Tunneling between a topological superconductor and a Luttinger liquid

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We study the quantum point contact between the topological superconductor and the helical Luttinger liquid. The effects of the electron-electron interactions in the helical Luttinger liquid on the low-energy physics of this system are analyzed by the renormalization group. Among the various couplings at the point contact which arise from the tunneling via the Majorana edge channel, the induced backscattering in the helical Luttinger liquid is the most relevant for repulsive interactions. Hence, at low temperatures, the helical Luttinger liquid is effectively cut into two separated half wires. As a result, the low-temperature physics is described by a fixed point consisting of two leads coupled to the topological superconductor, and the electrical transport properties through the point contact at low temperature and low bias are dominated by the tunneling via the Majorana edge channel. We compute the temperature dependence of the zero-bias tunneling conductance and study the full counting statistics for the tunneling current at zero temperature.

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I. INTRODUCTION

One of the central issues in the study of condensed matter physics in recent years is to verify the existence of Majorana fermions in various condensed matter systems. This tide of research has been partly triggered by the suggestion that the Majorana modes have great potential in the applications of fault-tolerant quantum computations.\textsuperscript{12} Theoretically, it was found that the Majorana edge states are supported by certain quantum Hall states as well as the $p_x + i p_y$-wave superconductors and superfluids.\textsuperscript{3–6} More recently, Fu and Kane\textsuperscript{3} proposed that Majorana fermions can be created in the vortices of a superfluid.\textsuperscript{13} In particular, chiral Majorana fermion edge states can be created at the interface between a superconductor and the area gapped by ferromagnetic materials. Motivated by these theoretical proposals, the transport properties between the Majorana edge states and the (noninteracting) normal metal were studied in a pioneering work\textsuperscript{14} and it was found that the Majorana fermions induce resonant Andreev reflection from the lead to the grounded superconductor. The resulting linear tunneling conductance is 0 or 2e$^2$/h, depending on whether there is an even or odd number of vortices in the superconductor.

Another interesting proposal for the realization of Majorana modes was made in two seminal works in Refs. 15 and 16, motivated by a model proposed by Kitaev.\textsuperscript{17} It was found that the localized Majorana modes can exist at the end of a spin-orbit coupled nanowire subjected to a magnetic field and proximate to an $s$-wave superconductor. Following these proposals, experimental evidence for such a Majorana edge mode was obtained in indium antimonide quantum wires.\textsuperscript{18} These developments initiated further theoretical studies on the electrical transport properties of this system.\textsuperscript{19–26} In particular, the effects of the electron-electron interactions in the metallic leads have been analyzed from a new perspective based on the renormalization-group (RG) method.\textsuperscript{24,25} The main conclusion is that among all possible local interactions in the tunneling junction, the tunneling via the Majorana edge mode is the most relevant, and it drives the system all the way from the perfect normal reflection fixed point to the perfect Andreev reflection fixed point. The latter controls the low-temperature electrical transport properties, characterized by a universal zero-bias conductance at zero temperature (2e$^2$/h for a single lead with spinless fermions).

