Adaptive Mesh Simulations of Multi-Physics Processes During Pellet Injection in Tokamaks

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Abstract. We present the results of fully 3D adaptive mesh MHD simulations of fueling pellets injected into tokamaks. The Chombo framework for block structured local adaptive mesh refinement (AMR), extended to use the equilibrium magnetic coordinates, is employed to mitigate the problems due to the large range of spatial scales. Generalized upwinding techniques are employed to deal with sharp gradients. The modeling includes a semi-analytical kinetic treatment of the transport of electron energy flux which drives the ablation. We discuss the phenomenology of the mass redistribution processes involving the density equilibrating along field lines and transport across surfaces (in the large-major-radius direction).

1. Introduction

ITER ("The Way" in Latin), a joint international research and development project that aims to demonstrate the scientific and technical feasibility of fusion power, is now under construction at Cadarache, France. Refueling of ITER is a practical necessity due to the burning plasma nature of the experiment, and longer pulse durations ($\mathcal{O}(100) - \mathcal{O}(1000)$ seconds). An experimentally proven method of refueling tokamaks is by pellet injection \cite{1, 2}. Pellet injection is currently seen as the most likely refueling technique for ITER. Furthermore, the use of "killer" pellets has been suggested as a method for fast shutdown of large tokamaks \cite{8}. Thus it is imperative that pellet injection phenomena be understood via simulations before very expensive experiments are undertaken in ITER. The emphasis of the present work is to understand the large-scale macroscopic processes involved in the redistribution of mass into a tokamak during pellet injection. Arguably, such large scale processes are best understood using magnetohydrodynamics as the mathematical model.

Pellet Injection: One can loosely classify the launch trajectories of pellet injection into two categories: HFS (or High-Field-Side) launch in which the pellet is injected from the inside of the tokamak; and LFS (or Low-Field-Side) launch in which the pellet is injected from the outside of the tokamak. It was experimentally observed that the HFS launches of the pellet lead to a significantly better fueling efficiency of the tokamak core compared with LFS launches. Identifying and quantifying the causal MHD mechanisms in HFS vs. LFS pellet launches are the goals of this research.

As a frozen pellet of hydrogen or deuterium is injected into the hot plasma tokamak fusion reactor it is rapidly heated by long mean-free-path electrons streaming along magnetic field
Table 1. Resolution requirements for pellet injection for three tokamaks. $N$ is the number of spatial points and $N_{steps}$ is the number of time steps required for an explicit method.

| Tokamak            | Major Radius (m) | $N$            | $N_{steps}$ | Spacetime Points |
|--------------------|------------------|----------------|-------------|------------------|
| CDXU (Small)       | 0.3              | $2 \times 10^9$ | $2 \times 10^{12}$ | $4 \times 10^{12}$ |
| DIII-D (Medium)    | 1.75             | $3.3 \times 10^{9}$ | $7 \times 10^6$ | $2.3 \times 10^{12}$ |
| ITER (Large)       | 6.2              | $1.5 \times 10^{11}$ | $9 \times 10^7$ | $1.4 \times 10^{16}$ |

lines, leading to ablation at the frozen pellet surface, with a shield of neutral gas and an ionized high density plasma cloud around it. This forms a local high $\beta$ plasmoid, i.e., a localized region of high pressure, which can trigger MHD instabilities [14]. Furthermore, the high-$\beta$ plasmoid expands along the magnetic field lines, and the plasma cloud drifts in the direction of the major radius. This drift was observed in experiments on ASDEX Upgrade tokamak, as well as DIII-D and JET [11]. Thus, HFS pellet injection showed a much better penetration and fueling efficiency than LFS injection.

**Numerical Challenges:** The physical dimensions of the pellet typically range from a sub-millimeter to a few millimeters in radius, which is three orders of magnitude smaller than the device size (for ITER the minor radius is 2m). Assuming uniform meshing, we estimate the computational resources required to simulate pellet injection in tokamak ranging from small (CDXU), medium (DIII-D) to large (ITER) in Table 1. It is clear that some form of adaptive meshing is necessary to overcome the large resolution requirements to adequately resolve the high-density cloud region around the pellet.

The large range of spatial scales is somewhat mitigated by the use of Adaptive mesh refinement (AMR). Our approach is to employ block structured hierarchical meshes as championed by the seminal work of Berger and Oliger [3] and Berger and Colella [4]. We employ the Chombo library for AMR developed by the APDEC SciDAC Center at LBNL. Refer to the Chombo documentation [5] for further details. The effectiveness of local mesh adaptiveness has been demonstrated in the context of pellet injection by Samtaney et al. [13].

