Summary of theoretical contributions

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Abstract. Results from various theoretical approaches and ideas presented at this exciting meeting are reviewed. I also point towards future directions, in particular hydrodynamic behaviour induced by jets traveling through the quark-gluon plasma, which might be worth looking at in more detail.

1. Theoretical overview

We have witnessed an exciting conference with an excellent program, heated scientific discussions and lots of new data and theoretical ideas. Our topics ranged from astrophysics to field theory, from heavy-ion reaction phenomenology to big-bang cosmology.

My task, i.e. to review all in all about 30 theory talks, is combined in this paper with a cross-disciplinary analysis of experiments, which verify - pardon, falsify the theoretical conjectures in many cases - there is an old saying that a theory can never be verified: even if lots of data support the theory, at some point the theory will always go astray...

Let me rearrange the order of the theory talks on the topics of our meeting:\textsuperscript{‡}:

- Equation of State
- Collective Dynamics
- Jets: Production and Quenching
- Results from $p + p$, $p(d) + A$ and $A + A$ collisions
- Signatures of Quark Gluon Plasma
- QCD at Finite Temperature and Density
- Multiparticle production, fluctuations and correlations
- Cosmological Implications of the QCD Phase Transition
- QCD Phenomenology
- Low $x$ behaviour of QCD

\textsuperscript{‡} Instead of giving explicit references I refer the reader to the electronic proceedings available on the Web
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- Strangeness and heavy flavor production

The common interest is given by the title of the conference: "Physics and astrophysics of the quark gluon plasma".

- Astrophysics
- Lattice
- Colored Glass
- Fluctuations & DCCs
- J/Psi & EM Probes
- Strangeness
- Transport Theory
- Hydro & Jets

John Ellis gave a beautiful survey of the common issues in both heavy-ion physics and the big bang cosmology: We do in both cases study a very fast expansion of dense/hot strongly interacting matter, and do have the task to reconcile whatever happened in the first few nanoseconds of the big bang from the sparse debris found nowadays. The connection to the matter-anti matter asymmetry problems is particularly exciting for future topical studies at the LHC. This is quite analogous to the transient $6-8\text{ fm}/c \approx 2.5 \times 10^{-23}\text{ s}$ timescale of the collision processes at RHIC.

The intense astrophysics discussions between Bombaci and Bandyopadhyay about the possible occurrence of massive strange quark stars (SQS), the transition of neutron stars to strange hyperon-, hybrid- and quark stars, and the relation to the gamma ray bursts (quark-deconfinement nova-model) has been of particular interest - this transition is predicted to yield a radius-collapse of several kilometers.

The first observation of the "double delight pulsar" psr-j0737-3039 will enable us to pin down the mass-radius curves by the spin-orbit effect with high precision. D. Bandyopadhyay showed that soft equations of state (EOS) are ruled out by EXO 0748-676. The connection of conjectured different color superconducting phases to the cooling curves of SQS have been pointed out in the paper by Mishra and Mishra.

The lattice-QCD (lQCD) discussions between Gavai and Laermann centered about the questions on the order of the phase transition and on the speed of sound. Laermann stated that there is no indication for criticality, while Gavai and friends showed that the critical endpoint is at $T = 0.95 \ T_c$, $\mu_B T = 1.1 - 1.3$, i.e. less than half of the $\mu_B=400$ MeV values given by the Swansea-Bielefeld and Wuppertal-Budapest collaborations (cf. Fig 1). Gavai showed also that all the way up to $T = 2 T_c$, the speed of sound is much less, $c_s^2 = 0.15$ at $T = 1.1 \ T_c$, than that of a noninteracting ultrarelativistic (massless) gas, $c_s^2 = 1/3$.

Somewhat in this summary, also pardon the fact that some of the many interesting items have not been taken up here, because I did not witness the first days of the conference.
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Figure 1. The phase diagram with the critical end point at $\mu_B \approx 400$ MeV, $T \approx 160$ MeV as predicted by the Swansea-Bielefeld and Wuppertal/Budapest collaborations [1]. In addition, the time evolution in the $T-\mu$-plane of a central cell in UrQMD calculations [2] is depicted for different bombarding energies. Note, that the calculations indicate that bombarding energies $E_{\text{LAB}} \lesssim 40$ A·GeV are needed to probe a first order phase transition as predicted by the Swansea-Bielefeld and Wuppertal-Budapest collaborations. At RHIC (see insert at the $\mu_B$ scale) this point is accessible in the fragmentation region only (taken from [3]). The new conjecture by Gavai and friends is that the critical endpoint is moving to the left to $T = 0.95 T_c$, $\mu_B = 1.1 - 1.3 T_c \approx 190-220$ MeV. In this case the top SPS energy range would be best suited to explore the endpoint in central Au+Au collisions.

