Magnetic Anisotropy in Quantum Hall Ferromagnets

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Abstract

We show that the sign of magnetic anisotropy energy in quantum Hall ferromagnets is determined by a competition between electrostatic and exchange energies. Easy-axis ferromagnets tend to occur when Landau levels whose states have similar spatial profiles cross. We report measurements of integer QHE evolution with magnetic-field tilt. Reentrant behavior observed for the $\nu = 4$ QHE at high tilt angles is attributed to easy-axis anisotropy. This interpretation is supported by a detailed calculation of the magnetic anisotropy energy.

73.40Hm, 75.10Lp, 75.30Gw
In the quantum Hall effect (QHE) regime, two-dimensional electron systems (2DES) can have ferromagnetic ground states in which electronic spins are completely aligned by an arbitrarily weak Zeeman coupling \([1]\). However, spin-independence of the Coulombic electron-electron interaction leads to isotropic Heisenberg ferromagnetism, and therefore to loss of ferromagnetic order at any finite temperature \([2]\). Richer physics occurs when the two Landau levels that are nearly degenerate differ by more than a spin index. For example, double-layer QHE systems can be regarded as easy-plane (XY) two-dimensional ferromagnets \([3]\) and exhibit a variety of effects which have received considerable experimental \([4]\) and theoretical \([5]\) attention in recent years. Idealized single-layer QHE systems have a phase transition \([6]\) in tilted magnetic fields between unpolarized and spin-polarized states, and as we show below, can be regarded as easy-axis (Ising) ferromagnets. In this Letter we report experimental data for a 43 nm wide unbalanced GaAs quantum well in which a loss of the QHE at \(\nu = 4\) is observed over a finite range of magnetic-field tilt-angles. We derive a general expression for the magnetic anisotropy energy and propose that its sign is responsible for this observation. We show that in realistic quantum wells either easy-axis or easy-plane anisotropy can occur, depending on spatial profiles of the orbitals of the crossing Landau levels.

We discuss the anisotropy energy first from a general point of view, then specialize to two illustrative idealized examples before presenting realistic results for the quantum well of interest. In a strong magnetic field, the single-particle states of a 2DES are grouped into Landau levels with orbital degeneracy \(N_\phi = AB/\Phi_0\), where \(A\) is the system area, \(B\) is the field strength, and \(\Phi_0\) is the magnetic flux quantum. We consider the case where the Landau level filling factor \(\nu \equiv N/N_\phi\) is an integer \([7]\) and two different groups of \(N_\phi\) orbitals are close to degeneracy. We assume that other Landau levels are far enough from the Fermi energy to justify their neglect \([8]\). Using a pseudospin language \([3]\) to represent the Landau level index degree of freedom, the class of Hamiltonians we consider can be expressed, up to an irrelevant constant, in the form
\[ H = -b\sigma(\vec{q} = 0) + \frac{1}{2A} \sum_{\vec{q}} \{ V_{\rho,\rho}(\vec{q})\rho(-\vec{q})\rho(\vec{q}) + V_{\sigma,\sigma}(\vec{q}) \times \sigma(-\vec{q})\sigma(\vec{q}) + V_{\rho,\sigma}(\vec{q}) [\rho(-\vec{q})\sigma(\vec{q}) + \sigma(-\vec{q})\rho(\vec{q})] \} \] \( (1) \)

In Eq. (1), \( b \) is half the energy separation between the nearly degenerate Landau levels, and \( \rho(\vec{q}) \) and \( \sigma(\vec{q}) \) are respectively the sum and difference of the density operators projected onto the up and down pseudospin Landau levels. Note that \( b \) is half the single-particle energy difference and does not include mean-field contributions from Coulomb or exchange interactions with electrons in the Landau levels of interest. For simplicity, we have limited the present discussion to cases for which the total number of electrons with each pseudospin index is conserved. The effective interactions that appear in Eq. (1) are related to the effective interactions between pseudospins by the following relations: \( V_{\rho,\rho} = (V_{\uparrow,\uparrow} + V_{\downarrow,\downarrow} + 2V_{\uparrow,\downarrow})/4 \), \( V_{\sigma,\sigma} = (V_{\uparrow,\uparrow} + V_{\downarrow,\downarrow} - 2V_{\uparrow,\downarrow})/4 \), and \( V_{\rho,\sigma} = (V_{\uparrow,\uparrow} - V_{\downarrow,\downarrow})/4 \).