In the present work, we study a quantum point contact (QPC) between a (chiral) Majorana liquid at the edge of a topological superconductor (TSC) and a helical (spinless) Luttinger liquid (HLL),\textsuperscript{22} as shown in Fig. 2. Our work intervenes between Refs. 14 and 24 in the sense that we investigate the interplay between the electron-electron interaction effect in the metallic lead and the role of the propagating Majorana mode, instead of a localized mode. The main results of our work are summarized in Fig. 2. Our analysis focuses on the intermediate temperature range below the superconducting gap such that the finite-size effects of both the HLL and the Majorana liquid can be ignored. Under this condition, our RG analysis in the weak-tunneling limit shows that, unlike the case with a localized Majorana end mode, the tunneling between the Majorana liquid and the HLL is inhibited at low energy and the system is driven to a new fixed point (referred to as the fixed point $I$ in the main text) in which the HLL is effectively cut into two separated pieces by a local backscattering potential at the QPC. (This local backscattering potential is produced by the proximity to the TSC, and is allowed because the time-reversal symmetry is broken by the $p_x + i p_y$-wave TSC.)
we also performed a scaling analysis regarding the stability of the perfect Andreev reflection fixed point (referred to as the fixed point \( A \)) in the strong-tunneling limit and found that it is unstable toward the weak-tunneling regime. Thus, the possibility of the existence of a nontrivial quantum critical point in the intermediate-tunneling-strength regime, which separates fixed point \( I \) from fixed point \( A \), is unlikely. Because the low-temperature and high-temperature physics of this system are controlled by different fixed points, we obtain a two-stage scaling of the tunneling conductance with temperature, dictated by different exponents as shown in Fig. 2. Since in real systems, the Majorana liquid is most likely to be of the mesoscopic length scale, we expect that at temperatures below the scale \( T_L \) set by the level spacing of the Majorana modes, the Majorana liquid will behave like a localized mode and the system will eventually experience a crossover to the model studied in Ref. 25, i.e., two metallic leads coupled to a single Majorana mode. Therefore, the tunneling via the Majorana mode will become a relevant perturbation, and the tunneling conductance of the system at zero temperature will saturate at a universal value depending on the LL parameter, as long as there is a Majorana edge state whose energy matches the Fermi level of the lead.  

In the remaining sections, we present systematical theoretical analysis which supports the above conclusions and compare our results with previous works in the final section.

II. THE MODEL

We consider a QPC between the grounded TSC and the HLL as shown in Fig. 1. This system can be described by the Hamiltonian \( H = H_{LL} + H_\eta + \delta H \), where

\[
H_{LL} = \sum_{m=\pm} \int dx \left[ \psi_m^\dagger (-i m v_F \partial_x \psi_m) + g_1 J_m J_m \right] + g_2 \int dx J_+ J_- \tag{1}
\]

is the Hamiltonian of the HLL, and

\[
H_\eta = \frac{v_M}{2} \int_{-L/2}^{L/2} dx \eta(-i \partial_x \eta) \tag{2}
\]

is the Hamiltonian of the chiral Majorana liquid. In the above, \( v_F \) and \( v_M \) are the Fermi velocity in the HLL and the speed of the Majorana fermion at the edge of the TSC, respectively. The operators \( \psi_+ \) and \( \psi_- \), which describe the right and leftmover in the HLL, obey the canonical anticommutation relations. \( J_m = \psi_{m+}^\dagger \psi_m \) with \( m = \pm \) are the current operators. Moreover, we take the normalization of the Majorana fermion \( \eta \) so that it satisfies the anticommutation relation

\[
\{ \eta(x), \eta(y) \} = \delta(x-y) . \tag{3}
\]

In the following, we shall focus on the limit \( L \to +\infty \) and comment on the finite \( L \) effects later.

\( \delta H \) describes various couplings at the point contact. By keeping the most relevant terms and setting the QPC to be located at \( x = 0 \), \( \delta H \) can be written as

\[
\delta H = \sum_{m=\pm} \left\{ -i \partial_\eta(0) [\xi_m \psi_m(0) + H.c.] + 2 u_1 \psi_{m+}^\dagger \psi_m(0) \right\} + \left[ u_2 \psi_+^\dagger \psi_-(0) + \Delta \psi_+ \psi_- (0) + H.c. \right] . \tag{4}
\]

FIG. 1: (Color online) A schematic picture of quantum point contact between the HLL and an island of TSC. At the edge of the TSC, there is a branch of chiral Majorana fermions denoted by \( \eta \). Electrons can tunnel between the HLL and the TSC through the point contact. The superconductor is grounded. The chemical potentials at the two leads are chosen to be identical, i.e., \( \mu_1 = \mu_2 = V \).

