Sample-Efficient Model-Free Reinforcement Learning with Off-Policy Critics

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Abstract

Value-based reinforcement-learning algorithms are currently state-of-the-art in model-free discrete-action settings, and tend to outperform actor-critic algorithms. We argue that actor-critic algorithms are currently limited by their need for an on-policy critic, which severely constraints how the critic is learned. We propose Bootstrapped Dual Policy Iteration (BDPI), a novel model-free actor-critic reinforcement-learning algorithm for continuous states and discrete actions, with off-policy critics. Off-policy critics are compatible with experience replay, ensuring high sample-efficiency, without the need for off-policy corrections. The actor, by slowly imitating the average greedy policy of the critics, leads to high-quality and state-specific exploration, which we show approximates Thompson sampling. Because the actor and critics are fully decoupled, BDPI is remarkably stable and, contrary to other state-of-the-art algorithms, unusually forgiving for poorly-configured hyper-parameters. BDPI is significantly more sample-efficient compared to Bootstrapped DQN, PPO, A3C and ACKTR, on a variety of tasks. Source code: https://github.com/vub-ai-lab/bdpi.

1. Introduction and Related Work

State-of-the-art stochastic actor-critic algorithms, used with discrete actions, all share a common trait: the critic $Q^*$ they learn directly evaluates the actor (Konda & Borkar, 1999; Schulman et al., 2017; Wu et al., 2017; Mnih et al., 2016). Some algorithms allow the agent to execute a policy different from the actor, which the authors refer to as off-policy, but the critic is still on-policy with regards to the actor (Haarnoja et al., 2018, for instance). ACER (Wang et al., 2016) uses off-policy corrections (Munos et al., 2016) to learn $Q^*$ from past experiences, DDPG learns its critic with an on-policy SARSA-like algorithm (Lillicrap et al., 2015), Q-prop (Gu et al., 2017a) uses the actor in the critic learning rule to make it on-policy, and PGQL (O’Donoghue et al., 2017) allows for an off-policy V function, but requires it to be combined with on-policy advantage values. Notable examples of algorithms without an on-policy critic are AlphaGo Zero (Silver et al., 2017), that replaces the critic with a slow-moving target policy learned with tree search, and the Actor-Mimic (Parisotto et al., 2016), that minimizes the cross-entropy between an actor and the Softmax policies of critics (see Section 4.2). The need of most actor-critic algorithms for an on-policy critic makes them incompatible with state-of-the-art value-based algorithms of the Q-Learning family (Arjona-Medina et al., 2018; Hessel et al., 2017), that are all highly sample-efficient but off-policy. In a discrete-actions setting, where off-policy value-based methods can be used, this raises two questions:

1. Can we use off-policy value-based algorithms in an actor-critic setting?
2. Would the actor bring anything positive to the agent?

In this paper, we provide a positive answer to these two questions. We introduce Bootstrapped Dual Policy Iteration (BDPI), a novel actor-critic algorithm. Our actor learning rule, inspired by Conservative Policy Iteration (see Sections 2.4 and 3.2), is robust to off-policy critics. Because we lift the requirement for on-policy critics, the full range of value-based methods can now be leveraged by the critic, such as DQN-family algorithms (Hessel et al., 2017), or exploration-focused approaches (Arjona-Medina et al., 2018; Burda et al., 2018). To better isolate the sample-efficiency and exploration properties arising from our actor-critic approach, we use in this paper a simple DQN-family critic. We learn several Q-Functions, as suggested by Osband et al. (2016), with a novel extension of Q-Learning (see Section 3.1). Unlike other approaches, that use the critics to compute means and variances (Nikolov et al., 2019; Chen et al., 2017), BDPI uses the information in each individual critic to train the actor. We show that our actor learning rule, combined with several off-policy critics, can be compared to bootstrapped Thompson sampling.
Our experimental results in Section 4 show that BDPI significantly outperforms state-of-the-art actor-critic and critic-only algorithms, such as PPO, ACKTR, ACER, A3C and Bootstrapped DQN, on a representative set of medium-complexity discrete-action tasks. Our ablative study shows that BDPI’s actor significantly contributes to its performance and exploration. To the best of our knowledge, this is the first time that, in a discrete-action setting, the benefit of having an actor can be clearly identified. Finally, and perhaps most importantly, BDPI is highly robust to its hyper-parameters, which mitigates the need for endless tuning (see Section 4.5). BDPI’s ease of configuration and sample-efficiency are crucial in many real-world settings, where compute power is not the bottleneck, but data collection is.

2. Background

In this section, we introduce and review the various formalisms on which Bootstrapped Dual Policy Iteration builds. We also discuss the shortcomings of current actor-critic algorithms (Section 2.3), and contrast them to Conservative Policy Iteration and Dual Policy Iteration (Scherrer, 2014; Sun et al., 2018).

2.1. Markov Decision Processes

A discrete-time Markov Decision Process (MDP) (Bellman, 1957) with discrete actions is defined by the tuple \((S, A, R, T, \gamma)\): a possibly-infinite set \(S\) of states; a finite set \(A\) of actions; a reward function \(R(s_t, a_t, s_{t+1}) \in \mathbb{R}\) returning a scalar reward for each state transition; a transition function \(T(s_{t+1}|s_t, a_t) \in [0, 1]\) determining the dynamics of the environment; and the discount factor \(0 \leq \gamma < 1\) defining the importance given by the agent to future rewards.

A stochastic stationary policy \(\pi(a_t|s_t) \in [0, 1]\) maps each state to a probability distribution over actions. At each time-step, the agent observes \(s_t\), selects \(a_t \sim \pi(s_t)\), then observes \(r_{t+1}\) and \(s_{t+1}\), which produces an \((s_t, a_t, r_{t+1}, s_{t+1})\) experience tuple. An optimal policy \(\pi^*\) maximizes the expected cumulative discounted reward \(E_{\pi^*}[\sum_{t} \gamma^t r_t]\). The goal of the agent is to find \(\pi^*\) based on its experience within the environment, with no \textit{a-priori} knowledge of \(R\) and \(T\).

