1. Introduction

The conception *zitterbewegung* (ZB) stands for an outlandish quantum motion, of a Dirac particle in vacuum, having length scale of the order of Compton wave length. It was originally envisioned by Schrödinger in 1930 [1]. The main obstruction to establish the existence of ZB in vacuum experimentally is its ultra-short length scale. However, a ray of hope was shown in 2005 when Zawadski [2] argued that a narrow gap semiconductor not only can host the intriguing phenomenon ZB but also the associated length scale can be enhanced up to five orders higher than that in vacuum. As a result, subsequent years witnessed immense interest in ZB in numerous systems [3] including spin–orbit coupled two dimensional (2D) electron/ hole gases [4–11], superconductors [12], sonic crystal [13], photonic crystal [14, 15], carbon nanotube [16], graphene [17–21], other Dirac materials [22–24] and ultra-cold atomic gases [25–28].

There exists several understandings behind the cause of ZB. It is believed that ZB happens as an outcome of the interference between positive and negative energy solutions of the Dirac equation. Huang [29] put forward a theory to establish a connection between ZB and electron’s intrinsic magnetic dipole moment. Later, Schliemann [6] et al interpreted that...
ZB in a quantum well occurs as a consequence of spin rotation due to spin–orbit interaction (SOI). It is also mentioned [25] that the ZB can be interpreted as a measurable aftermath of Berry phase in momentum space. Moreover, for a multiband quantum system, an explicit relation between Berry curvature and amplitude of ZB was also established [30].

In general ZB has permanent character i.e. oscillations do not die out in time. When an electron is illustrated by a wave packet the resulting ZB undergoes a transient nature according to Lock [31]. It is also proposed recently [9] that the permanent behavior of ZB in a spin–orbit coupled 2D electron gas can be restored by considering a time dependent Rashba SOI.

Furthermore, an intriguing quantum transport related phenomenon such as minimal conductivity [32] of graphene was understood in the light of ZB. Very recently, Iwasaki et al [33] demonstrated experimentally that conductance fluctuations in InAs quantum wells occur as a possible consequence of ZB.

From the perspective of ZB, most of the studies in electronic systems are mainly concerned about the ZB of quasiparticles with spin/pseudospin \( S = 1/2 \). However, there exists an example which portrays ZB of spin-1 ultra-cold atom [26]. To the best of our knowledge no such example of ZB of a quasiparticle with spin/pseudospin beyond \( S = 1/2 \) exists in typical condensed matter systems. Hence, in this article we consider the ZB effect in a relatively new model named as \( \alpha \)-T\(_3\) model in which quasiparticles are characterized by an enlarged pseudospin \( S \geq 1/2 \). The sole motivation behind adopting this model is due to growing interest in systems which are described by the generalized Dirac–Weyl equation with arbitrary pseudospin \( S \) [34–36]. Via the variation of a parameter \( \alpha \), the \( \alpha \)-T\(_3\) model reveals a smooth changeover from graphene (\( \alpha = 0 \)) to dice or T\(_3\) lattice (\( \alpha = 1 \)). There has been a lot of studies in T\(_3\) lattice from the standpoint of topological localization [37, 38], magnetic frustration [39, 40], Klein tunneling [41], minimal conductivity [42], plasmon [43] etc. Moreover, the existence of the T\(_3\)-model within the framework of optical lattice [44] and semiconductor structure [45] is also predicted recently. In addition, the interest in \( \alpha \)-T\(_3\) model [46] is growing rapidly nowadays. According to Malcolm and Nicol [47] a Hg\(_{1-x}\)Cd\(_x\)Te quantum well can be considered as \( \alpha \)-T\(_3\) model with \( \alpha = 1/\sqrt{3} \) at a particular doping. The connection of the parameter \( \alpha \) with the Berry phase makes the \( \alpha \)-T\(_3\) model more interesting. In recent years, a number of studies have been performed on this model within the context of orbital magnetic response [46], magneto-transport [48], optical conductivity [49–51], quantum tunneling [52], Wiess oscillations [53] etc.

In this work we have investigated the problem of ZB of a quasiparticle in \( \alpha \)-T\(_3\) model. We choose an initial Gaussian wave packet with definite pseudospin polarization to represent a quasiparticle. For \( 0 < \alpha < 1 \), the quasiparticle undergoes ZB which is transient in nature and consists of two frequencies, namely, \( 2\Omega_\alpha \) and \( \Omega_\phi \). The interference between conduction and valence band leads to occur ZB with frequency \( 2\Omega_\alpha \) whereas the \( \Omega_\phi \)-frequency ZB occurs as a result of interference between either conduction and flat band or flat and valence band. The nature of ZB depends significantly on the type of the initial pseudospin polarization. Particularly, when the initial pseudospin polarization was along \( z \)-direction, various interesting features of ZB emerge. For example, when the initial wave packet was concentrated entirely in any of the \( rim \) sites the resulting ZB has two above mentioned frequencies for a finite \( \alpha \). In this case we reveal a transition from \( 2\Omega_\phi \)-frequency ZB to \( \Omega_\phi \)-frequency ZB as \( \alpha \) is tuned from 0 to 1. When the wave packet was located initially in the hub site, the resulting ZB has only one frequency \( 2\Omega_\alpha \) for all possible values of \( \alpha \). In the limit of large width of wave packet we obtain analytical expression for the expectation value of velocity operator from which one can extract the timescale over which the ZB dies out. We also consider the effect of other possible pseudospin polarization on ZB. In addition we incorporate the effect of an external quantizing magnetic field on the ZB. In this case the temporal behavior of ZB is permanent. The effect of different pseudospin polarization has also been discussed. Using Fourier transformation we obtain frequency components involved in ZB for a particular choice of pseudospin polarization. We also find that the number of frequencies present in ZB depends significantly on the parameter \( \alpha \).

Rest of the present paper is organized in the following fashion. In section 2, we discuss zero field ZB by incorporating the basic informations about physical system, time evolution
of wave packet, and rigorous calculations of the expectation values of physical observables. The effect of perpendicular magnetic field on ZB is considered in section 3. We summarize the obtained results in section 4.

2. In absence of magnetic field

2.1. Lattice structure and low lying energy states

As depicted in figure I(a), α-T3 model has honeycomb lattice structure with an additional site at the center of each hexagon. Each unit cell (shown by the dashed rhombus) contains three sites. With respect to the co-ordination number (CN), those sites are classified into two categories, namely rim and hub sites. As evident from figure I(a) sites B and C are both rim sites with CN 3 whereas site A is known as hub site with CN 6. Note that each nearest-neighbor pair consists of one rim and one hub sites. The sites A and B are connected through hopping parameter $t$ while hopping energy between A and C is $ct$.

