1. Introduction

Modern day digital communication channels are subjected to Inter-Symbol Interference (ISI) which occurs due to the limited bandwidth allocated to channels. Multipath propagation further contributes constructively or destructively to other types of frequency selective channel distortions. Equalization to deal with these effects is a serious concern for designers of digital communication systems.

A physical channel response can be approximately represented by a finite length impulse response (FIR) filter. If the length of FIR filter is assumed to be infinite, ideal equalization can be accomplished. But, such filters are not feasible and therefore, length of filter is approximated according to the position of zeros of channel transfer function. Realistic channel responses are mostly characterized by Non-Minimum Phase (NMP) system whose one of the z-plane zeros lie outside the unit circle. Adaptive linear transversal filters are widely used to compensate for channel distortions in the received message signal as it elucidates the derivation of gradient based adaptive algorithms. Since, channel equalization can be seen as a problem of inverse modeling, filter in cascade with the channel response approximates the identity operation. Adaptive filters are necessary when the statistical characteristics of the input data are inadequate or known to change with time. They have the virtue of self-optimization and consist predominantly of a time varying filter, characterized by a set of adaptable coefficients and an iterative algorithm which modifies the coefficients, as more statistical information concerning the relevant signal is learnt.

Conventional gradient based optimization techniques such as Least Mean Square (LMS) and Recursive Least Square (RLS) algorithms are computationally too expensive and unsuitable for a fast dynamically changing channel as they require a latent time to collect the training data. Artificial neural network and multilayer perception based equalizer are marked by slow convergence and
unsatisfactory performance for nonlinear channel equalization. They can at times produce unpredictable solutions during the training phase. Grievances incurred by these gradient based algorithms resulted in using evolutionary soft computing tools for linear and nonlinear channel equalization in recent years. Genetic Algorithm (GA), Bacteria Foraging Optimization (BFO), Particle Swarm Optimization (PSO), Differential Evolution (DE), Artificial Immune System (AIS) are some of these techniques have their own distinctive way to exploit and explore the search space of the problem. Most of these methods are more effective when dealing with minimum phase channels.

Here, we focus on the application of WDO algorithm for effective equalization of NMP channels under nonlinear and noisy conditions. This approach has already been applied successfully in areas of antenna design and electromagnetics. We have presented some preliminary results in where in performance of WDO based channel equalizer for easy minimum phase channels was assessed with the help of extensive simulation work. Encouraged by its excellent performance, we extend the investigation on the suitability of WDO algorithm for equalization of NMP channels as well. It obtains better optimal results as compared to PSO and GA, from Normalized Mean Square (NMSE) and Bit Error Rate (BER) viewpoint, widely-used criteria in digital communication.

Remaining paper is organized as follows. Fundamentals of adaptive equalizer are discussed briefly in section II. The objective of this paper is also formulated in this section. WDO algorithm is reviewed and steps to implement it for NMP channel equalization is postulated in section III. Section IV contains the problem specifications and vigorous discussions on simulation results. The paper is finally concluded in section V with section VI devoted to references cited in this paper.

2. Problem Formulation and Preliminaries

Conventionally, an adaptive equalizer learns the channel attributes through an initial training period during which the transmitter sends out a known and synchronized data sequence. The receiver utilizes this sequence as the desired reference signal to update the equalizer parameters or coefficients. The equalizer switches to a decision directed mode at the instant channel attributes or characteristics have been learnt sufficiently well i.e. when the error function is minimized. This decision directed compliance permits an equalizer to track variations in the channel parameters during realistic data transmission.

