Simulation-Checking of Real-Time Systems with Fairness Assumptions

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Abstract—We investigate the simulation problem in of dense-time system. A specification simulates a model if the specification can match every transition that the model can make at a time point. We also adapt the approach of Emerson and Lei and allow for multiple strong and weak fairness assumptions in checking the simulation relation. Furthermore, we allow for fairness assumptions specified as either state-predicates or event-predicates. We focus on a subclass of the problem with at most one fairness assumption for the specification. We then present a simulation-checking algorithm for this subclass. We propose simulation of a model by a specification against a common environment. We present efficient techniques for such simulations to take the common environment into consideration. Our experiment shows that such a consideration can dramatically improve the efficiency of checking simulation. We also report the performance of our algorithm in checking the liveness properties with fairness assumptions.

Keywords: branching simulation, fairness, verification, Büchi automatas, concurrent computing, timed automata, algorithms, experiment

I. INTRODUCTION

Modern real-time systems have incurred tremendous challenges to verification engineers. The reason is that a model process running in a modern real-time system can be built with support from many server processes in the environment. Moreover, the model may also have to respond to requests from several user processes. The fulfillment of a computation with support from many server processes in the environment. The deadline constraints $x_1 < 20$ and $x_2 < 15$ can be too restrictive and hurt the extensibility of the model in the future. Another approach in this regard is using fairness assumptions [10], [22]. For example, for the model and specification processes in figure 1, we may want to check whether $S$ simulates $M$ under the fairness assumption that the environment functions reasonably. Such an assumption can be captured with the fairness assumption that there will always be infinitely many occurrences of event serve. Under this assumption, the $S$ in figure (b) actually simulates the $M$ in figure (a).

In such a setting, the model and the specification are both generalized Büchi timed automata (GBTA) [3] with communication channels and dense-time behaviors. We want to check whether the specification GBTA can simulate the model GBTA with multiple fairness assumptions. Following the approach of [9],

\[ \text{Example 1: } \]

In figure (a) we have the state-transition diagrams of two timed automata (TA) [3]. The one in figure (a) is for a model $M$ while the one in figure (b) is for a specification $S$. We use ovals for the control locations of the TAs while arcs for the transition rules. In each oval, we label the invariance condition that must be satisfied in the location. For example, in location $\text{wait}_1$, $M$ can stay for at most 20 time units. By each transition rule, we stack its synchronization event, triggering condition (guard), and actions. For convenience, tautology triggering conditions and nil actions are omitted. An event starting with a ‘?’ represents a receiving event while one with a ‘!’ represents a sending event. For example, for the transition from location idle to $\text{wait}_1$, $M$ must send out an event request, be in a state satisfying $x_1 > 5$, and reset clock $x_1$ to zero. The specification in figure (b) does not simulate the model in figure (a) since event end of $M$ cannot be matched by any event of $S$. Moreover, $S$ can neither receive a serve event 15 time units after issuing a request event while $M$ can.

However, the concept of simulation described in the last paragraph can be too restrictive in practice. Developers of a project usually cannot make too much assumption on the environment. The deadline constraints $x_1 < 20$ and $x_2 < 15$ can be too restrictive and hurt the extensibility of the model in the future. Another approach in this regard is using fairness assumptions [10], [22]. For example, for the model and specification processes in figure 1, we may want to check whether $S$ simulates $M$ under the fairness assumption that the environment functions reasonably. Such an assumption can be captured with the fairness assumption that there will always be infinitely many occurrences of event serve. Under this assumption, the $S$ in figure (b) actually simulates the $M$ in figure (a).

In this work, we propose the simulation with fairness assumptions for the processes in a dense-time setting. In such a setting, the model and the specification are both generalized Büchi timed automata (GBTA) [3] with communication channels and dense-time behaviors. We want to check whether the specification GBTA can simulate the model GBTA with multiple fairness assumptions. Following the approach of [9],

\[ \text{Keywords: branching simulation, fairness, verification, Büchi automata, concurrent computing, timed automata, algorithms, experiment} \]
and weak fairness assumptions. A strong fairness assumption intuitively means something will happen infinite many times. A weak fairness assumption means something will hold true eventually forever. For convenience, we use two consecutive index names and state variables. For convenience, we use two consecutive index formulas in the description of fairness assumptions. State formulas are constructed with a precondition, a event and a post-condition in sequence.

Example 2: For the system in figure [1], we may have the following fairness assumptions.

\{wait_1\}|{idle_1 \lor \text{wait}_1}\}

The fairness assumptions in the above say that a valid computation of the system must satisfy the following two conditions.

- For the strong fairness assumption of \{wait_1\}: For every \(t \in \mathbb{R}^\geq\), there exists a \(t' \in \mathbb{R}^\geq\) with \(t' > t\) such that in the computation at time \(t'\), the model process is in location \text{wait}_1.\) This in fact says that the model must enter location \text{wait}_1 infinitely many times along any valid computation.

- For the weak fairness assumption of \{idle_1 \lor \text{wait}_1\}: There exists a \(t \in \mathbb{R}^\geq\) such that for every \(t' \in \mathbb{R}^\geq\) with \(t' > t\), the model process is in either locations \text{idle}_1 or \text{wait}_1.\) This in fact says that the model will stabilize in locations \text{idle}_1 and \text{wait}_1.

The two types of fairness assumption complement with each other and could be handy in making reasonable assumptions.

Furthermore, we also allow for both state formulas and event formulas [25] in the description of fairness assumptions. State formulas are Boolean combinations of atomic statements of location names and state variables. For convenience, we use index 1 for the model and index 2 for the specification. Event formulas are then constructed with a precondition, a event name with a process index, and a post-condition in sequence.

Example 3: For the system in figure [1], we may write the following strong event fairness assumption.

\{(\text{wait}_1)?\text{serve}\oplus(1)(\text{true})\}\{}

The event specification of \?\text{serve}@\(\oplus\)(1)(\text{true})\} means there is an event \text{serve} received by process 1. The precondition for the event is \text{wait}_1 while the post-condition is \text{true}. The strong fairness assumption says that there should be infinite many events \text{serve} received by process 1 in location \text{wait}_1. \]

In general, an event specification can be either a receiving or a sending event. Such event formulas can be useful in making succinct specifications. Without such event formulas, we may have to use auxiliary state variables to distinguish those states immediately before (or after) an event from others. Such auxiliary variables usually unnecessarily exacerbate the state space explosion problem.

One goal of our work is to develop a simulation-checking algorithm based on symbolic model-checking technology for dense-time systems [15], [24]. To achieve this, we focus on a special class of simulations with the restriction of at most one fairness assumption for the specification. For convenience, we call this class the USF (unit-specification-fairness) simulations. Then we propose a symbolic algorithm for this special class of simulations. To our knowledge, this is the first such algorithm for GBTAs. Also unlike the fair simulation checking algorithm based on ranking function in the literature, our algorithm is based on symbolic logic formulas manipulation, which has been proven useful in symbolic model checking [6]. Thus, our algorithm style can be interesting in itself.

We also present a technique for the efficient simulation checking of concurrent systems by taking advantage of the common environment of a model and a specification. To
apply the simulation checking algorithms mentioned in the above and in the literature [8], [23], we need first construct a product automata of the environment \( E \) and the model \( M \), in symbols \( E \times M \). Then we construct a product of \( E \) and the specification \( S \), in symbols \( E \times S \). Then we check if \( E \times S \) simulates \( E \times M \). As a result, such algorithms incur duplicate recording of the state information of \( E \) while manipulating representations for the simulation of \( E \times M \) by \( E \times S \). Moreover, different transitions in \( E \) with the same observable events can also be matched in the simulation-checking. Such matching is not only counter-intuitive in simulation against the same environment, but also incur explosion in the enumeration of matched transitions between \( E \times M \) and \( E \times S \). Our technique is embodied with the definition of a new simulation relation against a common environment. We have implemented this technique and experimented with benchmarks with and without fairness assumptions.

We have the following presentation plan. Section II is for related work. Section III reviews our system models [3], [20]. Sections IV presents our simulation for dense-time systems with fairness assumptions. Section V presents a characterization of the simulation when the specification is a Büchi with fairness assumptions. Section VI presents our simulation checking algorithm based on the characterization derived in section V. Section VII presents the simulation against a common environment and techniques for performance verification in this context. Sections VIII and IX respectively report our implementation and experiment. Section X is the conclusion.

II. RELATED WORK

Cerans showed that the bisimulation-checking problem of timed processes is decidable [8]. Taşran et al showed that the simulation-checking problem of dense-time automata (TAs) [3] is in EXPTIME [23]. Weise and Lenzkes reported an algorithm based on zones for timed bisimulation checking [22]. Cassez et al presented an algorithm for the reachability games of TAs with controllable and uncontrollable actions [7].

Henzinger et al presented an algorithm that computes the time-abstraction simulation that does not preserve timed properties [13]. Nakata also discussed how to do symbolic simulation checking with integer-time labeled transition systems [19]. Beyer has implemented a refinement-checking algorithm for TAs with integer-time semantics [4].

Lin and Wang presented a sound proof system for the bisimulation equivalence of TAs with dense-time semantics [17]. Aceto et al discussed how to construct such a modal logic formula that completely characterizes a TA [1].

Larsen presented a similar theoretical framework for bisimulation in an environment for untimed systems [15]. However no implementation that takes advantage of the common environment information for verification performance has been reported.

Proposals for extending simulation with fair states have been discussed in [12], [14], [18]. Our simulation game of GBTAs stems from Henzinger et al’s framework of fair simulation [14]. Techniques for simulation checking of GBAs were also discussed in [10], [22].

III. PRELIMINARY

We have the following notations. \( \mathbb{R} \) is the set of real numbers. \( \mathbb{R}^{\geq 0} \) is the set of non-negative reals. \( \mathbb{N} \) is the set of nonnegative integers. Also ‘if’ is “if and only if.” Given a set \( P \) of atomic propositions and a set \( X \) of clocks, we use \( \mathbb{B}(P, X) \) as the set of all Boolean combinations of logic atoms of the forms \( q \) and \( x \sim c \), where \( q \in P \), \( x \in X \), ‘\( \sim \)’ \( \in \{=, <, =, >, \geq \} \), and \( c \in \mathbb{N} \). An element in \( \mathbb{B}(P, X) \) is called a state-predicate.

A. Timed automata

A TA [3], [20], [31] is structured as a directed graph whose nodes are modes (control locations) and whose arcs are transitions. Please see figure 1 for examples. A TA must always satisfy its invariance condition. Each transition is labeled with events, a triggering condition, and a set of clocks to be reset during the transitions. At any moment, a TA can stay in only one mode. If a TA executes a transition, then the triggering condition must be satisfied. In between transitions, all clocks in a TA increase their readings at a uniform rate.

Definition 1: Timed automata (TA) A TA \( A \) is a tuple \( \langle Q, P, X, I, \lambda, E, \Sigma, \epsilon, \tau, \pi \rangle \). \( Q \) is a finite set of modes (locations). \( P \) is a finite set of propositions. \( X \) is a finite set of clocks. \( I \in \mathbb{B}(P, X) \) is the initial condition. \( \lambda: Q \rightarrow \mathbb{B}(P, X) \) is the invariance condition for each mode. \( E \subseteq Q \times Q \) is the set of process transitions. \( \Sigma \) is a finite set of events. \( \epsilon: E \rightarrow 2^\Sigma \) is a mapping that defines the events at each transition. \( \tau: E \rightarrow \mathbb{B}(P, X) \) and \( \pi: E \rightarrow 2^X \) respectively define the triggering condition and the clock set to reset of each transition.

Without loss of generality, we assume that for all \( q, q' \neq Q \) with \( q \neq q', \lambda(q) \land \lambda(q') \) is a contradiction. We also assume that there is a null transition \( \perp \) that does nothing at any location. That is, the null transition transits from a location to the location itself. Moreover, \( \tau(\perp) = true \), \( \pi(\perp) = \emptyset \), and \( \epsilon(\perp) = \emptyset \).

