EXPLORING THE ROLE OF SUB-MICRON-SIZED DUST GRAINS IN THE ATMOSPHERES OF RED L0–L6 DWARFS

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ABSTRACT

We examine the hypothesis that the red near-infrared colors of some L dwarfs could be explained by a “dust haze” of small particles in their upper atmospheres. This dust haze would exist in conjunction with the clouds found in dwarfs with more typical colors. We developed a model that uses Mie theory and the Hansen particle size distributions to reproduce the extinction due to the proposed dust haze. We apply our method to 23 young L dwarfs and 23 red field L dwarfs. We constrain the properties of the dust haze including particle size distribution and column density using Markov Chain Monte Carlo methods. We find that sub-micron-range silicate grains reproduce the observed reddening. Current brown dwarf atmosphere models include large-grain (1–100 μm) dust clouds but not sub-micron dust grains. Our results provide a strong proof of concept and motivate a combination of large and small dust grains in brown dwarf atmosphere models.

Key words: brown dwarfs – dust, extinction

Supporting material: figure set

1. INTRODUCTION AND PROBLEM

Brown dwarfs are substellar objects with intermediate masses bridging the stellar and planetary regimes. Brown dwarfs do not have a high enough mass to sustain hydrogen fusion in their cores, so they keep cooling with time. As brown dwarfs age, they also contract and their surface gravities increase. The cool temperatures of brown dwarfs allow condensates to form in their atmospheres, which shape their emergent spectra (Burrows et al. 2001).

There is a wide spread in \( J – K_s \) colors of L dwarfs of a given spectral type. So-called “red” L dwarfs have redder-than-average near-infrared (NIR) colors, and have notably redder spectral slopes through the NIR than typical L dwarfs (Faherty et al. 2013). Low-gravity objects tend to be systematically redder but some field-aged L dwarfs also have redder colors. Red, low-gravity L dwarfs can be considered “exoplanet analogs” and are of particular interest (Kirkpatrick et al. 2008; Cruz et al. 2009; Faherty et al. 2009, 2013).

While the range of NIR colors of L dwarfs has not been fully explained, it is commonly attributed to variation in metallicity, gravity, and/or cloud properties (Saumon & Marley 2008; Marley et al. 2012). Brown dwarfs are thought to have refractory like grains that can account for the observed reddening in the NIR. Model spectral energy distribution (SED) fitting sometimes gives unrealistically low gravities and small \( f_{\text{sed}} \) (cloud sedimentation efficiency parameter) values (Ackerman & Marley 2001; Cushing et al. 2008) and radii too small compared to evolutionary models (Liu et al. 2013) for red L dwarfs. A better dust treatment is needed that can account for the observed reddening in L dwarf spectra.

In efforts to understand the red objects empirically, it has been found that dust described by the interstellar reddening law can de-redden red L dwarf spectra to look like standard objects (Looper et al. 2010; Marocco et al. 2014). Interstellar reddening is the extinction of starlight caused by the interstellar dust. The interstellar grains, which have radii less than 1 \( \mu m \), suppress blue light more effectively than red light. As a result, distant stars look redder than they actually are. The interstellar reddening law (extinction law) is an empirical relationship between the extinction at any wavelength \( A(\lambda) \) and the visual extinction \( A(V) \) (Cardelli et al. 1989). We do not expect significant interstellar reddening in brown dwarfs since they are so close to the Sun on a galactic scale, so the use of the interstellar reddening law to de-redden brown dwarf spectra is not physically motivated. However, interstellar-medium-(ISM-) like grains (<1 \( \mu m \)) in the atmospheres of red L dwarfs might produce results similar to the interstellar reddening law and explain the observed reddening.

Looper et al. (2010) de-redden the optical spectra of TWA 30 (young M5 star) using the interstellar reddening law described by Cardelli et al. (1989). Although the reddening seen toward TWA 30 may be due to dust in the disk within the line of sight, Looper et al. (2010) demonstrated that the interstellar reddening law can be successfully used in this case and it can be useful for dealing with reddening due to small grains other than the ISM. Marocco et al. (2014) de-redden the spectra of several L dwarfs including ULAS J222711 – 004547.
(L7pec) using two interstellar extinction curves (Cardelli et al. 1989; Fitzpatrick 1999) and found that sub-micron-sized dust grains can explain the reddening effect. Furthermore, Cushing et al. (2006) ascribe a flattening in the spectra of mid-type L dwarfs seen at 9–11 μm to a population of small silicate grains above the main cloud deck. These three results imply that brown dwarfs with red SEDs may have small grains like the ISM that are smaller than 1 μm in addition to larger grains ranging from 1 to 100 μm currently included in the models in their atmospheres that scatter and absorb the emergent light.

In this work, we develop a prescription for a dust haze in L dwarf atmospheres and test whether it can account for the characteristics of red L dwarf spectra. By constraining the nature of this dust haze we aim to better understand the physical cause of the reddening in brown dwarfs. Independently, two previous studies explored a similar dust haze analysis, Marocco et al. (2014) for brown dwarfs and Bonnefoy et al. (2015) for directly imaged exoplanets. Marocco et al. (2014), Bonnefoy et al. (2015), and this work are all motivated by the success of the interstellar reddening law in de-reddening unusually red L dwarf spectra and aim to explain the observed reddening by introducing a layer of small dust grains in the upper atmospheres. Despite the similar ideas and concepts, the method we describe here is distinct from those previous studies as described in Sections 4 and 6.3.

