Characteristics of electric quadrupole and magnetic quadrupole coupling in a symmetric silicon structure

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Keywords: Mie theory, quadrupole, Fano resonance, EIT, coupling

Abstract

Multipole interferences have attracted a lot of interests in last decade due to extraordinary performance on beam control and scattering shaping. However, most of previous works focused on the dipole-based interferences while the quadrupole modes and other high-order multipole modes with unique properties were of less attention. In this work, we aim to expand the present dipole-based multipole-interference regime to the quadrupole-interference regime. We study the interference between an electric quadrupole (EQ) and a magnetic quadrupole (MQ) in both isolated and periodically arranged homogeneous cross dielectric structure. Through structural parametric control, the EQ and MQ can be precisely tuned to share the same resonant intensity at a specific wavelength, resulting in a generalized Kerker effect. Moreover, a dark MQ mode, which is orthogonal with the original MQ mode, arises when we increase the interaction between structure. We find that the spectral approaching between dark MQ and original bright EQ results in an EIT effect and Fano-shaped spectral reflection response. The induced Fano spectrum possesses tunable quality factors varying from ~10 to >10⁵ with the variation of EQ–MQ coupling efficiency. The numerically derived maximum quality factor (238, 618) of the dielectric EQ–MQ coupling system even exceeds the quality factors of many plasmonic resonant systems. We also prove that such spectrum can be adopted to refractive index sensing. Besides, we show that EQ–MQ coupling can bring about rapid 2π phase change, which can be applied in metasurface designs. These results and conclusions about the EQ–MQ interference systems can provide a promising avenue for advanced optical devices.

1. Introduction

Optical resonances are the cornerstone of nowadays photonics and plasmonics. They are widely employed in real applications as the building blocks of sophisticated optical devices. Among different categories of optical resonances, Mie-type resonance [1] is a fundamental resonance that exists in all natural objects in spite of their shapes, sizes, and materials. Mie theory reveals the resonant nature of particles. It shows that electric multipole modes, as well as magnetic multipole modes, can be induced in particles by external electromagnetic environments. These electric and magnetic multipole modes produce confined fields and determine the radiation pattern of a particle. It has been found that particular optical phenomena, like directional scattering, could be derived by controlling the shape, material and exciting electromagnetic environment of a particle. Based on this idea, functional nano-particles and structures have been designed for various fundamental applications, like fluorescence enhancement, nanoscale photodetectors, high-harmonic generation, and single-molecule detection [2].

Among various studies about multipole modes, the interferences between two multipole modes are of particular interest. Spatial and spectral overlapping between different types of Mie modes leads to tunable scattering patterns. It is well known that directional scattering occurs when the resonating intensities of an
electric multipole and a same-ordered magnetic multipole are equal. Zero-backward scattering condition, first proposed by Kerker et al in 1983 [3], requires a similar electric and magnetic property, namely \( e = \mu \). Multipole interference induced directional scattering performances are of crucial importance to functional optical devices and structures like directional nano-antennas [4–9], generalized Brewster effect [10], Huygens metasurfaces [9, 11–13], and sensing devices [14]. Present publications have achieved much progress in realizing multipole interferences and directional scatterings. Theoretical and experimental results have already shown that interferences between an electric dipole (ED) and a magnetic dipole (MD) [11–13, 15], an ED and an electric quadrupolar (EQ) [5], a MD and an EQ [16, 17], are possible to realize in dielectric or plasmonic structures.

Although multipole interferences have been theoretically and experimentally studied a lot in recent decades [9, 11–21], the realization of quadrupole interference, especially the interference between two different quadrupoles in homogeneous optical structures can still be challenging. Like dipole interferences, high-ordered multipole interferences requires a same intensity for the two resonating multipoles at same wavelength. It is well known that high-ordered multipoles are often more sensitive to the geometries of nano-structures, because the electromagnetically induced currents and charges for exciting these multipole modes need 2-dimensional or even 3-dimensional spacial distributions. Present works about EQ–magnetic quadrupole (MQ) interferences often adopt material-design strategies. Spherical particles with inhomogeneous material [22], or radial-anisotropy material [23] are designed to achieve the EQ–MQ interference. However, applicable optical structures often prefer flat geometries and consistent homogeneous materials. These non-flat structures are difficult to be applied in real optical systems. Material designs can lead to more producing complexity. It’s meaningful to realize EQ–MQ interference in the flat homogeneous structures.

