Solar Wind Turbulence Studies Using MMS Fast Plasma Investigation Data

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Received 2018 July 16; revised 2018 August 29; accepted 2018 August 31; published 2018 October 15

Abstract

Studies of solar wind turbulence traditionally employ high-resolution magnetic field data, but high-resolution measurements of ion and electron moments have been possible only recently. We report the first turbulence studies of ion and electron velocity moments accumulated in pristine solar wind by the Fast Plasma Investigation (FPI) instrument on board the Magnetospheric Multiscale Mission. Use of these data is made possible by a novel implementation of a frequency domain Hampel filter, described herein. After presenting procedures for processing of the data, we discuss statistical properties of solar wind turbulence extending into the kinetic range. Magnetic field fluctuations dominate electron and ion-velocity fluctuation spectra throughout the energy-containing and inertial ranges. However, a multispacecraft analysis indicates that at scales shorter than the ion inertial length, electron velocity fluctuations become larger than ion-velocity and magnetic field fluctuations. The kurtosis of ion-velocity peaks around a few ion inertial lengths and returns to a near Gaussian value at sub-ion scales.

Key words: magnetohydrodynamics (MHD) – methods: data analysis – methods: statistical – plasmas – solar wind – turbulence

1. Introduction

The solar wind is hot, diffuse, supersonic plasma flow from the Sun. The solar wind provides a natural laboratory for the study of plasma turbulence (Tu & Marsch 1995; Bruno & Carbó 2005). In recent years, the nature of kinetic scale turbulence in the solar wind has been of considerable interest (Bale et al. 2005; Alexandrova et al. 2009; Sahraoui et al. 2010; Alexandrova et al. 2012; Salem et al. 2012; Roberts et al. 2017). Availability of high time-resolution magnetometer instruments has prompted the community to carry out most of these studies using magnetic field data. Due to practical constraints, kinetic scale studies with velocity data remain rare. This factor is particularly inconvenient from the perspective of turbulence studies, since a full understanding of a turbulent plasma requires multiscale information of both the magnetic field and ion and electron distribution function moments. In fact, the equations of magnetohydrodynamics (MHD) become symmetric and physically revealing when written in terms of Elsasser variables (e.g., see Politano & Pouquet 1998a, 1998b), thus calling for high-resolution measurements of both the magnetic field and velocity field.

The Magnetospheric Multiscale (MMS) Mission was launched in 2015 with the primary objective of studying magnetic reconnection in the terrestrial magnetopause and magnetotail (Burch et al. 2016). The very high time resolution of the plasma instruments on board MMS along with the availability of simultaneous measurements from four spacecraft, separated by sub-ion scale distances, make MMS observations an excellent case for the study of inertial and kinetic scale turbulence in near-Earth space. We note in passing that there have been numerous studies of turbulence in the magnetospheric environment, using Cluster (Retino et al. 2007; Sundkvist et al. 2007; Chasapis et al. 2015; Breuillard et al. 2016), MMS (Stawarz et al. 2016; Yordanova et al. 2016; Chasapis et al. 2017a; Chen & Boldyrev 2017; Breuillard et al. 2018; Roberts et al. 2018), as well as some earlier single-spacecraft studies (Borovsky et al. 1997). Here, we carry out analogous studies in the distinct turbulence environment of the solar wind using high-resolution MMS data.

