Removal of closed time-like curves by supertube domain walls

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Key words  domain-walls, closed time-like curves, supertubes

PACS  04.20.Gz, 11.25.-w, 11.25.Uv

We discuss how closed time-like curves can be eliminated from certain supergravity backgrounds by inclusion of domain-walls made of supertubes. Special emphasis is given to the mechanism by which the supertubes spread into domain walls, which is similar to the enhançon mechanism.

Lecture notes from my talk at the RTN meeting in Kolymbari, Crete, September 2004.

1 Introduction

Closed time-like curves, or “time machines” pose an intriguing puzzle in theoretical physics. While generally considered to be unphysical (including by this author), they are common in solutions of general relativity. The standard practice in the field has usually been to simply discard such solutions.

In recent years quite a few solutions of supergravity were found that resemble Gödel’s universe and are highly supersymmetric. This led to a small industry of finding more such solutions, and there were some proposals on how string theory may deal with spaces suffering from this pathology.

Given that string theory incorporates general relativity with the rest of physics, including quantum mechanics, it seems like the correct laboratory to address the issue of time machines. So the question we address here is whether those spaces are correct solutions of string theory, or is there a mechanism in string theory that changes those metrics in a way that removes the pathology.

The main claim is that, in the few examples studied, those supergravity solutions are not legitimate solutions of string theory. This is demonstrated by considering certain D-brane objects (supertubes) in these background. Those supertubes may be taken as the building blocks of those geometries, and their dynamics cease to make sense when their compact direction wraps a closed time-like curve. In a close analogy with the enhançon mechanism we conclude that the supertubes go into a new phase before hitting the closed time-like curves. In that phase they delocalize and form a domain wall. Across the domain wall the metric is non-differentiable, so the original metrics, which are analytic, and their time machines are just the naive continuation of the metric ignoring the domain walls.

These notes are a summary of a talk given at the RTN meeting in Kolymbari, Crete, September 2004. More details and a full list of references can be found in the original papers.

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2 Metrics

Our starting point is a family of metrics that solve the equations of motion of type IIA supergravity \[9\]

\[
d s^2 = - U^{-1} V^{-1/2} (d t - A)^2 + U^{-1} V^{-1/2} d y^2 + V^{1/2} \sum_{i=1}^{8} (d x_i)^2
\]

\[
B_2 = U^{-1} (d t - A) \wedge d y - d t \wedge d y,
\]

\[
C_1 = - V^{-1} (d t - A) + d t,
\]

\[
C_3 = - U^{-1} d t \wedge d y \wedge A,
\]

\[
e^\Phi = U^{-1/2} V^{3/4}.
\]

Here \(U\) and \(V\) are harmonic functions in the eight \(x^i\) directions and \(A\) is a one-form satisfying the Maxwell equation. From the harmonic functions we can see that these metrics describe a collection of fundamental strings stretched in the \(y\) direction as well as D0-branes. For appropriate values of the one-form, \(A\), the metric will carry angular momentum and the three-form potential in the Ramond-Ramond sector indicates the existence of a D2-brane. Those are exactly the ingredients that make up supertubes \([5, 9]\), and indeed this is the supergravity description of those objects.

Let us consider two examples of metrics of this class. Of the eight directions labeled by \(x^i\) above we use radial coordinates in the subspace spanned by \(x^1, \cdots, x^4\), and Cartesian coordinates in the other four. The radial coordinate is \(r\) and the tree Euler angles on \(S^3\) are \(\theta, \psi\) and \(\phi\). Throughout we will use the left one-forms

\[
\sigma_1^L = \sin \phi \, d \theta - \cos \phi \sin \theta \, d \psi,
\]

\[
\sigma_2^L = \cos \phi \, d \theta + \sin \phi \sin \theta \, d \psi,
\]

\[
\sigma_3^L = d \phi + \cos \theta \, d \psi,
\]

in terms of which the metric on \(S^3\) is

\[
ds_{S^3}^2 = \frac{1}{4} \left( (\sigma_1^L)^2 + (\sigma_2^L)^2 + (\sigma_3^L)^2 \right).
\]

The first solution is given by

\[
U = C_1, \quad V = C_2, \quad A = \frac{1}{2} C_3 \, r^2 \sigma_3^L,
\]

Where \(C_1, C_2\) and \(C_3\) are constants.

