TOWARD A COMPLETE ACCOUNTING OF ENERGY AND MOMENTUM FROM STELLAR FEEDBACK IN GALAXY FORMATION SIMULATIONS

Oscar Agertz\(^1\), Andrey V. Kravtsov\(^1,2,3\), Samuel N. Leitner\(^1,2,4\), and Nickolay Y. Gnedin\(^1,2,5\)

\(^1\) Kavli Institute for Cosmological Physics and Enrico Fermi Institute, The University of Chicago, Chicago, IL 60637, USA
\(^2\) Department of Astronomy and Astrophysics, The University of Chicago, Chicago, IL 60637, USA
\(^3\) Enrico Fermi Institute, The University of Chicago, Chicago, IL 60637, USA
\(^4\) Department of Astronomy, University of Maryland, College Park, MD 20742-2412, USA
\(^5\) Particle Astrophysics Center, Fermi National Accelerator Laboratory, Batavia, IL 60510, USA

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ABSTRACT

We investigate the momentum and energy budget of stellar feedback during different stages of stellar evolution, and study its impact on the interstellar medium (ISM) using simulations of local star-forming regions and galactic disks at the resolution affordable in modern cosmological zoom-in simulations. In particular, we present a novel subgrid model for the momentum injection due to radiation pressure and stellar winds from massive stars during early, pre-supernova (pre-SN) evolutionary stages of young star clusters. Early injection of momentum acts to clear out dense gas in star-forming regions, hence limiting star formation. The reduced gas density mitigates radiative losses of thermal feedback energy from subsequent SN explosions. The detailed impact of stellar feedback depends sensitively on the implementation and choice of parameters. Somewhat encouragingly, we find that implementations in which feedback is efficient lead to approximate self-regulation of the global star formation efficiency. We compare simulation results using our feedback implementation to other phenomenological feedback methods, where thermal feedback energy is allowed to dissipate over timescales longer than the formal gas cooling time. We find that simulations with maximal momentum injection suppress star formation to a similar degree as is found in simulations adopting adiabatic thermal feedback. However, different feedback schemes are found to produce significant differences in the density and thermodynamic structure of the ISM, and are hence expected to have a qualitatively different impact on galaxy evolution.

Key words: galaxies: evolution – galaxies: ISM – ISM: structure – methods: numerical – stars: formation

Online-only material: color figures

1. INTRODUCTION

Galaxy formation remains one of the most important, unsolved problems in modern astrophysics. In large part, this is because galaxy evolution depends on small-scale star formation and feedback processes, which are still poorly understood. For instance, it is now well established that stellar feedback from young massive stars can significantly affect the interstellar medium (ISM) by regulating star formation (Mac Low & Klessen 2004; McKee & Ostriker 2007, and references therein), driving turbulence (Kim et al. 2001; de Avillez & Breitschwerdt 2004; Joint & Mac Low 2006; Agertz et al. 2009a; Tamburro et al. 2009), and generating galactic scale outflows (Martin 1999, 2005; Oppenheimer & Davé 2006).

One of the most salient and longstanding problems of galaxy formation modeling is the overprediction of baryon masses and concentrations of galaxies compared to expected values derived using dark matter halo abundance matching (Conroy & Wechsler 2009; Guo et al. 2010), satellite kinematics (Klypin & Prada 2009; More et al. 2011), and weak lensing (Mandelbaum et al. 2006); see Behroozi et al. (2010) for a comprehensive discussion. It is widely thought that the low baryon masses of galaxies are due to galactic winds driven by stellar feedback at the faint end of the stellar mass function (Dekel & Silk 1986; Efstathiou 2000) and by the active galactic nuclei and the bright end (Silk & Rees 1998; Benson et al. 2003).

Pioneering numerical galaxy formation studies by Katz (1992), Navarro & White (1993), and Katz et al. (1996) demonstrated that thermal energy from SNe inefficiently coupled to the simulated ISM due to efficient radiative cooling in dense star-forming regions. To avoid such radiative losses, a delay of gas cooling in regions of recent star formation is often adopted (Gerritsen 1997; Thacker & Couchman 2000, 2001; Stinson et al. 2006; Governato et al. 2007; Agertz et al. 2009b, 2011; Colín et al. 2010; Guedes et al. 2011; Scannapieco et al. 2012), which is justified by the fact that the multiphase structure of star-forming regions with pockets of low-density hot gas is not resolved. While excessive radiative losses may indeed be partially due to resolution effects, one may also argue that such losses should increase with increasing resolution, as star-forming regions can collapse to higher densities at higher resolution. Indeed, simulations of supernova (SN) driven blast waves with sub-parsec resolution show that most of thermal energy of hot gas is radiated away during the blast wave expansion (Thornton et al. 1998; Cho & Kang 2008). It is thus necessary to consider other mechanisms of stellar feedback in addition to the energy-driven expansion of SN bubbles.

Observations of the giant molecular clouds (GMCs) hosting young star clusters indicate that gas is often dispersed well before the first SNe explode ($t \lesssim 4 \text{ Myr}$). This can be partly due to the fact that natal GMCs are not gravitationally bound (Mac Low & Klessen 2004; Padoan et al. 1997; Li et al. 2004; Kritsuk et al. 2007; Dobbs et al. 2011). However, there is also plenty of evidence that dense star-forming regions are destroyed by their H II regions, driven by ionization at low cluster masses (Walch et al. 2012) and radiation pressure, i.e., momentum transfer from radiation emitted by young massive stars to gas and dust (Murray et al. 2005), and stellar winds at high (Matzner 2002; Krumholz & Matzner 2009a). In the Milky Way (MW), the majority of star formation is concentrated in a few hundred
massive GMCs ($M_\text{cl} \sim 10^6 M_\odot$) containing the majority of the Galaxy’s molecular gas reservoir (Murray 2011). Fall et al. (2010a) presented arguments favoring radiation pressure as a gas removal mechanism to explain the similarity in the shapes of the molecular clump and stellar cluster mass functions. Murray et al. (2010) argued that radiation pressure is the dominant force in driving the expansion of Galactic H\textsc{ii} regions and stellar cluster mass functions. Murray et al. (2010) argued that radiation pressure is the dominant force in driving the expansion of Galactic H\textsc{ii} regions and stellar cluster mass functions. Murray et al. (2010) argued that radiation pressure is the dominant force in driving the expansion of Galactic H\textsc{ii} regions and stellar cluster mass functions. Murray et al. (2010) argued that radiation pressure is the dominant force in driving the expansion of Galactic H\textsc{ii} regions and stellar cluster mass functions. Murray et al. (2010) argued that radiation pressure is the dominant force in driving the expansion of Galactic H\textsc{ii} regions and stellar cluster mass functions.

Multi-wavelength data of the evolved H\textsc{ii} region around the dense R136 star cluster in the 30 Doradus region of LMC showed that 10% of the radiation from a young stellar population indeed carries a large amount of energy and momentum, as illustrated in Figure 1, which shows the specific luminosity and mechanical powers. However, it is not clear how such a scheme relates to the actual processes of early feedback, which are thought to be momentum- rather than energy-driven.

Cosmological zoom-in simulations of individual galaxies adopting a force resolution of $\lesssim 50$–$100$ pc, while reaching $z = 0$, are becoming increasingly common (Agertz et al. 2009b; Governato et al. 2010; Guedes et al. 2011). At such resolution, the largest sites of star formation can be identified in simulations directly, although their internal structure would not be resolved. It is hence crucial to understand how well we can capture the global effect of stellar feedback from star-forming regions taking into account all plausible sources and mechanisms of stellar feedback at the resolution level affordable in modern cosmological simulations.

In this paper we discuss the available energy and momentum budget from stellar winds, SNe, and radiation pressure. The latter is implemented using a novel empirically based subgrid model. Using the AMR code RAMSES (Teyssier 2002), we study the impact of these feedback sources in idealized simulations of star-forming clouds and isolated disk galaxies. The simulations are performed at spatial resolution $\sim 50$–$100$ pc, comparable to that of modern state-of-the-art cosmological simulations. We investigate how the detailed impact of stellar...
feedback depends on the implementation and choice of parameters of feedback schemes. We also compare a "straight-injection" approach, where energy and momentum are deposited directly onto the grid, to widely used phenomenological methods where thermal feedback energy is allowed to dissipate over longer timescales than expected by radiative cooling.

The paper is organized as follows. In Section 2, we discuss the feedback budget from SNe and stellar winds and radiation pressure. Section 3 outlines the numerical implementation of stellar feedback in the AMR code RAMSES. In Section 4, we present idealized cloud and galactic disk simulations, and discuss how the different sources of feedback affect global properties of star formation. We conclude by summarizing our results and conclusions in Section 5. We detail the empirically based subgrid model used to compute momentum due to radiation pressure in Appendix A, and implementation of the second energy variable in Appendix B.

2. STELLAR FEEDBACK AND STAR FORMATION

2.1. Stellar Feedback

Several processes are contributing to stellar feedback, as stars inject energy, momentum, mass, and heavy elements over time via SNII, SNIa, stellar winds from massive stars, radiation pressure, and secular mass loss into surrounding interstellar gas. The feedback terms we aim to quantify in this section are

\[
\begin{align*}
\text{Energy: } & \quad E_{\text{tot}} = E_{\text{SNII}} + E_{\text{SNIa}} + E_{\text{wind}} \\
\text{Momentum: } & \quad P_{\text{tot}} = P_{\text{SNII}} + P_{\text{wind}} + P_{\text{rad}} \\
\text{Mass loss: } & \quad m_{\text{tot}} = m_{\text{SNII}} + m_{\text{SNIa}} + m_{\text{wind}} + m_{\text{loss}} \\
\text{Metals: } & \quad m_{Z,\text{tot}} = m_{Z,\text{SNII}} + m_{Z,\text{SNIa}} + m_{Z,\text{wind}} + m_{Z,\text{loss}}.
\end{align*}
\]

We choose to calculate and include the contribution of all feedback processes at every simulation time step \(\Delta t\) for every star particle formed by our star formation recipe (see Section 2.3). Feedback is thus not done instantaneously but continuously in specific time periods when a given feedback process operates, taking into account the lifetime of stars of different masses in a stellar population. We assume that each star particle formed in our numerical simulations represents an ensemble of stars with a given IMF. For stellar masses \(M \in [0.1-100] M_\odot\), we assume the IMF form of Kroupa (2001),

\[
\Phi(M) = A \begin{cases} 
2 M^{-1.3} & \text{for } 0.1 \leq M < 0.5 M_\odot \\
2 M^{-2.3} & \text{for } 0.5 \leq M < 100 M_\odot.
\end{cases}
\]

where \(A\) normalizes \(\Phi(M)\) such that total mass of stars is equal to the initial mass of a star particle, \(m_{\text{star}}\). Note that the choice of IMF can significantly affect the amount of stellar feedback, especially the total energy and momentum output from massive stars. For example, the IMF of Equation (3) has more than twice as many massive stars exploding as Type II supernovae (assuming SNII mass range of 8–40 \(M_\odot\)) and a Chabrier IMF (Chabrier 2003) three times as many, compared to the more bottom heavy IMF of Kroupa et al. (1993).

\[\Phi(M) \approx 0.3 + 0.7 M\text{ for } M > 0.8.\] For the purpose of stellar feedback, accounting for the low-mass range has a negligible effect for the feedback energy budget presented in this paper; the total number of available SNII events are reduced by only ~6%.

Figure 2. Cumulative momentum from stellar winds from STARBURST99 (black lines) compared to the subgrid approximation in Equation (4) (dashed red line).

(A color version of this figure is available in the online journal.)

2.1.1. Stellar Winds from Massive Stars

Massive stars \((M \geq 5 M_\odot)\) can radiatively drive strong stellar winds from their envelopes during the first 6 Myr of stellar evolution, reaching terminal velocities of 1000–3000 km s\(^{-1}\) (Lamers & Cassinelli 1999). The kinetic energy of these winds is expected to thermalize via shocks. To account for the energy, momentum, mass, and metal injection by such winds, we use calculations done with the STARBURST99 code. We find that the dependence of energy and momentum injection on metallicity can be approximated by a simple function\(^7\) and we use such a functional form in our simulations. Although the fit is approximate, its accuracy is sufficient given the uncertainties in the underlying wind models (see discussion in Leitherer et al. 1992).

Specifically, we approximate the cumulative energy, momentum, and mass injection, in CGS units, for a stellar population of age \(t_\text{w}\) (in Myr), birth mass \(m_{\text{im}}\) (in \(M_\odot\)), and stellar metallicity \(Z_\odot\) (in units of solar metallicity \(Z_\odot = 0.02\)), as

\[
\begin{align*}
E_{\text{wind}} &= m_{\text{im}} \int_0^{t_\text{w}} \left( \frac{Z_\odot}{Z_\odot + 1} \right)^{1/2} \frac{t_\w}{t_\text{w}} \, d t_\w, \\
P_{\text{wind}} &= m_{\text{im}} \int_0^{t_\text{w}} \left( \frac{Z_\odot}{Z_\odot + 1} \right)^{1/2} \frac{t_\w}{t_\text{w}} \, d t_\w, \\
m_{\text{wind}} &= m_{\text{im}} \int_0^{t_\text{w}} \left( \frac{Z_\odot}{Z_\odot + 1} \right)^{1/2} \frac{t_\w}{t_\text{w}} \, d t_\w, \\
m_{Z,\text{wind}} &= m_{Z,\text{wind}} \int_0^{t_\text{w}} \left( \frac{Z_\odot}{Z_\odot + 1} \right)^{1/2} \frac{t_\w}{t_\text{w}} \, d t_\w,
\end{align*}
\]

where \(e_{1,2,3} = [1.9 \times 10^{48} \text{ erg } M_\odot^{-1}]\), \(0.50, 0.38\), \(p_{1,2,3} = [1.8 \times 10^{40} \text{ g cm s}^{-1} M_\odot^{-1}]\), \(0.50, 0.38\), and \(m_{1,2} = (2.4 \times 10^{-2}, 4.6 \times 10^{-4})\). The wind duration is \(t_\text{w} = 6.5 \text{ Myr}\).

In Figure 2, we show an example of how the momentum injection for a \(10^6 M_\odot\) star cluster, calculated using this approximation, compares to the STARBURST99 calculation

\[\Phi(M) \approx 0.3 + 0.7 M\text{ for } M > 0.8.\] For the purpose of stellar feedback, accounting for the low-mass range has a negligible effect for the feedback energy budget presented in this paper; the total number of available SNII events are reduced by only ~6%.

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for different metallicities. The momentum injection agrees quite well for \( Z \gtrsim 0.1 \) \( Z_\odot \), although we do oversimplify the time evolution, especially at early time \( (t_s \lesssim 3 \) Myr). A similar conclusion holds for the wind energy injection and mass loss.

