MTPA- and MSM-based Vibration Transfer of 6-DOF Manipulator for Anchor Drilling

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An anchor drilling for a coal mine support system can liberate an operator from heavy work, but will cause serious vibration, which will be transmitted to the pedestal from the roof bolter along a manipulator. Based on the multi-level transfer path analysis (MTPA) and modal superposition method (MSM), a vibration transfer model for the subsystem composed of the joints of a manipulator with six degrees of freedom (DOF) was established. Moreover, its frequency response function matrix was also built. The 6-DOF excitation of the roof bolter was deduced. The exciting force on the roof bolter transmitted to the pedestal along the 6-DOF manipulator was analysed with a force Jacobian matrix, to identify the external loading on the pedestal. A case in engineering practice shows that the amplitude of each DOF of the pedestal from large to small is as follows: bending vibration (component 1), longitudinal vibration, torsional vibration, bending vibration (component 2), rotational vibration around z-axis, rotational vibration around y-axis. The pedestal is mainly in the form of bending vibration. The theory of vibration transfer along the 6-DOF manipulator for anchor drilling proposed in this article can provide a theoretical foundation for the development of vibration-damping techniques and the design of absorbers.

Keywords: manipulator, multi-level transfer path analysis, modal superposition method, vibration transfer, force Jacobian matrix

0 INTRODUCTION

Roof bolters are key mechanical equipment for a coal mine supporting system. In the past, drilling was done manually. Working in an area with high concentrations of dust for a long time, workers’ physical and mental health will be seriously threatened. At present, the development of a coal mine tunnel support tends to be automatic and intelligent. The manual labour of roof bolters has been gradually replaced by mechanical clamping [1]. An operator can control the roof bolter to drill automatically by human-computer interaction, which can liberate the operator from heavy work, and improve the stability and safety of the coal mine support. Due to the comprehensive excitation of different geotechnical parameters, axial thrust, torque and other factors, there are complex vibrations on drill strings during construction. The main forms include bending vibration, longitudinal vibration, and torsional vibration, which interact with each other to form a nonlinear coupled vibration [2]. The excitation vibration of each degree of freedom (DOF) of the roof bolter is transmitted to the pedestal along a manipulator, which causes the manipulator to vibrate violently, shorten its service life, and then affect the support effect.

Transfer path analysis (TPA) is a tool to study vibration transfer [3] and [4]. There are several methods, such as operational transfer path analysis (OTPA) [5] and [6], global transmissibility direct transmissibility (GTDT) [7] and [8], inverse substructure TPA (ITPA) [9], multi-level transfer path analysis (MTPA) [10] and [11], and so on. Lee and Lee [5] proposed the OTPA method using an emerging deep neural network model, which can successfully predict the path contributions using only operational responses. Yoshida and Tanaka [6] attempted to calculate the vibration mode contribution by modifying OTPA, and then considered the relationship between the principal component and the vibration mode, as well as the associated the principal components with the vibration modes of a test structure. High contributing vibration modes to the response point have been found. It is easily disturbed by factors such as excitation coupling and noise employing this method when calculating the transfer matrix. Wang [7] developed further the prediction capabilities of the GTDT method, which can predict a new response using measured variables of an original system, even though operational
forces are unknown. Guasch [8] addressed some issues concerning the prediction capabilities of the GTDT method when blocking transfer paths in a mechanical system and outlined differences with the more standard force TPA. Wang [9] developed the SDD method further by considering the mass effect of resilient links, which can identify decoupled transfer functions accurately, whilst eliminating the mass effect of resilient links. However, its manoeuvrability is poor for a serial system with many substructures. Gao [11] used MTPA to find the critical paths of seat jitter caused by dynamic unbalance excitation of the drive shaft. The key technology of this method is to identify the external excitation loading, which has good operability for series system.

