Determination of hadronic partial widths for scalar-isoscalar resonances \( f_0(980), f_0(1300), f_0(1500), f_0(1750) \) and the broad state \( f_0(1530 \pm 90_{-250}) \)

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In the article of V.V. Anisovich et al., Yad. Fiz. 63, 1489 (2000), the \( K \)-matrix solutions for the wave \( I^{PC} = 00^{++} \) were obtained in the mass region 450–1900 MeV where four resonances \( f_0(980), f_0(1300), f_0(1500), f_0(1750) \) and the broad state \( f_0(1530 \pm 90_{-250}) \) are located. Based on these solutions, we determine partial widths for scalar-isoscalar states decaying into the channels \( \pi\pi, K\bar{K}, \eta\eta, \eta\eta', \pi\pi\pi\pi \) and corresponding decay couplings.

I. INTRODUCTION

In [1] the combined \( K \)-matrix analysis was performed for the meson partial waves \( I^{PC} = 00^{++}, 10^{++}, 02^{++}, 12^{++} \) on the basis of GAMS data on \( \pi^- p \to \pi^0\pi^0 n, \eta\eta, \eta\eta' n \); BNL data on \( \pi^- p \to K\bar{K} n \) and Crystal Barrel data on \( pp(\text{at rest}) \to \pi^0\pi^0\pi^0, \pi^0\eta, \pi^0\pi\eta \). The positions of amplitude poles (physical resonances) were determined together with the positions of the \( K \)-matrix poles (bare states) and bare-state couplings to the two-meson channels. The nonet classification of the bare \( q\bar{q} \) states was suggested, and the possibilities for the location of the lightest scalar glueball in the mass region 1200–1700 MeV were discussed.

The \( K \)-matrix technique has an advantage of taking account of the unitarity condition constraints and, in this way, of a correct incorporation of threshold singularities into the scattering amplitude. Since the search for resonances is always related to the investigation of analytical structure of the amplitude in the complex-mass plane by using data at real masses, it is important to perform analytical continuation of the amplitude into the lower half-plane of the complex mass, with correctly taken singularities on the real axis.

However, the \( K \)-matrix amplitude does not include resonance parameters in an explicit form, so additional calculations are needed to determine masses and couplings of real resonances. In paper [2] the pole positions have been found for the considered partial wave amplitudes (i.e. masses and total widths of resonances were found), but more complicated calculation of couplings has not been done yet. The decay coupling constants are to be determined as residues of the pole singularities of the multi-channel amplitude. In the present paper we overcome this deficiency and calculate coupling constants to the channels \( \pi\pi, K\bar{K}, \eta\eta, \eta\eta' \) and \( \pi\pi\pi\pi \) for the resonances \( f_0(980), f_0(1300), f_0(1500), f_0(1750), f_0(1530 \pm 90_{-250}) \). This procedure provided us with partial widths for the decays of these resonances. The choice of the scalar-isoscalar sector for primary study follows from the interest in pursuing the destiny of the lightest scalar glueball after its mixing with neighbouring states; one needs a knowledge of the couplings \( f_0 \to \pi\pi, K\bar{K}, \eta\eta, \eta\eta' \) for all resonances over the mass region 1000–1800 MeV.

The paper is organized as follows.

In Section 2 the coupling constants are presented for the decays \( f_0 \to \pi\pi, K\bar{K}, \eta\eta, \eta\eta', \pi\pi\pi\pi \), and partial widths for the mesons \( f_0(1300), f_0(1500), f_0(1750) \) and \( f_0(1530 \pm 90_{-250}) \) are determined.

In Section 3 the resonance \( f_0(980) \) is considered in detail: the results of the analysis [1] tell us that standard formulae for the description of resonances, such as Breit-Wigner or Flatté ones, in the case of \( f_0(980) \) are unable to give simultaneously the values of the decay coupling constants and the position of the amplitude pole. We suggest an alternative form of resonance amplitude for \( f_0(980) \) in which the important role is played by the prompt transition \( \pi\pi \to K\bar{K} \). In this Section another low-mass state, namely, the \( \sigma \)-meson, is also discussed.

The results are summarized in the Conclusion.

II. DECAY COUPLINGS AND PARTIAL DECAY WIDTHS

We determine the coupling constants and partial decay widths using the following procedure. The \( 00^{++} \)-amplitude for the transition \( a \to b \),

\[
A_{a \to b}(s) , \quad a, b = \pi\pi, K\bar{K}, \eta\eta, \eta\eta', \pi\pi\pi\pi
\]

is considered as a function of the invariant energy squared \( s \) in the complex-\( s \) plane near the pole related to the resonance \( n \). In the vicinity of the pole the amplitude reads:

\[
A_{a \to b}(s) = \frac{g_a^{(n)} g_b^{(n)}}{\mu_n^2 - s} e^{i\theta_{ab}^{(n)}} + B_{ab} ,
\]
Here $\mu_n$ is the resonance complex mass $\mu_n = M_n - i\Gamma_n/2$; $g_a^{(n)}$ and $g_b^{(n)}$ are the couplings for the transitions $f_0 \to a$ and $f_0 \to b$. The factor $\exp(i\theta_{ab}^{(n)})$ is due to a background contribution which can be the non-resonance terms or tails of neighbouring resonances. We also write down in (3) the non-pole background term $B_{ab}$.

