A new mechanism of neutron star radiation

Anatoly A. Svidzinsky
Bartol Research Institute, University of Delaware, Newark, DE 19716, USA
(January 13, 2022)

We find a new mechanism of neutron star radiation wherein radiation is produced by the stellar interior. The source of radiation is oscillating neutron vortices in the superfluid core of a rotating neutron star. Thermally excited helical waves of vortices generate fast magnetosonic waves at the stellar crust. Near the crust bottom such waves reduce to a collisionless zero sound in an electron liquid, while near the stellar surface they behave as electromagnetic waves in a medium. The magnetosonic waves propagate across the crust and transform into electromagnetic radiation at the surface. The vortex contribution has the spectral index \( \alpha \approx -0.45 \) and can explain nonthermal radiation of middle-aged pulsars observed in infrared, optical and hard X-ray bands. Detection of vortex radiation allows direct determination of the core temperature. Comparing the theory with available spectra observations we find that the core temperature of the Vela pulsar is \( T \approx 8 \times 10^6 \)K, while the core temperature of PSR B0656+14 exceeds \( 2 \times 10^8 \)K. This estimate excludes exotic equation of states incorporating Bose condensations of pions or kaons and quark matter in these objects. In principle, zero sound can also be emitted by other mechanisms, rather than vortices, which opens a perspective of direct spectroscopic study of superdense matter in the neutron star interiors.

Properties of matter at densities much larger than nuclear are poorly known and constitute a challenging problem of modern science. They have broad implications of great importance for cosmology, the early universe, its evolution, for compact stars and for laboratory physics of high-energy nuclear collisions. Present searches of matter properties at high density take place in several arenas. One of them is investigation of neutron stars.

Neutron stars (NSs) are compact objects of radius about 10km and the mass of the order of solar mass. Cores of NSs consist of superfluid neutrons with some (several per cent by particle number) admixture of protons and electrons. More dense NSs also possess an inner core. Its radius may reach several kilometers and central density can be much larger than the nuclei density. Several hypotheses about composition and equation of state of the inner core are discussed in the literature. One of them is appearance of \( \Sigma \)- and \( \Lambda \)-hyperons. The second hypothesis assumes the appearance of pion or kaon condensation. The third hypothesis predicts a phase transition to strange quark matter composed of almost free \( u \), \( d \) and \( s \) quarks with a small admixture of electrons.

The cooling rate and temperature of the central part of a NS substantially depends on the properties of dense matter. In this paper we find that NS radiation can be produced in its interior, rather then at the surface. Detection of such radiation in the X-ray band allows direct determination of the core temperature. We find that the core temperature of the Vela pulsar is \( T \approx 8 \times 10^6 \)K, while the core temperature of PSR B0656+14 exceeds \( 2 \times 10^8 \)K. This estimate excludes exotic equation of states incorporating Bose condensations of pions or kaons and quark matter in these objects. A systematic study of pulsar radiation in the X-ray band can locate objects with fast cooling core which could be candidates for NSs with exotic states of matter.

Zero sound in electron liquid — For middle aged NSs the effective frequencies of electron-electron and electron-phonon collisions in the stellar crust are less than \( 10^{12} \)Hz. In the core the collisions of electrons with baryons are suppressed by neutron superfluidity and proton superconductivity. As a result, at large frequencies the electrons in the stellar interior can be treated as an independent system of Fermi particles which move in collisionless regime. In such regime a zero sound is a possible collective excitation of Fermi liquid \[ \Pi \]. Speed of zero sound \( u_s \) is larger than the electron Fermi velocity \( v_F \). In the presence of magnetic field \( H \) equations of electron motion reduce to equations of magnetic hydrodynamics in which \( u_s \) enters the equations instead of speed of usual sound and the Alfvén velocity \( u_A \) is estimated in terms of the electron density \( \rho_e \): \[ u_A = H/\sqrt{4\pi\rho_e}. \] Magnetic field at the stellar crust couples electron sound and electromagnetic wave into one excitation known as a fast magnetosonic wave (FMW). Near the crust bottom \( u_A \ll u_s \approx c \), where \( c \) is the speed of light, and the FMWs reduce to zero sound. However, near the stellar surface (at small density) \( u_A > u_s \) and the FMWs behave as electromagnetic waves in a medium with dielectric constant \( \varepsilon = 1 + c^2/u_A^2 \). As a result, at the stellar surface the FMWs transform into electromagnetic radiation in vacuum the same way as refraction of usual electromagnetic waves at the dielectric-vacuum interface. Typically near the stellar surface \( u_A \gg c \) and, hence, the transmission coefficient is close to 1.

