Looking for magnetic monopoles at LHC

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Abstract

Magnetic monopoles have been a subject of interest since Dirac established the relation between the existence of monopoles and charge quantization. The intense experimental search carried thus far has not met with success. We study the observability of monopoles at the Large Hadron Collider in the $\gamma\gamma$ channel and show that LHC is an ideal machine to discover monopoles with masses below 1 TeV at present running energies and with less than 1 fb\(^{-1}\) of integrated luminosity.

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1 Introduction

The theoretical justification for the existence of classical magnetic poles, hereafter called monopoles, is that they add symmetry to Maxwell’s equations and explain charge quantization [1, 2]. Dirac showed that the mere existence of a monopole in the universe could offer an explanation of the discrete nature of the electric charge. His analysis leads to the Dirac Quantization Condition (DQC),

\[ e g = \frac{N}{2}, \quad N = 1, 2, \ldots \tag{1} \]

where \( e \) is the electron charge, \( g \) the monopole magnetic charge and we use natural units \( \hbar = c = 1 \). In Dirac’s formulation, monopoles are assumed to exist as point-like particles and quantum mechanical consistency conditions lead to Eq. (1), establishing the value of their magnetic charge. Their mass, \( m \), is a parameter of the theory.

Monopoles and their experimental detection have been a subject of much study since many believe in Dirac’s statement [1],

“...one would be surprised if Nature had made no use of it [the monopole].”

At present, despite intense experimental search, there is no evidence of their existence [3, 4]. LHC is opening a new frontier in the search for monopoles and, as we show here, should find them at present energies, with less than 1 fb\(^{-1}\) integrated luminosity, if their mass is below 1 TeV.

Although monopoles symmetrize Maxwell’s equations in form there is a numerical asymmetry arising from the DQC, namely that the basic magnetic charge is much larger than the smallest electric charge. This led Dirac himself in his 1931 paper [1] to state,

“... the attractive force between two one quantum poles of opposite sign is \((1372)^2 \approx 46921\frac{1}{4}\) times that between the electron and the proton. This very large force may perhaps account for why the monopoles have never been separated.”

This statement by Dirac is fundamental in our investigation. If instead of monopoles we deal with monopole-antimonopole pairs, we expect that due to this very strong interaction many of them annihilate into photons inside the detector. We study the production of a monopole-antimonopole pairs via photon fusion at LHC and their subsequent annihilation giving rise to two extremely energetic photons.

Another proposal, influenced also by the strength of the interaction, assumes that the produced pair of monopole-antimonopole, before decaying, forms a monopolium bound state [5, 6]. This bound state will also desintegrate producing two photons, a process which is being analyzed at present [7] in connection with LHC potentialities.
In the next section we describe the dynamics of monopoles and review the production of monopole-antimonopole pairs. Section 3 discusses the annihilation of the virtual monopole-antimonopole pair into photons. In section 4 we describe how to incorporate the elementary process into \( p - p \) scattering. In section 5 we present our results in the context of the present running features of LHC and in the last section we draw conclusions of our studies.

2 Monopole dynamics

The theory of monopole interactions was initially formulated by Dirac [8] and later on developed in two different approaches by Schwinger [9] and Zwanziger [10]. The formalism of Schwinger can be cast in functional form as a field theory for monopole-electron interaction which is dual to Quantum Electrodynamics (QED) [11]. The monopoles are considered fermions and behave as electrons in QED with a large coupling constant as a result of the DQC. In this formulation the conventional photon field is Dirac string dependent. Due to the large coupling constant and the string dependence, perturbative treatments à la Feynman are in principle not well defined. However, non perturbative high energy treatments, like the eikonal approximation, have rendered well defined electron-monopole cross sections [11, 12].

For the case of monopole production at energies higher than their mass the above procedure is not applicable, and being the treatment non perturbative, there is no universally accepted prediction from field theory. However, the study of the classical interaction of a monopole passing by a fixed electron leads to an interaction for the monopole which is associated with the electric field felt. If one compares this interaction with the that of a positron passing by an electron, one realizes that the difference between QED and dual QED is simply to change the electric charge by the magnetic charge times the velocity, i.e.

\[
e \rightarrow \beta g.
\]

(2)

Figure 1: Elementary processes of monopole-antimonopole production via photon fusion.
Thus a monopole behaves as a passing particle with electric charge $\beta g$ \[13\].

