Quantum Whispers

Lucien Hardy
Clarendon Laboratory, University of Oxford*

Wim van Dam
Centre for Quantum Computation, University of Oxford†
Quantum Computing and Advanced Systems Research, CWI

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Abstract

It is shown that with the use of entanglement a specific two party communication task can be done with a systematically smaller expected error than any possible classical protocol could do. The example utilises the very tight correlation between separate spin measurements on a singlet state for small differences in the angles of these two measurements. An extension of this example to many parties arranged in a row with only local, one–to–one communication (whispering) is then considered. It is argued that in this scenario there exists no reliable classical protocol, whereas in the quantum case there does.

*Clarendon Laboratory, Department of Physics, University of Oxford, OX1 3PU Oxford, United Kingdom. Email address: hardy@mildred.physics.ox.ac.uk
†Centre for Quantum Computing, Clarendon Laboratory, Department of Physics, University of Oxford, OX1 3PU Oxford, United Kingdom. Quantum Computing and Advanced Systems Research, C.W.I., P.O. Box 94079, NL–1090 GB Amsterdam, The Netherlands. Email address: wimvdam@mildred.physics.ox.ac.uk
1 Introduction

Quantum entanglement is used in various ways, both to illustrate fundamental aspects of quantum mechanics and in information processing applications. In the first category are the Einstein Podolsky Rosen argument and Bell’s theorem. In the latter category are teleportation, quantum cryptography, and quantum computing. Recently, another such application has been found by Cleve and Buhrman [4], namely that of ‘entanglement enhanced classical communication’ or ‘quantum communication’ for short. It has been shown by various authors [2, 3, 4, 7] that the communication complexity of certain communicational tasks can be reduced with the use of entanglement. This has to be understood as follows.

Several spatially separated parties $A, B, C, \ldots$ are each given their personal data $x_1, x_2, x_3, \ldots$, which is initially unknown to the other parties. Next, the parties communicate among each other by means of classical bits with the objective that one party (say $A$) can announce the value of a previously given function $f$ on the input values $x_1, x_2, \ldots$ after a minimal amount of communication. That is, $A$ wants to know the specific function value $f(x_1, x_2, \ldots)$ after the protocol with the least possible number of bits communicated during the protocol. The following stages are to be distinguished in the whole procedure:

**stage 1:** Before being given the data $x_i$ the parties are allowed to communicate as much as they like (for example to share random numbers).

**stage 2:** After they have been given the data $x_i$, the parties must communicate as little as possible.

The communication complexity of a function $f$ is the minimum amount of communication (measured in bits) in stage 2 that is necessary to determine the value of $f(x_1, x_2, \ldots)$. The quantum communication complexity is defined in the same way as the classical communication complexity except that now, during stage 1, the parties are also allowed to share quantum particles (qubits) that are entangled.

By considering an example involving three parties, Cleve and Buhrman [4] showed that for a specific function $f$, less communication is required in the quantum case than in the classical case. This is surprising because quantum entanglement cannot be used to send a signal from one place to another,
nor can it even compress information when using a classical communication channel. However, when embedded into an appropriate task which involves communication, the nonlocal properties of an entangled state can allow a reduction in the amount of communication required. Unlike the other applications of quantum mechanics mentioned above, this application really does make explicit use of the Bell type nonlocality of entangled states. Indeed, one can say that whenever we have an example of a function where a quantum protocol beats the best possible classical protocol (in terms of communication complexity), it can be re-interpreted as a proof of the non locality of quantum mechanics (the converse statement does not appear to be true though).

After the initial article by Cleve and Buhrman, several other results on quantum communication were obtained. Some of them expanded the difference in communication complexity between the quantum and the classical case, whereas others showed that for certain functions there can be no essential difference between the two scenario’s. See also the notion of telecomputation for related work.

In this paper we will take a slightly different approach to the problem. Rather than calculating the amount of communication required to compute the function value, we will allow only a certain amount of communication and calculate the expected error rate under this restriction. (A similar approach to probabilistic communication is taken in Section 3.) At first we will analyse a task for two parties $A$ and $B$. It will be shown that if $A$ and $B$ are allowed to share a singlet state the expected error rate can be systematically lower than the classical case when they are not allowed to share such an entangled state. The essence of this result relies heavily on the correlation between spin measurements that differ either by small angles, or by approximately $\pi$. This typical behaviour has been considered by various authors, and can be used to run a proof of Bell’s theorem by invoking an effect mathematically analogous to the quantum Zeno effect.

