1. Introduction

There are two principal ways to construct a coronal mass ejection (CME) model. The first one is to solve the magnetohydrodynamical equations for coronal and/or heliospheric plasma with appropriate boundary and initial conditions numerically. This approach requires considerable computing resources, moreover, the accurate accounting for initial conditions still remains a subtle matter. The other way is to use phenomenological CME models which are not completely based on rigorous solution of the magnetohydrodynamical equations but also on the additional considerations. The early models of this type being rough and “non-realistic” were based on simple exact solution of magnetohydrodynamical equations (see e.g., Burlaga, 1988). The nowadays trend is to use “more and more realistic” models which are simultaneously less and less supported by dynamical equations.

Consider recently published 3DCORE model (Möstl et al., 2018a) which states that it describes three-dimensional magnetic field of the coronal ejection. Elementary checkup shows that the magnetic field suggested by 3DCORE violates both Maxwellian equations: the one for absence of magnetic monopoles and the other one for Faraday induction (“frozen-in” condition). We recall basic heliospheric magnetohydrodynamical equations and show that they lead to a number of conservation laws. These conservation laws should be taken into account when developing a phenomenological CME model. We show that conservation laws take extremely simple form when written in Lagrange coordinates frozen into plasma. Thus, these are easy to conform and practically, there are no reasons not to conform them. We propose a modification of 3DCORE magnetic field that satisfies Maxwellian equations.

2. A Brief Review of 3DCORE Model

2.1. Magnetic Field at Fixed Time

The original study (Möstl et al., 2018a) does not contain technical details on the construction of 3DCORE model. These can be found in section “Supporting Information” on page https://agupubs.onlinelibrary.wiley.com/doi/10.1029/2017SW001735 and in the Python code 3dcore_v2.py available on the same page.
The construction of 3DCORE is based on transformation from Cartesian coordinate system \((x, y, z)\) with the Sun at the origin to some auxiliary coordinate system \((r, \phi, \psi)\)

\[
\begin{align*}
x &= \rho_0 - (\rho_0 + \rho_1 r \sin \frac{\psi}{2} \cos \phi) \cos \psi, \\
y &= (\rho_0 + \rho_1 r \sin \frac{\psi}{2} \cos \phi) \sin \psi, \\
z &= \rho_1 r \sin \frac{\psi}{2} \sin \phi.
\end{align*}
\]  

(1)

New coordinates obey the inequalities \(0 < r < 1\), \(0 < \phi < 2\pi\), \(0 < \psi < 2\pi\) and map the area called tapered torus (see Figure 1): the line \(r = 0\), \(0 < \psi < 2\pi\) (the circle of radius \(\rho_0\) passing the Sun) is the axis of the torus, the torus cross section radius varies as \(\rho_1 \sin \frac{\psi}{2}\), and \((r, \phi)\) are polar coordinates in the cross section.

Here we use different, though equivalent, definition of \(r\) than that of original Python code (Möstl et al., 2018b), lines 138–156. The value of \(r\) used in Python code corresponds to \(\rho_1 r\) in our notation. We find it more convenient to use variable \(r\) which boundary values \(0 < r < 1\) are independent of \(\rho_1\).

We should mention that contrary to CME models of cylindric and (non-tapered) toroidal shapes the tapered torus coordinates \((r, \phi, \psi)\) are not orthogonal: the \(r = \text{const}\) surfaces are tapered tori and are not orthogonal to \(\psi = \text{const}\) planes.

Components of magnetic field along \((r, \phi, \psi)\) coordinate lines are equal to

\[
\begin{align*}
B_r &= 0, & B_\phi &= \pm \frac{B_0 \alpha \rho_1 r}{1 + \alpha^2 \rho_1^2 r^2}, & B_\psi &= \frac{B_0}{1 + \alpha^2 \rho_1^2 r^2}.
\end{align*}
\]  

(2)

Magnetic lines have helix shape, \(\pm\) stands for right/left-hand helicity, \(\alpha\) determines the pitch of the helix.

