COMPOSITE SUPERSTRING MODEL FOR HADRONS AND EXTENDED VIRASORO SUPERCONFORMAL SYMMETRY WITH SUPERCURRENTS CONSTRAINTS.

V. A. Kudryavtsev
Petersburg Nuclear Physics Institute

Abstract

Hadron dynamics is formulated in terms of interacting composite strings. Consistent composite string model with extended Virasoro superconformal symmetry is found. These composite strings carry flavour and chiral degrees of freedom on edging two-dimensional surfaces. Necessary correct description of string amplitudes without states of negative norms in the spectrum of physical states is reached when special supercurrents conditions are fulfilled.
1 Introduction. Some problems of string models for hadrons.

An essential interest in string description of hadron interactions has arisen as far back as forty years ago (the Nambu string [1] and dual resonance models initiated by Veneziano’s work [2]) due to the remarkable universal linearity of Regge trajectories $\alpha(t)$ for meson and baryon resonances [3]: $J = \alpha(M^2) = \alpha(0) + \alpha' M^2$ ($\alpha' \approx 0.85 \text{GeV}^{-2}$); where $J, M$ are spin and mass of a resonance. Now we have these trajectories up to $J = 5$ and states for not only leading ($n=0$) but for second ($n=1$), for third ($n=2$) and even for fourth ($n=3$) daughter trajectories $J_n = \alpha(0) - n + \alpha' M^2$ $n=0,1,2,3...$. See [4].

However attempts to build the string model for hadron interactions have not been successful since consistent models for relativistic quantum strings [5] have required the intercept of leading meson trajectory $\alpha(0)$ to be equal to one. A shift of this value from one has led to contradiction with unitarity. The real leading $\rho$-meson trajectory has the intercept to be equal one half approximately. Just this reason has led to superstring models with massless gluons (open strings) on the trajectory $J = 1 + \alpha'_\rho M^2$ and for massless gravitons (closed strings) on the trajectory $J = 2 + 1/2\alpha'_\rho M^2$.

Here we are facing other problem for hadron dynamics in the framework of ordinary string description. In the consistent critical case of string models non-planar loop string diagrams lead to appearance of closed string states as bound states of open strings (Fig.1). Then we shall obtain the slope $\alpha'_{\text{closed}}$ for closed strings as half of $\alpha'_{\text{open}}$ for open strings following usual string models. But a natural scale parameter $\alpha'$ for the graviton trajectory would be corresponding to the Planck mass $10^{19}$ Gev i.e. ($\alpha'_{\text{closed}} \equiv \alpha'_\rho \sim 10^{-38}\text{GeV}^{-2}$). Certainly it is beyond reach of hadron interactions scale corresponding to $\alpha'_{\text{H}} \approx 0,85\text{GeV}^{-2}$. As we shall see both problems find a solution in the context of composite string model.

2 New class of relativistic quantum strings: composite strings

Traditional consistent models for open strings have $N=1$ superconformal Virasoro symmetry on two-dimensional world surface. This superconformal Virasoro algebra leads us in critical case to the intercept of leading boson trajectory
to be equal to one and to massless vector bosons in the spectrum of physical states correspondingly. For closed strings as bound states of two open strings we have a pair of two-dimensional surfaces and the N=2 superconformal Virasoro symmetry. These quantum superconformal symmetries are a reflection of classical conformal symmetries on two-dimensional world surfaces for string actions [6].

A generalization of classical multireggeon (multistring) vertices [7] has been suggested by author in 1993 [8]. These string amplitudes have been used for description of interaction of many π-mesons [9]. New string vertices give a new geometric picture for interactions of strings which has a natural description in terms of composite strings and three two-dimensional surfaces for moving open string instead of usual one (see Fig.2.).

These additional edging two-dimensional surfaces carry quark quantum numbers (flavour, spin, chirality). This composite string construction reminds a gluon string with two pointlike quarks at ends of this string or a simplest case of a string ending at two branes when they are themselves some strings (Fig.3).

It is not surprisingly as we shall see further that we have here a possibility for N=3 extended superconformal Virasoro symmetry for these composite strings. Let us note that we have no supersymmetry in the Minkovsky (target) space for this model. The topology of interacting composite strings allows to solve the problem of the intercept $\alpha(0)$ for leading meson trajectory and to shift the value of this intercept to one half without breakdown of the extended
superconformal Virasoro symmetry for composite strings due to non-vanishing conformal weights for fields on both edging two-dimensional surfaces.

