Efficient Exact Verification of Binarized Neural Networks

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Table 1. Preview of Results for MNIST and CIFAR10

|                  | Mean Solve Time (s) | PGD Accuracy | Verifiable Accuracy |
|------------------|---------------------|--------------|---------------------|
| MNIST $\epsilon = 0.1$ | EEV 0.0009          | 95.35%       | 84.46%              |
|                  | Xiao et al. 0.49    | 95.13%       | 94.33%              |
| MNIST $\epsilon = 0.3$ | EEV 0.0023          | 90.97%       | 36.41%              |
|                  | Xiao et al. 2.78    | 92.05%       | 80.68%              |
| CIFAR10 $\epsilon = \frac{2}{255}$ | EEV 0.0019          | 39.47%       | 13.48%              |
|                  | Xiao et al. 13.50   | 49.92%       | 45.93%              |
| CIFAR10 $\epsilon = \frac{8}{255}$ | EEV 0.0017          | 26.78%       | 10.79%              |
|                  | Xiao et al. 22.33   | 26.78%       | 20.27%              |

EEV is exact verification with EEV. Xiao et al. is exact verification for real-valued networks, with data taken from (Xiao et al., 2019). Both use the conv-small network architecture (binarized for EEV). See Table 4 for more results. While PGD accuracy is comparable, verifiable accuracy is significantly lower for EEV, reflecting the unavailability of a robust training algorithm for BNNs. See Section 6.2 for more discussion.

EEV incorporates novel codesigned SAT solver and training strategies. We deploy EEV to verify adversarial robustness against input perturbations bounded by the $\ell_\infty$ norm. Compared to the fastest previously existing exact verification methods for this task, including methods for either binarized or real-valued DNNs, our results show that, for our set of MNIST and CIFAR10 benchmarks, our techniques improve the verification performance by a factor of between ten to ten thousand times depending on the dataset and network architecture. This paper makes the following contributions:

1. We incorporate native support for reified cardinality constraints into a SAT solver, improving the performance of BNN verification by more than a factor of one hundred compared to an unmodified SAT solver (Section 4.4).

2. We identify that sparse weights induced by ternarization (Narodytska et al., 2020) cause unbalanced sparsity between layers of convolutional networks. While ternarization achieves sufficient overall sparsity, our results show that it also induces high verification complexity. We propose a new strategy (BinMask), which produces more balanced sparsity. Our results show that

Abstract

We present a new system, EEV, for verifying binarized neural networks (BNNs). We formulate BNN verification as a Boolean satisfiability problem (SAT) with reified cardinality constraints of the form $y = (x_1 + \cdots + x_n \leq b)$, where $x_i$ and $y$ are Boolean variables possibly with negation and $b$ is an integer constant. We also identify two properties, specifically balanced weight sparsity and lower cardinality bounds, that reduce the verification complexity of BNNs. EEV contains both a SAT solver enhanced to handle reified cardinality constraints natively and novel training strategies designed to reduce verification complexity by delivering networks with improved sparsity properties and cardinality bounds. We demonstrate the effectiveness of EEV by presenting the first exact verification results for $\ell_\infty$-bounded adversarial robustness of nontrivial convolutional BNNs on the MNIST and CIFAR10 datasets. Our results also show that, depending on the dataset and network architecture, our techniques verify BNNs between a factor of ten to ten thousand times faster than the best previous exact verification techniques for either binarized or real-valued networks.

1. Introduction

Deep learning has achieved impressive success in many application fields including image understanding, speech recognition, natural language processing, and game playing (Goodfellow et al., 2016). While the intrinsic complexity of deep neural networks (DNNs) enables them to learn difficult tasks, this complexity also hinders the understanding of their behavior. Moreover, the existence of adversarial examples (Szegedy et al., 2014) directly exposes the fragility of DNNs. Such fragility raises concerns for applying DNNs in safety-critical environments such as autonomous driving or aircraft control.

We present new techniques and a new system, EEV, for exact verification of binarized neural networks (BNNs).
BinMask improves the performance of our verification system by a factor of between one hundred to ten thousand times compared to its performance on ternarized networks (Section 5.1).

3. We further reduce verification complexity by introducing a cardinality bound decay regularizer with a tunable tradeoff between accuracy and solving time, leading to an additional speedup of up to a factor of thousands of times (Section 5.2).

4. We present the first exact verification of $\ell_\infty$-bounded adversarial robustness of convolutional BNNs on CIFAR10 (Table 1).

5. We present experimental results comparing EEV against the best previously existing exact robustness verification systems (for either binarized or real-valued networks). These results show that, for the MNIST and CFAR10 benchmarks, our system verifies exact network robustness of given inputs between ten to ten thousand times faster than these previous systems.

2. Background and Related Work

We formulate the problem of DNN exact verification (a.k.a. complete verification) as checking whether a DNN satisfies given properties, for which the answer should either be guaranteed satisfaction or a counterexample that violates the properties. Researchers have developed a range of techniques for verifying various properties of DNNs, mostly for real-valued ReLU networks. They are largely based on SMT solvers (Scheibler et al., 2015; Huang et al., 2017; Katz et al., 2017; Ehlers, 2017) or Mixed Integer Linear Programming (MILP) (Lomuscio & Maganti, 2017; Cheng et al., 2017; Fischetti & Jo, 2018; Dutta et al., 2018; Tjeng et al., 2019; Yang & Rinard, 2019). Another line of research delivers guaranteed robustness via incomplete verification (a.k.a. certification) that may fail to prove or disprove the desired properties in certain cases (Wong & Koller, 2017; Weng et al., 2018; Gehr et al., 2018; Zhang et al., 2018; Raghunathan et al., 2018; Dvijotham et al., 2018; Mirman et al., 2018; Singh et al., 2019). This research often explores the idea of over-approximation to improve scalability, where the verifier considers a relaxed form of the actual computation in a DNN. In this paper we focus on exact verification.