The original Python code (Möstl et al., 2018b), lines 177–188 (see also Möstl et al., 2018c, Equation 2) is again a little bit different due to different definition of \(r\). Surprisingly, the subsequent transformation of magnetic field from torus to Cartesian components is performed as if torus coordinates are orthogonal, see lines 201–205 of code and Equation 9 in (Möstl et al., 2018c).
The anzats for magnetic field comes from exact force free Gold-Hoyle solution (Gold & Hoyle, 1960) for cylindric CME. The force free condition is

\[ \mathbf{B} \times (\nabla \times \mathbf{B}) = 0. \]  

(3)

It should be mentioned that being rewritten in tapered torus coordinates the Gold-Hoyle anzats does not satisfy force free condition any more.

### 2.2. Time Dependence

The magnetic field time dependence is introduced via the dependences of \( \rho_{0,1}, B_0 \) quantities on time (see Möstl et al., 2018c, Equations 3–5, and Möstl et al., 2018b, lines 575–590). All of these are expressed through \( r_{\text{ apex}}(t) \)

\[ r_{\text{ apex}}(t) = \pm \ln(1 \pm y(V_0 - w)t) + wt + R_0, \quad \rho_1(t) = \frac{D_{\text{AU}} 1.14}{2} r_{\text{ apex}}(t), \]

\[ 2\rho_0(t) = r_{\text{ apex}}(t) - \rho_1(t), \quad B_0(t) = B_{\text{AMU}} (2\rho_0(t))^{-1.64}. \]

(4)

here \( R_0, V_0 \) stand for initial position and velocity of the apex (the point with the furthest heliocentric distance), \( w \) – ambient solar wind velocity, \( \pm \) corresponds to \( V_0 > w \) and \( V_0 < w \).

The expression for apex position comes from drag-based model (Vršnak et al., 2013). The exponents 1.14 and \(-1.64 \) come from statistical analysis of satellite data at different positions from the Sun (Leitner et al., 2007). These are specific assumptions of 3DCORE model.

Authors state that plasma velocity \( \mathbf{v} \) at space point \( \mathbf{r} = (x, y, z) \) at time \( t \) equals (see Möstl et al., 2018c, Equation 10)

\[ \mathbf{v} = \mathbf{r} \frac{r_{\text{ apex}}(t)}{r_{\text{ apex}}(t)}, \]

(5)

though this formula contradicts with the expressions for \( \rho_1(t) \) and \( \rho_0(t) \).

Equation 5 supposes that the evolution of CME is self-similar. The self-similar evolution of CME is of special interest since the corresponding self-similar evolution of magnetic field

\[ \mathbf{B}(r, t) = g_1(t) \mathbf{B}(\mathbf{r} / g_1(t)) \]

(6)

(with arbitrary \( g_1, g_2(t) \)) preserves force free condition. This is not the case for the 3DCORE model. Due to exponent 1.14 in 4 the torus cross section grows faster than the torus major radius, thus the evolution is not self-similar. Moreover, even if it were the magnetic field dependence is not of self-similar type, since 2 involves \( \rho_1 \) explicitly.

In what follows, we assume that plasma velocity should be obtained from 1 by differentiating with respect to \( t \) at fixed \( (r, \phi, \psi) \).

### 3. General Physical Restrictions on \( B \) and \( v \)

It is usually assumed that heliospheric plasma is perfectly conducting. The magnetohydrodynamical equations then read (Landau et al., 1984)

\[ \frac{\partial n}{\partial t} + \nabla (nv) = 0, \]

(7)

\[ n \left( \frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v} \nabla) \mathbf{v} \right) = -\nabla p - \frac{1}{4\pi} \mathbf{B} \times (\nabla \times \mathbf{B}), \]

(8)

\[ \frac{\partial s}{\partial t} + (\mathbf{v} \nabla) s = 0, \]

(9)
\[ \nabla \cdot \mathbf{B} = 0, \quad (10) \]
\[ \frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{v} \times \mathbf{B}), \quad (11) \]

