Basic MAC Scheme for RF Energy Harvesting Wireless Sensor Networks: Throughput Analysis and Optimization

Cheon Won Choi  
Department of Applied Computer Engineering  
Dankook University  
Korea  
cchoi@dku.edu

Abstract — Traditionally, how to reduce energy consumption has been an issue of utmost importance in wireless sensor networks. Recently, RF energy harvesting technologies, which scavenge the ambient RF waves, provided us with a new paradigm for such networks. Without replacement or recharge of batteries, a RF energy harvesting wireless sensor network may live an eternal life. Against theoretical expectations, however, energy is scarce in practice and, consequently, naïveté has to be within a MAC scheme that supports a sensor node to deliver its data to a sink node. In this paper, we thus consider a basic MAC scheme, rooted in ALOHA, in which a sensor node repeats harvesting energy, backing off for a while and transmitting a packet. Then, we develop an analytical method to exactly calculate the throughput that the basic MAC scheme can attain. In a case study, we also find an optimal distribution for the back-off time which maximizes the throughput. Especially, in a practical situation, we prove that an optimal back-off time is not characterized by the distribution but only by the mean value.

Keywords — wireless sensor network, RF energy harvesting, basic MAC scheme, throughput, optimal back-off time

I. INTRODUCTION

A wireless sensor network consists of sink nodes and sensor nodes [1]. In the network, a sensor node gathers information in the vicinity and delivers it to a sink node. To carry out such a mission, a sensor node is equipped with a battery that powers the sensor node itself. A wireless sensor network is often deployed in a harsh environment, where batteries are hardly recharged or replaced. Consequently, issues on energy consumption have arisen in designing a wireless sensor network. Then, many efforts have been made to solve the energy problems in a wireless sensor network, most of which were focused on devising efficient communication schemes to reduce energy consumption and ultimately to extend the lifetime of the network.

Recently, radio frequency (RF) energy harvesting technologies provided us with a new paradigm for wireless sensor networks. By scavenging the ambient RF waves, a wireless sensor network can overcome the battery constraint and is, at least theoretically, able to live eternally. However, harvested energy is not enough to directly power a sensor node but merely capable of charging an internal capacitor. Moreover, it takes quite a long time to charge the capacitor compared to the time for which the sensor node actually participates in communicating with a sink node [2][3]. As a result, energy is still scarce in practice and a sensor node should repeat the transition between the long harvesting state and the short communicating state.

A medium access control (MAC) scheme is needed for a sensor node to deliver information to a sink node, especially, in a wireless sensor network in which a number of sensor nodes attempt to send their data to a common sink node. In an RF energy harvesting wireless sensor network, which suffers from a scarcity of energy, signaling is not easily provided between sink and sensor nodes. Furthermore, a sink node is hardly able to exchange information with another sensor node. Thus, a sophisticated MAC scheme of scheduling-type is not suitable for supporting a sensor node to deliver data to a sink node. Naïveté has to be within the MAC scheme in an energy harvesting wireless sensor network.

Perceiving such a difficult situation, we consider a basic contending-type MAC scheme based on ALOHA [4] for an RF energy harvesting wireless sensor network. In the basic MAC scheme, a sensor node harvests energy, charges a capacitor, senses the environment, generates a packet, takes a back-off time, and transmits the packet. Since two or more sensor nodes may simultaneously contend for packet delivery, a collision is unavoidable and the throughput is degraded inevitably. Using the renewal theory, we thus develop an analytical method to exactly calculate the throughput that the basic MAC scheme can attain. In a case study, we also find an optimal distribution for the back-off time which maximizes the throughput.

In section 2, we describe a basic MAC scheme for supporting sensor nodes to deliver their data to a sink node in an RF energy harvesting wireless sensor network. In section 3, we present an analytical method to exactly calculate the throughput attained by the basic MAC scheme. In section 4, we find an optimal distribution for the back-off time, which maximizes the throughput, in a case study.

