Age of Information in Multi-hop Networks with Priorities

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Abstract—Age of Information is a new metric used in real-time status update tracking applications. It measures at the destination the elapsed time since the generation of the last received packet. In this paper, we consider the co-existence of critical and non-critical status updates in a two-hop system, for which the network assigns different scheduling priorities. Specifically, the high priority is reserved to the packets that traverse the two nodes, as they experience worse latency performance. We obtain the distribution of the age and its natural upper bound termed peak age. We provide tight upper and lower bounds for priority updates and the exact expressions for the non-critical flow of packets with a general service distribution. The results give fundamental insights for the design of age-sensitive multi-hop systems.

Index Terms—AoI, Peak AoI, IoT, multi-hop networks, priority

I. INTRODUCTION

The Age of Information (AoI) [1, 2] characterizes the freshness of the information from the receiver’s perspective, and it has been proved to be a proper metric in many real-time and context-aware Internet of Things (IoT) applications [3]. In these applications, the end receiver is interested in a time and context-aware Internet of Things (IoT) applications and it has been proved to be a proper metric in many real-time and context-aware Internet of Things (IoT) applications [3]. Specifically, the high priority is reserved to the packets that traverse the two nodes, as they experience worse latency performance. We obtain the distribution of the age and its natural upper bound termed peak age. We provide tight upper and lower bounds for priority updates and the exact expressions for the non-critical flow of packets with a general service distribution. The results give fundamental insights for the design of age-sensitive multi-hop systems.

II. RELATED WORKS

A system design similar to ours has been considered in [9]. Authors investigate the average AoI when the status update can be delivered either over the less reliable direct link or over the two-hop relay link with better reliability. However, all packets at the second node have been treated equally. In [5] only average PaIoI is given for the two-hop tandem exponential queues with multiple sources. Authors in [10] study the average AoI of a two-hop system with packet arrivals...
only at the first node and zero-waiting policy at the second node.

In [11] authors derive a general formula for the stationary distribution of the AoI in terms of the system delay and the PAoI for a wide class of $G/G/1$ systems with a single source under the general FCFS and Last Come First Serve (LCFS) packet management policies with various preemption and packet discarding options. However, LCFS policy cannot be applied to the systems where packets carry incremental information and cannot be discarded.

The idea of assigning different priorities to the update packets has been discussed for the first time in [12]. The average AoI is given for an exponential single-server system with a shared queue and LCFS discipline, where the arrived packet preempts another packet either in service or in waiting only if it has higher priority. In [13] authors focus on a queuing system with $k$ classes of priorities, different buffer sizes and queuing disciplines. In particular, the different combinations of infinite queues with FCFS and LCFS disciplines and queues with a single place to wait are considered. The exact expressions of the expected PAoI are given for the general service time distribution if the queues are infinite and for the exponential service time if the queue size is one, while the tight bounds have been calculated for the remaining scenarios. The above-mentioned works with the packet’s prioritization are limited to the single-node systems.

III. System model

We consider a two-hop network with intermediate traffic. Sources generate packets with status updates according to a Poisson process with rate $\lambda$. With probability $p$ priority packets arrive at the first node and with probability $1-p$ all remaining non-priority packets arrive directly to the second node, $\lambda_1 = p\lambda$ and $\lambda_2 = (1-p)\lambda$. Such a network is modeled as two tandem queues connected in series with packet prioritization in the second queue. In particular, both queues apply the general FCFS discipline but in the queue of the second node all packets is the mean service time at the first node.

Let $\lambda_1$ and $\lambda_2$ be the mean service times of priority and non-priority packets at the second node. The total system utilization equals to the second node utilization $\rho = \rho_1 + \rho_2$, where $\rho_2 = \lambda_2/\mu_2$, $j = (1, 2)$. Utilization of the first node $\rho_{11} = \lambda_1/\mu$, $\mu^{-1}$ is the mean service time at the first node.

