Homogeneous and isotropic charge densities to accelerate the universe

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Abstract

Various theoretical obstacles are associated with a homogeneous and isotropic distribution of ‘charge’ which is subject to a repulsive, long-range force. We show how these can be overcome, for all practical purposes, by the simple device of endowing the particle which carries the force with a small mass. The resulting situation may be relevant to a phase of cosmological acceleration which is triggered by the approach to masslessness of such a force carrier.

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Recent observations of type Ia supernovae with high redshift indicate that the universe is entering a phase of cosmological acceleration [1, 2]. Identifying the causative agent is both a challenge and an opportunity for fundamental theory. It could be a cosmological constant, the need for which was suggested on the basis of other evidence even before the supernovae results [3–5]. Scalars will also work because one can construct a potential to support any homogeneous and isotropic geometry for which the Hubble constant does not increase\(^4\). Minimally coupled scalars becoming dominant at late times was also suggested before the supernovae results [7–10]. Since then such models have been dubbed, ‘quintessence’ [11] and have received extensive study [12–16]. Non-minimal couplings have also been explored [17] and recent inspiration has been derived from string theory [18, 19] and extra dimensions [20,21]. It has even been suggested that quantum effects may be responsible [22].

In the absence of compelling observational or theoretical support for any of the existing scenarios it is worth considering what else might be driving the late-time acceleration we seem to be seeing. Since different portions of an accelerating universe seem to be pushing one another apart an obvious alternative is that this may actually be the case. That is, suppose

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4 For the construction see section 2 of [6].
some constituent of the current energy density carries a charge—for example, baryon number—which couples to a repulsive force that only became long-range late in evolution.

Powerful objections seem to preclude the realization of this scenario within the context of a homogeneous and isotropic cosmology. If the charge density is homogeneous then the total charge must be the 3-volume times this density. On a closed 3-manifold the total charge of any infinite range force field must be zero, so the density would have to vanish. The general phenomenon is known as a linearization instability [23]. One can impose a non-zero, homogeneous charge density on an open manifold, but not without breaking isotropy through the selection of a direction for the lines of force. Furthermore, the charge density could only be instantaneously homogeneous because different regions would necessarily feel different force fields.

These objections can be made more concrete within the context of scalar electrodynamics, the Lagrangian for which is,

\[
\mathcal{L} = -\frac{1}{4} F_{\mu \nu} F^{\mu \nu} g^{-1/2} - \frac{1}{4} g^{\mu \nu} F_{\alpha \beta} F^{\alpha \beta} - V(\phi) g^{-1/2}.
\]  

(1)

where the covariant derivative is \(D_\mu \equiv \partial_\mu - ie A_\mu\). The Euler–Lagrange equations for the scalar and vector potential are,

\[
\frac{\delta S}{\delta \phi^*} = -D_\mu \left( g^{\mu \nu} D_\nu \phi \right) - \phi V'(\phi^* \phi) g^{-1/2} = 0,
\]  

(2)

\[
\frac{\delta S}{\delta A_\mu} = \partial_\nu \left( g^{\nu \rho} g^{\mu \sigma} F_{\rho \sigma} \right) + ie g^{\mu \nu} \left( \phi^* D_\nu \phi - \phi D_\nu \phi^* \right) = 0.
\]  

(3)

The two sources for this system are the current density,

\[
J_\mu \equiv ie \left( \phi D_\mu \phi^* - \phi^* D_\mu \phi \right),
\]  

(4)

and the stress–energy tensor,

\[
T_{\mu \nu} \equiv \frac{2}{g^{-1/2}} \frac{\delta S}{\delta g^{\mu \nu}} = -F_{\mu \rho} F_{\nu \sigma} g^{\rho \sigma} + \frac{1}{2} g_{\mu \nu} F_{\sigma \tau} F_{\sigma \tau} + 2(D_\rho \phi)^* D_\rho \phi - V(\phi^* \phi).
\]  

(5)

Assuming spatial flatness in addition to homogeneity and isotropy, the metric can be put in the form,

\[
g_{\mu \nu} \, d\mathbf{x}^\mu \, d\mathbf{x}^\nu = dt^2 - a^2(t) \mathbf{d}\mathbf{x} \cdot \mathbf{d}\mathbf{x}.
\]  

(6)