2.2. Q-Learning, Experience Replay and Clipped DQN

Value-based reinforcement learning algorithms, such as Q-Learning (Watkins & Dayan, 1992), use experience tuples and Equation 1 to learn an action-value function \(Q^*\), also called a \textit{critic}, which estimates the expected return for each action in each state when the optimal policy is followed:

\[
Q_{k+1}(s_t, a_t) = Q_k(s_t, a_t) + \alpha \delta_{k+1}
\]

\[
\delta_{k+1} = r_{t+1} + \gamma \max_{a'} Q_k(s_{t+1}, a') - Q_k(s_t, a_t)
\]

with \(0 < \alpha < 1\) a learning rate. At acting time, the agent selects actions having the largest Q-Value, plus some exploration. To improve sample-efficiency, experience tuples are stored in an \textit{experience buffer}, and are periodically re-sampled for further training using Equation 1 (Lin, 1992).

Before convergence, Q-Learning tends to over-estimate the Q-Values (van Hasselt, 2010), as positive errors are propagated by the \(\max\) operator of Equation 1. Clipped DQN (Fujimoto et al., 2018), that we use as the basis of our critic learning rule (Section 3.1), addresses this bias by applying the \(\max\) operator to the minimum of the predictions of two independent Q-functions, such that positive errors are removed by the minimum operation. Addressing this over-estimation has been shown to increase sample-efficiency and robustness (van Hasselt, 2010).

2.3. Policy Gradient and Actor-Critic Algorithms

Instead of choosing actions according to Q-Values, Policy Gradient methods (Williams, 1992; Sutton et al., 2000) explicitly learn an \textit{actor} \(\pi_\theta(a_t|s_t) \in [0, 1]\), parametrized by a weights vector \(\theta\), such as the weights of a neural network. The objective of the agent is to maximize the expected cumulative discounted reward \(E_\pi[\sum_{t} \gamma^t r_t]\), which translates to the minimization of Equation 2 (Sutton et al., 2000):

\[
L(\pi_\theta) = -\sum_{t=0}^{T} R_t \log(\pi_\theta(a_t|s_t))
\]

\[
L(\pi_\theta) = -\sum_{t=0}^{T} Q^{\pi_\theta}(s_t, a_t) \log(\pi_\theta(a_t|s_t))
\]

with \(a_t \sim \pi_\theta(s_t)\) the action executed at time \(t\), and \(R_t = \sum_{\tau=t}^{T} \gamma^\tau r_\tau\) the Monte-Carlo return from time \(t\) onwards. At every training epoch, experiences are used to compute the gradient \(\frac{\partial L}{\partial \theta}\) of Equation 2, then the weights of the policy are adjusted by a small step in the opposite direction of the gradient. A second gradient update requires fresh experiences (Sutton et al., 2000), which makes Policy Gradient quite sample-inefficient. Three approaches have been proposed to increase the sample-efficiency of Policy Gradient: trust regions, that allow larger gradient steps to be taken (Schulman et al., 2015), surrogate losses, that prevent divergence if several gradient steps are taken (Schulman et al., 2017), and stochastic actor-critic methods (Barto et al., 1983; Konda & Borkar, 1999), that replace

\textsuperscript{1}Deterministic actor-critic methods are slightly different and outside the scope of this paper.
the Monte-Carlo $R_t$ with an estimation of its expectation, $Q^\pi(s_t, a_t)$, an on-policy critic, shown in Equation 3.

The use of $Q^\pi$-Values instead of Monte-Carlo returns leads to a gradient of lower variance, and allows actor-critic methods to obtain impressive results on several challenging tasks (Wang et al., 2016; Gruslys et al., 2017; Mnih et al., 2016). However, conventional actor-critic algorithms may not provide any benefits over a cleverly-designed critic-only algorithm, see for example O’Donoghue et al. (2017), Section 3.3. Actor-critic algorithms also rely on $Q^\pi$ to be accurate for the current actor, even if the actor itself can be distinct from the actual behavior policy of the agent (Degris et al., 2012; Wang et al., 2016; Gu et al., 2017b). Failing to maintain a healthy relationship between the actor and the critic may cause divergence (Konda & Borkar, 1999; Sutton et al., 2000).

2.4. Conservative and Dual Policy Iteration

Approximate Policy Iteration and Dual Policy Iteration are two approaches to Policy Iteration. API repeatedly evaluates a policy $\pi_k$, producing an on-policy $Q^\pi_k$, then trains $\pi_{k+1}$ to be as close as possible to the greedy policy $\Gamma(Q^\pi_k)$ (Kakade & Langford, 2002; Scherrer, 2014). Conservative Policy Iteration (CPI) extends API to slowly move $\pi$ towards the greedy policy (Pirotta et al., 2013). Dual Policy Iteration (Sun et al., 2018) formalizes as CPI several modern reinforcement learning approaches (Anthony et al., 2017; Silver et al., 2017), by replacing the greedy function with a slow-moving target policy $\pi'$:

$$\Gamma(Q^\pi_k) \quad \text{(API)}$$

$$\pi_{k+1} \leftarrow (1 - \alpha)\pi_k + \alpha\Gamma(Q^\pi_k) \quad \text{(CPI)}$$

$$\pi_{k+1} \leftarrow (1 - \alpha)\pi_k + \alpha\pi'_k \quad \text{(DPI)}$$

with $0 < \alpha \leq 1$ a learning rate, set to a small value in Conservative Policy Iteration algorithms (0.01 in our experiments). Among CPI algorithms, Safe Policy Iteration (Pirotta et al., 2013) dynamically adjusts the learning rate to ensure (with high probability) a monotonic improvement of the policy, while Thomas et al. (2015) propose the use of statistical tests to decide whether to update the policy.

While theoretically promising, CPI algorithms present two important limitations: their convergence is difficult to obtain with function approximation (Wagner, 2011; Böhmer et al., 2016); and their update rule and associated set of bounds and proofs depend on $Q^\pi_k$, an on-policy function that would need to be re-computed before every iteration in an on-line setting. As such, CPI algorithms are notoriously difficult to implement, with Pirotta et al. (2013) reporting some of the first empirical results on CPI. Our main contribution, presented in the next section, is inspired by CPI but distinct from it in several key aspects. Our actor learning rule follows the Dual Policy Iteration formalism, with a target policy $\pi'$ built from off-policy critics (see Section 3.2). The fact that the actor gathers the experiences on which the critics are trained can be compared to the guidance that $\pi$ gives to $\pi'$ in the DPI formalism (Sun et al., 2018).

2.5. Uncertainty-based Exploration

Measuring the uncertainty of an agent has the potential to lead to high-quality exploration (Strens, 2000). Two approaches to uncertainty estimation exist. The first one fundamentally changes the learning algorithm to produce uncertainty measures, and is as such difficult to combine with existing algorithms. Examples of it include Distributional Reinforcement Learning (Bellemare et al., 2017), that models the distribution of Q-Values, Bayes-by-Backprop (Blundell et al., 2015), that allows a neural network to express a distribution over outputs, or the addition of stochastic noise in the parameters of a neural network (Plappert et al., 2017).

