High performance telescope system design for the TianQin project

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Abstract

China is planning to construct a new space-borne gravitational-wave (GW) observatory, the TianQin project, in which the spaceborne telescope is an important component in laser interferometry. The telescope is aimed to transmit laser beams between the spacecrafts for the measurement of the displacements between proof-masses in long arms. The telescope should have ultra-small wavefront deviation to minimize noise caused by pointing error, ultra-stable structure to minimize optical path noise caused by temperature jitter, ultra-high stray light suppression ability to eliminate background noise. In this paper, we realize a telescope system design with ultra-stable structure as well as ultra-low wavefront distortion for the space-based GW detection mission. The design requirements demand extreme control of high image quality and extraordinary stray light suppression ability. Based on the primary aberration theory, the initial structure design of the mentioned four-mirror optical system is explored. After optimization, the maximum RMS wavefront error is less than $\lambda/300$ over the full field of view, which meets the noise budget on the telescope design. The stray light noise caused by the back reflection of the telescope is also analyzed. The noise at the position of optical bench is less than $10^{-10}$ of the transmitted power, satisfying the requirements of space GW detection. We believe that our design can be a good candidate for TianQin project, and can

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also be a good guide for the space telescope design in any other similar science project.

Keywords: gravitational waves, spaceborne telescope, telescope system design

(Some figures may appear in colour only in the online journal)

1. Introduction

Gravitational-wave (GW) is the product of the perturbation of the gravitational field, which was originally predicted by Einstein soon after proposing his theory of general relativity. In 1974, an orbiting binary system of two neutron stars roughly the mass of the Sun is discovered by Taylor and Russell Hulse [1]. In 1981, GW was indirectly demonstrated by the measurement of orbital-period decay from the orbiting pulsar [2, 3]. In 2015, the GW signal GW150914 was detected by the ground-based laser interferometer GW observatory (LIGO–VIRGO), this discovery directly confirmed Einstein’s prediction [4].

Ground-based interferometric GW observatories, such as LIGO, Virgo, and GEO-600 [5–7], have arm lengths on the order of kilometers and suffer from many sources of terrestrial noise, making it difficult to detect low-frequency GW signals below 10 Hz. In order to break through the limitation of the ground-based system, the space GW detection schemes with the length of the interference arm in the range of $10^3–10^5$ km were proposed [8–10]. The laser interferometer space antenna (LISA), which is a cooperative mission led by European Space Agency and National Aeronautics and Space Administration. It was originally proposed in the 1990s and is currently scheduled to launch in 2034 [11]. The LISA mission consists of a constellation of three identical spacecrafts that follow the earth around the Sun in a flat equilateral triangle. So far, the three spacecrafts have been chosen to be separated from each other by distances of about 2.5 million kilometers. The spacecraft sends laser beams to and receives laser beams from its matching spacecraft via spaceborne telescopes to establish a laser link for the precise measurement of the change in the distance between the test masses. As the space-based GW observatory consists of an equilateral triangle of spacecraft with pairs of proof masses at the end of each arm, the six telescopes are required to play the role in enabling the mission’s long arm interferometry [10, 12].

Recently, Luo et al proposed a new space-borne GW observatory, the TianQin project. The TianQin project aims to study a plethora of GW sources in the millihertz (mHz) range such as ultra-compact galactic binaries and coalescing massive black holes [13]. The space-based GW observatory consists of an equilateral triangle of three spacecrafts which orbit around the earth. The geocentric orbits have certain advantages in engineering such as satellite deployment and easy communication with the ground. But for spaceborne telescopes, the stability (optical path stability and stray light stability) and wavefront quality must be further improved compared with the orbit around the Sun due to the thermal stability [14]. The accuracy of the GW measurement is affected directly by these disturbances, as it will introduce the phase noise to the interferometric beat signal.

Plenty of researches on the telescopes have been carried out by LISA [10, 12, 15]. The telescope is designed based on the Cassegrain telescope, but the on-axis or off-axis design has not been decided. The initial LISA design used an on-axis Ritchey–Chretien telescope, where the light is reflected by the secondary mirror (SM) and passes through a matching lens directly into the optical bench. Compared with off-axis options, these on-axis designs have several major advantages in terms of thermal stability, volume, mass, and cost. Unfortunately, stray
light of the optical system, especially the back-scattering from the SM, made LISA later turn to an off-axis design. Even though some efforts are proposed to suppress the backscattering, it is obvious that the off-axis design is more reliable [16]. Therefore, the off-axis design was the preferred option, and the off-axis design was also adopted in the test prototype for LISA [17]. The current LISA telescope baseline design consists of a two-mirror telescope with a two-lens ocular, with a six-mirror design as an alternative [18].

