Optimization of the energy complex “NPP-accumulator” in case of force majeure

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Abstract. We consider a problem of optimization of NPP with accumulator operation mode in case of force majeure. A mathematical formulation and solving of problem of energy output’s time behavior is provided. A mathematical formulation and solving of problem of energy’s optimum allocation to consumers with different priorities. Mathematically, the problem reduces to linear programming problem. We received that optimal time behavior is uniform energy output, and one should start with consumer with highest priority.

Introduction
There are special Nuclear Power Plants for remote areas, where NPP is the only source of energy for quite a big area (for example, Bilibinskaya NPP in Russia [1]). In case of force majeure, there is a need in an instant stop of NPP. This will lead to economic and social losses.

We suppose to establish an accumulator in NPP to solve these problems. A need to store energy for a long time without big losses requires using of accumulators with low energy loss, such as described in [2-5]

Here we consider the problem of optimal distribution of energy in case of force majeure, when the NPP is shut down but still there are consumers that need electricity.

Fig. 1. Energy consumption in relation to time.

Fig. 1 illustrates energy consumption in relation to time [6]. Energy is demanded more in day time, than at night, therefore, power units have to work in a varying duty. This leads to waste of fuel and
power units’ reduction of serviceability [7]. Accumulator introduction into the system can solve this problem: power units will work at constant duty, accumulator will be charged with energy excess at night, and it will discharge during day time to cover peak ratings.

The problem is posed this way: there are a NPP with an accumulator and several consumers of energy provided by NPP (for example, factories etc.) A threat is announced at the time zero \( t=0 \) for indefinite period \( \tau \). This period is a random value, which has density function \( f(\tau) \). Therefore, the NPP is stopped, but there is some charge in accumulator. It is required to find the best way to provide energy to consumers, considering that lack of energy leads to economic loss determined by loss function.

This problem’s solution was divided on two parts: optimization of energy output’s time behavior and searching of energy’s optimum allocation to consumers depending on their loss functions.

### 1. Optimization of energy output’s time behavior

Minimum of average loss is searched to solve the first problem.

Total loss of one consumer is

\[
c(\tau) \int_0^\tau (W_n - W(t)) \, dt
\]

\( c(\tau) \) is some loss function here, its general form is shown on fig.2).

![Figure 2. General form of loss function.](image)

Therefore, considering that \( f(\tau) \) is density function of duration of force majeure, we obtain average loss (it is average value of (1)):

\[
\int_0^\tau f(\tau)c(\tau) \int_0^\tau (W_n - W(t)) \, dt \, d\tau
\]

In addition to this, we have to set some limitations on \( W(t) \):

\[
\begin{align*}
0 & \leq W(t) \leq W_0, \\
\int_0^\tau W(t) \, dt & \leq \int_0^\tau W(t) \, dt \leq Q_A.
\end{align*}
\]

\( W_0 \) here is power that was provided to consumer before force majeure, \( Q_A \) is accumulator’s charge.

\[
Q_A = \eta(t_2 - t_1)(1 - \alpha)W_n
\]

\( \eta \) – accumulator’s coefficient of efficiency, \( t_2 - t_1 \) – system’s working time at night, \( \alpha \) – portion of NPP’s power at night time, \( W_n \) – NPP’s power.
It is shown that if we expand $W(t)$ into power series, this problem is reduced to linear programming problem, which has an optimal solution $W(t) = \text{const}$ (uniform energy output). This result will be taken in account while solving the second problem.

2. Searching of energy’s optimum allocation to consumers

2.1. Problem definition

To solve the second problem, we have to search for

$$\min_{W_j} \sum_{j=1}^{M} (W_{sj} - W_j) \int_0^T C_j(\tau)f(\tau) \tau d\tau$$

(5)

considering that limitations on $W(t)$ are:

$$0 \leq W_j \leq W_{sj},$$

$$\sum_{j=1}^{M} W_j \tau \leq \sum_{j=1}^{M} W_j T \leq Q_A.$$  

(6)

This problem reduces to problem of searching

$$\max_{W_j} \sum_{j=1}^{M} W_j R_j$$

(7)

Where

$$R_j = \int_0^T C_j(\tau)f(\tau) \tau d\tau$$

(8)

and ratio between this values is determined by loss function.

