Abstract. Probabilistic models (developed by workers such as Boltzmann, on foundations due to pioneers such as Bayes) were commonly regarded merely as approximations to a deterministic reality before the roles were reversed by the quantum revolution (under the leadership of Heisenberg and Dirac) whereby it was the deterministic description that was reduced to the status of an approximation, while the role of the observer became particularly prominent. The concomitant problem of lack of objectivity in the original Copenhagen interpretation has not been satisfactorily resolved in newer approaches of the kind pioneered by Everett. The deficiency of such interpretations is attributable to failure to allow for the anthropic aspect of the problem, meaning a priori uncertainty about the identity of the observer. The required reconciliation of subjectivity with objectivity is achieved here by distinguishing the concept of an observer from that of a perceptor, whose chances of identification with a particular observer need to be prescribed by a suitable anthropic principle. It is proposed that this should be done by an entropy ansatz according to which the relevant micro-anthropic weighting is taken to be proportional to the logarithm of the relevant number of Everett type branch-channels.
1. Introduction.

As a prescription for ascribing a priori probability weightings to the eventuality of finding oneself in the position of particular conceivable observers, the anthropic principle was originally developed for application to problems of cosmology \(^1\) and biology \(^2\). The purpose of the present article is to provide a self contained introductory account of the motivation and reasoning underlying the recent development \(^3\) of a more refined version of the anthropic principle that is needed for the provision of a coherent interpretation of quantum theory.

In order to describe ordinary laboratory applications, it is commonly convenient, and entirely adequate, to use a “Copenhagen” type representation in terms of a Hilbert state vector that undergoes “collapse” when an observation is made. However from a broader perspective it is rather generally recognised that such a collapse can not correspond to any actual physical process.

A leading school of thought on this subject was founded by Everett \(^4\), who maintained the principle of the physical reality of the Hilbert state, and deduced that – in view of the agreement that no physical collapse process occurs – none of the ensuing branch channels can be “more real than the rest”, despite the paradox posed by the necessity that they be characterised by different (my italics) “weightings”, of a nature that was never satisfactorily explained. This intellectual flaw in the Everett doctrine was commonly overlooked, not so much by its adherents, who were seriously concerned about it \(^5\), as by its opponents, who were upset by its revolutionary “multi-universe” implications.

The main alternative line of development was based on the (widely accepted) principle – which will be adopted as the starting point for the present work – that neither the specialised pure Hilbert space vector, nor the von Neumann probability operator that replaces it under in more general circumstances, is of an objective physical nature, but that they are merely mathematical prediction tools of an entirely subjective nature, as also is the collapse to which they are subjected if and when the relevant information becomes available. However this approach also came up against a paradox, which was exemplified by the parable of “Wigner’s friend” \(^6\) (who, in the more detailed discussion below, I shall suppose to have been Schroedinger, the owner of the legendary cat). The problem – which became particularly acute in the context of cosmology – was how independent observers (such as Wigner and Schroedinger) can be dealt with objectively, on the same footing, by a probabilistic theory of an intrinsically subjective nature.

The longstanding problem of reconciling objectivity with subjectivity is solved here by the anthropic abstraction, which distinguishes a material observer (such as Wigner) from that of an abstract perceptor who may or may not perceive himself to be Wigner. The probability of such a perception must be attributed by some appropriate micro-anthropic principle, of the kind \(^3\) that will be presented below.

2. Eventualities and observables

Although their ultimate purpose is to account for (and even predict) events, i.e. things that actually happen, physical (and other) theories are mainly concerned with what I shall refer to as eventualities, meaning things that may or not actually happen.

Eventualities are subject to partial ordering, as expressible by a statement of the form
$e_1 \subset e_2$, which is to be understood as meaning that if an eventuality $e_1$ happens as an actual event then so does $e_2$. On the understanding that the concept of eventuality formally includes the special case of the null eventuality, $\emptyset$, which by definition never happens, it can be taken that any pair of eventualities $e_1$ and $e_2$ say will define a corresponding combined eventuality $e_1 \cap e_2$ whose occurrence as an actual event implies, and is implied by, the occurrence, both at once, of $e_1$ and $e_2$, so that we always have $e_1 \cap e_2 \subseteq e_1$. In particular, the condition for $e_1$ to be incompatible with $e_2$ will be expressible as $e_1 \cap e_2 = \emptyset$.

The kinds of (classical and quantum) theory that I know about are all additive in the sense that for each pair of eventualities $e_1$ and $e_2$ there will be a well defined sum $e_1 \oplus e_2$ that is an admissible eventuality such that $e_1 \cap (e_1 \oplus e_2) = e_1$. In such a case it is commonly useful to introduce a corresponding concept of complementarity whereby a set \{e\} say of eventualities $e_1, ..., e_N$ will be describable as complementary in cases for which the sum $s = e_1 \oplus ... \oplus e_N$ is an event that must necessarily happen.

An important related concept – on which (though it is less fundamental than that of an eventuality) discussions of quantum theory are commonly based – is that of an observable, a term that is used to describe a set \{e\} of non-null eventualities that is subject to a condition not just of complementarity but also of what may be termed mutual exclusivity. An awkward feature of this concept (one of the reasons why I prefer to attribute the primary role to eventualities rather than to observables) is that it is difficult to formulate in a manner that transcends the technicalities of the particular kind of theory under consideration.

For a theory that is classical, in the sense whose meaning will be recapitulated in the next section, a pair of eventualities $e_1$ and $e_2$ can be considered to be mutually exclusive if they simply satisfy the incompatibility condition $e_1 \cap e_2 = \emptyset$, but for a quantum theory such incompatibility is merely necessary, but not sufficient, for exclusivity in the strong sense – as defined further on below – that is required for what is meant by observability.

3. The classical paradigm

Some of the simplest and most commonly used theories are of the kind describable as deterministic, which means that they consist of rules whereby appropriate input data (such as initial conditions) can be use to single out a restricted subclass of events that actually happen within a much broader class of conceivable eventualities. However a much more widely applicable category of theories consists of those that are probabilistic. Instead of providing rules that clearly distinguish events that happen from other eventualities that do not, such theories merely provide prescriptions for ascribing what is usually called a probability (but what some people prefer to call a propensity) – meaning a real number $P$ in the range $0 \leq P \leq 1$ – to each of the relevant eventualities, in a manner that must naturally be consistent with the partial ordering, so that one has $e_1 \supset e_2 \Rightarrow P\{e_1\} \geq P\{e_2\}$ and in particular $P\{\emptyset\} = 0$. The category of probabilistic theories evidently includes deterministic theories as the special case for which the range of probabilities is restricted to the two extreme values, namely $P = 1$ characterising events, and $P = 0$ characterising other eventualities that do not actually happen.

A particularly important subcategory of probabilistic theories is that of classical theories. In a classical theory for the description of a system, $A$ say, the admissible eventualities will be identifiable as subsets of a corresponding set $I\{A\}$ that is endowed with
an ordinary probability measure whose restriction to a subset, \( e \subset I \), gives the corresponding probability, \( P\{e\} \), while the complete set \( I \) can be interpreted as representing an eventuality that is certain, meaning that \( P\{I\} = 1 \). Such a theory will automatically be endowed with an additive structure whereby any pair of eventualities \( e_1 \) and \( e_2 \) will not only have a combination given by the intersection \( e_1 \cap e_2 \), but it will furthermore have a well defined sum that is defined as the corresponding union \( e_1 \oplus e_2 = e_1 \cup e_2 \) so that (unlike what may happen in a quantum theory) its probability will be given by
\[
P\{e_1 \oplus e_2\} = P\{e_1\} + P\{e_2\} - P\{e_1 \cap e_2\}.
\]

The simplest example of a classical theory applies to the system consisting of a tossed coin, which can be described in terms of a total of four eventualities. Two of these eventualities are the independent possibilities \( e_1 \) say for the tail to turn up, and \( e_2 \) say for the head to turn up, while the other two (trivial) eventualities are their sum \( I = e_1 \oplus e_2 \), representing the certain event of something turning up, and finally of course the null eventuality \( \emptyset = e_1 \cap e_2 \) representing the impossible case of nothing turning up. The latter (trivial) eventualities must always be characterised by \( P\{I\} = 1 \) and \( P\{\emptyset\} = 0 \). The non-trivial part of the probability distribution will be given in the unbiased version of the theory by \( P\{e_1\} = P\{e_2\} = \frac{1}{2} \), but could be different in biased versions. In such a (biased or unbiased) theory the only non trivial observable consists of the complementary pair of alternatives \( \{e\} = \{e_1, e_2\} \), but of course there is also the trivial observable consisting just of \( I \) by itself.

4. The Dirac - von Neumann paradigm

As in a classical theory, the admissible eventualities in a quantum theory for the description of a system, \( A \) say, will be identifiable with subsets of a corresponding set \( I\{A\} \). The essential new feature distinguishing a quantum theory is that \( I \) is endowed with a Hilbert space structure, and that the admissible eventualities are identifiable, not with arbitrary subsets, but only with those that are Hilbert subspaces.

