Color superfluidity and trionic state of three-component lattice fermionic atoms

Shin-ya Miyatake\textsuperscript{1}, Kensuke Inaba\textsuperscript{2,3}, and Sei-ichiro Suga\textsuperscript{4}

\textsuperscript{1} Department of Applied Physics, Osaka University, Suita, Osaka 565-0871, Japan
\textsuperscript{2} NTT Basic Research Laboratories, NTT Corporation, Atsugi 243-0198, Japan
\textsuperscript{3} JST, CREST, Sanbancho, Chiyoda-ku, Tokyo 102-0075, Japan
\textsuperscript{4} Department of Materials Science and Chemistry, University of Hyogo, Himeji 671-2280, Japan

E-mail: suga@eng.u-hyogo.ac.jp

Abstract. We investigate three-component (colors) fermionic atoms with anisotropic attractive interactions in optical lattices at half filling using the dynamical mean field theory. In the weakly interacting region the color superfluid (CSF) state emerges, where atoms with two of three colors form the Cooper pairs. As the interaction is increased, a first-order quantum phase transition to the trionic state occurs, where singlet bound states of three different color atoms are formed. We show that the trionic state survives up to a fairly large anisotropic region. The phase diagram for the CSF and trionic states is obtained.

1. Introduction

Studies of ultracold fermionic atoms have attracted considerable attention of both theorists and experimentalists. Feshbach resonances provide the means for controlling both the strength of the interaction between fermionic atoms and its sign \cite{1}. By loading fermionic atoms into optical lattices, diverse interaction configurations can be introduced. Using these techniques, fascinating phenomena caused by many-body effects have been observed. Recently, a balanced mixture of three different hyperfine states of $^6$Li fermionic atoms was succeeded in creating \cite{2, 3}. The enhanced three-body loss coefficient was observed at a certain magnetic field. Its origin was discussed in connection with the formation of the trionic state which is singlet bound states of three fermionic atoms with different internal degrees of freedom (colors). For the three-component fermionic atoms in optical lattices, it was shown theoretically that for the small attractive interaction a color superfluid (CSF) emerges, where atoms with two of three colors form the Cooper pairs and the third ones remain a Fermi liquid \cite{4, 5, 6, 7}. As the attractive interaction becomes strong, a quantum phase transition from the CSF to the trionic state occurs \cite{6, 7}. In experiments, it is difficult to realize the isotropic interactions between atoms with different colors. In fact, at the magnetic field where the enhanced three-body loss coefficient was observed, two scattering lengths are nearly the same, while the remaining one takes a different enhanced value \cite{2}. These results mean the anisotropy of the attractive interactions between atoms with three colors. These findings motivated us to investigate the stability of the CSF and trionic state in the three-component fermionic atoms in optical lattices with anisotropic interactions.
2. Model and method
In this paper, we investigate the effects of anisotropic attractive interactions in the three-component fermionic atoms in optical lattices. The system is assumed to keep a balanced population of fermionic atoms with three colors. The fermionic optical lattice systems can be well described by the following Hubbard-type Hamiltonian [8]:

\[ \mathcal{H} = -t \sum_{\langle i,j \rangle, \alpha} c_{i\alpha}^\dagger c_{j\alpha} + \sum_{i, \alpha \neq \beta} \frac{U_{\alpha\beta}}{2} n_{i\alpha} n_{i\beta} - \sum_{i, \alpha} \mu_{\alpha} n_{i\alpha}, \]  

(1)

where \( c_{i\alpha} \) annihilates a fermionic atom with color \( \alpha (= 1, 2, 3) \) at the \( i \)th lattice site, \( n_{i\alpha} = c_{i\alpha}^\dagger c_{i\alpha} \), the subscript \( \langle i, j \rangle \) indicates the sum over the nearest-neighbor sites, and \( U_{\alpha\beta}(<0) \) is the attractive interaction between two atoms with different colors \( \alpha \) and \( \beta \). We set \( U_{12} \equiv U \) and \( U_{23} = U_{31} \equiv U' \) with \( U'/U < 1 \).

We make use of a dynamical mean field theory (DMFT). In the DMFT, the lattice model is mapped onto a single impurity model connected dynamically to a bath. The Green’s function is obtained via the self-consistent solution of this impurity problem. This retains nontrivial local quantum fluctuations missing in conventional mean-field theories. We apply here the two-site DMFT method [9], which allows us to study quantum phase transitions of lattice fermions qualitatively. To study the color superfluid of lattice fermions, we extend this method to the case when the superfluid order exists [10]. It is expected that the fermionic atoms for the strongest attractive interaction are paired to form the \( s \)-wave Cooper pair with an order parameter \( \Delta \equiv \langle c_{i1} c_{i2} \rangle \). We thus choose the gauge \( \Delta_{12} = \Delta \) and \( \Delta_{23} = \Delta_{31} = 0 \). The order parameter of the color superfluid is thus \( \Delta \). We calculate the quasiparticle weights \( Z_\alpha \), which are defined by \( Z_\alpha = [1 - d \text{Re}[\Sigma_\alpha(\omega)]/d\omega|_{\omega=0}]^{-1} \) in terms of the self-energy \( \Sigma_\alpha(\omega) \) of the respective color. In our case, \( Z_1 = Z_2 \equiv Z \) and \( Z_3 \equiv Z' \). The quasiparticle weight \( Z \) represents the coherent spectral weight of the Bogoliubov quasiparticle for \( \alpha = 1, 2 \). We also calculate the single-particle excitation spectra (SPES) \( \rho_\alpha(\omega) \). The density of states for an infinite-dimensional Bethe lattice is adopted, which is independent of color. The chemical potentials are set to be \( \mu_1 = \mu_2 = (U + U')/2 \) and \( \mu_3 = U' \) so that particle-hole symmetry can be satisfied. In this case, the system is half filling. We take no account of the possibility of the density wave state. The hopping integral \( t \) is used in units of energy.

