Model Investigation of Non-Thermal Phase Transition in High Energy Collisions

Wang Qin    Li Zhiming    Liu Liangshou
(Institute of Particle Physics, Huazhong Normal University, Wuhan, 430079)

Abstract

The Non-thermal phase transition in high energy collisions is studied in some detail in the framework of random cascade model. The relation between the characteristic parameter $\lambda_q$ of phase transition and the rank $q$ of moment is obtained using Monte Carlo simulation, and the existence of two phases in self-similarly cascading multiparticle systems is shown. The relation between the critical point $q_c$ of phase transition on the fluctuation parameter $\alpha$ is obtained and compared with the experimental results from NA22. The same study is carried out also by analytical calculation under central limit approximation. The range of validity of the central limit approximation is discussed.

Keywords random cascade multifractal anomalous scaling non-thermal phase transition
Recently, the prediction\cite{1} that there exist the property of self-affine fractal in the anisotropic phase space of multiparticle final states in high energy hadron-hadron collisions has been confirmed by experiments\cite{2,3}. This breakthrough in the nonlinear study of high energy physics places the further study of nonlinear property of multiparticle final states on the agenda.

In this respect, the non-thermal phase transition\cite{4,5} is a problem worthy while further study. In the presently available experiments\cite{6}, due to the restriction of energy, the average multiplicity is very low, and the rank of the factorial moments could not be high. So, no clear evidence of non-thermal phase transition has been seen. The new Large Hadron Collider (LHC), which is being built and will be put into operation in the beginning of next century, will dramatically raise the collision energy and multiplicity, providing perfect condition for the study of non-thermal phase transition. For a theoretical preparation it is necessary to carry on detailed discussion on this phase transition and to clarify its property.

The aim of this short paper is to make a model study of the non-thermal phase transition, especially to make clear of the relation between the critical point of non-thermal phase transition and the strength of dynamical fluctuations.

The random cascading $\alpha$ model is widely used in the study of nonlinear property of multiparticle final states in high energy collisions. Using this model, it is easy to get a system possessing the property of intermittency and fractal. We will show that non-thermal phase transition does exist in this system and the relation between the critical point of phase transition and the parameter $\alpha$ of fluctuation strength in the model can thus be obtained and compared with the experimental data.

Firstly, let us briefly remind the random cascading $\alpha$ model\cite{7} with probability conservation.

Consider a region $\Delta$ of one-dimensional phase space. Devide it into $\lambda$ cells. The probability of particles falling into the $i$th cell is

$$p_i = p_0 \omega_i,$$

(1)

where $p_0 = 1$ is the probability in the phase space region $\Delta$, $\omega_i$ is the probability of the elementary partition. Next, we divide each sub-bin into $\lambda$ even smaller sub-bins. The probability in the $ij$th bin ($i = 1, 2, \ldots, \lambda; j = 1, 2, \ldots, \lambda$) is

$$p_{ij} = p_i \omega_j,$$

(2)

After $\nu$ steps, the probability in a sub-bin is

$$p_{i_1 i_2 \ldots i_\nu} = \prod_{k=1}^{\nu} \omega_{i_k}.$$

(3)
The total number of intervals is $M = \lambda^\nu$.

In order to guarantee the conservation of probability in each step of cascading, we choose the elementary probability $\omega_i$ for $\lambda = 2$ as:

$$\omega_1 = \frac{1 + \alpha r}{2}, \quad \omega_2 = \frac{1 - \alpha r}{2}. \quad (4)$$

where $r_i$ is a random number distributed uniformly in the interval $[-1, 1]$, $\alpha$ is a model parameter describing the strength of nonlinear dynamical fluctuations ($0 < \alpha < 1$).