If \( e_1 \) and \( e_2 \) are the Hilbert subspaces representing a pair of admissible eventualities, their intersection \( e_1 \cap e_2 \) will also be a Hilbert subspace, representing the corresponding conjoint eventuality, but their union \( e_1 \cup e_2 \) will in general not have the structure of a Hilbert subspace and thus (unlike the classical case) will not represent an admissible eventuality. The eventualities of a quantum theory do nevertheless have an additive structure that is naturally induced by the Hilbert space structure: the sum \( e_1 \oplus e_2 \) is defined to be the Hilbert subspace that is spanned by the separate Hilbert subspaces \( e_1 \) and \( e_2 \). What this means, using the standard notation scheme originally developed by Dirac [7] (whose lectures on the subject I attended as an undergraduate at Cambridge) is that \(|\Psi\rangle \in e_1 \oplus e_2 \) if and only if \(|\Psi\rangle \) is a Hilbert space vector having the form \(|\Psi\rangle = |\Psi_1\rangle + |\Psi_2\rangle \) for some pair of Hilbert space vectors such that \(|\Psi_1\rangle \in e_1 \) and \(|\Psi_2\rangle \in e_2 \). In the particular case for which every such pair of vectors satisfies the orthogonality condition \( \langle \Psi_1 | \Psi_2 \rangle = 0 \), the corresponding subspaces \( e_1 \subset I \) and \( e_2 \subset I \) will be describable as mutually orthogonal.

Orthogonality in the sense of the preceding paragraph is what characterises the kind of exclusivity required for the definition of what is generally known as an observable in the context of quantum theory. Thus an observable (or to be more precise a qualitative observable, as distinct from a quantitative observable of the related kind to be discussed
below) in a quantum theory for the system $A$ can be formally defined to consist of a complete set $\{e\}$ of mutually orthogonal Hilbert subspaces $e_1, \ldots, e_N$, where the condition of completeness means that they span the entire Hilbert space $I\{A\}$, i.e. that $e_1 \oplus \cdots \oplus e_N = I$.

For any particular eventuality, the corresponding subspace $e \subset I$ will determine and be determined by an associated Hilbert space projection operator $e = e^2$ that is defined – in such a way as to be automatically Hermitean – by the conditions that $e |\Psi\rangle = |\Psi\rangle$ whenever $|\Psi\rangle$ lies in $e$, and that $e |\Psi\rangle = 0$ whenever $|\Psi\rangle$ is orthogonal to the subspace $e$. The condition for a set $\{e\}$ of eventualities, $\{e_i\}$ ($i = 1, \ldots, n$) to constitute an observable is thus expressible as the condition that the corresponding operators should satisfy the orthogonality requirement $e_i e_j = 0$ for $i \neq j$ and that they should satisfy the completeness condition $\sum_i e_i = I$, where $I$ is the unit operator on $I$.

In the earliest versions of quantum theory it was postulated that the relevant probabilities would be given just by the specification of a single state vector $|\Psi\rangle \in I\{A\}$, subject to the normalisation condition $\langle \Psi | \Psi \rangle = 1$, according to a prescription expressible in the familiar form

$$P_{[\mathcal{O}]}\{e_i\} = \langle \Psi | e_i | \Psi \rangle .$$

(1)

It is to be noted that this is just a conditional probability, subject to the requirement that the relevant observation, $\mathcal{O}_e$ say, be actually carried out.

Soon after the original development of this Dirac-Heisenberg paradigm, it came to be recognised that a prescription of the simple form (1) is too restrictive for applicability to typical cases in which the system $A$ under consideration may interact with another (internal or external) system, $B$ say. The extended system $\hat{A}$ say consisting of the combination of $A$ and $B$ will be characterised by a Hilbert space $\hat{I} = I\{\hat{A}\}$ that is constructed as the tensor product of $I\{A\}$ and $I\{B\}$. What this means is that a state vector $|\hat{\Psi}\rangle \in \hat{I}$ for the extended system will be expressible in terms of a basis of vectors $|\Phi_a\rangle \in I\{B\}$ satisfying the orthonormality condition $\langle \Phi_a | \Phi_b \rangle = \delta_{ab}$ in the form

$$|\hat{\Psi}\rangle = \sum_a |\Phi_a\rangle |\Psi_a\rangle$$

(2)

for some corresponding set of vectors $|\Psi_a\rangle \in I\{A\}$ that will not in general be orthonormal, but that must satisfy the condition $\sum_a \langle \Psi_a | \Psi_a \rangle = 1$ in order for the unit normalisation condition $\langle \hat{\Psi} | \hat{\Psi} \rangle = 1$ to be satisfied. If $e_i$ is a subspace of dimension $\mathcal{R}_i$ within the original Hilbert space $I\{A\}$ of dimension $\mathcal{N}\{A\}$ say, then it will determine a corresponding subspace $\hat{e}_i$ of dimension $\mathcal{R}_i \mathcal{N}\{B\}$ in the tensor product Hilbert space $\hat{I}$, where $\mathcal{N}\{B\}$ is the dimension of $I\{B\}$. Within the original Hilbert space $I = I\{A\}$ the corresponding projection operator will have rank given by its trace, namely $\mathcal{R}_i = \text{tr}\{e_i\}$ while the corresponding operator $\hat{e}_i$ of projection onto $\hat{e}_i$ in $\hat{I}$ will have rank $\mathcal{R}_i \mathcal{N}\{B\}$. According to the natural extension of the rule (1), a unit state vector $|\hat{\Psi}\rangle$ in $\hat{I}$ will specify a (conditional) probability distribution given by

$$P_{[\mathcal{O}]}\{e_i\} = \langle \hat{\Psi} | \hat{e}_i | \hat{\Psi} \rangle .$$

(3)

In order to express such a prescription within the simpler framework of the original Hilbert space $I\{A\}$ of the subsystem $A$ with which we are particularly concerned, it is
necessary to use a prescription of the kind whose development was attributed by Dirac to von Neumann.

In the Dirac-von Neumann paradigm, instead of being specified just by a single state vector $|\Psi\rangle$, the (conditional) probability distribution (for the outcome of an observation $O_e$ if actually performed) is specified by a hermitian probability density operator $P$ say with unit trace $\text{tr}\{P\} = 1$ on $I$ according to the prescription

$$P[O_e\{e_i\} = \text{tr}\{P e_i\}.$$ (4)

This prescription is compatible with the original pure state paradigm, as specified just by a single vector satisfying the unit normalisation $\langle \Psi | \Psi \rangle = 1$, according to the formula (1) whose effect can be seen to be the same as that of simply taking $P = |\Psi\rangle\langle\Psi|$ in the general formula (4). The advantage of the von Neumann type formulation (4) is that it can also express the result of the more general prescription (3), whose effect can be seen to be the same as that of taking

$$P = \sum_a |\Psi_a\rangle\langle\Psi_a|,$$ (5)

where the (in general non orthonormal) set of vectors $|\Psi_a\rangle$ is as specified by the decomposition (2).

Many authors – particularly those influenced by the Everett doctrine – have been continued to hanker after the original Heisenberg type paradigm – meaning the supposition that the probabilities should ultimately be determined by a pure state in a very large all embracing Hilbert space characterising the universe as a whole. Such authors – notably including Hawking – have been inclined to regard the use of a von Neumann operator as a rather unsatisfactory approximation device that may be made necessary by our ignorance due to the regrettable loss of some of the relevant information in for example a black hole. However my own attitude is like that of the distrustful insurance agent who doubts whether what was alleged to have been lost was ever actually possessed. I personally see no reason why – to encompass more and more detailed microstructure and more and more extended macrostructure – the process of construction of successively larger and larger Hilbert spaces should ever come to an end. In other words the search for a single ultimate all embracing “Wave function of the universe”, or even of an ultimate all embracing von Neumann operator, may be like the pursuit of the proverbially elusive “Will o’ the wisp”. It seems more reasonable to accept that any system sufficiently simple to be amenable to our mathematical analysis can only be a model of an incomplete subcomponent of something larger, and that it is therefore unreasonable to demand that it be describable by a pure state rather than a more general von Neumann operator. However that may be, these authors would agree that there can in any case be no harm in working throughout in terms of the von Neumann paradigm, as will be done here, because it includes the more restricted Heisenberg type pure state paradigm as a special case.

