ASPECTS OF NEUTRALINO DARK MATTER

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ABSTRACT

The possible solution of dark matter problem with neutralinos of supersymmetric models within the supergravity framework is reviewed. A novel correlation between the neutralino relic abundance $\Omega_\chi$ and the soft supersymmetry breaking patterns is demonstrated. It is explained that, this generic result together with the proton-decay constraint could significantly reduce the allowed parameter space of the minimal $SU(5)$ supergravity model, and therefore makes this model more easily testable. The prospect of obtaining further cosmological constraints from underground experiments for the minimal $SU(5)$ supergravity model is also briefly discussed.

1. Introduction

Low-energy supersymmetry is now widely believed to be one of the most appealing ideas likely responsible for new physics beyond the Standard Model [1], as such, it has a very good chance to reveal itself at next generation of $pp$ or $e^+e^-$ colliders, one way or another [2]. If we indeed live in a “SuperWorld”, whatever that might be, then low-energy supersymmetry must also have cosmological consequences. This motivated the study of “SuperCosmology”, of which many exciting topics have been covered in previous talk by Prof. Schramm [3]. In this talk, I will confine myself to the proposal [4, 5] of solving the dark matter problem with neutralinos of supersymmetric models, as one typical example which ties up low-energy supersymmetry with cosmology.

Let me begin with some remarks about the so-called dark matter problem. First of all, it is a problem caused by the huge discrepancy between the amount of directly observed visible matter and the total amount of matter in the Universe indirectly inferred based on general relativity (mostly Newtonian dynamics) [4]. One should keep an open mind that, if at large scale the law of gravity is in fact somehow modified, the dark matter problem itself could cease to exist [7]. I will not entertain such a possibility here, but instead just follow the majority point of

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view. In terms of the present-day cosmological density parameter $\Omega \equiv \rho_0/\rho_{\text{crit}}$, the visible matter of the Universe only amounts to $\Omega_{\text{vis}} \lesssim 0.01$. Big-bang nucleosynthesis indicates that the fraction of critical density contributed by ordinary baryons is $0.011 \lesssim \Omega_B \lesssim 0.12$. However, current theoretical prejudice prefers a flat universe today, in particular, inflation predicts $\Omega = 1$ to a very high precision. Furthermore, without dark matter as an essential ingredient, the mechanism of structure formation in the Universe, based on the gravitational growth of primeval density inhomogeneities (Jeans instability), does not seem to work for galaxy formation. There are also observational evidences for dark matter coming from various measurements of the present-day mass density of the Universe, performed at different distance scales, among them the flat rotation curves of spiral galaxies appears to be most compelling. Based on these arguments, one concludes that dark matter must exist and be mostly in non-baryonic form.

Accepting the dark matter problem as a fact, now the question is: what is dark matter made of? So far, we do not have a definite answer, but we do have many speculations (maybe too many!). Instead of going through the list of all proposed dark matter candidates, which can be found elsewhere [4], I will only consider one type of hypothetical particle dark matter which is closely associated with low-energy supersymmetry, i.e., the neutralino dark matter [5, 6]. It should be noted that, from particle physics point of view, massive neutrinos with mass around 30 eV and axions are also particularly attractive dark matter candidates in their own right.

We know that low-energy supersymmetry predicts many new particles yet to be discovered, it would be extremely fortunate if some of these particles in fact constitute the dark matter out there in the Universe. To be a possible dark matter candidate, a supersymmetric particle has to satisfy at least three requirements: (a) to be stable or at least have a life-time comparable with the age of the Universe so that it can be around today as a cosmological relic; (b) to have only gravitational and weak interactions; (c) to have right properties such that its relic abundance comes close to the critical value $\Omega = 1$. In most supersymmetric models, to eliminate the disastrous explicit baryon and lepton number violating interaction terms which lead to proton decay at unacceptable level, a discrete symmetry known as $R$-parity is often invoked, such that all ordinary particles are $R$-even, but all superparticles are $R$-odd. Therefore, if $R$-parity is conserved, superparticles always couple to the ordinary particles in pair, and as a result the lightest superparticle (LSP) is stable. The LSP is also likely to be colorless and electrically neutral, since otherwise it would have formed heavy anomalous isotope with ordinary matter which would have been observed already [3, 4]. These considerations tell us that, with unbroken $R$-parity, the LSP in fact could make a viable dark matter candidate, if requirement (c) is also satisfied. Unfortunately, no general remarks can be made about this requirement for LSP, mainly because we do not really know what is the LSP. The lightest neutralino $\chi$, a mixture of neutral gauginos and higgsinos, which I will simply call neutralino in the rest of this talk, currently stands as a front runner for the LSP. The basic argument in favor of this theoretical prejudice is essentially that,
so far there have been no cosmological, experimental or theoretical reasons against it. In any case, as a working hypothesis, I will assume that the lightest neutralino is indeed the LSP. As we will see later, in the supergravity models, since the complete particle mass spectrum of the model can be specified rather economically in terms of only a few parameters, it becomes possible to consistently check the validity of this very assumption.

