Influence of CNTs grading and reinforcement on dynamic characteristics of composite Plates in thermal environment

Vasuraj Garg¹, Sanidhya Saxena², Gaurav Goyal³, Sourabh Kumar Soni⁴ and Benedict Thomas⁵

¹, ², ³, ⁴, ⁵ School of Mechanical Engineering, Vellore Institute of Technology, Vellore - 632014, Tamil Nadu, India.
E-mail: benedict.thomas@vit.ac.in

Abstract - This analytical work deals with the dynamic analysis of the functionally graded composite plate reinforced with single walled carbon nanotubes is discussed in this article. By using the extended rule of mixtures, the material properties were computed. The dimensionless frequency parameter was discovered by generating ABAQUS-solved eigenvalues and eigenvectors. The subspace process was used to produce buckling Eigenvalues for the plate. The purpose of the current research work was to analyse the influence of varying temperatures, CNTs and CNTs grading volume fractions on dimensionless frequencies and buckling factors or parameters. From the findings available in the literature, the results of this analysis have been established. CNTs volume percentage and its distribution were found to have a substantial impact on the dimensionless fundamental frequencies. Moreover, similar effect of these parameters on plate buckling were also been observed. Results reveal that the addition of CNT enhances the stiffness of the plate.

Keywords: Buckling, CNTs, FG, CNTRCs, Vibration, Temperature.

1. Introduction

Carbon nanotubes (CNTs) have been replacing various reinforcing materials in nanocomposites because of its salient characteristics. It possesses high strength, extreme stiffness, low weight and corrosion resistance [1]. Functionally graded CNTs (FG-CNTRC) constitute polymer or matrix (metallic) with CNTs reinforced with dispersal pattern could be uniform (UD) or could vary (FG-X, FG-V, FG-O and FG-Ł). The distribution of volume fraction provides extremely desired materials for most of the applications. Dey et al. [2] investigated the vibration characteristics of the FG composite rotor shaft. They found that the vibration response of the FG composite rotor shaft are significantly influenced by radial thickness, material properties and power law index. In earlier literature significant amount of research work has been observed in the characterization of the functionally-graded carbon nanotube-reinforced composites (FG-CNTRCs) structures such as plates, panels, beams and shells [3]. Yas and Samadi [4] discussed the critical buckling loads and natural frequencies reduces with increasing slenderness ratio. It has also been unveiled that while ‘X’ type distribution has higher, CeC beams have the highest frequency values. Kamarian et al. [5] investigated the vibrational responses and concluded that parameters μ and η which are associated with agglomeration have an adverse effect on vibrational responses but not so much on the wavenumber. Mirzaei and Kiani [6] found that the panel frequencies are directly proportionate to the percentage of CNT and significantly depends on the pattern of distribution across the depth.

Keleshteri et al. [7] reveal that results of the CNTRC sector plate can be improved by variation in the percentage of CNTs and type of distribution. Vibrational redistribution is the cause of reduced vibration amplitude while nonlinear frequency increases. Fantuzzi et al. [8] have investigated the problem of arbitrary shapes and agglomeration in the present work. The former is done by Non-Uniform Rational B-splines while the latter is done by Generalized Differential Quadrature. Li and Hu [9] given emphasis on stiffness-hardening and stiffness-softening effects that are shown by FG material and conclude that the non-linearities are due to stretching effects. The asymmetric formulation is provided in order to study the vibrational responses under thermal loading which is performed on the annular FG-CNTRC plates by Torabi and Ansari [10]. Zhu et al. [11] used an independent finite element formulation method, which was very novel and the results are used as a reference in the present work. The fact that FE formulated results converge well with the software ANSYS became the basis for choosing the work as a benchmark. Lei et al. [12] tabulated the response due to buckling of FGCNT plates in different loading conditions, mesh size and aspect ratio. Thomas
and Roy [13] studied the vibration damping response of FG-CNTRC shell structures. They found that the addition of nanotubes in the conventional carbon fiber matrix results in good damping and vibration response of the resulting composites structures. Thomas and Roy [14] investigated the vibration characteristics of the FG-CNTRC shell structures (ellipsoidal, spherical, cylindrical and doubly curved). Their findings demonstrated that the vibration and damping characteristics of the FG-CNTRC shell structures were significantly affected by the $V_{fcnt}$ and CNT distribution. Thomas et al. [15] investigated the thermal analysis of FG-CNTRC Timoshenko beam. By employing the power law temperature distribution they examined the influence of $V_{fcnt}$ and CNT orientation on the stresses and strains.

