Exploring $\Lambda$CDM extensions with SPT-3G and Planck data:

$4\sigma$ evidence for neutrino masses, full resolution of the Hubble crisis by dark energy with phantom crossing, and all that

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Abstract: We present new cosmological constraints in a set of motivated extensions of the ΛCDM model using the polarization and gravitational lensing measurements from the South Polar Telescope and the Planck CMB temperature observations at large angular scales. It was shown in Ref. [1] that this CMB setup is free from the Planck anomalies which could affect the cosmological inference in extended models. Combining the SPT-3G, SPTPol and Planck large-scale temperature data with the latest full-shape BOSS and BAO measurements and information from the weak lensing surveys we find a $4\sigma$ evidence for nonzero neutrino mass, $\sum m_\nu = 0.23 \pm 0.06 \text{eV}$. In ΛCDM+$N_{\text{eff}}$ scenario we demonstrate that the Planck anomalies do not propagate into cosmological constraints. Then we examine two promising scenarios with modified late-time cosmology: one introduces a phantom crossing in dark energy equation of state, another provides with a sharp transition in the dark energy evolution. We find that the scenario with a phantom crossing completely alleviate the cosmological tensions yielding $H_0 = 73.92 \pm 1.09 \text{km s}^{-1}\text{Mpc}^{-1}$ and $S_8 = 0.774 \pm 0.010$. The Bayesian approach indicates that the model with phantom crossing is favoured by data as compared to ΛCDM. For the transitional dark energy we find no strong evidence for a rapid change in its equation of state after including the BAO from the BOSS DR12. Our proof-of-concept analysis shows that the Planck anomalies severely obscure the cosmological inference in some extensions of the ΛCDM model. More precise CMB data is needed to validate our conclusions on neutrino masses and phantom crossing in the dark energy sector.

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1 Introduction

Modern cosmology demonstrated a significant progress in the last decade. The most outstanding results came from the cosmic microwave background (CMB) which remains the most precise cosmological probe to date. The Planck measurements of
CMB anisotropies allow us to determine parameters of the minimal ΛCDM model with a percent precision (and some parameters like θ_s at the sub-percent level) [2]. Such pinpoint accuracy is facing increasing requirements to quality and internal agreement of observation data to ensure the robust cosmological bounds. The possible systematic errors and/or internal inconsistencies within the CMB data can hinder the inference of cosmological parameters and lead to artificial tensions with other cosmological probes.

Intriguingly, there are currently several statistical-persisting tensions between the Planck measurements based on the primary CMB anisotropies and direct probes in the late Universe. The first and most significant tension is the difference between the values of the Hubble constant (H_0) directly measured in the low-redshift Universe and extracted from the CMB assuming the ΛCDM cosmology. Different measurements in the local Universe based on various techniques bring persistently higher values of H_0 [3]. Distance ladder approach utilizing photometry of 75 Milky Way Cepheids and Gaia EDR3 parallaxes yields H_0 = 73.2 ± 1.3 km s^{-1} Mpc^{-1} [4] which exhibits a 4.2σ discrepancy with the number extracted from the Planck CMB observations under ΛCDM, H_0 = 67.36 ± 0.54 [2]. And the local measurement is in remarkable agreement with the other Cepheid-SN Ia distance ladder result which uses the improved distance to NGC 4258, H_0 = 73.5 ± 1.4 km s^{-1} Mpc^{-1} [5]. This discrepancy is conventionally treated as the Hubble tension, or even the Hubble crisis. The second problem consists in the moderately low clustering amplitude σ_8 directly extracted from various large-scale structure (LSS) surveys as compared to that measured by Planck from CMB [6]. This tension has been recently supported by new results from Dark Energy Survey (DES) and Kilo-Degree Survey (KiDS), S_8 = 0.776 ± 0.017 [7] and S_8 = 0.759^{+0.024}_{-0.021} [8], respectively, where S_8 = σ_8√Ω_m/0.3 is the late-time parameter and Ω_m is the relative contribution of matter to the present energy density of the Universe. Being combined DES-Y3 and KiDS-1000 measurements are in tension with the Planck baseline analysis at 3.3σ [1]. Recent analysis of full-shape galaxy power spectra and bispectrum [11] along with traditional measurements of redshift-space distortions [12] also bring consistently low values of S_8. The H_0- and S_8-tensions can indicate cracks in the standard cosmological paradigm and/or be driven by internal inconsistencies in observational data.

Indeed, the Planck measurements themselves suffer from multiple internal inconsistencies that can not be explained within the ΛCDM paradigm [2, 13].

\[^1\]Very similar level of disagreement comes from the combination of the Kilo-Degree Survey (KV450) and DES-Y1 results [9] and BOSS+KV450 analysis [10].
most significant feature in the Planck data is so-called "lensing anomaly". This effect consists in the oscillatory high-\(\ell\) residuals of the temperature power spectrum relatively to the \(\Lambda\)CDM prediction that resembles the extra smoothing of acoustic CMB peaks generated by gravitational lensing [14]. Modern analysis based on model-independent decomposition of the gravitation lensing signal [15] reveals a high statistical significance of this anomaly at the level of 2.8\(\sigma\). Even though this feature does not significantly affect the cosmological constraints in the base \(\Lambda\)CDM model [16], its effect becomes crucial in various extensions. Indeed, after marginalizing over the lensing information in the CMB power spectra the Planck constraint on the total neutrino mass, \(\sum m_\nu < 0.26\,\text{eV} (2\sigma)\) [2], degrades by more than 3 times to \(\sum m_\nu < 0.87\,\text{eV} (2\sigma)\) [15]. In the cosmological model with extra relativistic degrees of freedom in the plasma, traditionally described as effective number of neutrinos \(N_{\text{eff}}\), the marginalization over the lensing anomaly opens up a new degeneracy direction between \(H_0\) and \(N_{\text{eff}}\) parameters thereby introducing an interesting avenue to reduce (and possibly even resolve) the \(H_0\) tension [15]. These findings indicate that the existing lensing-like anomaly in the Planck power spectra substantially affects cosmological inference in valuable extensions of the \(\Lambda\)CDM model. This motivates us to consider alternative CMB measurements on small angular scales to obtain reliable cosmological constraints in models beyond \(\Lambda\)CDM.

Recently, an alternative approach which benefits from both full-sky surveys and ground-based observations of the CMB has been developed [1]. This analysis combines the Planck temperature and SPTPol polarization and lensing measurements in the \(\Lambda\)CDM model and brings the CMB estimate of \(S_8\) into full accordance with the direct measurements in the local Universe. This research reveals that the tension between LSS and CMB probes is solely driven by the lensing-like anomaly in the Planck data on small scales. This analysis also brings substantially higher values of \(H_0\) thereby lifting the Hubble tension down to 2.5\(\sigma\) statistical significance. The same methodology has been applied in the Early Dark Energy (EDE) scenario to explore the Hubble tension in the presence of LSS data [18]. Generally, it has been shown that the combined data approach yields robust measurements of cosmological parameters with only modestly larger error bars compared to the baseline Planck analysis [1, 18]. The successive implementation of this strategy in the \(\Lambda\)CDM and EDE models paves the way towards its application in other \(\Lambda\)CDM extensions.

In the first part of the paper we explore two physically well-motivated extensions:
ΛCDM with massive active neutrinos (ΛCDM+$\sum m_\nu$) and ΛCDM with extra relativistic degrees of freedom (ΛCDM+$N_{\text{eff}}$). The main goal of this study is to extract reliable cosmological information and explore the real potential of ΛCDM+$\sum m_\nu$ and ΛCDM+$N_{\text{eff}}$ models to alleviate the either or both cosmological tensions. We confront our results to that in the standard analysis and highlight the importance of the combined data approach for obtaining robust cosmological constraints.

In the second part of our research we examine two promising scenarios with modified late-time cosmology. The first one is associated with a phantom crossing in dark energy sector. Previous analyses [19–23] based on model-independent reconstruction of the expansion history of the Universe point towards possible crossing of the $\omega_{\text{DE}} = -1$ divide. Such behaviour generically occurs in non-minimally coupled dark energy models where dark energy and dark matter interact through an additional scalar, see for details Ref. [24, 25]. We consider a model-independent realization of the phantom crossing scenario, the Phantom Dark Energy (PDE) model [26], which was argued to be capable of alleviating the tension between the early and late Universe determinations of $H_0$. We extend this analysis by including the latest LSS data and utilizing the combined data approach. On this path we revisit the possibility of the PDE model to fully resolve the Hubble tension.

The second appealing scenario assumes a rapid appearance of dark energy in the late Universe. Recent analyses [27, 28] based on a model-independent reconstruction of the Universe evolution at late times revealed a non-trivial dark energy dynamics at intermediate redshifts. The dark energy component with a sharp transition in its equation of state naturally occurs in the scalar-tensor gravity theories, see for details Refs. [29, 30]. We consider a concrete parametric model for this dark energy evolution dubbed Transitional Dark Energy (TDE). The recent research [27] argues that the TDE scenario is capable to alleviate the $H_0$ and $S_8$ tensions simultaneously. We access the possibility of the proposed TDE model to alleviate the cosmological tensions using the different CMB setup and up-to-date LSS measurements.

