Parity–violating electron scattering from the pion–correlated relativistic Fermi gas

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Abstract

Parity–violating quasielastic electron scattering is studied within the context of the relativistic Fermi gas and its extensions to include the effects of pionic correlations and meson–exchange currents. The work builds on previous studies using the same model; here the part of the parity–violating asymmetry that contains axial–vector hadronic currents is developed in detail using those previous studies and a link is provided to the transverse vector–isovector response. Various integrated observables are constructed from the differential asymmetry. These include an asymmetry averaged over the quasielastic peak, as well as the difference of the asymmetry integrated to the left and right of the peak — the latter is shown to be optimal for bringing out the nature of the pionic correlations. Special weighted integrals involving the differential asymmetry and electromagnetic cross section, based on the concepts of $g$–scaling and sum rules, are constructed and shown to be suited to studies of the single–nucleon form factor content in the problem, in particular, to determinations of the isovector/axial–vector and electric strangeness form factors. Comparisons are also made with recent predictions made on the basis of relativistic mean–field theory.

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1 Introduction

In this paper we investigate the theory behind the inclusive scattering of longitudinally polarized electrons from nuclei in the kinematical regime of the quasielastic peak (QEP). Our specific purpose is that of studying the parity-violating (PV) effects arising from the interference between the neutral weak and electromagnetic currents which can be explored through measurement of the helicity asymmetry. This is a theme that has recently received considerable attention theoretically [1-8] and promises to be an active theme experimentally as well (see, for example, ref. [9] for a review covering past, present and future experimental perspectives, together with discussions of the connected theoretical issues).

Notwithstanding the fact that such PV experiments are difficult to carry out, requiring in particular high luminosity, high electron polarization, frequently specialized detectors and great care in controlling systematic errors, the results obtained heretofore are quite promising and lead one to expect that the new generation of studies planned at MIT/Bates, Mainz and CEBAF will bring about significant improvements in our knowledge of subtle and new aspects of nuclear and nucleonic physics. In addition, given sufficiently fine information on such hadronic physics issues, one might even expect that accurate tests of the standard model in an energy domain far away from the one explored with high-energy accelerators and yet complementary to atomic PV studies [1] will be achieved.

In the present work we draw upon some of our past studies of quasielastic electron scattering [3, 10, 11], especially on recent work [8] in which the electromagnetic (EM) and weak neutral current (NC) vector responses were explored in detail. The current study extends that work now to include in-depth discussion of the axial-vector response and combines the results of all of our nuclear physics modeling to yield predictions for the PV asymmetry.

We focus our attention on the QEP energy region not only because here the cross sections are large enough to yield significant figures-of-merit and hence measurable asymmetries, as discussed in ref. [3], but especially because this region is a (perhaps unique) one in which it should prove possible to unfold the two facets of the reaction that we wish to explore, namely those involving nuclear and nucleonic structure and dynamics. For it is in the QEP region that the process is essentially “quasi-free”, implying that the nuclear structure effects are not overwhelming and suggesting that studies of nucleonic physics (axial–vector and possibly strangeness form factors of the nucleon, in particular) can be undertaken. Our study spans a range of momentum transfers extending roughly from 300 MeV/c up to 1 GeV/c. Below 300 MeV/c we do not trust the relativistic Fermi gas (RFG) as a reliable model for describing nuclear phenomena; for momenta larger than 1 GeV/c severe difficulties are met in fulfilling Lorentz and gauge invariance in the extended model. In addition, here at large momentum and energy transfers $\Delta$ and $N^*$ production become hard to distinguish from quasielastic scattering and the associated reaction mechanism is

2
not yet sufficiently under control to allow for a reliable extraction of the single-nucleon form factors.

For the inputs needed to calculate the PV asymmetry, namely the electromagnetic and the weak neutral current nuclear responses, we draw, as already mentioned, on our past work [8]. Indeed in ref. [8] the nuclear responses have been calculated within the framework of the covariant RFG model — of course, this model, while representing a good starting point for nuclear structure studies, needs to be improved upon if it is to be brought into closer touch with physical reality. With this in mind, the basic model was extended in ref. [8] by adding to the RFG the pion in its dual role of force and current carrier through the incorporation of selected classes of perturbative diagrams, always taking great care to respect gauge invariance. While we believe that it is important to pursue the idea of employing baryons and mesons as effective degrees of freedom in describing nuclear phenomena, at least in some limited, intermediate–energy kinematical domain (in fact, an \textit{ab initio} treatment in terms of more fundamental degrees of freedom, namely quarks and gluons, while desirable is as yet beyond reach in this strong–coupling regime), the restriction of considering the pion alone among the mesons is certainly questionable. Yet we also believe, and in this respect we have, hopefully convincingly, argued in ref. [8], that the pion at least plays an important role in the QEP region and can be consistently treated at the level of both currents and forces. Importantly its impact on the nuclear responses is usually only strongly felt for not too large momentum transfers, defining the critical value of momentum transfer beyond which such effects become only modest corrections to the RFG predictions, at least for some of the responses. Not all observables behave this way, however, and as discussed in ref. [8] the PV longitudinal response in particular, is dramatically modified by pionic isospin–dependent correlations, producing detectable consequences for the asymmetry and thus yielding a potentially useful window on nuclear dynamics. This circumstance is exceptional and when it comes to addressing instead the physics of the nucleon in the nuclear medium, our previous studies indicate that the pionic correlations are sufficiently well understood to allow us to select with good confidence the kinematical domains where nucleonic physics inside nuclei can be safely addressed; that is to say, it is possible to find situations where the contamination arising from nuclear structure effects is relatively small, at least as far as this can be ascribed to pions.

In line with the above considerations, the plan of this paper is the following. In sect. 2 we summarize the basic formalism involved in PV electron scattering and begin our discussion of the axial–vector PV response (which was not dealt with in ref. [8]), exploiting for this purpose its close relationship with the isovector transverse EM response in leading order in the non–relativistic reduction. In sect. 3 we address the question of the impact of pionic correlations on the inclusive scattering of polarized electrons from nuclei. This issue appears to be explored best through certain frequency integrals of the asymmetry which are introduced in sects. 3 and 4 as they are needed. A particularly important one of these discussed in sect. 3 has
the range of integration divided into two parts, to the left– and right–hand sides of the QEP: the right–hand integral is then subtracted from the left–hand integral to form what is called $\Delta A$. As we shall see, this observable has the property of emphasizing the role of the (pionic) correlations in nuclear matter while minimizing the effects of the single–nucleon form factors. In sect. 4 several other integrated quantities are introduced. First, in sect. 4.1 the energy–averaged asymmetry $\overline{A}$ is considered — this has the merit of suppressing the pionic correlation effects while bringing out the dependences on the single–nucleon form factors, importantly, on the axial–vector form factor. However, the degree to which this is accomplished can be improved upon and, to this end, in sect. 4.2 we introduce two additional quantities called $R_1$ and $R_2$ based on previous treatments of scaling [10, 12] and sum rules [10, 13]. As we shall see in sect. 4.2, these observables will permit us to extract information on the single–nucleon axial–vector form factor largely without uncertainties from nuclear correlation effects and from meson–exchange current (MEC) corrections. Indeed, in quantifying the uncertainties from the former we consider not only the pionic correlation effects discussed in ref. [8] but also those treated by Horowitz et al. in ref. [7] and find that at large scattering angles where the axial–vector response has its greatest influence on the asymmetry in both cases the nuclear–physics–based uncertainties appear to be well under control. Finally, in sect. 4 we also consider the roles played by electric and magnetic single–nucleon strangeness form factors and then end with our conclusions in sect. 5.

2 Parity–violating electron scattering

The observable that plays the central role in PV electron scattering is the asymmetry, defined as

$$A = \frac{d^2\sigma^+ - d^2\sigma^-}{d^2\sigma^+ + d^2\sigma^-},$$

where $d^2\sigma^+$ and $d^2\sigma^-$ indicate the nuclear double–differential cross sections for the scattering of right– and left–handed longitudinally polarized electrons. The asymmetry can be cast in the following form [1, 2, 3, 5, 9]

$$A = A_0 \frac{v_L R_{AV}^L (q, \omega) + v_T R_{AV}^T (q, \omega) + v_T' R_{AV}^T (q, \omega)}{v_L R^L (q, \omega) + v_T R^T (q, \omega)},$$

where

$$A_0 = \frac{G |Q^2|}{2\pi \alpha \sqrt{2}},$$

having introduced the lepton kinematical factors [14, 15]

$$v_L = \left( \frac{Q^2}{q^2} \right)^2$$
\[ v_T = \frac{1}{2} \frac{Q^2}{q^2} + \tan^2 \frac{\theta}{2} \]  \hspace{1cm} (4)

\[ v_{T'} = \sqrt{\frac{Q^2}{q^2} + \frac{1}{2} \tan \frac{\theta}{2} \tan \frac{\theta}{2}}. \]

In the above formulae \( q = |q| \) is the transferred momentum, \( \omega \) the transferred energy, \( Q^\mu = (\omega, q) \), \( \alpha \) the fine-structure constant, \( G \) the Fermi constant and \( \theta \) the scattering angle. In (2) \( R^L \) and \( R^T \) are the usual (purely vector) EM longitudinal and transverse response functions, respectively. The responses \( R^L_{AV}, R^T_{AV} \) and \( R^T_{VA} \) all involve interferences between the EM and NC hadronic matrix elements; the two labeled \( AV \) arise from leptonic axial–vector/hadronic vector contributions, whereas the one labeled \( VA \) arises from the reverse; the two labeled \( L \) and \( T \) contain only purely vector hadronic currents (longitudinal and transverse projections, respectively), whereas the one labeled \( T' \) arises from an interference between the vector EM current and the axial–vector part of the NC and is purely transverse, although of a different class than the responses labeled \( T \) (see refs. [14, 15]). For brevity we shall refer to these last three PV responses as PV longitudinal, PV transverse and axial–vector, respectively.

