Progress on cubic interactions of arbitrary superspin supermultiplets via
gauge invariant supercurrents

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ABSTRACT

We consider cubic interactions of the form $s - Y - Y$ between a massless integer
superspin $s$ supermultiplet and two massless arbitrary integer or half integer superspin $Y$
supermultiplets. We focus on non-minimal interactions generated by gauge
invariant supercurrent multiplets which are bilinear in the superfield strength of the
superspin $Y$ supermultiplet. We find two types of consistent supercurrents. The first
one corresponds to conformal integer superspin $s$ supermultiplets, exist only for even
values of $s$, $s = 2\ell + 2$, for arbitrary values of $Y$ and it is unique. The second one,
corresponds to Poincaré integer superspin $s$ supermultiplets, exist for arbitrary values
of $s$ and $Y$.

Keywords: cubic interactions, higher spin, supersymmetry

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1 Introduction

The theory of massless higher spin fields can be understood as an attempt to understand and classify the potential list of symmetries emerging from string theory at high enough energy scales. As such, it is only natural to enhance higher spin symmetry with supersymmetry which is another key ingredient of strings. This line of thought inevitably leads to the study of higher spin irreducible representations of the super-Poincaré group. The free theory of massless, higher superspins for flat spacetime, its AdS and conformal versions has been developed [1–9].

The problem of finding consistent interactions involving these higher spin supermultiplets is non-trivial as in the case of non-supersymmetric higher spin theories. At present, a wide class of cubic interactions of the type $Y - 0 - 0$, between arbitrary integer or half-integer superspin ($Y$) supermultiplets and various matter supermultiplets ($Y = 0$) is known [11–19]. A more general class of non-minimal cubic interactions of the type $(s + \frac{1}{2}) - Y - Y$ has been discovered in [20]. For these interactions, the corresponding higher spin supercurrent is quadratic in the superfield strengths of the superspin $Y$ supermultiplets and thus gauge invariant. It was found that such interactions exist for all values of $s$, but superspin $Y$ (can be integer or half-integer) is bounded by $s^2$ $[Y \leq \frac{s}{2}]$. This was understood as a supersymmetric higher spin generalization of the Weinberg-Witten theorem.

In this letter, we continue the investigation of non-minimal cubic interactions among massless higher spin supermultiplets which are generated by gauge invariant supercurrents that can be expressed in terms of the superfield strengths. We consider interactions of the type $s - Y - Y$ between an integer superspin ($s$) supermultiplet and two arbitrary (integer or half integer) superspin $Y$ supermultiplets. We find two types of such interactions which is related with the conformal or Poincaré nature of the integer superspin $s$ supermultiplet. For the first one, the integer superspin supercurrent satisfies conservation equations that correspond to a conformal integer superspin $Y = s$ supermultiplet, it is unique and exist only for even values of $s$ ($s = 2\ell + 2$). For the second one, the integer superspin supercurrent satisfies conservation equations that correspond to a Poincaré integer superspin $s$ supermultiplet, it is not unique and there is no selection rule, it exist for all values of $s$. Most importantly and in a big contrast with the results of [20], there is no constraint on the values of $Y$. These interactions exist for arbitrary $Y$.

This letter is organized as follows. In section 2, we review the various conservation laws a higher spin supercurrent multiplet must satisfy in order to be a valid generator of cubic interactions that involve the corresponding higher spin supermultiplet. Sections 3 and 4 include the construction of the various higher spin supercurrents for the conformal and Poincaré cases. Finally, section 5 presents a summary of our results.

2 Higher spin supermultiplets, superfield strengths and conservation equations

We consider cubic interactions of irreducible, $4D$, $\mathcal{N} = 1$, higher spin supermultiplets. The off-shell, superspace, description of their free theory was given first in [3,4] by proposing a set of various superfields, including constrained ones, and their gauge transformations. Based on this proposition, the action principle was uniquely determined and led to the correct on-shell equations of motion for the various field strength supertensors. It was also commented that the various constrained superfields could be expressed in terms of unconstrained prepotentials with appropriate gauge transformations, thus solving the constraints. Build upon these foundational results, in [8] later it was shown an alternative path exists. A careful consideration

