String Driven Cosmology
and its Predictions

N. Sánchez *

Observatoire de Paris-DEIMIRM/LERMA. 61, Avenue de l’Observatoire,
75014 Paris, FRANCE

Abstract

We present a minimal model for the Universe evolution fully extracted from
effective String Theory. This model is by its construction close to the standard
cosmological evolution, and it is driven selfconsistently by the evolution of the
string equation of state itself. The inflationary String Driven stage is able to
reach enough inflation, describing a Big Bang like evolution for the metric.

By linking this model to a minimal but well established observational
information, (the transition times of the different cosmological epochs), we
prove that it gives realistic predictions on early and current energy density
and its results are compatible with General Relativity. Interestingly enough,
the predicted current energy density is found $\Omega = 1$ and a lower limit $\Omega \geq \frac{4}{9}$ is
also found. The energy density at the exit of the inflationary stage also gives
$\Omega|_{inf} = 1$. This result shows an agreement with General Relativity (spatially
flat metric gives critical energy density) within an inequivalent Non-Einsteinian
context (string low energy effective equations). The order of magnitude of
the energy density-dilaton coupled term at the beginning of the radiation
dominated stage agrees with the GUT scale.

The predicted graviton spectrum is computed and analyzed without any
free parameters. Peaks and asymptotic behaviours of the spectrum are a
direct consequence of the dilaton involved and not only of the scale factor
evolution. Drastic changes are found at high frequencies: the dilaton produces
an increasing spectrum (in no string cosmologies the spectrum is decreasing).

Without solving the known problems about higher order corrections and
graceful exit of inflation, we find this model closer to the observational Uni-
verse properties than the current available string cosmology scenarii.

*E-mail: Norma.Sanchez@obspm.fr
1 Introduction

The very early stages of the Universe must be described with physics beyond our current models. At the Planck time scale, energy and sizes involved require a quantum gravity treatment in order to account accurately for the physics at such scale. String Theory appears as the most promising candidate for solving the first stages evolution. Until now, one does not dispose of a complete string theory, valid at the very beginning of the Universe neither the possibility of extracting so many phenomenological consequences from it. Otherwise, effective and selfconsistent string theories have been developed in the cosmological context in the last years [1]-[5]. These approaches can be considered valid at the early stages immediately after the Planck epoch and should be linked with the current stages, whose physics laws must be expected as the very low energy limits of the laws in the early universe. Matters raise in this process. The Brans-Dicke frame, emerging naturally in the low energy effective string theories, includes both the General Relativity as well as the low energy effective string action as different particular cases. The former one takes place when the Brans-Dicke parameter \( \omega_{BD} = \infty \) while the last one requires \( \omega_{BD} = -1 \). Because of being extracted from different gravity theories, the effective string equations are not equivalent to the Einstein Equations. Since current observational data show agreement with General Relativity predictions, whatever another fundamental theory must recover it at its lowest energy limit, or at least give results compatible with those extracted in Einstein frameworks. The great difficulties to incorporate string theory in a realistic cosmological framework are not so much expected at this level, but in the description of an early Universe evolution (string phase and inflation) compatible with the observational evolution information.

The scope of this paper is to present a minimal model for the Universe evolution completely extracted from selfconsistent string cosmology [2]. In the following, we recall the selfconsistent effective treatments in string theory and the cosmological backgrounds arising from them. With these backgrounds, we construct a minimal model which can be linked with minimal observational Universe information. We analyse the properties of this model and confront it with General Relativity results. Although its simplicity, interesting conclusions are found about its capabilities as a predictive cosmological description. The predicted current energy density is found compatible with current observational results, since we have \( \Omega \sim 1 \) and in any case \( \Omega \geq \frac{4}{9} \). The energy density-dilaton coupled term at the beginning of radiation dominated stage is found compatible with the order of magnitude typical of GUT scales \( \rho e^\phi \sim 10^{90} \text{erg cm}^{-3} \). On the other hand, by defining the corresponding critical energy density, the energy density around the exit of inflation gives \( \Omega_{\text{inf}} = 1 \). This result agrees with the General Relativity statement for which \( k = 0 \rightarrow \Omega = 1 \), but it is extracted in a Non-Einsteinian context (the low energy string effective equations). No use of observational information neither further evolution Universe properties are needed in order to find this agreement; only the inflationary evolution law for
the scale factor, dilaton and density energy are needed.

Our String Driven Model is different from previously discussed scenarii in String Cosmology [3]. Until now, no complete description of the scale factor evolution from immediately post-Planckian age until current time had been extracted in String Cosmology. The described inflationary stage, as here presented and interpreted, is also a new feature among the solutions given by effective string theory. The String Driven Model does not add new problems to the yet still open questions, but it provides a description closer and more naturally related to the observational Universe properties. The three stages of evolution, inflation, radiation dominated and matter dominated are completely driven by the evolution of the string equation of state itself. The results extracted are fully predictive without free parameters.

This paper is organized as follows: In Sections 2 and 3 we construct the Minimal String Driven Model. In Section 4 we discuss its main features and the energy density predictions. We also discuss its main properties, differences and similarities with other string cosmology scenarii. In Section 5 we present our Conclusions.

2 Minimal String Driven Model

The String Driven Cosmological Background is a minimal model of the Universe evolution totally extracted from effective String Theory. We find the cosmological backgrounds from selfconsistent solutions of the effective string equations. Physical meaning of the model is preserved by linking it with a minimal but well established information about the evolution of the observational Universe. Two ways allowing extraction of cosmological backgrounds from string theory have been used. The first one is the low energy effective string equations plus the string action matter. Solutions of these equations are an inflationary inverse power evolution for the scale factor, as well as a radiation dominated behaviour. On the other hand, selfconsistent Einstein equations plus string matter, with a classical gas of strings as sources, give us again a radiation dominated behaviour and a matter dominated description. From both procedures, we obtain the evolution laws for an inflationary stage, a radiation dominated stage and a matter dominated stage. These behaviours are asymptotic regimes not including strictly the transitions among stages. By modelizing the transitions in an enoughly continuous way, we construct a step-by-step minimal model of evolution. In this whole and next sections, unless opposite indication, the metric is defined in length units. Thus, the (0,0) component is always time coordinate $T$ multiplied by constant $c$, $t = cT$. Derivatives are taken with respect to this coordinate $t$. 
2.1 The Low Energy Effective String Equations

