The doublet of Dirac fermions in the field of the non-Abelia monopole and parity selection rules

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Abstract

The paper concerns a problem of Dirac fermion doublet in the external monopole potential arisen out of embedding the Abelian monopole solution in the non-Abelian scheme. In this particular case, the Hamiltonian is invariant under some symmetry operations consisting of an Abelian subgroup in the complex rotational group \(SO(3.C) : [\hat{H}, \hat{F}(A)]_\pm = 0, \hat{F}(A) \in SO(3.C)\). This symmetry results in a certain freedom in choosing a discrete operator entering the complete set \(\hat{H}, \hat{J}^2, \hat{J}_3, \hat{N}_A\). The same complex number \(A\) represents a parameter of the basis wave functions \(\Psi_{\epsilon j m \delta \mu}^A(x, t, \theta, \phi)\) constructed. The generalized inversion-like operator \(\hat{N}_A\) implies its own \((A\)-dependent\) definition for scalar and pseudoscalar, and further affords some generalized \(N_A\)-parity selection rules.

It is shown that all different sets of basis functions \(\Psi_{\epsilon j m \delta \mu}^A(x)\) determine the same Hilbert space. In particular, the functions \(\Psi_{\epsilon j m \delta \mu}^A(x)\) decompose into linear combinations of \(\Psi_{\epsilon j m \delta \mu}^{A*}(x)\). However, the bases considered turn out to be nonorthogonal ones when \(A^* \neq A\); the latter correlates with the non-self-conjugacy property of the operator \(\hat{N}_A\) at \(A^* \neq A\).

(This is a shortened version of the paper).
1. Introduction

Together with geometrically topological way of classifying monopoles, another approach to studying various monopole configurations is possible, which concerns manifestations of monopoles playing the role of external potentials. Moreover, from the physical standpoint the latter method can be thought of as a more visualizable one in comparison with less obvious and more complicated topological language. So, the basic frame of the present investigation is an analysis of particles in the external monopole potentials. Here, both Abelian and non-Abelian cases will be discussed (see also [1-8, 9-13]), although the non-Abelian case is of primary interest to us.

Instead of the so-called monopole harmonics [4-8], the more conventional formalism of the Wigner’s D-functions is used. In contrast with the wide-spread approach, the general relativity tetrad formalism of Tetrode-Weyl-Fock-Ivanenko [14] is applied to this problem. This enable us to reveal explicitly a connection between the monopole topic and the early Pauli’s investigation [15] about the problem of allowable spherically symmetric wave functions in quantum mechanics; his results bear on the Dirac’s eg-quantization condition [16] (e and g are respectively the electric and magnetic charges). In addition, this Pauli’s work is of interest to us because it, among other things, used (heuristically) a special tetrad basis which can be associated with the unitary isotopic gauge in the non-Abelian monopole problem.

So, the major content of the work reported in this article is motivated by this Pauli’s paper [15] and foregoing special tetrad basis (it was introduced into the literature by Schrödinger [17]; see also [18]). It should be noted that some generalized analog of the Schrödinger’s basis may be successfully used, whenever in a linear problem there exists a spherical symmetry, irrespective of the concrete embodiment of this symmetry.

2. The Dirac and Schwinger unitary gauges in the isotopic space and monopole potentials

It is well-known that the usual Abelian monopole potential generates a certain non-Abelian potential being a solution of the Yang-Mills (Y-M) equations. First, such a specific non-Abelian solution was found out in [19]. A procedure itself of that embedding the Abelian monopole 4-vector $A_\mu(x)$ in the non-Abelian scheme: $A_\mu(x) \rightarrow A_\mu^{(a)}(x) \equiv (0, 0, A_\mu^{(3)} = A_\mu(x))$ ensures automatically that $A_\mu^{(a)}(x)$ will satisfy the free Y-M equations. Thus, it may be readily verified that the vector $A_\mu(x) = (0, 0, A_\phi = g \cos \theta)$ obeys the Maxwell general covariant equations in every curved space-time with the spherical symmetry: $dS^2 = [c^2(d\tau)^2 - c^2(d\rho)^2 - r^2((d\theta)^2 + \sin^2 \theta (d\phi)^2)]$; $A_\phi = g \cos \theta \rightarrow F_{\theta\phi} = -g \sin \theta$; here we get essentially a single equation $\frac{\partial}{\partial \theta} \frac{\sin \theta}{\sin^2 \theta} \frac{\sin(-g \sin \theta/\sin^2 \theta)}{\sin^2 \theta} = 0$. In turn, the non-Abelian tensor $F_\mu^{(a)}(x)$ defined by $F_\mu^{(a)}(x) = \nabla_\mu A_\mu^{(a)} - \nabla_\nu A_\mu^{(a)} + e\epsilon_{abc} A_\mu^{(b)} A_\nu^{(c)}$ and associated with the $A_\mu^{(a)}$ above has a very simple isotopic structure: $F_\theta\phi^{(3)} = -g \sin \theta$ and all other $F_\nu^{(a)}$ are equal to zero. So, this substitution $F_\nu^{(a)} = (0, 0, F_\theta\phi^{(3)} = -g \sin \theta)$ leads the Y-M equations to the single equation of the Abelian case. Strictly speaking, we cannot state that $A_\mu^{(a)}(x)$ obeys a certain set of really nonlinear equations (it satisfies linear rather than nonlinear equations). Thus, this monopole potential may be interpreted as a trivially non-Abelian solution of Y-M equations (this holds in every space-time with the spherical symmetry, but the case of ordinary flat space will be of primary interest to us).
Supposing that such a sub-potential is presented in the well-known monopole solutions of t’Hooft-Polyakov (and Julia-Zee) [20-22]:

\[
\Phi^{(a)}(x) = x^a \Phi(r), \quad W^{(a)}_0(x) = x^a F(r), \quad W^{(a)}_i(x) = \epsilon_{abc} x^b K(r) \quad (1)
\]

we can try to establish explicitly that constituent structure. The use of the spherical coordinates and special gauge transformation allows us to separate the trivial and non-trivial parts of the potentials (1) into different isotopic components \(W^1_\mu, W^2_\mu, W^3_\mu:\)

\[
W^S_\theta^{(a)} = \begin{pmatrix} 0 & 0 \\ (r^2 K + 1/e) & 0 \end{pmatrix}; \quad W^S_\phi^{(a)} = \begin{pmatrix} -(r^2 K + 1/e) \sin \theta \\ 0 \end{pmatrix} \quad (2)
\]

the symbol S. stands for the Schwinger unitary gauge in isotopic space. To the above-mentioned special monopole field there corresponds the \(K(r) = -1/er^2,\) so that the relations from (2) turn out to be very simple and related to the Abelian potential embedded into the non-Abelian scheme.

