Electrical response of S-F-TI-S junctions on magnetic texture dynamics

D. S. Rabinovich, I. V. Bobkova, and A. M. Bobkov

1 Skolkovo Institute of Science and Technology, Skolkovo 143026, Russia
2 Moscow Institute of Physics and Technology, Dolgoprudny, 141700 Russia
3 Institute of Solid State Physics, Chernogolovka, Moscow reg., 142432 Russia

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We consider a hybrid structure consisting of superconducting or normal leads with a combined ferromagnet-3D topological insulator interlayer. We compare responses of a Josephson junction and a normal junction on the magnetization dynamics. In both cases the electromotive force resulting from the magnetization dynamics generates a voltage between the junction leads. For an open circuit this voltage is the same for normal and superconducting leads and allows for electrical detection of magnetization dynamics and a structure of a time-dependent magnetization. However, under the applied current the electrical response of the Josephson junction is essentially different due to the strong dependence of the critical Josephson current on the magnetization direction. We propose a setup, which is able to detect a defect motion and to provide detailed information about the structure of magnetic inhomogeneity. The discussed effect can be of interest for spintronics applications.

I. INTRODUCTION

At present the magnetic field-driven and current-driven switching in spin valves and related systems, as well as domain-wall (DW) and skyrmion motion are in the focus of research activity, which is in part motivated by a promising application potential in spintronics. The inverse effect of electromotive forces (emf) induced by the time-dependent magnetization has also been actively discussed in recent years.

Due to the existence of the electromotive force the DW motion leads to appearance of an additional voltage drop at the region occupied by the moving wall. This voltage can vary in the range from $nV$ to $\mu V$ and in special situations can be used for electrical detection of the presence of magnetization dynamics. The emf can be considered as a consequence of the presence of a time-dependent gauge potential in the local spin basis of a spin-textured system. The spin-orbit coupling can also be described in terms of SU(2) gauge potential, which becomes time-dependent in the local spin basis in the presence of a magnetization dynamics and, therefore, also results in the appearance of an emf.

It is well-known that the property of spin-momentum locking of surface states of a 3D TI also leads to the appearance of a gauge vector potential, which becomes time-dependent in the presence of magnetization dynamics. It results in the appearance of the emf in the system, which is determined by time derivatives of the in-plane magnetization component. Motivated by this fact here we consider a hybrid structure in a Josephson junction geometry $L/(F/TI)/L$, where $L$ is a lead electrode, which can be as normal (N), so as superconducting (S), and $F/TI$ is an interlayer consisting of a ferromagnet (F) and a 3D topological insulator (TI). We investigate electrical manifestations of the magnetization dynamics in such a system and their prospects for electrical probing of time-dependent magnetization structure.

It is timely to study magnetization dynamics in such a system because at present there is a great progress in experimental realization of F/TI hybrid structures. In particular, to introduce the ferromagnetic order into the TI, random doping of transition metal elements, e.g., Cr or V, has been employed. The second option, which has been successfully realized experimentally, is a coupling of a nonmagnetic TI to a high $T_c$ magnetic insulator to induce strong exchange interaction in the surface states via the proximity effect. The spin injection into a TI surface states via the spin pumping techniques has also been realized and the resulting spin-electricity conversion has been measured.

Here we compare the responses of a Josephson junction and a normal junction on the magnetization dynamics. In both cases the electromotive force resulting from the magnetization dynamics generates a voltage between the junction leads. In the open circuit geometry the voltage is the same as for normal, so as for superconducting leads and allows for electrical detection of magnetization dynamics. However, under the applied current the electrical response of the Josephson junction is essentially different due to the strong dependence of the critical Josephson current on the magnetization direction.

The investigated electrical response of the Josephson junction on the magnetization dynamics can be of interest for spintronics applications because it provides a way of electrical reading of information encoded in the magnetization. We propose a setup, which is able not only to detect a defect motion but also to provide detailed information about the structure of magnetic inhomogeneity.

