Quantum teleportation and computation with Rydberg atoms in an optical lattice

Huaizhi Wu, Zhen-Biao Yang, Li-Tuo Shen and Shi-Biao Zheng

Department of Physics, Fuzhou University, Fuzhou 350002, People’s Republic of China
E-mail: huaizhi.wu@fzu.edu.cn

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Abstract
Neutral atoms excited to Rydberg states can interact with each other via dipole–dipole interaction, which results in a physical phenomenon called the Rydberg blockade mechanism. The effect attracts much attention due to its potential applications in quantum computation and quantum simulation. Quantum teleportation has been the core protocol in quantum information science playing a key role in efficient long-distance quantum communication. Here, we first propose the implementation of a teleportation scheme with neutral atoms via Rydberg blockade, in which the entangled states of qubits can readily be prepared and the Bell state measurements just require single qubit operations without precise control of Rydberg interaction. The rapid experimental progress of coherent control of Rydberg excitation, optical trapping techniques and state-selective atomic detection promise the application of the teleportation scheme for scalable quantum computation and many-body quantum simulation using the protocol proposed by Gottesman and Chuang (1999 Nature 402 390) with Rydberg atoms in an optical lattice.

(Some figures may appear in colour only in the online journal)
2010, Han et al 2010), quantum simulation of the interacting Rydberg gas (Weimer et al 2010, Lee et al 2011, Ji et al 2011, Qian et al 2012) and studying interaction-induced optical nonlinearity (Gorshkov et al 2011, Petrosyan et al 2011, Parigi et al 2012, Pritchard et al 2012). Experimental demonstrations for preparation of entangled states and implementation of quantum logic gates has been reported (Isenhower et al 2010, Wilk et al 2010). The blockade effect is found to be significant as well for other quantum systems, such as electron spins (Koppens et al 2006, Shaji et al 2008) and cold polar molecules (Jin and Ye 2011).

In this paper, we first propose a scheme for implementing quantum teleportation with neutral atom qubits, which interact with each other via strong and long-range dipole–dipole or van der Waals interaction. The merits of our protocol include (1) easy preparation of EPR states, (2) quantum logic operations without precise control of Rydberg–Rydberg interaction, (3) Bell state measurements involving only single qubit operations and state-selective detection based on conditional state transfer. We then outline a protocol where teleportation can be implemented in a Rydberg atom array trapped in a periodic optical potential to complete teleportation-based quantum computation and quantum simulation. The experimental demonstration of single-site-resolved optical control in an optical lattice paves the way for the implementation of our protocol (Bloch et al 2012).

The quantum circuit representation of teleportation is shown in figure 1(a) (Bennett et al 1993). There are three qubits involved. The quantum information to be teleported is carried by qubit 1. Qubits 2 and 3 are initially prepared in an entangled EPR state. Performing a Bell measurement on qubits 1 and 2 by the sender yields two classical bits of information, according to the outcomes, the receiver can apply a suitable single-qubit operation which makes long-range interaction while they are excited to the Rydberg state, are employed as quantum bits (see figure 1(b)).

Neutral atoms are trapped in optical lattices or optical tweezers and are frozen to the motional ground state. The structure of the relevant energy levels of the atoms is schematically shown in figure 2(a). Two ground states denoted by $|0\rangle$ and $|1\rangle$ are coupled to the Rydberg excited state $|r\rangle$ by two lasers with Rabi frequencies $\Omega_{0r}$ and $\Omega_{1r}$, respectively.

While two neutral atoms are exposed to the common laser beams used for excitation of the Rydberg state $|r\rangle$, the two-atom Rydberg–Rydberg interaction gives rise to an energy shift denoted by $\Delta_r$, which is determined by the principle quantum number of the Rydberg state and the interatomic distance. In experiment, the Rydberg excitation can be realized by two-photon transitions or by short wavelength single photon transition.

Suppose the system consisting of three atomic qubits is initially in the state $(\alpha |0\rangle_1 + \beta |1\rangle_1) |0\rangle_2 |0\rangle_3$. Qubit 1 has been prepared in an unknown quantum state (see the left-hand side, figure 2(b)). The teleportation scheme in general consists of three procedures.

