Evaluating Galaxy Dynamical Masses From Kinematics and Jeans Equilibrium in Simulations

Michael Kretschmer,1,2,* Avishai Dekel,2 Jonathan Freundlich,2,3 Sharon Lapiner,2
Daniel Ceverino,4,5 Joel Primack6
1Institute for Computational Science, University of Zurich, Winterthurerstrasse 190, 8057 Zurich, Switzerland
2Center for Astrophysics and Planetary Science, Racah Institute of Physics, The Hebrew University, Jerusalem 91904, Israel
3School of Physics and Astronomy, Tel Aviv University, Tel Aviv 69978, Israel
4Departamento de Física Teórica, Módulo 8, Facultad de Ciencias, Universidad Autónoma de Madrid, 28049 Madrid, Spain
5CIAM, Facultad de Ciencias, Universidad Autónoma de Madrid, 28049 Madrid, Spain
6Physics Department, University of California, Santa Cruz, 1156 High Street, Santa Cruz, CA 95064, USA

Accepted XXX. Received YYY; in original form ZZZ

ABSTRACT

We provide prescriptions to evaluate the dynamical mass \( M_{\text{dyn}} \) of galaxies from kinematic measurements of stars or gas using analytic considerations and the VELA suite of cosmological zoom-in simulations at \( z = 1 - 5 \). We find that Jeans or hydrostatic equilibrium is approximately valid for galaxies of stellar masses above \( M_\star \sim 10^{9.5} \, M_\odot \) out to 5 effective radii \( R_e \). When both measurements of the rotation velocity \( v_\phi \) and of the radial velocity dispersion \( \sigma_r \) are available, the dynamical mass \( M_{\text{dyn}} \approx G^{-1} V_\phi^2 r \) can be evaluated from the Jeans equation \( V_\phi^2 = v_\phi^2 + \alpha \sigma_r^2 \) assuming cylindrical symmetry and a constant, isotropic \( \sigma_r \). For spheroids, \( \alpha \) is inversely proportional to the Sérsic index \( n \) and \( \alpha \approx 2.5 \) within \( R_{e,\text{stars}} \) for the simulated galaxies. The prediction for a self-gravitating exponential disc, \( \alpha = 3.36 (r/R_e) \), is invalid in the simulations, where the dominant spheroid causes a weaker gradient from \( \alpha \approx 1 \) at \( R_{e,\text{gas}} \) to 4 at \( 5R_{e,\text{gas}} \). The correction in \( \alpha \) for the stars due to the gradient in \( \sigma_r (r) \) is roughly balanced by the effect of the aspherical potential, while the effect of anisotropy is negligible. When only the effective projected velocity dispersion \( \sigma_e \) is available, the dynamical mass can be evaluated as \( M_{\text{dyn}} = K G^{-1} R_e \sigma_e^2 \), where the virial factor \( K \) is derived from \( \alpha \) given the inclination and \( v_\phi/\sigma_r \). We find that the standard value \( K = 5 \) is approximately valid only when averaged over inclinations and for compact and thick discs, as it ranges from 4.5 to above 10 between edge-on and face-on projections.

Key words: galaxies: kinematics and dynamics – galaxies: evolution – galaxies: formation – galaxies: high-redshift

1 INTRODUCTION

Estimating the total mass of a galaxy is an important challenge in astrophysics, in particular to assess the dark matter (DM) fraction within given radii and its potential evolution with cosmic time. Recent kinematic observations by Wuyts et al. (2016) and Genzel et al. (2017, 2020) indicate low central DM fractions in massive star-forming disc galaxies at \( z = 0.6 - 2.6 \) within their effective radius. If confirmed, this result would have important implications for our understanding of galaxy formation and evolution, since it requires to rapidly drive DM out of the initial central cusps while baryons move inwards. Mechanisms such as dynamical friction (e.g. El-Zant et al. 2001, 2004) and feedback from stars and AGN (e.g. Pontzen & Governato 2012; El-Zant et al. 2016; Freundlich et al. 2020) or the combination of both may account for the observed low DM fractions in the early universe (Dekel et al., in prep.). More generally, estimating the total mass of a galaxy enables us to better understand the interplay between baryons and DM and hence to test the predictions of the LCDM model of structure formation.

Using kinematic data, it is possible to infer the dynamical mass from simple mass-estimation models based on Jeans equilibrium. However, the derived total mass from observed kinematics is uncertain and sometimes even turns out to be smaller than the stellar mass, indicating that the method used is flawed. It is therefore crucial to verify the validity of the Jeans equation and to quantify the associated expression for mass estimation during the different phases of galaxy evolution and for the different components – spheroid and disc, stars and gas.

Observationally, galaxy kinematics can be derived from spatially-resolved emission line maps. To measure gas rotation and dispersion profiles, position-velocity cuts along the galaxy’s major axis are fitted assuming Gaussian line profiles, such that both the rotation curve and the velocity dispersion are measured simultaneously (Dicaire et al. 2008; Kassin et al. 2007, 2012; Barat et al. 2019; Wuyts et al. 2016; Ho et al. 2014).

When both, the rotation velocity \( v_\phi \) and velocity dispersion \( \sigma_r \) are available, the dynamical mass can be assessed from the Jeans equation, which implies for cylindrically symmetric systems with a constant radial velocity dispersion (Jeans 1915; Binney & Tremaine 1987).

© 2020 The Authors
where $V_c$ is the circular velocity associated with the gravitational potential and $\alpha$ is a dimensionless parameter. The right hand side is valid for each component of the galaxy separately, disc or bulge, gas or stars, potentially with different $\alpha(r)$. One needs estimates of $\alpha(r)$ for the different galactic components, either derived analytically in simple cases or computed from simulations. Observationally it is often difficult to obtain independent reliable values for rotation and dispersion such that only a line-of-sight (los) velocity dispersion $\sigma_\text{r}$, is available. Usually this is done by fitting a Gaussian to a given spectrum (Cappellari & Emsellem 2004; Falcón-Barroso et al. 2011; Peralta de Arriba et al. 2014). In this case, the dynamical mass within the effective radius $R_e$ is often simply estimated as:

$$M_{\text{dyn}} = K \frac{R_e \sigma_\text{r}^2}{G}$$

(2)

where $G$ is the gravitational constant, $\sigma_\text{r}$ the los-velocity and $K$ a dimensionless parameter (Bertin et al. 2002; Pettini et al. 2001; Ciotti et al. 1991; Pettini et al. 1998; Erb et al. 2006; Cappellari et al. 2006; Walker et al. 2009; Wolf et al. 2010; Ferré-Mateu et al. 2012; Courteau et al. 2014; Peralta de Arriba et al. 2014; Wynts et al. 2016; Campbell et al. 2017). The value of $K$ is uncertain and different values have been used (Bertin et al. 2002; Cappellari et al. 2006; Walker et al. 2009; Wolf et al. 2010; Campbell et al. 2017). A commonly used value is $K = 5$, or a similar constant value (Pettini et al. 1998, 2001; Cappellari et al. 2006; Walker et al. 2009), which in some cases lead to non-physical results where the estimated total mass was smaller than the stellar mass: $M_{\text{dyn}} < M_*$ (Erb et al. 2006; Ferré-Mateu et al. 2012; Peralta de Arriba et al. 2014).

In this paper we use analytic arguments and high-resolution cosmological zoom-in simulations to estimate the parameters $\alpha$ and $K$, necessary for mass-estimates from observations where reliable values for rotation and dispersion are obtained and for observations where only a line-of-sight velocity dispersion is available. Since all of these mass-estimates rely on the assumption of dynamical equilibrium we first have to validate Jeans equilibrium for the stars and the equivalent hydrostatic equilibrium for the gas. Then we measure for the simulated galaxies the velocities. Combining these with the measured total mass within the relevant radius, we obtain $\alpha$ and $K$ as a function of observables. We investigate the importance of additional terms including a gradient in $\sigma_\text{r}(r)$, a non-spherical potential and anisotropic velocity dispersion. We do this for different structural components of the galaxies, namely the disc and the spheroid, gas and stars.

Galaxies are highly perturbed and gas rich at high redshifts. In many cases the disc is intensely fed by incoming intense streams (Dekel et al. 2009). Through episodes of dissipative gas contractions, the gas can be quickly pushed towards the central region of the galaxies, leading to the formation of compact, gas-rich, star-forming blue nuggets (Zolotov et al. 2015; Tacchella et al. 2016a,b; Dekel et al. 2020a,b). This phenomenon has been been referred to as wet compaction (Dekel & Burkert 2014). Those objects are small in size but very massive, star forming systems. Through intense star formation, the inner gas supply will be exhausted and the Blue Nuggets will gradually turn into quenched Red Nuggets with low star formation. Cosmic gas streams feed the galaxy at the same time with fresh cold gas. A galactic disc is slowly formed (Tacchella et al. 2016a,b). These characteristic episodes from compaction events into Nuggets and finally disc like systems, surrounding the compact spheroids, are

a robust phenomena that are accompanied by distinctive changes in morphology and kinematics. It has been shown that one very distinct feature is a clear critical stellar mass of $M_* \sim 10^{9.5} M_\odot$ for the major event of compaction followed by quenching.

Galactic gas discs are likely to survive only in dark-matter halos of mass above a threshold of $\sim 2 \times 10^{11} M_\odot$, corresponding to a stellar mass of $\sim 10^{10} M_\odot$. This is mostly because in the low-mass regime, the angular momentum is predicted to flip on a timescale shorter than the orbital timescale due to mergers (Dekel et al. 2020a). Additionally, violent disc instability exerts torques that shrink the disc by removing angular momentum. Lastly supernova feedback plays an important role in disrupting discs below the critical mass (Dekel & Silk 1986). Above the threshold mass, disruptive merger events are less frequent and not necessarily associated with a change in the pattern of the feeding streams. At the same time, the effects of supernova feedback are reduced (e.g. Kretschmer & Teyssier 2020; Kretschmer et al. 2020). All these effects allow gas discs to survive. Other changes are the transition from diffuse to compact with an extended disc and envelope, from prolate to oblate, from pressure to rotation support, from low to high metallicity, and from supernova to AGN feedback (Zolotov et al. 2015; Dekel et al. 2019a).

