LHC Phenomenology of $Z'$ and $Z''$ bosons in the $SU(4)_L \times U(1)_X$ little Higgs model

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We examine direct limits on masses of the extra neutral gauge bosons in the $SU(4)_L \times U(1)_X$ model with a little Higgs mechanism confronted with the LHC data. There exist two extra neutral gauge bosons, calling $Z'$ and $Z''$, in this model. The lower exclusion limit of the mass of the lighter extra neutral gauge boson is about 3 TeV while that of the heavier one 5 TeV. For comparison, we examine the mass limit of $Z'_3$ boson in the $SU(3)_L \times U(1)_X$ model as well, and discuss the implication of our result in the $SU(4)_L \times U(1)_X$ model with a standard Higgs mechanism. We also discuss the discovery potential of $Z'$ and $Z''$ at the future LHC with the center-of-momentum energy of 14 TeV.
I. INTRODUCTION

One of the major goals of the Large Hadron Collider (LHC) experiment is the search for heavy resonances. Since a heavy resonance indicates a new heavy particle, its discovery is a clear evidence of the new physics (NP) beyond the standard model (SM). Especially the existence of an extra neutral gauge boson $Z'$ is a common feature of many NP models. Therefore, the LHC has been pushing ahead on the neutral heavy resonances in various NP models. Recently CMS and ATLAS collaborations have reported the search results for the $Z'$ through the dilepton channels at $\sqrt{s} = 7$ and 8 TeV with the data up to 20 fb$^{-1}$, and also through $t\bar{t}$, $\tau^+\tau^-$, and dijet channels with similar or less integrated luminosities [1]–[14].

In this work, we consider the $SU(4)_L \times U(1)_X$ model with little Higgs mechanism by embedding anomaly-free set of fermions as a continuation of our previous work [15]. Little Higgs models (LHMs) adopts the early idea that Higgs can be considered as a Nambu Goldstone boson from global symmetry breaking at some higher scale $\Lambda \sim 4\pi f_1$ and acquires a mass radiatively through symmetry breaking at the electroweak scale $v$ by collective breaking [17]. The noble feature of LHMs is that the Higgs mass is protected by a global symmetry which is spontaneously broken and so the one-loop quadratic divergences to the Higgs mass are canceled by particles of the same spin; i.e., a new fermion cancels a quadratic divergence from a SM fermion. The little Higgs mechanism has been implemented in $SU(4)_L \times U(1)_X$ gauge group earlier by Kaplan and Schmaltz (K-S) [18], but the K-S model has a set of anomalous fermions with a simple family-universal embedding which requires additional fermion multiplets at the scale $\Lambda$ due to nonvanishing quadratic divergences from light fermions and gauge anomalies. Alternatively, we consider the modified K-S model by embedding anomaly-free set of fermions as proposed in Ref. [19] so that one-loop quadratic divergences for the Higgs mass are canceled for all fermion flavors [15].

Due to the different gauge group structure and the multiple breaking of global symmetry by separate scalar fields, the neutral currents in this model are fundamentally different from those in other types of LHMs [20]. In this model with anomaly-free set of fermions there are two extra massive neutral gauge bosons, $Z'$ and $Z''$, which couple to ordinary SM fermions, while other extra gauge bosons only couple to new heavy fermions. Therefore we discuss the phenomenology of $Z'$ and $Z''$ at the LHC in this paper. Similar neutral gauge bosons appear in other $SU(4)_L \times U(1)_X$ models with regular Higgs mechanism [21], and their low-energy phenomenology is also similar if the fermion family is assigned to be the same [15]. As a result, the mass bounds and the LHC discovery potentials of $Z'$ and $Z''$ obtained in this paper can be applicable to the models with regular Higgs mechanism. For comparison, we also consider the $Z'$ boson appeared in the LHM with the $SU(3)_L \times U(1)_X$ gauge symmetry in which the anomaly-free set of fermions are embedded as well.

The generic structure of the model is discussed in the section II. We present the decays of $Z'$ and $Z''$ in the section III, and obtain the lower bounds on their masses from the present LHC data in the section IV. In the section V, we study the discovery potential of $Z'$ and $Z''$ at the next stage of the LHC with 14 TeV center-of-mass (CM) energy. Finally, we conclude in the section VI.

