Boundary layer numerical modeling in a strongly turbulized liquid flow

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Abstract. Already in the first Taylor and Dryden's works it was discovered that besides incoming flow turbulence intensity (measured in percent and defined as the velocity mean squared deviation ratio to the flow averaged velocity), the turbulence influence in an aerodynamic tube is connected with one more parameter, which characterizes the turbulence disturbances size in the flow at large Reynolds numbers (so called turbulence dissipative scale). Turbulence scale is the physical value, characterizing the large eddies size, obtaining its energy from the turbulent flow. Investigation results of the scale and the free-stream turbulence high intensity influence in the boundary layer on a smooth flat plate with a rounded off leading edge (experiment PP169–60) under zero pressure gradient are presented in the study. Using well-known experimental and calculated data the modeling problem of the initially laminar boundary layer transfer to the turbulent one was investigated by numerical methods on the basis of the near-wall modified turbulence model with two additional transfer equations for the turbulent kinetic energy and the turbulence dissipation rate. Turbulent flows modeling near the flat surface with the incoming flow turbulence high level is complicated by two general problems: the definition and description of the laminar-to-turbulent transfer along the surface and the viscous sublayer precise resolution under the developed turbulent mode. While flowing along the flat plate the inviscid liquid with high turbulence degree it is more than 1%, the turbulence scale and the free-stream turbulence intensity joint influence on the flow dynamic and integral characteristics in the boundary layer and turbulence parameters was studied in detail.

1. Introduction

The free-stream two determinant parameters $Tu_\infty$ and $L_\varepsilon\infty$ influence on the average velocity and turbulence energy profiles in the laminar, transitive and fully turbulent areas, on the boundary layer transfer beginning location (critical Reynolds transfer number), the surface friction local coefficient, the boundary layer integral characteristics and a shape factor were investigated. To explain the free-stream turbulence structure influence on the flow in the boundary layer of the flat plate, the turbulence intensity influence was limited by the big value $Tu_\infty = 4.86\%$ in accordance with the experiment data [1]. The free-stream kinetic energy dimensionless dissipation rate $\overline{\varepsilon}_\varepsilon = \varepsilon_\varepsilon D / V_\infty^3$ (or turbulence scale $L_\varepsilon\infty = C_d k_\infty^{3/2} / \varepsilon_\infty$ in accordance with Kolmogorov-Prandtl hypothesis, where the empiric constant

$C_d$...
$C_d = C_{f}^{0.75} = 0.16431$, $C_{f} = 0.09$, $k_{*}$ is the free-stream turbulence kinetic energy) was varied in the calculations by an order from $\varepsilon_{*} = 1.84\cdot10^{-2},1.84\cdot10^{-1},1.84$. Further on the plate length $D$ and non-disturbed flow velocity $V_{\infty}$ were used as the length and velocity scales.

2. Task assignment

Equations system for averaged (by Reynolds) characteristics of incompressible fluid’s quasi-steady two-dimensional turbulent boundary layer together with the initial and boundary conditions is given in [2], where turbulence intensity $T_{u\infty}$ influence on the transfer structure in the boundary layer ranging from small up to the big local turbulent Reynolds numbers $Re_{t} = k^{2} / (\nu \varepsilon)$, which is the ratio of the product of the kinetic energy $k^{2}$ square to the liquid molecules’ kinetic coefficient $\nu$ on the turbulence energy dissipation isotropic rate, is studied and measures turbulence level in the boundary layer. To investigate near-wall boundary layers and to study the perturbed external stream the modified turbulence model $k - \varepsilon$ for full dissipation rate is considered [3], which allows to calculate in a continuous mode the laminar, transitive and turbulent areas. The near-wall model modified variant allowed to obtain numerical results well consistent with experimental data.

