Chiral Lagrangian with higher resonances and flavour $SU(3)$ breaking

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Abstract

A chiral Lagrangian with $SU(3)$ breaking and higher resonances is proposed. In this model, the $SU(3)$ breaking structure in vector-pseudoscalar sector is realized with the decay constants of pseudoscalar mesons and the masses of vector mesons used as inputs. We examine whether the resulting $SU(3)$ breaking effect in the charge radii of pseudoscalar mesons is consistent with the experimental facts.
I. INTRODUCTION

Quantitative prediction of rare $K$ decay rates based on the standard model is a necessary ingredient in the program for understanding the origin of $CP$ violation. However, since the typical energy scale in $K$ decays $\simeq M_K (\simeq 500 \text{ MeV})$ is too low for perturbative QCD to be applicable, we need some method to estimate the long distance contributions from QCD. The chiral symmetry imposes strong constraints on low energy dynamics associated with $K$ decays \cite{1–7}.

In the usual formalism of chiral dynamics, the energy dependence of the amplitudes involving Goldstone bosons ($\pi, K$) is given as a power series expansion with respect to the external momenta and quark masses. The coefficients of $O(p^4)$ terms in the effective chiral Lagrangian are explicitly determined from the experimental data \cite{9}. Later it has been recognized that these coefficients of $O(p^4)$ terms are saturated by the contributions from higher resonances (vector, axial vector and scalar mesons) \cite{4}. The importance of incorporating higher resonances can also be seen in the result that the $\Delta I = \frac{1}{2}$ rule is explained if we take the propagation of scalar resonances into account \cite{7}.

These results motivate us to the present work: in this article we propose a model for the chiral Lagrangian with scalar and vector mesons.

In QCD the $SU(3)$ breaking effect is introduced by the quark mass term $\mathcal{L}_m = -\bar{q}_L M^{(0)} q_R + h.c.$. In chiral dynamics, the scalar field plays the key role; it develops the vacuum expectation value(VEV) and induces the decay constants for pseudoscalar mesons. We shall see that the explicit $SU(3)$ breaking originating from current quark masses result in the differences among decay constants. We introduce vector meson following the hidden local Lagrangian approach \cite{10}. The $SU(3)$ breaking effects in vector-pseudoscalar couplings
are generated by those of decay constants of pseudoscalar mesons and the masses of vector mesons. The explicit form of the potential terms which gives the connection between the VEV's of scalars and current quark masses will be shown in Sec. II.

Let us consider the expansion of the full Lagrangian in powers of derivatives. At the order $\partial^2$ level, we have the kinetic term of pseudoscalars and the mass term of vector mesons. There are an infinitely large number of terms which are consistent with chiral and hidden local symmetries. This is because arbitrary higher dimensional operators of order $\partial^2$ are allowed in our approach. In Sec. III we argue first that we can restrict ourselves to four-dimensional operators since we are constructing an effective Lagrangian which is valid below the cutoff scale $\Lambda \simeq 4\pi F_\pi \approx 1$ GeV [11].

For a phenomenological test of this model we calculate the electromagnetic form factors for pseudoscalar mesons and the $K_{e3}$ form factor in Sec. IV. There, the charge radii of $\pi^+, K^+$ and $K^0$ will be compared to their experimentally measured values. We will also find that the normalization of $K_{e3}$ form factor $F_+(0) \simeq 1$ under fairly natural assumptions. In Sec. V, we examine the possible effects to charge radii from the extra terms which have not been included tentatively in the reduced chiral Lagrangian. It is shown that these additional terms do not modify the results obtained above. Sec. VI is devoted to discussion and summary.

II. CHIRAL LAGRANGIAN

A. Preliminaries

Let us begin our discussion by recalling the way one derives the chiral perturbation theory.
First we consider $U(N)_L \times U(N)_R$ linear $\sigma$ model \cite{12} which is defined by the Lagrangian \cite{13,14}

\[
\mathcal{L} = \text{Tr}(\partial_\mu M^\dagger \partial^\mu M) + \mu^2 \text{Tr}(M^\dagger M) - \frac{\lambda_1}{4} \left\{ \text{Tr}(M^\dagger M) \right\}^2 - \frac{\lambda_2}{4} \text{Tr}\left((M^\dagger M)^2\right) + \frac{1}{4G_1} \text{Tr}\left(m^{(0)}(M + M^\dagger)\right),
\]

(2.1)

where $m^{(0)} = \text{diag}(m_u^{(0)}, m_d^{(0)}, \ldots)$ denotes current quark mass matrix, and the last term breaks chiral symmetry explicitly. Using a unitary matrix $\xi$ and a hermitian matrix $\Sigma$, we can decompose $N \times N$ complex matrix $M$ in the form

$$M = \xi \Sigma \xi.$$ 

The positivity of $\mu^2$ signals the occurrence of spontaneous breakdown of chiral symmetry. This can be seen by examining the vacuum expectation value (VEV) of $\Sigma$ which is determined by minimizing the potential

\[
V(\langle \Sigma \rangle) = -\mu^2 \text{Tr}(\langle \Sigma \rangle^2) + \frac{\lambda_1}{4} \left\{ \text{Tr}(\langle \Sigma \rangle^2) \right\}^2 + \frac{\lambda_2}{4} \text{Tr}\left(\langle \Sigma \rangle^4\right) - \frac{1}{2G_1} \text{Tr}\left(m^{(0)} \langle \Sigma \rangle\right).
\]

(2.2)

Then the associated Goldstone bosons $\pi^a(a = 0, 1, \ldots, N^2 - 1)$ are non-linearly realized in $\xi$ as \cite{13}

\[
\xi = \exp \left( \frac{i}{2} \sum_{a=0}^{N^2-1} \frac{\pi^a T^a}{F^a} \right),
\]

(2.3)

with dimension-one constants $F^a(a = 1, \ldots, N^2 - 1)$.

The lowest order chiral Lagrangian used in chiral perturbation theory can be obtained in the infinitely heavy $\Sigma'$ limit, where $\Sigma' = \Sigma - \langle \Sigma \rangle$ is the dynamical degree of freedom associated with the $\Sigma$ field. Remember that the systematic chiral expansion is a power series expansion in the external momenta and quark masses. From the potential (2.2) we can see that the splitting in the components of diagonal matrix $\langle \Sigma \rangle$ is higher-order. Hence $F^a = F_\pi(a = 1, \ldots, N^2 - 1)$, and by setting $\langle \Sigma \rangle = \frac{F_\pi}{2} 1$ we obtain
\[ \mathcal{L} = \frac{F_\pi^2}{4} \left\{ \text{Tr}(\partial_\mu U^\dagger \partial^\mu U) + 2B_0 \text{Tr}(m^0(U + U^\dagger)) \right\} + \ldots, \]  

where \( U = \xi^2, \ B_0 = \frac{1}{4G_1F_\pi}, \) and the ellipsis contains the higher-order terms in chiral expansion, the irrelevant constant terms and the terms involving physical scalar fields.

Now let the coupling \( \lambda_1 \) and \( \lambda_2 \) go to infinity with \( F_\pi \) kept fixed. Then scalar particles become so heavy that they decouple effectively. Hence, in this limit, only Goldstone bosons survive and their low energy behaviour is described by the well-known leading order Lagrangian

\[ \mathcal{L} = \frac{F_\pi^2}{4} \left\{ \text{Tr}(\partial_\mu U^\dagger \partial^\mu U) + 2B_0 \text{Tr}(m^0(U + U^\dagger)) \right\}. \]  

(2.4)

Consider the case of two quark flavours (\( u \) and \( d \)) and apply the corresponding chiral perturbation approach. Then the chiral expansion is in powers of external momenta and the masses of these quarks which are about 10 MeV. Since the expansion parameter must be dimensionless, we have to divide the typical momentum scale \( p \) and the masses of quarks by some constant \( \Lambda \) with mass dimension. This \( \Lambda \) is considered to be the cutoff scale under which our effective chiral description is valid, and is about \( 4\pi F_\pi \simeq 1.2 \text{ GeV} \) where \( F_\pi \simeq 93 \text{ MeV} \) is the decay constant of \( \pi \) meson. Hence we expect, in this case, that the convergence of this expansion will be good as long as all external momenta are sufficiently smaller than \( \Lambda \) \( \left( \frac{p^2}{\Lambda^2} \lesssim \frac{2m_u}{\Lambda} \simeq 10^{-2} \right) \).

