Heavy Sneutrinos as Dark Matter

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Abstract

We calculate the relic density of very heavy, stable scalar neutrinos in the minimal supersymmetric standard model. We include all two-body final states, as well as the effects of co-annihilation with scalar electrons. We find that the sneutrino relic density is in the cosmologically interesting region $0.1 \lesssim \Omega_{\tilde{\nu}} h^2 \lesssim 1.0$ for $550 \text{ GeV} \lesssim m_{\tilde{\nu}} \lesssim 2300 \text{ GeV}$. For nominal values of the parameters of our galactic halo, recent results from the Heidelberg–Moscow direct detection experiment set an upper limit on $\Omega_{\tilde{\nu}}$ which is lower by a factor ranging from two to ten, depending on $m_{\tilde{\nu}}$. 
A sneutrino which is the lightest supersymmetric particle (LSP) is a theoretically attractive dark matter candidate [1, 2]. Previous studies of sneutrino relic densities [1, 2] have been concerned with a sneutrino with a mass $m_{\tilde{\nu}}$ that is less than $m_W$. Heavier sneutrinos can annihilate into a much greater set of final states; for $m_{\tilde{\nu}} \simeq 100\,\text{GeV}$, the annihilation rate is very large, and the relic density of sneutrinos is much too small to be of cosmological or astrophysical interest. However, as the sneutrino mass is increased, the annihilation rate drops and the relic density rises. We show that heavy ($m_{\tilde{\nu}} \gtrsim 500\,\text{GeV}$) sneutrinos may have a cosmologically interesting relic density. For nominal values of the parameters of our galactic halo, direct detection experiments [3] - [8] already place constraints on this range of sneutrino masses, and recent results from the Heidelberg–Moscow direct detection experiment [8] may even rule out the values of $\Omega_{\tilde{\nu}} \equiv \rho_{\tilde{\nu}}/\rho_{\text{crit}}$ that we find.

The calculation of relic densities of heavy sneutrinos involves an additional complication not present for light sneutrinos, arising from the need to consider coannihilations [9] with selectrons. The masses of the sneutrino $\tilde{\nu}$ and left-handed selectron $\tilde{e}_L$ are related via [10]

$$m_{\tilde{e}_L}^2 = m_{\tilde{\nu}}^2 - m_Z^2 \cos 2\beta \cos^2 \theta_W,$$

(1)

where $\tan \beta$ is the ratio of the vacuum expectation values of the two Higgs doublets. We see that for $\tan \beta > 1$ we have $m_{\tilde{e}_L} > m_{\tilde{\nu}}$, as we require if the sneutrino is to be the LSP. However, for $m_{\tilde{\nu}} \gg m_W$, the mass splitting between $\tilde{\nu}$ and $\tilde{e}_L$ will be small. Consequently their number densities are approximately equal until well after the sneutrinos go out of chemical equilibrium with all other particles, and we must follow the selectron density as accurately as the sneutrino density.

We compute the tree level cross-section for annihilation to all allowed two particle final states. Table 1 lists the set of final states we consider. The $f_d$ and $f_u$ refer to $T_3 = -1/2$ and $T_3 = +1/2$ fermions, respectively. As intermediate states we allow sneutrinos, selectrons, the gauge bosons $\gamma, W^\pm$ and $Z^0$, the three Higgs bosons of the minimal supersymmetric standard model (MSSM), $H_i^0$ ($i = 1, 2, 3$), the charginos $\tilde{\chi}_i^\pm$ ($i = 1, 2$), and the neutralinos $\tilde{\chi}_i^0$ ($i = 1, 2, 3, 4$). There are also four-scalar point interactions. The couplings are those of the MSSM [10, 11]. For simplicity, we ignore sfermion (in particular selectron) mixing, and for convenience we use the relation $M_1 = \frac{5}{3} M_2 \tan^2 \theta_W$ between the $U(1)$ gaugino mass $M_1$ and the $SU(2)$ gaugino mass $M_2$ which follows from grand unification. At tree level, the masses of the scalar Higgs particles $H_1^0$ and $H_2^0$ can be written in terms of $m_Z, \tan \beta$, and the mass the pseudoscalar $H_3^0$. (Although the one-loop corrections to the Higgs masses can be large, they will not impact our results significantly, and so we do not include them.) We are then left with five free parameters: $\tan \beta, m_{H_2^0}, m_{\tilde{\nu}}, M_2$, and the Higgs mixing parameter $\mu$. 