Except for traditional Mie-type multipole interferences, there are also dark multipoles excited in some particular nano-structures, like non-concentric ring/disk cavities [14, 24, 25], dolmen-type slabs [20, 26, 27], dielectric metasurfaces [28–30], cut plane or split ring/sphere structures [31, 32], and aggregates. Most of these structures adopt symmetry breaking processings. Such arrangement produces dark dipole modes or quadrupole modes to couple with the super-radiant dipole continuums, resulting in EIT or Fano-like spectral responses [33]. With asymmetry structure design, extremely narrow-banded transmission spectrum has been realized [20, 28]. In these works, dark multipole modes can be designed to be orthogonal to the Mie-type multipoles. It enables the near-field coupling between two multipoles. For example, EQ–MQ coupling has been realized in plasmonic structure recently [19] to realize plasmonic EIT effect. Through elaborately design, dark quadrupoles can also arise in dielectric structures to couple with bright quadrupoles. In recent years, high-ordered multipoles, especially quadrupoles have been proved to possess impressive nano-scale beam control and radiation shaping abilities. Researches have shown that quadrupoles can be applied in magnetic mirror [34], wavelength router [35], absorber [36], etc. The researches about quadrupole interferences can pave new avenues for traditional optical devices.

In this paper, we introduce the realization of both the far-field and the near-field interference between EQ and MQ in homogeneous cross structures. Firstly, we show that the resonant intensities and wavelengths of EQ and MQ can be well controlled by adjusting the geometry of the structures to realize directional scattering. Further, we prove that the EQ–MQ spectral overlapping can also be realized in periodical array, characterized by the spectral EIT effect. We find that the array effect can also produce a dark MQ mode, based on which the EQ–MQ near-field coupling can be realized. We explain the reason why the reflecting responses are akin to EIT- or Fano-like shape in the view of the multipole interference. We reveal the underlying physics of the extremely high quality factors of the proposed structure using the coupled-oscillators model. In the end, we present that the EQ–MQ coupling can induce other effects like 2–\( \pi \) phase change, which can be used for many other applications, like metasurface, and sensing.

### 2. Structure and design

#### 2.1. Quadrupoles in isolated spherical particles

With high symmetries, spherical particles are used here to present multipolar resonant profiles. The EQ and MQ resonant wavelengths and intensities of spherical particles can be analytically derived from Mie theory [1]. For the electric and magnetic quadrupole modes excited in spheres, their polarizabilities can be analytically calculated though the following equations [37]

\[
\alpha_{\text{EQ}} = \frac{120\pi \varepsilon_0 \varepsilon_i}{k_0^3} d_2^3
\]

\[
\alpha_{\text{MQ}} = \frac{40\pi}{k_0^3} b_2^3
\]

(1)
where $a_1$ and $b_2$ are the Mie coefficients, $\varepsilon_r$ is the permittivity of the surrounding medium, $k_0$ is the wave number in the surrounding environment. Our calculation for the Mie coefficients related in this paper are operated in MATLAB through an open-accessed code [38]. EQ and MQ resonance occurs when $\text{Re}(\lambda_{EQ}) = 0$ and $\text{Re}(\lambda_{MQ}) = 0$, respectively. By inserting equation (1) into the EQ and MQ resonant conditions, one can derive a series of solutions for EQ and MQ resonant conditions with respect to the size parameter $x$. The first-ordered solutions, namely $x_{1E}$ and $x_{1M}$, represent the first-order EQ and MQ resonances, namely the EQ and MQ resonance which we often refer to. According to the definition of the size parameter $x$ [1], EQ and MQ resonant wavelengths can be expressed as:

$$
\lambda_{EQ} = \frac{2\pi}{x_{1E} a}
$$

$$
\lambda_{MQ} = \frac{2\pi}{x_{1M} a}
$$

Equation (2) indicates that the EQ and MQ resonant wavelengths $\lambda_{EQ}$ and $\lambda_{MQ}$ of a spherical particle are proportional to the sphere radius $a$ with a slope of $2\pi/x_{1E}$ and $2\pi/x_{1M}$, respectively. In figure 1(a), we plot the EQ and MQ resonant wavelengths $\lambda_{EQ}$ as a function of the radius of an artificial sphere with a refractive index of $n = 3.5$. It shows a linear relationship between the sphere radius and the quadrupole resonant wavelengths. The slopes of the EQ and MQ resonant wavelengths with respect to the sphere radius in figure 1(a) is 4.07 and 5.05, respectively, which indicates that the MQ resonance redshifts faster than the EQ resonance with the increment of the sphere radius. Hence the spectral overlapping between EQ and MQ cannot be realized for such homogenous spherical particles through size control. Indeed, other simple homogenous structures like cubes, and low-aspect cylinders, have close quadrupole resonant performances as the spherical particles when their volumes are close. At EQ and MQ resonances, the electric and magnetic field distributions are plotted in figures 1(b)–(e). The electric field and magnetic field distributions show the EQ and MQ resonant characteristics at the corresponding resonances, where the dipolar modes or other higher-ordered modes are weak and contribute little for the electric and magnetic field intensities.

