CONSTRANITS ON HIGHER DIMENSIONAL MODELS FOR VIABLE EXTENDED INFLATION

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Abstract
We consider two kinds of higher dimensional models which upon dimensional reduction lead to Jordan-Brans-Dicke type effective actions in four dimensions with the scale factor of the extra dimensions playing the role of the JBD field. These models are characterized by the potential for the JBD field which arises from the process of dimensional reduction, and by the coupling of the inflaton sector with the JBD field in the Jordan frame. Taking into account the fact that these models allow the possibility of enough inflation and dynamical compactification of the extra dimensions, we examine in the context of these models the other conditions which need to be satisfied for a viable scenario of extended inflation. We find that the requirements of conforming to general relativity at the present epoch, and producing suitable bubble spectrums during inflation lead to constraints on the allowed values taken by the parameters of these models. A model with a ten dimensional JBD field is able to satisfy the condition for appropriate density perturbations viewed in the conformal Einstein frame, with a stringent restriction on the initial value taken by the scale factor of the extra dimensions.

PACS number(s): 98.80.Cq, 11.10.Kk

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1 Introduction

Gravitational field theories in which the spatial dimensions exceed the usual three, have been widely studied in the last several years [1]. A primary reason for this interest is the fact that several of these higher dimensional models are obtained in the low energy (point particle) limit of string theories [2]. The former are used to investigate the cosmological consequences of the latter models, and several aspects of dilaton cosmology have been revealed recently [3]. A key common feature of higher dimensional models is that upon dimensional reduction they lead to scalar-tensor theories [4] of gravity in the four spacetime dimensions, of which the Jordan-Brans-Dicke [5] (JBD) model is one particular example.

It is well known that scalar-tensor theories in four dimensions act as candidates for extended inflation (EI) [6]. The scenario of EI restores the spirit of ‘old’ inflation [7] in the sense that inflation is driven by the vacuum energy of a scalar field (inflaton) trapped in a metastable state, which subsequently tunnels out of the potential barrier through a predominantly first order phase transition by the formation of bubbles of true vacuum [8]. Although there exist several models which can implement EI (to the extent of alleviating the typical problems of other versions of inflation like ‘old’ inflation [7], ‘new’ and ‘chaotic’ inflation [9]), this scheme encounters some of its own characteristic problems such as the ‘ω-problem’, which have created obstacles towards building a realistic model.

The ‘ω-problem’ of EI is described in a simple fashion in the context of the JBD theory where a scalar filed (JBD field) with coupling parameter ‘ω’ takes the place of the gravitational constant. The maximum departure from general relativity allowed by present observations forces the constraint $\omega > 500$ at the present epoch [10]. On the other hand, it is required for the phase transition of the inflaton field to proceed in such a manner that the nucleation rate of earliest bubbles which have the potential to cause large anisotropies in the microwave background, should be suppressed. The desiradatum for a suitable bubble distribution restrains $\omega$ to be $\omega < 25$ during inflation [11]. These two bounds on $\omega$ are incompatible for the case of the simplest JBD model where $\omega$ is a constant. Although it is possible to construct more sophisticated models with variable $\omega$, or with potentials for the JBD field [12,13], the difficulty of
implementing appropriate density perturbations remains in such models. It is widely believed that quantum fluctuations of the scalar fields during inflation leading to density perturbations should be able to provide the seeds of large scale structure formation in the universe. The analysis of the COBE results [14] on CMBR anisotropy and various other large scale structure observations have led to the imposition of stringent constraints on the potential used for inflation [15].

Recently, Green and Liddle [16] have summarized all the conditions required for the construction of successful EI models. They have shown the incompatibility of a large variety of existing EI models (except for some specially contrived ones) in meeting these requirements. Nevertheless, as we shall argue below, there exist a few models of EI obtained from compactification of higher dimensional theories, which lie outside the general category of models considered in [16]. For example, it has been observed for a model with a nonminimally coupled inflaton field in higher dimensions [17], that a four dimensional EI scenario emerges where the effective JBD field after the completion of inflation is anchored in a potential which follows naturally from the higher dimensional action. The rate of bubble nucleation is time dependent, a feature which is generic to such higher dimensional models [17,18]. Another type of Kaluza-Klein models can lead to four dimensional lagrangians with variable $\omega$. Such an example was provided in [19] where together with enough inflation, stable compactification of the extra spatial dimensions was achieved by considering the dynamics in the conformal Einstein frame. The next step is to study to what extent these models [17-19] are capable of accommodating the recently formulated constraints from density perturbations [15].

