Quantum communication protocols using the vacuum

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We speculate what quantum information protocols can be implemented between two accelerating observers using the vacuum. Whether it is in principle possible or not to implement a protocol depends on whether the aim is to end up with classical information or quantum information. Thus, unconditionally secure coin flipping seems possible but not teleportation.

I. INTRODUCTION

Entanglement is in the eyes of the beholder. Unitary transformations that affect the definition of one’s systems change, in general, the amount of entanglement in a given state. For instance, the Lorentz transformation of the spin degrees of freedom of a particle depends on its momentum. Thus the entanglement between the spin and momentum of a particle is not a Lorentz-invariant concept [1]. One observer may see a product state, while another believes there is entanglement. Similar discussions of the Lorentz invariance or the lack thereof of entanglement can be found in Refs. [2,3]. For another example, take a state containing a single circularly polarized photon. When written in terms of linear polarization, this state becomes \(|0\rangle + i|1\rangle/\sqrt{2}\), which seems to be maximally entangled (for similar but more complicated and less boring cases see [4]). In both types of examples, however, the apparent entanglement is always local. As such, it cannot be used for any nontrivial quantum communication protocols.

Here we discuss a well-known phenomenon that does produce nonlocal entanglement, the Unruh effect [5–7]. It involves just the vacuum, and the unitary transformations arise when one describes accelerating observers. We investigate which, if any, quantum communication protocols could be implemented using this resource. We consider protocols involving two accelerated observers, in contrast to Ref. [8], which considers quantum teleportation involving one inertial and one accelerating observer.

II. THE UNRUH EFFECT

Suppose Alice is accelerating at a uniform acceleration \(a\). As is well known [5], she will perceive the Minkowski vacuum state (of, say, the electromagnetic field) as a mixed thermal state with equivalent temperature \(k_B T = |a|/(2\pi c)\). Since the transformation from an inertial to an accelerating frame is unitary, however, the vacuum should be transformed into a pure state, not a mixed state. Indeed, the state of the modes inside Alice’s event horizon only appears mixed because it is entangled with modes that lie outside that horizon. More precisely, each mode is entangled with one “mirror” mode, a mode propagating along a trajectory that is the mirror image relative to the appropriate event horizon. For each pair of mirror modes of frequency \(\omega'\) (as measured by the accelerating observers), the entangled state is in fact a two-mode squeezed state of the form

\[
|\Psi\rangle = \sqrt{1 - \mu^2} \sum_n \mu^n |n\rangle |n\rangle,
\]

where \(\mu = \exp(-\pi \omega' c/a)\). Mirror modes are the appropriate modes for an observer Bob accelerating uniformly at the same acceleration \(a\) but in the opposite direction along a trajectory that is, again, a mirror image relative to the same event horizon. That the entanglement and nonlocal correlations present in the state (1) may be real in the sense that they can be measured and perhaps even exploited has been discussed before [2,9–11,8].

The Unruh effect can be understood by considering the transformation between creation and annihilation operators from Alice’s frame of reference to that of a Minkowski observer, Mork. The transformation is of the form

\[
a' = (a - \mu \dot{a}^\dagger) / \sqrt{1 - \mu^2} \]

\[
\dot{a}' = (\dot{a} - \mu a^\dagger) / \sqrt{1 - \mu^2}.
\]

where we absorb irrelevant phase factors in the definitions of the mode operators. Here we use notational conventions that primed operators and variables correspond to accelerating observers, and that operators with a tilde correspond to mirror modes. The modes here are assumed to be localized wave packet modes, as constructed in [6]. The fact that a creation operator appears in the transformation of an annihilation operator, distinguishes (2) from standard unitary transformations of modes [4].

III. QUANTUM COMMUNICATION PROTOCOLS

The questions we consider now are (i) what useful quantum information tasks might Alice and Bob perform with the entangled state (1)? (ii) how does a Minkowski observer, Mork, describe their actions and make sense of it? After all, according to Mork, Alice and Bob share nothing but the vacuum and are causally disconnected,
and so it may seem they should not be able to perform any interesting protocols. We will consider several quantum information processing protocols that are known to rely on entanglement and discuss to what extent they can be implemented by Alice and Bob.

