Lepton masses and mixings in a $T'$ flavoured 3-3-1 model with type I and II seesaw mechanisms

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We propose a renormalizable $T'$ flavor model based on the $SU(3)_C \times SU(3)_L \times U(1)_X \times U(1)_L$ gauge symmetry, consistent with the observed pattern of lepton masses and mixings. The small masses of the light active neutrinos are produced from an interplay of type I and type II seesaw mechanisms, which are induced by three heavy right-handed Majorana neutrinos and three $SU(3)_L$ scalar antisextets, respectively. Our model is only viable for the scenario of normal neutrino mass hierarchy, where the obtained physical observables of the lepton sector are highly consistent with the current neutrino oscillation experimental data. In addition, our model also predicts an effective Majorana neutrino mass parameter of $m_\beta \sim 1.41541 \times 10^{-2}$ eV, a Jarlskog invariant of the order of $J_{CP} \sim -0.032$ and a leptonic Dirac CP violating phase of $\delta = 238^\circ$, which is inside the $1\sigma$ experimentally allowed range.

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I. INTRODUCTION

Despite its striking consistency with experimental data, the Standard Model (SM) of the elementary particle physics cannot provide a satisfactory explanation of the fermion mass hierarchy and mixing angles. There is a huge gap of about 5 orders of magnitude between the electron and the top quark masses. In addition many experiments show that neutrinos have tiny masses, of about 8 orders of magnitude much smaller than the electron mass. Furthermore, the absolute neutrino mass scale as well as the sign of $\Delta m_{31}^2$ is still unknown. In addition quark mixing angles are small, whereas two of the leptonic mixing angles are large and the other is Cabibbo sized. In addition, the existence of three fermion families, which is not explained in the context of the SM, can be understood in the framework of $SU(3)_C \otimes SU(3)_L \otimes U(1)_X$ models (3-3-1 models), where $U(1)_X$ nonuniversal family symmetry distinguishes the third fermion family from the first and second ones [1–8]. These models are very important because of the following reasons: 1) The existence of three generations of fermions arise from the cancellation of chiral anomalies and the asymptotic freedom in QCD. 2) The non universal $U(1)_X$ symmetry allows to explain the large mass hierarchy between the heaviest quark family and the two lighter ones. 3) These models provide an explanation for the electric charge quantization [9, 10]. 4) CP violation is generated in the 3-3-1 models [11, 12]. 5) The 3-3-1 models predict the upper bound $\sin^2 \theta_W < \frac{1}{4}$ for the Weinberg mixing angle. 6) Third, these models include a natural Peccei-Quinn symmetry, which is crucial for addressing the strong-CP problem as explained in detail in Refs. [13–16]. 7) Models with heavy sterile neutrinos include cold dark matter candidates as weakly interacting massive particles (WIMPs) [17]. A concise review of WIMPs in 3-3-1 Electroweak Gauge Models is provided in Ref. [18].

The global fits of the available data from neutrino oscillation experiments Daya Bay [19], T2K [20], MINOS [21], Double CHOOZ [22] and RENO [23], set constraints on the allowed values of the neutrino mass squared splittings, the leptonic mixing parameters and the leptonic Dirac CP violating phase, as displayed in Table II (based on Ref. [24]) for the normal (NH) and inverted (IH) hierarchies of the neutrino mass spectrum. These facts might suggest that the tiny neutrino masses can be related to a scale of new physics that, in general, is not related to the scale of Electroweak Symmetry Breaking $v = 246$ GeV. Furthermore, the charged fermion masses can be accommodated in the SM, at the price of having an unnatural tuning among its different Yukawa couplings. All these unexplained issues within the context of the SM, suggest that new physics have to be invoked to address the fermion puzzle of the SM.

The unexplained flavor puzzle of the SM has stimulated work on flavor symmetries which
includes the $T'$ discrete groups, that are used in order to provide an explanation for the observed pattern of SM fermion masses and mixing angles. In this paper we propose a 3-3-1 model with neutral leptons based on $T'$ flavor symmetry consistent with the current neutrino oscillation experimental data of Ref. [24] for the scenario of normal hierarchy. The masses of the light active neutrinos are generated from an interplay of type I and type II seesaw mechanisms mediated by three heavy right-handed Majorana neutrinos and three $SU(3)_L$ scalar antisextets, respectively.