Before continuing, it is to be remarked that the term observable has been used here to designate what in a more pedantically explicit terminology would be called a qualitative observable, in order to distinguish it from the quantitative observables that are definable as functions thereof. Thus any qualitative observable, $\{e\}$ say, determines and is determined
by a corresponding equivalence class of quantitative variables, in which any particular member, $E$ say, is determined by a corresponding non-degenerate real valued function $E_i$ of the index labeling the admissible alternatives $e_i$ for $\{e\}$. The condition of non degeneracy of the function is to be understood as meaning that $E_i \neq E_j$ whenever $i \neq j$. In a quantum theory for a system characterised by a Hilbert space $I\{A\}$, such a quantitative observable will be identifiable with a corresponding Hermitian operator $E$ whose eigenspaces are the Hilbert subspaces $e_i \subset I\{A\}$, while the corresponding eigenvalues are the real numbers $E_i$, so that one has

$$E|\Psi\rangle = E_i|\Psi\rangle \iff |\Psi\rangle \in e_i.$$  

(6)

Such a quantitative variable $E$ will have a mean (expectation) value $\langle E \rangle$ that will be given by the formula

$$\langle E \rangle = \text{tr}\{PE\},$$

(7)

in which the operator $E$ will be expressible in terms of the relevant projection operators $e_i$ in the explicit form

$$E = \sum_i E_i e_i.$$ 

(8)

The simplest illustration is provided by the familiar Stern Gerlach example for which the observable $E$ represents the spin energy of an electron (with respect to its own rest frame) in a uniform magnetic field. For this application, the relevant Hilbert space $I$ has only two (complex) dimensions, being spanned by a subspace $e_1$ representing the eventuality that the spin be aligned with the magnetic field, and a subspace $e_2$ representing the eventuality that it be aligned in the opposite direction. Other eventualities, corresponding to alignment in other directions, will not be characterised by well defined energy values. The quantum analogue of the unbiased coin toss theory considered in the previous section is the unbiased spin theory that is specified simply by adopting the isotropic probability distribution given (as the high temperature limit of an ordinary thermal distribution) by $P = \frac{1}{2} I$.

5. Sensors and conditional probabilities.

Having thus completed a brief overview of the basic quantum mechanical principles that are generally accepted as a matter of consensus, it is now necessary to approach the much more controversial issue of how these rather abstract principles should be interpreted in practice – and more particularly how to relate what might observable in principle – in the abstract sense of the term as used above – to what may be actually observed in the ordinary sense of the word, taking it that the ordinary meaning of the word observation is the recognition of the actual occurrence of an eventuality in some particular system under consideration.

The first, relatively uncontroversial, point that needs to be made at this stage is that the notion of an actual observation of an eventuality in a generic system under consideration is generally taken to involve an interaction with a specialised kind of system that I shall refer to simply as a sensor, which might consist of an artificial measuring apparatus of a simple and easily understandable kind such as a Stern Gerlach spin orientation detector,
but might also consist of something more mysterious such as the brain of Schroedinger’s famous cat.

In order for an observable \( \{f\} \) say of a system \( B \) say under consideration to be really able to be (exactly or approximately) observed – i.e. for the recognition of the actual occurrence of a particular eventuality \( f_j \in \{f\} \) to be feasible in practice – it is generally considered to be necessary not just that \( \{f\} \) should be observable in the abstract sense formulated above, but more particularly that it should be adequately correlated with a corresponding sensor observable, \( \{e\} \) say, in an appropriate sensor system, \( A \) say. The subsets \( \hat{f}_j = f_j \otimes I\{A\} \) and \( \hat{e}_i = e_i \otimes I\{B\} \) in the tensor product space \( \hat{I} = I\{A\} \otimes I\{B\} \) of the combined system will naturally give rise to a conjoint observable \( \{c\} = \{e\} \otimes \{f\} \), whose eventualities \( \{c_{ij}\} \) are given by the intersection subspaces \( \hat{c}_{ij} = \hat{e}_i \cap \hat{f}_j \). The probabilities of these conjoint eventualities will evidently form a matrix with elements

\[
P_{ij} = P\{c_{ij}\}.
\]  

The first prerequisite for the desired correlation of \( \{e\} \) and \( \{f\} \) is that they have the same channel number, \( N_e = N_f \), i.e. the same number of alternative eventualities, so that the matrix \( P_{ij} \) will be square. The final requirement for them to be more or less adequately correlated is that (for a suitable index ordering) the matrix should be more or less exactly diagonal, i.e. that for \( i \neq j \) the probability \( P_{ij} \) should be zero or very small. (There is an extensive literature \[10\] on decoherence processes by which such diagonalisation can be brought about.)

The conditions of the preceding paragraph are applicable both to classical and quantum systems. In the particular case of ordinary quantum systems, the observables \( \{e\} \) and \( \{f\} \) will give rise (on the extended Hilbert space \( \hat{I} \)) to corresponding sets of projection operators \( \hat{e}_i \) and \( \hat{f}_i \) that will automatically commute, \([\hat{e}_i, \hat{f}_j] = 0\), and whose products

\[
\hat{c}_{ij} = \hat{e}_i \hat{f}_j = \hat{f}_i \hat{e}_j
\]  

will be the projection operators specified by the corresponding subspaces \( \hat{e}_i \cap \hat{f}_j \) so that (whether it is satisfactorily diagonal or not) the probability matrix \([9]\) will be obtainable from the von Neumann operator \( \hat{P} \) on \( \hat{I} \) in the form

\[
P_{ij} = \text{tr}\{\hat{P} \hat{e}_i \hat{f}_j\}.
\]  

It is to be remarked that the relation described in the preceding paragraphs is reflexive, in the sense that if an observable \( \{f\} \) of \( B \) is observable by \( A \) then the corresponding observable \( \{e\} \) of \( A \) will be similarly observable by \( B \). A graphic illustration is provided by the gedanken experiment in which Schroedinger put his cat in a box that was equipped with an anaesthetising mechanism triggered by a Stern Gerlach detector. (Schroedinger originally envisaged a lethal mechanism, but that would have conflicted with the Popperian desideratum of repeatability of the experiment.) One way of describing this is to take the detector to be the sensor \( A \), whose reading will tell us about the state of the cat, considered as system \( B \). However by opening the box one can see directly whether the cat is still awake, thereby using it as a sensor, \( A \), that will tell us whether the spin measured by the detector, now considered as system \( B \), was up or down. If one also reads
the detector as well as opening the box, one can check the validity of the theory: an inconsistency might remind us of the likelihood for the cat to fall asleep spontaneously, with the implication that resort to a less satisfactory probability distribution, with non-vanishing off diagonal elements, might be more realistic for a subsequent repetition of the experiment.

It is commonly convenient to rewrite the expression for a joint probability such as \( P \) in terms of the corresponding conditional probability \( P_{i \{ f \}} \) for \( f \) given \( e \) in the form

\[
P_{ij} = P_{i \{ e \}} P_{i \{ f \}}.
\]  

(12)

In the quantum context we are concerned with here, it can be seen that such a conditional probability for \( f \) will be given by the prescription whose form is analogous to that of (13), namely

\[
P_{i \{ f \}} = \text{tr}\{ \hat{P}_{i \{ f \}} \}
\]  

(13)

where \( \hat{P}_{i \{ f \}} \) is the reduced probability operator associated with the subspace \( \hat{e} \), as given in terms of the original (unreduced) probability operator \( \hat{P} \) (on the extended space \( \hat{I} \)) by the defining formula

\[
\hat{P}_{i \{ f \}} = P_{i \{ f \}}^{-1} \hat{e} \hat{P} \hat{e}.
\]  

(14)

This formula is such as to ensure automatically that the reduced probability operator has the properties required for qualification as a von Neumann density in its own right, meaning that it is Hermitean with unit trace,

\[
\text{tr}\{ \hat{P}_{i \{ f \}} \} = 1.
\]  

(15)

The desideratum that \( \{ e \} \) should provide an approximate observation of \( \{ f \} \) is equivalent to the more restrictive requirement that the reduced probability operators should satisfy an approximation of the form \( \text{tr}\{ \hat{P}_{i \{ f \}} \} \approx \delta_{ij} \).

6. The subjective nature of a probability operator.

The consensus about what is meant in quantum theory by a – qualitative or quantitative – observable, and by a suitably adapted sensor, in the abstract sense does not extend to the question of what is meant by the occurrence of an actual observation. There is however a rather general understanding that it is something that can be performed only by sensors of privileged class for which the title of observer is reserved. It would be rather generally agreed, in the context of the example referred to above, that this class would include Schrödinger himself, but not his (gedanken) Stern Gerlach detector. What is more litigious is the status of the cat: would its own discovery that it was still awake count as an actual observation?

Such awkward questions are particularly crucial in the context of what is commonly referred to as the naive Copenhagen interpretation ("naive" to distinguish it from other purportedly more sophisticated variants) according to which the von Neumann operator - or the state vector in the pure case - has the status of an objective physical entity that undergoes a (non unitary) collapse

\[
P \mapsto P_{i \{ f \}}
\]  

(16)
to the relevant reduced operator (or reduced state vector in the pure case) as constructed according to the procedure given by (14), when the outcome \( e_i \) is actually observed for an observable \( \{e\} \).

The problem with this naive Copenhagen doctrine is how to give a coherent prescription for deciding just when this collapse is supposed to occur. A relativity theorist would object at the outset that a question about when something occurs implicitly refers to the concept of time, a concept that is ultimately elusive and at best dependent on a subjectively arbitrary choice of reference system. However there also is a more basic problem that will arise even in a context for which a reasonably unambiguous Newtonian type temporal description is available as a good approximation, as would be the case for the cat experiment if not in other (e.g. cosmological) contexts.