Near the end of 2017, a few reasonably long (≈1 hr) MMS burst mode measurement intervals of the pristine solar wind, outside the bow shock, were made available. We present here the first study of high-resolution statistical turbulence properties of the solar wind using simultaneous measurements of ion and electron moments, and magnetic field data from MMS. In particular, we take advantage of the Fast Plasma Investigation (FPI) instrument which provides accurate measurements of ion and electron distribution moments with unprecedented time resolution. In order to exploit the advantages of the MMS/FPI instrumentation, we find it necessary to carry out some technical refinements in the form of filtering, to render the computed FPI moments to be of suitable quality for this study. Additionally, MMS multispacecraft capability allows us to directly observe sub-ion scale turbulence without the need of assuming Taylor’s frozen-in hypothesis. This affords an unusual opportunity to directly compare single-spacecraft Taylor-hypothesis correlations with two-spacecraft direct correlation measurements. We note that in ideal circumstances, when the turbulence is isotropic and the Taylor hypothesis is valid, the two-point, single-time measurements and the single-point, time-lagged measurements are theoretically equivalent.
Various factors cause this relationship to breakdown in part or even entirely (Sahraoui et al., 2010). For example, when the turbulence is anisotropic relative to the radial (or other) direction, spacecraft separated in the direction transverse to radial may not agree well with (radial) frozen-in flow correlations. Finite integration times for individual data points also can blur the interpretation. These issues will be discussed further.

### 2. Overview

In late 2017, the MMS apogee was at \(\sim 27 R_E\) on Earth’s dayside of the magnetosphere and outside the ion foreshock region. This orbit allowed the spacecraft to sample the pristine solar wind, outside the Earth’s magnetosheath, for extended periods of time. It should be noted that, for statistical studies of turbulence in the solar wind, relatively long continuous intervals of duration corresponding to at least a few times the typical correlation scale are needed. Here, we focus on an interval of approximately 1 hour length on 2017 November 24 from 01:10:03 to 02:10:03 UT. Figure 1 shows an overview of this MMS data interval.

The burst mode magnetic and electric field data are provided by the FIELDS instrument suite (Torbert et al., 2016). Specifically, the flux-gate magnetometer (FGM) measures the vector magnetic field at a resolution of 128 Hz (Russell et al., 2016). The FPI instrument (Pollock et al., 2016) measures the ion and electron distribution functions and calculates the moments of those distributions with a cadence of 150 ms and 30 ms, respectively.

An overview of the relevant parameters for the interval of interest can be seen in Table 1 which reports the following plasma parameters: the mean magnetic field strength \(\langle |B|\rangle\), the rms fluctuation value of the magnetic field \(\delta B = \sqrt{\langle (|B(t)| - \langle B \rangle)^2 \rangle}\), ion inertial length \(d_i\), electron inertial length \(d_e\), the solar wind speed \(V_{SW}\), and the proton plasma beta \(\beta_p\).

Due to the difficulties of FPI measurements in the solar wind, as discussed earlier, some systematic uncertainties may exist in the plasma moments. These uncertainties become more prominent in higher moments like temperature. We have cross-checked the parameters of Table 1 with Wind Faraday Cups (FC) and Magnetic Field Investigation (MFI) data from around 00:40:00 UTC (i.e., \(\sim 1\) hour before the midpoint of the period used in this study). While the density, velocity, and magnetic field values agree very well, significant deviations were present in the temperature and consequently the proton beta estimates by the FPI instrument and Wind estimates. The Wind measurements resulted in the proton beta of \(\beta_p = 0.43\). Given the limitations of FPI instruments in the solar wind, the Wind measurements of temperature and proton beta are more reliable.

### 3. FPI Moments in the Solar Wind

Contrary to the magnetosheath and the magnetopause, a number of issues arise when using FPI data in the solar wind. Many of these problems arise because the FPI detectors are optimized for magnetospheric response. However, when operating in the solar wind, almost all of the particles enter in just a few angular channels. This introduces problems such as periodicities (harmonics of the spacecraft rotation) that are due to, for example, the solar wind particles beam crossing between the detectors as the spacecraft rotates. The main implication for the present study is the occurrence of large-amplitude fluctuations in the time series of the plasma moments. These systematic features correspond to numerous very narrow, large-amplitude peaks in the frequency spectrum as seen in Figure 2. They are present in several burst resolution solar wind intervals, including the one used for the results presented here. These artifacts are thought to be associated, in part, with gaps in the angular coverage of the particle distribution functions. It is worth noting here that the first spike in the spectrum is due to the satellite spintone at 0.05 Hz.

