Simulations of photon detection in silicon photomultiplier number-resolving detectors

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Abstract
Number-resolving single-photon detectors are essential for the implementation of numerous innovative quantum information schemes. While several number-discriminating techniques have previously been presented, the silicon photo-multiplier (SiPM) detector is a promising candidate owing to its rather simple integration in optical setups. On the other hand, the photon statistics obtained with the SiPM detector suffer from inaccuracies due to inherent distortions which are dependent on the geometrical properties of the SiPM. We simulated the detection process in an SiPM detector and studied these distortions. We used the results of the simulation to interpret the experimental data and to study the limits in which available models prevail.

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Photon-number-discriminating detectors have attracted much interest over the last few years. Number-resolving capabilities are key to several quantum state preparation schemes [1–3] as well as for the implementation and analysis of quantum computation schemes [4–8]. Photon-number resolution also enables direct measurement of a state’s photon-number statistics, from which non-classical properties as well as classical-to-quantum transitions can be studied and characterized [9–11].

Standard single-photon detection techniques cannot resolve the number of photons, and number resolution is generally obtained by combining several number-insensitive detectors in a cascade or an array-like structure. One of the first photon-number-resolving detectors introduced is the visible light photon counter (VLPC) [12]. The VLPC has a high detection efficiency, but requires cryogenic cooling. Superconducting devices that provide number resolution are available but require low working temperatures [13–17]. Room-temperature-based solutions using standard single avalanche photodiode (APD) detectors offer limited number resolution [18], whereas other solutions based on spatial and temporal multiplexing are experimentally demanding [19–22].

A promising approach is offered by the silicon photo-multiplier (SiPM) [23]. The SiPM is composed of multiple silicon avalanche photo-diodes arranged on a single substrate. Each detection element acts as an independent avalanche photo-diode in Geiger mode, which can absorb a photon and generate a confined electric discharge. Signals from all the detecting elements are combined to a single readout port, so that the intensity of the output signal is proportional to the number of impinging photons. The SiPM detector offers a very good number resolution, operates at room temperature and is easily integrated into optical setups.

When using SiPMs, three main factors affect the measured photon statistics. These are the relatively low detection probability, determined by the quantum efficiency and geometrical configuration, internal noise caused by thermal excitations resulting in false detection and optical crosstalk (CT) in which a photon created by carrier relaxation in one detection element is detected in a neighbouring element [24, 25]. There exist several models [26–28] that take these phenomena into account. These models aid in the interpretation of experimental results and allow reconstruction of the original photon-number statistics. However, these available models handle the CT effect in a rather limiting
manner, by limiting the CT probability, by limiting the overall number of CT events or by limiting the number of CT stages (ignoring crosstalk events generated by crosstalks). Since these models were inaccurate in describing our experimental results, we have written a computational model that simulates the detection process in SiPM detectors. Using this model, we determine the conditions under which available models are accurate and examine the limits imposed by their assumptions with regard to the interpretation of the experimental data.

In the model, the detector is represented by a two-dimensional (2D) lattice where each cell represents an element of the detector. The impinging photons are distributed uniformly across the lattice. When a photon approaches a certain cell, the cell is triggered with some probability, $\eta$, the detection efficiency. If triggered, each of the cell’s nearest neighbours can also be triggered with a probability $\epsilon_{nn}$, the optical CT probability. These new triggered elements can continue to trigger their remaining nearest neighbours. This process continues until no new cells are triggered.

The dead time of detection elements in available SiPM detectors is longer than the photon propagation time between cells [24], so our model allows each element to be triggered only once in the process. The statistics were obtained on a $10 \times 10$ lattice with 100 elements, which corresponds to a typical commercially available SiPM detector (Hamamatsu Photonics, S10362-11-100U, http://www.hamamatsu.com). Nevertheless, the conclusions can be applied to other sized samples by translating the absolute number of detections into the fraction of occupancy (20 photons in a 100 pixel detector are equivalent to 80 photons in a 400 pixel detector). The samples are not entirely scalable in size since the number of cells along the borders does not scale with the number of elements. However, simulations performed on different-sized samples showed that this has little effect on the overall results in the range of the experimental parameters.

We begin our discussion with the optical CT process. Figure 1 shows representative runs, which portray the CT evolution. The detector is initially triggered by $N_{trg}$ detections of the impinging photons. These triggers initiate a CT process, which results in the triggering of additional cells. The number of CT events shown in these examples was chosen so that it corresponds to the average number of CTs produced over multiple runs. As expected, the number of triggered cells increases with the CT probability, $\epsilon_{trg}$. In figures 1(b) and (d), we present an example of the effect the finite number of elements has on the number of generated CTs. Increasing the value of $N_{trg}$ beyond some critical limit does not result in additional CT triggers. The majority of cells attempt to trigger neighbouring cells, which have already been triggered and do not contribute to the evolution of the process.

The limited number of cells can cause the number of CT events to decrease rather than increase with the addition of more initial triggers. This result is shown in figure 2. When the average number of triggered elements is small, the number of CTs grows linearly with $N_{trg}$. As $N_{trg}$ is increased, the effective number of neighbouring cells, which can also be triggered, is reduced, as many of these neighbouring cells have already been triggered. As a result, the average number of CT events deviates from linearity and starts decreasing after reaching some peak value. Note that this effect, which is caused by the finite size of the detector, comes into play even at relatively low numbers of detected photons.

