Neutral gluon polarization tensor in color magnetic background at finite temperature

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Submitted to PRD on Oct 30th

Abstract
In the framework of SU(2) gluodynamics, we derive the tensor structure of the neutral gluon polarization tensor in an Abelian homogeneous magnetic field at finite temperature and calculate it in one-loop approximation in the Lorentz background field gauge. The imaginary time formalism and the Schwinger operator method are used. The latter is extended to the finite temperature case. The polarization tensor turns out to be non transversal. It can be written as a sum of ten tensor structures with corresponding form factors. Seven tensor structures are transversal, three are not. We represent the form factors in terms of double parametric integrals and the temperature sum which can be computed numerically. As applications we calculate the Debye mass and the magnetic mass of neutral gluons in the background field at high temperature. A comparison with the results of other authors is done.

1 Introduction
The investigations of QCD at high temperature carried out in recent years have elucidated the important role of color magnetic fields. In Refs. [1], [2] it was discovered in lattice simulations that sufficiently strong constant Abelian magnetic fields described by the potential of the form $A_\mu^a = B \delta_\mu^2 \delta^a_3$, where $B$ is field strength, $a$ is the index of internal symmetry, $\mu$ - Lorentz index, shift the deconfinement phase transition temperature $T_c$. In particular, it was shown that an increase in the field strength decreases the transition temperature and for sufficiently strong field strengths $T_c(B)$ can be equal to zero. On the other hand, in Refs. [3], [4] from the analysis of lattice simulations and in Refs. [6], [7], [8] from perturbative resummations of daisy graphs in the background field at high temperature it was found that Abelian chromomagnetic fields of order $gB \sim g^4 T^2$, where $g$ is a the gauge coupling, are spontaneously created.

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These results are of interest not only for QCD but also for problems of the early universe where strong magnetic fields of different kind had likely been present \[9\]. They served as motivations for investigations began in our recent papers Refs.\[10\], \[11\], which goal is to determine the operator structure of the gluon polarization tensor in the constant Abelian chromomagnetic background field at finite temperature. This is necessary for investigations of the quark-gluon plasma (QGP), first of all, when resummations of perturbative series are carried out. As a preliminary step, the tensor structure of the gluon polarization tensor as well as the one-loop contributions to its form factors at zero temperature have been obtained and partially investigated therein. In the presence of the background field, for many reasons it is convenient to use the decomposition of the gauge fields in the internal space of the form $W_{\mu}^{\pm} = 1/\sqrt{2}(A_{1}^{\pm} \pm iA_{2}^{\pm})$, $A_{\mu} = A_{3}^{\mu}$ and consider the former "charged" and the latter "neutral" gluons separately because of sufficiently different properties of them. That concerns not only physics but also the calculation procedures required for investigations.

In the present paper in the framework of SU(2) gluodynamics, we derive the operator structure of the neutral gluon polarization tensor (PT) in the Abelian chromomagnetic background field at finite temperature. In actual calculations, as in Refs. \[10\], \[11\], we use the background Lorentz gauge and the Schwinger operator formalism based on the proper-time representation for propagators \[12\]. It is modified in order to account for the Matsubara imaginary time method at finite temperature and at the same time it preserves the Lorentz covariance when the momentum loop integrations are fulfilled. This generalizes the formulas derived earlier at zero temperature. As applications we calculate the one-loop contribution to the form factors. For instance, we derive the Debye mass and the "magnetic mass" of the neutral gluons in the background field at finite temperature. The limit of zero background field is also considered, to the correspondence with known results obtained already by other methods \[15\] is established.

Here to present the results we would like to note that in the limit $T \to \infty$ the form factors can be calculated in terms of the Riemann Zeta-function. We also find that the "fictitious" pole (see for details, for instance, the surveys \[15\], \[16\]) in the gluon Green’s function at finite temperature disappears in one-loop order if the external field is switched on. At the same time, the transversal neutral gluon field modes remain long range ones. They are not screened in the field at one-loop order, in contrast to the charged ones, as it was determined in Refs. \[13\], \[8\]. This difference is important. Its possible consequences will be discussed below in the main text.

The paper is organized as follows. In the next section we introduce the necessary notation and review in brief the basic formulas. In sections 3, 4 we derive the operator structure of the gluon polarization tensor and develop the calculation procedure to carry out the integration over internal momenta of Feynman diagrams in the field at finite temperature. Explicit formulas for the form factors in the form of two-parametric integrals are obtained in section 5. In contrast to the previous paper \[10\], where the tadpole diagrams were not discussed in detail and instead some arguments in fewer of the cancellation of this contributions in the total by the surface terms appearing in other parts of the polarization tensor were used. Here we consider this cancelation because of peculiarities appearing at finite temperature. In section 6 the transition to the zero temperature case is discussed. The Debye mass of the neutral gluon in the external field is calculated in section 7. In the next section we calculate the mean values of the operator in the physical states of the transverse modes and show that the gluon "magnetic mass" in the field is
zero in one-loop order although the fictitious pole of the Green function is eliminated. The discussion of the results obtained and further prospects are given in section 9.

Throughout the paper we use latin letters \( a, b, \ldots = 1, 2, 3 \) for the color indexes and Greek letters \( \lambda, \mu, \ldots = 1, \ldots, 4 \) for the Lorentz indices. Summation over doubly appearing indices is assumed. All formulas are in the Euclidean formulation. We put all constants including the coupling equal to unity. Since the present work is a continuation of investigations began in Ref. [10], we follow the notations, definitions and calculation procedures used therein as close as possible.

2 Basic Notations

In this section we collect the well known basic formulas for SU(2) gluodynamics to set up the notations which we will use. We work in the Euclidean version. Dropping arguments and indices, the Lagrangian is

\[
\mathcal{L} = -\frac{1}{4} F_{\mu\nu}^2 - \frac{1}{2\xi} (\partial_\mu A_\mu)^2 + \bar{\eta} \partial_\mu D_\mu \eta, \tag{1}
\]

where \( \xi \) is the gauge fixing parameter and \( \eta \) is the ghost field. The action is \( S = \int dx \mathcal{L} \) and the generating functional of the Green functions is \( Z = \int DA \exp(S) \). In the following we divide the gauge field \( A_\mu^a(x) \) into background field \( B_\mu^a(x) \) and quantum fluctuations \( Q_\mu^a(x) \),

\[
A_\mu^a(x) = B_\mu^a(x) + Q_\mu^a(x). \tag{2}
\]

The covariant derivative depending on a field \( A \) is

\[
D_{\mu}^{ab}[A] = \frac{\partial}{\partial x^\mu} \delta^{ab} + \epsilon^{acb} A_\nu^c(x) \tag{3}
\]

and the field strength is

\[
F_{\mu\nu}^a[A] = \frac{\partial}{\partial x^\mu} A_\nu^a(x) - \frac{\partial}{\partial x^\nu} A_\mu^a(x) + \epsilon^{abc} A_\nu^b(x) A_\mu^c(x), \tag{4}
\]

and

\[
[D_\mu[A], D_\nu[A]]^{ab} = \epsilon^{acb} F_{\mu\nu}^c[A] \tag{5}
\]

holds. For the field splited into background and quantum parts we note

\[
F_{\mu\nu}^a[B + Q] = F_{\mu\nu}^a[B] + D_{\mu}^{ab}[B] Q_\nu^b(x) - D_{\nu}^{ab}[B] Q_\mu^b(x) + \epsilon^{abc} Q_\mu^c(x) Q_\nu^b(x). \tag{6}
\]

The square of it is

\[
-\frac{1}{4} \left( F_{\mu\nu}^a[B + Q] \right)^2 = -\frac{1}{4} \left( F_{\mu\nu}^a[B] \right)^2 + Q_\nu^a D_{\mu}^{ab}[B] F_{\mu\nu}^b[B] + \frac{1}{2} Q_\mu^a K_{\mu\nu}^{ab} Q_\nu^b + \mathcal{M}_3 + \mathcal{M}_4. \tag{7}
\]

The second term in the r.h.s. is linear in the quantum field and disappears if the background fulfills its equation of motion which will hold in our case of a constant background field. The third term is quadratic in \( Q_\mu^a \) and it defines the 'free part' with the kernel

\[
K_{\mu\nu}^{ab} = -\delta_{\mu\nu} D_{\rho}^{ac}[B] D_{\rho}^{cb}[B] + D_{\mu}^{ac}[B] D_{\nu}^{cb}[B] - 2 \epsilon^{abc} F_{\mu\nu}^c[B]. \tag{8}
\]
The interaction of the quantum field is represented by the vertex factors

\[ M_3 = -\epsilon^{abc} (D^{ad}_\mu Q^d_\nu) Q^b_\mu Q^c_\nu, \]
\[ M_4 = -\frac{1}{4} Q^a_\mu Q^a_\nu Q^b_\mu Q^b_\nu + \frac{1}{4} Q^a_\mu Q^b_\nu Q^a_\nu Q^b_\mu. \]  

(9)

The complete Lagrangian

\[ \mathcal{L} = -\frac{1}{4} (F^a_{\mu\nu} [B + Q])^2 + \mathcal{L}_{gf} + \mathcal{L}_{gh} \]  

(10)

consists of \[ \mathcal{L}_{gf} = -\frac{1}{2\xi} (D^a_\mu [B] Q^a_\mu)^2 = \frac{1}{2\xi} Q^a_\mu D^{ac}_\mu [B] D^{cb}_\nu [B] Q^b_\nu, \]  

and the ghost term

\[ \mathcal{L}_{gh} = \eta^a D^{ac}_\mu [B] (D^{cb}_\mu [B] + \epsilon^{cde} Q^e_\mu) \eta^b. \]  

(11)

These formulas are valid for an arbitrary background field. Now we turn to the specific background of an Abelian homogeneous magnetic field of strength \( B \) which is oriented along the third axis in both, color and configuration space. An explicit representation of its vector potential is

\[ B^a_\mu(x) = \delta^{a3} \delta_\mu 1 x_2 B \]  

(13)

and the corresponding field strength is

\[ F_{ij}^a = \delta^{a3} F_{ij} = B \epsilon^{3ij}, \]  

(14)

where only the spatial components \((i, j = 1, 2, 3)\) are nonzero. Once the background is chosen Abelian it is useful to turn into the so called charged basis,

\[ W^\pm_\mu = \frac{1}{\sqrt{2}} \left( Q^1_\mu \pm iQ^2_\mu \right), \]

\[ Q_\mu = Q^3_\mu. \]  

(15)

with the interpretation of \( W^\pm_\mu \) as color charged fields and \( Q_\mu \) as color neutral field. This is in parallel to electrically charged and neutral fields. Note also that \( Q_\mu \) is real while \( W^\pm_\mu \) are complex conjugated one to the other. In the following we will omit the word color when speaking about charged and neutral objects. The same transformation is done for the ghosts,

\[ \eta^\pm_\mu = \frac{1}{\sqrt{2}} \left( \eta^1_\mu \pm i\eta^2_\mu \right), \]

\[ \eta_\mu = \eta^3_\mu. \]  

(16)

A summation over the color indices turns into

\[ Q^a_\mu Q^a_\nu = Q_\mu Q_\nu + W^+_\mu W^-_\nu + W^-_\mu W^+_\nu. \]  

(17)

All appearing quantities have to be transformed into that basis. For the covariant derivative we obtain

\[ D^{33}_\mu = \partial_\mu, \quad D^{-}_\mu = \partial_\mu - iB_\mu \equiv D_\mu \]
\[ D^+_\mu = \partial_\mu + iB_\mu \equiv D^*_\mu \]  

(18)
where $D^*_\mu$ is the complex conjugated to $D_\mu$. Starting from here we do not need any longer to indicate the arguments in the covariant derivatives.

Before proceeding with writing down the remaining formulas in the charged basis it is useful to turn into momentum representation. This can be done in a standard way by the formal rules. It remains to define the signs in the exponential factors. We adopt the notation

$$Q \sim e^{-ikx}, \quad W^- \sim e^{-ipx}, \quad W^+ \sim e^{ip'x}. \quad (19)$$

In all following calculation the momentum $k$ will denote the momentum of a neutral line and the momenta $p$ and $p'$ that of the charged lines whereby $k$ and $p$ are incoming and $p'$ is outgoing. In these notations the covariant derivative $D_\mu$ acts on a $W^-_\mu$ and turns into

$$D_\mu = -i(i\partial_\mu + B_\mu) \equiv -ip_\mu. \quad (20)$$

Note that the components of the momentum $p_\mu$ do not commute,

$$[p_\mu, p_\nu] = iBF_{\mu\nu}. \quad (21)$$

In these notations the quadratic term of the action turns into

$$-\frac{1}{2}Q_\mu K_{\mu\nu}Q_\nu = \frac{1}{2}Q_\mu K^{33}_{\mu\nu}Q_\nu + \frac{1}{2}W^+_\mu K^{-}_{\mu\nu}W^-_\nu + \frac{1}{2}W^-_\mu K^{+}_{\mu\nu}W^+_\nu \quad (22)$$

with

$$K^{33}_{\mu\nu} \equiv K_{\mu\nu} (k) = \delta_{\mu\nu}k^2 - k_\mu k_\nu \quad (23)$$

and

$$K^{-+}_{\mu\nu} \equiv K_{\mu\nu} (p) = \delta_{\mu\nu}p^2 - p_\mu p_\nu + 2iBF_{\mu\nu}. \quad (24)$$

We use the arguments $k$ and $p$ instead of the indices to indicate to which line a $K_{\mu\nu}$ belongs. The third term in the r.h.s. of Eq.(22) is the same as the second one due to the complex conjugation rules. In the Feynman rules $(K^{33}_{\mu\nu})^{-1}$ is the line for neutral gluons and is denoted by a wavy line and $(K^{-+}_{\mu\nu})^{-1}$ is the line for charged gluons and is denoted by a directed solid line. We remark that these lines represent propagators in the background of the magnetic field. Frequently they are denoted by thick or double lines. Because we have in this paper no other lines the notation with ordinary (thin) lines is unique.