Telescopes for GW detection are aimed to deliver laser light efficiently from one spacecraft to another for supporting precision metrology beyond the usual requirements for good image formation. As the wavefront of the laser beams will diffract to large angles in the far-field, aberrations from the designed telescope directly affect the optical intensity at the location of the spacecraft far away. Besides, the wavefront error from the telescope will be directly imparted onto the received laser beam, which will lead to a reduction of the signal-to-noise. This also explains why we choose the wavefront error as the performance metric rather than the encircled energy to evaluate the telescope during the design process. The wavefront error caused by the receiving end telescope will be coupled into the receiving beam, which interferes with the local reference laser beam on the optical bench. On account of the spacecraft jitter and the breathing of the constellation, the laser beam within the field of view (FOV) jitters on the detector. As a result, a way to eliminate the tilt-to-length (TTL) coupling noise in the transceivers is to minimize the wavefront error change with angle. As the off-axis aberrations caused by temperature gradient is of more concern on the geocentric orbit, a more perfect wavefront structure as well as a super lower stray light level is required in the optical design of the telescope for TianQin project.

In this work, we describe a high performance catoptric telescope aimed at the interferometric study of GWs. As the wavefront quality is of crucial importance for the telescope, a coaxial mirror design was developed based on Seidel aberration theory. Then the occlusion is eliminated by making both the FOV and the aperture off-axis, and the exit pupil direction of the system can be controlled by rotating the eyepiece. A progressive optimization is implemented to gradually decrease the wavefront error over the full FOV. The final design result is obtained through the comparison of two optimized routes. In addition, we performed a sensitivity analysis on the telescope to ensure that the telescope could be manufactured correctly within reasonable tolerances. The effect of residual wavefront error of telescope design on TTL noise is analyzed to control the measurement noise caused by jitter. We conduct the stray light analysis results in the commercially optical software ASAP, demonstrating the stray light performance of the telescope. The influence of these optical performance on the telescope design and some challenges needed to deal with in the future work are discussed.

2. Design specifications

The design specifications about the TianQin telescope and specially derivations are listed in table 1. Most of them reference the requirements for the LISA mission and other LISA-like space-based GW mission.

The working wavelength of the telescope is determined by the laser used in the interferometry system, where the frequency stabilisation of the lasers is an important consideration [13, 19]. The FOV is set by the angle requirement that the laser link can be established to receive a beacon signal for acquiring initial pointing and subsequent complete the normal science operation [9]. As the telescope’s aperture directly determines the size of acceptable area of the laser beam in far field, we must ensure that the laser power collected by the receiving telescope meet the requirement for the given sensitivity [20]. The afocal magnification
Table 1. Specifications of the TianQin telescope.

| Parameter                          | Specification                  | Rationale                      |
|------------------------------------|--------------------------------|--------------------------------|
| Wavelength                         | 1064 nm                        | Laser system                   |
| Wavefront quality (design residual)| \( \leq \lambda/300 \) RMS@1064 nm | Property of the optical prescription |
| FOV                                | \( \pm 200 \) \( \mu \)rad     | Acquisition time and orbits     |
| Entrance pupil diameter            | 220 mm                         | Radiometry                      |
| Afocal magnification               | 40X                            | Aperture and energy transfer    |
| Stop location                      | Primary mirror                 | Geometrical optics              |
| Scattered light power              | \(< 10^{-10}\) of laser power  | Displacement noise              |
| Optical path length stability      | 1 pm Hz\(-1/2\)@0.1 mHz–1 Hz   | Path length noise               |

is equal numerically to the ratio between the primary mirror (PM) diameter and laser beam diameter on the optical bench. When the aperture of the PM is determined, the afocal magnification is determined by the design chosen of laser beam size on the optical bench, which is not fundamental but need to be specified in the optical design concept [21]. The optical path stability requirement of the telescope is determined by the allocation of noise budget of the interferometric distance measurement system. The pathlength stability mainly depends on the spatial thermal stability of the supporting structure [22], which will be discussed in mechanical structural design elsewhere.

Specially, the requirement for wavefront quality is unusual and challenging for a space-based GW telescope. The Strehl ratio is a critical indicator for the efficiency of energy transfer, which can be approximately expressed as \( S \approx 1 - (2\pi\sigma_W/\lambda) \), where \( \sigma_W \) is the wavefront error of single-link measurement system [23]. Generally, \( S = 0.8 \) is considered as the criterion for the diffraction limit of the optical system. Since the measurement arm contains two telescopes, the wavefront error \( \sigma_W = \lambda/20 \) corresponds to the total Strehl ratio \( S = 0.92 = 0.81 \). Considering the existence of aberrations in the optical elements on the optical bench, leaving a certain margin for the assembly of the interferometer optical system, the overall far-field wavefront error goals of the telescope system are \( \lambda/30 \). The TianQin constellation orbits the earth, which puts forward more stringent requirements to pointing accuracy. Because of the jitter, high quality far-field wavefront within a larger FOV is needed to suppress TTL noise. The telescope design residual wavefront error is expected to be within \( \lambda/300 \) based on the analyses above.

