Properties of dibaryons in nuclear medium

M. Kakenov1,2,a, V. I. Kukulin3,b, V. N. Pomerantsev3,c, O. Bayakhmetov4,d

1 Laboratory of Information Technologies, Joint Institute for Nuclear Research, 6 Joliot-Curie, Dubna, Moscow Region 141980, Russia
2 Institute of Nuclear Physics, 1 Ibragimova Street, 050032 Almaty, Kazakhstan
3 Skobeltsyn Institute of Nuclear Physics, Lomonosov Moscow State University, 1(2) Leninskie Gory, Moscow 119991, Russia
4 L.N.Gumilyov Eurasian National University, 2 Satpaev Str, 010008 Nur-Sultan, Kazakhstan

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Abstract Properties of six-quark dibaryons in a nuclear medium are considered by the example of $A = 6$ nuclei within the three-cluster $\alpha + 2N$ model. Dibaryon production in nuclei leads to the appearance of a three-body force between the dibaryon and nuclear core. This non-conventional scalar force is shown to provide an additional attractive contribution to the three-body binding energy. This three-body contribution improves noticeably agreement between theoretical results and experimental data for the majority of observables. The most serious difference between the traditional $NN$-force models and the dibaryon-induced model is found for the nucleon momentum distribution, the latter model providing a strong enrichment of the high-momentum components, both for the $^6\text{Li}$ and the $^6\text{He}$ cases.

1 Introduction. On the dibaryon concept for the nuclear force

The history of the nuclear force, i.e., the force which holds the nucleons in a nucleus together, is very long, contradictory and dramatic. It started almost immediately after the discovery of the neutron by Chadwick in 1932 and even now it can in no way be considered as completed. The dominant paradigm suggested by Yukawa in 1935 [1] is based on the general idea of pion exchange between nucleons both in free space and inside nuclei. This basic concept, having been generalized later to include other meson exchanges [2–5], was able to explain many fundamental properties of nuclear interactions like their short-range character, the presence of tensor and spin–orbit forces, and the properties of the deuteron and light nuclei. But simultaneously this model met very numerous contradictions and inconsistencies when interpreting the individual nuclear phenomena [6–11]. Therefore, to put nuclear theory on more consistent and solid ground, the so-called realistic one-boson-exchange (OBE) $NN$ potentials, used conventionally for calculations as regards numerous nuclear phenomena, have been replaced in recent years by the potentials derived from Effective Field Theory (EFT), or Chiral Perturbation Theory (ChPT) [12–18]. Lately, the EFT approach has attracted strong interest of researchers in the field and it gave birth to the hope to explain systematically many puzzles and paradoxes existing throughout nuclear physics.

The most convenient and appropriate field to apply the refined and powerful EFT-based approaches to are the few-nucleon (and, in general, few-body) systems which could be treated accurately, without further approximations using the well-known Faddeev equation formalism.

It is well known that there are numerous and long-standing puzzles and paradoxes in this field, where approaches based on phenomenological and semi-phenomenological realistic $NN$ potentials have failed in explaining the experimental data. Among such puzzles one can enumerate the following ones:

- the $A_y$ puzzle in the vector analyzing power for low-energy scattering of polarized nucleons on deuterons $N + d$ and also puzzles in the deuteron tensor analyzing powers $T_{20}$, $T_{21}$ and $T_{22}$ for scattering of nucleons on polarized deuterons $N + d$ [6, 7];
- the Sagara puzzle in the $N + d$ elastic scattering cross section at low energies and large angles [9];
- the space-star three-nucleon breakup in $n + d$ and $p + d$ collisions;

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the differential cross section and vector and tensor analyzing powers for \( n + d \) and \( p + d \) backward scattering at intermediate energies, 200–400 MeV [10];

- Coulomb effects in the three-body breakup in \( N + d \) collisions.

None of these puzzles has found a satisfactory explanation when passing from the traditional meson-exchange realistic \( NN \) and \( 3N \) potentials to the modern EFT-based approaches\(^1\). So, the origin for the failure in explaining the above puzzling phenomena appears to be the absence of some key ingredients both in the traditional meson-exchange \( NN \) potentials and in the EFT-based interaction.

There is another class of phenomena where the traditional meson-exchange and the modern EFT approaches face serious conceptual problems. These are processes accompanied by high-momentum transfers or occurring at short internucleon distances, like:

- the so-called cumulative particle production, i.e., production in the kinematical region forbidden for a single-particle process [20–25];
- the sub-threshold particle production [26–28].

Here one should mention also a very interesting feature discovered only recently: a direct interrelation between the short-range \( NN \) correlations in nuclei and the single-quark momentum distributions found in such phenomena as Deep Inelastic Scattering (DIS), and the EMC effect [29–32].

A possible explanation for these phenomena can be related to the production of some intermediate strongly correlated two- and few-nucleon clusters in nuclei (the so-called di- and multibaryons) which behave like tightly bound quasi-particles and can absorb a very large momentum from a high-energy projectile. In turn, such a two- or few-nucleon clustering is tightly related to the well-known short-range correlations (SRCs) of nucleons in nuclei which have been studied extensively in the numerous experiments done in JLab for the last two decades (see, e.g., [29–32]). These SRCs are apparently a manifestation of the quark structure of the nucleon and some features of the interquark interactions, which are described in the framework of the Quantum Chromodynamics (QCD).

It is commonly accepted that the QCD gives the general basis for nuclear physics. However, the QCD operates with degrees of freedom (d.o.f.)—quarks, gluons, strings—that are completely different from those with which traditional nuclear physics deals. Hence, when describing the nuclear phenomena with a high-momentum transfer, one needs a bridge between the fundamental QCD and the traditional nuclear physical d.o.f. The conventional meson-exchange and EFT approaches do not provide such a bridge.

Thus, to proceed one needs a sort of hybrid model combining both quark–gluon and meson-exchange aspects of the \( NN \) interaction. The specific model for the nuclear force which unifies both quark–gluon and meson-exchange features—the dibaryon (or dibaryon-induced) model—was developed by Kukulin et al. in the last two decades [33–39] (in the early studies [33–36] it was called “the dressed-bag model”). The model allowed for a very good description of both elastic and inelastic \( NN \) scattering phase shifts from zero energy up to \( T_{lab} = 600–800 \) MeV (or even up to 1 GeV in some partial waves) [37–39] and the deuteron properties [36] using only a few basic parameters. Moreover, when considering the three-nucleon system within the framework of the dibaryon model, a specific three-particle force inevitably arises due to the interaction of an intermediate dibaryon formed from a pair of nucleons with the third nucleon [40–42]. Our first calculations of the ground states of the three-nucleon nuclei \( ^3H \) and \( ^3He \) have shown that the dibaryon model for \( 2N \) and \( 3N \) forces gives a good description of the basic properties of these nuclei, including the precise value of the Coulomb displacement energy \( \Delta E_C \) [40,41].

Nevertheless, since our approach is highly non-canonical, it should be tested carefully in other nuclear physics predictions. So, in the present paper we have chosen for such a test the properties of the \( A = 6 \) nuclei within the three-cluster \( \alpha + 2N \) model. These nuclei (\( ^6He \) and \( ^6Li \)) are very well suited for our purposes, because their structure can be described quite well within the model of two interacting nucleons in the field of the inert core [43–48].

So, here we suggest to compare three groups of predictions:

- properties of the \( A = 6 \) nuclei with the conventional \( NN \) potentials;
- properties of the \( A = 6 \) nuclei with the dibaryon model potential for the \( NN \) and respective three-body interactions;
- properties of the isolated \( NN \) system, i.e., when the \( \alpha \)-particle core is removed.

This comparison allows to shed light not only on the properties of dibaryons in nuclear systems but also on some key features of specific nuclear phenomena such as pairing and the nature of Cooper pairs in nuclei. We plan to discuss these interesting problems in a separate paper.

A few words should be said about the impact of some fine effects, such as relativistic effects and the possible excitation of the alpha cluster core. The main focus of the present study is to investigate the behavior and properties of dibaryons in a nuclear field by comparing the properties of the same nuclei.

\(^1\) According to a recent study [19] of the low-energy analyzing powers, the values of \( A_T, iT_{11} \) and \( T_{20} \) can be explained only by varying the contact \( P \)-wave terms within the N\(^3\)LO and N\(^4\)LO approximations.
within two alternative \( NN \)-force models, namely the traditional one and that based on the dibaryon concept. So, one may hope that the differences between the results obtained for these two types of force models are hardly sensitive to the above finer corrections. Moreover, the accurate 3\( NN \) calculations of the Bochum–Cracow group [49] and also our estimations showed that the relativistic effects for such differences are almost negligible.