For the vibration problem of pedestal from manipulator caused by the excitation of a roof bolter, a response amplitude matrix in pedestal is established by the modal superposition method (MSM) in this article. According to the excitation of the roof bolter, the external loading of the response point of the manipulator pedestal is analysed using the force Jacobian matrix. The 6-DOF frequency response function of each subsystem of the manipulator is derived by MTPA, and then the frequency response matrix is constructed, which can solve the problem of transfer path analysis with low accuracy and poor operability. It will provide a theoretical foundation for the development of vibration damping techniques and the design of absorbers.

1. MULTI-LEVEL TRANSFER PATH ANALYSIS

To reduce the influence of non-important factors while analysing the vibration transfer of manipulator for anchor drilling, some simplifications are made as follows. 1) Each linkage of the manipulator is equivalent to a bar with uniform mass; 2) some transfer mechanisms, such as belt driving and harmonic decelerator in the manipulator, are equivalent to linear massless springs; 3) the modal parameters of the manipulator are linear, namely, the output caused by any combined input are equal to the combination of respective outputs; 4) it satisfies the assumption of time-invariance [12], namely, the dynamic properties of the system are not vary with time. According to the above simplifications, a vibration transfer model of the manipulator for anchor drilling is established, as shown in Fig. 1.

This is a multi-input and multi-output system, in which the six joints of the manipulator are connected in series. The vibration source is the roof bolter that provides excitation; the vibration receiver is the pedestal. The manipulator can be divided into six subsystems by the rotating joint. The external loading of the J6-subsystem is the excitation from the roof bolter; that of other subsystems is the output of the previous subsystem. The output (response point) of the subsystem is the input (exciting point) of the next subsystem. Nevertheless, the output of the J1-subsystem acts on the pedestal. According to different excitations of spatial degrees of freedom, each subsystem has $m$ inputs and $n$ outputs.

According to MTPA, the transfer function of manipulator for anchor drilling can be expressed by the product of the transfer functions of all subsystems [11].

$$H = H_{J1}H_{J2} \cdots H_{J6}.$$ (1)

The vibration response of the excitation from the roof bolter transmitted to the pedestal is expressed as follows.

$$S = HF.$$ (2)

2 MODAL SUPERPOSITION METHOD

2.1 Response Amplitude Matrix

The dynamic equation of the manipulator for anchor drilling is as follows.

![Fig. 1. Vibration Transfer model of the manipulator](image-url)
\[
M \ddot{\mathbf{S}} + C \dot{\mathbf{S}} + K \mathbf{S} = \mathbf{F}.
\]  
(3)

According to the linear hypothesis [12], the displacement column vectors of each subsystem of the manipulator can be expressed as the linear addition of each order of modal shapes.

\[
\mathbf{S} = \sum_{a=1}^{n} q_a \lambda_a.
\]  
(4)

Substituting Eq. (4) into Eq. (3), then,

\[
M \sum_{a=1}^{n} \ddot{q}_a \lambda_a + C \sum_{a=1}^{n} \dot{q}_a \lambda_a + K \sum_{a=1}^{n} q_a \lambda_a = \mathbf{F}.
\]  
(5)

Both ends of Eq. (5) are multiplied by \( \lambda_b^T \), then,

\[
\lambda_b^T M \left( \sum_{a=1}^{n} \ddot{q}_a \lambda_a \right) + \lambda_b^T C \left( \sum_{a=1}^{n} \dot{q}_a \lambda_a \right) + \lambda_b^T K \left( \sum_{a=1}^{n} q_a \lambda_a \right) = \lambda_b^T \mathbf{F}.
\]  
(6)

According to the orthogonality of the main modal shape [13], Eq. (7) can be obtained from Eqs. (5) and (6).

\[
\begin{aligned}
\lambda_b^T M \lambda_a &= M_{a\cdot a} = \begin{cases} M_a, & a = b \\ C_a, & a = b \\ K_a, & a = b \\ 0, & a \neq b \end{cases} \\
\end{aligned}
\]  
(7)