### Table 1

| Parameter | Value |
|-----------|-------|
| \(\langle |B|\rangle\) (nT) | 6.6 |
| \(\delta B / |B|\) | 0.4 |
| \(\langle n_i \rangle\) (cm\(^{-3}\)) | 8.6 |
| \(d_i\) (km) | 75.9 |
| \(d_e\) (km) | 1.8 |
| \(V_{SW}\) (km s\(^{-1}\)) | 377 |
| \(\beta_p\) | 1.3 |

**Note.** Data obtained from MMS1 on 2017 November 24 from 01:10:03 to 02:10:03 UT.
Figure 2. The power spectrum of ion velocity from unfiltered level 2 burst mode data is shown in black. The noise floor is shown in translucent blue. The satellite spin frequency is 0.05 Hz, which corresponds to the first spike in the spectrum.

An in-depth analysis of the FPI spectrometers’ response in the solar wind is beyond the scope of this study. We apply a specially designed filter, based on a time series analysis, to mitigate the effects of these instrumental artifacts to proceed with this study.

For the present work, an algorithm was developed to filter out the artifacts depicted in Figure 2. This kind of scheme is feasible since, although these features have large amplitude, for most of the cases, they are very narrow in frequency space as seen in Figure 2. An exception is a broad peak at ≈1.625 Hz, which persists in the filtered signal as well, as shown later. It should be noted that this frequency is close to the 1/32 of the spin period (∼20 s) of the spacecraft. Given that the particle distributions are measured in 32 azimuthal bins on the spin plane of the spacecraft, this suggests that the broad peak near 1.625 Hz may be instrumental in origin. Since we employ a low-pass filter to remove the part of the spectrum where the signal is dominated by the noise, this feature does not impact the present study.

Implementation of a spectral Hampel filter and time series reconstruction. To eliminate these spikes, we adapt a decision-based filter known as the Hampel filter, a well-known technique in the signal-processing community for removing outliers from a signal (Davies & Gather 1993; Liu et al. 2004; Pearson et al. 2016). We apply the filter in the frequency spectra domain to remove the spikes, while maintaining the original phase information. We then reconstruct the time series via an inverse Fourier transform.10 The basic working mechanism of the filter is outlined in Figure 3. It is distinct from a window average or median filter, which have only one tuning parameter: the window width. The Hampel filter works as follows. For each point, \( x_i \), in a given series, the local median, \( m_i \), and the local standard deviation, \( S_i \), are calculated over a window of a predetermined size (say, \( k \)) around that point. If the absolute difference of the value of that point and the local median, \( |x_i - m_i| \), is above a threshold defined as \( n_\sigma \) times the local standard deviation, then the value is replaced by the median. If not, the algorithm leaves the current point unchanged and proceeds to the next point. Mathematically, the filter’s response can be described as

\[
y_i = \begin{cases} 
  x_i & \text{if } |x_i - m_i| \leq n_\sigma S_i, \\
  m_i & \text{if } |x_i - m_i| > n_\sigma S_i,
\end{cases}
\]

where \( x_i \) is the particular point under consideration, \( m_i \) is the local median value from the moving window,

\[
m_i = \text{median}(x_{i-k}, x_{i-k+1}, \ldots, x_{i-1}, x_i),
\]

and \( S_i \) is the local standard deviation estimated as

\[
S_i = \kappa \times \text{median}(|x_{i-k} - m_i|, \ldots, |x_{i+k} - m_i|),
\]

where \( \kappa = 1/\sqrt{2} \) erfc\(^{-1}(1/2) \approx 1.4826 \) is an estimate of the standard deviation for Gaussian noise. In the usual cases, the filter reduces to a standard median filter when the threshold parameter \( n_\sigma \) is set to zero, and becomes an identity operator as \( n_\sigma \to \infty \).

In our case however, the outliers exist in frequency space. Therefore, we apply the filter to the Fourier transform of the signal. Localized peaks with extreme values are replaced by the local median value, while the phase information is conserved. Therefore, we are able to perform an inverse Fourier transform, which allows us to reconstruct the original signal without the undesired artifacts. This approach is found to perform significantly better than the use of notch filters, due to the fact that the peaks are very localized in frequency space.

