Longitudinal Polarization at future $e^+e^-$ Colliders and Virtual New Physics Effects

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Abstract

The theoretical merits of longitudinal polarization asymmetries of electron-positron annihilation into two final fermions at future colliders are examined, using a recently proposed theoretical description. A number of interesting features, valid for searches of virtual effects of new physics, is underlined, that is reminiscent of analogous properties valid on top of $Z$ resonance. As an application to a concrete example, we consider the case of a model with triple anomalous gauge couplings and show that the additional information provided by these asymmetries would lead to a drastic reduction of the allowed domain of the relevant parameters.
1 Introduction

Among the several quantities that can be measured in the process of electron-positron annihilation into a fermion-antifermion couple, the longitudinal polarization asymmetry $A_{LR} \equiv \frac{\sigma_L - \sigma_R}{\sigma_L + \sigma_R}$ has represented in the last few years an example of, least to say, remarkable theoretical interest. This is due to the known fact that, as it was stressed in a number of dedicated papers [1], [2], [3], [4], the properties of this observable on top of Z resonance are indeed special. In particular one can stress two main facts i.e. that $A_{LR}$ is independent of the final produced state (this was shown in particular detail in Ref. [3]), and that it is particularly sensitive to possible virtual effects of a large number of models of new physics (this was exhaustively discussed in Refs. [2] and [4]). These features, that appear essentially unique, have deeply motivated the tough experimental effort at SLC [5] where $A_{LR}$ has been (in fact, it is still being) measured to an extremely high precision [6], fully exploiting the fact that at a linear electron-positron collider it is ”relatively” easy to produce longitudinally polarized electron beams with a high and accurately known polarization degree [7]. This is not the case of a circular accelerator, and for this reason neither at LEP1 (in spite of the several impressive experimental studies and efforts of recent years [8]) nor at LEP2 a measurement of $A_{LR}$ has been, or will be predictably performed.

The possibility that a linear electron-positron collider of an overall c.m. energy not far from 500 GeV is eventually built in a not too distant future has been very seriously investigated in the last few years, and the results of a remarkable combined experimental and theoretical effort have been published in several dedicated Proceedings [9]. At such a kind of machine it would be, again, ”relatively” easy to produce longitudinally polarized electron beams, which implies the possibility of measuring $A_{LR}$, for various possible final states. One might therefore wonder whether the special theoretical properties valid on top of Z resonance will still be true and, if not, how they would be modified at about 500 GeV.

The purpose of this paper is precisely that of investigating the general features of $A_{LR}$ at such a future linear collider (NLC) and to show that, from a theoretical point of view, this quantity still retains beautiful and interesting features, that make it particularly promising as a tool for investigating virtual effects of models of new physics. This will be shown in some detail in the following Section 2. Section 3 will be devoted to an illustrative example, i.e. to the case of a model with anomalous gauge couplings, for which the benefits of a measurement of $A_{LR}$ will be explicitly shown in a quantitative way. A short final discussion will then be given in Section 4, valid for a more general class of theoretical models.
2 Longitudinal polarization asymmetries at one loop.

2a. General features

The purpose of this section is that of deriving relatively simple and compact expressions for longitudinal polarization asymmetries in the process of electron-positron annihilation into two final fermions at arbitrary c.m. energy at one loop. With this aim, we shall follow a procedure that has been fully illustrated in two recent papers [10], [11] and has been called ”Z-peak subtracted” representation. In order, though, to make this paper, at least reasonably, self-contained, we shall sketch a quick derivation of all the fundamental formulae, deferring to refs.[10], [11] for a more complete discussion of several technical details.

The starting point of our derivation will be the expression of the longitudinally polarized cross sections \( \sigma_{L,f} \) and \( \sigma_{R,f} \) (left-handed and right-handed initial electrons) in Born approximation, where \( f \) denotes the final fermion (in the case that we shall consider, charged lepton or quark). In practice, though, it will be more useful to consider from the very beginning the difference and the sum of such cross sections, that appear directly as the numerator and the denominator of the longitudinal polarization asymmetry. Denoting by \( \sigma_{LR,f} \) and \( \sigma_{f} \) these quantities, one easily finds that

\[
\sigma_{LR,f}(q^2) = \sigma_{L,f}(q^2) - \sigma_{R,f}(q^2) = N_f \left( \frac{4\pi q^2}{3} \right) \times
\left\{ \frac{\sqrt{2} G_{\mu} M_{Z0}}{4\pi} \frac{2g_{vi}\alpha_0 Q_{vi}+g_{if}^{(0)}}{(q^2 - M_{Z0}^2)^2} - 2\alpha_0 Q_{f} \left[ \frac{\sqrt{2} G_{\mu} M_{Z0}^2}{4\pi} \frac{g_{vi} g_{vf}}{q^2 - M_{Z0}^2} \right] \right\}
\]

(1)

\[
\sigma_{f}(q^2) = \sigma_{L,f}(q^2) + \sigma_{R,f}(q^2) = N_f \left( \frac{4\pi q^2}{3} \right) \left\{ \frac{\alpha_0^2 Q_{f}^2}{q^4} + \left[ \frac{\sqrt{2} G_{\mu} M_{Z0}^2}{4\pi} \frac{2g_{vi}^{(0)} g_{vf}^{(0)} + g_{if}^{(0)} g_{Af}^{(0)}}{(q^2 - M_{Z0}^2)^2} - 2\alpha_0 Q_{f} \left[ \frac{\sqrt{2} G_{\mu} M_{Z0}^2}{4\pi} \frac{g_{vi} g_{vf}}{q^2 - M_{Z0}^2} \right] \right] \right\}
\]

(2)

In the previous formulae, \( q^2 \) is the total c.m. squared energy, \( N_f \) is the colour factor and the various couplings are defined in the conventional way, i.e \( g_{Af}^{(0)} = I_{Af}^{3L} \); \( g_{vi}^{(0)} = I_{vi}^{3L} - 2Q_{i,f} s_0^2 \), with \( Q_{i,f} \) the charge of the lepton \( l \) or fermion \( f \). Note that all the couplings and the \( Z \) mass (with index \( (0) \)) are, by definition, 'bare' ones.

From eqs.(1),(2) it is straightforward to derive the expression at Born level of the longitudinal polarization asymmetry \( A_{LR,f}^{(0)}(q^2) \) defined as \( \sigma_{LR,f}^{(0)} / \sigma_{f}^{(0)} \).

