One-loop Weak Dipole Form Factors and Weak Dipole Moments of Heavy Fermions

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Abstract

The one–loop weak–magnetic and weak–electric dipole form factors of heavy fermions in a generic model are derived. Numerical predictions for the \( \tau \) lepton and \( b \) quark Weak Anomalous Magnetic and Electric Dipole Moments (AWMDM and WEDM) in the SM and MSSM are reviewed. The MSSM contribution to the \( \tau \) (\( b \)) AWMDM could be, in the high \( \tan \beta \) scenario, four (thirty) times larger than the Electroweak SM one, but still a factor five below the QCD contribution (in the \( b \) case). More interesting is the CP–odd sector where the contribution to the \( \tau \) (\( b \)) WEDM in the MSSM could be up to twelve orders of magnitude larger than in the SM.

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**V ff effective vertex for on–shell fermions**

The most general V ff effective vertex describing the interaction between a neutral vector boson and two on–shell fermions can be conventionally written in terms of six independent form factors as:

\[
\Gamma_{\mu}^{Vff}(s) = i\left\{ \gamma_\mu \left[ F^V_\gamma - F^V_\lambda \gamma_5 \right] - (q + \bar{q})_\mu \left[ iF^V_\Sigma + F^V_\pi \gamma_5 \right] + \sigma_{\mu\nu}(q + \bar{q})^{\nu} \left[ iF^V_\rho + F^V_\bar{E} \gamma_5 \right] \right\}.
\]  

(1)

Here \( q \) and \( \bar{q} \) are respectively the outgoing momenta of the fermion and the antifermion and \( s \) is the square of the total momentum \( p = q + \bar{q} \). The form factors \( F^V_i \) are, in general, functions of the total energy \( s \) and of all the other possible kinematic invariants of the process. \( F^V_\gamma \) and \( F^V_\lambda \) are the usual vector and axial–vector form factors. Being related to \( D = 4 \) operators they are the only terms that can appear, at tree–level, in the Lagrangian of a renormalizable theory. \( F^V_\Sigma(s) \) and \( F^V_\pi(s) \) are the so–called scalar and pseudo–scalar form factors. They are usually negligible. Finally \( F^V_\rho(s) \) and \( F^V_\bar{E}(s) \) are known as magnetic and electric form factors. The Anomalous Magnetic Dipole Moment (AMDM) and the Electric Dipole Moment (EDM), associated to a neutral vector boson \( V \), are defined as:

\[
a^V_f = \frac{2m_f}{e} F^V_M(s = M_V^2) \quad \text{and} \quad d^V_f = -F^V_E(s = M_V^2).
\]  

(2)

Here \( e \) is the electron charge, \( m_f \) and \( M_V \) are the fermion and boson masses. If \( V = \gamma \) (\( M_\gamma = 0 \)) Eq. 2 reproduces the usual definitions of the photon AMDM and EDM. For \( V = Z \) Eq. 2 defines the Anomalous Weak Magnetic Dipole Moment (AWMDM = \( a^w_f \)) and the Weak Electric Dipole Moment (WEDM = \( d^w_f \)). Although the formulation could be completely general in the following we concentrate our analysis on the Weak Dipole Form Factors (WDFFs).

**One–loop generic expressions of the Weak Dipole Form Factors**

All the possible one–loop contributions to the \( a^Z_f(s) \) and \( d^Z_f(s) \) form factors can be classified in terms of the six classes of triangle diagrams depicted in Fig. (1). The vertices are labelled by generic couplings, according to the following interaction Lagrangian, for vector bosons \( V^{(k)}_\mu = A_\mu, Z_\mu, W_\mu, W^\dagger_\mu \), fermions \( \Psi_k \) and scalar bosons \( \Phi_k \):

\[
\mathcal{L} = ieJ(W^\dagger_{\mu\nu}W^\mu Z^\nu - W^\mu W^\dagger_{\mu\nu}Z_\nu + Z^{\mu\nu}W^\dagger_{\mu\nu}W_\nu) + eV^{(k)}_\mu \bar{\Psi}_j \gamma^\mu(V^{(k)}_j - A^{(k)}_{jl}\gamma_5)\Psi_l

+ \left\{ e\bar{\Psi}_j(S_{jk} - P_{jk}\gamma_5)\Psi_k \Phi_j + eK_{jk}Z^{\mu}\Psi_{k}^{(k)}\Phi_j + \text{h.c.} \right\} + ieG_{jk}Z^{\mu}\Phi_j^\dagger \partial_{\mu} \Phi_k.
\]  

(3)

Every class of diagrams is calculated analytically and expressed in terms of the couplings introduced in (3) and the one–loop three–point integrals. The computation is done in the ‘t
Figure 1: One-loop topologies for a general $V f f$ effective vertex.

