A new type of plasma wakefield accelerator driven by magnetowaves

Pisin Chen\(^1\), Feng-Yin Chang\(^2\), Guey-Lin Lin\(^3\), Robert J Noble\(^5\) and Richard Sydora\(^6\)

\(^1\) Kavli Institute for Particle Astrophysics and Cosmology, Stanford Linear Accelerator Center, Stanford University, Stanford, CA 94305, USA
\(^2\) Department of Physics and Graduate Institute of Astrophysics, National Taiwan University, Taipei 106, Taiwan
\(^3\) Leung Center for Cosmology and Particle Astrophysics, National Taiwan University, Taipei 106, Taiwan
\(^4\) Institute of Physics, National Chiao-Tung University, Hsinchu 300, Taiwan
\(^5\) Stanford Linear Accelerator Center, Stanford University, Stanford, CA 94305, USA
\(^6\) Department of Physics, University of Alberta, Edmonton, Alberta, Canada

E-mail: chen@slac.stanford.edu

Received 30 June 2008, in final form 8 September 2008
Published 7 January 2009
Online at stacks.iop.org/PPCF/51/024012

Abstract

We present a new concept for a plasma wakefield accelerator driven by magnetowaves (MPWA). This concept was originally proposed as a viable mechanism for the ‘cosmic accelerator’ that would accelerate cosmic particles to ultra-high energies in the astrophysical setting. Unlike the more familiar plasma wakefield accelerator (PWFA) and the laser wakefield accelerator (LWFA) where the drivers, the charged-particle beam and the laser, are independently existing entities, MPWA invokes the high-frequency and high-speed whistler mode as the driver, which is a medium wave that cannot exist outside of the plasma. Aside from the difference in drivers, the underlying mechanism that excites the plasma wakefield via the ponderomotive potential is common. Our computer simulations show that under appropriate conditions, the plasma wakefield maintains very high coherence and can sustain high-gradient acceleration over many plasma wavelengths. We suggest that in addition to its celestial application, the MPWA concept can also be of terrestrial utility. A proof-of-principle experiment on MPWA would benefit both terrestrial and celestial accelerator concepts.

Ultra-high energy cosmic rays (UHECRs) with energy beyond \(10^{20}\) eV have been observed, yet their astrophysical source and acceleration mechanism remain a mystery. This has been considered as one of the 11 most important questions in cosmology and particle astrophysics in the 21st century [1]. Plasma wakefield accelerators (PWFA) [2–4] are known to possess two salient features: (1) the energy gain per unit distance does not depend on the particle’s
instantaneous energy or momentum. This is essential to avoid the gradual decrease of efficiency in reaching ultra-high energies; (2) the acceleration is linear. Bending of the trajectory is not a necessary condition for this mechanism. This helps to minimize inherent energy loss which would be severe at ultra-high energy. These qualities suggest that plasma wakefield might be a good candidate for the cosmic accelerator. However, high-intensity, ultra-short photon or particle beam pulses that excite the laboratory plasma wakefields are not readily available in the astrophysical setting. In 2002 it was proposed \cite{5} that large amplitude plasma wakefields can instead be excited by the astrophysically more abundant plasma ‘magnetowaves’, whose field components are magnetic in nature (\( | B | > | E | \)). As astrophysical outflows are most likely random, protons may be accelerated beyond ZeV energies by riding on such wakefields with stochastic phases, which would explain the observed UHECR inverse-power-law energy spectrum. Recent particle-in-cell simulations have demonstrated that this MPWA mechanism is indeed valid \cite{6}.

In this paper we suggest that in addition to its application to the celestial accelerator, this new way of plasma wakefield excitation may also be applicable to high acceleration-gradient terrestrial accelerators. In the case of the electron-beam-driven PWFA \cite{3} and the laser-driven laser wakefield accelerator (LWFA) \cite{2}, the drivers are physical entities that can independently exist outside of plasma. In contrast the driver of the MPWA, the magnetowave, is a ‘medium wave’ which cannot exist without the plasma medium. The production of ultra-short and high-intensity laser and electron beams requires complex and expensive facilities. Magnetowaves, on the other hand, can be excited easily by transversely perturbing a background static magnetic field. This salient feature would hopefully be an attractive alternative to drive the PWFA for laboratory applications. A proof-of-principle experiment on MPWA would not only benefit the terrestrial accelerator technology but also contribute to the understanding of celestial acceleration.