An unsatisfactory feature of such higher dimensional models is the presence of several ‘free’ parameters, e.g., scale of curvature of the extra dimensions, strength of the cosmological term, etc. It is expected that the various considerations for implementing a successful EI scenario would impose restrictions on the choice of values for these parameters. With this aim in mind, in this paper we study the dynamics of EI in two types of dimensionally reduced Kaluza-Klein models. In Section II we consider a model with nonminimal coupling for the inflaton field in the conformally transformed Einstein frame. Solutions for this model was earlier worked out in
the Jordan frame [17]. It is true that a conformal transformation of the spacetime metric could
in principle lead to certain differences in the dynamics [20]. However, for the purpose of the
present analysis, such differences can be ignored [16] by requiring that the models considered
are rendered sufficiently close to the limit of general relativity [10] after the epoch of inflation.
We analyze all the required conditions for successful EI in context of the specific model. We find
that although the model of Section II obeys most of the other criteria for a suitable range of
parameters, it is incapable of meeting the requirements for successful density perturbations. In
Section III, we study a higher dimensional JBD model [19] in the Einstein frame. Our analysis
shows that this model can accommodate all the conditions by imposing tight constraints on
the parameters. We present a brief summary of our results in Section IV.

2 Model with nonminimally coupled inflaton field in
$4 + D$ dimensions

Before introducing our model, it needs to be stated that the model with the standard coupling
for the inflaton in ten ($D = 6$) dimensions, which was analyzed in [18], fails to simultaneously
provide enough inflation for the four dimensional scale factor and achieve compactification for
the extra dimensions. The utility of nonminimal coupling for scalar fields in higher dimensions
has been noted in [21] where it was observed that the choice of a certain range of values for
the coupling parameter could prohibit the isotropic expansion of all the $4 + D$ dimensions. Our
model in $4 + D$ dimensions is given by the action

$$\tilde{S} = \int d^{10}z(-\tilde{g})^{1/2}[\frac{\tilde{R}}{16\pi G} + \frac{1}{2}\tilde{g}^{MN}\partial_M\tilde{\chi}\partial_N\tilde{\chi} - \xi \tilde{R}(\tilde{\chi}^2 - \tilde{\chi}_0^2) - \tilde{U}(\tilde{\chi})]$$ (1)

Tildes are used throughout to describe $4 + D$ dimensional quantities. It is assumed that the
inflaton field $\tilde{\chi}$ is anchored in a metastable state, from where it tunnels out to the true vacuum
through the nucleation of bubbles. The line element in $4 + D$ dimensions is chosen of the form

$$d\tilde{s}^2 = dt^2 - a^2(t)d\Omega_3^2 - b^2(t)d\Omega_D^2$$ (2)

where $d\Omega_3^2$ is the line element of a maximally symmetric 3-space with scale factor $a(t)$ and $d\Omega_D^2$
corresponds to a $D$-sphere with scale factor $b(t)$. 

We follow the usual prescription of dimensional reduction \cite{1,17,18} with the following definitions

\[
\Omega_D = \frac{2\pi^{(D+1)/2}}{\Gamma(D + 1)/2}, \\
\frac{\Omega_D b_0^D}{G} = m_{pl}^2, \\
\alpha = \frac{D(D - 1)}{b_0^2} \left( \frac{m_{pl}^2}{16\pi} \right)^{2/D}, \\
\sigma = (\Omega_D b_0^D)^{1/2} \chi, \\
V(\sigma) = (\Omega_D b_0^D) U(\chi), \\
V_0 = \frac{8\pi}{m_{pl}^2} V(\sigma = 0), \\
\delta = 1 - \frac{16\pi}{m_{pl}^2} \xi \sigma_0^2, \\
\Phi = \left( \frac{b}{b_0} \right)^D \frac{m_{pl}^2}{16\pi} \chi (\sigma = 0)
\] (3)

where \( b_0 \) is a parameter with dimensions of length. \( \sigma \) is the inflaton field in four dimensions which undergoes phase transition from its false vacuum value (\( \sigma = 0 \)) to the true vacuum defined at (\( \sigma = \sigma_0 \)). We consider the ten dimensional (\( D = 6 \)) case of (1) which upon dimensional reduction to four dimensions yields an effective action of the JBD type given by

\[
S = \int d^4x (-g)^{1/2} \left[ -\delta \Phi R - \frac{5}{6} \delta g^\mu\nu \frac{\partial \Phi}{\partial x^\mu} \frac{\partial \Phi}{\partial x^\nu} + \delta \alpha \Phi^{2/3} - 2\Phi V_0 \right]
\] (4)