In the following, I will discuss some recent work regarding the prospects for neutralino dark matter within the supergravity framework, paying special attention to the correlation between the neutralino relic abundance $\Omega_\chi$ and the supersymmetry breaking patterns. Taking the minimal $SU(5)$ supergravity model as an explicit example, in which the physics at grand unification scale come into play, I will demonstrate the implications of this novel correlation, and in passing also address the technical issue of appropriately treating the thermal average cross section. In addition, assuming that neutralinos constitute the dark matter in the Galactic halo, and using the upward-going muon events in underground detectors as a possible signal from such neutralinos captured in the Earth and the Sun, I will also briefly discuss the prospect of further exploring the minimal $SU(5)$ supergravity model.

2. Neutralino Relic Abundance

Now let me examine the requirement (c) for neutralinos in more detail. Will neutralinos provide just right amount of mass to close the Universe? From earlier work [5, 6, 16, 17], we have learned that the answer to this question is YES and NO. Yes, because it is possible to find regions in the parameter space of the supersymmetric models where neutralino relic abundance is just what we would like it to be ($\Omega_\chi \sim 1.0$); No, because it is equally true that there are regions where either $\Omega_\chi \ll 1.0$ or $\Omega_\chi \gg 1.0$. Needless to say, regions with $\Omega_\chi \sim 1.0$ are cosmologically favored. The regions of $\Omega_\chi \gg 1.0$ should be excluded on cosmological grounds, since where the Universe is uncomfortably younger than about 10 billion years, and this argument alone often leads to interesting constraints on the models under consideration. On the other hand, if $\Omega_\chi \ll 1.0$, the best one can learn is that now neutralinos can not be the sole source of dark matter. Despite this, I want to stress that models with too small values of $\Omega_\chi$ are perfectly healthy, while on the contrary models which predict $\Omega_\chi \gg 1.0$ are definitely in trouble. Clearly, one would like to be able to identify these three distinct cases as precisely as possible.

2.1 Calculation Procedures

The basic physics involved in calculating relic abundance for any massive stable particles was well understood some time ago [9], which can be applied to the neutralino case. At very early time the Universe was radiation dominated, all particles would be in thermal equilibrium, neutralinos annihilate into other particle species, and vice versa. As the temperature drops down below the neutralino mass $m_\chi$, the annihilation process becomes dominant and the neutralino number begins to decrease due to Boltzmann suppression, until the interactions between neutralinos
“freeze out”, which happens when the annihilation is no longer able to keep pace with the expansion of the Universe. After “freeze out”, the number of neutralinos essentially remains constant and the number density only reduces as a consequence of the cosmological expansion. All of these are neatly embodied into the Boltzmann equation

\[
\frac{dn}{dt} = -3Hn - \langle \sigma v_{M\phi} \rangle (n^2 - n_0^2),
\]

where \(n\) is the actual number density of the neutralinos, \(n_0\) is the density they would have in thermal equilibrium at temperature \(T\), \(H = (dR/dt)/R\) is the Hubble expansion parameter, and \(\langle \sigma v_{M\phi} \rangle\) is the thermal-averaged product of annihilation cross section and the Møller velocity of the annihilating neutralinos in the cosmic comoving frame.

In practice, it is convenient to replace time \(t\) in Eq. (1) by the photon temperature \(T\), this is because the present-day cosmic background radiation (CBR) temperature can be measured rather accurately, while determining the age of the Universe is a quite different story (see e.g. Ref. [10]). Using the conservation of entropy, one can recast Eq. (1) into an convenient form involving \(T\). Although there exist slightly different approaches in the literature, I will closely follow Ref. [11] here. The “new” Boltzmann equation now reads

\[
\frac{dq}{dx} = \lambda(x)(q^2 - q_0^2)(x),
\]

with

\[
\lambda(x) = (\frac{4}{45} \pi^3 G_N)^{-1/2} \frac{m_\chi}{\sqrt{g(T)}}[h(T) + \frac{1}{3}m_\chi x h'(T)] \langle \sigma v_{M\phi} \rangle,
\]

where \(q \equiv n/(T^3 h(T))\), \(q_0 \equiv n_0/(T^3 h(T))\), \(x \equiv T/m_\chi\), and \(g(T), h(T) (h'(T) = dh/dT)\) are the effective degrees of freedom associated with energy and entropy density respectively [11]. Finally, the relic abundance is given by

\[
\Omega_\chi h_0^2 = 1.555 \times 10^8 (m_\chi/GeV) h(0) q(0),
\]

where \(h_0 = H_0/(100\text{kmsec}^{-1}\text{Mpc}^{-1})\) parameterize our ignorance of the present-day value of the Hubble parameter (0.5 \(\leq h_0 \leq 1.0\)); \(h(0), q(0)\) are the present-day values of \(h(T), q(T)\) respectively.

