Elastic Modulus Reduction Method Based Ultimate Bearing Capacity Analysis of CFST Truss Arch

Jian Zheng1,2, Jianjun Wang2, Lufeng Yang1, Weiwei Xie2 and Yufeng Zhao1

1. Key Lab of Engineering Disaster Prevention and Structural Safety of China Ministry of Education, School of Civil Engineering and Architecture, Guangxi University, Guangxi, Nanning 530004, China.
2. Guangxi Luqiao Engineering Group Co., Ltd., Guangxi, Nanning530011, China.
Email: 625445306@qq.com

Abstract. In order to evaluate the application of elastic modulus reduction method on the ultimate bearing capacity analysis of CFST truss arch, the elastic modulus reduction method was compared with the test data and the increment nonlinear finite element method. Firstly, the element bearing ratio was defined according to the homogeneous generalized yield function, and the principle of deformation energy conservation was adopted to derive the elastic modulus adjustment strategy, based on which the modulus reduction method was developed for the ultimate bearing capacity analysis of CFST structures. Then, combined with the incorporated strength capacity technique, the section strength was modified to consider the structural constant load effect. Finally, the ultimate bearing capacity of a CFST truss girder and 136m span truss arch was obtained on the basis of the proposed method, and the results were compared with the test data and the incremental nonlinear finite element method. The results show that the elastic modulus reduction method can achieve high calculation accuracy and wide application for the ultimate bearing capacity analysis of CFST truss arch.

Keywords: CFST truss arch; ultimate bearing capacity; Shitanxi bridge; model test; elastic modulus reduction method.

1. Introduction

Concrete-filled steel tubular (CFST) presents high compressive strength, good plasticity, and strong resistance to deformation and it is widely used in arch bridge structures. According to the statistical data of literature [1-2], the number of CFST bridges up to January 2015 are 415, of which the truss section CFST bridges accounts for 37% of the total, and they are mainly used in long-span arch bridge structures with spans greater than 200 m. Therefore, it is necessary to study the ultimate bearing capacity of CFST truss arch bridges.

The model test method and the numerical analysis method are the main methods for the ultimate bearing capacity analysis of CFST truss arch. Because the model test method is not affected by the simplified assumptions, it can reflect the mechanical behavior of the structure intuitively and truly: Cui Jun [3] carried out the whole bridge model test of CFST truss arch bridge and introduced the whole process from design to loading; Huang Fuyun et al. [4] carried out model test on CFST arch with four limbs, and applied vertical concentrated load test at the top of the arch. The results show that the CFST truss arch has good plastic deformation ability. Although the model test method can reflect the mechanical properties of the structure directly and reliably, it is difficult to be widely used in...
engineering practice because of its limited working conditions, high cost and time consumed. The numerical method can solve the above disadvantages of the test method. The common numerical methods mainly include the incremental non-linear finite element method (INFEM) and elastic modulus adjustment procedure (EMAP). Among them, according to the section modeling method, INFEM can be mainly divided into double-material model [5-7] and single-material model [8-9]. The former can achieve high calculation accuracy by accurately simulating the constitutive relations and mechanical properties of steel tube and concrete. However, the calculation efficiency is low due to the need for fine meshes and the deformation compatibility model of steel tube and concrete needs further study. The composite material model is based on the CFST unified theory, which considers steel and concrete as a homogeneous material so as to establish the finite element model of the structure. The composite model can greatly reduce the amount of calculation amount EMAP. While, EMAP is established based on the plastic limit theory and the ultimate bearing capacity of the structure is solved by adjusting the elastic modulus of the elements. Therefore, the theory of EMAP is simple and the calculation process is easy to carry out, and it has been adopted by the ASME Code [10]. As the latest achievement of EMAP, elastic modulus reduction method has been widely used in civil engineering, water conservancy project, transportation and other fields [11-15] and has been successfully applied in the analysis of ultimate bearing capacity of CFST arch rib structure, which has the advantages of high calculation accuracy and good calculation efficiency [14-15].

