Ghost Imaging of Space Objects

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Abstract. The term “ghost imaging” was coined in 1995 when an optical correlation measurement in combination with an entangled photon-pair source was used to image a mask placed in one optical channel by raster-scanning a detector in the other, empty, optical channel. Later, it was shown that the entangled photon source could be replaced with thermal sources of light, which are abundantly available as natural illumination sources. It was also shown that the bucket detector could be replaced with a remote point-like detector, opening the possibility to remote-sensing imaging applications. In this paper, we discuss the application of ghost-imaging-like techniques to astronomy, with the objective of detecting intensity-correlation signatures resulting from space objects of interest, such as exo-planets, gas clouds, and gravitational lenses. An important aspect of being able to utilize ghost imaging in astronomy, is the recognition that in interstellar imaging geometries the object of interest can act as an effective beam splitter, yielding detectable variations in the intensity-correlation signature.

1. Introduction

In observation astronomy, a direct intensity measurement technique is often complimented by other types of measurements, a well-known example of which is intensity interferometry. The first and perhaps the most famous application of this technique in astronomy is the measurement of the angular size of a star, which was first demonstrated by R. Hanbury Brown and R.Q. Twiss in 1957 [1]. Fifty years after the pioneering work, a space-deployable version of this technique has been proposed [2].

Intensity correlation measurements also find common use in a very different field, namely in quantum optics, where the field fluxes incident on the photodetectors are typically at single-photon levels, and the correlation processing simplifies to a photon-coincidence measurement [3]. The focus of our paper is an intensity-correlation imaging technique referred to as ghost imaging, which was invented in the quantum optics community, although since then it has found broader range of applicability. Ghost imaging refers to a two-arm imaging system in which an object of interest is located in one arm, as shown in Fig. 1. A single-pixel bucket detector is placed in the signal arm, which collects all light that has been transmitted through (or reflected off of) the object, offering no spatial resolution. A high-resolution detector array (or a scanning pinhole detector) is placed in the reference arm (the arm without the object), but the incident field has had no interaction with the object of interest.

Historically, both entangled photon pairs (biphotons) obtained from spontaneous parametric down conversion (see Fig. 2) [4], and classical sources with pseudo thermal [5, 6] or thermal [7] radiation have been used to perform ghost imaging. The crux of virtually all ghost imaging schemes is that a source with low spatial coherence is configured such that correlated
intensity fluctuations (speckle patterns) are incident on both the object and the reference-arm photodetector array. Then, by cross-correlating the photodetector outputs (i.e., the photon counts in the single-photon regime or the photocurrents in the high-flux-imaging regime) one is able to reconstruct the transverse spatial transmissivity profile of the object of interest.

Besides its significance in quantum optics, ghost imaging has a few apparent practical advantages. Since no spatial resolution in the object channel is required, a very primitive single-pixel optical sensor could be placed in this channel, with the more advanced optics responsible for the image quality placed in the reference channel. This is convenient for imaging objects that are hard to access. The ghost imaging has also been demonstrated with distinct center wavelengths in the two arms, rendering it beneficial for tuning the object-arm wavelength

**Figure 1.** A generic two-arm ghost imaging setup.

![Diagram of a generic two-arm ghost imaging setup](image)

**Figure 2.** Simplified illustration of the original ghost imaging experiment [4]. A mask with letters “UMBC” is placed in the object channel, where all light is collected by a “bucket detector”. Nonetheless, an image is reconstructed by correlating this detector’s photo counts with those from another detector, raster-scanning the empty reference channel. A sharp image is observed when a modified thin lens equation is fulfilled: \(1/(a_1 + a_2) + 1/b = 1/f\).
independently from that employed in the reference channel [8]. Furthermore, under certain background and photodetection-noise constraints, the coincidence measurement technique can offer higher robustness than a direct intensity measurement.

Despite the apparent potential advantages, the early laboratory demonstrations of ghost imaging have been far from the practical requirements arising in astronomy. However, several important advances in the past decade allow us to consider this application. First, the ability to perform with ghost imaging with thermal light [5, 6, 7, 9] was a critical breakthrough, as most natural illumination sources in our universe (e.g., stars) generate thermal light. Second, it was shown that the bucket detector in the reference arm—which is nominally expected to collect most of the light scattered from the object—could be replaced with one that collects a fraction of the field scattered by the object [10, 11]. This permits an object to be placed at a large distance from the observer, which is critical for an astronomical application, wherein the interstellar distances result in large speckles sizes on the detection planes, and subsequently the detectors in both arms resemble point-like detectors. However, using a thermal light source brings about a new complication: in general, a beam splitter has to be placed between the source and the object to create correlated speckle patterns in the signal and reference arms. While this is not a challenge in the laboratory environment, it becomes problematic when the source and object are located far in space.

In this work we investigate the possible application of ghost imaging in space astronomy, by addressing the beam splitter issue. Our approach is illustrated in Fig. 3. Its main idea is that an object which partially transmits (or diffracts) and partially reflects (or scatters) light, can itself play the role of the beam splitter. Moreover, even a perfectly opaque scattering object may under certain conditions create coherence between the transmitted and scattered light, which may be utilized for the intensity correlation imaging. Since the object is present in both channels, this approach leads us away from “canonical” ghost imaging towards intensity interferometry of Hanbury Brown - Twiss type. The analogy and distinctions between these two types of imaging have been discussed in literature [12]. In our case, an important distinction is that we will assume the source properties are known, instead focusing on the parametric variation of the intensity-correlation signature as a function of the object’s geometry and location relative to the source. We will attempt to restore these parameters from the correlation measurements and show that these results could provide important information in addition to conventional direct observations.

In addition to the configuration shown in Fig. 3(b), we will consider the case when both detectors receive transmitted and scattered light. This case is more realistic when both observers are ground-based and the distant source and object cannot be optically resolved. We will develop

![Figure 3. Conceptual schematic of conventional thermal light ghost imaging setup (a) and of our approach (b)). S is a thermal light source, M object, D$_1$, D$_2$, detectors and BS (not present in case (b)), a beam splitter.](image-url)
and test a simple analytical model that will allow us to study the intensity-correlation signatures of the simplest test objects. Based on this model we will make predictions concerning the observability of various space objects, and concerning their parameters that can be inferred from such observations. Potential space objects of interest will include Earth-like planets (including those near bright stars), gravitational lenses formed by black holes or other massive objects, dust or gas clouds.

