Table of integrals. Asymptotical expressions for non–collinear kinematics.

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Abstract

We present a set of Feynman integrals appearing in calculations of different QED processes to the one–loop accuracy. We consider scalar, vector, and tensor integrals with two, three, four and five denominators. The cases of equal and different fermion masses are considered. Results obtained are valid in the region where all kinematical invariants are large compared to the masses squared. Mass corrections for some scalar integrals in the case of different fermion masses are also given.

1 Introduction

This paper is an electronic version of our preprint [1] from year 1998. Since that time the Table of integrals was used by our group in several calculations of 1-loop QED radiative corrections to various processes at high energies. It might be useful also for some more applications, estimates and cross checks.

We give below a list of 4–dimensional integrals in momentum space which were used in calculation of 1–loop radiative corrections to the inelastic processes of kind $e^+e^- \rightarrow e^+e^-\gamma$ and $\mu^-e^- \rightarrow \mu^-e^-\gamma$ at high energies. For definiteness we consider the case when all the scalar products of external 4–momenta are large compared to the mass of external real particles squared $p_ip_j \gg p_i^2 = m_i^2$. We omit systematically terms of order $m_i^2/p_ip_j$ compared to ones of order unity and restrict ourselves to consideration of scattering type Feynman diagrams only. Remaining ones can be obtained by application of crossing and analytical continuation transformations. We introduce the ultraviolet, infrared cut–offs and do not use the dimensional regularization. In paper of one of authors [4] 1–loop integrals, related to the $2 \rightarrow 2$ type Feynman diagrams, were considered. Here we give a set of integrals for description of inelastic $2 \rightarrow 3$ type processes. Our paper is organized as follows. In the first part we consider the 4–momentum integrals, appearing in one–loop vertex and five–point diagrams. The cases when fermions have different and equal masses are considered.
They may be helpful in constructing the five–point diagrams with one or two external off–shell mass particles. Scalar, vector and tensor integrals are considered up to the case of four denominators. Tensor integrals are calculated up to a third rank.

2 Five–point Feynman diagrams

The case of equal fermion masses.

Typical process: \( e^+e^- \rightarrow e^+e^-\gamma \)

2.1 Notations

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{five_point_diagram.png}
\caption{Five–point Feynman diagram}
\end{figure}

\[ I_{ijklm}^{1,\mu,\nu} = \int \frac{d^4k}{i\pi^2} \frac{1}{(i)(j)(k)(l)(m)} \]

\[(1) = (p_1 - k)^2 - m^2, \ (2) = (p_1 - k_1 - k)^2 - m^2, \ (3) = (p_2 + k)^2 - m^2, \]
\[(4) = (q - k)^2 - \lambda^2, \ (5) = k^2 - \lambda^2. \]

Invariants

\[ \chi_1 = 2p_1k_1, \ \chi_1' = 2p_1'k_1, \ \chi_2 = 2p_2k_1 = s - s_1 - \chi_1, \]
\[ \chi_2' = 2p_2'k_1 = s - s_1 - \chi_1', \ s = (p_1 + p_2)^2, \ s_1 = (p_1' + p_2')^2, \]
\[ t = q^2, \ q = p_2' - p_2, \ t_1 = q'^2 = t + \chi_1 - \chi_1', \ q' = p_1' - p_1, \]
\[ \chi_1 + \chi_2 = s - s_1, \ p_1^2 = p_2^2 = p_1'^2 = p_2'^2 = m^2, \ k_1^2 = 0. \]

\[ L_t = \ln \left( \frac{t}{m^2} \right), \ L_s = \ln \left( \frac{s}{m^2} \right), \ L_\Lambda = \ln \left( \frac{\Lambda^2}{m^2} \right), \]
\[ L_\lambda = \ln \left( \frac{\lambda^2}{m^2} \right), \]
\[ \text{Li}_2(z) = - \int_0^z \frac{dx}{x} \ln(1 - x), \]
\[ p^2 = m^2 - x\bar{s} - i0, \ p_1^2 = m^2 - x\bar{s}_1 - i0, \ \text{where} \ \bar{x} = 1 - x \]

Throughout the paper \( \Lambda \) is an UV cut–off parameter and \( \lambda \) is a fictitious photon mass.
2.2 Two-propagator integrals

2.2.1 Scalar integrals

\[ I_{12} = -1 + L_\Lambda, \quad I_{13} = -1 - \int_0^1 dx \ln \left( \frac{P^2_1}{\Lambda^2} \right) = 1 + L_\Lambda - L_s + i\pi, \]

\[ I_{14} = - \int_0^1 dx \ln \left( \frac{x_m^2 - x_\chi_1'}{\Lambda^2} \right) = 1 + L_\Lambda - L_\chi_1 + i\pi, \]

\[ I_{15} = I_{24} = I_{34} = I_{35} = 1 + L_\Lambda, \quad I_{23} = -1 - \int_0^1 dx \ln \left( \frac{P^2_1}{\Lambda^2} \right) = 1 + L_\Lambda - L_s_1 + i\pi, \]

\[ I_{25} = - \int_0^1 dx \ln \left( \frac{x_m^2 + x_\chi_1}{\Lambda^2} \right) = 1 + L_\Lambda - L_\chi_1, \quad I_{45} = 1 + L_\Lambda - L_t. \quad (6) \]

2.2.2 Vector integrals

\[ I_{12}^\mu = \left( p_1 - k_1 \right)^\mu \left( L_\Lambda - \frac{3}{2} \right), \quad I_{13}^\mu = (p_1 - p_2)^\mu \left( \frac{1}{4} + \frac{1}{2} L_\Lambda - \frac{1}{2} L_s + \frac{i\pi}{2} \right), \]

\[ I_{14}^\mu = (p_1 + q)^\mu \left( \frac{1}{4} + \frac{1}{2} L_\Lambda - \frac{1}{2} L_\chi_1 + \frac{i\pi}{2} \right), \quad I_{15}^\mu = \frac{p_1^\mu}{2} \left( L_\Lambda - \frac{1}{2} \right), \]

\[ I_{24}^\mu = (p_1 - k_1)^\mu \left( \frac{1}{2} + L_\Lambda \right) - \frac{p_1^\mu}{2} \left( \frac{3}{2} + L_\Lambda \right), \]

\[ I_{34}^\mu = \frac{p_2^\mu}{2} \left( \frac{3}{2} + L_\Lambda \right) - p_2^\mu \left( \frac{1}{2} + L_\Lambda \right), \]

\[ I_{35}^\mu = \frac{p_2^\mu}{2} \left( \frac{1}{2} - L_\Lambda \right), \quad I_{23}^\mu = (p_1 - k_1 - p_2)^\mu \left( \frac{1}{4} + \frac{1}{2} L_\Lambda - \frac{1}{2} L_s_1 + \frac{i\pi}{2} \right), \]

\[ I_{25}^\mu = (p_1 - k_1)^\mu \left( \frac{1}{4} + \frac{1}{2} L_\Lambda - \frac{1}{2} L_\chi_1 \right), \quad I_{45}^\mu = q^\mu \left( \frac{1}{4} + \frac{1}{2} L_\Lambda - \frac{1}{2} L_t \right). \quad (7) \]

2.3 Three-propagator integrals

2.3.1 Scalar integrals

\[ I_{123} = \frac{1}{s - s_1} \int_0^1 \frac{dx}{x} \ln \left( \frac{P^2_1}{P^2} \right) = \frac{1}{s - s_1} \left[ \frac{1}{2} L_s^2 - \frac{1}{2} L_s^2_1 + i\pi (L_s_1 - L_s) \right], \]

\[ I_{345} = - \int_0^1 \frac{dx}{x^2 m^2 + xt} \ln \left( \frac{m^2 x^2}{-tx} \right) = \frac{1}{t} \left[ \frac{1}{2} L_t^2 + 4\zeta(2) \right], \]

\[ I_{124} = \int_0^1 \frac{dz dy}{x y \chi_1' - y m^2} = \frac{1}{\chi_1^2} \left[ \frac{1}{2} L_{\chi_1}^2 - \zeta(2) - i\pi L_{\chi_1} \right], \]

\[ I_{125} = - \int_0^1 \frac{dz dy}{x y \chi_1 + y m^2} = \frac{1}{\chi_1} \left[ -\frac{1}{2} L_{\chi_1}^2 - 2\zeta(2) \right], \]
\begin{align*}
I_{134} &= -\int_0^1 \frac{dx dy}{y p^2 - x y \chi_1} = \frac{1}{s - \chi_1'} \left[ \frac{3}{2} L_i^2 - \frac{1}{2} L_{\chi_1'}^2 - 2 L_s L_{\chi_1'} + 2 \text{Li}_2 \left( 1 - \frac{\chi_1'}{s} \right) \right] + i \pi \left( L_{\chi_1'} - L_s \right), \\
I_{235} &= -\int_0^1 \frac{dx dy}{y p_i^2 + x y \chi_1} = \frac{1}{s + \chi_1} \left[ \frac{3}{2} L_{s_i}^2 + \frac{1}{2} L_{\chi_1}^2 - 2 L_{s_i} L_{\chi_1} - 9 \zeta(2) \right] + 2 \text{Li}_2 \left( 1 + \frac{\chi_1}{s} \right) + i \pi \left( 2 L_{\chi_1} - 3 L_{s_i} \right), \\
I_{135} &= -\frac{1}{2} \int_0^1 \frac{dx}{p^2} \ln \left( \frac{p^2}{\lambda^2} \right) = \frac{1}{s} \left[ \frac{1}{2} L_s^2 - L_s L_\lambda - 4 \zeta(2) + i \pi \left( L_\lambda - L_s \right) \right], \\
I_{234} &= -\frac{1}{2} \int_0^1 \frac{dx}{p_i^2} \ln \left( \frac{p_i^2}{\lambda^2} \right) = \frac{1}{s_i} \left[ \frac{1}{2} L_{s_i}^2 - L_{s_i} L_\lambda - 4 \zeta(2) + i \pi \left( L_\lambda - L_{s_i} \right) \right], \\
I_{245} &= \int_0^1 dx \left. \frac{dx}{x \chi_1 - t - x^2 m^2} \ln \left( \frac{x^2 m^2}{x \chi_1 - t} \right) = \frac{1}{\chi_1 + t} \left[ \frac{1}{2} L_i^2 - \frac{1}{2} L_{\chi_1}^2 \right] \right|_{\chi_1 - t} + 2 \text{Li}_2 \left( 1 - \frac{\chi_1}{-t} \right), \\
I_{145} &= -\int_0^1 \frac{dx dy}{y x^2 m^2 - y x t - y x \chi_1} = \frac{1}{\chi_1' - t} \left[ \frac{1}{2} L_{\chi_1'}^2 - \frac{1}{2} L_i^2 - 3 \zeta(2) \right] + 2 \text{Li}_2 \left( 1 + \frac{\chi_1'}{t} \right) - i \pi L_{\chi_1'},
\end{align*}