Given a TA \( A = \langle Q, P, X, I, \lambda, E, \Sigma, \epsilon, \tau, \pi \rangle \), for convenience, we let \( Q_A = Q \), \( P_A = P \), \( X_A = X \), \( I_A = I \), \( \lambda_A = \lambda \), \( E_A = E \), \( \Sigma_A = \Sigma \), \( \epsilon_A = \epsilon \), \( \tau_A = \tau \), and \( \pi_A = \pi \). Also, for convenience, we let \( V_A \equiv \bigvee_{q \in Q_A} (\lambda_A(q)) \) be the invariance predicate of \( A \).

Example 4: We have already seen examples of TAs in figure 1. For the TA in figure 1(a), the attributes are listed in table I.

A valuation of a set is a mapping from the set to another set. Given an \( \eta \in \mathbb{B}(P, X) \) and a valuation \( \nu \) of \( X \cup P \), we say \( \nu \) satisfies \( \eta \), in symbols \( \nu \models \eta \), iff \( \eta \) is evaluated true when the variables in \( \eta \) are interpreted according to \( \nu \).

Definition 2: States of a TA Suppose we are given a TA \( A \). A state \( v \) of \( A \) is a valuation of \( X_A \cup P_A \) with the following constraints.

- For each \( p \in P_A \), \( \nu(p) \in \{false, true\} \). There exists an \( q \in Q_A \) such that \( \nu \models \lambda(q) \) and for all \( q' \neq q \), \( \nu \not\models \lambda(q') \).
- For each \( x \in X_A \), \( \nu(x) \in \mathbb{R}^{\geq 0} \).

In addition, we require that \( \nu \models V_A \). We let \( S(S) \) denote the set of states of \( A \).
Note that we define a state as a mapping instead of as a pair of control locations and a real mapping as in [2]. This is for the convenience of presentation when latter we want to discuss the state-pairs in simulation relations.

For any state \( \nu \) and real number \( t \in \mathbb{R}_{\geq 0} \), \( \nu + t \) is a state identical to \( \nu \) except that for every clock \( x \in X_A \), \((\nu + t)(x) = \nu(x) + t\). Also given a process transition \( e = (q,q') \in E_A \), we use \( \nu e \) to denote the destination state from \( \nu \) through the execution of \( e \). Formally, if \( \nu = \tau_A(e) \), then \( \nu e \) is a new state that is identical to \( \nu \) except that the following constraints are true:

- \( q = \text{mode}_A(\nu) \) and \( q' = \text{mode}_A(\nu e) \).
- For every clock \( x \in \pi_A(e) \), \( \nu e(x) = 0 \).
- For every clock \( x \notin \pi_A(e) \), \( \nu e(x) = \nu(x) \).

Given a \( t \in \mathbb{R}_{\geq 0} \) and a transition \( e \), we write \( \nu \xrightarrow{t,e} \nu' \) iff \( \nu + t = \tau_A(e), (\nu + t)e = \nu', \nu' = V_A \), and for each \( t' \in [0, t] \), \( \nu + t' = V_A \). For convenience, we use \( \nu \xrightarrow{t} \nu' \) to denote such a \( \nu' \) with \( \nu \xrightarrow{t,e} \nu' \).

**Definition 3: Runs** A run of a TA \( A \) is an infinite sequence of state-transition-time triples \( (t_0, e_0, t_1)(t_1, e_1, t_2) \ldots (t_k, e_k, t_{k+1}) \ldots \) with the following restrictions:

- **Non-Zeno requirement:** \( t_0 t_1 \ldots t_k \ldots \) is a non-decreasing and divergent real-number sequence. That is, \( \forall k \in \mathbb{N}, t_k \leq t_{k+1} \) and \( \forall c \in \mathbb{N}, \exists k > 1, t_k > c \).
- For all \( k \in \mathbb{N} \), either \( t_k + t_{k+1} - t_k = v_{k+1} \) or \( \nu_{k+1} = \nu_k + (t_k + t_{k+1} - t_k) \).

A run prefix is a finite prefix of a run. A run prefix or a run \( (v_0, e_0, t_0) \ldots \) of \( A \) is initial iff \( t_0 = I_A \).

**B. Generalized Büchi TAs**

Suppose we are given a TA \( A \). An event-predicate is of the form \( \eta_1\eta_2 \). Here \( \eta_1 \) and \( \eta_2 \) are two state-predicates in \( \mathbb{B}(P_A, X_A) \) respectively for the precondition and the postcondition of the event. \( a \in \Sigma_A \) is an event name. Event-predicate “\( \eta_1\eta_2 \)” specifies the observation of event \( a \) with precondition \( \eta_1 \) and postcondition \( \eta_2 \).

In this work, we allow **fairness assumptions** either as state-predicates or as event-predicates. A state fairness assumption is in \( \mathbb{B}(P_A, X_A) \). An event fairness assumption is an event-predicate of \( A \). Given two sets \( \Phi \) and \( \Psi \) of fairness assumptions, \( \Phi \Psi \) denotes a multi-fairness assumption (MF- assumption) for \( A \). All elements in \( \Phi \) are called **strong fairness assumptions** while all in \( \Psi \) are called **weak fairness assumptions**. A run \( (v_0, e_0, t_0) \ldots (v_k, e_k, t_k) \ldots \) of \( A \) satisfies \( \Phi \Psi \) iff the following constraints hold:

- For every state-predicate \( \eta \in \Phi \), there are infinitely many \( k \)'s such that for some \( t \in [0, t_k + t_0 - t_k] \), \( v_k + t = \eta \).
- For every event-predicate \( \eta_1\eta_2 \in \Phi \), there are infinitely many \( k \)'s such that \( v_k + (t_{k+1} - t_k) = \eta_1 \) and \( v_{k+1} = \eta_2 \).
- For every state-predicate \( \eta \in \Psi \), there is a \( k \) such that for every \( h > k \) and \( t \in [0, t_{h+1} - t_h] \), \( v_h + t = \eta \).
- For every event-predicate \( \eta_1\eta_2 \in \Psi \), there is a \( k \) such that for every \( h > k \), if \( v_h + (t_{h+1} - t_h) = \eta_1 \) and \( a \in \epsilon_A(e_{h+1}) \), then \( v_{h+1} = \eta_2 \).

Given a TA \( A \) and a state \( \nu \in \mathbb{S}(A) \), we let \( \Omega_A(\nu, \Phi \Psi) \) denote the set of runs of \( A \) from \( \nu \) satisfying \( \Phi \Psi \). The following definition shows how to formally model real-time systems with fairness assumptions.

**Definition 4: GBTAs and BTAs** A generalized Büchi TA (GBT) is a pair \( (A, \Phi \Psi) \) with a TA \( A \) and an MF-assumption \( \Phi \Psi \). If \( |\Phi| + |\Psi| \leq 1 \), the pair is also called a Büchi TA (BTA).

Example 5: For the model \( M \) in figure 1(a), we may have a GBT

\[ \{M, \{\text{wait}_1, \text{true}?, \text{serve} @ (2) \text{idle}_1\} \} \]

that assumes \( M \) should stay in location \( \text{wait}_1 \) infinitely many times and event \( \text{serve} \) should be received by \( M \) infinitely many times with post-condition \( \text{idle}_1 \).

We may also have the following GBT

\[ \{M, \emptyset \{\text{stop}_1\} \} \]

that assumes that \( M \) should eventually stabilize in location \( \text{stop}_1 \).

**IV. Simulation of GBTAs**

Suppose we are given two TAs \( A, B \). For any transitions \( e \in E_A \) and \( f \in E_B \), \( e \) and \( f \) are compatible iff \( \epsilon_A(e) \neq \epsilon_B(f) \). That is, the observable events of the two automatons on the two transitions must be nontrivially identical. For each \( e \in E_A \) with \( \epsilon_A(e) \neq \emptyset \), we use \( E_B(e) \) to denote the subset of \( E_B \) with elements compatible with \( e \). For each \( e \in E_A - \{\emptyset\} \) with \( \epsilon_A(e) = \emptyset \), \( E_B(e) = \{\emptyset\} \). Also, \( E_B(\emptyset) \) denotes the subset of \( E_B \) with elements \( f \) such that \( \epsilon_B(f) = \emptyset \).

In this section, from now on, we assume the context of two GBTAs \( \langle M, \Phi_A, \Psi_M \rangle \) and \( \langle S, \Phi_S, \Psi_S \rangle \) respectively for the model and the specification.
Given a state $\mu$ of $M$ and a state $\nu$ of $S$, we use $\mu \nu$ to denote the state-pair of $\mu$ and $\nu$. Operationally, $\mu \nu$ can be viewed as $\mu \circ \nu$, the functional composition of $\mu$ and $\nu$. A play between $M$ and $S$ is made of two matching runs, one of $M$ and the other of $S$. Conceptually, it is a sequence
\[(\mu \nu_0, e_0 f_0, t_0) \ldots (\mu \nu_k, e_k f_k, t_k) \ldots\]
of triples with the following restrictions.
- $(\mu_0, e_0, t_0) \ldots (\mu_k, e_k, t_k) \ldots$ is a run of $M$. For convenience, we denote this run as $\text{run}_M(\rho)$.
- $(\nu_0, f_0, t_0) \ldots (\nu_k, f_k, t_k) \ldots$ is a run of $S$. For convenience, we denote this run as $\text{run}_S(\rho)$.
- For each $k \in \mathbb{N}$, $f_k \in E_S^{(e_k)}$.

The play is initial iff $\mu_0 \models I_M$ and $\nu_0 \models I_S$. A play prefix is a finite prefix of a play. Given a play $\rho$, we let $\rho^{(k)}$ be the prefix represented as the given the first $k+1$ elements of $\rho$.

Given a run (prefix) $\theta = (\mu_0, e_0, t_0) \ldots (\mu_k, e_k, t_k) \ldots$ of $M$ and a play (prefix) $\rho = (\mu_0 \nu_0, e_0 f_0, t_0) \ldots (\mu_k \nu_k, e_k f_k, t_k) \ldots$ between $M$ and $S$, we say $\rho$ embeds $\theta$ iff there is a monotonically increasing integer function $\gamma()$ such that $\gamma(0) = 0$ and for each $k \in \mathbb{N}$, $\bar{\mu}_k \mu(k) = \mu_k$, $\bar{\nu}_k \nu(k) = \nu_k$, and for each $h \in (\gamma(k), (\gamma(k) + 1))$, $e_h = \bot$. Notationally, we let $\rho \triangleright_M \theta$ denote the embedding relation between $\rho$ and $\theta$. Similarly we can define $\rho \triangleright_S \theta'$ for the embedding relation between $\rho$ and a run $\theta'$ of $S$.

A strategy in a game tells a TA what to execute at a state-pair in a play that is developing. Specifically, a strategy $\sigma$ for $S$ is a mapping from play prefixes of $M$ and $S$ to event sets of $\Sigma_S$. Symmetrically, we can define strategies for $M$. Given a strategy $\sigma$ for $S$ and a play $\rho = (\mu_0 \nu_0, e_0 f_0, t_0) \ldots (\mu_k \nu_k, e_k f_k, t_k) \ldots$ between $M$ and $S$, we say that $\rho$ complies to $\sigma$ if the following constraints are satisfied.
- For each $k \in \mathbb{N}$ and $t \in [0, t_{k+1} - t_k)$,
  \[\sigma(\rho^{(k)}((\mu_k + t)(\nu_k + t), \bot \bot t_k + t)) = \bot \]
- For each $k \in \mathbb{N}$ and $t = t_{k+1} - t_k$ with either $t_{k+2} - t_{k+1} > 0$ or $k_{k+1} \neq \bot$,
  \[\sigma(\rho^{(k)}((\mu_k + t)(\nu_k + t), \bot \bot t_{k+1})) = f_{k+1} \]

Similarly, we can also define the compliance of plays to strategies of $M$. Given a state-pair $\mu \nu \in S(M) \times S(S)$, a run $\theta$ of $M$ from $\mu$, and a strategy $\sigma$ of $S$, we let $\rho \triangleright (\mu \nu, \theta, \sigma)$ be the play (prefix) from $\mu \nu$ with the following restrictions.
- $\rho$ complies to $\sigma$.
- If $\rho$ is of infinite length, then it embeds $\theta$.
- If $\rho$ is of finite length, then there is a finite prefix $\theta = (\mu_0, e_0, t_0) \ldots (\mu_k, e_k, t_k)$ of $\theta$ with the following restrictions.
  - $\rho$ embeds $\theta$.
  - Any prefix of $\theta$ that supersedes $\theta$ is not embedded by $\rho$.