\footnote{Recently, a manifestly supersymmetric description of continuous spin representations has been proposed [10] but for the purpose of this paper we will not include such representations under the label of higher spins.}
of the massless limit of massive higher spin supermultiplets will lead to a description of massless higher spin supermultiplets in terms of the unconstrained superfields and correctly generate their gauge transformations. Furthermore, a detailed analysis of the component structure of the theory was given. This includes the field spectrum of the theory, the component action and the set of supersymmetry transformations for all components, which leave the action invariant. For the purpose of our discussion we review the basic results:

1. The integer superspin \( Y = s \) \((s \geq 1)\) supermultiplets \((s+1/2, s)\)\(^5\) are described by a pair of superfields \( \Psi_{\alpha(s)}\dot{\alpha}(s-1)\)\(^6\) and \( V_{\alpha(s-1)}\dot{\alpha}(s-1)\) (real) with the following lowest order gauge transformations

\[
\delta_0 \Psi_{\alpha(s)}\dot{\alpha}(s-1) = -D^2 L_{\alpha(s)}\dot{\alpha}(s-1) + \frac{1}{(s-1)!} \bar{D}_{\dot{\alpha}(s-1)} A_{\alpha(s)}\dot{\alpha}(s-2) ,
\]

\[
\delta_0 V_{\alpha(s-1)}\dot{\alpha}(s-1) = D_{\alpha(s)} L_{\alpha(s)}\dot{\alpha}(s-1) + \bar{D}_{\dot{\alpha}(s-1)} A_{\alpha(s-1)}\dot{\alpha}(s) .
\]

Off-shell, this supermultiplet carries \(8s^2 + 8s + 4\) bosonic and equal number of fermionic degrees for freedom.

2. The half-integer superspin \( Y = s + 1/2 \) supermultiplets \((s + 1, s + 1/2)\) have two descriptions. The first one \((s \geq 1)\) uses the pair of superfields \( H_{\alpha(s)}\dot{\alpha}(s)\) (real) and \( \chi_{\alpha(s)}\dot{\alpha}(s-1)\) with the following lowest order gauge transformations

\[
\delta_0 H_{\alpha(s)}\dot{\alpha}(s) = \frac{1}{\alpha(s)} D_{\alpha(s)} \bar{L}_{\alpha(s-1)}\dot{\alpha}(s) - \frac{1}{\alpha(s)} \bar{D}_{\dot{\alpha}(s-1)} L_{\alpha(s)}\dot{\alpha}(s-1) ,
\]

\[
\delta_0 \chi_{\alpha(s-1)}\dot{\alpha}(s-1) = \bar{D}^2 L_{\alpha(s)}\dot{\alpha}(s-1) + D^{s+1} L_{\alpha(s-1)}\dot{\alpha}(s) .
\]

This supermultiplet, off-shell describes \(8s^2 + 8s + 4\) bosonic and equal number fermions. The second formulation \((s \geq 2)\) has the same \( H_{\alpha(s)}\dot{\alpha}(s)\) as previously but a different compensating superfield \( \chi_{\alpha(s-1)}\dot{\alpha}(s-2)\) with gauge transformations

\[
\delta_0 H_{\alpha(s)}\dot{\alpha}(s) = \frac{1}{\alpha(s)} D_{\alpha(s)} \bar{L}_{\alpha(s-1)}\dot{\alpha}(s) - \frac{1}{\alpha(s)} \bar{D}_{\dot{\alpha}(s-1)} L_{\alpha(s)}\dot{\alpha}(s-1) ,
\]

\[
\delta_0 \chi_{\alpha(s-1)}\dot{\alpha}(s-2) = \bar{D}^{s-1} L_{\alpha(s)}\dot{\alpha}(s-1) + \frac{s-1}{s} D^{s+1} \bar{D}^{s-1} L_{\alpha(s)}\dot{\alpha}(s-1)
\]

\[
\bar{D}_{\dot{\alpha}(s-2)} J_{\alpha(s-1)}\dot{\alpha}(s-3) .
\]

This supermultiplet carries \(8s^2 + 4\) off-shell bosonic and equal number of fermionic degrees of freedom.