2.2. Quadrupoles in isolated cross particles

To realize EQ–MQ spectrally overlapping, other structures should be taken into consideration. In this section, we will analysis the quadrupole excitations in a cross particle. The cross structure used here is plotted in figure 2(a). The geometric parameters have been marked on the sketch. Of particular importance is the height $h$ of the cross. Generally, quadrupole moments can be considered as a couple of anti-parallel dipole moments [39, 40], which requires enough space in the $z$-direction to accommodate such arrangement. In this view, the height $h$ should be high enough to support the quadrupole modes.

The cross structure can be regarded as the superposition of two orthogonal rectangular arms parallel to the $x$- and $y$-axis, respectively. Here, we use the term X-arm and Y-arm to differentiate the two arms, shown in figure 2(b). The incident light is set as a plane wave polarized along the $x$-axis, propagating along the negative $z$-axis. With these arrangements, the EQ moment is supposed to be excited in the X-arm of the cross, while the MQ moment is supposed to be excited in the Y-arm of the cross, plotted in figure 2(b). The orthogonality of the two arms ensures relatively independent variation for the EQ moment and the MQ moment when structure changes. For example, when only the X-arm length increases, it influences mainly for the excited EQ moment while the MQ moment, which is excited in the Y-arm, changes slightly. Consequently, the EQ resonance spectrally changes prominently while the MQ resonance spectrally stays barely unmoved. Such effect can lead to the overlapping between EQ and MQ resonance. This is the main reason why the cross structure are chosen here.

![Figure 1](image1.png)

**Figure 1.** (a) EQ and MQ resonant wavelengths as a function of sphere radius. The refractive index of the silicon sphere is set as $n = 3.5$. (b) Electric field distribution and (c) magnetic field distribution at EQ resonance. (d) Electric distribution at and (e) magnetic field distribution at MQ resonance. The incident electric field is along the $x$-direction. The inner circle in (b)–(e) depicts the profiles of the spherical particles, out of which is air. Results (b)–(e) are derived from COMSOL in the scattering field environment.
Indeed, other geometries, like cuboid, and cylinder, can also have the same effect, only that such effect is relatively implicit in these structures.

For the proposed structure, analytical Mie theory is no longer efficient in numerically estimating quadrupole resonant intensities. Here, we use an electromagnetic multipole expansion (EME) method \[41\] to acquire the multipole coefficients. Detailed calculation process can be found in our former work \[36\]. The current density distribution and the integral operations required in the EME process are calculated by the numerical simulation software COMSOL Multiphysics.

Figure 3 shows the calculated quadrupole resonant wavelengths and intensities with the variation of structural parameters. In the figures 3(a) and (b), the quadrupole intensities and wavelengths are plotted with the variation of the X-arm length \(l_x\). Apparently, EQ shows a more active response to the parameter with a fast increment in intensity and a rapid redshift in resonant wavelength. MQ mode, however, changes slowly in both intensity and wavelength. The inconsistent variation results in an intersection for both the quadrupole intensity and wavelength at around \(l_x = 600 \text{ nm}\), marked on figures 3(a) and (b). The coincidences of both resonating intensities and resonating wavelengths indicate that the general Kerker condition resulting from EQ–MQ interference is satisfied. In figures 3(c) and (d), we further plot the quadrupole intensities and wavelengths varying with another parameter \(w_y\). Apparently, this parameter effectively controls the resonant wavelength of the MQ mode, which is near invariant when parameter \(l_x\) changes. These results prove that quadrupole excitations in the cross structure can be flexibly controlled to the desired situations. The EQ–MQ resonating

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overlapping can also be realized in other wavelength ranges. The method is effective for controlling the excitations of dipole and maybe other high-order multipoles.

In the figure 4(a), we plot the multipole scattering components of the structure at the EQ–MQ overlapping wavelength. Apparently, the two quadrupoles perfectly coincide at around 1150 nm, where ED contribution is negligible while MD contributes a little for the total scattering cross section. Figure 4(b) shows the 2D total scattering patterns at both the xz plane and the yz plane. More intuitive 3D scattering patterns are plotted at the right side of the figure 4(b). These patterns coincide with the theoretical results [42]. It is worth noting that the side lobes of the scattering pattern in the y–z plane is relatively larger than those in the x–z plane. It results from the superposition of a weak MD mode, namely the dotted cyan line in figure 4(b). The weak MD moment occurs in the Y-arm. Hence it mainly influences the scattering pattern in the y–z plane.

