Angular Distribution of $\gamma$-rays from Neutron-Induced Compound States of $^{140}$La

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Angular distribution of individual $\gamma$-rays, emitted from a neutron-induced compound nuclear state via radiative capture reaction of $^{139}$La(n,$\gamma$) has been studied as a function of incident neutron energy in the epithermal region by using germanium detectors.

An asymmetry $A_{LH}$ was defined as $(N_L - N_H)/(N_L + N_H)$, where $N_L$ and $N_H$ are integrals of low and high energy region of a neutron resonance respectively, and we found that $A_{LH}$ has the angular dependence of $(A \cos \theta + B)$, where $\theta$ is emitted angle of $\gamma$-rays, with $A = -0.3881 \pm 0.0236$ and $B = -0.0747 \pm 0.0105$ in 0.74 eV p-wave resonance.

This angular distribution was analyzed within the framework of interference between s- and p-wave amplitudes in the entrance channel to the compound nuclear state, and it is interpreted as the value of the partial p-wave neutron width corresponding to the total angular momentum of the incident neutron combined with the weak matrix element, in the context of the mechanism of enhanced parity-violating effects. Additionally we used the result to quantify the possible enhancement of the breaking of the time-reversal invariance in the vicinity of the p-wave resonance.

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I. INTRODUCTION

The magnitude of parity violating effects in effective nucleon–nucleon interactions is $10^{-7}$, as observed in the helicity dependence of the total cross section between nucleons [4]. Extremely large parity violation (P-violation) was found in the helicity dependence of the neutron absorption cross section in the vicinity of p-wave resonance of $^{139}$La+n [2]. The helicity dependence was measured as the ratio of the helicity-dependent cross section to the p-wave resonance cross section, referred to as longitudinal asymmetry, which amounts to $(9.56 \pm 0.35)\%$. The large P-violation was explained as the interference between the amplitudes of the p-wave resonance and the neighboring s-wave resonance [2, 6]. Longitudinal asymmetry was intensively studied in neutron transmission and in (n,$\gamma$) measurements [2, 10]. The $\gamma$-ray energy dependence of the asymmetry was not found, which implies that the interference occurs in the entrance channel to the compound state and not in the exit channel [11]. Under this assumption, the longitudinal asymmetry $A_L$ is given by

$$A_L \simeq -\frac{2xW}{E_p - E_n}\sqrt{\frac{\Gamma_n^p}{\Gamma_n^s}}$$

assuming s-wave and p-wave resonances, where $E_n$ and $E_p$ are their respective energies, $\Gamma^p_n$ and $\Gamma^s_n$ are the corresponding neutron widths and $W$ is the weak matrix element. The value of $x$ is defined as $x^2 = \frac{\Gamma^p_n}{\Gamma^s_n} = \frac{1}{2}$, where $\Gamma^p_n = \frac{1}{2}$ is the partial neutron width for the total angular momentum of the incident neutron $j = \frac{1}{2}$.

The value of $x$ have not yet been measured individually. The value of $x$ can be extracted from the energy dependence of the angular distribution of $\gamma$-rays from a p-wave resonance in a neutron induced compound nucleus, which has not yet been measured. Theoretically, it can be deduced assuming interference between partial waves in the entrance channel [13].

In this paper we report measurement results of the angular distribution of individual $\gamma$-rays emitted from 0.74 eV p-wave resonance of $^{139}$La+n as a function of incident neutron energy.
II. EXPERIMENT

A. Experimental Setup

The angular distribution of individual $\gamma$-rays through the radiative capture reactions induced by epithermal neutrons was measured by introducing a pulsed neutron beam into the Accurate Neutron–Nucleus Reaction Measurement Instrument (ANNRI) installed at the beamline BL04 of the Material and Life science experimental Facility (MLF) of the Japan Proton Accelerator Research Complex (J-PARC), as shown in Fig. 1 [16]. The primary proton beam pulses were injected to the neutron production target in a single-bunch mode with a repetition rate of 25 Hz and an average beam power of 150 kW during the measurement. The disk chopper was operated synchronously with the proton injection for the suppression of low energy neutrons, to avoid frame overlap. The beam collimation was adjusted to define the neutron beam in a 22 mm diameter circle on the target, placed at 21.5 m from the moderator surface [17]. A lead plate (thickness: 37.5 mm) was placed in the upstream optics to suppress the $\gamma$-ray background. The $z$-axis is defined in the beam direction, the $y$-axis is the vertical upward axis, and $x$-axis is perpendicular to them, thus $xyz$ forms a right-handed frame.

An assembled set of high-purity germanium spectrometers were used to detect $\gamma$-rays emitted from the target [18]. The configuration of the germanium spectrometer assembly is shown in Fig. 2.

The assembly consisted of two types of detector units: Type-A (Fig. 3) and Type-B (Fig. 4).

Two combined seven Type-A detectors were placed above and below the target. The shape of the Type-A detector was hexagonal to enable clustering as shown in Fig. 3. The polar angles between the center of the target and the center of the crystal surface facing the target were $\theta = 71^\circ$, $90^\circ$, and $109^\circ$.

Eight Type-B detectors were placed on the $xz$-plane at $\theta = 36^\circ$, $72^\circ$, $108^\circ$, and $144^\circ$, as shown in Fig. 4. The central crystals of the upper and lower Type-A detectors are denoted as d1 and d8, respectively, and the other surrounding six detectors are denoted d2-d7 (d9-d14). The names of each Type-B detectors are shown in Fig. 4. In our measurement, d16 and d17 were not used.

All germanium crystals were operated at a temperature of 77 K. The typical energy resolution for the 1.332 MeV $\gamma$-rays was 4.2 keV.