Before exploring how the asymmetry is affected by nuclear correlations (or by nucleonic form factors) we briefly comment on the level of accuracy that one can expect to attain in measuring the asymmetry itself. We start by noting that the statistical precision that one can achieve in determining \( A \) with a PV helicity–difference experiment is given by

\[ \frac{\delta A_{\text{expt}}}{A_{\text{expt}}} = \left[ \frac{p_e^2 \mathcal{L} T}{\sigma} \left( \frac{A\sigma}{\bar{A}\bar{\sigma}} \right)^2 \right]^{-\frac{1}{2}}, \]  \hspace{1cm} (5)

where \( p_e \) is the (longitudinal) polarization of the incident electron, \( \mathcal{L} \) the luminosity and \( T \) the runtime. Here \( \sigma \) and \( \bar{\sigma} \) are integrals of the EM cross section and that cross section times the asymmetry, respectively, over the angular and energy acceptances in the experiment, as well as over any range in \( \omega \) that we wish to consider (see below). We shall be discussing the energy–averaged asymmetry \( \bar{A} \) in sect. 4.1 and, to the extent that \( \bar{A}\sigma \approx \bar{A} \times \bar{\sigma} \), one obtains

\[ \frac{\delta A_{\text{expt}}}{A_{\text{expt}}} = \left[ \frac{p_e^2 \mathcal{L} T}{\bar{F}} \right]^{-\frac{1}{2}}, \]  \hspace{1cm} (6)

involving the average figure–of–merit \( \bar{F} \equiv \bar{\sigma} \times \bar{A}^2 \). In ref. [5] we have shown that by choosing for these quantities values that are perhaps presently a little optimistic, but likely to be attained in the not–too–distant future, and by extending the integrations over \( \omega \) to include the whole QEP, a precision of even 1% can be contemplated for not too large \( q \).
An issue that must be dealt with in studying PV quasielastic electron scattering is the question of how properly to select the range of the integrations above. One must generally avoid the very low and the very high energy tails of the quasielastic spectrum in order, on the one hand, not to have to deal with specific inelastic nuclear excitations for which a description using the RFG is ill-suited, while on the other, not to become entangled with pion production. These requirements (in fact the first one is not so serious when an energy integration is involved) are clearly met by not considering momentum transfers that are too small or too large. It is also clear that an integration encompassing most of the QEP leads to the best statistical precision for $A$, although, as we shall see in the discussions to follow, in future studies it might become desirable to explore in some detail the $\omega$–dependence of the asymmetry, accepting the unavoidable sacrifice in statistical precision that this entails.

On the one hand, in sect. 3 we shall investigate the roles played by pionic correlations and MEC contributions and so it is useful to recall [8] that, while the pionic MEC generally play a minor role in the QEP region, the correlations brought in by the pion are dominated by the so-called exchange term, with a characteristic oscillatory behaviour as a function of $\omega$. Furthermore, the self–energy contribution, although of minor importance, also displays the same behaviour. As a consequence, if the focus is on the correlations, then it appears that the appropriate procedure is to split the energy integral in two parts — from the low–energy end–point to $\omega = \omega_{QEP} \equiv |Q^2|/2m_N$ and from there to the high–energy end–point. Then, by subtracting the two contributions, it is clear that an observable is obtained which maximally emphasizes the impact of the correlations on the asymmetry. This yields the quantity $\Delta A$ discussed in sect. 3. It is possible to show that the precision to be expected for $\Delta A$ is much less than for $A$ (or for the other observables $R_{1,2}$ employed in sect. 4.2 which could be determined to the same precision as $A_{\text{expt}}$ or $A$):

$$\left| \frac{\delta \Delta A}{\Delta A} \right| \approx \left| \frac{2A}{\Delta A} \right| \times \left| \frac{\delta A}{A} \right|.$$  

(7)

Since $|2A/\Delta A|$ ranges from about 1–7 in the forward direction and 10–20 in the backward direction as $q$ goes from 300 MeV/c to 1 GeV/c, the fractional precision in $\Delta A$ will typically be around a few % in the forward direction and 10–20% in the backward direction at best. Although this is much poorer than for the integrated quantities discussed below, it nevertheless will be shown in sect. 3 to be good enough to provide an interesting window on the pionic physics issues.

On the other hand, in sect. 4 we shall address the issue of the sensitivity in the PV asymmetry to variations in the isovector/axial–vector and magnetic and electric strangeness form factors of the nucleon (see sect. 2.1). For this purpose we wish to suppress the sensitivity to pionic correlation and MEC effects in order to reveal the dependences on the form factors. Motivated by concepts of $y$–scaling and electroweak sum rules where these effects can be de–emphasized, accordingly
we shall perform various integrals involving the differential asymmetry and the EM cross section (usually with specific weighting factors) to define new observables, $\mathcal{A}$ in sect. 4.1 and $\mathcal{R}_{1,2}$ in sect. 4.2. There the integrations will extend over the entire region of the quasielastic response, even though, in the future in the context of a particular experiment, it will probably be necessary to address the problem of pion production in the high-$\omega$ part of this region.

2.1 Single–nucleon form factors

As has already been mentioned above, the responses appearing in (2) have been calculated in a companion paper [8] for a pion–correlated RFG, except for the axial–vector response $R_{V,A}^{T}$ which will be addressed in the next subsection. For this purpose, we start by recalling that the nucleonic electromagnetic and weak neutral vector ($V$) and axial–vector ($A$) currents read

$$ J_{\mu}^{\text{EM}}(Q,P) = \bar{u}(P+Q,s') \left[ F_{1}(Q^2)\gamma^{\mu} + i \frac{F_{2}(Q^2)}{2m_{N}} \sigma^{\mu\nu} Q_{\nu} \right] u(P,s) $$

$$ J_{\mu}^{\text{NC,V}}(Q,P) = \bar{u}(P+Q,s') \left[ \tilde{F}_{1}(Q^2)\gamma^{\mu} + i \frac{\tilde{F}_{2}(Q^2)}{2m_{N}} \sigma^{\mu\nu} Q_{\nu} \right] u(P,s) $$

$$ J_{\mu}^{\text{NC,A}}(Q,P) = \bar{u}(P+Q,s') \tilde{G}_{A}(Q^2) \gamma^{5}\gamma^{\mu} u(P,s) $$

where $s$ and $s'$ are the third components of the nucleon spin, $F_{1}$ and $F_{2}$ the standard Dirac and Pauli nucleon form factors (and $\tilde{F}_{1,2}$ their vector NC analogs), $\tilde{G}_{A}$ the axial–vector NC form factor and $P^\mu = (E_\mathbf{p}, \mathbf{p})$ the on–shell nucleon's four–momentum in the Fermi sphere. In fact, it is common practice in EM studies of the nucleus to use the Sachs [16] electric and magnetic form factors $G_{E}$ and $G_{M}$ rather than $F_{1}$ and $F_{2}$: $G_{E} = F_{1} - \tau F_{2}$ and $G_{M} = F_{1} + F_{2}$, with $\tau \equiv |Q^2|/4m_{N}^2$. Analogous expressions can, of course, be written for NC Sachs–like form factors $\tilde{G}_{E}$ and $\tilde{G}_{M}$.

All form factors in addition can be labeled with $p$ or $n$ to denote which are for the proton and which for the neutron, respectively, or with $T = 0, 1$ to denote which are isoscalar or isovector, respectively.

For the convenience of the reader and for sake of completeness we briefly summarize the parameterization of the single–nucleon form factors employed in the present research. For a broader discussion of the subject we refer the reader to refs. [3, 4]. For the electric and magnetic nucleonic EM form factors we have used the standard forms

$$ G_{Ep}(\tau) = G_{DV}^{p}(\tau) $$

$$ G_{Mp}(\tau) = \mu_{p}G_{DV}^{V}(\tau) $$

$$ G_{Mn}(\tau) = \mu_{n}G_{DV}^{V}(\tau) $$

$$ G_{En}(\tau) = -\mu_{n}\tau G_{DV}^{V}(\tau)\xi_{n}(\tau) $$

7
where $G^V_D(\tau) = \left[1 + \lambda^V_D \tau\right]^{-2}$ is the vector dipole form factor with $\lambda^V_D \approx 4.97$ and where $\mu_p \approx 2.793$ and $\mu_n \approx -1.913$ are the proton and neutron magnetic moments, respectively. Formula (14), with $\xi_n(\tau) = \left[1 + \lambda_n \tau\right]^{-1}$ and $\lambda_n = 5.6$, is usually referred to as the Galster parameterization [7]. For the isovector weak axial–vector form factor we use

$$G_A^{(1)}(\tau) = g_A^{(1)} G_D^A(\tau) \ ,$$

(15)

where $G_D^A(\tau) = \left[1 + \lambda_D^A \tau\right]^{-2}$ is the axial–vector dipole form factor, $g_A^{(1)} = 1.26$ from neutron $\beta$–decay and we have taken $\lambda_D^A \approx 3.53$. The NC (strong isospin 1 channel = isovector) form factors are then

$$\tilde{G}_E^{(1)}(\tau) = \beta_V^{(1)} G_E^{(1)} \ ,$$

(16)

$$\tilde{G}_M^{(1)}(\tau) = \beta_V^{(1)} G_M^{(1)} \ ,$$

(17)

$$\tilde{G}_A^{(1)}(\tau) = \beta_A^{(1)} G_A^{(1)} \ .$$

(18)

In the standard model the tree–level electroweak hadronic couplings are given by $\beta_V^{(1)} = 1 - 2 \sin^2 \theta_W$, $\beta_A^{(1)} = 1$ (isovector), and for use below, $\beta_V^{(0)} = -2 \sin^2 \theta_W$, $\beta_A^{(0)} = 0$ (isoscalar), using the notation of ref. [8]. Here $\theta_W$ is the weak mixing angle ($\sin^2 \theta_W \approx 0.227$).