We will also consider an extension of this situation in which a large number of parties, $A, B, \ldots$ are arranged in a row and communicate pairwise (i.e. $A$ and $B$, $B$ and $C$, etc.) It is argued that in this scenario the quantum case allows for a correct protocol (with high probability), whereas in the classical case no reliable protocol exists to calculate the function value. The similarity of this situation to the well known game of Chinese whispers gives this paper its title.
Figure 1: Explanation of the communication problem: “no jump” means that $x$ equals $y$ or is adjacent to it, and “jump” stands for the situation where $x$ is either opposite $y$ or adjacent to that opposite position.

2 A Two Party Communication Problem

Now let us introduce the communication problem for the two parties $A$ and $B$, or Alice and Bob. During stage 1 they are allowed to exchange information and, in the quantum case, quantum entanglement. After this stage they are each given a number $x$ and $y$ respectively, where $x, y \in \{0, 1, 2, \ldots, 2N - 1\}$. We will employ modular arithmetic so that these numbers can be thought of as being on a circle with $2N$ dots around it. The person providing these numbers makes a promise. He promises that either

$$x - y \in \{-1, 0, +1\} \quad \text{called “no jump”}$$

or

$$x - y \in \{N - 1, N, N + 1\} \quad \text{called “jump”},$$

where arithmetic is understood to be modulo $2N$ as already stated. In the case of “no jump” $x$ is either equal or immediately adjacent to $y$. In the case of “jump” $x$ is either opposite $y$ on the circle or either side of that opposite position (see Figure 1). During stage 2 party $A$ is allowed to send one classical bit of information (say either $+1$ or $-1$) to party $B$ after which Bob has to announce whether there has been no jump or a jump. This situation is depicted in Figure 2. (We could write this task down as being equivalent to evaluating an appropriately defined function on a restricted domain.) We assume that $N \geq 3$ such that the cases of jump and no jump are mutually
exclusive. As is customary in communication complexity theory, we define the 'expected error-rate' as the the expected error for the worst possible probability distribution over allowed input numbers $x$ and $y$ (worst case scenario assumption). An error occurs when $B$ states incorrectly whether there has been a jump or not. We will see that

1. For a described quantum protocol, the expected error-rate is proportional to $1/N^2$.

2. In the classical case, the expected error-rate will always be equal to or greater than $k/N$ where $k$ is a constant.

This means that for sufficiently large $N$, the quantum protocol outperforms all possible classical protocols.

### 2.1 The Quantum Protocol

The quantum case is easier to prove so we will start there. During stage 1 of the protocol the two parties share a singlet state:

$$|\psi\rangle = \frac{1}{\sqrt{2}}(|+\rangle_A|\rangle_B - |\rangle_A|+\rangle_B)$$.  

(2)

Then, at the start of the second stage, the two parties receive their respective numbers $x$ and $y$. According to those numbers, they define $\theta = 2\pi x/2N$ and $\phi = 2\pi y/2N$. Now Alice measures the spin on her particle along the angle $\theta$ to the $z$-axis in the $xz$ plane. At the same time Bob measures the spin on his particle along the angle $\phi$ to the $z$-axis in the $xz$ plane. Each of them will
get a result ±1 as the outcome of this measurement. To simplify matters for further analysis, Bob changes the sign of his outcome so that ±1 becomes ±1 (this means that if $\theta = \phi$ then Alice and Bob have the same outcome). Given this we can easily show that for the singlet state we have:

$$\text{Prob(same}|\theta,\phi) = \frac{1}{2}(1 + \cos(\theta - \phi)) \quad (3)$$

$$\text{Prob(opposite}|\theta,\phi) = \frac{1}{2}(1 - \cos(\theta - \phi)), \quad (4)$$

where Prob(same) is the probability that Alice and Bob’s outcomes are the same and Prob(opposite) is the probability that their outcomes are opposite.

