Neutron star inner crust: effects of rotation and magnetic fields

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We study the role of the pasta phases on the properties of rotating and magnetized neutron stars. In order to investigate such systems, we make use of two different relativistic mean-field unified inner-crust-core equations of state, with a different density dependence of the symmetry energy, and an inner-crust computed within a Thomas-Fermi calculation. Special attention is given to the crust-core transition density, and the pasta phases effects on the global properties of stars. The effects of strong magnetic fields and fast rotation are computed by solving the Einstein-Maxwell equations self-consistently, taking into account anisotropies induced by the centrifugal and the Lorentz force. The location of the magnetic field neutral line and the maximum of the Lorentz force on the equatorial plane are calculated. The conditions under which they fall inside the inner crust region are discussed. We verified that models with a larger symmetry energy slope show more sensitivity to the variation of the magnetic field. One of the maxima of the Lorentz force, as well as the neutral line, and for a certain range of frequencies, fall inside the inner crust region. This may have consequences in the fracture of the crust, and may help explain phenomena associated with star quakes.

I. INTRODUCTION

Neutron stars (NS) are not only extremely dense objects, but they are known to be associated with strong magnetic fields, and fast rotation as well. At present, it is commonly accepted that the huge range of densities inside NS can be naturally divided into several regions. Typically, the neutron star structure can be divided into an outer crust, an inner crust and a core. The outer crust region of neutron stars has an equation of state relatively well-known [1–3]. The same is not true for the inner crust. This region of the star begins when neutrons start dripping out of the nuclei at densities of about $\rho_{\text{drip}} \sim 4.3 \times 10^{11} \text{ g/cm}^3$. As a result, the inner crust is formed by very neutron-rich nuclei, immersed in a gas of neutrons and electrons. Heavy clusters, the pasta phases, form due to the competition between the nuclear and Coulomb forces [4–7]. This may affect the cooling of the proton neutron star.

Pulsars rotate extremely fast, which is related to their formation [8]. As the star core collapses, its rotation rate increases as a result of conservation of angular momentum, hence, pulsars rotate up to several hundred times per second. In the case of millisecond pulsars, they are thought to achieve such high speeds because they are gravitationally bound in a binary system with another star. During part of their life, matter flows from the companion star to the pulsar. Over time, the impact of the accreted matter spins up the pulsar’s rotation.

In addition, classes of neutron stars known as magnetars have strong surface magnetic fields that span the range $\sim 10^{12–15} \text{ G}$. Such fields are usually estimated from observations of the star’s period, and period derivative. One expects to find even stronger magnetic fields inside these stars. According to the virial theorem, which gives an upper estimate for the magnetic field inside neutron stars, they can possess stronger central magnetic fields, of the order of $\sim 10^{18} \text{ G}$ [9, 10].

The main objective of the present work is to understand how the distribution of the poloidal magnetic field lines affect the inner crust of a neutron star. We will concentrate our attention on the inner crust-core transition, and will not investigate the outer crust and transition to the inner crust of a strongly magnetized star. Several studies have already shown the important effects of the magnetic field on the outer crust and neutron drip line [11–14]. In Ref. [15], the authors have shown that including magnetic field effects in the EoS did not affect much the magnetized neutron star structure, therefore, in the following, we consider a non-magnetized EoS.

In order to describe the neutron star interior, the complete stellar matter EoS will be constructed by taking a standard EoS for the outer crust [1], with an adequate inner crust EoS that matches the outer crust EoS at the neutron drip line, and the core EoS at the crust-core transition density [6]. Between the neutron drip density and the crust-core transition density, we employ an inner crust EoS, that we have determined from a Thomas-Fermi calculation for the NL3 family [6], with the inclusion of the $\omega \rho$ meson coupling terms. There, the authors addressed the effect of the nonlinear $\omega \rho$ coupling terms on the crust-core transition density and pressure, and on the macroscopic properties of hadronic stars. We will also consider that the magnetic field affects the extension of the inner crust, as proposed in [16–19]. The complete EoS will be used as input to determine the star properties, such as the mass and radius, from the integration of the Einstein-Maxwell equations, in order to obtain both rotating and magnetized stellar models [20–23].