The scalar can be written, \(\phi(t) = f(t) e^{i\theta(t)}\), in terms of its magnitude \(f(t)\) and its phase \(\theta(t)\). The only non-zero vector potential can be \(A_0(t)\) and (3) can be solved for it uniquely to give,

\[
A_0(t) = \frac{ie}{2e^2 \phi^* \phi} \left( \phi \partial_0 \phi^* - \phi^* \partial_0 \phi \right) = \frac{\theta(t)}{e}.
\]  

(7)

This seems to be representing a self-interaction of non-zero charge density but one must bear in mind that the scalar current density (4) involves the vector potential. From the scalar’s covariant time derivative,

\[
D_0 \phi = \tilde{f}(t) e^{i\theta(t)},
\]  

(8)

we see that the actual charge density vanishes, while the stress tensor depends only upon the scalar magnitude \(f(t)\).

Of course the frustrating result we have just found merely confirms the general objections: the only charge density consistent with homogeneity and isotropy is zero for an infinite range
force. One cannot evade this fact mathematically, but it can be circumvented for practical purposes by the simple device of giving the force a finite range which can still be much larger than the Hubble radius. The problem really derives from Gauss’s law, and it can be understood in its simplest form on the flat, \((1 + 1)\)-dimensional manifold \(R^1 \times S^1\). The equation for the Coulomb–Green function is,

\[
-\frac{d^2}{dx^2} G(x, x') = \delta(x - x').
\]  

(9)

There is no solution with \(x = \pm L\) identified, so one cannot have a non-zero total charge on \(S^1\) or, it turns out, on any closed spatial manifold.

The usual procedure is to subtract the zero mode and solve for the restricted Green’s function appropriate to a charge density with zero total charge. The relevant equation is,

\[
-\frac{d^2}{dx^2} G_r(x, x') = \delta(x - x') - \frac{1}{2L}.
\]  

(10)

and, up to a constant, the solution is,

\[
G_r(x, x') = \frac{(L + x - x')^2}{4L} \theta(x' - x) + \frac{(L - x + x')^2}{4L} \theta(x - x').
\]  

(11)

What we are advocating instead is to add a small mass to attain the equation,

\[
\left\{-\frac{d^2}{dx^2} + m^2\right\} G_m(x, x') = \delta(x - x').
\]  

(12)

The unique periodic solution is,

\[
G_m(x, x') = \frac{\cosh[m(L + x - x')]}{2m \sinh(mL)} \theta(x' - x) + \frac{\cosh[m(L - x + x')]}{2m \sinh(mL)} \theta(x - x').
\]  

(13)

Note that the mass can be very much smaller than \(1/L\); as long as \(m \neq 0\) the equation can be solved. Furthermore, the solution makes physical sense for small \(m\). Expanding \(G_m(x, x')\) for small \(m\) gives a constant term which diverges like \(1/m^2\), followed by \(G_r(x, x')\) and terms which vanish with \(m\). It is this first term which is relevant for a homogeneous and isotropic cosmology.

The preceding discussion suggests that we wish to give the vector a mass. This entails breaking gauge invariance but one can at least preserve general coordinate invariance with the Lagrangian:

\[
\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{1}{2} m^2 A_{\mu} A^{\mu} + (D_{\mu} \phi^*) (D^{\mu} \phi) - V(\phi^* \phi),
\]  

(14)

Within the context of homogeneity and isotropy the unique solution for the vector potential is,

\[
A_0(t) = \frac{ie}{m^2 + 2e^2 \phi^* \phi} \frac{(\phi \partial_0 \phi^* - \phi^* \partial_0 \phi)}{m^2 + 2e^2 \phi^* \phi} = \frac{2e f^2(t)}{m^2 + 2e^2 f^2(t)} \dot{\theta}(t) + \frac{2e f^2(t)}{m^2 + 2e^2 f^2(t)} \dot{\theta}(t).
\]  

(15)

The scalar’s covariant time derivative becomes,

\[
e^{-i\theta} D_\phi \phi = \dot{f} \frac{im^2 f \dot{\theta}}{m^2 + 2e^2 f^2 \dot{\theta}}.
\]  

(16)

\footnote{In a model with two scalars one could preserve gauge invariance by generating the photon mass through spontaneous symmetry breaking.}
and the charge density (4) is,

$$J_0(t) = -m^2 A_0(t) = \frac{-2em^2 f^2(t) \dot{\theta}(t)}{m^2 + 2e^2 f^2(t)}.$$  \hfill (17)

