Introduction: Quarkonium suppression in relativistic heavy-ion collisions has been the subject of numerous theoretical and experimental studies since the pioneering work of Matsui and Satz \cite{1}, who proposed the phenomenon as a signature of quark-gluon plasma formation in the early stages of the collisions. While the $J/\psi$ is expected to melt away at sufficiently high temperature above the quark-hadron transition temperature $T_c$, its exact melting point, thermal width and mass shift above the transition temperature have been the focal point of intense theoretical studies often with varying results \cite{2,3,4,5}. These properties are essential inputs to quantitatively understand the charmonium suppression in heavy ion collisions (see, for instance, the recent review in Ref. \cite{6}) and are also related to the transport coefficients for heavy quarks near high temperatures \cite{7}.

From a potential model point of view, obtaining the correct mass shift and width is respectively related to identifying the correct real and imaginary parts of the potential to be used for the heavy quark system near $T_c$ \cite{8,9,10}. Employing the free-energy potential, the $J/\psi$ dissolves around $1.1T_c$, whereas it survives up to a higher temperatures when the internal energy is used \cite{11}. As for the imaginary part of the potential, even though the high temperature limit can be calculated using the hard-thermal-loop (HTL) resummed perturbation theory \cite{12}, applying the same formula near $T_c$ might be problematic. Lattice QCD will in all likelihood eventually be used to calculate the complex potential \cite{13} and the corresponding properties of quarkonium near $T_c$, but the present uncertainties in the imaginary potential appear to be still large \cite{14}.

Some of us have previously used QCD sum rules to calculate the mass shift of $J/\psi$ near $T_c$ \cite{15,16}. The advantage of this approach is that the temperature dependence of the operator product expansion (OPE) for the charmonium current correlator can be reliably obtained near $T_c$ \cite{17} through lattice calculations of the energy momentum tensor. Unfortunately, in contrast to the vacuum case, the $J/\psi$ pole in the spectral density will acquire a thermal width above $T_c$ so that the changes in the OPE can either be related to a mass change, an increase in the width \cite{19}, or a combination of these effects. Nevertheless, it was shown that this approach leads to a constraint for the changes of mass $\delta m$ and width $\Gamma$ for the $J/\psi$ at each temperature, which can be approximated as $\Gamma = \Gamma_0 + \delta \Gamma$.

It is important to note, however, that the overlap of the charmonium current with the $J/\psi$ appears as the strength of the ground state pole in the QCD sum rule approach and can be reliably determined at each temperature. At the same time, the overlap strength can be identified as the charmonium wave function at the spatial origin, which is sensitive to the potential through the normalization condition. Comparing the temperature dependencies of the overlap strength obtained from QCD sum rules to those obtained by solving the Schrödinger equation for a given potential, three of us were able to show that the potential for the charmonium system at short distance should be dominated by the free energy near $T_c$ \cite{17}, while at larger separation distance the potential will have a fraction of about 20% of the internal energy \cite{20}, hence called a transitional potential.

Given the real part of the potential, one can add an effective imaginary part that is composed of the leading order perturbative form derived in Ref. \cite{12}, multiplied by a free parameter $K$. Solving the Schrödinger equation with this potential, one obtains the corresponding real and imaginary eigenvalues, which can be related to $\delta m$ and $\Gamma$ for each value of $K$. This leads to another constraint equation involving these two quantities near $T_c$. 

$J/\psi$ near $T_c$

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We calculate the mass shift and thermal decay width of the $J/\psi$ near the QCD transition temperature $T_c$ by imposing two independent constraints on these variables that can be obtained first by solving the Schrödinger equation and second from the QCD sum rule approach. While the real part of the potential is determined by comparing the QCD sum rule result for charmonium and the D meson to that from the potential model result, the imaginary potential is taken to be proportional to the perturbative form multiplied by a constant factor, which in turn can be determined by applying the two independent constraints. The result shows that the binding energy and the thermal width becomes similar in magnitude at around $T = 1.09T_c$, above which the sum rule analysis also becomes unstable, strongly suggesting that the $J/\psi$ will melt slightly above $T_c$. 