Hooft-Feynman gauge. Similar expressions are also derived in Ref.[1,2,3].

- [Class I]: vector boson exchange contribution.

\[
\frac{a^2}{2m_f} \langle I \rangle = \frac{\alpha}{4\pi} \left\{ 4 \sum_{jkl} m_k \text{Re} \left[ V_{jk}^{(Z)} (V_{fj}^{(l)} V_{fl}^{(k)*) - A_{fj}^{(l)} A_{fl}^{(k)*}) - A_{jk}^{(Z)} (V_{fj}^{(l)} A_{fl}^{(k)*} - A_{fj}^{(l)} V_{fl}^{(k)*)} \right] [2C_1^+ - C_0]_{kjl} \right. \\
+ 4m_f \sum_{jkl} \text{Re} \left[ V_{jk}^{(Z)} (V_{fj}^{(l)} V_{fl}^{(k)*)} + A_{fj}^{(l)} A_{fl}^{(k)*}) + A_{jk}^{(Z)} (V_{fj}^{(l)} V_{fl}^{(k)*)} + A_{fj}^{(l)} A_{fl}^{(k)*}) \right] \left[ 2C_2^+ - 3C_1^+ + C_0 \right]_{kjl} \right\},
\]

(4)

\[
\frac{d^2}{e} \langle I \rangle = \frac{\alpha}{4\pi} \left\{ 4m_f \sum_{jkl} \text{Im} \left[ V_{jk}^{(Z)} (V_{fj}^{(l)} A_{fl}^{(k)*} + A_{fj}^{(l)} V_{fl}^{(k)*)} + A_{jk}^{(Z)} (V_{fj}^{(l)} V_{fl}^{(k)*)} + A_{fj}^{(l)} A_{fl}^{(k)*}) \right] [2C_1^+ - C_0]_{kjl} \right\}.
\]

(5)

A particular but relevant subcase of Class I is the gluon exchange contribution. The derivation of the corresponding formula is straightforward once one performs the substitutions $\alpha \to \alpha_s$, $V_{fj}^{(l)} \to T_i$ (the $SU(3)$ generators) and $A_{fj}^{(l)} \to 0$.

- [Class II]: fermion exchange contribution with two internal vector bosons.

\[
\frac{a^2}{2m_f} \langle II \rangle = \frac{\alpha}{4\pi} \left\{ 2m_f \sum_{jkl} \text{Re} \left[ J(V_{fj}^{(j)} V_{fl}^{(k)*} + A_{fl}^{(j)} A_{fl}^{(k)*}) \right] [4C_1^+ + C_1^+]_{kjl} \right. \\
- 6 \sum_{jkl} m_l \text{Re} \left[ J(V_{fj}^{(j)} V_{fl}^{(k)*} - A_{fl}^{(j)} A_{fl}^{(k)*}) \right] [C_1^+]_{kjl} \right\},
\]

(6)

\[
\frac{d^2}{e} \langle II \rangle = -\frac{\alpha}{4\pi} \left\{ 2m_f \sum_{jkl} \text{Im} \left[ J(V_{fj}^{(j)} A_{fl}^{(k)*} + A_{fl}^{(j)} V_{fl}^{(k)*)} \right] [4C_2^+ - C_1^-]_{kjl} \right. \\
+ 6 \sum_{jkl} m_l \text{Re} \left[ J(V_{fj}^{(j)} A_{fl}^{(k)*} - A_{fl}^{(j)} V_{fl}^{(k)*)} \right] [C_1^+]_{kjl} \right\}.
\]

(7)
- [Class III]: scalar boson exchange contribution.

\[
\frac{a_f^Z}{2m_f}^{(III)} = \frac{\alpha}{4\pi}(2m_f \sum_{jkl} \text{Re}[V_{jik}^{(Z)}(S_{jik} S_{ik}^* + P_{jik} P_{ik}^*) + A_{jik}^{(Z)}(S_{jik} P_{ik}^* + P_{jik} S_{ik}^*)][2C_2^+ - C_1^+])_{kjl} - 2 \sum_{jkl} m_k \text{Re}[V_{jik}^{(Z)}(S_{jik} S_{ik}^* - P_{jik} P_{ik}^*) - A_{jik}^{(Z)}(S_{jik} P_{ik}^* - P_{jik} S_{ik}^*)][C_1^+ + C_1^-])_{kjl},
\]