Pasta phases impact not only the structure of NS, but also may affect their rotation behavior, and the magnetic field distribution. The effects of rotation and
strong magnetic fields in the inner crust region, where the pasta phases appear, are going to be analysed, for two model with different slopes of the symmetry energy. For this purpose, we are going to use the Lorene C++ library for numerical relativity\(^1\) to self-consistently study the effects of strong magnetic fields and rotation on neutron stars. We will solve numerically the coupled Maxwell-Einstein equations by means of a pseudospectral method, taking into consideration the anisotropy of the energy-momentum tensor due to the magnetic field, and also the effects of the centrifugal force induced by rotation.

If the NS has a poloidal field with closed lines inside, instabilities will appear in the neighborhood of the neutral line characterized by a zero magnetic field [24, 25], and a mixed poloidal-toroidal configuration will stabilize the NS [25–29]. However, the relative magnitude of each field component depends on the boundary conditions imposed on the magnetic field [26, 27], and these will certainly depend on the properties of matter at the NS surface. In this paper, we want to address this issue, and the local-

\[ A_\mu = \frac{\partial \phi}{\partial x^\mu} = \frac{\partial \ln N}{\partial x_i} + \frac{F^\mu_\nu j_\nu}{E + P} = 0 , \tag{3} \]

with the spatial coordinates \(x_i = (r, \theta)\). The first term in Eq. (3) corresponds to the purely matter contribution, the second represents the gravitational potential, the third accounts for the centrifugal effects due to rotation, and the last one is the Lorentz force \((F^\mu_\nu J_\nu)\) induced by magnetic fields, which, in our case, are generated by the four-electric current \(j_\nu\). Since \(A_\mu = (A_r, 0, 0, A_\phi)\), then \(J_\nu = (j_r, 0, 0, j_\phi)\), which comes from the assumption of circularity condition. In other words, there are no meridional currents.

Eq. (3) is the relativistic version of the Euler equation. One can show, by taking the rotational, that the Lorentz term in Eq. (3) can be written as

\[ \frac{\partial M}{\partial x_i} = \frac{F^\mu_\nu j_\nu}{E + P} = \left( \frac{j^\phi}{E + P} - \Omega j_r \right) \frac{\partial A_\phi}{\partial x_i} . \tag{4} \]

Note that Eq. (4) represents also the integrability condition of Eq. (3). The term in parenthesis in Eq. (4) can be a constant, or a function of the magnetic vector potential, \(g(A_\phi)\). The arbitrary function \(M\) can then be chosen such that:

\[ \frac{\partial M}{\partial A_\phi} = g(A_\phi) . \tag{5} \]

In other words,

\[ M = M(A_\phi(r, \theta)) = \int_{u_0}^{A_\phi} g(u) du . \tag{6} \]

The function \(g(u)\) is called the current function, and \(M\) is the magnetic potential. Here, the magnetic star models are obtained by assuming a constant value for the dimensionless current function, also referred to as current function amplitude (CFA), and denoted by \(k_0\). In Ref. [21], other choices for \(g(u)\) were considered, other than constants functions, but the general conclusions remain the same.

\(^1\) www.lorene.obspm.fr
For higher values of the current function, the magnetic field in the star increases proportionally. In addition, \( k_0 \) is related to the macroscopic electric current via:

\[
j^\phi = \Omega j^j + (E + P) k_0 \ ,
\]

which is obtained relating Eq. (5) with Eq. (4). Here, \( E \) is the energy density and \( P \) is the pressure.

Finally, the integral form of the equation of motion for a fluid in the presence of magnetic fields, Eq. (3), reads:

\[
H(r, \theta) + \ln N(r, \theta) - \ln \Gamma(r, \theta) + M(r, \theta) = \text{const.} \ ,
\]

where \( M \) is the magnetic potential, see Eq. (6), and \( H \) is the dimensionless log-enthalpy (also called pseudo-enthalpy or heat function) defined as

\[
H(P) = \int_0^P \frac{dP'}{E(P') + P} \ ,
\]

which can be cast in terms of the specific enthalpy \( h \)

\[
h(P) = \frac{E(P) + P}{m_B n_B} \ ,
\]

as

\[
H(P) := \ln h(P) = \ln \left( \frac{\mu}{m_B} \right) \ ,
\]

where \( m_B = 939 \text{ MeV} \) is the baryonic mass, and \( \mu \) the baryonic chemical potential.

