Time Modulation of K–Shell Electron Capture Decay Rates of H–Like Heavy Ions and Neutrino Masses

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We calculate the neutrino masses, using the experimental data on the periods of the time modulation of the K–shell electron capture (EC) decay rates of the H–like heavy ions, measured at GSI. The corrections to neutrino masses, caused by interaction of massive neutrinos with the strong Coulomb field of the daughter ions, are taken into account.

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Introduction

Recently [1] we have analysed a process by which massive neutrinos, produced in weak decays of the H–like heavy ions [2], can acquire non–trivial mass–corrections caused by the interaction of massive neutrinos with a strong Coulomb field of the daughter ions. We have shown that due to the dissociation of massive neutrinos into ℓ + W + pairs in a strong Coulomb field at EC, can acquire non–trivial mass–corrections dependent on the relative distance between the neutrinos and the daughter ions. According to this hypothesis, massive antineutrinos, produced in the continuous–state and bound–state β−–decays of heavy ions, should acquire mass–corrections with a sign opposite to the mass–corrections for massive neutrinos due to the dissociation into ℓ + W + pairs at EC, where ℓ + is a positron (e +) and positively charged μ + and τ +.

The K–shell electron capture (EC) decay rates of the H–like ions 142Pm 60+, 140Pm 58+, and 122I 52+ (preliminary) at GSI in Darmstadt [2] showed a time modulation of exponential decays with periods $T_{EC} = 7.10(22) s, 7.06(8) s$ and $6.11(3) s$ and modulation amplitudes $a_{EC}^{d} = 0.23(4), 0.18(3)$ and $0.22(2)$ for 142Pm 60+, 140Pm 58+ and 122I 52+, respectively, defined by

$$\lambda_{EC}^{d}(t) = \lambda_{EC} (1 + a_{EC}^{d} \cos(\omega_{EC} t + \phi_{EC})),$$

(1)

where $\lambda_{EC}$ is the EC–decay constant, $a_{EC}^{d}$, $T_{EC} = 2\pi/\omega_{EC}$ and $\phi_{EC}$ are the amplitude, period and phase of the time–dependent term $\lambda_{EC}^{d}$. Furthermore it was shown that the β−–decay rate of 142Pm 60+, measured simultaneously with its modulation rate of EC–decay rate, is not modulated with an amplitude upper limit $a_{β−} < 0.03 [2]$. As has been proposed in [3–6], such a periodic dependence of the EC–decay rate can be explained by the interference of neutrino mass–eigenstates. The period of the time modulation $T_{EC}$ has been obtained as

$$T_{EC} = 4\pi\gamma M_{m} / (\Delta m_{21}^{2})_{GSI},$$

(2)

where $M_{m}$ is the mass of the mother ion, $\gamma = 1.432$ is the Lorentz factor of the ions in the experimental storage ring [2]. According to [1], $(\Delta m_{21}^{2})_{GSI}$ should be defined as follows

$$(\Delta m_{21}^{2})_{GSI} = (m_{2} + \Delta m_{2}^{2}) - (m_{1} + \Delta m_{1}^{2}),$$

(3)

where $m_{2}$ and $m_{1}$ are neutrino masses and $\Delta m_{2}$ and $\Delta m_{1}$ are neutrino mass–corrections, caused by the interaction of massive neutrinos with the strong Coulomb field of the daughter ion as proposed in [1].