### 2.1.2. Radiation Pressure

The momentum injection rate from radiation can be written as

\[
p_{\text{rad}} = (n_1 + n_2 \tau_{IR}) \frac{L(t)}{c},
\]

where \( \tau_{IR} \) is the infrared optical depth and \( L(t) \) is the luminosity of the stellar population. The first term describes the direct radiation absorption/scattering, and should in principle be \( \propto [1 - \exp(-\tau_{UV})] \). However, given the very large dust and \( \text{H}_2 \) opacities in the UV present in dense star-forming regions, \( n_1 \approx 1 \). The second term describes momentum transferred by infrared photons re-radiated by dust particles, and scattered multiple times by dust grains before they escape, where \( n_2 \) is added to scale the fiducial value of \( \tau_{IR} \). As discussed in Appendix A.1, we adopt a fiducial value of \( n_1 = n_2 = 2 \) to account for effects of grid smearing of injected momentum, the value based on results of numerical tests of momentum injection on a grid cell scale in a cloud of gas moving translationally with respect to the computational grid.

A simple, but crude, approach to account for radiation pressure feedback would be to assume that each star particle of mass \( m_e \) is a single star cluster with luminosity \( L(t) = L_1(t)m_\text{star} \), where the specific luminosity \( L_1(t) \) is shown in Figure 1, and that the infrared optical depth \( \tau_{IR} \) is a constant on the order of \( \approx 1-10 \). The total momentum injected into the ISM at every time step is then simply \( p_{\text{rad}} = \rho \Delta \). However, this oversimplifies the impact of radiation pressure, as the effect is not expected to be of uniform strength in star clusters of different masses (e.g., Krumholz & Matzner 2009b). This fact is illustrated in Figure 3 where we estimate \( \tau_{IR} = \kappa_{IR} \Sigma_{cl} \) using observational data for cluster/clump masses and radii, assuming \( \kappa_{IR} \approx 3 \text{ cm}^2 \text{ g}^{-1}(T_d/100 \text{ K})^2 \) (Semenov et al. 2003) at solar dust-to-gas ratios (for dust temperatures of \( T_d \gtrsim 200 \text{ K} \) and \( \kappa_{IR} \gtrsim 5 \text{ cm}^2 \text{ g}^{-1} \)). Although the scatter is significant, this rough estimate illustrates that very large values of the infrared optical depth are plausible in massive star clusters; e.g., the observed densities of the star clusters in M82 allows for \( \tau_{IR} \approx 10-100 \). In less massive star clusters (\( M_{cl} \approx 10^2-10^3 \)), \( \tau_{IR} \) is of order unity and photoionization is the dominant source of radiative feedback (see, e.g., recent numerical work by Walch et al. 2012), although radiation pressure may be an important source of momentum, even the single scattering (\( \tau_{IR} = 0 \)) regime (e.g., Murray et al. 2011; Wise et al. 2012). Note that these estimates assume a homogeneous and static distribution of dense gas around the young star clusters, and the effective values of \( \tau_{IR} \) around young clusters are quite uncertain (e.g., Hopkins et al. 2011; Kuiper et al. 2012; Krumholz & Thompson 2012).

In our fiducial simulations we use a subgrid model of radiation pressure, based on conservative empirical estimates of \( \tau_{IR} \), as discussed above. This approach differs from recent work by Hopkins et al. (2011) where attempt is made to calculate the optical depth directly from the density structure of the numerical simulations. The resolution of our simulations is matched to the typical resolution of modern state-of-the-art cosmological simulations and at such resolution the density field on the scale of star clusters is not resolved. As the injected radiation momentum depends nonlinearly on the mass of the star cluster, a recipe that assumes that the star particle mass \( m_e \) equals the star cluster mass \( M_{cl} \) will ultimately be sensitive to the choice of this particle mass and how the star formation recipe samples the underlying star formation law (see Section 2.3). For the purpose of radiation pressure feedback, we here define \( m_e \) to be the total mass of star particles in a computational cell that has formed within a timescale \( t_{\text{young}} \), typically chosen to be several Myr. We adopt \( t_{\text{young}} = 6 \) Myr, as this allows us to sample stars for twice the assumed timescale of embedded star formation; see below. Adopting \( t_{\text{young}} \) in the range 3–10 Myr does not change the results significantly.

The mass of young stars in a computational cell is the basis of the subsequent calculations within our subgrid model. In essence, this population is assumed to consist of an ensemble of star clusters embedded in an ensemble of natal molecular clumps. Via the time evolution of the bolometric luminosity of each star cluster, calculated using STARBURST99, we obtain the momentum injection rate \( p_{\text{rad}} \) as the integral over each molecular clump. By adopting a cluster/clump mass–size relation and mass function compatible with observations, we then compute the total momentum injection rate \( p_{\text{rad}} \) as the integral over all star cluster masses represented by the local young stars at each simulation time step. The star clusters are assumed to be embedded for a time \( t_{cl} \) (assumed to be 3 Myr throughout this work), during which infrared trapping of photons is assumed to be efficient, and the radiation momentum injection is enhanced by a factor of \( \tau_{IR} \), as discussed above. As \( t > t_{cl} \), the natal star-forming clumps are assumed to be disrupted and only direct radiation pressure, i.e., \( \dot{p} = L/c \), is assumed to operate. The full description of the subgrid model, and the adopted fiducial parameters, is presented in Appendix A.
2.1.3. Supernovae Type II

We calculate the time at which a star of mass $M$ ends its H- and He-burning phases, and leaves the main sequence, using the stellar age–mass–metallicity fit given by Equation (3) in Raiteri et al. (1996). By inverting this equation, we obtain the stellar masses exiting the main sequence at a given age and metallicity. At each simulation time step, $\Delta t$, we calculate the stellar masses $M_\ast$ and $M_\ast + M_{\text{end}}$ that bracket the stellar masses exiting the main sequence over the current $\Delta t$. If the masses are in range of 8–40 $M_\odot$, we assume they undergo core-collapse and end up as SNII events. The number of SNII events is hence given by

$$N_{\text{SNII}} = \int_{M_\ast}^{M_{\ast + M_{\text{end}}}} \Phi(M) dM. \quad (6)$$

Initially, the SNII explosion energy is in the form of kinetic energy, with a typical average value of $E_{\text{SNII}} = 10^{51}$ erg, which is thermalized via shocks. The total thermal energy injected by SNII is thus

$$E_{\text{SNII}} = N_{\text{SNII}} E_{\text{SNII}}. \quad (7)$$

The SNII ejecta also carry momentum initial momentum, which should be accounted for explicitly. At each time step, we inject

$$p_{\text{SNII}} = N_{\text{SNII}} m_{ej} v_{ej}, \quad (8)$$

where $m_{ej}$ is defined in Equation (9) below, and $v_{ej}$ is chosen to generate $p_{\text{SNII}} = 3.6 \times 10^5 M_\odot$ km s$^{-1}$ per SNII event. The amount of energy and momentum injected over $\approx 40$ Myr, as computed above, are in good agreement with the total momentum and energy injection computed using the STARBURST99 code.

Following Raiteri et al. (1996), we adopt the following fits to the results of Woosley & Weaver (1995) for the total ejected mass ($m_{ej}$), as well as the ejected mass in iron and oxygen ($m_{Fe}$ and $m_{O}$), as a function of stellar mass $M$ (in $M_\odot$):

$$m_{ej} = 0.77 M^{1.06} \quad (9)$$

$$m_{Fe} = 2.8 \times 10^{-4} M^{1.86} \quad (9)$$

$$m_{O} = 4.6 \times 10^{-4} M^{2.72} \quad (9)$$

The total and enriched amount of ejecta released at a given time step becomes

$$m_{\Delta t | ej, Fe, O} = \int_{M_\ast}^{M_{\ast + M_{\text{end}}}} m_{ej, Fe, O} \Phi(M) dM. \quad (10)$$

In the RAMSES implementation, we do not track separate variables of metal species, but simply one averaged metal density variable. The total mass of metals returned to the ISM, accounting for the pre-existing metallicity $Z_\ast$ of the stellar population, is

$$m_{Z, \text{SNII}} = (m_{Fe} + m_{O})(1 - Z_\ast) + m_{ej} Z_\ast. \quad (11)$$

After each feedback step the ejecta and metal mass is returned to the ISM, and the star particle mass is updated accordingly. A more sophisticated numerical treatment of chemical enrichment must ultimately include contributions from all relevant species, e.g., C, N, Ne, Mg, Si, Ca, and S (see, e.g., Wiersma et al. 2009; Few et al. 2012), which we leave for a future investigation. Note that oxygen dominates the ejected heavy elements by mass.

For the IMF given in Equation (3), a stellar population of birth mass $m_{\ast, \text{ini}} = 10^4 M_\odot$ and $Z = Z_\odot$ produces $\sim 101.4$ SNII, ejects $m_{ej} \sim 899.4 M_\odot$ of material and expels $m_{Z, \text{SNII}} \sim 143.8 M_\odot$ of metals into the ISM (of which newly produced iron and oxygen accounts for $\sim 128.4 M_\odot$).

In addition to the SNII feedback budget discussed above, which can be regarded as initial injections of energy and momentum into the ISM, late time evolution of SN remnants can in principle inject significantly more momentum. During the first $\sim 10–100$ yr after an SNII explosion, when SN ejecta move ballistically, the adiabatic Sedov–Taylor (S-T) stage sets in (e.g., Ostriker & McKee 1988), as the swept up interstellar material greatly exceeds the ejecta. The shock velocity is high, leading to an approximately adiabatic, energy conserving evolution. After $\sim 10^5$ yr, the shock wave slows down sufficiently for the cooling time of post-shock gas to be of the order of or less than the age of the remnant, and an adiabatic assumption is no longer valid. Blondin et al. (1998) calculated the transition time at which the cooling time equals the age of the remnant ($t_{\text{cool}} = t_{\text{SN}}$) to be $\approx 2.9 \times 10^4 E_{51}^{-2/17} n_{0}^{-9/17}$ yr, where $n_0$ is the ambient density and $E_{51}$ is the thermal energy in units of $10^{51}$ erg. At this time, the momentum of the expanding shell is approximately

$$p_{\text{ST}} = M_{ST} v_{\text{ST}} \approx 2.6 \times 10^5 E_{51}^{16/17} n_{0}^{-2/17} M_\odot \text{ km s}^{-1}. \quad (12)$$

Note that $p_{\text{ST}}$ depends very weakly on the surrounding gas density and linearly on $E_{51}$ and may hence be $\sim 5–15$ times greater than the initial ejecta momentum $p_{\text{SNII}}$ in the density range $n = 100–0.01$ cm$^{-3}$. We regard $p_{\text{ST}}$ as an upper limit to what a single SN explosion can generate, as a substantial portion of the energy is lost in shocks (see Section 2.2.1), and the classical S-T solution assumption of a perfectly intact thin shell expanding into a homogeneous medium is almost certainly a simplification. If stellar winds and radiation pressure are sufficiently effective in expelling gas from young star clusters during the first 3–4 Myr, hot gas may simply escape the natal cloud via the cleared channels. A spherical model for blast-wave evolution is clearly incorrect in such cases. Keeping this in mind, a scenario of maximally efficient S-T momentum generation can be modeled by replacing our fiducial choice $p_{\text{SNII}}$ by $p_{\text{ST}}$ (e.g., as is done by Shetty & Ostriker 2012).

2.1.4. Type Ia Supernovae

Following Raiteri et al. (1996), we assume that progenitors of SNIa are carbon plus oxygen white dwarfs that accrete mass from their binary companions. Stellar evolution theory predicts that the binary masses that can give rise to white dwarfs exceeding the Chandrasekhar limit to be in the range of $\sim 3–16 M_\odot$. The number of SNIa events within a star particle, at a given simulation time with an associated time step $\Delta t$, is then

$$N_{\text{SNIa}} = \int_{M_\ast}^{M_{\ast + M_{\text{end}}}} \Phi(M_2) dM_2, \quad (13)$$

where $\Phi(M_2)$ is the IMF of the secondary star (Greggio & Renzini 1983; Raiteri et al. 1996).

$$\Phi(M_2) = A' \int_{M_{\text{inf}}}^{M_{\text{sup}}} \left( \frac{M_2}{M_B} \right)^2 M_B^{-2.3} dM_B, \quad (14)$$

where $M_B$ is the mass of the binary, $M_{\text{inf}} = \max(2M_2, 3 M_\odot)$, and $M_{\text{sup}} = M_2 + 8 M_\odot$. The normalization parameter is set to $A' = 0.24$ A (see Equation (3)). Each explosion is assumed
to release $\bar{E}_{\text{SNII}} = 10^{51}$ erg as thermal energy, hence injecting a total of $E_{\text{SNII}} = N_{\text{SNII}} \bar{E}_{\text{SNII}}$ at each time step. We assume each SNII to be at the Chandrasekhar limit ($M_{\text{ch}} = 1.38 M_\odot$), and that this is the ejected mass upon explosion leading to $m_{\text{SNII}} = N_{\text{SNII}} M_{\text{ch}}$.

We allow each SNII event to produce 0.76 $M_\odot$ of metal-enriched material (0.13 $M_\odot$ of $^{16}$O and 0.63 $M_\odot$ of $^{56}$Fe; Thielemann et al. 1986). Note that we explicitly account for late-time mass loss of low-mass stars ($M < 8 M_\odot$) until the point they exit the main sequence; see Section 2.1.5.

For the assumed value of $A$, approximately 15% of all SNe are of Type Ia over the lifespan of a 1 $M_\odot$ star (10 Gyr). This is compatible with the notion that 10%–20% of the SN rates in galaxies with ongoing star formation, such as late-type spirals (Sbc-Sd), are due to Type Ia events (van den Bergh & McClure 1994).

2.1.5. Stellar Mass Loss by Low-mass Stars

Although low-mass stars ($M \lesssim 8 M_\odot$) contribute a negligible amount to the total momentum and energy budget, they shed a considerable amount of mass during the asymptotic giant branch phase of their evolution (e.g., Hurley et al. 2000). Kalari et al. (2008) provides relation between the initial stellar mass and final mass of the remnant in the relevant mass range:

$$M_{\text{final}} = (0.109 \pm 0.007)M_{\text{initial}} + 0.394 \pm 0.025 M_\odot.$$  

Using the average values, the fraction of mass lost from a star during its lifetime is

$$f_{\text{loss}}(M_{\text{initial}}) = 0.891 - 0.394/M_{\text{initial}}.$$  

Given a star particle of birth mass $m_{\text{star}}$ and age $t$, we calculate, at each time step $\Delta t$, the expelled stellar mass$^8$ as

$$m_{\text{loss}} = \int_{M_{\text{star}}}^{M_{\text{final}}} M f_{\text{loss}}(M) \Phi(M) dM.$$  

The lost stellar mass is added to the gas mass in the corresponding cell. The gas metallicity is also updated to take into account metals added as part of the stellar material, $m_{\text{Z,loss}} = Z_* m_{\text{loss}}$. The mass loss is accompanied by momentum transfer corresponding to the relative velocity of the star relative to the gas in surrounding cells. No additional momentum is transferred, i.e., we assume that the mass loss “quiescent.” For the IMF in Equation (3), a stellar population loses $\sim 25\%$ of its mass from stars in the mass range 0.5–8 $M_\odot$ during 10 Gyr of evolution.