The definitions of the probability moments and factorial moments are:

$$C_q = \frac{1}{M} \left\langle \sum_{m=1}^{M} p_m^q \right\rangle = M^{q-1} \left\langle \sum_{m=1}^{M} p_m^q \right\rangle. \quad (5)$$

$$F_q(M) = \frac{1}{M} \sum_{m=1}^{M} \frac{\left\langle n_m (n_m - 1) \cdots (n_m - q + 1) \right\rangle}{\langle n_m \rangle^q}. \quad (6)$$

It can easily be proved that under the assumption of Poisson or Bernoulli type of statistical fluctuations the normalized factorial moments $F_q$ are equal to the normalized probability moments $C_q$.

The character of dynamical fluctuations can be expressed as the anomalous scaling of probability (or factorial) moments:

$$C_q(M) \propto M^{\varphi_q},$$

or equivalently

$$\ln C_q(M) = A + \varphi_q \ln M \quad (M \to \infty). \quad (7)$$

where $\varphi_q$ is called intermittency index.

In order to see the anomalous scaling of probability moments more clearly, we choose the fluctuation-strength parameter $\alpha = 0.5$, the elementary partition number $\lambda = 2$, the division step $\nu = 12$, the ranks of moment $q = 5, 10, 15, 20, 25, 30$, and make use of Eq.(5) to simulate the relation $\ln C_q \sim \ln M$. The results are shown in Fig.1.

The intermittency parameters $\varphi_q$ are obtained through linear fit. We can see from the figure that the higher the rank $q$ is, the larger the slope $\varphi_q$ is.

A parameter $\lambda_q$ has been introduced [4,5] in the multifractal analysis to characterise the non-thermal phase transition in the multiparticle systems. It is related to the intermittency index $\varphi_q$ by the relation

$$\lambda_q = (\varphi_q + 1)/q. \quad (8)$$

We will try to evaluate this parameter both through analytic calculation and by using Monte Carlo simulation.
In the random cascading α model, the probability moment is:

\[ C_q (M) = \frac{\langle \omega^q (1) \cdots \omega^q (\nu) \rangle}{\langle \omega \rangle^{q\nu}}. \quad (9) \]

It can be rewritten as:

\[ C_q (M) = \lambda^{q\nu} \langle \omega^q (1) \cdots \omega^q (\nu) \rangle = \lambda^{q\nu} \left( \exp \left( -\sum_{i=1}^{\nu} q \varepsilon_i \right) \right), \quad (10) \]

where \( \varepsilon_i = -\ln \omega (i) \). The parameter \( \zeta = \sum_{i=1}^{\nu} q \varepsilon_i \) in the above equation is the sum of \( \nu \) random numbers. Under the central limit approximation \( \zeta \) approaches to Gaussian distribution:

\[ C_q (M) = \lambda^{q\nu} \langle e^{-q\zeta} \rangle = \exp \left( q\nu \ln \lambda + \frac{\nu q^2 \sigma^2}{2} - q \zeta \right). \quad (11) \]

Using \( C_1 (M) = 1 \), we get

\[ C_q (M) = e^{\nu q^2 \sigma^2 / 2}. \quad (12) \]

The intermittency indices can be deduced as:

\[ \varphi_q = \frac{(q - 1) q \sigma^2}{2 \ln 2}. \quad (13) \]

We have also the relation

\[ \sigma^2 = \langle \ln^2 \omega \rangle - \langle \ln \omega \rangle^2 = \frac{1}{3} \sigma^2 + \frac{2}{3} \sigma^4 + \cdots. \]

Under linear approximation\(^9\) it becomes \( \sigma^2 = \alpha^2 / 3 \). Substituting into Eq.(13) we get

\[ \varphi_q = \frac{q (q - 1) \alpha^2}{6 \ln 2}. \quad (14) \]

from eq.(8):

\[ \lambda_q = \frac{\varphi_q + 1}{q} = \frac{(q - 1) \alpha^2}{6 \ln 2} + \frac{1}{q}. \quad (15) \]

The resulting \( \lambda_q \sim q \) are plotted in Fig.2(a) for \( \alpha = 0.2, 0.3, 0.4, 0.5 \) respectively.