2.2. Problem solving

This problem has analytical solution: first, we have to provide energy to consumer, which loss function is maximum [8].

We consider this problem:

$$\begin{cases}
    f = \max_{W_j} \sum_{j=1}^{M} W_j R_j , \\
    0 \leq W_j \leq W_{sj}, \sum_{j=1}^{M} W_j = W_A;
\end{cases}$$

(9)

where $W_A = Q_A/T$. We consider that $R_1 > R_2 > \cdots > R_M$ (10) without loss of generality.

Let us prove that if $W_A \leq W_{s1}$ then optimum solution is

$$W_1 = W_A , W_2 = \cdots = W_M = 0$$

(10)

and

$$f = W_1 R_1$$

(11)

Now we consider any other allocation of $W_j$.

Let $W_1' = W_A - \Delta W , W_2' \neq 0, \ldots, W_M' \neq 0, \text{ and } W_1' + \cdots + W_M' = \Delta W$ (which follows from (9)).

Then

$$f' = (W_A - \Delta W' + R_2 W_2' + \cdots R_M W_M')$$

(12)

We have to compare $f$ and $f'$; if $f > f'$ it means that allocation (11) is optimum.

Let us consider $f - f'$:

$$f - f' = R_1 \Delta W - R_2 W_2' - \cdots - R_M W_M'$$

(13)

It follows from (10) that:

$$R_M W_M' < R_2 W_2' + \cdots + R_M W_M' < R_2 W_2' + \cdots + R_2 W_M'$$

(14)

And, considering (15), we have:
\[ f - f' > R_1 \Delta W - R_2 (W_1' + \cdots + W_M') = R_1 \Delta W - R_2 \Delta W = (R_1 - R_2) \Delta W > 0 \quad (15) \]

It follows from (16) that any other allocation gives us \( f > f' \), and original allocation (11) is optimum. Then, if there is some charge remained, it has to be allocated between the rest of consumers according to aboon: we have to start from consumer with maximum loss function.

3. Optimization effect
To count optimization effect, we have to compare to energy allocations: optimal and non-optimal. Let us sort consumers from lowers \( R_j \) to greatest. Then losses in optimal allocation will be:
\[ \nu_1 = \left( W_A - \sum_{i=1}^{M-1} W_i \right) R_M + \sum_{j=M+1}^{N} W_j R_j \quad (16) \]
(accumulator’s power is enough for \( M \) consumers)

And in case of non-optimal allocation, if accumulator’s power is enough for \( P \) consumers, total loss is:
\[ \nu_2 = \left( W_A - \sum_{i=N}^{N-P+2} W_i \right) R_{N-P+1} + \sum_{j=1}^{N-P} W_j R_j \quad (17) \]

Then optimization effect can be counted as
\[ \nu = \frac{\nu_2 - \nu_1}{\nu_2} = 1 - \frac{\nu_1}{\nu_2} \quad (18) \]

This formula is difficult to analyse, because we have too many parameters: we can simplify it considering that accumulator’s power is only enough for consumers with numbers \( 1 \div M \) or with numbers \( N - P \div N \), and \( W_j = W \). Then
\[ \nu = 1 - \frac{\sum_{j=M+1}^{N} W_j R_j}{\sum_{j=1}^{N} W_j R_j} = 1 - \frac{\sum_{j=M+1}^{N} R_j}{\sum_{j=1}^{N} R_j} \quad (19) \]

For example, if we consider that there are only two consumers, and \( W_1 = W_2 = W_A \), then
\[ \nu = 1 - \frac{(W_A - W_1) R_1 + W_2 R_2}{(W_A - W_2) R_2 + W_1 R_1} = 1 - \frac{R_2}{R_1} \quad (20) \]

4. Future works and conclusions
To summarize, we received that optimal time behavior is uniform energy output, and one should start with consumer with highest priority. We will continue solving the problem optimization of the energy complex “NPP-accumulator” in case of force majeure, and we are going to solve this problem considering that accumulator can be used to satisfy NPP’s own needs (for example, cooling water outlet, if energy generator is broken, as it was at Fukushima ([9-10])). Also, we have to take in account that the value of accumulator’s charge can also be a random value, because we do not know exactly, in what moment the NPP was stopped. It would be interesting to understand, how big should be accumulator’s capacity, that it would be enough to cover possible needs in case of force majeure, but not very expensive in regular regime.

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