The US Supreme Court throughout the 20th century has been characterized as being divided between liberals and conservatives, suggesting that justices with similar ideologies would have voted similarly had they overlapped in tenure. What if they had? I build an empirical, quantitative model of this counterfactual hypothesis using pairwise maximum entropy. I infer how 36 justices from 1946-2016 would have all voted on a Super Supreme Court. The model is strikingly consistent with a standard voting model from political science despite using $10^5$ less parameters and fitting the observed statistics better. As with historical courts, the Super Court is dominated by consensus. The rate at which consensus decays as more justices are included is extremely slow, nearly 100 years, and indicates that the modern Supreme Court is an extremely stable institution. Beyond consensus, I discover a rich structure of dissenting blocs that are distributed along a heavy-tailed Zipf’s law. The heavy tail means that dominant dissenting modes fail to capture the entire spectrum of dissent. Thus, I find that Supreme Court voting over time is not low-dimensional despite implications to the contrary in historical analysis of Supreme Court voting. Although it has been long presumed that strong higher order correlations are induced by features of the cases, the institution, and the justices, I show that such complexity can be expressed in a minimal model relying only on pairwise correlations. From the perspective of model selection, this minimal model may generalize better and thus be useful for prediction of Supreme Court voting over time.

The modern US Supreme Court is often described as being divided between liberal and conservative wings balanced on the fulcrum of one or two swing voters \[^{11,13}\]. In the 1930s, a similar dynamic was at work where a conservative bloc known as the “Four Horsemen” relied on a fifth swing vote to countermand policy designed to ameliorate the economic impact of the Great Depression \[^{4}\]. Over time, political issues and the composition of the Court have changed, but the idea that a left and right divide characterizes voting on the Court is a widespread observation in the literature \[^{2,5,6,8}\]. The implicit hypothesis is that ideologically similar justices on different courts would have voted similarly on the same cases, but in reality they never faced the same cases nor voted with the same set of colleagues. Although I could take similar cases and then compare two justices’ votes \[^{7,9}\], a direct approach would be to compare how the two vote with a shared cohort to infer how the two might have voted together.

The Supreme Court is the highest court in the US, consisting of up to 9 sitting justices at any given time with short periods where a seat is vacant. Each set of sitting justices is known as a natural court. Unlike Congress where all seats come up for reelection every few years, Supreme Court justices are appointed for life and typically one justice is replaced at a time. Although a retiring and an incoming justice may never have voted together, they may both accumulate a number of votes with the same 8 justices over many years and potentially many more years with a smaller cohort. In a typical year, 90-150 cases are decided with a vote (depending on the era) \[^{10,11}\]. After choosing to consider a case, the Opinion of the Court is decided by a majority and—although there may be subtleties in the Opinion—the voting justices must in the end choose to join the majority or not, and this binary code is used in the political science literature \[^{12}\].

The long overlapping stints are informative about how justices vote relative to each other. If the two pairs out of three voters were highly correlated, then I would by transitivity expect that the third pair likewise vote together. If, however, the two pairs were anticorrelated, then the proverbial logic “the enemy of my enemy is my friend” implies that the third pair is again correlated. In general, there are an infinite number of models that would match the observed pairs while filling in the missing ones. If I insist that the model match the observed pairwise disagreements but otherwise make no further assumptions, I have specified pairwise maximum entropy (maxent) models. Maxent models by design make minimal assumptions about the structure of the data and so are not guaranteed to capture the statistics of political voting with few parameters. I show, however, that they do while remaining consistent with highly parameterized spatial voting models from political science \[^{13}\].

I build a pairwise maxent model of the joint voting record of a Super Supreme Court of all 36 justices in the modern Supreme Court Database from 1946-2016 ($K = 8,737$) \[^{8,10}\]. Instead of ideological divisions, the Super Court is dominated by unanimity even across temporally separated justices, showing it to be an extremely stable institution. Just from pairwise correlations, the model generates a rich structure of dissenting blocs that cannot be captured by a low-dimensional, ideological picture. This comparison of Supreme Court justices across time is just one example of the comparative study of political institutions. I show how minimal models from statistical physics make quantitative predictions that could...
be rigorously tested against alternative models or new data.

