Small $x$ resummation in collinear factorisation

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Abstract

The summation of the small $x$ corrections to hard scattering QCD amplitudes by collinear factorisation method is reconsidered and the K-factor is derived in leading $\ln x$ approximation with a result differing from the corresponding expression by Catani and Hautmann (1994). The significance of the difference is demonstrated in the examples of structure function $F_L$ and of exclusive vector meson electroproduction. The formulation covers the channels of non-vanishing conformal spin $n$ paving the way for new applications.

1 Introduction

Semi-hard processes are characterized by two essentially different large momentum scales, the hard-scattering scale $Q^2$ and the large c.m.s. energy squared $s$, $x$ being the small ratio of these scales. The QCD calculation of the hard processes involving the factorization of collinear singularities has to be improved by including the corrections enhanced by the large logarithm of $x$. The results of the QCD Regge asymptotics [1] provide the basis for the resummation of these large corrections. The method of fitting the BFKL solution consistently into the collinear factorization, called also $k_T$ factorization, has been developed by M. Ciafaloni and collaborators starting in 1990 [2, 3, 5]. The idea of $k_T$ factorization and of unintegrated parton distributions appeared also elsewhere (e.g.[9, 10, 11]), but the question of factorization scheme dependence was treated in this work. The scheme has been worked out and presented in detail in [3] and has been reanalyzed in [5]. The resummed small $x$ corrections affect the hard-scale evolution of the parton distributions in terms of the anomalous dimension of two-gluon composite operators and generate a K-factor that can be viewed as an improvement of the coefficient function. Quite a number of papers is relying on this scheme in general and on the results given in [3] in particular.

In the present paper we reconsider the small $x$ resummation. We follow the known factorization scheme. A peculiar impact factor representing the scattering off a parton induces the collinear singularities. We rely on the factorisation of these singularities in the small $\varepsilon$ asymptotics. In these details we differ from the procedure of [3] and this results in a different K-factor. Our expression has the angular momentum singularity of the BFKL solution. In examples of the structure function $F_L$ and vector meson electroproduction we demonstrate the significance of the discrepancy.

Small $x$ resummation was of great importance for analysing physics at HERA and it will be even more important for LHC physics. The resummation has been applied first

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of all to structure function evolution [12, 13, 14, 22] and to hard inclusive processes like heavy quark production [3, 15], central production of Drell-Yan pairs [17] or of Higgs [18] and the production of prompt photons [16]. Recently, the relevance for exclusive semi-hard processes like electroproduction of vector mesons has been pointed out [19]. The resummed K-factor is of particular importance here, because it cures the prediction instability appearing when going from LO to NLO. There is no change in the basic scheme when going from the inclusive case, where forward kinematics applies, to the exclusive case as long as the momentum transfer remains much smaller than the hard scale. Parton distributions have to be substituted by generalized parton distributions (GPD), but still the BFKL solution in the forward limit applies.

The resummed gluonic anomalous dimension and the K-factor are universal in the sense, that they do not depend on the details of the process and also do not necessarily change when going from inclusive to exclusive cases. They depend on the exchange channel, merely on the quantum number \( n \) appearing in the BFKL solution as conformal spin.

The application considered so far concern the channel \( n = 0 \) only. In our formulation the extension to other values of \( n \) is straightforward and in the following the main steps are done for the general case. In this way we prepare new applications of the small \( x \) resummation, which may concern both inclusive (e.g. polarized structure functions) and exclusive (e.g. vector meson electroproduction) cases.

As a general remark we would like to remind that we are discussing the approximation to perturbative QCD appropriate in the situation if moving from the Bjorken asymptotics gradually towards to Regge region. The logs of the hard scale \( Q^2 \) are primary and summed first (eventually with NLO correction). The logs of \( x \) are included in the second step as further corrections. The applicability is limited and will be lost if \( \ln x \) becomes much larger than \( \ln Q^2 \).

1.1 Resummation scheme

Consider a hard-scattering amplitude or structure function calculated in (leading) collinear approximation, in particular the contribution of singlet-exchange (vacuum quantum numbers).

\[
A = C_A^{(0)} \otimes GPD, \quad GPD = GPD_0 + P^{(0)} \otimes GPD
\]

(1.1)

\( GPD \) stands for the generalized parton distribution function. The symbol \( \otimes \) can be read as convolution by integrations over longitudinal \( (x) \) and transverse momenta \( (\kappa) \) or multiplications, if double-Mellin representation \( (\omega, \gamma) \) is used. \( P^{(0)} \) stands for the DGLAP/ERBL [6, 7] evolution kernel (or its forward counterpart) and \( C_A^{(0)} \) for the coefficient function.

The resummation of the leading \( \alpha_S \ln \frac{1}{x} \sim \frac{\alpha_S}{\omega} \) contribution can be introduced as corrections to the coefficient function and to the kernel: \( P^{(0)} \rightarrow P, \quad C_A^{(0)} \rightarrow C_A \). However, the dominant small \( x \) contribution corresponds to the configuration in the s-channel intermediate state where a single two-particle sub-energy squared compares to the full energy squared, \( s_{i,i+1} \sim s \). Thus the corrections arise from the particular iteration loop \( i \) only, with a sum over \( i \).

In the Mellin representation the small \( x \) corrections amount to a universal factor \( R_\omega \) and
a correction to the leading-order anomalous dimension \( \gamma^{(0)}_\omega \),

\[
R_\omega = 1 + \sum_{N=1}^{\infty} \left( \frac{\alpha_S}{\omega} \right)^N r_N, \quad \gamma_\omega = \gamma^{(0)}_\omega + \sum_{N=1}^{\infty} \left( \frac{\alpha_S}{\omega} \right)^N b_N, \tag{1.2}
\]

Only the latter is a modification of the gluon DGLAP kernel. The first can be regarded as the improvement of the coefficient function,

\[
C^{(0)}_A \to C_A = C^{(0)}_A R_\omega \tag{1.3}
\]

The K-factor \( R_\omega \) does not depend on the kind of scattering particles but merely on the exchange channel in terms of the BFKL quantum number \( n \).

For calculating \( R_\omega \) we consider the hard-scattering amplitude

\[
A^{(0)} = C^{(0)}_A \otimes GPD^{(0)} \tag{1.4}
\]

disregarding the DGLAP evolution. Along with this amplitude we consider the BFKL amplitude with the same particles as coupling by \( C^{(0)}_A \) in high-energy scattering off a parton,

\[
A^{BFKL} = \Phi_A \otimes g \otimes \Phi^{part}. \tag{1.5}
\]

By convolution with a distribution of partons we obtain an amplitude describing the same scattering in the small \( x \) asymptotics,

\[
A^{(x)} = A^{BFKL} \otimes GPD^{(0)}. \tag{1.6}
\]

Here \( g \) stands for the Green function of BFKL two-gluon exchange and \( \Phi_A \) is the impact factor coupling the same particles as in the hard scattering. \( \Phi^{part} \) is the unusual partonic impact factor: since its projection onto the channel isotropic in the azimuthal angle is constant in the transverse momentum, it does not obey the condition of vanishing with the transverse momenta which for colourless hadronic impact factors follows from gauge invariance. The convolution of the partonic impact factor with the BFKL Green function \( g \) results in the collinear singularities which are factorized to all order of the coupling constant into an universal transition function \( \Gamma(\omega, \varepsilon) \),

\[
F^{(0)} = g \otimes \Phi^{part} = F \cdot \Gamma(\omega, \varepsilon). \tag{1.7}
\]

The factor \( \Gamma(\omega, \varepsilon) \) carrying the collinear divergencies is absorbed by redefining the bare parton distribution \( GPD^{(0)} \)

\[
GPD = \Gamma(\omega, \varepsilon) GPD^{(0)}, \quad A^{(x)} = \Phi_A \otimes F \otimes GPD. \tag{1.9}
\]

As an essential step for a consistent improvement, the factorisation prescription should match the one adopted in the collinear calculation of the hard amplitude, e.g. \( \overline{MS} \) scheme. The resulting convolution of \( F \) with the parton distribution, sometimes called unintegrated parton gluon distribution, is simply related to the gluon distribution at small \( x \),

\[
F \otimes GPD = \gamma_\omega GPD^{(x)}. \tag{1.10}
\]

In transverse momenta we expect the form (no running coupling)

\[
F = \gamma_\omega R_\omega \left( \frac{\kappa^2}{\mu_F^2} \right) \gamma_\omega. \tag{1.11}
\]
We compare now the structure of the original hard-scattering amplitude $A$ with the small $x$ amplitude

$$A^{(x)} = \Phi_A \otimes F \otimes GPD^{(x)}.$$  

(1.12)

Identifying the factorisation scale with the hard scale this results in the wanted improvement of the coefficient function,

$$C_A = C_A^{(0)} R(\omega).$$  

(1.13)

Simultaneously we learn that in the same approximation the impact factor and the bare coefficient function are related by

$$C_A^{(0)} = \Phi_A \gamma_\omega.$$  

(1.14)

2 BFKL in $2 + 2\varepsilon$ transverse dimensions

Consider the BFKL equation in the forward limit in $d = 2 + 2\varepsilon$,

$$\omega \ g(\omega, \vec{\kappa}, \vec{\kappa}_0) = \delta^{(2+2\varepsilon)}(\vec{\kappa} - \vec{\kappa}_0) + \frac{\tilde{\alpha}_S}{\mu^{2\varepsilon}} \hat{K} \cdot g(\omega, \vec{\kappa}, \vec{\kappa}_0).$$  

(2.1)

Where we have defined the (bare) dimensionless coupling constant $\tilde{\alpha}_S = \frac{g^2 N_C}{4\pi^2}$ and $\mu$ is the fixed scale introduced by dimensional regularisation. The inhomogeneous term is specified in such a way that the solution $g(\omega, \vec{\kappa}, \vec{\kappa}_0)$ is the Green function of the reggeized two-gluon exchange and the BFKL amplitude is composed with impact factors $\Phi_{A/B}$ as

$$A_{BFKL} = \int \frac{d^{2+2\varepsilon} \kappa}{\kappa^2} \int \frac{d^{2+2\varepsilon} \kappa_0}{\kappa_0^2} \Phi_A(\vec{\kappa}) g(\omega, \vec{\kappa}, \vec{\kappa}_0) \Phi_B(\vec{\kappa}_0).$$  

(2.2)

In one-loop approximation the operator $\hat{K}$ acts as

$$\hat{K} \cdot g(\vec{\kappa}, \vec{\kappa}_0) = \frac{1}{\pi} \int \frac{d^{2+2\varepsilon} \kappa' \kappa_0'}{(2\pi)^{2\varepsilon}} \frac{<\vec{\kappa}|\vec{\kappa}'|g(\vec{\kappa}', \vec{\kappa}_0)}{<\vec{\kappa}'|\vec{\kappa}'>}.$$  

(2.3)

with

$$<\vec{\kappa}|\vec{\kappa}'|g(\vec{\kappa}', \vec{\kappa}_0) = \frac{1}{(\vec{\kappa} - \vec{\kappa}')^2} - \frac{1}{2} \delta^{(2+2\varepsilon)}(\vec{\kappa} - \vec{\kappa}') \int \frac{d^{2+2\varepsilon} \kappa'' \kappa_0''}{(\vec{\kappa}'' - \vec{\kappa}''')^2}.$$  

(2.4)

For solving this equation we shall rely on rotation symmetry in $d = 2 + 2\varepsilon$ dimensions. For $d > 2$ representations besides of the trivial one have dimension larger that 1 and for $d > 3$ more than one quantum number is needed for specifying a generic representation. We can avoid complications by restricting to the class of representations that would appear as symmetric traceless tensors of rank $n$. Instead of working with all spherical harmonics we can restrict to the ones representing the highest or lowest weight states in these representations. This means we consider functions

$$\psi_{\gamma,n}(\vec{\kappa}) = (\vec{\kappa}^2)^{\gamma - \frac{n}{2}} (\hat{e} \vec{\kappa})^n, \quad \psi_{\gamma,n}^*(\vec{\kappa}) = (\vec{\kappa}^2)^{-\gamma - \frac{n}{2}} (\hat{e}^* \vec{\kappa})^n$$  

(2.5)

where $\hat{e}$, $\hat{e}^*$ are two null vectors (with complex-valued components) such that $\hat{e} \cdot \hat{e}^* = 1$ and $\hat{e}^2 = e^{*2} = 0$. They are the $2 + 2\varepsilon$ dimensional extensions of the two-dimensional vectors $\frac{1}{\sqrt{2}}(\hat{e}_1 \pm i \hat{e}_2)$. Note that in this paper we only consider positive values of the conformal spin $n$, thus identifying $n$ with $|n|$.
In a partial channel of a given \( n \) we substitute the inhomogeneous term by the projector \( \hat{\Pi}_n \) onto the highest weight states of representation \( n \) and consider the equation which now deals only with dimensionless quantities

\[
\omega g_n(\omega, \vec{\kappa}, \vec{\kappa}_0) = <\vec{\kappa}|\hat{\Pi}_n|\vec{\kappa}_0> + \tilde{\alpha}_S^r \hat{K} \cdot g_n(\omega, \vec{\kappa}, \vec{\kappa}_0). \tag{2.6}
\]

The dimensionless strong coupling which was frozen at \( d = 2 \) dimensions is now running as

\[
\tilde{\alpha}_S^r(\mu_R^2) = \tilde{\alpha}_S(\frac{\mu_R^2}{\mu^2})^\varepsilon \tag{2.7}
\]

with the renormalization scale \( \mu_R \) and \( \tilde{\alpha}_S = \tilde{\alpha}_S^r(\mu^2) \) is the bare dimensionless strong coupling defined previously. We have to calculate the action of the kernel on functions (2.5) (see Appendix A)

\[
\hat{K} \cdot \psi_{\gamma-1,n}(\vec{k}) = \lambda(\gamma, n, \varepsilon) \psi_{\gamma-1+\varepsilon,n}, \tag{2.8}
\]

\[
\lambda(\gamma, n, \varepsilon) = \frac{1}{(4\pi)^2} \left[ b(\gamma, n, \varepsilon) - \frac{1}{2} b(0, 0, \varepsilon) \right], \tag{2.9}
\]

\[
b(\gamma, n, \varepsilon) = \Gamma^{-1}(\varepsilon) B(\varepsilon, 1 + \frac{n}{2} - \gamma - \varepsilon) B(\varepsilon, \frac{n}{2} + \gamma + \varepsilon). \tag{2.10}
\]

The BFKL kernel can be viewed as the matrix elements of the operator \( \hat{K} \) which can be represented in terms of the quasi-eigenvalues and a shift operator in \( \gamma \).

\[
<\gamma,n|\hat{K}|\gamma_0,n> = \lambda(\gamma_0,n,\varepsilon) e^{\varepsilon \partial_{\gamma_0}} \delta(\gamma - \gamma_0). \tag{2.11}
\]

The \( \kappa \) and \( \gamma \) representations are related by the transition kernels

\[
<\vec{\kappa}|\gamma,n> = \psi_{\gamma,n}(\vec{k}), \quad <\gamma,n|\vec{\kappa}) = \psi_{\gamma,n}(\vec{k}). \tag{2.12}
\]

The completeness relation has to be formulated in such a way that the inhomogeneous term, i.e. \( <\kappa|\hat{\Pi}_n|\kappa'> \) appears.

\[
\int \frac{d^{2+2\varepsilon} \kappa}{|S^{1+2\varepsilon}|} \left< \gamma', n'|\vec{\kappa} \right> <\vec{\kappa}|\gamma,n> = \delta(\gamma' - \gamma) \delta_{n',n}, \tag{2.13}
\]

\[
<\vec{\kappa}|\hat{\Pi}_n|\vec{\kappa}' > = \frac{1}{2\pi i} \int_{\frac{1}{2} - i\infty}^{\frac{1}{2} + i\infty} d\gamma <\vec{\kappa}|\gamma,n> <\gamma,n|\vec{\kappa}' >
\]

\[
= \delta(\ln(\frac{\kappa'^2}{\kappa^2})) (\vec{\kappa}')^{-n} (\vec{\kappa})^n. \tag{2.14}
\]

Here \( |S^{1+2\varepsilon}| = 2\pi^{1+\varepsilon} \Gamma^{-1}(1 + \varepsilon) \) is the area of the unit hypersphere in \( d = 2 + 2\varepsilon \) dimensions.

