Quantum kinetic theory for dense Coulomb systems in strong electromagnetic fields

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A quantum kinetic theory for correlated charged–particle systems in strong time–dependent electromagnetic fields is developed. Our approach is based on a systematic gauge–invariant nonequilibrium Green’s functions formulation. We concentrate on the selfconsistent treatment of dynamical screening and electromagnetic fields which is applicable to arbitrary nonequilibrium situations. Numerical results for the nonlinear plasma heating by the laser field and the electron–ion collision frequency including multi–photon absorption (inverse bremsstrahlung) are presented.

1 Introduction

With the progress in short–pulse laser technology, high intensity electromagnetic fields are now allowing to create strongly correlated quantum plasmas in extreme nonequilibrium conditions with unprecedented applications [1]. At the same time, optical techniques for time–resolved diagnostics are improving remarkably [2] creating the need for a quantum kinetic theory of dense non–ideal plasmas in intense laser fields. A central problem is the simultaneous account of correlations (scattering) between the particles and of the influence of the electromagnetic field. On the other hand, such a theory is of interest for quantum transport phenomena in solids subject to intense THz fields, e.g. [3], where the common rotating wave approximation cannot be applied.

In this paper we take advantage of the Kadanoff-Baym formalism to treat charged particles and plasmons in a fully symmetric way. The time–dependent electromagnetic field is treated classically. Using a gauge–invariant extension of the generalized Kadanoff-Baym ansatz, we obtain a closed equation for the Wigner distribution which is solved for various limiting cases.
2 Quantum kinetic equations for charged particles and longitudinal photons

We consider charged particles (electrons and ions/holes/positrons etc.) interacting via the longitudinal Coulomb force which is equivalently described in terms of emission and absorption of longitudinal photons. The nonequilibrium state of this system is conveniently characterized by correlation functions of fermions of species \( a, g_a \) and longitudinal photons (screened potential) \( V_s(1,1') \), which are defined as averages of the respective field operator pairs \( \langle \quad \rangle \), where we use the notation \( 1 = r_1, t_1 \). In particular, the density matrices of carriers and longitudinal photons follow from the “\( \langle \quad \rangle \)” functions, 
\[
\rho_a(r_1, r_1', t_1) = -i\hbar^{-1} g_a(1,1')|t_1,t_1'\rangle \rangle, \quad \langle \rho_a(1,1')| = -i\hbar^{-1} V_s(1,1')|t_1,t_1'\rangle \rangle.
\]

The transverse electromagnetic field is given by the vector potential \( A \) and will be treated classically, i.e. it obeys Maxwell’s equations (3). For completeness, we include also the longitudinal field which is due to external sources, \( \phi^{ext} \).

The time evolution of the correlation functions is determined by the Kadanoff–Baym equations [7]
\[
[i\hbar \frac{\partial}{\partial t_1} - \frac{1}{2m_a} \left( \frac{\hbar}{i} \nabla_1 - \frac{e_a}{c} A(1) \right)^2 - e_a \phi^{ext}(1)] g_a^{\pm}(1,1')
= \int \theta(1,t_1-t_1') \left[ \Sigma_a^{\pm}(1,1')g_a^{\pm}(1,1') - \Sigma_a^{\pm}(1,1')g_a^{\pm}(1,1') \right],
\]
\[
\Delta V_{ab}^{\pm}(1,1') = \sum_c \int_{-\infty}^{\infty} dI \left[ \Pi^{R}(1,1)V_{cb}^{\pm}(1,1') - \Pi^{R}(1,1)V_{cb}^{\pm}(1,1') \right],
\]
\[
\left\{ \Delta - \frac{1}{c^2} \frac{\partial^2}{\partial t^2} \right\} A(1) = -\frac{4\pi}{c} j(1).
\]
\[ V_{ab}^{R/A}(1, 1') = V_{ab}(1 - 1')\delta(t_1 - t_1') + \]
\[ + \sum_{cd} V_{ac}(1 - 1') \int d2 \Pi_{cd}^{R/A}(1, 2) V_{db}^{R/A}(2, 1'). \quad (6) \]

\( V_{ab} \) denotes the Coulomb potential between charged particles of species “a” and “b”). Eq. (6) has to be supplemented by its adjoint. Also, we will not consider correlated initial states here, for the corresponding generalization of the Kadanoff-Baym equations, see [8].