In view of this, this paper establishes a numerical analysis method for the ultimate bearing capacity analysis of CFST truss arch structures based on CFST unified theory and EMRM. Firstly, the element bearing ratio was defined according to homogeneous generalized yield function (HGYF), then the elastic modulus adjustment strategy was established according to the element bearing ratio and principle of deformation energy conservation, and the constant load effect is considered by combining the incorporated strength capacity technique, then EMRM for the analysis of ultimate bearing capacity of CFST truss arch structure is developed. To verify the computational effect of the proposed method, the calculation accuracy of EMRM was verified by CFST truss girder model test and INFEM results.

2. EMRM for Ultimate Bearing Capacity Analysis of CFST Truss Arch

2.1. Element Bearing Ratio

The CFST truss arch is usually composed of the ventral steel tube member and the arch rib CFST member, therefore, the element bearing ratio should be defined according to the steel pipe and CFST:

(1) Steel tube member

When the component is of steel tube, the HGYF in lit. [16] was adopted to define element bearing ratio as:

\[ r_e^l = \sqrt[4]{f_4(n_x, m_y)} \]  

(1)

Where \( f_4(n_x, m_y) \) represents fourth order homogeneous generalized yield function of steel tube members, and the definition and value of each variable is described in the literature [16].

(2) CFST component

When the component is of CFST, the HGYF in Ref. [14] was adopted to define element bearing ratio as:

\[ r_e^c = \sqrt[4]{f_4(n_x, n_v, m_y)} \]  

(2)

Where, the definitions and values of the variables are described in the literature [14].

Then according to the element bearing ratio of the component, the reference bearing ratio \( r_k^0 \) is obtained.

\[ r_k^0 = r_k^\text{max} - d_k(r_k^{\text{max}} - r_k^{\text{min}}) \]  

(3)
Where, \( r_k^{\text{max}} \) and \( r_k^{\text{min}} \) denote the maximum and minimum values of the element bearing ratio in the \( k \)-th iteration, respectively, and \( d_k \) indicates the uniformity of bearing ratio:

\[
d_k = \frac{r_k^{\text{max}} + r_k^{\text{min}}}{r_k^{\text{max}} - r_k^{\text{min}}} , \quad r_k = \frac{1}{N_e} \sum_{e=1}^{N_e} r_k^e
\]

\[(4a, b)\]

Where, \( N_e \) and \( \overline{r_k} \) represent the total number of discrete elements of the structure and the mean value of the element bearing ratio, respectively.

2.2. Adjustment Strategy of Elastic Modulus

Based on the principle of deformation energy conservation, the adjustment strategy of elastic modulus is established [17-18]:

\[
E_{k+1}^e = \begin{cases} 
E_k^e + \frac{2(r_k^e)^2}{(r_k^e)^2 + (r_k^0)^2}, & r_k^e > r_k^0 \\
E_k^e, & r_k^e \leq r_k^0 
\end{cases}
\]

\[(5)\]

Where \( E_k^e \) and \( E_{k+1}^e \) are the elastic moduli of element \( e \) in the \( k \)-th and \( (k+1) \)-th iterations, respectively, when \( k = 1 \), \( E_k^e \) takes its initial value.

2.3. Ultimate Bearing Capacity Analysis

At every iteration step, the linear elastic finite element method is implemented to obtain the \( r_k^e \) of each element, the limit load \( P_k^L \) for \( k \)-th iterative step is determined according to the maximum element bearing ratio, and reads:

\[
P_k^L = P_0 / r_k^{\text{max}}
\]

\[(6)\]

Where \( P_0 \) denotes the initial load.

The above iteration process is repeated until the following convergence criterion is satisfied:

\[
|P_k^L - P_{k-1}^L| / P_{k-1}^L \leq \varepsilon, \quad k \geq 2
\]

\[(7)\]

Where \( \varepsilon \) is the allowable error which takes 0.001.