We have structured this paper as follows: First, in section 2, we describe the simulation setup and derive analytic expressions that we will use to investigate the assumptions for Jeans- and hydrostatic-equilibrium. Secondly, in section 3, we focus on the case when both rotation velocity and velocity dispersion $(v_g$ and $\sigma_\text{r}$) are measured to obtain $\alpha$. We focus on Jeans- and hydrostatic equilibrium in different components and we investigate if models for self-gravitating discs are valid and applicable. In section 4 we focus on cases where only $\sigma_\text{r}$ is available to obtain $K$ both for stars and gas. We finally summarise our results and conclude in section 5.

2. JEANS EQUILIBRIUM IN THE VELA SIMULATIONS

Our methodology is as follows: In the first subsection, we describe the simulation setup that we will use in our analysis. Then we recall the Jeans equation and derive analytic expressions for $\alpha$ for specific cases. We describe how we obtain the quantities of interest from the simulation and how we use them to infer if Jeans or hydrostatic equilibrium is valid. Finally, we derive three correction terms in the Jeans equation.

2.1 Simulation Method and Subgrid Physics

Alongside our analytical modelling, we use a series of VELA zoom-in hydro-cosmological simulations at $z = 1–5$, which are described in Appendix A, Table A1 and previous works (e.g. Ceverino et al. 2014; Zolotov et al. 2015; Dekel et al. 2019b, 2020a,b).

The simulations were performed using the Adaptive Refinement Tree (ART) code (Kratovskv et al. 1997; Ceverino & Klypin 2009). The suite comprises 34 galaxies evolved to $z \sim 1$, with a maximum spatial resolution of 17.5 to 35 pc at any given time. The dark matter halo masses at $z = 2$ span from $10^{11}$ to $10^{12} M_\odot$. The choice of galaxies was made such that their dark matter halos have not undergone a major merger close to $z = 1$.

Additionally, the code contains a set of subgrid physics models that describe many relevant processes of galaxy formation that are not directly calculable because of the limited resolution (Ceverino & Klypin 2009; Ceverino et al. 2012; Zolotov et al. 2015; Mandelker et al. 2014). Those processes include gas cooling by atomic hydrogen
and helium, metal and molecular hydrogen cooling, photoionization heating by the UV background with partial self-shielding, stochastic star formation, stellar feedback, metal enrichment, stellar mass loss, thermal feedback from supernovae, stellar winds, gas recycling and an implementation of feedback from radiation pressure as described in Ceverino et al. (2014).

2.2 Analytic expressions for \( \alpha \)

Consider a cylindrically symmetric galaxy that is rotating. Assume that along the radial direction the stellar component is in Jeans equilibrium and the gas component is in hydrostatic equilibrium (Jeans 1915; Weijmans et al. 2008; Burkert et al. 2010; Capelo et al. 2010; Wellons et al. 2020). This implies that at any given radius \( r \), the gas and stars each obey

\[
V_c^2(r) = \psi^2(r) - \frac{1}{\rho(r)} \frac{d(r(\sigma_r^2(r)))}{d \ln r},
\]

where the circular velocity \( V_c \) represents the gravitational force through the potential gradient

\[
V_c^2(r) = r \frac{\partial \Phi(r)}{\partial r} = \frac{GM(<r)}{r},
\]

and where the last expression is accurate for a spherical system, with \( M(<r) \) the total mass within a sphere of radius \( r \). The velocity \( \psi^2(r) \) is the actual angular-averaged rotation speed in the disc plane (perpendicular to the angular-momentum vector), representing the centrifugal force. The last term represents the pressure gradient in the disc, where \( \rho(r) \) is the 3D density profile of the given component and \( \sigma_r \) is the radial velocity dispersion of that component (with the thermal pressure assumed to be negligible for the gas). We additionally assume that the velocity dispersion is isotropic. Equation 3 is valid for the stars and the gas separately.

2.2.1 The Jeans Equation with Constant Dispersion

Assuming the radial velocity dispersion \( \sigma_r^2 \) to be constant with radius, Equation 3 can be written:

\[
V_c^2 = \psi^2 + \alpha \sigma_r^2
\]

with \( -\alpha \) the logarithmic slope of the density profile,

\[
\alpha = -\frac{d \ln \rho}{d \ln r}.
\]

For example, for an isothermal sphere \( \alpha = 2 \), but different values for different 3D density profiles within the disc are obtained.

2.2.2 Self-Gravitating Exponential Disc

For a self-gravitating disc, following Burkert et al. (2010), if the velocity dispersion \( \sigma \) is isotropic and constant also in the \( z \) direction, the vertical density distribution \( \rho(z) \) is given by the vertical hydrostatic Spitzer solution (Spitzer 1942; Binney & Tremaine 2008, Section 4):

\[
\rho(z) = \rho_0 \text{sech}^2(z/h)
\]

where \( \rho_0(r) \) is the density at the mid-plane (\( z = 0 \)) and the scale height is

\[
h = \frac{\sigma}{\sqrt{2\pi G \rho_0(r)}}.
\]

The surface density \( \Sigma(r) \) for such a scenario is given by

\[
\Sigma(r) = 2 \rho_0(r) h(r).
\]

Combining the two Equations above yields for the density

\[
\rho_0(r) = \frac{\pi G \Sigma^2(r)}{2 \sigma^2}.
\]

After inserting in the definition of \( \alpha \) (Equation 6), we get

\[
\alpha(r) = -2 \frac{\ln \Sigma(r)}{\ln r}.
\]

For the special case of an exponential disc with \( \Sigma(r) = \Sigma_0 \exp(-r/r_d) \), where the half-mass radius is related to the exponential radius \( r_d \) by \( r_e = 1.68 r_d \), one obtains

\[
\alpha(r) = 2 - \frac{r}{r_d} = 3.36 \frac{r}{r_e}.
\]

The value of \( \alpha \) at the effective radius is larger than that of an isothermal sphere, indicating that one should not expect the same value of \( \alpha \) for all galaxy types or all components.

2.2.3 Sérsic Profile Model

One can determine \( \alpha \) for a Sérsic profile as a function of the Sérsic index \( n \). Consider the Sérsic profile that describes the two-dimensional surface density \( \Sigma(r) \) as a function of the two-dimensional radius \( r \), with the parameter \( n \), the Sérsic index describing the steepness of the profile (Sérsic 1963). The profile is given by

\[
\Sigma_v(r, n) = \Sigma_e \exp \left[ -b(n) \left( \frac{r}{R_e} \right)^{1/n} - 1 \right],
\]

where \( b(n) \) is chosen such that \( R_e \) is the half-mass radius. Therefore \( \Sigma_v \) is the surface density at the half-mass radius. The function \( b(n) \) can be approximated as (Graham & Driver 2005; Gerbrandt et al. 2015):

\[
b(n) \approx 1.9992 n - 0.3271.
\]

(See Ciotti & Bertin (1999) for an alternative approximation.)

Choosing \( n = 1 \) yields the surface density profile of an exponential disc. A de Vaucouleurs profile, characteristic of an elliptical galaxy, is obtained if \( n = 4 \) (de Vaucouleurs 1948). Integrating the two-dimensional Sérsic profile gives the three-dimensional density profile within the disc:

\[
\rho(r', n) = -\frac{1}{\pi} \int_{r'}^{\infty} \frac{\Sigma_v(r, n)}{r'} \frac{1}{(r'^2 - r^2)^{1/2}} \, dr,
\]

where \( r' \) is the spherical radius. Here we have inverted the formula for the projection of the density to a surface density with Abel’s formula (Ciotti et al. 1991; Binney & Tremaine 2008). From this \( \alpha(n) \) can be obtained numerically as a function of Sérsic index \( n \) with Equation 6:

\[
\alpha(n) = -\frac{d \ln \rho(r, n)}{d \ln r}.
\]

This is shown in Figure 6 for different radii.

2.3 Testing Jeans/hydrostatic equilibrium: \( \alpha_v \) and \( \alpha_p \)

Rearranging Equation 5, we first measure from the velocities

\[
\alpha_v = \frac{V_c^2 - \psi^2}{\sigma_r^2}.
\]
The rotational and radial velocities $v_\phi$ and $v_r$ are calculated using the coordinates $x, y$ in the plane of rotation

$$v_\phi = \frac{(x v_y - y v_x)}{r}, \quad v_r = \frac{(x v_x + y v_y)}{r}, \quad (18)$$

with $r = \sqrt{x^2 + y^2}$. The circular velocity $V_c$ is calculated with the total mass enclosed in a sphere. The radial velocity dispersion is calculated as $\sigma_r = \sqrt{\langle v_r^2 \rangle - \langle v_r \rangle^2}$ where the average is computed in a cylindrical ring. We note that if $\sigma_r$ is measured over segments of cylindrical rings, a smaller value is obtained. However, it has been shown (see section 4.2 in Wellons et al. 2020) that such an estimate under-predicts the pressure term in the Jeans equation. In comparison, using Equation 6, we measure from the density profile

$$\alpha_\rho = \frac{d \ln \rho}{d \ln r}, \quad (19)$$

where the density $\rho(r) = (4\pi r^2)^{-1} dM(r)/dr$ is calculated from the smoothed cumulative mass profile $M(r)$. If Jeans or hydrostatic equilibrium is valid and the velocity dispersion is constant, Equation 5 is valid and $\alpha_\rho$ and $\alpha_\sigma$ are identical. Therefore we can validate the assumption of equilibrium based on the agreement between $\alpha_\rho$ and $\alpha_\sigma$.

### 2.4 Correction Terms in the Jeans Equation

Deviations may arise if one of the above stated assumptions are not valid. We therefore include the following corrections.

#### 2.4.1 Non-Constant Velocity Dispersion

First, we include the term

$$\gamma = -\frac{d \ln \sigma_r^2}{d \ln r}, \quad (20)$$

to account for non-constant velocity dispersion in Equation 3.