The sample of L dwarfs studied in this analysis is presented in Section 2. In Section 3, we present our method of using spectral observations to estimate the reddening. We explain our dust haze model in detail in Section 4, and model fitting method in Section 5. Finally, we present our results in Section 6 and conclusion in Section 7.

2. SAMPLE AND SPECTRAL OBSERVATIONS

In order to study the observed reddening, we compiled a sample of low-resolution NIR spectra of 23 L dwarfs with low-gravity features in the optical (Cruz et al. 2009) and 23 red field L dwarfs (Kirkpatrick et al. 2010). Our red field objects have spectral features indicative of field gravity with $J − K$ colors redder than the spectral standards (Kirkpatrick et al. 2010). This definition of “red” is specific to the purpose of this analysis. For example, Faherty et al. (2013) defined “red” as having a redder $J − K$ color than the mean $J − K$ of normal objects in the spectral type as opposed to comparing it to the spectral standard. Therefore, red objects in our sample may be different from objects that are defined red by Faherty et al. (2013) or other papers. The objects in our sample are listed in Table 1.

Our sample includes 18 new spectra of red L dwarfs obtained with the SpeX spectrograph on the Infrared Telescope Facility (IRTF; Rayner et al. 2003). Observations were obtained over 28 nights during 2003–2011. The targets and observation dates are listed in Table 1. All objects were observed in clear and dry conditions. The targets were observed dithered pairs (ABBA) to enable pair-wise subtraction. We used the 0′′5 slit and prism-dispersed mode to obtain $\lambda/\Delta \lambda \approx 120$ spectra covering 0.7–2.5 μm. The data were reduced using the SpeXtool package (Cushing et al. 2004), nearby A0 V stars were observed for flux calibration and telluric correction (Vacca et al. 2003), and internal flat field and Ar arc lamp exposures were obtained for pixel response and wavelength calibration.

3. ESTIMATING THE OBSERVED REDDENING

In order to estimate the observed reddening, we compared red L dwarfs (including young and field) to the field spectral standards. The observed reddening was obtained by dividing the spectrum of the field standard by the spectrum of the red L dwarf. The top panel of Figure 1 shows spectra of a red L dwarf and a spectral standard, illustrating the redder spectral slope of the red object. The bottom panel is the ratio of the two spectra and visualizes the estimated observed reddening of the red object. The small-scale features seen in the observed reddening are due to gravity-sensitive spectral features such as FeH, VO, and the triangular-shaped H band (Kirkpatrick et al. 2006; Allers & Liu 2013). We assume that these features are not caused by reddening, so we treat the overall shape of the observed reddening as a smooth curve.

4. MODELING THE OBSERVED REDDENING

In this paper, we develop a prescription for a hypothesized dust haze of small particles in the atmospheres of the so-called “red” L dwarfs to explain the observed reddening in their SEDs. In order to model the observed reddening, we used Mie theory to calculate the “raw” extinction coefficients due to forsterite grains. Then we averaged the raw extinction coefficients over various particle size distributions to calculate the “effective” extinction coefficients and to generate a model grid to compare with the observed reddening.

4.1. Dust Haze Prescription

In Figure 2, we show an illustration of the proposed dust haze in the upper atmospheres of red L dwarfs. Our model prescribes that the dust haze must be high in the atmospheres so that it is too cool (since the temperature decreases as the altitude increases) to radiate significantly in the NIR. The prescribed dust haze lies above the main cloud deck, so the dust grains in the haze affect the emergent spectra. Our prescription does not have thickness or height, therefore we are not constraining the position or dimensions of the dust haze any further than lying above the main cloud deck.

The dust haze grains were modeled for forsterite grains. As shown by Lodders & Fegley (2006), forsterite (Mg$_2$SiO$_4$) is thought to exist in L dwarf atmospheres among other dust species such as corundum (Al$_2$O$_3$), enstatite (MgSiO$_3$), and iron. Corundum condenses at higher temperatures in the atmospheres of late M dwarfs. Liquid iron and silicates condense in early L dwarfs between 1600 and 1840 K. Since iron grains form deeper in the atmosphere, the dust is most likely silicate. The silicate grains in the L dwarf atmospheres are thought to be a mixture of forsterite and enstatite. Since the extinction curves of forsterite and enstatite have similar shapes, forsterite was used in our analysis.

Extinction is the sum of absorption and scattering, and is the fraction of incoming light that gets affected by interactions with particles. Reddening is a type of extinction, which occurs when extinction is more effective at shorter (bluer) wavelengths than at longer (redder) wavelengths, and has the effect of making the spectral slope redder. We used Mie theory to model the reddening effect of the proposed dust haze on the emergent spectra of red L dwarfs. Mie theory applies when the scattering particle is spherical and its size is similar to the wavelength of the scattered light, which is appropriate for sub-micron-sized grains in the NIR. For larger particles, Mie scattering is
| 2MASS Designation | Sp. Type | J, Ks | Δ(J – Ks) | Discovery Reference | Spectral Type | SpEx Prism Reference | SpeX Prism Observation Date |
|-------------------|----------|------|----------|---------------------|--------------|---------------------|---------------------------|
| 0345432+254023    | L0       | 1.33 | 0        | Kirkpatrick et al. (1999) | Kirkpatrick et al. (1999) | Burgasser & McElwain (2006) | ... |
| 2130446-084520    | L1       | 1.33 | 0        | Kirkpatrick et al. (2008) | Kirkpatrick et al. (2008) | Kirkpatrick et al. (2010) | ... |
| 13054019-2541059  | L2       | 1.67 | 0        | Ruiz et al. (1997) | Kirkpatrick et al. (1999) | Burgasser (2007) | ... |
| 1506544+132106    | L3       | 1.63 | 0        | Gizis et al. (2000) | Gizis et al. (2000) | Burgasser (2007) | ... |
| 21580457-1550998  | L4       | 1.86 | 0        | Kirkpatrick et al. (2008) | Kirkpatrick et al. (2008) | Kirkpatrick et al. (2010) | ... |
| 1507476-162738    | L5       | 1.46 | 0        | Reid et al. (2000) | Kirkpatrick et al. (2000) | Burgasser (2007) | ... |
| 10101480-0406499  | L6       | 1.89 | 0        | Cruz et al. (2003) | Cruz et al. (2003) | Reid et al. (2006) | ... |