1. Entanglement preparation. So far, entanglement of individual neutral atoms via Rydberg blockade can be experimentally realized in two different ways. One is deterministic generation of entangled states using a Rydberg blockade mediated controlled-NOT (CNOT) gate (Isenhower et al 2010, Zhang et al 2010). The other is direct generation of an EPR pair depending on Rydberg-excitation competition between two ground-state neutral atoms (Wilk et al 2010). Here, we assume entanglement of the atom pair is prepared by the latter method. Qubits 2 and 3 within the Rydberg interaction range are manipulated by a common laser pulses sequence (sketch map shown in the right-hand side, figure 2(b)). The time evolution of the system, in the interaction picture, can be described by the Hamiltonian

$$H_{23} = \sum_{j=2,3} (\Omega_{0r} |r\rangle_j \langle 0| + \Omega_{1r} |0\rangle_j \langle r|) + \Delta_r |r\rangle_2 |r\rangle_3 |r\rangle_2 |r|, \quad (1)$$

2. Quantum State Transfer. Qubits 2 and 3 are initially prepared in an entangled state $\{\psi\}$, which is determined by the principle quantum number of the Rydberg state and the interatomic distance. When two neutral atoms are exposed to the common laser beams used for excitation of the Rydberg state $|r\rangle$, the two-atom Rydberg–Rydberg interaction gives rise to an energy shift denoted by $\Delta_r$, which is determined by the principle quantum number of the Rydberg state and the interatomic distance. In experiment, the Rydberg excitation can be realized by two-photon transitions or by short wavelength single photon transition.

3. Conditional State Transfer and Blockade of qubit 1.
where \( j = 2, 3 \), and we assume the interaction of the atoms with laser fields are identical so that they have the identical Rabi frequency \( \Omega_{\nu} \), which we suppose to be real for simplification later. For \( \Delta \gamma \gg \Omega_{\nu} \), only one of the qubits will be excited to the Rydberg state with resonant pulse length, the strong Rydberg blockade effect prevents the other qubit from excitation, namely, it remains in the ground state \( |0\rangle \). The effective Hamiltonian for the process is given by \( H_{\text{eff}} = \Omega_{\nu} \sqrt{2}|0\rangle \langle 2| + |2\rangle \langle 0| \) + h.c.. Thus, choosing appropriate interaction time, the system dominated by \( H_{\text{eff}} \) evolves into
\[
|\alpha|0\rangle + \beta|1\rangle \otimes \frac{1}{\sqrt{2}}(|0\rangle|2\rangle + |2\rangle|0\rangle).
\]
(2)

The fidelity of the entangled state \( |\text{EPR}_{23}\rangle = (|0\rangle|2\rangle + |2\rangle|0\rangle)/\sqrt{2} \) is essentially limited by the interaction strength \( \Delta \). In the dispersive regime, the transition from \( |\text{EPR}_{23}\rangle \) to \( |rr\rangle \) can be basically inhibited. We show the population of atomic states \( |0\rangle|2\rangle, |\text{EPR}_{23}\rangle \) and \( |rr\rangle \) as a function of rescaled time \( \Omega_{\nu}t \) in figure 3. The time-dependent population \( P_{\text{EPR}} \) of the entangled state \( |\text{EPR}_{23}\rangle \) oscillates with a varied time period revised by \( \Delta \). \( P_{\text{EPR}} \) increases as Rydberg interaction \( \Delta \) becomes stronger. The increasing energy shift \( \Delta \) lowers the probability for detecting double-excitation state \( |rr\rangle \) correspondingly, which can be suppressed to 0.05 when the blockade strength reaches \( \Delta \gamma /\Omega_{\nu} = 10 \). In addition, we note that the third peak value of \( P_{\text{EPR}} \) indicated by purple arrows in each sub-figure is larger than other extrema. It means that we can prepare the EPR pair with higher fidelity in a longer time limit. However, this would not be the case if the spontaneous decay from excited state \( |r\rangle \) is taken into account. Study of the open system including spontaneous emission can be done by using the master equation with the Lindblad form
\[
\dot{\rho}_{23} = \frac{1}{i\hbar}[H_{23}, \rho_{23}] + \frac{\gamma}{2} \sum_{j=2}^{3} (2S_{j}(0)\rho_{23}S_{j}^{\dagger}(0) - S_{j}^{\dagger}(0)S_{j}(0)\rho_{23} - \rho_{23}S_{j}(0)S_{j}^{\dagger}(0)\rho_{23}),
\]
(3)

