Wilson Lines Near the Light Cone

Hans J. Pirner\textsuperscript{a,b}

\textsuperscript{a} Institut für Theoretische Physik der Universität, Philosophenweg 19, D-69120 Heidelberg, Germany
\textsuperscript{b} Max-Planck-Institut für Kernphysik, Postfach 103980, D-69029 Heidelberg, Germany

Abstract

We study the dynamics of Wilson lines near the light cone in QCD. Lattice simulations of the near light cone Hamiltonian in $SU(2)$ show that the correlation mass at strong coupling vanishes at intermediate coupling, which signals a continuum transition on the light cone. We point out the possible relevance of this mass for high energy scattering.

1 Introduction

Many empirical data point towards a nontrivial vacuum structure in QCD. The question arises what is the effect of vacuum condensates on the dynamics near the light cone. High energy reactions have a very elegant formulation in light cone coordinates which present an enormous simplification, by reducing the number of relevant diagrams in perturbation theory. In recent years the extension of light cone theory to a calculation of wave functions and bound state spectra, i.e. soft physics has been presented in previous light cone conferences and reviews [1]. In light cone theory a trivial vacuum is the starting basis. Naively spoken, with only positive momentum fractions complicated many particle configurations cannot be contained in the vacuum. In this theory on the light cone with a trivial vacuum, the difficult object is not the vacuum, but the Hamiltonian itself which may contain terms beyond the naive Lagrangian which result from the elimination of one half of the Hilbert space, namely the negative energy states.

We propose an approach [2,3] which starts near the light cone, preserving the nonperturbative features of QCD, but aims at the derivation of an effective light cone Hamiltonian in a controlled limiting procedure. The Cauchy problem is well defined in near light cone coordinates, since the initial data are given
on a space-like surface. Such a formulation avoids the solution of constrained equations, which on the quantum field theoretic level may be very complicated. The problem of a nontrivial vacuum appears in a solvable form related to the transverse dynamics. This is physically very appealing, since in high energy reactions the incoming particles propagate near the light cone and interact mainly through particle exchanges with transverse momenta.

Near light cone QCD has a nontrivial vacuum which cannot be neglected even in the light cone limit. Generally, if the zero mode theory has massive excitations, then these masses diverge like $\frac{1}{\eta}$ in the light cone limit $\eta \to 0$. Massless excitations in the zero mode theory, however, do not decouple in the light cone limit. Genuine nonperturbative techniques must be used to investigate the behaviour of this limit. In principle the additional parameter which labels the coordinate system can be chosen arbitrarily. The zero mode Hamiltonian depends on an effective coupling constant containing this parameter and evolves towards an infrared fixed point. We choose the following near light cone coordinates which smoothly interpolate between the Lorentz and light front coordinates:

$$
\begin{align*}
    x^t &= x^+ = \frac{1}{\sqrt{2}} \left\{ \left( 1 + \frac{\eta^2}{2} \right) x^0 + \left( 1 - \frac{\eta^2}{2} \right) x^3 \right\}, \\
    x^- &= \frac{1}{\sqrt{2}} \left( x^0 - x^3 \right).
\end{align*}
$$

(1)

The transverse coordinates $x^1, x^2$ are unchanged; $x^t = x^+$ is the new time coordinate, $x^-$ is a spatial coordinate. As finite quantization volume we will take a torus and its extension in spatial “$-$” direction, as well as in “1, 2” direction is $L$. The scalar product of two 4-vectors $x$ and $y$ is given with $\vec{x}_\perp \vec{y}_\perp = x^1 y^1 + x^2 y^2$ as

$$
\begin{align*}
    x_{\mu} y^\mu &= x^- y^+ + x^+ y^- - \eta^2 x^- y^- - \vec{x}_\perp \vec{y}_\perp \\
    &= x_- y_+ + x_+ y_- + \eta^2 x_+ y_+ - \vec{x}_\perp \vec{y}_\perp.
\end{align*}
$$

(2)

Obviously, the light cone is approached as the parameter $\eta$ goes to zero. The gauge fixing procedure in the modified light-cone gauge $\partial_- A_- = 0$ allows zero modes dependent on the transverse coordinates only. These zero mode fields carry zero linear momentum $p_-$ in near light cone coordinates, but finite amount of $p_0 + p_3$. They correspond to “wee” or low $x$- partons in the language of Feynman. In color $SU(2)$ the zero-mode fields $a_3^\pm(x_\perp)$ can be chosen color diagonal proportional to $\tau^3$. The use of an axial gauge is very natural for the light-cone Hamiltonian even more so than in the equal-time Hamiltonian. The asymmetry of the background zero mode naturally coincides with the asymmetry of the space coordinates on the light cone. The zero-mode fields describe disorder fields. Depending on the effective coupling the zero mode transverse
system will be in the massive or massless phase. Our main objective has been to derive the precise relation between the nearness to the light cone $\eta$ and the transverse resolution $a$. In principle these are two independent parameters. A considerable simplification can be achieved if the limit towards the light cone is synchronized with the continuum limit.

