Balanced approach for the two-dimensional rectangular guillotine cutting stock problem with setup cost

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Abstract. The main optimization objective of two-dimensional rectangular guillotine cutting stock problem (2DRGCSP) is minimizing stock cost. However, setup cost is also needed to be considered as an important factor in the real manufacturing process. A sequential pattern generation algorithm with rectangular homogenous strips (SPGA-RHSs) is presented to generate a cutting plan based on each single stock. A better cutting pattern considering both stock cost and setup cost is obtained with the cost balance utilization (CBU). The balanced approach combining SPGA-RHSs and CBU is used to achieve a current optimal cutting plan for 2DRGCSP with the minimum sum of stock and setup cost. The computational results prove that the proposed balanced approach has good optimization performance for 2DRGCSP with setup cost.

1. Introduction

The object’s shape of two-dimensional cutting stock problem includes regular shape such as rectangle, circle, sector and so on, and some irregular shape. Cui et al.[1] proposed a cutting stock optimization method for circle and sector arranging on metal plate oriented generator manufacturing industry. MirHassani and Jalaeian[2] applied a greedy randomized adaptive search procedure meta-heuristic algorithm to solve cutting stock problem about two-dimensional irregular shape. The two-dimensional rectangular guillotine cutting stock problem (2DRGCSP) [3, 7, 14, 16] is always the most focused two-dimensional cutting stock problem in glass, paper and wood industry.

In the real glass or paper manufacturing industry, a cutting plan including different types of cutting patterns can be achieved by some optimization methods. Considering that switching between different patterns always involves a cost of setting up the cutting machine, each type of new pattern in the cutting plan would produce the corresponding setup cost [3]. As different cutting plans consist of different number of types of cutting patterns, each cutting plan possesses different setup cost. Moreover, it affects the total production cost of cutting process according to a cutting plan. So considering both stock cost and setup cost are necessary during optimization process for 2DRGCSP.

Wascher et al. [4] had introduced the classic two-dimensional rectangular multiple stock size cutting stock problem (2DRMSSSCSP). The feature of 2DRMSSSCSP is as follows: (1) The dimension of stocks and items is two dimension and their shape is all rectangle. (2) The length, width and supply of m-type stocks are \( \{L_1, L_2, \ldots, L_m\} \), \( \{W_1, W_2, \ldots, W_m\} \), \( \{S_1, S_2, \ldots, S_m\} \), respectively. (3) The length, width and demand of n-type items are \( \{l_1, l_2, \ldots, l_n\} \), \( \{w_1, w_2, \ldots, w_n\} \), \( \{d_1, d_2, \ldots, d_n\} \), respectively.
respectively. (4) The sum area of items on each pattern is not bigger than the area of the current stock used. Meanwhile, the item cannot overlap each other and exceed boundary of the current stock used. Therefore, the cutting stock problem discussed in this article can be expressed as two-dimensional rectangular guillotine multiple stock size cutting stock problem with setup cost (2DRGMSSCSP-S).

This article below is organized as follows. The relevant literatures are described in the next section. In the “Balanced Approach for 2DRGMSSCSP-S” section, we propose a balanced approach to solve 2DRGMSSCSP-S. The performance analysis and comparison analysis are given in the “Computational results” section, followed by conclusions in the final section.

2. Literature Review

From reducing setup (cutting pattern) point of view, cutting stock problem with setup cost (CSP-S) is also referred to as cutting stock problem with pattern reduction (CSP-PR) or pattern minimization problem (PMP) in previous literature. To our knowledge, there are mainly three categories of solution methods for CSP-S, CSP-PR and PMP.

The first category is called exact algorithm. Degraeve and Vandebroek [5] presented a mixed integer programming model with the objective of minimizing the number of setups and with little or no waste in the fashion clothing industry. They used general purpose software to search for an optimal set of cutting patterns. Vanderbeck [6] proposed an integer programming formulation for minimizing the amount of waste and the number of different cutting patterns. He combined column generation technique and multiple bounded integer knapsack problems to solve his IP formulation. Kallrath et al. [7] used branch-and-price technique and exhaustion method to deal with 1D and 2D real-world cutting problems for the simultaneous minimization of the number of rolls and the number of patterns. Cui et al. [8] proposed pattern-set generation algorithm for the one-dimensional cutting stock problem with setup cost.

