The convolution model of unstable particles

V.I. Kuksa

Institute of Physics, Rostov State University,
pr. Stachki 194, Rostov-on-Don, 344090 Russia,
E-mail address: kuksa@list.ru

Abstract
Quantum field model of unstable particles with random mass is suggested to describe the finite-width effects in decay rate. Within the framework of this model we derive the convolution formula for a width of the channels with unstable particle in a final state. The distribution function of random mass is considered for unstable particles of arbitrary type.

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1. Introduction
Quantum field description of the unstable particles (UP) with a large width runs into some problems, which are under considerable discussions [1]. These problems have both the conceptual and technological status and arise due to UP lie somewhat outside the traditional formulation of quantum field theory [2]. Unstable field can not be treated as asymptotic state and perturbative approach is unfit in the resonance neighborhood. This conceptual problems is connected with methodological difficulties, such as ambiguity in definition of mass and width. Therefore, the new quantum field approach [2] (Bohm et al), phenomenological models [3] and effective theories of UP [4] are actual now.

Convolution method [5] is convenient and clear phenomenological way to evaluate the instability or finite-width effects (FWE). This method describes FWE in the processes of type $\Phi \rightarrow \phi_1 \phi \rightarrow \phi_1 \phi_2 \phi_3 \ldots$, where $\phi$ is UP with a large width. The intermediate unstable state $\phi$ is simulated by the final state $\phi$ in the decay $\Phi \rightarrow \phi_1 \phi$ with invariant mass, described
by Breit-Wigner-like (Lorentzian) distribution function. The phenomenological expression for a decay rate has convolution form [5]:

$$\Gamma(\Phi \to \phi_1 \phi) = \int_{q_1^2}^{q_2^2} \Gamma(\Phi \to \phi_1 \phi(q)) \rho(q) dq^2, \quad (1)$$

where $\rho(q) = MT_\phi(q)/\pi|P(q)|^2$. In Eq.$(1)$ $\rho(q)$ is probability density of invariant mass distribution, $P(q) = q^2 - M^2 + iMT_\phi(q)$ [5](Altarelli et al), $\Gamma(\Phi \to \phi_1 \phi(q))$ and $T_\phi(q)$ are partial width of $\Phi$ and total width of $\phi$ in the stable particle approximation, when $m_\phi^2 = q^2$. The formula for a decay rate, which has a close analogy to the Eq.$(1)$, was applied first to describe FWE in $B$ and $\Lambda$ decay channels with $\rho(770)$ and $a_1(1260)$ in the final states, which have large total widths [6]. It was shown that the contribution of FWE to the decay rates of these channels are large (20-30%) and that the account of it significantly improves a conformity of experimental data and theoretical predictions. Analogous results were obtained in Ref. [3] for the dominant decay channels of $\Phi(1020)$, $\rho(770)$ and $K^*(892)$. The decay rates of the near-threshold decay channels $t \to WZb, cWW, cZZ$ were calculated with help of convolution formula (CF) in Ref. [5]. It was shown in these works, that the FWE play a significant role in the near-threshold processes.

The convolution formula $(1)$ was derived by direct calculation from the decay-chain method in Ref. [7], where in analogy with inclusive processes the contribution of all decay channels of UP is described by function $\rho(q^2) = q^2 \Gamma(q^2)/|P(q)|^2$. The essential elements of this derivation for vector and spinor UP was the expressions $\eta_{mn} = -g_{mn} + q_m q_n/q^2$ and $\hat{\eta} = \nabla + q$ as numerators of vector and spinor propagators ($\nabla = q \gamma^i$). The convolution formula was derived for the decay chain $t \to bW \to bf_i f_j$ in the limit of massless fermions $f$ [5](Galderon and Lopez-Castro). Quantitative analysis of convolution and decay-chain calculations of the $t \to WZb$ decay rate was fulfilled in Ref. [5] (Altarelli et al). The formula for a decay rate, which is in close analogy with $(1)$, was received in Ref. [3] for the scalar UP within the framework of the ”random mass” model. An UP is described in this model as quantum field with a ”smeared” (fuzzy) random mass in accordance to uncertainty principle for energy and lifetime of unstable quantum system [8]. The FWE is connected with fundamental principle, which gives the relation $\delta m * \tau \approx 1$, that is $\delta m \approx \Gamma$ in the rest frame of reference ($\delta E = \delta m, c = \hbar = 1$)[3]. So, uncertainty principle leads to the interpretation of kinematic value $q^2$ in Eq.$(1)$ as random mass square. Thus, the intermediate states of UP, which are traditionally defined as virtual, in the neighborhood of $q^2 = M^2$ are not differ from real ones in accordance to uncertainty principle. This interpretation is connected with a smearing of mass shell and with above mentioned definition of $\eta_{mn}$ and $\hat{\eta}$, which are proportional to the polarization matrix for vector and spinor UP (see section 3). As was noted
in \[7\], this proportionality leads to the factorization of expression for width in decay-chain method, and, as consequence, to the CF \[1\]. Thus, the suggested model is theoretical basis of the convolution method on the ground of uncertainty principle.

