A Framework for On-Line Devanagari Handwritten Character Recognition

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Abstract

The main challenge in on-line handwritten character recognition in Indian language is the large size of the character set, larger similarity between different characters in the script and the huge variation in writing style. In this paper we propose a framework for on-line handwritten script recognition taking cues from speech signal processing literature. The framework is based on identifying strokes, which in turn lead to recognition of handwritten on-line characters rather than the conventional character identification. Though the framework is described for Devanagari script, the framework is general and can be applied to any language.

The proposed platform consists of pre-processing, feature extraction, recognition and post processing like the conventional character recognition but applied to strokes. The on-line Devanagari character recognition reduces to one of recognizing one of 69 primitives and recognition of a character is performed by recognizing a sequence of such primitives. We further show the impact of noise removal on on-line raw data which is usually noisy. The use of Fuzzy Directional Features to enhance the accuracy of stroke recognition is also described. The recognition results are compared with commonly used directional features in literature using several classifiers.

Keywords: On-line handwriting recognition, Pre-processing, smoothing, knotless spline, curvature points, Fuzzy Directional Features, Dynamic Time Warping, Support Vector Machine

1. Introduction

In recent times PDAs, palms and handheld PCs are more frequently being used for composing short messages and e-mails. While composing messages on these devices using conventional keypads is difficult because of their small form factor, these devices come equipped with significant computing power making recognizing handwritten message composition on board the device possible. While use of keyboard for composing English language may be difficult
but feasible, message composition through handwritten characters becomes the only feasible way to compose messages in most Indian languages. Compactness of devices are paving way to electronic pen (e-pen) and/or a stylus touching a pressure sensitive surface are some of the popular non-keyboard data entry devices that is gaining popularity [34, 35, 23]. These input devices capture the pen lifts and the trace of the pen movement thus capturing handwritten strokes which is essentially a trace of the pen between a pen down and a pen up. These traced strokes can then be converted into electronically transferable character string using On-line Handwritten Character Recognition (OHCR) algorithms. Typically a trace of a pen between a pen-down and a pen-up is a set of $x, y$ points which is uniformly sampled in time (typically at 100 Hz). These set of $x, y$ points are non-uniformly sampled in space.

An OHCR system (see Fig. 1) generally consists of a learning phase and a testing phase. In the learning phase the system learns and builds reference models for all possible characters that need to be recognized. In the testing phase the character is compared with the reference models of the character using a classifier to determine the best matching reference character.

The choice of the feature set to represent the on-line character determines the ability of OHCR algorithm to distinguish one character from another while being able to cluster together the same characters written at different times and by different people. A typical OHCR recognition process would consist of a pre-processing module followed by feature extraction and a suitable classifier.

On-line $x, y$ data acquired for the purpose of OHCR is most often uniformly sampled in time, making the captured $x, y$ data non-uniform in space. The outcome of this is that the data points representing a character is dependent on the time taken to write the character, making the number of data points
representing a character different for the same character and of the same size. To overcome this non-uniform number of \(x, y\) points representing a character many popular OHCR approaches make sure that the on-line data is pre-processed so as to obtain data that is uniformly sampled in space [2, 42, 18]. While there are benefits in terms of the type of classification tools that one can use, it has a fundamental problem of not being able to exploit the crucial curvature information that is embedded in the on-line data that is uniformly sampled in time. In this paper, we stick to the original uniformly sampled in time data produced by an e-Pen. The difference between uniformly sampled in space and uniformly sampled in space is that in the later case every stroke or character has the same number of \((x, y)\) points, while when uniformly sampled in time the number of \((x, y)\) points are different.

Devanagari script is a widely used Indian script being used by more than 500 million people. It consists of vowels and consonants as shown in Fig. 2. It is used as the writing system for over 28 languages including Sanskrit, Hindi, Kashmiri, Marathi and Nepali [44]. Devanagari is written from left to right in horizontal lines and the writing system is alphasyllabary. In English script barring a few alphabets, all the alphabets can be written in a single stroke. In contrast, in most Indian languages, characters are made up of two or more strokes. This writing requirement makes it necessary to analyze a sequence of adjacent strokes to identify a character.

In Devanagari, like in most Indian languages, for a consonant vowel combination, the vowels are orthographically indicated by signs called *matras*. These modifier symbols are normally attached to the top, bottom, left or right of the base character which is highly dependent on the consonant vowel pair. In Indian languages, the consonants, the vowels, the matras and the consonant/vowel

\[1\text{A stroke is defined as the resulting trace between a pen-down and its adjacent pen-up}\]
modifiers constitute the entire alphabet set. These composite characters are then joined together by a horizontal line, called *shirorekha* (see Fig. 3) to form words.

Devanagari character[^2] is made up of multiple strokes, so the identification of an alphabet can be achieved by recognizing multiple strokes that make an alphabet. We earmarked, through visual inspection of handwritten Devanagari script, a basis like set of 69 strokes which we call them as *primitives*. The set of all *primitives* are shown in Fig. 4. The set of 50 primitives (Fig. 4(a)) are good enough to construct the entire character set in Devanagari. But we found that an additional set of 19 *primitives* (Fig. 4(b)) are often used by writers[^3] in Devanagari script. These set of 69 *primitives* are sufficient to generate Devanagari script. Note that these *primitives* can be combined together so as to form all the characters in Devanagari script. Fig. 5 shows an example where *primitives* (m, ou, R, A, Ab, ···) are combined together to form characters, resulting into words.

