A Bayesian Analysis for Circular Galaxies Using a Bose–Einstein Condensate as a Dark Matter Halo

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Abstract. In this paper we present an extension of a Bayesian study for a Bose–Einstein Condensate type halo surrounding a Schwarzschild-type black hole. The predictions obtained via the Thomas–Fermi approximation are tested using observations from SPARC galaxy rotation curves sampler with 10 circular galaxies and whose inclination (angle) is less that 80 deg. The statistical analysis is confronted with the Navarro–Frenk–White density profile model where we found that at the centre of galaxies is require a supermassive black hole with a constrain of \( \log_{10} M/M_\odot = 12.00 \pm 0.73 \) to allow a BEC configuration at this stage. Also, we show that the proposed scenario can fit the galaxy rotation curves on average rate \( \chi^2_{BEC}/\chi^2_{NFW} = 1.49 \).

1. Introduction

According to the current standard cosmological model a cold dark matter (CDM) component dominates the formation of structures (see [1] and references therein). Around 4% of the cosmological density is accounted for by baryons, 23% by the dark matter, with the remainder percent being the dark energy, which is responsible for the observed accelerated cosmic expansion. Dark matter has been successful in explaining cosmic structure over a considerably large redshift, but due its inherent properties it has confronted some difficulties from observations that explore the innermost regions of halos around a dwarf galaxies. Using rotation curve measurements it is possible to compute the mass profile of a dark matter halo. Regardless of the uncertainties in some scenarios, there has been some advances in the observational surveys, which can be used to compute a better statistical fit with several dark matter profiles. Usually, these profiles are compared with the standard Navarro-Frenk-White (NFW) model, but there are some particular galaxies that cannot be fitted well with it. This inconsistency between fits are the so-called cusp-core problem [2] and therefore they require modifying the halo profiles of typical galaxies away from the profiles that are standard ones.

Recently, attention has focused in that dark matter could consist on some type of generic scalar fields of spin zero, e.g Weakly Interacting Massive Particles (WIMPs), axions, neutrinos, Mev dark matter, WIMPzillas, and the list is by no means exhaustive [3]. Moreover, due that these kind of particles have been not observed yet, scalar field(s) models interpreted as dark matter can offer a realistic possibility to confront with observations. An interesting proposal is the Bose-Einstein condensate (BEC), which can offer an interpretation of dark matter in the form of a condensate of generic bosonic particles [4, 5]. To achieve this scenario we need a scalar field stable configuration surrounded a black hole [6]. If this configuration is stable
enough we can obtain a reasonable candidate to describe dark matter galactic halos. When self–interactions and quasi–bound states are present, the system can be analysed as a BEC configuration [7]. Furthermore, this idea is reinforced if we consider that a large population of galaxies have supermassive black hole at their center.

In this proposal, a Gross–Pitaevskii equation can be calculated from a Klein–Gordon equation of the system. After that, a Thomas–Fermi approximation can be performed over the resulting equation and be accurate when the interaction among the particles in the system is smaller in comparison to the mean inter–particle spacing for large dimension clouds [8, 9]. Our proposal will focus on the use of this solution as the density distribution of a dark matter galactic halo. Consequently, we will confront a group of valid solutions with the corresponding rotation curves of some set of galaxies given by the SPARC sampler [10].

This paper is organised as follows: we constrain the parameters of a proposed BEC–DM model by using a total of 10 circular galaxies type with observational characteristics of 3.6µm photometry and whose inclination (angle) is less that 80 deg from SPARC sampler. The mass density of the BEC is given in terms of the mass of the bosonic particles $M_\Phi$, a frequency $\omega$, a coupling constant $\lambda$ and, the only astrophysical parameter: the mass of the black hole $M$. Thenceforth, we will use Bayesian inference to update our evidence about unknown parameters, e.g., mass boson, with information from the sampler and performing the comparison with a NFW density profile.