Composite origin of objects under consideration brings to solution of second problem for hadron strings. Indeed there is a new parameter for composite open strings in addition to usual parameter $\alpha'$ for classical open strings. This new parameter defines the part of momentum of hadron composite string which flows in central (basic) two-dimensional surface in reference to the rest of momentum on edging surfaces. If this part is vanishing then unitarity breaks down due to appearance of some nonunitary fixed singularity without any massless tensor state (graviton) for nonplanar loop diagrams. The non-vanishing value of this ratio defines the value of other ratio $\alpha'_{\text{closed}}/\alpha'_{\text{open}}$. It follows from nonplanar one-loop diagram for composite strings which lead to states of closed strings without edging surfaces (Fig.4).

So we have a possibility to choose this ratio as it is necessary for the correct graviton pole as $\alpha'_{\text{closed}}/\alpha'_{\text{open}} \sim 10^{-38}$.

This circumstance solves one more problem of asymptotic behaviour of hadron string amplitudes in deep inelastic region. Here we have Regge trajectories with the very small slope for closed string sector. Such Regge trajectories lead us to approximately power behaviour in deep inelastic region and at high energies $s$ for fixed ratio $s/t$. 

Fig.4.
3 Formulation of composite string model and vertices for interacting composite strings.

Many-string vertices of interacting composite open strings are natural generalizations of corresponding many-string (many-reggeon) vertices for classical string models. The classical Lovelace-Olive-Alessandrini multi-string vertices [7] define an interaction of $N$ arbitrary string states in the Veneziano or the Neveu–Schwarz-Ramond models [6] in terms of two-dimensional string fields: $X_\mu^{(i)}(z_i)$ (the coordinate of $i$-th string) for the vanishing conformal spin $j$ and $H_\mu^{(i)}(z_i)$ (the anticommuting superpartner of $X^{(i)}_\mu(z_i)$) for the conformal spin $j$ to be equal to $\frac{1}{2}$:

\[ X^{(i)}_\mu(z_i) = X^{(i)}_{o\mu} + P^{(i)}_\mu \ln z_i + \sum_n \frac{a^{(i)}_{n\mu}}{in} z^n, \]

\[ a^n_{-\mu} = a^{n\mu}, \]

\[ a_n|0\rangle = \langle 0|a_{-n} = 0; n > 0, \]

\[ \left[ a_{n\mu}^{(i)}, a_{m\nu}^{(j)} \right] = -n g_{\mu\nu} \delta_{n,-m}, \]

\[ H^{(i)}_\mu(z_i) = \sum b^{(i)}_\mu z^r, \]

\[ \left\{ b^{(i)}_{\mu}, b^{(j)}_{\nu} \right\} = -g_{\mu\nu} \delta_{r,-s}, \]

\[ b^+_r = b_{-r}, b_r|0\rangle = \langle 0|b_{-r} = 0. \]

The $N$-string amplitudes are represented by some integrals over $z_i$ variables of the vacuum expectation value for a product of a vertex operator and wave functions of string states:

\[ A_N = \int \prod dz_i \langle 0|V_N \prod \hat{\Psi}^i|0\rangle, \quad (1) \]

where $\hat{\Psi}^i|0\rangle$ is a wave function of $i$-th string state.

The $N$-string vertex operator $V_N$ for the Neveu–Schwarz model is given by the following exponent:

\[ V_{NS}^{\nu} = \exp \left( \frac{1}{2} \sum_{n,m,p} a^{(i)}_{n\mu} (U^{(i)}_{\mu\nu})_{nm} (V^{(j)}_{\nu\rho})_{mp} \frac{a^{(j)}_{p\rho}}{\sqrt{P}} \right. + \]

\[ + \left. \frac{1}{2} \sum_{n,m,p} b^{(i)}_{n+1/2} (U^{(i)}_{1/2\nu})_{nm} (V^{(j)}_{1/2\rho})_{mp} \frac{b^{(j)}_{p+1/2}}{\sqrt{P}} \right), \quad (2) \]
where \((U^{(i)})_{nm}\), \((V^{(i)})_{mp}\) are the special infinite matrices which depend on 
z_{i-1}, z_i, z_{i+1} complex variables. They are some representations of the SU(1,1) group
for the conformal spin \(j\).

These vertices \(V^{NS}_N\) have the necessary factorization and conformal properties.

