The possible effect of scalar meson on the symmetry energy curve in the viewpoint of the quark-meson-coupling model

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Abstract. We give the energy density functional of quark-meson-coupling model within the Hartree-Fock approximation and make comparison with relativistic-mean-field and Skyrme-Hartree-Fock models. We find that the discrepancy of these models is mainly from the isovector channel of the energy density functional, which is also the major cause of different descriptions of symmetry energy against density. In the viewpoint of quark-meson-coupling model, the scalar meson through Fock exchange makes a density dependent form of the isovector channel which can soften the symmetry energy curve.

1. Introduction

In the quark-meson-coupling (QMC) model, a baryon in nuclear medium is assumed to be a non-overlapping static spherical bag in which quarks are coupled to meson fields in the mean-field approximation [1, 2]. For the modeling, one can choose one isoscalar-scalar field ($\sigma$) for medium-range attraction, one isoscalar-vector field ($\omega$) for the short-range repulsion, and one isovector-vector field ($\rho$) for isospin channel. In order to calculate the Dirac mass of the nucleon for QMC model, one can solve the bag equation. If one considers the polarization effect of nuclear medium, the Dirac mass of nucleons will have a complicated density-dependent form caused by the scalar field [2]. For simplicity, one can use a quadratic expansion to simulate this medium effect, so that the effective mass $M_f^*$ can be derived as [3],

$$M_f^* = M_f - g_\sigma \sigma + \frac{d}{2} (g_\sigma \sigma)^2,$$

where $d$ is called the scalar polarizability of the nucleon [3].

The improvement of the QMC model to include the exchange contribution has been achieved in Refs. [4, 5] with numerical calculations done in the recent works [3, 6]. The energy density of nuclear matter can be given as [3, 6],

$$\mathcal{E}_{\text{QMC}} = \rho M + \frac{\tau}{2M} - \frac{G_\sigma \rho^2}{2(1 + dG_\sigma \rho)} + \frac{G_\omega \rho^2}{2} - \frac{G_\rho \rho^2}{2M^2(1 + dG_\sigma \rho)} + \frac{G_{\rho}}{8} (\rho_n - \rho_p)^2 + \mathcal{E}_{\text{Fock}},$$

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where the non-relativistic approximation has been used in order to get the analytical formula. The non-relativistic approximation of the Fock exchange term in the QMC energy density functional can be written as,

$$\mathcal{E}_{\text{Fock}}^{\text{nr}} = \left[-\frac{3G_\rho}{32} + \frac{G_\sigma}{8(1 + dG_\rho)^3} - \frac{G_\omega}{8}\right] \rho^2 + \left[\frac{G_\rho}{32} + \frac{G_\sigma}{8(1 + dG_\rho)^3} - \frac{G_\omega}{8}\right] (\rho_n - \rho_p)^2 + \left(\frac{3G_\rho}{16m_\rho^2} + \frac{G_\omega}{4m_\omega^2} - \frac{G_\sigma}{8M^2}\right) \rho^\tau + \left(-\frac{G_\rho}{16m_\rho^2} + \frac{G_\omega}{4m_\omega^2} - \frac{G_\sigma}{8M^2}\right) \times (\rho_n - \rho_p)(\tau_n - \tau_p).$$ (3)

The non-relativistic approximation of the RMF energy density functional can be derived in a quite similar way (see, e.g., Ref. [7]). For RMF model of Ref. [8], the non-relativistic Hartree approximation of the energy density functional can be given as,

$$\mathcal{E}_{\text{RMF}}^{\text{nr}} = \rho M + \frac{\tau}{2M} - \frac{1}{2} G_\sigma \rho^2 + \frac{1}{2} G_\omega \rho^2 + \frac{G_\sigma}{2M^2} \rho^\tau + \frac{G_\rho}{8} (\rho_n - \rho_p)^2 + \frac{G_\delta}{2M^2} (\rho_n - \rho_p)(\tau_n - \tau_p),$$ (4)

where $G_\delta = g_\delta^2/m_\delta^2$, and $g_\delta$ is the meson-nucleon coupling constant for the isovector-scalar $\delta$ meson [9]. In RMF models without the $\delta$ meson considered, the last two terms of the equation above do not appear.