The partial width for the decay $f_0 \to a$ is determined as a product of the coupling constant squared, $g_a^{(n)}$, and phase space, $\rho_a(s)$, averaged over resonance density:

$$\Gamma_a(n) = C_n \int_{s > s_{th}} ds \frac{g_a^{(n)^2} \rho_a(s)}{\pi (Re \mu_n^2 - s)^2 + (Im \mu_n^2)^2}.$$  \hspace{1cm} (3)

Following (3), we write down the phase space factor as follows:

$$\rho_a(s) = \frac{2k_a}{\sqrt{s}},$$  \hspace{1cm} (4)

where $k_a$ is the relative momentum of mesons in the decay channel (for example, for the $\pi\pi$ channel $\rho_{\pi\pi}(s) = \sqrt{(s - 4m_\pi^2)/s}$). For the $\pi\pi\pi$ channel the phase space factor was chosen in (3) to be the same as for the two-$\rho$-meson state at $s < 1$ GeV$^2$ or be equal to 1 at $s \geq 1$ GeV$^2$. The integration over $s$ in (3) is carried out in the region above the $a$-channel threshold, $s > s_{th}$ (for the $\pi\pi$ channel it is $s > 4m_\pi^2$). The resonance density factor, $(Re \mu_n^2 - s)^2 + (Im \mu_n^2)^2)^{-1}$ guarantees rapid convergence of the integral (3). The normalization constant $C_n$ is determined by the requirement that the sum of all hadronic partial widths is equal to the total width of the resonance:

$$\Gamma(n) = \sum_a \Gamma_a(n).$$  \hspace{1cm} (5)

In paper (3) three solutions for the wave $IJ^{PC} = 00^{++}$ have been found; they are labelled as I, II-1 and II-2 (see Tables 4 and 5 in (3)). In practice Solutions II-1 and II-2 give the same physical parameters of resonances, though they differ from parameters found for the $K$-matrix elements. In particular, in Solution II-2 the state $f_0^{bare}(1600)$ may be identified as a gluonium, for the decay couplings satisfy all the requirements inherent to glueball state; in Solution II-1 such a state is $f_0^{bare}(1230)$. For Solution I, the same bare state, $f_0^{bare}(1230)$, should be considered as a gluonium.

In Table 1 we show the values of partial widths for the resonances $f_0(980)$, $f_0(1300)$, $f_0(1500)$, $f_0(1750)$ and broad state $f_0(1530 \pm 90)$, $f_0(1570 \pm 90)$, $f_0(1590 \pm 90)$, $f_0(1630 \pm 90)$, $f_0(1750 \pm 90)$ are calculated within standard formulae for the Breit-Wigner resonances (3), (3) and (3). The resonance $f_0(980)$ being located near the strong $K\bar{K}$ threshold needs a special consideration that is presented below.

The decay coupling constants squared, $g_a^{(n)^2}$, are shown in Table 2 for $a = \pi\pi, K\bar{K}, \eta\eta, \eta'\eta'$. The couplings are determined with the normalization of the amplitude used in (4); for example, we write the $\pi\pi$ scattering amplitude $A_{\pi\pi\to\pi\pi}(s) = \eta_0^\pi \exp(2i\delta_0^\pi) - 1)/2i\rho_{\pi\pi}(s)$, where $\eta_0^\pi$ and $\delta_0^\pi$ are the inelasticity parameter and phase shift for the $0^{++} \pi\pi$-wave. The coupling constants $g_a^{(n)}$ are found by calculating the residues of the amplitudes $\pi\pi \to \pi\pi, K\bar{K}, \eta\eta, \eta'\eta'$. Also we check the factorization property for the pole terms by calculating residues for other reactions, such as $K\bar{K} \to K\bar{K}$.

