Flavor changing neutral currents in $SU(3) \otimes U(1)$ models

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Abstract

We consider flavor changing neutral current effects coming from the $Z'$ exchange in 3-3-1 models. We show that the mass of this extra neutral vector boson may be less than 2 TeV and discuss the problem of quark family discrimination.

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Recently it was proposed an electroweak model based on the \( SU(3)_L \otimes U(1)_N \) gauge symmetry \([3,4]\). The leptons are treated democratically with the three generations transforming as \((3,0)\) but with one quark generation (it does not matter which one) transforming differently from the other two. This condition arises since the model must contain the same number of triplets and antitriplets in order to be anomaly free. Hence, the number of generations is related to the number of quark colors.

In Ref. \([1]\) the first generation is the one which transforms differently from the second and the third ones. On the other hand, in Ref. \([2]\) it was the third generation which was treated differently. It was claimed that neutral currents could discriminate between both choices of representation content \([3]\). In fact, as we will see below, this assessment is true only when further assumptions are made about the quark mass matrices.

The GIM mechanism in several 3-3-1 models was consider in Ref. \([4]\). Here we turn back to the problem of flavor changing neutral currents, showing in particular that the difference in the choice of the quark representations is less important than it was thought at first sight. The lower bound for the mass of \(Z'\) was overestimated in Ref. \([1]\).

We will use the notation of Ref. \([4]\), but our results are trivially written in the notation of Ref. \([4]\). We also do not consider the lepton sector here because there is no difference in it.

Let us start writing the quark representations: one of the generations transforms as \((3,2/3)\), denoting the second entry \(U(1)_N\) charges,

\[
Q_{1L} = \begin{pmatrix} u_1 \\ d_1 \\ J \end{pmatrix}
\]

and the other two as \((3^*,-1/3)\):

\[
Q_{2L} = \begin{pmatrix} j_1 \\ u_2 \\ d_2 \end{pmatrix}, \quad Q_{3L} = \begin{pmatrix} j_2 \\ u_3 \\ d_3 \end{pmatrix}.
\]
The exotic quark $J$ has charge $5/3$ and $j_i, i = 1, 2$ have both charge $-4/3$. Eq. (1) denotes the first or the third generation. At this stage it does not matter this choice.

Denoting $U' = (u_1, u_2, u_3)^T$ and $D' = (d_1, d_2, d_3)^T$ the symmetry eigenstates of charge $2/3$ and $-1/3$ respectively, we can write in this basis the neutral currents coupled with the extra neutral vector boson $Z'$

$$\mathcal{L}_{Z'} = -\frac{g}{2\cos\theta_W}(\bar{U}'_L\gamma^\mu Y^U_L U'_L + \bar{U}'_R\gamma^\mu Y^U_R U'_R + \bar{D}'_L\gamma^\mu Y^D_L D'_L + \bar{D}'_R\gamma^\mu Y^D_R D'_R)Z'^\mu, \quad (3)$$

where

$$Y^U_L = Y^D_L = -\frac{1}{\sqrt{3}h(x)} \begin{pmatrix} 1 & 0 & 0 \\ 0 & -(1 - 2x) & 0 \\ 0 & 0 & -(1 - 2x) \end{pmatrix} \quad (4)$$

and

$$Y^U_R = -\frac{4x}{\sqrt{3}h(x)} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad Y^D_R = \frac{2x}{\sqrt{3}h(x)} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}. \quad (5)$$

Here we have defined $h(x) \equiv (1 - 4x)^\frac{1}{2}$, with $x \equiv \sin^2\theta_W$.

In order to generate the quark masses, it is necessary to introduce the following Higgs multiplets

$$\eta \sim (3, 0), \quad \rho \sim (3, 1), \quad \chi \sim (3, -1), \quad (6)$$

and a sextet $(6, 0)$ which is necessary in order to give masses to the leptons [5]. Since this sextet does not couple to the quarks, we need not to consider it here.