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By combining the abovementioned two independent constraints among $\delta m$ and $\Gamma$, it becomes possible to identify the mass shift and thermal width separately for the $J/\psi$ near $T_c$, which is known to be a non-perturbative and strongly interacting region. Furthermore, the result allows us to quantify the strength of the imaginary potential for the heavy quark system near $T_c$.

$J/\psi$ from heavy quark potential: The strong interaction between a heavy quark and its anti-quark in our approach is modeled as a combination of the free energy obtained from lattice QCD calculations and the internal energy derived from thermodynamic relations. Three of us recently found that the long distance behavior of the heavy quark real potential is composed of 80% of the free energy and 20% of the internal energy in order to reproduce the $D$ meson mass near $T_c$ from QCD sum rules, while the short distance behavior should be close to the free energy potential. The behavior of such a potential can be parametrized as

$$V_R(r, T) = V_S(r, T) + \{V_L(r, T) - V_S(r, T)\} \times \frac{\tanh[(r - r_0)/\delta] + 1}{2},$$

where $V_S(r, T) = F(r, T)$ and $V_L(r, T) = 0.8 F(r, T) + 0.2 U(r, T)$ are the short and long distance potentials, respectively. The potential transits from $V_S$ to $V_L$ at $r_0 = 1$ fm with the width $\delta = 0.25$ fm. Changing $r_0$ and $\delta$ within our constraints on the overlap of the wave function at the origin, we find that our final result on the dissociation temperature do not change significantly. We adopt the imaginary heavy quark potential $V_I(r, T)$ calculated in HTL resumed perturbation theory [12]:

$$V_I(r, T) = -\frac{g^2 T C_F}{4\pi} \phi(m_D r),$$

where $g^2$ and $m_D$ are respectively the strong coupling constant squared and Debye mass parametrized in [23], and

$$\phi(x) = 2 \int_0^\infty \frac{dz z}{(z^2 + 1)} \left[ 1 - \frac{\sin(zx)}{zx} \right],$$

which has the limiting values $\phi(0) = 0$ and $\phi(\infty) = 1$. We have taken $V(r, T) = V_R(r, T) + i K V_I(r, T)$, where the multiplicative constant $K$ is varied from 0 to 4. The heavy quark potential is then used in the Schrödinger equation of a charm and anti-charm pair as

$$\left[2m_c - \frac{1}{m_c} \nabla^2 + V(r, T)\right] \psi(r, T) = M^{J/\psi}(T) \psi(r, T) \delta,$$

where $m_c = 1.25$ GeV is the bare charm quark mass, $\psi(r, T) = \psi_R(r, T) + i\psi_I(r, T)$ and $M^{J/\psi}(T) = M^J_{R}(T) + i M^I_{R}(T)$ the charmonium wave function and mass, respectively, at temperature $T$. Fig. 1 shows the binding energy $\epsilon_R = 2m_c + V_R(\infty, T) - M^{J/\psi}_{R}$ and imaginary eigenvalues obtained by numerically solving the coupled real and imaginary parts of the OPE with $K$ integers shown by different symbols. As one can see, the binding energies have only weak dependence on the strength of the imaginary potential, while the imaginary eigenvalue scales with the multiplicative strength of the imaginary potential. Assuming that the strength of the imaginary potential is not known, we can obtain a constraint equation between the mass shift $\delta m = M^J_{R}(T) - M^J_{I}(0)$ and the thermal width $\Gamma = M^I_{R}(T)$ at a given temperature. The results are shown as thin lines with filled circles in Fig. 2.