This research improves the previous analyses [1, 18] in the following directions. First, we analyse the latest measurements of South Pole Telescope (SPT) [31] collected by SPT-3G instrument which substantially improves the previous SPTPol observations. Second, we perform a consistent full-shape analysis of the BOSS DR12 data set including information from the power spectrum multipoles [32], the real-space power spectrum [33], the reconstructed power spectrum [34] and the bispectrum monopole [11]. In addition, we consider multiple BAO measurements based on catalogs of emission-line galaxies, quasars, Lyα absorption and cross-correlation.
between the last two that allows us to trace the cosmological evolution back to earlier times. Finally, we explore two promising late-time modifying scenarios which drastically change the dark energy sector, and revisit the status of the cosmological tensions in these models.

The outline of this paper is as follows. In Section 2 we describe our methodology and introduce all data sets used in the analysis. In Section 3 we brief our main results. In Section 4 we present cosmological constraints in the ΛCDM scenario. In Section 5 we fit the parameters of ΛCDM+$\sum m_\nu$ and ΛCDM+$N_{\text{eff}}$ models to cosmological data and highlight the importance of the combined CMB data approach. In Section 6 we examine the PDE scenario against up-to-date cosmological data. In Section 7 we analyze the implication of the TDE model for the $H_0$ and $S_8$ tensions. We conclude in Section 8.

Two appendices contain supplementary materials. In Appendix A we perform the comparison of our approach and the previous PDE analysis [26] based on the full Planck likelihood. In Appendix B we closely examine the difference between our TDE analysis and that of Ref. [27].

## 2 Methodology

We obtain cosmological parameter constraints using the modified Einstein–Boltzmann code CLASS-PT [35] interfaced with the Montepython Monte Carlo sampler [36, 37]. We exploit the Markov Chain Monte Carlo (MCMC) approach to sample the posterior distribution. The plots and marginalized constraints are generated with the latest version\(^3\) of the getdist package [38].

In the ΛCDM model we vary the following set of parameters ($\omega_{cdm}$, $\omega_b$, $H_0$, $A_s$, $n_s$, $\tau$), where $H_0$ is the Hubble constant, which value can be recast as $H_0 \equiv h \times 100 \text{ km s}^{-1} \text{ Mpc}^{-1}$. Then $\omega_{cdm} \equiv \Omega_{cdm} h^2$, $\omega_b \equiv \Omega_b h^2$ with $\Omega_{cdm}$ and $\Omega_b$ standing for the relative contribution of cold dark matter and baryons to the present energy density of the Universe. $A_s$ and $n_s$ are the amplitude and the tilt of the primordial spectrum of scalar perturbations, $\tau$ denotes the reionization optical depth. In ΛCDM we assume the normal neutrino hierarchy with the total active neutrino mass $\sum m_\nu = 0.06 \text{ eV}$ and fix $N_{\text{eff}}$ to the default value 3.046. Additionally, we run $\sum m_\nu$ in ΛCDM+$\sum m_\nu$ and $N_{\text{eff}}$ in ΛCDM+$N_{\text{eff}}$ models, respectively. In ΛCDM+$\sum m_\nu$ model we approximate the neutrino sector with three degenerate massive states to boost the evaluation of the Einstein-Boltzmann code. In ΛCDM+$N_{\text{eff}}$ model we al-

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\(^3\)https://getdist.readthedocs.io/en/latest/
low \( \Delta N_{\text{eff}} < 0 \) for completeness, corresponding to standard neutrinos having a lower temperature than expected. In the PDE and TDE models we extend the dark energy sector so as to implement the late-time dynamics in these models, see for details Secs. 6 and 7.

In what follows we performed many fits to cosmological data sets and present the inferred values of the model parameters in Tables and Figures often omitting the units of measure for the dimensional quantities to reduce the corresponding labels. Here we state that in all that cases the Hubble parameter \( H_0 \) is measured in units of \( \text{km} \, \text{s}^{-1} \, \text{Mpc}^{-1} \), the sum of neutrino masses \( \sum m_\nu \) is in units of eV, the present size of the horizon at the drag epoch \( r_{\text{drag}} \) is in Mpc, the angular diameter distance \( D_A \equiv 1/(1+z) \int_0^z \frac{dz'}{H(z')} \) is in units inversed of the Hubble parameter, \( \text{km}^{-1} \, \text{s} \, \text{Mpc} \).

### 2.1 Data sets

Here we describe all data sets involved in this analysis.

**PlanckTT-low\( \ell \):** We employ the Planck Plik likelihood for TT power spectrum in the multipole range \( 30 \leq \ell < 1000 \) in combination with the Commander low-\( \ell \) TT likelihood [2].

**SPT-3G:** We accommodate the recent polarization measurements collected by the SPT-3G detector in 2018 [31]. This data includes the EE and TE power spectra over the angular multipole range \( 300 \leq \ell < 3000 \) from observation of a 1500 deg\(^2\) survey field at three frequency bands. To reliably predict CMB estimates we follow the original analysis [31] and marginalize posteriors over super-sample lensing parameter, numerous terms representing Galactic dust emission, polarized dust and noise from radio galaxies, along with all-sky calibrations of temperature and polarisation. We impose reasonable priors on some of these parameters along the lines of Ref. [31]. To compute the binned power spectrum estimates we apply the SPT-3G bandpower window functions \(^4\).

**Lens:** As the last CMB-based observable we use the SPTPol measurement of the lensing potential power spectrum, \( C_{\ell \phi \phi} \), over the multipole range \( 100 < \ell < 2000 \) extracted from the connected 4-point correlation function [39] of CMB fields (temperature and polarization modes). The direct measurement of \( C_{\ell \phi \phi} \) from the SPTPol survey agrees well with SPTPol measurements of TE and EE spectra as shown in [40].

\(^4\)The full SPT-3G likelihood for the Montepython environment is publicly available at https://github.com/ksardase/SPT3G-montepython.
In all analyses we use a recent measurement of the reionization optical depth $\tau$ reported in [41]. Thus we impose a Gaussian prior

$$\tau = 0.0581 \pm 0.0055$$  \hspace{1cm} (2.1)

determined from the likelihood approximation scheme momento using only the Planck polarization (EE) data. We do not mention it in data set names for brevity.

We combine all the above CMB measurements into one data set Base.

To validate the result we also use the Planck estimate of the gravitational lensing potential [2] instead of the Lens and refer to this combination as Base$\prime$.

**Planck 2018:** For the standard CMB analysis we use the official Planck TT, TE, EE+lensing and low-$\ell$ TT likelihoods [2]. Note that we do not include low-EE data from Planck, choosing instead to constrain the optical depth $\tau$ via a Gaussian prior following [41], as described above.

**LSS:** We analyze the final BOSS catalogue DR12 of luminous red galaxies. The galaxy were observed in the North and South Galactic Caps (NGC and SGC, respectively). We divide each sample into the two non-overlapping redshift slices with effective redshifts $z_{\text{eff}} = 0.38$ and $z_{\text{eff}} = 0.61$, giving a total of four data chunks. We exploit window-free approach [42, 43] which allows one to measure the unwindowed power spectrum and bispectrum directly from the observational data. We analyze the following full-shape data $^5$:

- **Redshift-Space Power Spectrum:** We use the pre-reconstructed power spectrum monopole, quadrupole and hexadecapole in the mode range $k \in [0.01, 0.2] \, h\text{Mpc}^{-1}$ as presented in Ref. [45].

- **Real-Space Power Spectrum:** We use the analog to real space power spectrum for $k \in [0.2, 0.4] \, h\text{Mpc}^{-1}$ introduced in Ref. [33]. It allows us to avoid limitations related to fingers-of-God modeling and access significantly smaller scales.

- **BAO:** We include the BAO measurements extracted from the post-reconstructed power spectra via a joint covariance matrix, as discussed in Ref. [34].

- **Bispectrum:** We include the bispectrum monopole in the range $k \in [0.01, 0.08] \, h\text{Mpc}^{-1}$ with step $\Delta k = 0.01 \, h\text{Mpc}^{-1}$ following [11]. In total, it generates 62 bispectrum bins.

$^5$The previous full-shape BOSS analyses were affected by an error in the public BOSS power spectra, for details see Ref. [44]. In our approach we do not calculate the mask explicitly, so our analysis is not affected by this problem.
To model the above statistics we compute the power spectrum (bispectrum) up to one-loop (tree-level) order in cosmological perturbation theory. Our analysis features a full treatment of all necessary components: nonlinear corrections, galaxy bias, UV counter terms, infrared resummation (to address the effects of large-scale bulk flows) and stochastic bias. We marginalize the posteriors over all nuisance parameters along the lines of Ref. [11]. The theoretical calculations are performed with the Boltzmann code CLASS-PT [35] that embodies an end-to-end calculation of one-loop power spectra and incorporates all observation effects required for the direct application to the data.

We complement the full-shape measurements described above with the following BAO data:

- 6dFGS at \( z_{\text{eff}} = 0.106 \) [46]
- SDSS DR7 MGS at \( z_{\text{eff}} = 0.15 \) [47]
- eBOSS quasar sample at \( z_{\text{eff}} = 1.48 \) [48]
- Auto-correlation of Ly\( \alpha \) absorption and its cross correlation with quasars at \( z_{\text{eff}} = 2.33 \) from the final eBOSS data release [49]
- eBOSS emission line galaxy sample at \( z_{\text{eff}} = 0.845 \) [50] \(^6\).

