Magnetization of an elastic ferromagnet

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Abstract. At the macroscopic level, ferromagnetism is a quantum mechanical phenomenon. To describe magnetic materials, it is necessary to create a heuristic model that in terms of continuum mechanics describes the interaction between the lattice continuum, which is a carrier of deformations, and the magnetization field, which is associated with the spin continuum through the gyromagnetic effect. According to the laws of quantum mechanics, each individual particle is associated with a magnetic moment and an internal angular momentum – spin. Electrons mainly contribute to the magnetic moment of the atom. Therefore, the continuum is continuously associated with the discrete distribution of individual spins in a real ferromagnetic body known as the electron spin continuum. In addition, it is necessary to formulate field equations that, together with Maxwell's equations, describe the electron spin continuum. After that, it is necessary to consider the interaction between the lattice continuum and the electron spin continuum. Elastic ferromagnets should be described with due regard to the spin density and couple stresses. The spin system is a carrier of the magnetic properties, and the mechanical properties are associated with the lattice. Thus, spin–lattice interactions indicate the relationship between magnetic and mechanical properties.

1. Introduction

Electric charge $q$ with mass $m_0$ and radius vector $r$ rotates around the center $O$, and acquires momentum $mr$ to generate a convection current $j = qr$. The orbital angular momentum $S(0)$ and the magnetic moment $m(0)$ relative to the center $O$ can be determined by the following equations:

$$ S(0) = r \times p, \quad m(0) = \frac{1}{2c} r \times j. $$

From the equations

$$ m(0) = \gamma \cdot S(0), \quad \gamma = \frac{q}{2m_0c}, $$

where $\gamma$ is gyromagnetic ratio.

The electron magnetic moment can be created due to its spin (attributive property of the electron, its internal angular momentum). This moment is about twofold greater than the magnetic moment calculated using equation (2). Therefore, electrons determine the magnetic moment of the atom almost
completely. It is necessary to distinguish between magnetism of nuclei, nuclear magnetism, and magnetism of electrons, electron magnetism. The most evident confirmation of this division is nuclear magnetic resonance and electron paramagnetic resonance [3]. Figure 1 schematically shows the mutual orientation of the orbital angular momentum $S(0)$ and the magnetic angular momentum $m(0)$.

\[
\begin{align*}
S(0) & \quad \quad m(0)
\end{align*}
\]

**Figure 1.** Mutual orientation of orbital angular momentum $S(0)$ and magnetic moment of impulse $m(0)$

2. Results and Discussion

2.1. Spontaneous magnetization in the absence of external magnetic field

The phenomenon of spontaneous magnetization is observed in ferromagnet samples with characteristic dimensions of $10^{-5} - 10^{-2}$ cm, which are rather large compared to the lattice period, but much less than the linear dimensions of a macroscopic sample. Imagine that a macroscopic ferromagnet sample is a loaf of bread with raisins. ‘Raisins’ are domains with spontaneous magnetization, which are called Weiss domains.

For further studies, it is necessary to consider the main elements of the theory of ferromagnetism. The Curie temperature differs the disordered magnetic phase ($\theta > \theta_c$) from the ordered magnetic phase at ($\theta < \theta_c$). P. Weiss postulated the presence of a strong molecular field in ferromagnets due to the interaction of electrons, which tends to align all magnetic moments parallel to each other. The action of this field known as the action of an equivalent magnetic field $H^\alpha$. It should be noted that the field $H^\alpha$ is not a real magnetic field as it is not included in Maxwell's equations and no currents can be associated with it.

P. Weiss used the idea of coupling $H^\alpha$ and magnetization: the more ordered the neighbors of a given magnetic spin, the stronger the tendency of this spin to align itself parallel to others. Therefore, the molecular field is postulated by an increasing function of magnetization. Let $\lambda$ be the temperature independent proportionality factor. Then, $H^\alpha = \lambda M$. The magnetization observed in this equation is at thermal equilibrium in the field $H^\alpha$.

It can be seen [1] that the interaction energy between two atoms located at points $\alpha$ and $\beta$ that carry spins $S^\alpha$ and $S^\beta$ (in $\hbar$ units) contains the following term:

\[
W^{(\alpha,\beta)} = -2jS^{(\alpha)} \cdot S^{(\beta)},
\]

where $j$ is the exchange integral associated with the overlap of the charge distributions of atoms $\alpha$ and $\beta$.

$j$ is a function of the radius vector $r(\alpha, \beta)$ that connects points $\alpha$ and $\beta$. Unlike the Coulomb interaction, which decays relatively slowly, the exchange interaction drops sharply as the distance between the nuclei increases. Therefore, $j$ has a significant value only for the nearest neighbors.