Binarized neural networks (BNNs) (Hubara et al., 2016) constrain activations and weights to be binary, resulting in significant speed gain and energy saving during inference (Hubara et al., 2016; Rastegari et al., 2016; Moss et al., 2017) with tolerable accuracy degeneration (Darabi et al., 2018). Moreover, binarization facilitates analysis because the combinatorial nature of BNNs enables close interaction with logical reasoning, allowing a rich set of properties to be encoded in conjunctive normal form (CNF). Examples include queries on adversarial robustness, trojan attacks, fairness, network equivalence, and model counting (Narodytska et al., 2018; Baluta et al., 2019). The exact SAT encoding of BNNs is quite straightforward, compared to MILP methods which usually need to estimate the bounds of hidden neurons during verification for a given input. Moreover, it has been shown that exact verification of real-valued neural networks suffers from numerical error present in both the verifier and the inference implementation that allows adversarial examples to be constructed for networks with verified robustness (Jia & Rinard, 2020), while a BNN satisfies the verified properties on any correct inference implementation. Analysis techniques for BNNs include efficient encoding (Shih et al., 2019) and exploiting decomposability between neurons or layers (Cheng et al., 2018; Khalil et al., 2019).

Adversarial attack and defense of DNNs is a developing field where most research focuses on real-valued networks (Carlini & Wagner, 2017; Athalye et al., 2018; Madry et al., 2018; Kannan et al., 2018; Tramer et al., 2020). BNNs can also be attacked by gradient-based adversaries (Galloway et al., 2018) or specialized solving algorithms (Khalil et al., 2019).

Until recently, exact verification of DNNs was too computationally expensive to scale beyond a few hundred neurons. (Tjeng et al., 2019) present the first exact verification result for convolutional neural networks (CNNs) on MNIST by tightening the MILP formulation. A subsequent improvement induces stability of ReLU neurons during training (Xiao et al., 2019). (Narodytska et al., 2020) verify a non-trivial binarized multilayer perceptron on MNIST.

3. Preliminaries

3.1. The Boolean Satisfiability Problem (SAT)

SAT is the problem of deciding whether there exists a satisfying variable assignment for a given Boolean expression (Biere et al., 2009). We consider Boolean expressions in conjunctive normal form (CNF) defined over a set of Boolean variables $x_1, \ldots, x_n$. A CNF $e$ is a conjunction of a set of clauses $e = c_1 \wedge \cdots \wedge c_m$ where each clause $c_i$ is a disjunction of some literals $c_i = l_{i1} \lor \cdots \lor l_{is_i}$, and a literal $l_{ij}$ is either a variable or its negation: $l_{ij} = x_k$ or $l_{ij} = \neg x_k$.

Despite the well known fact that 3-SAT is NP-complete (Cook, 1971), efficient heuristics have been developed to enable SAT solvers to scale to industrial problems (Balyo et al., 2017).
3.2. Binarized Neural Networks (BNNs)

Binarization of neural networks is a special case of network quantization, proposed as a method to reduce the computation burden and speed up inference and possibly also training (Rastegari et al., 2016; Zhou et al., 2016; Jacob et al., 2018). We follow the framework of (Hubara et al., 2016), but modify the activation values from \([-1, 1]\) to \([0, 1]\).

The basic building block of a BNN is a \textit{linear-BatchNorm-binarize} operation that maps an input tensor \(x \in \{0, 1\}^n\) to an output tensor \(y \in \{0, 1\}^m\) with a weight parameter \(W \in \mathbb{R}^x\):

\[
y = \text{bin}_{\text{act}}(\text{BatchNorm}(\text{linear}(x, \text{bin}_{w}(W)))) \tag{1}
\]

where \(\text{bin}_{\text{act}}(x) = (\text{sign}(x) + 1)/2 \in \{0, 1\}\)

\[
\text{bin}_{w}(x) = \text{sign}(x) \in \{-1, 1\}\]

\text{linear} \in \{\text{convolution, matmul}\}

Note that the use of \([0, 1]\) rather than \([-1, 1]\) for activations does not impact network capacity, because it is a linear transformation on the activations and can be cancelled by the following batch normalization. Besides simplifying the conversion to SAT formulas, using a \([0, 1]\) encoding also makes zero padding for convolutional layers trivial.

Although the sign function has zero gradient almost everywhere, we can still train a BNN using gradient based optimizers by adopting the straight-through estimator (Bengio et al., 2013) which treats the sign function as an identity function during backpropagation.

**First layer:** The first layer of a BNN is usually applied on float or 8-bit fixed point inputs since it has many fewer channels and would not be a major issue for performance. However encoding floating-point or integer arithmetics in SAT incurs high complexity, and we add an extra quantization layer to process the input:

\[
x^q = \left\lfloor \frac{x}{s} \right\rfloor \cdot s \tag{2}
\]

where \(x \in [0, 1]^n\) is the real valued input, \(x^q\) is the quantized input to be fed into the BNN, and \(s\) is the quantization step size which can be set to \(s = 1/255\) for emulating 8-bit fixed point values, or \(2\epsilon\) for adversarial training with \(\ell_\infty\) norm bounded by \(\epsilon\).

**Last layer:** We consider the layer before softmax as the last layer of the BNN, whose output can be interpreted as classification score. We remove the \(\text{bin}_{\text{act}}\) in (1) to obtain a real valued score. To enable direct conversion into SAT, we also restrict the running variance and scale parameter in BatchNorm of the last layer to be a scalar computed on the whole feature map rather than per-channel statistics.