For the relativistic-mean-field (RMF) model with nonlinear scalar potential [10], the energy density functional in the non-relativistic Hartree approximation can be written as,

$$\mathcal{E}_{\text{RMF}-\text{NL}}^{\text{nr}} = \rho M + \frac{\tau}{2M} - \frac{1}{2} G_\sigma \rho^2 + \frac{\kappa}{6} G_\delta^2 \rho^3 + \frac{\lambda}{24} G_\sigma^2 \rho^4 + \frac{1}{2} G_\omega \rho^2 + \frac{G_\sigma}{2M^2} \rho^\tau + \frac{G_\rho}{8} (\rho_n - \rho_p)^2,$$ (5)

where $\kappa$ and $\lambda$ denote the corresponding coupling constants of the self-interacting terms. The QMC and RMF models have similar density functionals within the Hartree approximation, except for the density-dependent form of the scalar field which is brought by the difference of the Dirac mass. The Fock exchange term contributes to both isoscalar and isovector channels.

The QMC and RMF models have similar density functionals within the Hartree approximation, except for the density-dependent form of the scalar field which is caused by the difference of the Dirac mass. The Fock exchange term contributes to both isoscalar and isovector channels. The terms corresponding to $\rho^\tau$ are from the Dirac mass in the Hartree approximation. In the Hartree-Fock approximation, the $\omega$ and $\rho$ mesons also contribute to the $\rho^\tau$ terms. It is seen that the Fock exchange leads to a $(\rho_n - \rho_p)(\tau_n - \tau_p)$ term which does not appear in the Hartree approximation. Usually the $\delta$ meson in the Hartree approximation contributes to the isovector-scalar channel (see Eq. (4)), but with unnaturally small strength, while in the Hartree-Fock approximation the strength is instead comparable to the isovector-vector strength corresponding to the meson exchange [11].

2. Relations between the parameters of Skyrme force and effective meson-exchange force

It has been shown by Guichon and Thomas that the SIII and SkM* parameters of the Skyrme force give similar density functional to that of QMC model [3, 12]. The comparison between

2
Table 1. Skyrme parameters.

| Skyrme  | Sqmc     | SkM*    | SKP     | Ref. [3]  |
|---------|----------|---------|---------|-----------|
| \(t_0\) | -2648.19 | -2645   | -2931.7 | -2429.0   |
| \(t_1\) | 371.07   | 410     | 320.6   | 370.0     |
| \(t_2\) | -121.644 | -135    | -337.4  | -96.0     |
| \(t_3\) | 15553.495| 15595   | 18709.0 | 13773.0   |
| \(x_0\) | 0.60146  | 0.09    | 0.292   | 0.1       |
| \(x_1\) | 0.2697   | 0       | 0.653   | 0.0       |
| \(x_2\) | -0.23701 | 0       | -0.537  | 0.0       |
| \(x_3\) | 0.6968   | 0       | 0.181   | 0/0       |
| \(\alpha\) | 1/6     | 1/6     | 1/6     | 1/6       |
| \(W_0\) | 104.498  | 130     | 100     | 104.6     |

Skyrme and QMC models has been done in the case of \(N = Z\). As a result, the spin-exchange parameters, \(x_1\), \(x_2\) and \(x_3\), in the Skyrme force were set to zero for simplicity in the works [3, 12]. It is insufficient to restrict \(N = Z\) in the study of the isovector potential of the energy density functionals of these two models. The isovector potential is important in extrapolation to the calculations of any mass regions of nuclei. The parameters, \(x_1\) and \(x_2\), allow the adjustment of the contributions from protons and neutrons separately and therefore provide a powerful handle on the nuclear density distribution in heavy nuclei where protons and neutrons oscillate in opposite phase [13]. The parameter, \(x_3\), can significantly affect the symmetry energy [13]. Thus in our work, we have relax the restriction of \(N = Z\), and we obtain a full set of Skyrme parameters with spin-exchange parameters in Ref. [14] through comparison with energy density functionals similar as in Ref. [3].

For comparison, we list several sets of Skyrme parameters in Table. 1. We denote our results of Skyrme parameters as ”Sqmc”. We also list the parameters obtained in Ref. [3]. We can see that parameter \(t_0\), \(t_1\), \(t_2\), \(t_3\) in Sqmc and in Ref. [3] have similar values with SkM* while parameter \(x_1\), \(x_2\), \(x_3\) in Sqmc have similar values with SKP. With spin exchange parameters included, we can expect that the isovector channel of energy density functional has been improved in Sqmc. Thus we calculate binding energies of double magic nuclei without pairing to see to what extent the improvement can reach. We calculate binding energies for \(^{16}\text{O}, ^{40}\text{Ca},\) and \(^{208}\text{Pb}\) as in Table. 2. The results of Sqmc have better agreement with experimental data than those with parameters in Ref. [3], but are worse than those with SkM* and SLy4. The parameters in SkM* and SLy4 are obtained by fitting to the properties of finite nuclei while those in Sqmc are got through the QMC model whose parameters are from fitting the nuclear matter properties. Still, we can see the important contributions from spin-exchange parameters. We should also stress that our aim is not to give a new set of Skyrme parameters. There are already many sets with different advantages to describe the properties of finite nuclei. We want to show that a relation can be found between the Skyrme force and effective-meson-nucleon-exchange force.

