A New 3D Surface Reconstruction Method

Wei Fu1,3,4,* and Lushen Wu2
1 School of Software, Nanchang Hangkong University, Nanchang, 330063, China
2 School of Mechanical and Electrical Engineering, Nanchang University, Nanchang, 330031, China
3 Department of Physics, Nanchang Normal University, Nanchang, 330022, China
4 School of Flight Technology, Jiangxi Institute of Economic Administrators, 330088, China
* Email: fuwei75@163.com

Abstract. The existing complete orthogonal function system cannot introduce the spectrum analysis into geometry information processing field. This paper proposes complete orthogonal V system reconstruction method based on the 3D data model, and this method is to map surface data model to parametric triangular domain. The complete orthogonal V system in triangular domain is used to expand the surface data model and the 3D model reconstruction with piecewise polynomial is realized, and it has simple operation and can handle any topology grid. The experimental results show that the error of three experimental models is controlled in 0.25 mm, Compared with Voronoi-based method and Yoshimoto’s method, the accuracy of the proposed method is improved obviously, and the orthogonal V system under reconstruction time compared with Voronoi-based method reconstruction time is shortened by 13%. Therefore, reconstruction precision and speed can meet the requirements of industrial design.

1. Introduction
Surface reconstruction has been a hot topic of research in the past 20 years or so. For the unknown surface, We usually gets the 3D point cloud data by 3D scanning equipment. With the continuous development of three-dimensional scanning, three-dimensional models have been widely used in aerospace industry, visualization, virtual reality, geometry, 3D game video and entertainment, molecular biology and other fields. This greatly improves the demand for digital processing algorithms. In computer graphics, spatial geometry models usually use parametric surfaces and polygonal meshes to represent. Because of its simplicity and flexibility, triangulation has become one of the mainstream representations of 3D geometric models. As a very powerful mathematical tool, a complete orthogonal function system plays a very important role in signal analysis and image processing. It can be represented by piecewise polynomials composed of several planar graphs. In order to further study and process graph groups, it can be represented by finite terms of series. Therefore, we can introduce spectrum analysis into the field of geometric information processing. However, the existing complete orthogonal function systems, such as the Fourier transform function system, the Chebyshev polynomial system, the Walsh function, the shad horse function, the wavelet function and the orthogonal Legendre system, can not be realized. This is also the reason why there are very few studies on reconstruction using orthogonal systems. Therefore, this paper studies the complete orthogonal function V system to represent and process geometric object models.

Generally speaking, the existing algorithms for processing digital geometric signals are mainly based on subdivision mesh and relaxation operator method. Lounsbery uses wavelet analysis to extend the classic multi-resolution technology to grid signal processing, but the algorithm can only handle...
grids with subdivision topology. Taubin generalize the classical discrete Fourier transform to any grid, and define the frequency as the eigenvector of the discrete Laplacian operand that extends to the irregular grid. The algorithm is perfect in theory, but the huge amount of data that decomposes the eigenvector limits its use. It is usually used only for the smooth modeling of the three-dimensional model.

2. Orthogonal V-System of Degree K

The V system of degree K is orthogonal complete function system of piecewise polynomials, which is a complete orthogonal system constructed by Professor Song Ruixia on the basis of Professor Qi Dongxu's U system in 2005, and has carried out the corresponding application research in the geometric information and the orthogonal expression of the pattern recognition of the pattern recognition, and so on. The following brief introduction is to the construction of V system and its related characteristics.

Let the first set of functions of the V system of degree K be the first (K+1) Legendre polynomials on [0, 1].

\[ V_{k,i}(x) = \begin{cases} \sqrt{2^{-k}} V_{k,i+1} \left( 2^{-k} \left( x - \frac{j-1}{2^{n-k}} \right) \right), & x \in \left( \frac{j-1}{2^{n-k}}, \frac{j}{2^{n-k}} \right) \\
0, & i = 1, 2, \ldots, k+1; j = 1, 2, \ldots, 2^{n-k} 
\end{cases} \]

The graph of partial functions is shown in figure 1.