Apart from the wide range of spatial scales, there is a wide range of temporal scales ranging from the heat exchange time between electrons and singly charged ions to the slower drift of the ablated mass across magnetic flux surfaces. At present the electron heating is computed by a semi-analytical model (described later), which requires evaluating integrals of density along field lines. This poses technical challenges on hierarchical adaptive meshes. Other numerical challenges stem from the large gradients in density and pressure at the edge of the pellet cloud. Traditional non-dissipative finite difference schemes have large dispersive errors. These large spatial gradients are effectively treated with high-resolution upwind methods which have been effectively used in shock-capturing in computational fluid dynamics.

### 2. Mathematical Models

We begin by writing the equations of compressible resistive MHD in near-conservation form in cylindrical coordinates. These equations describe the conservation of mass, momentum and energy; coupled with Maxwell’s equations for the evolution of the magnetic field.

\[
\frac{\partial U}{\partial t} + \left( \frac{1}{R} \right) \frac{\partial}{\partial R} \left( RU \right) + \frac{1}{\partial z} \left( Zu \right) + \frac{1}{\partial \phi} \left( u_{\phi} \right) = \left\{ \begin{array}{l}
1 \frac{\partial R F}{\partial R} + \frac{1}{R} \frac{\partial}{\partial z} \left( \frac{HF}{R} \right) + \frac{1}{\partial \phi} \left( \frac{D}{R} \right) \\
S_D + S_{pellet} + S_B \end{array} \right.
\]

where $U \equiv U(R, \phi, Z, t) = \{\rho, u, e\}$ and $u_R, u_Z, u_{\phi}$ are the radial, axial and azimuthal components of velocity, $B_R, B_Z, B_{\phi}$ are the components of magnetic field. Here $\rho$ is the density, $u_R, u_Z, u_{\phi}$ are the radial, axial and azimuthal components of velocity, $B_R, B_Z, B_{\phi}$ are the components of magnetic field.
of the magnetic field, and e is the total energy per unit volume. The hyperbolic fluxes, and source terms are given by

\[ F = F(U) = \begin{bmatrix} 
\rho u_R \\
\rho u_R \phi - B_R^2 \\
\rho u_R u_\phi - B_R B_Z \\
0 \\
u_R B_\phi - u_\phi B_R \\
u_R B_Z - u_Z B_R \\
e + p t) u_R - (B_k u_k) B_R
\end{bmatrix}, \quad S(U) = \begin{bmatrix} 
0 \\
B_\phi^2 - \rho u_\phi^2 - p t \\
\rho u_R u_\phi - B_R B_\phi \\
0 \\
u_\phi B_R - u_R B_\phi \\
0
\end{bmatrix}.
\]

The diffusive fluxes and source terms are given by the following

\[ F_D(U) = \begin{bmatrix} 
0 \\
T_{RR} \\
T_{R\phi} \\
T_{RZ} \\
0 \\
\eta_{kZ} \\
-\eta_{k\phi}
\end{bmatrix}, \quad S_D(U) = - \frac{1}{R} \begin{bmatrix} 
0 \\
-T_{\phi\phi} \\
T_{\phi R} \\
0 \\
0 \\
0 \\
0
\end{bmatrix}.
\]

In the interest of brevity, we have not written expressions for \( H(U), G(U), H_D(U) \) etc. In the above equations \( p_t \) includes the magnetic energy \( (p_t = p + 1/2B_k B_k) \). The equation of state gives \( e = p/(\gamma - 1) + 1/2B_k B_k + 1/2\rho u_k u_k \), where \( \gamma = 5/3 \) is the ratio of specific heats. In the above equations we have implicitly made use of resistive MHD form of Ohm’s law \( \mathbf{E} = -\mathbf{v} \times \mathbf{B} + \eta \mathbf{J} \), where \( \mathbf{E} \) is the electric field, and \( \mathbf{J} = \mathbf{v} \times \mathbf{B} \) is the current density. The stress tensor \( \mathbf{T} \) is related to the strain as \( \mathbf{T} = \mu(\nabla \mathbf{u} + (\nabla \mathbf{u})^T) - 2/3\mu \nabla \cdot \mathbf{u} \), where \( \mu \) is the viscosity. Other plasma properties are the resistivity denoted by \( \eta \), and heat conductivity denoted by \( \kappa \). The source terms denoted as \( S_{\text{pellet}} \), due to pellet-specific ablation and heating physics, are described later. In addition to the above we have the constraint of no magnetic monopoles expressed as \( \nabla \cdot \mathbf{B} = 0 \). An additional source term, \( S_{\nabla \cdot \mathbf{B}} \) (as suggested by Falle et al. \[6\]) and others) is included as an option to control \( \nabla \cdot \mathbf{B} \) in the absence of divergence cleaning.