| 2MASS Designation | Sp. Type | J, Ks | Δ(J – Ks) | Discovery Reference | Spectral Type | SpEx Prism Reference | SpeX Prism Observation Date |
|-------------------|----------|------|----------|---------------------|--------------|---------------------|---------------------------|
| 01415823-4633574  | L0γ      | 1.74 | 0.410    | Kirkpatrick et al. (2006) | Cruz et al. (2009) | Kirkpatrick et al. (2006) | ... |
| 00325584-4405058  | L0γ      | 1.51 | 0.178    | Goldman et al. (1999) | Cruz et al. (2009) | Bardalez Gagliufi et al. (2014) | ... |
| 02103857-3015313  | L0γ      | 1.57 | 0.236    | A. Cruz et al. (2016, in preparation) | A. Cruz et al. (2016, in preparation) | This Paper | 2003 Sep 04 |
| 0241115-032658    | L0γ      | 1.76 | 0.434    | Cruz et al. (2007) | Cruz et al. (2009) | This Paper | 2006 Aug 21 |
| 03231002-461237   | L0γ      | 1.69 | 0.357    | Reid et al. (2008) | Cruz et al. (2007) | This Paper | 2007 Nov 13 |
| 2213449-213607    | L0γ      | 1.62 | 0.290    | Cruz et al. (2007) | Cruz et al. (2009) | Bardalez Gagliufi et al. (2014) | ... |
| 23153135+0617146  | L0γ      | 1.80 | 0.466    | A. Cruz et al. (2016, in preparation) | A. Cruz et al. (2016, in preparation) | This Paper | 2007 Nov 14 |
| 171113+232633     | L0γ      | 1.44 | 0.113    | Cruz et al. (2007) | A. Cruz et al. (2016, in preparation) | This Paper | 2008 Jul 13 |
| 0536199-192039    | L2γ      | 1.91 | 0.244    | Cruz et al. (2007) | Gagne et al. (2015) | This Paper | 2003 Sep 04 |
| 1515237+0941148   | L3γ      | 2.01 | 0.149    | Reid et al. (2008) | Reid et al. (2008) | This Paper | 2008 Jul 13 |
| 1726000+153819    | L3.5γ    | 1.81 | 0.380    | Kirkpatrick et al. (2000) | Allers & Liu (2013) | Bardalez Gagliufi et al. (2014) | ... |
| 05012406-0010452  | L4γ      | 2.02 | 0.159    | Reid et al. (2008) | Cruz et al. (2009) | Filipuzzi et al. (2015) | ... |
| 22495345+0404406  | L4γ      | 2.22 | 0.369    | Geballe et al. (2002) | Gagne et al. (2015) | Allers et al. (2010) | ... |
| 05120636-2949540  | L5γ      | 2.18 | 0.718    | Cruz et al. (2003) | Cruz et al. (2003) | Bardalez Gagliufi et al. (2014) | ... |
| 03264225-2102072  | L5 β/γ   | 2.21 | 0.752    | Gizis et al. (2003) | Gagne et al. (2015) | This Paper | 2007 Nov 13 |
| 21543454-1055308  | L5 β/γ   | 2.24 | 0.381    | Gagne et al. (2014) | Gagne et al. (2014) | This Paper | 2003 Aug 11 |
| 03552337+1133437  | 0355-typeb | 2.52 | 1.06     | Reid et al. (2008) | Reid et al. (2008) | Faherty et al. (2013) | ... |
| 1615425+495321    | 0355-typeb | 2.48 | 0.623    | Cruz et al. (2007) | Cruz et al. (2007) | Gagne et al. (2015) | This Paper |

| 2MASS Designation | Sp. Type | J, Ks | Δ(J – Ks) | Discovery Reference | Spectral Type | SpEx Prism Reference |
|-------------------|----------|------|----------|---------------------|--------------|---------------------|
| 0235593-2331205   | L1       | 1.48 | 0.154    | Burgasser (2008) | Gizis et al. (2001) | Burgasser (2008) | ... |
| 05431887+6422528   | L1       | 1.52 | 0.187    | Reid et al. (2008) | Reid et al. (2008) | Bardalez Gagliufi et al. (2014) | ... |
| 06022216+6336391   | L1:      | 1.58 | 0.263    | Reid et al. (2008) | Reid et al. (2008) | Bardalez Gagliufi et al. (2014) | ... |
| 00165953-4056541   | L3       | 1.88 | 0.254    | Kirkpatrick et al. (2008) | Kirkpatrick et al. (2008) | Burgasser & McElwain (2006) | ... |
| 23392527+3507165   | L3.5     | 1.77 | 0.144    | Reid et al. (2008) | Reid et al. (2008) | This Paper | 2003 Sep 04 |
| 11009956+4957470   | L3.5     | 1.81 | 0.178    | Reid et al. (2008) | Reid et al. (2008) | This Paper | 2004 Nov 08 |