These are continuity, Euler, adiabatic equations, the equation for absence of magnetic monopoles and the condition that magnetic field lines are “frozen” into the plasma. For perfectly conducting plasma, electric field vanishes in plasma rest frame and equals

\[ \mathbf{E} = -\frac{\mathbf{v}}{c} \times \mathbf{B} \quad (12) \]

in celestial reference frame. The “frozen-in” condition is just Faraday law with electric field from 12 substituted. Since \( \rho \) already stands for radius in 3DCORE, plasma density is designated with \( n \), \( p \) stands for pressure, \( s(n, p) \) for entropy (say, for ideal gas \( s(n, p) = \frac{1}{\gamma - 1} \ln (p n^{-\gamma}), \) here \( \gamma \) is adiabatic exponent).

Note that it is sufficient to satisfy Equation 10 at \( t = 0 \): it follows from 11 that \( \frac{\partial (\nabla \cdot \mathbf{B})}{\partial t} = 0 \), thus, if 10 is satisfied at \( t = 0 \) and 11 is satisfied at \( t > 0 \), then 10 is satisfied at \( t > 0 \).

The Equations 7–11 are written in Euler coordinates, \( \mathbf{r} \) stands for fixed space point independently of \( t \). CME models, in particular, 3DCORE model, are usually formulated using Lagrange coordinates, “frozen” into plasma. Thus, to check if the model satisfies magnetohydrodynamical equations it is convenient to rewrite these in Lagrange coordinates. Let \( q_{1,2,3} \) be Lagrange coordinates and let

\[ x_i = f_i(q_1, q_2, q_3, \tau), \quad t = \tau \quad (13) \]

be the transformation from Lagrange to Euler coordinates (the time in Lagrange set is designated \( \tau \) to avoid confusion). Plasma velocity is equal to

\[ v_i = \frac{\partial f_i}{\partial \tau} \quad (14) \]

One can treat \( q_k \) at fixed \( \tau \) as some non-orthogonal in general coordinate system. Using non-orthogonal coordinate systems is not common in most areas of physics. This technique is best developed in General Relativity. A brief introduction can be found, for example, in (Landau & Lifshitz, 1987), chapter X. We need very little of this technique and I try to present all the necessary things here to make the study self-contained.

The quantity

\[ M_{ik} = \frac{\partial f_i}{\partial q_k} \quad (15) \]

is called the transformation matrix from Lagrange to Cartesian coordinates. It follows directly from the definition that

\[ dx_i = \sum_k M_{ik} dq_k. \quad (16) \]

The distance between two close points is

\[ d \mathbf{r}^2 = \sum_{ik} g_{ik} dq_i dq_k, \quad g_{ik} = \sum_j M_{ij} M_{jk}. \quad (17) \]
Quantity $g_{ik}$ is called the metric tensor in Lagrange coordinates. Its diagonal elements $g_{ik} = h_i^2$ are usually called Lamé coefficients, but for non-orthogonal Lagrange coordinates the metric tensor also has non-diagonal components.

Contravariant components of magnetic field in Lagrange coordinates $b_i$ are defined with equation

$$B_i = \sum_k M_{ik} b_k.$$  

(18)

here $B_i$ are Cartesian components. In other words, the contravariant components transform by the same rule that coordinates differentials themselves. Comparing 18 and 16, we see that $b_k$ are components along $q_k$ coordinate lines, but normalized in a special way

$$B^2 = \sum_i B_i^2 = \sum_{i,k,l} (M_{ik} b_k)(M_{il} b_l) = \sum_{k,l} g_{kl} b_k b_l.$$  

(19)

It should be mentioned that contravariant components $b_i$ may not have direct physical meaning, in particular, may not even have a dimension of magnetic field. The physical meaning of contravariant components is that they can be recalculated into Cartesian ones with formula 18.

