3D Anderson transition for two electrons in 2D

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It is shown that the Coulomb interaction can lead to delocalization of two electron states in two-dimensional (2D) disordered potential in a way similar to the Anderson transition in three dimensions (3D). At fixed disorder strength the localized phase corresponds to low electron density and large value of parameter \( r_s \).

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Contrary to the well established theoretical result [1], according to which noninteracting electrons are always localized in 2D disordered potential, the pioneering experiment by Kravchenko et al. [2] demonstrated the existence of metal-insulator transition for real ensemble of electrons in 2D. The ensemble of experimental data obtained by different groups [3–8] clearly indicates the important role played by interaction. In the majority of experiments the Coulomb energy of electron - electron interaction \( E_{ee} \) is significantly larger than the Fermi energy \( E_F \), estimated for noninteracting electron gas in absence of disorder. The ratio of these energies is characterized by the dimensionless parameter \( r_s = 1/\sqrt{\pi n_s a_B^2} \approx E_{ee}/E_F \), where \( n_s \) is the electron density in 2D, and \( a_B^2 = \hbar^2 e_0/m^*\epsilon^2 \), \( m^*, e_0 \) are the effective Bohr radius, electron mass and dielectric constant respectively. Such large \( r_s \) values as 10 - 30 have been reached experimentally [2–8]. At these \( r_s \) the electrons are separated far from each other and it is natural to assume that in this regime the interaction effects will be dominated by pair interaction. The important role of the residual two-body interaction is also clear from the fact that in the Hartree-Fock (mean field) approximation the problem is again reduced to the one-particle 2D disordered potential with localized eigenstates [1–3].

The problem of two electrons interacting in the localized phase is rather nontrivial. Indeed, recently it has been shown that a short range repulsive/attractive interaction between two particles can destroy one-particle localization and lead to creation of pairs propagating on a distance much larger than their size [1–3]. The pair size is of the order of one-particle localization length \( l_1 \). Inside this length the collisions between particles destroy the quantum interference that results in their coherent propagation on a distance \( l_c \gg l_1 \). The important point is that only pairs can propagate on a large distance. Indeed, the particles separated by a distance \( R \gg l_1 \) have exponentially small overlap, the interaction between them is weak and such states are localized as in the noninteracting case. According to the theoretical estimates [1–3] in 2D the localization length \( l_c \) grows exponentially with \( l_1 \) according to the relation \( \ln(l_c/l_1) \sim \kappa > 1 \). Here \( \kappa \sim \Gamma_2/\rho_2 \), where \( \Gamma_2 \sim U^2/(Vl_1^2) \) is the interaction induced transition rate between localized states in e.g. 2D Anderson model, \( \rho_2 \sim l_1^3/V \) is the density of two-particle states directly coupled by interaction, \( V \) is the hopping between nearest sites, \( U \) is on (nearest) site interaction, and energy is taken in the middle of the band. In a sense the above estimate is similar to the case of one-particle localization in 2D where \( \ln l_1 \sim k_F \ell \sim (V/W)^2 \) and the product of the Fermi wave vector \( k_F \) on mean free path \( \ell \) is proportional to a local diffusion rate [1]; \( W \) is the strength of on site disorder. Indeed, in the same manner the interaction induced diffusion rate of a pair is given by \( D_2 \sim l_1^2 \Gamma_2 \sim \kappa/l_1^2 \propto \ln l_c \). According to the above estimates \( l_c \) should vary smoothly with the effective interaction strength characterized by the dimensionless parameter \( \kappa \). However, this consideration is valid only for a short range interaction while the analysis of the long range Coulomb interaction requires a separate study. The investigation of this case is also dictated by the experiments [2–8] where the electrons are not screened and are located far from each other (\( r_s \gg 1 \)). On a qualitative grounds one can expect that the effect of Coulomb interaction will be stronger since electrons are always interacting in a difference from the case of short range interaction. As we will see later the interaction effects will play an important role even at low density when the electrons are far from each other (\( R \gg l_1 \)) and where the interaction can lead to the delocalization transition similar to one in the 3D Anderson model. It is convenient to study this transition by the means of level spacing statistics as it was done for 3D one-particle case in [3].

To analyze the effect of Coulomb interaction between two electrons let us consider the 2D Anderson model with the diagonal disorder (\( -W/2 < E_i < W/2 \)), hopping \( V \), the lattice constant \( a = 1 \) and the interaction \( U/|r_1 - r_2| \). In these notations \( r_s = U/(2V\sqrt{\pi n_s}) \) and it is convenient to introduce another dimensionless parameter \( r_L = U l_1/2\sqrt{\pi V} \) which is equal to \( r_s \) value at \( n_s = 1/l_1^2 \). We will consider the case with \( U \sim V \) and \( r_s \gg 1 \) when the average distance between electrons \( R = |r_1 - r_2| \) is much larger than their noninteracting localization length: \( R \sim 1/\sqrt{n_s} \sim r_s \gg l_1 \gg 1 \). In this case the two-body interelectron interaction has a dipole-dipole form and is of the order of \( U_{dd} \sim U \Delta r_1 \Delta r_2/R^3 \sim U \Delta r_1 \Delta r_2/r_s^3 \sim U \Delta r_1 \Delta r_2/r_s^3 \);
transition induced matrix elements mix two-electron states if the stronger is the \( r_s \) each term in the sum has a random sign. According to the common lore according to which the larger is \( r_s \) disorder. In the presence of not very weak disorder contradiction with (2) is simply due to the fact that \( r_s \) are localized one-electron states, namely:

\[
\psi_{1b} \text{ are localized one-electron states}
\]

