Study of High Energy Heavy-Ion Collisions in a Relativistic BUU-Approach with Momentum-Dependent Mean-Fields*

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Abstract

We introduce momentum-dependent scalar and vector fields into the Lorentz covariant relativistic BUU- (RBUU-) approach employing a polynomial ansatz for the relativistic nucleon-nucleon interaction. The momentum-dependent parametrizations are shown to be valid up to about 1 GeV/u for the empirical proton-nucleus optical potential. We perform numerical simulations for heavy-ion collisions within the RBUU-approach adopting momentum-dependent and momentum-independent mean-fields and calculate the transverse flow in and perpendicular to the reaction plane, the directivity distribution as well as subthreshold $K^+$- production. By means of these observables we discuss the particular role of the momentum-dependent forces and their implications on the nuclear equation of state. We find that only a momentum-dependent parameter-set can explain the experimental data on the transverse flow in the reaction plane from 150 – 1000 MeV/u and the differential $K^+$- production cross sections at 1 GeV/u at the same time.

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1 Introduction

The main aim of high energy heavy-ion physics is to determine the equation of state (EOS) of nuclear matter under extreme conditions far from the ground state. To achieve this aim experiments between $100 \sim 2100 \text{ MeV/u}$ have been performed at the BEVALAC \cite{1} providing first information on the properties of hot and dense nuclear matter. A new generation of experiments is presently being performed at the SIS \cite{2} in Darmstadt delivering more precise and exclusive data about collective flow, multifragmentation or subthreshold particle production.

Any conclusion on the properties of hot and dense matter must rely on the comparison of the experimental data with theoretical predictions based on nonequilibrium models. Among these, the BUU-approach \cite{3, 4} is a very successful approach in describing the time-dependent evolution of the complex system especially with respect to single-particle observables. As a genuine feature of transport theories it has two important ingredients: the mean-fields or self-energies for nucleons and an in-medium nucleon-nucleon cross-section that accounts for the elastic and inelastic channels. By varying the mean-fields — which reflect a certain EOS — and comparing the theoretical calculations with the experimental data, one expects to be able to determine the nuclear EOS \cite{5}.

Within the framework of BUU-simulations we have succeeded to predict/reproduce particle production data in heavy-ion collisions and to clarify their reaction processes \cite{6, 7}. In spite of this success it has not yet been possible to determine the nuclear EOS because the mean fields are not uniquely fixed by the equation of state alone. In addition, it is so far not possible to extract the nuclear EOS from the results of the BUU calculations without ambiguities for other model inputs.

The most important model input besides the nuclear incompressibility is the momentum-dependence of the mean-fields; the nucleon-nucleus potential at normal density is attractive below and repulsive above about 200 MeV, respectively. Gale et al. \cite{8} were the first to take the momentum-dependence of the mean-fields into account in the BUU approach, but have not found any significant effect on the transverse flow in the reaction plane. Aichelin et al. \cite{9}, using the Quantum Molecular Dynamics (QMD) model, have found that the momentum-dependence plays a dom-
nant role e.g. for $K^+$ production [10]. In addition, Li et al. [11] also have obtained similar results in particle production calculations for light nuclei within the QMD approach.

However, there are problems with momentum-dependent (MD) mean-fields in these treatments that stem from their behavior above several hundred MeV/u where relativistic effects should become significant. Above 1 GeV/u, e.g., the Lorentz contraction of the phase-space distribution as well as relativistic kinematics largely influence the observables of interest. The Lorentz covariance of the interaction also influences the results such as the transverse flow [12] because the interaction range is Lorentz contracted, too. In addition, the Lorentz covariant vector-coupling produces a Lorentz force directed sideways from the vector fields which is not accounted for in present nonrelativistic transport simulations. Thus the Lorentz covariance of the interaction also generates a qualitatively new (i.e. directed) momentum-dependence of the mean-fields in the fast moving system. In the energy regime considered here we thus have to employ a Lorentz covariant transport approach with realistic MD mean-fields to examine the role of the MD forces as well as to determine the nuclear EOS.

The relativistic $\sigma-\omega$ model [13] and the Dirac phenomenology [14], which are based on the same picture, have successfully explained various properties of ground state nuclei [13] as well as high energy proton-nucleus scattering [14, 15], and thus provide a realistic basis for our investigation. The analysis of the experimental data clearly shows that we have to deal with large attractive scalar and repulsive vector fields in the medium. The relativistic BUU- (RBUU-) approach [4, 16, 17] is constructed in a Lorentz covariant way by combining the BUU-approach with the $\sigma-\omega$ model along the line of time-dependent Dirac-Brueckner theory [4, 18].

Even without any explicit momentum-dependence of the scalar and vector mean fields a momentum-dependence of the Schrödinger equivalent potential automatically emerges as a consequence of the Lorentz transformation properties of the vector-fields (see discussion below). Blättel et al. [17, 19] have shown in the RBUU approach, that this momentum-dependence (or the related effective mass) has a dominant effect on the transverse flow in the reaction plane through the Lorentz force from the vector fields.
Because of these large effects of a proper description of relativity on the flow observables it is interesting to look also for possible effects on particle production. In fact, Lang et al. [21] have shown in the RBUU approach that the nuclear incompressibility $K$ is more important than the effective mass for subthreshold $K^+$-production in Au + Au collisions whereas the conclusion is opposite in light systems such as Ne + Ne.

In all of these calculations, however, the strength of the momentum-dependence of the Schroedinger-equivalent optical potential has been overestimated at high relative energies because these calculations were based on the original $\sigma - \omega$ model which contains no explicit momentum-dependence of the nucleon self-energies [4, 17]. This shortcoming is essential because, as discussed above, the transverse momentum distribution in heavy-ion collisions has been found to be dominated by the MD potentials and to be less sensitive to the incompressibility $K$ [17, 19]. Thus a proper treatment of these fields is a necessary prerequisite on the way to determine the EOS of nuclear matter.

In preceding publications [22, 23] we have addressed the latter problem more generally and proposed a Fock-term like ansatz as well as suitable parametrizations for the MD interaction in line with the empirical optical potential or results from Dirac-Brueckner calculations. Sehn et al. [24] have attempted another approach for the MD mean-fields. They have introduced configuration-dependent couplings and fitted them to DBHF results for thermalized nuclear matter as well as two-Fermi-sphere configurations. However, this latter approach is strictly applicable only to these limiting configurations whereas our concept can be used for arbitrary phase-space distributions.