For the purpose of describing \( K \) mesons within the framework of chiral dynamics, we have to extend the above framework such as to include strange quark. In this case one more expansion parameter \( \frac{2m_s}{\Lambda} = 0.18 \sim 0.36 \) for \( m_s = 100 \sim 200 \text{ MeV} \) has to be taken into account. Also, in \( K \) decays, the magnitudes of the typical momenta are the order of \( K \) mass \( \simeq 500 \text{ MeV} \). Thus we have \( \left( \frac{p}{\Lambda} \right)^2 \simeq \left( \frac{M_K}{\Lambda} \right)^2 \simeq 0.21 \). Hence the expansion may not converge so rapidly as in the case of \( SU(2)_L \times SU(2)_R \).
Let us give one such example. One of the successful challenge is the explanation of the large enhancement of \( \Delta I = \frac{1}{2} \) component in the amplitude for \( K^0 \to 2\pi \) by incorporating scalar resonances [7]. There, the enhancement factor arises from the scalar (\( \sigma \)) propagator [8]

\[
\frac{M_\sigma^2 - M_\pi^2}{M_K^2 - M_K^2} = \frac{F_K}{3F_\pi - 2F_K} \simeq 2.2.
\]

(2.5)

In the systematic expansion of the chiral perturbation theory, this factor will be obtained as the series

\[
\frac{F_K}{3F_\pi - 2F_K} = 1 + \frac{3(F_K - F_\pi)}{F_K} + \ldots \ldots \\
= 1 + 0.54 + (0.54)^2 + \cdots.
\]

The convergence of this series is very slow. Note that the \( SU(3) \) breaking gets amplified when it comes into the numerator. Hence it is crucial to understand how to include the \( SU(3) \) breaking.

To do this we must go back to what we know about the low energy dynamics. One of the important facts is the vector meson dominance structure which can be seen in the pion-electromagnetic form factor \( F_\pi(s) \) (\( s \) is the invariant mass squared of virtual photon). Under this hypothesis \( F_\pi(s) \) is given by

\[
F_\pi(s) = \frac{M_\rho^2}{M_\rho^2 - s} = 1 + \frac{1}{6} \langle r^2 \rangle + s + \cdots,
\]

(2.6)

where \( M_\rho \) is the mass of \( \rho \) meson. Thus the charge radius \( \langle r^2 \rangle \) is obtained as

\[
\langle r^2 \rangle = 6\frac{1}{M_\rho^2} \simeq 0.41 \text{ fm}^2,
\]

which is in good agreement with its experimental value 0.44 fm\(^2\). It is also known that the behaviour caused by the infinite series in Eq.(2.6) recapitulates the experimentally measured momentum dependence of \( F_\pi(s) \).
In the systematic chiral expansion, QCD dynamics appears in the next-to-leading order terms, i.e., $\mathcal{O}(p^4)$ terms which accompany unknown constants $L_1, \cdots, L_{10}$ \[9\]. Each of these constants requires an experimental input. For example, $L_9$ has been determined by using the charge radius $\langle r^2 \rangle_{\pi^+}$ of $\pi^+$ as its input. However, as was shown before, $\langle r^2 \rangle_{\pi^+}$ could be obtained from the presence of $\rho$ meson with the vector meson dominance hypothesis. More generally, $L_1, \cdots, L_{10}$ are all determined from the higher resonance contributions, consistent with their proper values \[4\]. We shall, therefore, take an approach in which all higher-order effects in the chiral perturbation theory can be reproduced by the introduction of vector and scalar mesons \[4\].

**B. Model**

We propose a chiral Lagrangian \[16\] based on the hidden local symmetry approach \[10\]

$$
\mathcal{L}_{\text{chiral}} = \mathcal{L}_2 + \mathcal{L}_{\text{pot}} - \frac{1}{2g_V^2} \text{Tr}(F_{\mu\nu}V^\mu V^\nu),
$$

where $\mathcal{L}_2$ consists of the terms which contain two powers of derivatives. The explicit form of which will be shown in Sec.\[11\]. $F_{\mu\nu}^V$ is the field strength corresponding to vector meson $V^\mu$

$$
F_{\mu\nu}^V = \partial^\mu V^\nu - \partial^\nu V^\mu - i [V^\mu, V^\nu],
$$

and $g_V$ is the hidden local gauge coupling constant. $\mathcal{L}_{\text{pot}}$ determines the VEV $\Sigma^{(0)}$ of scalar nonet denoted by $S$ and takes the form \[17\]

$$
\mathcal{L}_{\text{pot}} = \mu^2 \text{Tr}(S^2) + \frac{1}{4G_1} \text{Tr} \left( S \left\{ \xi_L M^\dagger \xi_R^\dagger + \xi_R M^\dagger \xi_L^\dagger \right\} \right)
- \frac{\lambda_1}{4} \left\{ \text{Tr}(S^2) \right\}^2
- \frac{\lambda_2}{4} \text{Tr}(S^4). \tag{2.8}
$$

In this expression $M$ denotes the $3 \times 3$ matrix field which couples to quarks in QCD Lagrangian \[3\] as:
\[ - \left( \bar{q}_L \mathcal{M} q_R + \bar{q}_R \mathcal{M}^\dagger q_L \right). \]

Hence, under chiral transformation \( \{g_L, g_R\} \in [U(3)_L \times U(3)_R]_{\text{global}} \), \( \mathcal{M} \) is considered to “transform” as

\[ \mathcal{M} \rightarrow g_L \mathcal{M} g_R^\dagger. \]

The vacuum configuration is determined through Eq.(2.8) by setting \( \mathcal{M} \) to be current quark mass matrix \( m_i^{(0)} \): \( \mathcal{M} = m^{(0)} = \text{diag}(m_1^{(0)}, m_2^{(0)}, m_3^{(0)}) \).

The differences in current quark masses \( m_i^{(0)} (i = 1, 2, 3) \) induce the splitting among the VEV’s taken by the neutral components of scalar \( S \). Throughout this paper the isospin breaking effect due to quark masses will be neglected: \( m_1^{(0)} = m_2^{(0)} \). Hence the VEV( \( \Sigma^{(0)} \) ) of \( S \) takes the form

\[
\Sigma^{(0)} = \begin{pmatrix}
\Sigma_1^{(0)} \\
\Sigma_2^{(0)} \\
\Sigma_3^{(0)}
\end{pmatrix}.
\]

Since we do not perform an expansion in powers of \( m_i^{(0)} \), this splitting induces the differences in the decay constants of pseudoscalar mesons and the masses of vector mesons.

