1. Introduction

In the LHC era of precision QCD, which entails predictions for QCD processes at the total precision \(1\%\) or better, we need resummed \(\mathcal{O}(\alpha_s^2 L^n), \mathcal{O}(\alpha_s L^{n'}), \mathcal{O}(\alpha_s^2 L^{n''})\), \(n = 0, 1, 2, n' = 0, 1, 2, n'' = 1, 2\) corrections, in the presence of parton showers, on an event-by-event basis, with double counting and with exact phase space. The roles of QED and EW effects [2, 3] are integral parts of the discussion with which we deal by the simultaneous resummation of QED and QCD large infrared (IR) effects, QED \(\otimes\) QCD resummation [4] in the presence of parton showers, to be realized on an event-by-event basis by MC methods; for, as shown in Refs. [3], no precision prediction for a hard LHC process at the 1\% level can be complete without taking the large EW corrections into account.

In what follows, we first review our approach to resummation and its relationship to those in Refs. [5, 6]. Section 3 contains a summary of the attendant new IR-improved DGLAP-CS [7, 8] theory [9, 10]. Section 4 presents the implementation of the new IR-improved kernels in the framework of HERWIG6.5 [11] to arrive at the new, IR-improved parton shower MC HERWIRI1.0. We illustrate the effects of the IR-improvement first with the generic 2 \(\rightarrow\) 2 processes at LHC energies and then with the specific single Z production process at LHC energies. We compare with the specific single Z production at LHC energies and then with the specific single Z production process at LHC energies.

2. QED \(\otimes\) QCD Resummation

We make use of the discussion in Refs. [4, 9, 10], wherein we have derived the following expression for the hard cross sections in the SM \(SU_{2L} \times U_1 \times SU_3\) EW-QCD theory:

\[
d\hat{\sigma}_{\text{exp}} = e^{\text{SUM}_{\text{IR}}(\text{QCD})} \sum_{\alpha, m=0}^{\infty} \frac{1}{\alpha m} \int \prod_{j=1}^{3} \frac{d^3k_{1j}}{2 \pi} e^{i y (p_1 + q_1 - p_2 - q_2 - \sum k_{1j} - \sum k_{2j}) + D_{\text{QCD}}} \\
\prod_{j=1}^{m} \frac{d^3k_{1j}}{2 \pi} \int \frac{d^3k_{2j}}{2 \pi} e^{i y (p_2 + q_2 - p_2 - q_2 - \sum k_{1j} - \sum k_{2j}) + D_{\text{QCD}}} \\
\prod_{j=1}^{m} \frac{d^3k_{1j}}{2 \pi} \int \frac{d^3k_{2j}}{2 \pi} e^{i y (p_2 + q_2 - p_2 - q_2 - \sum k_{1j} - \sum k_{2j}) + D_{\text{QCD}}}
\]

where the new YFS-style [14] residuals \(\hat{\beta}_{n,m}(k_1, \ldots, k_n; k_1', \ldots, k_m')\) have \(n\) hard gluons and \(m\) hard photons and we show the final state with two hard final partons with momenta \(p_2, q_2\) specified for a generic \(2f\) final state for definiteness. The infrared functions SUM_{IR}(QCD), D_{QCD} are defined in Refs. [4, 9, 10]. This is the exact, simultaneous resummation of QED and QCD large IR effects.

One can see that our approach to QCD resummation is fully consistent with that of Refs. [5, 6] as follows. First, Ref. [15] has shown that the latter two approaches are equivalent. By using the color-spin density matrix realization of our residuals, we show in Refs. [9, 10] that our approach is consistent with that of Refs. [5] by exhibiting the transformation prescription from the resummation formula for the theory in Refs. [5] for the generic \(2 \rightarrow n\) parton process as given.

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\(^1\)By total precision of a theoretical prediction we mean the technical and physical precisions combined in quadrature or otherwise as appropriate.
in Ref. [16] to our theory as given for QCD by restricting (1) to its QCD component. In this way, we capture the respective full quantum mechanical color-spin correlations in the results in Ref. [16].