We work with the low energy effective string action (that means, to the lowest order in expansion of powers of $\alpha'$), which in the Brans-Dicke or string frame can be written as [1], [2], [3]:

$$S = -\frac{e^3}{16\pi G_D} \int d^{d+1}x \sqrt{|g|} e^{-\phi} \left( R + \partial_{\mu} \phi \partial^{\mu} \phi - \frac{H^2}{12} + V \right) + S_M$$

where $S_M$ is the corresponding action for the matter sources, $H = dB$ is the antisymmetric tensor field strength and $V$ is a constant vanishing for some critical dimension. The dilaton field $\phi$ depends explicitly only upon time coordinate and its potential will be considered vanishing. $D$ is the total spacetime dimension. We consider a spatially flat background and we write the metric in synchronous frame ($g_{00} = 1, g_{0i} = 0 = g_{0a}$) as:

$$g_{\mu\nu} = \text{diag}(1, -a^2(t) \delta_{ij})$$

here $\mu, \nu = 0 \ldots (D - 1)$ and $i, j = 1 \ldots (D - 1)$. The string matter is included as a classical source which stress energy tensor in the perfect fluid approximation takes the form:

$$T_{\mu}^{\ 
u} = \text{diag}(\rho(t), -P(t)\delta_{ij})$$

where $\rho$ and $P$ are the energy density and pressure for the matter sources respectively.

The low energy effective string equations are obtained by extremizing the variation of the effective action $S$ ([1] with respect to the metric $g_{\mu\nu}$, the dilaton field $\phi$ and the antisymmetric field $H_{\mu\nu\alpha}$ and taking into account the metric ([2] and matter sources ([3]). We will consider that the antisymmetric tensor $H_{\mu\alpha\beta}$ as well as the potential $B$ vanish. By defining $H = \frac{\dot{a}}{a}$ and the shifted expressions for the dilaton $\tilde{\phi} = \phi - \ln \sqrt{|g|}$, matter energy density $\tilde{\rho} = \rho a^4$ and pressure $\tilde{p} = Pa^4$, we obtain the low energy effective equations L.E.E. [2],[3]:

$$\ddot{\tilde{\phi}} - 2\dot{\tilde{\phi}} + dH^2 = 0$$

$$\ddot{\phi} - dH^2 = \frac{16\pi G_D}{c^4} \tilde{\rho} e^{\tilde{\phi}}$$

$$2(\dot{H} - H\dot{\phi}) = \frac{16\pi G_D}{c^4} \tilde{p} e^{\tilde{\phi}}$$

The shifted expressions have the property to be invariants under the transformations related to the scale factor duality symmetry ($a \rightarrow a^{-1}$) and time reflection ($t \rightarrow -t$). Following this, if $(a, \phi)$ is a solution of the effective equations, the dual expression $(\hat{a}, \hat{\phi})$ obtained as:

$$\hat{a}_i = a_i^{-1}, \quad \hat{\phi} = \phi - 2 \ln a_i$$

is also a solution of the same system of equations.
2.2 String Driven Inflationary Stage

The inflationary stage appears as a new selfconsistent solution of the low energy effective equations (4) sustained by a gas of stretched or unstable string as developed in [2] and also in [3]. This kind of string behaviour is characterized by a negative pressure and positive energy density, both growing in absolute value with the scale factor [2],[4],[7],[8]. Strings in curved backgrounds satisfy the perfect fluid equation of state $P = (\gamma - 1)\rho$ where $\gamma$ is different for each one of the generic three different string behaviours in curved spacetimes [2]. For the unstable (stretched) string behaviour, it holds $\gamma_u = \frac{D-2}{D-1}$ [4]. Thus, the equation of state for these string sources in the metric (3) is given by [2]:

$$P = -\frac{1}{d} \rho$$

We find the following selfconsistent solution for the set of effective equations (4) with the matter sources eq.(6):

$$a(t) = A_I(t_I - t)^{-Q} \quad 0 < t < t_b < t_I \quad Q = \frac{2}{d+1}$$

$$\phi(t) = \phi_I + 2d \ln a(t)$$

$$\rho(t) = \rho_I (a(t))^{(1-d)}$$

$$P(t) = -\frac{1}{d} \rho(t) = -\frac{\rho_I}{d} (a(t))^{(1-d)}$$

Notice that here $t$ is the cosmic time coordinate, running on positive values such that the parameter $t_I$ is greater than the end of the string driven inflationary regime at time $t_b$, $d$ is the number of expanding spatial dimensions; $\rho_I$, $\phi_I$ are integration constants and $A_I$, $t_I$ parameters to be fixed by the further evolution of scale factor.

Although the time dependence obeys a power function, this String Driven solution is an inflationary inverse power law proper to string cosmology. This solution describes an inflationary stage with accelerated expansion of scale factor since $H > 0$, $\dot{H} > 0$ and can be considered superinflationary, since $\ddot{a}(t)$ increases with time. However, notice the negative power of time and the decreasing character of the interval $(t_I - t)$. Notice also that the string energy density $\rho(t)$ and the pressure $P(t)$ have a decreasing behaviour when the scale factor grows.

The properties of string driven inflation are discussed in section 4, (particularly in 4.4 and 4.5). More details are given in [19].