Despite the $T'$ has been previously studied in Refs. [25–41], to the best of our knowledge, this discrete group has not been considered before in this kind of 3-3-1 model.

Table I: The experimental best fit values of neutrino mass squared splittings and leptonic mixing parameters, taken from Ref. [24]. Here, NH and IH stand for Normal Hierarchy and Inverted Hierarchy, respectively.

| Parameter   | $\Delta m^2_{31}(10^{-5}\text{eV}^2)$ | $\Delta m^2_{21}(10^{-5}\text{eV}^2)$ | $\sin^2\theta_{12}$ | $\sin^2\theta_{23}$ | $\sin^2\theta_{13}$ | $\delta/\pi$ |
|-------------|--------------------------------------|--------------------------------------|-----------------------|-----------------------|-----------------------|--------------|
| Best fit $\pm 1\sigma$ NH | $7.55^{+0.20}_{-0.16}$ | $2.50 \pm 0.03$ | $0.320^{+0.020}_{-0.016}$ | $0.547^{+0.020}_{-0.030}$ | $0.0216^{+0.00083}_{-0.00069}$ | $1.39^{+0.21}_{-0.15}$ |
| IH          | $7.55^{+0.20}_{-0.16}$ | $-2.42^{+0.03}_{-0.04}$ | $0.320^{+0.020}_{-0.016}$ | $0.551^{+0.018}_{-0.030}$ | $0.0222^{+0.00074}_{-0.00075}$ | $1.56^{+0.13}_{-0.15}$ |

Table II: The lepton and scalar assignments under $SU(3)_L \times U(1)_X \times U(1)_L \times T'$. 

II. THE MODEL

We consider a model based on the $SU(3)_C \times SU(3)_L \times U(1)_X \times U(1)_L$ gauge symmetry, which is supplemented by the $T'$ discrete group, introduced to generate a viable pattern of lepton masses and mixings consistent with the current neutrino oscillation experimental data. The lepton assignments of the model, under the $[SU(3)_L, U(1)_X, U(1)_L, T']$ symmetries, are given in Tab. II where $\alpha = 2, 3$
is a family index of the last two lepton generations, which defines the components of the $T'$ doublet representations.

To generate masses for the charged leptons, we need two $SU(3)_L$ scalar multiplets, namely $\phi$ and $\phi'$, whose assignments under the different discrete group factors of the model are given in Table. With the particle content and symmetries specified in Table, the following Yukawa interactions for charged leptons arise:

$$- L_l = h_1 \bar{\psi}_1 L l_1 R + h_2 (\bar{\psi}_\alpha L \phi) l_\alpha R + h_3 (\bar{\psi}_\alpha L \phi') l_\alpha R + H.c.$$  (1)

In this work, we impose the $T' \to Z_2$ symmetry breaking chain, which gives rise to the VEV pattern $\langle \phi' \rangle = (\langle \phi'_1 \rangle, 0, 0)$ for the $T'$ triplet scalar $\phi'$. In addition, the VEVs of the $SU(3)_L$ scalars $\phi'_1$ and $\phi$ are given by:

$$\langle \phi'_1 \rangle = (0 \ v' \ 0)^T, \quad \langle \phi \rangle = (0 \ v \ 0)^T.$$  (2)

Then, after electroweak symmetry breaking, the following charged lepton mass terms are obtained:

$$- L_l^{\text{mass}} = h_1 v_1 \bar{l}_1 L l_1 R - (h_2 v - h_3 v') \bar{l}_2 L l_2 R + (h_2 v + h_3 v') \bar{l}_3 L l_3 R + H.c.$$  (3)

From the mass terms given above, we find that the SM charged lepton mass matrix is diagonal and the masses for the SM charged leptons are given by:

$$m_e = h_1 v, \quad m_\mu = -h_2 v + h_3 v', \quad m_\tau = h_2 v + h_3 v',$$  (4)

and thus the diagonalization matrices are $U_{1L} = U_{1R} = 1$. This means that the charged lepton fields $l_{1,2,3}$ by themselves are physical mass eigenstates, and the Pontecorvo–Maki–Nakagawa–Sakata (PMNS) leptonic mixing matrix is the rotation matrix that diagonalizes the light active neutrino mass matrix. The masses of muon and tau leptons are explicitly separated by $\phi'$ resulting from the breaking $T' \to Z_6$. This is why we introduce $\phi'$ in accompanying with $\phi$. For the charged leptons masses at the electroweak scale we use the values given in Particle Data Group 2018:

$$m_e \simeq 0.511 \text{ MeV}, \quad m_\mu \simeq 105.66 \text{ MeV}, \quad m_\tau \simeq 1776.86 \text{ MeV}.$$  

