Temporal ghost Fourier compressive inference camera

CHENGYANG HU,¹,²† HONGHAO HUANG,¹,²† MINGHUA CHEN,¹,² SIGANG YANG,¹,² AND HONGWEI CHEN¹,²,*

¹Department of Electronic Engineering, Tsinghua University, Beijing 100084, China
²Beijing National Research Center for Information Science and Technology (BNRist), Beijing 100084, China
*Corresponding author: chenhw@tsinghua.edu.cn

Abstract

The need for real-time processing fast moving objects in machine vision requires the cooperation of high frame rate camera and a large amount of computing resources. The cost, high detection bandwidth requirements, data and computation burden limit the wide applications of high frame rate machine vision. Compressive Video Sensing (CVS) allows capturing events at much higher frame rate with the slow camera, by reconstructing a frame sequence from a coded single image. At the same time, complex frame sequence reconstruction algorithms in CVS pose challenges for computing resources. Even though the reconstruction process is low computational complexity, image-dependent machine vision algorithms also suffers from a large amount of computing energy consumption. Here we present a novel CVS camera termed Temporal Ghost Fourier Compressive Inference Camera (TGIC), which provides a framework to minimize the data and computational burden simultaneously. The core of TGIC is co-design CVS encoding and machine vision algorithms both in optical domain. TGIC acquires pixel-wise temporal Fourier spectrum in one frame, and applies simple inverse fast Fourier transform algorithm to get the desired video. By implementing pre-designed optical Fourier sampling schemes, specific machine vision tasks can be accomplished in optical domain. In fact, the data captured by TGIC is the results of traditional machine vision algorithms derived from the video, therefore the computation resources will be greatly saved. In the experiments, we can recover 160 frames in 0.1s single exposure with 16x frame rate gain (periodic motion up to 2002 frames, 1000x frame rate gain), and typical machine vision applications such as moving object detection, tracking and extraction are also demonstrated.
Introduction

Machine vision applications such as autonomous driving, robotics, security and industrial automation have fueled great interest in fast move object imaging. Capturing fast events with high frame rate cameras is the straightforward solution, but the cost of camera, high detection bandwidth requirements and large amount of data generated when capturing the video poses challenges for transmission and storage. With the development of computational photography, Compressed Video Sensing (CVS) has been proposed to solve the problems of bandwidth and data overhead. CVS is mainly used to compress video during capture process and are optimized to produce a high frame rate video with low frame rate camera from a coded single image\(^1\). Under this framework, various computational photography schemes have been proposed\(^2-8\). However, the reduction in data brings huge cost of increased computations. Although these schemes are quite effective in high speed imaging, CVS video reconstruction requires complex optimization and iterative algorithms making it only suitable for offline video reconstruction applications\(^1\). None of the current CVS solutions can be used for real-time high-speed video reconstruction, nor can it implement various machine vision tasks in real-time applications.

CVS uses low speed image sensors to detect rapidly varying signals, which is similar to ghost imaging. Ghost imaging (GI) also can recover fast varying signals with low resolution detectors, both in spatial and temporal domains, called spatial ghost imaging\(^9\) (SGI) and temporal ghost imaging\(^10,11\) (TGI) respectively. Hadamard and Fourier spatial ghost imaging are two representative deterministic compressive sensing models in SGI\(^12,13\). These techniques allow one to acquire the object image in a transformation domain and reconstruct the image via the corresponding inverse transform. Since complex optimization algorithms are not required in the reconstruction process, it is suitable for real-time applications. The difference between GI and CVS is that GI only cares about temporal or spatial signals, while CVS considers both spatial and temporal varying signals simultaneously. By taking into account joint spatio-temporal coding, some ideas of GI can be introduced into CVS. Furthermore, in many applications, video reconstruction is not the eventual goal, such as objects detection, tracking and extraction etc.. Even though the reconstruction process is fast, the following image or
video dependent algorithms also consume extra computation time and a large amount of computing resources.