**THE MAXENT MODEL**

I represent the vote of Justice i to be $\sigma_i$ where the binary nature of majority voting is represented by $\sigma_i \in \{-1, 1\}$. The average agreement between two justices who voted $K_{ij}$ times together is measured by the pairwise correlation

$$\langle \sigma_i \sigma_j \rangle_{\text{data}} = \frac{\sum_{n=1}^{K_{ij}} \sigma_i^{(n)} \sigma_j^{(n)}}{K_{ij}}. \tag{1}$$

I specify that the model match the observed pairwise correlations,

$$\langle \sigma_i \sigma_j \rangle_{\text{data}} = \langle \sigma_i \sigma_j \rangle = \sum_{\sigma} p(\sigma) \sigma_i \sigma_j \tag{2}$$

where the model is the probability distribution $p(\sigma)$ over a vector of votes $\sigma$ of all included justices. I must also specify the average vote of each justice $\langle \sigma_i \rangle$. For nearly all votes, the sign of the vote corresponds to whether to affirm or to reverse the previous court’s ruling, but the orientation is determined by the history of the case. Since I focus on the pattern of internal agreement and do not expect that to depend on whether the question is formulated as “Is A constitutional?” or as “Is A unconstitutional?”, I remove any bias by symmetrizing the data set such that \[8, 12\]

$$\langle \sigma_i \rangle_{\text{data}} = \langle \sigma_i \rangle = 0 \tag{3}$$

Eqs 2 and 3 along with the principle of maximum entropy specify the values of the remaining unobserved pairwise correlations.

Entropy is the unique measure of uncertainty that obeys simple rules for consistency [13, 15]. Maximizing entropy $S[p] = -\sum_\sigma p(\sigma) \ln p(\sigma)$ while constraining the distribution to match the pairwise correlations is equivalent to minimizing the Helmholtz free energy [8, 16, 17]

$$-\ln Z = \langle E \rangle - S \tag{4}$$

$$E(\sigma) = -\sum_{i<j}^{36} J_{ij} \sigma_i \sigma_j \tag{5}$$

where the Hamiltonian is formally equivalent to the Ising spin glass and the couplings $J_{ij}$ can be thought of as Langrangian multipliers enforcing Eq 2. The resulting pairwise maximum entropy model has Boltzmann form

$$p(\sigma) = e^{-E(\sigma)}/Z. \tag{6}$$

Despite this equivalence, the maxent formalism requires no assumptions about equilibrium, only that the pairwise correlations are representative of some stationary distribution $p(\sigma)$ I wish to characterize.

For each $\langle \sigma_i \sigma_j \rangle$ I fix, I have a corresponding coupling $J_{ij}$ that, in the strictest sense, only has probabilistic implications. A positive $J_{ij}$ lowers the energy $E$ when the two justices i and j vote together, increasing the probability that I observe such a vote by a factor $e^{J_{ij}}$. Disagreement between i and j is suppressed by a factor $e^{-J_{ij}}$. Since justices who never voted together have no correlations to constrain, the corresponding coupling are not specified, effectively setting $J_{ij} = 0$. Although one might interpret this to indicate the absence of real interaction, the precise statement is that the predicted correlations are mediated through the structure implied by the constrained correlations. The couplings do not imply causal interaction between the justices. If there were causal interactions, then the consequences would be hidden in the pairwise correlations along with any other effects like sampling biases. In other words, the only information in the model is that available in the pairwise correlations and no more as is explicitly imposed by the maximum entropy approach.

In the maxent model for the Super Court, the pairwise correlations are almost exclusively positive, confirming that the Supreme Court has always been dominated by consensus. Out of the 248 observed pairwise correlations, only 2 are negative—between WO Douglas and WE Burger and between WO Douglas and WH Rehnquist—hinting that WO Douglas is an unusual justice in the history of the Court. After solving the model I can predict the hypothetical correlations between justices who never served together, and even in this enlarged set of possibilities the only further negative correlations are between WO Douglas and conservatives SD O’Connor, W Rehnquist, A Scalia, and C Thomas. Additionally, the ideologically opposed pairs WJ Brennan and C Thomas, T Marshall and WH Rehnquist, and T Marshall and C Thomas are negatively correlated. Although these are all small negative values, they are nevertheless prominent because they are unusual in a political body that prizes public agreement [4, 18]. Although the remaining correlations are positive, the left-right divide is indicated by stronger correlations within and weaker correlations between ideological sides when the justices are ordered by ideology (Appendix Figure 4). This tension between competing blocs is reflected in the interaction matrix $J_{ij}$, where couplings of both signs are recovered from our model of majority vs. minority voting.