2. Bose–Einstein condensate as a dark halo configuration

We start by considering a static and spherical symmetric spacetime ansatz with pure dark matter in Schwarzschild coordinates with a spherically symmetric solution [11]:

$$ds^2 = -f(r)dt^2 + \frac{dr^2}{f(r)} + r^2(d\theta^2 + \sin^2\theta d\phi^2).$$ (1)

The dynamics of a scalar test field $\Phi$, with a scalar self–interacting potential has the following form

$$V(\Phi \Phi^*) = \mu^2 \Phi^* \Phi + \frac{\lambda}{2} (\Phi^* \Phi)^2,$$ (2)

where $\mu = M_\Phi c/\hbar$, is the scalar mass parameter and $M_\Phi$ is the mass of the bosonic particle. In order to obtain the time independent form of the Klein–Gordon equation, we use the monopolar component of the scalar field with harmonic time dependence: $\Phi = e^{i\omega t} u(r)r^{-1}$. Thus, the Klein–Gordon equation reduces to

$$\left(-\frac{d^2}{dr^2} + V_{\text{eff}} + \lambda_{\text{eff}} \rho_n\right) u = \mu_{\text{eff}} u,$$ (3)

which is the so–called Gross–Pitaevskii equation, where the effective chemical potential is $\mu_{\text{eff}} = \omega^2/c^2$ and we have introduced $dr^* = dr/f$, as a function $r = r(r^*)$. Consequently, the effective trapping potential can be written as

$$V_{\text{eff}}(r) = f \left( \sigma^2 + \frac{f'}{r} \right),$$ (4)

and the effective self–interaction parameter can be defined by $\lambda_{\text{eff}} = \lambda f$, with

$$f(r) = \left( \frac{r^2}{r_0^2} + 1 \right)^{-\frac{2\sigma_0 (1 - \frac{1}{r_0^2}) r_0^2}{r}} \left( \frac{r}{r_0} + 1 \right)^{-\frac{4\sigma_0 (\frac{1}{r_0} + 1) r_0^2}{r}} e^{\frac{4\sigma_0 (\frac{1}{r_0} + 1) r_0^2}{r} \tan^{-1}(\frac{r}{r_0})}.$$ (5)
From the latter equation we can obtain the effective potential

\[ V_{\text{eff}}(r) = \left( \frac{r^2}{r_0^2} + 1 \right) \frac{2\pi \rho_0^2 (r - r_0)}{r} \left( \frac{r + r_0}{r_0} \right)^{-\frac{4\pi \rho_0^2 (r + r_0)}{r}} e^{\frac{4\pi \rho_0^2 (r + r_0)}{r} \tan^{-1} \left( \frac{r}{r_0} \right)} \times \]

\[ \sigma^2 - \frac{2\pi \rho_0^3 (r_0^2 - 1)}{r_0} \left( \frac{r^2}{r_0} + 1 \right) \frac{2\pi \rho_0^2 (r - r_0)}{r} \left( \frac{r + r_0}{r_0} \right)^{-\frac{4\pi \rho_0^2 (r + r_0)}{r}} e^{\frac{4\pi \rho_0^2 (r + r_0)}{r} \tan^{-1} \left( \frac{r}{r_0} \right)} \]

\[ \left( -\log \left( \frac{r^2}{r_0} + 1 \right) - 2 \log \left( \frac{r + r_0}{r_0} \right) + 2 \tan^{-1} \left( \frac{r}{r_0} \right) \right) \right] . \]

(6)

In this scenario the form of (3) can be given by

\[ (V_{\text{eff}} + \lambda_{\text{eff}} \rho_n) u = \mu_{\text{eff}} u, \]

with an exact solution of the form

\[ \rho_n = \lambda^{-1} \left[ \mu - \left( \frac{r^2}{r_0^2} + 1 \right) \frac{2\pi \rho_0^2 (r - r_0)}{r} \left( \frac{r + r_0}{r_0} \right)^{-\frac{4\pi \rho_0^2 (r + r_0)}{r}} e^{\frac{4\pi \rho_0^2 (r + r_0)}{r} \tan^{-1} \left( \frac{r}{r_0} \right)} \right] \]

