Analysis for Gas in Transformer Oil based on improved GM (1,N) Grey

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Abstract—Based on the traditional gray GM (1, N) model, with seven kinds of gas fusion in transformer oil predicted, the actual detection times for the gas is less, the sequence accuracy is greatly reduced with a large error, and it will not conform to the standard judgment for the smooth curve. Therefore, an adaptive regression algorithm is proposed to revise each calculation result of its GM model to modify the model error. Through simulation, the sequence curves of the N data by error correction are obviously closer with a greater correlation degree. The improved gray prediction model can effectively improve the accuracy of traditional GM (1, N).

1. Introduction
In traction power systems, power transformers work as important core equipment, which are responsible for converting, distributing and transmitting electrical energy. With a fault occurring, the power transformer will directly affect the safe and stable operation in power systems[5]. Being not interfered by external electromagnetic field factors, dissolved gas analysis can not only reflect the types of transformer faults effectively, but also be made an effective means to discover transformer defects and potential faults[6]. Moreover, the transformer failure will lead to a change of its internal gas components[7]. By diagnosing and predicting the mass concentration of the dissolved gas in the transformer oil, it can realize the online real-time monitoring, find the fault immediately, eliminate the danger in order to guarantee safety in power systems[8].

Currently, the commonly used prediction models include time series model, fuzzy model, gray model, artificial neural network, support vector machine (SVM), Kalman filter, interval prediction[1]. Among them, the regression analysis prediction model is simple and easy to implement, but the prediction accuracy is relatively low; The neural network prediction model requires a large number of samples for training, which is difficult to implement[13]. Considering that the transformer itself can be regarded as a gray system containing known components and unknown connections, the gray prediction model is widely used due to its simple principle and low data requirements[3]. The traditional GM(1,1) model considers the change of a single variable over time, and there is a certain relationship between the dissolved gas components in transformer oil, so the GM(1,N) gray
multivariate model can be used to extract multiple related feature variables for comprehensive prediction[14], and the prediction accuracy is higher. Coupled with the adaptive regression algorithm to correct the error, improve the accuracy of prediction.

2. Grey multivariate GM (1, N) prediction model

Grey correlation degree is one of the gray theory analysis, represents the system factors or factors on the uncertainty of a connection between the main behavior. The variation in content of dissolved gas in transformer oil, analysis of the correlation with the content of a gas as the main body, to predict factor analysis of other gas content the extent of the impact on the gas content changes. Essence is the history curve shape of gas content in the sequence of similar level as the standard, namely judging other gases and closer proximity to predict gas curve, the closer the shows the greater the degree of correlation between the corresponding sequence, the opposite correlation degree is smaller.

The characteristic data sequence with a high correlation is

\[ x^{(0)}_i = (x^{(0)}_1, x^{(0)}_2, \ldots, x^{(0)}_n) \quad i = 2, 3, \ldots, n, \quad n \text{ is a factor sequence}[2]. \]

The matrix can be obtained separately from \( x^{(0)}_i, x^{(0)}_{i-1}, \ldots, x^{(0)}_1 \).

\[
\begin{align*}
X^{(1)}_j &= (x^{(1)}_j(1), x^{(1)}_j(2), \ldots, x^{(1)}_j(n)) \\
x^{(1)}_j(i) &= \sum_{k=1}^{i} x^{(0)}_j(k)
\end{align*}
\]

(1)

The matrix form of GM (1, N) model is

\[ dX^{(1)}(t) / dt = AX^{(1)}(t) + B \]

(2)

The response formula of equation (2) is

\[ X^{(1)}(t) = e^{A(t-1)}(X^{(0)}(1) + A^{-1}B) - A^{-1}B \]

(3)

Discretize equation (2) to obtain

\[ x^{(0)}_j(k) = \sum_{i=1}^{k} a_j z_i(k) + b_j \]

(4)

In equation (4): \( z_i(k) = 0.5(x^{(0)}_i(k-1) + x^{(0)}_j(k)) \)

Obtained by the least square method

\[ \hat{a}_j = (a_{j1}, a_{j2}, \ldots, a_{jn}, \hat{a}_j)^T = (P^T P)^{-1} P^T Y_j \]