3. IR-Improved DGLAP-CS Theory

We show in Refs. [9, 10] that the result (1) allows us to improve in the IR regime \(^2\) the kernels in DGLAP-CS [7, 8] theory as follows, using a standard notation:

\[
P_{\gamma\gamma}^{\text{exp}}(z) = C_F F_{VFS}(\gamma_0) e^{\frac{1}{2} \delta_q} \left[ \frac{1 + z^2}{1 - z} (1 - z)^{\gamma_q} - f_q(\gamma) \delta(1 - z) \right],
\]

\[
P_{G\gamma}^{\text{exp}}(z) = C_F F_{VFS}(\gamma_0) e^{\frac{1}{2} \delta_q} \frac{1 + (1 - z)^2}{z} z^{\gamma_q},
\]

\[
P_{G_G}^{\text{exp}}(z) = 2 C_G F_{VFS}(\gamma_G) e^{\frac{1}{2} \delta_G} \left[ \frac{1}{z} (1 - z)^{\gamma_G} + \frac{z}{1 - z} (1 - z)^{1 + \gamma_G} + \frac{1}{2} (z + 1) (1 - z) + z (1 - z)^{1 + \gamma_G} \right] - f_G(\gamma_G) \delta(1 - z),
\]

\[
P_{qG}^{\text{exp}}(z) = F_{VFS}(\gamma_G) e^{\frac{1}{2} \delta_G} \frac{1}{2} z^2 (1 - z)^{\gamma_G} + (1 - z)^2 z^{\gamma_G},
\]

(2)

where the superscript “\(\text{exp}\)” indicates that the kernel has been resummed as predicted by (1) when it is restricted to QCD alone – see Refs. [9, 10] for the corresponding details. These results have been implemented by MC methods as we exhibit in what follows.

Let us first note that a number of illustrative results and implications of the new kernels have been presented in Refs. [9, 10, 19]. Here, we call attention to the new scheme [10] which we now have for precision LHC theory: in an obvious notation, we utilize the jet evolution equation is

\[
\sigma = \sum_{i,j} \int dx_1 dx_2 F_i(x_1) F_j(x_2) \sigma(x_1 x_2 s)
\]

\[
= \sum_{i,j} \int dx_1 dx_2 F'_i(x_1) F'_j(x_2) \sigma'(x_1 x_2 s),
\]

where the primed quantities are associated with (2) in the standard QCD factorization calculus. We have [4] an attendant shower/ME matching scheme, wherein, for example, in combining (1) with HERWIG 11, PYTHIA 27, MC@NLO 28 or new shower MC’s 29, we may use either \(p_T\)-matching or shower-subtracted residuals \(\{\beta_{n,m}(k_1, \ldots, k_n; k'_1, \ldots, k'_m)\}\) to create a paradigm without double counting that can be systematically improved order-by-order in perturbation theory – see Refs. [4].

The stage is set for the full MC implementation of our approach. We turn next to the initial stage of this implementation – that of the kernels in (2).

4. MC Realization of IR-Improved DGLAP-CS Theory

In this section we describe the implementation of the new IR-improved kernels in the HERWIG6.5 environment, which results in a new MC, which we denote by HERWIRI1.0, which stands for “high energy radiation with IR improvement”\(^3\).

Specifically, our approach can be summarized as follows. We modify the kernels in the HERWIG6.5 module HWBRAN and in the attendant related modules [30] with the following substitutions:

\[
\text{DGLAP-CS } P_{AB} \Rightarrow \text{IR-I DGLAP-CS } P_{AB}^{\text{exp}}
\]

while leaving the hard processes alone for the moment. We have in progress [31] the inclusion of YFS synthesized electroweak modules from Refs. [32] for HERWIG6.5, HERWIG++ [33] hard processes, as the CTEQ [34] and MRST(MSTW) [35] best (after 2007) parton densities do not include the precision electroweak higher order corrections that do enter in a 1% precision tag budget for processes such as single heavy gauge boson production in the LHC environment [3].

