Alternative Approach to $B^- \rightarrow \eta'K^-$ Branching Ratio Calculation

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Short title:
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Abstract
Since the calculation of $BR(B^- \rightarrow \eta'K^-)$ in the framework of QCD improved factorization method, developed by Beneke et al., leads to numerical values much below the experimental data, we include two different contributions, in an alternative way. First, we find out that the spectator hard-scattering mechanism increases the $BR$ value with almost 50%, but the predictions depend on the combined singularities in the amplitude convolution. Secondly, by adding SUSY contributions to the Wilson coefficients, we come to a $BR$ depending on three parameters, whose values are constrained by the experimental data.

1 Introduction
As a first evidence of a strong penguin, the $B^- \rightarrow \eta'K^-$ decay has become of a real interest after CLEO announced its large numerical value $BR(B^- \rightarrow \eta'K^-) = (6.5^{+1.5}_{-1.4} \pm 0.9) \times 10^{-5}$ [1], which could not be explained by the existent theoretical models. As improved measurements followed, providing even larger values, $(80^{+10}_{-9} \pm 7) \times 10^{-6}$ (CLEO [2]), $(76.9 \pm 3.5 \pm 4.4) \times 10^{-6}$
(BaBar [3]) and $(79^{+12}_{-11} \pm 9) \times 10^{-6}$ (Belle [4]), the inclusion of new contributions for accommodating these data has quickly become a real theoretical challenge. In this respect, perturbative QCD mechanisms [5], with different $\eta'g^*g^*$ vertex function [5, 6], have been considered as main candidates for significantly increasing the $BR(B^- \to \eta'K^-)$ value. On the other hand, while searching for physics beyond the Standard Model (SM), supersymmetry has been employed in processes like $B \to J/\psi K^*$ [7], $B \to \phi K$ [8], $B \to \pi K$ [9, 10], $B \to X_s\gamma$ [11], and deviations from the SM predictions for the values of branching ratios and $CP$ asymmetries have been the main targets.

The present paper is organized as follows: in Section 2, we compute the $BR(B^- \to \eta'K^-)$ in the improved factorization approach developed by Beneke et al. [12]. Since we get a $BR$ much below the experimental values, we incorporate two alternative contributions. The first one, presented in Section 3, comes from the so-called spectator hard scattering mechanism. Following a similar approach as in [13], we give a detailed calculation of the gluonic transition form factor which plays an important role in the evaluation of this contribution. Although it has been concluded that this mechanism could provide large $BR$ values [13], we show that the presence of combined singularities in the amplitude convolution is a source of large uncertainties. In Section 4, we employ a supersymmetric approach and include exchanges of gluino and squark with left-right squark mixing. Working in the mass insertion approximation [14], the values of the Wilson coefficients $c_{8g}$ and $c_{7\gamma\gamma}$ can be significantly increased, by adding the SUSY contributions, and this has a strong numerical impact in the branching ratio estimation. Finally, one may use the experimental data to impose constraints on the flavor changing SUSY parameter $\delta_{sL}^{bs}$.
2 Improved QCD Factorisation

The relevant decay amplitude for $B^- \rightarrow \eta'K^-$, in the improved QCD factorization approach [12], is given by [5, 15]

$$A(B^- \rightarrow \eta'K^-) = -i \frac{G_F}{\sqrt{2}} (m_B^2 - m_{\eta'}^2) F_0^{B-\eta'}(m_K^2)f_K[V_{ub}V_{us}^*a_1(X)$$
$$+ V_{pb}V_{ps}^*[a_{\eta'}^p(X) + a_{\eta'}^u(X) + r^K_{\chi}(a_5^u(X) + a_5^p(X)))]$$
$$- i \frac{G_F}{\sqrt{2}} (m_B^2 - m_{\eta'}^2) F_0^{B-\tau}(m_{\eta'}^2)f_{\eta'}[V_{ub}V_{us}^*a_2(Y)$$
$$+ V_{pb}V_{ps}^*[a_3(Y) - a_5(Y)) (2 + \sigma)$$
$$+ a_p^p(Y) - \frac{1}{2} a_{\eta'}^p(Y) + r'_{\chi}(a_6^u(Y) - \frac{1}{2} a_6^p(Y))] \sigma$$
$$+ \frac{1}{2} (a_9(Y) - a_7(Y)) (1 - \sigma)],$$