II. $SU(4)_L \times U(1)_X$ MODEL WITH LITTLE HIGGS

The $SU(4)_L \times U(1)_X$ model with little Higgs mechanism is characterized by the scalar sector based on the non-linear sigma model describing $[SU(4)/SU(3)]^4$ global symmetry breaking with the diagonal $SU(4)$ subgroup gauged and four non-linear sigma model field $\Phi_i$ parameterized as

$$
\Phi_1 = e^{+i\mathcal{H}_u f_1} \begin{pmatrix} 0 \\ 0 \\ f_1 \\ 0 \end{pmatrix}, \quad \Phi_2 = e^{-i\mathcal{H}_u f_2} \begin{pmatrix} 0 \\ 0 \\ f_2 \\ 0 \end{pmatrix},
$$

$$
\Phi_3 = e^{+i\mathcal{H}_d f_3} \begin{pmatrix} 0 \\ 0 \\ f_3 \\ 0 \end{pmatrix}, \quad \Phi_4 = e^{-i\mathcal{H}_d f_4} \begin{pmatrix} 0 \\ 0 \\ f_4 \\ 0 \end{pmatrix},
$$

where

$$
\mathcal{H}_u = \begin{pmatrix} \Pi_u + \begin{pmatrix} 0 & 0 & h_u & 0 \\ 0 & 0 & 0 & h_u \end{pmatrix} \\ 0 & 0 & 0 & 0 \end{pmatrix} \big/ f_{12}, \quad \mathcal{H}_d = \begin{pmatrix} \Pi_d + \begin{pmatrix} 0 & 0 & h_d & 0 \\ 0 & 0 & 0 & h_d \end{pmatrix} \\ 0 & 0 & 0 & 0 \end{pmatrix} \big/ f_{34},
$$

(1)

(2)
with $f_{ij} = \sqrt{f_{i}^2 + f_{j}^2}$. Here we only show the two complex Higgs doublets $h_{u,d}$ and discard the other singlets in $\Pi_{q} \ (q = u, d)$ whose contributions to fermion and gauge boson masses are negligible. The two doublet Higgs fields $h_{u,d}$ shown in Eq. (2) are of the following form

$$h_u = \frac{1}{\sqrt{2}} (H^0_u \ H^+_{u}), \quad h_d = \frac{1}{\sqrt{2}} (H^0_d \ H^+_{d}),$$

where the neutral components of the two Higgs fields develop vacuum expectation values (VEVs) such that

$$\langle H^0_u \rangle = v_u, \quad \langle H^0_d \rangle = v_d. \quad (4)$$

The Higgs vacua introduced above give masses to the SM fermions after the electroweak symmetry breaking (EWSM).

In this model, the SU(4) breaking is not aligned and only the gauged SU(2) is linearly realized. The SM gauge group $SU(2)_L \times U(1)_Y$ can be embedded into the theory with an additional $U(1)_X$ group. Since this LHM has a gauged SU(4)$_L$, the SM doublets must be expanded to SU(4)$_L$ quadruplets. The extra fermions in the quadruplets should cancel the quadratic divergence from the SM fermion, especially from the top quark. Taking into account this requirement, we embed the SM doublet $(t, b)$ into the following type of SU(4)$_L$ quadruplet as

$$\psi_L = (t, b, T, B)^T_L, \quad (5)$$

so that the duplicated extra heavy fermions $T$ and $B$ remove the quadratic divergences due to their SM fermion partners $t$ and $b$, respectively, as discussed in Ref. [15]. After the EWSB, among the 15 gauge fields $A_{\mu}^{(4)}$ associated with SU(4)$_L$, the three neutral gauge bosons $A^3, A^8$ and $A^{15}$ mixed with the $U(1)_X$ gauge boson $A^X$ are associated with a $4 \times 4$ non-diagonal mass matrix. After the mass matrix is diagonalized, a zero eigenvalue corresponds to the photon $A$, and the three physical neutral gauge bosons $Z, Z'$ and $Z''$ get the masses as follows

$$M_Z^2 = \frac{g^2 v^2}{4c_w^2} \left(1 - \frac{t_w^4 v^2}{4 f^2} \right), \quad M_{Z'}^2 = (g^2 + g_X^2) f^2 - M_Z^2, \quad M_{Z''}^2 = \frac{1}{2} g^2 f^2, \quad (6)$$