Another vital requirement is the power of the back-scattering light which must be kept below \( 10^{-10} \) of transmit laser power to suppress the phase noise. Super-low stray light requirement is mainly set by two factors. One is that the signal power received by the telescope is about \( 10^{-10} \) of the transmitted, while the back-scattering light is produced by local high power laser beam from optical bench. Another is that coherent detection scheme is implemented by creating an interference signal with small incoming signal and higher power local reference laser beam, but it also implies that the detector is very sensitive to stray light. For the purpose of mitigating the consequences of stray light, an off-axis telescope design is preferred as the back-reflection from the SM of an on-axis design transmits directly back to the detector on the optical bench.
3. Initial structure design

3.1. Design of PM and SM

The spaceborne telescopes for GW detection are directly in a long path measurement layout to expand and compress the wavefront, which lead to the output of the telescope is a nominally collimated laser beam. The TianQin telescope is designed based on the off-axis four mirrors configuration, which can be temporarily considered as two separate telescopes in the preliminary design phase. When placed with the rear focal points of these two focused imaging systems aligned, it can form an unfocused telescope system. Their respective FOV are linked by paraxial magnification to ensure that the entire system can expand the wavefront correctly. Figure 1 shows a ray tracing of an on-axis Cassegrain design suitable for the proposed the TianQin mission. The PM is parabolic and the SM is aspherical. The even aspherical surface is defined by the following equation:

\[ z(x, y) = \frac{c(x^2 + y^2)}{1 + \sqrt{1 - (1 + \kappa)c^2(x^2 + y^2)}} + \sum c_k(x^2 + y^2)^{k+1}, \quad (1) \]

where \( \kappa \) is the conic constant, \( c \) represents the curvature, \( c_k \) stands for the coefficients of the higher-order term. Here is a rotationally symmetric system, \( r^2 = x^2 + y^2 \). If the primary wave aberrations are described in terms of the Seidel coefficients, the well-known equation can be obtained [24]:

\[
W(H, \rho, \theta) = \frac{1}{8} S_1 \rho^4 + \frac{1}{2} S_{II} H \rho^3 \cos \varphi + \frac{1}{2} S_{III} H^2 \rho^2 \cos^2 \varphi + \frac{1}{4} \left( S_{IV} + S_{V} \right) H^2 \rho^2 + \frac{1}{2} S_{V} H^3 \rho \cos \varphi, \quad (2)
\]

where \( H \) is the normalized field coordinate, \( \rho \) is the normalized pupil coordinate, and \( \varphi \) is the azimuth angle. The Seidel coefficients \( S_{Iph} - S_{Vph} \) of a given optical system is calculated by
the sum of contributions from each surface [25]:

\[
\begin{align*}
S_{\text{Isph}} &= -\sum A^2 \cdot y \cdot \Delta \left( \frac{u}{n} \right), \\
S_{\text{II} \text{sph}} &= -\sum \bar{A} A \cdot y \cdot \Delta \left( \frac{u}{n} \right), \\
S_{\text{III} \text{sph}} &= -\sum \bar{A} \cdot y \cdot \Delta \left( \frac{u}{n} \right), \\
S_{\text{IV} \text{sph}} &= -\sum \left\{ \frac{\bar{A}}{A} A \cdot y \cdot \Delta \left( \frac{u}{n} \right) + \bar{A} \cdot L^2 \cdot c \cdot \Delta \left( \frac{1}{n} \right) \right\},
\end{align*}
\]

\[S_{\text{Vph}} = \sum \left\{ \frac{\bar{A}}{A} A \cdot y \cdot \Delta \left( \frac{u}{n} \right) + \bar{A} \cdot L^2 \cdot c \cdot \Delta \left( \frac{1}{n} \right) \right\} \]

where \(y\) is the height of the marginal ray, \(u\) denotes the slope of the marginal ray, \(n\) represents the refractive index, \(c\) is surface curvature, \(A\) is the refraction invariant of the marginal ray, \(\bar{A}\) is the refraction invariant of chief ray, and \(L\) represents the Lagrange invariant of the system. The refraction invariants on the current surface and Lagrange invariants can be obtained by the following formula:

\[
\begin{align*}
A &= n(yc + u) \\
\bar{A} &= n(\bar{y}c + \bar{u}) \\
L &= \bar{n} \bar{y} - n\bar{u}
\end{align*}
\]

where \(\bar{y}\) represents the height of the chief ray, and \(\bar{u}\) is the slope of the chief ray. The relationship between the ray height and the ray slope on different mirrors can be calculated according to paraxial \(y-u\) method:

\[
\begin{align*}
&n_i' \mu_i' = n_i u_i - y_i \varphi_i \\
y_{i+1} = y_i + u_i' \mu_i' \\
\ell_i' = \ell_{i+1} \\
u_i' = u_{i+1}
\end{align*}
\]

where \(\varphi_i\) represents the optical power of the \(i\)th mirror, which can be determined by:

\[\varphi_i = (n_i' - n) c_i.\]

The surface with conic coefficient can be expressed by surface sag \(z\) [26]:

\[
z = \frac{cr^2}{2} + \frac{(\kappa + 1)c^3r^4}{2^2!} + \frac{1.3(\kappa + 1)c^5r^6}{2^3!} + \frac{1.3.5(\kappa + 1)c^7r^8}{2^4!} + \frac{1.3.5.7(\kappa + 1)c^9r^{10}}{2^5!} + \cdots
\]