We apply this technique to the power spectral density of relevant quantities with a local window of 101 points in the frequency domain, choosing a threshold value of \( n_\sigma = 3 \). The window length of 101 points, corresponding to a frequency of 0.0028 Hz, is heuristically determined here. For this particular interval, this choice of parameters helps to eliminate most of the spikes in the spectra while leaving the underlying signal unchanged. For other data intervals the parameters may need to be changed. Applying the filter effectively removes the undesired spectral features that we believe are associated with instrumental artifacts induced by the operation of FPI instrument in the solar wind. A diagnostic of the filter algorithm employed and an evaluation of its performance is detailed in the Appendix.

We calculated the noise floor as described in Gershman et al. (2018) and determined that the ion-velocity spectrum becomes noise dominant near \( \approx 1 \) Hz (see Figure 2). Therefore, we applied a low-pass filter to the signal at the noise floor and set all higher frequencies to zero. Finally, after applying the Hampel filter, edge effects create some oscillations at the start and end of the time series. So we have removed the first and

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10 This procedure differs from a typical application of the Hampel filter in which the outlying points in the time domain are trimmed to have amplitude similar to their neighbors. We adapt this idea here to frequency space.
last 500 points of the time series from the analysis. It is worth mentioning here that the interval of these edge effects depend on the total length of the time series. For shorter intervals the edge oscillations are confined to narrower regions. Figure 4 schematically outlines all of the steps required to obtain the final signal sufficiently free of instrumental artifacts.

4. Turbulence Results

In Figure 5, we show the spectral power density (trace of the spectral density matrix) of magnetic field, ion velocity, and electron velocity. The magnetic field has been converted to Alfvén units; \( B \rightarrow B/\sqrt{\mu_0 m_p n_0} \), where \( \mu_0 \) is the vacuum permeability, \( m_p \) is the proton mass, and \( n_0 = \langle n_e \rangle \) is the mean number density of electrons \( \approx \langle n_p \rangle \), the average number density of protons, assuming quasineutrality. While the electron density tends to be more reliable than the ion density for MMS observations, in this interval both were found to be within \( \approx 5\% \) of each other and in agreement with the corresponding values obtained from the Wind data.

The spectral noise floors were determined by the procedure described in Gershman et al. (2018). Figure 5 shows that for the electron velocity measurements the noise becomes significant near \( \approx 0.15 \text{ Hz} \), while for the ion velocity this occurs at \( \approx 1 \text{ Hz} \). Note that, for burst mode MMS data in the Earth’s magnetosheath, usually ion-velocity spectra encounters the noise floor before electron velocity (see Gershman et al. 2018). So one can conclude that for MMS measurements in the pristine solar wind, the ion velocity effectively covers a better frequency range than the electron velocity while the situation is reversed for the magnetosheath. The magnetic field spectrum obtained by FGM crosses the instrumental noise floor near \( \approx 5 \text{ Hz} \) (Russell et al. 2016).

An approximate Kolmogorov scaling of \( \sim f^{-5/3} \) can be seen in the inertial range for all three variables, followed by a steepening in both the magnetic field and ion-velocity field spectra in Figure 5. To investigate the behavior of each quantity more clearly, we proceed to calculate the equivalent spectra, as shown in Figure 6. For the single-spacecraft analysis the time increments of the velocity (or any other vector field) are defined as

\[
\Delta V_i^r(t) = V_i(t + \tau) - V_i(t),
\]

where \( i \) represents \( R(\text{radial}) = X_{\text{GSE}}, T(\text{tangential}) = Y_{\text{GSE}} \) or \( N(\text{normal}) = Z_{\text{GSE}} \) component of the velocity field, referring to the geocentric solar ecliptic (GSE) coordinate (Franz & Harper 2002). For the multispacecraft technique, we define the increment of the \( i \)th component as

\[
\Delta V_{ab}^i(t) = V_i^a(t) - V_i^b(t),
\]