**Figure 1.** Kth V system schematic

K=0, K=1(The first 16 functions), K=2(The first 24 functions), K=0(The first 32 functions)

3. Construct V System on Triangle Domain

3.1. Triangulated Domain and Triangulation

The V system on the triangulated domain is a complete orthogonal function system with a hierarchical Subdivision structure. As coordinate transformation from one triangular domain to another doesn’t change the orthogonality, we choose a right triangle G with vertices \( P_1 (0, 0), P_2 (0, 1), P_3 (2, 0) \) as base triangle, it is a triangle with area 1. The inner product of G is defined as:

\[ \langle f, g \rangle = \iint_G f(x,y) g(x,y) dx dy \]

We divided G as follows: Firstly, based on the G triangle, the middle point of each side is taken as the new vertex, and three vertices are connected, and G is divided into four sub triangles, as shown in Figure 2 (a), denoted by \( G_{1,1}, G_{2,1}, G_{2,2}, G_{2,3} \), this process is called triangulation at level one. Secondly,
each of these four sub triangles is subdivided into four smaller sub triangles denoted by
\[ G_{i,j}, i = 1,2,3,4, j = 1,2,\ldots, 2^l - 1, \] shown in Figure 2 (b). Accordingly, this process is called triangulation at level two.  
\[ \text{G}_{i,j} \] indicates that the triangle area after the division is located at the \( i \) line \( j \) column. Go on with this triangulation process, in the triangulation at level \( m \) there are \( 4^m \) subtriangles.

![Figure 2. Triangular region subdivision](image)

(a) First subdivision, (b) Secondary subdivision

### 3.2. Generation of Orthogonal \( V \) System

**Second group:**

\[ V_{1,1}^1 = \begin{cases} -3y + 5(x,y) \in G_{1,1} \\ -3y + 1(x,y) \in \text{others} \end{cases}, \quad V_{1,2}^1 = \begin{cases} 6x + 3y - 1(x,y) \in G_{1,1} \\ 6x + 3y - 5(x,y) \in \text{others} \end{cases}, \]

\[ V_{1,1}^2 = \begin{cases} -6x + 5(x,y) \in G_{2,3} \\ -6x + 1(x,y) \in \text{others} \end{cases}, \quad V_{1,2}^2 = \begin{cases} \sqrt{2}(6x-2),(x,y) \in G_{2,2} \\ \sqrt{2}(6x-4),(x,y) \in G_{2,2} \end{cases}, \]

\[ V_{1,3}^3 = \begin{cases} \sqrt{8}(4x+4y-6),(x,y) \in G_{i,1} \\ \sqrt{8}(-4x-4y+2),(x,y) \in G_{i,1} \end{cases}, \quad V_{1,4}^3 = \begin{cases} -\sqrt{8}(4x+4y-4),(x,y) \in G_{i,1} \\ -\sqrt{8}(4x+4y-4),(x,y) \in G_{i,1} \end{cases}, \]

\[ V_{1,5}^4 = \left\{ \begin{array}{ll} \sqrt{8}(4x+4y-2),(x,y) \in G_{i,1} \\ \sqrt{8}(4x+4y-2),(x,y) \in G_{i,1} \end{array} \right\}, \quad V_{1,6}^4 = \left\{ \begin{array}{ll} \sqrt{8}(4x+4y-1),(x,y) \in G_{i,1} \\ \sqrt{8}(4x+4y-1),(x,y) \in G_{i,1} \end{array} \right\}. \]

These nine functions constitute the second group of V systems on the triangulated domain \( G \). It is called a wavelet function.

**Third group:** The function generator \( V_i^j(x,y), (j = 1,2,\ldots,9) \) is compressed and translated respectively. Duplicate squeezed generator onto each of the sub triangles \( G_{i,1}, G_{i,2}, G_{i,3}, G_{i,4} \).