The second category is heuristic algorithm. Haessler [9] proposed a sequential heuristic procedure to find cutting patterns with meeting aspiration level (trim loss and patterns usage) and determined the number of the selected pattern as many as possible. Foerster and Wascher [10] gave a two-step approach: generated an input-minimal cutting plan regardless of number of cutting patterns in the first step, and applied an iterative method to reduce the number of different patterns in the second step. Yanasse and Limeira [11] developed a hybrid heuristic to reduce the number of different patterns in cutting stock problem. Dikili et al. [12] used a successive elimination method for one-dimensional cutting stock problem (1DCSP) in ship production. Wu and Yan [13] proposed a sequential pattern generation approach for 1DCSP with considering stock cost and setup cost, simultaneously.

The third category is meta-heuristic algorithm. Teghem et al. [14] put forward a simulated annealing algorithm with the grouping of book covers on offset plates in order to minimize the total production cost. An iterated local search algorithm with adaptive pattern generation was applied to attain the minimization of the number of different cutting patterns in Umetani et al. [15]. Qi and Rao [16] came up with an integrated approach on cut planning and nesting for metal structures manufacturing. A genetic algorithm was developed to automatically find the proper number of each item on patterns in Bonnevay et al. [17].

In summary, although the above three categories of solution methods mention that their methods or algorithms can consider the balance both stock cost and setup cost, no method is given to introduce a detail standard about how to measure the balance between stock cost and setup cost for 2DCSP-S exactly. We come up with a balanced approach for 2DRGMSSCSP-S in the next section.

3. Balanced Approach for 2DRGMSSCSP-S

3.1. Rectangular Homogenous Strips.

Rectangular homogenous strips (RHSs) [18] including the same item type are commonly used to solve 2DRCSP. RHSs have some benefit aspects as follows: (1) Reducing time consuming of optimization computing as it is fast to accomplish a cutting plan accurately meeting all item demands. (2)
Possessing good manufacturability and reducing manufacturing time because it is easy to separate each RHS from cutting pattern with guillotine cutting and divide each RHS into each item, such as stamping along the arrangement direction of each RHS. Therefore, we also apply RHSs as the basic element of each cutting pattern for 2DRGMS CSP-S in this article.

Each type of item along its same arrangement direction can be used to constitute a series of RHSs paralleling to the length direction or width direction of each type of stock. The direction of each type of item can also be rotated by 90°. According to the direction of RHSs, they can be divided into two types of RHSs (horizontal RHSs and vertical RHSs), shown in Fig. 1. The leftovers of horizontal RHSs and vertical RHSs are expressed by $L_{oH}$ and $L_{oV}$ in Fig. 1.

![Fig. 1 horizontal RHSs and vertical RHSs](image)

3.2. Cost Balance Utilization.

The proposed cost balance utilization (CUB) consists of the first cost balance utilization (1st-CBU) and the second cost balance utilization (2nd-CBU). The 1st-CBU is suitable for two-dimensional rectangular single stock size cutting stock problem (2DRSSCSP), and the 2nd-CBU is fit for two-dimensional rectangular multiple stock size cutting stock problem (2DRMSSCSP). Some basic variables from the 2DRSSCSP and 2DRMSSCSP are assumed as self-defined notations for convenience in Table 1. The detail definitions of the 1st-CBU and the 2nd-CBU are given, respectively.

