Quench-induced trapping of magnetic flux in annular Josephson junctions

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Abstract. The aim of the project is to investigate spontaneous symmetry breaking in non-adiabatic phase transitions (Kibble-Zurek processes). A long and narrow annular Josephson tunnel junction is subjected to repeated thermal quenches through the normal-superconducting transition. The quench rate is varied over 4 orders of magnitude. After the quench the result of the spontaneous production of topological defects, trapped fluxons, is unambiguously observed as zero-field steps in the DC I-V characteristic of the junction. A power-law scaling behavior of trapping probability versus quench rate is found with a critical exponent of 0.5 (within experimental error). The main experimental challenges are to generate many identical quenches with accurate cooling rate, to automate data analysis and acquisition, and to suppress external magnetic fields and noise by passive magnetic shielding and compensation.

1. Introduction
The purpose of the project is to experimentally clarify the effects of non-adiabatic cooling of physical systems through continuous phase transitions. Because all phase transitions are completed within a finite time, causality ensures that the correlation lengths in the system remain finite - even for continuous phase transitions. This reflects the fact that a growing domain represents information transfer from its initial seed to the media around it, and as such is ultimately limited by the information velocity in the medium. The Kibble-Zurek (KZ) scenario [1, 2, 3] proposes that this maximum velocity of domain boundaries is also the actual velocity, thus predicting that the final domain structure directly reflects the causal horizons of each nucleation site. This prediction is directly available for experimental verification in systems in which the domain boundaries carry a topological charge.

As a solid state model system, the annular Josephson tunnel junction has been selected. Josephson tunnel junctions (JTJs) in general have fluxons as topological defects carrying the topological charge i.e., a supercurrent vortex carrying magnetic flux \( \Phi_0 = h/2e \) in the plane of the oxide layer between the two superconductors that make up the JTJ. For the annular geometry considered in this paper, this corresponds to a magnetic flux line threading either one of the superconducting annuli making up the junction an odd number of times. The motivation for using long annular Josephson junctions is that - because of the annular geometry - any
fluxons created during the quench, will be trapped in the junction until it is reheated, because of the Meissner effect of the superconducting annuli. This is schematically illustrated in figure 1(a).

![Simplified illustration of a trapped fluxon in an annular Josephson tunnel junction.](image)

(a) Simplified illustration of a trapped fluxon in an annular Josephson tunnel junction. It is seen that the magnetic flux loop threads only one of the rings.

![Actual geometry of annular Josephson tunnel junction no. 4, designated JJ4.](image)

(b) Actual geometry of annular Josephson tunnel junction no. 4, designated JJ4. The geometry of no. 3 (JJ3) is similar, only the ground plane is without circular hole.

**Figure 1.** Model and real geometry of the annular Josephson junctions. In both cases, the perpendicular dimension is exaggerated.

The creation of an fluxon, and thereby a closed, oriented magnetic loop, into the system is an example of a spontaneous symmetry breaking event; the perpendicular up-down symmetry of the system is broken by the directionality of the B-field. A key point to make is that the symmetry-breaking mechanism itself (ie. the cooling) is symmetrical, and thus is not the direct cause of the break in symmetry.

## 2. Theory

An idealised model was proposed in 2000 [4] to test the KZ scenario for annular JTJs, assuming that causal horizons are constrained only by the velocity of electromagnetic waves in the junction, the Swihart velocity [5, 6]. As a result, the probability $f_1$ for spontaneously producing one fluxon in the thermal quench of a symmetric annular Josephson junction of circumference $C$ was predicted to scale with the quench time $\tau_Q$ (the inverse quench rate) as

$$f_1 \simeq \frac{C}{\xi} = \frac{C}{\xi_0} \left( \frac{\tau_Q}{\tau_0} \right)^{-\sigma},$$

(1)

where the scaling exponent $\sigma$ depends on the system. In Eq.(1), $\xi$ is the Kibble-Zurek causal length, the correlation length of the relative Josephson phase angle at the time of defect formation. The relative Josephson phase angle is the space-dependent difference in phase between the superconducting wavefunctions of the two rings making up the annular Josephson junction. The trapping probability is defined through Eq.(1) in terms of the zero-temperature correlation length $\xi_0$, the relaxation time $\tau_0$ of the long wavelength modes of the superconducting wavefunction, and the quench time $\tau_Q$, in turn defined by: $T_C/\tau_Q = -(dT/dt)_{T=T_C}$. Eq.(1) holds for $C < \xi$, where $\xi$ is the frozen-in correlation length, roughly equivalent to the size of fluctuations in the superconducting wavefunction at the time of freezing. The probability $f_1$ is the sum of the probabilities of all the different ways of threading the geometry by a closed magnetic field loop, in such a way, that the field loop intersects the tunnel barrier oxide just once, regardless of direction.
In theory, the creation of a fluxon with direction in through the oxide towards the center of the annulus, and the creation of one with opposite direction should be equally probable. However, because of the detection mechanism inherent to this setup, there is no obvious way to discriminate between the two states. Using the current detection scheme (see section 3.3) results in the same outcome regardless of the directionality of the fluxon.

3. Measurements
The actual experimental procedure consists of a large number of heating-cooling-measurement cycles for different cooling rates. For each cycle, the sample is electrically heated, passively cooled without electrical connections and finally the actual detection takes place.

3.1. Thermal control
The thermal part of the cycle is performed by electrical heating by resistive strips in either end of the sample, as seen in figure 2. In order to avoid electromagnetic noise induced by current noise in the heaters, the heating pulse is mechanically switched off outside of the cryoprobe during cooling and measurements. Cooling takes place passively through the helium exchange gas (approximately 7-8 mbar of pressure) as well as through the copper sample holder. The cooling rate can be controlled by the exchange gas pressure and the heating time. For longer heating times, more of the copper sample holder is heated along with the silicon sample. This gives a larger effective heat capacity of the total system, and thus a slower cool down.