Let $j, i$ denote packet $i$ of priority class $j$. Let $t_{j,i}$ and $t'_{j,i}$ be the time instances of packet $j, i$ arrival to the system (generation of a new status at source) and its departure from the system (updating the status at the monitor). Then $Y_{j,i} = t_{j,i} - t_{j,i-1}$ denotes the random variable (RV) of packet $j, i$ interarrival time and $T_{j,i} = t'_{j,i} - t_{j,i}$ corresponds to the RV of the packet’s system delay. The AoI $\Delta_{j,i}$ at time $t > 0$ consists of the AoI $Z_{j,i-1}$ immediately after the departure of the packet $j, i-1$ and the time from $t'_{j,i-1}$ to $t$, i.e. $\Delta_{j,i} = Z_{j,i-1} + (t - t'_{j,i-1})$. In general FCFS systems $Z_{j,i}$ equals to the system delay $T_{j,i}$ if all packets are time-stamped on their arrival. Therefore the PAoI $A_{j,i} = t'_{j,i} - t_{j,i-1} = Y_{j,i} + T_{j,i}$.

In the ergodic system ($\rho < 1$), the probability density function (pdf) of the AoI can be defined as $f_{\Delta_{j,i}}(x) = \lambda_1 F_{T_1}(x) - FA_1(x)$, $x \geq 0$, where $F_{T_1}(x)$ and $FA_1(x)$ stand for the Probability Distribution Functions (PDFs) of the system delay and PAoI, respectively [11]. The Laplace-Stieltjes Transform (LST) $\delta_{j,i}(s)$ of the AoI distribution therefore yields:

$$
\delta_{j,i}(s) = \frac{\lambda_1}{s} \left( \tau_j(s) - \alpha_j(s) \right), \quad s > 0,
$$

where $\tau_j(s) = \int_0^\infty e^{-sx}dF_{T_j}(x)$ and $\alpha_j(s) = \int_0^\infty e^{-sx}dFA_j(x)$.

Priority and non-priority packets arrive to the system independently, their interarrival times are exponentially distributed holding the LST $\lambda_j/\left(\lambda_j + s\right)$. System delay $T_{j,i}$ depends on the packets interarrival time $Y_{j,i}$ and the system delay $T_{j,i-1}$, it also depends on the arrival and departure processes of packets of another class. The RV $T_{1,i} = T_{11,i} + T_{12,i}$ while $T_{2,i}$ and $T_{12,i}$ are not independent. In the next section we define the PAoI for packet $j, i$ and then obtain the general distribution of $A_{j,i}$ for both classes of packets, the similar approach is applied for calculation of the total system delay $T_1$ of priority packets.

Let us give the known distributions of the system delays at each node as preliminaries for further analysis. The system delay $T_{11}$ at the first node ($M/M/1$) is exponentially distributed with parameter $\theta = \mu - \lambda_1$, the corresponding LST $\tau_{11}(s)$ equals to $\theta/(\theta + s)$. The LST of the system delay $T_{12}$ of priority packets and system delay $T_2$ of non-priority packets at the second node are given in [14] chapter 8.6:

$$
\tau_{12}(s) = \frac{s(1 - \rho) + \lambda_2(1 - \beta_2(s))}{s - \lambda_1 + \lambda_2 \beta_1(s)} \beta_1(s),
$$

$$
\tau_2(s) = \frac{(1 - \rho)(s + \lambda_1 - \lambda_1 \gamma(s))}{s - \lambda_2 + \lambda_2 \beta_2(s)} \beta_2(s),
$$

where $\beta_1(s)$ and $\beta_2(s)$ are the LSTs of the service time distributions of priority and non-priority packets at the second node, $\gamma(s)$ stands for the LST of the distribution of the interval $G_1$, which elapses from the arrival of a priority packet in the empty queue of the second node until the end of continuous service of priority packets arriving afterwards. This interval is known as a busy period generated by a priority packet and its LST $\gamma(s) = \beta_1(s + \lambda_1 - \lambda_1 \gamma(s))$. The busy period $G_2$ starts
illustrated in Fig. 2. Let also η priority packets at the second node hinders the derivation of the 1 from the moment when a non-priority packet arrives to the 1