FIG. 2: (Color online) A schematic temperature dependence of the tunneling conductance \( G_0 \) (in units of e^2/h) at zero bias. For \( T > T_L = |v_M|/L \), \( G_0 \) is a monotonously decreasing function with decreasing temperature \( T \), where \( L \) is the length of the Majorana liquid. At high temperature, it behaves like \( G_0 \propto T^{-(K+1/K)-1} \), where \( K \) is the LL parameter. At low temperature, it is \( G_0 \propto T^{1/K-1} \) for \( T > T_L \). For \( T < T_L \), \( G_0 \) increases as decreasing \( T \) and reaches a universal value \( g \) in units of e^2/h at \( T = 0 \) provided that there exists a Majorana zero-energy state, where \( g \) is a function of \( K \).
where \( t, u_2 > 0, u_1 \) is real, and \( |\xi_m| = 1 \). The \( \tilde{t} \) term describes the tunneling between the TSC and the HLL via the Majorana edge channel, the \( u_1 \) term describes the local chemical potential variation due to the presence of the QPC, the \( u_2 \) term accounts for the backscattering in the HLL arising from the contact to the TSC (which is allowed in the present case on account of the breaking of time-reversal symmetry by the TSC), and the \( \Delta \) (Cooper-pairing) term is induced by the proximity to the superconductor.

III. THE SCALING ANALYSIS

We now study the effects of \( \delta H \) on the decoupled fixed point, which corresponds to \( \delta H = 0 \). We first ignore the electron-electron interactions in the HLL, i.e. setting \( g_1 = 0 = g_2 \). Then, the scaling dimensions of the various terms in \( \delta H \) around the decoupled fixed point are \( D[\tilde{t}] = 1 = D[u_1] = D[u_2] = D[\Delta] \). That is, all are marginal operators. This is different from the coupling of a lead to the TSC via a single Majorana edge mode. For the latter, the \( \tilde{t} \) term is already a relevant perturbation for free electrons. This distinction arises from the nontrivial scaling dimension \( D[\eta] = 1/2 \) acquired by the Majorana fermion \( \eta \) in the limit \( L \to +\infty \).

A. The weak-tunneling regime

To take into account the effects of the electron-electron interactions (the \( g_1 \) and \( g_2 \) terms), we employ the method of bosonization. Using the bosonization formula, \( \psi_{\pm}(x) = \frac{1}{\sqrt{2\pi a_0}} \exp[\pm i\sqrt{4\pi\phi_{\pm}}(x)] \), where \( a_0 \) is a short-distance cutoff and \( \gamma \) is the Klein factor with the normalization \( \gamma^2 = 1 \), \( H_{LL} \) can be written as

\[
H_{LL} = \frac{\nu}{2} \int dx \left[ K (\partial_x \Theta)^2 + \frac{1}{K} (\partial_x \Phi)^2 \right],
\]

where \( \Phi = \phi_- + \phi_+ \), \( \Theta = \phi_- - \phi_+ \), and \( K \) is the LL parameter. For the repulsive (attractive) interactions, \( K < 1 \) (\( K > 1 \)). On the other hand, the bosonized form of \( \delta H \) is given by

\[
\delta H = -\frac{i\eta(0)\gamma}{\sqrt{2\pi a_0}} \left[ \xi_+ e^{i\sqrt{4\pi\phi_+(0)}} + \xi_- e^{-i\sqrt{4\pi\phi_-}(0)} + \text{H.c.} \right] + \frac{2u_1}{\sqrt{\pi}} \frac{\partial_x \Phi(0)}{\pi a_0} - \frac{u_2}{\pi a_0} \sin \left[ \sqrt{4\pi}\Phi(0) \right] + \frac{|\Delta|}{\pi a_0} \sin \left[ \sqrt{4\pi}\Theta(0) - \alpha \right],
\]

where we have written \( \Delta = |\Delta|e^{i\alpha} \). The \( u_1 \) term can be removed by the transformation \( \Phi(x) \to \Phi(x) - \frac{K\nu\sqrt{\pi}}{\pi a_0} \text{sgn}(x) \), and \( \delta H \) becomes

\[
\delta H = -\frac{i\eta(0)\gamma}{\sqrt{2\pi a_0}} \left[ \xi_+ e^{i\sqrt{4\pi\phi_+(0)}} + \xi_- e^{-i\sqrt{4\pi\phi_-}(0)} + \text{H.c.} \right] - \frac{u_2}{\pi a_0} \sin \left[ \sqrt{4\pi}\Phi(0) \right] + \frac{|\Delta|}{\pi a_0} \sin \left[ \sqrt{4\pi}\Theta(0) - \alpha \right].
\]