Table 1
L. Dwarfs Used in this Paper

Spectral Standard L Dwarfs

Low-gravity L Dwarfs
| 2MASS Designation | Sp. Type | 2MASS J – Ks | 2MASS Δ(J – Ks)$^a$ | Discovery Reference | Spectral Type | SpeX Prism Reference | SpeX Prism Observation Date |
|------------------|----------|--------------|-----------------|--------------------|--------------|-------------------|-----------------------------|
| 00511078-1544169 | L3.5     | 1.81         | 0.181           | Kirkpatrick et al. (2000) | L3.5         | Kirkpatrick et al. (2000) | Burgasser et al. (2010)     |
| 23174712-4838501 | L4pec    | 1.97         | 0.109           | Reid et al. (2008)     | L4pec        | Reid et al. (2008)       | Kirkpatrick et al. (2010)   |
| 0337036-175807   | L4.5     | 2.04         | 0.180           | Kirkpatrick et al. (2000) | L4.5         | Kirkpatrick et al. (2000) | Bardalez Gagliuffi et al. (2014) |
| 0208236+2737400  | L5       | 1.84         | 0.382           | Kirkpatrick et al. (2000) | L5           | Kirkpatrick et al. (2000) | Burgasser et al. (2010)     |
| 0835425-081923   | L5       | 2.03         | 0.573           | Cruz et al. (2003)     | L5           | Cruz et al. (2003)       | This Paper                  |
| 03582255-4116060 | L5       | 2.01         | 0.548           | Reid et al. (2008)     | L5           | Reid et al. (2008)       | This Paper                  |
| 09054654+5623117 | L5       | 1.67         | 0.205           | Reid et al. (2008)     | L5           | Reid et al. (2008)       | This Paper                  |
| 12281523-1547342 | L5       | 1.61         | 0.151           | Delfosse et al. (1997) | L5           | Delfosse et al. (1997)   | Burgasser et al. (2010)     |
| 12392727+5515371 | L5       | 1.92         | 0.459           | Kirkpatrick et al. (2000) | L5           | Kirkpatrick et al. (2000) | Burgasser et al. (2010)     |
| 03101401-275652  | L5       | 1.84         | 0.376           | Cruz et al. (2007)     | L5           | Cruz et al. (2007)       | Bardalez Gagliuffi et al. (2014) |
| 06244595-4521548 | L5       | 1.89         | 0.425           | Reid et al. (2008)     | L5           | Reid et al. (2008)       | Bardalez Gagliuffi et al. (2014) |
| 0652307+471034   | L5       | 1.82         | 0.357           | Cruz et al. (2003)     | L5           | Cruz et al. (2003)       | This Paper                  |
| 14382359+5722168 | L5       | 1.59         | 0.129           | Zhang et al. (2009)    | L5           | Zhang et al. (2009)      | Bardalez Gagliuffi et al. (2014) |
| 1326298-003831   | L5       | 1.90         | 0.435           | Fan et al. (2000)      | L5           | Fan et al. (2000)        | Geballe et al. (2002)       |
| 22120703+3430351 | L5       | 1.95         | 0.486           | Reid et al. (2008)     | L5           | Reid et al. (2008)       | This Paper                  |
| 21481633+4003594 | L6       | 2.38         | 0.49            | Looper et al. (2008)   | L6           | Looper et al. (2008)     | Looper et al. (2008)        |
| 22443167+2043433 | L6.5     | 2.45         | 0.56            | Dahn et al. (2002)     | L6.5         | Dahn et al. (2002)       | Looper et al. (2008)        |

Notes.

$^a$ ΔJ – Ks is the difference in J – Ks colors between the red L dwarf and the corresponding spectral standard object.

$^b$ 0355-type objects are defined by Gagne et al. (2015). A conservative estimate of L3–L6γ is adopted for the spectral type range. For our analysis, 0355-type objects are compared to the L6 spectral standard object.
independent of wavelength while for smaller particles, it is wavelength dependent. Mie scattering reduces to strongly wavelength-dependent Rayleigh scattering when particle sizes are much smaller than wavelength. We employed a Mie code, described in Toon & Ackerman (1981), and the refractive indices of forsterite (G. Sloan, 2016, private communication) to compute the “raw” extinction coefficients, $Q_{\text{ext}}(r, \lambda)$, for particles of radii between 0.01 and 10 $\mu$m.

4.2. Particle Size Distributions

In order to calculate effective extinction coefficients from the “raw” extinction coefficients directly computed from the Mie code, we need to choose a particle size distribution $n(r)$. We considered three different particle size distributions commonly used to model grains, clouds, and hazes in various settings: power-law, Gaussian, and Hansen distributions. Figure 3, compares the shape of the three distributions. We describe the motivation behind using these particle size distributions and results below.