For definiteness, let us illustrate the implementation by an example [36, 37], which for pedagogical reasons we will take as a simple leading log shower component with a virtuality evolution variable, with the understanding that in HERWIG6.5 the shower development is angle ordered [36] so that the evolution variable is actually \(\sim E \theta\) where \(\theta\) is the opening angle of the shower as defined in Ref. [36] for a parton initial energy \(E\). In this pedagogical example, which we take from Ref. [36], the probability that no branching occurs above virtuality cutoff \(Q_0^2\) is \(\Delta_a(Q^2, Q_0^2)\) so that

\[
d\Delta_a(t, Q_0^2) = \frac{-dt}{t} \Delta(t, Q_0^2) \sum_b \int dz \frac{\alpha_s}{2\pi} P_{ba}(z),
\]

which implies

\[
\Delta_a(Q^2, Q_0^2) = \exp \left[- \int_{Q_0^2}^{Q^2} \frac{dt}{t} \sum_b \int dz \frac{\alpha_s}{2\pi} P_{ba}(z) \right].
\]

The attendant non-branching probability appearing in the evolution equation is

\(^3\)We thank M. Seymour and B. Webber for discussion on this point.
\[ \Delta(Q^2, t) = \frac{\Delta_a(Q^2, Q_0^2)}{\Delta_a(t, Q_0^2)} \quad t = k_a^2 \equiv \text{virtuality of gluon } a. \] (6)

The respective virtuality of parton \( a \) is then generated with

\[ \Delta_a(Q^2, t) = R, \] (7)

where \( R \) is a random number uniformly distributed in \([0, 1] \). With

\[ \alpha_s(Q) = \frac{2\pi}{b_0 \log \left( \frac{Q^2}{\Lambda^2} \right)}. \] (8)

we get for example

\[ \int_0^1 dz \frac{\alpha_s(Q^2)}{2\pi} P_{qG}(z) = 4\pi \int_0^1 dz \frac{1}{2} \left[ z^2 + (1 - z)^2 \right] \frac{2}{3} b_0 \ln \left( \frac{Q^2}{\Lambda^2} \right) = \frac{2}{3} b_0 \ln \left( \frac{Q^2}{\Lambda^2} \right). \] (9)

so that the subsequent integration over \( dt \) yields

\[ I = \int_0^{Q_0^2} \frac{1}{3} \frac{dt}{t} \frac{2}{b_0 \ln \left( \frac{Q^2}{\Lambda^2} \right)} = \frac{2}{3} b_0 \ln \left( \frac{Q^2}{\Lambda^2} \right) = \frac{2}{3} b_0 \left[ \ln \left( \frac{Q^2}{\Lambda^2} \right) \right]. \] (10)

Finally, introducing \( I \) into (5) yields

\[ \Delta_a(Q^2, Q_0^2) = \exp \left[ -2 \frac{b_0}{3} \ln \left( \frac{\ln \left( \frac{Q^2}{\Lambda^2} \right)}{\ln \left( \frac{Q_0^2}{\Lambda^2} \right)} \right) \right] \]

\[ = \left[ \ln \left( \frac{Q^2}{\Lambda^2} \right) \right]^{-2 b_0} = R. \] (11)

If we now let \( \Delta_a(Q^2, t) = R \), then

\[ \left[ \ln \left( \frac{Q^2}{\Lambda^2} \right) \right]^{-2 b_0} = R\] (12)

which implies

\[ t = \Lambda^2 \left( \frac{Q^2}{\Lambda^2} \right)^{-2 b_0}. \] (13)

Recall in HERWIG6.5 [11] we have

\[ b_0 = \left( \frac{11}{3} n_c - \frac{2}{3} n_f \right) = \frac{1}{3} (11 n_c - 10), \quad n_f = 5 \]

\[ = \frac{2}{3} \text{BETAF}. \] (14)

where in the last line we used the notation in HERWIG6.5. The momentum available after a \( q\bar{q} \) split in HERWIG6.5 [11] is given by

\[ QQBAR = QCDL3 \left( \frac{QLST}{QCDL3} \right)^{R\text{BETAF}}, \] (15)

in complete agreement with (13) when we note the identifications \( t = QQBAR^2 \), \( \Lambda = QCDL3 \), \( Q = QLST \).