4. Combinatorial Analysis of BNNs

4.1. Encoding BNNs with Reified Cardinality Constraints

We discuss techniques for encoding a trained BNN as a SAT formula, focusing on the details of encoding a single layer. During inference the Batch Normalization becomes a linear transformation (Ioffe & Szegedy, 2015):

\[
x^{\text{BN}} = k^{\text{BN}} x + b^{\text{BN}}
\]

where

\[
k^{\text{BN}} = \frac{\gamma}{\sqrt{\text{Var}[x] + \epsilon}}
\]

\[
b^{\text{BN}} = \beta - k^{\text{BN}} \mathbb{E}[x]
\]

With \(W^{\text{bin}} = \text{bin}_w(W)\) being a fixed parameter, we can rewrite (1) as the following, where \(x \in \{0, 1\}^n\) is the layer input and \(y \in \{0, 1\}^m\) is the layer output with 0 interpreted as FALSE and 1 interpreted as TRUE:

\[
y = (k^{\text{BN}} \text{linear}(x, W^{\text{bin}}) + b^{\text{BN}} \geq 0) \tag{3}
\]

To convert \(\text{linear}(x, W^{\text{bin}})\) into a Boolean expression, we consider the simple case of dot-product \(\text{dot}(x, W^{\text{bin}}) = \sum_{i=1}^{n} x_i W_i^{\text{bin}}\), which can be easily extended to handle convolutional or fully connected layers. If \(W_i^{\text{bin}} = 1\), we have \(x_i W_i^{\text{bin}} = x_i\); and if \(W_i^{\text{bin}} = -1\), we rewrite \(x_i W_i^{\text{bin}} = -x_i = (1-x_i) - 1 = (-x_i) - 1\). Therefore

\[
\text{dot}(x, W^{\text{bin}}) = \sum_{i=1}^{n} l_i + b^{\text{SAT}}
\]

where

\[
l_i = \begin{cases} x_i & \text{if } W_i^{\text{bin}} = 1 \\ -x_i & \text{if } W_i^{\text{bin}} = -1 \end{cases}
\]

\[
b^{\text{SAT}} = \sum_{i=1}^{n} \min(W_i^{\text{bin}}, 0)
\]

Now (3) can be rewritten as a \textit{reified cardinality constraint}, where \(\leq\) acts as \(\geq\) or \(\leq\) according to the sign of \(k^{\text{BN}}\) and \(b\) can be rounded to an integer accordingly:

\[
y = \left( \sum_{i=1}^{n} l_i \leq b \right) \tag{4}
\]

where

\[
b = \frac{b^{\text{BN}}}{k^{\text{BN}}} - b^{\text{SAT}}
\]

Cardinality constraints belong to a more general class called \textit{pseudo-Boolean constraints}, which allow literals to be multiplied by integer coefficients. They are usually converted to CNF formulas by encoders such as sequential counters (Sinz, 2005; Hölldobler et al., 2012) or binary decision diagrams (Abio et al., 2011). Rather than using a standard SAT solver on the encoded formula, we extend the SAT solver to handle such constraints natively.
4.2. Encoding for Adversarial Attacks

Input Perturbation Encoding: We discuss how to constrain the solver space within a $\ell_\infty$ bound around the given input $x_0$. We will focus on a single scalar in the input tensor for simplicity, but it easily applies to the whole tensor under $\ell_\infty$ norm. Recall that we quantize an input $x$ with step size $s$ in (2), allowing us to rewrite the first layer as

$$y = \left(k_{\text{BN}} \text{ linear } \left[ \frac{x}{s} \right], W_{\text{bin}} \right) + b_{\text{BN}} \geq 0$$

$$= \left(k_{\text{BN}} s \text{ linear } \left[ \frac{x}{s} \right], W_{\text{bin}} \right) + b_{\text{BN}} \geq 0$$

This formulation suggests that we can treat $k_{\text{BN}} s$ as the coefficient for Batch Normalization in (4) during inference so that the input $\left[ \frac{x}{s} \right]$ is an integer. For adversarial attacks on $x_0$ with $\|x - x_0\|_\infty \leq \epsilon$, we encode the attack space as $[\frac{x}{s}] = L(x_0) + \sum_{i=1}^{k} t_i$ where $L(x_0) = \left[ \frac{\max(x_0 - \epsilon, 0)}{s} \right]$ and $U(x_0) = \left[ \frac{\min(x_0 + \epsilon, 1)}{s} \right]$ are the bounds of allowed input, $k = U(x_0) - L(x_0)$ is the possible range, and $\{t_1, \cdots, t_k\}$ are Boolean variables whose sum corresponds to the value of adversarial inputs. We further restrict the search space by enforcing the thermometer encoding (Buckman et al., 2018) on $\{t_1, \cdots, t_k\}$, via adding additional clauses $t_i \lor \neg t_j$ for $1 \leq i < j \leq k$.

Untargeted Attack Encoding: Assume there are $m$ output classes and $C$ is the target class, such that the adversary tries to cause the network to output a classification other than $C$. Let $x \in \{0, 1\}^n$ and $y \in \mathbb{R}^m$ denote the input and output of the last layer respectively. Similar to the analysis in Section 4.1, we can rewrite $y$ in the form $y_j = k_{\text{BN}} \left( \sum_{i=1}^{n} l_{ij} + b_j \right)$. Note that we have required $k_{\text{BN}}$ to be a scalar in the last layer. To ensure that the network makes a wrong prediction, we add a clause $\lor_{1 \leq j \neq C \leq m} (r_{jC})$ where $r_{jC} = (y_j - y_C \geq \delta)$ is a decision variable indicating whether the confidence of class $j$ exceeds that of class $C$ by a margin of $\delta$. Note that $r_{jC} = (y_j - y_C \geq \delta) = (k_{\text{BN}} \left( \sum_{i=1}^{n} (l_{ij} - l_{iC}) + b_j - b_C \right) \geq \delta)$ is also a refined cardinality constraint, except that the weight on some $x_i$ may be up to 2, which can be handled by duplicating the literal. We set $\delta = 10^{-6} |k_{\text{BN}}|$ in our experiments.