From Eqs. (2)-(3) it is clear that the calculation of relic abundance typically involves three procedures: (I) Computing \(h(T), g(T)\), in particular, \(h(0)\); (II) Evaluating \(\langle \sigma v_{M\phi} \rangle\); (III) Solving Eq. (2) for \(q(x)\) and find \(q(0)\). Again, one can often find different ways of implementing these procedures in literature. For a detailed discussion of our approach to (I) and (III), see the Appendix of Ref. [12]. It is worthy of mentioning that, in solving Eq. (2), a WKB approximation [13] was used in Ref. [12], which matches the solution \(q(x)\) with \(q_0(x)\) and provides the initial condition for Eq. (2) at a point \(x_0\) where the WKB solution fails. Although, it is the same in spirit to determine \(x_0\) here as to determine the so-called freeze-out temperature in the standard Lee-Weinberg method [5, 6], however, one advantage of this new approach is that the accuracy of the solution can be easily controlled as one desires.
The most accurate treatment so far available for the thermal average factor \( \langle \sigma v_{Mbi} \rangle \) was given in Ref. [14]. In this remarkable paper, the authors were able to reduce the multiple integrals associated with the thermal average into a single integral, which can be rewritten in the following convenient Lorentz invariant from [41]

\[
\langle \sigma v_{Mbi} \rangle = \frac{1}{4m^2_M xK_2^2(x-1)} \int_{4m^2_M}^{\infty} ds (s - 4m^2_M)^{1/2} K_1(\sqrt{s}/x m) w(s),
\]

in terms of the Lorentz invariant function [11]

\[
w(s) = \frac{1}{4} \int dLP S |A(\chi \chi \rightarrow all)|^2.
\]

In Eq. (5), \( K_i \) (\( i = 1, 2 \)) are the modified Bessel functions of order \( i \). Eq. (2) is only sensitive to the value of \( \lambda(x) \) for \( x \leq 0.1 \) which roughly corresponds to the freeze-out temperature, and the center-of-mass energy \( \sqrt{s} \geq 2m_\chi \), as a result the argument of \( K_1 \) in Eq. (5) \( \sqrt{s}/x m \chi \geq 20 \) so that \( K_1 \) dies away quickly with increasing \( \sqrt{s} \), owing to the asymptotic behavior of the Bessel function \( (K_1(y) \sim \sqrt{\pi/2ye^{-y}}, y \ll 1) \). Therefore, replacing \( w(s) \) in (6) by its series expansion around \( \sqrt{s} = 2m_\chi \) and the Bessel functions by their asymptotic expansions for large argument, the resulting series of integrals can be readily carried out analytically leading to [11]

\[
\langle \sigma v_{Mbi} \rangle = \frac{1}{m^2_\chi} \left[ w - \frac{3}{2} (2w - w') x + \frac{3}{8} (16w - 8w' + 5w'' + O(x^3)) \right]_{s=4m^2_M} \\
\equiv a + bx + cx^2 + O(x^3),
\]

where primes denote derivatives with respect to \( s/4m^2_M \) rather than \( s \) itself.

Contrary to Eq. (5) which inevitably requires numerical integration, the expansion such as Eq. (7) or its non-relativistic counterpart is simple to use and gives overall fairly good results. In fact, most of the relic abundance calculations were carried out using such thermal average expansion [3, 6, 16, 17]. Recently, however, the degree of accuracy of such expansions has been reexamined, and now it is clear that such treatment actually fails badly near the s-channel resonances and/or new-channel thresholds, basically because the expansion of \( w(s) \) (or equally, cross section for this matter) at \( \sqrt{s} = 2m_\chi \) becomes inappropriate in those cases [14, 15]. In addition, in Ref. [14] it is shown with explicit examples that, when close to the s-channel resonances the first-order expansion \( a + bx \) is much better behaved than the second-order expansion \( a + bx + cx^2 \), in fact latter renders the thermal average factor negative right above the poles which doesn’t make sense. Of course, in this case, one should abandon the expansions and use the exact integral expression (5) to get reliable results. Later in subsection 2.4, I will discuss a practical situation where a proper treatment of the thermal average factor is needed and present the results calculated with Eq. (5) and with first-order expansion \( a + bx \).