It turns out there is another operator \(W_N\) with these properties:

\[
W_N = \sum_{n,m,k} \bar{\Psi}_{n+\frac{1}{2}} (U^{(1)}_{\frac{1}{2}})_{nm} (V^{(2)}_{\frac{1}{2}})_{mk} \Psi_{k+\frac{1}{2}} (U^{(2)}_{\frac{1}{2}})_{lp} (V^{(3)}_{\frac{1}{2}})_{ps} \Psi_{s+\frac{1}{2}} \ldots \times
\]

\[
\times \sum_{l,p,s} \bar{\Psi}_{l+\frac{1}{2}} (U^{(N)}_{\frac{1}{2}})_{lp} \Psi_{p+\frac{1}{2}} \Psi_{s+\frac{1}{2}} \ldots \times
\]

\[
\prod_{i=1}^{N} \sum_{n,m,p} \bar{\Psi}_{n+\frac{1}{2}} (U^{(i)}_{\frac{1}{2}})_{nm} (V^{(i+1)}_{\frac{1}{2}})_{mp} \Psi_{p+\frac{1}{2}} \Psi_{s+\frac{1}{2}} \ldots \times
\]

\[
\equiv \sum_{r} \Psi_{r+\frac{1}{2}} (z_i) = \sum_{r} \Psi_{r+\frac{1}{2}} (z_i).
\]

The generalized \(N\)-string vertex operator for composite strings is the product
of the old \(V^{NS}_N\) and the new operator \(W_N\):

\[
V^{\text{comp}}_N = V^{NS}_N W_N. \quad (4)
\]

It is evident that the operators \(V^{NS}_N\) and \(W_N\) have the different structures.
The matrices \(U^{(i)}\) and \(V^{(i)}\) in \(V^{NS}_N\) connect all possible fields \(X^{(i)}(a^{(i)}_n)\) with
each other. In the operator \(W_N\) the matrices \(U, V\) connect only neighboring
fields \(\Psi^{(i)}\) and \(\Psi^{(i+1)}\). That is why this operator \(W_N\) reproduces the structure
of dual quark diagrams here and the operator \(V^{NS}_N W_N\) leads us to composite
objects i.e. composite strings.

It is possible to use other similar operators instead of (3) with \(Y^{(i)}\)-fields
of \(j=1\) and \(f^{(i)}\)-fields of \(j = \frac{1}{2}\) in this cyclic operator.
Namely

\[ W_N \hat{V}^{(1)} \exp \left( \sum_{n,m,p} \frac{Y_n^{(1)}}{\sqrt{n}} (U^{(1)}_\xi)_{nm}(V^{(2)}_\xi)_{mp} \frac{Y_p^{(2)}}{\sqrt{p}} + \sum_{n,m,p} f_{n+1/2}^{(1)}(U^{(1)}_{1/2})_{nm}(V^{(2)}_{1/2})_{mp} f_{p+1/2}^{(2)} \right) \psi^{(2)} \]

\[ \hat{V}^{(2)} \exp \left( \sum_{n,m,p} \frac{Y_n^{(2)}}{\sqrt{n}} (U^{(2)}_\xi)_{nm}(V^{(3)}_\xi)_{mp} \frac{Y_p^{(3)}}{\sqrt{p}} + \sum_{n,m,p} f_{n+1/2}^{(2)}(U^{(2)}_{1/2})_{nm}(V^{(3)}_{1/2})_{mp} f_{p+1/2}^{(3)} \right) \psi^{(3)} \ldots \]

\[ \hat{V}^{(N)} \exp \left( \sum_{n,m,p} \frac{Y_n^{(N)}}{\sqrt{n}} (U^{(N)}_\xi)_{nm}(V^{(1)}_\xi)_{mp} \frac{Y_p^{(1)}}{\sqrt{p}} + \sum_{n,m,p} f_{n+1/2}^{(N)}(U^{(N)}_{1/2})_{nm}(V^{(1)}_{1/2})_{mp} f_{p+1/2}^{(1)} \right) \psi^{(1)} \]

\[ \equiv \prod_{i=1}^N \hat{V}^{(i)} \exp \left( \sum_{n,m,p} \frac{Y_n^{(i)}}{\sqrt{n}} (U^{(i)}_\xi)_{nm}(V^{(i+1)}_\xi)_{mp} \frac{Y_p^{(i+1)}}{\sqrt{p}} + \sum_{n,m,p} f_{n+1/2}^{(i)}(U^{(i)}_{1/2})_{nm}(V^{(i+1)}_{1/2})_{mp} f_{p+1/2}^{(i+1)} \right) \psi^{(i+1)}. \]  

For the composite string model under consideration we shall use the operator \( V^{comp}_N = V^{NS}_N W_N \) with \( W_N \) of type (5). So far as just this structure has a symmetrical description of two-dimensional fields both on the basic and on the edging surfaces. It provides the necessary extended superconformal symmetry of composite string amplitudes as we shall see further.