The derivatives with respect to $\tau$ and $q_k$ in Lagrange coordinates are expressed as follows

$$\frac{\partial}{\partial \tau} = \frac{\partial}{\partial t} + (\mathbf{v} \nabla), \quad \frac{\partial}{\partial q_k} = \sum_i M_{ik} \frac{\partial}{\partial x_i}.$$  

(20)

Now, it is easy to rewrite magnetohydrodynamical equations in Lagrange coordinates. We will rewrite only those equations which impose restrictions on $\mathbf{v}$ and $\mathbf{B}$, these are continuity equation, the equation for absence of magnetic monopoles and the condition that magnetic field lines are “frozen” into the plasma.

To rewrite continuity Equation 7, let us first write it in alternative form

$$\left( \frac{\partial}{\partial t} + (\mathbf{v} \nabla) \right) n + n(\nabla n) = 0.$$  

(21)

Then, making use of 20, we obtain

$$\frac{\partial n}{\partial \tau} + n \sum_k \left( \sum_i (M^{-1})_{ik} \frac{\partial}{\partial q_i} \right) \frac{\partial f_k}{\partial \tau} = \frac{\partial n}{\partial \tau} + n \sum_i (M^{-1})_{ik} \frac{\partial}{\partial \tau} M_{ki} = 0.$$  

(22)

Recalling well-known relation $\sum_{i,k} (M^{-1})_{ik} dM_{ki} = d \ln \det M$ (see e.g., Landau & Lifshitz, 1987, Equation 86.4), we find

$$\frac{\partial}{\partial \tau} \sqrt{g n} = 0.$$  

(23)

here $g$ stands for determinant of metric tensor $g_{ik}$ which is equal to the square of $\det M$ according to 17. In other words, the product $\sqrt{g n}$ remains constant upon plasma motion.

The expression for divergence of vector field in arbitrary coordinates reads (Landau & Lifshitz, 1987, Equation 86.9)

$$\nabla \mathbf{B} = \sum_k \frac{1}{\sqrt{g}} \frac{\partial}{\partial q_k} \sqrt{g} b_k.$$  

(24)

Thus, Equation 10 takes the form

$$\sum_k \frac{\partial}{\partial q_k} \sqrt{g} b_k = 0.$$  

(25)
To transform the “frozen in” Equation 11, we first rewrite it in equivalent form using Equations 7 and 10 (see Landau et al., 1984, Equation 65.15)

\[
\left( \frac{\partial}{\partial \tau} + (\mathbf{v} \nabla) \right) \frac{\mathbf{B}}{n} = \left( \frac{\mathbf{B}}{n} \nabla \right) \mathbf{v}.
\]  

(26)

Then using 20, we obtain

\[
\frac{\partial}{\partial \tau} \sum \mathbf{M}_k b_n = \left( \sum \mathbf{M}_k \frac{b_n}{n} \sum (M^{-1})_{j} \frac{\partial f_i}{\partial q_j} \right) \frac{\partial f_i}{\partial \tau} = \sum \frac{b_n}{n} \frac{\partial}{\partial \tau} \mathbf{M}_k,
\]

and finally

\[
\frac{\partial}{\partial \tau} b_k = 0.
\]  

(27)

Recalling that the product \( g_n \) is independent of \( \tau \), Equation 23, we can write

\[
\frac{\partial}{\partial \tau} \sqrt{g} b_k = 0.
\]  

(28)

In other words, the quantity \( \sqrt{g} b_k \) remains constant upon plasma motion. In particular, the direction of magnetic field remains unchanged and magnetic field lines in Lagrange coordinates are independent of \( \tau \), “frozen” into plasma.

Note that Equations 25 and 29 have the same property as Equations 10 and 11: it is sufficient to demand that Equation 25 is correct at \( \tau = 0 \), then it will be correct at any \( \tau > 0 \) automatically.