where \( t \) is the time of measurement and the indices \( a, b \) represent any of the four MMS spacecraft. The increment in this case corresponds to a spatial separation or lag of \( r_{ab} = x_b - x_a \), where \( x_a \) and \( x_b \) are the positions of the two spacecraft. This enables one to evaluate the actual spatial increment without needing to invoke the Taylor hypothesis.
The second-order longitudinal structure function of the vector field is defined as

\[ D_R^{(2)}(\lambda) = \langle (\Delta V_R^k(t))^2 \rangle, \]

where \( \langle \cdots \rangle \) represents an average over the full time series. The timescales of the nonlinear processes we are interested in are much larger than the timescales of the flow of the plasma past the spacecraft. For single-spacecraft solar wind data, therefore, it is assumed that the plasma is frozen-in and the available time series data are equivalent to one-dimensional spatial data along the plasma system. Operationally, to a good approximation, the associated spatial increments \( \lambda \) are given by the Taylor hypothesis (Taylor 1938) as

\[ \lambda \hat{R} = -V_{sw} \tau \hat{R}, \]

where \( \hat{R} \) is the unit vector along the radial direction and \( \tau \) is the time increment. The multispacecraft version of the calculation of the structure function reads similarly as

\[ D_R^{(2)}(r_{ab}) = \langle (\Delta V_R^k(t))^2 \rangle. \]

The quantity \( S^2(\lambda) = \lambda D_R^{(2)}(\lambda) \) then behaves as an equivalent spectrum viewed as a function of an effective wavenumber \( k^* = 1/\lambda \). If the omnidirectional wavenumber spectra exhibits a power-law scaling of \( E(k) \sim k^{-\alpha} \) for a range of bandwidth, one expects the equivalent spectrum to scale similarly, \( S^{(2)}(\lambda) \sim (1/\lambda)^{-5/3} \), for a similar range of \( k^* = 1/\lambda \). Equivalent spectra are useful for analyzing statistics of data sets that are relatively limited in time duration, as in the present one (Monin & Yaglom 1971, 1975). Figure 6 illustrates the equivalent spectra of magnetic, ion velocity, and electron velocity field, computed from the longitudinal second-order structure functions. The ion and electron velocities have been processed as described in Section 3. Only parts of the spectra unaffected by the instrumental noise are plotted. The ion and electron velocities follow each other almost perfectly in the large-scale and inertial-scale ranges. A Kolmogorov scaling is clearly seen for the magnetic field equivalent spectrum in the inertial range. The ion and electron velocity spectra show a shallower scaling, as was observed in Podesta et al. (2006).

Due to the effect of noise, we cannot extend the electron velocity spectrum further into the kinetic range, so we adopt a multispacecraft technique (Horbury 2000). For the present interval, the average separation of the four MMS spacecraft is about 15.6 km. These separation lengths are smaller than the inertial length \( d_{in} \), but still larger than the electron inertial length \( d_e \) (see Table 1).

The reader should note that the time for convection of the solar wind over a distance comparable to the spacecraft separation (~20 km), assuming a ~400 km s\(^{-1}\) wind speed, is about 0.05 s. This advection time is less than the integration time for an ion velocity distribution function (150 ms), but greater than the integration time for the electron distributions (30 ms). Therefore, the blurring of the spatial information due to finite integration time is a greater concern for the ion distributions. Even though the spacecraft pairs are not aligned in the radial direction (flow direction) for this interval, this influence causes us to have most confidence in the magnetic field two-spacecraft structure functions, and least confidence in the ion-velocity structure functions.

Further, the Taylor hypothesis needs to be modified from its single-spacecraft version \( (V_{sw} \gg V_a) \) to a multispacecraft analysis (Horbury 2000; Osman & Horbury 2007, 2009; Chen et al. 2010). For two spacecraft, separated by a displacement of \( d_{12} \), the frozen-in condition is satisfied when

\[ \frac{V_{sw} \Delta t}{|d_{12} - V_{sw} \Delta t| V_{sw}} < 1, \]

where \( V_A \) is the Alfvén speed and \( \Delta t \) is the sampling time of each spacecraft. We checked the criterion of Equation (9) for all the measurements used here. The left side of Equation (9) always remains below 0.12, so the condition imposed by Equation (9) is well satisfied for the measurements employed here.