For investigation of composite superstrings it is more appropriate to move from multi-string vertices (4) to more simple vertices \( \hat{V}_i \) corresponding to emission of ground states. An amplitude \( A_N \) of interaction of \( N \) ground string states is represented by integral of vacuum expectation of product of \( \hat{V}_i \) vertices (Fig.5.)

\[ A_N = \int \prod dz_i \langle 0 | \hat{V}_1(z_1) \hat{V}_2(z_2) \hat{V}_3(z_3) \ldots \hat{V}_{N-1}(z_{N-1}) \hat{V}_N(z_N) | 0 \rangle \]

\[ \hat{V}_i(z_i) = z_i^{-L_0} \hat{V}_i(1) z_i^{L_0}, \]

(6)

These vertices \( \hat{V}_i \) have the well-known expressions for the Neveu-Schwarz model:

\[ \hat{V}_i(z_i) = z_i^{-L_0} [G_r, \exp ip_i X(1) :] z_i^{L_0}, \]

: \( \exp (ip_i X(1)) : = \exp (ip_i X^{(+)}(1)) \exp (ip_0 X(0)) \exp (ip_i X^{(-)}(1)) \]

(7)

\[ G_r^{NS} = \frac{1}{2\pi} \int_0^{2\pi} d\tau \left( H^\mu \frac{d}{d\tau} X_\mu + \hat{P}_\nu H^n \right) e^{-i\nu} \]

(8)

8
\[ \hat{V}_i(1) = (p_i H(1)) : \exp(i p_i X(1)) : \equiv \]
\[ \left( \sum_r p_i b_r \right) \exp(i p_i X^{(+)}(1)) \exp(i p_i X(0)) \exp(i p_i X^{(-)}(1)) \equiv \]
\[ \left( \sum_r p_i b_r \right) \exp(-i p_i \sum_n a_n) \exp(i p_i X(0)) \exp(i p_i \sum_n a_n) \]  
(9)

For composite strings operator structures (5) correspond to two-dimensional fields on edging surfaces. The corresponding operator vertices \( \hat{V}_i \) will contain as in (5) additional edging fields: \( Y_{\mu} \mu = 0,1,2,3 \) and its superpartner \( f_{\mu} \) with Lorentz indices. In addition to them we include similar fields (J with superpartner Φ) which carry internal quantum numbers (isospin and other flavours) on edging surfaces and corresponding I,Θ fields on the basic two-dimensional surface. As we shall see some relations between fields on the basic (central) surface and fields on edging two-dimensional surfaces (\( \partial X_\mu, H_\mu \) and \( Y_\mu, f_\mu \) for Lorentz indices; I,Θ and J,Φ for internal numbers) play an important role for the afore-mentioned extended Virasoro superconformal symmetry of the composite superstring model under consideration.

Since the edging fields are propagating only on the corresponding surfaces it is convenient to introduce vacuum states for the fields on the separate edging surfaces and to write (6) in equivalent form with help of these vacuum states:

\[ A_N = \int \prod dz_i \langle 0(1,2)|\hat{V}_{i+1}(z_i)|0(3)|\hat{V}_{23}(z_2)|0(4)|0(5)|\cdots \hat{V}_{34}(z_3)|0(6)|\cdots \hat{V}_{i+1,i}(z_i)|0(i+2)|0(i+1)|0|0|0(2)|0(1,2)|0(3)|0(4)|0(5)|\cdots \]  
(10)

This form (10) excludes this amplitude from the set of additive string models of the Lovelace’s paper [5] and leads to the topology of composite string models [8,9]. Now we are ready to formulate the vertex operator \( \hat{V}_{i+1,i}(z_i) \) (Fig.6) for
this composite string model:

\[ \hat{V}_{i,i+1}(z) = z_i^{-L_0} [G_r, \hat{W}_{i,i+1}] z_i^{L_0}, \]
\[ \hat{W}_{i,i+1} = \hat{R}^{\text{out}}_i \hat{R}_{i+1} \hat{R}^{\text{in}}_i \]  