**W-NOMINATE MODEL**

I compare the maxent model with a standard spatial voting model from political science W-nominate (Appendix Section IV) [13]. Canonical spatial voting models assume that each justice is located in some low-dimensional, policy space along with the positions for ev-
For the maxent model, over 97% of the 197 fit correlations are within two standard errors of the mean given over other variations of spatial voting models. Despite this large disparity in parameterization, the maxent model fits the observed statistics of the data better as shown in Figure 1. Nevertheless, both models show very similar collective patterns.

**Strong Consensus**

Despite consisting of members from 24 nine-member natural courts, the probability of unanimity on the Supreme Court is an astounding 10% of the time. In comparison, independent justices would vote unanimously with a relatively minuscule probability of $p_u(k = 36) = 2^{-36}$, implying that the interactions impose strong collective structure. As a stricter test of the strength of cohesion, I might consider that justices already have an idea of how the Court is leaning before they cast their vote. If I instead imagine that each justice independently defects from unanimity with probability $p_{\text{def}}$, I can measure for a natural court of $k' = 9$ that typically votes unanimously about 36% of the time, $p_{\text{def}} = 0.89$. Extrapolating this I find that the Super Court would vote unanimously with probability $p_u(k = 36) = 0.015$ [19]. This is an order of magnitude below what I find on the Super Court. Thus,
the Super Court shows a strong tendency to consensus that spans the many natural courts it represents.

Even in a chain of tightly correlated voters the probability that justices agree with others far away becomes dominated by drift \[20\]. Looking at Figure 1 I see that pairwise correlations far from the diagonal tend to be smaller than those closer, indicative of weaker correlations spanning longer periods of time. To test this more systematically, I consider the probability of unanimity in blocs of size \(k\), \(p_u(k)\), and compare justices that sat together with random subsets in Figure 2. It is clear that both models of independent justices decay much more rapidly than what I find in the Super Court. In the limit of large blocs of 20 or more justices, or roughly two different Supreme Courts, the unanimous mode begins to decay like an exponential, indicating the regime of finite correlation length. Here, adding a new justice is like adding an independent voice to the system. The decay length \(l\), however, is 43 justices, roughly equivalent to a century of justices. W-Nominate predicts a similar decay length of 77 justices. Although the W-Nominate curve is lifted because I included unanimity in Eq 5, its shape which is not given is remarkably similar. In this quantitative sense, the Supreme Court is an extremely stable and conservative institution.

The force quelling dissent, a “norm to consensus” \[18\], is even stronger for justices that sit on the same court. I again calculate the probability of unanimity for blocs of size \(k\), but only for justices who sat on the same natural court. As \(k\) increases from 2 to 9, the log-ratio of the probability, or energy difference, of finding a unanimous vote on a natural court \(p_u^{\text{nat}}\) compared to justices from different courts \(p_u^{\text{diff}}\) increases linearly with the bloc size \(\log(p_u^{\text{nat}}(k)/p_u^{\text{diff}}(k)) = E^{\text{diff}}(k) - E^{\text{nat}}(k) \propto 0.035k\) as I show in Figure 2. This reveals a force encouraging the formation of blocs and especially for consensus specific to natural courts. Notably, the energy term for consensus grows faster than the inverse decorrelation length in the limit of large courts \(l^{-1} = -0.023\) per justice. This suggests that consensus would dominate even when random justices who had never voted together are made to vote together which is akin to the situation when new justices are nominated into the Supreme Court.