3. Diagonalized operators and separation of variables

In Section 3 we enter on analyzing the isotopic doublet of Dirac fermions in the external t’Hooft-Polyakov monopole field. We are going to reexamine this problem all over again, using the general relativity formalism [14]. Given the specified tetrad basis (Schrödinger basis of spherical tetrad [17,18]) and the unitary Schwinger frame in the local isotopic space (see (2)), the matter equation takes the form

\[
\left[ \gamma^0 (i\partial_t + er F t_3) + i\gamma^3 (\partial_r + \frac{1}{r}) + \frac{1}{r} \Sigma^S_{\theta,\phi} + \frac{er^2 K + 1}{r} (\gamma^1 \otimes t^2 - \gamma^2 \otimes t^1) - (m + \kappa r F t_3) \right] \Psi^S = 0; \quad (3)
\]

\[
\Sigma^S_{\theta,\phi} = \left( i\gamma^1 \partial_\theta + \frac{i\partial_\phi + (i\sigma^{12} + t_3) \cos \theta}{\sin \theta} \right)
\]

A characteristic feature of such a correlated choice of frames in both these spaces is the explicit form of the total angular momentum operator (the sum of orbital, spin, and isotopic ones) \(J^S_1 = (l_1 + \frac{(i\sigma^{12} + t_3) \cos \phi}{\sin \theta}), J^S_2 = (l_2 + \frac{(i\sigma^{12} + t_3) \sin \phi}{\sin \theta}), J^S_3 = l_3;\) so that the present case entirely comes under the situation considered by Pauli in [15]. The Pauli criterion allows here the following values for \(j: j = 0, 1, 2, 3, \ldots\) In agreement with a general procedure [8], the \(\theta, \phi\)-dependence of composite multiplet wave function \(\Psi_{jm}\) is to be built up from the Wigner \(D\)-functions: \(D^j_{-m,\sigma}(\phi, \theta, 0),\) where the lower right index \(\sigma\) takes the values from \((-1, 0, +1),\) which correlates with the explicit diagonal structure of the matrix \((i\sigma^{12} + t^3)\):

\[
\Psi_{ejm}(x) = e^{-i\sigma m} \left[ T_{+1/2} \otimes F(r) + T_{-1/2} \otimes G(r) \right] \quad (4)
\]

here (the fixed symbols \(j\) and \(-m\) in \(D^j_{-m,\sigma}(\phi, \theta, 0)\) are omitted).

\[
F(r) = \begin{pmatrix} f_1(r) D_{-1} \\ f_2(r) D_0 \\ f_3(r) D_{-1} \\ f_4(r) D_0 \end{pmatrix}; \quad G(r) = \begin{pmatrix} g_1(r) D_0 \\ g_2(r) D_{+1} \\ g_3(r) D_0 \\ g_4(r) D_{+1} \end{pmatrix}
\]
further throughout the paper the temporal factor will be omitted. Another essential feature of the given frame in the \((\text{Lorentz}) \times (\text{isotopic})\)-space is the appearance of the very simple expression for the term that mixes up together two distinct components of the isotopic doublet (see eq. (3)). Moreover, it is evident at once, that both these features are preserved, with no substantial variations, when extending this particular problem to more complex ones (with other given Lorentz and isotopic spins).

The separation of variables in the equation is accomplished by a conventional \(D\)-function recursive relation fashion. Moreover, only two relationships from the enormous \(D\)-function technique (see, for instance, in [23]) are really needed in doing such a separation; they are

\[
\frac{\partial}{\partial \beta} D^j_{mm'}(\alpha, \beta, \gamma) = + \frac{1}{2} \sqrt{(j + m')(j - m' + 1)} e^{-i\gamma} D^j_{m,m'-1} - \\
\frac{1}{2} \sqrt{(j - m')(j + m' + 1)} e^{+i\gamma} D^j_{m,m'+1};
\]

\[
\frac{m - m' \cos \theta}{\sin \theta} D^j_{mm'}(\alpha, \beta, \gamma) = - \frac{1}{2} \sqrt{(j + m')(j - m' + 1)} e^{-i\gamma} D^j_{m,m'-1} - \\
\frac{1}{2} \sqrt{(j - m')(j + m' + 1)} e^{+i\gamma} D^j_{m,m'+1}.
\]

As known, an important case in theoretical investigation is the electron-monopole system at the minimal value of the quantum number \(j\), so the case \(j = 0\) should be considered in an especially careful way. In the chosen frame, it is the independence on \(\theta, \phi\)-variables that sets the wave functions of minimal \(j\) apart from all other particle multiplet states (certainly, functions \(f_1(r)\), \(f_3(r)\), \(g_2(r)\), \(g_4(r)\) in the substitution (4) must be equated to zero at once). Besides, the angular term \(\Sigma_{\theta, \phi}\) in the wave equation is eliminated effectively due to the identity \((i\sigma_{12} + t^3)\Psi_{j=0} \equiv 0\).

The system of radial equations found by separation of variables (it is 4 and 8 equations in the cases of \(j = 0\) and \(j > 0\) respectively) are rather complicated (they cannot be considered here in detail). They are simplified by searching a suitable operator that could be diagonalized simultaneously with the \(\vec{j}^2, j_3\). As well known, the usual space reflection (\(P\)-inversion) operator for a bispinor field has to be followed by a certain discrete transformation in the isotopic space, so that a required quantity could be constructed. Indeed, the solution of this problem, which has been established up to date, is not general as much as possible. For this reason, the question of reflection symmetry in the doublet-monopole system is reexamined here all again. As a result we find that there are two different possibilities depending on what type of external monopole potential is analyzed. So, in case of the non-trivial potential, the composite reflection operator with required properties is (apart from an arbitrary factor)

\[
\hat{N}^S = \hat{\pi} \otimes \hat{P}_{\text{bisp.}} \otimes \hat{P}, \quad \hat{\pi} = +\sigma_1.
\]  

(5a)

A totally different situation occurs in case of the simplest monopole potential. Now, a possible additional operator suitable for separating the variables depends on a certain arbitrary complex parameter \(A\) \((e^{iA} \neq 0)\):

\[
\hat{N}^S_A = \hat{\pi}_A \otimes \hat{P}_{\text{bisp.}} \otimes \hat{P}, \quad \hat{\pi}_A = e^{iA\sigma_3}\sigma_1.
\]  

(5b)
The same quantity $A$ appears also in expressions for the wave functions $\Psi^A_{\epsilon jm}(t, r, \theta, \phi)$ (the eigenvalues $N_A = \delta(-1)^{j+1}; \delta = \pm 1$) :

$$\Psi^A_{\epsilon jm}(x) = [ T_{+1/2} \otimes F(x) + \delta \ e^{iA} T_{-1/2} \otimes G(x) ] \quad (5c)$$

now the $g_i(r)$ from (4) are to be $g_1 = \delta f_4, g_2 = \delta f_3, g_3 = \delta f_2, g_4 = \delta f_1$.

