Power-law fluid flow in a T-shaped channel with slip boundary conditions on the solid walls

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Abstract. Planar flow of shear-thinning fluid in a T-shaped channel is investigated assuming slip occurs along the wall. Motion of the fluid is caused by pressure difference between inlet/outlet boundaries. On the solid walls, two models of fluid interaction with solid walls are employed: no slip boundary condition and Navier slip boundary condition relating the wall shear stress to the slip velocity. The problem is solved numerically using a finite difference method based on the SIMPLE procedure. The parametric studies of the flow pattern depending on the main parameters of the problem have been performed. Dependences describing kinematic and dynamic characteristics of the flow have been shown.

Introduction

In numerous experimental studies [1-3], it was found that flows of polymers can be accompanied by instabilities due to the transition from sticking of the polymers on solid walls to their slipping along the walls. This phenomenon should be taken into account when formulating and solving of such problems using correctly defined boundary conditions [4].

At present time, many works describing the slip phenomenon are available. The results of experiments based on the Near Field Laser Velocimetry technique to determine the slip velocity on a solid wall are presented in [5]. Numerical studies of Newtonian fluid flow with Navier slip boundary condition on the solid wall are demonstrated in [6-8]. The research of viscous fluid flow with model of non-monotonic slip of the fluid along solid walls is performed in [9-11]. The flows of non-Newtonian fluids under assumption that slip occurs along the wall with non-zero slip yield stress are studied in [12]. The flow of a Herschel-Bulkley fluid in a horizontal channel with a slip along the upper wall described by generalized Navier slip law is considered in [13].

The slip occurs along the wall in channels of different geometry including a T-shaped channel [14]. The T-shaped channel is a common used element of pipelines networks to transport fluids and gases. The fluid flow in a T-shaped channel is characterized by separation of the flow into two parts [15-16]. Nowadays, a large number of investigations of the flows of both Newtonian [15-19] and non-Newtonian [20-24] fluids with given flow rate at the boundaries of a T-shaped channel were carried out. Among the results of the studies in which pressures are given at the boundaries of a T-shaped channel, we can note those presented in [25-27].

The objective of the present work is to investigate the flow of power-law fluid moving in a T-shaped channel under the given pressure difference between inlet/outlet sections employing two
interaction models of the fluid with a solid wall, namely no slip boundary condition and Navier slip boundary condition.

**Problem formulation**

The planar steady-state flow of an incompressible non-Newtonian fluid in a T-shaped channel is researched. The flow region is limited by solid walls MKF, EDC and AB (figure 1). The considered flow is described by the momentum and continuity equations which in dimensionless form are written as follows:

\[ u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = - \frac{\partial p}{\partial x} + \eta \Delta u + 2 \frac{\partial \eta}{\partial x} \frac{\partial u}{\partial x} + \frac{\partial \eta}{\partial y} \left( \frac{\partial v}{\partial x} + \frac{\partial u}{\partial y} \right), \]

(1)

\[ u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} = - \frac{\partial p}{\partial y} + \eta \Delta v + 2 \frac{\partial \eta}{\partial y} \frac{\partial v}{\partial y} + \frac{\partial \eta}{\partial x} \left( \frac{\partial v}{\partial x} + \frac{\partial u}{\partial y} \right), \]

(2)

\[ \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0. \]

(3)

Here \( u \) and \( v \) are components of the velocity vector \( \mathbf{U} \) in the Cartesian coordinate system \((x, y)\), \( p \) is the dimensionless pressure.

**Figure 1. Flow region.**

The apparent viscosity of non-Newtonian fluid is determined by the Ostwald–de Waele power law [28]:

\[ \eta = (A)^{m-1}, \]

(4)

where \( A = \left( 2 \left( \frac{\partial u}{\partial x} \right)^2 + \left( \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right)^2 + 2 \left( \frac{\partial v}{\partial y} \right)^2 \right)^{1/2} \) is the dimensionless intensity of the strain rate tensor, \( m \) is the power-law index. Setting \( m=1 \) in equation (4) leads to rheology of Newtonian fluid.

The system of equations (1)-(4) is reduced to dimensionless form by using width of the boundary section \( AM, L \), as the characteristic length, and the value \( U_0 = \left( \frac{k}{\rho L_m} \right)^{1/(2-m)} \) as the characteristic velocity. Dimensionless pressure is prescribed by the expression
\[ p = \left( P - P_{FE} \right) f \left( \frac{k^2}{P^{m}L^{m}} \right)^{1/(2-m)}, \]

where \( k \) is the power-law consistency index, \( \rho \) is the fluid density, \( P \) is dimension pressure, \( P_{FE} \) is dimension pressure given in the cross-section \( FE \).

In the through-flow sections \( AM, FE, \) and \( BC \), zero tangential components of the velocity vector and the pressure are specified as follows:

\[
\begin{align*}
v &= 0, \quad p_{AM} = p_1, \quad x = 0, \quad 0 \leq y \leq 1 \\
u &= 0, \quad p_{FE} = 0, \quad L_1 \leq x \leq L_1 + 1, \quad y = L_2 + 1 \\
v &= 0, \quad p_{BC} = p_3, \quad x = L_1 + L_2 + 1, \quad 0 \leq y \leq 1 \\
\end{align*}
\]

(5)

On the solid walls, two models of fluid interaction with solid walls are considered. Model 1: no slip boundary condition according to which tangential and normal components of the velocity vector are equal to zero,

\[
\begin{align*}
\{ u_s &= 0, \\
u_n &= 0. \\
\end{align*}
\]

(6)

Model 2: Navier slip boundary condition supposes that the tangential component of the velocity vector on the solid wall is linearly proportional to the shear stress and the normal component is equal to zero,

\[
\begin{align*}
\{ u_s &= \beta \frac{du_s}{dn}, \\
u_n &= 0. \\
\end{align*}
\]

(7)

Here \( \beta \) is the slip coefficient, \( u_s \) and \( u_n \) are tangential and normal components of the velocity vector, respectively.

