Dynamics of glass phases in the three-dimensional gauge glass model

Q. H. CHEN

Center for Statistical and Theoretical Condensed Matter Physics, Zhejiang Normal University, Jinhua 321004, PRC and
Department of Physics, Zhejiang University - Hangzhou 310027, PRC

received 5 June 2008; accepted in final form 5 November 2008
published online 17 December 2008

PACS 47.32.C- – Vortex dynamics
PACS 64.70.Q– – Theory and modeling of the glass transition
PACS 68.35.Rh – Phase transitions and critical phenomena

Abstract – Large-scale simulations have been performed in the current-driven three-dimensional XY gauge glass model with resistively-shunted-junction dynamics. It is observed that the linear resistivity at low temperatures tends to zero, providing a strong evidence of a finite-temperature glass transition. Dynamical-scaling analysis demonstrates that a perfect collapse of current-voltage data can be achieved. The obtained critical exponents agree quite well with those in equilibrium Monte Carlo simulations. A genuine continuous depinning transition is found at zero temperature. For low-temperature creep motion, critical exponents are evaluated, and a non-Arrhenius creep motion is observed in the glass phase.

Introduction. – The phase diagram in high-$T_c$ cuprates has attracted considerable attention both experimentally and theoretically [1,2] in the past two decades. For weak disorder, a dislocation-free Bragg glass with quasi-long-range order was proposed [3], which was observed convincingly in a neutron experiment [4]. It undergoes a first-order melting to the vortex liquid. The situation is much less clear, however, for strong disorder. Analogous to the spin-glass [5], Fisher et al. suggested that the system freezes into a genuine thermodynamic vortex glass phase, full of dislocation loops [6,7]. It is a true superconducting state with vanishing linear resistivity by diverging energy barriers. The evidences to support the existence of a vortex glass phase have been reported in many experiments by the dynamic scaling of the measured current-voltage data [8,9]. However, the conclusion remains very controversial to date [10–12].

The gauge XY glass model [13], where the disorder is introduced as a random vector potential added to the phase difference of the superconducting order parameter, has been extensively employed to study the vortex glass transition. In three dimensions (3D), there is a general consensus that the gauge glass model exhibits the finite-temperature glass transition [14–16], although the value of the stiffness exponent is not mutually consistent. Interestingly, the generally accepted value for the correlation length critical exponent $\nu \approx 1.39$ in the 3D gauge glass model is close to those obtained in the vortex glass transition in certain recent experiments [8,9], suggesting a common universality class.

To the best of our knowledge, all the previous works in 3D gauge glass model are subject to Monte Carlo simulations without external currents, the dynamical simulations in the presence of applied currents are still lacking to date, which are more relevant to the experiments. As is well known, in the random pinning environment, the energy landscape for the vortex motion is highly nontrivial. The theoretical understanding for the nonlinear dynamics response has been advanced in many years [17]. But due to the dislocations in the vortex glass phase, the full theoretical study is still very challenging. Therefore, computer simulations are expected to provide useful insights complementary to the previous analytic works.

In this paper, by resistively-shunted-junction (RSJ) dynamics, we perform large-scale dynamical simulations in the 3D gauge glass model, both the glass transition temperature $T_g$ and the critical exponents are estimated. The depinning transition at zero temperature and creep motion far below $T_g$ are also investigated. The rest of the paper is organized as follows. The second section describes the model and dynamic method. The third section presents our main results, where some discussions
are also performed. Finally, a short summary is given in the last section.

**Model and dynamic method.** – The Hamiltonian of the 3D XY gauge glass model is given by [13]

\[ H = -J_0 \sum_{i,j} \cos(\phi_i - \phi_j - A_{ij}), \]  

(1)

where the sum is over all nearest-neighbor pairs on a 3D square lattice, \( \phi_i \) specifies the phase of the superconducting order parameter on grain \( i \), \( J_0 \) denotes the strength of Josephson coupling between neighboring grains, and the quenched variable \( A_{ij} \) is distributed uniformly on the interval \([−\pi, \pi]\). The present simulations are performed with the system size \( L = 64 \) for all directions, considerably larger than those in literature.

