Unitary Supermultiplets of $OSp(1/32, R)$ and M-theory

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Abstract

We review the oscillator construction of the unitary representations of non-compact groups and supergroups and study the unitary supermultiplets of $OSp(1/32, R)$ in relation to M-theory. $OSp(1/32, R)$ has a singleton supermultiplet consisting of a scalar and a spinor field. Parity invariance leads us to consider $OSp(1/32, R)_L \times OSp(1/32, R)_R$ as the "minimal" generalized AdS supersymmetry algebra of M-theory corresponding to the embedding of two spinor representations of $SO(10, 2)$ in the fundamental representation of $Sp(32, R)$. The contraction to the Poincare superalgebra with central charges proceeds via a diagonal subsupergroup $OSp(1/32, R)_{L-R}$ which contains the common subgroup $SO(10, 1)$ of the two $SO(10, 2)$ factors. The parity invariant singleton supermultiplet of $OSp(1/32, R)_L \times OSp(1/32, R)_R$ decomposes into an infinite set of "doubleton" supermultiplets of the diagonal $OSp(1/32, R)_{L-R}$. There is a unique "CPT self-conjugate" doubleton supermultiplet whose tensor product with itself yields the "massless" generalized $AdS_{11}$ supermultiplets. The massless graviton supermultiplet contains fields corresponding to those of 11-dimensional supergravity plus additional ones. Assuming that an AdS phase of M-theory exists we argue that the doubleton field theory must be the holographic superconformal field theory in ten dimensions that is dual to M-theory in the same sense as the duality between the $N = 4$ super Yang-Mills in $d = 4$ and the $IIB$ superstring over $AdS_5 \times S^5$.

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1 Introduction

In a recent work Maldacena [1] conjectured that the large $\mathcal{N}$ limits of certain conformal field theories in $d$ dimensions are dual to supergravity (and superstring theory in a certain limit) on the product of $d+1$ dimensional AdS spaces with certain spheres. Maldacena’s conjecture was motivated by earlier work on $p$-branes [2]. The prime example of this duality is between the $\mathcal{N}=4$ super Yang-Mills in $d=4$ and the IIB superstring over $\text{AdS}_5 \times S^5$ in the large $\mathcal{N}$ limit. In [2] it was pointed out how the conjecture of Maldacena can be understood naturally within the framework of work done long time ago on Kaluza-Klein supergravity theories. Referring to [2] for details and references to the earlier work we shall summarize briefly the most relevant results below. Since the work of Maldacena there has been an explosion of interest in AdS supergravity theories, $p$-branes and their relations to superconformal field theories ([3]-[24]).

In [25] the unitary supermultiplets of the $d=4$ AdS supergroups $OSp(2N/4, R)$ were constructed and the spectrum of the $S^7$ compactification of eleven dimensional supergravity was fitted into an infinite tower of unitary supermultiplets of $OSp(8/4, R)$. The ultra-short singleton supermultiplet sits at the bottom of this infinite tower of Kaluza-Klein modes and decouple from the spectrum as local gauge degrees of freedom [25]. However, even though the singleton supermultiplet decouples from the spectrum as local gauge modes, one can generate the entire spectrum of 11-dimensional supergravity over $S^7$ by tensoring the singleton supermultiplets repeatedly and restricting oneself to "CPT self-conjugate " vacuum supermultiplets [25].

The compactification of 11-d supergravity over the four sphere $S^4$ down to seven dimensions was studied in [26, 27]. The spectrum of 11-dimensional supergravity over $S^4$ fall into an infinite tower of unitary supermultiplets of $OSp(8^*/4)$ with the even subgroup $SO(6,2) \times USp(4)$ [26]. Again the doubleton supermultiplet of $OSp(8^*/4)$ decouples from the spectrum as local gauge degrees of freedom. It consists of five scalars, four fermions and a self-dual two form field [26]. The entire physical spectrum of 11-dimensional supergravity over $S^4$ can be obtained by simply tensoring the doubleton supermultiplets repeatedly and restricting oneself to the vacuum supermultiplets [26].

The spectrum of the $S^5$ compactification of ten dimensional IIB super-
gravity was calculated in [28, 29]. The entire spectrum falls into an infinite tower of massless and massive unitary supermultiplets of $N = 8$ AdS$_5$ superalgebra $SU(2, 2/4)$ [28]. The "CPT self-conjugate" doubleton supermultiplet of $N = 8$ AdS superalgebra decouples from the physical spectrum as local gauge degrees of freedom. By tensoring the doubleton supermultiplet with itself repeatedly and restricting oneself to the CPT self-conjugate vacuum supermultiplets one generates the entire spectrum of Kaluza-Klein states of ten dimensional IIB theory on $S^5$.

In [28, 30] it was pointed out that the $N = 8$ AdS$_5$ doubleton supermultiplet does not have a Poincare limit and its field theory exists only on the boundary of AdS$_5$ which can be identified with the $d = 4$ Minkowski space. Hence the doubleton field theory of $SU(2, 2/4)$ is the conformally invariant $N = 4$ super Yang-Mills theory in $d = 4$. Similarly, the singleton supermultiplet of $OSp(8/4, R)$ and the doubleton supermultiplet of $OSp(8^*/4)$ do not have a Poincare limit in $d = 4$ and $d = 7$, respectively, and their field theories are conformally invariant theories in one lower dimension. Thus we see that the proposal of Maldacena follows directly from the above mentioned results if we assume that the spectrum of the superconformal field theories fall into ("CPT self-conjugate") vacuum supermultiplets. Remarkably, this is equivalent to assuming that the spectrum consists of color singlet supermultiplets! Furthermore, taking the large $\mathcal{N}$ limit allows one to tensor arbitrarily many doubleton supermultiplets so as to be able to obtain the entire infinite tower of Kaluza-Klein states.

Our goal in this paper is to extend these results to the maximal possible dimension. In particular, we would like to find out if there exists an AdS phase of M-theory in the maximal space-time dimension that is dual to some superconformal quantum field theory. We are not able to give a definitive answer to this question. However, we find evidence from the supermultiplet structure of generalized AdS supergroups in 11 dimensions that M-theory can indeed have an AdS phase that is dual to a superconformal doubleton field theory in ten dimensions. In sections 2 and 3 we review briefly the oscillator construction of the unitary lowest weight representations of non-compact groups and supergroups. In section 4 we study the unitary lowest weight representations of the supergroup $OSp(1/2m, R)$, in general, and $OSp(1/32, R)$ in particular. In section 5 we discuss the unitary supermultiplets of $OSp(1/32, R)$ in relation to M-theory and physics in ten and eleven dimensions. The parity invariance of M-theory requires the extension of