where the Weinberg mixing angle $\theta_W$ is

$$c_w \equiv \cos \theta_W = \frac{\sqrt{g^2 + g_X^2}}{\sqrt{g^2 + 2g_X^2}}, \quad (7)$$

and the VEV $v$ is given by $v^2 = v_1^2 + v_2^2$ with

$$v_1^2 = v_2^2 - \frac{v^4 u}{3f^2} \left( f_1^2 + f_2^2 - 1 \right), \quad v_2^2 = v_3^2 - \frac{v_4^4}{3f^2} \left( f_3^2 + f_4^2 - 1 \right). \quad (8)$$

Note that the following simplifying assumption $f_{12} = f_{34} = f$ is used as done in Ref. [15] [24]. Under this assumption, $Z''$ does not mix with $Z$ or $Z'$ but still couples to the ordinary SM fermions. There are also extra flavor-changing neutral gauge bosons other than $Z'$ and $Z''$ in this model, but they only couple to new heavy fermions. So we discard their contributions by assuming that the new fermions are too heavy to be seen below a few TeV. The neutral currents contributions involving $Z'$ and $Z''$ are given by

$$\mathcal{L}_{NC}' = -\frac{g}{2c_w} \sum_{\psi} \left[ \bar{\psi} \gamma^\mu (g_{\mu}^{(4)} - g_{A}^{(4)} \gamma_5) \psi \right] Z'_{\mu} + \frac{g}{4\sqrt{2}} \sum_{\psi} \left[ \bar{\psi} \gamma^\mu (1 - \gamma_5) \psi \right] Z''_{\mu}, \quad (9)$$

where $\psi$ are the SM fermions, and $g_{\mu}^{(4)}$ and $g_{A}^{(4)}$ are corresponding coupling constants of which values are listed in Table I. The couplings in the table are family-universal while those in the K-S model with anomaly-free fermion embedding are not. One can see from the table that the couplings contain additional new physics (NP) contributions proportional to the mixing angle $\theta$ between $Z$ and $Z'$ where $s_{\theta} \equiv \sin \theta = t_w^2 \sqrt{1 - f_{12}^2 - v^2/(2c_w f^2)}$. Note that the masses of all heavy gauge bosons and the mixing angle $\theta$ are uniquely determined by the single parameter $f$. In the general case of $f_{12} \neq f_{34}$, we obtain the following condition: $f_{12}^2 - f_{34}^2 = v_1^2 - v_2^2 \ll f_{ij}^2$, from the fact that there must

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1 The contributions of the flavor mixings between the SM and the new heavy fermions to $g_{\mu}^{(4)}$ and $g_{A}^{(4)}$ are suppressed by $1/f^2$, so we neglect such flavor mixing effects.
be one zero eigenvalue corresponding to the photon. Therefore, \( f_{12} \) could not be much different from \( f_{34} \), and our simplifying assumption is legitimate as a first approximation.

Instead of the little Higgs mechanism, one can achieve the symmetry breaking for fermion and gauge boson masses by introducing the four SM type Higgs scalar fields with VEVs aligned as [21] to obtain

\[
\langle \phi_i^T \rangle = (v_u, 0, 0, 0) \sim [1, 4, -1/2], \quad \langle \phi^A_i \rangle = (0, 0, V_u, 0) \sim [1, 4, -1/2],
\]

\[
\langle \phi^Z_i \rangle = (0, v_d, 0, 0) \sim [1, 4, 1/2], \quad \langle \phi^Z_i \rangle = (0, 0, v_d) \sim [1, 4, 1/2],
\]

where \( V_u(V_d) \) corresponds to \( f_{12}(f_{34}) \) in the LHM. With this choice of scalar sector, the theory becomes renormalizable but the Higgs mass fine-tuning issue by the one-loop quadratic divergences remains intact in the model. The symmetry breaking pattern of this model with the hierarchy \( V_u \sim V_d \gg v_u, v_d \) is similar to that of the LHM, due to the same gauge structure. After EWSB with the VEVs in Eq. (10), three physical neutral gauge bosons \( Z, Z', \) and \( Z'' \) have the following masses (squared) similarly to Eq. (6):