2.2. Feedback Budget Comparison

In Section 1, we stated that radiation pressure, stellar winds, and SNe have roughly the same momentum injection rate $\dot{p}$. This is shown explicitly in the left-hand panel of Figure 4, where we plot the time evolution of the integrated specific momentum injected into the gas by a single star particle of mass $m_{\text{star}}$ born at $t = 0$, i.e., $\int p(t)/m_{\text{star}} dt$, due to radiation, SNe, and stellar winds for $Z = 0.01 Z_\odot$ and $Z = 1 Z_\odot$ calculated using formulae described above. Note that stellar winds and radiation pressure inject momentum into the ISM immediately after star cluster birth, while SNIIe inject momentum during $t \sim 4.6$–38 Myr. The cumulative contribution of stellar winds alone dominates over SNIIe in the first $\sim 14$ Myr (6 Myr) for $Z = 1 Z_\odot$ (0.01 $Z_\odot$).

In the low-metallicity case, $\sim 5$ times less momentum is injected.
via winds into the ISM. As we parameterize the energy release in a similar fashion, the same trends are found for the shocked wind and SN energy shown in the right-hand panel of Figure 4.

At solar metallicity, the dominant source of momentum is radiation pressure, reaching the equivalent total specific SN momentum after only 3 Myr (see Equation (A13)), assuming a stellar population of mass $m_* = 10^5 M_\odot$. The result weakens by a factor of $\sim 2$ for $Z = 0.01 Z_\odot$ as infrared trapping becomes negligible (note that we only assume photon trapping for $t \leq t_d = 3$ Myr during which cluster stars are assumed to be fully embedded in their natal gas clumps). The nonlinear behavior of the strength of radiation pressure with the mass of the stellar population is evident, as shown by comparing results for $m_* = 10^5 M_\odot$ and $10^6 M_\odot$, where our model (via Equation (A13)) gives us $\tau_{fr} \approx 26$ for the latter. This illustrates how radiation pressure can be an important, and even dominant, source of feedback in dense gas associated with young massive star clusters, as it operates at early times before the first SNII explode.

Recall that the wind and SN momenta in Figure 4 refer to the initial ejecta momentum and not any late stage momentum generated by an expanding bubble. The momentum expected from the ideal adiabatic S-T phase (Equation (12)) is greater than that generated by an expanding bubble. The momentum expected from the initial SNII ejecta can be illustrated as follows. Following Sutherland & Dopita (1993), we define the cooling timescale as $t_{cool} \equiv \frac{U}{\Lambda_{net}}$, where the thermal energy density $U = 3nkT/2$ and the net cooling function $\Lambda_{net} = n_e n_i \Lambda_N$. Here, $n = n_e + n_i$, where $n_e$ and $n_i$ are the number density of electrons and ions, respectively, and $\Lambda_N$ is the normalized cooling rate in units of erg s$^{-1}$ cm$^{-3}$.

In a fully ionized primordial plasma at $T = 10^6$ K, the normalized cooling rate is $\log(\Lambda_N) \approx -23.2$ and $\approx -22$ for gas at $Z = 1 Z_\odot$; see Figure 5 where we plot the cooling function used by RAMSES in the absence of a UV background. In the latter case, the cooling time is

$$t_{cool} \approx 10^3 \left( \frac{100 \text{ cm}^{-3}}{n_h} \right) \text{yr}, \hspace{1cm} (18)$$

and roughly 20 times greater for a pristine plasma. Clearly, the cooling time is very short at average densities relevant for GMCs, and hot gas is quickly radiated away, unless a strong local heating source can maintain it.

The criterion for heating to dominate over cooling can be written as

$$n_h^2 \Lambda_N \leq \rho_s \tilde{\Gamma}, \hspace{1cm} (19)$$

where $\rho_s$ is the mass density of stars and $\tilde{\Gamma}$ is the specific heating rate of the gas in units of erg s$^{-1}$ M$^{-1}$  $\odot$, $\Lambda_N$ for at least $m_*/m_{gas} \sim 10$ is required for heating to overcome cooling. As argued by Ceverino & Klypin (2009), gas cooling rates drop by orders of magnitude at lower gas temperatures, making it possible for thermal feedback to maintain greater pressure gradients between dense star-forming regions and the ambient ISM. This leads to expansion of the star-forming region that lowers the average density, eventually bringing the medium into a regime where heating can overcome cooling.

The density where the cooling rate $\Lambda$ is in units of erg s$^{-1}$ cm$^{-3}$ and the specific heating rate $\tilde{\Gamma}$ is in units of erg s$^{-1}$ M$^{-1}$  $\odot$. In a cell of size $\Delta x = 40$ pc, a stellar-to-gas fraction of at least $m_*/m_{gas} \sim 10$ is required for heating to overcome cooling (at $T \sim 10^4$ K), which is unachievable via star formation alone unless at least 90% of the original cell mass was converted into stars. This is an order of magnitude greater than what is observed in massive GMCs (Evans et al. 2009; Murray et al. 2011). As argued by Ceverino & Klypin, gas cooling rates drop by orders of magnitude at lower gas temperatures, making it possible for thermal feedback to maintain greater pressure gradients between dense star-forming regions and the ambient ISM. This leads to expansion of the star-forming region that lowers the average density, eventually bringing the medium into a regime where heating can overcome cooling.

The estimates made above are subject to many caveats. While relevant to understanding the fate of thermal energy injected into gas in galaxy formation simulations, the real ISM is multiphase and highly inhomogeneous on the scale of the resolution elements of such simulations. This means that pockets of tenous hot gas may exist within dense gas in a simulation cell, but it also means that estimates of the cooling time are optimistic as they need to include a clumping factor that is expected to be significant in star-forming regions. However, it is unclear how efficiently thermal energy should couple to the ISM in realistic settings; Cho & Kang (2008) demonstrated, using high-resolution simulations of SN explosion in pre-existing wind-blown bubbles, that less than $\sim 10%$ of the shocked thermal energy could be converted into kinetic energy, as the rest is lost in radiative shocks within the bubble.

Keeping these issues in mind, the effect of gas clearing due to pre-SNII momentum feedback may in many situations enhance the effect of feedback, which is one of the main motivations of this work.
2.3. The Star Formation Recipe

In this work, we employ a fairly standard prescription of star formation based on the SFR given by

\[ \dot{\rho}_* = \frac{\rho_g}{t_{SF}} \quad \text{for} \quad \rho > \rho_*, \]  

(21)

where \( \rho_g \) is the gas density, \( \rho_* \) is the threshold of star formation, and \( t_{SF} \) is the star formation, or equivalently gas depletion, time. Observations indicate that in the local universe \( t_{SF} \sim 2 \text{ Gyr} \) (Bigiel et al. 2011), which is a manifestation of the fact that observed galaxies convert their gas into stars quite inefficiently.

In this work we assume that \( t_{SF} = t_{ff}/\epsilon_{ff} \), where \( t_{ff} = \sqrt{3\pi/32G\rho} \) is the local gas free-fall time and \( \epsilon_{ff} \) is the star formation efficiency per free-fall time. With this assumption Equation (21) enforces \( \dot{\rho}_* \propto \rho^{1.5} \), which is close to the observed projected density relation \( \Sigma_{\text{gas}} \sim \Sigma_{\text{SFR}}^{n} \), where \( n \sim 1.4 \) (Kennicutt 1998). As noted above, the efficiency of star formation is globally observed to be low, \( \epsilon_{ff} \sim 1\% \) (Krumholz & Tan 2007), and we discuss our adopted values of \( \epsilon_{ff} \) in Section 4.

For now, we would like to note that the efficiency of star formation per free fall is usually kept fixed in galaxy formation simulations. However, it is likely that this is not the case in observations. In fact, there is ample observational and theoretical evidence for \( \epsilon_{ff} \) to depend on scale and environment (e.g., Murray et al. 2011; Padoan et al. 2012), which will manifest as stochasticity of star formation efficiency. Such stochasticity can potentially have a strong impact on feedback, because it implies that \( \epsilon_{ff} \) can be high in some regions and low in others. The overall star formation would thus be concentrated in fewer star-forming sites that have high star formation efficiency, even as the global star formation efficiency averaged over a large patch of ISM is low. We leave an investigation of the effects of such stochastic efficiency on the effects of feedback for future work (O. Agertz et al., in preparation), and note that this caveat should be kept in mind when interpreting the numerical result presented below.

Recent work by Gnedin et al. (2009) and Gnedin & Kravtsov (2011) relate star formation to molecular gas, hence \( \rho_g \rightarrow \rho_{H_2} \) in Equation (21), which can explain why metal/dust poor galaxies at \( z \sim 3 \), which physically should be more prone to \( \text{H}_2 \) destruction via UV dissociation, show deviations from the \( z = 0 \) Kennicutt–Schmidt (KS) relation (Gnedin & Kravtsov 2010). Gnedin & Kravtsov (2011) demonstrated that the density at which molecular fraction reaches 50\% can be approximated as

\[ n_\text{a} \approx 25(Z/Z_\odot)^{-1} \text{cm}^{-3}, \]

(22)

which we adopt in all of our simulations as the threshold for star formation. In addition to the density threshold, we also use the temperature threshold by only allowing star formation to occur in cells of \( T < 10^4 \text{ K} \). No other conditions or thresholds are used.

To ensure that the number of star particles formed during the course of a simulation is tractable, we sample Equation (21) stochastically at every fine simulation time step \( \Delta t \). For a cell eligible for star formation, the number of star particles to be formed, \( N \), is determined using a Poisson random process (Rasera & Teyssier 2006; Dubois & Teyssier 2008)

\[ P(N) = \frac{\lambda \rho}{N!} \exp(-\lambda \rho), \]

(23)

where the mean is

\[ \lambda = \left( \frac{\rho \Delta x^3}{m_*} \right) \Delta t. \]

(24)

Here, \( \rho_* \) is the adopted SFR (Equation (21)), and \( m_{*\text{min}} \) is the chosen unit mass of star particles. In this work we adopt \( m_{*\text{min}} = \eta \rho_* \Delta x_{\text{max}}^3 \), where \( \eta = 0.1 \), and \( \rho_* \) is taken from Equation (22) at solar metallicity. This yields \( m_{*\text{min}} \sim 10^4 M_\odot \) for a typical resolution of \( \Delta x_{\text{max}} = 50 \text{ pc} \). When the Poisson process produces \( N > 1 \) star particles in a cell at a single star formation event, we bin these into one stellar particle of mass \( N m_* \).

3. NUMERICAL IMPLEMENTATION OF FEEDBACK

The efficiency of stellar feedback depends not only on its magnitude, but also on specifics of implementation in a given numerical code (see, e.g., Scannapieco et al. 2012 and references therein). In this work, we are mainly interested in gauging the impact of stellar feedback at the state-of-the-art resolution of modern galaxy formation simulations without resorting to suppression of cooling (Gerritsen 1997; Thacker & Couchman 2000; Stinson et al. 2006; Governato et al. 2007; Agertz et al. 2011) or hydrodynamical decoupling of gas elements (Scannapieco et al. 2006; Oppenheimer & Davé 2006).

We choose to inject energy and momentum directly into computational cells as follows. Over a simulation time step \( \Delta t \), we calculate the thermal energy release \( (E_{\text{hot}}) \), as well as the associated mass of ejecta \( (m_{\text{ej}}) \) and metals \( (m_{\text{Z,hot}}) \). These quantities are deposited in the 27 cells surrounding the star particle, although we have also carried out most of our experiments using nearest grid point approach without significant differences in the final results. \(^9\) We explore two different methods to deposit momentum.

1. Momentum “kicks”. Over a simulation time step \( \Delta t \), the momentum \( p_{\text{hot, total}} = m_{\text{hot}} \Delta t \) is directly deposited isotropically in the 26 cells surrounding the grid cell nearest the star particle.

2. Non-thermal pressure. The momentum injection rate \( \dot{p}_{\text{hot}} \) can be thought of as a non-thermal pressure corresponding to momentum flux through cell surface \( P_{\text{tot}} = p_{\text{hot}}/A \), where the area \( A \) is the surface area of a cell \((A \approx 6\Delta x^2)\), or an arbitrary computational region, containing a young star particle. This pressure is calculated at every time step and is added to the thermal pressure, \( P_{\text{thermal}} \), to give the total pressure \( P_{\text{tot}} = P_{\text{thermal}} + P_{\text{hot}} \) that enters in the Euler equation. We describe this technique in detail in Appendix B.

The first method is qualitatively similar to what was considered by Navarro & White (1993), although these authors compute the momentum corresponding to a fraction of injected SNII energy, while we specifically compute the momentum injection due to various specific processes that generate momentum. The advantage of the first implementation method is its simplicity, as the second method requires minor modifications to the Riemann solver in the case of the MUSCL–Hancock scheme (Toro 1999) adopted by the RAMSES code. On the other hand, the first method does not explicitly affect the cell containing the feedback producing a star particle, which will be evacuated in

\(^9\) One may add further sophistication to this approach by considering supernovae explosions as discrete events, hence only applying \( E_{\text{SNII}} + E_{\text{SNIa}} \) when an integer number of explosions occur during over the time step (see, e.g., Hopkins et al. 2012).
the case of a pressure approach. We adopt the first method as our fiducial choice, but we present results of both implementations in Section 4.

Strong heating and/or momentum deposition in diffuse regions can lead to extremely large temperatures and velocities. To avoid this, we disallow feedback if cell temperature is \( T \geq 5 \times 10^4 \) K and limit momentum feedback to deliver maximum kicks of \( v = 1000 \) km s\(^{-1}\).

The effect of momentum feedback is weakened when star particles occur in neighboring regions, or even computational cells, as momentum cancellations will occur (see, e.g., Socrates et al. 2008). Hopkins et al. (2011) discussed this effect in their SPH simulations (see their Figure A1 and associated text). They maximized the effect of feedback by depositing momentum isotropically from the cloud’s center of mass found by a friends-of-friends (FOF) technique. In addition, momentum was deposited in a probabilistic way that ensured that each affected SPH particle would receive a velocity kick at the local cloud escape velocity. If momentum is added gradually around each stellar particle, akin to our current method, Hopkins et al. found that feedback limited star formation less efficiently (by factor of \( \sim 5 \) in the measured star formation histories (SFHs)). This effect should be kept in mind as an implementation uncertainty.

3.1. Increasing the Impact of Hot Gas by Delayed Cooling

As we demonstrate in Section 4.1, momentum feedback aids in clearing gas out from star-forming regions, and runaway heating can occur in some regions. However, it is still not guaranteed that the evolution of hot gas is accurately captured due to resolution effects (see discussion in Section 2.2.1). In addition to relying solely on early momentum feedback to clear out dense gas, we also consider the two following methods to capture the maximum effect that thermal energy from SNI II may have on their surroundings.