This idea has been used to define an effective field theory to lowest order \[14\] which has been applied to Drell-Yan like monopole, or monopole-antimonopole production mechanisms \[14, 15\] and to monopole-antimonopole production by photon fusion \[16, 17\]. In Fig. 1 we show the corresponding diagrams that have been calculated for the latter process.

We note that this approximation can be shown to be almost equivalent to the low energy effective theory of Ginzburg and Schiller \[18, 19\]. The theory of Ginzburg and Schiller was derived from the standard electroweak theory in the one loop approximation leading to an effective coupling of the order of $g_{\text{eff}} \sim \frac{\varepsilon}{m} g$, where $m$ is the monopole mass and $\varepsilon$ a kinematical energy scale of the process which is below the monopole production threshold, thus rendering the effective theory perturbative. In a photon fusion diagram the dynamical scale is $\sqrt{E^2 - 4m^2}$, where $E$ is the center of mass energy, thus

$$\frac{\varepsilon}{m} \sim \frac{\sqrt{E^2 - 4m^2}}{2m} \sim \frac{E\beta}{2m}, \quad (3)$$

where $\beta = \sqrt{1 - \frac{4m^2}{E^2}}$ is the monopole velocity. Therefore if $E \sim 2m$, i.e. the kinetic energy is small, and both schemes coincide \[6\].

The aim here is to study possible signals of magnetic monopoles at LHC. According to previous studies \[17\], the most promising mechanism is photon fusion. The elementary diagrams contributing to pair production are those in Fig. 1 where the explicit couplings have been shown.

![Figure 2: Elementary photon-fusion cross section for electron-positron (solid) and that of the monopole-antimonopole (dashed) as a function of $\omega$, to show the importance of the $\beta$ effect.](image)

The photon-fusion elementary cross section is obtained from the well-known QED electron-positron pair creation cross section \[20\], simply changing the coupling constant
(e → gβ) and the electron mass by the monopole mass $m_e → m$, leading to

$$\sigma(\gamma \gamma → m\bar{m}) = \frac{\pi g^4 (1 - \beta^2) \beta^4}{2 m^2} \left( \frac{3 - \beta^4}{2 \beta} \log \left( \frac{1 + \beta}{1 - \beta} \right) - (2 - \beta^2) \right).$$

where $E$ is the center of mass energy and $\beta$, a function of $E$, is the monopole velocity. In Fig. 2 we show the $\omega = E/2m$ dependence of Eq. (4). The solid curve corresponds to the electron-positron case, the dashed one to the monopole case which contains the $\beta^4$ factor. One should notice the large effect associated with this factor in the vicinity of the threshold.

LHC detectors, apart from the MoEDAL experiment [21], have not been designed specifically to see monopoles and therefore even those which do not annihilate inside the detectors will be difficult to detect. However, all detectors are well designed to measure low cross sections of photons, since the two photon decay is one of the favorable channels to detect a low mass Higgs, thus we would like to benefit from this capability and proceed to investigate how monopole-antimonopole annihilation into photons is described in our scheme.

3 Monopole-antimonopole annihilation into $\gamma \gamma$

It is natural to think that the enormous strength and long range of the monopole interaction leads to the annihilation of the pair into photons very close to the production point. Thus one should look for monopoles through their annihilation into highly energetic photons, a channel for which LHC detectors have been optimized.

In order to calculate the annihilation into photons we assume that our effective theory, a technically convenient modification of Ginzburg and Schiller’s, agrees with QED at one loop order, and therefore we apply light-by-light scattering with the appropriate modifications as shown in Fig. 3. An interesting feature of the calculation is that the

![Figure 3: Elementary processes for monopole-antimonopole production and annihilation into photons.](image)
additional magnetic coupling will increase the cross section dramatically and therefore this measurement should lead to a strong restriction on the monopole mass. However, as we have seen, in $m - \overline{m}$ production, the large magnetic coupling is always multiplied by the small electric one, leading to effective couplings $e g$, and the same will happen in detection. Thus, the effective coupling of the process is $e g$, and therefore has strengths similar to the strong interaction, not more. The extreme sensitivity of LHC detectors to photons, as a consequence of the hope to detect the Higgs in the $\gamma \gamma$ channel makes them ideal for the purpose of detecting monopoles, even if the monopole mass is large, as will be shown.