This explicit momentum-dependence is not very effective for nuclear matter problems at zero temperature. In matter-on-matter collisions at relativistic energies, however, it dominantly affects properties of this system such as the binding energy, the longitudinal and the transverse pressure [23]. Especially the transverse pressure is found to be reduced at high bombarding energies with respect to momentum-independent (MID) fields. Thus the momentum-dependence plays an important role if the local momentum distribution differs significantly from a spherical, equilibrated configuration.
In a previous letter [25] we have presented the first time-dependent covariant simulations with MD scalar and vector fields and have studied especially the transverse flow in nucleus-nucleus collisions at bombarding energies from $E_{\text{LAB}} = 150 - 800$ MeV/u. Our new approach has been found to reproduce the experimental flow data [26, 27] quite successfully. In that work, however, we have not obtained any definite information about the nuclear EOS.

In this paper, therefore, we calculate further independent observables with the same model input and compare them with experimental data. The most promising candidate is at present the subthreshold $K^+$-production in heavy nucleus-nucleus collisions. In fact, the previous RBUU [21] and QMD calculations [28] have predicted a high sensitivity to the incompressibility in $\text{Au} + \text{Au}$ collisions at $E_{\text{lab}} = 1$ GeV/u; however, reliable MD mean-fields have not been employed so far. In order to reduce the ambiguity of the input mean-field parametrizations, we also calculate flow phenomena, in particular the directivity distribution as suggested by Alard et al. [29, 30], using the same theoretical input for all observables.

The present paper is arranged as follows. In Sec. 2 we briefly explain the RBUU approach and the methods to introduce the MD scalar and vector mean-fields into this approach. In Sec. 3 we calculate various observables: the transverse flow in the reaction plane and perpendicular to this plane, the directivity distribution as well as differential $K^+$-production cross sections for $\text{Ne} + \text{NaF}$ and $\text{Au} + \text{Au}$. The transverse flow and the subthreshold $K^+$- yields are compared with the most recent experimental data from the SIS [31]. In order to minimize the ambiguities we aim at a set of mean-fields which reproduce the experimental data on the transverse flow and the $K^+$- production at the same time. We summarize and conclude our work in Sec. 4.

2 Relativistic BUU Approach with Momentum Dependent Mean-Fields

In this section we give a brief description of the RBUU-approach with the MD mean-fields and present the actual simulation method used for heavy-ion collisions.
In Ref. \[22\] we have formulated the Relativistic BUU equation with MD mean-fields as
\[
\{[\Pi_\mu - \Pi_\nu (\partial_\mu U_\nu) - M^* (\partial_\mu U_\mu) \} \partial_\mu \partial_\mu - [\Pi_\nu (\partial_\mu U_\nu) + M^* (\partial_\mu U_\mu) \} \partial_\mu \partial_\mu \} f(x, p) = I_{\text{coll}},
\] (1)
where \(f(x, p)\) is the Lorentz covariant phase-space distribution function, \(I_{\text{coll}}\) is a collision term given in Ref. \[4\], and \(U_s\) and \(U_\mu\) are MD scalar and vector mean-fields, respectively. Then the kinetic momentum \(\Pi_\mu\) and effective mass \(M^*\) become local quantities and are defined in terms of the fields by
\[
\Pi_\mu(x, p) = p_\mu - U_\mu(x, p)
\] (2)
\[
M^*(x, p) = M + U_s(x, p),
\] (3)
which satisfy the mass-shell condition:
\[
V(x, p) f(x, p) = 0
\] (4)
with
\[
V(x, p) \equiv \frac{1}{2} (\Pi^2(x, p) - M^2(x, p)),
\] (5)
where \(M\) and \(p_\mu\) are the baryon mass and the canonical momentum, respectively. The above mass-shell condition \([\Pi]\) shows that \(f(x, p)\) is nonvanishing only for \(V = 0\). The usual Wigner function \(\tilde{f}(t; x, p)\), which is a Lorentz scalar, is then obtained as
\[
f(x, p) = \tilde{f}(t; x, p) \ 2\Theta(\Pi_0) \ \delta(\Pi^2 - M^*^2),
\] (6)
where the step function \(\Theta(\Pi_0)\) restricts the phase space to the positive energy states.

In actual simulations the RBUU eq. \([\Pi]\) is solved with the test-particle method \([34]\) as follows: the Wigner function \(\tilde{f}(t; x, p)\) is expanded as
\[
\tilde{f}(t; x, p) = \frac{(2\pi)^3}{4N_T} \tilde{N}_T^{-A} \sum_{i=1}^{\tilde{N}_T^{-A}} \delta(x - x_i(t)) \delta(p - p_i(t)),
\] (7)
with the total nucleon number \(A\) and the number of test-particles per nucleon \(\tilde{N}_T\); the limit \(\tilde{N}_T \rightarrow \infty\) then provides the exact solutions.

Substituting the above eq. \([7]\) into the RBUU eq. \([\Pi]\), we can obtain the equations of motion for the test-particles as
\[
\frac{dx_i}{dt} = D_{p_i} \varepsilon(x_i, p_i) = \frac{\tilde{\Pi}_i}{\tilde{\Pi}_{0i}}
\]
\[
\frac{dp_i}{dt} = -D_{x_i} \varepsilon(x_i, p_i).
\] (8)
where \( \varepsilon(x_i, p_i) \) is the on-mass-shell energy of the i-th test-particles, and \( \Pi_\mu(x, p) \) is defined as

\[
\Pi_\mu(x, p) \equiv \frac{\partial}{\partial p^\mu} V(x, p). \tag{9}
\]

In the above equations the partial derivative \( \partial \) is defined by using \( p_0 \) as an independent variable whereas the total derivatives \( \partial \) are defined using \( p_0 \equiv \varepsilon(x, p) \).

