In combat with nonperturbative QCD

Eugene Levin

School of Physics and Astronomy
Raymond and Beverly Sackler Faculty of Exact Science
Tel Aviv University, Tel Aviv, 69978, ISRAEL
and
Theory Department, Petersburg Nuclear Physics Institute
188350, Gatchina, St. Petersburg, RUSSIA

Abstract. This talk is the second part of the summary of the theory section at DIS’97. We demonstrated that theory of the “hard” processes, based on QCD, has achieved a better understanding of available experimental data during the past year. This was due to the remarkable success achieved in calculating of high order corrections in perturbative QCD (see the summary talk of S. Forte) as well as progress in nonperturbative QCD approaches.

INTRODUCTION

In this talk we briefly outline the main ideas, approaches and results of the calculations of the “hard” processes that were discussed in the theory working group at DIS97. The main message, which we wish to convey to a reader, is that the development of theory during the past year has been exciting and solid, mostly because of remarkable breakthrough in the calculation of the high orders in perturbative QCD (pQCD) (see the summary of S. Forte) and because of the new ideas and methods for the nonperturbative contributions in QCD (npQCD).

To have a better understanding of what was achieved, we start with basics of our knowledge on QCD.

Everyone knows, that:

1. “hard” processes occur at small distances and they are the most suitable ones to apply the pQCD approach especially for such inclusive observables as the deep inelastic structure functions or/and the cross sections of “hard” production;

2. for “hard” processes we can use a very general formalism of the renormalization group approach which gives a deeper insight in the main properties of QCD than obtained from any sophisticated calculations in pQCD;

3. the renormalization group approach leads to the DGLAP [1] evolution equations, which play a role of the Coulomb law in QCD;
4. for “hard” processes we have a regular procedure which takes into account a power - like corrections with respect to the “hard” scale, so called Wilson Operator Product Expansion (WOPE)) [2];
5. for “hard” processes the factorization theorem was proven [3], which allows us to separate the “hard” and “soft” contributions.

All these properties make the “hard” processes a good training ground for all theoretical methods. Unfortunately, the main theoretical method for “hard” processes is still pQCD. However, if one wants to characterize all contributions to our section in one sentence, it is we have to start to discuss nonperturbative contributions to DIS. These nonperturbative approaches have been widely presented in our section as well as in proceedings, and we tried to include not only the results of the past year but also the minireviews of the situation. We are happy to report that methods of npQCD such as renormalons, instantons, semiclassical field approach, high parton density QCD and lattice calculations have been covered in detail, giving the up to date picture of the situation in npQCD at small distances.

The goal of this talk is to review the status of the understanding of nonperturbative QCD at small distances, that has been reached in our working group.

PERTURBATIVE QCD

The goal of this section is to recall the main ideas and terminology of pQCD to help the reader in understanding our strategy in dealing with the nonperturbative contributions to the “hard” processes. One can find an up to date review of the progress in pQCD in the Forte’s talk [4].

Two devils, that we are struggling with

In DIS, where the typical transverse distances ($r_\perp$) are small, we have a natural small parameter, $\alpha_s(r^2) \ll 1$. Therefore, at first sight, we have to calculate a couple of the Feynman diagrams to obtain the answer. However, the situation is much more complicated and, doing the pQCD calculation, we are always struggling with two major problems:

1. the real parameter of pQCD is not $\alpha_s(r^2)$, but $\alpha_s(r^2) L$, where $L$ is a large log. For example, a gluon structure function $xG(x, Q^2)$ can be written as a perturbative series in the form:

$$xG(x, Q^2) = \sum_{n=1}^{\infty} C_n \alpha_s^n (L^n + a_{n-1} L^{n-1} + \ldots a_0) ,$$  \hspace{1cm} (1)

where $L$ could be:

1. $L = \ln(Q^2/Q^2_0)$ at $Q^2 \gg Q^2_0$ and $x \approx 1$ ;

2. $L = \ln(1/x)$ at $Q^2 \approx Q^2_0$ and $x \to 0$ ;
3. $L = \ln(Q^2/Q_0^2) \ln(1/x)$ at $Q^2 \gg Q_0^2$ and $x \to 0$;
4. $L = \ln(1-x)$ at $Q^2 \approx Q_0^2$ and $x \to 1$.