**ZIPP'S LAW: MANY WAYS TO DISSENT**

Besides consensus, the pattern of dissenting blocs that emerge from pairwise interactions are consistent with notions of ideological blocs, but show much richer structure than binary ideological division. Ordering votes by frequency rank \(r\), I find that Zipf’s law holds after the strong unanimous mode \[21\] \[22\]

\[
p(r) = (1 - \alpha)r^{-\alpha}/(r_{\min} - r_{\max})
\]

where \(\alpha = 0.85\). The power law form with is clear over 4 orders of magnitude. This is consistent with previous work on the Second Rehnquist Court, where we observed a wide distribution of votes as measured from the data over a limited range of \(\sim 10^2\) unique votes. There, a power law fit yields \(\alpha \approx 1\) \[8\].

The slow decay in the distribution with \(r\) signals that there is no typical set of voting behavior. The Super Court shows an extremely heavy-tailed distribution over votes where even “rare” events happen relatively often. In this sense, the intuitive notion that I could consider an average ideological behavior of the Super Court is misleading because even votes that defy this intuition are probable.

If I were, however, to group votes together such that all votes almost along ideological lines—where a few individuals buck the line—I might remove the heavy tail and find dominant clusters of “noisy” ideological votes that serve as a useful, compressed description of the Super Court \[8\] \[23\] \[24\]. A natural way of clustering votes is given by the energy landscape constructed using Eq 6. By taking a vote and flipping the single voter most likely to change his or her vote at a time, I will eventually reach an energy minimum where no vote flip will lower the energy of the vote. This defines an energy landscape where states can be grouped by local energy minima. Since the energy is directly related to the probability, \(\log p(\sigma) \propto -E(\sigma)\), this procedure is equivalent to searching for local maxima in probability where single voter flips climb hills in
The eigenvalue spectrum of the covariance matrix is strongly mistaken. This pattern is reflected in the idea that the number of maxima scales linearly with system size \([17, 25]\), so the interactions as with the Super Court, the number of maxima grows exponentially with system size \([17, 25]\), so the interactions as with the Super Court \([8]\). This comparison, however, is misleading.

The large number of energy minima is in stark contrast with the Second Rehnquist Court where there are only six meaningful clusters despite being a quarter of the size \([3, 5]\). Yet a low-dimensional representation of collective voting over time misses the heavy tail generating a rich range of behavior.

**DISCUSSION**

Although voting in a political context is complex, such collective phenomena are natural to approach from the perspective of statistical physics \([17, 24, 26–28]\). Voting on the Supreme Court shows strong collective effects—often obscured by the focus on individual ideology \([2, 7, 18]\)—that I exploit by using observations of justices that voted together to infer how justices who never did might have (Figure 1). I find that the hypothetical Super Court is dominated by strong consensus that nearly spans a century (Figure 2), an observation that may be surprising to Supreme Court scholars \([29]\).

Hiding beneath this dominant mode is a rich range of dissenting coalitions (Figure 3), illustrating the hypothetical behavior of historically disjoint groups of justices. The remarkable similarity in the distribution of states beyond the unanimous mode of the maxent and W-Nominate models suggests that they are largely capturing the same collective structure encoded in the maxent energy landscape.

Consideration of such hypothetical situations has long played a significant role in legal research. Legal scholars study the influence of past precedents by using written opinions as sources of legal reasoning for justices in the future \([30]\). Political scientists compare justices by considering similarities and differences between the issues brought up in cases \([30]\). These alternative approaches for finding correlations across time are ways of indirectly testing the predictions of this model. A direct quantitative test might be for forecasting where the temporal structure captured by the pairwise maxent model might find immediate application in the prediction of Supreme Court votes \([31]\).

In contrast to binary ideological intuition obtained from news about the Supreme Court, the collective behavior of the Court over time reveals an institution focused on consensus and insulated from the seemingly rapid pace of political change. When I look beyond consensus, ideological intuition fails again if I account for the changing composition of the Court: ideological modes are lost amongst the cacophony of dissenting voters.
voices. As we know from statistical physics, even simple models can generate an incredibly rich range of behavior when there are many competing, frustrated interactions. From the perspective of statistical physics, I show that this intuition is expressed in the complex voting patterns of the Supreme Court.

I thank Paul Ginsparg, Bryan Daniels, and Colin Clement for invaluable feedback and Bill Bialek, Guru Khalsa, and Ti-Yen Lan for comments on previous versions of the manuscript. I acknowledge funding from the NSF Graduate Research Fellowship under grant DGE-1650441.

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