Experimentally exploiting the violation of the Lagrange invariant for resolution improvement

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Abstract: It is experimentally demonstrated that even though the numerical aperture in the object space is fixed, the resolution of an imaging system still can be improved by adjusting the parameters in the image space. This strategy cannot be realized until the discovery of the violation of the Lagrange invariant in a kind of self-interference holography. With the violation, parameters in the image space can escape the constraint of the object space for resolution improvement. Experiments that directly confirm this new ability are implemented and results agree well with the theoretical prediction. Additionally, better performance on frequency recording and finer details beyond the diffraction limit have been recorded with this method.

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1. Introduction

According to the diffraction theory [1], two identical emitters within $\lambda/(2N_A_o)$ cannot be resolved, where $\lambda$ and $N_A_o$ represent the wavelength of light and the numerical aperture in the object space, respectively. Thus, people usually focus on using smaller $\lambda$, higher $N_A_o$ (such as oil immersion or synthetic aperture), or frequency modulation method (such as structured or tilted illumination [2, 3]) to get better resolution. In recent decades, an alternative way to improve the resolution has been proposed. However, in this kind of method, the physical or chemical properties of label have been used to shatter the diffraction barrier [4–7].

It is worth noting that in an optical imaging system there actually exists two spaces, the object space and the image space, where objects and images located respectively. However, as mentioned above once the wavelength is settled methods for resolution improvement are usually focused on the object space [2, 3], or for another solution by combining with other technology [4–7]. However, little efforts are made in the image space for resolution improvement. That is because once the numerical aperture in the object space $N_A_o$ fixed, changes in the parameters in image space (i.e. the lateral magnification, $M_T$, and the numerical aperture in the image space, $N_A_i$) cannot improve the resolution.

Actually, this conclusion is valid because of the constraint of the Lagrange invariant (or the sine condition [1]). According to the Lagrange invariant [1], it can be derived that the product of the parameters in the image space is equal to that in the object space (i.e. the numerical aperture in the object space), as $M_T N_A_i = N_A_o$. Therefore, once the numerical aperture in the object space is fixed, changes in one of $M_T$ and $N_A_i$ will cause the other one change inversely and proportionally so that their product keeps unchanged as $N_A_o$ [8], resulting in no alteration in resolution. Thus, in conventional imaging system, because of this constraint, it is impossible to improve the resolution by adjusting the parameters in the image space. Therefore, people usually focused on the object space, ignoring the possibility in the image space.

This traditional knowledge will be altered in a kind of self-interference holography (SH) systems [9–14] in which the Lagrange invariant is violated [11]. The SH system is a kind of new holography in which the reference wave also contains the object information as the object wave, while in conventional holographic system the reference wave is usually a plane wave or a spherical wave without object information. This new property of SH system introduces new characters to this system, such as the violation of the Lagrange invariant. In our last paper [11], we presented why and how the Lagrange invariant is violated. To find the operating rules of resolving ability after the violation of the Lagrange invariant, further study is needed.

After the violation of the Lagrange invariant, the lateral magnification and the angular magnification can be altered independently [11], which means it is promising that the parameters in the image space can be altered independently, escaping the constraint of the Lagrange invariant. Joseph Rosen et al. used the violation of the Lagrange invariant to found its effect on resolution by studying the unconventional ratio between the transverse magnification of the image-object and the transverse magnification of the spot [8]. In their work, these suppositions have not been verified with convincible experimental results. Until now, there still exists uncertainty about the new capability of resolution improvement when the Lagrange invariant is violated and thorough experimental studies are required.

Here, we also study the effect of the violation of the Lagrange invariant on the resolution. We find out that the resolution of an imaging system can be improved by altering the parameters in the image space even though the numerical aperture in the object space is fixed. This cannot be realized in the past. We explain how and why this becomes possible both theoretically and experimentally. More importantly, this strategy is directly confirmed with our two-dot experiments. Furthermore, it is experimentally observed that this method has better performance on frequency recording and finer details beyond the diffraction limit can
be resolved. These experimental results convinced the new capability of resolution improvement when the Lagrange invariant is violated and present valuable results for optical imaging system research.