### III. RESULTS

In the following, we present the main results of our study. We consider the effect of the magnetic field on the NS crust for a non-rotating star in Sec. III A, and, for a rotating star, in Sec. III B.

#### A. Magnetised neutron stars

As already discussed in Ref. [17], the presence of strong magnetic fields originates a region, at the boundary between the inner crust and the core, where homogeneous and non-homogeneous matter (matter with the presence of clusters) coexist – the extended crust – identified by the densities \( \rho_1 \) and \( \rho_2 \) (cf. Fig. 1). We shall denote the radii that correspond to each of these densities as \( R_1 \) and \( R_2 \), respectively. In this notation, the thickness of the extended crust is defined as \( \Delta R_t = R_1 - R_2 \), whilst the total size of the crust is given by the difference \( \Delta R_c = R - R_2 \) (with \( R \) being the coordinate radius of the star). The difference \( \Delta R_1 = R - R_1 \) corresponds to the size of the crust without the extended region.

For the region between the surface and the boundary defined by \( R_1 \) and the density \( \rho_1 \), which coincides with the crust-core transition of a non-magnetized star, we take the EoS of non-magnetized matter. In [18], it has been shown that the magnetic field does not affect much the value of \( R_1 \), and the results of [32] concerning the inner crust seem to indicate that the structure of the pasta phases inside \( R_1 \) are not influenced by the magnetic field, if the intensity of the field satisfies \( B < 10^{18} \text{ G} \), as expected in the crust region. The authors of [32] did not consider the possibility that at densities above \( \rho_1 \) new non-homogeneous regions would exist, as calculated in [18], using a dynamical spinodal approach. For the region bounded by \( \rho_1 \) and \( \rho_2 \) in Fig. 1, we will take results of Refs. [17,18] to define the location of the non-homogeneous regions, since presently no other results are available that identify these regions.

We consider two models which only differ in the isovector properties: NL3\( \omega \rho \) with the symmetry energy slope \( L = 55 \) and 88 MeV at saturation [6]. These values lie at the average and top limit obtained in [33] for the symmetry energy slope at saturation from constraints for nuclear properties and neutron star observations, \( L = 58.7 \pm 28.1 \text{MeV} \).

In order to study the effects of the magnetic field on the star crust, we first analyze how the three quantities, \( \Delta R_1, \Delta R_2 \) and \( \Delta R_c \), vary with the radial component of the magnetic field measured at the surface (poles), \( B_s \). These results are presented in Figure 2 for stars with baryon masses 1.2, 1.4 and 1.8 \( M_\odot \). On the top panel, we show how the size of the crust is affected by the presence of the magnetic field. We note that for \( B_s = 0 \), the results for the two models considered do not differ much from each other in comparison with the case where \( B_s \neq 0 \). However, a difference does exist, as discussed in [34], where it was shown that the larger the slope \( L \), the smaller the transition density to the core. This, in turn, may reflect itself on the thickness of the crust: in [35], it was found that a thinner crust corresponds to a larger \( L \), when comparing NL3 \( L = 118 \text{ MeV} \) with NL3\( \omega \rho \) with
even though both models predict the same properties for symmetric nuclear matter, they will respond differently with the inclusion of the magnetic field due to their different symmetry energy properties. As a result, the model with \( L = 88 \text{ MeV} \) shows a much bigger sensitivity to the increase of the magnetic field. The reason lies in the fact that, for densities below saturation density, the fraction of protons is smaller for larger values of \( L \), and, therefore, more sensitive to a given value of the magnetic field. It is also clear that the smaller the star mass, the larger the effect of the magnetic field.

In the middle panel of Figure 2, it is shown how the size of the crust without the extended zone varies with the magnetic field. Here, the overall trend is a reduction on the size of the region, as the magnetic field increases. Again we note that the model with the larger \( L \) is more affected by the increase of the magnetic field, a conclusion that can also be reached by looking at the bottom panel of the same figure, where we present the behavior of the extended crust alone. It is important to note that this behavior is not monotonic, which is a consequence of the discrete feature of the Landau levels introduced by the magnetic field [17].