In the framework of hidden local symmetry, \( \xi_L \) and \( \xi_R \) contain not only the nonet of pseudoscalar mesons but also a set of unphysical scalar degrees of freedom (\( \sigma \))

\[
\xi_L = \xi(\sigma) \xi(\Pi)^\dagger, \\
\xi_R = \xi(\sigma) \xi(\Pi),
\]

where \( \xi(\sigma) = e^{i\sigma} \) and \( \xi(\Pi) = e^{i\Pi} \). The vector mesons are the gauge bosons of hidden local symmetry (in our case it is \([U(3)_V]_{\text{local}}\)). The unphysical \( \sigma \) will be absorbed by the vector mesons and give them nonzero masses through Higgs mechanism [10]. The transformation properties of \( \xi_L, \xi_R \) and \( S \) under chiral and hidden local symmetries are
\[ \xi_L \rightarrow \xi_L' = h \xi_L g_L^\dagger, \]
\[ \xi_R \rightarrow \xi_R' = h \xi_R g_R^\dagger, \]
\[ S \rightarrow S' = h S h^\dagger, \]

where \( h \) is an element of \([U(3)]_\text{local}\). Then the Lagrangian \( \mathcal{L}_{\text{pot}} \) in Eq.(2.8) can be seen to be invariant under \( G \equiv [U(3)]_\text{local} \times [U(3)_L \times U(3)_R]_\text{global} \).

When we discuss the terms of \( \mathcal{O}(p^2) \) in Sec.III, it is sufficient to use the following blocks, each of which transforms as \( A \rightarrow h A h^\dagger \) under \( G \);

\[ S, \, \alpha_{\parallel \mu}, \, \alpha_{\perp \mu}, \, D_\mu S, \, F^\mu_\nu, \, \frac{1}{G_1} \xi_L M \xi_R^\dagger. \]

Here \( \alpha_{\parallel \mu} \) and \( \alpha_{\perp \mu} \) are

\[ \alpha_{\parallel \mu} = \frac{D_\mu \xi_L \xi_L^\dagger + D_\mu \xi_R \xi_R^\dagger}{2i}, \]
\[ \alpha_{\perp \mu} = \frac{D_\mu \xi_L \xi_L^\dagger - D_\mu \xi_R \xi_R^\dagger}{2i}. \]

If we write the external vector fields as \( \mathcal{L}_\mu \) and \( \mathcal{R}_\mu \), associated with the gauging of chiral symmetry \([U(3)_L \times U(3)_R]_\text{global}\), then various covariant derivatives are given by

\[ D_\mu \xi_L = \partial_\mu \xi_L - i V_\mu \xi_L + i \xi_L \mathcal{L}_\mu, \]
\[ D_\mu \xi_R = \partial_\mu \xi_R - i V_\mu \xi_R + i \xi_R \mathcal{R}_\mu, \]
\[ D_\mu S = \partial_\mu S - i [V_\mu, S]. \]

In constructing \( \mathcal{L}_2 \), it is also necessary to know the operation of charge conjugation(\( \mathcal{C} \)), in particular, on \( \alpha^\mu_{\parallel} \) and \( \alpha^\mu_{\perp} \)

\[ \mathcal{C} : \alpha^\mu_{\perp} \rightarrow (\alpha^\mu_{\perp})^T, \quad \alpha^\mu_{\parallel} \rightarrow -(\alpha^\mu_{\parallel})^T. \] 

(2.10)

Hence the terms such as \( \text{Tr} \left( \alpha^\mu_{\parallel} \{ S, D_\mu S \} \right) \) are forbidden by \( \mathcal{C} \) symmetry.
III. $O(p^2)$ TERMS

Here we discuss the $O(p^2)$ terms with respect to the momentum expansion.

The counting rule for momentum order is

- $\alpha_\parallel^\mu$ and $\alpha_\perp^\mu$ are $O(p)$.
- $V_\mu, L_\mu$ and $R_\mu$ are $O(p)$. $F_V^{\mu\nu}$ is $O(p^2)$.
- $S$ is $O(p^0)$.
- $\frac{1}{G_1} M$ is $O(p^2)$.

We remark on the counting rule assigned to the scalar field $S$. Recall that in the original hidden local symmetry approach \cite{10} the mass of vector meson is generated by the term

$$aF_\pi^2 \text{Tr}(\alpha_\parallel^\mu \alpha_\parallel^\mu),$$

which is called $O(p^2)$ term. In our present scheme the origin of the decay constants of pseudoscalars is traced to the the nonzero VEV’s of scalar $S$. Since $F_\pi$ is $O(p^0)$, consistency will demand that $S$ is $O(p^0)$. Then $O(p^2)$ terms, such as

$$\text{Tr} \{ S, \alpha_\parallel^\mu \} \{ S, \alpha_\parallel^\mu \},$$

contribute to the masses for vector meson.

According to the above counting rule, the fields which can participate in $O(p^2)$ terms are $\alpha_\parallel^\mu, \alpha_\perp^\mu, \frac{1}{G_1} M$ and $S$. The operator with dimension less than 4 requires compensation by the multiplication of constant(s) with mass dimension when involved in the Lagrangian. In our scheme we consider the constants with mass dimension available to us are only the VEV of $S$ and $\frac{1}{G_1} M$. The latter appears only in the pseudoscalar mass terms which have
been already included in the potential term in Eq.(2.8). Thus we do not need to be concerned with the operators with dimension less than 4.

The operator with dimension \( \equiv 4 + d \) more than 4 has a coefficient of the form \( \frac{a}{\Lambda^d} \) where \( a \) is a constant of order 1, and \( \Lambda \) is the cutoff under which our effective chiral Lagrangian is expected to describe the low energy behavior of QCD. Problematic thing is that higher dimensional operators, such as

\[
\frac{a}{\Lambda^d} \text{Tr}(S^dO^{(4)}),
\]

where \( O^{(4)} \) is some four-dimensional and \( O(p^2) \) operator, can induce four-dimensional operators by taking the VEV’s of scalars. To see that we do not need to add these contributions, write the operator in Eq.(3.3) as

\[
a \left\{ \left( \frac{\Sigma_1^{(0)}}{\Lambda} \right)^d \text{Tr} O^{(4)} + \left( \frac{\Sigma_3^{(0)}}{\Lambda} \right)^d - \left( \frac{\Sigma_1^{(0)}}{\Lambda} \right)^d \right\} O^{(4)}_{33}.
\]

(3.4)

The first term in this expression can be absorbed by the renormalization of the operator \( \text{Tr} O^{(4)} \). In the context where \( \Lambda \approx 4\pi F/\rho \), \( \Sigma_1^{(0)} \approx F/\rho \). As will be shown later, the splitting in the VEV’s of \( S \) has the form: \( \Sigma_3^{(0)} = \frac{M_{\phi}}{M_{\rho}} \Sigma_1^{(0)} \).

Hence the second term in Eq.(3.4) is of order

\[
\left( \frac{\Sigma_3^{(0)}}{\Lambda} \right)^d - \left( \frac{\Sigma_1^{(0)}}{\Lambda} \right)^d \sim 0.013 \quad (d = 1),
\]

\[
\sim 0.0012 \quad (d = 2),
\]

(3.5)

which is smaller for larger \( d \). Hence, to order 1% accuracy, this second term can be dropped. Since we will not consider any processes with scalar particles on the external lines, this type of higher dimensional operators is not important. Therefore, we can concentrate ourselves with the dimension-four operators which can be constructed from \( \alpha^\mu_\parallel, \alpha^\mu_\perp \) and \( S \).