#### 2.4.2 Non-Spherical Potential

The above given expression $V_c^2 = GM/r$ is only true for a spherical mass distribution. To allow for a more accurate expression we expand the potential $\Phi$ in multipoles to find non-spherical correction terms. We write the potential at position $\mathbf{r}$ as

$$\Phi(\mathbf{r}) = -\frac{G}{r} M - \frac{G}{r^3} \sum_\alpha D_\alpha \mathbf{r}_\alpha + \frac{G}{2r^5} \sum_{\alpha \beta} Q_{\alpha \beta} \mathbf{r}_\alpha \mathbf{r}_\beta - \ldots \quad (21)$$

where $r = |\mathbf{r}|$ and summation is implied over double occurring coordinates $\alpha, \beta = x, y, z$. The terms $M, D_\alpha$, and $Q_{\alpha \beta}$ are the monopole, the dipole and the (traceless) quadrupole:

$$M = \sum_i m_i,$$

$$D_\alpha = \sum_i m_i \mathbf{r}_{i, \alpha}, \quad (22)$$

$$Q_{\alpha \beta} = \sum_i m_i (3 \mathbf{r}_{i, \alpha} \mathbf{r}_{i, \beta} - \delta_{\alpha \beta} r_i^2).$$

Since the monopole represents the total mass and the dipole vanishes, the quadrupole is the first term that contributes as a non-spherical correction to the potential.

To obtain the quadrupole correction in the symmetry plane of the disc at the effective radius $R_e$, we calculate the eigenvalues of $Q_{ij}$. The correct eigenvalues are identified using the properties that the quadrupole is by construction trace-less and the trace is invariant under rotation. Including contributions from the non-spherical part of the potential, the circular velocity $V_c(Q)$ is changed and therefore also the resulting $\alpha_\nu(Q)$. We define the difference as

$$\Delta Q = \alpha_\nu(Q) - \alpha_\nu(Q_e). \quad (23)$$

#### 2.4.3 Anisotropic Velocity Distribution

Lastly, we include corrections for anisotropic dispersion where we define the anisotropy parameter as

$$\beta = 1 - \frac{\sigma_\phi^2}{\sigma_r^2}. \quad (24)$$

Note that $\beta$ as defined here is actually twice the standard $\beta$ in the cylindrical case (see Eq. 4.61 and 4.224 in Binney & Tremaine 2008).

Taking the three corrections mentioned above into account, Equation 3 becomes

$$V_c^2 = v_\phi^2 + \sigma_r^2 (\alpha_\rho + \gamma + \Delta Q - \beta). \quad (25)$$

From this, together with the definition of $\alpha_\nu$ (Equation 17), we obtain

$$\alpha_\nu = \alpha_\rho + \gamma + \Delta Q - \beta. \quad (26)$$

By comparing the two sides of this equation (see for example Figure 1) we can evaluate the degree of validity of the Jeans equation or the equation for hydrostatic equilibrium respectively, and the relative contribution of each correction. If equilibrium is approximately valid, we can estimate $M_{\text{dyn}}$ by using the values for $\alpha_\nu$ in Equation 25.

$$V_c^2 = v_\phi^2 + \alpha_\nu \sigma_r^2. \quad (27)$$

### 3 MEASURING THE DYNAMICAL MASS FROM DECOMPOSED KINEMATICS

We now report on our findings for equilibrium in different components of the galaxies. First we focus on bulges, secondly we investigate equilibrium in galactic discs and lastly we analyse equilibrium in whole galaxies. Using the VELA suite of high resolution cosmological simulations we test if stars and gas in high-redshift galaxies are in Jeans or hydrostatic equilibrium. We compare values for $\alpha_\nu$ obtained from the velocities and $\alpha_\rho$, obtained from the density slope. As described above, equilibrium is indicated if those values agree. The analysis is done in steps of scale factor $\Delta a = 0.01$ using the entire VELA suite in the redshift range $z = 1 - 5$, which includes 933 snapshots. We do not show the evolution of the measured quantities with redshift explicitly, but report instead on the evolution with stellar mass, since the two quantities are strongly correlated. Furthermore, the stellar mass provides an important indication of the evolutionary phase of galaxies, since below a stellar mass of $10^{10.5} \text{M}_\odot$ galaxies are highly perturbed and only above this critical mass long lived discs are expected.

#### 3.1 Jeans Equilibrium in Galactic Bulges

First, we are interested in verifying the assumption of Jeans equilibrium for stars in bulges of galaxies. We focus on Jeans equilibrium of the stars only, since there is little gas left in the central part of galaxies. To separate the galactic bulge from the disc we fit a double Sérsic profile to the surface density profile of the stars face-on, where the Sérsic index for the outer disc part is kept constant at $n_{out} = 1$.
are measured and mass weighted averaged in rings centred on $R_{\text{e,in}}$ with additional corrections. Shown is the median of the obtained values and the 68% confidence level for $\alpha_\rho$ and $\alpha_v$. Curves agree after compaction ($M_\star > 10^{10.5}M_\odot$) which indicates Jeans equilibrium with $\alpha = 2.55$. The corrections for a non-constant dispersion $\gamma$, non-spherical potential $\Delta Q$ and anisotropic dispersion $\beta$ are included. The Jeans equation is valid for $M_\star > 10^{9.5}M_\odot$.

(see subsection 3.5). Although stars from the bulge dominate at $R_{\text{e,in}}$, there are also stars belonging to the disc present. However, disc stars (as defined in subsection 3.3) inside $R_{\text{e,in}}$ on average only make up 8.5% of the total bulge mass and therefore will not significantly change our results.

At the resulting effective radius $R_{\text{e,in}}$ of the bulge, the velocities are measured and mass weighted averaged in rings centred on $R_{\text{e,in}}$ with size $\Delta R = \pm R_{\text{e,in}}/2$. The plane of rotation is determined by the angular momentum inside $R_{\text{e,in}}$. To verify if the assumption of Jeans-equilibrium is valid, we show the values for $\alpha_v$ obtained from the velocities and $\alpha_\rho$ obtained from the density slope in Figure 1. For $M_\star > 10^{10.5}M_\odot$ the similarity between $\alpha_v$ and $\alpha_\rho$ indicate the validity of Jeans equilibrium with a scatter of $\sim 20\%$ around a value of $\alpha_v = 2.63 \pm 0.53$, (28)

constant in stellar mass. This is consistent with the finding that long-term discs form in these simulations above a threshold mass of $M_{\text{vir}} \sim 10^{11}M_\odot$ due to the infrequent merger-driven spin flips and compaction-driven bulges (Dekel et al. 2020a).

Including $\gamma$ as a correction for a non-constant dispersion does not significantly improve the validity of Jeans equilibrium at low masses, and generates a slight overestimate by $\sim 0.4$ of $\alpha_\rho$ at high masses ($\gamma = 0.38 \pm 0.25$). This discrepancy is alleviated if the correction for a non-spherical potential are included ($\Delta Q = -0.31 \pm 0.18$). This is because a deeper potential well will be counteracted by a larger dispersion. Accounting for anisotropic dispersion through $\beta$ does not change the results significantly ($\beta = -0.01 \pm 0.16$).

In galaxies with stellar masses below $10^{9.5}M_\odot$, before compaction and where discs are disrupted by frequent merger-driven spin flips, $\alpha_v$ systematically underestimates $\alpha_\rho$, by an amount that is comparable to the 1-σ scatter among galaxies. The corrections for a non-spherical potential or anisotropic dispersion and non-constant dispersion are not significant ($\Delta Q = -0.05 \pm 0.13$, $\beta = -0.03 \pm 0.20$, $\gamma = -0.10 \pm 0.66$). Below $M_\star = 10^{9.5}M_\odot$, the value for $\alpha$ can be crudely approximated with

\begin{equation}
\alpha_v(M_\star) = 1.8\log(M_\star/10^{9.5}M_\odot) + 2.63. \tag{29}
\end{equation}

For galaxies with stellar masses $M_\star < 10^{8.5}M_\odot$ the discrepancy between the curves is large and therefore the assumption of Jeans equilibrium is invalid.

3.2 Equilibrium in the Plane of Galactic Discs

We expand our study of equilibrium to galactic discs. We are especially interested in the validity of Jeans and hydrostatic equilibrium in the plane of the disc. Therefore we choose a small height of $h = \pm R_{\text{e}}/4$ and use the 3D half-mass radius $R_\text{e}$ of the analysed component (computed inside 0.1R$_{\text{vir}}$). The discs are orientated relative to the angular momentum of the analysed component, stars or cold gas ($T < 5 \times 10^4K$). The velocities are mass weighted averaged over rings of width 0.5$R_{\text{e}}$ from 0.5$R_{\text{e}}$ to 5.5$R_{\text{e}}$.

Figure 2a shows the values of $\alpha_\rho$ and $\alpha_v$ for the stars as a function of stellar mass at $R_{\text{e,stars}}$. For the stars there is good agreement between $\alpha_\rho$ and $\alpha_v$ for $M_\star > 10^{9.5}M_\odot$, which indicates Jeans equilibrium with $\alpha = 2.6$, constant for galaxies with large stellar masses. The small but systematic deviations for low mass systems indicate that deviations from Jeans equilibrium are not too large and the presented approach to $\alpha$ is still usable for crudely estimating the total mass. These discrepancies are alleviated if corrections are included. For galaxies above the threshold mass the contribution from non-spherical potentials $\Delta Q = -0.3 \pm 0.1$ is counteracted by the non-constant dispersion term $\gamma = 0.4 \pm 0.2$. Anisotropic dispersions are not significant ($\beta = 0.0 \pm 0.2$).

Figure 2c refers to the gas, where we show values of $\alpha_\rho$ and $\alpha_v$ as a function of stellar mass at $R_{\text{e,gas}}$. We see large fluctuations of the radial velocity in rings, possibly reflecting variations along the rings and between the rings. However the average value for the radial velocity is consistent with zero ($\langle v_r \rangle = -0.7 \pm 11.7\text{ km/s}$). (Note that in contrast, for the stars we find $\langle v_r \rangle = -0.8 \pm 2.0\text{ km/s}$). These errors propagate to $\alpha_v$ and the large size of the errors may indicate that there are deviations from equilibrium. We see that the median values for $\alpha_v$ agree with $\alpha_\rho$, especially if non-spherical potentials and non-constant dispersions are taken into account ($\Delta Q = -0.47 \pm 0.28$, $\gamma = -0.20 \pm 0.26$). We find that the velocity dispersion for the gas is not isotropic ($\beta = 0.52 \pm 0.14$) and should be taken into account (Agertz et al. 2009; Chemin et al. 2020).