This metric was first described in \([2]\), and is one of the large class of Gödel like solutions of supergravity. While the general metric of the class \([1]\) preserves eight supersymmetries, this specific ansatz preserves 20.

To study the causal structure of this space it’s instructive to look at the angular components of the metric

\[
\frac{\sqrt{V}}{4} r^2 \left[ (\sigma_1^L)^2 + (\sigma_2^L)^2 + \left( 1 - \frac{C_3^2}{UV} r^2 \right) (\sigma_3^L)^2 \right].
\]

Thus for radii \(r > UV/C_3^2\) there are closed time-like curves, given by the integral curves of the vector field \(\partial_\phi\) dual to \(\sigma_3^L\), or the Hopf fibres.

The second example is given by

\[
U = 1 + \frac{Q_s}{r^2}, \quad V = 1 + \frac{Q_0}{r^2}, \quad A = \frac{J}{2 r^2} \sigma_3^L,
\]

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Where $Q_s$, $Q_0$ and $J$ are constants. These harmonic functions indicate a source localized at the origin of this $\mathbb{R}^4$ and smeared in the transverse $\mathbb{R}^4$. This metric is the type IIA dual of the two charge black hole of type IIB, where the charges are usually of D1 and D5-branes. Here the three constants correspond to the fundamental string charge, the D0-brane charge density and angular momentum respectively.

Again we want to look at the angular part of the metric

$$
\sqrt{\frac{V}{4}} r^2 \left[ (\sigma_1^L)^2 + (\sigma_2^L)^2 + \left( 1 - \frac{J^2}{UVr^6} \right) (\sigma_3^L)^2 \right].
$$

(7)

So now there will be closed time-like curves for very small radii, $r^6 < J^2/(Q_0 Q_s)$. For large charges this simplifies to $r^2 < J^2/(Q_0 Q_s)$.

Of those two examples the second one is asymptotically flat and the problem of closed time-like curves arises only near the origin. In the case of Gödel there was no problem near the origin, but at large radii the space does not make sense. This suggests the possibility of patching the two metrics; replacing the singular center of the black hole with a piece of the Gödel space.

This can be done for any radius $r = R$ and the metric will be continuous provided we take the constants

$$
C_1 = 1 + \frac{Q_s}{R^2}, \quad C_2 = 1 + \frac{Q_0}{R^2}, \quad C_3 = \frac{J}{R^4}.
$$

(8)

### 3 The domain wall

We chose the constants describing the interior geometry such that the metric will be continuous, but it is not differentiable at the interface, indicating the existence of a source at $r = R$. It is easy to see what the source is, either by using the Israel matching conditions [10], or just by Gauss’s law. The Gödel space solves the vacuum equations, while the black hole carries charges. Therefore the charges associated with the black hole are now located at the domain wall.

So at $r = R$ we have a domain wall that has $Q_0$ D0-brane density, $Q_s$ fundamental strings as well as $J$ units of angular momentum. For the D0-branes and fundamental strings to carry angular momentum they have to form a bound state, a supertube [5]. The simplest description of this composite object is as a D2-brane extended in the $t$ and $y$ directions and wrapping the Hopf fibres of the $S^3$. To get the domain wall we have to smear the source along all the transverse directions: the $S^2$ base of the fiber and the transverse $\mathbb{R}^4$.

We have not specified yet the radius of the domain wall, $R$, and for certain choices the metric will still include closed time-like curves. In fact since the source is made up of supertubes there are a few restrictions. First the radius of the supertube is related to the charges it carries by $R^2 = Q_0 Q_s$. In addition there is a bound on the angular momentum $J \leq Q_0 Q_s = R^2$. This is enough to guarantee that there are no closed time-like curves.

The analysis so far indicates that if we assume that the source of the black-hole is not singular and localized at the origin it will be made of supertubes who will have a radius $R^2 \geq J$ that will prevent the appearance of closed time-like curves. But is there a preferred radius?

To answer that question we look at this background from the point of view of the supertubes themselves. i.e we probe the metric with another supertube. As mentioned, a supertube is best described as a D2-brane, and the fact that it carries string charge is manifested by an electric field $E = 1$ and the D0-brane charge density is proportional to the magnetic field $B$.