Below we list those operators that are Lorentz, \( \mathcal{C}, \mathcal{P} \) (Parity operation) invariant and have chiral and hidden local symmetries:
\[
\begin{array}{l}
\text{(Category 1)} \\
\begin{cases}
\text{Tr}([S, \alpha_\mu][S, \alpha_{\parallel \mu}]) \\
\text{Tr}\{S, \alpha_\mu\}\{S, \alpha_{\parallel \mu}\} \\
\text{Tr}([S, \alpha_\mu^\dagger][S, \alpha_{\perp \mu}]) \\
\text{Tr}\{S, \alpha_\mu^\dagger\}\{S, \alpha_{\perp \mu}\} \\
i \text{Tr}(\alpha_\mu^\dagger [S, D_{\mu} S]) \\
\text{Tr}(D_{\mu} S D_{\mu} S),
\end{cases}
\end{array}
\]

\[(3.6)\]

\[
\begin{array}{l}
\text{(C.2)} \\
\begin{cases}
\text{Tr}(S \alpha_\mu \alpha_{\parallel \mu}) \text{Tr}(S) \\
\text{Tr}(S \alpha_\mu \alpha_{\perp \mu}) \text{Tr}(S),
\end{cases}
\end{array}
\]

\[(3.7)\]

\[
\begin{array}{l}
\text{(C.3)} \\
\begin{cases}
\text{Tr}(\alpha_\mu \alpha_{\parallel \mu}) \text{Tr}(S) \text{Tr}(S) \\
\text{Tr}(\alpha_\mu \alpha_{\parallel \mu}) \text{Tr}(S^2) \\
\text{Tr}(\alpha_\mu \alpha_{\perp \mu}) \text{Tr}(S) \text{Tr}(S) \\
\text{Tr}(\alpha_\mu \alpha_{\perp \mu}) \text{Tr}(S^2),
\end{cases}
\end{array}
\]

\[(3.8)\]

\[
\begin{array}{l}
\text{(C.4)} \\
\begin{cases}
\text{Tr}(S^2 \alpha_\mu^\dagger) \text{Tr}(\alpha_{\parallel \mu}) \\
\text{Tr}(S^2 \alpha_\mu^\dagger) \text{Tr}(\alpha_{\perp \mu}),
\end{cases}
\end{array}
\]

\[(3.9)\]

\[
\begin{array}{l}
\text{(C.5)} \\
\begin{cases}
\text{Tr}(S \alpha_\mu^\dagger) \text{Tr}(S \alpha_{\parallel \mu}) \\
\text{Tr}(S \alpha_\mu^\dagger) \text{Tr}(S \alpha_{\perp \mu}) \\
\text{Tr}(D_{\mu} S) \text{Tr}(D_{\mu} S),
\end{cases}
\end{array}
\]

\[(3.10)\]

\[
\begin{array}{l}
\text{(C.6)} \\
\begin{cases}
\text{Tr}(S \alpha_\mu^\dagger) \text{Tr}(\alpha_{\parallel \mu}) \text{Tr}(S) \\
\text{Tr}(S \alpha_\mu^\dagger) \text{Tr}(\alpha_{\perp \mu}) \text{Tr}(S),
\end{cases}
\end{array}
\]

\[(3.11)\]

\[
\begin{array}{l}
\text{(C.7)} \\
\begin{cases}
\text{Tr}(\alpha_\mu) \text{Tr}(\alpha_{\parallel \mu}) \text{Tr}(S) \text{Tr}(S) \\
\text{Tr}(\alpha_\mu) \text{Tr}(\alpha_{\parallel \mu}) \text{Tr}(S^2) \\
\text{Tr}(\alpha_\mu) \text{Tr}(\alpha_{\perp \mu}) \text{Tr}(S) \text{Tr}(S) \\
\text{Tr}(\alpha_\mu) \text{Tr}(\alpha_{\perp \mu}) \text{Tr}(S^2).
\end{cases}
\end{array}
\]

\[(3.12)\]
We divided operators into classes according to the flavour $SU(3)$ breaking structure which will be generated by the splitting among the VEV’s of scalar $S$. We first concentrate our attention on the following reduced Lagrangian which consists of the operators belonging to the Category-1:

$$\mathcal{L}_2 = \text{Tr}(\{S, \alpha^\mu\}_\perp \{S, \alpha_{\perp \mu}\}) + f \text{Tr}([S, \alpha^\mu][S, \alpha_{\perp \mu}])$$

$$+ a \text{Tr}(\{S, \alpha^\mu\}_\parallel \{S, \alpha_{\parallel \mu}\}) + b \text{Tr}([S, \alpha^\mu][S, \alpha_{\parallel \mu}])$$

$$+ d \text{Tr}(D^\mu SD^\mu S) + 2c_i \text{Tr}(\alpha^\mu [S, D^\mu S]) \tag{3.13}$$

The effects of operators in the other categories will be discussed later.

We first express fundamental quantities in the pseudoscalar sector. Since isospin breaking is ignored here, isospin nonsinglet mesons are always in their mass eigenstates. We did not concern $\eta$ and $\eta'$ explicitly here. So it is convenient to work in the nonet basis

$$\Pi = \frac{1}{\sqrt{2}} \begin{pmatrix} \pi^0 + \eta & \pi^+ & K^+ \\ \sqrt{2}F_\pi & F_\pi^- & -\pi^0 + \eta \\ \frac{\pi^-}{F_\pi} & \frac{-\pi^0}{F_\pi} + \eta & F_K^- \\ K^- & \sqrt{2}F_\pi & F_\pi^0 \\ \frac{K^-}{F_K} & \frac{F_\pi^0}{F_K} & \eta_{33} \\ \frac{\eta_{33}}{F_3} & \frac{F_\pi^0}{F_3} & F_{33} \end{pmatrix}.$$ 

Then we have

$$F_\pi = 2\Sigma_1^{(0)},$$

$$F_{33} = 2\Sigma_3^{(0)},$$

$$F_K^2 = \frac{1}{4} \left\{ (F_\pi + F_{33})^2 - f (F_{33} - F_\pi)^2 \right\}. \tag{3.14}$$

The masses of $\pi$ and $K$ can be read off from Eq.(2.8)

$$M_\pi^2 \equiv \frac{m_{11}^{(0)}(2\Sigma_1^{(0)})}{2G_1F_\pi^2} = \frac{m_{11}^{(0)}}{2G_1F_\pi},$$

$$M_K^2 \equiv \frac{(m_{11}^{(0)} + m_{33}^{(0)})(\Sigma_1^{(0)} + \Sigma_3^{(0)})}{4G_1F_K^2} = \frac{(m_{11}^{(0)} + m_{33}^{(0)})(F_\pi + F_{33})}{8G_1F_K^2}. \tag{3.15}$$

Next we turn our attention to the scalar-vector sector. There are transitions among vector meson $V_\mu$, scalar $\sigma$ and $\Sigma$ where the field $\Sigma$ is defined as
\[ S = \Sigma^{(0)} + \frac{1}{\sqrt{d}} \Sigma. \]

We define the unitary gauge as follows. First we set \( \sigma = 0 \) and redefine the fields \( V_\mu \) and \( \Sigma \) as

\[ V_\mu \rightarrow V_\mu + i \frac{1}{\sqrt{2} \sqrt{1 - C^2}} \frac{g_V}{M_{K^*}} \partial_\mu K^S, \]
\[ \Sigma^j_i \rightarrow \frac{1}{\sqrt{1 - C^2}} \Sigma^j_i \text{ for } (i, j) = (1, 3), (2, 3), (3, 1), (3, 2). \] (3.16)

In these expressions \( M_{K^*} \) is the mass of \( K^* \) (see Eq.(3.20)), and \( K^S \) consists of scalar components \( \kappa \) each of which has strange number

\[ K^S \equiv \begin{pmatrix} 0 & 0 & -\kappa^+ \\ 0 & 0 & -\kappa^0 \\ \kappa^- & \kappa^0 & 0 \end{pmatrix}, \] (3.17)

and

\[ C \equiv \frac{d + c}{\sqrt{d}} \frac{g_V \Sigma_3^{(0)} - \Sigma_1^{(0)}}{M_{K^*}}. \] (3.18)