The scaling dimensions of various terms in \( \delta H \) around the decoupled fixed point are given by \( D[\tilde{t}] = \frac{1}{2}(1/K + 1) + 1/2 \), \( D[u_1] = 1 \), and \( D[\Delta] = 1/K \). Hence, we reach the following conclusions: (i) for repulsive interactions (\( K < 1 \)), the \( u_2 \) term is relevant while the \( \Delta \) term is irrelevant; (ii) for attractive interactions (\( K > 1 \)), the \( \Delta \) term is relevant while the \( u_2 \) term is irrelevant. In both cases, the \( \tilde{t} \) term is irrelevant and the \( u_1 \) term is marginal. In this paper, we focus on the case with \( K < 1 \).

For \( K < 1 \), the \( u_2 \) term will flow to the strong-coupling regime, and we expect that the low-energy physics is determined by the fixed-point Hamiltonian

\[
H_1 = H_{LL} + H_\eta - \frac{u_2}{\pi a_0} \sin \left[ \sqrt{4\pi}K\Phi(0) \right],
\]

where \( \Phi = \Phi/\sqrt{K} \). \( H_1 \) takes the form of the Hamiltonian of a spinless LL with an impurity backscattering term at \( x = 0 \). Since \( K < 1 \), this term is always relevant and cuts the wire into two separate pieces at \( x = 0 \). However, we still have to examine the stability of the fixed point described by \( H_1 \) with \( u_2 \to +\infty \), which will be denoted by \( I \).

To examine the stability of the fixed point \( I \), we consider the possible perturbations around it. They are the \( \tilde{t} \) term, which becomes \( -i\eta(0)\gamma \left[ \xi_+ e^{i\sqrt{4\pi\phi_+(0)}} + \text{H.c.} \right] \) by taking into account the boundary condition at the fixed point \( I \), the \( \Delta \) term, as well as the tunneling between the two separated half wires (denoted by \( A \) and \( B \)) \( \lambda \hat{O} \), where \( \hat{O} = \Psi_A^\dagger(0)\Psi_B(0) + \text{H.c.} \) and \( \Psi_{A(B)} \) is the fermion field referred to region \( A \) (\( B \)). The scaling dimensions of the operators involving the \( \Theta \) field can be calculated by the action for the half wire in the imaginary-time formulation

\[
S_1 = \frac{K}{2\nu} \int_0^\beta d\tau \int_0^{+\infty} dx \left[ (\partial_x \Theta)^2 + v^2 (\partial_x \Theta)^2 \right],
\]

yielding \( D[\tilde{t}] = 1/(2K) + 1/2 \), \( D[\Delta] = 2/K \), and \( D[\lambda] = 1/K \). Hence, all terms are irrelevant for \( K < 1 \). That is, the fixed point \( I \) is stable for repulsive interactions. Moreover, the leading irrelevant operator (LIO) around this fixed point is the \( \tilde{t} \) term.

To sum up, the low-energy physics of the QPC between the TSC and the HLL is controlled by the fixed point \( I \), which is composed of two half wires and the chiral Majorana liquid on the edge of the TSC. The tunneling between the two half wires and the Majorana liquid gives rise to the leading temperature dependence of thermodynamics at low temperatures. We shall see later that
We may define the fermion operator

In the above, the LL is described by the repulsive Hub-

The previous analysis holds only in the weak-tunneling

The action of the system in the real-time formulation

Applying a bias $V$ between the HLL and the edge of the TSC will generate a tunneling current $I_t$ through the point contact. The quantity we would like to compute is the tunneling conductance $G = dI_t/dV$. This can be done as follows.