The output signal from each crystal was processed independently. The block diagram of the signal processing is shown in Fig. 5. The output signals from the preamplifier were fed into the signal processing module CAEN V1724 [19], which stored the combination of the pulse
height digitized using the peak-sensitive ADC and the timing of the zero-cross point measured from the timing pulse of the injection of the primary proton beam bunch $t^m$. The CAEN V1724 module transferred the stored data to the computer when 1024 event data are accumulated in the local buffer. Two pulses temporarily closer than 0.4 $\mu$s were processed as a single event, while their pulse heights were recorded as a zero when their time difference was in the range of 0.4 $\mu$s to 3.2 $\mu$s.

Their response functions were simulated using GEANT4.9.6. Definitions of the symbols to describe the detector characteristics and results of the simulation are discussed in detail in Appendix A.

The relation between the pulse height of photo peaks and the deposit $\gamma$-ray energy was determined by observing $\gamma$-rays emitted in neutron capture reactions by aluminum. The effective photo-peak efficiency including the solid angle coverage of each detector unit was determined relatively based on the assumption that prompt $\gamma$-rays from $^{14}$N(n, $\gamma$) of a melamine target were emitted isotropically. The relative photo-peak efficiencies are shown in Table III.

### B. Measurement

The target was a natural-abundance lanthanum plate at room temperature, with the dimensions of 40 mm $\times$ 40 mm $\times$ 1 mm, and with a purity of 99.9%. The total number of $\gamma$-ray events detected in the experiment are denoted $I_\gamma$. Here, the corresponding neutron energy $E_n^m$ is defined as

$$E_n^m = \frac{m_n}{2} \left( \frac{L}{r^m} \right)^2,$$

where $m_n$ is the neutron mass and $L$ is the distance between the target and the moderator surface. The deposited $\gamma$-ray energy $E_\gamma^m$ obtained from the calibration of the pulse height is defined as well. The obtained results are shown as a 2-dimensional histogram corresponding to $\partial^2 I_\gamma/\partial t^m \partial E_\gamma^m$ in Fig. 6. The histograms projected on $t^m$ and $E_\gamma^m$ are shown in Fig. 7 and Fig. 8 respectively. In Fig. 7 $\gamma$-ray events with $E_\gamma^m$ are integrated and relatively corrected by the incident beam spectrum for $t^m$. The incident beam spectrum was obtained by measuring the 477.6 keV $\gamma$-rays in $^{10}$B(n, $\alpha\gamma$)$^7$Li reactions, with a boron target placed at the detector center. The small peak at $t^m \sim 1800$ $\mu$s is a p-wave resonance, and the $1/v$ component is the tail of an s-wave resonance in the negative energy region as listed in Table III.

The neutron energy in the center-of-mass system $E_n$ is given as

$$E_n = \frac{m_n m_A}{m_n + m_A} \left( \frac{p_n}{m_n} - \frac{p_A}{m_A} \right)^2,$$

where $m_n$, $m_A$, $p_n$, and $p_A$ are the neutron, target, and projectile masses and momenta, respectively.
The fitted result is shown in Fig. 9.

FIG. 8. Pulse height spectrum of γ-rays $\partial I_\gamma/\partial E_\gamma$ from the (n,γ) reaction with lanthanum target as a function of γ-ray energy, the s-wave resonance, which are obtained by fitting $\partial I_\gamma/\partial t_m$ with Eq. (21) are shown in Table I, together with the published values. The formalism of the neutron absorption cross section is described in Appendix B. The pulse shape of the neutron beam and the Doppler effect of the target nucleus are considered, as shown in Appendix C and Appendix D, respectively. As the neutron width of the p-wave resonance is negligibly smaller than γ-ray width of the p-wave resonance, the total width of p-wave resonance was used as the γ-ray width of p-wave resonance. The fitted result is shown in Fig. 9.

The level scheme related to $^{139}$La(n,γ)$^{140}$La reaction is schematically shown in Fig. 10. The γ-ray transitions to the ground state and low excited states of $^{140}$La were observed as shown in the expanded $\partial I_\gamma/\partial E_\gamma$ (Fig. 11). The highest peak at $E_m^\gamma=5161$ keV corresponds to the γ-ray direct transition to the ground state of $^{140}$La (spin of the final state: $F=3$), the middle peak corresponds to the overlap of two transitions at $E_m^\gamma=5131$ keV and $5126$ keV to the first and second excited states at excited energy $30$ keV ($F=5$), $35$ keV ($F=2$), and the lower peak at $E_m^\gamma=5098$ keV corresponds to the excited state at $63$ keV ($F=4$).

Figure 12 shows the magnified 2-dimensional histogram of $\partial^2 I_\gamma/\partial t_m \partial E_\gamma$ in the vicinity of the p-wave resonance and the γ-ray transition to the ground state of $^{140}$La. The p-wave resonance was selectively observed only for two ridges corresponding to the transition at $E_m^\gamma=5161$ keV and the sum of transitions at $E_m^\gamma=5131$ keV, $5126$ keV, but not for the ridge at $E_m^\gamma=5098$ keV. According to the dependence of $\partial^2 I_\gamma/\partial t_m \partial E_\gamma$ on $t_m$, and therefore on the incident neutron energy, the s-wave resonance in the negative region contributes to all three γ-ray transitions, and the p-wave resonance contributes to the $5161$ keV transition.

FIG. 7. γ-ray counts relatively corrected by the incident beam intensity as a function of $t_m$ for $E_m^\gamma \geq 2$ MeV, which is referred as $\partial I_\gamma/\partial t_m$.

FIG. 8. Pulse height spectrum of γ-rays $\partial I_\gamma/\partial E_\gamma$ from the (n,γ) reaction with lanthanum target as a function of $E_\gamma$.

FIG. 9. Fitted result of the p-wave resonance. The curve shows the best fit.