At this stage a comparison with the action in the Jordan frame representing the general class of models considered in \cite{16} is in order. The main differences are (i) the kinetic term of the JBD field \( \Phi \) (representing the scale factor of the six dimensional internal manifold) comes with an opposite sign, (ii) a potential for \( \Phi \) occurs naturally and cannot be set to zero unlike as done for the subsequent analysis in the Einstein frame in \cite{16}, (iii) the inflaton sector gets coupled to the JBD field \( \Phi \) in the Jordan frame itself, and (iv) the last two terms in (4) represent the total effective potential of the inflaton field in the Jordan frame. It is clear that the analysis of \cite{16} does not exhaust all the possibilities provided by models of the type (4).
Equations of motion following from (4) were solved numerically in [17] where it was seen that inflation accrues for the scale factor \( a(t) \), whereas the field \( \Phi \) (\( \sim \) internal scale factor \( b(t) \)) rolls down the hill of its potential \( V(\Phi) \). An extra Maxwell type field was introduced to maintain stability of compactification at a nonvanishing value of \( b(t) \), since it induces a local minimum of \( V(\Phi) \) at the corresponding value. The above analysis was carried out in the Jordan frame. Nevertheless, a conformal transformation to the Einstein frame not only simplifies the gravitational sector of the theory, but is also essential for studying some key details of EI [16,22]. Henceforth we shall concentrate on these aspects in the conformal Einstein frame.

We define the Einstein frame metric \( \bar{g}_{\mu\nu} \) as
\[
\bar{g}_{\mu\nu} = \Omega^{-2} g_{\mu\nu} = \frac{16\pi\delta\Phi}{m^2_{pl}}
\]
and a new scalar field \( y \) as
\[
y = \frac{m_{pl}}{\sqrt{12\pi}} \ln(\Phi/\Phi_0)
\]
where \( \Phi_0 \) is a parameter having dimensions of \((mass)^2\). Using (5) and (6) in (1), the action in the Einstein frame takes the form
\[
\bar{S} = \int d^4x (-\bar{g})^{1/2} \left[ -\frac{m^2_{pl}}{16\pi} \bar{R} + \frac{1}{2} \bar{g}^{\mu\nu} \partial_{\mu} y \partial_{\nu} y - V(y) \right]
\]
with \( V(y) \) given by
\[
V(y) = \left( \frac{m^2_{pl}}{16\pi} \right)^2 \left[ 2 \frac{V_0}{\delta^2\Phi_0} \exp\left( -\frac{\sqrt{12\pi}y}{m_{pl}} \right) - \frac{\alpha}{\delta\Phi_0^{4/3}} \exp\left( -\frac{4\sqrt{12\pi}y}{3m_{pl}} \right) \right]
\]

Having defined the model in the Einstein frame we shall now analyze the various criteria which need to be satisfied for a viable scenario of extended inflation. These conditions can be stated as (a) recovering general relativity after the end of inflation within the freedom allowed by present experiments, (b) reproducing the correct strength of the gravitational coupling, (c) obtaining a bubble spectrum which agrees with CMBR isotropy, and (d) generating density...
perturbations that are compatible with large scale structure observations. Let us now consider
the specific details of these separate desiradata in the context of the present model.

To begin with, note that the potential for this model \( V(y) \) (8) has no stable minimum,
thus allowing the field \( y \) to role down unhindered. (See Fig.1 where the dimensionless function
\( U(y) = (16\pi b_0^2/m_{pl}^2)V(y) \) is plotted versus \( y/m_{pl} \) for certain typical values of the parameters.)
As stated earlier, it is known [17,18] that this defect can be rectified by the addition of a
Maxwell-type term in the higher dimensional action (1). Such a term plays the role of intro-
ducing a local minimum in \( V(y) \) at a small value of \( y \), but has negligible effect on the dynamical
evolution for large values of \( y \). A similar role can be performed by the introduction of a Casimir
term in higher dimensions, as we shall do in Section III. However, to keep the analysis as simple
as possible in the present case, one can assume that \( y \) can be anchored at a local minimum of
the potential, without explicitely writing the required extra term in the Lagrangian. With this
assumption, general relativity is exactly recovered once \( y \) stops rolling down. From (4) it is
clear that reproducing the present value of the gravitational coupling enforces the condition

\[
y_{\text{now}} = \frac{m_{pl}}{\sqrt{12\pi}} \ln \left( \frac{m_{pl}^2}{16\pi \delta \Phi_0} \right)
\]  