(8)

### 2.3.2 Vector integrals

The parameterization reads

\[ I_{ijk}^\mu = a_{ijk} p_i^\mu + b_{ijk} p_i^\mu + c_{ijk} k_i^\mu + d_{ijk} p_i^\mu. \]  

(9)

\begin{align*}
\alpha_{245} &= -c_{245} = \frac{1}{t + \chi_1} \left[ \frac{1}{2} L_i^2 - \frac{1}{2} L_{\chi_1}^2 + L_{\chi_1} - L_t + 2 \text{Li}_2 \left( 1 - \frac{\chi_1}{-t} \right) \right], \\
\beta_{245} &= 0, \\
\alpha_{45} &= \frac{1}{t + \chi_1} \left[ L_{\chi_1} - L_t + 2 i \pi + \frac{t}{\chi_1'} - t \left[ \frac{1}{2} L_i^2 - \frac{1}{2} L_{\chi_1}^2 \right] \right], \\
\beta_{45} &= 0, \\
\alpha_{145} &= \frac{1}{\chi_1' - t} \left[ 2 L_{\chi_1'} - L_t - 2 i \pi + \frac{t}{\chi_1'} - t \left[ \frac{1}{2} L_i^2 - \frac{1}{2} L_{\chi_1}^2 \right] \right] + 2 L_{\chi_1'} - 2 L_t + 3 \zeta(2) + 2 \text{Li}_2 \left( 1 + \frac{\chi_1'}{t} \right) + i \pi \left( L_{\chi_1'} - 2 \right), \\
\beta_{145} &= 0, \\
\gamma_{145} &= d_{145} = \frac{1}{\chi_1' - t} \left[ L_t - L_{\chi_1'} + i \pi \right].
\end{align*}

(10)
\[ a_{345} = -c_{345} = -d_{345} = -\frac{1}{t} L_t, \quad b_{345} = \frac{1}{t} \left[ -\frac{1}{2} L_t^2 + 2L_t - 4\zeta(2) \right]. \] (12)

\[ a_{125} = \frac{1}{\chi_1} \left[ -\frac{1}{2} L_{\chi_1}^2 + L_{\chi_1} - 2\zeta(2) \right], \quad b_{125} = d_{125} = 0, \quad c_{125} = \frac{1}{\chi_1} \left[ L_{\chi_1} - 2 \right]. \] (13)

\[ a_{235} = -c_{235} = \frac{1}{s_1 + \chi_1} \left[ L_{s_1} - L_{\chi_1} - i\pi \right], \quad d_{235} = 0, \]
\[ b_{235} = \frac{1}{s_1 + \chi_1} \left[ -L_{s_1} + i\pi \right. \]
\[ + \frac{\chi_1}{s_1 + \chi_1} \left[ 2L_{s_1} - 2L_{\chi_1} - \frac{3}{2} L_{s_1}^2 - \frac{1}{2} L_{\chi_1}^2 + 2L_{s_1}L_{\chi_1} + 9\zeta(2) - 2Li_2 \left( 1 + \frac{\chi_1}{s_1} \right) \right. \]
\[ \left. + i\pi \left( -2 - 2L_{\chi_1} + 3L_{s_1} \right) \right]. \] (14)

\[ a_{135} = -b_{135} = \frac{1}{s} \left[ L_s - i\pi \right], \quad c_{135} = d_{135} = 0. \] (15)

\[ a_{234} = -c_{234} = -b_{234} - d_{234} = \frac{1}{s_1} \left[ \frac{1}{2} L_{s_1}^2 - L_{s_1}L_{\chi_1} - L_{s_1} - 4\zeta(2) \right. \]
\[ + i\pi \left( 1 + L_{\chi_1} - L_{s_1} \right), \quad b_{234} = a_{234} - I_{234} = \frac{1}{s_1} \left[ -L_{s_1} + i\pi \right], \]
\[ d_{234} = \frac{1}{s_1} \left[ -\frac{1}{2} L_{s_1}^2 + L_{s_1}L_{\chi_1} + 2L_{s_1} + 4\zeta(2) + i\pi \left( -2 + L_{s_1} - L_{\chi_1} \right) \right]. \] (16)

\[ a_{134} = \frac{1}{s - \chi_1} \left[ L_s - 2L_{\chi_1} + i\pi - \frac{2s}{s - \chi_1} \left[ L_s - L_{\chi_1} \right] + sI_{134} \right], \]
\[ b_{134} = a_{134} - I_{134} = \frac{1}{s - \chi_1} \left[ -L_s + i\pi - \frac{2\chi_1'}{s - \chi_1'} \left[ L_s - L_{\chi_1'} \right] + \chi_1'I_{134} \right], \]
\[ c_{134} = d_{134} = \frac{1}{s - \chi_1} \left[ L_{\chi_1'} - i\pi + \frac{2s}{s - \chi_1'} \left[ L_s - L_{\chi_1'} \right] - sI_{134} \right]. \] (17)

\[ a_{124} = I_{124} = \frac{1}{\chi_1} \left[ \frac{1}{2} L_{\chi_1'}^2 - \zeta(2) - i\pi L_{\chi_1'} \right], \quad d_{124} = \frac{1}{\chi_1} \left[ -L_{\chi_1'} + i\pi \right], \]
\[ c_{124} = \frac{1}{\chi_1} \left[ -\frac{1}{2} L_{\chi_1'}^2 + L_{\chi_1'} + \zeta(2) - 2 + i\pi \left( L_{\chi_1'} - 1 \right) \right], \quad b_{124} = 0. \] (18)

\[ a_{123} = b_{123} - I_{123} = \frac{1}{s - s_1} \left[ \frac{1}{2} L_{s_1}^2 - \frac{1}{2} L_{s_1}^2 - L_s + L_{s_1} + i\pi \left( L_{s_1} - L_s \right) \right], \]
\[ b_{123} = -\frac{1}{s - s_1} \left[ L_s - L_{s_1} \right], \quad d_{123} = 0, \quad c_{123} = \frac{1}{s - s_1} \left[ L_{s_1} - 2 - i\pi \right. \]
\[ + \frac{s}{s - s_1} \left[ -\frac{1}{2} L_s^2 + \frac{1}{2} L_{s_1}^2 + 2L_s - 2L_{s_1} + i\pi \left( L_s - L_{s_1} \right) \right]. \] (19)
2.4 Four-propagator integrals

2.4.1 Scalar integrals

\[
I_{1245} = \int_0^1 \frac{dx \, dy}{[xy \chi_1 - \bar{y} t | y m^2 - x \bar{y} \chi_1]} = \frac{1}{\chi_1 \chi_1} \left[ -L_{2x}^2 - L_{2x_1}^2 - L_{t}^2 - 2L_{x_1}L_{x_1} \right] \\
+ 2L_{x_1}L_t + 2L_{x_1}L_t + 4\zeta(2) + i\pi \left( 2L_{x_1} + 2L_{x_1} - 2L_t \right),
\]

\[
I_{2345} = \frac{1}{t} \int_0^1 \frac{dx}{P_1^2} \left[ -\frac{1}{2} \ln \left( \frac{P_1^2}{\lambda^2} \right) + \ln \left( \frac{x \chi_1}{-t} \right) \right] = \frac{1}{s_t} \left[ L_{s_1}^2 - L_{s_1}L_\lambda - 2L_{s_1}L_{x_1} \right] \\
+ 2L_{s_1}L_t - 5\zeta(2) + i\pi \left( 2L_{x_1} - 2L_t - 2L_{s_1} + L_\lambda \right),
\]

\[
I_{1345} = \frac{1}{t} \int_0^1 \frac{dx}{P_2^2} \left[ -\frac{1}{2} \ln \left( \frac{P_2^2}{\lambda^2} \right) + \ln \left( \frac{-x \chi_1}{-t} \right) \right] \\
= \frac{1}{s_t} \left[ L_{s}^2 - L_{s}L_\lambda - 2L_{s}L_{x_1} + 2L_{s}L_t + 7\zeta(2) + i\pi \left( 2L_{x_1} - 2L_t + L_\lambda \right) \right],
\]

\[
I_{1235} = \frac{1}{\chi_1} \int_0^1 \frac{dx}{P_2^2} \left[ \frac{1}{2} \ln \left( \frac{P_2^2}{\lambda^2} \right) - \ln \left( \frac{P_1^2}{x \chi_1} \right) \right] = \frac{1}{s_t} \left[ L_{s_1}^2 - 2L_{s_1}L_{x_1} + L_{s_1}L_{x_1} \right] \\
- 5\zeta(2) + 2Li_2 \left( 1 - \frac{s_1}{s} \right) + i\pi \left( -2L_{s_1} + 2L_{x_1} - L_\lambda \right),
\]

\[
I_{1234} = \frac{1}{\chi_1} \int_0^1 \frac{dx}{P_1^2} \left[ -\frac{1}{2} \ln \left( \frac{P_1^2}{\lambda^2} \right) + \ln \left( \frac{P_2^2}{-x \chi_1} \right) \right] = \frac{1}{s_t} \left[ -L_{s}^2 - 2L_{s}L_{x_1} \right] \\
- L_{s_1}L_\lambda - 7\zeta(2) - 2Li_2 \left( 1 - \frac{s_1}{s} \right) + i\pi \left( 2L_{s_2} - 2L_{s_1} - 2L_{x_1} + L_\lambda \right). \tag{20}
\]