In this paper, we present a novel CVS camera termed Temporal Ghost Fourier Compressive Inference Camera (TGIC), which provides an elegant framework to co-design CVS encoding and machine vision algorithms both in optical domain to develop new solutions to traditional camera technology. The traditional camera only records the light intensity data about the scene, and the purpose of machine vision is to extract the information that ones need from the data. The process from data to information is termed ‘inference’\textsuperscript{1}. The unique capability of TGIC is performing ‘inference’ while compressing data in the optical domain. In detail, TGIC not only acquires compressed video, but also can directly extract information from the scene, such as moving object detection, tracking and extraction. The concept of TGIC is to consider each scene pixel as a temporal channel, use the fast spatial light modulator (SLM) and slow image detector to acquire each pixel temporal Fourier spectrum in a single exposure. The desired video can be perfectly reconstructed by applying the one-dimensional (1D) inverse fast Fourier transform (IFFT) algorithm to each temporal channel. Due to low computational complexity, IFFT can be used to reconstruct video from a single exposure in real-time. Thanks to the clear physical meaning of the Fourier transform, compression and inference can be realized based on the same optical coding principles. By implementing specific Fourier sampling schemes, the data captured by the camera is the result that the machine vision algorithms need to calculate based on the video. In this way, machine vision algorithms processing of some common tasks can be avoided. To demonstrate the capability of TGIC, we present three applications, which cover compressed video reconstruction, moving object detection and tracking, and object extraction. These TGIC applications can be easily switched only by adjusting the coding pattern without changing the system structure. As a flexible framework, TGIC can be integrated with existing imaging systems, and is suitable for micro to macro imaging.
**Principle and results**

TGIC is suitable designed for processing machine vision tasks in fast dynamic scenarios (Fig. 1a). The principle of TGIC is illustrated in Fig. 1b and Fig. 1c. A spatial light modulator, which is digital micro-mirror device (DMD) in experiment, is set between the camera lens and image sensor (CCD). The DMD is utilized to encode the scene during a single exposure of the imaging sensor. The scene is imaged on the digital micro-mirror device (DMD) for light amplitude distribution modulation and the light from DMD is then focused onto an image sensor. Fig. 1c illustrated the coding strategy of TGIC. The DMD is spatially divided into m x n coding groups (marked as CG) to perform parallel detection and each of them serve as an independent temporal channel. During a single exposure, a series of patterns are projected on DMD to encode the scene frame sequence. As DMD is pixel-matched to the image sensor and a corresponding pattern is created on the frame. Every CG consists of p x q coding elements (marked as CE), each of them contains 4 pixels and these pixels are modulated in a pre-determined sinusoid fashion with 4 different phases (2 x 2, phases: 0, 0.5pi, pi, 1.5pi). Since a single exposure of the image sensor integrates the Hadamard product between sinusoid signal and temporal waveform of the scene, we can extract the Fourier coefficient for one specific frequency by means of 4-step phase-shifting (see Methods) in one CE and further acquire the temporal spectrum in one CG. Through an inverse Fourier transform, we can reconstruct the temporal waveform. With the same operation applied to all CGs, the video of the whole scene can be recovered. However, the DMD only modulates the light in a binary form. Here we utilize pulse-width modulation (PWM) mode to realize equivalent temporal sinusoid modulation, which is validated by experiments. The PWM mode reduces the DMD refresh rate. Zhang et al.\textsuperscript{14} used error diffusion dithering techniques to binarize the Fourier basis patterns in space, which can be used in TGIC to maintain the refresh rate of DMD.
**Fig. 1 Overview of TGIC.**

a Capture a dynamic scene using TGIC.  
b Schematic and photo of TGIC.  
c The coding strategy of TGIC. The real scene is coded by spatial light modulator and integrated during a single exposure of the image sensor. The DMD was spatially divided into coding groups (5x5 coding groups are shown here, marked as CG) and each CG contains coding elements (4x4 coding elements are shown here, marked as CE) to extract the Fourier coefficients of the pixel.
temporal vector. The Fourier coefficients can be used to reconstruct video by applying inverse Fourier transform or extract information of the scene. Compressed data captured by TGIC (left) and three demonstrative applications of TGIC, including cover compressed video reconstruction, moving object detection and tracking, and object extraction.

Modern imaging sensors come with a large scale of pixels, which may prove useful for still photography but superfluous in most other imaging situations. TGIC makes use of the camera’s redundant spatial resolution for further frame rate improvement by transferring temporal spectrum into spatial pixels. Here we define a ratio to evaluate the spatio-temporal transfer efficiency $T = K/L$. By acquiring $K/2$ Fourier coefficients, one can improve the frame rate by $K$ times at the cost of $L$ times reduction in spatial resolution. Each Fourier coefficient is in need of 4 pixels for 4-step phase-shifting, thus $L = 2^4K$. As a result, $T$ equals to $1/2$ for 4-step phase-shifting scheme in TGIC. Although the 4-step phase-shifting offers better measurement performance while one can also utilize 3-step phase-shifting ($T = 2/3$) or 2-step phase-shifting ($T = 1$) for a higher spatio-temporal transfer efficiency. We can choose proper phase-shifting schemes for different scenarios, which shows the flexibility of TGIC. Moreover, the pitch size of the DMD is larger than the pitch size of the image sensor. We adjust the paraxial magnification of the zoom lens to match one DMD mirror with 3x3 image sensor pixels (i.e., larger effective image sensor pixel size) to ensure accurate DMD and image sensor alignment (see Supplementary Information Section S1 for details).