Note that it may happen that $\text{play}(\mu \nu, \theta, \sigma)$ is of only finite length. This can happen when at the end of the finite play, a player chooses a transition with an event set that the other player (opponent) cannot choose a transition to match. This can also happen when at the end of the finite play, a player can only execute matching transitions with post-condition falling outside the invariance predicate.

**Definition 5:** Simulation of GBTAs A simulation $F$ of $(M, \Phi_M, \Psi_M)$ by $(S, \Phi_S, \Psi_S)$ is a binary relation $F \subseteq S(M) \times S(S)$ such that for every $\mu \nu \in F$ and every run $\theta$ of $M$ from $\mu$ that satisfies $\Phi_M \Psi_M$, there exists a play $\rho$ from $\mu$ such that $\rho$ embeds $\theta$ and $\text{run}_S(\rho)$ satisfies $\Phi_S \Psi_S$.

We say that $(S, \Phi_S, \Psi_S)$ simulates $(M, \Phi_M, \Psi_M)$, in symbols $(M, \Phi_M, \Psi_M) \sim (S, \Phi_S, \Psi_S)$, if there exists a simulation $F$ of $(M, \Phi_M, \Psi_M)$ by $(S, \Phi_S, \Psi_S)$ such that for every $\mu \models I_M \land V_M$, there exists a $\nu \models I_S \land V_S$ with $\mu \nu \in F$.

**Example 6:** For the TAs in figure 1 we have that $(S, \emptyset)$ does not simulate $(M, \emptyset)$. Also, $(S, \{\text{true}\searrow\text{true}\})$ does not simulate $(M, \emptyset\{\text{stop}_{P_1}\})$. However, $(S, \emptyset)$ simulates $(M, \{\text{wait}_{i_1}\})$.

If $(S, \Phi_S, \Psi_S)$ simulates $(M, \Phi_M, \Psi_M)$, then for every initial states $\mu$ and runs $\theta$ of $M$ from $\mu$ satisfying $\Phi_M \Psi_M$, there exists a strategy $\sigma$ such that $\text{play}(\mu \nu, \theta, \sigma)$ satisfies $\Phi_S \Psi_S$. We call such a $\sigma$ a simulating strategy for $\theta$ by $S$.

If $(S, \Phi_S, \Psi_S)$ does not simulate $(M, \Phi_M, \Psi_M)$, then there exists an initial run $\theta$ of $M$ such that $\theta$ satisfies $\Phi_M \Psi_M$ and for all initial states $\mu$ and all strategies $\sigma$ of $S$, all initial runs $\rho$ of $S$ embedded by $\text{play}(\mu \nu, \theta, \sigma)$ do not satisfy $\Phi_S \Psi_S$. We call such a run $\theta$ a refuting run of $M$.

A strategy $\sigma$ of a TA $S$ is memory-less iff for any two plays $\rho$ and $\rho'$ that end at the same triple, $\sigma(\rho) = \sigma(\rho')$. It is well known that parity games and reachability games all have memory-less winning strategies for either player $P$. The following lemma shows that the simulation of TAs may need finite-memory revealing strategies.

**Lemma 7:** There is a simulation of TAs with a simulation strategy for the specification but without a memory-less simulation strategy for the specification.

**Proof:** In figure 2 we have the TAs of two TAs $(M, \{m_0, m_1\})$ and $(S, \{s_1, s_2\})$. Suppose we have a state-pair $\mu \nu$ with $\text{mode}_M(\mu) = m_0$ and $\text{mode}_S(\nu) = s_0$. As can be seen, for any memory-less strategy $\sigma$, either transition $(s_0, s_1)$ will always be chosen for any initial play prefix that ends at $\mu v$ or transition $(s_0, s_2)$ will always be. But such plays do not satisfy the strong fairness assumption of $(S, \{s_1, s_2\})$ and cannot be used to fulfill the strong fairness assumptions of $S$. Thus we know there is no memory-less simulation strategy.
for $\langle S, \{s_1, s_2\}\rangle$.

On the other hand, we can devise a strategy for $S$ that chooses $(s_0, s_1)$ and $(s_0, s_2)$ alternately. It is clear that such a strategy fulfills the strong fairness assumptions of $\{s_1, s_2\}$. ■

V. CHARACTERIZATION OF USF-SIMULATION

In this work, we focus on characterization of the simulation of a model GBTA by a specification BTA. That is, we restrict that the specification $\langle S, \Phi_S\rangle$ is a BTA with $|\Phi_S| + |\Psi_S| \leq 1$.

For convenience, given an MF-assumption $\Phi_S$ and a play $\eta = (\mu_0v_0, e_0f_0, t_0) \ldots (\mu_kv_k, e_kf_k, t_k) \ldots$, we may also define the satisfaction of $\Phi_S$ by $\eta$ in a way similar to the satisfaction of $\Phi_S$ by runs.

According to definition 5, a state-pair $\mu\nu$ is not in any simulation if there exists a run $\theta$ of $M$ from $\mu$, satisfying $\Phi_M\Psi_M$ such that for every strategy $\sigma$ for $S$ and play $\rho$ from $\mu\nu$ complying to $\sigma$ and embedding $\theta$, $\rho$ does not satisfy $\Phi_S\Psi_S$.

Put this description in a structural way, we have the following presentation.

$$(\mu \text{ starts a run } \theta \text{ of } M \text{ satisfying } \Phi_M\Psi_M) \wedge \forall \rho \left( \begin{array}{c} \rho \text{ starts from } \mu\nu \text{ and embeds } \theta. \\ \Rightarrow \rho \text{ does not satisfy } \Phi_S\Psi_S. \end{array} \right)$$

According to the composition of $\Phi_S\Psi_S$, this can be broken down to cases described with the following four lemmas.

**Lemma 8:** In case $\Phi_S = \{\eta\}$ for a state-predicate $\eta$, a state-pair $\mu\nu$ is not in any simulation of $\langle M, \Phi_M\Psi_M \rangle$ by $\langle S, \Phi_S\Psi_S \rangle$ iff

$$(\mu \text{ starts a run } \theta \text{ of } M \text{ satisfying } \Phi_M\Psi_M) \wedge \forall \rho \left( \begin{array}{c} \rho \text{ starts from } \mu\nu \text{ and embeds } \theta. \\ \Rightarrow \rho \text{ satisfies } \Phi_M(\Psi_M \cup \neg\eta). \end{array} \right)$$

is true.

**Proof:** According to the argument in the beginning of the subsection, we only have to prove that the following two statements are equivalent in the context that $\rho$ embeds $\theta$.

- $\rho$ does not satisfy $\{\eta\}\emptyset$.
- $\rho$ satisfies $\Phi_M(\Psi_M \cup \neg\eta)$.

Assume that $\rho = (\mu_0v_0, e_0f_0, t_0) \ldots (\mu_kv_k, e_kf_k, t_k) \ldots$.

We can prove this equivalence in two directions.

$(\Rightarrow)$ We assume that $\rho$ does not satisfy $\{\eta\}\emptyset$. According to the definition of strong fairness, we know that there are only finitely many $k$’s with a $t \in [0_t, k_{k+1} - t_k]$ such that $\mu_kv_k + t = \eta$. We let $m$ the maximum of such $k$’s. Then it is clear that for every $h > m$ and $t \in [0_t, h_{h+1} - t_h]$, $\mu_ht_h + t \neq \eta$. This means that $\rho$ satisfies $\theta(\neg\eta)$. Then the embedding of $\theta$ by $\rho$ implies that $\rho$ satisfies $\Phi_M(\Psi_M \cup \neg\eta)$.

$(\Leftarrow)$ We assume that $\rho$ satisfies $\Phi_M(\Psi_M \cup \neg\eta)$. Then according to the definition of weak fairness, we know that there exists an $m$ such that for every $h > m$ and $t \in [0_t, h_{h+1} - t_h]$, $\mu_ht_h + t = \eta$. Thus it is not true that there are infinitely many $k$’s with a $t \in [0_t, k_{k+1} - t_k]$ such that $\mu_kv_k + t = \eta$. According to the definition of strong fairness, $\rho$ does not satisfy $\{\eta\}\emptyset$.

With the proof of the two directions, we know the lemma is proven. ■

**Lemma 9:** In case $\Phi_S = \{\eta_1 \land \eta_2\}$ for an event-predicate $\eta_1 \land \eta_2$, a state-pair $\mu\nu$ is not in any simulation of $\langle M, \Phi_M\Psi_M \rangle$ by $\langle S, \Phi_S\Psi_S \rangle$ iff

$$(\mu \text{ starts a run } \theta \text{ of } M \text{ satisfying } \Phi_M(\Psi_M)) \wedge \forall \rho \left( \begin{array}{c} \rho \text{ starts from } \mu\nu \text{ and embeds } \theta. \\ \Rightarrow \rho \text{ satisfies } \Phi_M(\Psi_M \cup \{\eta_1 \land \neg\eta_2\}). \end{array} \right)$$

is true.

**Proof:** Suppose we are given $\rho = (\mu_0v_0, e_0f_0, t_0) \ldots (\mu_kv_k, e_kf_k, t_k) \ldots$.

The proof is similar to the one for lemma 8 except that we need to show that for a $k > 0$, the equivalence between the following two statements.

- It is not true that $(\mu_k + t_{k+1} - t_k)(\nu_k + t_{k+1} - t_k) = \eta_1$, $a \in \epsilon_M \cap \epsilon_S$, and $\mu_k + t_{k+1} = \eta_2$.
- If $(\mu_k + t_{k+1} - t_k) = \eta_1$ and $a \in \epsilon_M \cap \epsilon_S$, then $\mu_k + t_{k+1} = \eta_2$.

This equivalence follows from the semantics of propositional logic. By treating the event-predicate as a state-predicate, we can prove the lemma as we have proved lemma 8. ■

**Lemma 10:** In case $\Psi_S = \{\eta\}$ for a state predicate $\eta$, a state-pair $\mu\nu$ is not in any simulation of $\langle M, \Phi_M\Psi_M \rangle$ by $\langle S, \Phi_S\Psi_S \rangle$ iff

$$(\mu \text{ starts a run } \theta \text{ of } M \text{ satisfying } \Phi_M(\Psi_M)) \wedge \forall \rho \left( \begin{array}{c} \rho \text{ starts from } \mu\nu \text{ and embeds } \theta. \\ \Rightarrow \rho \text{ satisfies } (\Phi_M \cup \{\neg\eta\})(\Psi_M). \end{array} \right)$$

is true.

**Proof:** By replacing $\eta$ with $\neg\eta$, we can use a proof similar to the one for lemma 8 for this lemma.

**Lemma 11:** In case $\Psi_S = \{\eta_1 \land \eta_2\}$ for a state-predicate $\eta$, a state-pair $\mu\nu$ is not in any simulation of $\langle M, \Phi_M\Psi_M \rangle$ by $\langle S, \Phi_S\Psi_S \rangle$ iff

$$(\mu \text{ starts a run } \theta \text{ of } M \text{ satisfying } \Phi_M(\Psi_M)) \wedge \forall \rho \left( \begin{array}{c} \rho \text{ starts from } \mu\nu \text{ and embeds } \theta. \\ \Rightarrow \rho \text{ satisfies } (\Phi_M \cup \{\eta_1 \land \neg\eta_2\})(\Psi_M). \end{array} \right)$$

is true.

**Proof:** By replacing $\eta_1 \land \eta_2$ with $\neg\eta_1 \land \neg\eta_2$, we can use a proof similar to the one for lemma 9 for this lemma.

For convenience, given two sets $\Delta$ and $\Delta’$ of fairness assumptions, we let $(\Delta \land \Delta’)$ denote $\Delta \cup \{\neg\eta \mid \eta \in \Delta’\} \cup \{\eta \mid \eta \land \neg\eta_2 \in \Delta’\}$. According to lemmas 8, 9, 10, and 11, we conclude with the following lemma.