Thus, we get

$$h_1 v = 0.511 \text{ MeV}, \quad h_2 v = 835.6 \text{ MeV}, \quad h_3 v' = 941.26 \text{ MeV}.$$  

Consequently, to explain the SM charged lepton mass hierarchy, it is required that $|h_1| \ll |h_2|$, and if $|v'| \sim |v|$, then $|h_2|$ and $|h_3|$ have to be of the same order of magnitude.

To generate the masses of the light active neutrinos we introduce six $SU(3)_L$ scalar antisextets, namely, $\sigma, \sigma'_k (k = 1, 2), s_j (j = 1, 2, 3)$ and one $SU(3)_L$ scalar triplet $\rho$. The $SU(3)_L$ scalar antisextet
\( \sigma \) is assigned as a \( T' \) trivial singlet, whereas the \( \sigma'_k (k = 1, 2) \), \( s_j (j = 1, 2, 3) \) scalar fields are grouped into a \( T' \) doublet and a \( T' \) triplet, respectively. Furthermore, the \( SU(3)_L \) scalar triplet \( \rho \) is assigned as a trivial \( T' \) singlet. The lepton and scalar field assignments under the different group factors of the model are shown in Table I. In this work we assume that both \( T' \to Z_6 \) and \( Z_6 \to \{ \text{identity} \} \) breakings must take place in the neutrino sector. The breakings \( T' \to Z_6 \) can be achieved by the scalar antisextet \( s \) whose VEV pattern is set as \( (\langle s \rangle, 0, 0) \) under \( T' \), where

\[
\langle s \rangle = \begin{pmatrix} \lambda_s & 0 & v_s \\ 0 & 0 & 0 \\ v_s & 0 & \Lambda_s \end{pmatrix} .
\]  

(5)

To achieve the direction of the \( Z_6 \to \{ \text{identity} \} \) breaking chain, we additionally introduce another scalar assigned as \( Z_2 \) under \( T' \). We can therefore understand the misalignment of the VEVs as follows. The \( T' \) discrete group is spontaneously broken via two stages, the first stage is \( T' \to Z_6 \) and the second one is \( Z_6 \to \{ \text{identity} \} \). The second stage can be achieved by adding a new \( SU(3)_L \) anti-sextet \( \sigma' \), transforming as \( Z_2 \) under \( T' \) as shown in Table I with VEVs chosen as

\[
\langle \sigma'_1 \rangle = \langle \sigma'_2 \rangle = \begin{pmatrix} \lambda'_\sigma & 0 & v'_\sigma \\ 0 & 0 & 0 \\ v'_\sigma & 0 & \Lambda'_\sigma \end{pmatrix} .
\]

On the other hand, the neutrino Yukawa interactions invariant under the symmetries of our model are given by:

\[
-\mathcal{L}_\nu = \frac{1}{2} \overline{\psi}^c L \sigma \psi L + y (\overline{\psi}^c L \sigma')_2 \psi\alpha L + \frac{1}{2} z (\overline{\psi}^c L \sigma) \overline{\psi} \alpha L + \frac{1}{2} t (\overline{\psi}^c \rho) \overline{\psi} \alpha L + \text{H.c.}
\]  

(6)

After electroweak symmetry breaking, we find the following neutrino mass terms:

\[
-\mathcal{L}_\nu^{\text{mass}} = \frac{1}{2} x \lambda^c \overline{\nu}^c L \nu^c L + \frac{1}{2} x v_s \overline{\nu}^c L N^c_1 R + \frac{1}{2} x v_s \overline{\nu}^c L N^c_1 R + \frac{1}{2} x \lambda^c \overline{N}_1 R \nu^c L + \frac{1}{2} x \lambda^c \overline{N}_1 R \nu^c L R + \frac{1}{2} x \lambda^c \overline{N}_1 R \nu^c L R + \frac{1}{2} x \lambda^c \overline{N}_1 R \nu^c L R + \frac{1}{2} x \lambda^c \overline{N}_1 R \nu^c L R + \frac{1}{2} x \lambda^c \overline{N}_1 R \nu^c L R + \frac{1}{2} x \lambda^c \overline{N}_1 R \nu^c L R .
\]