(5)

In equation (5):

\[
P^T = \begin{bmatrix}
z^{(0)}_1(2) & z^{(0)}_1(3) & \cdots & z^{(0)}_1(n) \\
z^{(0)}_2(2) & z^{(0)}_2(3) & \cdots & z^{(0)}_2(n) \\
\vdots & \vdots & \ddots & \vdots \\
z^{(0)}_n(2) & z^{(0)}_n(3) & \cdots & z^{(0)}_n(n) \\
1 & 1 & \cdots & 1
\end{bmatrix}
\]

\[ Y_j = (x^{(0)}_j(2), x^{(0)}_j(3), \ldots, x^{(0)}_j(n))^T \]

It can be obtained that the identifications of \( A \) and \( B \) are

\[ \hat{A} = (\hat{a}_j)^{\text{max}}, \hat{B} = (\hat{b}_1, \hat{b}_2, \ldots, \hat{b}_n). \]

The response formula for GM (1, N) is equation (6).

\[ \hat{X}^{(1)}(k) = e^{A(k-1)}(X^{(1)}(1) + A^{-1}B) - A^{-1}B \]

(6)

Model data restored to equation (7).

\[ \hat{X}^{(0)}(k) = \hat{X}^{(1)}(k) - \hat{X}^{(1)}(k-1) \]

(7)

In equation (7): \( k = 2, 3, \ldots, n \).
2.1 Deformation to approximate method

From the above, the grey differential equation can be obtained as equation (8).

\[
x_i^{(0)}(k) + ax_i^{(0)}(k) = b_2 x_2^{(1)}(k) + b_3 x_3^{(1)}(k) + \cdots + b_N x_N^{(1)}(k)
\]

(8)

From the generated sequence, the whitening equation can be obtained as equation (9).

\[
\frac{dx_i^{(1)}(t)}{dt} + ax_i^{(1)}(t) = b_2 x_2^{(1)}(t) + b_3 x_3^{(1)}(t) + \cdots + b_N x_N^{(1)}(t)
\]

(9)

In equation (8): If the change of \( x_i(i=2,3,\cdots,N) \) is small, Take the gray constant as \( \sum_{i=2}^{N} b_i x_i^{(0)}(k) \).

The approximate response equation can be obtained from the grey differential equation (8) and transformed into equation (10)

\[
\Delta x_i^{(1)}(k + 1) = x_i^{(0)}(0) - \frac{1}{a} \sum_{i=2}^{N} b_i x_i^{(1)}(k + 1) e^{-ak} + \frac{1}{a} \sum_{i=2}^{N} b_i x_i^{(1)}(k + 1)
\]

(10)

This method of treating variables as constants is called approximation method.

2.2 Trapezoidal method

By using the ordinary differential method, the solution of the whitening equation in any interval can be obtained as equation (11).

\[
x_i^{(1)}(t) = e^{-at} x_i^{(0)}(t_a) + e^{-at} \int_{t_a}^{t} e^{at} \sum_{i=2}^{N} b_i x_i^{(1)}(t)dt
\]

(11)

In equation (11): \( t = t_{k+1} \), According to the characteristics of time series, When \( t_a = k \).

\[
x_i^{(1)}(k + 1) = e^{-ak} x_i^{(0)}(k) + e^{-ak} \int_{k}^{k+1} e^{at} \sum_{i=2}^{N} b_i x_i^{(1)}(t)dt
\]

(12)

In equation (12): use trapezoidal formula, we use computer programs to simplify the calculation process. the corresponding time expression of the grey differential equation can be obtained.

\[
\Delta x_i^{(1)}(k + 1) = e^{-ak} x_i^{(0)}(k) + \sum_{i=2}^{N} b_i \left( x_i^{(1)}(k) e^{-ak} + x_i^{(1)}(k + 1) \right)
\]

(13)

This method is called trapezoidal method.