| Variables                               | 2DRSSCSP          | 2DRMSSCSP          |
|-----------------------------------------|-------------------|-------------------|
| Number of stock type                    | 1                 | $m$               |
| Length of stock(s)                      | $L$               | $\{L_1, L_2, \ldots, L_m\}$ |
| Number of stock(s) supply               | $S$               | $\{S_1, S_2, \ldots, S_m\}$ |
| Number of item type                     | $n$               | $n$               |
| Lengths of items                        | $\{l_1, l_2, \ldots, l_n\}$ | $\{l_1, l_2, \ldots, l_n\}$ |
| Widths of items                         | $\{w_1, w_2, \ldots, w_n\}$ | $\{w_1, w_2, \ldots, w_n\}$ |
| Number of item demands                  | $\{d_1, d_2, \ldots, d_n\}$ | $\{d_1, d_2, \ldots, d_n\}$ |
| Each better cutting pattern             | $\{P_1, P_2, \ldots, P_n\}$ | $\{P_1^{1st-CBU}, P_2^{1st-CBU}, \ldots, P_n^{1st-CBU}\}$ |
| Sets of better cutting patterns frequency in a cutting plan based on each size stock | $\{P\}$ | $\{P_1^{1st-CBU}, P_2^{1st-CBU}, \ldots, P_n^{1st-CBU}\}$ |
| Maximum frequency of pattern in a cutting plan | $f_{max}$ | $f_{max}^{1st-CBU}$ |
| Stock cost per unit area                | $C_s$             | $C_s$             |
| Each new setup cost                    | $C_p$             | $C_p$             |
In the 2DRSSSCSP, the formula of the 1st-CBU can be derived as follows:

The stock utilization of each cutting pattern is \( U_s = \left( \sum_{j=1}^{n} l_j w_j p_j \right) / (LW) ; \)

The setup utilization of each cutting pattern is \( U_p = f / f_{\text{max}} ; \)

The weight of stock cost of each cutting pattern is \( W_s = (LWC_s f) / (LWC_s f + C_p) ; \)

The weight of setup cost of each cutting pattern is \( W_p = (C_p) / (LWC_s f + C_p) ; \)

The 1st-CBU of each cutting pattern is \( U_{1CB} = U_s W_s + U_p W_p ; \)

Actually, 2DRMSSCSP changes into 2DRSSSCSP when there is only one size stock in the two-dimensional rectangular cutting stock problem (2DRCSP). Therefore, 2DRMSSCSP can be seen as the combination of m-type 2DRSSSCSP. As a better cutting pattern is obtained with the maximum value of the 1st-CBU in the 2DRSSSCSP, m-type better cutting patterns are achieved with the maximum value of the 1st-CBU based on each size stock in the 2DRMSSCSP.

For 2DRMSSCSP, m-type better cutting patterns are generated by using the 1st-CBU. So the superscript with 1st-CBU is added to the notations of four basic variables in Table 1. Such as \( \{ P_1^{1st-CBU}, P_2^{1st-CBU}, \ldots, P_n^{1st-CBU} \} \) stands for each better cutting pattern. \( \{ P_1^{1st-CBU}, P_2^{1st-CBU}, \ldots, P_m^{1st-CBU} \} \) represents a set of better cutting patterns. \( \{ f_1^{1st-CBU}, f_2^{1st-CBU}, \ldots, f_m^{1st-CBU} \} \) stands for a set of frequencies of better cutting pattern in a cutting plan based on each size stock. \( f_{\text{max}}^{1st-CBU} \) represents maximum frequency of cutting pattern in a cutting plan. In the 2DRMSSCSP, the formula of the 2st-CBU can be derived as follows:

The stock utilization of each better cutting pattern is \( U_{s}^{1st-CBU} = \left( \sum_{j=1}^{n} l_j w_j p_j^{1st-CBU} \right) / (L^{1st-CBU} W^{1st-CBU}) ; \) 

The setup utilization of each better cutting pattern is \( U_{p}^{1st-CBU} = f^{1st-CBU} / f_{\text{max}}^{1st-CBU} ; \)

The weight of stock cost of each better cutting pattern is \( W_{s}^{1st-CBU} = (L^{1st-CBU} W^{1st-CBU} f^{1st-CBU} + C_p) ; \)

