Ungravity realization in fractional extra dimensions

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Abstract

Ungravity by tensor unparticles is realized in AdS_{4+N} space through deconstruction. It is shown that ungravity is equivalent to the gravity in extra dimensions. There is a close relation between the scaling dimension of the unparticle operator and the number of extra dimensions. Consequently it is possible to discover the fractional extra dimensions, which would be a stringent signal for unparticles.

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I. INTRODUCTION

With the reoperation of the Large Hadron Collider (LHC) at CERN very recently, we are now entering a new era of physics in history. The LHC will unveil many mysteries of high energy physics such as electroweak symmetry breaking, dark matter, and new symmetries, to name a few. It seems quite true that the standard model (SM) of particle physics is only an effective theory at low energy, and there must be some new physics behind it. Many kinds of new physics — supersymmetry or extra dimensions, etc. — involve some new sets of particles. But recently a totally different type of new physics was suggested by Georgi [1]. In this scenario, there is a scale-invariant hidden sector which couples to the SM particles very weakly. When seen at low energy, the hidden sector behaves in different ways from ordinary particles, hence dubbed as "unparticles."

Consider a ultraviolet (UV) theory in the hidden sector with the infrared (IR)-stable fixed point. The theory interacts with the SM sector at a scale of $M_{\mathcal{U}}$. Below $M_{\mathcal{U}}$, the interaction between a UV operator $\mathcal{O}_{\text{UV}}$ and an SM operator $\mathcal{O}_{\text{SM}}$ is described as $\mathcal{O}_{\text{SM}} \mathcal{O}_{\text{UV}} / M_{\mathcal{U}}^{d_{\text{SM}} + d_{\text{UV}} - 4}$. Here $d_{\text{UV(SM)}}$ is the scaling dimension of $\mathcal{O}_{\text{UV(SM)}}$. When the scale goes down via the renormalization flow, a scale $\Lambda_{\mathcal{U}}$ appears through the dimensional transmutation where the scale invariance emerges. Below $\Lambda_{\mathcal{U}}$ the theory is matched onto the above interaction with the new unparticle operator $\mathcal{O}_{\mathcal{U}}$ as

$$C_{\mathcal{U}} \frac{\Lambda_{\mathcal{U}}^{d_{\text{UV}} - d_{\mathcal{U}}}}{M_{\mathcal{U}}^{d_{\text{SM}} + d_{\text{UV}} - 4}} \mathcal{O}_{\text{SM}} \mathcal{O}_{\mathcal{U}} ,$$

where $d_{\mathcal{U}}$ is the scaling dimension of $\mathcal{O}_{\mathcal{U}}$ and $C_{\mathcal{U}}$ is the matching coefficient. Because of the scale invariance, $d_{\mathcal{U}}$ can have nontrivial values. This unusual behavior is reflected on the phase space of $\mathcal{O}_{\mathcal{U}}$. To see it, consider the spectral function of the unparticle which is given by the two-point function of $\mathcal{O}_{\mathcal{U}}$:

$$\rho_{\mathcal{U}}(P^2) = \int d^4 x \, e^{iP \cdot x} \langle 0 | \mathcal{O}_{\mathcal{U}}(x) \mathcal{O}_{\mathcal{U}}^\dagger(0) | 0 \rangle = A_{d_{\mathcal{U}}} \theta(P^0) \theta(P^2)(P^2)^{d_{\mathcal{U}} - 2} ,$$

where

$$A_{d_{\mathcal{U}}} = \frac{16\pi^2}{(2\pi)^2} \frac{\Gamma(d_{\mathcal{U}} + \frac{1}{2})}{\Gamma(d_{\mathcal{U}} - 1) \Gamma(2d_{\mathcal{U}})} ,$$

is the normalization factor. The corresponding phase space is

$$d\Phi_{\mathcal{U}}(P) = \rho_{\mathcal{U}}(P^2) \frac{d^4 P}{(2\pi)^4} = A_{d_{\mathcal{U}}} \theta(P^0) \theta(P^2)(P^2)^{d_{\mathcal{U}} - 2} \frac{d^4 P}{(2\pi)^4} .$$
Since $d_U$ can be any real number, it looks like a phase space for a fractional number of particles.