2.2 Organizing Model Parameters: Supergravity Models

So far, I have kept my discussion fairly model-independent, obviously, to go further we need the information of specific supersymmetric model. Primarily, the
details of the particle physics model enter the function $w(s)$, which is related to the annihilation cross section of neutralino pair into all kinematically accessible final states. What makes the analysis of these cross section fairly complicated is mainly the fact that supersymmetric models often contain many free parameters which to certain degree are rather arbitrary. For instance, in the minimal supersymmetric standard model (MSSM) [18], which happens to be the playing ground of almost all previous studies on neutralino dark matter [3, 4, 16, 17], even if the mixing between generations are neglected and only the Yukawa terms for the third generation are kept, there are still at least 21 free parameters, many of which describe the soft supersymmetry breaking at low energies. With so many parameters, any thorough analysis becomes hopeless. Nevertheless, to make life easy, in all these earlier works, several additional assumptions about these parameters are made, notably, the GUTs relation among different gaugino masses, and a common mass parameter for all sleptons and squarks. This situation is hardly satisfactory. We know that supersymmetry must be broken, however, just as we do not really understand the gauge symmetry breaking mechanism, our knowledge about supersymmetry breaking is even more limited. The phenomenology of low-energy supersymmetry should reflect how supersymmetry is broken, and therefore could in principle provides us with some information on the pattern of supersymmetry breaking. Since any such information is precious, it would thus be very important if one can also obtain constraints on the soft supersymmetry breaking terms from the cosmology of neutralino dark matter. Clearly, in the usual approach, particularly due to the ad-hoc assumption about the masses of sleptons and squarks, no “fine structure” of supersymmetry breaking is left to be found. On the other hand, as I mentioned earlier, to address such problem with all the soft-breaking parameters being arbitrary simply is not possible. One way out is provided by $N = 1$ supergravity [19]. Although it was realized long ago that local supersymmetry, instead of global supersymmetry, should be the natural framework in which to construct realistic low-energy supersymmetric models, only very recently has the study of neutralino dark matter in supergravity models attracted enough attention [21, 22, 12, 23, 24, 25, 26, 27, 28].

In supergravity models, in addition to the observable sector, which contains quarks, leptons, Higgs bosons, gauge bosons as well as their superpartners, and in case of a grand unified model also the extra particles, there is also a so-called hidden sector responsible for the spontaneous breaking of $N = 1$ local supersymmetry. At low energies, the breaking of supergravity taking place in the hidden sector transmits into the observable sector via gravitational interaction, and therefore leads to a globally supersymmetric effective theory with explicit soft-breaking terms. One of the virtue of the supergravity models is that the number of soft-breaking parameters are greatly reduced. In fact, the supersymmetry breaking pattern now can be specified mainly in terms of only three universal parameters at some unification scale $M_U$: (I) the scalar mass $m_0$; (II) the Majorana gaugino mass $m_{1/2}$; and (III) the trilinear $A$ and bilinear $B$ scalar couplings. Furthermore, in supergravity models the breaking of the electroweak gauge symmetry can be realized as a consequence of the supersymmetry breaking through radiative correction [20], with this radiative
In the model with minimal particle content same as that of MSSM, up to the sign of Higgs mixing parameter $\mu$, one only needs two more parameters to describe the whole model, these can be chosen as the unknown top quark mass $m_t$ and the ratio of Higgs vacuum expectation values $\tan \beta = v_2/v_1$, and finally all the couplings and superparticle masses are determined dynamically via the relevant renormalization group equations (RGEs) as functions of the above five parameters [29].

2.3 Supersymmetry Breaking and Dark Matter

I am now ready to describe the correlation between the neutralino relic abundance $\Omega_\chi$ and the supersymmetry breaking patterns, first found in Ref. [23], as one of the most interesting features of the supergravity models we are considering.

The idea is very simple. Since now all the low-energy couplings and superparticle masses entering $w(s)$ depend explicitly upon soft-breaking parameters $m_0$, $m_{1/2}$ and $A$, the relic abundance $\Omega_\chi$ itself becomes a function of these parameters as well. The most crucial factors here are the masses of the gauginos, squarks and sleptons at low energies, which are determined as functions of soft-breaking parameters through the renormalization equations. First, the gaugino masses ($M_i, i = 1, 2, 3$) are given by $M_i = (\alpha_i/\alpha_U)m_{1/2}$, where $\alpha_U = 0.0409$ is the gauge coupling at the unification scale $M_U \approx 10^{16}$ GeV (determined from following input low-energy values: $\alpha_3 = 0.111$, $\sin^2 \theta = 0.233$ and $\alpha = 1/127.9$). Second, neglecting the Yukawa coupling contributions, the renormalization equations of the sfermion mass can be solved exactly giving [30]

$$m_f^2 = m_f^2 + m_0^2 + m_{1/2}^2 c_f - M_0^2 \frac{\tan^2 \beta - 1}{\tan^2 \beta + 1} [(T_{3f} - Q_f) \tan^2 \theta_W + T_{3f}^2]$$

$$= m_f^2 + (1.22M_2)^2(c_f + \xi_0^2) + D_f$$

(8)

where $\xi_0 = m_0/m_{1/2}$, and the coefficients $c_f$ are determined to be: $c_{\tilde{e}_L, \tilde{\nu}_L, \tilde{\nu}} = 0.514, c_{\tilde{e}_R, \tilde{\nu}_R} = 0.150, c_{\tilde{\nu}_L, \tilde{e}_L} = c_{\tilde{d}_L, \tilde{s}_L} = 6.134, c_{\tilde{u}_R, \tilde{e}_R} = 5.720, c_{\tilde{d}_R, \tilde{s}_R} = 5.670$. The parameter $A$ enters the off-diagonal terms of the third generation scalar masses. In Ref. [23] only the left-right mixing for stop quark masses is considered, and the diagonal contributions for all the third generation sfermions are approximated with the $\xi$'s given above and Eq. (8). It is found that the major effect of $A$ is to restrict the allowed parameter space, otherwise $A$ does not change the neutralino abundance $\Omega_\chi$ significantly. In what follows, I choose $A = m_0$.