(8)

\[
\frac{d_f^Z}{e}^{(III)} = \frac{\alpha}{4\pi}(2m_f \sum_{jkl} \text{Im}[V_{jik}^{(Z)}(P_{jik} S_{ik}^* + S_{jik} P_{ik}^*) + A_{jik}^{(Z)}(S_{jik} S_{ik}^* + P_{jik} P_{ik}^*)][2C_2^- - C_1^-])_{kjl} + 2 \sum_{jkl} m_k \text{Im}[V_{jik}^{(Z)}(P_{jik} S_{ik}^* - S_{jik} P_{ik}^*) + A_{jik}^{(Z)}(S_{jik} P_{ik}^* - P_{jik} S_{ik}^*)][C_1^+ + C_1^-])_{kjl}.
\]

(9)

- [Class IV]: fermion exchange contribution with two internal scalar bosons.

\[
\frac{a_f^Z}{2m_f}^{(IV)} = -\frac{\alpha}{4\pi}(2m_f \sum_{jkl} \text{Re}[G_{jik}(S_{jik} S_{ik}^* + P_{jik} P_{ik}^*)][2C_2^+ - C_1^+])_{kjl} + \sum_{jkl} m_k \text{Re}[G_{jik}(S_{jik} S_{ik}^* - P_{jik} P_{ik}^*)][2C_1^+ - C_0^-])_{kjl},
\]

(10)

\[
\frac{d_f^Z}{e}^{(IV)} = -\frac{\alpha}{4\pi}(2m_f \sum_{jkl} \text{Im}[G_{jik}(S_{jik} P_{ik}^* + P_{jik} S_{ik}^*)][2C_2^- - C_1^-])_{kjl} - \sum_{jkl} m_k \text{Im}[G_{jik}(S_{jik} P_{ik}^* - P_{jik} S_{ik}^*)][2C_1^+ - C_0^-])_{kjl}.
\]

(11)

- [Class V+VI]: fermion exchange contribution with one vector and one scalar internal boson.

\[
\frac{a_f^Z}{2m_f}^{(V+VI)} = \frac{\alpha}{4\pi}2 \sum_{jkl} \text{Re}[K_{jik}(V_{jl}^{(k)} S_{jl}^* + A_{jl}^{(k)} P_{jl}^*)][C_1^+ + C_1^-])_{kjl},
\]

(12)

\[
\frac{d_f^Z}{e}^{(V+VI)} = -\frac{\alpha}{4\pi}2 \sum_{jkl} \text{Im}[K_{jik}(V_{jl}^{(k)} P_{jl}^* + A_{jl}^{(k)} S_{jl}^*)][C_1^+ + C_1^-])_{kjl}.
\]

(13)

In the Eqs. (4–13) the shorthand notation $[C]_{kjl}$ stands for $C(-\bar{q}, q, M_k, M_j, M_l)$. The definition of the $C$ integrals used and the relations with the conventional set of three-point function integrals can be found in [4]. All the expressions (4–13) are, at least, proportional to a fermion mass (either internal or external), consistently with the chirality flipping character of the dipole integrals used and the relations with the conventional set of three-point function integrals. In class $V$ and $VI$ diagrams the fermion mass proportionality arises in the product of the Yukawa couplings $S$ and $P$ and the mass-dimension term $K$. From this fact follows that heavy fermions seem, in general, to be the best candidates for an experimental analysis. Hence, for on-shell $Z$ bosons, the $b$ quark and $\tau$ lepton are the most promising options. Eqs. (4–13) show also that, in general, the DMs for massless fermion are not vanishing but proportional to final fermion masses running in the loop. The SM cancellation of the massless neutrino DMs is only “accidental”. Finally notice that all the contributions to the WEFFs are proportional to the imaginary part of a certain combination of couplings. A theory with only real couplings has, manifestly, vanishing (W)EFs.
The $\tau$ and $b$ WDMs in the SM