The structure of the paper is as follows. In Sect. 2, we discuss briefly the three-cluster \( \alpha + 2N \) model for the \( A = 6 \) nuclei and present the variational formalism for the calculations of these nuclei. In Sect. 3 we describe the dibaryon model for the \( NN \) interaction and its employment for the basic \( NN \) channels in the \( \alpha + 2N \) system. Here we also introduce the interactions between the intermediate dibaryon and the third nucleon in the \( 3N \) system or the \( \alpha \)-core in the \( \alpha + 2N \) system, which result in three-body forces for both systems. In Sect. 4 we present the results of our detailed calculations for the ground states of the \( A = 6 \) nuclei within the dibaryon-induced model for the \( NN \) and \( NN\alpha \) forces in comparison with the results for the traditional Reid Soft Core (RSC) model. Here we discuss some observables for the \( A = 6 \) nuclei with the focus on the momentum distributions at high momenta and discuss the relation of our results to some fundamental general properties of nuclei, such as pairing of neutrons in the \( ^1S_0 \) channel. In Sect. 5 we study the dependence of the dibaryon admixture on nuclear density by changing the binding energy of the \( NN \) pair in the nucleus due to changing the coupling constant for the three-body force. In Sect. 6 we summarize the results and present some concluding remarks.

2 Three-body \( \alpha + 2N \) cluster model for \( A = 6 \) nuclei

The three-body cluster model for the \( A = 6 \) nuclei in its modern form was developed first in a series of papers [43–48]; see also Refs. [50–53]. This model was able to explain in a quantitative manner many properties of the \( A = 6 \) nuclei, though some minor disagreement with the data still remained, e.g., for the binding energies of \(^6\)He and \(^6\)Li (disagreement with the data was about 0.5–0.8 MeV) and for the rms charge radii (at the 5% error level). The physical justification for the three-cluster \( \alpha + 2N \) model for the \( A = 6 \) nuclei (\(^6\)Li–\(^6\)He–\(^6\)Be) can be their low binding energy in the channel \( \alpha + N + N \) or \( \alpha + 2N \) as compared to the binding energy of the \( \alpha \)-cluster. Another important hint for the validity of the three-cluster model for the \( A = 6 \) nuclei is the rather large interparticle distance in the ground and excited states of these nuclei, larger than the interparticle potential range (see the detailed discussion for these problems in Sect. 5). So, it is plausible to assume that when combining three particles \( \alpha + N + N \) into an eigenstate of the above \( A = 6 \) nuclei, the \( \alpha \)-cluster distortion is small and can be ignored.

Recently, within similar approximations, an interesting EFT-based model for the \(^6\)He nucleus has been developed [55]. The approach employs basically the halo structure for \(^6\)He with three well-separated particles. Unfortunately, the authors of Ref. [55] studied only the binding energy and the gross structure of the \(^6\)He ground state within their EFT-Halo approach. Contrary to this, in the present work we study the basic properties of the \( A = 6 \) nuclei, spin–orbit structure of the wave functions, their geometrical forms, nucleon momentum distributions, etc.

The three-body Hamiltonian for the \( A = 6 \) nuclei includes the \( \alpha N \) potential with the 0\( s \) forbidden (by the Pauli principle) state [48] and some realistic \( NN \) potentials which are chosen here in two alternative forms:

- in the form of the traditional \( NN \) potential with a repulsive core (here we have chosen the RSC potential as in Ref. [48]);
- in the form of the dibaryon-induced \( NN \) potential [36], which can describe the \( NN \) scattering phase shifts from zero energy up to \( T_{lab} \approx 1 \) GeV or even higher.

Keeping in mind that the RSC potential can fit the empirical \( NN \) phase shifts until \( T_{lab} = 300 \) MeV only, while the dibaryon-induced \( NN \) interaction can describe \( NN \) scattering in a much broader energy range, one can assume that the off-shell properties of the dibaryon-induced potential should be much more adequate than those for the RSC model.

The main formal difference between the dibaryon-induced interaction and the traditional \( NN \)-force models is its energy dependence, which arises due to the exclusion of the internal (dibaryon) channel. In the three-particle system \( N + N + \alpha \), this energy dependence also leads to the dependence of the pair \( NN \) potential on the momentum of the \( \alpha \)-particle relative to the center of mass of the \( NN \) pair. Another feature of the dibaryon model is the presence of the three-particle force due to the interaction of the dibaryon with the \( \alpha \)-particle. This three-particle force in the effective Hamiltonian also depends on the energy of the system.

We use the following notation for the coordinates and momenta (see Fig. 1): \( r \) is the relative coordinate of two nucleons, while \( \rho \) is the Jacobi coordinate of the \( \alpha \)-particle relative to the center of mass of the nucleon pair; \( p \) and \( q \) are the momenta canonically conjugated to the coordinates \( r \) and \( \rho \), respectively. The composite index \( \gamma = [\lambda, l, L, S] \) represents the set of quantum numbers for the basis functions: the orbital angular momenta \( \lambda \) and \( l \) correspond to the Jacobi coordinates \( r \) and \( \rho \), respectively, \( L = l + \lambda \) is the total

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orbital angular momentum and $S$ is the spin of the system, which in this case is equal to the total spin of two nucleons. The total angular momentum is $J = L + S$.

For the case of the dibaryon-induced $NN$ potential the total effective three-body Hamiltonian for the $\alpha + 2N$ system takes the form

$$H_{\text{tot}}^{\text{eff}}(E) = T + V_{N\alpha} + V_{N\alpha} + V_{N\alpha}^{\text{eff}}(E) + W_{3BF}(E).$$ (1)

Here $T$ is the kinetic energy, $V_{N\alpha}$ is the potential of the interaction of the $i$th nucleon with the $\alpha$-core, $V_{N\alpha}^{\text{eff}}(E)$ is the effective pair $NN$ potential of the dibaryon-induced model, and $W_{3BF}(E)$ is the effective three-body force induced by one-sigma exchange between the dibaryon and $\alpha$-core. The form of $V_{N\alpha}^{\text{eff}}(E)$ and $W_{3BF}(E)$ is described below. The $NN$ pair potential $V_{N\alpha}^{\text{eff}}(E)$ acting in the three-body system is obtained from the pure two-nucleon effective dibaryon-induced potential $V_{N\alpha}^{\text{eff}}(E)$ (see Eq. (24)), according to a general recipe for the transition from the two-particle system to the three-particle one, i.e., by replacing the two-body energy $E$ with the difference $E - q^2/2\tilde{m}$ in all expressions containing the energy dependence:

$$V_{N\alpha}^{\text{eff}}(E) = V_{N\alpha}^{\text{eff}}(E - q^2/(2\tilde{m}))\delta(q - q'),$$ (2)

where $q$ is the relative $\alpha - NN$ momentum and $\tilde{m} = 2m_Nm_\alpha/(2m_N + m_\alpha)$ is the reduced mass of the $\alpha$-particle and the nucleon pair.

As the $N\alpha$ potential, we employed the quasipotential with even–odd splitting and with the forbidden state $|\Phi_{N\alpha}\rangle$, which was proposed and used in Ref. [48] (the so-called MS potential). This quasipotential includes the projector onto the forbidden $0S$ state with a large constant $\mu$ and can be written in the following form:

$$V_{N\alpha} = V_{C} + V_{\lambda}(l) + \mu|\Phi_{N\alpha}\rangle\langle\Phi_{N\alpha}|,$$ (3)

$$V_{C} = V_{1}^{C}(r) + P_{v}V_{2}^{C}(r),$$ (4)

$$V_{\lambda} = V_{1}^{L}(r) + P_{v}V_{2}^{L}(r).$$ (5)

The total angular momentum is $J = L + S$.

To find the energies and eigenfunctions for the $A = 6$ nuclei within the cluster $\alpha + 2N$ model, we used the variational formalism on the non-orthogonal multiscale Gaussian basis which was described in detail in a series of studies of the structure of the $A = 6$ nuclei [43–45,48]. Here we give only the form of the basis functions used in this work and the corresponding notation for the quantum numbers.

The Schrödinger equation for the wave function of the $\alpha + 2N$ system with the effective energy-dependent Hamiltonian $H_{\text{tot}}^{\text{eff}}(E)$ was solved by the variational method using a Gaussian basis. To solve this equation with the explicit energy dependence of the effective Hamiltonian (1), we used an iterative procedure with respect to the total energy $E$:

$$H_{\text{tot}}^{\text{eff}}(E^{(n-1)})\Psi^{(n)} = E^{(n)}\Psi^{(n)}.$$

Such iterations can be shown to converge if the energy derivative of the effective interaction is negative. For bound states, i.e., for $E < 0$, this condition is always valid. In our calculations, 5–10 iterations provided usually the accuracy of 5 decimal digits for the binding energy.