We look now on the Green function as on matrix elements of the operator \( \hat{g}_n \), where \( g_n(\vec{\kappa}, \vec{\kappa}') = <\vec{\kappa}|\hat{g}_n|\vec{\kappa}' >. \) The operator equation reads

\[
\omega \hat{g}_n = \hat{\Pi}_n + \tilde{\alpha}_S^r \hat{K} \cdot \hat{g}_n \tag{2.15}
\]

and has a simple formal solution. In the \( \gamma \) representation where the operator \( \hat{K} \) has the simple form (2.11) we obtain

\[
<\gamma,n|\hat{g}_n|\gamma_0,n> = \frac{1}{\omega - \tilde{\alpha}_S^r \lambda(\gamma_0,n,\varepsilon)} e^{\varepsilon \partial_{\gamma_0}} \delta(\gamma - \gamma_0). \tag{2.16}
\]

Here \( \omega = \tilde{\alpha}_S^r \lambda(\gamma_0,n,\varepsilon) <\gamma_0,n|\hat{g}_n|\gamma_0,n> \).
Changing to the original transverse momentum representation we obtain
\[ g_n(\kappa, \kappa_0) = \frac{1}{2\pi i} \int_{\frac{1}{2}-i\infty}^{\frac{1}{2}+i\infty} \frac{d\gamma(\kappa^2)\gamma^{-\frac{2}{3}}(\kappa\bar{\epsilon})^n}{\omega - \tilde{\alpha}_S e^{-\epsilon\partial_\gamma} \lambda(\gamma, n, \epsilon)(\kappa_0^2)\gamma^{-\frac{2}{3}}(\epsilon^* \kappa_0)^n}. \tag{2.17} \]

3 Factorisation of collinear singularities

3.1 Unintegrated GPD

The reggeized gluon Green function is not singular in $\varepsilon$, the singularities appearing in the action of the bare kernel and in the gluon trajectory cancel. The collinear singularities appear from the convolution with a parton impact factor. No singularity would appear with a hadronic impact factor. Therefore, we study the convolution with dimensional regularisation
\[
F(\omega, n, \kappa, \mu_F, \varepsilon) = \int \frac{d^{2+2\varepsilon} \kappa_0}{\kappa_0^2} g_0(\kappa, \kappa_0, \varepsilon) \theta(\mu_F^2 - \kappa_0^2) \Phi_B(\kappa_0, \mu, \varepsilon, n)
+ \int \frac{d^2 \kappa_0}{\kappa_0^2} g_0(\kappa, \kappa_0, \varepsilon = 0) \theta(\kappa_0^2 - \mu_F^2) \Phi_B(\kappa_0, \mu, 0, n) \tag{3.1}
\]
with the partonic impact factor $\Phi_B(\kappa_0, \mu, \varepsilon, n)$. In the partial channel $n = 0$ it is just constant
\[
\Phi_B(\kappa_0, \mu, \varepsilon, 0) = \frac{\bar{\alpha}_S}{\mu_F^2} \frac{2}{|S(1+2\varepsilon)|}. \tag{3.2}
\]
This convolution defines the unintegrated gluon density or GPD of the parton B denoted by $F(\omega, \kappa, n)$ in the following. We shall include the case of general $n$ below. Its convolution with some input hadronic GPD results in the unintegrated small-$x$ improved GPD.

For $Re\gamma > 0$ divergencies appear in the first term and they are regularised by $\varepsilon$ having a real part larger than that. The wanted result will be obtained by separating the leading singularity factor. Less singular contributions, in particular the regular contribution from the second term (3.1), will not affect this result. The form of the second term is specified such that in the double-log approximation only the term with the leading singular factor appears with no non-leading remainder. This specification is actually not essential, however the introduction of the factorisation scale $\mu_F$ in the definition of a parton impact factor is unavoidable.

It is known that for hadronic impact factors no divergencies appear, because they vanish in the limit $\kappa_0 \to 0$. In the BFKL Green function the integration line is $Re\gamma = \frac{1}{2}$. The asymptotics in $\kappa^2$ (leading twist) receives its contributions from the singularities in the vicinity of $\gamma = 0$. In the partonic case the condition of regularisation $Re(\varepsilon - \gamma) > 0$ has to be preserved and therefore the pole $(\varepsilon - \gamma)^{-1}$ arising from the integration over $\kappa_0$ is to be kept to the right of the contour. We consider the collinear divergent part
\[
F_c(\omega, \kappa) = \frac{\bar{\alpha}_S}{\mu_F^2} \int \frac{d\gamma}{2\pi i} (\kappa^2)\gamma \int_0^{\mu_F^2} \frac{1}{\omega - \tilde{\alpha}_S e^{-\epsilon\partial_\gamma} \lambda(\gamma, 0, \varepsilon)(\kappa_0^2)^{\varepsilon - \gamma - 1}}.
+ \frac{\bar{\alpha}_S}{\omega} (\mu_F^2)^{\varepsilon} \int \frac{d\gamma}{2\pi i} (\kappa^2)\gamma \frac{1}{\omega - \tilde{\alpha}_S e^{-\epsilon\partial_\gamma} \lambda(\gamma, 0, \varepsilon) e^{-\epsilon\partial_\gamma} \varepsilon - \gamma}. \tag{3.3}
\]
Owing to the explicit form of $\lambda(\gamma, \varepsilon)$ given in Appendix A eq.(A.5) we have substituted it by $\frac{1}{\gamma + \varepsilon} \lambda_1(\gamma + \varepsilon, \varepsilon)$. An important remark has to be done at this point: as we discussed previously, the strong coupling $\tilde{\alpha}_S$ in the denominator comes from the BFKL Green function,
therefore runs with the yet unknown renormalization scale $\mu_R$. But we explicitly show in the Appendix B how this scale dependance actually disappears, hence we replace from now $\tilde{\alpha}_S^\gamma$ by $\tilde{\alpha}_S$, and postpone the discussion on fixing $\mu_R$ at the end of the calculation.

We first study the simplified version of $F(\omega, \kappa, 0)$ where we put just $\lambda_1 = 1$ for explaining the essential steps, denoting the result by $F_0(\omega, \kappa)$. This corresponds to the double-logarithmic approximation. Indeed, the dependence on $(\gamma, \omega)$ enters essentially as $\tilde{\alpha}_S(\omega) e^{-\varepsilon \partial_{\gamma}}$. The leading contribution at large $\kappa^2$ resulting in the large $Q^2$ asymptotics after convolution with the impact factor involving the virtual photon, is obtained from the residue in the pole $\gamma = \hat{\gamma}_0^{(0)}$, with

$$\hat{\gamma}_0^{(0)} = \frac{\tilde{\alpha}_S}{\omega} e^{-\varepsilon \partial_{\gamma}}, \quad (3.4)$$

as

$$F_{0<}(\omega, \kappa, 0) = \left( \frac{\tilde{\alpha}_S}{\mu^2} \right)^\varepsilon \int \frac{d\gamma}{2\pi i} (\kappa^2)^\gamma \int_0^{\mu_F^2} d\kappa_0^2 \frac{1}{\gamma - \hat{\gamma}_0^{(0)}} \frac{\gamma}{\omega} (\kappa_0^2)^{-\gamma - 1} \bigg|_{\gamma' = 0}. \quad (3.5)$$

The factor involving the shift operator can be expanded in geometric series and then in each term the shift operator is moved to one side. The calculation is given in Appendix C, where it is shown that the result is

$$F_{0<}(\omega, \kappa, 0) = \tilde{\alpha}_S(\mu^2)^\varepsilon (\frac{\kappa^2}{\mu^2_F})^{\hat{\gamma}_0^{(0)}} \frac{1}{\omega} \left( \exp(\frac{1}{\varepsilon \hat{\gamma}_0^{(0)}}) - 1 \right). \quad (3.6)$$

Also an alternative way is described in Appendix C because it is convenient for treating the general case $\lambda_1 \neq 1$. The integral over $\gamma$ can be done before operator ordering. To avoid interference with the action of the shift operator we replace in the integrand $\gamma$ by $\gamma + \gamma'$, let the shift operator act on $\gamma'$ only and put finally $\gamma' = 0$.

We add the contribution from $k_0^2 > \mu_F^2$,

$$F_{0>}(\omega, \kappa, 0) = \tilde{\alpha}_S \int \frac{d\gamma}{2\pi i} (\kappa^2)^\gamma \int_0^{\infty} d\kappa_0^2 \frac{1}{\gamma - \hat{\gamma}_0^{(0)}} \frac{\gamma}{\omega} (\kappa_0^2)^{-\gamma - 1} = \tilde{\alpha}_S(\frac{\kappa^2}{\mu^2_F})^{\hat{\gamma}_0^{(0)}} \quad (3.7)$$

and obtain

$$F_0(\omega, \kappa, 0) = \left( \frac{\kappa^2}{\mu^2_F} \right)^{\hat{\gamma}_0^{(0)}} \frac{\tilde{\alpha}_S}{\omega} \exp(\frac{1}{\varepsilon \hat{\gamma}_0^{(0)}}). \quad (3.8)$$

Our way of factorizing the collinear singularities in the convolution of the BFKL Green function with the partonic impact factor follows the standard BFKL method and differs technically from the ways followed in [3, 5]. In order to show that the reason for the discrepancy in the results is not in these technical details we treat also the form given in [3] as the first solution. This solution of BFKL equation is obtained by iteration and has been presented by Catani and Hautmann in the form

$$F = \sum_{k=0}^{\infty} \left( \frac{\tilde{\alpha}_S}{\omega} C_\varepsilon(\frac{k^2}{\mu^2})^\varepsilon \right)^{k+1} C_{k+1}(\varepsilon). \quad (3.9)$$

In [3] the iteration starts from a Born term representing what we have called parton impact factor, so the notation $F$ refers to the same quantity as above. $C_\varepsilon$ is to specify the $\overline{MS}$ prescription. We shall use temporarily the abbreviation $z_\varepsilon = C_\varepsilon(\frac{k^2}{\mu^2})^\varepsilon$. The BFKL equation results in the iterative relation for the coefficients,

$$C_{k+1}(\varepsilon) = C_k(\varepsilon) I_k(\varepsilon), \quad (3.10)$$
with $C_1(\varepsilon) = 1$. The notation $I_k(\varepsilon)$ in [3] is related to ours as $I_k(\varepsilon) = I(k\varepsilon, \varepsilon)$ and

$$I(\gamma, \varepsilon) = \frac{\lambda_1(\gamma, 0, \varepsilon)}{\gamma} = \lambda(\gamma - \varepsilon, 0, \varepsilon).$$

(3.11)

Note that only the case $n = 0$ was considered in [3] and we restrict ourselves to this case now. With the help of the shift operator $T_\varepsilon = e^{\varepsilon \partial_\gamma}$ the coefficients can be written compactly,

$$C_{k+1}(\varepsilon) = \prod_{j=1}^k \frac{\lambda_1(j\varepsilon, \varepsilon)}{j\varepsilon} = \left. \left( T_\varepsilon \frac{\lambda_1(\gamma', \varepsilon)}{\gamma'} \right)^k \right|_{\gamma' = 0}. \quad (3.12)$$

Defining $\hat{\gamma}(0)^{\omega} = \frac{\alpha_S}{\omega} e^{\varepsilon \partial_\gamma}$ this allows to do the sum

$$F = \left( \alpha_S \frac{\varepsilon}{\mu^2} \right) \left. \frac{1}{1 - \frac{\hat{\gamma}(0)^{\omega}}{\gamma}} \frac{1}{z_\varepsilon} \frac{\lambda_1(\gamma', \varepsilon)}{\gamma'} \right|_{\gamma' = 0}. \quad (3.13)$$

The double log case where $\lambda_1(\gamma, \varepsilon) = 1$ is easily done

$$\left. \frac{1}{1 - \gamma(0)^{\omega}} \frac{1}{z_\varepsilon} \frac{1}{\mu^2} \right|_{\gamma = 0} = \sum_{N=0}^\infty z_\varepsilon^N \gamma(0)^{\omega} \frac{1}{\gamma} \frac{1}{\gamma} \frac{1}{\gamma} \frac{1}{\gamma} \right|_{\gamma = 0} = \exp\left( \frac{1}{\varepsilon} \gamma(0)^{\omega} z_\varepsilon \right)

= \left( \frac{k^2}{\mu^2} \right) \frac{\gamma(0)^{\omega}}{\varepsilon} \frac{1}{\varepsilon} \gamma(0)^{\omega}. \quad (3.14)$$

The result for this simplified case is equivalent to (3.8) as expected, beside on the $\mu_F$ dependance which will be further discussed in details for the general case. The details of an alternative method, which starts from implementing the condition $\gamma = 0$ by a contour integral are also given in Appendix C.

### 3.2 Asymptotics in $\varepsilon$

We consider now $F_<(\omega, \kappa)$ (3.3) without omitting $\lambda_1(\gamma, \varepsilon)$. We substitute the integration variable $\gamma$ by

$$\tilde{\gamma} = \frac{\gamma}{\lambda_1(\gamma, \varepsilon)} \quad (3.15)$$

and obtain

$$F_<(\omega, \kappa) = \frac{\alpha_S}{\omega} \left( \frac{\mu_F^2}{\mu^2} \right)^\varepsilon \int \frac{d\tilde{\gamma}}{2\pi i} \frac{\lambda_1(\gamma, \varepsilon)}{1 - \tilde{\gamma}} \frac{1}{\mu_F^2} \frac{1}{\gamma - \gamma(0)^{\omega}} \frac{1}{\varepsilon - \gamma}. \quad (3.16)$$

Now $\gamma$ is to be considered as a function of $\tilde{\gamma}$ and $\lambda'_1(\gamma, \varepsilon) = \partial_\gamma \lambda_1(\gamma, \varepsilon)$. We proceed in close analogy to the above simplified case treated in detail in Appendix C. We do first the $\gamma$ integral by residue following the second calculation in Appendix C. The singularities originate from the last factor in (3.16) and it is sufficient to account for the shift operator action on this particular factor. We have to release the integration variable from its role as the operator conjugate to the infinitesimal shift. This we do by substituting $\gamma$ by $\gamma + \gamma'$ in the last pole factor and redefining that now $\gamma(0)^{\omega}$ is acting on $\gamma'$. The corresponding
substitution in the remaining factors is unimportant, i.e. amounts to corrections non-leading in $\varepsilon \to 0$.