We consider a particular dynamic model, i.e., the RSJ dynamics. In this case, the current through a nearest-neighbor link (or bond) from grain \( i \) to grain \( j \) contains three contributions,

\[ J_{ij} = J_{ij}^{(s)} + J_{ij}^{(n)} + \eta_{ij}, \]  

(2)

where \( J_{ij}^{(s)} = (2eJ_0/\hbar) \sin(\phi_i - \phi_j) \) is the Josephson supercurrent, \( J_{ij}^{(n)} = \sigma(V_i - V_j) \) is the normal current with \( \sigma \) the conductance of the resistive shunt, and \( \eta_{ij} \) is a thermal noise current, independent from bond to bond, with zero mean and a correlator \( \langle \eta_{ij}(t)\eta_{i'j'}(t') \rangle = 2\sigma k_B T \delta(t - t')\delta_{ij,i'j'} \). The voltage on grain \( i \) is given by \( V_i = (\hbar/2e)d\phi_i/dt \).

The dynamical equations for the \( \phi \)'s are readily derived by requiring the sum of currents into each grain to vanish. The conservation law is satisfied, so RSJ dynamics may be close to the real dynamics. The RSJ dynamics can be there fore described as

\[ \frac{\sigma h}{2e} \sum_{j \in \text{nn of } i} (\frac{d\phi_j}{dt} - \frac{d\phi_i}{dt}) = -\frac{2e}{\hbar} \frac{\partial H}{\partial \phi_i} + J_{\text{ext},i} - \sum_{j \in \text{nn of } i} \eta_{ij}. \]  

(3)

Here \( J_{\text{ext},i} \) is the external current which vanishes except for the boundary grains. In the following, the units are taken as \( 2e = J_0 = \hbar = \sigma = k_B = 1 \).

In the present simulation, a uniform external current \( I_\alpha \) along the \( x \)-direction is fed into the system. The fluctuating twist boundary condition [18] is applied in the \( xy \)-plane to maintain the current, and the periodic boundary condition is employed in the \( z \)-axis. In the \( xy \)-plane, the supercurrent between sites \( i \) and \( j \) is now given by \( J_{ij}^{(s)} = L_{ij} \sin(\theta_i - \theta_j - A_{ij} - r_{ij} \cdot \Delta) \), with \( \Delta = (\Delta_x, \Delta_y) \) the fluctuating twist variable, \( \theta_i = \phi_i + r_i \cdot \Delta \), and \( r_{ij} = r_i - r_j \). Since the new phase angle \( \theta_i \) is periodic, we obtain the voltage difference across the sample \( V_\alpha = -L \Delta_\alpha, \alpha = x, y \). To achieve a given flow \( I_\alpha \) through the sample, we require

\[ \Delta_\alpha = \frac{1}{L^2} \sum_{(i,j)_\alpha} [J_{ij}^{(s)} + \eta_{ij}] - I_\alpha, \]  

(4)

\[ \dot{\Delta}_\alpha = \sum_{(i,j)_\alpha} \mathbf{e}_{ij} J_{ij}^{(s)} + \sum_{(i,j)_\alpha} \mathbf{e}_{ij} \eta_{ij} - \mathbf{e}_{ij} I_\alpha. \]  

\[ \dot{\Delta}_\alpha \]

(5)

The above equations can be solved efficiently by a pseudo-spectral algorithm [18] due to the periodicity of phase in all directions. The time stepping is done using a second-order Runge-Kutta scheme with \( \Delta t = 0.05 \). Our runs are typically \((4–8) \times 10^7 \) time steps and the latter half-time steps are for the measurements. For the data points presented in the following figures, the statistical error is smaller or comparable to the symbol size. The detailed procedure in the simulations was described in ref. [18]. Our results are based on one realization of disorder. The present system size is much too larger than those reported in the literature, so it is impossible to perform any serious disorder averaging. Fortunately, a good self-averaging effect is expected for the present very large systems. Similarly, in a recent study in two-dimensional Josephson junction arrays with positional disorder, Um et al. [19] confirmed that a well-converged disorder averaging for the measurement is not necessary, and well-converged data for large systems at a single disorder realization leads to a convincing result. We have performed an additional simulation with a different realization of disorder for further confirmation, and observed qualitatively the same behavior. Actually, the results from dynamic simulations in the 3D XY model in the recent literature were also for a single disorder realization [20,21], mainly due to the large system simulated.

**Simulation results and discussions.** – First, we study the vortex glass phase transition. In the literature [14–16], the transition temperature is found to be in the range 0.4–0.5. The current-voltage characteristic is simulated at various temperatures in the range [0.3, 0.6]. At each temperature, we try to probe the system at a current as low as possible. The voltage is determined when a steady state is reached. Figure 1 demonstrates the resistivity \( R = V/I \) as a function of the current \( I \) at various temperatures. It is clear that, at lower temperatures,
$R$ tends to zero as the current decreases, which follows the fact that there is a true superconducting phase with zero linear resistivity, while $R$ tends to a finite value at higher temperatures, corresponding to an ohmic resistivity in the vortex liquid. These observations provide a strong evidence of the existence of the vortex glass phase in the 3D gauge glass model in the dynamical sense.