Given a BNN and an input image, a formula can be obtained by encoding the input constraints, the BNN itself and the output constraints using the techniques outlined above. If a SAT solver finds a solution for the formula, then an adversarial input can be recovered from the solution. Otherwise, the network is proven to be robust for this input.

4.3. Extending the CDCL Algorithm

Modern SAT solvers typically utilize the conflict-driven clause learning (CDCL) algorithm (Marques-Silva et al., 2009), which tries to reduce the search space by learning new clauses from conflicts. There are three key procedures in this algorithm:

1. Branching: Pick an undecided variable and assign a value to it. The order of branching is usually decided by heuristics like VSIDS (Moskewicz et al., 2001).

2. Propagation: Given current branching and propagation decisions, try to infer values of undecided variables. Such inference is based on the crucial concept of unit clause that contains only one unassigned literal: if there is a clause $c = l_1 \lor \cdots \lor l_n$ in the clause database and $l_1, \ldots, l_{n-1}$ are all known to be false, then $l_n$ must be true so the whole clause could be satisfied.

3. Clause Learning: When a conflict is encountered, a new clause is constructed and inserted into the clause database by summarizing the reasons that lead to the conflict. The learning is performed on the implication graph, whose nodes correspond to assignments of variables. For a node $[x = a]$, it has incoming edges from $[y_1 = b_1], \ldots, [y_k = b_k]$ such that they are the conditions to imply $x = a$ (i.e., there is a clause containing exactly $y_1, \ldots, y_k, x$ and all literals corresponding to $\{y_i\}$ are false in the clause given the assignment $y_i = b_i$). Branching variables have no incoming edges in the graph. Starting from a special node representing the conflict, the graph is traversed in reverse order to enumerate all branching variables that lead to the conflict. Disjunction of negation of those variables are added to the set of learned clauses.

The propagation and clause learning processes can be generalized to handle clauses not in disjunctive form, as long as each clause permits inferring values of undecided variables. This idea has been explored in the literature to extend SAT solvers to domain-specific problems (Soos et al., 2009; Lifitton & Maglalang, 2012; Ganesh et al., 2012).

Given a refined cardinality constraint $l = (\sum_{i=1}^{n} l_i \leq b)$, there are two types of propagations:

- **Operand-infering**: If $l$ is known and enough of the $\{l_i\}$ are known, then the remaining $\{l_i\}$ can be inferred. For example, if $l$ is known to be true and there are already $b$ literals in $\{l_i\}$ known to be true, then the other literals must be false.

- **Target-infering**: If enough of the $\{l_i\}$ are known, then $l$ can be inferred. For example, if the number of false literals in $\{l_i\}$ reaches $n - b$, then $l$ can be inferred to be true.
4.4. MiniSatCS: An Efficient Implementation

We present MiniSatCS, an novel system with native support for reified cardinality constraints. MiniSatCS is based on MiniSat 2.2 (Eén & Sörensson, 2003). An important design in modern SAT solvers is the use of watched literals (Moskewicz et al., 2001), which allows rapid detection of unit clauses. In the case of reified cardinality constraints, we keep similar watchers for each variable, and maintain counters for the current number of known true or false literals for each clause, so the situations that allow propagation can be detected without scanning the whole clause every time a variable changes. We use random polarity and turned off phase saving (Pipatsrisawat & Darwiche, 2007) in the solver since it is faster for BNN verification.

The SAT formula encoding of a BNN is constant for different input values, in contrast with MILP-base methods that need to estimate the bounds of hidden neurons for each input. Therefore we have designed a model cache mechanism in MiniSatCS to reuse the set of formulas corresponding to the BNN for different test cases, reducing model build time by ten times for the large networks.

We compare the performance of MiniSatCS on the MNIST–MLP network against two other solvers: the unmodified MiniSat 2.2 using a sequential counter encoder for reified cardinality constraints, and an SMT solver Z3 (De Moura & Björner, 2008) that has native pseudo-Boolean logic support. From Figure 1 we can see that our system is significantly more efficient than previous ones on verification of BNNs, and hundreds of times faster than the prior state-of-the-art result on the same task (Narodytska et al., 2020).

5. Training Solver-friendly BNNs

5.1. BinMask: Balanced Weight Sparsifying

It has been observed that sparse weights facilitate verification of neural networks (Tjeng et al., 2019; Xiao et al., 2019; Narodytska et al., 2020). A common sparsifying method (Narodytska et al., 2020) for BNNs is to use ternary weights, i.e. setting $\text{bin}_w(W) = 0$ when $|W| < T$. However this technique suffers from two drawbacks: (i) The threshold $T$ and the penalty coefficient for $\ell_1$ regularization are two coupled parameters that need tuning, and (ii) For convolutional networks, the sparsity of convolutional layers is usually lower than that of fully connected layers, which has also been observed during pruning real-valued networks (Han et al., 2015).

Such unbalanced sparsity complicates verification because convolutional layers bear most of the computation burden. In this case their low sparsity reduces the verification speedup. While it is possible to prune each layer with a fixed rate and retrain the network iteratively (Frankle & Carbin, 2019), such methods are especially costly when we consider adversarial training.