The rest of this paper is organized as follows. In Section 2 we will discuss the simplified geometry of the physical system under study and introduce its physical model. In Section 3 we will narrow this model to a particular but very important case of equal distances from the object to the detectors. This will allow us to carry out this analysis in the analytical form and gain the insight in its physical aspects by considering two simple model objects. We will also test some of our theory predictions by comparing them to the actual astronomy observation data, available from the Kepler mission. In Section 4 we briefly discuss the consequences of departing from the equal arms limit. In Section 5 we will discuss the signal to noise ratio in the correlation measurement and compare it to a direct intensity measurement. The results of our analysis will be summarized in Section 6.

2. The model and approach

We will consider a transverse (2D) extended source and a similar object placed in the source and object planes in which we introduce the local transverse coordinates $\vec{\rho}$ and $\vec{\rho}_o$, respectively. Let $L_s$ be the distance between the source and object planes, and let $L_{1,2}$ denote the distances between the object plane and the planes of point-like detectors 1 and 2. The local transverse positions of these detectors are $\vec{\rho}_1$ and $\vec{\rho}_2$, respectively, as shown in Fig. 4.

![Figure 4. Relative position of the source, object and detectors in the flat paraxial model.](image)

Let us further assume that the source field is bound by a Gaussian envelope with the width $R_s$ and can be written as $E(\vec{\rho}, t)e^{-\frac{\rho^2}{2R_s^2}}$. This model approximates the source with a diameter (intensity-distribution width) equal to $\sqrt{2}R_s$. In the paraxial approximation, the field at detector 1 or 2 is related to the field of the source as the following:

$$E_{1,2}(\vec{\rho}_1, t) = \int \int d^2\rho d^2\rho_o e^{-\frac{\rho^2}{2R_s^2}} E(\vec{\rho}, t - \frac{L_{1,2}}{c})h_{L_s}(\vec{\rho}_1 - \vec{\rho}_o)h_{L_{1,2}}(\vec{\rho}_o - \vec{\rho}_{1,2}).$$

(1)

Although it looks complicated, expression (1) is actually straightforward. It is expressed in
terms of the field propagation functions

\[ h_Z(\vec{x}) = \frac{e^{ikZ}}{i\lambda Z} e^{ik\frac{\vec{x}^2}{2Z}} \]  

(2)

relating the electric field at two spatial points separated by a distance Z in the longitudinal direction and by a transverse displacement \( \vec{x} \) (assuming that \( |\vec{x}| \ll Z \)) in the transverse directions. A field produced by an extended source at a remote point is then given by a convolution of (2) with the source-field distribution. This way we find the field in the object plane. We multiply this field by the object’s transmission function \( T(\vec{\rho}) \) which may be real for a purely absorbing object and imaginary for a phase object, e.g. a lens. Then we repeat the propagation and integration steps to obtain the fields at the detectors (1).

The observable in case of the correlation-based ghost imaging is proportional to the product of local intensities, i.e. to

\[ \langle I_1(\vec{\rho}_1, t_1)I_2(\vec{\rho}_2, t_2) \rangle = \langle E_1^\dagger(\vec{\rho}_1, t_1)E_2(\vec{\rho}_2, t_2)E_1(\vec{\rho}_1, t_1)E_2(\vec{\rho}_2, t_2) \rangle, \]  

(3)

where the fields can be treated as quantum-mechanical operators, or as classical values. For thermal light, the phase-sensitive term in (3) vanishes [13], and we arrive at

\[ \langle I_1(\vec{\rho}_1, t_1)I_2(\vec{\rho}_2, t_2) \rangle = |\langle E_1^\dagger(\vec{\rho}_1, t_1)E_2(\vec{\rho}_2, t_2) \rangle|^2 + \langle E_1^\dagger(\vec{\rho}_1, t_1)E_1(\vec{\rho}_1, t_1)\rangle \langle E_2^\dagger(\vec{\rho}_2, t_2)E_2(\vec{\rho}_2, t_2) \rangle, \]  

(4)

where the first term describes the possible ghost image and the second term gives the uncorrelated “background” intensity product, which also describes the object’s shadow. To separate these effects it is convenient to introduce the normalized Glauber correlation function

[14]

\[ g^{(2)}(\vec{\rho}_1, t_1; \vec{\rho}_2, t_2) = 1 + \frac{|\langle E_1^\dagger(\vec{\rho}_1, t_1)E_2(\vec{\rho}_2, t_2) \rangle|^2}{\langle E_1^\dagger(\vec{\rho}_1, t_1)E_1(\vec{\rho}_1, t_1)\rangle \langle E_2^\dagger(\vec{\rho}_2, t_2)E_2(\vec{\rho}_2, t_2) \rangle} = 1 + \frac{|G_{12}|^2}{G_{11}G_{22}}. \]  

(5)

Glauber correlation function will be our main observable. However let us mention that other types of measurements are possible. In particular, one can measure higher-order correlation functions \( g^{(m,n)} \), or the variance of intensity difference (rather than a product) [15, 16, 17].

The analysis based on the field propagation equation (1) can be easily extended to these measurements. Such measurements will have different dependencies on the optical mode structure and on the detector’s quantum efficiencies, and may offer interesting resolution-SNR trade-off opportunities. These measurement strategies depart from conventional intensity interferometry, which exclusively focuses on inferences from the \( g^{(2)} \)-function measurement.