Mean system delay at the first node

Node index

Packet (j, i) arrival time

Packet (j, i) departure time

Arrival rate for class j

Mean service time for class j

Second node utilization by class j

Mean service time at the first node

Mean system delay at the first node

First node utilization

| RV | LST | Definition |
|----|-----|------------|
| Yj,i | \( s_{j,i} \) | Intervals contributing to system delay |
| Wj,k,i | \( \omega_{j,k}(s) \) | Service time of packet \((j,i)\) at node k |
| Tj,k,i | \( \tau_{j,k}(s) \) | System delay of packet \((j,i)\) at node k |
| Dj,i | \( \eta_{j}(s) \) | Supplementary to PAoI of packet \((j,i)\) in interval as defined in Fig. 2 |
| Xj,i | \( \xi_{j}(s) \) | Supplementary to system delay of packet \((j,i)\) interval as defined in Fig. 2 |
| Gj,i | \( \gamma_{j}(s) \) | Busy period generated by a packet \((j,i)\) |
| Zj,i | \( \zeta_{j,i}(s) \) | Residual time of interval \((j,i)\) |
| Aj | \( \alpha_{j}(s) \) | PAoI of class j |
| \( \Delta_{j} \) | \( \delta_{j}(s) \) | AoI of class j |

The LST of the joint distribution of intervals contributing to \( D_{1,i} \) and \( X_{1,i} \) for a case \( C_{m}, m = \{1, \ldots, 6\} \), respectively.

We define the LST of the system delay \( \tau(s, C_{m}) \) and the PAoI \( \alpha(s, C_{m}) \) for each case. The resulting distributions will be given as a sum of LSTs of the six joint distributions namely \( \tau_{1}(s) = \sum_{m} \tau_{1,i}(s, C_{m}) \) and \( \alpha_{1}(s) = \sum_{m} \alpha_{1,i}(s, C_{m}) \).

C1: Packet \( i \) does not experience any queuing at nodes, therefore the PAoI \( A_{1,i} = D_{1,i} + S_{12,i} \) and system delay \( T_{1,i} = X_{1,i} + S_{11,i} \). This happens if \( T_{11,i-1} < Y_{1,i}, T_{12,i} < Y_{1,i} + S_{11,i} \) and if during the interval \( Y_{1,i} + S_{11,i} - T_{11,i-1} - T_{12,i} \) all unserved non-priority packets complete their service and no new non-priority packets arrive. Since we assume that packet \( i \) finds the second node free of non-priority packets with the probability \( 1 - \rho_{2} \) and service time \( S_{12,i} \), is independent of other intervals, the LST of both metrics can be given as \( \alpha(s, C_{1}) = (1 - \rho_{2}) \eta_{1}(s,C_{1}) \beta_{1}(s) \) and \( \tau(s, C_{1}) = (1 - \rho_{2}) \xi_{1}(s,C_{1}) \beta_{1}(s) \).

C2: Packet \( i \) finds the second node busy with packet \( i - 1 \), but its queuing delay at the first node \( W_{11,i} = 0 \), therefore PAoI \( A_{1,i} = D_{1,i} + S_{2} + S_{12,i} \) and system delay \( T_{1,i} = X_{1,i} + S_{12,i} \) like in the case C1, but \( D_{1,i} = T_{1,i} - T_{12,i-1} \), \( X_{1,i} = T_{11,i-1} + T_{12,i-1} - Y_{1,i} \). This is true if \( T_{11,i-1} < Y_{1,i} \) and \( T_{12,i-1} > Y_{1,i} + S_{11,i} \). The PAoI and system delay distributions in this case give \( \alpha(s, C_{2}) = \eta_{1}(s,C_{2}) \beta_{1}(s) \) and \( \tau(s, C_{2}) = \xi_{1}(s,C_{2}) \beta_{1}(s) \).