The low energy excitations near the Dirac point in a particular valley are described by the following Hamiltonian [46]

$$
H(p) = \begin{pmatrix}
0 & f_0 \cos \varphi & 0 \\
f_0 \cos \varphi & 0 & f_0 \sin \varphi \\
0 & f_0 \sin \varphi & 0
\end{pmatrix},
$$

(1)

where $f_0 = v_F (\xi p_x - ip_y)$ with $v_F$ being the Fermi velocity. The valley index $\zeta$ takes a value $+1$ or $-1$ for K(K') valley. Finally, following relation $\alpha = \tan \varphi$ holds between $\alpha$ and $\varphi$. The energy spectrum corresponding to the Hamiltonian given in equation (1) consists of three branches. Out of them two are linearly dispersing $E_{\pm} = \lambda \hbar v_F k$ with $\lambda = \pm 1$ and $k = |p|/\hbar$, known as conic band. The conic band itself consists of conduction band (CB) and valence band (VB) corresponding to $\lambda = +1$ and $\lambda = -1$, respectively. The other energy branch is dispersionless and is known as flat band (FB). All these energy branches are depicted in figure 1(b). The wave functions corresponding to conic band and FB around the K-valley are, respectively, given by

$$
\psi_k(r) = \frac{1}{\sqrt{2}} \begin{pmatrix}
\cos \varphi e^{-i\theta} \\
\sin \varphi e^{i\theta}
\end{pmatrix} \frac{e^{ikr}}{2\pi},
$$

(2)

and

$$
\psi_0(r) = \frac{1}{\sqrt{2}} \begin{pmatrix}
0 \\
-i \cos \varphi e^{i\theta}
\end{pmatrix} \frac{e^{ikr}}{2\pi},
$$

(3)

where $\theta$ is the polar angle of the wave vector $k$.

2.2. Velocity and pseudospin operators

From equation (1), using the relation $v_i = \partial H/\partial p_i$ with $i = x, y$, one can obtain the following components of the velocity matrix for the K-valley as

$$
v_x = v_F \begin{pmatrix}
0 & \cos \phi & 0 \\
\cos \phi & 0 & \sin \phi \\
0 & \sin \phi & 0
\end{pmatrix},
$$

(4)

and

$$
v_y = v_F \begin{pmatrix}
0 & -i \cos \phi & 0 \\
i \cos \phi & 0 & -i \sin \phi \\
0 & i \sin \phi & 0
\end{pmatrix}.
$$

(5)

The $x$ and $y$ components of the pseudospin operator are governed from the velocity operators as $S_i = \hbar v_i/\tau_F$ with $i = x, y$. The $z$-component of the pseudospin operator is obtained through the commutation relation $[S_x, S_z] = i \hbar S_z$ as

$$
S_z = 2\hbar \begin{pmatrix}
\cos^2 \phi & 0 & 0 \\
0 & -\cos(2\phi) & 0 \\
0 & 0 & -\sin^2 \phi
\end{pmatrix}.
$$

(6)

2.3. Time evolution of an initial wave packet

To study the dynamics of a quasiparticle, it is important to know the corresponding wave function at a later time $t$. In the following, we consider the initial wave function representing the quasiparticle to be a plane wave modulated by a Gaussian wave packet

$$
\Psi(r, 0) = \frac{1}{2\pi C} \int dk a(k, 0) e^{ikr} \begin{pmatrix}
c_1 \\
c_2 \\
c_3
\end{pmatrix},
$$

(7)

where the wave packet $a(k, 0) = (d/\sqrt{\pi}) e^{-d^2(|k-k_0|^2)/2}$ is centered at some wave vector $k = k_0$ and $d$ is its width. Note that the wave packet was polarized initially along any arbitrary direction characterized by the constants $c_1$, $c_2$, and $c_3$ which are, in general, complex numbers. The normalization constant $C$ is defined as $C = \sqrt{|c_1|^2 + |c_2|^2 + |c_3|^2}$. The wave function, at a later time $t$, can be obtained by applying appropriate time evolution operator onto $\Psi(r, 0)$. We find the time evolved state as

$$
\Psi(r, t) = \frac{1}{2\pi} \int dk a(k, 0) e^{ikr} \begin{pmatrix}
k_1(k, t) \\
k_2(k, t) \\
k_3(k, t)
\end{pmatrix},
$$

(8)

where $k_\mu(k, t)$'s, with $\mu = 1, 2, 3$, are obtained from the following matrix equation

$$
\begin{pmatrix}
k_1(k, t) \\
k_2(k, t) \\
k_3(k, t)
\end{pmatrix} = \frac{1}{C} \begin{pmatrix}
\sin^2 \varphi + \cos^2 \varphi \cos(\Omega_k t) & -i \cos \varphi e^{-i\theta} \sin(\Omega_k t) & \sin \varphi \cos \varphi e^{-i\theta} [\cos(\Omega_k t) - 1] \\
-i \cos \varphi e^{i\theta} \sin(\Omega_k t) & \cos(\Omega_k t) & -i \sin \varphi e^{-i\theta} \sin(\Omega_k t) \\
\sin \varphi \cos \varphi e^{i\theta} [\cos(\Omega_k t) - 1] & -i \sin \varphi e^{i\theta} \sin(\Omega_k t) & \cos^2 \varphi + \sin^2 \varphi \cos(\Omega_k t)
\end{pmatrix} \begin{pmatrix}
c_1 \\
c_2 \\
c_3
\end{pmatrix}.
$$

(9)
It is noteworthy that equation (8) is the Fourier transformation of the following function

$$
\Phi(\mathbf{k}, t) = a(\mathbf{k}, 0) \left( \begin{array}{c} \kappa_1(\mathbf{k}, t) \\ \kappa_2(\mathbf{k}, t) \\ \kappa_3(\mathbf{k}, t) \end{array} \right) .
$$

(10)

Equation (10) represents the time evolved state in the momentum space which can be used to calculate the expectation values of various physical observables.

### 2.4. Expectation value of the velocity operator

The expectation value of a physical observable corresponding to an operator $\hat{O}$ is defined as $\langle \hat{O}(t) \rangle = \int d\Phi \hat{O}(\mathbf{k}, t) \Phi(\mathbf{k}, t)$. Instead of evaluating the expectation value of position operator we prefer here to calculate that of velocity operator because the former is always obtained by integrating the latter with suitable initial conditions.