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4 see [3] for references
$OSp(1/32, R)$ to a larger supergroup that admits parity invariant representations. The "minimal" parity symmetric supergroup is $OSp(1/32, R)_L \times OSp(1/32, R)_R$. The two factors of $OSp(1/32, R)_L \times OSp(1/32, R)_R$ correspond to the embedding of left-handed and right-handed spinor representations of $SO(10, 2)$ in the fundamental representation of $Sp(32, R)$. The contraction to the Poincare superalgebra with central charges proceeds via a diagonal subalgebra $OSp(1/32, R)_{L-R}$ which contains the common subgroup $SO(10, 1)$ of the two $SO(10, 2)$ subgroups. The parity invariant tensor product of the singleton supermultiplets of the two factors decomposes into an infinite set of "doubleton" supermultiplets of the diagonal $OSp(1/32, R)_{L-R}$. There is a unique "CPT self-conjugate" doubleton supermultiplet whose tensor product with itself leads to "massless" supermultiplets. The "CPT self-conjugate" massless graviton supermultiplet contains fields corresponding to those of 11-dimensional supergravity plus additional ones. We conjecture that the doubleton field theory is a superconformal field theory in ten dimensions that is dual to an AdS phase of M-theory in the same sense as the duality between the $N = 4$ super Yang-Mills in $d = 4$ and the $IIB$ superstring over $AdS_5 \times S^5$.

2 Unitary Lowest Weight Representations of Non-compact Groups

A representation of the lowest weight type of a simple non-compact group is defined as a representation in which the spectrum of at least one of the generators is bounded from below. A non-compact simple group $G$ admits unitary lowest weight representations (ULWR) if and only if its quotient space $G/H$ with respect to its maximal compact subgroup $H$ is an hermitian symmetric space [33]. Thus the complete list of simple non-compact groups that have unitary representations of the lowest weight type follows from the list of irreducible noncompact hermitian symmetric spaces. Below we give the complete list of such simple non-compact groups $G$ and their maximal compact subgroups $H$ [33]:

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5That this supergroup is the "minimal" parity symmetric generalized AdS group in eleven dimensions was argued by Horava [31] in his recent work on generalized topological Chern-Simons theory in eleven dimensions [32].
The Lie algebra \( g \) of a non-compact group \( G \) that admits ULWR’s has a Jordan structure (3-grading) with respect to the Lie algebra \( h \) of its maximal compact subgroup \( H \).

\[
g = g^{-1} \oplus g^0 \oplus g^{+1}
\]

(2 - 1)

where \( g^0 = h \) and we have the formal commutation relations

\[
[g^{(m)}, g^{(n)}] \subseteq g^{(m+n)} \quad m, n = \mp 1, 0
\]

and \( g^{(m)} \equiv 0 \) for \( |m| > 1 \).

In [34] a general method of construction of unitary lowest weight representations (ULWR) of non-compact groups over the Fock space of an arbitrary number of bosonic oscillators was given. Some very special cases of this construction had appeared in the physics literature previously. For the groups \( SU(p, q) \), \( Sp(2n, R) \) and \( SO^*(2n) \) the oscillator method yields directly the irreducible ULWR’s. For the non-compact groups \( SO(n, 2) \), \( E_{6(-14)} \) and \( E_{7(-25)} \) the naive application of the method leads to reducible representations and one then needs to project out the irreducible representations.

To construct the ULWR’s one realizes the generators of the noncompact group \( G \) as bilinears of bosonic oscillators and in the corresponding Fock space \( \mathcal{F} \) one chooses a set of states \( |\Omega> \), referred to as the “lowest weight vector”, which transforms irreducibly under the Lie algebra \( h \) of the maximal compact subgroup \( H \) and which are annihilated by the generators belonging to the \( g^{-1} \) space. Then by acting on \( |\Omega> \) repeatedly with the generators belonging to the \( g^{+1} \) space one obtains an infinite set of states

\[
|\Omega>, \; g^{+1}|\Omega>, \; g^{+1}g^{+1}|\Omega>, ... \quad (2 - 2)
\]

which forms the basis of an irreducible unitary lowest weight representation of \( g \). Irreducibility of the representation of \( g \) follows from the irreducibility of
the representation $|Ω>$ under $h$. Generically the generators of $g$ are realized as bilinears of bosonic oscillators $a_i(r)$ satisfying the canonical commutation relations

$$[a_i(r), a_j(s)] = δ_{ij} δ_{rs} \quad i, j = 1, ..., n$$
$$[a_i(r), a_j(s)] = 0 \quad r, s = 1, ..., p$$

(2 - 3)

where the upper indices $i, j, k, ...$ are the indices in the representation $R$ of $h$ under which the oscillators transform and $r, s, = 1, 2, ..p$ label the different sets of oscillators. We denote the creation operators with upper indices $i, j, ..$ which are hermitian conjugates of the annihilation operators.

$${a_i(r)}^\dagger \equiv a^i(r)$$

Generally, $R$ is the fundamental representation of $h$. We shall refer to $p$ as the number of colors. The generators are written as color singlet bilinears but the lowest weight vector $|Ω>$ and hence the infinite tower of vectors belonging to the corresponding ULWR can carry color. Depending on the non-compact group the minimal number $p$ of colors required to realize the generators can be one or two. If $p_{min} = 1$, then the corresponding unitary irreducible representations will be called singletons and if $p_{min} = 2$, then the corresponding unitary irreducible representations are referred to as doubletons. The non-compact groups $Sp(2n, R)$ admit singleton unitary irreducible representations \[34, 25, 36, 37\] while the groups $SO^*(2n)$ and $SU(n, m)$ admit doubleton unitary irreducible representations \[34, 26, 28, 38, 37\]. The above definition of singleton representations coincides with the definition of singleton representations of the four dimensional AdS group $SO(3,2)$ with the covering group $Sp(4, R)$ discovered by Dirac \[38\]. While there exist only two singleton unitary irreducible representations of $Sp(2n, R)$, one finds infinitely many doubleton unitary irreducible representations of non-compact groups. The two singleton unitary irreducible representations of $Sp(4, R)$ can be associated with the spin zero and spin $\frac{1}{2}$ fields in AdS background. On the other hand the $d = 7$ AdS group $SO^*(8) = SO(6,2)$ admits infinitely many doubleton unitary irreducible representations corresponding to fields of arbitrarily large spin \[26\]. However, we should note that the doubleton fields are not of the form of the most general higher spin fields in $d = 7$. Their decomposition with respect to the little group $SU(4) \equiv Spin(6)$ in $d = 7$ correspond to those representations of $SU(4)$ whose Young-Tableaux have only one row \[26\]. Whereas the general massive higher spin fields correspond to the representations of the little group with arbitrary Young-Tableaux.
If one replaces the bosonic oscillators in the above construction with fermionic ones, then one obtains the unitary representations of the compact forms of the corresponding groups. Extending the definition given above for singleton and doubleton representations to compact groups, one finds that $USp(2n)$ admits doubleton unitary irreducible representations (finitely many) while the group $SO(2n)$ admits two singleton unitary irreducible representations \[^{[10]}\]. The singleton unitary irreducible representations of $SO(2n)$ are the two spinor representations.