We pick up the residue located at the pole $\tilde{\gamma} = \hat{\gamma}_\omega^{(0)}$ equivalent to the operatorial equation

$$\hat{\gamma}_\omega(\varepsilon) = \lambda_1(\hat{\gamma}_\omega(\varepsilon), \varepsilon) \hat{\gamma}_\omega^{(0)},$$

(3.17)

whose solution $\hat{\gamma}_\omega(\varepsilon)$ can be expanded in powers of $\hat{\gamma}_\omega^{(0)}$ as shown in Appendix B eq.(B.3). A crucial feature of our approach is that the location of this pole, or equivalently the explicit expression of $\hat{\gamma}_\omega(\varepsilon)$ is not affected by the running of the strong coupling as we show in the Appendix B, therefore justifying the replacement done previously of $\tilde{\alpha}_S^\gamma$ by $\tilde{\alpha}_S$, also hidden in $\hat{\gamma}_\omega^{(0)}$. Using the relation (3.17), we get

$$F_{<}(\omega, \kappa) = \frac{\tilde{\alpha}_S \mu_F^2}{\omega} \left[ \frac{1}{1 - \hat{\gamma}_\omega^{(0)} \Lambda_1(\hat{\gamma}_\omega^{(0)}, \varepsilon)} \right] \frac{1}{\mu_F^2} \hat{\gamma}_\omega(\varepsilon) \frac{1}{1 - \hat{\gamma}_\omega^{(0)} - \hat{\gamma}_\omega^{(0)}} \bigg|_{\gamma' = 0} \right.$$ \hspace{1cm} (3.18)

Now we evaluate the product

$$A(\hat{\gamma}_\omega^{(0)}, \varepsilon) B(\hat{\gamma}_\omega^{(0)}, \varepsilon, \gamma') \bigg|_{\gamma' = 0} = A(\hat{\gamma}_\omega^{(0)}, \varepsilon) \sum_{N=1}^{\infty} \left( \hat{\gamma}_\omega(\varepsilon) \frac{1}{\gamma - \gamma'} \right)^N \bigg|_{\gamma' = 0} \right.$$

(3.19)

We write (for $\gamma' < 0$) $\frac{1}{\gamma - \gamma'} = \int_0^1 \frac{d\alpha}{\alpha} \alpha^{-\gamma'}$ and notice that

$$f(\hat{\gamma}_\omega^{(0)}) \alpha^{-\gamma'} = \alpha^{-\gamma'} f(\hat{\gamma}_\omega^{(0)} \alpha^\varepsilon).$$

(3.20)

Therefore we have

$$(\hat{\gamma}_\omega(\varepsilon) \frac{1}{\gamma - \gamma'})^N = \prod_{i=1}^{N} \int_0^1 \frac{d\alpha_i}{\alpha_i} \alpha_i^{-\gamma'} \prod_{i=1}^{N} \hat{\gamma}_\omega(\varepsilon) \alpha_i \ldots \alpha_i \varepsilon, \varepsilon).$$

(3.21)

We substitute $\beta_i = \alpha_i \alpha_i \ldots \alpha_i \varepsilon$ and obtain

$$A(\hat{\gamma}_\omega^{(0)}, \varepsilon) B(\hat{\gamma}_\omega^{(0)}, \varepsilon, \gamma') = \sum_{N=1}^{\infty} \frac{1}{\varepsilon^N} \prod_{i=1}^{N} \hat{\gamma}_\omega(\varepsilon) \beta_i \alpha_i \ldots \alpha_i \varepsilon \varepsilon) \times \prod_{i=1}^{N} \hat{\gamma}_\omega(\varepsilon) \beta_i A(\hat{\gamma}_\omega^{(0)} \beta_i, \varepsilon).$$

(3.22)

After having acted on the whole $\gamma'$ dependent terms, all the involved shift operator are moved to the right and act on a function constant in $\gamma'$; then the shift operators can be substituted by unit operator. We can now do the limit $\gamma' \to 0^-$. This means to replace

$$\hat{\gamma}_\omega^{(0)} \to \gamma_\omega^{(0)} = \frac{\tilde{\alpha}_S}{\omega}$$

(3.23)

and also $\hat{\gamma}_\omega(\varepsilon) \to \gamma_\omega(\varepsilon)$ where $\gamma_\omega(\varepsilon)$ is the solution of

$$\gamma_\omega(\varepsilon) = \lambda_1(\gamma_\omega(\varepsilon), \varepsilon) \gamma_\omega^{(0)}.$$

(3.24)
After this we get
\[ A(\hat{\gamma}_0, \varepsilon) B(\hat{\gamma}_0, \varepsilon, \gamma') \bigg|_{\gamma'=0} = \frac{1}{\varepsilon} \int_0^1 \frac{d\beta}{\beta} \gamma_\omega(\gamma_0 \beta, \varepsilon) A(\gamma_0 \beta, \varepsilon) \exp \left( \frac{1}{\varepsilon} \int_\beta^1 \frac{d\beta_1}{\beta_1} \gamma_\omega(\gamma_0 \beta_1, \varepsilon) \right). \]  
(3.25)

We compute in the Appendix D the asymptotics of this product and we get
\[ F_\varepsilon(\omega, \kappa) = \frac{\bar{\alpha}_S}{\omega} \frac{1}{1 - \gamma_\omega(0)} \left( \frac{\kappa^2}{\mu_F^2} \right)^\gamma_\omega \exp \left( \frac{1}{\varepsilon} \int_0^1 \frac{d\alpha}{\alpha} \gamma_\omega(\gamma_0 \alpha, \varepsilon) \right) - 1 \} . \]  
(3.26)

Outside the exponential \( \gamma_\omega(\varepsilon) \) is evaluated at \( \varepsilon = 0 \). The subtraction term in the bracket is actually overestimating accuracy. The latter as well as the contribution \( F_\varepsilon > 0 \) are nonleading in the asymptotics \( \varepsilon \to 0 \) and cancel each other. We rewrite the preexponential factors in terms of the usual BFKL eigenvalue function \( (A.12) \) in 2 dimensions \( \chi(\gamma) \), using the relation \( (B.15) \), and obtain the final result by restoring the explicit expression \( \bar{\alpha}_S(\mu_F^2) \) of the dimensionless strong coupling in the argument of the anomalous dimension appearing in the exponential term since it leads to a factor contributing to the order \( \varepsilon^0 \), and also with the factor \( S_\varepsilon = \exp\{ -\varepsilon[\psi(1) + \ln 4\pi] \} \) which characterizes the MS-scheme, as
\[ F(\omega, \kappa) = \gamma_\omega \left( \frac{1}{1 - \gamma_\omega(0)} \right) \left( \frac{\kappa^2}{\mu_F^2} \right)^\gamma_\omega \exp \left( \frac{1}{\varepsilon} \int_0^1 \frac{d\alpha}{\alpha} \gamma_\omega(\gamma_0 \alpha, \varepsilon) \right) \]  
(3.27)

with
\[ \gamma_\omega(\gamma_0, \varepsilon) = \gamma_\omega(\gamma_0) + \varepsilon \gamma_\omega(\gamma_0) + \mathcal{O}(\varepsilon^2) \]  
(3.28)

where the explicit expressions of the BFKL gluon anomalous dimension \( \gamma_\omega \) and of its \( \varepsilon \)-correction \( \gamma_\omega(\varepsilon) \) are given in Appendix B, eq.(B.11) and (B.12).

The analysis of the \( \varepsilon \) asymptotics works with modification also for the discrete sum solution (3.9). We show that this solution results in the same asymptotics. We follow the steps of the double log calculation as in Appendix C. We include the factor \( z_\varepsilon \) into \( \lambda_1 \). We use the notation \( \tilde{\gamma}(\gamma) = \frac{\gamma}{\lambda(\gamma)} \).

After this we get
\[ \frac{1}{\gamma - \hat{\gamma}_0(\gamma, \varepsilon)} \bigg|_{\gamma'=0} = \frac{1}{2\pi i} \oint_{C_0} \frac{d\gamma}{\gamma} \frac{1}{\gamma(\gamma + \gamma')} \bigg|_{\gamma'=0} = 1 + \frac{1}{2\pi i} \oint_{C_0} \frac{d\gamma}{\gamma} \frac{1}{\gamma(\gamma + \gamma') - \hat{\gamma}_0(\gamma, \varepsilon)} \bigg|_{\gamma'=0} = 1 + \frac{1}{2\pi i} \oint_{C} \frac{d\gamma}{\gamma'' - \gamma'} \frac{1}{\gamma'' - \gamma'} \hat{\gamma}_0(\gamma'|T) \bigg|_{\gamma'=0}. \]  
(3.29)

We change the integration variable \( \gamma'' \) to \( \tilde{\gamma} \); now \( \gamma'' \) is to be considered as a function of \( \tilde{\gamma} \).

\[ \frac{1}{\gamma - \hat{\gamma}_0(\gamma, \varepsilon)} \bigg|_{\gamma'=0} = 1 + \frac{1}{2\pi i} \oint_{C} \frac{d\tilde{\gamma}}{\tilde{\gamma}'' - \gamma'} \frac{1}{\tilde{\gamma}'' - \gamma'} \hat{\gamma}_0(\gamma'|T) \frac{\lambda_1(\gamma'', \varepsilon)}{1 - \gamma_0(\gamma', \varepsilon)} \bigg|_{\gamma'=0}. \]  
(3.30)

We have
\[ \hat{\gamma}_0(\gamma'|T) = \hat{\gamma}_0(\gamma)|_{\gamma = \hat{\gamma}_0(\gamma'|T)} , \]  
(3.31)
for the solution of the equation (3.17) and continue the calculation as

\[
\frac{1}{1 - \gamma_0(\omega) \lambda_1(\gamma, \epsilon)} \bigg|_{\gamma = 0} = 1 + \frac{1}{\gamma' - \gamma_0 T} \frac{\gamma_T}{1 - \gamma_0 T} \lambda_1(\gamma_T, \epsilon) \bigg|_{\gamma' = 0}
\]

\[
= 1 + \sum_{N=1}^{\infty} \frac{1}{\gamma' - \gamma_0 T} \frac{\gamma_T}{1 - \gamma_0 T} \frac{1}{\lambda_1(\gamma_T, \epsilon)} \bigg|_{\gamma' = 0}
\]

\[
(3.32)
\]

We write \( \frac{1}{\gamma'} = \int_0^1 \frac{d\alpha}{\alpha} \alpha^{\gamma'} \) and use the commutation relation (3.20). The \( N \)th term of the series results in

\[
\prod_{i=1}^{N} \int_0^1 \frac{d\alpha_i}{\alpha_i} \prod_{i=1}^{N} \gamma_\omega(\gamma_0(0) \alpha_1^\epsilon \alpha_2^\epsilon \ldots \alpha_N^\epsilon, \epsilon) A(\gamma_0(0) \alpha_1^\epsilon \alpha_2^\epsilon \ldots \alpha_N^\epsilon, \epsilon)(\alpha_1 \ldots \alpha_N)^{\gamma'}.
\]

\[
(3.33)
\]

We change to the integration variables \( \beta_i = \prod_i \alpha_i^\epsilon \) and obtain

\[
\varepsilon^{-N} \int_0^1 \frac{d\beta_1}{\beta_1} \gamma_\omega(\gamma_0(0) \beta_1, \epsilon) \int_0^\beta_1 \frac{d\beta_2}{\beta_2} \gamma_\omega(\gamma_0(0) \beta_2, \epsilon) \ldots \int_0^{\beta_{N-1}} \frac{d\beta_N}{\beta_N} \gamma_\omega(\gamma_0(0) \beta_N, \epsilon) A(\gamma_0(0) \beta_N, \epsilon) \beta_N^{\gamma'}
\]

\[
= \frac{1}{\varepsilon} \int_0^1 \frac{d\beta_N}{\beta_N} \gamma_\omega(\gamma_0(0) \beta_N, \epsilon) A(\gamma_0(0) \beta_N, \epsilon) \beta_N^{\gamma'} \left( \frac{1}{\varepsilon} \int_0^1 \frac{d\beta_1}{\beta_1} \gamma_\omega(\gamma_0(0) \beta_1, \epsilon) \right)^{N-1}
\]

\[
(3.34)
\]

The sum can be done and we obtain at \( \gamma' = 0 \)

\[
F = \gamma_0(0) \left\{ 1 + \frac{1}{\varepsilon} \int_0^1 \frac{d\beta}{\beta} \gamma_\omega(\gamma_0(0) \beta) A(\gamma_0(0) \beta, \epsilon) \exp \left( \frac{1}{\varepsilon} \int_0^1 \frac{d\beta_1}{\beta_1} \gamma_\omega(\gamma_0(0) \beta_1, \epsilon) \right) \right\}
\]

\[
(3.35)
\]

In Appendix D we prove that the asymptotics is obtained substituting in the above expression the function \( A(\beta) \) by \( A(1) \). With this asymptotic result and restoring also the scale dependence of the dimensionless strong coupling \( z_\epsilon = (\frac{\kappa}{\mu^2})^\epsilon C_\epsilon \) we get

\[
F = \gamma_0(0) \frac{1}{1 - \gamma_0(0) \lambda_1(\gamma, 0)} \left( \frac{\kappa^2}{\mu^2} \right)^\gamma \omega \exp \left( \frac{1}{\varepsilon} \int_0^C \frac{d\beta_1}{\beta_1} \gamma_\omega(\gamma_0(0) \beta_1, \epsilon) \right) (1 + \mathcal{O}(\epsilon))
\]

\[
(3.36)
\]

which is equivalent to eq.(3.27), the renormalization/factorization scale dependence being discussed in the next section.

### 3.3 Channels with \( n > 0 \)

Let us now consider the generalisation of the previous calculation for a non vanishing conformal spin \( n \). Notice first that the singularities of the eigenvalue function to the left of \( \mathcal{R} \epsilon \gamma = \frac{n}{2} \) are located in the vicinity of \( \gamma = -\frac{n}{2} \). For picking up the contributions to the asymptotics in \( \kappa^2 \) it is convenient to move the contour to this region and change the integration variable to \( \gamma' = \gamma + \frac{n}{2} \). Then inverse powers \( (\gamma')^{-m} \) result in \( (\ln \kappa^2)^m \). This shows that the collinear singularity generated by the partonic impact factor should supply just a pole in \( \gamma' \) regularised by \( \epsilon \), i.e. \( (\epsilon - \gamma')^{-1} \). For regularisation this pole is kept to the right of the contour.