To this end, we hypothesize that this unbalancing in BNNs is caused by a uniform setting of $T$ with coupled optimization of both weight sparsity and weight values. The zero in the ternary weights creates a gap between $-1$ and $1$, requiring the weight to go through the zero zone even when a sign change suffices. Since the convolutional and fully con-
While BinMask alone sparsifies the small network enough to be efficiently verified, it is not sufficient for a larger network. To further reduce verification complexity, we revisit the reified cardinality constraint \( y = (\sum_{i=1}^{n} l_i \leq b) \) and note the following facts:

1. If it is encoded into CNF using sequential counters (Sinz, 2005) by introducing auxiliary variables \( r_{ij} \) to encode whether \( \sum_{k=1}^{i} l_k \leq j \), then \( O(nb) \) variables and \( O(nb) \) clauses are needed for the encoding. Thus smaller \( b \) produces simpler encoding.

2. MiniSatCS can infer \( y \) to be false once the number of true literals in \( \{l_i\} \) exceeds \( b \), and a smaller \( b \) increases the likelihood of this inference.

3. If the literals \( \{l_i\} \) are drawn from independent Bernoulli distribution parameterized with probability 0.5, then the entropy of \( y \) is a symmetrical concave function with respect to \( b \) maximized when \( b = \frac{2}{2} \). Therefore the further \( b \) deviates from \( \frac{2}{2} \), the more predictable \( y \) becomes.

We are thus motivated to regularize the bound in reified cardinality constraints to reduce verification complexity. We propose a Cardinality Bound Decay (CBD) loss to achieve this goal, by adding an \( \ell_1 \) penalty of strength \( \eta \) on the bias term \( b \) in (4). We also introduce a parameter \( \tau \) so that bounds below \( \tau \) do not get penalized, and set \( \tau = 5 \) in all of our experiments. Meaningful setting of \( \tau \) should be non-negative because if \( b \) drops below zero, then \( \sum_{i=1}^{n} l_i \geq b \) becomes constantly true or false and the bound should not be penalized anyway. The CBD loss term is formally defined as:

\[
L_{CBD} = \eta \max \left( -\frac{b}{k_{BN}} - b^{SAT} - \tau, 0 \right) \tag{6}
\]

It is worth noting that since \( \sum_{i=1}^{n} l_i \leq b \) is equivalent to \( \sum_{i=1}^{n} l_i \geq n - b \), we only consider the value of \( b \) rather than \( |b - \frac{n}{2}| \) in this loss. Table 3 summarizes our empirical evaluation of the performance of CBD loss. Our proposed method effectively reduces the bounds in cardinality constraints and speeds up verification significantly, and the parameter \( \eta \) can be tuned to control the tradeoff between accuracy and verification speed. Notably although CBD also introduces weight sparsity, it is not an alternative for weight sparsifying, which can be observed from the last two experiments on CIFAR10.

### 5.2. Cardinality Bound Decay

While BinMask alone sparsifies the small network enough to be efficiently verified, it is not sufficient for a larger network.

| Mean Solve Time (s) | MNIST | η = 0 | η = 1e−5 | η = 5e−4 | CIFAR10 | η = 0 | η = 1e−5 | η = 1e−4 |
|---------------------|-------|-------|----------|----------|---------|-------|----------|----------|
|                     |       |       |          |          |         |       |          |          |
| 2200.503            | 1332.398 | 0.318 | 3.343    | 3.642    | 0.048              |
| 3600.014            | 7595   | 93.188 | 54.192   | 0.127    |
| Test Accuracy       | 99.01% | 99.01% | 97.05%   | 95% / 81%| 95% / 81%          |
|                      |        |        |          |          |         |        |          |          |
| Provable Accuracy (Timeout%) | 9% (60%) | 3% (30%) | 5% (0%) | 0% (0%) | 2% (0%) |
|                      |        |        |          |          |         |        |          |          |
| Mean / Max Cardinality Bound | 235.4 / 561.6 | 11.6 | 235.4 / 561.6 | 11.6 |
|                      |        |        |          |          |         |        |          |          |
| First Layer/Total Sparsity | 84% / 72% | 84% / 83% | 88% / 88% | 94% / 86% | 94% / 86% | 50% / 33.9 |

We conduct our experiments on a workstation equipped with two GPUs (NVIDIA Titan RTX and NVIDIA GeForce RTX 2070 SUPER), 128 GiB of RAM and an AMD Ryzen Threadripper 2970WX 24-core processor. We used the PyTorch (Paszke et al., 2019) framework to train all the networks. We evaluated our methods using adversarial attack as an example task on two datasets: MNIST (LeCun et al., 1998) and CIFAR10 (Krizhevsky et al., 2009). Unless stated otherwise, we limit the execution time to 120 seconds for
the MiniSatCS solver per input image as in (Xiao et al., 2019).

Network Architecture: We adopt three network architectures from the literature for the evaluation of EEV:

1. MNIST-MLP: This is a binarized multilayer perceptron with hidden layers having [500, 300, 200, 100, 10] units (Narodytska et al., 2020). It is trained with an input quantization step \( s = 0.1 \) and sparsified by Bin-Mask.

2. Small-conv: This is a network with two convolutional layers of 16 and 32 channels, followed by two fully connected layers with 100 and 10 units. The convolutional layers have \( 4 \times 4 \) filters and \( 2 \times 2 \) stride with a padding of 1. The architecture is the same as in (Xiao et al., 2019) except that we binarize the network.