From a glance to eqs.(1),(2) one can derive a rather important and well-known fact. On top of \( Z \) resonance, where the pure \( Z \) exchange term largely dominates, the dependence on the final state completely disappears, so that \( A_{LR,f}^{(0)} \) becomes only dependent on the initial electron-\( Z \) couplings. But when one moves away from the \( Z \) peak this peculiar feature disappears, and other terms become competitive. As a result of this, \( A_{LR,f}^{(0)} \) will
now effectively depend on products of $Z$ couplings to the initial and to the final considered fermion, and several different observables will therefore become potentially relevant.

Concerning the final fermion, we shall be limited in this paper to the case of "light" charged ones ($f = l, u, d, s, c, b$). Moreover, the considered $q^2$ values will always be (much) larger than $M_Z^2$. In terms of the final masses, this means that they will be safely negligible, $m_f \simeq 0$. For what concerns calculations within the Standard Model framework, this will have the consequence that at the one loop level the independent Lorentz structures of the invariant scattering amplitude will be of only four types corresponding to initial and final axial and vector "currents". Equivalently, one shall have, following the definitions of ref.[11], a "$\gamma\gamma$", a "$ZZ$", a "$\gamma Z$" and a "$Z\gamma$" structure, that will appear as the four independent combinations of the elementary $\gamma$, $Z$ "currents" defined as

$$v^{(\gamma)}_{\mu f} = e_0 Q_f \bar{u}_f \gamma_{\mu} v_f$$

$$v^{(Z)}_{\mu f} = \frac{e_0}{2c_0 s_0} \bar{u}_f \gamma_{\mu} (g_{V f}^{(0)} - \gamma^5 g_{A f}^{(0)}) v_f$$

For instance, the "$\gamma Z$" structure will correspond in our notations to the product of $v^{(\gamma)}_{\mu l} v^{(Z)}_{\mu f}$, while the "$Z\gamma$" structure will correspond to $v^{(Z)}_{\mu l} v^{(\gamma)}_{\mu f}$.

In this paper, we shall focus our attention on three cases that we consider to be realistic at a future 500 GeV electron-positron collider, i.e. those of production of two final charged leptons ($A_{LR,l}$), of a final $b\bar{b}$ couple ($A_{LR,b}$), and of production of all possible light final quark couples ($A_{LR,5}$). This should cover all the meaningful possibilities for two final light fermions production.

The previous equations (1),(2) were strictly valid at Born level. To make more rigorous statements, one has now to move to the one loop expressions. This implies a redefinition of the various bare quantities and also a consideration of the potentially dangerous QED radiation. For what concerns the latter point, a rigorous treatment of $A_{LR,f}$ at NLC (on $Z$ resonance an exhaustive discussion is available [12]), does not yet exist to our knowledge (and is, in fact, under examination). We shall assume that, as it happens in all other cases, a proper apparatus-dependent calculation allows to eliminate the unwanted difficulties and we proceed from now on to the treatment of the purely "non QED" component. In the latter one we shall leave aside, and consider it as a separate and fixed component, the contribution to the considered observables originated by standard strong interactions that, in the conventional treatment, will be denoted as the "QCD" term \( \simeq \alpha_s(q^2) \). Our interest will be concentrated on the purely electroweak components of the various $A_{LR,f}$, computed at one loop. The color factor of all these quantities will consequently continue to cancel exactly in the ratio, as it did at pure Born level.

To illustrate the philosophy and the main features of our approach with the simplest example, we shall consider the modifications at one loop of the "pure $Z$" Born exchange term \( \simeq \frac{1}{(q^2-M_Z^2)^2} \) in the denominator of a general $A_{LR,f}^{(0)} = \sigma_{LR,f}^{(0)} / \sigma_f^{(0)}$. As it has been shown in full detail in ref.[11], sect.2, in the discussion leading to eq.(38), and as one can easily derive, the Born expression becomes at one loop:

3
\[
\sigma_{lf}^{(Z)}(q^2) = N_f \frac{4\pi q^2}{3} \left[ \frac{3\Gamma_f}{M_Z} \right] [\frac{3\Gamma_f}{N_f^{QCD} M_Z}] (q^2 - M_Z^2)^2 + M_Z^2 \Gamma_Z^2 [1 - 2R^{(lf)}(q^2)]
- 8s_{lf} c_{lf} \left[ \frac{\tilde{\nu}_l}{1 + \tilde{\nu}_l^2} V_{\gamma Z}^{(lf)}(q^2) + \frac{\tilde{\nu}_f [Q_f]}{1 + \tilde{\nu}_f^2} V_{Z\gamma}^{(lf)}(q^2) \right]
\]

(5)

We want to stress now again the main features of this equation. As one sees, the squared Fermi coupling \(G_{\mu}^{(0)}\) has been replaced by the product of the two \(Z\) widths \(\Gamma_f \Gamma_f\), for which one is supposed to take the experimental values measured on top of \(Z\) resonance (in fact, for \(f \neq l\), \(\Gamma_f\) appears divided by the quantity \(N_f^{QCD} \simeq 3(1 + \alpha_s(M_Z^2))/\pi\), where also \(\alpha_s(M_Z^2)\) is supposed to be measured on top of \(Z\) resonance). As a consequence of this bargain, the one loop ”form factors” \(R^{(lf)}(q^2), V^{(lf)}(q^2)\) are subtracted at \(q^2 = M_Z^2\). More precisely, they will be given by integrations over the angular variable of the following expressions:

\[
R^{(lf)}(q^2, \theta) = \bar{I}_Z^{(lf)}(q^2, \theta) - \bar{I}_Z^{(lf)}(M_Z^2, \theta)
\]

(6)

\[
V_{\gamma Z}^{(lf)}(q^2, \theta) \equiv \bar{F}_{\gamma Z}^{(lf)}(q^2, \theta) - \bar{F}_{\gamma Z}^{(lf)}(M_Z^2, \theta)
\]

(7)

\[
V_{Z\gamma}^{(lf)}(q^2, \theta) \equiv \bar{F}_{Z\gamma}^{(lf)}(q^2, \theta) - \bar{F}_{Z\gamma}^{(lf)}(M_Z^2, \theta)
\]