In Figs. 1–3, the neutralino relic abundance $\Omega_\chi$ is shown in the $(\mu, M_2)$ plane for $\tan \beta = 8$, $\mu > 0$ ($\mu < 0$ case is similar) [1] and three representative values of $\xi_0$: (a) $\xi_0 = 10.0$, (b) $\xi_0 = 1.0$, (c) $\xi_0 = 0.1$. Here only the tree-level Higgs masses are used, and the lightest CP-even Higgs mass is taken to be $m_h = 45$ GeV, different choice of this parameter leads to similar results. I also take $h_0 = 0.5$, a favorite choice of cosmologists [14], and divide the allowed parameter space into three types of distinct regions: (1) regions represented by stars corresponds to $\Omega_\chi > 1.0$, which are excluded cosmologically; (2) regions represented by vertical crosses corresponds to $0.1 < \Omega_\chi < 1.0$, which are cosmologically favored; (3) regions represented by dots

\footnote{Here the sign convention of $\mu$ is opposite to that of Refs. [12, 23].}
corresponds to $\Omega_\chi < 0.1$, where neutralinos can not even account for enough dark matter in Galactic halos.

In these figures, the shape of the allowed parameter space in each case was determined by several constraints. The direct experimental constraints used are: (1) The LEP lower bound on the chargino mass $m_{\chi^\pm} > 45$ GeV, and on the slepton mass $m_{\tilde{l}} > 43$ GeV; (2) The CDF lower bound on the gluino mass $m_{\tilde{g}} > 150$ GeV, which translates into $M_2 > 45$ GeV, and on the squark mass $m_{\tilde{q}} > 100$ GeV. In addition, the assumption about neutralino being the LSP imposes important consistency constraints on the allowed parameter space as well. For example, in Fig. 3 a portion of the upper right corner is excluded since there right-handed sleptons become lighter than the neutralino. Also, in all these figures, the allowed minimal value of $M_2$ is in fact bigger than that required by the gluino mass lower bound; and when $\xi_0$ decreases, this lower bound on $M_2$ increases. It is because that we do not want the sneutrinos to be lighter than the neutralino. Eq. (8) clearly shows that this constraint becomes stronger for small value of $\xi_0$. Finally, in the lower right corner of these figures, there is a triangle region where the left-right mixing term for stop quark masses could drive the $\tilde{t}_1$ mass below either the CDF bound or $m_{\chi}$, and thus it should be excluded. This effect is only barely visible in Fig. 2, but it becomes more pronounced for small values of $\tan \beta$.

As in the case of MSSM [17], Figs. 1–3 once again exhibits the fact that the neutralinos relevant to cosmology should either contain a dominant higgsino component or a dominant bino component. In addition, two very interesting new features emerge in Figs. 1–3, due to the supergravity squark and slepton relation (8). First, in the pure higgsino region (upper left portion of the ($\mu, M_2$) plane), we see that the relic abundance $\Omega_\chi$ does not change with the different values of $\xi_0$. This is because there the contributions to the annihilation cross section due to the exchange of sfermion have already been considerably suppressed, so the variation of squark and slepton masses with supersymmetry breaking patterns essentially has no effect on $\Omega_\chi$ for nearly pure higgsinos. Independent of how supersymmetry is broken, pure higgsino dark matter candidate will have a mass of roughly the order of $M_W$, but there the gluino would be heavier than about 1.5 TeV. However, if one insists that $m_{\tilde{g}} \lesssim 1$ TeV to insure the fine-tuning of the parameters occurs at two-orders-of-magnitude or less [29], this possibility of nearly pure higgsino dark matter would be eliminated. Second, in the pure bino region (lower right portion of the ($\mu, M_2$) plane), however, $\Omega_\chi$ changes with $\xi_0$ significantly, mainly because the sfermion exchange contribution is dominant in this region. From Eq. (8) we see that large value of $\xi_0$ implies all sfermions are heavy, so this contribution is suppressed, which leads to a large value of $\Omega_\chi$. When $\xi_0$ decreases, the annihilation due to sfermion exchange becomes increasingly efficient, and therefore $\Omega_\chi$ reduces. Note that $m_f$ now vary throughout the ($\mu, M_2$) plane due to their $M_2$ dependence, and in the case of stop quark $\tilde{t}_{1,2}$ their $\mu$ dependence as well.