\( S_8 \): We streamline the results of photometric surveys KiDS-1000 [8], DES-Y3 [7] and Subaru Hyper Suprime-Cam (HSC) [52] in the form of the Gaussian prior

\[
S_8 = 0.772 \pm 0.013. 
\]  

(2.2)

We treat it separately from the other LSS data for the following reasons. First, it allows one to assess the consistency of different likelihoods before combining them into a single set. Second, we can examine the \( S_8 \) tension in the most direct way.

\( H_0 \): Finally, we adopt the SH0ES measurement [4] of the Hubble constant

\[
H_0 = 73.2 \pm 1.3 \text{ km s}^{-1}\text{Mpc}^{-1} 
\]  

(2.3)

based on the Cepheid-SN Ia distance ladder approach and Gaia EDR3 parallaxes. This measurement exhibits the highest accuracy and most severe tension with the Planck estimate among the others, based on the late-time observations.

\(^6\)We do not include the full-shape measurements of emission line galaxies because their impact on the eventual parameter constraints is rather limited as shown in [51].
3 Summary of our Main results

Let us briefly summarize our main results before going into the technical details. We fit the model parameters to the cosmological data considering five different cosmological models: ΛCDM, ΛCDM+$\sum m_\nu$, ΛCDM+$N_{\text{eff}}$, PDE and TDE.

Our results for the ΛCDM+$\sum m_\nu$ model are shown in Fig. 1. The Base data analysis brings a substantially weaker constraint on $\sum m_\nu$ compared to the full Planck result. It happens because in the standard analysis the Planck lensing-like anomaly strengthens the constraints on neutrino masses making higher values of $\sum m_\nu$ implausible [2]. The Base+LSS+S analysis suggests a 4σ detection of nonzero neutrino masses, $\sum m_\nu = 0.23 \pm 0.06$ eV. Using the Planck measurement of the gravitational lensing potential we find a consistent constraint $\sum m_\nu = 0.19 \pm 0.06$. Our results indicate that the strong preference for nonzero neutrino masses is robust against re-
placement of CMB lensing data set. We also show the result after marginalizing over
the principal components of the CMB gravitational lensing potential from Ref. [15].
This analysis removes any effect of the extra peak smoothing and other lensing ef-
fects in the CMB power spectra on constraints of the cosmological parameters. Its
result shows that the baseline Base + LSS + S8 constraint is fully consistent with the
Planck data after marginalizing over lensing information.

Our findings demonstrate the importance of robust measurements of the small-
scale CMB anisotropies to reliably predict the posterior probability distribution of
cosmological parameters. We found that the Planck lensing-like anomaly propagates
into the $\sum m_\nu$ constraint thus obscuring the cosmological inference. Our measure-
ment of the total neutrino mass is fully consistent with the results of Ref. [53]
which analyzes the SPT-3G and ACT-DR4 data when combined with WMAP. This
analysis shows a mild preference for $\sum m_\nu \sim 0.22\,\text{eV}$ at about just one standard
deviation. More precise CMB data is needed to validate the strong preference for
nonzero neutrino masses suggested by our data analysis.

We summarize the results of multiple models considered in this paper in the
form of posterior distributions of $H_0$ in Fig. 2. Let us begin with the $\Lambda$CDM
model. The Base data analysis predicts a moderately higher value of $H_0$ alleviating
the Hubble tension to a 2.7$\sigma$ level. Adding the large-scale structure information
provides a twice stronger constraint on $H_0$ which exacerbates the tension with the
SH0ES measurement to 3.5$\sigma$. The model with free relativistic degrees of freedom
($\Lambda$CDM+$N_{\text{eff}}$) only partially alleviates this disagreement to a 3$\sigma$ level. The late-time
scenario with a phantom crossing in the dark energy sector (PDE) solves the Hubble
tension, e.g. the Base+LSS+S8+H0 analysis yields $H_0 = 73.92 \pm 1.09\,\text{km}\,\text{s}^{-1}\text{Mpc}^{-1}$.
We emphasize that even without any information from the SH0ES and cosmic shear
measurements the PDE prediction is fully consistent with the direct measurements
of $S_8$ and $H_0$, for details see Sec. 6.2. In contrast, the scenario with a fast transition
in the dark energy evolution at late times (TDE) provides a lower value of $H_0$, once
the BAO measurements are taken into account. This result is in full agreement with
the value reported by Planck collaboration.

Our analysis reveals that the $H_0$ tension can be resolved by the non-trivial dy-
namics in the dark energy sector associated with the phantom crossing. This finding
agrees with the result of Ref. [22] showing that a phantom like component is capable
to address the $H_0$ tension while keeping the CMB acoustic scale intact. Compared
to the previous PDE analysis [26] the present study predicts a lower value of $S_8$,
thereby eliminating the $S_8$ tension, for details see App. A. The statistical analysis
Figure 2. Marginalized 1d posterior distributions of $H_0$ for the ΛCDM Planck 2018 (green), ΛCDM Base (blue), ΛCDM Base+LSS+S8 (purple), ΛCDM+$N_{\text{eff}}$ Base+LSS+S8 (dot-dashed purple), PDE Base + LSS + S8 (red) and TDE Base + LSS + S8 (orange).

based on the Bayesian approach shows that the PDE scenario is definitely preferred against ΛCDM.

4 ΛCDM model

In this Section we present the results for the ΛCDM model. First, we revisit the combined data approach introduced in Ref. [1]. Second, we present the cosmological constraints in the ΛCDM cosmology using up-to-date large-scale structure data. Finally, we compare our parameter estimates with those in the standard Planck analysis.

4.1 Combined CMB data approach

We combine CMB data sequentially to justify the statistical agreement between them. The final one-dimensional (1d) marginalized parameter constraints for various data set combinations are given in Tab. 1. The corresponding two-dimensional (2d) posterior distributions are shown in Fig. 3.
Our findings demonstrate that the parameter constraints inferred from the PlanckTT-lowℓ and SPT-3G data are remarkably consistent. It justifies the combining of these likelihoods into one data set. The combined likelihood significantly improves constraints on all cosmological parameters except for τ which is primarily controlled by the Gaussian prior (2.1). The cosmological measurements given by SPT-3G+PlanckTT-lowℓ moderately improve upon the previous analysis [1]. Note that the mean value of ln(10^{10} A_s) decreases by 1σ compared to that in Ref. [1]. This effect is attributed to a more precise (two times) measurement of the optical depth provided by Ref. [41] which pulls the amplitude of the scalar primordial perturbations down. Generally, the analysis based on the SPT-3G+PlanckTT-lowℓ likelihood agrees well with the results of previous study [1].

We eventually include the measurement of the lensing power spectrum from the SPTPol survey. We find that the accuracy of inferred parameters S_8 and H_0 improve by 15% upon adding the Lens data. Specifically, the CMB-only analysis based on the Base data yields

$$S_8 = 0.780 \pm 0.020 \quad H_0 = 69.09 \pm 0.84 \text{ km s}^{-1} \text{Mpc}^{-1}$$

(4.1)

We also found that the mean values of σ_8 and S_8 increase by 1σ compared to that

| Parameter | SPT-3G | PlanckTT-lowℓ | SPT-3G+PlanckTT-lowℓ | Base |
|-----------|--------|----------------|-----------------------|------|
| 100ω_b   | 2.243 ± 0.033 | 2.264 ± 0.039 | 2.258 ± 0.021 | 2.255 ± 0.020 |
| 10ω_{cdm} | 1.147 ± 0.036 | 1.141 ± 0.032 | 1.147 ± 0.021 | 1.151 ± 0.018 |
| H_0      | 68.98 ± 1.51 | 69.87 ± 1.68 | 69.29 ± 0.96 | 69.09 ± 0.84 |
| τ        | 0.058 ± 0.006 | 0.058 ± 0.006 | 0.058 ± 0.006 | 0.058 ± 0.005 |
| ln(10^{10} A_s) | 3.016 ± 0.023 | 3.035 ± 0.014 | 3.035 ± 0.013 | 3.036 ± 0.012 |
| n_s      | 1.004 ± 0.019 | 0.979 ± 0.011 | 0.977 ± 0.007 | 0.977 ± 0.006 |
| r_{drag} | 148.47 ± 0.98 | 148.38 ± 0.59 | 148.28 ± 0.48 | 148.18 ± 0.43 |
| Ω_m     | 0.290 ± 0.020 | 0.282 ± 0.019 | 0.288 ± 0.012 | 0.290 ± 0.010 |
| σ_8      | 0.791 ± 0.016 | 0.789 ± 0.013 | 0.791 ± 0.009 | 0.793 ± 0.008 |
| S_8      | 0.778 ± 0.041 | 0.766 ± 0.038 | 0.775 ± 0.024 | 0.780 ± 0.020 |

Table 1. Marginalized 1d constraints on cosmological parameters in the standard ΛCDM model for four data sets. Recall that the Base data set includes SPT-3G+PlanckTT-lowℓ+ Lens.
Figure 3. Marginalized 2d posterior distributions of the cosmological parameters in the ΛCDM model for SPT-3G (red), PlanckTT-lowℓ (blue), combined SPT-3G+PlanckTT-lowℓ (solid black) and adding the Planck lensing, SPT-3G+PlanckTT-lowℓ+Lens (dashed black) data sets. The Gaussian prior on \( \tau \) (2.1) advertised in [41] is always adopted. The yellow bands represent 1σ and 2σ constraints on \( S_8 \) (2.2) coming from the photometric surveys (DES-Y3, KiDS, HSC), whereas the green bands refer to the \( H_0 \) measurement (2.3) reported by the SH0ES collaboration.

in Ref. [1]. This effect is attributed to the latest SPT-3G data which favours higher values of the mass fluctuation amplitude [31]. Our constraint on \( S_8 \) is perfectly
consistent with the weak lensing measurements. In turn, the statistical difference between the CMB-based estimate of $H_0$ and the local measurement of this parameter reported by SH0ES decreases from 4.2σ to 2.7σ level. So, the Hubble tension is reduced compared to that in the standard analysis but still remains statistically implausible. We examine this tension in various extensions of the ΛCDM model afterwards.