We considered a power-law particle size distribution $n(r) \propto r^p$ with $p = -3$ and particle radius $r$ ranging between 0.01–10 $\mu$m to model theoretical extinction due to the proposed small dust grains. Power-law particle size distributions with $p \approx -3.5$ are typically used to characterize interstellar dust and grains in the circumstellar disks around young brown dwarfs (Mathis et al. 1977; Luhman et al. 2005; Draine 2006;
Figure 3 shows two different power-law particle size distributions with $p = -3$ and $-3.5$.

We also considered a Gaussian particle size distribution

$$n(r) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(r-\mu)^2}{2\sigma^2}}$$

with the mean radius $\mu = 0.5 \ \mu m$ and a width $\sqrt{2\sigma} = 0.1 \times \mu$. This particle size distribution is adopted from Marocco et al. (2014), who de-reddened ULAS J222711–004547 using corundum and enstatite, and other red L dwarfs using corundum with Gaussian particle size distributions and found $\mu \sim 0.5 \mu m$.

We also considered the Hansen particle size distribution, which is a variation of the gamma distribution and is expressed as follows.

$$n(r) = r^{1+b/2} e^{-r/\alpha}$$

where $\alpha$ is the mean effective radius and $b$ is the effective variance. Following Hansen (1971), $a$ and $b$ are defined as

$$a = \frac{\int_0^\infty r^2\pi^2 n(r)dr}{\int_0^\infty \pi^2 n(r)dr}$$

$$b = \frac{\int_0^\infty (r-a)^2\pi^2 n(r)dr}{\int_0^\infty \pi^2 n(r)dr}$$

respectively.

The Hansen particle size distribution successfully reproduces the observed particle size distributions of different types of water clouds in Earth’s atmosphere (fair weather cumulus, altostratus, and stratus clouds) as shown in Figure 1 of Hansen (1971). Figure 3 shows two Hansen particle size distributions with the mean effective radius $a = 0.2 \ \mu m$ for effective variances $b = 0.1$ and 0.5. The Hansen particle size distribution with a large variance (dashed green, $b = 0.5$) is similar to power-law particle size distributions in the regime of small particle size ($\lesssim 0.1 \mu m$).

4.3. Computing Modeled Extinction Curves and Comparing to Observations

In order to account for a range of particle sizes and to smooth over the small-scale interference patterns in forsterite extinction coefficients, we computed effective extinction coefficients by averaging the raw extinction coefficients over a particle size distribution. Effective extinction coefficients are defined as

$$Q_{ext}(\lambda) = \frac{\int_{min}^{max} \pi^2 n(r)\lambda Q_{ext}(r,\lambda)n(r)dr}{\int_{min}^{max} \pi^2 n(r)dr}$$

where $n(r)dr$ is the number of particles per unit volume with radius between $r$ and $r + dr$. The integration limits we employed are 0.01–10 $\mu m$. Particles smaller than this range would be too small (a few atoms) to scatter light and particles that exceed 10 $\mu m$ tend to be gray at all wavelengths (i.e., the extinction they cause will be independent of wavelength).

In order to compare the observed reddening to the modeled extinction, we assume that the observed flux, $I$, from a red L dwarf can be modeled as

$$I(\lambda) = f I_0(\lambda)e^{-\tau(\lambda)}$$

where $I_0(\lambda)$ is the flux of the field standard L dwarf, $f$ is a scaling factor, and $\tau(\lambda)$ is the optical depth of the dust haze in the red L dwarf atmosphere, assuming that the red L dwarf can be described by the field L dwarf surrounded by the dust haze as in Figure 2. The scaling factor $f$ is determined by the distances and sizes of the objects.

$$f = \frac{d_0^2}{d^2} \frac{R_0^2}{R^2}$$

where $d_0$ and $d$ are the distances to the field and the red L dwarfs, and $R$ and $R_0$ are the radii of the red and the field L dwarfs, respectively.

Solving Equation (1) for the optical depth we get

$$\tau(\lambda) = \frac{\ln f + \ln l_0(\lambda)}{I(\lambda)}$$

The optical depth $\tau(\lambda)$ is related to the Mie extinction coefficient $Q_{ext}(\lambda)$ as

$$\tau(\lambda) = N\pi a^2 Q_{ext}(\lambda)$$

where $N$ is the column density of the haze layer and $a$ is the effective radius of the scattering grains that we assume are composed of forsterite. $Q_{ext}(\lambda)$ is the wavelength dependent forsterite grain extinction coefficient, which we calculated using the Mie code (Toon & Ackerman 1981). We averaged the forsterite coefficients over a particle size distribution $n(r)$ as described in Equation (4).

Combining Equations (7) and (8), we get

$$\ln \frac{l_0(\lambda)}{I(\lambda)} = N\pi a^2 Q_{ext}(\lambda) + C$$

where $N$, $a$, and $Q_{ext}(\lambda)$ are the column density of the dust haze, the mean effective radius found from the particle size distribution $n(r)$, and the effective forsterite extinction coefficients of small grains from Equation (4), respectively. The constant term $C$ accounts for the scaling factor $f$ and any differences in gray atmospheric opacity between the L dwarfs. Since the reddening we observe is wavelength dependent, we relegate any gray component to the $C$ term. $C$ is independent of wavelength because $Q_{ext}(\lambda)$ approaches a constant for $a \gg \lambda$. The right-hand side of Equation (9) is the modeled extinction and the left-hand side is the observed reddening. As explained later in Section 5, we used MCMC methods to constrain the parameters in the modeled extinction ($N, a$) that best reproduce the observed reddening.