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Since our results are largely insensitive to tan $\beta$, we fix tan $\beta = 2$ in our numerical work.

| Initial State | Final States |
|---------------|--------------|
| $\bar{\nu}\bar{\nu}^*$ | $f\bar{f}$, $ZZ$, $W^+W^-$, $ZH^{0}_{(1,2,3)}$, $W_{\pm}H_{\mp}^+$, $H_{\pm}^+$, $H_{\mp}^-$, $H_i^0H_i^0 (i = 1, 2, 3)$, $H_1^0H_2^0$, $H_{(1,2)}^0H_3^0$ |
| $\bar{e}_L\bar{e}_L^*$ | same as above, plus $\gamma\gamma$, $\gamma Z$ |
| $\bar{\nu}\bar{e}_L^*$ | $ZW^+$, $\gamma W^+$, $ZH^+$, $\gamma H^+$, $W^+H^{0}_{(1,2,3)}$, $H_1^+H_1^0H_1^0 (1, 2, 3)$, $f_uf^d$ |
| $\nu\bar{\nu}^*$ | $\nu\nu$ |
| $\bar{e}_L\bar{e}_L$ | $ee$ |
| $\bar{\nu}\bar{e}_L$ | $\nu e$ |

To derive a thermally averaged cross section, we make use of the technique of ref. [12]. We expand $\langle \sigma v_{rel} \rangle$ in a Taylor expansion in powers of $x = T/m_\nu$:

$$\langle \sigma v_{rel} \rangle = a + bx + O(x^2).$$

(2)

Repeating the analysis of ref. [12] for initial particles with different masses $m_1$ and $m_2$ yields

$$\langle \sigma v_{rel} \rangle = \frac{1}{m_1m_2} \left( 1 - \frac{3(m_1 + m_2)T}{2m_1m_2} \right) w(s)|_{s \rightarrow (m_1+m_2)^2 + 3(m_1+m_2)T} + O(T^2).$$

(3)

Here

$$w(s) = \frac{1}{4}\int dLIPS |M|^2$$

(4)

$$= \frac{1}{32\pi s^{1/2}} \int_{-1}^{+1} d\cos\theta_{CM} |M|^2,$$

(5)

where $dLIPS$ is the Lorentz Invariant Phase Space element, $p(s)$ is the magnitude of the three-momentum of one of the initial particles in the center-of-mass frame as a function of the total center-of-mass energy squared $s$, $\theta_{CM}$ is the center-of-mass scattering angle, and $|M|^2$ is the absolute square of the reduced matrix element for the annihilation, summed over final spins. The $a$ and $b$ coefficients may be read off the right-hand side of Eq. (3) after expanding in powers of $x$. 

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The sneutrinos and selectrons remain in chemical equilibrium with each other through freeze-out. We consider the total density \( n = \sum_i n_i \), where the index runs over \( \tilde{\nu}, \tilde{\nu}^*, \tilde{e}_L \), and \( \tilde{e}_L^* \). We write the rate equation for \( n \):

\[
\frac{dn}{dt} = -3Hn - \langle \sigma_{\text{eff}} v_{\text{rel}} \rangle (n^2 - n_{\text{eq}}^2).
\]

Here

\[
\sigma_{\text{eff}} = \sum_{ij} \sigma_{ij} r_i r_j,
\]

where \( H \) is the Hubble parameter, \( r_i = n_{i}^{\text{eq}} / n_{\text{eq}} \), \( n_{i}^{\text{eq}} \) is the equilibrium density of particle species \( i \), \( \sigma_{ij} \) is the total cross-section for the \( i \)th particle to annihilate with the \( j \)th particle. Long after freeze-out, the selectrons decay into sneutrinos, leaving a number density \( n \) of sneutrinos. We rewrite the derivative in eq. (6) in terms of the temperature \( T \) and numerically integrate the resulting equation down to \( T = 2.75 \) K to get the relic number (and hence mass) density of sneutrinos.