In order to identify the critical point where finite-size effects begin to dominate the CT process, we consider the average number of triggered elements generated by one initial trigger. In a detector of infinite size, the CT expansion is only suppressed by the probability for CT. When the number of elements is limited, the progression is also suppressed due to overlaps with previously triggered cells. The average number of triggered elements should therefore be constant in
the absence of finite size effects and should decrease when the finite size begins to impose a limitation. Figure 3 shows that for large CT probabilities, the finite size imposes an immediate restriction, whereas the process expands rather freely when the CT probability is below \( \epsilon_{nn} \approx 0.025 \).

We define the critical number of initial triggers above which the size effects must be considered as the point where the average number of elements triggered by a single detection decreases by more than 10% compared to the unaffected value. In figure 4, we show this critical value as a function of the CT probability, \( \epsilon_{nn} \). The number of CT-triggered cells increases with the value of \( \epsilon_{nn} \), and the value of the critical value is therefore lower. Figure 4 also presents the typical CT values we experimentally measured for a range of bias voltages. The higher the bias voltage, the higher the gain and the CT probability [24]. For the typical range of experimental values, the maximal number of detected photons for which finite size effects are not significant is in the range 20–100. These numbers are significantly higher than the number of photons observed in previously published works [26–29]. This explains why the modelling of the CT effect without the geometrical features of the detector was sufficient in these works.

We now turn to evaluating the number of CT stages involved in the process before it stops. The number of stages is an indication of the number of cells triggered due to CT generated by CT events and is an important factor in the modelling of the CT process. The average number of CT stages is shown in figure 5. For small CT probabilities only a small number of neighbouring elements is triggered due to CT, and the process ceases naturally after one stage even for large initial values of \( N_{eg} \). For large values of \( \epsilon_{nn} \), the number of CTs generated by other CT events increases and we observe a rise in the number of stages. It is interesting to note that the work [26] had a CT probability equivalent to \( \epsilon_{nn} \approx 0.025 \) and used a one-stage CT model. From figure 5, we can see that for this value of \( \epsilon_{nn} \), the number of stages indeed does not exceed 1 even for photon numbers higher than what was detected in this work. On the other hand, we measured a CT value of \( \epsilon_{nn} \approx 0.07 \) and detected up to 14 photons. In this regime, the number of stages exceeds 1. In fact, a simplistic one-stage model would result in an error of over 20% in the evaluation of the number of CT events. The finite-size effects are also apparent in this graph. Similar to the behavior observed in figure 2, the number of CT stages begins to decrease slowly after reaching some peak value.

We now consider the second effect that governs the behaviour of SiPM detectors, the inherent loss mechanism. Apart from the low detection efficiency, when two or more photons approach the same detection element, only one avalanche can be generated and the number resolution is lost. This nonlinear response of the detector affects the detection probabilities. The average number of detected photons can be generally written as (http://www.hamamatsu.com)

\[
\langle N_{\text{detected}} \rangle = N_{\text{elements}} \times \left[ 1 - \exp \left( -\eta \cdot \frac{N_{\text{photons}}}{N_{\text{elements}}} \right) \right],
\]

where \( N_{\text{photons}} \) is the number of impinging photons, \( \eta \) the detection efficiency and \( N_{\text{elements}} \) the number of elements. In
the limit $\eta \cdot N_{\text{photons}} \ll N_{\text{elements}}$, where the finite-size effects can be neglected, the expression reduces to the linear relation

$$\langle N_{\text{detected}} \rangle = \eta \cdot N_{\text{photons}}.$$  

We use this relation to determine a critical condition below which the finite number of detection elements does not affect the detection probabilities. In figure 6 we show the average number of detected photons as a function of detection efficiency. We identify deviations of over 10% from a linear slope when the fraction of triggered elements, $N_{\text{elements}}$, exceeds 20%. Interestingly, the deviations from linearity depend on the detection efficiency and this behaviour may be used to obtain the absolute detection efficiency of the detector.

In order to demonstrate the usefulness of our model in reconstructing the original photon statistics from measured data, we present in figure 7 thermal photon-number statistics measured using an SiPM detector. The measurements were carried out on a single polarization mode from a collinear type-II parametric down-conversion source [30] with a pulsed pump at a repetition rate of 250 kHz. The detector was operated using two different bias voltage values, which changed the CT probability between the two measurements. The photon-number distribution that should appear as a straight line in a semi-log plot ($p(n) \sim \langle n \rangle^{n} e^{-\langle n \rangle}$) experiences a change in slope due to the CT effect. Zero photons cannot generate CT, and thus the probability of zero photons is not affected. However, the probability of measuring two or more photons grows considerably due to CTs. We fitted the experimental data to the photon-number statistics obtained by our computational model, the one-stage CT model of [26] and the recursive model of [28]. The CT probability defined in previously presented models is defined as the overall probability that CT will be generated, rather than the probability of triggering a specific neighbour. We associate this CT value with our defined $\epsilon_{\text{rec}}$ through the relation $\epsilon = 1 - (1 - \epsilon_{\text{rec}})^{4}$, four being the number of nearest neighbours in the square lattice. When the CT effect is weak, all models fitted the data. When the CT effect is strong, the one-stage model and the recursive model show large deviations from the data. An analytic model that provides a good description for all CT values is currently under development.

In conclusion, we have simulated the detection process in SiPM detectors and have shown that it is highly affected by the finite number of detection elements. These effects must be taken into account when the photon-number statistics of impinging photons is reconstructed. We have shown that simplistic modelling that does not account for the geometrical properties of the detection elements is applicable provided the number of detected photons is below some threshold, which depends on the fraction of triggered elements and on the CT probability. We have also shown the number of CT stages that must be taken into account in order to properly model the CT effect. Finally, we demonstrated the application of our computational model to the experimental data.

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