(7)

The neutrino mass term \( -\mathcal{L}_\nu^{\text{mass}} \) in Eq. (7) can be rewritten in a matrix form as follows:

\[
-\mathcal{L}_\nu^{\text{mass}} = \frac{1}{2} \chi^c L M_\nu^{\chi L} + \text{H.c.},
\]  

(8)
where
\[
\chi_L = \begin{pmatrix} \nu_L^c \\ N_R \end{pmatrix}, \quad M_\nu = \begin{pmatrix} M_L & M_D^T \\ M_D & M_R \end{pmatrix}, \quad (9)
\]
\[
\nu_L = (\nu_{1L}, \nu_{2L}, \nu_{3L})^T, \quad N_R = (N_{1R}, N_{2R}, N_{3R})^T,
\quad (10)
\]
and
\[
M_{L,R,D} = \begin{pmatrix} a_{L,D,R} & -b_{L,D,R} & b_{L,D,R} \\ -b_{L,D,R} & c_{L,D,R} + d_{L,D,R} & 0 \\ b_{L,D,R} & c_{L,D,R} - d_{L,D,R} & 0 \end{pmatrix}, \quad (11)
\]
with
\[
a_L = \lambda_\sigma x, \quad b_L = y\lambda'_\sigma, \quad c_L = z\lambda_s, \quad d_L = 0,
\]
\[
a_D = v_\sigma x, \quad b_D = yv'_\sigma, \quad c_D = zv_s, \quad d_D = tv_\rho,
\]
\[
a_R = \Lambda_\sigma x, \quad b_R = y\Lambda'_\sigma, \quad c_R = z\Lambda_s, \quad d_R = 0.
\quad (12)
\]
Three light active neutrinos gain masses from a combination of type I and type II seesaw mechanisms as follows from Eqs. (9) and (11). Then, the light active neutrino mass matrix takes the form:
\[
M_{\text{eff}} = M_L - M_D^T M_R^{-1} M_D = M_{\text{eff}}^0 + dM_1 + dM_2,
\quad (13)
\]
where
\[
M_{\text{eff}}^0 = \begin{pmatrix} A & -B & B \\ -B & D + H & D \\ B & D & D + H \end{pmatrix}, \quad dM_1 = \begin{pmatrix} 0 & p \\ p & q & 0 \\ p & 0 & -q \end{pmatrix}, \quad dM_2 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & s & r \\ 0 & r & s \end{pmatrix},
\quad (14)
\]
with
\[
A = \lambda_{si} x + \frac{x [2v'_\sigma (\Lambda_{si} v'_\sigma - 2\Lambda'_\sigma v_{s}) y^2 - \Lambda_s v_{s}^2 x z]}{2\Lambda'_\sigma y^2 + \Lambda_s \Lambda_s x z},
\]
\[
B = \lambda'_\sigma y - \frac{2\Lambda'_\sigma v'_\sigma y^3 + (\Lambda_\sigma v'_\sigma v_s + \Lambda_s v'_\sigma v_{s} - \Lambda'_\sigma v_{s} v_{s}) y z}{2\Lambda'_\sigma y^2 + \Lambda_s \Lambda_s x z},
\]
\[
D = \lambda_s z - \frac{z [(\Lambda^2_{s} v_{s}^2 - 2\Lambda_\sigma \Lambda_\sigma v_{s} v_s - \Lambda'_\sigma v_{s} v_{s}) y^2 - \Lambda_s \Lambda_\sigma v_{s}^2 x z]}{\Lambda_s (2\Lambda'_\sigma y^2 + \Lambda_s \Lambda_s x z)},