**Lemma 12:** In case $|\Phi_S| + |\Psi_S| \leq 1$, a state-pair $\mu\nu$ is not in any simulation of $\langle M, \Phi_M\Psi_M \rangle$ by $\langle S, \Phi_S\Psi_S \rangle$ iff

$$(\mu \text{ starts a run } \theta \text{ of } M \text{ satisfying } \Phi_M(\Psi_M)) \wedge \forall \rho \left( \begin{array}{c} \rho \text{ starts from } \mu\nu \text{ and embeds } \theta. \\ \Rightarrow \rho \text{ satisfies } (\Phi_M \land \neg\Psi_S)(\Psi_M \land \neg\Phi_S). \end{array} \right)$$

is true.

A procedure to construct a formula for states $\mu$ that starts a run of $M$ satisfying $\Phi_M\Psi_M$ can be found in [25]. Lemma 12 suggests that we still need to implement a procedure that constructs formulas for state-pairs that start all plays $\rho$ satisfying the following constraints.

$$\forall \rho \left( \begin{array}{c} \rho \text{ starts from } \mu\nu \text{ and embeds } \theta. \\ \Rightarrow \rho \text{ satisfies } (\Phi_M \land \neg\Psi_S)(\Psi_M \land \neg\Phi_S). \end{array} \right)$$

Such a play $\rho$ eventually stabilizes into a cycle of state-pairs.
Definition 6: CSR A state-pair $\mu \nu$ is CSR (Cyclically simulation-refuting) with $(\Phi_M, \Psi_S)$ if for every $\phi \in (\Phi_M, \Psi_S)$, there exists a run $\theta$ of $M$ with the following two constraints.

C1: For every strategy $\sigma$ of $S$ with $\rho = \text{play}(\mu \nu, \theta, \sigma)$, if $\rho$ is of infinite length, then the following four constraints are satisfied.

C1a: All state-pairs along $\rho$ satisfy state-predicates in $(\Phi_M, \Psi_S)$.

C1b: All transition-pairs along $\rho$ satisfy event-predicates in $(\Phi_M, \Psi_S)$.

C1c: For every state-predicate $\eta$ in $(\Phi_M, \Psi_S)$, there is a CSR state-pair in $\rho$ satisfying $\eta$ in more than 1 time units from the start of $\rho$.

C1d: For every event-predicate $\eta$ in $(\Phi_M, \Psi_S)$, there is a transition-pair in $\rho$ satisfying $\eta$ in more than 1 time units from the start of $\rho$.

C2: There exists a strategy $\sigma$ of $S$ with an infinitely long play$(\mu \nu, \theta, \sigma)$. The 1-time-unit requirement at condition C1c is for making sure that the play is non-Zeno.

A state-pair $\mu \nu$ is inevitably SR (ISR) with $(\Phi_M, \Psi_S)$ if there exists a run $\theta$ of $M$ from $\mu$ such that for all strategies $\sigma$ of $S$, if play$(\mu \nu, \theta, \sigma)$ is infinite, then play$(\mu \nu, \theta, \sigma)$ visits a CSR state-pair.

The following lemma is important for our algorithm development.

Lemma 13: Suppose we are given a GBTA $M$ and a BTA $S$. For any state-pair $\mu \nu \in S(\langle M \rangle \times S(S))$, the following two statements are equivalent.

R1: $\mu$ starts a run $\theta$ of $M$ satisfying $\Phi_M \Psi_M$ and for all plays $\rho$ from $\mu \nu$ embedding $\theta$, $\rho$ satisfies $(\Phi_M, \Psi_S)(\Psi_M, \Psi_S)$.

R2: There exist an $e \in E_M$, a $t \in \mathbb{Z}^+$, and a $\mu' \in S(\langle M \rangle)$ with the following constraints.

R2a: $\mu \xrightarrow{t_e} \mu'$.

R2b: $\mu'$ starts a run satisfying $\Phi_M \Psi_M$.

R2c: For every $f \in E(\langle S \rangle)$ and $\nu' \in S(\langle S \rangle)$ with $\mu \xrightarrow{t_{e f}} \mu' \nu'$, $\mu' \nu'$ is an ISR state-pair with $(\Phi_M, \Psi_S)(\Psi_M, \Psi_S)$.

Proof: We prove the lemma in two directions.

$(\Rightarrow)$ We assume that R1 is true. Conditions R2a and R2b are automatically true since $\theta$ must begin with a timed transition step $\mu \xrightarrow{t_e} \mu'$ for some $t \in \mathbb{Z}^+$, $e \in E_M$, and $\mu' \in S(\langle M \rangle)$.

As for condition R2c, we establish it in the following. The truth of R1 means that for every strategy $\sigma$ of $S$, if $\rho = \text{play}(\mu \nu, \theta, \sigma)$ embeds $\theta$, then $\rho$ must satisfy $(\Phi_M, \Psi_S)(\Psi_M, \Psi_S)$. This means that there exists a $b \in \mathbb{Z}^+$ such that for every such infinite $\rho$, after $b$ time units from the start of $\rho$, all predicates in $(\Phi_M, \Psi_S)$ are satisfied and all predicates in $(\Phi_M, \Psi_S)$ are satisfied infinitely and divergently many times. If such a $b$ does not exist, then we can construct a play that violates $(\Phi_M, \Psi_S)(\Psi_M, \Psi_S)$ and the assumption of R1. We claim that all state-pairs $\mu \nu$ happening after $b$ time units from the start in all infinite plays are CSR state-pairs in definition 6. The reasons are the following.

- Since $\mu \nu$ happens $b$ time units after the start of the play, it must satisfy conditions C1a and C1b in definition 6.
- Moreover, along every infinite play from $\mu \nu$, for every predicate $\eta$ in $(\Phi_M, \Psi_S)$, there are infinitely and divergently many state-pairs or transition-pairs that satisfies $\eta$.

Thus we can find the first state-pair $\mu \nu$ in the tail with the following restrictions.

- $\mu \nu$ is at least one time unit from the start of the play.
- Either $\mu \nu$ satisfies $\eta$ as a state-predicate or the transition-pair right before $\mu \nu$ satisfies $\eta$ as an event-predicate.

This implies that conditions C1c and C1d in definition 6 are satisfied at $\mu \nu$.

- The assumption that leads to the satisfaction of $(\Phi_M, \Psi_S)(\Psi_M, \Psi_S)$ by $\rho$ then implies that there exists such a play. This implies that condition C2 in definition 6 is satisfied.

The argument in the above establishes that $\mu \nu$ is indeed a CSR state-pair. Thus we know that along every infinite play from $\mu \nu'$, we can reach such a $\mu \nu$. This implies that $\mu \nu'$ is an ISR state-pair and condition R2c is satisfied. Thus the lemma is proven in this direction.

$(\Leftarrow)$ We assume that R2 is true. This implies that there exist an $e \in E_M$, a $t \in \mathbb{Z}^+$, and a $\mu' \in S(\langle M \rangle)$ with $\mu \xrightarrow{t_e} \mu'$ and $\mu'$ starting a run satisfying $\Phi_M \Psi_M$. There are two cases to analyze.

- By letting $\theta$ start with $(\mu, \perp, 0)(\mu' e, t)$ and followed by the tail from $\mu'$ that satisfies $\Phi_M \Psi_M$, we deduce that $\mu$ also starts a run $\theta$ that satisfies $\Phi_M \Psi_M$.

- Then for all strategies $\sigma$ of $S$ with $\rho = \text{play}(\mu \nu, \theta, \sigma)$, we can go to an ISR state-pair $\mu \nu'$ with $(\Phi_M, \Psi_S)(\Psi_M, \Psi_S)$. This implies that for all infinite plays from $\mu \nu'$, we can visit a CSR state-pair $\mu \nu$. Then according to the definition of CSR state-pairs, for each predicate $\eta \in (\Phi_M, \Psi_S)$, we can go from $\mu \nu'$ along a play with all state-pairs and transition-pairs satisfying the predicates in $(\Phi_M, \Psi_S)$. Moreover, the play visits a CSR state-pair $\mu \nu$ that either satisfies $\eta$ as a state-predicate or satisfies with the transition-pair immediately before $\mu \nu$ as an event-predicate. Since $\mu \nu$ is also CSR, we can then repeat the same argument to fulfill another predicate assumption in $(\Phi_M, \Psi_S)$.

By repeating this procedure for all predicates in $(\Phi_M, \Psi_S)$ infinitely many times, we can construct every infinite plays from $\mu \nu$ that embeds $\theta$ mentioned in the last item. This construction then leads to the conclusion that all plays from $\mu \nu$ embedding $\theta$ satisfy $(\Phi_M, \Psi_S)(\Psi_M, \Psi_S)$.

This completes the proof of this direction. Since both directions of the proof are done, we know the lemma is true.

Lemma 13 suggests the development of evaluation algorithm for CSR state-pairs for the solution of USF-simulations of GBTAs. In the following, we explain how to do this.
VI. A SYMBOLIC ALGORITHM FOR USF-SIMULATION

In this work, we focus on the simulation algorithm for a model GBTA by a specification BTA. Our algorithm is based on the construction of formulas for CSR and ISR state-pairs. In the following, we assume the context of a model GBTA \( \langle M, \Phi_M, \Psi_M \rangle \) and a specification BTA \( \langle S, \Phi_S, \Psi_S \rangle \).

In subsection VI-A we present some symbolic procedures from model-checking technology of dense-time systems as our basic building blocks. In subsection VI-B we present algorithms for state-pairs that can be forced to a goal in one timed transition step. In subsection VI-C we use the procedures in subsection VI-B to construct a algorithms for state-pairs that can be forced to a goal in zero or more timed transition steps. In subsection VI-D we present the algorithm for simulation-checking. In subsection VI-E we analyze the complexity of our algorithm.

A. Building blocks from model-checking technology

In this subsection, we adapt procedures for TCTL model-checking [2] for the evaluation of simulation-checking.

Given a formula \( \eta \), a run prefix \( \langle \mu_0, e_0, t_0 \rangle, \ldots, \langle \mu_k, e_k, t_k \rangle \) of \( M \) is called an \( \eta \)-Prefix if for every \( h \in [0, k) \) and \( t \in [0, t_{h+1} - t_h] \), \( \mu_h + t \models \eta \). Similarly, a play prefix \( \langle \mu_0, e_0, t_0 \rangle, \ldots, \langle \mu_k, e_k, t_k \rangle \) of \( M \) is called an \( \eta \)-PPrefix if for every \( h \in [0, k) \) and \( t \in [0, t_{h+1} - t_h] \), \( \mu_h + t \models \eta \).

A state-pair set \( D \), we let \( \exists S(D) = \{ \mu | \mu \in D \} \). Given a TA \( S \) with \( P_S = \{ p_1, \ldots, p_n \} \) and \( X_S = \{ x_1, \ldots, x_n \} \), we let \( \exists S(\eta) \) be the following formula.

\[
\exists p_1 \ldots \exists p_n \exists x_1 \ldots \exists x_n (\eta).
\]

Also given a set \( P = \{ p_1, \ldots, p_m \} \) and a set \( X = \{ x_1, \ldots, x_n \} \), we let \( \exists S(P, X)(\eta) \) be the following formula.

\[
\exists p_1 \ldots \exists p_m \exists x_1 \ldots \exists x_n (\eta \land \bigwedge_{x \in X} x = 0).
\]

Standard procedures for constructing state-predicates of existentially quantified formulas can be found in [15, 24].

Given a transition-pair \( e \in E_M \times E_S^e \) with \( e = (q_1, q_1') \) and \( f = (q_2, q_2') \), we let \( e(f)(\eta) \) be the formula of state-pairs that may go to state-pairs in \( \eta \) through the simultaneous execution of \( e \) and \( f \) respectively. Specifically, \( e(f)(\eta) \) is defined as follows.

\[
\begin{align*}
&q_1 \land q_2 \land \lambda_M(q_1) \land \lambda_S(q_2) \land \tau_M(e) \land \tau_S(f) \\
&\land \text{reser}[P_M \cup P_S, X_M \cup X_S]
\end{align*}
\]

We also need the formulas for the precondition of time-progress to a state-pair satisfying \( \eta_2 \) through intermediate state-pairs satisfying \( \eta_1 \). Procedures for such formulas can be found in [15, 24, 28, 29]. We present the formula, denoted \( T(\eta_1, \eta_2) \), for the readers’ convenience in the following.

\[
\eta_1 \land \exists t \left( t \geq 0 \land \eta_2 + t \land \forall t' \left( t' < t \land t' \geq 0 \rightarrow \eta_1 + t' \right) \right)
\]

Here \( t \) represents a formula obtained from \( \eta \) by replacing every clock variable \( x \) with \( x + t \).