(8)

where the ”auxiliary” quantity \(\bar{I}_Z\) is defined as

\[
\bar{I}_Z^{(lf)}(q^2, \theta) = \frac{q^2}{q^2 - M_Z^2} \left[ \frac{\bar{F}_Z^{(lf)}(q^2, \theta) - \bar{F}_Z^{(lf)}(M_Z^2, \theta)}{\pi} \right]
\]

(9)

and all the quantities denoted as \(\bar{F}^{(ij)}\) in the previous equations are conventional, gauge invariant combinations of self-energies, vertices and boxes (defined following the conventions of De-grassi and Sirlin [13]) that belong to the previously defined ”\(Z\) \(Z\)”,”\(\gamma \gamma\)” and ”\(Z\gamma\)” Lorentz structures. To fix the normalization, the self-energy (\(\cos \theta\) independent) component of \(\bar{F}^{(ij)}\) is the one appearing in the usual definition of the transverse self-energies:

\[
A_i(q^2) \equiv A_i(0) + q^2 F_i(q^2)
\]

(10)

The generalization of the given example to the complete expressions of the asymmetries will now be a trivial one. In the final ”\(\gamma \gamma\)” component, for instance, new quantities measured on \(Z\) resonance will appear. One will be the longitudinal polarization asymmetry itself, defined as ;

\[
A_{LR}(M_Z^2) = \frac{2\tilde{\nu}_l(M_Z^2)}{1 + \tilde{\nu}_l^2(M_Z^2)}
\]

(11)

where \(\tilde{\nu}_l(M_Z^2) = 1 - 4s^2_f(M_Z^2)\) and \(s^2_f(M_Z^2)\) is the effective (leptonic) Weinberg-Salam angle measured at \(M_Z^2\). Also, the corresponding hadronic variables \(\tilde{\nu}_f(M_Z^2)\) will enter, whose
exact definition is provided by the so called polarized forward-backward asymmetries originally [14] defined as:

\[ A_{b,c} = \frac{2\tilde{v}_{b,c}(M_Z^2)}{1 + \tilde{v}_{b,c}(M_Z^2)} \]  

(In practice, \(\tilde{v}_f \simeq 1 - 4|Q_f|s_f^2(M_Z^2)\).)

One should also say at this point that, for what concerns the photon contribution, the treatment is of strictly conventional type, with the bare \(\alpha^{(0)}\) replaced by the physical coupling computed at zero momentum transfer \(\alpha_{QED} \equiv \alpha(0)\) and the form factor

\[ \tilde{\Delta}^{(l,f)}(q^2, \theta) = \tilde{F}^{(l,f)}(0, \theta) - \tilde{F}^{(l,f)}(q^2, \theta) \]  

where \(\tilde{F}_\gamma\) is a proper projection on the ”\(\gamma\gamma\)” structure of the usual photon self-energy with corresponding vertices and boxes.

After this long but, we hope, useful discussion we shall now write the required one loop expressions of the electroweak component of the considered asymmetries. Using the previous notations, we have that:

\[ \sigma_{LR,f}^{(1)} = N_f \frac{4\pi q^2}{3} q^4 \frac{[3\Gamma_f(1 + \frac{3\Gamma_f}{M_Z^2} + \frac{3\Gamma_f}{N_f QCDF M_Z}] [1 - 2\tilde{\Delta}^{(l,f)}(q^2)]}{(q^2 - M_Z^2)^2 + M_Z^2 \Gamma_Z^2} \]

\[ - \frac{4s_t c_t}{\tilde{v}_t} V_{\gamma Z}^{(l,f)}(q^2) - \frac{8s_t c_t}{1 + \tilde{v}_f} |Q_f| V_{Z\gamma}^{(l,f)}(q^2) + 2\alpha(0)|Q_f| q^2 ((q^2 - M_Z^2)^2 + M_Z^2 \Gamma_Z^2) \]

\[ = \frac{3\Gamma_f}{M_Z^2} \frac{3\Gamma_f}{N_f QCDF M_Z} \frac{1/2}{(1 + \tilde{v}_f^2)^{1/2}} \frac{1 + \tilde{v}_f^2}{1 + \tilde{v}_f^2} \frac{1 + \tilde{v}_f}{1 + \tilde{v}_f^2} [1 + \tilde{\Delta}^{(l,f)}(q^2) - R^{(l,f)}(q^2)] \]

\[ - \frac{4s_t c_t}{\tilde{v}_f} V_{\gamma Z}^{(l,f)}(q^2) \]  

\[ \sigma_{f}^{(1)} = N_f \frac{4\pi q^2}{3} q^4 \frac{[3\Gamma_f(1 + \frac{3\Gamma_f}{M_Z^2} + \frac{3\Gamma_f}{N_f QCDF M_Z}] [1 - 2\tilde{\Delta}^{(l,f)}(q^2)]}{(q^2 - M_Z^2)^2 + M_Z^2 \Gamma_Z^2} \]

\[ - 8s_t c_t \{ \frac{1}{1 + \tilde{v}_t^2} V_{\gamma Z}^{(l,f)}(q^2) + \frac{\tilde{v}_f |Q_f| V_{Z\gamma}^{(l,f)}(q^2)}{1 + \tilde{v}_f^2} \} + 2\alpha(0)|Q_f| q^2 ((q^2 - M_Z^2)^2 + M_Z^2 \Gamma_Z^2) \]

\[ = \frac{3\Gamma_f}{M_Z^2} \frac{3\Gamma_f}{N_f QCDF M_Z} \frac{1/2}{(1 + \tilde{v}_f^2)^{1/2}} \frac{1 + \tilde{v}_f^2}{1 + \tilde{v}_f^2} \frac{1 + \tilde{v}_f}{1 + \tilde{v}_f^2} [1 + \tilde{\Delta}^{(l,f)}(q^2) - R^{(l,f)}(q^2)] \]

\[ - \frac{4s_t c_t}{\tilde{v}_t} V_{\gamma Z}^{(l,f)}(q^2) + \frac{|Q_f|}{\tilde{v}_f} V_{Z\gamma}^{(l,f)}(q^2) \]  

Eqs.(14) and (15) conclude our introductory discussion. In the forthcoming part of this section we shall consider in more detail the various cases corresponding to the three chosen different final states.
2b. Discussion of different final states