2.3. EQ–MQ interference in particle array

2.3.1. EQ–MQ interference

In periodical arrays, multipole modes exist in the form of coupled multipole modes. The couplings of multipoles consist of two kinds of interactions. One is the self-interacting effect, which denotes the self-coupling behaviour of a certain type of multipole. The other is the mutual-interference mode, which refers to the interactions between two different-type multipoles. Both the self-interacting and mutual-interacting effect affect the total spectral response of the arrays. Since multipole modes are spectrally separated, the mutual-interacting effects are often shown between spectrally neighboured multipoles, and they leads to little spectral shift for multipoles when the two neighbouring multipoles are spectrally separated. Hence, the self-interacting mode are considered to be the major interference effect to evaluate the spectral change of multipoles here. In this study, quadrupole modes are the main resonant modes in the operating wavelength (around 1200 nm), while dipole modes and the higher ordered multipole modes are omitted due to their destructive self-interacting effects, which lead to little spectral reflecting and transmitting contribution for quadrupole modes at the operating wavelength.

In a simpler case when the particles are spherical, all particles in the array will have the same quadrupole polarizabilities under the normal incidence. The EQ and MQ self-coupling modes can be modelled by the multipole coupling theory, where the multipoles are treated as point multipoles with the same electric or magnetic polarizabilities. The effective quadrupole polarizabilities in the spherical particle array and the quadrupole polarizabilities in an isolated particle have the following relationship [37]

\[
\begin{align*}
\frac{\varepsilon_0}{\alpha_{\text{eff}}^\text{EQ}} &= \frac{\varepsilon_0}{\alpha^\text{EQ}} - \frac{k^2 G^\text{Q}}{2}, \\
\frac{1}{\alpha_{\text{eff}}^\text{MQ}} &= \frac{1}{\alpha^\text{MQ}} - \frac{k^2 G^\text{Q}}{2},
\end{align*}
\]

where \(\alpha^\text{EQ}\) and \(\alpha^\text{MQ}\) are the quadrupole polarizabilities in an isolated spherical particle, \(\alpha_{\text{eff}}^\text{EQ}\) and \(\alpha_{\text{eff}}^\text{MQ}\) are the effective quadrupole polarizabilities in the spherical particle array. The term \(k^2 G^\text{Q}/2\) counts the quadrupole sum in the particle array and is determined by the array geometry and the surrounding medium, both of which are independent on individual particle properties. The EQ and MQ resonances in the spherical particle array occurs when \(\text{Re}(\varepsilon_0/\alpha_{\text{eff}}^\text{EQ})) = 0\), and \(\text{Re}(1/\alpha_{\text{eff}}^\text{MQ}) = 0\), respectively. The spectral reflection and transmission coefficients are related with the effective EQ and MQ polarizabilities in the array. In general, the coupling effect in the array will leads to spectral shifting for the quadrupoles [43]. Ignoring the influence of other multipoles, the relation can be written as [44]...
Equation (4) indicate that the EQ and MQ arisen in the array bring about spectral reflection responses in spherical particle arrays. However, such conclusion is valid only for spherical particles, since its quadrupole polarizabilities can be treated as scalar values. In the cross particles, the quadrupole polarizabilities are tensors, which leads to a more complicated expression for the multipole coupling.

To get an intuitive understanding for the multipoles in the cross array, multipole contributions of an isolated cross particle and the reflection of its array is plotted in figure 5(a). In the figure, the dotted lines show the multipole excitations in the isolated particle, while the solid line denotes reflection spectrum resulting from the multipole excitations in the particle array. Numerical studies of the spectral reflection is performed in the FEM-based CST Microwave Studio. The cross particle is placed in a unit cell with a periodicity of 1000 nm in both x- and y-direction. TM-polarized plane wave are imposed on the structure along the negative z-direction. Meshes are generated through the build-in algorithm with a density order of 14, ensuring that the result is independent with the mesh size.

In figure 5(a), the reflection spectrum have four obvious resonant peaks, corresponding to four types of multipoles marked on the figure, namely MD, ED, MQ, and EQ. The result reveals that the spectral reflection peaks stem from the multipoles inside the cross particle. From the multipole excitation spectrums to the reflecting spectrum, MQ and EQ have a little spectral shift, which indicates the self-interacting effects of quadrupoles have very limited influence on the quadrupole resonant wavelengths. However, large spectral shifting for the dipole resonances reveals that dipole modes suffer from strong self-interaction influences. The conclusion coincides with our former work [36]. In figures 6(a) and (b), we plot the electric and magnetic field vector of the periodical structure at EQ and MQ resonance, respectively. The EQ electric vector in figure 6(a) shows obvious coupling effect between two neighbouring elements, as noted by the red dotted box. However, the MQ magnetic vectors in the neighbouring elements barely couple with each other. Consequently, the self-coupling effect leads to slight difference between the EQ resonant wavelength in the particle and the EQ resonant wavelength inside the particle array, while it barely leads to spectral difference for the two MQ resonances. The result coincides with the quadrupole spectral differences in figure 5(a).
Since the quadrupoles spectrally changes slightly with the influence of array effect, structural parameter control for realizing EQ–MQ spectral overlapping in the periodical array is possible. In the figure 5(b), the spectral reflection of the array varying with the x-arm length of the structure is plotted. The outstanding two peaks, corresponding to the EQ and MQ resonance, redshift at different speeds with the variation of the structural parameter. Consequently, the two peaks merge, and a redshift at different speeds with the variation of the MQ moment in the y-axis direction at position A in figure 7. The 2D field distributions here are plotted at the x = 0 plane.