II. BASIC MAC SCHEME

Consider an RF energy harvesting wireless sensor network which consists of a single sink node and many sensor nodes. In the network, sensor nodes send collected data to the sink node.
For such delivery of data, we consider a MAC scheme as follows:

Time is divided into frames and a frame is partitioned into harvest period, back-off period and transmission period. When a frame starts, a harvest period also begins. During the harvest period, a sensor node harvests energy in the vicinity and charges the internal capacitor. Note that the harvest period lasts until the sensor node accumulates as much energy as it can transmit a packet. As the harvest period ends, the sensor node senses the environment, collects information and encapsulates the information into a packet. Then, a back-off period starts. The sensor node intentionally waits for the back-off period in the expectation for reducing the possibility of packet collision. When the back-off period is over, a transmission period begins and the sensor node finally transmits the packet to the sink node. As the transmission period ends, the next frame starts.

Note that the sink node sends no acknowledgement message to any sensor node. Thus, a sensor node has no need to retransmit a packet. Consequently, a sensor node immediately discards a packet as it finishes transmitting the packet. Figure 1 summarizes the basic MAC scheme considered in the paper.

Consider a wireless sensor network which consists of a single sink node and \( M \) sensor nodes, denoted by \( s_1, \ldots, s_M \). At sensor node \( s_m \), time is divided into frames and a frame is again partitioned into harvest period, back-off period and transmission period as stated in section 2. For \( k \in \mathbb{N} \), let \( H_k^{(m)} \), \( B_k^{(m)} \) and \( T_k^{(m)} \), respectively, denote the harvest time, back-off time and transmission time (i.e., the lengths of harvest period, back-off period and transmission period) in the \( k \)th frame at sensor node \( s_m \). Set

\[
S_k^{(m)} \triangleq H_k^{(m)} + B_k^{(m)} + T_k^{(m)}
\]

for \( k \in \mathbb{N} \). Assume that the time for sensing the environment, gathering information and generating a packet is negligible. Then, \( S_k^{(m)} \) represents the length of the \( k \)th frame at sensor node \( s_m \). Also, define \( A_0^{(m)} = 0 \) almost surely and set

\[
A_n^{(m)} \triangleq \sum_{k=1}^{n} S_k^{(m)}
\]

for \( n \in \mathbb{N} \). Then, \( A_n^{(m)} \) indicates the time that the \( n \)th frame starts. Note that \( \{A_n^{(m)}, n = 0, 1, \ldots\} \) forms a point process.

Suppose that a sensor node needs a fixed amount of energy, denoted by \( \epsilon \), to transmit a packet. Recall that a sensor node keeps harvesting energy until it accumulates a necessary amount of energy for transmitting a packet. Let \( f_t^{(m)} \) represent the amount of energy that sensor node \( s_m \) is able to harvest in \((0, t]\) for \( t \in [0, \infty) \). Then, the harvest time \( H_t^{(m)} \) can be expressed by

\[
H_t^{(m)} = \min\{u \in (0, \infty) : f_{A_k^{(m)} + u}^{(m)} - f_{A_k^{(m)}}^{(m)} \geq \epsilon \}
\]

for \( k \in \mathbb{N} \). Set \( f_0^{(m)} = 0 \) almost surely and assume that \( \{f_t^{(m)}, t \geq 0\} \) is a non-decreasing Lévy process, i.e., a subordinator [5]. Then, the harvest times \( H_1^{(m)}, H_2^{(m)}, \ldots \) are mutually independent and identically distributed. Let \( H^{(m)} \) denote a random variable such that \( H^{(m)} = H_t^{(m)} \) in distribution. Let \( F_H^{(m)} \) denote the distribution function of \( H^{(m)} \), i.e.,

\[
F_H^{(m)}(x) = P(H^{(m)} \leq x)
\]
for \( x \in (0, \infty) \). At sensor node \( s_m \), suppose that the back-off times \( B^{(m)}_1, B^{(m)}_2, \ldots \) are mutually independent and identically distributed. Let \( B^{(m)} \) denote a random variable such that \( B^{(m)} = B^{(m)}_k \) in distribution and \( F_B^{(m)} \) be the distribution function of \( B^{(m)} \), i.e.,

\[
F_B^{(m)}(x) = P(B^{(m)} \leq x)
\]

for \( x \in (0, \infty) \). In addition, assume that the transmission times \( T_1^{(m)}, T_2^{(m)}, \ldots \) are mutually independent and identically distributed as a random variable \( T^{(m)} \). Let \( F_T^{(m)} \) denote the distribution function of \( T^{(m)} \), i.e.,

\[
F_T^{(m)}(x) = P(T^{(m)} \leq x)
\]

for \( x \in (0, \infty) \).