To correctly recover the video, we design a corresponding reconstruct algorithm. First, the captured coded image is split into blocks based on CGs. Then within one CG, every CE is extracted and performed 4-step phase-shifting to calculate Fourier coefficients. Next, we combine Fourier coefficients to a spectrum and reconstruct temporal waveform by inverse Fourier transform. With same operations to all CGs, the temporal characteristics of the encoded image are analyzed, and one image in single exposure can generate different time series sub-images, which can form a video. (see Supplementary Information Section S2 for quantitative analysis on the reconstruction results of TGIC).

Since TGIC offers the unique capability to acquire compressive video at low computation cost and directly extract information from the scene, it enables us to record dynamic scenes using low-
speed image sensor, sense information of moving objects or pre-process the image in the optical domain. Here, we introduce three applications to demonstrate these advantages of TGIC (illustrated in Fig. 1d). Firstly, for both aperiodic motion and periodic motion, we utilize TGIC to reconstruct compressive video. Secondly, moving object can be detected and the trace of moving object is obtained directly from the captured coded data without reconstruct the video. Finally, the objects of interest with specific moving speed or texture object can be extracted from the static background.

Example application I: compressive video reconstruction

Aperiodic motion video

For ordinary aperiodic moving objects or natural varying scenes, the energy is mainly concentrated at low frequencies in temporal spectrum. This observation is exploited to realize compressive video in time domain by only acquired the Fourier coefficients of low frequencies using TGIC. With one additional Fourier coefficient acquired, K can be increased by 2 (see Supplementary Information Section S3 for details).

To demonstrate compression characteristics of TGIC, the experiment setup and the corresponding coding signal of DMD is illustrated in Fig. 2a. With applying nine frequencies ranging from 0Hz (DC component) to 80Hz to encode the scene within 0.1 second exposure (corresponding 10fps), we reconstructed a video of the scene at an equivalent 160Hz frame rate with 16 times speed up compared to the original frame rate. To acquire nine frequencies we need 3 x 3 CEps in each CG, resulting in a resolution of 235 x 157 in reconstructed video. The frequency interval of the encoded signal satisfies the frequency domain sampling theorem and is determined by the exposure time (see Supplementary Information Section S3 for details).

The first scene in this application is a toy car running in the field of view. A capture of the static toy car is shown in Fig. 2b (top left) as ground truth. The coded data acquired by TGIC is shown in Fig. 2b (bottom left) in which the scene is blurred and features of the toy car cannot be visually distinguished. After decoding, a high frame rate video was reconstructed and Fig. 2b (right)
displays three frames from the video. The results show TGIC clearly resolves the shape and features of the toy car. We also provide reconstructed video of this case as Supplementary Video 1.

In addition, we test aperiodic motion video of TGIC on rotating object compared to aforementioned toy car which is mainly a translating object. This scene is a panda pattern on a rotating disk with an angular velocity of ~20rad/s. In Fig. 2c, the static capture of the object (top left), coded data (bottom left) and three frames (right) from the recovered video are shown respectively. Our results clearly show the status of the rotating object at different time during a single exposure.

**Fig.2 Capturing aperiodic motion video using TGIC.** **a** Illumination of experiment setup and coding pattern on DMD. Each CG contains 9 CEs (3 x 3, ranging from 0Hz to 80Hz) to encode the scene. **b** A toy car is used as target. Top left: static object as groups truth. Bottom left: coded data
captured by TGIC with motion blur. Right column: three frames from the reconstructed video. A red dotted line is shown as reference. c Results for a rotating disk with a panda pattern. These results show that TGIC also works for aperiodic rotating object.