Solution of the problem is reduced to finding the velocity and pressure fields which satisfy the equations (1)-(4) at the given boundary conditions (5)-(7).

**Numerical method**

The problem is solved numerically. An asymptotic time solution of the unsteady flow equation is used to obtain steady-state velocity and pressure fields [29]. Such method of solution assumes the addition of time derivatives of functions \( u \) and \( v \) in the equations (1) and (2), respectively. The received system is discretized by the finite difference method based on the SIMPLE procedure [30]; at that, rectangular staggered grid is used.

For shear-thinning fluid \((n<1)\), there is peculiarity of «infinite» viscosity, as \( A \to 0 \). To ensure the stability and accuracy of calculations in the regions of small values of \( A \), the modified model of the rheological equation is used [31]. According to this model, the viscosity is determined by expression

\[ \eta = \left( A + \epsilon \right)^{-n}, \]

where \( \epsilon \) is the regularization parameter. The approximate convergence of the calculation method using regularized rheological model is presented in [27].

**Results and discussion**

The following main parameters affect the flow characteristics: the power-law index \((m)\), the pressure given at boundaries \( AM \) and \( FE \) \((p_1 \) and \( p_3, \) respectively), and the slip coefficient \((\beta)\). The objective of this research is to illustrate influence of the slip phenomenon on the flow pattern. Parametric studies of
the flow characteristics have been performed at $m=0.8$, $p_1=-150$, $p_3=-200$, and different values of the slip coefficient changing within the range $0 \leq \beta \leq 0.5$. The geometry of the T-shaped channel is schematically represented in figure 1. All branches of the channel have the same width equal to one dimensionless unit and the same length $L_1=L_2=L_3=3$.

Figure 2 demonstrates distribution of the flow characteristics at $m=0.8$, $p_1=-150$, $p_3=-200$, and $\beta=0.3$. The fluid flow entering through the boundary $FE$ separates in the vicinity of the junction of the branches and leaves the channel through the boundaries $AM$ and $BC$. Fully developed laminar flow of the power-law fluid is realized near the through-flow sections $AM$, $FE$, and $BC$. Recirculation zones are formed in the vicinity of the sections with corner points $K$ and $D$.

Figure 2. Distribution of the flow characteristics at $m=0.8$, $p_1=-150$, $p_3=-200$, and $\beta=0.3$
(a – the streamlines, b – the pressure field, c and d – the fields of velocity $u$ and $v$, respectively)

Figure 3. Distribution of the streamlines at different values of the slip coefficient $\beta$
($m=0.8$, $p_1=-150$, $p_3=-200$, a – $\beta=0$, b – $\beta=0.4$)

The results of the conducted research have shown that increase of the slip coefficient leads to qualitative and quantitative change of the flow pattern. It can be seen from figure 3 (a) that the flow without recirculation zones is observed at $\beta=0$ (no slip boundary condition on the solid walls is
applied). Maximum flow rate of the fluid is equal to the flow rate through the boundary $FE$, $Q_{FE}=5.049$. The recirculation regions appear as the slip coefficient increases. Two recirculation zones with length of the order of one dimensionless unit exist in the vicinity of the sections with corner points $K$ and $D$ at $\beta=0.4$ (figure 3, b). It was found that the average velocity of the flow and, consequently, the flow rate through the boundaries $AM$, $FE$, and $BC$ enhance as the parameter $\beta$ is increased. Maximum flow rate of the fluid at $\beta=0.4$ is equal to 24.861.

Dependence of the flow rate through the sections $AM$, $FE$, and $BC$ on the slip coefficient $\beta$ is presented in figure 4. Positive values of the flow rate $Q_i$ correspond to the case where the fluid enters into the channel, while negative values correspond to the case – where the fluid leaves the channel. Growth of the absolute flow rate through the boundaries $AM$, $FE$, and $BC$ is observed when increasing the parameter $\beta$.

**Figure 4.** Flow rate through the sections $AM$, $FE$, and $BC$ as a function of the slip coefficient $\beta$ ($m=0.8$, $p_{AM}=-150$, $p_{FE}=0$, $p_{BC}=-200$, $1-Q_{FE}$, $2-Q_{AM}$, $3-Q_{BC}$, $4-Q=0$)

**Conclusion**

The planar steady-state flow of the incompressible power-law fluid in the T-shaped channel has been studied. In the through-flow sections $AM$, $FE$, and $BC$, zero tangential components of the velocity vector and the pressure are specified. On the solid walls, two models of fluid interaction with solid walls are considered, namely no slip boundary condition and Navier slip boundary condition.

Parametric studies of the flow depending on the slip coefficient $\beta$ have been performed at the power-law index $m=0.8$ and fixed values of the pressure in through-flow sections $AM$, $FE$, and $BC$ ($p_{AM}=-150$, $p_{FE}=0$, $p_{BC}=-200$). The range of the slip coefficient variation has been chosen so that the planar-parallel flow of the non-Newtonian fluid with the fully developed velocity profile has been realized in the vicinity of the through-flow sections $AM$, $FE$, and $BC$ ($0 \leq \beta \leq 0.5$).

As a result, we estimated the influence of the slip phenomenon on the flow pattern. Distributions of the kinematic and dynamic characteristics of the flow at different values of the parameter $\beta$ have been presented. Dependences of the flow rate on the slip coefficient have been obtained.

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