Modeling and performance analyses of fin-and-tube heat exchanger operating under frost conditions

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Abstract

As far as the frost accumulation in fin-and-tube heat exchangers is concerned (regarding this process as a quasi-dynamic), a time-dependent distributed parameter model of fin-and-tube heat exchangers in the presence of frosting is presented. The model consists of two different sub-models: frosting and heat exchanger. The model results are compared with the experimental data, and an average error between them of ~20% was found. In addition, the performances of the fin-and-tube heat exchangers under frost conditions, obtained from the experimental results, have been analyzed. These experimental results reveal that the inlet air velocity, temperature and relative humidity strongly affect frost thickness, frost mass accumulation, evaporating pressure and pipe wall temperature.

Keywords: fin-and-tube heat exchanger; frost formation; performance

1 INTRODUCTION

Frost is a well-known phenomenon occurring on a cold surface when water vapor in the surrounding air freezes on cold surfaces producing a layer of porous ice (frost). So the fin-and-tube heat exchangers which operate below freezing temperature are subjected to frost deposition and progressive thickening of frost layer on their surface. Frost deposition and accumulation on the fin-and-tube heat exchanger surface behaves as a thermal insulator between the surface and the humid air of the ambient and reduce the airflow passage area, thus leading to a significant reduction in the fin-and-tube heat exchanger performances.

Literature survey shows that a relatively large amount of effort has been done to explain frost formation. Yao and Jiang [1] report the development of a distributed parameter mathematical model of airside heat exchanger under frosting in an air source heat pump water heater/chiller (ASHPWHC) unit and the impacts of frosting on the operational performance of an ASHPWHC unit were also evaluated. The design and thermal selection of the heat exchangers have been extensively studied by Kakac and colleagues [2, 3]. Mao et al. [4] show that frost thickness and mass concentration varied with time and space. Yan et al. [5] investigate the performance of flat plate finned tube heat exchangers operating under frosting conditions. In the Seker’s studies [6, 7], the heat and mass transfer characteristics of heat exchangers during frost formation process are analyzed numerically, and the unsteady heat and mass transfer coefficients of the air side, the heat transfer coefficient of the refrigerant side, the air–frost layer interface temperature, the surface efficiency of the heat exchanger and the mass flow rate of the frost accumulated on the heat exchanger surface are calculated. Senshu and Yasuda [8, 9] predict the performance of a cross-finned tube heat exchanger on which frost occurred using a mathematical model. Oskarsson et al. [10, 11] develop a finite model, a three-region and a parameter model to examine the performance of heat exchangers with dry, wet and frosted surfaces.

In spite of a large amount of studies on the performance of the heat exchangers under frost conditions, there are still gaps and inconsistencies due to different empirical correlations used in models, different experimental conditions and different heat exchanger geometries used in the experiments.

Therefore, in the present paper, a dynamic distributed parameter model of an outdoor heat exchanger under frosting is presented, and experimental data are used to validate the accuracy of the model. On the basis of the referred experimental results, the performance of the fin-and-tube heat exchange under frosting is analyzed.
2 MATHEMATICAL MODEL

The model can be divided into two sub-models, frosting and heat exchanger, which consists of two modules for refrigerant side and air side, respectively. In order to simplify the analysis, the following assumptions have been made:

1. Only one-dimensional heat and mass transfer over the heat exchanger is assumed.
2. The axial heat conduction of the pipe wall is ignored.
3. Frost formation is assumed to be quasi-dynamic state.
4. All surface temperatures resulting from local heat transfer are below frost point.
5. The top of the fin is adiabatic.
6. Frost distribution is homogeneous over the each infinitesimal element.
7. Average thermophysical properties are used for the frost layer

2.1 Frost growth model

Because of the complexity of the process and the mechanism of frost formation, it is difficult to use a numerical method to simulate the frost deposition and thickening. Therefore, in the present paper, empirical formulas were used to describe frost formation. The process of frost accumulation is considered as a quasi-dynamic process, that is, in a single time step of the calculation, the process is seen as the primary conditions of the next step.