2.3 String Driven Radiation Dominated Stage

This stage is obtained by following the same procedure above described, but
by considering now a gas of strings with dual to unstable behaviour. Dual strings propagate in curved spacetimes obeying a typical radiation type equation of state [2, 3].

\[ P = \frac{1}{d} \rho \]  

(8)

This string behaviour and the dilaton “frozen” at constant value (\( \phi = \) constant) gives us the scale factor for the radiation dominated stage:

\[ a(t) = A_{II} t^R \quad \quad R = \frac{2}{d+1} \]  

(9)

\[ \phi(t) = \phi_{II} \]

\[ \rho(t) = \rho_{II}(a(t))^{-(1+d)} \]

\[ P(t) = \frac{1}{d} \rho(t) = \frac{\rho_{II}}{d} (a(t))^{-(1+d)} \]

(11)

2.4 String Driven Universes in General Relativity

As shown in ref.[3, 4] and [5], string solutions in curved spacetimes are self-consistent solutions of General Relativity equations, in particular in a spatially flat, homogeneous and isotropic background:

\[ ds^2 = dt^2 - a(t)^2 dx^2 \]  

(10)

where the Einstein equations take the form:

\[ \frac{1}{2} d(d-1) H^2 = \rho \quad , \quad (d-1) \dot{H} + P + \rho = 0 \]  

(11)

As before, the matter source is described by a gas of non interacting classical strings (neglecting splitting and coalescing interactions). This gas obeys an equation of state including the three different possible behaviours of strings in curved spacetimes: unstable, dual to unstable and stable. Let be \( \mathcal{U}, \mathcal{D} \) and \( \mathcal{S} \) the densities for strings with unstable, dual to unstable and stable behaviours respectively. Taking into account the properties of each behaviour [2], the density energy and the pressure of the string gas are described by:

\[ \rho = \frac{1}{(a(t))^{d}} \left( \mathcal{U} a(t) + \frac{\mathcal{D}}{a(t)} + \mathcal{S} \right) \quad , \quad P = \frac{1}{d} \frac{1}{(a(t))^{d}} \left( \frac{\mathcal{D}}{a(t)} - \mathcal{U} a(t) \right) \]  

(12)

Equations (12) are qualitatively correct for every \( t \) and become exact in the asymptotic cases, leading to obtain the radiation dominated behaviour of the scale.
factor, as well as the matter dominated behaviour. In the limit $a(t) \to 0$ and $t \to 0$, the dual to unstable behaviour dominates in the equations (12) and gives us:

$$\rho(t) \sim D(a(t))^{-(d+1)} , \quad P(t) \sim \frac{1}{d} D(a(t))^{-(d+1)}$$

(13)

This behaviour is characterized by positive string density energy and pressure, both growing when the scale factor approaches to 0. Dual to unstable strings behave in similar way to massless particles, i.e. radiation. Solving selfconsistently the Einstein equations (11) with sources following eqs.(13), the scale factor solution takes the form:

$$a(t) \sim \left( \frac{2D}{d(d-1)} \right)^{\frac{1}{d+1}} \left( \frac{d+1}{2} \right)^{\frac{d}{d+1}} (t - t_{II})^R , \quad R = \frac{2}{d+1}$$

(14)

This describes the evolution of a Friedmann-Robertson-Walker radiation dominated Universe, the time parameter $t_{II}$ will be fixed by further evolution of the scale factor.

On the other hand, studying the opposite limit $a(t) \to \infty$, $t \to \infty$ and taking into account the behaviour of the unstable density $U$ which vanishes for $a(t) \to \infty$ [2], the stable behaviour becomes dominant and the equation of state reduces to:

$$\rho \sim S(a(t))^{-d} , \quad P = 0$$

(15)

The stable behaviour gives a constant value for the string energy, that is, the energy density evolves as the inverse volume decreasing with growing scale factor, while the pressure vanishes. Thus, stable strings behave as cold matter. Again, from solving eqs.(11) with eqs.(15), the solution of a matter dominated stage emerges:

$$a(t) \sim \left( \frac{d}{(d-1)2} \right)^{\frac{1}{d}} (t - t_{III})^M , \quad M = \frac{2}{d}$$

(16)

We construct in the next sections a model with an inflationary stage described by the String Driven solution (see eq.(7)), followed by a radiation dominated stage (see eq.(14)) and a matter dominated stage (see eq.(16)). We will consider the dilaton field remain practically constant and vanishing from the exit of inflation until the current time, as suggested in the String Driven Radiation Dominated Solution. It must be noticed that the same solution for the radiation dominated stage emerges from the treatment with dilaton field and without it (general relativity plus string equation of state), allowing us to describe qualitatively the evolution of the universe by means of these scale factor asymptotic behaviours.

### 3 Scale Factor Transitions

Taking the simplest option, we consider the “real” scale factor evolution mini-
mally described as:

\[
\begin{align*}
a_I(t) &= A_I(t_I - t)^{-Q} \quad t \in (t_i, t_r) \\
a_{II}(t) &= A_{II} t^R \quad t \in (t_r, t_m) \\
a_{III}(t) &= A_{III} t^M \quad t \in (t_m, t_0)
\end{align*}
\]

with transitions at least not excessively long at the beginning of radiation dominated stage \(t_r\) and of matter dominated stage \(t_m\). We also define a beginning of inflation at \(t_i\), and \(t_0\) is the current time. It would be reasonable do not have instantaneous and continuous transitions at \(t_r\) and \(t_m\) for the stages extracted in the above section, since the detail of such transitions is not provided by the effective treatments here used. One can suspects the existence of very brief intermediate stages at least at the end of the inflationary stage \((t \in (t_b, t_r))\), as we will discuss in the next section, and also at the end of radiation dominated stage \((t \sim t_m)\). The dynamics of these transitions is unknown and not easy to modelize, it introduces in any case free parameters. In order to construct an evolution model for the scale factor, it is compatible with the current level of knowledge of the theory to suppose the transitions very brief. We will merge our lack of knowledge on the real transitions by means of descriptive temporal variables for which the modelized transitions are instantaneous and continuous. We link this descriptive scale factor with the minimal evolution information of the observational Universe, the standard values for cosmolological times: the radiation-matter transition held about \(T_m \sim 10^{12}\) s, the beginning of radiation stage at \(T_r \sim 10^{-32}\) s and the current age of the Universe \(T_0 \sim H_0^{-1} \sim 10^{17}\) s (The exact numerical value of \(T_0\) turns out not crucial here). We impose also to our description satisfy the same scale factor expansion (or scale factor ratio) reached in each one of the three stages considered \((\mathbb{L})\). It is also convenient to fix the temporal variable of the third (and current) stage \(t\) with our physical time (multiplied by \(c\)). This leads finally to the following scale factor in cosmic time-type variables:

\[
\begin{align*}
\bar{a}_I(\bar{t}) &= \bar{A}_I(\bar{t}_I - \bar{t})^{-Q} \quad \bar{t}_i < \bar{t} < \bar{t}_1 \\
\bar{a}_{II}(\bar{t}) &= \bar{A}_{II}(\bar{t} - \bar{t}_{II})^R \quad \bar{t}_1 < \bar{t} < \bar{t}_2 \\
\bar{a}_{III}(\bar{t}) &= \bar{A}_{III}(\bar{t})^M \quad \bar{t}_2 < \bar{t} < \bar{t}_0
\end{align*}
\]

with continuous transitions at \(\bar{t}_1\) and \(\bar{t}_2\) of both the scale factor and its first derivatives with respect to the variables \(\bar{t}, \bar{t}_1\) and \(\bar{t}_2\). The parameters \(\bar{t}_1, \bar{A}_I, \bar{t}_{II}, \bar{A}_{III}\) can be written in function of \(\bar{A}_{II}\) and the transition times using the matching conditions. In terms of the standard observational values, the transitions \(\bar{t}_1, \bar{t}_2\) and the beginning of the inflationary stage description \(\bar{t}_i\) are expressed as:

\[
\begin{align*}
\bar{t}_1 &= \frac{R}{M} t_r + \left(1 - \frac{R}{M}\right) t_m \\
\bar{t}_2 &= t_m \\
\bar{t}_i &= \left(\frac{R}{M} - \frac{Q}{M} \frac{t_r - t_i}{t_I - t_r}\right) t_r + \left(1 - \frac{R}{M}\right) t_m
\end{align*}
\]
The parameters of the scale factor \( t_I \) can be written also in terms of the observational values \( t_r, t_m \) and the global scale factor \( \bar{A}_{II} \):

\[
\begin{align*}
\bar{t}_I &= t_r \left( \frac{R}{M} + \frac{Q}{M} \right) + t_m \left( 1 - \frac{R}{M} \right) , \\
\bar{A}_I &= \bar{A}_{II} \left( \frac{Q}{M} \right)^{R} t_r^{R+Q} , \\
\bar{A}_{II} &= \bar{A}_{III} \left( \frac{R}{M} \right)^R t_m^{R-M}
\end{align*}
\]

The time variable \( \bar{t} \) of inflationary stage and \( \bar{t} \) of radiation stage are not a priori exactly equal to the physical time coordinate at rest frame (multiplied by \( c \)), but transformations (dilatation plus translation) of it. The low energy effective action equations from where the scale factor, dilaton and energy density have been extracted, allow these transformations. With this treatment of the cosmological scale factor, we will attain computations free of “by hand” added parameters and with full predictability as can be seen in the next sections.

The last point is to make an approach for the dilaton field. This is considered practically constant from the beginning of the radiation dominated stage until the current time. The dilaton field increases during the inflationary string driven stage. Its value can be supposed coincident with the value at exit inflation time in a sudden but not continuous transition, since its temporal derivatives cannot match this asymptotic behaviour. \( \phi_{II} = \phi(t_r) \equiv \phi_1 \). Recall the expression for the dilaton in inflation dominated stage,

\[
\phi_{II} = \phi_I + 2d \ln a(t_r)
\]

The scale factor can be written also in terms of the conformal time variable \( d\eta = \frac{dt}{a(t)} \) defined for each stage.

\section{Properties of the String Driven Model}

Enough inflation of the model and evolution of the Hubble factor are discussed elsewhere \cite{[19]}.

\subsection{Energy Density Predictions}

As can be easily seen, from eq.(4) we have:

\[
\dot{\phi}^2 - 2\dot{\phi} + dH^2 = 0 , \quad \ddot{\phi}^2 - dH^2 = \frac{16\pi G_D}{c^4} \bar{\rho} \ e^{\phi}
\]
Both equations give:
\[ \ddot{\phi} - dH^2 = \frac{8\pi G_D}{c^4} \tilde{\rho} e^{\phi} \]

By substituting the shifted expressions for the dilaton \( \tilde{\phi} = \phi - \ln|\sqrt{g}| \) and matter energy density \( \tilde{\rho} = \rho a^d \), the above equation yields:
\[ \ddot{\phi} - d\left(\dot{H} + H^2\right) = \frac{8\pi G_D}{c^4} \rho e^{\phi} \quad (22) \]

Eq. (22) can be considered as the generalization of the Einstein equation in the framework of low energy effective string action. This equation will allow us extract some predictions on the energy density evolution in our minimal model.

### 4.2 Energy Density at the Exit of Inflation

By introducing the String Driven Solution for \( a(t), \phi(t), \rho(t) \) (eqs. 7) in eq. (22), we obtain a relation for the integration constants \( \rho_I \) and \( \phi_I \):
\[ \rho_I e^{\phi_I} = \frac{c^4}{8\pi G_D} \frac{2d(d-1)}{(d+1)^2} A_I^{(1+d)} A_{\bar{I}}^{(1+d)} \quad (23) \]

Now, is easy to find the relation between \( \rho_I, e^{\phi_I} \) and the values of the energy density \( \rho_1 \) and dilaton field \( \phi_1 \) at the end of inflation stage. We can compute these values either with the real scale factor \( a_I(t) \) or with the description \( \bar{a}_I(\bar{t}) \). By defining \( \rho_1 = \rho(a_I(t_r)) = \rho(\bar{a}_I(\bar{t}_1)) \) and \( \phi_1 = \phi(a_I(t_r)) = \phi(\bar{a}_I(\bar{t}_1)) \) and making use of eqs. (7), we can write:
\[ \rho_I e^{\phi_I} = \rho_1 e^{\phi_1} A_{1+d}(t_I - t_r)^{-Q(1+d)} \quad (24) \]

