Algorithm synthesis for tertiary information processing of distance measuring channel in information and measurement system of air traffic control

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Abstract. The issue of improving the efficiency of the information and measurement system of automated air traffic control systems on the example of the distance measuring channel algorithm synthesis for following up the aircraft is considered. The choice of state and observation models is justified, computer simulation is performed, and the results of the studied algorithm are obtained.

1. Introduction

Tracking accuracy and interference immunity are the most important components of information and measurement systems (IMS) for air traffic control (ATC), support of proper level of aircraft flight safety, and increase in the system throughput at a given factor of safety are impossible without improving the accuracy characteristics of all IMS ATC subsystems. The problem of low accuracy in determining the distance to the maneuvering aircraft of the IMS ATC using secondary radar information processing stems from the fact that aircraft movements at the aerodrome does not meet the existing classical models of motion. According to the results of research, in such maneuvers of aircrafts as takeoff, banking, turn-in, landing, etc., there are significant errors in the assessment of aircraft coordinates that do not meet modern requirements for the system throughput of flight control towers while executing the specified factor of aircraft safety [1, 2, 3, 4].

Modern IMS ATC systems for determining the distance to an aircraft under secondary information processing based on α-β-γ filtering algorithms are employed [5]. However, the use of secondary information processing with existing algorithms does not reveal a true air situation due to the only one radar station used in the system.

The preferred solution to this problem is to synthesize the distance measuring channel algorithm of the IMS ATC using tertiary information processing with dynamic weight coefficients.

2. System models selection and rationale

The algorithms of α-β and α-β-γ filtering in secondary information processing do not always correspond to reality, especially when performing maneuvers, since in practice the speed of the aircraft mainly changes according to a nonlinear law (variable acceleration), and the relative movement of the aircraft is not straight-line, which results in significant errors in measuring (evaluating) the range. When developing IMS ATC based on tertiary information processing in radar stations, it is possible to
perform secondary information processing using the Singer state model. The main feature of the Singer model is that in the equation of state, acceleration has components of maneuver and perturbation noise, which allows describing all possible trajectories of aircrafts [6, 7]. The Singer model of the distance measuring channel has the following form:

the state model:

\[ D(k) = D(k - 1) + v(k - 1)T + 0.5a(k - 1)T^2; \]  

\[ v(k) = v(k - 1) + a(k - 1)T; \]  

\[ a(k) = (1 - \alpha_F T)a(k - 1) + \xi_a(k - 1); \]  

the observation model:

\[ D_1(k) = D(k) + \xi_D(k). \]  

In expressions (1) – (4): \( D(k) \) is the distance to the aircraft; \( V(k) \) is the aircraft speed; \( a(k) \) is the aircraft acceleration; \( \alpha_F \) is the aircraft maneuver coefficient; \( \xi_a \) is the Gaussian noise with variance \( \sigma_a^2 \); \( T \) is the sampling interval; \( k \) is the number of time discrete; \( \xi_D(k) \) is the discrete centered Gaussian noise of distance measurements with a known variance \( \sigma_D^2 \).

Data on the aircraft (targets) represent a group of marks that are received from multiple radar stations. Ideally, the marks in radar stations from aircrafts should coincide on the monitor, but in practice this does not happen, since there are systematic and random errors to form a generalized aircraft coordinate. The averaging method is used to form a generalized aircraft mark.

The simplest method of averaging can be used to obtain the arithmetic mean of coordinates. This method is simple, but does not take into account the errors of radar stations, while the accuracy of coordinates averaging is low. To improve the accuracy of coordinates averaging, methods that take into account weight coefficients mapping the accuracy of the information source are used. Methods of applying weight coefficients are divided into static and dynamic ones. Static weight coefficients that characterize the efficiency and accuracy of posts and centers for receiving radar information are set at the initial time and do not change in processing radar information. This method is more accurate than the method of finding the arithmetic mean, but does not take into account changes of the system in processing radar information. The solution to this problem is in adapting the method of dynamic weight coefficients that map the effectiveness of target detection.