Therefore, to obtain first approximation to a physical observable we have to sum all contributions of the order $(\alpha_s L)^n$, not only a couple of diagrams. This is a rather difficult technical problem, which we usually call resummation.

2. The second and more serious problem is the fact that $C_n \to n!$ at large values of $n$. This means that this series is the asymptotic one. We have no general approach for a summation of such a series, occasionally, we can guess an analytic function which has the same perturbative series. Sometimes, we develop a general approach for summation (mostly when $C_n \to (-1)^n n!$), but mostly it is an open problem. We know, at the moment, at least three sources for $n!$ behaviour: infrared and ultraviolet renormalons and instantons. All three give us a window to npQCD and we will discuss them later. Our practical strategy of dealing with asymptotic series is as follows:

1. Instead of the full series of Eq.(1), we first sum a simpler series, so called, leading log approximation (LLA):

\[
xG^{LLA}(x,Q^2) = \sum_{n=1}^{\infty} C_n \left(\alpha_s L\right)^n ,
\]

for which we have evolution equations. Solving them we have an analytic function. Actually, there are three evolution equations for the deep inelastic structure functions on the market: the celebrated Dokshitser - Gribov - Lipatov -Altarelli - Parisi (DGLAP) evolution equation [1] for region 1 in Eq.(2), the Balitsky - Fadin - Kuraev - Lipatov equation or so called the BFKL Pomeron [5] for region 2 in Eq.(2) and the Ciafaloni-Catani - Fiorani - Marchesini (CCFM) evolution equation [6] which provides a correct matching between region 1 and region 2. The current situation with the GLAP evolution equation is reviewed in Ref. [4], while the other two we will discuss here. We want to emphasize that we need to develop the LLA approach for each “hard” process with the careful analysis of the value of scale $L$ in different kinematic regions. This is a very difficult technical problem, in which remarkable progress has been made (see Forte’s talk [4]).

2. We calculate the ratio

\[
R(x,Q^2) = \frac{xG(x,Q^2)}{xG^{LLA}} = \sum_{n=1}^\infty r_n = \sum_{n=1}^\infty C_n \left(L^{n-1} + \ldots + a_0\right).
\]

The situation today in computation of the terms $r_n$ in Eq.(4) is reviewed in Ref. [4], in most processes we know $r_1$ and $r_2$, sometimes even $r_4$ has been calculated.

3. Our hope is that $\frac{r_n}{r_{n-1}} \ll 1$ for sufficiently large $n = N$.

4. The result of the calculation should be given in the form:

\[
R(x,Q^2) = \sum_{n=1}^{N-1} r_n \pm r_N.
\]
Therefore, inside pQCD we have an intristic accuracy which makes all calculations difficult. Practically, Eq.(5) means that, strictly speaking, we have to use calculations in the next order in $\alpha_s$, only as an error for the result of calculation in the previous order.

5. The accuracy of the pQCD approach depends crucially (i) on our skill in defining the value of scale $L$ for the process of interest, (ii) on the value of $N$ for particular process, (iii) on our success in solving the LLA equations and (iv) on our understanding of $n!$ behaviour of $C_n$ in our perturbative series.

The factorization theorem

Any calculation of "hard" processes is based on the factorization theorem [3], which allows us to separate the nonperturbative contribution from large distances (parton densities, $F_A^i(\mu^2)$) from the perturbative one ("hard" cross section, $\sigma^{hard}$). For example, the cross section of the high $p_t$ jet production in hadron-hadron collisions can be written schematically in the form:

$$\sigma(A + B \rightarrow jets(p_t) + X) = F_A^i(\mu^2) \otimes F_A^j(\mu^2) \otimes \sigma^{hard}(\text{partons with } p_t \geq k_t \geq \mu) . \quad (6)$$

As we have discussed, we need to calculate $\sigma^{hard}$ not only in the leading order of pQCD but also in high orders so as to specify the accuracy of our calculation. Practically, we need to calculate the "hard" cross section at least in the next to leading order, to reduce the scale dependence, which appears in the leading order calculation, as a clear indication of the low accuracy of our calculation. Of course, we have to adjust the accuracy in the calculation of $F(\mu^2)$ and $\sigma^{hard}$. Solid progress has been achieved in this program [4].