This more basic problem \[6\] is that of what is known as “Wigner’s friend”. Let us suppose that the friend in question was Schrödinger himself, and that Wigner was interested in the fate of the cat. Wigner would have had no direct access to the Stern Gerlach detector, but would have been able to telephone to Schrödinger to ask what had happened, thus using Schrödinger himself as the sensor, which prior to the opening of the box would have been in a mixed state. As far as Wigner was concerned the relevant collapse process (16) would not have been applicable until the time of the telephone call, whereas from Schrödinger’s point of view it would have occurred at the earlier time when the box was opened, while the cat itself would have already known even sooner if it had not been put to sleep. One might resolve the discrepancy between Schrödinger’s point of view and that of the cat by taking the line (which might be that of a theologian such as former Cambridge physics professor, John Polkinghorne \[11\]) that the subhuman status of the cat disqualifies it from membership of the privileged class of genuine “observers”, but no such specious evasion of the issue is available for discrepancy between Wigner’s point of view and that of Schrödinger, whose equivalent status can not so easily be denied.

The implication of the well known example recapitulated in the preceding paragraph is that the naive Copenhagen interpretation can not be coherently applied to cases in which several independent (human or other qualified) observers are involved, which means that it can ultimately be acceptable only to a (deliberate or subconscious) solipsist.

The obvious conclusion to be drawn from this is that a probability operator (or state vector in the pure case) should not be thought of as an objective physical entity, and that – as would be agreed even by followers of the Everett doctrine, who refuse such subjectivity – its collapse (16) should not be thought of as a physical process, but just as a mathematical step whose application will be appropriate whenever the necessary information, namely the observation of the particular eventuality \( e_i \), becomes available. The operator collapse process (16) is thus merely the quantum analogue of the ordinary Bayesian reduction process \( P \mapsto P_i \) for an ordinary classical probability distribution, whereby its \textit{a priori} value is to be replaced by the corresponding \textit{a posteriori} – i.e. conditional – value when the relevant information is supplied. Like the classical probability distributions \( P \) and \( P_i \), the corresponding \textit{a priori} and \textit{a posteriori} von Neumann operators \( P \) and \( P_i \) should be considered to have a status that is not objective but intrinsically subjective.

A corollary of the foregoing conclusions is the anticipation that observers with different personal historical backgrounds should use \textit{different} von Neumann operators, particularly \textit{a priori}, although there will of course be a tendency toward agreement \textit{a posteriori} when
observational information is shared. In discussions of their (different) opinions about what is appropriate in cosmological contexts, authors such as Hawking and Vilenkin [9] tend to use the definite article for what they call “the” state of the universe, but the reasoning I am developing here would suggest that such definiteness is unjustifiable, and that the most that is reasonable would be to propose “an” (not “the”) a priori probability operator.

7. Everett’s concept of branch-channels.

Having recognised the incoherence of the naive Copenhagen interpretation, a newer school of thought founded [4] by Everett has emphasised – correctly according to the reasoning I am developing here – that there is no physical process of collapse of the probability operator. What is not so clearly correct or even meaningful is Everett’s concomitant conclusion that all the ensuing “branches of the universe” remain equally real.

Before the validity of this doctrine can be discussed, it is necessary to explain what is meant by the branches – or to be more precise branch-channels – in question. The origin of the idea dates back to the pre von Neumann epoch when it was assumed that the relevant probability distribution would be provided by a pure state, as specified by a unit Hilbert space vector that could of course be represented as a sum, $|\Psi\rangle = \sum_i |\Psi_i\rangle$, of eigenvectors $|\Psi_i\rangle \in e_i$ of the observable $\{e\}$ under consideration. The observation process was commonly described as having a first step consisting of a splitting of $|\Psi\rangle$ into the set of alternative projections $|\Psi_i\rangle = e_i |\Psi\rangle$ (17) onto the relevant eigenspaces, which were referred to (rather misleadingly) as branches.

According to the naive Copenhagen doctrine, the observation process would be completed by a second step consisting of a collapse, whereby the set would be replaced by a single appropriately renormalised branch vector, $P_i^{-1/2} |\Psi_i\rangle$ that would turn up with the corresponding conditional probability $P_i = \langle \Psi_i | \Psi_i \rangle$. On the other hand the Everett doctrine denied the occurrence of the collapse as a physical process, with the implication that the system would be subsequently describable [12] as being in a mixed state, for which the corresponding von Neumann operator would have the form

$$P = \sum_i |\Psi_i\rangle \langle \Psi_i|,$$  

representing what I shall refer to as the provisional probability operator, in order to distinguish it from the relevant (pure) a priori probability operator

$$P_{(0)} = |\Psi\rangle \langle \Psi|,$$  

and whichever a posteriori probability operator

$$P_{[i]} = P_i^{-1} |\Psi_i\rangle \langle P_{\text{a priori}}|,$$  

may turn out to apply.

If the presumption that the system was initially in a pure state is replaced by the more general supposition that it was in an initial state describable by an a priori probability
operator, \( P_0 \) say, consisting of an arbitrary sum of pure state operators, then by considering the effect on each member of such a sum it can be seen that the effect of the first step of the observation process described in the preceding paragraph will be to provide a provisional probability operator given no longer by the simple formula (18) but by the more general prescription

\[
P = \sum_i P_i P_i^{\dagger},
\]

that is known \cite{13} as Luder’s rule, in which the operators \( P_i \) are the \textit{a posteriori} probabilities for the relevant output channels, i.e. the relevant eventualities \( e_i \), which are what Everett referred to as “branches”. In accordance with the formula (14), these \textit{a posteriori} probability operators, and the corresponding probabilities, are given in terms of the \textit{a priori} probability operator \( P_0 \) by

\[
P_i = P_i^{-1} e_i P_0 e_i, \quad P_i = \text{tr} \{ P_0 e_i \},
\]

and it is to be noted that they are also recoverable, using expressions of the same form

\[
P_i = P_i^{-1} e_i P e_i, \quad P_i = \text{tr} \{ P e_i \},
\]

from the ensuing provisional probability operator (21).

8. The deficiency of the Everett interpretation

Before exposing the essential deficiency of the Everett doctrine, I would like to rectify an accessory misconception to which it has given rise. In its usual presentation, the use of the term branch is motivated by the notion that the number of relevant channels increases whenever an observation is made. It is important to recognise that this idea – of perpetual multiplication of the relevant number of branch-channels – is, as a general rule, misguided. It is based on the – rarely realistic – presumption that the \textit{a priori} state of the system under consideration is pure, consisting just of a single branch-channel, whereas in a generic case (for the reasons discussed above) \( P_0 \) will already be mixed, involving as many branch channels as \( P_0 \), so no actual increase occurs. It is thus more appropriate as a metaphor to speak of channels rather than branches, which is why I have chosen, as a compromise, to use the term branch-channel. In any case (even if the initial state really was pure) the commonly accepted idea that – as more and more information is obtained by successive observations – the number of branches will go on increasing is also unrealistic for a different reason, which is that a given finite system cannot continue to acquire more and more information without limit. After a certain amount of information has been acquired, the system will saturate, so that further information will be able to be taken into account only by a (Landauer type \cite{14}) process involving the erasure of previously recorded information in order to release the necessary memory space. The number of channels available for useful observation can at best be only a small fraction of the number of dimensions needed for a complete physical representation of the sensor, which in practice (if he, she, or it is a system constituted from a finite number of molecules with a finite total energy in a finite volume) will of course be limited.

Bounded though it must be, the number of branch-channels – meaning the number of eventualities that may be observationally distinguished – in a given (human or other)
sensor system can indeed be very large. It is this consideration that has exposed the Everett proposal [4] that all the branches are “actual, none any more real than the rest” to the criticism [15] that it entails a “bloated ontology”. However, as I have previously remarked [16] as far as the scientific desideratum of Ockham’s razor (meaning economy of formulation) is concerned it does not matter how extensive or otherwise the ensuing “ontology” may be.

A more serious reason for dissatisfaction with the Everett doctrine of quantum theory is its failure to apply its own declared rules in a coherent manner, which has made the question of the interpretation of this “interpretation” the subject of much discussion [17, 18]. The assertion that the branches are “actual” seems to imply their ontological reality, but Everett’s categorical denial that any one is “more real than the rest” is followed by the Orwellian admission [4] in a subsequent paragraph that “in order to obtain quantitative results” the branches must be given “some sort of quantitative measure (weighting)”.

The aim of the Everett program, as expressed by De Witt [17], is to construct a theory “in which it makes sense to talk about the state vector of the whole universe. This vector never collapses, and hence the universe as a whole is deterministic”. The troublesome problem [18, 19, 20] is how to use such an ultimately deterministic model to obtain the probabilistic predictions that work so well in local applications of quantum theory. As Graham [5] puts it “Everett attempts to escape from this dilemma by introducing a numerical weight for each world”. The work of Graham and of Hartle [21] has shown that Everett’s “weighting” scheme does successfully reproduce the usual probabilistic predictions, so much so that indeed the distinction between the terms “weighting” and “probability” can be seen to be merely semantic. Changing its name to “weighting” (or “propensity”, which is another traditionally favoured alternative) does not solve the problem of interpreting the meaning of the “probability” that is involved.

It is clear that Everett and his followers have so far failed to achieve their declared objective. Their bold attempt to solve the – originally local – interpretation problem by reintroducing determinism at a global level has been helpful for providing a deeper understanding of many of the issues involved, but the question of how much “reality” should be attributed to the probabilistically “weighted” branch-channels has nevertheless remained unsolved until now.