Here we note that the spectrum of the operator (24) in a constant magnetic field,

$$E^2_n = p_3^2 + B(2n + 1), \quad n = -1, 0, 1, \ldots, \quad (25)$$

contains a tachyonic mode at $n = -1$. $p_3$ is momentum along the field direction $B = B_3$. This state is a peculiar of non Abelian gauge fields as it is discussed in different aspects in the literature (see, for instance Ref.[10],[7] and references therein).

For later use we introduce here the set of eigenstates for the operator (24). For the color neutral states we take exactly the same polarizations $| k, s >$ as known from
electrodynamics,

\[ |k, 1 \rangle_\mu = \frac{1}{h} \begin{pmatrix} -k_2 \\ k_1 \\ 0 \\ 0 \end{pmatrix}_\mu, \quad |k, 2 \rangle_\mu = \frac{1}{k h} \begin{pmatrix} k_1 k_3 \\ k_2 k_3 \\ -k^2 \\ 0 \end{pmatrix}_\mu, \quad |k, 3 \rangle_\mu = \frac{1}{k} \begin{pmatrix} k_1 \\ k_2 \\ k_3 \\ 0 \end{pmatrix}_\mu, \quad |k, 4 \rangle_\mu = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}_\mu \]  

(26)

with \( h = \sqrt{k_1^2 + k_2^2}, \ k = \sqrt{k_1^2 + k_2^2 + k_3^2} \). Here the polarizations \( s = 1, 2 \) describe the two transversal gluons \( (k_\mu | k, s = 1, 2 \rangle_\mu = 0) \), \( s = 3 \) is the longitudinal one and \( s = 4 \) after rotation into Minkowski space becomes the time like one. For the transversal gluons

\[ K_{\mu\nu}(k) | k, s = 1, 2 \rangle_\nu = (k_4^2 + k^2) | k, s = 1, 2 \rangle_\mu \]  

(27)

holds.

After discussing the free part of the Lagrangian (10), (7), in the 'charged basis' we note that for the vertex factor \( \mathcal{M}_3 \) in (9) we obtain

\[ \mathcal{M}_3 = W^-_\mu \Gamma_{\mu\nu\lambda} W^+\nu Q_\lambda \]  

(28)

with

\[ \Gamma_{\mu\nu\lambda} = \delta_{\mu\nu}(k - 2p)_\lambda + \delta_{\lambda\mu}(p + k)_\nu + \delta_{\lambda\nu}(p - 2k)_\mu. \]  

(29)

The notations are shown in Fig.1. It should be remarked that all graphs and combinatorial factors are exactly the same as in the well known case without magnetic field. On this level the only difference is in the meaning of the momentum \( p_\mu \) which in our case depends on the background magnetic field, see Eq.(20).

The vertex factor describing the interaction of the neutral gluons with charged ghost fields is

\[ \mathcal{M}^{gh}_3 = \eta^* \Gamma^*_\lambda \eta^+ Q_\lambda \]  

(30)

where

\[ \Gamma^*_\lambda = p'_\lambda = (p + k)_\lambda. \]  

(31)

We also need in the four particle vertexes which are momentum independent and have the same form as at zero external field.
3 Operator Structures

The neutral PT is denoted by \( \Pi_{\lambda\lambda'}(k) \) where the argument \( k \) is an ordinary momentum. Its one-loop diagram representation is shown in Fig. 2. In this section we discuss the general tensor structure of it at zero and finite temperature. As it was shown in Refs. [10], [11] it is not transversal in a magnetic background field. That means that the condition \( k_\lambda \Pi_{\lambda\lambda'}(k) = 0 \) does not hold. This follows either from the Slavnov-Taylor identity for the gluon Green function or from an explicit one-loop calculation. So, we are left with the weaker condition

\[
k_\lambda \Pi_{\lambda\lambda'}(k) k_{\lambda'} = 0.
\] (32)

At zero temperature, it can be combined with the remaining Lorentz symmetry which results in a dependence of \( \Pi_{\lambda\lambda'}(k) \) on two vectors, \( l_\lambda \) and \( h_\lambda \), and on the magnetic field.

We use the notations

\[
l_\mu = \begin{pmatrix} 0 \\ 0 \\ k_3 \\ k_4 \end{pmatrix}, \quad h_\mu = \begin{pmatrix} k_1 \\ k_2 \\ 0 \\ -k_1 \end{pmatrix}, \quad d_\mu = \begin{pmatrix} k_2 \\ -k_1 \\ 0 \\ 0 \end{pmatrix},
\]

\[
F_{\mu\lambda} = \begin{pmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}.
\] (33)

The third vector is \( d_\mu \equiv F_{\mu\nu} k_\nu \). Note that here and further in actual calculations the magnetic field strength \( B \) is put equal to unity. For the vectors \( k_\lambda = l_\lambda + h_\lambda \) holds.

The general structure of \( \Pi_{\lambda\lambda'}(k) \) at \( T = 0 \) allowed by (32) and the vectors \( l_\lambda \) and \( h_\lambda \) is determined by the set of tensor structures

\[
T^{(1)}_{\lambda\lambda'} = i^2 \delta^{||}_{\lambda\lambda'} - l_\lambda l_{\lambda'},
T^{(2)}_{\lambda\lambda'} = h^2 \delta^\perp_{\lambda\lambda'} - h_\lambda h_{\lambda'} = d_\lambda d_{\lambda'},
T^{(3)}_{\lambda\lambda'} = h^2 \delta^{||}_{\lambda\lambda'} + i^2 \delta^\perp_{\lambda\lambda'} - l_\lambda h_{\lambda'} - h_\lambda l_{\lambda'},
T^{(4)}_{\lambda\lambda'} = i(l_\lambda d_{\lambda'} - d_\lambda l_{\lambda'}) + i^2 F_{\lambda\lambda'},
T^{(5)}_{\lambda\lambda'} = h^2 \delta^{||}_{\lambda\lambda'} - i^2 \delta^\perp_{\lambda\lambda'},
T^{(6)}_{\lambda\lambda'} = i F_{\lambda\lambda'}
\] (34)

together with the identity \( d_\lambda h_{\lambda'} - h_\lambda d_{\lambda'} = h^2 F_{\lambda\lambda'} \). Further we introduced the notations \( \delta^\perp_{\mu\lambda} = \text{diag}(1, 1, 0, 0) \) and \( \delta^{||}_{\mu\lambda} = \text{diag}(0, 0, 1, 1) \). The first four operators are transversal, \( k_\lambda T^{(i)}_{\lambda\lambda'} = T^{(i)}_{\lambda\lambda'} k_{\lambda'} = 0 \) with \( i = 1, 2, 3, 4 \), the last two fulfill (32), only. The sum of the first three operators is just the transversal part of the kernel of the quadratic part of the action, Eq. (23),

\[
T^{(1)}_{\lambda\lambda'} + T^{(2)}_{\lambda\lambda'} + T^{(3)}_{\lambda\lambda'} = K_{\lambda\lambda'}(k).
\] (35)

At finite temperature, an additional vector \( u_\mu \) - the velocity of the thermostat - must be accounted for and used in construction of the tensors \( T^{(i)} \). Therefore new tensor structures appear. We chose them in the form:

\[
T^{(7)}_{\lambda\lambda'} = (u_\lambda l_{\lambda'} + l_\lambda u_{\lambda'})(u k) - \delta^{||}_{\lambda\lambda'}(u k)^2 - u_\lambda u_{\lambda'} t^2
\]
\[ T^{(8)}_{\lambda\lambda'} = (u_{\lambda} h_{\lambda'} + h_{\lambda} u_{\lambda'}) (uk) - \delta^{(1)}_{\lambda\lambda'} (uk)^2 - u_{\lambda} u_{\lambda'} h^2 \]
\[ T^{(9)}_{\lambda\lambda'} = u_{\lambda} id_{\lambda'} - id_{\lambda} u_{\lambda'} + iF_{\lambda\lambda'} (uk), \]
\[ T^{(10)}_{\lambda\lambda'} = k^2 \delta_{\lambda\lambda'} - \frac{u_{\lambda} u_{\lambda'} (k^2)^2}{(uk)^2}. \]  

Obviously the sum
\[ T^{(7)}_{\lambda\lambda'} + T^{(8)}_{\lambda\lambda'} = (u_{\lambda} k_{\lambda'} + k_{\lambda} u_{\lambda'}) (uk) - \delta_{\lambda\lambda'} (uk)^2 - u_{\lambda} u_{\lambda'} k^2 = B_{\lambda\lambda'} \]  

equals to one of two transversal tensor structures commonly used at zero field \[15\]. In that case the first structure is given by the sum of tensors Eqs. (35), (23). Below in actual calculations we use the reference frame of the thermostat, so only one component of \( u_{\mu} \) is nonzero: \( u_{\mu} = (0, 0, 1, \mu) \). We mention that the first two tensors in Eq.(36) are transversal and the other two satisfy the weaker condition (32).

We adopt the following way for the representation of our expressions. The dimensionality of the polarization tensor is implemented in the tensors \( T^{(i)} \). To restore the dimensionality for the tensors in Eqs.(34) and (36), one has to multiply the operator \( T^{(6)} \) by the factor \( B \), and the operator \( T^{(9)} \) by \( \sqrt{B} \). The form factors are dimensionless functions of dimensionless momenta \( l^2, h^2, \) and temperature \( T \). That means, in fact they are measured in units of \( B \). To restore the correct dimensionality one has to replace \( l^2 \rightarrow l^2 / B, h^2 \rightarrow h^2 / B, \) and \( T \rightarrow T / \sqrt{B} \). Correspondingly, the arguments of all functions appearing in the actual calculations are also dimensionless.

Knowing the operators (34) and (36), which may appear at zero and finite temperature, the polarization tensor can be represented in the form
\[ \Pi_{\lambda\lambda'} (k) = \sum_{i=1}^{10} \Pi^{(i)} (k) T^{(i)}_{\lambda\lambda'}, \]  

where the form factors \( \Pi^{(i)} (k) \) depend on the external momentum \( k_{\mu} \) through the variables \( l^2 \) and \( h^2 \) at zero temperature and \( h^2, k_4 \) and \( k_5 \) at finite temperature. In the former case, the polarization tensor \( \Pi_{\lambda\lambda'} (k) \) is real and symmetric in its indices, so the form factors \( \Pi^{(4)} (k) \) and \( \Pi^{(6)} (k) \) are zero. In the latter case all structures will contribute in general.

In the Matsubara formalism, it is possible to add to the set of tensors Eq.(36) a structure of a special type which contributes in the static limit, \( k_4 = 0 \), only (remember, \( k_4 \) takes discrete values). It should be of the form \( u_{\mu} u_{\nu} \phi (k_4) \) with \( \phi (k_4) \) possesses the following property: it is non zero for \( k_4 = 0 \), only. This term is obviously transversal for itself. So, the set of tensors in Eq.(38) could be extended due to this structure. On the other side this structure is a linear combination of \( T^{(7)}_{\lambda\lambda'} \) and \( T^{(8)}_{\lambda\lambda'} \) at \( k_4 = 0 \).

4 Calculation of the Neutral Polarization Tensor

In this section we calculate the one-loop contribution to the PT at finite temperature. The imaginary time formalism is used. That means that in loops the integration over momentum component \( p_4 \) is substituted by an infinite series in discrete values \( p_4 = 2\pi NT \):
\[ \int_{-\infty}^{+\infty} \frac{dp_4}{2\pi} f (p_4) \rightarrow T \sum_{N=-\infty}^{+\infty} f (2\pi NT). \]
The neutral polarization tensor has the following representation in momentum space (see Fig. 2)

\[ \Pi_{\lambda\lambda'}(k) = T \sum_{N=-\infty}^{+\infty} \int_{-\infty}^{+\infty} \frac{d^3p}{(2\pi)^3} \Pi(p, p_4, k, k_4)_{\lambda\lambda'}, \]  

(39)

where in the integrand we noted explicitly the dependence on the external momentum and the momentum inside loops. In what follows, as in Ref. [10], at intermediate stage of computation we, for brevity, shall omit the general factors and the signs of integration and summation. That is, we relate these factors and operations with the momentum \( p \) standing in r.h.s. of equations. Within this convention we write

\[ \Pi_{\lambda\lambda'}(k) = \Gamma_{\mu\nu\lambda}(p) \Gamma^\mu_{\nu'}(p-k) \]

\[ - p_\lambda G(p)(p-k)_\lambda G(p-k) - (p-k)_\lambda G(p) p_\lambda G(p-k) \]

\[ + G_{\lambda\lambda'}(p) + G_{\lambda\lambda'}(p) - 2\delta_{\lambda\lambda'} Tr G(p). \]

The second line gives the contribution from the ghost loops and the third one is due to the tadpole diagram.