To compress the 9 functions of the second group four times, duplicate them to the subregion of two level dissection of \( G \) and get a function, which is divided into 9 classes, each class of 4 functions, and all 36 functions constitute the third groups of the V system.

\[ V_{3,1}^1(x,y) = \begin{cases} 2V_2^1(2x,2y-1),(x,y) \in G_{1,1} \\ 0, \text{others} \end{cases}, \quad V_{3,2}^1(x,y) = \begin{cases} 2V_2^1(2x,2y),(x,y) \in G_{1,1} \\ 0, \text{others} \end{cases}, \]

\[ V_{3,1}^0(x,y) = \begin{cases} 2V_2^0(-2(x-1/2),-2(y-1)),(x,y) \in G_{2,2} \\ 0, \text{others} \end{cases}, \quad V_{3,2}^0(x,y) = \begin{cases} 2V_2^0(2x-1,2y),(x,y) \in G_{2,3} \\ 0, \text{others} \end{cases}. \]
M group: Go on with above process over and over again. The mth group can be obtained by squeezing the (m-1)th group four times, and copying them to the corresponding sub triangles. That is, after having squeezed each function in the (m-1)th group four times, four new functions can be defined by duplicating them onto each of the sub triangles respectively. This process is depicted in Figure 3. The mth group is also divided into 9 classes and each one has $4^{m-2}$ functions, here $m = 3, 4, \ldots$.

This compression and replication process can be repeated. In this way, we obtain all the basis function of V system: $\{V^i_1, V^i_2, \cdots, V^i_9; V^j_1, V^j_2, \cdots, V^j_9; \cdots; V^j_4, \cdots, V^j_4\}_m, m = 3, 4, \cdots \ldots$ Where $V^i_j$ represents the jth function of the ith class of the mth group in the v system.

The jth function of ith class in mth group can be expressed as:

$$v^i_j(x, y) = \begin{cases} 2^{m-1}P_{ij}(x, (y - \frac{\beta - 1}{2^m}), 2^{m-1}(x - \frac{\beta - 1}{2^m})), (x, y) \in G_{i,j}; & v^i_j(x, y) = 2^{m-1}P_{ij}(-2^{m-1}(x - \frac{\beta - 1}{2^m}), -(y - \frac{\beta - 1}{2^m})), (x, y) \in G_{i,j}; \\
0, & \text{Others} \\
0, & \text{Others} \end{cases}$$

Where $m = 3, 4, \cdots, i = 1, 2, 3, \cdots j = (a-1)^2 + 2b + 1, a = 1, 2, \cdots, 2^{m-1}, \beta = 1, 2, \cdots a$.

4. Orthogonal Reconstruction of 3D Model in V System

4.1. 3D Model Reconstruction

In spatial geometry, the surface parameters of 3D space are represented as:

$$\begin{align*}
x &= x(u, v) \\
y &= y(u, v) \\
z &= z(u, v)
\end{align*}$$

$u, v$ changes in some region of the $(u, v)$ plane. According to the construction principle of V system, we can use piecewise linear function to represent the target triangular mesh model. The triangles in the model are defined in different subregions of the triangle domain. Define the function $f: \Omega \rightarrow M$, Mapping is directly established in parameter domain $\Omega \in R^2$ and triangular mesh $M = f(\Omega) \subset R^2$.

4.2. Model Parameterization

Let $(u, v) \in R^2$ be coordinate of a point in triangular domain G, and $(x, y, z) \in R^3$ be coordinate of a point in 3D space. As long as a 3D geometric model is mapped onto the triangulated domain G and expressed as a piecewise polynomial of degree one of two variables with all breaking points on the section line of G, it can be precisely represented using a finite number of basis functions of the V-system, due to the reproducibility property of the V-system.

(1) Obtain the vertex coordinates $P_i = (x_i, y_i, z_i); i = 1, 2, 3$ for each triangular patch $p_1, p_2, p_3$ using vertex index method.