FIG. 10. Transitions from $^{139}$La+n to $^{140}$La. Dashed line shows separation energy of $^{139}$La+n.
TABLE I. Resonance parameters of $^{139}$La. (a) taken from Ref. [20] and Ref. [21]. (b) taken from Ref. [22]. (c) calculated from Refs. [23] and [22]. $\Gamma_{nr}$ is a reduced neutron width.

|       | $E_r$ [eV] | $J_r$ | $I_r$ | $\Gamma_n [\text{meV}]$ | $g_r \Gamma_n [\text{meV}]$ | $E_r$ [eV] | $\Gamma_r [\text{meV}]$ |
|-------|------------|-------|-------|-------------------------|-----------------------------|------------|------------------------|
| 1     | -48.63$^{(a)}$ | 4$^{(a)}$ | 0     | 62.2$^{(a)}$             | (5.6 $\pm$ 0.5) $\times 10^{-5}$ | 0.740 $\pm$ 0.002 | 40.41 $\pm$ 0.76 |
| 2     | 0.758 $\pm$ 0.001$^{(b)}$ | 1     | 40.11 $\pm$ 1.94$^{(c)}$ | 11.76 $\pm$ 0.53$^{(c)}$ | 3     | 72.30 $\pm$ 0.05$^{(b)}$ | 75.64 $\pm$ 2.24$^{(c)}$ |

The photo-peak efficiency, including both the detection efficiency and the solid angle coverage, was readjusted using the photo-peak counts of the $\gamma$-rays at $E_m^\gamma=5262$ keV from the $^{14}$N(n,$\gamma$) reaction measured using the melamine target. It can be reasonably assumed that the $\gamma$-rays are emitted isotropically, as the $^{14}$N does not have any resonance below 400 eV and p-wave or higher angular momentum components of the incident neutron is negligibly small in this energy region.

The photo-peak counts of the 5161 keV transition were determined by subtracting the background counts caused by Compton scattering of the more energetic $\gamma$-rays from targets other than the lanthanum target. To evaluate the background, two energy regions were used: (I) $5200 \text{ keV} \leq E_m^\gamma \leq 5290 \text{ keV}$ and (II) $4900 \text{ keV} \leq E_m^\gamma \leq 4980 \text{ keV}$. The contribution of Compton scattering of $\gamma$-rays corresponding to the three photo-peaks are contained in region (II). The amount of this contribution from Compton scattering is estimated using the response function $\bar{\psi}$ given in Eq. A2 and obtained by simulation. The background in region (II) is estimated by subtracting the Compton contribution from the $\gamma$-ray counts in region (II). The background is estimated using a best-fit third-order polynomial of $E_m^\gamma$ in regions (I) and (II).

There still remains a possible contamination of prompt $\gamma$-rays from impurities overlapping with the 5161 keV photo-peak. The possible contamination was examined over the entire pulse height spectrum, and was determined to be less than 0.08% of the photo-peak. The possibility of contamination was neglected as the determined upper limit of 0.08% is smaller than the statistical error of the photo-peak.

According to the data acquisition system, two pulses detected within 3.2 $\mu$s did not have amplitude information, which amounted to 2% of the total $\gamma$-ray counts in the vicinity of the p-wave resonance. The 2% loss was corrected in the following analysis.

Two pulses detected within 0.4 $\mu$s were processed as a single pulse. The corresponding loss of the events were estimated as 0.2% of the total $\gamma$-ray counts in the vicinity of the p-wave resonance, which is negligibly small compared with the statistical error of the corresponding $\gamma$-ray counts, and is ignored in the following analysis.

Equation D1 was extended to describe the angular dis-
The angular dependence of the p-wave resonance is shown in Fig. 16. The value of $j = l + s$, where $s$ is the neutron spin. The value of $j$ is 1/2 for s-wave neutrons ($l=0$) and $j=1/2, 3/2$ for p-wave neutrons ($l=1$).

The p-wave resonance and two neighboring s-wave resonances are considered in the negative and positive energy region, listed in Table II in the following analysis. The resonance energy and resonance width measured in

\[
\gamma_{\text{ex}} \quad J_l \quad I_l \quad \Gamma^g_{\text{ex}} \quad \Gamma^r_{\text{ex}}
\]

\[
\begin{array}{cccc}
1 & -48.63^{(a)} & 4^{(a)} & 0 & 62.2^{(a)} & (571.8)^{(a)*} & s_1 \\
2 & 0.740 \pm 0.002 & 4 & 1 & 40.41 \pm 0.76 & (5.6 \pm 0.5) \times 10^{-5}^{(c)} & p \\
3 & 72.30 \pm 0.05^{(b)} & 3 & 0 & 75.8 \pm 5.4^{(c)} & 117.6 \pm 0.53^{(c)} & s_2
\end{array}
\]

TABLE II. Resonance parameters of $^{139}$La used in the analysis. (a) taken from Ref. 21 and Ref. 22. (b) taken from Ref. 22. (c) calculated from Refs. 23 and 22. * The neutron width for the negative resonance was calculated using $|E_1|$ instead of $E_1$.

III. ANALYSIS

Our experimental results are analyzed using the formulation of possible angular correlations of individual $\gamma$-rays, emitted in ($n, \gamma$) reactions induced by low energy neutrons according to s- and p-wave amplitudes [13]. The formalism of the differential cross section of the ($n, \gamma$) reaction induced by unpolarized neutrons is described in Appendix E. We use $I$ as the spin of the target nuclei, $J$ as the spin of the compound nucleus, $F$ as the spin of the final state of the $\gamma$-ray transition, and $l$ as the orbital angular momentum of the incident neutron. The total neutron spin is defined as $j = l + s$, where $s$ is the neutron spin. The value of $j$ is 1/2 for s-wave neutrons ($l=0$) and $j=1/2, 3/2$ for p-wave neutrons ($l=1$).