[^2]: we use character and alphabet interchangeably in this paper
[^3]: a set of 100 hand written paragraphs by different people were analyzed
In this paper we assume that we can recognize the Devanagari characters by recognizing the primitives and analyzing a sequence of primitives to identify a character.

In an unconstrained handwritten script these primitives exhibit large variability even for the same writer making the task of recognizing primitives and hence characters difficult. In this paper we use these primitives as the units for recognition taking parallel from the recognition of phone set used in speech recognition literature. The main contribution of this paper is

1. Identification of a set of primitives which encompass Devanagari script,
2. Development of a framework to enable Devanagari script recognition by recognizing primitives,
3. Use of pre-processing techniques to enhance primitive stroke recognition accuracies,
4. Use of fuzzy directional feature (FDF) set to represent the on-line characters [43] and
5. Use of relative spatial position of the primitives to enhance recognition.

The rest of the paper is organized as follows. We review the state of the art OHCR for Indian languages in Section 2 followed by the proposed OHCR framework in Section 3. The pre-processing techniques for noise removal of the on-line handwritten data are described in Section 4 which also discusses the procedure for identifying the critical points and gives an analysis of number of critical points identified on each primitive. We describe the feature extraction techniques including Directional Features (DF), Extended Directional Features (EDF) and our [43] Fuzzy Directional Feature (FDF) in Section 5. The recognition experiments conducted for stroke level recognition using different classifiers including second order statistics based classifier, Discrete Time Warping (DTW) and Support Vector Machine (SVM) are described in Section 6. We conclude in Section 7.

2. Indian language OHCR - An Overview

We give an overview of the recent advances, new trends and important contributions in the area of OHCR for Indian languages. On-line handwriting recognition is of prime importance especially in the context of Indian languages because of the fact that entering Indian language scripts is both difficult and time consuming. Currently, word processing in Indian languages can be a vexing experience, considering the restriction on use of the regular keyboard, designed for English. Elaborate keyboard mapping systems are normally used in case of Indian languages, which are not convenient to use. While a large amount of OHCR literature is available for on-line handwriting recognition of

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4 A SIG on Indian Language SMS has been working on various issues related to composing an SMS message in Indian languages on a mobile device; however the concentration has been on using the mobile keypad so far.
English, Chinese and Japanese languages, relatively very less work has been reported for the recognition of Indian languages. In the case of Indian languages, research work has been active for Devanagari \[18, 25\], Bangla[29, 6], Tamil \[28, 4, 19, 37, 1\] and Telugu \[3, 31\].

Joshi et al.\[18\] describe a system for the automatic recognition of isolated handwritten Devanagari characters obtained by linearizing consonant conjuncts. They used structural recognition techniques to reduce some characters to others. The residual characters are then classified using the subspace method. Finally the results of structural recognition and feature based matching are mapped to give a final output and the system is evaluated for writer dependent scenario. In another work\[30\], a set of stroke templates is derived from analysis of common writing styles of different Devanagari characters, and each character represented by a set of combinations of these stroke templates. In \[12\] the authors use a combination of two HMM classifiers trained with on-line features and three Nearest Neighbor classifiers each trained on different sets of on-line features for Devanagari character recognition. The combination of on-line and off-line classifiers is shown to improve the accuracy from 69.2% (on-line, HMM alone) to 86.5%.

Tamil on-line handwriting recognition has also been attempted with varying degree of success. In \[38\] the problem of representation of Tamil characters is considered. In another work \[40\] the subspace based on-line recognition of Tamil and other Indian languages is described. Joshi et al. \[19\] use template based elastic matching algorithms for on-line Tamil handwriting recognition. They use the advantage of elastic matching algorithms which do not require a very large amount of training data, making them suitable for writer dependent recognition. Aparna et al. \[2\], represent Tamil character strokes as strings of shape features. In order to recognize an unknown stroke, its equivalent feature string is computed. The test stroke is then identified by searching the database using a flexible string matching algorithm. Once all the strokes in the input are known, the character is determined using a Finite State Automaton. Prior knowledge about popular writing styles has been exploited to design a first stage classifier for Tamil characters in \[36\]. The authors observe that the start of any Tamil character is either a line, semi-loop or a loop. Accordingly, the candidate choices are pruned during recognition. Artificial Neural Network (ANN) based approach is also proposed \[37\] for the recognition of on-line Tamil characters. In another effort on Tamil character recognition \[28\], templates are identified from the training set using Agglomerative Hierarchical Clustering and Learning Vector Quantization (LVQ) with dynamic time warping (DTW) as the distance measure. A DTW-based Nearest Neighbor classifier is then employed for matching the test sample.