**periodic motion video**

Periodic motion widely exists in medical, industry and scientific research, such as heartbeat, rotating tool bit and vibration. Since a periodic signal contains energy only in the direct current, fundamental frequency and harmonics, it has a very sparse representation in the Fourier domain (see Supplementary Information Section S5 for details). By taking the temporal spectrum characteristics into account as prior information, we use TGIC to selectively acquire several principal frequencies and recover a video for periodic motion with higher compression ratio compared to the coding method for aperiodic motion, which allows less sacrifice of pixels and a higher spatial resolution.
Fig. 3 Capturing periodic motion video using TGIC. a To capture an periodic motion with 4 frequencies, each CG contains 4 CEs (2 x 2) to encode the scene. b A designed rotating disk is used as target. Top left: static object as groups truth. Top right: the zoom-in view of the captured data with and without coding, corresponding to normal slow cameras and TGIC respectively. Normal slow cameras blur out the details of moving objects while coded structure in TGIC capture provides sufficient information to reconstruct the video. Bottom: four frames from the reconstructed video. Red dotted lines are shown in each frame to indicate the direction of the disk.

As shown in Fig. 3b (top left), a rotating disk with periodic patterns is designed as target. The disk rotates at a speed as high as 5460 rpm. The disk contains four rings with 3, 5, 7, 11 spatial periods from inner to outer, thus the temporal frequencies of these four rings are 273, 455, 637, 1001Hz, respectively. We applying these frequencies to DMD to encode the scene (Fig. 3a) during a 0.5s exposure (2Hz frame rate) and further reconstruct a video of the rotating disk. Here, the equivalent maximum frame rate is 2002Hz, so the frame rate improvement is 1001 times (corresponding compression ratio is 0.1%). The acquiring of four frequencies needs 2 x 2 CEs in each CG, thus the resolution of reconstructed video is 353 x 235. Four frames from the video are shown in Fig. 3b (bottom). The reconstructed video is provided as Supplementary Video 2.

Example application II: moving object detection and tracking

Object detection and tracking for fast moving object has found important applications in various fields. In general, object detection is to determine the presence of an object and object tracking is to acquire the spatio-temporal coordinates of a moving object. For the temporal waveform of a pixel where the object would pass by, moving object takes the form of a pulse at a specific time. As the object moving, the temporal waveforms at different spatial positions are of different temporal pulse positions, resulting in phase shift in their temporal spectrums. Since Fourier transform is a global-to-point transformation, we can extract the information of presence and position of pulse in time domain from the amplitude and phase of a single Fourier coefficient. From this perspective, we can use TGIC to determine the presence or/and simultaneously acquired the spatial trajectory and temporal position of moving object. The aforementioned information can be obtained directly from the captured coded data without reconstruct the video and therefore the inference characteristics were demonstrated.
Fig.4 Moving object detection and tracking by TGIC. a Only one frequency is needed to encode the scene for moving object detection and tracking. The period of sinusoidal coding signal is equal to the exposure time. Thus, only one CE is contained in each CG. b Coded data captured by TGIC and tracking results. Left column: characters ‘T’, ‘H’, ‘U’, ‘EE’ sequentially displayed by a screen with a 0.25s duration for each. The color indicates the distribution of appearing time. Middle column: results for a displayed spot moving along a heart-shaped trajectory. Right column: results for two spots moving in circular trajectories with different radius. The spots are printed on a rotating disk driven by a motor.

To test this capability of TGIC, we capture several targets ranging from flash characters, single moving object and multiple objects. The image sensor exposure time is 1 second and the corresponding coding signal on DMD is 1Hz to ensure one period is contained by a single exposure to avoid 2-pi phase ambiguity due to the periodicity of the Fourier basis coding. Firstly, a screen displays ‘T’, ‘H’, ‘U’, ‘EE’ sequentially with a 0.25 second duration for each (see Supplementary Video 3). The raw capture and the extracted temporal position are shown in Fig. 4b (the left column), which indicates that TGIC is able to detect the objects via amplitude and distinguish different temporal position of objects via phase. Then a spot moving along a heart-shaped trajectory,
displayed on the screen, is used as target to test the tracking capability of TGIC (see Supplementary Video 4). This result (Fig. 4b, the middle column) shows TGIC can resolve the spatial and temporal position of the object. We also test TGIC on actual multiple objects, which are two spots moving in circular trajectories with different radius (Fig. 4b, the right column). The spots are printed on a rotating disk driven by a motor in a speed of 60 rpm. The scene is also recorded by a relatively high-speed camera for reference (see Supplementary Video 5). The temporal resolution is determined by both the exposure time and coding frequency (see Supplementary Information Section S6 for details), that is, the higher coding frequency is, the higher temporal resolution will be, but the temporal range also narrows at the same time. For the current setup, the temporal resolution is 3.9 millisecond. By applying phase unwrapping algorithms, the trade-off between temporal resolution and temporal range can be overcome to further improvement the tracking performance.