We begin with the (simplest) case of two final charged leptons. Since \( f = l \), only three independent form factors will remain \((V_{\gamma Z} \equiv V_{Z\gamma})\). To maintain the notations of refs. [10], [11] we shall put no fermion index on them, so that they will be labelled as \( \Delta \alpha \), \( R \) and \( V \). Also, we shall use here the quantity

\[
\kappa \equiv \frac{\alpha(0)}{3\Gamma_i/M_Z}
\]  

(16)

From eqs.(14),(15) one is then led to the desired expression. Here for simplicity we shall write it in an ”effective” way i.e. throwing away terms that are numerically irrelevant and only retaining the meaningful contributions. In this way we obtain the (relatively simple) formula:

\[
A_{\text{LR},l}^{(1)}(q^2) = \frac{q^2[\kappa(q^2 - M_Z^2) + q^2]}{\kappa^2(q^2 - M_Z^2)^2 + q^4}A_{\text{LR}}(M_Z^2) \times
\]

\[
\left\{ 1 + \frac{\kappa(q^2 - M_Z^2)}{\kappa(q^2 - M_Z^2) + q^2} - \frac{2\kappa^2(q^2 - M_Z^2)^2}{\kappa^2(q^2 - M_Z^2)^2 + q^4}\left[\tilde{\Delta}\alpha(q^2) + R(q^2)\right] - \frac{4c_l s_l}{v_l}V(q^2) \right\}
\]

(17)

A few comments are, at this point, appropriate. First of all, one sees that numerically the value of eq.(17) (more precisely, of its leading term, the first one in the r.h.s. of the equation) decreases when \( q^2 \) becomes larger than \( M_Z^2 \), pointing to an asymptotic value of about \( \frac{1}{2}A_{\text{LR}}(M_Z^2) \simeq 0.07 \). The one loop modifications to the leading term contain two quantities, the combination \( [\tilde{\Delta}\alpha + R] \) and the ”\( \gamma Z \)” term \( V \). The fact that the sum \( [\tilde{\Delta}\alpha + R] \) appears in eq.(17) is not accidental: it will be a general feature for the new physics effects in any ratio of cross sections. We shall return on this point in the next section. The point that deserves attention is the fact that the coefficient of \( V \) is relatively enhanced with respect to the coefficient of \([\tilde{\Delta}\alpha + R]\) by the factor \( \frac{1}{v_l} \), which makes it one order of magnitude larger. Note that this fact comes from the (accidental) smallness of the quantity \( \hat{\nu}_l(M_Z^2) \) and is generated by the contribution to the ”\( \gamma Z \)” structure in the ”pure Z” exchange component of \( \sigma_{\text{LR},l} \) at one loop, that is strongly reminiscent of the situation met on top of Z resonance. As a consequence of this, one expects a relative enhancement of the virtual effects for those models of new physics where the contribution to \( V(q^2) \) is not accidentally depressed. We shall provide one specific example in section 3.

The next case that we shall consider is that of a final \( b\bar{b} \) couple. From the relevant expressions, making the same numerical approximations as in the previous case i.e. only retaining the dominant contributions to the various coefficients, we obtain in this case:

\[
A_{\text{LR},b}^{(1)}(q^2) = \tilde{A}_{\text{LR},b}(q^2)\left[1 + a_b(q^2)[\tilde{\Delta}^{\text{lb}}\alpha(q^2) + R^{\text{lb}}(q^2)] + b_b(q^2)V^{\text{lf}}_{\gamma Z}(q^2) + c_b(q^2)V^{\text{lf}}_{Z\gamma}(q^2)\right]
\]

(18)

where

\[
\tilde{A}_{\text{LR},b}(q^2) = \frac{C_{\text{LR},b}(q^2)}{C_b(q^2)}
\]

(19)
 experimental quantities that will enter in the theoretical formulae will be the once the prescriptions of our approach have been made clear. In practice, the only new in the case of the leptonic asymmetry. More precisely, we define systematically, for any model of new physics and final state introducing a separation of the new physics effects that will be useful for our next analysis. For the moment we write the final expression for the asymmetry unpolarized quantities that contain them as an input. We shall return on this point at the end of the paper. For the experimental error would not produce any consequence in the theoretical formulae for been given and computed in Ref.[11], where it has also been shown that the related expression for production of the five light quarks \( A_{LR,5} = \sigma_{LR,5}/\sigma_5 \). To derive its expression is straightforward once the prescriptions of our approach have been made clear. In practice, the only new experimental quantities that will enter in the theoretical formulae will be the charge asymmetry on \( Z \) resonance and the overall \( Z \) width \( \Gamma_5 \). The various relevant expressions have all been given and computed in Ref.[11], where it has also been shown that the related experimental error would not produce any consequence in the theoretical formulae for unpolarized quantities that contain them as an input. We shall return on this point at the end of the paper. For the moment we write the final expression for the asymmetry introducing a separation of the new physics effects that will be useful for our next analysis. More precisely, we define systematically, for any model of new physics and final state \( f \):
\[ \tilde{\Delta}^{(f)}(q^2) = \tilde{\Delta}(q^2) + \delta \tilde{\Delta}^{(f)}(q^2) \]  \hspace{1cm} (30)

\[ R^{(f)}(q^2) = R(q^2) + \delta R^{(f)}(q^2) \]  \hspace{1cm} (31)

\[ V_{\gamma Z}^{(f)}(q^2) = V(q^2) + \delta V_{\gamma Z}^{(f)}(q^2) \]  \hspace{1cm} (32)

\[ V_{Z \gamma}^{(f)}(q^2) = V(q^2) + \delta V_{Z \gamma}^{(f)}(q^2) \]  \hspace{1cm} (33)

where the first bracket contains the "universal" (without index) effects, i.e. those that would be exactly the same for final leptons or quarks.

