Subsymmetry and asymmetry models for multiway square contingency tables with ordered categories

1 Introduction

Square contingency tables with the same categories occur frequently in applied sciences. Such tables arise from tabulating the repeated measurements of a categorical response variable. Some examples for these kind of tables are: for instance, when the subjects are measured at two different points in time (e.g., responses before and after experiments); the decisions of two experts are measured on the same set of subjects (e.g., the grading of the same cancer tumors by two specialists); two similar units in a sample are measured (e.g., the grades of vision of the left and the right eyes); matched pair experiments (e.g., social status of the fathers and sons) [1]. For square contingency tables, several models have been proposed (see, for example [2–8] but the models of symmetry (S), quasi-symmetry (QS), marginal homogeneity (MH) are classical and well known models [9, 10] and the applicability of these models is straightforward. The QS is less restrictive model than the S model [11–13].

Consider an RxR square contingency table with the same row and column classifications. Let $p_{ij}$ denote the probability that an observation will fall in the $i$th row and $j$th column of the table. Bowker [14] considered the symmetry (S) model for RxR tables defined by

$$p_{ij} = p_{ji} \quad (i \neq j).$$

The S model implies that the probability that an observation will fall in cell $(i, j)$ of the table is equal to the probability that it falls in cell $(j, i)$.

Multiway contingency table is obtained when a sample of $n$ observations is cross classified with respect to $T$ categorical variables having the same number of categories. Such tables are very popular in panel studies or matched pair examples. The symmetry model is denoted in multidimensional way.

Denote the $k$th categorical variable by $X_k$ ($k = 1, \ldots, T$) and consider an $R^T$ contingency table ($T \geq 3$). Let $p_{i_1 \ldots i_T}$ denote the probability that an observation will fall in the $(i_1, \ldots, i_T)$th cell of the table.
Agresti [1] defined the S model as

\[ p_{i_1...i_T} = p_{j_1...j_T}, \]

for any permutation \((j_1, \ldots, j_T)\) of \((i_1, \ldots, i_T)\) with \(i_t = 1, \ldots, r; t = 1, \ldots, T\).

For example, when \(T = 3\), let \(X, Y\) and \(Z\) denote the row, column and layer variables, the S model can be expressed as

\[ p_{ijk} = p_{ikj} = p_{jik} = p_{jki} = p_{kij} = p_{kji}. \]

The simplest possible model of interest is the model of complete independence, where the joint distribution of the three variables is the product of the marginals. The corresponding hypothesis is

\[ H_0: p_{ijk} = p_{i..j..k} \]

Symmetry model for multiway tables is given in general as follows:

\[ p_{i_1...i_T} = \left( \prod_{j=1}^{T} a_j \right) \left( \prod_{k=1}^{T} \beta_k \right) \left( \prod_{s=1}^{T} \gamma_s \right) \left( \prod_{l=1}^{T} \omega_l \right) \psi_{i_1...i_T}. \]  \hspace{1cm} (1.1)

The common schemes for representing contingency tables are based on the row column and layer variables that are independent. In three way contingency tables, the choice of predictor and control variable is of interest to many researchers. The purpose of this paper is to give some models which represent the subsymmetry and asymmetry for multiway contingency tables. We will concentrate on only three dimensional tables which are a cross-classification of observations by the levels of three categorical variables.

The models are defined in the sub symmetry and asymmetry context taking the first variable as a control variable. The models below are often used to analyze three dimensional tables.

| Model                          | Terms               |
|-------------------------------|---------------------|
| Saturated                     | (XYZ)               |
| Homogeneous associations      | (XY, XZ, YZ)        |
| Conditional independence      | (XY, XZ), (XY, YZ), (XZ, YZ) |
| Joint independence            | (XY, Z), (XZ, Y), (X, XZ) |
| Complete independence         | (X, Y, Z)           |

2 Subsymmetry and asymmetry models

We collect the triplet \((X, Y, Z)\) for each unit in a sample of \(n\) units, then the data can be summarized as a three-dimensional table. Let \(p_{ijk}\) be the probability of units having \(X = i, Y = j,\) and \(Z = k\). In what follows, we define some models that represent the subsymmetry and asymmetry.