(9)

where \( y_{\text{now}} \) denotes the present value of \( y \). If \( y \) is anchored at the local minimum of \( V(y) \) at
the end of inflation, then \( y_{\text{now}} = y_{\text{end}} \) (\( y_{\text{end}} \) denotes the value of \( y \) at the end of inflation). Since
\( y_{\text{end}} \) is approximately zero, the value of \( \Phi_0 \) is constrained to be

\[
\Phi_0 \simeq \frac{m_{pl}^2}{16\pi \delta}
\]  

(10)

In order to examine the constraints from bubble spectrum and density perturbations, it
is customary to define, in the slow role approximation, the parameters \( \epsilon(y) \) and \( \eta(y) \) [8,15]
associated with the potential \( V(y) \) as

\[
\epsilon(y) = \frac{m_{pl}^2}{16\pi} \left( \frac{V'}{V} \right)^2
\]

\[
\eta(y) = \frac{m_{pl}^2}{8\pi} \frac{V''}{V}
\]  

(11)
with the primes indicating derivatives with respect to $y$. The number of e-foldings of inflation between two values of $y$ is approximately given by

$$N(y_1, y_2) \simeq -\frac{8\pi}{m_{pl}^2} \int_{y_1}^{y_2} \frac{V}{V'} dy$$  \hspace{1cm} (12)

For the present model obtaining the expressions of $V$ and $V'$ from (8), it turns out that the right hand side of (12) is of a rather simple form. The expression for $N$ is given by

$$N(y_1, y_2) \simeq \frac{\sqrt{12\pi}}{2m_{pl}} (y_2 - y_1)$$ \hspace{1cm} (13)

The progress of phase transition of the inflaton field in an expanding background is determined by the quantity

$$E(t) = \frac{\Gamma(t)}{H^4(t)}$$ \hspace{1cm} (14)

which measures the percentage of false vacuum occupied by bubbles of true vacuum ($E = 1$ signals the end of phase transition). $H$ is the Hubble parameter and $\Gamma$ is the nucleation rate of true vacuum bubbles per unit time per unit volume. It needs to be emphasized that unlike some of the less complicated EI scenarios [6,11,16], $\Gamma(t)$ is not a constant here, but varies according to the time evolution of $y$. This feature is generic to dimensionally reduced Kaluza-Klein models [17,18,19] since the JBD field in four dimensions gets coupled to the inflaton sector in the Jordan frame itself. Although the calculation of nucleation rate in the presence of time dependent fields is a complicated problem [23], in the limit of weak gravity, it is possible to write down a closed form expression for $\Gamma(y)$ [17,18,24] which for the present model reduces to

$$\Gamma(y) = A_0 e^{\frac{-B_0}{m_{pl}}} \left( \frac{\sqrt{12\pi} y}{m_{pl}} \right) + 2\sqrt{12\pi} y \right]$$ \hspace{1cm} (15)

where $A_0 \sim \sigma_0^4$, and $B_0$ is the flat space bounce action.

The time evolution of $\Gamma(y)$ was computed in [18] using solutions for $y$ in the Jordan frame. It was found to be extremely favorable for the desired bubble distribution as the early (large $y$) formation of bubbles is exponentially suppressed. This point can be understood better by
considering the value of various quantities at an era when bubble formation has just begun to take place. At this instant (about 55 e-foldings from the end of inflation), we call the value of $y$ as $y_{55}$. In order that the unthermalyzed bubbles from this stage do not inflate up to lead to unobserved anisotropies in the CMBR, an upper bound is placed on the value of $E$ at this epoch ($E_{55}$) [16,25]

$$E_{55} = \frac{\Gamma_{55}}{H_{55}} < 10^{-5} \quad (16)$$

To calculate the value of $\Gamma_{55}$ (15) for our model one needs to first know the possible values taken by $y_{55}$. This can be easily fixed from (13) by substituting $y_2 = y_{55}$ and $y_1 = y_{end} \simeq 0$, to obtain

$$y_{55} \simeq \frac{110m_{pl}}{\sqrt{12\pi}} \quad (17)$$

Now substituting (17) in (15) and using the fact that the Jordan frame Hubble parameter is approximately proportional to the fourth root of the Einstein frame potential [16], the bound (16) translates into the condition (after the normalization $E_{end} = 1$)

$$E_{55} = \frac{\Gamma_{55}}{\Gamma_{end}} \left( \frac{H_{end}}{H_{55}} \right)^4 \simeq exp\left[-B_0 exp(110)\right] V_{end} V_{55} < 10^{-5} \quad (18)$$

By substituting the values of the various quantities from (8), (10) and (17), it can be seen that (18) is easily satisfied. The necessity of a suitable bubble distribution does not impose any additional constraints on the parameters of this model.