(2) Map 3D triangular patch $p_1, p_2, p_3$ onto subdomain $G_{i,j}$ of domain G by one to one linear transformation (See Figure 4). Let the vertices of $G_{i,j}$ be $P_i = (u, v); i = 1, 2, 3$. The linear
transformation is defined as:

\[
\begin{align*}
    x &= a_u + b_u v + c_u \\
    y &= a_v + b_v v + c_v \\
    z &= a_w + b_w w + c_w
\end{align*}
\]  

(3)

Where the coefficients \(a_u, b_u, c_u, a_v, b_v, c_v, a_w, b_w, c_w\) satisfy below equations:

\[
\begin{align*}
    x_i &= a_u + b_u v_i + c_u \\
    y_i &= a_v + b_v v_i + c_v \\
    z_i &= a_w + b_w w_i + c_w
\end{align*}
\]  

(i = 1, 2, 3)  

(4)

The 9 coefficients in formula (3) can be determined by the calculation of equation (6), and then the triangular patch \(P_1 P_2 P_3\) can be parameterized. That is: the triangular patch \(P_1 P_2 P_3\) is parameterized as above polynomial of degree one of two variables defined on \(G_{i,j}\), which is written as:

\[
P(u,v) = [a_u + b_u v + c_u, a_v + b_v v + c_v, a_w + b_w w + c_w]
\]  

(5)

3D geometric model consists of \(N\) triangular patches, and \(N\) is set as the minimum integer satisfying \(4^* \geq N\). The 3-D geometric model is expressed as a piecewise polynomial of degree one of two variables:

\[
P(u,v,w) = \begin{cases} 
P_0(u,v,w), & (u,v,w) \in G_0 \\
P_1(u,v,w), & (u,v,w) \in G_1 \\
P_2(u,v,w), & (u,v,w) \in G_2 \\
\vdots & \vdots \\
P_N(u,v,w), & (u,v,w) \in G_N \\
0 & (u,v,w) \not\in G, N
\end{cases}
\]  

(6)

Equation (6) gives the parameter expression of \(N\) sub triangles domain. The function values of the other sub triangles \(4^* \geq N\) is defined as 0. In this mapping, a 3D model triangular patch is projected into a sub triangle domain \(G\) by one-to-one correspondence, which means that any two different triangular patches can’t be mapped into the same sub triangle. The above parameterization process only depends on the triangular mesh of the 3d model and is not affected by the connectivity of the mesh. For a 3D model, once given its triangular grid, no matter whether the grid is connected or not, the parameterization process can be realized by mapping each triangular plane to the triangular domain of \(V\) system.

The triangular domain is divided into \(n\) levels, all the subregions obtained by the division are denoted as \(\{S_i\}_{i=1,2,\ldots,4^*}\). Let the area coordinates of the three vertices be \((u_i, v_i, w_i)\). For each point, mapping the \(i\) triangular patch \(P_1 P_2 P_3\) to \(G_1\) cause 3 vertices of \(P_1 P_2 P_3\) exactly map to the 3 vertices of \(G_1\). This process can be made up of two linear maps: first mapped to the 3 vertices that exactly mapped to the 3 vertices, and then mapped to the 3 vertices of the 3 vertices that are mapped to each other. In combination with these two steps, the mapping relation between \(P_1 P_2 P_3\) and \(G_1\) of the \(i\) th triangular patch is:

\[
\begin{bmatrix}
    x \\
    y \\
    z
\end{bmatrix} = \begin{bmatrix}
    u \\
    v \\
    w
\end{bmatrix} = X_{U_i} U_i \in \mathbb{R}^{3 \times 3}
\]  

(7)

Where \(X_{U_i} = \begin{bmatrix} x_1 & x_2 & x_3 \\ y_1 & y_2 & y_3 \\ z_1 & z_2 & z_3 \end{bmatrix}\) and \(U_i = \begin{bmatrix} u_1 & u_2 & u_3 \\ v_1 & v_2 & v_3 \\ w_1 & w_2 & w_3 \end{bmatrix}\).

So the first triangle is parameterized by, of course, a vector function with three components, each of which is a polynomial. Each of them has three components of a polynomial, which is, \(P(u,v,w) = [P_1(u,v,w), P_2(u,v,w), P_3(u,v,w)]^T\).
4.3. Model Reconstruction

Parameterize a spatial model P with N triangle patches to G domain that has been triangulated into $4^n$ sub triangles. According to the regenerative property, we reconstruct the geometric model by using the basis function of the first $3 \times 4^n$ V system. The number of $3 \times 4^n$ has been determined because any polynomial of two variables of degree one has 3 degrees of freedom and $4^n$ sub triangle domains, where n is the smallest integer to satisfy $4^n > N$. The basis functions of linear V systems on triangle domains are arranged by their grouping order.