The Yukawa couplings for the charged $2/3$ and $-1/3$ sector are

$$-\mathcal{L}_Y = \tilde{Q}_{1L}(G_{1a}U'_{aR}\eta + \tilde{G}_{1a}D'_{aR}\rho) + \tilde{Q}_{iL}(F_{ia}U'_{aR}\rho^* + \tilde{F}_{ia}D'_{aR}\eta^*) + \text{ H.c.}, \quad (7)$$

1 The neutral currents coupled with the $Z$ boson are diagonal in the flavor space and we will not consider them here.
with \( i = 2, 3 \) and \( \alpha = 1, 2, 3 \). \( SU(3) \) indices have been suppressed and \( \eta^*, \rho^* \) denote the respective antitriplets [6].

From Eq. (7) it is straightforward to write the mass term

\[- L_m = \bar{U}'_{\alpha L} \Gamma^U_{\alpha \beta} U'_{\beta R} + \bar{D}'_{\alpha L} \Gamma^D_{\alpha \beta} D'_{\beta R} + \text{H.c.}, \tag{8}\]

where we have introduced the mass matrices

\[
\Gamma^U = v_\eta \begin{pmatrix} G_{11} & G_{12} & G_{13} \\ F_{21r} & F_{22r} & F_{23r} \\ F_{31r} & F_{32r} & F_{33r} \end{pmatrix}, \quad \Gamma^D = v_\rho \begin{pmatrix} \tilde{G}_{11} & \tilde{G}_{12} & \tilde{G}_{13} \\ \tilde{F}_{21r} & \tilde{F}_{22r} & \tilde{F}_{23r} \\ \tilde{F}_{31r} & \tilde{F}_{32r} & \tilde{F}_{33r} \end{pmatrix}. \tag{9}\]

Here, \( v_\eta \) and \( v_\rho \) represent the vacuum expectation values of the neutral components of \( \eta \) and \( \rho \) respectively, and \( r \) is the ratio \( v_\rho/v_\eta \). The mass matrices can be diagonalized by making the biunitary transformations

\[
U'_L = V^U_L U_L, \quad U'_R = V^U_R U_R, \tag{10a}\]

\[
D'_L = V^D_L D_L, \quad D'_R = V^D_R D_R, \tag{10b}\]

where the mass eigenstates are \( U = (u, c, t)^T \) and \( D = (d, s, b)^T \) if we assume that the first generation is the one which transforms differently, or \( U = (t, u, c)^T, \ D = (b, d, s)^T \) if it is the third generation which is treated in a different way. Both choices will differ in the parameterization of the matrices in Eqs. (10). We will choose in the following the first alternative.

Using Eqs. (10), we see from (3) that

\[
Y^U_L \rightarrow V^U_L Y^U_L V^U_L, \quad Y^U_R \rightarrow V^U_R Y^U_R V^U_R = Y^U_R. \tag{11a}\]

\[
Y^D_L \rightarrow V^D_L Y^D_L V^D_L, \quad Y^D_R \rightarrow V^D_R Y^D_R V^D_R = Y^D_R. \tag{11b}\]

Notice that the right handed neutral currents remain diagonal, but not the left-handed ones.
Next, we want to consider possible constraints that arise from experimental data in the $K^0 - \bar{K}^0$, $D^0 - \bar{D}^0$ and $B^0 - \bar{B}^0$ systems. In particular, from Eqs. (3) and (11) we can write the flavor-changing vertices $\bar{d}\gamma^\mu s$, $\bar{u}\gamma^\mu c$ and $\bar{d}\gamma^\mu b$

$$\mathcal{L}_{ds} = \frac{g \cos \theta_W}{\sqrt{3} h(x)} [V_{L}^{D*S}_{L12}] \bar{d}_L \gamma^\mu s_L Z''_\mu + \text{H.c.} \quad (12a)$$

$$\mathcal{L}_{uc} = \frac{g \cos \theta_W}{\sqrt{3} h(x)} [V_{L}^{U'S}_{L12}] \bar{u}_L \gamma^\mu c_L Z''_\mu + \text{H.c.} \quad (12b)$$

$$\mathcal{L}_{db} = \frac{g \cos \theta_W}{\sqrt{3} h(x)} [V_{L}^{D*B}_{L13}] \bar{d}_L \gamma^\mu b_L Z''_\mu + \text{H.c.} \quad (12c)$$

