Significance of adding titanium dioxide nanoparticles to an existing distilled water conveying aluminum oxide and zinc oxide nanoparticles: Scrutinization of chemical reactive ternary-hybrid nanofluid due to bioconvection on a convectively heated surface

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Abstract: With the emphasis on the properties of titanium dioxide nanoparticles and numerous applications of chemical reactive distilled water due to bioconvection in the industries, nothing is known about the significance of adding titanium dioxide nanoparticles to an existing distilled water conveying aluminum oxide and zinc oxide nanoparticles when viscous dissipation, heat source, and higher buoyancy forces and thermal radiation are substantial. The governing partial differential equations that model the motion of both transport phenomena mentioned earlier were transformed into ordinary differential equations using appropriate similarity transmutations and solved with bvp4c (MATLAB built-in function). Multiple linear regression (i.e., a statistical tool used to explain outcomes related to engineering parameters of interest) was adopted for a deep scrutinization and exploration. The outcome of the analysis suggests that the thermal radiation parameter can be used to control the heat transferred via convection in the fluid flow. It is detected that the magnetic field parameter and volume fraction of nanoparticle parameters are useful to reduce the shear stress near the surface. The heat source ameliorates the fluid temperature, and the concentration of the fluid decreases with the rise in the chemical reaction parameter. Worthy to conclude that the Peclet and Schmidt number escalates the density number of motile microorganisms.

Keywords: ternary-hybrid nanofluid, higher buoyancy forces, convectively heated surface, bioconvection, aluminum oxide, zinc oxide, titanium dioxide

1 Background information

Titanium dioxide is one of the most fundamental elements in our daily lives. Titanium dioxide is one of the most studied crystalline oxides in the field of metal oxide surface science due to its capabilities as a photocatalyst with reasonably high efficiency for water breakdown et al. [1]. Titanium dioxide powders possess varieties of applications, owing to its capacity to impart whiteness and opacity to a variety of items, including paints, papers, and cosmetics. Titanium dioxide, for example, is approved as a food and pharmaceutical addition by Rowe et al. [2]. In the United States, titanium dioxide is considered an inert...
ingredient useful for oral capsules, cutaneous treatments, dental paste, nonparenteral medications, tablets, and suspensions by the Food and Drug Administration. The mechanistic toxicity investigations on titanium dioxide nanoparticles were discussed by Skocaj et al. [3]. The majority of the negative effects of titanium dioxide nanoparticles are caused by oxidative stress. Because the physicochemical qualities of particles are proportional to their size, the majority of materials including titanium dioxide nanoparticles are chemically more reactive at the nanometer level. Fujishima et al. [4] explicitly presented more information on the outstanding photocatalyst material of titanium dioxide for environmental purification. Although, according to the study by Kang et al. [5], high overpotential, slow migration, and quick recombination of photogenerated electron/hole pairs are important characteristics that limit the applicability of titanium dioxide. Ziental et al. [6] introduced titanium dioxide nanoparticles as photosensitizing agents for the treatment of malignant tumors and the photodynamic inactivation of antibiotic-resistant bacteria. Titanium dioxide nanoparticles, in particular, can be used as photosensitizers on their own or in composites and combinations with other chemicals or biomolecules. In a comparative analysis of water conveying copper oxide (CuO) nanoparticles, gold (Ag) nanoparticles, alumina (Al$_2$O$_3$) nanoparticles, copper (Cu) nanoparticles, and titania TiO$_2$ nanoparticles by Animasaun et al. [7], the thermal conductivity of titanium dioxide nanoparticles is 8.9538 W m$^{-1}$K$^{-1}$, while its density is 4,250 kg m$^{-3}$ and heat capacity is around 686.2 J kg$^{-1}$ K$^{-1}$.