Figure 4: The light-by-light scattering quantity $X(\omega)$, which determines the cross section up to factors related to the coupling constant. The solid curve corresponds to forward scattering, the dashed one to right-angle scattering, both as a function of the dimensionless quantity $\omega$.

Light-by-light scattering was studied by Karplus and Neuman [22, 23], who reproduced all previous low energy results by Euler [24] and high energy results by Achieser [25], and was later revisited and corrected by Csonka and Koelbig [26].

In the case of monopoles, with the appropriate changes, the expression for the cross section becomes,

$$
\sigma_{\gamma \gamma}(\theta, \omega) = \frac{(g\beta)^8}{8\pi^2m^2} X(\theta, \omega). \tag{5}
$$

where $X(\omega)$ is $\frac{<|M|^2>}{\omega^2}$ in the notation of ref. [23]. In Fig. 4 we reproduce these results for forward scattering and right-angle scattering. In Fig. 5 (left) we show the ratio of the forward scattering to the right-angle cross sections and therefore show how the cross section, which is basically isotropic close to threshold, becomes anisotropic as the energy increases. In the same figure (right) we plot $X(\omega)$ for several values of $\omega$ as a function of angle. We note that the forward cross section is larger and the right-
angle one smaller, than that for any other value of the scattering angle. As the energy
increases the drop in the $X$ function from $\theta = 0$ to $\pi/2$ increases.

Figure 5: Left: ratio of the light-by-light scattering quantity $X(\omega)$ for forward to right-
angle scattering as a function of $\omega$. Right: angular dependence of the cross section for
three values of $\omega$ (1 (solid), 5 (dashed), 10 (dotted)).

It is clear from these figures that close to the threshold the cross section is quite
isotropic and away from threshold the forward cross section, which is very difficult to
measure, is much larger than the right-angle one. Since the detectors cannot detect
all of the photons coming out, we take the right-angle cross section as a conservative
indication of the magnitudes to be expected.

Figure 6: The effect of the $\beta$ factor as a function of $\omega$. The solid curve corresponds to
electron scattering, while the dashed curve corresponds to the monopole case.

In Fig. 6 we show the effect of the $\beta$ factor which diminishes greatly the cross
section close to the monopole-antimonopole threshold.
Figure 7: Processes contributing to the $\gamma\gamma$ cross section. The blob contains the three cases described in the text.

4 Analysis of p-p scattering

LHC is a proton-proton collider, therefore, in order to describe the production and desintegration of the monopole-antimonopole pair, we have to study the following processes above the monopole threshold ($\beta > 0 \rightarrow E \geq 2m$),

\[ p + p \rightarrow p(X) + p(X) + \gamma + \gamma, \]  

shown globally in Fig.7, where $p$ represents the proton, $X$ an unknown final state and we will assume that the blob is due exclusively to monopoles. This diagram summarizes the three possible processes:

i) inelastic $p + p \rightarrow X + X + \gamma + \gamma \rightarrow X + X + m + \overline{m} \rightarrow X + X + m + \overline{m} + \gamma + \gamma$

ii) semi-elastic $p + p \rightarrow p + X + \gamma + \gamma \rightarrow p + X + m + \overline{m} \rightarrow p + X + m + \overline{m} + \gamma + \gamma$

iii) elastic $p + p \rightarrow p + p + \gamma + \gamma \rightarrow p + p + m + \overline{m} \rightarrow X + X + m + \overline{m} + \gamma + \gamma$.

In the inelastic scattering, both intermediate photons are radiated from partons (quarks or antiquarks) in the colliding protons.

In the semi-elastic scattering one intermediate photon is radiated by a quark (or antiquark), as in the inelastic process, while the second photon is radiated from the other proton, coupling to the total proton charge and leaving a final state proton intact.

In the elastic scattering both intermediate photons are radiated from the interacting protons leaving both protons intact in the final state.
In the blob we incorporate the elementary subprocess shown in Fig. 3 and described by Eq. (3).

We calculate \( \gamma \gamma \) fusion for monopolium-antimonopolium production following the formalism of Drees et al. [27].

In the inelastic scattering, \( p + p \rightarrow X + X + (\gamma \gamma) \rightarrow X + X + m + \bar{m} + \gamma + \gamma \), to approximate the quark distribution within the proton we use the Cteq6–1L parton distribution functions [28] and choose \( Q^2 = \hat{s}/4 \) throughout.

We employ an equivalent–photon approximation for the photon spectrum of the intermediate quarks [29, 30].