The key idea of the paper is that NS interior is transparent for zero sound (FMWs). We assume that the crust matter is in a crystal phase: \( T < T_m = 2.5 \times 10^7 \rho_0^{1/3} \)K, where \( T_m \) is the melting temperature, \( \rho_0 \) is the matter density in units \( 10^6\text{g/cm}^3 \). Near the stellar surface
there is a phase transition between a gaseous atmosphere and a condensed metallic phase and the atmosphere has negligible optical depth. The density of the metallic phase at the surface depends on magnetic field. E.g., for iron crust and $H = 10^{12}$Gs the surface density is $\rho \approx (10^{13} \text{g/cm}^3)$. 

For a weakly interacting Fermi gas, such as a gas of electrons in the NS interior, the attenuation of zero sound is exponentially small. However, in the presence of magnetic field the FMWs are accompanied by oscillation of electric field which results in partial damping of FMWs near the stellar surface. The attenuation increases with decreasing the matter density. However, at $\rho_0 \lesssim 0.6H_{12}^{3/2}$ the system as a whole occupies only a small number of Landau levels. At such densities the electron motion becomes effectively one dimensional and the electron scattering, as well as magnetosonic attenuation, is suppressed by magnetic field. The maximum damping occurs in a boundary region where magnetic field still does not suppress the electron scattering. FMWs pass the critical region and reach the surface without attenuation if in the region

$$|\sin \theta| < 0.004H_{12}^{2/3} T_6^{1/2} G_s^{5/2},$$

where $\theta$ is the angle between the wavevector and the magnetic field, $G_s$ is the acceleration of gravity in units $10^{15}$cm/s$^2$, and $T_6$ is the superfluid velocity. In the limit $\omega < \omega_c$, one can omit the first term in Eq. (4). Then solution of this equation which satisfies the boundary condition has the form

$$\Phi' = i\hbar k_{||} K_1(\xi r) [i(\omega t + k_{||} z - \phi)] / 2m_n r,$$

In the bulk the function $\Phi'$ should satisfy a linearized equation of superfluid hydrodynamics, which is, in fact, the wave equation of sound propagation

$$\partial^2 \Phi' / \partial t^2 - c_s^2 \nabla^2 \Phi' = 0,$$

where $c_s$ is the speed of sound in superfluid. In the limit $\omega \ll \omega_c$, one can omit the first term in Eq. (4). Then solution of this equation which satisfies the boundary condition has the form

$$\Phi' = i\hbar k_{||} K_1(\xi r) [i(\omega t + k_{||} z - \phi)] / 2m_n,$$

where $K_1$ is the modified Bessel function. The solution exponentially decreases at $r \gg 1/k_{||}$, that is perturbation in the superfluid velocity $V = \nabla \Phi$ is localized near the vortex core and no energy is radiated.

**Generation of zero sound at the core boundary** — The bulk of superconducting protons in NS core do not rotate by forming vortices. However, due to interactions between protons and neutrons, neutron superfluid velocity generates a superfluid current of protons in the vicinity of neutron vortices (drag effect). At the interface between the stellar crust and the outer core the matter undergoes a first order transition from an $\alpha en$ phase to a uniform liquid of neutrons, protons and electrons ($\mu en$ phase). Nuclei in the $\alpha en$ phase form a Coulomb crystal. In the $\mu en$ phase the proton liquid is superconducting, while in the $\alpha en$ phase protons constitute a part of nuclei. According to Eq. (4), helical vortex motion does not generate sound waves inside the stellar core. However, such waves are generated at the interface between the superconducting and $\alpha en$ phases.