The second approach to uncertainty estimation consists of using several actors or critics, leading to a bootstrap distribution whose statistics give sound measures of uncertainty (Efron & Tibshirani, 1994). This approach does not need the learning algorithm to be fundamentally changed, which makes it more suited to our BDPI algorithm, that aims at generality and simplicity. It is also used by A3C and modern implementations of PPO (Mnih et al., 2016; Schulman et al., 2017), that deploy several actors in replicas of the environment, and Bootstrapped DQN (Osband et al., 2016), that uses only one environment and bootstraps critics instead of actors.

3. Bootstrapped Dual Policy Iteration

Our main contribution, Bootstrapped Dual Policy Iteration (BDPI), consists of two original components. In Section 3.1, we introduce an aggressive off-policy critic, inspired by Bootstrapped DQN and Clipped DQN (Osband et al., 2016; Fujimoto et al., 2018). In Sections 3.2 to 3.3, we introduce an actor that leads to high-quality exploration, further enhancing sample-efficiency. We detail BDPI’s exploration properties in Section 3.4, before empirically validating our results in a diverse set of environments (Section 4). The complete pseudocode of the algorithm is available in Appendix A, and our implementation of BDPI is part of the supplementary material.

3.1. Aggressive Bootstrapped Clipped DQN

We begin our description of BDPI with the algorithm used to train its critics, Aggressive Bootstrapped Clipped DQN (ABCDQN). Like Double Q-Learning (van Hasselt, 2010), ABCDQN maintains two Q-functions per critic,
Every training iteration, \( Q^A \) and \( Q^B \) are swapped, then \( Q^A \) is trained with Q-Learning on a set of experiences sampled from an experience buffer, using targets from \( Q^B \). Inspired by Double Q-Learning, Clipped DQN (Fujimoto et al., 2018), an on-policy algorithm, uses \( V(s_{t+1}) = \min_{i=A,B} Q_i^A(s_{t+1}, \pi(s_{t+1})) \) as target value of the Q-Learning equation. The minimum prevents prediction errors from propagating through the max operator. Unlike Clipped DQN, we design our critic to be off-policy. As such, we remove the reference to \( \pi(s_{t+1}) \) and propose a new target value instead:

\[
V(s_{t+1}) \equiv \min_{i=A,B} \min_{\alpha} Q_i^A(s_{t+1}, \alpha) \tag{4}
\]

Inspired by Bootstrapped DQN (Osband et al., 2016), we maintain a set of \( N_c > 1 \) critics, each having its own \( Q^A \) and \( Q^B \) functions. Unlike Bootstrapped DQN, all our critics share the same experience buffer, and learn slightly different Q-Values thanks to each critic seeing their own batches of experiences. Every training epoch, a critic is chosen randomly and \( N_c > 1 \) training iterations of our modified Clipped DQN algorithm are applied to it. To summarize, Aggressive Bootstrapped Clipped DQN, the critic of BDPI, maintains \( N_c \) critics, each one having two Q-Functions, learned with our modified Clipped DQN algorithm. ABCDQN performs \( N_c \) training epochs per timestep, so that all the critics are updated as often as possible, and \( N_c > 1 \) training iterations per training epoch.

3.2. Training the Actor with Off-Policy Critics

To improve exploration, and further increase sample-efficiency, we now complement our ABCDQN critic with the second component of BDPI, its actor. The actor \( \pi \) is trained using a variant of Conservative Policy Iteration (Pirotta et al., 2013), tailored to off-policy critics. Every training epoch, after one of the critics \( i \) has been updated on a uniformly-sampled subset \( E \subset B \) of experiences, the same subset is used to update the actor towards the greedy policy of that \( i \)-th critic:

\[
\pi_{k+1}(s) = (1 - \lambda)\pi_k(s) + \lambda \Gamma(Q^A_{k+1}(s)) \tag{5}
\]

with \( \lambda = 1 - e^{-\delta} \) the actor learning rate, computed from the maximum allowed KL-divergence \( \delta \) defining a trust-region (see Appendix B), and \( \Gamma \) the greedy function, that returns a policy greedy in \( Q^A_{k+1} \), the \( Q^A \) function of the \( i \)-th critic. Pseudocode for the complete BDPI algorithm is given in Appendix A.

Contrary to Conservative Policy Iteration algorithms, and because our critics are off-policy, the greedy function is applied on a randomly-sampled estimate of \( Q^* \), the optimal Q-function, instead of \( Q^\pi \). The use of an actor, that slowly imitates approximations of \( \Gamma(Q^*) \equiv \pi^* \), leads to an interesting relation between BDPI and Thompson sampling (see Section 3.4). While expressed in the tabular form in Equations 4 and 5, the BDPI update rules produce Q-Values and probability distributions that can directly be used to train any kind of function approximator, on the mean-squared-error loss, and for as many gradient steps as desired.

3.3. Convergence Compared Conservative Policy Iteration

The standard Conservative Policy Iteration update rule (see Section 2.4) updates the actor \( \pi \) towards \( \Gamma(Q^\pi) \), the greedy function according to the Q-Values arising from \( \pi \). This slow-moving update, and the inter-dependence between \( \pi \) and \( Q^\pi \), allows several properties to be proven (Kakade & Langford, 2002), and the optimal policy learning rate \( \alpha \) to be determined from \( Q^\pi \) (Pirotta et al., 2013). Because BDPI learns off-policy critics, that can be arbitrarily different from the on-policy \( Q^\pi \) function, the Approximate Safe Policy Iteration framework of Pirotta et al. (2013) would infer an “optimal” learning rate of 0. Fortunately, a non-zero learning rate still allows BDPI to learn efficiently. In Section 3.4, we show that the off-policy nature of BDPI’s critics makes it approximate Thompson sampling, which CPI’s on-policy critics do not do. This theoretical result is validated in Section 4.