Example application III: object extraction

Subtracting background and extracting moving objects are significant techniques for video surveillance and other video processing applications. In the frequency domain, the background is concentrated on the DC component. By filtering the DC component, one can subtract background and extract moving objects. Many moving object extraction approaches performed in frequency domain\textsuperscript{16,17,18} have been proposed, which need to acquire the video first and then perform Fourier transform, thus suffer from relatively high computational cost and low efficiency. Thanks to the capability of TGIC to directly acquire specific spectral components in optical domain, it can overcome the drawbacks of aforementioned methods. In addition to subtracting background, pre-analysis on the temporal spectrum profile of the objects of interest gives the prior for one to design coding patterns for TGIC to realize specific object extraction.
**Fig. 5 Object extraction by TGIC.**

**a** Illumination of objects extraction. The coding frequencies are based on the spectrum of the objects of interest. In this demonstration, the four rings on the disk are regarded as four objects of interest. Each ring only contains one frequency so that one CE is used in one CG. **b** Left: reference static scene with a disk and a poker card. The disk is rotating when capturing and the four rings share the same rotating speed. Four right columns: TGIC captured data for four rings extraction and corresponding results. For each extracted ring, other rings and static poker card are neglected. **c** Results for two identical rings rotating at different speed (1980 rpm and 800 rpm, respectively). TGIC enables extraction of a specific one out of these two rings.
To demonstrate background subtraction capability of TGIC, we capture a scene that has a rotating disk identical to the one in application I as object and a static poker card as background (Fig. 5a, left). Same as previously introduced in application I, the exposure time is 0.5s and the temporal frequencies of these four rings are 273, 455, 637, 1001Hz, respectively. The main difference to application I is that only the frequency that corresponds to one ring is applied for coding (Fig. 5a) in this case. In this way, each ring can be separately extracted without the background static poker card (Fig. 5b). The results also indicate that we can distinguish objects with the same rotating speed but different textures. In comparison, objects with the same texture but different speeds can also be extracted separately. In Fig. 5c (left), two identical disks, both with six stripes, are present in the scene. They rotate at 1980 rpm (200Hz in temporal spectrum) and 800 rpm (80Hz in temporal spectrum) relatively and they appear the same in the capture of normal slow camera. With TGIC we can see the difference in the coding data and can extract a specific one out of them (Fig 5c). For simplicity, the above results are all one frame in the reconstructed video.

The above results show that TGIC enables background subtraction and object extraction based on the temporal spectrum difference. Although only one frequency was used in the experiment, in principle it is allowed to use multiple frequencies to reconstruct more complex scenes (as shown in application I), as long as the spectral difference are sufficiently obvious. It is worth noting that in some special cases, objects with different textures and speeds may have the same spectral features, making TGIC fails to distinguish them (see Supplementary Information Section S7 for details).

**Conclusion and discussion**

This paper introduces a novel CVS camera framework, not only acquires compressed video at low computation cost, but also can directly extract information from the scene. The strategy of compression inferences in the optical domain, while reducing the data and computational burden in the machine vision workflow as much as possible. TGIC has the basic capability of increasing frame rate with low speed camera as common CVS camera did. Furthermore, it can realize specific
machine vision applications in optical domain instead of complex post-processing of images and videos. Three kinds of applications have been introduced in experiment. Firstly, the compression characteristics of TGIC are demonstrated. We show that the frame rate of TGIC has been improved by 16 times, and it has the ability to detect periodic motion in a high compression ratio. Secondly, the inference characteristics were also demonstrated. We show moving object detection and tracking where the information is obtained directly from the captured coded data even without the inverse Fourier transform. Then, the static background is neglected while only extracted the objects of interest with specific moving speed or texture. Among these applications, prior knowledge is not required for aperiodic compressive video reconstruction, moving object detection and tracking, subtracting background and extracting moving object (see Supplementary Information Table 1 for details). These applications cover the most common scenarios and can be integrated with existing machine vision systems, especially autonomous driving and security. Fast move object imaging, moving object tracking, and subtracting background are all basic requirements in these systems\textsuperscript{19}. The emergence of prior knowledge makes TGIC lose some flexibility, but gains better performance. Applications that require prior knowledge (periodic compressive video reconstruction and specific object extraction) have special scenarios (e.g., modal analysis of vibrations). Several engineering disciplines rely on modal analysis of vibrations to learn about the physical properties of structures. Relevant areas include structural health monitoring\textsuperscript{20} and non-destructive testing\textsuperscript{21,22}. These special scenarios are usually stable (i.e., require less flexibility) and allow better performance at a higher cost.

