Nuclear temperatures from the evaporation fragment spectra and possible lifetime effect

A. Ray¹, A. De², A. Chatterjee³, S. Kailas³, S. R. Banerjee¹
¹Variable Energy Cyclotron Center, 1/AF, Bidhannagar, Kolkata – 700064, India
²Raniganj Girls’ College, Raniganj, West Bengal, India
³Nuclear Physics Division, Bhabha Atomic Research Center, Mumbai, India

E-mail: ray@vecc.gov.in

Abstract. The temperature of a compound nucleus should decrease for the higher values of its spin angular momentum and so the spectra of the heavier evaporation fragments emitted from the compound nucleus should have progressively steeper slopes. To test this idea, we formed the same compound nucleus \(^{105}\)Ag at the same excitation energy \((E_X=76 \text{ MeV})\) and with very similar spin distribution by \(^{16}\)O+\(^{89}\)Y and \(^{12}\)C+\(^{93}\)Nb reactions and studied back-angle alpha to carbon particle emissions. The compound nucleus character of the reaction was established from the angular distribution of the emitted fragments and the lack of any entrance channel dependence of the angle-integrated yields of the fragments. It was found that the temperatures of the ensembles of the residual nuclei as obtained from the slopes of alpha, lithium, boron, carbon spectra remain about 3 MeV in most cases and higher (about 4.2 MeV) for lithium emission, whereas the statistical model codes predict decrease of the corresponding temperatures from 3 MeV (for alpha) to 1.65 MeV (for carbon). The result could not be understood by adjusting the parameters of the statistical models and might imply the effect of the lifetime of the exit channel fragment plus residual dinuclear system.

1. Introduction
The temperature of a system is defined when it is in full statistical equilibrium which means that each possible state of the system is populated with equal probability. If \(E\) be the energy of the system and it comprises a fixed \(N\) number of particles, then \(\frac{1}{T} = \frac{S(E,N)}{k}\), where \(T\), \(S\) and \(\rho\) are the temperature, entropy and density of states of the system at energy \(E\) respectively. The atomic nucleus is a microscopic system and the temperature of a nucleus is defined when its excitation energy is much greater than the energy of the first excited state. Since no external probe can be used, so the information about the temperature of a nucleus has to be obtained from the emission of the small parts of the nucleus itself. The necessary conditions are that the emission process must be completely statistical and the emitted particles must be part of the equilibrium and the density of states of the whole system before emission. The statistical emission of particles usually takes place from a compound nucleus and the spectrum of the emitted particles shows an exponential tailing. The slope of the exponential tail of the statistical evaporation spectrum of the charged particles gives the...
temperature of the ensemble of the residual nuclei (produced with a distribution of the excitation energy) at the instant of the break-up of the exit channel dinuclear system (residual + emitted particle). The temperature of the ensemble of the residual nuclei corresponds to their average excitation energy.

Following Moretto [1], the statistical evaporation spectrum of charged particles from a compound nucleus can be written as

\[ P(x) \propto \exp \left( -\frac{x}{f} \right) \text{erfc} \left( \frac{p-2x}{2\sqrt{p_I}} \right) \]

\[ x = E_{\text{kin}}(c.m.) - V_C. \]

Here \( E_{\text{kin}}(c.m.), V_C, p, T, P(x) \) are the center of mass kinetic energy, Coulomb barrier, amplification parameter, temperature of the ensemble of the residual nuclei and the corresponding probability of the emission of the particle respectively. Usually the temperature is obtained by fitting evaporation proton and alpha spectra from a compound nucleus with eq(1). The effect of the sequential decay is considered by adding up several source terms (like eq(1)) with decreasing temperatures.