Under the assumption that \( \{H^{(m)}_k, k = 1, 2, \ldots\} \), \( \{B^{(m)}_k, k = 1, 2, \ldots\} \) and \( \{T^{(m)}_k, k = 1, 2, \ldots\} \) are independent and identically distributed sequences, the lengths of frames \( S^{(m)}_1, S^{(m)}_2, \ldots \) are also mutually independent and identically distributed as a random variable \( S^{(m)} \). Then, \( \{A^{(m)}_n, n = 0, 1, \ldots\} \) becomes a renewal point process [6]. For \( t \in (0, \infty) \), let \( V^{(m)}_t \) denote the time elapsed from \( t \) until the next frame starts. (See figure 2.) Then, \( V^{(m)}_t \), which is called the forward recurrence time [7] associated with the renewal point process \( \{A^{(m)}_n, n = 0, 1, \ldots\} \), can be expressed by

\[
V^{(m)}_t = A^{(m)}_{t^+} - t
\]

where

\[
Q^{(m)}_t = \min\{n \in \mathbb{N} : A^{(m)}_n > t\}.
\]

Recall that \( T^{(m)}_k \) is the time needed by sensor node \( s_m \) to transmit a packet in the \( k \)th frame. Define

\[
W^{(m)}_t = \begin{cases} V^{(m)}_t - T^{(m)}_k, & \text{if } V^{(m)}_t > T^{(m)}_k, \\ 0, & \text{otherwise} \end{cases}
\]

for \( t \in (0, \infty) \). Then, \( W^{(m)}_t \) represents the time elapsed from \( t \) until sensor node \( s_m \) begins to transmit a packet, if it happens in the current frame. (See figure 2.) Apparently, \( \{W^{(m)}_t, t \geq 0\} \) is a regenerative process [7] associated with the renewal point process \( \{A^{(m)}_n, n = 0, 1, \ldots\} \). Thus, \( W^{(m)}_t \) satisfies the renewal equation [6]. Note that

\[
P(W^{(m)}_t > y, A^{(m)}_1 > t) = P(H^{(m)}_1 + B^{(m)}_1 > t + y)
\]

for \( y \in (0, \infty) \). From the key renewal theorem [6], we then have

\[
P(W^{(m)}_t > y) \rightarrow \frac{1}{E(S^{(m)})} \int_0^\infty 1 - f_H^{(m)}(t) * F_G^{(m)}(t + y) \, dt \quad (11)
\]

as \( t \to \infty \).

Suppose that sensor node \( s_m \) starts transmitting a packet at time \( t \). Assume that the propagation delay is negligible. Then, the packet arrives at the sink node at time \( t \). The packet of sensor node \( s_m \) will not collide with a packet of sensor node \( s_m \) if and only if sensor node \( s_m \) starts transmitting no packet in the vulnerable period \( (t - T^{(m)}_k, t + T^{(m)}_k) \). Note that the necessary and sufficient condition for avoiding a collision with a packet of sensor node \( s_m \) is equivalent to the condition that

\[
W^{(m)}_t > T^{(m)}_k
\]

where \( W^{(m)}_t \) is defined in (9). Set

\[
\phi^{(m)} \triangleq \lim_{t \to \infty} P(W^{(m)}_t > T^{(m)}_k)
\]

for \( m \in \{1, \ldots, M\} \setminus \{\bar{m}\} \). Then, \( \phi^{(m)} \) represents the probability that a packet of sensor node \( s_m \) does not collide with a packet of sensor node \( s_m \) at steady state. By use of (11), an exact expression of the probability \( \phi^{(m)} \) is derived as follows:

\[
\phi^{(m)} = \frac{1}{E(S^{(m)})} \int_0^\infty \int_0^\infty 1 - F_H^{(m)}(t) * F_G^{(m)}(t + z) \, dt \, dz = \frac{1}{E(S^{(m)})} \int_0^\infty \int_0^\infty F_H^{(m)}(t + z) \, dt \, dz.
\]

