We study the mass spectrum of baryons with two and three charmed quarks. For double charm baryons the spin splitting is found to be smaller than standard quark-model potential predictions. This splitting is not influenced either by the particular form of the confining potential or by the regularization taken for the contact term of the spin-spin potential. We consistently predict the spectra for triply charmed baryons.

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I. INTRODUCTION

It has been recently reported the first observation of a candidate for a double charm baryon, \( \Xi_{cc}^+ \), in data from SELEX [1], the charm hadroproduction experiment at Fermilab. The data are compatible with a narrow state with a mass of 3520 MeV/c\(^2\) decaying through a weak Cabibbo-allowed process into \( \Lambda_c^+ K^- \pi^+ \). This observation has been confirmed through the measurement of a different weak decay mode that also involves a final state with baryon number and charm, \( \Xi_c^+ \to p D^+ K^- \) [2]. This production region had not been probed by other experiments and a big effort is being done by FOCUS and BELLE looking for doubly charmed baryons. SELEX data [3] suggest the existence of four \( ccq \) states (\( q \) being a light quark) in a mass region of 350 MeV.

The doubly and triply charmed baryons provide a new window for understanding the structure of all baryons. As pointed out by Bjorken [4] one should strive to study these systems because their excitation spectrum should be close to the perturbative regime. For their size scales the quark-gluon coupling constant is small and therefore the leading term in the perturbative expansion is enough to describe the system. Moreover, the spectroscopy of baryons containing two heavy quarks is of interest because of similarities both to a quarkonium state, \( QQ \), and to a heavy-light meson, \( \overline{Q}q \). On the one hand, the slow relative motion of the tightly bound color antitriplet \((cc)_3\) diquark in \( ccq \) is similar to quarkonium. On the other hand, for \( ccq \) the radius is dominated by the low mass \( q \) orbiting the tightly bound \( cc \) pair, and therefore is large. As a consequence, the relative \( (cc) - (q) \) structure may be described similar to \( \overline{Q}q \) mesons, where the \( cc \) pair plays the role of the heavy antiquark. The study of such configurations can help to set constraints on models of quark-quark forces [5,6]. For example, Ref. [7] emphasized how the \( QQq \) excitation spectra can be used to phenomenologically determine the \( QQ \) potential, to complement the approach taken for \( QQ \) quarkonium interactions.

Heavy-quark baryons are ideal systems to probe QCD dynamics in close connection to the structure of heavy-light mesons and the general structure of hadronic systems. While from the point of view of the interacting potential the analysis of light hadronic systems becomes complicated by nonperturbative effects, heavy-quark systems are rather simple. Heavy-quark current masses are clear signals of the explicit breaking of chiral symmetry and as a consequence there are no Goldstone-boson exchange contributions, the interacting potential being controlled by the perturbative short-ranged one-gluon exchange (OGE) and confinement. From a theoretical point of view, in the doubly charmed system one expects a \( J = 1/2 \) ground state isodoublet, termed \( \Xi_{cc}^{++} \) in PDG notation. The \( cc \) color antitriplet diquark has spin one. The spin of the third quark is either parallel, \( J = 3/2 \), or anti-parallel, \( J = 1/2 \), to the diquark. The \( J = 3/2 \) state has been predicted to be heavier than the \( J = 1/2 \) state by around 80 MeV/c\(^2\) [5–9]. For the \( ccc \) system the Pauli principle demands a \( J = 3/2 \) ground state.

Having in mind that the role of models in QCD is to build the simplest physical picture that connects the phenomenological regularities with the underlying structure, in this article we use a model that describes correctly the light baryon and the heavy and heavy-light meson spectra to study the spectra of doubly and triply charmed baryons. This procedure, that will have important consequences as we will see in the following, allows to make parameter-free predictions for the masses of doubly and triply charmed baryons. For this purpose we first make use of the model of Ref. [10] designed to describe the meson spectra from the light pseudoscalar mesons to bottomonium. The model is based on the assumption that the light quark constituent mass is a consequence of the spontaneous breaking of chiral symmetry generating Goldstone boson exchanges between light quarks. Besides it contains a confining term and a minimal one-gluon exchange potential. As mentioned above, for doubly and triply charmed baryons chiral symmetry is explicitly broken and therefore the interacting potential gets reduced to the one-gluon exchange and confinement. Let us revise the most important aspects of these two contributions.