Concerning Fig. 2, some comments are in order: a) there is a large increase of the crust size when the magnetic field increases from 0 to \( 4.4 \times 10^{15} \text{ G} \), but the size of the crust is practically the same for \( 4.4 \times 10^{15} < B < 4.4 \times 10^{16} \text{ G} \); b) the effect of the magnetic field is much stronger if the model has a large symmetry energy slope; c) stars with smaller masses are more strongly affected.

In Figure 3, we have used the results obtained with the stronger magnetic field intensity \( B_s = 4.41 \times 10^{16} \text{ G} \) at the surface’s pole) to show how the width of the crust varies along the polar angle \( \theta \). We have normalized the curves with the values obtained with \( B = 0 \); for both \( R_1 \) and \( R_2 \), we divided the values obtained with \( B_s = 4.41 \times 10^{16} \text{ G} \) by the corresponding values (i.e. same mass and same \( L \)) obtained at \( B = 0 \). Since the values \( \rho_1 \) and \( \rho_2 \) do not have any spatial dependency, the results that we observe here are only consequence of the overall deformation of the star induced by the magnetic field. In fact, the way the crust is deformed is quite similar to the deformation of the radius (i.e., coordinate radius) of the star itself, as shown on the bottom panel of the same figure, where the radius \( R \) of the star is plotted versus the polar angle. Nonetheless, it becomes clear from Figure 3 that the effect of the magnetic field is much stronger in the \( L = 88 \text{ MeV} \) model: the difference between the equatorial and polar radii is larger; and the extended crust extends much more into the interior of the star. As discussed before, the magnetic field has a stronger effect on the width of the crust of the less massive star: for \( 1.2 M_\odot \) star, the ratio between the equatorial radius and the polar radius, while for the 1.4 and 1.8 \( M_\odot \) stars, this difference is \( \approx 2 - 3\% \). It is also interesting to notice that the reduction of the radius at the pole is stronger than its increase at the equator. This is also true for the thickness of the crust. The middle

\[ \Delta R \]

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**FIG. 2.** (Color online) Effect of the magnetic field on the total size of the crust \( \Delta R_2 \) (top), on the crust without its extension, \( \Delta R_1 \), (middle), and on the extended region, \( \Delta R_t \) (bottom). Full lines correspond to the model with \( L = 55 \text{ MeV} \), whilst dashed lines are for the \( L = 88 \text{ MeV} \) model. The colours red, blue and green correspond to baryon masses \( 1.2M_\odot, 1.5M_\odot \) and \( 1.8M_\odot \), respectively.

\( L = 55 \text{ MeV} \).

On the other hand, a much greater difference is verified for \( B_s \neq 0 \). This is because the value \( \rho_2 \), which defines the crust size, depends on the proton fraction value considered. It was shown in [36] that the proton fraction at the crust-core transition is determined by the slope of symmetry energy, the smaller the \( L \) the larger the proton fraction. A similar conclusion was drawn in [37] for the average proton fraction at the inner crust. Therefore,
FIG. 3. (Color online) Normalised $R_1$, $R_2$ and $R$ as a function the polar angle, $\theta$. Full lines correspond to the model with $L = 55$ MeV, whilst dashed lines are for the $L = 88$ MeV model. The colours red, blue and green correspond to baryon masses $1.2M_\odot$, $1.5M_\odot$ and $1.8M_\odot$, respectively. In each panel, the results obtained with $B_s = 4.4 \times 10^{16}$ G are divided by the corresponding value at $B_s = 0$ (notice that $R_2(B = 0) = R_1(B = 0)$). See text for more details.

FIG. 4. (Color online) Baryon density as function of the radial coordinate. The top panel corresponds to $L = 55$ MeV whilst the bottom one corresponds to $L = 88$ MeV. The colours red, blue and green correspond to baryon masses $1.2M_\odot$, $1.5M_\odot$ and $1.8M_\odot$, respectively. The vertical bands correspond to the transition zone at the crust-core transition.

This is also evident in Figure 4, where we plot the profile of each star that we have considered and, in each of them, we identify the extended zone, the region delimited by $R_2$ and $R_1$. By doing this, we observe that the extended crust, which is itself a consequence of the inclusion of the magnetic field, is much bigger for the model with the larger $L$.