Substituting Eq. (7) into Eq. (6), then,

\[
M_a \ddot{q}_a + C_a \dot{q}_a + K_a q_a = \lambda_a^T \mathbf{F} = \lambda_b^T \mathbf{F}.
\]  
(8)

The excitation loading and displacement response in Eq. (8) are expressed in complex form as follows.

\[
\begin{aligned}
\mathbf{F} &= f e^{i \omega t} \\
q_a &= \bar{q}_a e^{i \omega t}.
\end{aligned}
\]  
(9)

Substituting Eq. (9) into Eq. (8),

\[
\bar{q}_a = \frac{\lambda_a^T f}{-\omega^2 M_a + j \omega C_a + K_a}.
\]  
(10)

Substituting Eqs. (10) and (9) into Eq. (4), \( \mathbf{S}^T \) can be obtained as Eq. (11).

\[
\mathbf{S}^T = \sum_{a=1}^{n} \lambda_a^T \frac{f e^{i \omega t} \lambda_a}{-\omega^2 M_a + j \omega C_a + K_a}.
\]  
(11)

It is assumed that the system has two points: \( o \) and \( p \), substitute Eqs. (11) and (9) into Eq. (4), the response amplitude of the point \( p \) can be expressed as follows.

\[
S_p = \sum_{a=1}^{n} \frac{\lambda_a^T F_{ap} \lambda_a}{K_a \left( 2 \frac{j \omega}{\omega^2 M_a + K_a} \right) - \frac{\omega^2 M_a}{K_a} + 1}.
\]  
(12)

Their frequency response function (FRF) is as Eq. (13).

\[
H_{po} = \frac{S_p}{F_o} = \sum_{a=1}^{n} \frac{\lambda_a^T \lambda_{pa}}{K_a \left( 2 \frac{j \omega}{\omega^2 M_a + K_a} \right) - \frac{\omega^2 M_a}{K_a} + 1}.
\]  
(13)

According to the linear superposition assumption [12], when \( \mathbf{F} = [F_1 F_2 \cdots F_n]^T \), the response amplitude of each point of the system is as Eq. (14).

\[
\mathbf{S} = \begin{bmatrix} S_1 \\ S_2 \\ \vdots \\ S_N \end{bmatrix} = \begin{bmatrix} H_{11} & H_{12} & \cdots & H_{1N} \\ H_{21} & H_{22} & \cdots & H_{2N} \\ \vdots & \vdots & \ddots & \vdots \\ H_{N1} & H_{N2} & \cdots & H_{NN} \end{bmatrix} \begin{bmatrix} F_1 \\ F_2 \\ \vdots \\ F_N \end{bmatrix},
\]  
(14)

### 2.2 Parameter Identification

Since the excitation of the roof bolter includes all DOF in space, and each subsystem has \( m(6) \) inputs and \( n(6) \) outputs, the frequency response of each subsystem of the manipulator is a \( 6 \times 6 \) th order matrix, as follows.

\[
\mathbf{H}_J = \begin{bmatrix} H_{11} (J_1) & H_{12} (J_1) & \cdots & H_{16} (J_1) \\ H_{21} (J_1) & H_{22} (J_1) & \cdots & H_{26} (J_1) \\ \vdots & \vdots & \ddots & \vdots \\ H_{61} (J_1) & H_{62} (J_1) & \cdots & H_{66} (J_1) \end{bmatrix}.
\]  
(15)

The parameters of the \( J_1 \) to \( J_6 \) frequency response curves are identified in the frequency domain [14] by the rational polynomial method [15]. Its mathematical model is a rational formula of frequency response function, as follows.

\[
H_{\text{fin}} (J_1) = \frac{\alpha_1 x^3 + \alpha_2 x^4 + \alpha_3 x^5 + \alpha_4 x^6 + \alpha_5 x + \alpha_6}{\beta_1 x^3 + \beta_2 x^4 + \beta_3 x^5 + \beta_4 x^6 + \beta_5 x + \beta_6}.
\]  
(16)
3 EXTERNAL EXCITATION LOADING