Now from (12) we obtain at first order in $G_F$ the effective Lagrangian

$$\mathcal{L}^{\text{eff}}_{\Delta S=2} = \frac{G_F}{\sqrt{2}} \frac{M_W^4}{M_Z^2 M_{Z'}^2} \left\{ \frac{4}{3h^2(x)} (V_{L11}^{D*D} V_{L12}^{D'})^2 [\bar{d}_L \gamma^\mu s_L Z''_\mu]^2 \right\}, \quad (13)$$

together with the respective expressions for the $\Delta C=2$ and $\Delta B=2$ operators. Moreover, defining $\Delta m_P = m_{P_1} - m_{P_2}$, where $P_1$ and $P_2$ represent the neutral $K$, $D$ and $B$ mass eigenstates, we obtain

$$\frac{\Delta m_P}{m_P} = \frac{G_F}{\sqrt{2}} \frac{M_W^4}{M_Z^2 M_{Z'}^2} \left\{ \frac{8}{3h^2(x)} f_P B_P \text{Re}[V_{L11}^{P*} V_{L12}^{P}] \right\} \quad (14)$$

Here, $j=2$ for the $K_L - K_S$ and $D^0_1 - D^0_2$ mass differences, and $j=3$ in the $B^0 - \bar{B}^0$ mixing case. The matrix $V$ has to be chosen as the $V_L^U$ or the $V_L^D$ one depending on the quark type involved in the corresponding mixing.

The experimental value for the mass differences are

$$\Delta m_K = (3.522 \pm 0.016) \times 10^{-15} \text{ GeV} \quad (15a)$$

$$\Delta m_D < 1.3 \times 10^{-13} \text{ GeV} \quad (15b)$$

$$\Delta m_B = (3.6 \pm 0.7) \times 10^{-13} \text{ GeV} \quad (15c)$$

Using $m_K \simeq 0.5 \text{ GeV}$, $m_D \simeq 1.86 \text{ GeV}$ and $m_B \simeq 5.28 \text{ GeV}$, we obtain
\[
\frac{\Delta m_K}{m_K} = 7.04 \times 10^{-15}, \quad \frac{\Delta m_D}{m_D} \approx 7.0 \times 10^{-14}, \quad \frac{\Delta m_B}{m_B} \approx 6.82 \times 10^{-14}. \tag{16}
\]

Assuming now that in all these cases the contribution (14) to \(\Delta m_P/m_P\), coming from the \(Z'\) exchange, is less than the experimental values given in (16), we obtain the bounds

\[
M_{Z'} > 1.39 \times 10^6 |\text{Re}(V_{L11}^U V_{L12}^D)^2|^{\frac{1}{2}} \, \text{GeV} \tag{17a}
\]

\[
M_{Z'} > 5.52 \times 10^5 |\text{Re}(V_{L11}^U V_{L12}^D)^2|^{\frac{1}{2}} \, \text{GeV} \tag{17b}
\]

\[
M_{Z'} > 6.15 \times 10^5 |\text{Re}(V_{L11}^U V_{L13}^D)^2|^{\frac{1}{2}} \, \text{GeV}. \tag{17c}
\]

We have used \(f_K \approx 0.16 \, \text{GeV}\) and \(f_D \approx 0.2 \, \text{GeV}\), \(x \approx 0.2325\) and all other values from Ref. [7]. In order to get the numerical estimations, we have taken the “bag constants” \(B_D\) and \(B_K\) equal to one, and used the lattice calculation \(\sqrt{B_{B_d}f_{B_d}} \simeq 0.22\) [8].