The above discussion is mainly concerned with the impact of the supergravity relation Eq. (8) for the squark and slepton masses on the neutralino relic abundance $\Omega_\chi$. In this analysis, the radiative electroweak gauge symmetry breaking [20] and
the radiative corrections to Higgs masses \cite{31} were not considered. These two issues also have important consequences on the cosmology of neutralino dark matter \cite{23, 27, 28}. The first new feature is that, after enforcing the requirement of radiative electroweak symmetry breaking, the allowed parameter space is considerably reduced \cite{27}. In the usual \((\mu, M_2)\) plane, the two coordinates can no longer vary independently. For example, for \(\xi_0 \lesssim 1.0\), this correlation between \(\mu\) and \(M_2\) could eliminate a rather large triangle region below the diagonal which are otherwise allowed, since \(\mu \lesssim m_{1/2} = 1.22 M_2\). In addition, since the one-loop radiative corrected Higgs masses vary continuously as one moves around the \((\mu, M_2)\) plane, and they are normally bigger than their tree-level values, the overall effect is a suppression of the relevant annihilation rate and hence an enhancement of the relic abundance \(\Omega_\chi\) relative to that shown in Figs.1–3, as long as the regions in comparison now are still allowed. Notice that pure binos only couple to Higgs very weakly, so such an enhancement of \(\Omega_\chi\) mainly occurs in the “mixed” regions. This makes the “mixed” neutralino a possible candidate to be the major component of the Galactic halo. Of course, the qualitative feature of the novel correlation between the neutralino relic abundance \(\Omega_\chi\) and the supersymmetry breaking patterns, as demonstrated in Figs.1–3, remains the same. In fact, the prospects for neutralino dark matter depend most strongly on the parameter \(\xi_0\) \cite{23, 27}, which can be summarized as follows: (1) For \(\xi_0 \sim 1.0\) there is a wide range of the other parameters such that \(\Omega_\chi \sim 1.0\); (2) For \(\xi_0 \ll 1.0\) the relic abundance normally is too small, but it is possible in this case that some “mixed” neutralinos may still be able to account for the dark matter in the Galactic halo; (3) For \(\xi_0 \gg 1.0\) the relic abundance is almost always much too large in conflict with current cosmological observations, except the accidental circumstances where the relic abundance could be locally diluted due to the presence of resonances and thresholds in the annihilation cross section.

2.4. The Minimal SU(5) Supergravity Models

In the supergravity models, one of the crucial assumptions is that a unification of gauge couplings and mass parameters takes place at some high-energy scale \(M_U\) not far from the Planck scale. One simple way to realize this is to invoke a grand unification type of symmetry. 

\footnote{However, in the context of superstring theories such unification arises naturally even in the absence of a grand unification group.}

Up to now, my discussion of the correlation between the neutralino relic abundance \(\Omega_\chi\) and the soft supersymmetry breaking patterns has not relied on the details of any specific grand unification models. Now, I would like to consider the implications of this generic result in the minimal \(SU(5)\) supergravity model \cite{32}.

At low-energies, the minimal \(SU(5)\) supergravity model consists of the normal light particles of the MSSM. Again, the masses and couplings of these light particles are completely specified in terms of a few parameters as described in subsection 2.2. The new feature of this model (as in all grand unified models) is the existence of the additional heavy degrees of freedom arisen around \(M_U \sim 10^{16}\) GeV, which would lead to new phenomena such as proton decay \cite{33}. In the minimal \(SU(5)\) super-
gravity model, due to its supersymmetric nature, the usual proton decays through dimension-six operators mediated by exchange of either heavy gauge bosons or heavy triplet Higgs bosons, which plague the ordinary non-supersymmetric $SU(5)$ model [33], are strongly suppressed and place no danger as compared with the experimental limit. However, the new proton decays through the genuinely supersymmetric dimension-five operators [34] mediated by exchange of heavy higgsinos are not very strongly suppressed. Note, for the dimension-five-induced amplitude the suppression factor is $1/(M_U M_W)$, while for the normal dimension-six amplitude it is $1/M_U^2$. These dimension-five-induced processes could lead to rather dangerous proton decays, typically in the modes $p \rightarrow \bar{\nu}_\mu, \tau K^+$ [35].

Recently, in light of the current experimental limit $\tau_{p \rightarrow \bar{\nu} K^+} > 10^{32}$ yr, detailed analysis of the dimension-five-induced proton decay constraints have been carried out [36, 37, 38] in the minimal $SU(5)$ supergravity model. Assuming the exchanged triplet higgsino mass to be bounded above by $M_{\tilde{H}_3} < 3M_U$ [36], it is found that the proton decay constraint is rather restrictive. In particular, as for the soft supersymmetry breaking patterns, the proton decay favors a large value of $\xi_0$ [36, 38]. Again, this result can be easily understood in view of Eq. (8). Since squarks and sleptons appear in the loops of the box diagrams responsible for the dimension-five-induced proton decays, large value of $\xi_0$ implies that such processes are suppressed. From the discussion in previous subsection, however, we see that the cosmology of neutralino dark matter, on the other hand, disfavors large value of $\xi_0$. Therefore, a delicate balance have to be attained in order to satisfy these two constraints simultaneously in the minimal $SU(5)$ supergravity model.