Till now there have been a lot of investigations about unparticles in every respect \[2, 3\]. Among them is the so called ungravity \[4, 5, 6\]. Ungravity is induced by a traceless tensor unparticle $O_{ \mu \nu }$ with the interaction

\[ \kappa_s \frac{1}{\Lambda_U} \sqrt{g} T^{ \mu \nu } O_{ \mu \nu }, \]  

where $\kappa_s = \Lambda_U^{-1} (\Lambda_U / M_U)^{d_U V}$. The most important result of ungravity is the power law correction to the Newtonian gravitational potential, of type $\sim (1/r)^{2d_U - 1}$. This type of power law correction reminds one of the extra dimensional scenarios \[4, 8\]. Typically for extra $N$ dimensions, the Newtonian gravity potential gets corrections $\sim (1/r)^{N+1} \[7, 9, 10, 11\]$. In fact, there are much stronger motivations to relate unparticles to extra dimensions. As already pointed out in \[2\], the unparticle and the Kaluza-Klein (KK) states of extra dimensions share analogous phase space integrals. The integral over mass spectrum of KK states behaves as $\left( m^2 \right)^{N/2 - 1} dm^2$. Comparing with Eq. (2), one has $d_U = 1 + N/2$, which is consistent with the results from comparing the gravitational potential corrections.

Furthermore, there is a transparent way of realizing unparticles, known as deconstruction \[12\], which looks much like the KK decomposition. In this scheme the unparticle is described by an infinite tower of particles with vanishing masses. A continuous spectrum of unparticles is simulated by a discrete sum over deconstructing states, which comes to an integral in the vanishing mass limit. One way of explicit realization of deconstruction is to use AdS/CFT correspondence \[13\] to build a 5-dimensional field theory. But it is also possible to build flat $4 + N$ dimensional theory for deconstructing unparticles \[14\]. In the framework of \[14\], it can be easily shown that the spectral function shifts as $\rho_U(P^2) \to \rho_U(P^2 - \mu^2)$ when the scale invariance is broken by a new scale $\mu^2$.

In this paper, we try to construct $4 + N$ dimensional theory in anti-de Sitter (AdS) space to realize tensor unparticles by deconstruction. The main idea is that the ungravity effects by tensor unparticles are equivalent to the effects of excited KK modes in $\text{AdS}_{4+N}$. Again one gets a relation between the scaling dimension of the unparticle operator and the number of extra dimensions, $d_U = 1 + N/2$. Since $d_U$ does not have to be integers in principle, it is possible that we would confront with the fractional extra dimensions (FXD)! In other words, if we find signals at the LHC telling that the number of extra dimensions is not an integer,
it would be a strong evidence for unparticles.

In the next section, it is given how to deconstruct ungravity. In sec. III, gravity in AdS$_{4+N}$ is introduced and related to the deconstructed ungravity of the previous section. Section IV contains discussions and conclusions.

II. DECONSTRUCTING UNGRAVITY

Consider first the ungravity due to the tensor unparticle operator $O_{\mu\nu}$:

$$L_{\text{ung}} \equiv \frac{\kappa_s}{\Lambda_{\text{dU}}^{d_U-1}} \sqrt{g} T_{\mu\nu} O_{\mu\nu}.$$  \hfill (6)

The two-point function of $O_{\mu\nu}$ is

$$\langle 0 | O_{\mu\nu}(x) O_{\alpha\beta}^\dagger(0) | 0 \rangle = \int \frac{d^4 P}{(2\pi)^4} e^{-i P \cdot x} \rho_{\mu\nu\alpha\beta}(P^2),$$  \hfill (7)

where the spectral function $\rho_{\mu\nu\alpha\beta}(P^2)$ is given by

$$\rho_{\mu\nu\alpha\beta}(P^2) = A_{d_U} \theta(P^2) \theta(P^2) d_{d_U-2} \Pi_{\mu\nu\alpha\beta}(P).$$  \hfill (8)