ON THE BORDER BETWEEN PERTURBATIVE AND NONPERTURBATIVE QCD

1 The BFKL and CCFM equations

The BFKL equation was derived in LLA of pQCD summing $\alpha_S \ln(1/x))^n$ terms in the kinematic region (see region 2 in Eq.(2)) without any "hard" scale [5]. The simple physics behind the infrared stable answer, related to new degrees of freedom at high energy (colour dipoles [7]), as well as the beauty of hidden symmetries, make this equation a hot subject of investigation for theorists, as a possible model of matching of the "hard" and "soft" processes in QCD. The fact that the BFKL dynamics has not been seen in HERA data, that have penetrated the BFKL kinematic region, brought an additional challenge to experts. For a long time the BFKL approach suffered from the lack of corrections in the next to leading order, without which it was impossible to estimate the accuracy of the calculation.
The **hot and good news** of this conference is that these corrections were calculated and presented for the first time in our section [8]. The bad news is the fact that we have not understood these corrections, and we are still not able to tell how they will change the energy dependence of the cross section and the value of the anomalous dimension (\(Q^2\) dependence) in DIS. The expectation is that they lead to a smoother energy behaviour and to an increase of the anomalous dimension. However, unfortunately, we have to postpone this discussion till DIS98.

Fortunately, we do know what to look for in the calculations of Ref. [8]. The beauty of the BFKL equation is based on two simple properties of the leading log \((1/x)\) approximation of pQCD: the separation of the longitudinal and transverse degrees of freedom, and conformal symmetry related to dimensionless coupling in QCD. Both of them are broken in the next order. The running coupling constant brings the scale to the physical observables and separation of the longitudinal and transverse degrees of freedom does not hold. However, one can see that the gluon BFKL kernel of the next to leading order (see Ref. [8]) as well as the quark one [9] confirms the suggestion [10] that we need to substitute \(\alpha_S \rightarrow \frac{\alpha_S(q_1^2)\alpha_S(q_1-q_2)^2}{\alpha_S(q_2^2)}\) in the leading order BFKL kernel to take into account the running coupling constant of QCD. \(q_1\) is the transverse momentum of radiating gluon, \(q_1-q_2\) is the transverse momentum of produced gluon, while \(q_2\) is the transverse momentum of the gluon after emission. The second way of introducing the scale into the BFKL equation is closely related to kinematic constrain taken into account in the CCFM equation (see Ref. [11]). The CCFM equation is an elegant way of including in the equation for the parton density the angle ordering in the parton cascade which holds in any order of the perturbation theory.

Therefore, we hope that if we write the CCFM equation instead of the BFKL one, and if we include the running QCD coupling as has been described above, the rest of the kernel will preserve conformal invariance. On the other hand, we have some arguments that the conformal invariant part of the next to leading BFKL kernel has a very definite form [12].

We notice that real progress in the CCFM equation has been achieved not only in the matching of two kinematic regions for the deep inelastic structure function but in creating a theoretical approach to multigluon production [11]. It gives one hope of building a theoretical description of the inclusive processes in the BFKL-like dynamics.

Most experts feel that the BFKL equation is more fundamental than the LLA of pQCD in which it was originally proven. This is the reason why we return to the proof of the BFKL equation using different approaches. It is well known that this strategy has produced results, namely, the new degrees of freedom (colour dipoles [7]) at high energy interaction. In our working group we discussed four new derivations: in string-based method of pQCD [13], in new renormalization group approach to longitudinal degrees of freedom (Ref. [14] and below), in the OPE-like formalism for high energy scattering [15] and in semiclassical field approach

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1) It should be stressed that the CCFM equation leads to the BFKL one in the LLA.
In Ref. [15] a new formalism was suggested which gives the OPE for high energy scattering (HEOPE). In the HEOPE, power-like corrections in energy squared $s$ has been separated and assigned to nonlocal operators (Wilsons lines). Each nonlocal operator has only $\ln s$ dependence on energy and the first one (one Wilson line) has the energy dependence given by the BFKL equation. This approach passed the first non-trivial test: in Ref. [15] the so-called triple Pomeron vertex was calculated and the answer coincides with the expression obtained in pQCD (see Ref. [17]). Much more work needs to be done to understand how general this approach is, but if it is a correct one, we have a chance to understand better the high energy asymptotic beyond of pQCD.