\[
M^2_{Z'} = \frac{g^2 v_u^4}{\cos^2 \theta} \left( 1 - 4 \frac{v_d^2}{v_u^2} \right), \quad M^2_{Z''} = \frac{g^2 + g^2}{2} v_d^2 - r^2 s_W^2 M^2_Z, \quad M^2_{Z'} = \frac{1}{2} g^2 (v_d^2 + v_u^2),
\]

where we use the following simplifying assumption \( v_u = V_d \equiv V_u \) and \( v_u = v_d \equiv v_4 \) as done in Ref. [21]. The \( Z'' \) mass can be expressed in terms of the lighter \( Z \) and \( Z' \) masses as

\[
M^2_{Z''} = 2(1 + r^2) M^2_{Z'} - M^2_{Z'}. \tag{12}
\]

One can clearly see from Eq. (6) that the above mass relationship holds in the LHM with the same gauge group. Once we introduce the same anomaly-free set of fermions given in Eq. (5), the gauge boson couplings to the SM fermions are also similar to those in the LHM so that the low energy phenomenology of \( Z' \) and \( Z'' \) shall appear very similarly in both models. Therefore, the bounds of \( Z' \) and \( Z'' \) masses obtained in the LHM can be applicable to the model with regular Higgs mechanism.

### III. DECAYS OF \( Z' \) AND \( Z'' \)

Since the neutral current interactions are identified, we can now proceed to analyze the \( Z' \) and \( Z'' \) decays and production rates. Other than the neutral currents discussed in the previous session, \( WWZ' \) and \( HZZ' \) gauge boson coupling arise in this model through \( Z - Z' \) mixing so that \( Z' \) can decay into \( WW \) and \( ZH \) final states while \( Z'' \) cannot. The \( Z' \) decay rates are then given by

\[
\Gamma(Z' \rightarrow \psi \bar{\psi}) = \frac{G_F^2}{6\sqrt{2} \pi} M^2_Z M_{Z'} \left[ (g_{\psi}^2 + g_{\eta}^2) \left( 1 - \frac{M_{\psi}^2}{M_{Z'}^2} \right) + 3 \left( g_{\psi}^2 - g_{\eta}^2 \right) \frac{M_{\psi}^2}{M_{Z'}^2} \right] \left( 1 - 4 \frac{M_{\psi}^2}{M_{Z'}^2} \right)^{1/2},
\]

\[
\Gamma(Z' \rightarrow W^+ W^-) = \frac{G_F^2}{24 \sqrt{2} \pi} c_W^2 s_W^2 M^2_Z M_{Z'} \left[ \frac{M_{\phi}^4}{M_{W}^2} + 20 \frac{M_{Z'}^2}{M_{W}^2} + 12 \right] \left( 1 - 4 \frac{M_{\phi}^2}{M_{Z'}^2} \right)^{3/2},
\]

\[
\Gamma(Z' \rightarrow ZH) = \frac{G_F^2}{6 \sqrt{2} \pi} s_W^2 M^2_Z M_{Z'} \left[ 2 \frac{M_{\phi}^2}{M_{Z'}^2} + 4 \left( 1 - \frac{M_{\phi}^2}{M_{Z'}^2} \right) \left( 1 - 4 \frac{M_{\phi}^2}{M_{Z'}^2} \right)^{1/2} \right] \left[ 1 - \left( \frac{M_{\phi}^2}{M_{Z'}^2} + \frac{M_{H}^2}{M_{Z'}^2} \right)^2 \right]^{1/2}.
\]

\[
\tag{13}
\]
The $Z''$ decay rates into fermions pairs are given similarly to the $Z'$ decay rates. The new fermion masses are comparable to the new gauge boson masses. In this work, we assume that those new fermions such as $T$ and $B$ are too heavy for the neutral gauge bosons to decay into them $[15]$. The decay rates of $Z' \to W^+W^-$ and $Z' \to ZH$ are suppressed by $Z - Z'$ mixing angle squared $s_{\theta}^2 \sim (v/f)^4$. However, the triple gauge boson coupling is proportional to the incoming and outgoing momentum, and the suppression by $Z - Z'$ mixing is compensated by the heavy $Z'$ mass contribution $(M_{Z'}/M_W)^4 \sim (f/v)^4$ so that the decay rate of $Z' \to W^+W^-$ is comparable to those of $Z' \to \psi \bar{\psi}$ as shown in Eq. (13). On the other hand, there is no enhancement in the decay of $Z' \to ZH$ so that the corresponding rate is negligible in this model.