The paper is organized as follows. In Sec.II we describe the system under consideration, formulate the necessary equations and calculate the general expression for the electric current in the presence of magnetization dynamics. In Sec.III we investigate the electrical response of the system on magnetization dynamics in two particular situations: for an open circuit and in the presence of the constant applied electric current. In this section we also demonstrate the possibilities of using the effect for detection of DWs motion and their structure.
II. MODEL AND METHOD

The sketch of the system under consideration is presented in Fig. 1. A ferromagnet is placed on top of the interlayer region of a S/3D TI/S Josephson junction. The magnetization dynamics is assumed to be induced in the ferromagnet by external means. We discuss particular examples below, and now we derive a formalism allowing for studying the electrical response on the magnetization dynamics of a general type. In principle, the ferromagnet can be as metallic, so as insulating. We assume that the transport between the leads occurs via the TI surface states. This is strictly the case for insulating ferromagnets. We believe that our results can be of potential interest mainly for systems based on Be2Se3/YIG or Be2Se3/EuS hybrids, which have been realized experimentally. For metallic ferromagnets the situation is a bit more complicated. If the ferromagnet is strong, that is its exchange field is comparable to the Fermi energy, then the Josephson current through the ferromagnet is greatly suppressed and indeed flows through the 3D TI surface layer. This provides an additional channel for the normal current and its influence on the results is discussed below.

![Sketch of the system under consideration.](image)

It is assumed that the magnetization \( \mathbf{M}(r) \) of the ferromagnet induces an effective exchange field \( h_{\text{eff}}(r) \sim -\mathbf{M}(r) \) in the underlying conductive TI surface layer and \( h_{\text{eff}} \) is small as compared to the exchange field in the ferromagnet. The Hamiltonian that describes the TI surface states in the presence of an in-plane exchange field \( h_{\text{eff}}(r) \) reads:

\[
\hat{H} = \int d^2 r \hat{\Psi}^\dagger(r) \hat{H}(r) \hat{\Psi}(r),
\]

\[
\hat{H}(r) = -i v_F (\nabla \times \mathbf{e}_z) \hat{\sigma} + h_{\text{eff}}(r) \hat{\sigma} - \mu,
\]

where \( \hat{\Psi} = (\Psi^\dagger, \Psi) \), \( v_F \) is the Fermi velocity, \( \mathbf{e}_z \) is a unit vector normal to the surface of TI, \( \mu \) is the chemical potential, and \( \hat{\sigma} = (\sigma_x, \sigma_y, \sigma_z) \) is a vector of Pauli matrices in spin space. It was shown that in the quasiclassical approximation \( (\hbar_{\text{eff}}, \varepsilon) \ll \mu \) the Green’s function has the following spin structure:

\[
\hat{g}(\mathbf{n}_F, \mathbf{r}, \varepsilon, t) = \hat{g}(\mathbf{n}_F, \mathbf{r}, \varepsilon, t)(1 + \mathbf{n}_\perp \sigma)/2,
\]

where \( \mathbf{n}_\perp = (n_{F,y}, -n_{F,x}, 0) \) is the unit vector perpendicular to the direction of the quasiparticle trajectory \( \mathbf{n}_F = p_F/p_F \) and \( \hat{g} \) is the spinless \( 4 \times 4 \) matrix in the direct product of particle-hole and Keldysh spaces. The spin structure above reflects the fact that the spin and momentum of a quasiparticle at the surface of the 3D TI are strictly locked and make a right angle.

Following standard procedures it was demonstrated that the spinless Green’s function \( \hat{g}(\mathbf{n}_F, \mathbf{r}, \varepsilon, t) \) obeys the following transport equations in the ballistic limit:

\[
-i v_F \mathbf{n}_F \nabla \hat{g} = \left[ \varepsilon \tau_z - \hat{\Delta}, \hat{g} \right] \otimes \mathbf{I},
\]

where the spin-momentum locking allows for including \( h_{\text{eff}} \) into the gauge covariant gradient

\[
\nabla \hat{g} = \nabla \hat{g} + (i/v_F)[(\mathbf{n}_F \cdot \hat{\sigma} - \mathbf{n}_\perp \cdot \hat{\sigma})] \hat{g} \otimes \mathbf{I}.
\]