Useful integrals

\[
\int_0^1 \frac{dx}{P_1^2} = \frac{2}{s} \left[ -L_s + i\pi \right], \quad \int_0^1 \frac{dx}{P_2^2} \ln x = \frac{1}{s} \left[ \frac{1}{2} L_s^2 - \zeta(2) - i\pi L_s \right],
\]

\[
\int_0^1 \frac{dx}{P_2^2} \ln \left( \frac{P_2^2}{m^2} \right) = \frac{1}{s} \left[ -L_s^2 + 8\zeta(2) + 2i\pi L_s \right],
\]

\[
\int_0^1 \frac{dx}{P_1^2} \ln \left( \frac{P_1^2}{m^2} \right) = \frac{1}{s} \left[ -L_{s_1}^2 + 8\zeta(2) - 2Li_2 \left( 1 - \frac{s_1}{s} \right) + 2i\pi L_{s_1} \right]. \tag{21}
\]

2.4.2 Vector integrals

Parameterization

\[
I_{ijkl}^\mu = a_{ijkl}p_1^\mu + b_{ijkl}p_2^\mu + c_{ijkl}k_1^\mu + d_{ijkl}p_1^\mu
\]

\[
a_{1245} = \frac{\Delta^{(1)}}{\Delta}, \quad b_{1245} = 0, \quad c_{1245} = \frac{\Delta^{(2)}}{\Delta}, \quad d_{1245} = \frac{\Delta^{(3)}}{\Delta}. \tag{23}
\]
\[
\begin{align*}
\Delta &= -2t_1\chi_1\chi'_1, \quad \Delta^{(1)} = \chi'_1[-\chi'_1 I_{124} + \chi_1 I_{125} + (\chi_1 + t)I_{245} + (\chi'_1 - t)I_{145}], \\
\Delta^{(2)} &= t_1 [-\chi'_1 I_{124} - \chi_1 I_{125} + (\chi_1 + t)I_{245} + (\chi'_1 - t)I_{145} + \chi_1\chi_1 I_{1245}], \\
\Delta^{(3)} &= \chi_1 [\chi'_1 I_{124} - \chi_1 I_{125} + (\chi_1 + t - 2\chi'_1)I_{245} + (\chi'_1 - t)I_{145} + \chi_1\chi_1 I_{1245}].
\end{align*}
\] (24)

\[
a_{1235} = \frac{\Delta^{(1)}}{\Delta}, \quad b_{1235} = \frac{\Delta^{(2)}}{\Delta}, \quad c_{1235} = \frac{\Delta^{(3)}}{\Delta}, \quad d_{1235} = 0,
\] (25)

\[
\begin{align*}
\Delta &= -2s\chi_1\chi_2, \\
\Delta^{(1)} &= \chi_2 [(\chi_1 + \chi_2)I_{123} - \chi_1 I_{125} - sI_{135} + (s - \chi_2)I_{235} - s\chi_1 I_{123}] , \\
\Delta^{(2)} &= \chi_1 [-(\chi_1 + \chi_2)I_{123} + \chi_1 I_{125} - sI_{135} + (s + \chi_2)I_{235} - s\chi_1 I_{123}], \\
\Delta^{(3)} &= s [(\chi_1 - \chi_2)I_{123} - \chi_1 I_{125} + sI_{135} - (s - \chi_2)I_{235} + s\chi_1 I_{123}].
\end{align*}
\] (26)

\[
a_{1345} = \frac{\Delta^{(1)}}{\Delta}, \quad b_{1345} = \frac{\Delta^{(2)}}{\Delta}, \quad c_{1345} = d_{1345} = \frac{\Delta^{(3)}}{\Delta},
\] (27)

\[
\begin{align*}
u &= -s - t + \chi'_1, \quad \Delta = 2stu, \quad \Delta^{(1)} = -(-su + t\chi'_1)I_{134} + s(s + t)I_{135} + t(s + t)I_{345} - (tu + s\chi'_1)I_{145} - st(s + t)I_{134}, \quad \Delta^{(2)} = -(-\chi'_1 u + st)I_{134} + t(2s + t - \chi'_1)I_{345} + (t - \chi'_1)^2I_{145} + st(t - \chi'_1)I_{1345}, \\
\Delta^{(3)} &= s [(s + 2t - \chi'_1)I_{134} - sI_{135} - tI_{345} - (t - \chi'_1)I_{145} + stI_{1345}].
\end{align*}
\] (28)

\[
a_{2345} = -c_{2345} = \frac{\Delta^{(1)}}{\Delta}, \quad b_{2345} = \frac{\Delta^{(2)}}{\Delta}, \quad d_{2345} = \frac{\Delta^{(3)}}{\Delta},
\] (29)

\[
\begin{align*}
u_1 &= -s_1 - t - \chi_1, \quad \Delta = 2s_1tu_1, \\
\Delta^{(1)} &= -u_1 [-(t + \chi_1)I_{245} - s_1 I_{234} + (s_1 + \chi_1)I_{235} + tI_{345} - s_1 tI_{234}], \\
\Delta^{(2)} &= -(t + \chi_1)^2I_{245} - s_1(t + \chi_1)I_{234} + (s_1^2 + s_1 t)I_{235} + t(2s_1 + t + \chi_1)I_{345} + s_1 t(t + \chi_1)I_{2345}, \\
\Delta^{(3)} &= (s_1^2 u_1 - s_1 t)I_{245} + s_1 (s_1 + 2t + \chi_1)I_{234} - (s_1 + \chi_1)^2I_{235} + t(s_1 + \chi_1)I_{345} + s_1 t(s_1 + \chi_1)I_{2345}.
\end{align*}
\] (30)

\[
\begin{align*}
a_{1234} &= I_{1234} + \frac{\Delta^{(2)}}{\Delta}, \quad b_{1234} = \frac{\Delta^{(2)}}{\Delta}, \quad c_{1234} = -I_{1234} - \frac{\Delta^{(2)}}{\Delta} + \frac{\Delta^{(3)}}{\Delta}, \\
d_{1234} &= -I_{1234} + \frac{\Delta^{(1)}}{\Delta} - \frac{\Delta^{(2)}}{\Delta}.
\end{align*}
\] (31)
\[ \Delta = 2s_1 \chi'_1 \chi'_2, \quad \chi'_2 = s - s_1 - \chi'_1, \]
\[ \Delta^{(1)} = \chi'_2 [(s - s_1)I_{123} + (s - \chi'_1)I_{134} + \chi'_1 I_{124} - s_1 I_{234} + s_1 \chi'_1 I_{1234}], \]
\[ \Delta^{(2)} = \chi'_1 [(s - s_1)I_{123} + (2s_1 - s + \chi'_1)I_{134} - \chi'_1 I_{124} - s_1 I_{234} + s_1 \chi'_1 I_{1234}], \]
\[ \Delta^{(3)} = s_1 [(\chi'_2 - \chi'_1)I_{123} - (s - \chi'_1)I_{134} + \chi'_1 I_{124} + s_1 I_{234} - s_1 \chi'_1 I_{1234}]. \]

### 2.4.3 Tensor

Parameterization

\[ I^{\mu\nu}_{ijkl} = g^{T}_{ijkl} g^{\mu\nu} + a^{T}_{ijkl} p_1^\mu p_1^\nu + b^{T}_{ijkl} p_2^\mu p_2^\nu + c^{T}_{ijkl} k_1^\mu k_1^\nu + d^{T}_{ijkl} p_1^\mu p_2^\nu \]
\[ + \alpha^{T}_{ijkl} (p_1^\mu p_2^\nu) + \beta^{T}_{ijkl} (p_2^\mu k_1^\nu) + \gamma^{T}_{ijkl} (p_1^\mu k_1^\nu) \]
\[ + \rho^{T}_{ijkl} (p_2^\mu k_1^\nu) + \sigma^{T}_{ijkl} (p_1^\mu k_1^\nu), \]  

where \( \{\ldots\} \) means symmetrization with respect to Lorentz indices: \( \{v_\mu u_\nu\} = v_\mu u_\nu + v_\nu u_\mu \).