The tensor structure of $\rho_{\mu\nu\alpha\beta}$ is encoded in $\Pi_{\mu\nu\alpha\beta}$. On the other hand, the structure of the two-point function is fixed by the scale invariance. In general, one can put

$$\langle 0 | O_{\mu\nu}(x) O_{\alpha\beta}^\dagger(0) | 0 \rangle = c_T \frac{1}{(2\pi)^2} \frac{1}{(x^2)d_{d_U}} \left\{ [I_{\mu\alpha}(x) I_{\nu\beta}(x) + \mu \leftrightarrow \nu] - \frac{1}{2} \delta_{\mu\nu} \delta_{\alpha\beta} \right\}$$

$$= c_T \frac{\Gamma(2 - d_{d_U})}{4(d_{d_U} - 1) \Gamma(d_{d_U} + 2)} \int \frac{d^4 P}{(2\pi)^4} e^{-i P \cdot x} (P^2)^{d_{d_U}-2} T_{\mu\nu\alpha\beta}(P),$$  \hfill (9)

where

$$I_{\mu\nu}(x) \equiv g_{\mu\nu} - \frac{4 x^\mu x^\nu}{x^2}.$$  \hfill (10)

Here the tensor structure is encoded in $T_{\mu\nu\alpha\beta}$ as

$$T_{\mu\nu\alpha\beta}(P) = d_{d_U}(d_{d_U} - 1)(g_{\mu\alpha} g_{\nu\beta} + \mu \leftrightarrow \nu) + \left[ 2 - \frac{d_{d_U}}{2}(d_{d_U} + 1) \right] g_{\mu\nu} g_{\alpha\beta}$$

$$- 2(d_{d_U} - 1)(d_{d_U} - 2) \left( g_{\mu\alpha} \frac{P_{\nu} P_{\beta}}{P^2} + g_{\mu\beta} \frac{P_{\nu} P_{\alpha}}{P^2} + \mu \leftrightarrow \nu \right)$$

$$+ 4(d_{d_U} - 2) \left( g_{\mu\alpha} \frac{P_{\alpha} P_{\beta}}{P^2} + g_{\mu\beta} \frac{P_{\alpha} P_{\nu}}{P^2} \right) + 8(d_{d_U} - 2)(d_{d_U} - 3) \frac{P_{\mu} P_{\nu} P_{\alpha} P_{\beta}}{(P^2)^2}.$$  \hfill (11)

Combining the two expressions one can fix

$$\Pi_{\mu\nu\alpha\beta} = T_{\mu\nu\alpha\beta},$$  \hfill (12)

$$c_T = \frac{A_{d_U}}{\Gamma(2 - d_{d_U})} d_{d_U} (d_{d_U} + 1) \frac{\sin \pi d_{d_U}}{\pi}.$$  \hfill (13)
The propagator of the tensor unparticle is

\[ \Delta_{\mu\nu\alpha\beta}(P) = \int d^4x \ e^{iP\cdot x} \langle 0| T \mathcal{O}_{\mu\nu}(x) \mathcal{O}_{\alpha\beta}^\dagger(0) |0 \rangle \]

\[ = \frac{1}{2\pi} \int dM^2 \frac{i}{P^2 - M^2 + i\epsilon} \rho_{\mu\nu\alpha\beta}(M^2) \]

\[ = \frac{A_{dU}}{2\sin(\pi dU)} (-P^2)^{dU-2} T_{\mu\nu\alpha\beta} . \]  

(14)

Note that the propagator above is different from that of \[4, 5\] in tensor structure. Consequently, the resulting ungravity effect on the modification of Newtonian gravity must be changed.