The conflict between these two types of constraints in the minimal $SU(5)$ supergravity model was first pointed out in Ref. [39], which has since spurred further investigations on this subject [38, 40, 41]. Considering the combined effect of these two constraints, it is found that the allowed parameter space of the minimal $SU(5)$ supergravity model is dramatically reduced (see below). However, there is still a region in the parameter space where both constraints are simultaneously satisfied. It turns out that, in this region the neutralino $\chi$ is actually very close to the lightest CP-even Higgs ($h$) and/or $Z$-boson resonances, i.e., $m_\chi \approx \frac{1}{2}m_{h,Z}$. This fact has cast doubts [40] on the accuracy of the results obtained in Refs. [34, 38], since there the first-order expansion of form (7) has been used to approximate the thermal average factor, which in general is not a valid approach (see subsection 2.1). Two groups [40, 41] have recently reassessed this problem by treating the thermal average factor properly.

In Ref. [41], an extensive search of the parameter space of the minimal $SU(5)$ supergravity model has performed. In practice, a data set of the five-dimensional parameter space is generated first, which gives adequate radiative electroweak gauge symmetry breaking and satisfies all known current phenomenological constraints as described in Ref. [29]. Then, the proton decay constraint is used to further reduce the allowed points in the data set. In this step, the effects of two-loop gauge coupling unification and light supersymmetric thresholds have also been included [42]. Finally, for all the remaining points in parameter space, which have $\tan \beta = 1.5, 1.75, 2.0,$
the neutralino relic abundance $\Omega_\chi$ is computed in two different approaches: (a) the thermal average factor is treated accurately using Eq. (5); (b) the thermal average factor is treated approximately using first-order expansion of form Eq. (7). The results are shown in Fig. 4 and Fig. 5 respectively. These figures indicate that indeed the cosmological constraint is very powerful: all the points above the solid (dashed) line, allowed by other constraints, are now excluded if $h_0 = 1.0$ ($h_0 = 0.5$). This result does not change even if the exact thermal average procedure is followed. From these two figures, it is clear that some shifts of the points are noticeable, and the actual structure around the resonances in Fig. 4, in particular that close to the Z-pole ($m_\chi \sim \frac{1}{2} M_Z$), is broader, shallower and asymmetric relative to Fig. 5. Moreover, the cosmologically allowed points in Fig. 4 are increased relative to that in Fig. 5. However, the qualitative distributions of the points in Fig. 4 and Fig. 5 are similar, and the difference between two cases appears to be not significant.

3. Indirect Search for Neutralinos with “Neutrino Telescope”

In this section, I wish to briefly report on our recent work [43] about the possibility of indirectly detecting neutralinos of some supergravity models with the so-called “Neutrino Telescope”, based on the assumption that neutralinos in these models constitute the Galactic halo. Here, I will only present the result for the minimal $SU(5)$ supergravity model, for detailed discussion see Ref. [43]. This subject has a rich history [46], more recent work in the context of MSSM can be find in Refs. [47, 48, 49, 50, 51, 52, 53].

If neutralinos populate the Galactic halo, some of them could be trapped [44, 45] when traveling through the Sun or Earth, after losing substantial amount of energy during their interactions with nuclei. The captured neutralinos sink to the core of the Sun or Earth, annihilate with one another therein, then produce a shower of ordinary particles which further decay or interact with solar or terrestrial material and finally lead to energetic neutrinos as one sort of ultimate products. The high-energy neutrinos coming from the Sun or the core of the Earth can be detected in underground detectors, via either (a) contained events, by looking for charged leptons within the detector (useful for detecting both $\nu_e$ and $\nu_\mu$); or (b) through-going events, by searching for upward-going muons which are the products of the interaction of neutrinos with rock below the detector (useful for detecting $\nu_\mu$). I will only consider the neutrino-induced upward-going muon events here, as it is most promising.

In the primary direct capture, a neutralino with mass $m_\chi$ passing through the Sun or Earth loses enough energy due to its elastic scattering with nuclei, so that its velocity falls below the escape velocity at one particular point inside the Sun or Earth. The capture rate can be written as

$$C = \left( \frac{2}{3\pi} \right)^{1/2} \frac{M_B \rho_\chi \bar{v}_\chi}{m_\chi} \sum_i \frac{f_i}{m_i} \sigma_i X_i,$$

where $M_B$ is the mass of the Sun or Earth, $\rho_\chi$ and $\bar{v}_\chi$ are the local neutralino density
and velocity in the halo respectively, $\sigma_i$ is the elastic scattering cross section of the neutralino with the nucleus of element $i$, and $X_i$ is a kinematic factor which can be obtained from Eq. (A10) of the second paper in Ref. [15]. The annihilation process normally takes a time scale much shorter than the age of the Sun or Earth to reach equilibrium with the capture process, in this case the neutralino annihilation rate, entering the high-energy neutrino flux, is just half of that given in Eq. (9).