### 4.2 Parameter constraints

It is instructive to compare our CMB-based parameter constraints with those in the standard full-likelihood analysis. To this end we consider the Planck 2018 likelihood [2]. To allow straightforward comparison we impose the same Gaussian prior on $\tau$ as used in our baseline analysis. The resulting 2d posterior distributions for the Planck 2018 and Base data are shown in Fig. 4. The corresponding 1d marginalized constraints on cosmological parameters are listed in Tab. 2.

| Parameter | Planck 2018 | Base | Base+LSS | Base+LSS+S$_8$ |
|-----------|-------------|------|----------|----------------|
| $100 \omega_b$ | 2.241 ± 0.015 | 2.255 ± 0.020 | 2.240 ± 0.018 | 2.247 ± 0.018 |
| $10 \omega_{cdm}$ | 1.197 ± 0.011 | 1.151 ± 0.018 | 1.174 ± 0.010 | 1.163 ± 0.008 |
| $H_0$ | 67.53 ± 0.50 | 69.09 ± 0.84 | 68.01 ± 0.46 | 68.49 ± 0.38 |
| $\tau$ | 0.060 ± 0.005 | 0.058 ± 0.005 | 0.055 ± 0.005 | 0.053 ± 0.005 |
| $\ln(10^{10} A_s)$ | 3.055 ± 0.011 | 3.036 ± 0.012 | 3.034 ± 0.012 | 3.028 ± 0.011 |
| $n_s$ | 0.967 ± 0.004 | 0.977 ± 0.006 | 0.971 ± 0.005 | 0.973 ± 0.005 |
| $r_{\text{drag}}$ | 147.12 ± 0.25 | 148.18 ± 0.43 | 147.75 ± 0.31 | 147.98 ± 0.28 |
| $\Omega_m$ | 0.313 ± 0.007 | 0.290 ± 0.010 | 0.304 ± 0.006 | 0.297 ± 0.005 |
| $\sigma_8$ | 0.815 ± 0.005 | 0.793 ± 0.008 | 0.799 ± 0.006 | 0.793 ± 0.005 |
| $S_8$ | 0.833 ± 0.013 | 0.780 ± 0.020 | 0.803 ± 0.012 | 0.789 ± 0.009 |

Table 2. Parameter constraints in the standard ΛCDM model with 1σ errors. The Gaussian prior on $\tau$ (2.1) is adopted from Ref. [41]. The Base data set includes PlanckTT-lowℓ+SPT-3G+Lens.

We found that resulting parameter constraints in the full Planck and combined data approach are substantially different. In particular, the full-likelihood approach predicts a $\sim 2.3\sigma$ higher values of $\sigma_8$ and $S_8$ compared to the Base results. This drives the 3.3σ tension with the local probes of $S_8$ in the baseline Planck analysis.
Figure 4. Marginalized 2d posterior distributions of the cosmological parameters in the ΛCDM model for Planck 2018 (black), Base (blue) and Base+LSS (red) data sets. The Gaussian prior on $\tau$ (2.1) is set from [41]. The yellow bands represent 1σ and 2σ constraints on $S_8$ (2.2) coming from the photometric surveys (DES-Y3, KiDS, HSC), whereas the green bands refer to the $H_0$ measurement (2.3) reported by the SH0ES collaboration.

This effect is attributed to the lensing-like anomaly in the Planck data that pulls the late-time amplitude to a higher value. We use the alternative CMB measurements at small angular scales that makes our analysis pipeline insensitive to this feature. We also found that the mean value of $H_0$ in the full-likelihood analysis is 1.6σ lower.
than the value suggested by the Base data. This shift can also be explained by the Planck lensing-like anomaly because $\sigma_8$ is anti-correlated with $H_0$ [14].

We complement the information from the Base data with up-to-date LSS measurements (without $S_8$). We found that the cosmological measurements drastically improve upon including the LSS information. In particular, it provides a twice more accurate measurement of $\omega_{cdm}$. This effect is attributed to the full-shape information encoded in the BOSS power spectra which primarily constrains this parameter. The LSS data also shrinks the error bars of $H_0$ and $S_8$ by 45% and 40%, respectively, when compared with the Base only results. It leads to the more severe 3.8σ tension with the SH0ES measurement. Remarkably, the Base + LSS data analysis is consistent with the direct probes of $S_8$ at 1.7σ level. It justifies further account for the $S_8$ data.

Finally, we add the information from the photometric surveys given by the Gaussian prior on $S_8$. We found that the $S_8$ data improves the $S_8$ and $H_0$ measurements by 25% and 20%, respectively. Specifically, our final estimates inferred from the complete Base + LSS + $S_8$ likelihood read

$$S_8 = 0.789 \pm 0.009, \quad H_0 = 68.49 \pm 0.38 \text{ km s}^{-1}\text{Mpc}^{-1}. \quad (4.2)$$

Our results demonstrate a perfect agreement with the direct measurements of $S_8$ (2.2) (at 1.1σ level). Interestingly, the mean value of $H_0$ raises up by 1σ upon including the $S_8$ information that alleviates the Hubble tension to 3.5σ level, cf. with (2.3).

Overall, our results demonstrate the importance of the combined CMB data approach for reliable parameter inference in the ΛCDM model. The CMB data free from the Planck lensing-like anomaly opens up new previously unexplored directions in parameter space which allows one to satisfy the direct constraints on $S_8$ (2.2) imposed by photometric surveys. To a lesser extent it alleviates the $H_0$ tension.

5 Minimal extensions of the base-ΛCDM model

In this Section we apply our combined CMB data approach in the ΛCDM+$\sum m_\nu$ and ΛCDM+$N_{\text{eff}}$ extensions. We examine parameter inference and assess the possibilities of both models to alleviate the cosmological tensions.

5.1 ΛCDM+$\sum m_\nu$

The resulting 2d posterior distributions for the Planck 2018 and Base analyses in the ΛCDM+$\sum m_\nu$ model are shown in Fig. 5. The corresponding 1d marginalized
Figure 5. Marginalized 2d posterior distributions of the cosmological parameters in the ΛCDM+$\sum m_\nu$ model for Planck 2018 (black), Base (blue) and Base+LSS (red) data sets. The Gaussian prior on $\tau$ (2.1) from Ref. [41] is adopted. The yellow bands represent 1σ and 2σ constraints on $S_8$ (2.2) coming from the photometric surveys (DES-Y3, KiDS, HSC), whereas the green bands refer to the $H_0$ measurement (2.3) reported by the SH0ES collaboration.

constraints on cosmological parameters are listed in Tab. 3.

It is worth to illuminate the main differences between our approach and the Planck 2018 analysis. First, the Base data predicts the 2.2σ lower value of $S_8$ when
Planck 2018

The upper limits on neutrino masses relax the gravitational lensing signal which exacerbates the tension at the level of 3.3σ. Second, we found that both approaches predict consistent constraints on $H_0$ but its error bar is two-times larger in the Base data analysis. Finally, we obtain the following CMB-based constraint on the total neutrino mass, $\sum m_\nu < 0.51$ eV (2σ). The error bar on $\sum m_\nu$ is significantly inflated with respect to that in the Planck 2018 analysis \(^7\). Note that the Planck lensing-like anomaly artificially strengthens the constraint on the total neutrino mass as the positive neutrino masses relax the gravitational lensing signal which exacerbates the tension.

It is instructive to compare our constraint on the total neutrino mass with the result of the analysis based on the model-independent reconstruction of the gravitothermal error in $\sum m_\nu$. The Gaussian prior on $\tau$ (2.1) is imposed following [41]. Recall, the Base data set includes PlanckTT-lowℓ+SPT-3G+Lens.