In general the compact group $USp(2n)$ admits $n$ non-trivial doubleton unitary irreducible representations. For $USp(4)$ they are the spinor representation $(4)$ and the adjoint representation $(10)$. The two singleton (spinor) representations of $SO(2n)$ combine into the unique singleton (spinor) representation of $SO(2n + 1)$.

### 3 Unitary Lowest Weight Representations of Non-compact Supergroups

The extension of the oscillator method to the construction of the ULWR’s of non-compact supergroups with a Jordan structure with respect to a maximal compact subsupergroup was given in \[^{[35]}\]. This method was further developed and applied to space-time supergroups and Kaluza-Klein supergravity theories in references \[^{[26, 25, 28, 41]}\]. The general construction of the ULWR’s of the noncompact supergroup $OSp(2n/2m, R)$ with the even subgroup $SO(2n) \times Sp(2m, R)$ was studied in \[^{[36]}\] and the ULWR’s of $OSp(2n^*/2m)$ with the even subgroup $SO^*(2n) \times USp(2m)$ in reference \[^{[38]}\].

Consider now the Lie superalgebra $g$ of a non-compact supergroup $G$ that has a 3-graded structure with respect to a compact subsuperalgebra $g^0$ of maximal rank

$$g = g^{-1} \oplus g^0 \oplus g^{+1}$$

To construct the ULWR’s of $g$ one realizes it as bilinears of a set of superoscillators $\xi_A(\xi^A)$ whose first $m$ components are bosonic and the remaining $n$ components are fermionic

$$\xi_A(r) = \begin{pmatrix} a_i(r) \\ \alpha_\mu(r) \end{pmatrix} \quad \xi^A(r) = \begin{pmatrix} a^i(r) \\ \alpha^\mu(r) \end{pmatrix}$$
\[ i, j = 1, ..., m \ ; \ \mu, \nu = 1, ..., n \]
\[ r, s = 1, ..., p. \]

They satisfy the supercommutation relations
\[
[\xi_A(r), \xi^B(s)] = \delta_A^B \delta_{rs}
\]
where \([ , ]\) means an anti-commutator for any two fermionic oscillators and a commutator otherwise. Furthermore we have
\[
[\xi_A(r), \xi_B(s)] = 0 = [\xi^A(r), \xi^B(s)]
\]

Typically \(g^0\) is the Lie superalgebra \(U(m/n)\) and \(\xi^A\) and \(\xi_A\) transform in its fundamental representation and its conjugate, respectively. Generally the operators belonging to the \(g^{-1}\) and \(g^{+1}\) spaces are realized as di-annihilation and di-creation operators respectively. Consider now a lowest weight vector \(|\Omega\rangle\), that transforms irreducibly under \(g^0\) and is annihilated by \(g^{-1}\) operators. Acting on \(|\Omega\rangle\) with the \(g^{+1}\) operators repeatedly one generates a ULWR of \(g\)

\[
g^{-1}|\Omega\rangle = 0 , \quad g^0|\Omega\rangle = |\Omega'\rangle
\]

\[
\{\text{ULWR}\} \equiv \{ |\Omega\rangle , g^{+1}|\Omega\rangle , g^{+1}g^{+1}|\Omega\rangle , ... \}
\]

uniquely labelled by \(|\Omega\rangle\). A supergroup \(g\) admits singleton or doubleton unitary irreducible representations depending on whether \(p_{\min} = 1\) or \(p_{\min} = 2\), respectively. For example the non-compact supergroup \(OSp(2n/2m,R)\) with even subgroup \(SO(2n) \times Sp(2m,R)\) admits singleton representations. The non-compact supergroup \(OSp(2n^*/2m)\) with even subgroup \(SO(2n)^* \times USp(2m)\) admits doubleton unitary irreducible representations, as does the supergroup \(SU(n,m/p)\) with even subgroup \(S(U(n,m) \times U(p))\). There exist only two irreducible singleton supermultiplets of the non-compact supergroup \(OSp(2n/2m,R)\) \cite{25, 26}. On the other hand, the supergroups \(OSp(2n^*/2m)\) and \(SU(n,m/p)\) admit infinitely many irreducible doubleton supermultiplets \cite{26, 38, 28}. 

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Contrary to the situation with noncompact groups, not all noncompact supergroups that have ULWRs admit a three grading with respect to a compact subsupergroup of maximal rank. The method of [35] was generalized to the case when the noncompact supergroup admits a 5-grading with respect to a compact subsupergroup of maximal rank in [42]. For example, the superalgebra of $OSp(2n + 1/2m, R)$ admits a 5-grading with respect to its compact subsuperalgebra $U(n/m)$, but it does not admit a three grading with respect to a compact subsuperalgebra of maximal rank for general $n$ and $m$. All finite dimensional non-compact supergroups do admit a 5-grading (Kantor structure) with respect to a compact subsupergroup of maximal rank [42].

$$g = g^{-2} \oplus g^{-1} \oplus g^{0} \oplus g^{+1} \oplus g^{+2} \quad (3 - 5)$$

The construction of the ULWR’s in this more general case proceeds in a similar manner. One realizes the Lie superalgebra $g$ in terms of superoscillators and chooses a set of states $|\Omega\rangle$ in the corresponding super Fock space transforming irreducibly under the maximal compact subsuperalgebra $g^{0}$ and annihilated by the operators in $g^{-1}$ subspace. The $by$ acting on $|\Omega\rangle$ repeatedly by the generators belonging to $g^{+1}$ one generates an infinite set of states that form the basis of an irreducible ULWR of $g$ [42].

3.1 Massless Supermultiplets of Anti-de Sitter Supergroups

The Poincaré limit of the singleton representations of the $d = 4$ AdS group $SO(3, 2)$ is singular [43]. However, the tensor product of two singleton unitary irreducible representations of $SO(3, 2)$ decomposes into an infinite set of massless unitary irreducible representations which do have a smooth Poincaré limit [43, 44, 45]. Similarly, the tensor product of two singleton supermultiplets of $N$ extended AdS supergroup $OSp(N/4, R)$ decomposes into an infinite set of massless supermultiplets which do have a Poincaré limit [44, 25, 36, 37]. The AdS groups $SO(d - 1, 2)$ in higher dimensions than four that do admit supersymmetric extensions have doubleton representations only. The doubleton supermultiplets of extended $AdS$ supergroups in $d = 5$ ($SU(2, 2/N)$) and $d = 7$ ($OSp(8^*/2N)$) share the remarkable features of the singleton supermultiplets of $d = 4$ AdS supergroups. The tensor product of any two doubleton supermultiplets decompose into an infinite set of massless supermultiplets [26, 28, 47]. In $d = 3$ the AdS group $SO(2, 2)$ is not simple and is isomorphic to $SO(2, 1) \times SO(2, 1)$. Because of this fact,
one has a rich variety of AdS supergroups in \( d = 3 \) \([41]\). Since locally we have \( SO(2, 1) \approx SL(2, R) \approx SU(1, 1) \approx Sp(2, R) \) the AdS supergroups in \( d = 3 \) (and hence in \( d = 2 \)) admit singleton representations \([11]\).