Formally the situation corresponds to a quasi-two dimensional case with \( M_{e,f} \approx \pi R/l_1 = \pi r_{1/3}^3 \gg 1 \) parallel planes (number of circles of size \( l_1 \) in the ring) so that the pair localization length \( l_c \) jumps from \( l_c \sim l_1 \) for \( \kappa_e < 1 \) to \( l_c \sim l_1 \exp(\pi \kappa_e r_{1/3}^3) \gg l_1 \) above the transition \( \kappa_e > 1 \). The transition is sharp and similar to 3D Anderson transition when \( r_s > r_L > 1 \). If electrons would be able to move inside the ring then \( M_{e,f} \) would be even larger (\( M_{e,f} \sim r_{L}^{2/3} \)).

It is important to stress that the parameter \( \chi_e \) which determines the delocalization border and measures the effective strength of two-body interaction decreases with the increase of \( r_s \). Apparently, this looks to be against the common lore according to which the larger is \( r_s \) the stronger is the e-e-interaction. The reason of this contradiction with (3) is simply due to the fact that \( r_s \) compares \( E_{ee} \) with \( E_F \) computed in the absence of disorder. In the presence of not very weak disorder \( (r_D = E_{ee}/W \ll 1 \) and \( r_L \gg 1) \) the one-electron states are localized and form the basis of Coulomb glass [10]. In this Coulomb glass phase the e-e-interaction becomes weaker and weaker with the growth of average distance between electrons \( R \sim n_s^{-1/2} \propto r_s \) in the natural agreement with (3). The transition border \( \delta \) was obtained for excited states. However, it is clear that if the interaction is not able to delocalize the excited states then the low energy states will also remain localized since two-electron density \( \rho_2 \) drops at low energy. In this sense \( \delta \) determines the upper border for \( r_s \).

To study the delocalization transition \( \delta \) the level spacing statistics \( P(s) \) is determined numerically for different system sizes \( L \). To follow the transition from the localized phase with the Poisson statistics \( P_L(s) \) to delocalized one with the Wigner-Dyson statistics \( P_WD(s) \) it is convenient to use the parameter \( \eta = \int_0^{s_0} (P(s) - PWD(s))ds \) for low energy states up to \( NS = 10^8 \) at high energies with larger density of levels. The matrix diagonalization is done in the one-electron eigenbasis truncated at high energies that allowed to study two electron low energy excitations (with energy \( E \)) at large system sizes \( L \leq 24 \). The periodic boundary conditions are used for one-electron states, the Coulomb interaction is taken between electrons in one cell of size \( L \) and with 8 charge images in nearby 8 cells. The Coulomb interaction periodic in one cell gave similar results. Only the triplet case was considered but the singlet case should give similar results [11,12].

The results of Fig.1a show that at fixed interaction and strong disorder \( W/V = 15 \) the \( P(s) \) statistics approaches to the Poisson distribution \( \eta = 1 \) at large system size \( L \) and large \( r_s = UL/(2\sqrt{2\pi}V) \). This means that all states are localized. For smaller disorder the situation becomes different (Fig. 1b,c). While near the ground state still \( \eta \rightarrow 1 \) for large \( L \), the tendency is inverted above some critical energy \( \epsilon_c \) where \( \eta \rightarrow 0 \). All curves \( \eta(\epsilon) \) for different \( L \) are crossed in one point in a way similar to the 3D Anderson transition studied in [13]. This result can be understood in the following way. At strong Coulomb interaction \( U \sim V \) the excitation energy \( \epsilon \) is related to the distance between electrons \( R : \epsilon \sim U/R \) (similar relation was used in [14] for the Coulomb glass). At higher \( \epsilon \) the distance \( R \) becomes smaller, the interaction is stronger and for \( \epsilon > \epsilon_c \) the delocalization border \( R \sim \epsilon \sim l_1 r_{L}^{1/3} \) (4) is crossed and the states...
FIG. 1. Dependence of $\eta$ on the rescaled one-electron energy $/B$ (with $B = 4V$) for different $W$, system size $L$ (a–c) and interaction strength $U$ (d). For (a–c): the size is $L = 6(+); 8(\triangle); 10(\circ); 12(\bullet); 16(\ast); 20(\ast);$ $W/V = 15(a); 10(b); 7(c)$. For (d): $W/V = 7, L = 16$ and $U/V = 2$ (full diamond); $1(\triangledown); 0.4(\circ); 0.2$ (full circle); 0.1 (full triangle).

