The phase diagrams of the ferromagnet-superconductor trilayers in external magnetic field

Maxim Avdeev, Yurii Proshin
Kazan Federal University, 420008, 18 Kremlyovskaya St., Kazan, Russia.
E-mail: avdeyev_mv@mail.ru

Abstract. We investigate the critical properties of asymmetrical trilayers $F_1F_2S$ and $F_1SF_2$ structures in an external parallel magnetic field. The different mutual orientations of the $F$ layers magnetizations are examined. At this condition the triplet component of the superconducting condensate is arisen. Assuming that all $F$ and $S$ layers are dirty, we solve the boundary value problem for the Usadel function. Then we use Gor'kov's self-consistency equation and calculate the critical temperature for both trilayers as function of the $F$ layers thicknesses in external magnetic field $H$. We predict the surprising appearance of re-entrant superconductivity with increasing magnetic field.

1. Introduction
At present, there is a great interest to study of the layered structures, consisting of alternating layers of ferromagnetic (F) and superconducting (S) metals (see reviews [1, 2, 3] and references therein). It is common knowledge that superconductivity and ferromagnetism are two competing physical phenomena and their coexistence in homogeneous materials is difficult to observe. However, this coexistence is possible in artificial structures FS due to the proximity effect [4].

In the case of good contact S and F metals the superconducting correlations can penetrate from the S layer into the F metal on depth of order coherence length $\sim \sqrt{a_F l_f}$ ($a_F = v_F/2I$ is spin stiffness length, $v_F$ is Fermi velocity, $I$ is exchange field and $l_f$ is free path length in the F metal). Note, that penetration depth in the ferromagnet is much smaller than that in the normal metal, since condition $I \gg T_c$ is satisfied in conventional ferromagnets.

As it was predicted Larkin, Ovchinnikov [5] and Fulde, Ferrell [6], the order parameter $\Delta(r)$, penetrating into F metal, is oscillating function of coordinate $r$ and in the simplest case $\Delta(r) = \exp[ik_F r]$. Such a nonuniform state with a nonzero momentum $k^2_F \approx 2I/D_f$ ($D_f$ is diffusion coefficient) is called LOFF state.

The interplay between superconducting and ferromagnets layers leads in particular to nonmonotonic dependence of the critical temperature $T_c$ on the F layers thickness $d_f$ (see [7, 2, 3]) even for the two- (F/S) and trilayered (F/S/F) systems.

In recent years more theoretical and experimental works were devoted to studying the properties of trilayer FS systems with noncollinear magnetizations direction in the F layers [8, 9, 10, 11]. This leads to the generation of spin-triplet correlations, decaying in the F metal at much greater distances in comparison with the spin-singlet component of the superconducting
condensate [12, 13]. This fact for the thin-film structures leads to strong dependence of the critical temperature $T_c$ on the angle $\phi$ between the directions of the magnetizations of layers F [11].

The unique properties of layered F/S structures make them attractive for technological applications. Thus, in late 90s, a model of a spin switch based on a three-layered F/F/S [14] and F/S/F [15, 16] systems was proposed. External magnetic field changes the relative direction of magnetizations $M$ of the F layers. As a result, the system can turn from the superconducting to the resistive state.

In the present paper we consider thin-films asymmetrical trilayers $F_1/F_2/S$ and $F_1/S/F_2$ in an external parallel magnetic field.

2. Method
The critical temperature $T_c$ at the second-order transition is obtained from the self – consistency equation for the gap

$$\Delta(r) \ln t = \pi T_c \sum_{\omega > 0} \text{Sp} \left( \hat{F}(r, \omega) - \frac{\Delta(r)}{\omega} \right), \tag{1}$$

where $t = T_c/T_{c0}$ is the reduced temperature ($T_{c0}$ is the transition temperature of isolated S layer), $\omega = \pi T_c(2n + 1)$ is Matsubara frequency.

