Loss of classicality in alternating Spin Chain, in the presence of next-neighbor couplings and Dzyaloshinskii-Moriya interactions

Abhiroop Lahiri and Swapan K Pati
Theoretical Sciences Unit, Jawaharlal Nehru Centre for Advanced Scientific Research, Bangalore, India
(Dated: October 14, 2020)

We have considered and alternating Heisenberg spin chain with nearest-neighbor ($J_1$), next-nearest neighbor ($J_2$) antiferromagnetic couplings along with $z$-component of the Dzyaloshinskii-Moriya(DM) ($D_z$) interactions. The Hamiltonian has been studied using (a) Linear Spin-Wave Theory(LSWT) and (b) Density Matrix Renormalization Group (DMRG). The system had been reported earlier as a classical ferrimagnet only when nearest neighbor exchange interactions are present. Both the antiferromagnetic next-nearest neighbor interactions and DM interactions introduce strong quantum fluctuations and due to which all the signatures of ferrimagnetism vanishes. We find that the nonzero $J_2$ introduces strong quantum fluctuations in each of the spin sites due to which the $z$-components of both spin-1 and spin-1/2 sites average out to be zero. The ground state becomes a singlet. The presence of $J_1$ along with $D_z$ introduces a short range order but develops long range order along the XY plane. $J_1$ along with $J_2$ produces competing phases with structure factor showing sharp and wide peaks, at two different angles reflecting the spin spiral structure locally as well as in the underlying lattice. Interestingly, we find that the $D_z$ term removes the local spin spiral structure in z-direction, while developing a spiral order in the XY plane.

PACS numbers: 02.70.-c, 67.80.Jd

I. INTRODUCTION

Low dimensional quantum spin systems are of considerable interest as they exhibit a wide range of exotic physical phenomena\cite{1,2}. Due to strong quantum fluctuations, in most of the quantum low dimensional systems, the long range order gets destroyed even at absolute zero temperature. Many such properties have theoretically and computationally been predicted\cite{3,4} and most of those have already been realized experimentally\cite{15-20}. The most popular model to explain these phenomena is the Heisenberg model along with various other terms such as anisotropy\cite{7,19}, spin-orbit coupling and Dzyaloshinskii-Moriya\cite{27,28} interactions. Most of these terms break the SU(2) symmetry and lead to exotic broken symmetry ground states.

The low-lying excitations in these systems vary dramatically depending on the site spins (whether they are integer or half-odd-integer) and nature of the superexchange interactions. The nearest neighbor integral spin or nearest and next-neighbor coupled half-odd-integer spins show a finite gap in the excitation spectrum and in effect a short range two-spin correlation functions in the ground state. Interestingly, the next-neighbors coupling induces frustration in the system and such an infinite degeneracy of the classical ground state of the spin system gets lifted when quantum fluctuations are introduced. The gapless spin system, the gap due to resonating valence bond or spontaneous dimerization due to frustration, single magnon state, multi-magnon states, spin-glass and spin-ice ground states in a large classes of magnetic systems have already been realized computationally and experimentally. There have been studies on the low-energy and low temperature properties of alternating spin chains with nearest neighbor Heisenberg interactions. Such ferrimagnetic systems have been shown to display a rich low energy spectrum with both antiferromagnetic (AFM) ground state and ferromagnetic excitations. Due to underlying non-compensating site spins with finite magnetization, these low dimensional systems show long range magnetic order with finite magnetization even with finite temperature. Since quantum fluctuations of the Heisenberg model cannot destroy the classical ferromagnetic order, these alternating spin-chains can be well explained in the limit of Linear Spin Wave Theory (LSWT).

In this work, we are interested in the effect of Dzyaloshinskii-Moriya (DM) interactions on the low energy spectrum of Heisenberg chain systems consisting of spin-$\frac{3}{2}$ and spin-1 at alternating sites with competing exchange interactions. We have carried out detailed studies on the ground state properties of these systems using perturbative LSWT and non-perturbative Density Matrix Renormalization Group (DMRG) methods. We have compared and contrasted the spin-density, two-spin correlation functions and various order parameters between the perturbative and non-perturbative methods and with change in the magnitude of alternating site spins in the low-dimensional systems.