In order to generate this pole the partonic impact factor should be of the form

\[
\Phi_\kappa(\kappa) = \frac{\alpha_S}{\mu^{2\epsilon}} \left( \frac{\kappa}{\mu} \right)^n (\kappa^2)^{-n} \frac{2}{|S^1 + 2\epsilon|}.
\]

(3.37)
In analogy to the $n = 0$ case we have from (2.17, 3.1)

$$F_< (\omega, \kappa, n) = \frac{d\gamma}{2\pi i} (\kappa^2)^{-\frac{n}{2}} (\varepsilon^\gamma)^n \int \frac{d^{2+2\varepsilon} \gamma_0}{\kappa_0^2} \theta (\mu_F^2 - \kappa_0^2) \frac{1}{\omega - \alpha_s e^{-\varepsilon \partial'}} \lambda (\gamma, n, \varepsilon) \times (\kappa_0^2)^{-\frac{n}{2}} (\varepsilon^\gamma)^{\Phi_n (\kappa_0)}$$

$$= \tilde{F} (\mu_F^2)^2 (\kappa^2)^{-n} (\varepsilon^\gamma)^n \int \frac{d\gamma'}{2\pi i} (\kappa^2)^{\gamma'} \frac{1}{\omega - \alpha_s 1/\gamma 1/\gamma - \gamma'} \frac{1}{\gamma - \gamma'}. \tag{3.38}$$

We did the substitution $\gamma' = \gamma + \frac{n}{2}$ and used the function $\lambda_1 (\gamma, n, \varepsilon) = \gamma' \lambda (\gamma - \varepsilon, n, \varepsilon)$ defined in eq.(A.19) in Appendix A. Indeed, we see that the integral in the second line is just the same as in (3.3) with the only replacement in the function $\lambda_1$. The calculation is therefore completely analogous but the change of integration variable reads now

$$\tilde{\gamma} = \frac{\gamma'}{\lambda_1 (\gamma' - \frac{n}{2}, n, \varepsilon)} \tag{3.39}$$

which leads to solve, after having picked up the residue in $\tilde{\gamma} = \tilde{\gamma}^{(0)}$, the operatorial equation

$$\tilde{\gamma} (n, \varepsilon) = \lambda_1 \left( \tilde{\gamma} (n, \varepsilon) - \frac{n}{2}, n, \varepsilon \right) \tilde{\gamma}^{(0)} \tag{3.40}$$

whose solution $\tilde{\gamma} (n, \varepsilon)$ now expands as power of $\gamma_0$ (see the Appendix B). We finally get, after restoring (as in the $n = 0$ case) the $\mu_R$ dependance in the strong coupling

$$F (\omega, \kappa, n) = \gamma_0 (n) \left[ \frac{1}{-\gamma_0^2 (n)} \chi_n (\gamma_0 (n) - \frac{n}{2}) \right] \left( \frac{\kappa^2}{\mu_F^2} \right) \gamma_0 (n) \exp \left( \frac{1}{\varepsilon} \int_0 S_\varepsilon \frac{d\alpha}{\alpha} \gamma_0 (\gamma_0 (\mu_F^2) \chi_0 (\gamma_0 (n), \varepsilon) \right) \tag{3.41}$$

where

$$\gamma_0 (\gamma_0 (n), \varepsilon) = \gamma_0 (\gamma_0 (n), \varepsilon) + \varepsilon \gamma_0 (\gamma_0 (\varepsilon), n) + O (\varepsilon^2) \tag{3.42}$$

and the explicit expressions for $\chi_n$, the BFKL anomalous dimension $\gamma_0 (n)$ for conformal spin $n$ and its $\varepsilon$-correction $\gamma_0 (n)$ are given respectively in Appendix eqs.(A.21), (B.23), and (B.25).

4 Analysis of the K-factor

4.1 Extraction of K-factor

In order to compare explicitly the result (3.27) with the result obtained in [3], we define in a similar way $R_\omega$ which absorbs the $O (\varepsilon^0)$ factor coming from the exponentiation of the $\varepsilon$-correction $\gamma_0 (\varepsilon)$ to the BFKL anomalous dimension,

$$F (\omega, \kappa, n) = \gamma_0 (n) R_\omega (\gamma_0 (n), \mu_F^2) \gamma_0 (n) \Gamma \left( \frac{\gamma_0 (\mu_F^2) \varepsilon}{\mu_F^2}, n, \varepsilon \right). \tag{4.1}$$

This expression singles out the explicit factorization of the collinear singularities (in agreement with [3], see also [4]), appearing in the Laurent series of poles in $\frac{1}{\varepsilon}$ contained in the $\overline{MS}$ scheme gluon transition function $\Gamma (\gamma_0 (n), n, \varepsilon)$ as a direct consequence of our calculation,

$$\Gamma (\gamma_0 (n), n, \varepsilon) = \exp \left( \frac{1}{\varepsilon} \int_0 S_\varepsilon \frac{d\alpha}{\alpha} \gamma_0 (\gamma_0 (\alpha, n), \varepsilon) \right). \tag{4.2}$$
In eq.(4.1) we identify the renormalization and factorization scales ($\mu_R = \mu_F$) in order that $F(\omega, \kappa, n)$ would be independent of this arbitrary scale, since
\begin{equation}
\Gamma(\gamma_0(\omega), n, \varepsilon) = \left(\frac{\mu_F^2}{\mu^2}\right)^{\gamma_0(n)} \Gamma(\gamma_0(0), n, \varepsilon) + O(\varepsilon) .
\end{equation}

The BFKL normalization factor $R_\omega(\gamma_0(0), n)$ encodes all the dynamics coming from the soft singularities resummed by the BFKL equation, and which is responsible for the singular behaviour of the perturbative QCD Pomeron at the saturation value $\gamma_0 - \frac{n}{2} = 1/2$ corresponding to extreme energies as we will see further,
\begin{equation}
R_\omega(\gamma_0(0), n) = \frac{1}{-\gamma_0^2(n) \chi_n(\gamma_0(n) - \frac{n}{2})} \exp \left( \int_0^{\gamma_0(0)} \frac{d\alpha}{\alpha} \gamma_0^{(e)}(\gamma_0(0), \alpha, n) \right)
\end{equation}

which can be rewritten explicitly as
\begin{equation}
R_\omega(\gamma_0(0), n) = \frac{1}{-\gamma_0^2(n) \chi_n(\gamma_0(n) - \frac{n}{2})}
\times \exp \left\{ \frac{1}{2} \int_0^{\gamma_0(n)} d\gamma \frac{2\psi'(1) - \psi'(1 + n - \gamma) - \psi'(\gamma)}{\chi_n(\gamma - \frac{n}{2})} + \chi_n(\gamma - \frac{n}{2}) \right\} .
\end{equation}

Let us now consider the particular case $n = 0$ to make an explicit comparison with the corresponding K-factor obtained by Catani and Hautmann, we have now
\begin{equation}
R_\omega(\gamma_0(0)) = \frac{1}{-\gamma_0^2 \chi(\gamma_0)} \exp \left( \int_0^{1} \frac{d\alpha}{\alpha} \gamma_0^{(e)}(\gamma_0(0), \alpha) \right)
\end{equation}

and the gluon transition function
\begin{equation}
\Gamma(\gamma_0, \varepsilon) = \exp \left( \frac{1}{\varepsilon} \int_0^{\gamma_0} \frac{d\alpha}{\alpha} \gamma_0(\gamma_0(0), \alpha) \right) .
\end{equation}

For doing this, let us write $R_\omega$ in a more explicit form by the use of eq.(B.16) and a change of variable:
\begin{equation}
R_\omega(\gamma_0(0)) = \frac{1}{-\gamma_0^2 \chi'(\gamma_0)} \exp \left\{ \frac{1}{2} \int_0^{\gamma_0} d\gamma \frac{2\psi'(1) - \psi'(1 + n - \gamma) - \psi'(\gamma)}{\chi(\gamma)} + \chi(\gamma) \right\} .
\end{equation}

4.2 Comparison of K-factor results

We write now the analytical expression of the same quantity, obtained in Ref.[3] by Catani and Hautmann:
\begin{equation}
R^{CH}(\gamma_0(0)) = \left\{ \frac{\Gamma(1 - \gamma_0) \chi(\gamma_0)}{\Gamma(1 + \gamma_0) [-\gamma_0^2 \chi'(\gamma_0)]} \right\}^{1/2} \exp \left\{ \gamma_0 \psi(1) + \int_0^{\gamma_0} d\gamma \frac{\psi'(1) - \psi'(1 + \gamma)}{\chi(\gamma)} \right\} .
\end{equation}

To make an explicit comparison with our result, we then use the following identity
\begin{equation}
\gamma_0 \psi(1) + \int_0^{\gamma_0} d\gamma \frac{\psi'(1) - \psi'(1 + \gamma)}{\chi(\gamma)} = \frac{1}{2} \int_0^{\gamma_0} d\gamma \frac{2\psi'(1) - \psi'(1 - \gamma) - \psi'(\gamma)}{\chi(\gamma)} + \chi(\gamma)
\end{equation}
\[-\ln \left\{ \gamma_\omega \chi(\gamma) \frac{\Gamma(1 - \gamma_\omega)}{\Gamma(1 + \gamma_\omega)} \right\}^{1/2} \]

to write finally

\[ R^{CH}(\gamma_\omega) = \frac{1}{\gamma_\omega \sqrt{-\chi'(\gamma_\omega)}} \exp \left\{ \frac{1}{2} \int_0^{\gamma_\omega} d\gamma \frac{2\psi'(1) - \psi'(1 - \gamma) - \psi'(\gamma)}{\chi(\gamma)} + \chi(\gamma) \right\} \]

\[ = \frac{1}{\gamma_\omega \sqrt{-\chi'(\gamma_\omega)}} \exp \left( \int_0^1 \frac{d\alpha}{\alpha} \gamma_\omega^{(\varepsilon)}(\gamma_\omega^{(0)} \alpha) \right). \tag{4.11} \]

We see that both results agree for the exponentiation of the \( \varepsilon \)-correction \( \gamma_\omega^{(\varepsilon)} \) to the anomalous dimension term, but there is a mismatching for the prefactor term. We plot these K-factors as a function of \( \gamma \) in the physical range \([0, 1/2]\):

**Figure 1:** K-factor \( R(\gamma) \), our result (solid) and Catani-Hautmann’s result (dashed)

We write the first terms in the coupling expansion for these factors:

\[ R^{CH}(\gamma_\omega^{(0)}) = 1 + \frac{8}{3} \zeta(3) \left( \gamma_\omega^{(0)} \right)^3 - \frac{3}{4} \zeta(4) \left( \gamma_\omega^{(0)} \right)^4 + \frac{22}{5} \zeta(5) \left( \gamma_\omega^{(0)} \right)^5 \]

\[ + \left( \frac{209}{9} \zeta^2(3) - \frac{5}{6} \zeta(6) \right) \left( \gamma_\omega^{(0)} \right)^6 + \mathcal{O} \left( (\gamma_\omega^{(0)})^7 \right) \tag{4.12} \]

and

\[ R_\omega(\gamma_\omega^{(0)}) = 1 + \frac{14}{3} \zeta(3) \left( \gamma_\omega^{(0)} \right)^3 - \frac{3}{4} \zeta(4) \left( \gamma_\omega^{(0)} \right)^4 + \frac{42}{5} \zeta(5) \left( \gamma_\omega^{(0)} \right)^5 \]

\[ + \left( \frac{419}{9} \zeta^2(3) - \frac{5}{6} \zeta(6) \right) \left( \gamma_\omega^{(0)} \right)^6 + \mathcal{O} \left( (\gamma_\omega^{(0)})^7 \right). \tag{4.13} \]

With the unintegrated gluon density \( F(\omega, \kappa) \) obtained in eq.(3.27), the \( k_T \) factorization of the physical cross-section writes (in Deep Inelastic Scattering for example)

\[ 4Q^2 \sigma_\omega(Q^2) = \int \frac{d^2+2\varepsilon \kappa}{k^2} - \hat{\sigma}_\omega(\kappa^2/Q^2, \hat{\alpha}_S(Q^2/\mu^2)^\varepsilon; \varepsilon) F(\omega, \kappa) \tilde{f}_g^{(0)}(\mu, \varepsilon) \]

\[ \tag{4.14} \]

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where the hard cross section (or impact factor) \( \hat{\sigma} \) is obtained in perturbation theory using \( Q^2 \) as perturbation scale (corresponding in the case of DIS to the photon virtuality), and \( \tilde{f}_{g,\omega}^{(0)} \) is the bare gluon distribution function. The factorization of the transition function shown in eq.(3.27) allows to define the renormalized gluon distribution which now depends on the arbitrary factorization scale \( \mu_F \)

\[
\tilde{f}_{g,\omega}(\mu_F) = \Gamma(\gamma_{\omega}^{(0)}(\mu_F^2 / \mu^2), \varepsilon) \tilde{f}_{g,\omega}(\mu, \varepsilon).
\]  (4.15)

After having defined the Mellin transform of the hard cross-section

\[
h_{\omega}(\gamma) = \gamma \int_0^\infty \frac{d\kappa^2}{\kappa^2} \left( \frac{\kappa^2}{Q^2} \right)^\gamma \tilde{\sigma}_\omega(\kappa^2/Q^2, \alpha_S; \varepsilon = 0)
\]  (4.16)

we finally obtain

\[
4Q^2 \sigma_{\omega}(Q^2) = h_{\omega}(\gamma_{\omega}(\gamma_{\omega}^{(0)})) R_{\omega}(\gamma_{\omega}^{(0)}) (Q^2 / \mu_F^2)^\gamma_{\omega} \tilde{f}_{g,\omega}(\mu_F^2).
\]  (4.17)

Note also that we can easily interpret the fact that the anomalous dimension appears in the expression (3.27) of the unintegrated gluon density \( F(\omega, \kappa) \), since we can from this Green function define the gluon density \( G(0) \) integrated up to a renormalization scale \( \mu_R \):

\[
G_{g\eta, \omega}(\alpha_S, \varepsilon) = \int \frac{d^{2+2\varepsilon} \kappa}{\kappa^2} F(\omega, \kappa) \Theta(\mu_R^2 - \kappa^2)
\]

\[
= R_{\omega}(\gamma_{\omega}^{(0)}) (\mu_F^2 / \mu^2)^{\gamma_{\omega}} \Gamma \left( \gamma_{\omega}^{(0)}(\mu_F^2 / \mu^2), \varepsilon \right) + \mathcal{O}(\varepsilon).
\]  (4.18)

If we now identify this renormalization scale with the factorization scale previously defined, \( \mu_R^2 = \mu_F^2 \), and neglect \( \mathcal{O}(\varepsilon) \) terms, we obtain the following simple expression of the factorized gluon density \( G^{(0)} \) integrated up to a renormalization scale \( \mu_R \):

\[
G_{g\eta, \omega}(\alpha_S, \varepsilon) = R_{\omega}(\gamma_{\omega}^{(0)}) \Gamma \left( \gamma_{\omega}^{(0)}(\mu_F^2 / \mu^2), \varepsilon \right).
\]  (4.19)