$J/\psi$ from QCD sum rules: The details of the sum rule calculations of this work are adapted from Ref. [20]. One starts with the Borel transformed energy dispersion relation for the invariant part of the correlator of the operator $j^\mu(x) = \bar{c}(x)\gamma^\mu c(x)$,

$$\tilde{\Pi}(M^2, T) = \int_0^\infty dse^{-s/M^2} \rho(s, T),$$

where $\tilde{\Pi}(M^2, T)$ represents the Borel transformed operator product expansion (OPE) of the correlator and $M$ stands for the Borel mass. $\rho(s, T)$ is the vector channel spectral function at temperature $T$. For the charmonium sum rule in medium, the OPE is well determined by the temperature dependencies of dimension 4 gluon operators through lattice calculations [27, 28] of the energy density $\epsilon(T)$ and pressure $p(T)$ [24]. Such a description of the OPE is valid up to temperatures slightly above $T_c$, where the temperature corrections are smaller than the vacuum values [13] and the contribution from higher dimensional operators are small [15].

The behavior of the spectral function is assumed to
have the following pole and continuum contribution
\[
\rho_{\text{pole}}(s, T) = \frac{1}{\pi} \frac{f \Gamma \sqrt{s}}{(s - m^2)^2 + s^2/T^2}, \quad s > 4m^2, \quad \text{(6)}
\]
\[
\rho_{\text{cont}}(s, T) = \frac{1}{\pi} \theta(s - s_0) \text{Im} \tilde{\Pi}^{\text{pert}}(s), \quad \text{(7)}
\]
where the perturbative spectral function \(\text{Im} \tilde{\Pi}^{\text{pert}}(s)\) is given for instance in Appendix B of Ref. [22].

To eliminate the dependence on the residue \(f\), which itself was critical in determining the potential at short distance [22], we consider the ratio
\[
\frac{\rho_{\text{pole}}(s, T)}{\rho_{\text{cont}}(s, T)} = \int_{4m^2}^\infty ds \frac{s/M^2}{e^{-s/M^2}} \rho_{\text{pole}}(s, T)
\]
\[
\int_{4m^2}^\infty ds \frac{s/M^2}{e^{-s/M^2}} \rho_{\text{pole}}(s, T) = \frac{1}{\pi} \frac{f \Gamma \sqrt{s}}{(s - m^2)^2 + s^2/T^2}, \quad \text{(8)}
\]
with
\[
\tilde{\Pi}^{\text{cont}}(M^2, T) = \int_0^\infty ds \frac{s}{e^{-s/M^2}} \rho_{\text{cont}}(s, T). \quad \text{(9)}
\]

For given \(s_0\) and \(\Gamma\), Eq. (8) can be solved for the \(m(M^2, T)\), appearing in Eq. (6), as a function of the Borel mass \(M^2\). The admissible range of \(M^2\) is fixed by the so-called Borel window. The minimum value \(M^2_{\text{min}}\) is determined by the condition that the dimension 4 OPE terms are smaller than 30% of the total OPE expression, the maximum \(M^2_{\text{max}}\) from the condition that the continuum term contributes less than 30% of the perturbative term to the sum rule.

Next, we define
\[
\chi^2 = \frac{1}{M^2_{\text{max}} - M^2_{\text{min}}} \int_{M^2_{\text{min}}}^{M^2_{\text{max}}} dM^2 \left[m(M^2, T) - m(M^2_0, T)\right]^2, \quad \text{(10)}
\]
where \(M^2_0\) is the value of \(M^2\), at which the derivative of \(m(M^2, T)\) by \(M^2\) vanishes: \(dm(M^2, T)/dM^2|_{M^2=M^2_0} = 0\). \(m\) is determined as \(m(M^2_0, T)\) and \(\Gamma\) is chosen such that \(\chi^2\) is minimal. Hence, starting from three independent parameters \(m, \Gamma, s_0\), Eq. (8) and Eq. (10) effectively lead to a constraint among \(\delta m\) and \(\Gamma\). The obtained constraint relations [26] are shown as solid thick lines without symbols in Fig. 2.