Next we give the parametrization of the MD mean-fields and explain our implementation in the RBUU-simulation. Following Ref. [22] we first separate the mean-fields into a local part and an explicit MD part, i.e.

\[
U_s(x, p) = U_s^H(x) + U_s^{MD}(x, p),
\]

\[
U_\mu(x, p) = U_\mu^H(x) + U_\mu^{MD}(x, p), \tag{10}
\]

where the local mean-fields are determined by the usual Hartree equation:

\[
U_s^H(x) = -g_s \sigma^H_H(x),
\]

\[
U_\mu^H(x) = g_v \omega_\mu^H(x) \tag{11}
\]

with

\[
\frac{\partial}{\partial \sigma^H_H} \tilde{U}[\sigma^H_H(x)] = g_s \rho_s(x)
\]

\[
m^2_v \omega_\mu^H(x) = g_v j_\mu^H(x), \tag{12}
\]

where

\[
\tilde{U}[\sigma^H_H] = \frac{\frac{1}{2} m^2_s \sigma^2_H + \frac{1}{2} B_s \sigma^3_H + \frac{1}{4} C_s \sigma^4_H}{1 + \frac{1}{2} A_s \sigma^2_H}. \tag{13}
\]

In the above equations the scalar density \( \rho_s(x) \) and the current \( j_\mu^H(x) \) are given in terms of the phase-space distribution function by

\[
\rho_s(x) = \frac{4}{(2\pi)^3} \int d^4p \; M^*(x, p) f(x, p) = \frac{4}{(2\pi)^3} \int d^3p \; \frac{M^*(x, p)}{\Pi_0(x, p)} \tilde{f}(x, p),
\]

\[
j_\mu^H(x) = \frac{4}{(2\pi)^3} \int d^4p \; \Pi_\mu(x, p) f(x, p) = \frac{4}{(2\pi)^3} \int d^3p \; \frac{\Pi_\mu(x, p)}{\Pi_0(x, p)} \tilde{f}(x, p). \tag{14}
\]

In eq. (13) we have introduced a new parameter \( A_s \) in the denominator of the sigma-field potential \( \tilde{U}(\sigma_H) \) to avoid numerical instabilities of the solution. In
previous applications, not including the denominator part (i.e. \( A_s \equiv 0 \)), the value of \( C_s \) sometimes could become highly negative, especially for low incompressibility and low effective mass. In this case the solution of eq. (12) cannot be uniquely determined and the numerical simulation becomes unstable.

Now we present the actual parametrizations of the MD parts in eq. (10). In order to be able to formulate a conserving theory (with respect to the energy-momentum tensor) these parts are constructed in analogy to Fock terms of nucleon self-energies. Instead of adopting an infinite sum of Fock-like terms as in Ref. [22], we use a simple parametrization for the MD parts as

\[
U_s^{MD}(x, p) = -\frac{4}{(2\pi)^3 m_s^2} \int d^4q \ f(x, q) D_s(p, q),
\]

\[
= -\frac{4}{(2\pi)^3 m_s^2} \int d^3q \ M^*(x, q) \frac{\tilde{f}(x, q)}{\Pi_0(x, q)} D_s(p, q),
\]

\[
U_\mu^{MD}(x, p) = \frac{4}{(2\pi)^3 m_v^2} \int d^4q \ \Pi_\mu(x, q) f(x, q) D_v(p, q)
\]

\[
= \frac{4}{(2\pi)^3 m_v^2} \int d^3q \ \frac{\Pi_\mu(x, q)}{\Pi_0(x, q)} \tilde{f}(x, q) D_s(p, q),
\]

with

\[
D_{s,v}(p, q) = \frac{\Lambda_{s,v}^2}{\Lambda_{s,v}^2 - (p - q)^2},
\]

where the effective coupling constants \( g_{s,v} \) and the effective meson masses \( \Lambda_{s,v} \) are treated as free parameters which are fixed approximately by the empirical optical potential.

For the effective approach above (15), (16) the MD parametrizations require some comments. First, the above parametrization gives a reliable momentum-dependence of the mean fields for general momentum distributions as well as for the special case of a single Fermi distribution because the expression (15) is general within the Hartree-Fock theorem. Second, our approach can reproduce the Dirac Brueckner results for the scalar and vector fields, too [22]. Third, comparing our parametrizations of Fock diagrams with vacuum polarization calculations [33], we find that we do not have any serious problem with the imaginary parts of the nucleon self-energies, except for off-mass-shell particles with \( p_0 \gg \varepsilon(p) \) which, however, do not appear in semi-classical transport approaches according to the condition (11). Therefore we can neglect a more complicated density dependence for the MD parts.
as well as vacuum polarization effects.

However, the above parametrization is still difficult to be applied in actual RBUU-simulations [23]. In principle we should use a six-dimensional grid in phase space for the fields to get exact results, but this method needs a large amount of memory and consumes tremendous CPU times. In order to avoid this difficulty, we propose an alternative polynomial approximation for the propagator $D_{s,v}$ as

$$D_{s,v}(p,q) = 1 + \frac{a_{s,v}}{m_{s,v}^2}(p-q)^2 + \frac{b_{s,v}}{m_{s,v}^4}(p-q)^4.$$  

(17)

In this way we obtain analytical expressions for the MD fields as

$$U_s(x,p) = -g_s\sigma_H(x) - \frac{\bar{g}_s^2}{m_s^2}\rho_s(x) + p^2U_s^{(1)}(x) + p^4U_s^{(2)}(x),$$

$$U_\mu(x,p) = \frac{(g_v^2 + \bar{g}_v^2)}{m_v^2}j_\mu(x) + p^2U_\mu^{(1)}(x) + p^4U_\mu^{(2)}(x).$$  

(18)

The spatial derivatives of $U_s$ and $U_\mu$ at fixed momentum are easily evaluated with the above expressions. The above polynomial approximation is also applicable for general configurations below a certain limiting energy as will be discussed below. Note that eq. (18) does not reduce to MD coupling parameters as in Ref. [24] since each term has a different $x$-dependence. The conserving character of the theory [22] is thus maintained.