Since we shall also be concerned with the strangeness content of the nucleon, we have introduced in the NC (strong isospin 0 channel = isoscalar) form factors

$$\tilde{G}_E^{(0)}(\tau) = \beta_V^{(0)} G_E^{(0)} - G_E^{(s)} \ ,$$

(19)

$$\tilde{G}_M^{(0)}(\tau) = \beta_V^{(0)} G_M^{(0)} - G_M^{(s)} \ ,$$

(20)

$$\tilde{G}_A^{(0)}(\tau) = \beta_A^{(0)} G_A^{(0)} - G_A^{(s)} \ ,$$

(21)

the following purely isoscalar strangeness form factors:

$$G_E^{(s)}(\tau) = \rho_s \tau G_D^V(\tau) \xi_E^{(s)}(\tau) \ ,$$

(22)

$$G_M^{(s)}(\tau) = \mu_s G_D^V(\tau)$$

(23)

$$G_A^{(s)}(\tau) = g_A^{(s)} G_D^A(\tau) \ ,$$

(24)

where again $G_D^V(\tau)$ and $G_D^A(\tau)$ are the dipole form factors used above and where $\xi_E^{(s)} \equiv \left[1 + \lambda_E^{(s)} \tau\right]^{-1}$. As an orientation, we shall let the strength of the electric and magnetic strangeness form factors vary in the ranges [3, 4]

$$\rho_s: 0 \rightarrow -3 \ \ \ \mu_s: 0 \rightarrow -1 \ ,$$

(25)

whereas for $\lambda_E^{(s)}$ we shall consider the two options $\lambda_E^{(s)} = 0$ and $\lambda_E^{(s)} = \lambda_n = 5.6$. Finally, in our analysis we shall set $g_A^{(s)} = 0$, since it plays only a minor role in quasielastic electron scattering. Note that these expressions have been stated at
the tree level; beyond tree level we may allow for additional contributions in the form $g_A^{(1)} \rightarrow g_A^{(1)} [1 + R_A^{(1)}]$, etc. to take into account radiative corrections (this is the approach followed in refs. [3, 4]) or we may consider the above choices for the parameters $\{g_A^{(1)} \ldots\}$ to be starting approximations in fixing our parameterizations of the effective form factors that arise when higher–order electroweak corrections are taken into account. Of course, the effective form factors then become process–dependent. Specifically, from modeling of such contributions [19], one expects that the effective $g_A^{(1)}$ could differ from 1.26 by as much as $\pm 20\%$. Indeed, putting the burden on the strength of the form factors is not the most general way of effectivizing the problem: for example, the $|Q^2|$–dependences in many cases are not known or are only known with limited precision. However, our interest in the present work when any aspect of the single–nucleon form factors is concerned is to explore whether or not variations ranging from a few $\%$ to as much as 10$\%$ away from the starting parameterizations would be manifest in specific observables. Whether those variations occur because of higher–order electroweak contributions or are due to differences in the $|Q^2|$–dependences, the relevant issue to address here is the level of sensitivity in quasielastic PV electron scattering to such variations.

### 2.2 The axial–vector response

Following refs. [3, 8, 10, 11], the above currents are to be inserted into the so–called hadronic interference tensor for the RFG

$$W_{\mu\nu}^{EM/NC} = \frac{3N m_N^2}{4\pi p_F^2} \int \frac{d^3p}{E(p) E(p + q)} \delta \{\omega - [E(p + q) - E(p)]\} \times \theta(p_F - |p|) \theta(|p + q| - p_F) f_{\mu\nu}^{EM/NC}(P + Q, P),$$

(26)

where $p_F$ is the Fermi momentum, $E(p) = [p^2 + m_N^2]^{1/2}$ the energy of a nucleon with momentum $p$, $N$ the particle number ($Z$ for a proton gas, $N$ for a neutron one) and

$$f_{EM/NC}^{\mu\nu}(P + Q, P) = \frac{1}{2} \text{Tr} \left[ J_{\mu}^{EM}(Q, P) J_{\nu,\lambda}^{\dagger}(Q, P) + J_{\mu,\lambda}^{NC,A}(Q, P) J_{\nu}^{\dagger}(Q, P) \right],$$

(27)

the single–nucleon interference tensor (i.e., only the part which contains axial–vector hadronic currents; the corresponding vector contributions have been discussed in ref. [8]). Then in symmetric nuclear matter ($N = Z = N = A/2$) one obtains the axial–vector, transverse, purely isovector PV nuclear response:

$$R_{VA}^{T} = -ia_V W_{\mu\nu}^{12,EM/NC}$$

$$= a_V \frac{3N}{4m_N^2 k_F^2} (\epsilon_F - \Gamma) \theta(\epsilon_F - \Gamma) \sqrt{\tau(1 + \tau)} G_M^{(1)}(\tau) G_A^{(1)}(\tau) \left\{1 + \tilde{\Delta}\right\},$$

(28)

1Actually the standard model beyond the tree level allows for an isoscalar component of the axial–vector current of the nucleon (see (21) above), which we shall disregard in the present study.
\[ a_V = -(1 - 4\sin^2\theta_W) \] being the standard model leptonic vector coupling constant (for completeness, \( a_A = -1 \) is the standard model leptonic axial–vector coupling constant). In (28) we employ the functions [3, 10]

\[ \Gamma(q, \omega) \equiv \max \left\{ (\epsilon_F - 2\lambda), \kappa \sqrt{1 + \frac{1}{\tau} - \lambda} \right\} \] (29)

and

\[ \bar{\Delta} \equiv \frac{1}{\kappa} \sqrt{\frac{\tau}{1 + \tau}} \left\{ \frac{1}{2}(\epsilon_F + \Gamma) + \lambda \right\} - 1 \] . (30)

having introduced the dimensionless variables:

\[ \begin{align*}
\kappa & \equiv \frac{q}{2m_N} \\
\lambda & \equiv \frac{\omega}{2m_N}
\end{align*} \] \[ \implies \tau = \kappa^2 - \lambda^2 \] (31)

\[ \begin{align*}
\eta_F & \equiv \frac{p_F}{m_N} \\
\varepsilon_F & \equiv \frac{E(\mathbf{k}_F)}{m_N} = \sqrt{1 + \eta_F^2}.
\end{align*} \]

Next we consider the leading order of the non–relativistic expansion of the space components of the electromagnetic and weak axial–vector currents. They read

\[ \begin{align*}
\mathbf{J}_{EM} & \approx -iG_M\chi_s^\dagger \frac{\left(\mathbf{\sigma} \times \mathbf{q}\right)}{2m_N}\chi_s \\
\mathbf{J}_{NC,A} & \approx -\bar{G}_A\chi_s^\dagger \mathbf{\sigma} \chi_s,
\end{align*} \] (32) (33)

where \( \chi_s \) is a 2–component (spin) spinor. By inserting these expressions in (27), one obtains for the space components of the single–nucleon interference hadronic tensor

\[ f_{ij}^{EM/NC} = \frac{i}{m_N}G_M\bar{G}_A\epsilon_{ijk}q_k . \] (34)

On the same footing, by calculating \( f_{ij}^{EM} \) with (32), one gets

\[ f_{ij}^{EM} = \frac{1}{4m_N^2}G_M^2(q^2\delta_{ij} - q_iq_j), \] (35)

which would yield, when embedded in the hadronic EM tensor, the non–relativistic expression for the transverse EM response according to

\[ R^T = W_{EM}^{11} + W_{EM}^{22} . \] (36)

Notably, it then turns out that the isovector component of the latter, \( R^{T(1)} \), and the PV axial–vector response in the leading order of the non–relativistic expansion (obtained via (32) and (33)) are exactly connected through the simple formula

\[ R_{VA}(q, \omega) = a_V \frac{G_A^{(1)} G_M^{(1)}}{\kappa} R^{T(1)}(q, \omega) . \] (37)
Actually, by comparing the exact expressions for \( R^T \) and \( R^T' \) (see ref. [3]), the link (37) can be extended into the relativistic regime. Indeed, it turns out that the prescription

\[
R^T_{VA}(q, \omega) \simeq a_V \frac{G_A^{(1)}}{G_M^{(1)}} \sqrt{\frac{T+1}{\tau}} R^T(q, \omega) \tag{38}
\]

holds to better than 2% in the momentum range 300 MeV/c < \( q < 1 \) GeV/c.