From the experimental data on the periods of the time modulation of the EC–decay rates, the masses $M_{m} \sim 931.494 A$ of mother ions and Eq.(2) we get the following values for quadratic mass difference $(\Delta m_{21}^{2})_{GSI}$ of massive neutrinos

$$\begin{align*}
(\Delta m_{21}^{2})_{GSI} &= \left\{ \begin{array}{l}
2.20(7) \times 10^{-4} \text{ eV}^{2}, 142\text{Pm}^{60+} \\
2.18(3) \times 10^{-4} \text{ eV}^{2}, 140\text{Pm}^{58+} \\
2.19(1) \times 10^{-4} \text{ eV}^{2}, 122\text{I}^{52+}.
\end{array} \right.
\end{align*}$$

(4)

The values are equal within their error margins and yield a combined squared neutrino mass difference $(\Delta m_{21}^{2})_{GSI} = 2.19 \times 10^{-4} \text{ eV}^{2}$. This confirms the proportionality of the period of time modulation to the mass number A of the mother nucleus $T_{EC} = \kappa A$, where $\kappa = 4\pi\gamma h M_{m} / (\Delta m_{21}^{2})_{GSI} = 0.050(4)$ s [3].

The value $(\Delta m_{21}^{2})_{GSI} = 2.19 \times 10^{-4} \text{ eV}^{2}$ is 2.9 times larger than that reported by the KamLAND $(\Delta m_{21}^{2})_{KL} = 7.59(21) \times 10^{-5} \text{ eV}^{2}$ [7]. In [1] such a discrepancy has been proposed to be explained by the neutrino mass–corrections in the strong Coulomb field of the daughter ions. Assuming that $(\Delta m_{21}^{2})_{KL} = 7.59(21) \times 10^{-5} \text{ eV}^{2}$, deduced from antineutrino - antineutrino oscillations $\bar{\nu} \rightarrow \bar{\nu}$ at KamLAND, represents difference of
squared proper neutrino masses \((\Delta m_{21}^2)_{KL} = m_2^2 - m_1^2\), and taking into account the GSI experimental value \((\Delta m_{21}^2)_{GSI} = 2.19 \times 10^{-4} \text{eV}^2\) one can estimate the magnitude of neutrino masses \(m_j \sim 0.11 \text{eV}\) [1].

However, such an estimate of neutrino masses is only qualitative, since nobody took into account the contribution of antineutrino mass–corrections to antineutrino masses, produced in the \(\beta^-\)-decays of fission fragments in nuclear reactors [3]. Indeed, as has been pointed out by Nakajima et al. [9], the energy spectrum of antineutrinos, produced in the \(\beta^-\)-decays of fission fragments, has been analysed with the probability of the electron antineutrino oscillations \(\bar{\nu}_e \leftrightarrow \bar{\nu}_e\), defined by

\[
P_{\bar{\nu}_e \leftrightarrow \bar{\nu}_e}(E_{\bar{\nu}_e}) = 1 - \sin^2(2\theta_{12}) \sin^2 \left( \frac{\Delta m_{21}^2 L}{4E_{\bar{\nu}_e}} \right),
\]

where \(\theta_{12}\) is the mixing angle [10], \(\Delta m_{21}^2 = m_2^2 - m_1^2\) is given by the proper neutrino masses, \(L\) is the distance between the source and the detector of antineutrinos and \(E_{\bar{\nu}_e}\) is the antineutrino energy.

However, one can show that the antineutrino mass–corrections, caused by interactions of massive antineutrinos with the Coulomb field of the daughter nuclei in the final state of \(\beta^-\)-decays of fission fragments, change the probability of the electron antineutrino oscillations \(\bar{\nu}_e \leftrightarrow \bar{\nu}_e\) as follows

\[
P_{\bar{\nu}_e \leftrightarrow \bar{\nu}_e}(E_{\bar{\nu}_e}) = 1 - \frac{1}{2} \sin^2(2\theta_{12})
\times \left\{ 1 - \cos \left[ \arctan \left( \frac{\delta m_j^2 - \delta m_{j'}^2}{2E_{\bar{\nu}_e} \lambda_{\beta^-}} \right) \right] \times \cos \left[ \frac{\Delta m_{21}^2 L}{2E_{\bar{\nu}_e}} + \arctan \left( \frac{\delta m_j^2 - \delta m_{j'}^2}{2E_{\bar{\nu}_e} \lambda_{\beta^-}} \right) \right] \right\},
\]