Nanofluids are generally employed as coolants in heat transfer equipment such as heat exchangers, electronic cooling systems, and radiators due to their increased thermal characteristics. In recent years, there has been a focus on heat transmission over a flat plate. Nano fluids have been introduced as a new interesting kind of heat transfer fluids to replace regular fluids in industrial processes. Nano fluids may diminish erosion and corrosion significantly owing to their small size. Nano fluids have a broad range of applications, including refrigeration, heat exchangers, and cooling electronic devices. The use of nanoparticles is becoming more fascinating as the demand for high thermal performance in industries grows Khan et al. [8]. The dynamics of non-Newtonian Eyring-Powell liquid substance conveying not only nanoparticles but also gyrotactic microorganisms through the porous medium on Riga surface was examined by Khan et al. [9]. Consequently, Algehyne et al. [10] explained Darcy–Forchheimer of radiative alumina–water nano fluid, and Wakif et al. [11,12] explored the dynamics of 40% water conveying alumina nanoparticles and the dynamics of 60% ethylene glycol conveying alumina nanoparticles, mixed convection flow of non-Newtonian Walters-b fluid conveying tiny particles experiencing haphazard motion, and thermo-migration when Lorentz force and movement of gyrotactic microorganisms is significant. In another report, in the case of metallic and metallic oxide nanomaterials, Nayak et al. [13] explored nano fluid flows over an isothermal thin needle due to mixed convection.

A hybrid nano fluid is a revised kind of a mono nano fluid in which more than one nanoparticle is present. As a result, hybrid fluids outperform mono fluids in terms of heat transfer. These are used in various applications, including military equipment and solar collectors. Mustafa et al. [14] investigated heat transfer along the vertical rough surface of the sinusoidal wall beneath the nano fluid made up of interactions between alumina and silver nanoparticles subjected to an external magnetic field in the presence of internal heat generation. Because of the increased concentration of silver and alumina nanoparticles, it was discovered that skin friction increases while the Nusselt number decreases. The dynamics of non-Newtonian micropolar fluid conveying magnesium oxide and molybdenum disulfide nanoparticles experiencing Cattaneo–Christov heat flux was studied by Reddy and Shehzad [15]. The motion of couple stress hybrid nano fluids experiencing electroosmosis-induced alterations and peristaltic pumping through microchannel was examined by Tripathi et al. [16]. In another related study, the non-Newtonian Carreau hybrid nano fluid experiencing convective heat and non-Newtonian micropolar fluid conveying ferromagnetic and titanium alloy nanoparticles was examined by Kumar et al. [17] and Shehzad et al. [18] respectively.

Ternary hybrid nano fluid (THNF) is best described as a blend of three nanomaterials and a base fluid; see the studies by Elnaqeeb et al. [19], Animasaun et al. [20], and Saleem et al. [21]. In other words, THNF is described as the colloidal mixture of three different types of nanoparticles with a single base fluid. It exhibits greater thermal conductivity compared to binary hybrid nano fluid (one base fluid + two nanomaterials), as the name suggests. Sahoo and Kumar [22] experimentally prepared a THNF (Al$_2$O$_3$ + CuO + TiO$_2$/H$_2$O) and discussed its viscosity with different nanoparticle volume fractions. A linear relationship was discovered between dynamic viscosity and nanoparticle volume fraction. Boroomandpour et al. [23] prepared a water-ethylene glycol-based THNF and concluded that thermal conductivity and nanoparticle volume fraction have a linear relationship. In a recent study by Cao et al. [24] on thermo-migration, the haphazard motion of platelet alumina nanoparticles, spherical carbon nanotubes, and cylindrical graphene in the dynamics of water induced by free convection, forced convection, and mixed convection, it was discovered that rising thermo-migration of spherical carbon nanotubes, cylindrical graphene, and platelet alumina nanoparticles cause the heat and mass.
transfer across the ternary-hybrid nanofluid to diminish. Several researchers like Yang et al. [25], Ahmed et al. [26], Sundar et al. [27], and Manjunatha et al. [28] have recently contributed to the experimental and theoretical analyses of various ternary-hybrid nanofluid flows. Within the scope of rheology, distilled water conveying aluminium oxide, zinc oxide, and titanium dioxide possess a thermal conductivity, that is, 69% more than the thermal conductivity of distilled water.