3.4. BDPI and Thompson Sampling

In a bandit setting, Thompson sampling (Thompson, 1933) is regarded as one of the best ways to balance exploration and exploitation (Agrawal & Goyal, 2012; Chapelle & Li, 2011). Thompson sampling consists of maintaining a posterior belief of how likely any given action is optimal, and drawing actions directly from this probability distribution. In a reinforcement-learning setting, Thompson sampling consists of selecting an action \( a \) according to:

\[
\pi(a|s) = P_{\text{belief}}(a = \arg\max_{a'} Q^*(s,a')) \tag{6}
\]

with \( Q^* \) the optimal Q-function. BDPI learns off-policy critics, that produce estimates of \( Q^* \). Sampling a critic and updating the actor towards its greedy policy is therefore equivalent to sampling \( Q \sim P(Q = Q^*) \) (Osband et al., 2016), then updating the actor towards \( \Gamma(Q) \), with \( \Gamma(Q)(s,a) = \mathbb{I}[a = \arg\max_{a'} Q(s,a')] \) and \( \mathbb{I} \) the indicator function. Over several updates, and thanks to a small \( \lambda \) learning rate (see Equation 5), the actor learns the expected greedy function of its critic, which (intuitively) folds the
indicator function into the sampling of $Q$, producing the Thompson sampling equation (6):

$$
\pi(a|s) = E_{Q \sim P(Q=Q^*)}[\mathbb{1}(a = \text{argmax}_{a'} Q(s, a'))] \\
= \int_Q \mathbb{1}(a = \text{argmax}_{a'} Q(s, a')) P(Q = Q^*) dQ \\
= \int_Q P(a = \text{argmax}_{a'} Q(s, a'), Q = Q^*) dQ \\
= \int_Q P(a = \text{argmax}_{a'} Q(s, a')|Q = Q^*) \rightarrow P(Q = Q^*) dQ \\
= P(a = \text{argmax}_a, Q^*(s, a'))
$$

The use of an explicit actor, instead of directly sampling critics and executing actions as Bootstrapped DQN does (Osband et al., 2016), positively impacts BDPI’s performance (see Section 4). Nikolov et al. (2019) discuss why Bootstrapped DQN, without an actor, leads to a higher regret than their Information Directed Sampling, and propose to add a Distributional RL (Bellemare et al., 2017) component to their agent. Osband et al. (2018) present arguments against the use of Distributional RL, and instead combines Bootstrapped DQN with prior functions. In the next section, we show that BDPI largely outperforms Bootstrapped DQN, along with PPO and ACKTR, without relying on Distributional RL nor prior functions. We believe that having an explicit actor changes the way the posterior is computed, which may positively influence exploration compared to actor-less approaches.

4. Experiments

To illustrate the properties of BDPI, we compare it to its ablations and a wide range of reinforcement learning algorithms, in three environments with completely different state-spaces and dynamics. Our results demonstrate the high sample-efficiency and exploration quality of BDPI. Moreover, these results are obtained with the same configuration of critics, experience replay and learning rates across environments, which illustrates the ease of configuration of BDPI. In Section 4.5, we carry out further experiments, that demonstrate that BDPI is more robust to its hyper-parameters than other algorithms. This is key to the application of reinforcement learning to real-world settings, where vast hyper-parameter tuning is often infeasible.

4.1. Algorithms

We evaluate the algorithms listed below. We also evaluated ACER and A3C (Wang et al., 2016; Mnih et al., 2016), conventional actor-critic algorithms available in the OpenAI baselines, but their sample-efficiency was too low for inclusion in our plots.

| Algorithm            | Reference                  |
|----------------------|----------------------------|
| BDPI                 | this paper                 |
| ABCDQN               | this paper                 |
| BDPI w/ AM           | this paper                 |
| BDQN, Bootstrapped DQN | Osband et al. (2016)       |
| PPO                  | Schulman et al. (2017)     |
| ACKTR                | Wu et al. (2017)           |

All algorithms use feed-forward neural networks to represent their actor and critic, with one (2 for PPO and ACKTR) hidden layers of 32 neurons (256 on the LunarLander environment). The state is one-hot encoded in the FrozenLake environment, and directly fed to the network in the other (continuous) environments. The neural networks are trained with the Adam optimizer (Kingma & Ba, 2014), using a learning rate of 0.0001 (0.001 for PPO, ACKTR uses its own optimizer with a varying learning rate). Several extensively-tuned implementations of PPO and ACKTR have been evaluated, to ensure the fairest comparison (parameters in Appendix E, we used implementations from pytorch-a2c-ppo-acktr on Github). BDPI required far less tuning than the other algorithms, as discussed later. Unless specified otherwise, BDPI uses $N_c = 16$ critics, all updated every time-step on a different 256-experiences batch, sampled from the same shared experience buffer, for 4 applications of our ABCDQN update rule. BDPI trains its neural networks for 20 epochs per training iteration.

4.2. BDPI with the Actor-Mimic loss

To the best of our knowledge, the Actor-Mimic (Parisotto et al., 2016) is the only actor-critic algorithm, along with BDPI, that learns critics that are off-policy with regards to the actor. We therefore compare BDPI to the Actor-Mimic in Section 4.4. These two algorithms perform extremely well, which demonstrates the potential of off-policy critics, with BDPI being more robust than the Actor-Mimic.

$$
\mathcal{L}(\pi_\theta) = - \sum_{s \in S, a \in A, i < N} S(Q_i)(a|s) \log(\pi_\theta(a|s))
$$

(7)

The Actor-Mimic is designed for transfer learning tasks. One critic per task is trained, using the off-policy DQN algorithm (Mnih et al., 2015). Then, the cross-entropy between the actor and the Softmax policies $S(Q_i)$ of all the critics is minimized, using the (simplified) loss of Equation 7. Applying the Actor-Mimic to a single-task setting is possible. We implemented an agent based on BDPI, that retains its ABCDQN critics, but replaces our actor learning rule of Equation 5 with the Actor-Mimic loss of Equation 7. Because we only change how the actor is trained, and still use our aggressive critics, we ensure the fairest comparison between our actor learning rule and the cross-entropy loss of the Actor-Mimic. In our experiments, the Actor-Mimic loss with Softmax policies fails to learn efficiently, even after extensive hyper-parameter tuning, probably because the
Softmax prevents the policies from becoming deterministic in states where this is necessary. We therefore replaced the Softmax with the greedy function, which led to the much better results that we present in Section 4.4. The high performance of the Actor-Mimic, while still less stable than BDPI, demonstrates that our use of the greedy function, and our actor learning rule, are sound.

4.3. Environments

Our evaluation of BDPI takes place in three environments that challenge the algorithms on different aspects of reinforcement learning: exploration with sparse rewards (Table), high-dimensional and not pixel-based state-space (LunarLander), and high stochasticity (FrozenLake).

