What can Gaia (with TMT) say about Sculptor’s Core?

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Walker et al.’s Magellan/MMFS Survey survey identified 1355 red giant candidates in the dwarf spheroidal galaxy Sculptor. We find that the Gaia satellite will be able to measure the proper motions of 139 of these with a precision of between 13 and 20 km/s. Using a Jeans analysis and 5-parameter density model we show that this allows a determination of the mass within the deprojected half-light radius to within 16% and a measurement of the dark matter density exponent γ to within 0.68 within that radius. If, even at first light, the TMT observes Sculptor then the combined observations will improve the precision on these proper motions to about 5 km/s, about 5 years earlier than would be possible without Gaia, further improving the precision of γ to 0.27. Using a bimodal stellar population model for Sculptor the precision of γ improves by about 30%. This suggests that Gaia (with TMT) is capable of excluding a cored profile of the kind predicted by CDM simulations with 2σ (4σ) of confidence.

According to the standard cosmological model, most of the matter in the universe is cold dark matter (CDM), which at long distances only interacts gravitationally. Direct and indirect dark matter detection experiments are a major industry and so far have rapidly excluded large regions of the CDM parameter space, as has the Large Hadron Collider. However such tests can only confirm CDM, any such attempt to falsify this paradigm can be evaded by fine-tuning. On the other hand, CDM makes falsifiable predictions for the density profiles of sufficiently dark matter dominated systems. They must fall as 1/r^3 at large radii and as at least 1/r at small radii, a scaling known as a cusp. While there is little consensus over just how much dark matter domination is sufficient, studies such as Ref. [1, 2] suggest that galaxies with stellar masses beneath about 10^{6.5–7} M_⊙ should be cusped while Ref. [3] argues that energy conservation implies that baryonic effects cannot remove cusps in the Milky Way’s dwarf spheroidal (dSph) satellites except under quite special circumstances. Thus the discovery that a dSph is cored, not cusped, would pose a strong challenge to CDM.

In this letter we attempt to determine just when it will be possible to determine whether the dSph Sculptor, with a stellar mass of 2.3 × 10^6 M_⊙, is cored. Common wisdom states that one need first wait 10 years for a 30 meter class telescope, such as the Thirty Meter Telescope (TMT), to observe such a galaxy. Then one measures the positions of the stars, waits 5 more years and observes again. This yields proper motions of hundreds or thousands stars at a precision of about 4 km/sec, allowing a clean resolution to the cusp/core problem around 2030. We will argue that the Gaia satellite can yield a 2σ hint of the cored/cusped nature of Sculptor within 5 years and then, when its position measurements are combined with TMT’s first observations of Sculptor in about 10 years, a definitive exclusion of CDM will already be possible.

In Ref. [4] the authors report the observation of 1541 objects in the part of the sky occupied by the dSph Sculptor, as part of their MMFS survey using the Magellan/Clay Telescope. Of these, we consider the 1355 objects for which they assign a 90% or greater membership probability in Sculptor. For each object they provide the V magnitude and the I magnitude, which we identify with the Johnson-Cousins magnitude I_C. We use these to calculate

![FIG. 1: The precision with which Gaia can measure the proper motion of a Sculptor member with Gaia magnitude G and color V – I_C = 1.35](image-url)
FIG. 2: Number of stars in Sculptor whose proper motion can be measured with a given precision.

This function of $G$ is drawn in Fig. 1 for a typical color $V - I$.

As is explained in Ref. [8], at Sculptor’s ecliptic latitude of 36.5° south this improvement is a combination of a geometric parallax factor and a higher than average number of transits. However, the number of focal plane transits is also unusually high for Sculptor’s ecliptic latitude, as a result of its longitude and Gaia’s transit pattern. Assuming a distance to Sculptor of 79 kpc [9], the resulting proper motion precisions are summarized in Fig. 2.

In this letter we will answer the following questions: Given the line of sight velocities of the 1355 members in the MMFS catalog and proper motions measured with the precisions shown in Fig. 2 (1) To what extent can Sculptor’s dark matter density profile be determined? (2) To what extent could the CDM paradigm in principle be falsified?