2. Theoretical analysis

The study is implemented on a SH system in which the Lagrange invariant is violated. The diagram of one kind of SH system, namely a two-lens Fresnel incoherent correlation holography system [10–12], is depicted in Fig. 1(a) for hologram recording and Figs. 1(b)-1(d) for image reconstruction. One kind of classical optical imaging system (in which the Lagrange invariant holds), namely wide-field microscopy, can also be achieved with the arrangement in Fig. 1(a) by displaying one lens on the spatial light modulator (SLM) and placing the CCD camera in the image plane.

In Fig. 1(a), the light from object is collected by an infinitely corrected micro-objective (MO, Olympus, UPLSAPO 4X) with a focal length $f_o$ of 45 mm. The collected light will be modulated by a SLM (Holoeye, Pluto VIS). A filter (F, Semrock, FF01-488/20-25) is inserted between the MO and the SLM to control the bandwidth of the light source. A polarizer (P) is also introduced to obtain polarization parallel to the optical axis of SLM. The SLM acts as two concentric lenses with adjustable focal lengths (lens L1 with focal length $f_1$ and lens L2 with focal length $f_2$). The multiplexing is realized by randomly choosing half of the pixels to simulate one lens while the other part of pixels to simulate the other one. For one point object S, the combination of the MO and the two lenses on SLM produces two magnified images, O1 and O2, one of which is used as the object wave while the other is used as the reference wave. The hologram plane $xy$ (the CCD plane) is located at a distance $d$ from the SLM, where $d$ can be changed flexibly. With phase shift methods [15], the wave front in the hologram plane can be decoded and then an image can be reconstructed with this wave front, as presented in Figs. 1(b)-1(d). The reconstruction distance $d_r$ can be equal to, smaller than, or larger than the recording distance (for example $d_1$), as presented in Figs. 1(b)-1(d), respectively.

![Fig. 1. Schematic of an SH system. (a), Hologram recording; (b), (c), (d), image reconstruction. The actual SLM is reflective but is illustrated as transmissive for clarity.](image)

2.1 Derivation of the parameters in the image space

The parameters in the image space and their relations to the object space will be studied both in the classical imaging system and the SH system by analyzing the point spread function (PSF). It is worth mentioning that here, “the parameters in the image space” stands for “the numerical aperture in the image space $NA_i$” and “the lateral magnification $M_T$”. As given later, in the image space, the former determines the size of the diffraction patterns while the latter
called the reconstruction distance. Assume that the first cross term, 

\[ U_1^* U_2 \] 

can be decoded and used to reconstruct the image. From the location of the image, the lateral magnification \( M_T \) of this holographic system can be calculated [16] as \( M_T = \frac{x_i}{x_{i0}} = \frac{d'}{d} \left( f_1 T_{i1} - f_2 T_{i2} \right) / \left( T_{i1} T_{i2} \right) \). Higher lateral magnification \( M_T \) will lead to larger distance between two point images. We should point out that in this paper
negative sign of $M_T$ is neglected, as the reverse of image is not concerned. With the expression for $d_i$, $T_{i1}$ and $T_{i2}$, and setting $z_s = 0$ for simplicity, $M_T$ becomes

$$M_T = \frac{d}{f_o} \tag{5}$$

As mentioned above, in the image space the radius of the diffraction pattern is $\delta_i$. As we know, the size of diffraction pattern in the object space is $M_T$ times smaller than $\delta_i$. Thus, the radius of the diffraction pattern in the object space is

$$\delta_o = 0.61\lambda / (M_T NA_i) \tag{6}$$

Equation (6) describes the resolving ability of an optical imaging system.

For wide field imaging system, there is only one wave, for example O1. Without loss of generality, set $z_s = 0$ for simplicity. Similarly, the numerical aperture in the image space of the wide-field system can be derived as $NA_i = L_o f_i$. On the other hand, the lateral magnification of the wide-field imaging system can be derived as $M_T = f_o / f_i$. When $z_s = 0$, the numerical aperture in the object space $NA_o$ is $NA_o = L_o f_o$.

