Disorder effects on the quantum coherence of a many-boson system

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The effects of disorders on the quantum coherence for many-bosons are studied in a double well model. For the ground state, the disorder enhances the quantum coherence. In the deep Mott regime, dynamical evolution reveals periodical collapses and revivals of the quantum coherence which is robust against the disorder. The average over variations in both the on-site energy and the interaction reveals a beat phenomenon of the coherence-decoherence oscillation in the temporal evolution.

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The ultracold atoms in an optical lattice have provided an unprecedented opportunity to explore the correlated many-body problems and nonlinear dynamics in condensed matter physics[1–6]. The atoms tunneling between lattice sites play the role of electrons in the solid. Due to the controllable manipulation of the optical lattice, the inter-particle interaction as well as the well-developed measurement technique, scientists can now precisely examine the coincidence of theoretical models, e.g., the Hubbard model in strongly correlated systems, with experiments[7–9]. On the other hand, experimental techniques also exploited the collapse and revival dynamics of ultracold atoms in the optical lattice[10, 11]. In a recent experiment, S. Will et al observed the beat phenomenon of quantum coherence for Bose atoms in the dynamical evolution[12]. This phenomenon was interpreted as the multi-body interactions due to multi-orbital virtual transitions.

The disorders usually play important roles in realistic materials. It profoundly affects the transport properties of a wide range of materials. The dilute atomic gas in disordered optical potentials is again an ideal candidate model to simulate various disorder effects in condensed matter physics[12–15]. Disorders may be created in the laser potential in different ways. One of them is to add a speckle pattern to the regular lattice[16]. The disorder strength is continuously tunable by controlling the intensity of the speckle field. The localization effects due to correlation and disorder compete against each other, resulting in a partial delocalization of the particles in the Mott regime, which in turn lead to increased phase coherence[17]. However, there is significant disagreement regarding features of the disordered Bose-Hubbard model[15–22]. Experiments are still some way from the appropriate regime to observe, e.g., Anderson localization, due to the large speckle size and the delocalizing effect of the interatomic interactions. The interplay between the interaction and the disorder also lead to a rich and complex arrangement of different insulating and superfluid states[23, 24].

In this paper, we explore the quantum coherence of a correlated Bose gas in a double well system[27, 29]. We show that the on-site energy disorder enhance the quantum coherence in the ground state. In the deep Mott regime, dynamical evolution reveals periodical collapses and revivals of the quantum coherence. The periodicity is robust against the disorder. In addition, the disorder in the interatomic interaction also plays an important role. We find that the revival oscillations of quantum coherence exhibit the beat phenomenon when a moderate interaction disorder is taken account of. The corresponding Fourier spectra of the coherence parameter exhibit a double-peak structure.

We are concerned with $N$ bosonic atoms confined in a double well potential. By considering only the lowest energy band, the Bose-Hubbard Hamiltonian is written as[29, 31]

$$\hat{H} = -t(\hat{a}_1^\dagger \hat{a}_2 + \hat{a}_2^\dagger \hat{a}_1) + \frac{U}{2} \sum_i \hat{n}_i(\hat{n}_i - 1) + \varepsilon(\hat{n}_1 - \hat{n}_2),$$ (1)

where $\hat{a}_i^\dagger$ and $\hat{a}_i$ ($i = 1, 2$) are the creation and annihilation operators in either side of the well. $\hat{n}_i$ are the number operators. $U$ and $t$ are the Hubbard energy and the atom hopping, respectively. $\varepsilon$ is the on-site energy difference between the two sites. The disorder is introduced into the model via the fluctuation of $\varepsilon$, which is assumed to randomly and uniformly distribute within the interval $-\Delta \varepsilon \leq \varepsilon \leq \Delta \varepsilon$.

The Hamiltonian (1) is explicitly represented in the Fock basis set $\{|N, 0\rangle, |N - 1, 1\rangle, \ldots, |0, N\rangle\}$. The general eigenstates are expressed as linear combinations of the occupation bases, $|\psi_j\rangle = \sum_{k=0}^{N} c_{jk} |N - k, k\rangle$ ($j = 0, 1, 2, \ldots, N$), which correspond to the eigenvalues $\omega_j$. The coefficients $c_{jk}$ satisfy the recursive relation

$$- t \sqrt{(N - k)(k + 1)} c_{j(k+1)} - t \sqrt{(N - k + 1)(k)} c_{j(k-1)} + \frac{U}{2} (N^2 - 2Nk - N + 2k^2) + \varepsilon(N - 2k) - \omega_j) c_{jk} = 0.$$ (2)
The temporal evolution of the state is governed by the Schrödinger equation for a given initial state $|\psi(0)\rangle$:

