Taking into Consideration the Work of Monolithic Reinforced Concrete Floor with Shaped Steel Profiles in Two Directions

S I Roshchina¹, A G Gonshakov¹, N G Gonshakov¹

¹The Department of Building Constructions, Institute of Architecture of Construction and Energy, Vladimir State University named after Alexander Grigorievich and Nikolai Grigorievich Stoletovs, Gorky Street 87, Vladimir, 600000, Russian Federation

E-mail: GAG37@yandex.ru

Abstract. The publication is devoted to the study of the work of monolithic reinforced concrete floor with shaped steel profiles in two directions. Theoretical and experimental studies on models of the plate were made. To evaluate the stress-strain state of the slab reinforced with profiled flooring during operation, under the action of a uniformly distributed load, the theory of structurally orthotropic slabs supported along the contour was used. A technique has been developed for determining the bearing capacity of a slab based on the method of limiting equilibrium. The field of application of the developed methodology for calculating square and quasiquadratic monolithic plates with steel profiled flooring is substantiated. Putting into practice the construction of these floors, designed taking into account their work in two directions, can reduce the consumption of steel (steel beams) up to 50%, concrete up to 8% and labor at the construction site up to 22% per 100 m² of the covered area compared to the calculation of the proposed structures according to the beam scheme. These structures are recommended for use in coatings and ceilings during the reconstruction of buildings, as well as in new construction.

1. Introduction

The use of monolithic reinforced concrete floor structures with profiled steel flooring is appropriate for high loads at $q = 30 \ldots 50$ kN/m² compared to monolithic floors. Fundamental in this direction are the works of A.A. Gvozdeva, A.R. Rzhanitsyna, A.M. Dubinsky, V.I. Murashova, S.M. Krylova, S.G. Lehnitsky, M.I. Dodonova, N.D. Hormiza, K.B. Baktygulova, A.P. Vasiliev, V.V. Zhukova, R.I. Rabinovich and others.

In 1987 under the leadership of A.P. Vasiliev (NIIZhB) and R.I. Rabinovich (Central Research Institute of Industrial Buildings) a number of studies were carried out, which results are presented in the recommendations for the design of monolithic reinforced concrete floors with steel profiled floor (SPN) [1]. Profiled steel floor used as external reinforcement had reliable adhesion to concrete, which was ensured by stamped reefs during rolling, forming dowels on its faces, and special anchor devices. Vertical bar anchors made of reinforcing steel, which are welded during installation through the flooring sheet to the upper shelf of the steel girder, are recommended as anchor devices. It is also recommended in the floors to install additional flexible reinforcement in the form of individual rods, frames and nets.
The calculation was carried out for two stages: construction and maintenance. At the construction stage, the calculation of the SPN as a supporting structure was carried out according to the scheme of a two-span beam with a length of each span 3 m. During maintenance, a four-span continuous beam with a length of each span 3 m was adopted as the design scheme. Calculation of two groups of limiting states confirmed the provision of strength and deformability of the floor to static load. However, this source presents the work of the plate in only one direction, namely, along the corrugations of the profiled flooring, but did not take into account the nature of the work in two directions in the form of square or quasi-square in terms of plates, supported along the contour.

In 1989 at MGSU [2, 3] M.I. Dodonov and N.D. Hormiz [18, 19] tested a monolithic concrete slab measuring 410x280 cm, articulated on the long sides and free of supports on the short, on the action of concentrated loads applied near the center at four points. To evaluate the strength of a slab operating in two directions, the kinematic method of limiting equilibrium was adopted [4, 5, 6], proposed by A.A. Gvozdev and A.R. Rzhanitsyn and described in the works of M. Porter, E. Ekberg [7]. The bearing capacity of the slab was determined from the condition of equality of work of external and internal forces.

The performed review - analysis of studies in the field of developing methods for calculating monolithic reinforced concrete floors with SPN allows us to conclude that, despite the proposed approaches, at present, calculation methods for determining the operation of these structures in two directions for the action of a uniformly distributed load do not exist.