Now, with the information about the evolution of the scale factor and the descriptions in each stage, it is possible to relate this expression with observational cosmological parameters. In fact, we must understand eq. (24) as one obtained in the description of inflationary stage:
\[ t_I - t_r \to \bar{t}_I - \bar{t}_1 = \frac{Q}{M} t_r \]

For the String Driven solution, the exponents have the values \( Q = \frac{2}{d+1}, R = \frac{2}{d+1}, M = \frac{2}{d} \). With these expressions we obtain the density-dilaton coupling at the end of inflation:
\[ \rho_1 e^{\phi_1} = \frac{c^4}{8\pi G_D} \frac{2(d-1)}{d} t_r^{-2} \quad (25) \]
With $t_r = cT_r$ where $T_r \sim 10^{-32} s$, this expression gives the numerical value:

$$\rho_1 e^{\phi_1} = 7.1 \times 10^{90} \text{ erg cm}^{-3}$$  \hspace{1cm} (26)

It must be noticed that the same result can be achieved from the Radiation Dominated Stage, due to the continuity of the scale factor, of the density energy and the dilaton field at the transition time $t_r$.

It must be noticed also an interesting property of this energy density-dilaton coupled term. We can extend the General Relativity treatment and define $\Omega_{\inf}$ as this coupled quantity in critical energy density units, where the critical energy density $\rho_c(t)$ for our spatially flat metric is $\rho_c(t) = \frac{3c^2}{8\pi G}H(t)^2$. We compute the corresponding $\rho_c$ at the moment of the exit of inflation:

$$\rho_c|_{\bar{t}_1} = \frac{3c^2}{8\pi G}H(\bar{t}_1) = \frac{3c^4}{8\pi G}M^2 t_r^{-2}$$  \hspace{1cm} (27)

With this, it is easily seen:

$$\Omega|_{\inf} = \frac{\rho_1 e^{\phi_1}}{\rho_c|_{\bar{t}_1}} = \frac{2}{3} \left( \frac{d - 1}{d} \right) M^2$$  \hspace{1cm} (28)

where $M$ is given by eq.(16). It gives for our model in the three dimensional case $\Omega|_{\inf} = 1$.

Here we have used the descriptive variables in order to link the scale factor transitions with observational transition times. But the conclusion $\Omega|_{\inf} = 1$ is independent from this choice. In fact, we have computed it again in the proper cosmic time of the inflationary stage and taking the corresponding values at the beginning of the radiation dominated stage $t_r$. From eq.(24) and $H(t) = Q(t_I - t)^{-1}$, we have:

$$\Omega|_{\inf} = \frac{1}{3Q} \left( \frac{2}{Q} - 1 - dQ \right)$$  \hspace{1cm} (29)

$Q$ is given by the String Driven solution eq.(7) and with it, we recover $\Omega|_{\inf} = 1$. In fact, the same result could be achieved also by evaluating $\Omega|_{\inf}$ exactly at the end of String Driven inflationary stage, whatever this time may be. That means, we have a prediction non-dependent of whenever the exit of inflation happens.

In the proper cosmic time coordinates, $(t_I - t_r)$ is a very little value. But independently from this, the coupled term energy density-dilaton at the exit of inflation gives the value one in the corresponding critical energy density units. The value of the critical energy density is computed following the Einstein Equations for spatially flat metrics, but solely the low energy effective string equations give the expression for the coupled term eq.(22) and the String Driven solution itself. In this last point, no use of further evolution, linking with observational results
neither standard cosmology have been made. This enable us to affirm that in the L.E.E. treatment, the relation between spatial curvature and energy density holds as in General Relativity, at least for the spatially flat case \((k = 0 \rightarrow \Omega = 1)\). This result means to recover a General Relativity prescription within a Non-Einsteinian framework.

### 4.3 Predicted Current Values of the Energy Density and Omega.

From the above sub-section, we can obtain the corresponding current value of \(\rho_0 e^{\phi_0}\) in units of critical energy density or contribution to \(\Omega\).

To proceed, we remember that the evolution of the density energy in the matter dominated stage follows \(\rho \sim a(t)^{-3} \sim t^{-2}\), therefore at the beginning of matter dominated stage we would have

\[
\rho_m = \left(\frac{T_m}{T_0}\right)^{-2} \rho_0
\]

On the other hand, in the radiation dominated stage, the density behaviour is: \(\rho \sim a(t)^{-4} \sim t^{-2}\) which gives for the energy density

\[
\rho_1 = \left(\frac{T_r}{T_m}\right)^{-2} \rho_m
\]

That is,

\[
\rho_0 = \left(\frac{T_r}{T_0}\right)^2 \rho_1
\]

Considering that the dilaton field has been remained almost constant since the end of inflation \(\phi_0 \sim \phi_r\), we have

\[
\Omega = \frac{\rho_0 e^{\phi_0}}{\rho_c} = \left(\frac{T_r}{T_0}\right) e^{\phi_1} \frac{\rho_1}{\rho_c}
\]

where the current critical energy density is expressed in terms of the current Hubble factor \(H_0 = H(T_0)\) as:

\[
\rho_c = \frac{3c^2}{8\pi G} H_0^2
\]

From eq.(30) and with eqs.(25) and (31), we obtain:

\[
\Omega = \frac{2(d-1)}{3d} \frac{T_0^{-2}}{H_0^2}
\]
Since $H_0 \sim T_0^{-1}$, we have finally
\[ \Omega = \frac{2(d - 1)}{3d} \]
In our three-dimensional expanding Universe, it gives $\Omega = \frac{4}{9}$.