In this paper we consider the generalization of the model \[3\], which includes vector and spinor fields. Within the framework of this generalized model the CF is derived for UP of arbitrary type. To determine the probability density \(\rho(m)\), which is an analogue of \(\rho(q)\) in Eq.\([1]\), we put a connection between the model and effective theory of UP with modified propagators, used in Ref. \[7\]. It was shown that this connection leads to Lorentzian probability density \(\rho(m)\), which was commonly used in convolution method. Suggested model is applicable to the decay processes of type \(\Phi \to \phi_1\phi(q)\) and gives the convolution formula \([1]\) for UP of arbitrary type.

2. The model of unstable particles with a random mass

The effect of mass smearing is described by the wave packet with some weight function \(\omega(\mu)\), where \(\mu\) is random mass parameter \[3\]. The model field function, which simulates UP in the initial, final or intermediate states, is represented by the expression:

\[
\Phi_\alpha(x) = \int \Phi_\alpha(x, \mu)\omega(\mu)d\mu. \tag{2}
\]

In Eq.\([2]\) \(\Phi_\alpha(x, \mu)\) are the components of field function, which are determined in the usual way when \(m^2 = \mu\) is fixed (stable particle approximation). The limits of integration will be defined in the sections 3 and 4.

The model Lagrangian, which determines ”free” unstable field \(\Phi(x)\), has the convolution form:

\[
L(\Phi(x)) = \int L(\Phi(x, \mu))|\omega(\mu)|^2 d\mu. \tag{3}
\]

In Eq.\([3]\) \(L(\Phi(x, \mu))\) is standard Lagrangian, which describes model ”free” field \(\Phi(x, \mu)\) with fixed mass \(m^2 = \mu\).

From Eq.\([3]\) and prescription \(\partial\Phi(x, \mu)/\partial\Phi(x, \mu') = \delta(\mu - \mu')\) it follows Klein-Gordon equation for each spectral component:

\[
(\Box - \mu)\Phi_\alpha(x, \mu) = 0. \tag{4}
\]

As a result we have standard momentum representation of field function for fixed mass parameter \(\mu\):

\[
\Phi_\alpha(x, \mu) = \frac{1}{(2\pi)^{3/2}} \int \Phi_\alpha(k, \mu)\delta(k^2 - \mu)e^{ikx}dk. \tag{5}
\]
All standard definitions, relations and frequency expansion take place for \( \Phi_\alpha(k, \mu) \), but the relation \( k_\mu^0 = \sqrt{k^2 + \mu} \) defines smeared (fuzzy) mass-shell due to random \( \mu \).

The expressions (2) and (3) define the model ”free” unstable field, which really is some effective field. This field is formed by interaction of ”bare” UP with it’s decay channels and includes nonperturbative self-energy contribution in the resonant region. Such an interaction leads to the spreading (smearing) of mass from \( \rho^{st}(\mu) = \delta(\mu - M^2) \) for the bare particles to some smooth density function \( \rho(\mu) = |\omega(\mu)|^2 \) with mean value \( \bar{\mu} \approx M^2 \) and \( \sigma_\mu \approx \Gamma \). So, the UP is characterized in the discussed model by the weight function \( \omega(\mu) \) or probability density \( \rho(\mu) \) with parameters \( M \) and \( \Gamma \) (or real and imaginary parts of pole). A similar approach has been discussed by Matthews and Salam in Ref. [8].