We illustrate the branching ratios of $Z'$ and $Z''$ bosons in Fig. 1 as a function of their masses and of $f$. The decay patterns of $Z'$ and $Z''$ are obviously different, so they are distinguishable at the collider detector. In Fig. 2, we show the total decay widths of $Z'$ and $Z''$ bosons as a function of their masses and of the scale parameter $f$. In the figure, as a comparison, we also show the total decay width of the extra neutral gauge boson denoted as $Z'_3$ which appears in the $SU(3)_L \times U(1)_X$ LHM $[22, 23]$. If $Z'$ or $Z''$ decays into new heavy fermion pairs, the slope of each curve in the Fig. 2 increases from the resonance masses of the new fermions. But we assume $M_{Z'}, M_{Z''} < 2M_F$ for all new fermions $F$, and Fig. 2 just shows uniform slopes. Taking consideration of all the possibilities for the new fermion masses is beyond the scope of this paper, and our results in the next sections are obtained with no new heavy fermionic resonances. Even if $M_{Z'}, M_{Z''} > 2M_F$ for some of new fermions $F$, still, branching ratios of $Z'$ and $Z''$ decaying into the new fermions would be only a few percent due to the small fermion mixing. Hence the decreases of the $Z'$ and $Z'$ cross-section times the SM fermion branching ratios should be very small $[25]$. Therefore, the bounds obtained in

![Branching ratios of (a) $Z'$ boson and of (b) $Z''$ boson as a function of their masses.](image1.png)

![Decay widths of $Z'$ and $Z''$ bosons as a function of (a) their masses and of (b) the scale parameter $f$. $Z'_3$ denotes the extra neutral gauge boson in the $SU(3)_L \times U(1)_X$ LHM.](image2.png)
the selected channels discussed here shall not change much.

IV. CURRENT BOUNDS AT $\sqrt{s} = 8$ TeV

In order to generate processes, we implemented the LHMs in the MadGraph5 package \[26\] using Feynrules \[27\]. For cross-section calculation and event generation, we used Pythia \[28\], and simulated the signal events of four different processes, dilepton, $\tau^+\tau^-$, dijet, and $t\bar{t}$, at the CM energy of 8 TeV. The $Z'$ and $Z''$ cross-section times branching ratios ($\sigma \times BR$) are calculated within a range of $\pm 3\Gamma$ around the $Z'$ and $Z''$ pole masses as done similarly in Ref. \[29\]. For experimental bounds, we used the 8 TeV collision data collected by the CMS and the ATLAS experiments. The combined dilepton data by the CMS group correspond to an integrated luminosity of 19.6 fb$^{-1}$ in the dielectron channel and 20.6 fb$^{-1}$ in the dimuon channel \[10\], and those by the ATLAS group correspond to an integrated luminosity of 20.3 fb$^{-1}$ in the dielectron channel and 20.5 fb$^{-1}$ in the dimuon channel \[12\]. The dijet data collected by the CMS and the ATLAS groups correspond to an integrated luminosity of 19.6 fb$^{-1}$ \[9\] and 13 fb$^{-1}$ \[11\], respectively. The most recent ditau and ditop resonance searches have been performed by the ATLAS group using 19.5 fb$^{-1}$ data in the fully hadronic channel \[14\] and 14 fb$^{-1}$ data in the lepton plus jets channel \[13\], respectively.

![FIG. 3. Observed upper cross-section times branching ratio ($\sigma \times BR$) limits at 95% CL for $Z'$ and $Z''$ bosons in the $SU(4)_L \times U(1)_X$ LHM and for $Z'_3$ bosons in the $SU(3)_L \times U(1)_X$ LHM using (a) combined dilepton, (b) $\tau^+\tau^-$, (c) dijet, and (d) $t\bar{t}$ channels.](image-url)