In semi–elastic scattering, \( p + p \rightarrow p + X + (\gamma \gamma) \rightarrow p + X + m + \bar{m} + \gamma + \gamma \), the photon spectrum associated with the interacting proton must be altered from the equivalent–photon approximation for quarks to account for the proton structure. To accommodate the proton structure we use the modified equivalent–photon approximation of [27].

The total cross section is obtained as a sum of the three processes. The explicit expressions for the different contributions can be found in [17].

In order to obtain the differential photon-photon cross section from the above formalism we develop a procedure which we exemplify with the elastic scattering case. In that case the \( pp \) cross section is given by [27],

\[
\sigma_{pp}(s) = \int_{4m^2/s}^{1} dz_1 \int_{4m^2/sz_1}^{1} dzz_2 f(z_1)f(z_2)\sigma_{\gamma\gamma}(z_1z_2s),
\]  

(7)

where \( \sqrt{s} \) is the center of mass energy of the \( pp \) system.

We perform the following change of variables

\[
v = z_1z_2, \quad w = z_2,
\]

which leads to

\[
\sigma_{pp}(s) = \int_{4m^2/s}^{1} dv \int_{v}^{1} dw f(\frac{v}{w})f(w)\sigma_{\gamma\gamma}(vs).
\]  

(8)

Note that to fix the center of mass energy of the photons is equivalent to fix \( v \). For fixed \( v \) we have,

\[
\frac{d\sigma_{pp}}{dv}(s) = \int_{v}^{1} dw f(\frac{v}{w})f(w)\sigma_{\gamma\gamma}(vs),
\]  

(9)

which can be rewritten in terms of \( E_\gamma \), the center of mass energy of the photons, and the elementary photon-photon cross section as,

\[
\frac{d\sigma_{pp}}{dE}(E_\gamma) = \frac{2E_\gamma}{s} \sigma_{\gamma\gamma}(s_{\gamma\gamma}) \int_{s_{\gamma\gamma}/s}^{1} dw f(\frac{s_{\gamma\gamma}}{w})f(w).
\]  

(10)

This procedure can be generalized easily to the semielastic and inelastic cases, where the appropriate change of variables are
Figure 8: The total monopole-antimonopole production cross section (solid line) from $\gamma \gamma$ fusion in fb as a function of monopole mass. The different components of the cross section are shown: elastic (dotted line), semielastic (small dashed line) and inelastic (dashed line).

$$v = z_1 z_2 x_1, \ w = z_2 x_1, \ u = x_1$$

and

$$v = z_1 z_2 x_1 x_2, \ w = z_2 x_1 x_2, \ u = x_1 x_2, \ t = x_2,$$

respectively.

## 5 Results

In Fig. 8 we show the total cross section for monopole-antimonopole production and the contribution from each of the individual processes described above as a function of monopole mass. In order to set the mass axes we have taken into account the previous lower mass limit for the monopole of ref. [15] which was set at 360 GeV. The cross section is of $\mathcal{O}(\text{fb})$ thus we limit the high mass values to those potentially observable at present, or in the near future, by the LHC with an integrated luminosity of 1 fb$^{-1}$. Our result differs from that of ref. [17] around threshold.

We see in the previous figure that monopoles of mass around 500 GeV should be produced abundantly, while those of mass around 1000 GeV have a cross section which makes them difficult to detect in the near future, at least directly.

Since, as already mentioned, the LHC detectors have not been tuned to detect monopoles but are excellent detectors for $\gamma$'s let us discuss next our annihilation cross section. In Fig. 9 we show the differential cross section for forward (solid) and right-angle (dotted) scattering given in fb/GeV as a function of the invariant mass of the $\gamma \gamma$ system. We have assumed a monopole of mass 750 GeV, chosen because the cross
sections turned out to be close to the expected magnitude of the Higgs to $\gamma \gamma$ cross section above background. The cross sections are wide, almost gaussian, structures rising softly just above threshold (1500 GeV). LHC detectors are blind for forward scattering and have black spots due to construction features in the non-forward regions which do not allow for a full detection of photons. Therefore in order to obtain an educated estimation of the observable cross section we take the right-angle cross section and multiply it by $4\pi$. This differential cross section is the smallest possible. However, away from threshold, it corresponds quite well to a realistic estimate, since, as we have seen, the elementary differential cross section drops fast with angle and moreover one should consider an efficiency factor for the various detectors.