The commutative relations for a model operators have an additional \( \delta \)-function:

\[
[\hat{\Phi}_-^\alpha(\vec{k}, \mu), \hat{\Phi}_+^\beta(\vec{q}, \mu')]_{\pm} = \delta(\mu - \mu')\delta(\vec{k} - \vec{q})\delta_{\alpha\beta},
\]

(6)

where subscripts \( \pm \) correspond to the fermion and boson fields. The presence of \( \delta(\mu - \mu') \) in Eq.(6) means an assumption - the acts of creations and annihilations of particles with various \( \mu \) (random mass square) don’t interfere. So, the parameter \( \mu \) has the status of physically distinguishable value as random \( m^2 \). This assumption directly follows from the interpretation of \( q^2 \) in Eq. (1) as random parameter \( \mu \). By integrating both side of Eq.(6) with weights \( \omega^*(\mu)\omega(\mu') \) one can get standard commutative relations

\[
[\hat{\Phi}_-^\alpha(\vec{k}), \hat{\Phi}_+^\beta(\vec{q})]_{\pm} = \delta(\vec{k} - \vec{q})\delta_{\alpha\beta},
\]

(7)

where \( \Phi_\pm^\alpha(\vec{k}) \) is full operator field function in momentum representation:

\[
\Phi_\pm^\alpha(\vec{k}) = \int \Phi_\pm^\alpha(\vec{k}, \mu)\omega(\mu)d\mu.
\]

(8)

It should be noted that Eq.(7) follows from Eq.(6) when \( \int |\omega(\mu)|^2d\mu = 1 \).

The expressions (2) and (6) are the principal elements of the discussed model. The weight function \( \omega(\mu) \) in Eq.(2) (or \( \rho(\mu) \)) is full characteristic of UP and the relations (6) define the structure of the model amplitude and of the transition probability (section 3). The probability density \( \rho(\mu) \) will be defined in the fourth section by matching the model propagator to renormalized one.

With help of traditional method one can get from Eqs.(2), (4) and (6) the expression for the unstable scalar Green function [3]:

\[
\langle 0|T(\phi(x), \phi(y))|0 \rangle = D(x - y) = \int D(x - y, \mu)\rho(\mu)d\mu.
\]

(9)
In Eq. (9) \( D(x, \mu) \) is defined in the standard way for the scalar field with \( m^2 = \mu \) and describes UP in an intermediate state:

\[
D(x, \mu) = \frac{i}{(2\pi)^4} \int \frac{e^{-ikx}}{k^2 - \mu + i\epsilon} dk .
\]

The right side of the Eq. (9) is Lehmann-like spectral (on \( \mu \)) representation of the scalar Green function, which describes the propagation of scalar UP. Taking into account the connection between scalar and vector Green functions, we can get the Green function of the vector unstable field

\[
D_{mn}(x, \mu) = -(g_{mn} + \frac{1}{\mu} \frac{\partial^2}{\partial x^m \partial x^n}) D(x, \mu) = \frac{-i}{(2\pi)^4} \int \frac{g_{mn} - k_m k_n/\mu}{k^2 - \mu + i\epsilon} e^{-ikx} dk .
\]

Analogously Green function of the spinor unstable field:

\[
\hat{D}(x, \mu) = (i\hat{\partial} + \sqrt{\mu}) D(x, \mu) = \frac{i}{(2\pi)^4} \int \frac{\hat{k} \pm \sqrt{\mu}}{k^2 - \mu + i\epsilon} e^{-ikx} dk ,
\]

where \( \hat{k} = k_i \gamma^i \). These Green functions in momentum representation have a convolution structure:

\[
D_{mn}(k) = \int D_{mn}(k, \mu) \rho(\mu) d\mu , \quad \hat{D}(k) = \int \hat{D}(k, \mu) \rho(\mu) d\mu .
\]