In \[7\], the authors describe a novel direction code based feature extraction approach for recognition of on-line Bangla handwritten basic characters. It is a 50-class recognition problem and they achieved 93.90 % and 83.61 % recognition accuracies on training and test sets respectively. For the problem of Telugu character recognition, PVS Rao et al. \[31\] perform coarse matching with the templates using the number of \(x-y\) extrema points in the test sample and
affine matching using DTW. HMMs have also been used for Telugu on-line handwritten character recognition. In [20], feed-forward Neural Networks with a single hidden layer have been used for the recognition of handwritten Kannada characters. The authors use approximation coefficients derived from Wavelet decomposition on the pre-processed \((x, y)\) as features for representing the characters.

3. Proposed System Architecture

We proposed a framework for Indian language OHCR system (Fig. 7) where we can use the stroke recognition for Devanagari handwritten script recognition. The stroke recognition would be language independent, though the shape of the strokes and the number of strokes might vary from one language script to another language script. Initially the on-line data (see Fig. 6) is acquired and a spatio-temporal analysis of the individual strokes is done. Typically, this analysis provides the ability to segment the paragraphs into words (based on shirorekha identification); identify matras by identifying the relative position of the strokes. This analysis can be used to improve the performance of the stroke recognition. For example, having identified a matra based on the spatial position of the stroke, we could constrain the recognition to only the reference matras. The individual strokes are then recognized to be one of the 69 primitives. The actual stroke recognition has several steps; the stroke is first pre-processing,
and features extracted from each stroke before stroke recognition. The stroke level recognition is modified based on the failure of character recognition to put together a sequence of strokes to form a character. The error analysis block helps in improving the stroke recognition. Rules for character formation from a sequence of strokes and the spatio-temporal knowledge help in ascertaining if the recognized sequence of strokes are valid or not. The framework has a word recognition module which is very general and based on a lexicon making it adaptable for any Indian language. Lexicons and other language models are an important aspect of achieving acceptable accuracy for OHCR. Barring a few reported methods that use lexicons, the use of language models has not been exploited substantially for Indian scripts. Word recognition uses the characters identified along with the spatio-temporal information and domain based lexicon word level knowledge.

The described framework in Fig. 7 is influenced by the speech recognition literature. The strokes are analogous to phonemes in speech. It is well known in speech literature that the phoneme recognition accuracies are poor, however the
final output of the speech recognition is significantly high. The poor phoneme recognition in speech recognition is enhanced by lexicons and language models. In a similar fashion, in the proposed framework we argue that the use of lexicon will improve the OHCR. In this paper, we restrict our discuss to stroke recognition.

4. Stroke Recognition

We analyzed handwritten script by 100 people and identified a set of 69 strokes (Fig. 4) which we will refer to as primitives) which encompassed the Devanagari script. These primitives are shown in Fig. 5. The rest of the paper deals with the recognition of Devanagari script by the recognition of these primitives.

Let a primitive be represented by a variable number of 2D points which are in a time sequence. For example an on-line script would be represented as

$$\{(x_{t_1}, y_{t_1}), (x_{t_2}, y_{t_2}), \cdots, (x_{t_n}, y_{t_n})\}$$  \hspace{1cm} (1)

$t$ denotes the time and assume that $t_1 < t_2 < \cdots < t_n$ and $n$ represents the total number of points. Equivalently we can represent the on-line data as

$$\{(x_1, y_1), (x_2, y_2), \cdots, (x_n, y_n)\}$$  \hspace{1cm} (2)

by dropping the variable $t$. The number of points denoted by $n$ vary depending on the size of the primitive and also the time taken to write the primitive. Most script digitizing devices (popularly called electronic pen) sample the script uniformly in time. For this reason, the number of sampling points is large when the writing speed is slow which is especially true at curvatures.

Noise in on-line script is inherent. As seen in Fig. 8 the handwritten character is far from smooth. The number of data points is very sparse especially when the pen movement is fast and when the pen movement is slow they are prone to be contaminated by high frequency noise.

Figure 8: Sample on-line Devanagari characters $u$ and $k$: Raw data
There are essentially two types of noise that contributes to the noisy data, (a) the inherent shake of the hand of the writer especially at the beginning and end of the stroke and (b) contribution by the noise creeping in due to digitization process. Fig. 8 (a) and (b) show samples on-line Devanagari primitives $u$ and $k$ collected from a writer using Mobile e-Note Taker [34]. We can observe that the characters are not smooth and contaminated by noise. The noise severely affects the performance of on-line character recognition algorithms. There are essentially two ways of taking care of the noise, (a) in the first case an appropriate noise removal algorithm is used on the raw noisy data and (b) in the second case one uses a feature extraction algorithm that can compensate for the noise.

Pre-processing is a necessary first step in OHCR. In the next sections we discuss noise removal pre-processing techniques and show that noise removal helps in improving stroke recognition.