The free theory actions (quadratic in the superfields and up to two spacetime derivatives) that describe the above irreducible representations are uniquely determined by the gauge symmetries. The physical and propagating degrees of freedom for massless integer and half-integer superspins are described by superfield strengths \( W_{\alpha(2s)} \) and \( W_{\alpha(2s+1)} \) respectively. They are defined in the following way:

\[
Y = s + 1/2 : \quad W_{\alpha(2s+1)} \sim \bar{D}^2 D_{\alpha_{2s+1}} \partial_{\alpha_{2s+1}} \dot{\alpha}_{2s+1} \dot{\alpha}_{2s-1} \ldots \partial_{\alpha_{2s+1}} \dot{\alpha}_{2s-1} H_{\alpha(s)}\dot{\alpha}(s) \quad (4a)
\]

\[
Y = s : \quad W_{\alpha(2s)} \sim \bar{D}^2 D_{\alpha_{2s}} \partial_{\alpha_{2s-1}} \dot{\alpha}_{2s-1} \dot{\alpha}_{2s-2} \ldots \partial_{\alpha_{2s+1}} \dot{\alpha}_{2s+1} \dot{\alpha}_{2s-1} \psi_{\alpha(s)}\dot{\alpha}(s-1) \quad (4b)
\]

and they are invariant with respect the respective gauge symmetries mentioned above. Their characteristic feature is to have a special index structure, i.e. they have only one type of index and \(2Y\) of them. Moreover, they are chiral

\[
\bar{D}_{\dot{\alpha}} W_{\alpha(2s+1)} = 0 \quad , \quad \bar{D}_{\dot{\alpha}} W_{\alpha(2s)} = 0
\]

\(^4\)We follow [8] and we use the conventions of “Superspace” [21].
\(^5\)On-shell they describe the propagation of helicities \(\pm(s + 1/2)\) and \(\pm s\).
\(^6\)The notation \(\alpha(k)\) is a shorthand for \(k\) undotted symmetric indices \(\alpha_1 \alpha_2 \ldots \alpha_k\). Similarly for dotted indices.
and on-shell they satisfy the following equations of motion

\[ D^3 W_{\beta\alpha(2s)} = 0 \quad , \quad D^3 W_{\beta\alpha(2s-1)} = 0 . \]  

At the component level they include the bosonic and fermionic higher spin field strengths.

Notice that in all case, we need two superfields to describe the corresponding higher superspin supermultiplets. The first one is associated with the superfield strength and plays the role of the (pre)potential and the second one (compensator) is required in order to write a two derivative, manifestly super-Poincaré, invariant action. However, one can consider conformal higher superspin supermultiplets. The Lagrangian description of such irreducible representations is given purely in terms of the superfield strengths, as described above, so it includes higher derivatives. Nevertheless, for such theories we require only one superfield, the (pre)potential which must be a primary superfield with appropriate weights. Its gauge transformation is determined by the largest symmetry that preserves the superfield strength:

1. The conformal integer superspin \( Y = s \), being described by superfield \( \Psi_{\alpha(s)\dot{\alpha}(s-1)} \), which is primary with conformal weights \( (-\frac{s}{2}, -\frac{s-1}{2}) \) and has a gauge transformation

\[ \delta_0 \Psi_{\alpha(s)\dot{\alpha}(s-1)} = \frac{1}{s!} D_{(\alpha_s \bar{\alpha}(s-1))\dot{\alpha}(s-1)} + \frac{1}{(s-1)!} \bar{D}_{(\dot{\alpha}(s-1) \Lambda(s-2))} . \]  

2. The conformal half-integer superspin \( Y = s + \frac{1}{2} \), being described by a real primary superfield \( H_{\alpha(s)\dot{\alpha}(s)} \) with conformal weights \( (-\frac{s}{2}, -\frac{s}{2}) \) and gauge transformation

\[ \delta_0 H_{\alpha(s)\dot{\alpha}(s)} = \frac{1}{s!} D_{(\alpha_s \bar{\alpha}(s-1))\dot{\alpha}(s)} - \frac{1}{s!} \bar{D}_{(\dot{\alpha}(s) \bar{\beta}(s-1))} . \]  

Notice that the gauge transformation (8) is identical to the corresponding gauge transformations (2a, 3a) of the super-Poincaré half-integer superspin representations. However for the integer superspin case, the gauge transformation (7) of the conformal representation is larger than the corresponding Poincaré one (1a). This difference will be the reason why for integer superspin interactions we find two sets of conserved supercurrents whereas for the half-integer case only one.