Although generally the temperature is determined from the slopes of the evaporation proton and alpha spectra, the temperatures of the corresponding residual nuclei can also be obtained by fitting heavier fragment (such as Li, Be, B, C) evaporation spectra with eq(1), provided those emissions are also completely statistical. The temperatures of the residual nuclei obtained by fitting heavier fragment evaporation spectra should be significantly lower compared to those obtained from proton or alpha spectra, because the heavier fragment such as Li, Be, B or C takes away significantly more orbital kinetic energy, thus lowering the available thermal energy.

\[ U_{\text{thermal}}(\text{residual}) = (E_{\text{c.m.}} - Q) - \frac{\ell(\ell + 1)h^2}{2I} - E_{\text{rot}}(\text{spin}) \]

Where \( E_{\text{c.m.}}, Q, E_{\text{rot}}(\text{spin}), I \) and \( \ell \) denote the center of mass energy, Q-value of the reaction and spin rotational energy of the nuclei, moment of inertia and orbital angular momentum and of the system respectively. In the case of heavy fragment emission, generally \( E_{\text{rot}}(\text{spin}) \ll \frac{\ell(\ell+1)h^2}{2I} \) and the orbital rotational energy shows up as the kinetic energy in the exit channel.

2. Statistical model calculations

The statistical model code ‘CASCADE’ [2] was used to calculate the spectra of neutron, proton, alpha and heavier fragments. In the ‘CASCADE’ code, the neutron, proton and alpha particles were the main particle emission channels and the lithium or boron or carbon particle was used as a fourth particle channel one at a time. The spectra of alpha, lithium, boron and carbon obtained from the calculations were fitted with eq(1) to obtain the corresponding slope temperatures which should indicate the temperatures of the corresponding ensembles of the residual nuclei. In Fig. 1(a), we show the calculated alpha spectrum (in the center of mass frame) and the fit using eq(1). The fit yields \( T=2.9 \) MeV, \( p=4.0 \) MeV and \( V_c=12.5 \) MeV. In Fig. 1(b), we show the calculated \(^{12}\text{C}\) spectrum and the corresponding fit yielding \( T=1.65 \) MeV, \( p=18.0 \) MeV and \( V_c=37.0 \) MeV. Similarly we have also calculated \(^{6}\text{Li}\) spectrum and the corresponding fit yielding \( T=2.35 \) MeV, \( p=8.0 \) MeV and \( V_c=19.2 \) MeV. The fit of the calculated \(^{11}\text{B}\) spectrum yielded \( T=2.1 \) MeV, \( p=16 \) MeV and \( V_c=33.5 \) MeV. So as expected, it was found that the temperature of the residual nucleus drops from \( T=2.9 \) MeV for the alpha spectrum to \( T=1.65 \) MeV for the carbon spectrum, because the heavier fragment takes away more rotational kinetic energy. On the other hand, the value of the amplification parameter (p) increases from 4.0 MeV for the alpha particle to 18.0 MeV for the carbon particle. The calculations have been done using the level density parameter \( a=A/8 \) and the extracted parameters are essentially independent of the transmission coefficients used in the calculations. The use of different optical model parameters resulting in a different set of transmission coefficients does not affect the slope of the exponential tail of the spectrum and the extracted temperature significantly, because the distribution of the excitation energy of the residual nuclei determines the temperature of the ensemble of the residual nuclei and this distribution should be essentially independent of the transmission coefficients of the ejectiles. The statistical model calculations have also been done considering the
emission of $^6\text{Li}$, and $^7\text{Li}$ in their excited states (below the particle emission threshold) and the slope temperatures obtained from the lithium spectra remain essentially unchanged and the same as that obtained considering only the ground state emission of $^6\text{Li}$. Similar results have also been obtained by considering the emission of $^{12}\text{C}$ in its ground and first excited states.