If we denote the vector meson matrix \( V_\mu \) as

\[ V_\mu = \frac{g_V}{\sqrt{2}} \begin{pmatrix} \frac{1}{\sqrt{2}} (\rho^0 + \omega)_\mu & \rho^+_\mu & K^{*+}_\mu \\ \rho^-_\mu & \frac{1}{\sqrt{2}} (-\rho^0 + \omega)_\mu & K^{*0}_\mu \\ K^{*-}_\mu & \bar{K}^{*0}_\mu & \phi_\mu \end{pmatrix}, \] (3.19)

the masses of vector mesons are given by

\[ M_{\rho}^2 = ag_V F_\pi^2 = M_{\omega}^2, \]
\[ M_{\phi}^2 = ag_V F_{33}^2, \]
\[ M_{K^*}^2 = \frac{ag_V^2}{4} \left\{ (F_{33} + F_\pi)^2 + \frac{d + 2c - b}{a} (F_{33} - F_\pi)^2 \right\}. \] (3.20)

We can see from Eq.(3.16) that \( C^2 < 1 \) is necessary for our model to become meaningful. Eq.(3.18) tells us that there is no mixing between \( V_\mu \) and \( \Sigma \) when \( c = -d \). Note that if \( \frac{d + 2c - b}{a} \) is order unity, we have
\[ M_{K^*} = \frac{1}{2}(M_\rho + M_\phi), \quad (3.21) \]

since
\[ \frac{(F_{33} - F_\pi)^2}{(F_{33} + F_\pi)^2} = \frac{(M_\phi - M_\rho)^2}{(M_\phi + M_\rho)^2} \approx 0.02. \quad (3.22) \]

The \( f \) term in the expression for \( F_K \) cannot be neglected: in fact, by considering \( f = \mathcal{O}(1) \) and neglecting the \( f \) term in Eq. (3.14), we obtain the relation
\[ \frac{F_K}{F_\pi} = \frac{1}{2} \left( 1 + \frac{M_\phi}{M_\rho} \right) \approx 1.16, \quad (3.23) \]

which is off by 6 \%.

We close this section by quoting the mass \( M_\kappa \) of \( \kappa \) since \( \kappa \) will contribute to the \( K_{e3} \) form factor. \( M_\kappa \) is given by
\[ M_\kappa^2 = \frac{1}{d} \frac{1}{1 - C^2} \frac{1}{\Sigma_3^{(0)} - \Sigma_1^{(0)}} \left( \frac{M_K^2 F_K^2}{\Sigma_3^{(0)} + \Sigma_1^{(0)}} - \frac{M_\pi^2 F_\pi^2}{2 \Sigma_1^{(0)}} \right). \quad (3.24) \]

Using Eq. (3.14), it can also be expressed as
\[ M_\kappa^2 = \frac{2}{d(1 - C^2)} \left( \frac{2 F_K^2}{F_{33}^2 - F_\pi^2} M_K^2 - \frac{1}{F_{33}^2 - F_\pi^2} \right). \quad (3.25) \]

IV. ELECTROMAGNETIC FORM FACTORS AND \( K_{e3} \) FORM FACTOR

We now explore the electromagnetic form factors and \( K_{e3} \) form factor based on our chiral Lagrangian. In the standard model the external gauge fields \( \mathcal{V}_\mu = \mathcal{R}_\mu + \mathcal{L}_\mu \) and \( \mathcal{A}_\mu = \mathcal{R}_\mu - \mathcal{L}_\mu \) take the form
\[ \mathcal{V}_\mu = \frac{2}{3} e \gamma_\mu \begin{pmatrix} 2 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix} \]
\[ \begin{pmatrix} \frac{1}{2} - \frac{4}{3} s_W^2 & 0 & 0 \\ 0 & -\frac{1}{2} + \frac{2}{3} s_W^2 & 0 \\ 0 & 0 & -\frac{1}{2} + \frac{2}{3} s_W^2 \end{pmatrix} \]

\[ + \frac{g}{c_W} Z_\mu \]

\[ \begin{pmatrix} 0 & c_1 & -s_1 c_3 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \]

\[ + \frac{g}{\sqrt{2}} W^+ \mu \]

\[ \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix} \]

\[ A_\mu = -\frac{g}{2c_W} Z_\mu \]

\[ \begin{pmatrix} 0 & c_1 & -s_1 c_3 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \]

\[ -\frac{g}{\sqrt{2}} W^+ \mu \]

\[ \begin{pmatrix} 0 & c_1 & -s_1 c_3 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \]

where \( c_W = \cos \theta_W \) with the Weinberg angle \( \theta_W \), and \( g \) is the \( SU(2) \) coupling constant. \( c_1 = \cos \theta_1, s_1 = \sin \theta_1, c_3 = \cos \theta_3 \) and \( \theta_i \)'s are the angles parametrizing Kobayashi-Maskawa matrix [19].

First we see that photon \( \gamma_\mu \) and neutral vector mesons mix

\[ \mathcal{L}_{\gamma - V} = -e \gamma^\mu \left[ g_\rho \rho_\mu + g_\omega \omega_\mu - g_\phi \phi_\mu \right], \quad (4.1) \]

where [10]

\[ g_\rho = a g V F_\pi^2 = \frac{1}{g V} M_\rho^2, \]

\[ g_\omega = \frac{1}{3} g_\rho, \]

\[ g_\phi = \frac{\sqrt{2}}{3} a g V F_3^2 = \frac{\sqrt{2} M_\phi^2}{3 g V} = \frac{\sqrt{2} M_\phi^2}{3 M_\rho^2 g_\rho}. \quad (4.2) \]

The \( W - V \) mixing is given by

\[ \mathcal{L}_{W - V}^{A_{S=1}} = s_1 c_3 g g_k^{\star} W^{-\mu} K^{++}_\mu. \quad (4.3) \]

Here the constant \( g_k^{\star} \) is calculated to be
\[ g_{K^*} = \frac{ag_V}{8} \left\{ (F_{33} + F_{\pi})^2 - \frac{-c + b}{a} (F_{33} - F_{\pi})^2 \right\} \]
\[ = \frac{1}{2g_V} \left( 1 - \frac{C^2}{1 + \bar{c}} \right) M^2_{K^*} = \frac{g_\rho M^2_{K^*}}{2 M^2_\rho} \left( 1 - \frac{C^2}{1 + \bar{c}} \right), \]  

(4.4)

with \( \bar{c} \equiv \frac{c}{d} \).

The \( V - P P \) coupling (\( P \) denotes pseudoscalar) takes the form

\[ \mathcal{L}_{V - P P} = -i \rho^\mu \left( g_{\rho\pi\pi} \bar{\pi} \gamma^\mu \pi - g_{\rhoKK} K^+ \gamma^\mu K^- - g_{\rhoKK} K^0 \gamma^\mu \bar{K}^0 \right) \]
\[ - i g_{\PhiKK} \bar{\phi} \gamma^\mu \left( K^+ \gamma^\mu K^- - \bar{K}^0 \gamma^\mu K^0 \right) \]
\[ + i \frac{g_{\PhiKK}}{\sqrt{2}} \phi^\mu \left( K^+ \gamma^\mu K^- + K^0 \gamma^\mu \bar{K}^0 \right) \]
\[ - i \frac{g_{\PhiKK}}{\sqrt{2}} K^+ \gamma^\mu \left( K^- \gamma^\mu K^0 - \sqrt{2} \bar{\pi} \gamma^\mu K^0 \right) \]
\[ - i \frac{g_{\PhiKK}}{\sqrt{2}} K^+ \gamma^\mu K^0 \left( K^- \gamma^\mu K^0 - \frac{1}{\sqrt{2}} \bar{K}^0 \gamma^\mu K^0 \right) \]
\[ + \cdots, \]  

(4.5)

and each coefficient is given by KSRF(I) relation

\[ g_{\rho\pi\pi} = \frac{g_\rho}{2F^2_\pi}, \]
\[ g_{\rhoKK} = g_{\omegaKK} = \frac{g_\rho}{4F^2_K}, \]
\[ g_{\PhiKK} = \frac{3g_\phi}{2\sqrt{2}F^2_K}, \]
\[ g_{K^*KK} = \frac{g_{K^*}}{F^2_\pi F_K}. \]  