A toroidal cross-section (i.e. constant \( \phi \) slice) of a tokamak is generally shaped by current-carrying coils leading to an elongated plasma with some triangularity. For such a shaped plasma, we employ a mapped grid in the \((R, Z)\) plane: \((\xi, \zeta) \rightarrow (R, Z)\) (this mapping is a diffeomorphism), where \( \xi \equiv \xi(R, Z), \zeta \equiv \zeta(R, Z) \). Conversely, \( R \equiv R(\xi, \zeta), \) and \( Z \equiv Z(\xi, \zeta) \). The Jacobian \( J \) and its inverse \( J^{-1} \) of the mapping are

\[ J = \left| \frac{\partial (R, Z)}{\partial (\xi, \zeta)} \right| = R_\xi Z_\zeta - R_\zeta Z_\xi, \quad J^{-1} = \left| \frac{\partial (\xi, \zeta)}{\partial (R, Z)} \right|. \]  \[ (2) \]

Note that \( R, Z, \xi \) and \( \zeta \) written with subscripts denote derivatives.

Equation (1) is then transformed to

\[ \frac{\partial \mathbf{U} J}{\partial t} + \frac{1}{R} \frac{\partial R \mathbf{F}}{\partial \xi} + \frac{1}{R} \frac{\partial \mathbf{R}}{\partial \xi} + \frac{1}{\rho} \frac{\partial \mathbf{G}}{\partial \phi} + \mathbf{S} = \frac{1}{R} \frac{\partial \mathbf{F}_D}{\partial \xi} + \frac{1}{R} \frac{\partial \mathbf{H}_D}{\partial \xi} + \frac{1}{R} \frac{\partial \mathbf{G}_D}{\partial \phi} + \mathbf{S}_D \]  \[ (3) \]

Here \( \mathbf{F} = J(\xi_R F + \xi_Z H) = Z_\zeta F - R_\xi H, \quad \tilde{H} = J(\zeta_R F + \zeta_Z H) = -Z_\xi F + R_\zeta H, \quad \tilde{G} = JG, \quad \tilde{S} = JS, \quad \tilde{F}_D = J(\xi_R F_D + \xi_Z H_D) = Z_\zeta F_D - R_\xi H_D, \quad \tilde{H}_D = J(\zeta_R F_D + \zeta_Z H_D) = -Z_\xi F_D + R_\zeta H_D, \quad \tilde{G}_D = JG_D, \) and \( \tilde{S}_D = JS_D. \)
Pellet Physics Models: In the present work, the pellet is described by a sphere of frozen molecular hydrogen with a prescribed trajectory. The source terms due to the pellet, $S_{\text{pellet}}$ are non-zero in the continuity and the energy equations, described below.

Electron heat flux model: A heat source $-\nabla \cdot \bar{q}$ is included in the energy equation. This arises from the energy deposition by hot long-mean-free-path plasma electrons streaming into the ablation cloud along magnetic field lines. It can be analytically approximated [7] as

$$-\nabla \cdot \bar{q} = \frac{q_{\infty} n(R, \phi, Z)}{\tau_{\infty}} [g(u_{+}) + g(u_{-})],$$

where, $n(R, \phi, Z)$ is the number density, $g(u) = u^{1/2} K_{1/2}(u^{1/2}) / 4$, and $K_{1}$ is the Bessel function of the second kind. $u_{\pm} = \tau_{\pm} / \tau_{\infty}$ is the dimensionless opacity, where

$$\tau_{+}(R, \phi, Z) = \int_{-\infty}^{R} n(r, \phi, z) \, ds,$$

$$\tau_{-}(R, \phi, Z) = \int_{R}^{\infty} n(r, \phi, z) \, ds,$$

are the respective line integrated densities penetrated by right-going (left-going) electrons arriving at the point $(R, \phi, Z)$ from infinity, and $\tau_{\infty} = \frac{T_{e}^{2}}{8\pi e^{3} m_{e} \Lambda}$, where $\Lambda$ is the Coulomb logarithm. The above model is derived by assuming half-Maxwellian distribution for electrons and neglecting pitch angle scattering.