The free parameters are determined in the following way: we fit the mean-fields using the ansatz (16) to reproduce the experimentally observed nucleon-nucleus potential and then determine the parameters of the polynomial approximation (17) to give the same results for the mean-fields up to about $1 - 1.2$ GeV kinetic energy of the nucleon. The resulting parameters are given in Table 1. All parameter-sets yield a saturation density $\rho_0 = 0.17$fm$^{-3}$ and a binding energy per nucleon of $E_B/A = -16$MeV at $\rho_0$. The incompressibility $K$ and the effective mass $M^*/M$ at the Fermi level and $\rho = \rho_0$ are also given in Table 1.

In order to compare our calculations with momentum-independent mean-fields, furthermore, we also present the parameter-set NL6, NL7, NL21 in Table 1 where the corresponding nuclear matter properties are also given in the bottom lines. Note that the above saturation density is different from that for the NL parameter-sets used in the previous RBUU-approach [10, 17, 19, 20, 21]. For the old NL parameter-sets the value of the saturation density had been taken as $\rho_0 = 0.145$fm$^{-3}$, which is too small...
in comparison to the bulk density of heavy nuclei. The old NL parameter-sets lead to vector fields which are about 10% larger than the present ones; this difference influences especially calculations at high density and/or high energy. Thus we have refitted the parameter-sets to reproduce the proper saturation density in all cases.

In Fig. 1 we show the resulting MD mean-fields $U_s$ (a) and $U_0$ (b) at normal nuclear matter density $\rho = \rho_0$ as a function of the nucleon kinetic energy $\varepsilon_K$. Solid, thick dashed and dashed lines indicate the results of POL6, MD6 and NL6, respectively. By construction the two results of POL6 and MD6 are almost the same up to 1.2 GeV/u while the polynomial approximation fails at higher energy. This limits the applicability of the polynomial approximation to energies up to about 1 GeV/u.

In Fig. 1c we present the Schrödinger equivalent potential defined by

$$ U_{SEP} = U_s + U_0 + \frac{1}{2M}(U_s^2 - U_0^2) + \frac{U_0}{M}\varepsilon_K. \quad (19) $$

The hatched area indicates the result of the experimental analysis by Hama et al. [15]. Again, the optical potentials for the parametersets MD6 and POL6 are almost identical and approximately reproduce the experimental result. The $U_{SEP}$ of NL6 is linear in $\varepsilon_K$ and not so different from the experimental result up to about 300 MeV, but yields too strong repulsion at higher energy (cf. Sec. 1). Since this strong repulsion is due to very strong vector fields, the NL6 parameter-set overestimates the Lorentz force on a fast moving particle in the medium.

In Fig. 2 we show the energy per nucleon for nuclear matter (a) and the effective mass at the Fermi surface (b) as a function of density for each parameter-set. The expression for the total energy has been given in Ref. [22]. Again, the EOS of POL6 (solid line) almost completely agrees with that of MD6 (thick dashed line). Besides, it is not very different from that for NL6 (dashed line) since the explicit momentum-dependence does not significantly change the nuclear EOS at moderate densities. In Fig. 2, furthermore, we display the results for POL7 (dash-dotted line), NL7 (dotted line) and NL21 (thick dotted line), which are discussed in Sec. 3. Here we only confirm again that the momentum-dependence does not strongly change the nuclear matter properties.

In Ref. [23], however, it was shown that the momentum-dependent forces have a large influence on matter-on-matter collisions where the momentum distribution is described by two shifted Fermi spheres. Before performing actual simulations, we
compare our polynomial approximation to the exact results for the above configuration. The negative binding energy (a), longitudinal (b) and transverse pressure (c) are shown for all parameter-sets in Fig. 3 as a function of the bombarding energy per nucleon.

First, the results of POL6 (solid line) reproduce those of MD6 (thick dashed line) up to the bombarding energy of about $E_{\text{lab}} = 1 \text{ GeV/u}$ for all three quantities; the biggest difference shows up in the longitudinal pressure at the highest energy ($\Delta p_L < 10\%$). Thus we can safely conclude that the polynomial approximation should be valid up to this energy in both proton-nucleus and nucleus-nucleus collisions. Second, the energy and the longitudinal pressure are almost completely determined by the momentum-dependence. Note especially the big difference in the binding energy between MD and MID parametrizations. Third, the negative binding energy is almost proportional to the bombarding energy $E_{\text{lab}}$ in the MID parameter-sets and saturates in the high-energy region above about $E_{\text{lab}} = 300 - 400 \text{ MeV/A}$ for the realistic MD parametrizations. Fourth, the MD parametrizations reduce the transverse pressure at high bombarding energies around 1 GeV/u quite significantly; on the other hand, below about 500 MeV/u a sizeable dependence on the incompressibility can be seen. Thus the dynamical effects of the MD mean-fields should show up in the early stage of a heavy-ion collision when the momentum distribution is almost equivalent to two shifted Fermi spheres.

The most drastic effects of the momentum-dependence arise for the negative binding energy and the transverse pressure where the results obtained with the correct momentum-dependence are much lower than those calculated with the original $\sigma - \omega$ model. The lowering of the transverse pressure and the negative binding energy is a direct consequence of the weakened strength of the Lorentz force at higher energy. From the above results we can already predict that the MD mean-fields in actual heavy-ion collisions will lead to a higher compression (as a consequence of the smaller total energy) and a suppression of the transverse flow (due to a smaller transverse pressure) as compared to the MID parametrizations.
3 Results and Discussion

In this section we report results of the actual numerical simulation of the RBUU approach with the MD mean-fields and calculate various observables for heavy-ion collisions. We use $150 - 1000$ testparticles per nucleon in the simulation and adopt the Cugnon parametrizations of the baryon-baryon cross-sections \cite{3}, \cite{35} for the stochastic baryon-baryon (BB) collisions including the elastic and inelastic channels with the $\Delta$ resonance in the “frozen $\Delta$” approximation. In the actual simulation the collision term is treated within the Local Ensemble Method described in Ref. \cite{36}, which gives a proper solution of the BUU equation.