Anomalous Green function $\hat{F}$ near the second-order transition into the superconducting state, for S and F layers, satisfies the Usadel equation ($\omega > 0$) written on the external magnetic field presence [17, 18]

$$\left[ |\omega| - \frac{1}{2} D_s \left( \nabla - \frac{2\pi i}{\Phi_0} A \right)^2 \right] F^{s}_{\alpha\beta}(r, \omega) = \delta_{\alpha\beta} \Delta(r),$$

$$\left[ |\omega| - i J \sigma^{s}_{\alpha\beta} - \frac{1}{2} D_{\alpha\beta} \left( \nabla - \frac{2\pi i}{\Phi_0} A \right)^2 \right] F^{f}_{\alpha\beta}(r, \omega) = 0, \tag{2}$$

$$D_{\alpha\beta} = \frac{D_f}{1 - i 2 J f \sigma^{s}_{\alpha\beta}}.$$
Here $\Phi_0$ is the quantum magnetic flux, $D_{s,f}$ is the diffusion coefficient in S and F layers, respectively and $\tau_f = l_f/v_F$ is the elastic scattering time. Eq. (2) should be solved with boundary conditions on the internal SF interfaces of the Kupriyanov–Lukichev type [19] modified with consideration of the external magnetic field $H$ (see, for example [20])

$$\frac{4D_s}{\sigma_s v_F} \left( \nabla - \frac{2\pi i}{\Phi_0} \right) n^{s}_{\alpha\beta} = \frac{4D_{\alpha\beta}}{\sigma_f v_F} \left( \nabla - \frac{2\pi i}{\Phi_0} \right) n^{f}_{\alpha\beta} = F^s_{\alpha\beta} - F^f_{\alpha\beta},$$

and on the free boundaries

$$\left( \nabla - \frac{2\pi i}{\Phi_0} \right) n^{s,f}_{\alpha\beta} = 0,$$

where $\sigma_s$ and $\sigma_f$ are the parameters of the transparency of the junction on the S and F sides, respectively, $\mathbf{n}$ is the unit vector of the normal to the contact surface. The boundary conditions (3) take into account the condition of detailed balance $\sigma_s v^s_F N_s = \sigma_f v^f_F N_f$ where $N_s$ and $N_f$ are the densities of state on the Fermi surface in the S and F metals, respectively. Note that, if the magnetization $\mathbf{M}$ in F layer has the form $\mathbf{M}(0, M \sin \phi, M \cos \phi)$ then the solution of the Eq. (2) is $F = U^T F U$. Here $U$ is unitary matrix, which has the form

$$U = \begin{pmatrix} \cos \phi/2 & i \sin \phi/2 \\ i \sin \phi/2 & \cos \phi/2 \end{pmatrix}.$$

The external magnetic field is parallel to the plane of contact. Because the thicknesses $d_s, d_f \ll \lambda_H$ ($\lambda_H$ is magnetic penetration depth), we may average the coefficients depending on magnetic field in Eq. (2) over the thicknesses of the S and F layers, i.e.

$$A^2(x) \approx \langle A^2 \rangle = \frac{1}{d_s} \int H^2(x - x_c)^2 dx \approx \frac{1}{12} d_s^2 H^2.$$

Similarly, we assume that $\Delta_s(x) \approx \langle \Delta \rangle$ and $\Delta_f(x) = 0$ in S and F layers respectively.

In Figure 1 the geometry of the problem is displayed for F$_1$/F$_2$/S (panel (a)) and F$_1$/S/F$_2$ (panel (b)) trilayers. The external magnetic field is parallel to the plane of contact.

3. Results and discussion

At first, we discuss an influence of asymmetry on the critical temperature value. For convenience, we have introduced the parameter $n_{s,f} = v^s_F N_s/v^f_F N_f$. In Figure 2 the difference $\Delta t = t^{AP} - t^P$ is displayed as function of the reduced thicknesses $d_{f1}/a_{f1}$ and $d_{f2}/a_{f2}$ for the F$_1$/S/F$_2$ (panel (a)) and F$_1$/F$_2$/S (panel (b)) trilayers without external magnetic field ($H = 0$). Here $t^{AP}$ and $t^P$ denote the reduced temperature for the antiparallel and parallel orientation of the magnetizations, respectively. Note, that the function $\Delta t(d_{f1}, d_{f2})$ maximum corresponds to the asymmetric case, i.e. $d_{f1} \neq d_{f2}$ for both cases. It is interesting fact, because the stable operation of the spin valve device based on three-layered structure FSF [15, 16, 21] (see also references cited in [11]) needs in the high $\Delta t$ values. Note also, $\Delta t$ for the F$_1$/F$_2$/S trilayer (Figure 2(b)) can be both positive or negative as opposed to above considered F$_1$/S/F$_2$ case (Figure 2(a)) [8, 11].