where \( S_{j}^{(0)} = |0\rangle\langle j|, \rho_{23} \) is the system’s density matrix, and \( \gamma \) is the rate of spontaneous emission. The probability for preparing two-atom maximally entangled states \( |\text{EPR}_{23}\rangle \) as functions of Rydberg interaction strength \( \Delta \), and spontaneous emission rate \( \gamma \) is shown in figure 4. Recession of the fidelity of \( |\text{EPR}_{23}\rangle \) is due to the spontaneously atomic transition to the ground state \( |0\rangle \) followed by a photon emitted at random directions as well as Rydberg interaction-induced double excitation. Thus, careful selection of atom–field interaction (Rabi frequency) and excited Rydberg energy levels can prompt a high fidelity preparation of entangled states. Besides, the trapping potential for ground and Rydberg states may be different regardless of the type of trap used to hold the atoms, which will lead to motional heating. The effect of the decoherence due to motional heating on the fidelity can be minimized by carefully selecting the frequency of the lattice, so that the ground states and the Rydberg state have the same polarizability, which has been studied by Safronova et al. (2003).

In terms of the Bell bases states for qubits 1 and 2 in the subspace spanned by \( |0\rangle_{1}|0\rangle_{2}, |0\rangle_{1}|r\rangle_{2}, |1\rangle_{1}|0\rangle_{2}, |1\rangle_{1}|r\rangle_{2} \)
\[
|\Phi_{\pm}^{1,2}\rangle = \frac{1}{\sqrt{2}}(|0\rangle_{1}|0\rangle_{2} \pm |1\rangle_{1}|r\rangle_{2})
\]
(4)

and
\[
|\Psi_{\pm}^{1,2}\rangle = \frac{1}{\sqrt{2}}(|0\rangle_{1}|r\rangle_{2} \pm |1\rangle_{1}|0\rangle_{2}).
\]
(5)

The system state in equation (2) can be rewritten as
\[
\frac{1}{\sqrt{2}}[|\Phi_{\pm}^{1,2}\rangle(|\alpha|3 + \beta|0\rangle) + |\Psi_{\pm}^{1,2}\rangle(|\alpha|3 + \beta|0\rangle) + |\Psi_{-}^{1,2}\rangle(|\alpha|3 - \beta|0\rangle)]
\]
(6)

The measurement of Bell states will collapse qubit 3 to the quantum states that contain quantum information from qubit 1.

(2) Bell state measurement. Based on the Rydberg blockade mechanism, conditional state transfer and blockade between ground states \( |0\rangle \) and \( |1\rangle \) for qubit 1 can be realized by addressing the qubit individually, which is utilized for disentangling the Bell basis states. Qubits 1 and 2 are separately measured eventually. In this step,

we first switch on the coupling of the Rydberg state
$|r\rangle_1$ to both ground states $|0\rangle_1$ and $|1\rangle_1$. The atom–field interaction is described by the Hamiltonian

$$H_{12} = (\Omega_{0r} |r\rangle_1 \langle 0| + \Omega_{1r} |r\rangle_1 \langle 1| + \text{h.c.}) + \Delta_r |r\rangle_1 \langle 1|_2, \tag{7}$$

where $\Omega_{kr} (k = 0, 1)$ are Rabi frequencies with respect to the interaction of laser fields with atomic transitions $|k\rangle_j \leftrightarrow |r\rangle_j$. Atoms in the excited Rydberg state $|r\rangle$ will spontaneously transit to the ground states via two independent channels, namely, $|r\rangle \rightarrow |0\rangle$ and $|r\rangle \rightarrow |1\rangle$. The corresponding spontaneous emission rates are $\gamma_{0r}$ and $\gamma_{1r}$, respectively. For simplicity we assume $\Omega_{0r} = \Omega_{1r} = \Omega_0$ and $\gamma_{0r} = \gamma_{1r} = \gamma$ in the following.