Before calculating observables we examine the time dependence of the maximum density in the local rest frame ($\rho_r = \sqrt{j_{\mu} j^\mu}$) in Au + Au collisions for the parameter-sets POL6, POL7, NL6, NL7 and NL21 at the bombarding energy $E_{\text{lab}} = 400$ MeV/u for the impact parameter $b = 3$ fm (a) and at $E_{\text{lab}} = 1$ GeV/u for $b = 0$ fm (b) (Fig. 4). The parametrizations POL6 and NL21 lead to much higher densities than NL6 and NL7 at $E_{\text{lab}} = 1$ GeV/u, whereas the parameter-set NL7 produces only a slightly higher density than NL6 because the EOS for NL7 is softer than that for NL6 (cf. Fig. 2a). These tendencies are also present at $E_{\text{lab}} = 400$ MeV/u, but cannot be identified so clearly. Since the MD mean-fields reduce the repulsion in the high energy region, the mean force between two nuclei is much weaker for POL6 and POL7 than for NL6 and NL7 at the beginning of the nucleus-nucleus collisions. In addition, we can understand the similarity of the results obtained with POL6 and NL21 at $E_{\text{lab}} = 1$ GeV/u from the fact that the MD parameter-set POL6 generates the same strength of the Schrödinger equivalent potential as NL21 around 1 GeV/u (cf. Fig. 1c). Thus the maximum density in the compression phase is determined by the property of the mean-fields at the early times of the reaction.
3.1 Collective Flow

We have already discussed that the explicit momentum-dependence of the mean fields decreases the transverse pressure in the system for the two-Fermi-sphere geometry which is realized in the early stage of a heavy-ion collision. As discussed in the previous section, this fact predicts also a suppression of the transverse flow. To demonstrate this we calculate the mean transverse momentum in the reaction plane \(< p_X/A > \) versus the normalized center-of-mass (CM) rapidity \(Y_{CM}/Y_{PR}\) (\(Y_{PR}\): the projectile rapidity), which has been shown in Ref. \([17, 19]\) to be very sensitive to the momentum-dependence, but almost insensitive to the incompressibility \(K\).

Fig. 5 shows the results in Ar + KCl collisions at \(E_{lab} = 800\) MeV/u (a), Au + Au collisions at \(E_{lab} = 400\) MeV/u (b) and 150 MeV/u (c) at \(b = 6\) fm for the parametrizations POL6 (solid lines), POL7 (dash-dotted lines), NL6 (dashed lines) and NL7 (dotted lines). For the calculation of Au + Au collisions at 400 MeV/u the (PlasticBall) filter routine and the coalescence model of \([17]\) are used.

The differences between our various parametrizations are negligible at \(E_{lab} = 150\) MeV/u in the interesting regime around midrapidity, but become larger with increasing beam-energy. At \(E_{lab} = 400\) MeV/u, both the momentum-dependence and the incompressibility lead to differences at high rapidity \(Y_{CM}/Y_{PR} > 0.6\), however, there are again no sizeable effects at midrapidity. The parameter-sets POL6 and NL7 give similar results whereas the flow angle (\(\theta_{flow}\)) for NL6 is bigger than for POL6 and that for POL7 smaller than for NL7; i.e. \(\theta_{flow}(POL7) < \theta_{flow}(NL7) \approx \theta_{flow}(POL6) < \theta_{flow}(NL6)\). Thus a low incompressibility and a low momentum-dependence clearly reduce the transverse momentum, as expected; in addition, the behavior of the above results is quite similar to that of the transverse pressure for the two-Fermi-sphere geometry around 400 MeV/u (cf. Fig. 3c). However, these differences are not very pronounced because all results almost agree with the experimental data of the plastic ball group \([27]\).

At higher energy (800 MeV/u) the MD mean-fields POL6, POL7 nicely reproduce the experimental data \([26]\) (a) while the flow for the MID parameter-sets NL6 and NL7 is overestimated at 800 MeV/u; the results are quite insensitive to the incompressibility \(K\). This behavior is again found to be similar to the transverse
pressure in the two-Fermi-sphere geometry around 800 MeV/u.

From the above results we learn that the transverse flow is strongly correlated with the transverse pressure for the two-Fermi-sphere geometry as it exists in the early stages of the collision. Both the momentum-dependence and the incompressibility contribute to the flow around $E_{lab} = 400$ MeV/u. At the highest energy, however, the momentum-dependence dominates. Note that this holds in spite of the fact that the densities reached here are even higher than at the lower energies. This demonstrates again (cf. Ref. [19]) that it is not the density compression but the Lorentz force which drives the transverse flow. At $E_{lab} = 800$ MeV/u the MD parametrization POL6 and POL7 can reproduce the experimental results whereas the results of the MID parametrization NL6 and NL7 overestimate the data. The latter result is due to the fact that our MD mean-fields (POL6 and POL7) yield the proper strength of the Lorentz force while the mean-field of NL6 and NL7 yield a too strong repulsion at high energy (cf. Fig. 1c).

We can thus not obtain any precise information about the nuclear EOS from $< p_x/A >$, which is too insensitive to the incompressibility in the high energy region. The reason for this insensitivity is that the quantity $< p_x/A >$ is essentially produced by the Lorentz force [19] in the surface region of the overlapping nuclei in the initial phase, where the density is not so high and the momentum-distribution is similar to the two-Fermi-sphere geometry. When increasing the bombarding energy, the distance between these two Fermi spheres becomes larger and the properties of the system are dominantly determined by the momentum-dependence of the mean-fields. In order to determine the nuclear EOS, therefore, we need to discuss also other observables from a heavy-ion collision.

Next we discuss the directivity distribution, which was suggested by Alard et al. [29] as a probe for the nuclear EOS. The directivity $D$ is defined by

$$D = \frac{\left| \sum_i p_T(i) \right|}{\sum_i |p_T(i)|} \quad \text{for} \quad Y_{CM}(i) > 0 \quad \text{and} \quad 7^\circ < \theta_{lab} < 30^\circ. \quad (20)$$

In Fig. 6 we show the yield distribution for the observable $D$ in a Au + Au collision at 400 MeV/u for the impact parameter average $b \leq 5$ fm. We find differences with respect to the incompressibility as well as to the momentum-dependence. A low incompressibility or a low momentum-dependence shift the results to smaller
directivity, as expected. The results for POL6 (hard EOS) and NL7 (soft EOS) agree with each other though the maximum yield is slightly different.

In order to examine the directivity in more detail we show the average value $< D >$ and its width $\Delta D$ in Fig. 7 as a function of impact parameter. With increasing impact parameter the average value increases first, has a peak at about $b = 5$ fm, and then decreases again for very peripheral reactions.