The total wave function $\Psi^{JM_j}(r, \rho)$ of the three-body system with the total angular momentum $J$ and its projection $M_j$ is expanded in a series of the basis functions $\Phi^{(l)}_{\gamma}$:

$$\Psi^{JM_j}(r, \rho) = \sum_{\gamma} \sum_{n} \sum_{l} C^{(l)}_{\gamma n} \Phi^{(l)}_{\gamma n}(r, \rho).$$ (7)

where $\gamma = \{\lambda, l, L, S\}$ and $C^{(l)}_{\gamma n}$ are the unknown coefficients or linear variational parameters. The six-dimensional basis

| Table 1 Parameter values for the MS $N\alpha$ interaction potential (from Ref. [48]) |
|--------------------------|---------------|--------------------------|
| Even waves               | Odd waves     |
| $g$(MeV)             | $\kappa$(fm$^{-1}$) | $g$(MeV)             | $\kappa$(fm$^{-1}$) |
| $V_{c}$             | -65.58        | 0.6203          | -46.303       | 0.43216        |
| $V_{d}$             | -12.169       | 0.8032          | -15.931       | 0.62816        |

where $P_\alpha$ is the Majorana operator which realizes the space reflection $r \rightarrow -r$, and the central and spin–orbit parts of the potentials $V_1$ and $V_2$ are determined via the potentials $V_{ev}$ and $V_{od}$ describing even and odd partial waves: $V_{1,2} = \frac{1}{2} [V_{ev} \pm V_{od}]$. The potentials $V_{ev}$ and $V_{od}$ having the Gaussian form

$$V(r) = g \exp(-\kappa r^2)$$ (6)

with the parameters given in Table 1 reproduce quite well the $S$-, $P$-, and $D$-wave phase shifts in $N-\alpha$ scattering at energies from 0 to 20 MeV.
functions $\Phi^{(i)}_{\gamma n}$ are constructed from the Gaussian functions and the corresponding spin–angular factors:

$$\Phi^{(i)}_{\gamma n}(r, \rho) = N^n r^\lambda \rho^\mu \exp\{-\alpha_{\gamma n} r^2 - \beta_{\gamma n} \rho^2\} \Omega^{J M_J}_{\gamma}(\hat{r}, \hat{\rho}).$$  

(8)

The spin–angular part $\Omega^{J M_J}_{\gamma}(\hat{r}, \hat{\rho})$ is taken in the following form:

$$\Omega^{J M_J}_{\gamma}(\hat{r}, \hat{\rho}) = \sum_{M_L M_S} \langle L M_L S M_S | J M_J \rangle \chi^{SM_S}(2, 3),$$

(9)

where $\chi^{LM_L}(\hat{r}, \hat{\rho})$ is the standard angular tensor and $\chi^{SM_S}(2, 3)$ is the spin function of the nucleon pair.

The nonlinear parameters of the basis functions $\alpha_{\gamma n}$ and $\beta_{\gamma n}$ are taken on the Chebyshev grid, which provides the completeness of the basis and fast convergence of the variational calculations [44, 56]:

$$\alpha_{\gamma n} = \alpha_{\gamma 0} \tan^{a_{\gamma}}_{\gamma}(\frac{\pi}{2n} - \frac{1}{2N_{\alpha_{\gamma}}}), \ n = 1, \ldots N_{\alpha_{\gamma}},$$

$$\beta_{\gamma n} = \beta_{\gamma 0} \tan^{b_{\gamma}}_{\gamma}(\frac{\pi}{2n} - \frac{1}{2N_{\beta_{\gamma}}}), \ n = 1, \ldots N_{\beta_{\gamma}},$$

(10)

where $N_{\alpha_{\gamma}}$ ($N_{\beta_{\gamma}}$) is the basis dimension for the variable $r$ ($\rho$) for the channel with the quantum numbers $\gamma$, and the parameters $a_{\gamma}$ ($b_{\gamma}$) are selected to be in the range 0.8–5 from the condition of optimality of the basis and stability of the calculation.

As was demonstrated earlier [44, 45, 48, 57], such a multidimensional Gaussian basis is very flexible and can reproduce even quite complicated correlations in few-body systems. An important advantage of the Gaussian basis in the few-body calculations is that it allows for a calculation of all the necessary matrix elements of the realistic Hamiltonian (including the spin–orbit and tensor forces) in the fully analytical form [43, 57]. Another advantage of such a basis is that the form of the Gaussian functions is the same in the coordinate and momentum representations. Thus the (normalized) basis functions $\Phi^{(i)}_{\gamma n}(\mathbf{p}, \mathbf{q})$ in the momentum representation have the same form as in Eq. (8):

$$\Phi^{(i)}_{\gamma n}(\mathbf{p}, \mathbf{q}) = N^n r^\lambda q^\mu \exp(-\tilde{\alpha}_{\gamma n} p^2 - \tilde{\beta}_{\gamma n} q^2) \Omega^{J M_J}_{\gamma}(\hat{p}, \hat{q})$$

(11)

where

$$\tilde{\alpha}_{\gamma} = \frac{1}{4\alpha_{\gamma}}, \quad \tilde{\beta}_{\gamma} = \frac{1}{4\beta_{\gamma}}.$$  

(12)

This feature of the Gaussian basis is especially significant when using the dibaryon-induced force model, since the effective pair $NN$ potentials and three-body forces explicitly depend on the momentum of the third particle.

3 Dibaryon model for $NN$ and $3N$ forces

3.1 Description of the basic $NN$ channels in $A = 6$ nuclei

The dibaryon model proposed initially in Ref. [33] and developed further in Refs. [35, 36] (where it was called “the dressed-bag model”) suggests the following picture of the interaction between nucleons. At a relatively long distance ($r_{NN} > 1$ fm) the nucleons interact by the conventional pion exchange. However, when the nucleons come closer to each other (to a distance $r_{NN} \lesssim 1$ fm), a compound state is formed: two nucleons fuse into a dibaryon state which is a six-quark bag dressed by the field of the light scalar $\sigma$-mesons. As a result of multiple transitions of two nucleons to the state of a dressed six-quark bag and vice versa, an effective $NN$ interaction arises, which gives the main attraction between the nucleons at intermediate distances.

To describe such an interaction mechanism, it is natural to use a two-channel formalism which assumes that a system of two nucleons can be in two different states (channels): an external $NN$ channel and an internal six-quark (dibaryon) channel. The full wave function of such a system consists of two components belonging to two different Hilbert spaces. The external component of the wave function depends on the relative coordinate (or momentum) of two nucleons and their spins and isospins, while the internal one depends on the quark and gluon (or string) variables. Two independent Hamiltonians, i.e., the external one $h^{ex}$ and the internal one $h^{in}$, are defined in these two spaces, respectively.

The external $NN$ Hamiltonian includes the kinetic energy $t$ and the peripheral part of the one-pion- and two-pion-exchange (OPE and TPE, respectively) interactions as well as the Coulomb interaction (for two protons):

$$h^{ex} = t + \{v^{OPE} + v^{TPE}\} + v^{Coul}.$$  

Excluding the internal dibaryon channel from the two-channel Schrödinger equation, one obtains the effective energy-dependent $NN$ Hamiltonian, which includes the resolvent of the internal channel $g^{in}(E) = (E - h^{in})^{-1}$ and the operators for transitions from the external channel to the internal one, $h^{in,ex}$, and backwards, $h^{ex, in} = (h^{in,ex})^\dagger$.

In the simplest version of the model [36], a single-pole approximation for the dibaryon (internal) resolvent $g^{in}$ was used:

$$g^{in}(E) = \sum_{\zeta} \int \frac{\langle \zeta, \mathbf{k} | \zeta, \mathbf{k} ]d^3k}{E - E_{\zeta}(k)}.$$  

(13)

Here $|\zeta\rangle$ is the six-quark part of the wave function for the dressed bag with the definite quantum numbers of the orbital angular momentum $\lambda$, spin $S$ and total angular momentum $J = \lambda + S$. For simplicity, we will denote this set of quantum numbers by the one symbol $\zeta = (\lambda, S, J)$. Note that, since
in this version of the model only the symmetric six-quark state \([\sigma^6]\) was included in the internal channel, the orbital angular momentum of the dressed dibaryon \(\lambda\) is equal to that of the \(\sigma\)-meson. The state \(|k\rangle\) represents the free wave of the \(\sigma\)-meson propagation. The total energy \(E_\zeta(k)\) of such a dressed state is

\[
E_\zeta(k) = m_D + \epsilon_\sigma(k),
\]

where

\[
\epsilon_\sigma(k) = k^2/2m_D + \omega_\sigma(k) \simeq m_\sigma + k^2/2\tilde{m}_\sigma,
\]

\[
\tilde{m}_\sigma = m_\sigma m_D/(m_\sigma + m_D), \quad \omega_\sigma(k) = \sqrt{m_\sigma^2 + k^2} \text{ is the relativistic energy of the } \sigma\text{-meson, } m_\sigma \text{ and } m_D \text{ are the masses of the } \sigma\text{-meson and the } 6q \text{ bag, respectively.}
\]

The effective \(NN\) interaction \(w(E)\) resulting from the coupling of the external \(NN\) channel to the intermediate dibaryon state is illustrated by the graph in Fig. 2. To derive the effective interaction \(w(E)\) for the \(NN\) channel in this approximation, one does not need to know the full internal Hamiltonian \(h^{\text{in}}\) of the dressed dibaryon, as well as the full transition operator \(h^{\text{ex, in}}\). One needs to know only how the transition operator \(h^{\text{ex, in}}\) acts on those dibaryon states \(|\zeta, k\rangle\), which are included into the resolvent (13). The calculation of this quantity within the microscopical quark–meson model results in a sum of the factorized terms [36]:

\[
h^{\text{ex, in}}|\zeta J^M, k\rangle = \sum_\lambda |\varphi^{J^M}_\lambda\rangle B^{J}_\lambda(k),
\]

where \(|\varphi^{J^M}_\lambda\rangle \in \mathcal{H}^{\text{ex}}\) is the \(NN\) transition form factor and \(B^{J}_\lambda(k)\) is the vertex function dependent on the \(\sigma\)-meson momentum. Here and below, where possible, to shorten the notation, we omit the index indicating the spin of the dibaryon. The quantum numbers of the dibaryon \((\lambda, S, J)\) are naturally equal to the quantum numbers of the \(NN\) system from which the dibaryon is formed.