The time-evolutional dynamics of the system occurs in two different ways conditionally depending on the excitation status of qubit 2. Single qubit operations act only on qubit 1. First, while atom 2 stays in the Rydberg state $|r\rangle_2$, the excitation of qubit 1 to $|r\rangle_1$ is impossible if the Rydberg–Rydberg interaction is strong enough. Thus, ideally, both the system states $|0\rangle_1 |r\rangle_2$ and $|1\rangle_1 |r\rangle_2$ will be kept unperturbed. However, due to the finite energy gap $\Delta_r$, the transitions out of these two states cannot be completely suppressed. To see this, we find the analytical solution for the system’s time evolution with the initial state $|0\rangle_1 |r\rangle_2$:

$$\begin{align*}
\frac{1}{2} \left[ 1 & + e^{-i \frac{\Delta_r}{\Omega} t} \left( \cos \left( \frac{\Omega^* t}{2} \right) \sin \left( \frac{\Omega^* t}{2} \right) \right) \right] |0\rangle_1 |r\rangle_2 \\
&+ \frac{1}{2} \left[ e^{-i \frac{\Delta_r}{\Omega} t} \left( \cos \left( \frac{\Omega^* t}{2} \right) \sin \left( \frac{\Omega^* t}{2} \right) \right) \right] |0\rangle_1 |r\rangle_2 \\
&- \frac{1}{2} e^{-i \frac{\Delta_r}{\Omega} t} \left( \cos \left( \frac{\Omega_0 t}{2} \right) \sin \left( \frac{\Omega_0 t}{2} \right) \right) |0\rangle_1 |r\rangle_2, \tag{8}
\end{align*}$$

where $\Omega^* = \sqrt{2\Omega_{01}^2 + \Delta_r^2}$. For $\Delta_r \gg \Omega_{01}$, the state evolution above approximates to $|0\rangle_1 |r\rangle_2 \rightarrow \cos(\Omega_{01} t/4\Delta_r) |0\rangle_1 |r\rangle_2 - i \sin(\Omega_{01} t/4\Delta_r) |1\rangle_1 |r\rangle_2$, from which we can derive the probability for the system staying in the initial state $(1 + \cos(\Omega_{01} t/2\Delta_r))/2$. Exact results can be alternatively obtained via numerical simulation, as shown in figure 5. The blockade effect strongly depends on the strength of the Rydberg–Rydberg interaction, which impacts the population of the highly excited Rydberg state giving rise to decoherence. Thus, selecting Rydberg states with a high principle quantum number can help to block double excitation (green, dashed lines) and reduce dissipation induced by spontaneous emission (blue, dashed lines). Similar behaviour happens for the case in which the system is initially in the state $|1\rangle_1 |r\rangle_2$.

Second, while atoms 1 and 2 are in states $|0\rangle_1 |0\rangle_2$ and $|1\rangle_1 |0\rangle_2$, the Rydberg blockade does not exist and conditional state transfer takes place. The laser pulses will lead to a resonant two-photon Raman transition for qubit 1. The time evolution can be found by setting $\Delta_r = 0$ in equation (8) with the initial state $|0\rangle_1 |0\rangle_2$ or $|1\rangle_1 |0\rangle_2$ and the related mediate system states. Choosing appropriate pulse length (see figure 5), we can realize the state transfer $|0\rangle_1 |0\rangle_2 \rightarrow |1\rangle_1 |0\rangle_2$ and $|1\rangle_1 |0\rangle_2 \rightarrow |0\rangle_1 |0\rangle_2$ perfectly without including spontaneous decay. The resonant transfer is fast compared with the dispersive interaction process. A practical spontaneous emission rate will slightly reduce the transferring efficiency. However, utilization of optimized pulse shapes will help to increase the fidelity.
Figure 5. Disentanglement of qubits 1 and 2 based on conditional state transfer and blockade. Parameters are (a) \( \gamma_1/\Omega_{01} = 0, \Delta_r/\Omega_{01} = 10, \) (b) \( \gamma_1/\Omega_{01} = 0.02, \Delta_r/\Omega_{01} = 10, \) (c) \( \gamma_1/\Omega_{01} = 0, \Delta_r/\Omega_{01} = 5, \) (d) \( \gamma_1/\Omega_{01} = 0.02, \Delta_r/\Omega_{01} = 5. \) Without dissipation, transferring from initial state \(|0\rangle|0\rangle\) to \(|1\rangle|0\rangle\) is perfect (solid). The system in the state \(|0\rangle|1\rangle\) is blocked depending on the Rydberg interaction strength \(\Delta_r\) (dash). Involving spontaneous emission reduces the fidelity of the disentangled operation.