In the case where there is no jump we have

$$|\theta - \phi| \leq \frac{\pi}{N} \quad (5)$$

and hence

$$\text{Prob(same|no jump)} \geq 1 - \left(\frac{\pi}{2N}\right)^2 \quad (6)$$

$$\text{Prob(opposite|no jump)} \leq \left(\frac{\pi}{2N}\right)^2 \quad (7)$$

In the case where there is a jump we have

$$\pi - \frac{\pi}{N} \leq |\theta - \phi| \leq \pi + \frac{\pi}{N} \quad (8)$$

and hence

$$\text{Prob(opposite|jump)} \geq 1 - \left(\frac{\pi}{2N}\right)^2 \quad (9)$$

$$\text{Prob(same|jump)} \leq \left(\frac{\pi}{2N}\right)^2 \quad (10)$$

Therefore Alice and Bob can adopt the following protocol. Alice sends the result of her spin measurement (±1) to Bob along the classical channel. This constitutes the one allowed bit of classical communication. Next, Bob multiplies this result by his own spin measurement outcome. If the result of this is +1 (i.e. Alice and Bob’s outcomes are the same) then it follows from Equations (6), (7), (9), and (10) that it is most likely that there was no jump. Hence Bob announces that there was no jump. If the result of this
multiplication is \(-1\) (i.e. the two results are opposite), Bob announces that there has been a jump.

For both cases the worst possible angle deviation is \(\pi/N\) (Equations (5) and (8)), and hence the expected error less than or equal to \((\pi/2N)^2\). The above described quantum protocol has therefore an expected error-rate of the order \(1/N^2\):

\[
\text{Error}_{\text{quantum}} \leq \left(\frac{\pi}{2N}\right)^2.
\]  

For large \(N\), this probability tends very quickly to zero.

### 2.2 The Best Possible Classical Protocol

In analysing the classical case we want to be sure that we have found the minimum possible error. This means that we must be more careful and the analysis is correspondingly more detailed than in the quantum case.

We will start by assuming that the provider of the numbers \(x\) and \(y\) distributes them evenly over the possibilities such that they satisfy the promise in Equation (4). The probability that \(y\) takes some particular value in the set \(\{0, \ldots, 2N-1\}\) is therefore \(1/2N\). For this particular value, Bob knows that there are six possible values for \(x\). Since the distribution is uniform, each of these possible values is equally probable with probability \(1/6\). Alice’s task is to communicate one bit (either +1 or -1) to Bob in such a way that Bob has a good chance of correctly saying whether there has been a jump or not. We assume that Alice behaves in a deterministic way (though later we will also discuss the indeterministic case). Thus, the best Alice can do is to look up the value of some function \(g(x) = \pm 1\) and communicate this to Bob via the one bit classical channel (this corresponds to the deterministic case because a for a given \(x\) Alice will always send the same message). Bob can also know the form of the function \(g\) and hence when he receives a +1 he knows which subset Alice’s \(x\) is in, and similarly for the disjunct set if he receives a -1. We will rephrase this into a colouring problem.

Each value of \(x\) gives rise to a +1 or a -1 for \(g(x)\) which we will colour as black and white respectively. Thus, we can colour each dot \(x\) on the circle representing the values of \(g(x)\). Except in the trivial case where all dots are either black or white, there must be at least one place on the circle where black and white dots are adjacent. Consider these two dots, one that is adjacent to the two and the three opposite dots making six dots in total.
Figure 3: Given the fact that Bob knows $y$, there are six equally likely possibilities for the Alice’s value $x$. Here we have chosen $y$ such that for some allowed values of $x$ Alice will send $+1$ (black) and for other possible values $-1$ (white).