To close this system of equations, we have to specify the carrier and longitudinal photon selfenergies \( \Sigma \) and \( \Pi \). The simplest ansatz is the random phase approximation

\[ \Sigma_\alpha^{<}(1, 1') = i\hbar g_\alpha^{<}(1, 1') V_{\alpha\alpha}^{<}(1, 1'), \]
\[ \Pi_\alpha^{<}(1, 1') = -i\hbar g_\alpha^{<}(1, 1') g_\alpha^{<}(1, 1'). \quad (7) \]

Despite its simplicity, the system (2) - (7) [9] describes remarkably complex physical processes: the evolution in space and time of charged carriers interacting via the full dynamic Coulomb potential which in turn evolves selfconsistently (screening build up) and may have nonequilibrium modes (including instabilities and nonlinear phenomena). Furthermore, the dynamics is influenced by the transverse electromagnetic field \( A \) which contains external fields (e.g. a laser field) and induced contributions and obeys Maxwell’s equations.

In this classical treatment, \( A \) does not modify the equations for carriers and plasmons explicitly, but only indirectly, via the particle propagators \( g_{R/A} \).

Thus, a direct solution of the system (2) - (7) would yield a tremendous amount of information. However, here we proceed differently - we derive equations of motion for the Wigner distributions of the charged particles \( f_{\alpha}(k, t) \) which turn out to be simpler. To this end we consider the correlation functions \( g_{\alpha}^{<} \) on the time diagonal. To make the derivations independent on a particular gauge, it is advantageous to make the Green’s functions explicitly gauge-invariant [3,10]. For the case of a spatially homogeneous field \( E(t) \) it is convenient to choose \( \phi^{ext} = 0 \) and \( A(t) = -c \int_{-\infty}^{t} dt' E(t') \). Then, the gauge-invariant Green’s functions are obtained by the transform

\[ g_{a}(k, \omega; t) = \int d\tau dr \exp \left[ i \omega \tau - \frac{i}{\hbar} r \left( k + \frac{e_a}{c} \int_{t_1}^{t_1'} \frac{d\bar{t}}{\tau} A(\bar{t}) \right) \right] g_{a}(r, \tau; t), \quad (8) \]

where \( r = r_1 - r_1' \), \( \tau = t_1 - t_1' \) and \( t = (t_1 + t_1')/2 \). The resulting quantum kinetic equation is given by [10]

\[ \frac{\partial}{\partial t} f_{\alpha}(k, t) + e_{\alpha} E(t) \cdot \nabla_{k} f_{\alpha}(k, t) = -2Re \int_{t_0}^{t} d\tau \left\{ \Sigma_{\alpha}^{>} g_{\alpha}^{<} - \Sigma_{\alpha}^{<} g_{\alpha}^{>} \right\} = I_{\alpha}, \quad (9) \]
where the full arguments of the functions on the r.h.s. are,

\[ \Sigma^a g^a = \Sigma^a \left[ k_a + K^A_a(t, t); t, \bar{t} \right] g^a \left[ k_a + K^A_a(t, \bar{t}); \bar{t}, t \right], \quad (10) \]

with \( K^A_a(t, \bar{t}) \equiv \frac{e_a}{c} \int_{t'}^t dt'' \frac{A(t) - A(t'')}{t - t''} \).

Finally, to obtain a closed equation for the Wigner functions, the two-time functions \( \Sigma^a \) and \( g^a \) in Eq. (9) have to be expressed in terms of \( f^a \). The appropriate solution is the generalized Kadanoff-Baym ansatz \([11]\) which, in the case of time-dependent fields, has the form \([10,9]\)

\[ \pm g^a(k; t_1, t'_1) = g^R_a(k; t_1, t'_1) f^a \left[ k - K^A_a(t', t_1); t'_1 \right] \\
- f^a \left[ k - K^A_a(t, t_1); t_1 \right] g^A_a(k; t_1, t'_1), \quad (11) \]

where the upper (lower) sign refers to \( g^> (g^<) \), and \( f^< \equiv f \) and \( f^> \equiv 1 - f \).