2.3.2. EQ–MQz coupling
In the periodical array, dark modes can arise due to the near-field coupling between elements. Near-field coupling can be quite interesting, since it leads to distinct different phenomena. To well excite the dark quadrupole mode, we adopt a smaller periodicity for the array geometry to enhance the interaction between neighbouring elements. After adjusting the geometry of the cross structure, the dark MQ mode can be excited in the structure. In figure 7(a), the normalized single-element multipole contributions and the spectral reflection of the element array are plotted. The parameters of the new cross structure are noted in the caption of figure 7.

In the figure 7(a), the Mie-type MQ resonance is supressed and it contributes little for the reflection spectrum. Meanwhile, an abnormally existed Fano peak occurs near the EQ reflection peak. In figure 7(c), the magnetic field direction is plotted at the resonance. It shows a MQ resonance resonant pattern with negligible coupling field distribution in the space between neighbouring elements. The occurrence of the dark MQ mode stems from the near-field coupling between neighbouring elements.

Indeed, this MQ resonant pattern is different from the original Mie-type MQ resonant pattern in figure 6(b), where the MQ moment is along the y-direction, namely the direction of the incident magnetic field. The MQ moment in the figure 7(c) is along the z-axis. Figure 7(b) is a sketch for a better understanding of the difference between the two MQ resonances. Both of the MQ moments are in the y–z plane, while their directions are

Figure 7. (a) Normalized multipole contribution of an isolated cross particle and the reflection of the particle array. Variable $l_x$ is set as 500 nm. Other parameters change into $h = 500$ nm, $w_x = w_y = 230$ nm, and $l_z = 460$ nm. The periodicity of the array here is set as 725 nm. (b) A sketch of the two MQ moments arising in two cross structures. (c) Magnetic field direction at MQz resonance. (d) Electric field direction at (d) position B and (e) position C in figure 7(a). (f) Electric field direction at position A in figure 5(b). The 2D field distributions here are plotted at the x = 0 plane.
To literally differentiate the two MQ resonance modes, we use the term MQ and MQz to respectively represent the two MQ resonances. The MQz mode has also been reported in other publications [40]. Compared with the original Mie-type EQ–MQ interference depicted in the figure 5, the EQ–MQz interference shows some distinct differences. First of all, their spectral reflection responses are quite different. Due to orthogonality, EQ mode cannot couple with the Mie-type MQ mode. The quadrupole reflection peak in the figure 5(b) are almost symmetric and do not interfere with each other. Destructive interference occurs only at the EQ–MQz overlapping position. However, the spectrum in the figure 7(a) shows prominent Fano-type characteristic. Obvious destructive interference occurs at the right side of the MQz peak, namely position C marked on the figure 7(b). It is often illustrated that such asymmetric Fano spectrum is accompanied by the near-field coupling effect [46].

Another crucial difference is that the internal field distributions of the two interference systems are different. In the figure 7(f), we depict the electric field directions at the EQ–MQ overlapping position, namely position A in the figure 5(b). Apparently, the electric currents distribution still shows an EQ pattern. The original EQ mode is reserved at the overlapping position, which confirms that the destructive interference in position A results from the far-field interference. However, the EQ–MQz interference possesses two states, the destructive interference and the constructive interference, separated by the MQz peak. In figure 7(a), the destructive interference leads to a reflection dip at the position C, while the constructive interference results in high reflection at the position B. The corresponding electric field directions are plotted in the figures 7(d) and (e). The directions of the current at the two positions are obviously opposite, indicating there are two opposite interference modes near the Fano peak, which is the common characteristics of the Fano resonance [47]. Besides, the electric currents distributions in the figures 7(d) and (c) is no longer akin to the EQ pattern. They are also different from the EQ–EQ coupling modes in the figure 6(a). The result means that the EQ mode truly couples with the MQ modes in the form of near-field coupling.

The reflection responses of the cross array varying with the X-arm length are plotted in figure 8(a). The EQ and MQz resonances induced reflection peaks are marked on the figure. The variation of the EQ and MQz resonant wavelengths shows a similar trend in the cross structure as in the figure 5(b). Both the EQ and MQz resonances redshift with the increment of the X-arm length l_x, while the EQ resonant peak shifts much faster than the MQz resonant peak. As a result, there is an intersection of the EQ and MQz resonances. Near the intersection area, a black circle, which roughly depicts the spectral strong-coupling (SC) area, is marked on figure 8(a).