2 A violation of the factorization theorem at high energy

We have two examples that the factorization theorem, which is the basis of all our calculations of the “hard” processes, has only limited accuracy at high energy (low $x$).

The first one was given in Ref. [18], where it was shown that, due to the BFKL dynamics, the WOPE can only be used safely for DIS at low $x$ (say $x_0 < x \ll 1$) in the limited region of $Q^2$, namely for $Q^2 \geq Q^2(x_0)$ where

$$\ln(Q^2(x_0)/\Lambda^2) \geq \left[ \frac{7N_c \zeta(3)}{\pi b} \right] \ln(x_0/x)$$

(7)

$\zeta$ is Riemann zeta function and all other notations are clear from $\alpha_s(q^2) = \frac{4\pi}{b \ln(q^2/\Lambda^2)}$. However, what is more important that this breakdown of the WOPE is not due to higher twist terms but rather to an inability to properly separate “hard” and “soft” scales at $x \to 0$. This means that the factorization theorem does not work at high energies. This statement has a very simple physical meaning. Indeed, in the BFKL kinematic region the mean transverse momentum of partons increases with energy. Therefore, we have a two scale problem even for the deep inelastic structure function. Only at a high value of $Q^2$ (given by Eq.(7)) when $Q^2$ is bigger than the mean parton transverse momentum, $Q^2$ become the obvious scale of hardness in DIS and we can use the WOPE.

The second example was discussed for the first time in our working group [19]. It was argued that some interference diagrams have been missed in the proof of the factorization theorem [3] these will give a sizable contribution for high energy “hard” inclusive production, especially for nucleus collisions.

Both of these examples show that we have to reconsider the formal proof of the factorization theorem to determine a correct region of its applicability.

3 Factorization for “hard” exclusive processes

As was recently shown [20] [21] the diffractive electroproduction of vector mesons can be treated in pQCD (see Ref. [22] for a general proof), since all nonperturbative
contributions can be factorised out in the wave function of the produced vector meson. In Ref. [22] it was shown, that the amplitude of diffractive production \( M \) of the vector mesons as well as other hadrons can be written in the factorizable form:

\[
M(\gamma^* + A \rightarrow V + A) = \sum F_A^i(x_B, x_M, \mu^2) \circledast H_{ij}(Q^2, \ldots) \circledast \varphi^V_j(\mu^2) + O(\frac{\mu^2}{Q^2}),
\]

where \( H_{ij} \) is the “hard” scattering function, \( \varphi \) is the light-cone wave function of the meson [23] and \( F_A^i(x_B, x_M, \mu^2) \) is the off-diagonal parton density. The new kinematic variable \( x_M \) is equal to \( M^2/W^2 \).

The off-diagonal parton densities were actually introduced long ago in Refs. [24] [25] where the diffractive production of \( Z \) in DIS was considered. The difference, between the off-diagonal parton densities and the conventional ones, is due to the fact that the longitudinal momentum transfer in the diffractive production in DIS is not equal to zero, as it is for the conventional deep inelastic structure functions. For the off-diagonal parton densities we have a new evolution equation which was suggested first in Ref. [25] (see also [26]). In our section two papers were presented [27] [28] where these new evolution equations were re-derived, checked, corrected and generalized. We think, that this is a closed chapter in the leading order of pQCD (see Ref. [27] for the most detail and rigorous proof).

4 High twist contribution

Due to the WOPE, a deep inelastic structure function can be written in the form:

\[
F_2(x, Q^2) = F_2^{LT}(x, Q^2) + \frac{m^2}{Q^2} F_2^{HT}. 
\]