Thus, the effective interaction in the \(NN\) channel \(w(E) = h^{\text{ex, in}} g^{\text{in}}(E) h^{\text{in, ex}}\) can be written as a sum of separable terms in each partial wave:

\[
w(E) = \sum_{J', \lambda, \lambda'} w^{J'}_{\lambda, \lambda'}(r, r', E),
\]

with

\[
w^{J'}_{\lambda, \lambda'}(r, r', E) = \sum_M \varphi^{J^M}_{\lambda}(r') \lambda^{J'}_{\lambda, \lambda'}(E) \varphi^{J^M\dagger}_{\lambda}(r').
\]

The energy-dependent coupling constants \(\lambda^{J'}_{\lambda, \lambda'}(E)\) appearing in Eq. (18) are directly calculated from the loop diagram shown in Fig. 2. They are expressed in terms of the loop integral of the product of two transition vertices \(B\) and the convolution of two propagators for the meson and six-quark bag with respect to the momentum \(k\):

\[
\lambda^{J'}_{\lambda, \lambda'}(E) = \int dk \frac{B^{J'}_{\lambda}(k) B^{J^M\dagger}_{\lambda}(k)}{E - \epsilon_\zeta(k)}.
\]

The vertex form factors \(B^{J'}_{\lambda}(k)\) and the transition form factors \(|\varphi^{J^M}_\lambda\rangle\) have been calculated using the microscopic quark–meson model [35, 36]. Within this microscopic model, the transition form factors \(|\varphi^{J^M}_\lambda\rangle\) are equal to \(|2\lambda\rangle\) and \(|2d\rangle\) harmonic oscillator wave functions with a radius \(r_0 = \sqrt{\frac{3}{2}} b\), where \(b = 0.5 \text{ fm}\). The explicit form of the vertex functions \(B^{J'}_{\lambda}(k)\) is not required to calculate the effective \(NN\) potential. However, these functions are necessary for calculating the three-body force due to two-sigma exchange (see below).

From the microscopic quark model they can be derived in Gaussian form [36]:

\[
B^{J'}_{\lambda}(k) = B_0^{J\lambda} e^{-b^2 k^2} / \sqrt{2 \omega_\sigma(k)},
\]

where \(k\) is the meson momentum and

\[
b^2 = \frac{5}{24} b_0^2, \quad b_0 = 0.5 \text{ fm}.
\]

The vertex constants \(B_0^{J\lambda}\) in Eq. (20) must satisfy Eq. (19), i.e.,

\[
\frac{1}{(2\pi)^3} \int dk \frac{B_0^{J\lambda} B_0^{J^{\prime}\dagger} e^{-2 b^2 k^2}}{(E - m_\sigma - \epsilon_\sigma(k)) \cdot 2 \omega_\sigma(k)} = \lambda^{J'}_{\lambda, \lambda'}(E),
\]

where \(\lambda^{J'}_{\lambda, \lambda'}(E)\) are the coupling constants found from the \(NN\) phase shifts fitted within the model.

Thus, the dibaryon concept leads to an effective energy-dependent \(NN\) Hamiltonian in the external channel:

\[
\mathcal{H}^{\text{eff}} = t + v^{\text{eff}}_{NN}(E),
\]

where

\[
v^{\text{eff}}_{NN}(E) = w(E) + V_{\text{OPE}} + V_{\text{TPE}} + \lambda_{\text{orth}} \Gamma.
\]

The effective \(NN\) interaction \(v^{\text{eff}}_{NN}(E)\) includes the peripheral OPE potential \(V_{\text{OPE}}\) with a soft dipole cut-off with parameter \(\Lambda = 700 \text{ MeV}\) and a peripheral \(2\pi\)-exchange contribution \(V_{\text{TPE}}\) which has been imitated in [36] in the form

\[
V_{\text{TPE}} = V_{\text{TPE}}^0 \left(\frac{\beta r}{r_0}\right)^2 \exp(-\beta r^2),
\]
with the parameter values given in Table 2. From this table one can conclude that the strength of this potential is quite small (ca. 4–8 MeV). However, this small contribution in the region \( r \sim 2 \text{ fm} \) is needed to reproduce exactly the effective-range parameters and the low-energy phase shifts.

The microscopic quark model also implies an orthogonality condition, which is provided by the term \( \lambda_{\text{orth}} \Gamma \) in the effective \( NN \) interaction (24):

\[
\lambda_{\text{orth}} \Gamma = \lambda_{\text{orth}} |0s\rangle \langle 0s|,
\]

with \( \lambda_{\text{orth}} \gtrsim 10^6 \text{ MeV} \) and the same value of the oscillator radius \( r_0 = \sqrt{\frac{3}{2}}b \) as in the transition form factors for the \( s \)-states.

Finally, we give the explicit formulas for the effective \( NN \) interaction \( w(E) \) in the lowest partial waves induced by the intermediate dressed dibaryon, which are used in the present calculations of the \( A = 6 \) nuclei.

- For the singlet \( ^1S_0 \) channel:

\[
w(E) = \lambda_{000}^0(E) |0s\rangle \langle 0s|,
\]

(27)

- for the triplet coupled \( ^3S_1 - ^3D_1 \) channels:

\[
w(E) = \left( \frac{\lambda_{10}^1(E) |2s\rangle \langle 2s| \lambda_{10}^0(E) |2s\rangle \langle 2d|}{\lambda_{20}^1(E) |2d\rangle \langle 2d| \lambda_{12}^0(E) |2d\rangle \langle 2d|} \right),
\]

(28)

where \( |0s\rangle, |2s\rangle \) and \( |2d\rangle \) denote the harmonic oscillator functions \( |N\lambda\rangle \) with the number of quanta \( N \) and orbital angular momentum \( \lambda \).

In Ref. [36], to reproduce the energy dependence of the constants \( \lambda_{\text{SAID}} \) derived from the above microscopic calculation (19), a Padé approximant [1,1] with two parameters \( E_0 \) and \( a \) was used:

\[
\lambda(E) = \lambda(0) + \frac{E_0 + aE}{E_0 - E}.
\]

(29)

The above dibaryon-induced model for the \( NN \) force (see Eqs. (24) and (25)–(29)) was employed in Ref. [36] to describe the lowest \( NN \) partial phase shifts until \( E_{\text{lab}} = 1 \text{ GeV} \) by adjusting the parameters \( r_0, \lambda(0), E_0, a \) and also the “\( 2\pi \)-exchange” parameters \( V_{\text{TPE}}^0 \) and \( \beta \). The parameter values found from that fit are given in Table 2.

The quality of predictions for the \( NN \) phase shifts in the singlet \( ^1S_0 \) and triplet \( ^3S_1 - ^3D_1 \) partial channels is shown in Fig. 3. As is clearly seen from the figure, the above model gives a very good description for the lowest partial phase shifts in the energy region from zero up to 1 GeV [36]. Agreement with experimental data for the \( NN \) phase shifts and also for the static deuteron properties found within this force model, in general, is better and spans a much broader energy interval than for the so-called realistic \( NN \) potentials such as the Nijmegen and Argonne models. The weight of the internal (dibaryon) component in the deuteron was obtained in Ref. [36] to be \( P_{\text{fm}} = 3.66\% \).

It is this version of the dibaryon model (with the parameters from Table 2) that is used in the present work to describe
the \( A = 6 \) nuclei within the framework of the cluster \( \alpha + 2N \) model.