It can be seen again that a weak momentum-dependence and a soft EOS reduce both the average value and the width. For larger impact parameters around $b = 7$ fm (comparing POL6 and POL7), moreover, the effect of the incompressibility becomes almost negligible and the momentum-dependence is dominant because there is no compressed participant zone in peripheral collisions [20]. Thus the directivity distribution might be useful to determine the nuclear EOS, but only after MD mean fields have been determined from a study of peripheral reactions.

In recalling: the directivity distribution is influenced both by the incompressibility and the momentum-dependence. The directed flow in the $x$-direction (cf. $< p_X/A >$) is still essential to build up the directivity; consequently there are no clear differences in the small impact parameter region. In addition, the behavior is also quite similar to that of $< p_X/A >$ for large rapidities. Hence the directivity is again determined by the transverse pressure in the early stage of the reaction, but the differences between various input quantities, that show up only weakly in the mean transverse momentum $< p_X/A >$, are magnified.

We leave the discussion on the directivity with a short comment: the authors of refs. [29, 32] have suggested to select events from central collisions using the restriction $D < 0.2$ together with a high multiplicity bin. Even taking into account the width in the directivity distribution, this restriction should select central events with $b < 2$ fm in all cases; our results thus strongly support their idea.

Next we discuss the mean absolute momentum perpendicular to the reaction plane $< |p_Y|/A >$ defined as

$$< |p_Y|/A > = \frac{1}{N_T A} \sum_i |p_y(i)|. \quad (21)$$

where the $y$--direction is defined as that perpendicular to the reaction plane ($x, z$). In Fig. 8 we show the $< |p_Y|/A >$ versus the rapidity ($a$) in Au + Au collisions
at $E_{lab} = 400$ MeV/u for central collisions ($b < 3$ fm). For reference we add the rapidity distribution in Fig. 8b. The results for the parameter-sets POL6 and NL6 ($K = 400$ MeV) are larger than those for POL7 and NL7 ($K = 200$ MeV) for small rapidities $Y_{CM}/Y_{PR} < 0.3$, while there is no significant difference among all cases in the rapidity distribution.

We thus find that the flow in $y$-direction is more sensitive to the incompressibility than to the momentum-dependence. This result can be understood as follows: On one hand, the spectators do not contribute significantly to the flow in $y$-direction since the Lorentz force - which is the dominant force for the spectators due to their high relative velocity - acts in the reaction plane ($x, z$). On the other hand, the nucleons in the participant zone are accelerated by the pressure in the high density participant region into all directions. These nucleons experience many baryon-baryon (BB) collisions and consequently populate the CM midrapidity region which involves only relatively small momenta. Thus, for these nucleons the Lorentz force is rather weak and their final momenta are mainly determined by the incompressibility. Unfortunately, the sensitivity of the flow perpendicular to the reaction plane with respect to the incompressibility is rather weak; the differences caused by the incompressibility $K$ are about $5 - 10\%$ of the total strength.

In summary, the results on $< p_x/A >$ show that our MD parametrizations describe the Lorentz forces in the initial stages of the collision correctly, but they also show that the in-plane transverse flow does not give any information on the nuclear EOS. This is in line with the results of Lang et al. [37] who showed that the in-plane transverse flow is determined by the transverse pressure, but does not depend sensitively on the EOS. On the other hand, the directivity distribution and the $< |p_y|/A >$ can be used as probes to determine the incompressibility and help to understand the global evolution of the heavy-ion collision. However, also these observables are not very sensitive to the properties of highly compressed matter. In the high energy region ($E_{lab} > 1$ GeV/u) the transverse pressure of the initial overlap area is almost entirely determined by the momentum-dependence of the mean-fields at $\rho = \rho_0$ and the contribution from the collision terms becomes large, too. Thus we have to study other observables such as particle production yields at subthreshold energies as a trigger on high baryon density.
3.2 Kaon Production

In this section we now turn to the study of particle production at subthreshold energies where the particles are predominantly produced in the high density zone \[^{21}\]. We study these reactions with the hope to narrow down the ambiguities in the nuclear EOS as obtained from the study of flow phenomena. In particular, we discuss in this section the $K^+$- production in the subthreshold energy region, which has been suggested by Aichelin and Ko to be a good probe for the nuclear EOS \[^{38}\]. There are no serious ambiguities in the $K^+$ production process: neither $N + \text{multi} \times N \[^{39}\]$ nor $\pi + N$ collisions \[^{40}\] play a significant role; the dominant contribution comes from binary BB collisions. In addition, $K^+$- production can be calculated perturbatively because of the small production probability in BB collisions and the negligible absorption in the medium due to strangeness conservation. In Ref. \[^{21}\] Lang et al. have found that this observable should be a good probe in Au + Au collisions at $E_{\text{lab}} = 1 \text{ GeV/u}$.

Since the detailed description of the production calculations is given in \[^{21}\], we directly show the results of our present calculations for the experimental angle $\theta_{\text{lab}} = 44^\circ$ in Ne + NaF (a) and Au + Au (b) collisions at $E_{\text{lab}} = 1 \text{ GeV/u}$ (Fig. 9). Also displayed in Fig. 9 are the experimental data from Au + Au collisions and the preliminary data from Ne + NaF collisions of the KAOS group at SIS \[^{31}\]. In the Ne + NaF reaction there is no significant difference among all cases except for the results for NL21 which are about 15 % larger than the others. The reason for this behavior has already been shown in Ref. \[^{21}\]: in Ne + NaF collisions only moderate densities in the medium ($\rho < 2\rho_0$) are achieved such that the results do not reflect the properties of high density nuclear matter. Furthermore, the Ne + NaF results quite strongly depend on the surface properties. For example, when varying the surface thickness by about 25 %, the inclusive $K^+$ yields change by about a factor of 5 in the RBUU simulation. This fact shows that the total cross-section is determined by the initial phase-space distribution and thus very sensitive to the initialization. The strong sensitivity to the initialization is a typical property of the deep subthreshold particle production in light systems where the surface plays a larger role. It does not occur for heavy systems; the different initializations change the results
for Au + Au only by a few percent.