Magnetized plasmas support a variety of wave modes propagating at arbitrary angles to the imposed magnetic field \cite{7}. For our purpose, we focus on wave propagation parallel to the external magnetic field to ensure the linear acceleration. In this case, the electromagnetic waves become circularly polarized and the dispersion relation in the non-relativistic limit is \cite{8}

\[
\omega^2 = k^2 c^2 + \frac{\omega_{ip}^2}{1 \pm \omega_c/\omega} + \frac{\omega_{ep}^2}{1 \mp \omega_c/\omega},
\]

where the upper (lower) signs denote the right-hand (left-hand) circularly polarized waves. \( \omega_{ip, ep} = \sqrt{4\pi e^2 n_p/m_i, e} \) is the ion and electron plasma frequency and \( \omega_{ec} = eB/m_i, e \) the ion and electron cyclotron frequency, respectively. Each polarization has two real solutions with high and low frequency branches and both have a frequency cutoff which forms a forbidden gap for wave propagation. The right-hand polarized, low frequency solution is called the whistler wave which propagates at a phase velocity less than the speed of light. When the magnetic field is sufficiently strong such that \( \omega_{ec} >> \omega_{ep} \), the dispersion of the whistler mode becomes more linear over a wider range of wavenumbers with phase velocity approaching the speed of light (see figure 1). The \( E \) and \( B \) components of the wave are now comparable in strength. In this regime the traveling wave pulses can maintain their shape over macroscopic distance, a condition desirable for plasma wakefield acceleration.

Applying the dispersion relation for the whistler wave, with the ion motion neglected, we obtain the ponderomotive force acting on an individual electron as \cite{9}

\[
F_z = -\frac{1}{2} \frac{e^2}{m_e \omega (\omega - \omega_c)} \left[ 1 + \frac{k v_g \omega_c}{\omega (\omega - \omega_c)} \right] \partial_\xi E_W^2 (\xi),
\]

where \( E_W(\xi) \) is the amplitude of the whistler wavepacket, and \( \xi \equiv z - v_g t \) the co-moving coordinate for the driving pulse. From here on, we drop all the indices for ion and electron.
Combining this equation with the continuity equation and the Poisson equation, the longitudinal electric field in the plasma, i.e. the plasma wakefield, can be solved and it reads

\[ E_z(\zeta) = -\frac{e k_p E_w}{m_e \omega (\omega - \omega_c)} \left[ 1 + \frac{k v_p \omega_c}{\omega (\omega - \omega_c)} \right] \chi(\zeta), \]

with

\[ \chi(\zeta) = \frac{k_p}{2 E_w} \int_\zeta^\infty d\zeta' E_w^2(\zeta') \cos \left[ k_p (\zeta - \zeta') \right], \]

where \( E_w \) is the maximum value of \( E_w(\zeta) \). An expression similar to equation (3) has been obtained for laser-induced wakefield in a magnetized plasma [10]. For a Gaussian driving pulse with \( E_w(\zeta) = E_w \exp(-\zeta^2/2\sigma^2) \), it can be shown that behind the driving pulse, i.e. \(|\zeta| \gg \sigma\),

\[ \chi(\zeta) = \frac{\sqrt{\pi}}{2} k_p \sigma e^{-k_p^2 \sigma^2 / 4} \cos k_p \zeta \equiv \chi \cos k_p \zeta. \]

It is customary to express the plasma wakefield in terms of the Lorentz invariant ‘strength parameter’ of the driving pulse, \( a_0 = e E_w / m_e c \omega \), and the ‘cold wavebreaking’ field, \( E_{wb} = m_e c \omega_p / e \). Assuming the driving pulse frequency is centered around \( \omega \) and its speed \( v_p \approx \omega / k \sim c \), the maximum wakefield, or the acceleration gradient, attainable behind the driving pulse is then

\[ G = \frac{k^2 c^2}{\omega^2} \frac{a_0^2}{(1 - \omega_c / \omega)^2} \chi e E_{wb} \approx \frac{a_0^2}{(1 - \omega_c / \omega)^2} \chi e E_{wb}. \]

