Exciton complexes in impurity doped quantum ring

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Abstract. We propose a simple unified method for theoretical description of free and bound to impurity neutral and charged excitons, confined in a heterostructure with ring-like geometry. In order to assess the experimentally relevant domain of parameters, we adopt a model of a narrow ring, when for both neutral and charged excitons 3D wave functions can be separated. By using the Fourier series method, we have calculated the energy spectra of excitons complexes in a quantum ring as a function of the electron-to-hole mass ratio, and the magnetic field strength. The size-dependent magnetic oscillations of energy levels of excitons’ complexes spectra have been revealed.

1. Introduction
Very recently, a number of experimental and theoretical studies of neutral \( X \), negatively \( X^- \) and positively \( X^+ \) charged excitons confined in quantum dots have been performed [1]. The possibility to control individual excitations, in quantum dot structures is an indispensable prerequisite for the new field of quantum information processing in a semiconductor environment, in which exciton complexes are considered as the intermediate states. Fabrication of nanometer-sized lithography defined semiconductor quantum rings (QR) generated interest in the Aharonov-Bohm (AB) effect induced by charged excitons [2]. As it was shown in [3], the AB effect is more pronounced under condition that the electron and hole move in a ring-like structure over different paths providing a non-zero angular momentum. It is clear that the separation between their paths is strongly related to the electron-to-hole mass ratio, the larger the ratio, the more distant are paths. There is an additional reason for studying the effect of the masses ratio, associated with a possibility of the participation in the formation of the exciton complexes of particles with different masses, such as light-hole and heavy-hole, or as a case of extremely heavy carriers a donor or an acceptor. Therefore, below we propose a simple method for solving the Schrödinger equation for a particular case when exciton complexes consist of one electron and two holes with arbitrary masses \( m_{h1} \) and \( m_{h2} \) confined in a narrow QR that allows us to analyze in the framework of the same formalism the properties of different types of the excitonic complexes such as free trions \( X^+ \) with participation of holes with different masses, free exciton when \( m_{h2} \rightarrow 0 \), or exciton bound to acceptor when \( m_{h2} \rightarrow \infty \). We apply the formalism for analyzing the AB effect for free and bound to an acceptor \( X \) and \( X^+ \) complexes confined in QR and show that a decrease of the electron-to-hole effective mass ratio provides a quenching of the AB oscillations of the lower energy levels when a free exciton is bound to an acceptor, similarly to one observed recently for QRs with single and coupled donors [4].
2. Theory

In order to assess the experimentally relevant domain of parameters, we adopt a simple model of a narrow ring, in which its height \( h \) and width \( w \) are significantly smaller than its centerline radius \( R \). In this case, which we call the adiabatic limit, 3D wave equations for exciton complexes can be separated and the analysis of their low-lying states is reduced to a simpler problem of 1D rotation of two holes with effective masses \( m_{h*}, m_{e*} \) and the electron with mass \( m_e \), whose positions are given by three polar coordinates \( J_{h1}, J_{h2} \) and of \( J_e \), respectively [5]. By using the exciton effective Bohr radius \( a_0^* = h^2/e/m_e^* \) as the unit of length, the effective Rydberg \( R^* = e^2/2e a_0^* = h^2/2m_e^* \), as the energy unit and \( g = eB/2m_e R^* \), as the unit of the magnetic field strength, being \( m = m_e^* m_h^* \left( m_e^* + m_h^* \right) \) the reduced mass, the Hamiltonian which describes simultaneously a free exciton for \( \eta = m_{h*}/m_{h1} \), free trion \( \chi \) for \( 0 < \eta \leq 1 \) and exciton bound to an acceptor for \( \eta = m_{h*}/m_{h1} \), can be written as:

\[
H(\eta) = (2 + \eta) \left[ \frac{\pi^2}{b^2} + \frac{\pi^2}{w^2} \right] - \frac{1}{I_{h1}} \frac{\partial^2}{\partial \theta_{h1}^2} + \eta \frac{\partial^2}{\partial \theta_{h2}^2} - \frac{1}{I_e} \frac{\partial^2}{\partial \theta_e^2} + \frac{\gamma^2 R^2}{4} \left[ \frac{1 + \eta}{\mu_h} + \frac{1}{\mu_e} \right] + \frac{2(\eta)}{I_{h1}} \frac{\partial^2}{\partial \theta_{h1}^2} + \frac{\partial^2}{\partial \theta_{h2}^2} + V_{\text{Coul}}(\theta_{h1}, \theta_{h2}) \Phi = (E - E_0) \Phi;
\]

Here the parameter \( \lambda \) is equal to zero for excitons and to one for trions. Additionally, in the expression (1), obtained in framework of the adiabatic approximation [6], are used the following notations \( R^2 = R^2 + w^2/12; V_{\text{e}}(\theta) = 2/\sqrt{w^2 + R^2} \sin^2 \theta/2 \), and \( m_i = m_i^2/m_e; I_{i} = m_i R^2; i = e, h \).