$$|\psi(\tau)\rangle = \sum_{j=0}^{N} f_j(\tau) |\psi_j\rangle = \sum_{k=0}^{N} g_k(\tau) |N - k, k\rangle,$$  \hspace{1cm} (3)

where $g_k(\tau) = \sum_{j=0}^{N} f_j(0) c_{jk} e^{-i\omega_j \tau}$, with $f_i(0) = \langle \psi_j | \psi(0) \rangle$. To depict the coherence degree of the system, we introduce a characteristic parameter $\alpha(\tau)$:

$$\alpha(\tau) = \frac{\lambda_1 - \lambda_2}{\lambda_1 + \lambda_2}$$  \hspace{1cm} (4)

where $\lambda_1$ and $\lambda_2$ are the two eigenvalues of the single-particle density $\rho_{\mu\nu}(\tau) = \langle \psi(\tau) | \hat{a}_\mu^\dagger \hat{a}_\nu | \psi(\tau) \rangle$ ($\mu, \nu = 1, 2$) [32, 33]. When $\alpha \rightarrow 1$, the system is in the coherent (quasicoherent) state since in this case there is only one large eigenvalue of the matrix $\rho_{\mu\nu}$. Accordingly, $\alpha \rightarrow 0$ indicates the system is in the decoherent or fragmented state because there are two densely populated natural orbits. In the weak-interaction or strong-tunneling limit ($U/t \ll 1$), each atom is in a coherent superposition of the left-well and the right-well states. The ground state of the system is a state with a mean number $N/2$ of atoms in each well. In the opposite limit of the strong-interaction or weak tunneling ($U/t \gg 1$), the tunneling term is negligible. In this case, the Hamiltonian is the product of number operators for the left and right wells. The eigenstates are products of Fock states and are referred as decoherent states. This regime is analogous to the Mott insulator (MI) phase in a lattice system.

In the following we consider the typical cases in which the initial state is a coherent state. We choose $N = 10$ and set $t = 1$ as the energy units. All results are taken 50 repetitions of the random choice of disorders, which is enough to average out the fluctuations.

Figures 1 display the coherence degree $\alpha$ versus the disorder in the ground states by taking average over the on-site energy within the range of $\Delta \varepsilon/t = 0, 5, 11, 20$, respectively. From Fig.1 (a), it can be seen that in all range of $U/t$ the disorder raises $\alpha$, in agreement with previous studies. The coherence increase by the disorder is most prominent in the regime of intermediate strength of the Hubbard interactions. Fig.1 (b) shows the disorder dependence of the coherence degree $\alpha$ at several values of the Hubbard energies $U/t = 2, 10, 20, 50$, respectively. For smaller $U/t$, the ground state has larger $\alpha$, indicating that it is in a coherent (superfluid) state. For all $U$, the on-site energy disorder helps to enhance the quantum coherence. Generally, the stronger disorder has the larger coherence degree.

Next we study the disorder effect on the quantum dynamics of the $N$-bosons system. Suppose the system is initially in a coherent state,

$$|\psi(0)\rangle = (\hat{a}_1^\dagger + \hat{a}_2^\dagger) N |0\rangle.$$

The lattice parameter is non-adiabatically changed to the deep MI regime. Figures 2 are the temporal evolution of the coherence degree $\alpha$ for two typical interactions $U/t = 10, 50$, respectively. It is clearly shown that $\alpha$ oscillates between one and zero, implying the system experiences revivals and collapses of the coherence. Fig.2 (a) and (b) are for the system without the disorder. Clearly, the periodicity is more distinct for larger $U/t$. The period of the oscillation is $T = \pi/U$. Fig.2(c) and (d) present temporal evolution of $\alpha$ with the on-site disorder $\Delta \varepsilon/U = 20\%$ for two different values of $U/t = 10$ and $U/t = 50$, respectively. For small $U/t$ (Fig.2(c)), the amplitude of the oscillation is small. The large amplitude of periodical oscillation of $\alpha(\tau)$ in Fig.2(d) implies the system experiences revivals and collapses of the coherence. In contrary to one’s intuition that the disorder may destroy the periodicity of the revival and collapse oscillation, they enhance the periodicity by damping the irregular fluctuations of the coherence degree $\alpha$ in the temporal evolution.

