RESONANCE PARAMAGNETIC RELAXATION AND ALIGNMENT OF SMALL GRAINS

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ABSTRACT

We show that the energy-level splitting that arises from grain rotation ensures that paramagnetic dissipation acts at its maximum rate, i.e., the conditions for paramagnetic resonance are automatically fulfilled. We refer to this process as “resonance relaxation.” The differences between the predictions of classical Davis-Greenstein relaxation and resonance relaxation are most pronounced for grains rotating faster than 1 GHz, i.e., in the domain in which classical paramagnetic relaxation is suppressed. This mechanism can partially align even very small grains, resulting in linearly polarized microwave emission that could interfere with efforts to measure the polarization of the cosmic microwave background.

Subject headings: atomic processes — cosmic microwave background — dust, extinction — polarization

1. INTRODUCTION

Experiments to study the cosmic background radiation have stimulated renewed interest in diffuse galactic emission. Recent maps of the microwave sky brightness have revealed a component of the 10–100 GHz microwave continuum that is correlated with 100 μm thermal emission from interstellar dust (see review by Draine & Lazarian 1999a). Draine & Lazarian (1998a, 1998b, hereafter DL98a and DL98b) attributed this emission to electric dipole radiation from small (<10⁻⁷ cm) rapidly rotating grains. Recent observations by de Oliveira–Costa et al. (1999) support this interpretation. The question now is whether these small grains are aligned and their emission polarized.

One process that might produce alignment of the ultrasmall grains is the paramagnetic dissipation mechanism suggested by Davis & Greenstein (1951) to explain the polarization of starlight. The Davis-Greenstein mechanism is straightforward: the component of interstellar magnetic field perpendicular to the grain angular velocity varies in grain coordinates, resulting in time-dependent magnetization, energy dissipation, and a torque acting on the grain. As a result, grains tend to rotate with angular momenta parallel to the interstellar magnetic field. Although recent research (Draine & Weinberger 1996, 1997; Lazarian & Draine 1999a, 1999b) suggests that paramagnetic alignment may not be the dominant alignment mechanism for a ≳ 10⁻⁵ cm grains, it may be effective for small (a ≲ 5 × 10⁻⁶ cm) grains.

In the present Letter, we claim that the traditional picture of paramagnetic relaxation is incomplete, since it disregards the splitting of energy levels that arises within a rotating body. Unpaired electrons having spin parallel and antiparallel to the grain angular velocity have different energies resulting in the Barnett effect (Landau & Lifshitz 1960)—the spontaneous magnetization of a paramagnetic body rotating in field-free space. Therefore, the implicit assumption in Davis & Greenstein (1951)—that the magnetization within a rotating grain in a static magnetic field is equivalent to the magnetization within a stationary grain in a rotating magnetic field—is clearly not exact.

In what follows, we show that a very important effect due to rotation has thus far been overlooked. This effect, which we term “resonance relaxation,” leads to energy dissipation—and grain alignment—which is much more rapid than the classical Davis-Greenstein estimate when the grain rotates very rapidly.

2. DAVIS-GREENSTEIN THEORY

Paramagnetic dissipation in a stationary grain depends upon the imaginary part of the magnetic susceptibility χ”, which characterizes the phase delay between the grain magnetization and the rotating magnetic field. Because of this delay, a grain rotating in a static magnetic field experiences a decelerating torque: the energy dissipated in the grain comes from rotational kinetic energy. The Davis-Greenstein alignment timescale is

\[ \tau_{DG} \approx 3 \times 10^3 \text{ yr} \left( \frac{a}{10^{-7} \text{ cm}} \right)^2 \left( \frac{10^{-13} \text{ s}}{K(\omega)} \right) \left( \frac{5 \text{ } \mu\text{G}}{B_0} \right)^2, \]  

where \( K(\omega) \equiv \chi''(\omega)/\omega \) (see Davis & Greenstein 1951). Following Draine & Lazarian (1999b), we estimate

\[ K(\omega) \approx \frac{\chi'' T_\phi}{[1 + (\omega T_\phi)^2]} \left( 10^{-13} \text{ s} \right) \approx \frac{(20 \text{ } K/T_\phi)}{[1 + (\omega T_\phi)^2]}. \]