In the colored glass condensate section, Venugopalan explored the demise of the structure function, in particular how the dipole and higher multipole operators may turn out to be the more relevant observables at high energies. Adding valence quark contributions, Kovchegov showed a quite satisfactory agreement of the Color-Glass-Condensate (CGC)-model to the observed rapidity dependence of the $p_T$-distributions. McLerran iterated the theme of the Color Glass Condensate as THE Medium: Pomerons, Odderonas, Reggeons as Quasiparticle excitations of the CGC - does this mean that the CGC is the initial phase for the QGP? Is the strong Quark-Gluon Plasma (sQGP) really the CGC? Is rapid ‘thermalization’ due to the CGC? Does flow arise largely from the CGC? Well, definitely LHC is THE CGC machine – according to McLerran.

Fluctuations and Disordered Chiral Condensates (DCC’s) were discussed by Koch, Csörgö, Chandrasekar and Randrup, among others. $K/\pi$ fluctuations increase towards lower beam energy with a significant enhancement over the hadronic cascade model UrQMD [5] (cf. Fig. 2)! On the other hand, $p/\pi$ fluctuations are negative – this indicates a strong contribution from resonance decays, as was shown by Koch in comparing NA49-data to UrQMD results.

Dileptons, J/Psi, and photons have been discussed by Lee, Mustafa and Koch
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Figure 2. Energy dependence of the event-by-event fluctuation signal of the $(K^+ + K^-)/(\pi^+ + \pi^-)$ ratio (left panel) and the $(p + \bar{p})/(\pi^+ + \pi^-)$ ratio (right panel). The systematic errors of the measurements are shown as grey bands (from Ref. [4]).

(among others). Large corrections on the QCD NLO Quarkonium- Gluon/hadron dissociation cross section have been reported even for the Ypsilon system, especially near threshold. The thermal width of the $J/\Psi$ should be $\sim 1$ GeV at $T=600$ MeV according to Lee’s estimates.

Strangegeeness and equilibration has been the main topic of Rafelski, Cleymans, Braun-Munzinger and Bleicher. The structure in the $K/\pi$ ratios reported by NA49 near $\sqrt{s} = 8$ GeV is not reproduced by any model (cf. Fig. 3), but Peter Braun-Munzinger notes: the natural smearing is 3 GeV near that energy - how can the 'horn' then be so steep? Hadron-string models work well globally, as Bleicher reports, but these models do NOT give MULTI-STRANGE BARYONS! Is the alternative a four parameter nonequilibrium thermal model, with $T, \mu, \Gamma_\mu, \Gamma_s$, by Rafelski et al.?

The extreme density/temperature dependence of the characteristic equilibration time, $\tau_{eq.} \sim T^{-60}$, was pointed out by Braun-Munzinger, which implies that all particles freeze out at about the hadronization time. According to Braun-Munzinger this might be due to Carsten Greiner’s conjecture of Hagedorn states as intermediate doorway states.

Deeply bound $\bar{p}$ and $K^-$ states as gateway to cold and dense matter were discussed by Walter Greiner: $\bar{p}$’s – due to $G$-parity in the strong interactions – and $K^-$ can suppress repulsive vector fields, thus predicting discrete bound states with binding energies of several 100th MeV and 20 fm/c life times [8]. Formation of such cold and highly dense nuclear system at densities $\rho 3 - 5\rho_0$ will be studied in dense $\bar{p}$ - nuclear systems at FAIR (GSI)and the $K$-nucleus collisions at J-PARC.

Jacak, Shuryak, Heinz and Chauduri discussed applications of hydrodynamics to RHIC-collisions. The reasons why hydro does reasonably well fit both, radial and elliptic
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Figure 3. The energy dependence of the $4\pi$-yields of strange hadrons, normalized to the pion yields, in central Pb+Pb/Au+Au collisions. The data are compared to string hadronic models UrQMD2.0 [5]: dotted lines; HSD [6]: dashed-dotted lines) and statistical hadron-gas models from Braun-Munzinger and Becattini and collaborators (with strangeness undersaturation: dashed line, assuming full equilibrium: solid line). The figure is taken from [7].

Figure 4. Differential elliptic flow $v_2(p_T)$ for several identified hadron species from minimum bias Au+Au collisions at $\sqrt{s} = 130$ GeV (right) and $\sqrt{s} = 200$ GeV compared with hydrodynamic predictions from [9]. The figure is taken from [10].
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Figure 5. Scaled elliptic flow $v_2/n(p_{T}/n)$ of baryons and mesons as calculated from the quark flow compared to data for $\Lambda$-hyperons and kaons. The figure is taken from [26].

flow for a large number of hadron species (cf. Fig. 4), is still not fully settled. The question of early thermalization and the unsatisfactory rapidity distributions from ideal hydrodynamics remain open.