$$V_{i_1}^1, V_{i_1}^2, V_{i_2}^2, \ldots, V_{i_m}^2$$, where, $m = 3, 4, \ldots ; i = 1, 2, \ldots 9$; $j = 1, 2, \ldots 4^{m-2}$

Therefore, 3D geometric model can be represented as

$$P(u, v) = \sum_{i=1}^{34^n} c_i V_i(u, v)$$  \hspace{1cm} (8)

Where, $c_i$ is the orthogonal expansion coefficient of V system, it is derived from the equation (8):

$$c_i = \iint_G P(u, v) V_i(u, v) dG$$  \hspace{1cm} (9)

The objective of 3D geometric model reconstruction is to use the spectrum of the model to restore its original space form. The steps of reconstruction include two parts. First, the parameters of the model are obtained by using the spectrum of the model.

The second step is to find the vertex coordinates of all triangle patches in the model by the parameters of the 3D geometric model.

5. Experiment Result

This algorithm is implemented on Intel Dual-Core 2GHZ, 2 GB memory Windows XP operating platform computers. Three instance models are adopted, namely blade, model and Armadillo.

The experimental results show that this method can reconstruct the original model well.

5.1. Model Reconstruction Speed

Through the reconstruction of V system, a large number of integral operations can be calculated in
advance and simplified, thus greatly shortening the reconstruction time. Table 1 shows the comparison of operation speed of three different models. Among them, the reconstruction time of V system is 13% shorter than the commonly used reconstruction method voronoi-based method.

| model        | Triangular slice number | Voronoi-based method /s | V system reconstruction /s |
|--------------|-------------------------|-------------------------|---------------------------|
| Blade        | 897                     | 7.68                    | 7.13                      |
| model        | 185                     | 11.85                   | 10.58                     |
| Armadillo    | 661                     | 9.97                    | 8.25                      |

The time complexity of voronoi-based method is the time complexity of the proposed algorithm is. Compared with them, the algorithm in this chapter has low time complexity and high efficiency.

5.2. Precision Evaluation of 3D Model Reconstruction

The evaluation of surface profile error of the reconstructed surface is actually translated into the problem of finding the vertical distance from the point to the surface. Error analysis of the three experimental models is as follows:

As shown in figure 7(a), the blade surface reconstruction error analysis renderings, whose maximum deviation is: 0.235mm; The average deviation is: 0.001; Standard deviation: 0.0009.

As shown in fig.7 (b), the maximum deviation of the surface reconstruction error analysis rendering diagram of the model is: 0.231mm; the average deviation is: 0.091. Standard deviation: 0.020

As shown in fig.7 (c), the maximum deviation of Armadillo is 0.246mm. The average deviation is: 0.045. Standard deviation: 0.025

As shown in figure 7, the small square on the left represents the detection point of the corresponding position, and the color of the small square represents the shortest distance between the detection point and the reconstructed surface. Therefore, the surface reconstruction error of the three models is controlled within 0.25mm.

![Image](a) blade  (b) model  (c) Armadillo

**Figure 6.** Three models surface reconstruction error analysis

6. Conclusion

In this paper, a method to reconstruct 3D surface model by using complete orthogonal V system is proposed. This method can be used to reconstruct the surface model and improve the precision of surface reconstruction. In this paper, the accuracy evaluation experiment of the 3D model surface reconstructed by V system is carried out. The experiment results prove that the reconstruction error of the model surface is controlled within 0.25mm, which meets the error requirement of the reconstructed surface and the orthogonal v system under reconstruction time compared with Voronoi-based method reconstruction time is shortened by 13%. This algorithm can reconstruct the three-dimensional model with piecewise polynomial. It has the advantages of simple operation, can handle the grid of arbitrary topology, and the operation speed is faster. In future work, in-depth analysis of spectrum and the three dimensional model is mapped to a triangular domain of parametric method, and discusses the method.
in the application of 3D model feature extracting, further improves the reconstruction precision of the model.

7. Reference

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