Second-harmonic plasma response in diffusion-controlled surface-wave-sustained discharges

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Abstract. The formation of nonlinear plasma response at the second harmonic frequency in diffusion controlled surface-wave-sustained discharges is studied theoretically. The study is aimed at estimating theoretically the ratio of the squared amplitudes of the wave field of fundamental frequency and of the resulting – from the nonlinear effects – electric field at the second harmonic frequency. The model presented is intended for further use in discharge diagnostics.

Introduction
The Surface-Wave-Sustained Discharges (SWSDs) are well known and reliable plasma sources [1,2] with many applications [3]. Their increasing use motivates the work on their modeling and diagnostics. Moreover, the SWSDs, being a self-consistent structure created of simultaneously interrelated plasma and wave field, are sensitive to the diagnostic techniques used and require non-disturbing methods for their study, such as optical spectroscopy [4] and radiophysical methods [5].

The use of radiophysical diagnostic methods for discharge diagnostics, based on the plasma-wave interrelation, requires that the measurements of the wave characteristics be compared with results from discharge modeling [6] in order to obtain the plasma parameters. The plasma of the SWSD is also a nonlinear medium where phenomena such as frequency doubling or second harmonic (SH) generation (as observed in [7]) and detection (zero-frequency response) occur. The use of such nonlinear phenomena for measuring the system parameters of electronic circuits is a well established technique in engineering practice [8]. Thus, provided there is a model, the measurement of the plasma SH response in a SWSD could be used as a diagnostic technique. This is why the aim of the paper presented is to study the formation of the plasma SH response and its relation to the plasma parameters in a SWSD.

Plasma response in the wave field
The study presented considers a diffusion controlled SWSD sustained by an azimuthally symmetric surface wave (sustained by an azimuthally symmetric TM surface wave with nonzero field components $E_z$, $E_r$ and $B_\phi$). A common assumption for the SWSD discharge modelling is used,
namely, a radially inhomogeneous plasma column with Bessel profile and inhomogeneity parameter $\mu$ [9-11].

Within the framework of the fluid plasma theory, the motion of the plasma in the wave field is governed by the fluid motion equation:

$$\frac{\partial}{\partial t} \big( n m_e \mathbf{v} \big) + \mathbf{v} \cdot \big[ (n m_e) \mathbf{v} \big] = -en \mathbf{E}(r,t) + \mathbf{v} \times \mathbf{B} - n(m_e) \mathbf{v},$$

(1)

where $e$ and $m_e$ are, correspondingly, the electron charge and mass, $\mathbf{v}$ is the electron velocity, $n$ is the plasma density, $\mathbf{E}$ and $\mathbf{B}$ are the electric and magnetic field intensities of the wave; $\nu$ is the electron-neutral elastic collision frequency. Equation (1), written in terms of the plasma current density $j$, becomes the modified nonstationary and nonlinear Ohm’s law:

$$\frac{\partial j}{\partial t} + \nabla \cdot (j \mathbf{v}) = \varepsilon_0 \nu \frac{e^2}{m} \big[ \mathbf{E}(r,t) + \mathbf{v} \times \mathbf{B}(r,t) \big] + vj.$$  

(2)

In order to obtain the plasma response at the fundamental frequency $\omega$ and SH-frequency $2\omega$, the wave electric field $\mathbf{E}$, the plasma current density $j$ and the electron velocity $\mathbf{v}$ are taken in the following form:

$$\begin{align*}
\mathbf{E}(r,t) &= \frac{1}{2} \left[ E_{w}(r)e^{-i\omega t} + E_{2\omega}(r)e^{-2i\omega t} + c.c. \right], \\
\mathbf{v}(r,t) &= \frac{1}{2} \left[ v_{w}(r)e^{-i\omega t} + v_{2\omega}(r)e^{-2i\omega t} + c.c. \right], \\
j(r,t) &= \frac{1}{2} \left[ j_{w}(r)e^{-i\omega t} + j_{2\omega}(r)e^{-2i\omega t} + c.c. \right],
\end{align*}$$

(3)

where $\mathbf{E}_{w,2\omega}$, $\mathbf{v}_{w,2\omega}$ and $\mathbf{j}_{w,2\omega}$ are, correspondingly, the slowly varying in space complex amplitudes of the above quantities.