In Fig. 1 we show the response \( R^T_{VA} \) for the free RFG (dashed curve) and for the model with pionic correlations and MEC effects included (solid curve) and so complete the picture of the five electroweak responses that we began in ref. [8]. We see a “hardening” (a shift to higher \( \omega \)) of the peak of the response at intermediate values of \( q \) which then fades away and even leads to a slight “softening” of the response at the highest momentum transfers considered. In computing the response with pionic effects included we have used (38) to provide a link between the response \( R^T(q, \omega) \) obtained previously, even though that relationship was derived for the free RFG. When one introduces such effects in \( R^T_{VA} \) the question arises whether this axial–vector/transverse EM link holds as well in this instance. In this connection an important simplifying result holds: by performing the appropriate spin algebra, one easily sees that in the leading order of the non–relativistic expansion for any Feynman diagram at whatever order one always has (schematically)

\[
R^T_{VA} \sim a_V (\sigma \times q)_1 \sigma_1 + (\sigma \times q)_2 \sigma_2 \sigma_2 \text{ and } R^T \sim (\sigma \times q)_1 \sigma_1 + (\sigma \times q)_2 \sigma_2 \sigma_2, \text{ so that (37) is an exact relation to all orders. Since the prescription given in our previous work to get approximate relativistic response functions from their non–relativistic counterparts does not involve their spin structure, the validity of (38) in the relativistic regime is then inferred. The door is thus open for the calculation of the nuclear axial–vector response to the level of accuracy of (38), since the necessary ingredients, namely the various contributions to the electromagnetic isovector transverse nuclear response, have been calculated in the work reported in ref. [8]. One caveat should, however, be added to the above considerations: because of the isovector, transverse nature of the axial–vector response, the MEC contributions of the type shown in Fig. 2, when the axial–vector part of the \( Z^0 \) coupling is involved, are of higher order in the non–relativistic expansion (Kubodera–Delorme–Rho theorem [20]) and have accordingly been disregarded. At high enough energy/momentum in future work it may be necessary to re-examine this approximation in more depth. Of course, as discussed in ref. [8], when the vector part of the \( Z^0 \) coupling is involved such MEC effects are taken into account in the present work.
3 Pionic correlations in the asymmetry

In accordance with the arguments given in the previous section here we discuss the observable

\[ \Delta A(q, \theta) \equiv \frac{1}{\Delta \omega} \left[ \int_{\omega_{\text{min}}}^{\omega_{\text{QEP}}} d\omega A(\theta; q, \omega) - \int_{\omega_{\text{QEP}}}^{\omega_{\text{max}}} d\omega A(\theta; q, \omega) \right], \]  

(39)

where \( \omega_{\text{min}} \) and \( \omega_{\text{max}} \) are the RFG response boundaries for a fixed \( q \),

\[ \omega_{\text{min}} = \sqrt{(p_F - q)^2 + m_N^2} - \sqrt{p_F^2 + m^2_N} \]  

(40)

\[ \omega_{\text{max}} = \sqrt{(p_F + q)^2 + m_N^2} - \sqrt{p_F^2 + m^2_N} \]  

(41)

and \( \omega_{\text{QEP}} = |Q^2|/2m_N \), as above. We shall make use of the energy interval

\[ \Delta \omega \equiv \omega_{\text{max}} - \omega_{\text{min}} \]  

(42)

\[ \approx 2p_Fq/\sqrt{q^2 + m^2_N} \]  

(43)

and so define it here. The expressions given here pertain in the case where \( q > 2p_F \); of course, when results are presented for \( q < 2p_F \) (the Pauli–blocked region) the correct equations are used.

In the present paper we have calculated \( \Delta A \) taking into account all Feynman diagrams with one pion line: these include the self–energy, the exchange and the MEC contributions, all of which have been extensively dealt with in our past work. For sake of illustration we have set the Fermi momentum \( p_F = 225 \text{ MeV/c} \), which roughly corresponds to a light nucleus such as \( ^{12}\text{C} \).

That the pionic correlations are particularly felt by \( \Delta A \), as previously anticipated, is clearly apparent from Fig. 3. Indeed, there we first observe that at \( q = 300 \text{ MeV/c} \) in the case of results obtained with the free relativistic Fermi gas (labeled RFG) \( \Delta A_{\text{RFG}} \) almost vanishes because of the nearly perfect cancellation between the contributions where \( \omega < \omega_{\text{QEP}} \) and those where \( \omega > \omega_{\text{QEP}} \); however, at larger \( q \) this cancellation becomes less complete, owing partly to the role played by the nucleonic form factors and partly to the RFG model itself, whose responses (in contrast to the non–relativistic case) become less and less symmetric as \( q \) increases. It is also clear from Fig. 3 that the correlations, in particular the exchange contributions, dramatically alter the prediction of the free RFG, yielding a huge \( \Delta A_\pi \) at small \( \theta \).

This outcome is simply interpreted: of the nuclear responses that enter in the asymmetry the pion has its greatest effect on \( R_{\text{AV}}^L \) (see ref. 8) and although the latter in the RFG model accounts only for at most about 10% of the total asymmetry (and this only in the forward direction), nevertheless the impact of the pionic correlations is violent enough to induce a large negative value of \( R_{\text{AV}}^L \) at small \( \omega \), which is in turn reflected in the large negative value of \( \Delta A_\pi \) at small \( \theta \) displayed in Fig. 3. We
deduce from this that the characteristic behaviour of $\Delta A$ with $\theta$ shown in the figure represents one of the most transparent signatures of pion–induced isoscalar correlations in nuclei (we recall that $R_{VA}^T$ is purely isovector and that in $R_{AV}^T$ the isoscalar contribution is strongly suppressed — see ref. [8]). We return to the families of curves in Fig. 3 in the discussions below.

Let us next examine the $\omega$–dependence of the asymmetry. Looking first at Fig. 4, we notice the significant effect occurring at moderate $q$ (say 300–500 MeV/c), small $\omega$ and forward angles that is responsible for the striking behaviour of $\Delta A$ previously commented upon. As discussed above, it is related to the large negative value assumed by the correlated $R_{AV}^L$, which leads to a pionic asymmetry that is an order–of–magnitude larger than the free RFG one. However, as $\omega$ increases $R_{AV}^L$ rapidly decreases until it changes sign, while $R_{AV}^T$ stays negative: accordingly, they largely cancel in the numerator of the ratio expressing $A$ and this becomes substantially lowered.

Interestingly, an energy is reached (about 60 MeV for $q = 300$ MeV/c) where the correlated and free RFG values of $A$ coincide. At still larger $\omega$ a further reduction of $A$ is seen to occur until, at about 90 MeV, it nearly vanishes. This constitutes an example of a dynamical restoration of a symmetry (here the left–right parity symmetry) and reflects the complex nature of the PV longitudinal response. Indeed, for the free RFG

$$R_{AV}^L = a_A \left[ \beta_V^{(0)} R_{EM}^L + \beta_V^{(1)} R_{EM}^L \right]$$

(45)

is very small owing to the internal fight between its isoscalar and isovector components brought about by the opposite signs and similar magnitudes of the standard model hadronic couplings, $\beta_V^{(0)} \approx -0.45$ and $\beta_V^{(1)} \approx 0.55$, as discussed in ref. [3]. In fact, the free RFG response $R_{AV}^L$ is negative at $q = 300$ MeV/c, becomes positive at 500 MeV/c and then stays so up to very large momenta, always remaining quite small in magnitude. When pionic correlations are switched on, this behaviour is profoundly altered. In particular, the magnitude of $R_{AV}^L$ is much increased, becoming comparable in magnitude to, for example, the EM response $R^L$. In addition, for a given moderate value of $q$ the response $R_{AV}^L$ changes sign at some $\omega$. The near vanishing of the asymmetry at $\omega \approx 90$ MeV in Fig. 4 thus stems from the cancellation between the positive contribution it gets from $R_{AV}^L$ and the negative one it gets from $R_{AV}^T$. This trend of course fades away at larger $\theta$, where the role of the longitudinal PV response gradually becomes irrelevant. Finally, from Fig. 4, we see that at larger momenta, where the impact of correlations is no longer so strongly felt, the nearly perfect restoration of the left–right symmetry does not show up anymore. It is, however, still true (even at 1 GeV/c) that an energy exists where the free and the correlated values of the asymmetry coincide.

We now conclude this section with a brief discussion of the influence of the nucleonic form factors on the asymmetry. As previously stated, $\Delta A$ has been specifically devised to enhance the signal for nuclear correlations. That this is indeed the case can be inferred from Figs. 3 and 4 where we display results allowing for a variation
of the strength $g_{A}^{(1)}$ of the isovector axial–vector form factor of $\pm 10\%$ around the canonical value $g_{A}^{(1)} = 1.26$ (Cf. the discussion on the uncertainty in the effective axial–vector form factor in the previous section). As one can see from Fig. 3, $\Delta A$ is totally insensitive to this variation of $g_{A}^{(1)}$ at low momenta; it becomes mildly so at larger momenta, but even in this case the impact on $\Delta A$ of pionic correlations remains an order–of–magnitude larger than the that arising from variations in the axial–vector form factor. Note that an angle $\Theta^{-}$ exists ($\Theta^{-} \sim 110^\circ$, relatively independent of $q$) such that for $500 \text{ MeV/c} < q < 1 \text{ GeV/c}$ the free and correlated $\Delta A$ coincide. A compensation is thus seen to occur inside $\Delta A$ among the transverse, axial–vector and longitudinal response functions in the two cases. In Fig. 4, where the $\omega$–dependence of the asymmetry is displayed, we again see the influence of variations in the effective axial–vector form factor: clearly the effects are greatest at large angles, where $R_{TVA}$ is weighted most heavily.