The position of poles in the complex-$M$ plane ($M \equiv \sqrt{s}$) is illustrated by Fig. 1. The complex-$M$ area, where the $K$-matrix fit (4) may reliably reproduce analyticity of the amplitude, is inside a semi-circle depicted by dashed line. The poles which are a subject of the $K$-matrix analysis and correspond to $f_0(980)$, $f_0(1300)$, $f_0(1500)$, $f_0(1750)$, $f_0(1530 \pm 90)$ are located on the 3rd, 4th, 5th and 6th sheets of the complex-$M$ plane. The resonance $f_0(980)$ is located near the strong $K\bar{K}$ threshold, therefore two poles are related to $f_0(980)$: the nearest one is on the 3rd sheet ($M \approx 1014 - i39$ MeV) and a remote pole on the 4th sheet ($M \approx 936 - i238$ MeV). Coupling constants for $f_0(980)$ are determined as residues of the nearest pole (on the 3rd sheet). The $\eta'\eta'$ threshold is weak for $f_0(1500)$, and because of that positions of poles on the 5th and 6th sheets are practically the same (note that couplings related to these poles nearly coincide).

The $K$-matrix fit (4) has been carried out in a broad mass interval, 450 MeV $\leq M \leq 1900$ MeV. This very fact allows us to believe that we deal with successfully reconstructed analytical amplitude which goes rather deeply into lower half-plane $M$, and this area is restricted by dashed line in Fig. 1. It is of crucial importance that the pole of the broad state $f_0(1530 \pm 90)$ is inside this area, for this broad state plays a key role in a mixing of $q\bar{q}$-mesons with the lightest glueball, see (4) for details.
III. THE LOW-MASS MESONS: $f_0(980)$ AND $\sigma$

The two low-mass mesons, $f_0(980)$ and $\sigma$-meson, need special consideration and comments.

The analysis \cite{1} shows us that $f_0(980)$ cannot be described either by standard Breit-Wigner formula or its modification for the case of the nearly located strong $K\bar{K}$ threshold, that is, Flatté’s formula \cite{7}. Here we suggest another resonance formula for $f_0(980)$ which agrees with the results of \cite{1}.

In the compilation of Particle Data Group \cite{3}, the $\sigma$-meson is denoted as $f_0(400 - 1200)$ that reflects a cumulative result obtained in a number of papers where the mass of $\sigma$ was found in this region or even higher. However, the analysis \cite{1} definitely demonstrates the absence of poles in the 00++-amplitude at $600 \leq Re\, M \leq 1200$ MeV, with an exception of poles for $f_0(980)$ – we will discuss the situation with $\sigma$-meson in this Section later on.

A. Description of $f_0(980)$

For $f_0(980)$, the $K$-matrix fit \cite{1} gives us the position of the pole and coupling constant values, see Tables 1 and 2. These parameters are sufficient to reconstruct the Breit-Wigner resonance amplitude. However, in case of $f_0(980)$ there exists a strong $K\bar{K}$ threshold near the pole, so the resonance term in the amplitude \cite{1} should be suggested not as the Breit-Wigner pole but in a more complicated form. For the $\pi\pi \to \pi\pi$ and $KK \to KK$ transitions near $f_0(980)$, the following resonance amplitudes can be written instead of the Breit-Wigner pole term $R_{f_0(980)}^{(ab)} = g^{(a)}_n g^{(b)}_n / (\mu^2_n - s)$ entering equation (2):

$$R_{f_0(980)}^{(ab)} = \frac{G^2 + i \frac{\sqrt{s - 4 m^2_{K\bar{K}}}}{m_0} F}{D}, \quad R_{f_0(980)}^{(KK, KK)} = \frac{(G_{K\bar{K}}^2 + i F)}{D},$$

(6)

where

$$F = 2 G G_{K\bar{K}} f + f^2 (m^2_0 - s), \quad D = m^2_0 - s - i G^2 - i \frac{\sqrt{s - 4 m^2_{K\bar{K}}}}{m_0} (G_{K\bar{K}}^2 + i F).$$

(7)

Here $m_0$ is the input mass of $f_0(980)$, $G$ and $G_{K\bar{K}}$ are coupling constants to pion channels ($\pi\pi + \pi\pi\pi\pi$) and $K\bar{K}$. The dimensionless constant $f$ stands for the prompt transition $KK \to \pi\pi$: the value $f/m_0$ is the “transition length” which is analogous to the scattering length of the low-energy hadronic interaction. The constants $m_0, G, G_{K\bar{K}}, f$ are parameters which are to be chosen to reproduce the $f_0(980)$ charactristics (position of pole $s \simeq (1.02 - i 0.08)$ GeV$^2$ and couplings to the channels $\pi\pi$ and $K\bar{K}$, $g^2_{\pi\pi} \simeq 0.070$ GeV$^2$ and $g^2_{K\bar{K}} \simeq 0.184$ GeV$^2$, see Tables 1 and 2).

The $\pi\pi$ scattering amplitude in the $f_0(980)$ region is now defined as

$$A_{\pi\pi \to \pi\pi} = e^{i \theta} R_{f_0(980)}^{(\pi\pi, \pi\pi)} + e^{i \delta} \sin \frac{\theta}{2}.$$  

(8)

This formula may be compared with equation (2): the background term in (8) is fixed by the requirement that $\pi\pi$ scattering amplitude below $K\bar{K}$ threshold has the form $\exp(i \delta) \sin \delta$.