\]
\[
H = -\lambda_s z + \frac{z [-2v'_\sigma (\Lambda_\sigma v'_\sigma - 2\Lambda'_\sigma v_{s}) y^2 + \Lambda_\sigma v_{s}^2 x z]}{2\Lambda'_\sigma y^2 + \Lambda_s \Lambda_s x z},
\quad (15)
\]
\[
p = \frac{tv_\rho (\Lambda_{si} v'_\sigma + \Lambda'_\sigma v_{s}) x y}{2\Lambda'_\sigma y^2 + \Lambda_s \Lambda_s x z}, \quad q = \frac{2\Lambda'_\sigma tv_\rho (\Lambda_\sigma v'_\sigma + \Lambda'_\sigma v_{s}) y^2}{\Lambda_s (2\Lambda'_\sigma y^2 + \Lambda_s \Lambda_s x z)},
\quad (16)
\]
\[
s = \frac{\Lambda_s^2 v_{s}^2 y^2}{2\Lambda_{si}^2 \Lambda_s y^2 z + \Lambda_s^2 \Lambda_s x z^2}, \quad r = \frac{t^2 v_\rho^2 (\Lambda'_\sigma y^2 + \Lambda_s \Lambda_s x z)}{\Lambda_s z (2\Lambda'_\sigma y^2 + \Lambda_s \Lambda_s x z)},
\quad (17)
\]
Let us note that, as indicated by Eq. (13), the light active neutrino mass matrix $M_{\text{eff}}$ receives a contribution from the three $SU(3)_L$ scalar antisextets, i.e., $\sigma, \sigma'$ and $s$, namely $M_{\text{eff}}^0$ as well as contributions $dM_1$ and $dM_2$ arising from the $SU(3)_L$ scalar triplet $\rho$. In the case where the $\rho$ contribution is forbidden, the two matrices $dM_1$ and $dM_2$ will vanish, and hence the matrix $M_{\text{eff}}$ in Eq. (13) reduces to $M_{\text{eff}}^0$. As will shown below, $M_{\text{eff}}^0$ can approximately fit the data with $\theta_{13} = 0$ that can be considered as a leading order approximation for the recent neutrino experimental data. The second and the third terms, which correspond to the contributions of the $\rho$ triplet will generate the Cabibbo sized deviation from $\theta_{13} = 0$, thus giving rise to the experimental value of the reactor mixing angle $\theta_{13}$. Thus, in this work we consider the $\rho$ contribution as a small perturbation ($d_D \ll d_R$) needed to generate the Cabibbo sized value of the reactor mixing angle measured by the neutrino oscillation experiments. On the other hand, since $p, q$ are proportional to $d_D$ whereas $s, r$ are proportional to $d_D^2$, we can work in the limit $r, s \ll 1$, and safely neglect the second order correction $dM_2$ to the light active neutrino mass matrix.