3. Model amplitude and the convolution formula for a decay rate

In this section we consider the model amplitude for the simplest processes with UP in a final state and get the CF (11) as direct consequence of the model. The expression for a scalar operator field [3]:

\[
\phi^\pm(x) = \frac{1}{(2\pi)^{3/2}} \int \omega(\mu) d\mu \int \frac{a^\pm(\bar{q}, \mu) e^{\pm iq x}}{\sqrt{2q_\mu^0}} d\bar{q} ,
\]

where \( q_\mu^0 = \sqrt{\bar{q}^2 + \mu} \) and \( a^\pm(\bar{q}, \mu) \) are creation or annihilation operators of UP with momentum \( q \) and mass square \( m^2 = \mu \). Taking into account Eq. (6) we can get:

\[
[\hat{a}^-(k, \mu), \phi^+(x)] = [\phi^-(x), \hat{a}^+(k, \mu)] = \frac{\omega(\mu)}{(2\pi)^{3/2} \sqrt{2k_\mu^0}} e^{\pm ikx} , \quad k_\mu^0 = \sqrt{\bar{k}^2 + \mu} .
\]

The expressions (15) differ from standard ones by the factor \( \omega(\mu) \) only. From this result it follows that, if \( \hat{a}^+(k, \mu)|0\rangle \) and \( \langle 0|\hat{a}^-(k, \mu) \) define UP with a mass \( m^2 = \mu \) and a momentum \( k \) in the initial and final states, then the amplitude for the decay of type \( \Phi \to \phi\phi_1 \) has the form:

\[
A(k, \mu) = \omega(\mu) A^{st}(k, \mu) ,
\]
where $A_{st}(k, \mu)$ is amplitude in a stable particle approximation when $m^2 = \mu$. This amplitude is calculated in a standard way and can include high corrections. Moreover, it can be effective amplitude for the processes with hadron participation \[3, 5\].

To define the transition probability of the process $\Phi \to \phi\phi_1$, where $\phi$ is UP with a large width, we should take into account the status of parameter $\mu$ as physically distinguishable value, which follows from Eq.\( (9)\). Thus, the amplitude at different $\mu$ don’t interfere and we have the convolution structure of differential (on $k$) probability:

$$d\Gamma_{\Phi \to \phi\phi}(k, \mu) = \int d\Gamma_{st}(k, \mu)|\omega(\mu)|^2d\mu .$$

In Eq.\( (17)\) the differential probability $d\Gamma_{st}(k, \mu)$ is defined in the standard way (stable particle approximation):

$$d\Gamma_{st}(k, \mu) = \frac{1}{2\pi} \delta(k_\Phi - k_\phi - k_1)|A_{st}(k, \mu)|^2dk_\phi dk_1 ,$$

where $k = (k_\Phi, k_\phi, k_1)$ denotes the momenta of particles. From Eqs.\( (17)\) and \( (18)\) it directly follows the known convolution formula for a decay rate

$$\Gamma(m_{\Phi}, m_{\phi_1}) = \int_{\mu_0}^{\mu_m} \Gamma_{st}(m_{\Phi}, \mu; \mu)\rho(\mu)d\mu ,$$

where $\rho(\mu) = |\omega(\mu)|^2$ and $\mu_0, \mu_m$ are defined in Refs. \[5, 7\] as threshold and maximal invariant mass square of unstable $\phi$.

An account of high corrections to the amplitude \( (16)\) and, hence, to Eq.\( (19)\) keeps convolution form \( (19)\). This form can be destroyed by the interaction between the products of UP ($\phi$) decay and initial $\Phi$ or final $\phi_1$ states. The calculation in this case can be fulfilled in a standard way, but UP in the intermediate state is described by the model propagator. However, a calculation within the framework of perturbative theory (PT) can not be applicable to the UP with large width, that is to the short-living particle. In any case, the applicability of PT, model approach or convolution method to the discussed decays should be justified by experiment. The correspondence of CM to the experimental data was demonstrated for some processes \[3, 4, 5, 6, 7\], but this problem needs in more detailed investigation. In this connection we should note the analysis of higher-order corrections for processes with UP \[4\]. The separation between factorizable and non-factorizable corrections make it possible to build the effective theory of UP \[4\].