![Graph](image)

Figure 9: Forward (solid) and the right-angle cross sections for a monopole of mass 750 GeV.

In Fig. 10 we plot the right-angle differential cross section for the same monopole mass and show the different contributions to the differential cross section. The structure of the cross section is a wide structure, rising softly after threshold, 1500 GeV, and extending for almost 2000 GeV. The structure is centered about 2300 GeV. Clearly the soft rise of the differential cross section is a signature of the two particle threshold, recall the $\beta$ factor. The width of the structure is associated to the mathematical form of the box diagram, as can be seen for both electron-positron annihilation and monopole-antimonopole, from the structure of the elementary cross section (Fig. 6).

In Fig. 11 we compare our total $\gamma \gamma$ cross section with Higgs process obtained from ref. [31]. We have extrapolated their background to our energies using an inverse polynomial fit to their data and their exponential fit. Both procedures give a negligible background for the signals obtained with monopoles masses up to 1 TeV and even higher. In the left figure we translate the monopole-antimonopole threshold to the origin in order to compare the two signals. In the right figure we show the actual energy scales in a LogLog plot. The Higgs signal above background has been multiplied by 50 to make it visible. The figures correspond to a monopole of mass 750 GeV. It is clear
Figure 10: The right-angle scattering cross sections for a monopole mass of 750 GeV. The smallest is the inelastic cross section (dotted), next comes the elastic (dashed) and the biggest is the semielastic (long-dashed). The total cross section, the sum of the three, is represented by the solid curve.

from the curves that monopole-antimonopole annihilation should appear as a soft rise of the cross section above the background over a large energy interval.

Figure 11: Comparison of the \( \gamma \gamma \) monopole-antimonopole annihilation cross section for a monopole of mass 750 GeV with the Higgs \( \gamma \gamma \) decay. The Higgs cross section above the background has been multiplied by 50. The monopole-antimonopole threshold has been set at the origin (100 GeV) (left). The right figure represents the same cross section drawn in a Log Log plot keeping the thresholds.

Finally we study the mass dependence of the differential cross section. Fig. 12 is a LogLog plot of the cross section for three masses, 500, 750 and 1000 GeV. In order to have a better comparison we have translated the thresholds to the origin. It is clear that the magnitude of the cross section and its extent falls rapidly with mass. For a low mass monopole, the magnitude of the differential cross section is of the order of \( \sim 1 \text{ pb/GeV} \), for an intermediate mass monopole the cross section is of the order of
\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{figure12.png}
\caption{The cross section for monopole-antimonopole annihilation into $\gamma\gamma$ for three masses: 500 GeV (solid), 750 GeV dashed 1000 GeV (dotted).}
\end{figure}

$\sim 10 \text{ fb/GeV}$ and for a heavy monopole is of the order of $\sim 1 \text{ fb/GeV}$. If the monopole has a low mass, monopoles should be seen within the initial period of LHC running.

6 Conclusions

The existence of Dirac monopoles would modify our understanding of QED. The DQC, implying a huge magnetic coupling constant, complicates matters from the theoretical point of view. Non perturbative methods are required. We have avoided the problem by using an effective theory valid at one loop order. The physical coupling turns out to be $e g$, equivalent in magnitude to the pionic couplings, and the expected high monopole masses provide convenient cut-offs which make the theory sensible. In the context of this theoretical scheme we are able to calculate monopole(antimonopole) production and monopole-antimonopole annihilation. Centering our description to LHC, a proton machine, we have described scenarios for the production of monopole-antimonopole pairs via photon fusion. We have described their annihilation into photons via the conventional box diagram, analogous to that of light-by-light scattering in conventional QED. The only difference in our approach is the introduction of a threshold factor in the form of a velocity. We have analyzed in detail the elementary process, light-by-light scattering at monopole energies, and have subsequently incorporated the description of the photon-photon elementary scattering into the initial proton-proton collision. One main assumption is that no other particle contributes to the box diagram in the region of interest. We have chosen in the present calculation a mass range which is above conventional particle masses and below the supersymmetric particle ranges. One could think of interference with elementary particle decays, however the latter would have a resonant structure which our box diagram does not have. Thus any interference would be avoided by their different geometric structure.
With all these preventions we have shown that monopoles up to 1 TeV in mass should be detected at LHC in the $\gamma \gamma$ channel at present running energies and with luminosities in reach within the next few months. The signature is a soft rise of the cross section which should stay for a long energy range, because no resonant peak structure is to be expected.