In Figure 3 we plot the phase diagrams for asymmetrical F$_1$/S/F$_2$ structures in the presence of external magnetic field, here $h = H/H_c$ ($H_c$ is critical field for isolated S metal). In the top panel (a) the dependence of the different $\Delta t$ on the reduced thickness F$_2$ layer is displayed. Without field this dependence has smooth maximum at the value $d_{f2}/a_{f2} \approx 0.5$ and then it reaches a plateau. With increasing field ($h = 0.2$) two peaks appear. Note, that
\[ \Delta t(h = 0.2) > \Delta t(h = 0) \]. In the panel (b) the \( t(d_{f2}/a_{f2}) \) dependences are displayed and in the panel (c) the \( t(h) \) curves for antiparallel state are shown. The magnetic field monotonically depresses the critical temperature, but it may also lead to qualitative change of dependence \( t(d_f) \) (see Figure 3(b)). The effect of magnetic field is especially strong in vicinity of the re-entrant superconductivity appearance. In other word, the magnetic field makes possible to observe re-entrant superconductivity.

In Figure 4 we display the phase diagrams for \( F_1/F_2/S \) system in the external magnetic field. In the panels (a, b) the dependences of the reduced critical temperature \( t_p, (AP) \) on the reduced thickness \( F_1 \) layer \( d_{f1}/a_{f1} \) for \( \phi = 0^0 \) (a) and \( \phi = 180^0 \) (b) are shown. In Figure 4(c) we plot the angular dependence of the critical temperature. We see that the temperature dependence \( t(\phi) \) is not monotonic and it has a minimum at \( \phi \approx 90^0 \). It is interesting to note that the surprising appearance of re-entrant superconductivity with increasing magnetic field (see Figure 4(a, b)).

In conclusion, we shortly discuss the model of the spin switch, based on the \( F_1/F_2/S \) trilayers (Figure 4). For technical realizations of a spin valve, the direction of the magnetization \( M_1 \) is fixed, while the magnetization of the adjacent \( F_2 \) layer can be rotated by the external magnetic field. If the initial state of the system to choose at \( t_{init}(\phi = 180^0, h = 0) = 0.28 \) on the phase diagram (see Figure 4(c)) then the system is in the superconducting state \( (t_{init} < t) \). However, under the influence of a weak magnetic field, we can change the direction of magnetization \( M_2 (\phi = 90^0) \). Thus, the system moves to the resistive state, since the condition \( t(\phi = 90^0, h \geq 0) > t \) is satisfied. Note, that, it is important to choose the “working point” for a stable work of a spin valve in an optimum manner: on the one hand, it should be located in the phase diagram with a maximally large temperature \( t \), but on the other hand, the difference \( \Delta t \) should remain larger than certain critical value \( \Delta t_c \).

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Figure 2. The $\Delta t(d_{f1}, d_{f2})$ dependence for the $F_1SF_2$ (a) and $F_1F_2S$ (b) trilayers without external field ($H = 0$). The common parameters of calculation are $l_s/\xi_0 = 0.2$, $I/\pi T_{cs} = 10$. In panel (a) $d_s/\xi_0 = 0.7$, $n_{sf} = 1.5$, $l_f/a_f = 0.25$, $\sigma_s = 0.8$. In panel (b) $d_s/\xi_0 = 0.6$, $n_{sf} = 1$, $l_f/a_f = 0.3$, $\sigma_s = 2$, $\sigma_f = 10$.

Figure 3. The phases diagrams displayed for asymmetrical $F_1/S/F_2$ system in external magnetic field $h = H/H_c$ ($H_c$ is critical field of the isolated S film). In panel (a) shown the different $\Delta t = t^{AP} - t^P$ dependence on the reduced thickness $d_{f2}/a_{f2}$ and fixed value $d_{f1}/a_{f1} = 0.4$. In panel (b) shown the dependence $t(d_{f2})$ for the antiparallel state. In panel (c) displayed dependence $t(h)$ for some values $d_{f1}/a_{f1}$ and $d_{f2}/a_{f2}$. Other parameters are the same as in Figure 2(a).
Figure 4. The phases diagrams displayed for asymmetrical F$_1$/F$_2$/S system in external magnetic field $h$. In panel (a) shown the dependence $t^P$ (for the parallel state) on the reduced thickness $d_{f1}/a_{f1}$ and fixed value $d_{f2}/a_{f2} = 0.5$. In panel (b) shown the dependence $t^{AP}(d_{f2})$ for the antiparallel state. In panel (c) displayed the angular dependence $t(\phi)$ for fixed values $d_{f1}/a_{f1} = 0.2$ and $d_{f2}/a_{f2} = 0.5$. Other parameters are the same as in Figure 2(b).