We see that in the non-relativistic limit, i.e. the linear regime of plasma perturbation, the wakefield in a magnetized plasma is enhanced over the unmagnetized case by a factor \( 1/(1 - \omega_c / \omega)^2 \), which can be substantial when \( \omega \) approaches \( \omega_c \). We stress that the above acceleration gradient is derived under the condition \( \omega_c \gg \omega_p \). In the opposite limit, the phase

![Figure 1. (a) Frequency and (b) phase velocity versus wavenumber for different magnetic field strengths. The vertical solid lines are the mean values of the pulse wavenumbers that were chosen for the PIC simulation for case 1 and case 2, and the shaded regions their ranges.](image-url)
velocity of the whistler wave becomes much less than $c$ (see figure 1), which is not as effective for high energy particle acceleration.

We have conducted computer simulations to study the MPWA process driven by a Gaussian whistler pulse described above. Our simulation model integrates the relativistic Newton–Lorentz equations of motion in the self-consistent electric and magnetic fields determined by the solution to Maxwell’s equations [11, 12]. The four-dimensional phase space $(z, p_x, p_y, p_z)$ is used for the charged particle dynamics and a uniform external magnetic field, $B_0$, is imposed in the $z$-direction. In order to investigate the influence of magnetization on the wakefield excitation, we study two cases with the normalized physical parameters $\omega_c/\omega_p = 6$ (case 1) and 12 (case 2). We used a wavepacket with Gaussian width $\sigma = 80/\Delta_1/\sqrt{2}$, where $\Delta_1$ is the cell size taken to be unity, and we assigned the wavenumber $k = 2\pi/60/\Delta_1$ and $2\pi/40/\Delta_1$ to case 1 and case 2, respectively. All other basic parameters are common, i.e. we set $m_i/m_e = 2000$ and imposed a uniform background plasma with electron collisionless skin depth, $c/\omega_p = 30/\Delta_1$. This gives $\omega/\omega_p = 2.98$ and $v_g/c \simeq \omega/ck = 0.95$ for case 1 and $\omega/\omega_p = 4.64$ and $v_g/c \simeq \omega/ck = 0.99$ for case 2. Other numerical parameters used were: total number of cells in the $z$-direction, $L_z = 2^{14}\Delta \simeq 546c/\omega_p$, average number of particles per cell was 10, and the time step $\omega_p\Delta t = 0.1$ for case 1 and 0.05 for case 2. The fields were normalized by $(1/30)E_{wb}$.

To compare the two cases on the equal footing, we choose to fix the gradient $G$. This is done by setting the maximum amplitude for case 1 at $E_W = 8.15$ and, commensurate with the increase in $\omega$, the maximum amplitude for case 2 at $E_W = 20$. These give the normalized vector potential $a_0 = eE_W/m_e\omega = 0.09$ for case 1 and 0.14 for case 2. Thus the wakefield in our simulation is in the linear regime. The pulse was initialized at $z_0 = 500\Delta = 16.66c/\omega_p$. To avoid spurious effects, we gradually ramped up the driving pulse amplitude until $t = 100\omega_p^{-1}$ for case 1 and $t = 200\omega_p^{-1}$ for case 2, during which the plasma feedback to the driving pulse was ignored. After these times, the driving pulse–plasma interaction was tracked self-consistently. As the dispersion relation in this regime is not perfectly linear, there was a gradual spread of the pulse width. Thus $\chi$ and $E_W$ of the driving pulse decreased accordingly. As a result, the maximum wakefield amplitude, $E_z$, declined in time. We shall see, however, that such degradation becomes much milder as the ratio of $\omega_c/\omega_p$ increases. Figure 2 is a snapshot of $E_x$ and $E_z$ in case 1 at $\Delta t = 100\omega_p^{-1}$ after the pulse was released. The initial wakefield...
amplitude agrees well with the theoretical maximum of $E_z \sim 0.0059 E_{wb}$. Figure 3 shows the snapshot of the same run at the later time $\Delta t = 300 \omega_p^{-1}$. Here we see the obvious dispersion of the driving pulse. We note, however, that while the driving pulse continues to disperse, the wakefield remains coherent albeit with a much lower amplitude.