Assuming that the vortex glass transition is continuous and characterized by the divergence of the characteristic length and time scales $t \sim \xi^z$ ($z$ is the dynamic exponent), Fisher, Fisher, and Huse [7] proposed the following dynamic-scaling ansatz to analyze the glass transition from a vortex liquid with ohmic resistance to a superconducting glass state,

$$TR^{d-2-z} = \Psi_d((Ic/I)^{(d-1)/T}),$$

where $d$ is the dimension of the system ($d = 3$ in this paper), and $\xi \propto [T/T_g]^{-1/\nu}$ is the correlation length which diverges at the transition. $\Psi(x)$ is a scaling function. The + and − signs correspond to $T > T_g$ and $T < T_g$. Equation (5) was often used to scale measured current-voltage data experimentally [8,9].

To extract the critical behavior from the numerical results of the current-voltage characteristics, we will also perform a dynamical-scaling analysis. As shown in fig. 2, using $T_g = 0.46 \pm 0.02$, $z = 4.05 \pm 0.15$, and $\nu = 1.4 \pm 0.1$, an excellent collapse is achieved according to eq. (5). The obtained $T_g$ and static exponent $\nu$ are consistent well with those in equilibrium Monte Carlo simulations [15,16].

The dynamical exponent is closer to $z = 4.2 \pm 0.6$ in ref. [15] than $z = 4.7 \pm 0.1$ in [16]. The finite-size effect is particularly significant at temperatures sufficiently close to $T_g$ when the correlation length exceeds the system size. The temperatures at which the current-voltage characteristics are used in fig. 2 are not very close to $T_g$, so it is expected that the finite-size effect is negligible in the present simulations in a very large system $L = 64$. This is confirmed by no deviations from the scaling shown in fig. 2. The convexity-concavity behavior in the $RI$ curves at these temperatures is clearly shown in fig. 1, which may demonstrate that the asymptotic scaling regime is reached. Therefore, a new evidence of a finite-temperature glass transition is provided in the 3D gauge glass model in the dynamical sense.

In the vortex glass prediction, the resistivity just above the transition should scale as $R \propto \left(\frac{T}{T_g}\right)^{d-2-z}$, where $s = \nu(z - 1)$ is the resistivity exponent. In the present model, the obtained static and dynamic exponents give $s \approx 4.27$, which compares well with $s \approx 4.0$ found in the cubic (K,Ba)BiO$_3$ superconductor, $s \approx 5.3$ in untwined proton-irradiated YBa$_2$Cu$_3$O$_{7-\delta}$, and $s \approx 5.12$ in low-$T_c$ superconducting Nb thin films [9]. So we propose that this model gives an effective description of the transport properties near the vortex glass transition in various superconductors.

With the low-temperature glass phase in hand, in the remaining part of the paper, we will study the depinning and creep phenomena in this phase.

To study the depinning transition at zero temperature, we start from high currents with random initial phase configurations. The current is then lowered step by step. The steady-state phase configuration obtained at higher currents is chosen to be the initial phase configurations of the lower currents in the next step. It becomes more difficult to measure the voltage with the lower currents. In the vicinity of the critical current, a huge amount of computer time is consumed to get accurate results. Figure 3 presents the current-voltage characteristics at $T = 0$. Interestingly, we observe continuous depinning transitions with unique depinning currents [22], which can be described as $V \propto (I - I_c)^3$ with $I_c = 0.125 \pm 0.001$, $\beta = 2.069 \pm 0.02$. Note that the depinning exponent $\beta$ is greater than 1, consistent with the mean-field studies on charge density wave models [22].

At low temperatures, the current-voltage characteristics is rounded near the zero-temperature critical current due to thermal fluctuations, indicating a crossover between

---

Fig. 2: Dynamic scaling of current-voltage data at various temperatures according to eq. (5).

Fig. 3: (Colour on-line) Log-log plots of the $V$-$I$ curve at zero temperatures.
the depinning and creep motion around $I_c$ at the lowest accessible temperatures. In order to address the thermal rounding of the depinning transition, Fisher first suggested to map this system to the ferromagnet in fields where the second-order phase transitions occur. This mapping was latter extended to the random-field Ising model [23] and flux lines in type-II superconductors [24]. If the voltage is identified as the order parameter, the current and temperature are identified as the inverse temperature and the field in the ferromagnetic system, respectively, analogous to the second-order phase transitions, a scaling relation among the voltage, current, and temperature in the present model should satisfy the following form:

$$V(T, I) = T^{1/\delta} S[T^{-1/\beta}(1 - I_c/I)],$$

(6)

where $S(x)$ is a scaling function.