To evaluate Eq. (5) above, we substitute \( E_1(\vec{\rho}_1, t_1) \) and \( E_2(\vec{\rho}_2, t_2) \), given by (1), into (5) and we take into account the correlation property of the source field \( E(\vec{\rho}, t) \)

\[ \langle E_1^\dagger(\vec{\rho}, t)E(\vec{\rho}', t') \rangle \propto \delta(\vec{\rho} - \vec{\rho}')\Gamma(t - t'), \]  

(6)

where \( \Gamma \) is a \( \delta \)-like function whose width corresponds to the optical coherence time. The latter may be determined by the spectral filters’ bandwidth. Unless we are interested in color imaging or spectroscopy, using narrow-band filters is undesirable because they reduce the signal. On the other hand, short coherence time requires fast optical detectors and electronics in order to ensure single longitudinal mode detection. Therefore to carry out a fair comparison between the direct intensity measurement and the correlation measurement, we need to take into account the photon flux reduction due to that minimal spectral filtering, required in the latter case. Let us assume a 1 ps timing accuracy (which exceeds the state of the art but may be attainable in the future) and
the central wavelength of 1 micron. At this wavelength, the 1 ps coherence time corresponds to a 3.3 nm wide spectral band. Comparing the optical power detected within this band to the total power within the typical band of a silicon photo detector (see Fig. 5), we find that for a correlation measurement we have 0.5% of the power available for the broadband intensity measurement. This reduces the signal to noise ratio (SNR) in a shot-noise limited narrowband measurement. The relative SNR will be more favorable for the correlation measurement when compared to a color-resolved intensity measurement. In particular, no flux loss will be suffered in a very narrowband measurement, e.g. a measurement with a specific spectral line. A more detailed discussion of the direct-intensity vs. correlation measurement SNRs will be given below in Section 5.

Let us point out that in addition to the high speed and low jitter requirements put on the photodetectors and correlation electronics, a broadband correlation measurement places stringent requirements on the clock synchronization between the two detectors, as well as on the knowledge of their relative position $L_1 - L_2$.

To continue our analysis we will assume that perfect synchronization between the detectors has been achieved and $\Gamma(t - t') = 1$ in (6). We then suppress the temporal part of the problem. For the numerator in (5) we derive

$$G_{12}(\vec{\rho}_1, \vec{\rho}_2) = \int d^2 \rho_1 d^2 \rho_1' e^{-\frac{\rho_1^2}{R^2}} T_2(\vec{\rho}_1') T_1^{*}(\vec{\rho}_1) h_{L_1}(\vec{\rho}_1 - \vec{\rho}_1') h_{L_1}^{*}(\vec{\rho}_2 - \vec{\rho}_1) h_{L_2}(\vec{\rho}_2 - \vec{\rho}_2').$$ (7)

In (7) we have introduced $T_{1,2}(\vec{\rho}_o)$ to allow the transmission functions to be different for detectors 1 and 2, which will be important in the following discussion.

For the following analysis it will be convenient to introduce a correlation function

$$G^{(R_s)}_{12}(Z_a, \vec{\rho}_a; Z_b, \vec{\rho}_b) = \int d^2 \rho e^{-\frac{\rho^2}{R^2}} h_{R_s}^{*}(\vec{\rho} - \vec{\rho}_a) h_{R_s}(\vec{\rho} - \vec{\rho}_b)$$ (8)

for the fields emitted by an extended Gaussian source of thermal light that has a width $R_s$ and is located at $Z = 0$, propagating to locations $(Z_a, \vec{\rho}_a)$ and $(Z_b, \vec{\rho}_b)$. Equivalently, from the advanced wave perspective [19], it describes time-reversed propagation of a photon from $(-Z_a, \vec{\rho}_a)$ to the...
source, and then forward in time to \((Z_b\tilde{\rho}_b)\). If the source is infinitely large, \(R_s \to \infty\), the aperture-limited propagation function \((8)\) becomes equal to a point-source propagation function from one detector to the other:

\[
G_{12}^{(\infty)}(Z_a, \tilde{\rho}_a; Z_b, \tilde{\rho}_b) = h_{Z_a-Z_b}(\tilde{\rho}_b - \tilde{\rho}_a).
\] (9)

The two-point propagation function arises in \((5)\):

\[
G_{12}(\tilde{\rho}_1, \tilde{\rho}_2) = \int \int d^2\rho d^2\rho' h^L_{Z_a}(\tilde{\rho}_a - \tilde{\rho}_2) h^L_{Z_b}(\tilde{\rho}_a - \tilde{\rho}_1) T_2(\tilde{\rho}_a) T_1^*(\tilde{\rho}_a) G_{12}^{(R_s)}(Z, \tilde{\rho}_0; Z, \tilde{\rho}_0),
\] (10)

and likewise for \(G_{11}(\tilde{\rho}_1)\) and \(G_{22}(\tilde{\rho}_2)\). To evaluate \(G_{12}^{(R_s)}(Z_a, \tilde{\rho}_a; Z_b, \tilde{\rho}_b)\) in a general form, we introduce polar coordinates such that

\[
\int d^2\rho = \int_0^\infty \rho \, d\rho \int_0^{2\pi} d\varphi \quad \text{and} \quad |\tilde{\rho}_a - \tilde{\rho}_b|^2 = \rho^2 + \rho'^2 - 2\rho_a\rho_b\cos(\varphi_a - \varphi_b). \tag{11}
\]

The angular integration in Eq. \((8)\) yields

\[
G_{12}^{(R_s)}(Z_a, \tilde{\rho}_a; Z_b, \tilde{\rho}_b) = 2\pi\frac{e^{ik(Z_a-Z_b)}}{\lambda^2 Z_aZ_b} e^{\frac{i}{2}(\rho^2/Z_a - \rho'^2/Z_b)} \int_0^\infty \rho \, d\rho e^{-\rho^2} \left[ \frac{\tilde{\rho}_a - \tilde{\rho}_b}{R_s} \right] J_0 \left( k\rho \left| \frac{\tilde{\rho}_b}{Z_b} - \frac{\tilde{\rho}_a}{Z_a} \right| \right). \tag{12}
\]

Then integrating over the radius we obtain

\[
G_{12}^{(R_s)}(Z_a, \tilde{\rho}_a; Z_b, \tilde{\rho}_b) = 2\pi\frac{R_s^2}{\lambda^2 2Z_a Z_b + ikR_s^2(Z_b - Z_a)} e^{\frac{i}{2}(\rho^2/Z_a - \rho'^2/Z_b)} e^{-\frac{1}{2}\rho^2/Z_a} e^{-\frac{1}{2}\rho'^2/Z_b} \frac{\tilde{\rho}_a - \tilde{\rho}_b}{2Z_a Z_b + ikR_s^2(Z_b - Z_a)}. \tag{13}
\]

In the case of interest \((10)\) we have \(Z_a = Z_b = L_s\), which leads to

\[
G_{12}^{(R_s)}(L_s, \tilde{\rho}_0^a; L_s, \tilde{\rho}_0^b) = q^2\pi^{-1} e^{\frac{q}{2}(\rho^2 - \rho'^2)} e^{-q^2|\tilde{\rho}_0 - \tilde{\rho}_0'|^2}, \tag{14}
\]

where \(q^{-1} = 2L_s/(kR_s)\) is the speckle size.