In next section, we have carried out detailed analysis of the low-energy spectrum of an alternating spin-$\frac{3}{2}$/spin-1 chain ($S_1, S_2$) with nearest neighbor AFM interactions and the $z$-component of the DM interactions using LSWT. In the subsequent section, DMRG calculations have been performed on the alternating spin sys-
tems (and at times with next-nearest neighbor frustrated term). The ground state energy, spin density and two-point equal time correlation functions and various other order parameters have been calculated to characterize the ground state.

Interestingly, with spin-wave spectrum and low-energy spectrum of DMRG calculations, we have calculated the low temperature properties, namely the magnetization and the magnetic susceptibility to see whether any new thermodynamic phases appear in low temperature. We analyze the effects of DM and next-neighbor frustrations on the low energy and low temperature properties of these alternating spin chains.

We conclude with summary of all the results in the last section of the manuscript.

II. RESULTS WITH NEAREST-NEIGHBOR INTERACTIONS

A. Linear Spin Wave Theory analysis

The Heisenberg Hamiltonian for an alternating chain of spins of $s_1$ and $s_2$ with Dzyaloshinskii-Moriya (DM) interactions can be written as

$$H = J \sum_i \{S_{1,i}S_{2,i} + S_{2,i}S_{1,i+1}\} + D_s \sum_i \{S_{1,i} \times S_{2,i} + S_{2,i} \times S_{1,i+1}\} \quad (1)$$

A detailed calculation has been carried out, which can be found in the Supplementary Information.

We apply the Holstein-Primakoff transformations

$$S_{1,i}^+ = S_1 - a_i^+ a_i,$$

$$S_{1,i}^- = \sqrt{2S_1 - a_i^+ a_i},$$

$$S_{1,i}^z = a_i^+ \sqrt{2S_1 - a_i^+ a_i}, \quad (2)$$

for the spin $S_1$ sites and

$$S_{2,i}^+ = -S_2 + b_i^+ b_i,$$  

$$S_{2,i}^- = b_i^+ \sqrt{2S_2 - b_i^+ b_i},$$  

$$S_{2,i}^z = \sqrt{2S_2 - b_i^+ b_i} \quad (3)$$

for the spin $S_2$ sites. Substituting the above in Eq.\[1\] followed by a Fourier transformation, we get,

$$H_{1k} = 2\sqrt{S_1 S_2} \sum_k \cos(\frac{k}{2}) \left( J^{(+)} a_k b_{-k} + J^{(-)} a_k^+ b_{-k}^+ \right) + J \sum_k \left\{ S_2 \left( a_k^+ a_k + a_{-k}^+ a_{-k} \right) + S_1 \left( b_k^+ b_k + b_{-k}^+ b_{-k} \right) \right\} \quad (4)$$

In the basis $A_k = \{a_k, b_k^+, a_{-k}, b_{-k}\}$, this can be written in matrix form

$$H_{1k} = \sum_k A_k^\dagger H_k A_k \quad (5)$$

where

$$H_k = \begin{pmatrix}
JS_2 & 0 & 0 & J \sqrt{S_1 S_2} \cos(\frac{k}{2}) \\
0 & JS_1 & 0 & J \sqrt{S_1 S_2} \cos(\frac{k}{2}) \\
0 & 0 & J^+ \sqrt{S_1 S_2} \cos(\frac{k}{2}) & JS_2 \\
J^+ \sqrt{S_1 S_2} \cos(\frac{k}{2}) & J^+ \sqrt{S_1 S_2} \cos(\frac{k}{2}) & JS_2 & 0
\end{pmatrix} \quad (6)$$