At $f \to 0$ the resonance amplitudes \cite{1} turn into Flatté’s formula \cite{7}, which is used rather often for the description of $f_0(980)$. Still, it happened that position of the pole (complex mass value) as well as the amplitude residue in the pole, which have been determined in \cite{1} and shown in Tables 1, 2, do not obey the Flatté formula but require $f \neq 0$.

We obtained two sets of parameters, with sufficiently correct values of the $f_0(980)$ pole position and couplings. They are equal (in GeV units) to:

**Solution A:** $m_0 = 1.000$, $f = 0.516$, $G = 0.386$, $G_{K\bar{K}} = 0.447$,

**Solution B:** $m_0 = 0.952$, $f = -0.478$, $G = 0.257$, $G_{K\bar{K}} = 0.388$.

(9)

The above parameters provide us with a reasonable description of the $\pi\pi$ scattering amplitude. The phase shift $\delta^0_0$ and inelasticity parameter $\eta^0_0$ are shown in Fig. 2; the angle $\theta$ for the background term in Solutions A and B, determined as

$$\theta = \theta_1 + \left( \frac{\sqrt{5}}{m_0} - 1 \right) \theta_2,$$

(10)
is numerically equal to

\[
\text{Solution } A : \quad \theta_1 = 189^\circ, \quad \theta_2 = 146^\circ, \\
\text{Solution } B : \quad \theta_1 = 147^\circ, \quad \theta_2 = 57^\circ. 
\]  

(11)

Solutions A and B give significantly different predictions for \(\eta_0\); however, the existing data do not allow us to discriminate between them.

Partial widths of \(f_0(980)\) are calculated with the expression similar to (3) — with the replacement of the integrand denominator as follows:

\[
(\text{Re } \mu_n^2 - s)^2 + (\text{Im } \mu_n^2)^2 \rightarrow |D|^2. 
\]  

(12)

For both sets of parameters the calculated partial widths are close to each other. For example, using Solution II-2 and the \(A\)-set of parameters we have \(\Gamma_{\pi\pi} = 62\) MeV, \(\Gamma_{K\bar{K}} = 14\) MeV, while for solution II-2 and the \(B\)-set of the parameters one has \(\Gamma_{\pi\pi} = 66\) MeV, \(\Gamma_{K\bar{K}} = 10\) MeV. The values of partial widths for \(f_0(980)\) averaged over Solutions A and B are presented in Table 1.

The total hadron width of \(f_0(980)\) is defined in the same way as for the other \(f_0\)-mesons, namely, by using the position of pole in the complex-\(M\) plane: the imaginary part of the mass is equal to a half-width of the resonance. For the Breit–Wigner resonance this definition is in accordance with what is observed from resonance spectrum (provided there is no interference with the background). If the resonance is located in the vicinity of a strong threshold, the observed resonance width can differ significantly from what is given by the pole position. In Fig. 3 one can see the magnitudes \(|R_i^{(\pi\pi,\pi\pi)}(s)|^2 \rho_{\pi}(s)|, \quad |R_i^{(KK,\bar{K}\bar{K})}(s)|^2 \rho_K(s)|\) and \(g^2_{\pi\pi} \Gamma_{\pi\pi} |D(s)|^2\) for the parameter sets A and B: the width of peaks does vary, being less than the value determined by the complex mass of the resonance.

The only objective characteristic of the position of pole in the complex-\(M\) plane, due to this reason we employ such a definition of total hadron width. Multiple variations of total width in the compilation are just due to the absence of a proper definition of \(\Gamma_{\pi\pi}\) for resonances near the strong threshold.

B. The light \(\sigma\)-meson

The light \(\sigma\)-meson reveals itself as a pole on the 2nd sheet: it is shown in Fig. 1 at \(M = (431 - i325)\) MeV (or, in terms of \(s\) which is more appropriate variable for light particles, at \(s = (4 - i14) m_{\pi}^2\)) that corresponds to the magnitude obtained in \([1]\). Although this pole does not appear in the area of complex \(M\), where the \(K\)-matrix fit reconstructs the amplitude rather reliably, it still deserves detailed comments.