The propagators are given by

\[ G(p) = \frac{1}{p^2} = \int_0^{+\infty} ds \ e^{-sp^2}, \]

\[ G(p-k) = \frac{1}{(p-k)^2} = \int_0^{+\infty} dt \ e^{-t(p-k)^2} \]  

(42)

for the scalar lines and by

\[ G_{\lambda\lambda'}(p) = \left( \frac{1}{p^2 + 2iF} \right)_{\lambda\lambda'} = \int_0^{+\infty} ds \ e^{-sp^2} E_{\lambda\lambda'}^{s}, \]

\[ G_{\lambda\lambda'}(p-k) = \left( \frac{1}{(p-k)^2 + 2iF} \right)_{\lambda\lambda'} \]

\[ = \int_0^{+\infty} dt \ e^{-t(p-k)^2} E_{\lambda\lambda'}^{-t}. \]  

(43)

Figure 2: The neutral polarization tensor

The vertex factor is given in Eq. (29). For a convenient grouping of terms it is useful to rearrange it,

\[ \Gamma_{\mu\nu\lambda} = g_{\mu\nu}(k-2p)_\lambda + 2(g_{\lambda\mu} k_\nu - g_{\lambda\nu} k_\mu) + g_{\lambda\mu}(p-k)_\nu + g_{\lambda\nu} p_\mu, \]

\[ \equiv \Gamma_{\mu\nu\lambda}^{(1)} + \Gamma_{\mu\nu\lambda}^{(2)} + \Gamma_{\mu\nu\lambda}^{(3)}, \]

(41)

where in the last line a subdivision into three parts is done.

The propagators are given by
for the vector lines (in the Feynman gauge, $\xi = 1$) with

$$E_{\lambda'}^s \equiv \left( e^{2isF} \right)_{\lambda'}^\lambda = \delta_{\lambda'}^\parallel + iF_{\lambda\lambda'} \sinh(2s) + \delta_{\lambda\lambda'}^\perp \cosh(2s). \quad (44)$$

At zero temperature, the momentum integration can be carried out by means of Schwinger’s algebraic procedure [12] and converted into an integration over two scalar parameters, $s$ and $t$. Here we deduce the known results in order to present their modifications at $T \neq 0$. The basic exponential is

$$\Theta = e^{-sp^2} e^{-t(p-k)^2} \quad (45)$$

and the integration over the momentum $p$ is denoted by the average $\langle \ldots \rangle$. The following formulas hold:

$$\langle \Theta \rangle = \exp \left[ -k \left( \frac{s}{s+t} \delta_{\parallel}^\parallel + \frac{ST}{S+T} \delta_{\perp}^\perp \right) k \right]$$

$$\langle p_\mu \Theta \rangle = \left( \frac{A}{D} \right)_\mu \langle \Theta \rangle, \quad (47)$$

$$\langle p_\mu p_\nu \Theta \rangle = \left( \left( \frac{A}{D} \right)_\mu \left( \frac{A}{D} \right)_\nu - i \left( \frac{F}{D^\top} \right)_\mu \right) \langle \Theta \rangle. \quad (48)$$

The notation $A \equiv E^t - 1$ and $D \equiv E^{s+t} - 1$ is used. Explicit formulas are

$$\frac{A}{D} = \delta_{\parallel}^\parallel \frac{t}{s+t} - iF \frac{\sinh(s) \sinh(t)}{\sinh(s+t)} + \delta_{\perp}^\perp \frac{\cosh(s) \sinh(t)}{\sinh(s+t)} \quad (49)$$

along with

$$-2iFE^{-s}D^{-\top} = \delta_{\parallel}^\parallel \frac{t}{s+t} - iF \frac{\sinh(s-t)}{\sinh(s+t)} + \delta_{\perp}^\perp \frac{\cosh(s-t)}{\sinh(s+t)}, \quad (50)$$

where we dropped the indices. It should be remarked that all these matrices, i.e., $E$, $F$, $D$ and $A$ commute. In addition we need the relation

$$p(s)_\mu \equiv e^{-sp^2} p_\mu e^{sp^2} = E_{\mu\nu}^s p_\nu$$

for commuting a factor $p_\mu$ with the propagator $G(p)$,

$$p_\mu G_{\mu\nu'}(p) = G(p)p_{\nu'} . \quad (51)$$

Now we turn to the finite temperature case. Our goal is to account for the temperature dependence within the above representation in a natural way. Usually in the imaginary time formalism the summation over $p_4$ and the integration over three-momenta are carried out separately. To restore the equivalence of these variables and to make use of the
formulas (46)-(48) we proceed in the following way. First we note that any function  
\[ f(p_4 = 2\pi NT) = \int dp_4 f(p_4) \delta(p_4 - 2\pi NT) \]
where \( \delta(x) \) is Dirac’s delta-function. Then we change the order of integration in variables \( \lambda \) and \( p_4 \) and fulfill the momentum integration as at zero temperature. The factor \( 1/(2\pi)^4 \) coming from the delta-function and the factor \( 1/(2\pi)^3 \) coming from the three-momentum integration give in the product the factor \( 1/(2\pi)^4 \) appearing at zero temperature. The only new factor, \( e^{i\lambda p_4} \), appears as another exponential in Eq.(45). Since the fourth component is not related with the magnetic field, the calculation is actually the same as at zero temperature. Remind that in course of it the factor \( 1/(2\pi)^4 \) results in the factor \( 1/(4\pi)^2 \) in the function \( \langle \Theta(s, t) \rangle \)

At the next step, we integrate the function \( \Theta(s, t, \lambda) \) over \( \lambda \) and after the restoration of the sum over \( N \) we obtain the basic expression at finite temperature:

\[
\langle \Theta(s, t) \rangle_T = T \sum_{N=-\infty}^{+\infty} \int_{-\infty}^{+\infty} d\lambda \Theta(s, t, \lambda) e^{-i\lambda(2\pi NT)}
\]

where we marked the averaging procedure with \( \lambda \)-dependence by the subscript \( \lambda \). Here \( k_4 \) is the discrete fourth component of the external momentum and we introduced \( q = s + t \) as a convenient variable. The expression in the angle brackets in the r.h.s. is the zero temperature value given in Eq.(46). Below we will denote the function (53) as \( \Theta(s, t, \lambda) \).

At the next step, we integrate the function \( \Theta(s, t, \lambda) \) over \( \lambda \) and after the restoration of the sum over \( N \) we obtain the basic expression at finite temperature:

\[
\langle \Theta(s, t) \rangle_T = T \sum_{N=-\infty}^{+\infty} \int_{-\infty}^{+\infty} d\lambda \Theta(s, t, \lambda) e^{-i\lambda(2\pi NT)}
\]

To obtain the result of the bracket procedure with the momentum \( p_4 \) entering, one has to differentiate Eq.(53) with respect to \( i\lambda \) and then to calculate the integral over \( \lambda \). This can be done also by means of differentiation of Eq.(54) with respect to the parameter \( b_N \equiv 2\pi NT \). In this way the all integrals of interest can be computed.

The expressions with the spatial indexes \( i, j = 1, 2, 3 \) are given by Eqs.(46)-(48), where the function (54) must be substituted.

The average with one spatial component and \( p_4 \) is

\[
\langle p_4 \Theta(s, t) \rangle_T = T \sum_{N=-\infty}^{+\infty} \int_{-\infty}^{+\infty} d\lambda e^{-i\lambda(2\pi NT)} \frac{k_4 + i\lambda/2}{q} \left( \frac{A}{D} \right)_i \Theta(s, t, \lambda)
\]

where \( \Theta(s, t, \lambda) \) must be substituted.

The average with one spatial component and \( p_4 \) is

\[
\langle p_4 \Theta(s, t) \rangle_T = T \sum_{N=-\infty}^{+\infty} \int_{-\infty}^{+\infty} d\lambda e^{-i\lambda(2\pi NT)} \frac{k_4 + i\lambda/2}{q} \left( \frac{A}{D} \right)_i \Theta(s, t, \lambda)
\]
The bracket procedure with $p_4^2$ results in the following expression:

$$
\langle p_4^2 \Theta(s,t) \rangle_T = T \sum_{N=\infty}^{+\infty} \int_{-\infty}^{+\infty} d\lambda \; e^{-i\lambda(2\pi NT)} \left( \left( \frac{k_4 + i\lambda/2}{q} \right)^2 + \frac{1}{2q} \right) \Theta(s,t,\lambda)
$$

$$
= \sum_{N=\infty}^{+\infty} \Theta_T(s,t) \ (2\pi NT)^2. \quad (56)
$$

Note as an interesting fact that the $k_4$-dependence in the last two equations comes in through the exponential, only.

Thus, we collected the necessary integrals over internal momentum which appear in the magnetic background field at finite temperature. These expressions are useful when the high temperature limit is investigated because due to the exponential factor in Eq.(54) a few first terms in the series ($N = 0, 1, 2, ...$) give the leading contributions at $T \to \infty$.

The factor $\sqrt{q}$ entering the integrals over $s$ and $t$ parameters ensures the convergence at $s, t \to 0$. This corresponds to the superficial divergence degree. In the four dimensional theory the form factors have zero superficial divergence degree, hence the three dimensional theory which effectively appears in the high temperature expansion is ultraviolet finite. The convergence at infinity is due to the exponential factors except for the complications resulting from the tachyonic mode which will be discussed later. However, the low temperature limit is less trivial.

Now we consider another representation for the integrals, convenient at low temperature and carry our the ultraviolet renormalization. For this purpose we make resummations of the form

$$
\sum_{N=-\infty}^{+\infty} e^{-(zN^2+aN)} = \sqrt{\pi z} \sum_{N=-\infty}^{+\infty} e^{(a+2\pi iN)^2/4z} \quad (57)
$$

of the series in $N$ in the Eq.(54) with the parameters $a = 4\pi T k_4 t$ and $z = 4\pi^2 T^2 q$ chosen. In this case the dependence on $k_4^2$ in the exponential disappears and we obtain for $\langle \Theta(s,t) \rangle_T$:

$$
\langle \Theta(s,t) \rangle_T = \sum_{N=-\infty}^{+\infty} \langle \Theta(s,t) \rangle \ exp \left( -\frac{N^2}{4T^2 q} + \frac{ik_4 t N}{q T} \right). \quad (58)
$$

This is just the representation of interest. As above, the function standing under the sign of the sum will be denoted by $\Theta_T$.

To obtain the functions $\langle \Theta(s,t)p_4 \rangle_T$ and $\langle \Theta(s,t)p_4^2 \rangle_T$ we have to calculate the integrals over the variable $\lambda$ with the powers of $\lambda$ and $\lambda^2$ in the Eqs.(55) and (56) by means of differentiation of the expression (58) with respect to the parameter $b_k = ik_4 t/q$ one and two times, correspondingly. In this way we obtain in the first case,

$$
\langle p_4 \Theta(s,t) \rangle_T = \sum_{N=-\infty}^{+\infty} \left( \frac{k_4 t}{q} + \frac{iN}{2qT} \right) \langle \Theta(s,t) \rangle \ exp \left( -\frac{N^2}{4qT^2} + \frac{ik_4 t N}{q T} \right). \quad (59)
$$

This expression can be combined with the bracket operation for the spatial momenta $p_i$. In fact, the function $(\partial / \partial k)_\mu$ for $\mu = 4$ equals to $(t/q)k_4$ that coincides with the first term in the bracket in the above equation. To account for the second term, we assume that the thermostat is at rest and therefore the vector $u_\mu$ has only one non zero component,
\[ u_\mu = \delta_{\mu 4}. \] To write down Eq. (59) in short, we introduce the notation \( \tilde{u}_\mu = \frac{iN}{2qT} u_\mu. \) Then, the average operation with one internal momentum yields

\[
\langle p_\mu \Theta(s, t) \rangle_T = \sum_{N=-\infty}^{+\infty} \left( \left( \frac{A}{D} k \right)_\mu + \tilde{u}_\mu \right) \Theta(s, t)_T. \tag{60}
\]

In the same manner, in case of two internal momenta the result can be presented in the form,

\[
\langle p_\mu p_\nu \Theta(s, t) \rangle_T = \sum_{N=-\infty}^{+\infty} \left[ \left( \left( \frac{A}{D} k \right)_\mu + \tilde{u}_\mu \right) \left( \left( \frac{A}{D} k \right)_\nu + \tilde{u}_\nu \right) - i \left( \frac{F}{D^T} \right)_{\mu \nu} \right] \Theta_T(s, t). \tag{61}
\]

Thus, we have calculated all integrals needed for what follows.