C3: Packet \( i \) finds the second node busy with a non-priority packet and its waiting time \( W_{11,i} = 0 \), thus the PAoI \( A_{1,i} = D_{1,i} + S_{2} + S_{12,i} \) and system delay \( T_{1,i} = X_{1,i} + S_{2} + S_{12,i} \), where \( S_{2} \) stands for the LST of the residual service time of a non-priority packet. This happens when \( T_{11,i-1} < Y_{1,i}, T_{12,i-1} < Y_{1,i} + S_{11,i} \) like in the case C1, but packet \( i \) sees a non-priority packet in service with the probability \( \rho_{2} \). The LST of the PAoI in the case C3 yields \( \alpha(s, C_{3}) = \rho_{2} \eta_{1}(s,C_{3}) \beta_{2}(s) \beta_{1}(s) \) and LST of the \( T_{1,i} \) gives \( \tau(s, C_{3}) = \rho_{2} \xi_{1}(s,C_{3}) \beta_{2}(s) \beta_{1}(s) \), where \( \beta_{2}(s) = (1 - \beta_{2}(s)) e^{S_{2}} \).

C4: Packet \( i \) is queued at the first node, but it finds the second node empty upon the arrival. The PAoI \( A_{1,i} \) and system delay \( T_{1,i} \) are defined as in the case C1, but in the case \( C_{4} T_{11,i} > Y_{1,i} \), and \( T_{12,i} < S_{11,i} \), in particular \( \alpha(s, C_{4}) = (1 - \rho_{2}) \eta_{1}(s,C_{4}) \beta_{1}(s) \) and \( \tau(s, C_{4}) = (1 - \rho_{2}) \xi_{1}(s,C_{4}) \beta_{1}(s) \).

C5: Packet \( i \) is delayed by the packet \( i - 1 \) in both nodes, if \( T_{11,i} > Y_{1,i} \), and \( T_{12,i} > S_{12,i} \). Given that \( A_{1,i} = D_{1,i} + S_{12,i} \), and \( T_{1,i} = X_{1,i} + S_{11,i} + S_{12,i} \), the distribution of PAoI \( \alpha(s, C_{2}) = \eta_{1}(s,C_{2}) \beta_{1}(s) \), the distribution of system delay \( \tau(s, C_{2}) = \xi_{1}(s,C_{2}) \beta_{1}(s) \) in terms of LST.

C6: Packet \( i \) is queued at the first node and finds the second node busy with a non-priority packet, then like in the case \( C_{3} \) \( \alpha(s, C_{6}) = \rho_{2} \eta_{1}(s,C_{6}) \beta_{2}(s) \beta_{1}(s) \) and \( \tau(s, C_{6}) = \rho_{2} \xi_{1}(s,C_{6}) \beta_{2}(s) \beta_{1}(s) \) given that \( T_{11,i} > Y_{1,i} \), and \( T_{12,i} < S_{12,i} \).

We now need to calculate the LST of \( D_{1,i} \) and \( X_{1,i} \) for each case. These intervals are equally defined for the cases C1 and
C3, and C4 and C6, therefore we give their derivations with double indexes \{13\} and \{46\}.

a) Cases C1 and C3: we denote the PDF of $D_{1,i}$ as $F_{D_{1,i}}(z, C_{13}) = P\{D_{1,i} < z, C_{13}\}$. Given that $T_{1,i-1} < Y_{1,i}$ and $T_{1,i-1} < Y_{1,i} + S_{11,i}$ we calculate it as follows:

$$F_{D_{1,i}}(z, C_{13}) = \int_0^z dF_{Y_{1}}(y) \int_0^y dF_{T_{11}}(t) \int_0^t dF_{S}(x) \int_{0}^{z-y-t} dF_{F_{12}}(u). \quad (4)$$

The LST $\eta_1(s, C_{13}) = \int_0^\infty e^{-sz}dF_{D_{1,i}}(z, C_{13})$ yields:

$$\eta_1(s, C_{13}) = \frac{\lambda_1}{\lambda_1 + s + \beta_1(s)\tau_{12}(\lambda_1 + s) - \rho_1 \beta_2(s)\tau_{12}(\mu + s)}.$$  

(5)