3. Experiments and results
We formed the same compound nucleus $^{105}\text{Ag}$ at the same excitation energy ($E_X=76$ MeV) and with very similar spin distributions ($\ell_{\text{crit}}$ equal within 10%) by $^{16}\text{O}+^{89}\text{Y}$ and $^{12}\text{C}+^{93}\text{Nb}$ reactions. A 10 pna $^{16}\text{O}$ beam with 95.9 MeV energy from BARC-TIFR pelletron machine in Mumbai, India was incident on a 1 mg/cm$^2$ thick $^{89}\text{Y}$ target and all the emitted particles from $^{16}\text{O}$ to oxygen were detected using four $\Delta E$-E solid-state telescopes placed between 145° to 175° in the center of mass frame. Similarly a 10 pna $^{12}\text{C}$ beam with 85.5 MeV energy from IUAC, New Delhi, India was incident on a 1 mg/cm$^2$ thick $^{93}\text{Nb}$ target and all the emitted particles were detected using three $\Delta E$-E solid-state telescopes placed between 130° and 160° in the center of mass frame. In the center of mass frame, the angular distributions of the emitted particles show a back-angle rise that can be approximated by a $1/\sin\theta_{c.m.}$ function [3]. We have studied the angle-integrated ratios of boron to carbon, beryllium to carbon and lithium to carbon yields as a function of the exit channel excitation energy for both $^{16}\text{O}+^{89}\text{Y}$ and $^{12}\text{C}+^{93}\text{Nb}$ reactions and find that the data sets for $^{16}\text{O}+^{89}\text{Y}$ and $^{12}\text{C}+^{93}\text{Nb}$ overlap with each other reasonably well, implying no significant entrance channel effect and hence their statistical origin from an equilibrated compound nucleus. In Fig. 2(a) and 2(b), we show the plots for boron to carbon and beryllium to carbon yields for $^{16}\text{O}+^{89}\text{Y}$ and $^{12}\text{C}+^{93}\text{Nb}$ reactions showing no entrance channel dependence. So we have established that the emissions of the heavier fragments such as lithium, beryllium, boron and carbon from $^{16}\text{O}+^{89}\text{Y}$ and $^{12}\text{C}+^{93}\text{Nb}$ reactions are statistical, because of their observed approximate $1/\sin\theta_{c.m.}$ angular distribution and the lack of any entrance channel dependence. Hence it should be possible to extract the temperatures of the corresponding ensembles of the residual nuclei from the experimental angle-integrated spectra of lithium, beryllium, boron and carbon and compare them with the predictions of the statistical model code ‘CASCADE’.

The fittings of the alpha spectra with eq(1) were discussed earlier [4] and it was found that the alpha spectra can be fitted well (within 1%) by using three sources at temperatures $T=2.96$ MeV, 2.51 MeV and 2.31 MeV to take into account the sequential neutron, proton and alpha emissions from the compound nucleus. A one source fitting with temperature of temperature $T=2.90\pm0.15$ MeV and $p=1.9$ MeV also gives a reasonable fit [4]. In Fig. 3(a) and 3(b), we show the angle-integrated boron and carbon spectra (in the center of mass frame) and the corresponding fits using eq(1). The extracted temperatures are 3.3 MeV and 3.5 MeV from the boron and carbon spectra respectively.

Table 1 shows the kinetic energies and the corresponding $^{16}\text{O}+^{89}\text{Y}$ and $^{12}\text{C}+^{93}\text{Nb}$ reactions.
extracted temperatures (T) and amplification parameters (p) as obtained from the fittings of the experimental and calculated (using ‘CASCADE’ code) alpha, lithium, boron and carbon spectra along with the estimated statistical uncertainties and find that the p-parameters extracted from the experimental and statistical model spectra follow qualitatively similar systematic. However the temperatures obtained from the experimental heavy fragment spectra do not show the expected trend of decreasing temperature. The experimental observations show the same or higher temperatures (T ≥3 MeV) for the residual nuclei from the spectral analysis of the heavier fragments compared to those obtained from the analysis of the alpha spectra. It is not possible to obtain such high temperatures (T ≥3 MeV) from the calculated heavy ion spectra by adjusting the parameters of the statistical model code ‘CASCADE’. We used the same value of the level density parameter a=A/8 for alpha, lithium, boron and carbon spectra calculations. Actually the value of the level density parameter should increase for the heavy ion emissions, as the nucleus becomes more deformed [5,6], thus decreasing the calculated temperature further. The effect of the deformation of the nucleus has also been considered in the statistical model calculations and it does not have any significant effect on the extracted temperature.