The weight of setup cost of each better cutting pattern is \( W_{s}^{1st-CBU} = (C_p) / (L^{1st-CBU} W^{1st-CBU} f^{1st-CBU} + C_p) ; \)

The 2st-CBU of each better cutting pattern is \( U_{CB}^{2nd} = U_{s}^{1st-CBU} W_{s}^{1st-CBU} + U_{p}^{1st-CBU} W_{p}^{1st-CBU} ; \)

3.3. Sequential Pattern Generation Algorithm with RHSs.

The sequential pattern generation algorithm with RHSs (SPGA-RHSs) is used to obtain a cutting plan satisfying all the current items demand based on the current stock. To clearly describe SPGA-RHSs, the representation of the relevant parameters comes from Table 1 and the following notations.
Structure of current pattern stock \[ L_C \otimes W_C \otimes S_C \]

Structure of remaining pattern stock \[ L_R \otimes W_R \otimes S_R \]

Arrangement direction of current RHS (0: horizontal, 1: vertical) \[ s_C \]

Number of current RHS \[ N_C \]

Structure of current better RHS \[ j_c(s_C : L_C \otimes W_C \otimes N_C) \]

Current cutting pattern (Set of RHSs) \[ \{ p_1, p_2, ..., p_n \} \]

Number of item in current cutting pattern \[ N_p \]

Frequency of current cutting pattern \[ f \]

Current cutting plan (Set of current cutting pattern) \[ (P_C, F_C) \]

The main steps of SPGA-RHSs are as follows:

Step 1: Initialize the basic parameters as \( L_R = L_C \), \( W_R = W_C \), \( S_R = S_C \), \( \{ p_1, p_2, ..., p_n \} = \{ \} \), \( f = 0 \), \( (P_C, F_C) = \{ \} \);

Step 2: If \( S_R \neq 0 \) and \( \sum_{j=1}^{n} d_j \neq 0 \), then go to Step 3; otherwise, go to Step 11;

Step 3: For \( j = 1 \) to \( n \);

Step 4: find out the number of \( j \)-type item of horizontal RHS and the number of \( j \)-type item of vertical RHS for current pattern stock, then compute the leftover length \( Lo_{jh} \) and the leftover width \( Lo_{jv} \);

Step 5: Compare the leftover length \( Lo_{jh} \) with the leftover width \( Lo_{jv} \), if \( Lo_{jh} \leq Lo_{jv} \), then select horizontal RHS for \( j \)-type item and determine the direction \( s_C = 0 \) and the number of item \( N_C \); otherwise select vertical RHS for \( j \)-type item and determine the direction \( s_C = 1 \) and the number of item \( N_C \);

Step 6: Compare the leftover of all the selected RHSs for each-type item, then select one RHS \( j_c(s_C : L_C \otimes W_C \otimes N_C) \) with the minimum leftover as a better RHS and save it into \( \{ p_1, p_2, ..., p_n \} \), save \( N_C \) into \( N_p \);

Step 7: Update the remaining length and width of current stock, if \( s_C = 0 \), then \( W_R = W_R - w_c \); else if \( s_C = 1 \), then \( L_R = L_R - l_c \);

Step 8: if \( W_R < \min(\forall l_j, \forall w_j) \) or \( L_R < \min(\forall l_j, \forall w_j) \), then go to step 9; otherwise, go to Step 3;

Step 9: Acquire a cutting pattern \( \{ p_1, p_2, ..., p_n \} \) and determine its frequency \( f \), then update the remaining supply of current stock \( S_R = S_R - f \), and the remaining item demand \( d_j = d_j - f \cdot N_p \), \( j = 1, 2, ..., n \);

Step 10: Save \( \{ p_1, p_2, ..., p_n, f \} \) into \( (P_C, F_C) \), and go to Step 2;

Step 11: Obtain a cutting plan \( (P_C, F_C) \) as a current better cutting plan.