The frost accumulation mass \( M_{fr} \) in the period of \( \Delta t \) is determined by the loss of water vapor in the air, as water vapor condenses on the cold coil surface of an airside heat exchanger. It is expressed by:

\[
\Delta M_{fr} = \frac{m_x \times \Delta t}{1 + d_{x,1}} (d_{x,1} - d_{x,0})
\]

Because of the molecular diffusion and porosity of the frost layer, when the finned coil surface temperature is below the freezing point, the total amount of water vapor reaching the frost layer and solidifying is separated into two terms: the first represents the amount of condensed vapor contributing to the increase in frost thickness, and the second the increasing in frost density. In this paper, the frost layer density and the thermal conductivity of the frost layer at any time were calculated using an empirical expression developed by Hosoda et al. [12].

\[
\rho_{fr} = 340|t_e|^{-0.455} + 85u_a
\]

\[
\lambda_{fr} = 0.0289(1 + \rho_{fr}^2 \times 10^{-4})
\]

For each time step \( \Delta t \), the increase in the frost layer thickness over an interval of specified extent \( (\Delta t) \) is calculated according to:

\[
\Delta \delta_{fr} = \frac{\Delta M_{fr}}{\rho_{fr} \times A_t}
\]

2.2 The heat exchanger model

The heat exchanger model has been divided into two sub-modules, the refrigerant side and the air side. Their detailed descriptions are reported in the following paragraphs.

2.2.1 Refrigerant side model

Two refrigerant flow regions exist inside the fin-and-tube heat exchanger: a two-phase region and a superheated region. In the first, annular flow corresponds to a refrigerant dryness fraction between 0 and \( x_{cr} \) \((0 \leq x \leq x_{cr})\), while the flow is mist flow when the refrigerant vapor fraction varies between \( x \) and 1. The refrigerant vapor fraction of critical point \( (x_{cr}) \) is calculated by a correlation suggested by [13]

\[
x_{cr} = 7.94 [Re_v (2.03 \times 10^4 Re_v^{-0.81} \Delta T - 1)]^{-0.161}
\]

Two-phase region

The governing equations for the two-phase region can be derived from the following relations:

Mass conservation:

\[
\frac{\partial}{\partial z} [a \rho_x u_x + (1-a) \rho_u u_l] = 0
\]

Momentum conservation:

\[
\frac{\partial}{\partial z} [a \rho_x u_x^2 + (1-a) \rho_u u_l^2] = - \frac{\partial P}{\partial z} - \frac{\rho_m \tau_w}{A}
\]

Energy conservation:

\[
\frac{\partial}{\partial z} [a \rho_x u_x h_x + (1-a) \rho_u u_l h_l] = - \left( \frac{\pi d}{A} \right) \times \dot{q}_{tp}
\]

Where: \( \dot{q}_{tp} = \alpha_{tp}(T_{wi} - T_l) \alpha_{tp} \) is the heat transfer coefficient in the two-phase region. Because of the difference between the annular flow \((0 \leq x \leq x_{cr})\) and mist flow \((x_{cr} < x \leq 1)\), the heat transfer coefficients are evaluated, respectively, from [13, 14]

\[
\alpha_{tp} = \begin{cases} \frac{3.0}{(X_{nt})^{2/3}} \times \left[ 0.023 \left( \frac{A}{D} \right) \frac{GD}{\mu_t} \right]^{0.8} Pr_t^{0.3} & 0 \leq x \leq x_{cr} \\ \alpha_{cr} \times \sin^2 \left( \frac{\pi(1 - x)}{2} \right) + \alpha_{nh} \times \cos^2 \left( \frac{\pi(1 - x)}{2} \right) & x_{cr} < x \leq 1 \end{cases}
\]

Where:

\[
X_{nt} = \left( \frac{1 - x}{X} \right)^{0.9} \left( \frac{\mu_l}{\mu_t} \right)^{0.1} \left( \frac{\rho_l}{\rho_u} \right)^{0.5}
\]

\[
\alpha_{cr} = \frac{x - x_{cr}}{1 - x_{cr}}
\]

\[
\alpha_{nh} = \frac{x_{cr} - x}{1 - x_{cr}}
\]
Four unknown variables $u_\alpha$, $u_t$, $P$ and $a$ are present in Equations (6–8). In order to solve these equations, a further slip ratio equation must be added.