To make this clear, let’s consider cubic interactions of type \( Y = Y_1 - Y_2 \) between supermultiplets with superspin values \( Y, Y_1, Y_2 \). Assuming that such interactions exist, they are local and manifestly super-Poincaré or super-conformal then they can be written in the following form:

1. For \( Y = s \)

\[ S_{s - Y_1 - Y_2} = \int d^8 z \left\{ [\Psi_{\alpha(s)\dot{\alpha}(s-1)} J_{\alpha(s)\dot{\alpha}(s-1)} + c.c.] + V^{\alpha(s-1)\dot{\alpha}(s-1)} T_{\alpha(s-1)\dot{\alpha}(s-1)} \right\} \]  

\[ S_{s - Y_1 - Y_2} = \int d^8 z \left\{ [\Psi_{\alpha(s)\dot{\alpha}(s-1)} J_{\alpha(s)\dot{\alpha}(s-1)} + c.c.] + V^{\alpha(s-1)\dot{\alpha}(s-1)} T_{\alpha(s-1)\dot{\alpha}(s-1)} \right\} \]

where the higher spin supercurrent \( J_{\alpha(s)\dot{\alpha}(s-1)} \) and the real higher spin supertrace \( T_{\alpha(s-1)\dot{\alpha}(s-1)} \) are bilinear in the \( (Y_1, Y_2) \) supermultiplets and they must satisfy the conservation equations as they are determined by the gauge transformations (1) and (7) respectively

\[ D^2 J_{\alpha(s)\dot{\alpha}(s-1)} = \frac{1}{s!} D_{(\alpha_s T_{\alpha(s-1)})\dot{\alpha}(s-1)} , \quad \bar{D}^{\dot{\alpha}(s-1)} J_{\alpha(s)\dot{\alpha}(s-1)} = 0 \]  

\[ D^2 J_{\alpha(s)\dot{\alpha}(s-1)} = \frac{1}{s!} D_{(\alpha_s T_{\alpha(s-1)})\dot{\alpha}(s-1)} , \quad \bar{D}^{\dot{\alpha}(s-1)} J_{\alpha(s)\dot{\alpha}(s-1)} = 0 . \]

Additionally for the conformal case the supercurrent must be primary with weights \( (1 + \frac{s}{2}, 1 + \frac{s-1}{2}) \).

\[ ^7 \text{A quick review of primary superfields can be found in [11].} \]
2. For $Y = s + \frac{1}{2}$

Poincaré I : $S_{(s+\frac{1}{2})-Y_{1}-Y_{2}} = \int d^{8}z \left\{ H^{(s)}J_{(s)}^{(s)} \right\}$ (11a)

Poincaré II : $S_{(s+\frac{1}{2})-Y_{1}-Y_{2}} = \int d^{8}z \left\{ H^{(s)}J_{(s)}^{(s)} \right\}$ (11b)

conformal : $S_{(s+\frac{1}{2})-Y_{1}-Y_{2}} = \int d^{8}z \left\{ H^{(s)}J_{(s)}^{(s)} \right\}$ (11c)

where the real higher spin supercurrent and the higher spin supertrace satisfy the following conservation equations

Poincaré I : $\hat{D} \alpha J_{\alpha} = \hat{D}^{2}T_{\alpha} \hat{\alpha} + \hat{D}(\alpha+1)T_{\alpha} \hat{\alpha} = 0$ (12a)

Poincaré II : $\hat{D} \alpha J_{\alpha} = -\frac{1}{s(s-1)}D(\alpha)\hat{D} \alpha T_{\alpha} \hat{\alpha} - \frac{s-1}{s} \hat{D} \alpha T_{\alpha} \hat{\alpha} = 0$ (12b)

conformal : $\hat{D} \alpha J_{\alpha} = 0$ (12c)

Additionally for the conformal case the supercurrent must be primary with weights $(1 + \frac{s}{2}, 1 + \frac{s}{2})$.

One has to keep in mind that the supercurrent and supertrace pair which generates the cubic interaction, in general is not unique. One can consider improvement terms and produce an infinite family of equivalent \{J, T\} pairs. For example, using this freedom one can exchange conservation equations (12a) and (12b) \cite{16} and reveal the duality that exist between the two super-Poincaré half-integer superspin supermultiplets. In other cases, it is possible to use the improvement terms in order to make the supertrace vanish \(T = 0\).