In the center-of-mass coordinates

\[
Q = \left( I_{h1}(J_{h1} + h J_{h2}) + I_e J_e \right)/I; \quad J_1 = J_{h1} - J_e; \quad J_2 = J_{h2} - J_e; \quad I = (1 + h) I_{h1} + I_e
\]

The Hamiltonian (1) is separated completely in the case of the exciton \( (\eta = 0) \) and partially for trion \( (\eta = 1) \). Being \( M = 0, \pm 1, \pm 2 \), the angular momentum corresponding to the center-of-mass rotation, the eigenfunctions of the Hamiltonian (1) can search in the form:

\[
H(h)Y(Q,J_1,J_2,h) = E(h)Y(Q,J_1,J_2,h); \quad Y(Q,J_1,J_2,h) = e^{iMQF}(J_1,hJ_2)
\]

Here unknown envelope function \( F \) describes only the relative particles motion. By using definitions (1) and (2) in equation (3) one can derive after simple algebraic manipulations the following differential equation for the envelope function \( F \):

\[
E_{\text{e}} = (2 + \eta) \left( \frac{\pi^2}{h^2} + \frac{\pi^2}{w^2} \right) + \frac{M^2}{I} + \frac{\gamma^2 R^2}{4} \left[ \frac{1 + \eta}{\mu_h} + \frac{1}{\mu_e} \right] - \gamma M \left[ \left( 1 + \eta \right) I_{h1} + I_e \right] \left( m_e^* + m_h^* \right);
\]

Here we denote by \( V_{\text{Coul}}(J, J) = 1V_e(J_{h1} - J_{h2}); V_e(J_{h1} - J_e); V_e(J_{h2} - J_e) \) the total Coulomb interaction energy and \( I_m = I_{h1} I_e / (I_{h1} + I_e) \) the reduced electron-hole moment of inertia. In our numerical work we solve the equation (4) by using the single and the double Fourier series expansion methods for the exciton and the trion, respectively.

3. Results

We analyse the influence of the magnetic field on the energies of exciton complexes in a ring with material parameters typical for InAs material, \( \varepsilon = 12.71, \quad m_e^* = 0.026m_0, \quad m_h^* = 0.026m_0 \) for light-hole...
and \( m_{h'} \approx 0.4 m_0 \) for heavy hole. Below we present in figures 1 and 2 some results for low-lying energies of neutral exciton \((h = 0, l = 0)\), formed by one electron and one light-hole, positively charged exciton complexes formed by one electron and two light \((h = 1, l = 1)\) or two heavy holes \((h = 16, l = 1)\) and also for light-hole exciton bound to an acceptor \((h = 0, l = 1)\). In all cases we consider a ring with the centerline radius \( R = 5 a_0^*\), height \( h = 0.2 a_0^*\) and width \( w = 0.2 a_0^*\). In figure 1 we show some low-lying levels as functions of the magnetic field for charged excitons, formed by one electron and a pair of heavy or light-holes.

It is seen that in both cases the energy levels are distributed in three groups with different responses to the variation of the external magnetic field. The lower group is formed by doubly degenerated levels whose energies are remarkably spaced and almost insensitive to any variation of the magnetic field. The states of this group remain bound in the presence of the magnetic field however the energy levels corresponding to these states suffer a weak double splitting due to exchange symmetry presented in the three-particle complexes. On the contrary, in the intermediate part of the energy spectrum one can observe typical for AB effect oscillations and reordering of energy levels with multiple crossovers between them. The appearance of the AB oscillations for heavy- and light-hole trions is different. In the first case, when the electron mass is 16 times less than the mass of holes, the crossovers are multiple while in the second case, when the masses of all three particles are similar, there is additional symmetry and the multiple crossovers split in doublet’s.

**Figure 1.** Energies of lower levels of charged excitons, formed by one electron and a pair of heavy or light holes confined in quantum ring, as functions of the magnetic field.

Both a weak double splitting and the multiple crossovers of the lower energy levels in figure 1 we attribute to interaction of the tunnel currents, generated by relative motion of the three-particle system through inter-particle interaction potential, with the external magnetic field. The higher the system energy the larger is the probability for the tunnelling and the upper is the amplitude of the AB oscillation. When the energy increasing becomes higher than barrier of the interaction potential the particles relative motion is transformed in a free rotation. Corresponding to these states energies dependencies in the upper part of figure 1 describe an independent rotation of three particles in their unbound state. Therefore the upper energies form quasi-continuum levels with multiple crossovers and reordering between them.

In figure 2 we compare energies of (a) the unbound light-hole exciton and (b) the exciton bound to an acceptor in quantum ring, as functions of the magnetic field. It is seen a vast difference between the energy spectrum of a free exciton and an exciton bound to an acceptor. The exciton bound to an acceptor loses almost completely its individual properties. One can observe in figure 2 (a) that the unbound exciton has only few energy levels insensitive to the external magnetic field with usual for

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Coulomb attraction a large spacing between the ground and the first excited states while the upper part of the exciton energy spectrum shows a typical for AB oscillation dependence on the external magnetic field. On the contrary, the exciton bound to an acceptor have many low lying levels that are completely independent on the external magnetic field. Also, comparing corresponding curves in figure 1 and 2, one can see that there is a general similarity between energy spectra of the heavy-hole trion and the exciton bound to an acceptor, but the energies in the second case are slightly lower.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure2.pdf}
\caption{Energies of lower levels of (a) unbound and (b) bound to an acceptor the light-hole exciton confined in quantum ring, as functions of the magnetic field}
\end{figure}

4. Conclusion
A unified differential equation that describes in the adiabatic limit the energy spectra of two and three carriers confined in narrow quantum ring is derived and the double Fourier series expansion method is applied in order to analyze the Aharonov-Bohm oscillations of the lower energy levels in the presence of magnetic field. The energies of low-lying states of the neutral X, positively heavy- and light–hole charged excitons \(X^+\), and exciton bound to an acceptor, as functions of the magnetic field strength have been calculated. A high sensibility of the energy spectrum on the electron-to-hole mass ratio has been revealed. It is shown that a decrease of this ratio conduces to quenching of the oscillation of the lower energy levels under the variation of the external magnetic field.

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