\[ \sigma^2 - \frac{2\pi \rho_0^3 (r_0^2 - 1)}{r_0} \left( \frac{r^2}{r_0} + 1 \right) \frac{2\pi \rho_0^2 (r - r_0)}{r} \left( \frac{r + r_0}{r_0} \right)^{-\frac{4\pi \rho_0^2 (r + r_0)}{r}} e^{\frac{4\pi \rho_0^2 (r + r_0)}{r} \tan^{-1} \left( \frac{r}{r_0} \right)} \]

\[ \left( -\log \left( \frac{r^2}{r_0} + 1 \right) - 2 \log \left( \frac{r + r_0}{r_0} \right) + 2 \tan^{-1} \left( \frac{r}{r_0} \right) \right) \right] . \]

(8)

3. Bayesian analysis for the density rotation curves

We describe our sampler with 10 circular galaxies and whose inclination (angle) is less that 80 deg from SPARC with 30 observables data points for each one. The main characteristics of these selections were based on observations near to the galactic center, with continuous rotation curves, with not wiggles and extended to large radii and without – or small – bulge. All these properties provide a good estimate of the free parameters of our model. The SPARC sampler offers robust mass models for the complete sample of galaxies using Spitzer 3.6 μm photometry. In most cases the stellar disk can be described by a single exponential disk. Data as the distance \( D \), inclination (angle – deg –), stellar mass-to-light ratio \( \Upsilon \) are given by the sampler.

For our approach, we use the observed rotation curve, stellar, and gas component as priors for the numerical analysis. The best fit relative to a given galaxy is defined as the value that minimizes the function [12]:

\[ \chi^2(r, \rho_0, r_s, r_c) = \sum_{i=1}^{N_g} \frac{(V(r, \rho_0, r_s, r_c) - V_{\text{obs},i})^2}{\sigma_i^2}, \]

(9)

where \( N_g \) is the number of data points, \( V_{\text{obs},i} \) is the velocity measured at \( r \) with the error \( \sigma \), \( r_s \) is the characteristic radius of the halo and \( \rho_0 \) is the scale radius. The model proposed include:
Figure 1. Best-fit rotational curves of one galaxy from SPARC sampler. Blue dots denote the total observed rotational velocity with error-bars. The NFW (top left plot) and BEC (top right plot) models are represented by the black solid line, respectively. The red dashed line stands for the contribution of the bulge, the orange solid line shows the contribution of the stars, and the dotted green line denotes the contribution of the gas. Bottom left: MCMC best fits and the corresponding posterior distribution. The dark yellow and light yellow bands show the 68% and 95% confidence regions, respectively, considering the posterior distribution of $\Upsilon_{\text{disk}}$; in this approach we do not include additional uncertainties on the inclination (angle) and the distance $D$.

The bulge – when is present – the gas disk $V_\text{g}$, the stellar disk $V_\star$, and the BEC halo $V_\text{bec}$. The observed rotation velocity for such model is given by

$$V_\text{tot}^2 = V_\text{g}^2 + V_\text{bec}^2 + \Upsilon_b V_\text{b}^2 + \Upsilon_\star V_\star^2,$$

where the free parameters $\Upsilon_\star$ and $\Upsilon_b$ are the mass-to-light ratio of the star and bulge disk, respectively, hold the relation $\Upsilon_b = 1.4 \Upsilon_\star$.

We compare our BEC–DM density profile with a NFW model [13]: $\rho_{\text{NFW}} = \rho_0 \left[ \frac{r}{r_s} (1 + r/r_s)^2 \right]^{-1}$. The mass distribution gives rise to a halo rotation curve $v_{\text{NFW}}^2 = V_{200}^2 \left[ \ln(1+c x) - c x (1+c x) \right]^{-1}$, with a concentration parameter $c \equiv r_{200}/r_s$, where $r_{200}$ is the virial radius within the mean density is 200 times the critical density $\rho_{\text{crit}}$, $x$ is the radius scaled with $r_{200}$. For a given virial mass $M_{200}$, its virial radius $r_{200}$ is fixed and the profile has only one free parameter $c$.