The concept of allowing for an adiabatic feedback phase in galaxy scale simulations has been proposed by several authors (see, e.g., Gerritsen 1997; Stinson et al. 2006), and is widely utilized in the community (Governato et al. 2007; Agertz et al. 2011; Brook et al. 2012). However, the specific implementations assume the duration of this phase to be much longer than the \( 10^4 \) yr expected from analytical arguments. Stinson et al. (2006) proposed a scheme in which SN energy is deposited in a region of size \( R_{SPH} = 10^{1.74} E_5^{0.32} n_0^{-0.16} \rho_0^{-0.20} \) pc, where \( E_5 = 10^{-4} P_0 k_B^{-1} \) and \( P_0 \) and \( n_0 \) are the ambient pressure and density, and cooling is disabled for \( t_{\text{max}} = 10^{8.85} E_5^{0.32} n_0^{-0.34} \rho_0^{-0.70} \) yr. However, the timescale \( t_{\text{max}} \) corresponds to the survival time of the low-density cavity (McKee & Ostriker 1977), not the adiabatic phase of SNe. Furthermore, as SN energy in the Stinson et al. implementation is delivered at every time step, as in the method described in Section 2.1.3, most of the gas in the star-forming region will behave adiabatically for \( \sim 40 \) Myr, assuming a minimum SNII mass of \( 8 M_\odot \). What the appropriate dissipation timescale of thermal energy for multiple SN explosion inside super bubbles should be is under debate (see, e.g., Krause et al. 2013).

Having noted that cooling delay models typically maximize the effect of SNII energy they are meant to mimic, we consider the effects of one such model below in a subset of our simulations, and compare it with results of simulations with no delay of cooling. When a star particle forms, we assign the time variable \( t_{\text{cool}} \) to a scalar in the cell containing the particle. This scalar field is passively advected with the hydro flow. At every time step, the variable is updated as \( t_{\text{cool}}' = t_{\text{cool}} - \Delta t \). For every cell where \( t_{\text{cool}} > 0 \), cooling is disabled. This method approximates the delay of cooling implemented in SPH codes (e.g., Stinson et al. 2006) within the Eulerian hydrodynamics context.\(^{10}\) We explore the effects of delayed cooling using two values of \( t_{\text{cool}} \): 10 and 40 Myr. The latter is, as argued above, the duration of SN feedback for stars \( \geq 8 M_\odot \).

3.2. Feedback Energy Variable

We also investigate a scenario in which some fraction of the feedback energy is evolved as a separate energy variable \( E_{fb} \), which is passively advected with the hydro flow and only couples directly to the hydrodynamic flow as an effective pressure in the Euler equations. We assume that this energy dissipates over a timescale \( t_{\text{dis}} \), which is assumed to be longer than the cooling time \( t_{\text{cool}} \) predicted by the cooling rates in dense star-forming gas; see Equation (18). This approach can be viewed as accounting for the effective pressure from a multiphase medium, where local pockets of hot gas exert work on the surrounding cold phase. Alternatively, it may be viewed as feedback-driven turbulence (Springel 2000), magnetic fields (Wang & Abel 2009), and cosmic rays (e.g., Jubelgas et al. 2008; Ublig et al. 2012). For example, it is known that these components contribute almost equally to the pressure of the gaseous disk of the MW.

In practice, at every time step we assume that a fraction \( f_{fb} \) of the total thermal feedback energy \( E_{tot} \) is added to the feedback energy \( E_{fb} \), and the remaining \((1 - f_{fb})\) enters the thermal energy of the gas. We experiment with \( f_{fb} = 0.1 \) and 0.5, where the lower value is motivated by the radiative SN-driven bubble simulations of Cho & Kang (2008). The pressure associated with \( E_{fb} \) enters into the sound-speed calculation, as well as in the Riemann solver. During the cooling step, dissipation is modeled as \( E_{fb}^{\text{dis}} = E_{fb}^{\prime} \exp(-\Delta t/t_{\text{dis}}) \) in every gas cell. The retention of feedback energy is here rather different than in the delayed cooling method described above; in the latter, the cooling delay operates over fixed time only in the gas present in a star particle’s birth region.

\( E_{fb} \) will only be important in local dense star-forming gas, where most of the thermal energy is radiated away due to high average density. In diffuse regions, the energy budget will be dominated by the surviving thermal energy which dissipates consistently on its proper cooling timescale. We assume the dissipation timescale to be comparable to the decay time of supersonic turbulence, i.e., of order of the flow crossing time (e.g., Ostriker et al. 2001). Massive GMCs typically have sizes of \( \sim 10^2 \) pc and velocity dispersions of \( \sim 10 \) km s\(^{-1}\), leading to a crossing time of \( t_c \sim 1 \) Myr. At the scale of the disk, where the cold gas layer thickness is an order of magnitude thicker, one may argue for \( t_c \sim 10 \) Myr. We hence consider feedback energy dissipation time in the range \( t_{\text{dis}} = 1-10 \) Myr.

Teyssier et al. (2013) recently demonstrated that a feedback scheme employing a separate energy variable, similar to what is described above, is quite efficient and has a significant effect on

\(^{10}\) Note however that we assign the cooling delay time \( t_{\text{cool}} \) to the gas present in the local cell at star particle birth, while Stinson et al. (2006) assign \( t_{\text{cool}} \) to SPH particles available within the blast wave radius \( R_{SPH} \) at every \( \Delta t \) for the duration of the SNII explosions. The gas particles affected by delayed cooling at the end of the SNII phase in the Stinson et al. approach are not necessarily the same particles that were present at birth. Furthermore, our \( t_{\text{cool}} \) variable is allowed to mix, leading to delayed cooling in cells previously not associated with the young star particle’s birth cell.
ALL simulation for a different fraction of gas mass converted into stars. The black solid lines show, from right to left, stellar mass fraction enabling runaway heating, but considerably slower than in the ALL run. Right panel: gas density and temperature evolution in the cell containing a star particle in the simulations. Purely thermal energy feedback (ENERGY run) has almost no effect on gas density and temperature. SN feedback with momentum can clear out the gas, the ejecta momentum for SNIIe. Note that simulations including momentum injection from stellar wind and radiation pressure can clear out enough gas for runaway galaxy.

Figure 6. Left panel: evolution of gas density and temperature in computation cell containing star particles for the ALL (black solid line), MOMENTUM (blue solid line), SNMOM (magenta solid line), ENERGY simulations (red solid line); see Table 1. Dashed lines indicate if Sedov–Taylor momentum was used instead of simply the ejecta momentum for SNIIe. Note that simulations including momentum injection from stellar wind and radiation pressure can clear out enough gas for runaway heating before the first SN explosion occurs (~4.6 Myr). The predicted transition to a runaway heating regime (Equation (20)) agrees well with the results of the simulations. Purely thermal energy feedback (ENERGY run) has almost no effect on gas density and temperature. SN feedback with momentum can clear out the gas, enabling runaway heating, but considerably slower than in the ALL run. Right panel: gas density and temperature evolution in the cell containing a star particle in the ALL simulation for a different fraction of gas mass converted into stars. The black solid lines show, from right to left, stellar mass fraction \( f = 1\%, 5\%, 10\%, \) and 20%. The predicted transition to the regime of runaway heating occurs before the first SN explosion for \( f > 10\% \). Note that runaway heating and efficient clearing of gas occur even when only 1% of gas is turned into stars.

(A color version of this figure is available in the online journal.)

the SFH, and dark matter density profile, of an isolated dwarf galaxy.

4. SIMULATIONS

In this section, we use idealized simulations of gas clouds and isolated galactic disks to gauge the effect of different prescriptions for stellar feedback described in the previous sections on the local and global efficiency of star formation (the \( \Sigma_{\text{SFR}} - \Sigma_{\text{gas}} \) relation), as well as on the structural properties of galactic disks. A more extensive analysis of processes such as outflows and study of feedback implementations in cosmological galaxy formation simulations will be presented in future work (O. Agertz et al., in preparation). The simulations considered here have resolution similar to the resolution of state-of-the-art cosmological simulations, and the results should hence be directly applicable to interpretation of results in cosmological runs. Specifically, we restrict the spatial resolution to reach minimum cell sizes of \( \Delta x \sim 10-100\text{ pc} \).

4.1. Effect of Feedback at the Resolution Scale

It is instructive to first study the impact of feedback in a typical star-forming computational cell. To this end, we place a single star particle of mass \( m_\star \) in a cell of size \( \Delta x = 40\text{ pc} \) within a periodic box of fixed resolution and homogeneous gas density \( \rho_{\text{gas}} = 100m_\text{H} \text{cm}^{-3} \) of solar metallicity. The initial gas temperature matters little, as it settles to a few 10 K after one time step. We adopt a series of (cell) stellar mass fractions \( f_\star = m_\star/m_{\text{tot}} \in \{1, 5, 10, 20\}\% \), consistent with observations of massive GMCs (Evans et al. 2009; Murray et al. 2011). In the case of \( f_\star = 10\% \), \( m_\star = 1.6 \times 10^4 M_\odot \). We are interested in studying the effect of feedback on the scale of individual simulation cells, where it will be applied in actual galaxy simulations. At such scales, the gas self-gravity is weak due to the softening of forces on the scale of a couple of cells, and is hence not calculated properly in the actual simulations. For simplicity, we choose not to include self-gravity in these tests.

This setup is evolved for 30 Myr using the different feedback implementations shown in Table 1. In all tests, we employ momentum deposition via a non-thermal pressure term in the Riemann solver (method 2 in Section 3) rather than via “kicks,” as we want to measure the impact on the central cell containing the star. The infrared optical depths relevant for the above stellar mass fraction, as approximated via Equation (A11), become \( \tau_{\text{IR}} \approx 0.39, 0.49, 0.55, 0.96 \), i.e., at most a factor of two boost compared to the single scattering “\( L/c\)-regime.”

In the left panel of Figure 6, we show the gas density and temperature evolution for the runs with \( f_\star = 10\% \). The evolution strongly depends on the form of feedback employed; while all momentum-based feedback sources can evacuate the
cell, energy-only feedback (ENERGY and SN run) has no effect, illustrating the common overcooling problem. If the initial SN momentum, \( p_{\text{SNi}} \), is included (SNMOM), gas is pushed out of the cell and the heating criterion of Equation (20) is satisfied after \( \sim 20 \) Myr, leading to temperatures \( > 10^4 \) K. When all momentum sources of feedback (MOMENTUM) are included, the final efficiency approaches \( \epsilon_f \sim 10\% \), while in cases of the cloud virial parameter \( \alpha_{\text{vir}} \gtrsim 1 \) and vice versa. Note that we do not attempt to model details of star formation in GMCs, which requires more advanced simulation setups.

Our main goal is simply to gauge systematic differences between different feedback implementations at the resolution level that should be affordable in cosmological simulations in the near future.

4.2. Isolated Cloud

In this idealized test, a spherical cloud of dense cold gas \( (n_{\text{cl}} = 100 \, m_\text{H} \, \text{cm}^{-3} \) and \( T_{\text{cl}} = 10 \) K) of radius \( r_{\text{cl}} = 50 \) pc is placed in pressure equilibrium with a diffuse ambient medium \( (n_{\text{ISM}} = 0.1 \, m_\text{H} \, \text{cm}^{-3} \) and \( T_{\text{ISM}} = 10^4 \) K). Star formation is then allowed to proceed, as described in Section 2.3 with \( \epsilon_f = 10\% \). As we are interested in the behavior of a marginally resolved ISM, we adopt a maximum resolution of \( \Delta x = 10 \) pc.

At this resolution, the cloud consists of 552 cells at exactly \( n_{\text{cl}} = 100 \, H \, \text{cm}^{-3} \), having a total initial gas mass of \( M_{\text{cl}} = 1.25 \times 10^6 \, M_\odot \). In the following tests, we evolve the cloud with and without self-gravity, which in a very crude way can be seen as limiting cases of the cloud virial parameter \( \alpha_{\text{vir}} \); no self-gravity simply means that unresolved turbulence supports the cloud \( (\alpha_{\text{vir}} \gtrsim 1) \) and vice versa. Note that we do not attempt to model details of star formation in GMCs, which requires more advanced simulation setups.

4.2.1. No Self-gravity

In Figure 7, we show evolution of star formation efficiency within the cloud, defined as \( \epsilon_{\text{cl}}(t) = M_\ast(t)/M_{\text{cl,ini}} \), where \( M_\ast(t) \) is the total stellar mass formed at time \( t \) and \( M_{\text{cl,ini}} \) is the initial cloud gas mass, in the simulations without self-gravity.

For \( Z = 1 \, Z_\odot \) without any feedback, the cloud forms stars unhindered until \( \epsilon_{\text{cl}} \sim 0.75 \) when the cell densities fall below the star formation threshold. SN feedback alone can reduce the overall efficiency to \( \epsilon_{\text{cl}} \sim 0.2 \). When no momentum from SNe is accounted for, the efficiency is somewhat larger: \( \epsilon_{\text{cl}} \sim 0.25 \). The same conclusion holds when all thermal energy (and no momentum) sources of stellar feedback are present.

The stellar fractions differ significantly when pre-SN momentum feedback is included. Radiation pressure alone sets \( \epsilon_{\text{cl}} \sim 0.125 \), and the conversion efficiency decreases somewhat when momentum from wind and SN feedback is added. When momentum and energy deposition from all feedback mechanisms is included, the final efficiency approaches \( \epsilon_{\text{cl}} \sim 0.1 \), although with significantly more hot gas present in comparison to other scenarios.

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11 This result is somewhat at odds with the results of Ceverino & Klypin (2009), who found that purely thermal feedback from winds and SNe could effectively overpressurize gas of similar characteristics, leading to gas evacuation. Part of the difference stems from how thermal energy is injected; in our simulations, energy is deposited to the gas at every time step, heating it to \( \sim 10^4 \) K. When the new gas state is passed to the cooling routine, all energy is lost over one time step, bringing the dense gas back to a few \( \sim 10 \) K. Ceverino & Klypin considered thermal feedback via a heating term in the cooling routine, which when balanced against cooling led to a larger equilibrium temperature.
Figure 8. Mass-weighted projected density (top) and temperature (bottom) of the gas in the ALL simulation of a non-gravitating isolated cloud. Each panel is 1.5 kpc across. Pre-SN feedback via stellar winds and radiation pressure pushes the cloud apart, thereby increasing cloud porosity. SNe later explode in a more tenuous medium capable of maintaining greater gas temperatures. Hot gas at temperatures of $T > 10^7$ K (shown in dark red) pushes on the ambient cold, dense phase.

(A color version of this figure is available in the online journal.)

to pure momentum feedback. The hot gas causes vigorous late time expansion of the star-forming region, which is illustrated in the time evolution of the projected density and temperature in Figure 8.

The results are different in the case of low-metallicity gas\textsuperscript{12} shown in the right-hand side of Figure 7. As the gas cooling rates are lowered, a purely energy-based feedback scheme can lower the efficiency of star formation to $\epsilon_{\text{cl}} \sim 0.1$. The effect of radiation pressure is, however, not much different from the simulation adopting $Z = 1 Z_\odot$, despite $\tau_{IR}$ being 100 times smaller ($\kappa_{IR} \propto Z$). This is because $\tau_{IR}$ plays a minor role in both cases, as stellar masses in the local cells are small ($m_\star \lesssim 10^4 M_\odot$).

4.2.2. With Self-gravity

In Figure 9, we show the cloud star formation efficiency for the self-gravitating cloud. We do not enforce hydrostatic equilibrium as the (unresolved) temperature profiles would immediately be erased by cooling. As the cloud now contracts, the global cloud star formation efficiency becomes greater by more than a factor of three in all simulations. However, the systematic trends measured in the non-self-gravitating tests are recovered; pre-SN feedback, and specifically momentum, limits star formation by roughly a factor of two more efficiently than SN feedback.