3. Large-conv: This is a network extending the small-conv, where each convolutional layer is preceded by another \( 3 \times 3 \) convolution with a padding of 1. The convolutional layers have \([32, 32, 64, 64]\) channels and there are three fully connected layers with \([512, 512, 10]\) output units. The architecture is the same as in (Xiao et al., 2019) except that we binarize the network.

Training Method We train the networks using the Adam optimizer (Kingma & Ba, 2014) for 200 epochs with a mini-batch size of 128. Due to fluctuations of test accuracy between epochs, we select from the last five epochs the model having the highest accuracy on the first 40 training minibatches. Learning rate is initially \( 1 \times 10^{-4} \) and decayed by a factor of two for the last 50 epochs. We use projected gradient descent (PGD) to generate adversarial examples for robust training as in (Madry et al., 2018), where \( \epsilon \) is increased linearly from 0 to the desired value in the first 100 epochs and the number of PGD iteration steps grows linearly from 0 to 10 in the first 50 epochs. All weights are initialized from a Gaussian distribution with standard deviation \( 0.01 \), and the mask weights \( M_W \) in (5) are enforced to be positive by taking the absolute value at initialization. We apply a weight decay of \( 1 \times 10^{-7} \) on binarized \( M_W \) in all experiments. For training on the MNIST dataset the input quantization step \( s \) is set to be 0.61. We set \( s = 0.064 \approx 16.3/255 \) for CIFAR10. These input quantization steps \( s \) are slightly greater than twice the largest perturbation bound we consider for each dataset. The CBD loss is applied on large-conv networks only and \( \eta \) is set to be \( 5 \times 10^{-4} \) for MNIST and \( 1 \times 10^{-4} \) for CIFAR10. We do not use any data augmentation techniques for training. Due to limited computing resource and significant differences between the settings we considered, data in this paper are reported based on one evaluation run.

6.2. Evaluating Adversarial Robustness

We evaluate the performance of EEV on the MNIST and CIFAR10 benchmarks. We train MNIST-MLP on MNIST only for comparison against (Narodytska et al., 2020), the previous fastest exactly verified BNN on MNIST. Figure 1 presents the results. For the other two network architectures, we train an undefended network on natural images and two robust networks against the PGD adversary with different \( \ell_\infty \) bounds on both MNIST and CIFAR10. We evaluate the robustness of each network against two adversaries, a 100-step PGD and our exact verifier, with three settings of the \( \ell_\infty \) bound. We also compare our results with the state-of-the-art exact verifier for real-valued networks (Xiao et al., 2019). Table 4 presents detailed results of verifier performance and test accuracy, showing that our verifier exhibits solving times 16.13 to 12815.62 times faster than (Xiao et al., 2019).

We highlight an interesting observation. An undefended CIFAR10 network has only 0.06% PGD accuracy on the largest \( \ell_\infty \) bound we considered, and adversarial training improves the number to 26.78%, comparable with real-valued networks. However if we evaluate the true adversarial robustness by applying our exact verifier, the undefended network totally fails (0.00% adversarially robust) while the seemingly robust network achieves only 10.79% adversarial accuracy. This suggests that first-order adversaries like PGD may be insufficient to explore the adversarial space of BNNs for robust training. We remark that the gap between PGD accuracy and verifiable accuracy is unlikely to be caused by obfuscated gradients (Athalye et al., 2018) because (i) success rate of PGD attack is higher when perturbation bound is increased and PGD training does improve PGD accuracy significantly, suggesting that gradient information is still useful for attacks, and (ii) we use the straight-through-estimator to compute gradients of the activation binarization and input quantization functions, which is the same method for training the networks, and gradients are unlikely to be shattered in this way.

6.3. Extensibility Case Study

We evaluate the extensibility of our system by considering an ensemble of \( M \) models that rejects the input if they do not fully agree on the classification. We are interested in how easily this ensemble can be attacked by requiring the adversary to cause all of the components to output the same wrong classification. The goal can be easily formulated in CNF: Let \( n \) be the number of classes, \( C \) be the correct class and \( r_{ij}^m \) denote whether score of class \( i \) is higher than \( j \) in model \( m \) as defined in Section 4.2. Let \( f_i^m = \bigwedge_{1 \leq j \neq i \leq n} (r_{ij}^m) \) denote whether class \( i \) has the highest score by model \( m \) and \( g_i = \bigwedge_{1 \leq m \leq M} (f_i^m) \) denote whether all models agree on class \( i \). Then the attack goal is simply \( \forall 1 \leq i \leq n. \neg g_i \). Such an encoding would not be so straightforward if we were
We present results of our system on BNNs and compare against real-valued networks of the same architecture in (Xiao et al., 2019). “Xiao et al. S” and “Xiao et al. L” correspond to real-valued networks conv-small and conv-large respectively, with data taken from Xiao et al. We conduct a complete evaluation of the large architecture while Xiao et al. evaluate only the first 1000 images due to long build time. We set $\epsilon_0$, $\epsilon_1$, $\epsilon_2$ = (0, 1, 2, 0.3) for MNIST and $\epsilon_0$, $\epsilon_1$, $\epsilon_2$ = (2/255, 5/255, 8/255) for CIFAR10.