Pellet Ablation Model: The electron heat flux incident on the pellet surface is computed, and at present we assume that the entire heat flux is used to ablate the pellet, i.e., that portion of the heat flux which contributes to ionization and dissociation is neglected. The pellet ablation rate is determined by dividing the heat flux by the sublimation energy of Deuterium. The pellet mass source term is expressed as $S_{p} = \rho \delta(x - x_{p})$, where $x_{p}$ denotes the surface of the pellet. The delta function is regularized as a Gaussian distribution with a characteristic size equal to a few pellet radii.

Initial and Boundary Conditions: The initial conditions correspond to a flow-free ideal MHD equilibrium obtained by solving the $\nabla p = J \times B$. The magnetic field is expressed as $B = \nabla \phi \times \nabla \psi + g(\psi) \nabla \phi$, where $\psi$ is the poloidal magnetic flux, and $g(\psi) / R$ is the toroidal component of the magnetic field. The pressure $p \equiv p(\psi)$ is expressed as a function of the poloidal flux. This leads to a nonlinear elliptic equation called the Grad-Shafranov (GS) equation. In this paper, we use Solovev’s analytical solution of the GS equation as the initial equilibrium. The domain of investigation is a one-eighth sector of a tokamak. The boundary conditions in the poloidal plane are that of a perfectly conducting wall, and zero-gradient in the $\phi$ direction.

3. Numerical Method
We use a finite volume technique wherein each variable is stored at the cell center. Presently we employ a method-of-lines approach and evolve the equations in time using a second order TVD Runge-Kutta method.

Hyperbolic fluxes: At each stage of the Runge-Kutta time integration the numerical fluxes of conserved quantities are obtained at the cell faces using upwinding methods. We first compute a vector of “primitive” variables $W = \{ \rho, u_{R}, u_{\phi}, u_{Z}, B_{R}, B_{\phi}, B_{Z}, p \}^{T}$, in each cell and use these to fit a linear profile in each cell subject to standard slope-limiting techniques. Given left and right states ($W_{L}$ and $W_{R}$, respectively) at each cell interface we use a seven-wave linearized Riemann problem to compute the solution at the cell interface as follows:

$$W_{rp} = W_{L} + \sum_{k, \lambda_{k} < 0} \alpha_{k} r_{k},$$

(5)
where $\alpha_k = l_k \cdot (W_R - W_L), \lambda_k, l_k$ and $r_k$ are the eigenvalues, left and right eigenvectors, respectively, of the Jacobian of the ideal MHD flux vector with respect to $W$. The solution to the Riemann problem is then used to compute the fluxes through the cell face. This method, as described above, is not guaranteed to be positivity preserving, i.e., negative pressures or densities may develop. If this occurs, we use the more robust and generally positivity preserving HLL flux as given below.

$$
F = \lambda_{\max} F(W_L) - \lambda_{\min} F(W_R) + \lambda_{\max} \lambda_{\min} (U_R - U_L) \over \lambda_{\max} - \lambda_{\min} 
$$

$\text{if } \lambda_{\min} < 0 < \lambda_{\max}$,

$$
F = F(W_L) \text{ if } \lambda_{\min} > 0, \quad F = F(W_R) \text{ if } \lambda_{\max} < 0, \quad (6)
$$

where $F$ is notationally a generic flux through a cell face, $U_L$ and $U_R$ are the conserved quantities on the left/right side of the cell face, and $\lambda_{\min}$, and $\lambda_{\max}$ are the minimum and maximum eigenvalues evaluated at the arithmetic average of the left and right state.

**Diffusive fluxes**: The diffusive fluxes in the resistive MHD equations are evaluated using second order central differences. These are evaluated explicitly in the present implementation which can have an adverse effect on the time step because the time step restriction based on diffusion is quadratic in the mesh spacing. In the future, we plan to treat these terms implicitly in a fashion similar to Samtaney et al. [12].

**Preserving the solenoidal property of $B$**: The source term $S_x B$, when included, advects divergence errors out of the domain, and ensure some measure of numerical stability. Alternatively, we use the the central difference version of the constrained transport algorithm proposed by Toth [15].