4.3.1. Evaluation of Particle Size Distributions and Creating a Model Grid

Figure 4 shows model fits to the observed extinction using the three different particle size distributions. The top panel shows the observed reddening (black, same as the bottom panel in Figure 1), and theoretical extinction curves using the Hansen, power-law, and Gaussian particle size distributions. The $\chi^2$ value and the degrees of freedom for each fit are reported in the legend. The bottom panel shows the residual between the observed reddening and the theoretical extinction curve as a percentage of the observed flux. Figure 4 demonstrates that all three modeled extinction curves fit the observed reddening reasonably. The Hansen and power-law particle size distributions reproduce the smooth shape of the
observed reddening, while the Gaussian particle size distribution results in a less smooth extinction curve.

For the remainder of our analysis, we adopt the Hansen particle size distribution because of the favorable behavior shown in Figure 4, the fact that it reflects the microphysics and structure of Earth’s clouds, and because it has various helpful properties that are useful for algebraic manipulation (Hansen 1971).

In Figure 5, we show our model grid of forsterite extinction coefficients for various Hansen particle size distributions. The effective forsterite extinction curves for small mean particle sizes ($a \lesssim 0.4 \mu m$) more closely resemble the observed reddening than those with a larger mean particle size. For these small mean particle sizes, extinction coefficients are wavelength dependent and resemble the curved shapes of the observed reddening (Figure 1). For particles larger than 0.4 $\mu m$, extinction coefficients are less wavelength dependent and the resulting extinction shapes are flat ("gray"), which do not fit the observed reddening. The curves for $a = 1.0 \mu m$ are shown as representatives of the curves with $a > 0.4 \mu m$.

The range of effective variance $b$ was determined based on the shape of the Hansen particle size distribution. Small effective variances ($b < 0.1$) result in a particle size distribution concentrated at the mean effective radius, and large effective variances ($b > 1$) make the particle size distribution wide and resemble a power-law particle size distribution. Thus, we decided to use Hansen particle distribution with a parameter grid of mean effective radius $a$ between 0.05 and 0.4 $\mu m$, and effective variance $b$ between 0.1 and 1.0 as priors for the rest of our analysis.

5. METHODS: FITTING THE MODELS TO THE OBSERVED REDDENING

We use Markov Chain Monte Carlo (MCMC) fitting to estimate the best-fit parameters and their uncertainties. MCMC is a Bayesian inference method that provides a sampling approximation of the posterior probability distribution function (PDF). An MCMC run produces a chain of positions in parameter space, and a histogram of these positions provides the approximation of the posterior PDF. MCMC allows for more in-depth probabilistic data analysis than $\chi^2$ minimization; for example, it efficiently approximates the full posterior PDF, which in turn provides uncertainties on and illustrates covariances between model parameters.

The Goodman–Weare (G-W) algorithm improves upon the Metropolis–Hastings (M-H) algorithm by changing the method for choosing trial positions (Goodman & Weare 2010). The G-W algorithm deploys an ensemble of chains, known as "walkers," instead of a single chain. The trial position for each walker is chosen from the ensemble’s location in parameter space, with some probability for choosing a position outside the occupied region. This method does not require hand-tuning the step size for each parameter, and the selection of trial positions can be parallelized. The G-W algorithm more efficient than the M-H algorithm in both human working hours and computation time (Goodman & Weare 2010; Foreman-Mackey et al. 2013).

We use the open-source python implementation of the G-W algorithm, emcee (Foreman-Mackey et al. 2013), to fit the observed reddening with our model grid described in Section 4.3. The modeled extinction curves are parameterized by the mean particle size $a$ and effective variance $b$ for the Hansen distribution and the column density $N$ of forsterite grains. We assume an unnormalized flat prior probability distribution for each parameter. The effective extinction coefficients are linearly interpolated and the modeled extinction at each wavelength point is calculated as Equation (9).

We also model the vertical offset between the observed reddening and the extinction curve with a constant $C$, and include a tolerance parameter $s$. The tolerance $s$ estimates the uncertainty in the model as a single value across the extinction curve; it accounts for the fact that the photon-noise uncertainties are smaller than the typical difference between each observed reddening point and the corresponding point on the extinction curve. If we denote observed reddening points as $r = \{r_i\}$ and the corresponding uncertainties as $\sigma_r = \{\sigma_{r_i}\}$, we...
compute the natural logarithm of the likelihood function as
\[
\ln L(r|a, b, N, C, s, \sigma_0) = -\frac{1}{2} \sum_i \left[ \frac{(r_i - (N\pi a^2 Q_i^2 + C)}{(\sigma_{0,i}^2 + s^2)) + \ln(2\pi (\sigma_{0,i}^2 + s^2)) \right].
\]

(10)

The natural logarithm of the posterior PDF is given by
\[
\ln(PDF)(a, b, N, C, s|r, \sigma_0) = \ln L(r|a, b, N, C, s, \sigma_0) + \ln P(a, b, N, C, s, \sigma_0).
\]

(11)

We assume an unnormalized flat prior on each parameter, so \(P(a, b, N, C, s, \sigma_0) = 1\) and \(\ln P(a, b, N, C, s, \sigma_0) = 0\).

We pass a function for \(\ln(PDF)\) to emcee, which uses that function to determine acceptance of each step in parameter space. We typically use 100 walkers. After we iterate for 200 steps to generate a new set of initial positions for the walkers, we reset the walkers and restart from the new initial positions. We iterate for 2000 steps after a burn-in period of 200 steps.