Following standard procedures it was demonstrated that the spinless Green’s function \( \hat{g}(\mathbf{n}_F, \mathbf{r}, \varepsilon) \) obeys the following transport equations in the ballistic limit:

\[
-j_x = -e N_F v_F / 4 \int_{-\infty}^{\infty} dz \int_{-\pi}^{\pi} \frac{d\phi}{2\pi} \cos \phi \hat{g}^K,
\]

where \( \phi \) is the angle the quasiparticle trajectory makes with the \( x \)-axis, \( \hat{g}^K \) is the Keldysh part of the Green’s function, which can be expressed via the retarded, advanced parts and the distribution function \( \varphi \) as follows:

\[
\hat{g}^K = g^R \otimes \varphi - \varphi \otimes g^A.
\]

In general, the electric current through the junction consists of two parts: the Josephson current \( j_x \) and the normal current \( j_n \). The Josephson current is connected to the presence of the nonzero anomalous Green’s functions in the interlayer and exists even in equilibrium. Below we calculate the both contributions to the current microscopically. It is assumed that the effective exchange field in the interlayer of the junction is spatially homogeneous.
The Josephson current for the system under consideration has already been calculated. Here we present the main calculation steps just for completeness. To obtain the simplest sinusoidal form of the current-phase relation we linearize Eq. (4) with respect to the anomalous Green’s function. In this case the retarded component of the Green’s function $g^R = \tau_+ + f^R \tau_+ + f^R \tau_-$. The anomalous Green’s function obeys the following equation:

$$-\frac{1}{2} i v_F \partial_x f^R + h_{eff} n_{\perp} f^R = \varepsilon f^R - \Delta(x). \quad (7)$$

Equation for $f^R$ is obtained from Eq. (7) by $v_F \to -v_F$, $\Delta \to -\Delta$ and $x \to -x$. The solution of Eq. (7) satisfying asymptotic conditions $f^R \to (\Delta/\varepsilon) e^{\pm ix/2}$ at $x \to \pm \infty$ and continuity conditions at $x = \mp d/2$ takes the form [the solution is written for $x \in (-d/2, d/2)$, the solution in the superconducting leads is also found, but it is not required for finding the Josephson current] :

$$f^R_{\pm} = \frac{\Delta e^{\pm ix/2}}{\varepsilon} \text{exp}\left[\mp 2i(h_{eff} n_{\perp} - \varepsilon)(d/2 \pm x)\right],$$

$$j^R_{\pm} = -\frac{\Delta e^{\pm ix/2}}{\varepsilon} \text{exp}\left[\mp 2i(h_{eff} n_{\perp} - \varepsilon)(d/2 \pm x)\right], \quad (8)$$

where the subscript $\pm$ corresponds to the trajectories sgn $v_x = \pm 1$. Here we assume the voltage, induced due the magnetization dynamics at the junction to be small $eV/(k_B T_s) \ll 1$. In this case the deviation of the distribution function from equilibrium is weak and can be disregarded in calculation of the Josephson current: $\varphi_+ = \varphi_- = \tanh(\varepsilon/2T)$. Exploiting the normalization condition one can obtain $g^A_{\pm} \approx 1 - f^R_{\pm} f^R_{\mp}/2$. Taking into account that $g^A_{\pm} = -g^R_{\pm}$ we find the following final expression for the Josephson current:

$$j_s = j_c \sin(\chi - \chi_0), \quad (9)$$

$$j_c = e v_F N_F T \sum_{\varepsilon_n > 0} -\pi/2 \int_{-\pi/2}^{\pi/2} d\phi \cos \phi \frac{\Delta^2}{\varepsilon_n^2} \times$$

$$\exp[-2\varepsilon_n d/v_F \cos \phi] \cos[2h_{\perp} d \tan \phi/v_F], \quad (10)$$

$$\chi_0 = 2h_0 d/v_F, \quad (11)$$

where $\varepsilon_n = \pi T (2n + 1)$. At high temperatures $T \approx T_c \gg \Delta$ the main contribution to the current comes from the lowest Matsubara frequency and Eq. (10) can be simplified further