**AMR Implementation**: We now briefly describe the main issues in implementing the above algorithm using block structured adaptive meshes using the Chombo framework [5]. Each of the blocks is surrounded by a layer of guard cells which are filled either by exchanging data from sibling meshes at the same level or by inter-level interpolation from coarse to fine meshes. In the calculation of the second order accurate hyperbolic fluxes, linear interpolation is sufficient, while the diffusive fluxes require a quadratic interpolation. We employ the Berger-Oliger time stepping technique in which the time steps are determined by the CFL condition imposed by the ideal MHD wave speeds and are computed at the finest level and then appropriately coarsened by the refinement ratio to determine the larger stable time step for coarser levels. We maintain flux registers which are used during synchronization when disparate levels reach the same physical time. During the refluxing procedure the coarse level fluxes at coarse fine boundaries are replaced by the sum of the fine level fluxes. At the end of the time step on a coarser level, the solution on any part of the coarser mesh covered by a finer level mesh is replaced by averaging down the finer-level solution. So far what we have described is fairly standard practice for structured hierarchical AMR meshes. We will now state a few of the difficulties stemming from AMR on generalized curvilinear meshes. Generally, one has to be careful when refining or coarsening on curvilinear meshes because the volume and surface areas are not conserved upon coarsening or refinement [9] and special care is required to maintain conservation. We have circumvented this issue by using the following method. We have the curvilinear representation only in the poloidal section. We precompute the equilibrium on the mesh at the finest level. Recall that the equilibrium determines the flux surfaces and the corresponding mesh. For all the coarser levels, we coarsen the flux surfaces in a manner which preserves the volume and the areas of the cell faces. The metric terms and the Jacobian used in the equations are thus predetermined at all the levels and stored. While this may be deemed excessive, we remind the reader that this is only required for the two-dimensional poloidal cross-section.

**Electron heat flux on AMR Meshes**: The electron heat flux term in the energy equation involves the computations of opacities by integrating the density along magnetic field lines.
These integrals are first broken into two parts: the part inside the ablation cloud, and the part outside. We make the approximation that the medium beyond the region of the ablation cloud is transparent, relative to the opacity inside the cloud. We use $\kappa$ to designate this point on the integral curve.

$$
\int_{-\infty}^{0} (\rho - \rho_{amb}) \, ds = \int_{\kappa}^{0} (\rho - \rho_{amb}) \, ds + \int_{-\infty}^{\kappa} (\rho - \rho_{amb}) \, ds \\
\int_{-\infty}^{0} (\rho - \rho_{amb}) \, ds \approx \int_{\kappa}^{0} (\rho - \rho_{amb}) \, ds + 0
$$

(7)

The grid refinement strategy employed in this code ensures that the pellet cloud is completely contained within the finest refinement level. On the boundary of this refinement level $\tau_{\pm}$ is initialized to zero.

In this section we refer to Figure 1. The cells on the edge of the fine-grid boundary are initialized to $\tau_{\pm} = 0$. Each regular grid region contains a single layer of ghost cells that extends beyond the valid region of grid points. The upstream values for $\tau$ and $\rho$ ($T\tau, \rho$) are computed using linear interpolation between the upstream cells at point $T$, which is the computed using a linear approximation of the magnetic field from point $a$. We will also assume that $\rho_{amb}$ is very small compared to the density inside the pellet cloud.
The value of $\tau$ is initialized on the grid boundary. This computation is then iterated at each grid point until the algorithm converges. Points become updated with the correct value in a traveling wave for each of $\tau_+$ and $\tau_-$. In Figure 1 points \([a, c]\) have $\tau_+$ updated on the first iteration, while points \([b, d]\) are left un-updated. On the next iteration points \([b, d]\) can have $\tau_+$ updated. Eventually from the other boundaries the $\tau_-$ wave arrives and computes $\tau_-$ for points \([b, d]\), followed by points \([a, c]\) on the next iteration. In a parallel multi-block AMR calculation each Box has one cell wide ghost cells. After each Box performs integration of $\tau$, the values of $\tau$ in the ghost cells are filled from neighboring Boxes. Then Box-by-Box integration is repeated. This process is iterated until convergence.