For these cases, there is no distinction between the Poincaré and conformal supercurrents if $Y = s + \frac{1}{2}$ at the level of cubic interactions (11) and conservation equations (12). Of course one also has to check the primary nature of the minimal\(^{8}\) supercurrent. However using arguments similar to \cite{22}, one may connect the proper transformations under conformal symmetry with the conformal conservation equations (12c). On the other hand for $Y = s$, one can still distinguish between the Poincaré and conformal supercurrents since the left hand sides of conservation equations (10a) and (10b) are different.

In previous works \cite{11–17, 19, 18} a variety of cubic interactions between arbitrary, massless, integer or half-integer superspin supermultiplets and massless or massive matter supermultiplets \([s+\frac{1}{2} - 0 - 0, s-0-0]\) have been found either by solving the corresponding conservation equations or using Noether’s method with appropriate transformations in order to generate consistent supercurrent multiplets. Another step was made in \cite{20} where new cubic interaction between arbitrary massless half-integer superspin supermultiplets and massless integer or half-integer superspin $Y$ supermultiplets \([s + \frac{1}{2} - Y - Y]\) were found. These interactions have two characteristic properties. The first one is that the higher spin supercurrent can be written in terms of the superfield strengths $W_{\alpha(2Y)}$ of the two superspin $Y$ supermultiplets, hence it is a non-minimal class of interactions and the supercurrent is gauge invariant. The second one is that these types of interactions do not exist for arbitrary $Y$ but only if $Y \leq \frac{s}{2}$. In this work, we investigate similar type of interactions for the integer superspin supermultiplet \([s - Y - Y]\). We find that such interactions are possible for both the conformal (10b) and Poincaré cases (10a) with a vanishing supertrace. A surprising distinction from previous results is that there is no upper bound in the value of superspin $Y$. However there is an even values of $s$ selection rule for the conformal case.

\(^{8}\)This is the new supercurrent acquired by the addition of the improvement terms that make the supertrace vanish: \{J, T\} $\sim$ \{J\_minimal, 0\}
3 Conformal integer superspin $s$ with arbitrary superspin $\gamma$: $s - Y - Y$

Now let’s consider the cubic interaction $s - Y - Y$ between a conformal integer superspin $s$ and two arbitrary, massless superspin $Y$ supermultiplets. The interaction, if it exists and assuming locality and manifest invariance, must take the form:

$$S_{s-Y-Y} = \int d^8z \Psi^{\alpha(s)\dot{\alpha}(s-1)} J_{\alpha(s)\dot{\alpha}(s-1)} + c.c.$$  \hspace{1cm} (13)

where the higher spin supercurrent must satisfy the conservation equations (10b). Additionally, the supercurrent must be a composite object, quadratic to the superspin $Y$ supermultiplets. Similarly to [20], we further assume that the supercurrent is gauge invariant under the gauge transformations of superspin $Y$ and can be written in terms of the superfield strength $W_{\alpha(2Y)}$. These interactions are interesting, despite their non-minimal nature, because if they exist they are unique as has been demonstrated in [23, 24] for non-supersymmetric theories. A general ansatz that one can write for the supercurrent is:

$$J_{\alpha(s)\dot{\alpha}(s-1)} = \sum_{p=0}^{s-1} a_p \partial^{(p)} D W^{\gamma(2Y)} \partial^{(s-1-p)} W_{\gamma(2Y)}$$  \hspace{1cm} (14)

where for clarity we have suppressed all free $\alpha$ and $\dot{\alpha}$ indices originating from the strings of partial spacetime derivatives and the spinorial derivative. Also we have suppressed the symmetrization of all these indices together with the appropriate symmetrization factors. However we explicitly indicate the indices of the two superfield strengths which are contracted to each other and do not contribute to the set of free indices. Using the chiral condition (5) it is straightforward to show that

$$D^a s J_{\alpha(s)\dot{\alpha}(s-1)} \approx D^2 \left\{ \sum_{p=0}^{s-1} \left[ \frac{p+1}{2s} a_p + \frac{s-p}{2s} a_{s-1-p} \right] \partial^{(p)} W^{\gamma(2Y)} \partial^{(s-1-p)} W_{\gamma(2Y)} \right\}$$  \hspace{1cm} (15)

where the equality symbol “$\approx$” means modulo terms that depend on the equations of motion (6). When we go on-shell, as we always do when we calculate conservation equations, this symbol can be replaced with the usual equality symbol. The conclusion is that in order for this supercurrent to satisfy the conservation equation $D^a s J_{\alpha(s)\dot{\alpha}(s-1)} = 0$ we must choose the coefficients $a_p$ such that

$$a_p \ (p+1) + a_{s-1-p} \ (s-p) = 0 \ , \ p = 0, 1, 2, ..., s-1$$  \hspace{1cm} (16)