We find that the dominant contribution to \( \sigma_{\text{eff}} \) comes from annihilations of \( \tilde{\nu} \tilde{\nu}^* \), \( \tilde{e}_L \tilde{e}_L^* \), and \( \tilde{\nu} \tilde{e}_L \) to gauge bosons, and \( \tilde{\nu} \), \( \tilde{e}_L \tilde{e}_L \), and \( \tilde{\nu} \tilde{e}_L \) to neutrinos and electrons via neutralino and chargino exchange.

In fig. 1 we show the relic density \( \Omega_{\tilde{\nu}}h^2 \) as a function of sneutrino mass; \( h \) is the Hubble parameter today in units of 100 km/s/Mpc. The dashed line assumes that two of the sneutrino generations are much heavier than the third, and that the lightest neutralino is just 10% heavier than the sneutrino. (We set \( \mu = M_1 \), but since the higgsinos do not couple significantly to the sneutrinos, the value of \( \mu \) does not affect our results. Also, the pseudoscalar Higgs mass was taken to be \( m_{H^0} = 500 \) GeV, but changing this parameter also has a negligible effect.) Note that \( \Omega_{\tilde{\nu}}h^2 \) passes through the cosmologically interesting region \( 0.1 \lesssim \Omega_{\tilde{\nu}}h^2 \lesssim 1.0 \) for \( 550 \) GeV \( \lesssim m_{\tilde{\nu}} \lesssim 2300 \) GeV. Increasing \( M_1 \) raises the masses of the gauginos and eventually turns off the gaugino-mediated annihilations of \( \tilde{\nu} \tilde{\nu}^* \), \( \tilde{e}_L \tilde{e}_L^* \), and \( \tilde{\nu} \tilde{e}_L \) to neutrinos and electrons. The dot-dashed line in fig. 1 is for this case, with \( M_1 \simeq 10 m_{\tilde{\nu}} \). We can also consider the case where all three generations of sneutrinos have very similar masses, and so are all in chemical equilibrium with each other at the time of freeze-out. Although one might expect that this case simply results in a value of \( \Omega_{\tilde{\nu}}h^2 \) which is three times larger, in fact it is a little more complicated because some annihilation channels (\( \tilde{\nu}_i \tilde{\nu}_j \rightarrow \nu_i \nu_j \), \( \tilde{\nu}_i \tilde{e}_{Lj} \rightarrow \nu_i e_j \), \( \tilde{e}_{Li} \tilde{e}_{Lj} \rightarrow e_i e_j \)) can now occur in nine different ways, whereas all the others (\( \tilde{\nu}_i \tilde{\nu}_i^* \rightarrow f \bar{f} \), etc.) can occur in three different ways. In fig. 1, the solid line corresponds to three equal-mass generations of LSP sneutrinos, but with the neutralinos again as light as possible (\( M_1 = \mu \simeq 1.10 m_{\tilde{\nu}} \)). When the gauginos are heavy, then \( \Omega_{\tilde{\nu}}h^2 \) is very close to what it would be for one generation of LSP sneutrinos.
The choice of masses for the sfermions here may seem a bit ad hoc. In the simplest grand unified model where all the sfermions share a common scalar mass parameter $m$, radiative corrections drive the mass of the sneutrino above the mass of the right-handed selectron [13]. To keep the sneutrino as the lightest sfermion, and to prevent the sneutrino mass from being driven above the mass the lightest neutralino, we must invoke new high-energy physics to alter the running of the sneutrino mass. The GUT relation between $M_1$ and $M_2$ may then be affected, but we have seen that the effect of changing the gaugino masses by a factor of a few is small.