The multispacecraft estimates reveal that the electron velocity exceeds the ion velocity and magnetic field at the spacecraft separation scales, which are deep inside the kinetic range. This kind of comparison is a significant advantage of the current approach in which we employ both single and multispacecraft techniques. Similar energy partitioning was found for low-beta plasma in the magnetosheath by Gershman et al. (2018). Since plasma \( \beta \) for the present interval is lower than one (according to the Wind measurements \( \beta_p = 0.43 \)), our results are qualitatively consistent with Gershman et al. (2018).

We now proceed to calculate third-order statistics of the fields, related to turbulent dissipation of the plasma. The Kolmogorov–Yaglom law, generalized to isotropic MHD (Politano & Pouquet 1998a, 1998b), in the inertial range reads as

\[ Y^{\pm}(r) = -(4/3) \epsilon \pm r, \]

where

\[ Y^{\pm}(r) = \langle (\hat{r} \cdot \Delta Z^+(x, r)|\Delta Z^\pm(x, r)|^2 \rangle \]

and

\[ Z^\pm = V \pm B. \]

Here \( \langle \cdots \rangle \) represents the ensemble average and \( B \) is in Alfvén units. The variables \( \Delta Z^\pm(x, r) = Z^\pm(x + r) - Z^\pm(x) \) are the increments of the Elsasser variables along a spatial lag \( r \). The quantities \( \epsilon^+ \) and \( \epsilon^- \) are the mean dissipation rates of the corresponding Elsasser variables. The total (magnetic+kinetic) energy dissipation rate is given by

\[ \epsilon = \frac{\epsilon^+ + \epsilon^-}{2}. \]

In order to calculate the Elsasser variables (Equation (12)) from the MMS data, the ion and magnetic field measurements of each spacecraft were synchronized, and the latter were resampled to match the 150 ms cadence of the former. The ion velocity was processed according to the procedure described in Section 3. So for the present context, Equation (10) can be written as

\[ Y^{\pm}(\tau) = (4/3) \epsilon \pm V_{sw} \tau, \]

where now \( Y^{\pm}(\tau) = \langle (|\Delta Z^{\pm}(\tau, t)|^2 |\Delta Z^\pm(\tau, t)|^2 \rangle \). Note that the ensemble average in Equation (10) is now replaced by a time average of \( \langle \cdots \rangle_{\tau} \). Henceforth, unless explicitly mentioned, we shall use \( \langle \cdots \rangle \) to represent the time average over the total data set interval. In Figure 7, we have plotted the absolute values of the mixed third-order structure functions. A linear scaling is
indeed observed in the inertial range and interpreted here according to \( |Y^\pm(\tau)| = (4/3)\epsilon^\pm V_{SW} \tau \). By fitting a straight line in the inertial range we obtain

\[
Y^+(\tau) \simeq 0.55\tau, \\
\frac{4}{3}\epsilon^+V_{SW}\tau \simeq 0.55\tau, \\
\implies \epsilon^+ \simeq 1.1 \times 10^3 \text{ J kg}^{-1} \text{s}^{-1}.
\]  

A similar calculation gives \( \epsilon^- \simeq 0.9 \times 10^3 \text{ J kg}^{-1} \text{s}^{-1} \). These values are comparable to the values reported in the literature for solar wind (Sorriso-Valvo et al. 2007; MacBride et al. 2008).