In this setup, a stronger impact of feedback is found when momentum feedback is generated via a non-thermal pressure in the Riemann solver (method 2 in Section 3). In the left panel of Figure 10, we show the ALL simulation adopting free-fall star formation efficiencies in the range $\epsilon_{\text{ff}} = 0.5\% - 10\%$. The 10\% case is here lower by a factor of two compared to momentum feedback via “kicks.” Even though we vary the star formation efficiency by a factor of 20, the final global conversion stays within $\epsilon_{\text{cl}} \sim 7\% - 25\%$, in agreement with observed GMCs (Evans et al. 2009; Murray et al. 2011), compared to >70\% when feedback is ignored.

\textsuperscript{12} We here adopt a metallicity-independent star formation threshold of $\rho_\star = 25 \text{ cm}^{-3}$ to facilitate a comparison with the $Z = 1 Z_\odot$ case.
model is hence qualitatively in agreement with observed star cluster forming regions in local galaxies, e.g., 30 Doradus, where the stars in the massive compact star cluster are younger than \(~4\) Myr, while the peripheral stars may be as old as \(~30\) Myr (De Marchi et al. 2011); see also the conclusions by Murray et al. (2011) regarding MW GMCs.

It is plausible that effective self-regulation only occurs when simulated star-forming gas clouds are resolved sufficiently for self-gravity to allow for some degree of collapse/contraction. At a cosmological resolution \(\Delta x \sim 100\) pc, such collapse may not occur to the same degree as observed in the experiments here, especially as the gas is pressurized at the scale of resolution to prevent spurious fragmentation (Truelove et al. 1997).

4.3. Disk Galaxy

Following Hernquist (1993) and Springel (2000; see also Springel et al. 2005), we create a particle distribution representing a late-type, star-forming spiral galaxy embedded in a Navarro–Frenk–White dark matter halo (Navarro et al. 1996, 1997). The halo has a concentration parameter \(c = 10\) and virial circular velocity, measured at overdensity \(200\rho_{\text{crit}}\), \(v_{200} = 150\) km s\(^{-1}\), which translates to a halo virial mass \(M_{200} = 1.1 \times 10^{12} M_{\odot}\). The total baryonic disk mass is \(M_{\text{disk}} = 4.5 \times 10^{10} M_{\odot}\) with 20% gas. The bulge-to-disk mass ratio is \(B/D = 0.1\). We assume exponential profiles for the stellar and gaseous components and adopt a disk scale length \(r_d = 3.6\) kpc and scale height \(h = 0.1 r_d\) for both. The bulge mass profile is that of Hernquist (1990) with scale length \(a = 0.1 r_d\).

We initialize the gaseous disk analytically on the AMR grid assuming an exponential profile. The galaxy is embedded in a hot \((T = 10^6\) K), tenuous \((n = 10^{-5} \text{ cm}^{-3})\) gas halo enriched to \(Z = 10^{-2} Z_{\odot}\), while the disk has solar abundance. We conduct all simulations at a maximum AMR cell resolution of \(\Delta x = 70\) pc, typical of current state-of-the-art galaxy formation simulations carried out to \(z = 0\).

We systematically vary the different sources of stellar feedback operating in the simulations, and conduct additional tests which include thermal feedback via phenomenological approaches described in Section 3.1. Table 2 presents details of all simulations considered in the following analysis. All runs adopt the standard star formation prescription outlined in Section 2.3, and we generally adopt a star formation efficiency per free-fall time of \(\epsilon_f = 10\%\). We note that this value of efficiency is an order of magnitude larger than the average values derived globally for kiloparsec patches of gas or in individual clouds (e.g., Krumholz & Tan 2007; Bigiel et al. 2008). However, as we show below, runs with feedback and large free-fall efficiencies produce normalizations of the KS relation quite close to observations (see also Hopkins et al. 2011).

4.3.1. Star Formation Histories

Figure 11 shows the SFHs in disk simulations presented in Table 2. The top-left panel presents the impact of direct feedback injection, i.e., without any phenomenological approach to thermal energy. We find the same trend in SFR as in the isolated cloud test. Simulations that include only thermal energy or SNe have a minor effect on the SFH compared to no feedback, while the inclusion of momentum lowers the SFRs by up to a factor of three. This process is mainly due to early, pre-SN feedback, especially radiation pressure. After a few orbital times all simulations regulate to roughly the same SFRs, although at different gas fractions.

It is instructive to compare our results with the recent work by Hopkins et al. (2011). These authors reported average infrared optical depths of \(\langle \tau_{\text{IR}} \rangle \sim 10–30\) in their simulated MW-like galaxy.\(^{13}\) Our model, on the other hand, predicts more modest...
average values in the range $\langle T_{\text{IR}} \rangle \sim 2$–6. The actual values of $T_{\text{IR}}$ in dense gas surrounding young, embedded star clusters are highly uncertain both because we do not know covering fraction of absorbing dusty gas (see, e.g., Krumholz & Thompson 2012) and because dust temperatures used in calculations of $T_{\text{IR}}$ are assumed to be high, $T_D > 100$ K, while the optical depth can be much lower if dust temperatures are much lower because $T_{\text{IR}} \propto T_D^2$ (Semenov et al. 2003).

To understand how significantly larger values of $T_{\text{IR}}$ affect our results, we perform two ALL simulations using fixed optical depths $T_{\text{IR}} = 10$ and 30. As shown in the right panel of Figure 11, increasing $T_{\text{IR}}$ further suppresses SFR by $\sim 30\%$ for $T_{\text{IR}} = 10$, and by a factor of 2–3 for $T_{\text{IR}} = 30$. The latter case renders SFRs $\sim 5$–10 times lower than in the case of no feedback. Another model uncertainty is the duration of the embedded phase of star cluster formation, $t_{\text{cl}}$, where we assume IR trapping to be important. By assuming $t_{\text{cl}} = 6$ Myr instead of the fiducial 3 Myr (see Appendix A.2), the rate of star formation can be suppressed by a factor of two compared to the fiducial ALL model.

In the bottom-left panel, we present the impact of disabling cooling in the gas surrounding newly born star particles. The SFR in runs with $t_{\text{cool}} = 10$ and 40 Myr is suppressed by amount similar to the runs with high $T_{\text{IR}}$ values discussed above. A significant suppression (by a factor of two) can be achieved via SNe alone, provided gas cooling is disabled for extended periods of time, $t_{\text{cool}} = 40$ Myr.

The effect of treating a fraction $f_{\text{fb}}$ of the feedback energy as an auxiliary energy variable $E_{\text{fb}}$ that dissipates on a timescale $t_{\text{dis}}$, longer than expected from cooling in the dense gas, is shown in the bottom-right panel of Figure 11. Even for a modest $f_{\text{fb}} = 10\%$ dissipating over $t_{\text{dis}} = 1$ Myr, SFRs can be affected by $\sim 30\%$. As the energy fraction is increased to $f_{\text{fb}} = 0.5$, and/or dissipation occurs over longer timescale $t_{\text{dis}} \gtrsim 10$ Myr, we find a significant impact on the SFHs, and SFRs approach a steady $\sim 1 M_\odot \text{yr}^{-1}$. As discussed in Section 2.2.1, up to 90% of SN energy may be lost in radiative shocks within wind-blown bubbles (Cho & Kang 2008) in a few Myr. However, as the above simulations indicate, even this amount of preserved energy has a non-negligible effect on SFR. We view this as an indication that some form of subgrid treatment of feedback energy may be required, even in the presence of pre-SN feedback sources, due to the unresolved ISM phases and gas motions.

We note that these results should only be viewed as indicative, as the effect of feedback can in general depend on metallicity, ISM pressure, depth of potential well, accretion rates, etc., which we plan to explore in future work. The intent of this study is to explore the sensitivity of simulations to the details of specific implementations of feedback and their associated parameters. The SFR in our fiducial model shown in Figure 11 is higher than in the MW today. At this point, this should not be taken as a preference for a particular model as gas fraction in our simulated galaxy is about two times larger than the gas fraction of the MW. The ultimate test of specific feedback implementation and parameter choices will be determined by contrasting predictions of self-consistent cosmological simulations with the full range of available observations.

### 4.3.2. The $\Sigma_{\text{SFR}}$–$\Sigma_{\text{cold}}$ Relation

Figure 12 shows how the KS relation is affected by the change of star formation efficiency per free-fall time in the presence, and absence, of feedback. All data points refer to

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**Table 2**

| Run                        | Description                                |
|----------------------------|--------------------------------------------|
| Direct injection runs      |                                            |
| nob(f001)                  | No feedback, $\epsilon_{\text{ff}} = 10\% (1\%) |
| Energy                     | Only energy: $E_{\text{tot}}, \epsilon_{\text{ff}} = 10\%$ |
| Prad                       | Only radiation pressure: $p_{\text{prad}}, \epsilon_{\text{ff}} = 10\%$ |
| SNnomom                    | Only SN energy: $E_{\text{SNII}}, \epsilon_{\text{ff}} = 10\%$ |
| all(f001)                  | All feedback processes: $E_{\text{tot}}$ and $p_{\text{prad}}, \epsilon_{\text{ff}} = 10\% (1\%)$ |
| all_tcl6Myr                | All feedback processes: $E_{\text{tot}}$ and $p_{\text{prad}}, \epsilon_{\text{ff}} = 10\%$ (fiducial $t_{\text{cl}} = 6$ Myr) |
| all_tau10                  | All feedback processes: $E_{\text{tot}}$ and $p_{\text{prad}}, \epsilon_{\text{ff}} = 10\% (1\%)$ |

| Simulations adopting delayed cooling |
|-------------------------------------|
| SN_dc10                             | Only SN energy: $E_{\text{SNII}}$, delayed cooling $t_{\text{cool}} = 40$ Myr, $\epsilon_{\text{ff}} = 10\%$ |
| SN_dc40                             | Only SN energy: $E_{\text{SNII}}$, delayed cooling $t_{\text{cool}} = 40$ Myr, $\epsilon_{\text{ff}} = 10\%$ |
| energy_dc10                         | Only energy: $E_{\text{tot}}$, delayed cooling $t_{\text{cool}} = 10$ Myr, $\epsilon_{\text{ff}} = 10\%$ |
| energy_dc40                         | Only energy: $E_{\text{tot}}$, delayed cooling $t_{\text{cool}} = 40$ Myr, $\epsilon_{\text{ff}} = 10\%$ |
| all_dc10                            | All feedback processes: $E_{\text{tot}}$ and $p_{\text{prad}}$, delayed cooling $t_{\text{cool}} = 10$ Myr, $\epsilon_{\text{ff}} = 10\%$ |
| all_dc40(e001)                      | All feedback processes: $E_{\text{tot}}$ and $p_{\text{prad}}$, delayed cooling $t_{\text{cool}} = 40$ Myr, $\epsilon_{\text{ff}} = 10\% (1\%)$ |

| Runs adopting a feedback energy variable |
|------------------------------------------|
| energy_f05_t1                           | Only energy: $E_{\text{tot}}$, feedback energy fraction $f_{\text{fb}} = 0.5$, dissipation time $t_{\text{dis}} = 1$ Myr, $\epsilon_{\text{ff}} = 10\%$ |
| energy_f05_t10                          | Only energy: $E_{\text{tot}}$, feedback energy fraction $f_{\text{fb}} = 0.5$, dissipation time $t_{\text{dis}} = 10$ Myr, $\epsilon_{\text{ff}} = 10\%$ |
| all_f05_t1                             | All feedback: $E_{\text{tot}}$ and $p_{\text{prad}}, f_{\text{fb}} = 0.5$, $t_{\text{dis}} = 1$ Myr, $\epsilon_{\text{ff}} = 10\%$ |
| all_f05_t10(e001)                       | All feedback: $E_{\text{tot}}$ and $p_{\text{prad}}, f_{\text{fb}} = 0.5$, $t_{\text{dis}} = 10$ Myr, $\epsilon_{\text{ff}} = 10\% (1\%)$ |
| all_f01_t1                             | All feedback: $E_{\text{tot}}$ and $p_{\text{prad}}, f_{\text{fb}} = 0.1$, $t_{\text{dis}} = 1$ Myr, $\epsilon_{\text{ff}} = 10\%$ |
| all_f01_t10                            | All feedback: $E_{\text{tot}}$ and $p_{\text{prad}}, f_{\text{fb}} = 0.1$, $t_{\text{dis}} = 10$ Myr, $\epsilon_{\text{ff}} = 10\%$ |
| all_f01_t40                            | All feedback: $E_{\text{tot}}$ and $p_{\text{prad}}, f_{\text{fb}} = 0.1$, $t_{\text{dis}} = 40$ Myr, $\epsilon_{\text{ff}} = 10\%$ |

*Note. See Equation (2) for notation.*
Figure 11. Star formation histories for the isolated galactic disk simulation. Top left: the impact of various feedback sources, in the "straight injection" implementation of feedback. The strongest suppression of star formation is in simulations that include early momentum injection, especially momentum due to radiation pressure. Thermal energy feedback has a sub-dominant effect due to short cooling times in dense gas. Top right: the impact of increasing the infrared optical depth to a fixed value of $\tau_{IR} = 10$ or 30. $\tau_{IR} = 10$ reduces the SFR by $\sim 30\%$, while boosting radiation pressure using $\tau_{IR} = 30$ suppresses the SFR by another factor of 2–3 compared to the fiducial ALL run, and a factor of $\sim 5$–10 compared to the case of no feedback. If the embedded IR trapping phase is prolonged from $t_{cl} = 3$ Myr to 6 Myr, the asymptotic SFRs are suppressed by a factor of $\sim 2$ compared to the fiducial model. Bottom left: the impact of delaying cooling in the local gas around newborn star particles for $t_{cool} = 10$ and 40 Myr. Note that delaying cooling for such values of $t_{cool}$ results in a similar suppression of SFR compared to the radiation pressure momentum injection with large values of $\tau_{IR}$. For example, SFR $\approx 2 M_\odot$ yr$^{-1}$ for $t_{cool} = 40$ Myr, which is similar to the SFR for run with $\tau_{IR} = 30$ shown in the top-right panel. Bottom right: the impact of assigning some fraction $f_{fb}$ of the feedback energy to an energy variable $E_{fb}$ that dissipates on longer timescales $t_{dis}$ than expected from cooling in the dense gas. Even if only 10% of the energy is assumed to dissipate over $t_{dis} = 1$ Myr, SFR is suppressed by $\sim 30\%$. If the energy fraction is increased to $f_{fb} = 0.5$, and/or dissipation occurs over longer timescale $t_{dis} \gtrsim 10$ Myr, we find a significant impact on star formation, as the SFR approaches a steady rate of $\sim 1 M_\odot$ yr$^{-1}$.

A color version of this figure is available in the online journal.

The quantities averaged over azimuthal bins of width $\Delta r = 720$ pc, and are calculated from simulation snapshots in the time range 240–300 Myr. Shown also is the THINGS data from Bigiel et al. (2008) and the galaxy-scale average relation from Kennicutt (1998). The Bigiel et al. (2008) relation is derived for kiloparsec-sized patches, and is hence a more comparison to our simulated data.