where $X = \eta'K$ and $Y = K\eta'$, $p$ is summed over $u$ and $c$, $r'_{\chi} = 2m_{\eta'}^2/(m_b - m_s)(2m_s)$, $r^K_{\chi} = 2m_K^2/m_b(m_u + m_s)$, $\sigma = f_{\eta'}/f_{\eta'}$, and [12]

$$a_1(M_1M_2) = c_1 + \frac{c_2}{N_c} \left[1 + \frac{C_F\alpha_s}{4\pi}(V_{M_2} + H)\right],$$
$$a_2(M_1M_2) = c_2 + \frac{c_1}{N_c} \left[1 + \frac{C_F\alpha_s}{4\pi}(V_{M_2} + H)\right],$$
$$a_3(M_1M_2) = c_3 + \frac{c_4}{N_c} \left[1 + \frac{C_F\alpha_s}{4\pi}(V_{M_2} + H)\right],$$
$$a_p^p(M_1M_2) = c_4 + \frac{c_3}{N_c} \left[1 + \frac{C_F\alpha_s}{4\pi}(V_{M_2} + H)\right] + \frac{C_F\alpha_s}{4\pi N_c} P_{M_2,2}^p,$$
$$a_5(M_1M_2) = c_5 + \frac{c_6}{N_c} \left[1 + \frac{C_F\alpha_s}{4\pi}(-12 - V_{M_2} - H)\right],$$
$$a_{\eta'}^p(M_1M_2) = c_6 + \frac{c_5}{N_c} \left(1 - 6 \frac{C_F\alpha_s}{4\pi}\right) + \frac{C_F\alpha_s}{4\pi N_c} P_{M_2,3}^p,$$
$$a_{\eta'}^u(M_1M_2) = c_7 + \frac{c_8}{N_c} \left[1 + \frac{C_F\alpha_s}{4\pi}(-12 - V_{M_2} - H)\right],$$
$$a_8^p(M_1M_2) = c_8 + \frac{c_7}{N_c} \left(1 - 6 \frac{C_F\alpha_s}{4\pi}\right) + \frac{\alpha}{9\pi N_c} P_{M_2,3}^{p,EW},$$
$$a_9(M_1M_2) = c_9 + \frac{c_{10}}{N_c} \left[1 + \frac{C_F\alpha_s}{4\pi}(V_{M_2} + H)\right],$$

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\[ a_{10}^p(M_1 M_2) = c_{10} + \frac{c_9}{N_c} \left[ 1 + \frac{C_F \alpha_s}{4\pi} (V_{M_2} + H) \right] + \frac{\alpha}{9\pi N_c} P_{M_2}^{\text{EW}} , \] (2)

where \( C_F = (N^2_c - 1)/2N_c \) and \( N_c = 3 \) is the number of colors. The vertex, the hard gluon exchange with the spectator and the penguin contributions, at \( \mu = m_b \), are:

\[ V_M = -18 + \int_0^1 dx g(x) \phi_M(x) , \]

\[ P_{M,2}^p = c_1 \left[ \frac{2}{3} + G_M(s_p) \right] + c_3 \left[ \frac{4}{3} + G_M(0) + G_M(1) \right] + (c_4 + c_6) \left[ (n_f - 2)G_M(0) + G_M(s_c) + G_M(1) \right] - 2c_{8g}^{\text{eff}} \int_0^1 \frac{dx}{1-x} \phi_M(x) , \]