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FIG. 13. $\partial N/\partial t_m$ in the vicinity of p-wave resonance for each $\bar{\theta}_d$. The central figure shows degrees in the direction of neutron momentum of the type-A detectors and the type-B detectors. The hexagons and the circles in the center of the figure denote each crystal of the type-A detector and the type-B detector, respectively.

FIG. 14. Visualization of the definition of $N_L$ and $N_H$.

FIG. 15. Angular dependences of $N_L$ and $N_H$. The white point and black point show $N_L$ and $N_H$, respectively.

small. We define $(\bar{u}_0)_L$, $(\bar{u}_1)_L$, $(\bar{u}_3)_L$, $(\bar{u}_0)_H$, $(\bar{u}_1)_H$, and
of the centrifugal potential by the factor $\lambda$. According to the centrifugal potential by the factor $\lambda$.

The p-wave resonance ($E_p \approx 0.6-0.9$ eV; shaded area with diagonal line), and the s-wave resonance ($E_s = 70-75$ eV: solid shaded area).

Here, the $a_3$ term is ignored as it is proportional to $\lambda$, and it is suppressed relative to the s-wave neutron width, according to the centrifugal potential by the factor of $(kr)^2$. Under this approximation, Eq. (13) is reduced to

$$\frac{d\sigma_{\gamma\gamma}}{d\Omega} = \frac{1}{2} (a_0 + a_1 \cos \theta_\gamma),$$

(13)

Substituting Eq. (13) into Eq. (14), the angular dependence of the $\gamma$-ray counts in the neutron energy regions $E_n < E_p$ and $E_p < E_n < E_p + 2\Gamma_p$ can be written as

$$\left( \frac{\partial^2 I_\gamma}{\partial \theta_\gamma^2} \right)_{L} = \frac{I_0}{2} \left( (\bar{a_0})_L + (\bar{a_1})_L P_1(\cos \theta_\gamma) \right),$$

$$\left( \frac{\partial^2 I_\gamma}{\partial \theta_\gamma^2} \right)_{H} = \frac{I_0}{2} \left( (\bar{a_0})_H + (\bar{a_1})_H P_1(\cos \theta_\gamma) \right).$$

(14)

By convoluting with Eq. (6), the $\gamma$-ray counts $(I_{\gamma,d})_{L,H}$ to be measured by the $d$-th detector can be written as

$$(I_{\gamma,d})_{L,H} = \frac{I_0}{2} \left( (\bar{a_0})_{L,H} + (\bar{a_1})_{L,H} P_1(\theta_\gamma) \right).$$

(15)

As the energy dependence of $x_2$ and $y_2$ is negligibly small in the vicinity of the p-wave resonance ($r_p = 2$), $\bar{a}_L$ and $\bar{a}_H$ are linear functions of $x_2$ and $y_2$, thus a function of $\phi_2$. The value of $\phi_2$ is determined by comparing $(I_{\gamma,d})_{L,H} / (I_{\gamma,d})_{L} + (I_{\gamma,d})_{H}$ with the measured values $A$ in Eq. (16)

$$A = \frac{(\bar{a}_L - \bar{a}_H)}{(\bar{a}_0)_L + (\bar{a}_0)_H} = 0.295 \cos \phi_2 - 0.345 \sin \phi_2. \quad (16)$$

Two solutions can be obtained as

$$\phi_2 = (99.2\pm 3.3)^\circ, \quad (161.9\pm 3.3)^\circ. \quad (17)$$

The visualization of $\phi_2$ is shown in Fig. (18)

IV. DISCUSSION

As the value of $\phi_2$ was obtained in the previous section, the T-violation sensitivity is discussed in this section. We obtain $x_2$ from Eq. (17) and Eq. (18) with resonance number $r_p = 2$ as

$$x_2 = -0.16^{+0.09}_{-0.11}, \quad -0.95^{+0.04}_{-0.03}. \quad (18)$$

This leads to the value of $W$ which is given by Eq. (19) as

$$W = (13.2^{+18.1}_{-3.3}) \text{ meV}, \quad (2.21^{+0.10}_{-0.06}) \text{ meV}. \quad (19)$$

The published value of $A_1 = (9.56\pm 0.35) \times 10^{-2}$ in Ref. [7] and resonance parameters in Table. I are used in the calculation. Note that the neutron width of the negative s-wave resonance $\Gamma_s$ at the resonance energy of the p-wave resonance is adopted.
The equations of a section is given as a unit circle. The solid line and shaded area show the central values of $\phi_2$ and 1σ area, respectively. $\phi_2$ denotes an angle of a point on a unit circle.

The ratio of P-odd T-odd cross sections to P-odd cross sections is given as

$$\frac{\Delta \sigma_{PT}}{\Delta \sigma_P} = \kappa(J) \frac{W_T}{W},$$

(20)

where $\Delta \sigma_{PT}$ is the P-odd T-odd cross section, $\Delta \sigma_P$ the P-odd cross section, $W_T$ the P-odd T-odd matrix element and $W$ the P-odd matrix element. The calculations of these matrix elements were performed in Ref. [25] and [26]. The spin factor $\kappa(J)$ is defined as

$$\kappa(J) = \begin{cases} (-1)^{2J} \left( 1 + \frac{1}{2} \sqrt{\frac{2J - 1}{J + 1}} \right) & (J = I - \frac{1}{2}) \\ (-1)^{2J+1} \left( 1 - \frac{1}{2} \sqrt{\frac{2J + 3}{J + 1}} \right) & (J = I + \frac{1}{2}) \end{cases}.$$  

(21)

The magnitude of $\kappa(J)$ indicates the sensitivity to the P-odd T-odd interaction. The $J = I + \frac{1}{2}$ case corresponds to the p-wave resonance of the $^{139}$La+n at $E_n = E_2$. The value of $\kappa(J)$ corresponding to the $\phi_2$ obtained is

$$\kappa(J) = 4.84^{+5.58}_{-1.69}, \quad 0.99^{+0.08}_{-0.07}$$

(22)

and $|\kappa(J)|$ is shown in Fig. 19.