Step 1 is used to initialize the relative parameters. Step 2 is applied to judge whether the supply of the remaining stock is used up or the remaining item demands are all completed. A cutting pattern is achieved with RHSs through Step 3 to Step 8. The remaining supply of stock used and each item demand are updated for the next iterative computation in Step 9. A cutting pattern is saved into a
cutting plan in Step 10. Step 11 obtains a better cutting plan satisfying all the current item demands based on the stock used.

3.4. Procedure of Balanced Approach with CBU
The SPGA-RHSs is used to achieve a better cutting plan satisfying all the current item demands arranging on the stock used. Meanwhile, the CBU in Section 3.2 introduces how to use the 1st-CBU to select a cutting pattern from a cutting plan for 2DRSSSCSP and the 2nd-CBU to select a better cutting pattern from 2DRMSSCSP. The proposed balanced approach in this article combining the SPGA-RHSs and the CBU is applied to solve 2DRGMSSCSP-S. The detail procedure of balanced approach is introduced in the following steps. The denotations of same variables are from Table 1.

Assumption: The available supplies of all stocks are sufficient.

Step 1: Initialize a temporary cutting plan \((P_T, F_T) = \{\}\), an optimal cutting pattern \(\{p_{o1}, p_{o2},..., p_{om}\} = \{\}\), \(f_o = 0\) and an optimal cutting plan \((P_o, F_o) = \{\}\);

Step 2: If \(\sum_{j=1}^{n} d_j \neq 0\), then go to Step 3; otherwise, go to Step 9;

Step 3: For \(i = 1\) to \(m\);

Step 4: Call the SPGA-RHSs to deal with all item demands based on \(i\)-type stock, and achieve a cutting plan \((P_i, F_i)\);

Step 5: Compute the 1st-CBU of \((P_T, F_T)\), and select an better cutting pattern \((P_{1\text{-st-CBU}}^i, f_{1\text{-st-CBU}}^i)\) with maximum value of the 1st-CBU;

Step 6: Repeat Step 3 and Step 4 to obtain a set of better cutting patterns \(\{P_{1\text{-st-CBU}}^1, P_{2\text{-st-CBU}}^1,\ldots, P_{m\text{-st-CBU}}^1\}\) and its corresponding a set of frequencies \(\{f_{1\text{-st-CBU}}^1, f_{2\text{-st-CBU}}^1,\ldots, f_{m\text{-st-CBU}}^1\}\);

Step 7: Compute the 2nd-CBU of a set of better cutting patterns \(\{P_{1\text{-st-CBU}}^1, P_{2\text{-st-CBU}}^1,\ldots, P_{m\text{-st-CBU}}^1\}\), and select an optimal cutting pattern \((\{p_{o1}, p_{o2},..., p_{om}\}, f_o)\) with maximum value of the 2nd-CBU;

Step 8: Save an optimal cutting pattern \((\{p_{o1}, p_{o2},..., p_{om}\}, f_o)\) into the optimal cutting plan \((P_o, F_o)\), then update number of stock used and each item demand, and then go to Step 2;

Step 9: Achieve an optimal cutting plan \((P_o, F_o)\).

In Step 1, three sets for saving cutting pattern are initialized for optimal computation. Step 2 determines whether all item demands are fulfilled. A set of better cutting patterns is obtained by combining the SPGA-RHSs and the 1st-CBU through Step 3 to Step 6. An optimal cutting pattern is acquired by optimizing set of better cutting patterns with the 2nd-CBU in Step 7. Step 8 saves the selected optimal cutting pattern and updates the remaining of stock used and item demands. An optimal cutting plan solving 2DRMSSCSP is achieved in Step 9.

4. Computational results
Two computational tests are given to verify good optimization performance of the proposed balanced approach for 2DRGMSSCSP-S. The initial values of all relative parameters are generated from the following ranges: stocks type \(i \in [1, 5]\), stocks length \(L(i) \in [1000, 2000]\), stocks width \(W(i) \in [500, 1000]\), stocks supply \(S(i) \in [50, 100]\), items type \(j \in [1, 20]\), items length \(l(j) \in [1000, 2000]\), items width \(w(j) \in [500, 1000]\), items demand \(d(j) \in [100, 200]\).