The leading log exercise leads to the same algebraic relationship that HERWIG6.5 has between \( QQBAR \) and \( QLST \) but we stress that in HERWIG6.5 these quantities are the angle-ordered counterparts of the virtualities we used in our example, so that the shower is angle-ordered.

When we repeat the above calculation for the IR-Improved kernels in (2), we have

\[ P_{qG}^{\text{IR}}(z) = F_Y F_S (\gamma G) e^{\frac{1}{2} (1 - z)^2 \frac{1}{2} \left[ z^2 (1 - z)^{\gamma G} + (1 - z)^2 z^{\gamma G} \right]} \] (16)

so that

\[ \int_0^1 dz \frac{\alpha_s(Q^2)}{2\pi} P_{qG}(z) = \frac{4 F_Y F_S (\gamma G) e^{\frac{1}{2} (1 - z)^2 \frac{1}{2} \left[ z^2 (1 - z)^{\gamma G} + (1 - z)^2 z^{\gamma G} \right]}}{b_0 \ln \left( \frac{Q^2}{\Lambda^2} \right)} \frac{1}{(\gamma G + 1)(\gamma G + 2)} \] (17)

This leads to the following integral over \( dt \)

\[ I = \int_0^{Q_0^2} \frac{dt}{t} \frac{4 F_Y F_S (\gamma G) e^{\frac{1}{2} (1 - z)^2 \frac{1}{2} \left[ z^2 (1 - z)^{\gamma G} + (1 - z)^2 z^{\gamma G} \right]}}{b_0 \ln \left( \frac{Q^2}{\Lambda^2} \right)} \frac{1}{(\gamma G + 1)(\gamma G + 2)} \] (18)

Here we have used

\[ \delta_C = \frac{\gamma G}{2} + \frac{\alpha_s C_G}{\pi} \left( \frac{\pi}{3} - \frac{1}{2} \right). \] (19)

We finally get the IR-improved formula

\[ \Delta_a(Q^2, t) = \exp \left[ - (F(Q^2) - F(t)) \right], \] (20)

where

\[ F(Q^2) = \frac{4 F_Y F_S (\gamma G) e^{0.25 \gamma G}}{b_0 (\gamma G + 1)(\gamma G + 2)(\gamma G + 3)} \] (21)

\[ = \frac{1}{b_0 \ln \left( \frac{Q^2}{\Lambda^2} \right)} \] (21)
and $Ei$ is the exponential integral function. In Fig. 1 we show the difference between the two results for $\Delta a(Q^2, t)$. We see that they agree within a few % except for the softer values of $t$, as expected. We look forward to determining definitively whether the experimental data prefer one over the other. This detailed study will appear elsewhere [38] but we begin the discussion below with a view on recent FNAL data. Again, we note that the comparison in

![Graph of $\Delta a(Q^2, t)$ for the DGLAP-CS and IR.Imp.DGLAP-CS kernels (11, 20). $Q^2$ is a typical virtuality closer to the squared scale of the hard sub-process – here we use $Q^2 = 25\text{GeV}^2$ for illustration.](image)

Figure 1: Graph of $\Delta a(Q^2, t)$ for the DGLAP-CS and IR.Imp.DGLAP-CS kernels (11, 20). $Q^2$ is a typical virtuality closer to the squared scale of the hard sub-process – here we use $Q^2 = 25\text{GeV}^2$ for illustration.

Fig. 1 is done here at the leading log virtuality level, but the sub-leading effects we have suppressed in discussing it will not change our general conclusions drawn therefrom.