Table simulates a tiny robot on a large table that has to locate its charging station and dock (see Figure 1a). Because the robot moves slowly on the table, and obtains rewards only every few hundred time-steps, Table is more difficult to explore than many Gym tasks (Brockman et al., 2016), several Atari games included. The table itself is a 1-by-1 square. The goal is located at (0.5, 0.5), and the robot always starts at (0.1, 0.1), facing away from the goal. A fixed initial position makes exploration more challenging, as the robot never spawns close to the goal. The robot observes its current \((x, y, \theta)\) position and orientation, with \(\theta \in [-\pi, \pi]\). Three actions allow the robot to either move forward 0.005 units, or turn left/right 0.1 radians. A reward of 0 is given every time-step. The episode finishes with a reward of -50 if the robot falls off the table, 0 after 200 time-steps, and 100 when the robot successfully docks, that is, its location is \((0.5 \pm 0.05, 0.5 \pm 0.05, \frac{\pi}{4} \pm 0.3)\).

LunarLander is a high-dimensional continuous-state physics-based simulation of a rocket landing on the moon (see Figure 1b). The agent observes the location and velocities of various components of the lander, and has access to four actions: doing nothing, firing the left/right side engines for one time-step, and firing the main engine. The reward signal for this task is quite complicated but informative, as it directly maps the distance between the rocket and the landing pad to a reward, on every time-step. The environment is considered solved when a cumulative reward of 200 or more is achieved per episode (Brockman et al., 2016).

FrozenLake is a \(8 \times 8\) grid composed of slippery cells, holes, and one goal cell (see Figure 1c). The agent can move in four directions (up, down, left or right), with a probability of \(\frac{2}{5}\) of actually performing an action other than intended. The agent starts at the top-left corner of the environment, and has to reach the goal at its bottom-right corner. The episode terminates when the agent reaches the goal, resulting in a reward of +1, or falls into a hole, resulting in no reward.

4.4. Results

Figure 2 shows the cumulative reward per episode obtained by various agents in our three environments. These results are averaged across 8 runs per agent, with the shaded regions representing the standard error. The plots compare BDPI to the algorithms detailed in Section 4.1, and display the effect of varying key hyper-parameters of BDPI.

Algorithms BDPI is the most sample-efficient of all the algorithms. BDPI with the Actor-Mimic matches BDPI with our actor learning rule on Table, but fails to learn LunarLander. ABCDQN (BDPI without its actor) does not match BDPI, and fails on Table, an environment where exploration is key. These results show that both having an explicit actor, and training it with our update rule of Section 3.2, are necessary components of BDPI. Bootstrapped DQN is highly sample-efficient on FrozenLake, but does not explore well enough on Table and LunarLander. PPO and ACKTR, after extensive tuning and with several implementations tested, are not as sample-efficient as BDPI and Bootstrapped DQN, two off-policy algorithms using experience replay. Even with per-environment hyper-parameters, PPO and ACKTR need about 5K episodes to learn FrozenLake, and 1K episodes on Table. BDPI is the only algorithm that, with a single configuration for all the environments, automatically adjusts to the complexity of a task to achieve maximum sample-efficiency.

Critics Increasing the number of critics leads to smoother learning curves in every environment, at the cost of sample-efficiency in Table, where a higher variance in the bootstrap distribution of critics seems to help with exploration. Having only one critic seriously degrades BDPI’s performance, and having less than 16 critics is detrimental on LunarLan-
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Figure 2. Results on Table (left), LunarLander (center) and FrozenLake (right). Top: BDPI (16 critics, updated for 4 iterations per time-step) outperforms ABCDQN (BDPI without an actor), Boostrapped DQN, PPO, ACKTR, and BDPI with the Actor-Mimic loss. Middle and Bottom: Varying the number of critics and how often they are trained, major hyper-parameters of BDPI, only has minimal impact on its performance, which demonstrates BDPI’s robustness. These results still provide valuable insights about BDPI, detailed in the text.

Aggressiveness At each training epoch, every critic samples a different batch of 256 experiences, then applies its learning rule (Equation 4) \( N_t \) times based on these experiences. Changing \( N_t \) only has a limited effect on the performance of the agent. On Table, where the reward is extremely sparse, a larger \( N_t \) positively impacts sample-efficiency, as it allows values to flow between states more rapidly. On LunarLander, where the reward is much denser, \( N_t = 1 \) performs slightly better than the alternatives, as it reduces potential overfitting.

Off-Policy noise BDPI’s actor learning equations do not refer to any behavior policy or on-policy return. We show that this makes BDPI an off-policy algorithm. In this experiment, training episodes have, at each time-step, a probability of 0.2 that the agents executes a random action, instead of what the actor wants (0.05 on Table, where docking requires precise moves). Testing episodes do not have this noise. The agent learns only from training episodes. Off-policy noise does not reduce BDPI’s learning performance at all, as we graphically show in Appendix D due to space constraints. Such robustness is a clear advantage of BDPI compared to on-policy approaches, especially in settings where backup policies are required for safety.

The performance of BDPI, obtained with a single set of hyper-parameters for all the environments\(^2\), demonstrate BDPI’s sample-efficiency, high-quality exploration, and strong robustness to hyper-parameters, as rigorously detailed in the next section.

### 4.5. Robustness to Hyper-Parameters

Hyper-parameters often need to be tweaked depending on the environment. Therefore, it is highly desirable that an algorithm provides good performance even if not optimally configured (Henderson et al., 2017), as BDPI does. To objectively measure an algorithm’s robustness to its hyper-parameters, we draw inspiration from sensitivity analysis. Thousands of runs of the algorithm are performed on randomly-sampled configurations of hyper-parameters, with each configuration evaluated on the total reward obtained over 800 episodes on LunarLander. Then, we compute the average absolute difference of total reward between random pairs of configurations, weighted by their distance in configuration space. This measures how much changing hyper-parameters affects performance. More details, and the complete list of hyper-parameters we consider for each algorithm, are given in Appendix C.

We evaluated numerous algorithms available in the OpenAI baselines. The algorithms, sorted by ascending sensitivity, are DQN with Prioritized ER (930), BDPI (1167), vanilla DQN (1326), A2C (2369), PPO (2452), then ACKTR (5815). Our plot in Appendix C shows that the apparent

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\(^2\)On LunarLander, the number of hidden neurons is 256 instead of 32, a trivial change.
Bootstrapped Dual Policy Iteration

The robustness of DQN-family algorithms comes from them performing equally badly for every configuration. 35% of BDPI’s configurations outperform the best configuration among all the other algorithms.