$$j_c = j_b \int_{-\pi/2}^{\pi/2} d\phi \cos \phi \times$$

$$\exp[-2\pi T d/v_F \cos \phi] \cos[2h_{\perp} d \tan \phi/v_F], \quad (12)$$

where $j_b = e v_F N_F \Delta^2/(\pi^2 T)$. Similar expression has already been obtained for Dirac materials. It is seen from Eqs. (9)-(11) that the Josephson current manifest strong dependence on the orientation of the ferromagnet magnetization. It is sensitive to the $y$-component of the magnetization via the anomalous phase shift, therefore this component does not lead to superconductivity suppression in the interlayer and does not influence the amplitude of the critical current. At the same time $x$-component of the magnetization does not couple to superconductivity via the anomalous phase shift, but causes the superconductivity depairing in the interlayer leading to the suppression of the critical current. The suppression of the critical current as a function of $m_x = M_x/M_s$ is presented in Fig. 2. For estimates we take $d = 50 nm$, $v_F = 10^5 m/s$ and $T_c = 10 K$, what corresponds to the parameters of Nb/Bi$_2$Te$_3$/Nb Josephson junctions. In this case $\xi_N = v_F/2\pi T_c \approx 12 nm$. We have also plotted $j_c(m_x)$ for $T_c = 1.8 K$, what corresponds to the Josephson junctions with Al leads.

It is difficult to give an accurate a-priori estimate of $h_{eff}$ because there are no reliable experimental data on its value. However, basing on the experimental data on the Curie temperature of the magnetized TI surface states, where the Curie temperature in the range $20 – 150 K$ was reported, we can roughly estimate $h_{eff} \sim 0.01 – 0.1 h_{eff}$. We assume $h_{eff} \sim 20 – 100 K$ in our numerical simulations, what corresponds to the dimensionless parameter $r = 2h_{eff} d/v_F = 2.6 – 13.2$. The both parts of the strong dependence of the Josephson current on the magnetization orientation (the dependence via anomalous phase shift as well as the dependence via the amplitude of the critical current) manifest themselves in the electrical response of the junction on the magnetization dynamics, as we demonstrate below.
netic equation due to superconductivity, because they lead to the corrections in the final expression for the normal current of the order of $(\Delta/T_c)^2$, which can be safely neglected near the critical temperature. In this approximation we neglect the terms of the same order $(\Delta/T_c)^2(eV/T_c)$ as in the Josephson current, so as in the normal current. The interlayer of the junction is assumed to be shorter than the inelastic energy relaxation length, therefore all the inelastic relaxation processes are neglected in Eq. (13). We have also expanded $\varphi$-products up to the lowest order with respect to time derivatives: $A \otimes B \approx AB + (i/2)(\partial_t A \partial_t B - \partial_t A \partial_t B)$.

We solve Eq. (13) neglecting the term $\dot{\varphi}$ and assuming that the deviation of the distribution function from equilibrium is small $\varphi = \tanh(\varepsilon/2T) + \delta \varphi$. The solution should also satisfy the asymptotic values $\varphi_\pm = \tanh[(\varepsilon \mp eV/2)/2T] \approx \tanh(\varepsilon/2T) \mp (eV/4T) \cosh^{-2}(\varepsilon/2T)$ at $x = \mp d/2$ if we assume that the leads are in equilibrium except for the voltage drop $V$ between them. In this case the solution of Eq. (13) can be easily found and takes the form:

$$\varphi_\pm = \tanh(\varepsilon/2T) \mp \frac{eV}{4T} \cosh^{-2}(\varepsilon/2T) \frac{(x \pm d/2)}{2T \cosh^2(\varepsilon/2T)} v_{F,x}. \tag{14}$$

It is seen that the term $\dot{\varphi}$ can be neglected if $d/(v_F t_d) \ll 1$, where $t_d$ is the characteristic time of magnetization variations. For estimates we take $d = 50 \text{nm}$, $v_F = 10^6 \text{m/s}$ and $t_d = 0.5 \times 10^{-8} \text{s}$ (this value of $t_d$ corresponds to material parameters of YIG thin films, see below). In this case $d/(v_F t_d) \sim 10^{-4}$ and the term $\dot{\varphi}$ can be safely neglected.

Substituting Eq. (14) into Eq. (6), for the quasiparticle contribution to the current we finally obtain

$$j_n = \frac{e^2 N_F v_F}{\pi} (V - \frac{h_y d}{e v_F}). \tag{15}$$

It is seen that in the presence of magnetization dynamics there is an electornative force $\mathcal{E} = \hbar_d d/(e v_F)$ in the TI resulting from the emergent electric field induced due to the simultaneous presence of the time-dependent exchange field and spin-momentum locking.