Figure 6. The per trigger particle normalized $\Delta \phi$ distributions for p+p (a) and 5% most central Au+Au collisions (b). The figure is taken from [24].

Bass showed in his talk, however, that the recombination/quark coalescence models (cf. Fig. 5) can help analyze the participant scaling and even the charm flow. However,
as Bleicher showed, even the hadron/string model UrQMD may exhibit “recombination” and participant scaling.

Jacak showed the PHENIX jet-pair distributions, which clearly give a novel signal to the away-side jet suppression (cf. Fig. 6 for STAR results), i.e. the recent topic of Mach-cones induced by stopped jets in the quark-gluon liquid [12]. This is most important as an observable, because it links the parton dynamics and collective flow and the jet tomography to the measurement of the speed of sound in the medium - be it a weakly or strongly coupled plasma: the opening angle of the Mach-shock-wakes directly gives the speed of sound in the medium, which is linked to both, the appearance of vector potentials and the parton/constituent mass parameters.

2. Interlude on Mach shocks

Sideward peaks around the away-side jet have been predicted recently [12] as a signature of Mach shock waves created by stopping partonic jets propagating through a QGP formed in an ultrarelativistic heavy–ion collision. Analogous Mach shock waves were studied long ago for heavy-ion induced Mach shocks travelling through cold hadronic matter [14, 15] as well as in nuclear Fermi liquids [16, 17]. It has been argued that Mach–like motions of quark–gluon matter can appear via the excitation of collective plasmon waves by the moving color charge associated with the leading jet [12, 22].

Point–like perturbations (a small body, a hadron or parton etc.) moving with a supersonic speed in the spatially homogeneous ideal fluid produce the Mach region of the
perturbed matter \cite{18}. In the fluid rest frame (FRF) the Mach region has a conical shape (cf. Fig. 7) with an opening angle with respect to the direction of particle propagation given by

\[ \bar{\theta}_M = \sin^{-1}\left(\frac{c_s}{\tilde{v}}\right), \]

where \(c_s\) denotes the sound velocity of the unperturbed (upstream) fluid and \(\tilde{v}\) is the particle velocity with respect to the fluid. In the FRF, trajectories of fluid elements (perpendicular to the surface of the Mach cone) are inclined at the angle \(\Delta\theta = \pi/2 - \bar{\theta}_M\) with respect to \(\tilde{v}\). Strictly speaking, formula (1) is applicable only for weak, sound–like perturbations and certainly not valid for space–time regions close to a leading particle. Nevertheless, it suffices for a qualitative analysis of flow effects. Following Refs. \cite{12, 13, 19} one can estimate the angle of preferential emission of secondaries associated with a fast jet in the QGP. Substituting \(\tilde{v} = 1, c_s = 1/\sqrt{3}\) into Eq. (1) gives the value \(\Delta\theta \simeq 0.96 \text{ rad} = 61^\circ\). This agrees well with positions of maxima of the away–side two–particle distributions observed by the STAR Collaboration (cf. \textit{6}) in central Au+Au collisions at RHIC energies (cf. also B. Jacak’s talk).

Let us consider \cite{13} the case when the away–side jet propagates with velocity \(\mathbf{v}\) parallel to the matter flow velocity \(\mathbf{u}\). Assuming that \(\mathbf{u}\) does not change with space and time, and performing the Lorentz boost to the FRF, one sees that a weak Mach shock has a conical shape with the axis along \(\mathbf{v}\). In this reference frame, the shock front angle \(\bar{\theta}_M\) is given by (1). Transformation from the FRF to the c.m. frame (CMF) shows that the Mach region remains conical, but the Mach angle becomes smaller in the CMF:

\[ \tan \theta_M = \frac{1}{\gamma_u} \tan \bar{\theta}_M, \]

Here and below the quantities in the FRF are marked by a tilde.
where $\gamma_u \equiv (1 - u^2)^{-1/2}$ is the Lorentz factor corresponding to the flow velocity $u$. The resulting expression for the Mach angle in the CMF is

$$\theta_M = \tan^{-1} \left( c_s \sqrt{\frac{1 - u^2}{\nu^2 - c_s^2}} \right),$$

(3)

where

$$\tilde{v} = \frac{v \mp u}{1 \mp vu},$$

(4)

and upper (lower) sign corresponds to the jet’s motion in (or opposite to) the direction of collective flow. For ultrarelativistic jets ($v \to 1$) one can take $\tilde{v} \simeq 1$ which leads to a simpler expression

$$\theta_M \simeq \tan^{-1} \left( \frac{c_s \gamma_s}{\gamma_u} \right) = \sin^{-1} \left( c_s \sqrt{1 - u^2 - c_s^2} \right),$$