Table 1. Comparison of experimental and statistical model results for $^{16}$O+$^{89}$Y reaction.

| Ejectile | Experiment | Statistical model calculations |
|----------|------------|--------------------------------|
|          | p(MeV)     | T(MeV)                      | p(MeV) | T(MeV)         |
| $\alpha$ | 1.9±0.15   | 2.9±0.15                    | 4.0    | 2.9            |
| Li       | 4.9±0.4    | 4.2±0.2                     | 8.0    | 2.35           |
| B        | 13.1±1.0   | 3.3±0.2                     | 16.0   | 2.1            |
| C        | 15.0±1.2   | 3.5±0.3                     | 18.0   | 1.65           |

So we find that the temperatures obtained from the slopes of the heavier ion spectra do not show the expected trend of decrease. In order to check whether this is a peculiarity of this particular reaction or a more general trend, we also studied the experimental data [7,8] regarding the back-angle yields of alpha, carbon and oxygen from $^{35}$Cl+ $^{12}$C reaction at $E_{lab}^{(35}\text{Cl})=260$ MeV. The experimental alpha, carbon and oxygen spectra [7,8] from the reaction were fitted with eq(1). The amplification parameter p and the spectral temperature T were extracted and compared with the corresponding statistical model ‘CASCADE’ code calculations. We find from our analysis that the p-parameters extracted from both the experimental and statistical model spectra qualitatively follow similar trend of increase for the
However the temperatures (T) extracted from the slopes of the experimental alpha, carbon and oxygen spectra remain about 4 MeV and do not show any trend of decrease for the heavier fragment spectra, although the temperatures extracted from the calculated statistical model spectra show a continuous decrease (T=4.2 MeV from the alpha spectrum, T=2.2 MeV from the carbon spectrum and T=1.4 MeV from the oxygen spectrum) for the heavier fragment spectra. During the fitting procedure of the experimental heavy ion spectra with eq(1), we tried to decrease the temperature parameter (T) by increasing the value of the amplification parameter (p) so that the extracted temperature might be lower and agree with the statistical model predictions. In the case of $^{16}$O+$^{89}$Y (at E$_{\text{c.m.}}$=81.4 MeV) and $^{12}$C+$^{93}$Nb (at E$_{\text{c.m.}}$ = 75.7 MeV) reactions, we could not get any reasonable fit by lowering the temperature and increasing p-parameter. In the case of $^{35}$Cl+$^{12}$C reaction at E$_{\text{c.m.}}$=105.7 MeV, although in some cases (such as for the carbon spectrum) it was possible to get a good fit by lowering the temperature parameter (T) and increasing the amplification parameter (p), the best fit obtained in this way gave p=76 MeV and T=2 MeV for the carbon spectrum, but the oxygen spectrum from the same reaction could not be fitted by using such a large value of p-parameter. The acceptable fit for the oxygen spectrum gave p=44 MeV and T=4 MeV. Since according to the p-parameter systematic, the value of p should increase for the heavier fragment spectrum, the fit of the carbon spectrum using such a large p-parameter (p=76 MeV) is not acceptable.

So we conclude from our analysis that the temperatures of the residual nuclei extracted from the alpha and heavier fragment spectra from $^{16}$O+$^{89}$Y reaction at E$_{\text{c.m.}}$=81.4 MeV, $^{12}$C+$^{93}$Nb reaction at E$_{\text{c.m.}}$ = 75.7 MeV and $^{35}$Cl+$^{12}$C reaction at E$_{\text{c.m.}}$=105.7 MeV do not show any trend of decrease for the heavier fragment emissions, although the heavier fragments in the exit channel dinuclear system carry much higher orbital kinetic energy and so the available thermal energies and the corresponding temperatures should be lower.