The difference between a conic and a sphere is given by:

\[
\Delta z = \frac{[(\kappa + 1) - 1]c^3r^4}{2^2!} + \frac{1.3[(\kappa + 1)^2 - 1]c^5r^6}{2^3!} + \frac{1.3.5[(\kappa + 1)^3 - 1]c^7r^8}{2^4!} + \frac{1.3.5.7[(\kappa + 1)^4 - 1]c^9r^{10}}{2^5!} + \cdots
\]
In the initial structural design round, we only consider the fourth-order aspheric coefficient \( c_4 \). In this case, the change of the vector height of the aspheric surface can be expressed as:

\[
\Delta z = \left( \frac{1}{8} \kappa c_3 + c_4 \right) r^4 + \cdots \tag{9}
\]

The terms of the aspheric surface only affect the fourth power of the aperture and the terms of higher order. To the level of third-order aberrations, aspherisation only affects the spherical aberration if the aperture stop is located at the aspheric surface. The contribution to spherical aberration \( S_1 \) by the aspheric part can be expressed as [27]:

\[
\delta S_{1\text{asph}} = \sum 8 \cdot \left( \frac{1}{8} \kappa c_3 + c_4 \right) y^4 \Delta n. \tag{10}
\]

The coordinates of the chief ray on each surface will vary with the location of the pupil. The value of the field curvature \( S_{IV} \) is independent of the height of the chief ray and the refraction invariant of the chief ray, so the movement of the aperture does not change the field curvature of the system. The extra coma, astigmatism, and distortion introduced by the aspheric surface when the aperture stop is shifted away from the aspheric surface can be calculated by means of the stop shift formulae:

\[
\begin{align*}
\delta S_{1\text{asph}} &= \frac{\tau}{\gamma} \delta S_{1\text{asph}} \\
\delta S_{II\text{asph}} &= \frac{\tau^2}{\gamma} \delta S_{1\text{asph}} \\
\delta S_{III\text{asph}} &= \frac{\tau^3}{\gamma} \delta S_{1\text{asph}}
\end{align*} \tag{11}
\]

where \( S_{1\text{asph}} \), \( S_{II\text{asph}} \) and \( S_{III\text{asph}} \) indicate the Seidel sums by the aspheric part after a stop shift. Then the Seidel coefficients of the optical system with conic coefficient and fourth-order aspheric coefficient \( S_I–S_V \) can be acquired as:

\[
\begin{align*}
S_I^* &= S_{I\text{asph}} + \delta S_{1\text{asph}} = S_{I\text{asph}} + \delta S_{1\text{asph}1} + \delta S_{1\text{asph}2} \\
S_{II}^* &= S_{II\text{asph}} + \delta S_{II\text{asph}} \\
S_{III}^* &= S_{III\text{asph}} + \delta S_{III\text{asph}} \\
S_{IV}^* &= S_{IV\text{asph}} \\
S_{V}^* &= S_{V\text{asph}} + \delta S_{V\text{asph}}
\end{align*} \tag{12}
\]

where the \( S_{1\text{asph}1} \) represents the contribution of conic coefficient \( \kappa_1 \) of PM to spherical aberration, and \( S_{1\text{asph}2} \) represents the contribution of conic coefficient \( \kappa_2 \) and fourth-order aspheric coefficient \( c_4 \) of SM to spherical aberration. As the aperture is located on the PM, the conic coefficient of the PM only affects the spherical aberration term in the primary aberration.

According to the specifications, \( y_1 = 110 \text{ mm}, u_1 = 0, \gamma_1 = 0, \) and \( \pi_1 = 2.1 \times 10^{-4} \). The height of the marginal ray \( y_1 \) is determined by the aperture and the property that the Gaussian object distance equals infinity. Similarly, we can obtain the paraxial aperture angle \( u_1 = 0 \). The value of \( \gamma_1 \) and \( \pi_1 \) are derived from the definition of the principal ray and the geometric relationship described in figure 1. To keep the reasonable space between the surfaces, we set \( d_1 = d_2 = -300 \text{ mm} \). The corresponding optical distances in paraxial ray tracing are \( \tau_1 = d_1/n_2 = 300 \text{ mm} \) and \( \tau_2 = d_2/n_3 = 300 \text{ mm} \). The optical invariant of the
Table 2. Coaxial initial structure parameters of PM and SM.

| Surface       | Radius (mm) | Distance (mm) | Conic | 4th-order  |
|---------------|-------------|---------------|-------|------------|
| PM (stop)     | −666.668    | −300.000      | −1.000| −4.689 × 10⁻⁸ |
| SM            | −75.001     | 300.000       | −1.404| −4.689 × 10⁻⁸ |
| Image plane   | —           | —             | —     | —          |