\begin{align*}
    g^{T}_{1245} &= \frac{1}{2} [2I_{124} - a_{124} - \chi_1 c_{1245} + (t + \chi_1) d_{1245}], \\
    a^{T}_{1245} &= \frac{1}{\chi_1} \left[ \chi'_1 (-I_{124} + a_{124} - c_{145}) + t_1 a_{145} - (t + \chi_1) a_{245} \\
    &\quad + t_1 \chi_1 a_{124} - \chi'_1 (t + \chi_1) d_{1245} \right], \\
    c^{T}_{1245} &= \frac{1}{\chi_1} \left[ t_1 (I_{124} + a_{124}) + \chi_1 c_{12} + (t_1 - \chi_1) c_{145} - \chi_1 \chi'_1 c_{1245} \right], \\
    d^{T}_{1245} &= \frac{1}{t_1 \chi_1} \left[ \chi_1 (I_{124} - a_{124} - a_{245}) + (t_1 - \chi_1) c_{145} - t_1 d_{1245} - \chi_1 \chi'_1 d_{1245} \right], \\
    \beta^{T}_{1245} &= \frac{1}{\chi_1} [-I_{124} + a_{124} + c_{145} + \chi_1 c_{1245}], \quad b^{T}_{1245} = \alpha^{T}_{1245} = \rho^{T}_{1245} = \sigma^{T}_{1245} = 0, \\
    \gamma^{T}_{1245} &= \frac{1}{t_1} \left[ I_{124} - a_{124} + a_{245} + c_{145} + (t + \chi_1) d_{1245} \right], \\
    \tau^{T}_{1245} &= \frac{1}{t'_1} \left[ -I_{124} + a_{245} + \chi_1 c_{1245} - (t + \chi_1) d_{1245} \right].
\end{align*}
\[ c_{1235}^T = \frac{1}{\chi_1 \chi_2} [s(I_{123} + b_{123}) - (s - \chi_2)a_{235} + \chi_2 c_{123} - s \chi_1 c_{123}] , \quad d_{1235}^T = 0 , \]
\[ a_{1235}^T = \frac{1}{s} [-I_{123} + a_{123} - a_{235} - b_{123}] , \quad \beta_{1235}^T = \frac{1}{\chi_1} [-I_{123} + a_{123} + \chi_1 c_{123}] , \]
\[ \sigma_{1235}^T = \frac{1}{\chi_2} [-I_{123} + a_{235} - b_{123} + \chi_1 c_{123}] , \quad \gamma_{1235}^T = \rho_{1235}^T = \tau_{1235}^T = 0. \quad (35) \]

\[ g_{1345}^T = \frac{1}{2} [I_{134} + tc_{1345}] , \quad b_{1345}^T = \frac{1}{s} [b_{134} - b_{345} - (\chi'_1 - t) \rho_{1345}^T] , \]
\[ a_{1345}^T = \frac{1}{st(\chi'_1 - s - t)} [(s + t)^2 I_{134} + t(\chi'_1 - s - t)a_{145} - (s(t + t) + t \chi'_1)a_{134} \]
\[ + \chi'_1(s + t)(c_{145} - c_{134}) + t(s + t)^2 c_{1345}] , \]
\[ c_{1345}^T = d_{1345}^T = \tau_{1345}^T = \frac{1}{t(\chi'_1 - s - t)} [(\chi'_1 - t)(c_{145} - c_{134}) - s(b_{134} - tc_{1345})] , \]
\[ \alpha_{1345}^T = \frac{1}{st(\chi'_1 - s - t)} [-t(\chi'_1 - s - t)a_{345} + \chi'_1(\chi'_1 - t)(c_{145} - c_{134}) \]
\[ - s \chi'_1(a_{134} - I_{134}) + st \chi'_1 c_{1345}] , \]
\[ \beta_{1345}^T = \gamma_{1345}^T = \frac{1}{t(\chi'_1 - s - t)} [(s + t)(b_{134} - tc_{1345}) - \chi'_1(c_{145} - c_{134})] , \]
\[ \rho_{1345}^T = \sigma_{1345}^T = \frac{1}{st(\chi'_1 - s - t)} [(\chi'_1(\chi'_1 - t) - st)c_{134} - (\chi'_1 - t)^2 c_{145} \]
\[ + t(\chi'_1 - s - t)a_{345} + s(\chi'_1 - t)b_{134} - st(\chi'_1 - t)c_{1345}] . \quad (36) \]

\[ g_{2345}^T = \frac{1}{2} [I_{234} + \chi_1 a_{2345} + (t + \chi_1)d_{2345}] , \quad a_{2345}^T = -\sigma_{2345}^T = \frac{1}{s_1 t} [-\chi_1 a_{235} \]
\[ - t a_{234}] , \quad a_{2345}^T = c_{2345}^T = -\beta_{2345}^T = \frac{1}{s_1 t} [-t a_{234} - (s_1 + \chi_1)a_{2345} + s_1 t a_{2345}] , \]
\[ b_{2345}^T = \frac{1}{s_1 t(\chi_1 + s_1 + t)} [s_1 t(b_{235} - b_{345}) - \chi_1(t + \chi_1)a_{235} \]
\[ - t(t + \chi_1)a_{345} - s_1 t(t + \chi_1)b_{235}] , \]
\[ d_{2345}^T = \frac{1}{(\chi_1 + s_1 + t)} [d_{245} - d_{234} - \frac{(\chi_1 + s_1)}{s_1 t(\chi_1 + s_1 + t)} [s_1 t(a_{245} - a_{234}) \]
\[ + t(\chi_1 + s_1)a_{345} + (\chi_1 + s_1)^2 a_{235} - s_1 t(\chi_1 + s_1)a_{2345}] ] , \]
\[ \gamma_{2345}^T = -\tau_{2345}^T = \frac{1}{s_1 t(\chi_1 + s_1 + t)} [s_1 t(a_{245} - a_{234}) + t(\chi_1 + s_1)a_{345} \]
\[ + (\chi_1 + s_1)^2 a_{235} - s_1 t(\chi_1 + s_1)a_{2345}] , \]
\[ \rho_{2345}^T = \frac{1}{s_1 t(\chi_1 + s_1 + t)} [-s_1 t a_{234} + \chi_1(\chi_1 + s_1)a_{235} + t(\chi_1 + s_1)a_{345} \]
\[ - s_1 t \chi_1 a_{2345} - s_1 t(\chi_1 + t)d_{2345}] . \quad (37) \]
\[ g_{1234}^T = \frac{1}{2} \left[ I_{123} - \chi_1' \Delta^{(3)} \right], \quad a_{1234}^T = \frac{2 \Delta^{(2)}}{\Delta} + I_{1234} + \tilde{b}_{1234}, \quad b_{1234}^T = \tilde{b}_{1234}, \]

\[ c_{1234}^T = \frac{2 \Delta^{(2)}}{\Delta} - 2 \frac{\Delta^{(3)}}{\Delta} + I_{1234} + \tilde{b}_{1234} + \tilde{c}_{1234} - 2 \tilde{\gamma}_{1234}, \]

\[ d_{1234}^T = \frac{2 \Delta^{(2)}}{\Delta} - 2 \frac{\Delta^{(1)}}{\Delta} + I_{1234} + \tilde{a}_{1234} + \tilde{b}_{1234} - 2 \tilde{\alpha}_{1234}, \]

\[ \alpha_{1234}^T = \frac{\Delta^{(1)}}{\Delta} + \tilde{b}_{1234}, \quad \beta_{1234}^T = \frac{\Delta^{(3)}}{\Delta} - \frac{2 \Delta^{(2)}}{\Delta} - I_{1234} - \tilde{b}_{1234} + \tilde{\gamma}_{1234}, \]

\[ \gamma_{1234}^T = \frac{\Delta^{(1)}}{\Delta} - 2 \frac{\Delta^{(2)}}{\Delta} - I_{1234} - \tilde{b}_{1234} + \tilde{\alpha}_{1234}, \]

\[ \rho_{1234}^T = - \frac{\Delta^{(2)}}{\Delta} - \tilde{b}_{1234} + \tilde{\alpha}_{1234}, \quad \sigma_{1234}^T = - \frac{\Delta^{(2)}}{\Delta} - \tilde{b}_{1234} + \tilde{\gamma}_{1234}, \]

\[ \tilde{\gamma}_{1234}^T = 2 \frac{\Delta^{(2)}}{\Delta} - \frac{\Delta^{(1)}}{\Delta} - \frac{\Delta^{(3)}}{\Delta} + I_{1234} + \tilde{b}_{1234} - \tilde{\alpha}_{1234} + \tilde{\beta}_{1234} - \tilde{\gamma}_{1234}, \quad (38) \]

where

\[ \tilde{a}_{1234} = \frac{1}{s_1 \chi_1'} \left[ \chi_1'(I_{124} - I_{123} + d_{124}) - (\chi_1' + \chi_2')b_{123} - \chi_1' \chi_2' \frac{\Delta^{(3)}}{\Delta} \right], \]

\[ \tilde{b}_{1234} = \frac{1}{s_1 \chi_2'} \left[ -\chi_1'(I_{134} - I_{123} + c_{134}) + (\chi_1' + \chi_2')(b_{123} - b_{134}) - \chi_1' \chi_2' \frac{\Delta^{(3)}}{\Delta} \right], \]

\[ \tilde{c}_{1234} = - \frac{1}{\chi_1' \chi_2'} \left[ (s_1 + \chi_2')(I_{123} - I_{134} + b_{123} - b_{134} - c_{134}) + \chi_2' c_{123} \right. \]

\[ - \frac{1}{s_1 \chi_1'} \Delta^{(3)} \left], \right. \]

\[ \tilde{\alpha}_{1234} = \frac{1}{s_1} \left[ -b_{134} - c_{134} - I_{134} \right], \quad \tilde{\beta}_{1234} = \frac{1}{\chi_1'} \left[ b_{123} + \chi_1' \Delta^{(3)} \right], \]

\[ \tilde{\gamma}_{1234} = \frac{1}{\chi_2'} \left[ -b_{123} + b_{134} + c_{134} + I_{134} - I_{123} + \chi_1' \Delta^{(3)} \right]. \quad (39) \]

### 2.4.4 Pentagon

Following ref. [2] we express the pentagon diagram in terms of box graphs.

\[ I_{12345} = - \frac{1}{\Delta} \left[ \Delta^{(1)} I_{2345} + \Delta^{(2)} I_{1345} + \Delta^{(3)} I_{12345} + \Delta^{(4)} I_{1235} + \Delta^{(5)} I_{1234} \right], \quad (40) \]

where

\[ \Delta = 2 s s_1 t \chi_1' \chi_1', \quad \Delta^{(1)} = s_1 t [-(s - s_1)t - s \chi_1 - s_1 \chi_1' - \chi_1' \chi_1'], \]

\[ \Delta^{(2)} = s t [(s - s_1)t + s \chi_1 + s_1 \chi_1' - \chi_1' \chi_1'], \]

\[ \Delta^{(3)} = \frac{1}{s_1} [(s - s_1)t + s \chi_1 + s' \chi_1']^2, \]

\[ \Delta^{(4)} = \frac{1}{s_1} [(s - s_1)t + s' \chi_1 + s \chi_1']^2, \]

\[ \Delta^{(5)} = \frac{1}{s_1} [s' \chi_1 + s \chi_1']^2. \]
\[
\Delta^{(3)} = \chi_1 \chi'_1 \left[ -(s + s_1)t - s \chi_1 + s_1 \chi'_1 + \chi_1 \chi'_1 \right]; \\
\Delta^{(4)} = s \chi_1 \left[ (s - s_1)t + s \chi_1 - s_1 \chi'_1 - \chi_1 \chi'_1 \right]; \\
\Delta^{(5)} = s_1 \chi'_1 \left[ (s - s_1)t - s \chi_1 + s_1 \chi'_1 + \chi_1 \chi'_1 \right].
\] (41)

The case of different fermion masses.