Without feedback, simulations adopting $\epsilon_{ff} = 1\%$ are consistent with the Kennicutt (1998) relation. However, at high $\Sigma_{gas}$, the adopted nonlinear star formation relation ($\dot{\rho}_s \propto \rho^{1.5}$) overshoots the observed, less steep relation of Bigiel et al. (2008). In runs with no feedback, the normalization of the $\Sigma_{SFR} - \Sigma_{gas}$ relation scales linearly with the assumed value of $\epsilon_{ff}$, while in runs with feedback (the ALL model) the amplitude of the relation changes by a factor of at most two for values of $\epsilon_{ff}$ that differ by a factor of 10. However, data points at the largest values of $\Sigma_{gas}$, corresponding to the galactic center in the analyzed simulation snapshots, are less affected by feedback and the difference in amplitude for runs with different $\epsilon_{ff}$ persists in these regions. We note that in runs with $\epsilon_{ff} = 1\%$, the KS relation with and without feedback is similar.

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14 Surface densities are corrected by a factor of 1.36 to account for helium.
Figure 12. Impact of feedback implementation and assumed star formation efficiency on the \( \Sigma_{\text{SFR}} - \Sigma_{\text{gas}} \) relation. The left panel shows runs with no feedback, while the right panel shows runs with ALL feedback implementation (see Table 2). In both panels, the two sets of points show runs with two different assumed star formation efficiencies \( \epsilon_{\text{ff}} = 1\% - 10\% \). The points correspond to the average disk values in azimuthal bins of width \( \Delta r = 720 \) pc, and are calculated from simulation snapshots in the time range 240–300 Myr. The black solid line shows the galactic scale averaged data from Kennicutt (1998) and the contour lines the distribution of sub-kpc-sized patches in the sample of nearby galaxies by Bigiel et al. (2008). In runs with no feedback, the normalization of the relation scales linearly with the assumed value of \( \epsilon_{\text{ff}} \), while in runs with feedback the amplitude of the relation changes by a factor of at most 2 for values of \( \epsilon_{\text{ff}} \) that differ by a factor of 10. Star formation in the central parts of the galaxy, here points with the largest values of \( \Sigma_{\text{gas}} \), is not affected by feedback to the same extent as the rest of the disk and the difference in amplitude for runs with different \( \epsilon_{\text{ff}} \) persists in these regions.

(A color version of this figure is available in the online journal.)

Figure 13. Impact of different implementations of feedback on the \( \Sigma_{\text{SFR}} - \Sigma_{\text{gas}} \) relation. The left panel shows the effect of varying the strength of radiation pressure momentum injection, the middle panel shows effect of delaying cooling around newly born star particles, and right panel shows effect of treating a fraction of thermal feedback energy as a separate energy variable. Data points show azimuthally averaged values adopting bin sizes of \( \Delta r = 720 \) pc, and are calculated from simulation snapshots in the time range 240–300 Myr. The observational data points are described in the caption of Figure 12. Larger values of \( \tau_{\text{IR}}, t_{\text{cool}}, \text{ or } t_{\text{dis}} \) can lead to a similar suppression of normalization of the \( \Sigma_{\text{SFR}} - \Sigma_{\text{gas}} \) relation. The investigated feedback methods show a factor of \( \sim 20 \) spread in the normalizations of the \( \Sigma_{\text{SFR}} - \Sigma_{\text{gas}} \) relation, which shows that this relation can be a useful tool in constraining parameters of feedback models.

(A color version of this figure is available in the online journal.)

The dependency of feedback model parameters on the KS relation is shown in Figure 13, in which different panels show the effect of increasing the strength of radiation pressure, delaying cooling for longer times, and increasing the contribution/duration of feedback energy using a second energy variable. Overall, the sensitivity to the parameters is fairly weak: the KS relation is similar for models in which dissipation of SNII energy is slowed down by delay of cooling or via using second energy variable for \( t_{\text{cool}} \leq 10 \) Myr or \( t_{\text{dis}} \leq 1 \) Myr, and for models with early momentum injection with optical depth up to \( \tau_{\text{IR}} = 10 \). As parameters are dialed up to even larger values (\( \tau_{\text{IR}} = 30 \), \( t_{\text{cool}} = 40 \) Myr, or \( t_{\text{dis}} \geq 10 \) Myr), normalization of the KS relation is significantly suppressed.
These results show that our fiducial feedback model (ALL) at the adopted resolution level, results in SFRs comparable to the runs in which cooling is delayed or SN energy is dissipated on a controlled timescale. The results also show that normalization of the KS relation can be used to constrain the plausible range of values of parameters, or at least exclude the most extreme values.

4.3.3. Visual Comparison

In Figure 14, we show face-on and edge-on maps at $t = 200$ Myr of the gas surface density, mass-weighted average temperature within $z = \pm 150$ pc of the disk, and stellar surface density of five of the simulations from Table 2: “nofb,” “all,” “all_tau30,” “all_dc40,” and “all_f05_t10.” The two former runs are our fiducial runs with and without feedback, and the latter three represent efficient feedback implementations.

In runs without feedback, dense star-forming clumps of gas form out of spiral arms, and remain intact throughout the simulation until star formation depletes most of their gas, or the clumps sink to the disk center. This run thus produces very massive star clusters clearly visible in the stellar surface density map. In the “all” simulation, gas clumps do not form or are effectively dispersed and gas distribution in this run is considerably less clumpy. Consequently, massive star clusters are not produced, and this effect is even more pronounced in the three example of efficient feedback.

All simulations feature a highly multiphase medium. Large holes filled with hot coronal gas at $T \sim 10^8$ K forms between the cold gas associated with the spiral arms in all simulations. This effect is less prominent in the simulations incorporating feedback, as cold gas is pushed out of star-forming regions, resulting in a larger filling factor of cold material. This effect is especially apparent in the face-on temperature map of the “all_f05_t10.” The edge-on maps of density and temperature in all feedback runs show that fountains and outflows of both cold and warm gas ($T \sim 10^4–10^5$ K) and hot gas ($T > 10^7$ K) are present close to the disk plane. The efficient feedback runs all feature a more porous ISM, with prominent pockets of hot gas forming within spiral arms, as seen in the face-on density and temperature maps.

This illustrates that specific details of feedback implementations do matter in determining qualitative structural properties of the ISM and even stellar distribution. We quantify the differences in density and temperature structure of the ISM in these runs by considering the corresponding probability distributions in the next section.

4.3.4. Structure of the Interstellar Medium

The visual differences discussed above are quantified in Figure 15, where we show the cumulative mass fraction above a given density and temperature at $t = 200$ Myr. All simulations are analyzed in the regions shown in Figure 14 within a distance of $\pm 0.5$ kpc of the disk plane. In the case of no feedback, the existence of dense gas clumps is manifested in the tail of the density distribution at $n > 100$ cm$^{-3}$. The density and temperature distributions in runs with feedback are qualitatively similar; the high-density tail at $n \approx 100$ cm$^{-3}$ is suppressed as gas in star-forming regions is efficiently dispersed. Simulation with a second feedback energy variable has the least amount of dense gas, as could be deduced from its SFR in the bottom-right panel of Figure 11. We note that the details of the high-density tail, as well as the dispersal process, likely depend on the choice of star formation density threshold and numerical resolution.

The distributions presented here are useful in interpreting trends of the KS relation normalization discussed above. For example, it is clear that runs with efficient feedback have SFR comparable to the run with no feedback and 10 times lower $\epsilon_\text{ff}$ because they simple have less dense gas.

The temperature structure in the right panel also reveals significant differences between feedback schemes. In runs with delayed cooling, $\sim 10\%$ of the disk’s gas mass is at $T \gtrsim 10^5$ K, which is two orders of magnitude greater than in the other runs. This can be seen in the temperature map in Figure 14, where the central region features a hole of hot, ionized, but dense, gas formed out of percolating star-forming regions of feedback ejecta. However, all runs have a comparable fraction of gas in the hot coronal phase ($T > 10^6$ K). For comparison, in the MW disk $\lesssim 1\%$ of the gas mass is thought to be in the hot phase (e.g., Ferrière 2001). The prominent bump in the “all” run around $T \sim 10^7$ K is associated with embedded star particles heating the ISM to warm temperatures. In the strong feedback models “allTau30” and “all_f05_t10,” feedback disperses dense gas more efficiently, and heating occurs in the diffuse rather than dense phase, which is why there is no significant mass contribution in the warm or hot phase from these runs in this figure.

Figure 16 shows the density, $(dV/d\log n)/V_{\text{tot}}$, and temperature probability distribution functions (PDFs), $(dV/d\log T)/V_{\text{tot}}$, defined as a fraction of disk volume in a given density or temperature range. A log-normal PDF is not a good description to the density PDF in our simulations contrary to results of Wada & Norman (2007), although it may be possible to describe the PDFs as superpositions of several log-normal distributions corresponding to different gas phases (Robertson & Kravtsov 2008). The figure shows that the run without feedback has the most dense gas, but the smallest amount of tenuous gas at $n < 0.1$ cm$^{-3}$. Interestingly, the run with delayed cooling has less tenuous gas of density $n \sim 10^{-2}–10^{-3}$ cm$^{-3}$ than our fiducial run. This indicates that feedback models with early feedback injection can efficiently create both a diffuse ionized warm phase and a tenuous coronal phase without resorting to delaying gas cooling.

The multiphase structure of the ISM is apparent in the temperature PDF, where all simulations show signatures of a three-phase ISM (McKee & Ostriker 1977), connected by gas at intermediate temperatures. Without feedback, the gas cools down to a very thin disk (only a few cells in vertical height) with a substantially lower contribution to the volume in the cold phase ($T < 10^4$ K) compare to runs with feedback, which all feature thicker cold gas disks due to feedback-driven turbulence. In addition, more cold gas is lost in star formation events when feedback is absent. This discrepancy is especially apparent when comparing to the most efficient feedback run, “all_f05_t10.” The hot ($T \sim 10^6$ K) tenuous gas phase is present in all runs, although vigorous heating in “all_dc40” and “all_f05_t10” creates pockets of gas at $\sim 10^2$ K, which vent out of the disk to the surrounding corona. As can be seen in Figure 14, the circumgalactic medium is more structured in “all” and “all_tau30,” which is evident from the wider distribution of gas at $T \sim 10^4–10^8$ K.

4.3.5. Velocity Dispersion Profiles

We quantify the level of turbulent gas motions in the disks via the mass-weighted, vertical line-of-sight velocity dispersion profile $\sigma_z(r)$, shown in Figure 17 for the gas cold component ($T < 10^4$ K). Such profiles can be observed in real galaxies.
Figure 14. Face-on and edge-on maps of the galactic disk at $t = 200$ Myr showing gas surface density (top), mass-weighted average gas temperature (middle), and stellar surface density (bottom). The face-on plots are calculated within $z \pm 1.5$ kpc of the disk to avoid excess halo material. Each panel is 24 kpc across. The temperature is calculated as $\int \rho T / \int \rho$, where the integral is performed along each pixel sightline within $z \pm 150$ pc. The map of stellar distribution only includes star particles formed after the start of the simulation, and does not include the star particles present in the initial conditions.

(A color version of this figure is available in the online journal.)

and comparisons of model results and observations can help to constrain parameters of feedback models. Indeed, we could expect that models with the most efficient feedback generate stronger gas motions, which should be manifested in larger velocity dispersions. The figure shows that significant velocity dispersion declining with increasing radius is produced in all runs. Such declining dispersion profiles are indeed observed in spiral galaxies for the neutral H$\text{I}$ gas (e.g., Meurer et al. 1996; Petric & Rupen 2007; Tamburro et al. 2009). The fact that significant velocity dispersion is observed in the run with
no feedback, indicates that most of the motions are due to disk instabilities and not due to feedback per se. In fact, velocity dispersion in the inner regions is even somewhat smaller in our fiducial run with the ALL feedback model. This difference is probably due to formation of massive gas clumps which can more efficiently stir the gas as they move around and merge with each other in the weaker feedback runs. Nevertheless, the largest velocity dispersions, in the inner 10 kpc of the disk, are observed in runs with delayed cooling and large \( \tau_{\text{IR}} \), i.e., models with the most efficient feedback.

Using the THINGS galaxy sample, Tamburro et al. (2009) analyzed the radial H\( \text{i} \) velocity dispersion, \( \sigma_{\text{H}\text{i}} \), and SFR surface density profiles and found positive correlation between the kinetic energy of H\( \text{i} \) and the SFR. The increase in \( \sigma_{\text{H}\text{i}} \) at smaller radii indeed correlates with an increase in star formation activity, both in observations and simulations, but so does the level of shear and strength of disk self-gravity. Gravitational instabilities can generate a significant baseline level of turbulence even without any contribution from feedback (Agertz et al. 2009a), as illustrated in Figure 17. Observations indicate a characteristic plateau of \( \sigma_{\text{H}\text{i}} \sim 10 \text{ km s}^{-1} \) in galaxies with a globally averaged \( \langle \Sigma_{\text{SFR}} \rangle \lesssim 10^{-3} - 10^{-2} M_{\odot} \text{ yr}^{-1} \text{ kpc}^{-2} \) (Dib et al. 2006), above which stellar feedback becomes the more dominant driver of the observed H\( \text{i} \) velocity dispersions (as shown numerically by Agertz et al. 2009a). The propensity for different feedback models to generate turbulent velocity dispersions in ISM gas.

**Figure 15.** Cumulative mass fraction of the ISM above a given density \( n \) (left panel) and temperature \( T \) (right panel). The analyzed region is a thin slab of dimensions \( 24 \times 24 \times 0.5 \text{ kpc} \) centered on the disk. The prominent tail at densities in excess of \( n > 100 \text{ cm}^{-3} \) in the simulation without feedback is due to a population of dense, long-lived gas clumps. In the presence of feedback, such gas clumps are effectively dispersed, which significantly reduces the fraction of gas at such densities. The temperature structure shows significant differences between different runs. In the run with delayed cooling, a significant \( \approx 10\% \) of the disk gas is heated to temperature in excess of \( T \sim 10^5 \text{ K} \), while this fraction is only \( \sim 0.1\% \) in other runs. The “all” run has more mass around \( T \sim 10^3 \text{ K} \), which is associated with embedded star particles heating the ISM to warm temperatures. In the case of strong feedback, dense gas is not heated, but dispersed, and diffuse gas is heated to very high temperatures. This mechanism has little effect on the cumulative mass function, as the hot phase is negligible by mass.

(A color version of this figure is available in the online journal.)

**Figure 16.** Probability distribution functions (PDFs) of density (left panel) and temperature (right panel) at \( t = 200 \text{ Myr} \). The PDFs are measuring a fraction of volume at a given density or temperature. The line types are the same as in Figure 15. The multiphase structure of the ISM is evident in all simulations.

(A color version of this figure is available in the online journal.)
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may therefore manifest more strongly in starbursting systems. We leave an investigation of the velocity dispersion dependence on feedback parameters and star formation surface density for a future study.