In the last, we have taken $T_0 \leq H_0^{-1}$ following the usual computation. In General Relativity framework, it holds:
\[ T_0 = \frac{2}{3}H_0^{-1} \]
if the deceleration parameter $q_0 = -\ddot{a}(t_0)\frac{a(t_0)}{a(t_0)} > \frac{1}{2}$. For our model, the deceleration parameter is found:
\[ q_0 = \frac{1 - M}{M}, \]
which for the standard matter dominated behaviour gives exactly $q_0 = \frac{1}{2}$. For this value, (and observations give as well $q_0 \sim 1$,) we compute the value of $\Omega$ in this framework. From eq.(32) and eq.(34) we obtain:
\[ \Omega = \frac{2}{3}d - \frac{1}{d}\left(\frac{2}{3}\right)^{-2} \]
which in the three dimensional case gives exactly:
\[ \Omega = 1 \]
We have obtained that a spatially flat metric $k = 0$ leads to a critical energy density $\Omega = 1$. This result, is well known in General Relativity, but here it has been extracted in a no General Relativity Framework, since low energy effective string equations are not at priori equivalent to the General Relativity equations. From the point of view of the Brans-Dicke metric-dilaton coupling, General Relativity is obtained as the limit of the Brans-Dicke parameter $\omega \to \infty$, while the low energy effective string action (see eq.(1)) is obtained for $\omega = -1$.

Notice that $\Omega = 1$ is obtained as a result of combining General Relativity (for the matter dominated stage), and eq.(25) for the inflationary stage in the low energy effective string framework. Thus, a result from this string treatment is compatible with, and leads to similar predictions that, standard cosmology.

Since in any case $T_0 \leq H_0^{-1}$, eq.(25) can be seen as giving a lower limit for $\Omega$:
\[ \Omega \geq \frac{2}{3} \frac{(d - 1)}{d} \]
This is the prediction for the current energy density from String Driven Cosmology and this is not in disagreement with the current observational limits for $\Omega$.
4.4 String Driven Cosmology is Selfconsistent

Matter dominated stages are not an usual result in string cosmology backgrounds. The standard time of matter dominated beginning $T_m \sim 10^{12}\, s$ is supposed too late in order to account for string effects.

In the framework of General Relativity equations plus string sources the metric evolves following classical General Relativity and the string effect is accounted by the classical matter sources [2]. Backreaction happens since the string equation of state has been derived from the string propagation in curved backgrounds. The result is selfconsistent because Einstein equations returns the correct selfsustained curved background. The current stage appears selfconsistently as the asymptotic limit for large scale factor. Similarly, the radiation dominated stage is obtained from both effective treatments, this is coherent with having such stage previous to the current stage (where string effects must be not visible) and successive to a inflationary stage where string effects are stronger.

The intermediate behaviour between radiation and matter dominated stages is not known. Because this and current knowledge, this effective treatment is not able to describe suitably the radiation dominated-matter dominated transition. A sudden but continuous and smooth transition among both stages is not possible without an intermediate behaviour. Other comment must be dedicated to the inflation-radiation dominated transition: in the String Driven Model, this transition requires a brief temporaly exit of the low energy effective regime for comeback within it at the beginning of radiation dominated, this could be understood as the conditions neccesary to modify the leading behaviour from unstable strings to dual strings. Further knowledge about the evolution of strings in curved backgrounds is necessary, multistring solutions (strings propagating in packets) are present in cosmological backgrounds and show different and evolving behaviours [12]-[14]. Research in this sense could aid to overcome the transition here considered, asymptotically rounded by low energy effective treatments.

4.5 String Driven Inflation realizes a Big-Bang

During the inflationary stage, the scale factor suffers enough expansion for solv- ing the cosmological puzzles. The almost amount of expansion is reached around the exit time. In fact, the beginning of this stage is characterized by a very slow evolution of the scale factor and dilaton. This evolution increases speed in approaching the exit inflation, since it is approaching also the pole singularity in the scale factor. From a phenomenological point of view, the inflationary scale factor describes a very little and very calm Universe emerging from the Planck scale. The evolution of this Universe is very slow at the beginning, but the string coupling with the metric
and the string equation of state drive this evolution, leading to a each time more increasing and fast dynamics. In approaching the exit of inflation, the scale factor and the spacetime curvature increase. The metric “explodes” around the exit of inflation. The last part of this explosion is the transition to radiation dominated stage in a process that breaks transitorily the effective treatment of strings. This transition is supposed brief, the transition to standard cosmology happens at the beginning of radiation dominated stage.

At priori, this model seems privilege an unknown parameter $t_f$, playing the role usually assigned to singularity at $t = 0$ in string cosmology. But this is not unnatural, since in order to reach an enough amount of inflation, this parameter is found to be very close to the standard radiation dominated beginning $t_r$ and so, to the beginning of standard cosmology. On the other hand, this value appears related to the GUT scale, which is consistent with the freezing of the dilaton evolution and the change in the string equation of state.

In this way, the Universe starts from a classical, weak coupling and small curvature regime. Driven by the strings, it evolves towards a quantum regime at strong coupling and curvature.

The argument above mentioned, where a brief transition exits and comebacks among stages in a low energy effective treatment, is not an exceptional feature in String Cosmology. Pre-Big Bang models deal with two branches described in low energy effective treatments. Both branches are string duality related, but the former one runs on negative time values. The intermediate region of high curvature is supposed containing the singularity at $t = 0$ and consequently, the Big Bang. Our String Driven Cosmology presents also this intermediate point of high curvature, but it is found around the inflation-radiation dominated transition, near to but not on the Planck scale. In our model, negative proper times are never considered and the instant $t = 0$ remains before the inflationary stage (is not consistent include $t = 0$ in the effective description, since before the Planck time a fully stringy regime is expected which can not be considered within the effective equations). There are not predicted singularities, neither at $t = 0$ nor at $t = t_f$, at the level of the minimal model here studied.

Another difference with the “Pre-Big Bang scenario” is the predicted dynamics of the universe. The Pre-Big Bang scenario includes a Dilaton Driven phase running on negative times. Not such feature is found here. Although the curvature does not obey a monotonic regime, time runs always on positive values and the scale factor always expands in our String Driven Cosmology.

The Pre-Big Bang scenario assumes around $t \sim 0$ a “String Phase” with high almost constant curvature. Our minimal String Driven Cosmology does not assume such a phase, but a state of high curvature is approached (and reached) at the
end of inflation. The low energy effective regime (L.E.E.) breaks down around the inflation exit, both due to increasing curvature (the scale factor approaches the pole singularity) and to the increasing dilaton. The exit of inflation and beginning of radiation dominated stage must be described with a more complete treatment for high curvature regimes.