It can be seen that in wide-field imaging system in which the Lagrange invariant holds, as expected the product of the parameters in image space is equal to the numerical aperture in the object space, as $M_T NA_i = NA_o$. Therefore, for the imaging system in which the Lagrange holds, the resolution given in Eq. (6) is equal to $0.61\lambda / NA_o$ showing consistency with the Abbe diffraction limit. On the other hand, it can be seen that when the $NA_o$ is fixed which means the $L_o$ and $f_o$ are fixed, smaller $f_i$ will not only lead to higher $NA_i$ but also smaller $M_T$. That means because of the constraint of the Lagrange invariant, parameters in the image space will change simultaneously and inversely.

However from Eqs. (4) and (5), it can be seen that in the SH system the parameters in the image space can change independently, escaping the constraint of the Lagrange invariant. More importantly, the product of the parameters in the image space is no longer $NA_o$. After the violation of the Lagrange invariant, as given in Eq. (6), the resolution is determined by $0.61\lambda / (M_T NA_i)$ rather than the commonly used Abbe diffraction limit $0.61\lambda / NA_o$.

When $M_T NA_i$ is larger than $NA_o$, the resolution will be improved. On the other hand, when $M_T NA_i$ is smaller than $NA_o$, the resolution will be degraded. It can be derived that the value of $M_T NA_i$ will be larger than $NA_o$ when $\min\{l_{i1}, l_{i2}\} < d < \max\{l_{i1}, l_{i2}\}$ otherwise it will become smaller than or equal to $NA_o$. It should be noted that when one of $l_{i1}$ and $l_{i2}$ is infinite, the size of object wave will be cut off by the reference wave when $d > 2\min\{l_{i1}, l_{i2}\}$ [13]. This will cause loss of information of the object wave, which is not appreciated. Considering this, in such case, the condition for larger $M_T NA_i$ is $\min\{l_{i1}, l_{i2}\} < d < 2\min\{l_{i1}, l_{i2}\}$. Moreover, in this system, when there is a perfect overlap between the reference and the object wave [12], it can be derived that the largest $M_T NA_i$ is $2NA_o$. That means, comparing with a system of same coherence property, the resolution can be improved two times at best even with same $NA_o$.

2.2 Theoretical prediction

Based on the theoretical analysis, a simple theoretical prediction about the effect of the Lagrange invariant on the spatial resolution is given in Fig. 2. In conventional imaging system in which the Lagrange holds, as mentioned above there exist a relation as $M_T NA_i = NA_o$. The blue line in Fig. 2 shows this relationship between $NA_i$ and $M_T$ when $NA_o$ is fixed. It can be seen that the $NA_i$ decreases with the increase of $M_T$. Therefore, as shown in cases w#1 and w#2, in the image space the radii of the point diffraction patterns (determined by $NA_i$) and the distance between them (determined by $M_T$) will change simultaneously and proportionally resulting in no alteration in resolution.

However, when the Lagrange invariant is violated, the numerical aperture in the image space $NA_i$ and the lateral magnification $M_T$ can be altered independently, which means that the radii of the point diffraction patterns and their distance can be adjusted independently. Ultimately, this can be used to modulate the resolution of an imaging system, as shown in cases #1-#4 in Fig. 2.
For example, for two dots beyond the Abbe diffraction limit, in cases #1 and #2, $M_T$ remains unchanged while $NA_i$ can be changed, therefore the peak-to-peak distance fixes while the radii of diffraction patterns can be reduced (case #1) or increased (case #2) to alter the resolution. On the other hand, in cases #3 and #4, $NA_i$ remains unchanged while $M_T$ can be changed. Therefore, the radii fixes while the distance can be increased (case #3) or reduced (case #4) to alter the resolution. Specifically, the resolving power can be improved in cases #1 and #3 even without higher $NA_o$.

![Fig. 2. Relationship between $NA_i$ and $M_T$ when $NA_o$ is fixed. The blue line is for a classical imaging system in which the Lagrange invariant holds.](image)

In general, the resolution will keep unchanged when $NA_i$ and $|M_T|$ located on the blue line. However, the resolution will be improved if they located at the area above the blue line. On the other hand, the resolution will be ruined if they located at the area below the blue line.

### 3. Experimental results

Experiments were implemented to verify the theoretical prediction. In experiments, the 1951 USAF target (Edmund, F55-622) and two-point resolution dots on a test target (Thorlabs, R1L3S5P) were used as the object. When using two-dot as object, the aperture of the objective was decreased so that the two dots could just be resolved. The targets were located at the front focal plane of the objective ($z_s = 0$) and illuminated with an LED light source (Thorlabs, M455L2). All of the lenses displayed on SLM had the same radius, as 4.32 mm.