We have carried out the corresponding changes for all of the kernels in (2) in the HERWIG6.5 environment, with its angle-ordered showers, resulting in the new MC, HERWIRI1.0, in which the ISR parton showers have IR-improvement as given by the kernels in (3). In the original release of the program, v. 1.0, we stated that the time-like parton showers had been completely IR-improved in a way that suggested the space-like parton showers had not yet been IR-improved at all. Then, in release 1.02, we stated that the only part of the space-like parton showers without IR-improvement in v. 1.0 is that associated with HERWIG6.5’s space-like module HWSQGG for the space-like branching process $G \rightarrow q\bar{q}$, a process which is not IR divergent and which is, in any case, a sub-dominant part of the shower. The module HWSQGG was IR-improved as well in the release HERWIRI1.02. This was in fact an oversight, as the module HWSFBR which controls the remainder of the space-like branching processes was also still in need of IR-improvement in versions 1.0 and 1.02\(^4\). We have done the required IR-improvement of the latter module as well in version 1.031. While the effect in going from version 1.0 to version 1.02 is small, that in going from version 1.02 to 1.031 is not in general and we recommend version 1.031 for the best precision. We now illustrate some of the results we have obtained in comparing ISR showers in HERWIG6.5 and with those in HERWIRI1.0(31) at LHC and at FNAL energies, where some comparison with real data is also featured at the FNAL energy. Specifically, we compare the $z$-distributions, $p_T$-distributions, etc., that result from the IR-improved and usual DGLAP-CS showers in what follows\(^5\).

First, for the generic $2 \rightarrow 2$ hard processes at LHC energies ($14 \text{ TeV}$) we get the comparison shown Figs. 2, 3 for the respective ISR $z$-distribution and $p_T$ distribution at the parton level. Here, there are no cuts placed on the MC data and we define $z$ as $z = E_{\text{parton}}/E_{\text{beam}}$ where $E_{\text{beam}}$ is the cms beam energy and $E_{\text{parton}}$ is the respective parton energy in the cms system. The two quantities $z$ and $p_T^2$ for partons are of course not directly observable but their distributions show the softening of the IR divergence as we expect. Turning next to the

![Figure 2: The $z$-distribution(ISR parton energy fraction) shower comparison in HERWIG6.5.](image)

Figure 2: The $z$-distribution(ISR parton energy fraction) shower comparison in HERWIG6.5.

similar quantities for the $\pi^+$ production in the

\(^4\)We thank Profs. B. Webber and M. Seymour for discussion here.

\(^5\)Note that similar results for PYTHIA and MC@NLO are in progress in general; for MC@NLO we have some initial results already in particular cases – see the discussion below.
generic $2 \to 2$ hard processes at LHC, we see in Figs. 4, 5 that spectra in the former are similar and spectra in the latter are again softer in the IR-improved case. These spectra of course would be subject to some "tuning" in a real experiment and we await with anticipation the outcome of such an effort in comparison to LHC data.

We turn next to the luminosity process of single $Z$ production at the LHC, where in Figs. 6,7,8 we show respectively the ISR parton energy fraction distribution, the $Z-p_T$ distribution, and the $Z$-rapidity distribution with cuts on the acceptance as $40\text{GeV} < M_Z, p_T^Z > 5\text{GeV}$ for $Z \to \mu\bar{\mu}$ – all lepton rapidities are included. For the energy fraction distribution and the $p_T$ distributions we again see softer spectra in the former and we see similar spectra in the latter in the IR-improved case. For the rapidity plot, we see the migration of some events to the higher values of $|\eta|$, which is not inconsistent with a softer spectrum for the IR-improved case. We look forward to the confrontation with experiment, where again we stress that in a real experiment, a certain amount of "tuning" with affect these results. The question will always be which set of distributions gives a better $\chi^2$ per degree of freedom.