Based on the above and other arguments the following definition of a massless representation (or supermultiplet) of an AdS group (or supergroup) was proposed in \([40]\):

A representation (or a supermultiplet) of an AdS group (or supergroup) is massless if it occurs in the decomposition of the tensor product of two singleton or two doubleton representations (or supermultiplets).

The tensor product of more than two copies of the singleton or doubleton supermultiplets of AdS supergroups decompose into an infinite set of massive supermultiplets in the respective dimensions as has been amply demonstrated within the Kaluza-Klein supergravity theories \([26, 25, 28]\). A noncompact group that admits only doubleton representations can always be embedded in a larger noncompact group that admits singleton representations. In such cases the singleton representation of the larger group decomposes into an infinite tower of doubleton representations of the subgroup.

### 4 Unitary Supermultiplets of \( OSp(1/2m, R) \)

Simple AdS supergroups with an even subgroup \( SO(d - 1, 2) \times K \), where \( K \) is a compact internal symmetry group, exists in \( d \leq 7 \). Embedding AdS groups in simple supergroups in \( d > 7 \) requires enlarging \( SO(d - 1, 2) \) to a larger simple group such as \( Sp(2^{d/2}, R) \). For example the supergroup \( OSp(1/32, R) \) was proposed as the generalized AdS supergroup in \( d=11 \) \([3]\). As mentioned above the noncompact supergroup \( OSp(2n + 1/2m, R) \) has a five grading with respect to its compact subsupergroup \( U(n/m) \) of maximal rank. For the case \( n = 0 \) the five grading of \( OSp(1/2m, R) \) is with respect to the compact subgroup \( U(m) \). In this section we shall apply the results of \([12]\) to construct the unitary lowest weight representations of \( OSp(1/2m, R) \), in particular, those of \( OSp(1/32, R) \). Denoting the Lie superalgebra of \( OSp(1/2m, R) \) as \( g \) we have the following decomposition in a split basis:

\[
g = g^{-2} \oplus g^{-1} \oplus g^0 \oplus g^{1} \oplus g^{2} \quad \quad (4 - 1)
\]

\[
g = L_{ij} \oplus L_i \oplus L^i \oplus L^i \oplus L^{ij}
\]

where \( i, j ... = 1, 2, ... m \). The supersymmetry generators \( L^i \) and \( L_i \) transform in the fundamental representation and its conjugate under the subal-
gebra \( U(m) \) generated by \( L_i^j \), respectively. The generators of the grade \( \pm 2 \) subspaces are symmetric tensors
\[
L_{ij} = L_{ji} \quad (4 - 2)
L^{ij} = L^{ji}
\]
We have the following oscillator realization of \( OSp(1/2m, R) \) [12]:
\[
L_i = \vec{\psi} \cdot \vec{\alpha}_i + \psi^\dagger \cdot \vec{\beta}_i + \frac{\epsilon}{\sqrt{2}}(\gamma + \gamma^\dagger)c_i \quad (4 - 3)
L_{ij} = \vec{\alpha}_i \cdot \vec{\beta}_j + \vec{\alpha}_j \cdot \vec{\beta}_i + \frac{\epsilon}{2}c_i c_j
L^i_j = \vec{\alpha}^i \cdot \vec{\alpha}_j + \vec{\beta}^i \cdot \vec{\beta}_j + \epsilon(c^i c_j + c_j c^i)
L^i = \vec{\psi}^\dagger \cdot \vec{\alpha}^i + \psi \cdot \vec{\beta}^i + \frac{\epsilon}{\sqrt{2}}(\gamma + \gamma^\dagger)c^i
L^{ij} = \vec{\alpha}^i \cdot \vec{\beta}^j + \vec{\alpha}^j \cdot \vec{\beta}^i + \frac{\epsilon}{2}c^i c^j
\]
The bosonic oscillators satisfy the commutation relations
\[
[c_i, c^j] = \delta^j_i \quad (4 - 4)
[a_i(r), a^j(s)] = \delta^j_i \delta_{rs}
[b_i(r), b^j(s)] = \delta^j_i \delta_{rs}
[a_i(r), a_j(s)] = [a_i(r), b_j(s)] = 0
[a_i(r), b^j(s)] = [b_i(r), b_j(s)] = 0
\]
where \( i, j, ... = 1, 2, ...m \); \( r, s, ... = 1, 2, ...f \) and \( \epsilon = 0, 1 \). The non-vanishing anticommutation relations of the fermionic oscillators \( \psi(r) \) and \( \gamma \) are
\[
\{\psi(r), \psi^\dagger(s)\} = \delta_{rs} \quad (4 - 5)
\{\gamma, \gamma^\dagger\} = 1
\]
We define the bilinears appearing above as \( \vec{\alpha}^i \cdot \vec{\beta}^j = \sum_{r=1}^{f} a^i(r)b^j(r) \) etc.. The supercommutation relations of the generators of \( OSp(1/2m, R) \) take on a very simple form in the above basis:
\[
\{L_i, L_j\} = L_{ij} \quad (4 - 6)
\{L^i, L^j\} = L^{ij}
\]
\[
\{L_i, L^j\} = L^j_i \\
[L_i, L_j] = L_{ij} \\
[L^i, L^j] = L^{ij} \\
[L_{ij}, L^{kl}] = \delta^j_k L^i_l + \delta^k_l L^i_j + \delta^l_i L^k_j
\]

Choosing \( f = 0 \) and \( \epsilon = 1 \) yields the singleton supermultiplet and \( f = 1 \) and \( \epsilon = 0 \) leads to the massless supermultiplets \([40]\). If the number of colors \( 2f + \epsilon \) is greater than two one gets massive supermultiplets. The Fock vacuum is defined as the state \( |0\rangle \) annihilated by all the bosonic and fermionic annihilation operators. A lowest weight vector \( |\Omega\rangle \) of a unitary irreducible representation of \( OSp(1/2m, R) \) is also a lowest weight vector of its even subgroup \( Sp(2m, R) \). Acting on the lowest weight vector \( |\Omega\rangle \) by the supersymmetry generators \( L^i \) one generates new lowest weight vectors of \( Sp(2m, R) \). The action of \( L^{ij} \) on such a lowest weight vector generates the higher modes within a unitary irreducible representation of \( Sp(2m, R) \).