The parameters used for each experiment are given in Table 1. For each object, all cases had the same numerical aperture in the object space $NA_o$. By using different experimental parameters, different values of the lateral magnification $M_T$ and the numerical aperture in the image space $NA_i$ were obtained according to Eqs. (4) and (5). As shown in Table 1, cases #1 and #2 shared the same value of $M_T$ as case w#1, but case #1 had higher $NA_i$ while case #2 had lower $NA_i$. On the other hand, cases #3 and #4 shared the same value of $NA_i$ as case w#1, but case #3 had larger $M_T$ while case #4 had smaller $M_T$.

|    | w#1       | w#2       | #1        | #2        | #3        | #4        |
|----|-----------|-----------|-----------|-----------|-----------|-----------|
| $NA_i$ | $b^*$     | $b$       | $b$       | $b$       | $b$       |           |
| Experimental parameters | $d = 1,000$ | $d = 592$ | $d = 1,000$ | $d = 1,000$ | $d = 1,779$ | $d = 592$ |
| $f = 1,000$ | $f = 900$ | $f = 1,100$ | $f = 1,000$ | $f = 1,000$ |
| $f_1 = 1,100$ | $f_2 = 1,200$ | $f_2 = \infty$ | $f_2 = \infty$ |
| $L = 1,000$ | $L = 592$ | $L = 550$ | $L = 2,200$ | $L = 1,000$ | $L = 1,000$ |
| $M_T$ | 22.2      | 13.2      | 22.2      | 13.2      |

* $b$, the value of the numerical aperture in the object space

First, the strategy was examined in a classical wide-field imaging system in which the Lagrange invariant holds. For this system, the imaging results in cases w#1 and w#2 are presented in Figs. 3(a), 3(c) and Figs. 3(b), 3(d), respectively. Figs. 3(a) and 3(b) for two-point resolution dots and Figs. 3(c) and 3(d) for the USAF target. For each image, the...
normalized intensity profile (or the normalized projected intensity profile of group 8 for the USAF target) is also presented with the same case number. It should be noted that the intensity profiles are scaled in the image space while the scale bars are scaled in the object space.

Comparing Figs. 3(a) and 3(b), we can see that from case w#1 to case w#2, the numerical aperture in the image space \( N_A \) becomes 1.7 times larger (see Table 1), leading to smaller radii of diffraction patterns, as shown in Fig. 3(b). Meanwhile, the lateral magnification also becomes 1.7 times smaller, leading to smaller distance between the two diffraction patterns, as given in Fig. 3(b). Therefore, the barely resolvable two-point object remains barely resolved. In addition, for the USAF target, there is no change in resolving power by the comparison of Figs. 3(c) and 3(d). For both cases, in the image of the USAF target, element 1 of group 8 can just be resolved, corresponding to a resolution of \( \sim 1.95 \mu m \), which is close to the diffraction limit of \( 1.86 \mu m \). It means that the resolution in case w#1 is the same as case w#2.

Therefore, as described by the Abbe diffraction limit, in the classical optical imaging system with certain wavelength, when the numerical aperture in the object space \( N_A \) is fixed, the resolving power is also fixed. Changes in the parameters in the image space will not improve the resolution.

![Image of intensity profiles and diffraction patterns](image)

Fig. 3. Changes in the resolving power by altering \( N_A \) or \( M_T \) in a wide-field system where the Lagrange invariant holds. (a), (c), Case w#1, \( N_A = L_s/1000, M_T = 22.2 \); (b), (d), case w#2, \( N_A = L_s/592, M_T = 13.2 \). Scale bar, 50 \( \mu m \).

However, when the Lagrange invariant is violated, as in cases #1 and #2 in Fig. 2, \( N_A \) can be changed without any change in \( M_T \). This enables the radii of diffraction patterns to be changed while the peak-to-peak distances remain unchanged. Figure 4 presents the influence of \( N_A \) on the resolving power when \( M_T \) stays unchanged.