In Fig. 3 we show the observed exclusion limits on the cross-section times branching ratio at 95% CL for $Z'$ and $Z''$ bosons in the $SU(4)_L \times U(1)_X$ LHM and for $Z'_3$ bosons in the $SU(3)_L \times U(1)_X$ LHM for the dilepton, $\tau^+\tau^-$, dijet, and $t\bar{t}$ channels. Note in the figures that $\sigma \times BR$ for $Z'$ are depicted as a function of the $Z''$ mass since the $Z'$ and $Z''$ masses are dependent each other and determined by a single parameter $f$. One can see from the figures that the dilepton measurements give the strongest bounds on the $Z'$ and $Z''$ boson masses as well as on the $Z'_3$ mass. In the dilepton channel, the CMS limit in Ref. \[10\] was given on the production ratio $R_\sigma$ of cross-section times branching fraction for $Z'$ bosons to the same quantity for $Z$ bosons, but we converted it to $\sigma \times BR$ using the $Z$ cross-section obtained within the range of 60–120 GeV \[30\], for a clear comparison with the other measurements. As shown in the
figures, $Z''$ in the $SU(4)_L \times U(1)_X$ model is excluded below 2980 GeV by the CMS measurements and 2730 GeV by the ATLAS measurements. Note that the corresponding ATLAS result has been recently published \[12\] while the CMS result is not \[10\], as of now. The bounds on the $Z'_3$ mass obtained from the dilepton measurements are about 100 GeV lower than those of $Z''$. Using the mass relationship given in Eq. \[12\], the lower mass bound of $Z'$ boson is also obtained as 5040 (4620) GeV with respect to the CMS (ATALS) dilepton measurement. For reference, if we use the best previous published results of up to 5 fb$^{-1}$ data collected at $\sqrt{s} = 7$ and 8 TeV \[5, 6\], we obtain the lower mass limit of $Z''$ to be 2500 GeV which corresponds to the $Z'$ mass of 4230 GeV. Instead of the arbitrarily chosen 3Γ interval used in the present analysis for cross-section calculations, if we choose the events in a range of 40% of the $Z''$ mass as done in Ref. \[10\], the mass bound rises about 70 GeV.

V. DISCOVERY POTENTIALS AT $\sqrt{s} = 14$ TEV

The LHC is expected to restart its operation at a CM energy of 13 TeV in early 2015, and will reach at the designed energy of 14 TeV in a few years. In this section, we investigate the LHC potential to find a $Z''$ as well as $Z'_3$ at 14 TeV, especially in the dilepton channel. Fig. 4 shows the total cross-sections calculated at tree level for the processes $pp \rightarrow Z''(Z', Z'_3) \rightarrow \ell^+ \ell^-$ at 14 TeV, with $\ell = e, \mu$. One can see from the figures that depending on the masses of $Z''(Z', Z'_3)$, the cross-sections increase by a factor of 10 to 10$^2$ at 14 TeV in comparison with their values at 8 TeV shown in Fig. 3(a).

The decays $Z''(Z', Z'_3) \rightarrow \ell^+ \ell^-$ provide a clean signature in the mass spectrum. The dominant background in
this case is the irreducible Drell-Yan (DY) process. The contribution of the other backgrounds is less than 30% of
the DY cross-section and can be heavily suppressed by isolation cuts at high masses, so we consider only the DY
background [31]. Since the interference between the $Z''(Z', Z'_3)$ and the $Z/\gamma^*$ processes is minimal, we also
treat signals independent of backgrounds. In Fig. 5 we show the invariant distribution for the dielectron system,
as expected for the $SU(4)_L \times U(1)_X$ LHM with $M_{Z''} = 3.5$ TeV and for the SM background using a luminosity of 100
fb$^{-1}$ data at 14 TeV. We also show the case for the $SU(3)_L \times U(1)_X$ LHM with $M_{Z''} = 3.5$ TeV as a comparison.
One can see from the figure that the width of $Z'_3$ boson is narrower than that of $Z''$ as also shown in Fig. 2(a). Although
only the invariant mass distribution for the electron channel is shown in the figure, the distribution for the muon
channel looks nearly the same.

![Graph showing integrated luminosity needed for 5 $\sigma$ discovery](image)

**FIG. 6.** Integrated luminosity needed for 5 $\sigma$ discovery of $Z''(Z', Z'_3) \rightarrow e^+e^-$ as a function of the $Z''$ and $Z'_3$ masses at $\sqrt{s} = 14$
TeV. The plot for $Z'$ is depicted as a function of the $Z''$ mass.