4. Case of special monopole field

Now, the problem of simplest monopole field is examined more closely. The system of radial equations specified for this monopole potential is basically simpler than in general case, so that the whole problem including the radial functions can be carried to its complete conclusion:

$$\Psi^A_{\epsilon jm\delta \mu}(x) = T_{+1/2} \begin{pmatrix} f_1(r)D_{-1} \\ f_2(r)D_0 \\ \mu f_2(r)D_{-1} \\ \mu f_1(r)D_0 \end{pmatrix} + \mu \delta \ e^{iA} \ T_{-1/2} \begin{pmatrix} f_1(r)D_0 \\ f_2(r)D_{+1} \\ \mu f_2(r)D_0 \\ \mu f_1(r)D_{+1} \end{pmatrix} \quad (6)$$

where $\delta = \pm 1$ and $\mu = \pm 1$, which are independent of each other; the quantum number $\mu$ relates to the so-called generalized Dirac operator $[17,18] \ K = i\gamma^0\gamma^3\Sigma_{\theta\phi}$ ; the functions $f_1$ and $f_2$ satisfy a system of first order differential equations. The $A$-ambiguity in the expansion (6) indicates that this parameter $e^{iA}$ may be regarded as a quantity measuring a violation of Abelicity in the composite non-Abelian wave functions $\Psi^A_{\epsilon jm\delta \mu}(x)$. Significantly, that the $e^{iA}$ must not be equal to zero; in other words, the operator sets above never lead to basis states without the A-violation. The two purely Abelian multiplet states are formally obtained from (5c) or (6) too: it suffices to put $e^{iA} = 0$ or $\infty$, but these singular cases are not covered by operator sets under consideration. These two bound values for $A$ provide us, in a sense, with the singular transition points between the Abelian and non-Abelian theories.

On relating the expressions (6) with the Abelian analogous solutions, it follows that these non-Abelian functions are directly associated with Abelian monopole ones (those were investigated by many authors [1-8]):

$$\Psi^A_{\epsilon jm\delta \mu}(x) = T_{+1/2} \otimes \Phi^{eg=-1/2}_{\epsilon jm\mu}(x) + \mu \delta \ e^{iA} \ T_{-1/2} \otimes \Phi^{eg=+1/2}_{\epsilon jm\mu}(x)$$

$$\Psi^A_{\epsilon 0\delta}(x) = T_{+1/2} \otimes \Phi^{eg=-1/2}_{\epsilon 0\delta}(x) + \delta \ e^{iA} \ T_{-1/2} \otimes \Phi^{eg=+1/2}_{\epsilon 0\delta}(x) .$$

5. On physical distinctions between manifestations of the Abelian and non-Abelian monopoles

A key question of Section 5 concerns distinctions between the Abelian and non-Abelian monopoles. We draw attention to that, for the distinguishable physical systems, namely, a free isodoublet with no external potentials and an isodoublet in the external monopole field (whether a trivial or non-trivial one is meant), their spherical symmetry operators $\vec{J}_2, J_3$ identically coincide. In a sequence, these isodoublet wave functions do not vary at all in their dependence on angular variables $\theta, \phi$. This non-Abelian wave function’s property
sharply contrasts with the Abelian one when both electronic spherically symmetric wave functions and all the symmetry operators undergo a significant transformation:

\[
\begin{align*}
    j_1^{eg} &= (l_1 + \frac{(i\sigma^{12} - eg) \cos \phi}{\sin \theta}), \\
    j_2^{eg} &= (l_2 + \frac{(i\sigma^{12} - eg) \sin \phi}{\sin \theta}), \\
    j_3^{eg} &= l_3, \\
    \Phi_{ejm\mu}^{eg}(t, r, \theta, \phi) &= \frac{e^{-i\xi t}}{r} = \left( \begin{array}{c}
        f_1 D_l^{\phi^{eg-1/2}} \\
        f_2 D_l^{\phi^{eg+1/2}} \\
        \mu f_2 D_l^{\phi^{eg-1/2}} \\
        \mu f_1 D_l^{\phi^{eg+1/2}}
    \end{array} \right)
\end{align*}
\]

the value \( eg = 0 \) relates to the free electronic function.

We may note that one of the fundamental ideas underlying the theory of the non-Abelian monopole is probably following: it being considered as the external potential does not destroy the isotopic angular structure of the particle multiplet wave functions. From this point of view, it represents a certain analog of a spherically symmetric Abelian potential \( A_\mu = (A_0(r), 0, 0, 0) \) rather than an analog of the Abelian monopole potential \( A_\mu = (0, 0, A_\phi = g \cos \theta) \). Here we may draw attention to that the designation “monopole” in the non-Abelian terminology anticipates an interpretation of \( W^{(a)}_\mu(x) \) as carrying, in a new situation, the essence of the well-known Abelian monopole, although a real degree of their similarity is going to be probably less than one should expect.

By the way, in both (Abelian and non-Abelian) cases, monopole terms involved in wave equations vanish at long distances (far away from \( r = 0 \)), but in the non-Abelian problem there is no remaining monopole manifestation through \( \theta, \phi \) - dependence, whereas in the Abelian problem such an implicit monopole presence is still conserved (as a result of \( eg \) - displacement in Wigner \( D \)-functions involved in wave functions). So we cannot get rid of the Abelian monopole manifestation up to infinitely distance points, and such a property is removed far from what is familiar when a situation is less singular (for instance, of electric charge or non-Abelian monopole problem). In other words, the non-Abelian monopole, being considered as external potential, provides a localized object (irrespective of whether the trivial or non-trivial monopole is meant); this property is motivated only by its isotopic structure. In contrast to this, the Abelian monopole is obviously non-localizable object, and this finds its natural corollary in giving rise the well-known difficulties on boundary conditions (both in classical and quantum mechanical scattering theory [24-30]). One should emphasize that possible mutual series expansions cannot be completely correct (in Abelian case) at \( \theta = 0 \) and \( \theta = \pi \), so that the whole situation on the axis \( x_3 \) does not conform to the basis superposition principle, whereas such a problem does not arise at studying the analogous non-Abelian problem. The intimate explanation of this behavior involves the detailed examination of the boundary properties of corresponding Wigner \( D \)-functions and cannot therefore be considered here in detail.

