Efficiency limit of nonuniform grid setting in two-dimensional cochlear model

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Abstract: In this study, a two-dimensional (2D) mechanical model of the cochlea, discretized by a nonuniform grid, is applied to investigate the mechanisms that limit the increase in the computational efficiency. To account for experimental findings, cochlear models have become complicated. A cochlear model consists of micro- and macro mechanical models. Many types of micro mechanical model have been proposed. However, macro mechanical models are described by the Laplace equation and show various patterns of the cochlear response depending on the location. Therefore, an efficient step width depends on the location in the cochlea. To resolve this issue, a numerical calculation has been applied to divide the space of a cochlear model into a nonuniform grid and to achieve improved efficiency of the model. However, the limitation of this method remains unclear. To investigate this point, we develop a state space model for 2D cochlear mechanics with a nonuniform grid. Stability analysis and simulations are conducted for the cochlear model with nonuniform and uniform grids. As a result, the number of segments is reduced by 29%. In addition, the execution time is reduced by 10-fold. Therefore, it is shown that a nonuniform grid can efficiently divide the space for cochlear modeling.

Keywords: Cochlear model, Nonuniform grid, State space model, Computational efficiency

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1. INTRODUCTION

In physiological experiments, cochlear processing involves sound frequency analysis [1]. To improve our understanding of cochlear mechanics, prediction of cochlear responses and corresponding mechanisms is important. However, it is difficult to estimate these responses and mechanisms in live humans because the cochlea is embedded in the temporal bone and has a complicated, small-scale structure [1]. Thus, to improve our understanding of the cochlea, an alternative approach is necessary, instead of the experimental approach.

A modeling approach is useful and powerful for cases in which the experimental approach is difficult. Here, cochlear mechanics have been described by a one-dimensional (1D) model of fluid and mechanical dynamics [2,3]. Recently, three-dimensional (3D) models have been shown to better match experimental results compared with 1D models [4]. In this regard, it has been reported that there is no substantial difference in response between 3D and two-dimensional (2D) models [5]. However, to fit the experimental data for living animals, it is clear that cochlear models have become more complicated than the initial models. Therefore, to fit new experimental findings, the computational efficiency of 2D cochlear models must be increased.

The cochlear model consists of a micro- and macro mechanical models. Numerous types of micro mechanical model have been proposed (e.g., Ref. [6]), and it is expected that more models will be developed in the future. In contrast, following the hydrodynamics for an ideal fluid, the macro mechanical model is commonly described by a differential equation, such as the Laplace equation [7]. Therefore, to increase the computational efficiency of cochlear models, optimization for the macro mechanical model is effective.

To improve the efficiency of cochlear models, some groups have attempted to develop specific solvers [8–11]. In this study, we consider a way to increase the efficiency by a different method. It has been reported that errors in the discretization process influence the efficiency [12]. Thus, they focused on rapid variation in the spatial pattern of...
cochlear responses, and applied a nonuniform grid property to fit the spatial pattern for decreasing the number of discretization errors. Furthermore, this approach is independent of the solver. Thus, the use of the nonuniform grid property is a compatible method for the improvement of efficiency, and can reduce the number of segments for fast calculation. However, the calculated result divergences when the number of segments is much smaller. Therefore, there is a limit to increase in the efficiency upon applying a nonuniform grid. For this problem, we introduce a state-space formulation that is commonly used for stability analysis of models.

A 2D cochlear model represented by a state space formulation has shown that errors in the discretized process have no influence on the stability of the model if the degree of discretization is sufficient [13]. Thus, the problem here is determining the sufficient degree of discretization. In previous studies, the degree of discretization was determined by trial and error; however, it can also be determined by conducting stability analysis. Therefore, it is possible to investigate the efficiency from the based on results of stability analysis.

In this study, we apply a 2D mechanical model of the cochlea discretized by a nonuniform grid and investigate the mechanisms that limit of computational efficiency. The structure of this paper is as follows: in Sect. 2, we introduce a 2D cochlear model, apply a nonuniform grid [14] to the state space represented model [13], and investigate the correct step width for the model. In Sect. 3, the number of segments is determined, and the model is evaluated. Finally, discussions and conclusions are presented in Sects. 4 and 5, respectively.

2. MODEL

2.1. Mathematical Description of the Cochlear Model

The cochlea is a coiled duct divided into two chambers by the cochlear partition (CP), as shown in Fig. 1. An incoming sound wave causes the cochlear entrance to vibrate and the wave propagates through a fluid from the base to the apex. Because of the fluid propagation, a pressure difference between the two chambers acts on the CP, and causes the BM to vibrate; therefore, the BM resonates at a frequency dependent on its natural frequency, termed the characteristic frequency (CF).

