Two-parameters electromagnetic hysteresis of a subwavelength nonlinear \( \epsilon \)-near-zero slab

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We consider propagation of transverse magnetic electromagnetic waves through a nonlinear metamaterial slab, of subwavelength thickness and very small average dielectric permittivity, made up of alternating metal and dielectric nonlinear layers. Exploiting the effective-medium approach, we prove that the output field intensity is a multi-valued function of both the input field intensity and incidence angle so that the transmissivity exhibits a novel kind of multi-stability and a pronounced two-parameter hysteresis behavior. The predicted two-parameter hysteresis physically stems from the fact that, since the linear and nonlinear contributions to the overall dielectric response can be comparable, the slab can host more than one electromagnetic configuration compatible with the incident field and the boundary conditions, regardless of the slab thickness. We investigate slab transmission also through full wave simulations proving the robustness of the two-parameter hysteresis beyond the effective-medium approximation.

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Hysteresis is probably one of the most intriguing features a nonlinear system can exhibit both theoretically and for its potential applications for the designing of memory devices. Optical bistability and related hysteresis have attracted a large research interest in the last decades [1, 2] since they are the basic ingredients for devising optical memories, logic gates and optical computing devices [3]. The advent of metamaterials, with the related possibility of engineering the medium electromagnetic response [4], has triggered a renewal of interest towards optical bistability [5]. One-dimensional photonic crystals consisting of alternating layers of positive-index and negative-index materials index [7] allow, in the presence of a nonlinearity, bistable switching and tunable nonlinear transmission [6], omnidirectional bistability [8] and bistability at very low values of input intensity [9]. Optical multistability have also been predicted in the presence of metal-dielectric multilayer structures [10] and it has been shown that, since for these structures the sign of the effective dielectric constant depends on the field intensity, step-like transmission of light can be achieved [11]. On the other hand, media with very small permittivity have been shown to support the regime where linear and nonlinear polarizations are comparable [12] and this has led to the prediction of a pronounced transmissivity directional hysteresis exhibited by nonlinear metamaterial slabs with very small linear permittivity [13].

In this Letter, we investigate the electromagnetic transmission of transverse magnetic (TM) waves through a metamaterial slab of subwavelength thickness made up of alternating metal and dielectric nonlinear Kerr layers stacked along the direction orthogonal to the thickness. The layers dielectric properties and filling fractions are chosen so as to yield a negative and very small effective linear permittivity. Exploiting the effective medium approach, we numerically solve Maxwell equations for the problem of reflection and transmission of an inclined incident TM plane wave and we show that the slab transmissivity is a multi-valued function of both the input field intensity and incidence angle. The predicted multistability physically stems from the fact that, since linear and nonlinear dielectric polarizations can be comparable (extreme nonlinear regime), the overall nonlinear dielectric response supports more than one field vector configuration compatible with the incoming plane wave and the boundary conditions, regardless of the slab thickness. Therefore, the proposed multistability fundamentally differs from standard optical bistability where the multiple field configurations are produced by a nonlinear feedback process. In order to support our results beyond the effective medium approximation, we perform full wave simulations whose results are in good agreement with those of the effective medium approach thus proving the robustness of the predicted phenomenology.

Consider a TM monochromatic field (with time de-
dependence \( \exp(-i\omega t) \) \( \mathbf{E} = E_z(x,z)\mathbf{e}_x + E_z(x,z)\mathbf{e}_z \) and \( \mathbf{H} = H_y(x,z)\mathbf{e}_x \) propagating through the metamaterial slab, embedded in vacuum, reported in Fig. 1(a) of sub-wavelength thickness (along the z-axis) \( L = 2/k_0 \) \((k_0 = \omega/c)\) and consisting of alternating, along the y-axis, two nonlinear isotropic and non-magnetic Kerr layers of thicknesses \( d_1 \) and \( d_2 \). We here consider the situation where the layers alternation period is much smaller than the vacuum wavelength (i.e. \( d_1 + d_2 \ll 2\pi/k_0 \)) so that, exploiting the effective-medium approach, the slab behaves as an homogeneous medium characterized by the dielectric response (constitutive relations)

\[
D_x = \varepsilon_0 \left\{ \chi E_x + \left( \mathbf{E} \cdot \mathbf{E}^* \right) E_x + \frac{1}{2} \left( \mathbf{E} \cdot \mathbf{E} \right) E_x^* \right\},
\]

\[
D_z = \varepsilon_0 \left\{ \chi E_z + \left( \mathbf{E} \cdot \mathbf{E}^* \right) E_z + \frac{1}{2} \left( \mathbf{E} \cdot \mathbf{E} \right) E_z^* \right\},
\]