We next analyze how the magnetic field potential varies inside the star, and we discuss the localization of the points where its gradient, proportional to the Lorentz force, is extreme and zero.

On the top panel of Fig. 5, we plot the radial component of the gradient of the magnetic potential measured along the plane $\theta = 0$ as a function of the radial coordinate. This quantity gives us the shape of the radial component of the Lorentz force inside the star, since $F_L \sim \nabla M(r, \theta)$. At the equator, the gradient of the magnetic potential function is zero at the neutral line of the poloidal magnetic field [25]. For polar angles close to the equator, the Lorentz force verifies a sign change inside the star, as discussed in [38], because the lines of field are closed. It was shown in [25], where the authors have...
studied instabilities in NS with poloidal magnetic fields, that the most unstable perturbations develop around the neutral line.

We wanted to ascertain whether the neutral line coincides with the extended crust region, which should be taken into account when one considers strong magnetic fields. For the models considered, we verified that that does not occur, and, in fact, we obtained the neutral line at \( r = R_n \), with \( R_n/R \sim 0.8 \), as predicted in [25]. It has, however, been shown that stability in a magnetized star is attained with both a poloidal and a toroidal component, with the last one embedded inside the region defined by the poloidal closed lines [26, 27, 29]. We have included in Table I the position of the neutral line \( R_n \), and the extension of the crust \( R_2 \) and \( R_1 \), as well as the NS radius \( R \), for stars with masses 1.2, 1.4 and 1.8 \( M_\odot \), described by models with \( L = 55 \) and \( L = 88 \) MeV, and the surface magnetic field \( B_s = 4.41 \times 10^{15} \text{G} \). In the next section, we will discuss the effect of rotation on the neutral line.

Besides the neutral line, the Lorentz force has two local extrema inside the NS, one located at the core and the other one in the crust. We may assume that a maximum of the Lorentz force inside the non-homogeneous region of the star may cause more easily matter to fracture or break. On the middle and bottom panels of Fig. 5, we identify the location of the pasta phases in the crust region. In the case of the \( L = 88 \) MeV model, no inner crust configurations besides droplets exist. However, for the \( L = 55 \) MeV, the maximum of the Lorentz force occurs near the region of rod-like configurations, which may be easier to deform. The localization of the transition between pasta phases in our work is only indicative, since they have been obtained in a calculation that considered the possible formation of only five different configurations. In a calculation that allows the appearance of any kind of geometry as in [39–41], the extreme of the Lorentz force would most probably fall in a region of non-spherical pasta phases.

![FIG. 5. (Color online) Gradient of the magnetic potential as function of the radial coordinate for a \( M_b = 1.5M_\odot \) star with \( B_s = 4.4 \times 10^{15} \text{G} \). Full lines correspond to the model with \( L = 55 \) MeV, whilst dashed lines are for the \( L = 88 \) MeV model. The two bottom panels show in more detail the region inside the crust, and the transition between the different pasta phases is signaled.](image-url)

| \( L \) (MeV) | \( M_b \) (M\( \odot \)) | \( R_1 \) (km) | \( R_2 \) (km) | \( R \) (km) | \( R_n \) (km) |
|-----------|-----------|-------------|-------------|-----------|-------------|
| 55        | 1.2       | 9.987       | 9.752       | 11.79     | 8.660       |
|           | 1.5       | 10.14       | 9.944       | 11.59     | 8.640       |
|           | 1.8       | 10.15       | 9.991       | 11.36     | 8.522       |
| 88        | 1.2       | 10.60       | 9.067       | 12.25     | 8.946       |
|           | 1.5       | 10.64       | 9.402       | 11.96     | 8.820       |
|           | 1.8       | 10.59       | 9.545       | 11.65     | 8.632       |

\( B_s = 5B^* \)

| \( L \) (MeV) | \( M_b \) (M\( \odot \)) | \( R_1 \) (km) | \( R_2 \) (km) | \( R \) (km) | \( R_n \) (km) |
|-----------|-----------|-------------|-------------|-----------|-------------|
| 55        | 1.2       | 10.06       | 9.729       | 11.83     | 8.691       |
|           | 1.5       | 10.17       | 9.907       | 11.62     | 8.658       |
|           | 1.8       | 10.19       | 9.955       | 11.38     | 8.536       |
| 88        | 1.2       | 10.72       | 9.131       | 12.30     | 8.982       |
|           | 1.5       | 10.74       | 9.444       | 11.99     | 8.842       |
|           | 1.8       | 10.65       | 9.573       | 11.67     | 8.646       |