The convergence of results with previous literature help in referring to the tabulated outcomes and hence forms a reference for the work presented here. The present work provides an insight by varying temperature, $V_{fcnt}$ and CNT grading on dimensionless frequencies and buckling factors (parameters) and what effects it has on them.

2. Formulations and Techniques

2.1 Modeling of Material Properties

In the present analysis, it is supposed that the composite plate under consideration is comprised of epoxy resin and CNTs as fibers. Also, the material characteristics are smoothly distributed over the thickness of the plate.

2.2 Distribution of Material

Here, Uniform type of distribution (UD) and graded distributions (FG-X type and FG-O type) of CNTs over the thickness direction of the nano-composite plate were used to evaluate the effect of various CNTs distributions on the dynamic behaviour of the plate. The $V_{fcnt}$ of different types of nanocomposite plate can be written as:

$$V_{fcnt} = \begin{cases} V_{f\text{UD}} & (UD), \\ \frac{4}{h}V_{f\text{FG-X}} & (FG-X), \\ \frac{5}{2h} - \frac{2}{h}V_{f\text{FG-O}} & (FG-O), \end{cases}$$

At this point, $V_{f\text{UD}}$ is a function of CNTs volume percentage, its mass portion and its density. $V_{fcnt}$ is the volume fraction of CNTs.

2.3 Extended rule of mixture

The material properties of proposed composite plates were found by applying the extended rule of mixture (EROM), which is expressed as:

$$E_{11} = h_{1}V_{f11}E_{11}^{\text{CNT}} + V_{m}E_{m},$$

$$E_{22} = \frac{V_{f12}}{E_{22}^{\text{CNT}}} + \frac{V_{m}}{E_{m}},$$

$$h_{1} = \frac{V_{f12}}{G_{12}^{\text{CNT}}} + \frac{V_{m}}{G_{m}},$$

Since Poisson’s ratio does not depend strongly on the position, $u_{12}$ can be assumed as:

$$u_{12} = V_{f12}u_{12}^{\text{CNT}} + V_{m}u_{m}.$$
Figure 1. Distribution of CNT in various kinds of CNTRC plates. (a) UD type; (b) FG-X type; (c) FG-O type

**Figure 1(a)** depicts the UD grading, where the percentage/fraction of CNTs is throughout constant. **Figure 1(b)** presented the FG-X type distribution in which has variation similar to alphabet X type fraction (i.e. highest on the top and bottom surfaces while least at the middle). **Figure 1(c)** depicts the FG-O type which is exactly opposite of X type (i.e. CNTs are plentifully present in the mid but lightly at the top regions).

### 2.4 Vibration Investigation

After evaluating the effective elastic properties of the proposed composite plate, finite element modelling and investigation has been performed using FEA software (ABAQUS). Finally, the vibration examination is performed using the below mentioned governing equation of motion:

$$\left[ M_e \right]\ddot{x} + \left[ K_e \right]x = \{ F \}$$  \hspace{1cm} (3)

here $\left[ M_e \right]$ signifies the global mass matrix, $\left[ K_e \right]$ represents the global stiffness matrix.

In this research work, the natural frequency of the FGCNT plate is converted to a dimensionless number (parameter) by employing the formula:

$$-w = w_0 \frac{\frac{E_m A}{h}}{K \frac{A}{h} E_m}$$  \hspace{1cm} (4)

Where $E_m$ = matrix elastic modulus at different temperatures.

### 2.5 Investigation of Buckling

After the vibrational analysis, buckling response of proposed composite plates are computed by solving the following equation of eigenvalue:

$$\gamma K - \lambda K g \frac{\partial^2}{\partial x^2} = 0$$

Where $\gamma$ = critical buckling loads, $X$ = modes of buckling , $Kg$ = Geometric stiffness matrix, obtained using the relation:

$$K_g = \frac{\partial^4}{\partial x^4} - \frac{\partial}{\partial x} \frac{\partial^3}{\partial x^3}$$  \hspace{1cm} (5)
In this work, a dimensionless critical buckling factor (parameter) is given by:

\[ \bar{N}_{cr} = \frac{N_{cr} b^2}{E_h h} \]  

(6)

2.6 Modelling using FEA software (ABAQUS)

Figure 2 presents the proposed model of the laminated composite plate with 0° orientation angle with 10 layers (plies), which explains the modeling, orientation at which the investigation has been conducted.