From the Au + Au collisions we get some information about the high density matter. The calculated yield is sensitive both to the incompressibility and the effective mass at the Fermi surface. However, the explicit momentum-dependence does not change the spectrum very much except for the slope which is slightly smaller for POL6 than for NL6. This shows that the MD mean-fields generate higher kinetic energies or higher temperatures than the MID fields. The yield is ordered as $(NL_{21}) > (NL_{7}) > (NL_{6}) \approx (POL_{6})$. This order is equivalent to that in the binding energy of high density nuclear matter. The results for the $K^{+}$ yield thus reflect the energy density of nuclear matter.

The MD parametrization POL6 can reproduce the experimental data for $K^{+}$-production as well as the transverse flow whereas none of the MID parametrizations can explain both sets of data at the same time. Hence we expect that the nuclear EOS corresponding to the parameter-set POL6 is very close to the exact one for $\rho \approx 2 - 3 \rho_{0}$.

Next we examine the contributions from the various production channels. In Fig. 10 we show the results for NL6 from the $N + N$, $N + \Delta$ and $\Delta + \Delta$ collisions separately. We find that 90 % of the kaons are produced from the $N + \Delta$ or $\Delta + \Delta$ channels while the $N - N$ channel contributes only 10 % to the total yield (cf. Ref. [21]). Furthermore, the slopes of each partial cross-section do not show any remarkable differences; apparently the nucleon and the $\Delta$ feel a similar temperature inside the compression zone.

In order to understand these results in more detail, we investigate the dependence of the $K^{+}$ yield on the density in the local rest frame and on the baryon effective mass for Ne + NaF and Au + Au collisions; the results are shown in Fig. 11 as a function of the ratio $r = \rho_{r}/\rho_{0}$ and $m = M^{*}/M$. A first observation is that the shape of the density dependence — not the absolute magnitude — is dominantly determined by the momentum-dependence and the effective mass at the Fermi surface; the NL6 and NL7 mean-fields yield similar results and also POL6 and POL7. Second, the results with the POL6 mean-fields show tails up to the high density region ($\rho > 3\rho_{0}$); in the Au + Au collision the maximum density is about $4\rho_{0}$ for POL6 and almost the same as that for NL21 (cf. Fig. 4). As shown in Fig. 1, the
momentum-dependence reduces the repulsion in the initial stage and this permits higher densities than the MID mean-fields. However, the peak position for POL6 agrees with those for NL6 and NL7, while the peak position for NL21 is higher than the others. The reason for these dependences is as follows: The EOS of POL6 is almost equivalent to that of NL6. The parameter-set POL6 leads to a higher density than NL6, but nevertheless there is not enough energy to produce more $K^+$-mesons because the bombarding energy is consumed by the build-up of the vector field; consequently, $K^+$-mesons cannot be easily produced in the compression zone.

Fig. 11b shows the distribution of the effective masses of the baryons involved in the $K^+$ production. All curves exhibit two maxima, one at a value significantly below $\rho_0$ and the other one approximately 0.3 GeV above. While the lower one reflects events from nucleon-nucleon collisions taking place at high densities, the higher one stems from the $\Delta$ resonances in a $\Delta - N$ collision event, respectively. The curves shown here do not correlate with the magnitude of the cross sections; there is thus no obvious connection between the number of kaons produced and the momentum-dependence of the interaction.

We briefly summarize the effects of the MD mean fields for $K^+$ production. The kaon yield is determined both by the collision rate and by the momentum distribution in the participant zone. The former is directly related to the maximum density reached and this, in turn, depends on the behaviour of the mean fields at the initial high energy. The latter, on the other hand, is determined by the low-energy behaviour of the mean fields which becomes relevant after many BB collisions have taken place. The MD parametrizations are much less repulsive than the MID ones in the early stage of the collision and thus lead to higher densities, but the incoming energy has partly been used to build up the correspondingly higher vector field. Both effects counteract each other to a large extent such that the kaon yield becomes rather insensitive to the momentum-dependence. Thus $K^+$ production around 1 GeV/u is a good tool to study the EOS.

Comparing with the results of ref. [21] we see that the kaon production cross sections are about a factor of two larger than those obtained there (for NL21 versus NL2). This difference is due to the fact that a lower saturation density was used in [21]; this affects the absolute yield but leaves the slope of the spectra nearly...
unchanged. Finally we would like to note that our conclusion of a small effect of the momentum dependence on the $K^+$ yield might seem to be at variance with the results of QMD calculations in refs. [9, 11], where a large effect was obtained. However, the results of ref. [9] show large statistical errors and have been obtained in an approach that lacks manifest covariance in the mean fields thus limiting their reliability in the relativistic energy domain treated here. The results of ref. [11] on the other hand are not inconsistent with ours; they were performed only for light systems where the $K^+$ yields depend strongly on the surface properties and on the initialization.

4 Summary

In this paper we have introduced momentum-dependent (MD) mean-fields into the RBUU approach in order to correct for the too repulsive potentials at high energies in the original approach. We have performed actual simulations of heavy-ion collisions from $E_{\text{lab}} = 150$ MeV/u to 1000 MeV/u employing two momentum-dependent parametrizations (POL6 and POL7) with a polynomial approximation for the nucleon-nucleon interaction and three momentum-independent parametrizations (NL6, NL7 and NL21). The MD parameter-sets (POL6, POL7) are fitted to the empirical optical potential for proton-nucleus scattering and lead to less repulsive mean fields in the high energy region than the MID parameter-sets NL6 and NL7. As a consequence, also in nucleus-nucleus collisions the explicit momentum-dependence suppresses the repulsion in the high energy region $E_{\text{lab}} > 300$ MeV/u; this effect is most pronounced in the early stage of the collision process. Due to the reduced repulsion from the MD mean-fields the system achieves higher densities and a lower transverse pressure in comparison to the results for the MID mean fields. The effect becomes even more important with increasing bombarding energy.