My purpose here is to present a recent clarification [3] whereby this issue is not so much decided as transcended, in conformity with the precept that questions of ontology are of a theological nature that is beyond the scope of ordinary science (whose modest ambition is to account for appearances, and not for ultimate reality, whatever that may mean). The anthropic approach described below provides a framework in which an intellectually coherent interpretation can be provided in a manner that leaves plenty of scope for adjustment, and that is compatible not only with an (unbloated) “oriental” option, in which hardly any of the relevant branches need be considered to be “real”, but also with a (scientifically indistinguishable, but theologically very different) “occidental” option in which they might all be describable as “actual”.

9. The side issue of the provisional distribution.

Whereas zealous adherents of the Everett doctrine – and a fortiori of the naive version of the Copenhagen interpretation that was discussed above – would have it that some
sort of objective reality can be attributed to the state vector on a sufficiently large scale, and hence to the probability operator that would be relevant on a more local scale, on the other hand most other schools of thought, including less naive versions of the dualistic Copenhagen interpretation, would concur with the supposition adopted here to the effect that such entities are essentially of a subjective nature. This contrasts with the status of the Hilbert space operator algebra of eventualities and observables, which have a more objectively well defined nature. According to this principle, the amplitudes (and corresponding “weightings”) of Everett type “branches” should be considered as ultimately subjective, whereas the branches themselves can be considered to be objective – which does not of course entail that such mathematical structures are “real” in any ontological sense.

Before leaving the subject of the “branching” process (misnamed because the number of branches involved in the description of a subsystem need not increase, and might even decrease, when an interaction occurs) it is worth commenting further on the nature of the process whereby an *a priori* probability operator $P_{(0)}$ is replaced by the corresponding provisional probability operator $P$ as given by (21) and (22). The original discussions of this process were formulated in terms of what Dirac [7] referred to as the Schroedinger picture, wherein states are considered to have a time dependence whereby the evolution from an initial time $t_{(0)}$ say to a later time $t$ is given by an operator transformation $P_{(0)} \mapsto P$ that will be given, in the special case of a pure state for an isolated system, by a corresponding vector transformation $|\Psi_{(0)}\rangle \mapsto |\Psi\rangle$. In the special case of an isolated system, such a transformation will be given by a unitary operator $U$ (that is continuously generated by some Hermitean Hamiltonian) according to prescriptions of the standard form $|\Psi\rangle = U|\Psi_{(0)}\rangle$ and $P = U P_{(0)} U^{-1}$. However the transformation will in general be of a less simple (non-unitary) type when interaction with an external system is involved. The idea, as discussed by von Neumann, was that the preparation of an actual experimental observation should involve an arrangement whereby a transformation of this latter (non-unitary) type produced a provisional probability operator $P$ of the required form, as given by the Luder formula (21).

As originally pointed out by Dirac [7], a representation in terms of such a Schroedinger picture can be translated into an equivalent representation in terms of the kind of Heisenberg picture that has been implicitly adopted throughout the present discussion. In this kind of representation, the relevant state vector $|\Psi\rangle$ or probability operator $P$ is considered to be time independent, and the effect of Schroedinger type time translations is allowed for by corresponding transformations of the relevant observables and their constituent eventualities. In the special case of an isolated system these transformations will be of the standard unitary type, so that for example if $e_{(0)}$ is the projection operator corresponding to some particular eventuality at a time $t_{(0)}$ then the corresponding time transposed eventuality at a later time $t$ will be given by

$$e = U^{-1} e_{(0)} U.$$  

(24)

The essential advantage of using a picture of this kind is that there is no impediment to its extension to (General Relativistic and other) applications for which no globally well define Newtonian type time parametrisation may be available, so that the concept of a
time translation relation of the form $e_{(0)} \mapsto e$ might make sense only for very particular locally related eventualities.

As seen from this Heisenberg (as opposed to Schroedinger) point of view, the process of preparation of an experimental observation in the manner prescribed by von Neumann should be thought of, not as the replacement of an \emph{a priori} probability operator $P_{(0)}$ by a different provisional probability operator $P$, but as the the replacement of an initially envisaged, but perhaps maladapted, observable $\{e_{(0)}\}$ by an appropriately adjusted observable $\{e\}$ with respect to which the probability distribution already has the required Luderian form (21).

From this point of view, there is no need to bother about any distinction between \emph{a priori} and provisional probability operators (which – in view of the possibility of using (23) instead of (22) – were in any case equivalent for the practical observational purpose under consideration). What matters for the purpose of making what von Neumann would consider to be a satisfactory observation is the choice of a suitably adjusted observable $e$. However the main point I wish to emphasize at this stage is that although it may be of technical interest in particular applications, the importance of the issue of obtaining a satisfactory observation in the sense specified by Luder’s rule has been greatly exaggerated, in so far as its relevance to the ultimate interpretation of the meaning of the observations process is concerned. To start with there is the consideration that the Luderian desideratum is obtainable not only by the non-trivial process described above, whereby $\{e\}$ is adjusted to a previously chosen probability operator, but also by the trivial process whereby the subjective \emph{a priori} choice of $P$ is adjusted \emph{ad hoc} to fit a prescribed observable $\{e\}$, an adjustment that in no way diminishes the credibility of its implications, as can be seen from the equivalence of the prescriptions (22) and (23).

A more fundamental reason why the question of the Luderian transition is irrelevant is that when an observation has been actually carried out (not merely planned) one will be left just with a single confirmed eventuality, $e_i$. Such a single eventuality might be incorporated with others to constitute a complete observable set (spanning the entire Hilbert space) in many different ways, whose substitution in the Luder formula (21) would provide many different results. Nevertheless, however that might be, and regardless of any distinction that may or may not have been made between an \emph{a priori} probability distribution $P_{(0)}$ and a provisional probability $P$, one will be left with an unambiguously specified \emph{a posteriori} probability distribution $P_{(i)}$, which is all that matters for the purpose of subsequent predictions one may wish to make.

The upshot is quite simply that someone (such as Wigner when concerned about Shroedinger’s cat) should use the \emph{a posterior} distribution when the relevant information has become available, and until then should just continue to use the ordinary \emph{a priori} distribution. One should avoid getting sidetracked (as so many of Everett’s followers have been) by intermediate Luderian technicalities, whose analysis is of little relevance to the two outstanding issues that remain. In addition to the question of interpretation, which will be addressed from an anthropic point of view below, the other outstanding issue is of course the usual practical Bayesian dilemma of how to decide quantitatively what \emph{a priori} distribution should be used in a particular context – something that can sometimes be resolved just by symmetry considerations (as in the coin tossing example described above).
10. Perceptions and perceptibles.

An important idea that was latent in much of the preceding discussion is that some privileged eventualities and observables are more naturally significant than others.

In the discussion of Luder’s rule it was remarked that this rule can be interpreted as selecting a privileged class of observables, but I would emphasize before continuing that privilege of that kind is not what I am concerned with here, because it is ultimately dependent on an arbitrary subjective choice of the relevant *a priori* probability distribution.

The kind of privilege I am concerned with here is something that depends on the essential nature of the system under consideration in a manner that is independent of the choice of the probability distribution. This is something that could be said about Bohm’s idea \cite{13} of privileging position with respect to its dynamical conjugate, namely momentum, but that particular choice is something that would not seem very natural to the numerous physicists whose mental life is based in Fourier space.

The kind of privilege that seems to me more relevant for the interpretation question is something that would be rather generally recognised as being imposed by the circumstances in particular cases. It is exemplified most simply by the existence of a privileged choice (determined by the background magnetic field) for the the particular spin eventualities characterised as “up” and “down” in the Stern Gerlach experiment that has been discussed above. It is exemplified by many familiar kinds of apparatus, such as can be found in scientific laboratories, and increasingly in ordinary homes, whose output is typically presented in terms of what, at the highest resolution usually turns out to consist of simple integer valued observables, such as the alternative eventualities in the range from 0 to 9 for a digit in a counter output, or the binary alternatives for a particular pixel on a screen to be “on” or “off”. It is mathematically possible to use other bases for a Hilbert space description of such systems, for example by working with eventualities defined as linear superpositions of “on” and “off” states of screen pixels, but that is evidently not the kind of treatment for which such an apparatus was intended by its designer.

Although the degree of complexity of the systems involved is very different, it seems to me that there is a rather strong analogy between the special role of the “on” and “off” states for a pixel on a screen and the “awake” and “sleeping” states of Schroedinger’s cat. The privileged status of the particular eventualities in question can be accounted for as the result of a process of design that is attributable in the first case, not just to an individual engineer, but to the collective activity of a scientific community, while it is attributable in the second case to a very long history of biological evolution by Darwinian selection. Having said this about the cat, the next thing to be said is of course that the same applies to Schroedinger and Wigner, for whom the relevant privileged eventualities are states of mind corresponding to the realisation that the cat is awake or not as the case may be.