The growth reached by the dilaton field during the inflationary stage would not be so large, at least while the low energy effective treatment holds.\[1\] The exact amount depends mainly on the initial inflationary conditions, the parameter $\phi_I$ being constrained by the effective equations (4). At the end of inflation the scale factor increases and the $e$-folds number $f$ increases very quickly with time, but this is not the case for the dilaton ratio. Comparatively, the dilaton ratio increases in a much slower way $\frac{\phi(t_r)}{\phi(t_i)} \sim f$ than the scale factor $\frac{a(t_r)}{a(t_i)} \sim e^f$. As a consequence, corrections due to the high curvature regime are needed much earlier than the corresponding to dilaton growth.

4.6 The Gravitational Wave Background

We have studied the production of a primordial stochastic gravitational wave background in a cosmological model fully extracted in the context of selfconsistent string cosmology.\[9\]

The variable in the power spectrum and the proper frequency $\omega$ are related in a way totally determined by the cosmological scale factor evolution. The factor relating them depends on the expansion ratios, the exit time of inflation and the coefficients of inflationary and current epochs. Being all them fixed in our cosmological background by the observational times, no free parameters are introduced at this level. None of the remaining unknown parameters, like the global scale factor $A_{II}$, appears on the results of our computation. Differently from almost all string cosmology computations in literature, firm predictions on precise frequencies ranges can be extracted in our case.

In this way, we have computed exact, fully predictive and free-parameter expressions for the power spectrum $P(\omega)d\omega$ and contribution to energy density $\Omega_{GW}$ of the primordial gravitational waves background. We have not considered the graviton production at the radiation dominated-matter dominated transition. The graviton contribution due to this transition is expected to be neglectely small, as compared to the first transition. It is expected that the second transition will have a role only on the low frequencies regime, not so important in anycase for our results.

For the same scale factor evolution, drastic differences in the stochastic gravitational wave background appear depending on the role of the dilaton; 1) The
simplest case, without accounting of the effect of the dilaton either on the perturbation equation or on the amplitude perturbation. 2) The second case, is a partial account of the dilaton, with the proper perturbation equation but still matching the reduced amplitude perturbation. 3) In the lastest case, a full account of the dilaton is taken by working with the total tensorial amplitude perturbation and perturbation equation

The background of gravitational waves is characterized in their shape by a parameter \( \nu \) which depends of the inflationary description, the inflation-radiation dominated transition and the role played by the dilaton. The expressions for \( \nu \) have been found in the three cases [9]. We obtain an exact expression for the power spectrum and energy density contribution [9] in terms of Hankel functions of order \( \nu \), formally equal in the No Dilaton and partial Dilaton cases; the differences among them are due to \( \nu \). The formal expressions in the full dilaton case are different both in parameter \( \nu \) as in the coefficients involved.

The low frequency and high frequency asymptotic regimes are given in [9]. In the No Dilaton case, asymptotic behaviours for the power spectrum are both vanishing at low and high frequencies as \( \omega^{-1} \) and \( \omega^{-1} \) respectively. This gives a gravitational wave contribution to the energy density asymptotically constant at high frequencies of magnitude \( \Omega_{GW} \sim 10^{-26} \). There is a slope change that produces a peak in the power spectra around a characteristic frequency totally determined by the model of \( \omega_x \sim 1.48 \text{ Mhz} \).

The Partial Dilaton case introduces the effect of the dilaton only in the tensorial perturbation equation (which is not longer equivalent to the massless real scalar field propagation equation), but not in the perturbation itself. The general characteristics are very similar to the No dilaton case. Both asymptotic regimes for \( P(\omega) d\omega \) vanish again, but with dependences \( \omega^{\frac{3}{2}} \) and \( \omega^{-1} \). The peak appears around the same characteristic frequency, with value one order of magnitude lower than in the No Dilaton case, as well as the asymptotic constant contribution to energy density.

In contrast, when the full dilaton role is accounted, general characteristics as well as orders of magnitude of the spectrum are drastically modified. It has similar values for the frequencies below the Mhz, with power spectrum vanishing again as \( \omega^{\frac{3}{2}} \). For high frequencies, in contrast to the former cases, both \( P(\omega)d\omega \) and \( \Omega_{GW} \) are increasing at high frequencies. For \( P(\omega)d\omega \), an asymptotic divergent behaviour proportional to \( \omega \) is found. It gives values much higher than the no dilaton and partial dilaton cases. The contribution to \( \Omega_{GW} \) is equally divergent at high frequencies as \( \omega^2 \). The change of slope is less visible and no clear peaks are found. The transition from the low frequency to the high frequency regime is slower than in the previous case and the full analytical expressions are needed on a wider range \( 10^6 \sim 10^9 \text{ Hz} \).

The existence of an upper cutoff must be considered in the Full Dilaton Case,
an end-point not predicted by the current minimal model considerations could be
introduced in the spectrum as made in the literature [20]. Divergent high frequencies
behaviour of the graviton spectra and introduction of an upper cutoff is an usual
feature in the string cosmology contexts. This is discussed in the next section.

5 Remarks and Conclusions

5.1 The Model

We have considered a minimal model for the evolution of the scale factor totally
selfsustained by the evolution of the string equation of state. The earliest stages (an
inflationary power type expansion and a radiation dominated stage) are obtained
from the low energy effective string equations, while the radiation dominated stage
and for the matter dominated stage are obtained as selfconsistent solutions of the
Einstein equations selfsustained by the strings. Such solutions suggest the low energy
effective action is asymptotically valid at earliest stages, around and immediately
after Planck time $t_P \sim 10^{-43}s$, when the spacetime and string dynamics would be
strongly coupled. The radiation dominated stage is extracted from both treatments,
coherently with being an intermediate stage among the two regimes: inflation and
matter dominated stage. On the other hand, since the stable string behaviour
describes cold matter, the current matter dominated stage can be also described in
a string matter treatment. Notice that in string theory, the equation of state of the
string matter is derived from the string dynamics itself and not given at hand from
outside as in pure General Relativity.