Having obtained the solution \( \psi^{\text{ex}} \) for the Schrödinger equation with the effective Hamiltonian (23), one can recover the excluded internal component \( \psi^{\text{in}} \) of the total wave function unambiguously:

\[
\psi^{\text{in}} = g^{\text{in}}(E)\mu^{\text{in,ex}}\psi^{\text{ex}}.
\]

Here, we give only the explicit expression for the weight of internal dibaryon component \( P_{\text{in}} \) in the bound-state (in our case, the deuteron) wave function. The norm of the internal component (with the given \( J \)) is

\[
\|\psi^{\text{in}}_J\|^2 = \|\alpha^{JM}\|^2\sum_{\lambda\lambda'}\langle \psi^{\text{ex}}_J | \psi^{\text{ex}}_{J'} \rangle \lambda\lambda' \times \int \frac{B^{J}_{\lambda\lambda'}(k)B^{J*}_{\lambda'\lambda'}(k)}{(E - E_{\xi}(k))^2}d^3k.
\]

(31)

It is easy to see from the comparison of Eqs. (19) and (31) that the integral \( I^{JM}_{\lambda\lambda'} \), in Eq. (31) is equal to the energy derivative (with an opposite sign) of the coupling constant \( \lambda^{JM}_{\lambda\lambda'}(E) \):

\[
I^{JM}_{\lambda\lambda'} = -\frac{d\lambda^{JM}_{\lambda\lambda'}(E)}{dE},
\]

and therefore

\[
\|\psi^{\text{in}}_J\|^2 \sim \frac{d\lambda^{JM}_{\lambda\lambda'}(E)}{dE},
\]

i.e., the weight of the internal dibaryon state is proportional to the energy derivative (with an opposite sign) of the coupling constant of the effective \( NN \) interaction. In other words, the stronger the energy dependence of the interaction in the \( NN \) channel, the larger the weight of the channel corresponding to the non-nucleonic degrees of freedom.

Assuming that the total bound-state wave function \( \psi \) must be normalized to unity, and the external (nucleonic) part of the wave function \( \psi^{\text{ex}} \) found from the effective Schrödinger equation also has the standard normalization \( \|\psi^{\text{ex}}\| = 1 \), one obtains the weight of the internal dibaryon component as follows:

\[
P_{\text{in}} = \frac{\|\psi^{\text{in}}\|^2}{1 + \|\psi^{\text{in}}\|^2}.
\]

(32)

3.2 Three-body force in \( 3N \) and \( \alpha + 2N \) systems within the dibaryon model

In a system of several nucleons, each pair of nucleons can form an intermediate dibaryonic state. Therefore, to describe such a system, it is necessary to use the multichannel formalism that includes one external (nucleon) channel and several internal (dibaryon) channels in accordance with the number of available nucleon pairs. In the internal channels, a new interaction between the dibaryon and other nucleons is possible.

3.2.1 Three-body force in \( 3N \) system

In the case of the three-nucleon system, one has a four-component space and thus one is dealing with a \((4 \times 4)\)-matrix Hamiltonian [42, 59]. Moreover, the dibaryon concept inevitably leads to the appearance of a new three-body force (3BF) which arises mainly due to the interaction of the third nucleon with the \( \sigma \)-meson field surrounding the dressed dibaryon in each internal channel (see Fig. 4).

In this respect, the 3BF in the dibaryon model which is the immediate consequence of the basic two-nucleon force is in sharp contrast to the conventional 3BF like that of the Fujita–Miyazawa type, where the 3BF operator is not related directly to the traditional meson-exchange two-nucleon force and, moreover, includes constants (e.g., the cut-off parameters) other than those used for the initial two-nucleon force. Therefore, in principle, one can add some 3BF contribution to the calculation with the conventional force model like RSC or AV18, but such a hybrid model should be considered as a fully phenomenological one.

The EFT approach where two- and three-nucleon forces are derived within the same paradigm is more consistent. It should also be noted that the role of three-particle forces turns out to be important in the halo–cluster EFT approach, e.g., in the description of the \( \alpha \) \( N \) + \( N \) + \( N \) system.

In Refs. [40–42], a dibaryon model for the \( 3N \) system was developed which included two types of three-nucleon forces caused by the \( \sigma \)-meson exchanges: one-meson exchange (OSE) between the dressed bag and the third nucleon (see Fig. 4(a)) and the exchange by two \( \sigma \)-mesons (TSE), where the third-nucleon propagator breaks the \( \sigma \)-loop of the two-body force (Fig. 4(b)).

These three-body forces can be included in the effective Hamiltonian for the external \( 3N \) channel as some integral operators in the momentum space with the factorized kernels:

\[
W^{3\text{BF}}_{ij}(\mathbf{p}_i, \mathbf{p}_j, \mathbf{q}_i; E) = \sum_{M, M', \lambda, \lambda'} \psi^{JM*}_{\lambda}(\mathbf{p}_i) \times W^{3\text{BF}}_{ij}(M, M', \lambda, \lambda') \psi^{JM*}_{\lambda'}(\mathbf{p}_j), \quad i = 1, 2, 3,
\]

(33)

where \( \mathbf{p}_i \) is the relative momentum of the nucleon pair \((ij)\) which generates the dibaryon, \( \mathbf{q}_i \) is the momentum of the relatively to this pair external (spectator) nucleon \( i \) and \( \psi^{JM}_{\lambda}(\mathbf{p}_i) \) is the transition form factor entering the effective \( NN \) interaction (18). Here, the conventional numbering of particles in the three-body system is used: \((ijk) = (123), (231), (312)\). Thus, the matrix elements for the 3BF include only the overlap functions, and therefore the contribution of the 3BF is
The graphs corresponding to two types of the three-body force induced by $\sigma$ exchange between the dibaryon and the third nucleon.

Fig. 4

proportional to the weight of the internal $6qN$ components in the total $3N$ wave function.

The operator for exchange by a scalar meson does not include any spin–isospin variables, therefore in the case of the one-sigma exchange, the kernel of the 3BF operator is simplified and takes the form

$$W_{3\text{BF}}^{J\sigma J'}(q_i, q_i'; E) = \delta_{J J'} \int \frac{d^3k}{2\pi^3} \frac{B_{J'}^{q_i}(k)}{E - E_i(k) - q_i^2/2m} \times V_{\text{OSE}}^{q_i}(q_i', q_i) \times \frac{B_{J}^{q_i}(k)}{E - E_i(k) - q_i^2/2m},$$

(34)

where

$$V_{\text{OSE}}^{q_i}(q_i', q_i) = \frac{-g_{\sigma NN}^2}{(q_i - q_i')^2 + m_{\sigma}^2}$$

(35)

is the standard scalar-meson-exchange potential. The integral over the $\sigma$-meson momentum $k$ in Eq. (34) can be shown to be reduced to a difference in the values for the constant $\lambda(E - q_i^2/2m)$, so that the vertex functions $B(k)$ can be excluded from the formulas for the OSE 3BF matrix elements. The details of the calculations for such matrix elements are given in the appendix of Ref. [42].

The operator of the TSE interaction includes explicitly the vertex functions $B(k)$ for the transitions $(NN \leftrightarrow 6q + \sigma)$, so that these functions cannot be excluded similarly to the case of the OSE interaction. In the calculations of the $3N$ nuclei [40–42], the vertex functions (20) with the parameter (21) were used. Here we do not give the formulas for the 3BF induced by TSE, since this contribution is not taken into account in the present calculations of the $A = 6$ nuclei.

Table 3 presents the results of calculations for the ground states of the $^3\text{H}$ and $^3\text{He}$ nuclei within the dibaryon-induced model for $NN$ and $3N$ forces (version designated as DBM(I) in Ref. [42]) in comparison with the results obtained with the Argonne $NN$ potential AV18 and the Urbana–Illinois three-body force UIX in Refs. [60,61].

The AV18 interaction includes the charge symmetry breaking (CSB) by providing the $nn$ force, which is fitted to an experimental $nn$ scattering length different from the $pp$ one. The electromagnetic part of the AV18 potential includes the one- and two-photon Coulomb terms, the Darwin–Foldy term, the vacuum polarization, the magnetic moment interactions, and a Coulomb term due to the neutron charge distribution. All these terms take into account the finite size of the nucleon charge distributions. The calculations in Ref. [61] have been done using the Faddeev equations for three identical fermions within the isospin formalism including total isospin states $T = 1/2$ and $3/2$. The small contribution of the $n$–$p$ mass difference from the ground-state energy was estimated perturbatively to be $-7 \text{ keV}$ for $^3\text{H}$ and $+7 \text{ keV}$ for $^3\text{He}$.

The results presented in Table 3 for the dibaryon model have been also obtained within the isospin formalism for three identical fermions with the use of the effective pair $NN$ potential (23)–(29) with the parameters from Table 2 and the OSE and TSE 3BF with the following parameters:

$$g_{\sigma NN} = 9.577, m_{\sigma} = 400 \text{ MeV}, m_D = 2240 \text{ MeV}.$$  (36)

The value of the $\sigma NN$ coupling constant in the $^3\text{H}$ calculations has been chosen to reproduce the exact binding energy of the $^3\text{He}$ nucleus.