4.1. Smoothing based Noise Removal

Smoothing and filtering are the two important techniques used for noise reduction. Smoothing usually averages a data point with respect to its neighboring data points such that there is no large variation between adjacent data points (for example [15, 8]). Filtering on the other hand eliminates duplicate data points and reduces the number of points [39]. Some filtering techniques force a minimum distance between adjacent data points (for example [8, 45]) which tend to produce data points that tends to be equally spaced. According to the writing speed, the distance between the points may vary significantly and interpolation can be used to obtain uniformly spaced points. In some filtering, minimum change in the direction of the tangent to the drawing for consecutive points are maintained [5]. This produces more points in the greater curvature regions. Filtering can also be done by the use of convolution with one dimensional Gaussian kernels [27], which reduces the noise due to pen movement and errors in the sensing mechanism. Joshi et al. [18] reduced the effect of noise with the help of 5-tap low pass Gaussian filter, where each stroke is filtered separately. Malik et al. [21] proposed time domain filter by using the convolution of input sequence with a finite impulse response for smoothing a jitter appeared in the sequence. Although many techniques exist in literature to suppress noise, it is very difficult to select a technique such that it can work equally for all types of strokes. In some other studies smoothing and filtering are performed as part of single operation. An example of this is piecewise-linear curve fitting [32, 17].

4.1.1. Wavelet based noise removal

Here as part of noise removal we performed smoothing on the raw data using Discrete Wavelet Transform (DWT) based decomposition using Daubechies wavelet\textsuperscript{6}. The DWT decomposition helps in removing noise due to small undulation caused due to the sensitiveness of the sensors on the electronic pen and

\textsuperscript{6}We do not dwell on this since this is well covered in Digital Signal Processing literature
inherent shake while writing. Fig. 12 (a) and (b) show the noise removal due to DWT on the Devanagari characters \( u \) and \( k \).

4.1.2. Knotless Spline for noise removal

This noise removal technique is based on splines. Different spline ([13, 10]) based smoothing have been successfully applied for denoising noise contaminated signal for example, [20, 16]. However, the degree of smoothness depends on the number and position of the control points and chosen knots. If the knots are close to each other, the smooth curve between the two knots would be linear. If the knots are far apart, a higher order polynomial would be needed for fitting a smooth curve between the two knots. The knotless spline technique uses a cubic spline based polynomial approximation with the knots being selected automatically. Hence, the smoothing technique becomes a cubic polynomial curve fitting with a variable span.

Let the sequence \((x_i^r, y_i^r)_{i=0}^n\) represents a handwritten stroke made up on \( n \) points. For the purpose of noise removal we treat the sequence \( x_i^r \) and \( y_i^r \) separately and remove noise from each of these sequences independently. The noise removal process is described below for the sequence \( x_i^r \).

1. Set the span to be \( n/2 \) data points (consider only \( n/2 \) of the original \( n \) points, namely, \( \{x_i^r\}_{i=0}^{n/2} \))
2. Fit a cubic spline in the span compute and a mean squared error (MSE) is calculated between the fitted spline and the actual data points, namely, find \( \{a_i\}_{i=1}^3 \) such that \( f(x_i^r) = a_0 + a_1 x_i^r + a_2 x_i^r^2 + a_3 x_i^r^3 \) such that

\[
MSE = \sum_{i=1}^{n/2} (f(x_i^r) - x_i^r)^2
\]

is minimum
3. Reduce the span by 25% (namely, consider \( \{x_i^r\}_{i=0}^{n/2-n/8} \) and repeat Step 1 and 2 until the span is 20% of the initial span.
4. The span with smallest MSE is selected as the optimum span with the starting and end points of the span are the chosen knots and a cubic spline is fitted in this span.
5. Repeat on the remaining data points until all points are covered.

It is to be noted that this process automatically selects the number and the location of the knots unlike other spline denoising techniques which requires the user to specify the number of knots. We compare the effect of noise removal using (a) wavelet denoising technique and (b) the method proposed above; the results of smoothing the \( x \) and \( y \) sequence separately for the Devanagari character \( u \) are shown visually in Fig. 9, 10 and 11.

\[7\] both the number of knots and the location of the knots
\[8\] span is defined as the distance between the two consecutive knots
Figure 9: (a)-(b) $x$ and $y$ sequences of Devanagari characters $u$: Raw data

Figure 10: (a)-(b) $x$ and $y$ sequences of Devanagari characters $u$: Smoothed using DWT

Figure 11: (a)-(b) $x$ and $y$ sequences of Devanagari characters $u$: Smoothed using the proposed procedure
Fig. 12 and Fig. 13 shows on-line Devanagari characters $u$ and $k$: Filtered using DWT and smoothed using proposed method.

Fig. 12 and Fig. 13 shows on-line Devanagari characters $u$ and $k$: Filtered using DWT and smoothed using proposed method.

We conducted a separate recognition experiment for evaluating the noise removal technique based on (a) Raw data (without pre-processing) (b) DWT based denoised data and (c) the knotless spline based pattern smoothing technique. The average stroke recognition accuracies obtained for 69 Devanagari characters were about 51.21% for raw data, 60.86% for DWT based smoothed data and about 70.29% for the data pre-processed using the knotless spline method. This improvement in recognition clearly demonstrates that noise removal is a necessary for improved performance in OHCR.

5. Direction Based Feature Extraction

Several temporal features have been used for script recognition in general and for on-line Devanagari script recognition in particular [22, 14, 33, 11]. We discuss feature set based on directional properties of the curve connecting two
consecutive critical points identified on a Devanagari primitive. In particular we discuss three different types of features based on directional properties.