Let us gather once again the restrictions on the phenomenological CME model

\[
\sum \frac{\partial}{\partial q_k} \sqrt{g} b_k = 0, \quad \frac{\partial}{\partial \tau} \sqrt{g} b_k = 0.
\]  

(30)

The first equation here is the condition for absence of magnetic charges, the second is “frozen in” condition for magnetic field. We see that it is extremely simple to set the model magnetic field: one has to set solenoidal \( \sqrt{g} b_k \) at \( \tau = 0 \), this quantity is merely independent of \( \tau \). There are absolutely no reasons to violate these conditions when constructing the particular CME model.

If one of Lagrange components of magnetic field, say, \( b_3 \) is set to zero, then the two other components are of the form

\[
\sqrt{g} b_2 = \frac{\partial \chi}{\partial q_3}, \quad \sqrt{g} b_1 = -\frac{\partial \chi}{\partial q_2}
\]

(31)

where the “magnetic potential” \( \chi \) is independent of \( \tau \) to satisfy the right equation of 30.

Equation 30 also implies that there is little sense in fitting experimental data separately for CME expansion and magnetic field. Since time dependence of the magnetic field is completely determined by the expansion of CME, the data should be fitted collectively in the framework of some CME model.

4. Analysis of 3DCORE Model

Let us now analyze the 3DCORE model from the standpoint of the previous section. It is natural to assume (although the authors have not stated it explicitly) that the coordinates \((r, \phi, \psi)\) are Lagrange coordinates, and formula 1 represents the transformation to Euler coordinates (with \( \tau \)-dependent radii \( \rho_{0,1} \)).

The coordinate system \((r, \phi, \psi)\) is not orthogonal
Lamé coefficients are equal to

$$h_r = \rho_0 \sin \psi/2, \quad h_\phi = \rho_0 r \cos \phi/2, \quad h_\psi = \sqrt{(\rho_0 + \rho_1 r) \sin \psi/2 \cos \phi}^2 + \frac{1}{4} \rho_1^2 r^2 \cos^2 \psi/2.$$  \hspace{1cm} (33)$$

and the determinant of the metric tensor equals

$$\sqrt{g} = \rho_1^2 r \sin \psi/2 (\rho_0 + \rho_1 r \sin \psi/2 \cos \phi).$$  \hspace{1cm} (34)$$

The equation expressing the absence of magnetic charges takes the form

$$\frac{\partial}{\partial \phi} b_\phi \rho_1^2 r \sin \psi/2 (\rho_0 + \rho_1 r \sin \psi/2 \cos \phi) + \frac{\partial}{\partial \psi} b_\psi \rho_1^2 r \sin \psi/2 (\rho_0 + \rho_1 r \sin \psi/2 \cos \phi) = 0.$$  \hspace{1cm} (35)$$

However, 2 are not contravariant components and we have to make a supposition how they are related to the contravariant ones. The most natural supposition is $B_\phi = h_\phi b_\phi$, $B_\psi = h_\psi b_\psi$, at least if torus coordinates were orthogonal (and the authors of 3DCORE transform components from torus to Cartesian coordinates as if they are orthogonal), the relation would be of this type.

Rewriting the equation for absence of magnetic charges via $B_\phi, \psi$, we obtain

$$\frac{\partial}{\partial \phi} B_\phi \rho_1^2 r \sin \psi/2 (\rho_0 + \rho_1 r \sin \psi/2 \cos \phi) + \frac{\partial}{\partial \psi} B_\psi \rho_1^2 r \sin \psi/2 (\rho_0 + \rho_1 r \sin \psi/2 \cos \phi) \sqrt{\rho_0 + \rho_1 r \sin \psi/2 \cos \phi}^2 + 14 \rho_1^2 r^2 \cos^2 \psi/2 = 0.$$  \hspace{1cm} (36)$$

The factors arising in this formula can be understood from the geometrical point of view. Recall that $\nabla \mathbf{B} = 0$ condition means zero magnetic flux over any closed surface. If we consider the part of tapered conus cut off by two $\psi = \text{const}$ planes (see Figure 2) then we find that the flux over $\psi = \text{const}$ cross section, equal to...
the product of $B_\psi$ value and the cross section square, should be the same for any $\psi = \text{const}$ cross section, in other words, $B_\psi \sin^2 \frac{\psi}{2} = \text{const}$.