Model 1:

\[ p_{ijk} = \left( \prod_{j=1}^{C} \beta_j \right) \left( \prod_{k=1}^{K} \gamma_k \right) \left( \prod_{s=1}^{S} \psi_s \right) \left( \prod_{l=1}^{L} \omega_l \right) \psi^{\delta.v.n}. \]

\[ j = 1, \ldots, C; k = 1, \ldots, K; s = 1, 2; l = 1, 2, 3, 4. \]

\[ \sum_{j=1}^{C} \beta_j = \sum_{k=1}^{K} \gamma_k = 0 \]
Model 2:

\[ p_{ijk} = \left( \prod_{j=1}^{C} \beta_j \right) \left( \prod_{k=1}^{K} \gamma_k \right) \left( \prod_{s=1}^{S} \psi_s \right) \left( \prod_{l=1}^{L} \omega_l \right) \cdot v \cdot \eta \]

\[ j = 1, \ldots, C; k = 1, \ldots, K; s = 1, 2; w = 1, 2, 3, 4. \]

\[ \sum_{j=1}^{C} \beta_j = \sum_{k=1}^{K} \gamma_k = 0 \]

Model 3:

\[ p_{ijk} = \left( \prod_{j=1}^{C} \beta_j \right) \left( \prod_{k=1}^{K} \gamma_k \right) \left( \prod_{s=1}^{S} \psi_s \right) \left( \prod_{l=1}^{L} \omega_l \right) \cdot \tau \cdot \eta \cdot \nu \]

\[ i = 1, \ldots, R; j = 1, \ldots, C; k = 1, \ldots, K; s = 1, 2; l = 1, 2, 3, 4. \]

\[ \sum_{j=1}^{C} \beta_j = \sum_{k=1}^{K} \gamma_k = 0 \]

Model 4:

\[ p_{ijk} = \left( \prod_{j=1}^{C} \beta_j \right) \left( \prod_{k=1}^{K} \gamma_k \right) \left( \prod_{s=1}^{S} \psi_s \right) \left( \prod_{l=1}^{L} \omega_l \right) \cdot u \cdot \eta \]

\[ i = 1, \ldots, R; j = 1, \ldots, C; k = 1, \ldots, K; s = 2, 3, 5; l = 2, 3, 5. \]

\[ \sum_{j=1}^{C} \beta_j = \sum_{k=1}^{K} \gamma_k = 0 \]

Model 5:

\[ p_{ijk} = \left( \prod_{j=1}^{C} \beta_j \right) \left( \prod_{k=1}^{K} \gamma_k \right) \left( \prod_{s=1}^{S} \psi_s \right) \left( \prod_{m=1}^{M} \theta_m \right) \cdot \tau \cdot \delta \cdot \nu \]

\[ i = 1, \ldots, R; j = 1, \ldots, C; k = 1, \ldots, K; s = 2, 3, 5; m = 2, 3, 5. \]

\[ \sum_{j=1}^{C} \beta_j = \sum_{k=1}^{K} \gamma_k = 0 \]

Model 6:

\[ p_{ijk} = \left( \prod_{i=1}^{R} \alpha_i \right) \left( \prod_{j=1}^{C} \beta_j \right) \left( \prod_{k=1}^{K} \gamma_k \right) \left( \prod_{s=1}^{S} \psi_s \right) \left( \prod_{m=1}^{M} \theta_m \right) \left( \prod_{l=1}^{L} \omega_l \right) \]

\[ i = 1, \ldots, R; j = 1, \ldots, C; k = 1, \ldots, K; s = 2, 3, 5; m = 2, 3, 5; l = 2, 3, 5. \]

\[ \sum_{i=1}^{R} \alpha_i = \sum_{j=1}^{C} \beta_j = \sum_{k=1}^{K} \gamma_k = 0 \]