Throughout the paper, we have used an equalization scenario in a baseband discrete time digital communication system depicted in Figure 1. FIR filter $h(z)$ is adopted to model a dispersive nonlinear channel which proliferates the transmitted energy throughout several symbol intervals of data. The transmitted symbols $u(k)$ are

![Figure 1](image-url)

Figure 1. Equalization scenario in a baseband discrete time digital communication systems.
convolved with the transfer function of channel model to get the kth sequence at the output of the channel.

\[ a(k) = \sum_{i=0}^{L} h_i u(k-i) \quad 0 < k \leq n \]  

where L is the length of channel, \( h_i \) is the channel taps and is the number of transmitted symbols. The input to the receiver after including nonlinear effects of amplifiers and converters in the transceiver can be modelled as

\[ r(k) = b(k) + N(k) \]  

where \( b(k) = g(.) \) is the nonlinearity added before the input faces the equalizer at the receiver end and \( N(k) \) is the white Gaussian noise with zero mean and variance \( \sigma_s^2 \). Symbol sequence \( u(k) \) is Independently Identically Distributed (IID) with a variance \( \sigma_u^2 \). The signal to noise ratio (SNR) of the system is defined as

\[ \text{SNR} = \frac{\sigma_u^2}{\sigma_s^2 \left( \sum_{i=0}^{L} k_i^2 \right)} \]  

Each transmitted symbol is reconstructed by the equalizer installed at the receiver and is represented by \( \hat{y}(k) \),

\[ \hat{y}(k) = \sum_{i=0}^{Q} r(K-Q)^* w(k) \]  

where \( w = [w_0, w_1, \ldots, w_7] \) is the associated weight vector and \( Q \) is the length of equalizer. An error signal defined by \( e(k) = y(k) - \hat{y}(k) \) is used to modify the internal parameters or the weights of the equalizer according to the optimization algorithm adopted for training. Mean Square Error (MSE) \( \xi \) is precisely a quadratic function of the components of the weight vector when the input components and desired response are stationary stochastic variables. Here, \( \xi \) is used as the cost or objective function,

\[ \xi = \frac{1}{n} \sum_{k=1}^{n} e^2(k) \]  

The objective of this paper is to minimize the above cost function (5) using WDO algorithm with special focus on non-minimum phase channel impulse response and to compare the results by two similar class of population based evolutionary algorithm, PSO and GA. PSO is based on the constructive cooperation between the particles whereas GA works on the theory of survival of the fittest. Thereafter, the actual performance of the system is tested by evaluating Bit Error Rate (BER) which is a figure of merit that allows different designs to be compared in a fair manner.

### 3. WDO for Channel Equalization

Wind driven optimization algorithm is a population based, nature inspired iterative heuristic global optimization algorithm for multi-dimensional and multimodal problems. It has the potential to implement constraints on the search domain as reported in14. Motion of wind in the earth’s atmosphere is the main source of inspiration. Wind in the form of group of air parcels blow in the troposphere attempting to balance the air pressure horizontally. The population of air parcels travel within the search space randomly within the bounds of [-1, 1]. The velocity and position of each air parcel is upgraded every iteration as the parcels move towards the best pressure location to obtain the optimum position at the end of the last iteration. The velocity and position update equation is postulated as:

\[ U_{\text{new}} = (1-a)U_{\text{cur}} - g x_{\text{cur}} + \left( \frac{RT}{j-1} \right) (x_{\text{opt}} - x_{\text{cur}}) + \frac{c u_{\text{other dim}}}{j} \]  

\[ x_{\text{new}} = x_{\text{cur}} + U_{\text{new}} \Delta t \]  

where \( x_{\text{new}} \) and \( x_{\text{cur}} \) are the new velocity and new position of air parcel for the next iteration respectively. Time step of \( \Delta t=1 \) is assumed in (7).

The first term of (6) states that the air parcels will resume its current path at a velocity proportionally lessened by frictional force, if other forces are not acting on the air parcel. \( \alpha \) is the friction coefficient. Second term portrays that the gravitation force draws the air parcel persistently from its current location, \( x_{\text{cur}} \) towards the centre of the coordinate system in which search space is juxtaposed. The magnitude of the force is proportional to the gravitational constant, \( g \). This gravitational force prevents any air parcel from sticking at the boundaries by eventually pulling them back into the search space. This factor is an added advantage over other nature-inspired optimization algorithms.