4. Simulation Results

Pellet Injection: In this section we present results of pellet injection simulations using the methods described in the previous sections. The parameters of the simulation studies are as follows: the magnetic axis toroidal field is $B_T = 1 T$; the ambient plasma number density and temperature are, respectively, $n_\infty = 1.0 \times 10^{19} \text{ / m}^3$, and $T_\infty = 4 \text{ KeV}$; the major (minor) radius of the tokamak is $R_0 = 1 m$ ($a = 0.3 m$). The initial pellet radius is $r_p = 2.5 \text{ mm}$. The Reynolds and Lundquist numbers are: $Re = S = 10^5$.

The mesh parameters for this run are the following. The coarsest or 'level 0' mesh is $64^3$ which is refined by 3 levels using a refinement factor of two (i.e., effective mesh resolution is $512^3$), where the density exceeds a user specified threshold. This results in a mesh which can effectively resolve the pellet ablation cloud. We note that unless the pellet ablation cloud is resolved properly the heating due to the electrons, which is directly proportional to the local density, drops precipitously and results in an inaccurate transport of the ablated mass. The density and pressure during a HFS pellet launch are shown in Figure 2(a-b) in computational space. The pressure shows the characteristic signature due to the electron heating depicting the formation of a high-$\beta$ plasmoid. This is accompanied by a rapid local decrease in the toroidal magnetic field (not shown). The pellet mass moves along the field lines as expected. However, the radial velocity during HFS and LFS launches, (Figure 2(c-d)) show the outward radial migration of the heated pellet cloud. This observed motion of the pellet mass towards the low field side in these adaptive mesh three-dimensional simulations of pellet injection is in qualitative agreement with experimental observations.

Motion of a “fully ablated” pellet: To better understand the MHD mechanisms for the cross-flux surface transport, we conducted a series of simulations in which the initial conditions were chosen to mimic a fully ablated pellet imposed on a background equilibrium. The temperature of high-density region is varied from $T_b = T_0$ (“Hot” or “High-$\beta$” case) to $T_b = p_0 / \rho_0$ (“Cold”) and toroidal beta $\beta_T \in [0.06, 2.18]$. The pellet source terms are dropped for this set of simulations. In these simulations, we observe a rapid decrease in the local toroidal field followed by a translation of the high-density region in the radial direction (see Figure 3). The most likely explanation of this phenomena is that it is a nonlinear manifestation of an interchange instability. Necessary conditions for this instability to occur are $dB_T^2/dR > 0$ and $dp/dR < 0$ [10], which are both adequately satisfied where the high-$\beta$ plasmoid is created.

The radial migration of the high-$\beta$ plasmoid and a time history of the peak density radial location are plotted in Figure 4.
5. Summary and Future Work
Pellet injection is a viable method to refuel a tokamak, and will most likely be the fueling mechanism for ITER. We presented an adaptive mesh method for resistive MHD, including the electron heat flux models, to simulate pellet injection into a tokamak. We employed mapped curvilinear grids to handle realistic tokamak geometry. Without AMR resolved simulations of pellet injection become prohibitively expensive. It is conservatively estimated that AMR simulations saved upwards of two orders of magnitude in CPU time compared with uniform mesh simulations. Simulations using this code were used to investigate the differences and similarities between HFS and LFS pellet injection. It was found that, in qualitative agreement with experiments, that HFS leads to better core fueling than LFS launches. Furthermore, the most likely MHD mechanism for radial transport is identified to be an interchange instability. On the algorithmic side, in the future, we expect to use a unsplit multi-dimensional upwind method for the hyperbolic fluxes, and implicit treatment of the diffusion terms. On the physics side, we expect to conduct more numerical experiments, validate our results against existing experiments and make prediction for ITER-like parameters.

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Figure 2. Density (a) and pressure (b) during the HFS pellet launch. Radial velocity during HFS (c) and LFS (d) pellet launches.
Figure 3. (a-b) Density in computational space with AMR mesh blocks shown at early and late stages in the simulation. (b) $B_\phi$ shown at $t = 0.7, 2.7$ shows a strong local decrease in the toroidal field. (c) The radial velocity shown at times $t = 2.7, 7$.

Figure 4. (a) Average density profiles shown at approximately unity time intervals for the high-$\beta$ case. (b) Time history of radial position of peak density. At early times, the peak actually shifts left. The average speed of the highest $\beta$ region is $\approx 0.038V_A$, and the next one is $0.019V_A$. The high-$\beta$ case is also plotted at higher and lower resolutions.