In the previous section, the $a_3$ term was ignored, as the centrifugal potential of the p-wave resonance is small. Hereafter we discuss the case when the $a_3$ term in Eq. [21] is activated. We analyze the angular dependences of $N_L - N_H$ and $N_L + N_H$ fitted by the functions of $f(P_{d,1}/P_{d,0}) = A'P_{d,1}/P_{d,0} + B'$ and $g(P_{d,2}/P_{d,0}) = C'P_{d,2}/P_{d,0} + D'$, respectively, with fitting parameters $A'$, $B'$, $C'$, and $D'$. The equations of $a_3$ can be written as

$$\frac{A'}{D'} = \frac{(\bar{a}_1)_L - (\bar{a}_1)_H}{(a_0)_L + (a_0)_H} = 0.295 \cos \phi_2 - 0.345 \sin \phi_2,$$

(23)

$$\frac{C'}{D'} = \frac{(\bar{a}_3)_L + (\bar{a}_3)_H}{(a_0)_L + (a_0)_H} = -0.295 \cos \phi_2 \sin \phi_2 + 0.050 \sin^2 \phi_2.$$  

(24)

The fitted results of $C'/D'$ and $A'/D'$ are

$$\frac{C'}{D'} = 0.191 \pm 0.028, \quad \frac{A'}{D'} = -0.409 \pm 0.024.$$  

(25)

The value of $\phi_2$ is determined by combining the equations of $a_1$ (Eq. 23) and $a_3$ (Eq. 24) on the $xy$-plane. The result is shown in Fig. 21. The restriction from the $a_3$ term is not consistent with that of the $a_1$ term. The $a_3$ term deviates from the requirement of $x_2^2 + y_2^2 = 1$ by more than 2σ.

In this analysis, $J_1 = J_2 = 4, J_3 = 3$ are assumed. However, there is a possibility of the case of $J_1 = J_2 = J_3 = 3$. As the effect of the $s_2$-wave is negligibly small in this discussion, we discuss combinations of $J_1$ and $J_2$ only. The result of the case of $J_1 = J_2 = J_3 = 3$ is shown in Fig. 22. Both $a_1$ and $a_3$ in the case of $J_1 = J_2 =
FIG. 21. $a_1$(straight lines) and $a_3$(curved lines) on the $xy$-plane for the cases of $J_1 = J_2 = 4, J_3 = 3$. The solid line, shaded area, dashed line, and dotted line show the central values of $\phi_2$, $1\sigma$ areas, $2\sigma$ contours and $3\sigma$ contours, respectively.

$J_3 = 3$ have no solution. As both $a_1$ and $a_3$ in the case of $J_1 = J_2 = 4, J_3 = 3$ have solution in $3\sigma$, we support $J_1 = J_2 = 4$.

The origin of the inconsistency has not been identified in the present study. The inconsistency may be due to possible incompleteness of the reaction mechanism based on the interference between s- and p-wave amplitudes with the Breit–Wigner approximation.

V. CONCLUSION

We observed clear angular distribution of emitted $\gamma$-rays in the transition from the p-wave resonance of $^{139}$La+n to the ground state of $^{140}$La as a function of incident neutron energy. The angular distribution was analyzed by assuming interference between s- and p-wave amplitudes, and the partial neutron width of the p-wave resonance was obtained. This result suggests that the T-violating effect can be enhanced on the same order of the P-violating effect for 0.74 eV p-wave resonance of $^{139}$La+n. Therefore an experiment to explore T-violation in compound nuclear states is feasible. In addition, the analysis under this assumption leads to results that are consistent with theoretical expectation, and we therefore believe the assumption of s-p mixing is correct.

ACKNOWLEDGMENTS

The authors would like to thank the staff of ANRRI for the maintenance of the germanium detectors, and MLF and J-PARC for operating the accelerators and the neutron production target. We would like to thank Dr. K. Kino for the calculation of the pulse shape of neutron beam. We also appreciate the continuous help by Prof. V. P. Gudkov for the interpretation of the measured results. The neutron scattering experiment was approved by the Neutron Scattering Program Advisory Committee of IMSS and KEK (Proposal No. 2014S03, 2015S12). The neutron experiment at the Materials and Life Science Experimental Facility of the J-PARC was performed under a user program (Proposal No. 2016B0200, 2016B0202, 2017A0158, 2017A0170, 2017A0203). This work was supported by MEXT KAKENHI Grant number JP19GS0210 and JSPS KAKENHI Grant Number JP17H02889.

Appendix A: Definitions of symbols describing detector characteristics and the results of the simulation

In this section, we describe the definition of characteristics of the germanium detectors and the simulation results. Herein, we use $\psi_d(E_\gamma, \Omega_\gamma, (E^m_\gamma)_d)$ to denote the probability of the case where the energy of $(E^m_\gamma)_d$ is deposited in the $d$-th detector, when a $\gamma$-ray with an energy of $E_\gamma$ is emitted in the direction of $\Omega_\gamma=(\theta_\gamma, \varphi_\gamma)$. The polar angle and the azimuthal angle of the direction of the emitted $\gamma$-ray are denoted by $\theta_\gamma$ and $\varphi_\gamma$, respectively.
The $\psi_d(E_\gamma, \Omega_\gamma, (E^m_\gamma)_d)$ satisfies

$$\int_0^{E_\gamma} \psi_d(E_\gamma, \Omega_\gamma, (E^m_\gamma)_d) d(E^m_\gamma)_d = 1. \quad (A1)$$