When there are two UP with large widths in a final state $\Phi \to \phi_1\phi_2$, then in analogy with the previous case one can get double convolution formula:

$$\Gamma(m_{\Phi}) = \int \int \Gamma_{st}(m_{\Phi}; \mu_1, \mu_2)\rho_1(\mu_1)\rho_2(\mu_2)d\mu_1d\mu_2 .$$

\[20\]
The derivation of CF for the cases when there is vector or spinor UP in a final state can be done in analogy with the case of scalar UP. However, in Eqs. (14), (15) and (16) we have a polarization vector $e_m(q)$ or spinor $u_{\alpha}^{\nu,\pm}(q)$, where $q$ is on fuzzy mass-shell. As a result we get polarization matrix with $m^2 = \mu$. For the vector UP in a final state:

$$\sum e_m(q)e_{n}^*(q) = -g_{mn} + \frac{q_{m}q_{n}}{\mu}.$$ \hspace{1cm} (21)

For the spinor UP in a final state:

$$\sum_{\nu} u_{\alpha}^{\nu,\pm}(q)\bar{u}_{\beta}^{\nu,\mp}(q) = \frac{1}{2q_{\mu}^0}(\hat{q}^{\pm}\sqrt{\mu})_{\alpha\beta}. $$ \hspace{1cm} (22)

In Eqs. (21) and (22) sum run over polarization and $q_{\mu}^0 = \sqrt{q^2 + \mu}$.

The formulae (19) and (20) describe FWE in full analogy with the phenomenological convolution method [5] and with some cases of the decay-chain method [5, 7]. Thus, we consider the quantum field basis for CM, which takes into account the fundamental uncertainty principle and is in good agreement with experimental date on some decays. To evaluate FWE for the case, when UP is in an initial state, we must account the process of UP generation. When UP is in an intermediate state, then the description of FWE is equivalent to the traditional one, but the model propagators are determined by Eqs. (9) - (13).

4. Determination of $\rho(\mu)$ from renormalized propagator

The possibility of $\rho(\mu)$-determination directly follows from the connection of the decay-chain method (DCM) and convolution method [7]. As was shown in Ref. [7], this connection leads to the convolution formula (11), where in accordance with uncertainty principle $q^2$ is interpreted as smeared mass square parameter $\mu$, which distribution is described by the expression:

$$\rho(\mu) = \frac{1}{\pi} \sqrt{\frac{\pi}{\Gamma(\mu)}} \frac{1}{|P(\mu)|^2}. $$ \hspace{1cm} (23)

In Eq. (23) $\Gamma(\mu)$ is $\mu$-dependent full width and $P(\mu)^{-1}$ is propagator’s denominator. It should be noted, that the convolution structure of Eq. (11) and universal structure of Eq. (23) don’t depend on the definition of $P(\mu)$. It has a complex pole structure $\mu - \mu_R$ and can be approximated by the Breit-Wigner $\mu - M^2 + iM\Gamma(\mu)$ or another phenomenological approximation. The expression (23) is very simple and convenient in practical calculations of decay rate, where the error of approximation is small.

Here we’ll consider the definition of $\rho(\mu)$ from the matching model propagators to standard dressed ones [3]. This consideration is rather methodological than practical and demon-
strates the connection between model and traditional descriptions. Let us associate the model propagator of scalar unstable field \( \mathcal{F} \) with standard one:

\[
\int \frac{\rho(\mu)d\mu}{k^2 - \mu + i\epsilon} \longleftrightarrow \frac{1}{k^2 - m_0^2 - \Pi(k^2)}, \tag{24}
\]

where \( \Pi(k^2) \) is conventional self-energy of scalar field. With help of an analytical continuation of the expressions (24) on complex plane \( k^2 \to k^2 \pm i\epsilon \) and prescription [9]:

\[
\Pi(k^2 \pm i\epsilon) = \text{Re}\Pi(k^2) \mp i\text{Im}\Pi(k^2), \tag{25}
\]