We use adapted TCTL formulas \( \exists \eta_1 \Upsilon_S \eta_2 \) in our presentation of the algorithm. Specifically, \( \exists \eta_1 \Upsilon_S \eta_2 \) characterizes those state-pairs \( \mu \) with the following restrictions.

- \( \mu \) starts an \( \eta_1 \)-Prefix \( \rho \) that ends at a state-pair satisfying \( \eta_2 \).
- Along the \( \rho \) mentioned in the above, all the transitions are of the form \( (\bot, f) \) with \( f \in E_S^e \).

Following the techniques in [15, 24], we can construct a formula in \( B(P_M \cup P_S, X_M \cup X_S) \) that characterizes state-pairs satisfying \( \exists \eta_1 \Upsilon_S \eta_2 \). Specifically, the formula is as follows.

\[
\exists \eta_1 \Upsilon_S \eta_2 \equiv \exists \eta \left( \eta_2 \lor T \left( \eta_1, \forall f \in E_S^e (f(Z)) \right) \right)
\]

Here \( \exists \eta \) is the least fixpoint operator and \( \exists \eta \left( \eta_2 \lor T \left( \eta_1, \forall f \in E_S^e (f(Z)) \right) \right) \) represents the smallest solution to \( Z = \beta(Z) \).

Another type of formulas that we want to use is for states \( \mu \) of \( M \) that start runs satisfying \( \Phi_M \Psi_M \). We denote this formula as \( \exists \Phi_M \Psi_M \) for convenience. The construction of this formula can be found in [20].

B. One-step timed inevitabilities by \( M \)

Given a set \( D \) of states (or state-pairs), we use \( \langle D \rangle \) to denote a formula that characterizes \( D \). Given a formula \( \eta \), we use \( \lbrack \eta \rbrack \) to represent the set of states (or state-pairs) that satisfies \( \eta \). Given an \( e \in E_M \), a set \( \Psi \) of event weak fairness assumption, and a \( t \in \mathbb{R}^{\geq 0} \), we use \( (M)[D_1 \bigcirc_{\Psi}^\top D_2] \) to denote the set of state-pairs \( \mu \) with the following restrictions.

\[
\begin{align*}
M_1: \ & \langle D_1 \rangle \text{-} \text{PRePrefix} \\
& (\mu_0, \bot, t_0) \in (D_2) \\
M_2: \ & \langle D_1 \rangle \text{-} \text{PRePrefix}
\end{align*}
\]

with

- \( \mu_0 \eta_0 = \mu \),
- \( t_k - t_0 = t \),
- \( e_k = e \), and
- \( \forall h \in [0, k) (e_h = \bot) \),

For every event weak fairness assumption \( \eta_3 \eta_4 \in \Psi \), if \( a \in \epsilon_M(e) \) and \( \mu \models \eta_3 \), then \( \mu \models \text{\footnotesize{\textbf{\textit{t}}}}(\eta \land e' \text{\footnotesize{\textit{t}}}(-\eta_4)) \).

Note that in the just-mentioned \( \langle D_1 \rangle \)-PRePrefix, the strategy of \( S \) can only use the internal transitions of \( S \).

We can use the following TCTL formula to help us characterize \( (M)[D_1 \bigcirc_{\Psi}^\top D_2] \). Given two state-predicates \( \eta_1, \eta_2 \), and a set \( \Psi \) of event formulas for weak fairness assumption, we let \( \circ_{\Psi}^\top(\eta_1, \eta_2) \) be defined as follows.

\[
\begin{align*}
&T \left( \exists \Psi(\eta_1), z \in C_M^S \land e \left( \exists \Psi(\eta_2) \land \exists \Phi_M \Psi_M \right) \right) \\
&\land \neg \exists \eta_1 \Upsilon_S \eta_2
\end{align*}
\]

Here \( z \) is an auxiliary clock variable not used in \( X_M \cup X_S \). The conjunction

\[
\begin{align*}
&\neg \exists \eta_1 \Upsilon_S \eta_2 \\
&\land \text{\footnotesize{\textbf{\textit{t}}}}(\eta_3 \land e' \text{\footnotesize{\textit{t}}}(-\eta_4))
\end{align*}
\]

is the least fixpoint operator and \( \exists \eta \left( \eta_2 \lor T \left( \eta_1, \forall f \in E_S^e (f(Z)) \right) \right) \) represents the smallest solution to \( Z = \beta(Z) \).
in the post-condition is used to make sure that no event weak fairness assumptions in $\Psi$ is violated. It is used to eliminate all state-pairs violating an event weak fairness assumption. The following lemma shows how to use the above formulas to help us evaluating $\langle M \rangle D_1$ for $D_2$.

**Lemma 14:** For every $\mu, \nu \in S(M) \times S(M)$, $t \in [0, C_{SM}]$, formulas $\eta_1, \eta_2$ of state-pairs, and set $\Psi$ of event weak fairness assumptions, $\mu, \nu \in \langle M \rangle (\eta_1 \cap \Psi) [\eta_2]$ iff $\mu, \nu \models \Omega_M(\eta_1, \eta_2)$.

**Proof:** We can rewrite condition $M_2$ of $\langle M \rangle (\eta_1 \cap \Psi) [\eta_2]$ as follows.

$M_2'$: There is no $\eta_1$-PPrefix

$$(\mu_0, \nu_0, e_0, f_0, t_0) \ldots (\mu_k, \nu_k, e_k, f_k, t_k)$$

with

- $\mu_0, \nu_0 = \mu, \nu$,
- $t_k = t_{k-1} - t$,
- $e_k = e$,
- $\forall h \in [0, k](e_k = 1)$,
- $\mu_k, \nu_k \models \neg \eta_2$, and
- for every $\eta_3 \eta_4 \in \Psi$ and $e' \in E_M$, $a \in \epsilon_3(e)$, and $0 < e' \\
v' \in E_S(e')$, it is not true that $[\mu_{k-1} \neg \eta_3 \eta_4] \models \neg \eta_4$.

It is clear that a state-pair satisfies $M_1$ and $M_2$ if and only if it satisfies $M_1$ and $M_2'$. By renaming $t_k$ as a clock variable $z$ and $t_k$ as constant $C_{SM}$, we can use $C_{SM} - z$ to represent $t$. This means that $M_1$ and $M_2'$ can be rewritten as $z = C_{SM} - t$ and the following two conditions.

$M_1$: There exists an $\exists S(\eta_1)$-PPrefix

$$(\mu, \eta_1, z)([\mu \in C_{SM} - z, e], C_{SM})$$

with $\exists S(\eta_1) \models \exists S(\eta_2) \land \exists \Phi_{MA} \Psi_{MA}$.

$M_2'$: There is no $\eta_1$-PPrefix

$$(\mu_0, \nu_0, e_0, f_0, t_0) \ldots (\mu_k, \nu_k, e_k, f_k, t_k)$$

with $e_k = e$, $\forall h \in [0, k](e_k = 1)$, $\mu_0, \nu_0 = \mu, \nu$, $\mu_k = \neg \eta_2$, and for every $\eta_3 \eta_4 \in \Psi$ and $e' \in E_M$, $a \in \epsilon_3(e)$, and $0 < e' \\
v' \in E_S(e')$, it is not true that $[\mu_{k-1} \neg \eta_3 \eta_4] \models \neg \eta_4$.

$M_1$ means the following.

$$\mu \models T(\exists S(\eta_1), z = C_{SM} - e \land (\exists S(\eta_2) \land \exists \Phi_{MA} \Psi_{MA}))$$

$M_2'$ means the following.

$$\mu \models \neg \exists \eta_1 \Psi_S$$

$$\mu \models \neg \exists \eta_1 \Psi_S$$

$$\mu \models \neg \exists \eta_1 \Psi_S$$

$$\mu \models \neg \exists \eta_1 \Psi_S$$

Combining these two formulas together and reduce them with the definition of $\Omega_M(\eta_1, \eta_2)$, we find that $\mu$ must satisfy $t = C_{SM} - z \land \exists \Phi_{MA}(\eta_1, \eta_2)$. Thus the lemma is proven.

Based on lemma [14] we can define the following notations for those state-pairs that can be forced into either certain destination or a transition of $M$ that $S$ cannot match. Specifically, we let

$$\langle M \rangle D_1 \cap \Psi D_2 = \supseteq \bigcup_{t \in [0, C_{SM}] \times S(M) \times S(M)} \langle M \rangle D_1 \cap \Psi D_2$$

Correspondingly, given two formulas $\eta_1$ and $\eta_2$, we can construct $\Omega_M(\eta_1, \eta_2)$, defined as follows.

Then according to lemma [14] we can establish the following lemma.

**Lemma 15:** For every $\mu, \nu \in S(S) \times S(S)$, formulas $\eta_1, \eta_2$ of state-pairs, and set $\Psi$ of event weak fairness assumptions, $\mu, \nu \models \langle M \rangle [\eta_1 \cap \Psi] [\eta_2]$ iff $\mu, \nu \models \Omega_M(\eta_1, \eta_2)$.

**Proof:** We have the following deduction.

$$\forall \mu, \nu \in \langle M \rangle [\eta_1 \cap \Psi] [\eta_2]$$

According to lemma [14] this implies the following.

$$\forall \mu, \nu \models \langle M \rangle [\eta_1 \cap \Psi] [\eta_2]$$

$$\forall \mu, \nu \models \langle M \rangle [\eta_1 \cap \Psi] [\eta_2]$$

Since $\Omega_M(\eta_1, \eta_2)$ does not contain variable $t$, the above formulas are equivalent to the following.

$$\forall \mu, \nu \models \langle M \rangle [\eta_1 \cap \Psi] [\eta_2]$$

The last step is from the definition of $\Omega_M(\eta_1, \eta_2)$. Thus the lemma is proven.

Note that before the fulfillment of $\eta_2$, $\Omega_M(\eta_1, \eta_2)$ is satisfied with play prefixes with only transitions internal to $S$.

**C. Multi-step timed inevitabilities by $M$**

In general, we want to characterize state-pairs from which $M$ can force the fulfillment of $\eta_2$ through zero or more timed transition steps of $M$ that do not violate the weak fairness assumptions in $\Psi$. We denote the set of such state-pairs as $\langle M \rangle (\eta_1 \cup \Psi) [\eta_2]$. For convenience, given two formulas $\eta_1, \eta_2$ for sets of state-pairs, we let

$$\Omega_M(\eta_1, \eta_2) = \supseteq \text{Ifp} \Psi(\eta_2 \cup \Omega_M(\eta_1, Y))$$

Here Ifp is the least fixpoint operator $\text{Ifp} \Psi(\eta_2 \cup \Omega_M(\eta_1, Y))$ specifies a smallest solution to equation $Y \models \eta_2 \cup \Omega_M(\eta_1, Y)$. The procedure to construct formulas for such least fixpoints can be found in [15, 24].

**Lemma 16:** For every state-pairs $\mu, \nu$ and formulas $\eta_1, \eta_2$ for state-pairs, $\mu, \nu \in \langle M \rangle (\eta_1 \cup \Psi) [\eta_2]$ iff $\mu, \nu \models \Omega_M(\eta_1, \eta_2)$.