Similarly we can show that

$$D^{\dot{\alpha}s-1} J_{\alpha(s)\dot{\alpha}(s-1)} \approx i(-1)^{2Y} D^2 \left\{ \sum_{p=0}^{s-2} \left[ \frac{s-1-p}{2(s-1)} a_p - \frac{p+1}{2(s-1)} a_{s-2-p} \right] \partial^{(p)} D W^{\gamma(2Y)} \partial^{(s-2-p)} D W_{\gamma(2Y)} \right\}$$  \hspace{1cm} (17)

hence in order to satisfy the second conservation equation we must choose the coefficients $a_p$ such that

$$a_p \ (s-1-p) - a_{s-2-p} \ (p+1) = 0 \ , \ p = 0, 1, 2, ..., s-2$$  \hspace{1cm} (18)

The system of recursive relations (16) and (18) can be solved only for even values of $s$. For that case the solution is unique

$$a_p = (-1)^p \binom{s-1}{p} \binom{s}{p+1} \ , \ p = 0, 1, ..., s-1 \ , \ s = 2\ell + 2 \ , \ \ell = 0, 1, 2,...$$  \hspace{1cm} (19)
The conclusion is that there is a cubic interaction \( s - Y - Y \) between a conformal integer superspin \( s \) and two massless, arbitrary integer or half-integer superspin \( Y \) supermultiplets but only for even values of \( s, s = 2\ell + 2 \). The supercurrent which generates the cubic interaction is

\[
\mathcal{J}_{\alpha(2\ell+2)\dot{\alpha}(2\ell+1)} = \sum_{p=0}^{2\ell+1} (-1)^p \binom{2\ell + 1}{p} \binom{2\ell + 2}{p+1} \partial^{(p)} D W^{(2\gamma)} \partial^{(2\ell+1-p)} W_{\gamma(2\alpha)}
\]

(20)

and on-shell it satisfies conservation equations (10b). An interesting observation is that there is no constraint on the value of \( Y \). Another interesting remark is about the \( Y \to 0 \) limit of (20). If we set by hand \( Y = 0 \) then \( W \) no longer has the interpretation of the superfield strength of a higher spin supermultiplet and expressions (4) are no longer valid. However \( W \) remains a chiral superfield and as such describes a matter supermultiplet. Therefore by setting \( Y = 0 \) in expression (20) we recover precisely the conformal integer superspin supercurrent of a chiral superfield [17, 20] which also has the even value selection rule for \( s \) and generates the \((2\ell + 2) - 0 - 0\) interaction.

4 Poincaré integer superspin \( s \) with arbitrary superspin \( Y \): \( s - Y - Y \)

Now let’s consider the possibility of a cubic interaction \( s - Y - Y \) between a Poincaré integer superspin \( s \) and two arbitrary, massless superspin \( Y \) supermultiplets. With the same assumptions as previously, the interaction, if it exists, must take the form

\[
S_{s - Y - Y} = \int d^8 z \left\{ [\Psi^{(s)(s-1)}] \mathcal{J}_{\alpha(s)\dot{\alpha}(s-1)} + \text{c.c.} + V^{(s-1)(s-1)} \mathcal{J}_{\alpha(s-1)\dot{\alpha}(s-1)} \right\}
\]

(21)

with the conservation equations (10a) for the higher spin supercurrent and the supertrace. The ansatz for the supercurrent is the same as (14). Therefore, due to (15) we immediately find that such a supercurrent tautologically satisfies \( D^2 \mathcal{J}_{\alpha(s)\dot{\alpha}(s-1)} \approx 0 \) for arbitrary values of \( a_p \). Hence, the supertrace must vanish

\[
D^2 \mathcal{J}_{\alpha(s)\dot{\alpha}(s-1)} \approx 0, \quad \forall \ a_p \Rightarrow T_{\alpha(s-1)\dot{\alpha}(s-1)} = 0
\]