5.1. Critical Point Extraction

The curvature points (also called critical point) are extracted from the smoothed handwritten data. The denoised sequence \((x_i, y_i)^n_{i=0}\) represents a noiseless handwritten stroke. We treat the sequence \(x_i\) and \(y_i\) separately and calculate the critical points for each of these sequence. For the \(x\) sequence, we calculate the first difference \((x'_i)\) as

\[ x'_i = \text{sgn}(x_i - x_{i+1}) \]

where

\[
\text{sgn}(k) = +1 \quad \text{if} \quad x_i - x_{i+1} > 0 \\
\text{sgn}(k) = -1 \quad \text{if} \quad x_i - x_{i+1} < 0 \\
\text{sgn}(k) = 0 \quad \text{if} \quad x_i - x_{i+1} = 0
\]

We use \(x'\) to compute the critical point in \(x\) sequence. The point \(i\) is a critical point \(i\)ff \(x'_i - x'_{i+1} \neq 0\). Observe that \(x'_i - x'_{i+1}\) is the second difference. Similarly we calculate the critical points for the \(y\) sequence. The final list of critical points is the union of all the points marked as critical points in both the \(x\) and the \(y\) sequence (see Fig. 14, 15 and 16). It must be noted that the position and number of critical points computed for different samples of the same strokes may vary significantly especially in the presence of noise. Fig. 14 shows the critical points identified from the original samples of on-line Devanagari characters \(u\) and \(k\).

Fig. 15(a) and (b) shows the critical points identified from the DWT filtered on-line Devanagari characters \(u\) and \(k\) respectively, while Fig. 16(a) and (b) shows the critical points identified from the on-line Devanagari characters \(u\) and \(k\) smoothed using the knotless spline method. It is clear from the critical points identified that the number and position of the critical points are more consistent and occur at the points where there is a change in curvature when we apply the
Figure 15: (a)-(b) Curvature points on on-line Devanagari characters $u$ and $k$: Filtered using DWT

Figure 16: (a)-(b) Curvature points on on-line Devanagari characters $u$ and $k$: Smoothed using knotless spline method
Table 1: Mean (\( \mu \)) and Variance (\( \sigma^2 \)) of the Number of critical points

| Primitive | Raw | DWT | Spline |
|-----------|-----|-----|--------|
|           | \( \mu \) | \( \sigma^2 \) | \( \mu \) | \( \sigma^2 \) | \( \mu \) | \( \sigma^2 \) |
| \( u \)   | 22.4 | 259.44 | 10.5 | 16.45 | 7.7 | 1.61 |
| \( i \)   | 19.9 | 103.29 | 9.7 | 1.81 | 7.7 | 0.41 |
| \( e \)   | 13.9 | 63.69  | 5.2 | 2.56 | 3.4 | 0.84 |
| \( k \)   | 29.3 | 113.21 | 14.9 | 7.49 | 11.9 | 1.49 |
| \( R \)   | 12   | 23.8   | 5.6 | 0.44 | 4.4 | 0.24 |
| \( v \)   | 18.4 | 64.64  | 9.2 | 3.96 | 7.6 | 1.44 |
| \( g \)   | 14.4 | 53.44  | 6.8 | 1.76 | 5.5 | 0.45 |
| \( gh \)  | 19.6 | 167.56 | 13.1 | 4.29 | 9.8 | 2.36 |
| \( D \)   | 32.7 | 88.44  | 7.1 | 6.89 | 5.6 | 3.24 |
| \( c \)   | 20.3 | 136.01 | 13  | 10.4 | 9.6 | 8.44 |

knotless spline based smoothing when compared to the critical point extraction on the raw data or the DWT.

To understand the effect of smoothing on the extraction of critical points we conducted an experiment on the primitive data. The number of critical points for 100 samples of 10 character primitives were computed and compared. Table 1 shows the mean and variance of the number of critical points obtained for a sample 10 primitives (a) without smoothing (raw data) (b) with DWT based smoothing (c) with knotless spline based smoothing. It is clear that the mean values obtained for knotless spline based smoothed data are very close to the expected values of the number of critical points, for respective primitives, compared to the mean values obtained for the raw data and the DWT smoothed data. The variance in the number of critical points also very less in the case of knotless spline based smoothed data. Which means that knotless smoothing helps in identifying close to the actual number of critical points in a primitive. The distribution curves obtained for the primitive \( u \) for (a) raw data (\( \mu = 22.4 \) and \( \sigma^2 = 259.44 \)), (b) DWT smoothed data (\( \mu = 10.5 \) and \( \sigma^2 = 16.45 \)) and (c) knotless spline based smoothed data (\( \mu = 7.7 \) and \( \sigma^2 = 1.61 \)) are shown in Fig. 17. From the distribution plot it is clear that the knotless spline based smoothing can be effectively used for noise removal and hence improve the efficiency of the feature extraction process.