3. Adaptive regression residual correction GM (1, N) model

When establishing GM (1, N) model for prediction, each number has a predicted value at a stage[11]. The residual value is obtained by comparing the fitting results with the actual results, regression analysis of a series of residual values can correct the prediction results of GM ((1, N) on the test set, avoid the continuous accumulation of residuals and increasing the prediction error, so as to improve the accuracy[10].
The residual between the fitted value and the actual value is

\[ \sigma_i^{(0)}(k) = \chi^{(0)}_i(k) - x_i(k) \]  

Since the fitting result may be greater than or less than the actual result, that is, the residual value may have a negative value, in order to facilitate the subsequent cumulative generation, a number \( a \) can be appropriately added to make all the residual values non-negative[4]. Then, the one-time accumulation generation method of grey system theory is adopted to obtain:

\[ \delta_i^{(1)}(k) = \sum_{j=1}^{n} \delta_j^{(0)}(j) = \begin{bmatrix} \delta_1^{(1)}(1) & \delta_2^{(1)}(1) & \cdots & \delta_n^{(1)}(1) \\ \delta_1^{(1)}(2) & \delta_2^{(1)}(2) & \cdots & \delta_n^{(1)}(2) \\ \vdots & \vdots & \ddots & \vdots \\ \delta_1^{(1)}(m) & \delta_2^{(1)}(m) & \cdots & \delta_n^{(1)}(m) \end{bmatrix} \]  

The change trend of \( \delta_i^{(1)}(k) \) discrete point diagram, combined with several commonly used curve regression equations, the fitting results are compared by using the determination coefficient, root mean square difference and standard deviation of the equation[15], select the best one as the regression equation of \( \delta_i^{(1)}(k) \). Then the residual correction value of the original data is obtained according to the progressive reduction processing:

\[ \sigma_i^{(0)}(k) = \sigma_i^{(1)}(k) - \sigma_i^{(1)}(k-1) - a \]  

The revised forecast results are:

\[ x_i^{(0)}(k) = x_i^{(0)}(k) + \xi_i^{(0)}(k) \]  

4. Grey correlation analysis of GM (1, N) model variables

When using GM (1, N) model for fault prediction, it is necessary to clarify the relationship between various variables[12]. After processing \( n \) original data sequences, the relationship between these sequences is judged according to the geometric similarity of the sequence curves[4]. The closer the curves are, the greater the correlation degree between the corresponding sequences, and vice versa. This is the principle of grey correlation analysis[9].

The parent sequence of grey correlation analysis is made as \( \chi_1^{(0)}, \chi_2^{(0)}, \cdots, \chi_n^{(0)} \). The remaining sequences are subsequences \( x_i(k) \).
In equation (19): $\xi_i(k)$ is the grey correlation coefficient of sequence $x_i$ and $x_i$ at point $K$. $\rho = 0.5$.

The grey correlation degree between subsequence and parent sequence is

$$r_i = \frac{1}{M} \sum_{k=1}^{M} \xi_i(k) \quad (20)$$

$M$ is the number of sequences. If $r_i \geq 0.5$, it is considered that there is correlation between child and parent sequences.

5. Accuracy test of prediction model

There is always a prediction error when predicting the state parameters of power transformer with grey prediction model. The reason is that there is always a certain difference between the prediction model and the objective reality it describes. In order to measure whether the prediction model is reasonable and whether the prediction results are credible, it is necessary to test its accuracy.

5.1 Mean residual

Residual test is a accuracy test method in grey theory. Set original data sequence $X^{(0)} = (x^{(0)}(1), x^{(0)}(2), ..., x^{(0)}(n))$.

The corresponding model simulation sequence is $\hat{X}^{(0)} = (\hat{x}^{(0)}(1), \hat{x}^{(0)}(2), ..., \hat{x}^{(0)}(n))$. The residual of the grey model is

$$\xi^{(0)}(k) = \frac{x^{(0)}(k) - \hat{x}^{(0)}(k)}{x^{(0)}(k)} \times 100\% \quad (21)$$

Sequence is $\xi^{(0)}(k) = (\xi^{(0)}(1), \xi^{(0)}(2), ..., \xi^{(0)}(n))$, The average residual is equation (22).

$$\varepsilon(\text{avg}) = \frac{1}{n-1} \sum_{k=2}^{n} |\xi(k)| \quad (22)$$

5.2 Then verify the relative error

Verifying the relative error is the relative error between the model prediction data and the actual data. That is

$$\delta(k) = \frac{1}{p} \sum_{i=1}^{p} \left| \frac{y_i(k) - \hat{y}_i(k)}{y_i(k)} \right| \times 100\% \quad (23)$$

In equation (23): Verifying the actual value used to $y_i(k)$; the predicted value of the model is $\hat{y}_i(k)$; the number of data inspected is $p$.