The situation with \(\sigma\)-meson is as follows. The \(K\)-matrix representation allows us to reconstruct correctly the analytical structure of the partial amplitude in the physical region, at \(s \geq 4m_{\pi}^2\) by taking account of the threshold and pole singularities. The singularities related to forces (or left singularities, at \(s \leq 0\), are not included directly into the \(K\)-matrix machinery. This does not allow us to be quite sure about the results of the \(K\)-matrix approach at \(s \leq 4m_{\pi}^2\). Concerning the low-mass region, \(s > 4m_{\pi}^2\), an important result of the \(K\)-matrix fit is the absence of the pole singularity in the 00-++ amplitude at 500–800 MeV. Here the \(\pi\pi\)-scattering phase \(\delta_0\) increases smoothly reaching 90° at 800–900 MeV. A straightforward explanation of such a behaviour of \(\delta_0\) could consist in the existence of a broad resonance, with the mass about 600–900 MeV and width \(\sim 800\) MeV (for example, see discussion in \([2,3]\) and references therein). However, as was stressed above, the \(K\)-matrix amplitude does not contain pole singularities at 500 \(\leq Re M \leq 900\) MeV: the \(K\)-matrix amplitude has a low-mass pole only, which is located near the \(\pi\pi\) threshold or below it. In \([1]\) the presence of the pole near the \(\pi\pi\) threshold was not emphasized, since the \(K\)-matrix solution does not guarantee a reliable reconstruction of the amplitude at \(s \sim 4m_{\pi}^2\). In \([1]\), in order to restore analytical structure at \(s \sim 4m_{\pi}^2\), the left-hand-side singularities were accounted for on the basis of the dispersion relation \(N/D\)-method. The \(\pi\pi\) scattering \(N/D\)-amplitude was represented at \(M < 900\) MeV being sewed with the \(K\)-matrix solution \([1]\) at 450 \(\leq M \leq 900\) MeV. The \(N/D\)-amplitude reconstructed in this way has a pole near the \(\pi\pi\) threshold, thus proving that qualitatively the results of \([1]\) are also valid for the region \(s \sim 4m_{\pi}^2\). The pole of the \(N/D\)-amplitude \([1]\) is shown in Fig. 1.

It is worth mentioning that the low-mass location of the \(\sigma\)-meson pole was also obtained in a set of papers, where the low-energy \(\pi\pi\) amplitude has been investigated by taking into account the left-hand cut as a set of meson exchanges. These papers include: (i) dispersion relation approach, \(s \simeq (0.2 - i22.5) m_{\pi}^2\) \([4]\), (ii) meson exchange models, \(s \simeq (3.0 - i17.8) m_{\pi}^2\) \([5]\), \(s \simeq (0.5 - i13.2) m_{\pi}^2\) \([6]\), \(s \simeq (2.9 - i11.8) m_{\pi}^2\) \([7]\), (iii) linear \(\sigma\)-model, \(s \simeq (2.0 - i15.5) m_{\pi}^2\) \([8]\). At the same time, in \([9,10]\) the pole position was found in the region of higher \(s\), at \(s > 7m_{\pi}^2\), that reflects the ambiguities of approaches which treat the left-hand cut as a known quantity.
As to finding out the location of $\sigma$-meson on the basis of the available experimental data, one should make general remark. Since the width of the $\sigma$-meson is rather large, it is necessary to fit to data in the energy interval which is much larger than a total width of the $\sigma$-meson. For example, to speak about $\sigma$-meson with a mass $M_\sigma \sim 900$ MeV and half-width $\Gamma/2 \simeq 400$ MeV, one should fit to data in the interval $300$ MeV $\leq M \leq 1400$ MeV and at the same time to take a correct account of the nearest singularities, which are poles corresponding to $f_0(980)$, $f_0(1300)$ and presumably $f_0(1500)$ as well as threshold singularities $\pi\pi$, $K\bar{K}$, $\eta\eta'$ and $\pi\pi\pi\pi$. Concerning $\pi\pi\pi\pi$, one should have in mind that the contribution of this channel is significant starting from $1300$ MeV, so this channel is absolutely necessary. Such demands towards the fit of experimental data have been fulfilled in no paper under discussion, with an exception for [1].

IV. CONCLUSION

We have obtained partial decay widths for five scalar-isoscalar states $f_0(980)$, $f_0(1300)$, $f_0(1500)$, $f_0(1750)$, $f_0(1530^{+90}_{-250})$ by calculating the decay couplings as residues of pole singularities in the $K$-matrix amplitude [1]: positions of poles in the complex-$M$ plane are shown in Fig. 1. The pole which corresponds to the light $\sigma$-meson is also shown in Fig. 1: it was not included into the $K$-matrix calculation procedure directly, being close to the left-hand cut; the discussion of its status can be found in [1] and references therein.