Using the previous definitions, it is relatively easy to derive the expression of \( A_{LR,5} \) for models of new physics that are of universal type. Working in the usual spirit of only retaining the important contributions we would obtain the following formula

\[ A^{(1)}_{LR,5}(q^2) = \tilde{A}_{LR,5}(q^2)\{1 + a_5(q^2) [\tilde{\Delta}(q^2) + R(q^2)] + [b_5(q^2) + c_5(q^2)] V(q^2)\} \]  \hspace{1cm} (34)

where

\[ \tilde{A}_{LR,5}(q^2) = \frac{C_{LR,5}(q^2)}{C_5(q^2)} \]  \hspace{1cm} (35)

\[ a_5(q^2) = C_{LR,5}^\gamma - 2C_5^\gamma - C_5^Z = -2C_{LR,5}^ZZ - C_{LR,5}^\gamma Z + 2C_5^ZZ + C_5^Z \]  \hspace{1cm} (36)

\[ b_5(q^2) + c_5(q^2) = -4s_l c_l \{[p_{LR,5}C_{LR,5}^ZZ + p'_{LR,5}C_{LR,5}^\gamma Z] - [p_5C_5^ZZ + p_5C_{LR,5}^\gamma Z]\} \]  \hspace{1cm} (37)

and

\[ C_{LR,5} = N_{LR,5}^ZZ + N_{LR,5}^\gamma Z \]  \hspace{1cm} (38)

\[ N_{LR,5}^ZZ = \frac{2\bar{v}_l}{1 + \bar{v}_l^2} \left( \frac{3\Gamma_{l}}{M_Z^2} \right) \left( \frac{3\Gamma_{l}}{M_Z^2} \right) \]  \hspace{1cm} (39)

\[ N_{LR,5}^\gamma Z = \frac{2\alpha}{3(1 + \bar{v}_l^2)^{1/2}M_Z^2} \sum q^2 (q^2 - M_Z^2)^2 + M_Z^2 \Gamma_Z^2 \]  \hspace{1cm} (40)

\[ C_{LR,5}^ZZ = N_{LR,5}^ZZ/C_{LR,5} \hspace{1cm} C_{LR,5}^\gamma Z = N_{LR,5}^\gamma Z/C_{LR,5} \hspace{1cm} C_5 = 1 + N_5^ZZ + N_5^\gamma Z \]  \hspace{1cm} (41)

\[ N_{5}^ZZ = \frac{9}{33} \left( \frac{3\Gamma_{l}}{M_Z^2} \right) \left( \frac{3\Gamma_{l}}{M_Z^2} \right) \left( \frac{3\Gamma_{l}}{M_Z^2} \right) \frac{q^4}{(q^2 - M_Z^2)^2 + M_Z^2 \Gamma_Z^2} \]  \hspace{1cm} (42)

\[ N_{5}^\gamma Z = \frac{2}{11} \left( \frac{\bar{v}_l}{1 + \bar{v}_l^2} \right)^{1/2} \left( \frac{3\Gamma_{l}}{M_Z^2} \right) \left( \frac{3\Gamma_{l}}{M_Z^2} \right) \frac{q^4}{(q^2 - M_Z^2)^2 + M_Z^2 \Gamma_Z^2} \]  \hspace{1cm} (43)
\[ \Sigma_5 = \sum_q \frac{3|Q_q|\tilde{v}_q}{(1 + \tilde{v}_q^2)^{1/2}} \left( \frac{3N_q \Gamma_q}{M_Z} \right)^{1/2} \] (44)

\[ C_{5}^{\gamma\gamma} = \frac{1}{C_5} \quad C_{5}^{\gamma Z} = \frac{N_5^{\gamma Z}}{C_5} \quad C_{5}^{Z Z} = \frac{N_5^{Z Z}}{C_5} \] (45)

\[ p_5 = \frac{\tilde{v}_l}{1 + \tilde{v}_l^2} + \sum_q \left( \frac{\tilde{v}_q}{1 + \tilde{v}_q^2} \right) \frac{|Q_q| \Gamma_q}{\Gamma_5} \quad p'_5 = \frac{1}{\tilde{v}_l} + p'_{LR,5} \] (46)

\[ p'_{LR,5} = \sum_q \left( \frac{3|Q_q|^2}{\Sigma_5 (1 + \tilde{v}_q^2)^{1/2}} \right) \left( \frac{3N_q \Gamma_q}{M_Z} \right)^{1/2} \quad p_{LR,5} = \frac{1}{\tilde{v}_l} + \sum_q \left( \frac{\tilde{v}_q}{1 + \tilde{v}_q^2} \right) \frac{2|Q_q| \Gamma_q}{3\Gamma_5} \] (47)

The coefficient \( \bar{A}_{LR,5}(q^2) \), in this particular notation, contains both the leading ("effective" Born) terms and the one-loop corrections \( \delta \bar{A}_{LR,5}^{SM} \) of the pure SM. The latter ones will not be, in general, of universal type, since they involve vertices and boxes. Neglecting their numerical value for a first estimate of the leading term gives us the expected large \( q^2 \) value of the asymmetry, that is approximately \( \bar{A}_{LR,5}(q^2) \simeq 0.50 \). For what concerns the remaining coefficients, one easily sees that, once again, that of \( [\Delta \alpha + R] \) is more than one order of magnitude smaller than that of \( V \). The latter one, in turn, comes mostly from the universal component of \( V_{\gamma Z} \) reproducing the situation that we have already met in the two previous examples.

This recurrent feature of "\( V_{\gamma Z} \)" dominance of the one loop effects of new physics survives, in the last considered asymmetry, even in the most general case of non universal type of effects, as one can see if one writes the full expression that generalizes eq.\( (34) \) to this case. This can be done in a straightforward way, and leads to the rather lengthy expression that we write here for completeness:

\[
A_{LR,5}^{(1)}(q^2) = \bar{A}_{LR,5}(q^2) \{ 1 + a_5(q^2)[\Delta \alpha(q^2) + R(q^2)] + [b_5(q^2) + c_5(q^2)]V(q^2) \}
+ \sum_q \delta \bar{A}_{\gamma \gamma}(q^2) \left[ (C_{LR,5}^{\gamma \gamma}(q^2) - C_5^{\gamma \gamma}(q^2)) \frac{3|Q_q|\tilde{v}_q}{\Sigma_5 (1 + \tilde{v}_q^2)^{1/2}} \left( \frac{3N_q \Gamma_q}{M_Z} \right)^{1/2} - C_{5}^{\gamma \gamma}(q^2) \frac{18}{11} (Q_q)^2 \right]
+ \sum_q \delta R_{\gamma}(q^2) \left[ 2(C_{LR,5}^{\gamma Z}(q^2) - C_5^{\gamma Z}(q^2)) \frac{\Gamma_q}{\Gamma_5} + (C_5^{\gamma Z}(q^2) - C_{LR,5}^{\gamma Z}(q^2)) \frac{3|Q_q|\tilde{v}_q}{\Sigma_5 (1 + \tilde{v}_q^2)^{1/2}} \left( \frac{3N_q \Gamma_q}{M_Z} \right)^{1/2} \right]
+ \sum_q \delta V_{\gamma Z}(q^2) \left[ \left( \frac{8s_{\gamma}c_1}{1 + \tilde{v}_q^2} \right) C_{LR,5}^{\gamma Z}(q^2) \frac{\Gamma_q}{\Gamma_5} - \frac{4s_{\gamma}c_1}{\tilde{v}_q} \left( C_{LR,5}^{\gamma Z}(q^2) \frac{\Gamma_q}{\Gamma_5} \right)^2 - C_5^{\gamma Z} \frac{3|Q_q|\tilde{v}_q}{\Sigma_5 (1 + \tilde{v}_q^2)^{1/2}} \left( \frac{3N_q \Gamma_q}{M_Z} \right)^{1/2} \right]
+ \sum_q \delta V_{\gamma Z}(q^2) \left[ \left( \frac{8s_{\gamma}c_1}{1 + \tilde{v}_q^2} \right) (C_5^{\gamma Z}(q^2) - C_{LR,5}^{\gamma Z}(q^2)) \frac{\Gamma_q}{\Gamma_5} \right]
+ \sum_q \delta V_{\gamma Z}(q^2) \left[ \left( \frac{8s_{\gamma}c_1}{1 + \tilde{v}_q^2} \right) (C_{LR,5}^{\gamma Z}(q^2) - C_5^{\gamma Z}(q^2)) \frac{\Gamma_q}{\Gamma_5} \right] \] (48)

and, this time, the one-loop corrections contain both the SM and the new physics effects. Note that the \( [\Delta \alpha(q^2) + R(q^2)] \) combination only appears for the universal term. Leaving aside a more quantitative discussion in this non-universal case, we only remark that, as we
said previously, the weight of both the universal and the non-universal $V_{\gamma Z}^{lf}$ components remain essentially enhanced by the typical $1/\bar{v}_l$ effect, that remains in conclusion the relevant feature of all the considered longitudinal polarization asymmetries.

This characteristic feature should be compared now with those of other specific unpolarized observables. We have done this for the following relevant quantities, exploiting their theoretical expressions in our approach that can be found in refs.\[10\],\[11\]:

I) $A_{FB,\mu}$, the muon forward-backward asymmetry. Here the size of the coefficient of the sum $[\Delta \alpha + R]$ that still appears as a unique block is approximately three times bigger than that of $V$.

II) $\sigma_\mu$, the muon cross-section. Here the dominant effect is by far (one order of magnitude) concentrated in the correction $\Delta \alpha$ (that now is no more related to $R$ as in the previous asymmetries).

III) $\sigma_5$, the five light quark cross section. For the case of universal effects, the coefficients of all the three form factors $\Delta \alpha$, $R$ and $V$ are now roughly equal (this remains qualitatively true for general non universal effects).

IV) $\sigma_b$, the $b\bar{b}$ cross section. Here, the leading coefficients of nearly equal size are those of $R$ and $V$.

This short analysis shows that, indeed, longitudinal polarization asymmetries are much more sensitive to one specific one-loop effect $\simeq V_{\gamma Z}$ and therefore to all those models that contribute this quantity in a sensible way. One the contrary, unpolarized leptonic observables are a better place for looking at effects generated by either the combination $[\Delta \alpha + R]$ (e.g. $A_{FB,\mu}$) or by the separate quantity $\Delta \alpha$ (e.g. $\sigma_\mu$). In unpolarized hadronic quantities, the three form factors $\Delta \alpha$, $R$ and $V$ all appear with coefficients of similar size.

We still have to discuss three specific points. The first one is that, as previously stressed, it is the combination $[\Delta \alpha + R]$ that appears systematically in ratios of cross sections. This can be understood from the general ($\gamma$, $Z$) structure if we write the general expression of any cross section in the following way

$$\sigma_1^{lf} \equiv c_{\gamma}(1 + 2\tilde{\Delta}^{lf} \alpha) + c_{ZZ}^{lf}(1 - 2R^{lf}) + c_{\gamma Z}^{lf}(1 + \Delta^{lf} \alpha - R^{lf}) + V \text{ terms} \quad (49)$$

defining $c_1 \equiv c_{\gamma} + c_{ZZ} + c_{\gamma Z}$, one can write

$$\sigma_1^{lf} \equiv c_1[1 + \tilde{\Delta}^{lf} \alpha - R^{lf} + \frac{c_{\gamma}^{ol} - c_{ZZ}^{ol}}{c_1}(\tilde{\Delta}^{lf} \alpha + R^{lf})] + V \text{ terms} \quad (50)$$

Keeping only first order terms in $\tilde{\Delta}^{lf} \alpha$, $R^{lf}$ and $V$, the ratio of two such cross sections $\sigma_1^{lf}$ and $\sigma_2^{lf}$, is given by:

$$\frac{\sigma_1^{lf}}{\sigma_2^{lf}} = \frac{c_1}{c_2}[1 + (\frac{c_{\gamma}^{ol} - c_{ZZ}^{ol}}{c_1} - \frac{c_{\gamma}^{ol} - c_{ZZ}^{ol}}{c_2})(\tilde{\Delta}^{lf} \alpha + R^{lf})] + V \text{ terms} \quad (51)$$

in which only the combination $[\Delta^{lf} \alpha + R^{lf}]$ appears. Note that this property in general disappears if one considers ratios of sums of different flavors $(\sum_f \sigma_1^{lf})/(\sum_f \sigma_2^{lf})$, as we have seen in the case of $A_{LR,5}^{(i)}$, eq.(48).
The second point is the statement that, in order to exploit the properties of \( A_{LR,f} \), the contribution of the model of new physics to \( V \) must not be accidentally depressed with respect to that of \( \tilde{\Delta} \alpha + R \). Although we cannot prove this fact in general, we shall provide now in the next section a specific example of a model where this is actually the case, and for which the role of \( A_{LR,f} \) will consequently be very useful.