If the criterion of convergence is satisfied at the \( m \)-th iterative step, the ultimate load bearing capacity of the CFST structure writes:

\[
P_L = P_L^m
\]

\[(8)\]

2.4. The Treatment of Constant Loading [19]

For CFST arch bridge, the dead weight usually occupies a large proportion, and the incorporated strength capacity technique [19] is used to consider the constant load effect of the ultimate bearing capacity of CFST truss arch structure. First, apply a constant load to the arch structure to solve the internal forces (including axial force, bending moment, shear force, etc.) of each component in the structure, and then the member’s cross-section resistance is subtracted from the internal force generated by the constant load (when the internal force direction of the constant load is the same as that of the live load, the direction of internal force is positive, whereas the negative), so as to consider the influence of the constant load on the ultimate bearing capacity of the structure. The specific calculation expression is as follows:
\begin{equation}
\begin{aligned}
N'_{px} &= N_{px} - N_{cx} \cdot \text{sign}(N_{lx} \cdot N_{cx}) \\
N'_{pxz} &= N_{pxz} - N_{cz} \cdot \text{sign}(N_{lz} \cdot N_{cz}) \\
M'_{py} &= M_{py} - M_{cy} \cdot \text{sign}(M_{ly} \cdot M_{cy})
\end{aligned}
\end{equation}

Where $N_{lx}, N_{lz}, M_{l}$ indicates the axial force, shear force and bending moment generated by the live load (in-plane force, the same as below), $N_{cx}, N_{cz}, M_{c}$ represent the axial force, shear force and bending moment caused by live load, $N'_{px}, N'_{pxz}$ and $M'_{py}$ represent the axial force, shear force and bending moment (in-plane force) section strength of the component after the constant load effect; sign $(x)$ is a symbolic function, when $x \geq 0$, signs $(x) = 1$, and when $x \leq 0$, sign $(x) = -1$.

3. Case 1: Truss Girder

The most common type of CFST truss arch ribs is the “N”-shaped joint. Therefore, in order to simulate the mechanical properties of CFST truss arch, the ultimate bearing capacity test of Pratt truss specimens with “N”-shaped joints was carried out in literature [20]. As shown in Figure 1, the spatial truss structure is composed of two planar truss beams connected by a flat linkage, and the spacing of the flat connecting rods is 0.18 m, the total length of $S=3.008$ m, span length of $L=2.88$ m, height of $H=0.4$ m, width of $B=0.133$ m. The dimensions of upper and lower chord steel tubes, abdominal rod, flat connecting rod and flat connecting rod steel pipe at both ends of the truss are $\phi 89 \times 1.8$ mm, $\phi 48 \times 1.5$ mm, $\phi 60 \times 1.6$ mm and $\phi 108 \times 7.0$ mm, respectively. The yield strength and elastic modulus of steel are 428 MPa and 209 GPa respectively, the upper and lower chords and the lower flat connecting rods were filled with C40 concrete with a compressive strength of 46.5 MPa.

(1) Calculation model
(2) Sectional dimensions and boundary conditions

Figure 1. Structure calculation model.

The EMRM calculation results are compared with the literature test results and the INFEM based on the two-element model, and the results are shown in Table 1.
Table 1. Results of ultimate bearing capacity.

| Test results [20] | INFEM  | EMRM  |
|-------------------|--------|-------|
| 142.5             | 156.75 | 154.63|

The EMRM is used to calculate the ultimate bearing capacity of the structure. The iterative process is shown in Figure 2.

![Figure 2. Ultimate bearing capacity iterative process.](image)

It can be seen from Table 1 that the relative errors of EMRM and INFEM are of 8.5% and 10.0% compared with the experimental data, respectively, and both of them have good calculation accuracy. In addition, it can be seen from Figure 2 that when the ultimate bearing capacity of the truss girder structure is calculated by EMRM, the whole iterative process is stable and smooth.