where \(\lambda_{\beta^-}\) is the decay rate of the \(\beta^-\)-decay of the fission fragment and \(\delta m_j^2 = 2m_j \delta m_j\) are antineutrino mass–corrections. For \(\delta m_j \rightarrow 0\) the probability Eq. (5) reduces to Eq. (4). The probability Eq. (6) should be averaged over all fission fragments unstable under \(\beta^-\)-decays. In thermal neutron fission [8], the fission fragments have a well known double hump mass and nuclear charge (Z) distribution (see Fig. 1).

Thus, we can argue that the value \((\Delta m_{21}^2)_{KL} = 7.59(21) \times 10^{-5} \text{eV}^2\), deduced from antineutrino - antineutrino oscillations \(\bar{\nu}_e \rightarrow \bar{\nu}_e\) by the KamLAND experiment, does not correspond to the difference of squared proper antineutrino masses \((\Delta m_{21}^2)_{KL} \neq m_2^2 - m_1^2\). This means that neutrino mass–corrections should be used for a quantitative explanation of the discrepancy between \((\Delta m_{21}^2)_{GSI} = 2.19(3) \times 10^{-4} \text{eV}^2\) and \((\Delta m_{21}^2)_{KL} = 7.59(21) \times 10^{-5} \text{eV}^2\).

In this letter we propose a model–independent calculation of neutrino masses. We use the experimental data on the periods of the time modulation of the \(EC\)-decay rates of the H–like like heavy ions with different nuclear charge Z, measured at GSI, and corrections to neutrino masses, caused by interactions of massive neutrinos with the strong Coulomb fields of the daughter nuclei.

**Neutrino Masses**

For the calculation of neutrino masses we use the experimental data on the time modulation of the \(EC\)-decay rates of a couple of ions \(A^* X^+(Z'+1)\) and \(A^* X^+(Z'+2)^+\). The neutrino masses we obtain as a solution of the system of two algebraical equations

\[
\begin{align*}
(m_2 + \delta m_j^2)^2 - (m_1 + \delta m_{j'}^2)^2 &= (\Delta m_{21}^2)_{GSI}^2, \\
(m_2 + \delta m_j^2)^2 - (m_1 + \delta m_{j'}^2)^2 &= (\Delta m_{21}^2)_{GSI}^2,
\end{align*}
\]

where \(\delta m_j^2\) and \(\delta m_{j'}^2\) are corrections to neutrino masses in the Coulomb fields of the daughter ions of the \(EC\)-decays of \(A^* X^+(Z'-1)^+\) and \(A^* X^+(Z'-2)^+\) ions, respectively. We can combine the GSI experimental data in three systems of two algebraical equations Eq. (4). The solutions of these systems of algebraical equations give neutrino masses. The Coulomb mass–corrections, required for the solution of the Eq. (4) for every pair of the H–like heavy ions, are given in Table I. The neutrino masses are summarised in Table II.

**TABLE I:** Numerical values for corrections to neutrino masses in the strong Coulomb field of daughter nuclei, calculated for \(R = 1.1 \times A^{-1/3}\).

| Ion | \(\Delta m_{21}^2\)| | \(\delta m_1\) | | \(\delta m_2\)| |
|-----|-----------------|-----------------|-----------------|-----------------|
| \(^{122}Xe\) | \(2.19(3)\times10^{-4}\) | \(-10.680\) | \(-5.990\) |
| \(^{130}Te\) | \(-8.967\) | \(-5.068\) |
| \(^{238}U\) | \(-11.610\) | \(-6.507\) |

**FIG. 1:** Distribution of fission fragment masses from the fission of \(^{235}U\) induced by thermal neutrons [8].
which we do not take into account in the system of algebraical equations Eq. (7). In the last line of Table II we calculate \( \Delta m_{32}^2 \), \( \Delta m_{21}^2 \) and \( \sum_j m_j \) without applying Coulomb corrections to the antineutrino masses.