**Figure 2.** Ray tracing in initial coaxial system of TM and QM.

The system can be calculated as \( L = 0.0230 \) mm. The objective of optimization is to control the primary aberration and the effective focal length of the system, so the evaluation function is set as:

\[
F(r_1, r_2, \kappa_1, \kappa_2, c_4) = \sqrt{w_1S_I^2 + w_2S_{II}^2 + w_3S_{III}^2 + w_4S_{IV}^2 + w_5S_{V}^2 + w_6(f - f_0)^2}, \quad (13)
\]

where \( w_i \) is the weight coefficients, \( f \) is the focal length of the system, and \( f_0 \) is the target focal length [28]. Then the curvature radii of PM and SM and the distances between them can be derived from the analysis of Seidel coefficients. The detailed parameters are listed in table 2. The Seidel coefficients are respectively \( S_I = 5.32 \times 10^{-6}, S_{II} = -1.56 \times 10^{-5}, S_{III} = 1.60 \times 10^{-6}, S_{IV} = 1.82 \times 10^{-6}, \) and \( S_{V} = -4.07 \times 10^{-8}. \)

3.2. Design of tertiary mirror and quaternary mirror

As shown in figure 2, we trace the rays in the opposite direction to design the tertiary mirror (TM) and quaternary mirror (QM). The pair of mirrors functions as a collimator, and the FOV and the size of the entrance pupil (in fact exit pupil of the whole system) are determined by the magnification of the PM and SM. The height of the marginal ray is scheduled to be 2.75 mm. Since the focal points of the two systems overlap, the sum of the back focal distance (BFD) of system consists of PM and SM and the BFD of the system consists of TM and QM systems should be less than the specified 350 mm. For TM and QM, the value of BFD is 50 mm.

The light ray is traced as a sequence manner, ignoring the occlusion problem, which will be addressed by the operation of off-axis in later designs. The aperture and the FOV of this sub-system are small. Therefore, the TM is configured as a sphere, and the QM is set as a conical surface. The primary aberrations are also calculated by equations (3)–(12). The detailed parameters are listed in table 3. The Seidel coefficients are respectively \( S_I = 2.38 \times 10^{-5}, S_{II} = 1.17 \times 10^{-5}, S_{III} = 7.33 \times 10^{-6}, S_{IV} = 8.98 \times 10^{-6}, \) and
Table 3. Coaxial initial structure parameters of tTM and qQM.

| Surface   | Radius (mm) | Distance (mm) | Conic   |
|-----------|-------------|---------------|---------|
| Stop      | —           | 200           | —       |
| QM        | −690.084    | −114.983      | −10.000 |
| TM        | 127.791     | 50.000        | 0       |
| Image plane| —           | —             | —       |

$S_V = -6.86 \times 10^{-7}$. Because the FOV of the telescope is very small, the aberrations of the whole coaxial design is easy to be corrected. When combining the two subsystems, the lens data of the telescope are basically the same as those in the initial design, and the overall layout is almost unchanged.

4. Design and optimization

Firstly, the coaxial afocal system is transformed to an off-axis system with conic surfaces [29]. The imaging quality of TianQin telescope is evaluated by RMS wavefront error during the optimization process, which is different from a conventional imaging telescope. In addition, the wavefront error at the sampled FOVs in all directions should be kept as low as possible. The RMS wavefront error is often taken as the performance metric to evaluate the variation of an actual wavefront map from a perfect spherical wavefront, and is defined as [30]:

$$W_{\text{rms}} = \sqrt{\frac{1}{A} \int \int [W(x, y) - W_{\text{mean}}(x, y)]^2 dx \, dy}, \quad (14)$$

where $A = \int \int dx \, dy$, and $W_{\text{mean}}$ is the average wavefront deviation over the pupil area, which is expressed as:

$$W_{\text{mean}}(x, y) = \frac{1}{A} \int \int W(x, y) dx \, dy. \quad (15)$$

Preliminary analysis indicates that the straightforward optical design is very difficult to build in an optical design software. Therefore, a progressive strategy is adopted during our design process, and the number of the aspheric terms gradually increases.

In the first step of the optimization, proper constraints should be selected and applied [12, 30, 31]. The axial optical length between the SM and TM is constrained up to 370 mm to satisfy the fabrication requirement. Take the layout plan of the refraction-type optical system into consideration, the distance between the primary and secondary mirrors is limited to less than 300 mm. At the same time, in order to control the weight of the supporting structure of the system, the off-axis amount in the $y$-direction should not be too large while eliminating the mirror occlusion. To accommodate the interface to the optical bench, the coordinates of the rays striking the image plane are constrained to control the incidence of light from the exit pupil being approximately perpendicular to the optical bench.

In the following steps, the commercial optical design software is employed for the further optimization. PM surface is set as the global coordinate reference surface. An off-axis configuration in the $y$ direction is set to eliminate the occlusion based on the coaxial parameters of two Cassegrain systems obtained in the previous section. By controlling the combined system power $\phi_{M1M2}$ and $\phi_{M3M4}$, the reduction ratio of laser beam can be determined as
Table 4. Off-axis initial structure parameters.

| Surface | Radius (mm) | Distance (mm) | Conic | Decenter Y (mm) |
|---------|-------------|---------------|-------|-----------------|
| PM (stop) | -666.661 | -300.000 | -1.000 | -130 |
| SM | -74.994 | 352.711 | -1.561 | — |
| TM | -150.357 | 74.592 | 3.673 | -5 |
| QM | 492.640 | — | -8.641 | -25 |

Table 5. Structure parameters of the design with SM asphere.