\( B_s = 10B^* \)

| \( L \) (MeV) | \( M_b \) (M\( \odot \)) | \( R_1 \) (km) | \( R_2 \) (km) | \( R \) (km) | \( R_n \) (km) |
|-----------|-----------|-------------|-------------|-----------|-------------|
| 55        | 1.2       | 10.17       | 9.873       | 11.94     | 8.775       |
|           | 1.5       | 10.25       | 10.01       | 11.69     | 8.711       |
|           | 1.8       | 10.24       | 10.02       | 11.43     | 8.576       |
| 88        | 1.2       | 10.98       | 9.267       | 12.44     | 9.084       |
|           | 1.5       | 10.91       | 9.528       | 12.07     | 8.905       |
|           | 1.8       | 10.77       | 9.625       | 11.73     | 8.688       |
B. Magnetised and rotating neutron stars

The effects of rotation on the geometry of neutron stars are already well documented [21], the major result being the flatness of the star on the polar regions, an effect similar to that of the polar magnetic fields discussed in the previous section. In [25], the authors showed that rotation stabilizes the instabilities developed in neutron stars with a poloidal magnetic field due to perturbations. Here we analyze how the profile of the Lorentz force in the equatorial plane is affected, relatively to the crust, when we take into account the effects of rotation. In particular, we will determine the frequency above which the neutral line does not exist.

It is important to notice that even though we fix the magnitude of the magnetic field on the star by choosing the current function amplitude (CFA), which is equivalent to fixing the magnetic dipole moment, the magnitude of the magnetic field measured at the pole surface $B_z$ is going to vary as we increase the frequency. This stems from the fact that the angular velocity of the fluid and the magnetic field are related by the fluid’s conservation equation (8). The behavior of $B_z$ with the rotation frequency is shown in Fig. 6, where the radial component of the magnetic field at the pole surface is plotted for a fixed CFA value (the one that gives, for each star, a field magnitude of $2.2 \times 10^{16}$ G when there is no rotation), using the two models of the present study, and considering stars with masses 1.2, 1.4 and 1.8 $M_\odot$. We conclude that larger magnetic field intensities are attained for the smaller mass stars, and with a larger symmetry energy slope. This happens because the proton fraction is bigger for the model with the larger $L$.

As already discussed in [23] and shown in the last section, the magnetic potential function, $M$, may present a concave shape and, thus, a local minimum. Since the Lorentz force is proportional to the gradient of $M$, this minimum, at the equator, corresponds to a line of points where the Lorentz force changes sign, and defines the neutral line. This means that there is a region in which the magnetic field acts towards the center of the star, and another one where the Lorentz force pushes outwards. A change of the direction of the Lorentz force, if occurring in a fragile region as the crust, could be associated to the breaking of the stellar crust and leading to flares. As discussed in the previous section, in [25] it was argued that in the neighborhood of the neutral line large instabilities could develop in a star with a poloidal field.

Taking into account the effects of rotation, for each model and mass, there is a frequency (hereafter referred to as critical frequency, and designated by $f_{\text{crit}}$) at which the Lorentz force sign changes. This effect is shown in Figure 7, where we present the radial component of $\nabla M$, measured along the equatorial plane with $\theta = \pi$. It is seen that for a frequency $f \gtrsim f_{\text{crit}}$, the Lorentz force is always pointing outwards. The larger the frequency, the stronger the Lorentz force.

In Table II we present the values of the critical frequency, $f_{\text{crit}}$, for the two models considered, and for stars with $M_b = 1.2M_\odot$, $M_b = 1.5M_\odot$, and $M_b = 1.8M_\odot$. We note that this so called critical frequency does not depend a lot on the model considered, but only on the baryonic mass of the star, and the magnitude of the magnetic field. The critical frequencies obtained are all above 90 Hz. As shown, for instance, in Ref. [42], pulsars with strong magnetic fields have periods of the order of 1 or larger. This means that the poloidal field inside these pulsars will always have a neutral magnetic line and closed lines.