Very small grains are expected to be paramagnetic owing to the presence of free radicals, paramagnetic carbon rings (see Altschuler & Kozyrev 1964), and captured ions. The spin–spin coupling time is

\[ \tau_\phi \approx \frac{\hbar}{3.8 \mu_0 g \mu_n} \approx 2 \times 10^{-9} \left( \frac{10^{21} \text{ cm}^{-3}}{n_p} \right) \text{ s} \]  

(see DL98b), where \( \mu_0 \) is the Bohr magneton and \( n_p \approx 10^{21} \text{ cm}^{-3} \) is the concentration of unpaired electrons, greater than in coals (Tsvetkov, Dzuba, & Gulin 1993), but less than the concentration of free radicals envisaged by Greenberg (1982). Equation (3) then predicts a cutoff frequency \( \nu_{\omega m} = \frac{(2\pi T_\phi)^{-1}}{1} \approx 0.1 \text{ GHz} \). In the extreme case in which ~10% of the atoms are paramagnetic, one can get \( \nu_{\omega m} \) as large as 1 GHz but hardly any higher.

3. BARNETT EFFECT AND BLOCH EQUATIONS

The Barnett effect states that a body rotating with velocity \( \omega \) develops a magnetization

\[ M = -\frac{\hbar}{g \mu_0} \chi_0 H_{BE}, \]

where

\[ \chi_0 \equiv \chi_{\text{static}}, \]

\[ H_{BE} \equiv \frac{\mu_0}{\hbar} \tau_\phi \]
where \( \chi_0 \) is the static susceptibility and \( H_{\text{BE}} \) is the “Barnett equivalent” field. The essence of the Barnett effect is easily understood: a rotating body can decrease its energy while keeping its angular momentum constant if some of the angular momentum is taken up by its unpaired spins. By flipping one spin of angular momentum \( h/2 \), the system can reduce its rotational kinetic energy by \( h\omega \).

Although the Barnett effect has been long known in physics (see Landau & Lifshitz 1960), its importance in the context of interstellar grains was only appreciated recently (Dolginov & Mytrophanov 1976; Purcell 1979; Lazarian & Roberge 1997; Lazarian & Draine 1997; Mytrophanov 1976; Lazarian & Draine 1997). In the present Letter, we discuss a hitherto unrecognized aspect of the Barnett effect, namely its influence on paramagnetic dissipation in a rapidly rotating grain.

Rotation removes the spin degeneracy of the electron energy levels. The energy difference between electron spin parallel or antiparallel to \( \omega \) provides a level splitting corresponding to \( h\omega = g\mu_B H_{\text{BE}} \). Insofar as the energy levels and magnetization are concerned, rotation of the grain is analogous to application of the Barnett equivalent field.

Now consider a (weak) static magnetic field \( H_i \) at an angle \( \theta \) to \( \omega \). In grain coordinates, this appears like a static field \( H_i \cos \theta \) plus a field \( H_i \sin \theta \) rotating with frequency \( \omega \). The rotating field can be resonantly absorbed, since the energy level splitting is exactly \( \omega_0 \).

The Bloch equations (Bloch 1946) are useful for describing both resonant and nonresonant absorption (see Pake 1962). These phenomenological equations reflect the tendency of the magnetization \( M \) to precess and to tend exponentially toward both resonant and nonresonant absorption (see Pake 1962).