Typical process: \( \mu^- e^- \rightarrow \mu^- e^- \gamma \)

2.5 Notations (see fig.1)

\[
I_{ijklm}^{1,\mu,\mu} = \int \frac{d^4k}{i\pi^2} \frac{1, k^\mu, k^\nu}{(i)(j)(k)(l)(m)}
\] (42)

\[
(1) = (p_1 - k)^2 - m^2, \quad (2) = (p_1 - k_1 - k)^2 - m^2, \quad (3) = (p_2 + k)^2 - \mu^2, \\
(4) = (q - k)^2 - \lambda^2, \quad (5) = k^2 - \lambda^2.
\] (43)

Invariants

\[
\chi_1 = 2p_1 k_1, \quad \chi'_1 = 2p'_1 k_1, \quad \chi_2 = 2p_2 k_1 = s - s_1 - \chi_1, \\
\chi'_2 = 2p'_2 k_1 = s - s_1 - \chi'_1, \quad s = (p_1 + p_2)^2, \quad s_1 = (p'_1 + p'_2)^2, \\
t = q^2, \quad q = p'_2 - p_2, \quad t_1 = q'^2 = t + \chi_1 - \chi'_1, \quad q' = p'_1 - p_1, \\
\chi_1 + \chi_2 = s - s_1, \quad p_1 = p'_1 = m^2, \quad p_2 = p'_2 = \mu^2, \quad k_1^2 = 0.
\] (44)

\[
L_{\Lambda_m} = \ln \left( \frac{\Lambda^2}{m^2} \right), \quad L_{\Lambda_\mu} = \ln \left( \frac{\Lambda^2}{\mu^2} \right), \quad L_{\lambda_m} = \ln \left( \frac{\lambda^2}{m^2} \right), \quad L_{\lambda_\mu} = \ln \left( \frac{\lambda^2}{\mu^2} \right), \\
L_{t_m} = \ln \left( \frac{-t}{m^2} \right), \quad L_{t_\mu} = \ln \left( \frac{-t}{\mu^2} \right), \quad L_{s_m} = \ln \left( \frac{s}{m^2} \right), \quad L_{s_\mu} = \ln \left( \frac{s}{\mu^2} \right), \\
\text{Li}_2(z) = -\int_0^z \frac{dx}{x} \ln(1-x).
\] (45)

\[
\mathcal{P}^2 = m^2 x + \mu^2 (1-x) - x\bar{x}s - i0, \quad \mathcal{P}_1^2 = m^2 x + \mu^2 (1-x) - x\bar{x}s_1 - i0, \quad \text{where } \bar{x} = 1-x
\] (46)
2.6 Two-propagator integrals

2.6.1 Scalar

\[
I_{12} = -1 + L_{\Lambda m}, \quad I_{13} = 1 + L_{\Lambda \mu} - L_{s\mu} + i\pi,
I_{14} = 1 + L_{\Lambda m} - L_{\chi'_{1m}} + i\pi, \quad I_{15} = I_{21} = 1 + L_{\Lambda m},
I_{34} = I_{35} = 1 + L_{\Lambda \mu}, \quad I_{23} = 1 + L_{\Lambda \mu} - L_{s1\mu} + i\pi,
I_{25} = 1 + L_{\Lambda m} - L_{\chi_{1m}}, \quad I_{45} = 1 + L_{\Lambda m} - L_{t_{1m}}.
\]

(47)

2.6.2 Vector

\[
I_{12}^\mu = \left( p_1 - \frac{k_1}{2} \right)^\mu \left( L_{\Lambda m} - \frac{3}{2} \right), \quad I_{13}^\mu = (p_1 - p_2)^\mu \left( \frac{1}{4} + \frac{1}{2} L_{\Lambda m} - \frac{1}{2} L_{s m} + \frac{i\pi}{2} \right),
I_{14}^\mu = (p_1 + q)^\mu \left( \frac{1}{4} + \frac{1}{2} L_{\Lambda m} - \frac{1}{2} L_{\chi'_{1m}} + \frac{i\pi}{2} \right), \quad I_{15}^\mu = \frac{p_1^\mu}{2} \left( L_{\Lambda m} - \frac{1}{2} \right),
I_{24}^\mu = (p_1 - k_1)^\mu \left( \frac{1}{2} + L_{\Lambda m} \right) - \frac{p_2^\mu}{2} \left( \frac{3}{2} + L_{\Lambda m} \right),
I_{34}^\mu = \frac{p_2^\mu}{2} \left( \frac{3}{2} + L_{\Lambda \mu} \right) - \frac{p_2^\mu}{2} \left( \frac{1}{2} + L_{\Lambda m} \right), \quad I_{35}^\mu = \frac{p_2^\mu}{2} \left( \frac{1}{2} - L_{\Lambda \mu} \right),
I_{23}^\mu = (p_1 - k_1 - p_2)^\mu \left( \frac{1}{4} + \frac{1}{2} L_{\Lambda m} - \frac{1}{2} L_{s_{1m}} + \frac{i\pi}{2} \right),
I_{25}^\mu = (p_1 - k_1)^\mu \left( \frac{1}{4} + \frac{1}{2} L_{\Lambda m} - \frac{1}{2} L_{\chi_{1m}} \right), \quad I_{45}^\mu = q^\mu \left( \frac{1}{4} + \frac{1}{2} L_{\Lambda m} - \frac{1}{2} L_{t_{1m}} \right).
\]

(48)

2.7 Three-propagator integrals

2.7.1 Scalar

\[
I_{123} = \frac{1}{s - s_1} \left[ \frac{1}{2} L_{s_{1\mu}}^2 - \frac{1}{2} L_{s_{1m}}^2 + i\pi \left( L_{s_{1m}} - L_{s_{1m}} \right) \right], \quad I_{345} = \frac{1}{L_{s_{1\mu}}} \left[ \frac{1}{2} L_{s_{1\mu}}^2 + 4\zeta(2) \right],
I_{124} = \frac{1}{s - \chi_1} \left[ \frac{1}{2} L_{\chi'_{1m}}^2 - \zeta(2) - i\pi L_{\chi'_{1m}} \right], \quad I_{125} = \frac{1}{s - \chi_1} \left[ \frac{1}{2} L_{\chi'_{1m}}^2 - 2\zeta(2) \right],
I_{134} = \frac{1}{s - \chi_1} \left[ \frac{3}{2} L_{s_{1\mu}}^2 + \frac{1}{2} L_{\chi'_{1\mu}}^2 - 2 L_{s_{1\mu}} L_{\chi'_{1\mu}} + 2 L_{12} \right] \left( 1 - \frac{\chi_1}{s} \right) + i\pi \left( L_{\chi'_{1\mu}} - L_{s_{1\mu}} \right),
I_{235} = \frac{1}{s_1 + \chi_1} \left[ \frac{3}{2} L_{s_{1\mu}}^2 + \frac{1}{2} L_{\chi'_{1\mu}}^2 - 2 L_{s_{1\mu}} L_{\chi'_{1\mu}} - 9\zeta(2) \right]
+ 2 L_{12} \left( 1 - \frac{\chi_1}{s_1} \right) + i\pi \left( 2 L_{\chi'_{1\mu}} - 3 L_{s_{1\mu}} \right),
I_{135} = \frac{1}{s} \left[ \frac{1}{2} L_{s_{1m}}^2 - L_{\Lambda m} L_{s_{1m}} + i\pi \left( L_{\Lambda m} - L_{s_{1m}} \right) - 4\zeta(2) + 2 \ln^2 \frac{\mu}{m} + 2 \ln \frac{\mu}{m} L_{\Lambda m} \right],
\]

12
\[ I_{234} = \frac{1}{s_1} \left[ \frac{1}{2} I_{s1m}^2 - L_{\lambda m} L_{s1m} + i\pi (L_{\lambda m} - L_{s1m}) - 4\zeta(2) + 2 \ln^2 \frac{\mu}{m} \right. \\
+ \left. 2 \ln \frac{\mu}{m} L_{\lambda m} \right] , \]

\[ I_{245} = \frac{1}{\chi_1 + t} \left[ \frac{1}{2} I_{t m}^2 - \frac{1}{2} I_{\chi_{1m}}^2 + 2 \text{Li}_2 \left( 1 - \frac{\chi_1}{-t} \right) \right] , \]

\[ I_{145} = \frac{1}{\chi_1 - t} \left[ \frac{1}{2} I_{\chi_{1m}}^2 - \frac{1}{2} I_{t m}^2 - 3\zeta(2) - 2 \text{Li}_2 \left( 1 + \frac{\chi_1}{-t} \right) - i\pi L_{\chi_{1m}} \right] . \] (49)