6. Case analysis

6.1 Data prediction

Taking the chromatographic data of the measured mass concentration of dissolved gas in oil of a 220kV transformer as an example, five groups of data were obtained for error analysis.
The improved GM (1, 5) prediction model is used to analyze the oil chromatographic data in the oil of multiple power transformers to verify that the improved GM (1, 5) prediction model is better than GM (1, 1) and traditional GM (1, 5). Superiority. Before the experiment, the original data sequence was normalized to reduce the difference between the various gas levels and the prediction results.

In order to verify the effectiveness of the improved GM (1, 5) model prediction, the first 3 data in the original data sequence are used to establish an equal-dimensional and new-information model, that is, \( m = 2 \) and the remaining 2 data are used to verify the effectiveness of the prediction. The results obtained are shown in Table 2.

### Table 2 predicted gas composition and concentration

| date       | \( \text{H}_2 \) (μ L/L) | \( \text{CH}_4 \) (μ L/L) | \( \text{C}_2\text{H}_6 \) (μ L/L) | \( \text{C}_2\text{H}_4 \) (μ L/L) | \( \text{C}_2\text{H}_2 \) (μ L/L) |
|------------|--------------------------|--------------------------|--------------------------|--------------------------|--------------------------|
| 2020-05-14 | 28.5                     | 80.2                     | 40.2                     | 135.1                    | 1.0                      |
| 2020-06-07 | 41.4                     | 95.4                     | 54.1                     | 162.7                    | 1.1                      |
| 2020-07-18 | 70.1                     | 114.5                    | 71.1                     | 200.4                    | 1.8                      |
| 2020-08-14 | 80.4                     | 121.3                    | 85.1                     | 220.4                    | 1.9                      |
| 2020-09-20 | 101.4                    | 150.2                    | 100.5                    | 270.2                    | 1.9                      |
| 2020-10-05 | 114.2                    | 190.2                    | 130.4                    | 350.1                    | 2.1                      |
| 2020-11-15 | 150.4                    | 220.1                    | 131.1                    | 415.2                    | 2.5                      |

Through the state interval, can directly see the adjusted error is very small. All state in the smallest range, and is distributed evenly on both sides. The correction results accord with theoretical analysis, practical engineering requirements.

6.2 Result analysis

Based on GM (1, N) grey multivariable model prediction and analysis method, multiple characteristic variables can be used for comprehensive prediction, but the error is large and the accuracy is low. In view of this problem, this paper uses adaptive regression method to correct the formula and data results from the perspective of residual correction. The limitation of grey prediction caused by
fluctuation of original data is improved. The example shows that the adaptive regression algorithm can significantly improve the traditional GM (1, N) grey prediction model and greatly improve the prediction accuracy.

7. Conclusions
In view of the rich data of dissolved gas content in transformer oil, and the characteristics of a certain trend, the transformer interior is regarded as one. The GM(1, N) grey multivariable model was established for prediction analysis.

To improve the model prediction effect, Based on the adaptive regression algorithm provides a new algorithm to predict. Using residual error correction forecast results. As a case study of dissolved gas in transformer oil are analyzed.

The predictive analysis method is simple based on GM (1, N) grey multivariable model in dealing with various data, on the other hand, can extract related multiple characteristic variables for comprehensive prediction, but the accuracy of the series will still be reduced with a long time span.

The adaptive regression method is used from the Angle of residual correction, which improves the limitation of grey prediction caused by the fluctuation of original data, and determines the difference of data with the help, which is different grey model formula. Correcting the residual result. The algorithm makes the prediction of gas composition more accurate and expands the application range of grey model. More in line with the transformer oil interaction and influence of multidimensional gas component in the reality.

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