Missing Transverse-Doppler Effect in Time-Dilation Experiments with High-Speed Ions

Santosh Devasia
U. of Washington, Seattle, WA 98195-2600

Recent experiments with high-speed ions have investigated potential deviations from the time-dilation predicted by special relativity (SR). The main contribution of this article is to show that the SR predictions are matched by the experimental results only when the transverse Doppler effect in the observed emissions from the ions are neglected in the analysis. However, the Doppler effect in the emission cannot be neglected because it is similar to the time dilation effect. Thus, the article highlights the need to consider Doppler emission effects when validating SR time dilation using high-speed ion experiments.

PACS numbers: 03.30.+p, 06.30.Ft, 42.62.Fi

INTRODUCTION

The interest in potential small Lorentz violations has led to recent interest in experimentally measuring the time-dilation predicted by special relativity (SR) using high-speed ions [1]. It is noted that time-dilation predicted by special relativity (SR) has been verified with a number of experiments starting with the classical experiment using Hydrogen canal rays by Ives and Stilwell [2]. More recent experiments have evaluated the time-dilation effect by using Doppler-shifted lasers to excite transitions in high-speed Neon (Ne) and Lithium (Li) ions [1, 3]-[6]. A challenge in using high-speed ions is the presence of substantial Doppler broadening, which is resolved by using two lasers with different Doppler shifts that affect ions at a unique speed. Emissions from the ions are used to identify when the Doppler-shifted laser frequencies match the ion transitions, e.g., using the Lamb dip in the fluorescence spectrum [6] and to optimize the experiment [4, 5]. Then, the frequencies of the lasers used in the experiments and the known transition frequencies of the ions are used to validate SR predictions.

The main contribution of this article is to show that the SR predictions are matched by the experimental results only when the transverse Doppler effect in the observed emissions from the ions are neglected in the analysis. In particular, we review the Lamb-dip-based saturation spectroscopy experiments in [6] and show that the Doppler effect in the emission cannot be neglected because it is similar to the time dilation effect. Thus, the article highlights the need to consider Doppler emission effects when validating SR time dilation using high-speed ion experiments.

SATURATION SPECTROSCOPY WITH LAMB DIP

A transition at frequency ($\nu_0$) in an ion moving with speed $v$ with respect to a laboratory frame $F_L$ can be excited by using parallel (co-propagating) or anti-parallel (counter-propagating) lasers as in Fig. 1. The associated laser frequencies $\nu_p, \nu_a$ (parallel and anti-parallel in the laboratory frame) are given by the relativistic Doppler as [6]

\[
\nu_0 = \nu_p \gamma (1 - \beta) \approx \nu_p (1 - \beta + \frac{\beta^2}{2})
\]

\[
\nu_0 = \nu_a \gamma (1 + \beta) \approx \nu_a (1 + \beta + \frac{\beta^2}{2})
\]

where the normalized speed $\beta = v/c$, the speed of light is $c$ and

\[
\gamma = 1/\sqrt{1 - \beta^2}.
\]

The ratio $R$, given by

\[
R = \frac{\nu_p \nu_a}{\nu_0^2} = 1,
\]

is independent of the speed of the ions. Potential dependence of the ratio $R$ on speed $\beta$ is evaluated to test SR. The transitional frequency $\nu_0$ of the ions is known a-priori, and the frequencies $\nu_p, \nu_a$ are experimentally measured.

The main challenge is to ensure that the lasers interact with ions that have the same speed $\beta$. For example, in saturation spectroscopy [6], one of the laser frequencies is kept constant and the other frequency is varied to observe the Lamb dip in the fluorescence spectrum at frequency $\nu_0$, which indicates that both lasers are acting on ions with the same speed.
THE PROBLEM: DOPPLER EFFECT IN EMISSION

The problem is that previous analysis \[6\] does not consider the Doppler effect in the emissions observed from the ion. In particular, a photon observed in the laboratory frame \(F_L\) at a specific frequency is emitted from the ion at a different frequency with respect to a frame \(F_i\) moving with the ion as in Fig. 2.

\[
\begin{align*}
\nu_o^* &= \nu_o \gamma (1 - \beta) \\
\nu_a &= \nu_a \gamma (1 + \beta)
\end{align*}
\]

FIG. 2. SR with transverse Doppler effect in emissions.