A uniform distribution generator is used to generate a group of random instance. The detail data of each parameter is as follows. The number of stock types is 3 and the length, width, supply of stocks are \(L(i) = \{1801, 1142, 1422\}\), \(W(i) = \{958, 895, 980\}\), \(S(i) = \{57, 71, 96\}\). The number of item
types is 9. The length of items is \( l(j) = \{259, 255, 292, 228, 276, 276, 238, 257, 207\} \). The width of items is \( w(j) = \{105, 153, 178, 194, 113, 157, 147, 101, 134\} \). The supply of items is \( d(j) = \{116, 180, 131, 153, 116, 160, 126, 166, 169\} \). Without loss of generality, each item can be allowed to rotate 90°, which means that item \( l(j) \otimes w(j) \) and item \( w(j) \otimes l(j) \) are considered to be same item for creating RHSs.

### 4.1. Performance Analysis of CBU

In order to analyze the effect of CBU in the optimization procedure, the proposed balanced approach is applied to solve the above given instance. The stock cost per unit area and each new setup cost are also from Table 2. Three cutting plans (Plan 1, Plan 2 and Plan 3) in Table 3 are achieved according to three groups of cost settings. It is noted that the data structures of Stock 1, Stock 2 and Stock 3 are 1801 \( \otimes \) 958 \( \otimes \) 57, 1142 \( \otimes \) 896 \( \otimes \) 71 and 1422 \( \otimes \) 980 \( \otimes \) 96 in Table 3, respectively.

**Table 2. The information of three groups of cost settings**

| Cost setting | Stock cost per unit area | Each new setup cost |
|--------------|--------------------------|---------------------|
| Cost setting 1 | 0.001                   | 200                 |
| Cost setting 2 | 0.002                   | 200                 |
| Cost setting 3 | 0.002                   | 1000                |

In Table 3, comparing the stock utilization of each corresponding pattern in Plan 1 and Plan 2, we can see that two-type patterns (Patten 3 and Patten 10) from Plan 1 are higher stock utilization than corresponding patterns from Plan 2. While five-type patterns (Patten 2, Patten 8, Patten 9, Patten 11 and Patten 12) from Plan 2 are higher stock utilization than corresponding patterns from Plan 1. This result verifies the previous proposed performance of the CBU that each pattern comes close to stock utilization when setup cost is constant and stock cost is increased. In the same way, comparing the frequency of each pattern in Plan 2 with the frequency of each pattern in Plan 3, one-type pattern (Patten 8) from Plan 3 is more frequency than corresponding patterns from Plan 1 and the frequency of each pattern from Plan 2 is in descending order. This result proves another proposed performance of the CBU that each pattern comes close to setup utilization when stock cost is constant and setup cost is increased.
Table 3. The data information of three cutting plans

| Patten | Stock utilization | Frequency | Patten | Stock utilization | Frequency | Patten | Stock utilization | Frequency |
|--------|-------------------|-----------|--------|-------------------|-----------|--------|-------------------|-----------|
| Patten 1 | 98.72%            | 10        | Patten 1 | 98.72%            | 10        | Patten 1 | 98.72%            | 10        |
| Patten 2 | 97.55%            | 8         | Patten 2 | 99.15%            | 4         | Patten 2 | 97.55%            | 8         |
| Patten 3 | 99.15%            | 4         | Patten 3 | 97.55%            | 8         | Patten 3 | 96.71%            | 7         |
| Patten 4 | 96.71%            | 7         | Patten 4 | 96.71%            | 7         | Patten 4 | 99.15%            | 4         |
| Patten 5 | 97.02%            | 3         | Patten 5 | 97.02%            | 3         | Patten 5 | 97.02%            | 3         |
| Patten 6 | 97.98%            | 2         | Patten 6 | 97.98%            | 2         | Patten 6 | 97.98%            | 2         |
| Patten 7 | 95.82%            | 2         | Patten 7 | 95.82%            | 2         | Patten 7 | 95.82%            | 2         |
| Patten 8 | 97.98%            | 2         | Patten 8 | 98.35%            | 1         | Patten 8 | 87.70%            | 2         |
| Patten 9 | 97.79%            | 1         | Patten 9 | 98.09%            | 1         | Patten 9 | 97.98%            | 1         |
| Patten 10 | 98.22%           | 1         | Patten 10 | 97.98%           | 1         | Patten 10 | 97.55%           | 1         |
| Patten 11 | 96.65%            | 1         | Patten 11 | 97.88%            | 1         | Patten 11 | 96.91%            | 1         |
| Patten 12 | 66.25%            | 1         | Patten 12 | 95.89%            | 1         | Patten 12 | 97.46%            | 1         |
| Patten 13 | 33.96%            | 1         | Patten 13 | 91.85%            | 1         | Patten 13 | 91.85%            | 1         |