The derived expressions (46)-(48) and (58), (60), (61) solve the problem on the momentum integration in the Schwinger operator formalism at finite temperature. The results are expressed in terms of two-parametric integrals, as at zero temperature, and a sum over \( N \) which takes into account the temperature dependence.

So, all further steps of calculations necessary to obtain the expression for the polarization tensor actually coincide with that in Ref. [10]. Within this formalism, the polarization tensor becomes an expression of the type

\[
\Pi_{\lambda \lambda'}(k) = \int_0^\infty \int_0^\infty ds \, dt \, \langle M_{\lambda \lambda'}(p, k) \Theta \rangle_T, \tag{62}
\]

where in \( M_{\lambda \lambda'}(p, k) \) we collected all factors appearing from the vertexes and from the lines except for that which go into \( \Theta \). By using (60) and (61) the average (the momentum integration over \( p \)) can be transformed into

\[
\langle M_{\lambda \lambda'}(p, k) \Theta \rangle_T = M_{\lambda \lambda'}(s, t) \Theta_T, \tag{63}
\]

where now \( M_{\lambda \lambda'}(s, t) \) collects all factors except for \( \Theta_T \).

As in Ref. [10], we break the whole polarization tensor into parts according to the division introduced in Eq. (41). We write

\[
\Pi_{\lambda \lambda'}(k) = \sum_{i,j} \Pi_{i j, \lambda \lambda'}(k) + \Pi_{\lambda \lambda', \text{ghost}}(k) + \Pi_{\lambda \lambda', \text{tadpole}}(k) \tag{64}
\]

with \( i, j = 1, 2, 3 \) and

\[
\Pi_{i j, \lambda \lambda'}(k) = \Gamma_{\mu \lambda \lambda'}^{(i)}(p) G_{\mu \mu'}(p-k) G_{\nu \nu'}(p-k) \tag{65}
\]

including corresponding subdivisions of \( M \).

To organize our calculation, we remind that the PT is calculated in terms of double parametric integrals in \( s \) and \( t \). The function \( \langle \Theta(s, t) \rangle \) is symmetric whereas \( \Theta_T(s, t) \) is not, because of the factor \( e^{iN N T} = e^{2(\tilde{u}k)t} \) in the temperature dependent exponential. To restore the \( s \leftrightarrow t \) symmetry, we write

\[
e^{2(\tilde{u}k)t} = S_T + A_T \tag{66}
\]

where

\[
S_T = \frac{1}{2} \left( e^{2(\tilde{u}k)t} + e^{2(\tilde{u}k)s} \right), \quad A_T = \frac{1}{2} \left( e^{2(\tilde{u}k)t} - e^{2(\tilde{u}k)s} \right). \tag{67}
\]
Then the function $\Theta_T(s, t)$ can be split in symmetric and antisymmetric parts, $\Theta_T = \Theta_s^T + \Theta_a^T$, with

$$
\Theta_s^T = \langle \Theta(s, t) \rangle S_T e^{-\frac{N^2}{4t^2}},
$$
$$
\Theta_a^T = \langle \Theta(s, t) \rangle A_T e^{-\frac{N^2}{4t^2}}.
$$

With these definitions introduced, the terms entering the integral in Eq.(62) have the following general structure. The expressions in $M_{\lambda\lambda'}(s, t)$ which are symmetric under $s \leftrightarrow t$ go multiplied by the symmetric temperature factor $\Theta_s^T$, and vice versa, the antisymmetric functions go multiplied by $\Theta_a^T$. This observation gives us the possibility to make use of the results in Ref.[10], where the symmetric form factors for the operators $T^{(1)} - T^{(4)}$ were calculated. At finite temperature, they should be multiplied by the factor $\Theta_s^T$ in the total expression. The antisymmetric terms as well as the terms giving contributions to the remaining form factors must be calculated separately.

In fact, this procedure is rather simple and it is described in the previous work. To make the present paper self-contained, we deduce the details of this calculation in the Appendix. In the main text, we restrict ourselves to the description of it in general words. For the first six operators, i.e., for that which do not contain the vectors $u_\mu$, the terms appearing after the momentum integrations have the following forms. First,

$$
\Pi_{\lambda\lambda'} = P_\lambda P_{\lambda'}^T + a \delta_{\lambda\lambda'}^{||} + b \delta_{\lambda\lambda'}^{\perp} + ic F_{\lambda\lambda'},
$$

(69)

where $P_\lambda$ is given in terms of the vectors (33),

$$
P_\lambda = r l_\lambda + \alpha id_\lambda + \beta h_\lambda
$$

(70)

and $P_{\lambda'}^T$ is the transposed expression, $r, \alpha, \beta, a, b, c$ are some functions specific for different parts of $\Pi_{\lambda\lambda'}$.

The second type of expressions has a slightly more complicated form

$$
\Pi_{\lambda\lambda'} = P_{\lambda} Q_{\lambda'} + Q_{\lambda} P_{\lambda'}^T + a \delta_{\lambda\lambda'}^{||} + b \delta_{\lambda\lambda'}^{\perp} + ic F_{\lambda\lambda'},
$$

(71)

with $P_\lambda$ from (A.2) and

$$
Q_\lambda = s l_\lambda + \gamma id_\lambda + \delta h_\lambda,
$$

(72)

where $s, \gamma, \delta$ are some other functions. Then, from the requirement that the weak transversality condition (32) holds, one can derive the expressions standing in front of the operators $T^{(1)} - T^{(6)}$. This procedure is efficient but, of course, not obligate.

The form factors standing in front of the operators $T^{(7)} - T^{(10)}$ can be obtained after integration in accordance with Eqs.(60), (61) as that part of the expressions which are proportional to one or to two powers of $u_\mu$. To make clear the structure of the expressions which appear after the averaging procedure, we note that it results in the formal substitution of $p_\mu$ in the initial expressions by $p_\mu \to \tilde{p}_\mu + \tilde{u}_\mu = [(\frac{\partial}{\partial y}) + \tilde{u}]_\mu$ in the final ones. So, the $u$-dependent parts are easily determined. The necessary details for different parts of the polarization tensor are given in the next section.

We complete this section with the description of the renormalization procedure adopted. As it is well known, the divergent parts of the polarization tensor do not depend on the temperature and field. So, each form factor can be written as follows

$$
\Pi^{(i)}(B, T) = [\Pi^{(i)}(B, T) - \Pi^{(i)}(B = 0, T = 0)] + \Pi^{(i)}(B = 0, T = 0).
$$

(73)
Then, the expression in the brackets is finite whereas the last term is divergent and must be renormalized by the standard procedure in quantum field theory. In terms of the resummed series \( \Gamma^{(i)}_{\mu \nu \lambda} \) the term with \( N = 0 \) just corresponds to the zero temperature case. So, actually the above procedure has relevance to these terms, only. As a result, we obtain the renormalized polarization tensor at finite temperature in the field presence which is the object of interest.

5 Calculation of the form factors

In this section we calculate the contributions to the form factors stemming from individual terms \( \Gamma^{(i)}_{\mu \nu \lambda} \) introduced in Eq. (41). At zero temperature that was done in Ref. [10]. So, here we present mainly that part of calculations having relevance to the temperature dependence of these expressions. The first is

\[
\Pi_{11}^{\lambda \lambda'} = (k - 2p)_{\lambda} G_{\mu \nu} (p) (k - 2p)_{\lambda'} G_{\mu' \nu'} (p - k)
\]

and it transforms into

\[
M_{11}^{\lambda \lambda'} (s, t) \Theta_T = \left( (k - 2p)_{\lambda} (k - 2p(s))_{\lambda'} E^{-s}_{\mu \nu} E^{-t}_{\mu' \nu'} \Theta \right) T
\]

\[
= \left[ \left( \left( 1 - 2 \frac{A}{D} \right) k - 2 \tilde{u} \right)_{\lambda} \left( \left( 1 - 2 E^s \frac{A}{D} \right) k - 2 \tilde{u} \right)_{\lambda'} - 4i \left( \frac{E^{-s} F}{D^\top} \right)_{\lambda \lambda'} tr E^{-s-t} \right] \Theta_T.
\]

We note the property \( E^s \frac{A}{D} = \left( \frac{A}{D} \right)^\top \). The trace is

\[
tr E^{-s-t} = 2 [1 + \cosh(2q)].
\]

Remind the variable \( q = s + t \).

Here we also introduce the variable \( \xi = s - t \) which is antisymmetric with respect to the replacement \( s \leftrightarrow t \) and assume \( s \) and \( t \) to be replaced, \( s = (q + \xi)/2 \) and \( t = (q - \xi)/2 \).

Using the notation of Eq. (A.1) we define

\[
P_\lambda = \left( \left( 1 - 2 \frac{A}{D} \right) k \right)_{\lambda}
\]

\[
= \frac{\xi}{q} l_\lambda + 2id_\lambda \frac{\beta \sinh(t)}{\sinh(q)} + h_\lambda \frac{\sinh(\xi)}{\sinh(q)}
\]

\[
= rl_\lambda + i\alpha d_\lambda + \beta h_\lambda
\]

and below we will express the obtained results for different form factors in terms of \( r, \alpha, \beta \).

The second part in the expression (75) has to be integrated by parts. This procedure at finite temperature requires a special explanation. We represent

\[
-2iF E^{-s} \frac{D^\top}{D} = \left( \frac{\partial}{\partial \xi} \right) \left[ r \delta^{(1)} - iF \frac{\cosh(\xi)}{\sinh(q)} + \delta^{1/2} \beta + C \right],
\]

where \( C \) is a constant, i.e., it must be independent on \( \xi \). We include it in the integration procedure in order to make use of this parameter, see below. The derivative with respect
to $\xi$ will be integrated by parts. We should note that expressions which are symmetric under an exchange of $s$ and $t$, i.e., which depend on $q$ only, are not affected in Eq. (78).

So, in $\Theta_T$ we have to differentiate only the terms which depend on $\xi$,

$$\frac{\partial}{\partial \xi} \Theta_T = \left( \frac{1}{2} B_1 - 2(\tilde{u}k) \right) \Theta_T,$$

with the notation

$$B_1 \equiv rl^2 + \beta h^2.$$  \hfill (80)

Then, after integration by parts, the last term in the Eq. (75) gives the contribution

$$-4i \left( \frac{E^{-s} F}{D^+} \right)_{\lambda\lambda'} = 2(1 + \cosh(2q))(2(\tilde{u}k) - B_1)$$

$$\left[ r\delta^{ll} - iF \frac{\cosh(\xi)}{\sinh(q)} + \delta^{\perp} \beta + C \right] \Theta_T,$$

that has to be added to the first part of the equation. It contains a term $4\tilde{u}_\mu \tilde{u}_\nu \Theta_T$ which is is the quadratic in $\tilde{u}_\mu$. Now we chose the constant $C = -2\frac{\tilde{u}_\mu \tilde{u}_\nu}{(\tilde{u}k)}$ in a way that this term cancels and considerably simplifications in further calculations appear. In this a way we arrive at the final expression at finite temperature. As concerns the surface term, it is canceled by the contribution of the tadpole diagram, see below.

Applying formula (A.3) from the Appendix for the function (A.1) with the parameters $a = rB_1$ and $b = \beta B_1$, giving the contributions to the first six form factors, and gathering the factors at the $u$-dependent structures giving rise to the operators $T^{(7)} - T^{(10)}$, we obtain for $M_{\lambda\lambda'}^{11}$:

$$M_{\lambda\lambda'}^{11}(s, t) = \left\{ -r^2 T^{(1)} + \left( \alpha^2 - \beta^2 \right) T^{(2)} - r\beta T^{(3)} - \alpha r T^{(4)} + [B_1 \coth(q) - 2(\tilde{u}k)]T^{(6)} + 2(\tilde{u}k)[-rT^{(7)} - \beta T^{(8)} + \alpha T^{(9)}] \right\} 2(1 + \cosh(2q)).$$  \hfill (82)

This rather simple expression includes the symmetric and antisymmetric with respect to the $\leftrightarrow t$ terms. Since the integral in $s, t$ is symmetric, actually, the factor $\Theta_T^x$ stands at the former terms, and $\Theta_T^z$ at the latter ones. This remark concerns all the expressions written below for other parts of $\Pi_{\lambda\lambda'}$.