Based on the aforementioned facts, it is important to discuss the impact of quadratic thermal radiation on the slip flow of THNF by an elongating sheet with higher order chemical reaction after carefully considering the preceding writing. The results are provided for two cases, ternary and binary hybrid nanofluids. Multiple linear regression is used to determine the effects of various parameters on surface drag force and heat transfer rate. Furthermore, a high level of agreement is observed in the validation between the current results and previous outcomes. The purpose of this study was to find answers to the following connected research questions:

1. At different levels of Lorentz force, volume fraction, and mixed convection, how does the velocity of the dynamics of chemical reactive distilled water conveying aluminium oxide and zinc oxide nanoparticles differ from the case of water conveying aluminium oxide, zinc oxide, and titanium dioxide nanoparticles?

2. At various levels of thermal radiation, volume fraction of the overall ternary-hybrid nanofluid, viscous dissipation, and heat source, to what extent does the addition of titanium dioxide nanoparticles to the existing chemical reactive distilled water conveying aluminium oxide and zinc oxide nanoparticles affects the distribution of temperature across the domain?

3. What is the difference in the concentration of chemical reactive distilled water conveying aluminium oxide, zinc oxide nanoparticles and chemical reactive distilled water conveying aluminium oxide, zinc oxide, and titanium dioxide nanoparticles?

## 2 Research methodology

This study investigates the laminar and two-dimensional chemically reactive and bioconvective flow of the THNF through a stretching sheet with quadratic thermal radiation and heat source.

### 2.1 Mathematical formulation

It is assumed that the stretching sheet is positioned along x-axis and y-axis is perpendicular to it. The sheet is assumed to be stretching at a speed of \( u_0(x) = bx \). The strength of the magnetic field \( B_0 \) was applied perpendicular to the flow; see Figure 1. Ambient temperature and concentrations are denoted by \( T_w \) and \( C_{w0} \), respectively. Values of the thermophysical properties of base fluid and nanomaterials are presented in Table 1. Furthermore, it is assumed that the induced magnetic field is insignificant. By using these assumptions, the governing equations for this study are as follows (Al-Kouz et al. [29], Al-Hossainy and Eid [30], Wakif et al. [31], and Ali et al. [32]):

\[
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{1}
\]

\[
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = \frac{\mu_{thnf}}{\rho_{thnf}} \frac{\partial^2 u}{\partial y^2} + g(\beta_T(T - T_w) + \beta_{1_f}(T - T_w)^2) + \beta_C(C - C_{w0}) + \beta_k(N - N_{w0}) - \frac{1}{\rho_{thnf}} g B_0^2 u, \tag{2}
\]

### Table 1: The thermo-physical properties of distilled water conveying aluminium oxide, zinc oxide, and titanium dioxide

| S.No. | DW (f) | Al₂O₃ \( (s_2) \) | ZnO \( (s_2) \) | TiO₂ \( (s_3) \) |
|-------|--------|-----------------|----------------|----------------|
| 1     | \( \rho \) (kg m\(^{-3}\)) | 998.203 | 3,970 | 5,600 | 4,250 |
| 2     | \( \rho \) (kg m\(^{-3}\)) | 3,970 | 5,600 | 4,250 |
| 3     | \( k \) (W m K\(^{-1}\)) | 0.613 | 40 | 13 | 8,953 |

Source: Ahmed et al. [33].

![Figure 1: Graphical illustration of the transport phenomenon.](Image)
\[
\begin{align*}
(pC_p)_{	ext{heat}} & \left( \frac{\partial T}{\partial x} + \nu \frac{\partial T}{\partial y} \right) \\
& = k_{\text{heat}} \frac{\partial^2 T}{\partial y^2} - \frac{\partial q_R}{\partial y} + Q_0(T - T_{\infty}) + \mu_{\text{heat}} \left( \frac{\partial u}{\partial y} \right)^2,
(3) \\
\rho C \frac{\partial T}{\partial x} + \nu \frac{\partial C}{\partial y} = D_m \frac{\partial^2 C}{\partial y^2} - k_0(C - C_{\infty})^2, \\
(4) \\
\frac{\partial N}{\partial x} + \frac{\partial N}{\partial y} = D_{m} \frac{\partial^2 N}{\partial y^2} - \frac{b W_c}{C_w - C_{\infty}} \frac{\partial}{\partial y} \left( N C \frac{\partial C}{\partial y} \right),
(5)
\end{align*}
\]

with the conditions

\[
\begin{align*}
\text{at } y = 0 : u = u_w, \nu = 0, & -k_{\text{heat}} T_y' = h_f(T_w - T), \\
C = C_{\infty}, N = N_w, & \text{as } y \to \infty : u = 0, T \to T_{\infty}, C \to C_{\infty}, N \to N_{\infty}.
(6)
\end{align*}
\]