Bogoliubov transformation, followed by diagonalization (see Supplementary Information) gives 2 doubly degenerate modes:

$$\omega_{1k} = \frac{1}{2} \left\{ (S_1 - S_2) \right\} - \Omega_k$$

$$\omega_{2k} = \frac{1}{2} \left[ (S_1 - S_2) \right] + \Omega_k \quad (7)$$

Two low-energy spin wave dispersion curves are shown in Fig.\[1\] corresponding to the two sublattices with two different bosonic modes. In the case of $D = 0$, the lower dispersion mode (colored red) is the gapless mode, due mainly to the antiferromagnetic interactions, while the
higher energy dispersion mode is gapped and has features of ferromagnetic interactions. This same relation was obtained by Pati et al [11]. As $D$ is increased, within the linear spin-wave theory, the dispersion $\omega_{1k}$ becomes negative near $k = 0$. This implies that there is an instability with respect to static spin-wave formalism[13]. This instability can be removed by applying an external magnetic field as there will be an extra tunable coefficient $B$ of $a_k^z a_k$ due to the field, which will shift the value of $\omega_{1k}$ back to zero. We will see below how the non-perturbative method accounts for such instability when the quantum fluctuations is properly accounted for.

For $D > 0.3535$, the discriminant, $\Omega_k^2$ [8] becomes negative. Thus, both the $\omega$s, namely, $\omega_{1k}$ and $\omega_{2k}$ become complex. Hence, the spin wave dispersion curves become unrealistic and are thus no longer remain valid. The sub-lattice magnetizations can be calculated as the expectation values of $S_{1z}$ and $S_{2z}$.

$$ M_a = S_1 - \langle a_i^z a_i = 1 \rangle = S_1 - \frac{1}{N_{wc}} \sum_{k,k'} \langle a_k^z a_{k'} \rangle $$ (9)

and

$$ M_b = \langle b_i^z b_i = 1 \rangle - S_2 = \frac{1}{N_{wc}} \sum_{k,k'} \langle b_k^z b_{k'} \rangle - S_2 $$ (10)

We can calculate (Suppl.)

$$ \frac{1}{N_{wc}} \sum_{k,k'} \langle b_k^z b_{k'} \rangle = \frac{1}{N_{wc}} \sum_{k,k'} \langle a_k^z a_{k'} \rangle $$

$$ = \left( S_1 + \frac{1}{2} \right) + \frac{1}{2} \int_{-\pi}^{\pi} \frac{J(S_1 + S_2) + \Omega_k}{4(J^2 + D^2)S_1 S_2 \cos^2(k/2)} dk $$ (11)

As can be seen from linear spin-wave results, for each spin-$\frac{1}{2}$ spin-1 dimer, the total magnetization is $(S_1 - S_2) = \frac{1}{2}$ since the fluctuations in each of the site spins are exactly opposite and thus gets cancelled out to give finite magnetization value for every dimer. Thus, the system behaves as a classical ferrimagnet with alignment of finite magnetizations of dimers in a lattice. However, this result is valid only for $D = 0$, as evident from non-perturbative DMRG calculations, discussed in the next subsection. For $D \neq 0$, the non-perturbative quantum fluctuations make the spin of each site as well as the dimer to be zero.

The $S^z-S^z$ correlation function is given by

$$ C^z (|i - j|) = \langle S^z_i S^z_j \rangle - \langle S^z_i \rangle \langle S^z_j \rangle $$ (12)

These correlation functions comprise of 3 types, $(S^z_1 S^z_{1j})$, $(S^z_2 S^z_{2j})$ and $(S^z_1 S^z_{2j})$. These are plotted in Fig. X Since the system behaves as a ferrimagnet with finite magnetization for every dimer, the correlation functions should be calculated after subtracting the product of the averages of each $S^z$ values, given by

$$ (S^z_{1i} S^z_{1j}) - \langle S^z_{1i} \rangle \langle S^z_{1j} \rangle = \frac{1}{2\pi} \int_{-\pi}^{\pi} \left( 1 + \frac{\Omega_k \cos(k|i - j|)}{J(S_1 + S_2)} \right) dk $$ (13)

for the first type of correlations.