The first term in Eq. (13) has three exact eigenvalues given by

$$m_{1,2} = \frac{1}{2} \left( A + H \mp \sqrt{(A - H)^2 + 8B^2} \right), \quad m_3 = 2D + H,$$

(18)

and the corresponding eigenstates included in the lepton mixing matrix take the form:

$$U_0 = \begin{pmatrix} K \sqrt{K^2 + 2} & \sqrt{2} \sqrt{K^2 + 2} & 0 \\ -\frac{1}{\sqrt{K^2 + 2}} & \frac{1}{\sqrt{2}} \sqrt{K^2 + 2} & \frac{1}{\sqrt{2}} \sqrt{2} \\ \frac{1}{\sqrt{K^2 + 2}} & -\frac{1}{\sqrt{2}} \sqrt{K^2 + 2} & \frac{1}{\sqrt{2}} \end{pmatrix}, \quad K = \frac{A - H - \sqrt{(A - H)^2 + 8B^2}}{2B}. \quad (19)$$

At the first order of perturbation theory, the matrix $dM_1$ in Eq. (13) does not contribute to the eigenvalues of the matrix $M_{\text{eff}}$, however, it changes the corresponding eigenvectors. Indeed, the three eigenvalues of the light active neutrino mass matrix $M_{\text{eff}}$ are obtained as follows:

$$m'_1 = m_1, \quad m'_2 = m_2, \quad m'_3 = m_3,$$

(20)

where $m_{1,2,3}$ are given by Eq. (18). The corresponding perturbed leptonic mixing matrix takes the form:

$$U = U_0 + \Delta U = U_0 + \begin{pmatrix} 0 & 0 & \Delta U_{13} \\ \Delta U_{21} & \Delta U_{22} & \Delta U_{23} \\ \Delta U_{31} & \Delta U_{32} & \Delta U_{33} \end{pmatrix}, \quad (21)$$
where $U_0$ is defined by Eq. (19), and the $\Delta U_{ij}$ $(i, j = 1, 2, 3)$ matrix elements are given by

$$
\Delta U_{11} = \Delta U_{31} = \frac{K_p - q}{\sqrt{K^2 + 2(m_1 - m_3)}}, \quad \Delta U_{22} = \Delta U_{32} = \frac{2p + K_q}{\sqrt{2K^2 + 2(m_2 - m_3)}},
$$

$$
\Delta U_{12} = -\frac{\sqrt{2}}{2} \{2m_1 + K^2(m_2 - m_3) - 2m_3p + K(m_1 - m_2)q\},
$$

$$
\Delta U_{23} = -\frac{2K(m_1 - m_2)p + K^2(m_1 - m_3)q + 2(m_2 - m_3)q}{(K^2 + 2)(m_1 - m_3)(m_2 - m_3)},
$$

$$
\Delta U_{33} = \frac{2K(m_1 - m_2)p + K^2(m_1 - m_3)q + 2(m_2 - m_3)q}{(K^2 + 2)(m_1 - m_3)(m_2 - m_3)},
$$

with $p, q, m_i = \lambda_i$ $(i = 1, 2, 3)$ and $K$ are given in Eqs. (16), (18) and (19), respectively.

**III. NUMERICAL RESULTS**

The matrix $U_0$ given by Eq. (19) can be parameterized in terms of three Euler’s angles, satisfying the relations $\theta_{13} = 0$, $\theta_{23} = \pi/4$ and $\tan \theta_{12} = \sqrt{2}/K \equiv \sqrt{2B/(m_1 - H)}$. In the case $A - H = B < 0$, with $B$ being a real number, $\theta_{12} = \frac{\pi}{4}$ and $U_0$ becomes an exact Tri-bimaximal mixing matrix $U_{HPS}$ which can be considered as a zero order approximation for the Pontecorvo–Maki–Nakagawa–Sakata (PMNS) leptonic mixing matrix constrained by the recent neutrino oscillation experimental data. The recent data imply that $\theta_{13} \neq 0$, however, the contribution arising from $dM_1$ will generate the experimentally observed deviation from $\theta_{13} = 0$, thus giving rise to the measured value of the reactor mixing angle. It is easy to show that our model is consistent with the current neutrino oscillation experimental data since the experimental values of the six physical observables of the neutrino sector, namely, the leptonic Dirac CP violating phase, the leptonic mixing angles and the neutrino mass squared splittings can successfully be reproduced for appropriate values of the neutrino sector model parameters as shown below. Indeed, in the standard parametrization of the PMNS leptonic mixing matrix, the three leptonic mixing angles $\theta_{12}, \theta_{23}$ and $\theta_{13}$ can be defined in terms of the elements of the leptonic mixing matrix as follows:

$$
t_{12} = U_{12}/U_{11}, \quad t_{23} = U_{23}/U_{33}, \quad s_{13} e^{-i\phi} = U_{13} \tag{23}
$$

Then, using from Eqs. (19), (21), (22) and (23) we get:

$$
p = \sqrt{2} D s_{13} e^{-i\phi} + B s_{13} e^{-i\phi} \left( t_{12} - \frac{1}{t_{12}} \right) + B \left( \frac{t_{23} - 1}{t_{23} + 1} \right),
$$

$$
q = \sqrt{2} B s_{13} e^{-i\phi} + 2D \left( \frac{t_{23} - 1}{t_{23} + 1} \right), \quad H = A + \sqrt{2B} \left( t_{12} - \frac{1}{t_{12}} \right). \tag{24}
$$