Finally, in Figs. 5 and 6 we show results like those above, but now for variations of the magnetic (Fig. 5) and electric (Fig. 6) strangeness form factors of the nucleon. We have limited our focus to $q = 500 \text{ MeV/c}$ and to rather forward and backward angles, although the results are representative of other kinematical situations. Clearly, as expected (see ref. [3]) the sensitivity to variations in the amount of magnetic strangeness is very weak (Fig. 5). Similarly, while not quite as weak, the dependence on electric strangeness in Fig. 6 as represented by the rather liberal variations employed here (see the discussions in refs. [3, 4]) is still overwhelmed by the effects of correlations. At higher values of $q$ than those displayed here and for forward–angle scattering, the sensitivity to variations in $G_E^{(s)}$ grows sufficiently to compete with the effects of the correlations and consequently the observables discussed in this section become less well suited to use in attempting to disentangle the single–nucleon from the many–body effects. In the next section, we shall discuss observables that are better designed to accomplish this.

4 The parity–violating asymmetry: isovector/axial–vector and strangeness form factors of the nucleon

In this section we introduce and discuss several observables that can be constructed from the PV asymmetry and the EM cross section which are designed to minimize the effects of pionic correlations (and, as we shall see, apparently other types of many–body effects as well) and hence allow us access to the single–nucleon form factors. Specifically, when our focus is the isovector/axial–vector single–nucleon form factor we shall be interested in large scattering angles where longitudinal effects fade away and where $R_{TVA}$ has its largest effect (see ref. [3]). Under these
circumstances the asymmetry becomes
\[
\mathcal{A} \rightarrow \mathcal{A}_0 \frac{v_T R_{AV}^T(q, \omega) + v_T, R_{VA}^T(q, \omega)}{v_T R^T(q, \omega)}.
\]  \tag{46}

Now, although the isoscalar piece of \( R^T \) is generally small, involving as it does the square of the isoscalar magnetic moment, it is still important to take it into account when attempting high–precision determinations of \( R_{VA}^T \) (see refs. [7, 8]). To explore at little further why this is an issue, let us divide both numerator and denominator of (46) by \( v_T R^T(1) \) to obtain
\[
\mathcal{A} = -\frac{1}{2} \mathcal{A}_0 \left\{ 1 + (1 - 4 \sin^2 \theta_W) - \frac{2}{1 + \rho} (\rho + \rho') \right\} \tag{47}
\]
for large \( \theta \), where we have defined the following ratios of response functions:
\[
\rho \equiv \frac{R^{T(0)}}{R^{T(1)}} \tag{48}
\]
\[
\rho' \equiv \frac{R^{T'_{VA}}}{R^{T(1)}}. \tag{49}
\]
The problem is the competition between \( \rho' \), which we wish to determine, and \( \rho \) in the second term in (47) involving the combination \( \rho + \rho' \). The fractional uncertainty in the former may be expressed in the following way:
\[
\left| \frac{\delta \rho'}{\rho} \right| \approx \left\{ \left[ 1 \left( \frac{\mathcal{A}}{\mathcal{A}_0} \right) \frac{\delta \mathcal{A}}{\mathcal{A}} \right]^2 + \left[ \frac{\rho}{\rho'} \frac{\delta \rho}{\rho' \rho} \right]^2 \right\}^{1/2} \tag{50}
\]
\[
\approx \left\{ \left[ 5 \frac{\delta \mathcal{A}}{\mathcal{A}} \right]^2 + \left[ 0.5 \frac{\delta \rho}{\rho} \right]^2 \right\}^{1/2}, \tag{51}
\]
where the last form results from using actual values for the responses and asymmetry at \( q \sim 500 \text{ MeV}/c \) yielding \( \rho \sim 0.05 \) and \( \rho' \sim -0.1 \). An uncertainty of \( \sim 1\% \) in the asymmetry or \( \sim 10\% \) in the ratio \( \rho \) then produces a corresponding uncertainty of \( \sim 5\% \) in the quantity of interest, \( \rho' \). Thus we see that several strategies are suggested as ways to proceed: one is to use parity–conserving (EM) electron scattering to limit the freedom in the model to the extent that this can be done (see below); another is to use PC and PV electron scattering to learn more about the correlation effects — this was discussed in the last section and there we saw considerable sensitivity to the pionic effects which could serve to make the uncertainty in \( \rho \) rather small using measured values for \( \Delta \mathcal{A} \); a third strategy, the one adopted in this section, is to form specific weighted integrals involving the asymmetry and the EM cross section that suppress the pionic correlation effects embodied in \( \rho \) and hence obtain new observables that are especially suited to determining the single–nucleon dependences. We begin with the energy–averaged asymmetry.
4.1 The energy–averaged asymmetry

First, we examine the physical observable

$$\bar{A}(q, \theta) \equiv \frac{1}{\Delta \omega} \int_{\omega_{\text{min}}}^{\omega_{\text{max}}} d\omega \ A(\theta; q, \omega)$$  \hspace{1cm} (52)$$

with the limits given in (40) and (41), where \(\Delta \omega\) is given in (42). Being an average over some range in energy, one might hope that the exact values taken for the end-point energies will not be crucial, although it should be realized that the energy interval involved is not dictated by compelling physical arguments, but simply reflects our theoretical framework which is restricted to particle–hole (ph) excitations and ignores electroproduction of mesons or internal excitations of the nucleon. The excitation of the \(\Delta\), for example, can indeed affect \(\bar{A}\) via the low–energy tail of its response function. As stated above, we shall not allow \(q\) to be so large that consideration of such effects becomes inevitable and thus we set aside these problems in the present work and leave them for future research.

As already anticipated above, in contrast to \(\Delta A\) discussed above, \(\bar{A}\) should be rather insensitive to the pionic correlations as these tend to cancel out in such a symmetrical integral. The MEC contribution, on the other hand, does not average out in \(\bar{A}\), although in the ph sector of the nuclear excitations this turns out to be rather puny. Typical results are shown in Fig. 7 for \(q = 500\ \text{MeV/c}\). In particular, as seen in the expanded view in Fig. 7b, at very backward scattering angles where one might hope to determine the effective axial–vector coupling \(g_A^{(1)}\) (again variations of \(\pm 10\%\) around 1.26 are shown in the figure) the free RFG and pionic correlated results for the energy–averaged asymmetry come together (for this momentum transfer at \(\theta \approx 147^{\circ}\)). At other momentum transfers the angle at which the two models come together is different from 147\(^{\circ}\) — since this special condition is presumably model–dependent, it is unlikely that one can count on using such particular kinematics to effect a determination of \(g_A^{(1)}\) through the variations shown in the figure. As we shall see in sect. 4.2, other observables are better suited for that purpose in any event.

In contrast to the backward–angle situation, at forward scattering angles where the pionic correlations induce drastic modifications in \(R_{\text{AV}}^L\), as we have seen, here the two families of curves differ, although certainly not as much as in the case of \(\Delta A\) discussed in the previous section. In other words, the observable \(\bar{A}\) has some of the properties that we are looking for when we construct quantities that suppress the effects of correlations while bringing out the dependences on the single–nucleon form factors; however, this particular observable appears not to be entirely optimal. Since the PV longitudinal response is so strongly affected by the presence of pionic correlations, it is necessary to adopt an alternative approach to minimize these effects and this is the subject of the next subsection.
4.2 The scaling and sum rule approaches

In order to free ourselves from the dependence on correlation effects, here we operate on the differential asymmetry with a more elaborate procedure than in the previous subsection. In fact, instead of performing a simple integration over $A(\theta; q, \omega)$, we shall now integrate separately over the numerator and the denominator of the asymmetry including specific weighting factors before taking their ratio: in this way we are naturally led to consider standard scaling relations and sum rules for the basic nuclear EM and PV responses. A well–known theorem [21] of many–body theory then insures that the frequency integrals we are considering are not going to be affected by self–energy terms and accordingly we should clearly obtain physical observables that are largely independent of such effects. These new observables will be seen to be well–suited for studies of the single–nucleon form factors.

The guiding principle in forming the new observables from the differential asymmetry and EM cross section is to remove the dependence on the single–nucleon form factors that varies most rapidly with $\omega$ before performing the integrals. One possibility is to proceed as in considerations of $y$–scaling [10, 12] where a prescription has been devised in the past that allows one to accomplish this task, if not exactly (actually, this is impossible in a relativistic context), at least to a very good level of approximation. Since we already know that the $R_{AV}^T$ response is dominant in the numerator of the ratio we will be forming [3] and, moreover, we wish to design that ratio to work best for backward–angle electron scattering in order to be able to extract information about the effective coupling $g_A^{(1)}$, we choose weighting factors that accomplish this ideally for $R_{AV}^T$ (numerator) and $R^T$ (denominator).