Despite the breaking of gauge invariance, the scalar equations of motion still imply that this charge density is conserved,

$$\frac{d}{dt}(a^3(t) J_0(t)) = 0.$$  \hfill (18)

If $n_0$ is the current number density of charges (when $a(t_0) = 1$) then we can isolate the time dependence as follows,

$$J_0(t) = \frac{en_0}{a^3(t)}.$$  \hfill (19)

Since general coordinate invariance was maintained the stress tensor is still conserved. In considering how things change with $m$ and $e$ it is useful to express the phase using relations (17) and (19),

$$\ddot{\theta}(t) = -\left(\frac{1}{2f^2(t)} + \frac{e^2}{m^2}\right) \frac{n_0}{a^3(t)}.$$  \hfill (20)

In these variables the photon and scalar kinetic terms are,

$$\frac{1}{2}m^2 A^2_0 + (D_0 \phi)^* D_0 \phi = f^2 + \frac{m^2 f^2 \dot{\theta}^2}{m^2 + 2e^2 f^2} = f^2 + \left(\frac{1}{4f^2} + \frac{e^2}{2m^2}\right) \left(\frac{n_0}{a^3}\right)^2.$$  \hfill (21)

It follows that the energy density and pressure are,

$$\rho = f^2 + \left(\frac{1}{4f^2} + \frac{e^2}{2m^2}\right) \left(\frac{n_0}{a^3}\right)^2 + V(f^2),$$  \hfill (22)

$$p = f^2 + \left(\frac{1}{4f^2} + \frac{e^2}{2m^2}\right) \left(\frac{n_0}{a^3}\right)^2 - V(f^2).$$  \hfill (23)

The extra term due to the vector interaction is the one proportional to $e^2/2m^2$. Note that it makes physical sense. Turning on the interaction, with fixed charge density per unit charge raises the energy, as one expects for a repulsive interaction. Similarly, the energy density diverges like $1/m^2$ for small $m$, just as the massive Coulomb–Green function (13) does.

The obvious phenomenological application for this trick is to give a model in which some constituent of the current energy density contains a uniformly distributed charge coupled to a force field that has recently been driven nearly massless. In this case $m^2$ would be the norm of some other complex scalar, call it $\psi$, whose phase is negligible. Suppose that the minimum of the total potential is at $\psi = 0$. Then as $\psi$ tries to roll down to this minimum the interaction gives rise to a peculiar sort of electromagnetic barrier which pushes $\psi$ back up its potential. By reducing the kinetic energy (which obeys $p_K = \rho_K$) and enhancing the potential energy (which obeys $p_P = -\rho_P$) this must favour cosmological acceleration.

For certain models the effect can be enough to make the time average deceleration parameter negative and, in fact, close to $-1$. What actually happens is that the field oscillates about the minimum of the effective potential obtained from adding the electromagnetic barrier (which is time dependent through the factor of $1/a^6$) to the original potential. Since the electromagnetic barrier is very steep and Hubble friction is low there, the field recoils sharply off and spends most of its time at high potential where Hubble friction is greatest. Therefore,
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The time average of the deceleration parameter is dominated by the period spent at high potential with low kinetic and electromagnetic contributions to the energy density. Detailed numerical simulations have been done and will be presented in a subsequent paper [24].

It is also interesting to note that the effective range of the force changes instantaneously with the mass. The virtual vector quanta which carry the repulsive interaction from one patch of charge density to another lose or acquire mass in route. This has the curious consequence that, even though the electromagnetic barrier forms due to an interaction becoming long range, one does not have to wait for distant regions to come into causal contact with one another after the range changes.

Note added. After the completion and release of this work we learned of a paper treating repulsive interactions in general terms from the context of phenomenological models [25]. It seems to us that our technique may provide a class of Lagrangian field theories which explicitly realize these ideas. We also became aware of a somewhat related class of models which exploit the kinetic energy in the phase of a complex scalar field [26, 27]. The energy and pressure in these theories agree with (22) and (23) for $\epsilon = 0$. Our models include a long range repulsive potential, in addition to the kinetic energy of the phase. This allows a wider range of possibilities in which the mass of the force carrier is generated by another complex scalar, or by some dynamical mechanism. It may also be relevant to the tendency for these models to decay into Q-balls [28]. Note that the repulsive potential must survive, and must continue to push the universe apart, even if the charge has been bundled into Q-balls.

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