We noticed that there are some impulse coding methods with simple reconstruction process, in which pixels in a spatial block are turned on at a certain time\textsuperscript{23,24}. However, these strategies cannot achieve data compression, and the light throughput is much lower than TGIC (see Supplementary Information Section S4 for details), and they do not have inference capabilities. One limitation of the current framework is that the system requires sacrificing the number of pixels to improve the frame rate. Although the reduction in spatial resolution is a common problem of CVS technology, and currently camera pixels are redundant, one would expect fewer pixel sacrifices. The
reason for the pixel sacrifice is that in one CG (temporal channel), multiple CEs (pixels) are used to obtain the temporal Fourier coefficients in parallel. Taking a closer look at the process, one can notice that the principle of TGIC is similar to the color camera based on Bayer Color Filter Array (CFA)\textsuperscript{25}. CFA and TGIC use different pixels to collect different wavelength and temporal Fourier coefficients in parallel, respectively. Therefore, the demosaicing algorithm in CFA can be introduced into TGIC to improve the spatial resolution\textsuperscript{26,27}. Combining high-level encoding methods and video reconstruction algorithms, the video quality of TGIC can be further improved. The encoding methods and the reconstruction algorithms presented considered the data at every CG as independent. In fact, adjacent CG have similar temporal waveforms, which depends upon the local speed and direction of motion object. By exploiting the spatio-temporal regularity and redundancy, TGIC can be further improved to deal with arbitrary high-speed events. If the computational complexity and real-time are ignored in order to obtain the highest quality video, conventional video reconstruction algorithms can also be used in TGIC. For example, there is potentially performance gain in formulating the reconstruction algorithm to enforce temporal smoothness over multiple reconstruction frames, such as optical flow method given by Brox et al\textsuperscript{28}. Although a monochrome image detector is used in the experiments, the possibility of combining TGIC with a color image detector is obvious. As long as the coding structure of the TGIC needs to be adjusted according to the distribution of CFA. In addition, proposing a more compact and lightweight design will help develop a commercial TGIC. One can borrow compact optical design from miniaturized DMD based projectors, the other is to integrate the modulator on sensor chip, which is still challenging with current technology. And in some applications with loose frame rate requirements, a commercial liquid crystal modulator can be used instead of DMD to reduce costs. Beyond machine vision, we believe that the inference properties of TGIC can play a role in other fields. For example, use TGIC to perform frequency division multiplexing demodulation in space optical communication, or to extract specific signals in voice signal detection.
Methods

Experimental setup

In the experimental setup, the scene is imaged on a virtual plane through a camera lens (CHIOPT HC3505A). A relay lens (Thorlabs MAP10100100-A) transfers the image to the DMD (ViALUX V-9001, 2560 x 1600 resolution, 7.6μm pitch size) for light amplitude distribution modulation. The reflected light from DMD is then focused onto an image sensor (FLIR GS3-U3-120S6M-C, 4242 x 2830 resolution, 3.1μm pitch size) by a zoom lens (Utron VTL0714V). Due to one DMD mirror is matched with 3x3 image sensor pixels, the effective resolution is one-third to the resolution of the image sensor in both horizontal and vertical direction (i.e. 1414 x 943).

Fourier coefficient obtention

The principle of the proposed TGIC system is spatially splitting the scene into independent temporal channels and acquiring the temporal spectrum by corresponding CG for each channel. Every CG contains some CEs to obtain Fourier coefficients for frequencies of interest. During one exposure time $t_{\text{expo}}$, the detected value $D_{jk\varphi}$ in CE $k$, CG $j$ is equivalent to an inner product of pixel temporal vector $I_j(t)$ and pixel temporal sampling vector $S_{jk\varphi}(t)$:

$$D_{jk\varphi} = \left< I_j(t), S_{jk\varphi}(t) \right> = \int_{t_{\text{expo}}} I_j(t) \times [A + B \cos(2\pi f_k t) + \varphi] dt$$

(1)

Where $S_{jk\varphi}(t)$ is the sinusoidal pixel temporal sampling vector with frequency $f_k$ and phase $\varphi$ in CE $k$, CG $j$. $A$ and $B$ denote the average intensity and the contrast of $S_{jk\varphi}(t)$, respectively. The Fourier coefficient $F_{jk}$ of $f_k$ can be extracted by 4-step phase-shifting as

$$2BC \times F_{jk} = \left( D_{jk0} - D_{jk\pi} \right) + i \left( D_{jk\frac{\pi}{2}} - D_{jk\frac{3\pi}{2}} \right)$$

(2)

Where $C$ depends on the response of the image sensor. The DC term $A$ can be cancelled out simultaneously by the 4-step phase-shifting.