The laser pulse sequence actually implements a CNOT-like quantum logic gate in only one step. Qubit 1 flips when qubit 2 is in the ground state \(|0\rangle\). While qubit 2 is in the Rydberg state \(|r\rangle\), the state transfer of qubit 1 is blocked. The transformation is ideally given by

\[
|0\rangle_1|r\rangle_2 \rightarrow -|0\rangle_1|r\rangle_2, \\
|1\rangle_1|r\rangle_2 \rightarrow -|1\rangle_1|r\rangle_2, \\
|0\rangle_1|0\rangle_2 \rightarrow |1\rangle_1|0\rangle_2, \\
|1\rangle_1|0\rangle_2 \rightarrow |0\rangle_1|0\rangle_2,
\]

which can be described by an amplitude matrix:

\[
M_{\text{CNOT}} = \begin{bmatrix}
-1 & 0 & 0 & 0 \\
0 & -1 & 0 & 0 \\
0 & 0 & 0 & 1 \\
0 & 0 & 1 & 0
\end{bmatrix}.
\]

It should be noted that the transformation can easily convert to CNOT gate operation with the help of the single qubit operation \(|r\rangle_2 \rightarrow -|r\rangle_2\). In fact, due to the finite Rydberg–Rydberg interaction strength, the state transfer cannot be completely inhibited. Thus, the gate operation \(M_{\text{CNOT}}\) is imperfect. As an example, we numerically calculate the gate fidelity by \(F = \text{Tr}(\rho_{\text{ideal}}\rho_{12}(t))\) with the initial two-atom state \(|0\rangle_1 + \sqrt{2} |1\rangle_1\rangle_1/\sqrt{3} \otimes (|0\rangle_2 + |r\rangle_2)/\sqrt{2}\), where \(\rho_{\text{ideal}}\) is the density operator of the system after an ideally perfect gate operation \(M_{\text{CNOT}}\) and \(\rho_{12}\) is the density operator after the realistic transformation including imperfections. The time-dependent density operator \(\rho_{12}(t)\) is obtained by numerically simulating the master equation with the form \(\frac{d\rho_{12}}{dt} = \frac{1}{\hbar}[\hat{H}_{12}, \rho_{12}] + \frac{\gamma_1}{2} \sum_{k=0,1} (2S^{(k)}_j \rho_{12} S^{(k)*}_j - \rho_{12} S^{(k)*}_j S^{(k)}_j - S^{(k)}_j S^{(k)*}_j)\), here \(S^{(k)}_j = |k\rangle_j\langle j|\). As shown in figure 6(a), excluding the influence of dissipation, the population transfer of the two-atom states \(|0\rangle_1|0\rangle_2\) and \(|1\rangle_1|0\rangle_2\) are nearly perfect for \(\Delta_r/\Omega_{01} = 0\). The double-exciton state \(|0\rangle_1|r\rangle_2\) is successfully suppressed. We show the fidelity of the transformation as a function of state transfer time in figure 6(b). For the Rydberg states of the lifetime \(\tau > 100 \mu s\) and interaction strength \(\Delta_r > 500\) MHz, the gate fidelity can reach 0.97 with the atom–laser coupling strength \(\Omega_{01}\) around 1 MHz. Thus, implementation of an accurate quantum computation is still in reach with the gate error less than 0.03 (Knill 2005).