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**References**

[1] P. A. M. Dirac, Proc. Roy. Soc. Lond. A 133 (1931) 60.

[2] J. D. Jackson, Classical Electrodynamics, de Gruyter, N.Y. (1982).

[3] K. A. Milton, Rept. Prog. Phys. 69 (2006) 1637 [arXiv:hep-ex/0602040].

[4] W. M. Yao *et al.* [Particle Data Group], J. Phys. G 33 (2006) 1.

[5] L. N. Epele, H. Fanchiotti, C. A. Garcia Canal and V. Vento, Eur. Phys. J. C 56 (2008) 87 [arXiv:hep-ph/0701133].

[6] L. N. Epele, H. Fanchiotti, C. A. G. Canal and V. Vento, Eur. Phys. J. C 62 (2009) 587 [arXiv:0809.0272 [hep-ph]].

[7] L. N. Epele, H. Fanchiotti, C. A. G. Canal, V. Mitsou and V. Vento, (work in preparation).

[8] P. A. M. Dirac, Phys. Rev. 74 (1948) 817.

[9] J. S. Schwinger, Phys. Rev. 144 (1966) 1087.

[10] D. Zwanziger, Phys. Rev. D 3 (1971) 880.

[11] L. P. Gamberg and K. A. Milton, Phys. Rev. D 61 (2000) 075013 [arXiv:hep-ph/9910526].
[12] L. F. Urrutia, Phys. Rev. D 18 (1978) 3031.

[13] M. J. Mulhearn, “A Direct Search for Dirac Magnetic Monopoles,” Ph.D. Thesis, Massachusetts Institute of Technology 2004, FERMILAB-THESIS-2004-51.

[14] G. R. Kalbfleisch, K. A. Milton, M. G. Strauss, L. P. Gamberg, E. H. Smith and W. Luo, Phys. Rev. Lett. 85 (2000) 5292 [arXiv:hep-ex/0005005].

[15] A. Abulencia et al. [CDF Collaboration], Phys. Rev. Lett. 96 (2006) 011802 [arXiv:hep-ex/0508051].

[16] Yu. Kurochkin, I. Satsunkevich, D. Shoukavy, N. Rusakovich and Yu. Kulchitsky, Mod. Phys. Lett. A 21 (2006) 2873.

[17] T. Dougall and S. D. Wick, Eur. Phys. J. A 39 (2009) 213 [arXiv:0706.1042 [hep-ph]].

[18] I. F. Ginzburg and A. Schiller, Phys. Rev. D 57 (1998) 6599 [arXiv:hep-ph/9802310].

[19] I. F. Ginzburg and A. Schiller, Phys. Rev. D 60 (1999) 075016 [arXiv:hep-ph/9903314].

[20] C. Itzykson and J. B. Zuber, New York, USA: McGraw-Hill (1980) 705 P. (International Series In Pure and Applied Physics)

[21] J. L. Pinfold, AIP Conf. Proc. 1304 (2010) 234; J. L. Pinfold [MOEDAL Collaboration], CERN Cour. 50N4 (2010) 19.

[22] R. Karplus and M. Neuman, Phys. Rev. 80 (1950) 380.

[23] R. Karplus and M. Neuman, Phys. Rev. 83 (1951) 776.

[24] H. Euler Ann. Phys. 26 (1936) 398.

[25] A. Achieser, Physik Z. Sowjetunion 11 (1937) 263.

[26] P. L. Csonka and K. S. Koelbig, Phys. Rev. D 10 (1974) 251.

[27] M. Drees, R. M. Godbole, M. Nowakowski and S. D. Rindani, Phys. Rev. D 50 (1994) 2335 [arXiv:hep-ph/9403368].

[28] The Cteq6 parton distribution functions can be found at [http://www.phys.psu.edu//cteq/].

[29] E. J. Williams, Phys. Rev. 45 (1934) 729.
[30] C. F. von Weizsacker, Z. Phys. 88 (1934) 612.

[31] G. Aad et al. [Atlas Collaboration], Phys. Rev. Lett. 106 (2011) 121803
arXiv:1012.4272 [hep-ex].

[32] D. Binosi and L. Theussl, Comput. Phys. Commun. 161 (2004) 76
arXiv:hep-ph/0309015.