5. DISCUSSION AND CONCLUSIONS

In this paper, we have presented a new model for stellar feedback that explicitly considers the injection of both momentum and energy in a time-resolved fashion. In particular, we have calculated the time-dependent momentum and energy budget from radiation pressure, stellar winds, Type II and Ia SNe, as well as the associated mass and metal loss for all relevant processes. We present a novel prescription for modeling the early (pre-SNII) injection of momentum due to stellar winds and radiation pressure from massive young stars. These stellar feedback processes were implemented and tested in the AMR code RAMSES. We have also examined and compared the effects of feedback in this new implementation and other popular recipes on properties of simulated galactic disks.

Using idealized simulations of star-forming patches of gas and star-forming spiral galaxies, we study how each stellar feedback source affects the overall rate of galactic star formation, as well as density, temperature, and velocity structure of the ISM. We find that early pre-SN injection of momentum is an important ingredient, which qualitatively changes the effectiveness of stellar feedback. In a given stellar population, SNe explode only after ~4 Myr, while essentially all momentum and energy associated with radiation pressure and stellar winds are deposited in the first 3–4 Myr. We show that such momentum injection can disperse dense gas in star-forming regions, leading to SNe exploding in a more tenuous medium, hence increasing the survivability of shock heated gas. Our simulations of massive ($M \sim 10^6 M_\odot$) star-forming clouds indicate that momentum-based feedback alone can limit the global cloud star formation efficiency to $\epsilon_{\text{cl}} \sim 10\%$. In absence of the pre-SN momentum feedback, we recover the classical overcooling problem for stellar feedback (Katz 1992; Navarro & White 1993), as gas cooling times are short in the dense star-forming ISM ($t_{\text{cool}} \sim 10^3$ yr), and star formation is less affected by feedback.

In a simulated MW-like galaxy, we find that SFRs, and the normalization of the KS relation, are significantly affected by inclusion of stellar feedback. Interestingly, we find that the normalization of the KS relation is less sensitive to the assumed star formation efficiency per free-fall time ($\epsilon_{\text{ff}}$) in schemes with efficient feedback due to self-regulating effect of feedback on density and temperature PDFs within the ISM of simulated galaxies. An order of magnitude change in $\epsilon_{\text{ff}}$ only results in only a factor of two increase in the KS relation normalization.

Our results illustrate the importance of not only accounting for the entire momentum and energy budget of stellar feedback, but also to inject momentum and energy at the appropriate stages of stellar evolution. A similar conclusion was recently reached by Hopkins et al. (2011; see also Hopkins et al. 2012) based on high-resolution SPH simulations. In this paper, we show how this effect can be incorporated at the resolution typical for state-of-the-art galaxy formation simulations.

Although the qualitative trends illustrated by our results are clear, it is not obvious whether the effects of feedback, especially the survival and impact of shocked winds and SN ejecta, are modeled correctly. This is because any subgrid feedback scheme by necessity is implemented at scales close to the resolution of the simulations, where numerical effects play a role. In our experiments, we find that even if only ~10% of thermal feedback energy is retained for 1–10 Myr (as suggested by, e.g., Thornton et al. 1998; Cho & Kang 2008), stored and followed using a separate energy variable, this energy has a significant effect on SFRs, the ISM density structure, and turbulent velocity dispersions. Furthermore, we have not explored the effect of cosmic-ray propagation in this work. Cosmic-ray pressure-driven winds may be an important ingredient in galaxy formation (e.g., Socrates et al. 2008), which we plan to explore in future work (C. M. Booth et al., in preparation).

Comparing different feedback prescriptions, we find that the recipe presented in this paper results in effects on galactic SFR and ISM structure similar to the results of feedback schemes with a delay of feedback energy dissipation if the infrared optical depth in star-forming regions is sufficiently high ($\tau_{\text{IR}} \gtrsim 10$), or it the embedded phase of star formation, where photon trapping is significant, is assumed to last for $t_{\text{fg}} > 3$ Myr. This conclusion is consistent with the results of Hopkins et al. (2011). Hopkins et al. reported average values of $\langle t_{\text{fg}} \rangle \sim 10-30$ in their isolated “Milky Way” SPH simulation. This value refers to the typical $t_{\text{fg}}$ adopted as an SPH particle is kicked by their feedback scheme. In the empirically motivated subgrid model for radiation pressure momentum presented in this paper, such high values of $t_{\text{fg}}$ are achieved only around massive star clusters, $M_\odot \gtrsim 10^6$, which are rare in our simulations of galactic disks. The actual values of IR optical depth around young clusters are quite uncertain. If large values, $\tau_{\text{IR}} \gtrsim 30$, indeed are appropriate, this can be incorporated as a normalization constant in our relation in Appendix A (i.e., $\eta_2$ in Equation (A11)). We note that in some situations, even momentum from single scattering of photons (i.e., without IR trapping, $t_{\text{fg}} \sim 0$) can have a significant effect (Wise et al. 2012; Chattopadhyay et al. 2012). The regimes in which such feedback is efficient remain to be explored and clarified.
Hopkins et al. (2012) found that for an analogous setup as investigated in this work, various stellar feedback sources interacted in a complex fashion, and their prescription for radiation pressure would not alone dominate the regulation of star formation. We find that our fiducial ALL model (see Table 2) impacts global SFRs, but not to the degree of being completely independent of the adopted star formation efficiency per free-fall time. To achieve stronger regulation, excursions from this model, be it via boosting parameters of the model or prolonging the dissipation timescale in gas affected by thermal feedback, is required.

The above conclusions are based on the simulations conducted at spatial resolutions typical of what is affordable by current cosmological simulations of galaxy formation, i.e., \( \sim 10–100 \) pc. We have not demonstrated numerical convergence in this work, and we do not necessarily expect this to be easily achieved; as resolution improves, the density PDF changes as self-gravitating gas can collapse to higher densities and the gas dissipates energy at a higher rate. This leads to a shorter star formation timescale (as \( t_{\text{SF}} \sim \rho^{-0.5} \); Equation (2.3)) and hence an increased rate of star formation. Numerical convergence can in principle be achieved by imposing a pressure floor via a polytropic equation of state, \( P \propto \rho^y \), where \( y = 2 \), similar to what is necessary to avoid numerical fragmentation (Truelove et al. 1997). In this case we impose, by hand, a floor to the allowed minimum Jeans mass, for which convergence in principle is achievable. However, in this case simulations may converge to an incorrect (and arbitrary) result, if the polytropic equation of state does not capture the actual thermodynamic properties of ISM realistically.

It is clear that any implementation of the star formation–feedback loop requires thorough testing against observations, such as the KS relation, velocity dispersion profiles of gas, etc. We plan to carry out such tests using the implementations of the feedback models described in this paper, as well as different implementations of star formation recipes, in self-consistent cosmological galaxy formation simulations in future work.

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APPENDIX A

THE SUBGRID MODEL FOR RADIATION PRESSURE MOMENTUM

One of the main difficulties in modeling momentum transferred to gas by radiation pressure is in proper accounting for contribution of momentum due to multiple scatterings of infrared photons by dust grains. In the implementation of Hopkins et al. (2011), an iterative clump finding algorithm was used to identify star-forming clouds. All stars within the cloud radius transfer momentum to the gaseous components according to Equation (5), where the infrared optical depth \( \tau_{\text{IR}} \) is \( \kappa_{\text{IR}} \Sigma_{\text{gas}} \).

Here, \( \Sigma_{\text{gas}} \) is the gas surface density and Hopkins et al. adopt a constant opacity \( \kappa_{\text{IR}} \approx 5 \, \text{cm}^2 \, \text{g}^{-1} \), which is appropriate for dust temperatures of \( T_d \gtrsim 100 \, \text{K} \).

The surface density of gas in star-forming clumps was calculated directly from simulations as \( \Sigma_{\text{cl}} = M_{\text{cl}}/(\pi R_{\text{cl}}^2) \), where \( R_{\text{cl}} \) is the radius given by the clump finding routine. Hopkins et al. report average values (at gas particle launch) of \( \tau_{\text{IR}} \sim 10–30 \) in the MW environment, and \( \sim 30–100 \) in high-redshift disk analogs. Values of this magnitude are a direct outcome of clouds collapsing to the point where densities are high enough for \( \tau_{\text{IR}} \) to halt the process, leading to cloud collapse. However, this process is highly sensitive to simulation resolution and other numerical effects. Moreover, surface density of real star-forming clumps depend on internal processes within these regions, such as supersonic turbulence and feedback, which will not be resolved even with the \( \sim \) parsec resolution. This uncertainty hence propagates into the calculation of the momentum transfer via reprocessing of IR radiation by dust.

Below we discuss an alternative way of estimating \( \tau_{\text{IR}} \) based on observed properties of young star clusters and molecular clumps, which can readily be implemented in simulations adopting a spatial resolution of \( \Delta x \sim 10–100 \) pc.

Observed molecular clouds have sizes of \( \sim 5–100 \) pc, average densities of \( \sim 100 \, \text{cm}^{-3} \), and surface densities of \( \Sigma_{\text{GMC}} \sim 50–200 \, \text{M}_\odot \, \text{pc}^{-2} \) (e.g., Bolatto et al. 2008; Fukui & Kawamura 2010). They have complex internal structure with gas density ranging over several orders of magnitude. This structure is thought to arise due to supersonic turbulent flows and gravitational contraction during cloud formation (e.g., Padoan et al. 1997; Li et al. 2004; Kritsuk et al. 2007).

Star clusters form in clumps which have densities of \( >10^3–10^4 \, \text{cm}^{-3} \), while individual stars form in high-density cores of even higher density within the clumps (\( >10^5 \, \text{cm}^{-3} \), e.g., Lada & Lada 2003). Clumps in the MW molecular clouds (e.g., Williams et al. 1994, 2000; Lada & Lada 2003) have masses of up to \( \times 10^5 \, \text{M}_\odot \) and their surface densities range from \( \sim 150 \, \text{M}_\odot \, \text{pc}^{-2} \) to \( \sim 15,000 \, \text{M}_\odot \, \text{pc}^{-2} \) (or \( \sim 0.03–3 \, \text{g} \, \text{cm}^{-2} \)), based on the structural parameters of more massive young star clusters, their parent gas clumps had even higher surface densities. Young \( (t < 10 \, \text{Myr}) \) star clusters MW clusters with masses of order \( \sim 10^4 \, \text{M}_\odot \) are have half-light radii of \( r_h \sim 0.5–2 \) pc (see the compilation by Portegies Zwart et al. 2010). Observations of young clusters in external galaxies find \( r_h \sim 2 \) pc independent of luminosity or cluster mass (M51; Scheepmaker et al. 2007) and young super star clusters in starburst galaxies, e.g., M82 show similar properties (McCrady & Graham 2007a).

The bottom panel of Figure 18 shows the relation between half-mass radius and mass for molecular clumps in the MW (from the compilation shown in Figure 1 of Fall et al. 2010b) and for young \( (\sim 2 \times 10^7 \, \text{yr}) \) star clusters in the MW and other nearby galaxies from the compilation of Portegies Zwart et al. (2010) as a function of their mass. Different lines show relations derived for star clusters and clumps in several recent studies, as described in the figure caption. The corresponding surface densities of the clumps and star clusters are plotted in the upper panel, and shows that although scatter is substantial, clumps and clusters in the MW follow a similar relation at masses \( \lesssim 10^3 \, \text{M}_\odot \); \( R \propto \alpha^{0.5} \) with \( \alpha \approx 0.3–0.5 \). This implies that it is reasonable to assume that radii and masses of young clusters are a good reflection of the corresponding properties of their parent molecular clumps.

For clusters of mass \( \gtrsim 10^5 \, \text{M}_\odot \), the relation flattens \( (\alpha \approx 0) \), although the scatter is large. The broken dashed line shows...
clusters (stars, from the compilation by Portegies Zwart et al. 2010). The green stars show clusters in the LMC (Mackey & Gilmore 2003) and SMC, while cyan stars show star clusters within the Milky Way. The other star symbols show clusters in other galaxies, including starbursts such as M82 (solid red stars; Krumholz & Matzner 2009b) and the Antennae galaxies. Note that surface densities are estimated within the half-mass radius: \( \Sigma = M_{\text{cl}}/(2\pi R_{\text{cl}}^2) \). The broken magenta dashed lines show power-law approximation to the clusters given by Equation (A2). The solid line in the bottom panel is a power-law fit to the MW low-mass star clusters from Lada & Lada (2003), while the dotted line is a fit to the mass–radius of MW clumps from Dib et al. (2010).

(A color version of this figure is available in the online journal.)

| log \( \Sigma \) (g cm\(^{-2}\)) | log \( R_{\text{cl}} \) (pc) |
|-----------------|-----------------|
| 0.4             | 2.5             |

**Figure 18.** Surface density vs. mass (top panel) and half-mass radius vs. mass (bottom panel) for the molecular clumps in the Milky Way (blue squares, from the compilation by Fall et al. 2010b, in their Figure 1) and young star clusters (stars, from the compilation by Portegies Zwart et al. 2010). The green stars show clusters in the LMC (Mackey & Gilmore 2003) and SMC, while cyan stars show star clusters within the Milky Way. The other star symbols show clusters in other galaxies, including starbursts such as M82 (solid red stars; Krumholz & Matzner 2009b) and the Antennae galaxies. Note that surface densities are estimated within the half-mass radius: \( \Sigma = M_{\text{cl}}/(2\pi R_{\text{cl}}^2) \). The broken magenta dashed lines show power-law approximation to the clusters given by Equation (A2). The solid line in the bottom panel is a power-law fit to the MW low-mass star clusters from Lada & Lada (2003), while the dotted line is a fit to the mass–radius of MW clumps from Dib et al. (2010).

(A color version of this figure is available in the online journal.)

\[
R_{\text{cl}} = \left( \frac{M_{\text{cl}}}{3000 M_\odot} \right)^{0.4} \text{pc}, \quad \text{for } M_{\text{cl}} < 3 \times 10^4 M_\odot, \quad (A1)
\]

\[
R_{\text{cl}} = 2.5 \text{ pc}, \quad \text{for } M_{\text{cl}} \geq 3 \times 10^4 M_\odot. \quad (A2)
\]

The corresponding data and lines for surface densities defined as \( \Sigma_{\text{cl}} = M_{\text{cl}}/(2\pi R_{\text{cl}}^2) \) are shown in the upper panel of the figure. For \( M_{\text{cl}} < 3 \times 10^4 M_\odot \), the surface densities are generally \( \Sigma \lesssim 1 \text{ g cm}^{-2} \), while for more massive clusters they can reach \( \Sigma \gtrsim 10 \text{ g cm}^{-2} \). For the latter values of surface densities, the optical depth \( \tau_{\text{IR}} \gtrsim 50 \) if dust temperatures are warm \( (T_d \gtrsim 200 \text{ K}) \), as illustrated in Figure 3.

One complication to considering radii and masses of observed young clusters is that they evolve with time from the time of their birth, when most of the radiative pressure feedback has occurred. Indeed, observations show a clear relation between cluster sizes and age (e.g., see Portegies Zwart et al. 2010 for a recent review, in particular their Figure 8). However, how the age–size relation scales with cluster mass is not obvious, especially for very massive young star clusters deeply embedded in dust (e.g., Gilbert et al. 2000; Gilbert & Graham 2007), where radiation pressure is thought to be particularly efficient. Due to these uncertainties, we refrain from applying an age correction to the data shown in the Figure 18.