After the transformation, the Bell basis states, referred to as maximally entangled states, are disentangled,

\[
|\Phi^{\pm}_{12}⟩ → \frac{1}{\sqrt{2}} (|1⟩_1(|0⟩_2 ± |r⟩_2)),
\]

\[
|\Psi^{1}_{12}⟩ → \frac{1}{\sqrt{2}} (|0⟩_1(|r⟩_2 ± |0⟩_2)).
\]

A single qubit operation with a \(\pi/2\) pulse can transform \((|0⟩_2 + |r⟩_2)/\sqrt{2}\) and \((|0⟩_2 - |r⟩_2)/\sqrt{2}\) to \(|0⟩_2\) and \(|r⟩_2\), respectively (Johnson et al 2008). Thus, the Bell states finally become

\[
|\Phi^{\pm}_{12}⟩ → \begin{cases} 
|1⟩_1|r⟩_2 \\
|1⟩_1|0⟩_2,
\end{cases}
\]

\[
|\Psi^{\pm}_{12}⟩ → \begin{cases} 
-|0⟩_1|r⟩_2 \\
|0⟩_1|0⟩_2,
\end{cases}
\]

The joint measurement can be achieved by detecting qubits 1 and 2 separately. The lossless state-selective detection of individual neutral atom qubits trapped in an optical lattice (Gibbons et al 2011) or optical tweezer (Fuhrmanek et al 2011) can be realized with high accuracy by probing laser-induced fluorescence. To avoid the influence of spontaneous decay from Rydberg state, qubit 2 can be firstly transferred to the ground state \(|1⟩\). Then, the target atomic qubits can be addressed by using an off-resonantly polarized laser beam focused onto the individual atoms. The laser beam induces differential energy shifts for the relevant hyperfine ground states and tunes the target atoms into resonant with an external microwave field (Weitenberg et al 2011). The transfer between hyperfine ground states can be realized via the Landau–Zener transition leaving the other atomic qubits unaffected. The state-selective detection is finally applied to the target atoms.

(3) Single qubit operation. The outcomes of the measurement for qubit 1 and 2 is transmitted to the receiver. A corresponding local rotation \(x, y, z\) for the measurement outcomes \(|1⟩_1|r⟩_2, |1⟩_1|0⟩_2, |0⟩_1|r⟩_2, |0⟩_1|0⟩_2\) respectively, is then made on qubit 3 to reconstruct the initial state of qubit 1.
The teleportation scheme can be implemented with rubidium atoms. Using polarized laser beams, we can couple ground states in $5s_{1/2}$ to an $ns$ or $nd$ Rydberg excited state mediated by the $5p_{1/2}$ state based on the transition selection rule. Alternatively, we can in principle manipulate the atoms in the $5s$ ground states by illuminating them with a UV pulse resonant with a transition to an $np$ Rydberg state (Farooqi et al. 2003). High-fidelity quantum teleportation needs strong Rydberg–Rydberg interaction, which arises from the large dipole moments of Rydberg atoms. The strength of the interaction can be tuned by using external electric fields (Jaksch et al. 2000). The extreme sensitivity to the electric field makes it possible to control the Rydberg–Rydberg interaction via a mechanism referred to as Förster interaction ( Förster 1948), which can be realized in the absence of applied electric fields. The interaction energy for two Rydberg-excited $^{87}\text{Rb}$ atoms (with the principal quantum number $n = 58$) separated by $4 \mu\text{m}$ can reach $\Delta_r/2\pi = 50 \text{ MHz}$ (Wilk et al. 2010). It is worthwhile to note that the interaction strength does not need to be precisely controlled for the implementation of the teleportation scheme. Moreover, the fidelity of the scheme is limited by the radiative lifetime of the Rydberg state. The spontaneous emission rate is determined by the principal and azimuthal quantum number of the Rydberg energy level and the temperature of the surrounding (Saffman and Walker 2005). For $n > 65$, the lifetimes of the s, p, d and f states are greater than 100 $\mu$s at room temperature (Saffman and Walker 2005).