We define the distribution of the energy deposit as

$$\tilde{\psi}_d(E_\gamma, (E^m_\gamma)_d) = \int_{\Omega_d} \psi_d(E_\gamma, \Omega_\gamma, (E^m_\gamma)_d) d\Omega_\gamma, \quad (A2)$$

where $\Omega_d$ is the geometric solid angle of the $d$-th detector. The photo-peak efficiency of $d$-th detector for $\gamma$-rays with the energy of $E_\gamma$ is defined as

$$\tilde{c}^{pk,w}_d(E_\gamma) = \int(E^m_\gamma)_d^{w^+} \tilde{\psi}_d(E_\gamma, (E^m_\gamma)_d) d(E^m_\gamma)_d, \quad (A3)$$

where $(E^m_\gamma)_d^{w^+}$ and $(E^m_\gamma)_d^{w^-}$ are the upper and lower limits of the region of the energy deposit for defining the photo-peak region, as schematically shown in Fig. 23. For the definition of the photo-peak efficiency $w = 1/4$ was used. The relative photo-peak efficiency is also defined as

$$\tilde{\psi}_d(E_\gamma, (E^m_\gamma)_d) = \tilde{c}^{pk,w}_d(E_\gamma) / \tilde{c}^{pk,w}_{d,\gamma_\gamma}(E_\gamma). \quad (A4)$$

The value of $\tilde{\psi}_d(E_\gamma, (E^m_\gamma)_d)$ was obtained using the simulation to reproduce the pulse height spectra for $\gamma$-rays from the radioactive source of $^{137}$Cs ($E_\gamma = 0.662$ MeV), as shown in Fig. 24. Subsequently, the reproducibility was checked by comparing the pulse height spectra for $^{60}$Co at $E_\gamma = 1.173$ MeV, $1.332$ MeV and $^{22}$Na at $E_\gamma = 1.275$ MeV. Finally, we confirmed that the simulation program is applicable to higher energies by comparing the numerical simulation with the pulse height spectrum for prompt $\gamma$-rays emitted by the $^{14}$N($n,\gamma$) reaction at $E_\gamma = 10.829$ MeV, as shown in Fig. 25. The photo-peak efficiency of the detector assembly for 1.332 MeV $\gamma$-ray is determined to be $3.64\pm0.11\%$ [27]. In the case of angular distribution of $\gamma$-rays this is expanded using Legendre polynomials as $\sum_{p=0}^{\infty} c_p P_p(\cos \theta_d)$, and the photo-peak counts of the $d$-th detector can be written as

$$N_d(E_\gamma) = N_0 \sum_{p=0}^{\infty} c_p \bar{P}_{d,p},$$

$$\bar{P}_{d,p} = \frac{1}{4\pi} \int_{(E^m_\gamma)_d^{w^+}} \int d(E^m_\gamma)_d \int d\Omega_\gamma P_p(\cos \theta_d) \tilde{\psi}_d(E_\gamma, \Omega_\gamma). \quad (A5)$$

The determined values of $\bar{P}_{d,p}$ are listed in Table III for $p = 0, 1, 2$. The quantity $\bar{P}_0$ corresponds to $\tilde{c}^{pk,w}_d$. The table also contains the weighted average of the viewing angle of each detector $\bar{\theta}_d$ determined as $P_{p}(\cos \bar{\theta}_d) = \bar{P}_{d,p} / \bar{P}_{d,0}$.

**Appendix B: Neutron absorption cross section**

The formula used to describe the neutron cross section $\sigma_t$ is given as a function of the neutron energy in the center-of-mass system:

$$\sigma_t(E) = \sigma_s + \sigma_{n\gamma}(E_n), \quad (B1)$$
\[ r \text{ represents the orbital angular momentum of the incident resonance, and } \Gamma \text{ the inelastic capture cross section,} \]

Appendix C: Pulse shape of the neutron beam

The pulse shape of neutron beam depends on the neutron energy \( E_n \) according to the time delay during the moderation process. The double differential of the flux of the pulsed neutron beam \( I_n \) as a function of \( E_n \) and the time measured from the primary proton beam injection \( t \) is known to be well reproduced by the Ikeda–Carpenter function, defined as

\[
\frac{\partial^2 I_n}{\partial E_n \partial t}(E_n,t) = \frac{\alpha C}{2} \left\{ \left[ 1 - (R - \alpha \beta)^2 e^{-\alpha t} + 2R\alpha^2 \beta \right] \times \left[ e^{-\beta t} - e^{-\alpha t} \left( 1 + (\alpha - \beta) t + \frac{(\alpha - \beta)^2}{2} t^2 \right) \right] \right\},
\]

where parameters \( \alpha, \beta, C \), \( R \) depend on \( E_n \). The Ikeda–Carpenter function was originally proposed to explain the pulse shape of the cold source of polyethylene moderator at the Intense Pulsed Neutron Source of the Argonne National Laboratory [28]. The double differential of the neutron beam flux on the moderator surface of the J-PARC spallation source was calculated using MCNPX [29], and was fitted with the Ikeda–Carpenter function. The dependence of neutron energy on the fitting parameters \( t_0, \alpha, \beta, R \), and \( C \) were obtained by fitting of the pulse shape of neutron beam with a polynomial function [17].