The results of our calculations of partial decay widths are presented below (the magnitudes are given in MeV units):

| Resonance       | $\Gamma_{\pi\pi}$ | $\Gamma_{K\bar{K}}$ | $\Gamma_{\eta\eta}$ | $\Gamma_{\eta\eta'}$ | $\Gamma_{\pi\pi\pi\pi}$ | $\Gamma_{\text{tot}}/2$ |
|-----------------|-------------------|----------------------|----------------------|-----------------------|--------------------------|-------------------------|
| $f_0(980)$      | $64 \pm 8$        | $12 \pm 1$           | --                   | --                    | $4 \pm 2$                | $40 \pm 5$              |
| $f_0(1300)$     | $46 \pm 12$       | $5 \pm 3$            | $4 \pm 2$            | --                    | $171 \pm 10$             | $113 \pm 10$            |
| $f_0(1500)$     | $37 \pm 2$        | $7 \pm 2$            | $4 \pm 1$            | $0.2 \pm 0.1$         | $70 \pm 6$              | $62 \pm 3$              |
| $f_0(1750)$     | $74^{+15}_{-30}$   | $11^{+17}_{-9}$      | $7 \pm 1$            | $3 \pm 1$             | $91^{+30}_{-60}$         | $93^{+20}_{-40}$         |
| $f_0(1530^{+90}_{-250})$ | $380 \pm 15$   | $185 \pm 10$         | $40 \pm 5$           | $1 \pm 1$             | $560^{+260}_{-125}$      | $590^{+120}_{-200}$      |

The values shown for partial widths as well as decay coupling constants of Table 2 need some comments. The comparison of the hadron decays $f_0(980) \to K\bar{K}$ and $f_0(980) \to \pi\pi$ points to a large $s\bar{s}$ component in $f_0(980)$. The analysis of radiative decays $\phi(1020) \to \gamma f_0(980)$ and $f_0(980) \to \gamma\gamma$ [23] shows also that the $s\bar{s}$ component in $f_0(980)$ is large: with the $f_0(980)$ flavour wave function written as $u\bar{u} \cos \varphi + s\bar{s} \sin \varphi$, the radiative decay widths give either $\varphi \simeq -48^\circ$ or $\varphi \simeq 86^\circ$ (solution with negative $\varphi$ is more preferable). When the decay processes are switched off, $f_0(980)$ transforms into $f_0^{\text{bare}}(720 \pm 100)$, with $\varphi^{\text{bare}} \simeq -70^\circ$ (corresponding pole trajectory in the complex-$M$ plane is shown in Fig. 10 of [1]). We see that the decay processes and related change of the state do not diminish the $s\bar{s}$ component strongly.

An opposite situation takes place with $f_0(1750)$. After switching off the decay channels, this resonance transforms into $f_0^{\text{bare}}(1810 \pm 30)$ which is dominantly $s\bar{s}$: $\varphi^{\text{bare}} \simeq 90^\circ$ for Solution I and $\varphi^{\text{bare}} \simeq -60^\circ$ for Solution II. However, partial decay widths of $f_0(1750)$ (or decay coupling constants given in Table 2) unambiguously prove that $s\bar{s}$ component in $f_0(1750)$ decreased strongly due to a mixing with other states after the onset of the decay processes. It is possible to guess that this $s\bar{s}$ component has flown into the broad state $f_0(1530^{+90}_{-250})$: the ratio $\Gamma_{K\bar{K}}/\Gamma_{\pi\pi}$ for $f_0(1530^{+90}_{-250})$ does not contradict such an assumption. Such a scenario looks rather intriguing, in particular when taking account of the fact that the broad state $f_0(1530^{+90}_{-250})$, according to [1], is a descendant of a pure glueball (see also [22, 23, 24]). However, the study of the mixing of $q\bar{q}$-state with the glueball is beyond the frame of this article; it will be investigated elsewhere.

For $f_0(980)$, the obtained magnitudes for the complex mass and decay couplings $g_s^2$ and $g_K^2$, demonstrate a failure of the Flatté formula. We suggest an alternative description of $f_0(980)$ which explores, as an addition to the pole term, the amplitude for the prompt transition $\pi\pi \to K\bar{K}$.