where the effective dielectric permittivity \( \varepsilon = f\varepsilon_1 + (1-f)\varepsilon_2 \) and effective nonlinear susceptibility \( \chi = f\chi_1 + (1-f)\chi_2 \) are the averages of their layers counterparts (here \( f = d_1/(d_1 + d_2) \) is the volume filling fraction of layer 1) [12]. Therefore, employing both nonlinear negative and positive dielectrics (\( \Re(\varepsilon_1) < 0 \) and \( \Re(\varepsilon_2) > 0 \)), conditions can be found so that the effective dielectric permittivity \( \varepsilon \) has a negative real part (with a very small imaginary part) and very close to zero, and \( \chi > 0 \) (see below for an explicit example) which is the situation we focus on in this Letter. A TM plane wave \((\mathbf{E}_t,\mathbf{H}_t)\) incoming from vacuum \((z < 0)\) and impinging on the slab at \( z = 0 \) with incidence angle \( \theta \), produces a TM reflected plane wave \((\mathbf{E}_r,\mathbf{H}_r)\) for \( z < 0 \) and a TM transmitted plane wave \((\mathbf{E}_t,\mathbf{H}_t)\) for \( z > L \), as reported in Fig. 1(b). The fields of the three waves are \( \mathbf{E}_s = E_s(\mathbf{e}_y \times \mathbf{k}_s/k_0) \exp(i\mathbf{k}_s \cdot \mathbf{r}) \), \( \mathbf{H}_s = E_s/\varepsilon_0/\mu_0/\mathbf{e}_x \exp(i\mathbf{k}_s \cdot \mathbf{r}) \) where \( s = (i,r,t) \), \( E_i \), \( E_r \) and \( E_t \) are the amplitudes of the three waves and \( \mathbf{k}_i = \mathbf{k}_0 \), \( \mathbf{k}_r = k_0(\sin \theta \mathbf{e}_x + \cos \theta \mathbf{e}_z) \), \( \mathbf{k}_t = k_0(\sin \theta \mathbf{e}_x - \cos \theta \mathbf{e}_z) \) are the wave vectors of the three waves. The TM electromagnetic field within the slab is of the form \( \mathbf{E} = E_x(z)\mathbf{e}_x + E_z(z)\mathbf{e}_z \exp(ik_0x \sin \theta) \), \( \mathbf{H} = H_y(z)\mathbf{e}_y \exp(ik_0x \sin \theta) \) so that, Maxwell equations \( \nabla \times \mathbf{E} = i\omega\mu_0\mathbf{H} \), \( \nabla \times \mathbf{H} = -i\omega\mathbf{D} \) yields the system

\[
\begin{align*}
\frac{dE_x}{dz} &= i(k_0 \sin \theta)E_x + i\omega\mu_0H_y, \\
\frac{dH_y}{dz} &= i\omega D_x, \\
(k_0 \sin \theta)H_y &= -\omega D_z,
\end{align*}
\]

where \( D_x \) and \( D_z \) are given by Eqs. (1). At the slab input \( z = 0 \) and output \( z = L \) faces we require the continuity of the tangential components of the electric and magnetic fields. In order to evaluate the slab transmissivity \( T = |E_t|^2/|E_i|^2 \), we specify the amplitude of the transmitted field \( E_t \) at the output face \( z = L \) and successively solve Eqs. (2) all the way to the input face at \( z = 0 \). For our numerical evaluations we have chosen a slab with \( \varepsilon = -0.05 \) and \( \chi > 0 \) and we have discarded the solutions of Eqs. (2) for which the profile of \( E_z \) is discontinuous within the slab \( 0 < z < L \) (a possibility stemming from the fact that the third of Eqs. (2) is a nonlinear algebraic equation and not a differential equation for \( E_z \)). In Fig. 2(a) we report the evaluated transmissivity \( T \) (solid line) as a function of the normalized input field amplitude \( |E_i|^{1/2} \) at the fixed incidence angle \( \theta = 0.17 \) rad. In order to evaluate the transmissivity for a given input field amplitude as a function of incidence angle \( \theta \).
we have first evaluated a number of amplitudes $E_0$ for various $E_i$ and $\theta$ and subsequently collected all the configurations corresponding to the same $|E_i|$. In Fig.3(a) we report the resulting slab transmissivity $T$ as a function of $\theta$ for $|\chi|^{1/2}E_i = 0.05$ (solid line). In Fig.2(a) and 3(a) we have also reported, for comparison purposes, the linear ($\chi = 0$) slab transmissivity (dashed line) obtained in the same electromagnetic excitation conditions. Note that both the transmissivities (solid lines) of Fig.2(a) and 3(a) are multi-valued functions or, more precisely, there are ranges of both the normalized input field amplitude $(0.025 < |\chi|^{1/2}E_i < 0.035)$ and the incidence angle $(-0.1 \text{ rad} < \theta < 0.1 \text{ rad})$ at which the transmissivity is threefold.