The energy spectrum at a given time \( t^m \) at the distance of \( L \) from the moderator surface is given as

\[
\frac{\partial I_n}{\partial t^m(E_n)} = \int dE' \frac{\partial^2 I_n}{\partial E_n \partial t} \left( E', t^m - L \sqrt{\frac{m_n}{2E'}} \right).
\]
Appendix D: Thermal motion of the target nuclei

We adopted the free gas model for the thermal motion of the target nuclei, which leads to the γ-ray yield in the form of

\[
\frac{\partial I_n}{\partial t} = I_0 \int dE' d^3p_{\Lambda} \frac{\partial^2 I_n}{\partial E_n \partial t} \left( E', t^m - L \frac{m_n}{2E'} \right) \\
\times \frac{1}{(2\pi m_A k_B T)^3/2} e^{-\frac{p^2_\Lambda}{2m_A k_B T}} \\
\times \frac{\sigma_{n\gamma}(E_n)}{\sigma_n(E_n)} \left( 1 - e^{-n\sigma(E) \Delta z} \right),
\]
(D1)
as long as the target is sufficiently thin, such that multiple scattering is negligible. \( I_0 \) is the normalization constant, \( n \) is the number density of target nuclei, \( k_B \) is the Boltzmann constant, and \( T \) is the effective temperature of the target, which can be used as a fitting parameter.

Appendix E: Formula describing the \((n, \gamma)\) angular dependence

The differential cross section of the \((n, \gamma)\) reaction induced by unpolarized neutrons can be written as

\[
\frac{d\sigma_{n\gamma}}{d\Omega_\gamma} = \frac{1}{2} \left( a_0 + a_1 \cos \theta_\gamma + a_3 \left( \cos^2 \theta_\gamma - \frac{1}{3} \right) \right),
\]

\[
a_0 = \sum_{rs} |V_{rs}|^2 + \sum_{rp} |V_{rp}|^2,
\]

\[
a_1 = 2 \text{Re} \sum_{rs,ij} V_{rs}V_{rp}^* z_{rp}^i P(J_{ij}, J_{rp} \frac{1}{2}) J_{ij}^I, F J J',
\]

\[
a_3 = 3\sqrt{10} \text{Re} \sum_{rs,ij,ij'} V_{rp}V_{rp}^* z_{rp}^i z_{rp}^{i'} P(J_{ij}, J_{ij'} \frac{1}{2}) J_{ij}^I J_{ij'}^I,
\]

\[
\times P(J_{rp}, J_{rp}^*, J_{rp}^* \frac{1}{2}) J_{rp}^I J_{rp}^* J_{rp}^* \frac{1}{2}
\]

where the absolute value of \( E_1 \) is adopted simply to avoid the imaginary neutron width. The terms \( a_0, a_1, \) and \( a_3 \)

\[
V_{rs,f} = \frac{\sqrt{g_{rs}}}{2k_n} \frac{\Gamma_{rs,f}^n (1 + \alpha_{rs})}{E_n - E_{rs} + i\Gamma_{rs,f}^n/2}
\]

\[
V_{rp,f} = \frac{\sqrt{g_{rp}}}{2k_n} \frac{\Gamma_{rp,f}^n (1 + \alpha_{rp})}{E_n - E_{rp} + i\Gamma_{rp,f}^n/2}
\]

\[
z_{rp,j} = \sqrt{\frac{\Gamma_{rp,j}^n}{\Gamma_{rp}^n}} \left\{ \begin{array}{c}
x_{rp} \\
y_{rp}
\end{array} \right. \left\{ \begin{array}{l}
(j = 1/2) \\
(j = 3/2)
\end{array} \right.
\]

\[
P(JJ' j') = (-1)^{J + J' + J' + I + F} \times \frac{3}{2} \sqrt{(2J + 1)(2J' + 1)(2j + 1)(2j' + 1)}
\]

\[
\times \left\{ \begin{array}{c}
k \ j \ j' \\
I \ J' \ J
\end{array} \right\} \left\{ \begin{array}{c}
k \ 1 \ 1 \\
F \ J' \ J'
\end{array} \right\}
\]

(E2)

\[
x_{rp} = \sqrt{\frac{\Gamma_{rp,j=1/2}^n}{\Gamma_{rp}^n}}, \quad y_{rp} = \sqrt{\frac{\Gamma_{rp,j=3/2}^n}{\Gamma_{rp}^n}}.
\]

(E3)

\[
x_{rp} \quad \text{and} \quad y_{rp} \quad \text{satisfy}
\]

\[
x_{rp}^2 + y_{rp}^2 = 1
\]

(E4)
due to the relation \( \Gamma_{rp}^n = \Gamma_{rp,j=1/2}^n + \Gamma_{rp,j=3/2}^n \). The resonance energy and the resonance width obtained from this experiment, the published values listed in Table IV and \( I = 7/2 \) are used to determine the value of \( \phi_{rp} \), defined as

\[
x_{rp} = \cos \phi_{rp}, \quad y_{rp} = \sin \phi_{rp}.
\]

(E5)

In the case of \(^{139}\text{La}\), negative s-wave amplitude \( V_1 \), p-wave amplitude \( V_2 \), and positive s-wave amplitude \( V_3 \) can be written as,

\[
V_{1f} = -\lambda_{1f} \left( \frac{E_1}{E_n} \right)^{1/4} \frac{\Gamma_{1/2}}{E_n - E_1 + i\Gamma_{1/2}},
\]

\[
V_{2f} = -\lambda_{2f} \left( \frac{E_2}{E_n} \right)^{1/4} \frac{\Gamma_{2/2}}{E_n - E_2 + i\Gamma_{2/2}},
\]

\[
V_{3f} = -\lambda_{3f} \left( \frac{E_3}{E_n} \right)^{1/4} \frac{\Gamma_{3/2}}{E_n - E_3 + i\Gamma_{3/2}}.
\]