the conformity (24) can be represented by the equality

\[
\int_0^\infty \frac{\rho(\mu \pm i\epsilon)d\mu}{k^2 - m^2(k^2) \pm i\text{Im}\Pi(k^2)} = \frac{1}{k^2 - m_0^2 - \Pi(k^2)}, \tag{26}
\]

where \( m^2(k^2) = m_0^2 + \text{Re}\Pi(k^2) \). With account of round pole rules and \( d\mu = d(\mu \mp i\epsilon) \), \( \rho(\mu \mp i\epsilon) = \rho(\mu) \mp O(i\epsilon) \) two Eqs. (26) can be combine into the equality \( (\mu \pm i\epsilon \to z) \):

\[
\int \frac{\rho(z)dz}{z - k^2} = \frac{1}{k^2 - m^2(k^2) - i\text{Im}\Pi(k^2)} - \frac{1}{k^2 - m^2(k^2) + i\text{Im}\Pi(k^2)}. \tag{27}
\]

The left side of Eq. (27) is Cauchy integral, which equal to \( 2\pi i\rho(k^2) \) and after a change \( k^2 \to \mu \) in the final expression for \( \rho \) we have:

\[
\rho(\mu) = \frac{1}{\pi} \frac{\text{Im}\Pi(\mu)}{[\mu - m^2(\mu)]^2 + [\text{Im}\Pi(\mu)]^2}. \tag{28}
\]

The expression (28) for \( \rho(k^2) \) in Breit-Wigner approximation is usually exploited within the framework of convolution method. From Eq. (28) and definition \( \rho(\mu) = |\omega(\mu)|^2 \) it follows:

\[
\omega(\mu) = \frac{1}{\sqrt{\pi} \mu - m^2(\mu) \pm i\text{Im}\Pi(\mu)}. \tag{29}
\]

The ambiguity of sign in (29) is not essential because the expression \( |\omega(\mu)|^2 \) only enters into the physical values. In the parametrization \( \text{Im}\Pi(\mu) = \sqrt{\Gamma(\mu)} \) we have relativistic Breit-Wigner \( \omega(\mu) \) and Lorentzian \( \rho(\mu) \), which coincides with the expression (23) for renormalized \( P(q^2) \). Inserting the expression (28) into the left side of Eq. (24) one can check with help of Cauchy method the self-consistency of Eqs. (24) and (28).

Thus, we have put the correspondence between the model [2] - [6] and some effective theory of UP with renormalized propagator of scalar UP. To establish such a correspondence for the vector UP we insert \( \rho(\mu) \) into the model propagator (13) with \( D_{mn}(k, \mu) \), defined by (11) for vector unstable field:

\[
\int_0^\infty \frac{-g_{mn} + k_mk_n/\mu}{k^2 - \mu + i\epsilon} \frac{1}{\pi [\mu - m^2(\mu)]^2 + [\text{Im}\Pi(\mu)]^2} d\mu = \frac{1}{2i\pi} \int_0^\infty \frac{-g_{mn} + k_mk_n/\mu}{k^2 - \mu + i\epsilon} \frac{1}{[\mu - m^2(\mu) - i\text{Im}\Pi(\mu)] - [\mu - m^2(\mu) + i\text{Im}\Pi(\mu)]} d\mu. \tag{30}
\]
With help of Eq.(25) and above used method we can represent the second part of Eq.(30) in the form ($\mu \rightarrow z = \mu \pm i\epsilon$):

$$\frac{1}{2i\pi} \int \frac{dz}{z - k^2} \frac{-g_{mn} + k_m k_n / z}{z - m^2(z) - i\text{Im}\Pi(z)} = \frac{-g_{mn} + k_m k_n / k^2}{k^2 - m^2(k^2) - i\text{Im}\Pi(k^2)}.$$  \hspace{1cm} (31)