The final point is that of whether the bargain introduced in our approach by the replacement of \( G_\mu \) with \( Z \)-peak quantities does not generate a dangerous theoretical input error (in the case of unpolarized observables, this was shown not to be the case for future \( e^+e^- \) colliders at their realistically expected accuracy in refs. [10], [11]). Let us start with the leptonic asymmetry eq.(17). In our approach, its new theoretical expression at the "effective" Born level is the first member on the r.h.s. of eq.(17). This contains the \( Z \) leptonic width \( \Gamma_l \) and \( A_{LR} \) measured on top of \( Z \) resonance. With the available errors on these quantities, one computes a theoretical error in eq.(17) of approximately 0.0018 which is mostly coming from \( A_{LR} \). Assuming an (optimistic) experimental error on \( A_{LR} \) at a 500 GeV NLC [14] of 0.007 (purely statistical) one sees that the induced theoretical error is completely negligible. This statement will also be made more accurate by future improvements on the measurement of \( A_{LR} \) at SLD [7], [16].

In the case of eq.(18), one easily sees that the major source of error in the expression of the "effective" Born terms comes from the quantity \( \tilde{v}_b \sqrt{1+\tilde{v}_b^2} \). To compute this error, we have used the definition eq.(12), from which we obtain:

\[
\delta \tilde{v}_b = \frac{(1 + \tilde{v}_b^2)^2}{2(1 - \tilde{v}_b^2)} \delta A_b
\]

Using the experimental LEP+SLD value [7], [17]

\[
A_b = 0.867 \pm 0.022
\]

we derive \( \delta \tilde{v}_b \simeq 0.04 \). A standard calculation then gives for the theoretical input error:

\[
\delta^{(th)} \tilde{A}_{LR,b}(q^2) \simeq 0.02
\]

Note that this numerical result is directly proportional to the experimental error on \( A_b \), and will be correspondingly reduced by future improved measurements of this quantity. This final error should be compared to the expected experimental precision on \( A_{LR,b} \). Although a detailed study does not exist yet to our knowledge, we can reasonably foresee a picture for \( b\bar{b} \) detection similar to that found for previous LEP2 studies [18], that would lead to an overall error of at least a few percent, sufficiently larger than our theoretical input error.

To conclude, we have considered the case of \( A_{LR,5} \). This case can be treated in a reasonably simple way since in the theoretical expression of the leading term \( \tilde{A}_{LR,5}(q^2) \) the only relevant theoretical uncertainty affects the "\( \gamma Z \)" component of the numerator (for the denominator, a previous discussion given in ref. [11] has shown that the main error is coming from \( \Gamma_h \), the \( Z \) hadronic width measured on \( Z \) peak, and is completely negligible i.e. much smaller than a relative one percent). The \( \gamma Z \) component contains the
Z peak quantities $\Gamma_{u,d,s,c,b}$ and the related quantities $A_{u,d,s,c,b}$ defined by a generalization of eq.(12). In fact, no experimental information is available on the $(u, d, s)$ variables. A reasonable attitude seems to us to be that of assuming a universality property, i.e.

$$\Gamma_u = \Gamma_c \quad \Gamma_d = \Gamma_s = \Gamma_b(m_t = 0)$$

and to derive $\Gamma_b(m_t = 0)$ from its knowns experimental value where the theoretical top quark contribution has been subtracted. Analogously, we shall assume that $A_u = A_c$ and $A_d = A_s = A_b(m_t = 0)$ and for the latter quantity we have again subtracted the known (and relatively small) top quark contribution. With these assumptions, one easily sees that the major theoretical error is coming from that of $\tilde{v}_c$ and $\tilde{v}_b$ (the induced error by the widths is much smaller). Using the experimental SLD results for $A_{b,c}$ then leads to an error of $A_{LR,5}$:

$$\delta^{th} A_{LR,5}(q^2) \simeq 0.02$$

This is not a very comfortable result, since one would expect an experimental error on $A_{LR,5}$ at NLC not far from the purely statistical one, that is around one percent. In order to reduce the theoretical error of our input to such values, an extra effort from SLD that reduces to the one percent level the error on $A_b$ and to the three percent level that on $A_c$ would be requested. Such a desirable goal seems to be reachable in future SLD measurements [6]. In the rest of this paper, we shall illustrate as an application the consequences of having been able to reduce the overall error on $A_{LR,5}$ to the one percent level. This will be done immediately in the next Section 3.

### 3 A model with anomalous gauge couplings

To illustrate the previous considerations with a concrete example, we shall now consider the case of a model where anomalous gauge couplings (AGC) are present. To be more precise, we shall discuss the consequences of our approach for the study of a model proposed by Hagiwara et al [19], to whose paper we defer for a full discussion of various theoretical aspects. Briefly, the model assumes that physics below a scale $\Lambda$ of supposed order 1 TeV can be described by an “effective” Lagrangian obtained by adding to the conventional SM component an extra $SU(2) \times U(1)$ invariant, C and CP conserving, dimension six piece. The latter contains, a priori, eleven parameters of which four enter at the one loop level for production of two final massless fermions from electron-positron annihilation. In the notation of ref.[19] these are called $f_{DW}$, $f_{DB}$, $f_{BW}$ and $f_{\Phi,1}$. In a conventional treatment that does not use our $Z$-peak representation they would all contribute this process at one loop. The treatment of this model in our approach turns out to be particularly convenient. As it has already been shown in ref.[10], the number of effective parameters that appear in the subtracted form factors is reduced to two ($f_{DW}$ and $f_{DB}$) since $f_{BW}$ and $f_{\Phi,1}$ are fully reabsorbed in the used input parameters $\Gamma_l$ and $s_l^2(M^2_Z)$. Another welcome feature of this model is that its effects for massless fermions are of universal type, so that the same two parameters will enter both leptonic and quark observables. This allows to
determine informations on bounds on these parameters in a greatly simplified way, using several measurements of different experimental quantities. This was done in a very recent paper [20] where the bounds that would be obtained from negative searches both at LEP2 and at NLC without polarization were derived. In Fig.1 the results of that investigation are presented showing the region within which the two parameters \((f_{DW}, f_{DB})\) would be constrained by negative searches in the unpolarized case. Numerically, we would find in this case:

\[
\Delta f_{DB} = \pm 0.16 \quad (57)
\]

\[
\Delta f_{DW} = \pm 0.025 \quad (58)
\]