Model 7:

\[ p_{ijk} = \left( \prod_{i=1}^{R} \alpha_i \right) \left( \prod_{j=1}^{C} \beta_j \right) \left( \prod_{s=1}^{S} \psi_s \right) \left( \prod_{m=1}^{M} \theta_m \right) \cdot v \cdot \tau \cdot \eta \]

\[ i = 1, \ldots, R; j = 1, \ldots, C; k = 1, \ldots, K; s = 2, 3, 5; m = 2, 3, 5. \]

\[ \sum_{i=1}^{R} \alpha_i = \sum_{j=1}^{C} \beta_j = \sum_{k=1}^{K} \gamma_k = 0 \]
Model 8:

\[ p_{ijk} = \left( \prod_{j=1}^{C} \beta_j \right) \left( \prod_{k=1}^{K} \gamma_k \right) \left( \prod_{s=1}^{S} \psi_s \right) \left( \prod_{m=1}^{M} \theta_m \right) \eta \cdot \xi \]

\[ i = 1, \ldots, R; j = 1, \ldots, C; k = 1, \ldots, K; s = 2, 3, 5; m = 2, 3, 5. \]

\[ \sum_{j=1}^{C} \beta_j = \sum_{k=1}^{K} \gamma_k = 0 \]

Model 9:

\[ p_{ijk} = \left( \prod_{j=1}^{C} \beta_j \right) \left( \prod_{k=1}^{K} \gamma_k \right) \left( \prod_{s=1}^{S} \psi_s \right) \left( \prod_{m=1}^{M} \theta_m \right) \eta \cdot \xi \cdot \nu \]

\[ i = 1, \ldots, R; j = 1, \ldots, C; k = 1, \ldots, K; s = 2, 3, 5; m = 2, 3, 5. \]

\[ \sum_{j=1}^{C} \beta_j = \sum_{k=1}^{K} \gamma_k = 0 \]

Model 10:

\[ p_{ijk} = \left( \prod_{l=1}^{L} \omega_s \right) \left( \prod_{s=1}^{S} \psi_s \right) \left( \prod_{m=1}^{M} \theta_m \right) \]

\[ l = 1, \ldots, 6; s = 1, \ldots, 6; m = 1, \ldots, 6 \]

Parameters in the models and the corresponding symbols in design matrices are defined as:

- \( \alpha \): row parameter (\( X \));
- \( \beta \): column parameter (\( Y \));
- \( \gamma \): layer parameter (\( Z \));
- \( \psi \): symmetry parameter (\( S \));
- \( \omega \): sub-symmetry parameter for \( X \times Z \) (\( B \));
- \( \theta \): sub-symmetry parameter for \( X \times Y \) (\( W \));
- \( \tau \): conditional symmetry parameter for \( Y \times Z \) (\( CS \));
- \( \delta \): inverse diagonal matrix for \( X \times Z \) (\( SSS \));
- \( \xi \): diagonal asymmetry parameter (\( DA \));
- \( \eta \): upper triangle parameter (\( CCS \));
- \( \nu \): main diagonal parameter (\( V \)).

Each model is in the log-linear form, therefore each has its associated degrees of freedom. The number of parameters to be fit are, for instance, the degrees of freedom for Model 1, which are:

\[ 27 - \left[ 1 + 2 + 2 + 2 + 4 + 1 + 1 + 1 \right] = 13. \]

Subsymmetry matrices are defined by each dimension as:

For \( X \times Y \), \( W = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 5 \\ 3 & 5 & 6 \end{bmatrix} \), for \( X \times Z \), \( B = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 5 \\ 3 & 5 & 6 \end{bmatrix} \), for \( Y \times Z \), \( S = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 5 \\ 3 & 5 & 6 \end{bmatrix} \).