The right side of Eq.(31) coincides with the expression for propagator of vector UP, which leads to the convolution formula \cite{1} in the decay-chain method \cite{7}. The numerator of this effective propagator coincides with $\eta_{mn}(k)$, which was used in \cite{7}. In Eqs.(30) and (31) the value $\Pi(k^2)$ is defined for vector field as transverse part of polarization matrix \cite{1}. The calculations of $\Pi(k^2)$ in effective theory (unstable hadrons) or in gauge theory (Z,W-bosons) can run into some difficulties. In the first case loop calculation can be ambiguous and we should use traditional Breit-Wigner approximation $m^2(\mu) \approx M^2$ and $\text{Im}\Pi(\mu) \approx \mu \Gamma(\mu)$. To escape the gauge-dependence in the second case we can use pole definitions of mass and width \cite{1}.

The description of $\rho(\mu)$ by the universal function (28) for scalar and vector fields can be justified by the general structure of parametrization for bosons:

$$m^2(q^2) = m^2_0 + \text{Re}\Pi(q^2), \quad \text{Im}\Pi(q^2) = q\Gamma(q^2).$$  \hspace{1cm} (32)

In the case of unstable fermion we have another parametrization scheme:

$$m(q^2) = m_0 + \text{Re}\Sigma(q^2), \quad \text{Im}\Sigma(q^2) = \Gamma(q^2).$$  \hspace{1cm} (33)

So, we need in additional analysis to define fermion function $\rho(\mu)$. If we choose for fermion UP the universal density function (28), which follows from convolution method \cite{7}, then we must do exchange $\text{Im}\Pi(\mu) \rightarrow \sqrt{\mu}\text{Im}\Sigma(\mu)$ in the Eq.(28). Inserting the result into Eq.(13) with $\hat{D}(x,\mu)$, defined by Eq.(12), we can get the correspondence between the model propagator of fermion unstable field and the effective theory one:

$$\int \frac{\hat{k} + \sqrt{\mu}}{k^2 - \mu + i\epsilon} \rho(\mu) d\mu \rightarrow \frac{\hat{k} + k}{k^2 - m^2(k^2) - ik\Sigma(k^2)},$$  \hspace{1cm} (34)

where $k = \sqrt{kk}$. The numerator of the right side of Eq.(34) coincides with the expression $\hat{\eta}$ \cite{7}.

The transitions (24), (31) and (34) establish the correspondence between the discussed model and some effective theory of UP within the framework of traditional QFT approach. These transitions follow from the determination of $\rho(\mu)$, that is from the accounting of interaction, which forms the wave packet (2) and mass-smearing. The most important feature of the effective theory, chosen in such a way, is the possibility to connect the decay-chain
method and convolution method within the framework of this theory \[7\]. So, we have some self-consistency of the discussed model, effective theory, convolution and decay-chain method. However, due to some difficulties, which arise in traditional approach, the search of alternative $\rho(\mu)$ - definition is actual now.

5. Conclusion

The finite width effects in the processes with participation of UP can be described by renormalized propagator, decay-chain method, convolution method and effective theory of UP. The convolution formula is convenient instrument for calculations of decay rate and gives the results in accordance with experiment. In this paper we have considered the model of UP with a random mass and derived the convolution formula as a direct consequence of the model. The model operator function and Lagrangian have a convolution structure, which describes mass-smearing in accordance with uncertainty principle.

The principal element of suggested model is probability density function $\rho(\mu)$, which describes the main properties of UP. Traditional description of UP in the intermediate state by resonance line with complex pole (or by dressed propagator with mass and width as parameters) corresponds to the model description of UP in arbitrary state by function $\rho(\mu)$ with the same parameters. We have considered the determination of $\rho(\mu)$ from DCM and by matching the model propagator to renormalized one. This approach is equivalent to the convolution method or truncated decay-chain method.

The second $\rho(\mu)$ - determination has some restrictions, caused by propagator renormalization peculiarities. The question arises, also, concern the possibility to describe mass-smearing of bosons and fermions by the universal function $\rho(\mu)$. Moreover, as the mass-smearing effect follows from the fundamental uncertainty principle, then the search of $\rho(\mu)$ from the first principles is reasonable. It should be noted also, that the model erases a difference between the real and virtual states of UP at peak region.

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