The plots of the correlation function with distance between the spin sites are shown in Fig ???. This clearly shows very short range order up to only a few sites, which is consistent with the earlier work [11] for $D = 0$. But for $D \neq 0$, in the LSWT regime, the correlation length is also very small and there is not much significant difference between the correlation function for different values of $D$ until the function becomes complex at $D \approx 0.36$. Thus LSWT fails to explain this case, as it assumes that there is primarily antiferromagnetic order with fluctuations, even in presence of $D$. This is contrary to all the DMRG results, especially with nonzero $D$, presented later.

### B. DMRG Results

Finite size DMRG calculations have been performed in the Matrix Product State (MPS) formalism for a chain of 120 and 240 sites with nearest neighbor (NN) as well as next-nearest neighbor (NNN) magnetic exchange interactions and the z-component of the Dzyaloshinskii-Moriya interactions. In this paper we present the results for 240 sites. Those for 120 sites are given in the Supplementary Information. The cut off for bond dimension of the MPS has been kept to be 500 and the finite size sweeps
FIG. 2: Spin correlation function plots from linear spin wave theory for (a) nearest-neighbour and (b) next-nearest-neighbour interactions in a spin-$\frac{1}{2}$/spin-1 alternating chain of 30 have been used to obtain the converged ground state. Without DM interactions, the spin density (shown is Fig.x) of each site is less than the classical value, but the difference between spin density for each dimer is 1/2 as expected, which is also confirmed from our LSWT calculations and previous work[11]. This means that each site has quantum fluctuations, but each spin-$\frac{1}{2}$-spin-1 dimer has classical magnetization value. Hence the $S_z$-$S_z$ correlation function, given by

$$C_z(|i-j|) = \langle S^z_i S^z_j \rangle - \langle S^z_i \rangle \langle S^z_j \rangle$$  \hspace{1cm} (14)

sharply falls to zero after a length of two sites, as the product of the averages have finite values. This is characteristic of a magnetic chain with long range order, which in this case is due to the formation of a ferrimagnetic chain with finite dimer magnetization in the lattice.

However, the moment we turn on the nearest-neighbor DM interactions, it introduces strong quantum fluctuations in each of the spin sites, making the dimer move away from classical magnetic state. Although, the quantum fluctuation was present in each site spin (when $D_z = 0$), the dimer did not have the quantum fluctuation, because oppositely oriented z-component of the site spins had exactly opposite fluctuations, thus cancelled each other. However, with $D_z \neq 0$, the situation is very different; it not only introduces strong quantum fluctuation in each of the site spins, their z-components vanishes thereby the dimer z-component also vanishes. This makes each spin density at individual sites zero even though there are different site spins in every alternate sites. Interestingly, the quantum fluctuations are more for larger magnitude spins and LSWT to explain any of these even with large magnitude of DM interactions. This suggests that although in a ferrimagnet, the interactions are antiferromagnetic, any non-local interactions can destroy the site magnetization of different magnitude magnetic ions as well as the dimeric magnetization of classically non-compensating spin dimers. However, we will see below how spin structure and ordering of the spin sites in a ferrimagnet change when DM interactions are introduced.

For non-zero $D_z$, the correlation function $C_z(|i-j|)$ decays at a much slower rate, thus introducing quasi long range order as the product of the averages are zero. Moreover the cross-correlation given by

$$C^{(+)}(|i-j|) = \langle S^+_i S^-_j \rangle - \langle S^+_i \rangle \langle S^-_j \rangle$$  \hspace{1cm} (15)

starts building up with finite values for nonzero $D_z$, which on the other hand was decaying for $D = 0$. In fact, such differences arise primarily due to quantum fluctuation in a low dimensional lattice system, which LSWT can not capture completely.