4.6. Value-Based Methods on Atari

Atari 2600 games are a common benchmark in the reinforcement-learning literature, due to their challenging pixel-based state-space. However, Atari-compatible value-based methods of the DQN family (Mnih et al., 2015) differ from conventional Q-Learning in several different key aspects, as can be seen by comparing the value loss used by DQN to Q-Learning (see Equation 1):

\[ L_{DQN} = \sum_{e \in B_{32}} (r_{t+1} + \gamma \max_{a'} Q^{-}(s_{t+1}, a') - Q(s_t, a_t))^2 \]

with \( e = (s_t, a_t, r_t, s_{t+1}) \) experience tuple. Compared to Q-Learning, DQN introduces a target network \( Q^{-} \), removes the learning rate \( \alpha \), and instead performs single small minimization steps of \( L_{DQN} \) per batch of experiences. Finally, DQN and almost all its extensions (Anschel et al., 2017; Hessel et al., 2017; Nikolov et al., 2019), use a batch size of 32. Changing any of these hyper-parameters prevents DQN from learning even the simplest games.\(^3\) Configuring our ABCDQN algorithm, the critic of BDPI, according to these Atari-specific rules, reduces it to vanilla DQN. With 16 vanilla DQN critics and one actor, BDPI performs comparably to DQN, except on Montezuma’s revenge, where BDPI consistently manages to obtain the key, leading to 200 points more than DQN.

Policy-based methods, such as A3C and PPO, achieve good results on Atari, while allowing most of their hyper-parameters to be changed without catastrophic results. For value-based methods, Ape-X allows batch sizes above 32 under certain circumstances (Horgan et al., 2018, see Appendix E). We therefore believe that there is no fundamental reason why Atari games would require overly precise hyper-parameter configuration. In future work, we will seek to explain the sensitivity of DQN to its hyper-parameters, in the hope of discovering a non-degenerate way of applying BDPI to pixel-based environments. Because BDPI largely outperforms PPO in our experiments, pixel-based BDPI could greatly push the state of the art forward. We also advocate for a more diverse set of environments on which algorithms are evaluated, as motivated by Whiteson et al. (2011). The environments we evaluate BDPI on are not pixel-based, but are stochastic, have continuous states and sparse reward signals, like many real-world tasks. Evaluating algorithms on such compute-

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\(^3\)We evaluated implementations from the OpenAI baselines and RL-Adventure.

5. Conclusion

In this paper, we proposed Bootstrapped Dual Policy Iteration (BDPI), an algorithm where a bootstrap distribution of aggressively-trained off-policy critics provides an imitation target for an actor. Multiple critics, combined with our actor learning rule, lead to high-quality exploration, comparable to bootstrapped Thompson sampling. Off-policy critics can be learned with any state-of-the-art value-based algorithm, depending on the application domain. BDPI is easy to implement, and remarkably robust to its hyper-parameters. The hyper-parameters we used for the highly-stochastic FrozenLake gridworld allowed BDPI to largely outperform the state of the art on two other environments, much more challenging and difficult to explore. This, complemented by the availability of BDPI’s full source code and experimental scripts, makes it one of the first plug-and-play reinforcement-learning algorithm that can easily be applied to new tasks, even by non-experts.

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A. Bootstrapped Dual Policy Iteration Pseudocode

The following pseudocode provides a complete description of the BDPI algorithm. To keep our notations simple and general, the pseudocode is given for the tabular setting, and does not refer to any parameter for the actor and critics. An implementation of BDPI based on function approximation, such as the neural networks we use in our experiments, uses the equations below to produce batches of state-action or state-value pairs. The function approximator is then trained on these batches, minimizing the mean-squared-error loss, for several gradient steps.

**Algorithm 1** Bootstrapped Dual Policy Iteration

```plaintext
Require: A policy π
Require: N_c critics. Q_A,i and Q_B,i are the two Clipped DQN networks of critic i.

procedure BDPI
    for t ∈ [1, T] do
        if t a multiple of K then
            LEARN
        end if
    end for
end procedure

procedure ACT
    Observe s_t
    Draw a_t ~ π(s_t)
    Execute a_t, observe r_{t+1} and s_{t+1}
    Add (s_t, a_t, r_{t+1}, s_{t+1}) to the experience buffer
end procedure

procedure LEARN
    for every critic i ∈ [1, N_c] (in random order) do
        Sample a batch E of N experiences from the experience buffer
        for N_t iterations do
            for all (s_t, a_t, r_{t+1}, s_{t+1}) ∈ E do
                Q(s_t, a_t) ← r_{t+1} + γ \min_{l=A,B} Q^l (s_{t+1}, \argmax_{a'} Q^{l,i} (s_{t+1}, a'))
            end for
            Train Q_A,i towards \hat{Q} with learning rate α
            Swap Q_A,i and Q_B,i
        end for
        Train π'(a|s) towards π with learning rate α
        π ← (1 - λ)π + λπ'
    end for
end procedure
```

B. The CPI Learning Rate Implements a Trust-Region

A trust-region, successfully used in a reinforcement-learning algorithm by (Schulman et al., 2015), is a constrain on the Kullback-Leibler divergence between a policy \( \pi_k \) and an updated policy \( \pi_{k+1} \). In BDPI, we want to find a policy learning rate \( \lambda \) such that \( D_{KL}(\pi_k || \pi_{k+1}) \leq \delta \), with \( \delta \) the trust-region.

While a trust-region is expressed in terms of the KL-divergence, Conservative Policy Iteration algorithms, the family of algorithms to which BDPI belongs, naturally implement a bound on the total variation between \( \pi \) and \( \pi_{k+1} \):

\[
\pi_{k+1} = (1 - \lambda)\pi_k + \lambda\pi'
\]

see Equation 5 in the paper

\[
D_{TV}(\pi_{k+1}(s)||\pi_{k}(s)) = \sum_a |\pi_{k+1}(a|s) - \pi_k(a|s)| \leq 2\lambda
\]
The total variation is maximum when π′, the target policy, and πk, both have an action selected with a probability of 1, and the action is not the same. In CPI algorithms, the target policy is a greedy policy, that selects one action with a probability of one. The condition can therefore be slightly simplified: the total variation is maximized if πk assigns a probability of 1 to an action that is not the greedy one. In this case, the total variation is 2λ (2 elements of the sum of (8) are equal to λ).