Concerning question (i), there is an important distinction between two types of quantum communication protocols: those that terminate with at least one party holding a quantum state, and those that terminate with all parties holding purely classical information. In the former case the desired quantum states typically exist only in the eyes of Alice and Bob, and thus, in the scenarios considered here, only as long as they keep accelerating. Clearly this does not allow Alice and Bob to ever communicate, not even classically. This does restrict at least the usefulness of the protocol and sometimes it prevents the protocol from being executed at all. In the latter type of protocols, however, Alice and Bob are both free to decelerate after having performed the required quantum operations (since we presume, hopefully correctly, that classical information, unlike quantum information, is not affected by deceleration), and thus may communicate afterwards. This then may lead, apart from practical considerations, to useful implementations of certain quantum protocols. Our primary goal here is to discuss in detail an example of each type, to demonstrate both the potential and the limitations of vacuum entanglement for quantum communication protocols. We also briefly mention various other protocols.

Concerning question (ii) we note that Alice’s and Bob’s local operations appear nonlocal to Mork, and vice versa. The parameter $\mu$, which measures the strength of the Unruh effect and the amount of nonlocality, written in Mork’s coordinates is equal to (using Ref. [6])

$$\mu = \exp(-\pi^2 D/\lambda),$$

where $D$ is the distance between the mirror trajectories and $\lambda$ the wavelength. (In contrast, recall that for the accelerating observers $\mu$ does not depend on the distance between the relevant modes.) In order to have any appreciable effect, at the moment Alice and Bob wish to use their entanglement, they must be within a distance $D \sim \lambda/\pi^2$ of each other, that is, within the coherence length of the vacuum fluctuations [10]. This is how Mork can make some physical sense out of the nonlocal character of Alice’s and Bob’s actions and of the fact that, counter to Mork’s expectations, some of their protocols seem to work.

### A. Teleportation

Let us start with one of the more famous protocols, quantum teleportation [12]. Indeed, a two-mode squeezed state of the same form (1) can be used for exactly that purpose [13,14]. We also note that teleportation with the resource (1) is briefly discussed in [2]. A later paper by the same authors considers teleportation involving one inertial and one accelerating observer [8].

Teleportation is a clear example where the aim is to end up with a quantum state, and where classical communication is necessary. Thus the prospects for Alice and Bob are bleak. Indeed, standard teleportation is not possible, but a weaker variant of it is. This weaker variant is in essence an example of quantum steering [15]—the process whereby a local choice of measurements by Alice can steer Bob’s half of an entangled state to any ensemble of his local density operator.

We first describe the experiment from Alice’s frame of reference. Alice uses two modes, one mode $T$ contains the state she wishes to teleport, the other, $E$ contains half of the entangled state. Similarly, Bob’s corresponding mirror modes are denoted by $\tilde{T}$ and $\tilde{E}$. Let us assume Alice wishes to teleport a coherent state. In order to prepare that state, Alice first cools down the mode $T$, i.e., removes all (Rindler) photons from it. Subsequently she applies a displacement operation. (Cooling is not a necessary part of the protocol. Alice could just teleport the action of the displacement operation on the thermal state as it is. In that case the description becomes more tedious and less clear.) For convenience, we assume that Bob, too, cools down his mode $\tilde{T}$ although this is not necessary at all for the teleportation protocol. The most convenient way to describe teleportation then is by using the Wigner function [14], as at all times the states of the modes involved will be Gaussian. Moreover, it is customary to use Hermitian quadrature variables $X$ and $P$ instead of $a$ and $\alpha$, defined through $a = X + iP$, and corresponding eigenvalues $x$ and $p$. For the two-mode squeezed state one has (leaving out irrelevant normalization factors)

$$W_{E,\tilde{E}} \sim \exp(-\frac{1 + \mu}{1 - \mu} |(x'_E - x'_\tilde{E})|^2 + (p'_E + p'_\tilde{E})^2)$$

$$-\frac{1 - \mu}{1 + \mu} |(x'_E + x'_\tilde{E})^2 + (p'_E - p'_\tilde{E})^2|.$$  

Similarly, a coherent state $|\alpha_0\rangle_T$ is described by

$$W_T \sim \exp(-2(x'_T - x_0)^2 - 2(p'_T - p_0)^2),$$

where $\alpha_0 = x_0 + ip_0$.