Following Manjunatha et al. [28], the heat capacity, viscosity, density, and thermal conductivity of the ternary-hybrid nanofluid (distilled water conveying aluminum oxide, zinc oxide, and titanium dioxide nanoparticles) are expressed as follows:

\[
\begin{align*}
(pC_p)_{\text{heat}} & = [(1 - \phi_3)(1 - \phi_3)(pC_p) + \phi_3(pC_p)_{\phi_3} + \phi_3(pC_p)_{\phi_3}](1

- \phi_3) + (pC_p)_{\phi_3} \phi_3, \\
\mu_{\text{heat}} & = \frac{\mu_1}{(1 - \phi_3)(1 - \phi_3)(1 - \phi_3)}, \\
p_{\text{heat}} & = [(1 - \phi_3)(1 - \phi_3)\mu_1 + \phi_3\mu_{\phi_3} + \phi_3\mu_{\phi_3}](1

- \phi_3) + \phi_3\mu_{\phi_3}, \\
k_{\text{heat}} & = k_{\text{heat}} \times \frac{k_{\phi_3} + 2k_{\text{heat}} - 2k_{\text{heat}}\phi_3 + 2k_{\phi_3}\phi_3}{k_{\phi_3} + 2k_{\text{heat}} - 2k_{\phi_3} - k_{\phi_3}\phi_3}, \\
k' & = k_{\text{heat}} \times \frac{k_{\phi_3} + 2k_{\phi_3} - 2k_{\phi_3}\phi_3 + 2k_{\phi_3}\phi_3}{k_{\phi_3} + 2k_{\phi_3} - 2k_{\phi_3} - k_{\phi_3}\phi_3}, \\
k_{\phi_3} & = k_f \times \frac{k_{\phi_3} + 2k_f - 2k_{\phi_3}\phi_3 + 2k_{\phi_3}\phi_3}{k_{\phi_3} + 2k_f - 2k_{\phi_3}\phi_3 - k_{\phi_3}\phi_3},
\end{align*}
\]

In agreement with the theory by Nasr et al. [34], the radiative heat flux \( q_R \) can be defined as follows:

\[
q_R = -\frac{4}{3k^*} \frac{\partial}{\partial y} (\sigma^* T^\alpha).
(7)
\]

With the aid of Taylor's series expansion about \( T_{\infty} \), \( T^\alpha \) can be expressed as follows:

\[
T^\alpha \approx T_{\infty}^\alpha + 4T_{\infty}^3(T - T_{\infty}) + 6T_{\infty}^2(T - T_{\infty})^2 + ....
\]

Considering truncating the series after the second-order term and substituting it in Eq. (7), we can obtain the quadratic thermal radiation term. Then

\[
T^\alpha \approx T_{\infty}^\alpha + 4T_{\infty}^3(T - T_{\infty}) + 6T_{\infty}^2(T - T_{\infty})^2 = 3T_{\infty}^4 - 8T_{\infty}^3T + 6T_{\infty}^2T^2.
(8)
\]

Substituting Eq. (8) into Eq. (7) to obtain

\[
q_R = -\frac{4\sigma^*}{3k^*} \frac{\partial}{\partial y} (3T_{\infty}^4 - 8T_{\infty}^3T + 6T_{\infty}^2T^2)
\Rightarrow -\frac{\partial q_R}{\partial y} = -\frac{32\sigma^* T_{\infty}^3}{3k^*} \frac{\partial T^2}{\partial y^2} + \frac{24\sigma^* T_{\infty}^2}{3k^*} \frac{\partial^2}{\partial y^2}(T^2).
\]