(E6)
are given as

\[ a_0 = \lambda_{1f}^2 \sqrt{\frac{|E_1|}{E_n}} \frac{\Gamma_1^2/4}{(E_n - E_1)^2 + \Gamma_1^2/4} + \lambda_{2f}^2 \sqrt{\frac{E_n}{E_2}} \frac{\Gamma_2^2/4}{(E_n - E_2)^2 + \Gamma_2^2/4} + \lambda_{3f}^2 \sqrt{\frac{E_3}{E_n}} \frac{\Gamma_3^2/4}{(E_n - E_3)^2 + \Gamma_3^2/4} \]

\[ a_1 = \lambda_{1f} \lambda_{2f} \left( \sqrt{\frac{|E_1|}{E_2}} \right)^4 \times \frac{\Gamma_1 \Gamma_2 (E_n - E_1)(E_n - E_2) + \Gamma_1^2 \Gamma_2^2/4}{2((E_n - E_1)^2 + \Gamma_1^2/4)((E_n - E_2)^2 + \Gamma_2^2/4)} \times \frac{5}{8} \left(-x + \sqrt{\frac{7}{5} y}\right) + \lambda_{3f} \lambda_{2f} \left( \sqrt{\frac{E_3}{E_2}} \right)^4 \times \frac{\Gamma_3 \Gamma_2 (E_n - E_3)(E_n - E_2) + \Gamma_3^2 \Gamma_2^2/4}{2((E_n - E_3)^2 + \Gamma_3^2/4)((E_n - E_2)^2 + \Gamma_2^2/4)} \times \frac{3\sqrt{3}}{8} \left(x + \sqrt{\frac{5}{7} y}\right) \]

\[ a_3 = \lambda_{2f}^2 \sqrt{\frac{E_n}{E_2}} \frac{\Gamma_2^2/4}{(E_n - E_2)^2 + \Gamma_2^2/4} \frac{33}{280} \left(-\sqrt{35xy} + y^2\right). \quad (E7) \]

It can be assumed that the energy dependence of the neutron width of the \( r \)-th resonance is given as

\[ \Gamma^n_r = (k_n R)^{2ir} \sqrt{\frac{E_n}{1\text{eV}}} \Gamma^{nl}_r, \]  

(E8)

for \( E_n > 0 \) where \( R \) is the radius of target nuclei and \( \Gamma^{nl}_r \) is the reduced neutron width. This energy dependence is implemented as

\[ \Gamma^n_r = \left( \frac{E_n}{|E_r|} \right)^{l_r + \frac{1}{2}} \frac{\Gamma_r}{\Gamma_r}, \]  

(E9)

where \( \Gamma_r^{nl} \) is a constant independent of the energy. As the phase shift due to the optical potential is negligibly small, each amplitude can be written as

\[ V_{rf} = -\lambda_{rf} \left( \frac{E_n}{|E_r|} \right)^{l_r + \frac{1}{2}} \frac{\Gamma_r/2}{E_n - E_r + i\Gamma_r/2} \]  

(E10)

where \( \lambda_{rf} \) is defined as

\[ \lambda_{rf} = \frac{\hbar}{2} \sqrt{\frac{2g_r \Gamma_{r}^{\gamma} \Gamma_{r}^{l}}{m_n |E_r| \Gamma_{r}^{2}}}. \]  

(E11)

\( \Gamma_{r}^{\gamma} \) is the \( \gamma \)-width from the \( r \)-th resonance to the final state.
ERRATUM

We correct the values of the spin factor $\kappa(J)$ in our previous paper. The spin factor $\kappa(J)$ was originally given by Eq. 23 in Ref. [14] as a function of the nuclear spin and the neutron partial widths defined by $x$ and $y$. In the original paper, the values of $x$ and $y$ were obtained from the analysis result based on the formalism by Flambaum et al. [12]. However, due to the different order of summation of the neutron spin and neutron orbital angular momentum, the sign of $y$ defined by Gudkov et al. [14] was different to that defined by Flambaum et al. [13]. Therefore, $x$ and $y$ in Eq. 22 in the original paper should be replaced with $x \rightarrow x$ and $y \rightarrow -y$, and Eq. 22 in the original paper should be corrected as,

$$\kappa(J) = \begin{cases} 
(1 - \frac{1}{2} \sqrt{\frac{2I+1}{I+1} \frac{y}{x}}) & (J = I - \frac{1}{2}) \\
\frac{1}{I+1} \left(1 + \frac{1}{2} \sqrt{\frac{2I+3}{I+3} \frac{y}{x}} \right) & (J = I + \frac{1}{2})
\end{cases} \quad (1)
$$

Consequently, the $\kappa(J)$ values in Eq. 23 in original paper are corrected as

$$\kappa(J) = -3.28^{+1.69}_{-5.58} \quad 0.56^{+0.07}_{-0.08} \quad (2)$$

Similarly, Fig. 19 in the original paper should be replaced as Fig. 26.

Additionally, there was a typo: a coefficient of $2/3$ should be added in Eq. 25 in the original paper like so

$$\frac{C'}{D'} = \frac{(\mu_1^L + (\mu_1^R)_H)}{(\mu_0^L + (\mu_0^R)_H)} = \frac{2}{3} \left(-0.295 \cos \phi_2 \sin \phi_2 + 0.050 \sin^2 \phi_2 \right) \quad (3)$$

This typo does not affect the results of the original paper because $a_3$, the curved lines in Fig. 21 in the original paper, was calculated using the correct expression found in Eq. 3 in the erratum.

FIG. 26. Value of $|\kappa(J)|$ as a function of $\phi_2$. The solid line and shaded area show the central values of $\phi_2$ and the $1\sigma$ area from central value, respectively.

\[\begin{array}{c}
\begin{array}{c}
\text{ERRATUM}
\end{array}
\end{array}\]

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\end{array}
\end{array}\]

\[\begin{array}{c}
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k(J) = \begin{cases} 
(1 - \frac{1}{2} \sqrt{\frac{2I+1}{I+1} \frac{y}{x}}) & (J = I - \frac{1}{2}) \\
\frac{1}{I+1} \left(1 + \frac{1}{2} \sqrt{\frac{2I+3}{I+3} \frac{y}{x}} \right) & (J = I + \frac{1}{2})
\end{cases} \quad (1)
\end{array}
\end{array}\]

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\end{array}\]

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