Due to the drag effect of neutrons the superfluid vortex produce density oscillation of protons in the superconducting phase. Free electrons screen the electric field and oscillate together with protons. In $\alpha en$ phase there is no drag effect and electron motion is not coupled with neutron vortices. To estimate the power of radiated zero sound we note that kinetic energy of ultrarelativistic electrons at the crust bottom is much larger than their interaction energy. Hence, the radiated power is approximately the same as in the limiting case of non interacting electrons. In this limit, the electrons move in a ballistic regime and the energy flux across the crust-core interface is given by the surface integral $Q = \int E_v v dv dS$, where $v_F \approx c$ and $E_v$ is the contribution to the electron energy density due to the helical vortex motion. Density oscillation of electrons (which is accompanied by large change in their Fermi energy) determine the helix energy in the stellar core which is equal to $\int E_v LdS$. As a result, the average energy flux of sound waves produced by a helical wave of a single vortex is

$$Q = \frac{c}{L} \int E_v LdS = \frac{c}{L} \left[ \frac{\hbar \omega}{\exp(\hbar \omega/k_B T) - 1} \right].$$
where $T$ is the core temperature. One should note that the wave length of sound is much larger than the size of the area from which it is generated and, hence, the sound waves have approximately spherical front. Taking into account that the number of vortex modes within the interval $dk_∥ = Ldk_∥/2\pi$ and the number of vortices $N = 2m_nR^2\Omega/\hbar$, we obtain the spectral density of sound waves power radiated from the oscillating vortex lattice in a half space

$$P_v(\omega) = \frac{\sqrt{2cm_n^{3/2}R^2\Omega\sqrt{\omega}}}{\pi h^{3/2}\hbar \ln(\omega_c/\omega)} \left[\exp(h\omega/k_BT) - 1\right].$$  \hspace{1cm} (7)

Fig. 1 shows vortex mechanism of NS radiation.

One should note that electromagnetic radiation produced by vortices can not exceed black body radiation produced by the surface $4\pi R^2$ with the same temperature $T$ (Kirchhoff’s law). We assume that vortices are excited thermally and if at some frequency the vortex radiation becomes comparable with radiation of a black body this means that at such frequency the rate of thermal excitation imposes the main restriction on the radiated power: vortices radiate maximum power which can be pumped from the thermal reservoir. Kirchhoff’s law results in the following limitation at which Eq. (7) describes vortex radiation

$$\omega/2\pi > 0.24\hbar^{-3/5}c^{6/5}m_n^{3/5}\Omega^{2/5}\ln^{-1/5}(\omega_c/\omega).$$ \hspace{1cm} (8)

For $\Omega/2\pi = 4s^{-1}$ we obtain $\omega/2\pi > 1.5 \times 10^{14}$Hz. At lower frequencies the spectrum of vortex radiation follows Planck’s formula.

To compare our theory with observations it is convenient to represent the spectral density of vortex radiation $P_v(\omega)$ as a power law $P_v(\omega) \propto \omega^\alpha$, where $\alpha$ is the spectral index. The $\sqrt{\ln(\omega_c/\omega)}$ function in Eq. (7) shifts the spectral index by a small value $1/2 \ln(\omega_c/\omega)$. The shift depends weakly on $\omega$ and changes the spectral index from $\alpha \approx -0.46$ in the optical band to $\alpha \approx -0.43$ in the X-ray band. Apart from vortex contribution there is thermal radiation from the NS surface. We estimate thermal radiation assuming helium atmosphere and use numerical results obtained by Romani [9]. The model fits well the thermal spectrum with only one parameter, the effective surface temperature $T_{\text{eff}}$. The total radiation from a NS in unit solid angle is given by the sum of vortex and thermal $P_{\text{th}}$ components:

$$P(\omega) = aP_v(\omega)/2\pi + P_{\text{th}}(\omega).$$ \hspace{1cm} (9)

Here we introduced a dimensionless free parameter $a \approx 0.1 \div 1$ which takes into account partial absorption of the vortex radiation near the stellar surface and geometrical effects related to unknown magnetic field distribution and position of the line of site.