We know almost everything about \( F_2^{LT} \) where the DGLAP evolution equations apply. We know a lot about the next order twist structure function \( F_2^{HT} \): the physical meaning [29], the evolution equations [30] and even the solution of the evolution equations in the region of low \( x \) [31]. We know enough to look with smile, at numerous attempts of experimentalist to fit the data assuming that \( F_2^{HT} \) has the same \( Q^2 \) dependence as \( F_2^{LT} \). However, nobody has yet given a numerical estimate of \( Q^2 \) and \( x \) dependence of the high twist contribution based on what we know. This has been done [32] and the result is to some extent surprising. The extra power of \( Q^2 \) does not give the feeling that this contribution should be negligibly small at least at \( Q^2 \approx 10 GeV^2 \). Indeed, if one wants to try a simple parameterization for \( F_2^{HT} \) it is better to take \( F_2^{HT} \propto (F_2^{LT})^2 \) [31] [32] and the \( Q^2 \) dependence due to anomalous dimension of \( F_2^{LT} \) compensates to large extent for the \( 1/Q^2 \) suppression. The importance of this result is obvious, since in all solutions of
the DGLAP equations that there are on the market, it has been assumed that the higher twist contribution is small and can be neglected at \( Q^2 = Q_0^2 \) where \( Q_0^2 \) is the initial virtually of the photon from which we start the DGLAP evolution. Notice that in practice the value of \( Q_0^2 \) is rather small (about 4 GeV\(^2\)).

5 Back to shadowing corrections

The HERA data on deep inelastic structure function lead to puzzling result. On one hand they can be successfully described in the framework of the DGLAP evolution equations without any new ingredients like the BFKL equation and/or the shadowing corrections (SC). On the other hand the parameter \((\kappa)\) which gives the estimate for the strength of the SC turns out to be large \((\kappa \geq 1)\) in the HERA kinematic region. This parameter \(\kappa\) was estimated in Refs. [25] [33] and it is equal to

\[
\kappa = xG(x, Q^2) \frac{\sigma(GG)}{\pi R^2} = xG(x, Q^2) \frac{3\pi \alpha_s}{Q^2 R^2}.
\]

To understand what is going on, we have to develop a theoretical approach in which we can treat the region of \( \kappa \approx 1 \) in DIS. It should be stressed that the previous attempts to develop a theory for the SC [25] only had a guaranteed theoretical accuracy for small \( \kappa \approx \alpha_s \ll 1 \). Two of such approaches were discussed in the working group: in the first one\(^2\) [34] pQCD was used at the edge of its validity \((\alpha_s \kappa \leq 1)\), while in the second (see Refs. [35] [16] and below) the new approach was developed in the kinematic region of high parton density QCD.

In Ref. [34] a new evolution equation was derived which describes that each parton in the parton cascade interacts with the target in Glauber-Mueller approach [36]. The results are the following: (i) \(\kappa\) is the correct parameter that determines the strength of the SC; (ii) the SC to the gluon structure function are big even in the HERA kinematic region, but nevertheless the value of the shadowed gluon structure function is still within the experimental errors or, another way of putting it, the difference between the shadowed and nonshadowed gluon structure functions does not exceed the difference between the gluon structure functions in the different parameterizations such as the MRS, GRV and CTEQ ones; (iii) the SC to \(F_2(x, Q^2)\) in HERA kinematic region are so small that can be neglected; (iv) the SC enter the game before the BFKL equation and, therefore, the BFKL Pomeron cannot be seen in the deep inelastic structure function since it is hidden under substantial SC; and (v) in the region of low \(x\) the asymptotic behaviour of \(xG(x, Q^2)\) is

\[
xG(x, Q^2) \rightarrow \frac{2 R^2 Q^2}{\pi x^2} \ln(1/x) \ln(\ln(1/x)).
\]

This means that the gluon density does not saturate [25] unlike in the GLR equation.

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\(^2\) E. Levin gave a talk on this subject but he did not write a contribution to the proceedings.
6 New ideas

A new idea of how to calculate the high energy amplitude in QCD was presented in Ref. [14]: i.e. to use the renormalization group approach for the longitudinal degrees of freedom. The arguments are based on the $k_t$ - factorization [37] and on similarity between $\ln Q^2$ and $\ln s$. The answer is, roughly speaking, the BFKL amplitude, but with running QCD coupling which depends on energy. Certainly, this answer does not contradict unitarity and, perhaps, even the experimental data, but, of course, it is in strong contradiction with the BFKL approach, that has been discussed in subsection 6. During the discussion in our working group we did not reach agreement and more time is needed to clarify the situation. However, the fact that none of the experts has seen a diagram which survives at high energy and in which the running coupling constant depends on $s$, makes the whole approach rather suspicious. On the other hand, this approach is sure to stimulate a more detailed study of this problem.