Now the expectation values of the components of velocity operator can be obtained as

$$
\langle v_\nu \rangle = 2\nu_F \int d\mathbf{k} |a(\mathbf{k}, 0)|^2 \text{Re} \left( \cos \varphi \kappa_1^* \kappa_2 + \sin \varphi \kappa_3^* \kappa_3 \right)
$$

(11)

and

$$
\langle v_\mu \rangle = 2\nu_F \int d\mathbf{k} |a(\mathbf{k}, 0)|^2 \text{Im} \left( \cos \varphi \kappa_1^* \kappa_2 + \sin \varphi \kappa_3^* \kappa_3 \right).
$$

(12)

By defining $\sum_{\mu\sigma}(t) = \int d\mathbf{k} |a(\mathbf{k}, 0)|^2 \kappa_\mu(\mathbf{k}, t) \kappa_\sigma(\mathbf{k}, t)$ with $\mu, \sigma = 1, 2, 3$, equations (11) and (12) can be further written in the following compact form

$$
\begin{align*}
\langle \dot{v}_\nu(t) \rangle &= 2\nu_F \left( \text{Re} \left( \cos \varphi \Sigma_{12}(t) + \sin \varphi \Sigma_{23}(t) \right) \right), \\
\langle \dot{v}_\mu(t) \rangle &= 2\nu_F \left( \text{Im} \left( \cos \varphi \Sigma_{12}(t) + \sin \varphi \Sigma_{23}(t) \right) \right).
\end{align*}
$$

(13)

Without any loss of generality, we consider the initial wave packet was moving along $+\mathbf{x}$ direction with wave vector $\mathbf{k}_0 = k_0 \mathbf{x}$. After doing the angular integration, we obtain $\Sigma_{12}(t)$ and $\Sigma_{23}(t)$ as

$$
\begin{align*}
\Sigma_{12}(t) &= \frac{2}{C_2} e^{-\alpha^2} \int_0^\infty dq q e^{-q^2} \left\{ c_1^* c_2 \sin^2 \varphi I_0(2aq) - c_2^* c_2 \sin \varphi \cos \varphi I_2(2aq) \right\} \cos(\Omega t) \\
&\quad + i \left\{ c_1^* c_2 \sin \varphi \cos^2 \varphi I_2(2aq) + \left( (|c_3|^2 - |c_1|^2) \sin^2 \varphi \cos \varphi - c_1^* c_3 \sin^2 \varphi \right) \right\} \sin(\Omega t),
\end{align*}
$$

and

$$
\begin{align*}
\Sigma_{23}(t) &= \frac{2}{C_2} e^{-\alpha^2} \int_0^\infty dq q e^{-q^2} \left\{ c_2^* c_3 \cos^2 \varphi I_0(2aq) - c_2^* c_1 \sin \varphi \cos \varphi I_2(2aq) \right\} \cos(\Omega t) \\
&\quad + i \left\{ - c_1^* c_3 \sin^2 \varphi \cos \varphi I_1(2aq) + \left( (|c_3|^2 - |c_1|^2) \sin \varphi \cos \varphi + c_1^* c_3 \cos^2 \varphi \right) \right\} \sin(\Omega t),
\end{align*}
$$

(14)

where $a = k_0 d$, $q = kd$, $\Omega_q = \nu_F d$, and $I_\nu(x)$ is the $\nu$th order modified Bessel function of first kind. Note that in equations (14) and (15) there exist two frequencies namely $\Omega_q$ and $2\Omega_q$. The former is a result of interference between either FB and VB or CB and FB while the latter arises due to the coupling between CB and VB.

### 2.5. Various types of pseudospin polarization

We are merely interested in the dynamics of the Gaussian wave packet of different types of initial pseudo-spin polarization. Here, we will discuss all the possibilities corresponding to various combinations of $c_1$, $c_2$, and $c_3$.

#### 2.5.1. z-polarization

Here, we consider the wave packet was polarized initially along $+z$ direction. In this case the possible combinations of $(c_1, c_2, c_3)$ are $(1, 0, 0)$, $(0, 1, 0)$, and $(0, 0, 1)$ corresponding to the eigen states of $\hat{S}_z$ operator. Physically for the choices $(1, 0, 0)$ and $(0, 0, 1)$ the initial wave function was concentrated solely in any of the rim sites while the choice $(0, 1, 0)$ corresponds to the case in which the wave function was entirely located in the hub site initially. Let us now discuss all the possibilities one by one.

(i) First we consider $c_1 = 1$, $c_2 = 0$, and $c_3 = 0$. i.e. the initial wave packet was located in the rim site B only. From equations (13)–(15), after a straightforward calculation, we find $\langle \dot{v}_\nu(t) \rangle = 0$ whereas $\langle \dot{v}_\mu(t) \rangle$ takes the following form

$$
\langle \dot{v}_\mu(t) \rangle = -2\nu_F \sin^2 \varphi e^{-\alpha^2} \int dq q e^{-q^2} \left[ \cos(2\varphi) \sin(2\tau) + 2 \left( 1 - \cos(2\varphi) \right) \sin(q\tau) \right],
$$

(16)

where $\tau = \nu_F t$. Note that equation (16) is an example of two-frequency ZB for a finite $\alpha$. The interference between CB and VB leads to the first term in the square.
bracket in equation (16) while the second term is appeared as a consequence of the interference between either CB and FB or VB and FB. When $\alpha = 0$ i.e. $\varphi = 0$ the second term in the square bracket of equation (16) contributes nothing to $\langle v_y(t) \rangle$. In this case the velocity average exhibits single frequency ZB determined from the interference between CB and VB which basically reminiscences the case of graphene. In the opposite limit i.e. for $\alpha = 1$ we have $\varphi = \pi/4$. In this case, equation (16) retains only the second term in square bracket. Here, $\langle v_y(t) \rangle$ performs single frequency ZB, determined from the coupling between either CB and FB or FB and VB. In other words, for $\alpha = 1$, the interference between CB and VB is completely prevented by FB.

The expectation value of position operator is obtained by integrating equation (16) with the initial condition: $\langle y(t) \rangle = 0$ at $t = 0$. Evaluating the $q$-integral in equation (16) numerically for $a = 5$, we show the behavior.

Figure 2. Time dependence of the expectation value of velocity operator for $a = 5$. We consider initial pseudospin polarization along $z$-direction with components $c_1 = 1$, $c_2 = 0$, and $c_3 = 0$.

Figure 3. Time dependence of the expectation value of position operator for $a = 5$. We consider initial pseudospin polarization along $z$-direction with components $c_1 = 1$, $c_2 = 0$, and $c_3 = 0$.

Figure 4. Time dependence of the expectation values of velocity and position operator for different choices of $(c_1, c_2, c_3)$. Here, we consider $a = 5$ and $\alpha = 0.5$. 
of \(\langle v_y(t)\rangle\) in figures 2 and 3, respectively for different values of \(\alpha\).