Assume also that $y$ is one of the two non-equally coloured pair, as shown in Figure 3. In this figure there are two dots of known colour and four of unstated colour. There are therefore 16 ways of colouring these remaining dots. We can consider each way individually and calculate the expected error for Alice and Bob in determining whether there has been a jump or not. One example of such a colouring is shown in Figure 3. Imagine that the value of $y$ is as indicated by the arrow in this picture (this occurs with probability $1/2N$). The probability that Alice sends a $+1$ (black) is $2/6$ (because two of the six dots are black) and the probability that she sends a $-1$ (white) is $4/6$. If Bob receives a $-1$ (i.e. white) from Alice then he cannot be sure whether $x$ corresponds to a jump or not, though it is more likely that it corresponds to a jump so he will announce that there has been a jump. The probability of error corresponds to the one white on the no jump side. As one of the four whites are on the no jump side the probability of error is $1/4$. If Bob receives a $+1$ (i.e. black) from Alice then he can be sure that $x$ corresponds to no jump since both of the blacks are on the no jump side. Hence this probability of error is zero. Taking all this into account, the contribution of
Figure 4: A specific colouring of the four grey dots in Figure 3. The analysis in the text shows that for this example, Alice and Bob have an error probability of $1/6$ when trying to decide if $x$ and $y$ make a jump or not.

this colouring combination with this specific value of $y$ to the overall error is:

$$\text{error}_{\text{Figure 4}} = \frac{1}{2N} \left[ \left( \frac{4}{6} \right) \left( \frac{1}{4} \right) + \left( \frac{2}{6} \right) \left( \frac{0}{2} \right) \right].$$  \hspace{1cm} (12)

Having performed one analysis of this kind, it is a simple matter to go through the remaining 15 colouring possibilities. When doing this, one sees that the particular example given above is one of the ways of colouring that gives rise to the smallest error.

When colouring the $2N$ dots around the circle there are the following two possible cases. If the dots are coloured uniformly black (or white) then Bob does not receive any information from Alice, hence the probability of error will be $1/2$. It is also clear that the related case of just one black (or white) dot will lead to a high error-rate. In the case where there are at least two black and two white dots there will be at least 4 instances of situation similar to Figure 3 and the above analysis. Each with a least expected error as expressed in Equation (12). Hence, we can conclude that the overall error-rate of any classical protocol will be of the order $1/N$:

$$\text{Error}_{\text{classical}} \geq \frac{1}{3N}. \hspace{1cm} (13)$$
The reason for this classical error can be intuitively understood in the following way. Alice’s strategy will involve partitioning the circle into at least two parts where each part is coloured either black or white. As long as the values of $x$ and $y$ are well inside these sections there will be no error. However, if $x$ and $y$ are at the boundary between these two parts then there is likely to be an error. The probability of being at such a boundary on the circle goes as $1/N$, which explains the above result.

It is interesting to note that if Alice is allowed to send one trit of communication (black, white, or blue) then an appropriate colouring will lead to no errors. For example the circle can be coloured in the following way: For $x \in \{2N - 1, 0, 1, 2\}$ put a blue dot; for $x \in \{3, \ldots, N\}$ put a white dot, and for $x \in \{N + 1, \ldots, 2N - 2\}$ put a black dot. A little thought shows that this arrangement will lead to no errors. It can be shown that one trit corresponds to an average of approximately 1.5 bits of communication by using the unambiguous set of code-words ‘0’, ‘10’, and ‘11’ \[^6\]. Thus, we can say that, in some sense, the quantum state is substituting for an average of not more than approximately half a bit of classical information.

### 2.3 Remarks on Randomisation

There are two subtleties concerning randomisation which should be discussed before honestly accepting the results of Equations (11) and (13).

First we have to ask whether it is possible that Alice and Bob could do better if Alice were to send +1 and −1 according to some randomised protocol. The answer to this question is in short: this can not be the case because we fixed the distribution of $x$ and $y$ (the homogeneous distribution) before we proved the error-rate. The reasoning behind this statement goes as follows.

If Alice and Bob would employ some randomisation in their protocol, then the description of the protocol \textit{in situ} will depend on some random numbers $r$. For a fixed distribution on the input data, some values of $r$ (denoted by $r'$) will give \textit{at least} the expected error-rate. Therefore Alice and Bob might have well shared those particular beneficial random numbers $r'$ in advance during stage 1 of the protocol. But knowing $r'$ yields again a deterministic protocol, so the error-rate can also be reached by a non-randomised (because determined by $r'$) procedure. Hence, the bound in Equation (13) on deterministic protocols translates directly to non-deterministic protocols as well.