Let $F_{X_{1,i}}(z, C_{13}) = P\{X_{1,i} < z, C_{13}\}$ be the PDF of $X_{1,i}$, it can be calculated as:

$$F_{X_{1,i}}(z, C_{13}) = \int_0^z dF_{Y_{1}}(y) \int_0^y dF_{T_{11}}(t) \int_0^t dF_{S}(x) \int_{0}^{z-y-t} dF_{F_{12}}(u). \quad (6)$$

and its LST $\xi_1(s, C_{13}) = \int_0^\infty e^{-sz}dF_{X_{1,i}}(z, C_{13})$ yields:

$$\xi_1(s, C_{13}) = \tau_{11}(s)\tau_{12}(\lambda_1) - \rho_1 \tau_{11}(s)\beta_1(s)\tau_{12}(\mu + s). \quad (7)$$

b) Case C2: the PDF of interval $D_{1,i}$ and its LST $\eta_1(s, C_2)$ in the case C2 are given as follows:

$$F_{D_{1,i}}(z, C_{2}) = \int_0^z dF_{T_{11}}(t) \int_0^t dF_{T_{12}}(u) \int_0^t dF_{Y_{1}}(y) \int_0^z dF_{S}(z). \quad (8)$$

$$\eta(s, C_{2}) = (1 - \rho_1)\beta_1(s)\tau_{12}(s) - \beta_1(s)\tau_{12}(s + \lambda_1) + \rho_1 \beta_1(s)\tau_{12}(\mu + s). \quad (9)$$

The define the PDF of interval $X_{1,i}$ as:

$$F_{X_{1,i}}(z, C_{2}) = \int_0^z dF_{Y_{1}}(y) \int_0^y dF_{T_{12}}(u) \int_0^t dF_{Y_{1}}(y) \int_0^z dF_{S}(z). \quad (10)$$

while its LST $\xi(s, C_{2})$ gives:

$$\xi(s, C_{2}) = \frac{\lambda_1}{s - \lambda_1} - \theta = \frac{\lambda_1}{s - \lambda_1} - \tau_{12}(s) + \rho_1 \tau_{11}(s)\tau_{12}(\mu + s). \quad (11)$$

c) Cases C4 and C6: we define the PDF $F_{D_{1,i}}(z, C_{46})$ and $F_{X_{1,i}}(z, C_{46})$ in the cases C4 and C6 as:

$$F_{D_{1,i}}(z, C_{46}) = \int_0^z dF_{T_{11}}(t) \int_0^y dF_{Y_{1}}(y) \int_0^t dF_{S}(x) \int_0^z dF_{T_{12}}(u). \quad (12)$$

$$F_{X_{1,i}}(z, C_{46}) = \int_0^z dF_{Y_{1}}(y) \int_0^y dF_{T_{11}}(t) \int_0^t dF_{S}(x) \int_0^z dF_{T_{12}}(u). \quad (13)$$

The LSTs of $D_{1,i}$ and $X_{1,i}$ give:

$$\eta(s, C_{46}) = \rho_1 \tau_{11}(s)\beta_2(s)\tau_{12}(s + \mu). \quad (14)$$

$$\xi(s, C_{46}) = \rho_1 \tau_{11}(s)\beta_1(s)\tau_{12}(s + \mu). \quad (15)$$

d) Case C5: the PDFs of the intervals $D_{1,i}$ and $X_{1,i}$ in the case C5 can be calculated as:

$$F_{D_{1,i}}(z, C_{5}) = \int_0^z dF_{T_{11}}(t) \int_0^y dF_{Y_{1}}(y) \int_0^t dF_{T_{12}}(u) \int_0^z dF_{S}(z). \quad (16)$$

$$F_{X_{1,i}}(z, C_{5}) = \int_0^z dF_{Y_{1}}(y) \int_0^y dF_{T_{11}}(t) \int_0^t dF_{T_{12}}(u) \int_0^z dF_{S}(z). \quad (17)$$