**2.7.2 Vector Parameterization**

\[ I^\mu_{ijk} = a_{ijk} p_1^\mu + b_{ijk} p_2^\mu + c_{ijk} k_1^\mu + d_{ijk} p_1^\mu \] (50)

\[ a_{245} = -c_{245} = \frac{1}{t + \chi_1} \left[ \frac{1}{2} I_{t m}^2 - \frac{1}{2} I_{\chi_{1m}}^2 + L_{\chi_{1m}} - L_{t m} + 2 \text{Li}_2 \left( 1 - \frac{\chi_1}{-t} \right) \right] , \]

\[ b_{245} = 0, \quad b_{245} = \frac{1}{t + \chi_1} \left[ -L_{t m} - \frac{\chi_1}{t + \chi_1} \left[ \frac{1}{2} I_{t m}^2 - \frac{1}{2} I_{\chi_{1m}}^2 + 2 L_{\chi_{1m}} - 2 L_{t m} \right. \right. \right. \]
\[ + \left. \left. 2 \text{Li}_2 \left( 1 - \frac{\chi_1}{-t} \right) \right] . \] (51)

\[ a_{145} = \frac{1}{\chi_1 - t} \left[ 2 L_{\chi_{1m}} - L_{t m} - 2i\pi + \frac{t}{\chi_1 - t} \left[ \frac{1}{2} I_{t m}^2 - \frac{1}{2} I_{\chi_{1m}}^2 + 2 L_{\chi_{1m}} \right. \right. \right. \]
\[ - 2 L_{t m} + 3\zeta(2) + 2 \text{Li}_2 \left( 1 + \frac{\chi_1}{-t} \right) + i\pi \left( L_{\chi_{1m}} - 2 \right) \right] , \]

\[ b_{145} = 0, \quad c_{145} = d_{145} = \frac{1}{\chi_1 - t} \left[ L_{t m} - L_{\chi_{1m}} + i\pi \right] . \] (52)

\[ a_{345} = -c_{345} = -d_{345} = \frac{1}{t} \left[ \frac{1}{2} I_{t m}^2 + 2 L_{t m} - 4\zeta(2) \right] . \] (53)

\[ a_{125} = \frac{1}{\chi_1} \left[ \frac{1}{2} I_{\chi_{1m}}^2 + L_{\chi_{1m}} - 2\zeta(2) \right] , \quad b_{125} = d_{125} = 0 , \]

\[ c_{125} = \frac{1}{\chi_1} \left[ L_{\chi_{1m}} - 2 \right] . \] (54)

\[ a_{235} = -c_{235} = \frac{1}{s_1 + \chi_1} \left[ L_{s_{1m}} - L_{\chi_{1m}} - i\pi \right] , \quad d_{235} = 0 , \]

\[ b_{235} = \frac{1}{s_1 + \chi_1} \left[ -L_{s_{1m}} + i\pi + \frac{\chi_1}{s_1 + \chi_1} \left[ 2 L_{s_{1m}} - 2 L_{\chi_{1m}} - \frac{3}{2} I_{s_{1m}}^2 - \frac{1}{2} I_{\chi_{1m}}^2 \right. \right. \right. \]
\[ + 2 L_{s_{1m}} L_{\chi_{1m}} + 9\zeta(2) + 2 \text{Li}_2 \left( 1 + \frac{\chi_1}{s_1} \right) + i\pi \left( -2 - 2 L_{\chi_{1m}} + 3 L_{s_{1m}} \right) \right] . \] (55)
\[ a_{135} = -b_{135} = \frac{1}{s} [L_{s_m} - i\pi], \quad c_{135} = d_{135} = 0. \]  
\[ a_{234} = -c_{234} = -b_{234} - d_{234} = \frac{1}{s_1} [-L_{s\mu} + i\pi + s_1 I_{234}], \]
\[ b_{234} = a_{234} - I_{234} = \frac{1}{s_1} [-L_{s\mu} + i\pi], \]
\[ d_{234} = \frac{1}{s_1} [L_{s_{1m}} + L_{s_{1\mu}} - 2i\pi - s_1 I_{234}]. \]  
\[ a_{134} = \frac{1}{s - \chi_1} \left[ L_{s\mu} - 2L_{\chi_{1\mu}}' + i\pi - \frac{2s}{s - \chi_1} \left( L_{s\mu} - L_{\chi_{1\mu}}' \right) + sI_{134} \right], \]
\[ b_{134} = a_{134} - I_{134} = \frac{1}{s - \chi_1} \left[ -L_{s\mu} + i\pi - \frac{2\chi_1'}{s - \chi_1} \left( L_{s\mu} - L_{\chi_{1\mu}}' \right) + \chi_1' I_{134} \right], \]
\[ c_{134} = d_{134} = \frac{1}{s - \chi_1} \left[ L_{\chi_{1\mu}}' - i\pi + \frac{2s}{s - \chi_1} \left( L_{s\mu} - L_{\chi_{1\mu}}' \right) - sI_{134} \right]. \]  
\[ a_{124} = I_{124} = \frac{1}{\chi_1} \left[ \frac{1}{2} L_{\chi_1\mu}^2 - \zeta(2) - i\pi L_{\chi_1\mu} \right], \quad d_{124} = \frac{1}{\chi_1} \left[ -L_{\chi_1\mu}' + i\pi \right], \]
\[ c_{124} = \frac{1}{\chi_1} \left[ \frac{-1}{2} L_{\chi_1\mu}^2 + L_{\chi_1\mu}' + \zeta(2) - 2 + i\pi \left( L_{\chi_1\mu} - 1 \right) \right], \quad b_{124} = 0. \]  
\[ a_{123} = b_{123} - I_{123} = \frac{1}{s - s_1} \left[ \frac{1}{2} L_{s_m}^2 - \frac{1}{2} L_{s_{1m}}^2 - L_{s_m} + L_{s_{1m}} \right. \]
\[ + \ i\pi \left( L_{s_{1m}} - L_{s_m} \right), \quad b_{123} = -\frac{1}{s - s_1} \left[ L_{s_m} - L_{s_{1m}} \right], \quad d_{123} = 0, \]
\[ c_{123} = \frac{1}{s - s_1} \left[ L_{s_{1m}}^2 - 2 - i\pi + \frac{s}{s - s_1} \left[ -\frac{1}{2} L_{s_m}^2 + \frac{1}{2} L_{s_{1m}}^2 \right. \right. \]
\[ + \ 2L_{s_m} - 2L_{s_{1m}} + i\pi \left( L_{s_m} - L_{s_{1m}} \right) \right]. \]  

2.8 Four-propagator integrals

2.8.1 Scalar

\[ I_{1245} = \int_0^1 dx dy \left[ \frac{dx dy}{[xy\chi_1 - \tilde{g}t][ym^2 - \tilde{x}y\chi_1]} \right] = \frac{1}{\chi_1\chi_1'} \left[ -L_{\chi_1\mu}^2 - \frac{1}{2} L_{\chi_1\mu}'^2 - \frac{1}{2} L_{t_m}^2 \right. \]
\[ - 2L_{\chi_1\mu} L_{\chi_1\mu}' + 2L_{\chi_1\mu} L_{t_m} + 2L_{\chi_1\mu}' L_t + \frac{4}{3} \zeta(2) + 2i\pi \left( L_{\chi_1\mu} + L_{\chi_1\mu}' - L_{t_m} \right) \right], \]
\[ I_{2345} = \frac{1}{t} \int_0^1 dx \left[ -\frac{1}{2} \ln \left( \frac{P_1^2}{\chi_1^2} \right) + \ln \left( \frac{x\chi_1}{-t} \right) \right] \]
\[ = \frac{1}{s_1 t} \left[ L_{s_1\mu}^2 - L_{s_1\mu} L_{\lambda_\mu} - 2L_{s_1\mu} L_{\chi_1\mu} + 2L_{s_1\mu} L_{t_\mu} - 5\zeta(2) \right] \]
In this section the masses are taken into account.

3 Self–energy and real–photon vertex corrections

The vector, tensor four–propagator and pentagon integrals are the same as in the case of equal fermion masses.

\[ I_{1345} = \frac{1}{t} \int_0^1 \frac{dx}{P^2} \left[ -\frac{1}{2} \ln \left( \frac{P^2}{\chi^2} \right) + \ln \left( \frac{-x\chi_1'}{t} \right) \right] \]

\[ I_{1325} = \frac{1}{\chi_1} \int_0^1 \frac{dx}{P^2} \left[ -\frac{1}{2} \ln \left( \frac{P^2}{\chi^2} \right) + \ln \left( \frac{P^2}{-x\chi_1'} \right) \right] = \frac{1}{s\chi_1} \left[ -\frac{1}{2} L_{s\mu}^2 - \frac{1}{2} L_{sm}^2 + \frac{1}{2} L_{s_1m}^2 \right] \]

\[ I_{1234} = \frac{1}{\chi_1} \int_0^1 \frac{dx}{P^2} \left[ -\frac{1}{2} \ln \left( \frac{P^2}{\chi^2} \right) + \ln \left( \frac{P^2}{m^2} \right) \right] = \frac{1}{s} \left[ -\frac{1}{2} L_{s\mu}^2 + 8\zeta(2) + 2\pi L_{s\mu} + 2 \ln \frac{\mu}{m} \right] \]

Useful integrals

\[ \int_0^1 \frac{dx}{P^2} = \frac{2}{s} \left[ -L_{sm} + \ln \frac{\mu}{m} + i\pi \right] \]

\[ \int_0^1 \frac{dx}{P^2} \ln x = \frac{1}{s} \left[ \frac{1}{2} L_{s\mu}^2 - \zeta(2) - i\pi L_{s\mu} \right] \]

\[ \int_0^1 \frac{dx}{P^2} \ln \left( \frac{P^2}{m^2} \right) = \frac{1}{s} \left[ -\frac{1}{2} L_{s\mu}^2 + \frac{1}{2} L_{s_1m}^2 + 8\zeta(2) - 2\pi L_{s\mu} \right] 
- 2 \ln \frac{\mu}{m} L_{s\mu} + i\pi \left( L_{s_1m} + L_{s_1\mu} + 2 \ln \frac{\mu}{m} \right) \]
Figure 2: Self–energy and real–photon vertex corrections to the incoming fermion

\[ \mathcal{L}_1 = \left[ \frac{\not{\phi}_1 - \not{k}_1 + m}{-\chi_1} \Gamma_\mu e^\mu + \Sigma(\not{\phi}_1 - \not{k}_1) \frac{1}{(\not{\phi}_1 - \not{k}_1 - m)^2} \not{\varphi} \right] u(p_1) \]  