2. Topicality

The issues of saving building materials and labor costs are currently important. Especially if it is associated with the use of reserves of existing structures, while improving not the structure itself, which entails additional costs, but only the calculation method. Such structures are monolithic reinforced concrete floors with profiled steel flooring. Putting into practice the construction of these floors, designed taking into account their work in two directions, allows to reduce the consumption of steel (steel beams) up to 50%, concrete up to 8% and labor at the construction site up to 22% per 100 m² of the covered area in comparison with the calculation of the proposed structures according to the beam pattern (along the corrugations). Therefore, the task of determining the overlap in two directions is relevant.

The aim of the work is to study the work of monolithic reinforced concrete flooring in profiled steel flooring in two directions based on experimental and theoretical studies and to develop a methodology for calculating such structures.

In accordance with the purpose of the work, the following tasks were considered:
- development of a methodology for calculating the strength and deformability of monolithic reinforced concrete floors with profiled steel flooring as slabs supported along the contour for the action of a uniformly distributed load;
- an experimental study of the operation of the structure under load to failure and the establishment of a fracture diagram of the plate, as well as the identification of the distribution of deformations and displacements over various sections;
- comparison and assessment of experimental and theoretical data;
- development of recommendations for the calculation of these plates.

The scientific novelty of the work is:
- the methodology and results of calculating the strength and deformability, taking into account the features of structures made of monolithic reinforced concrete with steel profiled flooring;
- the results of experimental studies of the bearing capacity, crack resistance and deformability of a plate with corrugations;
- the results of comparing the calculation of the slab according to the beam scheme and taking into account the work of the slab in two directions.
3. Research methodology

Theoretical studies were carried out in two directions: determination of the stress-strain state at the operational stage for calculating the slab according to the second group of limiting states and assessment of the bearing capacity at the fracture stage.

**Determination of the stress-strain state of the plate in the elastic stage on the action of a uniformly distributed load.** The methodology for calculating monolithic flooring with profiled steel flooring is based on the theory of structurally orthotropic thin rigid plates [8, 9, 10, 11, 12, 13, 14, 15, 16, 17]. Therefore, to determine the work of the plate in two directions, the double row method was adopted in the interpretation of S. G. Lehnitsky [14]. The calculation begins with determining the geometric characteristics for the three types of sections into which the plate was divided:

- $A_1$, $S_1$, $I_1$ – for a beam-strip of constant cross section with a width of 1 m at a height of $h_1 = 0.0258$m, taking into account the profiled floor construction;
- $A_2$, $S_2$, $I_2$ – for a beam-strip of T-section with a width of 1 m at a height of $h_2 = 0.0458$m, taking into account the profiled flooring;
- $A_3$, $S_3$, $I_3$ – for a beam-strip of constant cross section with a width of 1 m at a height of $h_3 = 0.0458$m, taking into account the profiled floor construction;

where $A_j$ – area of the beam-strip;

- $S$ – the static moment of the beam-strip relative to the y axis;
- $I$ – moment of inertia of the beam-strip relative to the y axis.

Then the strip was conditionally divided into high and low sections in the direction perpendicular to the corrugation. After that, a fictitious deflection is determined for a beam-strip having a variable section along the length, according to the formula:

$$f_{fic} = \sum_{j=1}^{m} \frac{M_{qj} \cdot M_{pj}}{E_p \cdot I_j} \cdot dx + \sum_{j=1}^{n} \frac{M_{p} \cdot M_{qj}}{E_h \cdot I_j} \cdot dx,$$

(1)

where $c_1$ and $c_2$ – lengths of sections with the height of the rib of the brand $h_2$ and the height of the shelf $h_1$;

- $M_{qj}$ – moment in the $j$-th section of the beam from the action of a uniformly distributed load $q$;
- $M_{pj}$ – moment in the $j$-th section of the beam from the action of a single concentrated force $P=1$, applied in the middle of the span.

After conversion: $M_{xu} = (l - x) \cdot q \cdot 2 / 3$;

$$f_{fic} = \frac{\alpha_j}{6E_b I} \left[ \frac{2M_{pj} + M_{p(j-1)}}{M_{qj} + 2M_{p(j-1)}} \cdot M_{qj} + \frac{M_{pj} + M_{p(j-1)}}{M_{qj} + 2M_{p(j-1)}} \cdot M_{qj(j-1)} \right],$$

(2)

where $l$ – plate length; $x$ – distance from the left support to the beam-strip section under consideration; $\alpha_j$ – size of high and low sections; $E_b$ – initial modulus of elasticity of concrete; $u$ – amount of sections; $j$ – number of section.