In this paper, the slip ratio model was selected from previous studies, as suggested by Premoli [15], the model can be expressed as follows:

\[
\frac{u_\alpha}{u_t} = S = 1 + F_1 \left( \frac{y}{1 + yF_2} - yF_2 \right)^{1/2}
\]

(10)

Where:

\[
F_1 = 1.578 \text{Re}_f^{0.19} \left( \frac{\rho_f}{\rho_\alpha} \right)^{0.22}
\]
\[
F_2 = 0.0273 \text{We}_f \text{Re}_f^{-0.51} \left( \frac{\rho_f}{\rho_\alpha} \right)^{-0.08}
\]

\[
y = \frac{\beta}{1 - \beta}
\]
\[
\beta = \left( 1 + \frac{1 - x \rho_\alpha}{x \rho_f} \right)^{-1} = \left( 1 + \frac{1 - a}{a} \right)^{-1}
\]

Superheated region The governing equations for this region derive as follows:

Mass conservation equation:

\[
\frac{\partial}{\partial z} (\rho_z u_z) = 0
\]

(11)

Momentum conservation equation:

\[
\frac{\partial}{\partial z} (\rho_z u_z^2) = -\frac{\partial P}{\partial z} - \frac{\tau_w S_w}{A}
\]

(12)

Energy conservation equation:

\[
\frac{\partial}{\partial z} (\rho_z u_z h_z) = -\left( \frac{m_d}{A} \right) \dot{q}_{sh}
\]

(13)

Where:

\[
\dot{q}_{sh} = \alpha_{sh} (T_w - T_v)
\]
\[
\alpha_{sh} = 0.023 \text{Re}_{sh}^{0.8} \text{Pr}_f^{0.4} \left( \frac{\lambda_{sh}}{D} \right)
\]

The values of the three unknown variables $u_\alpha$, $P$ and $T_v$ are obtained through a numerical solution of Equations (11–13).

2.2.2 Airside module

Neglecting the airside pressure drop, the governing equations for the airside are simplified as follows:

Mass conservation equation:

\[
G_{a,i} = G_{a,o}
\]

(14)

Energy conservation equation:

\[
\frac{dQ_a}{dz} = \alpha_{it} \times \left( \eta_{it} \Delta_f + A_p \right) \times (t_a - t_w)
\]

(15)

where $A_p$ is the tube surface area and $A_f$ the finned surface area. The total heat exchange coefficient $\alpha_{it}$ during frosting is calculated from [16]

\[
\alpha_{it} = \frac{1}{(1/1.25 \alpha_s \xi_{it}) + (\delta_{it}/\lambda_{ft})}
\]

(16)

Where:

\[
\xi_{it} = 1 + \frac{2835 + 1.86t_a - 2.05t_w}{C_p \text{a}} \times \frac{d_a - d_w}{l_a - t_w}
\]

(17)

\[
\alpha_s = 0.203 \frac{\lambda_{sh}}{e} \left( \frac{D}{e} \right)^{-0.54} \left( \frac{l}{e} \right)^{-0.14} (\text{Re}_s)^{0.65}
\]

(18)

The fin efficiency under frosting was derived from the literature [17]

\[
\eta_{it} = \frac{\text{th}(m_{it} \times l_{eq})}{m_{it} \times l_{eq}}
\]

(19)

\[
m_{it} = \sqrt{\frac{2 \alpha_{it}}{\delta_{it} \lambda_{ft}}}
\]

(20)

With the fin equivalent height $l_{eq}$ calculated from [18]

\[
l_{eq} = \left( 1.065 \frac{D}{2} - r_o \right) \left[ 1 + 0.805 \log \left( \frac{1.065D}{2r_o} \right) \right]
\]

\[
r_o = \frac{1}{2(D + 2 \delta_{it})}
\]

Equations (1–20) form the complete distributed mathematical model for the fin-and-tube heat exchanger under frosting. A Runge–Kutta method was employed to solve the partial differential equation system.