Due to magnetic ordering, ferromagnetic materials can transmit excitations of a purely magnetic nature, which are similar to the effects of propagation of electromagnetic and acoustic excitations. These excitations discovered by Bloch in 1930 are called spin waves. There are longitudinal and transverse spin waves. Spin waves are fluctuations of the density of the magnetic moment propagating through a magnetically ordered crystal.

Magnetic ordering is mainly of a quantum mechanical nature. Therefore, it is necessary to formulate precisely the ideas in order to carry out a macroscopic phenomenological investigation. The
spin wave energy must correspond to the crystal excitation energy required to change the spin orientation. Let \( \omega_s(k) \) be the frequency spectrum of a spin wave as a function of its wave vector \( k \).

The scalar \( k \) determined by the ratio \( k^2 = k \cdot k \) is known as the wave number. The wave number introduces into consideration an imaginary particle – magnon. The excitation energy of a ferromagnet is insignificant and correspond to low temperatures, i.e. \( \theta < \theta_c \).

### 2.2. Interaction model

Let a ferromagnet has the polarization vector \( \mathbf{P} = 0 \) at all points \( \mathbf{x} \in \mathbf{B} \) at any time moment \( t \). In this case, a certain form of generalized motion of the total continuum ‘lattice + spin system’ can be considered. Each material point of the body is associated with magnetization or spin vector so that the electron spin continuum cannot move relative to the lattice continuum at this point. The spin continuum ‘expands’ or ‘contracts’ together with the lattice continuum and occupies the same volume; its volumetric behavior is described by the continuity equation.

Assume that the lattice continuum can respond to the actions of volumetric and surface forces, and volumetric moments of forces. There is no mechanism for generating surface moments of forces. The law of conservation of momentum states that a ponderomotive magnetic force applied to a point of the spin continuum is completely transferred to the lattice continuum at this point. Whatever type of interaction between the lattice continuum and the spin continuum – spin–lattice interaction – is considered, it must have a momentary nature, since the spin continuum is sensitive to this type of interaction. In [1, 2], it is assumed that this moment of forces is generated due to a local magnetic induction field denoted by \( \mathbf{B}^L \), and the moment acting from the lattice continuum (LC) on the spin continuum (SC) calculated per the unit volume is defined as follows:

\[
\mathbf{C}_{(LC/SC)} = \mathbf{M} \times \mathbf{B}^L = \rho \mathbf{\mu} \times \mathbf{B}^L .
\]

Due to this moment of forces, the moment of forces equal in magnitude but opposite in sign acts on the unit volume of the lattice continuum

\[
\mathbf{C}_{(LC/SC)} = -\mathbf{C}_{(LC/SC)} = \mathbf{B}^L \times \rho \mathbf{\mu} .
\]

where \( \mathbf{B}^L \) is the unknown field value which needs a defining equation.

To consider ferromagnetic exchange effects, it is necessary to take into account that each particle of the electron spin continuum is subject to exchange forces of a quantum mechanical nature from its nearest neighbors. At the phenomenological level, they lead to the appearance of contact (surface) forces such as the stress vector in the lattice continuum. In particular, it is necessary to introduce the surface exchange contact force \( \mathbf{F} \), which generates the moment \( \mathbf{M} \times \mathbf{F} \) per unit area that acts on the spin continuum. The force \( \mathbf{F} \) will be associated with spatial inhomogeneities of magnetization. \( \mathbf{F} \) is the field axial vector with the dimension of the magnetic field multiplied by the length, or the dimension of the surface density of the magnetic dipole moment.

Consider the types of interaction [6–12] in deformable ferromagnets. Spin–lattice interaction is the interaction between the thermal vibrations of atoms of the crystal lattice (phonons) and the magnetic moments of these atoms (spins). It is due to the fact that the energy equation contains terms that depend on both the arrangement of atoms and the orientation of their spins.

The theory of spin–lattice relaxation considers two main mechanisms:

1) spin wave excitation or individual spin deviations due to a change in the exchange electrostatic and dipole (spin–spin) energy during thermal vibrations of the lattice;

2) modulation effect of electric fields in the crystal, which arise from thermal vibrations of the lattice, on the electron spins through the spin–orbit interaction. Spin–lattice relaxation plays a crucial role in dilute paramagnetic crystals at a low concentration of the magnetic component.
Quantum mechanics assumes a mechanism that causes transitions between the levels of a spin system with populations $N_+$ and $N_-$ and interacts with another system. Let $W \uparrow [3]$ be the probability of transition per unit time from the state (+) to the state (-) under the impact of this interaction, and $W \downarrow$ be the probability of the reverse transition. Assume that there is a reservoir with two energy levels with the distance between them similar to that of an atomic system. If the atom and the reservoir are initially in opposite states, the simultaneous transition to opposite states satisfies the law of conservation of energy. The atomic levels will be characterized by the population ratio similar to that for the lattice levels, i.e. the populations of atomic levels will be in thermal equilibrium with the lattice. Solution of the differential equation [3]

$$\frac{dn}{dt} = \frac{n_0 - n}{T_1}, \quad (6)$$

has the form:

$$n = n_0 + Ae^{-\frac{t}{T_1}}, \quad (7)$$

where $A$ is constant of integration, $n_0$ is equilibrium population difference, which establishment rate is characterized by time interval $T_1$, which is spin–lattice relaxation time required to magnetize the sample.