The determination of neutrino flux or final detection rate is normally complicated, basically because the differential energy spectra of neutrinos depend on various subsequent physical processes that take place, see e.g Ref. [50]. The rather limited current knowledge about these processes remains the major source of the uncertainties in this type of analysis. Nevertheless, some reasonable approximations can be made to render this part of the problem tractable. For example, in Ref. [47] the neutrino spectrum from injected quarks and leptons was calculated by using the Lund Monte Carlo, the same procedure then was followed and refined in Ref. [50]. It turns out that, for the neutrino-induced upward-going muons events, the detection rate can be approximated as [47, 50]

$$\Gamma = \kappa_B \left( \frac{C}{\text{sec}^{-1}} \right) \frac{m_\chi}{\text{GeV}}^2 \sum_i a_i b_i \sum_F B_F \langle N z^2 \rangle_{F_i} m^{-2} \text{yr}^{-1},$$

(10)

where $\kappa_B = 1.27 \times 10^{-29}$ ($= 7.11 \times 10^{-21}$) for neutrinos from the Sun (Earth); the $i$-sum is over muon neutrinos and anti-neutrinos, and $a_i = 6.8$ (3.1), $b_i = 0.51$ (0.67) for neutrinos (anti-neutrinos); the $F$-sum is over all annihilation final states that produce high-energy neutrinos ($\tau\bar{\tau}, c\bar{c}, b\bar{b}, t\bar{t}, WW, ZZ, hA$ and $HA$ in the minimal $SU(5)$ supergravity model.), each with branching ratio $B_F$, and $\langle N z^2 \rangle_{F_i}$ is the second moment of the spectrum of type-$i$ neutrino from final state $F$ scaled by $m_\chi^2$.

In Fig. 6, the predicted detection rate of upward-going muon events in the minimal $SU(5)$ supergravity model prediction is shown as a function of $m_\chi$. The top and bottom row correspond to the event rate resulting from the capture of halo neutralinos by the Sun and Earth respectively. Also shown in Fig. 6 as solid lines are the current Kamionkande 90% C.L. upper limits [54] of $6.6 \times 10^{-14}\text{cm}^{-2}\text{sec}^{-1}$ (Sun) and $4.0 \times 10^{-14}\text{cm}^{-2}\text{sec}^{-1}$ (Earth). From this figure, we see that the minimal $SU(5)$ supergravity model is not further constrained by this type of underground experiments. However, it is interesting to note, if the experiment limits can be improved in the near future by a factor of 100, then the region above the dashed lines could be explored.

4. Conclusions

The neutralino dark matter problem within the supergravity framework is considered. In the supergravity models, all the particle masses and couplings can be specified in terms of three universal soft supersymmetry breaking parameters $m_0$, $m_{1/2}$ and $A$ at the unification scale $M_U$, along with low-energy parameters $m_t$ and $\tan \beta$. Also, the electroweak gauge symmetry can be broken radiatively. As a result, the neutralino relic abundance becomes strongly depends upon the way
supersymmetry is broken. If the soft supersymmetry breaking seed is primarily a universal scalar mass \((\xi_0 \gg 1.0)\), then the relic abundance is almost always much too large, and the cosmological constraint is very strong. The exception occurs if resonances or thresholds are present. On the other hand, if the soft supersymmetry breaking seed is a dominant universal gaugino mass \((\xi_0 \ll 1.0)\), the cosmological constraint is rather weak. In the case of \(\xi_0 \sim 1.0\) there is normally a wide range of the other parameters where neutralinos can provide a closure relic density \(\Omega_\chi \sim 1.0\). In the minimal \(SU(5)\) supergravity model, since the dimension-five-induced proton decay also imposes very restrictive constraint which is somewhat in conflict with the cosmological constraint, the parameter space of this model is dramatically reduced, therefore, makes this model more easily testable. Assuming neutralinos constitute the dark matter in the Galactic halo, the upward-going muon events in underground detectors provide yet another probe to explore supergravity models. However, in order to obtain further useful constraints for the minimal \(SU(5)\) supergravity model, the current experimental limits would have to improve by a factor of about 100 in the future.

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Figure 1: The neutralino relic abundance distribution for \( \tan \beta = 8, h_0 = 0.5, \xi_0 = 10 \). The meaning of the three different symbols: (1) stars (\( \Omega_\chi > 1.0 \)); (2) crosses (\( 0.1 < \Omega_\chi < 1.0 \)); (3) dots (\( \Omega_\chi < 0.1 \)).
Figure 2: Same as Fig. 1, but with $\xi_0 = 1.0$. 
Figure 3: Same as Fig. 1, but with $\xi_0 = 0.1$. 


Figure 4: The neutralino relic abundance in the minimal $SU(5)$ supergravity model as a function of $m_\chi$ calculated using the exact thermal average procedure. The points above the solid (dashed) line are excluded if $h_0 = 1.0$ ($h_0 = 0.5$).
Figure 5: Same as Fig. 4, but the thermal average factor is approximated with the first-order expansion.
Figure 6: The detection rate for the neutrino-induced upwards-going muon events in the minimal $SU(5)$ supergravity model as a function of $m_{\chi}$. The top and bottom row show the event rate resulting from the capture of halo neutralinos by the Sun and Earth respectively. The solid lines in the figures are the corresponding 90% C.L. upper limits of Kamiokande. The dashed lines indicate where such upper limits will be in the future if the experiment capability is improved by a factor of 100.