The LSTs $\eta(s, C_{5})$ and $\xi(s, C_{5})$ in the case C5 yield:

$$\eta(s, C_{5}) = \rho_1 \tau_{11}(s)\beta_1(s)\tau_{12}(s + \mu). \quad (18)$$

$$\xi(s, C_{5}) = \rho_1 \tau_{11}(s)\tau_{12}(s + \mu). \quad (19)$$

The resulting LST of the PAoI distribution of priority packets yields:

$$\alpha_1(s) = \left[\frac{\lambda_1 \nu}{\lambda_1 + s} \beta_1(s)\tau_{12}(\lambda_1 + s) - \frac{s}{s + \theta} \rho_1 \beta_1(s)\tau_{12}(s + \mu) \times (1 - \nu \beta_3(s))\right] \beta_1(s), \quad (20)$$

where $\nu = \lambda_2 + \rho_2 \beta_3(s).$

The LST of system delay is given as follows:

$$\tau_1(s) = \left[\tau_{11}(s)\tau_{12}(\lambda_1) \left(\nu - \frac{\lambda_1}{\lambda_1 - s}\right) + \tau_{12}(s) \times (1 - \rho_1)\frac{\lambda_1}{\lambda_1 - s} + \rho_1 \tau_{11}(s)\right] \beta_1(s). \quad (21)$$

Given (1) and (20–21), the LST of $\Delta_1$ yields:

$$\delta_1(s) = \beta_1(s)\tau_{11}(s)\tau_{12}(\lambda_1) + \frac{\lambda_1}{\lambda_1 + s} \beta_1(s)\tau_{12}(s + \lambda_1 + s) - \rho_1 \beta_1(s)\tau_{12}(\mu + s) \times \tau_{12}(s + \mu)\tau_{11}(s)(1 - \nu \beta(s) + \nu \rho_2 \beta_3(s)) \beta_1(s). \quad (22)$$

Having the LSTs (20–22), we can calculate the average system delay, PAoI and AoI as $E[T_1] = \tau_1(0), E[A_1] = -\alpha_1'(0),$ and $E[\Delta_1] = -\beta_1'(0)$:

$$E[T_1] = b + \frac{\lambda_1 b_{1}^{(2)} + \lambda_2 b_{2}^{(2)}}{2(1 - \rho_1)} + b_1 + \frac{s}{2(1 - \rho_1)}, \quad (23)$$

where $b_{1,2}^{(k)}$ denote the k-th moments of packet j service time.

$$E[A_1] = \left(1 + \frac{1}{\lambda_1} + \rho_1 b_1 + \rho_2 b_2\right)\tau_{12}(\lambda_1 - \rho_1(b + \beta_1(\mu)) \times (1 - \rho_1 + \rho_2 b_2) + (1 - \rho_1)\left(b_1 + E[T_{12}]\tau_{12}(\mu)\right) + \rho_1(1 - \rho_1 + \rho_2 b_2)\left(b_1 + E[T_{12}]\tau_{12}(\mu)\right) + \rho_1\tau_{11}(b_1 + E[T_{12}]\tau_{12}(\mu)). \quad (24)$$

where $b_{2}^{(2)} = b_{2}^{(2)}/2b_2$ is the average residual service time of non-priority packets.

We give lower bound $E[\Delta_1]$ for the average AoI:

$$E[\Delta_1] = b_1 + \frac{1}{\lambda_1} \tau_{12}(\lambda_1 - \tau_{12}(\lambda_1)E[T_1] + \rho_2 E[T_1] + \rho_1\left[\frac{1}{\theta} - \frac{\rho_1}{\mu} + \frac{1}{\lambda_1} + \frac{1}{\theta} - \frac{1}{\rho_1} - \frac{1}{\rho_1}\right]. \quad (25)$$
B. Non-priority packets

Non-priority packet $i$ can start service only if the second node is free of priority packets, i.e., at the end of the busy period $G_{2,i}$ or if priority is empty. Let us introduce the interval $\Psi_{2,i-1} = W_{2,i-1} + G_{2,i-1}$, where $W_{2,i-1}$ stands for the waiting time of non-priority packet $i-1$. Intervals $W_{2,i-1}$ and $G_{2,i-1}$ are independent, therefore the LST of $\Psi_{2,i-1}$ can be given as $\psi_2(s) = \omega_2(s)\beta_2(s + \lambda_1 - \lambda_1\gamma(s))$. We consider three cases to define the P AoI $\Psi$