3 EXPERIMENTAL APPARATUS

The layout of the entire experiment system is shown in Figure 1. The apparatus consists of three independent parts: the psychrometric chambers, the refrigeration system and the measurement devices. The psychrometric chamber, 30 m³ in volume, is used as constant temperature–humidity ambient, in which the air
temperature and relative humidity are controlled by a computer program performing a PID control algorithm. An R22 vapor compression system is used as the refrigeration system. The fin-and-tube heat exchanger parameters are shown in Table 1. The measurement system detects temperature, pressure, air velocity, frost mass accumulation and frost layer.

### 3.1 Parameters measurement of the ASHP unit

The pipe wall temperatures of the heat exchanger were measured by 40 pre-calibrated T-type copper-constantan thermocouples (accuracy: ±0.1°C) stuck on the copper elbows of the evaporator. The refrigerant-side pressures and temperatures were measured by precision pressure transducers (model: MPM480, accuracy: ±0.25%) and PT 100Ω RTD sensors (accuracy: ±0.1°C). The air velocity, dry-bulb temperature and relative humidity were measured with 12 velocity sensor (hot film anemometers model: TSI8465, accuracy: ±0.5%) and 12 dry-bulb temperature/relative humidity multi-sensors (model: HUMOR10, accuracy: 2% RH, ±0.1°C), located on the 4 by 3 sensor grid in the upstream of the fin-and-tube heat exchanger to test outlet/inlet air velocity, dry-bulb temperature and relative humidity. The method of reducing heat exchanger exhaust area is adopted to change the face velocity of air on the heat exchanger in this paper. The measurement system of the frost layer consists of a CCD camera (model: COOLPIX4500, Nikon) and microscope (model: SZM, magnification: 90). The frost accumulation mass in the period \( \Delta \tau \) was calculated from:

\[
M_{fr} = \frac{m_a \times \Delta \tau}{1 + d_{a, in} \times (d_{a, in} - d_{a, out})}
\]  

### 3.2 Experimental procedure

In order to evaluate the performance of fin-and-tube heat exchanger under frosting, a series of tests were carried out: the inlet temperature could vary in the range \(-5 \) to \(5^\circ C\); using a humidifier, the humidity of the inlet air was changing from 65 to 85%, and the air velocity was controlled from 1.1 to 1.6 m/s. Data were recorded by a data acquisition system every 5 s during frosting.

### 4 RESULTS AND DISCUSSION

#### 4.1 Experimental verification of the model

In order to validate the accuracy of the frost formation model, calculations are compared with the experimental data while the outdoor air temperature, relative humidity and air velocity were fixed at \(0^\circ C\), 85%, 1.6 m/s. Results are shown in Figures 3–6. From pictures, we can see that the trends of experimental and computed behaviors well agree under the same frosting condition, but the average error between them is \(\sim 20\%\). It can be concluded that the model and the calculation methods are correct.
4.2 The performance of the fin-and-tube heat exchanger under frost conditions

4.2.1 Effects of outdoor air parameters on frost mass accumulation

Effects of air relative humidity, air temperature and air velocity on the frost mass accumulation on heat exchanger operating under frosting conditions were experimentally investigated. For the explored range, the frost mass accumulation increases linearly with the frosting time, and the slope of change is in agreement with other existing experimental and simulated results [1, 19, 20]. It is possible to note from Figure 8 that the frost mass increased as the air relative humidity increased at fixed air temperature and velocity. When the air temperature is 0°C, the rate of frost formation is the greatest (Figure 7). The trend of frost thickness growth with frosting time is consistent with it (Figure 10). As shown in Figures 7 and 8, the rate of frost formation falls down at the end of the frost cycle: a possible explanation of this phenomenon is that at the end of the frost cycle, the growth of frost thickness (Figures 10 and 11) meaningfully narrows the flow channel leading to a rapid decrease in the airflow, and resulting in a reduction in the moist content in the air condensing on the fin surfaces. From Figure 9, it is interesting to note that the minimum frost mass accumulation are formed at the air velocity of 1.3 m/s, the reason may be that the density of frost is the smallest at air velocity of 1.3 m/s. Because of the shortage of effective equipments of testing, the qualitative result of frost density can only be obtained.
4.2.2 Effects of outdoor air parameters on frost thickness