Now we deconstruct the unparticle operator \( O_{\mu\nu} \) into the infinite tower of states \( |\lambda_n\rangle \) with infinitesimal mass. One can write

\[ O_{\mu\nu} \equiv \sum_n F_n t_{\mu\nu}^{(n)}, \]  

(15)

where

\[ \epsilon_{\mu\nu} = \langle 0| t_{\mu\nu}^{(n)}(0)|\lambda_n\rangle \]  

(16)

is the polarization tensor.

With the deconstruction, the spectral function is given by

\[ \rho_{\mu\nu\alpha\beta} = 2\pi \sum_\lambda \delta(P^2 - p_\lambda^2) F_\lambda^2 P_{\mu\nu\alpha\beta}(p_\lambda), \]  

(17)

where

\[ P_{\mu\nu\sigma\rho} = \frac{1}{2}(P_{\mu\sigma} P_{\nu\rho} + P_{\mu\rho} P_{\nu\sigma} - \alpha P_{\mu\nu} P_{\rho\sigma}), \]  

(18)

and \( P_{\mu\nu}(p) \equiv -\eta_{\mu\nu} + p_\mu p_\nu/p^2 \). For a massive graviton, one has \( \alpha = 2/3 \). The corresponding propagator is

\[ \Delta_{\mu\nu\alpha\beta}(P) = \sum_\lambda \frac{iF_\lambda^2}{P^2 - p_\lambda^2 + i\epsilon} P_{\mu\nu\alpha\beta}(p_\lambda). \]  

(19)

The “decay constant” \( F_\lambda \) is matched as

\[ 2\pi \sum_\lambda \delta(P^2 - p_\lambda^2) F_\lambda^2 P_{\mu\nu\alpha\beta}(p_\lambda) = A_{dU} \theta(P^0) \theta(P^2)(P^2)^{dU-2} \Pi_{\mu\nu\alpha\beta}(P). \]  

(20)

The ungravity Lagrangian \( \mathcal{L}_{\text{ung}} \) is now

\[ \mathcal{L}_{\text{ung}} = \frac{\kappa^*}{\Lambda_{dU}^{dU-1}} \sqrt{g} T_{\mu\nu} \sum_n F_n t_{\mu\nu}^{(n)}. \]  

(21)
The presence of $\mathcal{L}_{\text{unq}}$ modifies the Newtonian gravitational potential through the exchange of unparticles. The amount of modification is

$$V_{ul}(r) = -\frac{m_1 m_2 G}{r} \left( \frac{R_G}{r} \right)^{2 d_{ul} - 2},$$

where

$$R_G = \frac{1}{\pi \Lambda_U} (\kappa_s M_{Pl})^{1/(d_{ul} - 1)} \left[ \frac{2(2 - \alpha)}{\pi} \frac{\Gamma(d_{ul} + 1/2)\Gamma(d_{ul} - 1/2)}{\Gamma(2d_{ul})} \right]^{1/(2d_{ul} - 2)}.$$  (23)

### III. GRAVITY IN AdS$_{4+N}$

The form of Eq. (15) or (21) reminds one of the KK decomposition of higher dimensional gravitions. As a concrete example, we consider 4 + $N$-dimensional AdS space with the metric (in the Poincare parametrization) [9]

$$ds^2_{4+N} = \frac{L^2}{z^2} (\eta_{\mu \nu} dx^\mu dx^\nu + d\vec{w}_{N-1}^2 + dz^2),$$

After some reparametrizations one can arrive at

$$ds^2_{4+N} = \Omega^2 \left( \eta_{\mu \nu} dx^\mu dx^\nu + \sum_{i=1}^{N} (d\bar{z}^i)^2 \right),$$

where

$$\Omega \equiv \frac{1}{k \sum_j |\bar{z}^j| + 1},$$

$$k \equiv \frac{1}{\sqrt{N}L}.$$  (26)