Video reconstruction

By using the above method, we assemble the Fourier coefficient $F_{jk}$ of $f_k$ in CG $j$. We can combine all Fourier coefficients in CG $j$ to form its temporal spectrum as

$$F_j = \{F_{jh}, F_{jh-1}, ..., F_{jh-\nu}, F_{jh}\}, h = p \times q$$

(3)
Where $h$ is the number of CEs in a CG, $p \times q$ is the number of CEs in every CG and $F_{jh}^\ast$ denotes the complex conjugate of $F_{jh}$. The pixel temporal vector $I_j(t)$ can be reconstructed by applying inverse Fourier transform:

$$2BC \times R_j = \mathcal{F}^{-1}\{F_j\}$$ (4)

Where $\mathcal{F}^{-1}$ denotes the inverse Fourier transform operator. The result of the inverse transform $R_j$ is proportional to the pixel temporal vector $I_j(t)$ in CG $j$. By applying the same operation to all CGs, we can reconstruct the video of the scene.

**Moving object detection and tracking**

To detect and track moving object, only one frequency is needed to encode the scene. In this case, we let

$$p = q = 1$$ (5)

and

$$f = f_0 = \frac{1}{t_{\text{expo}}}$$ (6)

Thus, $f_0$ is the lowest resolvable frequency and its Fourier coefficient $F_{j0}$ provides sufficient knowledge of presence or/and motion of object. The amplitude $A_{j0}$ of $F_{j0}$ is as follow:

$$A_{j0} = \text{abs}(F_{j0})$$ (7)

Where $\text{abs}(\ast)$ denotes the absolute operation. As a static scene does not contain the $f_0$ component in temporal spectrum, moving object detection can be achieved by applying a threshold on $A_{j0}$ that an $A_{j0}$ larger than the threshold indicates the presence of moving objects.

For moving object tracking, since the long exposure has already given the trace of the object, the phase $P_j$ of $F_{j0}$ is utilized to further extract the temporal information.

$$P_j = \text{arg} (F_{j0})$$ (8)

Where $\text{arg}(\ast)$ denotes the argument operation. A temporal waveform with a displacement of $t_j$ in time domain results in a linear phase shift of $-2\pi f_0 t_j$ in temporal spectrum:

$$I_j(t - t_j) = \mathcal{F}^{-1}\{F_{j0} \times \exp(-i 2\pi f_0 t_j)\}$$ (9)

Therefore, the temporal displacement can be derived through:
\[ t_j = t_{\text{expo}} \times \frac{p_j}{2\pi} \] (10)

By applying the same operation to all CGs, we can extract the temporal information for all CGs and acquire the spatio-temporal coordinates of a moving object in the scene.

Acknowledgments
This work was supported by National Natural Science Foundation of China (NSFC) (61771284); Beijing Natural Science Foundation (L182043); Beijing Municipal Science & Technology (Z181100008918011).

Conflict of Interest
There is no conflict of interest.

Author contributions
H.C., C.H. and H.H. conceived the TGIC concept and its three applications. C.H., and H.H. conducted the experiment and analysed the results. All authors wrote the manuscript. H.C. supervised the research.

†These authors contributed equally to this Letter.