The final check of testing the viability of this model comes from the requirement of generating density perturbations conforming to observations. This part of the analysis depends upon the specific cosmological model used (i.e., contributions to the energy density from different forms of dark matter, cosmological constant, etc.) [8]. Without going into these details, we adopt the criteria developed by Liddle et al.[15,16] which places constraints on the parameters $\epsilon$ and $\eta$ (11) at the epoch of 55 e-foldings from the end of inflation. It is required that

$$4\epsilon_{55} - \eta_{55} < 0.2$$
$$\eta_{55} - \epsilon_{55} < 0.1 \quad (19)$$
To see whether the above two conditions are satisfied, we first substitute the values of $y_{55}$ (17) and $\Phi_0$ (10) into $V(y)$ and its derivatives obtained from (8). Then, upon using (11) we find that both $\epsilon_{55}$ and $\eta_{55}$ can be made to possess values of $O(10^{-1})$ by demanding that

$$\frac{b_0^2 V_0}{\delta} \simeq 10^{-15}$$

(20)

This constraint on the parameters $b_0$ (scale of the internal manifold), $V_0$ (3), and $\delta$ (nonminimal coupling parameter) is in fact a requirement for enough inflation ($\epsilon, \eta \ll 1$). If the phase transition for the inflaton field is assumed to take place around the GUT scale, a particular choice which satisfies (21) is $b_0 \sim O(10(m_{pl})^{-1})$, $\delta \sim O(1)$. However, a closer scrutiny of the relations (19) by substituting the appropriate numbers, shows that both of them can never be satisfied together. The reason for this model failing to generate appropriate density perturbations is because of the form of the potential (8). Any tuning of the parameters is unable to salvage the scenario.

### 3 Higher dimensional JBD model

The action in $4 + D$ dimensions is

$$\tilde{S} = \frac{1}{16\pi} \int d^{4+D}z (-\tilde{g})^{1/2} \left[ \tilde{\Phi} \tilde{R} + \tilde{\omega} \tilde{g}^{MN} \frac{\partial_M \tilde{\Phi} \partial_N \tilde{\Phi}}{\tilde{\Phi}} - \tilde{\Lambda} + \mathcal{L}(\tilde{\chi}) \right]$$

(21)

where $\tilde{\Phi}$ and $\tilde{\Lambda}$ are the $4+D$ dimensional JBD field and cosmological constant respectively. $\mathcal{L}(\tilde{\chi})$ is the Lagrangian for the inflaton field which is caught in the metastable state of its potential. The line element is again assumed to be of the form (2). Employing the usual procedure of dimensional reduction one obtains the four dimensional action in the Jordan frame

$$S = \int d^4x (-g)^{1/2} \left[ -\Phi \left( \frac{b}{b_0} \right)^D R - \Phi \left( \frac{b}{b_0} \right)^D g^{\mu\nu} \frac{\partial_\mu \Phi \partial_\nu \Phi}{\Phi} \right. \\
+ \Phi \left( \frac{b}{b_0} \right)^D g^{\mu\nu} \frac{\partial_\mu \Phi \partial_\nu \Phi}{\Phi} \\
\left. + \left( \frac{b}{b_0} \right)^D \left( \mathcal{L}(\sigma) - \Lambda \right) \right]$$

(22)

where

$$\Phi = \frac{\Omega_D b_0^D}{16\pi} \Phi$$
\[ \omega = \frac{\tilde{\omega}}{\Omega_D} \]
\[ \Lambda = \frac{\Omega_D b_0^D}{16\pi} \tilde{\Lambda} \quad (23) \]

with \( \Omega_D \) given by (3).