3.1 Excitation from Roof Bolter

Force and moment on roof bolter: \( \mathbf{F}_g, \mathbf{F}_a, \mathbf{F}_z, \mathbf{F}_c, \) and \( \mathbf{M}_d. \) The direction of \( \mathbf{F}_g, \mathbf{F}_a \) and \( \mathbf{F}_z \) is along the shaft of the roof bolter, and their vector expressions is as follows.

\[
\begin{align*}
\mathbf{F}_g & = \begin{bmatrix} F_g \end{bmatrix} \quad \mathbf{0} \quad \mathbf{0} \quad \mathbf{0} \quad \mathbf{0} \quad \mathbf{0} \\
\mathbf{F}_a & = \begin{bmatrix} F_a \end{bmatrix} \quad \mathbf{0} \quad \mathbf{0} \quad \mathbf{0} \quad \mathbf{0} \quad \mathbf{0} \\
\mathbf{F}_z & = \begin{bmatrix} F_z \end{bmatrix} \quad \mathbf{0} \quad \mathbf{0} \quad \mathbf{0} \quad \mathbf{0} \quad \mathbf{0} \quad \mathbf{0} \\
\mathbf{F}_c & = \begin{bmatrix} F_c \end{bmatrix} \quad \mathbf{0} \quad \mathbf{0} \quad \mathbf{0} \quad \mathbf{0} \quad \mathbf{0} \quad \mathbf{0} \\
\mathbf{M}_d & = \begin{bmatrix} 0 & 0 & 0 & M_d & 0 & 0 \end{bmatrix}.
\end{align*}
\]

(17)

The direction of \( \mathbf{M}_d \) is along the shaft of the roof bolter, and its vector expression is as follows.

The excitation from the roof bolter at the tip of the manipulator is as Eq. (21).

\[
\mathbf{F}_q = \begin{bmatrix} F_g + F_z - F_a \quad F_z \quad M_d \quad 0 \quad 0 \quad 0 \end{bmatrix},
\]

(21)

3.2 Excitation to Pedestal

Force and torque on pedestal: \( \mathbf{F}_{g2}, \tau_1. \) Each joint of the manipulator can rotate independently. To accurately describe the mechanical properties of the excitation transmitted to the pedestal through the joints of the manipulator, a force Jacobian matrix of the manipulator is introduced [17]. The transfer relationship between the excitation and the joint generalized driving force is as follows [18].

\[
\mathbf{\tau} = \mathbf{J}(\mathbf{q})^T \mathbf{F}
\]

(22)

The torques of each joint of the manipulator is as follows (Eq. (23)).

\[
\mathbf{F}_s = F \sin \gamma \cos \phi; \quad \mathbf{F}_y = F \sin \gamma \cos \phi; \quad \mathbf{F}_z = F \cos \gamma.
\]

(23)
4 CASE IN ENGINEERING PRACTICE

4.1 Essential Parameters

A 6-DOF manipulator for anchor drilling in a coal mine in Huainan, China, is taken as the research object. The size of the two-wing drill adopted is \( \phi 32 \) mm; the length of drill string is 86 mm. The drilling object is sandstone, and its mechanical parameters [19] are as follows: \( \rho = 2600 \text{ kg/m}^3; R = 38 \text{ MPa}; R_m = 0.34 \text{ MPa}; E = 12 \text{ GPa}; \mu = 0.25; F_a = 6000 \text{ N}; M_d = 130 \text{ N}; \Omega = 0 \text{ mm}; k = 10^9 \text{ N·m}^{-1}; \) self-weight of the roof bolter is 40 kg; self-weight of the manipulator is 550 kg. Moreover, the parameters of the linkages of the manipulator are shown in Table 1 [20].

The three-dimensional model of the manipulator is shown in Fig. 2. Some finite element models of the manipulator are established, as shown in Fig. 3. The rotating joints of the manipulator are divided into some subsystems, and its exciting points and response points are determined.