3. Mathematical model

Quasi-steady equations of the incompressible liquid’s two-dimensional turbulent motion, written for averaged values, in the boundary layer approximation theory relatively to the arbitrary coordinates system, normally tied up with the flat body surface, look as [4]

$$\frac{\partial u}{\partial \xi} + \frac{\partial v}{\partial \zeta} = 0, \tag{1}$$

$$u \frac{\partial u}{\partial \xi} + v \frac{\partial u}{\partial \zeta} = -\frac{1}{\rho} \frac{\partial p}{\partial \xi} + \frac{1}{\rho} \frac{\partial}{\partial \zeta} \left( \mu \frac{\partial u}{\partial \zeta} - \rho \langle u'v' \rangle \right), \tag{2}$$

$$\frac{\partial p}{\partial \zeta} = 0. \tag{3}$$

Here (1) is the continuity equation and two equations of motion quantity (2)–(3) in projections on axis $\xi$ and $\zeta$ (in Einstein notation, $x', i = 1,3$). In the equations (1)–(3), $\xi$ and $\zeta$ are the body surface coordinates ($\zeta = 0$), $\xi$ is the coordinate along the surface, $\zeta$ is the coordinate, measured from the surface in the external normal direction and $\eta$ is the transversal coordinate ($x^{2}$), the alongside, transversal and normal velocity components, correspondingly, will be $u, w$ and $v$ (in tensor notation, $u', i = 1\ldots3$); $p$ is the static pressure; $\rho$ is the liquid density; $\mu = \nu \rho$ is the dynamic coefficient of the liquid’s molecular viscosity; corner brackets mean time averaging, comma refer to the velocity fluctuating components.

Turbulence models use Boussinesq hypothesis [5] introducing turbulent (eddy) viscosity and Reynolds stresses linear relation with the deformation velocity. In conformity with the generalized Boussinesq hypothesis turbulent viscosity is usually introduced by the following [6]:

$$-\langle u'v' \rangle = -\frac{2}{3} \delta^{ij}k + v_{t} \left( \frac{\partial u}{\partial x^{i}} + \frac{\partial u}{\partial x^{j}} \right), ~ i, j = 1\ldots3,$$

where $\delta^{ij}$ are the Kroneker symbols (symmetric unit tensor); $v_{t}$ is the molecular viscosity kinetic coefficient; index $t$ below refer to the turbulent mode. The tensor analysis usual agreements are accepted upon for mute indices (the summation over the repeated index) [7]. It is assumed that Latin indices run the values from 1 to 3, and Greek – from 1 to 2.
The use of the turbulent viscosity coefficient, Boussinesq hypothesis regarding the gradient mechanism transfer for turbulent tension \(-\rho \langle uu'\rangle\) allows to represent the frictions full tension \(\tau\) as follows

\[
\tau = \mu \frac{\partial u}{\partial \xi} - \rho \langle uu'\rangle, \quad \tau_i = -\rho \langle uu'\rangle = \mu_i \frac{\partial u}{\partial \xi}.
\]

Introducing an effective dynamic coefficient of the liquid's viscosity (or diffusion coefficient) \(\Gamma = \nu + \nu'_t\), transfer equation (2) for the incompressible liquid acquires the form analogous to the laminar boundary layer equation

\[
u \frac{\partial u}{\partial \xi} + v \frac{\partial u}{\partial \zeta} = -\frac{1}{\rho} \frac{\partial p}{\partial \xi} + \frac{\partial}{\partial \zeta} \left( \Gamma \frac{\partial u}{\partial \zeta} \right).
\]

The term dependent on pressure, is excluded from the formula (2) with Euler equation help, written for the boundary layer external boundary

\[
u \frac{\partial u}{\partial \xi} + v \frac{\partial u}{\partial \zeta} = -\frac{1}{\rho} \frac{\partial p}{\partial \xi}.
\]

To close up the equations system (1)–(3) it is necessary to choose the turbulence model. Further two-parameter turbulence model \(k - \varepsilon\) is employed.