Thus, the whole multiplicity of the Abelian monopole manifestations seems to be much more problematical than non-Abelian monopole’s. Strictly speaking, these two mathematical situations are not related to each other and an examination for non-Abelian case will not lead up to solving Abelian problems, but only being associated heuristically and thereby thrown a further light on each other. By way of illustration, we consider the question of \( P \)-parity in both these theories (see also in [31-37]).
6. N-parity selection rules

In the Abelian case (when $eg \neq 0$), the monopole wave functions cannot be proper functions of the usual space reflection operator for the bispinor field. There exists only the following relationship

$$\hat{P}_{\text{bisp.}} \otimes \hat{P} \Phi_{\epsilon j m \delta}^e(x) = e^{-i t} \frac{\epsilon}{r} (\epsilon - 1)^{j+1} \left( \begin{array}{c} f_1 D_{m, -e g - 1/2}^j \\ f_2 D_{m, -e g + 1/2}^j \\ \mu f_1 D_{-e g - 1/2}^j \\ \mu f_1 D_{-e g + 1/2}^j \end{array} \right)$$

compare it with analogous one for the free wave functions:

$$(\hat{P}_{\text{bisp.}} \otimes \hat{P}) \Phi_{\epsilon j m \delta}^0(x) = \delta (\epsilon - 1)^{j+1} \Phi_{\epsilon j m \delta}^0(x)$$

A certain diagonalized on these functions $\Phi_{\epsilon j m \delta}^e$ discrete operator obtains through multiplying the usual $P$-inversion bispinor operator by a formal one $\hat{\pi}$ which affects the $eg$-parameter in the wave functions: $\hat{\pi} \Phi_{\epsilon j m \delta}^e(x) = \Phi_{\epsilon j m \delta}^{-e g}(x)$. Thus, we have

$$\hat{M} = \hat{\pi} \otimes \hat{P}_{\text{bisp.}} \otimes \hat{P}, \quad \hat{M} \Phi_{\epsilon j m \delta}^e(x) = \mu (\epsilon - 1)^{j+1} \Phi_{\epsilon j m \delta}^e(x)$$

but the latter fact does not allow us to obtain $M$-parity selection rules. Indeed, a matrix element for some physical observable $\hat{G}^0(x)$ is to be

$$\int \Phi_{\epsilon j m \delta}^e(x) \hat{G}^0(x) \Phi_{\epsilon j m' \delta'}^e(x) dV = \int r^2 dr \int f(\vec{x}) d\omega$$

First we examine the case $eg = 0$, in order to compare it with the situation at $eg \neq 0$. Let us relate $f(-\vec{x})$ with $f(\vec{x})$. Considering the equality (and the same with $j'm'\delta'$)

$$\Phi_{\epsilon j m \delta}^0(-\vec{x}) = \hat{P}_{\text{bisp.}} \delta(-1)^{j+1} \Phi_{\epsilon j m \delta}^0(\vec{x}) \quad (7a)$$

we get

$$f(-\vec{x}) = \delta\delta'(1)^{j+j'+1} \Phi_{\epsilon j m \delta}^e(\vec{x})(\hat{P}_{\text{bisp.}} \delta' \hat{G}^0(\vec{x}) \hat{P}_{\text{bisp.}}) \Phi_{\epsilon j m' \delta'}(\vec{x})$$

If $\hat{G}^0(\vec{x})$ obeys the equation

$$\hat{P}_{\text{bisp.}} \hat{G}^0(-\vec{x}) \hat{P}_{\text{bisp.}} = \omega^0 \hat{G}^0(\vec{x}) \quad (7b)$$

here $\omega^0$ defined to be $+1$ or $-1$ relates to the scalar and pseudoscalar respectively, then the expression for $f(-\vec{x})$ above comes to $f(-\vec{x}) = \omega \delta\delta'(1)^{j+j'+1} f(\vec{x})$ that generates the well-known $P$-parity selection rules.

In contrast to everything just said, the situation at $eg \neq 0$ is completely different because any equality in the form $(7a)$ does not exist there. So, there is no $M$-parity selection rules in the presence of the Abelian monopole. In accordance with this, for instance, an expectation value for the usual operator of space coordinates $\vec{x}$ need not to equal zero and it follows this (see in [36-37]).

Let us return to the non-Abelian problem when there exists a needed relationship

$$\Psi_{\epsilon j m \delta}^e(-\vec{x}) = (\sigma^2 \otimes \hat{P}_{\text{bisp.}}) \delta(-1)^{j+1} \Psi_{\epsilon j m \delta}(\vec{x}) \quad (8a)$$
owing to the $N$-reflection symmetry; so that

$$f(-\vec{x}) = \delta \delta' (-1)^{j+j'} \bar{\Psi}_{ejm\delta}(\vec{x}) \otimes [ (\sigma^2 \otimes \hat{P}_{\text{bisp.}}^+) \hat{G}(-\vec{x}) (\sigma^2 \otimes \hat{P}_{\text{bisp.}}) ] \, \bar{\Psi}_{ej'm'\delta'}(\vec{x}).$$

If a certain quantity $\hat{G}(\vec{x})$ which in comparison with a previous one depends on isotopic coordinates, obeys the condition

$$(\sigma^2 \otimes \hat{P}_{\text{bisp.}}^+) \hat{G}(-\vec{x}) (\sigma^2 \otimes \hat{P}_{\text{bisp.}}) = \omega \hat{G}(\vec{x})$$

$\omega$ defined to be $+1$ or $-1$, the relationship above converts into $f(-\vec{x}) = \omega \delta \delta' (-1)^{j+j'} f(\vec{x})$, that results in the evident $N$-parity selection rules. For instance, applying this rule to $\hat{G}(\vec{x}) \equiv \vec{x}$, we found $< \Psi_{ejm\delta}(x) | \vec{x} | \bar{\Psi}_{ejm\delta}(x) > \sim [1 - \delta^2(1)^2j] \equiv 0$. That vanishing may be also readily understood from the following expansion of the matrix element

$$< \Psi_{ejm\delta}(x) | \vec{x} | \bar{\Psi}_{ejm\delta}(x) > = < \Psi_{ejm\delta}^{-1/2}(x) | \vec{x} | \Phi_{ejm}^{-1/2}(x) > + < \Phi_{ejm}^{+1/2}(x) | \vec{x} | \Phi_{ejm}^{+1/2}(x) >$$

(where both isotopic components contribute equally to the matrix element) and fitting relationships $< \Phi_{ejm}^{+1/2}(-\vec{x}) | -\vec{x} | \Phi_{ejm}^{+1/2}(-\vec{x}) > = -< \Phi_{ejm}^{+1/2}(\vec{x}) | \vec{x} | \Phi_{ejm}^{+1/2}(\vec{x}) >$. From the preceding it is evident that the solvable non-Abelian problem of $N$-parity selection rules does not guarantee that another problem of $M$-parity (for the Abelian case) can automatically be solved in analogous way.