Discussion — Fig. 2 compares the observed radiation spectrum of middle-aged pulsars PSR B0656+14 ($\Omega/2\pi = 2.6s^{-1}$) and Vela ($\Omega/2\pi = 11.2s^{-1}$) with the spectrum predicted by our theory. Thermal radiation of the stellar surface dominates in the ultraviolet and soft X-ray bands. The effective surface temperature is $T_{\text{eff}} = 4.9 \times 10^{5}$K for PSR B0656+14 and $T_{\text{eff}} = 7.8 \times 10^{5}$K for Vela. The NS radius $R_s$ (in km) is related to the pulsar distance $D$ (in kpc) as $R_s = 53\sqrt{1+z}D$ for PSR B0656+14 and $R_s = 41\sqrt{1+z}D$ for Vela ($z$ is the NS redshift). The vortex contribution (dash line) dominates in infrared, optical and hard X-ray bands. In the far infrared band the radiation spectrum changes its behavior and follows Planck’s formula. The sum of the vortex and thermal components is displayed with the solid line. For PSR B0656+14 we take the core temperature $T = 6.4 \times 10^{9}$K and the geometrical factor $a = 0.18$, while for Vela $T = 8 \times 10^{9}$K and $a = 0.33$. The observed broadband spectrum is consistent with our model for typical NS parameters, which suggests that the vortex mechanism of radiation operates in a broad frequency range from IR to hard X-rays. Also in the optical band the radiation of middle-aged pulsars was found to be highly pulsed with pulses similar to those in the hard X-ray band [8,9,10]. This agrees with the vortex mechanism which predicts pulsed radiation.

FIG. 1. Mechanism of a NS radiation. Thermaly excited helical waves of neutron vortices in the superfluid core produce FMWs in the stellar crust. FMWs propagate across the crust and transform into electromagnetic radiation at the star surface. Mainly the radiation comes out from regions with strong magnetic field (near magnetic poles).

For $\Omega/2\pi = 4s^{-1}$ we obtain $\omega/2\pi > 1.5 \times 10^{14}$Hz. At lower frequencies the spectrum of vortex radiation follows Planck’s formula.

To compare our theory with observations it is convenient to represent the spectral density of vortex radiation $P_v(\omega)$ as a power law $P_v(\omega) \propto \omega^\alpha$, where $\alpha$ is the spectral index. The $\sqrt{\ln(\omega_c/\omega)}$ function in Eq. (7) shifts the spectral index by a small value $1/2 \ln(\omega_c/\omega)$. The shift depends weakly on $\omega$ and changes the spectral index from $\alpha \approx -0.46$ in the optical band to $\alpha \approx -0.43$ in the X-ray band. Apart from vortex contribution there is thermal radiation from the NS surface. We estimate thermal radiation assuming helium atmosphere and use numerical results obtained by Romani [9]. The model fits well the thermal spectrum with only one parameter, the effective surface temperature $T_{\text{eff}}$. The total radiation from a NS in unit solid angle is given by the sum of vortex and thermal $P_{\text{th}}$ components:

$$P(\omega) = aP_v(\omega)/2\pi + P_{\text{th}}(\omega).$$ \hspace{1cm} (9)

Here we introduced a dimensionless free parameter $a \approx 0.1 \div 1$ which takes into account partial absorption of the vortex radiation near the stellar surface and geometrical effects related to unknown magnetic field distribution and position of the line of site.