In this model we include the Casimir energy contribution which may arise due to the compact nature of the internal space, which for a \( D \)-sphere takes the form \( A/b^{4+D} \) [26] where \( A \) is a constant. This term can provide the repulsive pressure at small values of the internal scale factor \( b(t) \) needed to balance the rolling down to zero value of \( b(t) \) [19,26]. The conformal Einstein frame can be defined with the transformation

\[ \bar{g}_{\mu\nu} = \frac{16\pi}{m^2_{pl}} \left( \frac{b}{b_0} \right)^D g_{\mu\nu} \quad (24) \]

The equations for motion in the Einstein frame have been solved numerically [19] where inflationary behaviour for the four dimensional scale factor \( a(t) \) is obtained together with stable compactification of the internal manifold. The average time variation of the higher dimensional JBD field \( \Phi \) can be made negligible with a suitable choice of parameters. It is required that the total cosmological constant should vanish at the end of the inflationary phase transition for emergence of the radiation dominated era. The above desiderata enable the elimination of two of the parameters in terms of the others, i.e.,

\[ A = -\frac{8\pi}{m^2_{pl}} D(D-1)\Phi_0 b_0^{D+2} \]
\[ \Lambda = \frac{D(D-1)\Phi_0}{2b_0^2} \quad (25) \]

where the present value of \( \Phi \) is taken as \( \Phi_0 \) which is required to match the strength of the gravitational coupling, and is thus given by

\[ \Phi_0 = \frac{m^2_{pl}}{16\pi} \quad (26) \]

for this model. From (22) it can be seen that the bound \( \omega > 500 \) ensures that the model stays within the present experimental limits of allowed departure from general relativity after the internal scale factor \( b(t) \) settles down at the minimum of its potential.
In order to apply the other conditions for a viable EI scenario, one has to consider the potential function for this model. In terms of the dimensionless scalar field $y$ defined as

$$y = D \ln \left( \frac{b}{b_0} \right)$$  \hspace{1cm} (27)

the Einstein frame potential (for $D = 6$) is given by

$$V(y) = \left( \frac{m^2_{pl} V_0}{8\pi} + \frac{15\Phi_0}{b_0^2} \right) \exp \left[ -(y + 2\theta) \right] - \frac{15m^2_{pl}}{8\pi b_0^2} \exp \left[ -\left( \frac{4}{3}y + \theta \right) \right] + \frac{15\Phi_0}{b_0^2} \exp \left[ -\left( \frac{8}{3}y + 2\theta \right) \right]$$  \hspace{1cm} (28)

where $V_0$ is given in (3) and $\theta = \ln(16\pi\Phi/m^2_{pl})$. The potential for this model in terms of the dimensionless function $W(y) = 16\pi V(y)/m^2_{pl}V_0$ is plotted in FIG.2. It was confirmed by numerical integration of the equations of motion [19] that $y$ undergoes oscillations about the minimum of the potential with decreasing amplitude. The contribution to the total energy density by these oscillations is approximately constant in time, thus aiding the inflationary behaviour of the scale factor $a(t)$.

To check the density perturbation constraint (19), we first define the parameters $\epsilon(y)$ and $\eta(y)$ (11) for this model. It can be checked that the requirement of inflation ($\epsilon, \eta \sim 10^{-1}$) leads to the constraint

$$V_0 b_0^2 \simeq 10^{-2}$$  \hspace{1cm} (29)

for the present model. (Note that the relation (13) between the number of e-foldings and $y$, which was derived for the potential (8) of the model considered in Section II, is no longer valid in the present case. Here the number of e-foldings is proportional to the number of oscillations undertaken by $y$ before it settles down at the minimum of its potential.) The density perturbation constraint (19) can be satisfied if the quantity $y_{55}$ lies within a narrow range of values. With the choice (25), (26) and (29) of the parameters, we find using (28) and (11) that to generate appropriate density perturbations, $y$ should start from a suitable initial value such that

$$0.2 \leq y_{55} \leq 0.6$$  \hspace{1cm} (30)
Furthermore, (19) imposes an additional condition on the behaviour of the $\Phi$ field, i.e.,

$$\frac{\Phi_{55}}{\Phi_0} \simeq 10^2$$  \hspace{1cm} (31)

Finally, let us examine the bubble spectrum of this model. Similar to the case of the model of Section II, Here again, the nucleation rate is time dependent [17,18,19,24], since in the dimensionally reduced action (22) the inflaton sector is coupled to the internal scale factor $b(t)$ in the Jordan frame itself. As in Section II, we consider the condition (16) which, after some algebra, and upon substitution of the relevant parameters translates into the constraint (for Einstein frame quantities)

$$\frac{V_{55}}{V_{\text{end}}} > 10^5 e^{\exp\left[-B_0 \left( e^{\exp(y_{55})} - 1 \right) \right]}$$  \hspace{1cm} (32)

Taking into account the fact that the range of allowed values of $y_{55}$ from density perturbations (30) is already rather narrow, it can be seen that (32) does not lead to any new constraint on the parameters of the higher dimensional model. For a typical choice of parameters, e.g., those used in plotting FIG.2, one obtains $V_{55} \approx V_{\text{end}}$ and $y_{55} \approx 0.3$. Substituting in (32), it is easy to see that the inequality holds if the flat space bounce action $B_0 \geq 30$.