\[ P_{M,3}^p = c_1 \left[ \frac{2}{3} + \hat{G}_M(s_p) \right] + c_3 \left[ \frac{4}{3} + \hat{G}_M(0) + \hat{G}_M(1) \right] + (c_4 + c_6) \left[ (n_f - 2)\hat{G}_M(0) + \hat{G}_M(s_c) + \hat{G}_M(1) \right] - 2c_{8g}^{\text{eff}} , \]

\[ P_{M,2}^{\text{EW}} = (c_1 + N_c c_2) \left[ \frac{2}{3} + G_M(s_p) \right] - 3c_{7g}^{\text{eff}} \int_0^1 \frac{dx}{1-x} \phi_M(x) , \]

\[ P_{M,3}^{\text{EW}} = (c_1 + N_c c_2) \left[ \frac{2}{3} + \hat{G}_M(s_p) \right] - 3c_{7g}^{\text{eff}} , \]

\[ H = \frac{4\pi^2 f_{B^0 M_1}}{N_c m_B^2 F_{0 \rightarrow M_1}(0)} \times \int_0^1 \frac{dx}{x} \phi_B(x) \int_0^1 dy \phi_{M_2}(y) \int_0^1 dy \left[ \phi_{M_1}(y) + \frac{2\mu_{M_1}}{m_b} \bar{x} \phi_{M_1}(y) \right] , \]

(3)

where \( \bar{x} = 1 - x \), \( \bar{y} = 1 - y \) and the parameter \( 2\mu_M/m_b \) coincides with \( r_x \).

The functions \( g(x) \), \( G_M(x) \) and \( \hat{G}_M(x) \) are given by

\[ g(x) = 3 \left( 1 - \frac{2x}{1-x} \ln x - i\pi \right) \]

\[ + \left[ 2\text{Li}_2(x) - \ln^2 x + \frac{2\ln x}{1-x} - (3 + 2i\pi) \ln x - (x \rightarrow \bar{x}) \right] , \]

\[ G(s, x) = 4 \int_0^1 du u\bar{u} \ln [s - u\bar{u}x] \]

\[ = -\frac{10}{9} + \frac{2}{3} \ln s - \frac{8s}{3x} + \frac{4}{3} \left( 1 + \frac{2s}{x} \right) \left[ \frac{4s}{x} - 1 \arctan \frac{1}{\sqrt{\frac{4s}{x} - 1}} \right] , \]
\[ G_M(s) = \int_0^1 dx \, G(s - i\epsilon, \bar{x}) \phi_M(x), \]
\[ \hat{G}_M(s) = \int_0^1 dx \, G(s - i\epsilon, \bar{x}) \phi^p_M(x), \]  
(4)

where \( s_i = m_i^2/m_b^2 \) are the mass ratios for the quarks involved in the penguin diagrams, namely \( s_u = s_d = s_s = 0 \) and \( s_c = (1.3/4.2)^2 \).

As it can be noticed, except for the hard contribution where the wave functions for both \( M_1 \) and \( M_2 \) are involved, the coefficients \( a_i \) are different for the \( X \) and \( Y \) final states, since they depend on the twist-2 and twist-3 wave functions of the \( M_2 \) meson. Thus, the twist-2 distribution amplitude \( \phi_K(x) \) has the following expansion in Gegenbauer polynomials [12, 16]

\[ \phi_K(x) = 6x(1-x)[1 + \alpha_{K1} C_{1/2}((2x - 1) + \alpha_{K2} C_{3/2}(2x - 1) + ...], \]  
(5)

with \( C_{1/2}(u) = 3u, C_{3/2}(u) = (3/2)(5u^2 - 1), \alpha_{K1} = 0.3 \pm 0.3, \) and \( \alpha_{K2} = 0.1 \pm 0.3. \) The corresponding twist-3 amplitude, \( \phi_{K}^{p*} \), is 1.