3. Numerical results

![Figure 1](image_url)

Figure 1. (Color Online) (a) The CSF order parameter \( \Delta \) and (b) the quasiparticle weights \( Z \) and \( Z' \) as a function of \( U \) for \( U'/U = 0.6 \).

In Fig. 1, we show the order parameter \( \Delta \), and the quasiparticle weights \( Z \) and \( Z' \) for the fixed ratio \( U'/U = 0.6 \) as a function of \( U \). For the weak interaction, \( \Delta \) takes a very small
value and increases with increasing $|U|$. The quasiparticle weights $Z$ and $Z'$ significantly decrease from unity in different ways, reflecting the difference in the renormalization of the attractive interaction. As $U$ becomes strong, the quasiparticle weights jump to 0 with vanishing $\Delta$ simultaneously. These results indicate that a discontinuous quantum phase transition occurs at $U = -3.0$. In correlated lattice fermion systems, the zero quasiparticle weight means a transition to the Mott insulating state. For the strong attractive interaction, it is considered that atoms with different three colors form the trion. In the DMFT, the trion is caused by local correlation effects and thus localized on a site, yielding a Mott insulator.

![Figure 2](image)

*Figure 2.* (Color Online) The single-particle excitation spectra for $U = -2.1$, $-2.6$ and $-3.1$ with $U'/U = 0.6$. (a) $\rho_1(\omega)(=\rho_2(\omega))$ and (b) $\rho_3(\omega)$.

We investigate the quantum phase transition in a spectral point of view. In Fig. 2, the SPES are shown for several values of $U$ with $U'/U = 0.6$. For the CSF phase ($U = -2.1$, $-2.6$), we observe the increase of the superfluid gap around $\omega = 0$ in $\rho_1(\omega)(=\rho_2(\omega))$ and the narrowing of the coherent peaks, as the interaction becomes strong. These results mark the significant renormalization of the Bogoliubov quasiparticle in agreement with the decrease in $Z$. In $\rho_3(\omega)$, the quasiparticle peak at $\omega = 0$ is seen, indicating the Fermi liquid. The results demonstrate that atoms with colors 1 and 2 form the Cooper pair, while atoms with color 3 remain the Fermi liquid. For $U = -3.1$, the SPES show qualitatively the same profile which is typical of the Mott insulator. Actually, the spectral gap of the trionic state measured between the two peaks is $\sim 4.96$ ($\sim 3.72$) for $\rho_1(\omega)$ ($\rho_3(\omega)$). This value is the same as $|U + U'| = 4.96$ ($2|U'| = 3.72$) which is the energy needed to take one atom of color 1 (3) away from the trionic bound state. The results for the quasiparticle weights, the CSF order parameter, and the SPES are consistent with each other. In this way, we have confirmed that the CSF-trion quantum phase transition occurs even in the anisotropic interaction system.

Performing the same two-site DMFT calculations for various values of $U'/U$ and $U$, we obtain the phase diagram at half filling as shown in Fig. 3. Note that the CSF-trion transition is of the first order irrespective of $U'/U$. We find that the trionic state is robust against the anisotropy of the attractive interaction and survives in a fairly large anisotropic region.

4. Discussions and Summary
In this study, we have shown that the CSF-trion transition is of the first order. This result is in contrast with that in Refs. [6, 7], where the CSF-trion transition below half filling is of...
Figure 3. Phase diagram of the three-component attractive fermionic atoms in optical lattices at half filling.

the second order. Recently, two of the present authors have investigated the zero- and finite-temperatures properties of the present system using the self-energy functional approach [11]. In Ref. [11], it was shown that for half filling the CSF-trion transition is of the first order. The discrepancy may originate in the difference of the accuracy for the calculation of correlation effects. In Refs. [6, 7], the trionic ground-state energy was estimated as $3Un$, where $n$ is the filling. In the present calculation and also in Ref [11], we obtain the lower trionic ground-state energy than $3Un$. When the ground state energy in our calculations is expanded in terms of $1/U$, the zeroth order corresponds to $3Un$. The higher order corrections lower the ground state energy, which may yield a discontinuous change of the ground state energy between the CSF and the trionic state, resulting in the first-order quantum phase transition.

In summary, we have investigated the CSF-trion quantum phase transition of the three-component fermionic atoms in optical lattices at half filling using the DMFT. We have shown that the trionic state is robust against the anisotropy of the attractive interactions.

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