3. Effective Mass
In mean-field models, the concept of effective mass is usually introduced to describe effects related to the nonlocality of the underlying nuclear interactions and the Pauli exchange effects in nuclear system [15]. The longstanding problem remains about the neutron-proton effective mass splitting, which is crucial for extracting the density dependence of symmetry energy from nuclear reaction using transport models [16, 17].
Table 2. Calculated binding energies for $^{16}$O, $^{40}$Ca, and $^{208}$Pb with different sets of Skyrme parameters.

| Skyrme force | $^{16}$O (MeV) | $^{40}$Ca (MeV) | $^{208}$Pb (MeV) |
|--------------|---------------|-----------------|-------------------|
| SLy4         | 128.457       | 343.741         | 1635.433          |
| SkM*         | 127.659       | 342.878         | 1636.874          |
| Ref. [3]     | 121.888       | 328.52          | 1563.12           |
| Sqmc         | 129.83        | 344.569         | 1610.862          |
| Expt.        | 127.616       | 342.052         | 1636.43           |

In Ref. [18], the authors have estimated the neutron-proton effective mass splitting in neutron-rich matter of isospin asymmetry at normal density derived from global nuclear optical potentials,

$$\frac{m_n^*-m_p^*}{m} = (0.32 \pm 0.15) \frac{N-Z}{A}. \quad (6)$$

The dominant contribution to the neutron-proton effective mass splitting is from the isovector channel of the energy density functional. In the case of QMC model, the non-relativistic effective mass can be extracted when one collects the \(\tau_f\) terms,

$$\frac{\hbar^2}{2m_f} \tau_f = \tau_f \left\{ \frac{\hbar^2}{2M} + \frac{G_\sigma}{2M^2} \left[ \frac{\rho}{1 + dG_\sigma \rho} + \frac{G_\rho}{4m_\rho^2} \right] + \frac{G_\rho}{8m_\rho^2} - \frac{G_\sigma}{2m_\omega^2} + \frac{G_\omega}{2m_\omega^2} - \frac{G_\sigma}{4M^2} \right\}. \quad (7)$$

The first two terms are the Hartree contribution while the remaining terms come from the Fock exchange. The label ”F” denotes neutron or proton. Thus we can derive the neutron-proton mass splitting from the above equation. We show the result at the saturation density of nuclear matter which is 0.16 fm$^{-3}$ in Fig. 1. In the figure we also show the upper and lower limit of neutron effective mass given by Eq. 6. We can see the neutron-proton effective mass splitting given by QMC-HF model is in good agreement with the estimation in Ref. [18]. It seems more acceptable recently that the effective mass of neutron is bigger than that of proton [18, 19].

4. Symmetry energy

The nuclear symmetry energy in dense nuclear matter remains to be understood [16, 20, 21, 22]. Energy functional calculations for the density dependence of the symmetry energy depend markedly on models and parameters employed [16, 23, 24]. It has been checked by Stone [25] that 27 of 87 sets of Skyrme parameters increase monotonically while 60 sets decreases to negative values. The RMF [26] and Brueckner-Hartree-Fock approaches [27] predict that the symmetry energy increases monotonically at all densities, while Dirac-Brueckner-Hartree-Fock [28] predicts that the symmetry energy can decrease at certain supra-saturation densities depending on the interactions used. So it is still encouraging to investigate the origin for the different behaviors of the symmetry energy at high densities.

In the Hartree approximation, the symmetry energy for the nuclear matter derived from QMC and RMF model have similar form. For example, symmetry energy of QMC model is,

$$e_{\text{sym}}^{\text{QMC-H}} = \frac{1}{6M} \left( \frac{3\pi^2}{2} \right)^{\frac{2}{3}} \rho^{\frac{5}{3}} + \frac{G_\sigma}{6M^2} \left( \frac{3\pi^2}{2} \right)^{\frac{2}{3}} \frac{\rho^{\frac{5}{3}}}{1 + dG_\sigma \rho} + \frac{G_\rho}{8} \rho. \quad (8)$$
Figure 1. (Color online) Calculated non-relativistic effective masses ($m^*/m$) of the proton and neutron against the isospin in the QMC calculations, compared with the upper and lower limit of neutron effective mass given by the estimated splitting of effective masses in Ref. [18]. The calculations are done at the saturation density of $\rho = 0.16$ fm$^{-3}$.