For massive galaxies, we show radial profiles in Figure 2b for the stars and Figure 2d for the gas. For the stars it is apparent that there is equilibrium up to radii of $\sim 5R_{\text{e,stars}}$, where $\alpha$ increases from $\approx 2.5$ to $\approx 3.5$. For the gas, the agreement between the curves imply that the assumptions of hydrostatic equilibrium are valid in the plane of the disc for radii up to $\sim 5R_{\text{e,gas}}$, where $\alpha$ increases from $\approx 1$ to $\approx 4$. The large errors however may indicate certain deviations from equilibrium. Increasing values of $\alpha$ with radius from $\approx 1$ to $\approx 4$ were also obtained by Wellons et al. (2020) where four massive galaxies of the FIRE-2 suite were analysed. These findings imply that a radial gradient of the pressure is significantly contributing. We see that the radial gradient of the velocity dispersion $\gamma$ is insignificant in comparison to the slope of the density profile $\alpha_\rho$. This demonstrates that pressure support is mostly provided through the slope of the density profile.

There are large deviations from the predictions for an exponential disc (Equation 12) where $\alpha$ was supposed to grow linearly with radius and should be e.g. 3.36 at $R_{\text{e,gas}}$ or 6.72 at $2R_{\text{e,gas}}$. These deviations

Figure 1.Validity of the Jeans equation for the bulge as a function of stellar mass. Values for $\alpha_\rho$ obtained from the density profile and $\alpha_v$ obtained from the velocities for the galactic bulge at the effective radius $R_{\text{e,in}}$ with additional corrections. Shown is the median of the obtained values and the 68% confidence level for $\alpha_\rho$ and $\alpha_v$. Curves agree after compaction ($M_\star > 10^{10.5}M_\odot$) which indicates Jeans equilibrium with $\alpha = 2.55$. The corrections for a non-constant dispersion $\gamma$, non-spherical potential $\Delta Q$ and anisotropic dispersion $\beta$ are included. The Jeans equation is valid for $M_\star > 10^{9.5}M_\odot$. 

**Equation 12:** $\alpha_v(M_\star) = 1.8\log(M_\star/10^{9.5}M_\odot) + 2.63$. 

**Equation 29:** $\alpha_v(M_\star) = 1.8\log(M_\star/10^{9.5}M_\odot) + 2.63$. 

**Equation 28:** $\alpha_v = 2.63 \pm 0.53$. 

MNRAS 000, 1–17 (2020)
The approximation of self-gravitating disc is problematic. To investigate this, we first measure \( \alpha_z \), the specific force exerted by each component in the \( z \) direction at a distance \( z = -H_d \) from the disc plane, where \( H_d \) is half the gas-disc height, as defined in Mandelker et al. (2014). The specific forces are calculated by direct summation for test-points lying in the plane of \( z = -H_d \) at various radii out to \( 5R_{e,\text{gas}} \). We separately compute the contributions from dark-matter, stars and gas. Furthermore we divide stars into disc stars and spheroid stars, where for the disc we adopt a threshold of \( J_z/J_c > 0.7 \) for each star particle where \( J_c \) is the angular momentum component parallel to the \( z \)-axis and \( J_c \) the angular momentum of a co-rotating circular orbit with the same energy. Figure 3 shows the specific force exerted by different components \( \alpha_z \) in the \( z \) direction as a function of \( R/R_{e,\text{gas}} \) for galaxies with \( M_\bullet > 10^{9.2}M_\odot \) where gas discs are likely to survive. When comparing contribution of the spheroidal component (bulge stars and dark matter) to the contribution of the disc components (gas and disc stars), we see that the spheroidal components dominates over the disc component by a factor of larger than two at \( R < R_e \), and by a larger factor at large radii, implying that the approximation of self-gravitating disc is problematic.

Figure 3 shows the specific force exerted by different components \( \alpha_z \) in the \( z \) direction as a function of \( R/R_{e,\text{gas}} \) for galaxies with \( M_\bullet > 10^{9.2}M_\odot \) where gas discs are likely to survive. When comparing contribution of the spheroidal component (bulge stars and dark matter) to the contribution of the disc components (gas and disc stars), we see that the spheroidal components dominates over the disc component by a factor of larger than two at \( R < R_{e,\text{gas}} \). For \( R > R_{e,\text{gas}} \), this factor increases linearly in radius from \( \sim 2 \) at \( R_{e,\text{gas}} \) to \( \sim 6 \) at \( 5R_{e,\text{gas}} \). It is apparent that the gas disc contributes only little. This is consistent with the \( \alpha \) values that we obtain, which are very different from the self-gravitating disc prediction. We see that

### 3.3 Self-Gravitating Discs

As discussed above, we have seen that values obtained for \( \alpha \) in the disc deviate from the predictions for an exponential disc (Equation 12). To investigate this we first measure \( \alpha_z \), the specific force exerted by each component in the \( z \) direction at a distance \( z = -H_d \) from the disc plane, where \( H_d \) is half the gas-disc height, as defined in Mandelker et al. (2014). The specific forces are calculated by direct summation for test-points lying in the plane of \( z = -H_d \) at various radii out to \( 5R_{e,\text{gas}} \). We separately compute the contributions from dark-matter, stars and gas. Furthermore we divide stars into disc stars and spheroid stars, where for the disc we adopt a threshold of \( J_z/J_c > 0.7 \) for each star particle where \( J_c \) is the angular momentum component parallel to the \( z \)-axis and \( J_c \) the angular momentum of a co-rotating circular orbit with the same energy. Figure 3 shows the specific force exerted by different components \( \alpha_z \) in the \( z \) direction as a function of \( R/R_{e,\text{gas}} \) for galaxies with \( M_\bullet > 10^{9.2}M_\odot \) where gas discs are likely to survive. When comparing contribution of the spheroidal component (bulge stars and dark matter) to the contribution of the disc components (gas and disc stars), we see that the spheroidal components dominates over the disc component by a factor of larger than two at \( R < R_{e,\text{gas}} \). For \( R > R_{e,\text{gas}} \), this factor increases linearly in radius from \( \sim 2 \) at \( R_{e,\text{gas}} \) to \( \sim 6 \) at \( 5R_{e,\text{gas}} \). It is apparent that the gas disc contributes only little. This is consistent with the \( \alpha \) values that we obtain, which are very different from the self-gravitating disc prediction. We see that
the dark matter is the dominating component at radii $r > 0.5R_{e,\text{gas}}$. Only inside $0.5R_{e,\text{gas}}$ the contribution from bulge stars becomes more dominant.

Figure 4 shows the obtained $\alpha$ from the simulations, divided by the prediction $\alpha_{\text{self}} = 3.36 (R/R_e)$ for self-gravitating discs, versus the specific force in the $z$ direction $a_z$ exerted by the disc with respect to the total $a_z$. Colours indicate radial bins, where the half mass radius refers to the gas. We learn that at $1 - 2R_e \alpha$ is about $0.45 \alpha_{\text{self}}$ (consistent with Figure 2d).

At larger radii, the relative contribution of the disc drops even further, and $\alpha \sim 0.3\alpha_{\text{self}}$ at $R \sim 5R_{e,\text{gas}}$. The pressure term grows slower than linearly with radius. We conclude that the approximation derived from a self-gravitating disc for the asymmetric-drift pressure term in the Jeans equation is not valid in our simulated high-$z$ discs.

### 3.4 Jeans and Hydrostatic Equilibrium in Whole Galaxies

Up until now, we only analysed the validity of equilibrium for stars and gas in the bulge and in the plane of the galactic disc. Often it is however not possible to obtain kinematic measurements for gas or stars separately in the bulge or only in the thin plane of the disc. Therefore we now expand our study to galactic systems, namely where we use all the stars or gas inside a sphere with radius $R_e$, the half-mass radius of the analysed component. The resulting values for $\alpha$ for stars and gas are shown in Figure 5.

For the stars, we find for $M_* > 10^{9.5}M_\odot$, where there are long-lived discs,

$$\alpha_v = 2.61 \pm 0.35,$$

(30)

in-line with the values that we obtained for the bulge only and for the disc only. The correction for non-constant dispersion $\gamma = 0.30 \pm 0.27$ is partly counteracted by the correction for non-spherical potential $\Delta Q = -0.26 \pm 0.16$ and the correction for anisotropy $\beta = -0.04 \pm 0.15$ is negligible. For galaxies below the threshold mass ($M_* < 10^{9.5}M_\odot$), $\alpha_v$ increases with $M_*$ from $\sim 1.7$ to $\sim 2.25$, $\gamma = -0.1 \pm 0.3$ is negligible but $\beta = 0.14 \pm 0.16$ together with $\Delta Q = -0.12 \pm 0.07$ alleviate discrepancies with $\alpha_p$. If all corrections are applied, we find Jeans equilibrium over the whole analysed mass-range. Polynomial approximations to the values of $\alpha$ can be found in Table 1. These findings are consistent with results from a different suite of simulations (NIHAO), where the anisotropy $\beta$ was close to zero except very close to the centre and towards the virial radius such that anisotropy is negligible when assessing the dynamical mass and that the Jeans equation was valid with a $\sim 13\%$ RMS error between 0.02 $R_{\text{vir}}$ and 0.56 $R_{\text{vir}}$ (Freundlich et al. 2020).

For the gas, $\alpha_v \approx 1.2$ for massive galaxies with $M_* > 10^{9.5}M_\odot$ (see the approximation in Table 1). There is a discrepancy with $\alpha_p$ which is alleviated if corrected for a non-constant dispersion $\gamma = -0.18 \pm 0.48$, and for non-spherical potentials ($\Delta Q(M_*) = -0.307 \log(M_*/10^{9.5}M_\odot) - 0.228$ which evolves with mass. Including corrections for anisotropy $\beta = 0.48 \pm 0.16$ leads to slightly lower predictions of $\alpha$. Below the threshold mass, the typical value for $\alpha_v \approx 1.4$ is in good agreement with $\alpha_p$, and the corrections are smaller ($\gamma = -0.07 \pm 0.9$, $\Delta Q = -0.18 \pm 0.09$, $\beta = 0.27 \pm 0.31$).
3.5 Sérsic profile

In addition to $a$ as a function of stellar mass we provide $n$ as a function of Sérsic index. Figure 6 shows the theoretical curves that we have derived in section 2.2.3. For an exponential disc, $\alpha(n = 1) = 2$ at the effective radius $R_e$. For larger Sérsic indices, namely around $n = 4$, which reflect de Vaucouleurs profile, $\alpha(n) \approx 2.7$ at $R_e$. For a fixed Sérsic index, $\alpha$ gets larger with increasing radius. For $n > 0.5$ the theoretical curves can be approximated at any given radius by the functional fit

$$\alpha(n) = \frac{a}{n} + b,$$

where for $r = (0.5, 1, 2, 4)R_e$ the parameters are approximated by:

$$a = -1.54, -0.86, 0.98, 4.08,$$

$$b = 2.87, 2.98, 2.9, 2.69.$$  

We provide $\alpha(n)$ separately for the bulge and for whole galaxies. We measure the Sérsic indices $n$ for the bulge by fitting a two-component prescription to the surface density profile of the stars, where the Sérsic index for the outer disc part is kept constant at $n_{\text{out}} = 1$. Secondly we measure global Sérsic indices for fitting a single Sérsic component to the stellar surface-density profiles using Equation 13.