We found that the inverted hierarchy scenario of our model cannot accommodate the experimental data on neutrino oscillations, however, the model predictions in the lepton sector are in good
agreement with the recent neutrino oscillation experimental data for the case of normal hierarchy, which favors the normal hierarchy over the inverted one at 3.4σ. Indeed, for the normal neutrino mass spectrum, taking the best fit values of the leptonic mixing angles and Dirac CP violating phase as well as the neutrino mass-squared differences given in Ref. [24] as displayed in Tab. I, we find the following solution:

$$\Delta m^2_{31} = 7.55 \times 10^{-5} \text{eV}^2, \quad \Delta m^2_{21} = m_2^2 - m_1^2 = 2.50 \times 10^{-3} \text{eV}^2,$$

with their corresponding values obtained in Tab. III, we get:

$$A = 0.545705 B - \frac{0.0000124518}{B},$$

$$D = 0.272853 B + \frac{6.22589 \times 10^{-6}}{B}$$

$$+ 7.39148 \times 10^{-6} \sqrt{1.1267 \times 10^7 + \frac{0.70948}{B^2} + 1.05145 \times 10^{10} B^2}. \quad (25)$$

In the scenario of normal hierarchy, the range of the elements $U_{ij}$ ($i, j = 1, 2, 3$) in Eq. (21) are displayed in Fig. 1 with $B \in (-10^{-3}, -5 \times 10^{-4}) \text{eV}$. The values of the light active neutrino masses $m_{1,2,3}$ as functions of the effective $B$ parameter with $B \in (-10^{-3}, -5 \times 10^{-4}) \text{eV}$ are plotted in Fig. 2 for the scenario of normal neutrino mass hierarchy.

The effective neutrino mass $\langle m_{ee} \rangle$ governing neutrinoless double beta decay takes the form $\langle m_{ee} \rangle = \left| \sum_{i=1}^{3} U_{ei}^2 m_i \right|$, whereas $m_\beta = \left\{ \sum_{i=1}^{3} |U_{ei}|^2 m_i^2 \right\}^{1/2}$ where $m_i$ and $U_{ei}$ are defined by Eqs. (19), (20), (21), and (22). We plot the parameters $\langle m_{ee} \rangle$ and $m_\beta$ in Fig. 3 with $B \in (-10^{-3}, -5 \times 10^{-4}) \text{eV}$. In the case $B = -10^{-3} \text{eV}$, the physical neutrino masses and the other parameters are explicitly given in Table. $\text{II}$ The value of the Jarlskog invariant $J_{CP}$ which determines the magnitude of CP violation in neutrino oscillations, in the model under consideration, is determined as

$$J_{CP} = \text{Im} [U_{23}(U_{13})^*U_{12}(U_{22})^*] = -3.21528 \times 10^{-2}.$$

In the 3-3-1 models, the parameters $\lambda_{s,\sigma} \sim \frac{v_{s,\sigma}^2}{\Lambda_{s,\sigma}}$, $\lambda_{s,\sigma}' \sim \frac{v_{s,\sigma}'}{\Lambda_{s,\sigma}}$ and $\lambda_{s,\sigma}, \lambda_{s,\sigma}'$ are at the eV scale. Hence, in order to have explicit values for the model parameters, we assume $\Lambda_{s,\sigma} = v_{s,\sigma}^2$, $\Lambda_{s,\sigma}' = v_{s,\sigma}'^2$ and $v_{s,\sigma} = v_s$, $v_{s,\sigma}' = -v_s$, $\lambda_{s,\sigma} = \lambda_s$. By comparing the expressions of the parameters $A, B, D, H, p, q$ with their corresponding values obtained in Tab. $\text{III}$ we get:

$$x = 0.0714657 + 0.286867i, \quad y = 0.0423168 - 0.0545414i,$$

$$z = -0.0319395 - 0.000219628i, \quad t = (0.00190228 - 0.000161596i) \frac{v_s}{v_p},$$

$$\lambda_s = -0.999572 - 0.0252618i, \quad \lambda_s = -0.602386 + 0.0110186i. \quad (26)$$
IV. CONCLUSIONS