To estimate the total fidelity of the teleportation process, we have $|r = 58d_{3/2}\rangle$ excited by a two-photon transition process and set the strengths for the coupling of the ground states to the Rydberg state $\Omega_0$ by $\Omega = 2\pi \times 2.5 \text{ MHz}$ (Wilk et al. 2010), which implies the conditions $\Delta_r/\Omega = 20$ and $\gamma/\Omega \approx 10^{-3}$. Then, the time needed for the preparation of the EPR state, the implementation of the CNOT gate followed by a $\pi/2$ pulse, and the recovery of the original state on qubit 3 (a statistical average over the four different cases) are $t_1 = \pi/2\sqrt{2\Omega}$, $t_2 = \sqrt{2\pi/\Omega + \pi/4\Omega}$ and $t_3 = \pi/\Omega$, respectively. Thus, the total time consumption of the scheme is about $T \approx 600 \text{ ns}$. Based on the above parameters, we also find that the fidelity of the teleportation process can reach 0.976 by assuming a fast and exact state selective measurement.

So far, we only consider the intrinsic errors, that is, the finite blockade shift and finite Rydberg lifetime, neglecting all other errors due to technical imperfection, such as the errors induced by spontaneous emission from the intermediate state $5p_{1/2}$ (with the rate $\Gamma = 2\pi \times 3 \text{ MHz}$) and the atomic motion. Now we assume that the two laser beams used for Rydberg excitation (two-photon resonance) are detuned by $\delta/2\pi = 1 \text{ GHz}$ from the intermediate level $5p_{1/2}$ and have identical coupling strengths ($\sim 2\pi \times 50 \text{ MHz}$) to the atoms, then the fidelity of the teleportation scheme reduces to 0.92 by including the spontaneous decay from the $5p_{1/2}$ state and assuming frozen motion of the atoms during the laser sequence (Zhang et al. 2012, Gàetan et al. 2009). Although we assume that the atoms are in the motional ground state, the sub-Doppler temperatures at the level of 50 $\mu$K are, however, sufficient for high-fidelity preparation of the EPR states and implementation of the quantum gates (Saffman and Walker 2005). We have numerically checked that the fidelity of the teleportation process is only slightly modified by introducing a deviation induced by atomic motion to the desired condition $\Delta_r/\Omega \sim (1 \pm 0.1) \times 20$. A relevant problem was recently discussed for two Rydberg-blockaded atom clouds by Möbius et al. (2013). Another important problem is the Rydberg photoionization induced by trapping light, which presents a limit to the usable Rydberg lifetime and thus further decreases the fidelity of the scheme (Saffman and Walker 2005).

Quantum teleportation can be used for construction of quantum gates (Gottesman and Chuang 1999). Making use of single qubit operations, Bell state measurements and Greenberger–Horne–Zeilinger (GHZ) states, the teleportation-based method is sufficient to build a universal quantum computer. The protocol proposed in (Gottesman and Chuang 1999) has proved that the Hadamard gate which commuted through a Pauli gate produced a Pauli gate and a similar property happens for the CNOT and Toffoli gates. Thus, a nice merit of the teleportation construction is that the quantum gates only operate on specific known states, instead of operating on unknown states, before the Bell-basis measurements. A modified correction is made with single qubit operations at the end of the protocol to achieve preset quantum operation. Since the fidelity of the quantum gates can be tested before they actually work on the unknown quantum states, the failed operations can then be discarded intentionally (Gottesman and Chuang 1999). Although building a fault-tolerant quantum computer is still out of reach in the near future.
future, performing quantum simulation with a many-body system is definitely of experimental interest (Bloch et al 2012). A teleportation-based simulation method can be utilized for simulation of an interacting high-dimensional quantum system and precise generation of many-body interaction terms (Dür et al 2008). Quantum simulation can be regarded as an intermediate step to a full scale quantum computer. Quantum computation or quantum simulation using teleportation can relax the experimental qualifications by reducing the resource requirement (Gottesman and Chuang 1999).