### 4.3 Discussion of the discrepancy

Let us now summarize how this formulation leads to the factorization of the whole BFKL dynamics into the K-factor, build from two different pieces: the prefactor term and the exponent of the \( \mathcal{O}(\varepsilon) \) part of the BFKL anomalous dimension. The prefactor emerges by picking up the leading twist residue \( \gamma_{\omega}(\varepsilon) \) of the BFKL green function in \( d = 2 + 2\varepsilon \) dimensions and is equivalent in the \( \varepsilon \) asymptotics to the one we usually get in exactly two dimensions [2, 8]. Two ingredients leads to the second term: solving the leading logarithm BFKL equation in \( d = 2 + 2\varepsilon \) transverse dimensions, we notice that \( \tilde{\alpha}_S \) is appearing with the shift operator \( e^{-\varepsilon b_0} \) in the BFKL Green function. Note that this feature can be interpreted closely to the formulation of [13]. The action of these involved shift operators on the singularity in \( \gamma \) emerging after the convolution of the Green function with the partonic impact factor, leads directly to the exponentiation of the BFKL anomalous dimension. And since the eigenfunctions \( \lambda_1(\gamma, \varepsilon) \) of the BFKL kernel gets an \( \varepsilon \) correction in \( d \) dimensions, it is also the case for the BFKL anomalous dimension \( \gamma_{\omega}(\varepsilon) \), whose \( \mathcal{O}(\varepsilon) \) term \( \gamma_{\omega}^{(0)}(\varepsilon) \) appears at \( \mathcal{O}(\varepsilon^0) \) in the K-factor.
Focusing the discussion on the gluon density, we see that these two ingredients are in agreement with the renormalisation group requirement going from \(d = 2\) to the \(d = 2 + 2\varepsilon\) case: in two dimensions, the naive scaling property of the gluon density being
\[
\left(\frac{\mu^2}{\mu_0^2}\right)\gamma_{\omega},
\]
must be replaced in \(d = 2 + 2\varepsilon\) transverse dimensions where the BFKL strong coupling is now running, see eq.(2.7), by
\[
G_{gg,\omega}(\mu_R^2, \mu^2, \varepsilon) = \tilde{G}_\omega(\mu^2, \varepsilon) \exp\left(\int_{\mu^2}^{\mu_R^2} \frac{dk^2}{k^2} \tilde{\gamma}_{\omega}(\tilde{\alpha}_S^\varepsilon(k^2), \varepsilon)\right).
\]
(4.20)

The collinear singularities emerge, if the integration is extended to \(\kappa = 0\). Indeed
\[
\int_0^{\mu_R^2} \frac{dk^2}{k^2} \tilde{\gamma}_{\omega}(\tilde{\alpha}_S^\varepsilon(k^2), \varepsilon) = \frac{1}{\varepsilon} \int_0^1 \frac{d\beta}{\beta} \tilde{\gamma}_{\omega}(\tilde{\alpha}_S \beta, \varepsilon) + \int_{\mu^2}^{\mu_R^2} \frac{dk^2}{k^2} \tilde{\gamma}_{\omega}(\tilde{\alpha}_S^\varepsilon(k^2), \varepsilon).
\]
(4.21)

Thus expanding the anomalous dimension obtained from BFKL in \(d\) dimensions as
\[
\tilde{\gamma}_{\omega}(\alpha_{\varepsilon}) = \tilde{\gamma}_{\omega}(\alpha) + \varepsilon \tilde{\gamma}_{\omega}^{(1)}(\alpha) + O(\varepsilon^2),
\]
we obtain the bare distribution involving the singular gluon transition function,
\[
G_{gg,\omega}^{(0)}(\mu_R^2, \mu^2, \varepsilon) = \tilde{G}_\omega(\mu_R^2, \mu^2, \varepsilon) \Gamma(\tilde{\gamma}_{\omega}^{(0)}(\varepsilon)),
\]
(4.22)

with
\[
\tilde{G}_\omega(\mu_R^2, \mu^2, \varepsilon) = \tilde{G}_\omega(\mu^2, \varepsilon) \left(\frac{\mu_R^2}{\mu^2}\right)^\gamma_{\omega} \exp\left(\int_0^1 \frac{d\beta}{\beta} \tilde{\gamma}_{\omega}^{(1)}(\tilde{\alpha}_S \beta)\right).
\]
(4.23)

The last factor combined with the prefactor coming from the K-factor in 2 dimensions give \(R_{\omega}\), see eq.(4.6).

We could also observe, that whereas the prefactor in the Catani-Hautmann result leads near the saturation region (located at the singularity in the twist 2 BFKL anomalous dimension \(\gamma_{\omega} \simeq 1/2\) or equivalently to the square-root singularity in \(\omega = \omega_0 = 4\tilde{\alpha}_S \ln 2\)) to a behaviour
\[
R_{\chi} \simeq \frac{1}{(\omega - \omega_0)^{\frac{1}{2}}}
\]
(4.24)
related to a \(\left(\frac{1}{\varepsilon}\right)^{\omega_0}(\ln(\frac{1}{\varepsilon}))^{-\frac{1}{2}}\) behaviour in the x-space, the prefactor in our approach is fully compatible with the square-root branch point singularity typical of the BFKL solution
\[
R_{\omega} \simeq \frac{1}{(\omega - \omega_0)^{\frac{1}{2}}}
\]
(4.25)
corresponding to the well-known leading behaviour \(\left(\frac{1}{\varepsilon}\right)^{\omega_0}(\ln(\frac{1}{\varepsilon}))^{-\frac{1}{2}}\).

The discrepancy in the K-factor results is caused by non-commutative limits, the perturbative \((\alpha_S \to 0)\) and the regularisation \((\varepsilon \to 0)\) limits. \(F(\omega, \kappa) \Gamma^{-1}(\gamma_{\omega}^{(0)}, \varepsilon)\) has a finite limit for \(\varepsilon \to 0\). In [3] this limit has been calculated resulting in
\[
\lim_{\varepsilon \to 0} F(\omega, \kappa) \Gamma^{-1}(\gamma_{\omega}^{(0)}, \varepsilon) = \gamma_{\omega} R_{\chi}.
\]

On the other hand \(\Gamma(\gamma_{\omega}^{(0)}, \varepsilon)\) is also the singular factor in the \(\varepsilon \to 0\) asymptotics of \(F(\omega, \kappa),
\[
F(\omega, \kappa) = \gamma_{\omega} R_{\omega}(\gamma_{\omega}^{(0)}) \Gamma(\gamma_{\omega}^{(0)}, \varepsilon) (1 + O(\varepsilon)).
\]
As we have seen in the calculation, the prefactor in the asymptotics does not coincide with the results of the limit, \( R_\omega \neq R_\omega^{CH} \).

For illustration consider instead of the actual BFKL equation the simplified case without extra \( \varepsilon \) dependence in the eigenvalue function \( \frac{\lambda_1(\gamma)}{\gamma} = \chi(\gamma) \) and substitute \( \lambda_1(\gamma) \) by simple expressions. The results (4.6, 4.11) reduce to

\[
\gamma_\omega R_\omega(\gamma_\omega^{(0)}) = \frac{1}{-\gamma_\omega \chi'(\gamma_\omega)}, \quad \gamma_\omega R_\omega^{CH}(\gamma_\omega^{(0)}) = \left( \frac{1}{-\chi'(\gamma_\omega)} \right)^{\frac{3}{2}},
\]

where as above \( \gamma_\omega \) is the solution of \( 1 = \gamma_\omega^{(0)} \chi(\gamma) \).

In the particular example \( \lambda_1(\gamma) = 1 + a \gamma^3 \) and small coefficient \( a \ll 1 \) we write the first terms in the perturbative expansion of \( F \)

\[
F = \gamma_\omega^{(0)} \left\{ 1 + \gamma_\omega^{(0)} \left( \frac{1}{\varepsilon} + a \varepsilon^2 \right) + \gamma_\omega^{(0)} \left( \frac{1}{2 \varepsilon^2} + \frac{9}{2} a \varepsilon \right) + \gamma_\omega^{(0)} \left( \frac{1}{6 \varepsilon^3} \right) + a \varepsilon + \ldots \right\}
\]

Factorise the singular terms related to the expansion of \( e^{\frac{1}{2} \gamma_\omega^{(0)}} \),

\[
\gamma_\omega^{(0)} (1 + \gamma_\omega^{(0)} \frac{1}{\varepsilon} + \gamma_\omega^{(0)} \frac{7}{2 \varepsilon^2} + 2 \gamma_\omega^{(0)} a + \ldots) (1 + \frac{\gamma_\omega^{(0)}}{\varepsilon} + \gamma_\omega^{(0)} \frac{1}{2 \varepsilon^2} + \gamma_\omega^{(0)} \frac{1}{6 \varepsilon^3} + \ldots)
\]

After separating the singularities in this way the \( \varepsilon^0 \) terms in the factor in front of the singular one approximate \( \gamma_\omega R_\omega^{CH} \).

Our calculation is focussed on the asymptotics. In this case the terms with positive powers of \( \varepsilon \) are to be neglected from the start. This leads to a difference in separating the singularities order by order

\[
\gamma_\omega^{(0)} (1 + \gamma_\omega^{(0)} \frac{1}{\varepsilon} + 6a + \ldots) (1 + \gamma_\omega^{(0)} \frac{1}{\varepsilon} + \gamma_\omega^{(0)} \frac{1}{2 \varepsilon^2} + \gamma_\omega^{(0)} \frac{1}{6 \varepsilon^3} + \ldots).
\]

The relation of the discrepancy to the positive power \( \varepsilon \) terms is confirmed by the example \( \lambda_1(\gamma) = 1 + a \gamma^3 \) (\( a \) not small). No terms with positive powers of \( \varepsilon \) appear here. The sum in \( F \) can be done here. Thus the K-factor expressions are easily checked and we obtain \( R_\omega = R_\omega^{CH} \) for this particular case.

The extraction of the collinear singularities along the scheme by Catani and Hautmann has been reanalysed by Ciafaloni and Colferai [5] confirming the previous result with the preexponential factor \( \sim (\chi'(\gamma_\omega))^{-\frac{3}{2}} \).

The \( Q_0 \) regularisation is used there. It means to introduce in the convolution with the parton impact factor besides of the factorisation scale \( \mu_F \) an infrared cut-off \( Q_0^2 = \mu_F^2 e^{-T} \) for large \( T \). In that paper first the Green function of the BFKL equation is derived. This step is analogous to our approach, but there the solution is derived by iteration in steps of size \( \varepsilon \) and the result is written in Mellin transformation with respect to \( \kappa, \kappa_0 \). The iteration starts from a Born term involving the cut-off \( Q_0 \) (eq.(2.9) in [5]),

\[
f_{Q_0}^{(0)} = \gamma_\omega^{(0)} \frac{e^{\gamma T}}{\gamma}, \quad T = \ln \frac{\mu^2}{Q_0^2}.
\]

The calculation of [5] can be easily adapted to our scheme allowing to see the discrepancy appearing from another side. We should replace this Born term by the one calculated from the parton impact factor (3.2) with the result

\[
f_{Q_0}^{(0)} = \frac{1}{\varepsilon - \gamma} = \int_0^\infty dT e^{-T} \gamma_\omega^{(0)} e^{\gamma T}.
\]
Notice that now $T$ is an integration variable with no relation to $Q_0$. The dominant contribution arises from large values $T \sim \frac{1}{\varepsilon}$.

This replacement leads to the following modification of the solution (eq (2.29) in [5], where the $\varepsilon$ dependence in the eigenvalue function is suppressed now),

$$
\tilde{F}(t) = \int_0^\infty dT e^{-\varepsilon T} \int \frac{d\gamma}{2\pi i} \int_{\gamma}^{\gamma} d\gamma \frac{\exp(\gamma t + \frac{1}{\varepsilon} \int_{\gamma}^{\gamma} L_0(z) dz - \gamma T)}{\varepsilon \chi(\gamma) \sqrt{\chi(\gamma')} \sqrt{\chi(\gamma')}} \frac{L_0(\gamma') - \varepsilon T}{1 - e^{-L_0(\gamma') + \varepsilon T}}.
$$

The notation $L_0(\gamma) = \ln(\gamma\omega(\gamma))$ is used here. The estimate of the asymptotics at $t = \ln \kappa^2 \mu^2 \gg 1$ and at $T \sim \frac{1}{\varepsilon} \gg 1$ is done by applying the saddle-point approximation twice. The saddle-point equations are similar,

$$t + \frac{1}{\varepsilon} L_0(\gamma) = 0, \quad T - \frac{1}{\varepsilon} L_0(\gamma) = 0$$

with the solutions

$$\gamma : \bar{\gamma}_t = \gamma(\gamma_0(0) e^{\varepsilon t}), \quad \gamma' : \bar{\gamma}_T = \gamma(\gamma_0(0) e^{-\varepsilon T}).$$

$\gamma(\gamma_0(0))$ denotes the solution of $1 = \gamma(\gamma_0(0)) \chi(\gamma)$. The integral over the fluctuation results in two factors $(\frac{\varepsilon \chi(\gamma)}{-\chi(\gamma)} )^{\frac{1}{2}}$. The estimate for $\tilde{F}$ is therefore

$$
\tilde{F}(\omega) = \int_0^\infty dT e^{-T(\varepsilon - \bar{\gamma}_T)} \frac{\exp(\bar{\gamma}_t t + \frac{1}{\varepsilon} \int_{\bar{\gamma}_t}^{\bar{\gamma}_T} L_0(z) dz)}{\sqrt{\chi(\bar{\gamma}_t)} \sqrt{\chi(\bar{\gamma}_T)}} \frac{L_0(\bar{\gamma}_T) - \varepsilon T}{1 - e^{-L_0(\bar{\gamma}_T) + \varepsilon T}}.
$$

The second saddle point does not tend to zero in the regularisation limit and both fluctuation factors remain.

In the analysis of [5] with $Q_0$ the second fluctuation factor turns to 1, if the limit $Q_0 \to 0$ is taken before the $\varepsilon$ regularisation limit because then the corresponding saddle point tends to zero. To match with our asymptotic factorisation the $\varepsilon$ asymptotics is to be considered, where $\varepsilon = 0$ in all regular terms. At vanishing $\varepsilon$ the saddle point does not tend to zero and no fluctuation factor turns to 1.