\(^7\)Note that our Planck 2018 constraint on the total neutrino mass is somewhat weaker than the Planck baseline estimate, $\sum m_\nu < 0.24$ eV (2σ) [2]. This effect can be explained as follows. First, we use the Gaussian prior on $\tau$ (2.1) which inflates the error in $\sum m_\nu$ as compared to the baseline Planck analysis which employs the low multipole polarization likelihood. Second, we exploit a 0.6σ higher value of $\tau$ (2.1) [41] which raises up $A_s$ and requires somewhat higher values of $\sum m_\nu$. 

| Parameter | Planck 2018 | Base | Base + LSS | Base + LSS + $S_8$ |
|-----------|-------------|------|------------|---------------------|
| $100\omega_b$ | $2.239 \pm 0.015$ | $2.246 \pm 0.022$ | $2.246 \pm 0.018$ | $2.247 \pm 0.017$ |
| $10\omega_{cdm}$ | $1.200 \pm 0.013$ | $1.163 \pm 0.021$ | $1.162 \pm 0.012$ | $1.159 \pm 0.008$ |
| $H_0$ | $67.03_{-0.71}^{+1.47}$ | $67.02_{-1.54}^{+2.54}$ | $67.15 \pm 0.59$ | $67.15 \pm 0.60$ |
| $\tau$ | $0.060 \pm 0.005$ | $0.058 \pm 0.005$ | $0.057 \pm 0.005$ | $0.057 \pm 0.005$ |
| $\ln(10^{10}A_s)$ | $3.057 \pm 0.011$ | $3.040 \pm 0.012$ | $3.037 \pm 0.012$ | $3.037 \pm 0.012$ |
| $n_s$ | $0.966 \pm 0.004$ | $0.973 \pm 0.007$ | $0.974 \pm 0.005$ | $0.975 \pm 0.005$ |
| $\sum m_\nu$ | < 0.30 | < 0.51 | 0.22 $\pm$ 0.07 | 0.23 $\pm$ 0.06 |
| $r_{drag}$ | 147.07 $\pm$ 0.28 | 147.93 $\pm$ 0.49 | 147.98 $\pm$ 0.34 | 148.03 $\pm$ 0.28 |
| $\Omega_m$ | $0.320 \pm 0.016$ | $0.316 \pm 0.027$ | $0.313 \pm 0.007$ | $0.313 \pm 0.007$ |
| $\sigma_8$ | $0.806 \pm 0.019$ | $0.760 \pm 0.031$ | $0.761 \pm 0.018$ | $0.758 \pm 0.013$ |
| $S_8$ | $0.832 \pm 0.013$ | $0.778 \pm 0.021$ | $0.777 \pm 0.018$ | $0.774 \pm 0.010$ |

**Table 3.** Parameter constraints in the standard $\Lambda$CDM+$\sum m_\nu$ model with 1σ errors. The upper limits on neutrino masses are given at 95% CL. The Gaussian prior on $\tau$ (2.1) is imposed following [41]. Recall, the Base data set includes PlanckTT-lowℓ+SPT-3G+Lens.
tational lensing potential \[15\]. That approach performs the marginalizing over the lensing principal components that removes any effect of the peak smoothing in the temperature and polarization power spectra on constraints of the cosmological parameters. Allowing arbitrary gravitational lensing in the Planck TT,TE,EE maps the constraint on the total neutrino mass significantly degrades, $\sum m_\nu < 0.87 \text{eV} \ (2\sigma) \ [15]$, which is 40% weaker than the limit suggested by our Base data. The improvement found in our approach is simply explained by the SPT-3G data which independently constrains $\sum m_\nu$ through the CMB gravitation lensing at small angular scales. Note that the lensing information delivered by SPT-3G is fully consistent with the $\Lambda$CDM expectation \[31\].

Next, we assess the impact of the LSS data on cosmological constraints. Let us illuminate the key features of the Base + LSS analysis. First, the LSS data tremendously improves (more than 3 times) the accuracy of $H_0$ measurement. This effect is driven by the distance information encoded in the BOSS full-shape spectra and anisotropic BAO measurements at intermediate redshifts. The error bar of $\sigma_8$ also shrinks by 40% upon including the LSS likelihood. Note that our analysis does not employ any information from photometric surveys but its result is perfectly consistent with the direct probes of $S_8$. Intriguingly, we found the 3.1$\sigma$ evidence for nonzero neutrino masses, namely $\sum m_\nu = 0.22 \pm 0.07 \text{eV}$. The LSS data effectively breaks the parameter degeneracies present in the CMB data that allows one to pinpoint neutrino masses.

We finally add the $S_8$ data. Our analysis reveals that the including $S_8$ likelihood substantially improves the accuracy in measurements of $\sigma_8$ and $S_8$, by 30% and 45%, respectively. Striking, the final estimate of the total neutrino mass remains largely unchanged. This result indicates that the information on neutrino masses comes from breaking the degeneracies between the LSS and CMB rather than from a late-time measurement of the fluctuation amplitude itself. All other constraints do not change significantly which demonstrates an excellent agreement between Base + LSS and $S_8$ data sets.

Our results are summarized in Fig. 1. We found the 4$\sigma$ preference for nonzero $\sum m_\nu$ from the Base + LSS + $S_8$ data,

$$\sum m_\nu = 0.23 \pm 0.06 \text{eV}. \ (5.1)$$

Our measurement is consistent with both neutrino mass hierarchies. We demonstrated that the information gain comes from breaking of the degeneracies between the LSS and Base data. In the standard approach, the Planck lensing-like anomaly
The resulting 2d posterior distributions for the Planck 2018 and Base analyses in the $\Lambda$CDM+$N_{\text{eff}}$ model are shown in Fig. 6. The corresponding 1d marginalized constraints on cosmological parameters are listed in Tab. 4.

Let us highlight the key differences between our approach and the Planck 2018 analysis. As previously, the Base data suggests a significantly lower $S_8$ compared to the Planck 2018 analysis that makes the combined data analysis completely consistent with the weak lensing measurements (2.2). Then, the Base data analysis favours

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Table 4. Parameter constraints in the $\Lambda$CDM+$N_{\text{eff}}$ model with 1σ errors. The Gaussian prior on $\tau$ (2.1) is imposed from [41]. Recall, the Base data set includes PlanckTT-lowℓ+SPT-3G+Lens.
Figure 6. Marginalized 2d posterior distributions of the cosmological parameters in the $\Lambda$CDM+$N_{\text{eff}}$ model for Planck 2018 (black), Base (blue) and Base+LSS (red) data sets. The Gaussian prior on $\tau$ (2.1) is set from results of Ref. [41]. The yellow bands represent $1\sigma$ and $2\sigma$ constraints on $S_8$ (2.2) coming from the photometric surveys (DES-Y3, KiDS, HSC), whereas the green bands refer to the $H_0$ (2.3) measurement reported by the SH0ES collaboration.

larger values of $H_0$ with a 40% larger error bar. Finally, for the effective number of relativistic degrees of freedom we found $N_{\text{eff}} = 3.00 \pm 0.28$. This constraint is consistent with the Planck 2018 result [2].
It is also instructive to compare our outcomes with the result of the analysis based on the model-independent decomposition of the gravitation lensing signal from CMB power spectra [15]. Allowing arbitrary gravitational lensing in the Planck maps one gets $H_0 = 68.2 \pm 1.6 \text{ km s}^{-1}\text{Mpc}^{-1}$ [15]. This estimate agrees well with both the Base and Planck 2018 data analyses. Unlike the $\sum m_\nu$ limit, the error bar of $H_0$ is only moderately inflated upon marginalization over lensing signal. This effect can be attributed to the fact that the CMB gravitational lensing barely affects the CMB angular acoustic scales and therefore contains a very little information on $H_0$ which is mainly controlled by the position of the first acoustic peak. It explains why the final constraint on $H_0$ does not degrade significantly after marginalization over lensing information in the CMB. More precise polarization data or reliable estimate of temperature power spectrum on small scales is needed to achieve a higher accuracy in the measurement of $H_0$.

Next, we complement our CMB-based analysis with the LSS data. Adding the LSS information provides more refined estimates of all cosmological parameters. In particular, it reduces the uncertainty in values of $S_8$ and $H_0$ by 40%. We also found a 1σ lower value of $H_0$ which is more consistent with the Planck 2018 constraint. Adding the LSS likelihood also shrinks the error bar of $N_{\text{eff}}$ by 30% resulting in $N_{\text{eff}} = 2.95 \pm 0.22$.

Finally, we add the $S_8$ data. We found that the adding of $S_8$ information significantly improves only the $S_8$ measurement, namely by 30%. All other constraints remain largely unchanged.

Let us summarise the main results. We do not detect any significant difference in cosmological constraints between our approach and the standard full-likelihood analysis. In the Base + LSS + $S_8$ analysis we find

$$N_{\text{eff}} = 2.87 \pm 0.21$$

Our results indicate that the Planck lensing-like anomaly does not propagate into the cosmological constraints in the $\Lambda$CDM+$N_{\text{eff}}$ model and using the full Planck likelihood is safe there. This result can be readily understood. Allowing arbitrary $N_{\text{eff}}$ alters the value of sound horizon at recombination which changes the angular distribution of CMB peaks and troughs. In turn, the smoothing effect of acoustic peaks at the first approximation is insensitive to geometric information encoded in CMB. It explains why the Planck lensing-like anomaly does not affect the $N_{\text{eff}}$ and other parameter constraints. Our analysis also does not recognize any deviations from the standard cosmological model prediction $N_{\text{eff}} \approx 3.044$. 


6 Phantom Dark Energy

In this Section we explore one late-time cosmological scenario with a phantom crossing in dark energy sector.

6.1 Model description

We parametrize the phantom crossing behaviour in the dark energy through its energy density $\rho_{\text{PDE}}$ following [26]

$$\rho_{\text{PDE}}(a) = \rho_0 [1 + \alpha(a - a_m)^2 + \beta(a - a_m)^3].$$  \hspace{1cm} (6.1)

where $\rho_0$ normalizes the dark energy density, $a_m$ defines the moment when the dark energy density passes the extremum, $\alpha, \beta$ control the shape of phantom crossing and we choose the present scale factor to be $a = 1$. We ignore the first order derivative term in the Taylor expansion (6.1) because $\rho_{\text{PDE}}$ has an extremum at $a = a_m$.