Alice then performs a joint measurement on her modes $T$ and $E$. She measures the commuting observables $X_E + X'_T$ and $P_E - P'_T$, for example, by homodyne detection with a strong local oscillator field. After this measurement, which we assume to have outcomes $X$ and $P$ respectively, Alice then ascribes the following quantum state to mode $E$ on Bob’s side

$$W'_E \sim \exp\bigg(-\frac{2 - 2\mu}{3 + \mu} |(x'_E + x_0 - X)|^2 + (p'_E + p_0 - P)^2\bigg)$$

$$-\frac{2 + 2\mu}{3 - \mu} |(x'_E - x_0 - X)^2 + (p'_E - p_0 - P)^2|.$$  

When $\mu$ approaches unity, the state on Bob’s side reduces to a coherent state, but displaced by an amount
\[ \beta = X + iP. \] In the standard teleportation protocol Alice would send Bob the classical outcomes \( X \) and \( P \) and Bob would displace his state by an amount \(- \beta\) to retrieve the state Alice teleported. Clearly, this step is not possible. According to Bob, his state will always remain a mixed thermal state, but based on Alice’s information she assigns Bob’s system the state \( (5) \).

It is easy, albeit somewhat tedious, to write down the Wigner functions Mork observes, by using the inverse transformations of \( (2) \). Clearly, they will remain Gaussians for Mork as well. Here, however, we just focus on the local vs. nonlocal aspects of Alice’s actions. Alice and Bob start out with the vacuum. Then both Al-
sians for Mork as well. Here, however, we just focus on transformations of \( (2) \). Clearly, they will remain Gaussian, with the same outcomes \( X \) and \( P \), respectively. This will create an entangled state of all 4 modes involved, according to Mork. This is in contrast to Alice’s description, who believes her 2 modes have been disentangled from Bob’s modes. According to Mork, no teleportation takes place.

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**B. Secure coin flipping**

Two-party cryptographic protocols involve two antagonistic parties, Alice and Bob, who wish to complete some information processing task. In classical information theory it has been proven that no two-party protocols exist which have “information theoretic” security [16–18]. In quantum cryptography protocols do exist with various degrees of quantum information theoretic security, and thus examining these protocols provides a readily quantifiable way of distinguishing classical from quantum information theory.

It is standard to assume in two-party quantum cryptography that the initial state of systems held by Alice and Bob is separable, i.e. of the form \( |0\rangle_A |0\rangle_B \). However it is interesting to note that if Alice and Bob share prior trusted entangled states then some (otherwise impossible) arbitrarily secure quantum cryptographic protocols become possible, while others remain impossible. For example, if they share a maximally entangled state of two qubits, then an arbitrarily secure coin flip is trivially possible - the coin flip outcome is simply the result each party obtains by measuring their half of the entangled pair in an orthogonal basis. By contrast, as can be deduced from [18], the sharing of a prior trusted entangled state does not give Alice and Bob the ability to perform an arbitrarily secure bit commitment. Thus there is an intricate hierarchy of the security obtainable in these protocols with respect to any initially trusted entanglement resources.

What we are proposing here is that the (Minkowski) vacuum state \( |0\rangle_A |0\rangle_B \) can also be considered a “prior trusted” state. Since, as discussed above, this is in fact also a (Rindler) entangled state, we surmise that it can, in fact, be used to implement a secure coin flip.

We first define the task of coin flipping more precisely:

**Strong Coin Flipping**: Alice and Bob implement a protocol, at the end of which each infers the outcome of the protocol to be one of ‘0’, ‘1’ or ‘fail’. If both are honest, then they agree on the outcome and find it to be 0 or 1 with equal probability. If party \( X \) cheats, while his or her opponent is honest, then \( X \) cannot make the probability of the opponent finding the outcome 0 to be greater than \( 1/2 + \epsilon_A \) and cannot make the probability of the opponent finding the outcome 1 to be greater than \( 1/2 + \epsilon_B \). The parameters \( \epsilon_A, \epsilon_B \), which specify the security of the protocol, must each be strictly less than 1/2.

The protocol is considered arbitrarily secure if, and only if, the parameters \( \epsilon_A, \epsilon_B \) can simultaneously be made to approach zero.

Intuitively speaking, such a coin flipping protocol is meant to result in a random bit outcome, and neither Alice nor Bob should be able to bias the bit value towards either 0 or 1. It has been shown by Kitaev, that within the standard quantum communication paradigm wherein Alice and Bob start with a state of the form \( |0\rangle_A |0\rangle_B \) and build up an entangled state via rounds of communication, all coin flipping protocols satisfy \( (1/2 + \epsilon_A)(1/2 + \epsilon_B) \geq 1/2, b = 0, 1 \). Thus, arbitrarily secure quantum coin flipping within this paradigm is impossible. The best known protocols [17,18] do not even saturate Kitaev’s lower bound.