\( V \) matrix corresponds to the cells on the main diagonal for \( X \times Y \times Z \).

\[ V = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \]
The conditional factor variables are defined for the asymmetric associations as follows:

Conditional symmetry matrix:
For \( YxZ / i = 1,2 \), \( CS = \begin{bmatrix} 1 & 2 & 2 \\ 3 & 1 & 2 \\ 3 & 3 & 1 \end{bmatrix} \),

Upper triangle matrix:
For \( YxZ / i = 3 \), \( CCS = \begin{bmatrix} 1 & 2 & 2 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix} \),

Diagonal asymmetry matrix:
For \( YxZ / i = 1,2 \), \( DA = \begin{bmatrix} 5 & 1 & 2 \\ 3 & 5 & 1 \\ 4 & 3 & 5 \end{bmatrix} \),

Inverse diagonal matrix:
For \( YxZ / i = 1,2,3 \), \( SSS = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix} \) Using these factors we analyze the models by GLM approach.

3 Numerical example

The data in Table 1 are taken directly from Yamamoto et al. [15] and give results of the treatment group only in randomized clinical trials conducted by a pharmaceutical company in anemic patients with cancer receiving chemotherapy. The response is the patient’s hemoglobin (HB) concentration at baseline (before treatment) and following 4 and 8 weeks of treatment. Hb response is classified as \( \geq 10 \) g/dl, 8–10 g/dl and < 8 g/dl. The reference ranges for hemoglobin concentration in adults are as: for men: 14.0–17.5 g/dL, for women: 12.3–15.3 g/dL.

Table 1: Hemoglobin concentration at baseline, 4 weeks and 8 weeks in carcinomatous anemia patients from a randomized clinical trial.

| Baseline | 4 weeks | 8 weeks | 8 weeks |
|----------|---------|---------|---------|
| \( \geq 10 \) g/dl | \( \geq 10 \) g/dl | 77 | 7 | 1 |
| 8–10 g/dl | \( \geq 10 \) g/dl | 43 | 7 | 0 |
| < 8 g/dl | \( \geq 10 \) g/dl | 3 | 0 | 0 |
| \( \geq 10 \) g/dl | 8–10 g/dl | 3 | 8 | 1 |
| 8–10 g/dl | 8–10 g/dl | 17 | 16 | 5 |
| < 8 g/dl | 8–10 g/dl | 3 | 8 | 1 |
| \( \geq 10 \) g/dl | < 8 g/dl | 1 | 1 | 1 |
| 8–10 g/dl | < 8 g/dl | 0 | 2 | 3 |
| < 8 g/dl | < 8 g/dl | 0 | 4 | 3 |

The Models (1–10) proposed here attempt to analyze what is the relationship between X, Y and Z taking “Baseline” as the control.

The example of the design matrix is given for Model (8) in Table 2.
Correspondingly, denote \( m_{ijk} \) expected frequencies, the Model (8) is represented as

\[
p_{ijk} = \left( \prod_{j=1}^{C} \beta_j \right) \left( \prod_{k=1}^{K} \gamma_k \right) \left( \prod_{s=1}^{S} \psi_s \right) \left( \prod_{m=1}^{M} \theta_m \right) \eta \cdot \xi.
\]

Correspondingly, denote \( m_{ijk} \) expected frequencies, the Model (8) is represented as

\[
\text{Log}(m_{ijk}) = Y + Z + S2 + S3 + S5 + W2 + W3 + W5 + DA + CCS.
\]

In this model representation, “Baseline” is the control variable therefore it is not included in the parameters.

Model (8) tests the \( p_{ijk} = \beta_1 \gamma_k \psi_2 \psi_3 \psi_4 \theta_3 \theta_5 \eta \cdot \xi \) hypothesis and takes the table \( Y \times Z \) frequencies. The probability that a subject at baseline has hemoglobin level \( \geq 10 \text{ g/dl} \) is 13.10 more likely being \( \geq 10 \text{ g/dl} \) at 4 and 8 consecutive weeks instead of 8–10 g/dl.