An important difference between paramagnetic resonance in an external magnetic field and the resonance relaxation discussed here is that the Barnett equivalent magnetic field is different for species with different magnetic moments. Therefore, unlike paramagnetic resonance, resonance relaxation happens simultaneously to species with completely different magnetic moments and \( g \)-factors. For example, the conditions for electron spin resonance and nuclear magnetic resonance are satisfied simultaneously when a grain rotates in a static weak magnetic field.

4. SPIN-LATTICE RELAXATION

For a spin to flip in a rotating grain, it is necessary for the total energy in lattice vibrations to change by \( h\omega \). Because the density of states is finite, it may not be possible for the lattice vibrations to absorb an energy \( h\omega \).

To estimate the lowest vibrational frequency \( \omega_{\text{min}} \), we note that the lowest frequency bending mode of the coronene molecule \( (C_{24}H_{12}) \) is \( \omega_{\text{min}} = 1.9 \times 10^3 \text{s}^{-1} \) (Cyvin 1982). Coronene has an effective radius \( a = 4.3 \text{Å} \) for an assumed density \( \rho = 1.5 \text{g cm}^{-3} \); if \( \omega_{\text{min}} \approx a^{-1} \), then \( \omega_{\text{min}} = 8.3 \times 10^2 \text{cm}^{-1} \) large compared to \( kT/h \). Thus, for ultrasmall grains one cannot use spin-lattice relaxation times \( \tau_i \) measured for macroscopic samples.

We obtain an upper estimate for \( \tau_i \) by appealing to the Raman scattering of phonons (see Waller 1932; Pake 1962): annihilation of a vibrational quantum \( h\omega \) and creation of a quantum \( h/2\omega - \omega \). Elastic lattice vibrations of all frequencies participate, and therefore the relaxation is present for small grains. After probability of the spin-lattice transition via Raman scattering (Altshuler & Kozyrev 1964)

\[
\tau_i^{-1} \approx K_s (kT/h)^{n+1} J_n / p c_s^{n+1},
\]

where \( \rho \) is the density, \( c_s \) is the sound speed, \( K_s \) is a function that depends on the density of states, and \( J_n \) \((m = 6 \text{ or } 8)\) is...
an integral over the body’s phonon frequencies:

\[ J_m = \int_{T_m/T}^\infty \frac{x^m e^x}{(e^x - 1)^2} \, dx, \]

where \( T_B \) is the Debye temperature. The conventional treatment assumes the body to be infinite with the integration extending down to \( T_B = 0 \). In this case, and for \( T \ll T_B/m \), we have \( J_m \approx m! \zeta(m) \), where \( \zeta \) is the Riemann zeta function. For a grain of size \( a \),

\[ T_B = \frac{\hbar \omega_{\text{min}}}{k} \approx \frac{63}{a \gamma} \text{ K,} \quad (10) \]

For \( T \ll T_B/m \), we have \( J_m \approx (T/T_B)^m \exp(-T/T_B) \). The ratio of the Raman spin-lattice relaxation in a small grain at temperature \( T \ll T_B/m \) to such relaxation in an infinite body at \( 77 \text{ K} \leq \theta_B/m \) is then

\[ \frac{\tau_1(T)}{\tau_1(77 \text{ K})} \approx \left( \frac{77 \text{ K}}{T} \right)^{m+1} \left( \frac{T}{T_B} \right)^m \exp \left( \frac{T}{T_B} \right) m! \zeta(m). \quad (11) \]

Data in Altshuler & Kozyrev (1964) suggest that ionic crystals have a spin-lattice relaxation time \( \tau_{1, \text{se}}(77 \text{ K}) \approx 10^{-6} \text{ s} \). If due to the Raman process, then we would estimate that a macroscopic sample at \( T = 4 \text{ K} \) would have \( \tau_1 \approx 28 \text{ s} \) if \( m = 6 \), or \( 2 \times 10^4 \text{ s} \) if \( m = 8 \). From equation (11), a grain with \( a = 10^{-7} \text{ cm} \) at \( T = 4 \text{ K} \) would have \( \tau_1 \approx 3.1 \times 10^5 \text{ s} \) for \( m = 6 \), or \( 2.7 \times 10^7 \text{ s} \) for \( m = 8 \). Grains with \( a < 10^{-7} \text{ cm} \) would have even larger values of \( \tau_1 \).