Substituting the calculated data in the formula (1), the fictitious deflection for a beam of variable cross section is determined.

The fictitious moment of inertia $I_{fic}$ and the corresponding fictitious thickness $h_{fic}$ were determined from the equality of the deflections of a strip of variable cross section, selected in the direction perpendicular to the corrugations and some fictitious strip having a constant cross section with the moment of inertia of $I_{fic}$. The beam has a T-section in the transverse direction, and a constant thickness in the longitudinal direction $h_{fic} = \sqrt{\frac{I_{f}^2}{f^2}}$.

Hence, the fictitious moment of inertia is determined from the expression:

$$I_{f} = (l_2 \cdot f_2) / f_1,$$

(3)

where $f_2 = \frac{5qh^3}{384E_p l_2}$ – deflection of a beam-strip of a T-section with a width of 1 m.

For a plate ($a=b$), pivotally supported along a contour and loaded with a distributed load $q$, bending moments $M_x$ and $M_y$ in two directions and deflection $f_{max}$ is determined by the formulas:
\[ M_x = D_1 \left[ 16qg^2 \pi^4 \left( \sum_{m} \sum_{n} \left( m^2 + n^2 \right) S_{mx} S_{ny} \right) \right], \quad (4) \]

\[ M_y = D_2 \left[ 16qg^2 \pi^4 \left( \sum_{m} \sum_{n} \left( m^2 + n^2 \right) S_{mx} S_{ny} \right) \right], \quad (5) \]

\[ f_{\text{act}} = \frac{16qg^4}{\pi^6 \sum_{m} \sum_{n} \left( S_{mx} S_{ny} \right)}, \quad (6) \]

\[ B_{mn} = D_1 m^4 l^4 + 2D_2 n^2 l^2 + D_2 n^4, \]
where \( a \) and \( b \) – the lengths of the sides of the plate in plan (\( a = b \));
\( m = 1,3,5... \), \( n = 1,3,5... \) - harmonic numbers of the expansion of moments and deflection into double trigonometric rows in the longitudinal direction (along the corrugation) \( 0 \leq x \leq a \) and in the transverse direction (along the corrugation) \( 0 \leq y \leq b \);
\[ S_{mx} = \sin \frac{m \pi x}{a}, \quad S_{ny} = \sin \frac{n \pi y}{b}, \quad i = b/a, \]
\( D_1 = I_{hume} E' b \) – bending stiffness per 1 m section width (\( y = \text{const} \));
\( I_b = h_{hume}^3 / 12 \) – fictitious moment of inertia for \( y = \text{const} \); \( E' \) – modulus of elastic - plastic deformation of concrete;
\( h_{hume} \) – fictitious plate thickness for the section \( y = \text{const} \) in the direction \( x \);
\( D_2 = I_2 E' \) – bending stiffness in the transverse direction in the direction \( y \);
\( I_2 = I_1 \) – moment of inertia of the cross section of the corrugations as a tee per 1 m of width;
\( D_3 = 2D_b; \quad D_b = 0.4 E' I_3 \) – plate torsional rigidity.

To determine the work of overlap in two directions, simultaneously with the calculation according to the theory of structurally-orthotropic plates, a calculation was performed according to the beam scheme. At each loading stage, the moment \( M_6 \) and deflection \( f \) were determined using the formulas:
\[ M_6 = q l^2 / 8, \quad (7) \]
\[ f = \frac{5 q l^4}{384 E_b I_2}, \quad (8) \]
where \( q \) – load on the stage per \( 1 \, \text{m}^2 \) of slab,
\( l \) – plate length.

**Assessment of the bearing capacity of the plate.** To assess the bearing capacity of structurally orthotropic reinforced concrete plates, the method of ultimate equilibrium proposed by A. A. Gvozdev is used [20, 21]. For a square or quasi-square orthotropic plate pivotally supported along the contour, plastic hinges (fracture lines) are accepted by envelope as for rectangular isotropic plates. The only difference is that for isotropic plates, the angle of inclination of the plastic hinge to the reference line along the corrugation \( \varphi = 45^\circ \), and for orthotropic plates – the angle \( \varphi < 45^\circ \) depends on the ratio of the ultimate bending moments perceived by the plate in two mutually perpendicular directions.