The third term in (6) signifies that to diminish the repercussions of pressure gradient, location closer to the optimum position, \( x_{\text{opt}} \) is most likely occupied by higher
ranked air parcel. A rank based approach is used to rank (i) the air parcels in descending order depending on their pressure values. \( RT \) involved in the term denotes the universal gas constant and temperature. The last term in (6) allows the velocity direction to be adjusted by other dimensions with a greater impact on higher ranked air parcels. Contribution of Coriolis force is represented by \( C = -2|\Omega|RT \). Rotation of the earth represented by \( \Omega \) introduces indiscriminate effect from other dimensions providing vigour to the motion of the parcel.

Flow chart of the basic algorithm is shown in Figure 2 and Table 1 describes the algorithm to implement WDO based channel equalizer.

Table 1. WDO algorithm for channel equalizer

| Step # | Description |
|--------|-------------|
| 1.     | Determination of output of the channel: Uniformly distributed random binary signal, \( u(k) \) is considered as the input to the channel with 'n' number of samples. Channel produces an output \( a(k) \) given by (1). |
| 2.     | Equalizer input: The output of the channel \( a(k) \) is passed through nonlinear mathematical function to incorporate nonlinearity in the output signal and this resultant signal \( r(k) \) acts as input to equalizer. |
| 3.     | Initialization of air parcels: Since it is an evolutionary algorithm, we begin with a group of random solutions, which is a group of weight vectors of equalizer. Each weight vector consists of Q number of elements and is represented by air parcel which is basically a binary string of definite length. So, a set of binary strings equal to the population of air parcels is initialized to represent corresponding weight vector. Each weight vector in the set can be a probable optimum solution. |
| 4.     | Calculation of desired output of the equalizer: The desired signal \( y(k) \) is created by delaying the input sequence \( u(k) \) by \( m \) samples, where \( m = Q/2 \) or \( (Q+1)/2 \). |
| 5.     | Fitness evaluation: For each \( k^{th} \) weight vector, MSE is determined using (5) and is used as a cost/fitness function. |
| 6.     | Ranking of air parcel: The air parcel population is ranked based on their pressure value (cost function) and velocity is updated according to (6) with following limitation. \[ u_{\text{new}} = \begin{cases} u_{\text{max}} & \text{if } u_{\text{max}} > u_{\text{max}} \\ -u_{\text{max}} & \text{if } u_{\text{max}} < u_{\text{max}} \end{cases} \] (8) |
| 7.     | Weight updation: Weight vector is updated using (7) |
| 8.     | Steps 5, 6 and 7 are repeated until maximum number of iterations is reached. |
| 9.     | At the end of stipulated iterations, almost all air parcels occupy same position and the best solution i.e. least ranked weight vector is considered as optimum solution. |

4. Simulation Results and Discussion

In this section, two sets of experiments are conducted...
with the help of computer simulations to investigate the performance enhancement of WDO based channel equalizer over GA and PSO. We have focused our attention on NMP model\textsuperscript{16} of the channel because majority of physical channels are not minimum phase. Throughout the simulation, noise is modelled as having white Gaussian distribution with 10dB and 20dB SNR. Simulation results are given in terms of MSE, rate of convergence and BER. The defining parameters of WDO, PSO and GA are recorded in Table 2.

Table 2. WDO, PSO and GA PARAMETERS used for simulation

| Parameters                      | WDO | PSO | GA |
|--------------------------------|-----|-----|----|
| Population size                | 60  | 60  | 60 |
| Iteration cycles/ Maximum no.  | 200 | 200 | 200|
| Number of input samples        | 500 | 500 | 500|
| Constant, RT                   | -   | -   | 1.5|
| Gravitational constant, g      | -   | -   | 0.1|
| Friction coefficient, α        | -   | -   | 0.8|
| Coriolis Force                 | -   | -   | 0.4|
| Maximum allowed speed, umax    | -   | -   | 0.3|
| Lower dimension boundary, dimMin| -   | -   | - 1|
| Upper dimension boundary, dimMax| -   | -   | 1  |
| Acceleration constant, C1      | -   | 1   | -  |
| Acceleration constant, C2      | -   | 1   | -  |
| Inertia weight,                | -   | 0.2 | -  |
| Mutation rate                  | -   | -   | 0.1|
| Crossover rate                 | -   | 0.75| -  |
| Selection probability          | -   | -   | 1/3|

Experiment 1: The true NMP model is:
NMP1: \( h(z)=1-2.333z^{-1}+0.667z^{-2} \) (9) with zeros at 2 and 1/3.