(4.6)

We explore the numerical values of various quantities by using the measured values of \( F_\pi, F_K, M_\rho, M_\phi, M_{K^*} \) and \( g_{\rho\pi\pi} \). The results are shown in Table I and II. There, experimental uncertainties \( \sqrt{2}F_\pi = 130.8 \pm 0.3 \text{ MeV}, \)
\( \sqrt{2}F_K = 159.8 \pm 1.4 \text{ MeV} \) and \( 5.9 \leq g_{\rho\pi\pi} \leq 6.1 \) were taken into account. For the values of \( M_\rho \) and \( M_{K^*} \), there is some theoretical uncertainties due to our approximation. Here we used \((M_\rho)_{\text{exp}} \leq M_\rho \leq (M_\omega)_{\text{exp}} \) for \( M_\rho \) and \((M_{K^*})_{\text{exp}} \leq M_{K^*} \leq (M_{K^*})_{\text{exp}} \) for \( M_{K^*} \). The value of \( g_{K^*KK} \) shown in Table II was obtained under the assumption that \( d + c \) is at most of order unity: in fact, we have from Eq.(3.18)
\[ C \simeq \frac{d + c}{\sqrt{d}} \times 0.10, \]

so that the quantity dependent on \((d + c)\) in the parenthesis in Eq.(4.4) becomes \((d + c) \times 0.01\), and this correction is within the experimental errors accompanied with it.

The direct \(\gamma - PP\) and \(W - PP\) coupling are

\[
\mathcal{L}_{\gamma PP} = -i e \gamma^\mu \left\{ (1 - r_\rho) (\pi^+ \partial_\mu \pi^- - \partial_\mu \pi^+ \pi^-) + (1 - r_\phi) (K^+ \partial_\mu K^- - \partial_\mu K^+ K^-) - r_\rho \frac{F_{33}^2 - F_\pi^2}{3 F_K^2} (K^0 \partial_\mu K^0 - \partial_\mu K^0 \bar{K}^0) \right\},
\]

\[
\mathcal{L}_{W - PP}^{\Delta S = 1} = -i s_1 c_3 \frac{g}{4} W^{-\mu} \left\{ \left( \frac{F_K}{F_\pi} - r_{K^*} \right) (K^0 \partial_\mu \pi^0 + \sqrt{2} \partial_\mu \pi^+ K^0) - \left( \frac{F_K}{F_\pi} - r_{K^*} \right) (\partial_\mu K^+ \pi^0 + \sqrt{2} \partial_\mu K^- K^0) \right\},
\]

where \(r_\rho\) and \(r_\phi\) characterize the vector dominance and are given by

\[
r_\rho \equiv \frac{a}{2} = \frac{2 g_{\rho \pi \pi}^2 F_\pi^2}{M_\rho^2}, \]

\[
r_\phi = \frac{2 g_{\phi KK}^2 F_K^2}{M_\phi^2} \left( \frac{F_{33}^2 + 2 F_\pi^2}{3 F_{33}^2} \right),
\]

\[
r_{K^*} \equiv \frac{1}{2 F_\pi F_K} a \left\{ (F_{33} + F_\pi) \right\}^2 - \frac{b}{a} (F_{33} - F_\pi) \right\} \]

\[
= \frac{2 g_{K^* K \pi}^2 F_\pi F_K}{M_{K^*}^2} \left( 1 - \frac{1 + 2 \bar{c}}{(1 + \bar{c})^2} C^2 \right) \left( \frac{1}{1 + \bar{c}} \right)^2. \tag{4.7}
\]

The expression for \(r_\rho\) has been obtained before [10]. This originates from the fact that when ignoring the \(SU(3)\) breaking, the first term and \(a\) term in Eq.(3.13) induces the same terms as in the original hidden local Lagrangian [11]. However the expression for \(r_\phi\) is a peculiar one arising from our model. Table [11] shows the numerical consequences for these quantities in our model.

With these preparations, the electromagnetic form factors can be obtained:
\[ F_{\pi^+}(s) = 1 + r_{\rho} \frac{s}{M_{\rho}^2 - s}, \]
\[ F_{K^+}(s) = \frac{1}{3} \left\{ 1 + 2r_{\rho} \left( \frac{F_\pi}{F_K} \right)^2 \frac{s}{M_{\rho}^2 - s} + \tilde{r}_\phi \frac{s}{M_{\phi}^2 - s} \right\}, \]
\[ F_{K^0}(s) = -\frac{1}{3} \left\{ r_{\rho} \left( \frac{F_\pi}{F_K} \right)^2 \frac{s}{M_{\rho}^2 - s} - \tilde{r}_\phi \frac{s}{M_{\phi}^2 - s} \right\}, \] (4.8)

where \( s \) is the invariant mass squared of virtual photon, and
\[
\tilde{r}_\phi \equiv \frac{2 g_\phi^2 K K}{M_{\phi}^2}.
\]

We can deduce the charge radius for each particle
\[
\left\langle r^2 \right\rangle_{\pi^+} = 6 \frac{r_{\rho}}{M_{\rho}^2},
\]
\[
\left\langle r^2 \right\rangle_{K^+} = 6 \frac{\tilde{r}_\phi}{M_{\phi}^2} = \left( \frac{F_K}{F_\pi} \right)^2 \left\langle r^2 \right\rangle_{\pi^+},
\]
\[
\left\langle r^2 \right\rangle_{K^0} = 0.
\] (4.9)

In deriving these expressions we use the relation
\[
\tilde{r}_\phi = r_{\rho} \left( \frac{M_{\phi}}{M_{\rho}} \right)^2 \left( \frac{F_\pi}{F_K} \right)^2.
\]

The numerical results for the charge radii are summarized in Table IV. The \( SU(3) \) breaking in the charge radii appears in the simple form
\[
\frac{\left\langle r^2 \right\rangle_{K^+}}{\left\langle r^2 \right\rangle_{\pi^+}} = \left( \frac{F_\pi}{F_K} \right)^2,
\]
the value of which is about 0.67 (its experimental value is 0.64) so that in this aspect our model is not inconsistent with the experiment.

Table IV shows that our model gives slightly smaller values for both \( \left\langle r^2 \right\rangle_{\pi^+} \) and \( \left\langle r^2 \right\rangle_{K^+} \). Also, as shown in Eq.(4.3), the charge radius \( \left\langle r^2 \right\rangle_{K^0} \) for \( K^0 \) becomes exactly 0, in contrast to its being experimentally small but having a finite value \((-0.054 \pm 0.026fm^2\) ). We will reexamine in Sec.V whether there is some modification to these results by the addition of \( 1/N_C \) non-leading but \( O(p^2) \) terms, which are not included in the reduced Lagrangian in Eq.(3.13).
To calculate the $K_{e3}$ form factor, we further need the transition term between $\Sigma$ and $W_{\mu}$, and the interaction among $\kappa^-$, $\pi^0$ and $K^+$

$$\mathcal{L}_{S-W} = -i \sqrt{d(1-C^2)} (F_{33} - F_{\pi}) s \epsilon_3 \frac{g}{4} W_{\mu} \partial^\mu \kappa^- + (\text{h.c.}),$$

$$\mathcal{L}_{\kappa^-\pi^0K^+} = \frac{i v_{\sigma}}{\sqrt{d}} F_{\kappa^+} \partial^\mu \pi^0 \partial_\mu K^+ - \frac{\lambda g_{K^+K^0\pi}^{\kappa^-}}{2M_{K^+}} \partial^\mu \kappa^- (K^+ \partial_\mu \pi^0),$$

where the constants $v_{\sigma}$ and $\lambda$ are

$$v_{\sigma} \equiv \frac{1}{2\sqrt{1-C^2}} \left\{ \frac{1 + \frac{F_{33} + F_{\pi}}{2F_{\pi}} - f \frac{F_{33} - F_{\pi}}{2F_{\pi}}}{\frac{F_{K}}{F_{\pi}}} \right\},$$

$$\lambda \equiv \frac{C\sqrt{1-C^2}}{1+\bar{c}-C^2}.$$ (4.10)