According to the procedure given in ref. [10], in order to obtain quantities that have good $y$–scaling behaviour (called $\psi$–scaling in that reference for the reasons presented there) we are to divide the EM and PV responses

$$ W_{EM} = v_L R_L^L + v_T R_T^T $$

$$ W_{PV} = v_L R_L^L + v_T R_T^T + v_T' R_T' $$

by the transverse projection of the function $X(\theta, \tau, \psi; \eta_F)$ defined in that work for the EM case, denoted $X_T$, and by its PV analog, denoted $\tilde{X}_T$, respectively:

$$ X_T(\theta, \tau, \psi; \eta_F) = v_T U_T $$

$$ \tilde{X}_T(\theta, \tau, \psi; \eta_F) = a_A v_T \tilde{U}_T $$

where, using the notation of refs. [10, 3]

$$ U_T = 2W_1 + W_2 \Delta $$

$$ \tilde{U}_T = 2\tilde{W}_1 + \tilde{W}_2 \Delta $$

In the above

$$ W_1(\tau) = \tau G_M^2(\tau) $$

17
\[
W_2(\tau) = \frac{1}{1 + \tau} \left[ G^2_E(\tau) + \tau G^2_M(\tau) \right]
\]

\[
\tilde{W}_1(\tau) = \tau G_M(\tau) \tilde{G}_M(\tau)
\]

\[
\tilde{W}_2(\tau) = \frac{1}{1 + \tau} \left[ G_E(\tau) \tilde{G}_E(\tau) + \tau G_M(\tau) \tilde{G}_M(\tau) \right],
\]

so that

\[
(1 + \tau)W_2(\tau) - W_1(\tau) = G^2_E(\tau)
\]

\[
(1 + \tau)\tilde{W}_2(\tau) - \tilde{W}_1(\tau) = G_E(\tau)\tilde{G}_E(\tau).
\]

The quantity \(\Delta\), the vector current analog of (30), was defined in ref. [10]:

\[
\Delta = \tau \kappa^2 \left[ \frac{1}{3} (\varepsilon_F^2 + \varepsilon_F \Gamma + \Gamma^2) + \lambda (\varepsilon_F + \Gamma) + \lambda^2 \right] - (1 + \tau).
\]

It is straightforward to show that \(\Delta\) and \(\tilde{\Delta}\) may be written in terms of \(\tau, \eta_F\) and the scaling variable \(\psi\) introduced in ref. [10]:

\[
\psi = \left[ \frac{1}{\xi_F} (\gamma_1 - 1) \right]^{1/2} \times \left\{ \begin{array}{ll} +1, & \lambda \geq \lambda_0 \\ -1, & \lambda \leq \lambda_0 \end{array} \right.
\]

where

\[
\gamma_1 = \kappa \sqrt{1 + 1/\tau - \lambda}, \quad \xi_F = \varepsilon_F - 1, \quad \text{and} \quad \lambda_0 = \left( \sqrt{1 + 4\kappa^2} - 1 \right)/2.
\]

Since \(\Delta\) and \(\tilde{\Delta}\) are both of order \(\eta_F^2 \ll 1\), excellent approximations for \(X_T\) and \(\tilde{X}_T\) may be obtained by dropping the terms containing them in the expressions above. Of course, as usual it is intended that we take one copy of these expressions for the proton contribution (\(N = Z\)) and add it to another for the neutron contribution (\(N = N\)). We are thus led to consider the first of our new observables:

\[
\mathcal{R}_1(q, \theta) \equiv \frac{\int_{\omega_{\min}}^{\omega_{\max}} d\omega \ W_{PV}(q, \omega) \tilde{X}_T(\theta, \tau, \psi; \eta_F)}{\int_{\omega_{\min}}^{\omega_{\max}} d\omega \ W_{EM}(q, \omega)/X_T(\theta, \tau, \psi; \eta_F)}.
\]

While the above ideas represent one way to proceed, other approaches can also be taken: let us turn to a second of these before presenting detailed results using several nuclear models. Motivated by sum rules instead of \(y\)-scaling (especially by the Coulomb Sum Rule (CSR) — see ref. [13]), let us recall the general expression of the inclusive cross section for inelastic electron scattering in the free RFG model:

\[
\frac{d^2\sigma}{d\Omega de} = \frac{N\sigma_M}{4m_N} S(\psi) \left\{ v_L U^L + v_T U^T \right\},
\]
where we now have the longitudinal contribution (the analog of (57))

\[ U_L = \frac{\kappa^2}{\tau}(G_E^2 + W_2\Delta) \]  

(see ref. [3]) and \( X_L = v_L U_L \). In (59) the \( \psi \)-scaling function is given by

\[ S(\psi) = \frac{3\xi_F}{\eta_F} (1 - \psi^2) \theta(1 - \psi^2). \]  

Moreover, to insure that the asymptotic value of unity for the sum rules is reached at large \( q \), we notice that

\[ \int_{-1}^{1} d\psi \ (1 - \psi^2) = 4 \frac{3}{3}. \]  

Accordingly, one has

\[ \frac{3}{8m_N} \int_{0}^{\infty} d\omega \left( \frac{\partial \psi}{\partial \lambda} \right) (1 - \psi^2) \theta(1 - \psi^2) = 1. \]  

It is thus clearly apparent that the convenient choice is to introduce the following EM longitudinal and transverse normalized nuclear responses

\[ S^{L,T} \equiv v_{L,T} R^{L,T}/X'_{L,T}, \]  

where the normalizing factors are given by

\[ X'_{L,T} \equiv \frac{N X_{L,T}}{(\kappa \eta_3 F/2 \xi_F)(\partial \psi/\partial \lambda)}. \]  

Similarly, in discussing the PV responses, following ref. [3] we need in addition to (58) two more functions

\[ \tilde{U}^L = \frac{\kappa^2}{\tau} (G_E \tilde{G}_E + \tilde{W}_2 \Delta) \]  

\[ \tilde{U}^T = \sqrt{\tau(1 + \tau)} \tilde{W}_3 [1 + \tilde{\Delta}], \]  

where \( \tilde{\Delta} \) is given in (30) and \( \tilde{W}_3(\tau) = 2G_M(\tau)G_A(\tau) \). The corresponding normalized nuclear responses are

\[ \tilde{S}^{L,T} \equiv v_{L,T} R_{AV}^{L,T}/\tilde{X}'_{L,T} \]  

\[ \tilde{S}^{T} \equiv v_T R_{TAV}^{T}/\tilde{X}'_{T}, \]  

where, as above, the normalizing factors are given by

\[ \tilde{X}'_{L,T} \equiv \frac{N \tilde{X}_{L,T}}{(\kappa \eta_3 F/2 \xi_F)(\partial \psi/\partial \lambda)} \]  

\[ \tilde{X}'_{T} \equiv \frac{N \tilde{X}_{T}}{(\kappa \eta_3 F/2 \xi_F)(\partial \psi/\partial \lambda)}. \]
with $\tilde{X}_L = a_A v_L \tilde{U}^L$, $\tilde{X}_T = a_A v_T \tilde{U}^T$ and $\tilde{X}_{T'} = a_{T'} v_T \tilde{U}^{T'}$. The derivative in the above equations is given by

$$\frac{\partial \psi}{\partial \lambda} = \frac{\kappa}{\tau} \frac{\sqrt{1 + \xi_F \psi^2/2}}{\sqrt{2\xi_F}} \left[ \frac{1 + 2\lambda + \xi_F \psi^2}{1 + \lambda + \xi_F \psi^2} \right]$$

(82)

Finally, we obtain a set of five sum rules:

$$\int_0^\infty d\omega S^K(q, \omega) = 1, \ K = L, T \quad (84)$$

$$\int_0^\infty d\omega \tilde{S}^K(q, \omega) = 1, \ K = L, T, T' \quad (85)$$

for $\kappa > \eta_F \leftrightarrow q > 2p_F$ and are now in a position to define the second of our new observables:

$$R_2(q, \theta) \equiv \frac{\int_{\omega_{\text{min}}}^{\omega_{\text{max}}} d\omega \ W^{PV}(q, \omega) / \tilde{X}_T'(\theta, \tau, \psi; \eta_F)}{\int_{\omega_{\text{min}}}^{\omega_{\text{max}}} d\omega \ W^{EM}(q, \omega) / X_T'(\theta, \tau, \psi; \eta_F)} \quad (86)$$

Let us begin the discussion of our results by considering the Coulomb sum rule. For clarity, retaining only the leading dependence in expansions in powers of $\eta_F$ (although in the results to follow we use the exact expressions), we obtain

$$\Sigma^L(q) \equiv \int_0^\infty d\omega S^L(q, \omega)$$

$$\approx \int_0^\infty d\omega \left( \frac{1 + \lambda}{1 + 2\lambda} \right) \left[ ZG_{Ep}^2 + NG_{En}^2 \right] \quad (88)$$

$$\rightarrow 1 + O[\eta_F^2] \quad \text{RFG, } q > 2p_F \quad (89)$$

in agreement with eq. (3.31) of ref. [13] and with ref. [22] at the QEP where $\lambda = \tau$. Note that at $q = 2p_F = 550 \text{ MeV}/c$, the factor $(1 + \lambda)/(1 + 2\lambda) \approx 0.94$ and therefore a naive sum rule obtained by neglecting this factor (as is sometimes done in analyzing experimental data) will be about 6% too low at $2p_F$ and even further below unity at higher $q$. In Fig. 8 we show the CSR as a function of $q$ computed using the exact expression (74); as expected by the way that we have constructed it, the free

\footnote{In fact, it is not practical experimentally to integrate over the full range of $\omega$; the sum rules used in the present work, as well as ref. [3], should be understood to involve integrations over the usual quasielastic response region. While this is well defined in our model, there must always be some doubt as to whether this has been achieved experimentally. In the latter case, the longitudinal response is reasonably confined to the region defined by the RFG model and yet could have strength extending to high $\omega$ that is essentially unmeasurable. Consequently, when we give the range of integration as extending up to infinity, we actually mean extending to a high enough value of $\omega$ that the response function has peaked and fallen back essentially to zero.}
RFG answer becomes exactly unity at \( q = 2p_F \) and remains so for all higher values of \( q \). The CSR for our more sophisticated model that includes pionic correlation and MEC contributions approaches unity from below, falling about 9% below the asymptotic answer at \( q = 2p_F \). Thus, the above relativistic effect involving \( \lambda \) and the pionic effects together yield a 15% decrease of the CSR from the naive answer at \( q = 2p_F \). Indications are that experiment \([23]\) yields a result that is even smaller, although the comparison with our pionic model is quite encouraging. The other curves in Fig. 8 will be discussed below.