4. The near-wall turbulence model \(k - \varepsilon\)

The near-wall \(k - \varepsilon\) turbulence model has two additional differential equations of the turbulent kinetic energy transfer \(k\) and its dissipation average rate. For the first time the turbulence mode \(k - \varepsilon\) was introduced in [8] and was used far from the wall in the fully developed turbulent flow area under large Reynolds numbers. In general tensor representation the equations are given in the study [9]. In the boundary layer approximation theory in the coordinate system, the equations system looks as:

\[
u \frac{\partial k}{\partial \xi} + v \frac{\partial k}{\partial \zeta} = \frac{\partial}{\partial \zeta} \left( \Gamma \frac{\partial k}{\partial \zeta} \right) + P_k - \bar{\varepsilon},
\]

\[
u \frac{\partial \varepsilon}{\partial \xi} + v \frac{\partial \varepsilon}{\partial \zeta} = \frac{\partial}{\partial \zeta} \left( \Gamma \frac{\partial \varepsilon}{\partial \zeta} \right) + C_{\mu} \frac{P_k}{k} - C_{\varepsilon} f_1 \frac{k}{k} + E_k,
\]

\[
\begin{align*}
k &= \frac{1}{2} \left( \langle uu'^2 \rangle + \langle vv'^2 \rangle + \langle vv'^2 \rangle \right) - \frac{1}{2} \left( \langle uu'^2 \rangle \right), \\
P_k &= -\langle uu'\rangle \frac{\partial u}{\partial \zeta}, \\
\bar{\varepsilon} &= 2 \nu \langle s^i s^i \rangle, \\
s^{ij} &= \frac{1}{2} \left( \frac{\partial u^i}{\partial \xi} + \frac{\partial u^j}{\partial \xi} \right), \\
\Gamma_k &= \nu + \frac{\nu'_t}{\sigma_k}, \\
\Gamma_\varepsilon &= \nu + \frac{\nu'_t}{\sigma_\varepsilon},
\end{align*}
\]

where \(k\) is the turbulence kinetic energy (kinetic energy is the sum of velocity fluctuating components kinetic energies on all over the three space directions), \(\bar{\varepsilon} = \varepsilon + D_\varepsilon\) is an isotropic part rate of the energy full dissipation; \(P_k\) is determines turbulent kinetic energy generation, conditioned by average motion; \(D_\varepsilon\) and \(E_k\) are the additional empiric members in the equations for kinetic energy and dissipation rate correspondingly, introduced by different authors for \(k - \varepsilon\) correct modeling near the wall; \(s^{ij}\) is the tensions fluctuations symmetric tensor; \(\Gamma_k, \Gamma_\varepsilon\) full (effective) viscosities; \(\sigma_k, \sigma_\varepsilon\) are the corresponding Prandtl numbers for \(k\) and \(\varepsilon\); \(C_{\mu}, C_{\varepsilon 1}, C_{\varepsilon 2}, \sigma_k, \sigma_\varepsilon\) are the turbulence model constant coefficients.

Kinetic coefficient of the turbulent viscosity is determined by the second Kolmogorov-Prandtl formula [10, 11]

\[
\nu_t = C_{\mu} f_\mu \frac{k^2}{\varepsilon}.
\]
Damping function $f_\mu$ serves to correct value $v_t$ in the formula (6) near the flat surface. The following $k-\varepsilon$ turbulence model, considering the near-wall influence and small Reynolds numbers was investigated in the study. Model Myong-Kasagi [3] is written for the turbulent kinetic which energy full dissipation rate $\varepsilon$. Damping functions and additional source members, provide viscous and near-wall effects modeling in the transfer equations, in the model [3] were following:

$$f_\mu = \left[1 + \frac{3.45}{Re_{t}^{1/2}} \left(1 - \exp\left(-\frac{\zeta^+}{70}\right)\right)\right], \quad f_t = 1,$$

$$f_Z = \left[1 - \frac{2}{9} \exp\left(-\frac{Re_t}{6}\right)\right]\left[1 - \exp\left(-\frac{\zeta^+}{5}\right)\right]^2,$$