Based on MTPA and MSM, the computation flow chart of the vibration transfer of 6-DOF manipulator for anchor drilling is shown Fig. 4.

![Image](image1)

**Table 1. Parameters of the linkages of the manipulator**

| Linkages | Angle variable variable $a_i$ | $d_i$ | Angle range [rad] |
|----------|-------------------------------|-------|-------------------|
| 1        | 0                             | 0.56  | -3.14 to 3.14     |
| 2        | 0.90                          | 0     | -2.27 to 1.22     |
| 3        | 0.16                          | 0     | -1.40 to 3.05     |
| 4        | 0                             | 1.01  | -6.28 to 6.28     |
| 5        | 0                             | 0     | -2.09 to 2.09     |
| 6        | 0                             | 0.2   | -6.28 to 6.28     |

4.2 Frequency Response Curves

The excitation of the roof bolter is high-frequency vibration; the frequency range in practice is 0 Hz to 200 Hz [21]. Substituting Eqs. (19) and (21) into Eq. (13), the frequency responses of subsystems are analysed using ABAQUS [22]. The acceleration frequency response curves of $J_1$ to $J_6$ are shown in Figs. 5 to 10.

According to Figs. 5 to 10, there are resonance peaks [23] in the frequency response curves of each subsystem of the manipulator for anchor drilling in the...
Fig. 4. Computation flow chart

Fig. 5. Frequency response curves of $J_1$; a) translational acceleration, and b) rotational acceleration

Fig. 6. Frequency response curves of $J_2$; a) translational acceleration, and b) rotational acceleration

Fig. 7. Frequency responses curve of $J_3$; a) translational acceleration, and b) rotational acceleration
frequency range of 0 Hz to 200 Hz. The frequencies corresponding to the peak values of the 6-DOF frequency response curves is shown in Table 2.

4.3 FRF Matrixes of Subsystems

The frequency response curves of each subsystem are imported into MATLAB, which are fitted according to Eq. (16) with the “Curve Fitting Tool” toolbox [24]. The coefficients of each element in the frequency response matrixes are shown in Tables 3 to 8.

Table 2. Frequencies corresponding to peak values

| Joints | DOF Frequencies |
|--------|-----------------|
|        | $x$  | $y$  | $z$  | $x^R$ | $y^R$ | $z^R$ |
| $J_1$  | 191.3 | 191.3 | 192.2 | 197.3 | 197.4 | 197.2 |
| $J_2$  | 88.95 | 90.32 | 90.32 | 91.55 | 91.34 | 91.24 |
| $J_3$  | 42.19 | 42.19 | 91.77 | 86.2  | 82.9  | 88.1  |
| $J_4$  | 137.5 | 135.2 | 137.9 | 135.3 | 137.8 | 137.8 |
| $J_5$  | 178.8 | 178.4 | 178.8 | 178.4 | 178.7 | 178.8 |
| $J_6$  | 91.27 | 91.34 | 91.54 | 81.66 | 73.24 | 95.54 |
4.4 Torques of Joints

Substituting the data in Table 1 into Eq. (23), the values of $\tau_1$ change with $\theta_1$, $\gamma$ and $\phi$ are obtained by MATLAB, as shown in Fig. 11. $\tau_1$ is distributed symmetrically with the change of joint angle of the manipulator. When $\theta_2$ is at the ultimate angle of $-2.27$ rad and $\theta_4$ is at that of $-6.28$ rad, $\tau_1$ is only $-2209$ N·m as $\theta_1$ and $\theta_3$ change. While $\theta_4 \in (-1.24 \sim 1.13)$ rad, $\tau_1$ shows a trend of decay, when $\theta_3 \in (1.13 \sim$
2.30) rad, $\tau_1$ shows a steady trend; and $\theta_4 \in (2.30 \sim 3.04)$ rad, $\tau_1$ showed a slight upward trend. When $\theta_6$ is at the ultimate angle of $-6.28$ rad, $\tau_1$ is distributed symmetrically with the change of $\theta_6$; and $\tau_{\max} = 6018$ N·m. As $\gamma$ and $\varphi$ change, $\tau_1$ is symmetrically distributed obviously; and $\tau_{1\ max} = 9117$ N·m. The maximum of $\tau_1$ applied to the pedestal is 9117 N·m, which is transmitted along the 6-DOF manipulator with any position and posture in space.
4.5 Vibration Response of the Pedestal