**3. Balanced arms configuration**

In this section we consider the special balanced case when \(L_1 = L_2 = L\). While this case limits possible observation scenario, it allows us to carry out exact analytical calculations in many cases of interest, and to assess the practical utility of our approach. To carry out these calculations it will be convenient to introduce the new coordinates \(\bar{x} = (\tilde{\rho}_0^a + \tilde{\rho}_0^b)/\sqrt{2}\) and \(\bar{y} = (\tilde{\rho}_0^a - \tilde{\rho}_0^b)/\sqrt{2}\). Then substituting \((14)\) into \((10)\) we obtain

\[
G_{12}(\tilde{\rho}_1, \tilde{\rho}_2) = A \int \int S(\bar{x}, \bar{y}) e^{-q^2\rho^2} e^{i(\bar{x} \Delta \bar{x} + i\bar{y} \bar{\Sigma} \bar{y})} d\bar{x} d\bar{y}, \tag{15}
\]

where

\[
A = \pi \left( \frac{R_s}{\lambda^2 L_s} \right)^2 e^{\frac{i}{2\pi}(\rho^2 - \rho'^2)},
\]

\[
S(\bar{x}, \bar{y}) = T_2(\frac{\bar{x} + \bar{y}}{\sqrt{2}}) T_1^*(\frac{\bar{x} - \bar{y}}{\sqrt{2}}),
\]

\[
\bar{\Delta} = \frac{k}{\sqrt{2}L} (\tilde{\rho}_1 - \tilde{\rho}_2),
\]

\[
\bar{\Sigma} = \frac{k}{\sqrt{2}L} (\tilde{\rho}_0^a + \tilde{\rho}_0^b),
\]

\[
\gamma = k(1/L + 1/L_s).
\]
The Gaussian term in (15) arises from Fourier transform of the source field distribution. This suggests that (15) could be generalized for any such distribution. However at this stage we will limit our consideration to a Gaussian source.

As a sanity check we notice that if we “turn off” the object by setting \( S(\vec{x}, \vec{y}) = 1 \), the integral over \( d^2x \) in (15) yields \((2\pi)^2\delta(\Delta + \gamma\vec{y})\). Then the \( d^2y \) integral yields, quite expectedly, the correlation function of a Gaussian source (14) with increased free-space propagation length \( L_s \rightarrow L + L_s \):

\[
G_{12}(\vec{\rho}_1, \vec{\rho}_2) \rightarrow G_{12}^{(R_s)}(L + L_s, \vec{\rho}_1; L + L_s, \vec{\rho}_2).
\]

Let us now consider a few example objects and discuss their possible relevance for the astronomy applications.

3.1. A Gaussian absorber
This case can represent, e.g., a spherical dust or gas cloud of roughly uniform density. It also can be used as a crude model for a planet occluding a star. The transmission function of such an object can be modeled as

\[
T(\vec{\rho}_o) = 1 - T_0 e^{-\frac{\rho_o^2}{2R_o^2}},
\]

which gives rise to four terms in the two-channel transmission function:

\[
S(\vec{x}, \vec{y}) = S_0 + S_{1a} + S_{1b} + S_2 = 1 - T_0 e^{-\frac{(\vec{x} + \vec{y})^2}{4R_o^2}} - T_0 e^{-\frac{(\vec{x} - \vec{y})^2}{4R_o^2}} + T_0^2 e^{-\frac{x^2 + y^2}{2R_o^2}}.
\]

In (18) and (19) \( T_0 \) is the amplitude transmission of the central part of the object. Consequently, the correlation function also will consist of four terms: \( G_{12} = G_{12}^{(0)} + G_{12}^{(1a)} + G_{12}^{(1b)} + G_{12}^{(2)} \), where the zero-order term corresponds to free-space propagation (17): \( G_{12}^{(0)} = G_{12}^{(R_s)}(L + L_s, \vec{\rho}_1; L + L_s, \vec{\rho}_2) \). A straightforward but cumbersome calculations gives both the first and the second order terms in the form

\[
G_{12}^{(1, 2)}(\vec{\rho}_1, \vec{\rho}_2) = \frac{(-T_0)^n}{\pi(q^2 + R_o^2)} \left( \frac{k^2 R_o R_s}{2 L L_s} \right)^2 e^{-\frac{i q^2 R_o^2}{2 L^2}} e^{-\frac{i q^2 R_o^2}{2 L^2}} e^{-\frac{i q^2 R_o^2}{2 L^2}} e^{-\frac{i q^2 R_o^2}{2 L^2}} \cdot (20)
\]

To obtain \( G_{12}^{(1a), (1b)} \), we substitute in (20)

\[
n = 1, \quad q^2 = 2q^2 + \frac{1}{4R_o^2}, \quad \gamma = \gamma \pm \frac{i}{2R_o^2}.
\]

For \( G_{12}^{(2)} \), we substitute

\[
n = 2, \quad q^2 = 2q^2 + \frac{1}{2R_o^2}, \quad \gamma = \gamma.
\]

Note that in the multi-mode case when the speckle size on the object greatly exceeds the object size, we have \( q^2 \approx 2q^2 \). However we will proceed without this approximation.