Discussion — Fig. 2 compares the observed radiation spectrum of middle-aged pulsars PSR B0656+14 ($\Omega/2\pi = 2.6s^{-1}$) and Vela ($\Omega/2\pi = 11.2s^{-1}$) with the spectrum predicted by our theory. Thermal radiation of the stellar surface dominates in the ultraviolet and soft X-ray bands. The effective surface temperature is $T_{\text{eff}} = 4.9 \times 10^{5}$K for PSR B0656+14 and $T_{\text{eff}} = 7.8 \times 10^{5}$K for Vela. The NS radius $R_s$ (in km) is related to the pulsar distance $D$ (in kpc) as $R_s = 53\sqrt{1+z}D$ for PSR B0656+14 and $R_s = 41\sqrt{1+z}D$ for Vela ($z$ is the NS redshift). The vortex contribution (dash line) dominates in infrared, optical and hard X-ray bands. In the far infrared band the radiation spectrum changes its behavior and follows Planck’s formula. The sum of the vortex and thermal components is displayed with the solid line. For PSR B0656+14 we take the core temperature $T = 6.4 \times 10^{9}$K and the geometrical factor $a = 0.18$, while for Vela $T = 8 \times 10^{9}$K and $a = 0.33$. The observed broadband spectrum is consistent with our model for typical NS parameters, which suggests that the vortex mechanism of radiation operates in a broad frequency range from IR to hard X-rays. Also in the optical band the radiation of middle-aged pulsars was found to be highly pulsed with pulses similar to those in the hard X-ray band [8,9,10]. This agrees with the vortex mechanism which predicts pulsed radiation.
vortex peak in the energy interval 1keV–1MeV. If we assume that the amplitude of the vortex peak follows Eq. (7), then we can find total (integrated over frequency) radiation intensities of vortices in different energy bands. Then we compare the detected integrated intensities of the vortex peak in the Rossi and OSSE bands [14] with those predicted by Eq. (7). We found that if there was no temperature decay of the vortex peak the total vortex intensity in the OSSE band should be 3 times larger than those actually observed. This indicates on the decay of the vortex spectrum with the core temperature $T \approx 8 \times 10^8$K.

The estimate of the core temperature allows us to make a conclusion about the interior constitution of the NSs. A hot NS cools mainly via neutrino emission from its core. Neutrino emission rates, and hence the core temperature, depend on the properties of dense matter. If the direct Urca process for nucleons is allowed such NS cools to $10^8$K in weeks [17]. However, the characteristic age of PSR B0656+14 is about $10^5$ yrs, while the Vela pulsar is about $10^4$ years old. So, the NS core cools down to $10^8$K at least $10^8$ times slower then the rate predicted by the direct Urca process. This estimate excludes equation of states incorporating Bose condensations of pions or kaons and quark matter. The point is that the neutrino emission processes for these states may be regarded as variants of the direct Urca process for nucleons [17]. As a result, all these states give rise to neutrino emission, though generally smaller, comparable to that from the direct Urca process for nucleons which inconsistent with the discrepancy of the cooling rate in the factor $10^6$.

Detection of vortex radiation opens a possibility to study composition of NS crust. Since FMWs generated by vortices propagate through the stellar interior the spectrum of vortex radiation should contain (red-shifted) absorption lines which correspond to low energy excitation of nuclei that form NS crust. E.g., the $^{57}$Fe nucleus has an excited state with the energy $14.4kev=3.5 \times 10^{18}Hz$, which would produce an absorption line in the hard X-ray band if the core temperature is greater than $1.7 \times 10^9K$. Bottom layers of the crust may contain exotic nuclei with the mass number up to 600 and the core radiation creates a perspective to study their properties. Another challenging problem is interaction of zero sound with exotic states of matter. The point is that generation of zero sound by vortices is only one of the possible mechanisms. If an exotic state of matter absorbs zero sound at some frequency, then, according to Kirchhoff’s law, it will radiate zero sound at this frequency. As a result, spectrum of stellar radiation must contain characteristic emission lines corresponding to such processes which opens a perspective of direct spectroscopic study of superdense matter. A detailed version of our theory is available in [1].