Following de Rújula et al. [11] the OGE is a standard color Fermi-Breit interaction containing a coulomb term plus a spin-spin interaction:

\[
V_{OGE}(r_{ij}) = \frac{1}{4} \alpha_s \vec{X}_i \cdot \vec{X}_j \left\{ \frac{1}{r_{ij}} - \frac{1}{6m_i m_j} \vec{\sigma}_i \cdot \vec{\sigma}_j \frac{e^{-r_{ij}/r_0(\mu)}}{r_{ij}^2 r_0^2(\mu)} \right\},
\]

where \( \vec{X} \) are the \( SU(3) \) color matrices, \( r_{ij} \) is the interquark distance, \( m_i \) the constituent mass of quark \( i \), \( \vec{\sigma} \) are the spin Pauli matrices, and \( \alpha_s \) is the quark-gluon coupling constant. The nonrelativistic reduction of the OGE diagram in QCD for point-like quarks presents a contact term that, when not treated perturbatively, leads to collapse [12]. This is why the structure of the OGE is maintained but the \( \delta \) function is regularized in a suitable way. Such regularization takes into account the finite size of the constituent quarks and should be therefore flavor dependent [13] \( r_0(\mu) = r_0(\mu_q/\mu_{q_1q_2}) \), where \( \mu_q \) is the reduced mass of two light quarks and \( \mu_{q_1q_2} \) is the reduced mass of the two quarks under consideration. The typical size of the system scales with its reduced mass as expected for a confombic system.

The wide energy covered to describe hadrons made of light and heavy quarks requires an effective scale-dependent strong coupling constant [10,14] that cannot be obtained from the usual one-loop expression of the running coupling
constant because it diverges when $Q \to \Lambda_{QCD}$. The freezing of the strong coupling constant at low energies studied in several theoretical approaches [15,16] has been used in different phenomenological models [17]. The momentum-dependent quark-gluon coupling constant is frozen for each flavor sector. For this purpose one has to determine the typical momentum scale of each flavor sector that, as explained in Ref. [18], can be assimilated to the reduced mass of the system. As a consequence, we make use of the effective scale-dependent strong coupling constant of Ref. [10], giving rise to the following values of $\alpha_s$: $\alpha_s(qq) = 0.54$, $\alpha_s(qc) = 0.44$, and $\alpha_s(cc) = 0.29$. Such scaling generates for the light-quark sector a value consistent with the one used in the study of the nonstrange hadron phenomenology [19,20], and it also has an appropriate high $Q^2$ behavior, $\alpha_s \sim 0.127$ at the $Z_0$ mass [21]. For the sake of consistency we compare in Fig. 1 the parametrization of Ref. [10] to the experimental data [22,23] and the parametrization obtained in Ref. [15] from an analytical model of QCD.

Regarding confinement, lattice calculations in the quenched approximation derive, for heavy quarks, a confining interaction linearly dependent on the interquark distance,

$$V_{CON}^L(r_{ij}) = \frac{8}{3} \alpha_s r_{ij}. \quad (2)$$

This form of strict confinement has been widely used for light and heavy quarks when studying the meson and baryon spectra within a quark model framework. The consideration in the lattice of sea quarks apart from valence quarks (unquenched approximation) suggests a screening effect on the potential when increasing the interquark distance [24]. Creation of light $q\bar{q}$ pairs out of vacuum in between the quarks becomes energetically preferable resulting in a complete screening of quark color charges at large distances. In the 80's a specific parametrization of these effects was given in the form of a screening multiplicative factor in the potential reading $[(1 - e^{-\mu r_{ij}})/\mu r_{ij}]$ where $\mu$ is a screening parameter [25],

$$V_{CON}^S(r_{ij}) = \frac{8}{3} \alpha_s' r_{ij} \left( 1 - e^{-\mu r_{ij}} / \mu r_{ij} \right). \quad (3)$$

Screened confining potentials have been analyzed in the literature obtaining significant improvement both for the baryon [26] and for the heavy-meson spectra [10,27].

To get the baryon spectrum we have solved exactly the Schrödinger equation by the Faddeev method. The Faddeev equations for the case when two particles are identical and the third is different decouple into two sets corresponding to the two possibilities that the wave function be either symmetric or antisymmetric with respect to the exchange of the two identical ones [28,29]. Thus, since the Faddeev formalism includes only the space, spin, and isospin degrees of freedom one must choose in the case of the three-quark problem the set which is symmetrical under the exchange of space, spin and isospin, since the color part of the wave function is already assumed to be antisymmetrical under the exchange of any two quarks. In order to assure convergence we shall include $(l, \lambda, s, t)$ configurations ($l$ is the orbital angular momentum of a $2q$ pair, $\lambda$ is the orbital angular momentum of the third quark with respect to the center of mass of the $2q$, and $s$ and $t$ are the spin and isospin of the $2q$ respectively) up to $l=5$ and $\lambda=5$ [20]. For the case of three identical particles we have also calculated the spectra by means of the hyperspherical harmonic (HA) expansion method [30]. The HA treatment allows a more intuitive understanding of the wave functions in terms of the hyperradius of the whole system. As a counterpart one has to go to a very high order in the expansion to get convergence. To assure this we shall expand up to $K = 24$ ($K$ being the great orbital determining the order of the expansion). Differences in the results for the $3q$ bound state energies obtained by means of the two methods turn out to be at most of 5 MeV.