Let \( \psi^{(m)} \) denote the probability that a packet of sensor node \( s_m \) never collides with any other packet at steady state. Then, the probability \( \psi^{(m)} \) can be expressed by

\[
\psi^{(m)} = \prod_{m \in \{1, \ldots, M\} \setminus \{\bar{m}\}} \phi^{(m)}.
\]

Let \( \eta^{\bar{m}} \) denote the nodal throughput that sensor node \( s_m \) can attain. Since sensor node \( s_m \) transmits a packet only once and succeeds in delivering the packet to the sink node with probability \( \psi^{(m)} \), we have

\[
\eta^{\bar{m}} = \frac{\psi^{(m)}}{E(S^{(m)})}.
\]

IV. OPTIMIZATION: CASE STUDY

The back-off time is a critical factor in designing the basic MAC scheme. Suppose that harvest times and transmission times are deterministic. Once packets of some sensor nodes collide, their collision will be repeated forever unless the sensor nodes take back-off times. In this section, we focus on a case in which the harvest times and the transmission times are
degenerated and obtain an optimal distribution for the back-off time that maximizes the throughput.

Suppose that the harvest time \( H^{(m)} \) and the transmission time \( T^{(m)} \) are degenerated into positive numbers \( \alpha \) and \( \gamma \) respectively, i.e.,

\[
H^{(m)} = \alpha \\
T^{(m)} = \gamma
\]  

(17)

almost surely for all \( m \in \{1, \ldots, M \} \). In addition, assume that the back-off times \( B^{(1)}, \ldots, B^{(M)} \) are positive random variables which are governed by a same proper distribution with mean \( \beta \). Let \( F_\beta \) denote the distribution function of the back-off time \( B^{(m)} \). Then, the probability in (10) is expressed by

\[
P(W_t^{(m)} > y, A_1^{(m)} > t) = 1 - F_\beta(y + t - \alpha)
\]

(18)

for \( y \in (0, \infty) \). Also, we have

\[
E( P( W_t^{(m)} > T^{(m)}, A_1^{(m)} > t)) = 1 - F_\beta(t - \alpha + \gamma)
\]

(19)

for \( t \in (0, \infty) \). Thus, the probability that a packet of sensor node \( m \) does not collide with any packet of sensor node \( m \) at steady state, denoted by \( \phi^{(m)} \), is yielded by

\[
\phi^{(m)} = \frac{\beta + \alpha - \gamma}{\beta + \alpha + \gamma}
\]

(20)

if \( \alpha > \gamma \) and

\[
\phi^{(m)} = \frac{1}{\beta + \alpha + \gamma} \int_{y-\alpha}^{\infty} 1 - F_\beta(x) \, dx
\]

(21)

if \( \alpha < \gamma \). Since the probability \( \phi^{(m)} \) is identical for all \( m \in \{1, \ldots, M \} \), the probability that a packet of sensor node \( m \) does not collide with any other packet at steady state, denoted by \( \psi_m \), is obtained by

\[
\psi_m = \frac{(\beta + \alpha - \gamma)^{M-1}}{(\beta + \alpha + \gamma)^M}
\]

(22)

if \( \alpha > \gamma \) and

\[
\psi_m = \frac{(\beta + \alpha - \gamma)^{M-1}}{(\beta + \alpha + \gamma)^M} \left[ \int_{y-\alpha}^{\infty} 1 - F_\beta(x) \, dx \right]^{M-1}
\]

(23)

if \( \alpha < \gamma \). Note that

\[
\frac{(\beta + \alpha - \gamma)^{M-1}}{(\beta + \alpha + \gamma)^M} \leq \psi_m \leq \frac{\beta^{M-1}}{(\beta + \alpha + \gamma)^M}
\]