The action of the system in the real-time formulation can be written as

$$S = \int dt \left[ \sum_m \int dx \psi_m^\dagger (i\partial_t + \mu) \psi_m - H \right],$$

where $\mu = -eV$. We perform a time-dependent gauge transformation $\psi_m \rightarrow e^{i\mu t} \psi_m$, leading to $\Phi \rightarrow \Phi$ and $\Theta \rightarrow \Theta + \frac{\partial \Phi}{\partial \Phi}$. Consequently, the dependence on $V$ will appear only in the $\tilde{t}$ and the $\Delta$ terms. The tunneling current operator is given by $\tilde{I} = -\frac{\delta H}{\delta a} = \tilde{I}_1 + \tilde{I}_2$, where $a = -Vt$ and

$$\tilde{I}_1 \propto i \gamma \psi(0) e^{-i\tilde{t}V} \left[ \xi_+ e^{i\pi \tilde{t}} \Phi^+ (0) + \xi_- e^{-i\pi \tilde{t}} \Phi^- (0) \right] + \text{H.c.},$$

$$\tilde{I}_2 \propto \cos \left[ \sqrt{4\pi} \Theta (0) - \alpha + eVt \right].$$

The leading perturbation $\delta H$ to the fixed point $A$ is produced by $V$. With the help of the Schrieffer-Wolff transformation\(^3\) we find that

$$\delta H = \frac{w_1 w_2}{2\sqrt{2}} \sum_{\ell=-\infty}^{\infty} \xi (c_\ell + c_{-\ell}) - \text{H.c.} \left[ n_\ell - n_{-\ell} \right] + \text{H.c.}.$$
The tunneling current is then given by \( I_t = \langle \hat{I} \rangle \).

The scaling dimensions \( D_1 \) and \( D_2 \) for the operator \( \hat{I}_1 \) and \( \hat{I}_2 \) are given by \( D_1 = D[\chi] \) and \( D_2 = D[\Delta] \), respectively. Since \( D_1 < D_2 \) for \( K < 1 \), we have \( G(T = 0) \propto V^{2D_1-2} \) at low bias, and \( G_0 \equiv G(V = 0) \propto T^{2D_1-2} \) at low temperatures. Therefore, the temperature dependence of \( G_0 \) reveals the RG flow of the \( T \) term. Since the high-energy and low-energy physics of the system are controlled by the decoupled fixed point and the fixed point \( I \), we conclude that \( G_0 \propto T^{2(1+1/K)-1} \) at high temperature and \( G_0 \propto T^{1/K-1} \) at low temperature. Moreover, \( G_0 \) is a monotonously decreasing function with decreasing temperature.

In practice, the Majorana liquid has a finite length \( L \), which introduces a new characteristic energy scale \( T_L = |v_M|/L \). The above results hold only when \( T_L < T < \min\{E_F, \Delta_\pi\} \), where \( E_F \) is the Fermi energy of the HLL and \( \Delta_\pi \) is the gap of the TSC. For \( T < T_L \), this problem becomes one with two leads coupled to a single Majorana mode. It turns out that the physics at low temperatures is controlled by a nontrivial critical point \( \pi \) which gives rise to a universal conductance \( G_0 = g(K)e^2/h \), at \( T = 0 \), where \( g \) is a universal function of \( K \). A schematic temperature dependence of \( G_0 \) is shown in Fig. 2.

**B. Full counting statistics**

It has been known that a measurement of current noise reveals information on the studied system not present in the average current. Further information can be obtained by studying the higher moments (cumulants) of charge transfer, or in general, the full counting statistics (FCS).