With this, Eq. (3) becomes,

\[
(pC_p)_{\text{heat}} \left( \frac{\partial T}{\partial x} + \nu \frac{\partial T}{\partial y} \right) = k_{\text{heat}} \frac{\partial^2 T}{\partial y^2} - \frac{32\sigma^* T_{\infty}^3}{3k^*} \frac{\partial T^2}{\partial y^2} + \frac{24\sigma^* T_{\infty}^2}{3k^*} \frac{\partial^2}{\partial y^2}(T^2) + Q_0(T - T_{\infty}).
(9)
\]

Similarity transformations (Ali et al. [32] and Pal et al. [35]) are expressed as follows:

\[
\begin{align*}
\eta_f = \frac{u}{u_w}, \quad \eta = y \sqrt{\frac{u_w}{v}}, & \quad \nu = -\sqrt{bu_f}(\eta), \quad \frac{T - T_{\infty}}{T_{\infty} - T_{\infty}} \theta(\eta), \\
C = C_{\infty} + (C_w - C_{\infty})\Phi(\eta), & \quad N = N_{\infty} + (N_w - N_{\infty})\chi(\eta),
\end{align*}
\]

satisfies continuity Eq. (1), and Eq. (2), Eqs. (4)–(5), and Eq. (9) are transformed as follows:

\[
\begin{align*}
\frac{1}{N_2} \frac{\partial}{\partial \eta} \frac{f^{\alpha}}{f^{\alpha}} - \frac{1}{N_2} \frac{\partial}{\partial \eta} f^{\alpha} & + \delta(\theta + \lambda_3 \theta^2 + \lambda_3 \Phi + \lambda_3 \chi) = 0, \\
\frac{1}{N_2} \frac{1}{\text{Fr}} \frac{R_s(1 - 8\eta_0)\theta^m + \frac{1}{12} \frac{R_s(1 + \lambda_2 (\theta - 1))\theta^m}{\frac{1}{N_2}}}{\frac{1}{\text{Fr}}} & + \frac{1}{N_2} \left( \frac{R_s(1 - 8\eta_0)\theta^m + \frac{1}{12} \frac{R_s(1 + \lambda_2 (\theta - 1))\theta^m}{\frac{1}{N_2}}}{\frac{1}{\text{Fr}}} \right) \theta^m + \lambda_1 \frac{\partial \theta}{\partial \eta} + \frac{\partial f}{\eta} + \frac{\partial f}{\eta} &= 0,
\frac{1}{N_2} \frac{1}{\text{Le}} \frac{\partial \Phi}{\partial \eta} + \frac{\partial f}{\partial \eta} - \frac{k_4}{\nu_4} \frac{\partial \Phi}{\partial \eta} = 0, \\
\frac{1}{\nu_4} \frac{1}{\text{Le}} \frac{\partial \chi}{\partial \eta} + \frac{f}{\nu_4} - \frac{P_r}{\nu_4} \left( \Theta^m + (N_2 + \chi) \Phi^m \right) = 0, \\
\frac{1}{\nu_4} \frac{1}{\text{Le}} \frac{\partial \chi}{\partial \eta} + \frac{f}{\nu_4} - \frac{P_r}{\nu_4} \left( \Theta^m + (N_2 + \chi) \Phi^m \right) = 0,
\end{align*}
\]

and the conditions in Eq. (6) are changed as follows:

\[
\begin{align*}
\text{at } \eta = 0 : f = 0, & \quad f' = 1, \quad \theta' = -\frac{1}{N_3} \frac{\partial \Theta(1 - \theta)}{\partial \eta}, \\
\Phi = 1, \chi = 1, & \quad \text{as } \eta \to \infty : f' \to 0, \quad \theta \to 0, \quad \Phi \to 0, \chi \to 0,
\end{align*}
\]

where
and transfer rate of species, heat transfer rate \( \text{Nu} \), and mass transfer rate of motile microorganisms, mass transfer rate of reactive species, \( \text{Sh}_w \), and \( \text{Nu}_w \), and surface drag force \( C_f \), are indicated as follows:

\[
\text{Nu}_x = \frac{xq_w}{k(T_w - T_{\infty})} \bigg|_{y = 0} \quad \text{and} \quad C_f_x = \frac{\tau_w}{2 \rho u_w^2} \bigg|_{y = 0}.
\]

\[
\text{Sh}_w = \frac{xq_w}{D_m(C_w - C_{\infty})} \bigg|_{y = 0} \quad \text{and} \quad \text{Nu}_w = -D_n \frac{\partial N_n}{\partial y} = -D_n \frac{\partial c}{\partial y} = -k_{thf} \frac{\partial T}{\partial y}, \quad \tau_w = \mu_{thf} \frac{\partial u}{\partial y}. \tag{16}
\]

By using Eq. (10), we can rewrite Eq. (16) as follows:

\[
(\text{Re}_x)^{0.5} \text{Nu}_x = -\chi'(\eta)|_{\eta = 0}, \quad (\text{Re}_x)^{1/2} \text{Sh}_x = -\Phi'(\eta)|_{\eta = 0},
\]

\[
(\text{Re}_x)^{1/2} \text{Nu}_w = -N_l \theta'(\eta)|_{\eta = 0}, \quad (\text{Re}_x)^{1/2} C_f = \frac{2}{N_l} f''(\eta)|_{\eta = 0}.
\]

### 2.2 Numerical procedure and validation

In-built function MATLAB, bvp4c, is utilized to solve the transmuted Eqs. (11)–(14) with the conditions (15). It is simple to use because it is a built-in function. It is imperative first to make the following assumptions before we can write code (Waini et al. [36]):

\[
t_1 = f, \quad t_2 = f', \quad t_3 = f'' , \quad t_4 = \theta, \quad t_5 = \theta', \quad t_6 = \Phi, \quad t_7 = \Phi', \quad t_8 = \chi, \quad t_9 = \chi'.
\]

Then, using Eqs. (11)–(14) and conditions (15), we can construct a subsequent system of first-order ordinary differential equations:

\[
t'_1 = t_2, \quad t'_2 = t_3, \quad t'_3 = t_4, \quad t'_4 = -N_l t_5 \left( \delta(t_4 + \lambda t_4^2 + \lambda t_6 t_5) - t_2^2 - \frac{1}{N_l} M t_2 + t_3 t_4 \right), \quad t'_5 = t_6, \quad t'_6 = \left[ \frac{1}{N_l} \left( N_l - 8 R_a \right) + \frac{1}{N_l} R_a \left( \delta(t_4 - 1) t_2^2 + t_4 t_5 + \frac{1}{N_l} R_a (1 + (\delta t_4 - 1) t_6) \right) \right], \quad t'_7 = t_8, \quad t'_8 = \left( k_{thf} u_0^2 / \rho \right) \left( N_l + t_6 t_9 \right) - S_d t_9^2, \quad t'_9 = P(t_9 t_5 + (N_l + t_6) t_9^2) - S_d t_9.
\]

The conditions

\[
ta(1) = 0, \quad ta(2) = 1, \quad ta(5) = -\frac{1}{N_l} B(1 - ta(4)), \quad ta(6)
\]

\[
= 1, \quad ta(8) = 1, \quad tb(2) = 0, \quad tb(4) = 0, \quad tb(6) = 0, \quad tb(8) = 0.
\]

Next stage is to run the aforementioned system after conversion to MATLAB code and obtain the desired results in graphs. Table 2 presents the verification of our results with the predicted outcomes under special circumstances, which revealed a good concordance.
Table 2: Verification of our results with previously published results in exceptional circumstances, such as $\phi_1 = \phi_2 = \phi_3 = 0$

| $Pr$  | $-\theta'(0)$ $Manjunatha et al. [28]$ | Current outcomes |
|-------|--------------------------------------|------------------|
| 2     | 0.9113                               | 0.911320         |
| 6.13  | 1.7597                               | 1.759701         |
| 7     | 1.8954                               | 1.895449         |
| 20    | 1.3539                               | 1.353909         |