REAL NONPERTURBATIVE STUFF

7 Semiclassical gluon field approach

The real nonperturbative contributions are closely related to the “soft” gluonic field with small transverse momenta ($q_t \leq \Lambda$) and small frequency ($\omega_s = q_\perp = \frac{q^2}{q_t} \leq \Lambda$) where $\Lambda$ is QCD scale ($\alpha_s(k^2) = \frac{4\pi}{b \ln(k^2/\Lambda^2)}$). This is an old idea [38] [39], which led to certain understanding of nonperturbative behaviour of the deep inelastic structure functions. What is new is that we can say a lot about “hard” processes (DIS), using only a general form of the QCD Lagrangian [40]. The physical picture of this approach is extremely transparent: the colour dipole of a small size ($r_t \approx \frac{1}{Q}$ for DIS) penetrates the cloud of the “soft” gluon field which can be treated semiclassically. The amazing result is that we can obtain very definite predictions without a detailed knowledge of the structure of this cloud. For example, this picture leads to the following prediction for diffractive charm production in DIS:

$$\sigma_L^{DDcharm} \approx \frac{1}{Q^2}$$
$$\sigma_T^{DDcharm} \approx \frac{\ln(Q^2/m_c^2)}{Q^2},$$

which have to be confronted with the pQCD results [41]:

$$\sigma_L^{DDcharm} \approx \frac{\Lambda^2}{Q^2} \ln(Q^2/m_c^2)$$
$$\sigma_T^{DDcharm} \approx \frac{\Lambda^2}{Q^2 m_c^2}. $$

In Ref. [40] one can find more predictions for future experiments which will help to differentiate the nonperturbative and perturbative contributions to DIS.

It turns out that this idea of semiclassical gluon field is very useful in obtaining the effective Lagrangian for high parton density QCD (see Refs. [42], [35] and [16]
The physical problem has been pointed out a long ago (see Ref. [25] and Ref. [43] for updated review): at high energy (low $x$) and/or for DIS with heavy nucleus we are dealing with the system of partons so dense that conventional methods of pQCD does not work. However, the typical distances are still small for DIS and this fact results in weak correlations between partons due to small the coupling constant of QCD.

The revolutionary idea, suggested in Ref. [42] and developed in Refs. [35] and [16], is: in the Bjorken frame for DIS we can replace the complex QCD interaction between parton in such a system by the interaction of a parton ($i$) with energy fraction $x_i$ with the classical field created by all partons with energy fraction $x$ bigger than $x_i$. Indeed, in leading $\log(1/x)$ approximation of QCD all parton with $x > x_i$ live for a much longer time than parton $i$, therefore, they create a gluon field which only depends on their density. Using this idea and Wilson renormalization group approach, in Refs. [35] and [16] (see also Ref. [44] for elucidating remarks), the effective Lagrangian was obtained.

It has been demonstrated that this effective Lagrangian correctly reproduces the DGLAP evolution equations [35] and even the BFKL Pomeron [16] in the limit of a sufficiently weak gluon field. However, the main problem in matching the two approaches: one which we discussed in subsection 5 and this one, is still an open problem. The equation which was originally obtained in Ref. [35], does not match, but in our working group we were instructed by A.Kovner and H.Wegert that the equation is not complete. We hope that this problem will be solved soon and we will have a reliable theoretical approach for high parton density QCD which will allow us to search for new collective phenomena in QCD at HERA, Tevatron, RHIC and at LHC.

8 Renormalons and Instantons

As mentioned previously, renormalons and instantons are the known sources of the $n!$ behaviour of coefficients of the perturbative series which give estimates for the value and $Q^2$ dependence of $r_N$ term in Eq.(5).