Equations (9) and (10) are valid for any choice of the selfenergies. In particular, we can use the RPA expressions (7), which leads to a collision integral \( I^a \) which generalizes the result of Haug and Ell \([12,13]\) to the case of an external electric field. A particularly interesting phenomenon is that the dielectric and screening properties of the plasma are directly modified by the electromagnetic field:

\[ \Pi^R_{aa}(q; t, t') = -\frac{i}{\hbar} \Theta(t - t') e^{i q R_a(t, t')} \int \frac{d^3k}{(2\pi\hbar)^3} e^{-\frac{i}{\hbar}(\epsilon^a_k + \epsilon^a_{k+q})(t - t')} \\
\times \left\{ f_a \left[ k + Q_a(t, t'); t' \right] - f_a \left[ k + Q_a(t, t'); t' \right] \right\}, \quad (12) \]

where we used for \( g^{R/A} \) in Eq. (11) the propagators of a free particle with energy \( \epsilon \) in an electromagnetic field \([14]\). Further, \( Q_a \) and \( R_a \) are, respectively, the momentum gain and displacement of a free particle in a field \( E(t) \) during the time interval \([t', t]\) given by

\[ Q_a(t, t') = -e_a \int_{t'}^{t} dt'' E(t''), \quad R_a(t, t') = \frac{e_a}{m_a} \int_{t'}^{t} dt \int_{t'}^{t} d\tilde{t} E(\tilde{t}). \quad (13) \]

Obviously, the plasmon spectrum is modified in two ways: first, by the field-dependent prefactor of the standard RPA-polarization and, second, by the field-dependent momentum arguments of the distribution functions.

For the practical issue of plasma heating it is important to understand the scattering process of charged particles in the presence of the field \( E(t) \) in detail. Analytical and numerical progress can be made if the RPA selfenergies (7) are approximated by their static limit, which in Eq. (6) leads to the
substitution of the dynamic potential by the statically screened Debye potential, \( V_s^{R/A}(q, t_1, t'_1) \rightarrow V_{st}(q, t_1) \delta(t_1 - t'_1) \). As a result, the collision integral becomes

\[
I_a(k_a, t) = 2 \sum_b \int \frac{d\mathbf{k}_a d\mathbf{k}_b d\mathbf{k}_b}{(2\pi \hbar)^3} |V_{ab}^t(k_a - \mathbf{k}_a)|^2(2\pi \hbar)^3 \delta(k_a + \mathbf{k}_b - \mathbf{k}_a - \mathbf{k}_b) \\
\times \int_{t_0}^{t} d\bar{t} \cos \left\{ \frac{1}{\hbar} \left[ (\epsilon_{ab} - \bar{\epsilon}_{ab})(t - \bar{t}) - (\mathbf{k}_a - \bar{\mathbf{k}}_a) \mathbf{R}_{ab}(t, \bar{t}) \right] \right\} \\
\times \left\{ \bar{f}_a \bar{f}_b [1 - f_a] [1 - f_b] - f_a f_b [1 - \bar{f}_a] [1 - \bar{f}_b] \right\} |_t ,
\]

where we denoted \( \epsilon_{ab} = \epsilon_a + \epsilon_b, \epsilon_a = p_a^2/2m_a, f_a = f(K_a, \bar{t}), \bar{f}_a = f(\bar{K}_a, \bar{t}), \mathbf{K}_a \equiv \mathbf{k}_a + \mathbf{Q}_a \) and \( \mathbf{R}_{ab} \equiv \mathbf{R}_a - \mathbf{R}_b \). Although plasmon and screening dynamics are no longer included, Eq. (13) with the collision term (14) still contains important physics reflecting the influence of the electromagnetic field: (I) field-induced change of the arguments of the distribution functions, i.e. time-dependent generalization of the intra-collisional field effect; (II) modification of the energy balance in two-particle scattering [argument of the cosine in Eq. (14)] - a strong field may essentially modify the energy broadening in the electron-ion integral; (III) nonlinear (exponential) dependence of the collision integral on the field strength - this leads to the generation of higher field harmonics in the scattering processes. Furthermore, it gives rise to scattering processes which involve emission (absorption) of photons, i.e. (inverse) bremsstrahlung. Indeed, it is straightforward to show (10) that, for an electric field \( E(t) = E_0 \cos \Omega t \) transport quantities computed from (14) will contain contributions proportional to \( \int_0^\infty dz J_n^2(z) \delta(\epsilon_{ab} - \epsilon_a + q \omega_{ab}(t) - n\hbar \Omega), (-\infty < n < \infty) \), where the amplitude of an n-photon process is given by the Bessel functions \( J_n \). The argument \( z \) of \( J_n \) is determined by the field strength and frequency, \( z = q|v_a^0 - v_b^0|/\hbar \Omega \), where \( v_a^0 = e_a E_0/m_a \Omega \) and \( \omega_{ab} = |v_a^0 - v_b^0| \sin \Omega t \).