Since photodetectors detect emissions at frequency \(\nu_o\) [6] perpendicular to the moving ions (in the laboratory frame \(F_L\)), the corresponding emission frequency \(\nu_o^*\) in a frame \(F_i\) moving with the ions is (from SR, e.g., [7])

\[
\nu_o^* = \nu_o \gamma, \tag{5}
\]

with angle \(\cos \theta_I = -\beta\) in the ion’s frame \(F_i\). Even though counts in the photodetectors (used to measure the emission) might be insensitive to the frequency of the photon, the observations will be sensitive to transverse Doppler effects if filters are used before the photodetectors. For example, the emission is observed using an interference filter (before the photodetector) centered at \(\nu_o\) with a narrow (10nm) halfwidth in [8] to precisely detect the laser frequency where the Lamb dip occurs — this makes the observations sensitive to the Doppler effect.

The Doppler effect on emissions implies that the observed Lamb dip at frequency \(\nu_o\) in the laboratory frame \(F_L\) corresponds to frequency \(\nu_o^*\) in frame \(F_i\) moving with the ions. Therefore, equations (1, 2) should be modified to

\[
\begin{align*}
\nu_o^* &= \nu_p \gamma (1 - \beta) \tag{6} \\
\nu_o^* &= \nu_a \gamma (1 + \beta) \tag{7}
\end{align*}
\]

which yields a different expression for the ratio \(R\) than Eq. 4

\[
R = \frac{\nu_p \nu_a}{\nu_o^2} = \frac{\nu_p \nu_a}{(\nu_o^*)^2} = \frac{1}{1 - \beta^2} \approx 1 + \beta^2 \tag{8}
\]

Thus, if the Doppler effect is included in the emissions, then, from SR, the ratio \(R\) is no longer independent of speed \(\beta\). Current experimental results [6] show that the ratio \(R\) is independent of speed \(\beta\), which would contradict SR prediction (in Eq. 3) if the emission Doppler effect is considered. Moreover, arguments (e.g., shift due to electrical potentials in the ion beam) would be needed to clarify why the transition frequency shifts from \(\nu_o\) to \(\nu_o^*\) in the reference frame \(F_i\) moving with the ion.

DOPPLER EMISSION EFFECT IS NOT NEGLIGIBLE

The Doppler effect on emission \(\gamma\) (in Eq. 5) is the same as the time dilation term \(\gamma\) (in Eqs. 11, 12) that is being measured. Note that there are nonlinear second-order \(\beta^2\) terms in Eqs. 11, 12 if observations are at frequency \(\nu_o\) in the frame \(F_i\) moving with the ions. With the addition of Doppler effect in emission (i.e., observations at frequency \(\nu_o\) in the laboratory frame \(F_L\)), the expressions in Eqs. 11, 12 change to (by using Eqs. 6, 7)

\[
\begin{align*}
\nu_o &= \nu_p (1 - \beta) \tag{9} \\
\nu_o &= \nu_a (1 + \beta) \tag{10}
\end{align*}
\]

that are linear in the speed \(\beta\) since the time-dilation effect \(\gamma\) cancels out. Therefore, the emission Doppler effect is not negligible, and should be included in analysis of results from high-speed ion experiments.

EFFECT OF OBSERVATION ANGLE

Additional work is needed to resolve the apparent inconsistency of experimental observations of the ratio \(R\) and prediction from SR with Doppler effects in the emission. It is shown, below, that expressions for the ratio \(R\) appear to be closer to the observed results if the angle of observation is not perpendicular to the motion of the ion. Note that previous work has shown that resonance fluorescence can be affected by the observational angle [9]. In the following analysis, time dilation is not included, however, it is possible that similar arguments could be developed with the time-dilation effect. Additionally, predictions using such modifications of the observation angle would need to be validated experimentally. The linear Doppler shifts of the ion transition frequencies (without time dilation) are

\[
\begin{align*}
\nu_o^* &= \nu_p (1 - \beta) \tag{11} \\
\nu_o^* &= \nu_a (1 + \beta) \tag{12}
\end{align*}
\]

with the Doppler shift in the emission given by

\[
\nu_o = \nu_o^* \sqrt{1 + \beta^2} \tag{13}
\]

where the emissions are observed at an angle

\[
\theta = \tan^{-1}(\beta)
\]