Next, we compute higher-order turbulence statistics. We estimate single-spacecraft values using the Taylor frozen-in hypothesis, as well as multispacecraft increments which allow us to estimate the values at the spacecraft separation independent of the Taylor hypothesis. The structure functions defined earlier can be generalized to higher orders as

\[
D^{(p)}_R(l) = \langle [\Delta V^\prime_R(l)]^p \rangle, \\
D^{(p)}_{Rab}(r_{ab}) = \langle [\Delta V_{Rab}^\prime(r_{ab})]^p \rangle,
\]

for each single-spacecraft, Taylor-hypothesis based estimate, and

for the two-spacecraft estimate. Here \( l \) is the spatial lag computed from temporal lag using the Taylor hypothesis. We show the second-, fourth-, and sixth-order structure functions for the radial (\( R \)) component of the ion velocity in panel (a) of Figure 8. The \( p \)-th order structure functions are normalized by the \( p \)-th power of the rms velocity fluctuation. The lines are single-spacecraft, Taylor-hypothesis based calculations and the solid symbols represent multispacecraft evaluations, computed at the spatial lags given by the corresponding spacecraft separation scales. Second-, fourth-, and sixth-order results are plotted as red, blue, and green lines, respectively.

The kurtosis, or normalized fourth-order moments, quantifies the deviation from gaussianity, and therefore provides insights into the intermittency or patchiness of the turbulence (Matthaeus et al. 2015). For a Gaussian process the kurtosis is equal to three. The scale-dependent kurtosis (SDK) can be calculated using the single-spacecraft time series as

\[
\kappa_i(l) = \frac{D^{(4)}_i(l)}{[D^{(2)}_i(l)]^2},
\]

where \( i \) is the component in the GSE coordinate. Similarly, SDK can be computed from multispacecraft measurements as

\[
\kappa_i(r_{ab}) = \frac{D^{(4)}_i(r_{ab})}{[D^{(2)}_i(r_{ab})]^2}.
\]

We plot the SDK at varying lags for all three components of the ion velocity in panel (b) of Figure 8. For SDK, the radial (\( R \)), tangential (\( T \)), and normal (\( N \)) components are plotted in red, blue, and green, respectively. The lines represent single-spacecraft estimates, while the symbols are evaluated using the multispacecraft technique.

It is important to note that the kurtosis reaches a peak at scales of a few \( d_i \). Similar results were obtained for magnetic field increments by Chasapis et al. (2017b). Therefore, the presence of intermittent magnetic field structures at scales of a few \( d_i \) coincides with intermittency in ion velocity at the same scale. This suggests that such structures have an important contribution to kinetic effects, including heating, temperature anisotropies, and production of suprathermal particles in solar wind turbulence, consistent with previous studies (Greco et al. 2012; Servidio et al. 2012; Karimabadi et al. 2013; Parashar & Matthaeus 2016).

Previous studies have observed ion-scale current sheets in the solar wind, and have reported particle energization due to...
magnetic reconnection in the form of reconnection outflow jets (Gosling et al. 2005; Gosling 2007, 2012; Osman et al. 2014). Such case studies of individual events would be consistent with the observed increase of intermittency of the ion velocity at exactly those scales. Regardless of the underlying mechanisms producing the energization suggested by the increased intermittency of the ion velocity, this observation suggests that intermittent magnetic field ion-scale structures, such as current sheets, play a dominant role in dissipation of turbulent energy in the solar wind.

5. Discussion

In this paper we have provided a first look at turbulence statistics using MMS FPI data at scales ranging from energy-containing scales down to proton kinetic scales. This is made possible by the development and implementation of a time series analysis procedure based on a spectral domain Hampel filter followed by a phase-preserving time series reconstruction. Specific results include first a demonstration that the electron and proton fluid velocity spectra track the magnetic spectrum down to the ion kinetic scales very well. However we find, from multispacecraft estimates, that the fluctuation velocity of electrons exceed that of ions, which further exceed the fluctuations of magnetic field at the kinetic scales. We then showed an application of the Politano–Pouquet third-order law, based on Elsasser variables (which simultaneously require both magnetic field and velocity data), also extending to kinetic scales. At small scales the mixed, third-order structure functions deviate from a linear scaling, clearly indicating the prominence of kinetic effects at those scales. Finally we explore the higher-order statistics for the solar wind proton velocities, up to sixth-order increments, finding good qualitative agreement with previously published results based only on the magnetic field data by Chasapis et al. (2017b).