The work of internal forces is expressed by the formula:
\[ A_m = 4eM_{x} \varphi, + 4M_{y} b \frac{b}{2} \varphi_{1} + M_{y} (b - 2c) 2 \varphi_{1}, \quad (9) \]

The work of external forces is expressed by the formula:
\[ A_q = q \left[ \frac{b}{2} (b - 2c) + \frac{1}{3} 2 cb \right], \quad (10) \]
where \( \varphi_{1} = 2/b, \quad \varphi_{2} = 1/c \)
From the equality of the work of external and internal, we determine the ultimate load \( q \):
\[ q = \frac{12(M_{x} b + 2M_{y} c)}{bc(3b - 2c)}, \quad (11) \]
where \( M_y = R_s \cdot A_{xy} \cdot z_y \) and \( M_x = R_s \cdot A_{xx} \cdot z_x \) - bending moments perceived by the cross-section of the plate, respectively, in two mutually perpendicular directions with the area of reinforcement crossing the plastic joints, \( A_{xy} \) and \( A_{xx} \) and 1 m of width or length;

- \( R_s \) - design resistance of SPN to axial tension;
- \( z_y = h - y_c - x/2 \) - shoulder of the inner pair, taking into account the corrugations;
- \( h \) - plate height;
- \( y_c \) - distance from the center of gravity of the floor to the bottom;
- \( x = R_s A_{xy} / R_p b \) - height of the compressed zone of concrete in section with corrugations;
- \( R_p \) - design resistance of concrete to axial compression (prismatic strength);
- \( z_y = h_f + t/2 - x/2 \) - shoulder of an internal pair of a section of a plate without corrugations;
- \( h_f \) - shelf height of the design brand;
- \( t \) - thickness of steel profiled floor;
- \( x = R_s A_{xx} / R_p b \) - the height of the compressed zone of concrete in the section without corrugations;
- \( b \) - plate width;

In accordance with the kinematic method of limiting equilibrium, it is necessary to find the minimum load. For this, we determine \( c \) from the condition:

\[
\frac{dq}{dc} = 0
\]

Finally, from this relation we find \( c \):

\[
c = \frac{3bM_y}{2(M_y + \sqrt{M_y^2 + 3M_xM_y})}
\]

Substituting \( c \) into formula (11) we find the true value of the load.

To determine the effect on the plate in two directions, aspect ratio, as well as the relationship \( h_f/h_p \), a numerical experiment was made. As a calculation model, a slab with dimensions in terms of 6x6 m was adopted. The calculation was performed using the double-row method in the interpretation of S.G. Lehntskey. The aspect ratio \( i = a/b \) was taken equal to 1; 0,75; 0,5. The height of the rib \( h_p = 0,08 \) m, and the height of the shelf \( h_f = 0,04 \) m and 0,08m. The obtained theoretical data of moments and deflections were compared with the results of a slab operating according to a beam scheme.

To study the influence of constructive orthotropy on floor slabs, and to verify the proposed method for their calculations, experimental studies were carried out on models.

When testing models, the following tasks were set:

- get a picture of changes in the stress-strain state of the models under the action of a uniformly distributed load;
- confirm the reliability of theoretical studies by the proposed calculation method by comparing the results of the experiment and the calculation of models;
- get a diagram of the fracture of the plate and determine the actual bearing capacity;
- compare how the stiffness changes along and across the edges of the profiled flooring in a given floor slab during the loading process.

Experimental studies were carried out on five models.

For reinforcing and anchoring, we used ordinary steel wire of class Vr-1. As fixed formwork and external reinforcement, galvanized steel flooring with a rib height of 0.02 m was used. The material of the models is fine-grained concrete of class B 20.
In the process of testing the models were determined: deflection in the center of the plate; horizontal movement of faces; concrete deformation in the compressed zone; deformation of steel corrugated board; crack opening width.

All tested models were brought to destruction.

The deflections obtained during testing of five models in the center of the plate differed from each other from 5% to 25%. With an increase in the load, the deflections in the center of the slab of each model increased in proportion to the load. At a load of 0.82 from the disruptive displacement models began to grow more intensively, i.e. proportionality (linearity) was violated. Thus, the deflections of the models changed in proportion to the change in the load, to a certain level of loading, and then the influence of plastic deformations was manifested.