The symmetry energy of RMF model in the non-relativistic Hartree approximation is,

$$a^\text{RMF-H}_{\text{sym}} = \frac{1}{6M} \left( \frac{3\pi^2}{2} \right)^\frac{3}{2} \rho^\frac{3}{2} + \frac{1}{6} \left( \frac{3\pi^2}{2} \right)^2 \frac{G_\sigma}{M^2} \rho^\frac{5}{2} + \frac{G_\rho}{8} \rho. \quad (9)$$

In the above two equations, the first two term are from momentum dependence of isoscalar potential which can be restricted well by experiments [29, 30], thus they both have similar form. The last term is from the isovector channel of the potential. In the Hartree approximation, the only contribution to the isovector channel is from the $\rho$ meson. If more isovector mesons is considered, for example, the $\delta$ meson, the symmetry energy in RMF model becomes,

$$a^\text{RMF'-H}_{\text{sym}} = \frac{1}{6M} \left( \frac{3\pi^2}{2} \right)^\frac{3}{2} \rho^\frac{3}{2} + \frac{1}{2} \left( \frac{3\pi^2}{2} \right)^2 \frac{G_\sigma}{3M^2} + \frac{G_\sigma}{M^2} \rho^\frac{5}{2} + (\frac{G_\rho}{8} - \frac{G_\delta}{2}) \rho. \quad (10)$$

Still, the density dependence of the last term is linear. We can add more correlations into the potential to see whether density dependence can be changed. Thus if one add the Fock exchange in QMC model, the symmetry energy is,

$$a^\text{QMC-HF}_{\text{sym}} = \frac{1}{6M} \left( \frac{3\pi^2}{2} \right)^\frac{3}{2} \rho^\frac{3}{2} + \frac{1}{3} \left( \frac{3\pi^2}{2} \right)^2 \left( \frac{G_\omega}{m_\omega^2} - \frac{G_\sigma}{m_\sigma^2} \right) \rho^\frac{5}{2}.$$
Figure 2. Symmetry energy curves calculated by the QMC model in the Hartree and Hartree-Fock approximations. We also show symmetry energy curves with dashed lines obtained by $E_{\text{sym}}(\rho) = E_{\text{sym}}(\rho_0) \left( \frac{\rho}{\rho_0} \right)^\gamma$ using different values of parameter $\gamma$.

\[
E_{\text{sym}}(\rho) = E_{\text{sym}}(\rho_0) \left( \frac{\rho}{\rho_0} \right)^\gamma + \left[ \frac{5G_\rho}{32} + \frac{G_\sigma}{8(1 + d\rho G_\sigma)^3} - \frac{G_\omega}{8} \right] \rho. \tag{11}
\]

The last term becomes a complicated density-dependent form, which is caused by the scalar meson. From Eq. (1), we can learn that if the substructure of nucleons is considered, the effective mass becomes non-linear with the scalar field, which is also the main source to bring a nonlinear density-dependence into the symmetry energy through the Fock exchange.

To describe the trend of symmetry energy curve, one can use the single-parameter formula to characterize the softness of symmetry energy curve [16],

\[
E_{\text{sym}}(\rho) = E_{\text{sym}}(\rho_0) \left( \frac{\rho}{\rho_0} \right)^\gamma, \tag{12}
\]

where the parameter $\gamma$ is used for adjusting the softness of the curve. It is soft if $0 < \gamma < 1$ and it is stiff if $\gamma \geq 1$. In Fig. 2, we show the symmetry energy curve given by QMC model in the Hartree and Hartree-Fock approximation. We also show the symmetry energy curve given by Eq. (12). We can see that for the symmetry energy curve of the QMC-Hartree model, $\gamma \approx 1$ and for that of QMC-Hartree-Fock model, $\gamma \approx 0.5$ at subsaturation density and become even softer at supersaturation density. Thus the inclusion of the Fock exchange gives a strongly density-dependent symmetry energy. A soft symmetry energy curve has been supported by the recent analysis of experimental data [24]. Combined with the analysis of the formulas of symmetry energy curve, Eqs. (8)-(11), we could conclude that because of the polarization effect
of the nuclear medium the scalar meson can soften the symmetry energy curve through the Fock exchange.

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