Here the new coordinates $\bar{z}^j$ are obtained by a rotation such that $z = \sum_{j=1}^{N} \bar{z}^j / \sqrt{N}$. The linearized perturbation $h_{\mu \nu}(x, \bar{z})$ around $\eta_{\mu \nu}$ satisfies the field equation [9]

$$\left[ \frac{1}{2} \Box_4 - \frac{1}{2} \nabla_\bar{z}^2 + V(\bar{z}) \right] \hat{h} = 0,$$  (27)

where $\hat{h} = \Omega^{(N+2)/2}h$ with $\mu \nu$ indices dropped, and

$$V(\bar{z}) = \frac{N(N + 2)(N + 4)}{8} \Omega^2 - \frac{(N + 2)k}{2} \Omega \sum_j \delta(\bar{z}^j).$$

If we decompose $\hat{h}(x, \bar{z}) = e^{i p x} \hat{\psi}(\bar{z})$, the 4D mass $m_\lambda = \sqrt{p^2}$ for the $\lambda$-th level is determined through

$$\left( -\frac{1}{2} \nabla_\bar{z}^2 + V(\bar{z}) \right) \hat{\psi}_\lambda = \frac{1}{2} m_\lambda^2 \hat{\psi}_\lambda.$$  (29)
For a test mass $M$ located at $x = \bar{z} = 0$, the gravitational potential $U(r = |\vec{x}|)$ is

$$\frac{U(r)}{M} = \sum_\lambda G_{N(4+N)} |\hat{\psi}_\lambda(0)|^2 \frac{e^{-m_\lambda r}}{r},$$

$$\sim G_{N(4)} \frac{1}{r} + \sum_{\text{continuum}} G_{N(4+N)} |\hat{\psi}_\lambda(0)|^2 \frac{e^{-m_\lambda r}}{r}, \quad (30)$$

where $G_{N(4)} \sim G_{N(4+N)}/L^N$. The continuum contribution produces the $(4+N)$-dimensional potential $\sim G_{N(4+N)}/r^{1+N}$.

At this stage, one can find a strong similarity between ungravity and the $(4+N)$-dimensional gravity. The action for the $(4+N)$-dimensional gravity is

$$S \sim \int d^{4+N}x \sqrt{g_{4+N}} M_s^{N+2} R_{4+N} \sim \int d^4x \sqrt{g_4} T^{\mu\nu} \int d^N\bar{z} \sqrt{g_N} M_s h_{\mu\nu}, \quad (31)$$

which is the same in form as Eq. (21) because $h(x, \bar{z}) \sim \sum_\lambda e^{ip_\lambda x} \hat{\psi}_\lambda(\bar{z})$. The resulting modification of the Newtonian potential is $\sim 1/r^{1+N}$ in $(4+N)$-dimensional theory while $\sim 1/r^{2d_U-1}$ for ungravity, as given in Eq. (22). Thus one can identify $1+N = 2d_U - 1$, or

$$N = 2(d_U - 1). \quad (32)$$

Naively, we expect (neglecting the overall dimension for the moment)

$$O_{\mu\nu} = \sum_n t^{(n)}_{\mu\nu} F_n \sim \sum_{n\neq 0} h^{(n)}_{\mu\nu}(x) \left[ M_s^N \int d^N\bar{z} \sqrt{g_N} \psi_n(\bar{z}) \right]. \quad (33)$$

The zero mode of KK excitation is excluded since it is just the usual 4-dimensional gravity. The continuum modes of KK states correspond to the deconstructed continuous states of the unparticle. Since $M_s \sim L^{-1}$, the factor of $M_s^N$ in front of the $\bar{z}$-integral plays the role of normalization with the "volume" $L^N$. More specifically, if we consider the spectral function of $O_{\mu\nu}$,

$$\rho_{\mu\nu\alpha\beta}(P) = \int d^4xe^{iP\cdot x} \langle 0 | O_{\mu\nu}(x) O_{\alpha\beta}^\dagger(0) | 0 \rangle$$