To extract the cumulants of charge transfer through the QPC, we may calculate the cumulant generating function \( W(\chi) = \ln[Z(\chi)] \). In the present case, \( Z(\chi) \) is defined as

\[
Z(\chi) = \exp \left[ \int_{-\infty}^{\infty} dt \chi(t) \int_{-\infty}^{+\infty} dx \sum_{m=\pm} \psi_{m+}^{\dagger} \psi_{m-} \right],
\]

where the time integral is taken over the Keldysh contour and \( \chi(t) \) is the counting field. In order that \( Z(\chi) \neq 1 \), we must choose \( \chi_+(t) \neq \chi_-(t) \), where the subscripts + and − refer to the forward and the backward branches of the closed time contour, respectively. Here, we choose \( \chi_+(t) = \chi_0(t - T) \) and \( \chi_-(t) = 0 \), where \( T \) is the measurement time. For large \( T \), \( W \) is proportional to \( T \) so that

\[
I_t = i e \lim_{T \to +\infty} \frac{1}{T} \int_{-\infty}^{+\infty} \delta W \biggr|_{\chi = 0},
\]

\[
S(0) = -e^2 \lim_{T \to +\infty} \frac{1}{T} \int_{-\infty}^{+\infty} \delta^2 W \biggr|_{\chi = 0},
\]

where \( I_t \) is the tunneling current and \( S(0) \) is the noise power of \( I_t \) at zero frequency.

To proceed, we perform a time-dependent gauge transformation \( \psi_m \to e^{i\theta(t)} \psi_m \), where \( \theta_+(t) = -\chi(T - t) \) and \( \theta_-(t) = 0 \). Then, \( Z(\chi) \) can be written as

\[
Z(\chi) = \int D[\eta] D[\Theta] \exp \left[ \int dt \left( L_0 + L_t \right) \right],
\]

where \( \Theta = \sqrt{K} \Theta \),

\[
L_0 = \sum_{l=1,2} \frac{1}{2v} \int_{0}^{+\infty} \left[ (\partial_t \Theta_l)^2 - v^2 (\partial_x \Theta_l)^2 \right] + i \frac{1}{2} \int_{-\infty}^{+\infty} dx \eta (\partial_t + v_M \partial_x) \eta + i \frac{1}{4} \partial_t \gamma,
\]

describes the fixed point \( I \), and

\[
L_t = -i e \ell \eta(t,0) \gamma(t) \sum_{l=1,2} \cos \left[ \sqrt{\frac{n}{K}} \Theta_l (t,0) + \alpha(t) \right],
\]

gives the LIO. In Eq. (14), \( \alpha(t) = eV t - \theta(t) \).

At low temperatures (bias), we may calculate \( W \) in terms of the perturbative expansion in \( \ell \). To \( O(\ell^2) \), we find that

\[
W = \frac{a_0^{1/K} v^{1-1/K} K}{2 |v_M| \Gamma(1/K)} T^2 \left[ \omega_0 \right]^{1/K} \left[ e^{-i \gamma} \chi - 1 \right],
\]

at \( T = 0 \), where \( \omega_0 = eV \) and \( a_0 \) is the short-distance cutoff. Inserting Eq. (13) into Eqs. (10) and (11), we get the tunneling current at \( T = 0 \):

\[
I_t = \frac{e a_0^{1/K} v^{1-1/K} K}{2 |v_M| \Gamma(1/K)} \Delta_{\pi} \left[ \omega_0 \right]^{1/K},
\]

and the noise power of the tunneling current at zero frequency:

\[
S(0) = \frac{e^2 a_0^{1/K} v^{1-1/K} K}{2 |v_M| \Gamma(1/K)} T^2 \left[ \omega_0 \right]^{1/K}.
\]

A few comments regarding the above results are in order. First of all, the exponents of \( \omega_0 \) in Eqs. (10) and (11) are determined by the scaling dimension of \( I_t \) around the fixed point \( I \). For the usual metal-superconductor junction, the current at \( T = 0 \) due to the single-electron tunneling is suppressed by the superconducting (SC) gap \( \Delta_\pi \) such that \( I_0 = 0 \) for \( |\omega_0| < 2\Delta_\pi \).