The physical states \( \eta \) and \( \eta' \) are mixtures of SU(3)-singlet and octet components \( \eta_0 \) and \( \eta_8 \) and therefore the corresponding decay constants, in the two-angle mixing formalism, are given by

\[ f_{\eta'}^u = \frac{f_8}{\sqrt{6}} \sin \theta_8 + \frac{f_0}{\sqrt{3}} \cos \theta_0 , \]
\[ f_{\eta'}^s = -2 \frac{f_8}{\sqrt{6}} \sin \theta_8 + \frac{f_0}{\sqrt{3}} \cos \theta_0 , \]  
(6)

with \( \theta_8 = -22.2^\circ, \theta_0 = -9.1^\circ, \) \( f_8 = 168 \) MeV, and \( f_0 = 157 \) MeV [17]. These lead to \( f_{\eta'}^u = 63.5 \) MeV, \( f_{\eta'}^s = 141 \) MeV and to the relevant form factor for the \( B \to \eta' \) transition

\[ F_{0}^{B\to\eta'} = F_0^\pi \left( \frac{\sin \theta_8}{\sqrt{6}} + \frac{\cos \theta_0}{\sqrt{3}} \right) = 0.137 \]  
(7)

Even though the \( \eta' \) flavor singlet meson has a gluonic content which could bring a contribution to the wave function, this is supposed to be small [18]
and therefore we employ, in the calculation of $V_{q'}$, $P^p_{q',2}$ and $P^{p,EW}_{q',2}$ in $a_i(Y)$, only the leading twist-2 distribution amplitude
\[ \phi_{q'} = 6x\bar{x}. \] (8)

Also, since the twist-3 quark-antiquark distribution amplitude do not contribute, due to the chirality conservation, the penguin parts in $a_6^p(Y)$ and $a_8^p(Y)$ are missing. As for the $B$ meson wave function, we shall work with a strongly peaked one, around $z_0 = \lambda_B/m_B \approx 0.066 \pm 0.029$, for $\lambda_B = 0.35 \pm 0.15$ GeV.

Putting everything together, we get, within the SM improved factorization approach [12], the numerical value $BR_{SM}(B \to \eta'K) = 3.65 \times 10^{-5}$, which although is in accordance with other theoretical estimations [5, 15, 17], yet lay below the experimental data [1-4]. Hence, in spite of the “conservative” prediction that the conventional mechanism should be the dominant one, it has been getting clear that new contributions are needed in order to account for the existent data.

3 Spectator Hard-Scattering Mechanism

It has been considered that the spectator hard-scattering mechanism (SHSM), depicted in Figure 1, is a reliable framework for this process, which significantly increases the value of $BR(B \to \eta'K)$ [5, 13]. Following this idea, let us write down the corresponding di-gluon exchange amplitude for the $b$ quark decaying into an $s$ quark and a hard gluon
\[
A_{hs} = -iC_Fg_s^3 \frac{f_B}{2\sqrt{6}} \frac{f_K}{2\sqrt{6}} \int dz dy \phi_B(z)\phi_K(y)
\times \text{Tr} [\gamma_5 P_k \Gamma_\mu(P_B + m_B)\gamma_5\gamma_\nu] \frac{\varepsilon^{\mu\nu\alpha\beta}Q_1Q_2}{Q_1^2Q_2^2} F_{\eta'g^*g^*}(Q_1^2, Q_2^2, m_{{\eta'}}^2) \] (9)
in terms of the effective $b \to sg$ vertex [19]
\[
\Gamma^a_\mu = \frac{G_F}{\sqrt{2}} \frac{g_s}{4\pi^2} V_{ps}^* V_{pb} t^a \left[ F^p_1 \left( Q_1^2\gamma_\mu - Q_1\gamma_1 \right) - F^p_2 i\sigma_{\mu\nu}Q_1^\nu m_b R \right] \] (10)
and the transition form factor \[6\]

\[
< g_a^* g_b | \eta' > = - i \delta_{abc} \varepsilon^{\mu\nu\alpha\beta} \varepsilon^{a*}_{\mu} \varepsilon^{b*}_{\nu} Q_{1a} Q_{2b} F_{\eta' g^* g^*}(Q_1^2, Q_2^2, m_{\eta'})
\]  