Our results for the excitation energy of the low-lying $ccq$ states for the model of Ref. [10] are presented in Table I together with the corresponding results for the $qqq$ system and their experimental values. As can be observed, the first radial excitation lies above the first negative parity state. This is the same situation found in the light baryon spectra where OGE quark-model based potentials predict the first negative parity excitation below the so-called Roper resonance, the first radial excitation. For double charmed baryons (DCBs) this is, however, expected to correspond to the experimental situation opposite to the light baryon case. The reverse of the ordering between the positive and negative nucleon excited states has been explained in terms of the combination of two different effects: relativistic kinematics and the pseudoscalar Goldstone-boson exchange interaction between light quarks. The nucleon Roper resonance is particularly sensitive to the form of the kinetic energy operator [31,32], its energy being decreased with respect to the first negative parity state when a relativistic kinetic energy operator is used. Once this energy difference has been diminished, the pseudoscalar Goldstone boson exchange potential produces the desired inversion [32,33]. In DCBs kinetic energy relativistic effects are expected to be much smaller due to the presence of two heavy quarks that makes the system analogous to a hydrogen-like atom [5]. On the other hand, chiral symmetry is explicitly broken and the interaction between heavy-light or heavy-heavy quarks does not present a Goldstone-boson pseudoscalar term.
These two effects combined should recover the normal ordering between positive and negative parity states in the case of doubly charmed baryons.

Another striking result appearing in the obtained spectra concerns the $J^P = 1/2^+ - J^P = 3/2^+$ spin splitting, it appears to be much smaller than the one predicted based on potential models directly obtained from the $c\bar{c}$ spectra [5-9]. Such a small spin splitting seems to be related with our scale-dependent quark-gluon coupling constant and the fact that the $\Delta - N$ mass difference is correctly reproduced.

As has been discussed above the interacting potential for doubly and triply charmed baryons may depend on i) the specific form used for the confining term, and ii) the regularization taken for the contact term of the one-gluon exchange potential. To judge the model dependence of our results on these items, we have recalculated the baryon spectra with four different set of parameters that are given in Table II. Two of them, sets B and D, have an infinite linear confining potential as the one of Eq. (2), while sets A and C present a screened confining potential, as the one of Eq. (3). For both types of confining potentials we have used two different values of the regularization parameter for the spin-spin interaction, $r_0$. In all cases the $\Delta - N$ mass difference is asked to be correctly described.

The results for the $ccq$ system are presented in Table III and the corresponding ones for the $qqq$ system in Table IV. As can be seen on these tables the same conclusions as before hold for any of the set of parameters used. All set of parameters give comparable radial and orbital excitation energies, the relative position of the first positive and negative parity excited states being preserved. Besides the spin-splitting appears to be almost constant (see the third column of Table III) independently of the set of parameters used and much smaller than the one predicted on the existing literature (one should have in mind at this point that most existing calculations are of rather exploratory nature, since made when double charm physics was considered far future). This result is independent of the type of confinement used, screened or infinite, and also of the strength of the spin-spin force controlled by the values of $\alpha_s$ and $r_0$, as it is illustrated in Table V. Such a small spin splitting resembles the situation with the $\Upsilon - \eta_b(1S)$ mass difference, being very difficult to disentangle from the experimental point of view, but it should stress efforts in order to try to find evidence for the existence of two peaks in the same energy region. It also does not appear affected by the form of the confining interaction, in agreement with conclusions of Ref. [26], that observes that only the high-energy excited states seem to be "confinement states", and therefore influenced by the explicit form of the confining potential.

The mass of the $ccq$ $J^P = 1/2^+$ ground state can be tuned to reproduce the measured experimental data by means of the constituent charm quark mass. This has been done by means of the set of parameters E in Table II, that is exactly the same as the set A except for the constituent charm quark mass (the same could have been done for any of the other sets of Table II). The results are shown in Table VI. As can be seen there are no variations in the predicted structure of the excited states. The spin splitting is exactly the same as before as well as the general structure of the spectra that remains also independent on modifications of the charm constituent quark mass, the excitation energies being almost the same as those presented in Table III for the set of parameters A, but reproducing in this case the experimental energy of the double charm baryon, $\Xi_{cc}^+$, reported by SELEX, 3520 MeV/$c^2$ [1,2].

Using the set of parameters E we have also calculated the spectra for the $ccc$ system. The results are given in Table VII. In this case it is the $J^P = 3/2^+$ the lighter state. As in the $ccq$ system the first negative parity state appears below the first radial excitation. The spin splitting increases up to values close to the results obtained for the light-quark baryons. Therefore, the small spin-splitting found in this work is a characteristic feature of double-heavy baryons.