The next contribution is $M^{22}$. From (41) we get

$$\Pi_{\lambda\lambda'}^{22} = 4 \left( \delta_{\lambda\mu} k_{\nu'} - \delta_{\lambda\nu} k_{\mu'} \right) G_{\mu\nu'}(p) \left( \delta_{\lambda'\mu'} k_{\nu'} - \delta_{\lambda'\nu'} k_{\mu'} \right) G_{\nu\nu'}(p - k)$$  \hfill (83)

and

$$M_{\lambda\lambda'}^{22}(s, t) = 4 \left( E_{\lambda\lambda'}^{-s} (kE^{-1}k) + E_{\lambda\lambda'}^{t} (kE^{-s}k) \right.$$

$$- \left( E^{-s}k \right)_{\lambda'} \left( E^{-t}k \right)_{\lambda'} - \left( E^{t}k \right)_{\lambda} \left( E^{s}k \right)_{\lambda'}).$$  \hfill (84)

Here we have expression of the type of (A.4). The parameters are

$$r = 1, \quad \alpha = \sinh(2t), \quad \beta = \cosh(2t),$$

$$s = 1, \quad \gamma = \sinh(2s), \quad \delta = \cosh(2s).$$  \hfill (85)
and

\begin{align*}
a &= -2l^2 - h^2(\cosh(2s) + \cosh(2t)), \\
b &= -l^2(\cosh(2s) + \cosh(2t)) - 2h^2 \cosh(2s) \cosh(2t).
\end{align*}

Using formula (A.6) from the Appendix we obtain

\begin{equation}
M_{22}^{22}(s, t) = 8T^{(1)}(1) + 8 \cosh(2(s + t))T^{(2)} + 4(\cosh(2s) + \cosh(2t))T^{(3)} - 4(\sinh(2s) - \sinh(2t))T^{(4)}.
\end{equation}

Note that the last form factor is antisymmetric and does not contribute at zero temperature.

Now we consider \(\Pi_{12}^1\) and \(\Pi_{21}^1\). From (41) we get

\begin{equation}
\Pi_{12}^1 + \Pi_{21}^1 = \delta_{\lambda\mu}(k - 2p)\lambda G_{\mu\nu}(p)2(\delta_{\lambda\mu'}k_{\nu'} - \delta_{\lambda\nu'}k_{\mu'})G_{\nu\nu'}(p - k) + 2(\delta_{\lambda\mu}k_{\nu} - \delta_{\lambda\nu}k_{\mu})G_{\mu\nu'}(p)\delta_{\mu'\nu'}(k - 2p)\lambda G_{\nu\nu'}(p - k),
\end{equation}

and further

\begin{equation}
(M_{12}^{12} + M_{21}^{21}) \Theta_T = \left\{2 \left( (k - 2p)\lambda \left( (E^{s+t} - E^{-s-t})k \right)_{\lambda'} + \left( (E^{s+t} - E^{-s-t})^\top k \right)_{\lambda} (k - 2p)\lambda \right) \Theta_T \right\}.
\end{equation}

We use the averages (60) and obtain,

\begin{equation}
(M_{12}^{12} + M_{21}^{21}) \Theta_T = 2 \left\{ \left( 1 - 2\frac{A}{D} \right) k - 2\tilde{u} \right\}_{\lambda} \left( Qk \right)_{\lambda'} + \left( Q^\top k \right)_{\lambda} \left( 1 - 2\frac{A^\top}{D^\top} \right) k - 2\tilde{u} \right\}_{\lambda} \Theta_T.
\end{equation}

This is an expression of the form of (A.4) with \(P_{\lambda}\) from Eq. (77) and with additional \(u\)-dependent terms, where

\begin{equation}
Q \equiv E^{s+t} - E^{-s-t} = 2iF \sinh(2q).
\end{equation}

Then from Eq. (A.6) and accounting for the \(u\)-dependent structures we find

\begin{equation}
M^{12}(s, t) + M^{21}(s, t) = 4 \sinh(2q) \left[ -2\alpha T^{(2)} + r T^{(4)} + (2(\tilde{u}k) - B_1)T^{(6)} - \frac{iN}{qT} T^{(9)} \right].
\end{equation}

Next we consider the contribution of \(\Pi^{33}\) together with the contribution from the ghosts, \(\Pi^{\text{ghost}}\). We get

\begin{equation}
\Pi^{33} = (\delta_{\lambda\mu}(p - k)\nu + \delta_{\lambda\nu}p_{\mu}) G_{\mu\nu'}(p) (\delta_{\lambda\nu'}(p - k)\nu' + \delta_{\lambda\nu'}p_{\nu'}) G_{\nu'\nu}(p - k).
\end{equation}
We use the property of the propagator

\[ p_\mu G_{\mu\nu'}(p) = G(p)p_\mu' \]  

and obtain after simple calculation, using, for instance, the cyclic property of the trace,

\[ \Pi_{\lambda\nu'}^{33} = G_{\lambda\nu'}(p)G(p-k)(p-k)^2 + p^2G(p)G_{\lambda\nu'}(p-k) + p_\lambda G(p)(p-k) + (p - k)_\lambda G(p)p_{\nu'}G(p-k). \]

In the first two lines in the r.h.s. one line collapses into a point by means of, e.g., \( p^2G(p) = 1 \) and the corresponding graph becomes a tadpole like contribution which will be considered below separately together with the tadpole diagram. The last two lines in the r.h.s. are just equal to the contribution from the ghosts, second line in (40), with opposite sign and cancel. So we obtain

\[ \Pi_{\lambda\nu'}^{33} + \Pi_{\lambda\nu'}^{\text{ghost}} = 0. \]  

Let us turn to the \( \Pi_{\lambda\nu'}^{13} \) and \( \Pi_{\lambda\nu'}^{31} \) contributions. We start from

\[ \Pi_{\lambda\nu'}^{13} = \delta_{\mu\nu'}(k - 2p)_\lambda G_{\mu\nu'}(p) (\delta_{\lambda\nu'}(p - k)_\nu + \delta_{\lambda\nu'}p_{\mu'}) G_{\nu\nu'}(p - k), \]

\[ \Pi_{\lambda\nu'}^{31} = (\delta_{\lambda\mu}(p - k)_\nu + \delta_{\lambda\mu}p_\mu) G_{\mu\nu'}(p)\delta_{\mu\nu'}(k - 2p)_\lambda G_{\nu\nu'}(p - k) \]

and arrive at

\[ M_{\lambda\nu'}^{13}(s, t) = -\langle (k - 2p)_\lambda (E^{sy}(k - 2p(s)) + E^{as}k)_\nu \Theta \rangle_T, \]

\[ M_{\lambda\nu'}^{31}(s, t) = -\langle (E^{sy}(k - 2p) - E^{as}k)_\lambda (k - 2p(s))_\nu \Theta \rangle_T, \]

where the notation

\[ E^{sy} = \frac{1}{2}(E^{s+t} + E^{-s-t}) = \delta^\parallel + \delta^\perp \cosh(2q), \]

\[ E^{as} = \frac{1}{2}(E^{s+t} - E^{-s-t}) = iF \sinh(2q) = \frac{1}{2}Q \]

is introduced. Remind that the contributions appearing after the averaging procedure can be obtained by the substitution \( p_\mu \rightarrow \bar{p}_\mu + \bar{u}_\mu = [(\frac{1}{16} k) + \bar{u}]_\mu \) in the final expressions.

Doing so we obtain for the averages

\[ M_{\lambda\nu'}^{13}(s, t) = \left\{ -\bar{P}_\lambda \tilde{Q}_\lambda + 4i \left( \frac{E^{sy}E^{-s}F}{D^\dagger} \right)_{\lambda\nu'} \right\} \Theta_T, \]

\[ M_{\lambda\nu'}^{31}(s, t) = \left\{ -\tilde{Q}_\lambda^\dagger \bar{P}_\lambda^\dagger + 4i \left( \frac{E^{sy}E^{-s}F}{D^\dagger} \right)_{\lambda\nu'} \right\} \Theta_T, \]

where \( \tilde{P}_\lambda \) is given by Eq.(77) with the mentioned replacement being done, and \( \tilde{Q}_\lambda = Q_\lambda - 2\bar{u}_\lambda \) with

\[ Q_\lambda = \left( \begin{pmatrix} E^{as} + E^{sy} \left( 1 - \frac{2A}{D} \right)^\top \\ \end{pmatrix} k \right)_\lambda \]

\[ = rl_\lambda + id_\lambda \left( \sinh(2q) - \alpha \cosh(2q) \right) + \beta \cosh(2q) h_\lambda \]

\[ \equiv s l_\lambda + \gamma id_\lambda + h_\lambda. \]
The second contributions to the both lines in the r.h.s. in Eq. (95) must be integrated by parts.

This expression differs from that of in Eq. (78) by the factor \( E^{sy} \). That results in the following replacements in the Eqs. (78), (81), \( \beta \rightarrow \beta \cosh(2q) \equiv \delta, \cosh(\xi) \rightarrow \cosh(q) \cosh(2q) \equiv \gamma. \) With these substitutions done and accounting for the overall factor 8, we calculate this contribution,

\[
8i \frac{E^{sy} E^{-s} F}{D^*} = [2B_1 - 4(\bar{u}k)](r\delta^1 + \delta^\perp - i\gamma F).
\] (97)

Again, we have chosen the constant \( C = -2\bar{u}u/(\bar{u}k) \) and omitted the surface term. The latter will be considered separately. The above expression gives the parameters \( C \) and \( \gamma \) of the \( u \)-dependent tensors, we obtain

\[
\Pi^{13} + \Pi^{31} = 2r^2T^{(1)} + 2 \left\{ [\beta^2 - \alpha^2] \cosh(2q) + \alpha \sinh(2q) \right\} T^{(2)} + r\beta (1 + \cosh(2q)) T^{(3)} - 2r \sinh(q) \sinh(\xi) T^{(5)} + [2(\bar{u}k)(\alpha - \gamma + 2\gamma)
\]

\[
+l^2 r(\gamma - \alpha - 2\gamma) + h^2 \beta(\gamma - \alpha \cosh(2q) - 2\gamma)] T^{(6)} + \frac{iN}{qT} \left[ r \frac{T^{(7)}}{k_4} - \frac{\beta(1 + \cosh(2q)) T^{(8)}}{k_4} + r(\alpha - \gamma) T^{(4)} + (\gamma - \alpha) T^{(9)} + k_4 \beta(1 - \cosh(2q)) T^{(10)} \right].
\] (98)

Finally, we need \( \Pi^{23} \) and \( \Pi^{32} \). Proceeding in the same way as before, we derive from (40) and (11)

\[
\Pi^{23}_{\lambda\lambda'} = 2(\delta_{\lambda\mu} k_{\nu} - \delta_{\lambda\nu} k_{\mu}) G_{\mu\mu'}(p)(\delta_{\lambda\mu'}(p - k)_{\nu} + \delta_{\lambda\nu'} k_{\mu'}) G_{\nu\nu'}(p - k),
\]

\[
\Pi^{32}_{\lambda\lambda'} = 2(\delta_{\lambda\mu}(p - k)_{\nu} + \delta_{\lambda\nu} p_{\mu}) G_{\mu\mu'}(p)(\delta_{\lambda\mu'} k_{\nu'} - \delta_{\lambda\nu'} k_{\mu'}) G_{\nu\nu'}(p - k),
\] (99)

which gives

\[
M^{23}_{\lambda\lambda'}(s, t)\langle \Theta \rangle = \left\langle 2 \left\{ - E^{-s}_\lambda (kE^t(k-p(s))) - E^t_{\lambda\lambda'} (kE^{-s}p(s)) \right. \right.
\]

\[
+ \left. \left. (E^{-s}p(s))_\lambda (E^{-t}k)_{\lambda'} - (E^t(k-p(s)))_\lambda (E^{s}k)_{\lambda'} \right\} \right. \Theta \right\rangle_T,
\]

\[
M^{32}_{\lambda\lambda'}(s, t)\langle \Theta \rangle = \left\langle 2 \left\{ - E^{-s}_\lambda (kE^{-t}(k-p)) - E^t_{\lambda\lambda'} (pE^{-s}k) \right. \right.
\]

\[
+ \left. \left. (E^{-s}k)_\lambda (E^{-t}(k-p))_{\lambda'} - (E^t k)_\lambda (E^{s}p)_{\lambda'} \right\} \right. \Theta \right\rangle_T.
\] (100)

Now the average (60) must be used. As it was noted, this results in the substitutions \( p_{\mu} \rightarrow \bar{p}_{\mu} + \bar{u}_{\mu} \) in Eq. (100). For the operator \( \bar{p}(s) \) the matrix \( (A^T k) \) must be substituted. In this way the \( u \)-dependent part is calculated. The \( u \)-independent part can be rearranged according to

\[
M^{23}_{\lambda\lambda'} + M^{32}_{\lambda\lambda'} = 2(A + B)
\] (101)
with

\[ A = -2E_{\lambda \lambda'}^t \left( k \frac{E^t - 1}{D} k \right) + (E^t k)_{\lambda} \left( \left( \frac{E^t - 1}{D} \right)^\top k \right)_{\lambda'}, \]
\[ + \left( \frac{E^t - 1}{D} k \right)_{\lambda} \left( E^{-t} k \right)_{\lambda'}, \]
\[ B = -2E_{\lambda \lambda'}^s \left( k \frac{E^s - 1}{D} k \right) + (E^{-s} k)_{\lambda} \left( \frac{E^s - 1}{D} \right)^\top k \right)_{\lambda'}, \]
\[ + \left( \left( \frac{E^s - 1}{D} \right)^\top k \right)_{\lambda} \left( E^s k \right)_{\lambda'}, \]

\[ A \text{ and } B \text{ have the structure of (A.4). For } A \text{ we obtain} \]
\[ r' = 1, \quad \alpha' = \sinh(2t), \quad \beta' = \cosh(2t) \]
\[ s' = \frac{t}{q}, \quad \gamma' = \frac{1}{2} \alpha, \quad \delta' = \frac{\cosh(s) \sinh(t)}{\sinh(q)} \quad (102) \]

and

\[ a' = -2 \left( \frac{t}{q} l^2 + \delta' h^2 \right), \]
\[ b' = \cosh(2t) a', \quad c' = \sinh(2q) a', \quad (103) \]

and for \( B \)

\[ r'' = 1, \quad \alpha'' = -\sinh(2s), \quad \beta'' = \cosh(2s) \]
\[ s'' = \frac{s}{q}, \quad \gamma'' = -\gamma', \quad \delta'' = \frac{\sinh(s) \cosh(t)}{\sinh(q)} \quad (104) \]

and

\[ a'' = -2 \left( \frac{s}{q} l^2 + \delta'' h^2 \right), \]
\[ b'' = \cosh(2s) a'', \quad c'' = -\sinh(2s) a''. \quad (105) \]

For convenience, in the formulas (102) and (104) we used apostrophes for denoting similar parameters having the same structure in different parts of Eq. (A.4).