from the perpendicular to the ion velocities as shown in Fig. 3, e.g., [10]. This leads to nonlinear (second-order)
terms of speed $\beta$ in the Doppler shifted frequencies

$$
\nu_o = \nu_p (1 - \beta) \sqrt{1 + \beta^2} \approx \nu_p (1 - \beta + \frac{\beta^2}{2}) \quad (14)
$$

$$
\nu_o = \nu_a (1 + \beta) \sqrt{1 + \beta^2} \approx \nu_a (1 + \beta + \frac{\beta^2}{2}) \quad (15)
$$

that are similar to the second-order terms in Eqs. (12), which will therefore match (upto second order in $\beta$) experimental results for ion transitions \[4, 5, 11\].

The resulting ratio $R$ (using Eqs. 13,15)

$$
R = \frac{\nu_p \nu_a}{\nu_o^2} = \frac{\nu_p \nu_a}{(\nu_o^*)^2 (1 + \beta^2)} = \frac{1}{1 - \beta^4} \approx 1 + \beta^4
$$

is still not a constant — however, it does not contain second-order terms of speed $\beta$ as in the results with SR and Doppler emission effect in Eq. (8). In this sense, the results with an observation angle $\theta$ from the perpendicular (as in Fig. 3) appear closer to the experimentally-observed, constant-ratio $R$ in Eq. (4). Thus, it is possible, that the observation angle might be important in the interpretation of high-speed ion experiments.

CONCLUSIONS

This article showed that time-dilation predictions of special relativity (SR) are matched by high-speed experimental results when the transverse Doppler effect in the observed emissions from the ions are neglected in the analysis. However, the Doppler effect in the emission should not be neglected because it is similar to the time dilation effect. Thus, the article highlighted the need to consider Doppler emission effects when using high-speed ion experiments to validate SR time-dilation predictions.

[1] C. Novotny, G. Huber, S. Karpuk, and S. Reinhardt et. al., “Sub-Doppler laser spectroscopy on relativistic beams and tests of Lorentz invariance,” Physical Review A \textbf{80}, 1–5, #022107 (2009).

[2] Herbert E. Ives and G. R. Stilwell, “An experimental study of the rate of a moving atomic clock,” Journal of the Optical Society of America \textbf{28} (7), 215–219 (1938).

[3] J. J. Snyder and J. L. Hall, \textit{A new measurement of the relativistic Doppler shift}, in Laser Spectroscopy, Lecture Notes in Physics, Vol. 43 (Springer, Berlin / Heidelberg, 1975) pp. 6–17.

[4] Matti Kaivola, Ove Poulsen, Erling Riis, and Siu Au Lee, “Measurement of the relativistic doppler shift in neon,” Physical Review Letters \textbf{54}(4), 255–258 (January 28 1985).

[5] Roger W. McGowan, David M. Giltner, Scott J. Sternberg, and Siu Au Lee, “New measurement of the relativistic doppler shift in neon,” Physical Review Letters \textbf{70}(3), 251–254 (January 18 1993).

[6] G. Saathoff, S. Karpuk, U. Eisenbarth, G. Huber, S. Krohn, R. M. Horta, S. Reinhardt, D. Schwalm, A. Wolf, and G. Gwinner, “Improved test of time dilation in special relativity,” Physical Review Letters \textbf{91} (19), 1–4, #190403 (November 7 2003).

[7] M. Mansuripur, “Doppler shift, stellar aberration, and convection of light by moving media,” Optics and Photonics News, 52–56 (April 2003).

[8] Sascha Benjamin Reinhardt, \textit{Measurement of Time Dilation by Laser Spectroscopy on Fast Stored Lithium Ions}, PhD dissertation, Ruperto-Carola University of Heidelberg, Germany (2005).

[9] De-Zhong Wang and Jin-Yue Gao, “Effect of Doppler broadening on resonance fluorescence,” Physics Letters A \textbf{228}, 25–28 (March 31 1997).

[10] Santosh Devasia, “Nonlinear models for relativity effects in electromagnetism,” Zeitschrift fur Naturforschung A \textbf{64a} (5-6), 327–340 (May-June 2009).

[11] David Cyganski, “Comment on test of the complementary special relativity theory,” The old and new Concepts of Physics \textbf{4}(4), 633–639 (December 2007).