The Pinsker inequality (Pinsker, 1960) provides a lower bound on the KL-divergence based on the total variation. The inverse problem, upper-bounding the KL-divergence based on the total variation, is known as the Reverse Pinsker Inequality. It allows to implement a trust-region, as \( D_{KL} \leq f(D_{TV}) \) and \( D_{TV} \leq 2\delta \), with \( f(D_{TV}) \) a function applied to the total variation so that the reverse Pinsker inequality holds. Upper-bounding the KL-divergence to some \( \delta \) then amounts to upper-bounding \( f(D_{TV}) \leq \delta \), which translates to \( \lambda \leq \frac{1}{2} f^{-1}(\delta) \).

The main problem is finding \( f^{-1} \). The reverse Pinsker inequality is still an open problem, with increasingly tighter but complicated bounds being proposed (Sason, 2015). A tight bound is important to allow a large learning rate, but the currently-proposed bounds are almost impossible to inverse in a way that produces a tractable \( f^{-1} \) function. We therefore propose our own bound, designed specifically for a CPI algorithm, slightly less tight than state-of-the-art bounds, but trivial to inverse.

If we consider two actions, we can produce a policy \( \pi_k(s) = \{0, 1\} \) and a greedy target policy \( \pi′(s) = \{1, 0\} \). The updated policy \( \pi_{k+1} = (1-\lambda)\pi_k + \lambda\pi′ \) is, for state \( s \), \( \pi_{k+1}(s) = \{\lambda, 1-\lambda\} \). The KL-divergence between \( \pi_k \) and \( \pi_{k+1} \) is:

\[
D_{KL}(\pi_k||\pi_{k+1}) = 1 \log \frac{1}{1-\lambda} + 0 \log \frac{0}{\lambda} = \log \frac{1}{1-\lambda}
\]

if we assume that \( \lim_{x \to 0} x \log x = 0 \). Based on the reverse Pinsker inequality, we assume that if the two policies used above are greedy in different actions, and therefore have a maximal total variation, then their KL-divergence is also maximal. We use this result to introduce a trust region:

\[
D_{KL}(\pi_k||\pi_{k+1}) \leq \delta \\
\log \frac{1}{1-\lambda} \leq \delta \\
\frac{1}{1-\lambda} \leq e^\delta \\
\lambda \leq 1 - e^{-\delta}
\]

Interestingly, for small values of \( \delta \), as they should be in a practical implementation of BDPI, \( 1 - e^{-\delta} \approx \delta \). The trust-region is therefore implemented by choosing \( \lambda = \delta \), which is much simpler than the line-search method proposed by (Schulman et al., 2015).

B.1. State-Dependent Exploration

Compared to Bootstrapped DQN, well-known for its high-quality exploration, BDPI lacks an important component: explicit deep exploration. Deep exploration consists of performing a sequence of directed exploration steps, instead of exploring in a random direction at each time-step (Osband et al., 2016). Bootstrapped DQN achieves deep exploration by greedily following a single critic, sampled at random, for an entire episode. BDPI trains its actor towards a randomly-selected critic at every time-step, which is incompatible with deep exploration. We empirically show in Section 4.4 that BDPI outperforms Bootstrapped DQN, so the loss of explicit deep exploration does not seem to negatively affect performance. In Figure 3, we provide a likely explanation in the Table environment. At the early stages of training, the agent regularly falls off the table, which resets the episode. This can be observed as dips in the entropy of the actor. We believe that this is caused by a sort of novelty-based exploration, probably more limited than what highly-advanced algorithms produce (Burda et al., 2018), but still present. After a few episodes, the individual runs learn different policies, which breaks the correlation between them and explains the flat portion of Figure 3. The emergence of such an interesting exploration strategy, leading to higher-quality exploration than Bootstrapped DQN, from the simple use of an actor with several
Figure 3. Entropy of the policy per time-step, on the Table environment (running average and standard deviation of 8 runs). The entropy oscillates as the agent falls off the table, which resets the environment to familiar states. After some time (blue bar), runs start learning distinct policies, whose entropies cannot be observed anymore on an averaged plot.

off-policy critics, illustrates how amenable the architecture of BDPI is to relatively advanced features. We believe that further work will allow more features to naturally emerge, or be easily implemented, on top of the BDPI algorithm we present in this paper.

C. Robustness to Hyper-Parameters

Evaluating the robustness of an algorithm to its hyper-parameters is challenging, and typically not done in Deep RL research. We propose a simple approach, that we designed to be easy to understand and intuitive, and that provides two measures of robustness.

C.1. Data Collection

For each algorithm, namely BDPI, DQN, Prioritized and Dueling DQN, A2C, PPO and ACKTR, we define a configuration space that consists of all the combinations of the most relevant hyper-parameters of the algorithms. We then randomly sample configurations, run the algorithm on LunarLander for 800 episodes, and compute the total reward obtained during these 800 episodes. We used the OpenAI Baselines implementations of all the algorithms (but BDPI), to ensure that no implementation error on our side invalidates the results.

The hyper-parameters evaluated for each algorithms are listed below. We ensured that all the known-good configurations of all the algorithms, for various environments in the literature, are covered.

All algorithms
- Neural network learning rate: 0.00001, 0.00005, 0.0001, 0.0005, 0.001
- Neurons in the hidden layer of the neural network: 32, 64, 96, 128, 256

All but BDPI
- Number of parallel environments: 1. BDPI is single-threaded, so, to avoid artificially increasing the sensitivity of the other algorithms, we chose to keep this highly-sensitive parameter to 1.
- Entropy regularization: 0, 0.01, 0.03, 0.05

BDPI
- Experience buffer size: 5K, 10K, 20K, 50K, 100K
- Batch size: 64, 128, 256, 512
- Critics trained per time-step: 1, 4, 8
- Number of critics: 1, 4, 8, 16, 32
- Clipped DQN iterations per critic-time-step: 1, 2, 4, 8
- Epochs used to fit the neural networks: 1, 4, 8, 16. The absolute best performance of BDPI is achieved with 20-50+ epochs, but our computing resources did not allow us to increase this parameter as much. We ensure that the best-known configuration of the other algorithms is included in our configuration space.
Figure 4. Total reward per configuration, sorted by descending total reward. This plot shows that more than 35% of BDPI's randomly-sampled configurations perform better than the best (PPO) configuration. The worst BDPI configuration is also better than most of the configurations of the other algorithms. On LunarLander, for 800 episodes, the random policy achieves a total reward of about -240K.