| Surface | Radius (mm) | Distance (mm) | Conic | Asphere coefficient of SM |
|---------|-------------|---------------|-------|---------------------------|
| PM (stop) | -666.661 | -300.000 | -1.000 | Item Coefficient |
| SM | -74.994 | 356.374 | -0.736 | 4th order term $2.443 \times 10^{-7}$ |
| TM | -155.3009 | 68.091 | 4.320 | 6th order term $-5.868 \times 10^{-12}$ |
| QM | 540.424 | — | -8.669 | 8th order term $3.451 \times 10^{-17}$ |

$m = \phi_{M3M4}/\phi_{M1M2}$. Only an initial structure with potential has been constructed here, as the ability to correct off-axis aberrations by configurations with only conic surfaces is limited. The parameters are listed in table 4, including the curvature radii, distance, conic coefficient, and the decenter value. The PM is about F/1.5, and the parameters of the PM will not be optimized in the subsequent design process. The RMS wavefront error of the marginal FOV is 0.00162 $\lambda$.

Then we first introduce even aspheric surface on SM. Higher-order aspheric coefficient are gradually set to be the optimized parameters for further optimization. In addition, the weight coefficients of the marginal FOVs should be properly improved to make the aberration evenly distribute across the full FOV. Because the TTL noise generated by the non-uniform wavefront error cannot be eliminated by the calibration method, and it will affect the measurement results inevitably. At this stage, TM and QM have no tilt, as the tilt will introduce additional asymmetric aberrations. After optimization, the aberrations induced by the decenter are partly corrected. The basic design parameters and aspheric coefficients of SM are listed in table 5.

In the subsequent stage, optimization is used surface by surface to minimize the design residual aberrations at each surface. The parameters including the radius, the aspherical coefficients of the TM and QM, decenter in Y direction, tilt about x axis are set as variables in the optimization. When the configuration becomes stable, the additional constraint about the exit pupil should be considered, where the exit pupil must be positioned between the telescope and the optical platform, the same size as the local laser beam. According to the characteristics of the afocal optical system, the exit pupil is controlled by ray tracing from the central FOV. With the optimization being conducted, the location and size of the exit pupil are restricted to the specification values, and the higher order terms of aspheric coefficients are gradually added to improve the imaging quality.

The whole design procedure adopts two optimized strategies, as illustrated in figure 3. It is clear that the large off-axis aberrations introduced by the off-axis mirrors are difficult to control with the traditional conic surfaces. Therefore, it is necessary to introduce higher order aspherical coefficients into these surfaces. However, only two aspheric surfaces with 4th-order, 6th-order, and 8th-order coefficients cannot entirely compensate for these aberrations, no matter they are introduced into the SM and TM, or into the SM and QM. The results suggest that
the final design is inevitable to use three aspherical mirrors simultaneously. After the number of used aspherical surfaces and the order of each aspherical coefficient are determined, these coefficients are set as variables at the same time for final optimization in the software, and the final design result is obtained.

5. Design results and performance

The final design system is illustrated in figure 4, which consists of a large diameter paraboloid mirror and three aspheric surface mirrors. The structural layout of the system is adjusted reasonably, where occlusion is completely eliminated. The basic structural parameters and even-order aspheric coefficients are detailed in tables 6 and 7. The total optical length is 328.151 mm, and the transverse length is 260.044 mm in the Y direction. The exit pupil is located 100 mm behind the QM along the z-axis. An intermediate image plane is arranged
### Table 6. Off-axis structure parameters of recommended design.

| Surface | Radius (mm) | Distance (mm) | Conic | Decenter Y (mm) | Tilt (°) |
|---------|-------------|---------------|-------|----------------|---------|
| PM (stop) | −666.661 | −300.000 | −1.000 | −130 | — |
| SM | −74.994 | 328.155 | 0.032 | — | — |
| TM | −94.556 | −114.983 | −1.893 | −3.998 | — |
| QM | −366.344 | — | −7.904 | −24.485 | −5.306 |

### Table 7. Detailed even-order aspheric coefficients of recommended design.

| Surface | 4th-order term | 6th-order term | 8th-order term |
|---------|---------------|----------------|----------------|
| SM | $4.714 \times 10^{-7}$ | $2.139 \times 10^{-11}$ | $2.959 \times 10^{-15}$ |
| TM | $-2.446 \times 10^{-6}$ | $3.491 \times 10^{-8}$ | — |
| QM | $-6.765 \times 10^{-8}$ | $1.755 \times 10^{-10}$ | $-1.431 \times 10^{-13}$ |

Airy Radius : 0.2378mrad

![Spot Diagram](image)

**Figure 5.** The spot diagram of the recommended TianQin telescope design.

Between SM and TM so that the field stop can be placed to shield large amounts of stray light.