(11)

The quark contribution to the \(\eta' g^* g^*\) vertex

\[
F_{\eta' g^* g^*}(Q_1^2, Q_2^2, m_{\eta'}) = 4 \pi \alpha_s \sum_{q=u,d,s} f_q \frac{1}{2N_c} F(y, a),
\]

(12)

with

\[
F(y, a) = \int_0^1 dx \frac{\phi_{\eta'}(x)}{xQ_1^2 + xQ_2^2 - x\bar{x}m_{\eta'}^2 + i\varepsilon} + (x \leftrightarrow \bar{x}), \quad a^2 = m_{\eta'}/m_B^2,
\]

(13)

will play an important role in the evaluation of the amplitude \(A_{hs}\). Performing the calculations in (9), we come to the following expression of the hard scattering amplitude:

\[
A_{hs} = -2 i \frac{G_F}{\sqrt{2}} V_{ps}^* V_{pb} \frac{\alpha_s^2}{N_c} f_B f_K (2f_{\eta'}^u + f_{\eta'}^s) \int_0^1 dz \phi_B(z) \int_0^1 dy \phi_K(y) \\
\times \left[ F_1^p Q_1^2 ((P_B \cdot Q_1)(P_K \cdot Q_2) - (P_K \cdot Q_1)(P_B \cdot Q_2)) + \\
+ F_2^p m_B m_b ((P_K \cdot Q_2)Q_1^2 - (P_K \cdot Q_1)(Q_1 \cdot Q_2)) \right] \frac{F(y, a)}{Q_1^2 Q_2^2}
\]

(14)

With the gluon momenta

\[
Q_1 = zP_B - \bar{y}P_K, \quad Q_2 = zP_B - yP_K,
\]

(15)

and neglecting, for the moment, both \(m_{\eta'}^2\) and \(m_K^2\), the amplitude (14) becomes

\[
A_{hs} = -2 i \frac{G_F}{\sqrt{2}} V_{ps}^* V_{pb} \frac{\alpha_s^2}{2N_c} f_B f_K (2f_{\eta'}^u + f_{\eta'}^s) \frac{1}{z_0} \int_0^1 dz \phi_B(z) \int_0^1 dy \phi_K(y) \\
\times \left[ m_B^2 F_1^p + m_B m_b \frac{F_2^p}{y - z_0} \right] F(y, a)
\]

(16)

where, for the dominant contribution coming from the insertion of the \(O_u,c\) and the magnetic-penguin \(O_{8g}\) operators, one has [13]

\[
F_1^p = c_1 \left[ \frac{2}{3} + G[s_p, (1 - z_0)(y - z_0)] \right], \quad F_2^p = -2c_{8g}
\]

(17)
In what it concerns the $F(y, a)$ function, which is an essential input in the calculations, it can be first written as

$$F(y, a) = 4 \int_0^1 dx \frac{6x(1-x)}{[Q_1^2 + Q_2^2 - 2x(1-x)m_{\eta'}^2]^2 - [(x - \bar{x})(Q_1^2 - Q_2^2)]^2}$$

and it comes, after algebraic computations, to the following form

$$F(y, a) = -\frac{12}{m_{\eta'}^2} \left[ 1 - \frac{Q_1^2 - Q_2^2}{2m_{\eta'}^2} \log \left| \frac{Q_1^2}{Q_2^2} \right| + \frac{(Q_1^2 - Q_2^2)^2 - m_{\eta'}^2(Q_1^2 + Q_2^2)}{2m_{\eta'}^2 \sqrt{p^2 - 4Q_1^2Q_2^2}} \right] \times \log \left| 1 + 2 \frac{\sqrt{p^2 - 4Q_1^2Q_2^2}}{p^2 - \sqrt{p^2 - 4Q_1^2Q_2^2}} \right|,$$

where we have introduced the notation $p^2 = Q_1^2 + Q_2^2 - m_{\eta'}^2$. The logarithmic nature of the $F(y, a)$ function makes it very sensitive to the values of $Q_1^2$, $Q_2^2$, $m_{\eta'}^2$. We recommend [6] for a detailed discussion of the $\eta'g^*g^*$ vertex in the case of arbitrary gluon virtualities in the time-like, $Q_1^2 > 0$, $Q_2^2 > 0$, $p^2 - 4Q_1^2Q_2^2 > 0$, and space-like, $Q_1^2 < 0$, $Q_2^2 < 0$, $p^2 - 4Q_1^2Q_2^2 < 0$, regions.