This phenomenon can be understood as follows. The initial state can be expanded as $|\psi(0)\rangle \sim \sum_{k=0}^{N} C_{N}^{k} (\hat{a}_1^\dagger)^{N-k} (\hat{a}_2^\dagger)^{k} |0\rangle$. When $U/t \gg 1$, the Fock states $(\hat{a}_1^\dagger)^k |0\rangle$ in each well are the eigenstates, and the corresponding eigenenergies are $Uk(k - 1)/2$. The system is then a superposition of products of the Fock states, which evolves independently as,
FIG. 2: (color online) The temporal evolution of the coherence degree $\alpha(\tau)$ without the disorder for (a) $U/t=10$ and (b) $U/t=50$, and with the on-site disorder $\Delta\varepsilon/U = 20\%$ for (c) $U/t=10$ and (d) $U/t=50$. The dashed line in (d) is for the independent evolution as explained in the text.

\begin{equation}
|\psi(\tau)\rangle \sim \sum_{k=0}^{N} C_{N}^{k} e^{-i[\frac{1}{2}U(N-k)(N-k-1)+\varepsilon_{1}(N-k)]\tau} \times (\hat{a}_{1}^\dagger)^{(N-k)} \times e^{-i[\frac{1}{2}uk(k-1)+\varepsilon_{2}k]\tau} \times (\hat{a}_{2}^\dagger)^{k} |0\rangle,
\end{equation}

where a time-dependent phase factor is attached in each term. If there are no disorders ($\Delta\varepsilon = 0$), the system will recover its initial state when the time evolves an integer multiples of $\pi/U$. In between this period, superposition form various terms cancels and the coherence is destroyed\[31]. In the presence of the disorder, there are additional phases associating to the $\Delta\varepsilon$ in each term that seem destroy the coherence of the quantum state. However, numerical calculations show that it is not true. The origin is that in the deep Mott regimes, particle tunneling is suppressed. All Fock states have almost the same oscillating periods which only depends on the interaction strength $T = \pi/U$. Hence average over onsite energy $\varepsilon$ does not reduce the amplitude. The dashed line in Fig.2(d) reveals that the periodicity are perfectly reserved. We conclude that the phenomenon of the revivals and collapses of the coherence is robust against the disorder\[3].

We explore the system by further taking account of the interaction disorders with the Hubbard energy $U$ in a range of uniformly distributed interval $U \pm \Delta U$. The on-site disorder is fixed at $\Delta\varepsilon/U = 20\%$ with $U/t = 50$. Figures 3 show the temporal evolution of $\alpha$ for various interaction disorders (left panels). The right panels (d-f) are the corresponding Fourier spectra which exhibit double-peak structures as signals of beat effects. In the absence of the interaction disorders ($\Delta U/U = 0$), Fig.3(a) shows that the beat effect is smeared by additional oscillations in the temporal evolution of $\alpha$, which is evidently shown in Fig.2(b). These oscillations give rise to an additional peak at two times of the double-peak frequency. When interaction disorders are counted in (Fig.3(b) and (c)),
the additional oscillations in Fig.2(b) are damped, as shown in Fig.2(d). Consequently, the interaction disorder enhances the beat effect of the coherence-decoherence oscillations.

Figures 3 show the temporal evolution of $\alpha$ for various interactions $U/t = 30, 40, 50$, respectively, with $\Delta \varepsilon/U = 20\%$ and $\Delta U/U = 2\%$. As a comparison, Fig.3(d) displays the result from independent evolution described by equation (6). Figs.3 (e)-(g) are the corresponding Fourier spectra. It shows that in deeper MI regime, the beat effects become more prominent.

In a recent paper, it has been observed the beat phenomenon of the revivals and collapses of the quantum coherence in an optical lattice in a longer time interval[1]. The beat signal of the revival and collapse oscillation was attributed to the multi-body interactions, emerging through virtual transitions of particles from the lowest energy band to the higher energy bands. In our model, the atoms are assumed to occupy a single spatial orbital and only the two-body interaction Hubbard energy $U$ is independent of the filling at the lattice site. The disorder can enhance the periodicity of the revival and collapse oscillation by damping the higher-order frequencies. It is not clear yet if there are inherent relations between the beat phenomena observed by S. Will et al and our theoretical model.

In summary, we have investigated the disorder effect on the quantum coherence of a many-boson system. We find that the disorder enhance quantum coherence in the ground states. Dynamical evolution of the cold Bose atoms exhibits collapses and revivals of the coherence which is robust against the disorder. In particular, the interaction disorders generate the beat phenomenon of coherence in the temporal evolution. It may provide a signal to observe the effects of the interaction disorders experimentally.

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