We shall identify the particle states of a unitary irreducible representation of the ”generalized AdS group” \( Sp(2m, R) \) with the Fourier modes of a field on that generalized AdS space. As in references \([26, 25, 28]\) these fields will be uniquely identified by the labels of their lowest weight vectors under the maximal compact subgroup of the AdS group which is \( U(m) \) in this case. For labelling fields in AdS space with respect to the \( U(m) \) transformation properties of the corresponding lowest weight vectors we shall use the Young tableaux notation. A field in AdS space whose lowest weight vector transforms in a representation of \( U(m) \) with the Young tableau \((n_1, n_2, \ldots, n_m)\) will be labelled as

\[ \Xi_{(n_1, n_2, \ldots, n_m)_p} \]  

where \( p = 2f + \epsilon \) is the number of colors and \( n_i \) denotes the number of boxes in the \( i \)th row of the Young tableau. In some suitably chosen units the AdS energy \( E \) of this field will be given by

\[ E = s + m(f + \frac{\epsilon}{2}) = s + \frac{1}{2}mp \]

where \( s = n_1 + n_2 + \cdots + n_m \)

This field is a fermion if \( s \) is an odd integer, and a boson when it is an even integer.
4.1 Unitary Supermultiplets of $OSp(1/32, R)$

Let us now study the unitary supermultiplets of $OSp(1/32, R)$ in detail. For a single color i.e $f = 0$ and $\epsilon = 1$ there exists only a single lowest weight vector of $OSp(1/32, R)$ in the super Fock space, namely the vacuum state

$$L_i|0> = 0 \implies L_{ij}|0> = 0$$ (4 - 10)

Acting on $|0>$ by the generators $L^{ij}$ repeatedly one generates the infinite set of states corresponding to the Fourier modes of a scalar field $\Phi$ with eight units of AdS energy. The AdS energy is given by the eigenvalue of the $U(1)$ generator $L_i^i$ that determines the 5-grading. Let us denote an irreducible bosonic (fermionic) field in our generalized AdS space as $\Phi_d(E)$ ($\Psi_d(E)$) where $d$ labels the $SU(16)$ representation of the lowest weight vector of $Sp(32, R)$ and $E$ stands for the AdS energy. Thus the scalar field defined by the Fock vacuum $|0>$ with a single color is $\Phi_1(8)$:

$$|0> \implies \Phi_1(8) = \Xi_{(0,0,\ldots,0)}$$ (4 - 11)

By acting with the supersymmetry generator $L^i$ on the lowest weight vector of $OSp(1/32, R)$ we generate another lowest weight vector of $Sp(32, R)$:

$$L_{ij}L^k|0> = L_{ij}\gamma^i\epsilon^j|0> = 0$$ (4 - 12)

The corresponding AdS field is a fermion $\Psi_{16}(9)$ .

$$\gamma^i\epsilon^j|0> \implies \Psi_{16}(9) = \Xi_{(1,0,\ldots,0)}$$ (4 - 13)

There are no other lowest weight vectors of $Sp(32, R)$ for a single color. Hence the singleton supermultiplet consists of a scalar field and a spinor field:

$$\Phi_1(8) \oplus \Psi_{16}(9)$$ (4 - 14)

By our definition [40] the massless supermultiplets are obtained by choosing two colors i.e $f = 1$ and $\epsilon = 0$. The vacuum vector is always a lowest weight vector of $OSp(1/32, R)$ . Acting on it with supersymmetry generators we obtain two additional lowest weight vectors of $Sp(32, R)$. These lowest weight vectors and the corresponding fields in AdS space are

$$|0> \implies \Phi_1(16) = \Xi_{(0,0,\ldots,0)}$$ (4 - 15)

$$L^i|0> = \psi^i a^i|0> \implies \Psi_{16}(17) = \Xi_{(1,0,\ldots,0)}$$ (4 - 16)
The lowest weight vector $a^i |0>$ of $OSp(1/32, R)$ leads to the following massless supermultiplet of fields

$$a^i |0> \rightarrow \Psi_{16}(17) = \Xi(1,0,0,\ldots)_2$$  \hspace{1cm} (4 - 18)

$$L^i a^i |0> \rightarrow \Phi_{136}(18) = \Xi(2,0,0,\ldots)_2$$  \hspace{1cm} (4 - 19)

For two colors the other unitary massless supermultiplets of fields are of the form $(n > 1)$.

$$\Xi(n,0,\ldots,0)_2 \oplus \Xi(n+1,0,\ldots,0)_2$$  \hspace{1cm} (4 - 20)

As one increases the number of colors the possible unitary supermultiplets become much richer and for more than two colors they become massive supermultiplets. Typically, the shortest supermultiplet for a given number of colors is obtained by choosing the vacuum $|0>$ as the lowest weight vector of an extended AdS supergroup \cite{23, 24, 28}. We shall refer to these supermultiplets as the vacuum supermultiplets. The massless vacuum supermultiplets of AdS supergroups in $d = 4$ go over to CPT self-conjugate supermultiplets in the Poincare limit. Therefore, in analogy with the situation in $d = 4$ we may sometimes refer to the vacuum supermultiplets as CPT self-conjugate supermultiplets in other dimensions. For three colors the vacuum supermultiplet consists of the following fields:

$$|0> \rightarrow \Phi_1(24) = \Xi(0,0,\ldots,0)_3$$  \hspace{1cm} (4 - 21)

$$L^i |0> \rightarrow \Psi_{16}(25) = \Xi(1,0,\ldots,0)_3$$  \hspace{1cm} (4 - 22)

$$L^i L^j |0> \rightarrow \Phi_{120}(26) \oplus \Phi_{136}(26) = \Xi(1,1,0,\ldots,0)_3 \oplus \Xi(2,0,\ldots,0)_3$$  \hspace{1cm} (4 - 23)

$$L^i L^j L^k |0> \rightarrow \Psi_{560}(27) \oplus \Psi_{1360}(27) = \Xi(1,1,1,0,\ldots,0)_3 \oplus \Xi(2,1,0,\ldots,0)_3$$  \hspace{1cm} (4 - 24)

The vacuum supermultiplet for four colors consist of the following fields:

$$\Phi_1(32) \oplus \Psi_{16}(33) \oplus \Phi_{120}(34) \oplus \Phi_{136}(34) \oplus \Phi_{1360}(35) \oplus \Psi_{560}(35) \oplus \Phi_{5440}(36) \oplus \Phi_{1820}(36) \oplus \Phi_{7140}(36)$$

$$= \Xi(0,\ldots,0)_4 \oplus \Xi(1,0,\ldots,0)_4 \oplus \Xi(1,1,0,\ldots,0)_4 \oplus \Xi(2,0,\ldots,0)_4 \oplus \Xi(2,1,0,\ldots,0)_4 \oplus \Xi(1,1,1,0,\ldots,0)_4 \oplus \Xi(1,1,1,1,0,\ldots,0)_4 \oplus \Xi(2,1,1,0,\ldots,0)_4 \oplus \Xi(2,1,1,1,0,\ldots,0)_4 \oplus \Xi(2,2,0,\ldots,0)_4$$

By increasing the number of colors one obtains more generalized AdS supermultiplets. The rules for determining the $U(16)$ labels of the fields of these supermultiplets are simple and have been given in \cite{25, 26, 36, 38, 42}.  