Information about the object range with the residual coefficients from several radar stations, is sent to the air traffic control center, where the weight of the data is estimated based on the residual coefficients. If there are two radar stations in the tertiary information processing system, the range measurement equation is written as:

\[ D_{TIP} = \frac{b_1 D_1 + b_2 D_2}{b_1 + b_2}; \]  

\[ b_1 = \frac{\sum_{i=1}^{2} \frac{1}{2} \exp \left\{ -\frac{\Delta D_1^2}{2D_1} \right\}}{\sum_{j=1}^{2} \frac{1}{2} \exp \left\{ -\frac{\Delta D_1^2}{2D_1} \right\}}; \]  

\[ b_2 = \frac{\sum_{j=1}^{2} \frac{1}{2} \exp \left\{ -\frac{\Delta D_1^2}{2D_1} \right\}}{\sum_{j=1}^{2} \frac{1}{2} \exp \left\{ -\frac{\Delta D_1^2}{2D_1} \right\}}. \]
\[
\begin{align*}
\mathbf{b}_2 &= \frac{1}{2} \sum_{i=1}^{2} t_2^2 (2\pi D_1) \exp \left\{ -\frac{\Delta D_2^2}{2D_2^2} \right\} \\
\mathbf{b}_1 &= \frac{1}{2} \sum_{j=1}^{2} t_2^2 (2\pi D_1) \exp \left\{ -\frac{\Delta D_2^2}{2D_2^2} \right\} 
\end{align*}
\]

In equations (5) – (7) \(b_1\) and \(b_2\) are the weight coefficients of radar stations; \(\Delta D_1\) and \(\Delta D_2\) are the range residual coefficients of the distance measuring channel for radar stations; \(D_1\) and \(D_2\) are the residual variance.

3. Algorithm for the range measurement device operation

To develop the algorithm of the distance measuring channel operation for estimating phase coordinates, the Kalman algorithm is used, which takes the form [1, 8, 9, 10]:

\[
x_{es}(k+1) = x_{es}(k+1) + K_F(k+1)[z_u(k+1) - H(k+1)x_{es}(k+1)]
\]

\[
x_{e}(k+1) = F(k+1,k)x_{es}(k), x_{e}(0) = x_{es}(0);
\]

\[
P_{e}(k+1) = F(k+1,k)P_{es}(k)F^T(k+1,k) + Q_x(k);
\]

\[
K_F(k+1) = P_e(k+1) + H^T(k+1) \left[ H(k+1)P_e(k+1)H^T(k+1) + R(k+1) \right]^{-1} ;
\]

\[
P_{es}(k+1) = P_e(k+1) - K_F(k+1)H(k+1)P_e(k+1)
\]

In equations (8) – (12): \(x_{e}(k+1)\) – the extrapolation (forecast) of the state vector \(x(k+1)\); \(K_F(k+1)\) is the optimal filter transfer gain coefficients; \(F(k+1,k)\) is the transition matrix; \(P_e(k+1)\) is the a posteriori covariance matrix of error filtering; \(P_{es}(k+1)\) is the a priori covariance matrix of error filtering; \(R(k+1)\) is the measurement variance matrix; \(Q_x(k)\) is the perturbation variance matrix; \(H(k+1)\) is the measurement matrix that relates state variables \(x(k+1)\)and measurements (observations) \(z_u(k+1)\).

Electronic tracking system operation algorithms to estimate the phase coordinates (coordinates and parameters) of aircrafts depend on the state and observation models. Taking into account the state models (1) – (3) and observation models (4), and using the Kalman filtering algorithm (8) – (12), an algorithm for distance measuring channel operation is obtained for estimating range, speed, and acceleration:

\[
D_{es}(k+1) = D_e(k+1) + K_F3\Delta D(k+1);
\]

\[
V_{es}(k+1) = V_e(k+1) + K_F3\Delta D(k+1);
\]

\[
A_{es}(k+1) = A_e(k+1) + K_F3\Delta D(k+1);
\]

\[
D_e(k+1) = D_{es}(k) + V_{es}(k)\tau + A_{es}(k)\tau^2 / 2;
\]

\[
V_e(k+1) = V_{es}(k) + A_{es}(k)\tau;
\]

\[
A_e(k+1) = (1 - \alpha)A_{es}(k);
\]
\Delta D(k + 1) = D_i(k + 1) - D_e(k + 1).

(19)

4. Study of real accuracy characteristics of the distance measuring channel

Actual accuracy characteristics of the distance measuring channel were computer-simulated. For this purpose, the aircraft maneuver "Big box" was used, where the aircraft range was measured relative to the radar [1]. Computer simulation modeling is a simulation of input signals $D_i$ and $\Delta D$ and the processing of these signals using algorithms (13) - (19).

The simulation of the input signal $D_i$ represents a changing true value of the range to the aircraft from the radar station $1 - D_1$ (radar station $2 - D_2$) coming from each of the radar stations (Figure 1 and Figure 2) and measurement noise. Figures 1, 2 show that the aircraft movement relative to each of the spaced diverse radar stations is nonlinear due to maneuvering. This complicates their assessment using only secondary information processing.