(22)

Lastly, we must check the second conservation equation \( \bar{D} \dot{\mathcal{J}}_{\alpha(s)\dot{\alpha}(s-1)} = 0 \). Using (17) we conclude that coefficients \( a_p \) must obey (18). This is the only constraint that for coefficients \( a_{-p} \) for the Poincaré case. This condition is not enough to uniquely fix everything. For example, notice that these recursion relations do not include \( a_{s-1} \), which remains unconstrained. A general solution of (18) is

\[
a_p = d \binom{s-1}{p} \binom{s+2\kappa-2}{p+\kappa} \quad , \quad p = 0, 1, ..., s-2
\]

\[
a_{s-1} = c
\]

for arbitrary \( c, d, \kappa, n \) and \( s \). Hence, there exist a family of such supercurrents that can generate the cubic interactions between Poincaré integer superspin \( s \) and two arbitrary superspin supermultiplets. They take the following form:

\[
\mathcal{J}_{\alpha(s)\dot{\alpha}(s-1)} = c \partial^{(s-1)} D W^{(2\gamma)} W_{\gamma(2\alpha)} + d \sum_{p=0}^{s-2} \binom{s-1}{p} \binom{s+2\kappa-2}{p+\kappa} \partial^{(p)} D W^{(2\gamma)} \partial^{(s-1-p)} W_{\gamma(2\alpha)}
\]

(23)

Unlike the previous result for the conformal case, there is no \( s \)-selection rule and the supercurrent exist for all values of \( s \). Moreover, this result holds for all values of \( Y \), similar to the conformal result. Following the arguments of previous section we can take the \( Y \to 0 \) limit in order to recover the integer superspin supercurrent of a chiral supermultiplet. By setting \( Y = 0 \) and interpreting \( W \) as a chiral superfield \( \Phi \), we get precisely the result found in [20]

\[9\]

In [20] only the corresponding to the first term of (23) was considered \( \partial^{(s-1)} D \Phi \Phi \). That is because the second term would correspond to an improvement term.
5 Summary

To summarize our results we consider cubic interactions $s - Y - Y$, between one massless integer superspin $s$ supermultiplet and two massless arbitrary superspin $Y$ supermultiplets. Specifically, we focus on cubic interactions that are generated by gauge invariant supercurrent multiplets with respect to the gauge symmetry of the two superspin $Y$ supermultiplets. For this reason we consider higher spin supercurrents and supertraces that are composite objects, written in terms of the superspin $Y$ superfield strength $W_{\gamma(2Y)}$.

A general ansatz for such an integer superspin supercurrent $J_{\alpha(s)\dot{\alpha}(s-1)}$ can be written (14) and we checked its compatibility with the appropriate conservation equations. The integer superspin $Y = s$ supermultiplet can be either conformal (7) or Poincaré (1) hence the cubic interactions could be of the form (9b) or (9a) and the supercurrent multiplet must satisfy the conservation equations (10b) or (10a). For both cases we find a non-trivial supercurrent:

1. For the conformal case, we find that the integer superspin supercurrent is uniquely fixed (20) by the conservation equations. Furthermore, the supercurrent and therefore the cubic interaction exist for all values of superspin $Y$ but only for even values of $s$, $s = 2 \ell + 2$. Moreover, by setting $Y = 0$ we recover the result of a conformal integer superspin supercurrent of a chiral supermultiplet [17,20].

2. For the Poincaré case, we find that the supertrace vanishes and there is a family of consistent supercurrents given by (23). Similar to the conformal case, the supercurrent exist for all values of $Y$ but now there is no selection rule for $s$. It holds for all values of $s$. Also, one can set $Y = 0$ and recover the result for Poincaré integer superspin supercurrent of chiral supermultiplet as described in [20].

In a previous work [20], similar types of interactions were studied for the half-integer superspin supermultiplet $[(s + \frac{1}{2}) - Y - Y]$. It is interesting to notice that consistent interactions for that case have in a sense an “opposite” behavior to what we find for the integer superspin case. For the half-integer superspin case the value of $s$ is arbitrary, whereas the superspin $Y$ had an upper bound $Y \leq \frac{s}{2}$.

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