The $K_{e3}$ form factors $F_\pm(s)$ ($s \equiv (p_K - p_\pi)^2$) is defined by

$$\langle \pi^0(p_\pi) | \bar{s}(1-\gamma_5)u | K^+(p_K) \rangle = -\frac{1}{\sqrt{2}} \{ F_+(s)(p_K + p_\pi)_\mu + F_-(s)(p_K - p_\pi)_\mu \}. $$

Direct calculation shows that $F_+(s)$ and $F_-(s)$ are given in our model by

$$F_+(s) = \frac{1}{2} \left( \frac{F_{K}}{F_{\pi}} + \frac{F_{\pi}}{F_{K}} \right) - (r_{K^*} - \bar{r}_{K^*}) + \bar{r}_{K^*} \frac{s}{M_{K^*}^2 - s},$$

$$F_-(s) = \frac{1}{2} \left( \frac{F_{K}}{F_{\pi}} - \frac{F_{\pi}}{F_{K}} \right) - \bar{r}_{K^*} \frac{M_{K}^2 - M_{\pi}^2}{M_{K^*}^2 - s}$$

$$+ \frac{1}{M_{K}^2 - s} \left\{ \frac{1}{2} \left( \frac{F_{K}}{F_{\pi}} - \frac{F_{\pi}}{F_{K}} \right) \left( M_{K}^2 + M_{\pi}^2 - s \right) + (r_{K^*} - \bar{r}_{K^*}) \left( M_{K}^2 - M_{\pi}^2 \right) \right\}. $$ (4.11)

Here $\bar{r}_{K^*}$ is

$$\bar{r}_{K^*} = \frac{2g_{K^*K^0\pi}^{F_K}F_{\pi}}{M_{K^*}^2}. $$ (4.12)

Hence the linear slope $\lambda_{e3}$ defined by

$$F_+(s) = F_+(0) \left( 1 + \lambda_{e3} \frac{s}{M_{\pi}^2} \right) + \mathcal{O} \left( \left( \frac{s}{M_{\pi}^2} \right)^2 \right),$$

takes the form

$$\lambda_{e3} = \bar{r}_{K^*} \frac{M_{\pi}^2}{M_{K^*}^2 F_+(0)}, $$ (4.13)
where

\[ F_+(0) = \frac{1}{2} \left( \frac{F_\pi}{F_\pi} + \frac{F_K}{F_K} \right) - (r_{K^*} - \bar{r}_{K^*}), \]

\[ r_{K^*} - \bar{r}_{K^*} = \bar{r}_K \cdot \frac{C^2(1 - C^2)}{(1 + \bar{c} - C^2)^2} \simeq d \bar{r}_{K^*} \times 0.01 \simeq d \times 0.01. \]  (4.14)

In obtaining the first approximate relation in Eq.(4.14), we made the same assumption \( d + c \lesssim O(1) \) as used for evaluating the numerical value of \( g_{K^*K\pi} \). The second one follows by using the explicit value of \( \bar{r}_{K^*} \) in Table[II].

There is still an ambiguity due to the presence of factor containing \( d \). The value of \( d \) would be determined if we knew the mass of strange scalar \( \kappa \) because from Eq.(3.25)

\[ M_\kappa \simeq \sqrt{\frac{2}{d}} \times 984 \text{ MeV}. \]  (4.15)

However we only know that the strange scalar may be rather heavy. We require that the mass of \( \kappa \) be greater than 980 MeV. Also, following to the spirit of the effective Lagrangian approach, the mass of the particles contained explicitly in the Lagrangian should be less than the cutoff \( \Lambda \) of the theory (\( \Lambda \simeq 4\pi F_\pi \simeq 1.2 \text{ GeV} \)). These considerations lead to \( 980 \text{ MeV} \lesssim M_\kappa < \Lambda (\simeq 1.2 \text{ GeV}) \). This constraint to \( M_\kappa \) yields \( r_{K^*} - \bar{r}_{K^*} \simeq 0.02 \) through the Eqs.(1.14) and (4.15). Now we have

\[ F_+(0) \simeq \frac{1}{2} \left( \frac{F_K}{F_\pi} + \frac{F_\pi}{F_K} \right) \]  (4.16)

to the 2 % level. Then the value of \( \lambda_{e3} \) is determined which is shown in Table[IV].

V. EFFECTS OF EXTRA TERMS

In this section, we examine the effects of the operators in Eqs.(3.6) – (3.12) to the the results obtained in the previous two sections. We make a few
remarks before adding these operators to the reduced Lagrangian in Eq. (3.13) as extra terms.

First note that only operators in the Category 1 are allowed at the leading order of \(1/N_C\) expansion (\(N_C\) is the number of colours) if it is admitted as a proper approximation. This is because a trace in flavour space closes the flow of flavour so that we have one quark loop corresponding to one trace, while the number of quark loops must be one at the leading order of \(1/N_C\) [21]. Hence at this order only terms with just one trace are allowed. In this respect, the extra terms are non-leading.

The terms in which two \(\alpha_\parallel\)'s are contained in separate traces will induce the deviation from \(SU(3)\) nonet basis for vector mesons. Since we know that the ideal mixing of the octet and singlet vector mesons is fairly good experimentally, we do not consider such operators any more. On the other hand, \(\text{Tr}(D_\mu S)\text{Tr}(D^\mu S)\) (= \(\partial_\mu \text{Tr}(S)\partial^\mu \text{Tr}(S)\)) and the operators which contain two \(\alpha_\perp\)'s in different traces such as \(\text{Tr}(S\alpha_\perp^\mu)\text{Tr}(S\alpha_\perp^\mu)\) do not affect our interested quantities. Hence these operators will not be explicitly included in the renewed Lagrangian.

Now we discuss the possible effects to the charge radii of pseudoscalars and the linear slope of \(K_{e3}\) form factor, by the addition of the following Lagrangian to the reduced Lagrangian in Eq. (3.13):

\[
L_{\text{extra}} = \tilde{\delta}^{(0)}_\perp \text{Tr}(S\alpha_\perp^\mu \alpha_\perp^\mu)\text{Tr}(S) + \{ \tilde{\delta}^{(1)}_\perp \text{Tr}(S^2) + \tilde{\Delta}^{(1)}_\perp \text{Tr}(S)\text{Tr}(S) \} \text{Tr}(\alpha_\perp^\mu \alpha_\perp^\mu) \\
+ \tilde{\delta}^{(0)}_\parallel \text{Tr}(S\alpha_\parallel^\mu \alpha_\parallel^\mu)\text{Tr}(S) + \{ \tilde{\delta}^{(1)}_\parallel \text{Tr}(S^2) + \tilde{\Delta}^{(1)}_\parallel \text{Tr}(S)\text{Tr}(S) \} \text{Tr}(\alpha_\parallel^\mu \alpha_\parallel^\mu).
\] (5.1)

Each coefficient denoted by \(\tilde{\delta}\) includes a possible suppression factor associated with \(1/N_C\) expansion. The similar remark is also made for \(\tilde{\Delta}\), but with a doubled suppression factor.
From the modified Lagrangian, Eq. (5.1),

$$F_\pi^2 = 4(\Sigma_{1}^{(0)})^2 + \bar{\delta}^{(0)} \cdot \text{Tr}(\Sigma^{(0)}) \cdot \Sigma_{1}^{(0)} + T_\perp,$$

$$F_K^2 = (\Sigma_{1}^{(0)} + \Sigma_{3}^{(0)})^2 - f(\Sigma_{3}^{(0)} - \Sigma_{1}^{(0)})^2 + \frac{1}{2} \bar{\delta}^{(0)} \cdot \text{Tr}(\Sigma^{(0)})(\Sigma_{1}^{(0)} + \Sigma_{3}^{(0)}) + T_\perp,$$