Our numerical evaluation of the SM $\tau$ and $b$ AWMDM are in perfect agreement with Ref. [5]. Taking as input parameters $m_\tau = 1.777$ GeV, $m_b = 4.5$ GeV, $m_t = 175$ GeV, $M_Z = 91.19$ GeV, $s_W^2 = 0.232$, $\alpha = 1/128$ and $\alpha_s = 0.118$, the pure electroweak contribution are $a^{\tau(\text{EW})}_w = (2.10 + 0.61 i) \times 10^{-6}$ and $a^b_w(\text{EW}) = ([1.1; 2.0; 2.4] - 0.2 i) \times 10^{-6}$ for three different values of the Higgs mass (respectively $M_{H^0} = M_Z$, $2M_Z$ and $3M_Z$). $V_{tb}$ equal to one and off-diagonal entries equal to zero are assumed in the CKM matrix. But also QCD affects the $Zb\bar{b}$ vertex at one loop. The gluon exchange dominates being almost a factor one hundred larger than the weak contributions. The whole SM $b$ AWMDM is then $a^b_w[\text{SM}] = (-2.98 + 1.56 i) \times 10^{-4}$.

The only phase present in the SM, the $\delta_{CKM}$, it is not sufficient for generating one-loop contributions to the (W)EDM\[1\]. It is also proved that such CP violating terms cannot appear even at two-loop level. A very crude estimate of the three-loop SM contribution, using simple power counting arguments, gives the indicative limits for the $\tau$ and $b$ (W)EDM: $d^{\tau}/\mu_\tau \leq 1.7 \times 10^{-19}$ and $d^b/b \leq 1.4 \times 10^{-18}$. The “magnetons” $\mu_\tau = 1.7 \times 10^{-15}$ and $\mu_b = 0.7 \times 10^{-15}$ ecm are useful for rendering the (W)EDMs dimensionless.

The $\tau$ and $b$ WDMs in the MSSM

The particle content of the MSSM comes from the SM spectrum with two substantial modifications: the enlargement of the Higgs sector from one to two doublets and the appearance of all the SUSY partners of the standard particles. In a R–parity conserving formulation the sets of new “genuine” MSSM diagrams are: i) diagrams with the MSSM two-doublet Higgs sector; ii) diagrams with charginos (and sfermions); iii) diagrams with neutralinos (and sfermions); iv) diagrams with gluinos (and sfermion) in the $b$ case.

The MSSM contribution to the imaginary part of the $\tau$ or $b$ AWMDM is generally negligible. Only strong MSSM threshold effects (typically originated by light neutralinos ) can produce contributions comparable to the SM ones.

The real part of the $\tau$ AWMDM is dominated by the chargino contribution and is roughly proportional to $\tan \beta$. Neutralino and Higgs sector contributions are negligible in most of the experimentally allowed MSSM parameter space [1]. The total contribution can reach the value of $|\text{Re}(a^\tau_w[\text{MSSM}])| \sim 0.5 (8) \times 10^{-6}$ for $\tan \beta = 1.6 (50)$, so being, in the most favourable case, four times larger than in the SM.

The most important MSSM contributions to the real part of the $b$ AWMDM is provided by charginos and gluinos in the high $\tan \beta$ scenario. The total contribution to the real part can

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\[1\] We implicitly assume here vanishing $\theta_{QCD}$ phase.
reach the value of $|\text{Re}(a^w[MSSM])| \sim 2 \times 10^{-6}$ for $\tan \beta = 1.6 (50)$. The high tan $\beta$ values are one order of magnitude higher than the pure electroweak SM contribution, but still a factor five below the standard QCD contribution [1].

A full scan of the MSSM parameter space has been performed in search for the maximum effect on the WEDM of the $\tau$ lepton and the $b$ quark [3]. The Higgs sector does not contribute and chargino diagrams are more important than neutralino ones. Gluinos are also involved in the $b$ case and compete in importance with charginos. In the most favourable configuration of CP-violating phases and for values of the rest of the parameters still not excluded by experiments, these WEDMs can be as much as twelve orders of magnitude larger than the SM predictions $|\text{Re}(d^w_\tau)| \lesssim 0.2 (6) \times 10^{-6} \mu_\tau$ and $|\text{Re}(d^w_b)| \lesssim 2 (35) \times 10^{-6} \mu_b$. There may be a contribution to the imaginary part if the neutralinos are light but this contribution is at least one order of magnitude less than the real part of the $\tau$ or bottom WEDM.

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