First, let us investigate the result (20) for a set of parameters that can be easily implemented on an optical bench. We simulate an opaque \( (T_0 = 1) \) object placed between the source \( (R_s = 1 \text{ cm}) \) and the detector plane, so that \( L_s = L = 50 \text{ cm} \). The object size \( R_o \) is varied from zero to 1, 2 and 3 mm. In Fig. 6(a) we show the correlation function \( q(2)(|\vec{\rho}_1 - \vec{\rho}_2|) \) measured at the line of sight (i.e., for \( \vec{\rho}_1 + \vec{\rho}_2 = 0 \)). In Fig. 6(b) we show the intensity profile, i.e. the object’s shadow. We see that as the object becomes larger its shadow becomes deeper, and the speckle size becomes smaller. For larger objects the speckle shape also becomes distorted. However since we are interested in detection and characterization of small objects that distort the speckle.
shape only by a little amount, the *speckle width* can be conveniently introduced and used as the main observable in the following discussion. In practice, one can measure the speckle width by a linear array of photo detectors. Depending on the system’s parameters and the desired measurement resolution, as few as two detectors could be sufficient.

The speckle width undergoes a non-intuitive evolution as an object crosses the line of sight, imitating a planet passing across the star. In Fig. 7(a) we see that the speckle width measured along the direction of the transient first gets broader, and then narrower, reaching the minimum when the object is exactly on the line of sight. In Fig. 7(b) we show the corresponding variation of the intensity, or the photon flux. Remarkably, the fractional variation of both values due to the transient object is approximately 9%.

Let us now apply our model to an actual astronomical observation carried out at Kepler space telescope [20, 21]. Substituting the Kepler-20f parameters [21] into our model we find relative intensity variation on the order of $2 \times 10^{-4}$, which is consistent with the actual observation [21], see Fig. 8. Based on the earlier discussion, we set $R_o$ equal to $\sqrt{2}$ times the planet radius.

![Figure 6](image_url)

**Figure 6.** (a) The correlation function $g^{(2)}(\tilde{\rho}_1, \tilde{\rho}_2)$ vs. the distance $\Delta \rho = |\tilde{\rho}_1 - \tilde{\rho}_2|$; and (b) the intensity profile, for a lab parameter set ($T_0 = 1$, $L_s = L = 50$ cm and $R_s = 1$ cm). The object size $R_o$ is varied from zero to 1, 2 and 3 mm.

![Figure 7](image_url)

**Figure 7.** Dependence of the correlation function $g^{(2)}(\Delta \rho)$ width (a) and of the normalized photon flux (b) on displacement of the object from the line of sight $\rho_o$, for a lab parameter set ($T_0 = 1$, $L_s = L = 50$ cm, $R_s = 1$ cm, $R_o = 1$ mm).
Figure 8. The intensity variation for Kepler-20e observed in [21] (a) and computed based on our model (b).

The disagreement in the dip shape shown in Fig. 8(a) and Fig. 8(b) arises from using the Gaussian absorber model while the actual planet is, of course, better approximated by an opaque disk. However the numerical agreement with the experiment shows that our simplistic, yet fully analytical, Gaussian model is sufficiently powerful. Following this model we predict that the speckle size will increase from 3,603.6 m to 3,604.2 m when the planet is centered on in the line of sight, which corresponds to a relative variation of $1.7 \times 10^{-4}$. Again, the magnitude of this variation is very close to the magnitude of the photon flux variation. Of course, a detailed SNR analysis is required in order to conclude which type of planetary detection will be more efficient. Let us however point out that the measurement of the speckle width does not preclude conventional intensity measurement, such as has been carried out in the Kepler experiment, and should be considered as an extension of such a measurement rather than its substitute.

### 3.2. Phase objects.

Another model object allowing for the fully analytical treatment is a thin lens. The motivation for studying of this example is the possibility to observe a phase object where direct intensity measurements may not be as efficient. Examples of such objects in space may be gravitational lenses or dilute gas clouds.

For an infinite thin lens with focal distance $f$,

$$S(\vec{x}, \vec{y}) = e^{-\frac{ik}{2f}(\rho_0^2 - \rho_2^2)} = e^{-i\frac{\rho_0^2}{2f}x^2+y^2}. \quad (21)$$

The effect of such a lens can be absorbed into $\gamma \rightarrow \gamma_f = k(1/L + 1/L_s - 1/f)$. Then we quickly arrive at

$$G_{11} = G_{22} = \frac{q^2}{\pi D^2},$$

$$G_{12}(\rho_1, \rho_2) = \frac{q^2}{\pi D^2} e^{\frac{i2q}{2}D} e^{-2(q/D)^2|\vec{\rho}_2 - \vec{\rho}_1|^2},$$

$$g_{12}(\rho_1, \rho_2) = 1 + e^{-2(q/D)^2|\vec{\rho}_2 - \vec{\rho}_1|^2}, \quad (22)$$

where $D \equiv L(1/L + 1/L_s - 1/f)$ is a dimensionless “out-of-focus” factor. If the lens images the source plane onto the detector plane then $D = 0$ and the speckles become infinitely small while
the intensity goes to infinity. Both observations are a consequence of our assumption in (6) of a delta-correlated source field. If the source is in focus then the light propagates as a collimated beam and consequently we have $D = 1$. In this case, the speckle size as well as the intensity in the detector plane will be the same as in the lens plane.

Let us consider another example of a lens-like phase and amplitude object. This object has a finite extent and allows for the analytical treatment. Its transmission function is

$$T(\rho_o) = 1 - e^{-\frac{\rho_o^2}{2R_o^2}}(1 - e^{-\frac{ik\rho_o^2}{2f}}).$$

(23)

This object can produce shadows very similar to those from a Gaussian absorber, see Fig. 9 (a) and (b). Therefore it would be very difficult to distinguish these two objects based on the intensity measurement. However the behavior of the speckle widths in these two cases is clearly distinct, as we see from Fig. 9 (c) and (d) (notice the difference in the signal magnitude, as well as in its qualitative behavior). This example demonstrates the advantages offered by a correlation measurement for object characterization, beyond its mere detection.

![Figure 9](image-url)

**Figure 9.** (a): A shadow from a test object (23) with $R_o = 3$ mm and $f = 1$ m inserted between the source and detectors as a function of its distance from the line of sight. (b) A shadow from a Gaussian absorber with $R_o = 5$ mm and $T_0 = 0.62$. (c) and (d): The speckle width for (a) and (b), respectively. In this simulation $L_s = L_1 = L_2 = 50$ cm, $R_o = 1$ cm.