The dibaryon model with the same parameters was used to calculate the $^3\text{He}$ nucleus in [42]. In the $^3\text{He}$ nucleus, treated within the framework of the dibaryon model, in addition to the Coulomb interaction between protons, a new Coulomb three-particle force appears due to the Coulomb interaction between the proton and the charged dibaryon. When calculating the Coulomb interactions in [42], the finite size of the charge distributions of the proton and dibaryon was taken into account. Thus, the calculations for $^3\text{He}$ have been carried out without any free parameters.

In Table 3 the following characteristics are given: the bound-state energy $E$, the weight $P_{\text{in}}$ of the internal dibaryon channel, the weight $P_D$ of the $D$-wave component in the total $3N$ wave function as well as the weight $P_{\Sigma'}$ of the mixed-symmetry $\Sigma'$ component (only for the $3N$ channel); the average individual contributions from the kinetic energy $T$, the
two-body interactions $V^{(2N)}$ plus the kinetic energy $T$, and the 3BF $V^{(3N)}$ due to OSE and TSE in the total Hamiltonian expectation. The contributions $V^{(2N)}$ and $V^{(3N)}$ for the AV18 + UIX model are taken from [60].

Note that the TSE contribution for the $^3$H and $^3$He nuclei is substantial: it is about 40% of the OSE contribution. However, our calculations showed that the TSE-induced interaction, when the coupling constant $g_{\sigma NN}$ is increased from 9.577 to 14.259, reproduces the basic properties of the ground states for the $^3$H and $^3$He nuclei just as well as the original model that takes into account both types of three-nucleon forces (OSE and TSE). Similarly to this, in our present calculations for the $A = 6$ nuclei we fit the $g_{\sigma NN}$ constant to account effectively for both OSE and TSE contributions.

### 3.2.2 Three-body force in the $\alpha + 2N$ system

In the $\alpha + 2N$ system, there are one pair of nucleons, therefore, within the framework of the dibaryon concept, the system must be described as two-channel one with one external and one internal channel. In the external channel, the system consists of two nucleons interacting with each other and with the $\alpha$-particle, and in the internal channel one has a dressed $6q$ bag (dibaryon) interacting with the $\alpha$-particle. This interaction between the dibaryon and the $\alpha$-particle in the internal channel, $V_{DA}^{\alpha}$, leads to a three-particle force in the external channel, acting along the Jacobi coordinate of the $\alpha$-particle $\rho$ relative to the center of mass for the nucleon pair.

In the present work, such a dibaryon-$\alpha$ potential in the internal channel, we use the folding potential obtained by averaging the OSE potential (35) between the $6q$ bag and the nucleon from the $\alpha$-cluster over the single-particle nucleon density of the $\alpha$-particle $\hat{\rho}(r_N)$ [62]:

$$\hat{\rho}(r_N) = 4N_\rho \exp\left(-\frac{(r_N/r_\alpha)^2}{2}\right), \quad \int \hat{\rho}(r_N)d^3r_N = 1,$$

where $r_N$ is the single-nucleon coordinate with respect to the center of mass of the $\alpha$-particle, $N_\rho$ is the normalization constant and $r_\alpha = 1.325$ fm. It is easy to show that averaging the Yukawa potential (35) with the density (37) results in a scalar-meson-exchange potential (in the internal channel) with the standard exponential cut-off:

$$V_{D\alpha}^{\text{OSE}}(\mathbf{q}, \mathbf{q}') = -\frac{g_{\sigma NN}^2}{8\sigma_{NN}} \exp\left(-\frac{(\mathbf{q} - \mathbf{q}')^2}{\Lambda^2}\right) \left(|\mathbf{q}|^2 + m_{\sigma}^2\right),$$

where the cut-off parameter $\Lambda = 2/r_\alpha$.

Thus, the three-body force in the $2N + \alpha$ system can be determined by Eq. (34) with a replacement of the $\sigma$-meson-exchange potential (35) by the cut-off potential (38):

$$W_{3\text{BF}}(\mathbf{q}, \mathbf{q}'; E) = \int dk \frac{B_{\lambda}(k)}{E - E_{\epsilon}(k) - q^2/2\bar{m}} \times \frac{B_{\lambda'}(k)}{E - E_{\epsilon}(k) - q'^2/2\bar{m}},$$

where $\bar{m} = m_Dm_{\alpha}/(m_D + m_\alpha)$, $m_\alpha$ is the mass of the $\alpha$-particle, and $m_D$ is the mass of the dibaryon.

As in the case of the three-nucleon system, the integral in Eq. (39) over the meson momentum $k$ reduces to the difference of the coupling constants:

$$\int \frac{B_{\lambda'}(k) B_{\lambda}(k)}{(E - E_{\epsilon}(k) - \frac{q^2}{2\bar{m}})(E - E_{\epsilon}(k) - \frac{q'^2}{2\bar{m}})} dk \equiv \Delta \lambda_{\lambda'}(q', q, E).$$

For the energy dependence $\lambda(E)$ taken according to Eq. (29), this difference is

$$\Delta \lambda(q', q, E) = \lambda(0)E_0(1 + a) \frac{1}{E - E_0 - \frac{q^2}{2\bar{m}}} - \frac{1}{E - E_0 - \frac{q'^2}{2\bar{m}}}.$$
Thus, the three-particle interaction in the $2N + \alpha$ system (in the external channel) due to the $\sigma$-exchange averaged over the $\alpha$-particle density in the internal channel takes the form

$$W_{3BF}^{ij\lambda}(q, q'; E) = \lambda \frac{1}{E - E_0 - q^2/2m} \times \left[ -g_{\sigma NN}^2 \exp\left(-\frac{(q - q')^2}{2\Lambda^2}\right) \frac{1}{E - E_0 - q^2/2m} \right].$$

(42)

In the present calculations for the $A = 6$ nuclei, we used just this form for the three-body force with the parameters $m_D$ and $m_\sigma$ from Eq. (36). We consider the coupling constant $g_{\sigma NN}$ as the only adjusted parameter. However, it turned out that the value $g_{\sigma NN} = 9.577$ adopted in the $^3$H calculations reproduces the $^6$Li binding energy very well. Therefore, we decided not to change this value.

4 Properties of $A = 6$ nuclei within the dibaryon-induced model for $NN$ and $NN\alpha$ forces

We performed three series of complete variational calculations within the $\alpha + 2N$ cluster model using the same $\alpha N$ potentials which include the odd–even splitting and the Pauli-projection operator (see Eqs. (3)–(6) and Table 1) but with different $NN$-interaction models: (i) the conventional $NN$ potential from the RSC model; (ii) the dibaryon-induced $NN$ potential (see Eqs. (2), (24), (25)–(29) and Table 2 for the parameter values); (iii) the same dibaryon-induced $NN$ potential as in point (ii) but with added OSE three-body (42) with the constant $g_{\sigma NN}$ chosen to reproduce the binding energy of $^6$Li.

We should emphasize that using the above fixed parameter value $g_{\sigma NN}$, the calculation for the $^6$He nucleus was done without any new fit parameters.

Table 4 presents the basis configurations (the dimension and quantum numbers) used in all versions of the calculations for the ground states of the $^6$He and $^6$Li nuclei.

| Nuclei | $J^\pi$ | $\gamma = \lambda L S$ | $J = \lambda + S$ | $N_{\alpha\gamma} \times N_{\beta\gamma}$ |
|-------|---------|----------------|----------------|----------------------------------|
| $^6$He | $0^+\, 1$ | 0000 | 0 | 12×10 |
| | 1111 | 1 | 10×10 |
| | 2200 | 2 | 10×10 |
| | 3311 | 3 | 10×10 |
| $^6$Li | $1^+\, 0$ | 0001 | 1 | 15×7 |
| | 2021 | 1 | 15×7 |
| | 1110 | 1 | 9×7 |
| | 2201 | 1,2,3 | 9×7 |
| | 2211 | 1,2,3 | 9×7 |
| | 2221 | 1,2,3 | 9×7 |
| | 0221 | 1 | 9×7 |

1. For the conventional $NN$ potential (RSC) the ground states are seriously underbound both for the $^6$He and $^6$Li nuclei. This is well known from Refs. [44,45,48] and is also in agreement with Ref. [63] where authors tested a few different $NN$ potentials in similar $\alpha + 2N$ calculations.

2. Replacement of the conventional pairwise $NN$ interaction with the dibaryon-induced interaction (keeping the $\alpha + N$ interaction to be the same) results in even less binding for both $^6$He and $^6$Li. This decrease of the binding energies is due to the strong enhancement of the average kinetic energy in the dibaryon model.

3. Inclusion of the three-body force due to $\sigma$-meson exchange between the dibaryon and the nucleons in the $\alpha$-core results in an increase of the binding energies and a significant improvement in agreement of the calculated binding energies and their experimental values.

4. The weights of the dibaryon components in the $n - n$ and $n - p$ subsystems are also increased when the three-particle force is included in the calculation.

5. When replacing the traditional model for the $NN$ force with the dibaryon-induced model, the average kinetic energy in both nuclei is increased remarkably.

6. General agreement with experimental data for the rms radii of the charge and matter distributions calculated within the DM+3BF model (see Table 6) gets also better than that for the case without 3BF.