At low energies the dynamics of the supertube will be best described by a non-commutative field theory with open string metric $G$ and non-commutativity parameters $\Theta$ calculated from the closed string metric $g$, the pullback of the Neveu-Schwarz 2-form $B_2$ and the world-volume field strength $F$ and given by [11].
(the lines and columns correspond to the \(t, \phi\) and \(y\) directions)

\[
G^{ab} + \Theta^{ab} = \left( \frac{1}{y + B_2 + 2\pi \alpha' F} \right)^{ab} = \frac{1}{B} \begin{pmatrix}
-m & -V^{1/2} & f + V r^2 / B \\
-V^{1/2} & 0 & 1 \\
-(f + V r^2 / B) & -1 & V^{1/2} r^2 / B
\end{pmatrix},
\]

where

\[
m = \frac{V g_{\phi \phi}}{B} + \frac{V^{1/2} U}{B} \left( B + \frac{f}{U} \right)^2,
\]

and \(A = f \sigma_5 \).

Slow rigid motion at velocity \(v\) of the supertube in the transverse directions is governed by an action with kinetic term

\[
\mathcal{L} = \frac{1}{2} M v^2,
\]

where \(M = \int d\phi m\) is the metric on moduli space. This quantity is positive if the supertube wraps a space-like curve, but for a certain choice of \(B\) will be negative when wrapping a closed time-like curve, or zero when wrapping a closed null curve.

This behavior is very similar to the enchainment mechanism \([6]\), where the metric on the moduli space of a D6-brane wrapping a \(K3\) surface vanishes at a finite radius. There it was interpreted as a sign that the brane cannot continue further in to smaller radii, and instead goes into a new phase, where a \(U(1)\) gauge symmetry is enhanced to \(SU(2)\).

We wish to apply the same reasoning to the case at hand. The parameter \(M\) corresponds to the inertial mass of the supertube for motion in the transverse directions. At the radius where \(M = 0\) there could be big fluctuations in those directions, and the description of the supertubes as localized objects breaks down. The gauge theory describing them should enter new phase and the supertubes are naturally smeared into the domain wall.

This calculation of the moduli space metric is based on the supergravity solution but we can trust it due to the high degree of supersymmetry, the supertubes preserve eight supercharges in this background. It would be very interesting to understand the new phase of the non-commutative field theory in which the supertubes are delocalized, but until we have this description we can trust the results from supergravity.

## 4 Discussion

The preceding discussion summarizes the main claim in \([8]\), that supertube naturally form domain walls to shield against the creation of closed time-like curves in those backgrounds. Another example studied there is the case of the BMPV black hole \([12]\), or its type IIA dual, which carries three charges, of D0 and D4-branes and fundamental strings.

The same kind of domain wall geometry can be constructed in that example, where from the outside it looks like the BMPV black hole. But inside (behind the Cauchy horizon) the metric is replaced by a certain Gödel-like space. This construction was also found independently by Gimon and Hořava \([13]\). The necessary source for making up the domain wall in this case would be a bound state of the three objects whose charges are carried by the black hole. Such a three-charge generalization of the supertube was indeed found by Bena and Kraus \([14]\).

Since those objects preserve only four supercharges the moduli space may be corrected by quantum effects. So an argument similar to the one above would not be rigorous. Still we find it likely that the general picture will still hold, and those objects will experience an enchainment like phenomenon and form the appropriate domain wall.

In the example studied here both spaces, the Gödel-like metric and the two-charge black hole, had closed time-like curves. But we argued that neither space is a complete solution of string theory, and the
correct solution includes a domain wall that patches those two metrics together. Thus each of these spaces could be considered the wrong analytical continuation of a legitimate solution, ignoring the domain wall. Such a mechanism is a possible solution to many other metrics that have closed time-like curves. Locally those solutions are usually perfectly good, and problems arise only when extending the solutions to global metrics. Such continuations are unique assuming the metric is analytic, which was not the case here. Domain walls are a natural way to break analyticity and thus averting this global problem.

Another beautiful example where a similar mechanism is at play was found by Israel [15]. In that case a Gödel-like deformation of $AdS_3$ develops closed time-like curves, but those are naturally cured after the inclusion of a domain wall made of large fundamental strings. It would be very interesting to see how general this mechanism is.

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