Our calculation of the pseudospin anisotropy energy is based on the following single Slater determinant wavefunction:

\[ |\Psi[\hat{n}]\rangle = \prod_{m=1}^{N_\rho} c^\dagger_{m,\hat{n}} |0\rangle \] \( (2) \)

Here \( m \) labels the orbital states within a Landau level and \( \hat{n} \) denotes the pseudospinor aligned in the \( \hat{n} = [\sin(\theta)\cos(\phi), \sin(\theta)\sin(\phi), \cos(\theta)] \) direction. This many-particle state is fully pseudospin polarized \( [10,11] \). A straightforward calculation yields the following result for the dependence of energy on pseudospin orientation:

\[ \frac{\langle \Psi[\hat{n}] | H | \Psi[\hat{n}] \rangle}{N} = -b^* \cos(\theta) + \frac{U_{\sigma,\sigma}}{2} \cos^2(\theta) \] \( (3) \)

Here \( b^* = b - U_{\rho,\sigma} \) and for all indices \( U_{s,s'} = \int \frac{d\vec{q}}{(2\pi)^2} [V_{s,s'}(\vec{q} = 0) - V_{s,s'}(\vec{q})] \exp(-q^2\ell^2/2) \), \( (4) \)

where \( \ell = \sqrt{\hbar c/eB} \) is the magnetic length. In Eq. (3) we have dropped terms in the energy that are independent of pseudospin orientation. The right hand side of this equation is independent of \( \phi \) because the \( \hat{z} \) component of total pseudospin is a good quantum number.
For each effective field strength $b^*$, the pseudospin orientation is determined by minimizing the total energy. For $U_{\sigma,\sigma} > 0$, easy-plane anisotropy, $\cos(\theta) = 0$ at $b^* = 0$ and the pseudospin evolves continuously with effective field as illustrated in Fig. 1(a), reaching alignment for $|b^*| > U_{\sigma,\sigma}$. For $U_{\sigma,\sigma} < 0$, easy-axis anisotropy, local minima occur at both $\cos(\theta) = 1$ and $\cos(\theta) = -1$ for $|b^*| < |U_{\sigma,\sigma}|$. If only global pseudospin rotation processes were possible, macroscopic energy barriers would separate these two locally stable states, resulting in hysteretic behavior (see Fig. 1(a)). The sign of $U_{\sigma,\sigma}$ is determined by competition between the two terms in square brackets on the right hand side of Eq. (4). The $V_{\sigma,\sigma}(\vec{q} = 0)$ term is an electrostatic energy which is present when the two pseudospin states have different charge density profiles perpendicular to the electron layers. This term favors easy-plane anisotropy. The $V_{\sigma,\sigma}(\vec{q})$ term is the exchange energy which favors easy-axis anisotropy which will always occur when $V_{\sigma,\sigma}(\vec{q})$ is an increasing function of wavevector.

Transport measurements in the QHE regime are extremely sensitive to the energy gap for charged excitations. Generally, large energy gaps give rise to well developed Hall plateaus and deep minima in the dissipative resistivity. In the Hartree-Fock approximation, the quasiparticle energy gap of anisotropic QHE ferromagnets can be written quite generally as

$$\Delta_{HF} = I_{\uparrow\downarrow} - 2U_{\sigma\sigma} + \frac{2b^*}{\cos(\theta)} ,$$

where $I_{\uparrow\downarrow} = \int \frac{dq^2}{(2\pi)^2} \exp(-q^2\ell^2/2)(V_{\rho\rho} - V_{\sigma\sigma})$. For the easy-plane case, $\Delta_{HF}$ is a continuous function of the effective field $b^*$, decreasing linearly for $b^*/U_{\sigma\sigma} < -1$, constant for $|b^*|/U_{\sigma\sigma} < 1$ and increasing linearly for $b^*/U_{\sigma\sigma} > 1$. In contrast, if the system has easy-axis anisotropy, $\Delta_{HF}$ decreases to $I_{\uparrow\downarrow}$ at the extremes of the hysteresis loop ($b^*/U_{\sigma\sigma} = \pm 1$) before jumping to $I_{\uparrow\downarrow} + 4|U_{\sigma\sigma}|$ when the pseudospin magnetization reverses. In Fig. 1(b) we summarize the above results by plotting the renormalized Hartree-Fock gap $\Delta_{HF}^* = (\Delta_{HF} - I_{\uparrow\downarrow})/2|U_{\sigma\sigma}|$ as a function of $b^*/|U_{\sigma\sigma}|$. In the Hartree-Fock approximation this quantity depends only on the sign of the anisotropy energy.