4.2. Comparison Analysis of Balanced Approach.
To explain the good optimization performance of the proposed balanced approach, the given random instance is solved by three optimization approaches including balanced approach, the improved SPGA-RHSs and general two-dimensional column generation algorithm (2D CGA) [19] for 2DRGMS CSP. The above SPGA-RHSs is applied to obtain a cutting plan that satisfying all item demands based on the above stock. The SPGA-RHSs with the objective of maximizing stock utilization is improved to deal with 2DRGMS CSP. Three groups of setting costs are used in Table 2. The proposed balanced approach is used to achieve three cutting plans (Plan 1, Plan 2 and Plan 3 in Table 3) for cost settings in Table 2. The sum of stock cost and setup cost for those optimization approaches are computed in Table 4.

Table 4. The sum of stock cost and setup cost for those optimization approaches

| Cost setting | Plan 1 | Plan 2 | Plan 3 | SPGA-RHSs | 2D CGA |
|-------------|-------|-------|-------|-----------|--------|
| Cost setting 1 | 51848.998 | 52048.998 | 51667.978 | 52722.008 | 53235.382 |
| Cost setting 2 | 101297.996 | 101497.996 | 100735.956 | 102644.016 | 103270.764 |
| Cost setting 3 | 110897.996 | 111897.996 | 111135.956 | 113844.016 | 116070.764 |

In Table 3, the last cutting pattern of both Plan 1 and Plan 2 has leftover (used in the future), while the last cutting pattern of Plan 3 has almost no leftover. It is hard to compare the sum cost of stock cost and setup cost in each cutting plan based on each cost setting. But we can include that regardless of
each of cost setting in Table 3, the sum cost of stock cost and setup cost of a cutting plan solved by the balanced approach is less than other two optimization approaches in Table 4. This proves the proposed optimization approach in this article has good effectiveness in dealing with 2DRGMSSCSP-S.

5. Conclusion
In this article, the balanced approach for 2DRGMSSCSP-S is presented, which takes into account both stock cost and setup cost, simultaneously. The proposed balanced approach is used to select an optimal cutting pattern from a current cutting plan with the CBU. The optimal cutting plan satisfying current item demands is changing with the different proportion of stock cost and setup cost. The computational results also prove that the proposed balanced approach has good optimization performance for solving 2DRGMSSCSP-S.

The optimization approach mainly includes the CBU, RHSs, SPGA-RHSs and the balanced approach. The CBU consists of 1st-CBU and 2nd-CBU. The 1st-CBU is used to choose a better cutting pattern from a cutting plan based on current single stock, and the 2nd-CBU is applied to select an optimal cutting pattern from all better cutting patterns chosen by 1st-CBU. The CBU plays an important role to balance stock cost and setup cost. The RHSs provide the fundamental structure for each cutting pattern and meeting the demand of guillotine cutting. The SPGA-RHSs is used to achieve a cutting plan using RHSs based on the single stock. The balanced approach combining SPGA-RHSs and CBU is applied to solve 2DRGMSSCSP-S.

In future research, the authors will focus on 2DMSSCSP about other two-dimensional regular shape such circle, triangle and sector with setup cost.

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