Table 2: Design matrix of Model (8).

| \( X \) | \( Y \) | \( Z \) | Parameter |
|--------|--------|--------|-----------|
| \( 1 \) | \( 1 \) | \( 1 \) | \( Y = 1 \) |
| \( 1 \) | \( 2 \) | \( 1 \) | \( Y = 2 \) |
| \( 1 \) | \( 3 \) | \( 1 \) | \( Z = 1 \) |
| \( 1 \) | \( 1 \) | \( 0 \) | \( Z = 2 \) |
| \( 1 \) | \( 2 \) | \( 1 \) | \( S2 \) |
| \( 1 \) | \( 3 \) | \( 1 \) | \( S3 \) |
| \( 1 \) | \( 1 \) | \( 0 \) | \( S5 \) |
| \( 1 \) | \( 0 \) | \( 1 \) | \( DA \) |
| \( 1 \) | \( 0 \) | \( 1 \) | \( W2 \) |
| \( 1 \) | \( 0 \) | \( 1 \) | \( W3 \) |
| \( 1 \) | \( 0 \) | \( 1 \) | \( W5 \) |
| \( 1 \) | \( 0 \) | \( 1 \) | \( CCS \) |

Design matrices are generated for each model. Likelihood ratio chi-square values with associated degrees of freedom, AIC and BIC are given in Table 3. Model comparisons, here in addition to the goodness of fit tests, tend to give better information on what model represents the data better.

The results show that all models fit the data well. The smallest value for both AIC and BIC is obtained for Model (8). Note that Model (8) and Model (9) are the conditional models that collapsed the baseline variable. Recall that Model (8) is

\[
p_{ijk} = \left( \prod_{j=1}^{C} \beta_j \right) \left( \prod_{k=1}^{K} \gamma_k \right) \left( \prod_{s=1}^{S} \psi_s \right) \left( \prod_{m=1}^{M} \theta_m \right) \eta \cdot \xi.
\]
Table 3: Model results under various models.

| Model | Terms                          | Likelihood ratio chi-square | Degrees of freedom | P-value | BIC  | AIC  |
|-------|--------------------------------|-----------------------------|--------------------|---------|------|------|
| 1     | Y, Z, S1, S2, B1, B2, B3, B4, CCS, SSS, V | 18.043                      | 13                 | 0.156   | -51.771 | -7.957 |
| 2     | Y, Z, S1, S2, B1, B2, B3, B4, V, CCS    | 19.531                      | 14                 | 0.146   | -55.658 | -8.469 |
| 3     | Y, Z, S1, S2, B1, B2, B3, B4, CC, CCS, V| 19.443                      | 13                 | 0.110   | -50.059 | -6.557 |
| 4     | Y, Z, S2, S3, S5, B2, B3, B5, V        | 20.268                      | 14                 | 0.122   | -54.922 | -7.73  |
| 5     | Y, Z, V2, S2, S3, S5, B2, B3, B5, CS, CCS, SSS | 19.953                      | 13                 | 0.096   | -49.865 | -6.047 |
| 6     | B, Y, Z, S2, S3, S5, B2, B3, B5, W2, W3, W5, V | 12.943                      | 10                 | 0.227   | -40.763 | -7.057 |
| 7     | B, Y, S2, S3, S5, W2, W3, W5, V, CS, CCS | 20.825                      | 13                 | 0.076   | -48.990 | -5.175 |
| 8     | Y, Z, S2, S3, S5, DA, W2, W3, W5, CCS  | 17.623                      | 14                 | 0.225   | -57.565 | -10.38 |
| 9     | Y, Z, S2, S3, S5, DA, W2, W5, CCS, V   | 15.694                      | 13                 | 0.266   | -54.124 | -10.306 |
| 10    | B1, B2, B3, B4, B5, B6, S1, S2, S3, S4, S5, S6, W1, W2, W3, W4, W5, W6 | 13.293                      | 11                 | 0.275   | -45.780 | -8.707 |

Table 4: Parameter estimates under Model (8).