The second subtlety is on the data distribution side. Imagine that the
person providing the numbers $x$ and $y$ is malicious and tries to make the expected error-rate as high as possible. Since he knows Alice and Bob’s protocol he can always choose $x$ and $y$ so that an error will occur. Our above result is still safe since it is a lower bound, and for the quantum case we already assumed in our analysis the worst possible distribution on $x$ and $y$. However, it is instructive to see that Alice and Bob can take measures to keep their error low if they are able to share random numbers secretly. There is a large number of different ways of colouring the circle for $x$ (i.e. of defining the function $g$). We can select those ways that give rise to a minimum error and label them with a number $\lambda$. This number can now serve as a random number which is to be shared secretly by Bob and Alice at stage 1. In this situation the provider of $x$ and $y$ does not know where the boundaries of the black/white colouring are, so he cannot choose his numbers $x$ and $y$ to guarantee an error. Indeed, his actions will be averaged out and so once again the error will be of the order $1/N$.

In the above described reasonings, two opposing forces are at work. i) If Alice and Bob know the probability distribution over the input values, they can optimise their protocol for this case. ii) For a given probabilistic protocol, the person providing the input data can choose the distribution that gives the worst possible error-rate for the two parties. Analysing the trade-off between these two forces in a general setting finds its origin in game theory and is now also an important part of probabilistic computation and communication theory[11].

3 A Multiparty Communication Problem

We now come to quantum whispers. This name is derived from the well known game of Chinese whispers in which a row of people pass a message from one to next. Typically, this message gets distorted so that the message arriving at the end in no way resembles the original message. Thus consider a row of $M$ people $A, B, C, \ldots, Z$ (we do not mean to say here that $M = 26$). Let the number of people be of the order of $N$:

$$M = cN,$$  

(14)

where $c$ is a constant (we have in mind a typical value of about $c = 10$). Now consider the following communication protocol. In stage 1 of the protocol the parties are allowed to share information pairwise. Thus, $A$ and $B$ share
information, $B$ and $C$ share information, and so on. In the quantum case they are also allowed to share entangled states pairwise. As they did in the two party problem, they will share singlet states. After this $A, B, \ldots, Z$ are given numbers. $A$ is given $x_1$ and $B$ is given $y_2$. Also $B$ is given $x_2$ and $C$ is given $y_3$, and so on, so that each party has two numbers $y_i$ and $x_i$ except for $A$ (Alice) who has only $x_1$ and $Z$ (Zarah) who has only $y_M$ (see Figure 5). All these numbers belong to the set \{0, 1, \ldots, 2N−1\}. Furthermore the pairs $x_i$ and $y_{i+1}$ satisfy the jump or no jump condition of Equation (1). (We do not impose any condition on the pairs $x_i$ and $y_i$.) In stage 2, $A$ sends one bit (+1 or −1) to $B$, then $B$ sends one bit to $C$, and so on. Thus we have the same situation as before but repeated many times (Figure 5). The communication problem is for Zarah, at the end of the row, to announce whether there are an even or odd number of jumps.

### 3.1 The Quantum Protocol

Consider first the quantum case where the parties share singlet states pairwise. These singlet states consist of a left particle $L$ and a right particle $R$. This means Alice has only an $L$ particle, Zarah has only an $R$ particle, and all the other parties have both an $R$ and $L$ particle. Alice measures $2\pi x_1/2N$ on her $L$ particle and Zarah measures $2\pi y_M/2N$ on her $R$ particle. Each of the other parties measures the spin along the direction $2\pi y_i/2N$ on their $R$ particle and $2\pi x_i/2N$ on their $L$ particle. As before the sign of the outcome for the measurements on the $R$ particles is redefined so that $\pm 1$ becomes $\mp 1$. Alice sends the result of her measurement to Bob. Bob multiplies this result by both of his results and communicates the result of this to $C$ (Carol).
C multiplies the message she receives from Bob by the results of her two spin measurements and communicates this result to D. This procedure is repeated until Zarah receives a message. As a last step she multiplies the message she received from Y by the result of her own measurement. Zarah will now have a number ±1. Every jump along the row contributes a factor −1, hence if the end result is +1 then Zarah can announce that there has been an even number of jumps. If the end result is −1 then Zarah can announce that there has been an odd number of jumps. For big N (that is, small \(1/N^2\)) the overall expected error is the sum of the \(M-1\) pairwise error probabilities (which are less than \((\pi/2N)^2\)). Hence we can say that under that assumption, the overall error is at most \(M(\pi/2N)^2\) or, using \(M = cN\),

\[
\text{Error}_{\text{quantum whispers}} \leq \frac{c\pi^2}{4N}.
\]  

(15)

Which gives us a useful protocol if \(c/N\) is sufficiently small.