The values that we report in the following are measured face-on, i.e., the $z$-axis of the simulations are aligned with the AM-vector of the stars. We fit the logarithm of the stellar surface-density profile of each simulated galaxy using a least-square minimisation. We use logarithmically spaced bins from 100pc to 0.1 $R_{\text{vir}}$. Of all snapshots less than 7% per cent did not converge for the two-component fit and less then 5% per cent did not converge for the single-component fit. Obtained values for $R_e$ from the global fit are in agreement with the true half-mass radius with deviations of less than 9 per cent.

The measured values of $\alpha$ and $\sigma_\rho$ as a function of Sérsic index $n$ are shown in Figure 6a for the bulge and 6b for the global fit. This is because agreement between $\alpha$ and $\sigma_\rho$ indicates Jeans equilibrium for all radii in the bulge and the disc as discussed above. Furthermore, measured values agree well with the theoretical expectations. Only for systems with small $n$ we find small discrepancies at radii $\gtrsim 2R_e$. This may be because at large radii there are deviations from spherically symmetric density distributions to more prolate systems. With Equation 31 for $\alpha(n)$, the dynamical mass is recovered within 20%. It is apparent that for galaxies with $n \gtrsim 2$ the value for $\alpha$ is $\approx 2.5 - 2.7$ at the effective radius.

3.6 Prescriptions and Tests

We approximate the values of $\alpha$ separately for galactic discs and for whole galaxies as a function of either $M_*$ or $R/R_e$ with parametric third order polynomials of the form

$$\alpha(x) = ax^2 + bx + c$$

where $x = \log(M_*/10^{9.5}M_\odot)$ or $x = R/R_e - 1$ and $R_e$ is the half-mass radius of the analysed component. The coefficients for the different components are given in Table 1. We estimate the dynamical mass using these prescriptions together with Equation 1 and compare it to the true value of the dynamical mass in the simulations. From the distribution of the ratio between the estimated dynamical mass and the true dynamical mass $M_{\text{dyn,estimated}}/M_{\text{dyn,true}}$ we can estimate the statistical $1\sigma$ confidence interval within which our prescriptions will successfully recover the dynamical mass. For the stars the typical scatter is $\lesssim 16\%$ for all prescriptions. For the gas the scatter is larger but above the threshold mass and at $R_e,\text{gas}$, the error is $\lesssim 20\%$.

### Table 1. Coefficients for polynomial approximations to the different measured values of $\alpha$ for gas and stars in different volumes. $\alpha(x) = ax^2 + bx + c$, where $x$ is the observable (where $m = \log(M_*/10^{9.5}M_\odot)$) together with corresponding $1\sigma$ confidence intervals, where the number in brackets is for $M_*>10^{9.5}M_\odot$ (the radial profiles were analysed above the threshold mass).

| $x$ | type | volume | $a$ | $b$ | $c$ | $\sigma [\%]$ |
|-----|------|--------|-----|-----|-----|---------------|
| $m$ | stars | disc   | -0.078 | 0.411 | 2.304 | 16 (11)       |
| $m/R_e - 1$ | stars | disc   | -0.075 | 0.591 | 2.544 | 16          |
| $m$ | gas   | disc   | -0.298 | 0.061 | 1.411 | 37 (18)      |
| $m/R_e - 1$ | gas   | disc   | -0.146 | 1.204 | 1.475 | 40          |
| $m$ | stars | sphere | -0.047 | 0.356 | 2.412 | 12 (9)       |
| $m$ | gas   | sphere | -0.243 | -0.147 | 1.486 | 20 (15)      |

Furthermore, we validate the prescriptions for whole galaxies by applying them to two different sets of zoom-in simulations, namely the MIGA suite detailed in Appendix B and the NIHAO suite detailed in Appendix C. These simulations use different codes, numerical resolutions and models for galaxy formation physics where in particular different implementations for star formation and stellar feedback are used. The resulting relative errors are shown in Figure 7. Using the stars, we find that the estimated dynamical mass is slightly underestimated by $\lesssim 15\%$ in MIGA and NIHAO. Using the gas in NIHAO, the estimated dynamical mass is underestimated by a systematic bias of $\sim 20\%$. This is likely because NIHAO galaxies have more radial outflows caused by strong supernova feedback. Using the gas in the MIGA galaxies, the estimated dynamical mass is overestimated by $\sim 5\%$. For all simulations sets, the true value for the dynamical mass is recovered within the $1\sigma$ scatter except for the stars in MIGA where it is recovered within $2\sigma$. The average dynamical mass is recovered to better than $20\%$ in all cases which demonstrates the validity of our prescriptions.

4 MEASURING THE DYNAMICAL MASS FROM THE LOS VELOCITY DISPERSION

When only a line-of-sight velocity dispersion $\sigma_\ell$ is available, the dynamical mass can be estimated using the virial factor $K$.

4.1 The Virial Factor

4.1.1 Isotropic Case

In order to estimate dynamical masses from the line-of-sight velocity dispersion $\sigma_\ell$ alone, the virial factor $K$ in Equation 2 should be evaluated. We define the dynamical mass $M_{\text{dyn}}$ to be twice the total mass inside the effective radius $R_e$, which using Equation 4 gives

$$M_{\text{dyn}} = 2G^{-1}R_eV_e^2.$$  

(34)

We wish to express this in terms of observables, namely the effective radius $R_e$ and the line-of-sight (los) velocity dispersion $\sigma_\ell$.

$$M_{\text{dyn}} = KG^{-1}R_e\sigma_\ell^2,$$  

(35)

and find the value of the virial factor $K$ under different conditions,

$$K = \frac{2V_e^2}{\sigma_\ell^2} = \frac{GM_{\text{dyn}}}{R_e\sigma_\ell^2}.$$  

(36)

For a given line-of-sight with an inclination $i$ relative to the rotation
axis \((i = 0\) for the face-on view) of the observed galaxy, the los velocity dispersion can be written as:

\[
\sigma_i^2 = \psi (\sin i \, v_\phi)^2 + \sigma_T^2,
\]

where \(\psi\) represents the projection of the rotation velocity at each point onto the direction of the line-of-sight. \(\sigma_T\) is the actual velocity dispersion while \(\sigma_i\) includes additional contribution from the rotation in the foreground and the background with respect to the galaxy centre line-of-sight distance to the line-of-sight dispersion. For a transparent cylindrical disc extending to radius \(R\) and viewed with an aperture of radius \(R\),

\[
\psi = \frac{1}{2} \int_0^R \Sigma(r) \, dr \int_0^R \Sigma(r) \, dr,
\]

where \(\Sigma(r)\) is expressed in units of the characteristic value \(\Sigma \psi\) that appears in Equation 37. The value of \(\psi\) depends on the shape of the rotation curve. For a flat rotation curve \(\psi = 1/2\) independent

\[
\text{of } \Sigma(r). \text{ For an exponential disc with a rising rotation curve } v_\phi \propto r^{1/2} \text{ evaluated at the exponential radius } R_{\exp} \text{ gives } \psi \approx 0.3 \text{ and at } R \gg R_{\exp} \text{ it approaches unity. With a decreasing rotation curve } v_\phi \propto r^{-1/2}, \text{ the value at } R_{\exp} \text{ is } \psi \approx 1.2 \text{ and at } R \gg R_{\exp} \text{ it approaches } \psi = 1/2.
\]

In the face-on view \(\sigma_i = \sigma_r\) and in the edge-on view \(\sigma_i\) is an average of \(\sigma_r\) and \(\sigma_\phi\). In the simplest case we assume isotropy such that \(\sigma_i\) is the same for every \(i\).

Combining Equation 5, 36 and 38 and with

\[
\xi \equiv \frac{v_{\phi}}{\sigma_r},
\]

we obtain both for gas and stars separately the virial factor:

\[
K = \frac{2(\alpha + \xi^2)}{1 + \psi \xi^2 (\sin i)^2},
\]

with \(\alpha\) from the pressure term in Equation 5. We can find approximations for various cases. For example, in the dispersion-dominated case \((\xi = 0)\) we have \(K = 2\alpha\), which for a de Vaucouleurs spheroid \((n = 4)\) is \(K = 5.54\). In the face-on case we obtain

\[
K = 2(\alpha + \frac{\xi^2}{\psi}),
\]

for rotation-dominated systems \((\xi^2 \gg \alpha)\) this becomes \(K = 2\xi^2\), which may yield values up to \(~30\). In the edge-on case

\[
K = 2(\alpha + \frac{\xi^2}{\psi}) + \psi^2 \xi^2,
\]

which for rotation dominated systems becomes \(K = 2/\psi\).

4.1.2 Anisotropic Case

In practice, the assumption of isotropic velocity dispersion may need to be modified, especially for the gas. This may represent a true anisotropy in the velocity dispersion, e.g., due to small-scale anisotropy in feedback-driven outflows or recycled inflows. Alternatively, \(\sigma_r\) as measured in the selected face-on view may include additional contributions from coherent motions. Such motions can arise from not dealing with the best face-on view of the gas, which may occur if the face-on view is determined from the stars, and the gas and stellar discs are misaligned. A similar contamination can
3. Measuring $K$

Using Equation 36 we can directly measure $K$ in the simulations. We measure line-of-sight velocities along cylinders of radius $R_e$ and depth $3.4 R_e$, where the cylinder is rotated to 64 randomly chosen orientations. For each orientation, the line-of-sight velocities is calculated as

$$
\sigma_l = \sqrt{\langle v_z^2 \rangle - \langle v_z \rangle^2},
$$

where $v_z$ is the velocity-component of the analysed component along the $z$-axis of the cylinder. Finally, the average over the 64 orientations is computed.