Whatever doubts we may have about the status of the cat, we must recognise that Schroedinger and Wigner are closely analogous to ourselves (meaning the author and presumed readers of this essay) which means that insight into the working of their minds can be obtained from our own experience. Since the only eventualities about whose reality we can be sure are the conscious perceptions in our own minds (of which some, namely those occurring in dreams, are evidently uncorrelated with anything outside)
corresponding to the “mind states” whose essential role has been recognised by several authors, such as Donald [22], Lockwood [23], and in particular by Page [24], whose line of approach is followed here. It seems reasonable to postulate the validity of Page’s principle according to which conscious perceptions are the only eventualities that can be considered to actually happen. It also seems reasonable to make the concomitant postulate that these perceptions must belong to some restricted class of privileged eventualities of the kind discussed in the preceding paragraph. I shall refer to the eventualities of this subclass as perceptibles.

In his “sensible quantum theory” [24], Page has attributed a privileged role to a class of observables that he refers to as “awareness operators”, which I interpret to mean observables whose individual constituent eventualities are the perceptibles introduced in the previous paragraph. Page has used these particular operators to develop a refined version of the Everett interpretation, in which the branches – or as I would prefer to say, channels – that matter are specified with respect to these awareness operators. Thus whereas Everett’s original version might attribute “actuality” to branches defined with respect to observables of a rather arbitrary kind, Page’s more refined version would attribute “actuality” only to branches of an appropriately restricted kind, namely the channels that are specified by perceptibles. Having thus provided a much clearer idea of which channels are actually needed, Page was still left with the problem of interpreting what, following Everett’s evasive example, he referred to as their “weighting”. The point at which Everett stumbled was in trying to reconcile his recognition that the weighting was needed with his preceding claim that all the branches were equally real. Page came up against the same problem with respect to the claim to the effect that all the perceptibles are actually perceived.

11. The anthropic abstraction

A corresponding paradox is reached from a rather different angle in the approach I am developing here, which is in agreement with that of Page [24] in so far as the special role of perceptions is concerned, but differs in affirming that the weighting in question must be considered to have an essentially subjective and probabilistic nature. The intrinsically probabilistic nature of models of the kind advocated here raises the problem of what it can mean to attach a probability to the actuality of an eventuality in the mind of someone else if the only events one can actually observe are those occurring in one’s own mind.

Before presenting what I think is the only acceptable way of dealing with this paradoxical problem, I would mention two less satisfactory ways of resolving the issue that have been suggested in the past. The first way is of course that of the solipsist, who would deny the existence of any conscious perceptions other than his (or her [20]) own, with the implication that the apparent analogy between oneself and others such as Schroedinger is merely a superficial illusion. The second way (which unlike that of the solipsist has been followed up deliberately by many physicists, starting with de Broglie) is to revert to a deterministic description of the world, providing a theoretically well defined answer to the question of what really happens by denying the (experimentally well established) validity of the essentially probabilistic description provided by orthodox quantum theory. Neither the first nor the second of these ways of solving the problem can be said to actually resolve the paradox: they merely evade the issue by dropping one or other of the essential (exp-
mentally motivated) elements of the problem, which is that of providing an essentially probabilistic treatment of perceived reality that respects the apparent symmetry between different people.

A historical analogy is provided by the incompatibility between Maxwellian electromagnetism and Newtonian gravity, which was ultimately resolved by their unification in Einstein’s General Relativity. The problem to be dealt with here is that of reconciling subjective probability with objective reality. The only way that I know of for solving this problem in a satisfactory manner is the anthropic approach, which faces the issue head on without denying the validity of the considerations that lead to the paradox.

It is worth emphasising, by the way, that the problem is not specifically a problem of quantum theory, but also arises in probabilistic versions of classical theory, as was recognised, I suspect, by many of those who were hostile to anything associated with the name of Bayes. The importance in this context of the quantum revolution is that it changed the status of Bayesian theorists from that of radicals (because they were willing to abandon determinism) to that of reactionaries (because they continued to use old fashioned Boolean logic).

The situation, as I understand it, is as follows. Suppose that to describe a system that includes ourselves (but, for the sake of finiteness, perhaps not the whole of the universe) we have set up some (classical or quantum) theory that provides probabilities for an extensive class of eventualities. This class includes a specially privileged subclass of eventualities that I shall refer to as perceptibles, which are the only ones that can be actually observed as conscious perceptions. The set of such perceptions (not just yours and mine, but also those of everyone else) can be described as objective, and it is the only thing in the theory that can be considered to be real.

You have an objective model attributing probabilities to perceptibles, not only your own but those of other people. But what sense can it make to attribute a probability to an observation you cannot make? If you are Wigner, what sense can it make – even in a classical theory – to use an objective distribution attributing probability to something that can only be known by Schroedinger? The contradiction arises when Schroedinger makes the Bayesian transition to the relevant a posteriori distribution, while Wigner continues for the time being to use the a priori distribution. How in these conditions can either of these distributions be considered to be objective?

The resolution to this paradox is provided by what may be called the anthropic abstraction (so called because it underlies of what I designated – perhaps inaptly – as the anthropic principle [20]). The paradox that arises in this case (as in many others) can be attributed to an unnecessary assumption that has been consciously or subconsciously taken for granted. The unnecessary assumption is that of knowing in advance who one is. The anthropic abstraction consists in refraining from assuming in advance that one has the identity of some particular sensorial observer in the model, so that one’s status a priori is that what I shall refer to as an abstract perceptor. It is not until the actual happening of the perception that one can know whether one is Schroedinger, or Wigner, or whoever else may be included in the model.

It is of course to be understood that the perceptible eventualities that are involved in this anthropic approach cannot just be of the elementary type exemplified by the observation that someone else is awake, but need to include eventualities of the more
complicated kind known as consistent histories [25]. The sort of eventuality that needs to be envisaged is not simply that of finding oneself to be Schroedinger, but that of finding oneself to be Schroedinger at a particular instant in his life, with all the memories he would have had at that moment.

The use (which I see no satisfactory way of avoiding without reverting to determinism) of the anthropic abstraction entails the need to adopt some kind of anthropic principle, by which I mean some kind of prescription for attributing appropriate probabilities to the relevant perceptible eventualities. The rather crude kind of anthropic principle that I have put forward on previous occasions [2] was concerned with the attribution of probability to entire observer systems, (such as those associated with the names of Schroedinger or Wigner) without getting into the details of particular moments in their lives. For the applications I was then considering, it was sufficient to use a crude statistical treatment attributing equal weight to all terrestrial or extraterrestrial observers who can be considered to be sufficiently like ourselves to be describable as “anthropic”. However – as several authors have already remarked [24, 26, 27] – the more detailed applications I have been considering here (particularly those involving quantum effects) require the use of a more refined kind of anthropic principle [3] that will distinguish not just between anthropic individuals but between different instants in the lives of such individuals.

The question that naturally arises at this point in this line of reasoning is whether it can suffice to use just the probability weightings that are directly provided by orthodox quantum theory (such as has been discussed above) in conjunction with some prescription for deciding which of the many mathematically defined eventualities in the model should be considered to have the privileged status of perceptibility?

12. Uniqueness of the perceptor?

In the subsequent subsections I shall address the scientifically important question of the attribution of the required anthropic probability. However before doing so, I would like to digress by mentioning another question of a less scientific nature that might also be a subject of philosophical discussion in the future.

This is the question of the nature of what I have referred to as a perceptor, whose actual perceptions are the only entities within the model that are considered to be real (which is not to deny the reality, in some theological sense, of other entities beyond the scope of the model). The perceptor acquires an a posteriori identity (of an ephemeral nature) as a material observer (such as Schroedinger) on the occasion of an actual perception, but what about the immaterial identity the perceptor might have a priori?

Is the perceptor unique? The notion that all anthropic observers might just be avatars of a single perceptor will not seem strange to anyone familiar with oriental (Hindu-Buddhist) religious tradition. (A scientific analogy that comes to mind is Feynmann’s idea that the universe is inhabited only by a single electron, which is able to follow all the world lines that we usually attribute to distinct elections by also following – but in a time reversed sense – the other world lines that we attribute to positrons.) The obvious Wheelerian epithet for the succinct encapsulation of this idea – namely that we all share the same abstract identity – is solipsism without solipsism.

The postulate of a unique perceptor has the advantage of being particularly economical in the sense required by Ockham’s razor. Nevertheless, in the framework of the occiden-
tal (Judaean-Christian-Islamic) religious tradition it might seem more natural to suppose that there are many distinct perceptrons. What is not permissible, however tempting it may seem, is to suppose that distinct perceptrons are correlated with distinct anthropic observers, such as Shroedinger and Wigner: the essence of the anthropic abstraction is that a perceptor has the potential for actualisation in any observer state that has a non zero probability amplitude. The only way you, as a material observer, can claim an exclusive monopoly of the potential for actualisation of your own perceptor, is by adopting an a priori probability distribution that attributes no weight to anyone other than yourself, in other words by adopting the (unacceptable) autocentric attitude describable as solipsism.

For someone whose objection to the Everett doctrine was based not on not on its failure to follow its own declared rules, but on the ontological bloating \[15\] implicit in the many universe doctrine, the present idea that one might adopt a many perceptor doctrine might be felt to even worse. Whereas the number of Everett branch-channels is restricted, as I have remarked above, by the limited information content for any finite system, on the other hand there is no limitation at all on the number of distinct perceptrons that might be conceived to exist, and that might all have a chance of undergoing the experience of being Shroedinger at some moment in his life.