It is implied in eq. (5) that right at $I = I_c$ the voltage shows a power law behavior $V(T, I = I_c) \propto T^{1/\delta}$, which provides a technique to determine the critical exponent $1/\delta$. The log-log $V$-$T$ curves are plotted in fig. 4 at three currents. We can see that the critical current is between 0.115 and 0.135. The values of voltage at other currents within (0.115, 0.135) can be evaluated by quadratic interpolation. The square deviations from the power law can be calculated. The critical current at which the square deviation is minimum is defined. We obtain $I_c = 0.125 \pm 0.02$, consistent with those obtained at zero temperature. The temperature dependence of voltage at the critical current, which is obtained by interpolations, is also plotted in fig. 4 with a red line, yielding $1/\delta = 1.3328 \pm 0.0010$. It is an accident that the critical current is very close to one of the three currents at which we have actually simulated the $V$-$T$ curves.

With the critical exponent $\delta$ and the critical current $I_c$, we can adjust the depinning exponent $\beta$ to achieve the best-data collapse according to the scaling relation, eq. (6), for $I \leq I_c$. In fig. 5, an optimal data collapse of the current-voltage data at various temperatures below $T_g$ provides an estimate of $\beta = 2.07 \pm 0.01$, which is in excellent agreement with those derived at the $T = 0$ depinning transition. Actually, we can also extract all these critical values directly from the scaling (5) and get the nearly same results for the $I_c$, $\beta$, and $\delta$, which can be in principle confirmed by the good collapse shown in fig. 5.

Moreover, the scaling function with the form $V \propto T^{1/\delta} \exp[A(1 - I_c/I)/T^{\beta \delta}]$ is used to fit well the current-voltage data in the creep regime, which is also listed in the legends of fig. 5. Note that the product of the two exponents $\beta \delta$ describes the temperature dependence of the creeping law. Interestingly, $\beta \delta \approx 1.55$ deviates from unity, demonstrating that the creep law is of non-Arrhenius type. The value of $\beta \delta$ is close to $3/2$, which may motivate further analytical work.

The non-Arrhenius-type creep behaviors have been previously observed in charge density waves [25]. By simulations of the overdamped London-Langevin model, Luo and Hu observed an Arrhenius law for the creep motion with a linearly suppressed energy barrier for strong pinning [24], inconsistent with the present study. In the same model, the stable vortex glass phase is not found [12], whereas within the present 3D gauge glass model, the existence of a stable glass phase is well established through both previous equilibrium studies [14–16] and present dynamical simulations. We believe that the different nature of the phases is the possible reason for the discrepancy.

It is also interesting to note that the combined exponent $\beta \delta$ in this model is close to that for weak pinning in a Bragg glass in the London-Langevin model [24]. The glass phase in the present model is as random as the liquid phase, and essentially different from the Bragg glass with quasi-long-range order. Further work is needed in order to clarify this observation.

Conclusions. – We have performed large-scale dynamical simulations in the 3D gauge glass model within the RSJ dynamics for the first time. The strong evidence

Fig. 4: (Colour on-line) Log-log plots of the $V$-$T$ curves at three currents around $I_c$.

Fig. 5: Scaling plot of the current-voltage data at various temperatures below $T_g$ according to eq. (6).
for the low-temperature glass phase is provided in dynamical sense. By the dynamical-scaling analysis, a perfect collapse of simulated current-voltage data is achieved by using $T_g = 0.46 \pm 0.02$, $z = 4.05 \pm 0.15$, and $\nu = 1.4 \pm 0.1$. These critical values are in good agreement with those in the previous equilibrium Monte Carlo simulations. The obtained resistivity exponent $s \approx 4.27$ is compared well with those observed on various superconductors experimentally, which underlines the significance of this model. We also study the depinning transition at zero temperature and creep motion at low temperatures in detail. A genuine continuous depinning transition is observed and the depinning exponent is evaluated. With the notion of scaling, the critical exponents are estimated, which are consistent with those from independent simulations at zero temperature and at critical current. The value of $\beta \delta$ is close to $3/2$ and the scaling curve is fitted well by an exponential function, suggesting a non-Arrhenius-type creep motion in the glass phase of the 3D gauge glass model.

Finally, it should be pointed out that the results on the glass transition temperature and exponents are complementary and confirmatory of previous studies, although the present technique had not been applied before to the 3D gauge glass model, whereas the results on the depinning and creep transition are obtained for the first time, to the best of our knowledge. Further experimental and theoretical works are clearly motivated.