4 Conclusions

We have studied two kinds of higher dimensional models which lead to JBD type theories upon dimensional reduction to four dimensions. The numerical solutions of these models had been worked out earlier [17,19] which showed that enough inflation together with dynamical compactification of the extra dimensions is possible, thus bolstering their \textit{a priori} feasibility as candidates for EI. A crucial common feature of these models is that the time dependent scale factor of the internal manifold is coupled to the inflaton sector of the four dimensional effective action in the Jordan frame. This causes the nucleation rate of the true vacuum bubbles to be time dependent [17,18,19,24]. Furthermore, this feature leads to a different form for the effective inflaton potential and the corresponding slow role parameters in the conformal Einstein frame,
from that obtained in the usual models of EI [6,11,12,13]. The models considered by us, thus clearly lie outside the ambit of the general class of models examined in [16].

In this paper we have applied to the framework of these models four essential conditions [16] required for the viability of any EI model, which are (a) recovering general relativity within the present day experimental limits, (b) reproducing the present value of the gravitational coupling, (c) producing a bubble spectrum conforming to CMBR isotropy [25], and (d) generating density perturbations compatible with present large scale structure observations [15]. The model considered in Section II is able to satisfy the first three of these desiradata which in turn impose the constraints (9), (10) and (17) on the parameters used. However, it fails to meet the criterion of appropriate density perturbations. The latter model (Section III) can be made compatible with all the above conditions, albeit only for a narrow range of parameters (25), (26), and (29)—(32). Our analysis shows that the simpler higher dimensional models [17,18] are not viable as candidates of successful EI, although more complicated ones [19] (e.g., with the inclusion of extra fields, and quantum effects in the lagrangian) are just able to squeeze through the conditions imposed by present observations through stringent constraints on parameters.

The author wishes to acknowledge the financial support provided through a project by the Department of Science and Technology, Government of India.
REFERENCES

[1] See, for example, “Modern Kaluza-Klein theories”, edited by T.Appelquist, A.Chodos and P.G.O.Freund (Addison-Wesley, Reading, MA, 1987).

[2] P.G.O.Freund, Nucl. Phys. B 209, 146 (1982); P.G.O.Freund and P.Oh, *ibid.* 255, 688 (1985); G.F.Chapline and G.W.Gibbons, Phys. Lett. B 135, 43 (1984); K.Maeda, *ibid.* 138, 269 (1984); *ibid.* 166, 59 (1986); D.Bailin and A.Love, *ibid.* 163, 135 (1985); *ibid.* 165, 270 (1985); A.S.Majumdar, A.Mukherjee and R.P.Saxena, Mod. Phys. Lett. A 7, 3647 (1992).

[3] I.Antoniadis, C.Bachas, J.Ellis and D.V.Nanopoulos, Phys. Lett. B 211, 393 (1988); Nucl. Phys. B 328, 117 (1989); Phys. Lett. B 257, 278 (1981); M.Mueller, Nucl. Phys. B 337, 37 (1990); B.A.Campbell, M.J.Duncan, N.Kaloper and K.A.Olive, *ibid.* 351, 779 (1991); A.A.Tseytlin, Int. Jour. Mod. Phys. D 1, 223 (1992); A.A.Tseytlin and C.Vafa, Nucl. Phys. B 372, 443 (1992); R.Brustein and P.J.Steinhardt, Phys. Lett. B 302, 196 (1993).

[4] P.G.Bergman, Int. J. Teor. Phys. 1, 25 (1968); R.V.Wagoner, Phys. Rev. D 1, 3209 (1970); K.Nordtvedt, Ap. J. 161, 1059 (1970).

[5] P.Jordan, Z. Phys. 157, 112 (1959); C.Brans and R.H.Dicke, Phys. Rev. 124, 925 (1961).

[6] C.Mathiazhagan and V.B.Johri, Class Quant. Grav. 1, L29 (1984); D.La and P.J.Steinhardt, Phys. Rev. Lett. 62, 376 (1989); E.W.Kolb, Phys. Scr. 36, 199 (1991).

[7] A.H.Guth, Phys. Rev. D 23, 347 (1981).