3 Analysis of results and discussion

The results of this study are shown for two cases: THNF (DW + Al$_2$O$_3$ + ZnO + TiO$_2$) and binary hybrid nanofluid (DW + Al$_2$O$_3$ + ZnO). The motion of the fluid is affected by the applied magnetic field. The particles of the liquid structure a chain turn toward the course of the applied attractive field. During this time, the particles are collided with one another, forming a barrier to the fluid flow. Fluid velocity decreases due to the increased viscosity, which is illustrated by Figure 2. It is clear from Figure 3 that the fluid velocity enriches with the increase in $\phi_1$. Note that the velocity profiles are looking high in the case of THNF flow compared to binary hybrid nanofluid flow. An increase in the mixed convective parameter minimizes the viscous forces. As a result, the fluid velocity increases, as shown in Figure 4. An increase in the Biot number increases the convective heat transfer to the fluid flow. As a result, the fluid temperature increases (Figure 5). Thermal radiation can be transmitted without the use of an intermediate medium. Growing thermal radiation parameter as shown in Figure 6, on the other hand, quickly indicates a higher process in which energy is radiated in all directions by a heated surface and travels at the speed of light to its absorption location. The temperature of the fluid rises as a result of particle collisions, as shown in Figure 6. More collisions between fluid particles occur as the volume fraction of the nanoparticles improves. Consequently, the fluid temperature rises, as shown in Figure 7. More kinetic energy is converted as internal energy in the fluid flow as the Eckert number improves. As a result, the temperature increases as shown in Figure 8. Figure 9 shows how the heat source changes the temperature profile. When the heat source increases, the fluid absorbs more heat energy from the source, which causes the escalation in the fluid temperature. The temperature profiles in the case of THNF flow appear to be higher than in the case of binary hybrid nanofluid flow, as seen in these graphs.
As shown in Figure 10, as the chemical reaction parameter goes up, the concentration usually goes down. This behavior could be explained by the increased entropy generation.

The effect of Le on the concentration profile is shown in Figure 11. When Le increases, momentum diffusivity far exceeds mass diffusivity, implying that the fluid moves at a high velocity and the fluid concentration decreases. Figures 12 and 13 show that the concentration of motile microorganisms minifies with the rise in $S_c$ and $P_e$.

3.1 Multiple linear regression

Multiple linear regression is a statistical technique for estimating the correlation among at least two independent and one dependent variables. The formula for multiple linear regression is expressed as follows:

$$ y = a_0 + a_1 x_1 + a_2 x_2 + \ldots + a_n x_n, $$

where $y$ is the dependent variable, $x_i$ is the independent variable, $a_0$ is the intercept, and $a_i$ is the regression coefficient of $x_i$ for $i = 1, 2, \ldots, n$. While obtaining the values of $a_i$’s, if $p$ value is less than 0.05, then the relation between $y$ and $x_i$ is significant. In this work, we used the following models to know the relationship among the engineering parameters of concern including the heat transfer rate and the parameters including radiation.

$$ C_{\varphi} = a_0 + a_1 M + a_2 \phi_1, $$

$$ Nu_e = b_0 + b_1 E_{ck} + b_2 R_a + b_3 Q_t, \quad (18) \quad (19) $$

![Figure 5: Increasing effects of Bi on $\theta(\eta)$](image)

![Figure 6: Increasing effects of $R_a$ on $\theta(\eta)$](image)

![Figure 7: Increasing effects of $\phi_1$ on $\theta(\eta)$](image)

![Figure 8: Increasing effects of $E_{ck}$ on $\theta(\eta)$](image)
With the aid of 21 sets of values for each equation, we obtain the following results:

\[ C_{k} = -0.983251 - 0.691169M - 15.939798\phi_1, \]  
\[ N_{n} = d_0 + d_5S_c + d_2P, \]

With the aid of 21 sets of values for each equation, we obtain the following results:

\[ Sh_1 = c_0 + c_jk_r + c_jLe, \]  
\[ Nn_1 = d_0 + d_5S_c + d_2P. \]

It is noticed from Eq. (22) that \( M \) and \( \phi_1 \) are having a negative impact on surface drag force. This means that
surface drag force minimizes with the amelioration in $M$ and $\phi_i$. Eq. (23) exhibited that the radiation, heat source, and viscous dissipation parameters have a negative influence on the Nusselt number. This means that the heat transfer rate minimizes with the escalation in those parameters. From Eq. (24), it is clear that there is a positive relation among Sherwood number and $k_i$, i.e. It is clear from Eq. (25) that the escalation in Schmidt number and Peclet numbers enriches motile microorganisms’ mass transfer rate.