***

This work was supported by the National Natural Science Foundation of China under Grant Nos. 10574107 and 10774128, PCSIRT (Grant No. IRT0754) in University in China, National Basic Research Program of China (Grant Nos. 2006CB601003 and 2009CB929104) and Zhejiang Provincial Natural Science Foundation under Grant No. Z7080203.

REFERENCES

[1] Blattner G. et al., Rev. Mod. Phys., 66 (1994) 1125; Crabtree G. W. and Nelson D. R., Phys. Today, 50, issue No. 4 (1997) 38.
[2] Nattermann T. and Scheidl S., Adv. Phys., 49 (2000) 607.
[3] Nattermann T., Phys. Rev. Lett., 64 (1990) 2454; Giarmarchi T. and Le Doussal P., Phys. Rev. Lett., 72 (1994) 1530; Phys. Rev. B, 55 (1997) 6577.
[4] Klein T. et al., Nature (London), 413 (2001) 404.
[5] Young P., Spin Glass Random Fields (World Scientific, Singapore) 1998.
[6] Fisher M. P. A., Phys. Rev. Lett., 62 (1989) 1415.
[7] Fisher D., Fisher M. P. A. and Huse D., Phys. Rev. B, 43 (1991) 130.
[8] Koch R. H. et al., Phys. Rev. Lett., 63 (1989) 1511; Koch R. H., Foglietti V. and Fisher M. P. A., Phys. Rev. Lett., 64 (1990) 2586; Gammel P., Schneemeyer L. and Bishop D., Phys. Rev. Lett., 66 (1991) 953.
[9] Klein T. et al., Phys. Rev. B, 58 (1998) 12411; Petrean A. M. et al., Phys. Rev. Lett., 84 (2000) 5852; Villegas J. E. and Vicent J. L., Phys. Rev. B, 71 (2005) 144522.
[10] Vester gren A. et al., Phys. Rev. Lett., 88 (2002) 117004; Lidmar J., Phys. Rev. Lett., 91 (2003) 097001; Olsson P., Phys. Rev. Lett., 91 (2003) 077002.
[11] Strchan D. R. et al., Phys. Rev. Lett., 87 (2001) 067007; Landau I. L. and Ott H. R., Phys. Rev. B, 65 (2002) 064511.
[12] Reichhardt C. et al., Phys. Rev. Lett., 84 (2000) 1994; Bustingorry S. et al., Phys. Rev. Lett., 96 (2006) 027001.
[13] Huse D. A. and Seung H. S., Phys. Rev. B, 42 (1990) R1059; Reger J. D. et al., Phys. Rev. B, 44 (1991) 7147.
[14] Kosterlitz J. M. and Akino N., Phys. Rev. Lett., 81 (1998) 4672; Akino N. and Kosterlitz J. M., Phys. Rev. B, 66 (2002) 054536.
[15] Olsson T. and Young A. P., Phys. Rev. B, 61 (2000) 12467.
[16] Katzgraber H. G. and Campbell I. A., Phys. Rev. B, 69 (2004) 094413; 72 (2005) 014462.
[17] Larkin A. I. and Ovchinnikov Yu. N., J. Low Temp. Phys., 34 (1979) 409; Ioffe L. B. and Vinokur V. M., J. Phys. C, 20 (1987) 6149; Nattermann T., Europhys. Lett., 4 (1987) 1241; Feigel’man M. V. et al., Phys. Rev. Lett., 63 (1989) 2303; Chauve P., Giarmarchi T. and Le Doussal P., Phys. Rev. B, 62 (2000) 6241.
[18] Chen Q. H. and Hu X., Phys. Rev. Lett., 90 (2003) 117005; Phys. Rev. B, 75 (2007) 064504.
[19] Um J., Kim B. J., Minnhagen P., Choi M. Y. and Lee S. I., Phys. Rev. B, 74 (2006) 094516.
[20] Olsson P., Phys. Rev. Lett., 98 (2007) 097001.
[21] Hernández A. D. and Domínguez D., Phys. Rev. Lett., 92 (2004) 117002.
[22] Fisher D. S., Phys. Rev. Lett., 50 (1983) 1486; Phys. Rev. B, 31 (1985) 1396.
[23] Roters L. et al., Phys. Rev. E, 60 (1999) 5202.
[24] Luo M. B. and Hu X., Phys. Rev. Lett., 98 (2007) 267002.
[25] Middleton A. A., Phys. Rev. B, 45 (1992) 9465.