**Proof:** We can prove this lemma in two directions.

$\Rightarrow$ Assume that $\mu, \nu \models \Omega_M(\eta_1, \eta_2)$. Then it is clear that $\mu, \nu$ also satisfies every formula of the form
Now we assume that this direction of the lemma is true for every state-pairs with maximum number no greater than $k$ with $k \geq 0$. Now we have a state-pair $\mu \nu$ with maximum number $k + 1$ of timed transition steps to reach state-pairs in $[\eta_2]$ through state-pairs in $[\eta_1]$. This implies that there exist an $e \in E_M$ and a $t \in \mathbb{R}^\geq 0$ with

$$
\mu \nu \models (\langle M \rangle [\eta_1] \bigcirc_i^{\psi_0} (\langle M \rangle [\eta_1] U^{\psi_0} [\eta_2])).
$$

This means that in one timed transition step of $e$ and time units by $M$, we end up in a state-pair $\mu \nu'$ such that within $k$ timed transition of $M$ steps through state-pairs in $[\eta_1]$, we can go from $\mu \nu'$ to state-pairs in $[\eta_2]$. According to the inductive hypothesis, we know that $\mu \nu'$ satisfies $U^{\psi_0}_M (\eta_1, \eta_2)$. Together, this implies the following deduction.

$$
\mu \nu \models (\bigcirc_i^{\psi_0} (\langle M \rangle [\eta_1] U^{\psi_0} [\eta_2]))
\quad \equiv \quad \mu \nu \models (\bigcirc_i^{\psi_0} (\langle M \rangle [\eta_1] \bigcirc_i^{\psi_1} (\langle M \rangle [\eta_1] U^{\psi_1} [\eta_2])))
\quad \equiv \quad \mu \nu \models (\bigcirc_i^{\psi_0} (\langle M \rangle [\eta_1] \bigcirc_i^{\psi_1} (\langle M \rangle [\eta_1, \eta_2])))
\quad \equiv \quad \mu \nu \models (\bigcirc_i^{\psi_0} (\langle M \rangle [\eta_1] U^{\psi_0} [\eta_2]))
\quad \equiv \quad \mu \nu \models (\bigcirc_i^{\psi_0} (\langle M \rangle [\eta_1] U^{\psi_0} [\eta_2])).
$$

According to the definition of least fixpoint, the last step implies $\mu \nu \models \epsilon Y [\eta_2, \langle M \rangle [\eta_1, \eta_2])$. Thus, by definition, this implies that $\mu \nu \models U^{\psi_0}_M (\eta_1, \eta_2)$. Thus this direction of the lemma is proven by induction.

(⇐) We assume that there exist $Y_0, Y_1, \ldots, Y_n$ such that $Y_0 = \eta_2$, $Y_n = \eta_2 \vee \bigcirc^{\psi_0}_M (\eta_1, Y_n)$, and for every $i \in [0, n]$, $Y_i = \eta_2 \vee \bigcirc^{\psi_0}_M (\eta_1, Y_i)$. We prove by induction on $k \in [0, n]$ that $\mu \nu \models Y_i$ implies $\mu \nu \models U^{\psi_0}_M [\eta_2]$. The base case is that $k = 0$ and $\mu \nu \models \eta_2$. This implies that $\mu \nu \models \eta_2$ and $\mu \nu \models [\eta_2]$. Thus the base case is proven.

Now we assume that the lemma in this direction is true for all $i \in [0, k]$. Now we have a $\mu \nu \models Y_{k+1}$. This means that $\mu \nu \models \eta_2 \vee \bigcirc^{\psi_0}_M (\eta_1, Y_k$). There are two cases to analyze. The first is $\mu \nu \models \eta_2$ and coincides with the base case. Thus the first case is already proven.

The second case is $\mu \nu \models \eta_2 \vee \bigcirc^{\psi_0}_M (\eta_1, Y_k$). According to lemma [13], this implies that we can force in one timed transition step through state-pairs in $[\eta_1]$ to state-pairs $\mu \nu'$ in $[Y_k]$. Moreover, the inductive hypothesis says that all such $\mu \nu' \in \langle M \rangle [\eta_1] U^{\psi_0} [\eta_2]$. According to the definition of $\langle M \rangle [\eta_1] U^{\psi_0} [\eta_2]$, this implies that $\mu \nu \models \eta_2$. Thus the lemma is proven in this direction.

Thus the lemma is proven.

### D. Simulation checking algorithm

Our plan is first to use the procedures in subsections VI-A and VI-B to construct a procedure for evaluating CSR state-pairs. Then we use this procedure to evaluate ISR state-pairs. For convenience, we denote

$$
SP^M_S \equiv V_M \land \bigwedge_{\text{state-predicate } \psi \in (\Phi_M \land \Phi_S)} \psi.
$$

Conceptually, $SP^M_S$ denotes the state-predicates that a play satisfying $(\Phi_M \land \Phi_S) (\psi_S \land \Phi_M)$ must stabilize with. Also we let $EP^M_S$ be the set of event-predicates in $(\Phi_M \land \Phi_S)$.

We present a greatest fixpoint characterization, denoted $UF^M_S (\eta)$, of the CSR state-pairs with an MF-assumption $\Phi_S$. A state-pair $\mu \nu$ satisfies $UF^M_S (\eta)$ if there is a fair run from $\mu$ such that all plays embedding the run from $\mu \nu$ cannot be fair for $S$. The characterization follows.

$$
UF^M_S \equiv \text{gfp} W. \left( \bigwedge_{\phi \in \Phi} U^{EP^M_S} (SP^M_S (W \land \phi)) \right).
$$

Here $\text{gfp}$ is the greatest fixpoint operator. $\text{gfp} W. (\beta(W))$ is a largest solution $W$ to $W \equiv \beta(W)$. The procedure to construct formulas for greatest fixpoints can be found in [15], [24]. The following lemma establishes the correctness of the characterization.

Lemma 17: A state-pair $\mu \nu$ is CSR with $(\Phi_M \land \Phi_S) (\psi_S \land \Phi_M)$ iff $\mu \nu \models UF^M_S$.

**Proof:** Following definition 6 lemma 16 the definition of $SP^M_S$, and the semantics of greatest fixpoint, $UF^M_S$ is actually a rewriting of the CSR definition with logic formulas, the greatest fixpoint procedure, and the $U^{\psi_0}_M$ procedure. Thus the lemma is proven.

Now we use $UF^M_S$ to evaluate ISR state-pairs. Given a fair run $\theta$ of $M$, there are two classes of ISR state-pairs. The first class contains state-pairs that start no play embedding $\theta$. The second class contains state-pairs with a strategy $S$ to drive a play to stabilize to CSR state-pairs. The former can be evaluated with the traditional procedures for branching simulation [8], [23], [32]. Specifically, state-pairs is the first class can be characterized with the following lemma.

Lemma 18: A state-pair $\mu \nu$ is a first-class ISR state-pair iff $\mu \nu \models U^{\theta}_M (V_M \land \bigwedge_{\text{state-predicate } \psi \in (\Phi_M \land \Phi_S)} \psi).

**Proof:** $\mu \nu$ is first class iff for all strategies $\sigma$ of $S$, play$(\mu \nu, \theta, \sigma)$ is of finite length. There can only be two causes for the termination of the plays.

- Along a time progress operation, $M$ moves to a valid state while $S$ cannot. This is captured by formula $T (V_M \land V_S, V_M \land \neg V_S)$.
- At a transition $e \in M$, no compatible $f \in E^c$ can result in a valid state of $S$. This is captured by $e f (V_M \land \neg V_S)$.

If and only if $M$ can drive all plays to state-pairs with these two causes, then it is clear all plays are finite in length. Thus the lemma is proven.

The state-pairs in the second class can be forced into infinite plays that stabilize in CSR state-pairs. Specifically, we have the following lemma.

Lemma 19: A state-pair $\mu \nu$ is a second-class ISR state-pair iff $\mu \nu \models U^{\psi}_M (V_M \land \bigwedge_{\text{state-predicate } \psi \in (\Phi_M \land \Phi_S)} \psi).

**Proof:** This lemma follows from the definition of the second class state-pairs, lemma 16 and lemma 17.

Combining lemmas [13], [18] and [19] we present the following lemma for the characterization of state-pairs that is in no simulation of a GBTA by a BTA.

Lemma 20: A state-pair $\mu \nu$ is in no simulation of a GBTA $(M, \Phi_M \land \Phi_M)$ by a BTA $(S, \Phi_S \land \Phi_S)$ iff $\mu \nu$ satisfies either $U^{\theta}_M (V_M \land \bigwedge_{\text{state-predicate } \psi \in (\Phi_M \land \Phi_S)} \psi)$ or $U^{\psi}_M (V_M \land \bigwedge_{\text{state-predicate } \psi \in (\Phi_M \land \Phi_S)} \psi)$.
E. Complexity

The complexity of our algorithm relies on the implementation of the basic manipulation procedures of zones. Like in [15], we argue that we can implement the formulas as sets of pairs of proposition valuations and regions [2]. In such an implementation, basic operations like subsumption, intersection, union, complement, time progression, and variable quantification can all be done in \( \text{EXPTIME} \).

Lemma 21: Proper implementations of the formulation in lemma 20 can be done in \( \text{EXPTIME} \).

Proof: According to [2], a zone can be implemented as a set of regions. The number of regions is exponential to the input size of \((M, \Phi_M, \Psi_M)\) and \((S, \Phi_S, \Psi_S)\). All precondition calculations need at most polynomial numbers of region set operations and can all be done in \( \text{EXPTIME} \). The numbers of iterations of the least and greatest fixpoint procedures are at most the number of regions. Thus, summing everything up, we conclude that our algorithm can be executed in \( \text{EXPTIME} \).

VII. SIMULATION-CHECKING AGAINST A SHARED ENVIRONMENT

In real-world, we may usually want to check whether a system component satisfies its specification. In such a context, the simulation-checking is carried out against the same behavior of the environment of the component. Such a context can usually make room for verification efficiency if we carefully represent the common environment state information. In this section, we extend the simulation defined in section IV to simulation of a model by a specification against a common environment. Then we propose a technique to take advantage of the common environment information for simulation-checking efficiency.

In figure 3(b) there are two TAs for two environment processes. Note that location \( \lambda \text{comp} \) in figure 3(b) is labeled with a deadline \( x_3 < 10 \). This means that the environment process in figure 3(b) can only stay in location \( \lambda \text{comp} \) for at most 10 time units. Thus the environment process in figure 3(a) may deliver late service while the one in figure 3(b) always deliver service in 10 time units. Against the environment described by figure 3(a), the \( S \) in figure 3(b) does not simulate the \( M \) in figure 3(a) since the \( M \) terminates the computation on late service while the \( S \) never terminates the computation. In comparison, against figure 3(b), the \( S \) simulates the \( M \) since the service is always in time.

A. CTA

We use CTAs (communicating timed automata) to model the interaction between an environment and a model (or a specification). The formal definition is in the following.

Definition 7: CTA A CTA of two TAs \( A \) and \( B \), in symbols \( A \times B \), is a TA with the following constraints.

- For each \((q_1, q_2) \in Q_A \times B, \lambda_A((q_1, q_2)) = \lambda_A(q_1) \land \lambda_B(q_2)\).
- For simplicity, we assume that \( P_A \cap P_B = \emptyset, Q_A \cap Q_B = \emptyset \), and \( X_A \cap X_B = \emptyset \). Moreover, the transitions of a product TA needs to consider the synchronization between the two process TAs. Specifically, we let \( E_A \times B \subseteq E_A \times E_B \).
- \((e, f) \) represents the autonomous execution of a process TA with a transition without any events. Formally speaking, this means at least one of \( e \) and \( f \) is \( \bot \), i.e., no operation. We have the following two cases to explain.
- If \( e \neq \bot \) and \( f = \bot \), then \( e \in E_A, \epsilon_{A \times B}(e, f) = \epsilon_A(e) = 0, \tau_{A \times B}((e, f)) = \tau_A(e), \pi_{A \times B}((e, f)) = \pi_A(e). \)
- If \( e = \bot \) and \( f \neq \bot \), then \( f \in E_B, \epsilon_{A \times B}(e, f) = \epsilon_B(f) = 0, \tau_{A \times B}((e, f)) = \tau_B(f), \pi_{A \times B}((e, f)) = \pi_B(f). \)

\((e, f) \) represents the synchronized execution of the two process TAs respectively with a receiving event and a sending event of the same type. Formally speaking, this means that there is an \( a \in \Sigma_{A \times B} \) with the following restrictions.