In the CP, the tectorial membrane (TM) and two types of sensor cell, called the inner hair cell (IHC) and outer hair cell (OHC), are located on the BM. Both of the hair cells sense the gap between the TM and BM, but, they have different functions. The IHC transmits the gap information to the brain. This action can be considered as encoding sound in neural information. On the other hand, the OHC feeds back the obtained information to itself, and controls their motility. Furthermore, it is suggested that this OHC motion amplifies the BM vibration, widens the dynamic range, and produces nonlinear characteristics [15]. However, the mechanics involved in the OHC motion have not yet been experimentally understood.

A modeling approach is complementary to experiments. The cochlear model consists of macro mechanical and micro mechanical models. The macro model simulates an incoming sound wave, which generates a pressure difference \( p \) between the two chambers, and causes the CP to vibrate the BM. Most macro mechanical models are uniquely described. On the other hand, the micro mechanical model shows that the interactions between BM and OHC enhance BM motion. Although the specifics of this interaction are unclear, it is caused by a complicated process that can be probed by fitting the cochlear model to experimental data [4]. To summarize the modeling studies, macro and micro-cochlear mechanics are equivalent and dependent on each of the proposed models. Therefore, in this study, we attempt to improve the computational efficiency of the macro-cochlear model.

For macro mechanics in most of the 2D cochlear models, \( p \) is required to satisfy Laplace’s equation:

\[
\frac{\partial^2 p(t)}{\partial x^2} + \frac{\partial^2 p(t)}{\partial y^2} = 0, \quad 0 < x < L, \quad 0 < y < H, \quad (1)
\]

where \( t \) is a time series; \( x \) and \( y \) are the longitudinal location and height location, respectively; and \( L \) and \( H \) are the length and height of the cochlear canal, respectively. The boundary conditions are given as

\[
\frac{\partial p(t)}{\partial x}_{|x=0} = -2p\bar{\mu}_{SR}(t) = 2p\bar{\mu}_{SO}(t), \quad (2)
\]
where \( \bar{w}_{SR} \) and \( \bar{w}_{SO} \) are the linearly superposing components of the acceleration of the stapes; they are the acceleration due to the pressure in the ear canal and the internal pressure responses in the cochlea at \( x = 0 \), respectively. \( \bar{w} \) is the acceleration of the CP, and \( \rho \) is the density of the cochlear fluid.

### 2.2. State Space Formulated Cochlear Model with a Nonuniform Grid

In this section, we apply the finite difference method with a nonuniform grid [14] to the 2D cochlear model represented by the state space formulation [13]. To discretize the 2D cochlear model, we introduce the central difference operator \( \Delta_0 \), which acts on individual elements of a sequence \( z \):

\[
(\Delta_0 z)_k = z_{k+1} - z_{k-1}.
\]

The operators are defined for all \( k = 0, \pm 1, \pm 2, \ldots \) with a given step size. By applying this operator for the governing equation in Eq. (1), we derive the following equations

\[
\frac{\Delta_0^2}{(\Delta x)^2} p_{n,m} + \frac{\Delta_0^2}{(\Delta y)^2} p_{n,m} = 0.
\]

Equation (7) is divided into \( N \) and \( M \) segments with varying step widths, \( \Delta x \) and \( \Delta y \), respectively.

A procedure for constructing a state space model of 2D cochlear mechanics using a finite difference scheme with a uniform grid has been proposed [13]. In this study, we develop a state space model of 2D cochlear mechanics with a nonuniform grid [14]. By applying a nonuniform grid to the discretized model represented as Eq. (7) with the boundary conditions in Eqs. (4)–(5), we can write the relationship between the displacement of the cochlear partition \( w_n \) and the pressure \( p_n \) at the \( n \)-th segment as

\[
\Delta_0^2 p_n + \Omega p_n = \bar{w}_n,
\]

where

\[
p_n = (p_{n,1}, \ldots, p_{n,M})^T,
\]

\[
\bar{w}_n = \frac{4\rho}{\Delta y_0} (w_n, 0, \ldots, 0),
\]

\[
\Omega = \begin{pmatrix}
-\frac{2}{(\Delta y_1)^2} & \frac{2}{(\Delta y_1)^2} & 0 \\
\frac{b_2^y a_1^y}{(\Delta y_1)} & -\frac{b_2^y(1 + a_2^y)}{\Delta y_1} & b_2^y \\
0 & \frac{2}{(\Delta y_m)^2} & -\frac{2}{(\Delta y_m)^2}
\end{pmatrix},
\]

\[
a_m^y = \frac{\Delta y_m}{\Delta y_{m-1}},
\]

\[
b_m^y = \frac{2}{\Delta y_m \Delta y_{m-1}(1 + a_m^y)}.
\]