### 3.2 Analysing Possible Classical Protocols

For the final answer of the protocol, the parties have to solve \(M-1\) independent “jump or no jump?” questions. The questions are independent in the sense that Alice’s value \(x_1\) and Bob’s value \(y_2\) are uncorrelated with the other \(x\) and \(y\) values. Therefore with the one bit of communication from Alice to Bob, the two parties have to solve the same problem as we discussed in the previous section. Hence Alice and Bob will have an expected error-rate of the order \(1/N\). This holds in general for all \(M-1\) pairs \((x_i, y_i+1)\) for which the jump/no jump question has to be answered. These errors are uncorrelated with each other. For small \(c\) (meaning \(M/N \ll 1\)), the overall error-rate is the sum of the \(M-1\) individual error probabilities, giving

\[
\text{Error}_{\text{small } c \text{ classical whispers}} \geq \frac{M-1}{3N}.
\]

(16)

For large \(c\) this formula is no longer valid. If we assume (like we did in the quantum case) that \(N\) is big, the expected error-rate will then be:

\[
\text{Error}_{\text{big } N \text{ classical whispers}} \geq \frac{1}{2} - \frac{e^{-4c/3}}{2}.
\]

(17)

In the case where \(c\) is about 10 this error effectively equals 1/2. Thus implying that the protocol is as effective as a random coin flip by Zarah when choosing her announcement.
The error-rate given for small $c$ (Equation (16)) can actually be achieved by the following protocol. Each pair agrees on a function $g$, thus $g_{AB}, g_{BC}, \ldots$. Alice sends the result $g_{AB}(x_1)$ to Bob. Bob must decide whether he thinks there has been a jump or not. He uses a function $h_{AB}(g_{AB}(x_1), y_2)$ to decide, where this function has value $+1$ (jump) or $-1$ (no jump). The other pairs can define similar functions $h_{BC}, h_{CD}, \ldots$ which give the best guess as to whether there has been a jump or not. Now, Bob multiplies $h_{AB}$ by $g_{BC}$ and sends that result to Carol. This proceeds until Zarah receives a message from Y which she then multiplies by $h_{YZ}$. As in the quantum case, Zarah will announce that there have been an even number of jumps if the end outcome is $+1$ and an odd number of jumps if the end outcome is $-1$. This protocol becomes unreliable for large $c$ for the reasons given above.

4 Conclusions

We have seen how quantum entanglement is useful from a communication complexity point of view. This result employed the very tight quantum correlations of a singlet state for small angles. First we considered a two party example. The two parties, Alice and Bob, are given the numbers $x$ and $y$ respectively, where $x, y, \in \{0, \ldots, 2N - 1\}$. These two numbers satisfy either a jump or a no-jump condition (defined in Equation (1)). Alice is allowed to communicate one bit of classical information to Bob and Bob has to announce whether he thinks there has been a jump or not. In the classical case the error rate goes as $1/N$ whereas in the quantum case, where the parties are allowed to share an entangled quantum state beforehand, the error goes as $1/N^2$. Thus as $N$ gets big the quantum error is significantly smaller.

We also considered an extension of this scheme to a situation in which many parties are arranged in a row. They are given numbers which, from one party to the next in the row, satisfy the jump or no jump condition. Each party can communicate one bit of classical information to the next party down the row. The object is for the person at the end of the row to announce whether there have been an odd or an even number of jumps. We considered the case where the number of parties in the row is about $10N$. In the quantum case, in which the parties are allowed to share singlet states pairwise, the error-rate goes as $1/N$ so is small for large $N$. However, in the analogous classical scheme the error-rate is approximately $1/2$. 
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