I am very grateful to A. Fetter, G. Shlyapnikov, M. Binger, S. T. Chui, V. Ryzhov for valuable discussions and the Aspen Center for Physics where part of the re-

FIG. 2. Broadband spectrum of PSR B0656+14 [12] and the Vela pulsar [13,14]. Solid line is the fit by the sum of the vortex and thermal components. Thermal radiation of the stellar surface dominates in the ultraviolet and soft X-ray bands, while the vortex contribution (dash line) prevails in infrared, optical and hard X-ray bands, where its spectrum has a slope $\alpha \approx -0.45$. In the far infrared band the spectrum changes its behavior and follows Planck’s formula with $P \propto \omega^2$.

It is worth to note that the spectral index of vortex radiation is a fixed parameter in our theory. If a single mechanism of nonthermal radiation operates in the broad range, then the continuation of the X-ray fit to the optical range determines the spectral index of the nonthermal component with a big accuracy. Good quantitative agreement of our theory with the observed spectral index serves as a strong evidence that the vortex mechanism is responsible for radiation of middle-aged NSs in IR, optical and hard X-ray bands. The vortex contribution exponentially decreases at $\hbar \omega > k_B T$. Observation of such spectrum behavior allows direct determination of the core temperature $T$. For PSR B0656+14 and the Vela pulsars the power law spectrum in the hard X-ray band shows no changes up to the highest frequency $2 \times 10^{18}$Hz at which the data are available (see Fig. 2). This indicates that temperature of the NSs core is larger than $2 \times 10^8K$. Measurements of spectra of middle-aged pulsars in the $10^{18} - 10^{19}$Hz range are needed to search for possible manifestations of the core temperature. Recent observation of the Vela pulsar with RXTE has covered the frequency band $(0.49 - 7.3) \times 10^{18}Hz$. These data in combination with OSSE observations, $(1.7 - 13.8) \times 10^{19}Hz$, allows us to estimate the temperature of the Vela core. Light curves of the Vela pulsar have several peaks. We associate the vortex radiation with the second optical peak (see Fig. 1 in [14] and Fig. 4 in [14]). To estimate the core temperature one should trace the evolution of the vortex peak in the energy interval 1keV–1MeV.

\[ T \approx 8 \times 10^8 K \]
results has been obtained. This work was supported by NSF, Grant No. DMR 99-71518 and by NASA, Grant No. NAG8-1427.

[1] L. D. Landau, Zh. Eksp. Teor. Fiz. 32, 59 (1957) [Sov. Phys. JETP 5, 101 (1957)].
[2] L. Hernquist, ApJs 56, 325 (1984).
[3] D. Lai, Rev. Mod. Phys. 73, 629 (2001).
[4] A. Thorolfsson et al., ApJ 502, 847 (1998).
[5] K. Gottfried & L. Picman, K. Dan. Vidensk Selsk. Mat. Fys. Medd. 32, no. 13 (1960).
[6] A.A. Svidzinsky, E-print astro-ph/0212367.
[7] E.M. Lifshitz & L.P. Pitaevskii, Statistical Physics, Part 2, 3rd edition (Pergamon, Oxford, 1980).
[8] M.A. Alpar et al., ApJ 282, 533 (1984).
[9] R.W. Romani, ApJ 313, 718 (1987).
[10] A. Shearer et al., A&A 335, L21 (1998).
[11] A. Shearer et al., ApJ 487, L181 (1997).
[12] A.B. Koptsevich et al., A&A 370, 1004 (2001).
[13] G.G. Pavlov et al., ApJ 552, L129 (2001).
[14] F.P. Nasuti et al., A&A 323, 839 (1997).
[15] A.K. Harding et al., E-print astro-ph/9911263.
[16] M.S. Strickman et al., E-print astro-ph/9904357.
[17] C.J. Pethick, Rev. Mod. Phys. 64, 1133 (1992).