Now, using

$$Q_1^2 \approx z \left[ (y - z)m_B^2 + \bar{y}m_{\eta'}^2 \right], \quad Q_2^2 \approx z \left[ -(y - z)m_B^2 + ym_{\eta'}^2 \right],$$

where we have neglected $m_K^2$, the dominant term in (19) is:

$$F(y, a) \approx -\frac{12}{m_{\eta'}^2} \left[ 1 - \frac{1}{2} \left( \frac{y}{a^2} + (1 - y - z) \right) \log \left| \frac{a^2 + y - z}{z(y - z)} \right| + \frac{(y - z)a^2 + (y - z)^2}{2a^2 |y - z|} \log \left| \frac{y(1 - a^2) - z + |y - z|}{y(1 - a^2) - z - |y - z|} \right| \right].$$

On the other hand, by comparing the expressions in (20), it clearly results that we are in the limit where $|Q_1^2| \gg |Q_2^2|$. So, the function $F(y, a)$ can be computed in this approximation and it simply yields

$$F(y, a) = -\frac{12}{m_{\eta'}^2} \left[ 1 + \left( \frac{y - z_0}{a^2} + \bar{y} \right) \log \left| 1 - \frac{1}{\frac{y - z_0}{a^2} + \bar{y}} \right| \right]$$

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As it can be seen from (20), the term \((y - z_0)/a^2 + \bar{y} = Q_1^2/m_{\eta'}^2\) takes a whole range of values, from \(-0.87\) to 26.5, as \(Q_1^2\) goes from the space-like to the time-like regions. Consequently, a logarithmic singularity develops as \(y \to z_0/(1 - a^2)\), i.e. for \(Q_1^2 \to m_{\eta'}^2\). Inspecting (16), we also notice the pole at \(y = z_0\) in the \(F_2^p\) contribution. In addition, while \(G[s_r,(1 - z_0)(y - z_0)]\) is divergence free for all \(s > 0\), the \(G[0,(1 - z_0)(y - z_0)]\) gets a logarithmimic singularity at \(y = z_0\). Hence, in the course of numerically evaluating the scattering contribution, one must be careful about dealing with these combined singularities in the convolution (16).

As in the case of other hard-scattering theoretical estimations [5, 13], the amplitude of this contribution contains, as main uncertainty, the peaking position, \(z_0\), in the \(B\) meson distribution function and accordingly, the branching ratio is extremely sensitive to it. For \(z_0 \in [0.063, 0.068]\) and the average value \(\alpha_s(Q_1^2) = 0.28\), the total branching ratio, including besides the improved factorization approach, the spectator hard-scattering mechanism with the vertex function (22), is in the range from \(BR(B \to \eta'K) = 6.58 \times 10^{-5}\), for \(z_0 = 0.063\), to \(BR(B \to \eta'K) = 5.8 \times 10^{-5}\), for \(z_0 = 0.068\).

Comparing these results with the experimental data [1 – 3], we notice that they are still below the lowest limit. An alternative way which increases the \(BR\) and avoids the uncertainties coming from the combined singularities in the convolution (16), would presumably look more reliable.