We should emphasize that in two-party cryptographic protocols there is a basic presumption that each party feels secure about their own laboratory. In fact, it is desirable that this need be the only thing they feel secure about - i.e. we presume that the parties should not have to feel secure about things outside their own lab. We note
here explicitly that, for instance, changing boundary conditions on the field in one of the two Rindler wedges does not modify the thermal spectrum seen by an observer in the other wedge, as was pointed out by Pringle [7]. The protocol we consider here is as follows:

**Unruh based coin flipping:**

From Alice’s point of view, an instance of the protocol is specified by two points in spacetime chosen to be located inside Alice’s lab. At time $t = 0$ Alice, who is uniformly accelerating with acceleration $a$, is instantaneously at rest (see Figure 1) in Mork’s reference frame. At that moment, Alice turns on a detector $D_2$ (see Fig. 1), if she records a photon then the outcome of the coin flip is 1, otherwise it is 0. In order for her to trust this result she must have checked before $t = 0$ whether the corresponding field mode was in fact in the Minkowski vacuum. She does that by using an inertial detector $D_1$, which must be inside her lab for a sufficient amount of time that she can verify the detector and the state of the relevant localized wavepacket.

Bob uses a similar procedure by traveling on the “mirror trajectory” (accelerating in the opposite direction) such that he is detecting the other half of the Unruh entangled state.

![FIG. 1. Upside-down spacetime diagram indicating coin flipping using the Unruh effect.](image)

The above protocol is slightly unconventional. However, intuitively speaking, it is simply relying on the fact that what Minkowski observers consider to be a separable vacuum state $|0\rangle_A|0\rangle_B$, transforms to the entangled state (1) for accelerating observers. The purpose of the first detector measurement at time $t = 0$ is to ensure that the mode $k$ (see Fig. 1) which will determine the coin flip outcome is in the correct Minkowski initial state. (A cheating Bob could have his friend try and populate this mode prior to it entering Alice’s laboratory, for example).

Several other technical issues arise. Clearly $a$ must be chosen to ensure that the probability of detector $D_2$ registering 0 photons is the same as that of registering one or more photons – i.e. such that $\mu^2 = 1/2$. We also wish this detector to be sensitive to a localized, travelling Unruh wavepacket. A formal quantization of Rindler space in terms of such modes can be found in [6], and from these results one can infer the appropriate detector mode function responses required.

Another issue is that localized detectors necessarily will register “dark counts”, with some (small) probability. This would affect measurements done at small accelerations, but not at the impractically large accelerations we are considering.

We conclude with a few observations. Firstly we must assume that Alice’s laboratory is large enough to contain both detectors at the appropriate spacetime points. (Alice does not, however, require other guarantees about the whereabouts of Bob). Also, the further Alice and Bob are apart, the larger the acceleration has to be. Thus there are nontrivial tradeoffs between the various physical requirements of such a protocol.

### C. Other protocols

Here we briefly consider other protocols of both types mentioned previously:

1. **Dense coding**

Dense coding [19] is a protocol that allows one to send 2 classical bits of information by sending one qubit, provided one prepared an entangled state in advance. Here, however, at least the receiver would have to keep accelerating so as to keep the entangled state, but in that case the receiver cannot receive anything from the sender. Thus, it seems not possible to use the Unruh effect for dense coding.

2. **Quantum Key Distribution**

In Quantum Key Distribution Alice and Bob wish to share secret classical bits. The way they can agree on classical bits is very much as in the coin flipping protocol. This time, though, they do trust each other, but not a possible eavesdropper Eve. Just as in the Ekert protocol [20], they can check for Eve’s existence by performing the appropriate Bell measurements. Indeed, it is well-known Bell inequalities are violated (to the maximum extent, in fact) in the vacuum of any quantum field theory [9]. They do have to communicate classically, in order to
check their measurement results, but they can do that afterwards.

3. Bit commitment

As mentioned above, bit commitment [21] would require even more than a trusted entangled state. We suspect this protocol is not possible with the Unruh effect, even though only classical information is needed in the end.

IV. SUMMARY

Certain two-party quantum communication protocols that require entanglement can be performed with just the vacuum. Typically, protocols whose goal it is to produce a quantum state will not work in any useful way, but two-party protocols aimed at establishing purely classical information may work. In particular, unconditionally secure coin flipping is possible, so is unconditionally secure key distribution. On the other hand, teleportation between two accelerating observers is only possible in a weaker version.

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