Both figures clearly demonstrate a crossover from a ZB of frequency \(2\Omega_2\) to a ZB of frequency \(\Omega_2\) with the smooth evolution of \(\alpha\) from 0 to 1. In other words, as \(\alpha\) approaches 1, the frequency of ZB gets reduced to half of the value corresponding to \(\alpha = 0\). It is difficult to comment on the frequency of ZB for \(0 < \alpha < 1\). Due to the interplay of two frequencies, namely, \(2\Omega_2\) and \(\Omega_2\) a complicated pattern is obtained for \(\alpha = 0.4\). It is worthy to notice that only one frequency either \(2\Omega_2\) or \(\Omega_2\) roughly dominates when \(\alpha\) is below or above the value \(\alpha = 0.4\). As evident from figures 2 and 3, the resultant ZB is transient in character. To understand this transient behavior, it is possible to find an approximate analytical expression of equation (16) when the width of the wave packet is large enough i.e. for \(a \gg 1\). In this limit the modified Bessel function \(I_0(2aq)\) can be approximated as \(I_0(2aq) \approx e^{2aq}/\sqrt{4\pi aq}\). With the help of the stationary phase approximation we obtain

\[
\langle v_y(t)\rangle \approx -\frac{\nu}{2}\cos^2\varphi \left[ \cos(2\varphi)e^{-r^2} \sin(2\tau) \right] + 2\left[ 1 - \cos(2\varphi) \right] e^{-r^2/4} \sin(ar).\]  

Note that the presence of decaying exponential terms in equation (17) clearly explains the transient behavior of ZB. For \(\alpha = 0\) the ZB decays rapidly due to the presence of the term \(e^{-r^2}\) in equation (17). Here, the characteristic time scale corresponding to the decay in ZB amplitude is of the order of \(d/\nu\). When \(\alpha = 1\) ZB decays slowly in comparison to the \(\alpha = 0\). This behavior is justified by the presence of \(e^{-r^2/4}\) term in equation (17). Here the characteristic decay time scale is \(2d/\nu\). It is also evident from figures 2 and 3, the second term within the square bracket in equation (17) dominates over the first term in the intermediate range of \(\alpha\) i.e. \(0 < \alpha < 1\).

(ii) Next, we consider another choice, namely, \(c_1 = 0, c_2 = 0,\) and \(c_3 = 1\). In this case initially the electronic wave function was concentrated only in the other rim site C. Here we find \(\langle v_y(t)\rangle = 0\) and

\[
\langle v_y(t)\rangle = -2\nu e^{-2\varphi} \sin^2 \varphi \int dq qe^{-q^2} I_0(2aq) \left[ \cos(2\varphi) \sin(2\tau) - 2\left[ 1 + \cos(2\varphi) \right] \sin(q\tau) \right].
\]

It is obvious from equation (18) that \(\langle v_y(t)\rangle = 0\) when \(\alpha = 0\). This clearly contradicts the established results corresponding to graphene. However, the discrepancy arose here is apparent because we are dealing with pseudospin-1. For graphene the pseudospin component \(c_3\) is completely absent. Hence, the results corresponding to graphene can not be correctly interpreted from equation (18). For a finite \(\alpha\), comparing equations (16) and (18), one may say that choices (i) and (ii) impose two-fold differences in ZB. Firstly, the amplitudes differ by an \(\alpha\)-dependent factor, namely, \(\sin^2 \varphi\) and \(\cos^2 \varphi\). Secondly, the second terms in the square brackets in equations (16) and (18) are different which may bring a phase difference in the oscillations. In the \(a \gg 1\) limit, we obtain

\[
\langle v_y(t)\rangle \approx -\frac{\nu}{2}\cos(2\varphi)e^{-r^2} \sin(2\tau) - 2\left[ 1 + \cos(2\varphi) \right] e^{-r^2/4} \sin(ar).\]  

(iii) Finally, we consider \(c_1 = 0, c_2 = 1,\) and \(c_3 = 0\). The physical meaning of this particular choice corresponds to the initial localization of the wave function at the hub site A. In this case, we obtain \(\langle v_y(t)\rangle = 0\) and

\[
\langle v_y(t)\rangle = 2\nu e^{-2\varphi} \int dq e^{-q^2} I_0(2aq) \sin(2\tau).\]  

It is transparent from equation (20) that ZB consists of only one frequency governed by the energy difference between CB and VB for all values of \(\alpha\). For this specific case, the FB contributes nothing to ZB. In the large \(\alpha\) limit, we find

\[
\langle v_y(t)\rangle \approx \frac{\nu}{2}\cos(2\varphi)e^{-r^2} \sin(2\tau).\]  

For all the choices of \((c_1, c_2, c_3)\), we obtain \(\langle v_y(t)\rangle = 0\) but \(\langle v_x(t)\rangle \neq 0\). This implies that ZB occurs in a direction perpendicular to the direction of initial wave vector and pseudospin of the wave packet. Interestingly, we also note that the behavior of ZB is significantly dependent on the type of the initial pseudospin polarization of the wave packet. More specifically, when the initial wave packet was located entirely in any of the rim sites, as understood from equations (16) and (18), corresponding ZB consists of two frequencies for \(0 < \alpha < 1\). But when the initial wave packet was in the hub site, we obtain single frequency ZB for all \(\alpha\) as evident from equation (20). To establish all qualitative arguments made earlier we portray the time evolution of the expectation values of velocity and position operators in figure 4 for different choices of \((c_1, c_2, c_3)\). For the plots we consider \(\alpha = 0.5\) and \(a = 5\). As illustrated in figure 4 the ZBs in position and velocity corresponding to different choices clearly differ in phase and amplitude. The ZB corresponding to the choice \((c_1, c_2, c_3) = (0, 1, 0)\) decays more rapidly than the other choices due to the structure of the equation (21). Moreover, the ZB in position for \((c_1, c_2, c_3) = (1, 0, 0)\) is negative while the ZBs for other choices are positive.

2.5.2. x-polarization. Now, we seek to study the wave packet dynamics by considering that initial pseudospin polarization was in the x-direction. The operator \(S_y\) has three eigenstates corresponding to the eigenvalues, namely, \(0, \pm 1\) in units of \(\hbar\). For those states we have following choices of \((c_1, c_2, c_3)\), namely, \((\sin \varphi, 0, -\cos \varphi), (\cos \varphi, 1, \sin \varphi),\) and \((-\cos \varphi, 1, -\sin \varphi)\). In figure 5 the time dependence of the expectation values of position and velocity operators are shown corresponding to the above mentioned choices of \((c_1, c_2, c_3)\) for a fixed \(\alpha = 0.5\). In contrast to case 1, here both \(\langle x(t)\rangle\) and \(\langle v_x(t)\rangle\) do not vanish for all choices of \((c_1, c_2, c_3)\). When \((c_1, c_2, c_3) = (\sin \varphi, 0, -\cos \varphi)\), both \(\langle x(t)\rangle\) and \(\langle v_x(t)\rangle\) are zero. Corresponding to the remaining choices of \((c_1, c_2, c_3)\),
namely, \((\cos \varphi, 1, \sin \varphi)\), and \((-\cos \varphi, 1, -\sin \varphi)\) both \(\langle x(t) \rangle\) and \(\langle v_x(t) \rangle\) are mirror image of each other individually. Although \(\langle v_x(t) \rangle\) shows a tiny oscillation as depicted in the insets of figure 5(a) but \(\langle x(t) \rangle\) exhibits a linear dependence with time (figure 5(b)). On the other hand the \(y\)-component of position and velocity exhibit the expected ZB oscillation. However, their amplitudes are much smaller in comparison with the case 1. Note that the amplitude of ZB in both \(\langle x(t) \rangle\) and \(\langle v_x(t) \rangle\) for \((c_1, c_2, c_3) = (\sin \varphi, 0, -\cos \varphi)\) are larger than that corresponding to the other choices. When \((c_1, c_2, c_3) = (\cos \varphi, 1, \sin \varphi)\) and \((\cos \varphi, 1, -\sin \varphi)\) the corresponding ZBs coincide with each other.