If we adopt \( \tau_2 \) from equation (3), then

\[ g_s^2 g_A^2 \tau_2 H_i^2 \sin^2 \theta = 8 \left( \frac{\tau_{1, \text{se}}}{10^6 \text{ s}} \right) \left( \frac{H_i}{5 \mu \text{G}} \right)^2 \sin^2 \theta \left( \frac{2}{3} \right)^3, \quad (12) \]

so we see from equation (8) that saturation may be important for \( a < 10^{-7} \) grains even in the \( ~5 \mu \text{G} \) fields in diffuse interstellar gas.

How reliable is our above estimate for \( \tau_1 \)? Our calculations were based on the so-called Waller theory, which frequently overestimates \( \tau_1 \) by a factor up to \( 10^6 \) (Pake 1962). If the dependence of the spin-lattice relaxation time on temperature is different from that given by equation (9), our estimates of \( \tau_1 \) at 4 K would be very different. Ultimately we require laboratory measurements of \( \tau_1 \) in small particles of appropriate composition.

5. GRAIN ALIGNMENT

Paramagnetic alignment of grains with a given axis ratio depends on two parameters: the ratio \( T/T_{\text{rot}} \) of grain vibrational and rotational temperatures and the ratio of the alignment time \( \tau_{\text{rot}} \) to the rotational damping time \( t_d \). The rotational damping time \( t_d \) depends on various processes of damping and excitation (e.g., collisions with ions and neutrals, plasma drag, and emission of photons) discussed in DL98b. Assuming that the paramagnetic torque only marginally reduces \( \omega \), we follow the analysis of DL98b to obtain \( \omega \) as a function of \( a \).

In our calculations, we assume that a grain spends most of its time between thermal spikes with a vibrational temperature \( T \approx 4 \text{ K} \) (see Rouan et al. 1992), since the time between photon absorptions is \( \approx 10^5 \text{ s} \), while the grain cools much more rapidly. Note that photons usually contribute marginally to the disorientation of the grain angular momentum \( J \) (see Fig. 5 in DL98b).

Let \( \theta \) be the angle between \( J \) and the interstellar magnetic field \( B \). Figure 1 presents the measure of alignment \( a = 3/2 (\cos^2 \theta - 1/3) \) for grains in the cold neutral medium. For our estimate, we used standard formulas for paramagnetic alignment of angular momentum with \( B \) (Lazarian 1997; Roberge & Lazarian 1999), which for weak alignment provide...
\[ \sigma \approx 2/15[1 - (1 + rt)/(1 + r)], \text{ where } r = \tau_{\text{rot}}/t_d \text{ and } t = T/T_{\text{rot}}. \]

The discontinuity at \( \approx 6 \times 10^{-3} \text{ cm} \) is due to the assumption that smaller grains are planar and larger grains are spherical. The degree of polarization of rotational electric dipole emission \( p \approx \sigma \cos^2 \psi \), where \( \psi \) is the angle between \( B \) and the plane of the sky. The value of \( p/\cos^2 \psi \) as a function of frequency is also shown in Figure 1. The dipole rotational emission predicted in DL98a and DL98b is sufficiently strong that polarization of a few percent may interfere with efforts to measure the polarization of the cosmic microwave background radiation. It worth noting that the degree of microwave polarization is sensitive to the magnetic \( B \) field intensity (through \( \tau_{\text{rot}} \)).

6. DISCUSSION

We have discussed a new gyromagnetic effect—resonance relaxation—that is closely related to normal paramagnetic resonance and arises naturally whenever a body rotates in a weak magnetic field. The standard assumption of the equivalence of relaxation when the magnetic field rotates about a grain or a grain rotates in a static magnetic field is incorrect; the difference is directly related to the spontaneous magnetization due to the Barnett effect.