In Table III we show how the neutral line position is altered by the inclusion of rotation for the models and masses previously considered. Similarly to what happens to the full coordinate radius of the star, the distance of the neutral line to the star centre increases with the fre-
TABLE II. Frequency, $f_{\text{crit}}$, at which the neutral line disappears for the two models considered, and stars with different baryonic masses. The surface magnetic field is set to $B_s \approx 2.2 \times 10^{16}$ G.

| L (MeV) | $M_b$ (M$_\odot$) | $f_{\text{crit}}$ (Hz) |
|---------|-------------------|----------------------|
| 55      | 1.2               | 127                  |
|         | 1.5               | 109                  |
|         | 1.8               | 96                   |
| 88      | 1.2               | 125                  |
|         | 1.5               | 108                  |
|         | 1.8               | 96                   |

TABLE III. The position of the inner crust boundaries $R_1$ and $R_2$, the NS radius $R$, and the neutral line $R_n$ measured along the equatorial plane, for different rotation frequencies, and stars with masses 1.2, 1.4 and 1.8 M$_\odot$, and described by models with $L = 55$ and $L = 88$ MeV. The surface magnetic field at the pole is $B_s \approx 2.2 \times 10^{16}$ G.

| L (MeV) | $M_b$ (M$_\odot$) | $R_1$ (km) | $R_2$ (km) | $R$ (km) | $R_n$ (km) |
|---------|-------------------|------------|------------|---------|------------|
| 55      | 1.2               | 10.06      | 9.729      | 11.83   | 8.691      |
|         | 1.5               | 10.17      | 9.907      | 11.62   | 8.658      |
|         | 1.8               | 10.19      | 9.955      | 11.38   | 8.536      |
| 88      | 1.2               | 10.72      | 9.131      | 12.30   | 8.981      |
|         | 1.5               | 10.74      | 9.444      | 11.99   | 8.842      |
|         | 1.8               | 10.65      | 9.573      | 11.67   | 8.646      |

As already mentioned in the previous section, stars endowed with a poloidal magnetic field may show instabilities around the neutral line, and it is believed that rotation might cease those instabilities [25]. Unlike the non-rotating case, Figure 8 shows that the neutral line can indeed fall inside the crust region, when the extended crust is taken into account. In the bottom panel of the same figure, we show, for the model with $L = 55$ MeV, how the neutral line is affected by the frequency increase for different masses. We conclude that lower mass stars are much more sensitive to the effects of the frequency. The results are analogous for $L = 88$ MeV, however, the neutral line enters the extended crust for smaller frequencies.

IV. CONCLUSIONS

In this paper, we analyse how strong magnetic fields and rotation affect the inner crust of a NS. The inner crust is complemented with an extended crust which, as reported in [16, 17], should be taken into consideration when strong magnetic fields are present. Part of our goal was to understand how models of the same family, but with different symmetry energy slope, $L$, compare when
subject to extreme magnetic fields and rotation. Our results show that the larger the slope of the symmetry energy $L$, the bigger the sensitivity of the model regarding variations of the magnetic field, which is consistent with the fact that below saturation density, the fraction of protons is smaller for larger values of $L$, and above it is larger. This is particularly evident on the difference in the size of the extended crust. The magnetic field may affect the different types of layers that exist in the crust. We verified that the Lorentz force has two local maxima, and one of them is localized in the region populated by pasta phases, including those that might be more susceptible to brake due to stresses.

Studies on the evolution of magnetic fields in neutron stars have reported the existence of a line inside the star, the neutral line, where the magnetic field is zero. These same studies indicate the existence of instabilities around this line [25], if a pure poloidal field is considered. If a mixed magnetic field configuration is assumed, the Lorentz force has two local maxima, and one of them is localized in the region populated by pasta phases, including those that might be more susceptible to brake due to stresses.

We wanted to ascertain whether this line falls inside the inner crust, when one takes into account the extended crust. This was not verified for non-rotating stars, but the situation changes when one includes rotation. Given the richness of phenomena that occur at the region of the neutral line, it is expected that they will depend on the properties of matter that is present in this region. It would be interesting to understand the role that pasta phases might have in connection to known astrophysical phenomena associated with magnetars.

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