The classical ferrimagnetic ordering in the chain is also reflected in the structure factor, $S(q)$, for $D_z = 0$. The preferred direction of ordering of any two spin-$\frac{1}{2}$ or spin-1 at alternate sites is parallel. This is manifested in the sharp peak of $S(q)$ at $q = 0$ and at $q = \pi$. For $D_z \neq 0$, the peak at $q = 0$ or at $q = \pi$ vanishes, as there is no preferred ordering. Nevertheless, there are two broad peaks at $q < \pi$ and $q > -\pi$, referring to some canted ordering angle. In fact, the spiral ordering angle varies with the magnitude of $D_z$. As can be seen in Fig. 3d, the spiral angle increases quite non-linearly with the increase in $D_z$ magnitude.
III. RESULTS WITH NEXT-NEAREST-NEIGHBOR INTERACTIONS

A. Spin-Wave Theory Analysis

The Hamiltonian for next-nearest neighbor (NNN) exchange interaction in k-space is given by

\[
\mathcal{H}_{2k} = \mathcal{H}_{1k} + \mathcal{H}_{nn,k}
\]

where

\[
\mathcal{H}_{nn,k} = J_2 \sum_{\delta,k} (e^{2ik\delta} - 1) \left( S_1 a_k a_k^\dagger + S_2 b_k b_k^\dagger \right)
\]

Bogoliubov transformation, followed by diagonalization (see Supplementary Information) gives 2 modes as before (the other two are the same):

\[
\omega_{1k} = \frac{1}{4} \left[ (S_1 - S_2) \left( J_1 + J_2 (1 - 2\cos(2k)) \right) \right] + \Omega_k
\]

\[
\omega_{2k} = \frac{1}{4} \left[ (S_1 - S_2) \left( J_1 + J_2 (1 - 2\cos(2k)) \right) \right] - \Omega_k
\]

where

\[
\Omega_k = \sqrt{-4 (J_1^2 + D^2) S_1 S_2 \cos^2(k) + \left( J_1 - 2J_2 \left( \frac{1}{2} - \cos(2k) \right) \right)^2 (S_1 + S_2)^2}
\]

A similar expression without the DM interaction had been derived by Mohakud et. al [12]. The energy modes dispersion values for different parameters are plotted in Fig () and Fig ()

CASE-I: D = 0

On introducing the next-neighbor AFM coupling, one introduces spontaneous frustration in a one-dimensional antiferromagnetic lattice. This leads to the dispersion relation becoming flattened and as the J_2 is increased further, the spin group velocity reduces. Interestingly, on further increase of J_2, the AFM dispersion mode flattens more, until it becomes negative at J_2 = 0.25 and complex. The magnetization in this case (calculated similarly as in Eqs. [9][10][11]) reduces from its classical value, but for the dimer, it remains the same, (S_1 - S_2), which is classical.

The correlation function vs. length is plotted in Fig 2b. Clearly, the LSWT predicts short range order for both non-zero J_2 and nonzero D values, contrary to DMRG results.

B. DMRG Results

First, the DMRG calculations were performed for J_2/J_1 = 0.5, as for a spin-1/2 antiferromagnetic chain, this was predicted to have resonating valance bond doubly degenerate ground state way back in 1969[? ]. With next-neighbor interactions term as half of nearest neighbor, the dimer no longer behaves as classical magnetic dimer with finite magnetization value. The magnetization average at each site becomes zero again, thus making the magnetization value in the dimer to be zero. For D_2 = 0, the frustration propagates, spontaneously dimerizes with introduction of quasi long range order, as evident from the S_z - S_z correlation function plotted in Fig.x. As D_2 is turned on, unlike that of only nearest neighbor interactions, the quasi long range order is destroyed (shown in the correlation plot in Fig.x). Hence, in both the cases, the DM interaction term changes the spin vectors through quantum fluctuations and thereby the two point correlation function between them and effectively reverses completely or to some extent the quasi long range order or short range order set by the Heisenberg interaction terms. Interestingly, even with small values of DM interactions the spontaneous changes in quantum fluctuations in spin density and spin spin correlation functions occur.

The cross-correlation C^{(+)}(|i – j|) builds up more with increase in D.