As a summary, we have predicted the $ccq$ and $ccc$ spectra by means of a potential model that describes the light-baryon and the heavy and heavy-light meson spectra. The results are tested against different forms of the confining interaction and different values of the regularization parameter of the delta term in the OGE potential. Two results are specially relevant. First of all, the $J^P = 1/2^+ - J^P = 3/2^+$ spin-splitting is obtained to be almost constant independently of the model used and smaller than the results reported in the past, of the order of 20 MeV. It seems to be related with the correct description of the $\Delta - N$ mass difference. Secondly, the normal ordering between the positive and negative parity excited states is recovered, what should be the consequence of the absence of pseudoscalar forces and the reduced influence of relativistic effects in the DCBs spectra. The increasing interest and the actual experimental possibilities in the charm sector claims for an experimental effort to disentangle these questions.

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TABLE I. Relative energy $ccq$ and $qqq$ spectra, in MeV, for the model of Ref. [10]. For the $qqq$ system ‘Theor.’ stands for the results of the model of Ref. [10] and ‘Exp.’ for the experimental result.

| System         | $J^P(1/2^+)$  | $J^P(3/2^+)$  | $J^P(1/2^-)$  |
|----------------|---------------|---------------|---------------|
| $ccq$          | 287           | 25            | 206           |
| $qqq$ (Theor.) | 500           | 290           | 469           |
| $qqq$ (Exp.)   | 491–531       | 291–295       | 581–616       |
TABLE II. Different parameter sets used in the text.

|          | A     | B     | C     | D     | E     |
|----------|-------|-------|-------|-------|-------|
| $m_q$ (MeV) | 313   | 313   |       |       |       |
| $m_c$ (MeV) | 1752  | 1550  |       |       |       |
| $r_0$ (fm)  | 0.25  | 0.45  | 0.25  | 0.25  |       |

Linear confinement

| $a_c$ (MeV fm$^{-1}$) | 55    |       | 115   |       |       |

Screened confinement

| $a_c'$ (MeV) | 160   |       | 300   |       | 160   |
| $\mu$ (fm$^{-1}$) | 0.7   |       | 0.7   |       | 0.7   |

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TABLE III. Relative energy $ccq$ spectra, in MeV, for the first four sets of parameters of Table II.

| Set | $J^P(1/2^+)^*$ | $J^P(3/2^+)$ | $J^P(1/2^-)$ |
|-----|----------------|--------------|--------------|
| A   | 221            | 23           | 165          |
| B   | 221            | 22           | 156          |
| C   | 347            | 25           | 243          |
| D   | 332            | 21           | 217          |

TABLE IV. Relative energy $qqq$ spectra, in MeV, for the first four sets of parameters of Table II.

| Set | $J^P(1/2^+)^*$ | $J^P(3/2^+)$ | $J^P(1/2^-)$ |
|-----|----------------|--------------|--------------|
| A   | 408            | 291          | 403          |
| B   | 473            | 292          | 430          |
| C   | 591            | 299          | 548          |
| D   | 689            | 297          | 580          |

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TABLE V. Strength of the spin-spin force, $\alpha_s/r_0^2(\mu)$ in fm$^{-2}$, for the different quark-quark pairs and for the different sets of parameters of Table II.

| Quark pair | Sets A and B $r_0 = 0.25$ | Sets C and D $r_0 = 0.45$ | Set E $r_0 = 0.25$ |
|------------|-----------------------------|-----------------------------|-------------------|
| $qq$       | 8.64                        | 2.66                        | 8.64              |
| $qc$       | 20.36                       | 6.26                        | 19.11             |
| $cc$       | 149.79                      | 45.31                       | 119.2             |

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TABLE VI. $ccq$ spectra, in MeV, for the set of parameters E of Table II.

| $J^P(1/2^+)^*$ | $J^P(3/2^+)^*$ | $J^P(1/2^-)^*$ |
|----------------|----------------|----------------|
| 3524           | 3749 (225)     | 3692 (188)     |

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TABLE VII. $ccc$ spectra, in MeV, for the set of parameters E of Table II.

| $J^P(3/2^+)$ | $J^P(3/2^-)^*$ | $J^P(1/2^+)$ | $J^P(1/2^-)$ |
|--------------|----------------|--------------|--------------|
| 4632         | 4870 (238)     | 4915 (283)   | 4808 (176)   |

FIG. 1. Effective scale-dependent strong coupling constant $\alpha_s$ as a function of momentum. We plot by the solid line our parametrization. Dots and triangles are the experimental results of Refs. [22] and [23], respectively. For comparison we plot by a dashed line the parametrization obtained in Ref. [15] using $\Lambda = 0.2$ GeV.
Figure 1