In order to discuss the origin of the predicted multistability it is worth noting that the first two of Eqs. (2) are differential equations describing the spatial dynamics of the fields $E_x$ and $H_y$ whereas the third of Eqs. (2) is a nonlinear algebraic (cubic) equation yielding a field constraint which allows to evaluate $E_z$ (at least locally) for $E_x$ and $H_y$ prescribed. Therefore, it is evident that, in general, there are three values of $E_z$ compatible with given $E_x$ and $H_y$ so that, from a physical point of view, the overall nonlinear response of Eqs. (1) can generally support different electromagnetic field configurations for a given input intensity. In order to clarify this point and to discuss the impact of the boundary conditions, note that, from the third of Eqs. (2), it is evident that the continuity of $H_y$ across the slab boundaries is equivalent to the continuity of $D_z$, so that, after defining the real amplitude $A_z$ through the relation $E_z(L) = A_z(E_i/E_0)| \exp(2i\cos \theta)$ and exploiting the second of Eqs. (1) together with the continuity of $E_x$, the matching condition $D_z(L) = D_z(L^+)$ yields

$$
\left[ \epsilon + \frac{3}{2} \chi (|E_i|^2 \cos^2 \theta + A_z^2) \right] A_z + |E_i| \sin \theta = 0,
$$

which is a relation that, for a given $E_i$ and $\theta$, yields the possible values of $A_z$. Note that, in the linear regime ($\chi = 0$) Eq. (3) yields $A_z = -|E_i| \sin \theta / \epsilon$ i.e. the boundary conditions allow only one possible value of $A_z$. The possible values of $|\chi|^{1/2}A_z$ obtained by Eq. (3) belong to a surface which is plotted in Figs. 2(b) and 3(b), in the case $\epsilon = -0.05$ and $\chi > 0$, and we note that such surface is folded so that there are regions where, for a given pair $|\chi|^{1/2}E_i$ and $\theta$, there are more than one accessible value of $|\chi|^{1/2}A_z$. Therefore the multi-valuedness of the slab transmissivity corresponds to different fields with the same input intensity and incidence angle having different output longitudinal component $A_z$ belonging to different sheets of the folded surface. Note that, for $\theta = 0$, Eq. (3) admits the solutions $A_z^0 = 0$ and $|\chi|^{1/2}A_z^{(\pm)} = \pm \sqrt{-\text{sign}(\chi)}(2\chi/3 + |\chi|E_i^2)$ so that the surface can be folded only if $\text{sign}(\chi) = -1$ and its upper and lower sheets are such that $\chi A_z^2 \sim |\epsilon|$. This implies that, as opposite to the situation we are considering in this Letter where $|\epsilon| \ll 1$, in standard materials where $\epsilon$ is of the order of unity, the required nonlinearity would be so large to effectively forbid the folding of the surface (and the consequent feasibility of the directional multistability).

The predicted multistability is accompanied by a two-parameters transmissivity hysteresis. Note that each of the points of the curves (solid lines) of Figs. 2(a) and 3(a) corresponds to an electromagnetic configuration within the slab and hence these curves have an image on the surface $|\chi|^{1/2}A_z$ which are reported in Figs. 2(b) and 3(c) as solid lines as well. Suppose to switch on the incident plane wave with $\theta = 0.17 \text{ rad}$ and with an input intensity very small so that the state A of Fig.2(a) and 2(b) is excited since before the illumination the slab was not polarized. By increasing the input intensity and holding the incidence angle fixed, the transmissivity follows the curve joining the point A and B of Fig.2(a) while the electromagnetic state continuously varies as in Fig. 2(b). From the state B, if the intensity is further increased, it is evident that the electromagnetic state undergoes a sudden jump to the state C (see Fig.2(b)) on the lower surface sheet since there is no allowed state continuously joined to B. As a consequence, the transmissivity undergoes a sudden jump to a higher value (see Fig.2(a)). If now, starting from the state C, the input intensity is decreased, the electromagnetic state varies along the curve from C to D of Fig.2(b) whereas the transmissivity in Fig.2(a) assume the values along the curve from C to D which are different from those attained along the forward path. At the state D, if the intensity is further decreased, the state undergoes a jump to the state E of Fig.2(b), belonging to the central surface sheet, since the states from D to F of Fig.2(b) have, according to Fig.2(a), higher input intensities. An analogous but different hysteretic behavior is obtained if the input intensity is held fixed and the incidence angle is varied, as depicted in Fig.3. Since the presented multistability depends on two parameters, one can vary both the incidence angle and intensity on a prescribed path to obtain nontrivial hysteresis behaviors, as reported in Fig.4 where $\theta$ and $\chi^{1/2}|E_i|$ are varied along a closed path.