One should notice that in the literature there are several suggestions how obtain a certain formal covariance of the monopole situation with respect to $P$-symmetry. Such attempts would imply, for example, a pseudo scalar character of the isolated magnetic charge [31,33-35] or improvement [36] in understanding the $P$-symmetry, which in turn provides some genuine $P$-reflection operator, etc. But, admittedly, all these solutions do not permit to get over the non-existence of the discrete symmetry selection rules for matrix elements at considering a single-particle problem in a fixed monopole potential.

7. New $N_A$-parities selection rules

First, by simple calculation, we detail explicit forms of $\hat{N}_A$-operator in the unitary Dirac ($D.$) and Cartesian ($C.$) gauges of isotopic space (see Suppl. A):

$$\hat{\pi}_A^D = \left( \begin{array}{cc} 0 & -i e^{-i A} e^{-i \phi} \\ +i e^{i A} e^{i \phi} & 0 \end{array} \right); \quad \hat{\pi}_A^C = (-i)exp \left[ iA \sigma \bar{n}_{\theta,\phi} \right]. \quad (9)$$

Now, we turn to the question how this complex characteristic parameter $A$ can manifest itself. As a representative example, the above problem of parity selection rule is investigated again, but now depending on this $A$-background. For the composite physical observable having inclusive constituent structure (its isotopic content is separated out explicitly)

$$G(\vec{x}) = \left( \begin{array}{c} \hat{g}_{11}(\vec{x}) \\ \hat{g}_{21}(\vec{x}) \end{array} \right) \times G^0(\vec{x}) \quad (10a)$$

a natural definition of scalars and pseudoscalars relative to the $\hat{N}_A$-reflection occurs (compare it with (8b)): $(\hat{\pi}_A \otimes \hat{P}_{\text{bisp.}}^+) \hat{G}(-\vec{x}) (\hat{\pi}_A \otimes \hat{P}_{\text{bisp.}}) = \Omega A \hat{G}(\vec{x})$ or in more detailed form

$$\left( \begin{array}{cc} e^{+i(A-A^*)} \hat{g}_{22}(-\vec{x}) & e^{-i(A+A^*)} \hat{g}_{21}(-\vec{x}) \\ e^{+i(A+A^*)} \hat{g}_{12}(-\vec{x}) & e^{-i(A-A^*)} \hat{g}_{11}(-\vec{x}) \end{array} \right) \left[ \hat{P}_{\text{bisp.}}^+ \hat{G}^0(\vec{x}) \hat{P}_{\text{bisp.}} \right] = \Omega A \hat{G}(\vec{x}) \quad (10b)$$
where $\Omega^A = +1$ or $-1$. For every given $A$, the (10b) produces its own special limitations on composite scalars and pseudoscalars, which are individualized by this $A$. That is, the different values of $A$ lead to various concepts of scalars and pseudoscalars respectively. Correspondingly, $N_A$-parity selection rules arising in sequel for matrix elements (if an observable belongs to either the $\Omega_A = +1$ or $\Omega_A = -1$ type) differ basically from each other.

8. Parameter $A$ and isotopic “chiral” symmetry

Section 8 is interested in the following question: Where does the above $A$-ambiguity come from? Here, we can note that all different values for $A$ lead to the same whole functional space; each fixed $A$ governs only the basis states: $\Psi_{ijm}(x) = U(A, A)\Psi^{A}(x)$. The explicit form of $U(A', A)$ in $S$-gauge, is

$$U_S(A', A) = e^{+i(A' - A)/2} \begin{pmatrix} e^{-i(A' - A)/2} & 0 \\ 0 & e^{+i(A' - A)/2} \end{pmatrix}$$

(11a)

correspondingly, in $C$-gauge, it is

$$U_C(A', A) = e^{+i(A' - A)/2} \exp\left[-i \frac{A' - A}{2} \vec{\sigma} \hat{n}_\theta, \phi \right]$$

(11b)

In both cases (11a) and (11b), the second factor is the 2-spinor transformation lying in the 3-dimensional complex rotation group $SO(3.C)$. In addition, as readily verified, the operations $\hat{N}$ and $\hat{N}_A$ turn out to be connected by the relation

$$\hat{N}^S_A = U_S(A, 0) \hat{N}^S \hat{U}_S^{-1}(A, 0); \quad \hat{N}^C_A = U_C(A, 0) \hat{N}^C \hat{U}_C^{-1}(A, 0).$$

(12)

In this connection, the question arised is how to evaluate contrasting these two discrete operators $\hat{N}^S$ and $\hat{N}^S_A$. So, we have to give more attention to this relationship $\hat{N}^S_A$ and $\hat{N}^S$. To clearing up this matter, as turned out, it suffices to draw consistently distinction between two situations. The first one concerns the sets $[\hat{J}_i, \hat{N}^S]$ and $[\hat{J}_i, \hat{N}^S_A]$ when they are considered as different but equivalent realizations of the same given representation of the group $SO(3.R)$. The second relates to case when these two operator sets are regarded as the physical observables at the same physical system of fixed Hamiltonian: $[\hat{J}_i, \hat{N}^S]_H$, $[\hat{J}_i, \hat{N}^S_A]_H$; and then they are physically distinguishable as generating further different basis wave functions. An analogy with a more familiar example of the Dirac massless field can be called [18], when the complex chiral symmetry transformation

$$\begin{pmatrix} \xi'(x) \\ \eta'(x) \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & Z \end{pmatrix} \begin{pmatrix} \xi(x) \\ \eta(x) \end{pmatrix}; \quad \Psi'(x) = e^{iA/2} \exp(i \frac{A}{2} \gamma^5) \Psi(x)$$