Experiment 2: The true NMP model is:
NMP2: \( h(z)=1+0.9z^{-1}+0.385z^{-2}-0.771z^{-3} \) (10) with zeros at 0.6 and -0.75 ± j0.85

Both linear and nonlinear channels are taken to add versatility to the investigation.

Linear channel (NL0): When the channel is assumed to be linear time invariant the output is given by (1).

Nonlinear channel (NL1, NL2, and NL3): Three nonlinear functions \( g(.) \) are considered to incorporate nonlinearity in the channel.
NL1: \( b(k)=\tanh(a(k)) \)
NL2: \( b(k)=a(k) + 0.2 * (a(k)^2) - 0.1 * (a(k)^3) \)
NL3: \( b(k)=a(k) + 0.2 * (a(k)^2) - 0.1 * (a(k)^3) + 0.5 * \cos(\pi * a(k)) \)

The output of the nonlinear noisy channel \( b(k) \) is given by (2).
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Figure 4. Convergence characteristics of channel NMP1 at 20dB.

The simulation results shown in Figure 3-6 reveal that WDO based equalizer converges consistently with lesser number of iterations as compared to GA and PSO. For a linear channel NMP1 NL0 under moderate noise level of 20dB, MSE achieved by WDO is -17.84dB at 10 iterations whereas NMP2 achieved -13.02dB at 19 iterations. When noise level is further increased to 10dB, NMP1 takes 10 iterations to converge to -14.19dB and NMP2 converges to -11.87dB at 11 iterations. These results are much better than GA, and PSO, as can be compared from Table 3-6.

Figure 5. Convergence characteristics of channel NMP2 at 10dB.
Three models of nonlinear channel NL1, NL2, and NL3 in increasing order of complexity are considered for each experiment 1 and 2 at two different noise levels 10dB and 20dB. The results are documented from Table 3-6. It clearly depicts that the first channel NMP1 performs may well when non-linearity NL1 is considered for both cases of 10dB and 20dB SNR. MSE as low as -11.82dB at 18 iterations and -17.46dB at 10 iterations is obtained for this channel by WDO when SNR is 10dB and 20dB respectively. As the complexity of nonlinearity increases, the performance diminishes but WDO still performs better than GA and PSO.
**Table 3.** Convergence performance comparison of WDO with PSO and GA for channel NMP1 at 10dB SNR

|        | NL0          | NL1          | NL2          | NL3          |
|--------|--------------|--------------|--------------|--------------|
| MSE    | No. of iter. | MSE          | No. of iter. | MSE          | No. of iter. | MSE          | No. of iter. |
| WDO    | -14.19       | 10           | -11.82       | 18           | -9.64        | 14           | -9.799       | 14           |
| PSO    | -11.98       | 20           | -11.6        | 36           | -9.419       | 18           | -8.009       | 24           |
| GA     | -11.19       | 29           | -9.565       | 25           | -7.557       | 14           | -7.654       | 16           |

**Table 4.** Convergence performance comparison of WDO with PSO and GA for channel NMP2 at 10dB SNR

|        | NL0          | NL1          | NL2          | NL3          |
|--------|--------------|--------------|--------------|--------------|
| MSE    | No. of iter. | MSE          | No. of iter. | MSE          | No. of iter. | MSE          | No. of iter. |
| WDO    | -11.87       | 11           | -5.698       | 12           | -4.939       | 10           | -5.181       | 22           |
| PSO    | -10.78       | 13           | -4.928       | 12           | -4.803       | 20           | -4.833       | 13           |
| GA     | -9.562       | 20           | -4.484       | 17           | -4.256       | 27           | -3.876       | 18           |