| Parameter | Estimate | Std. Error | Z    | Sig. | Lower Bound | Upper Bound |
|-----------|----------|------------|------|------|-------------|-------------|
| Constant  | 1.377    | 0.560      | 2.460| 0.014| 0.280       | 2.474       |
| Y = 1     | 1.181    | 0.376      | 3.146| 0.002| 0.445       | 1.918       |
| Y = 2     | 0.502    | 0.343      | 1.462| 0.144| -0.171      | 1.175       |
| Y = 3     | 0\(^a\)  |            |      |      |             |             |
| Z = 1     | 1.375    | 0.374      | 3.674| 0.000| 0.641       | 2.108       |
| Z = 2     | 0.576    | 0.339      | 1.702| 0.089| -0.087      | 1.240       |
| Z = 3     | 0\(^a\)  |            |      |      |             |             |
| S2        | -1.026   | 0.331      | -3.097| 0.002| -1.675      | -0.377      |
| S3        | -3.283   | 0.769      | -4.269| 0.000| -4.790      | -1.776      |
| S5        | -0.607   | 0.422      | -1.439| 0.150| -1.433      | 0.220       |
| W2        | -0.679   | 0.156      | -4.361| 0.000| -0.985      | -0.374      |
| W3        | -2.298   | 0.511      | -4.499| 0.000| -3.299      | -1.297      |
| W5        | -0.669   | 0.317      | -2.107| 0.035| -1.291      | -0.047      |
| CCS       | -0.198   | 0.404      | -0.491| 0.624| -0.990      | 0.593       |
| DA        | 0.087    | 0.089      | 0.975| 0.329| -0.088      | 0.261       |
Table 5: Odds Ratios under Model (8).

| Parameter Estimates under Model (9). |
|--------------------------------------|
| **Baseline**                         |
| | 10 g/dl  | 8-10 g/dl | <8 g/dl |
| $\theta_{11}$                      | 13.10     | 13.10    | 7.78    |
| $\theta_{12}$                      | 3.37      | 3.37     | 4.24    |
| $\theta_{21}$                      | 3.99      | 4.02     | 6.35    |
| $\theta_{22}$                      | 5.69      | 5.66     | 3.36    |

The HB concentration tends to decrease from baseline throughout 8 weeks, since the maximum likelihood estimates are less than 1.

Table 6: Parameter estimates under Model (9).

| Parameter | Estimate | Std. Error | Z | Sig. | 95% Confidence Interval |
|-----------|----------|------------|----|------|-------------------------|
|            |          |            |    |      | Lower Bound | Upper Bound |
| Constant   | 1.638    | 0.609      | 2.690 | 0.007 | 0.445 | 2.832 |
| [Y = 1]    | 1.225    | 0.380      | 3.225 | 0.001 | 0.481 | 1.970 |
| [Y = 2]    | 0.529    | 0.347      | 1.524 | 0.127 | -0.151 | 1.210 |
| [Y = 3]    | 0$^a$    | .          | .    |      | .       | .       |
| [Z = 1]    | 1.392    | 0.366      | 3.799 | 0.000 | 0.674 | 2.110 |
| [Z = 2]    | 0.590    | 0.335      | 1.762 | 0.078 | -0.066 | 1.247 |
| [Z = 3]    | 0$^a$    | .          | .    |      | .       | .       |
| S2         | -1.252   | 0.376      | -3.332 | 0.001 | -1.988 | -0.515 |
| S3         | -3.502   | 0.792      | -4.422 | 0.000 | -5.054 | -1.949 |
| S5         | -0.800   | 0.458      | -1.749 | 0.080 | -1.697 | 0.096 |
| W2         | -1.065   | 0.325      | -3.277 | 0.001 | -1.701 | -0.428 |
| W3         | -2.642   | 0.573      | -4.607 | 0.000 | -3.765 | -1.518 |
| W5         | -0.916   | 0.371      | -2.471 | 0.013 | -1.642 | -0.189 |
| CCS        | -0.141   | 0.420      | -0.337 | 0.736 | -0.965 | 0.682 |
| DA         | 0.111    | 0.093      | 1.197 | 0.231 | -0.071 | 0.294 |
| V          | -0.479   | 0.350      | -1.371 | 0.170 | -1.165 | 0.206 |

Table 7: Odds Ratios under Model (9).