(63)

Using well known expressions for the off-shell vertex function \( \Gamma \) and mass operator \( \Sigma \), we obtain

\[ \mathcal{L}_1 = \frac{\alpha}{2\pi} \left[ \mathcal{A}_1 \left( \not{\varphi} - \frac{e p_1}{k_1 p_1} \right) + \mathcal{A}_2 \not{k}_1 \not{\varphi} \right] u(p_1), \]

(64)

where

\[ \mathcal{A}_1 = -\frac{m}{2(\chi_1 - m^2)} \left[ 1 - \frac{\chi_1}{\chi_1 - m^2} L \chi_1 \right], \]

\[ \mathcal{A}_2 = -\frac{1}{2(\chi_1 - m^2)} + \frac{2\chi_1^2 - 3m^2 \chi_1 + 2m^4}{2\chi_1(\chi_1 - m^2)^2} L \chi_1 + \frac{m^2}{\chi_1^2} \left[ -\text{Li}_2 \left( 1 - \frac{\chi_1}{m^2} \right) \right. \]

+ \ \zeta(2). \]

(65)

Figure 3: Self–energy and real–photon vertex corrections to the outgoing fermion

\[ \mathcal{L}'_1 = \frac{\alpha}{2\pi} \bar{u}(p'_1) \left[ \mathcal{B}_1 \left( \not{\varphi} - \frac{e p'_1}{k_1 p'_1} \right) + \mathcal{B}_2 \not{k}_1 \not{\varphi} \right], \]

(66)

where

\[ \mathcal{B}_1 = \frac{m}{2(\chi'_1 + m^2)} \left[ 1 - \frac{\chi'_1}{\chi'_1 + m^2} \left( L \chi'_1 - i\pi \right) \right], \]
\[
\mathcal{B}_2 = \frac{1}{2(\chi' + m^2)} - \frac{2\chi'^2 + 3m^2\chi' + 2m^4}{2\chi'(\chi' + m^2)^2} \left( L\chi' - i\pi \right) + \frac{m^2}{\chi'^2} \left[ -\text{Li}_2 \left( 1 + \frac{\chi'}{m^2} \right) \right] + \zeta(2).
\]

(67)

Figure 4: Self–energy and real–photon vertex corrections to the incoming anti-fermion

\[
\mathcal{L}_2 = \frac{\alpha}{2\pi} \bar{u}(-p_2) \left[ \mathcal{C}_1 \left( \phi' - \frac{\not{k}_1 e}{\not{k}_1 \not{p}_2} \right) + \mathcal{C}_2 \not{k}_1 \not{g} \right], \quad (68)
\]

where

\[
\begin{align*}
\mathcal{C}_1 &= -\frac{m}{2(\chi_2 - m^2)} \left[ 1 - \frac{\chi_2}{\chi_2 - m^2} L\chi_2 \right], \\
\mathcal{C}_2 &= -\frac{1}{2(\chi_2 - m^2)} + \frac{2\chi_2^2 - 3m^2\chi_2 + 2m^4}{2\chi_2(\chi_2 - m^2)^2} L\chi_2 + \frac{m^2}{\chi_2^2} \left[ -\text{Li}_2 \left( 1 - \frac{\chi_2}{m^2} \right) \right] + \zeta(2).
\end{align*}
\]

(69)

Figure 5: Self–energy and real–photon vertex corrections to the outgoing anti-fermion

\[
\mathcal{L}'_2 = \frac{\alpha}{2\pi} \left[ \mathcal{D}_1 \left( \phi' - \frac{\not{k}_1 e'}{\not{k}_1 \not{p}'_2} \right) + \mathcal{D}_2 \not{k}_1 \not{g} \right] u(-p'_2), \quad (70)
\]

where

\[
\begin{align*}
\mathcal{D}_1 &= \frac{m}{2(\chi'_2 + m^2)} \left[ 1 - \frac{\chi'_2}{\chi'_2 + m^2} \left( L\chi'_2 - i\pi \right) \right], \\
\mathcal{D}_2 &= -\frac{1}{2(\chi'_2 - m^2)} + \frac{2\chi'_2^2 - 3m^2\chi'_2 + 2m^4}{2\chi'_2(\chi'_2 - m^2)^2} L\chi'_2 + \frac{m^2}{\chi'_2^2} \left[ -\text{Li}_2 \left( 1 - \frac{\chi'_2}{m^2} \right) \right] + \zeta(2).
\end{align*}
\]

(71)
\[ D_2 = \frac{1}{2(\chi'_2 + m^2)} - \frac{2\chi'_2 + 3m^2\chi'_2 + 2m^4}{2\chi'_2(\chi'_2 + m^2)^2} (L_{\chi'_2} - i\pi) \]
\[ + \frac{m^2}{\chi'_2} \left[ -\text{Li}_2 \left( 1 + \frac{\chi'_2}{m^2} \right) + \zeta(2) \right]. \] (71)

4 Heavy–photon vertex diagrams

4.1 Notations

Figure 6: Heavy–photon vertex diagrams with real–photon emission

\[ J^{1,\mu,\nu}_{ijkl} = \int \frac{d^4 k \, 1, k^\mu, k^\mu k^\nu}{i\pi^2 (i)(j)(k)(l)} \] (72)

(0) = \( k^2 - \chi^2 \), (1) = \( (p_1 - k)^2 - m^2 \), (2) = \( (p'_1 - k)^2 - m^2 \),
(\( q \)) = \( (p_1 - k_1 - k)^2 - m^2 \). (73)

4.2 Two-propagator integrals

4.2.1 Scalar

\[ J_{01} = J_{02} = L_\Lambda + 1, \; J_{12} = L_\Lambda - L_{t_1} + 1, \; J_{0q} = L_\Lambda + 1 - L_{\chi_1}, \]
\[ J_{1q} = L_\Lambda - 1, \; J_{2q} = L_\Lambda - L_t + 1. \] (74)
4.2.2 Vector

\[ J_{01}^\mu = p_1^\mu \left[ \frac{1}{2} L_\Lambda - \frac{1}{4} \right], \quad J_{02}^\mu = p_1^\mu \left[ \frac{1}{2} L_\Lambda - \frac{1}{4} \right], \quad J_{1q}^\mu = \left( p_1 - \frac{1}{2} k_1 \right)^\mu \left[ L_\Lambda - \frac{3}{2} \right], \]

\[ J_{12}^\mu = \left( p_1' + p_1 - k_1 \right)^\mu \left[ \frac{1}{2} L_\Lambda + \frac{1}{4} - \frac{1}{2} L_t \right]. \] (75)

4.3 Three-propagator integrals

4.3.1 Scalar

\[ J_{012} = \frac{1}{2 L_t} \left[ -2 L_\Lambda L_{t_1} + L_{t_1}^2 - 2 \zeta(2) \right], \quad J_{12q} = \frac{1}{2 (\chi' - \chi_1)} \left[ L_{t_1}^2 - L_{t_1}^2 \right], \]

\[ J_{01q} = - \frac{1}{\chi_1} \left[ - L_{t_1} \left( 1 - \frac{\chi_1}{m^2} \right) + \zeta(2) \right], \]

\[ J_{02q} = \frac{1}{\chi_1 + t_1} \left[ L_t (L_t - L_{\chi_1}) + \frac{1}{2} (L_t - L_{\chi_1})^2 + 2 L_{t_1} \left( 1 + \frac{\chi_1}{t_1} \right) \right]. \] (76)

4.3.2 Vector

Parameterization

\[ J_{\mu}^\nu = a_{ijk} p_1^{\mu} + b_{ijk} p_1^{\mu} + c_{ijk} q^\mu. \] (77)

\[ a_{012} = b_{012} = \frac{1}{t_1} L_{t_1}, \quad c_{012} = 0, \]

\[ a_{01q} = \frac{1}{\chi_1} \left[ \chi_1 J_{01q} + 2 L_{\chi_1} - 2 \right], \quad b_{01q} = - c_{01q} = \frac{1}{\chi_1} \left[ - L_{\chi_1} + 2 \right], \]

\[ a_{02q} = 0, \quad b_{02q} = \frac{\chi_1}{\chi' + t_1} J_{02q} + \frac{2}{\chi' + t_1} L_t - \frac{t}{\chi' + t_1} L_{\chi_1}, \]

\[ c_{02q} = - \frac{1}{\chi' + t_1} L_t + \frac{1}{\chi' + t_1} L_{\chi_1}, \]

\[ a_{12q} = \frac{t}{\chi' - \chi_1} J_{12q} + \frac{t + t_1}{(\chi' - \chi_1)^2} L_{t_1} - \frac{t}{(\chi' - \chi_1)^2} L_t - \frac{2}{\chi' - \chi_1}, \]

\[ b_{12q} = J_{12q} - a_{12q}, \]

\[ c_{12q} = \frac{t_1}{\chi' - \chi_1} J_{12q} + \frac{2 t_1}{(\chi' - \chi_1)^2} L_{t_1} - \frac{t + t_1}{(\chi' - \chi_1)^2} L_t + \frac{2}{\chi' - \chi_1}. \] (78)
4.3.3 Tensor

\[ J_{ijk}^{\mu
u} = g_{ijk}^{T}g^{\mu\nu} + a_{ijk}^{T}p_{\mu}^{T}p_{\nu}^{T} + b_{ijk}^{T}p_{1}^{T}p_{1}^{T} + c_{ijk}^{T}q_{\mu}^{T}q_{\nu}^{T} + \alpha_{ijk}^{T}\{p_{1}^{T}p_{1}^{T}\} + \beta_{ijk}^{T}\{p_{1}^{T}q_{\nu}^{T}\} + \gamma_{ijk}^{T}\{p_{1}^{T}q_{\nu}^{T}\}. \] (79)

\[
g_{012}^{T} = \frac{1}{4}L_{\Lambda} - \frac{1}{4}L_{t_1} + \frac{3}{8}a_{012}^{T} = b_{012}^{T} = \frac{1}{2t_1}L_{t_1} - \frac{1}{2t_1},
\]