To confirm the operation of the structure in two directions in the elastic stage, bending moments Mx and My for each loading stage were calculated from the readings of the electric load cells, which were then compared with the theoretical calculation data.

When the reinforcement reached the yield strength, the strain growth rate increased significantly. Reinforcing deformations at the time of fracture averaged 177x10^5...195x10^5.

Measurement of concrete deformations performed in each slab made it possible to obtain a qualitative picture of the stress-strain state of the models. The deformation values of compressed concrete in various models had a small difference, their change in the experimental models occurred due to a reduction in the area of the compressed zone. The height of the compressed zone of concrete in all tested models changed almost the same.

4. Result

Determination of the stress-strain state of the plate in the elastic stage on the action of a uniformly distributed load. Based on the calculations, it was found that in the square in terms of structurally orthotropic plate there is a redistribution of forces, as evidenced by the values of bending moments Mx and My. It should also be noted that the bearing capacity and floor stiffness have significantly increased in comparison with the strength of the same plate, but working according to the beam scheme. As a result of this, the maximum bending moment decreased by 46%, and the deflection of the overlap decreased by 44%. This indicates that the overlapping work in two directions must be taken into account in the calculations for the case under consideration.

Assessment of the bearing capacity of the plate. Based on the analysis of the results, the following conclusions can be drawn: - with a decrease in the thickness of the plate shelf, namely, with the ratio h_i/h_o = 0.5 and i = 1 the plate works practically according to the beam scheme, that is, there is almost no redistribution of efforts;

- with a ratio of h_i/h_o = 1 and i = 1 a redistribution of forces occurred in the plate, that is, the M_i moment decreased by 72%, compared to the beam moment, the M_o moment decreased by M_o – 31,4%, and the structural deflection decreased by 26,54%;

- with a ratio of h_i/h_o = 1 and i = 0.75 the moment M_i decreased in comparison with the beam one by 68,2%, the moment M_o – 20,8%, and the deflection of the structure decreased by 66,67%;

- with a ratio of h_i/h_o = 1 and i = 0.5 the moment M_i decreased in comparison with the beam one by 79,8%, the moment M_o – 48,7%, and the deflection of the structure decreased by 94,2%.

When the a ratio of h_i/h_o=0,5 the bearing capacity of the plate decreased by 48,7% compared with the ratio h_i/h_o=1.

Experimentally obtained values of bending moments and deflections models compared with the calculation performed with the expectation of work structures in two directions.

Comparison of theoretical calculation, analytical calculation (software package STARK ES) and experimental data gives grounds to conclude the following:

- the difference in terms of spans moments in tested models slightly decreased with increasing load;

- calculation of slab models as structures, working in two directions, to reflect their actual work, although divergence compared with experience.
Thus, the results obtained when testing the models associated with the results of calculation show that the design models have the same scheme of the jog, and qualitative and quantitative picture stressedly-deformed state on stages of booting up, points to the need to take into account the work in two directions.

5. Conclusions
In studies, the most important results are as follows:

1. To assess the stress-strain state of the slab reinforced with profiled flooring during operation, under the action of a uniformly distributed load, using the theory of structurally-orthotropic slabs, supported along the contour, recommendations are developed for determining the stiffness characteristics.

2. It was found that work in two directions of monolithic reinforced concrete flooring is observed at a ratio of $h_f/h_p = 1$ and $i = 1-0.5$, and at a ratio of $h_f/h_p = 0.5$ and $i = 1$, the slab works almost according to the beam scheme.

3. The values of bending moments and deflections of the square plate, calculated by the proposed method, exceeded by 0.8% ... 26% the experimental values obtained by testing the models. This gives reason to recommend a method for calculating monolithic, contour-supported slabs with steel profiled flooring when calculating them according to the second group of limit states.

4. A technique has been developed for determining the bearing capacity of a slab based on the method of limiting equilibrium. An analogy was used with rectangular isotropic plates supported along a contour for which the angle of inclination of the plastic joints is formed at an angle of $45^0$, whereas for orthotropic ones, this angle is less than $45^0$ and depends on the ratio of the ultimate bending moments perceived by the plate in two mutually perpendicular directions.