$$\sim \int d^4xe^{iP\cdot x} \sum_{n,m} \langle 0 | h^{(n)}_{\mu\nu}(x) \chi_n h^{(m)*}_{\alpha\beta}(0) \chi_m^\dagger | 0 \rangle$$

$$= \int d^4xe^{iP\cdot x} \sum_{n,m} \left[ \sum_\lambda \int \frac{d^Np_\lambda}{(2\pi/L)^N} \right] \langle 0 | h^{(n)}_{\mu\nu}(x) \chi_n | \lambda \rangle \langle \lambda | h^{(m)*}_{\alpha\beta}(0) \chi_m^\dagger | 0 \rangle, \quad (34)$$

where

$$\chi_n \equiv M_s^N \int d^N\bar{z} \Omega^N \psi_n(\bar{z}). \quad (35)$$
Here we have used the periodic boundary condition for $p_\lambda$ with the spatial size $L$. To eliminate the explicit $L$-dependence, we can rescale $\chi_n(z)$ as

$$\chi_n \to \frac{\chi_n}{\sqrt{L^N}}.$$  \hfill (36)

So it is quite natural to write

$$S \sim \int d^4x \sqrt{g_4} T^{\mu\nu} M_*^N \int \frac{d^N\bar{z}}{\sqrt{g_N}} h_{\mu\nu}$$

$$= \int d^4x \sqrt{g_4} T^{\mu\nu} \frac{1}{M_*^{1+N/2}} \sum_n \left[ M_{P1} h_{\mu\nu}^{(n)}(x) \right] \left( \frac{\chi_n}{\sqrt{L^N}} \right). \hfill (37)$$

Now we can define the tensor unparticle operator $O_{\mu\nu}$ from above as

$$O_{\mu\nu} \equiv \sum_n M_{P1} h_{\mu\nu}^{(n)}(x) \left[ M_*^N \int \frac{d^N\bar{z}}{\sqrt{g_N}} \Omega^N \psi_n(z) \right]. \hfill (38)$$

In this prescription, one identifies

$$t_{\mu\nu}^{(n)} \equiv M_{P1} h_{\mu\nu}^{(n)}, \hfill (39)$$

$$F_n \equiv M_*^N \int \frac{d^N\bar{z}}{\sqrt{g_N}} \Omega^N \psi_n(z), \hfill (40)$$

where $F_n$ must satisfy the matching condition of Eq. (20) (see the discussions below).

### IV. DISCUSSIONS AND CONCLUSIONS

If the KK decomposition is to be the unparticle deconstruction, one must check whether the masses of intermediate states are vanishing, and $F_n$ defined in Eq. (40) satisfies the 'decay constant' matching condition of Eq. (20). This can be easily checked by inspecting the Eq. (29). For large $L \gg r$, $k \ll 1$ and the equation becomes

$$(\nabla_{\bar{z}}^2 + m_\lambda^2) \hat{\psi}_\lambda = 0.$$  \hfill (41)

With the periodic boundary conditions at $\bar{z} = 0$ and $\bar{z} = L$, the solution is $\hat{\psi}_\lambda(\bar{z}) \sim \prod_j \sin(\pi n_{\lambda,j} \bar{z}_j/L)$ where the integers $n_{\lambda,j}$ satisfy

$$m_\lambda^2 = \sum_j \frac{\pi^2 n_{\lambda,j}^2}{L^2}. \hfill (42)$$

Hence the masses are $m_\lambda \sim 1/L \to 0$ for large $L$. 

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Also in this limit,
\[ F_n \sim M_n^N \int d^N \bar{z} \frac{\psi_n(\bar{z})}{\sqrt{L_N^N}} \sim (p^2)^{N/4}, \]
where \( p_\lambda = \pi n_\lambda / L \). Hence
\[ \delta(p^2 - p_\lambda^2) F_\lambda^2 \sim (p^2)^{N/2 - 1} = (p^2)^{d_U - 2}, \]
showing the proper scaling behavior. Other coefficients can be adjusted by appropriate normalizations. In short, the vanishing mass spectra and matching conditions guarantee our deconstructing methodology for unparticles.