Non-detection of halo particles in direct detection experiments have put bounds on the mass and relic abundance of dark matter sneutrinos [3, 4, 5, 6, 8]. The dotted line in fig. 1 marks the most stringent reported upper bound [8] on the relic density $\Omega_{\tilde{\nu}}$ of sneutrinos for a given sneutrino mass $m_{\tilde{\nu}}$. We assume the total dark matter density $\Omega_{\text{DM}} = 1$, so that the fraction of the dark matter halo which is composed of sneutrinos is $\Omega_{\tilde{\nu}}$. As in [8], we take the local halo density of dark matter to be $0.3 \text{ GeV/cm}^3$; then $\rho_{\tilde{\nu}} = (0.3 \text{ GeV/cm}^3)\Omega_{\tilde{\nu}}$. The cross-section for sneutrino-nucleus interactions is four times greater than the cross-section for Dirac neutrino-nucleus interactions [14, 15] (not two times greater, as sometimes stated [3, 6]). In fig. 1, we take the published data of [7] to determine the upper bound on $\Omega_{\tilde{\nu}}$. We compare their cross section constraint to four times the Dirac neutrino cross section. We see that there are upper bounds of $m_{\tilde{\nu}} < \sim 600 \text{ GeV}$ and $\Omega_{\tilde{\nu}} h^2 < \sim 0.08 \text{ GeV}$ (for $h = 1$). Most cosmologically interesting relic densities are excluded. This assumes that the dark matter which is not composed of sneutrinos interacts less strongly with direct detection experiments than do sneutrinos. Also in fig. 1, the lower dotted line is found from $\Omega = \sigma_b/4\sigma_c$, where $\sigma_b$ and $\sigma_c$ refer to the curves from fig. 3 of [8]. From this experiment we see that our computed range of $\Omega_{\tilde{\nu}}$ is ruled out by a factor ranging from roughly two to ten (for $h = 1$). If we consider sneutrinos as the sole component of the dark matter, then we should fix $\rho_{\tilde{\nu}} = 0.3 \text{ GeV/cm}^3$ independent of $\Omega_{\tilde{\nu}} h^2$; in this case we get a lower bound on the sneutrino mass of $m_{\tilde{\nu}} \gtrsim 17 \text{ TeV}$, which we find requires a value of $\Omega_{\tilde{\nu}} h^2$ that is much bigger than one. However, we should keep in mind that the uncertainty in the local value of our halo density is at least a factor of two [16]. Furthermore, the momentum transfer in the interaction of a heavy sneutrino with a detector nucleus is large; a calculation of the interaction cross-section must therefore include nuclear form factors, which are not well known.

To summarize, we have calculated the relic density of sneutrinos in a model where a heavy ($\gtrsim 500 \text{ GeV}$) sneutrino is the LSP. We have found that sneutrino masses in the range $550 \text{ GeV} \lesssim m_{\tilde{\nu}} \lesssim 2300 \text{ GeV}$ may give cosmologically interesting relic densities, but that this range is ruled out for nominal values of the parameters of our galactic halo by a factor
ranging from roughly two to ten. Uncertainties in the halo parameters and nuclear form factors still leave a marginal chance for the heavy sneutrino to be a component of the halo dark matter.

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Fig. 1) The relic density $\Omega_{\tilde{\nu}} h^2$ as a function of $m_{\tilde{\nu}}$. The dashed line corresponds to $m_{H^0} = 500$ GeV and $m_{\tilde{\chi}_1^0} \simeq 1.10 m_{\tilde{\nu}}$, with one generation of LSP sneutrinos. The dot-dashed line is for $m_{\tilde{\chi}_1^0} \simeq 10 m_{\tilde{\nu}}$. The solid line corresponds to the same parameters as the dashed line, but with three generations of LSP sneutrinos. The Neuchatel-CIT [7] and the Heidelberg–Moscow [8] direct detection experiments exclude values of $\Omega_{\tilde{\nu}}$ (not $\Omega_{\tilde{\nu}} h^2$) above the upper and lower dotted lines respectively.