B1: if $Y_{2,i} > \Psi_{2,i-1}$ and packet $i$ finds the second node empty it immediately goes to service, therefore $A_{2,i} = Y_{2,i} + S_{2,i}$. At the end of interval $\Psi_{2,i-1}$ the node is empty, therefore the probability that packet $i$ finds the node empty upon arrival equals to $1 - \rho_1$.

B2: if $Y_{2,i} > \Psi_{2,i-1}$ and packet $i$ finds the node busy with a priority packet with probability $\rho_1$ it waits until the end of the ongoing busy period $G_1$, thus $A_{2,i} = Y_{2,i} + G_1 + S_{2,i}$, where $G_1$ denotes the residual time of interval $G_1$.

B3: if $Y_{2,i} < \Psi_{2,i-1}$ packet $i$ finds the second node busy with non-priority packet $i-1$, therefore $A_{2,i} = \Psi_{2,i-1} + S_{2,i}$.

The LST $\alpha_2(s)$ can be given as the sum of three LSTs namely $\alpha_2(s, B_1)$, $\alpha_2(s, B_2)$ and $\alpha_2(s, B_3)$ defined above.

a) Case B1: the LST of $Y_{2,i} + S_{2,i}$ if $Y_{2,i} > \Psi_{2,i-1}$ and the node is free of priority packets can be given as

$$\alpha_2(s, B_1) = (1 - \rho_1)\frac{\lambda_2}{\lambda_2 + s} \psi_2(s + \lambda_2)\beta_2(s).$$  (26)

b) Case B2: the LST of $Y_{2,i} + G_1 + S_{2,i}$ when $Y_{2,i} > \Psi_{2,i-1}$ and packet $i$ arrives during the busy period $G_1$ takes

$$\alpha_2(s, B_2) = \rho_1\frac{\lambda_2}{\lambda_2 + s} \psi_2(s + \lambda_2)\tilde{\gamma}(s)\beta_2(s),$$  (27)

where $\tilde{\gamma}(s) = (1 - \gamma(s))/E[G_1]s$ stands for the distribution of the residual time of the interval $G_1$.

c) Case B3: if $Y_{2,i} < \Psi_{2,i-1}$ the LST of the PAoI yields

$$\alpha_2(s, B_3) = (\psi_2(s) - \psi_2(s + \lambda_2))\beta_2(s).$$  (28)

The resulting LST of the PAoI distribution of non-priority packets gives

$$\alpha_2(s) = \left[(1 - \rho_1)\frac{\lambda_2}{\lambda_2 + s} \psi_2(s + \lambda_2) + \psi_2(s) - \psi_2(s + \lambda_2) + \rho_1\frac{\lambda_2}{\lambda_2 + s} \psi_2(s + \lambda_2)\tilde{\gamma}(s)\right]\beta_2(s).$$  (29)

Having (1), (3) and (29) we give the LST of the AoI distribution of non-priority packets as follows:

$$\delta_2(s) = \frac{\rho_2}{1 - \rho_1}\gamma_2(s)\beta_2(s + \lambda_1 - \lambda_1\gamma(s)) + \psi(\lambda_2 + s)\beta_2(s)\left(\frac{\lambda_2}{\lambda + s} + \rho_1\frac{\lambda_2}{\lambda + s} \psi_2(s + \lambda_1 - \lambda_1\gamma(s))\right),$$  (30)

where $\beta_2(s + \lambda_1 - \lambda_1\gamma(s))$ denotes the residual time of the busy period $G_2$ and equals to $(1 - \beta_2(s + \lambda_1 - \lambda_1\gamma(s)))/sE[G_2]$.