\[
\alpha_{012}^{T} = \frac{1}{2t_1}, \quad c_{012}^{T} = \beta_{012}^{T} = \gamma_{012}^{T} = 0. \] (80)

\[
g_{01q}^{T} = -\frac{1}{4}L_{\chi_1} + \frac{3}{8}a_{01q}^{T} = J_{01q} + \frac{3}{\chi_1}L_{\chi_1} - \frac{9}{2\chi_1},
\]

\[
b_{01q}^{T} = c_{01q}^{T} = -\gamma_{01q}^{T} = -\frac{1}{2\chi_1}L_{\chi_1} + \frac{1}{\chi_1}, \quad \beta_{01q}^{T} = -\alpha_{01q}^{T} = \frac{1}{2\chi_1}L_{\chi_1} - \frac{3}{2\chi_1}. \] (81)

\[
g_{02q}^{T} = -\frac{1}{4}\chi_1 \frac{L_{\chi_1}}{\chi_1 + t_1} - \frac{1}{4}\chi_1 \frac{L_{t_1}}{\chi_1 + t_1} + \frac{3}{8}a_{02q}^{T} = J_{02q} + \frac{3}{\chi_1}L_{\chi_1} + \frac{3}{2(\chi_1 + t_1)^2}J_{02q},
\]

\[
b_{02q}^{T} = \left[ -\frac{\chi_1(t - \chi_1)}{2(\chi_1 + t_1)^3} - \frac{1}{2}\left(\frac{t^2 + 2t \chi_1 - \chi_1^2}{\chi_1 + t_1}\right) \right] L_{\chi_1} + \frac{t(t + 4 \chi_1)}{2(\chi_1 + t_1)^3}J_{02q} + \frac{t - \chi_1}{2(\chi_1 + t_1)^2} + \frac{\chi_1^2}{(\chi_1 + t_1)^2}J_{02q},
\]

\[
c_{02q}^{T} = -\frac{1}{2\chi_1 + t_1}L_{\chi_1} + \frac{1}{2(\chi_1 + t_1)}L_{t_1}, \quad \alpha_{02q}^{T} = \alpha_{02q}^{T} = \beta_{02q}^{T} = 0,
\]

\[
\gamma_{02q}^{T} = \frac{t + 2 \chi_1}{2(\chi_1 + t_1)^3}L_{\chi_1} - \frac{t + 2 \chi_1}{2(\chi_1 + t_1)^3}L_{t_1} - \frac{1}{2(\chi_1 + t_1)}L_{t_1}. \] (82)

\[
g_{12q}^{T} = \frac{1}{4}\frac{t_1}{\chi_1 - \chi_1}L_{t_1} - \frac{1}{4}\frac{t_1}{\chi_1 - \chi_1}L_{t_1} + \frac{1}{4}\frac{L_{\Lambda}}{\chi_1 - \chi_1} + \frac{3}{8},
\]

\[
a_{12q}^{T} = \frac{t^2}{(\chi_1 - \chi_1)^2}J_{12q} + \frac{(3t^2 + 4t_1 t - t_1^2)}{2(\chi_1 - \chi_1)^3}L_{t_1} - \frac{3t^2}{(\chi_1 - \chi_1)^3}L_{t_1} + \frac{4t - t_1}{(\chi_1 - \chi_1)^2},
\]

\[
b_{12q}^{T} = \frac{t^2}{(\chi_1 - \chi_1)^2}J_{12q} + \frac{-t^2 + 4t_1 t + 3t_1^2}{2(\chi_1 - \chi_1)^3}L_{t_1} + \frac{t(t - 4t_1)}{(\chi_1 - \chi_1)^3}L_{t_1} + \frac{3t_1}{(\chi_1 - \chi_1)^2},
\]

\[
c_{12q}^{T} = \frac{t^2}{(\chi_1 - \chi_1)^2}J_{12q} + \frac{3t^2}{(\chi_1 - \chi_1)^3}L_{t_1} + \frac{t^2 - 4t_1 t - 3t_1^2}{2(\chi_1 - \chi_1)^3}L_{t_1} + \frac{4t_1 - t}{(\chi_1 - \chi_1)^2},
\]

\[
\alpha_{12q}^{T} = -\frac{t_1}{(\chi_1 - \chi_1)^2}J_{12q} - \frac{t^2 + 4t_1 t + t_1^2}{2(\chi_1 - \chi_1)^3}L_{t_1} + \frac{t(t + 2t_1)}{(\chi_1 - \chi_1)^3}L_{t_1} - \frac{2(t + t_1)}{(\chi_1 - \chi_1)^2},
\]

\[
\beta_{12q}^{T} = \frac{t_1}{(\chi_1 - \chi_1)^2}J_{12q} + \frac{t_1(5t + t_1)}{2(\chi_1 - \chi_1)^3}L_{t_1} - \frac{t(t + 5t_1)}{2(\chi_1 - \chi_1)^3}L_{t_1} + \frac{3}{2(\chi_1 - \chi_1)^2},
\]

\[
\gamma_{12q}^{T} = -\frac{t_1}{(\chi_1 - \chi_1)^2}J_{12q} - \frac{t_1(t + 5t_1)}{2(\chi_1 - \chi_1)^3}L_{t_1} + \frac{-t^2 + 5t_1 t + 2t_1^2}{2(\chi_1 - \chi_1)^3}L_{t_1}.
\]
+ \frac{t - 7t_1}{2(\chi'_1 - \chi_1)^2}. \quad (83)

4.4 Four-propagator integrals

4.4.1 Scalar

\[ J_{012q} = -\frac{1}{\chi_1 t_1} \left[ -L_\chi L_{t_1} + 2 L_{t_1} L_{\chi_1} - L_t^2 - 2 \text{Li}^2 \left( 1 - \frac{t}{t_1} \right) - \zeta(2) \right]. \quad (84) \]

4.4.2 Vector

Parameterization

\[ J_{012q}^\mu = a_{012q} p_1^\mu + b_{012q} p_1^\mu + c_{012q} q^\mu. \quad (85) \]

\[ a_{012q} = \frac{1}{d} \left[ -(t_1 \chi_1 + t \chi'_1) J_{12q} + (\chi'_1 + t_1)^2 J_{02q} - \chi_1 (\chi'_1 - t_1) J_{01q} 
- t_1 (\chi'_1 + t_1) (J_{012q} - \chi_1 J_{012q}) \right], \]

\[ b_{012q} = \frac{1}{d} \left[ (t_1 \chi'_1 + t \chi_1) J_{12q} - (\chi'_1 + t_1)^2 J_{02q} - \chi_1 (t_1 - \chi_1) J_{01q} 
+ t_1 (t_1 - \chi_1) (J_{012q} + \chi_1 J_{012q}) \right], \]

\[ c_{012q} = \frac{1}{d} \left[ -t_1 (\chi'_1 + t_1) J_{12q} + t_1 (\chi'_1 + t_1) J_{02q} + \chi_1 t_1 J_{01q} 
- t_1^2 (J_{012q} + \chi_1 J_{012q}) \right]. \quad (86) \]

where \( d = -2t_1 \chi_1 \chi'_1. \)

4.4.3 Tensor

Parameterization

\[ J_{012q}^{\mu \nu} = g_{012q}^{\mu \nu} + a_{012q}^{\mu \nu} p_1^\mu p_1^\nu + b_{012q}^{\mu \nu} p_1^\mu p_1^\nu + c_{012q}^{\mu \nu} q^\mu q^\nu 
+ \alpha_{012q}^{T} \{p_1^\mu q^\nu\} + \beta_{012q}^{T} \{p_1^\mu q^\nu\} + \gamma_{012q}^{T} \{p_1^\mu q^\nu\}. \quad (87) \]

\[ g_{012q}^{T} = \frac{1}{2} [J_{12q} - \chi_1 c_{012q}], \]

\[ a_{012q}^{T} = \frac{1}{d} \left[ (\chi'_1 + t_1)^2 (J_{12q} - \chi_1 c_{012q}) - (\chi_1 t_1 + \chi'_1 t) a_{012q} 
- \chi_1 (\chi'_1 - t_1) a_{01q} - t_1 (\chi'_1 + t_1) (a_{012q} + \chi_1 a_{012q}) \right], \]

\[ b_{012q}^{T} = \frac{1}{d} \left[ (t_1 - \chi_1)^2 (J_{12q} - \chi_1 c_{012q}) + (\chi'_1 t_1 + \chi_1 t) b_{12q} 
- (t_1 t + \chi_1 \chi'_1) b_{02q} + \chi_1 (\chi_1 - t_1) b_{01q} \right] \]
\[
\begin{align*}
&+ \ t_1 (t_1 - \chi_1)(a_{012} + \chi_1 b_{012q})]; \\
\alpha_{012q}^T &= \frac{1}{d} \left[ t_1^2 (J_{12q} - 2 \chi_1 c_{012q}) - t_1 (\chi'_1 + \chi_1) a_{12q} \\
&- t_1 \chi_1 b_{01q} + t_1 (t_1 + \chi'_1) c_{02q} \right], \\
\beta_{012q}^T &= \frac{1}{d} \left[ -(t_1 t + \chi_1 \chi_1') (J_{12q} - \chi_1 c_{012q}) + (\chi'_1 t_1 + \chi_1 t) a_{12q} \\
&+ \chi_1 (\chi_1 - t_1) a_{01q} + t_1 (t_1 - \chi_1)(a_{012} + \chi_1 a_{012q}) \right], \\
\gamma_{012q}^T &= \frac{1}{d} \left[ t_1 (\chi_1 - t_1) (J_{12q} - 2 \chi_1 c_{012q}) - (\chi'_1 t_1 + \chi_1 t) c_{12q} \\
&- (t_1 t + \chi'_1 \chi_1) c_{02q} - \chi_1 (\chi_1 - t_1) b_{01q} \right].
\end{align*}
\]

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