Next we repeat the exercise with case 2. Figure 4 shows the snapshot of case 2 at $\Delta t = 100 \omega_p^{-1}$. Comparing with case 1 at the same time step, we see that the wakefield amplitudes in case 2 had barely degraded and were more constant than that in case 1, which is indicative that the driving pulse had smaller dispersion for case 2. Figure 5 shows the snapshot of case 2 at $\Delta t = 300 \omega_p^{-1}$. Here the difference is more dramatic. Whereas the driving pulse had already widely spread and the corresponding wakefield had largely diminished in case 1 by this time step, which in case 2 were still well maintained.

In figure 6, we show the PIC simulations driving pulse intensity contours in the $\omega - k$ space for the two cases. There are superimposed with the theoretical curves for the whistler wave dispersion relations deduced from equation (1). We confirm that our driving pulses were indeed whistler waves.
Figure 5. A late time snapshot for case 2 at $\Delta t = 300\alpha_p^{-1}$.

Figure 6. The intensity contours of the driving pulse as a function of $(\omega, k)$ from PIC simulation. The light cone and the linear dispersion curves for the whistler wave with $\omega_c/\omega_p = 1, 6$ and 12 are superimposed.

In our previous paper [6], we have confirmed via PIC simulations the concept of plasma wakefield excited by a magnetowave in a magnetized plasma. In this paper we further demonstrate that by raising the $\omega_c/\omega_p$ ratio, the driving whistler pulse becomes less vulnerable to the dispersion. This aspect is especially important for MPWA to be a viable mechanism for terrestrial accelerator since it is essential for an accelerated particle to continuously gain energy from the plasma wakefield so as to attain high energy. Different from the laser and the particle beam, the magnetowave is a medium wave which cannot exist without the support
from the plasma. MPWA should thus be of interest as a fundamental phenomenon in plasma physics and an alternative approach to plasma wakefield acceleration.

Aside from the issues of dispersion of the driving pulse and sustainment of the plasma wakefield, we have not yet investigated other key accelerator physics issues in MPWA. It is curious whether MPWA would be more or less sensitive to plasma uniformity than PWFA and LWFA do. With regard to the phase slippage between the driving pulse and the accelerated particle, we see from our simulation results that by raising $\omega_c/\omega_p$ from 6 to 12, the phase velocity of the wakefield increases from 0.95 to 0.99. This indicates that phase velocities can approach the speed of light if this ratio is further raised. One challenge common to PWFAs is that, in order to concentrate the driving pulse energy, its transverse size is often as small as a few plasma skin depths. This is known to induce large transverse wakefield which in turn creates problems with undesirably large betatron motions for the accelerated particle. It would be interesting to know if this new type of plasma wakefield driver using the magnetowaves can help to ameliorate this difficulty.

Finally, it would be extremely exciting if proof-of-principle experiments on MPWA can be pursued. With regard to the possible physical mechanism to excite the whistler magnetowave driving pulse for experimentation, it has been shown that a fast ion-acoustic wave can decay into a whistler wave plus an ion-acoustic wave [13]. It is therefore conceivable that such a decay process, or conversely the fusion of two ion-acoustic waves, can produce whistler wave. Inspired by this, one wonders whether a similar process can occur between a light wave and a whistler wave. If so, then perhaps a laser pulses may convert into a whistler wave pulse in a magnetized plasma under suitable conditions.

Acknowledgments

This work is supported by the US DOE (Contract No DE-AC03-76SF00515), the National Science Council of Taiwan (Grant No 95-2119-M-009-026) and the Natural Sciences and Engineering Research Council of Canada.

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