Substituting the frequency response matrix of each subsystem into Eq. (1) and substituting Eqs. (1) and (24) into Eq. (2), the vibration responses on each DOF of the pedestal are shown in Fig. 12, in which the positive and negative values of the vibration response only represent the direction.

As shown in Fig. 12a, the response of the longitudinal vibration \( S_x \) of the pedestal reaches the maximum, being \( 1.65 \times 10^{-2} \) m, when the frequency is 45 Hz. While the frequencies are 90 Hz and 180 Hz, the responses of the two components \( S_y \) and \( S_z \) of the bending vibration of the pedestal reach the maximum, being \( 2.12 \times 10^{-2} \) m and \( 8.06 \times 10^{-3} \) m, respectively. At this time, the frequencies corresponding to the peaks are integer multiples of each other, so the phenomenon of resonance will happen.

As shown in Fig. 12b, the response of the torsional vibration \( S_{xR} \) of the pedestal reaches the maximum, being \( 9 \times 10^{-3} \) m, when the frequency is 190 Hz. The two components \( S_{yR} \) and \( S_{zR} \) of rotational vibration...
around $y$ and $z$ axes sharply increase in the frequency interval [166-181] Hz and [151-180] Hz, and reach the maximum, being $2.08 \times 10^{-4}$ m and $2.46 \times 10^{-4}$ m respectively, at the frequencies of 180 Hz and 181 Hz. At this time, the frequencies corresponding to the peaks are very similar, which is easy to induce a resonance. The amplitude of each DOF of the pedestal is from large to small is as follows: bending vibration (component 1) $S_{y1}$, longitudinal vibration $S_{x1}$, torsional vibration $S_{xz1}$, bending vibration (component 2) $S_{x2}$, rotational vibration around $z$-axis $S_{zrot}$, rotational vibration around $y$-axis $S_{yrot}$.

5 CONCLUSIONS

(1) Based on MTPA and MSM, a mathematical model of vibration transfer of 6-DOF manipulator for anchor drilling is established. The frequency response matrix of subsystems is derived under multi-DOF excitations. When the external excitation loading is determined, the frequency response function of each DOF at the response point can be calculated by the mathematical model, which is universal for series systems.

(2) Within the frequency range (0 Hz to 200 Hz) of the excitation of the roof bolter, the corresponding frequency ranges of the peak values in the 6-DOF vibration direction of each subsystem are [191.3, 197.4] Hz, [88.95, 91.55] Hz, [42.19, 91.77] Hz, [135.2, 137.8] Hz, [178.8, 187.4] Hz and [73.24, 95.54] Hz, respectively. In the above frequency ranges, the vibration response will be the largest, and the resonance can be avoided by changing the excitation frequency of the roof bolter.

(3) A case in engineering practice shows that the amplitude of each DOF of the pedestal is from large to small is as follows: bending vibration (component 1) $S_{y1}$ (2.12×10^{-2} m) at 90 Hz, longitudinal vibration $S_{x1}$ (1.65×10^{-2} m) at 45 Hz, torsional vibration $S_{xz1}$ (9×10^{-3} m) at 190 Hz, bending vibration (component 2) $S_{x2}$ (8.06×10^{-3} m) at 180 Hz, rotational vibration around $z$-axis $S_{zrot}$ (2.46×10^{-4} m) at 180 Hz, rotational vibration around $y$-axis $S_{yrot}$ (2.08×10^{-4} m) at 180 Hz. Obviously, the pedestal is mainly in the form of bending vibration.