3 Numerical results

The kinetic equation (3) with the collision integral (14) is a convenient starting point for numerical investigations. We underline that it fully contains quantum effects, therefore, in contrast to classical equations, no cutting procedures at large momenta are required. Before presenting results of direct numerical solutions, we outline the idea for an approximate treatment (9,10).

If in Eq. (3) the collision term is much smaller than the field term on the l.h.s., one can use an ansatz proposed by Silin (10), \( f_a = f_a^0 + f_a^0 \), where \( f_a^0 \) obeys the collisionless equation, with the solution \( f_a^0(k, t) = f_{a0} [k + \frac{e}{\epsilon_k} \mathbf{A}(t)] \), and
the correction \( f_a^1 \) follows from Eq. (9) with \( I_a[f_a] \rightarrow I_a[f_n^0] \). Further simplifications are possible if the distributions are Maxwellians. Then, most of the integrations in the collision term can be performed, and analytical results for field-dependent transport quantities can be derived such as the conductivity, Fig. 1, or the cycle-averaged energy gain per particle, \( \langle \mathbf{j} \cdot \mathbf{E} \rangle = \frac{1}{T} \int_{-T}^{T} dt' \mathbf{j}(t') \cdot \mathbf{E}(t') \), for which one obtains

\begin{align*}
\langle \mathbf{j} \cdot \mathbf{E} \rangle = & \frac{8\sqrt{2\pi^2}Z^2e^4n_e^2n_i\sqrt{m_e}}{(k_BT)^{3/2}} \Omega^2 \sum_{n=1}^{\infty} n^2 \int_0^{\infty} dk \frac{k}{(k^2 + k_D^2)^2} \exp \left\{ -\frac{n^2m_e\Omega^2}{2k_BTk^2} \right\} \\
& \times \exp \left\{ -\frac{\hbar^2k^2}{8m_ek_BT} \right\} \sinh \frac{\hbar\Omega}{2k_BT} \int_0^1 dz J_n^2 \left( \frac{eE_0k}{m_e\Omega^2} z \right),
\end{align*}

with the Debye screening wave number \( k_D^2 = \lambda_D^{-2} = 4\pi n_e e^2/k_BT \). We underline that quantum effects occur in (15) in two places: first, the exponential function in the second line automatically ensures the convergence of the integral for large \( k \) and, second, the Bose-Einstein statistics of photons is reflected by the factor with the sinh function. For a classical plasma, both factors approach 1, and one recovers the result of Klimontovich [17]. Fig. 1 shows results for the conductivity computed using the Silin ansatz with the statically screened and the dynamically screened e-i collision integral. Also, we show a fit formula of Silin which holds for classical plasmas, but fails at low temperatures.

Finally, we present results of direct numerical integration of the quantum kinetic equation (9) with the collision term (14). This has the advantage that no assumption on weak collisions or shape of the distributions has to be made. The heating of the electrons for three different densities is shown in Fig. 2, and Fig. 3 shows the evolution of the electron distribution during the first laser cycle. One clearly sees the anisotropy of the distribution and its broadening due to absorption of laser energy. A detailed analysis shows that, in fact, multi photon absorption (inverse bremsstrahlung) occurs which will be discussed elsewhere.

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Figure 1: Conductivity of a hydrogen plasma with density \( n = 5 \times 10^{21}\text{cm}^{-3} \) vs. temperature in a strong high frequency laser field with \( \lambda = 248.5\text{nm}, \Omega/\omega_{pl} = 1.9, v_0/v_\text{th} = 5 \).

Figure 2: Electron heating due to Coulomb collisions in a strong laser field for different densities: \( n_e = 10^{22}\text{cm}^{-3} \) (full line), \( n_e = 10^{23}\text{cm}^{-3} \) (dashes) and \( n_e = 10^{24}\text{cm}^{-3} \) (dots). Remaining parameters same as in Fig. 3. (The straight lines are a guide for the eye.)
Figure 3: Evolution of the electron distribution during the first laser period. $E(t=0) = 10^8 \text{V/cm}$, figures correspond to subsequent quarter periods. Parameters are for hydrogen, see inset.