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[1] V.V. Anisovich, A.A. Kondashov, Yu.D. Prokoshkin, S.A. Sadovsky, A.V. Sarantsev, Yad. Fiz. 60, 1489 (2000) [Physics of Atomic Nuclei, 60, 1410 (2000)].
[2] D. Alde et al., Zeit. Phys. C 66, 375 (1995);
Yu. D. Prokoshkin et al., Physics-Doklady 342, 473 (1995);
A. A. Kondashov et al., Preprint IHEP 95-137, Protvino, 1995;
F. Binon et al., Nuovo Cim. A 78, 313 (1983), 80, 363 (1984).
[3] S. J. Lindenbaum and R. S. Longacre, Phys. Lett. B 274, 492 (1992);
A. Etkin et al., Phys. Rev. D 25, 1786 (1982).
[4] V. V. Anisovich et al., Phys. Lett. B 323, 233 (1994);
C. Amsler et al., Phys. Lett. B 342, 433 (1995), 355, 425 (1995).
[5] A.V. Anisovich, V.V. Anisovich, A.V. Sarantsev, Z. Phys. A 359, 173 (1997); Phys. Lett. B 359, 123 (1997).
[6] V.V. Anisovich, D.V. Bugg, A.V. Sarantsev, Phys. Rev. D 58:111503 (1998); Yad. Fiz. 62, 1322 (1999) [Phys. Atom. Nuclei 62, 1247 (1999)].
[7] S.M. Flatté, Phys. Lett. B 63, 224 (1976).
[8] D.E. Groom et al. (Particle Data Group), Eur. Phys. J. C 15, 1 (2000).
[9] G. Grayer et al. Nucl. Phys. B 75, 189 (1974); W. Ochs, PhD Thesis, Münich University, (1974).
[10] V.V. Anisovich, A.V. Sarantsev, Phys. Lett. B 382, 429 (1996).
[11] V.V. Anisovich and V. A. Nikonov, Eur. Phys. J. A 8, 401 (2000).
[12] L. Montanet, Nucl. Phys. Proc. Suppl. 86, 381 (2000).
[13] M.R. Pennington, "Riddle of the scalars: Where is the \(\sigma\)?", Frascati Phys. Series XV, 95 (1999).
[14] J. L. Basdevant, C. D. Frogatt and J. L. Petersen, Phys. Lett. B 41, 178 (1972).
[15] J. L. Basdevant and J. Zinn-Justin, Phys. Rev. D 3, 1865 (1971); D. Iagolnitzer, J. Justin and J. B. Zuber, Nucl. Phys. B 60, 233 (1973).
[16] B. S. Zou and D. V. Bugg, Phys. Rev. D 48, 3942 (1994); 50, 591 (1994).
[17] G. Janssen, B. C. Pearce, K. Holinde and J. Speth, Phys. Rev. D 52, 2690 (1995).
[18] N. N. Achasov and G. N. Shestakov, Phys. Rev. D 49, 5779 (1994).
[19] S.D. Protopopescu et al., Phys. Rev. D 7, 1279 (1973).
[20] P. Estabrooks, Phys. Rev. D 19, 2678 (1979).
[21] K. L. Au, D. Morgan and M. R. Pennington, Phys. Rev. D 35, 1633 (1987).
[22] S. Ishida et al., Prog. Theor. Phys. 98, 1005 (1997).
[23] N. A. Törnqvist and M. Roos, Phys. Rev. Lett. 76, 1575 (1996).
[24] M. P. Locher, V. E. Markushin and H. Q. Zheng, Eur. Phys. J. C 4, 317 (1998).
[25] A.V. Anisovich, V. V. Anisovich, V. A. Nikonov, "Quark structure of \(f_0(980)\) from the radiative decays \(\phi(1020) \to \gamma f_0(980), \gamma \eta, \gamma \eta', \gamma \pi^0\) and \(f_0(980) \to \gamma \gamma\)" [hep-ph/0011191].
[26] A.V. Anisovich, A.V. Sarantsev, Phys. Lett. B 413, 137 (1997).
[27] V.V. Anisovich, UFN 168, 481 (1998) [Physics-Uspekhi 41, 419 (1998)].
TABLE I. Partial widths of scalar-isoscalar resonances (in MeV units) in hadronic channels $\pi\pi$, $K\bar{K}$, $\eta\eta$, $\eta'\eta'$ and $\pi\pi\pi\pi$ for different $K$-matrix solutions of Ref. [1].

|         | $\pi\pi$ | $KK$ | $\eta\eta$ | $\eta'\eta'$ | $\pi\pi\pi\pi$ | pole position | solution |
|---------|----------|------|-------------|--------------|----------------|--------------|----------|
| $f_0(980)$ |          |      |             |              |                |              |          |
| 71      | 13       | –    | –           | –            | 6              | 1006 - i 45  | I        |
| 56      | 10       | –    | –           | –            | 2              | 1020 - i 34  | II-1     |
| 64      | 12       | –    | –           | –            | 3              | 1015 - i 39.5 | II-2     |
| $f_0(1300)$ |          |      |             |              |                |              |          |
| 71      | 12       | 7    | –           | 170          | 3              | 1307 - i 130 | I        |
| 34      | 2        | 2    | –           | 171          | 3              | 1296 - i 104 | II-1     |
| 34      | 2        | 2    | –           | 171          | 3              | 1296 - i 104 | II-2     |
| $f_0(1500)$ |          |      |             |              |                |              |          |
| 38      | 9        | 4    | 0.3         | 83           | 3              | 1495 - i 67  | I        |
| 37      | 6        | 4    | 0.1         | 83           | 3              | 1498 - i 60  | II-1     |
| 37      | 6        | 4    | 0.1         | 72           | 3              | 1495 - i 60  | II-2     |
| $f_0(1750)$ |          |      |             |              |                |              |          |
| 45      | 28       | 6    | 4           | 29           | 3              | 1781 - i 56  | I        |
| 88      | 2        | 7    | 3           | 120          | 3              | 1817 - i 110 | II-1     |
| 90      | 2        | 7    | 3           | 123          | 3              | 1817 - i 113 | II-2     |
| $f_0(1530^{\pm 250})$ |          |      |             |              |                |              |          |
| 376     | 173      | 42   | 2           | 818          | 3              | 1609 - i 706 | I        |
| 377     | 196      | 39   | 1           | 434          | 3              | 1427 - i 526 | II-1     |
| 376     | 196      | 39   | 1           | 438          | 3              | 1430 - i 525 | II-2     |