If now we consider cylindric layer cut off by two $\psi = \text{const}$ planes, two $r = \text{const}$ tubes, and two $\phi = \text{const}$ surfaces, then we find that the flux over $\phi = \text{const}$ cross section, equal to the product of $B_\phi$ value and the cross section square, should be the same for any $\phi = \text{const}$ cross section, in other words

$$B_\phi (\rho_0 + \rho_1 \sin \frac{\psi}{2} \cos \phi) = \text{const}. $$

The geometrical factors mentioned here are already solidly incorporated in the space weather literature for non-tapered torii (see e.g., Fan, 2008).

It is easy to see that $2$ is not a solution of $36$. The same is true for the “frozen-in” condition

$$\frac{\partial}{\partial \tau} b_{\phi,\psi} \rho_1^2 r \sin^2 \frac{\psi}{2} (\rho_0 + \rho_1 \sin \frac{\psi}{2} \cos \phi) = 0,$$

or, via $B_{\phi,\psi}$ components,

$$\frac{\partial}{\partial \tau} B_{\phi} \rho_1 \sin \frac{\psi}{2} (\rho_0 + \rho_1 \sin \frac{\psi}{2} \cos \phi) = 0, \quad \frac{\partial}{\partial \tau} B_\psi \frac{\rho_1^2 r \sin^2 \frac{\psi}{2} (\rho_0 + \rho_1 \sin \frac{\psi}{2} \cos \phi)}{\sqrt{(\rho_0 + \rho_1 \sin \frac{\psi}{2} \cos \phi)^2 + \frac{1}{4} \rho_1^2 r^2 \cos^2 \frac{\psi}{2}}} = 0. \tag{38}$$

The “frozen-in” condition says that one cannot set the time dependence of magnetic field value $B_0(t)$ and CME expansion $\rho_{0,1}(t)$ independently as it was done by the authors of 3DCORE. Once the expansion of CME is set the time dependence of magnetic field is fixed.

Thus, the magnetic field of 3DCORE needs to be corrected to meet physical conditions. The Gold-Hoyle no-force condition is anyway destroyed already in original 3DCORE model; thus, there is no reason to try hard to preserve it. But the important topological field feature that all magnetic lines make the same number of revolutions over $\phi$ when $\psi$ changes from $0$ to $2\pi$ regardless of $r$ can be preserved. This corresponds to the relation $b_\phi/b_\psi$ independent of $r$. If we additionally require, that $b_\phi/b_\psi$ does not depend on $\phi$ and $\psi$, then the simplest field, in some sense similar to $2$, is

$$b_\phi = \frac{1}{\rho_1^2 \sin^2 \frac{\psi}{2} (\rho_0 + \rho_1 \sin \frac{\psi}{2} \cos \phi) \left(1 + (\alpha D_{\text{IAU}} / 2)^2 r^2\right)} \cdot b_\psi = (\alpha / 2) b_\psi. \tag{39}$$

here constant factors are determined by comparison with original 3DCORE field in the vicinity of the apex at the time it crosses 1AU sphere.

## 5. Conclusion

The magnetohydrodynamical equations impose a number of restrictions on phenomenological CME models. These restrictions have the form of conservation laws and should be accounted for when constructing the particular CME model. The restrictions on magnetic field take especially simple form in Lagrange coordinates commonly used to formulate CME model. Thus, there are absolutely no reasons not to conform them. The magnetic field that meets physical conditions can be used not only for simulation of K-indices or satellite measurements but also in true three-dimensional sense, say, for calculation of cosmic ray scattering.

The formalism developed in this article can be used in other domains, since flux ropes are broadly present in magnetized plasma. For example, it can be used to model flux ropes in the solar convective zone, which are emerging at the photospheric level (Fan & Gibson, 2003; Luoni et al., 2011; Poisson et al., 2016).
Data Availability Statement

Data were not used nor created for this research.

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