Table 5. Model Ensemble with Reject Option on MNIST

| Dataset Training $\epsilon$ | Method        | Test Accuracy | PGD Adversarial Accuracy | Verifiable Adversarial Accuracy |
|----------------------------|---------------|---------------|--------------------------|--------------------------------|
|                            |               | $\epsilon = \epsilon_0$ | $\epsilon = \epsilon_1$ | $\epsilon = \epsilon_2$ | $\epsilon = \epsilon_0$ | $\epsilon = \epsilon_1$ | $\epsilon = \epsilon_2$ |
| MNIST $\epsilon = 0.1$     | conv-small    | 97.23%        | 92.98%                   | 84.81%                        | 67.57%                       | 75.29%                     | 26.17%                     | 2.55%                        |
|                            | conv-small    | 97.16%        | 95.35%                   | 92.82%                        | 86.57%                       | 84.46%                     | 48.71%                     | 11.69%                       |
|                            | Xiao et al. S| 98.68%        | 95.13%                   | -                             | -                            | 94.33%                     | -                          | -                            |
| MNIST $\epsilon = 0.3$     | conv-small    | 95.53%        | 94.61%                   | 93.37%                        | 90.97%                       | 87.08%                     | 68.28%                     | 36.41%                       |
|                            | conv-large    | 97.33%        | -                        | -                             | -                            | -                          | -                          | -                            |
|                            | Xiao et al. L| 97.05%        | 96.22%                   | 95.07%                        | 92.40%                       | 88.24%                     | 62.38%                     | 21.52%                       |
| CIFAR10 $\epsilon = 8/255$ | conv-small    | 51.67%        | 15.13%                   | 0.83%                         | 0.06%                        | 0.36%                      | 0.02%                      | 0.00%                        |
|                            | conv-large    | 46.57%        | 39.47%                   | 27.79%                        | 19.00%                       | 13.48%                     | 1.63%                      | 0.26%                        |
|                            | Xiao et al. L| 61.12%        | 49.92%                   | -                             | -                            | 45.93%                     | -                          | -                            |

6.4. Validating Training Methods

We conduct comprehensive experiments to validate that our proposed training methods significantly reduce verification complexity without sacrificing much test accuracy.

For each dataset, we train the conv-small and conv-large networks under two training settings: undefended (i.e., $\epsilon = 0$) and PGD-based adversarial training with a large perturbation bound ($\epsilon = 0.3$ for MNIST and $\epsilon = 8/255$ for CIFAR10). The undefended network can be regarded as a reference of test accuracy for the architecture under specific sparsity. The results show that on all the data sets and adversarial training perturbation bounds that we have considered, our proposed solver MiniSatCS is consistently faster than MiniSat 2.2 and Z3, reach-

Working with other formulations such as MILP. We present in Table 5 the results of an ensemble of small-conv and large-conv networks on MNIST adversarially trained and tested with $\epsilon_\infty$ bound 0.3. It shows that our system can easily handle more complex queries.
Table 6. Comparison of Methods on MNIST Subset

| ϵ | Network Architecture | Training Method | Solver | Test Accuracy | Mean Solve Time | Median Solve Time | Timeout | Verifiable Accuracy | Overall Sparsity |
|---|----------------------|-----------------|--------|---------------|-----------------|-----------------|--------|-------------------|-----------------|
| 0 | conv-small           | Ternary         | MiniSatCS | 97.28% | 212.084 | 2.587 | 3% | 0% | 79% |
|   |                      | BinMask         | MiniSatCS | 97.06% | 0.001  | 0.001  | 0% | 75% | 86% |
|   |                      |                | MiniSat 2.2 | 97.06% | 0.944  | 0.536  | 0% | 75% | 86% |
|   |                      |                | z3 | 97.06% | 0.079  | 0.078  | 0% | 75% | 86% |
|   | conv-large           | Ternary         | MiniSatCS | 98.89% | 2732.030 | 3600.004 | 72% | 0% | 82% |
|   |                      | Ternary+CBD     | MiniSatCS | 97.50% | 1190.475 | 166.054 | 28% | 0% | 60% |
|   |                      | BinMask         | MiniSatCS | 99.01% | 1252.975 | 169.778 | 28% | 52% | 81% |
|   |                      |                | MiniSat 2.2 | 97.84% | 7.195  | 0.221  | 0% | 38% | 87% |
|   |                      |                | z3 | 97.84% | 0.079  | 0.078  | 0% | 75% | 86% |
| 0.3 | conv-small           | Ternary         | MiniSatCS | 96.18% | 704.245 | 0.376  | 18% | 12% | 73% |
|   |                      | BinMask         | MiniSatCS | 95.53% | 0.002  | 0.002  | 0% | 0% | 83% |
|   |                      |                | MiniSat 2.2 | 95.53% | 0.519  | 0.377  | 0% | 0% | 83% |
|   |                      |                | z3 | 95.53% | 0.062  | 0.061  | 0% | 0% | 83% |
|   | conv-large           | Ternary         | MiniSatCS | 98.35% | 2811.341 | 3600.003 | 78% | 0% | 87% |
|   |                      | BinMask         | MiniSatCS | 99.01% | 2200.503 | 3600.002 | 60% | 5% | 72% |
|   |                      |                | BinMask+CBD | 97.05% | 0.318  | 0.021  | 0% | 25% | 88% |
|   |                      |                | MiniSat 2.2 | 97.05% | 159.477 | 6.248  | 3% | 25% | 88% |
|   |                      |                | z3 | 97.05% | 219.558 | 1.482  | 0% | 25% | 88% |

All the methods are evaluated on a fixed subset containing 40 randomly sampled examples from the MNIST test set. Time limit is 3600 seconds.