The straightforward calculation of $\alpha_2'(0)$ and $\delta_2'(0)$ gives the average PAoI $\mathbb{E}[A_2]$ and AoI $\mathbb{E}[\Delta_2]$:

$$\mathbb{E}[A_2] = b_2 + \frac{\lambda_1 b_2(2) + \lambda_2 b_2(2)}{2(1 - \rho)(1 - \rho_1)} + \frac{b_1}{1 - \rho_1} + \psi(\lambda_2)\frac{1}{\lambda_2} + \frac{\rho_1\psi(\lambda_2)}{2(1 - \rho_1)^2},$$  (31)

$$\mathbb{E}[\Delta_2] = \frac{\rho_2}{1 - \rho_1}b_2 + \frac{\lambda_1 b_2(2) + \lambda_2 b_2(2) + b_1}{2(1 - \rho)(1 - \rho_1)} + \psi(\lambda_2)\left(\frac{1}{\lambda_2} + \frac{\rho_1\lambda_2^2}{\lambda_2} + \frac{\lambda_1 b_2(2)}{3(1 - \rho_1) + (1 - \rho_1)^2}\right) + \frac{\rho_1\rho_2}{2(1 - \rho_1)^2}(b_2 + \psi(\lambda_2)).$$  (32)

V. SELECTED NUMERICAL RESULTS

The results of our analysis have been validated by Monte Carlo simulation. All data collected during the transient state has been discarded. We model arrivals, service and departures of $10^5$ packets of the reference system. We calculate the average PAoI, AoI and system delay for different values of $p = \{0.1, 0.3, 0.5, 0.7, 0.9\}$ to capture the effect of the status updates generation rate on the AoI. The numerical results are given under the assumption of exponential service time with means $b = b_1 = b_2 = 1$ for variable utilization $\rho = \{0.1, \ldots, 0.9\}$ at the second node.

The metrics of interest of priority packets are depicted in Fig. 3. The simulation results of the PAoI illustrated in Fig. 3(a) show a perfect fit of our bound with the analytical curves, which justifies the assumption that priority packet $i$ finds non-priority packets at the second node with the given probability. The results for non-priority packets in Fig. 4 are instead exact. The given lower bound for AoI is tight when the system utilization is low and becomes more visible when $\rho$ increases. In our system, the PAoI is a tight upper bound of the AoI due to the low correlation between interarrival and delay intervals of consecutive packets. The average AoI of priority packets decreases when the status update rate increases if the priority system utilization $\rho_1 < 0.63$. If $\rho_1 \geq 0.63$ the AoI gradually increases demonstrating a wide U shape, the AoI of non-priority packets shows similar results in Fig. 3(b). This means that the optimal performance can be reached.

Besides the average AoI the average PAoI and system delay of non-priority packets are shown in Fig. 4(a) and Fig. 4(c) respectively. Again the average PAoI is a very tight upper bound for the AoI. Due to the non-priority packets preemption in waiting the average system delay rapidly increases when the utilization at the second node increases. Both PAoI and AoI of non-priority packets depend on the system delay more than that of priority packets. If non-priority packets may tolerate a certain packet error rate also due to the discarding of outdated packets the AoI could be improved if a newly arrived non-priority packet replaces the previously queued packet.
traffic. We have obtained the exact expressions for non-priority packets and tight bounds for their first moments. We have obtained the system delay in terms of LST and have given closed-form expressions for non-priority packets discarding, and age-aware packet management.

The extension to $N$ hops requires an exponential service time at first $N-1$ hops while the last hop that aggregates traffic from all previous hops holds general service time distribution. Such an assumption is in line with many multi-hop systems from the reference literature. Other possible research directions are the extension to more priority levels, LCFS discipline with packets discarding, and age-aware packet management.

VI. CONCLUSIONS

In this paper we have investigated the timeliness of the status updates in a multi-hop IoT tracking system with two nodes and different entry points for priority and non-priority traffic. We have derived the distribution of AoI, PAoI and system delay in terms of LST and have given closed-form expressions for their first moments. We have obtained the exact expressions for non-priority packets and tight bounds for priority flow of packets. In our system, PAoI is a tight upper bound for both classes of traffic.

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