Next, we expand the operator \( \Delta_0^2 \) in Eq. (8). The displacement term \( \bar{w} \), the pressure term \( p \), and the source term \( q \) obey

\[
F \cdot p(t) - \bar{w}(t) = q(t),
\]

where

\[
F = \begin{pmatrix}
-\frac{2}{\Delta x_1} & \frac{2}{\Delta x_1} & 0 \\
\frac{b_2^x a_1^x I}{\Delta x_1} & -\frac{b_2^x(1 + a_2^x)}{\Delta x_1} & b_2^x I \\
0 & \frac{2}{\Delta y_m} & -\frac{2}{\Delta y_m}
\end{pmatrix},
\]

\[
p = (p_1^T \cdots p_N^T)^T,
\]

\[
w = \frac{4\rho}{\Delta y} \left( \frac{\Delta y}{4\rho} \bar{w}_1^T, \bar{w}_2^T, \ldots, \bar{w}_{N-1}^T 0 \right)^T,
\]

\[
q = (\bar{w}_{SR}^T 0^T \cdots 0^T)^T.
\]

The model equation, Eq. (14), is equivalent to Eq. (11) of Ref. [13]. Therefore, the expansion of the following expressions is equivalent to their method [13], and the following state space equations are derived:

\[
\dot{x}(t) = Ax(t) + Bu(t),
\]

\[
y(t) = Cx(t) + Du(t),
\]

where \( x, u, \) and \( y \) are the state variable, input, and output of the model. To evaluate instabilities in the system, the eigenvalues of the system matrix \( A \) are calculated. When the real parts of these eigenvalues are positive, the system is unstable. The frequency response of the system for the input angular frequency \( \omega \) is obtained from the state space equation and can be described as

\[
Y(j\omega) = (D + C(j\omega I - A)^{-1}B)U(j\omega),
\]

where \( j \) is the unit imaginary number.

A simulation was conducted using the original parameters [13] that reproduce a cat’s cochlear response.
2.3. Fitting a Nonuniform Grid for Various Parameters in the Cochlear Model

Figure 2 shows the grid characteristics in the developed model for a nonuniform grid. The step widths at the base and for the state in contact with the micro mechanical cochlear model are smaller than those for the apex site and the state in contact with the cochlear shell. In this section, we explain why this nonuniform grid property is employed.

To determine efficient step widths, $\Delta x_n$ and $\Delta y_m$, where $n$ and $m$ are the segment numbers at each position, the discretized positions $x_n$ and $y_m$ were fitted into the wavelength of BM motion and the pressure gradient, respectively. The discretized position $x_n$ was fitted

$$x_n = \frac{A_x}{B_x} \exp(B_x \cdot n) + C_x \cdot n + D_x,$$  \hspace{1cm} (22)

where $A_x$, $B_x$, $C_x$, and $D_x$ are fitting constants. Figure 3 shows that the BM wavelength increases with distance from the cochlear entrance. To fit this curve with Eq. (22), the following parameters were used

$$B'_x = \exp(B_x),$$  \hspace{1cm} (23)

$$A_x = \frac{(r - 1)B_x}{(r - 1)(B'_x - 1) + B_x(B'_x - r)},$$  \hspace{1cm} (24)

$$C_x = 1 - \frac{A_x(B'_x + 1)}{B_x},$$  \hspace{1cm} (25)

$$D_x = -\frac{A_x}{B_x},$$  \hspace{1cm} (26)

here, the constants $B$ and $r$, respectively represent the steepness of the curve and the ratio of step widths between the base and the apex. Both of these values were set to 3. Thus, the step width $\Delta x_n$ is calculated as

$$\Delta x_n = -x_n + x_{n+1}.$$  \hspace{1cm} (27)

The results of the calculation in Fig. 3 show that the step width $\Delta x_n$ varies with the cochlear length and fits the wavelength of BM motion.