The discovery potential to find a new exotic particle is determined by the integrated luminosity needed for a 5 $\sigma$
excess. The statistical significance is obtained from the expected number of signal($N_s$) and background($N_b$) events
within a chosen exotic particle mass window ($3\sigma$ in our study), and defined as [32]:

$$S = \sqrt{2 ((N_s + N_b)ln(1 + N_s/N_b) - N_s)}, \quad (14)$$

which gives a good approximation to the likelihood ratio based significance in the low statistical regime. For more
realistic study, we consider an overall efficiency of 73% for the electron channel as determined by the ATLAS experiment [12]. In Fig. 6 we shows the expected integrated luminosity needed for 5$\sigma$ discovery in the electron channel
as a function of the $Z''$ mass for the $SU(4)_L \times U(1)_X$ LHM and of the $Z'_3$ mass for the $SU(3)_L \times U(1)_X$ LHM.
The figure shows that, even with a total integrated luminosity of a few fb$^{-1}$, it would be possible to reach the limits set by
previous searches of this type of neutral gauge boson. For $M_{Z''} \sim 3.5$ TeV in the $SU(4)_L \times U(1)_X$ LHM, it is required
to have about 12 fb$^{-1}$ data to discover this new heavy state, while about 5 fb$^{-1}$ more data are needed to observe $Z'_3$
boson expected in the $SU(3)_L \times U(1)_X$ LHM. It is not trivial to identify the origin of the observed neutral bosons
in the discussed decay channels until 100 fb$^{-1}$ data are collected above the obtained mass bound. As also shown in
the figure, if we observe a new neutral boson with a mass of 3.5 TeV for instance, we need to collect about $10^3$ fb$^{-1}$
data to see if there is another exotic boson in order to discriminate the models from a direct observation. Otherwise,
we should take into account to investigate other physical observables as well such as the forward-backward charge
asymmetry although it is non-trivial to measure such a quantity at the LHC because the original quark direction is
unknown [29], and we are not going that far in this paper. As discussed in Sec. II, our obtained results are valid on
the mass bounds and the LHC discovery potentials in the similar models with the standard Higgs mechanism as far
as the gauge structures and the fermion types are the same.

**VI. CONCLUSIONS**

In this paper, we obtained the lower bounds on the masses of $Z'$ and $Z''$ bosons appeared in the $SU(4)_L \times U(1)_X$
LHM with anomaly-free set of fermions using the present LHC data. In this model, $Z''$ boson is lighter than $Z'$ boson,
and their masses are determined by the single scale parameter $f$ with the relationship given in Eq. (12). The strongest
lower mass bound is obtained currently in the dilepton channel with about 20 fb$^{-1}$ data collected at $\sqrt{s} = 8$ TeV.
The CMS result from the analysis of the production ratio $R_\gamma$ excludes $Z''$ boson with mass below 2980 GeV while
the ATALS result from the analysis of $\sigma \times BR$ excludes $Z''$ boson with mass below 2730 GeV. For comparison, we also considered $Z_1'$ boson appeared in the $SU(3)_L \times U(1)_X$ LHM with anomaly-free set of fermions, and the exclusion limit of the $Z_1'$ mass is about 100 GeV lower than that of the $Z''$ mass.

The search for these new neutral bosons is an important issue of the experimental program of the LHC running at the designed CM energy of 14 TeV. In order to see the discovery potentials of $Z'$ and $Z''$ bosons, we presented in Fig. 6 the expected integrated luminosity needed for 5$\sigma$ discovery in the electron channel as a function of new exotic neutral boson masses. For $M_{Z''} \sim 3.5$ TeV, it is required to have about 12 fb$^{-1}$ data to discover this new heavy state, while about 5 fb$^{-1}$ more data are needed to observe $Z_1'$ boson with the same mass. With a luminosity of 100 fb$^{-1}$ data, one can search for $Z''(Z_1')$ boson with mass up to about 4650(4450) GeV in the electron channel. Of course, if the detector efficiency is improved, the search region of the exotic boson masses can be extended further.

As discussed earlier, our results can be applicable to the $SU(4)_L \times U(1)_X$ model with the standard Higgs mechanism as far as the fermion structure is the same. The different models could be discriminated either by observing the extra scalars appeared in the models or by investigating the other properties of the exotic bosons such as charge asymmetry and rapidity distribution simultaneously. This study shall be proceeded further as future experimental progress reveals a hint on an exotic state.

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