Rydberg atoms trapped in one- or two-dimensional optical lattice (figures 7(a) and (b)) (Anderson et al 2011) provides the possibility for doing quantum simulation (Weimer et al 2010) and adiabatic quantum computation (Keating et al 2012). Cubic lattices generated by superimposing three independent standing waves is the main way to trap atoms. Generalized lattice structures including triangular and hexagonal (Becker et al 2010), alternatively facilitate versatile control of Rydberg–Rydberg interaction. The simultaneous, state-insensitive confinement of the ground and Rydberg atoms in an optical lattice can be realized by using the near-resonant blue-detuned trap (Zhang et al 2011, Li et al 2013). For this purpose, laser beams with a wavelength of 1.012 μm are used to construct the optical lattice, and the atoms are driven in resonance between the ground atomic level 5s1/2 and the Rydberg level 90s1/2 (Li et al 2013). In this case, the Rydberg atom size ~0.4 μm is smaller than the lattice period, the interactions between the highly excited electron of a Rydberg atom and a neighbouring ground-state atom can be suppressed. The Rydberg–Rydberg interaction strength can reach several hundred megahertz with the interatomic distance around 4 μm for the Rydberg state 90s1/2. As shown in figure 7, the atom to be teleported (in blue) is able to interact with the atoms (in the optical lattice) within the Rydberg interaction range, according to which an interaction radius \( r_0 \) can be defined. Beyond \( r_0 \), the interatomic interaction is neglected. While the single-lattice-site-resolved addressing is experimentally achieved, the Rydberg–Rydberg interaction can be selectively controlled between atom pairs in the optical lattice. Suppose the distance between atoms 1 and 2 (or atoms 2 and 3) is less than \( r_0 \), while the interatomic distance between atoms 1 and 3 exceeds \( r_0 \). The excitation of atom 1 will only block atom 2 from excitation leaving atom 3 unaffected. In this case, the teleportation scheme can be easily realized. In the case where all of the qubits involved in the teleportation scheme are within the interaction range \( r_0 \), then it is convenient to first transfer atom 3 from the Rydberg state \( |r\rangle \) to the ground state \( |1\rangle \) before the Bell state measurement (i.e. step (2)). This will be useful to avoid many-body interaction and to reduce the influence of the finite radiative lifetime of the Rydberg state. Using the above rule, we can implement our teleportation protocol among all the lattice sites. A generalization of quantum teleportation can then be used for realization of fault-tolerant quantum computation (Gottesman and Chuang 1999). Compared to the quantum computing scheme where information is processed via a series of unitary gate operations, the teleportation-based method connected to the measurement-based quantum computing scheme possesses the advantages that the quantum information can be processed via a sequence of adaptive measurement on an initially prepared, highly entangled resources state, and therefore, the resource requirements of the quantum information processing can be reduced (Bloch 2008).

Recent progress in neutral atom experiments promises the successful implementation of the teleportation-based protocol. Single-site-resolved addressing and control of the individual atoms in a Mott insulator in an optical lattice has been demonstrated (Weitenberg et al 2011). The ground states of neutral atoms can couple to Rydberg excited states coherently (Johnson et al 2008), and quantum entangled states and the CNOT gate for two individual neutral atoms have been realized by using the Rydberg blockade method (Isenhower et al 2010, Wilk et al 2010). Preparation of GHZ states and implementation of multiqubit quantum phase gates based on the current architecture are theoretically feasible (Saffman and Mølmer 2009, Wu et al 2010). Therefore, useful transformations, including the set of gates in the Clifford group, can be performed with quantum teleportation in a fault-tolerant way (Gottesman and Chuang 1999), which leads to conceptual simplification for a universal fault-tolerant quantum computer. A Rydberg atom array referred to as a
Rydberg quantum simulator can reproduce the dynamics of the other many-body quantum systems (Weimer et al. 2010). Using entangled states as resources, teleportation-based gates are able to simulate the time evolution of the many-body Hamiltonian of the form $H = \sigma_1^{z\Delta_n} \sigma_c$ with $\sigma_c$ the Pauli matrix (Dür et al. 2008).

In conclusion, we have studied the implementation of quantum teleportation with Rydberg neutral atoms. The Rydberg-excited atoms interact via the Rydberg blockade mechanism. The Rydberg–Rydberg interaction-induced double-excitation energy shift and conditional state transfer are used for preparation of the EPR pair and disentanglement, respectively. Using trapped neutral atoms (in the optical lattice) as an architecture, quantum information can be teleported among the nodes of a quantum network. The teleportation-based quantum computation and quantum simulation (of an interacting many-body system) can be carried out using quantum entangled states as resources. The protocol is based on the current experimental techniques and provides a new way for testing fundamental protocols in quantum information science and studying basic phenomena in condensed matter physics.

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