We have examined a variety of observables in these RBUU simulations: the mean transverse momentum in the reaction plane $< p_X/A >$, the directivity distribution, the mean absolute transverse momentum perpendicular to the reaction plane $< |p_Y|/A >$ and the subthreshold $K^+$- production. The quantity $< p_X/A >$ is
found to be sensitive to the momentum-dependence, but much less sensitive to the nuclear incompressibility. The quantity $< |p_Y|/A >$, on the contrary, is found to be rather insensitive to the momentum-dependence. Finally, the directivity distribution is both sensitive to the incompressibility and the momentum-dependence; however, the differences between the results obtained with the incompressibilities $K = 200$ MeV and $K = 400$ MeV are too small to be observed experimentally. On the other hand, the subthreshold $K^+$- production in Au + Au collisions turns out to be a suitable probe for the nuclear EOS, thus confirming the original suggestion of ref. [38]. The differential yield is sensitive to the nuclear incompressibility but insensitive to the momentum-dependence for heavy systems like Au + Au.

From the comparison of the above results, furthermore, we obtain a clear picture of the effects of the MD mean-fields. The high energy behaviour of the mean-fields is significant in the initial stage and the strength at the initial energy determines the mean transverse flow and the density distribution in the participant zone. With increasing time, the matter is compressed and the bombarding energy is distributed among the nucleons in the high-density participant zone where the average relative momentum between two baryons becomes much smaller. In the most condensed phase the high energy behaviour of the mean-fields becomes less important and the low energy behaviour of the mean-fields essentially determines the energy-momentum distribution in the participant zone due to energy-momentum conservation. Thus the phase-space distribution in the participant zone is determined by both: the high energy behaviour at $\rho = \rho_0$ and the low energy behaviour of the mean-fields in the whole density region. Fortunately, the difference in the maximum densities between POL6 and NL6 does not influence the result for $K^+$-production. In addition, only the nuclear incompressibility $K$ affects the $K^+$ yield significantly; the scalar and vector fields have no separate effect on this quantity.

From the results of the present investigations and the comparison to various experimental data on flow phenomena and kaon production simultaneously we infer that the true nuclear EOS should be close to that of POL6 around $\rho \approx 2 - 3\rho_0$. Essential for this conclusion is the simultaneous analysis of unrelated sets of data within the same model since there might be unknown medium effects as e.g. suggested by Ko et al. [41] for the $K^+$ mesons and Jung et al. [42] for long-range
correlations induced by the vacuum polarization.

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Parameters for the various equations of state discussed in this paper. Also given are the incompressibility $K$ and the effective mass $M^*$. In all cases we have used $m_s = 550$ MeV and $m_v = 783$ MeV.
Figure captions

Fig. 1 The kinetic energy dependence of the scalar fields (a), time-component of vector fields (b) and the Schrödinger equivalent potential (c). The results for POL6 and MD6 are denoted by solid and thick dashed lines, respectively. The dashed lines show the results for the momentum-independent mean-field (NL6) and the thick dotted lines those for NL21.

Fig. 2 The equation of state for nuclear matter: total energy per nucleon versus baryon density (a) and the nucleon effective mass divided by the vacuum mass (b). The solid, dash-dotted, dashed, dotted, thick dotted and thick dashed lines show the results for POL6, POL7, NL6, NL7, NL21 and MD6, respectively.

Fig. 3 The negative binding energy per nucleon (a), the longitudinal pressure (b) and the transverse pressure (c) versus the kinetic energy per nucleon in the Lab. frame. for matter-on-matter configurations (see text). The solid, dash-dotted, dashed, dotted, thick dotted and thick dashed lines show the results for POL6, POL7, NL6, NL7, NL21 and MD6, respectively.

Fig. 4 The time dependence of the maximum density (in units of $\rho_0$) for Au + Au collisions at $E_{LAB} = 1\text{GeV MeV/u}$ and $b = 0\text{ fm}$ (a), and Au + Au at 400 MeV/u and $b = 3\text{ fm}$ (b). The solid, dash-dotted, dashed and dotted lines show the results for POL6, POL7, NL6 and NL7, respectively.

Fig. 5 The mean transverse momentum per nucleon $p_x/A$ versus the center-of-mass rapidity (per initial projectile rapidity) for Ar + KCl collisions at $E_{LAB} = 800\text{ MeV/u}$ (a), Au + Au at 400 MeV/u (b) and Au + Au at 150 MeV/u (c) at $b = 6\text{ fm}$. The solid, dash-dotted, dashed and dotted lines show the results for POL6, POL7, NL6 and NL7, respectively. The experimental data (asterisks) are taken from [26] for (a) and [27] for (b).

Fig. 6 The distribution of the directivity $D$ in Au + Au collisions at 400 MeV/u for $b < 5\text{ fm}$ and the experimental acceptance $7^\circ < \theta_{lab} < 30^\circ$. The solid, dash-dotted, dashed and dotted lines show the results for POL6, POL7, NL6, and NL7, respectively.
Fig. 7 The impact parameter dependence of the average directivity distribution (a) and the width of the directivity distribution (b). The solid, dash-dotted, dashed and dotted lines show the results for POL6, POL7, NL6 and NL7, respectively.

Fig. 8 The mean absolute transverse momentum perpendicular to the reaction plane versus the center-of-mass rapidity. The solid, dash-dotted, dashed and dotted lines show the results for POL6, POL7, NL6 and NL7, respectively.

Fig. 9 The inclusive cross-section for $K^+$- production for Ne + NaF (a) and Au + Au collisions (b) at $\theta_{Lab} = 44^\circ$ and $E_{lab} = 1$ GeV/u. The solid, dashed, dotted and thick dotted lines show the results for POL6, NL6, NL7 and NL21, respectively. The asterisks show the experimental data [31].

Fig. 10 The inclusive cross-section for $K^+$- production for Au + Au collisions at $\theta_{Lab} = 44^\circ$ and $E_{lab} = 1$ GeV/u for the parameter-set NL6. The thin solid, solid, dotted curves indicate the contributions from the $N + N$, $N + \Delta$ and $\Delta + \Delta$ collisions while the dashed curve shows the total result. The asterisks show the experimental data [31].

Fig. 11 The number of produced $K^+$- mesons versus the nuclear density (in units of $\rho_0$) (a) and versus the baryon effective mass (in units of the bare nucleon mass) (b) in Au + Au collisions at 1 GeV/u for $b = 0$ fm. The solid, dashed, dotted and thick dotted lines show the results for POL6, NL6, NL7 and NL21, respectively.