For concreteness, we mention two idealized models which we regard as paradigms for
the easy-plane and easy-axis anisotropy cases. For two arbitrarily narrow quantum wells separated by a distance \( d \) with full polarization of the true electron spin, we let pseudospin represent the layer index \([13]\). The “pseudospin” Zeeman field \( b \) is then proportional to the bias electric field, \( E_g \), created by a gate external to the electron system: \( b = e E_g d / 2 \). On the other hand, for a single arbitrarily narrow quantum well with \( \nu = 2m \) in which the real-spin Zeeman coupling has been increased \([6]\) so as to bring the up-spin \( n = m \) Landau level close to degeneracy with the down-spin \( n = m - 1 \) Landau level, we let the pseudospin represent the spin-index of the Landau level close to the Fermi energy. The pseudospin Zeeman coupling for this model is \( b = (g^* \mu_B B - \hbar \omega_c + I_0) / 2 \). Here the first term is the real-spin Zeeman coupling, the second term is the cyclotron energy and the last term is the contribution to \( b \) from exchange interactions with frozen Landau levels lying well below the Fermi energy \( (I_0 / \sqrt{\pi/2 e^2/\epsilon\ell}) = 1/2, 5/16, \) and \( 31/128 \) for \( m=0, 1, \) and \( 2 \) respectively \([6, 14]\)). The effective Coulomb interaction energies for the two models are summarized in Tab. I. For the ideal double-layer model, the electrostatic term \( V_{\sigma,\sigma}(q = 0) \) dominates, \( V_{\sigma,\sigma}(q) \) is a monotonically decreasing function of \( q \) and \( U_{\sigma,\sigma} \) is positive. On the other hand for the ideal tilted-field model, the pseudospin wavefunctions differ only in the plane of the 2DES, the electrostatic term is consequently absent, and the exchange term produces easy-axis anisotropy \( (U_{\sigma,\sigma} / (\sqrt{\pi/2 e^2/\epsilon\ell}) = -3/16, -33/256, \) and \(-107/1024 \) for \( m=0, 1, \) and \( 2 \) respectively).

Now we turn to the discussion of the measured QHE evolution with tilted field, shown in Fig. 2 \([15]\). In finite width quantum wells, the large tilt angles necessary to bring the up and down pseudospin Landau levels close to degeneracy result in substantial coupling of the in-plane component of the magnetic field to orbital degrees of freedom \([10]\). These orbital effects can be incorporated \([11]\) by adjusting the effective interactions appropriately. In particular, for real finite-width quantum wells, the perpendicular charge density profiles of the two pseudospin Landau levels differ, and the electrostatic contribution to \( U_{\sigma,\sigma} \) is no longer zero. The sign of the anisotropy energy depends in detail on the quantum well geometry, the tilt angle and the filling factor. The insets in Fig. 2(b) show charge-density
profiles in the studied quantum well for the relevant orbitals at high tilt angles obtained from self-consistent LDA calculations \[16\]: \( n = 0, \downarrow \) and \( n = 1, \uparrow \) at \( \nu = 2 \); \( n = 1, \downarrow \) and \( n = 2, \uparrow \) at \( \nu = 4 \). From these orbitals we obtain \[11\] that, for \( \nu = 2 \), \( U_{\sigma,\sigma} \) increases substantially with tilt angle, is only marginally negative for \( b^* = 0 \) which occurs at \( \theta = 72^\circ \) and becomes positive at larger \( \theta \) (see Fig. 2(b)). This results demonstrates that easy-plane anisotropy can occur in realistic single quantum wells. If so, referring to the quasiparticle gap predictions summarized in Fig. 1, a strong QHE may be expected throughout the region of tilt angles where the relevant Landau levels are close to degeneracy. Consistent with this expectation, the experimental data of Fig. 2 show a strong minimum at \( \nu = 2 \) at all angles near \( \theta = 72^\circ \) and no clear evidence for the disappearance of the QHE is observed up to the highest accessible tilt-angles for \( \nu = 2 \). Like the weak dependence of quasiparticle gap on bias potential, noted in experimental studies of double-quantum-well systems \[17\], this robustness of the QHE is a general property of easy-plane QHE ferromagnets.