We note that, as far as we are aware, these are the first such studies of turbulence statistics involving both velocity and magnetic field measurements at these scales in the solar wind. The results of the spectra suggest that ion and electron velocity cascade follows the magnetic field spectral cascade to the vicinity of the ion inertial length, as expected from various theoretical studies (Sonnerup 1979; Karimabadi et al. 2013), and inferred by observations that make use of electric field data (Bale et al. 2005). Previously, Safrankova et al. (2013, 2015, 2016) reported spectra of ion distribution moments extending to kinetic scales using data from the Faraday cups on board Spektr-R spacecraft, but no magnetic field measurements were available. A systematic comparison would be premature at this stage since the Faraday cups and the FPI instrument operate in fundamentally different ways. Further, we consider only one selected solar wind interval in this paper. We plan to take up a detailed study of a large sample of MMS solar wind data in future. The scale-dependent kurtosis of ion velocity shows a peak near $l \sim d_i$, suggesting enhanced intermittency at those scales. This result indicates the presence of kinetic processes and particle energization that are associated with the coherent structures (Tessein et al. 2013; Chasapis et al. 2018).

This research was partially supported by the MMS mission through NASA grant NNX14AC39G at the University of Delaware, by NASA LWS grant NNX17AB79G, and by the Parker Solar Probe Plus project through Princeton/IS-CIS subcontract SUB0000165, and in part by NSF-SHINE AGS-1460130. A.C. is supported by the NASA grants. W.H. M. is a member of the MMS Theory and Modeling team. The French involvement (SCM) on MMS is supported by CNES and CNRS. We are grateful to the MMS instrument teams, especially SDC, FPI, and FIELDS, for cooperation and collaboration in preparing the data. The data used in this analysis are Level 2 FIELDS and FPI data products, in cooperation with the instrument teams and in accordance with their guidelines. All MMS data are available at https://lasp.colorado.edu/mms/sdc/.

Appendix
Evaluation of the Filter

As described in Section 3, for solar wind FPI moments, a main feature is the presence of large-amplitude spikes at specific frequencies. These features are presumed to be instrumental in origin and probably arise from a combination of factors. Figure 9 demonstrates the effect of applying the Hampel filter to the ion velocity spectrum in frequency space. Panel (a) shows the original spectrum in black (as in Figure 2), superimposed on the spectrum corresponding to the Hampel-filtered signal in red. Most of the narrow spikes are eliminated except the broad peak near $\approx 1.625$ Hz. The low-frequency part of the spectrum remains immune to the filter, as desired. Panel (b) shows the real-space time series (only the radial component shown here) before this procedure and after filtering and
reconstruction of the time series. It is evident that the filter reduces the fluctuation amplitude of the signal by a considerable amount, leaving the global mean unchanged. The effect of presence of these spikes in the spectra becomes prominent in various standard quantities related to turbulence, e.g., the correlation function and SDK. SDK has been defined in the main text. The autocorrelation function is defined as

$$R(\tau) = \langle V(t + \tau) \cdot V(t) \rangle.$$  \hspace{1cm} (21)

We use the correlation functions and the SDK as diagnostic tools to check if the signal is sufficiently "clean" at different stages. Figure 10 shows these quantities at different stages of the procedure. As can be seen from the top panels of Figure 10, the unfiltered signal is dominated by the spurious signals which obscure the physical content of the signal, rendering it difficult to be used for statistical studies. After applying the Hampel filter, however, the middle panels show dramatic improvement in the quality of the signal. As discussed in Section 2, we further remove the high-frequency region of the signal, dominated by counting-statistics noise, by applying a sharp low-pass filter at 1 Hz. Finally, after removing the edges to eliminate artificial oscillations, the bottom panels of Figure 10 appear to be free of most of the artifacts and usable for studies of turbulence statistics, as the results in the main text demonstrate.

**Figure 10.** Normalized correlation functions and scale-dependent kurtosis for ion velocity at different stages of the filter procedure. Top panel: unfiltered signal; middle panel: Hampel-filtered; bottom panel: low-pass filtered at the noise floor (1 Hz), and then removing the edge effects.

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