Similar to equation (6), the equilibrium value $M_z$ is expected to be established according to the law

$$\frac{dM_z}{dt} = \frac{M_0 - M_z}{T_1}, \quad (8)$$

where $M_z = \frac{\gamma n}{2}$ and $M_0$ is the equilibrium value of magnetization, which can be expressed in terms of the magnetic susceptibility $\chi_0$ and the amplitude of the constant magnetic field $H_0$:

$$M_0 = \chi_0 H_0. \quad (9)$$

The combination of equation (8) and the equation that determines the change in $M$ under the action of the moment of forces yields the following:

$$\frac{dM_z}{dt} = \frac{M_0 - M_z}{T_1} + \gamma (M + H). \quad (10)$$

2.3. Relationship between the microscopic model and the continuous model

Consider an elastic ferromagnet subjected to large deformations with respect to its undeformed state, and find a phenomenological equation for the exchange energy through its representation at the microscopic level.

At temperatures below the Curie temperature, the magnetic moment is not a linear function of the field $H$. The molecular field overpowers the effect of thermal fluctuations. However, the Weiss molecular field gives an approximate equation for the exchange interaction.

For a hard ferromagnet, the exchange interaction in free energy is expressed by the term that contains spatial inhomogeneities of the magnetization density. This term can be written as [1]

$$W^{ex}(\Delta V) = \int_\Delta V \psi^{ex} dV, \quad \psi^{ex} = \frac{1}{2} a_{ij} M_{i,j} M_{k,j} = \frac{1}{2} a_{ij} M_{k,i} M_{k,j}, \quad (11)$$
where $\Delta V$ is the volume that is small in comparison with the sample size but rather large in comparison with the atomic scale; symmetric second-order tensor with the components

$$a_{ij}(r) = \frac{1}{2(2\mu_0)^2} \int_{\Delta V} F(r) \xi_i \xi_j \, d\varepsilon = a_{ij}.$$

is called the exchange-modulus tensor of the ferromagnet; $\xi_i$ are the coordinates of a point inside the volume $\Delta V$. $F$ is closely related to the exchange integral at Curie temperatures and rapidly decays with distance.

2.4. Results of saturation magnetization

If the temperature is uniform, spontaneous magnetization has a coordinate-independent magnitude, but the direction varies from point to point. This sample is said to be magnetized to saturation.

Van der Pol equation [4] was chosen as a nonlinear model of magnetization of a pipe metal by a strip magnet during its examination using an in-tube flaw detector. In this equation, the angle between the magnetization vector $p$ and the induction of the magnetic field $B$ designated as $\theta$ was chosen as a variable. Since the magnetic moment performs forced oscillations in the magnetic field $B$, the van der Pol equation will have the form

$$\ddot{\theta} - \mu(1 - \dot{\theta}^2)\dot{\theta} + \theta = B_0 \sin(\omega t),$$

where $B_0$ is the amplitude of the external magnetizing field; $\omega$ is its angular frequency; $\mu$ is the damping coefficient and nonlinearity of oscillations.

A detailed discussion of the solution of this equation is presented, for example, in [4].

From the solution of this equation, focus on the presence of a limiting cycle. At insignificant oscillations, nonlinear friction causes negative damping; while at strong oscillations, the amplitude of oscillations is limited by the nonlinear term proportional to $\dot{\theta}$.

Oscillations of such systems are often called limiting cycles. The trajectories of the van der Pol oscillator on the phase plane spirally contract to the limiting cycle. In terms of the magnetic moment behavior in a magnetized ferromagnet, this indicates that saturation magnetization is attained.

Thus, the limiting cycle is a geometric image of saturation magnetization from the perspective of nonlinear dynamics. Probably, broadening of the wave packet [5, 6] is associated with the nonlinear character of the ferromagnet magnetization.

3. Conclusion

The types of interaction in deformable ferromagnets are considered. Spin–lattice interaction is the interaction between the thermal vibrations of atoms of the crystal lattice and the magnetic moments of these atoms.

The relationship between the microscopic model and the continuous model can be determined using the phenomenological equation for the exchange energy.

The results of saturation magnetization during magnetization by a strip magnet examined using an in-tube flaw detector can be considered from the perspective of nonlinear dynamics.

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