7. However, the results for the rms radii of the charge and matter distributions show, unfortunately, some opposite trends, viz., the results within the dibaryon-induced model for $^6$Li get worse while those for $^6$He get better as compared with the RSC results. The worsening of the results for the rms radii for $^6$Li can be explained by ignoring some higher spin–angular components in the varia-
### Table 5 Properties of the ground states of the $^6$He and $^6$Li nuclei calculated with the RSC $NN$ potential and with the dibaryon-induced $NN$ potential without the three-body force (DM) and with the three-body force (DM+3BF)

| Nuclei | Model | $E$, MeV | $T$ | $V_{NN}$ | $W_{3BF}$ | $P_{\text{in},\%}$ | Configuration | $\gamma = \{\lambda L S\}$ | $P_{\gamma},\%$ |
|--------|-------|---------|-----|---------|---------|----------------|---------------|----------------|----------------|
| $^6$He | RSC   | $-0.2761$ | 26.084 | $-6.866$ | 0 | 0 | 0000 | 88.091 |
|        | DM    | $-0.0055$ | 33.322 | $-17.238$ | 0 | 2.32 | 0000 | 88.954 |
|        | DM+3BF | $-0.723$ | 43.235 | $-23.066$ | $-1.053$ | 2.99 | 0000 | 88.873 |
| $^6$Li | RSC   | $-3.225$ | 41.306 | $-28.225$ | 0 | 0 | 0001 | 86.815 |
|        | DM    | $-2.691$ | 53.420 | $-42.671$ | 0 | 4.34 | 0001 | 91.353 |
|        | DM+3BF | $-3.678$ | 63.896 | $-49.138$ | $-1.389$ | 4.90 | 0001 | 90.58 |
| $^6$He | RSC   | $4.555$ | 3.36 | 3.99 | 2.56 | 2.43 | 5.49 | 0.830 |
|        | DM    | 4.397 | 4.45 | 2.74 | 2.62 | 4.96 | 0.839 |
|        | DM+3BF | 3.323 | 3.70 | 2.46 | 2.32 | 4.59 | 0.835 |
| $^6$Li | RSC   | 4.555 | 3.877 | 2.085 | 2.55 |
|        | DM    | 4.837 | 4.468 | 2.212 | 2.80 |
|        | DM+3BF | 4.337 | 3.647 | 2.038 | 2.44 |

### Table 6 Static observables of $^6$He and $^6$Li nuclei

| Nuclei | Model | $\sqrt{r^2}$, fm | $\sqrt{\rho^2}$, fm | $\sqrt{r^2_c}$, fm | $\sqrt{r^2_m}$, fm | $Q$, mb | $\mu$, a.u. |
|--------|-------|-----------------|-----------------|-----------------|-----------------|-----|-----------|
| $^6$Li | RSC   | 3.36            | 3.99            | 2.56            | 2.43            | 5.49 | 0.830     |
|        | DM    | 3.497           | 4.45            | 2.74            | 2.62            | 4.96 | 0.839     |
|        | DM+3BF | 3.323          | 3.70            | 2.46            | 2.32            | 4.59 | 0.835     |
| Experiment | 5.289(39) | 2.42(4) | $-0.818(17)$ | 0.8220473(6) |
| $^6$He | RSC   | 4.555           | 3.877           | 2.085           | 2.55            |
|        | DM    | 4.837           | 4.468           | 2.212           | 2.80            |
|        | DM+3BF | 4.337          | 3.647           | 2.038           | 2.44            |
| Experiment | 2.054(14) | 2.48(3) |
tional basis used in our calculations. The impact of these higher components in the $^6\text{Li}$ wave functions should be much larger than in the $^9\text{He}$ case.

From the above results one can conclude that the binding energy of the two-nucleon cluster in the field of the $\alpha$-core becomes higher, i.e., the two-nucleon coupling in the field of the core gets stronger. This result is especially interesting for the two-neutron system: for the traditional $NN$ force the $nn$ pair is unbound in case of an isolated $nn$ system and is very weakly bound in the $\alpha + 2n$ system. In contrast to this, when replacing the traditional $NN$ force with the dibaryon-induced one plus three-body OSE interaction, the effective $nn$ coupling gets much stronger and resembles the Cooper pairing in solids. (It is worthwhile to remind the reader that the Cooper pairing between two electrons in solids appears due to the interaction of the electron pair with the lattice, i.e., with the third body, quite similarly to our case.) It is informative to note that the traditional $3N$-force mechanism of the Fujita–Miyazawa type does not work in the $\alpha$ distance in $^6\text{He}$. So, this novel dibaryon-induced mechanism in the contact three-body force also makes a very low contribution system due to the scalar–isoscalar nature of the $NN$ interaction. It is well known that the $S$-wave contribution—dashed curves, $D$-wave contribution—dot-dashed curves, and their sum—solid curves. Experimental data (squares) are taken from Ref. [68]

$$\hat{\rho}(\mathbf{p}) = \sum_{lL} \int |\Psi_{lL}(\mathbf{p}, \mathbf{q})|^2 \mathbf{q} \mathbf{d}\mathbf{q} \frac{d\mathbf{p}}{4\pi}. \quad (43)$$

First, we compare the momentum distributions of the $np$ relative motion in the deuteron calculated with the RSC $NN$ potential and the dibaryon-induced model. In the case of the deuteron, the corresponding distributions $\rho_\lambda$ are simply the squares of the $S$- and $D$-wave components of the wave function. In Fig. 5 these distributions are shown in comparison with the experimental data [68].

As can be seen from the figure, the dibaryon-induced model leads to good agreement with the experiment for the momentum distribution up to $p \sim 5 \text{ fm}^{-1}$, in contrast to the traditional RSC model for the $NN$ interaction. It is well known that the $S$-wave contribution to the $np$ momentum distribution in the deuteron has a dip at $p \sim 2 \text{ fm}^{-1}$ which is filled up by the $D$-wave component of the deuteron. When replacing the RSC potential with the dibaryon-induced inter-
action, the dip in the $S$-wave contribution shifts towards lower momenta and the values of the $S$-wave distribution at $p > 2 \text{ fm}^{-1}$ increases by almost an order of magnitude.

A similar picture is observed for the momentum distributions of the valence nucleons in $^{6}\text{Li}$ and $^{6}\text{He}$ nuclei shown in Fig. 6. It should be stressed that all results in this subsection, referred to as the dibaryon model results and denoted in Figs. 6, 7, 8 as DM, have been obtained within the dibaryon-induced model with taking the dibaryon-induced 3BF into account.

It can be seen from Fig. 6 that in the broad momentum range $p = 1.7 - 9 \text{ fm}^{-1}$ the momentum distributions for the dibaryon model are several times higher than those for the traditional RSC potential. It means that the replacement of the traditional $NN$ potential with the dibaryon-induced interaction leads to a strong increase of the short-range $NN$ correlations.

To estimate the magnitude of the high-momentum components in the momentum distributions, we calculated the probability for the relative momentum of two nucleons ($p \geq p_0$) in the components of the wave functions for the $^2\text{H}$, $^6\text{Li}$, and $^6\text{He}$ nuclei found with two interaction models, i.e., the RSC and dibaryon-induced ones:

$$\hat{W}_\lambda(p_0) = \int_{p_0}^{\infty} \hat{\rho}_\lambda(p) p^2 dp.$$  \hspace{1cm} (44)

These probabilities determine the magnitude of the short-range $NN$ correlations in the respective nuclei.

In Fig. 7 we present these probabilities for the $S$- and $D$-wave components of the wave functions of the $^6\text{Li}$ nucleus and the deuteron found with the dibaryon-induced interaction. It is seen from the figure that at $p_0 \geq 1.5 \text{ fm}^{-1}$ (300 MeV/c) the weight of the high-momentum $np$ components in $^6\text{Li}$ is $\sim 40\%$ greater than that in deuteron both for $S$- and $D$-waves.

The probabilities of the high-momentum $NN$ components at $p_0 = 1.5 \text{ fm}^{-1}$ ($W_\lambda(1.5)$) in the $^2\text{H}$, $^6\text{Li}$, and $^6\text{He}$ nuclei are presented in Table 7. The total weight ($w_s + w_d$) of the high-momentum components with $p > 300 \text{ MeV/c}$ in the free deuteron for the dibaryon-induced model is 6.5\% vs 4.3\% for the conventional (RSC) model.