**A.1. A Subgrid Model for Radiation Pressure Momentum**

As the above discussion illustrates, a direct implementation of radiation pressure momentum injection is not feasible in galaxy formation simulations, as it requires a resolved density structure of star-forming clouds at parsec scales. For more general application, it is useful to develop a subgrid model based on empirical knowledge of structure and physics of star-forming molecular clumps, which could be valid for different spatial resolutions. The proposed model is local in its nature, acting only in the local resolution elements surrounding young stars, and therefore does not account for continuous acceleration of gas far outside of galactic disks, as recently proposed by Murray et al. (2011), and modeled in the radiative transfer simulations by Wise et al. (2012).

In this model, any star particle formed by star formation recipe in simulations is regarded as an ensemble of star clusters, with an associated ensemble of natal molecular clumps onto which radiation pressure acts at early times. Such model can then be used to calculate the total momentum input from stars in all clumps within star-forming cell by integrating over the clump mass function.

We start off by defining the rate of momentum deposition imparted by radiation pressure from young stars on a molecular clump as

\[
\dot{\rho}_{\text{cl}} = (\eta_1 + \eta_2 \tau_{\text{IR}}) \frac{L(t)}{c}, \quad (A3)
\]

where \( L(t) \) is the bolometric luminosity of stars in a star cluster of age \( t \), and as before \( \tau_{\text{IR}} = \kappa_{\text{IR}} \Sigma_{\text{cl}} \) is the IR optical depth. The first term describes the direct radiation absorption/scattering and should in principle be \( \propto [1 - \exp(-\tau_{\text{UV}})] \), but given that UV optical depth is always very large in dense star-forming regions, \( \eta_1 \gg 1 \). To account for momentum injection by stellar winds, which is also of order \( L/c \) (see Section 1), we adopt \( \eta_2 = 2 \) as our fiducial value. The optical and UV photons heat dust particles in surrounding gas and IR photons radiated by dust can transfer additional momentum if gas is optically thick to the IR radiation. The second term thus specifies momentum transferred via multiple scatterings of IR photons re-radiated by dust particles (see, e.g., Gayley et al. 1995), where \( \eta_2 \) is added to parameterize possible modifications to the adopted \( \tau_{\text{IR}} \). As we discuss below, our fiducial choice is \( \eta_2 = 2 \), although factors of a few maybe be motivated due to grid smearing/cancellations for large advection velocities, as discussed below.

By integrating \( \dot{\rho}_{\text{cl}} \) over the clump mass function, we obtain the total imparted momentum rate from all star clusters onto their natal clumps,

\[
\dot{\rho}_{\text{tot}} = \int_{M_{\text{cl,min}}}^{M_{\text{cl,max}}} \dot{\rho}_{\text{cl}} \psi(M_{\text{cl}}) dM_{\text{cl}}, \quad (A4)
\]

where the minimum and maximum clump masses \( (M_{\text{cl,min}} \text{ and } M_{\text{cl,max}}) \) set the normalization of the cluster mass function. We approximate the observed mass function of molecular clumps by a power law

\[
\psi(M_{\text{cl}}) = A_{\text{cl}} M_{\text{cl}}^{-\beta}, \quad (A5)
\]

with \( \beta \approx 1.7 \pm 0.2 \) (Kramer et al. 1998) similar to the power-law slope of the molecular clouds themselves (e.g., Fukui & Kawamura 2010). The latter can be approximated by a Schechter-like function with exponential cutoff. The mass...
function of young star clusters can also be approximated by the Schechter form (Zhang & Fall 1999; de Grijs et al. 2003; Bik et al. 2003; Cresci et al. 2005; McCrady & Graham 2007b; see Portegies Zwart et al. 2010 for a review). Different studies that sample different parts of cluster mass function and often approximate it with a simple power law can get somewhat different values of the slope. Nevertheless, the mass function of star clusters is generally found to be quite similar in shape to that of the molecular clouds and clumps. The similarity of mass function slopes implies that star formation efficiency in clumps, $e_{cl} \equiv M_{*, cl}/M_{cl}$ (where $M_{*, cl}$ is the mass of a star cluster a given clump of mass $M_{cl}$ forms), is approximately independent of clump mass (Fall et al. 2010b).

We interpret a formed star particle of mass $m_{*}$ as the total mass of an ensemble of star clusters formed with the constant efficiency $e_{cl}$ from molecular clumps with a mass function given by Equation (A5). Although the actual mass function may have different parts of cluster mass function and often approximate it with a simple power law can get somewhat different values of the slope. Nevertheless, the mass function of star clusters is generally found to be quite similar in shape to that of the molecular clouds and clumps. The similarity of mass function slopes implies that star formation efficiency in clumps, $e_{cl} \equiv M_{*, cl}/M_{cl}$ (where $M_{*, cl}$ is the mass of a star cluster a given clump of mass $M_{cl}$ forms), is approximately independent of clump mass (Fall et al. 2010b).

Equation (A4) may now be evaluated. The relation has two terms; let us consider them in turn.

The first term is independent of surface density and, when integrated over the clump mass function, will simply give

$$\dot{P}_{tot, 1} = \eta_{1} \frac{L_{1}(t)}{c} m_{*},$$

(A9)

where $t$ is the age of a given stellar particle of mass $m_{*}$ in the cell under consideration. This contribution can simply be summed up for all young stellar particles in the cell with ages as old as it is deemed to be significant, typically for a few Myr.

The second term depends on the surface density. Before the parent molecular clumps are dispersed by their child star clusters, i.e., for time less than clump lifetime $t < t_{cl}$, radiation pressure operates on the surface density of the clumps,

$$\Sigma_{cl} = (1 - e_{cl}) \frac{M_{cl}}{2 \pi R_{cl}^{2}},$$

(A10)

where the $(1 - e_{cl})$ factor takes into account the fact that the fraction $e_{cl}$ of clump mass was turned into stars, while the factor of 0.5 takes into account that $R_{cl}$ in the assumed mass–radius relation is the half-mass radius. Thus, for $t < t_{cl}$, using the equations above, and integrating over the clump mass function, we obtain

$$\dot{P}_{tot, 2a} = \frac{\eta_{2} \kappa \Sigma_{gas,c} R_{cl} m_{*}}{2 \pi C_{R}^{2}} \left(1 - (M_{cl, min}/M_{cl, max})^{3 - 2\alpha - \beta} \right) L_{1}(t) \frac{L_{1}(t)}{c} m_{*}^{2(1-\alpha)}.$$

(A11)

For $t > t_{cl}$, the clump is dispersed and the radiation pressure will simply act on the surface density of the cell, $\Sigma_{gas,c}$, with a possible boosting by some clumping factor to account for a clumpy nature of parent molecular cloud. The latter can be introduced via $\eta_{2}$. The total momentum rate in this case will therefore be

$$\dot{P}_{tot, 2b} = \frac{\eta_{2} \kappa \Sigma_{gas,c} R_{cl} m_{*}}{2 \pi C_{R}^{2}} \left(1 - (M_{cl, min}/M_{cl, max})^{3 - 2\alpha - \beta} \right) L_{1}(t) \frac{L_{1}(t)}{c} m_{*}^{2(1-\alpha)}.$$

(A12)

Summarizing, the total momentum rate is

$$\dot{P}_{tot} = \begin{cases} \dot{P}_{tot, 1} + \dot{P}_{tot, 2a} & \text{if } t < t_{cl}, \\ \dot{P}_{tot, 1} + \dot{P}_{tot, 2b} & \text{if } t > t_{cl}. \end{cases}$$

(A13)

The clump lifetime $t_{cl}$ is a highly uncertain factor, but can reasonably be assumed to be a fixed multiple of the clump crossing time: $t_{cl} = N_{c} t_{c}$ with $N_{c} \sim 5–10$ (Palla & Stahler 2000), where the crossing time $t_{c}$ is given by

$$t_{c} \equiv \frac{R_{cl}}{\sigma} = \frac{M_{cl}^{\alpha-1/2}}{\sqrt{0.4G}}.$$

(A14)

where $R = C_{R} M_{cl}^{\alpha}$ relation was used. For massive clusters, $C_{R} \approx 2.5$ pc, $\alpha \approx 0$, and

$$t_{c} = 9.3 \times 10^{4} \left(\frac{M_{*, cl}}{10^{6} M_{\odot}}\right)^{-1/2} \text{yr.}$$

(A15)

Thus, for $N_{c} \sim 5–10$, the lifetime of a clump is $0.5–1 \times 10^{8}$ yr. However, the first stage of cluster life before clump dispersal
is highly uncertain (e.g., Portegies Zwart et al. 2010) and we do not know either the exact lifetime or its scaling with cluster mass. Recent observations of ongoing massive star formation (Martínez-Galarza et al. 2012) suggest that this embedded phase may last for $\gtrsim 4$ Myr, and that dust temperatures can reach $\sim 300$ K.

In the current work, we adopt the above model for radiation pressure feedback assuming $t_{\text{f}} = 3$ Myr, $\varepsilon_d = 0.2$, $\mu_{\text{max}} = 1$, $\epsilon_{\text{IR}} = 5 (Z/Z_{\odot})$ cm$^2$ g$^{-1}$, and $\eta_1 = \eta_2 = 2$ for our fiducial model, unless noted otherwise. We note that for the total amount of injected momentum into the ISM, multiplicative changes in $\mu_{\text{max}}$ and $t_{\text{f}}$ has roughly the same impact; $t_{\text{f}}$ simply increases the duration of a given momentum injection rate (during the embedded phase), while $\mu_{\text{max}}$ increases the injection rate at a fixed duration. In Section 4.3.1, we explore some of these dependencies.

The metallicity scaling on the opacity is a crude way of accounting for the varying dust-to-gas ratios. The values of $\eta$ account for the fact that the actual measured momentum injected by a star particle is found to be reduced during advection of gas through computational mesh.\footnote{This is due to two effects: smearing occurring at the grid level and momentum cancellations introduced by the nearest grid point approach. The latter occurs as a star particle discretely switches cells and reduces the momentum flux injected from a previous time step. We have measured this effect to be on the order of $\sim 15\%$--$25\%$ per spatial dimension for translational velocities (relative to the grid) up to $v \sim 1000$ km s$^{-1}$.}

A.2. Caveats Related to the Stellar Mass used for Radiation Pressure

The particle mass $m_\star$ entering the terms of Equation (A13) does not necessarily need to be interpreted as the mass of each star particle. In fact, this is not preferred as the strength of radiation pressure depends on $m_\star$ in a nonlinear fashion. As the particle mass usually is a function of numerical resolution, and the effect of radiation pressure would weaken at higher resolution when the numerical scheme allows for the formation of lower mass star particles. When calculating the effect of radiation pressure we therefore adopt

$$m_\star \rightarrow m_\star = \sum_{i=1}^n m_{\star,i}(t) \quad \text{for} \quad t \leq t_{\text{young}},$$

(A16)

i.e., $m_\star$ is the binned mass of all $n$ star particles in a cell younger than some given age $t_{\text{young}}$. Our fiducial choice of this timescale is twice the clump lifetime $t_{\text{cl}}$ defined above, i.e., $t_{\text{young}} = 6$ Myr is our fiducial value. This is a conservative choice, as star formation could proceed for longer, but is consistent with estimates of the duration of the embedded stage of young clusters (e.g., Portegies Zwart et al. 2010) for which most of the momentum is delivered to the gas. Adopting $t_{\text{young}}$ in the range 3–10 Myr does not change the results significantly.

Another caveat in our implementation of radiation pressure is that we only consider the young stars in one computational cell when estimating $t_{\text{f}}$. The actual cell has nothing to do with the physics of the problem, and is just a convenient implementation choice. At very high resolution, $\Delta x \sim$ few parsec, star formation will be spread over several cells in massive GMCs and the collective effect of radiation pressure will be underestimated. This issue can be avoided by searching neighboring cell for young stars, or by using a star cluster finder, as suggested by Hopkins et al. (2011). In the regime of $\Delta x \sim 50$–100 pc, adopted for the galactic disks in Section 4.3, where cell sizes matches observed sizes of massive GMCs, this is not an issue.

APPENDIX B

IMPLEMENTATION OF NON-THERMAL PRESSURE

As discussed in Section 3, feedback momentum can be injected directly to the computational grid via “kicks.” An alternative approach is to allow for the hydro scheme to generate momentum by appropriately pressurizing the local volume in which momentum is injected. In this approach, which we adopt in a subset of our simulations, we define the non-thermal pressure due to injected momentum as

$$P_{\text{nt}} = \dot{\rho}/A,$$

(B1)

where the area $A$ is an arbitrary computational region, here chosen to be the surface area of computational cell ($A = 6\Delta x^2$) containing a young star particle used to compute the momentum injection in the subgrid model described above. This pressure is added to the effective pressure,

$$P_{\text{eff}} = P_{\text{therm}} + P_{\text{nt}},$$

(B2)

where $P_{\text{therm}}$ is the thermal pressure. $P_{\text{eff}}$ replaces $P_{\text{therm}}$ in the sound speed definition when calculating the time step, and is otherwise only actively involved in the flux calculation (i.e., in the Godunov step). Here, $P_{\text{eff}}$ is consistently traced to the cell interfaces (here using the piecewise linear approximation) as a separate pressure variable, using its own Total-Variation-Diminishing (TVD) limited slopes. The left- and right-hand states of $P_{\text{eff}}$ then enter the Riemann solver, where it replaces the thermal pressure. Specifically, the momentum and energy equations that we aim to solve are

$$\frac{\partial}{\partial t} (\rho v) + \nabla \cdot (\rho v \otimes v + P_{\text{eff}}) = -\rho \nabla \phi$$

(B3)

and

$$\frac{\partial}{\partial t} (\rho E) + \nabla \cdot [\rho v (E + P_{\text{eff}}/\rho)] = -\rho v \cdot \nabla \phi,$$

(B4)

where $E$ is the specific total energy, $\phi$ is the gravitational potential, and $\otimes$ is the outer vector product. The effective pressure hence never enters into the specific total energy (as $P_{\text{eff}}/\rho (\gamma - 1)$) which is an important distinction to make in the MUSCL-scheme (van Leer 1979) adopted in the RAMSES code (Teyssier 2002), as it traditionally only stores one variable representing the total energy (the conservative variable), and pressure (the primitive variable) is derived from it after subtracting the kinetic energy. To universally replace $P_{\text{therm}}$ by $P_{\text{eff}}$ everywhere in the method would hence not be consistent with the equations we want to solve (as well as in the cooling routines). For our implementation of radiation pressure, the effective pressure is not advected, but is updated every finite time step in the feedback routine, and stored as a separate variable.

In the case of the second feedback energy variable $E_{\text{fb}}$, introduced in Section 3.2, we calculate a non-thermal pressure $P_{\text{nt}} = (\gamma - 1)\rho E_{\text{fb}}$, which enters the effective pressure as in Equation (B2). This quantity is then treated exactly as described above, although $E_{\text{fb}}$ is passively advected with the flow, i.e., it obeys

$$\frac{\partial}{\partial t} (\rho E_{\text{fb}}) + \nabla \cdot (\rho v E_{\text{fb}}) = 0.$$

(B5)
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