2.5.3. \(y\)-polarization. Finally, we consider the pseudospin associated with the initial wave packet was polarized along \(y\)-direction. Like \(S_y\), the operator \(S_y\) has also eigen values, namely, 0, ±1 (in units of \(\hbar\)). In this situation the options of \((c_1, c_2, c_3)\) are \((\sin \varphi, 0, \cos \varphi)\), \((-\cos \varphi, 1, i \sin \varphi)\), and \((i \cos \varphi, 1, -i \sin \varphi)\). Figure 6 shows the behavior of ZB in position and velocity corresponding to those choices of \((c_1, c_2, c_3)\). For this particular case, ZB appears only in \(y\)-direction. Different possibilities of \((c_1, c_2, c_3)\) introduce a phase in the oscillation.

3. In presence of a magnetic field

3.1. Energy spectrum

As a consequence of an external transverse magnetic field \(\mathbf{B} = B_x\), the continuous energy spectrum of the conic band is redistributed in the form of following Landau levels:

\[ E_{n,\zeta} = \lambda \gamma B \sqrt{n + \chi \zeta}, \]  

(22)

where \(\lambda = \pm 1\) denotes either CB or VB, \(n = 0, 1, 2, \ldots\) is the LL index, \(\gamma_B = \sqrt{2\hbar v_F l_0}\) with \(l_0 = \sqrt{\hbar/(eB)}\) being the magnetic length and \(\chi \zeta = \left[1 - \zeta \cos(2\varphi)\right]/2\). It is important to note that the magnetic field can not alter the fate of the FB. It still retains its zero energy states.

Choosing the vector potential \(\mathbf{A}\) corresponding to \(\mathbf{B}\) in the Landau gauge as \(\mathbf{A} = (-By, 0, 0)\), we find the conic band eigenfunctions corresponding to the K-valley as [48]

\[ \psi_{n,\zeta}^{\lambda}(\mathbf{r}) = \frac{1}{\sqrt{2}} \begin{pmatrix} \sqrt{\frac{1}{n + \chi \zeta}} \Phi_{n-1}(y) \\ \lambda \Phi_{n}(y) \\ \sqrt{\frac{1}{(n + 1)\chi \zeta}} \Phi_{n+1}(y) \end{pmatrix} e^{i k_x x} \]  

(23)
and
\[ \psi_{0,k}^{\lambda}(\bm{r}) = \frac{1}{\sqrt{2\pi}} \begin{pmatrix} 0 \\ \Phi_k(y) \end{pmatrix} e^{ikx} \sqrt{\frac{\lambda}{2\pi}} \tag{24} \]
for \( n > 0 \) and \( n = 0 \), respectively. Here, \( \Phi_n(y) = \frac{1}{\sqrt{\Gamma(n+1)}} e^{-\frac{(y-y_0)^2}{2\sigma}} H_n[(y-y_0)/\sigma] \), with \( y_0 = \frac{\lambda}{2\pi} \), is the standard harmonic oscillator wave function.

On the other hand, despite its zero energy the FB wave functions for K-valley are obtained as [48]
\[ \psi_{0,k}^{FK}(\bm{r}) = \begin{pmatrix} -\frac{\sqrt{n+1}x}{\sqrt{\pi N_0}} \Phi_{n-1}(y) \\ 0 \end{pmatrix} e^{\frac{ikx}{\sqrt{2\pi}}} \tag{25} \]
and
\[ \psi_{0,k}^{\lambda}(\bm{r}) = \begin{pmatrix} 0 \\ \Phi_k(y) \end{pmatrix} e^{\frac{ikx}{\sqrt{2\pi}}} \tag{26} \]
for \( n > 0 \) and \( n = 0 \), respectively. Note that the states in the FB is infinitely degenerate.

3.2. Time evolution

Here we attempt to study the cyclotron dynamics of a quasi-particle represented by a wave packet. It should be mentioned here that we have considered the momentum-space wave packet in the case of zero magnetic field, but here we prefer to adopt position-space wave packet to circumvent calculation difficulties.

The initial Gaussian wave packet is chosen as
\[ \Psi(\bm{r}, 0) = \frac{1}{\sqrt{\pi l_0^2}} e^{-\frac{\rho_0^2}{2l_0^2}} + i \frac{\rho_0}{\hbar} \begin{pmatrix} c_1 \\ c_2 \end{pmatrix}, \tag{27} \]
where \( \rho_0 \) is the initial momentum and the constants \( c_1, c_2 \) play the same role as mentioned in the case of zero external field.

To find the wave packet at a later time \( t \), we need to construct suitable propagator that can describe the desired time evolution. In this way we follow the Green’s function technique given in [7] with appropriate modifications.

The time evolved wave packet can be found as
\[ \Psi(\bm{r}, t) = \int d\bm{r}' G(\bm{r}, \bm{r}', t) \Psi(\bm{r}, 0), \tag{28} \]
where the propagator or the Green’s function is given by
\[ G(\bm{r}, \bm{r}', t) = \begin{pmatrix} G_{11} & G_{12} & G_{13} \\ G_{21} & G_{22} & G_{23} \\ G_{31} & G_{32} & G_{33} \end{pmatrix}. \tag{29} \]
The matrix elements of \( G(\bm{r}, \bm{r}', t) \) are defined as
\[ G_{lm}(\bm{r}, \bm{r}', t) = \sum_{\lambda=\pm 0}^{\infty} \psi_{n,k,l}(\bm{r}) \psi_{n,k,m}^{*}(\bm{r}', 0), \tag{30} \]
where the time evolved state is given by \( \psi_{n,k,l}(\bm{r}, t) = \psi_{n,k,l}(\bm{r}, 0) e^{-i\epsilon_n t}/\hbar \).