- Either of the following two is true.
  - \( \epsilon_{A \times B}(e, f) = \{ !a @ (A), !a @ (B) \}, \epsilon_A(e) = \{ !a \}, \) and \( \epsilon_B(f) = \{ !a \}. \)
  - \( \epsilon_{A \times B}(e, f) = \{ !a @ (A), !a @ (B) \}, \epsilon_A(e) = \{ !a \}, \) and \( \epsilon_B(f) = \{ !a \}. \)

Note here we blend the process names and the operations into the name of the new events. For example, \( ?a @ (A) \) and \( !a @ (A) \) respectively represent the receiving and the sending of event \( a \) by process \( A \).

Example 22: For the specification \( S \) in figure 1(b) and the environment \( E \) in figure 3(a), we have \( E \times S \) with attributes in table II.

Since a CTA is also a TA, we explain how to interpret the notations about TAs for CTAs. Given a state \( \alpha \) of \( A \) and a state \( \mu \) of \( B \), \( (\alpha, \mu) \) is called a state of \( A \times B \). We say a state \( (\alpha, \mu) \) satisfies a state predicate \( \eta \in B(\times A \times B), X_{A \times B} \), in symbols \( (\alpha, \mu) \models \eta \), with the following inductive rules.

- For any \( p \in P_A, (\alpha, \mu) \models p \) iff \( \alpha \models p \).
- For any \( p \in P_B, (\alpha, \mu) \models p \) iff \( \mu \models p \).
- For any \( x \in X_A, (\alpha, \mu) \models x \sim c \) iff \( \alpha \models x \sim c \).
- For any \( x \in X_B, (\alpha, \mu) \models x \sim c \) iff \( \mu \models x \sim c \).
- \( (\alpha, \mu) \models \neg \eta_1 \) iff it is not the case that \( (\alpha, \mu) \models \eta_1 \).
- \( (\alpha, \mu) \models \eta_1 \lor \eta_2 \) iff \( (\alpha, \mu) \models \eta_1 \) or \( (\alpha, \mu) \models \eta_2 \).

The state after a transition \( (e, f) \) from a state \( (\alpha, \mu) \) of CTA, denoted \( (\alpha, \mu)(e, f) \), can also be interpreted as \( (\alpha e, \mu f) \). A timed transition of \( t \) time units from a state \( (\alpha, \mu) \), denoted \( (\alpha, \mu) + t \), can be defined as \( (\alpha + t, \mu + t) \). In this way, we can also define the timed transition relation between two states \( (\alpha, \mu), (\alpha', \mu') \) through a transition \( (e, f) \) in \( t \) time units, denoted as \( (\alpha, \mu)^{t,(e,f)}(\alpha', \mu') \), with the following restrictions.
Fig. 3. A non-responsive and a responsive environment processes

(a) a non-responsive environment process $\mathcal{E}$

(b) a responsive environment process $\mathcal{E}$

TABLE II

| Attributes of the CTA of $\mathcal{S}$ in figure 3(b) and $\mathcal{E}$ in figure 3(a). |

- For all $t' \in [0, t]$, $(\alpha, \mu) + t' \models V_A \land V_S$.
- $(\alpha, \mu) + t \models \tau_A(e) \land \tau_B(f)$.
- $((\alpha, \mu) + t)(e, f) = (\alpha', \mu')$.

Then a run of $\mathcal{A} \times \mathcal{B}$ can also be defined as a sequence

$$((\mu_0, \nu_0), (e_0, f_0), 0) \ldots ((\mu_k, \nu_k), (e_k, f_k), t_k) \ldots$$

with $(\mu_k, \nu_k) \xrightarrow{t_k + 1} (\mu_{k+1}, \nu_{k+1})$ for all $k \geq 0$.

Given a CTA $\mathcal{A} \times \mathcal{B}$ and an MF-assumption $\Phi \Psi$ of $\mathcal{A} \times \mathcal{B}$, $(\mathcal{A} \times \mathcal{B}, \Phi \Psi)$ is called a GCBTA (Generalized communicating BTA). Similarly, $(\mathcal{A} \times \mathcal{B}, \Phi \Psi)$ is a CFTA (Communicating BTA) if $|\Phi| + |\Psi| \leq 1$.

**B. Simulation of GCBTAs against an environment**

**Definition 8: Simulation of GCBTAs against an environment** A simulation $F^\epsilon$ of a model GCBTA $\langle \mathcal{M}, \Phi_M \Psi_M \rangle$ by a specification GCBTA $\langle \mathcal{S}, \Phi_S \Psi_S \rangle$ against an environment GCBTA $\langle \mathcal{E}, \Phi_E \Psi_E \rangle$ is a binary relation $F^\epsilon \subseteq \mathbb{S}(\mathcal{E} \times \mathcal{M}) \times \mathbb{S}(\mathcal{E} \times \mathcal{S})$ such that for every $(\alpha, \mu)(\beta, \nu) \in F^\epsilon$, the following restrictions are satisfied.

- **SE1:** $\alpha, \beta \in \mathbb{S}(\mathcal{E})$ with $\alpha = \beta$.
- **SE2:** $\mu \in \mathbb{S}(\mathcal{M})$.
- **SE3:** $\nu \in \mathbb{S}(\mathcal{S})$.
- **SE4:** For every run $\theta$ of $\mathcal{E} \times \mathcal{M}$ from $(\alpha, \mu)$ that satisfies $(\Phi_M \cup \Phi_E)(\Psi_M \cup \Psi_E)$, there exists a play $\rho$ from $(\alpha, \mu)(\beta, \nu)$ with the following restrictions.
  - $\rho$ embeds $\theta$ and satisfies $(\Phi_E \cup \Phi_M \cup \Phi_S)(\Psi_E \cup \Psi_M \cup \Psi_S)$.
  - For every transition $(e, f)(e', g)$ along $\rho, e = e' \in \mathbb{E}_E$.

We say that $\langle \mathcal{S}, \Phi_S \Psi_S \rangle$ simulates $\langle \mathcal{M}, \Phi_M \Psi_M \rangle$ against environment $\langle \mathcal{E}, \Phi_E \Psi_E \rangle$, in symbols

$$\langle \mathcal{M}, \Phi_M \Psi_M \rangle \xrightarrow{\epsilon} \langle \mathcal{S}, \Phi_S \Psi_S \rangle : \langle \mathcal{E}, \Phi_E \Psi_E \rangle$$

if there exists a simulation $F^\epsilon$ of $\langle \mathcal{M}, \Phi_M \Psi_M \rangle$ by $\langle \mathcal{S}, \Phi_S \Psi_S \rangle$ against $\langle \mathcal{E}, \Phi_E \Psi_E \rangle$ such that for every $(\alpha, \mu) \models I_\mathcal{E} \land V_\mathcal{E} \land I_\mathcal{M} \land V_\mathcal{M}$, there exists an $(\alpha, \nu) \models I_\mathcal{E} \land V_\mathcal{E} \land I_\mathcal{S} \land V_\mathcal{S}$ with $(\alpha, \mu)(\beta, \nu) \in F^\epsilon$. 

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As can be seen, definition 8 is more restrictive than definition 5 in their presentations. However, we can prove that they are equivalent.

Lemma 23: Given an environment GCBTA $\langle \mathcal{E}, \Phi_E \Psi_E \rangle$, a model GCBTA $\langle M, \Phi_M \Psi_M \rangle$, and a specification GCBTA $\langle S, \Phi_S \Psi_S \rangle$, $(\mathcal{E} \times M, (\Phi_E \cup \Phi_M)(\Psi_E \cup \Psi_M)) \times (\mathcal{E} \times S, (\Phi_E \cup \Phi_S)(\Psi_E \cup \Psi_S))$ iff $(\Phi_M, \Phi_M \times \Phi_M) \times (S, \Phi_S \Psi_S) : (\mathcal{E}, \Phi_E \Psi_E)$. 

Proof: The backward direction of the proof is straightforward since every simulation against an environment in definition 8 is also a simulation in definition 5. Thus we only have to focus on the forward direction of the proof. We first assume that there is a simulation $F$ of $(\mathcal{E} \times M, (\Phi_E \cup \Phi_M)(\Psi_E \cup \Psi_M))$ by $(\mathcal{E} \times S, (\Phi_E \cup \Phi_S)(\Psi_E \cup \Psi_S))$. We can construct $F^e$ as follows.

$$F^e \triangleq \{ (\alpha, \mu) : (\alpha, \nu) \in F \}.$$ 

Given an $(\alpha, \mu)(\beta, \nu) \in F$, it is apparent that $(\alpha, \mu)(\beta, \nu)$ satisfies conditions SE1, SE2, and SE3 of definition 8. Then for every runs $t$ of $\mathcal{E} \times M$ from $(\alpha, \mu)$ satisfying $(\Phi_M \cup \Phi_S)(\Psi_M \cup \Psi_S)$, there exists a trace $\rho$ from $(\alpha, \mu)(\beta, \nu)$ such that $\rho$ embeds $\theta$ and satisfies $(\Phi_E \cup \Phi_M \cup \Phi_S)(\Psi_E \cup \Psi_M \cup \Psi_S)$. Suppose

$$\rho = (\alpha_0, \mu_0)(\beta_0, \nu_0), (\alpha_0, \mu_0)(\epsilon'(0, g_0), t_0) \ldots (\alpha_k, \mu_k)(\beta_k, \nu_k), (\epsilon_k, f_k)(\epsilon'_k, g_k), t_k) \ldots$$

This implies the following for all $k \geq 0$.

$$\begin{align*}
\alpha_k & \xrightarrow{t \to t_k(\epsilon_k, f_k)} \alpha_{k+1} \quad (v1) \\
\mu_k & \xrightarrow{t \to t_k(\epsilon_k, f_k)} \mu_{k+1} \quad (v2) \\
\beta_k & \xrightarrow{t \to t_k(\epsilon'_k, g_k)} \beta_{k+1} \quad (v3) \\
\nu_k & \xrightarrow{t \to t_k(\epsilon'_k, g_k)} \nu_{k+1} \quad (v3)
\end{align*}$$

Then we can construct a sequence $\rho^e$ as follows.

$$\rho^e \triangleq (\alpha_0, \mu_0)(\alpha_0, \nu_0), (\alpha_0, \mu_0)(\epsilon'(0, g_0), t_0) \ldots (\alpha_k, \mu_k)(\alpha_k, \nu_k), (\epsilon_k, f_k)(\epsilon'_k, g_k), t_k) \ldots$$

We have the following two claims to prove the lemma.

CL1: $\rho^e$ is a play of $(\mathcal{E} \times M)$ by $(\mathcal{E} \times S)$ and embeds $\theta$. 

CL2: $\rho^e$ satisfies $(\Phi_E \cup \Phi_M \cup \Phi_S)(\Psi_E \cup \Psi_M \cup \Psi_S)$. 

Claim CL1 relies on the validity that for all $k \geq 0$.

$$\begin{align*}
\alpha_k & \xrightarrow{t \to t_k(\epsilon_k, f_k)} \alpha_{k+1} \quad (v4) \\
\mu_k & \xrightarrow{t \to t_k(\epsilon_k, f_k)} \mu_{k+1} \quad (v5) \\
\nu_k & \xrightarrow{t \to t_k(\epsilon'_k, g_k)} \nu_{k+1} \quad (v5)
\end{align*}$$

The validity of the above three then follows from statements $(v1)$, $(v2)$, and $(v3)$ in the above. Thus we know that $\rho^e$ is indeed a play of $(\mathcal{E} \times M) \times (\mathcal{E} \times S)$. Furthermore, the validity of statements $(v4)$ and $(v5)$ implies that $\rho^e$ indeed embeds $\theta$. 

Now we want to prove claim CL2. For all assumptions in $\Phi_E \cup \Phi_M$, and $\Psi_E \cup \Psi_M$, they are automatically satisfied since $\rho^e$ also embeds $\theta$ and $\theta$ satisfies $(\Phi_E \cup \Phi_M)(\Psi_E \cup \Psi_M)$. For a strong fairness assumption $\phi \in \Phi_S$, we have the following two cases to analyze.