Next, we determine the discretized position $y_m$ from the pressure gradient for the $y$-axis. The pressure along the $y$-axis monotonically decreases from the BM to the cochlear wall, as shown in Fig. 4. To ensure that the segments in the pressure gradients for the $y$-axis are evenly spaced, the following equation was used:
where \( A_y \) and \( B_y \) are fitting constants. To fit the pressure curve shown in Fig. 4 with Eq. (28), the parameter \( A_y \) is set to

\[
A_y = -\frac{1}{H} \log B_y,
\]

where \( H \) is the height of the canal, and \( B_y \), which represents the ratio of maximum to minimum pressure is set to 10\(^{-3}\). Thus, the step width \( \Delta y_m \) is calculated as

\[
\Delta y_m = -y_m + y_{m+1}.
\]

The results of the calculation in Fig. 4 shows that the step width \( \Delta y_m \) is determined such that the pressure gradients are equal.

3. RESULTS

3.1. Pole Distribution

To determine a suitable number of segments \( N \), it is useful to analyze the stability of the state space model of 2D cochlear mechanics by calculating the eigenvalues of the system matrix \( A \), as described in Eq. (20). Figure 5 shows these eigenvalues for the state space model, where the feedback gain \( \gamma \) was set to 0.5. In this simulation, the numbers of segments \( N \) and \( M \) were set to 160 and 20, respectively. The pole calculation shows that the model with a nonuniform grid is stable since there are no poles with positive real parts, as shown in Fig. 5. In contrast, unstable poles are obtained for the model with a uniform grid.

Regarding the sensitivity of the model instability with respect to the number of segments \( N \), Fig. 6 shows that the rate of unstable poles depends on the number of segments \( N \). For the model with a nonuniform grid, unstable poles are obtained for a small number of segments \( N \), whereas no unstable poles are obtained when the number of segments exceeds 150. In contrast, for the model with a uniform grid, the number of unstable poles decreases with an increasing number of segments \( N \); however, unstable poles still remain when the number of segments is 150.

From the above consideration, we chose 160 and 220 as suitable numbers of segments \( N \) for the nonuniform and uniform grids, respectively. In the subsequent simulation, these values are used for each grid condition.

3.2. BM Responses in the Frequency Domain

To evaluate the influence of the number of segments \( M \) on the BM response, the frequency responses are calculated, as formulated in Eq. (21). Figure 7(a) shows the BM velocity along the cochlear length, calculated for incoming sounds of three different frequencies, with the number of segments \( M \) set to 8. In this case, the peak locations of the BM response, which depends on the input frequency, are similar for both grid properties. However, the peak value of the BM responses calculated from the model with the nonuniform grid is larger than that for that with the uniform grid. In contrast, the patterns of the BM responses are similar for both grid properties, with the number of segments \( M \) set to 20, as shown in Fig. 7(b).

The BM motion enhancement is realized through feedback via an active process with the gain \( \gamma \). The experimental results show that the BM enhancement is approximately 50 dB [16]. Figure 7 indicates that a suitable number of segments \( M \) is required to calculate the BM responses via the feedback process. Next, we evaluate the influence of the number of segments \( M \) on BM enhancement. This influence is calculated using the BM velocity at the CF position when the feedback gain \( \gamma \) is set to zero.

Fig. 5 Pole distribution of the cochlear model, where the numbers of segments \( N \) and \( M \) were set to 160 and 20, respectively. The black and gray poles were obtained from the models with nonuniform and uniform grids, respectively.

Fig. 6 Rate of unstable poles varies with the number of segments \( N \), where the number of segments \( M \) was set to 20.
Figure 8 shows: the BM enhancement for an incoming sound with three different frequencies under both grid conditions, the BM enhancement is maximum. In particular, these enhancements are responsible for the physiological experimental data [16] for 4 and 16 kHz tones at the base and middle turn of the cochlea. However, the model with a nonuniform grid reaches the maximum enhancement with a smaller number of segments $M$ than the model with a uniform grid.

On the basis of the above results, we chose 8 and 20 as suitable numbers of segments $M$ for the nonuniform and uniform grids, respectively. These values were used to maximize the BM enhancement. Therefore, the number of segments was changed from $220 	imes 20 = 4,400$ to $160 	imes 8 = 1,280$, which led to a 29% segment reduction. In the subsequent simulation, these values are used under each grid condition.

### 3.3. BM Responses in Time Domain

The pole distribution and BM responses were calculated in the frequency domain. In addition, the state space model formulated in Eq. (14) is defined in the time domain. To investigate the dynamics of the model, time domain simulations are necessary. Temporal features evoked by a click tone that includes a broad frequency of incoming sound is typical for such simulations. The models are simulated with a 30 μs click tone, and the temporal responses of the BM are shown at the base, middle, and apex sites for 5 ms, as shown in Fig. 9. Under both grid conditions with different numbers of segments $N$ and $M$, the BM vibrates from the base to the apex site with identical amplitudes and timings. Thus, with a smaller number of segments, the model with a nonuniform grid can...
simulate BM motion as well as the model with a uniform grid.