The measured frost thickness under various outdoor air conditions is shown in Figures 10–12. The growth rate of frost thickness does not increase linearly with time, but it could be divided into linearly increasing stage and sharply increasing stage, which agrees well with the experimental data reported by Xia et al. [21], but is different with the previous experimental data and numerical results reported by Yun and Kim [22] and Na and Webb [23]. The grown rate of the frost layer in the sharply increasing stage is ≏1.5 times of that in the linearly increasing stage. The reason is that the growth and densification of frost not only act as an insulation but also reduces the air flow passage area that reduce the heat transfer between air and refrigerant, when the frost layer up to a certain thickness, the evaporating pressure and evaporator surface temperature is dropping quickly (Figures 13–18), although the air flow rate is reduced, the temperature difference between import and export is increased, so the increase in moist is condensing from the per unit volume of air. On the other hand, compared with the early frosting stage, the density of frost layer is reduced in the final frosting stage. So, this will be accelerating the growth of the frost layer.

As shown in Figure 12, as the air velocity increases, the growth rate of the frost layer decreases. The reason could be the following: the main factors of effect on frosting are the wall temperature and air relative humidity, the air relative humidity of the windward side is almost distribution uniformity on the heat exchanger surface, but the air side convective heat transfer coefficient increases with the air velocity, resulting in an increase in the temperature of both the evaporating refrigerant and the fin
surface, so as the air velocity increases, the growth rate of the frost layer decreases.

4.2.3 Effects of outdoor air parameters on evaporating pressure
Figures 13–15 show the effects of the outdoor air parameters on evaporating pressure of the R22 refrigerant. From Figures 13–15, we can see that it could be divided into the growth stage, slow decline stage and dramatically dropping stage. In the growth stage, the evaporating pressure increases with the frosting time and reaches the peak point at the end of the stage. The higher the air temperature, relative humidity and air velocity are, the bigger the peak point reached by the evaporating pressure will be. In the slow decline stage, the evaporating pressure is slightly affected by frost formation in this stage. In the dramatically dropping stage, the evaporating pressure reduces rapidly due to the rapid increase in the frost thickness.

4.2.4 Effects of outdoor air parameters on pipe wall temperature
Figures 16–18 show the effects of the outdoor air parameters on pipe wall temperature. It could be divided into the slow decline stage and dramatically dropping stage. It is noted from Figures 16–18 that the greater are the air temperature, air relative humidity and air velocity, the larger pipe wall temperature will be. In the dramatically dropping stage, the pipe wall temperature reduces quickly due to the rapid decrease in the evaporating pressure (Figures 13–15).
CONCLUSIONS

The dynamic distributed parameter model of the fin-and-tube heat exchanger under frosting was presented, and experimental data were used to validate the accuracy of the model. Based on the experimental results, the performance of the fin-and-tube heat exchanger under frosting was analyzed. The following conclusions can be made:

1. The model is verified with the experimental result with an average error of approximately 20%. It can be concluded that the model and the calculation methods are correct.

2. The frost mass accumulation increases linearly with frosting time, but the growth rate of the frost layer could be divided into two phases, both with linear trend: the first with a lower slope, and the second higher.

3. The effect of frost formation on the evaporation refrigerant pressure could be divided into the growth stage, slow decline stage and dramatically dropping stage, and that of pipe wall temperature could be divided into the slow decline stage and dramatically dropping stage.

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