become delocalized. Since the distance $R$ is related with the two electron energy $E = 2/ \sim U/R$ the spacing statistics $P(s)$, which is local in energy and therefore also in $R$, is not influenced by states where particles are far from each other. In this sense the situation is different from the case of short range interaction. According to the above arguments $\epsilon_c = \epsilon_c \xi_1^{1/3}/B$ should remain constant when $l_1$ changes with disorder. The value of $l_1$ can be extracted from the average inverse participation ratio $\xi_1 = 1/ \sum |\psi|^4$ computed for one-particle states in the middle of the band ($l_1 \sim \sqrt{2}$). For $L = 24$ and $W/V = 10; 7; 5$ we have respectively $\xi_1 = 11.6; 36.7; 84.2$ that with $\epsilon_c/B \approx 0.6; 0.28; 0.16$ (the case $W/V = 5$ is not shown) gives $\epsilon_c = 3.08 \pm 0.01$ in a satisfactory agreement with the above expectations. The variation of $\eta$ with the interaction $U$ is shown in Fig. 1d. According to it $\eta$ increases with the decrease of $U$ (states become more localized) in agreement with the general estimate $\xi_1$. The analysis above allows to understand the dependence of $\eta$ on $\epsilon$ and $L$. Another reason for the decrease of $\eta$ at higher $\epsilon$ is related to the fact that the two-electron density of states $\rho_2$ grows with energy that allows to mix levels more easily. A more detail theory should take this fact into account but also to analyze the variation of the rate $\Gamma_\epsilon$ with $\epsilon$. The results in this direction will be published elsewhere [18].

The $P(s)$ statistics for two electrons in 2D near the critical point $\epsilon_c/B$ is shown in Fig. 2. Its comparison with the critical statistics in 3D Anderson model taken.
from [19] (see also [20]) demonstrates that both statistics are really very close in agreement with the arguments given above. At the critical point the value of $\eta_e$ is close to its value in the Anderson model ($\eta_e = 0.20$). The small deviations from this value in the case of 2D electrons ($\eta_e \approx 0.25(W/V = 10); 0.17(W/V = 7)$) can be attributed to the fact that the parameter $l_1^{1/3}$ was not sufficiently large. The investigation of the case with larger $l_1$ requires a significant increase of the system size $L > 24$. Indeed, for $L = 24$ and $W/V = 5$ the localization length becomes comparable with $L (l_1 \sim \sqrt{\xi_1} \approx 9$) that gives a decrease of $\eta_e \approx 0.13$.

![Poisson statistics](image)

**FIG. 2.** Level statistics $P(s)$ for two 2D electrons at the critical point: (+) $W/V = 10, L = 12$ ($0.55 \leq \epsilon/B \leq 0.65$), total statistics $NS = 4 \times 10^5$ (see Fig. 1b); (a) $W/V = 7, L = 16$ ($0.25 \leq \epsilon/B \leq 0.3$), $NS = 5 \times 10^5$ (see Fig. 1c). The full line shows the critical $P(s)$ in 3D Anderson model ($W/V = 16.5, L = 14$, taken from Ref. [19]); the dashed lines give Poisson statistics and Wigner surmise.

Of course, one cannot expect that the simple model of two electrons considered above will explain the variety of experimental results obtained by different groups [3, 4]. However, it shows some tendencies which are in agreement with the experiment. Indeed at large $r_s$ (density lower than some critical $n_c$) the experiments demonstrate the transition from metal to insulator. According to Fig. 4 in [3] the density at the transition $n_c \propto 1/\sqrt{r_s}$ drops exponentially with the increase/decrease of the mobility/disorder $\mu \propto 1/W^2$. This qualitatively agrees with the estimate [4] according to which near the transition $\log n_c \sim \log(1/r_s) \sim -\log r_L \sim -1/W^2$. However, the condition $r_s \gg r_L$ seems to be not well satisfied and apparently multi-electron effects should be also taken into account. Another interesting experimental result (Fig. 2 in [3]) shows that the conductivity $\sigma_c$ near the critical point grows with increase of density $n_c$ or disorder $W$. This is in qualitative agreement with the estimate [4] according to which $\sigma_c \sim D_e/V \sim 1/l_1^2 \propto r_L^{-2} \propto r_s^{-8/3}$ since near the critical point [2] $\kappa_e \sim 1$ and $r_s \sim l_L^{4/3}$. It is also interesting to remark that the scaling index $\nu \approx 1.5$ found in [2] is close to the index $\nu \approx 1.5$ near 3D Anderson transition (the fact that in 3D $\nu \approx s$ can be related to the observed symmetry of I-V curves).

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