($Z = e^{iA}$) leads us to use (alternatively) the generalized $P$-reflection bispinor operator: $e^{iA\gamma^5} \hat{P}_{bisp} \otimes \hat{P}$. Moreover, its explicit form remains basically unchanged at translating the spheric tetrad basis into Cartesian’s ($\hat{P}_{bisp} = i\gamma^0$) as the $\gamma^5$ and the gauge transformation involved are commutative with each other. Such a condition is not realized for the non-Abelian monopole-electron system, so the Schwinger expression of the $\hat{N}^S_A$, after transformation to the Cartesian isotopic basis, takes the form in which the $\theta, \phi$-dependence appears explicitly:

$$\hat{N}^S_A = (e^{iA\sigma_3} \hat{\tau} \otimes \hat{P}_{bisp}) \otimes \hat{P}; \quad \hat{N}^C_A = [(-i) \exp(iA \vec{\sigma} \hat{n}_\theta, \phi) \otimes \hat{P}_{bisp} \otimes \hat{P}].$$

(13)
As an additional remark, it should be mentioned that starting solely from the usual Cartesian isotopic formulation, we could not have found out (with great probability) such an $A$-ambiguity as in (13) and also we could not have disclosed the existence itself of a possible isotopic chiral symmetry in non-Abelian nonopole-electron system. Thus, an incidental choice of a basis (both in the isotopic and bispinor spaces) results in unexpected possibilities in the characterization of this system.

9. Complex values of the $A$ and a collision between the quantum mechanical superposition principle and self-conjugacy requirement

In this Sec. 9, let us look closely at some qualitative peculiarities of the above considered $A$-freedom placing special notice to the division of $A$-s into the real and complex ones.

It is convenient to work at this matter in the Schwinger unitary basis. Recall that the $A$-freedom tell us that simultaneously with $\hat{H}, \hat{j}_2, \hat{j}_3$, else one discrete operator $\hat{N}_A$ that depends generally on a complex number $A$ can be diagonalized on the wave functions.

Correspondingly, the basis functions associated with the complete set $(\hat{H}, \hat{j}_2, \hat{j}_3, \hat{N}_A)$ besides being certain determined functions of the relevant quantum numbers ($\epsilon, j, m, \delta$), are subject to the

$$\Psi^A_{\epsilon jm\delta}(x) = \left[ T_{+1/2} \otimes \Phi^+_{\epsilon jm}(x) + \delta e^{iA} T_{-1/2} \otimes \Phi^-_{\epsilon jm}(x) \right]. \tag{14a}$$

In other words, all different values of this $A$ lead to different quantum-mechanical bases of the system. There exists a set of possibilities, but one can relate every two of them by means of a respective linear transformation. For example, the states $\Psi^A_{\epsilon jm\delta}(x)$ decompose into the following linear combinations of the initial states $\Psi^0_{\epsilon jm\delta}(x)$ (further, this $A=0$ index will be omitted):

$$\Psi^A_{\epsilon jm\delta}(x) = \left[ \frac{1 + \delta e^{iA}}{2} \Psi_{\epsilon jm,+1} + \frac{1 - \delta e^{iA}}{2} \Psi_{\epsilon jm,-1} \right]. \tag{14b}$$

One should give heed to that, no matter what an $A$ is (either real or complex one), the new states (14b) being linear combinations of the initial states are permissible as well as old ones. This added aspect of the allowance of the complex values for $A$ conforms to the quantum-mechanical superposition principle, the latter presupposes that arbitrary complex coefficients $c_i$ in a linear combination of some basis states $\Sigma c_i \Psi_i$ are acceptable.

However, an essential and subtle distinction between real and complex $A$-s comes straightforward to light as we turn to the matter of normalization and orthogonality for $\Psi^A_{\epsilon jm\delta}(x)$. An elementary calculation gives

$$< \Psi^A_{\epsilon jm,\delta} | \Psi^A_{\epsilon jm,\delta} > = \frac{1 + e^{i(A-A^*)}}{2} ; \quad < \Psi^A_{\epsilon jm,\delta} | \Psi^A_{\epsilon jm,-\delta} > = \frac{1 - e^{i(A-A^*)}}{2} \tag{15}$$

i.e. if $A \neq A^*$ then the normalizing condition for $\Psi^A_{\epsilon jm\delta}(x)$ does not coincide with that for $\Psi_{\epsilon jm\delta}(x)$, and what is more, the states $\Psi^A_{\epsilon jm,-1}(x)$ and $\Psi^A_{\epsilon jm,+1}(x)$ are not mutually orthogonal. The latter means that we face here the non-orthogonal basis in Hilbert space and the pure imaginary part of the $A$ plays a crucial role in the description of its non-orthogonality property.
The oblique character of the basis $\Psi_{\epsilon jm\delta}(x)$ (if $A \neq A^*$) exhibits its very essential qualitative distinction from perpendicular one for $\Psi_{\epsilon jm\delta}(x)$. But, in a sense, the existence of the non-orthogonal bases in the Hilbert space represents a direct consequence of the quantum-mechanical superposition principle. By the way, for this reason, a prohibition against complex $A$-s could be partly a prohibition against the conventional superposition principle too; since all complex values for $A$, having forbidden, imply specific limitations on two coefficients in (14b); but those are not presupposed by the superposition principle itself.

Up to this point, the complex $A$-s seem to be good as well as the real ones. Now, it is the moment to point to some clouds handing over this part of the subject. Indeed, as readily verified, the operator $\hat{N}_A$ does not represent a self-conjugated (self-adjoint) one $\langle \hat{N}_A \Phi(x) | \Psi(x) \rangle = \langle \Phi(x) | e^{i(A-A^*)\frac{1}{2}} \hat{N}_A \Psi(x) \rangle$. It is understandable that this (nonself-conjugacy) property correlates with the above-mentioned nonorthogonality conditions (as well known, a self-conjugated operator entails both real its eigenvalues and the orthogonality of its eigenfunctions). As already noted, the eigenvalues of $\hat{N}_A$ are real ones and this conforms to the general statement that all inversion-like operators possess the property of the kind: if $\hat{G}^2 = I$ then $\lambda$ is a real number, as $\hat{G} \Phi_{\lambda} = \lambda \Phi_{\lambda}$).