4.2 Measuring $K$

Using Equation 36 we can directly measure $K$ in the simulations. We measure line-of-sight velocities along cylinders of radius $R_e$ and depth $3.4 R_e$, where the cylinder is rotated to 64 randomly chosen orientations. For each orientation, the line-of-sight velocities is calculated as

$$
\sigma_l = \sqrt{\langle v_z^2 \rangle - \langle v_z \rangle^2},
$$

where $v_z$ is the velocity-component of the analysed component along the $z$-axis of the cylinder. Finally, the average over the 64 orientations is computed.

Figure 8. Measured values of the virial factor $K$ for the stars. The inclination changes from top to bottom: face-on (red), average (black) and edge-on (blue). Different columns show $K$ as a function of $M_*$, $n$ and $v_\phi/\sigma_r$. Measured values are shown as solid lines with errors as shaded areas. Approximations are shown in dashed lines. Face-on, the model with anisotropy ($\mu$) successfully predicts $K$. For systems with low masses or small $v_\phi/\sigma_r$, the model without $\mu$ for $K_f$ is good. Edge-on, $K_e = 2/\psi$ is not a good approximation since the systems are not purely rotation dominated. The other two models predict $K_e$ more successfully. The last model, which includes anisotropic dispersion, does not give a better approximation. Additionally in the middle row, $K$ is measured at a fixed angle of 60$^\circ$ in yellow ($K_{avegaaged}^{60^\circ}$) and compared to the theoretical expectations at this orientation in green ($K_{expected}^{60^\circ}$). The similarity of the measured and predicted values for a fixed angle demonstrates the validity of our approach for $K$. In the face-on case $K$ increases with $M_*$ and $n$ from 4.5 to 7.5, and with $\xi$ from 4.5 to 10. In the edge-on case $K$ is ~ 4.5 except for $n < 2$. Averaged over inclinations, $K$ is consistent with $K = 5$ but only for $n > 2$ and $0.25 < \xi < 1.5$.

4.2 Measuring $K$

Using Equation 36 we can directly measure $K$ in the simulations. We measure line-of-sight velocities along cylinders of radius $R_e$ and depth $3.4 R_e$, where the cylinder is rotated to 64 randomly chosen orientations. For each orientation, the line-of-sight velocities is calculated as

$$
\sigma_l = \sqrt{\langle v_z^2 \rangle - \langle v_z \rangle^2},
$$

where $v_z$ is the velocity-component of the analysed component along the $z$-axis of the cylinder. Finally, the average over the 64 orientations is computed.
4.3 The Virial Factor in the Simulations

We measure the virial factor $K$ separately for stars and gas using Equation 36. From the discussion in subsection 4.1 it is apparent that the orientation of a galaxy influences the value for $K$ significantly. Therefore we separately measure $K$ for each snapshot in the face-on and edge-on cases as well as an average over 64 random orientations. Additionally, we measure $K$ at a fixed angle of $60^\circ$ and compare it to the expected values. The average values for the measured $K$ as a function of the stellar mass of the system, the Sérsic index and $v_\phi/\sigma_r$, are shown in Figure 8 for the stars and Figure 9 for the (cold) gas. We do not show an explicit redshift dependence of $K$ but instead note that $v_\phi/\sigma_r$ and $M_*$ are strongly correlated with $z$ and can therefore be interpreted as indicators of the evolutionary phase.

The measurement is done at the effective radius $R_e$ of the analysed component, where the reference dynamical mass is taken to be twice the total mass inside a sphere of radius $R_e$. The line-of-sight velocity is measured along cylinders of radius $R_e$ and depth $3.4R_e$. For the average case the cylinder is rotated to 64 randomly chosen orientations.

To compare with theoretical values and approximations discussed in subsection 4.1, we use $\alpha$, $\psi$ and the velocity components evaluated at $R_e$ inside the same volume where $K$ is measured. The values for $\psi$, $\xi$ and $n$ are shown in Figure 10. The values for $\alpha$ are taken from subsection 3.4.
4.4 $K$ for the Stars

Generally for the stars, we learn from Figure 10 that most systems have Sérsic indices $n \sim 1 - 2$, for $M_\star < 10^{9.5}M_\odot$ and $n \sim 2 - 6$ for $M_\star > 10^{10}M_\odot$. Pre-compaction, the ratio $v_\phi/\sigma_r < 0.5$ which increases post-compaction for more disc-like systems to $v_\phi/\sigma_r = 1.3 \pm 0.4$ (Figure 10). This implies that for low-mass systems the stellar components of galaxies are not discs and that massive galaxies do have a disc component, but the bulge dominates. It is apparent that $\psi \sim 0.5$ for low-mass systems, implying a flat rotation curve for the stars. In galaxies above the threshold mass we find $\psi \sim 0.5 - 0.7$ which implies that these systems have slightly decreasing rotation curves.

From Figure 8 the strong dependence of $K$ on the inclination is apparent. For high mass systems $K_{\text{stars}}$ is $\sim 4.5$ in the face-on case and $\sim 7.5$ in the face-on case. Furthermore, $K$ in the face-on case is increasing with mass and with $v_\phi/\sigma_r$. The reason for this is that high-mass systems form thin discs with large $v_\phi/\sigma_r$ such that the line-of-sight velocity in the face-on case gets smaller. Note that only in the average case $K \sim 5$, which is the value often used in the literature. However, $K \sim 5$ is only consistent with our measurements for $n > 2$ and for $0.25 < \xi < 1.5$, namely compact and thick discs. In a similar study, Frigo & Balcells (2017) demonstrated that for compact galaxies $K$ is systematically smaller than 5 which is consistent with our findings.

The measured values in the face-on case agree well with the approximation $K_1 = 2(\alpha + \xi^2)/\mu^2$ which accounts for anisotropic dispersion $\mu$. In the middle value, the ratio for the averaged $K$ is obtained with the measured value for galaxies oriented at $60^\circ$ in yellow ($K^{60}_{\text{averaged}}$) and the expectation from Equation 40 in green ($K^{60}_{\text{expected}}$).

The agreement between the average $K$ and the measured $K^{60}_{\text{averaged}}$ is consistent with the fact that the average orientation is near $60^\circ$. Furthermore, the agreement between the averaged measured $K^{60}_{\text{averaged}}$ and the expected $K^{60}_{\text{expected}}$ demonstrates the validity of Equation 40.

In the edge-on case the approximations $K = 2(\alpha + \xi^2)/(1 + \psi \xi^2)$ (Equation 42) and $K = 2(\alpha + \xi^2)/((\sigma_\phi^2 + \psi \xi^2)$ (Equation 43) yield good agreement with the measured values. Since the systems are not heavily rotation dominated, $K = 2/\psi \phi$ is not a good approximation.

Courteau et al. (2014) compared predictions for the virial factor from three different formulas to various idealised models (see their table 2). These formulas should be used a priori for average inclinations and we therefore limit our comparison to this case. We find that the formulas from Spitzer (1969) and Wolf et al. (2010)\footnote{We have doubled the original values to convert from the half-light mass to the dynamical mass, ignoring variations in the mass-to-light ratio.}, where $K = 7.5$ and $K = 8.0$ respectively, over-predict the dynamical mass. The predicted value of $K = 5$ from Cappellari et al. (2006) yields the closest approximation of the three formulas to our measured values, which is only valid for compact and thick discs as discussed above. The values for $K$ from Wolf et al. (2010) are larger than those from Cappellari et al. (2006) or ours because $\sigma_\phi$ is measured inside $R_{\text{vir}}$ instead of $R_e$. We compare our values to the model of Courteau et al. (2014) which features a Sérsic profile for the stars together with a fixed $m = 6$ Einasto profile for the DM. We use the values that were obtained following the definitions of Cappellari et al. (2006) which are the closest to our definitions. We see the same trend of $K$ decreasing with $n$. Their values however over-predict the dynamical mass by $30\% - 50\%$. The discrepancies likely originate from the fact that the simple models used fixed DM and stellar profiles, isotropic velocities ($\beta = 0$) and no rotation.

4.5 $K$ for the Gas

We extend the study for $K$ to the gas in galaxies. From Figure 10 it is apparent that for systems with $M_\star < 10^{9.5}M_\odot$ the gas typically has $v_\phi/\sigma_r = 0.9 \pm 0.3$. For high-mass systems ($M_\star > 10^{10}M_\odot$), post-compaction, thinner and more quiet gas discs are formed with $v_\phi/\sigma_r = 2.7 \pm 0.8$. The Sérsic index of the gas is typically $n \sim 0 - 1$, except during compaction when gas is driven to the centre into a compact object resulting in $n > 1$. It is apparent that $\psi \sim 0.5$ for low-mass systems, implying a flat rotation curve for the gas. In galaxies above the threshold mass we find $\psi \sim 0.5 - 0.9$ which implies that these systems have decreasing rotation curves.

The dependence of $K$ on the inclination is even stronger for the gas compared to the stars. Note the different scaling of the y-axis in the first row of Figure 9 compared to the other rows and compared to Figure 8. For massive systems we find that $K_{\text{gas}}$ is $\sim 4.5$ edge-on, $\sim 6$ for an average orientation and $\sim 10 - 30$ face-on. We find that the literature value $K = 5$ is only valid for the averaged case with $n > 1.5$ and $v_\phi/\sigma_r < 2$, namely for compact and thick gas discs.

In the face-on case, $K$ is crudely approximated by $K_1 = 2(\alpha + \xi^2)$ (Equation 41). If anisotropic velocity dispersion are taken into account through $\mu = \sigma_\phi/\sigma_r$, the values in the face-on case are well
approximated by \( K_I = 2(\alpha + \xi^2)/\mu^2 \) (Equation 43). This is expected since coherent motions, especially outflows from various feedback processes, can give rise to anisotropic gas velocity dispersions.

In the middle row of Figure 9, the measured value for galaxies orientated at 60° in yellow (\( K_{\text{expected}}^{60°} \)) and the expectation at \( i = 60° \) in green (\( K_{\text{expected}}^{60°} \)) agree well, which demonstrates the validity of Equation 40.