6. RESULTS AND DISCUSSION

6.1. Fitting Dust Haze Parameters

We used the MCMC method described in Section 5 to fit dust haze extinction models to the observed reddening curves for each object and constrain the physical properties of the proposed dust haze. We plot the one- and two-dimensional (1D and 2D) marginalized posterior PDFs for all parameters in the figure set of Figure 6. Models corresponding to 100 randomly drawn parameter sets from the posterior PDF are shown with the data in the figure set of Figure 7. The constrained properties of the dust haze include mean effective radius, effective variance of the Hansen distribution (Section 4.2), and column density of the dust haze.

In the figure set of Figure 6, we show the posterior distributions for each parameter. Each figure has 1D distributions for the parameters and 2D contours for each combination of parameters. Gaussian-like 1D distributions and round 2D contours indicate no covariances. Quantiles (16%, 50%, 84%) are shown with dashed lines and are used to report the uncertainties on the parameter fits. In many objects, the PDFs for the mean effective radius \(a\), column density \(N\), tolerance parameter \(\log s\), and vertical offset constant \(C\) have clear peaks.
and therefore are well constrained. The variance $b$, on the other hand, is not well constrained in most objects.

The column density values $N$ are comparable to the value of typical brown dwarf atmospheres ($\sim 10^8 \text{ cm}^{-2}$), which indicates that our results are reasonably realistic. However, there is a correlation between parameters $a$ and $N$ as seen in the 2D contours. The relationship between $a$ and $N$ is shown in Equation (9). In order to compute the optical depth, we multiply the column density $N$, scattering cross section $\pi a^2$, and effective extinction coefficient $Q_{\text{eff}}(\lambda)$. The effective extinction curves are similar over a small range of grain radii, so we are constraining the product $Na^2$. Therefore, $a^2$ and $N$ are inversely proportional and this relationship appears in the posterior distributions.

In the figure set of Figure 7, we show the resulting model fits to the observed reddening. The black line is the observed reddening (Section 3) and the green lines are 100 models randomly drawn from the posterior distributions. The models reproduce the overall shape of the observed reddening.

In the figure set of Figure 8, we compare a de-reddened spectrum of a red L dwarf, the spectrum of the field standard L dwarf, and the original red L dwarf spectrum. The de-reddened spectrum is the spectrum of a red L dwarf corrected by the best-fit forsterite extinction curve determined by the MCMC analysis. $\chi^2$ values between the red and standard spectra, and between the de-reddened and standard spectra are reported. The $\chi^2$ values are used simply to quantitatively demonstrate that the de-reddened spectrum is a better fit to the standard spectrum than the original red L dwarf spectrum. These results show that the sub-micron-sized dust haze prescription can successfully account for the red SED and $J - K_s$ colors of L dwarfs.

In Figure 10, we show the improvement in $\chi^2$ due to the proposed dust haze prescription. The ratio of $\chi^2$ before de-reddening to $\chi^2$ after de-reddening is plotted against $\Delta(J - K_s)$ color, which is the difference in $J - K_s$ color between the red L dwarf and the field standard L dwarf. We use $\Delta(J - K_s)$ because we compare the spectra of red L dwarfs to the standards to isolate the observed reddening. The value of $\chi^2$ is improved for all objects, which shows that the de-reddened spectra fit the field standards better, and in most cases substantially better ($>10\times$), than the original red L dwarf spectra.

### 6.2. Correlation with Gravity

It has been widely noted that low-gravity L dwarfs have redder NIR SEDs compared to the field-gravity spectral standards. A discussion of how clouds behave differently at low and moderate gravity is given in Marley et al. (2012). In addition to the cloud height differences discussed by Marley et al. (2012), one might expect to see a correlation between dust haze properties and low-gravity spectral features. We hypothesized that the proposed dust haze might dissipate over time due to grain growth by condensation. Large condensed particles are expected to fall out of the dust haze as a result of the sedimentation rate exceeding the remixing rate by eddy turbulence (Marley et al. 1999). Reduced convective velocities and perhaps less vigorous gravity wave excitation with age could also contribute to more efficient dust settling. Thus, young, low-gravity L dwarfs might have optically thicker dust hazes, which may explain their red NIR colors. The proposed dust haze could also explain the reddening within field L dwarfs and the properties of the dust haze might be correlated with age.
In Figures 11 and 12, we show scatter plots of mean effective radius $a$ and column density $N$ versus $\Delta (J - K)$ color, respectively. Green symbols denote low-gravity objects and magenta symbols denote field-gravity. Circles denote objects with PDFs with strong constraints, while squares and diamonds denote objects with weak constraints for some of the

Figure 9. (a) Example of de-reddened spectra (equivalent to the figure set of Figure 8) and model fits (equivalent to the figure set of Figure 7) for spectral types L0–L3. The left column shows the spectral standard (black), the red L dwarf (red), and the de-reddened (green) spectra. The right column shows the best-fit model (green) and the observed reddening (black). (b) Same as Figure 9(a) for spectral types L4–L6 including 0355-type. The left column shows the spectral standard (black), the red L dwarf (red), and the de-reddened (green) spectra. The right column shows the best-fit model (green) and the observed reddening (black).
Figure 9. (Continued.)
Figure 11. Scatter plot of mean effective radius $a$ [μm] against $\Delta(J - K)$ color. Green markers denote low-gravity L dwarfs and magenta markers denote field-aged red L dwarfs both ranging between L0 and L5. Circles denote objects with PDFs with clear peaks. Diamonds denote objects with PDFs for $a$ close to the limit. Squares denote objects with PDFs for both $a$ and $b$ close to the limits. Our sample includes object with $\Delta(J - K) > 0.1$. There appears to be a linear correlation between radius and $\Delta(J - K)$ for the field-gravity objects, while there is no noticeable trend for the low-gravity objects. The distributions of low-gravity and field-gravity objects are distinct.