![Figure 2. Proposed composite plate model in ABAQUS](image)

Figure 2 shows the stacking sequence comprises ten layers arranged composed of same thickness (each plies relative thickness is thus \( t=0.1 \) or 10%). Red lines reflected horizontally in the figure show the orientation angle of the respective plies. Though all the lines are analogous, the angle of orientation was 0/0/0/0/0/0/0/0/0. The end supporting condition of C-C-C-C depicted that the all four sides of the nanocomposite plates are clamped while doing the free vibration response investigation. In this study three kind of grading patterns (UD, FG-X and FG-O) have been employed for the preparation of the plates. The grading is supposed to over the entire thickness (i.e. \( +h/2 \) to \( -h/2 \)). The dimensions of the nanocomposite plate is length = \( a \) and width= \( b \), as square plates are considered in this analysis, so \( a=b \).

3. Results and Discussion

Table 1 depicts the validation and convergence behavior of natural frequency at 300K for a/h = 50 for UD and graded plates with different \( V_{\text{cnt}} \). The observed amount of the dimensionless frequency parameter agreed well with those available in the literature [11].

Table 1. Validation of dimensionless natural frequency at 300K for UD-CNT with different \( V_{\text{cnt}} \).

| Volume percentage of CNT | Natural frequency | M-1  | M-2  | M-3  | M-4  |
|-------------------------|-------------------|------|------|------|------|
|                         | Present           | 43.55| 49.32| 63.94| 88.98|
|                         | Ref. [11]         | 39.73| 43.88| 54.77| 74.49|
| 11                      | Present           | 47.94| 53.06| 67.04| 92.21|
|                         | Ref. [11]         | 43.58| 47.48| 57.97| 77.40|
| 14                      | Present           | 51.62| 56.42| 69.92| 94.77|
|                         | Ref. [11]         | 49.07| 54.32| 68.07| 92.97|
Here, M-1, M-2, M-3, M-4 represents the mode numbers and will be used to represent mode numbers in this article.

Table 2 represents the dimensionless frequency parameter for different $V_{\text{fcnt}}$, grading type for various mode numbers. It is evident from the Table 2 that as the $V_{\text{fcnt}}$ increases the frequency also increases. Variation of frequency parameter with different mesh sizes are shown in Table 3.

Table 2. The dimensionless frequency parameter for different $V_{\text{fcnt}}$, grading type for various mode numbers.

| Temp. | $V_{\text{fcnt}}$ % | CNT Grading | Mode Number |
|-------|-------------------|-------------|-------------|
|       |                   |             | 1 | 2 | 3 | 4 | 5 | 6 |
| 300   | 11                | UD          | 43.55 | 49.32 | 63.94 | 88.98 | 109.36 | 111.85 |
|       |                   | FG-O        | 27.55 | 36.19 | 54.74 | 69.86 | 74.48 | 83.24 |
|       |                   | UD          | 47.94 | 53.06 | 67.04 | 92.21 | 119.39 | 120.82 |
|       |                   | FG-O        | 27.96 | 36.52 | 54.99 | 71.08 | 75.61 | 83.45 |
|       |                   | UD          | 51.62 | 56.42 | 69.92 | 94.77 | 127.34 | 128.46 |
|       |                   | FG-O        | 29.60 | 37.74 | 55.78 | 75.25 | 79.38 | 83.98 |
|       | 14                | UD          | 43.20 | 49.03 | 63.74 | 88.86 | 108.60 | 111.14 |
|       |                   | FG-O        | 47.47 | 52.86 | 66.93 | 91.60 | 118.18 | 120.34 |
|       |                   | FG-O        | 27.32 | 36.08 | 54.50 | 75.25 | 79.04 | 82.33 |
|       | 17                | UD          | 51.18 | 56.26 | 69.88 | 94.26 | 126.30 | 128.18 |
|       |                   | FG-O        | 29.35 | 37.62 | 55.54 | 74.42 | 79.04 | 83.07 |
| 500   | 14                | UD          | 47.47 | 52.86 | 66.93 | 91.60 | 118.18 | 120.34 |
|       |                   | FG-O        | 27.32 | 36.08 | 54.50 | 69.09 | 74.18 | 82.33 |
|       |                   | UD          | 51.18 | 56.26 | 69.88 | 94.26 | 126.30 | 128.18 |
|       |                   | FG-O        | 29.35 | 37.62 | 55.54 | 74.42 | 79.04 | 83.07 |
|       |                   | UD          | 43.02 | 48.86 | 63.61 | 88.76 | 108.17 | 110.71 |
|       |                   | FG-O        | 24.99 | 34.38 | 53.39 | 62.79 | 68.53 | 81.52 |
|       |                   | FG-O        | 47.28 | 52.69 | 66.80 | 91.52 | 117.77 | 119.94 |
|       | 17                | UD          | 50.98 | 56.08 | 69.75 | 94.17 | 125.88 | 127.78 |
|       |                   | FG-O        | 29.24 | 37.54 | 55.48 | 74.13 | 78.78 | 83.04 |
| 700   | 14                | UD          | 47.47 | 52.86 | 66.93 | 91.60 | 118.18 | 120.34 |
|       |                   | FG-O        | 27.32 | 36.08 | 54.50 | 69.09 | 74.18 | 82.33 |
|       |                   | UD          | 51.18 | 56.26 | 69.88 | 94.26 | 126.30 | 128.18 |
|       |                   | FG-O        | 29.35 | 37.62 | 55.54 | 74.42 | 79.04 | 83.07 |
|       |                   | UD          | 43.02 | 48.86 | 63.61 | 88.76 | 108.17 | 110.71 |
|       |                   | FG-O        | 24.99 | 34.38 | 53.39 | 62.79 | 68.53 | 81.52 |
|       |                   | FG-O        | 47.28 | 52.69 | 66.80 | 91.52 | 117.77 | 119.94 |
|       | 17                | UD          | 50.98 | 56.08 | 69.75 | 94.17 | 125.88 | 127.78 |
|       |                   | FG-O        | 29.24 | 37.54 | 55.48 | 74.13 | 78.78 | 83.04 |