**PPO**
- Steps per batch: 64, 128, 256, 384, 512, 1024, 2048
- Lambda: 0.7, 0.8, 0.9, 1.0
- Optimization steps per epoch: 1, 2, 4, 8, 16

**A2C**
- Time-steps between learning epochs: 1, 2, 4, 6, 8
- Critic loss weight compared to the actor: 0.1, 0.3, 0.5, 0.7, 0.9
- Gradient norm clipping: 0.1, 0.3, 0.5, 0.8, 1.0

**ACKTR**
- Learning rate (specific to ACKTR, default of 0.25): 0.01, 0.10, 0.25, 0.50, 0.90
- Time-steps between learning epochs: 1, 5, 10, 20, 40, 80
- Critic loss weight: 0.1, 0.3, 0.5, 0.7, 0.9
- Fisher weight: 0.1, 0.3, 0.5, 0.7, 0.9
- Gradient norm clipping: 0.1, 0.3, 0.5, 0.8, 1.0
- Kronecker-Factored clipping: 0.0001, 0.001, 0.005, 0.01, 0.1

**DQN**
- Experience buffer size: 5K, 10K, 20K, 50K, 100K
- Batch size: 16, 32, 128
- Exploration fraction: 0.02, 0.05, 0.1, 1.0
- Final epsilon after exploration: 0.1, 0.05, 0.01, 0.001
- Time-steps between learning epochs: 1, 2, 4, 8, 16
- Time-steps before learning starts: 1, 500, 1000, 10000
- Target network update frequency: 1, 50, 100, 500, 1000

**Dueling DQN with Prioritized Experience Replay**
*All the same parameters as DQN, and:*
- Alpha parameter for Prioritized ER: 0.5, 0.6, 0.7, 0.8, 0.9
Figure 5. BDPI, Bootstrapped DQN and ACKTR with and without off-policy noise, on our three environments. BDPI’s performance is not impacted by the off-policy noise, which confirms its off-policy nature. Bootstrapped DQN is slightly more impacted by the noise, but in a positive way, which may point at it not exploring enough without the noise. ACKTR, an on-policy algorithm that we spent considerable amount of time configuring for our tasks, manages to overcome the 0.05 noise on Table, but not the 0.2 noise on LunarLander. ACKTR requires about 10K episodes before learning the highly-stochastic FrozenLake task, with or without the noise.

C.2. Data Processing

Hundreds of randomly-sampled configurations of the algorithms are evaluated, and we propose to use the total reward over 800 episodes on LunarLander as performance measure. Figure 4 graphically displays this dataset: for each algorithms, all the configurations are sorted by descending total reward, then the lines are stretched horizontally to compensate for the unequal amount of configurations that each algorithm was evaluated on, due to each algorithm having different computational resources requirements.

The measures that we report in Section 4.5 are slightly more advanced. While Figure 4 intuitively shows that BDPI produces a higher curve, sorting the configurations by performance remove any information about the locality of the configurations. It shows that many configurations are good, not that they are close together in configuration space. In order to better measure how slight changes in parameters influences performance, be introduce a second measure:

$$S = \frac{\sum_{a,b} w(a,b) \delta(a,b)}{\sum_{a,b} w(a,b)}$$

$$w(a,b) = |a_{params} - b_{params}|^{-1}$$

$$\delta(a,b) = |a_{score} - b_{score}|$$

with $a$ and $b$ two randomly-sampled configurations. In order to produce accurate scores, we evaluate each algorithm on more than 2000 configurations, and apply Equation 10 on 4000000 pairs of configurations. The resulting scores, also reported in Section 4.5, are DQN with Prioritized ER (930), BDPI (1167), vanilla DQN (1326), then, significantly larger, A2C (2369), PPO (2452) and ACKTR (5815).

D. Robustness to Off-Policy Noise

BDPI, in addition to having critics that are off-policy with regards to the actor, also allows the actions being executed in the environment to be distinct from what the actor wants. Theoretically, this can be seen in Equations 4 and 5. The critic is trained with a variant of Q-Learning, an off-policy algorithm. The actor only sees greedy policies emanating from the critics, and does not depend on any behavior policy or on-policy return. We now empirically demonstrate that these equations indeed lead to an agent that can act off-policy. Training episodes, that generate experiences added to the experience buffer, have a 0.2 (0.05 on Table) probability at each time-step that the agent takes a random action, not what the actor wants. Testing episodes, used to produce the plots of Figure 5, strictly follow the actor. Our results show that BDPI is not impacted by off-policy noise to any extend, while ACKTR, an on-policy algorithm, fails on LunarLander when off-policy noise is added.
|                        | ACKTR       | PPO        | BDQN       | ABCDQN     | BDPI       |
|------------------------|-------------|------------|------------|------------|------------|
| Discount factor $\gamma$ | 0.99        |            |            |            |            |
| Replay buffer size     | –           | –          | 20K        | 20K        |            |
| Experiences/batch      | 20          | 256/1024$^a$ | 256        | 256        |            |
| Training epoch every $K$ time-steps | 20          | 256/1024$^a$ | 1          | 1          |            |
| Policy loss            | PG+Fisher   | PPO        | –          | –          | MSE        |
| Trust region $\delta$  | –           | –          | –          | –          | 0.05       |
| Entropy regularization | 0.01        | 0.01       | –          | –          | 0          |
| Value loss coefficient | 0.5         | –          | –          | –          | –          |
| Critic count $N_c$     | 1           | 1          | 16         | 16         |            |
| Critic sampling frequency | –           | –          | episode    | –          |            |
| Critic learning rate $\alpha$ | 0.25       | 1.0 (on $R_t$) | 1.0     | 0.2        |            |
| Critic training iterations $N_t$ | –       | 1          | 1          | 4          |            |
| Gradient steps/batch   | 1           | 4          | 20         | 20         |            |
| Learning rate          | dynamic     | 0.001      | 0.0001     | 0.0001     |            |
| Activation function    | tanh        | tanh       | tanh       | tanh       |            |
| Hidden layers          | 2           | 2          | 1          | 1          |            |
| Hidden neurons         |             |            |            | 32/256$^a$ |            |

Table 1. Hyper-parameter of the various algorithms we experimentally evaluate. (a) Hyper-parameters that were required for LunarLander to perform well.

E. Experimental Setup

All the algorithms evaluated in Section 4 use feed-forward neural networks to represent their actor(s) and critic(s). They take as input the one-hot encoding of the current state, and are trained with the Adam optimizer (Kingma & Ba, 2014), using a learning rate of 0.0001 (0.001 for PPO, as it gave better results). We configured each algorithm following the recommendations in their respective papers, and further tuned some parameters to the environments we use. These parameters are given Table 1. They are kept as similar as possible across algorithms, and constant across the three environments, to evaluate the generality of the algorithms.