In practice, the determination of such bounds in Fig.1 is mostly provided by two quantities i.e. the muon cross section and the five light hadron production cross section \(\sigma_5\) (the forward-backward asymmetry \(A_{FB,\mu}\) plays a negligible role because of a weaker sensitivity). Their expression in the considered model are provided in ref.[20] and are fixed by the (AGC) content of the three form factors \(\tilde{\Delta}\alpha\), \(R\) and \(V\), that read respectively:

\[
\tilde{\Delta}^{(AGC)}(q^2) = -q^2 \frac{2 \alpha(0)}{\Lambda^2} (f_{DW} + f_{DB}) \quad (59)
\]

\[
R^{(AGC)}(q^2) = (q^2 - M_Z^2) \frac{2 \alpha(0)}{s_W c_W \Lambda^2} (f_{DW} c_l^4 + f_{DB} s_l^4) \quad (60)
\]

\[
V^{(AGC)}(q^2) = (q^2 - M_Z^2) \frac{2 \alpha(0)}{s_W c_W \Lambda^2} (f_{DW} c_l^2 - f_{DB} s_l^2) \quad (61)
\]

We have now added to the previous unpolarized information that derivable from longitudinal polarization asymmetries. To avoid problems related to \(b\) quark identification and to stick more rigorously to the massless quark configuration, we have only considered the leptonic and the full light hadronic asymmetry (where the weight of the \(b\) contribution is sufficiently depressed). For the latter ones we have assumed, following our previous discussion (and an optimistic attitude), an experimental error \(\delta A_{LR,l} = \pm 0.007\) and \(\delta A_{LR,5} = \pm 0.01\). This is based on an integrated luminosity of 20 \(fb^{-1}\) leading at \(\sqrt{q^2} = 500 GeV\) to about \(5 \times 10^4\) hadronic events and \(1.7 \times 10^4\) (muon + tau events). To give a hint of how the "V enhancement" mechanism works, we write the two corresponding theoretical expressions in the chosen configuration \(\sqrt{q^2} = 500 GeV\), that numerically read:

from \(A_{LR,5}\)

\[
\left| \frac{32 \pi \alpha(0) M_Z^2}{\Lambda^2} [-53.36 f_{DW} + 14.43 f_{DB}] \right| = \left| \frac{\delta A_{LR,5}}{A_{LR,5}} \right| \gtrsim 0.02 \quad (62)
\]

and from \(A_{LR,l}\)

\[
\left| \frac{32 \pi \alpha(0) M_Z^2}{\Lambda^2} [-342.65 f_{DW} + 92.55 f_{DB}] \right| = \left| \frac{\delta A_{LR,l}}{A_{LR,l}} \right| \gtrsim 0.1 \quad (63)
\]
(For $\Lambda = 1 \, TeV$, the coefficient $\frac{32\pi\alpha(0)M_Z^2}{\Lambda^2}$ is equal to 0.0061).

In eqs.(62) and (63) the last numbers on the r.h.s. represent the visibility threshold for the effect. Note that both equations involve the same type of combination of $f_{DW}$ and $f_{DB}$ couplings. With the expected accuracies, eq.(63) due to $A_{LR,4}$ is slightly more stringent than eq.(62) due to $A_{LR,5}$. This is fortunate because of the uncertainty on the final accuracy that will be reachable on $A_{LR,5}$. In the following numerical analysis we shall combine quadratically the informations coming from these two constraints and this reduces somewhat the importance of $A_{LR,5}$.

These expressions should be compared with those provided by the unpolarized observables. Taking for simplicity the two most sensitive quantities i.e. the muon and the hadron cross sections, the corresponding equations would be:

from $\sigma_\mu$

$$\left| \frac{32\pi\alpha(0)M_Z^2}{\Lambda^2}[-22.02f_{DW} - 13.07f_{DB}] \right| = \left| \frac{\delta\sigma_\mu}{\sigma_\mu} \right| \gtrsim 0.01$$  \hspace{1cm} (64)

and from $\sigma_5$

$$\left| \frac{32\pi\alpha(0)M_Z^2}{\Lambda^2}[-49.53f_{DW} - 5.45f_{DB}] \right| = \left| \frac{\delta\sigma_5}{\sigma_5} \right| \gtrsim 0.005$$  \hspace{1cm} (65)

Comparing eqs.(64)(65) with eqs.(62),(63) one actually sees that the combination of $f_{DW}$, $f_{DB}$ that appear in the two sets are almost orthogonal. This corresponds indeed, as we discussed in Section 2, to the fact that different form factors are selected in the two cases.

From a practical point of view, the additional improvements for future negative bounds derivable from the addition of the two extra asymmetries is shown in Fig.1. As one sees, the final limits would be:

$$\Delta f_{DB} = \pm 0.08$$  \hspace{1cm} (66)

$$\Delta f_{DW} = \pm 0.014$$  \hspace{1cm} (67)

In other words, the additional constraint provided by longitudinal polarization would lead, in this example, to an improvement in the bounds equal to, roughly, a factor of two.

### 4 Conclusions

We have shown in this paper that longitudinal polarization asymmetries of electron-positron annihilation into pairs of light fermion-antifermion at energies larger than $M_Z$ exhibit interesting theoretical features that might be useful for detection of a certain type of virtual effects of new physics at one loop, and that are due to a special enhancement of the subtracted $V$ form factor. This feature is analogous to that found on top of $Z$ resonance, showing that $A_{LR}$ continues to be a relevant observable even far from that privileged kinematical configuration.
We have presented in this paper only one concrete example of how this enhancement mechanism works, for the special case of one model of universal type. Other similar cases could be examined. For instance, general models of technicolour type (already qualitatively considered in ref.\[10\]) would probably benefit from a more detailed numerical calculation. This will be done in a separate work. Also, the more complicated case of models of non universal type would deserve consideration. An interesting case would be that of general supersymmetric models. Here the virtual effects are usually depressed on $Z$ resonance. Away from $Z$ resonance, there might be, though, unconventional effects of non universal type (we have in mind e.g. boxes, that are kinematically depressed on $Z$ peak but resuscitate when $(q^2 - M_Z^2)$ is sufficiently large). These would enter in our subtracted form factors at large energies since they would not be reabsorbed, by definition, in the $Z$ peak observables that are the new inputs of our procedure. The study of this possibility is by now in progress.

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Fig. 1 Constraints on AGC couplings from $e^+e^- \rightarrow f \bar{f}$ processes at 500 GeV. Without polarization: $\sigma_\mu$ (□), $\sigma_5$ (+), $\sigma_b$ (×). With polarization: $A_{LR}$ (◇) for which the band is obtained by combining quadratically the informations coming from $A_{LR,5}$ and $A_{LR,l}$. 
Fig. 1