We conjecture that our observations might imply the effect of the life-time of the exit channel dinuclear system (residual and ejectile system) on the measured temperature of the residual nucleus at the instant of its separation from the dinuclear system. The lifetime of the exit channel dinuclear system is inversely proportional to the transmission coefficient of the ejectile. Since the average transmission coefficient of $^{12}$C (emitted with a mean kinetic energy of 34 MeV and $t=40h$) is significantly larger than that of $^4$He (emitted with a mean kinetic energy of 15 MeV and $t=11-12h$), so the average lifetime of $^{12}$C+$^{93}$Nb dinuclear system should be much shorter than that of $^4$He+$^{101}$Ru dinuclear system. In the statistical model calculation, the lifetime of the exit channel dinuclear system has been implicitly assumed to be infinitely long and so although the fragment and the residual emissions.
nucleus separate out after a short time, the slope of the exponential tail of the fragment spectrum remains unaffected by the lifetime of the dinuclear system. The use of different sets of transmission coefficients obtained from different optical model parameters can drastically change the total yield of the emitted fragments, but they do not change much the slope of the fragment spectrum. Considering the quantum uncertainty principle, the slope of the fragment spectrum indicating the measured temperature of the ensemble of the residual nuclei might be affected by the short lifetime of the dinuclear system. If we consider an average width of the exit channel dinuclear states, then the exponential tail of the fragment spectrum might extend to higher energy implying a higher temperature for the corresponding residual nucleus at the time of the separation from the exit channel dinuclear system. On the other hand, the observed $1/\sin\theta_{c.m.}$ angular distribution of the fragment (such as $^{12}$C) at back angles in the center of mass frame implies a long lifetime of the exit channel dinuclear complex contradicting the earlier conjecture of the short lifetime. However one can also get a $1/\sin\theta_{c.m.}$ angular distribution in the case of the emission of the fragments in a plane perpendicular to the direction of the orbital angular momentum. Alternatively, if the initial decay of the heavy fragment and residual system (such as $^{12}$C+$^{93}$Nb) is assumed to be non-exponential [9,10], then also a relatively longer lifetime resulting in the observed $1/\sin\theta_{c.m.}$ angular distribution might be possible, even though the later exponential decay responsible for the width of the dinuclear state might be fast.

4. Conclusion
We have studied experimentally the statistical emission of alpha, lithium, boron, beryllium, carbon particles from $^{16}$O+$^{89}$Y and $^{12}$C+$^{93}$Nb reactions. The statistical compound nuclear origin of the emission of the particles was demonstrated from the observed back-angle $1/\sin\theta_{c.m.}$ angular distribution (in the center of mass frame) and the lack of any entrance channel dependence. It was found that the temperatures extracted from the spectra of the heavier fragments (such as lithium, boron, beryllium, carbon) remain about the same or higher compared to the temperature extracted from the spectrum of the alpha particles. Similar result was also obtained from the spectra of alpha, carbon and oxygen particles emitted from the reaction $^{35}$Cl+$^{12}$C at $E_{c.m.}$=105.7 MeV at back-angles in the center of mass frame. On the other hand, the statistical model code 'CASCADE' predicts that the temperature extracted from the evaporation fragment spectra should decrease continuously for the emission of the heavier fragments, because the heavier fragments carry higher orbital rotational kinetic energy, thus reducing the available thermal energy. It is not possible to explain the observed results by adjusting the parameters of the statistical model code. We have conjectured that the effect of the lifetime of the exit channel dinuclear system might increase the measured temperature of the residual nucleus.

5. References
[1] Moretto L G, Jing K X, Phair L and Wozniak G J 1997 J. Phys. G 23, 1323
[2] Puhlhofer F 1977 Nucl. Phys. A 280 267
[3] Ray A, Das P, Banerjee S R, De A, Kailas S, Chatterjee A, Dutta S K, Saha S and Roy S 2003 Phys. Rev. C 68 051602
[4] Das P et al. 2002 Phys. Rev.C 66 044612
[5] Lestone J P and McCalla S G 2009 Phys. Rev. C 79 044611
[6] Töke J and Swiatecki W J 1981 Nucl. Phys. A 371 141
[7] Mahboub D, Beck C, Djerroud B, Freeman R M, Haas F, Rousseau M, Papka P and Sánchez Zafra A 2004 Phys. Rev. C 69 034616
[8] Mahboub D 1996 PhD thesis Universite Louis Pasteur, Strasbourg, France
[9] Fonda L, Ghirardi G C and Rimini A 1978 Rep. Prog. Phys. 41 587
[10] Sudbery A 1984 Annals of Physics 157 512