In the edge-on case, the approximations for isotropic dispersion \( K = 2(\alpha + \xi^2)/(1 + \psi \xi^2) \) (Equation 42) underestimate \( K \) by \( \approx 1.0 \). However, there is good agreement if we account for anisotropic dispersion with \( K = 2(\alpha + \xi^2)/(\sigma_\alpha^2 + \psi \xi^2) \) (Equation 43).

### 4.6 Prescriptions and Tests

We approximate the values of \( K \) for the stars and the gas separately, as a function of either \( M_\star, n \) or \( \xi \) with parametric polynomials of the form

\[
\alpha(x) = ax^3 + bx^2 + cx + d
\]

where \( x = \log(M_\star/10^{9.5}M_\odot) \), \( n \), or \( \xi \). We fit \( K \) in the face-on, average and edge-on case individually, using a least-square minimisation. For each case we fit separately a cubic, square, linear and constant polynomial form is then chosen by evaluating the reduced \( \chi^2 \). The coefficients for the different components are given in Table 2.

We estimate the dynamical mass using the prescriptions and compare it to the true value of the dynamical mass. From the distribution of \( M_{\text{dyn, estimated}}/M_{\text{dyn, true}} \) we can estimate the statistical 1σ confidence interval within which our prescriptions will successfully recover the dynamical mass. Using these prescriptions the dynamical mass is recovered within relative errors of \( \lesssim 20\% \) using Equation 2. Only for the face-on cases using the gas, larger deviations occur.

Furthermore, we test our prescriptions on two different sets of simulated galaxies (see Appendix B for the MIGA suite and Appendix C for the NIHAO suite). Figure 11 shows the resulting relative errors using the stars or the gas for different simulations with different feedback strengths. Using \( K \) for the stars, we find for all orientations and observables that the dynamical mass is underestimated by a systematic bias of \( \approx 25\% \). The statistical error is comparable to the scatter obtained from the VELA simulations. For the gas, using the prescriptions for \( K \) in the NIHAO simulations under-predict the dynamical mass by \( \approx 35\% \). However, using the prescriptions for \( K \) in the MIGA simulations over-predicts the dynamical mass by \( \approx 7\% \). The larger scatter for \( K \) using the gas in NIHAO and MIGA originates mostly from face-on cases and is caused by stronger gas motions driven by the stronger feedback that is implemented in these simulations. The good agreement between the estimated and true dynamical masses in the different simulations demonstrates the accuracy of our prescriptions.

### 5 CONCLUSIONS

We derived recipes for evaluating the dynamical mass of a galaxy from kinematic measurements, based on the VELA suite of high-resolution zoom-in cosmological simulations. First, we studied the validity of Jeans and hydrostatic equilibrium for the stars and gas respectively. Assuming cylindrical symmetry and a constant, isotropic velocity dispersion \( \sigma_v \), the Jeans equation was written as

\[
V_c^2 = v_\phi^2 + \alpha \sigma_v^2
\]

with the circular velocity \( V_c \), the rotational velocity \( v_\phi \) and \( \alpha = -d \ln \rho/d \ln r \) the logarithmic slope of the density profile. We have compared values for \( \alpha_\phi \) obtained from the density slope (Equation 6) and \( \alpha_\phi \) obtained from the velocities (Equations 17 and 47). Equilibrium is indicated if both measurements yield consistent values. Otherwise we inspect corrections from aspherical potentials, non-constant and anisotropic velocity dispersions. Our analysis reveals that equilibrium is valid for stars and gas above a stellar mass of \( M_\star \sim 10^{9.5}M_\odot \), the threshold mass for long-lived discs associated with infrequent merger-driven spin flips (Dekel et al. 2020a).

---

**Table 2. Coefficients for polynomial approximations to the different measured values of \( K \) for the gas and stars. \( K(x, a\ldots d) = ax^3 + bx^2 + cx + d \), where \( x \) is the observable \( (n = \log(M_\star/10^{9.5}M_\odot), \, n, \, \xi = v_\phi/\sigma_v) \) and relative 1σ confidence interval on the estimated value for \( M_{\text{dyn}} \).**

| \( K \)  | \( x \) | \( a \) | \( b \) | \( c \) | \( d \) | \( \sigma [%] \) |
|--------|--------|--------|--------|--------|--------|----------|
| \( K_f \) | \( m \) | 0.00 | 0.00 | 1.18 | 6.70 | 20 |
| \( K_f \) | \( n - 4 \) | 0.00 | -0.11 | 0.13 | 6.96 | 25 |
| \( K_f \) | \( \xi - 1 \) | 0.00 | 0.00 | 2.53 | 7.23 | 16 |
| \( K_{\text{avg}} \) | \( m \) | 0.00 | 0.00 | 0.10 | 4.97 | 13 |
| \( K_{\text{avg}} \) | \( n - 4 \) | 0.05 | -0.07 | -0.31 | 5.09 | 13 |
| \( K_{\text{avg}} \) | \( \xi - 1 \) | 0.00 | 0.00 | 0.76 | 5.04 | 15 |
| \( K_g \) | \( m \) | 0.00 | 0.00 | -0.14 | 4.45 | 15 |
| \( K_g \) | \( n - 4 \) | 0.05 | -0.04 | -0.36 | 4.46 | 13 |
| \( K_g \) | \( \xi - 1 \) | 0.00 | 0.00 | 0.40 | 4.39 | 16 |

---

**Figure 11.** Relative errors on the dynamical mass using the prescriptions for \( K \) for different simulations. The distributions for a given component and simulation set include values obtained using the prescriptions for all orientations and observables. Shown is the median and the 1σ confidence interval. The dynamical mass is accurate to better than 20% in MIGA. It is underestimated by \( \approx 35\% \) for the gas in NIHAO, possibly because of the strong feedback inducing outflows.
compaction events (Zolotov et al. 2015; Tacchella et al. 2016a,b; Tomassetti et al. 2016; Dekel et al. 2020b) and less effective supernova feedback (Dekel & Silk 1986; Ceverino et al. 2015; Dekel et al. 2019b). The equilibrium is typically valid out to $\sim 5R_e$. We separately analysed the bulge component, the disc component and the two components together. For each component we provide functional fits for $\alpha$ as a function of mass and radius to enable measurements of the dynamical mass $M_{\text{dyn}}$ from stellar and gas kinematics. When only the line of sight velocity dispersion $\sigma_l$ within $R_e$ is available, we provide fitting functions for the virial factor $K$ which was defined as

$$K = \frac{GM_{\text{dyn}}}{R_e\sigma_l^2},$$

(48)
as a function of various observables. A summary of our results is as follows:

- For high-mass systems with $M_\star > 10^{9.5}M_\odot$, the stars are in fair Jeans equilibrium. We find the typical value to be $\alpha \approx 2.6$ for the stars at the effective radius. The effect of a non-constant velocity dispersion ($\gamma \sim 0.3$) is counteracted by the effect of a non-spherical potential ($\Delta Q \sim -0.3$), while the anisotropy has a negligible effect ($\beta \approx 0$).

- For the gas we find that hydrostatic equilibrium is valid above a similar mass threshold with a typical value of $\alpha \approx 1$. We find that the corrections from anisotropic velocity dispersions ($\beta \approx 0.5$) as well as contributions from non-spherical potentials and non-constant velocity dispersion ($\gamma \sim -0.2$, $\Delta Q \sim -0.5$) are not negligible.

- For galaxies below the mass threshold, we find small but systematic deviations from Jeans and hydrostatic equilibrium. However, these deviations are sufficiently small to allow crude estimates of the dynamical mass.

- For massive galaxies, the equilibrium is valid up to radii of $\sim 5R_e$ where $\alpha$ increases from $\approx 2.5$ below $R_{e,\text{stars}}$ to $\approx 5.5$ at $5R_{e,\text{stars}}$ for the stars, and from $\approx 1$ below $R_{e,\text{gas}}$ to $\approx 4$ at $5R_{e,\text{gas}}$ for the gas.

- We find large deviations from the predictions for self-gravitating exponential discs. By analysing the force exerted by each component in the vertical direction at a distance $z = -H_d$ from the disc plane, we learned that the contribution from the spheroidal components (dark matter and bulge stars) dominate over the contribution from the disc component (gas and disc stars) by a factor larger than two at $R < R_{e,\text{gas}}$, and by an even larger factor at large radii.

- We provided $\alpha$ as a function of Sérsic index. A simple estimator for the theoretical predictions is given by $\alpha(n) = a/n + b$, where we provide $a$, $b$ at various radii. For an exponential disc, $\alpha(n = 1) = 2$ at the effective radius $R_e$. For a de Vaucouleurs profile $\alpha(n = 4) \approx 2.7$ at $R_e$.

- When only an estimate of the line-of-sight velocity dispersion $\sigma_l$ within $R_e$ is available, we use the demonstrated validity of hydrostatic and Jeans equilibrium and provide the virial factor $K$ (Equation 36 and 48) for different inclinations and as a function of either $M_\star/n$ or $v_\phi/c_\phi$. For the stars, $K$ varies from 4.5 to 7.5 from edge-on to face-on views respectively. For the gas it varies from 4.5 to 30.

- For the stars, the standard value of $K = 5$ is valid in the simulations for values averaged over inclinations, but only for $n > 2$ and $0.25 < v_\phi/c_\phi < 1.5$. Similarly, for the gas we find that $K = 5$ is only valid for values averaged over inclinations with $n > 1.5$ and $v_\phi/c_\phi < 2$, namely for compact and thick gas discs.

ACKNOWLEDGEMENTS

We acknowledge stimulating discussions with Andi Burkert, Reinhard Genzel and Romain Teyssier. This work was partly supported by the Minerva foundation, the Swiss National Supercomputing Center (CSCS) project s890 - “Predictive models for galaxy formation”, the Swiss National Science Foundation (SNSF) project 172535 - “Multiphysics models of galaxy formation” and the grants France-Israel PICS, I-CORE Program of the PBC/ISF 1829/12, BSF 2014-273 and NSF AST-1405962, GIF I-1341-303.7/2016, DIP STE1869/2-1, ISF 861/20, GE625/17-1. DC is a Ramon-Cajal Researcher and is supported by the Ministerio de Ciencia, Innovación y Universidades (MICIU/FEDER) under research grant PGC2018-094975-C21.

DATA AVAILABILITY

The data underlying this article will be shared on reasonable request to the corresponding author.