5. For the beam fracture scheme, when the fracture line propagates in the middle of the plate across the corrugations along the entire length, the theoretical maximum load of the model $q_1 = 45,22 kN/m^2$. The actual (experimental) breaking load $q_2 = 68 kN/m^2$ turned out to be 28% more, which confirms the nature of the plate in two directions.

6. The results of experimental studies on models of monolithic plates with steel profiled flooring showed that, up to a load of 81% of the breaking load, a linear relationship between stresses and deformations (deflection) in the profiled flooring was observed. Accordingly, the theory of structural orthotropic plates is in good agreement with the test data.

7. Comparison of the experimental results ($M_x, M_y, f$) and calculation using the STARK ES software package showed satisfactory convergence and confirmed the work of the slab reinforced with corrugated board in two directions.

8. The field of application of the developed methodology for calculating square and quasiquadratic monolithic plates with steel profiled flooring is substantiated. These structures are recommended for use in coatings and ceilings during the reconstruction of buildings, as well as in new construction.

9. Introducing into practice the construction of monolithic reinforced concrete floors with profiled steel flooring, designed taking into account their work in two directions, reduces the consumption of building materials and, accordingly, the cost of structures up to 26%.

References
[1] Dyakhter A S, Rasskazov A O 1990 Bearing capacity of thin-walled structures (K.: Budivelnik) 152 p
[2] Gvozdev A A 1948 Calculation of the bearing capacity of structures by the method of limit equilibrium Issue 1 The essence of the method and its justification (M.:)
[3] Gvozdev A A 1948 On Ultimate Equilibrium Engineering Collection vol 5 1
[4] Bartenev I A, Batazhkova V N 1983 Essays on the History of Architectural Styles: A Study Guide (M.: Izob. Art) 384 p
[5] Borishansky M S, Schepotiev A S 1934 An experimental study of thin-walled spatial structures Project and standard 2 pp 19-29
[6] Vlasov V Z 1964 Selected Works T 3 *Thin-walled spatial systems* (M.: Ed. USSR Academy of Sciences) 472 p
[7] Porter M L, Ekberg E Ir 1978 Compendium of ISU Research Conducted on Cold - Formed Steel - desk. Rein forced Slab Systems *Engineering Research Institute* (Iowa, USA)
[8] SP 16.13330.2011 2010 Steel structures Ministry of Regional Development of Russia 28 p
[9] SP 63.13330.2012 2011 Concrete and reinforced concrete structures Ministry of Regional Development of Russia
[10] GOST 10180-90 (ST SEV 3978-83) Concretes Methods for determining the strength of the control samples
[11] GOST 10178-62 Reinforced concrete and concrete products General technical requirements. Cement.
[12] 1987 Recommendations for the design of monolithic reinforced concrete floors with profiled steel flooring (M.: NIIZHB) *Central Research Institute of Industrial Buildings*
[13] Murashov V I 1940 The theory of the appearance and opening of cracks, calculation of the stiffness of reinforced concrete bending elements *Construction industry* 11
[14] Nemirovsky Ya M 1968 Investigation of the stress-strain state of reinforced concrete elements, taking into account the work of stretched concrete over cracks, and the revision of the theory of deformations and crack opening on this basis *In the book. Strength, stiffness of reinforced concrete structures* (M.: Stroyizdat)
[15] Baykov V N, Hampé E, Raue E 1990 Design of reinforced concrete thin-walled spatial structures (M.: Stroyizdat) 232 p
[16] Zvorykin D N 1981 The development of building science (M.: Stroyizdat) 293 p
[17] Korobov L A 1975 Determination of the bearing capacity of spatial coatings under the action of a uniformly distributed load *Spatial structures of buildings and structures* vol 2 (M.: Stroyizdat) pp 131-136
[18] Dodonov M I, Khormiz N D 1991 Strength of monolithic slabs on profiled steel flooring at local loads *Concrete and reinforced concrete* 5
[19] Dodonov M I, Baktygulov K B 1988 Precast monolithic floors using profiled steel flooring *Concrete and reinforced concrete* 4
[20] Gvozdev A A 1949 The method of limit equilibrium as applied to the calculation of reinforced concrete structures *"Engineering Digest"* vol 5 2
[21] Gvozdev A A 1985 Calculation of reinforced concrete conventional and prestressed structures according to limit states *ASiS USSR* 1958