In Fig. 9 we show the energy shifts of the peaks of the \( R^L \) and \( R^T \) responses away from the free RFG value:

\[
\bar{\epsilon}_{L,T}(q) = \omega \left[ \text{peak in } R^{L,T} \right] - \omega \left[ \text{peak in RFG} \right],
\]

(90)
drawing upon the results presented in ref. \([8]\) for the model in which pionic correlations and MEC effects have been included (again, the other curves will be discussed below). A \( q \)-dependent hardening of the positions of the maxima of the responses is seen which, for the reasons presented in ref. \([8]\), is greater for \( R^L \) than for \( R^T \). As \( q \) increases, this hardening becomes weaker — again there is some evidence from experiment \([24]\) for this behaviour, which helps to substantiate the pionic approach that we are following.

Let us now turn to a discussion of the quantities \( \mathcal{R}_1 \) and \( \mathcal{R}_2 \) defined above. In Fig. 10 we show \( \mathcal{R} \) (actually \( \mathcal{R}_2 \) is shown; however, since \( \mathcal{R}_1 \) and \( \mathcal{R}_2 \) differ by a negligible amount for the kinematics chosen here, we shall consistently only show one of the two ratios) as a function of \( \theta \) for three values of \( q \). As in Figs. 3, 4 and 7 two families of curves are displayed, one for the free relativistic Fermi gas (RFG) and one for the model with pionic effects included (\( \pi \)), and each family has three curves \((g_A^{(1)} = 1.26, 1.26+10\% \text{ and } 1.26-10\%)\). Looking at Fig. 10b (and especially at the inset where the backward–angle region is expanded) we see a significant range of angles over which the pionic effects provide negligible modifications with respect to the RFG results and where the (merged) curves with the three values of the isovector/axial–vector strength can clearly be discerned. This behaviour is similar at \( q = 1 \text{ GeV/c} \), although not quite as nicely separated; even so the difference between the RFG and pionic families at backward scattering angles amounts to an effective change in \( g_A^{(1)} \) of only about 4%. It should also be realized that the difference between the families of curves is not the uncertainty in the curves. In the light of the successes seen reflected in Figs. 8 and 9, we believe the pionic model to represent the actual state of affairs better than the free RFG model. Even at low momentum transfer (Fig. 10a) there is still a range of angles where an axial–vector determination could be attempted, although somewhat higher values of \( q \) appear to be better suited for this (see also ref. \([3]\) where it is argued that the statistical precision would be best around 400–500 MeV/c).

In Fig. 11 similar results are shown, except now for variations in the magnetic strangeness single–nucleon form factor (Cf. the results in Fig. 5). Clearly,
as stated previously, PV quasielastic electron scattering is desensitized to the magnetic strangeness content for the reasons presented in detail in ref. [3]. At low-to-intermediate values of $q$ the curves computed with (dashed) and without (solid) magnetic strangeness almost coincide; importantly, they are close enough together that the above conclusions are not modified. By the time $q = 1$ GeV/c is reached (Fig. 11c) some separation of the curves begins to appear and thus, in principle, there is some degree of sensitivity to changes in $\mu_s$. On the other hand, it is also clear that PV elastic electron scattering from the proton as in the MIT/Bates SAMPLE experiment is more ideal for studying $G_M^{(s)}$. The sensitivity to electric strangeness is more pronounced, however, as can be seen in Fig. 12. As the momentum transfer increases (and accordingly the single-nucleon electric strangeness form factor grows, being proportional to $\tau$ at low momentum transfer — see (22)) the separation between results with different models for $G_E^{(s)}$ becomes quite noticeable and at $q = 1$ GeV/c overwhelms the spread that occurs in going from the RFG to the pionic correlated model. While again alternative approaches can be taken to determine $G_E^{(s)}$, such as PV elastic electron scattering from $^4$He (see refs. [5, 9]), in this case there may be some merit in employing forward-angle PV quasielastic electron scattering for this purpose as well (see also ref. [4]).

Finally, let us come to some comparisons with a different model for the nuclear physics content in the problem, namely one where the responses, cross section and PV asymmetry are calculated [7] using the relativistic mean-field theory (RMFT) of nuclear matter [25]. One of the variants presented in ref. [7] involves the introduction of an effective nucleon mass $m^*_N$ in specific places in the formalism (e.g., the Dirac contributions that involve $F_1$ and the Pauli contributions that involve $F_2$ are affected differently) — a value of $m^*_N \simeq 0.68m_N$ is favoured to fit the effective nuclear binding energy. We have repeated the calculations for this model presented in ref. [7] and applied the results to the observables discussed above. Let us start with the Coulomb sum rule: while the pionic model discussed above yields quite encouraging results for this quantity when compared with experiment, the RMFT CSR is not in good agreement with experiment, as can be seen by examining Fig. 8. Likewise in Fig. 9 we see that the RMFT model produces a shift in the QEP that is rather dramatic and, moreover, is in disagreement with experiment. Thus, the RMFT results should not be taken too seriously, but rather should be used to see how sensitive the various

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Note that here we present a relativistic CSR following the developments presented above. In ref. [25] where a CSR is also discussed a non-relativistic expansion is made, retaining only the square of the Dirac form factor in the dividing factor. The changes that result when terms involving the Pauli form factor are included are quite large; these are effectively incorporated in having used Sachs form factors, as we do in the present work (see the discussion in ref. [11]), and yield the result in Fig. 8 which then appears to differ significantly from that in ref. [25]. Naturally, it is largely irrelevant what prescription one adopts as long as the same is applied to experiment. Since the one presented in [88] is essentially the one used in analyzing experiment, we shall continue to adopt this approach.

22
observables that we have constructed are to rather extreme modeling differences. In
Fig. 13 we show results for a few representative cases. We include the RFG and the
pionic model results for comparison and, in addition to using \( m_N^* = 0.68m_N \) in the
RMFT, we also present results with \( m_N^* = 0.8m_N \) to see the trends as functions of
the effective mass. In Fig. 13a we observe that \( \Delta A \) could serve to differential between
the various models. Even a rather crude measurement of this quantity should be
capable of telling the difference between the pionic correlated model and the RMFT
model with \( m_N^* = 0.68m_N \), for instance. The quantity \( \bar{A} \), on the other hand, is more
ambiguous as can be seen in Fig. 13b. As discussed in ref. [7], if the \( m_N^* = 0.68m_N \)
RMFT model were viable (in which case the quantity \( \rho \) introduced in (47) would
be about twice as large as in the pionic model — in other words, there would be a
much more dramatic modification of the isoscalar responses in the former case than
in the latter), then there is more of a spread in the results for this observable and
hence more confusion in attempting to extract the dependence on the axial–vector
form factor (see above). However, such is not the case for the quantity \( R \), as can
be seen from Fig. 13c. There all models coalesce at large angles and the above
arguments in favour of using this observable to determine the isovector/axial–vector
single–nucleon form factor remain valid. It remains to be seen what the situation
will be for more sophisticated models when the observables we have constructed
are employed, although it would appear likely given the above arguments that, for
any model that is reasonably capable of reproducing the known behaviour of the
parity–conserving quasielastic responses (not to mention the PV observables such
as \( \Delta A \) that can serve to limit the model uncertainties to a very large extent), this
will continue to be an effective way to proceed.

5 Conclusions

In this paper we have applied our treatment of pionic correlations and meson–
exchange currents in quasielastic electron scattering developed in our past work
on the subject to studies of parity–violating electron scattering. Quasielastic PV
electron scattering was also previously investigated by us in earlier studies within
the context of the relativistic Fermi gas model and the present work extends that
treatment of the problem insofar as it incorporates the pionic effects and introduces
new observables, constructed as specific integrals involving the differential asymme-
try and electromagnetic cross section, whose purpose is to emphasize the various
physics issues of interest while suppressing unwanted model dependences. In par-
icular, on the one hand we have defined an observable \( \Delta A \) designed to be very
sensitive to fine details in the pionic correlations, while at the same time to be
rather insensitive to variations in some of the (as yet) unmeasured single–nucleon
form factors. On the other hand, observables denoted \( R_{1,2} \) have been constructed as
weighted integrals of the asymmetry and EM cross section, based on the concepts
of \( y \)-scaling and sum rules, which have the reverse properties: they are seen to be quite unaffected by pionic correlations under favourable circumstances and yet capable of being used to provide information on specific single–nucleon form factors, notably the isovector/axial–vector form factor at backward angles and the electric strangeness form factor at forward angles. As anticipated from previous studies, there is little sensitivity to the magnetic strangeness form factor of the nucleon in PV quasielastic electron scattering from nuclei.