### 5 Structure functions and exclusive electroproduction

#### 5.1 The longitudinal structure function $F_L$

As an application of the previous discussion, we can write the factorized expression of the longitudinal structure function $F_L$ in the Mellin space, in agreement with [3]

$$F_L(Q^2) = C_{L, \omega}(\alpha_S, Q^2/\mu_F^2) f^g_\omega(\mu_F^2),$$

where $f^g_\omega(\mu_F^2)$ is the $\omega-$moment of the renormalized gluon distribution function and we have defined the gluonic (improved) coefficient function

$$C_{L, \omega}(\alpha_S, Q^2/\mu_F^2) = h_{L, \omega}(\gamma) R_\omega(Q^2/\mu_F^2) \gamma_\omega,$$

with the Mellin transform of the corresponding hard cross-section

$$h_{L, \omega}(\gamma) = \frac{\bar{\alpha}_S}{2 \pi} N_f T_R \frac{4(1 - \gamma)}{3 - 2\gamma_\omega} \Gamma^3(1 - \gamma_\omega) \Gamma^3(1 + \gamma_\omega) \Gamma(2 - 2\gamma_\omega) \Gamma(2 + 2\gamma_\omega).$$
Then we easily obtain from (4.6) and (B.11) the all order perturbative expansion in power of \( \gamma^{(0)} \) of this coefficient function for the simpler case \( \mu_F^2 = Q^2 \)

\[
C^{g}_{L,N}(\bar{\alpha}_S, Q^2/\mu_F^2 = 1) = \frac{\bar{\alpha}_S}{2\pi} T_R N_f \frac{4}{3} \left\{ 1 - \frac{1}{3} \gamma^{(0)} + \left[ \frac{34}{9} - \zeta(2) \right] \left( \gamma^{(0)} \right)^2 + \left[ -\frac{40}{27} \right. \right.
\]
\[
+ \frac{1}{3} \zeta(2) + \frac{14}{3} \zeta(3) \right\} \left( \gamma^{(0)} \right)^3 + \left[ \frac{1216}{81} - \frac{34}{9} \zeta(2) - \frac{20}{9} \zeta(3) - 6 \zeta(4) \right] \left( \gamma^{(0)} \right)^4
\]
\[
\left. + \mathcal{O} \left( \left( \gamma^{(0)} \right)^5 \right) \right\}
\]
\[
\simeq \frac{\bar{\alpha}_S}{2\pi} T_R N_f \frac{4}{3} \left\{ 1 - 0.33 \gamma^{(0)} + 2.13 \left( \gamma^{(0)} \right)^2 + 4.68 \left( \gamma^{(0)} \right)^3 - 0.37 \left( \gamma^{(0)} \right)^4
\right.
\]
\[
+ \mathcal{O} \left( \left( \gamma^{(0)} \right)^5 \right) \right\}
\]

which has to be compared with eq.(5.24) of Catani-Hautmann’s paper [3]. Indeed, the results start to deviate at the fourth loop in the perturbative expansion. The calculation made by Moch, Vermaseren and Vogt [21] on the same coefficient function derived by complete loop calculations in pure collinear factorization scheme extends to three loops and is, therefore, still not sufficient to discriminate the small \( x \) resummation results.

**Figure 2:** \( C^{g}_{L,N}, \; Q^2 = \mu_F^2 \)

Going from \( (\omega, \gamma) \) to the \( (x, Q^2) \) space, we display in Fig.2 the longitudinal coefficient functions, the dotted curve corresponding to the Catani-Hautmann’s result and the solid one to our approach. We considered for that the case \( N_f = 4, \log Q^2 / \Lambda^2 = 6 \) in the strong coupling and \( \mu_F^2 = Q^2 \). We show this plot also for comparison with [22], in particular with the Fig.6 which gives the corresponding result for \( C_L \). Note that we restrict the comparison to the small-\( x \) \( (x \leq 10^{-1}) \) region to avoid the discussion of modelling the Born \( \delta(1-x) \) term. The analysis of [22] shows the impact of the running coupling (see [23]) and of the large corrections coming from NLL BFKL resummation which essentially tame the small-\( x \) growth: the authors consider the NLL BFKL equation [24] with leading order
running coupling and an estimation of the longitudinal NLL impact factor. The NLL BFKL extension of our approach goes beyond the scope of our present study; it would be needed to confirm whether those corrections imply a weaker small-$x$ growth like in the result of [22].

In order to estimate the order of the discrepancy of our approach with the one of Catani-Hautmann for the longitudinal structure function, we consider in the following $F_L(x, Q^2)$ as a function of $x$ for different values of the hard scale $Q^2$. We simply used for the convolution with the soft part the following parametrization of the gluon PDF $xg(x, Q_0^2)$ at $Q_0^2 = 30 GeV^2$ (cf. [21])

$$xg(x, Q_0^2) = 1.6 x^{-0.3} (1 - x)^{1.5} (1 - 0.6 x^{0.3})$$ (5.5)

and we have made it evolve through the DGLAP equation in the small $x$ limit at one loop accuracy to obtain its $Q^2$ dependence. We then obtain the following curves Fig.3 and Fig.4.

Figure 3: $F_L, Q^2 = 30 GeV^2$

The solid curve corresponds to the Born term in the high energy expansion, equivalently to the leading order accuracy. The dotted curve contains the Born term and the negative (see eq.(5.4)) next-to-leading order corrections, explaining the trend. Then we do the expansion of the small $x$ resummation result in $\bar{\alpha}_S \ln x$ up to 12th order which is quite sufficient because of good convergence. This allows to show the discrepancy for these phenomenological predictions between the two different K-factor expressions: the dashed curve corresponds to our result and the dashed-dotted one to the same analysis with the Catani-Hautmann result. The convergence is very convincing and it is increased with the value of the hard scale. All the curves are plotted with the factorization scale $\mu_F^2 = Q^2$. 
5.2 Exclusive electroproduction

We now turn to the application for exclusive vector meson (VM) electroproduction in Deeply Virtual Compton Scattering (DVCS) following the line of [19]: considering the gluon dominance in the Regge limit of the scattering, the amplitude reads

$$\text{Im} A^g \simeq H^g(\xi, \xi) + \int \frac{dx}{x} H^g(x, \xi) \sum_{n=1} C_{VM,n}^g \frac{\bar{\alpha}_S^n}{(n-1)!} \log^{n-1} \frac{x}{\xi},$$

(5.6)

where $H^g(\xi, \xi)$ is the Born contribution of the gluon GPD. The $C_{VM,n}^g$ are polynomials of $\log \frac{Q^2}{\mu_F^2}$ obtained as in the previous example in the perturbative expansion of the coefficient function in power of $\gamma_\omega^{(0)}$

$$C_{VM,\omega}^g(\bar{\alpha}_S, Q^2/\mu_F^2) = h_{VM,\omega}(\gamma_\omega) R_\omega(Q^2/\mu_F^2) \gamma_\omega$$

$$= \sum_{n=0} C_{VM,n}^g(\gamma_\omega^{(0)})^n,$$

(5.7)

with the following expression of the Mellin transform $h_{VM,\omega}$ of the hard cross-section: we define for that the properly normalized impact factor $\gamma^* \to VM$ written as a convolution of the hard scattering amplitude with the leading twist non-perturbative Distribution Amplitude (DA) [20], where both photon and vector meson are longitudinally polarized.

$$h_{VM}(k_T^2) = \frac{1}{\int_0^1 dz \frac{Q^2}{k_T^2 + z(1-z)Q^2} \phi_{VM}(z)} \int_0^1 dz \frac{\phi_{VM}(z)}{z(1-z)},$$

(5.8)
and its Mellin transform reads for an asymptotic vector meson DA $\phi_{VM}(z) = 6z(1 - z)$, 

$$
\begin{align*}
\phi_{VM}(z) &= 6z(1 - z), \\
h_{VM,\omega}(\gamma_\omega) &= \gamma_\omega \int_0^\infty \frac{dk_t^2}{k_t^2} \left( \frac{k_t^2}{Q^2} \right)^{\gamma_\omega} h_V(k_t^2) \\
&= \frac{\Gamma^3(1 + \gamma_\omega)\Gamma(1 - \gamma_\omega)}{\Gamma(2 + 2\gamma_\omega)}.
\end{align*}
$$

(5.9)

We replace in the high energy term the gluon GPD by its forward limit $H^g(x, \xi) \to xg(x)$. On the contrary to the previous study for $F_L$, we keep this expression for the soft part without doing any $Q^2$-evolution. We replace the Born term by a very simple model $H^g(\xi, \xi) = 1/\xi g(\xi)$. We obtain the following curves Fig.5 and Fig.6, in the same spirit as previously by doing the high energy expansion.

![Figure 5: VM electroproduction, $Q^2 = 30 GeV^2$](image)

The solid curve corresponds to the Born term, the dotted curve to the Born term with the NLO corrections. Note that the numerical value of the $C_{VM,1}^g$ coefficient is much larger than in the $F_L$ case, giving these stronger and very negative NLO corrections. This appears as an instability in the perturbative prediction for this process. Here the small $x$ resummation is really needed to obtain a reasonable and stable prediction [19]. After twelve iterations we get the dashed curve corresponding to our result and the dashed-dotted one corresponding to the same analysis with the Catani-Hautmann K-factor. Also here the convergence of the series expansion after twelve iterations is very good. Although the curves are only plotted here with the factorization scale $\mu_F^2 = Q^2$, we observe that the factorization scale dependence is reduced when taking into account the high energy resummations compared to the Born or even NLO case. We also note that the sensitivity of this choice is equivalent for both K-factor expressions.
The resummation of small $x$ corrections to hard scattering amplitudes calculated by the collinear factorization method has been reconsidered. The contributions enhanced by $\ln x$ are resummed by BFKL equation. The collinear singularities appear due to the convolution of the BFKL Green function with a parton impact factor. The factorization of these singularities is demonstrated in accordance with the scheme chosen in the collinear calculation. Besides of the resummed gluon anomalous dimension a correction factor is derived that amounts into the improved coefficient function. In this way we follow the known resummation scheme. However, our result for the K-factor differs from the one given in [3] and used in previous applications.

We have presented our arguments in all details in order to explain the origin of the difference related to the non-commutativity of the perturbative and the regularisation limits. The basic elements are the BFKL Green function in $2 + 2\varepsilon$ transverse dimensions, its convolution with the parton impact factor and the factorisation of the resulting collinear singularities in the small $\varepsilon$ asymptotics. For comparison we have applied the same asymptotic analysis to the iterative solution given in [3] and obtained the same result. Our result has the square-root singularity in the continued angular momentum $\omega$ typical for BFKL partial-wave amplitudes.

The significance of the difference in the K-factor has been illustrated in applications to structure function and to exclusive electroproduction of vector mesons. Hard exclusive processes as a new kind of applications of the small $x$ resummation has ben pointed out in [19]. The formulation presented here extends to channels corresponding to non-vanishing conformal spin $n$ in the BFKL solution. These appear in polarized structure functions and contribute to hard exclusive amplitudes.
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Appendices

A Calculation of $\lambda_1(\gamma, n, \varepsilon)$

A.1 Case $n = 0$

We now study the action of the full kernel in $2 + 2\varepsilon$ transverse dimensions on the pseudo eigenfunctions $(\vec{k}'^2)^{\gamma-1}$

$$
\hat{K} \cdot (\vec{k}'^2)^{\gamma-1} = \frac{1}{\pi} \int \frac{d^2 \vec{\kappa}' d^2 \vec{\kappa}}{(2\pi)^4} \left\{ \frac{1}{(\vec{\kappa}'^2 - \vec{\kappa}^2)^2} - \frac{1}{2} \alpha_G(\vec{k}'^2) \delta(\vec{\kappa}' - \vec{\kappa}) \right\} ((\vec{k}'^2)^{2\varepsilon})^{\gamma-1}. \quad (A.1)
$$

We need first to evaluate the gluon trajectory function

$$
\alpha_G(\kappa) = \int \frac{d^2 \vec{\kappa}' \vec{\kappa}'^2}{(\vec{\kappa}'^2 - \vec{\kappa}^2)^2} = \frac{1}{\varepsilon} \Gamma(1 + \varepsilon) \Gamma(1 - \varepsilon) \Gamma(1 + 2\varepsilon) \quad (A.2)
$$

then we calculate the action of the bare kernel

$$
\int \frac{d^2 \vec{\kappa}' (\vec{\kappa}'^2)^{\gamma-1}}{(\vec{\kappa}'^2 - \vec{\kappa}^2)^2} = \int \frac{d^2 \vec{\kappa}' \vec{\kappa}'^2}{(\vec{\kappa}'^2 - \vec{\kappa}^2)^2} \int_0^\infty d\lambda_1 d\lambda_2 \lambda_1^{-\gamma} e^{-\lambda_1 (\vec{\kappa}'^2)^{1-\gamma}} \lambda_2^{-\lambda_2 (\vec{\kappa}'^2)^{1-\gamma}} \quad (A.3)
$$

$$
= \frac{1}{\varepsilon} \Gamma(1 + \varepsilon) \Gamma(1 - \varepsilon) \Gamma(1 + 2\varepsilon) \quad (A.4)
$$

With these results we compose the action of the full kernel

$$
\hat{K} \cdot (\vec{k}'^2)^{\gamma-1} = (\gamma \varepsilon) (\vec{k}'^2)^{\gamma-1+\varepsilon}. \quad (A.5)
$$

We have defined this quasi-eigenvalue function:

$$
\lambda(\gamma, \varepsilon) = \frac{1}{(4\pi)^\varepsilon} \left\{ b(\gamma, \varepsilon) - \frac{1}{2} b(0, \varepsilon) \right\} \quad (A.6)
$$

with

$$
b(\gamma, \varepsilon) = \frac{1}{\Gamma(\varepsilon)} B(\varepsilon, 1 - \gamma - \varepsilon) B(\varepsilon, \gamma + \varepsilon). \quad (A.7)
$$

Since in our calculation we encounter the shifted quantity $\lambda(\gamma - \varepsilon, \varepsilon)$, we define the function

$$
\lambda_1(\gamma, \varepsilon) = \gamma \lambda(\gamma - \varepsilon, \varepsilon) \quad (A.8)
$$

$$
= \frac{1}{(4\pi)^\varepsilon} \Gamma(1 + \varepsilon) \left\{ \frac{\Gamma(1 - \gamma)}{\Gamma(1 - \gamma + \varepsilon) \Gamma(1 + \gamma + \varepsilon)} \frac{\Gamma(1 + \gamma)}{\Gamma(1 + \gamma + \varepsilon)} + \frac{\gamma}{\varepsilon} \left( \frac{\Gamma(1 - \gamma)}{\Gamma(1 - \gamma + \varepsilon) \Gamma(1 + \gamma + \varepsilon)} - \frac{\Gamma(1 + \varepsilon) \Gamma(1 - \varepsilon)}{\Gamma(1 + 2\varepsilon)} \right) \right\}. \quad (A.9)
$$
and give its expression in the $\overline{MS}$ renormalization scheme
\[
\lambda_1(\gamma, \varepsilon) = \gamma \, e^{\varepsilon \psi(1)} \frac{\Gamma(1 + \varepsilon)}{\varepsilon} \left( \frac{\Gamma(1 - \gamma) \, \Gamma(\gamma)}{\Gamma(1 - \gamma + \varepsilon) \, \Gamma(\gamma + \varepsilon)} - \frac{\Gamma(1 + \varepsilon) \, \Gamma(1 - \varepsilon)}{\Gamma(1 + 2 \varepsilon)} \right). \tag{A.8}
\]

We expand this function in $\varepsilon$ up to the first power (higher orders are suppressed in our approach) and get
\[
\lambda_1(\gamma, \varepsilon) = \lambda_1(\gamma, 0) + \varepsilon \lambda_1^{(\varepsilon)}(\gamma) + \mathcal{O}(\varepsilon^2) \tag{A.9}
\]
where the constant term is
\[
\lambda_1(\gamma, 0) = 1 + \gamma \left( 2 \psi(1) - \psi(1 - \gamma) - \psi(1 + \gamma) \right) \tag{A.10}
\]
and the contribution proportional to the first power of $\varepsilon$ is
\[
\lambda_1^{(\varepsilon)}(\gamma) = \frac{\gamma}{2} \left( 2 \psi'(1) - \psi'(1 - \gamma) - \psi'(1 + \gamma) + \chi^2(\gamma) \right) \tag{A.11}
\]
where $\chi(\gamma)$ is the well-known one-loop BFKL eigenvalue function also recovered in
\[
\chi(\gamma) = \frac{1}{\gamma} \lambda_1(\gamma, 0) = 2 \psi(1) - \psi(1 - \gamma) - \psi(1 + \gamma). \tag{A.12}
\]

Also the expansions in powers of $\gamma$ of the previous expressions (A.10) and (A.11) reads:
\[
\lambda_1(\gamma, 0) = 1 + \sum_{k=1}^{\infty} 2 \zeta(2k + 1) \gamma^{2k + 1} \tag{A.13}
\]
and similarly for $\lambda_1^{(\varepsilon)}$:
\[
\lambda_1^{(\varepsilon)}(\gamma) = \sum_{k=1}^{\infty} 2 \zeta(2k + 1) \gamma^{2k + 1} + \sum_{k=1}^{\infty} \left[ - (2k + 1) \zeta(2k + 2) + 2 \sum_{p=1}^{k-1} \zeta(2p + 1) \zeta(2(k - p) + 1) \right] \gamma^{2k + 1}. \tag{A.14}
\]

**A.2 Case $n > 0$**

We calculate now the action of the full kernel on an ansatz for the general conformal spin $n$ pseudo eigenfunctions $((\vec{\kappa})^2)^{\gamma - 1 - \frac{n}{2}} (\vec{b}, \vec{\kappa})^n$, in order to obtain an analytical expression of the first order $\varepsilon$ correction to the corresponding one-loop BFKL eigenvalue function $\chi_n$.