Putting these contributions together we obtain with (A.6)

\[ \Pi^{23} + \Pi^{32} = 2 \left\{ -2T^{(1)} \right. \]
\[ -2 \left[ \cosh(2t) \cosh(s) \sinh(t) + \cosh(2s) \beta \cosh(t) \right. \]
\[ \left. + 2 \cosh(\xi) \beta \sinh(t) \right] T^{(2)} \]
\[ - \left[ 1 + \frac{s \cosh(2s) + t \cosh(2t)}{q} \right] T^{(3)} \]

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\[+ \frac{s \sinh(2s) - t \sinh(2t)}{q} T^{(4)}
+ \left[-1 + \frac{s \cosh(2s) + t \cosh(2t)}{q}\right] T^{(5)}
+ \left[\frac{(s \sinh(2s) - t \sinh(2t))l^2 + 2 \cosh(q) \sinh(\xi) h^2}{q}\right] T^{(6)}
+ 2(\bar{u}k) \left[- \sinh(q) \cosh(\xi) T^{(6)} - \frac{\sinh(q) \sinh(\xi)}{h_4}\right] T^{(8)}
- \sinh(q) \cosh(\xi) T^{(9)} + \sinh(q) \sinh(\xi) T^{(10)}\right\}. \quad (106)

Now we consider the contribution of the tadpole diagram, the last line in Eq.(40). Accounting for the explicit form of the propagator,
\[G_{\mu\nu}(p) = \int_0^\infty dq e^{-q^2} E_{\mu\nu}^-,\] (107)
and calculating \(trG(p) = 2 \int_0^\infty dq e^{-q^2} (1 + \cosh(2q))\) and the bracket averages, we obtain
\[\Pi_{\mu\nu}^{t_p} = (G_{\mu\nu} + G_{\nu\mu} - 2trG\delta_{\mu\nu})_T
- 2 \frac{1}{(4\pi)^2} \sum_{N=-\infty}^{+\infty} \int_0^\infty dq e^{-\frac{N^2}{4q^2}} \left[\delta_{\mu\nu}(1 + 2 \cosh(2q)) + \delta_{\mu\nu}(2 + \cosh(2q))\right] T^{(8)}
- \sinh(q) \cosh(\xi) T^{(9)} + \sinh(q) \sinh(\xi) T^{(10)}\] (108)

The contribution from the tadpole like terms coming from \(\Pi_{\mu\nu}^{33}\), Eq.(90) equals just the first two terms in the Eq.(108). That adds the extra terms \(2(\delta_{\mu\nu}^|| + \delta_{\mu\nu}^\perp \cosh(2q))\) into the numerator of the above expression and cancels the first and the last terms in the total. Hence, the final contribution of the tadpoles and the tadpole like terms is
\[\Pi_{\mu\nu}^{t_p} = -4 \frac{1}{(4\pi)^2} \sum_{N=-\infty}^{+\infty} \int_0^\infty dq e^{-\frac{N^2}{4q^2}} \frac{\delta_{\mu\nu}^|| \cosh(2q) + \delta_{\mu\nu}^\perp (2 + \cosh(2q))}{q \sinh(q)}\] (109)

Now let us consider the surface contributions from the sum \(\Pi^{sf} = \Pi^{(11)} + \Pi^{(13)} + \Pi^{(31)}:\n\Pi^{sf} = 4 \int_{\text{surface}} dsdt \left[\delta_{\mu\nu}^|| q \cosh(2q)) + \delta_{\mu\nu}^\perp \frac{\sinh(\xi)}{\sinh(q)}\right] \Theta_T,
- iF \frac{\cosh(\xi)}{\sinh(q)} - 2 \frac{\bar{u} \cdot \bar{u}}{(ak) \cosh(2q)}\] (110)

where the explicit expressions for the parameters \(r, \beta\) from Eq.(77) are substituted. To relate the integrals in \(q\) and \(s, t\) we introduce an integration variable \(v = \xi/q\) in place of \(q\).

In this case \(\frac{\partial}{\partial \xi} = \frac{1}{q} \frac{\partial}{\partial v}\), and \(\int dsdt \rightarrow \frac{1}{2} \int_0^\infty dq \int_{-1}^1 dv\). Hence, the integration over \(\xi\) results

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in an integration by parts over \( v \). In the new variables the above equation reads,

\[
\Pi_{sf} = 2 \int_{0}^{\infty} dq \Theta_T \left[ \delta^{II} v \cosh(2q) + \delta^{\perp} \frac{\sinh(qv)}{\sinh(q)} - iF \frac{\cosh(qv)}{\sinh(q)} - 2 \frac{\tilde{u} \cdot \tilde{u}}{(\tilde{u} \cdot \tilde{k})} \cosh(2q) \right]_{v=+1}^{v=-1},
\]

with \( \Theta_T = \exp \left[ 2(\tilde{u} \cdot \tilde{k}) \frac{1}{2} q(1-v) - \frac{N^2}{4qT} \right] \langle \Theta \rangle \),

\[
\langle \Theta \rangle = \frac{1}{(4\pi)^2} \frac{e^{-k(\delta^{II} + \delta^{\perp}) \frac{Nk_4}{T}}}{q \sinh(q)}
\]

and \( \frac{ST}{S_{TT}} = \frac{\tanh[\frac{1}{4} q(1-v)] \tanh[\frac{1}{4} q(1+v)]}{\tanh[\frac{1}{4} q(1-v)] + \tanh[\frac{1}{4} q(1+v)]} \). Hence it follows that

\[
(\Theta_T)_{+1} = \frac{1}{(4\pi)^2} \frac{e^{-\frac{N^2}{4qT} Nk_4}}{q \sinh(q)}
\]

\[
(\Theta_T)_{-1} = \frac{1}{(4\pi)^2} \frac{e^{\frac{iNk_4}{T}}}{q \sinh(q)}
\]

After substitution of these functions in Eq. (111) we obtain

\[
\Pi_{sf} = 2 \int_{0}^{\infty} dq \sum_{N=-\infty}^{\infty} (\Theta_T)^{+1} \left[ (\delta^{II} \cosh(2q) + \delta^{\perp}) \left( 1 + e^{\frac{iNk_4}{T}} \right) - iF \coth(q) + 2 \frac{\tilde{u} \cdot \tilde{u}}{(\tilde{u} \cdot \tilde{k})} \cosh(2q) \right] \left( 1 - e^{\frac{iNk_4}{T}} \right),
\]

Now we note that in the imaginary time formalism the external momentum \( k_4 = 2\pi n_k T \) with \( n_k = 0, \pm 1, \pm 2, \ldots \). The phase factor in the exponential is \( \frac{Nk_4}{T} = 2\pi Nn_k \). So, the exponentials in the brackets equal to 1. This results in the factor 2 for the first two terms and zero for the third term in the integrand. To find the last term we also note that for \( k_4 \neq 0 \) the denominator \( (\tilde{u} \cdot \tilde{k}) \neq 0 \) and the bracket is zero too. For \( k_4 = 0 \), we expand the exponential in a series and find

\[
-2 \left( 1 - e^{\frac{iNk_4}{T}} \right) \frac{\tilde{u} \cdot \tilde{u}}{(\tilde{u} \cdot \tilde{k})} \mid_{k_4=0} = -\frac{N^2}{4qT}.
\]

Thus, we come to the conclusion that the last term contributes in the static limit \( k_4 = 0 \), only.

As a final step, we take the Eq. (111) and the tadpole contribution Eq. (109) together. The terms in front of \( \delta^{II} \) and \( \delta^{\perp} \) do cancel in the total. So, the only contribution is the \( u \)-dependent part coming from \( \Pi_{sf} \). We denote it as \( \Pi_{sf}^{tot} \),

\[
\Pi_{sf}^{tot} = -\frac{4}{(4\pi)^2} \int_{0}^{\infty} dq \sum_{N=-\infty}^{\infty} \frac{\tilde{u} \cdot \tilde{u}}{(\tilde{u} \cdot \tilde{k})} \frac{\cosh(2q)}{q \sinh(q)} e^{-\frac{N^2}{4qT^2}} \left( 1 - e^{\frac{iNk_4}{T}} \right).
\]

This term contributes in the static limit only and it is transversal by itself. This is because being multiplying by \( k_\lambda \) it is zero due to the difference in the curly brackets, if \( k_4 \neq 0 \). The integrand is nonzero if \( k_4 = 0 \), but for the product \( k_\lambda (\Pi_{sf}^{tot})_{\lambda \lambda} = 0 \) holds. In fact, it is
the limiting expression of the tensor $T^{(8)}$ for $k_4 = 0$. However, we will consider this part separately for convenience. Thus, we collect all the contributions coming from individual parts.

Now let us gather them together to obtain the one-loop form factors $M^{(i)}(s, t)$ for the polarization tensor. We present the results as the list of explicit functions of variables $q = s + t$ and $\xi = s - t$:

\[ M_1 = 4 - 2(\xi)^2 \cosh(2q), \]
\[ M_2 = 4\left(1 - \cosh(q)\cosh(\xi)\right) \left(\frac{\cosh(q)}{(\sinh(q))^2}\right) - 2 + 8 \cosh(q) \cosh(\xi), \]
\[ M_3 = -2 \cosh(2q) \frac{\xi \sinh(\xi)}{q \sinh(q)} - 2 + 6 \cosh(\xi) \cosh(q), \]
\[ M_4 = 2\frac{\xi}{q} \left(\sinh(2q) - \frac{\cosh(q) - \cosh(\xi)}{\sinh(q)}\right) - 6 \cosh(q) \sinh(\xi), \]
\[ M_5 = -2 + 2 \cosh(q) \cosh(\xi), \]
\[ M_6^{(1)} = 2 \left[\frac{\xi}{q} \cosh(q)(1 - 3(\sinh(q))^2) + \sinh(\xi) \cosh(q)\right] l^2 + 2 \left[\frac{\sinh(\xi)}{\sinh(q)} \cosh(q)(1 - 3(\sinh(q))^2) + 2 \sinh(\xi) \cosh(q)\right] h^2, \]
\[ M_6^{(2)} = \frac{iN}{qT} k_4 \left(\sinh(2q) - \coth(q)\right), \]
\[ M_7 = \frac{iN}{qT} \frac{1}{k_4 q} \left(-2 \cosh(2q)\right), \]
\[ M_8 = \frac{iN}{qT} \frac{1}{k_4} \left(-2 \frac{\sinh(\xi)}{\sinh(q)} - 4 \sinh(q) \sinh(\xi)\right), \]
\[ M_9 = \frac{iN}{qT} 2 \left[\frac{\cosh(q) - \cosh(\xi)}{\sinh(q)} - \sinh(2q) - 2 \sinh(q) \cosh(\xi)\right], \]
\[ M_{10} = 0. \] (117)

The symmetric form factors have to be multiplied by $\Theta_T^s$ and the antisymmetric ones - by $\Theta_T^a$, when the integration over $s, t$ is carried out. It is interesting that $M_{10}$ is zero in one-loop order.

Thus, according to Eq.(38), we presented the polarization tensor in the form

\[ \Pi^{(i)}(k) = \sum_{i=1}^{9} T^{(i)}_{\lambda\lambda'} \Pi^{(i)}(k) + (\Pi_{\text{tot}}^{(i)})_{\lambda\lambda'}, \]
\[ \Pi^{(i)}(k) = \sum_{N=-\infty}^{+\infty} \int_0^\infty dsdt M^{(i)}(s, t) \Theta_T \] (118)

as double parametric integrals over the proper time parameters $s, t$ and the temperature sum. The last term in the first line is written in Eq.(117). This representation is crucial for what follows. It preserves as much symmetries of the polarization tensor as possible.