5.2. Directional and Extended Directional Features

In the case of directional features (DF) we compute the directions between only the adjacent critical points. Suppose we have \( k \) critical points, then we will obtain the direction between the consecutive critical points by first determining the angles \( [\theta_{12}, \theta_{23}, \theta_{34}, \ldots, \theta_{k-1k}] \) and then \( [d_{12}, d_{23}, d_{34}, \ldots, d_{k-1k}] \) using Algorithm 1.

The extended directional feature (EDF) set is computed by computing all direction between one critical point and all other critical points following it.
Figure 17: Distribution curves for number of critical points obtained for (a) raw (b) DWT smoothed (c) knotless spline smoothed Devanagari character $u$.

Table 2: Extended Directional Features

| CP | $c_1$ | $c_2$ | $c_3$ | $c_4$ | $\cdots$ | $c_m$ | $\cdots$ | $c_k$ |
|----|------|------|------|------|---------|-------|---------|------|
| $c_1$ | 0    | $d_{12}$ | $d_{13}$ | $d_{14}$ | $\cdots$ | $d_{1m}$ | $\cdots$ | $d_{1k}$ |
| $c_2$ | $-$ | 0    | $d_{23}$ | $d_{24}$ | $\cdots$ | $d_{2m}$ | $\cdots$ | $d_{2k}$ |
| $c_3$ | $-$ | $-$ | 0    | $d_{34}$ | $\cdots$ | $d_{3m}$ | $\cdots$ | $d_{3k}$ |
| $c_4$ | $-$ | $-$ | $-$ | 0    | $\cdots$ | $d_{4m}$ | $\cdots$ | $d_{4k}$ |
| $\cdots$ | $\cdots$ | $\cdots$ | $\cdots$ | $\cdots$ | $\cdots$ | $\cdots$ | $\cdots$ | $\cdots$ |
| $c_l$ | $-$ | $-$ | $-$ | $-$ | $\cdots$ | $d_{lm}$ | $d_{lk}$ |
| $\cdots$ | $\cdots$ | $\cdots$ | $\cdots$ | $\cdots$ | $\cdots$ | $\cdots$ | $\cdots$ |
| $c_k$ | $-$ | $-$ | $-$ | $-$ | $-$ | $-$ | 0    |
These directions are indicated as the upper diagonal elements in Table 2. Where $d_{lm}$ corresponding to the angle $\theta_{lm}$ (computed using the Algorithm 1) and is the direction between the critical point $c_l$ and $c_m$. Given, $k$ critical points, we get an extended directional feature (EDF) vector of size

$$\frac{k(k-1)}{2}$$

(6)

while we get a directional feature (DF) vector of size $k$.

5.3. Fuzzy Directional Feature Extraction

We propose a simple yet effective feature set based on fuzzy directional feature set. Let $k$ be the number of critical points (denoted by $c_1, c_2, \ldots, c_k$) extracted from a stroke of length $n$; usually $k << n$. The $k$ critical points form the basis for extraction of the directional features and the FDF. We first compute the angle between two critical points, say $c_l$ and $c_{l+1}$, as

$$\theta_l = \tan^{-1}\left(\frac{y_l - y_{l+1}}{x_l - x_{l+1}}\right)$$

(7)

where $(x_l, y_l)$ and $(x_{l+1}, y_{l+1})$ are the coordinates corresponding to the critical point $c_l$ and $c_{l+1}$ respectively.

Note that we divide $2\pi$ into eight directions with overlap. Every $\theta_l$ (for example the angle $\theta$ that the blue dotted line makes with the horizontal axis in Fig. 18) has two directions (say $d^1_l = 1, d^2_l = 2$, note that the line making an angle $\theta$ with the dotted line in Fig. 18 lies in both the triangles represented by direction 1 and direction 2) associated with it having $m^1_l, m^2_l$ membership values respectively (represented by the green and the red dot respectively in Fig. 18. Also note

1. $m^1_l + m^2_l = 1$ and
2. $d^1_l, d^2_l$ are adjacent directions, for example if $d^1_l = 5$ then $d^2_l$ could be either 4 or 6.

We use $\theta$ in Algorithm 1 assisted by triangular membership function described in Algorithm 2 for computing the FDF set (refer Table 3). Here $\theta_{1,k-1}$ is the angle between two consecutive critical points (where $k$ is the total number of critical points) in a handwritten primitive and $d_{1,8}$ is the respective direction. The fuzzy membership values assigned to each direction are represented as $m^{12}_{1,k-1}$ and the corresponding feature vector values as $f_1, \ldots, f_8$. It should be noted that the sum of the membership functions of a particular row in Table 3 is always 1. We calculate the FDF by averaging across the columns, so as to form a vector of dimension eight. The mean is calculated as follows; for each direction (1 to 8), collect all the membership values and divide by the