The idea that there might be an unlimited number of distinct perceptrons may be abhorrent to anyone for whom ontological economy is a desideratum, but on the other hand it might be extremely attractive to those who still hanker after determinism. Indeed for those who consider that in order to be meaningful the concept of probability must be defined in terms of frequencies of the outcome of many identical performances of the same experiment, the many perceptor doctrine can provide what is desired. If the number of perceptrons is vastly larger than the number of anthropic observers in the model, then each observer state (even those that are relatively improbable) would actually be perceived by a large number (albeit a small fraction) of the perceptrons. This would provide the desired frequency interpretation for the probability distribution. By using the anthropic abstraction in this ontologically uninhibited manner, it is at last possible to deliver what the Everett program sought, which may be epitomised as probability without probability.

Multiplication of the number of sensors is not the only way of obtaining probability without probability, if that is what is desired. Another number whose magnification can achieve the same result is the number of perceptions that each particular perceptor is allowed to make. The supposition that there is a large number of perceptrons, each allowed to make only a small number of perceptions or, even restricted to a single perception, is ontologically equivalent to the supposition that there is just a single perceptor who is allowed to make a large number of perceptions. As far as ontology is concerned, all that counts is the total number of perceptions.

Whether – as in the oriental version of the anthropic interpretation – there is a unique perceptor, or whether – as in the occidental version – the number of perceptrons is large (even compared with the number of anthropic observers) – is an issue that belongs to the realm of theology rather than science. The same can be said about the (more ontologically relevant) number of total perceptions, which may seem important to those who believe in probability only when formulated in terms of frequencies, but which in no way affects the way the theory is actually applied in practice. All that matters for scientific purposes is...
the relative probability distribution for the perceptions, which will now be discussed.

13. Anthropic weighting: the proper ansatz?

On the basis of what precedes, it seems reasonable to suppose that, from the point of view of a perceptor, the “net” probability, $P$ say, of a particular perception $e_i$ within a particular subsystem (representing the part of the universe under consideration) should be given by an expression of the form

$$ P \{e_i\} = P_e P_{\{\mathcal{O}\}} \{e_i\}, \quad (25) $$

where $P_{\{\mathcal{O}\}} \{e_i\}$ is the ordinary ‘gross” classical or quantum mechanical probability (as calculated in the manner described above) for the particular perceptible eventuality $e_i$ to occur on the occasion when the relevant Page type awareness observable $\{e\}$, is actually observed, while the $P_e$ is the anthropic factor giving the probability for the perception to belong to that particular observable set. A sensor of the familiar macroscopic but localised kind – exemplified by an ordinary computer, or a human observer – will be characterisable by a fairly well defined world line with a proper time parametrisation $\tau$, in terms of which the anthropic probability factor will be expressible in the form

$$ P_e = \dot{P} \Delta_e \tau, \quad (26) $$

where $\Delta_e \tau$ is the relevant proper time duration, and $\dot{P}$ is a corresponding probability rate factor, whose integral

$$ P = \int \dot{P} \, d\tau, \quad (27) $$

will be interpretable as the giving the total probability for the perception to occur in at some stage in the life of that particular observer.

Whereas the conditional probability designated by a roman capital $P$ in (25) is of the ordinary kind that is provided by the relevant classical or quantum physical theory for the system under consideration, on the other hand the anthropic probability factor designated by a caligraphic $\mathcal{P}$ (which is also conditional in so much as it is subject to the condition of restriction to that particular system within the universe) can only be provided by what I call an anthropic principle.

In my earlier discussions of applications [2] that were not concerned with discrimination between individuals, but with averages over entire populations, it was good enough to suppose that provided they were sufficiently similar to ourselves (that was the motivation the – rather debatable – choice of the term anthropic) the relevant total probability $\mathcal{P}$ per observer could be taken to be the same for each one, in accordance with what Vilenkin has referred to as a postulate of mediocrity, and what I would refer to just as a postulate of approximate symmetry. The application of such a mediocrity postulate in the present context gave rise to what I called the weak anthropic principle, whose purport is that the anthropic probability factor should take a fixed value

$$ \mathcal{P} = \frac{1}{N}, \quad (28) $$

where $N$ is the number of anthropic observers that come into existence within the system under consideration (so that if the system were scaled up to include a larger chunk of the
universe, with a larger population number $N$, then the value of $\mathcal{P}$ would be correspondingly scaled down.)

The ordinary (weak) anthropic principle formulated in the preceding paragraph will evidently not be enough for more detailed purposes, such as comparison of the probability of finding oneself to be someone very short lived (as in the case of a child that dies in infancy) with that of finding oneself to be someone more long lived (as in the case of a normal adult). For such a purpose, the most naively obvious possibility is to adopt the ansatz what I would call the *proper* anthropic principle, meaning the postulate of a fixed universal value for the anthropic probability rate $\mathcal{P}$, which would be given numerically by

$$\dot{\mathcal{P}} = \frac{1}{\langle \tau \rangle N},$$

(29)

where $\langle \tau \rangle$ is the average total proper lifetime of an anthropic observer in the system. In so far as the total probability over the total lifetime $\tau$ of an observer is concerned, adoption of the proper anthropic principle (29) evidently entails that (28) should be replaced by

$$\mathcal{P} = \frac{\tau}{\langle \tau \rangle N}.$$  

(30)

The foregoing proper refinement of the original anthropic principle (28) should, I think, be good enough for a wide range of applications. However for the purpose of comparing observers of very different kinds (for which the qualification anthropic might not be so appropriate) such as extraterrestrials and cats, not to mention babies in our own species, the plausibility of (29) is much less obvious.

14. Micro-anthropic principle: the entropic ansatz.

A hint toward a more plausible (though not so easily applicable) alternative is discernible in the response to the eschatological problem posed by Islam (28) that was provided by Dyson, who suggested (29) that what really matters is not the proper time duration of an interval but how much information is effectively processed therein. There is of course room for discussion about how to quantify what is effectively processed (as opposed to what is merely stored in a memory) in the case even of an ordinary computer and hence much more so for in the case of a feline or human mind. Estimating that the duration of a human “moment of consciousness”, which presumably corresponds to what is denoted here by $\Delta e\tau$ has the same order of magnitude as was supposed in the more recent work of Page (24), namely a significant fraction of a second, Dyson deduced (from the fact that the heat production of an entire human body is typically about 200 Watts at a temperature of 300° K) that the corresponding entropy production, $Q_e$ say, is of the order of $10^{23}$ bits. However experience with the analogous problem for computers indicates that the amount of information $S_e$ that can be judged to have been effectively processed by the mind itself during the corresponding period of perception – and the associated Landauer entropy production (14) – must have a vastly smaller value $S_e \ll Q_e$ that is not so easy to evaluate.

A plausible prescription for the evaluation of the processed information $S_e$ will however be available if we have a sufficiently detailed (quantum not just classical) theory to
characterise the Hilbert space projection operator $e_i$ corresponding to a particular perception $e_i$ under consideration. If we suppose that this particular perception belongs to a complete set of eventualities having the same rank (i.e. subspace dimension) $\mathcal{R}_e = \text{tr}\{e_i\}$ constituting an observable $\{e\}$ in a Hilbert space of dimension $\mathcal{N} = \text{tr}\{I\}$, so that the corresponding number of Everett type branch channels is $\mathcal{N}_e = \mathcal{N}/\mathcal{R}_e$, then the associated information capacity will be given by

$$S_e = \log\{\mathcal{N}_e\} = \log\{\text{tr}\{I\}\} - \log\{\text{tr}\{e_i\}\},$$

using a logarithm with base 2 if one wants to use Shannon’s bit units, or using a natural (Naperian) logarithm if one wants to use the entropy units that are commonly preferred by physicists. This information capacity represents the maximum amount of information that can be given – for a probability distribution $P_i (i = 1, ..., \mathcal{N}_e)$ – by Shannon’s formula $S = -\sum_i P_i \log\{P_i\}$.