$$D_\xi = 0, \quad E_\xi = 0, \quad \zeta^+ = \frac{u_\tau \zeta}{v}, \quad u^* = \frac{u}{u_*}, \quad u_* = \left(\frac{\tau_\text{w}}{\rho}\right)^{1/2},$$

where $\tau_\text{w}$ is the dynamic velocity on the wall (velocity scale based on the friction tension); $\tau_\text{m}$ is the friction tension on the wall; $\zeta^+$ is the dimensionless distance up to the wall; hereinafter the subscript $w$ indicates wall value. We note that all the model functions are simple algebraic expressions and don’t include differential operators. The empiric constants values, which belong to such equations (4)–(5), were as follows:

$$C_\mu = 0.09, \quad C_{e1} = 1.44, \quad C_{e2} = 2, \quad \sigma_k = 1, \quad \sigma_\varepsilon = 1.3.$$

### 5. Boundary conditions

As boundary conditions for equations (1)–(5), reattachment and impermeability on the wall conditions were given. Velocity distributions $u_\varepsilon$ on the boundary layer external edge was determined from the corresponding task decision of the non-viscous flow past a body by the incompressible liquid. On the surface:

$$\zeta = 0: \quad u = 0, \quad v = 0, \quad k = 0, \quad \varepsilon_\varepsilon = \nu \frac{\partial^2 k}{\partial \zeta^2}\bigg|_w.$$

On the boundary layer external boundary:

$$\zeta \to \infty: \quad \begin{cases} u \to u_\varepsilon(\xi), & w \to w_\varepsilon(\xi), \\ k \to k_\varepsilon(\xi), & \varepsilon \to \varepsilon_\varepsilon(\xi). \end{cases}$$

$k_\varepsilon$ and $\varepsilon_\varepsilon$ distributions on the boundary layer external border are found from the numerical solution of non-viscous gas equations system when $u_\varepsilon \neq 0$ or $w_\varepsilon \neq 0$

$$u_\varepsilon \frac{\partial k_\varepsilon}{\partial \zeta} + w_\varepsilon \frac{\partial k_\varepsilon}{\partial \zeta} = -\varepsilon_\varepsilon,$$

$$u_\varepsilon \frac{\partial \varepsilon_\varepsilon}{\partial \zeta} + w_\varepsilon \frac{\partial \varepsilon_\varepsilon}{\partial \zeta} = -C_{e2} \frac{\varepsilon_\varepsilon^2}{k_\varepsilon}.$$

In case of flat plate flow ($u_\varepsilon = V_\varepsilon, \quad w_\varepsilon = 0, \quad d/d\eta = 0$) there is $k_\varepsilon$ and $\varepsilon_\varepsilon$ analytical representation as it is shown in [12]

$$k_\varepsilon = k_{\varepsilon0} \left[1 - (1 - C_{e2}) \frac{\varepsilon_{e0}}{k_{\varepsilon0}} \int_0^\zeta \frac{d\xi}{u_\varepsilon} \right]^{(1-C_{e2})}, \quad \varepsilon_\varepsilon = \varepsilon_{e0} \left[1 - (1 - C_{e2}) \frac{\varepsilon_{e0}}{k_{\varepsilon0}} \int_0^\zeta \frac{d\xi}{u_\varepsilon} \right]^{(1-C_{e2})},$$

where $k_{\varepsilon0}, \quad \varepsilon_{e0}$ are their initial values. The boundary layer and external flow measurements in the experiment were conducted in 11 sections, starting from $\zeta = 0.05m$ up to $\zeta = 0.6m$. 