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*Note that [24] talks of fuzzy feature set for Devanagari script albeit for off-line handwritten character recognition*
Figure 18: θ contributing to two directions (1, 2) with fuzzy membership values (green and red dot)
Algorithm 1 Angle between two critical point conversion into eight direction

int deg2dir(double θ)
int dir = -1;
if (θ > −π/8 & θ < π/8) then
    dir = 1;
end if
if (deg >= π/8 & θ < 3π/8) then
    dir = 2;
end if
if (θ >= 3π/8 & θ < 5π/8) then
    dir = 3;
end if
if (θ >= 5π/8 & θ < 7π/8) then
    dir = 4;
end if
if (((θ >= 7π/8 & θ < 9π/8) || (θ >= −9π/8 & θ < −7π/8)) then
    dir = 5;
end if
if (θ >= −7π/8 & θ < −5π/8) then
    dir = 6;
end if
if (θ >= −5π/8 & θ < −3π/8) then
    dir = 7;
end if
if (θ >= −3π/8 & θ < −π/8) then
    dir = 8;
end if
return(dir);

Algorithm 2 Triangular Fuzzy Membership Function

fuzzy-membership(θc, θ);
m = 1.0 − (|(θc−θ)|) / π/4
return(m);
number of occurrences of the membership values in that direction. For example, in Table 3, the mean for direction 1 is calculated as

$$f_1 = \frac{(m_2^1 + m_3^1)}{2}$$

In all our experiments we have used these mean values to construct 8 directional FDF to represent a stroke.

Clearly, the fuzzy aspect comes into picture due to the membership function which associates the angle between two critical points into two directions with different membership values. In the commonly used Directional Features (see Section 5.2) only one direction is associated with each \( \theta \) (the angle between two consecutive critical points).

### 6. Experimental Results

Experiments were carried out for two different datasets, namely, (a) the isolated primitive dataset (Dataset 1) where the identified 69 primitives were written by 100 users and (b) primitives extracted (Dataset 2) from the paragraph data (continuous text). Note that in Dataset 1 people were asked to just write the strokes and hence was not natural while in Dataset 2 the writers were asked to write a paragraph using e-pen. We used knotless spline noise removal in all our experiments because this performed the best as seen in earlier sections. Critical point were extracted from the smoothened primitive and three different set of features were extracted for each of the dataset, namely, Directional Features (DF), Extended Directional Features (EDF) and Fuzzy Directional Features (FDF) to build the reference models for the 69 primitives. In the testing (recognition) phase we used three different types of classifiers, namely (a)
Second order statistics based nearest neighborhood classifier (b) Dynamic Time Warping (DTW) based classifier and (c) Support Vector Machine (SVM) based classifier. The recognition results are then compared across the two dataset, three feature sets and three classifiers.

6.1. Recognition of Isolated primitive (Dataset 1)

The recognition experiments are conducted as follows. We collected 10 isolated samples of each 69 Devanagari primitive strokes from 100 different writers using Mobile e-Note Taker. We divide the collected data into two non-overlapping sets, namely the train dataset and test dataset. The training and test set were in the ratio of 80 : 20 and five-fold cross validation procedure is performed and the recognition accuracies computed.

During training, we calculate the three feature set (DF, EDF and FDF) for all the training samples corresponding to the same primitive. These features are extracted after knotless spline smoothing. The mean and variance feature vector is computed for each primitive from the feature vectors of that primitive from the training dataset.

For testing purpose, we took a stroke \( t \) from the test dataset and extracted the three features after noise removal. Then the distance between the test feature vector and all the 69 reference primitives is computed using second order statistics based classifier, Discrete Time Warping (DTW) and the Support Vector Machine (SVM) based classifier as described below.

6.1.1. Dataset 1 using second order statistics

In case of FDF for all stroke corresponding to the same primitive as described in Section 5.3. Then we computed the mean \( (\mu) \) and co-variance \( (\Sigma) \) for each of the 69 primitives to model the primitive. If there are \( \beta \) strokes corresponding to a primitive, then we have \( \beta \) Fs, say, \( F_1, F_2, \ldots, F_\beta \) then each primitive is modeled as

\[
\mu = \frac{1}{\beta} \sum_{i=1}^{\beta} F_i
\] (9)

\[
\Sigma = \frac{1}{(\beta - 1)} \sum_{i=1}^{\beta} (F_i - \mu)(F_i - \mu)^T
\] (10)

Here a primitive is represented by a mean vector \( (\mu) \) of size \( 1 \times 8 \) and covariance matrix of size \( 8 \times 8 \). The class reference models are represented as \( (\mu_i, \Sigma_i)_{i=1}^{69} \). For testing purpose, we took a stroke \( t \) from the test data set and extracted FDF as described in Section 5.3 and compared it with 69 reference models. The likelihood of the test primitive \( t \) with reference primitive \( k \) (where \( k = 1, 2, \ldots, 69 \)) is computed as

\[
P(t/k) = \frac{1}{(2\pi)^{\frac{8}{2}} \sqrt{|\Sigma_k|}} \exp\left(-\frac{1}{2} (t - \mu_k)^\Sigma_k^{-1} (t - \mu_k)^T\right)
\] (11)

The \( k \) for which (11) is maximum is identified as the recognized primitive.
Table 4: Recognition results for different $\alpha$

| Feature Set | Data | $\alpha = 1$ | $\alpha = 2$ | $\alpha = 5$ |
|-------------|------|--------------|--------------|--------------|
| FDF         | Train| 80.25%       | 85.46%       | 96.89%       |
|             | Test | 70.29%       | 74.86%       | 82.75%       |

Table 4 shows the experimental results obtained for both the train and the test data for FDF using second order statistics based classifier. Note that the values in Table 4 are computed as follows. For $N = \alpha$, the test stroke $t$ is recognized as the primitive $l$ if $l$ occurs at least at the $\alpha^{th}$ position from the best match (this is generally called the N-best in literature). It should be noted that the recognition accuracies are for writer independent unconstrained strokes. As expected the recognition accuracies are not very high (very similar to the phoneme recognition in speech literature) for $\alpha = 1$ but improves with increasing $\alpha$. It is also noted that similar experiments with Directional Features (DF) shows $\pm 10\%$ lower recognition efficiency. The recognition experiments are also performed using DTW, SVM classifiers which are explained in following sessions.