Table 7. Comparison of Methods on CIFAR10 Subset

| ϵ | Network Architecture | Training Method | Solver | Test Accuracy | Mean Solve Time | Median Solve Time | Timeout | Verifiable Accuracy | Overall Sparsity |
|---|----------------------|-----------------|--------|---------------|-----------------|-----------------|--------|-------------------|-----------------|
| 0 | conv-small           | Ternary         | MiniSatCS | 53.72% | 64.366 | 0.009  | 0% | 0% | 83% |
|   |                      | BinMask         | MiniSatCS | 53.72% | 352.748 | 93.883 | 3% | 0% | 83% |
|   |                      |                | MiniSat 2.2 | 53.72% | 1612.839 | 870.972 | 25% | 0% | 83% |
|   |                      |                | z3 | 53.72% | 0.034  | 0.003  | 0% | 2% | 81% |
|   | conv-large           | Ternary         | MiniSatCS | 66.15% | 774.398 | 2.506  | 18% | 0% | 89% |
|   |                      | BinMask         | MiniSatCS | 65.85% | 235.996 | 0.171  | 3% | 0% | 91% |
|   |                      |                | BinMask+CBD | 65.15% | 19.585 | 0.095  | 0% | 0% | 93% |
|   |                      |                | MiniSat 2.2 | 65.15% | 865.934 | 378.570 | 5% | 0% | 93% |
|   |                      |                | z3 | 65.15% | 3600.195 | 3600.190 | 100% | 0% | 93% |
| 0.3 | conv-small           | Ternary         | MiniSatCS | 35.84% | 0.305  | 0.005  | 0% | 0% | 89% |
|   |                      | BinMask         | MiniSatCS | 33.18% | 0.002  | 0.021  | 0% | 8% | 88% |
|   |                      |                | MiniSat 2.2 | 33.18% | 0.718  | 0.710  | 0% | 8% | 88% |
|   |                      |                | z3 | 33.18% | 0.071  | 0.070  | 0% | 8% | 88% |
|   | conv-large           | Ternary         | MiniSatCS | 39.27% | 112.048 | 0.359  | 3% | 0% | 82% |
|   |                      | BinMask         | MiniSatCS | 42.35% | 3.343  | 0.111  | 0% | 0% | 68% |
|   |                      |                | BinMask+CBD | 42.35% | 0.048  | 0.044  | 0% | 2% | 81% |
|   |                      |                | MiniSat 2.2 | 42.35% | 13.175 | 9.460  | 0% | 2% | 81% |
|   |                      |                | z3 | 42.35% | 152.297 | 26.479 | 0% | 2% | 81% |

All the methods are evaluated on a fixed subset containing 40 randomly sampled examples from the CIFAR10 test set. Time limit is 3600 seconds.

Our proposed training methods, BinMask and Cardinality Bound Decay (CBD), work together to significantly reduce verification time at the cost of some small degradation of test accuracy. Specifically, compared to ternary weights, BinMask with CBD delivers verification speedup by a factor of between 5.48 to 500.76 times compared to the fastest of the other two. A positive side effect of BinMask is that it improves robustness, possibly due to improved sparsity of convolutional layers. Note that CBD works better with BinMask, and combining ternary weights with CBD results in worse accuracy and longer verification time of the conv-large architecture.
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Table 8. MiniSatCS Results on Full Dataset

| Dataset | Network Architecture | Training Method | Test Accuracy | Mean Solve Time | Median Solve Time | Timeout | Verifiable Accuracy | Overall Sparsity |
|---------|----------------------|-----------------|---------------|-----------------|-------------------|---------|---------------------|-----------------|
| MNIST   | conv-small           | BinMask         | 97.06%        | 0.002           | 0.001             | 0.00%   | 75.29%             | 86%             |
|         | conv-large           | BinMask+CBD     | 97.84%        | 9.661           | 0.216             | 5.09%   | 39.27%             | 87%             |
| MNIST   | conv-small           | BinMask         | 95.53%        | 0.002           | 0.002             | 0.00%   | 36.41%             | 91%             |
| 0.3     | conv-large           | BinMask+CBD     | 97.05%        | 2.322           | 0.028             | 1.11%   | 21.52%             | 88%             |
| CIFAR10 | conv-small           | BinMask         | 51.67%        | 0.004           | 0.003             | 0.00%   | 0.36%              | 81%             |
| 0       | conv-large           | BinMask+CBD     | 65.15%        | 14.336          | 1.088             | 6.64%   | 0.00%              | 93%             |
| CIFAR10 | conv-small           | BinMask         | 33.18%        | 0.002           | 0.002             | 0.00%   | 10.79%             | 88%             |
| 8/255   | conv-large           | BinMask+CBD     | 42.35%        | 0.057           | 0.043             | 0.00%   | 0.15%              | 81%             |

All the methods are evaluated on the complete test sets. Time limit is 120 seconds.

Because of the relatively long verification times for some of the cases with other solvers or ternary weights, we evaluate all the methods on a fixed subset containing 40 randomly sampled examples from the complete test set and summarize the results in Table 6 and Table 7 for MNIST and CIFAR10 respectively. We evaluate MiniSat 2.2 and z3 on the easiest-to-verify models that are trained with BinMask or BinMask with CBD, but we also run the two solvers on a small ternary weight model for CIFAR10, to show that BinMask also benefits other solvers and MiniSatCS is also more efficient in the ternary weight case. In fact CBD also helps the other solvers, because when we try to verify the seemingly easiest-to-verify conv-largest network trained with only BinMask (i.e., the one adversarially trained on CIFAR10), MiniSat 2.2 fails due to out of memory error, and z3 exceeds the one hour time limit. We also present the corresponding results for MiniSat 2.2 and z3 on MNIST.

7. Conclusion

In this work we demonstrate that it is possible to significantly scale up the exact verification of binarized neural networks (BNNs) by equipping an off-the-shelf SAT solver with domain-specific propagation rules and simultaneously training solver-friendly BNNs. Although we focus on verifying adversarial robustness, our method could be generalized to verify other properties of BNNs. Our experimental results demonstrate the significant performance increases that our techniques deliver.

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