Our approach to these questions will be similar to that of Ref. [10]. However, while we still use an assumed stellar profile to determine the expected dispersion, we perform our fit using stars in the MMFS catalog with, as described above, proper motions determined with a precision given by Eq. (3). The dark matter profile $ρ(r)$ is assumed to lie in a 5-parameter family, corresponding to the generalization [11] of the Hernquist profile [12].

$$\rho(r) = \rho_0 \left( \frac{r_0}{r} \right)^a \left( 1 + \left( \frac{r}{r_0} \right)^b \right)^{(a-c)/b}$$

where $r$ is the distance from the center of Sculptor. Assuming a constant orbital anisotropy $β = 1 - (v_r^2)/(v_t^2)$, we obtain the radial-dependence of the radial velocity dispersion $σ_r^2 = ⟨v_r^2(r)⟩$ by integrating the Jeans equation

$$σ^2(r) = \frac{G}{ρ^*(r)r^{2β}} \int_r^{30} ρ^*(R)M(R)R^{2β-2}dR$$

where $M(R)$ is the integrated mass within the radius $R$, approximated to be just the dark matter mass. $ρ^*(R)$ is the 3d stellar density at a radius $R$, not just the density of the stars in the catalog, which is taken to follow a King profile [13] with core radius $r_k = 0.28$ kpc and tidal radius $r_t = 1.63$ kpc [14]. The observed line of sight, radial and tangential dispersions can be found by integrating (5) over the line of sight.

$$σ_{los}^2(r) = \frac{2}{ρ^2_2D(r)} \int_r^{30} \left( 1 - β \frac{r^2}{R^2} \right) \frac{Rρ^*(R)σ^2(R)dR}{\sqrt{R^2 - r^2}}$$

$$σ_r^2(r) = \frac{2}{ρ^2_2D(r)} \int_r^{∞} \left( 1 - β + β \frac{r^2}{R^2} \right) \frac{Rρ^*(R)σ^2(R)dR}{\sqrt{R^2 - r^2}}$$

$$σ_t^2(r) = \frac{2}{ρ^2_2D(r)} \int_r^{∞} \left( 1 - β \right) \frac{Rρ^*(R)σ^2(R)dR}{\sqrt{R^2 - r^2}}$$

where $ρ^2_2D(r)$ is the stellar density integrated along the line of sight at $r$, the distance from the center of Sculptor in the transverse plane.

To determine the precision with which the mass profile can be determined, we use the Fisher matrix...
where the index $i$ runs over the stars in the MMFS catalog, the index $n$ runs over the 3 directions $los$, $r$ and $t$, $r_i$ is the projected distance from the center of Sculptor to the $i$th star, $\theta_a$ are the 5 parameters of the dark matter mass model [14] and $\sigma_{i,n}^2$ is the measurement uncertainty on the $n$th component of the velocity of the $i$th star. The line of sight measurement uncertainties are taken to be 1 km/sec. The derivatives with respect to the parameters $\theta_a$ are evaluated at the fiducial NFW model [14]

$$F_{ab} = \frac{1}{2} \sum_{i,n} \left( \frac{1}{(\sigma_{i,n}^2(r_i) + \sigma_{i,n}^2)} \frac{\partial \sigma_{i,n}^2(r_i)}{\partial \theta_a} \frac{\partial \sigma_{i,n}^2(r_i)}{\partial \theta_b} \right)$$

(7)

where $a = b = 1$, $c = 3$, $r_0 = 0.5$, $\rho_0 = 8$, $\beta = 0$

(8)

where $r_0$ and $\rho_0$ are measured in units of kpc and $10^7 M_{\odot}/$kpc$^3$ respectively.

The parameter $\theta_a$ can be measured with a precision $\sqrt{(F^{-1})_{aa}}$. By the chain rule, a quantity $q$ which depends upon the $\theta_a$ may be measured with a precision

$$\delta q = \sqrt{(F^{-1})_{ab} \frac{\partial q}{\partial \theta_a} \frac{\partial q}{\partial \theta_b}}$$

(9)

We will consider two quantities $q$. The first is $M(r_{1/2})$, the mass within the deprojected half-light radius $r_{1/2} = 375$ pc. Using the Jeans equation, this can be determined directly from the stellar density profile and radial velocity dispersion at $r = r_{1/2}$, their derivatives, and $\beta$ and so is quite robust to different choices of dark matter density profile [15].

The second is

$$\gamma(r) = -3 + 4\pi r^3 \frac{\rho(r)}{M(r)}$$

(10)

which is essentially a weighted average of the exponents of the $r$-dependence of the density at radii between 0 and $r$. This depends on the derivative of $M(r)$, which is determined less precisely than $M(r)$ itself, but unlike standard definitions it does not directly depend upon the second derivative of $M(r)$ and so is nonetheless fairly well constrained. Below we will report the fractional uncertainty in $M(r_{1/2})$ and the total uncertainty in $\gamma(r_{1/2})$.

These quantities will be determined in 6 cases. The first case just uses the line of sight velocities.

In the second case we impose the condition $c = 3$ by adding a large number to $F_{cc}$. This condition is satisfied by both cold and warm dark matter models, and in fact by any particulate dark matter model with no nongravitational long range interactions. Thus, while it is certainly less general, if one is interested in testing CDM then it suffices to impose $c = 3$ and test to see if $\gamma \sim -1$ as is suggested by dark matter simulations with the low baryonic content of the Sculptor dwarf.