3.4. Effect of Segmentation Reduction on Execution Time

The aim of this study is to investigate methods of improving the computational efficiency of a 2D cochlear model with a finite difference scheme. To evaluate the effect of segmentation reduction, we compute the elapsed time in calculating the pole distribution, frequency response, and 5 ms click response shown in Figs. 5, 7, and 9, respectively. The models were implemented in Python and tested on a computer with 4.2 GHz, an Intel Core i3 processor, and an 8 GB RAM. Figure 10 shows the elapsed time under the two grid conditions. For all simulations, the elapsed time is reduced by approximately 10-fold by applying a nonuniform grid.

4. DISCUSSION

4.1. Mechanisms in Limit of Efficiency in Cochlear Model with Nonuniform Grid

In this study, we developed a 2D cochlear model with a nonuniform grid; the entrance and the regions near the BM sites have a fine grid, and the apex and the regions near the cochlear shell sites have a coarse grid, as shown in Fig. 2. This model can be solved with fewer segments in both the frequency and time domain than the model with a uniform grid, as shown in Figs. 7 and 9. Furthermore, the segment numbers are determined by the number of unstable poles and the cochlear amplifier gain, as shown in Figs. 5 and 8. Figure 10 shows that the execution time required for the model with a nonuniform grid is approximately 10-fold shorter than that for a uniform grid. Therefore, the developed model can increase computational efficiency beyond its limit.

The pole distribution indicates that the model with a nonuniform grid is more stable than that with a uniform grid, as shown in Fig. 5. This trend is particularly obvious at higher frequencies, whereas the effect is weaker at lower frequencies. Therefore, the higher-frequency site corresponding to the base site is required for a fine grid. In contrast, for the lower-frequency site corresponding to the apex site, the step width is much smaller in previous cochlear modeling studies, as shown in Fig. 2. Thus, Fig. 6 shows that the model with a nonuniform grid can be implemented with a smaller number of segments \( N \) than the model with a uniform grid. Furthermore, to improve computational efficiency, the space division on the log scale as the non uniform grid is more effective than that on the linear scale as the uniform grid BM vibration pattern dependence shown in Fig. 3.

The extent of BM enhancement increases with the number of segments \( M \), as shown in Fig. 8, and is affected by the pressure generated by the OHC models. As shown in Fig. 4, the pressure in a cross section reaches a maximum at the BM and exponentially decreases toward the cochlear shell. Therefore, a uniform grid cannot divide the space to obtain identical pressure variations. However, with a nonuniform grid, the space is evenly divided, as shown in Fig. 4. Therefore, the nonuniform grid can accurately produce exponentially decreasing pressure variations with a small number of segments.

4.2. Applying a Nonuniform Grid to Other Cochlear Models

The nonuniform grid scheme can be applied to most cochlear models. A cochlear model consists of micro- and macro mechanical models. Numerous types of micro mechanical model have been proposed [6]. In contrast, most macro mechanical models are described by the Laplace equation, where the cochlear fluid is assumed to be ideal [7]. In this study, as described in Sect. 2.2, we applied the nonuniform grid scheme to the macro mechanical model, which is independent of the micro mechanical model. Furthermore, specific fast solvers for cochlear models [8–11] can be applied to the nonuniform grid scheme. Therefore, this method can increase the efficiency of most cochlear models.

When the grid is transformed from being uniform to nonuniform, the issue arises that the resulting output from the model might be transformed. In this regard, variation in the spatial step width has little influence on the computational results [13]. Therefore, in this study, we consider the nonuniform grid scheme to be sufficiently robust for transforming.
5. CONCLUSION

In this study, we attempted to reduce the number of segments in a 2D cochlear model to improve the efficiency of numerical calculations. To investigate this problem, a finite difference scheme with a nonuniform grid [14] was applied to a 2D cochlear model represented by a state space formulation [13]. Simulations were performed for the developed model with a nonuniform grid and a model with a uniform grid. The simulation results show that the pole calculation and BM responses in both the frequency and time domains can be obtained using the model with a nonuniform grid with fewer segments and a shorter execution time than the model with a uniform grid. Therefore, a nonuniform grid can divide the space effectively for efficient computation in cochlear mechanics. Furthermore, the nonuniform grid scheme can be applied to most cochlear models to improve the computational efficiency.

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