References
1. Baraniuk, R. G. et al. Compressive Video Sensing: Algorithms, architectures, and applications. IEEE Signal Process. Mag. 34, 52–66 (2017).
2. Veeraraghavan, Ashok, Dikpal Reddy, and Ramesh Raskar. Coded strobing photography: Compressive sensing of high speed periodic videos. IEEE Transactions on Pattern Analysis and Machine Intelligence 33,4, 671-686 (2010).
3. Reddy, D., Veeraraghavan, A. Chellappa, R. P2C2: Programmable pixel compressive camera for high speed imaging. in CVPR 2011 329–336 (IEEE, 2011).
4. Hitomi, Y., Gu, J., Gupta, M., Mitsunaga, T. Nayar, S. K. Video from a single coded exposure photograph using a learned over-complete dictionary. in 2011 International Conference on Computer Vision 287–294 (IEEE, 2011).
5. Llull, P. et al. Coded aperture compressive temporal imaging. Opt. Express 21, 10526 (2013).
6. Dengyu Liu et al. Efficient Space-Time Sampling with Pixel-Wise Coded Exposure for High-Speed Imaging. IEEE Trans. Pattern Anal. Mach. Intell. 36, 248–260 (2014).
7. Koller, R. et al. High spatio-temporal resolution video with compressed sensing. Opt. Express 23, 15992 (2015).
8. Deng, C. et al. Sinusoidal Sampling Enhanced Compressive Camera for High Speed Imaging. IEEE Trans. Pattern Anal. Mach. Intell. 1–1 (2019).
9. Edgar, M. P., Gibson, G. M. & Padgett, M. J. Principles and prospects for single-pixel imaging. Nature Photon 13, 13–20 (2019).
10. Ryczkowski, P., Barbier, M., Friberg, A. T., Dudley, J. M. Genty, G. Ghost imaging in the time domain. Nature Photon 10, 167–170 (2016).
11. Devaux, F., Moreau, P.-A., Denis, S. Lantz, E. Computational temporal ghost imaging. Optica 3, 698 (2016).
12. Zhang, Z., Wang, X., Zheng, G. Zhong, J. Hadamard single-pixel imaging versus Fourier single-pixel imaging. Opt. Express 25, 19619 (2017).
13. Zhang, Z., Ma, X. & Zhong, J. Single-pixel imaging by means of Fourier spectrum acquisition. Nat Commun 6, 6225 (2015).
14. Zhang, Z., Wang, X., Zheng, G. & Zhong, J. Fast Fourier single-pixel imaging via binary illumination. Sci Rep 7, 12029 (2017).
15. Bian, L., Suo, J., Hu, X., Chen, F. & Dai, Q. Efficient single pixel imaging in Fourier space. J. Opt. 18, 085704 (2016).
16. Weiqiang Wang, Jie Yang & Wen Gao. Modeling Background and Segmenting Moving Objects from Compressed Video. IEEE Trans. Circuits Syst. Video Technol. 18, 670–681 (2008).
17. Tsai, D.-M. & Chiu, W.-Y. Motion detection using Fourier image reconstruction. Pattern Recognition Letters 29, 2145–2155 (2008).
18. Oh, T.-H., Lee, J.-Y. & Kweon, I. S. Real-time motion detection based on Discrete Cosine Transform. in 2012 19th IEEE International Conference on Image Processing 2381–2384 (IEEE, 2012). doi:10.1109/ICIP.2012.6467376.
19. Ojha, S. & Sakhare, S. Image processing techniques for object tracking in video surveillance- A survey. in 2015 International Conference on Pervasive Computing (ICPC) 1–6 (IEEE, 2015).
20. Ishii, I. et al. Real-time laryngoscopic measurements of vocal-fold vibrations. in 2011 Annual International Conference of the IEEE Engineering in Medicine and Biology Society 6623–6626 (IEEE, 2011).
21. Davis, A. et al. The visual microphone: passive recovery of sound from video. ACM Trans. Graph. 33, 1–10 (2014).
22. Davis, A. et al. Visual Vibrometry: Estimating Material Properties from Small Motions in Video. 9.
23. Bub, G., Tecza, M., Helmes, M., Lee, P. & Kohl, P. Temporal pixel multiplexing for simultaneous high-speed, high-resolution imaging. Nat Methods 7, 209–211 (2010).
24. Hutchison, D. et al. Flexible Voxels for Motion-Aware Videography. in Computer Vision – ECCV 2010 (eds. Daniilidis, K., Maragos, P. & Paragios, N.) vol. 6311 100–114 (Springer Berlin Heidelberg, 2010).
25. B. E. Bayer, “Color imaging array,” U.S. Patent No. 3,971,065 ~1976.
26. Malvar, H. S., Li-wei He & Cutter, R. High-quality linear interpolation for demosaicing of Bayer-patterned color images. in 2004 IEEE International Conference on Acoustics, Speech, and Signal Processing vol. 3 iii 485–8 (IEEE, 2004).
27. Ramanath, R., Snyder, W. E., Bilbro, G. L. & Sander, W. A. Demosaicking methods for Bayer color arrays. J. Electron. Imaging 11, 306 (2002).
28. Brox, T., Bruhn, A., Papenberg, N. & Weickert, J. High Accuracy Optical Flow Estimation Based on a Theory for Warping. in Computer Vision - ECCV 2004 (eds. Pajdla, T. & Matas, J.) vol. 3024 25–36 (Springer Berlin Heidelberg, 2004).