The keypoint of the equivalence between ungravity and FXD is that the spectral density functions for both cases behave in the same way. For ungravity, the modification of the Newtonian potential results from
\[ V_{\text{u}}(r) \sim \int d^3 \vec{p} \ e^{i \vec{p} \cdot \vec{x}} \Delta_{0000} \sim \int d^3 \vec{p} \ e^{i \vec{p} \cdot \vec{x}} (\vec{p}^2)^{d_U - 2} \sim (\frac{1}{r})^{2d_U - 1}. \]
Here the factor of \((\vec{p}^2)^{d_U - 2}\) originates from the unparticle spectral density \( \rho_{\mu \nu \alpha \beta}(p) \).

For FXD, the modification of the potential comes from the continuum KK excitations
\[ V_{\text{FXD}}(r) \sim \sum_{\lambda} \frac{e^{-r m_\lambda}}{r} \sim \int dm_\lambda (m_\lambda)^{N-1} \frac{e^{-r m_\lambda}}{r} \sim (\frac{1}{r})^{1+ N}, \]
which has the same power of \( V_u \) since \( 1 + N = 2d_U - 1 \). In the integration, the factor of \((m_\lambda)^{N-1}\) is the spectral density of states in \( N \) dimensions \[2,11\]. Note that \( \int dm_\lambda (m_\lambda)^{N-1} \sim \int dm_\lambda^2 (m_\lambda^2)^{N/2 - 1} = \int dm_\lambda^2 (m_\lambda^2)^{d_U - 2} \), ensuring that the spectral density functions are basically the same for ungravity and FXD.

It would be quite interesting to see whether there are other ways of realizing ungravity under different metrics. Or, on top of it, is it always possible to realize other unparticles in the context of FXD in general? Although there are no explicit realizations to date for both case, the answers are positive. The reason is that the phase space integrations over unparticles and KK states are very similar, as shown above. And the deconstruction of unparticles is much like the KK mode decompositon in extra dimensions. Both are sum over infinite tower of states with vanishing mass gap(this is true only for limiting case of extra dimensions), sharing the same type of spectral density function. For example, scalar unparticles can be easily realized within the context of FXD developed in this work. (One has only to ignore the spin structure and indices.) Vector unparticles can also be incorporated with this framework, though the spin structure might be quite different.
One way of realizing unparticles in the context of deconstruction is to use the AdS/CFT correspondence \[12\]. According to the AdS/CFT, for a given conformal theory in 4 dimensions there exists a gravity theory in AdS\(_5\). In this approach, there is no connection between the scaling dimension of the unparticle operator and the number of extra dimensions. Rather, the 5 dimensional mass \(m_5\) of the bulk scalar field is closely related to \(d_U\) via 
\[m_5^2 = d_U(d_U - 4)\]. The scalar unparticle operator is defined by
\[
O_{\Phi}(x) \equiv \lim_{z \to 0} z^{-d_U} \Phi(x, z),
\]
where \(z\) is the fifth coordinate and \(\Phi(x, z)\) is a scalar field living in the 5 dimensions. If both AdS\(_5\) theory and FXD describe the same unparticle physics of 4 dimensions, then there must be some kind of relations between them. But the issue is far beyond the scope of this paper.

In conclusion, we have deconstructed ungravity in terms of extra dimensional theory, realizing tensor unparticles in AdS\(_{4+N}\) for the first time. The main result is that the scaling dimension of unparticles is closely related to the number of extra dimensions, as already claimed in the early literatures. In this context, unparticle physics is equivalent to the FXD theory. Thus it is quite interesting and challenging to explore the LHC probes of extra dimensions to see whether they can be interpreted as unparticles when the number of extra dimensions turns out to be deviated from an integer. And it remains also as future studies to check whether there is a deeper connection between unparticle and FXD (or even with AdS/CFT) at much more fundamental level.

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