In order to quantitatively determine if the distributions of the low-gravity objects and the field-gravity objects are different, we performed a two-dimensional K-S test on the two distributions in both Figures 11 and 12. The probability of the two distributions in Figure 11 drawn from the same parent distribution is $2.32 \times 10^{-4}$, and the probability of the two distributions in Figure 12 drawn from the same parent distribution is $5.11 \times 10^{-6}$. These results show that the low-gravity and the field-gravity objects are most likely drawn from different distributions.

The lack of strong correlations between the dust haze properties and gravity might be due to a model grid not spanning a wide enough range of effective variance, $b$. We consider $0.1 < b < 1.0$ because Hansen particle size distributions for large $b$ look like a power-law size distribution (Figure 3; Equation (1)). This makes the results for these objects somewhat unreliable. In most of those objects, $b$ tends to be close to the upper limit and therefore is not well constrained (figure set of Figure 6).

Furthermore, our initial assumptions about low-gravity and field L dwarfs may be unrealistic. We assumed that low-gravity and field L dwarfs with the same base spectral type (e.g., L2 and L2-γ) have the same effective temperatures and the low-gravity L dwarf has the hypothetical dust haze of small grains in the upper atmosphere. However, recent evidence suggests that low-gravity L dwarfs do not necessarily share the same physical properties with field L dwarfs just because they share the same base spectral type (Luhman 2012; Filippazzo et al. 2015). Young, low-gravity L dwarfs might have cooler effective temperatures than field L dwarfs in the same spectral classification (Faherty et al. 2013). This suggests that an earlier-type standard would be a better comparison. We could be comparing objects with different effective temperatures and thus, not setting accurate estimates of the dust haze properties. Since the overall shape of a spectrum is very sensitive to
the effective temperature, comparing objects with the same effective temperatures might be more useful for future analysis.

Finally, the overlapping distributions of $a$ and $N$ may indicate that the dust hazes of low-gravity and red field L dwarfs are not different from one another. It might be the case that gravity does not play a major role in determining the properties of the dust haze and the same distribution of dust haze properties exist in both low-gravity and red field L dwarfs. Some red field L dwarfs might have a dust haze for reasons other than gravity, such as differing rotation rates, composition, or evolutionary history.

### 6.3. Comparison to Marocco et al. (2014)

Independently, Marocco et al. (2014) (hereafter, M14) present results from a similar analysis. Bonnefoy et al. (2015) closely followed the approach of M14 so we do not consider it further here. Like us, they are motivated by the utility of the interstellar extinction law in de-reddening red L dwarf spectra, use Mie theory to characterize a high-altitude population of sub-micron dust grains, and isolate the observed reddening by comparing red L dwarf spectra to spectral standards.

However, M14 consider corundum ($Al_2O_3$), enstatite ($Mg_SiO_3$), and iron while we use forsterite ($Mg_SiO_4$). We use forsterite as a test dust particle because extinction curves for forsterite, enstatite—both of which are silicates—and corundum all behave similarly in the near-infrared. We use silicate instead of iron because silicate grains form higher in the atmosphere than iron. Actual dust grains in the brown dwarfs' atmospheres are most likely a mixture of these species.

Our model includes more parameters than M14. They have two parameters: characteristic grain radius $r$ and normalization of the extinction curve at 2.20 $\mu$m while our model includes mean effective grain radius $a$, effective grain size variance $b$, column density $N$, vertical offset $C$, and tolerance factor $s$ as described in Section 4. The $N$ and $C$ parameters account for normalization as shown in Equation (9). M14 perform $\chi^2$ minimization to select the best-fit parameters and therefore do not have uncertainties, while we use MCMC to determine best-fit parameters and their uncertainties.

The ranges of the characteristic grain radius M14 found for corundum and enstatite are slightly larger than what we found. They obtained $r = 0.4–0.6$ $\mu$m for corundum and enstatite, while our results for the mean effective radius for forsterite are generally smaller ($0.15–0.35$ $\mu$m). Their results for the maximum radius of iron are $0.15–0.3$ $\mu$m closer to the values we found for forsterite grains.

M14 applied their method to five red L dwarfs, and we applied our method to 46 red L dwarfs including low-gravity field L dwarfs. They used objects with later spectral types (L5–L7), so there is only one common object between their sample and ours. 2MASS 0355+1133 is used in both studies but compared to different spectral standards. M14 used SDSS J0835+1953 as the L5 standard while we used 2MASS 1507-1627. They obtained the characteristic grain radius for 2MASS 0355+1133 to be $0.4$ $\mu$m and we found the mean effective radius to be $0.3^{+0.03}_{-0.02} \mu$m.

Regardless of the different methods, these two studies show similar results. The reproducibility of the results demonstrates the viability of sub-micron-sized dust grains, and warrants further study and inclusion of small dust grains in future atmosphere models.