4. Case 2: Engineering Example of CFST Truss Arch

4.1. Project Overview [8]

The Shitanxi Bridge is a mid-supported CFST arch bridge on the 316 National Highway on the line of Minqing County, Fujian Province, China. The net span of the bridge is 136m, the rise span ratio of $l/L=1/5$, and the arch axis is an inverted catenary with $m=1.167$. The main arch rib is a four-limbed space arch structure, with height of $H=3.0m$, width of $B=1.6m$. The top and bottom chord members are steel tube of $\phi 550 \times 8mm$ with outer diameter of 550 mm and wall thickness of 8 mm. The straight and diagonal brace members are steel tube of $\phi 219mm \times 8mm$, all of the steel tubes are made of A3 steel and filled with C40 concrete. Bridge design loads include vehicle loads, trailer loads, and crowd loads. The overall layout of the bridge and the corresponding section dimensions are shown in Figure 3:

![Figure 3. Bridge layout and corresponding section dimensions of Shitanxi Bridge.](image)
4.2. Ultimate Bearing Capacity Analysis of Shitanxi Bridge

Reference [8] takes Shitanxi Bridge in Fujian Province as the engineering background, the ultimate bearing capacity of the structure is analyzed based on INFEM, the load condition is shown in Figure 4, the constant load is loaded with full span and the live load is loaded with half span. The constant load effect coefficient and the live load effect coefficient are 1.2 and 1.4, respectively.

When EMRM is employed, the finite element model of the structure is established by means of ANSYS, and every member is simulated by BEAM189. Adjacent two nodes of the truss arch structure are connected into a line and each line is separated into two elements. The live load is applied on the structure as the same with that of the literature [8]. The constant load is considered based on the incorporated strength capacity technique [19]. The calculation model and its node number are shown in Figure 4.

![Figure 4. Calculation model of Shitanxi Bridge.](image)

Figure 5 illustrates the ultimate bearing capacity results under load conditions in Figure 4. The black and red lines in the figure indicate that the results of ultimate bearing capacity obtained from the EMMEM and INFEM of the reference [8], respectively. The comparison between them shows that the proposed EMRM has good calculation accuracy (the relative error with INFEM is 3.03%), and the calculation efficiency and stability are good. The ultimate bearing capacity of CFST arch structure can be obtained within 19 iteration steps.

![Figure 5. EMRM calculation iterative process.](image)

5. Conclusion

On the basis of the incorporated strength capacity technique and the HGYF defined element bearing ratio, the EMRM for the analysis of ultimate bearing capacity of CFST truss arch is proposed based on the elastic modulus adjustment strategy and linear elastic theory. The proposed EMRM can achieve high accuracy and efficiency compared with test data and traditional incremental nonlinear finite element method, and it can be applied to practical engineering for determining the ultimate bearing capacity of CFST truss arch.
Acknowledgement

Fund project: National Natural Science Foundation of China (51738004); Guangxi Graduate Education Innovation Program (YCBZ2017024); Guangxi Key Laboratory Systematic Research Project (2016ZDK006).

References

[1] Chen BC, Wei JG, Zhou J, et al. Application of Concrete-Filled Steel Tube Arch Bridges in China: Current Status and Prospects. China Civil Engineering Journal [J]. 2017; 50 (6): 50-61 (in Chinese).

[2] Teng QJ. Ultimate Load-carrying Capacity Study of Concrete-filled Steel Tubular Arch Bridge [D]. Dalian University of Technology, Dalian, 2007 (in Chinese).

[3] Cui J, Sun BN, Lou WJ, et al. Model Test Study on Concrete Filled Steel-tube Truss Arch Bridge. Engineering Mechanics [J]. 2004; 21(5): 83-86 (in Chinese).

[4] Huang FY, Qian HM, Chen BC, et al. Experimental Study on In-plane Mechanical Behavior of Concrete-filled Steel Tubular Truss Arch. Journal of Building Structures [J]. 2015; 36 (12): 120-128 (in Chinese).