The trajectory of the gluon is not changed, then we need to compute the action of the real (production) part of the kernel on these pseudo eigenfunctions
\[
\int \frac{d^{2+2\varepsilon} \kappa'}{(\vec{\kappa} - \vec{\kappa}')^2} ((\vec{\kappa}')^2)^{\gamma - 1 - \frac{n}{2}} (\vec{b}, \vec{\kappa}')^n
\]
\[
= \int d^{2+2\varepsilon} \kappa' \Gamma^{-1}(1 + \frac{n}{2} - \gamma) \int_0^\infty d\lambda_1 d\lambda_2 \, \lambda_1^{\frac{\gamma}{2} - \gamma} \exp(-\lambda_1 (\vec{\kappa}')^2 - \lambda_2 (\vec{\kappa} - \vec{\kappa}')^2) (\vec{b}, \vec{\kappa}')^n
\]
\[
= \sum_{i=0}^{n} \binom{n}{i} \pi^{\frac{1}{2} + \varepsilon} \left[ 1 + (-1)^i \frac{\Gamma(\frac{i+1}{2}) \Gamma(1 + \frac{n-i}{2} - \gamma - \varepsilon) \Gamma(\gamma + \varepsilon + \frac{n-i}{2}) \Gamma(\frac{n+1}{2})}{\Gamma(1 + \frac{n}{2} - \gamma) \Gamma(\gamma + 2\varepsilon + \frac{n}{2})} b_i \right]
\times ((\vec{\kappa})^2)^{\gamma + \varepsilon - 1 - \frac{n-i}{2}} (\vec{b}, \vec{\kappa})^{n-i}. \tag{A.15}
\]
We now keep only the $i=0$ term corresponding to the conformal spin $n$ representation of the pseudo-eigenfunctions, which means that we ignore the contributions coming from the mixing with the others representations. We then obtain the quasi-eigenvalue function $\lambda(\gamma, n, \varepsilon)$ of the bare kernel relative to this representation:

$$\hat{K} \cdot (\vec{\kappa}^2)^{\gamma-1-\frac{n}{2}} (\vec{b}, \vec{\kappa})^n = \lambda(\gamma, n, \varepsilon) (\vec{\kappa}^2)^{\gamma-1-\frac{n}{2}+\varepsilon} (\vec{b}, \vec{\kappa})^n$$  

(A.16)

with

$$\lambda(\gamma, n, \varepsilon) = \frac{1}{(4\pi)^{\varepsilon}} \left[ b(\gamma, n, \varepsilon) - \frac{1}{2} b(0, 0, \varepsilon) \right]$$  

(A.17)

where

$$b(\gamma, n, \varepsilon) = \frac{1}{\Gamma(\varepsilon)} B(\varepsilon, 1 + \frac{n}{2} - \gamma - \varepsilon) B(\varepsilon, \frac{n}{2} + \gamma + \varepsilon).$$  

(A.18)

As for the $n=0$ case we define

$$\lambda_1(\gamma, n, \varepsilon) = \left( \gamma + \frac{n}{2} \right) \lambda(\gamma - \varepsilon, n, \varepsilon)$$  

(A.19)

$$= \frac{1}{(4\pi)^{\varepsilon}} \Gamma(1 + \varepsilon) \left[ \frac{\Gamma(1 + \frac{n}{2} - \gamma) \Gamma(1 + \frac{n}{2} + \gamma)}{\Gamma(1 + \frac{n}{2} - \gamma + \varepsilon) \Gamma(1 + \frac{n}{2} + \gamma + \varepsilon)} + \frac{\gamma + \frac{n}{2}}{\varepsilon} \left( \frac{\Gamma(1 + \frac{n}{2} - \gamma) \Gamma(1 + \frac{n}{2} + \gamma)}{\Gamma(1 + \frac{n}{2} - \gamma + \varepsilon) \Gamma(1 + \frac{n}{2} + \gamma + \varepsilon)} - \frac{\Gamma(1 + \varepsilon) \Gamma(1 - \varepsilon)}{\Gamma(1 + 2\varepsilon)} \right) \right].$$

We expand in $\varepsilon$ eq.(A.19) up to the first power and write the result in the $\overline{MS}$ renormalization scheme

$$\lambda_1(\gamma, n, \varepsilon) = \lambda_1(\gamma, n, 0) + \varepsilon \lambda_1^{(\varepsilon)}(\gamma, n) + \mathcal{O}(\varepsilon^2).$$  

(A.20)

Where $\lambda_1(\gamma, n, 0)$ coincides with the known one-loop BFKL eigenvalue function $\chi_n$ for the corresponding $\{\gamma, n\}$ representation,

$$\chi_n(\gamma) = \frac{1}{\gamma + \frac{n}{2}} \lambda_1(\gamma, n, 0)$$  

(A.21)

$$= 2 \psi(1) - \psi(1 + \frac{n}{2} - \gamma) - \psi(\frac{n}{2} + \gamma)$$

and the contribution proportional to the firstpower of $\varepsilon$ is given by

$$\lambda_1^{(\varepsilon)}(\gamma, n) = \frac{\gamma + \frac{n}{2}}{\varepsilon} \left( 2 \psi'(1) - \psi'(1 + \frac{n}{2} - \gamma) - \psi'\left(\frac{n}{2} + \gamma\right) + \chi_n^2(\gamma) \right).$$  

(A.22)

Like in the $n=0$ case, we also write the expansion in powers of $\gamma$ of these functions:

$$\lambda_1(\gamma, n, 0) = 1 + [\psi(1) - \psi(1 + n)] \left( \gamma + \frac{n}{2} \right)$$

$$+ \sum_{k=1}^{\infty} [\zeta(2k, 1 + n) - \zeta(2k)] \left( \gamma + \frac{n}{2} \right)^{2k}$$

$$+ \sum_{k=1}^{\infty} [\zeta(2k + 1, 1 + n) + \zeta(2k + 1)] \left( \gamma + \frac{n}{2} \right)^{2k+1}$$  

(A.23)

and

$$\lambda_1^{(\varepsilon)}(\gamma, n) = \psi(1) - \psi(1 + n)$$

26
\[
+ \left[ \frac{1}{2} \left( \psi^2(1 + n) - \psi^2(1) + \psi'(1 + n) - \frac{\pi^2}{6} \right) - \psi(1) \left( \psi(1 + n) - \psi(1) \right) \right] \left( \gamma + \frac{n}{2} \right) \\
+ \left[ 2\zeta(3) + (\psi(1 + n) - \psi(1)) \left( \frac{\pi^2}{6} - \psi'(1 + n) \right) \right] \left( \gamma + \frac{n}{2} \right)^2 \\
+ \sum_{p=1}^{\infty} \left[ (\psi(1) - \psi(1 + n))(\zeta(2p + 1) + \zeta(2p + 1, 1 + n)) - (2p + 1)\zeta(2p + 2) + \left( p - \frac{1}{2} \right) \right] \\
\times H_n^{(2p+2)} + 2 \sum_{k=1}^{p-1} \zeta(2k + 1)\zeta(2(p - k) + 1, 1 + n) + \frac{1}{2} \sum_{k=1}^{2p-1} H_n^{(k+1)} H_n^{(2p-k+1)} \left( \gamma + \frac{n}{2} \right)^{2p+1} \\
+ \sum_{p=2}^{\infty} \left[ 2\zeta(2p + 1) + (\psi(1 + n) - \psi(1))H_n^{(2p)} + (p - 1)H_n^{(2p+1)} \right. \\
\left. - \sum_{k=1}^{p-1} \left[ \zeta(2k + 1) + \zeta(2k + 1, 1 + n) \right] H_n^{(2(p-k))} \right] \left( \gamma + \frac{n}{2} \right)^{2p} \\
\] (A.24)

where we have used the Hurwitz Zeta function (given here for a positive real number \( a \), since \( n \) is always positive)

\[
\zeta(s, a) = \sum_{k=1}^{\infty} \frac{1}{(k + a - 1)^s} \\
\] (A.25)

and the harmonic numbers of order \( r \) given by

\[
H_n^{(r)} = \sum_{k=1}^{n} \frac{1}{k^r} . \\
\] (A.26)

Note that we merely recover the expressions (A.13) from (A.23) and (A.14) from (A.24) in the \( n = 0 \) case, since \( \zeta(k, 1) = \zeta(k) \).

**B The BFKL anomalous dimension \( \gamma_\omega(n, \varepsilon) \)**

**B.1 Case \( n = 0 \)**

We now look at the solution \( \hat{\gamma}_\omega(\varepsilon) \) of the operatorial equation

\[
\hat{\gamma}_\omega(\varepsilon) = \lambda_1(\hat{\gamma}_\omega(\varepsilon), \varepsilon)\hat{\gamma}_\omega^{(0)} . \\
\] (B.1)

Doing the expansion of \( \lambda_1 \) in power of \( \gamma \)

\[
\lambda_1(\gamma, \varepsilon) = \sum_{N=0}^{\infty} a_N(\varepsilon)\gamma^N , \\
\] (B.2)

the coefficients \( a_N(\varepsilon) \) are known from (A.9), (A.13) and (A.14), in particular \( a_0(\varepsilon) = 1 \). Therefore we can get \( \hat{\gamma}_\omega \) as a power series in the operator \( \hat{\gamma}_\omega^{(0)} \)

\[
\hat{\gamma}_\omega(\hat{\gamma}_\omega^{(0)}, \varepsilon) = \sum_{N=0}^{\infty} b_N(\varepsilon)(\hat{\gamma}_\omega^{(0)})^{N+1} \\
\] (B.3)
and we know already that \( b_0(\varepsilon) = a_0(\varepsilon) \). We find the iterative relation

\[
b_N(\varepsilon) = \sum_{k=1}^{N} a_k(\varepsilon)c_{N-k}^{(k)}(\varepsilon)
\]

(B.4)

where

\[
c_0^{(n)}(\varepsilon) = (b_0(\varepsilon))^n
\]

(B.5)

and

\[
c_m^{(n)}(\varepsilon) = \frac{1}{mb_0(\varepsilon)} \sum_{k=1}^{m} (k(1+n) - m) b_k(\varepsilon)c_{m-k}^{(n)}(\varepsilon)
\]

(B.6)

for \( m \geq 1 \). Finally we shall need this solution up to first order in \( \varepsilon \) and with the shift operator removed, \( \hat{\gamma}_\omega^{(0)} \to \hat{\gamma}_\omega = \frac{\tilde{\alpha}_s}{\omega} \). We then can write

\[
\gamma_\omega(\hat{\gamma}_\omega, \varepsilon) = \gamma_\omega(\hat{\gamma}_\omega) + \varepsilon \gamma_\omega^{(\varepsilon)}(\hat{\gamma}_\omega) + \mathcal{O}(\varepsilon^2)
\]

(B.7)

where these quantities can be expanded in power series of \( \gamma_\omega^{(0)} \) as

\[
\gamma_\omega(\gamma_\omega^{(0)}) = \sum_{0}^{\infty} b_N(0)(\gamma_\omega^{(0)})^{N+1}
\]

(B.8)

and

\[
\gamma_\omega^{(\varepsilon)}(\gamma_\omega^{(0)}) = \sum_{0}^{\infty} b_N^{(\varepsilon)}(0)(\gamma_\omega^{(0)})^{N+1}
\]

(B.9)

with the following coefficients

\[
b_N(\varepsilon) = b_N(0) + \varepsilon b_N^{(\varepsilon)}(0) + \mathcal{O}(\varepsilon^2)
\]

(B.10)

obtained through the previous relation (B.4). We give the first terms of these series:

\[
\gamma_\omega(\gamma_\omega^{(0)}) = \gamma_\omega^{(0)} + 2\zeta(3)(\gamma_\omega^{(0)})^4 + 2\zeta(5)(\gamma_\omega^{(0)})^6 + 12\zeta^2(3)(\gamma_\omega^{(0)})^7 + \mathcal{O}\left((\gamma_\omega^{(0)})^8\right)
\]

(B.11)

which corresponds to the well-known BFKL anomalous dimension in the \( n = 0 \) gluon channel and to its \( \varepsilon \)-corrections:

\[
\gamma_\omega^{(\varepsilon)}(\gamma_\omega^{(0)}) = 2\zeta(3)(\gamma_\omega^{(0)})^3 - 3\zeta(4)(\gamma_\omega^{(0)})^4 + 2\zeta(5)(\gamma_\omega^{(0)})^5 + 22\zeta^2(3) - 5\zeta(6)\right)(\gamma_\omega^{(0)})^6 + \mathcal{O}\left((\gamma_\omega^{(0)})^7\right)
\]

(B.12)

Note that \( \gamma_\omega \), can directly be obtained from the order \( \varepsilon^0 \) of eq.(B.1) (with shift operator replaced by unity) as the solution of the usual implicit equation

\[
1 = \gamma_\omega^{(0)} \chi(\gamma_\omega)
\]

(B.13)

and the \( \varepsilon \)-correction \( \gamma_\omega^{(\varepsilon)} \) is given from the order \( \varepsilon \) terms by

\[
\gamma_\omega^{(\varepsilon)} = \lambda_1^{(\varepsilon)} \gamma_\omega^{(0)} + \gamma_\omega^{(\varepsilon)} \left[ 1 + \gamma_\omega^{(0)} \gamma_\omega \chi'(\gamma_\omega) \right]
\]

(B.14)
where we have used the relation
\[ \gamma_1^{(0)} \lambda_1^{(0)}(\gamma_\omega, 0) = 1 + \gamma_\omega^{(0)} \gamma_\omega \chi'(\gamma_\omega), \] (B.15)
which leads to the solution
\[ \gamma_\omega^{(e)} = \frac{\lambda_1^{(e)}(\gamma_\omega)}{-\gamma_\omega \chi'(\gamma_\omega)} \] (B.16)
with \( \lambda_1^{(e)} \) defined in (A.11).
We now consider the correcting terms to the previous eq.(B.14) coming from the running of the dimensionless strong coupling as \( \bar{\alpha}_S = \bar{\alpha}_S \left( \frac{\mu^2}{\bar{\mu}^2} \right)^\varepsilon \): it leads to write
\[ \gamma_\omega^{(e)} - \ln\left( \frac{\mu^2}{\bar{\mu}^2} \right) \frac{\chi(\gamma_\omega)}{\chi'(\gamma_\omega)} = \lambda_1^{(e)} \gamma_\omega^{(0)} + \gamma_\omega^{(e)} \left[ 1 + \gamma_\omega^{(0)} \gamma_\omega \chi'(\gamma_\omega) \right] + \ln\left( \frac{\mu^2}{\bar{\mu}^2} \right) \left[ -\frac{\chi(\gamma_\omega)}{\chi'(\gamma_\omega)} - \gamma_\omega \right] + \lambda_1(\gamma_\omega, 0) \gamma_\omega^{(0)} \] (B.17)
where we have used the relation
\[ \frac{\partial \gamma_\omega}{\partial \ln \bar{\alpha}_S} = -\frac{\chi(\gamma_\omega)}{\chi'(\gamma_\omega)}. \] (B.18)
These terms explicitly cancel, thus restoring the original equation (B.14), and leading to the same expression of \( \gamma_\omega^{(e)} \). Therefore, the renormalization scale dependence of the dimensionless strong coupling does not modify the solutions \( \gamma_\omega, \gamma_\omega^{(e)} \) of the equation (B.1), which legitimates to neglect it during the calculation in our approach. Note that the same cancellation occurs in the Catani-Hautmann calculation if such running effects are considered, then eq.(B.15) (from which the \( R_N \) factor is obtained) in [3] will be unchanged.