4. **Unbalanced arms configuration**

In this section we consider a more general case of $L_1 \neq L_2$ which may be important for an asymmetric configuration, e.g. when the correlation measurement is performed by a ground-based detector jointly with a distant space-based detector. We will continue to assume perfect time synchronization between the two detectors, or equivalently, a monochromatic light source. However we will not be able to carry out the analytical calculations without making some
reasonable approximations. From (13) we see that the large aperture approximation (9) holds if

\[
\alpha_s \equiv \frac{2Z_oZ_b}{kR^2_s|Z_b-Z_a|} \ll 1.
\] (24)

This approximation is appropriate for evaluation of the first-order terms in (10) where \(Z_b-Z_a = L_1\) is large. Indeed, for the optics lab geometry \(\lambda = 1 \mu m\) and \(R_s = L_s = 1 \ cm\), so we get \(\alpha_s = 1.6 \times 10^{-5}\). For the Solar system geometry with \(R_s = 7 \times 10^9\) km (Sun radius), \(L_s = 1.5 \times 10^8\) km (the distance from Earth to Sun), and \(\lambda = 1 \mu m\), we get \(\alpha_s = 5 \times 10^{-14}\). This parameter becomes even smaller for inter-stellar distances. Therefore when we calculate \(G_{12} (10)\) for a Gaussian absorber described by (18), the first-order terms can be approximated as

\[
G_{12}^{(1)} (\tilde{\rho}_1, \tilde{\rho}_2) \equiv G_{12}^{(1a)} (L_1, \tilde{\rho}_1; L_2, \tilde{\rho}_2) + G_{12}^{(1b)} (L_2, \tilde{\rho}_2; L_1, \tilde{\rho}_1) \approx -2 \int d^2 \rho e^{\frac{-\rho^2}{2kR_s^2} h^*_{L_1}(\tilde{\rho} - \tilde{\rho}_1) h_{L_2}(\tilde{\rho} - \tilde{\rho}_2)}.
\] (25)

The opposite case of (24) occurs when \(Z_a = Z_b\). Furthermore, if \(q^2\) is much greater than the sum of all coefficients for the \(\rho^2\) terms in the real and imaginary exponents in (10), then (14) can be proven to approach a \(\delta\)-function normalized to unity area. This is the small aperture approximation, applicable for the second-order terms of (10). Let us point out that within this approximation, the object cannot create coherence between the transmitted and scattered light unless the speckle size in the object plane approaches or exceeds the size of the object itself.

It is easy to see that for the optics-lab geometry as described above, \(q^2\) exceeds all relevant parameters by a factor of at least \(3 \times 10^3\). The excess factors are much greater in all reasonable astronomical geometries. Therefore we derive

\[
G_{12}^{(2)} (\tilde{\rho}_1, \tilde{\rho}_2) \approx \int d^2 \rho e^{-\frac{\rho^2}{2kR_s^2} h^*_{L_1}(\tilde{\rho} - \tilde{\rho}_1) h_{L_2}(\tilde{\rho} - \tilde{\rho}_2)}.
\] (26)

For a Gaussian absorber case with \(L_1 \neq L_2\) we obtain the following approximate expressions:

\[
\begin{align*}
G_{12}^{(0)} (\tilde{\rho}_1, \tilde{\rho}_2) &= G_{12}^{(R_s)} (L_1 + L_s, \tilde{\rho}_1; L_2 + L_s, \tilde{\rho}_2), \\
G_{12}^{(1)} (\tilde{\rho}_1, \tilde{\rho}_2) &= -2G_{12}^{(\sqrt{2}R_s)} (L_1, \tilde{\rho}_1; L_2, \tilde{\rho}_2), \\
G_{12}^{(2)} (\tilde{\rho}_1, \tilde{\rho}_2) &= G_{12}^{(2R_s)} (L_1, \tilde{\rho}_1; L_2, \tilde{\rho}_2).
\end{align*}
\] (27)

Let us first evaluate the correlation function \(g^{(2)}\) found by substituting (27) into (5) in the absence of the object, by setting \(R_o = 0\). In Fig. 10(a) we show this function for a typical set of optical lab parameters \((L_1 = 55 \ cm, L_2 = L_1 + \Delta L, L_s = 55 \ cm, \lambda = 1 \mu m)\), where we assumed that the detectors are coplanar with the line of sight and are placed symmetrically on its opposite sides: \(\tilde{\rho}_1 = -\tilde{\rho}_2\). This allows us to introduce a single scalar parameter \(\Delta \rho = |\tilde{\rho}_1 - \tilde{\rho}_2|\), in the same way it was done in Fig. 6.

The correlation loss due to \(\Delta L \neq 0\) is clearly visible. We would like to emphasize that this is not due to a limited longitudinal coherence of the source, but because of its transverse coherence properties. We can interpret this result as following. By placing the first detector in the plane \(L_1\) we define the speckle pattern in this plane as the transverse mode structure. These speckles may be further considered as mutually incoherent light sources. As light from these sources propagates further, the coherence areas expand as well as overlap. The expansion causes the widening of the correlation function while the overlap causes the contrast reduction due to multimode detection. Using the expression \(g^{(2)}(0) = 1 + 1/m\) relating the correlation peak for thermal light to the number of detected modes \(m\) we can determine that in our example the
longitudinal displacement of the detector by \(\Delta L = 6\) mm has lead to the number of detected modes \(m \approx 3\). Note that this interpretation differs from the speckle pattern behavior that one might observe, e.g. on a screen, in which case the speckles do not overlap and do not appreciably change in size for small longitudinal translations.

Now let us “turn on” the object and investigate its effect on the correlation function. If a small \((R_o = 1\) mm) object is placed half way between the source and the detectors, the correlation function becomes narrower, which means smaller speckles, see Fig. 10(b).

Let us point out that the \(\Delta L = 0\) case in Fig. 10 is consistent with the \(R_o = 1\) mm case from Fig. 6, which validates the large-aperture and small-aperture approximations for an absorbing object. On the other hand, we notice that when the small aperture approximation is utilized and (14) is treated as a \(\delta\)-function, a phase object such as a thin large lens is “erased” by taking the absolute-square of \(T(\rho_o)\).