[8] See, for instance, A.D.Linde, “Particle Physics and Inflationary Cosmology”, (Harwood Academic, Chur, Switzerland, 1990); E.W.Kolb and M.S.Turner, “The Early Universe”, (Addison-Wesley, Redwood City, CA, 1990); A.R.Liddle and D.H.Lyth, Phys. Rep. 231, 1 (1993).
[9] A.Albert and P.Steinhardt, Phys. Rev. Lett. 48, 1220 (1982); A.D.Linde, Phys. Lett. B 129, 177 (1983).

[10] R.D.Reasenberg, et al., Ap. J. 234, L219 (1979); C.M.Will, “Theory and Experiment in Gravitational Physics”, (Cambridge University Press, Cambridge, 1993).

[11] E.J.Weinberg, Phys. Rev. D 40, 3950 (1989); D.La, P.J.Steinhardt, and E.Bertschinger, Phys. Lett. B 231, 231 (1989).

[12] R.Holman, E.Kolb and Y.Wang, Phys. Rev. Lett. 65, 17 (1990); P.J.Steinhardt and F.S.Accetta, Phys. Rev. Lett. 64, 274 (1990); R.Crittenden and P.J.Steinhardt, Phys. Lett. B 293, 32 (1992); A.Layock and A.R.Liddle, Phys. Rev. D 49, 1827 (1994).

[13] F.S.Accetta and J.J.Trester, Phys. Rev. D 39, 2854 (1989); A.D.Linde, Phys. Lett. B 249, 18 (1990); A.Burd and A.Coley, ibid. 267, 330 (1991); J.D.Barrow and K.Maeda, Nucl. Phys. B 341, 294 (1991); A.R.Liddle and D.Wands, Phys. Rev. D 45, 2665 (1992); F.Occhionero and L.Amendola, Phys. Rev. D 50, 4846 (1994).

[14] G.F.Smoot et al., Ap. J. 396, L1 (1992).

[15] A.R.Liddle and D.H.Lyth, Phys. Lett. B 291, 391 (1992); Ann. N.Y. Acad. sci. 688, 653 (1993); J.Garcia-Bellido and D.Wands, Phys. Rev. D 52, 6739 (1995); A.R.Liddle, D.H.Lyth, R.K.Shaeffer, Q.Shafi and P.T.P.Viana, Mon. Not. R. Astron. Soc. 281, 531 (1996).

[16] A.M.Green and A.R.Liddle, Phys. Rev. D 54, 2557 (1996).

[17] A.S.Majumdar and S.K.Sethi, Phys. Rev. D 46, 5315 (1992).

[18] R.Holman, E.W.Kolb, S.L.Vadas and Y.Wang, Phys. Rev. D 43, 995 (1991).

[19] A.S.Majumdar, T.R.Seshadri and S.K.Sethi, Phys. Lett. B 312, 67 (1993).

[20] Y.M.Cho, Phys. Rev. Lett. 68, 3133 (1992).
[21] K.Sunahara, M.Kasai and T.Futamase, Prog. Theor. Phys. 83, 353 (1990).

[22] K.Maeda, Phys. Rev. D 39, 3159 (1989).

[23] F.S.Accetta and P.Romanelli, Phys. Rev. D 41, 3024 (1990).

[24] R.Holman, E.W.Kolb, S.L.Vadas, Y.Wang and E.J.Weinberg, Phys. Lett. B 237, 37 (1990); R.Holman, E.W.Kolb, S.L.Vadas, and Y.Wang, ibid. 250, 24 (1990).

[25] A.H.Guth and E.J.Weinberg, Nucl. Phys. B 212, 321 (1983); E.J.Copeland, A.R.Liddle, D.H.Lyth, E.D.Stewart and D.Wands, Phys. Rev. D 49, 6410 (1994).

[26] F.S.Accetta, M.Gleiser, R.Holman and E.W.Kolb, Nucl. Phys. B 276, 501 (1986); A.D.Linde and M.I.Zelnikov, Phys. Lett. B 215, 59 (1988); L.Amendola, E.W.Kolb, M.Litterio and F.Occhionero, Phys. Rev. D 42, 1944 (1990).
FIGURE CAPTIONS

**Fig.1** The potential for the model with nonminimal inflaton coupling is plotted versus the internal scale factor which has the form of a JBD field in four dimensions.

**Fig.2** The potential for the model of Sec.III shows that the JBD field $y$ can undergo oscillations before settling down at the minimum. For the values of parameters used in this plot, the choice of $y_{55} \approx 0.3$ satisfies the density perturbation constraints.
FIG. 1