The corresponding matrix elements i.e. \( G_{lm} \)’s can be found from the following equation
\[ \begin{pmatrix} G_{11} & G_{12} & G_{13} \\ G_{21} & G_{22} & G_{23} \\ G_{31} & G_{32} & G_{33} \end{pmatrix} = \frac{1}{2\pi} \int dk \epsilon(k)(t - \tau) \sum_{\lambda=\pm 0}^{\infty} \begin{pmatrix} f_{n+1}(t) \phi_n(y - y_0) \phi_n(y' - y_0) \\ g_{n+1}(t) \phi_{n+1}(y - y_0) \phi_{n+1}(y' - y_0) \\ h_{n+1}(t) \phi_{n+2}(y - y_0) \phi_{n+2}(y' - y_0) \end{pmatrix}, \tag{31} \]
where \( f_n(t) = A_n \cos(\delta_n t) + B_n \sin(\delta_n t), \phi_n(t) = D_n \cos(\delta_n t) - 1 \), \( g_n(t) = \cos(\delta_n t), \phi_n(t) = -i D_n \sin(\delta_n t) \), and \( r_n(t) = B_n e^{i \pi n} + A_n e^{-i \pi n} \) with \( A_n = (n^2 + \chi_+)/(n^2 + \chi_-), B_n = (n + 1) \chi_+/n \chi_+, C_n = n^2 (1 - \chi_-)/n^2 (1 - \chi_+), F_n = \sqrt{(n + 1) \chi_+/n \chi_+}, \) and \( \delta_n = \gamma n \sqrt{\chi_+/\chi_-}/\hbar \).

Now using equations (27), (28) and (31), we find the wave packet at a later time \( t \) as
\[ \begin{pmatrix} \psi_1(\bm{r}, t) \\ \psi_2(\bm{r}, t) \\ \psi_3(\bm{r}, t) \end{pmatrix} = \frac{1}{\sqrt{2\pi l_0}} \sum_{n=0}^{\infty} \frac{1}{2l_0} \int d\epsilon \Gamma(x, y, u) (-u)^n \begin{pmatrix} \Sigma_1(t, u) \\ \Sigma_2(t, u) \\ \Sigma_3(t, u) \end{pmatrix}, \tag{32} \]
where \( \Gamma(x, y, u) = iax/(l_0 - (p_0/l_0 h - u)^2/2 - u^4/4 - (y/l_0 - u)^2/2 + \Sigma_3(t, u) \)’s with \( t = 1, 2, 3 \) are given by the following matrix equation
\[ \begin{pmatrix} \Sigma_1(t, u) \\ \Sigma_2(t, u) \\ \Sigma_3(t, u) \end{pmatrix} = \frac{1}{\sqrt{2\pi l_0}} \sum_{n=0}^{\infty} \frac{1}{2l_0} \int d\epsilon \begin{pmatrix} f_{n+1}(t) \phi_n(u) - i \frac{e_n(t)}{\sqrt{(n+1)(n+2)}} \phi_n(u) \\ g_{n+1}(t) \phi_{n+1}(u) - i \frac{e_{n+1}(t)}{\sqrt{(n+1)(n+2)}} \phi_{n+1}(u) \\ h_{n+1}(t) \phi_{n+2}(u) - i \frac{e_{n+2}(t)}{\sqrt{(n+1)(n+2)}} \phi_{n+2}(u) \end{pmatrix} \begin{pmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_3 \end{pmatrix}. \tag{33} \]

3.3. Expectation values

The expectation value of the velocity operator can be written in the following form as
\[ \begin{pmatrix} \langle \dot{\psi}_1(t) \rangle \\ \langle \dot{\psi}_2(t) \rangle \\ \langle \dot{\psi}_3(t) \rangle \end{pmatrix} = 2\hbar \begin{pmatrix} \text{Re} I_{12}(t) + \sin \varphi I_{23}(t) \end{pmatrix}, \tag{34} \]
where, \( I_{\mu\sigma}(t) \) is defined as \( I_{\mu\sigma}(t) = \int \Psi_{\mu}^{*}(\bm{r}, t) \Psi_{\sigma}(\bm{r}, t) d\text{dy} \).

After a straightforward calculation we evaluate \( I_{12}(t) \) and \( I_{23}(t) \) as
Figure 7. Expectation values of position and velocity operators in presence of a transverse magnetic field for pseudo spin polarization along z-direction with components \((c_1, c_2, c_3) = (1, 0, 0)\). Here, we consider \(k_0 = 2 \times 10^9 \text{ m}^{-1}\) and \(l_0 = 8.11 \text{ nm}\).

Figure 8. Expectation values of position and velocity operators in presence of a transverse magnetic field for pseudo spin polarization along z-direction with components \((c_1, c_2, c_3) = (1, 0, 0)\). Here, we consider \(k_0 = 6 \times 10^9 \text{ m}^{-1}\) and \(l_0 = 8.11 \text{ nm}\).

\[
I_{12}(t) = \sqrt{\frac{2}{3}} \frac{1}{C^2} \sum_{n=0}^{\infty} \frac{1}{2^{2n+1} n!} \left[ c_1^* c_2 f_{n+1}^* g_n \xi_{2n} + \frac{1}{\sqrt{2(n+1)}} \left\{ |c_1|^2 f_{n+2}^* g_{n+1} + c_1^* c_3 f_{n+1}^* p_{n+1} + |c_2|^2 g_{n+1}^* q_n \right\} \xi_{2n+1} \right. \\
+ \frac{1}{2\sqrt{(n+1)(n+2)}} \left\{ c_1^* c_2 h_{n+1}^* q_n + c_2^* c_3 h_{n+2}^* g_{n+1} + c_3^2 c_1 \sqrt{\frac{n+2}{n+1}} g_{n+1}^* p_n \right\} \xi_{2n+2} \\
+ \left. \frac{1}{2\sqrt{2(n+1)(n+2)(n+3)}} \left\{ c_3^* c_1 h_{n+2}^* g_{n+1} + |c_3|^2 \sqrt{\frac{n+3}{n+1}} h_{n+1}^* p_n \right\} \xi_{2n+3} \right]
\]  

(35)
...with different combinations of \(c_1, c_2, c_3\). Here, we consider \(k_0 = 2 \times 10^8 \text{ m}^{-1}, l_0 = 8.11 \text{ nm}, \) and \(\alpha = 0.5\).