In our working group we covered the renormalon contributions in much detail (see Forte's talk [4] for a comprehensive review of the present situation). For the sake of completeness I list here the main achievements that have been made: (i) a successful phenomenology based on renormalons as the way to take into account nonperturbative corrections [45] [46] [47]; (ii) confirmation of the WOPE in processes with one scale of hardness and the possibility to calculate power-like corrections with respect to hard scale ($Q^2$) to processes without the WOPE [45] [47]; (iii) a powerful dispersion relation method which allows us to treat the nonperturbative corrections to a variety of processes; (iv) the general nonlocal OPE for processes without the WOPE [48] and (v) the possibility to discuss corrections to the WOPE due to high order terms in the perturbative series [47].
Renormalons give us the possibility to discuss power-like nonperturbative corrections to the BFKL equation (see Refs. [10] [49]). It turns out that the nonperturbative corrections from infrared instantons have different dependences on $Q^2$ for the gluon structure function ($\frac{1}{Q}$ [10] [49]), and for $F_2(x, Q^2)$ ($\frac{1}{Q^2}$ [49]).

Instantons were reviewed in Ref. [50]. The main result is that the instanton contributions can be theoretically calculated in inclusive production in DIS, and an experimental observation of a typical instanton event in the DIS, which has a clear signature, will open a new insight regarding the nonperturbative contribution. There is no QCD without instantons, therefore, an experimental observation of an instanton event is a great challenge to experimentalists.

\section{9 Lattice calculation}

For the first time the lattice calculation in DIS were discussed seriously. The experimental errors in the lattice experiment are still large but, nevertheless, it gives a convincing result that the initial quark distributions at $Q^2 = 2.5 \text{ GeV}^2$ differs from experimental one (see minireview [51]). Such a difference was expected since in the lattice experiment the leading twist contribution has been calculated while experimental data give the contribution of all twists at definite value of virtuality $Q^2$. It is interesting to note that the leading twist quark distributions derived on the lattice turn out to be closer to one that was expected in the constituent quark model, where the mean momentum of the quark about $\frac{1}{3}$.

We would like to emphasize that the lattice calculations give an unique opportunity to develop a self consistent theoretical approach to DIS. The following strategy is advocated. We first use the lattice parton leading twist densities as the initial conditions for the GLAP evolution equations and solve them. The difference between experimental initial parton densities and the lattice one should be treated as the high twist contribution. For them we should use the high twist evolution equations and the theoretical status of the high twist contribution was discussed. The above procedure will provide a theoretical approach to DIS and after it has been achieved we will be able to discuss DIS on the solid theory basis. However, the experts in lattice experiments need to calculate the initial gluon density which has not yet been done.

\section{CONCLUSIONS}

Going back over the titles of our subsections one can see the main subjects of our discussions. The first trivial conclusion is that the theory of “hard” processes is in a very good shape in the year 1997. Of course, we are still struggling with nonperturbative QCD but, obviously, we are in offensive phase, attacking a number of very difficult problems and making great progress in our understanding. An illustration of this could be our recommendation for phenomenology of “hard” processes. They are:
1. the calculation of the next order corrections to the BFKL equation which will allow us to calculate the accuracy of the new phenomenological approach based on the BFKL anomalous dimension [52] which provides a reasonable matching of “hard” and “soft” processes in QCD;

2. the theoretical discussion of the high twist contribution (see subsection 4) suggests the following formula to fit the deep inelastic structure function:

\[ F_2(x, Q^2) = F_2^{DGLAP}(x, Q^2) + \frac{m^2}{Q^2} c(x) \left[ F_2^{DGLAP}(x, Q^2) \right]^2, \]

where \( c(x) \) is a phenomenological function of \( x \);

3. using lattice calculations we can develop a self consistent theoretical approach to predict the deep inelastic structure function (see subsection 9);

4. the semiclassical gluon field approach (see subsection 7) gives a definite prediction for diffractive charm production which should be checked experimentally;

5. new studies on shadowing corrections (see subsections 5 and 7) show the urgent need to determine experimentally, or extract phenomenologically, the gluon parton density;

6. the factorization theorem has only a limited region of applicability and should be used with caution.

The last remark that we wish to make is., that the new physics beyond the Standard Model was not discussed here, as this item was discussed in detail in the structure function working group. Our theoretical speakers (in our common session with the structure function WG) demonstrated that new particles like leptoquark are needed in theory and, perhaps to the surprise of the non experts, the theory is now in such good shape that it can predict a lot, in particular, the manifestation of the same mechanism which could lead to new HERA data on high \( Q^2 \) production in a variety of other processes. Welcome to DIS’98 for more profound discussions on this issue, if at all.

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