According to these analyses, we have obtained an average $\mu = 4.06 \pm 0.97$ pc$^{-1}$, which corresponds to a boson mass of $M_{\Phi} = (3.47 \pm 1.43) \times 10^{-23}$ eV. For the self-interacting
parameter we get the values $\log_{10}(\lambda [\text{pc}^{-1}]) = -89.83 \pm 0.78$, which can be interpreted as a dark matter halo viewed as a BEC profile. The mass of the black hole at the galaxy centre gives $\log_{10} M/M_\odot = 12.00 \pm 0.73$ for the best fit galaxy IC2574 with $\ln B = 4.44$ in comparison with NFW profile with $\Upsilon_{\text{disk}} = 0.0.067^{+0.33}_{-0.10}$, $c = 0.84 \pm 0.05$ and $v_{200} = 180.01 \pm 0.19$.

4. Discussion

In this paper we notice that using a BEC configuration as a dark matter halo model is statistically enough on average $\chi^2_{\text{BEC}}/\chi^2_{\text{NFW}} = 1.49$, in comparison to NFW density profile in order to describe galaxy rotation curves. It is important to remark that this proposal comes from first principles, in where a Thomas–Fermi approximation can be employed by assuming only a compact central object that condensates the relativistic boson cloud. This proposal can be generalised in order to include bulges and central stars contributions that may reduce the value of the mass of the black hole, or the self-gravitating nature of the halo with rotation effects. A preliminary extension of this work has been reported in [14].

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References

[1] Magaña J and Matos T 2012 J. Phys. Conf. Ser. 378 012012; Salucci P 2018 Found. Phys. 48 1517; Salucci P 2019 Astron. Astrophys. Rev. 27 2
[2] de Blok W J G 2010 Adv. Astron. 2010 789293
[3] Boehm C, Hooper D, Silk J and Casse M 2004 Phys. Rev. Lett. 92 101301; Sikivie P 1982 Phys. Rev. Lett. 48 1156; Preskill J, Wise M B and Wilezczek F 1983 Phys. Lett. B 120 127; Abbott L F and Sikivie P 1983 Phys. Lett. B 120 133; Dine M and Fischler W 1983 Phys. Lett. B 120 137; Fukuda Y et al. [Super-Kamiokande Collaboration] 1998 Phys. Rev. Lett. 81 1562
[4] Boehmer C G and Harko T 2007 JCAP 025
[5] Li B, Rindler-Daller T and Shapiro P R 2014 Phys. Rev. D 89 083536
[6] Barranco J, Bernal A, Degollado J C, Diez-Tejedor A, Megevand M, Nunez D and Sarbach O 2012 Phys. Rev. Lett. 109 081102
[7] Barranco J, Bernal A, Degollado J C, Diez-Tejedor A, Megevand M, Nunez D and Sarbach O 2017 Phys. Rev. D 96 024049
[8] Castellanos E, Escamilla-Rivera C, Macias A and Nunez D 2014 JCAP 1411 034
[9] Castellanos E, Escamilla-Rivera C, Lammerzahl C and Macias A 2016 Int. J. Mod. Phys. D 26 1750032
[10] Lelli F, McGaugh S S and Schombert J M 2016 Astron. J. 152 157
[11] Jusufi K, Jamil M, Salucci P, Zhu T, and Haroon S 2019 Phys. Rev. D 100 044012
[12] Rodrigues D C, Marra V, del Popolo A and Davari Z 2018 Nat. Astron. 2 668
[13] Navarro J F, Frenk C S and White S D M 1996 Astrophys. J. 462 563; Navarro J F, Frenk C S and White S D M 1995 Mon. Not. Roy. Astron. Soc. 275 720; Navarro J F, Frenk C S and White S D M 1995 Mon. Not. Roy. Astron. Soc. 275 56
[14] Castellanos E, Escamilla-Rivera C and Mastache J arXiv:1910.03791 [gr-qc]