Table 3. Variation of frequency parameter with different mesh sizes.

| CNT % | CNT Grading | Mesh Size  |
|-------|-------------|------------|
|       |             | 8 × 8 | 16 × 16 | 24 × 24 | 32 × 32 |
| 11    | UD          | 43.63159 | 43.54778 | 43.4659 | 43.458 |
|       | FG-X        | 50.6053 | 50.51882 | 50.4995 | 50.4933 |
|       | FG-O        | 28.1 | 27.96348 | 27.932 | 27.9237 |
|       | UD          | 48.052 | 47.93973 | 47.9138 | 47.9093 |
| 14    | FG-X        | 55.2792 | 55.23794 | 55.2298 | 55.2241 |
|       | FG-O        | 27.6838 | 27.54566 | 27.5152 | 27.50731 |
|       | UD          | 51.7019 | 51.6191 | 51.6006 | 51.5946 |
| 17    | FG-X        | 59.2274 | 59.24614 | 59.2464 | 59.2464 |
|       | FG-O        | 29.7428 | 29.59782 | 29.5656 | 29.5579 |
After the free vibration investigation of the nanocomposite plate, buckling investigation has been conducted and the results of buckling parameters are validated in Table 4.

**Table 4.** Validation of buckling parameter at 500K for \( V_{f_{cnt}} \) (11\%) which is UD graded subjected to uniaxial compression.

| Buckling parameter | M-1     | M-2     | M-3     | M-4     |
|--------------------|---------|---------|---------|---------|
| Present Study      | 24.23   | 32.976  | 41.988  | 47.198  |
| Ref. [12]          | 24.95   | 33.4917 | 47.8115 | 50.0545 |

After the validation critical buckling factor (parameter) at diverse temperatures with \( a/h=10 \) and for changing \( V_{f_{cnt}} \), grading type under uniaxial compression are tabulated in Table 5.

**Table 5.** The critical buckling factor (parameter), \( N_{cr} \) at several temperatures for varying \( V_{f_{cnt}} \), grading type subjected to uni-axial compression.

| Temp. | CNT % | CNT Distribution | M-1 | M-2 | M-3 | M-4 |
|-------|-------|------------------|-----|-----|-----|-----|
| 300   | 11    | UD               | 24.55 | 33.26 | 42.19 | 47.33 |
|       |       | FG-O             | 10.25 | 10.51 | 16.32 | 23.69 |
| 14    |       | UD               | 28.89 | 37.39 | 45.70 | 50.24 |
| 17    |       | FG-O             | 10.46 | 10.99 | 17.53 | 25.01 |
| 11    |       | UD               | 32.80 | 40.93 | 48.70 | 52.80 |
| 17    |       | FG-O             | 10.91 | 11.67 | 17.16 | 24.72 |
| 11    |       | UD               | 24.23 | 32.98 | 41.99 | 47.20 |
| 17    |       | FG-O             | 9.18  | 9.44  | 15.27 | 22.43 |
| 500   | 14    | UD               | 28.46 | 37.04 | 45.46 | 50.09 |
|       |       | FG-O             | 10.19 | 10.40 | 16.24 | 23.61 |
| 17    |       | UD               | 32.41 | 40.68 | 48.60 | 52.81 |
|       |       | FG-O             | 10.85 | 11.55 | 17.09 | 24.64 |
| 11    |       | UD               | 24.06 | 32.81 | 41.85 | 47.09 |
|       |       | FG-O             | 9.13  | 9.41  | 15.24 | 22.40 |
| 700   | 14    | UD               | 28.28 | 36.89 | 45.36 | 50.03 |
|       |       | FG-O             | 10.16 | 10.35 | 16.34 | 23.58 |
| 17    |       | UD               | 32.21 | 40.53 | 48.51 | 52.75 |
|       |       | FG-O             | 10.82 | 11.49 | 17.06 | 24.61 |