Inserting (6.1) into the Friedman equation for the flat space

$$H^2 = \frac{8\pi G}{3} [\rho_m + \rho_{\text{rad}} + \rho_{\text{PDE}}],$$  \hspace{1cm} (6.2)

we get the following evolution for the Hubble parameter,

$$\frac{H^2(a)}{H_0^2} = \frac{\Omega_m}{a^3} + \frac{\Omega_{\text{rad}}}{a^4} + (1 - \Omega_m - \Omega_{\text{rad}}) \frac{1 + \alpha(a - a_m)^2 + \beta(a - a_m)^3}{1 + \alpha(1 - a_m)^2 + \beta(1 - a_m)^3},$$  \hspace{1cm} (6.3)

and for the dark energy equation of state,

$$w_{\text{PDE}}(a) = -1 - \frac{a[2\alpha(a - a_m) + 3\beta(a - a_m)^2]}{3[1 + \alpha(a - a_m)^2 + \beta(a - a_m)^3]}.$$  \hspace{1cm} (6.4)

The PDE model is parametrised with the set of three parameters, $(a_m, \alpha, \beta)$.

We implement (6.3) and (6.4) into the Boltzmann solver CLASS. We vary all PDE parameters $(\alpha, \beta, a_m)$ along with the six standard $\Lambda$CDM $(\omega_{\text{cdm}}, \omega_b, H_0, A_s, n_s, \tau)$. We impose the following flat uniform priors on PDE parameters

$$a_m \in [0, 1], \quad \alpha \in [0, 30], \quad \beta \in [0, 30].$$  \hspace{1cm} (6.5)

6.2 Parameter constraints

The final 2d posterior distributions in the PDE model are shown in Fig. 7. The corresponding 1d marginalized constraints on cosmological parameters are listed in Tab. 5. We found that the CMB data alone can not break degeneracies present in the PDE model. It makes the PDE parameters largely unconstrained in full accordance
with the results of Ref. [26]. Given this reason, we do not consider the Base data only.

We start with the Base+LSS analysis. We found a substantial preference for a phantom crossing in dark energy sector at late times, $a_m = 0.774_{-0.020}^{+0.037}$. For
the dark energy parameters we obtain $\alpha = 8.2^{+2.6}_{-3.7}$, $\beta = 14.2^{+6.7}_{-8.7}$. Since the distributions of these parameters are highly non-gaussian, the Base+LSS data suggests a 4\sigma indication for a transition in the dark energy evolution. We found $H_0 = 75.70^{+2.05}_{-2.32}$ km s$^{-1}$ Mpc$^{-1}$. The $H_0$ constraint is perfectly consistent within one standard deviation with the SHOES measurement and in the significant 3.7\sigma tension with the value reported by Planck collaboration. Intriguingly, we found $S_8 = 0.770 \pm 0.017$ which is in a remarkable agreement with the weak lensing measurements (2.2). We emphasize that our analysis does not include any priors on late-time parameters but its result is fully consistent with the direct measurements of $S_8$ (2.2) and $H_0$ (2.3) in the late Universe.

Next, we proceed with the $S_8$ data. We found that adding the $S_8$ information (2.2) does not affect the posterior distributions of the PDE parameters. But it improves the $S_8$ and $\omega_{cdm}$ measurements by 40\% and 30\%, respectively. Constraints on all other cosmological parameters remain largely unchanged that indicates a remarkable agreement between the Base+LSS and $S_8$ data sets.

**Table 5.** Parameter constraints in the PDE model with 1\sigma errors. The Gaussian prior on $\tau$ informed by [41] is always adopted.
The Base + LSS + S\(_8\) and \(H_0\) data are in agreement now, so we can combine them safely together. The combined data analysis shows a strong preference for a transition in the dark energy density, \(a_m = 0.773^{+0.044}_{-0.025}\). The \(H_0\) likelihood efficiently breaks the degeneracy between PDE and standard cosmological parameters. It results in significantly tighter constraints on the dark energy parameters, \(\alpha = 6.5^{+1.6}_{-2.8}\), \(\beta = 10.2^{+4.5}_{-6.9}\). After including the SH0ES information the error bar of \(\sigma_8\) gets reduced by 40% that is explained by breaking the degeneracy between \(\sigma_8\) and \(H_0\) parameters shown in Fig. 7. Note that the Base + LSS + S\(_8\) + \(H_0\) analysis predicts a 2\(\sigma\) higher value of \(\sigma_8\) compared to the Planck result [2]. Even so, the estimate \(S_8 = 0.774 \pm 0.010\) is perfectly consistent with the weak lensing measurements (2.2) due to a substantially lower matter density fraction \(\Omega_m\). Eventually, we obtain \(H_0 = 73.92 \pm 1.09 \text{ km s}^{-1}\text{Mpc}^{-1}\) which is two-times more accurate than the Base + LSS + S\(_8\) result (without SH0ES). A higher value of \(H_0\) in the PDE scenario can be explained by the positive correlations between \(\alpha\), \(\beta\) and \(H_0\) parameters shown in Fig. 7.

Constraints in Tab. 5 demonstrate that all data set combinations agree with a larger value of \(H_0\). Our proof-of-concept analysis shows that a late-time solution can bring the value of \(H_0\) into accordance with the SH0ES measurement (2.3) without necessity to alter the early-time physics. Importantly, the value of \(r_{\text{drag}}\) remains essentially the same as in the \(\Lambda\)CDM scenario \(^8\). This result may seem counter intuitive if the BAO data are taken into account. Indeed, BAO measurements directly constrain \(D_A(z)/r_{\text{drag}}\) and \(r_{\text{drag}}H(z)\), so a different Universe evolution in late times could alter the BAO distances and hence would require the proper shift in \(r_{\text{drag}}\) to preserve the BAO fit. For monotonic evolution of the dark energy density at late times the radial BAO scale can be translated to the present-day parameter combination \(r_{\text{drag}}H_0\) which sets up a certain relation between \(r_{\text{drag}}\) and \(H_0\) [54]. But if the behaviour of dark energy density is not-monotonic (akin to PDE) the final result strongly depends on a particular dynamics in the dark energy sector [55]. Our findings indicate that the model with a phantom crossing predicts essentially the same BAO distances as in the \(\Lambda\)CDM model.

It is instructive to demonstrate explicitly that the model with a phantom crossing in dark energy equation of state can fit to the BAO data at intermediate redshifts. To this end, we show the behaviour of the Hubble parameter and the inverse BAO distance for the Base + LSS + S\(_8\) + \(H_0\) analysis in Fig. 8. We found that the

\(^8\)A modification of early-time physics shifts the CMB-inferred sound horizon at radiation drag down relative to the \(\Lambda\)CDM prediction [54]. In contrast, the late-time solutions does not alter the Universe evolution at early times and hence \(r_{\text{drag}}\) remains unchanged.
Figure 8. Behaviour of $H/(1+z)$ (left panel) and $\log(1+z)/(1+z)/D_A$ (right panel) for the Base + LSS + $S_8$ data in the PDE model. Both quantities are measured in units of km s$^{-1}$ Mpc$^{-1}$. The absolute scale for the BAO measurements is set by the best-fit value of the sound horizon optimized to the Base likelihood $r_{\text{drag}} = 148.04$ Mpc.

PDE scenario agrees with all BAO measurements, as well as higher values of $H_0$ consistent with SH0ES, leaving unaltered $r_{\text{drag}}$. It means that the PDE solution is fully consistent with the BAO distances calibrated to the inferred from CMB value of $r_{\text{drag}}^9$.

We have also carried out the Bayesian model comparison between the PDE and $\Lambda$CDM scenarios. We compute the Bayesian evidence with the publicly available cosmological code MCEvidence $^{10}$ [56]. We present our result in the form of the Bayes factor, $\ln B \equiv \ln Z_{\text{PDE}} - \ln Z_{\Lambda\text{CDM}}$ where $Z$ is the Bayesian evidence. For the most complete data set Base + LSS + $S_8$ + $H_0$ we found

$$\ln B = 1.13$$ \hspace{1cm} (6.6)

The PDE model is found to have positive preference over $\Lambda$CDM one. According to Jeffreys scale [57] the preference is definite. Note that the PDE analysis [26] based on the full Planck likelihood revealed a negative value of $\ln B$ for the complete data set that suggests the preference on $\Lambda$CDM over PDE model. Detailed comparison of our approach and that of Ref. [26] is presented in Appendix A.

$^{9}$It agrees with the result of Ref. [22] showing that dynamical dark energy with phantom crossing is capable of matching BAO data completely.

$^{10}$github.com/yabebalFantaye/MCEvidence.
7 Transitional Dark Energy

In this Section we examine another late-time solution which introduces a fast transition in the dark energy evolution.