13
5 $OSp(1/32, R)$ and Physics in Ten and Eleven Dimensions

The superalgebra $OSp(1/32, R)$ has a contraction to the 11-dimensional Poincare superalgebra with two and five form central charges [46, 45] which can be written as [47]:

$$\{Q_A, Q_B\} = (\text{C} \Gamma^M)_{AB} P_M + \frac{1}{2} (\text{C} \Gamma^{MN})_{AB} Z_{MN} + \frac{1}{5!} (\text{C} \Gamma^{M_1..M_5})_{AB} Y_{M_1..M_5}$$

where $\Gamma^M(M, N = 0, 1, 2, .., 10)$ are the 11 dimensional Dirac matrices, $\Gamma^{M_1..M_p}$ $(p = 2, 5)$ their antisymmetrized products, and $C$ is the charge conjugation matrix. In the Majorana representation the 32 component spinors of the Lorentz group $SO(10, 1)$ are real and $C$ can be chosen to be $\Gamma^0$. The above superalgebra is to be interpreted simply as the translation superalgebra with central charges. The Lorentz group $SO(10, 1)$ acts as its automorphism group [47].

Now the group $SO(10, 2)$ is also the conformal group in ten dimensions and hence the group $Sp(32, R)$ can also be interpreted as a generalized conformal group in ten dimensions. Generalized spacetimes and superspaces with associated generalized Lorentz and conformal groups and supergroups were introduced and studied via the theory of Jordan algebras and Jordan superalgebras [48]. The conformal Lie algebras $g$ (Lie superalgebras) of Jordan algebras (superalgebras) can be given a 3-graded decomposition:

$$g = g^{-1} \oplus g^0 \oplus g^1$$

where the subspace $g^0$ is spanned by the generalized Lorentz group generators plus the generator of dilations. The subspaces $g^{-1}$ and $g^1$ are spanned by the generators of translations and special conformal transformations, respectively. Within this general framework the group $Sp(32, R)$ is then the conformal group of the Jordan algebra $J_{16}^R$ of $16 \times 16$ real symmetric matrices with the generalized Lorentz group $Sl(16, R)$. It has the decomposition

$$Sp(32, R) = K^{\alpha \beta} \oplus (M_\beta^\alpha + D) \oplus T_{\alpha \beta}$$

where the $T_{\alpha \beta} = T_{\beta \alpha}$ $\alpha, \beta = 1, 2, .., 16$ are translation generators, $K^{\alpha \beta} = K^{\beta \alpha}$ the generators of special conformal transformations, $D$ the dilation generator and $M_\beta^\alpha$ are the generators of the generalized Lorentz group $Sl(16, R)$. The $N = 1$ supersymmetric extension of this conformal algebra
by $Q$ and $S$ type supersymmetry is simply the superalgebra $OSp(1/32, R)$ with the 5-graded decomposition:

$$OSp(1/32, R) = K^{\alpha\beta} \oplus S^\alpha \oplus (M_\alpha^\beta + D) \oplus Q_\alpha \oplus T_{\alpha\beta}$$ (5-4)

The conformal supersymmetry generators satisfy the anticommutation relations

$$\{Q_\alpha, Q_\beta\} = T_{\alpha\beta}$$ (5-5)
$$\{S^\alpha, S^\beta\} = K^{\alpha\beta}$$
$$\{S^\alpha, Q_\beta\} = L_\alpha^\beta$$

where $L_\alpha^\beta$ are the generators of $Gl(16, R) = Sl(16, R) + D$. The decomposition of the generators of $OSp(1/32, R)$ in terms of $SO(9,1)$ covariant operators can be read off from the results of [47]:

$$T_{\alpha\beta} = (CT^m)_{\alpha\beta}P_m + \frac{1}{5!}(CT^{mnpqr})_{\alpha\beta}Z^+_{mnpqr}$$ (5-6)
$$K^{\alpha\beta} = (CT^m)_{\alpha\beta}K_m + \frac{1}{5!}(CT^{mnpqr})_{\alpha\beta}Z^-_{mnpqr}$$
$$L_\alpha^\beta = \delta_\beta^\alpha D + \frac{1}{2}(\Gamma^{mn})_\beta^\alpha M_{mn} + \frac{1}{4!}(\Gamma^{mnpq})_\beta^\alpha Y_{mnpq}$$

where $P_m, K_m, D, M_{mn}$ are the usual generators of ten dimensional translations, special conformal transformations, dilations and Lorentz transformations. The $Z^\pm$ are the selfdual and antiselfdual 5-form charge generators and $Y$ is a 4-form charge generator. Note that these generators are not central charges of the ten dimensional conformal superalgebra. When restricted to maximal parabolic subalgebras $g_0^{1/2} \oplus g^{\pm1/2} \oplus g^{\pm1}$ the generators $Z^\pm$ can be interpreted as central charges.

In the last reference of [47] the algebras 5 - 1 and 5 - 6 were referred to as the M-theory superalgebra and the membrane superalgebra, respectively, and the differences between them were stressed. The fact that both the uncontracted form of 5 - 1 and the membrane superalgebra turn out to be $OSp(1/32, R)$ was considered as a coincidence. As we shall explain below the fact that these two algebras are related via $OSp(1/32, R)$ is not a coincidence from our point of view and is the well-established connection between AdS supergroups in $d + 1$ dimensions and the conformal supergroups in $d$ dimensions.

15
Now the M-theory superalgebra $\text{OSp}(1/32, R)$ is obtained from the supergroup $\text{OSp}(1/32, R)$ via an Inönü-Wigner type contraction. An intermediate step in this contraction is to decompose the algebra $\text{OSp}(1/32, R)$ with respect to the $\text{SO}(10, 2)$ subgroup of $\text{Sp}(32, R)$. Interpreting this $\text{SO}(10, 2)$ as the AdS group in 11 dimensions one then takes the standard Inönü-Wigner contraction to the Poincare group. This embedding of AdS group $\text{SO}(10, 2)$ in $\text{Sp}(32, R)$ is achieved by identifying the fundamental representation of $\text{Sp}(32, R)$ with the spinor representation of $\text{SO}(10, 2)$. The fundamental representation of $\text{Sp}(32, R)$ decomposes as $16 \oplus 16$ under its maximal compact subgroup $\text{SU}(16) \times U(1)$. The fundamental representation 16 of $\text{SU}(16)$ is then identified with the Weyl spinor of the $\text{SO}(10)$ subgroup of $\text{SO}(10, 2)$. In contracting to the Poincare group, the fundamental representation of $\text{Sp}(32, R)$ goes over to the spinor representation of the Lorentz group $\text{SO}(10, 1)$ in eleven dimensions.