By using different experimental parameters (see Table 1), cases #1 and #2 had the same lateral magnification \( M_T \) as case w#1, but comparing with case w#1, case #1 had higher \( N_A \) (~2 times) while #2 had smaller \( N_A \) (~0.5 times). The imaging results under cases #1 and #2 are given in Figs. 4(a), 4(c) and Figs. 4(b), 4(d) respectively, Figs. 4(a) and 4(b) for the two-point resolution dots and Figs. 4(c) and 4(d) for the USAF target.
Comparing with case w#1 in Fig. 3(a), the \( N_A \) of case #1 becomes larger while \( M_T \) stays the same. Therefore, as shown in Fig. 4(a), the radii of diffraction patterns decreased while the distance between them stayed unchanged, and the change of radii agrees well with the theoretical prediction. As a result, the two barely resolvable points in case w#1 can be easily distinguished in case #1. Consequently, the resolution is improved and the unresolvable bar in Fig. 3(c) (marked by a blue circle) can be clearly distinguished in Fig. 4(c).

Obviously, comparing Figs. 3(c) and 4(c), the resolving ability in holographic system is better than that of the wide-field imaging system and fine details beyond the diffraction limit can be resolved. From the intensity profile of holographic image, group 8 element 6 can be just resolved corresponding to a resolution of 1.10 \( \mu m \). Comparing with Fig. 3(c), the resolution is improved about 1.8 times, which is same as the theoretical prediction (i.e. \( 1000/550 = 1.8 \)).

On the other hand, for case #2, as given in Fig. 4(b), when \( N_A \) becomes smaller, the radii of diffraction patterns become larger without any change in the distance between them. (We should point out that, in experiments the change in radius is a little larger than 2 times, resulting from the overlapping of two diffraction patterns). Therefore, the two barely resolvable points can no longer be distinguished. The resolution decreased, and the bars in Fig. 3(c) become more blurred in Fig. 4(d).

Thus, the resolution can be modulated by changing the radii of the point diffraction patterns (with different \( N_A \)) while the distance between them remain the same (with the same \( M_T \)). It means that the resolution can be modulated by changing the numerical aperture in the image space (\( N_A \)) with the lateral magnification (\( M_T \)) stays unchanged. Especially, the resolution can be improved with increased \( N_A \), even though \( N_A \) is fixed.

Similarly, when the Lagrange invariant is violated, as in cases #3 and #4 in Fig. 2, \( M_T \) can be altered with \( N_A \), keeping unchanged. This enables the distance between two diffraction patterns to be changed while their radii keep unchanged. Figure 5 presents the influence of \( M_T \) on the resolving power when \( N_A \) stays unchanged. In these experiments, the light source was
changed to one with a bandwidth of ~2 nm (by using another filter, Semrock, LL01-458-25) to satisfy the larger coherence length requirement [10]. Here the change of bandwidth has little effect on the resolution.

By using different experimental parameters (see Table 1), for each object, cases #3 and #4 had the same \(NA_i\) as case w#1, but comparing with case w#1, case #3 had larger \(MT\) (~1.8 times) while case #4 had smaller \(MT\) (~0.6 times). The imaging results under cases #3 and #4 are given in Figs. 5(a), 5(c) and Figs. 5(b), 5(d) respectively, Figs. 5(a) and 5(b) for the two-point resolution dots and Figs. 5(c) and 5(d) for the USAF target.

Comparing with case w#1 in Fig. 3(a), the \(MT\) of case #3 becomes larger while its \(NA_i\) stays the same. Therefore, as shown in Fig. 5(a), the two diffraction patterns get further away from each other while the radii of diffraction patterns stay unchanged. As a result, the two barely resolvable points in case w#1 can be easily distinguished in case #3. The resolution is improved, and the unresolvable bars in Fig. 3(c) marked by the blue circle become resolvable in Fig. 5(c). It should be noticed that this improvement is not as good as case #1 because of the smaller signal-to-noise ratio for large recording distance [10].

On the other hand, for case #4, in Fig. 5(b), when \(MT\) becomes smaller, the two diffraction patterns become closer to each other without any change in the radii of diffraction patterns. (It should be noted that the small change in radius results from the overlapping of two diffraction patterns). Accordingly, the two barely resolvable points in Fig. 3(a) can no longer be distinguished in Fig. 5(b). Consequently, the resolution decreased, and comparing Fig. 5(d) with Fig. 3(c), the unresolvable bars become more blurred.