7.1 Model description

We parametrize the rapid change in the dark energy density following [27]

$$\rho_{\text{TDE}} = \rho_{\text{TDE},0}(1 + z)^{3(1 + W(z))},$$

$$W(z) = \frac{1}{2} \left( (w_0 + w_1) - (w_1 - w_0) \tanh \left( \frac{z - z_{\text{tr}}}{\Delta_{\text{tr}}} \right) \right),$$

(7.1)

where $W(z)$ is related to the dark energy equation of state $w_{\text{TDE}}$ through

$$W(z) = \frac{1}{\ln(1 + z)} \int_0^z w_{\text{TDE}}(z') \frac{dz'}{1 + z'}$$

(7.2)

The dark energy equation of state tends towards $w_0$ at $z < z_{\text{tr}}$ and towards $w_1$ at $z > z_{\text{tr}}$ where $z_{\text{tr}}$ refers to the moment of transition. The width of this transition is parameterized by $\Delta_{\text{tr}}$. In essence, at early times the dark energy is completely absent and then rapidly turns on at the time interval $|z - z_{\text{tr}}| \lesssim \Delta_{\text{tr}}$\textsuperscript{11}.

We implement (7.1) into the standard Boltzmann solver CLASS. We vary all four TDE parameters ($w_0$, $w_1$, $z_{\text{tr}}$, $\Delta_{\text{tr}}$) along with the six standard ones $\Lambda$CDM ($\omega_{\text{cdm}}$, $\omega_b$, $H_0$, $A_s$, $n_s$, $\tau$). We impose the following flat uniform priors on TDE parameters:

$$w_0 \in [-\infty, +\infty], \quad w_1 \in [-4, 0],$$

$$z_{\text{tr}} \in [0, 10], \quad \Delta_{\text{tr}} \in [0, 10]$$

(7.3)

We adopt flat uninformative priors on $w_0$ to obtain a robust distribution of this parameter.

7.2 Parameter constraints

The resulting 2d posterior distributions in the TDE model are shown in Fig. 9. The corresponding 1d marginalized constraints on cosmological parameters are listed in Tab. 6. Here we also do not show the results of the Base data analysis because CMB alone can not break the degeneracies which present in the TDE sector.

\textsuperscript{11}We emphasize that the dark energy density in the TDE model is constrained to be positive. Reconstruction techniques which allow for negative dark energy have greater flexibility to fit cosmological data, see for instance Refs. [28, 58].
Let us start with the Base + LSS analysis. We do not find a decisive evidence for a sharp transition in the TDE equation of state at intermediate redshifts, but the posteriors are consistent with this scenario. We found that $z_{\text{tr}}$ and $\Delta_{\text{tr}}$ are largely un-
| Parameter | Base + LSS | Base + LSS + S_8 | Base + H_0 |
|-----------|------------|-----------------|------------|
| \(w_0\)   | \(-0.94^{+0.15}_{-0.20}\) | \(-0.98^{+0.12}_{-0.14}\) | \(-0.83^{+0.24}_{-0.47}\) |
| \(w_1\)   | \(-1.54^{+1.14}_{-0.34}\) | \(-1.29^{+0.88}_{-0.11}\) | \(-2.09^{+1.47}_{-0.73}\) |
| \(z_{tr}\) | > 4.6     | > 4.7           | –          |
| \(\Delta_{tr}\) | < 6.0     | < 5.1           | –          |
| \(100\omega_b\) | 2.244 ± 0.020 | 2.252 ± 0.018   | 2.256 ± 0.021 |
| \(10\omega_{cdm}\) | 1.170 ± 0.015 | 1.158 ± 0.012   | 1.151 ± 0.018 |
| \(H_0\)   | 67.62^{+1.38}_{-1.66} | 67.77^{+1.32}_{-1.70} | 73.30^{+1.31}_{-1.30} |
| \(\tau\)  | 0.057 ± 0.005 | 0.056 ± 0.006   | 0.057 ± 0.005 |
| \(\ln(10^{10}A_s)\) | 3.038 ± 0.012 | 3.033 ± 0.012   | 3.035 ± 0.012 |
| \(n_s\)   | 0.973 ± 0.006 | 0.976 ± 0.005   | 0.977 ± 0.006 |
| \(r_{drag}\) | 147.80 ± 0.37 | 148.06 ± 0.32   | 148.20 ± 0.43 |
| \(\Omega_m\) | 0.307 ± 0.013 | 0.303 ± 0.013   | 0.258 ± 0.010 |
| \(\sigma_8\) | 0.793 ± 0.019 | 0.781 ± 0.016   | 0.831 ± 0.018 |
| \(S_8\)   | 0.801 ± 0.016 | 0.785 ± 0.010   | 0.769 ± 0.018 |

Table 6. Parameter estimates (1σ error bars or limits at 1σ confidence level) in the transitional dark energy model. The Gaussian prior on \(\tau\) (2.1) (2.1) is adopted from [41]. The limits on \(z_{tr}\) and \(\Delta_{tr}\) for Base + H_0 are dominated by flat priors (7.3), so we do not show constraints on these parameters.

Constrained showing only a mild preference for lower and upper bounds, \(z_{tr} > 4.6\) and \(\Delta_{tr} < 6.0\) at 65% CL. The Base+LSS analysis predicts \(H_0 = 67.62^{+1.38}_{-1.66}\) km s\(^{-1}\)Mpc\(^{-1}\). This \(H_0\) estimate is fully consistent with the Planck result and disagrees with the SH0ES measurement (2.3) at the 2.8σ level. We find that the value of the clustering amplitude \(\sigma_8\) is smaller with respect to the ΛCDM value in full agreement with the claim of Ref. [27] that the TDE model predicts a slower growth of cosmic structures at late times. But this effect is entirely compensated by a positive shift in \(\Omega_m\), so the eventual estimate of \(S_8\) remains virtually unchanged, \(S_8 = 0.801 ± 0.016\). Even so, this result is entirely consistent with the direct probes (2.2) in the late Universe.

Next, we include the \(S_8\) data. We found that adding \(S_8\) information noticeably improves the \(w_0\) and \(w_1\) constraints. The posterior distributions still indicate no substantial preference for a transition in the dark energy equation of state. Upon
including the information from photometric surveys the accuracy in measurements of \( \omega_{cdm} \) and \( S_8 \) increases by 20% and 40%, respectively. Expectedly, the estimated value of \( S_8 \) is in a remarkable agreement with the weak lensing data (2.2).

The Base + \( S_8 \) and \( H_0 \) likelihoods are not consistent within the TDE framework, so one can not combined these data sets together. For illustrative purpose we discard all large-scale structure data (the LSS and \( S_8 \)) and examine the cosmological inference from the Base + \( H_0 \) alone. In this case, we found a mild preference for a transition in the dark energy density, \( w_0 = -0.83^{+0.24}_{-0.47} \) and \( w_1 = -2.09^{+1.47}_{-0.73} \). The limits on \( z_{tr} \) and \( \Delta_{tr} \) are prior-dominated but the best-fit values of these parameters (\( z_{tr}^{\text{best-fit}} = 1.88 \) and \( \Delta_{tr}^{\text{best-fit}} = 1.60 \)) are fully consistent with a sharp transition in the dark energy evolution at intermediate redshifts. We obtain \( H_0 = 73.30^{+1.31}_{-1.30} \) km s\(^{-1}\) Mpc\(^{-1}\) which is now perfectly consistent with the SH0ES measurement (2.3). Remarkably, the Base + \( H_0 \) analysis predicts lower values of the clustering parameter, \( S_8 = 0.770 \pm 0.018 \), without any input from low redshift observations.

To elucidate the source of strong constraints on the TDE scenario imposed by the large-scale structure data we show the behaviour of the Hubble parameter and the inverse BAO distance for the Base + LSS and Base + \( H_0 \) analyses along with the multiple BAO measurements and SH0ES constraint in Fig. 10. We found that the Base+LSS prediction is in a remarkable agreement with all BAO measurements but provides a low value of \( H_0 \) being more consistent with the value reported by the Planck collaboration. On the contrary, the Base+\( H_0 \) analysis predicts a higher

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**Figure 10.** Behaviour of \( H(z)/(1 + z) \) and \( \log(1 + z)/(1 + z)/D_A \) in the TDE model. Both quantities are measured in units of km s\(^{-1}\) Mpc\(^{-1}\). The absolute scale for the BAO measurements is set by the best-fit value of the sound horizon optimized to the Base likelihood \( r_{\text{drag}} = 148.04 \) Mpc.

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$H_0$ but fails to reproduce the transverse BAO distances inferred from the BOSS galaxy sample. Interestingly, the radial BAO separations are broadly consistent with the Base+$H_0$ prediction. It exemplifies the internal conflict between the BAO measurements and higher $H_0$ in the TDE paradigm: once the BAO information is discarded the global fit can accommodate a high value of $H_0$ consistent with SH0ES (2.3).

8 Conclusions

We have performed fitting the cosmological data within several physically motivated cosmological models: $\Lambda$CDM and four its extensions. We included the SPT-3G polarization, SPTPol gravitational lensing and Planck temperature CMB measurements along with the full-shape BOSS, BAO and weak-lensing measurements.

We start with revisiting the combined data approach originally introduced in Ref. [1] by including the recent SPT-3G measurements of $TE$ and $EE$ power spectra. Our analysis affirms the conclusions of the previous study [1]. In particular, this CMB setup accommodates a lower value $S_8$ in full agreement with the weak-lensing measurements and other direct probes in the late Universe. We also found that the Hubble tension is partially alleviated to $3.5\sigma$ after including the large-scale structure data.