Although present for all grains, resonance relaxation is most prominent for the smallest ones. When grains rotate very rapidly, as is the case for very small grains, the resonance relaxation effect ensures that \( \chi' \) does not plunge as the rotation frequency increases. As a result, we conclude that small grains (e.g., \( a \leq 10^{-2} \text{ cm} \)) should be paramagnetically aligned. The degree of their alignment depends on the particular phase of the interstellar medium and on the efficiency of spin-lattice relaxation. The latter factor is unfortunately uncertain for very small grains for which the existing laboratory data is not applicable.

If the ultrasmall grains are partially aligned, the implications are as follows: (1) The microwave radiation described in DL98a and DL98b will be polarized—by a few percent—and could have dramatic consequences for experiments (such as MAP or Planck) designed to measure polarization of the cosmic microwave background. (2) If the grain body axes are aligned with \( J \), then absorption by these small grains will contribute to starlight polarization in the ultraviolet, and (3) the infrared emission following absorption of starlight photons by these small grains will also be polarized. However, the contribution to starlight polarization is expected to be small owing to only partial alignment of the grain body axes with \( J \). The infrared emission will be even less polarized because of disorientation of the grain axes (Lazarian & Roberge 1997) during the thermal spike following a photon absorption, i.e., while the infrared emission is taking place.

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REFERENCES

Altschuler, S. A., & Kozyrev, B. M. 1964, Electron Paramagnetic Resonance (New York: Academic)
Atherton, N. M. 1973, Electron Spin Resonance (New York: Wiley)
Bloch, F. 1946, Phys. Rev., 70, 460
Cyvin, S. J. 1982, J. Mol. Struct., 79, 423
Davis, J., & Greenstein, J. L. 1951, ApJ, 114, 206
de Oliveira-Costa, A., et al. 1999, ApJ, 527, L9
Dolgino, A. Z., & Mytrophanov, I. G. 1976, Ap&SS, 43, 291
Draine, B. T., & Lazarian, A. 1998a, ApJ, 494, L19 (DL98a)
———. 1998b, ApJ, 508, 157 (DL98b)
———. 1999a, in ASP Conf. Ser. 181, Microwave Foregrounds, ed. A. de Oliveira-Costa & M. Tegmark (San Francisco: ASP), 133
———. 1999b, ApJ, 512, 740 (DL98b)
Draine, B. T., & Weingartner, J. C. 1996, ApJ, 470, 551
———. 1997, ApJ, 480, 633
Greenberg, J. M. 1982, in Comets, ed. L. L. Wilkening (Tucson: Univ. Arizona Press), 131

Landau, L. D., & Lifshitz, E. M. 1960, Electrodynamics of Continuous Media (Reading: Addison-Wesley), 144
Lazarian, A. 1997, MNRAS, 288, 609
———. 1999a, ApJ, 520, L67
———. 1999b, ApJ, 516, L37
Lazarian, A., & Roberge, W. G. 1997, ApJ, 484, 248
Morrish, A. H. 1980, The Physical Principles of Magnetism (Huntingdon: Krieger)
Pake, G. E. 1962, Paramagnetic Resonance (New York: Benjamin)
Purcell, E. M. 1979, ApJ, 231, 404
Roberge, W. G., & Lazarian, A. 1999, MNRAS, 305, 615
Rouan, D., Leger, A., Omont, A., & Giard, M. 1992, A&A, 253, 498
Tsvetkov, Y. D., Dzuba, S. A., & Gulin, V. I. 1993, in Magnetic Resonance in Carbonaceous Solids, ed. R. E. Botto & Y. Sanada (Washington, DC: American Chemical Society), 443
Waller, J. Z. 1932, Physik, 79, 370