| Parameter Estimates under Model (9). |
|--------------------------------------|
| **Baseline**                         |
| | 10 g/dl  | 8-10 g/dl | <8 g/dl |
| $\theta_{11}$                      | 14.76     | 14.76    | 12.21   |
| $\theta_{12}$                      | 2.44      | 3.94     | 3.75    |
| $\theta_{21}$                      | 3.07      | 4.93     | 4.91    |
| $\theta_{22}$                      | 9.67      | 5.98     | 3.07    |

Therefore, under the model (9), the conditional probability that when a patient’s Hb concentration at 4 week is $\geq$ 10 g/dl, the probability that a patient’s HB the probability that a patient’s level $\geq$ 10 g/dl at baseline
instead of 8 weeks and 4 weeks is 14.76 times higher than a patient’s Hemoglobin level $\geq 10$ g/dl instead of 8–10 g/dl at 8 weeks.

The odds ratios greater than one under model (8) and model (9) indicate that the HB concentration at level $\geq 10$ g/dl is more likely to occur at baseline instead of after 4 and 8 weeks.

Table 8: Expected frequencies under Model (8).

| Baseline | 4 weeks | 8 weeks |
|----------|---------|---------|
| $\geq 10$ g/dl | $\geq 10$ g/dl | 78.83 | 8.98 | 0.58 |
| 8–10 g/dl | $\geq 10$ g/dl | 6.10 | 9.12 | 1.97 |
| < 8 g/dl | $\geq 10$ g/dl | 0.08 | 0.5 | 0.6 |
| $\geq 10$ g/dl | 8–10 g/dl | 39.96 | 4.56 | 0.29 |
| 8–10 g/dl | 8–10 g/dl | 12.04 | 17.99 | 3.89 |
| <8g/dl | 8–10 g/dl | 0.43 | 2.56 | 3.13 |
| $\geq 10$ g/dl | < 8g/dl | 4.21 | 0.56 | 0.03 |
| 8–10 g/dl | < 8g/dl | 4.76 | 4.89 | 1.23 |
| < 8 g/dl | < 8 g/dl | 0.59 | 3.85 | 3.25 |

Table 9: Expected frequencies under Model (9).

| Baseline | 4 weeks | 8 weeks |
|----------|---------|---------|
| $\geq 10$ g/dl | $\geq 10$ g/dl | 76.19 | 10.11 | 0.66 |
| 8–10 g/dl | $\geq 10$ g/dl | 4.84 | 9.45 | 1.51 |
| < 8 g/dl | $\geq 10$ g/dl | 0.06 | 0.42 | 0.64 |
| $\geq 10$ g/dl | 8–10 g/dl | 42.44 | 3.49 | 0.23 |
| 8–10 g/dl | 8–10 g/dl | 14.05 | 17.04 | 4.39 |
| < 8 g/dl | 8–10 g/dl | 0.39 | 2.33 | 3.59 |
| $\geq 10$ g/dl | < 8 g/dl | 4.36 | 0.49 | 0.03 |
| 8–10 g/dl | < 8 g/dl | 4.02 | 5.45 | 1.18 |
| < 8 g/dl | < 8 g/dl | 0.62 | 4.17 | 2.77 |

4 Conclusions

We considered subsymmetry models for multiway square contingency tables in which the main diagonal is not of interest. The models are established to analyze square multidimensional contingency tables with ordered categories. We see from the results that the models described here can be applied to a multiway table. We applied models to the patient’s hemoglobin concentration data set to illustrate the proposed models. The response was the patient’s hemoglobin (Hb) concentration at baseline (before treatment) and following 4 weeks and 8 weeks of treatment. The primary goal was to compare the baselines levels to 4th and 8th weeks.
taking the baseline as a layer variable. We were interested in considering the changing status of patient’s Hb concentration from baseline through time. But one wished to see whether there was an asymmetric transition of those concentrations or not, when the value of those concentration at baseline was given. The advantages of the models proposed here are that they are capable of analyzing the conditional odds ratios as well as the parameter estimates. Extensions to k-way tables are straightforward.

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