In this area, the coupling between the two quadrupole modes are strong, since they are both in resonant or quasi-resonant state. Their coupling performs strong destructive interference, resulting in a forward scattering performance. Consequently, the two closely located near-unit quadrupole resonant peaks merge together and produce a near-zero reflection dip. The reflection spectrum at the intersection of two quadrupole modes is plotted in figure 8(b). There is a reflection dip appearing at the EQ resonant peak around 1.2 μm. The near-unit transparent effect, induced by the coupling between an EQ mode and an MQz mode, is also known as the electromagnetically induced transparency (EIT) effect.

When the EQ and MQz are relatively spectrally separated, the EQ mode is no longer in resonant state near the MQz resonance. This coupling mode is represented as the weak coupling (WC) mode. The weak-coupling area is depicted in the figure 8(a) with a red circle. Extracted reflection spectra varying with the X-arm length at the WC area are plotted in figure 8(c). Instead of the EIT effect, the reflection shows Fano-like spectral shapes with extremely narrow bandwidths (less than 1 nm). During simulation, a 0.001 nm wavelength resolution is adopted to well capture these extremely narrow-banded Fano-shaped spectra. Such resolution is unachievable for most of experimental devices.

The coupling effect in the WC area is of much difference with that in the SC area. In the figure 9, sketches are plotted to explain these coupling effects. In the SC area, the EQ and MQ resonant intensities are strong at their

**Figure 8.** (a) Reflection spectrum of the cross array varying with the X-arm length. SC and WC represent the strong coupling area and the weak coupling area, respectively. (b) EIT effect at X-arm length l_x = 0.55 μm in the strong coupling (SC) area. (c) Executed extreme narrow-banded Fano-shaped reflection spectra in the weak coupling (WC) area.
common resonant wavelength, shown in Figure 9(b). The phases of the both quadrupole modes changes by $\pi$ at the resonance, representing as ‘+’ and ‘−’ at the both sides of the resonant peaks. However, the phases of the EQ mode and the MQ mode is opposite, leading to a forward scattering performance near the resonance where their scattering intensities are close. The forward scattering performance gives rise to the enhancement to the transmission of the array. Consequently, the resonant coupling between EQ and MQ modes produces a destructive effect over the reflection of the structure, shown in Figure 9(e).

Unlike the resonant coupling mode in the SC area, the EQ mode in the WC area is in non-resonant state. The phase of the MQ mode changes by $\pi$ at the resonance, while the phase of the EQ mode changes slowly, as shown in Figure 9(c). As a result, backward scattering occurs on the one side of the MQ resonant peak where the EQ and MQ mode are in phase, while forward scattering occurs on the other side where the EQ and MQ mode are out of phase. The backward scattering promotes the reflection of the structure, while the forward scattering improves the transmission of the structure. Consequently, there is a constructive effect on the one side of the MQ reflection peak, while a destructive effect on the other side of the MQ reflection peak. Therefore, the MQ resonant reflection peak shows a Fano-like asymmetric shape, shown in Figure 9(f). At the weak coupling case, there is a dramatic variation from the backward to forward scattering with a small frequency change of the incident light, which is also a characteristic of the Fano resonance [46].

Compared with the EIT effects resulting from the resonant couplings between one dipolar mode and one multipolar modes [28], the EIT effects derived in this paper in Figure 8(b) have a relatively smaller quality factor (about 240). Figures 7(a) and (b) depicts the general scheme of the intensities of the bright and dark modes in two resonant coupling systems. At both cases, the signs of the bright mode and the dark modes are always opposite at the both sides of the resonant peak, indicating that there is only one directional scattering, namely the forward scattering performance, exists. For the bright dipolar modes, they often designed to be in the super-radiant state and have much broader resonant bands. Their damping rates are near zero at the dark-mode resonance. The bright quadrupole mode have a relatively narrow resonant band and it decays apparently like the dark quadrupole mode as shown in Figure 9(b). Therefore, the intensity of the bright mode and the dark mode is close in a broader wavelength region for the quadrupole case instead of the dipolar case, resulting in a broader transmission band for the bright quadrupole-based EIT effect, shown in Figures 9(d) and (e).