In the present case, the low-energy states are allowed at the edge of the TSC, which turns the suppression of the tunneling current into a power-law behavior. Next, the Fano factor for the tunneling current is of the value

\[
F = \frac{S(0)}{e^2 I_t} = 1,
\]

instead of 2 for the local Andreev reflection. It implies the single-electron tunneling through the QPC at low bias. This is possible in the present case because of the overlapping between the wavefunction of electrons in the wire and that of Majorana fermions in the TSC.
V. CONCLUSION

To sum up, we study a QPC between the chiral Majorana edge states of a TSC and the HLL. Our RG analysis predicts a nonmonotonic behavior of the zero-bias tunneling conductance with temperature and the power-law temperature dependence in the intermediate temperature range. Since tunneling into the usual nontopological superconductors (or insulators) will make the tunneling conductance decrease exponentially with decreasing temperature due to the bulk gap in the energy spectrum, the behavior of the tunneling conductance with temperature we found is an indication of the existence of gapless states at the edge of the TSC. Moreover, we evaluate perturbatively the FCS of the tunneling current at zero temperature, and use the corresponding result to extract the tunneling current and its noise power at zero frequency.

Previous studies on a distinct TSC-HLL tunneling junction shows that the low-temperature transport properties are controlled by a perfect Andreev reflection fixed point, while in our case, it is the fixed point which controls the low-energy physics. Experimentally, the distinction between these two fixed points can also be revealed by the corresponding Fano factors, which reflect the nature of the transported charges. Specifically, the Fano factor obtained in our case is 1 rather than 2, as expected for a tunneling process dominated by Andreev reflections. The origin of this difference is twofold: First of all, the scaling dimension of the tunneling via the Majorana edge states is increased in our case by the propagating nature of Majorana fermions. Next, the metallic lead in the system studied in Refs. 24 and 26 is essentially a half wire. Therefore, upon folding, the local potential scattering in the tunneling junction contains only the forward scattering of electrons, whose sole effect is to change the phase of the incident electron and does not affect the low-energy physics of the system qualitatively. On the other hand, the potential scattering term in our system also contains the backscattering of electrons, which effectively cuts the wire into two semi-infinite pieces at low energies. We would like to stress that such a term must be included not only because the time-reversal symmetry is broken by the $p_x + ip_y$-wave TSC, but also because it will inevitably be generated at the energy scale below the SC gap through the virtual tunneling between the electrons in the HLL and the gapped quasiparticles in the TSC. Another similar tunneling junction between two metallic leads and the end of a TSC wire via the localized Majorana mode was analyzed in Ref. 22. In that case, a local backscattering can also be generated by renormalization. However, the tunneling via the Majorana end mode is still the most relevant perturbation, which leads to different low-energy physics from those presented here.

Our analysis in the present work can be extended readily to study the electron-electron interaction effect on the system considered in Ref. 14, which is a tunneling junction between a chiral Majorana liquid of finite size and the metallic lead. According to the scaling analysis presented in Sec. IIIA it is easy to see that the zero-bias tunneling conductance in that system will also exhibit a nonmonotonic behavior with decreasing temperature. As the temperature is lowered, the zero-bias conductance will decrease toward zero in the form of $T^{1/k-1}$, as shown by the second stage behavior presented in Fig. 2. When $T \ll T_L$, the tunneling conductance will increase as the temperature decreases further and will eventually be saturated at the value of $2e^2/h$, as long as there is a zero-energy Majorana mode in the TSC, as predicted in Ref. 24. This nonmonotonic scaling behavior of the tunneling conductance with temperature is in sharp contrast to that for tunneling into a single Majorana end mode. In the latter case, the tunneling conductance is expected to rise immediately as the temperature is lowered. The subtle difference between these two cases should serve as a guide in future experiments in identifying the possible Majorana edge states of TSC. Moreover, a complete analysis regarding the interplay between the effects of finite bias (temperature) and the finite size of the Majorana liquid is a topic for future study.

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