TABLE II. Coupling constants squared (in GeV$^2$ units) of scalar-isoscalar resonances to hadronic channels $\pi\pi$, $K\bar{K}$, $\eta\eta$, $\eta'\eta'$ and $\pi\pi\pi\pi$ for different $K$-matrix solutions of Ref. [1].

|         | $\pi\pi$ | $KK$ | $\eta\eta$ | $\eta'\eta'$ | $\pi\pi\pi\pi$ | solution |
|---------|----------|------|-------------|--------------|----------------|----------|
| $f_0(980)$ |          |      |             |              |                |          |
| 0.076   | 0.180    | 0.075| –           | –            | 0.009          | I        |
| 0.076   | 0.180    | 0.072| –           | –            | 0.004          | II-1     |
| 0.076   | 0.180    | 0.072| –           | –            | 0.004          | II-2     |
| $f_0(1300)$ |          |      |             |              |                |          |
| 0.050   | 0.015    | 0.012| –           | –            | 0.124          | I        |
| 0.026   | 0.002    | 0.003| –           | –            | 0.132          | II-1     |
| 0.026   | 0.002    | 0.003| –           | –            | 0.132          | II-2     |
| $f_0(1500)$ |          |      |             |              |                |          |
| 0.032   | 0.010    | 0.005| 0.012       | 0.012        | 0.070          | I        |
| 0.038   | 0.009    | 0.007| 0.006       | 0.006        | 0.074          | II-1     |
| 0.038   | 0.009    | 0.007| 0.006       | 0.006        | 0.074          | II-2     |
| $f_0(1750)$ |          |      |             |              |                |          |
| 0.039   | 0.029    | 0.007| 0.030       | 0.025        | 0.764          | I        |
| 0.086   | 0.003    | 0.009| 0.028       | 0.028        | 0.117          | II-1     |
| 0.086   | 0.003    | 0.009| 0.028       | 0.028        | 0.117          | II-2     |
| $f_0(1530^{\pm 250})$ |          |      |             |              |                |          |
| 0.329   | 0.229    | 0.061| 0.022       | 0.764        | 0.764          | I        |
| 0.304   | 0.271    | 0.062| 0.016       | 0.382        | 0.382          | II-1     |
| 0.304   | 0.271    | 0.062| 0.016       | 0.382        | 0.382          | II-2     |
FIG. 1. Pole positions (full circles) in the complex-$M$ plane ($M = \sqrt{s}$). Solid lines stand for cuts related to the threshold singularities $(\pi\pi$, $\pi\pi\pi$, $KK$, $\eta\eta$ and $\eta\eta')$. Two poles, which correspond to $f_0(980)$, are shown: on the 3rd and 4th sheets. On the 5th sheet the poles for $f_0(1300)$, $f_0(1500)$ and broad state $f_0(1530\pm90\mp250)$ are located (for the broad state the pole stands at $(1430 - i525)$ MeV, that is the mass for Solution II of the fit [1]). On the 6th sheet there is a pole for $f_0(1750)$. The dashed semi-circle restricts the area where the $K$-matrix fit [1], which was carried out on the real axis in the interval $450$ MeV $\leq M \leq 1900$ MeV, can give a reliable reconstruction of analytical amplitude.

FIG. 2. Reaction $\pi\pi \rightarrow \pi\pi$: description of $\delta_0^0$ and $\eta_0^0$ in the region of $f_0(980)$. Solid and dashed curves correspond to the parameter sets A and B. Data are taken from [9] (full squares) and [10] (open circles).
FIG. 3. a,b) The magnitudes $|R_{I_0(980)}^{(\pi\pi,\pi\pi)}|^2\rho_{\pi}(s)$ (solid curve) and $|R_{I_0(980)}^{(K\bar{K},K\bar{K})}|^2\rho_{K}(s)$ (dashed curve) for the parameter sets A and B. Visible peaks in the $\pi\pi$ spectra have the total widths $\sim 60$ MeV (set A) and $\sim 45$ MeV (set B). c,d) Values $g_{\pi\pi}^2\Gamma_{tot}/|D(s)|^2$ (where $g_{\pi\pi}^2\Gamma_{tot} = 0.006$ GeV$^2$) for the parameter sets A and B. Visible total widths of the peaks are $\sim 55$ MeV (set A) and $\sim 45$ MeV (set B).