![Figure 10](image.png)

**Figure 10.** (a): the correlation functions \(g^{(2)}(\Delta \rho)\) in the absence of an object for \(R_s = 1\) cm \(L_1 = 55\) cm and \(L_2 = L_1 + \Delta L\). (b): the same correlation functions (solid lines) become narrower (dashed lines) when a small \((R_o = 1\) mm) Gaussian absorbing object is inserted in the line of sight at the distance \(L_s = 55\) cm from the source.

5. Signal-to-noise ratio analysis

The sensitivity of the correlation-based measurements can be ascertained by a careful study of the signal-to-noise ratio, which strongly depends on the object of interest and the imaging geometry. As a result, the value added by the correlation-based measurement can range from significant to negligible. The quantitative analysis of the signal-to-noise ratio (SNR) for both types of measurements can leverage SNR calculations for Hanbury Brown-Twiss intensity interferometry, because performing ghost imaging at interstellar distances often yields geometries wherein the detectors are point-like (rather than bucket-like, which is more common in laboratory demonstrations of ghost imaging).

The ghost imaging configuration shown in Fig. 4 consists of two point-like detectors, each measuring the incident far-field irradiance resulting from the thermal source and object combination. The correlation of the two photocurrent outputs isolates the common irradiance fluctuations at their respective transverse locations, yielding information on the mutual coherence function of the effective source created by the true source illuminating the object.

Suppose the baseband envelope of the field incident on detector \(m\), for \(m = 1, 2\), is denoted by \(E_m(t)\) for \(0 < t < T\), having units \(\sqrt{\text{photons/s}}\). Because we are considering thermal fields,
we assume that the field have zero mean (i.e., $\langle E_m(t) \rangle = 0$) and a complex degree of coherence

$$
\gamma_{1,2}(t-u) \equiv \frac{\langle E_m^*(t)E_m(u) \rangle}{\sqrt{\langle |E_m(t)|^2 \rangle \langle |E_m(u)|^2 \rangle}}
$$

(28)

that is nonzero. We assume that the the coherence time of the two fields (i.e., the time delay $\tau$ for which $\langle E_m^*(t+\tau)E_m(t) \rangle$ is appreciable) is equal and given by $T_c$.

We will carry the following analysis for the photocurrents $i_1(t)$ and $i_2(t)$ produced by the detectors in response to the incident optical field. This analysis can be easily generalized for the photon-counting detectors. We have

$$
\langle i_m(t)|E_m(t) \rangle = \int d\tau \left( \eta |E_m(\tau)|^2 + \lambda_d \right) h_B(t-\tau),
$$

(29)

and

$$
\langle \Delta i_m(t)\Delta i_m(u)|E_m(t,u) \rangle = \int d\tau \left( \eta |E_m(\tau)|^2 + \lambda_d \right) h_B(t-\tau) h_B(u-\tau).
$$

(30)

Here $\Delta i(t) \equiv i(t) - \langle i(t) \rangle$, $h_B(t)$ denotes the real-valued baseband impulse response of the photodetectors with time constant (i.e., inverse bandwidth) equal to $T_B$, and $\lambda_d$ is the mean dark photoelectron counts, which are assumed to obey homogeneous Poisson point process statistics. With little loss in generality, we assume that $\int dt h_B(t) = 0$, i.e., that the photodetectors are DC blocking, and that $T_B \ll T_c$, i.e., that the coherence time of the source is much longer than the response time constant of the photodetector. Note that the DC term is useful for direct intensity observations, thus it could be tapped off prior to the $h_B(t)$ filter.

The equal-time photocurrent correlation measurement performed at the receiver corresponds to

$$
C \equiv \int_0^T dt i_1(t)i_2(t),
$$

(31)

where $C$ is repeated for every position of the two detectors. Thus, strictly speaking $E_m$, $i_m$ and $C$ should have two vector arguments indicating the positions of the two detectors in 2D or 3D space. However, here we are concerned with the SNR at a particular position of these detectors, and therefore we are omitting these variables to avoid clutter.

The SNR of the $C$ measurement is defined then as

$$
\text{SNR} \equiv \sqrt{\frac{\langle C \rangle^2}{\langle (C-\langle C \rangle)^2 \rangle}}.
$$

(32)

Calculating the first and second moments is tedious, but straightforward [22, 13]. The SNR can be evaluated, in the $T_c \gg T_B$ limit, as [22]

$$
\text{SNR} = \sqrt{TT_B \frac{\left| \gamma_{1,2} \right|^2 N^2}{T_c^2 (1 + N_d/N)^2 + 2(1 + N_d/N)N(T_B/T_c) + N^2(T_B/T_c)^2 (|\gamma_{1,2}|^4 + 1 + |\gamma_{1,2}|^2)^2}},
$$

(33)

where $\gamma_{1,2} \equiv \gamma_{1,2}(0)$, $N \equiv \eta(T_c/T) \int_0^T dt \langle |E_m(t)|^2 \rangle$, for $m = 1, 2$, is the mean photoelectron number per temporal mode (or equivalently per coherence time) of the fields incident on the detectors, and $N_d = \lambda_d T_c$ is the mean dark photoelectron number per coherence time. The terms in the denominator have intuitive interpretations. The first term, which is independent of $N$, is due to the shot noise of the two detectors. The third term, with the $N^2$ dependence, is excess noise resulting from the statistical fluctuations of the incident power on the detectors. This term is sometimes referred to as relative intensity noise. The middle term, with the $N$ dependence, is a result of the beating between the intensity fluctuations and shot noise. Asymptotic SNR
expressions can be derived for different limiting cases, depending on the particular imaging configuration and photodetector properties. In the following discussions, we assume that the dark-counts from the detector are negligibly small relative to the incident photon counts. In the shot-noise limited regime $N_d \ll N \ll 1$ holds, i.e., the mean number of photoelectrons per mode is very small, and the SNR can be approximated as

$$\text{SNR} = \sqrt{TT_B/T_c} |\gamma_{1,2}|^2 N.$$  \quad (34)