2 Near Light Cone QCD Hamiltonian

In ref. [2] we have derived the near light cone Hamiltonian. We refer to this paper for further details. The light cone Hamiltonian on the finite light like $x^-$ interval of length $L$ has Wilson line or Polyakov operators similarly to QCD formulated on a finite interval in imaginary time at finite temperature.

$$P(\vec{x}_\perp) = \frac{1}{2} \tr \text{P} \exp \left( ig \int_0^L dx^- A_-(\vec{x}_\perp, x^-) \tau^3 / 2 \right).$$  \hfill (3)

The underlying Hamiltonian governing the dynamics of these Polyakov operators is a part of the full Hamiltonian $H$:

$$H = \int d^3x H(\vec{x}),$$  \hfill (4)

with

$$H = \tr \left[ \partial_1 A_2 - \partial_2 A_1 - ig [A_1, A_2] \right]^2 + \frac{1}{\eta^2} \tr \left[ \vec{\Pi}_\perp - \left( \partial_- \vec{A}_\perp - ig[a_-, \vec{A}_\perp] \right) \right]^2$$

$$+ \frac{1}{\eta^2} \tr \left[ \frac{1}{L} \vec{e}^3 - \vec{\nabla} a_- \right]^2 + \frac{1}{2L^2} p^+(\vec{x}_\perp) p^-(\vec{x}_\perp)$$

$$+ \frac{1}{L^2} \int_0^L dz^- \int_0^L dy^- \sum_{p,q,n} \left[ \frac{2\pi n}{L} + g(a_-,q(\vec{x}_\perp) - a_-,p(\vec{x}_\perp)) \right] e^{2\pi i (z^- - y^-) / L}. \hfill (5)$$

The prime indicates that the summation is restricted to $n \neq 0$ if $p = q$. The operator $G_\perp$ gives the right hand side of Gauss's law:

$$G_\perp(\vec{x}) = \vec{\nabla} \vec{\Pi}_\perp(\vec{x}) + g f^{abc} \frac{\lambda^a}{2} \vec{A}_\perp^b(\vec{x}) \left( \vec{\Pi}_\perp^c(\vec{x}) - \frac{1}{L} \vec{e}^3(\vec{x}_\perp) \right) + g \rho_m(\vec{x}), \hfill (6)$$

with $\rho_m$ the matter density. The above Hamiltonian shows rather clearly that a naive limiting procedure $\eta \to 0$ does not work. There are severe divergencies in this limit. The diverging terms reappear in the usual light cone Hamiltonian as constraint equations which are extremely difficult to solve on the
quantum level in $3 + 1$ dimensions. The zero mode part of the full Hamiltonian is coupled to the three-dimensional modes in the rest of the Hamiltonian. In the following we will concentrate on universal properties of the zero mode Hamiltonian which will survive the renormalization of the $(2 + 1)$ transverse dynamics due to the coupling to the $(3 + 1)$ dimensional rest. The zero mode Hamiltonian contains the Jacobian $J(a_-)$ which takes into account the Haar measure of $SU(2)$ $J(a_-(\vec{x}_\perp)) = \sin^2 \left( \frac{gL_2}{2} a_-(\vec{x}_\perp) \right)$. It stems from the gauge fixing procedure, effectively introducing curvilinear coordinates. It also appears in the functional integration volume element for calculating matrix elements. It is convenient to introduce dimensionless variables

$$\varphi(\vec{x}_\perp) = \frac{gL a_- (\vec{x}_\perp)}{2}, \quad (7)$$

which vary in a compact domain $0 \leq \varphi \leq \pi$. We regularize the above Hamiltonian $h_{\text{red}}$ by introducing a lattice constant $a$ on the lattice in the transverse directions. Next we appeal to the physics of the infinite momentum frame and factorize the reduced true energy from the Lorentz boost factor $\gamma = \sqrt{2/\eta}$ and the cut-off by defining $h_{\text{red}}$