The bounds in (17) have been found to depend on the matrix elements \(V_{L1j}^{U,D}\). As it is well known, in the Standard Model both matrices \(V_{L}^{U,D}\) appear only in the combination

\[
V_{L}^{U\dagger}V_{L}^{D} = V_{CKM}, \tag{18}
\]

and for this reason it is a usual convention to assume that \(V_{CKM} = V_{L}^{D}\), that is, \(V_{L}^{U} \equiv 1\). However, as we can see from (12), this is not the situation in the present case. Actually, in the model of refs. [1,2] both matrices survive in different pieces of the Lagrangian and it is too strong to set \(V_{L}^{U} = 1\). This assumption would be also not stable against radiative corrections since all matrix elements evolve with energy according to the renormalization group equation [10]. Hence, the upper bounds for \(M_{Z'}\) depend on new parameters, which have been introduced due to the special representation content of the model.

The complex numbers \(V_{L1j}^{U,D}\) cannot be estimated from the present experimental data. Indeed, the mixing matrices are only constrained by the relation (18). We see thus from (17) that it is possible to have a neutral boson \(Z'\) with a mass of about 1.5 TeV if

\[
\text{Re}(V_{L11}^{U,D} V_{L1j}^{U,D}) \sim 10^{-3} \quad \forall j. \tag{19}
\]
All the results, up to now, are common to both models of Refs. [1,2], since the labels “first” or “third” generation are completely meaningless. Indeed, it is possible to go from one choice to another just using the transformations

\[
V_{L}^{U,D} \rightarrow \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix} V_{L}^{U,D}
\] (20)

As it is pointed out above, we are not able to get any experimental information about the matrices \(V_{L}^{U,D}\), except for relation (18). However, the observed hierarchies among both fermion masses and mixing angles have induced the physicists to propose different Ansätze about the matrices \(\Gamma_{U,D}^{U,D}\) defined in (9). We will see that with this assumptions the equivalence between both \(SU(3)_{L} \otimes U(1)_{Y}\) models would not exist any more.

The hierarchy puzzle has given rise to many quark mass matrix models in the last fifteen years. In order to show how strongly the \(Z'\) bounds can be affected, let us consider the simple scheme proposed by H. Fritzsch [11], in which the mixing matrix elements respect the hierarchy

\[
V_{ij}^{U,D} \approx \left( \frac{m_i}{m_j} \right)^{\frac{1}{2}}, \quad i < j
\] (21)

This can be obtained assuming mass matrices \(\Gamma_{U,D}^{U,D}\) obeying \(\Gamma_{ij} \approx (m_{i}m_{j})^{\frac{1}{2}}\) [12].

Within this scenario, the experimental values for \(\Delta m_P\) \((P = K, D, B_{d}, B_{s}, ...)\) will imply respective bounds for \(M_{Z'}\) depending on which is the quark family treated in a different way from the other two. The results are summarized in Table I. The approximated values for the quark masses have been taken from [13].

In order to obtain numerical estimations, we have taken all the phases of the matrix elements equal to zero. This cannot be true if the quark mixing matrix is to be responsible for the observed CP violation in nature. The inclusion of complex phases would conduce to a reduction in the bounds appearing in Table I. However, the hierarchical picture should not be modified, unless we asked the phases \(V_{L1j}\) in (14) to yield the particular result
Re[(V_{L1}^* V_{Lj})^2] \simeq 0. In this way, as emerges from the table, now the election of the third family is favored if $Z'$ has to get a mass of $\mathcal{O}(1 \text{ TeV})$, as it is claimed in refs. \cite{4,5}. The strongest bound for this election can arise from the $B_s - \bar{B}_s$ mixing, whose suppression factor within the Fritzsch model is $|V_{23}^{D} V_{33}^{D}| \approx 0.2$.