After validation, the modal shapes have been plotted and depicted in Figure 3 for the first 6 fundamental frequencies.
Figures 4-6 depicts the dimensionless frequency w.r.t. mode number with different CNT volume percentages and for different CNT grading (UD and FG-O) for temperature 300 K. It is evident from these figures that as the $V_{\text{cnt}}$ the natural frequency also increases. It is also perceived that the CNT grading profile have a noteworthy impact on the vibrational features of the nanocomposite plate.

Figure 4. Dimensionless frequency variation with Mode Number of UD at 300 K
Figure 5. Dimensionless frequency variation with mode number of FG-O at 300 K

Dimensionless frequency variation with mode number of UD at various temperatures and volume contents of CNTs is shown in Figure 6. Also, dimensionless frequency variation with mode number of FG-O type distribution at at various temperatures and volume contents of CNTs is shown in Figure 7. It is evident from both figures that the temperature and percentage of CNT have an adverse effect on dimensionless frequency. As the percentage of CNT is enhanced, the frequency also gets enhanced, whereas it reduces with the rise in temperature.

Figure 6. The dimensionless frequency vs mode number of UD type with several temperatures and volume percentages of CNTs.
**Figure 7.** The dimensionless frequency vs Mode Number of FG-O type with several temperatures and volume percentages of CNTs.

Further dimensionless frequency are also plotted with respect to mesh size at 300 K, 500 K and 700 K in Figures 8-10, which reveals that frequency is highest for the higher volume fraction of CNT.

**Figure 8.** Dimensionless frequency vs Mesh Size at 300K

**Figure 9.** Dimensionless frequency vs Mesh Size at 500K
After vibration analysis for the plate under consideration, buckling analysis was also carried out and the variation of Buckling Parameter with mode number is shown in Figure 11 and Figure 12 for UD and FG-O kind of grading respectively. It is perceived from figures that for both type of grading as the $V_{f,\text{min}}$, buckling parameters also enhances for different temperature.

**Figure 10.** Dimensionless frequency vs Mesh Size at 700K

**Figure 11.** The Buckling parameter vs Mode number of UD
4. Conclusions

The aim of this work is to research the effect of different CNT distribution and volume fractions on the dynamic characteristics of the composite plate reinforced with CNTs. After finding the effective material properties and validation of results, vibrational analysis is conceded in order to analyse the effect of various parameters including CNTs volume (11%, 14% and 17%), distribution (UD, FG-X and FG-O) and temperatures (300 K, 500 K and 700 K) on the dynamic behaviour of the composite plate with constant a/h ratio (a/h=50). Considering the obtained results of vibrational study it is witnessed that FG-X type show the maximum values for dimensionless frequency parameters at any percentage of $V_{fcnt}$ and temperature, followed by UD and FG-O. For the same temperature and a / h ratio, the non-dimensional frequency values increase as with higher percentage of CNT and results in a stiffer plate. For a constant $V_{fcnt}$ and for the identical distribution type with increasing temperature, the natural frequency decreases. Meshing size does not show any prominent variation, but as mesh size decreases and mesh quantity increases, it results in reduction in frequency.

Buckling analysis on the identical plate keeping a/h =10 was performed to analyse the buckling effect. The original findings have been confirmed and the results were obtained. It is witnessed that the buckling factor for a specified CNTs fraction reduces as the temperature rises. For a given temperature, the buckling parameter value rises as the CNTs fraction rises. The buckling factor (parameter) for UD grading is found considerably greater than FG-O type at a particular temperature and CNTs fraction.

Finally, it can be inferred, based on the results obtained, that the CNTs volume and its grading show a significant role in modifying vibrational and buckling properties of resulting composite.

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Figure 12. Buckling Parameter vs Mode Number of FG-O
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