Finally, we turn the issue of the IR-cut-off in HERWIG6.5. In HERWIG6.5, there is are IR-cut-off parameters used to separate real and virtual effects and necessitated by the $+\text{-function}$ representation of the usual DGLAP-CS kernels. In HERWIRI, these parameters can be taken arbitrarily close to zero, as the IR-improved kernels are integrable [9, 10]. We now illustrate the difference in IR-cut-off response by comparing it for HERWIG6.5 and HERWIRI: we change the default values of the parameters in HERWIG6.5 by factors of .7 and 1.44 as shown in the Fig. 9. We see that the harder cut-off reduces

One might wonder why we show the $Z$ rapidity here as the soft gluons which we study only have an indirect affect on it via momentum conservation? But, this means that the rapidity predicted by the IR-improved showers should be close to that predicted by the un-improved showers and we show this cross-check is indeed fulfilled in our plots.
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Figure 6: The $z$-distribution(ISR parton energy fraction) shower comparison in HERWIG6.5.

Figure 7: The $Z_pT$-distribution(ISR parton shower effect) comparison in HERWIG6.5.

Figure 8: The $Z$ rapidity-distribution(ISR parton shower) comparison in HERWIG6.5.

We finish this initial comparison discussion by turning to the data from FNAL on the $Z$ rapidity and $p_T$ spectra as reported in Refs. [39, 40]. We show these results, for 1.96TeV cms energy, in Fig. 10. We see that, in the case of the CDF rapidity data, HERWIRI1.0(31) and HERWIG6.5 both give a reasonable overall representation of the data but that HERWIRI1.0(31) is somewhat closer to the data for small values of $Y$. The two $\chi^2$/d.o.f are 1.77 and 1.54 for HERWIG6.5 and HERWIRI1.031 respectively. The data errors in Fig. 10(a) do not include luminosity and PDF errors [39], so that they can only be used conditionally at this point.

Including the NLO contributions to the hard process via MC@NLO/HERWIG6.510 and MC@NLO/HERWIRI1.031[28]7 improves the agreement for both HERWIG6.5 and for HERWIRI1.031, where the $\chi^2$/d.o.f are changed to 1.40 and 1.42 respectively.

For the D0 $p_T$ data, we see that HERWIRI1.0(31) gives a better fit to the data compared to HERWIG6.5 for low $p_T$, (for $p_T < 12.5$GeV, the $\chi^2$/d.o.f. are ~ 2.5 and 3.3 respectively if we add the statistical and systematic errors), showing that the IR-improvement makes a better representation of QCD in the soft regime for a given fixed order in perturbation theory. Including the results of MC@NLO [28] improves the $\chi^2$/d.o.f for the HERWIRI1.031 in both the soft and hard regimes and it improves the HERWIG6.510 $\chi^2$/d.o.f for $p_T$ near 3.75 GeV where the distribution peaks. For $p_T < 7.5$GeV the $\chi^2$/d.o.f for the MC@NLO/HERWIRI1.031 is 1.5 whereas that for MC@NLO/HERWIG6.510 is worse.

7We thank S. Frixione for helpful discussions with this implementation.
Figure 9: IR-cut-off sensitivity in z-distributions of the ISR parton energy fraction: (a), DGLAP-CS, (b), IR-I DGLAP-CS – for the single Z hard sub-process in HERWIG-6.5 environment.

Figure 10: Comparison with FNAL data: (a), CDF rapidity data on \((Z/\gamma^*)\) production to \(e^+e^-\) pairs, the circular dots are the data; (b), D0 \(p_T\) spectrum data on \((Z/\gamma^*)\) production to \(e^+e^-\) pairs, the circular dots are the data, the blue triangles are HERWIRI1.031, the green triangles are HERWIG6.510. These are untuned theoretical results.

5. Conclusions

Our new MC HERWIRI1.0(31) sets the stage for the further implementation of the attendant [4] new approach to precision QED × QCD predictions for LHC physics by the introduction of the respective resummed residuals needed to systematically improve the precision tag to the 1% regime for such processes as single heavy gauge boson production, for example. Here, we already note that this new IR-improved MC, HERWIRI1.0(31), available at http://thep03.baylor.edu, is expected to allow for a better \(\chi^2\) per degree of freedom in data analysis of high energy hadron-hadron scattering for soft radiative effects and we have given evidence that this is indeed the case.

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