Errata for: Differential Equations for Sine-Gordon Correlation Functions at the Free Fermion Point

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We present some important corrections to our work which appeared in Nucl. Phys. B426 (1994) 534. Our previous results for the correlation functions \( \langle e^{i\alpha \Phi(x)} e^{i\alpha' \Phi(0)} \rangle \) were only valid for \( \alpha = \alpha' \), due to the fact that we didn’t find the most general solution to the differential equations we derived. Here we present the solution corresponding to \( \alpha \neq \alpha' \).
Appendix E. Errata

In section 3 we did not find the most general solution to the differential equations (3.37) when we imposed \( \partial_z a = \partial_z b = \partial_{\overline{z}} (b - a) = 0 \). We now understand that for \( \alpha \neq \alpha' \), the latter condition is not valid. In this errata we present the modifications for \( \alpha \neq \alpha' \). A corrected version of the paper which incorporates the modifications below is available\(^1\)

1. Equation 1.3 should be replaced with:

\[
\left( \partial_r^2 + \frac{1}{r} \partial_r \right) \Sigma(r) = \frac{m^2}{2} (1 - \cosh 2\varphi)
\]

\[
\left( \partial_r^2 + \frac{1}{r} \partial_r \right) \varphi = \frac{m^2}{2} \sinh 2\varphi + \frac{4(\alpha - \alpha')^2}{r^2} \tanh \varphi (1 - \tanh^2 \varphi),
\]

(E.1)

where \( r^2 = 4z\overline{z} \), and \( m \) is the mass.

2. In equation (3.31), \( \partial_\overline{z} B_+ = \frac{m}{2} \overline{C}_+ C_- \) should be replaced with \( \partial_\overline{z} \widehat{B}_+ = \frac{m}{2} \overline{\widehat{C}}_+ C_- \).

3. The end of section 3, beginning with the sentence after (3.38), should be replaced with the following:

Inserting this parameterization into the differential equations gives the following. The first two equations in (3.37) give

\[
\partial_z a = -\tanh^2 \varphi \partial_z b.
\]

(E.2)

Using this equation and its \( \partial_{\overline{z}} \) derivative the second 2 equations can be simplified to

\[
(\partial_{\overline{z}} \partial_\overline{z} a) \coth \varphi - (\partial_\overline{z} \partial_\overline{z} b) \tanh \varphi - 2\partial_\overline{z} \varphi \partial_\overline{z} (b - a) = 0
\]

\[
\partial_\overline{z} \partial_\overline{z} \varphi = \frac{m^2}{2} \sinh 2\varphi - \tanh \varphi \partial_\overline{z} b \partial_\overline{z} (b - a).
\]

(E.3)

The function \( b \) can be deduced using Lorentz invariance. Let \( z = re^{i\theta}/2, \overline{z} = re^{-i\theta}/2 \), and consider shifts of \( \theta \) by \( \gamma \). The functions \( e, \widehat{e} \) satisfy

\[
e(e^{i\gamma z}, e^{-i\gamma \overline{z}}, u) = e^{-i\gamma(1+\alpha'-\alpha)/2} e(z, \overline{z}, e^{i\gamma} u)
\]

\[
\widehat{e}(e^{i\gamma z}, e^{-i\gamma \overline{z}}, u) = e^{-i\gamma(1+\alpha-\alpha')/2} e(z, \overline{z}, e^{i\gamma} u).
\]

(E.4)

4 Recently, similar results were obtained using different methods in [2].
From the definition (3.21) of $C_+, \hat{C}_+$, and making the change of variables $u \to e^{-i\gamma}u$, one finds
\[ C_+ = e^{2i(\alpha-\alpha')\theta} f(r), \quad \hat{C}_+ = e^{-2i(\alpha-\alpha')\theta} \hat{f}(r), \tag{E.5} \]
for some scalar functions $f, \hat{f}$. Then, using
\[ e(z, \overline{z}, u) = u \hat{e}(\overline{z}, z, 1/u), \tag{E.6} \]
one can show $f = \hat{f}$ by making the change of variables $u \to 1/u$. Thus,
\[ e^{2b} = C_+ \hat{C}_+ = e^{4i(\alpha-\alpha')\theta}, \tag{E.7} \]
and
\[ b = (\alpha - \alpha') \log \left( \frac{z}{\overline{z}} \right). \tag{E.8} \]
Inserting this $b$ into (E.2) and taking the complex conjugate, one deduces
\[ \partial_{\overline{z}} a = \tanh^2 \varphi \partial_{\overline{z}} b. \tag{E.9} \]
The function $\varphi$ is only a function of $r$. Thus (E.3) can be written as
\[ \left( \partial_r^2 + \frac{1}{r} \partial_r \right) \varphi = \frac{m^2}{2} \sinh 2\varphi + \frac{4(\alpha - \alpha')^2}{r^2} \tanh \varphi(1 - \tanh^2 \varphi). \tag{E.10} \]
Finally, using the equation (3.36), and also (3.38), one obtains
\[ \left( \partial_r^2 + \frac{1}{r} \partial_r \right) \Sigma(r) = -m^2 \sinh^2 \varphi = \frac{m^2}{2} (1 - \cosh 2\varphi). \tag{E.11} \]
This is the result announced in the introduction. Notice that $\partial_{\overline{z}} \partial_z \Sigma$ is only parameterized by a single function $\varphi(r)$, and the differential equation for $\varphi$ involves only $\varphi$ itself.

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References

[1] D. Bernard and A. Leclair, hep-th/9402144.

[2] H. Widom, An Integral Operator Solution to the Matrix Toda Equations, solv-int 9702004.

[3] S. Lukyanov and A. Zamolodchikov, Exact expectation values of local fields in quantum sine-Gordon model, hep-th/9611238.