So, we have got into a clash: whether one has to reject all complex values for $A$ and thereby violate the one quantum mechanical principle of major generality (of superposition) or whether it is remain to accept all complex $A$-s as well as real ones and thereby, in turn, stretch another quantum-mechanical regulation about the self-conjugate character of physical quantities. Thus, the physical system under consideration exhibits insel a logical collision between two conventional quantum propositions of principal significance. In the author’s opinion, one should accord the primacy of the general superposition principle over the self-adjointness requirement. In support of this point of view, there exist some physical grounds. Indeed, recall the quantum-mechanical status of all inversion-like quantities: they serve always to distinguish two quantum-mechanical states. Moreover, to those quantum variables there not correspond any classical variables; the latter correlates with that any classical apparatus measuring those discrete variables does not exist at all. In contrast to this, one should recollect why the self-adjointness requirement itself was imposed on physical quantum operators. The reason is that such operators imply all their eigenvalues to be real. Besides, that limitation on physical quantum variables had been put, in the first place, for quantum variables having their classical counterparts (with the continuum of classical values measured). And after this, in the second place, the discrete quantities such as $P$-inversion and like it were tacitly incorporated into a set of self-adjoint mathematical operations, as a natural extrapolation. But one should notice (and the author inclines to place a special emphasis on this) the fact that the single relation $\hat{N}_A^2 = I$ is completely sufficient that the eigenvalues of $\hat{N}_A$ to be real. In the light of this, the above-mentioned automatic incorporation of those discrete operators into a set of self-adjoint ones does not seem inevitable. But admitting this, there is a problem to solve: what is the meaning of complex expectation value of such non self-adjoint discrete operators; since, evidently, the conventional formula $\langle \Psi | \hat{N}_A | \Psi \rangle$ provides us with a complex value. Indeed, let $\Psi(x)$ be $\Psi(x) = [m \Psi_{+1}(x) + n \Psi_{-1}(x)]$, then

$$\langle \Psi | \hat{N}_A | \Psi \rangle = \langle m \Psi_{+1}(x) + n \Psi_{-1}(x) | m \Psi_{+1}(x) - n \Psi_{-1}(x) \rangle = \langle 16 \rangle$$

1The author is grateful to Dr. E.A. Tolkachev for pointing out that it is so
\[
(m^*m - n^*n) \frac{1 + e^{i(A-A^*)}}{2} + (n^*m - nm^*) \frac{1 - e^{i(A-A^*)}}{2} \].

Must one be skeptical about those complex \(\bar{N}_A\), or treat them as physically acceptable quantities? Let us examine this problem in more detail. It is reasonable to begin with an elementary consideration of the measuring procedure of the \(\hat{N} = \hat{N}_{A=0}\). Let a wave function \(\Psi(x)\) decompose into the combination

\[
\Psi(x) = [ e^{i\alpha} \cos^2 \Gamma \Psi_{+1}(x) + e^{i\beta} \sin^2 \Gamma \Psi_{-1}(x) ]
\]

(17a)

where \(\alpha\) and \(\beta\) \(\in [0, 2\pi]\), and \(\Gamma \in [0, \pi/2]\). For the \(\hat{N}\) expectation value, one gets

\[
\bar{N} = <\Psi \mid \hat{N} \mid \Psi> = (-1)^{j+1} (\cos^2 \Gamma - \sin^2 \Gamma) = (-1)^{j+1} \cos 2\Gamma .
\]

(17b)

From (17b), one can conclude that the \(\bar{N}\) having measured, provides us only with the information about the parameter \(\Gamma\) at (17a), but does not furnish any information on the phase factors \(e^{i\alpha}\) and \(e^{i\beta}\) (or their relative factor \(e^{i(\alpha-\beta)}\)). Such an interpretation of measured \(\bar{N}\) as receptacle of the quite definite information about superposition coefficients in the decomposition (17a) represents one and only physical meaning of the \(\bar{N}\).

Now, returning to the case of \(\hat{N}_A\) operation, one should put an analogous question concerning the \(\bar{N}_A\). The required question is: what information about \(\Psi(x)\) can be extracted from the measured \(\bar{N}_A\). It is convenient to rewrite the above function \(\Psi(x)\) as a linear combination of functions \(\Psi_{\epsilon jm, +1}\) and \(\Psi_{\epsilon jm, -1}\). Thus inverting the relations (14b) we get

\[
\Psi_{\epsilon jm, +1} = \left[ \frac{1 + e^{-iA}}{2} \Psi_{\epsilon jm, +1} + \frac{1 - e^{-iA}}{2} \Psi_{\epsilon jm, -1} \right],
\]

\[
\Psi_{\epsilon jm, -1} = \left[ \frac{1 - e^{-iA}}{2} \Psi_{\epsilon jm, +1} + \frac{1 + e^{-iA}}{2} \Psi_{\epsilon jm, -1} \right]
\]

and then \(\Psi(x)\) takes the form (the quantum numbers \(\epsilon, j, m\) as fixed ones are omitted)

\[
\Psi(x) = \left[ ( e^{i\alpha} \cos \Gamma \frac{1 + e^{-iA}}{2} + e^{i\beta} \sin \Gamma \frac{1 - e^{-iA}}{2} ) \Psi_{+1}(x) + ( e^{i\alpha} \cos \Gamma \frac{1 - e^{-iA}}{2} + e^{i\beta} \sin \Gamma \frac{1 + e^{-iA}}{2} ) \Psi_{-1}(x) \right].
\]

(18a)

Although the quantity \(A\) enters the expansion (18a), but really \(\Psi(x)\) only contains three arbitrary parameters: those are \(\Gamma, e^{i\alpha}\), and \(e^{i\beta}\). After simple calculation one gets

\[
\bar{N}_A = <\Psi \mid \hat{N}_A \mid \Psi> = (-1)^{j+1} (\rho \cosh g + i\sigma \sinh g ),
\]

(18b)

where \(\rho = \cos 2\Gamma \cos f + \sin 2\Gamma \sin f \sin(\alpha - \beta)\), \(\sigma = -\cos 2\Gamma \sin f + \sin 2\Gamma \cos f \sin(\alpha - \beta)\)

\[
\rho + \sigma = \cos 2\Gamma (\cos f + \sin f \sin(\alpha - \beta))
\]

\[
\rho - \sigma = \cos 2\Gamma (\cosh g - \sinh g \sin(\alpha - \beta))
\]

where \(f\) and \(g\) are real parameters defined by \(A = f + ig\). Examining this expression, one may single out four particular cases for separate consideration. Those are:

1. \(g = 0, f = 0\) : \(\bar{N}_A = (-1)^{j+1} \cos 2\Gamma\)

(19a)
here, the $\bar{N}$ only fixes $\Gamma$, but $e^{i(\alpha-\beta)}$ remains indefinite.