$$h = \frac{1}{2\eta a} h_{\text{red}}. \quad (8)$$

For small lattice spacing we obtain the reduced Hamiltonian

$$h_{\text{red}} = \sum_{\vec{b}} \left\{ -g_{\text{eff}}^2 \frac{1}{J} \frac{\delta}{\delta \varphi(\vec{b})} J \frac{\delta}{\delta \varphi(\vec{b})} + \frac{1}{g_{\text{eff}}^2} \sum_\epsilon \left( \varphi(\vec{b}) - \varphi(\vec{b} + \vec{\epsilon}) \right)^2 \right\}. \quad (9)$$

with the effective coupling constant

$$g_{\text{eff}}^2 = \frac{g^2 L \eta}{4a}. \quad (10)$$

In the continuum limit of the transverse lattice theory we let $a$ go to zero. In ref. [3] we have done a Finite Size Scaling (FSS) [4] analysis obtaining a second order transition between a phase with massive excitations at strong coupling and a phase with massless excitations in weak coupling. The critical coupling is calculated as $g^* = 0.17 \pm 0.03$, which is, however, not a universal quantity and subject to renormalization from the $(3 + 1)$ dimensional modes. A calculation in the epsilon expansion [5] gives the zero of the $\beta$-function as an infrared stable fixed point. Therefore the limit of large transverse and longitudinal dimensions $L$ is well defined. Using the running coupling constant

$$g_{\text{eff}}^2 = g^2 + \left( \xi_0 g^2 \right)^{1/\nu} \left( \frac{a}{L} \right)^{1/\nu}, \quad (11)$$
the lattice constant over the correlation length \( L \) approaches zero at the critical coupling:

\[
\frac{a}{L} = \frac{1}{\xi_0 g^{*2}} \left( g_{\text{eff}}^2 - g^{*2} \right)^\nu,
\]

with \( \nu = 0.56 \pm 0.05 \). If we infer in addition that the coupling to the three-dimensional modes will produce the usual running gauge coupling \( g^2 \) of \( SU(2) \) QCD: 

\[
g^2 = \frac{g_0^2}{\log(L/a)},
\]

then we can synchronize the approach to the light cone, i.e. the limit \( \eta \to 0 \) with the continuum limit \( a \to 0 \). The condition that the three-dimensional evolution of \( g^2 \) has to be compatible with the two-dimensional evolution of \( g_{\text{eff}}^2 \) yields that for \( a \to 0 \) the light cone parameter \( \eta \) approaches zero as

\[
\eta(a) = \frac{4a g^{*2}}{L g^2}.
\]

3 High Energy Scattering and Wilson Lines

The dynamics of high energy reactions is characteristically different from a description of a single hadron. The fast projectile partons propagate near one light cone direction, whereas the target partons cut the trajectories of the projectile partons coming from the opposite light cone direction. Balitsky [6] has coined the phrase that a shock wave is encountered by the fast partons piercing through the target. In a theory of total cross sections the nonperturbative infrared dynamics in the transverse plane is essential. The model of the stochastic vacuum [7,8] allows to connect confinement with high energy diffractive scattering. The underlying idea is to approximate the projectile and target by two color dipoles. The color and anticolor components of these dipoles acquire phase factors running along the respective light cones. These phase factors are path ordered exponentials, i.e. Wilson lines along the light cone. Polyakov variables near the light cone enter the calculations of high energy cross sections in refs. [9]. After a transformation to Minkowski space time nonvanishing correlators of the stochastic vacuum model arise between gauge field strengths \( < F_{+i} F_{-j} > \) arising from the phases on opposite light cones. For these calculations correlations between Polyakov lines on lightlike lines along the projectile and target directions are important.

There are two inherent deficiencies of a naive translation of the Gaussian stochastic model to the light cone. Firstly, the correlation function of Wilson lines along the same light cone, i.e. correlators \( < F_{+i} F_{+j} > \) vanish. Here the more carefully derived Hamiltonian in this paper can make an important
contribution. It generates a transverse confining interaction for particles propagating along the same light cone. The correlations in transverse direction depend on the nearness to the light cone. Secondly, the model of soft scattering does lead to an energy independent high energy cross section. This is not so bad an approximation for large dipole-dipole scattering, like proton-proton scattering where the energy dependence is weak $s^{0.08}$, but for the scattering of small dipoles on the proton HERA experiments show a very strong energy dependence $s^{0.35}$. The weak energy dependence can be interpreted as a weakly increasing cloud of very soft sea partons in the proton which with increasing energy produces a growing total cross section. The increasing cross section of small dipoles may have hard gluons as its origin. These arise from the radiative corrections to the correlation function of the Wilson line operators along the same light cone under evolution towards the light cone, c.f. ref. [6]. This equation contains no external mass scale. Here the above treatment of the near light cone nonperturbative dynamics may come in to give such a mass scale which limits the diffusion into large transverse distances under evolution.

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