B.2 Case \( n > 0 \)
We now look at the solution of the operatorial equation in the general conformal spin \( n \) case
\[ \hat{\gamma}_\omega(n, \varepsilon) = \lambda_1 \left( \hat{\gamma}_\omega(n, \varepsilon) - \frac{n}{2}, \varepsilon \right) \hat{\gamma}_\omega^{(0)} \] (B.19)
whose expansion in power of \( \hat{\gamma}_\omega^{(0)} \) can easily be obtained from eqs.(A.20), (A.23) and (A.24) following the same procedure as previously for \( n = 0 \), see eqs.(B.2) to (B.6). After replacement of the shift operator by unity, we expand the solution in \( \varepsilon \) as
\[ \gamma_\omega(\gamma_\omega^{(0)}, n, \varepsilon) = \gamma_\omega(\gamma_\omega^{(0)}, n) + \varepsilon \gamma_\omega^{(e)}(\gamma_\omega^{(0)}, n) + \mathcal{O}(\varepsilon^2). \] (B.20)
The equation (B.19) writes now at \( \mathcal{O}(\varepsilon^0) \) accuracy
\[ \gamma_\omega(n) = \lambda_1 \left( \gamma_\omega - \frac{n}{2}, n, 0 \right) \gamma_\omega^{(0)}, \] (B.21)
which leads with the use of the relation (A.21), to the implicit equation defining the BFKL anomalous dimension \( \gamma_\omega(n) \) in the general conformal spin \( n \) gluon channel
\[ 1 = \gamma_\omega^{(0)} \chi_n \left( \gamma_\omega(n) - \frac{n}{2} \right). \] (B.22)
The result of the expansion of this quantity in power series of $\gamma_\omega^{(0)}$ reads now for the firsts coefficients

$$
\gamma_\omega(\gamma_\omega^{(0)}, n) = \gamma_\omega^{(0)} + [\psi(1) - \psi(1 + n)] \left( \gamma_\omega^{(0)} \right)^2 
+ \left[ (\psi(1) - \psi(1 + n))^2 + \zeta(2, 1 + n) - \zeta(2) \right] \left( \gamma_\omega^{(0)} \right)^3 + \mathcal{O} \left( (\gamma_\omega^{(0)})^4 \right). \quad (B.23)
$$

The $\mathcal{O}(\epsilon)$ terms of the equation (B.19) lead to the $\epsilon$-correction $\gamma_\omega^{(\epsilon)}(n)$ of the BFKL anomalous dimension with conformal spin $n$

$$
\gamma_\omega^{(\epsilon)}(n) = \frac{\lambda_1^{(\epsilon)}(\gamma_\omega(n) - \frac{\kappa}{2}, n)}{-\gamma_\omega(n) \chi'(\gamma_\omega(n) - \frac{\kappa}{2})} \quad (B.24)
$$

with $\lambda_1^{(\epsilon)}(\gamma, n)$ defined in the eq.(A.22). Its perturbative expansion reads

$$
\gamma_\omega^{(\epsilon)}(\gamma_\omega^{(0)}, n) = [\psi(1) - \psi(1 + n)] \gamma_\omega^{(0)} + \left[ \frac{3}{2} (\psi(1) - \psi(1 + n))^2 
+ \frac{1}{2} (\zeta(2, 1 + n) - \zeta(2)) \right] \left( \gamma_\omega^{(0)} \right)^2 + \mathcal{O} \left( (\gamma_\omega^{(0)})^3 \right). \quad (B.25)
$$

Whereas the Riemann Zeta function $\zeta(k)$ is evaluated at natural numbers $k$ (related to the order of the perturbation) in the coefficients of the series expansion (B.11) and (B.12) for the $n = 0$ case, it also appears as Hurwitz Zeta function $\zeta(k, 1 + n)$ in the $n \neq 0$ case. We also notice that the NLO and NNLO coefficients of the BFKL anomalous dimension are no more vanishing for the general $n$ case.

### C The treatment of shift operator

For $\lambda_1 = 1$ (3.3) simplifies to

$$
F_{0<}(\omega, \kappa) = \bar{\alpha}_S \left( \frac{\mu_F^2}{\mu_F^2} \right)^{\epsilon} \int d\gamma \left( \frac{\kappa}{\mu_F^2} \right)^\gamma \frac{1}{\omega - \bar{\alpha}_S e^{-\epsilon \partial_\gamma} \gamma \gamma - \bar{\gamma} - \epsilon} \quad (C.1)
$$

The pole in the last factor lies to the right of the contour.

In the first way calculation we decompose the factor involving the shift operator in powers of $\bar{\alpha}_S$ before doing the $\gamma$ integral. The the Nth term in the integrand is

$$
\frac{1}{\omega} \left( \frac{1}{\gamma \gamma_\omega^{(0)}} \right)^N \frac{1}{\epsilon - \gamma}. \quad (C.2)
$$

We should move $\gamma_\omega^{(0)}$ to one side. Here it is not convenient moving to the right because the pole of the right-most factor should be kept to the right of the contour. We conjugate the operator $\left( \frac{1}{\gamma \gamma_\omega^{(0)}} \right)^N$ so that now the shift operators are acting to the left and then we move the shift operators to the left.

$$
\frac{1}{\omega} \left( \frac{1}{\gamma \gamma_\omega^{(0)}} \right)^N \frac{1}{\gamma + (N-1)\epsilon} \frac{1}{\gamma + (N-2)\epsilon} \cdots \frac{1}{\gamma + \epsilon} \frac{1}{\epsilon - \gamma}. \quad (C.3)
$$
We calculate the residues in the poles to the left of the contour and replace the shift operators by unity

\[
\frac{1}{\omega} \left( \gamma^{(0)}_{\omega} \right)^N \varepsilon^{-N} \sum_{0}^{N-1} (-1)^k \frac{1}{(k+1)! (N-k-1)!}
\]  

(C.4)

The sum can be written as

\[
\frac{1}{(N-1)!} \int_0^1 dx (1-x)^{N-1} = \frac{1}{N!}.
\]  

(C.5)

Notice that the contribution of \( N = 0 \) vanishes, therefore the result is

\[
\frac{1}{\omega} \left( \exp \left( \frac{1}{\varepsilon} \gamma^{(0)}_{\omega} \right) - 1 \right).
\]  

(C.6)

As the alternative way of calculation we would like to do the \( \gamma \) integral first. Write the integral in (C.1) as

\[
\int d\gamma \frac{1}{\gamma - \gamma^{(0)}_{\omega}} \frac{\gamma}{\omega} \frac{1}{\varepsilon - \gamma}.
\]  

(C.7)

In order to release the integration variable from its role of the operator canonically conjugated to \(-i\partial_\gamma\) we substitute \( \gamma \) by \( \gamma + \gamma' \) in the factors besides of the pole factor and also \(-i\partial_\gamma\) by \(-i\partial'_{\gamma'}\). We should set \( \gamma' = 0 \) after all differentiations have been done. The residue is

\[
\frac{\tilde{\gamma}^{(0)}_{\omega} + \gamma'}{\omega} \bigg|_{\gamma' = 0}.
\]  

(C.8)

We write the second factor as

\[
\frac{1}{1 - \frac{\tilde{\gamma}^{(0)}_{\omega}}{-\gamma' + \varepsilon}} - \frac{1}{\gamma'}.
\]  

(C.9)

and expand in powers of \( \tilde{\alpha}_S \). The Nth term reads

\[
\frac{\tilde{\gamma}^{(0)}_{\omega} + \gamma'}{\omega} \left( \frac{1}{-\gamma' + \varepsilon} \gamma^{(0)}_{\omega} \right) N \frac{1}{-\gamma' + \varepsilon}.
\]  

(C.10)

Now we move the operators acting on \( \gamma' \) to the right. After this the shift operators can be replaced by unity and we get

\[
\frac{1}{\omega} \frac{1}{-\gamma' + 2\varepsilon} \frac{1}{-\gamma' + 3\varepsilon} \cdots \frac{1}{-\gamma' + (N+1)\varepsilon} \frac{1}{-\gamma' + (N+1)\varepsilon} \left( \gamma^{(0)}_{\omega} \right)^{N+1} + \frac{\gamma'}{\omega} \gamma^{(0)}_{\omega}.
\]  

(C.11)

We are advised to set \( \gamma' = 0 \), we obtain

\[
\frac{1}{\omega} \left( \frac{1}{(N+1)!} \gamma^{(0)}_{\omega} \right)^{N+1}
\]  

(C.12)

and the sum over \( N \) yields the same result as in the first approach.

Preparing for the discrete sum case the treatment of a non-trivial \( \lambda_1(\gamma, \varepsilon) \) we outline the alternative way of treating (3.14).
The condition $\gamma = 0$ is implemented by contour integral and then the contour is deformed to enclose the anomalous dimension instead of the origin in $\gamma$ plane.

$$\frac{1}{1 - \hat{\gamma}(0) \frac{1}{\gamma}} \bigg|_{\gamma=0} = \frac{1}{2\pi i} \oint_{C_0} \frac{d\gamma}{\gamma} \left( \frac{1}{1 - \hat{\gamma}(0) \frac{1}{\gamma}} \right) = \frac{1}{2\pi i} \oint_{C_0} \frac{d\gamma}{\gamma} \left( \frac{1}{1 - \hat{\gamma}(0) \frac{1}{\gamma + \gamma}} \right) \bigg|_{\gamma'=0}$$

$$= 1 + \frac{1}{2\pi i} \oint_{C_0} \frac{d\gamma}{\gamma} \gamma' \frac{1}{\gamma + \gamma' - \hat{\gamma}(0) \frac{1}{\gamma}} \bigg|_{\gamma'=0} \cdot (C.13)$$

The shift operator is now acting on $\gamma'$ and is disentangled from the integration variable. The subtraction removes the singularity at $\gamma = \infty$ in the integrand. After this the contour $C_0$ can be deformed to $\hat{C}$ encircling the pole of the second factor (with opposite orientation). It is convenient to change the integration variable to $\gamma'' = \gamma + \gamma'$. This is accompanied by a transposition of the operators acting now to the left. We evaluate the integral taking residue

$$\frac{1}{1 - \hat{\gamma}(0) \frac{1}{\gamma}} \bigg|_{\gamma=0} = 1 - \frac{1}{2\pi i} \oint_{\hat{C}} \frac{d\gamma''}{\gamma'' - \gamma' - \hat{\gamma}(0) \frac{1}{\gamma'}} \hat{\gamma}(0) \frac{1}{\gamma'} \bigg|_{\gamma'=0}$$

$$= 1 + \frac{1}{\gamma' - \hat{\gamma}(0) \frac{1}{\gamma'}} \hat{\gamma}(0) \frac{1}{\gamma'} \bigg|_{\gamma'=0} = 1 + \sum_{N=0}^{\infty} \frac{1}{\gamma' - \hat{\gamma}(0) \frac{1}{\gamma'}} \frac{1}{\gamma' - \hat{\gamma}(0) \frac{1}{\gamma'}} \cdots \frac{1}{\gamma' - \hat{\gamma}(0) \frac{1}{\gamma'}} \bigg|_{\gamma'=0}$$

$$= 1 + \sum_{N=0}^{\infty} \gamma(0)^{N+1} \frac{1}{(N+1)! \epsilon^{N+1}} = e^{\hat{\gamma}(0) \frac{1}{\epsilon}} \cdot (C.14)$$

### D The asymptotics $\epsilon \to 0$

Let us write the integral appearing in the results (3.25) and (3.35) as

$$I(\epsilon) = \frac{1}{\epsilon} \int_0^1 \frac{d\beta}{\beta} \gamma(\beta) A(\beta) \exp\left( \frac{1}{\epsilon} \int_0^1 \frac{d\beta_1}{\beta_1} \gamma(\beta) \right) \cdot (D.1)$$

We change the integration variable to $y(\beta) = \int_\beta^1 \frac{d\beta_1}{\beta_1} \gamma(\beta_1)$ and denote the resulting function from this substitution by $\tilde{A}(y) = A(\beta(y))$ and also $y_0 = y(0)$. Notice that the value $y = 0$ corresponds to $\beta = 1$.

$$I(\epsilon) = \frac{1}{\epsilon} \int_0^{y_0} dy \tilde{A}(y) \exp\left( \frac{1}{\epsilon} y \right) = \frac{y_0}{\epsilon} \int_0^1 dz \tilde{A}(y_0 z) \exp\left( \frac{y_0}{\epsilon} z \right)$$

$$= y_0 \int_0^{\epsilon^{-1}} dz_1 \tilde{A}(\epsilon y_0 z_1) \exp(\epsilon y_0 z_1) \cdot (D.2)$$

We apply the mean value theorem and obtain

$$I(\epsilon) = y_0 \tilde{A}(\epsilon y_0 z_0) \int_0^{\epsilon^{-1}} dz_1 \exp(\epsilon y_0 z_1)$$

$$= \tilde{A}(\epsilon y_0 z_0) \left( \exp\left( \frac{1}{\epsilon} y_0 \right) - 1 \right) \cdot (D.3)$$
for some $z_0$ in the integration range. The exponential function in the integrand ensures that $y_0 z_0 \sim 1$ is independent of $\varepsilon$. We have noticed above that at the argument $y = 0$ of the function $\tilde{A}(y)$ is equal to the function $A(\beta)$ at $\beta = 1$ Therefore, the asymptotics in $\varepsilon$ is

$$I(\varepsilon) = A(1) \left( \exp \left( \frac{1}{\varepsilon} \int_0^1 \frac{d\beta}{\beta} \gamma(\beta) \right) - 1 \right) (1 + O(\varepsilon)) . \quad (D.4)$$

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