$$2. \ g = 0, \ f \neq 0 : \quad \bar{N}_A = (-1)^{j+1} [\cos 2\Gamma \cos f + \sin 2\Gamma \sin f \sin(\alpha - \beta)] \quad (19b)$$

here, the $\bar{N}_A$ measured does not fixes $\Gamma$ and $(\alpha - \beta)$, but only imposes a certain limitation on both these parameters.

$$3. \ g \neq 0, \ f = 0 : \quad \bar{N}_A = (-1)^{j+1} [\cos 2\Gamma \cosh g + i \sin 2\Gamma \sin(\alpha - \beta) \sinh g] \quad (19c)$$

here, the $\bar{N}_A$ determines both $\Gamma$ and $(\alpha - \beta)$; and thereby this complex $\bar{N}_A$ is a physical quantity being quite interpreted. Finally, for the fourth case

$$4. \ g \neq 0, \ f \neq 0 : \quad \cos 2\Gamma = (\rho \cos f - \sigma \sin f),$$

$$\quad \sin 2\Gamma \sin(\alpha - \beta) = (\rho \cos f + \sigma \sin f) \quad (19d)$$

i.e. the complex $\bar{N}_A$ also gives some information about $\Gamma$ and $(\alpha - \beta)$ and therefore has character of a physically interpreted quantity.
References

[1] I. E. Tamm, Z. Phys. 71, 141 (1931).

[2] M. Fierz, Helv. Phys. Acta, 17, 27 (1944) ; P. P. Banderet, Helv. Phys. Acta. 19, 503 (1946); Harish-Chandra, Phys. Rev. 74, 883 (1948).

[3] H. J. Lipkin, W. I. Weisberger, M. Peshkin, Ann. of Phys. 53, 203 (1969); A. Frenkel, P. Hrasko, Ann. of Phys. 105, 288 (1977).

[4] T. T. Wu, C. N. Yang, Phys. Rev. D 12, 3845 (1975); Nucl. Phys. B 107, 365 (1976); Phys. Rev. D 16, 1018 (1977).

[5] Y. Kazama, C. N. Yang, Phys. Rev. D. 15, 2300 (1977); Y. Kazama, Int. J. Theor. Phys. 17, 249 (1978); A. P. Balach andran, S. M. Roy, Singh Vivendra, Phys. Rev. D 28, 2669 (1983); Dong Ming De, Phys. Lett. 155, 387 (1985).

[6] J. N. Goldberg, A. J. Macfarlane, E. T. Newman, F. Rohrlach, E. C. G. Sudarshan, J. Math. Phys. 8, 2155 (1967); T. J. Dray, Math. Phys. 26, 1030 (1985).

[7] D. V. Galtzov, A. A. Ershov, Yad. Phys. 47, 560 (1988) (in Russian);

[8] V.M. Red'kov, Generally relativistical Tetrode-Weyl-Fock-Ivanenko formalism and behaviour of quantum-mechanical particles of spin 1/2 in the Abelian monopole field. 25 pages; quant-ph/9812002

[9] J. H. Swank, L. J. Swank, Tekin Dereli, Phys. Rev. D 12, 1096 (1975).

[10] R. Jackiw, C. Rebbi, Phys. Rev. D 13, 3398 (1976).

[11] E. B. Prokhvatilov, V. A. Franke, Yad. Phys. 24, 856 (1976) (in Russian).

[12] H. Yamagishi, Phys. Rev. D 28, 977 (1983).

[13] S. M. Ajithkumar, M. Sabir, Ann. of Phys. 169, 117 (1986).

[14] S. Weinberg, Gravitation and Cosmology: Principles and Applications of the General Theory of Relativity (Wiley, New York, 1970).

[15] W. Pauli, Helv. Phys. Acta. 12, 147 (1939).

[16] P. A. M. Dirac, Proc. Roy. Soc. A 133, 60 (1931); 74, 817 (1948).

[17] E. Schrödinger, Commentatationes Pontif. Acad. Sci. 2, 321 (1938).

[18] D. Brill, J. Wheeler, Rev. Mod. Phys. 29, 465 (1957).

[19] F. A. Bais, R. J. Russeli, Phys. Rev. D 11, 2692 (1975).

[20] G. 'Hooft, Nucl. Phys. B 79, 276 (1974).

[21] A. M. Polyakov, Lett. J. E. T. P. 20, 430 (1974) (in Russian).

[22] B. Julia, A. Zee, Phys. Rev. D 11, 2227 (1975).
[23] D. A. Varshalovich, A. N. Moskalev, V. K. Khersonskii, *Quantum Theory of Angular Momentum* (Nauka, Leningrad, 1975; in Russian).

[24] K. Ford, J. A. Wheeler, Ann. of Phys. 7, 287 (1959).

[25] A. S. Goldhaber, Phys. Rev. B 140, 1407 (1965).

[26] J. Schwinger et al., Ann. Phys. 101, 451 (1976).

[27] D. Boulvare et al., Phys. Rev. D 14, 2708 (1976).

[28] Y. Kazama, C. N. Yang, A. S. Goldhaber, Phys. Rev. D 17, P.2287 (1977).

[29] W. Greub, H. R. Petry, J. Math. Phys. 216, 1347 (1975).

[30] A. G. Savinkov, Lett. J. E. T. P. 47, 13 (1988).

[31] N. F. Ramsey, Phys. Rev. 109, 225 (1958).

[32] D. Zwanziger, Phys. Rev. D 6, 458 (1972).

[33] L. M. Tomil’chik, J. E. T. P. 44, 160 (1963); see also in the book: V. I. Strazhev, L. M. Tomil’chik, *Electrodynamics with magnetic charge* (Nauka i Technika, Minsk, 1975; in Russian).

[34] L. M. Tomil’chik, Phys. Lett. B 61, 50 (1976).

[35] E. A. Tolkachev, L. M. Tomil’chik, Phys. Lett. B 81, 173 (1979).

[36] E. A. Tolkachev, L. M. Tomil’chik, Ya. M. Shnir, Vesti of Belarus Academy of Sciences. Ser. fiz.-mat. nauk. No. 5, 55 (1983) (in Russian); Yad. Phys. 38, 541 (1983) (in Russian); J. of Phys. G 14, 1 (1988); Yad. Phys. 50, 442 (1989) (in Russian).

[37] A. O. Barut, Ya. M. Shnir, E. A. Tolkachev, J. Phys. A.: Math.Gen. 26, L.101 (1993).

[38] V. M. Red’kov, On discrete symmetry for spin 1/2 and spin 1 particles in external monopole field and quantum-mechanical property of self-conjugacy; .......