6.1.2. Dataset 1 based on DTW

Being an elastic matching technique, Dynamic Time Warping (DTW) allows to compare two sequences of different lengths. This is especially useful to compare patterns in which rate of progression varies non-linearly, which makes similarity measures such as Euclidean distance and cross-correlation unusable [18]. Since different strokes corresponding to the same primitive had different feature vector length in the case of Directional Features (DF) and Extended Directional Features (EDF) we used DTW algorithm [11] to compute the distance between the test primitive and the reference primitives. We calculated the DTW distance of the test primitive from all the reference primitives. between all strokes corresponding to the same primitive in the training set with the test stroke. The stroke with minimum average stroke is identified as the recognized primitive. The recognition experiments are conducted based on DF and EDF (which are unequal length feature vectors) and recognition accuracies are tabulated [5].

6.1.3. Dataset 1 based on Support Vector Machine

The principle of an SVM is to map the input data onto a higher dimensional feature space non-linearly related to the input space and determine a separating hyper-plane with maximum margin between the two classes in the feature space [9]. This results in a nonlinear boundary in the input space. The optimal separating hyper-plane can be determined without any computations in the

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10 edit distance in Computer Science literature
11 We do not discuss this algorithm, in detail since it is well documented in literature
higher dimensional feature space by using kernel functions in the input space. We used a multi-class SVM classifier with a Radial Basis Function (Gaussian) Kernel function \[ \text{Gaussian} \] for our experimentation. Since data should have a constant length representation in the SVM-based approach, FDF (a fixed length feature vector) have been used in the recognition experiments. The recognition results are tabulated. The recognition results for DF, EDF and FDF are compared. It can be observed that the FDFs give better recognition accuracies and SVMs can be used effectively for primitive recognition (see Table 5).

### 6.2. Recognition of primitives in Paragraph (Dataset 2)

We used 80 % of 100 user paragraph data for training and the rest for the purpose of testing the performance of the ED feature set and the DF set. We initially extracted the primitives from the paragraph dataset and hand tagged each stroke as belonging to one of the 69 primitives (see Fig. 4). All the strokes corresponding to the given primitive in the training data were collected and clustered together. For each stroke we extracted the DF, EDF and FDF set as described in Section \(5.3\) and \(5.2\). All strokes corresponding to the same primitive which were within a distance of \(\tau\) were clustered together and only one representative stroke from the cluster was retained as the cluster representative.

For testing purpose, we took a stroke \(t\) from the test data, we first extracted EDF\[^{12}\]\(t\) and compared it with the EDF of the all reference strokes using DTW algorithm. We then took the average distance of \(t\) from all the references of a primitive. We arranged these average distances (69 in number) in the increasing order of magnitude. The primitive with the least average distance from the test stroke \(t\) is declared as being recognition of stroke \(t\).

The recognition results obtained for different features sets using different classification techniques are tabulated in Table 6. The results obtained for recognition of Devanagari primitive strokes show that reliable classification is possible using SVMs and FDF can be easily extended to other Indian scripts as well. The results also indicate the scope for further improvement. Future work is directed towards extending the stroke level recognition to character level recognition and further to world level recognition based on the proposed framework in Section \(3\).

\[^{12}\text{for experiments with DF, we extracted the corresponding directional features}\]
7. Conclusion

In this paper we have presented an on-line handwriting script recognition framework for Devanagari which can be extended to other Indian language scripts. The framework is motivated by speech recognition literature and in our opinion has not been reported in literature. Handwritten script is inherently noisy and we experimented with several noise removal techniques and identified that the noise removal is a necessary pre-processing step in OHCR, we further showed that knotless spline based denoising work better than the wavelet based denoising. We introduced a feature set called the Fuzzy Directional Features (FDF) which is able to incorporate directional variance in the handwritten primitives.

Experimental results show that the performance of the FDF for the two datasets is better that for other feature sets (DF and EDF). The recognition accuracies obtained for Dataset 1 (isolated primitive) was 73.91% (using FDF and SVM classifier) and for Dataset 2 (primitives extracted from unconstrained continuous script data) was 71.73 %. Experiments show that FDF performs much better than the commonly used directional features.

Character based OHCR reported in literature fail because of the increased number of characters in the Indian language script and the presence of a large number of complex compound characters. The advantage of the proposed framework based on stroke (primitive) recognition rather than character recognition is its ability to handle complex characters. It is because the number of primitives to be recognized do not increase with increased character set.

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