(24)

when \( \alpha < \gamma \) since

\[
0 \leq \int_{0}^{\gamma-\alpha} 1 - F_\beta(x) \, dx \leq \gamma - \alpha.
\]

(25)

In practice, the harvest time is usually much longer than the transmission time. Thus, we focus on the case that \( \alpha > \gamma \). In this case, the nodal throughput is given in (22). By differentiating the nodal throughput with respect to the expected back-off time, denoted by \( \beta \) and equating to zero, we obtain an critical point as follows:

\[
\hat{\beta} = (2M - 1)\gamma - \alpha.
\]

(26)

Since \( \beta \) should be strictly positive, the critical point \( \hat{\beta} \) is also a global maximum point as far as \( \alpha < (2M - 1) \). Note that the maximum nodal throughput that sensor node \( s_m \) can attain is yielded by

\[
\hat{\eta}_m = \frac{1}{2\gamma M} \left( \frac{1}{M} - 1 \right)^{M-1}
\]

(27)

if \( \alpha < (2M - 1) \). Otherwise,

\[
\eta_m \to \frac{(\alpha + \gamma)^{M-1}}{(\alpha + \gamma)^M}
\]

(28)

as \( \beta \to 0 \).

Fig. 3. Total throughput with respect to expected back-off time given number of sensor nodes.

Figure 3 shows the total throughput with respect to the expected back-off time. In this figure, the harvest time is fixed to 10 unit times while the transmission time is set to be 1 unit time. In figure 3, we observe that there exists an optimal expected back-off time which maximizes the total throughput. Also, we notice that the optimal expected back-off time increases as the number of sensor nodes increases.

Figure 4 shows the total throughput with respect to the expected back-off time. In this figure, the number of sensor nodes is set to be 10 while the transmission time is fixed 1 unit time. In figure 4, we observe that there exists an optimal
expected back-off time which maximizes the total throughput. Also, we notice that the optimal expected back-off time increases as the harvest time decreases.

Fig. 4. Total throughput with respect to expected back-off time given harvest time.

V. CONCLUSIONS

RF energy harvesting technologies provided us with a new paradigm for wireless sensor networks; a wireless sensor network may live an eternal life without replacement of recharge of batteries. Against theoretical expectations, however, an RF energy harvesting wireless sensor network suffers from a scarcity of energy in practice. Perceiving such a difficulty, we considered a basic contending-type MAC scheme for supporting sensor nodes to deliver packets to a sink node. Since the basic MAC scheme belongs to ALOHA clan, it inevitably brings about a collision between some packets, which in turn leads to a loss of the packets. Such packet collision and packet loss also degrade throughput performance. Using the renewal theory, we thus developed an analytical method to exactly calculate the throughput that the basic MAC scheme is able to attain. Then, we investigated the effect of packet collision and packet loss on the throughput. In a study of the case that harvest times and transmission times are deterministic, we found an optimal back-off time, which maximizes the throughput. Furthermore, we proved that an optimal back-off time is not characterized by the distribution but only by the mean value.

REFERENCES

[1] I. Akyildiz, W. Su, Y. Sankarasubramaniam, and E. Cayirci, “Wireless Sensor Networks: A Survey,” Elsevier Computer Networks, vol. 38, no. 4, pp. 393-422, March 2002.
[2] D. Bouchouicha, F. Dupont, M. Latrach, and L. Ventura, “Ambient RF Energy Harvesting,” Proceedings of International Conference on Renewable Energies and Power Quality, 2010.
[3] C. Mikeka and H. Arai, “Design Issues in Radio Frequency Energy Harvesting System,” A Sustainable Energy Harvesting Technologies – Past, Present, Future, InTech, 2011.
[4] R. Rom and M. Sidi, Multiple Access Protocols - Performance and Analysis. Springer-Verlag, 1990.
[5] K. Sato, Levy Processes and Infinitely Divisible Distributions. Cambridge University Press, 1999.
[6] S. Ross, Stochastic Processes. 2nd edition, John Wiley and Sons, 1995.
[7] E. Cinlar, Introduction to Stochastic Processes. Prentice Hall, 1975.