It is important to remark that the proposal of Fritzsch is certainly not the unique one which conduce to results like those of Table \[\text{I}\]. Actually, the matrix texture

$$\Gamma = \begin{pmatrix} \alpha_{11} & \alpha_{12} & \alpha_{13} \\ \alpha_{21} & \beta_{22} & \beta_{23} \\ \alpha_{31} & \beta_{32} & \gamma \end{pmatrix},$$

with $\alpha_{ij} \ll \beta_{ij} \ll \gamma$, is common to many quark mass matrix Ansätze \cite{14}. Although the numerical results in the table might be modified, assuming the structure (22) is enough to favor the differentiation of the third family in order to keep relatively low bounds for $M_{Z'}$.

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\(^2\)Exceptions of this type of models are those which treat with “democratic” mass matrices \cite{15}. Their mixing angles are in general not small, conducing to high bounds on $M_{Z'}$. 

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REFERENCES

[1] F. Pisano and V. Pleitez, Phys. Rev. D 46 (1992) 410.

[2] P. H. Frampton, Phys. Rev. Lett. 69 (1992) 2889.

[3] D. Ng, The electroweak theory of $SU(3) \otimes U(1)$, preprint TRI-PP-92-125, December 1993.

[4] J. C. Montero, F. Pisano and V. Pleitez, Phys. Rev. D 47 (1993) 2918.

[5] R. Foot, O. F. Hernandez, F. Pisano and V. Pleitez, Phys. Rev. D 47 (1993) 4158.

[6] See the Appendix of Ref. [4].

[7] Particle Data Group, Phys. Rev. D 45, Part II (1992).

[8] R. J. Morrison and M. S. Witherell, Annu. Rev. Nucl. Part. Sci. 39 (1989) 183.

[9] A. Abada et al., Nucl. Phys. B376 (1992) 172.

[10] E. Ma and S. Pakvasa, Phys. Lett. 86B (1979) 43; Phys. Rev. D 20, 2899 (1979); K. Sasaki, Z. Phys. C 32 (1986) 149; K. S. Babu, Z. Phys. C 35 (1987) 699; V. Barger, M. S. Berger and P. Ohmann, Phys. Rev. D 47 (1993) 2038.

[11] H. Fritzsch, Phys. Lett. 73B (1978) 317, Nucl. Phys. B155 (1979) 189.

[12] T. P. Cheng and M. Sher, Phys. Rev. D35 (1987) 3484.

[13] H. Gasser and H. Leutwyler, Phys. Rep. 87 (1982) 77.

[14] S. Ranfone, Phys. Rev. D 42 (1990) 3819; S. Dimopoulos, H. Hall and S. Raby, Phys. Rev. D 45 (1992) 4192; J. Rosner and M. Worah, Phys. Rev. D 46 (1992) 1131; A. Antaramian, L. Hall and A. Rašin, Phys. Rev. Lett. 69 (1992) 1871; L. Hall and S. Weinberg, Phys. Rev. D 48 (1993) R979.

[15] P. Kaus and S. Meshkov, Mod. Phys. Lett. A3 (1988) 1251; Y. Koide, Phys. Rev D 28 (1983) 252.
TABLE I. $M_Z'$ lower bounds within a Fritzsch-type Ansatz for the quark mass matrices

| “Different” family | $K - \bar{K}$ | $D - \bar{D}$ | $B_d - \bar{B_d}$ |
|-------------------|---------------|---------------|------------------|
| First (ref. [1])  | 315 TeV       | 35 TeV        | 25 TeV           |
| Second            | 315 TeV       | 35 TeV        | 25 TeV           |
| Third (ref. [2])  | 10 TeV        | 300 GeV       | 25 TeV           |