FIG. 4: Energy dispersion plots for spin-1/2/spin-1 alternating chain with next-neighbor frustration for D/J_1 = (a) 0.0, (b)0.2, (c) 0.35 and (d) 0.4
Interestingly, for $D_z/J_1 = 0$, and with nonzero $J_2$, both $C^z (|i−j|)$ and $C^{(+) (|i−j|)}$ show spiral ordering. However, the $J_2$ introduces frustration and due to this, the local as well as the chain spin structure and their ordering and change. As can be seen from the correlation functions, the antiferromagnetic short range correlations between two consequitive spin sites (from a given site) remain positive or negative, suggesting local frustrated ferromagnetic domains. This is manifest in the structure factor, $S(q)$, where we find two sharp peaks, each at $\pi/2 > q > 0$ and $0 < q < \pi$ (shown in Fig 5b). For $D_z \neq 0$, this local order vanishes, again giving two broad peaks at $−\pi > q > 0$ and $0 < q < \pi$. The structure factor reveals the manifestation of the competing nature of the frustrated interactions and $z$-component of the DM interactions. Note that, the DM interactions arise due to the local non-centrosymmetry in the spin systems [29, 31].

On the other hand, when we consider correlation functions, the antiferromagnetic short range correlations between two consequitive spin sites (from a given site) remain positive or negative, suggesting local frustrated ferromagnetic domains. This is manifest in the structure factor, $S(q)$, where we find two sharp peaks, each at $\pi/2 > q > 0$ and $0 < q < \pi$ (shown in Fig 5b). For $D_z \neq 0$, this local order vanishes, again giving two broad peaks at $−\pi > q > 0$ and $0 < q < \pi$. The structure factor reveals the manifestation of the competing nature of the frustrated interactions and $z$-component of the DM interactions. Note that, the DM interactions arise due to the local non-centrosymmetry in the spin systems [29, 31].

![Image](figure5.png)

**FIG. 5:** Plots of (a) correlation function $C^z (|i−j|)$, (b) structure factor $S(q)$ vs $q$ and (c) $C^{(+) (|i−j|)}$ for $J_2/J_1 = 0.4$ and $D/J_1 = 0.0, 0.25$ and 0.5 for a spin-1/2/spin-1 alternating chain of 240 sites with NNN interactions.

The reason behind this is that the DM interactions introduce strong quantum fluctuations at each site, thus making the spin at each site zero, and hence the total spin zero. This effect could be captured by cubic or quartic orderings of Spin Wave Theory [29], which will be addressed after further work. In presence of next-neighbor frustration, the system again goes away from classical limit, and the average of $z$-component of each site spin becomes zero and two point equal time correlations functions show quasi-long range order. In fact, when both next-neighbor frustration and DM interactions are present, the magnetic structure generates short range order and a local ordering accompanies it.

**IV. CONCLUSIONS**

We have investigated the ground state and low energy properties of an alternating spin-$1/2$/spin-1 chain in the presence of DM interactions and next-neighbor frustrations. Without DM and next-neighbor antiferromagnetic interactions, both Linear Spin Wave Theory and non-perturbative DMRG results predict the ground state to be a classical ferrimagnetic state with $i$th total spin $N(S_i−S_2)$. When DM interactions are present, however small it may be, LSWT again predicts a ground state with total spin $N(S_i−S_2)$, contrary to DMRG results, which show that the ground state to have total spin zero. The reason behind this is that the DM interactions introduce strong quantum fluctuations at each site, thus making the spin at each site zero, and hence the total spin zero. This effect could be captured by cubic or quartic orderings of Spin Wave Theory [29], which will be addressed after further work. In presence of next-neighbor frustration, the system again goes away from classical limit, and the average of $z$-component of each site spin becomes zero and two point equal time correlations functions show quasi-long range order. In fact, when both next-neighbor frustration and DM interactions are present, the magnetic structure generates short range order and a local ordering accompanies it.