An especially interesting comparison can be done for the weights $W_\lambda$ of the $S$- and $D$-wave components in the $^6\text{Li}$, $^6\text{He}$, and $^2\text{H}$ nuclei calculated within the conventional and
Fig. 8 Probability of the high-momentum components with \((p \geq p_0)\) in the nucleon momentum distribution for \(^6\text{Li}\) and \(^{6}\text{He}\) calculated with the RSC potential and the dibaryon model

dibaryon-induced force models (see Fig. 8 and Table 7). We see that \(W_{d,RSC}^{RSC} > W_{d,DM}^{DM}\) while \(W_{s,DM}^{DM} < W_{s,RSC}^{RSC}\) at \(p_0 = 300\text{ MeV/c}\). In other words, for the conventional force model the \(D\)-wave component gives the main contribution to the nucleon’s high-momentum distribution both in the deuteron and \(^6\text{Li}\) (this is a well-known fact [69]), while for the dibaryon-induced model the situation is opposite: the \(S\)-wave component gives the main contribution to the nucleon momentum distribution for both nuclei at \(p_0 \gtrsim 300\text{ MeV/c}\).

The enhancement of the short-range (\(S\)-wave) neutron–neutron correlations in the case of the dibaryon-induced interaction vs. the conventional (RSC) \(nn\) interaction can be most clearly seen in the \(^6\text{He}\) nucleus (see Fig. 8, bottom panel, and Table 7). Here the probability of the high-momentum \(nn\) components at \(p > 300\text{ MeV/c}\) for the dibaryon-induced model is seven times larger than that for the traditional RSC model. Such a strong enhancement of the short-range correlations is one of the most noticeable features of our approach to the \(NN\) interaction from the conventional ones.

4.3 Substantiation of the cluster model

Before passing to a general discussion of the results obtained it is worthwhile to consider some important arguments in favor of the validity of the three-cluster \(\alpha + 2N\) model for the description of the \(A = 6\) nuclei.

First of all, let us consider the geometrical forms and the matter distribution in the \(^6\text{Li}\) and \(^{6}\text{He}\) nuclei found in our calculations (see Figs. 9 and 10).

Figures 9 and 10 present the isodensity levels for the probability density

\[
W_\gamma (r, \rho) = \frac{|\Phi_\gamma (r, \rho)|^2 r^2 \rho^2}{P_\gamma},
\]

(45)
calculated for the main $S$-wave components of the ground states of $^6\text{Li}$ and $^6\text{He}$ within the dibaryon-induced model for $NN$ and three-body $\alpha NN$ forces. In Eq. (45) $P_\gamma$ is the weight of the $\gamma$-component, so the probability density is normalized to unity: $\int W(r, \rho)drd\rho = 1$. Therefore, the density value $W_C$ at maximum, marked by numerals on the chart of isolines, allows us to estimate the sharpness of the peaks.

In Refs. [46–48] the geometric shapes of the $A = 6$ nuclei were predicted and investigated in detail for the case of the traditional $NN$ force. It has been shown that there are three basic geometric structures, each of which can be described by the factorized wave function: the cluster $\alpha-d$ configuration (“dumb-bell”), the “cigar” shape with an $\alpha$-particle on the straight line between two nucleons, and the “helicopter” configuration with the double rotation. These predictions were completely confirmed both in the experiments [70, 71] and in a series of calculations [50] using $K$-harmonic method. In particular, it was found that the $S$-components ($\lambda = 0, l = 0$) always contain two structures: cluster and cigar-like with the peak height ratio $1.7 \div 2$.

It can be seen from Figs. 9 and 10 that replacing the traditional $NN$ potential by the dibaryon-induced one does not substantially change the geometrical shape of the nuclei. The figures also clearly show that the three particles ($N, N, \alpha$) are localized in the $^6\text{Li}$ and $^6\text{He}$ nuclei rather far from each other as compared with the own dibaryon size ($r_D \sim 0.6$–0.9 fm), and the distance between the $2N$ center of mass and the $\alpha$-particle in the cluster configuration is about 3 fm. This distance is much larger than the range of the OSE three-body force ($r_{\text{OSE}} \approx 0.56$ fm for $m_\sigma \approx 350$ MeV) and much larger than the range of the TSE interaction.

Therefore the three-body force contribution is very sensitive to the $\sigma NN$ coupling constant. This leads to a highly nonlinear effect for the dependence of the three-body binding energy on the dibaryon contribution. As a result of this nonlinear effect, the peak density in two basic configurations of $^6\text{Li}$ is greater than that of $^6\text{He}$ (see Figs. 9 and 10).

From the density profiles presented in Figs. 9 and 10, one can see that the average distances between three particles in the $\alpha-N-N$ system are much larger than the ranges of the respective interactions $V_{NN}$ and $V_{\alpha N}$, so that the overlap between the outer and $\alpha$-particle nucleons is very small (4–5%) and the possible distortion of the $\alpha$-cluster in these nuclei should be minimal and can be ignored. This conclusion can serve as an additional good confirmation for the applicability of the three-cluster model for the $A = 6$ nuclei used in this paper.

5 Dibaryon admixture at different nuclear densities

The weight of the dibaryon components in the nuclear wave functions is of great interest because many properties of nuclear matter can depend crucially upon the dibaryon admixture. The most natural way to change the dibaryon admixture is to change the binding energy of the $NN$ pair in the given nucleus. It can be done easily by increasing the $g_{\sigma NN}$ coupling constant, which controls the contribution of the three-body force due to OSE.

Within the framework of the cluster model considered in this paper, we studied the dependence of the dibaryon admixture on the average distance between the nucleons forming the dibaryon, i.e., on the density of the nuclear medium. To do this, we carried out a series of calculations, increasing the magnitude of the three-particle force between the pair of nucleons and the $\alpha$-core, which is determined by the coupling constant $g_{\sigma NN}$ of OSE.

The results of this study for both $^6\text{Li}$ and $^6\text{He}$ nuclei are presented in Figs. 11 and 12.

From Fig. 11 it is seen clearly that the behavior of $E_b$ as a function of $g_{\sigma NN}$ is rather similar for the $^6\text{Li}$ and $^6\text{He}$ nuclei at the moderate values of $g_{\sigma NN}$, but for high values of $g_{\sigma NN}$ the binding energy for two neutrons in the field of the $\alpha$-core rises faster than for the neutron–proton pair.

A similar trend is observed for the weight of the dibaryon component in the total three-cluster wave functions for $^6\text{Li}$ and $^6\text{He}$: the admixture $P_{nn}(nn)$ of the neutral $nn$ isovector dibaryon in $^6\text{He}$ is lower than the admixture $P_{nn}(np)$ of the charged $np$ isoscalar dibaryon in $^6\text{Li}$ at the “physical” value of the constant $g_{\sigma NN}$, but it increases faster than $P_{nn}(np)$ with increasing $g_{\sigma NN}$. At large values of $g_{\sigma NN}$ these admixtures become close to each other (see Fig. 12).

As is seen from Figs. 11 and 12, when the binding of the $NN$ pair inside the nucleus becomes stronger, the average $NN$ distance gets smaller and thus the admixture of the dibaryon component gets higher. If we assume that the binding energy of two nucleons in the middle-weight nuclei
In the two nuclei turn out to be very close to each other (see Table 5).

6 Conclusion

The present paper represents the first study in the literature of the properties of the low-lying dibaryon resonances, namely the \(np\) \(3S_1\) and \(nn\) (or \(pp\)) \(1S_0\) resonances predicted by Dyson and Xuong [72] as early as 1964, in a nuclear medium. We presented the results of two series of calculations for the \(A=6\) nuclei \(^6\text{Li}\) and \(^6\text{He}\) within the three-cluster \(\alpha + 2N\) model for two alternative types of the basic \(NN\) interaction: the conventional (RSC) \(NN\) potential and the dibaryon-induced model including the effective pair \(NN\) potential together with the new three-body force arising due to the interaction of the intermediate dibaryon with the nucleons in the \(\alpha\)-core. We compared the results for the properties of the \(^6\text{Li}\) and \(^6\text{He}\) nuclei obtained with the use of two above \(NN\)-force models with each other and with the experimental data.

Our main conclusion is as follows: in many aspects, the properties of the real \(^6\text{Li}\) and \(^6\text{He}\) nuclei, e.g., the binding energies and the rms radii of the charge and matter distributions, are reproduced better with the dibaryon-induced model than with the traditional RSC \(NN\) interaction. We found also that the neutron–neutron short-range correlations in \(^6\text{He}\) are enhanced in the case of the dibaryon-induced interaction as compared to the conventional force model. It may give a key to a better understanding of general \(nn\) pairing in nuclei.

In the present work, we also investigated the properties of the \(nn\) and \(np\) dibaryons in the field of the \(\alpha\)-core and their dependence on the magnitude of the three-particle force due to the interaction of the dibaryon with the \(\alpha\)-core.

There is another important aspect of the present study. A large series of recent publications worldwide have been devoted to the short-range correlations (SRCs) of nucleons in nuclei. It is now generally accepted that the SRCs are based on fluctuations in the density of nucleons in nuclei. The generation of \(di\)-, \(tri\)-, or multibaryons in nuclei resulting from the dibaryon concept of nuclear force can be considered as a source of such density fluctuations and, therefore, of the SRCs.

We end with the hope that the present study will motivate researchers in the field over the world to study in detail other manifestations of the dibaryon degrees of freedom in nuclei and nuclear medium.

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