REFERENCES

Agerz O., Lake G., Teyssier R., Moore B., Mayer L., Romeo A. B., 2009, MNRAS, 392, 294

Barat D., et al., 2019, MNRAS, 487, 2924

Bertin G., Ciotti L., Del Principe M., 2002, A&A, 386, 149

Binney J., Tremaine S., 2008, Galactic Dynamics: Second Edition

Bryan G. L., Norman M. L., 1998, ApJ, 495, 80

Burkert A., et al., 2010, ApJ, 725, 2324

Campbell D. J. R., et al., 2017, MNRAS, 469, 2335

Capelo P. R., Natarajan P., Coppi P. S., 2010, MNRAS, 407, 1148

Cappellari M., Emsellem E., 2004, PASP, 116, 138

Cappellari M., et al., 2006, MNRAS, 366, 1126

Ceverino D., Klypin A., 2009, ApJ, 695, 292

Ceverino D., Dekel A., Bournaud F., 2010, MNRAS, 404, 2151

Ceverino D., Dekel A., Mandelker N., Bournaud F., Burkert A., Genzel R., Primack J., 2012, MNRAS, 420, 3490

Ceverino D., Klypin A., Kline E. S., Trujillo-Gomez S., Churchill C. W., Primack J., Dekel A., 2014, MNRAS, 442, 1545

Ceverino D., Primack J., Dekel A., 2015, MNRAS, 453, 408

Chemin L., Braine J., Combes F., Kam Z. S., Carignan C., 2020, A&A, 639, A145

Ciotti L., Bertin G., 1999, A&A, 352, 447

Ciotti L., D’Ercole A., Pellegrini S., Renzini A., 1991, ApJ, 376, 380

Courteau S., et al., 2014, Reviews of Modern Physics, 86, 47

Danovich M., Dekel A., Hahn O., Ceverino D., Primack J., 2015, MNRAS, 449, 2087

Dekel A., Birnboim Y., 2006, MNRAS, 368, 2

Dekel A., Burkert A., 2014, MNRAS, 438, 1870

Dekel A., Silk J., 1986, ApJ, 303, 39

Dekel A., et al., 2009, Nature, 457, 107

Dekel A., Lapiner S., Dubois Y., 2019b, arXiv e-prints, p. arXiv:1904.08431

Dekel A., Sarkar K. C., Jiang F., Bournaud F., Krumholz M. R., Ceverino D., Primack J. R., 2019b, MNRAS, 488, 4753

Dekel A., Ginzburg O., Jiang F., Freudling J., Lapiner S., Ceverino D., Primack J., 2020a, MNRAS, 496, 5372

Dicaire I., et al., 2008, MNRAS, 385, 553

El-Zant A., Shlosman I., Hoffman Y., 2001, ApJ, 560, 636

El-Zant A. A., Hoffman Y., Primack J., Combes F., Shlosman I., 2004, ApJ, 607, L75

El-Zant A. A., Freudling J., Combes F., 2016, MNRAS, 461, 1745

Erb D. K., Steidel C. C., Shapley A. E., Pettini M., Reddy N. A., Adelberger K. L., 2006, ApJ, 646, 107

Falcón-Barroso J., et al., 2011, MNRAS, 417, 1787
Ferré-Mateu A., Vazdekis A., Trujillo I., Sánchez-Blázquez P., Ricciardelli E., de la Rosa I. G., 2012, MNRAS, 423, 632
Freundlich J., Dekel A., Jiang F., Ishai G., Cornuault N., Lapiner S., Dutton A. A., Macciò A. V., 2020, MNRAS, 491, 4523
Frigo M., Balcals M., 2017, MNRAS, 469, 2184
Genzel R., et al., 2017, Nature, 543, 397
Genzel R., et al., 2020, ApJ, 902, 98
Gerbrandt S. A. N., McConnachie A. W., Irwin M., 2015, MNRAS, 454, 1000
Graham A. W., Driver S. P., 2005, Publ. Astron. Soc. Australia, 22, 118
Ho I. T., et al., 2014, MNRAS, 444, 3894
Jeans J. H., 1915, MNRAS, 76, 70
Kassin S. A., et al., 2007, ApJ, 660, L35
Kassin S. A., et al., 2012, ApJ, 758, 106
Kravtsov A. V., Klypin A. A., Khokhlov A. M., 1997, ApJS, 111, 73
Kretschmer M., Teyssier R., 2020, MNRAS, 492, 1385
Kretschmer M., Agertz O., Teyssier R., 2020, MNRAS, 497, 4346
Mandelker N., Primack J. R., 2016a, MNRAS, 457, 2790
Mandelker N., Primack J. R., 2016b, MNRAS, 458, 242
Mandelker N., Primack J. R., 2013a, MNRAS, 428, 129
MNRAS, 373, 1074
Stinson G. S., Brook C., Macciò A. V., Wadsley J., Quinn T., 2006, MNRAS, 373, 1074
Stinson G. S., Brook C., Macciò A. V., Wadsley J., Quinn T. R., Couchman H. M. P., 2013a, MNRAS, 428, 129
Stinson G. S., et al., 2013b, MNRAS, 436, 625
Tacchella S., Dekel A., Carillo C. M., Ceverino D., DeGraf C., Lapiner S., Mandelker N., Primack Joel R., 2016a, MNRAS, 457, 2790
Tacchella S., Dekel A., Carillo C. M., Ceverino D., DeGraf C., Lapiner S., Mandelker N., Primack Joel R., 2016b, MNRAS, 458, 242
Teyssier R., 2002, A&A, 385, 332
Teyssier R., 2012, MNRAS, 421, 3464
Sérsic J. L., 1963, Boletín de la Asociación Argentina de Astronomía La Plata, 6, 41
Shen S., Wadsley J., Stinson G., 2010, MNRAS, 407, 1581
Spitzer Lyman J., 1942, ApJ, 95, 329
Spitzer Lyman J., 1969, ApJ, 158, L139
Stinson G., Seth A., Katz N., Wadsley J., Governato F., Quinn T., 2006, MNRAS, 373, 1074
Tacchella S., Dekel A., Carillo C. M., Ceverino D., DeGraf C., Lapiner S., Mandelker N., Primack Joel R., 2016a, MNRAS, 457, 2790
Tacchella S., Dekel A., Carillo C. M., Ceverino D., DeGraf C., Lapiner S., Mandelker N., Primack Joel R., 2016b, MNRAS, 458, 242
Teyssier R., 2002, A&A, 385, 337
Tomassetti M., et al., 2016, MNRAS, 458, 4477
Wadsley J. W., Veeravalli G., Couchman H. M. P., 2008, MNRAS, 387, 427
Wadsley J. W., Keller B. W., Quinn T. R., 2017, MNRAS, 471, 2357
Walker M. G., Mateo M., Olszewski E. W., Peiarrubia J., Evans N. W., Gilmore G., 2009, ApJ, 704, 1247
Wang L., Dutton A. A., Stinson G. S., Macciò A. V., Penzo C., Kang X., Keller B. W., Wadsley J., 2015, MNRAS, 454, 83
Weijmans A.-M., Krajnović D., van de Ven G., Oosterloo T. A., Morganti R., de Zeeuw P. T., 2008, MNRAS, 383, 1343
Wellons S., Faucher-Giguère C.-A., Anglés-Alcázar D., Hayward C. C., Feldmann R., Hopkins P. F., Kereš D., 2020, MNRAS, 497, 4051
Wolf J., Martinez G. D., Bullock J. S., Kaplinghat M., Geha M., Muñoz R. R., Simon J. D., Avedo F. F., 2010, MNRAS, 406, 1220
Wuyts S., et al., 2016, ApJ, 831, 149
Zolotov A., et al., 2015, MNRAS, 450, 2327
Genzel R., et al., 2020, MNRAS, 497, 117
Wadsley J. W., Keller B. W., Quinn T. R., 2015, MNRAS, 454, 83
MNRAS, 383, 1343
Wang L., Dutton A. A., Stinson G. S., Macciò A. V., Penzo C., Kang X., Keller B. W., Wadsley J., 2015, MNRAS, 454, 83
Weijmans A.-M., Krajnović D., van de Ven G., Oosterloo T. A., Morganti R., de Zeeuw P. T., 2008, MNRAS, 383, 1343
Wellons S., Faucher-Giguère C.-A., Anglés-Alcázar D., Hayward C. C., Feldmann R., Hopkins P. F., Kereš D., 2020, MNRAS, 497, 4051
Wolf J., Martinez G. D., Bullock J. S., Kaplinghat M., Geha M., Muñoz R. R., Simon J. D., Avedo F. F., 2010, MNRAS, 406, 1220
Wuyts S., et al., 2016, ApJ, 831, 149
Zolotov A., et al., 2015, MNRAS, 450, 2327
de Vaucouleurs G., 1948, Annales d’Astrophysique, 11, 247
APPENDIX C: THE NIHAO COSMOLOGICAL SIMULATIONS

The NIHAO suite (Wang et al. 2015) consists of ~90 cosmological zoom-in hydrodynamical simulations that were run with the Smoothed Particle Hydrodynamics (SPH) code gasoline2 (Wadsley et al. 2017).

The NIHAO sample consists of halo masses in the range of log $M_{\text{halo}}/M_\odot = 9.5 - 12.3$ chosen to be in isolation at $z = 0$. In the selection of the halos, merging histories, concentrations and spin parameters were not taken into account. The particle mass and force softening was chosen such that the mass profiles at 1 per cent of the virial radius are well resolved. The code contains a set of sub-grid physics models that describe the processes of turbulent mixing of metals and thermal energy (Wadsley et al. 2008), metal cooling, heating by the UV background (Shen et al. 2010) and ionising feedback from massive stars (Stinson et al. 2013a).
Stellar feedback is modelled using the blast-wave formalism (Stinson et al. 2006) where cooling is delayed for 30Myr to prevent spurious cooling. Additionally, stars inject thermal energy and metals into the surrounding ISM. Star-formation is modelled according to the Kennicutt-Schmidt relation with a constant star-formation efficiency per free-fall time $\epsilon_{ff} = 0.1$ if the gas temperature is below $T = 15000K$ and the gas density is above $n = 10.3cm^{-3}$ (Stinson et al. 2013b). The resulting galaxies range from dwarfs to Milky Way sized galaxies and reproduce a range of observational quantities.