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6. The initial conditions
The initial conditions on the longitudinal coordinate are usually set up as to satisfy the existence and unicity in a certain area (in the initial section when $\xi = \xi_0$). In the initial section $\xi_0 / D = 0.05$ the velocity profile is given $u = u_0(\xi)$, got from the corresponding Prandtl decision, and the velocity profile $v_0(\xi) = 0$. Though $Re_\tau$ is not enough, it is assumed that there is a pulsing motion in the laminar mode, that is why the initial profiles and $\epsilon_0$, as in the study [13] from the turbulence local equilibrium condition, were given as

$$k_0(\xi) = \left( \frac{v_t}{L_0(\xi)} \right)^2 + k_0 \frac{\xi}{\delta}, \quad e_0(\xi) = C_{\mu} f_\mu \left( \frac{k_0(\xi)^{3/2}}{L_0(\xi)} + e_{\infty} \frac{\xi}{\delta} \right), \quad k_\infty = 1.5(0.01Tu_\infty V_\infty)^2,$$

$$L_0(\xi) = C_{\mu}^{\nu-1} m, \quad v_t = \int_0^L \frac{du}{d\xi} \quad f_\mu = \exp\left( -\frac{2.5}{1 + Re_\tau / 50} \right), \quad m = C_\mu \delta \tanh\left( \frac{\kappa \xi}{C_\mu \delta} \right),$$

where $L_0(\xi)$ is the turbulence scale in the initial section; $l_m$ is the mixture path length; $\kappa = 0.4$ is the Karman constant; $v_t$ is the eddy’s viscosity, given in the initial section on Prandtl formula; $\delta$ is the boundary layer thickness, determined by $\xi$ coordinate, under which velocity $u$ would be different by 0.5% from $u_\infty$. While modeling the near-wall flow with boundary layer formation on the bypassed body surface, at the calculated area entrance the turbulent flow scale $L_0(\xi)$ is given as in the mixture path length Prandtl theory $l_m$.

7. Calculations results
The turbulence modeling is performed on the basis of the low-Reynolds $k-\epsilon$ model variant [3], modified with the near-wall presence effect and for the turbulent kinetic energy full dissipation rate $\dot{\epsilon}$. The calculating experiment was conducted for the following parameters $Re_\lambda = V_\infty D / \nu = 2.835 \cdot 10^5$, $V_\infty = 4.25 \text{m/s}$. The free-stream flow turbulence intensity $Tu_\infty = 4.86\%$. Further the dimensionless values are introduced (the line above means nondimensionalization on characteristic parameters):

$$\bar{\xi} = \xi / D, \quad \bar{\xi} = \xi \sqrt{Re_\lambda / D}, \quad \bar{u} = u / V_\infty, \quad \bar{v} = \bar{v} \sqrt{Re_\lambda / V_\infty},$$

$$\bar{k} = k \sqrt{Re_\lambda / V_\infty^2}, \quad \bar{\epsilon} = \epsilon D / V_\infty^3, \quad Re_\tau = V_\infty \bar{\xi} / \nu, \quad Re_\mu = V_\infty \nu / \nu.$$

Many studies is devoted to the problem of homogeneous free-stream low or high turbulence level effect on the surface friction local coefficient $c_f = 2x_\mu / (\rho V_\infty^2)$ in the turbulent boundary layer. It is shown that the higher turbulence intensity $Tu_\infty$ in the external flow under the permanent turbulence scale $L_\infty$ causes the earlier transfer from laminar to turbulent flow mode. However the turbulent mode advent extends in length. The transition beginning on the friction coefficient $c_f (Re_\tau)$ plot against the local Reynolds number (calculated over the longitudinal coordinate) is defined as a point where the friction coefficient minimum is achieved, the transition end – the local maximum point lying lower along the streamwise. The applied turbulence model testing results are described in detail in [2].

The local friction coefficient calculations comparison on the plain plate under different values of the mainstream turbulence energy dimensionless rate dissipation (which is tied up with the dissipative scale $L_\infty$ by the inverse proportion) is represented on figure 1(a). The straight line on the left (curve ______) is described by Blasius formula for laminar boundary layer under zero pressure gradient ($c_f = 0.664 / Re_\tau^{0.5}$) and the flow with the developed turbulence (the curve on the right - - - - -) for zero turbulence intensity $Tu_\infty$ is given by the empiric Prandtl ratio ($c_f = 0.0592 / Re_\tau^{0.2}$).