One can see that the neutrino masses, calculated from the experimental data on the periods of the time modulation of the EC–decays of the H–like heavy ions, reproduce better \( \Delta m_{32}^2 \), \( \Delta m_{21}^2 \), \( \sum_j m_j \) and the mass of the heaviest neutrino \( m_3 \), which is less sensitive to Coulomb corrections to neutrino masses. We notice that the neutrinos in our analysis are massive Dirac particles with masses, obeying a direct hierarchy \( m_1 < m_2 < m_3 \).

The values of the Coulomb corrections to neutrino masses depend on the values of the mixing angles \( \theta_{12} \) and \( \theta_{23} \). For the numerical analysis we have used \( \theta_{12} = 34^\circ \) and \( \theta_{23} = 45^\circ \) (see also [3]). Since the mixing angles can be obtained from the experimental data on the solar neutrino fluxes [10], for the analysis of the solar neutrino data by SNO [10] instead of \( \Delta m_{21}^2 \) and \( \Delta m_{32}^2 \) we propose to use the difference of squared proper neutrino masses \( \Delta m_{21}^2 = 2.155 \times 10^{-4} \text{eV}^2 \) (see Table II), which is less sensitive to Coulomb corrections to neutrino masses, and to find new mixing angles for massive neutrinos.

**Conclusion**

We have shown that taking into account the corrections to neutrino masses, caused by the interactions of neutrino mass–eigenstates with the Coulomb fields of the fission products, the probability of electron antineutrino oscillations \( \bar{\nu}_e \leftrightarrow \bar{\nu}_e \) differs from the probability of the electron antineutrino oscillations, calculated without Coulomb corrections to the antineutrino masses. This implies that the experimental data \( \Delta m_{21}^2 \) by the KamLAND [2] cannot be used for the determination of \( \Delta m_{21}^2 \) without applying Coulomb mass–corrections.

For the calculation of neutrino masses we have used the GSI experimental data on the time modulation of the EC–decay rates of the H–like heavy ions with different nuclear charges \( Z \) and the corrections to neutrino masses, caused by the Coulomb field of the daughter nuclei in the EC–decays of the H–like heavy ions. The calculated neutrino masses are given in Table II. They satisfy the strict cosmological constraints for the sum of neutrino masses \( 0.05 \text{eV}/c^2 < \sum_j m_j < 0.17 \text{eV}/c^2 \) [11] and the mass of the heaviest neutrino \( 0.02 \text{eV}/c^2 < m_3 < 0.40 \text{eV}/c^2 \) [10].

We notice that the neutrinos in our analysis are massive Dirac particles with masses, obeying a direct hierarchy \( m_1 < m_2 < m_3 \).

| \((X', X'')\) | (I, Pr) | (I, Pm) | (Pr, Pm) |
|----------------|--------|--------|--------|
| \(m_1\) (eV)   | 0.00657| 0.01345| 0.02982|
| \(m_2\) (eV)   | 0.01636| 0.01991| 0.03292|
| \(m_3\) (eV)   | 0.05165| 0.05288| 0.05735|
| \(\sum_j m_j\) (eV) | 0.07458| 0.08624| 0.12010|
| \(10^3\Delta m_{32}^2\) | 2.175 | 2.196 | 2.141 |
| \(10^4\Delta m_{21}^2\) | 2.245 | 2.155 | 1.945 |

TABLE II: Neutrino masses, calculated for \( R = 1.1 \times A^{1/3} \) [3]. The mass \( m_3 \) is calculated for \( \Delta m_{32}^2 = m_3^2 - m_2^2 = 2.40 \times 10^{-3} \text{eV}^2 \) [10]; \( \Delta m_{21}^2 = m_2^2 - m_1^2 \).

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