**Table 5.** Convergence performance comparison of WDO with PSO and GA for channel NMP1 at 20dB SNR

|        | NL0          | NL1          | NL2          | NL3          |
|--------|--------------|--------------|--------------|--------------|
| MSE    | No. of iter. | MSE          | No. of iter. | MSE          | No. of iter. | MSE          | No. of iter. |
| WDO    | -17.46       | 10           | -13.93       | 13           | -10.27       | 21           | -10.61       | 20           |
| PSO    | -16.08       | 17           | -12.07       | 23           | -9.652       | 12           | -10.15       | 11           |
| GA     | -15.32       | 37           | -9.338       | 28           | -8.259       | 42           | -8.835       | 50           |

**Table 6.** Convergence performance comparison of WDO with PSO and GA for channel NMP2 at 20dB SNR

|        | NL0          | NL1          | NL2          | NL3          |
|--------|--------------|--------------|--------------|--------------|
| MSE    | No. of iter. | MSE          | No. of iter. | MSE          | No. of iter. | MSE          | No. of iter. |
| WDO    | -13.02       | 19           | -6.28        | 16           | -5.861       | 10           | -5.665       | 10           |
| PSO    | -12.65       | 15           | -6.10        | 20           | -5.145       | 15           | -5.021       | 10           |
| GA     | -11.43       | 32           | -5.252       | 18           | -4.303       | 19           | -3.922       | 21           |
However, second channel NMP2 is a severe channel and poses more of a challenge for the equalizer. Minimum MSE achieved is -13.02dB at 19 iterations for the linear case NL0, when SNR is 20dB. Worst result is obtained for non-linear case NL2 when the learning characteristic converges to -4.939 dB at 10 iterations. But still these results are better than GA and PSO.
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The actual performance of the communication system is tested by assessing the bit error rate performance of the equalizer at receiver side. Improved BER performance shown from Figure 7-10 and recorded in Table 7-8 depicts that the equalizer based on WDO method yields more accurate reconstruction of transmitted data compared to its other counterparts, GA and PSO. Probability of error of WDO is as low as 3 bit in 105 samples (3*10^{-5}) for NMP1, NL0 at 20dB. It performs exceptionally well for non-linear channel NMP1, NL1 where its improvement is 12dB over PSO and 2dB over GA.

### Table 7. Improvement of WDO over PSO and GA at 4x10^{-5} (probability of error) for channel NMP1

| Noise condition | SNR=10dB | SNR=20dB |
|-----------------|----------|----------|
| Channel type    | PSO      | GA       | PSO      | GA       |
| Linear channel  | NL0      | 1dB      | 2dB      | 0dB      | 1dB      |
| Nonlinear channel | NL1    | 4dB      | 2dB      | 12dB     | 2dB      |

### Table 8. Comparison of probability of error at 20dB signal to noise ratio for channel NMP2

| SNR=10dB | SNR=20dB |
|----------|----------|
| Linear channel NL0 |        |
| WDO      | 0.0055   | 0.0163   | 0.0085   | 0.0052   | 0.0080   | 0.0094   |
| PSO      | 0.0601   | 0.0668   | 0.0788   | 0.0596   | 0.0709   | 0.0625   |
| GA       | 0.0584   | 0.0632   | 0.0754   | 0.0588   | 0.0698   | 0.0620   |
| Nonlinear channel NL1 |    |
| WDO      | 0.0574   | 0.0630   | 0.0752   | 0.0588   | 0.0698   | 0.0620   |
| PSO      | 0.0601   | 0.0668   | 0.0788   | 0.0596   | 0.0709   | 0.0625   |
| GA       | 0.0584   | 0.0632   | 0.0754   | 0.0588   | 0.0698   | 0.0620   |

### 5. Conclusion

As we begin to push the limits of the communication channel by increasing the data rate, linear distortions and non-minimum phase characteristics of channel often becomes the limiting factors. Thus, it is necessary to define new and robust algorithms to equalize the channel. It has been established in this work that WDO offers an excellent convergence characteristics and performance improvement over PSO and GA based equalizer. Hence, it is a well suited potential learning tool for severe channels such as non-minimum phase.

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