No detail on the transitions dynamics can be extracted in this framework,
too naive for accounting such effects. The inflation-radiation dominated transition
implies a transitory breaking of the low energy effective regime. The radiation
dominated-matter dominated stage can not be modelized in sudden, continuous and
smooth way. The three string behaviours, unstable,dual and stable, are present in
cosmological backgrounds and each cosmological stage is selfconsistently driven by
them. In this way, the transition from inflation to radiation dominated stage is re-
lated with the evolution from unstable to dual string behaviour, while the radiation
dominated-matter dominated transition would be driven by passing from the dual
to stable behaviour.

Phenomenological information extracted from this String Driven model is com-
patible with observational information. An amount of inflation, usually considered
as enough for solving the cosmological puzzles, can be obtained in the inflation-
ary stage. Energy ranges at the exit of inflation are found coherents with GUT
scales. The inflationary stage gives a value for the energy density-dilaton coupled
term equivalent to the corresponding critical energy density, computed as in General Relativity. That means, we have $\Omega_{\text{inf}} = 1$, whenever the end of inflationary stage be computed. Also, the contribution to current energy density is found $\Omega \geq \frac{4}{9}$, and taking account the validity of General Relativity in the current matter dominated stage, we find this contribution be exactly $\Omega = 1$.

Our main conclusion is to have proved that string cosmology, although being effective, is able to produce a suitable minimal model of Universe evolution. It is possible to place each effective context in a time-energy scale range. Energy ranges are found and General Relativity conclusions are coherently obtained too in a string theory context. We have extracted the General Relativity statement about spatial curvature and energy density, at least for the spatially flat case ($k = 0 \rightarrow \Omega = 1$) in a totally Non-Einsteinian framework, as the low energy effective string action giving rise to the inflationary String Driven stage.

In their validity range, no need of extra stages is found. Only the interval around the transitions and the very beginning epoch, probably the Planck epoch, will require more accurate treatments that hitohere considered. Since the behaviours above extracted are asymptotic results, it is not possible to give the detail of the transitions among the different stages. The connection among asymptotic low energy effective regimes through a very brief stage (requiring a more complete description of string dynamics) enables us to suppose this brief intermediate transition stage containing the evolution in the equation of state from unstable strings to dual strings. From the point of view of the scale factor evolution, this brief transition could be modelized as nearly instantaneous, provided curvature and scale factor expansion have attained nearly their maximun values. Similarly, the radiation dominated-matter dominated transition, should be driven by the subsequent evolution of strings from dual to stable behaviour. Again, a brief intermediate stage could take place among both asymptotic behaviours. But this is an open question in the framework of string cosmology both for inflation-radiation dominated as well as radiation dominated-matter dominated transition.

### 5.2 String and No-String Cosmologies

Among the spectra computed in string cosmology contexts, we must distinguish between those computed in Brans-Dicke frames (that we compare with our Full Dilaton Case) and those computed following usual quantum field theory, that is, similar to our No Dilaton Case. The shape of the spectra computed in string cosmology contexts are very similar. The principal features, as slope changings, are signal of the number of stages or transitions considered in the scale factor evolution. All the known cases, coherently treated in Brans-Dicke frames, present an increasing dependence at high frequencies.
We conclude that all gravitational wave computations on inflationary stages of
the type extracted in string cosmology, coherently made in the Brans-Dicke frame,
must give an increasing spectrum. The peaks are produced by slope changes and
they are signal of the transitions in the dynamics of the background evolution. We
consider that the Pre-Big Bang scenario does not predict a peak, but it is supposed
by defining a $\omega_1$ such that $|\beta(\omega_1)|^2 = 1$. This proper frequency is computed at
the beginning of radiation dominated stage, when the wave reenters the horizon
and it must suffer a redshift at current time, expressed as a function of unknown
parameters of a “string phase”. It acts as the end-point because waves with $\omega > \omega_1$
are supposed exponentially suppressed. Since the spectrum was increasing with
frequency, the same frequency $\omega_1$ constitutes a maximum (peak).

If we use the same argument in order to fix an upper limit, our spectrum must be
cutted at frequency $\omega_{max} \sim 3.85$ MHz where the power spectrum will have a value
around $P(\omega)d\omega \sim 5.68 \times 10^{-41}$ erg/s/cm$^3$ and $\Omega_{GW} \sim 3.40 \times 10^{-26} \rho_c$. These are the same
order of magnitude of the peaks atteined on the No Dilaton and partial Dilaton
Cases. No conflict with observational constraints look possible for such predicted
weak signals. But this argument could be too naive, since in the practical way is
equivalent to cancelate from the spectra the features introduced by the full dilaton
role.

In relation to the no-string inflationary cosmologies, the so called standard
inflation is usually intended as a De Sitter stage. Notice that the string driven inflationary
stage describes an evolution with inverse power dependence. It must not be
confused with the usual power law, although our model can be said superinflationary
too.

There is a radical difference among the string cosmology spectra and those ob-
tained with an exponential inflationary expansion. The divergence at low frequencies
that it supposes is not found in our String Driven Cosmological Background. The
high frequency behaviour is compatible with the No Dilaton Case, computed in a
totally equivalent way. But comparaison with the Full Dilaton Case shows a totally
different behaviour with respect to the obtained in De Sitter case. Notice that comparaison
among string comology inflationary models and standard inflation means
confrontate power and inverse power-type laws with De Sitter exponential infla-
tion, since until now no De Sitter type expansion have been coherently obtained in
String Cosmology. This difference in the inflationary scale factor, together with the
appropriated treatment of metric perturbations in each framework, cause the main
differences among the gravitational wave power spectra in both cases.

No explicit dependence on the beginning of the inflationary stage $t_i$ has been
found on the gravitational wave background. In anycase, further study must be
done in order to determine the influence on the power spectra if earliers inflation
stages are considered.
There are many points that deserve further study: from the point of view of string cosmological backgrounds and transitions dynamics, and from those of gravitational wave computations in the string appropriated frameworks. Better treatment than that applied here could take place in every phase of the problem, advising us against considering this model or its results as definitives. In any case, we have proven here some aspects of the way in which predictions and observational consequences can be extracted from string cosmology.

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