Numerical investigation of wave propagation in high-permeable cylindrical waveguide in porous medium

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Abstract. The features of waves propagating in a highly permeable cylindrical waveguide in a porous medium are numerically investigated. The influence of the ratio of permeability inside the cavity and the surrounding porous medium, as well as the signal frequency on the evolution of the pressure wave inside the cavity is studied. On the basis of the calculated data, estimates of the velocity and attenuation of the signal during its propagation in the waveguide in a wide frequency range are obtained. It is shown that with the increase in the waveguide permeability, the propagation velocity increases and the attenuation of waves decreases, the depth of penetration of perturbations into the surrounding space decreases. Increasing the frequency of the initial signal leads to an increase in the velocity and attenuation of the pulse. A comparison is made with the case of wave propagation in a liquid-filled cylindrical cavity in a porous medium. The results of the study can be used in the interpretation of well logging data.

1. Introduction
It is known that the composition and structure of micro-homogeneous rocks and fluids saturating them significantly affect the velocity and damping of elastic waves propagating in such media. Therefore, by changing the signals propagating in the internal waveguide in a porous medium, it is possible to determine the properties of rocks, such as porosity, permeability, fluid saturation, etc. [1, 2]. In [3-6], waves in a liquid-filled cylindrical cavity in a porous medium containing gas bubbles or gas hydrate were studied. This work is devoted to the study of the influence of the properties of the internal waveguide and the surrounding porous medium on the peculiarities of wave propagation in such a system.

2. Governing equations
To study numerically wave propagation and attenuation in a highly permeable cylindrical waveguide in a porous medium, we use a two-velocity porous medium model [7, 8].

\[
\frac{\partial \rho_f}{\partial t} + \nabla \cdot \left( \rho_f v_f \right) = 0, \quad \frac{\partial \rho_s}{\partial t} + \nabla \cdot \left( \rho_s v_s \right) = 0, \quad (1)
\]

\[
\rho_f \frac{d v_f^k}{dt} = -\alpha_f \nabla^k p_f - F^k, \quad \rho_s \frac{d v_s^k}{dt} = -\alpha_s \nabla^k p_f + \nabla^k \sigma^b + F^k, \quad (2)
\]
The features of waves propagating in a highly permeable cylindrical waveguide in a porous medium are numerically investigated in an axisymmetric approximation. The study was performed in the framework of a two-phase porous medium model using the Lax-Wendroff method. The results presented below are obtained for the computational domain $0 \leq z \leq 2m$, $0 \leq r \leq r_n + 1m$, $r_n = 0.1m$. The region $r > r_n$ is filled with a main porous medium (PM1), and a part of a cylindrical cavity $z > 0.1m$, $0 \leq r \leq r_n$ is filled with porous medium with a higher permeability (PM2). The region $0 \leq z \leq 0.1m$, $0 \leq r \leq r_n$ is filled with liquid.

The perturbation of finite duration was created by a liquid source located inside the cavity at $0 \leq \tau \leq \tau_0 = 0.04m$, $0 \leq z \leq z_0 = 0.04m$. The material of the porous medium skeleton is quartz, the liquid is water. The case of open pores at the porous medium–liquid interface and at the PM1 – PM2 interface is considered.

The values of the main parameters taken in the calculations are as follows: for PM1 porosity is 0.3, characteristic grain size of the porous medium is 0.1 mm, permeability $4.3 \cdot 10^{-11}$ m$^2$; for PM2 porosity 0.4, characteristic grain size of the porous medium 0.2 mm, permeability $2.7 \cdot 10^{-10}$ m$^2$.

Fig.1 shows the change in fluid pressure on the axis of the cavity at $r = 0$, $z = 0.2$, $0.4$, $0.6$, $0.8$, $1m$, for compression wave (carrier frequency is about 0.5 kHz) and for oscillating pulses of carrier frequency 1 kHz and 3 kHz. It can be seen that the propagation of the compression wave is characterized by a significant distortion, i.e. form-spread and attenuation. For oscillating pulses, attenuation is also noticeable, but the waveform does not change much. Analysis of the calculated data indicates that high-frequency signals attenuate significantly stronger than low-frequency ones and propagate almost without dispersion.
Based on the calculated data, estimates of the signal velocity and attenuation during its propagation in the waveguide in the considered frequency range are obtained. From the estimates it follows that an increase in the frequency of the original signal leads to an increase in the velocity and a more significant attenuation of the pulse. With increasing permeability of the waveguide, the propagation velocity increases and the attenuation of waves decreases, the depth of penetration of disturbances into the surrounding space decreases.

A comparison is made with the case of wave propagation in a liquid-filled cylindrical cavity in a porous medium. In contrast to this case, a wave in a porous waveguide contains two modes: deformation and filtration. In the limiting case, when the permeability of the porous medium inside the waveguide increases, the pressure curves coincide with the corresponding dependences for the waveguide filled with liquid.

4. Conclusions
The features of waves propagating in a highly permeable cylindrical waveguide in a porous medium are numerically investigated. The influence of the ratio of permeability inside the cavity and the surrounding porous medium, as well as the signal frequency on the evolution of the pressure wave inside the cavity is studied. In the considered frequency range with increase in the permeability of the
waveguide, the propagation velocity is increased, and the wave damping decreases, and the penetration depth of disturbances into the surrounding space decreases; increase in the frequency of the original signal causes an increased velocity and increased attenuation of the pulse.

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