Then, we examine the cosmological inference in the $\Lambda$CDM+$\sum m_\nu$ and $\Lambda$CDM+$N_{\text{eff}}$ models. We found that the Base + LSS + $S_8$ data exhibits a $4\sigma$ detection of neutrino masses $\sum m_\nu = 0.23 \pm 0.06\text{eV}$. This preference only moderately weakens to $3.1\sigma$ if one replaces the SPTPol measurement of the gravitational lensing potential with the Planck lensing data. In the conventional data analysis, the Planck lensing-like anomaly strengthens the constraints on neutrino masses making such higher values of neutrino masses implausible [2]. Our CMB setup is free from this feature and hence provides reliable inference of neutrino masses. In contrast, in the $\Lambda$CDM+$N_{\text{eff}}$ scenario we found that the Planck anomalies do not propagate into cosmological constraints, so using the full Planck data is safe there.

Finally, we analyze two late-time modifying scenarios which drastically modify the dark energy sector. First model introduces a phantom crossing in dark energy equation of state and was suggested to alleviate the $H_0$ tension between low and high redshift observations [26]. We found that the this scenario can accommodate a larger $H_0$ even not considering the local $H_0$ measurements. The complete Base + LSS + $S_8$ + $H_0$ analysis yields $H_0 = 73.92 \pm 1.09\text{km s}^{-1}\text{Mpc}^{-1}$ in full agreement
with SH0ES. In contrast, the standard analysis [26] after combining all data sets demonstrates a mild $2.3\sigma$ tension with the local $H_0$ measurement. The statistical analysis based on the Bayesian approach revealed that the PDE scenario is definitely preferred against ΛCDM. The second scenario introduces a fast transition in the dark energy evolution at late times which could alleviate cosmological tensions [27]. We found that the TDE model fails to reproduce both the BAO distances and local $H_0$ measurement simultaneously. Accordingly, the Base + LSS + S8 data suggests $H_0 = 67.77^{+1.32}_{-1.70}$ km s$^{-1}$ Mpc$^{-1}$ in full agreement with the value reported by Planck collaboration.

Our analysis emphasises the extreme importance of robust measurements of the small-scale CMB anisotropies to reliably predict the posterior probability distribution of cosmological parameters. We observe that using the full Planck likelihood obscures the cosmological inference in the valuable extensions of the base-ΛCDM model. First, our CMB setup suggests a $4\sigma$ detection of nonzero neutrino masses whereas the full Planck likelihood rules out this possibility. Second, the SPT and Planck data combined together suggest a complete alleviation of the $H_0$ tension within the PDE framework whilst the full-likelihood approach reduces this tension to $2.3\sigma$ only.

Our findings indicate that the Planck anomaly represents a severe problem. It pulls the late-time clustering amplitude to a higher value causing the tension with the large-scale structure measurements. This anomaly is responsible for stringent limits on the total neutrino mass imposed by Planck as the positive neutrino masses relax the gravitational lensing signal which exacerbates the tension. It also results in substantially lower values of $H_0$ in the PDE scenario. We conclude that the understanding of the ”lensing-like anomaly” in the Planck data becomes mandatory. Future high-resolution CMB data from the Simons Observatory [59] and CMB-S4 experiment [60] will provide the robust measurement of the CMB lensing effect down to very small angular scales which allows one to clarify the source behind this anomaly.

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Comparison with previous study of PDE

It is instructive to compare our results with those of the full Planck analysis reported in Ref. [26]. First, we confront the cosmological inference governed by the Base+LSS data with the CMB+BAO analysis from Ref. [26]. We found that the constraints on the PDE parameters are broadly consistent in these two approaches. But the Base + LSS analysis brings the two-fold improvement for \( a_m \) and marginally improves the constraints on \( \alpha, \beta \) parameters. As a result, the combined data approach demonstrates a decisive evidence for \( a_m < 1 \), whereas in the CMB+BAO analysis [26] predicts a much milder (\( \simeq 2\sigma \)) preference for a transition in the dark energy evolution. The full-likelihood approach [26] yields \( H_0 = 71.0^{+2.9}_{-3.8} \) \( \text{km s}^{-1}\text{Mpc}^{-1} \). The Base + LSS analysis improves this constraint by 35\% and shifts \( H_0 \) to higher values. This improvement over [26] can be explained by the full-shape measurements of large-scale structure utilized in our work, for details see Sec. 2.1. The full-likelihood analysis [26] leads to \( S_8 = 0.826 \pm 0.019 \) which demonstrates a 2.4\( \sigma \) tension with the weak lensing measurements (2.2). This difference can be attributed to the lensing-like anomaly in the Planck data which pulls the late-time amplitude to higher values. Our analysis is free from this feature and fully consistent with the direct probes of \( S_8 \) in the late Universe.

Second, we confront the cosmological constraints in the Base + LSS + \( S_8 + H_0 \) analysis with the joint result of CMB+lensing+BAO+Pantheon+R19 based on the full Planck likelihood from Ref. [26]. While the constraints on \( a_m \) and \( \beta \) are broadly consistent, the distributions of \( \alpha \) are substantially different in these two analyses. The Base + LSS + \( S_8 + H_0 \) data suggests more than 5\( \sigma \) deviation of \( \alpha \) from zero value expected in the \( \Lambda \)CDM, while in the full-likelihood analysis [26] one has \( \alpha < 3.32 \) (1\( \sigma \)), that is consistent with zero. The standard approach [26] yields \( H_0 = 70.25 \pm 0.78 \) \( \text{km s}^{-1}\text{Mpc}^{-1} \). This estimate is significantly lower than the Base + LSS + \( S_8 + H_0 \) result and exhibits a mild tension with the SH0ES measurement [61] at the 2.3\( \sigma \) level. The full-likelihood result for the late-time amplitude reads \( S_8 = 0.823 \pm 0.011 \) [26] being in the significant, 3\( \sigma \), tension with the weak lensing measurements (2.2).

Our findings unravel the main differences between our analysis and the full-likelihood approach. In essence, the PDE scenario in the standard analysis only partially alleviates the \( H_0 \) tension and does nothing with the \( S_8 \) problem. In contrast, the combined CMB data approach allows one to solve \( S_8 \) and \( H_0 \) tensions simultaneously. This effect is attributed to the robust measurement of CMB anisotropies at
small angular scales that makes smaller values of the mass fluctuation amplitude consistent with the CMB data. The latter provides a smaller value $S_8$ in full agreement with the direct measurement of $S_8$ (2.2) in the late Universe.

B Comparison with previous study of TDE

Our work continues the exploration of the TDE scenario put forward in Ref. [27]. In that study it was argued that a sharp transition in the dark energy density occurred at $z_{tr} \sim 2$ is the preferable solution of the $H_0$ tension. In contrast to Ref. [27] we found that the TDE model can not accomodate a large $H_0$ if the BAO measurements are taken into account. This result can be explained by several methodological differences between our approach and the analysis performed in Ref. [27] as we described below.

First and most important, the TDE analysis of Ref. [27] uses the SH0ES Cepheid data to calibrate the absolute magnitude of Type Ia supernova. The supernova distances calibrated to the local $H_0$ measurement pull the values of angular diameter distance ($D_A$) down at intermediate redshifts $z < 2$ which eventually leads to the preference for a rapid transition in the dark energy evolution between $z = 2$ and today [27]. However, this distance evolution at late times is in tension with the BAO measurements calibrated by the CMB-inferred value of $r_{\text{drag}}$. Indeed, the BAO angular distances disagree with the TDE prediction as shown in Fig. 1 and 10 of Ref. [27]. One way to bring the BAO and supernova distances calibrated by SH0ES into agreement is if the true value of $r_{\text{drag}}$ is smaller compared to ΛCDM. It means that the TDE scenario just recasts the Hubble tension into another tension, the tension in $r_{\text{drag}}$. So, the TDE model can not be considered as a solution of the Hubble tension. This corollary agrees with the result of our baseline analysis.

Second, the previous TDE analysis [27] adopts the local measurement of $H_0$ with the projected sensitivity of 1%. When using a 2.4% uncertainty from the official SH0ES release [62] the evolution of dark energy becomes consistent with a cosmological constant. It indicates that the strong preference for a transition in the dark energy evolution reported in Ref. [27] is driven by a tight prior on $H_0$.

Finally, the authors of [27] employ a numerically economical compression of the CMB data. It allows one to expresses the early-time information encoded in the CMB maps in the form of the angular diameter distance and Hubble distance at the redshift of last scattering.

This approximation has the two important drawbacks. First, it loosens the constraints of parameters compared to the full likelihood approach as shown in Ref.
Second, it does not include the information from the lensing potential power spectrum \( C_\ell^{\phi \phi} \) [27] that weakens the final cosmological constraints. Overall, to take advantage of the whole information encoded in the CMB one needs to employ the full CMB likelihood.

The analysis [27] also reported a slower growth of cosmic structures at late times in the TDE model which might explain a lower value of \( S_8 \) predicted by various large-scale structure data. In our analysis we found that the clustering amplitude \( \sigma_8 \) indeed goes down compared to the \( \Lambda \)CDM prediction, but its effect on \( S_8 \) is entirely compensated by a positive shift in \( \Omega_m \). It means that the TDE can not be responsible for a low \( S_8 \). The agreement with the large-scale structure measurements we observe in our analysis is attributed to the different CMB data set which accommodates a lower value of the clustering amplitude.

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