If $\text{OSp}(1/32, R)$ is a generalized AdS symmetry of M-theory then among its massless supermultiplets there must be one whose contraction includes the fields of 11 dimensional supergravity. A study of the massless supermultiplets given in the previous section makes evident that there do not exist such an irreducible supermultiplet. Thus $\text{OSp}(1/32, R)$ can not naively be identified with the generalized AdS supersymmetry of M-theory. This is actually not surprising since the embedding of $\text{SO}(10, 2)$ in $\text{Sp}(32, R)$ is not parity invariant. As was shown in [50, 31] M-theory and its low energy effective theory in eleven dimensions are parity invariant. In embedding $\text{SO}(10, 2)$ in $\text{Sp}(32, R)$ we identified its spinor representation with the fundamental representation of $\text{Sp}(32, R)$. However, this embedding is not unique as $\text{SO}(10, 2)$ has two spinor representations of dimension 32, i.e the left handed and right handed spinors that are related by parity. We shall denote the superalgebra in which the the fundamental representation of $\text{Sp}(32, R)$ is identified with the left-handed (right-handed) spinor representation of $\text{SO}(10, 2)$ as $\text{OSp}(1/32, R)_L$ ($\text{OSp}(1/32, R)_R$). If we are to have parity invariant supermultiplets we must embed the full 64 dimensional Dirac spinor of $\text{SO}(10, 2)$ in a larger supergroup. The minimal supergroup satisfying this requirement is $\text{OSp}(1/32, R)_L \times \text{OSp}(1/32, R)_R$ with the proviso that one restricts oneself to parity invariant representations [31]. In the contraction of $\text{OSp}(1/32, R)_L \times \text{OSp}(1/32, R)_R$ to obtain the M-theory algebra one identifies the two spinor representations hence breaking the $\text{SO}(10, 2)_L \times \text{SO}(10, 2)_R$ symmetry down to the diagonal $\text{SO}(10, 1)$ subgroup. The situation is rather similar to the situation in 2+1 dimensions
where the AdS group $SO(2, 2)$ decomposes as

$$SO(2, 2) = SO(2, 1)_+ \times SO(2, 1)_- = Sp(2, R)_+ \times Sp(2, R)_-$$ (5 - 7)

The spinorial generators transform in the representations $(\frac{1}{2}, 0)$ and $(0, \frac{1}{2})$ of the AdS group. The complete list of AdS supergroups in $d = 3$ was given in [1]. For our discussion and comparison with eleven dimensions we shall consider AdS supergroups of the form

$$OSp(p/2, R)_+ \times OSp(q/2, R)_-$$ (5 - 8)

There exist a one parameter family of Chern-Simons supergravity actions of the form [51, 52]

$$S = a_+ S_+ + a_- S_-$$ (5 - 9)

where $S_+$ and $S_-$ are the C-S actions for $OSp(p/2, R)_+$ and $OSp(q/2, R)_-$, respectively, and $a_\pm$ are some real constants. Not all these AdS supergravity theories in $d = 3$ have a Poincare limit. Those AdS gravity and supergravity theories that do not have a Poincare limit are generally referred to as “exotic” [2, 51]. To obtain an AdS supergravity which admits a Poincare limit one needs to take different forms of the two OSp factors such that they differ by an overall sign in the anticommutators of the supersymmetry generators. One crucial difference between the three dimensional AdS supergroups $OSp(p/2, R)_+ \times OSp(q/2, R)_-$ and the eleven dimensional generalized AdS supergroups of the form $OSp(1/32, R)_L \times OSp(1/32, R)_R$ is that the spinorial representations of the supersymmetry charges in the two factors are isomorphic in three dimensions and non-isomorphic in eleven dimensions.

Consider now the chiral component $OSp(1/32, R)_L$ of the eleven dimensional generalized AdS supergroup. It admits a singleton supermultiplet which consists of a scalar field $\Phi$ and a left-handed spinor $\Psi_L$ of $SO(10, 2)$. Since it is a singleton supermultiplet we expect its quantum field theory to be a conformally invariant theory in ten dimensions. The massless supermultiplets are obtained by tensoring two singleton supermultiplets and decomposing them into irreducible supermultiplets. By decomposing the maximal compact subgroup $U(16)$ representations of the lowest weight vectors with respect to the $SO(10) \times U(1)$ subgroup of $SO(10, 2)$ one can determine the field content of the supermultiplets in $AdS_{11}$. The massless vacuum supermultiplet of $OSp(1/32, R)$ include a scalar field $\Phi$, a left-handed spinor $\Psi_L$ and an anti-symmetric tensor field $A_{\mu\nu\rho}(\mu, \nu, .. = 0, 1, ...10)$ and does not include the graviton. The massless graviton supermultiplet of $OSp(1/32, R)_L$
include the graviton $g_{\mu\nu}$, a gravitino $\Psi_\mu$ a spinorial field whose lowest weight vector transforms in the $672$ representation of the little group $SO(10)$, and two bosonic fields whose lowest weight vectors transform in the $1050$ and $2772$ of $SO(10)$. To obtain an irreducible vacuum supermultiplet that contains the graviton one needs to consider four colors, which by our definition would correspond to a massive supermultiplet. The other chiral component $OSp(1/32, R)_R$ has similar supermultiplets with the left handed spinorial indices replaced by right handed spinorial indices.

Now if we consider the full generalized AdS supergroup $OSp(1/32, R)_L \times OSp(1/32, R)_R$ then to construct supersymmetric AdS quantum field theories that are parity invariant we need to consider left-right symmetric supermultiplets. In particular, we would like to construct an AdS supergravity that in the Poincare limit leads to the 11-dimensional supergravity theory of \[\text{[53]}\]. In the contraction to the 11 dimensional Poincare superalgebra we will consider the diagonal subgroup $OSp(1/32, R)_{L-R}$ of $OSp(1/32, R)_L \times OSp(1/32, R)_R$. Here we should stress the important point that the groups $SO(10,2)_L$ and $SO(10,2)_R$ intersect within their $SO(10,1)$ subgroups inside $OSp(1/32, R)_{L-R}$. The simplest parity invariant irreducible representation of $OSp(1/32, R)_L \times OSp(1/32, R)_R$ is the tensor product of the singleton supermultiplets of $OSp(1/32, R)_L$ and $OSp(1/32, R)_R$. This tensor product however is infinitely reducible with respect to the diagonal subsupergroup $OSp(1/32, R)_{L-R}$. The irreducible supermultiplets contained in this tensor product are the supermultiplets of $OSp(1/32, R)_{L-R}$ for two colors constructed in the previous section. ( For $OSp(1/32, R)_L$ and $OSp(1/32, R)_R$ they would be the massless multiplets.) However, for the diagonal supergroup $OSp(1/32, R)_{L-R}$ they are the shortest possible supermultiplets consisting of parity invariant irreducible ones plus those that come in parity conjugate pairs. Therefore by an abuse of terminology we shall refer to these supermultiplets as "doubleton supermultiplets" of $OSp(1/32, R)_{L-R}$ and the supermultiplets one obtains by tensoring two doubletons as "massless" supermultiplets. It is easy to show that the decomposition of the parity invariant irreducible supermultiplets of $OSp(1/32, R)_L \times OSp(1/32, R)_R$ with respect to $OSp(1/32, R)_{L-R}$ yields supermultiplets corresponding to an even number of colors. In other words they consist of doubletons and those supermultiplets that can be obtained by tensoring doubleton supermultiplets.