Fig.2(a) is the result under central limit approximation. The exact relation can not be calculated analytically. Therfore, we use Monte Carlo simulation. The resulting \( \lambda_q \sim q \) for \( \alpha = 0.2, 0.3, 0.4, 0.5 \) respectively, are shown in Fig.2(b).

It can be seen from the figures that the \( \lambda_q \sim q \) curves from both the Monte Carlo simulation and the analytical calculation under central limit approximation have the same trend, i.e. with the increasing of \( q \), \( \lambda_q \) arrive at a minimum at the point \( q_c \), which means that there really exists non-thermal phase transition in the self-similar cascading model and two different phases do indeed coexist, \( q_c \) is the critical point of phase transition.
In Fig. 2 we also draw the experimental data from NA22 (open circles). It stops at the rank $q = 5$ and is unclear whether there is a minimum at some higher rank as required by non-thermal phase transition. The open triangles in the figure is the result from the same experiment selecting only the particles with low transverse momenta ($p_t < 0.15$ GeV/c). In this case, with the increasing of $q$ (from 4 to 5), $\lambda_q$ increases. It seems to show that there is phase transition and the critical point $q_c < 5$. As is well known, choosing only the particles with low transverse momenta, the strength of intermittency increases\cite{1}. Therefore, this experimental phenomenon shows that the system with lower transverse momenta, which has larger intermittency strength, has lower critical point of non-thermal phase transition. This is qualitatively the same as the result of our model, where the phase transition point shifts left with the increasing of fluctuation strength (when $\alpha$ increases $q_c$ decreases).

In order to see more clearly the relation between the phase transition point $q_c$ and the fluctuation parameter $\alpha$ of the model, we draw the figure of $q_c \sim \alpha$, as shown in Fig. 3. We can see from the figure that the larger $\alpha$ is, the earlier $q_c$ appears.

Comparing the exact values of $q_c \sim \alpha$ from Monte Carlo simulation and the analytical results under central limit approximation, it can be seen that both have the same trend of continuously descending. However, the values of $q_c$ in central limit approximation are generally smaller than the exact values. This shows that the central limit approximation can reflect qualitatively the property of non-thermal phase transition but there is noticeable quantitative deviation.

References

[1] Wu Yuanfang and Liu Lianshou. Phys. Rev. Lett., 1993, **70**: 319; Science in China, 1995, **A38**: 4357
[2] EHS/NA22 Coll., Agababyan N M et al. Phys. Lett., 1996 **B382**: 305; ibid., 1998 **B431**: 451
[3] Wang Shaoahun, Wang Zhaomin and Wu Chong. Phys. Lett., 1997 **B410**: 323
[4] Peschanski R. Nucl. Phys., 1989 **B327**: 144
[5] Bialas A, Zalewski K. Phys. lett., 1990 **B238**: 413
[6] De Wolf E A, Dremin I M and Kittel W. Phys. Rep., 1996 **270**: 1
[7] Wu Yuanfang, Zhang Kunshi and Liu Lianshou. Chinese Science Bulletin, 1991 **36**: 1077
[8] Bialas A and Peschanski R. Nucl. Phys., 1986 **B273**: 703; ibid., 1988 **B308**: 857
[9] Liu Lianshou, Fu Jinghua and Wu Yuanfang. Phys. Lett., 1998 **B444**: 563
[10] Wu Yuanfangand Liu Lianshou. Phys. Lett., 1991 **B269**: 28
Fig. 1 Log-log plot of various rank probability moments versus partition number in $\alpha$ model
Fig. 2  Relation between the parameter $\lambda_q$ and the rank $q$ of moments. The vertical lines indicate the position of minima.  
(a) Analytical results under central limit approximation. The open circles are the experimental results from NA22. Open triangles are the results from the same experiments taking only low momentum particles with ($p_t < 0.15$ GeV/c) Data taken from Ref.[6].  
(b) Results of Monte Carlo simulation.
Fig. 3 The relation between phase transition point $q_c$ and fluctuation strength $\alpha$