In order to evaluate the image quality of an afocal system, we insert a perfect lens at the pupil with an ideal lens to focus the collimated rays. The standard spot diagrams on the image plane of all configurations are exhibited in figure 5. The figure shows that the RMS radii of the spots at all FOV are much smaller than that of the Airy spot, and the principal components of residual aberrations are spherical aberration and coma. Figure 6 demonstrates the wavefront aberration at the full FOV. The RMS wavefront error of the system is not greater than $\lambda/400$, ensuring that the laser beam after beam expansion has high quality of wavefront.
There are other lenses and optical devices on the optical bench that need to be coupled to the telescope. The pupil aberrations are usually used to guide this combination when the two parts of an interlinked optical system are designed independently. We used the RMS radius of the chief ray on the pupil plane to assess the level of pupil aberration, which can be defined as:

$$R_{\text{CRMS}} = \sqrt{\frac{\sum_{i=1}^{N} (x_i - \bar{x})^2 + (y_i - \bar{y})^2}{N}}$$  \hspace{1cm} (16)$$

where \(N\) is the number of sampling points, \(x_i\) is the \(x\) coordinate of \(i\)th sampling point on the pupil plane, \(y_i\) is the \(y\) coordinate of \(i\)th sampling point on the pupil plane, the notation with bar is the corresponding arithmetic mean. Including the central FOV, we selected 151 sampling points on the full FOV to calculate \(R_{\text{CRMS}} = 0.729\) mm. If there is a higher demand for pupil aberration in the future, TM may be replaced with freeform surfaces to further correct pupil aberration.

Another design goal is to eliminate the TTL coupling in the transceivers caused by the continuous jitter during the GWs measurement. After several hundred thousand kilometers of propagation, the wavefront at the far-end telescope can be treated as a flat-topped beam. The difference in the traveled path length between the tilted measuring beam and Gaussian beam from the local optical bench can be obtained by calculating the complex phase of the integral over the overlap term [32]:

$$\arg\left(\int_{S} E_{\text{meas}} E_{\text{ref}}^* \, d\mathbf{r}^2\right) = k \cdot s$$ \hspace{1cm} (17)$$

where the wavenumber \(k = 2\pi/\lambda\). The change in OPL with angle provide quantitative index to describe the TTL coupling noise with units of length/angle.

The first 25 terms of the fringe Zernike polynomials are used to fit the wavefront error in measuring beam. The measuring beam with a wavefront residual \(\Delta W\) can be expressed as:

$$E_{\text{meas}}(x, y, z) = A_{\text{meas}} e^{-i2(\alpha \sin \theta + \phi \cos \theta)} e^{2\pi i \Delta W}$$ \hspace{1cm} (18)$$
where $\alpha$ is the angle of jitter. Zhao et al provided a complete derivation and code for this interference model [33]. Then we can get the TTL noise change rate caused by design residual wavefront error. As shown in figure 7, the TTL noise of this design is about $0.5 \text{ nm} \mu\text{rad}^{-1}$ at most over the entire $\pm200 \mu\text{rad}$ FOV, which meets the noise budget of $0.6 \text{ nm} \mu\text{rad}^{-1}$. Considering the jitter of the spacecraft, the current maximum TTL noise within the FOV range of $\pm300 \mu\text{rad}$ is $0.62 \text{ nm rad}^{-1}$, which means that other schemes may be needed to suppress the TTL at the marginal FOV. From numerical simulations and experimental demonstration, it is shown that it is possible to decrease TTL coupling by using imaging systems, which suppress the beam walk on the photo detectors. Another way of suppressing the TTL noise is to place the beam waist in an appropriate position and use a suitably large detector without clipping. Further work is needed to complete these designs for the suppression of TTL noise on the optical bench and to consider some trade-off with the telescope design.

6. Sensitivity analysis and stray light analysis

In addition to the six flight telescopes aboard the three spacecrafts, prototypes need to be built for testing on the ground. Fabrication of these telescopes on a small scale requires verification of the telescope’s tolerance performance to minimize schedule risk and engineering costs. Using the optical design software’s wavefront tolerance analysis function, the sensitivity of the optical system to manufacturing and assembly errors can be evaluated.

The wavefront error of the telescope is dominated by the surface figure error of the mirrors and the assembly alignment. The surface error is caused by machining error as well as the deformation due to the assembly. After a rough sensitivity analysis, we know that the wavefront error sensitivities of the TM and QM positions in this design are low. They are actually considered to be mounted on a translation stage, which can at least partially compensate the defocus of the system. Therefore, the positions of TM and QM serve as the compensator in
sensitivity analysis to obtain the maximized performance at final assembly process. The compensation range is set to ±3 mm. The tolerance distribution of each term is listed in table 8, where the thickness represents the distance error of the current surface to the next surface. In addition to the radius of curvature error, figure error is added to the three aspheric mirrors, which is expressed in terms of the first 21 Zernike coefficients except for the piston term. The maximum figure error is set to λ/40.

A Monte Carlo tolerance analysis is executed to predict the optical performance. Each parameter is perturbed with a random value within the tolerance range for each variable. This procedure is repeated for 1000 times, and each result for the RMS wavefront error are derived in the sensitivity analysis. The overall analysis results are listed in table 9. There is an 98% probability that the RMS wavefront error is less than 0.03242λ, which is acceptable and leaves room for other possible error source.