Thus, the resolution can be modulated by changing the distance between point diffraction patterns (with different \(MT\)) while their size keeps the same (with the same \(NA_i\)). It means that the resolution can be modulated by changing the lateral magnification (\(MT\)) with the numerical aperture in the image space (\(NA_i\)) stays unchanged. Especially, the resolution can be improved with increased \(MT\) even though \(NA_o\) is fixed.

![Fig. 5. Influence of \(MT\) on the resolving power when \(NA_i\) keeps unchanged. (a), (c), Case #3, \(NA_i = L/1000, MT = 39.5\); (b), (d), case #4, \(NA_i = L/1000, MT = 13.2\). For each object, cases #3 and #4 have the same \(NA_i\), as case w#1, but case #3 has larger \(MT\) while case #4 has smaller \(MT\). Because of differences in magnification, the size and contrast of images have been adjusted and some margin areas have been cut for display. Scale bar, 50 \(\mu m\).](image)

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It should be pointed out that all the experimental results shown above were obtained when one of $NA_i$ and $MT$ changed while the other one fixed. Actually, $NA_i$ and $MT$ can be changed simultaneously to affect the resolution. Until now, we proposed and experimentally demonstrated a reliable concept that the resolving power of the imaging system can be changed by altering $NA_i$ or $MT$. Experiments with two-dot target were performed to directly confirm its effect on resolution.

In addition, important observations has been made in the Fourier transform of the images. The spectral distributions (on common logarithmic scale) of the wide-field image (shown in Fig. 3(c)) and the holographic image (shown in Fig. 4(c)) are given in Fig. 6(a) and Fig. 6(b), respectively. In Fig. 6, the background noise of holographic image is higher. This might be caused by the sharper transfer function of SH system.

![Fig. 6. Comparison of the spectrum. (a), Spectral distribution of the wide-field image given in Fig. 3(c). (b), Spectral distribution of the holographic image given in Fig. 4(c).](image)

In Fig. 6(a), it should be noted that the bright lines along the whole $x$ and $y$-axis were caused by the high brightness of Fig. 3(c). From Figs. 6(a) and 6(b), it is obvious that the image obtained with the SH system contains more complex frequencies than that of the wide-field imaging system. It means that the SH system can record more frequencies than the wide-field imaging system because finer details has been recorded. This also shows the better resolving ability of the SH system.

4. Discussion

It is worth emphasizing that although the proof-of-principle experiments presented here were performed under low $NA_o$ conditions because of the available resolution of target. This strategy can also work in high numerical aperture system because even in this kind of system, the image space is still under paraxial conditions. A relay system with small magnification can be adopted for second imaging. The Lagrange invariant holds in the relay system, and then the strategy presented here can be used to violate the Lagrange invariant for resolution improvement.

Moreover, it should be mentioned that although here the achievable $MTNA_i$ is two times at best, theoretically higher improvement could be obtained with further increase of the numerical aperture in the image space. Because the numerical aperture in the image space can be further increased with shorter reconstruction distances by utilizing multiplex-wave interference. This part of the research will be studied in detail in future work.

5. Conclusion

In summary, we have investigated that for a standard optical imaging system, as indicated by the Abbe diffraction limit, once the numerical aperture in the object space $NA_o$ is fixed, the resolution is also fixed and cannot be improved by increasing the parameters in the image space (i.e. $NA_i$ or $MT$), resulting from the Lagrange invariant. However, for one kind of holographic system, it is demonstrated that the parameters in the image space, $NA_i$ and $MT$, can be adjusted independently because of the violation of the Lagrange invariant. The
experiments with two-dot target directly show the subsequent ability that, in the image space, the radii of the point diffraction patterns and their distance can be adjusted independently. This ability enables the improvement on resolution by altering the parameters in the image space. Additionally, it is experimentally observed that this method has better performance on frequency recording, and details beyond the Abbe diffraction limit can be resolved.

This approach is particularly attractive because it requires no increase on the numerical aperture in the object space and it can be easily realized on the existing holographic microscopy by introducing object information into the reference wave. On the other hand, this shows a possibility that people can also focus on the image space for higher resolution, which indicates a promising way for resolution improvement and refreshes our knowledge about the optical imaging systems.

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