- $\phi$ is a state-predicate. We claim that along $\rho^e$, for every $k > 0$, there exists an $h > k$ and a $t \in [0, t_{h+1} - t_h]$ with $((\alpha_h, \nu_h) + t \models \phi$. This is true since along $\rho$, $((\alpha_h, \mu_h)(\beta_h, \nu_h) + t \models \phi$ which implies that $\nu_h + t \models \phi$ which in turn implies the claim.
- $\phi = \eta_1 \eta_2$ is an event-predicate. We claim that along $\rho^e$, for every $k > 0$, there exists an $h > k$ with $((\alpha_h, \nu_h) + t_{h+1} - t_h \models \eta_1$, $a \in \epsilon_e(\epsilon'_h, g_{h+1})$, and $(\alpha_h, \nu_h) + t_{h+1} - t_h \models \eta_2$. This is true since along $\rho$, $((\alpha_h, \mu_h)(\beta_h, \nu_h) + t_{h+1} - t_h \models \eta_1$, $a \in \epsilon_e(\epsilon'_h, g_{h+1})$ and $(\alpha_h, \nu_h) + t_{h+1} - t_h \models \eta_2$. This further implies that $\nu_h + t_{h+1} - t_h \models \eta_1$, $a \in \epsilon_e(g_{h+1})$, and $\nu_h + t_{h+1} \models \eta_2$. In the end, this implies the claim.

For a weak fairness assumption $\psi \in \Psi_S$, we have the following two cases to analyze.

- $\psi$ is a state-predicate. We claim that there exists a $k > 0$ such that for every $h > k$ and $t \in [0, t_{h+1} - t_h]$, $((\alpha_h, \nu_h) + t \models \psi$. This is true since along $\rho$, $((\alpha_h, \mu_h)(\beta_h, \nu_h) + t \models \psi$ which implies that $\nu_h + t \models \psi$ which in turn implies the claim.
- $\psi = \eta_1 \eta_2$ is an event-predicate. We claim that along $\rho^e$, there exists a $k > 0$ such that for all $h > k$, if $((\alpha_h, \nu_h) + t_{h+1} - t_h \models \eta_1$, and $a \in \epsilon_e(\epsilon'_h, g_{h+1})$, then $(\alpha_h, \nu_h) + t_{h+1} \models \eta_2$. This is true since along $\rho$, $((\alpha_h, \mu_h)(\beta_h, \nu_h) + t_{h+1} - t_h \models \eta_1$, and $a \in \epsilon_e(\epsilon'_h, g_{h+1}) = \epsilon_e(\epsilon_h, f_{h+1})$, then $(\alpha_h, \mu_h)(\beta_h, \nu_h) + t_{h+1} \models \eta_2$. This further implies that $\nu_h + t_{h+1} - t_h \models \eta_2$, and $a \in \epsilon_e(\epsilon'(h+1), g_{h+1})$, then $\nu_h + t_{h+1} \models \eta_2$. In the end, this implies the claim.

With the proof of claims CL1 and CL2, thus we conclude that the lemma is proven.

According to lemma 23 we can check the classic simulation in definition 5 by checking the one in definition 8. This can be helpful in enhancing the verification performance when the common environment between the model and the specification is non-trivial.

C. Efficiency techniques for simulation against an environment

Lemma 23 implies that we can use the following techniques to enhance the simulation algorithm against an environment.

- Based on condition SE1 of definition 5 we significantly reduce the sizes of the spaces of state-pairs by disregarding state-pairs of the form $(\alpha, \mu)(\beta, \nu)$ with $\alpha \neq \beta$. Since the number of different zones representing $\beta$’s can be exponential to the input size, the reduction can result in exponential speed-up.
- By mapping variables in $\beta$ in state-pairs $(\alpha, \mu)(\beta, \nu)$, to those in $\alpha$, we actually only have to record one copy of values for each variables in $\alpha$. Since the size of BDD-like diagrams 5 is exponential to the number of variables, this technique can also significantly reduce the memory usage in representations with BDD-like diagrams.
- In evaluating the precondition of state-pairs, we need to enumerate all the transition pairs of the form $(e, f)(e', g)$ with $e, e' \in E_E$, $f \in E_M$, and $g \in E_S$.
use the classic simulation, the enumeration is of size \(O(|E_2|^2 \cdot |E_M| \cdot |E_S|)\). But with the simulation against a common environment in definition 8 the enumeration is of size \(O(|E_2| \cdot |E_M| \cdot |E_S|)\). Thus significant reduction in time and space complexity can also be achieved with definition 8.

VIII. IMPLEMENTATION

We have implemented the techniques proposed in this manuscript in RED 8, a model/simulation-checker for CTAs and parametric safety analysis for LHAs based on CRD (Clock-Restriction Diagram) 24 and HRD (Hybrid-Restriction Diagram) technology 26. The state-pair spaces are explored in a symbolic on-the-fly style. To our knowledge, there is no other tool that supports fully automatic simulation checking with GBTAs.

We used parameterized networks of processes as our benchmarks. For a network of \(m\) processes, we use integer 1 through \(m\) to index the processes. Users supply two index lists, the first for the indices of the model processes and the second for indices of the specification processes. The process indices not in the two lists are treated as indices of the environment processes. For example, we may have a system of 10 processes. The following describes a simulation-checking task of process 1 (the model) by process 1 (the specification).

\[
\text{1;2;}
\]

Here processes 3 through 10 are the environment processes.

To support convenience in presenting fairness assumptions, we allow parameterized expressions. For example, in table III(a), we have a simulation requirement with parameterized strong fairness assumptions. Here \#PS is a parameter for the number of processes. Thus for a system of 10 processes, process 9 is the model, process 10 is the specification, while the others are the environment. The last \texttt{assume} statement is for the fairness assumption of the environment. The specification of event-predicates is in the following form.

\[
\text{type } [\eta_1] \text{ a } [\eta_2]
\]

Here type is either 'strong' or 'weak.' \([\eta_1]\) and \([\eta_2]\) are respectively the optional precondition and the optional postcondition. We may also use quantified expressions to present several fairness assumptions together. For example, in the above,

\[
\text{assume} \{ |k:2..\#PS-2,} \text{ strong true event } \{\text{execute@}(k)\};
\]

presents the following strong fairness assumptions.

\[
\text{strong true event } \{\text{execute@}(2)\}
\]
\[
\text{strong true event } \{\text{execute@}(3)\}
\]
\[
\ldots \ldots
\]
\[
\text{strong true event } \{\text{execute@}(8)\}
\]

IX. EXPERIMENTS

To our knowledge, there is no other tool that supports fully automatic simulation checking with fairness assumptions for TAs as ours. So we only experimented with our algorithms.

We report two experiments. The first is for timed branching simulation against a common environment without fairness assumptions in subsection IX-A. Especially, we report the performance enhancement of the simulation in definition 8 (without fairness assumption) over the simulation in definition 5.

The second experiment is for simulation against a common environment with fairness assumptions in subsection IX-B. Especially, we use liveness properties in the experiment.

A. Report of timed branching simulation

We used the following three parameterized benchmarks from the literature.

1. Fischer's timed mutual exclusion algorithm 24: The algorithm relies on a global lock and a local clock per process to control access to a critical section. Two timing constants used are 10 and 19.
2. CSMA/CD 43: This is the Ethernet bus arbitration protocol with collision-and-retry. The timing constants used are 26, 52, and 808.
3. Timed consumer/producer 21: There is a buffer, some producers, and some consumers. The producers periodically write data to the buffer if it is empty. The consumers periodically wipe out data, if any, in the buffer. The timing constants used are 5, 10, 15, and 20.

For each benchmark, we use one model process and one specification process. All the other processes are environment. Also for each benchmark, two versions are used, one with a simulation and one without. For the versions with a simulation, \(M\) and \(S\) are identical. For the version without, \(M\) and \(S\) differ in only one process transition or invariance condition. For example, for the Fischer’s benchmark, the difference is that...
TABLE IV: PERFORMANCE DATA OF SCALABILITY W.R.T. VARIOUS STRATEGIES

| benchmarks                  | versions | Definition 5 | Definition 8 |        |        |
|-----------------------------|----------|--------------|--------------|-------|-------|
|                             |          | m | time  | memory  | m | time  | memory  |
| Fischer’s mutual exclusion  | Simulation exists. | 4 | > 1800s | > 8M     | 5 | 31.3s | 320k   |
| (m processes)               | No simulation exists. | 6 |         |          | 6 | 281s  | 1319k  |
| CSMA/CD (1 bus+ m senders)  | Simulation exists. | 1 | 0.236s | 102k     | 2 | 0.098s | 41k    |
|                             | No simulation exists. | 3 | > 1800s | > 700M    | 3 | 12.3s  | 5303k  |
| Consumer & producer (1 buffer +1 producer +m consumers) | Simulation exists. | 3 |         |          | 4 | 0.30s  | 57k    |
|                             | No simulation exists. | 4 |         |          | 5 | 0.43s  | 63k    |
|                             |            | 5 |         |          | 6 | 0.53s  | 75k    |

The CPU time used and the total memory consumption for the data-structures in state-space representations are reported. As can be seen, the performance of our new simulation (definition 8) against a common environment is significantly better than the classic one (definition 5).

B. Report of simulation with fairness assumptions

We use a network of TAs as our benchmarks for liveness property verification. A network consists of m process TAs. Process 1 is a dispatcher process. Processes 2 through m − 1 are the environment processes. Process m is the model and process m + 1 is for the specification. The execution of a process depends on the incoming services by its peer processes. In figure 4 we draw example topologies of networks: linear, binary-tree, and irregular. The nodes represent the processes while the arcs represent service channels. Inside each node, we put down the name of the TA for the process. Note that the model (process m) and the specification (process m + 1) have the same channel connections to the other processes.

The connection relation of the service channels is given in a 2-dimensional Boolean array serve. For the linear networks, serve(i, j) is true iff i ∈ [2, m − 1] and j = i + 1. For the binary-tree networks, serve(i, j) is true iff j/2 = i with integer division. For the irregular networks, for all i, j ∈ [2, m], serve(i, j) is true iff (i+prime(i)%8)+prime(j%8)) is divisible by 7 where prime(i) is the ith prime and ‘%’ is the remainder operator. For example, in figure 4(c), processes 7 and 8, respectively the model and the specification, are served by both processes 4 and 5. Process 6 is only served by itself.

Templates of the state transition graphs of the processes can be found in figure 5. Figure 5(a) is the TA for the dispatcher process. Specifically, the dispatcher works as a scheduler that sends out execution signal, exec, to the other processes.

![](image_url)

Fig. 4. Network topologies of processes
Fig. 5. TA templates in a network of \( m \) processes

As can be seen from the performance data, our techniques show promise for the verification of fulfillment of liveness properties in concurrent computing.

X. CONCLUDING REMARKS

In this work, we investigate the simulation problem of TAs with multiple strong and weak fairness assumptions. For the succinct presentation of fairness assumptions, we also allow for event fairness properties. We then present an algorithm for the USF-simulation of GBTAs. The algorithm is based on symbolic model-checking and simulation-checking techniques and can be of interest by itself. We then propose a new simulation against a common environment between the model and the specification. We then present efficiency techniques for this new simulation. Our implementation and experiment shows the promise that our algorithm could be useful in practice in the future.
TABLE V

| benchmarks     | service by all incomings | m environment processes | service by one incoming |
|----------------|--------------------------|-------------------------|-------------------------|
|                | service by all incomings | m environment processes | service by one incoming |
|                | strong       | weak       | strong       | weak       | strong       | weak       |
|                | time         | memory     | time         | memory     | time         | memory     |
| linear networks| 1.16s        | 67k        | 0.87s        | 67k        | 0.44s        | 48k        | 0.44s        | 48k        |
|                | 1.46s        | 122k       | 1.88s        | 122k       | 0.98s        | 39k        | 0.98s        | 39k        |
|                | 2.03s        | 191k       | 4.22s        | 192k       | 0.93s        | 158k       | 1.14s        | 159k       |
|                | 2.08s        | 281k       | 9.70s        | 281k       | 1.46s        | 244k       | 1.46s        | 244k       |
|                | 3.46s        | 393k       | 20.3s        | 393k       | 1.50s        | 359k       | 1.48s        | 359k       |
|                | 6.22s        | 28.3M      | 43.46s       | 28.1M      | 1.91s        | 508k       | 2.24s        | 508k       |

For each benchmarks, there are a model process, a specification process, and data collected on a Pentium 4 1.7GHz with 380MB memory running LINUX;

s: seconds; k: kilobytes of memory in data-structure; M: megabytes of total memory

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