What emerges from our investigations are several possibilities: clearly some combination of parity–conserving (EM) quasielastic electron scattering and of observables such as \( \Delta A \) should help in limiting the level of nuclear model uncertainty in the problem. For instance, even relatively modest–precision measurements of \( \Delta A \) in the forward direction should leave little doubt about how well the model is doing. Concerning this observable, we have discussed how it is affected by the longitudinal PV response: in the free RFG model it is quite small owing to delicate cancellations which are dictated by the standard model hadronic couplings that weight the isoscalar and isovector components occurring in this response. However, the pion correlates the nucleons strongly in the isoscalar channel, while doing so only weakly in the isovector channel, and as a consequence, it turns out that for not too high momentum transfers the pion restores the longitudinal PV response back to a size that is comparable to the transverse and axial–vector PV responses and gives rise to an anomalous \( \omega \)-dependence (strongly negative at low– and strongly positive at high–energies). Because of the opposite sign of the longitudinal and transverse PV responses, the asymmetry then comes close to vanishing at some frequency. Actually, were it possible to disentangle the longitudinal contribution to the asymmetry, then experimental evidence for an interesting dynamical restoration of the left–right parity symmetry could be obtained.

Having limited the model uncertainty, one will be in a position to proceed to study the single–nucleon form factors using, for example, the energy–averaged asymmetry \( (\overline{A}) \) or the quantities \( R_{1,2} \) constructed from the asymmetry — since \( \overline{A} \), \( R_1 \) and \( R_2 \) all have been constructed from integrations across the quasielastic response region, they will have the maximal statistical precision attainable in such experiments. For such integrated observables it appears that a level of precision approaching 1\% might be feasible in future PV electron scattering experiments and hence determinations of the axial–vector single–nucleon form factor at the 5–10\% level can be contemplated. Alternatively, one may be most interested in the nuclear many–body responses entering into the asymmetry, in which case quantities such as \( \Delta A \) (and to some extent, particularly in the forward direction, \( \overline{A} \) as well) should provide a sensitive probe of these nuclear response functions. It should be remarked that, although the level of precision expected for \( \Delta A \) is typically a few times the 1\% that might be reached for \( \overline{A} \), \( R_1 \) or \( R_2 \), even a relatively crude measurement of \( \Delta A \) would provide interesting information about the nuclear responses.

In discussing the other side of the problem, namely studies of the single–nucleon
form factors, we have found the observables $\mathcal{R}_{1,2}$ defined in this work to be sufficiently uncontaminated by correlation effects for favourable kinematics that the above limiting of the nuclear model uncertainty, while desirable, may not even be required. These observables have specifically been designed to integrate out the anomalous $\omega$–dependence alluded to above and so to leave quantities that, for the most part, only retain sensitivities to variations in some (although not all) of the single–nucleon form factors. Indeed, in testing these ideas with other models, in particular, with responses obtained on the basis of relativistic mean–field theory, we have found all of the above conclusions to work in that case as well. It should be remarked that this last test is a rather stringent one, since we already know that results such as the Coulomb sum rule and the “hardening” of the EM responses are not well reproduced by the naïve RMFT description — the pionic model put forward in the present work does much better in this regard. It remains to be seen whether the observables advocated here serve as well for other models, although our expectation is that they will do so. In the final analysis we continue to be encouraged by the fact that it appears likely that interesting information about the isovector/axial–vector and electric strangeness form factors of the nucleon could be obtained from quasielastic PV electron scattering experiments even when nuclear many–body effects are taken into account.

References

[1] T.W. Donnelly, J. Dubach and I. Sick, Phys. Rev. C37 (1988) 2320.
[2] T.W. Donnelly, J. Dubach and I. Sick, Nucl. Phys. A503 (1989) 589.
[3] T.W. Donnelly, M.J. Musolf, W.M. Alberico, M.B. Barbaro, A. De Pace and A. Molinari, Nucl. Phys. A541 (1992) 525.
[4] E. Hadjimichael, G.I. Poulis and T.W. Donnelly, Phys. Rev. C45 (1992) 2666.
[5] M.J. Musolf and T.W. Donnelly, Nucl. Phys. A546 (1992) 509.
[6] M.J. Musolf and T.W. Donnelly, Z. Phys. C57 (1993) 559.
[7] C.J. Horowitz, Phys. Rev. C47 (1993) 826; C.J. Horowitz and J. Piekarewicz, Phys. Rev. C47 (1993) 2924.
[8] W.M. Alberico, M.B. Barbaro, A. De Pace, T.W. Donnelly and A. Molinari, preprint CTP#2194, in print in Nucl. Phys. A.
[9] M.J. Musolf, T.W. Donnelly, J. Dubach, S.J. Pollock, S. Kowalski and E.J. Beise, submitted to Phys. Reports.
[10] W.M. Alberico, A. Molinari, T.W. Donnelly, E.L. Kronenberg and J.W. Van Orden, Phys. Rev. C38 (1988) 1801.

[11] W.M. Alberico, T.W. Donnelly and A. Molinari, Nucl. Phys. A512 (1990) 541.

[12] D.B. Day, J.S. McCarthy, T.W. Donnelly and I. Sick, Ann. Rev. Nucl. Part. Sci. 40 (1990) 357.

[13] T.W. Donnelly, E.L. Kronenberg and J.W. Van Orden, Nucl. Phys. A494 (1989) 365.

[14] T.W. Donnelly and A.S. Raskin, Ann. Phys. 169 (1986) 247.

[15] A.S. Raskin and T.W. Donnelly, Ann. Phys. 191 (1989) 78.

[16] R.G. Sachs, Phys. Rev. 126 (1962) 2256.

[17] S. Galster, Nucl. Phys. B32 (1971) 221.

[18] T.W. Donnelly and R.D. Peccei, Phys. Reports 50 (1979) 1.

[19] M.J. Musolf and B.R. Holstein, Phys. Lett. B242 (1990) 461.

[20] K. Kubodera, J. Delorme and M. Rho, Phys. Rev. Lett. 40 (1978) 755.

[21] K. Takayanagi, Nucl. Phys. A510 (1990) 162.

[22] T. De Forest, Nucl. Phys. A414 (1984) 352.

[23] P. Barreau et al., Nucl. Phys. A402 (1983) 515.

[24] C.F. Williamson, private communication.

[25] D. Serot and J.D. Walecka, in: Advances in Nuclear Physics, Vol.16, eds. J.W. Negele and E. Vogt (Plenum, New York, 1986).
Figure captions

Fig. 1 The PV response $R_{V_A}^{P_T}$ versus $\omega$ for three values of momentum transfer, $q = 300$ (a), 500 (b) and 1000 MeV/c (c). The dashed curve is for the free RFG and the solid curve is for the pionic model discussed in the text.

Fig. 2 Typical MEC diagrams involving both a photon and a $Z^0$ for $1p$–$1h$ excitations. The contributions where the $Z^0$ involves only the axial–vector current are higher order and have been neglected in the present work.

Fig. 3 The quantity $\Delta A$ defined in the text shown as a function of $\theta$ for $q = 300$ (a), 500 (b) and 1000 MeV/c (c). The relativistic Fermi gas model results are labeled (RFG), while the ones with pionic effects included are labeled ($\pi$). The two families of curves have $g_A^{(1)} = 1.26$ (solid), $1.26 + 10\%$ (dashed) and $1.26 - 10\%$ (dash–dot) — see ($^{[1]}$).

Fig. 4 The $\omega$–dependence of $A$ for different kinematic conditions: $(q$ [MeV/c], $\theta$ [degrees]) = (300, 10):(a), (300, 150):(b), (500, 10):(c), (500, 150):(d), (1000, 10):(e) and (1000, 150):(f). The labeling of the curves is otherwise as in Fig. 3.

Fig. 5 As for Figs. 3 and 4, except now with two models for the magnetic strangeness form factor of the nucleon: (solid curves) no strangeness and (dashed curves) $\mu_s = -1$ in ($^{[2]}$). Only results for the case $q = 500$ MeV/c are displayed.

Fig. 6 As for Fig. 5, except now for three models for the electric strangeness form factor of the nucleon: (solid curves) no strangeness, (dash–dot curves) $(\rho_s, \lambda_E^{(s)}) = (-3, 5.6)$ and (dashed curves) $(\rho_s, \lambda_E^{(s)}) = (-3, 0)$ in ($^{[2]}$).

Fig. 7 As for Fig. 3, except now showing the energy–averaged asymmetry ($\overline{A}$). Only results for the case $q = 500$ MeV/c are displayed. Panel (a) shows the entire angular range, while panel (b) shows only the backward–angle region in greater detail.

Fig. 8 The Coulomb sum rule defined in ($^{[3]}$) for the free relativistic Fermi gas (RFG), the pionic model ($\pi$) and the relativistic mean–field theory (RMFT) with two effective masses, $m_N^* = 0.68m_N$ (dash–dot) and $m_N^* = 0.8m_N$ (dashed).

Fig. 9 The shift of the EM longitudinal ($L$) and transverse ($T$) response functions from the RFG peak (see ($^{[4]}$)) for the pionic model (solid) and the relativistic mean–field theory (RMFT) with two effective masses (labeled as in Fig. 8).

Fig. 10 The quantity $\mathcal{R}$ versus $\theta$ for $q = 300$ (a), 500 (b) and 1000 MeV/c (c). The curves are labeled as in Fig. 3.

Fig. 11 As for Fig. 10, except now for variations of magnetic strangeness. The curves are labeled as in Fig. 5.
Fig. 12 As for Fig. 10, except now for variations of electric strangeness. The curves are labeled as in Fig. 6.

Fig. 13 As for Figs. 3, 7 and 10, except now showing results at $q = 300$ MeV/c for the free relativistic Fermi gas (RFG), the pionic model ($\pi$) and the relativistic mean–field theory (RMFT) with two values for the effective mass, $m_N^* = 0.68m_N$ (dash–dot) and $m_N^* = 0.8m_N$ (dashed).