(5.2)

where $T_\perp$ is defined as

$$T_\perp \equiv \bar{\delta}^{(1)} \cdot \text{Tr}(\Sigma^{(0)})^2 + \bar{\Delta}^{(1)} (\text{Tr}(\Sigma^{(0)}))^2.$$

(5.3)

The vector meson masses are also calculated to give

$$M_{\rho}^2 = M_{\omega}^2 = g_V^2 \left\{ 4a(\Sigma_{1}^{(0)})^2 + \bar{\delta}^{(0)} \cdot \text{Tr}(\Sigma^{(0)}) \cdot \Sigma_{1}^{(0)} + T_\parallel \right\},$$

$$M_{K^*}^2 = g_V^2 \left\{ a(\Sigma_{3}^{(0)} + \Sigma_{1}^{(0)})^2 + (d + 2c - b)(\Sigma_{3}^{(0)} - \Sigma_{1}^{(0)})^2 + \bar{\delta}^{(0)} \cdot \text{Tr}(\Sigma^{(0)}) \cdot \frac{\Sigma_{1}^{(0)} + \Sigma_{3}^{(0)}}{2} + T_\parallel \right\},$$

(5.4)

$$M_{\phi}^2 = g_V^2 \left\{ 4a(\Sigma_{3}^{(0)})^2 + \bar{\delta}^{(0)} \cdot \text{Tr}(\Sigma^{(0)}) \cdot \Sigma_{3}^{(0)} + T_\parallel \right\},$$

where

$$T_\parallel \equiv \bar{\delta}^{(1)} \cdot \text{Tr}(\Sigma^{(0)})^2 + \bar{\Delta}^{(1)} (\text{Tr}(\Sigma^{(0)}))^2.$$

(5.5)

If the large $N_C$ argument can be applied, the meaning of the VEV of scalar $S$ remains unchanged in this extension ($\Sigma_{1}^{(0)} \simeq \frac{F_\pi}{2}$). Therefore the discussion which asserts that higher dimensional operators are less important than four-dimensional operators in Sec. III makes sense even in this case.

By using these expressions we can directly check that there is no modification to the expressions in Eqs. (4.2), (4.4), (4.6), (4.7) and (4.8) by the presence of terms in Eq. (5.1). Therefore the charge radii are the same as those obtained according to the reduced Lagrangian. We can also calculate the $K_{e3}$ form factor and find that the result is the same as in Eq. (4.11). Hence Eqs. (4.13) and (4.14) for the normalization and the linear slope of $F_+(s)$ remain true. However, in this extended model, we cannot express $\kappa$ mass in the
form of Eqs. (3.25) and (4.15) (but Eq. (3.24) is true). Since Eq. (4.14) holds even in this case, $F_+(0) \simeq 1$ if we further assume that $d$ is at most of order unity.

VI. DISCUSSION AND SUMMARY

We have proposed a chiral Lagrangian (Eqs. (2.7) and (3.13)) with higher resonances (scalars and vectors), paying a close attention to the flavour $SU(3)$ breaking structure which shall be crucial for the description of $K$ decay. From the observation of Eq. (3.21) the resulting $SU(3)$ breaking structure in vector meson sector seems to be well incorporated into our Lagrangian. Our model constructed here includes not only $O(p^2)$ operators but also the kinetic term of vector meson which is $O(p^4)$ in the lowest order Lagrangian. Hence the quantities which requires higher-order terms can be calculated. Our challenge with the use of our model is to ask whether it can give sufficient predictions consistent with the experimental facts only by taking the flavour $SU(3)$ breaking structure into account, with only such an $O(p^4)$ term.

The first test of our model was performed by confronting its prediction for charge radii of pseudoscalars $\pi^+, K^+$ and $K^0$ with their experimentally obtained values. As a result, the predicted value for $\langle r^2 \rangle_{\pi^+}$ is found to be slightly smaller. Also the charge radius of $K^0$ becomes exactly zero.

These consequences does not change even if we add the extra terms in Eq. (5.1) as was shown in Sec. V. From this fact we can say that the $SU(3)$ breaking structure in the pseudoscalar-vector sector in our model are determined only by the chiral and hidden local symmetries.

We finally remark on the application of our Lagrangian to the actual calculation of $K$ decays. The most familiar framework for it will be the effective Hamiltonian method [22] with the factorization hypothesis. There the long
distance contribution from QCD is considered to reside in the hadronic matrix elements of four-Fermi operators and is expected to be calculated by using our chiral Lagrangian. The unreliability to the results obtained by following this approach comes from the strong dependence of them on the renormalization point which is conceptually the matching scale between short and long distance physics. Hence we must at first reexamine this point in order to give definite prediction form our chiral Lagrangian.

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[16] When we consider the process which occurs through the existence of chiral anomalies, we add the appropriate anomaly terms.

[17] From the symmetry argument, we have more terms for the quartic coupling of $S$ ( $\text{Tr}(S) \text{Tr}(S^3)$, $\{\text{Tr}(S)\}^2 \text{Tr}(S^2)$ and $\{\text{Tr}(S)\}^4$ ). Even if they are included in $L_{\text{pot}}$, all the results to be obtained in this paper do not change. Thus we will not be concerned with this matter here.

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TABLES

TABLE I.  \( g_V(V = \rho, \omega, \phi) \) coupling with \( g_{\rho\pi\pi} \) used as input through KSRF(I) relation (in unit [GeV²])

|      | Prediction       | Experiment      |
|------|------------------|-----------------|
| \( g_\rho \) | 0.103 ± 0.003    | 0.117 ± 0.003   |
| \( g_\omega \) | 0.034 ± 0.001    | 0.036 ± 0.001   |
| \( g_\phi \) | 0.084 ± 0.003    | 0.081 ± 0.002   |

TABLE II.  \( g_{\rho\pi\pi} \) coupling with \( g_{\rho\pi\pi} \) used as input

|      | Prediction       | Experiment      |
|------|------------------|-----------------|
| \( g_{\rho\pi\pi} \) | 6.00 ± 0.10(input) | 6.00 ± 0.10     |
| \( g_{\phi KK} \) | 6.98 ± 0.39      | 6.58 ± 0.22     |
| \( g_{K^*K\pi} \) | 6.55 ± 0.34      | 6.47 ± 0.11     |

TABLE III.  \( r_\rho, r_\phi \) and \( \bar{r}_{K^*} \)

|      |                 |                 |
|------|-----------------|-----------------|
| \( r_\rho \) | 1.028 ± 0.058   |                 |
| \( r_\phi \) | 0.859 ± 0.074   |                 |
| \( \bar{r}_{K^*} \) | 1.069 ± 0.048   |                 |
TABLE IV. charge radii of pseudoscalars and $\lambda_{e3}$ (charge radii are in unit [fm$^2$].)

|                | Prediction       | Experiment      |
|----------------|------------------|-----------------|
| $\langle r^2 \rangle_{\pi^+}$ | $0.401 \pm 0.030$ | $0.439 \pm 0.008$ |
| $\langle r^2 \rangle_{K^+}$  | $0.269 \pm 0.026$ | $0.28 \pm 0.07$  |
| $\langle r^2 \rangle_{K^0}$   | $0.00$           | $-0.054 \pm 0.026$ |
| $\lambda_{e3}$               | $0.026 \pm 0.004$ | $0.0286 \pm 0.0022$ |