Matching the heavy quark potential to QCD sum rules:

Let us now study the consequences of combining the two independent constraints for \(\delta m\) and \(\Gamma\). In Fig. 3 we respectively plot \(\delta m\) and \(\Gamma\) from the intersection points of the two constraints given in Fig. 2. One notes that \(|\delta m| \sim \Gamma\) at \(T = 1.08 T_c\), a bit below the highest temperature considered in this work. Also plotted in Fig. 3 are results from a recent extraction of \(\delta m\) and \(\Gamma\) from lattice QCD data [28]. One notes that the temperature dependence of \(\delta m\) are within the bounds of the lattice results. For \(\Gamma\), our results show a stronger temperature dependence than those of Ref. [29], with a comparable magnitude just above \(T_c\).

From the intersection points, we can moreover determine the strength of the imaginary potential. As can be seen in Fig. 2, the multiplicative constant \(K\) starts from 0 at \(T = 0.98 T_c\) and rises to about 4 at \(T = 1.09 T_c\), demonstrating the highly non-perturbative behaviour near \(T_c\).

Combining the above observations and Fig. 1 one sees that the thermal width of the \(J/\psi\) exceeds the binding energy above 1.1\(T_c\), suggesting that that the dissociation will occur there because the \(J/\psi\) will lose its identity when the width becomes sufficiently larger than the binding energy [30]. This is consistent with the fact that the dissociation will occur around 1.1\(T_c\) when the free energy potential is used.

Within the potential nonrelativistic QCD (pNRQCD) approach, related transport coefficients can be obtained through the relation \(\delta m(1S) = 2a_0^2 \gamma\) and \(\Gamma(1S) = 3a_0^2 \kappa\), where \(\kappa\) is the diffusion coefficient of a heavy quark in the medium, \(\gamma\) its dispersive counterpart and \(a_0\) the Bohr radius. Assuming \(a_0 = 0.23\) fm and \(T_c = 155\) MeV, we find that our data yield \(-10.78 \leq \gamma/T^3 \leq -5.63\) and \(0 \leq \kappa/T^3 \leq 6.97\) for \(T_c \leq T \leq 1.09 T_c\), which is close to the findings of several methods summarized in Ref. [7].

In Fig. 4 we also plot the thermal width for \(J/\psi\) using a formula recently derived by a partonic description that includes the HTL resummation and reduces to low energy theorems obtained by pNRQCD in the relevant kinematical limit [31] so that it can be used in a wide temperature range applicable to heavy quark systems in heavy ion collisions [32]. The inputs in the calculations are the effective Bohr radius \(a_0^2 = \frac{1}{3} \int \frac{d^3 p}{(2\pi)^3} |\nabla \psi(p)|^2\) and the bind-
FIG. 3. (Color online) Temperature dependence of $J/\psi$ mass and width. The blue lines and surrounding shaded regions are adapted from Ref. [29]. The violet (orange) lines with filled rectangles (circles) are the results of this work, obtained from the intersection points of Fig. 2 (partonic HTL resummation).

Summary and Conclusions: Combining the two constraint equations for $\delta m$ and $\Gamma$ discussed in this work, we are now finally able to separately identify the temperature dependencies of these observables near $T_c$, showing that the $J/\psi$ will melt slightly above $T_c$. We also identified the strength of the imaginary potential for the heavy quark- anti-quark system near $T_c$, which is found to have a large multiplicative factor of about 4 relative to the leading perturbative results at $1.09 T_c$, which lies in the highly non-perturbative temperature domain. Our results are within the limits of a recent lattice calculation [29], and the thermal width is close to that obtained by using a partonic picture with resummed pQCD [32]. They are based on stringent constraints from two independent methods and form reliable nonperturbative results for the properties of $J/\psi$ as well as the real and imaginary potential near $T_c$ and will thus provide valuable input to our understanding of the heavy quark system in heavy ion collisions.

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