![Figure 2.](image)

For slow quenches, a solid-state germanium resistor mounted in the copper sample holder is used to determine $\tau_Q$. However, for faster quenches, in which $\tau_Q$ is less than a few seconds, thermal equilibrium is not obtained between the sample and the sample holder. For this, a much faster thermometer is required. This is supplied by the device itself, as the Josephson junction can be used as a transition edge thermometer in constant-current bias mode. The junction is biased at 25-30% of the maximum gap current, and the voltage response as a function of time is measured, see fig. 3(a). This can be directly converted into a temperature as long as $T < T_C$ for the junction [10]. As $\tau_Q$ is decided by the time-derivative of the temperature of the device at $T = T_C$ an extrapolation is performed to this using a simple thermal relaxation model. In general, the fit to the thermal model is excellent (correlation coefficient, $R^2 > 0.98$) as is seen in fig. 3(b).
3.2. Magnetic shielding and compensation

The magnetic shielding is implemented using a combination of cryoperm and superconducting shields. The cryostat has 2 layers of cryoperm built into it. The cryoprobe itself incorporates 3 more passive magnetic shields - first a superconducting Pb shield, then a cryoperm shield, and finally another superconducting Pb shield. The alternation of cryoperm and superconducting shields makes for a large suppression of both AC and DC magnetic fields.

In addition a superconducting coil has been built around the sample for compensation. The general approach to compensating magnetic fields is to assume the field is low. It is assumed that in this limit, any increase in B-field strength will increase trapping probability through induced trapping. Therefore, the compensation magnetic field, perpendicular to the sample, can be calibrated to the nearest minimum in fluxon trapping probability. A thorough theoretical treatment of this is in preparation [9].

3.3. Detection

The detection of trapped fluxons in the annular Josephson junction is performed by detection of zero-field steps. These are generated by internal resonances in the junction only available in the presence of trapped fluxons. For a theoretical description of this see [6, 7].

Zero-field steps in an annular Josephson junction can be seen in the DC I-V curve of the junction as voltage steps at voltages decided by the geometry of the junction itself. For the junctions in question, the voltages of the first zero-field step are on the order of a few to several tens of microvolts. The detection of zero-field steps is performed by an algorithm developed for the purpose. Detection accuracy is ≈95% measured against manual evaluation of the I-V curves.

4. Results

As expected, a minimum fairly close to zero is seen in the dependence of trapping rate, $f$, on B-field strength (fig. 4). However, the side peaks were not expected. It is seen, that when the magnetic field reaches a certain value, the trapping rate decreases with increasing magnetic field. This somewhat counterintuitive behaviour is currently being incorporated into a unified model of the KZ effect in the presence of a pre-existing symmetry-breaking field, and a paper is in preparation [9].
Figure 4. The variation of trapping rate with magnetic field. The central minimum is clearly expressed, as well as two side peaks.

Figure 5. A clear, systematic scaling of single-fluxon trapping probability as a function of quench time is found. The critical exponent is $0.51 \pm 0.03$.

The main result of the experiments is shown on figure 5. It represents the condensation of a few 100,000 thermal quenches. The points are very closely distributed around the prediction of eq. (1). The different markers indicate different samples and/or junctions used. They are, however, all of the same basic geometry as shown in figure 2.

5. Discussion
In order for this system to be a good model for a general continuous phase transition a number of high-level assumptions were made: (1) The phase transition of the annular Josephson junction
is continuous, (2) The temperature is at all times approximately constant over a length larger than the dimensions of the junction, (3) The perpendicular symmetry of the junction is not broken by residual magnetic fields.

It is possible to question all of the above assumptions on different grounds. The first assumption requires that there is no latent heat involved in the phase transition. This is true for bulk superconductor in zero magnetic field. However, for a multiply connected superconductor, such as an annulus, in which a magnetic flux loop is spontaneously generated, this is only approximately true. The kinetic energy of the cooper pairs in the screening current in the annulus should be taken into account as a rather exotic form of latent heat, if a fluxon is trapped, and it turns out that this energy is on the same order as the thermal energy $k_BT$ at the transition. The second assumption is essentially a question of the timescales involved in the heating as compared to the heat transfer along the sample from the heaters on either end. If the time constant for heat transfer along the length of the sample is significantly lower than the quench time, $\tau_Q$, the assumption is fulfilled. This can be verified by either heat-flow simulations or analytical approximations. Finally, the third assumption seems to be the hardest to fulfill. One has to find a logical limit to how much magnetic field should be allowed, without expecting it to interfere with the phase transition. The simplest choice is to require the magnetic flux through the ring to be significantly less than one magnetic flux quantum per ring area, $B < \Phi_0/A_{ring} = h/2e$. This has been tested by placing a SQUID device of higher sensitivity in the immediate vicinity of the sample. The SQUID has been rotated to measure the absolute value of the B-field as well as the noise level, and both have been seen to be below this limit. One final note should be to mention that the pulsed current running through the heaters will induce an oscillating magnetic field in the coil, and should therefore be temporally separated from the phase transition by a time larger than a few times the relaxation time of the coil system.

6. Conclusion
It has been shown that the probability of trapping a fluxon in an annular Josephson tunneling junction scales with a power law dependence on the inverse cooling rate at the time of transition in good agreement with the prediction of Zurek [1, 2]. In particular the critical exponent of this system is 0.5 to a great accuracy. It has been shown on different samples and on annular junctions of different sizes and detailed geometry.

This experiment remains to this day the only solid-state experiment to show reliable Kibble-Zurek scaling behaviour.

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