In order to discuss the feasibility of the above predicted two-parameters multistability we have performed 3D full-wave finite-element simulations for evaluating the transmissivity of the slab reported in Fig.1(a) in
and 3. We note that good agreement exists between the transmissivities (panel (a) and (b)) corresponding to those of Fig.2 obtained by solving Eqs.(2) (solid line) in the two situations full-wave simulations (dotted line) and the transmissivity evaluated through the slab nonlinear behavior. In Fig.6 we report the maximum values (within the medium) attained by the three field components for \( \theta = 0.17 \) rad as a function of the normalized input field amplitude corresponding to the evaluated full-wave transmissivities reported in Fig.5(a).

It is evident that, outside the shaded region, \( E_y \) (dotted line) is much smaller than both \( E_x \) and \( E_z \) (solid and dashed lines respectively) and in the corresponding regions of Fig.4(a) the agreement with the homogenization approach is very satisfactory. Within the shaded region of Fig.6, \( E_y \) is not negligible and this partially breaks the validity of the homogenization approach thus leading to the discrepancies of the transmissivities in the range 0.02 < |\( \chi |^{1/2} |E_i| | < 0.04. reported in Fig.5(a).

The intensity of the incident plane wave is \( I = (1/2)\sqrt{\epsilon_0 / \mu_0} |E_i|^2 \) which, for the amplitude range 0.02 < |\( \chi |^{1/2} |E_i| | < 0.04 where multistability occurs for \( \epsilon = -0.05 \) (see Figs.2(a) and 4(a)) and for the above effective nonlinear susceptibility \( \chi \), yields the values 1.2 MW/cm² < \( I < 4.8 \) MW/cm², which are intensities smaller than those normally required for observing the standard optical bistability. However it is evident that the smaller |\( \epsilon | the smaller the intensity required for observing the two-parameter hysteresis so that very smaller intensity thresholds (of the order of W/cm²) are very likely to be attained.

In conclusion, we have shown that a nonlinear metamaterial slab with a very small effective linear permittivity can host more than one electromagnetic configuration compatible with the incoming field and this leads to multistability and two-parameters hysteresis steered through the input intensity and incident angle. The two degrees of freedom potentially offers an infinite number of hysteresis loops. In addition, as a consequence of the small value of the permittivity, hysteresis is predicted at low input intensities and this suggests that our findings can have important applications in devising tunable and compact photonic devices for advanced optical control and memory functionalities.

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[1] A. Szoke, V. Daneu, J. Goldhar and N. A. Kurnit, Appl. Phys. Lett. 15, 376, 1969.
[2] W. Chen and D. L. Mills, Phys. Rev. B 35, 524 (1987).
[3] E. Abrahaham and S. D. Smith, Rep. Prog. Phys. 45, 815
(1982).

[4] D. R. Smith, J. B. Pendry and M. C. K. Wiltshire, Science 305, 788 (2004).

[5] N. M. Litchinitser, I. R. Gabitov and A. I. Maimistov and V. M. Shalaev, Opt. Lett. 32, 151 (2007).

[6] M. W. Feise, I. V. Shadrivov and Y. S. Kivshar, Appl. Phys. Lett. 85, 1452 (2004).

[7] J. Li, L. Zhou, C. T. Chan and P. Sheng, Phys. Rev. Lett. 90, 083901 (2003).

[8] R. S. Hegde and H. G. Winful, Micr. Opt. Tech. Lett. 46, 528 (2005).

[9] M. Z. Ali and T. Abdullah, Opt. Comm. 281, 3177 (2008).

[10] R. E. Noskov and A. A. Zharov, Opto-Electron. Rev. 14, 217 (2006).

[11] A. Husakou and J. Herrmann, Phys. Rev. Lett. 99, 127402 (2007).

[12] A. Ciattoni, C. Rizza and E. Palange, Phys. Rev. A 81, 043839 (2010).

[13] A. Ciattoni, C. Rizza and E. Palange, "Transmissivity directional hysteresis of a nonlinear metamaterial slab with very small linear permittivity", accepted for publication on Optics Letters. [arXiv:1004.2619v1]

[14] COMSOL, www.comsol.com

[15] E. D. Palik, Handbook of Optical Constants of Solids (Academic Press, San Diego, 1998).

[16] G. Yang, D. Guan, W. Wang, W. Wu, Z. Chen, Opt. Mater. 25, 439 (2004).

[17] W. Cai and V. Shalaev, Optical Metamaterials: Fundamentals and Applications (Springer, Dordrecht, 2010).