2.4. Coupled oscillators model for analyzing the Fano lineshape and quality factor
The Fano resonance caused by the EQ and MQz weak coupling in the cross array also can be qualitatively expressed by applying the linearly coupled Lorentzian oscillators described by the following matrix equation:

$$
\begin{pmatrix}
\omega_0 - \omega - i\gamma_{EQ} & \kappa \\
\kappa & \omega_0 - \omega - i\gamma_{MQz}
\end{pmatrix}
\begin{pmatrix}
\tilde{x}_{EQ} \\
\tilde{x}_{MQz}
\end{pmatrix}
= -i (gE_0)
$$

where $\omega_0$ is the central resonant frequency of the Fano resonance, $\gamma_{EQ}$ and $\gamma_{MQz}$ are the damping rates of the two quadrupolar modes, $x_{EQ}$ and $x_{MQz}$ are the quadrupole magnitudes, $E_0$ is the incident field, $g$ is a parameter indicating the coupling strength between the bright EQ mode and the incident field, and $\kappa$ is the coupling coefficient and denotes the interaction between the EQ and MQ mode. In the coupled-oscillators model, the EQ
resonance functions as the broadband continuum oscillating state while the MQz resonance functions as the narrowband discrete oscillating state. In a loss-less particle array, the damping $\gamma_{EQ}$ and $\gamma_{MQz}$ comes from only the scattering loss due to coupling to other modes. The Fano resonance occurs near the position of the MQz resonance. The amplitude of the MQz resonance can be presented as

$$|\tilde{x}_{MQz}|^2 = \left| \frac{-gE_0}{(\omega - \omega_0 + i\gamma_{EQ})(\omega - \omega_0 + i\gamma_{MQz}) - \kappa^2} \right|^2.$$  

(6)

When the coupling between the EQ and MQz resonance is weak and the system satisfies $\kappa^2 \ll \gamma_{EQ} < \gamma_{MQ} < \omega_0$, the amplitude of MQz resonance can be rewritten as

$$|\tilde{x}_{MQz}|^2 \approx \left| \frac{-gE_0}{\omega - \omega_0 + i\gamma_{MQz}} \right|^2,$$

(7)

which is akin to the Fano line shape in the form of [28]

$$f(\omega) \approx \left| \frac{a_1 + ja_2}{\omega - \omega_0 + i\gamma} \right|^2,$$

(8)

when the real constant $a_1 = a_2 = 0$. According to equation (4), the spectral reflection can be presented as

$$R = |r|^2 \propto |\tilde{x}_{MQ}|^2,$$

(9)

at the vicinity of the MQ resonance where the EQ mode is weak. Therefore, the reflection response of the proposed periodical array has the Fano-like line shape.

For the Fano model in the equation (8), the Q-factor is calculated as $Q \approx \omega_0 / 2\gamma$. $\omega_0$ and $\gamma$ for each line shape are calculated through a curve-fitting method in MATLAB. In figure 10(a), the Q-factors of the proposed cross array is plotted as a function of the X-arm length. The Q-factors vary from dozens to over tens of thousands with the variation of the X-arm length. Maximum quality factor is about 238, 618 when the $l_x = 390\, \text{nm}$. At the weak coupling regime (0.35 $\mu\text{m} < l_x < 0.45\, \mu\text{m})$, the Fano resonance possess extraordinarily high Q-factor, exceeding the theoretical Q-factors of many other coupling structures [19, 26, 28]. In the view of coupled-oscillator model, achieving high Q-factors Fano resonance requires both $\gamma_{MQz} \rightarrow 0$ and $\kappa \rightarrow 0$ [28]. In the proposed coupling systems, periodical arrangement of the cross particle suppresses the MQz radiative loss. The damping $\gamma_{MQz}$ of the MQz mode mainly results from the coupling damping. The condition $\gamma_{MQz} \rightarrow 0$ is satisfied at the weak coupling area, where the coupling between EQ and MQz mode is weak and the condition $\kappa \rightarrow 0$ is also satisfied.

Therefore, extremely high Q-factors are obtained in the weak coupling area. The adjustable spectral separation between EQ and MQz resonances controls the EQ–MQz coupling efficiency. In strong coupling area, the coupling coefficient $\kappa$ is on longer near zero. The strong coupling effect leads to a large $\gamma_{MQz}$ for the MQz mode. As a result, the condition $\gamma_{MQz} \rightarrow 0$ and $\kappa \rightarrow 0$ are invalid, and the Q-factors of the Fano spectra in the strong coupling area (0.5 $\mu\text{m} < l_x < 0.55\, \mu\text{m})$ is small as depicted in the figure 10(a).

In figure 10(b), we plot the sensing performance of the cross array at high-Q condition. Varying background index, the Fano-resonance peaks redshifts accordingly. The reflecting efficiency maintains near 100%. Index sensitivity is about 160 nm/RIU. Compared with plasmonic sensors, the all-dielectric structure here shows a weaker sensing performance. This can be attributed to that the Mie resonances is less sensitive to the background index than the surface plasmon polaritons. However, such structure can be applied in circumstances where dielectrics are preferred and low sensitivity performance is enough to meet the demand.