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The surface friction coefficient \( c_f(\text{Re}_x) \) calculations comparison on the plain plate, obtained in a wide range of Reynolds numbers with the corresponding experimental data showed that best agreement of the numerical modeling with the experiment is observed when calculating \( \text{Re}_x = 1.84 \). As it follows from the data presented the mathematical modeling results quite satisfactorily agree with the experimental data. The transfer beginning Reynolds number \( \text{Re}_{\text{tu}} \approx 5.41 \times 10^4 \) is close to the measurements data. The transfer end on calculations results is near the value \( \text{Re}_{\text{tu}} = 1.69 \times 10^3 \), on the experimental data this value equals \( \text{Re}_{\text{tu}} \approx 1.19 \times 10^5 \). The calculations data have some divergence with the experiment. The near-wall \( k - \varepsilon \) model equations have in fact precisely described the flow in laminar, transitive and nearly turbulent boundary layers. In \( \text{Re}_x \geq 1.2 \times 10^5 \) area this turbulence model marks down local friction coefficient \( c_f \) though further on values \( c_f \) with increasing \( \text{Re}_x \) asymptotically converge on the plot to a known empiric Prandtl dependence.

Let’s note the turbulence scale \( L_{\text{wu}} \) influence (or free-stream kinetic energy dissipation rate \( \varepsilon_x \) ) at the constant value \( Tu_{\text{wu}} \) on the longitudinal friction local coefficient. The less is the scale \( L_{\text{wu}} \) (more \( \varepsilon_x \) value), the more prolonged becomes the transition to turbulence, but the dependence is less expressed than for \( Tu_{\text{wu}} \).

The shape factor longitudinal distribution \( H = \delta' / \delta \) is shown on figure 1(b). Here displacement thicknesses \( \delta' \) and the impulse loss \( \theta \), correspondingly are determined by rations: \( \delta' = \int_0^\infty (1 - u/\bar{u}_x) d\zeta' \), \( \theta = \int_0^\infty u/\bar{u}_x (1 - u/\bar{u}_x) d\zeta' \). Experimental points ■ at \( \text{Re}_x > 10^5 \) lay somewhat below the calculated curve for \( \varepsilon_x = 1.84 \), however give the best fit to the experiment at \( \text{Re}_x \leq 10^5 \).

The Reynolds number dependence \( \text{Re}_{\text{tu}} \) by the impulse loss thickness from the local Reynolds number \( \text{Re}_x \) is given on figure 2(a) and compared with the calculation for \( \varepsilon_x = 1.84 \) The sufficient agreement of the numerical modeling data to the experimental points, marked by symbol □ is observed.

As a mathematical processing result with the help of Levenberg-Marquardt algorithm for least squares non-linear method is offered the following approximating dependence of the friction coefficient \( c_f \) on the Reynolds number \( \text{Re}_x \) by the longitudinal coordinate:

\[
\frac{c_f(\text{Re}_x)}{\text{Re}_x} \left(\frac{\text{Re}_x}{\text{Re}_x} + 1\right) = \left(a + c \ln(\text{Re}_x) + e \left(\ln(\text{Re}_x)\right)^2\right) / \left(1 + b \ln(\text{Re}_x) + d \left(\ln(\text{Re}_x)\right)^2 + f \left(\ln(\text{Re}_x)\right)^3\right),
\]

\[a = -0.0027, \quad b = -0.3139, \quad c = 0.0005, \quad d = 0.0317, \quad e = -2.0733, \quad f = -0.0010.
\]

The approximation result on formula (7) is shown on figure 2(b), here on the same figure the plots for Blazius, Prandtl analytic expressions and experimental points are provided. Few experimental measurements \( c_f \) give variation \( \text{Re}_x \) from \( 1.41 \times 10^4 \) to \( 2.06 \times 10^5 \).