III. ELECTRICAL RESPONSE ON MAGNETIZATION DYNAMICS

A. Open circuit

Now we consider a long ferromagnetic strip on top of the 3D TI surface. The domain structure of the strip can be moved along $x$-axis by an electric current or by a magnetic field. At first we consider an insulating ferromagnet. Let us consider a setup presented in Fig. 3. Two junctions discussed above are connected to the TI surface. For each of the junctions the total electric current $j$ through the junction is a sum of the supercurrent contribution Eq. (9) and the normal current Eq. (15) flowing via the TI surface states. It can be rewritten as follows:

$$j = j_e \sin(\chi - \chi_0) + \frac{1}{2eR_N} (\dot{\chi} - \dot{\chi}_0) \tag{16}$$

For the open circuit $j = 0$ and the solution of Eq. (16) is $\chi(t) = \chi_0(t)$. Therefore, the voltage generated at the junction due to magnetization dynamics in the ferromagnet is

$$V = \frac{\dot{\chi}}{2e} = \frac{\hbar_y d}{e v_F}. \tag{17}$$

The voltage is determined by the dynamics of the magnetization component perpendicular to the current direction. It is the same as for superconducting leads, so as for nonsuperconducting leads at $j_e = 0$ and is only determined by the emf. This is the consequence of the fact that as well as the emf so as the anomalous phase shift are manifestations of the same gauge vector potential, which is only determined by the spin-momentum locking and magnetization and is not influenced by superconductivity.

![FIG. 3. Sketch of the system under consideration. Detector 1 consists of superconducting(normal) leads L1 and L2, while detector 2 is formed by leads L2 and L3. For all the calculations we take $d_{12} = d_{23} = d$.](image)
magnetization. For the open circuit considered here this contribution is absent and we also neglect it in the next subsection, where we assume \( j \neq 0 \), because our goal is to demonstrate by the simplest example how the magnetization dynamics can be read by means of electrical detection. We find \( M(x,t) \) numerically from the Landau-Lifshitz-Gilbert (LLG) equation

\[
\frac{\partial M}{\partial t} = -\gamma M \times H_{\text{eff}} + \frac{\alpha}{M_s} M \times \frac{\partial M}{\partial t},
\]

where \( M_s \) is the saturation magnetization, \( \gamma \) is the gyromagnetic ratio and \( H_{\text{eff}} \) is the local effective field

\[
H_{\text{eff}} = \frac{H_K M_y}{M_s} e_y + \frac{2A}{M_s^2} \nabla_z^2 M - 4\pi M_z e_z + H_{\text{ext}} e_y.
\]

The voltage, measured by detector 1(2) is denoted by \( V_{y(x)} \) and probes \( M_{z(x)} \). The result of the numerical calculation of \( M_{x,y,z}(t) \) at the detectors is presented in panels (a) of Figs. 4-5, while the corresponding dependencies \( V_{x,y}(t) \), which should be measured at detectors 1 and 2 are presented at panels (b) of these figures. As the characteristic size of the region containing both detectors \( \sim 100nm \) is assumed to be much smaller than the characteristic scale of magnetic inhomogeneity \( d_W \), we can safely use our Eq. (17), which is strictly valid for a homogeneous magnetization, for calculation of the voltage at the detector.

Fig. 4 demonstrates the results for small enough external applied field \( H_{\text{ext}} = 0.003 M_s \). This field is below the Walker’s breakdown field and in this regime the DW moves keeping its initial plane structure. Having at hand \( V_{x,y}(t) \) it is possible to restore the time-dependent structure of the moving wall at the detector point according to the relation \( \Delta M_{x,y}(t)/M_s = \int_{t_i}^{t_f} V_{y,x} dt \), where \( V_0 = dh_{\text{eff}}/|e|v_F t_d \) is the natural unit of the voltage induced at a given detector. The characteristic time of magnetization variation can be obtained from LLG equation and takes the form \( t_d = 1/(4\pi \alpha \gamma M_s) \). Taking material parameters of YIG thin films we obtain that \( t_d \sim 0.5 \times 10^{-8} \text{s} \).