In general, the quadrupole modes possess weaker resonant intensities and narrower resonant bandwidths than the dipolar modes in regular dielectric structures. These intrinsic characters leads to a weaker coupling intensity for the quadrupole–quadrupole coupling system and result in an extremely narrow-band spectral response.

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**Figure 10.** (a) Quality factor of the reflection response of the proposed cross array as a function of the X-arm length. (b) Reflection responses varying with background index. The X-arm length is set as 385 nm.
Fano-like response. Besides, due to the bandwidth of bright quadrupole mode is also narrow, dozens of nanometers change over the structure geometric parameter can bring about the spectral transformation from an EIT effect to a high-Q Fano response, namely from a totally transmitting state to a totally reflecting state, in a small spectral variation. This property can be useful in functional metasurfaces, like optical switching.

Like the dipole-based overlapping system, the EQ–MQz resonant coupling produces EIT effect, accompanied with a swift phase change around the resonant peak. In figure 11, we plot the phase and transmission of the transmitting light at EQ–MQ resonant coupling. The transmission and phase-change rate reaches their maximum at the wavelength where EQ and MQ resonance overlaps. As depicts in figure 9(b), the EQ resonance have a broader bandwidth than the MQ resonance. As a result, the EIT effect happens only in very limited bandwidth. Therefore, the transmission of the EQ–MQ coupling system is near-unit in about tens of nanometers as depicted in figure 9. Such coupling system is not competent to design high-efficiency transmitting Huygens’ metasurface. However, it can be used for reflecting Huygens’ metasurface, as there is $2\pi$ phase change in the coupling system.

Besides, another characteristic of the quadrupole excitation is that the higher ordered Mie resonant modes are excited in the shorter wavelengths while the lower ordered Mie resonant modes occurs at the longer wavelength for dielectric structures. To be excited at the same wavelength as the lower wavelength, the higher ordered Mie resonances requires larger geometric sizes for exciting structures. Such character indicates that functional devices realized by the higher ordered Mie resonant couplings possess larger geometric sizes than their counterpart. The quadrupole-based structures can release dipole-based functional devices from the deep sub-wavelength sizes design, which reduces producing-robust requirement. In addition, the EQ and MQ modes can promote light-absorbing ability for a lossy particle \([36]\). The spectral control of the EQ and MQ excitation can be applied to intentionally manipulate the absorbing and scattering performance of a particle, which has been realized for dipole modes \([48]\).

3. Conclusion

In this paper, we show that the proposed cross structure can well control the excitation of quadrupole modes, through which we realize not only the far-filed interference between an electric quadrupole (EQ) resonance and a magnetic quadrupole resonance (MQ), but also the coupling between an EQ and a dark MQz mode in the silicon cross structure array. The resonating EQ–MQ far-field directional scattering effect is realized in nanostructures consisting of homogeneous material. The quadrupole induced directional scattering can be very substantial for advanced dielectric antenna design. Moreover, we realize the EQ–MQ spectral overlapping inside the cross particle array. The Mie-type quadrupole modes are insensitive to element interaction inside the array, which makes them promising in designing quadrupole-based Huygens’ metasurfaces. Furthermore, we successfully excite a dark MQz mode, coupling with the bright Mie-type EQ mode. Unlike most dipole-based EIT or Fano resonance systems in which symmetric breaking is adopted to derive a dark mode, the proposed EQ–MQz coupling system applies the array effect to obtain the dark MQz mode. We theoretically prove that, in spite of dipole-based couplings, the EQ–MQz resonant coupling can also produce the EIT effect. The coupling between non-resonant EQ modes with resonant MQ mode can result in tunable spectral Fano-like reflection responses. The corresponding quality factors range from $\sim 10$ to $>10^5$ with a maximum number of 238, 618 with the variation of structural parameter. We illustrate that the EIT effect, as well as the Fano resonance, results from...
the backward scattering and forward scattering caused by the interactions between the EQ and MQz modes. With the geometric variation of dozens of nanometers, one can realize the transformation from a total transmission (EIT effect) to a total reflection (Fano resonance). Besides, the EIT effect produced by EQ–MQz coupling can also leads to a $2\pi$ phase change, which can also be applied for designing metasurfaces. The studies about the excitations and spectral control of quadrupoles reveals a serial of unique quadrupole interaction and coupling characteristics, enabling a serial of advanced applications such as functional metasurfaces, high-efficiency directional antenna, and sensing.

Acknowledgments

Supported by National Key R&D Program of China (2016YFA0301300); National Natural Science Foundation of China (NSFC) (Nos.61671090 and 61875021); Fund of State Key Laboratory of Information Photonics and Optical Communications (IPOC20172204); The Beijing University of Posts and Telecommunications Excellent PhD Students Foundation (CX20173302); Guangxi Key Laboratory of Wireless Wideband Communication and Signal Processing; Natural Science Foundation of Beijing (2192036).

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