(4) The case also shows that a resonance will occur among the two components of the bending vibration at the frequencies of 90 Hz and 180 Hz; a resonance among the two components of rotational vibration around the $y$ and $z$ axes is highly likely to occur at the frequencies of 180 Hz and 181 Hz.

The theory of vibration Transfer along the 6-DOF manipulator for anchor drilling proposed in this article can provide a theoretical foundation for the development of vibration damping techniques and the design of absorbers.

6 NOMENCLATURES

- $\lambda_a$ the $a$th order modal shape
- $\lambda^T_b$ the $b$th order modal shape
- $\lambda_{oa}$ modal shape of the $o$th DOF of the $a$th modal vector
- $\lambda_{pa}$ modal shape of the $p$th DOF of the $a$th modal vector
- $q_a$ modal participation factors
- $o$ exciting point
- $p$ response point
- $n$ modal order
- $M_a$ modal mass coefficient
- $C_a$ modal damping coefficient
- $K_a$ modal stiffness coefficient
- $M$ mass matrix
- $C$ damping matrix
- $K$ stiffness matrix
- $\dot{S}$ vibration displacement
- $\ddot{S}$ vibration speed
- $\dddot{S}$ vibration acceleration
- $J(q)^T$ force Jacobian matrix
- $J$ joint
- $J$ elements in Jacobian matrix
- $d_i$ distance between two adjacent linkages along the common axis [m]
- $a_{i-1}$ common perpendicular length between joint $i$ and joint $i$ [m]
- $z$ moving DOF of $z$-axis
- $x$ moving DOF of $x$-axis
- $y$ moving DOF of $y$-axis
- $z^R$ rotational DOF of $z$-axis
- $x^R$ rotational DOF of $x$-axis
- $y^R$ rotational DOF of $y$-axis
- $S_{y1}$ bending vibration (component 1) [m]
- $S_{x1}$ longitudinal vibration [m]
- $S_{xz1}$ bending vibration (component 2) [m]
- $S_{x2}$ rotational vibration around $x$-axis (torsional vibration) [m]
- $S_{yrot}$ rotational vibration around $y$-axis [m]
- $S_{zrot}$ rotational vibration around $z$-axis [m]
- $F$ external loading [N]
- $F_y$ force projected on the $y$-axis [N]
- $F_o$ loading at $o$ point [N]
- $F_{i}$ excitation force transmitted to joints [N]
- $F_{x}$ force projected on the $x$-axis [N]
$F_{g2}$ gravity of manipulator [N]  
$F_g$ gravity [N]  
$F_a$ axial thrust [N]  
$F_z$ disturbing force [N]  
$F_c$ impact force [N]  
$M_d$ torque  
$M_x$ torque projected on the $x$-axis [Nm]  
$M_y$ torque projected on the $y$-axis [Nm]  
$M_z$ torque projected on the $z$-axis [Nm]  
$v$ bending vibration displacement (component 2) [m]  
$w$ bending vibration displacement (component 1) [m]  
$\mu$ Poisson’s ratio  
$D$ outer diameter of drill string [m]  
$\rho$ material density [kg·m$^{-3}$]  
$R$ compressive strength [MPa]  
$R_m$ tensile strength [MPa]  
$E$ elastic modulus [GPa]  
$k$ elastic coefficient of impact force [N·m$^{-1}$]  
$\gamma$ angle between force and $z$-axis [rad]  
$\phi$ angle between force and $x$-axis [rad]  
$\theta_i$ joint angle [rad]  
$\Omega$ clearance between drill string and rock-soil [mm]  
$\xi_a$ damping ratio  
$\tau$ torque matrix  
$\tau_i$ torque from the $J_i$-axis  
$\alpha_i$ molecular coefficients, $i = 1$ to 6  
$\beta_i$ denominator coefficients, $i = 1$ to 6

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