(5)

where $\gamma_s = (1 - c_s^2)^{-1/2}$. According to (5), in the ultrarelativistic limit $\theta_M$ does not depend on the direction of flow with respect to the jet. The Mach cone becomes more narrow as compared to jet propagation in static matter. This narrowing effect has a purely relativistic origin. Indeed, the difference between $\theta_M$ from (5) and the Mach angle in absence of flow ($\lim_{u \to 0} \theta_M = \sin^{-1} c_s$) is of second order in the collective velocity $u$. The Mach angle calculated from (5) is shown in Fig. 8 (from [13]) as a function of $u$ for different sound velocities $c_s$. Following Ref. [19], the value $c_s^2 = 1/5$ is identified with the hadronic matter and $c_s^2 = 1/3$ with ideal QGP composed of massless quarks and gluons. The value $c_s^2 = 2/3$ may be chosen to represent a strongly coupled QGP [20]. We see that precise measurements will provide valuable information on the properties of the quark-gluon liquid [11, 12].

3. Future directions

I propose future correlation measurements which can yield spectroscopic information on the plasma:

(i) Measure the sound velocity of the expanding plasma by the emission pattern of the plasma particles traveling sideways with respect to the jet axis: The dispersive wave generated by the wake of the jet in the plasma yields preferential emission to an angle (relative to the jet axis) which is given by the ratio of the leading jet particles’ velocity, divided by the sound velocity in the hot dense plasma rest frame. The speed of sound for a non-interacting gas of relativistic massless plasma particles is $c_s \approx \frac{\sqrt{3}}{4} \approx 57\% c$, while for a plasma with strong vector interactions, $c_s \approx c$, since strong shocks can yield larger speeds. They are also related – unlike the linearized sound waves – to strong matter flow with high flow velocities $v_f$ approaching the speed of light relative to the expanding medium. Hence, the emission angle measurement can yield information of the interactions in the plasma.
(ii) The NA49 collaboration has observed the collapse of both, $v_1$- and $v_2$-collective flow of protons (cf. Fig. 9), in Pb+Pb collisions at 40 A·GeV, which presents first evidence for a first order phase transition in baryon-rich dense matter. It should be possible to study the nature of this transition and the properties of the expected chirally restored and deconfined phase both at the forward fragmentation region at RHIC, with upgraded and/or second generation detectors, and at the new GSI facility FAIR.

(iii) A critical discussion of the use of collective flow as a barometer for the equation of state (EoS) of hot dense matter at RHIC showed that hadronic rescattering models can explain $< 30\%$ of the observed elliptic flow $v_2$ for $p_T > 2$ GeV/c [23, 28]. I interpret this as evidence for the production of superdense matter at RHIC with initial pressure way above hadronic pressure, $p > 1$ GeV/fm$^3$. 

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**Figure 9.** The flow $v_1$ and $v_2$ for protons from NA49 [27] for Pb+Pb at 40 A·GeV in comparison to the results of the hadron/string models HSD (red lines) and UrQMD (blue lines). Note the large $v_2$ deviations (at $y_{cm}$) of the data from the best conventional hadronic transport theories. The figure is taken from [25].
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Figure 10. Left: STAR data on near-side and away-side jet correlations compared to the HSD model for p+p and central Au+Au collisions at midrapidity for $p_T^{\text{rig}} = 4 \ldots 6 \text{GeV}/c$ and $p_T = 2 \text{GeV}/c \ldots p_T^{\text{rig}}$. Right: High $p_T$ correlations: in-plane vs. out-of-plane correlations of the probe (jet+secondary jet fragments) with the bulk ($v_2$ of the plasma at $p_T > 2 \text{GeV}/c$), prove the existence of the initial plasma state (STAR-collaboration, preliminary).

(iv) The fluctuations in the flow, $v_1$ and $v_2$, should be measured. Ideal Hydrodynamics predicts that they are larger than 50 % due to initial state fluctuations. The QGP coefficient of viscosity may be determined experimentally from the fluctuations observed and proof the conjecture of Ref. [11].

(v) The connection of $v_2$ to jet suppression has proven experimentally that the collective flow is not faked by minijet fragmentation and theoretically that the away-side jet suppression can only partially ($< 50\%$) be due to pre-hadronic or hadronic rescattering [23] (cf. Fig. 10).

(vi) I propose upgrades and second generation experiments at RHIC, which inspect the first order phase transition in the fragmentation region, i.e. at $\mu_B \approx 200 \ldots 400 \text{MeV}$ ($y \approx 3 \ldots 5$), where the collapse of the proton flow – analogous to the 40 A-GeV data – should be seen.

Let me finally express my birthday greetings to Bikash and thank him and his crew for decades of exciting physics conjectures, his strong involvement into our field and courage to built up such a great school of young successful scientists in India, which are highly competitive in the whole world.

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