4 SUSY Gluonic Dipole Contribution

Employing the Minimal Supersymmetric Standard Model (MSSM), we shall add to the effective SM Hamiltonian (1), the SUSY contribution

\[
H^{SUSY} = -i \frac{G_F}{\sqrt{2}} (V_{ub} V_{us}^* + V_{cb} V_{cs}^*) \left( c_{SUSY} O_{8g} + c_{SUSY}^{\gamma} O_{7\gamma} \right),
\] (23)
expressed in terms of the usual gluon and photon operators:

\begin{align}
O_{8g} &= \frac{g_s}{8\pi^2} m_b \bar{s} \sigma_{\mu\nu}(1 + \gamma_5) G^{\mu\nu} b, \\
O_{7\gamma} &= \frac{e}{8\pi^2} m_b \bar{s} \sigma_{\mu\nu}(1 + \gamma_5) F^{\mu\nu} b,
\end{align}

(24)

and of the Wilson coefficients \cite{10, 20}

\begin{align}
c_{8g}^{SUSY}(M_{SUSY}) &= - \frac{\sqrt{2}\pi\alpha_s}{G_F(V_{ub}V_{us}^* + V_{cb}V_{cs}^*) m_\tilde{g}^2} \delta_{bs} \frac{m_\tilde{g}}{m_b} G_0(x), \\
c_{7\gamma}^{SUSY}(M_{SUSY}) &= - \frac{\sqrt{2}\pi\alpha_s}{G_F(V_{ub}V_{us}^* + V_{cb}V_{cs}^*) m_\tilde{g}^2} \delta_{bs} \frac{m_\tilde{g}}{m_b} F_0(x),
\end{align}

(25)

where

\begin{align}
G_0(x) &= \frac{x}{3(1-x)^4} \left[ 22 - 20x - 2x^2 + 16x \ln(x) - x^2 \ln(x) + 9 \ln(x) \right], \\
F_0(x) &= - \frac{4x}{9(1-x)^4} \left[ 1 + 4x - 5x^2 + 4x \ln(x) + 2x^2 \ln(x) \right]
\end{align}

(26)

In the above expressions, \(x = m_\tilde{g}^2/m_\tilde{q}^2\), with \(m_\tilde{g}\) being the gluino mass and \(m_\tilde{q}\) an average squark mass, while the factor \(\delta_{bs} = \Delta_{bs}/m_\tilde{q}^2\), where \(\Delta_{bs}\) are the off-diagonal terms in the sfermion mass matrices, comes from the expansion of the squark propagator in terms of \(\delta\), for \(\Delta \ll m_\tilde{q}^2\). In principle, the dimensionless quantities \(\delta_{bs}\), measuring the size of flavor changing interaction for the \(\tilde{s}\tilde{b}\) mixing, are present in all the SUSY corrections to the Wilson coefficients in (1) and they are of four types, depending on the \(L\) or \(R\) helicity of the fermionic partners. In the followings, we focus on the \(\delta_{bs}^{LR}\) insertions because only the SUSY Wilson coefficients (25), being proportional to the large factor \(m_\tilde{g}/m_b\), are going to make an important contribution, even for small values of \(\delta\).

In (3), we replace the Wilson coefficients \(c_{8g}^{eff}\) and \(c_{7\gamma}^{eff}\), by the total quantities

\begin{align}
c_{8g}^{total}[x, \delta] &= c_{8g}^{eff} + c_{8g}^{SUSY}(m_b), \\
c_{7\gamma}^{total}[x, \delta] &= c_{7\gamma}^{eff} + c_{7\gamma}^{SUSY}(m_b),
\end{align}

(27)
where $c^{SUSY}(m_b)$ have been evolved from $M_{SUSY} = m_{\tilde{g}}$ down to the $\mu = m_b$ scale, using the relations [10, 19]

$$
c^{SUSY}_{8g}(m_b) = \eta c^{SUSY}_{8g}(m_{\tilde{g}}),
$$

$$
c^{SUSY}_{7\gamma}(m_b) = \eta^2 c^{SUSY}_{7\gamma}(m_{\tilde{g}}) + \frac{8}{3}(\eta - \eta^2)c^{SUSY}_{8g}(m_{\tilde{g}}),
$$

with

$$
\eta = \left(\frac{\alpha_s(m_{\tilde{g}})}{\alpha_s(m_t)}\right)^{2/21} \left(\frac{\alpha_s(m_t)}{\alpha_s(m_b)}\right)^{2/23}
$$