Here we only analyze the scattered light from inside the optical system, caused by the surface roughness and the particulates of the surface [34, 35]. The scattering model uses Harvey Shake model, and the simulation is completed in commercially software ASAP. The scatter model is applied to all of the mirror surfaces, and these surfaces are all are coated with a 0.99 reflectivity film. In the simulation model established, there is no other baffle except the field stop placed in front of the detector. Then the stray light received by the detector in the FOV of the telescope is simulated numerically. Cleanliness levels of CL200 and surface roughness of 0.5 nm can meet the design requirements. Here we set particles size range from 5 μm to 30 μm. Surface percent area coverage, which denotes the ratio of the sum of all particles projected area to surface area when incident wave is irradiated on mirror surface perpendicularly, is set as 10^{-3}. The analysis results we obtained are similar to the surface roughness requirements of LISA project, but the manufacturing still faces difficulties. Due to the limitations of the current fabrication technique, achievable roughness closely correlated with mirror design. It is necessary to take some measures in other areas to reduce the fabrication requirements for a lower the cost and schedule risk. The field stop and stray light shield will be explored in future work to block part of the stray light outside the FOV, leading to a looser requirement for surface roughness.

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**Table 8.** Tolerance setting in sensitivity analysis.

| Surface | Radius (μm) | Figure error | Thickness (μm) | Decenter in x (μm) | Decenter in y (μm) | Tilt in x (arc min) | Tilt in y (arc min) |
|---------|-------------|--------------|----------------|--------------------|--------------------|--------------------|--------------------|
| PM      | ±5          | —            | ±1.5           | ±0.5               | ±0.5               | 0.06               | 0.06               |
| SM      | ±5          | λ/40         | ±5             | ±0.5               | ±0.5               | 0.09               | 0.09               |
| TM      | ±10         | λ/40         | ±10            | ±0.5               | ±0.5               | 0.6                | 0.6                |
| QM      | ±10         | λ/40         | ±10            | ±0.5               | ±0.5               | 0.6                | 0.6                |

**Table 9.** Sensitivity analysis results.

| RMS WFE | 98%   | 90%   | 80%   | 50%   | 20%   | 10%   | 2%    |
|---------|-------|-------|-------|-------|-------|-------|-------|
| 0.04138 | 0.03242 | 0.02411 | 0.01989 | 0.01331 | 0.007215 | 0.004930 | 0.002954 |
7. Thermal analysis

A complete description of the thermal design is outside the scope of this article, thus we will focus only on surface deformations caused by ambient temperature changes that directly degrade the telescope’s optical properties. From the tolerance analysis above, we know that almost all tolerance items of the TM and QM has very little influence on the system. Therefore, we are concerned the wavefront error caused by PM and SM.

Put in more optical design terms, the surface of the telescope may lose rotational symmetry, with changes in the radius of curvature or sags of the surface. We will use the first 25 terms of the Zernike polynomial to fit the resulting grid sags, which can be obtained by the finite element analysis software. The material used for the reflector mirror is Zerdour in the current plan, as its linear expansion coefficient is small enough. The actual structural support is directly adopted as the displacement constraints, then the solved temperature field is used as a new temperature load and introduced into finite elements model to solve displacement nephogram. The deformation nephograms of PM and SM are showed as figure 8. Then the sampling points are fitted to the surface in the form of Zernike polynomials using the least square method in Matlab. The RMS wavefront error caused by surface deformation of PM is $1.0173 \times 10^{-7}$ mm, the PV of wavefront error is $4.2393 \times 10^{-7}$ mm. The RMS wavefront error caused by surface deformation of SM is $1.7869 \times 10^{-8}$ mm, the PV of wavefront error is $5.63 \times 10^{-8}$ mm. The results show that the degradation of the wavefront quality is acceptable to the design requirement of $\lambda/30$ for the telescope subsystem. The next step are to characterize the complete thermal stability performance and to consider its influence on our current telescope designs.

8. Conclusion

In this paper, we have successfully designed a high-performance space-borne telescope with ultra-low wavefront distortion, super-low stray light level as well as ultra-stable structure for space-based interferometric GW detectors, such as TianQin project. Our provided system design can enjoy an advantage of compactness as well as high efficiency with no occlusion for its full FOV. During the design process, we have conducted detailed discussion with regards to the initial structure design, structure constrains as well as optimization strategy. In addition, comprehensive performance analysis has been performed to verify our provided design, such as wavefront imaging quality, the exit pupil location, sensitivity...
analysis, and stray light suppression level, and so on. Our analysis demonstrates that the
design can have relatively high performance with the RMS wavefront errors of full FOV
less than \(\lambda/300\) as well as super low-level back reflection under normal optical process-
ning and cleaning level, which satisfies TianQin’s science requirements very well. Our per-
formed tolerance analysis results for the designed telescope also indicate that the telescope
can have very good manufacturability. In conclusion, our designed space-borne telescope can
basically satisfy the requirements of the space-borne GW detection mission. Further work
is needed to consider the supporting structure for mirrors, shading design for suppression
of stray light, dimensional stability performance and suppressing TTL noise. The provided
design strategy can also be a good guidance for any other similar science project like LISA
and TaiJi.

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Data availability statement

All data that support the findings of this study are included within the article (and any supple-
mentary files).

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