Fig. 5 represents the results for higher applied field \( H_{\text{ext}} = 0.02 M_s \), which exceeds the Walker’s breakdown field. In this regime the DW does not keep its initial plane shape during motion. It is reflected in the oscillations of all the magnetization components, as it is seen in Fig. 5(a). In this regime the magnetization at the detector point as a function of time can also be found as described above.

Observation of a single DW travelling through the detector region gives a way to find experimentally the effective exchange field \( h_{\text{eff}} \) induced by the FI in the surface layer of the TI according to the relation

\[
h_{\text{eff}} = \frac{ev_F}{2d} \int_{t_i}^{t_f} V_x dt,
\]

where \( V_z \) should be integrated over the whole time region, where the voltage is nonzero. Typical values of the voltage induced at the detectors are of order \( V_0 \). It is difficult to give an accurate a-priori estimate of \( V_0 \) because there are no reliable experimental data for \( h_{\text{eff}} \). However, basing on the experimental data discussed above and assuming \( h_{\text{eff}} \sim 20 - 1000 \Omega \) and \( d/(v_F t_d) \sim 10^{-4} \), as it was discussed above, we obtain \( V_0 \sim 0.2 - 10 \mu V \).

If the ferromagnet is metallic, there is also an additional normal current flowing via the ferromagnet. The resistance of the ferromagnet is typically much smaller than the resistance of the TI surface states. For this reason the voltage induced at the junction due to the presence of the emf in the TI is suppressed by the factor \( R_F/(R_F + R_N) \), where \( R_{F(N)} \) is the resistance of the ferromagnet (TI surface states). Therefore, ferromagnetic metals are not good candidates for measuring the discussed effect. In addition, as it was already mentioned, the time-dependent spin texture of a ferromagnet also gives rise to emergent spin-dependent electric and magnetic fields in it and, consequently, to an emf in it. It can interfere with the emf generated in the TI and the

![FIG. 4. (a) Magnetization components \( m_{x,y,z} = M_{x,y,z}/M_s \) in the region of the detectors and (b) voltages \( V_{x,y} \) at detectors 1,2 as functions of time. \( H_{\text{ext}} = 0.003 M_s \), for other parameters of the numerical calculation see text.](image-url)
FIG. 5. (a) Magnetization components $m_{x,y,z} = M_{x,y,z}/M_s$ at the detectors and (b) voltages $V_{x,y}$ at detectors 1,2 as functions of time. $H_{ext} = 0.02M_s$, the detectors are located in the spatial region, where the DW motion is steady.

resulting effect is quite complicated.

In principle, a similar structure of the emf can also be obtained for systems where a topological insulator is replaced by a material with Rashba spin-orbit coupling or if the Rashba spin-orbit coupling is an internal property of the ferromagnetic film arising, for example, from structural asymmetry in the $z$-direction. However, we consider the TI-based systems as a more preferable variant because here the effect should be stronger. The emf is also predicted to be proportional to $h_y$ in Rashba spin-orbit based junctions, but in addition it should contain a reducing factor $\Delta_\alpha/\varepsilon_F \sim \alpha_R/hv_F$. This factor can be estimated taking a realistic value of $\alpha_R \sim 10^{-10}eV_F$, for the interfaces of heavy-metal systems. Then $\Delta_\alpha/\varepsilon_F \sim 0.1$ if one assumes $v_F \sim 10^6 m/s$. Therefore, it reduces the value of the emf, which is important for the magnetization detection.

**B. Response under the applied electric current**

Now we turn to the case when the constant electric current $j$ is applied to the junction. In this case the voltage $V = \dot{\chi}/2e$ should be found from Eq. (16). If $j_c$ does not depend on time, the solution of this equation represents the well-known solution of the current-biased Josephson junction $V_0(t)$ shifted due to presence of magnetization dynamics $V = V_0(t) + \chi_0/2e$. This leads to appearance of

the nonzero resistance of the IV-characteristics at $j < j_c$ and an additional resistance at $j > j_c$. However, the specific feature of the TI-based system is the strong dependence of $j_c$ on $h_x$, what results in the strong dependence of $j_c$ on time in the presence of magnetization dynamics.