We choose for $m_{\tilde{q}}$ the value $m_{\tilde{q}} = 500$ GeV and write $m_{\tilde{g}}$ as $m_{\tilde{g}} = \sqrt{x} m_{\tilde{q}}$ and $\delta_{LR}^{bs} \equiv \rho e^{i\varphi}$. As the total branching ratio can be expressed in terms of three free parameters: $x$, $\rho$, $\varphi$, one is able to plot the $BR^{total}$, in units of $10^{-5}$, as a function of $(\rho, \varphi)$, for different values of $x$. By inspecting the 3D plots displayed in Figure 2, for $x = 0.3$ (the upper) and $x = 1$ (the lower surface), we notice that the SUSY contributions (25) to the Wilson coefficients have significantly increased the SM value, $BR_{SM} = 3.65 \times 10^{-5}$, represented by the horizontal plane. Using the experimental data, one is able now to determine the $\delta_{LR}^{bs}$ complex values, for each $x$.

Let us take, for example, $x = 1$, pointing out that the same discussion can be performed for any $x$-value. For $\rho = 0.005$, the $BR^{total}$ is increasing from $5.1 \times 10^{-5}$, for $\varphi \approx \pm \pi/3$, to the maximum value $BR^{total} = 6.24 \times 10^{-5}$, for $\varphi = 0$. As $\rho$ goes to bigger values, we find a better agreement with the large experimental data. For $\rho = 0.01$, the data can be accommodated for $\varphi \approx -\pi/4$, while, for $\rho = 0.02$, one has to impose $\varphi \approx -8\pi/15$.

### 5 Concluding Remarks

At first, we have analyzed the $B^- \rightarrow \eta'K$ decay and computed its branching ratio using the improved factorization method developed by Beneke al. [12]. Since the obtained result, $BR_{SM} = 3.65 \times 10^{-5}$, is much below the experimental data, [1-4], we have have added new contributions.
In this respect, the so-called spectator hard-scattering mechanism, which is depicted in Figure 1, has allowed us to compute the amplitude in terms of the effective $b \to sg$ vertex and the transition form factor (11) which contains the quark contribution to the $\eta'g^*g^*$, (21), as an essential input. The total $BR$ has, as a main uncertainty, the peaking position in the $B$ meson wave function, $z_0 = \lambda_B/m_B$, with $\lambda_B = 0.35 \pm 0.15$ GeV. Even the results are closer to the experimental data, we point out the combined singularities in the amplitude convolution (16) which must be treated carefully.

Secondly, we extend the SM to the MSSM and add SUSY contributions to the Wilson coefficients $c_{sq}^{eff}$ and $c_{7\gamma}^{eff}$. The total $BR$ is expressed in terms of the parameters $x = m_\tilde{g}^2/m_\tilde{q}^2$, and $\delta_{LR}^b = \rho e^{i\varphi}$ whose contribution turns out to be important, even for very small values of $\rho$. Finally, by inspecting the 3D-graphics (see Figure 2), representing the $BR_{total}$ for $x = 0.3$ (the upper surface) and $x = 1$ (the lower surface), one is able to find numerical values for $\rho$ and $\varphi$ that can account for the experimental data or other theoretical predictions [21].

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Figure Captions

Fig.1. Feynman diagrams of the hard scattering mechanism for $B^− \rightarrow \eta'K^−$. The gluons are represented by the dashed lines.

Fig.2. Total branching ratios (SM+SUSY) for $B^− \rightarrow \eta'K^−$, in units of $10^{-5}$, as functions of $(\rho, \varphi)$, for $x = 0.3$ (the upper plot) and $x = 1$ (the lower plot), compared to the SM estimation represented by the horizontal plane.
Fig. 1  Dariescu
Fig. 2

Dariescu