[1] Parkinson, John B., Farnell, Damian J. J., *An Introduction to Quantum Spin Systems*, Springer Lecture Notes in Physics (2010).
[2] Affleck, I., J. Phys. Condens. Matt. 1, 3047 (1989)
[3] Anderson, P. W., Science 235, 1196–1198 (1987)
[4] Anderson, P. W., Baskaran, G., Zou, Z., Hsu, T. Phys. Rev. Lett. 58, 26 (1987)
[5] Balents, L., Nature 464, 199-208 (2010)
[6] Sandvik, A. V., AIP Conf. Proc. 1297, 135 (2010)
[7] Takahashi, M., Suzuki, M., Prog. Theo. Phys., 48, 6, (1972)
[8] Binder, K., Landau, D.P., Phys. Rev. B 13, 1140 (1976)
[9] Dong, X.Y., Ahmed, S., McGurn, A.R., Phys. Rev. B 37, 13 (1988)
[10] Dmitriev, D.V., Krivnov, V.Ya., Ovchinnikov, A.A., Langari, A., JETP 95, 538 (2002)
[11] Pati, Swapan K., Ramasesha, S. and Sen, Diptiman, Phys. Rev. B 55, 8894 (1997).
[12] Mohakud, S, Pati, S.K., and Miyashita, S., Phys. Rev. B, 76, 144435 (2007)
[13] Wurstbauer, U., Majumder, D. and Mandal, S. S., Dujoine, L., Rhone, T. D., Dennis, B. S., Rigosi, A. F., Jain, J. K., Pinczuk, A., West, K. W. and Pfeiffer, L. N., Phys. Rev. Lett., 107, 6 (2011)
[14] Rousochatzakis, I. and Läuchli, A. M. and Moessner, R., Phys. Rev. B 85, 104415 (2012).
[15] Arjun, U. and Ranjith, K. M. and Koo, B. and Sichelschmidt, J. and Skourski, Y. and Baenitz, M. and Tsirlin, A. A. and Nath, R., Phys. Rev. B 99, 014421 (2019).
[16] Buyers, W. J. L and Morra, R. M. and Armstrong, R. L. and Hogan, M. J. and Gerlach, P. and Hirakawa, and K., Phys. Rev. Lett. 56, 371 (1986).
[17] Renard,J.P., Verdaguer, M., Regnault,L.P., Erkelens,W. A. C., Rossat-Mignod,J and Stirling, W.G., Europhys. Lett. 3, 945, (1986).
[18] Senko, C. and Richerme, P., Smith, J., Lee, A., Cohen, I., Retzker, A. and Monroe, C., Phys. Rev. X 5, 021026 (2015).
[19] Breunig, Oliver, Garst, Markus, Klümper, Andreas, Rohrkamp, Jens, Turnbull, Mark M., and Lorenz, Thomas, Science Advances 3, 12 (2017).
[20] Banerjee, A., Bridges, C.A., Yan, J.Q., Aczel, A.A., Li, L., Stone, M.B., Granroth, G.E., Lumsden, M.D., Yiu, Y., Knolle, J. and Bhattacharjee, S., Nat. Mater. 15, 733–740 (2016)
[21] Ghorbani E. and Shahbazi F. and Mosadeq H., J. Phys. Condens. Matter 28, 40 (2016).
[22] Chernyshev, A.L., Phys. Rev. B 72, 174414 (2005).
[23] Yi, Tian-Cheng, You, Wen-Long, Wu, Ning and Oles, AndrzeJ M., Phys. Rev. B 100 024423 (2019).
[24] White, S.R., Phys. Rev. Lett. 69, 2863 (1992).
[25] Schollwock, U., Ann. Phys. 326 1 (2011).
[26] Jin, Shangjian, Luo, Cheng, Datta, Trinjanand and Yao, Dao-Xin, Phys. Rev. B 100, 054410 (2019).
[27] Dzyaloshinskii, I.E. Sov. Phys. JETP , 5, 1259–1272 (1957)
[28] Moriya, T., Phys. Rev. Lett. 4, 228 (1960)
[29] Coffey, D., Rice, T. M., and Zhang, F.C., Phys. Rev. B 44, 10112 (1991)
[30] Daniel, M, and L, Kavitha, L., Phys. Rev. B 63, 172302 (2001)
[31] Mohakud,S., Hijii, K., Miyashita, K. S., and Pati, S. K. J.Phys.Chem. Sol. 73, 2 (2012)