In Figs. 6(a) we present results of the electrical response of the Josephson detectors on the moving DW under the constant applied current flowing via the detectors. These figures are plotted for $h_{eff} = 100K$, when the dependence of the critical current on the magnetization orientation is given by the blue solid curve in Fig. 2. We take the value of the applied current $j = 0.9j_\alpha$. Depending on the particular magnetization orientation this value of the applied current can be as below so as higher than the critical current of the junction for a given orientation, see Fig. 2. Figs. 6(a)-7(a) demonstrate the voltages induced at the detectors as functions of time, while the critical currents of the detectors are shown in panels (b).

We denote the critical current of the junction formed by electrodes of detector 2(1) as $j_{cx(cy)}$. Fig. 6 corresponds to the parameters of Fig. 4(a), when the magnetic field driven the DW motion is below the Walker’s breakdown, and Fig. 6 represents the case when it exceeds the Walker’s breakdown field and the magnetization of the moving DW oscillates, as in Fig. 5(a).

FIG. 6. (a) Voltages $V_{x,y}$ at detectors 1,2 as functions of time. (b) Critical currents $j_{cx}$ and $j_{cy}$ as functions of time. $j = 0.9j_\alpha$, $h_{eff} = 100K$, $j_\alpha R_N = 1\mu V$, $V_0 = 0.75\mu V$. The other parameters are the same as for Fig. 4.

According to Eq. (12) the critical current $j_{dy(x)}$ depends on time via the dependence of $h_{y(x)}$ for detector
The oscillating DW moving under the field higher than the Walker’s breakdown, presented in Fig. 7, is more complicated. The magnetization oscillations are clearly seen in the dependence of the critical current on time. These oscillations manifest themselves in the dependencies $V_{x,y}$ and for the chosen parameters interfere with Josephson oscillations.

When there is no a DW inside the detector region, $j_{cx}$ is fully suppressed by this effective field component, because for this detector it is parallel to the current. When a DW passes through the detector region the situation changes: $j_{cx}$ is suppressed by nonzero $h_{eff,x}$, and $j_{cy}$ is restored due to suppression of $h_{eff,y}$ in the region occupied by the DW.

The $j_{cx(y)}(t)$ dependence on time manifests itself in the voltage induced at the corresponding Josephson junction. As it is seen from Figs. 6(a), 7(a), voltage $V_y$ is nonzero when there is no DWs in the detector region. This detector is in the resistive state because its critical current is less when the externally applied current. When a DW travels via the detectors region we observe as $V_x$, so as $V_y$ voltage pulses. These pulses are the results of two different effects: (i) the contribution due to emf, the same as in the open circuit discussed above and (ii) the contribution due to the dependence of the critical current on time, which leads to the time dependence of the phase difference $\chi$ between the superconductors. For the example under consideration $j_{cx}$ becomes lower than the applied current when the DW passes through the detector region. This leads to appearance of the Josephson oscillations, which are seen in Fig. 6(a).
because the applied current does not exceed the critical current of the detectors.

In principle, the contributions to the voltage from the emf and from the time-dependence of the critical current can be separated. For example, if one considers the dynamics of a DW wall with perpendicular anisotropy located in the \((x, z)\)-plane, then for detector 2 there is no in-plane exchange field component, which is perpendicular to the current. Therefore, the emf contribution does not occur at this detector. At the same time, the parallel to the current component of \(h_{\text{eff}}\) is absent at detector 1. Consequently, the critical current does not depend on time for this detector, and the emf is the only contribution to the voltage.

IV. CONCLUSIONS

The electrical response of the S/3D TI-F/S Josephson junction on magnetization dynamics is studied and compared to the electrical response of the junction with nonsuperconducting leads. In 3D TI/F hybrid structures spin-momentum locking of the 3D TI conducting surface states in combination with the induced magnetization leads to appearance of a gauge vector potential. In the presence of magnetization dynamics the gauge vector potential becomes time-dependent and generates an electromotive force. In both cases, as for superconducting, so as for nonsuperconducting leads this emf generates a voltage between the leads. For an open circuit this voltage is the same for both normal and superconducting leads and allows for electrical detection of magnetization dynamics, a structure of a time-dependent magnetization and a measurement of the effective exchange field. In the presence of the applied current the electrical response of the Josephson junction contains additional contribution from the time dependence of the critical Josephson current, which comes from the strong dependence of the critical current on the magnetization orientation.

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