. Shenker, and I. Stoica. Resilient Distributed Datasets: A Fault-Tolerant Abstraction for In-Memory Cluster Computing. In *9th USENIX Symposium on Networked Systems Design and Implementation (NSDI 12)*, pages 15–28, 2012.