The experimental results represented as a graphic dependence \( \text{Re}_{\text{tu}} \) on \( \text{Re}_x \) are also nicely approximated by the following square formula, which is shown by the solid line on the left on figure 3(a)

\[
\text{Re}_{\text{tu}}(\text{Re}_x) = a_1 + b_1 \text{Re}_x + c_1 \text{Re}_x^2, \quad a_1 = 39.9329, \quad b_1 = 0.0025, \quad c_1 = 2.8064.
\]

The experimental points for the shape factor \( H(\text{Re}_x) \) are approximated with fine precision by the pulse peak function dependence broken line on the right on figure 3(a)

\[
H(\text{Re}_x) = a_2 + 4b_2 \text{h(Re}_x) (1 - h(\text{Re}_x)), \quad h(\text{Re}_x) = \exp(-(\text{Re}_x - c_2) / d_2),
\]

\[a_2 = 1.3756, \quad b_2 = 1.1218, \quad c_2 = -21642.2836, \quad d_2 = 43285.7306.
\]
The velocity profile in wall law coordinates is represented on figure 3(b) solid line (____) in boundary layer section $\xi = 0.3$. The dotted line (........) shows the linear wall law $u^* = \zeta^*$, the dashed line (- - - - - -) gives the logarithmic profile $u^* = 2.5\ln(\zeta^*) + 5.1$. The velocity profiles a good fit to the experimental data.

8. Conclusion

The numerical modeling on the base of boundary layer equations with near-wall turbulence model of the incompressible liquid flow near a flat plate with a rounded off leading edge was performed and in all the cases the sufficient agreements of calculation data to the natural experimental data were achieved [1]. While investigating the incompressible liquid boundary layer on a flat plate with zero pressure gradient it is found that the free-stream turbulence scale $L_\infty$ (or mainstream turbulence kinetic energy dissipation rate $\epsilon_\infty$) slightly effects on the longitudinal friction coefficient $c_f$ distribution as compared with the turbulence intensity parameter influence. At the permanent turbulence level, the less is the turbulence level, the more prolongated becomes the transition to turbulence.

The calculated velocity profiles $u^*(\zeta^*)$ in the terms of a wall law at different longitudinal Reynolds numbers and turbulence parameters.

The calculated dependencies on friction coefficients and a shape factor at dissipation rate different values and high intensity turbulence mainstream, which equals $Tu_\infty = 4.86\%$, in accordance with the experiment [1] are adduced.

The influence of dissipation rate at high intensity turbulence on the formation and location of the transition area on a plain plate for an incompressible flow is evaluated in the study.

The new analytical dependencies for boundary layer hydrodynamic and integral characteristics, which give a good agreement with the experimental data, are obtained.

Figure 1. The local friction coefficient calculations $c_f(Re_\xi)$ (a) at turbulence dissipation $\bar{\epsilon}_\infty$ different values in the external flow, the shape factor $H(Re_\xi)$ (b) and the comparison with the experiment;

(........) laminar flow (Blasius formula); (- - - - - -) developed turbulent flow (Prandtl formula);

●, ■ – experimental points [1].
**Figure 2.** Reynolds number $Re(Re_\infty)$ at $\bar{\varepsilon}_\infty = 1.84$ (a) and friction coefficient $c_f(Re_\infty)$ dependences (b); approximation by the formula (7) solid line (---); Blasius formula curve (.....); Prandtl formula, curve (- - - - -); □, ■ – experimental points [1].

**Figure 3.** Reynolds number $Re(Re_\infty)$ and shape factor $H(Re_\infty)$ dependencies (a): left side, approximation by the formula (8) solid line (---); right side, approximation by the formula (9) dashed line (- - - - -); comparison of longitudinal average velocity calculated and measured profile in wall law variables $u^* = \zeta^*$ at $\bar{\zeta} = 0.3$; solid line at $\bar{\varepsilon}_\infty = 1.84$; dotted line (.....), linear wall law $u^* = \zeta^*$; dashed line (- - - - -), logarithmic profile $u^* = 2.5\ln(\zeta^*) + 5.1$; ○, □, ■ – experimental points [1].
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