Equation (2) is solved via a perturbation scheme, where the zero-order approximation includes the terms at the fundamental frequency $\omega$, while the terms at the SH frequency $2\omega$ are the first order approximation. At zero-order approximation, equation (2) yields:

$$j_{\omega} = i\varepsilon_0 \frac{\omega_p^2}{\omega} E_{w},$$

(4)

where $\omega_p$ denotes the plasma frequency. Using the zero-order approximation, one obtains the SH current density:

$$j_{2\omega} = i\varepsilon_0 \frac{\omega}{m} \frac{\nabla \left( \omega_p^2 |E_{w}|^2 \right)}{(2\omega + iv)^2}. $$

(5)

In a SWSD [2] with the electric field having its maximum at the discharge wall and the plasma density having a steep drop there, the expression (5) for the SH current density is of a significant magnitude only in the vicinity of plasma edge. Bearing in mind the continuity relation

$$\nabla \cdot \mathbf{E} = \frac{1}{\varepsilon_0} \nabla \cdot \int_{-\infty}^{t} \int_{-\infty}^{t} j(r',t')d'r',$$

the SH current density (5) corresponds to a SH field amplitude:

$$E_{2\omega} = \frac{\omega}{2\omega} \frac{\nabla \left( \omega_p^2 |E_{w}|^2 \right)}{(2\omega + iv)^2}. $$

(6)

Expressing the average electric field amplitude at the fundamental frequency $\omega$ via the discharge parameter $\Theta$ – the averaged-per-electron power input into the plasma, [12] the second harmonic deviation (SHD) ratio is obtained:

$$\text{SHD} = \left| \frac{E_{2\omega}^2}{E_{\omega}^2} \right| \approx A \left( \frac{\Theta \pi^3}{T_e} \right),$$

(7a)
where \( T_e \) is the electron temperature and \( \bar{n} \) is the plasma density averaged over the discharge cross-section. The factor \( A \) is equal to:

\[
A = \frac{e^2 \omega^2 \nu^2 + \nu^2}{4 m e_0 n_e^2 k \mu \nu} \left( \frac{4 \omega^2 + \nu^2}{4 \omega^2 + \nu^2} \right)^2 + 16 \omega^2 \nu^2.
\] (7b)

Here \( n_{cr} \) is the critical density for the fundamental frequency \( \omega \), \( k \) is the Boltzmann constant and \( \mu = \mu J_1(\mu)/2 J_0(\mu) \) is expressed via the Bessel functions and \( \mu \) is the parameter of radial inhomogeneity. Thus, the SHD-ratio is expressed in terms of the plasma parameters \( \Theta \), \( T_e \) and \( \bar{n} \).

**Discussion**

The SHD-ratio is proportional to the quantity \( \left( \frac{\bar{n}^3 \Theta}{T_e} \right) \). The ratio \( \left( \frac{\Theta}{T_e} \right) \) varies slowly along the main-long-part of a diffusion controlled SWSD [2]. The strong dependence of the plasma density, however, relates the SHD to the plasma density profile (figure 1(a)), such as the ones obtained from the steady state SWSD modelling [6].

![Figure 1](image1.png)

**Figure 1.** Theoretical plasma density profiles (a) and wave phase diagrams (b) from the SWSD model in Ref. [6]. The parameters shown are for a discharge sustained in argon gas (at two pressures 0.1 and 1 Torr) by high frequency power at 133 MHz in a quartz tube with internal and external radii 1.4 and 1.5 cm respectively.

As equation (7) suggests, the axial variations of the SHD ratio follow the plasma density profile. Moreover, if the common – for the SWSD radiophysical diagnostics [5, 13-15] – assumption that the end of the discharge comes at a given plasma density corresponding to the point at the wave phase diagram (figure 1(b)) where the wavenumber equals the wave damping rate (the wave stops its propagation). Hence, the end of the discharge could be used as a reference point to recover the axial plasma density profile by means of tracking the changes of the SHD-ratio along the discharge length (figure 2).

![Figure 2](image2.png)

**Figure 2.** Axial changes of the SHD ratio \( k_2^2/k_2 \) (\( z=0 \)) along the discharge length for the two gas pressures on figure 1.
Conclusion
The nonlinear plasma response in a diffusion controlled SWSD at SH frequency is analyzed. The axial variations of the ratio of the amplitudes of the electric fields at the fundamental $\omega$ and SH $2\omega$ frequencies is related to the axial plasma density profile of the SWSD.

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