What I would propose is that the formula (31) be used as an estimate of the amount of information that can be considered to be processed during the perception $e_i$, and that the corresponding anthropic probability should be postulated to be proportional to this, i.e. the required factor in (25) should be taken to be given by

$$P_e = \alpha S_e,$$

where $\alpha$ is a fixed proportionality factor that is chosen so as to ensure satisfaction of the usual requirement that the total probability (over all the relevant world lines) should add up to unity. According to this micro-anthropic principle – which might appropriately be described by the term entropic principle – the probability rate factor will not have a fixed value (as was postulated by the proper anthropic principle formulated above) but will be given by

$$\dot{P} = \alpha \frac{S_e}{\Delta_e \tau}.$$

The advantage of using the term entropic principle for this ansatz is that it emphasizes its virtue of being applicable in principle not just to observers qualifiable as anthropic, in the sense of being sufficiently similar to ordinary adult humans, but also to very different kinds ranging from such familiar examples as babies and cats to the highly exotic extraterrestrial observers whose survival at extremely low temperatures was envisaged by Dyson. A rather obvious application of this entropic principle is its use as evidence against Dyson’s conjecture [29] that civilisations constituted by observers capable of surviving at the extremely low temperatures predicted [28] for a non compact universe in the far distant future would be able to survive indefinitely with respect not just to proper time but with respect to the relevant information processing measure. If this conjecture were correct it would mean that the probability measure defined according to (33) by the entropic principle would diverge toward the future. The contrary prediction by Islam [28] that “it is unlikely that civilisation in any form can survive indefinitely” is therefore overwhelmingly favoured by the fact that we do not observe ourselves to be incarnated in an asymptotically viable low temperature life forms (if any such can exist at all) but in carbon based life forms adapted to a (cosmologically ephemeral) conditions of moderate temperature.
It is to be emphasised that the preceding argument against the likelihood of long
term survival is entirely dependent on the acceptance of the kind of a priori probability
distribution proposed (as a matter of choice, not merely as a tautology) by the anthropic
principle and its entropic extension. Dyson’s writing in this and other analogous contexts
– notably that of the prospects for our own terrestrial civilisation in particular [27] –
give the impression that he personally prefers an a priori probability distribution of the
traditional kind based on what I would refer to as an autocentric (or preordination)
principle, to the effect that the attribution of non-zero weighting should be restricted
retroactively to wherever one already finds oneself to be. Although it may be logically
admissible as an alternative to principles of the anthropic kind, I would maintain that
such an autocentric attitude is scientifically unreasonable, in so much as it violates the
desideratum that comparable observers be treated objectively on the same footing. By
adopting such an attitude [30], Dyson implicitly assumes for himself a privileged position
to which other observers (such as Wigner and Schroedinger) are not admitted.

Before leaving the subject of logically admissible (even if not scientifically reasonable)
alternatives to principles of the anthropic kind, I would mention a conceptually possible
alternative that is quite the opposite of the autocentric deviation described in the previous
paragraph. Instead of prescribing an a priori probability deviation with weighting re-
stricted to material observers as in the anthropic case (or to a single privileged observers
in the autocentric case) one might go so far as to envisage the attribution of non-zero
weighting even to situations where no material observer is present at all. Such an un-
reasonably overextended weighting (as exemplified by the kind of ubiquity principle that
was implicit in Dirac’s original argument in favour of his now discredited theory [31, 2] of
varying gravitational coupling) might make logical sense if one could imagine perceiving
oneself to be some sort of disembodied spirit, but (as Dirac’s example shows) does not
deserve trust for scientific purposes.

Illustration

As a toy example to illustrate the application of this micro-anthropic principle, con-
sider a gedanken experiment in which Schroedinger’s cat, C, has equal chance of being
awake or dreaming, as also does its master, M, who, if awake can see whether the cat is
too, but if asleep has equal chances of dreaming that the cat is awake or asleep, whether or
not it actually is. The cat is unconcerned about its master, and so has only two relevant
mind states $e_1, e_2$ (awake or dreaming) with entropy $S = \log 2 = 1$. Schroedinger has four
relevant mind states, $e_3, e_4, e_5, e_6$ with $S = \log 4 = 2$, so his net probability is $2/3$ while
the cat’s is $1/3$. The conditional “gross” probability $P$, and absolute “net” probability $P$
for the relevant eventualities are tabulated as follows.

| C: gross → net | $e_1$: C awake | $e_2$: C asleep | M: gross→net |
|----------------|----------------|----------------|-------------|
| $e_3$: M awake, sees C awake | $P_{31} = 1/4$ | $P_{32} = 0$ | $1/4 \rightarrow 1/6$ |
| $e_4$: M awake, sees C asleep | $P_{41} = 0$ | $P_{42} = 1/4$ | $1/4 \rightarrow 1/6$ |
| $e_5$: M dreams C awake | $P_{51} = 1/8$ | $P_{53} = 1/8$ | $1/4 \rightarrow 1/6$ |
| $e_6$: M dreams C asleep | $P_{61} = 1/8$ | $P_{62} = 1/8$ | $1/4 \rightarrow 1/6$ |

$1/2 \rightarrow 1/6$  $1/2 \rightarrow 1/6$
13. Local Application

Whereas the term entropic principle has the advantage of avoiding any risk of misunderstanding that the range of applicability of the ansatz (32) extends beyond observers of narrowly anthropic type, on the other hand the alternative term micro-anthropic principle has the advantage of advertising the applicability of the principle (as of its proper predecessor) not just to the entire life of an observer but to particular parts thereof. The question of whether one is more likely to find oneself to be nearer the beginning or nearer the end of one’s life was raised in an epilogue by Leslie [27] who suggested, on the basis of Everett’s own (confusing) presentation of his doctrine [4], that its continual multiplication of the number \( N_e \) of relevant branches entailed a probability distribution that would be heavily biased towards the last moments of life, on the understanding that the dogma that all the branch channels are equally “real” implies that the corresponding anthropic probability factor should be given by \( P_e \propto N_e \) (rather than by an expression of the entropic form \( P_e \propto \log\{N_e\} \) that has been advocated here). Having safely survived, and thereby invalidated, this alarming prediction, Leslie arrived at the observational conclusion that – as was argued on purely theoretical grounds at the beginning of this essay – this particular interpretation of the Everett doctrine is untenable.

According to the present analysis, the correct answer to Leslie’s question is as follows. To start it is necessary to reject not only Everett’s claim that the relevant branches are “real” (which might be interpreted as meaning \( P_e \propto N_e \)) but also his attribution to them, nevertheless, of an ordinary non anthropic quantum probability weighting (which might be interpreted as implying the choice of a constant value for \( P_e \)). This contradiction between Everett’s preaching and his practice is resolved in the present approach by what is interpretable as a compromise, according to which the appropriate formula has the logarithmic form \( P_e \propto \log\{N_e\} \).

The replacement of a linear by a logarithmic dependence law merely moderates, but does not avoid, the unrealistic implication that the probability distribution would strongly disfavour the earlier stages of a lifetime if Everett’s branching metaphor were to be taken literally. It is therefore obvious that this aspect of what is commonly understood to be meant by the Everett’s interpretation is also misleading, and, as remarked above, it is very easy to see why. The idea of a rapidly increasing number of relevant branch channels is something that may make sense in the case when, for example, one has just taken delivery of a new computer with entirely empty memory banks, but it will soon cease to be valid when saturation sets in so that erasure becomes necessary, so as to release occupied space by a process whereby the relevant information is converted into Landauer entropy [14]. Concerning the human case, parents and primary school teachers know that even small children do a lot of forgetting as well as learning, while as adulthood progresses the ratio of what is learned to what is forgotten goes on decreasing, so that it may ultimately become quite small compared with unity as senility sets in. This means that the relevant number \( N_e \) of Everett branch channels should normally reach a maximum – not a peak but a plateau – in mid life. It is to be understood that this statement refers to a smoothed average over diurnal variations, because the number of channels involved in conscious perception presumably undergoes considerable reduction during sleep, particularly during deep dreamless phases.
For practical probabilistic purposes it is only relative values that matter. The intrinsically interesting question of the absolute height of the plateau is beyond the scope of the present investigation, but it is evident from physical considerations that $N_e$ it cannot be nearly as large as the (admittedly gigantic) value of $\exp\{Q_e\}$, where $Q_e$ is the Dyson entropy number discussed above, which exceeds the corresponding Landenauer entropy $S_e = \log\{N_e\}$ (representing the amount of useful information processed during the perception) by an enormous thermodynamical waste factor $W_e = Q_e/S_e > 1$. (In the days before valves were replaced by transistors, the relevant waste factors for computers were far worse even than those of their biological analogues, but the spectacular progress of engineering techniques in recent years has brought about an amazing rate of improvement.)

It is a noteworthy coincidence that Dyson’s evaluation of $Q_e$ in the human case gave a value of the same order as the Avrogadro number, which is interpretable as the number of molecules in a fraction of the order of $10^{-3}$ of the mass of a human body. If it is supposed that this fraction is comparable with the fraction of the molecules that are active in metabolic processes, then it can be deduced that the corresponding metabolic turnover time must have a value of the same order of magnitude as the mental time interval $\Delta_e \tau$, of the order of a fraction of a second, that was used by Dyson’s as basis for his evaluation.

This observation – that the estimated duration $\Delta_e \tau$ of a conscious perception is roughly comparable with a timescale characterising metabolic processes throughout the body – may offer a significant clue as to the nature of the (still largely mysterious) mental processes involved. One of the things that is rather clear is that the relevant value of $\Delta_e \tau$ can undergo considerable variation – lengthening in states of hibernation for example. In so far as the solution to the problem posed by Leslie is concerned, what is relevant is the age dependence of $\Delta_e \tau$. My impression, with which I think most people would agree, is that the typical duration of a moment of consciousness is relatively short in early childhood and that on average – modulo diurnal fluctuations through states of shallow or deep sleep – it increases monotonically throughout life. According to the formula, this means that the maximum of the anthropic probability distribution need not coincide with the summit of the midlife plateau where the relevant branch channel number $N_e$ and its logarithm $S_e$ is highest, but may actually occur at a more youthful stage.
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Figure 1: Some of the participants at the Clifford Centennial meeting organised by John Wheeler at Princeton in February, 1970, assembling many of the people whose thoughts contributed to the synthesis presented here, including, in the front at left, Bob Dicke with Eugene Wigner, and in the center Stephen Hawking with the present author, behind whom are Bryce DeWitt with Freeman Dyson.