Information about the range is sent to the air traffic control center of the IMS ATC, where tertiary information processing takes place.

![Figure 1](image1.png)

**Figure 1.** Dependence of the aircraft range on time relative to radar station 1.

![Figure 2](image2.png)

**Figure 2.** Dependence of the aircraft range on time relative to radar station 2.

The estimation of the real accuracy of the tertiary information processing system using Kalman filtering is performed by estimating the mean square error of the target phase coordinates by the
formula [2, 9, 10]

$$\sigma_{x_p}(k) = \sqrt{\frac{\sum_{j=1}^{N} (x(k) - x_{oj}(k))^2}{N - 1}}.$$ \hspace{1cm} (20)

In the expression (20): $\sigma_{x_p}(k)$ is the quadratic error of estimation for the phase coordinates of the target aircraft; $x(k)$ are the true values of the phase coordinates of the aircraft; $x_{oj}(k)$ is the estimation of the aircraft phase coordinates of the $j$-th realization; $N$ is the number of realizations.

Simulation modeling was performed for the range value of each of the radar stations separately and for the range value after the process of tertiary information processing. The modeling results of the mean square quadratic range error for radar stations 1 and 2 are shown in Figures 3 and 4. The mean quadratic range error of the tertiary information processing is shown in Figure 5.

The dependencies shown in Figures 3 –5 confirm that the increase in the mean square error values is due to the nonlinear changes in the aircraft trajectory at times of extremely sharp changes in the aircraft movement trajectory relative to spaced diverse radar stations.

Computer simulation substantiates noticeable improvement in the system accuracy characteristics when using tertiary information processing with dynamic weight coefficients in determining the range.
5. Evaluating the effectiveness of the synthesized algorithm

One of the main problems of the IMS ATC is security control of air traffic. This problem is solved through strict enforcement of flight rules by the air staff as well as reliable and high-quality operation of ATC and navigation facilities.

Based on the requirements for flight operations, the probability of flight security compromise should be no more than 10\(^{-6}\).

The aircraft safety area depends on the measuring accuracy of range coordinates, azimuth, and course.

For identical values of the root-mean-square error of angular coordinates, the security area is a sphere with a volume calculated by the formula:

$$V_{\text{ar}} = \frac{4}{3} \pi (6\sigma_\chi)^3.$$  \hspace{1cm} (21)

From the expression (22) \(\sigma_\chi\) is the mean square error of coordinates.

Estimation of operation efficiency for the synthesized algorithm is made by finding the dependence of security areas intersection probability on the number of aircrafts.

The security areas intersection probability for \(n\) number of aircrafts is determined by the formula:

$$P_{\text{ai}} = \frac{(n-1)V_{\text{ar}}}{V_{\text{ZS}} - (n-1)V_{\text{ar}}} P_{\text{ne}};$$ \hspace{1cm} (22)

$$P_{\text{ne}} = \frac{V_{\text{ar}} - V_{\text{ac}}}{V_{\text{ar}}};$$ \hspace{1cm} (23)

$$P_{\text{nj}} = \frac{V_{\text{ac}}}{V_{\text{ZS}} - V_{\text{ac}}}.$$ \hspace{1cm} (24)

In the expressions (22) – (24): \(P_{\text{nj}}\) is the probability of the \(i\)-th aircraft entering the security area of the \(j\)-th aircraft; \(P_{\text{ne}}\) is the probability of the aircraft not entering the security area; \(V_{\text{ac}}\) is the volume occupied by the aircraft.

Figure 6 shows the dependences of security areas intersection probability on the number of aircrafts for the synthesized algorithm and, the tracking algorithm based on \(\alpha-\beta-\gamma\) filtering for comparison.
Figure 6. Dependence of the probability of security areas intersection on the number of aircrafts for the developed algorithm and for the algorithm based on α-β-γ filtering.

The use of the synthesized algorithm of the distance measuring channel of tertiary information processing reduces the probability of security areas intersection by 3...5 times compared with tracking algorithms based on α-β-γ filtering.

6. Simulation results

Based on the results of computer simulation, it can be concluded that the accuracy characteristics of the IMS ATC are improved when using the synthesized algorithm of the distance measuring channel for tertiary information processing using dynamic weight coefficients.

Also, the results of the efficiency assessment and finding the dependence of security areas intersection probability on the number of aircrafts and comparison with the algorithm based on α-β-γ filtering allow us to infer that the IMS ATC is more effective using the developed algorithm.

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