The obtained expression for the polarization tensor could be used in various applications. In the next three sections we consider the zero field limit, the Debye mass and the magnetic mass of gluons in the magnetic background field at high temperature.
6 Limit of zero background field

To make a link of our formalism with a usual one, let us consider the limit of zero background field, \( B = 0 \). In our dimensionless variables this simply corresponds to the limit \( q, \xi \) go to zero. In this case, the most form factors and operators also go to zero. More precise, we have for form factors,

\[
M_1 = M_2 = M_3 = 4 - 2 \left( \frac{\xi}{q} \right)^2, \tag{119}
\]

and

\[
M_7 = M_8 = iN \frac{1}{qTk_4} \left( -2\frac{\xi}{q} \right). \tag{120}
\]

All the other operators or form factors are zero. In accordance with Eqs.(35),(37) this means that we obtain two transversal operators with the correct formfactors at finite temperature [15]. Hence, at \( B = 0 \), the polarization tensor can be written as

\[
\Pi_{\lambda\lambda'}(k, T = 1) = \frac{1}{(4\pi)^2} \sum_{N = -\infty}^{\infty} \int_{0}^{\infty} ds dt \left[ e^{-\frac{k^2}{4}st} e^{(2(\hat{u}k)t - \frac{N^2}{4qT^2})} \left( 4 - 2 \left( \frac{\xi}{q} \right)^2 \right) K_{\lambda\lambda'} - 2 \frac{iN}{qTk_4} \frac{1}{q} \int_{0}^{\infty} dq e^{\frac{iNk}{T}} \right] B_{\lambda\lambda'}, \tag{121}
\]

where we substituted the surface contribution Eq.(116) at \( B = 0 \).

In the representation in terms of a series resummed according to Ref. (57), the value \( N = 0 \) corresponds to the zero temperature case. In Eq.(121) this is

\[
\Pi_{\lambda\lambda'}(T = 0) = 1 \left( \frac{4\pi}{2} \right)^2 \int_{0}^{\infty} ds dt e^{-\frac{k^2}{4}st} \left( 4 - 2 \left( \frac{\xi}{q} \right)^2 \right) K_{\lambda\lambda'}, \tag{122}
\]

that coincides with Eq.(103) in Ref.[10]. This part must be renormalized in a standard way. Actually, normalizing to \( B = 0 \) at \( T = 0 \) it must be simplify skipped.

Now, let us calculate the Debye mass squared defined as the limit of the form,

\[
m_D^2 = -\Pi_{44}(T, k_4 = 0, \vec{k} \to 0)[15]. \]

Within the representation (121), only the last term contributes and we obtain

\[
m_D^2 = \frac{1}{4\pi^2} \sum_{N = 1}^{\infty} \frac{N^2}{T^2} \int_{0}^{\infty} dq \frac{e^{-\frac{N^2}{4qT^2}}}{q^3}. \tag{123}
\]

The integral is simply calculated, \( \int_{0}^{\infty} dq \frac{e^{-\frac{N^2}{4qT^2}}}{q^3} = \frac{4\pi^4}{N^2T^4} \), and the sum is expressed through Riemann’s Zeta-function, \( \zeta(2) = \frac{\pi^2}{6} \). Thus, we obtain \( m_D^2 = \frac{2}{3}T^2 \), which is the well known result [15].

The next important parameter is the ”magnetic” mass squared which can be determined as the limit for transversal with respect to the external field direction components,
\[ m^2_{\text{magn}} = -\Pi_{12}(T, k_4 = 0, \vec{k} \to 0). \] In this case, the operator \( K_{12} \) in Eq. (121) contributes (the component \( B_{12} = 0 \)). To calculate the form factor in the high temperature limit, it is convenient to make an inverse resummation according to Eq. (57). In the static limit, \( k_4 = 0 \), the parameter \( a = 0 \) and we have

\[
\sum_{N = -\infty}^{+\infty} e^{-\frac{N^2}{4\pi^2 T^2}} = 2\pi T \left( \frac{2}{\pi} \right)^{1/2} \sum_{N = -\infty}^{+\infty} e^{(-4\pi^2 T^2 N^2 q)}. \tag{124}
\]

For the components \( \Pi_{12} \) we then get

\[
\Pi_{12}(k, T) = \frac{1}{(4\pi)^2} \sum_{N = -\infty}^{+\infty} \int_0^\infty \frac{dq}{q} \int_0^1 du (2\pi T) \left( \frac{q}{\pi} \right)^{1/2} e^{-k^2 qu(1-u)} e^{-N^2 4\pi^2 T^2 q} \left( 4 - 2 \left( \frac{s}{q} \right)^2 \right) K_{12}, \tag{125}
\]

where new variables, \( s, t \to s = qu, t = q(1-u) \), were introduced. The high temperature limit corresponds to the value of \( N = 0 \) in the Matsubara sum. In that case the integrals can be calculated easily. First we compute the integral over \( q \) which delivers \( \Gamma \)-function, \( \frac{\Gamma(\frac{1}{2})}{(k^2 u(1-u))^{1/2}} \). Then the integration over \( u \) gives \( 3\pi \). So, for the form factor we obtain \( \Pi(k)_{12}^{(1)} = \frac{3}{8} \frac{T}{k} \). Hence for the Green function,

\[
G_{12}^0 = \left( -\frac{k_1 k_2}{k^2} \right) \frac{1}{k^2 - \frac{3}{8} k T} \tag{126}
\]

follows. Here \( k \) is the length of the three-momentum vector \( \vec{k} \). This expression has a fictitious pole that was an old problem of gauge theories at finite temperature [15].

From the above calculations we see how to proceed in the present formalism. The procedure remains actually the same when the field is switched on. Below, we investigate an influence of the field on the Debye mass and the existence of the fictitious pole.

### 7 Debye mass in the presence of the background field

The gluon Debye mass squared in the background field is defined as before, \( m^2_D = -\Pi_{44}(T, B, k_4 = 0, \vec{k} \to 0) \), where we have to use the expression Eq. (116) from the polarization tensor. In this limit it reads

\[
m^2_D(B) = \frac{1}{4\pi^2} \int_0^\infty dq \frac{q}{q T^2} \sum_{N = 1}^{+\infty} \frac{N^2}{q T^2} \frac{B \cosh(2Bq)}{\sinh(Bq)} e^{-\frac{N^2}{4\pi^2 T^2}}, \tag{127}
\]

where the dimensional parameters are restored. We investigate the case for the field and temperature ratio \( s = \frac{B}{4T^2} \ll 1 \) and present Eq. (127) in the form: \( m^2_D = \frac{2}{3} T^2 f(s) \), where the function \( f(s) \) is to be computed. It satisfies the condition \( f(0) = 1 \). With this parameter introduced we return again to the dimension less variable \( Bq \to q \) and write \( f(s) \) in the form,

\[
f(s) = \frac{6}{\pi^2} s^2 \int_0^\infty \frac{dq}{q^2} \sum_{N = 1}^{+\infty} N^2 \left[ \frac{1}{q} + \frac{\cosh(2q)}{\sinh(q)} - \frac{1}{q} \right] e^{-\frac{N^2 q}{s}}, \tag{128}
\]
where the first term in the square bracket delivers the zero field limit \( f(0) = 1 \). To calculate the other terms resummations will be done. By differentiating Eq.(57) with respect to \( Z \) with \( a = 0, Z = \frac{q}{s} \), we rewrite \( f(s) \) as follows

\[
f(s) = 1 + \frac{3}{2} \frac{1}{\pi^2} \sqrt{\pi s} \int_{0}^{\infty} \frac{dq}{\sqrt{q}} \sum_{N=-\infty}^{+\infty} \left[ e^q - e^{-q} \right] + \left( \frac{1}{\sinh(q)} - \frac{1}{q} \right) \left( 1 - 2 \frac{\pi^2 N^2 q}{s} \right) e^{-\frac{\pi^2 N^2 q}{s}}.
\]

(129)

Here we made a rearrangement of the integrand in the above equation and split it into three parts - the tachyonic one \( f_t(s) \) coming from \( e^q \), \( f_2(s) \), and \( f_3(s) \) coming from the curly brackets. Then we calculate the leading terms of the high temperature expansion which is given by the terms with \( N = 0 \) in the temperature sum, next to leading - by the terms with \( N \geq 1 \).

Now, let us calculate contributions from \( N = 0 \). For \( f_t(s, N = 0) \) we get

\[
f_t(s, N = 0) = \frac{3}{2} \frac{1}{\pi^2} \sqrt{\pi s} \int_{0}^{\infty} \frac{dq}{\sqrt{q}} e^q \]

(130)

This integral diverges at the upper limit that reflects the tachyonic mode in the spectrum of charged gluons (25). To obtain its value we make the inverse Wick rotation in the \( q \)-plain, that is, replace \( q \rightarrow q e^{-i\pi} \). After that a simple integration yields

\[
f_t(s, N = 0) = -i \frac{3}{2} \frac{1}{\pi^2} \sqrt{\pi s} \sqrt{\pi} = -i \frac{3}{2\pi} \sqrt{s}.
\]

(131)

The second term is

\[
f_2(s, N = 0) = -\frac{3}{2} \frac{1}{\pi^2} \sqrt{\pi s} \int_{0}^{\infty} \frac{dq}{\sqrt{q}} e^{-q}
\]

(132)

and can be easily computed, \( f_2(s, N = 0) = -\frac{3}{2\pi} \sqrt{s} \). The third term is

\[
f_3(s, N = 0) = \frac{3}{2} \frac{1}{\pi^2} \sqrt{\pi s} \int_{0}^{\infty} \frac{dq}{\sqrt{q}} \left( \frac{1}{\sinh(q)} - \frac{1}{q} \right),
\]

(133)

that can be expressed through a Zeta- function, \( f_3(s, N = 0) = \frac{3}{2} \frac{1}{\pi^2} \sqrt{\pi s} \sqrt{2\pi} \zeta \left( \frac{1}{2} \right) \). Finally we derive

\[
f_3(s, N = 0) = \frac{3}{\sqrt{2\pi}} \left( \sqrt{2} - 1 \right) \zeta \left( \frac{1}{2} \right) \sqrt{s}.
\]

(134)

Similar simple integrations for the value \( N = 1 \) result in

\[
f(s, N = 1) = \frac{25}{4} \frac{\zeta(3)}{\pi^4} s^2.
\]

(135)

Thus, for the function \( f(s) \) we obtain

\[
f(s) = 1 + \left[ \frac{3}{\sqrt{2\pi}} \left( \sqrt{2} - 1 \right) \zeta \left( \frac{1}{2} \right) - \frac{3}{2\pi} \right] \sqrt{s}
\]

\[
+ \frac{25}{4} \frac{\zeta(3)}{\pi^4} s^2 - i \frac{3}{2\pi} \sqrt{s}.
\]

(136)
Hence, for the Debye mass we derive
\[
m_D^2(B) = \frac{2}{3} T^2 \left[ 1 - 0.8859 \left( \frac{\sqrt{E}}{2T} \right) + 0.4775 \left( \frac{B^2}{16T^4} \right) \right.
- \left. i \, 0.4775 \left( \frac{\sqrt{B}}{2T} \right) + O \left( \frac{B^3}{T^6} \right) \right],
\]
where the numeric values of the coefficients in Eq. (136) are substituted. This expression is interesting in two respects. In presence of the field, the screening mass for color Coulomb forces is decreased as compared to the zero field case. This behavior was already determined in Ref. [7] in calculation of other type. The imaginary part of \( m_D^2 \) is signaling the instability of the states because of the tachyonic mode in the spectrum (25). However, the numeric value of it is small as compared to the real one. It is of order of the usual plasmon damping factor at finite temperature.

8 Transversal modes in the field presence

Let us investigate the behaviour of the transversal modes in the field at high temperature. We have to calculate the mean value of the polarization tensor in the states given by Eq. (26) for the polarizations \( s = 1 \) and \( s = 2 \) in the limit of \( k_4 = 0, \vec{k} \to 0 \). Accounting for Eqs. (34), (36), (26), we derive for the mean values
\[
\langle s = 1 | \Pi(k) | s = 1 \rangle = h^2 \Pi_2 + l^2 (\Pi_3 - \Pi_5),
\]
\[
\langle s = 2 | \Pi(k) | s = 2 \rangle = \frac{h^2 l_4^2}{k^2} \Pi_1 + \left( h^2 + \frac{l^2 + h^2}{k^2} l_3^2 \right) \Pi_3 + \frac{h^4 - l^2 l_3^2}{k^2} \Pi_5,
\]

where \( l_4 = k_4, k^2 = h^2 + l_3^2, l^2 = l_3^2 + l_4^2 \). To consider the behaviour of the static modes in the perpendicular with respect to the field plane we put \( l_3 = 0 \) and \( k_4 = 0 \) and get
\[
\langle s = 1 | \Pi(k) | s = 1 \rangle = h^2 \Pi_2,
\]
\[
\langle s = 2 | \Pi(k) | s = 2 \rangle = h^2 (\Pi_3 + \Pi_5).
\]
We have to calculate the form factors \( \Pi_2, \Pi_3 \) and \( \Pi_5 \) educed in Eqs. (117), (118) also for this case.