\[
I_{23}(t) = \sqrt{\frac{2}{3}} \sum_{n=0}^{\infty} \frac{1}{2^n n!} \left\{ c_1^* c_2 g_{n+1}^* p_n + c_2^* c_3 g_{n+1}^* r_n \right\} \xi_n + \frac{1}{\sqrt{2(n+1)}} \left\{ |c_1|^2 g_{n+2}^* h_{n+1} + c_1^* c_3 g_{n+1}^* r_{n+1} \right\} \\
+ |c_2|^2 q_{n+1}^* p_n + |c_3|^2 p_n^* r_n \right\} \xi_{n+1} + \frac{1}{2\sqrt{(n+1)(n+2)}} \left\{ c_2^* c_1 q_{n+1}^* h_{n+1} + c_3^* c_2 p_{n+1}^* p_n \right\} \xi_{n+2} \\
+ \frac{1}{2\sqrt{(n+1)(n+2)(n+3)}} c_3^* c_1 p_{n+2}^* h_{n+1} \right\} \xi_{n+3},
\]

where

\[
\xi_n = (-1)^m \left( \frac{2}{3} \right)^m \exp \left( -\frac{\rho_0^2 c_0^2}{3h^2} \right) \frac{H_n(i\sqrt{\frac{2\rho_0 c_0}{h}})}{(2i)^n}. \tag{37}
\]

### 3.4. Different choices of initial pseudospin polarization

Likewise the zero magnetic field case we discuss here the behavior of ZB for various choices of initial pseudospin polarization corresponding to the different values of \(c_1, c_2,\) and \(c_3.\)

#### 3.4.1. z-polarization

We consider the pseudospin associated with the initial wave packet is polarized along the \(z\)-direction. Similar to the zero field case, here we also have three distinct choices of \((c_1, c_2, c_3),\) namely, \((1, 0, 0), (0, 1, 0),\) and \((0, 0, 1).\) In the following we demonstrate how different choices of \((c_1, c_2, c_3)\) lead to modify the structure of ZB.

(i) For \((c_1, c_2, c_3) = (1, 0, 0)\) we find from equations (35) and (36)

\[
I_{12}(t) = \frac{1}{\sqrt{3}} \sum_{n=0}^{\infty} \frac{f_{n+2}^* g_{n+1}^*}{2^n n! \sqrt{n+1}} \xi_{n+1}, \tag{38}
\]

\[
I_{23}(t) = \frac{1}{\sqrt{3}} \sum_{n=0}^{\infty} \frac{g_{n+2}^* h_{n+1}^*}{2^n n! \sqrt{n+1}} \xi_{n+1}. \tag{39}
\]

Note that the products \(f_{n+2}^* g_{n+1}^*\) and \(g_{n+2}^* h_{n+1}^*\) are purely imaginary. As a result, we readily obtain from equation (34) that \(\langle \mathbf{\tilde{v}}(t) \rangle = 0\) and

\[
\langle \mathbf{\tilde{v}}(t) \rangle = \frac{2\eta}{\sqrt{3}} \sum_{n=0}^{\infty} \frac{\xi_{n+1}}{2^n n! \sqrt{n+1}} \text{Im} \left( \cos \varphi f_{n+2}^* g_{n+1} + \sin \varphi g_{n+2}^* h_{n+1} \right). \tag{40}
\]

(ii) For \((c_1, c_2, c_3) = (0, 1, 0)\) we find \(\langle \mathbf{\tilde{v}}(t) \rangle = 0\) and

\[
\langle \mathbf{\tilde{v}}(t) \rangle = \frac{2\eta}{\sqrt{3}} \sum_{n=0}^{\infty} \frac{\xi_{n+1}}{2^n n! \sqrt{n+1}} \text{Im} \left( \cos \varphi g_{n+2}^* g_{n} + \sin \varphi q_{n+1}^* p_n \right). \tag{41}
\]
Figure 10. Time dependence of the expectation values of $x$-component of velocity and position operators when the initial pseudospin was polarized along $x$-direction. Here, solid (red), dashed (green), and dot-dashed (blue) lines correspond to different values of $(c_1, c_2, c_3)$, namely, $(\sin \varphi, 0, -\cos \varphi)$, $(\cos \varphi, 1, \sin \varphi)$, and $(-\cos \varphi, 1, -\sin \varphi)$, respectively. We also consider $k_0 = 2 \times 10^8$ m$^{-1}$, $l_0 = 8.11$ nm, and $\alpha = 0.5$.

Figure 11. Time dependence of the expectation values of $y$-component of velocity and position operators when the initial pseudospin was polarized along $x$-direction. Here, solid (red), dashed (green), and dot-dashed (blue) lines correspond to different values of $(c_1, c_2, c_3)$, namely, $(\sin \varphi, 0, -\cos \varphi)$, $(\cos \varphi, 1, \sin \varphi)$, and $(-\cos \varphi, 1, -\sin \varphi)$, respectively. We also consider $k_0 = 2 \times 10^8$ m$^{-1}$, $l_0 = 8.11$ nm, and $\alpha = 0.5$.

(iii) When $(c_1, c_2, c_3) = (0, 0, 1)$, it is obtained that $\langle \xi_y(t) \rangle = 0$ and

$$\langle \xi_y(t) \rangle = \frac{2\psi}{\sqrt{3}} \sum_{n=0}^{\infty} \frac{1}{2n!\sqrt{n+1}} \text{Im} \left( \sin \varphi \, p_n^* r_n \xi_{2n+1} \right)$$

$$+ \frac{\cos \varphi}{2\sqrt{(n+1)(n+2)}} b_n^* \eta_{2n+3}. \tag{42}$$

A careful inspection of equations (40)–(42) reveals that the structure of the ZB appeared in velocity significantly depends on different choices of $(c_1, c_2, c_3)$. In all cases the velocity average undergoes multi-frequency transverse ZB governed by the interference among different Landau levels. In figures 7–9 we portray the time dependence of the expectation values of velocity and position operators. With appropriate initial conditions taken into account corresponding position expectation values can be obtained by integrating equations (40)–(42). The actual initial condition is $\langle y(t) \rangle = k_0 \xi_y$ at $t = 0$. To calculate $\langle y(t) \rangle$ we choose the following initial condition: $\langle y(0) \rangle = 0$ at $t = 0$. The expectation values of position operator as illustrated in figures 7–9 differ from their actual values at most by a constant shift $\langle y(0) \rangle$. We perform all the calculations for a constant magnetic field $B = 10$ T for which the magnetic length scale becomes $l_0 = 8.11$ nm. We also consider the width of the wave packet as $d = l_0$. Figure 7 illustrates the ZB appeared in velocity and position for various values of $\alpha$. In this case we choose $(c_1, c_2, c_3) = (1, 0, 0)$ and $k_0 = 2 \times 10^8$ m$^{-1}$. It is clear from figure 7, the ZBs appeared in both position and velocity undergo permanent oscillations and oscillatory patterns depend significantly on $\alpha$.

To plot figure 8 we repeat the calculations for a higher value of $k_0$, namely, $k_0 = 6 \times 10^8$ m$^{-1}$. In this case the ZB in position exhibits transient character. However, for ZB in velocity a highly oscillatory pattern is superimposed on the transient character. Note that the locations of maxima and minima are almost insensitive to $\alpha$. Only the amplitude of ZB changes with $\alpha$.