Based on the structure of the supermultiplets for two colors and the knowledge of currently known supersymmetric theories in ten and eleven dimensions \[\text{[49]}\] I expect the doubleton supermultiplets not to have a Poincare
limit in eleven dimensions and hence their field theories must be conformally invariant theories on the boundary of the $AdS_{11}$ which we can identify with the ten dimensional Minkowski space. Indeed, there exist conformally invariant supergravity theories with and without matter couplings in ten dimensions \cite{54,49}. It will be important to study their connection to the doubleton supermultiplets given above. Of all the supermultiplets for two colors, the vacuum doubleton supermultiplet consisting of a scalar field $\Phi$, a spinor field $\Psi$ and an antisymmetric tensor field $A_{\mu\nu\rho}$ ($\mu,\nu,.. = 0,1,..9$) seems to be of central importance as will be explained shortly.

Denoting a bosonic and a fermionic field corresponding to a lowest weight vector transforming in the representation $r$ of the little group $SO(10)$ as $B^{(r)}$ and $F^{(r)}$, respectively, the massless vacuum supermultiplet ($p=4$) of $OSp(1/32,R)_{L-R}$ has the decomposition:

$$
\begin{align*}
\Phi_1^{(32)} &= B_1 \\
\Psi_{16}^{(33)} &= F_{16} \\
\Phi_{120}^{(34)} &= B_{120} \\
\Phi_{136}^{(34)} &= B_{10} + B_{126} \\
\Psi_{560}^{(35)} &= F_{560} \\
\Psi_{1360}^{(35)} &= F_{16} + F_{136} + F_{1280} \\
\Phi_{1820}^{(36)} &= B_{770} + B_{1050} \\
\Phi_{5440}^{(36)} &= B_{1} + B_{54} + B_{210} + B_{1050} + B_{4125} \\
\Phi_{7140}^{(36)} &= B_{45} + B_{210} + B_{945} + B_{5940}
\end{align*}
$$

Since it contains the fields that in the Poincare limit go over to the fields of eleven dimensional supergravity we shall refer to it as the massless $AdS_{11}$ graviton supermultiplet. In addition to the fields of 11 dimensional Poincare supergravity, the above supermultiplet contains extra bosonic and fermionic fields. Furthermore, as is typical in AdS supersymmetric theories there is a mismatch of bosonic and fermionic degrees of freedom. One ought to keep in mind however that in taking the Poincare limit many of the generators of $OSp(1/32/R)_{L-R}$ will become central charges and some degrees of freedom of the fields will become gauge degrees of freedom. In any case, what is evident is that an AdS supergravity theory in eleven dimensions would require additional fields than those of ordinary Poincare supergravity. This may explain why the attempts to construct a gauged version of 11 dimensional supergravity with 128 bosonic and 128 fermionic degrees of freedom have so far produced negative results \cite{55}. We should also note that as in $d = 2 + 1$
dimensions it may be possible to construct exotic AdS supergravity theories in \( d = 10 + 1 \) dimensions that have no Poincare limit.

Assuming that the \( AdS_{11} \) supergravity corresponding to the above supermultiplet exists, then it is related to the superconformal field theory of the vacuum doubleton supermultiplet in ten dimensions as the gauged \( N = 8 \) supergravity in \( d = 5 \) theory is related to the \( N = 4 \) super Yang-Mills theory in \( d = 4 \) which is conformally invariant \([28, 30]\). In \([28]\) the entire spectrum of the \( S^5 \) compactification of \( IIB \) supergravity in \( d = 10 \) were fitted into massless and massive vacuum supermultiplets of the \( N = 8 \) AdS superalgebra \( SU(2, 2/4) \). The doubleton supermultiplet of \( SU(2, 2/4) \) is nothing but the \( N = 4 \) Yang-Mills supermultiplet living on the boundary of \( AdS_5 \) which is identified with the \( d = 4 \) Minkowski space. The entire spectrum of \( IIB \) can be obtained by tensoring the doubleton supermultiplet with itself repeatedly and restricting to the CPT self-conjugate color singlet supermultiplets. As explained in \([3]\) this is in complete parallel with the recent conjecture of Maldacena that the \( N = 4 \) supersymmetric Yang-Mills theory with the gauge group \( SU(n) \) is equivalent to the \( IIB \) superstring on \( AdS_5 \times S^5 \) in the large \( n \) limit.

If M-theory admits a phase whose low energy effective theory is an \( AdS_{11} \) supergravity theory corresponding to the above supermultiplet then I expect M-theory in that phase to be dual to the "CPT self-conjugate" doubleton field theory in ten dimensions. In particular the entire spectrum of the AdS phase of M-theory, massless as well as massive, should then be obtainable by tensoring the doubleton supermultiplet with itself repeatedly. In other words, the doubleton field theory would then be the holographic relativistic quantum field theory underlying M-theory in the sense of \([59, 60, 9]\).

As in \( d = 2 + 1 \) dimensions there may exist AdS supergravity theories based on chiral superalgebras of the form \( OSp(1/32, R)_L \times OSp(1/32, R)_L \) which do not have any Poincare limit in eleven dimensions. However, they may have a Poincare limit in ten dimensions which is related to the type IIB supergravity and/or conformal supergravity. \(^6\) Both the left-right symmetric superalgebra \( OSp(1/32, R)_L \times OSp(1/32, R)_R \) and the chiral superalgebra

\(^6\) We should note that the topological Chern-Simons supergravity theories in eleven dimensions based on the non-unitary adjoint representations of \( OSp(1/32, R) \) and \( OSp(1/32, R) \times OSp(1/32, R) \) were studied in \([56, 57, 31]\). In the first paper of \([57]\) it was shown that there is a contraction of \( OSp(1/32, R) \) that leads to a non-standard supergravity theory with a Poincare and parity invariant gravity sector. The connection of such non-standard supergravity theories to Cremmer-Julia-Scherk supergravity, if any, is not known \([58]\).
OSp\((1/32, R)_L \times OSp(1/32, R)_L\) can be embedded in the simple superalgebra \(OSp(1/64, R)\) which had been discussed in connection with the 11-dimensional supergravity theory \([46, 45]\) and more recently for unification of various duality groups in string/M-theory \([61]\). The unitary supermultiplets of \(OSp(1/64, R)\) can be written down easily using the results of section 4 above and those of \([37]\). The physical interpretation of these supermultiplets and their relation to M-theory as well as the construction of ten dimensional singleton and doubleton superconformal field theories will be discussed elsewhere \([62]\).

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