The procedure of calculations is quite similar to that of in the previous sections. We describe its steps by computing the form factor \( \Pi_5 \). To carry out integration over \( \xi \), we change variables, \( s = qu, t = q(1 - u) \), as in section 6. In the limit of interest, \( h^2 \to 0 \), we restrict ourselves to the leading in this parameter term when the function \( \langle \Theta \rangle \) Eq. (46) is substituted. More precise, we take into account the first term in the expansion,
\[
\langle \Theta \rangle = \frac{1}{(4\pi)^2} \frac{1 + O(h^2)}{q \sinh(q)}
\]

because, being substituted into Eq. (140), the \( O(h^2) \) terms result in a next to leading correction. Then we make resummation of the series in \( N \) and take into consideration the
term with $N = 0$, that gives the leading high temperature contribution. In this limit we obtain for the form factors
\[
\Pi_i^{(N=0)} = \frac{1}{(4\pi)^{3/2}} \frac{T}{\sqrt{B}} \int_0^1 du \int_0^\infty dq \sqrt{q} \frac{M_i(q,u)}{\sinh(q)}, \quad (142)
\]
where the functions from Eq. (117) should be substituted. Here we restored the overall factor $\frac{T}{\sqrt{B}}$. All other variables are dimensionless.

For the function $M_5$, integration over $u$ results in the expression
\[
M_5(q) = -2 + \frac{2 \cosh(q) \sinh(q)}{q} \quad (143)
\]
which after substitution in Eq. (142) leads to the integral
\[
I_5 = \int_0^\infty dq \left[-2 \sqrt{q} \sinh(q) + \frac{e^{-q}}{\sqrt{q}} + \frac{e^q}{\sqrt{q}}\right]. \quad (144)
\]
The second and the third terms are calculated in Eqs. (130)-(132) in section 7, $I_5^{(2)} = \sqrt{\pi}$, $I_5^{(3)} = -i \sqrt{\pi}$, and for the first one we compute $I_5^{(1)} = \frac{1}{2}(-4 + \sqrt{2}) \sqrt{\pi} \zeta(\frac{3}{2})$. Thus, we obtain in the total,
\[
\Pi_5 = \frac{1}{(4\pi)^{3/2}} \frac{T}{\sqrt{B}} [-4.21405 - 1.77245i], \quad (145)
\]
where numeric values of the integrals are substituted.

Similar calculations carried out for the form factors $\Pi_2$ and $\Pi_3$ yield
\[
\Pi_2 = \frac{1}{(4\pi)^{3/2}} \frac{T}{\sqrt{B}} [-5.79894 - 7.08982i],
\]
\[
\Pi_3 = \frac{1}{(4\pi)^{3/2}} \frac{T}{\sqrt{B}} [1.04427 - 8.86227i]. \quad (146)
\]
The above expressions have to be used in Eq. (140) to obtain final result. The sum of $\Pi_3 + \Pi_5$ equals, $\Pi_3 + \Pi_5 = \frac{T}{\sqrt{B}} [-3.16978 - 10.6347i]$. The imaginary part is signaling the instability of the state because of the tachyonic mode presenting in the vacuum, and the real part is responsible for the screening of transversal gluon fields.

As concerns the imaginary part. Since its origin is clear, one should conclude that the one-loop result needs in additional terms coming from some resummation at finite temperature in the sector of charged components similar to that of in Refs. [7], [8]. That we left for the future.

Let us turn to the real part and substitute it in the Schwinger-Dyson equation
\[
D^{-1}(k^2) = k^2 - \Pi(k) \quad (147)
\]
for the neural gluon Green function. We obtain for the mean values
\[
\langle s = 1 | D^{-1}(h^2) | s = 1 \rangle = h^2 - \text{Re}(\Pi_2) h^2 = h^2 \left(1 + 5.79894 \frac{T}{\sqrt{B}}\right) \quad (148)
\]
\[
\langle s = 2 \mid D^{-1}(h^2) \mid s = 2 \rangle = h^2 - Re(\Pi_3 + \Pi_5) h^2 \\
= h^2 \left( 1 + 3.16978 \frac{T}{\sqrt{B}} \right).
\] (149)

These are the expressions of interest.

Two important conclusions follow from Eqs. (148), (149). First, for the transversal modes in the field presence, there is no fictitious pole similar to that of in Eq. (126). Second, there is no the magnetic screening mass in one-loop order. The transversal components of gluon field remain long range in this approximation, as at zero external field [14]. These conclusions are in a complete correspondence with the results obtained in Ref. [14] in the Fujikawa gauge. In this paper only the static limit \( k_4 = 0 \) was investigated.

As it has been discovered in Refs. [7], [8], for the charged components the magnetic mass in the color magnetic field at high temperature is generated in one-loop order. So, there is an essential difference in this point. It is also interesting that the non transversal tensor structure \( T^{(5)} \) contributes to the transversal state \( s = 2 \).

9 Discussion

In the present paper the operator structure of the neutral gluon polarization tensor in a constant Abelian chromomagnetic field at finite temperature is derived. There are ten operator structures contributing in general case. We mention that these structures do not depend on the gauge group; for a SU(3) these are the same as for the SU(2) considered here. As investigated in details in Refs. [10], [11] at zero temperature, the polarization tensor is not transversal. The tensors \( T^{(5)}, T^{(6)}, T^{(9)} \) and \( T^{(10)} \) satisfy the weak transversality condition Eq. (32). As actual one-loop calculations showed, the form factors \( \Pi_5, \Pi_6, \Pi_9 \) are nonzero and \( \Pi_{10} = 0 \). So, only three non transversal operators contribute in the total in this approximation.

As at zero temperature Ref. [10], to carry out integration over internal momentum of diagrams in the field presence which is necessary in calculation of form factors, the Schwinger algebraic method [12] was applied. It has been extended to account for finite temperature. As a result, we have derived the explicit expressions for integrals which present the form factors in terms of two parametric integrals over the proper-time parameters \( s, t \), as at zero temperature, and extra series in discrete variable \( N \) accounting for the temperature dependence. This representation preserves gauge invariance and even Lorentz covariance at each step of calculations as far as possible. It is simple and convenient for investigations.

Within this representation, the one-loop form factors for the polarization tensor of neutral gluons have been obtained in the background Lorentz-Feynman gauge. Note here that in previous papers [17], [18], [14] this tensor in the field without and with temperature included has been computed in the Fujikawa gauge. In this non-linear gauge the structure of the neutral polarization tensor is simpler as compared to the Lorentz gauge and the tensor is transversal, but the charged sector has a much more complicated structure.

The tensors Eqs. (34), (35) and form factors (117), (118) present the structure of the neutral polarization tensor off shell and can be used in various applications. First of all, the spectra of gluon modes in the field at high temperature can be investigated. They
may also serve as the basis for different kinds of resumptions of perturbation series. This is because to make a resummation one needs in the Green functions of shell. As far as we know, the operator structure of the gluon tensor in the considered background was not educed before. To relate our approach with a common one at finite temperature, we have investigated in short the zero field case and recalculated the corresponding well known results.

As applications, in the present paper we restricted ourselves to the consideration of two important parameters - the Debye mass of neutral gluons, Eq. (137), and the magnetic mass of transversal neutral gluons, Eqs. (148), (149), in the field at high temperature, in one-loop approximation. It was found that all the asymptotic expansions for the high temperature limit are expressed in terms of Riemann’s \( \zeta \)-function. As it follows from Eq. (137), the screening temperature mass of plasmons is decreased as compared to the zero field case. That increases a radius of color Coulomb forces when the background magnetic field is present. This behavior may be important for quark-gluon plasma. As concerns the magnetic mass, from Eqs. (148), (149) it follows that it is zero in one-loop order, as at zero field. At the same time, the fictitious pole, appearing in one-loop order at \( B = 0 \) (see Eq. (126)), disappears in the field presence. This is important property of hot gluon plasma. Already earlier one believed that the elimination of the fictitious pole requires a resumptions of perturbation series \([15],[16]\) and it is removed by the gluon magnetic mass.

In connection with the last result we would like to speculate a little. If one takes the one-loop result seriously and the magnetic mass is zero, that means that neutral gluon fields remain long range like Abelian magnetic fields at high temperature. This is in contrast to the charged gluon fields which acquire the magnetic mass in the field at high temperature in one-loop order \([7],[13]\) and therefore are screened at long distances. This behaviour is important either for problems of quark gluon plasma or the early universe because at high temperature the stable Abelian magnetic fields are spontaneously created, as it has been derived in the daisy resummations in Refs. \([5],[6],[7],[8]\).

To make a final conclusion about this phenomenon, one has to apply the super daisy resummations which can be realized on the base of solution of the Schwinger-Dyson equations for the common system of the neutral and charged gluon Green’s functions. The general structure of the Green function for neutral gluon fields is derived in the present paper. The case of the charged gluon field will be considered separately elsewhere.

Acknowledgement

One of us (V.S.) was supported by DFG under grant number 436 UKR 17/24/05. Also he thanks the Institute for Theoretical Physics of Leipzig University for kind hospitality.

Appendix

In this appendix we collect formulas which are used to identify the contributions to the form factors, i.e., the contributions which go with the tensor structures \( T_{\mu\nu}^{(i)} \).

In the course of calculation, from the graph (Fig. 2) contributions appear which have
up to a constant the following structure. First,

$$F^1_{\lambda\lambda'} = P_\lambda P_{\lambda'}^T + a\delta^\parallel_{\lambda\lambda'} + b\delta^\perp_{\lambda\lambda'} + icF_{\lambda\lambda'}, \quad (A.1)$$

where $P_\lambda$ is given in terms of the vectors (33),

$$P_\lambda = r_\lambda l + \alpha id_\lambda + \beta h_\lambda \quad (A.2)$$

and $r$, $\alpha$ and $\beta$ are some functions of the variables $s$ and $t$. The transposition in $P_{\lambda'}^T$ changes the sign of $d_{\lambda'}$, $P_{\lambda'}^T = r_\lambda l - \alpha id_\lambda + \beta h_\lambda$.

It can be seen that the expression in Eq. (38) fulfills (32) if

$$(r_\lambda^2 + \beta h_\lambda^2)^2 + a l^2 + b h^2 = 0$$

holds. In that case it can be represented in terms of form factors according to

$$F^1_{\lambda\lambda'} = -r_\lambda T^{(1)}_{\lambda\lambda'} - (\alpha^2 - \beta^2)T^{(2)}_{\lambda\lambda'} - r_\lambda T^{(4)}_{\lambda\lambda'}$$

$$+ \frac{r(r_\lambda^2 + \beta h^2)^2 + a l^2 + \alpha \beta h^2 + c)}{h^2} T^{(5)}_{\lambda\lambda'} + (r_\lambda l^2 + \alpha^2 + \beta h^2 + c)T^{(6)}_{\lambda\lambda'} \quad (A.3)$$

which can be checked by inserting the explicit expressions (103).

A second type of expressions appears which has a slightly more complicated form,

$$F^2_{\lambda\lambda'} = P_\lambda Q_{\lambda'} + Q_\lambda P_{\lambda'}^T + a\delta^\parallel_{\lambda\lambda'} + b\delta^\perp_{\lambda\lambda'} + icF_{\lambda\lambda'} \quad (A.4)$$

with $P_\lambda$ from (A.2) and

$$Q_\lambda = s_\lambda l + \gamma id_\lambda + \delta h_\lambda, \quad (A.5)$$

and $s$, $\gamma$ and $\delta$ are also some functions of the variables $s$ and $t$. In parallel to the above case, if for (A.4) the condition (32) is fulfilled than $(a + 2 s l^2)^2 + (b + 2 \beta \delta h^2)h^2 + (r \delta + s \beta)2 l^2 h^2 = 0$ must hold. In that case the representation in terms of form factors is

$$F^2_{\lambda\lambda'} = -2rs T^{(1)}_{\lambda\lambda'} - 2(\beta \delta + \alpha \gamma)T^{(2)}_{\lambda\lambda'} - (r \delta + s \beta)T^{(3)}_{\lambda\lambda'}$$

$$+ (r \gamma - sa)T^{(4)}_{\lambda\lambda'} + \left( (a + 2 rs l^2) \frac{1}{h^2} + r \delta + s \beta \right) T^{(5)}_{\lambda\lambda'}$$

$$+ \left( c - (r \gamma - sa)^2 + (\alpha \delta - \beta \gamma)h^2 \right) T^{(6)}_{\lambda\lambda'}. \quad (A.6)$$

Formulas (A.3) and (A.6) are used in the section 5 for calculation of form factors.

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