We have constructed a $T'$ flavor model based on the $\text{SU}(3)_{C} \otimes \text{SU}(3)_{L} \otimes \text{U}(1)_{X} \otimes \text{U}(1)_{L}$ gauge symmetry responsible for lepton masses and mixings. We argue how flavor mixing patterns and
mass splitting are obtained with a perturbed $T'$ symmetry. In the model under consideration, the naturally small neutrino masses arise from a combination of type I and type II seesaw mechanisms mediated by three heavy right-handed Majorana neutrinos and three $SU(3)_L$ scalar antisextets, respectively. Our model predicts normal neutrino mass ordering with the inverted ordering disfavoured by our fit. In addition, we find an effective Majorana neutrino mass parameter of $m_\beta \sim 1.41541 \times 10^{-2}$ eV, a Jarlskog invariant $J_{CP} \sim -0.032$ and a leptonic Dirac CP phase $\delta = 238^\circ$ for the scenario of normal neutrino mass hierarchy.
Table III: The model parameters in the case $B = -10^{-3}$ eV in the normal hierarchy

| Parameters [eV] | The derived values [eV] |
|----------------|-------------------------|
| $A$            | $1.19061 \times 10^{-2}$|
| $D$            | $1.90922 \times 10^{-2}$|
| $H$            | $1.29975 \times 10^{-2}$|
| $K$            | $2.06155$               |
| $p$            | $(-2.23417 + 3.44627) \times 10^{-3}$|
| $q$            | $(1.91002 - 0.17549) \times 10^{-3}$|

$m_{\text{light}}^N \equiv m_1^N$ $1.09359 \times 10^{-2}$

$m_2^N$ $1.39676 \times 10^{-2}$

$m_3^N$ $5.1182 \times 10^{-2}$

$\sum m_i^N$ $7.60855 \times 10^{-2}$

$\langle m_{ee}^N \rangle$ $1.14354 \times 10^{-2}$

$m_3^N$ $1.41541 \times 10^{-2}$

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Appendix A: The breakings of $T'$ by triplet 3

Under $T'$ group, for triplets 3 we have the followings alignments:

1. The first alignment: $0 = \langle \phi'_2 \rangle = \langle \phi'_3 \rangle \neq \langle \phi'_1 \rangle$ or $0 = \langle \phi'_1 \rangle = \langle \phi'_2 \rangle \neq \langle \phi'_3 \rangle$ or $0 = \langle \phi'_1 \rangle = \langle \phi'_3 \rangle \neq \langle \phi'_2 \rangle$ then $T'$ is broken into $Z_2$.

2. The second alignment: $0 = \langle \phi'_1 \rangle \neq \langle \phi'_2 \rangle = \langle \phi'_3 \rangle \neq 0$ or $0 = \langle \phi'_3 \rangle = \langle \phi'_1 \rangle \neq \langle \phi'_2 \rangle \neq 0$ or $0 \neq \langle \phi'_1 \rangle = \langle \phi'_2 \rangle \neq \langle \phi'_3 \rangle = 0$ then $T'$ is broken into $Z_2$.

3. The third alignment: $0 = \langle \phi'_1 \rangle \neq \langle \phi'_3 \rangle \neq \langle \phi'_2 \rangle \neq 0$ or $0 = \langle \phi'_2 \rangle \neq \langle \phi'_1 \rangle \neq \langle \phi'_3 \rangle \neq 0$ or $0 \neq \langle \phi'_1 \rangle \neq \langle \phi'_2 \rangle \neq \langle \phi'_3 \rangle = 0$ then $T'$ is broken into $Z_2$.

4. The fourth alignment: $\langle \phi'_1 \rangle \neq \langle \phi'_2 \rangle \neq \langle \phi'_3 \rangle$ then $T'$ is broken into $Z_2$.

5. The fifth alignment: $0 \neq \langle \phi'_1 \rangle \neq \langle \phi'_3 \rangle = \langle \phi'_2 \rangle \neq 0$ or $0 \neq \langle \phi'_1 \rangle = \langle \phi'_3 \rangle \neq \langle \phi'_2 \rangle \neq 0$ or $0 \neq \langle \phi'_1 \rangle = \langle \phi'_2 \rangle \neq \langle \phi'_3 \rangle \neq 0$ then $T'$ is broken into $Z_2$. 
(6) The sixth alignment: $\langle \phi'_1 \rangle = \langle \phi'_2 \rangle = \langle \phi'_3 \rangle \neq 0$ then $T'$ is broken into $Z_4$.

(7) The seventh alignment: $0 \neq \langle \phi'_1 \rangle \neq \langle \phi'_2 \rangle = \langle \phi'_3 \rangle = 0$ then $T'$ is broken into $Z_6$.

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