[5] Deng JH, Zhou FL, Tan P. Study on Method for Calculating Spatial Ultimate Load of Circular CFST Arch. Journal of Building Structures [J]. 2014; 35 (11): 28-35 (in Chinese).

[6] Ding FX, Yu ZW, Jiang LZ. Nonlinear Finite Element Analysis of Concrete Filled Circular Steel Tubular Structures. Journal of Building Structures [J]. 2006; 27 (4): 110-115 (in Chinese).

[7] Chen BC, Sheng Y. Research on Load-carrying Capacity of Concrete Filled Steel Tubular Dumbbell Shaped Rib Arch under In-plane Loads. Engineering Mechanics [J]. 2009; (9): 94-104 (in Chinese).

[8] Chen BC. Concrete Filled Steel Tubular Arch Bridge 2nd [M]. Beijing: China Communications Press, 2007 (in Chinese).

[9] Xie XL, Zhao GF Zhou CJ. Computer Simulation of the Whole Process for Ultimate Load of Stability in the Plane of Ribbed Arches of Concrete-filled Steel Tubular Arch Bridge. China Civil Engineering Journal [J]. 2004; 37 (5): 54-58 (in Chinese).

[10] 2007 ASME BPVC. Section VIII: Rules for Construction of Pressure Vessels. New York, USA: ASME Boiler and Pressure Vessel Committee; 2007.

[11] Yang L F, Li Q, Zhang W, et al. Homogeneous Generalized Yield Criterion Based Elastic Modulus Reduction Method for Limit Analysis of Thin-Walled Structures with Angle Steel. Thin-Walled Structures [J]. 2014; 80: 153-158.

[12] Yang LF, Zhang W, Yu B, et al. Safety Evaluation of Branch Pipe in Hydropower Station Using Elastic Modulus Reduction Method. Journal of Pressure Vesse Technology [J]. 2012; 134 (4): 041202.

[13] Zhang W, Yang LF, Han XF. Safety Evaluation of Shell Type Bifurcated Pipes Using Elastic Compensation Finite Element Method. Journal of Hydraulic Engineering [J]. 2009; 40 (10): 1175-1183 (in Chinese).

[14] Yang LF, Zheng J, Zhang W, et al. Self-adaptive Approach for Analysis of Ultimate Load Bearing Capacity of Concrete-filled Steel Tubular Arch bridges. China Journal of Highway and Transport [J]. 2017; 30 (3): 191-199 (in Chinese).

[15] Yang LF, Xie WW, Zheng J, et al. Linear-elastic Analysis Method of Ultimate Bearing Capacity of Dumbbell-shaped CFST Arch Rib. Journal of Traffic and Transportation Engineering [J]. 2017; 17 (3): 25-35 (in Chinese).

[16] Yang LF, Li Q, Zhang W, et al. Homogeneous Generalized Yield Function of Circular Tube Section for Ultimate Bearing Capacity of Thin-walled Structures. Chinese Journal of Computational Mechanics [J]. 2013; 30 (5): 693-698 (in Chinese).

[17] Yang L, Yu B, Qiao Y. Elastic Modulus Reduction Method for Limit Load Evaluation of Frame Structures. Acta Mechanica Solida Sinica [J]. 2009; 22 (2): 109-115.

[18] Yu B, Yang L F. Elastic Modulus Reduction Method for Limit Analysis of Thin Plate and Shell
Structures. Thin-Walled Structures [J]. 2010; 48 (4-5): 291-298.

[19] Lin YH, Yang L, Zhou JJ. Elastic Modulus Reduction Method for Limit Analysis of Structures under Dead Load. Journal of Guangxi University (Natural Science Edition) [J]. 2011; 36(1): 45-52 (in Chinese).

[20] Huang WJ, Chen BC. Research on Influence of Web Member on Mechanical Behavior of CFST Truss Girders. Journal of Building Structures [J]. 2009; 30(1): 55-61 (in Chinese).