Therefore the SNR has a linear dependence on the incident photon flux per mode. In the opposite regime with many photoelectrons per mode, thus $N \gg 1$, the SNR saturates to its maximum value

$$\text{SNR} = \sqrt{T/T_B} |\gamma_{1,2}|^2 / \sqrt{|\gamma_{1,2}|^4 + (1 + |\gamma_{1,2}|^2)^2}.$$  \quad (35)

For direct intensity measurements, we assume that a simple photon bucket operation is performed over $T$-seconds of integration,

$$D = \int_0^T dt i(t),$$  \quad (36)

where the statistics in Eqs. (29) and (30) are still true, but we assume that $\int dt h_B(t) = 1$, such that the DC component is no longer filtered out. Again, defining

$$\text{SNR} \equiv \sqrt{\langle D^2 \rangle - \lambda_D^2} / \langle (D - \langle D \rangle)^2 \rangle,$$  \quad (37)

it is straightforward to derive the SNR as

$$\text{SNR} = \sqrt{\frac{T}{T_c} \frac{N}{1 + N_d/N + N}},$$ \quad (38)

where all parameters are as defined before. Thus, when $N_d \ll N \ll 1$, the system becomes shot-noise limited and we have

$$\text{SNR} = \sqrt{\frac{N T}{T_c}},$$ \quad (39)

whereas when $N \gg 1$ and $N > N_d$ excess noise dominates and the SNR saturates at

$$\text{SNR} = \sqrt{\frac{T}{T_c}}.$$  \quad (40)

To separate the SNR dependence on the integration time $T$, which is common to both the correlation and direct intensity measurements, it is convenient to normalize it to $\sqrt{T/T_c}$. Fig. 11 shows thus normalized SNR of a direct intensity measurement (38) and the asymptotic approximations (34) and (35) of correlation-measurement SNRs, as a function of the mean photoelectron number per mode for the case of $\gamma_{1,2} = 1$. As seen from this figure, the correlation-measurement SNR can approach the direct-intensity measurement SNR for the sources with high spectral brightness, $N \approx 1$. The correlation-based measurements typically have worse SNR than direct-intensity measurements when shot-noise limited due to the dependence on the square of the incident average photon number in correlation measurements rather than just the photon number in direct intensity measurements. However in the excess-noise limited regime,
the correlation measurements’ SNR may remain shot-noise limited (as also pointed out in [22])
due to the fact that such measurements can distinguish source fluctuations from those caused
by an object better than the direct-intensity measurement. Note, however, these plots compare
the SNRs when the photoelectrons per mode are equal in both methods. Whereas correlation-
based measurements require \( T_B < T_c \), thereby limiting the total flux incident on the detector,
the direct intensity measurement can integrate over a very wide optical bandwidth, without
penalty. We have briefly discussed this aspect of correlation imaging in Section 2, considering
the example of background light derived from the Solar spectrum.

Thus, in general, the SNR in the correlation measurement is worse than in the direct intensity
measurements. Nonetheless, in order to have a realistic assessment of the added value of
correlation-based measurements for imaging space objects, one must take into account stray
light, detector aging, natural variation of source brightness and other practical effects that are
usually omitted in the SNR analyses published to date. The detectors’ dark noise may be
particularly important in cases when the incident photons flux is low. On the other hand, the
correlation technique may be particularly beneficial for narrow-band imaging, e.g. imaging using
a specific spectral line. A narrow spectral feature will lead to the higher spectral brightness and
give the correlation imaging advantage according to Fig. 11.

![Figure 11. The normalized intensity SNR (red) and correlation measurement SNR for
\( T_B/T_c = 0.1 \) (the lowest), 0.5 and 1 (the highest).](image)

6. Conclusions and summary
In this paper we have analyzed intensity-correlation “ghost imaging” of dark objects in space
illuminated by thermal light sources (stars). Our approach hinges on replacing the beam splitter,
indispensable for thermal light ghost imaging but infeasible for space imaging, with the object
itself. The absorptive and refractive properties of the object are predicted to imprint themselves
on the intensity correlation properties of the transmitted and scattered light. Consequently,
in principle, they could be extracted from intensity correlation measurements. To provide
an insightful exposition of this concept we limited our discussion to a fully analytical model
relying on a two-dimensional source and an object with Gaussian distribution of luminosity,
absorption, or phase delay (the latter representing a thin lens) in the paraxial approximation.
We demonstrated the variation of the far-field speckle size due to the presence of the object. We
have shown that the speckle size variation is a non-trivial function of the object’s properties and
position. In some cases it allows us to distinguish different phase and amplitude objects even
when they produce very similar shadows and can hardly be distinguished by a direct intensity measurement. Thus the correlation measurement provides a complementary information to a direct observation.

This understanding has encouraged us to apply our analytical model to a realistic space object imaging scenario, such as the Kepler mission. Our prediction for the flux variation is very close to the actual observation. It also predicts a similar fractional change of the speckle size. We have carried out a preliminary SNR analysis for a correlation measurement, comparing it to a direct flux measurement. The analysis has shown that, for parameters typical of the Kepler mission, the correlation measurement SNR would be significantly worse than the intensity measurement SNR. This analysis however does not include instrument-generated technical noise, that may be detrimental for the intensity measurement more than for the correlation measurement, and could potentially balance or even reverse the SNRs inequality between the two schemes. Examples of such technical noises are dark noise and variation of detector’s responsivity (quantum efficiency) due to environmental fluctuations and aging. The ambient background light is another important factor that needs to be considered.

Regardless of the technical noises, the correlation measurement SNR improves relative to the intensity measurement SNR when the spectral brightness of the signal increases. Thus the correlation measurement may be especially advantageous in narrow-band imaging, e.g. imaging based on a selected spectral line. As a final note, comparison of the SNRs of two types of measurement is the decisive criterion when the measurements provide the same type of information. As we have seen, the correlation measurement can provide information additional to the intensity measurement. Moreover, a space object predominantly introducing phase (such as e.g. a gravitational lens or a dilute gas cloud) results in little or no tell-tale shadows, but has a more pronounced affect on the optical coherence. In such scenarios, intensity-correlation ghost imaging may be a preferable option.

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