Figure 9 describe the behavior of ZB corresponding to different possibilities of $(c_1, c_2, c_3)$. For the plots, we have taken $k_0 = 2 \times 10^8$ m$^{-1}$ and $\alpha = 0.5$. Here, the oscillatory pattern is significantly different for different initial pseudospin polarization.

3.4.2. $x$-polarization. Here, we consider that the initial wave packet was polarized along $x$-direction and the resultant behaviors are shown in figures 10 and 11. As mentioned
earlier we have following choices of \((c_1, c_2, c_3)\), namely, \((\sin \varphi, 0, -\cos \varphi)\), \((\cos \varphi, 1, \sin \varphi)\), and \((-\cos \varphi, 1, -\sin \varphi)\). In this case the values of parameters are taken as \(k_0 = 2 \times 10^8 \text{ m}^{-1}\) and \(l_0 = 8.11 \text{ nm}\).

Interestingly we find that the \(x\)-component of the expectation values of velocity and position operators are non-zero as evident from figure 10. However, for \((c_1, c_2, c_3) = (\sin \varphi, 0, -\cos \varphi)\), it is obtained that \(\langle \hat{v}_x(t) \rangle = 0\) and \(\langle \hat{x}(t) \rangle = 0\). The expectation values of \(x\) corresponding to other choices of \((c_1, c_2, c_3)\) are mirror images of each other. This particular feature was also reflected in the case of zero magnetic field. However, the values of \(\langle \hat{v}_y(t) \rangle\) corresponding to \((c_1, c_2, c_3) = (\cos \varphi, 1, \sin \varphi)\) and \((c_1, c_2, c_3) = (-\cos \varphi, 1, -\sin \varphi)\) are not exactly mirror images of each other. In fact, the amplitude of \(\langle \hat{v}_y(t) \rangle\) in the first case is greater than that in the second case.

On the other hand, the time dependence of the expectation values of \(y\) and \(v_y\) are portrayed in figure 11. Both \(\langle y(t) \rangle\) and \(\langle \hat{v}_y(t) \rangle\) exhibit regular oscillations for \((c_1, c_2, c_3) = (\sin \varphi, 0, -\cos \varphi)\). Irregularities in the oscillations appear for other choices of \((c_1, c_2, c_3)\).

### 3.4.3. \(y\)-polarization

Finally, we depict the time dependence of the expectation values of position and velocity operators in figure 12 by considering the initial pseudospin polarization was along \(y\)-direction. We have the following choices of \((c_1, c_2, c_3)\) such as \((\sin \varphi, 0, \cos \varphi)\), \((-i \cos \varphi, 1, i \sin \varphi)\), and \((i \cos \varphi, 1, -i \sin \varphi)\). We take same parameter values as considered for choice 2. Similar to choice 1, we obtain \(\langle \hat{x}(t) \rangle = 0\) and \(\langle \hat{v}_x(t) \rangle = 0\). For all possibilities of \((c_1, c_2, c_3)\), complicated irregular oscillatory patterns are obtained in both \(\langle y(t) \rangle\) and \(\langle \hat{v}_y(t) \rangle\).

### 3.5. Determination of frequencies involved in ZB

It would be interesting to find out the frequency components which are present in the complicated structure of ZB. As an example we consider the case in which the wave packet was polarized initially along \(z\)-direction with components \(c_1 = 1\), \(c_2 = 0\), and \(c_3 = 0\). The time dependence of \(\langle \hat{v}_z(t) \rangle\) and \(\langle y(t) \rangle\) are already shown in figure 7. We make fast Fourier transformation of \(\langle \hat{v}_y \rangle\) versus \(t\) data to find the frequencies involved in ZB. The corresponding results are shown in figure 13. The arrows in the right panels of figure 13 denote the frequencies involved in ZB. The revealed frequencies corresponding to different values of \(\alpha\) are given in table 1. Note that the number of frequency components depends on \(\alpha\) significantly. We find,

---

### Table 1. Frequency involved in ZB for different values of \(\alpha\)

| Freq. (THz) | \(\alpha = 0\) | \(\alpha = 0.5\) | \(\alpha = 1\) |
|-------------|----------------|----------------|----------------|
| \(f_1\)    | 10.99          | 9.76           | 9.76           |
| \(f_2\)    | 13.43          | 13.43          | 12.21          |
| \(f_3\)    | 80.59          | 36.63          | 40.29          |
| \(f_4\)    | 105.00         | 48.84          | 52.50          |
| \(f_5\)    | 124.50         | 59.83          | 62.27          |
| \(f_6\)    | 141.60         | 68.38          | 70.82          |
| \(f_7\)    | 156.30         | 75.70          | 78.14          |
| \(f_8\)    | 169.70         | 85.47          | 85.47          |
| \(f_9\)    | —              | 108.70         | 91.58          |
| \(f_{10}\) | —              | 128.20         | —              |
| \(f_{11}\) | —              | 144.10         | —              |
| \(f_{12}\) | —              | 158.70         | —              |
approximately, 8, 12, and 9 frequencies for $\alpha = 0$, $\alpha = 0.5$, and $\alpha = 1$, respectively. These frequencies are governed by all possible differences between Landau energy levels. In a similar way one can also find out the frequencies corresponding to the other choices of $c_1$, $c_2$, and $c_3$.

4. Summary

In summary, we have studied the ZB of a Gaussian wave packet which represents a quasiparticle in $\alpha$-T$_3$ model. We also consider the effect of an external transverse magnetic field on ZB. The manifestation of the ZB of the wave packet is shown in the expectation values of physical observables like position and velocity. For zero magnetic field case, we find that the ZBs appeared in position and velocity diminish with time. The problem studied in this article is an example of two frequency ZB for a finite values of $\alpha$ e.g. $0 < \alpha < 1$. One frequency is originating due to the interference between conduction and valence band whereas the other frequency is a result of interference between either conduction or flat band or flat and valence band. It is revealed that ZB depends significantly on the nature of the initial pseudospin polarization. Specifically, the case with initial pseudospin polarization along $z$-direction is more interesting. By considering this particular spin polarization we find that ZB consists of two aforesaid frequencies for $0 < \alpha < 1$ when the initial wave packet was completely located in any of the rim sites. A transition from $\Omega_q$-frequency ZB to $\Omega_{q'}$-frequency ZB is unveiled as $\alpha$ is varied from 0 to 1. On the contrary, the existence of a single $\Omega_q$-frequency ZB is realized for a finite $\alpha$ in the case of initial wave packet being situated in the hub site. The timescales over which the ZB persists can be extracted from the approximate results of expectation values obtained in the large width limit of the wave packet. Other choices of initial pseudospin polarization have produced some interesting features. In the presence of a finite magnetic field the ZB displays complicated permanent oscillations as a result of interference among large number of Landau levels. Similar to the zero magnetic field case, the oscillatory pattern depends on the type of initial pseudospin polarization.

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