4 Conclusion

In this report, the dynamics of chemical reactive distilled water conveying aluminum oxide, zinc oxide, and titanium dioxide due to bioconvection on a convectively heated surface when there is a significant viscous dissipation, heat source, and higher buoyancy forces and thermal radiation had been investigated. The engineering parameters, including heat transfer rate, have been discussed using multilinear regression. Based on the analysis of the new observations, it is worthy to conclude that

1. the volume fraction of nanoparticles lessens the shear stress near the surface.
2. heat source parameter ameliorates the temperature of the fluid.
3. radiation and Eckert number have a negative impact on the Nusselt number.
4. the mass transfer rate is positively influenced by the larger Lewis number and chemical reaction parameter.
5. the Peclet number enriches the density number of motile microorganisms.
6. the concentration of motile microorganisms minifies with bigger Schmidt number.
7. magnetic field parameter is used to control the fluid velocity.

Nomenclature

- $\mu$: dynamic viscosity of the fluid (kg m$^{-1}$s$^{-1}$)
- $\rho$: density of the fluid (kg m$^{-3}$)
- $\beta_T$: volumetric coefficient of thermal expansion (K$^{-1}$)
- $\beta_C$: volumetric coefficient of diffusion expansion of species (m$^{-3}$)
- $\beta_N$: volumetric coefficient of diffusion expansion of microorganisms (m$^{-3}$)
- $g$: acceleration of gravity (m s$^{-2}$)
- $\nu$: kinematic viscosity (m$^2$s$^{-1}$)
- $T$: dimensional temperature of fluid (K)
- $k$: thermal conductivity (W m$^{-1}$ K$^{-1}$)
- $\theta$: dimensionless temperature
- $C_p$: specific heat capacitance (J kg$^{-1}$ K$^{-1}$)
- $u, v$: velocity components in $x, y$ directions (m s$^{-1}$)
- $\eta$: similarity variable
- $Pr$: Prandtl number
- $E_{ck}$: Eckert number
- $\theta_w$: temperature ratio parameter
- $D_n$: microorganism diffusivity (m$^2$s$^{-1}$)
- $N_{\infty}$: difference parameter of microorganisms concentration (mol m$^{-3}$)
- $S_c$: Schmidt number
- $Le$: Lewis number
- $n$: order of chemical reaction
- $Q_0$: dimensional heat source parameter (W m$^{-3}$ K$^{-1}$)
- $Re_k$: Local Reynold’s number
- $Gr_T$: Grashoff number related to thermal
- $Gr_C$: Grashoff number related to concentration of species
- $Gr_N$: Grashoff number related to concentration of microorganisms
- $\lambda_1$: nonlinear convection parameter related to temperature
- $\lambda_2$: Buoyancy ratio parameter related to concentration of species
- $\lambda_3$: Buoyancy ratio parameter related to concentration of microorganisms
- $\delta$: mixed convection parameter
- $M$: magnetic field parameter
- $a^*$: Stefan–Boltzman constant (W m$^{-2}$ K$^{-4}$)
- $k^*$: mean absorption constant (m$^{-1}$)
- $f$: dimensionless Stream function
- $\chi$: dimensionless concentration of microorganisms
- $N$: dimensional concentration of microorganisms
- $W_c$: maximum speed of swimming cell (km h$^{-1}$)
- $h$: convective heat transfer Coefficient (W m$^{-2}$ K$^{-1}$)
- $P_c$: Peclet number
- $b^*$: Chemotaxis constant
- $B_i$: biot number subscripts
- $n_f$: nanofluid
- $hnf$: Binary hybrid nanofluid
- $thnf$: ternary hybrid nanofluid

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