At \( \nu = 4 \), our calculations predict that \( b^* = 0 \) occurs at \( \theta = 79^\circ \), and that the density profiles of the two pseudospin states are similar even at high tilt angles, as illustrated in Fig. 2. Hence, \( U_{\sigma,\sigma} \) is only weakly angle dependent and is still negative around \( \theta = 79^\circ \). We attribute the clear degradation of the measured QHE at \( \nu = 4 \) to easy-axis anisotropy. The tilt angle \( \theta = 80^\circ \) where the \( \nu = 4 \) QHE disappears is in a good quantitative agreement with the theoretically predicted angle \( \theta = 79^\circ \) at which the pseudospin Zeeman field \( b^* \) vanishes. We expect transport properties inside the hysteresis loop in the easy-axis case, to have a complicated disorder dependence. Spatially random potentials couple differently to different Landau levels and will produce a random pseudospin magnetic field. This is expected \[18\] to lead to the formation of large domains with particular pseudospin orientations. The dynamics of pseudospin reorientation is likely to be controlled by barriers to domain wall motion. If these are comparable to \( k_B T \), the pseudospin will achieve alignment with the effective field on laboratory time scales, \( \cos(\theta) \) will change from \(-1\) to \(1\) at \( b^* = 0 \), and the energy gap will have a cusp. This scenario appears to apply for recent experiments which study analogous Landau level crossings in the valence band of GaAs \[19\] and to some
tilted field driven transitions at fractional Landau level filling factors \cite{[15]}. On the other hand, when some domain wall motion barriers are much larger than $k_B T$, we expect that all physical properties will exhibit hysteretic behavior, and that the electronic state will have domain structure for $b^*$ close to zero. Dissipation due to mobile charges created in domain walls \cite{[11]} can then lead to a breakdown of the QHE. We expect that dissipative and Hall resistances will then depend on measuring current and sample history, as well as on temperature.

In the disorder free limit, easy-axis anisotropy in two-dimensions leads to a finite temperature continuous phase transition in the Ising universality class and stronger thermodynamic anomalies than for the Kosterlitz-Thouless phase transition of easy-plane systems. The transition temperature can be estimated \cite{[11]} by balancing energy and entropy terms in the free-energy of long domain walls:

$$k_B T_c \sim U_{\sigma,\sigma} (w R / \ell^2) ,$$

where $w$ is the domain wall width and $R$ is the domain wall orientation correlation length. The domain wall physics of these easy-axis ferromagnets is unconventional because the spin-stiffness is negative \cite{[11]}. Preliminary results from work presently in progress suggest that $w R / \ell^2$ is substantially larger than one and that the critical temperature should typically exceed $\sim 1$ Kelvin.

Note added: A recent experimental study \cite{[20]} we learned of after this work was completed finds hysteretic behavior in a narrow (25 nm) GaAs quantum well in vicinity of $\nu = 2/5$ and $4/9$ fractional QHE’s which correspond to integer QHE’s at composite fermion filling factors $\nu = 2$ and 4 respectively. In these experiments, Zeeman coupling strength was controlled both by applying hydrostatic pressure and by tilting the field. We believe that the theory developed in this paper explains the origin of the hysteresis found in Ref. \cite{[20]} at very low temperatures ($T \lesssim 200$ mK). We have not observed similar hysteresis in our data (Fig. 2); this may be because of our higher available base temperature (300 mK).

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TABLE I. Effective Coulomb interactions in units of $2\pi e^2\ell/\epsilon$ as a function of wavevector $q$ in units of $\ell^{-1}$ for ideal double-layer and tilted-field models. $L_n(x)$ is the Laguerre polynomial.
FIGURES

FIG. 1. Pseudospin orientation (a) and Hartree-Fock quasiparticle gap (b) as a function of the effective field $b^*/|U_{\sigma\sigma}|$ for the easy-axis (full line) and easy-plane (dashed line) broken symmetry states.

FIG. 2. (a): Measured longitudinal resistance vs perpendicular component of the magnetic field for different tilt angles ($\theta$ is measured from the normal to the plane of the 2DES) at $T = 300$ mK. The $\nu = 4$ QHE is lost at $\theta \approx 80^\circ$ and reappears at $\theta \approx 80.5^\circ$ while for $\nu = 2$ no loss of the QHE is observed near $\theta = 72^\circ$. The inset shows calculated charge distribution at $\theta = 0$ for the unbalanced GaAs quantum well studied here. The front-gate and back-gate voltages and the 2DES density ($N = 1.57 \times 10^{11}$ cm$^{-2}$) were fixed during the experiment. (b): Anisotropy energies calculated for the geometry of this sample at $\nu = 2$ and 4. The two Landau levels which are brought close to degeneracy by applying in-plane component of the magnetic field are indicated in the insets together with calculated density profiles for up (dashed line) and down (solid line) pseudospin orbitals at high tilt angles.
\[
\cos(\theta)
\]

\[
\frac{b^*}{|U_{\sigma\sigma}|}
\]

\[
\Delta^*_{HF}
\]
\[ \nu \sigma^2 (e/\varepsilon) \]

\[ \nu = 2, 4 \]

\[ \theta = 75^\circ, 79^\circ \]

\[ n = 0, 1, 2 \]