In the next two cases we include proper motion measurements from Gaia’s 5 year mission, as has been described above, with and without the condition $c = 3$. In the last two cases we add a survey of Sculptor by the TMT in 2026. Dividing the precision of Gaia’s true position measurement (plus that of TMT), available in the Science Performance document [6], by the time interval between 2026 and 2016, the middle of the Gaia mission, one obtains the expected precision of proper motion measurements of Sculptor’s members which are observed by both Gaia and TMT. This precision is about a factor of four better than that of Gaia alone. Note that without Gaia, a single TMT survey of Sculptor would only determine the positions of these stars and not their proper motions. Such a precise proper motion measurement with TMT alone would take roughly five more years, although then TMT will also be able to measure the velocities of even fainter stars.

These results are all summarized in Table I. We have also tried including estimates of $r_0$ and $\rho_0$ from CDM simulations, as summarized in Ref. [14]. With these constraints we find that Gaia can determine $\gamma(r_{1/2})$ with a precision of 0.57.

In the standard ΛCDM paradigm one expects [11, 2] that a dark-matter dominated galaxy with the luminosity of Sculptor will have $\gamma$ equal to about -1 whereas other dark matter models, such as a Bose-Einstein condensate or giant monopole model, suggest $\gamma$ closer to zero. If $\gamma$ indeed is close to zero, then by assuming the CDM condition $c = 3$ and measuring $\gamma \sim 0$ one may hope to exclude CDM with $2\sigma$ of confidence when the Gaia mission is completed in 5 years and with $4\sigma$ of confidence when TMT begins observing in a decade. One should note however that the CDM simulations on which these conclusions rest consider spherically symmetric field galaxies, of which Sculptor is not an example.
\[ \frac{\delta M(r_{1/2})}{M(r_{1/2})} = \frac{\delta \gamma(r_{1/2})}{\gamma} \]

| LOS only | 69% | 4.1 |
| -------- | ---- | ---- |
| LOS only, \( c = 3 \) | 18% | 1.6 |
| Gaia     | 16% | 0.68 |
| Gaia, \( c = 3 \) | 10% | 0.66 |
| Gaia+TMT | 10% | 0.27 |
| Gaia+TMT, \( c = 3 \) | 7%  | 0.27 |

TABLE I: The precision of measurements of the mass within \( r_{1/2} = 375 \) pc and the dark matter density slope within \( r_{1/2} \) that can be expected with only LOS data and also at Gaia with and without TMT. Precisions are given using a single component King profile for the stellar distribution with and without imposing that \( c = 3 \). The linearized Fisher matrix approach cannot be trusted for entries of order or greater than unity.

In Ref. [16] it was observed that the Sculptor dwarf contains at least two ancient populations of stars, a metal rich population near the center and a more spatially extended metal poor population. The determination of the shape of Sculptor’s dark matter profile may be further improved by separately considering the dynamics of these subpopulations, each of which is in equilibrium. In Ref. [17] the authors found that the stellar density and metallicity distribution is well fit by the sum of two Plummer profiles of projected half-light radii 167 pc and 302 pc, having 53% and 47% of the members respectively. We have redone our analysis using this bimodal stellar profile and have found a notable improvement in the precision with which \( \gamma \) can be measured, as is summarized in Table II.

| LOS only | 30% | 1.3 |
| -------- | ---- | ---- |
| LOS only, \( c = 3 \) | 25% | 0.86 |
| Gaia     | 13% | 0.47 |
| Gaia, \( c = 3 \) | 13% | 0.44 |
| Gaia+TMT | 9%  | 0.22 |
| Gaia+TMT, \( c = 3 \) | 6%  | 0.21 |

TABLE II: As in Fig. 1 but using a 2-component Plummer model for the stellar density profile.

In Ref. [17] the exponent of the dark matter density was found by comparing the masses within two radii, but we have found that our quantity \( \gamma \) can be determined more precisely than such a ratio when proper motion data is included. However, it is important to note that our analysis does not use the fact that the stars in each Plummer profile are in fact separate populations, with distinct metallicities, each in equilibrium with itself. The bimodal structure of the stellar population is only used in the calculation of the line of sight integrals of the luminosities and dispersions. It would be interesting in the future to perform a true multi-population analysis.

If one trusts the equilibrium approximation even for fourth moments of the velocities, then these may be included without introducing any new free parameters following the strategy of Ref. [18]. Alternately, as this strategy introduces more observables without increasing the number of unknowns, the proper motion measurements may be used to test the consistency of the fourth order equilibrium analysis, which may in turn shed light on Sculptor’s formation.

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