The operators $\hat{R}^{\text{out}}_i$ and $\hat{R}^{\text{in}}_{i+1}$ are defined by fields on $i$-th and $(i+1)$-th edging surfaces. The operator $\hat{R}_{NS}$ is defined by fields on the basic surface. They have the same structure as the operator $: \exp ip_i X(1) :$ in (9) for both $Y$ and $J$ fields:

\[ \hat{R}^{\text{out}}_i = \exp \left( \xi_i \sum_n \frac{j^{(i)}}{n} \right) \exp \left( k_i \sum_n \frac{y^{(i)}}{n} \right) \exp ik_i y_0^{(i)} \]
\[ \tilde{\lambda}^{(+)}_i \exp \left( -k_i \sum_n \frac{y^{(i)}}{n} \right) \exp \left( -\xi_i \sum_n \frac{j^{(i)}}{n} \right) \]  

\[ \hat{R}^{\text{in}}_{i+1} = \exp \left( -\xi_{i+1} \sum_n \frac{j^{(i+1)}}{n} \right) \exp \left( -k_{i+1} \sum_n \frac{y^{(i+1)}}{n} \right) \exp \left( -ik_{i+1} y_0^{(i+1)} \right) \]
\[ \lambda^{(-)}_{i+1} \exp \left( k_{i+1} \sum_n \frac{y^{(i+1)}}{n} \right) \exp \left( \xi_{i+1} \sum_n \frac{j^{(i+1)}}{n} \right) \]  

\[ \sum_i k_i = 0 \]
\[ \hat{R}^{(NS)}_{i,i+1} = \exp \left( -\zeta_s \sum_n \frac{I_s^n}{n} \right) \exp \left( -\zeta_{i,i+1} \sum_n \frac{I_{i,i}^n}{n} \right) \exp \left( -p_{i,i+1} \sum_n \frac{a_{i,i}^n}{n} \right) \exp \left( -ip_{i,i+1}X_0 \right) \]

\[ \Gamma_{i,i+1} \exp \left( p_{i,i+1} \sum_n \frac{a_{i,i}^n}{n} \right) \exp \left( (\zeta_{i,i+1} \sum_n \frac{I_{i,i+1}^n}{n} \right) \exp \left( (\zeta_s \sum_n \frac{I_s^n}{n} \right) \] (15)

Here we have introduced \( \lambda_\alpha \) operators to be carrying quark flavours and quark spin degrees of freedom.

\[ \langle 0 | \lambda^+ | 0 \rangle = 0; \langle - \lambda^- | 0 \rangle = 0 \] (16)

These operators obey simple equations:

\[ \{ \lambda^{(-)}_\alpha, \lambda^{(+)}_\beta \} = \delta_{\alpha,\beta}; \quad \tilde{\lambda} = \lambda T_0, \]

\[ T_0 = \gamma_0 \otimes \tau_2; \]

\[ \hat{p}_{i,i+1} = \hat{\beta}_{in}^{(i+1)} \hat{p}_{i+1} + \hat{\beta}_{out}^{(i)} \hat{p}_i \rightarrow \hat{\beta}_{in}^{(i+1)} k_{i+1} - \hat{\beta}_{out}^{(i)} k_i \] (17)

\[ (\hat{p}_i)_\mu = -\frac{1}{i} \frac{\partial}{\partial Y_{0\mu}}; \quad (\hat{p}_{i+1})_\mu = -\frac{1}{i} \frac{\partial}{\partial Y_{0\mu}^{(i+1)}} \] (18)

\[ \hat{\zeta}_{i,i+1} = \hat{\alpha}_{in}^{(i+1)} \hat{\zeta}_{i+1} + \hat{\alpha}_{out}^{(i)} \hat{\xi}_i \]

\[ \hat{\xi}_i = \tilde{\lambda}^{(+)}_i \xi\lambda^{(-)}_i; \quad \hat{\xi}_{i+1} = \tilde{\lambda}^{(i+1)}_{i+1} \xi\lambda^{(-)}_{i+1}; \] (19)

Here \( \xi \) is some universal matrix over quark flavours.

Let us notice that values \( k_i; k_{i+1}; \xi_i; \xi_{i+1}; p_{i,i+1}; \zeta_{i,i+1} \) in the vertex operator \( \hat{V}_{i,i+1}(1) \) and in (12),(13),(15) are eigenvalues of the corresponding operators (18), (17),(19). The quark spinor and isospinor operators \( \lambda_\alpha \) therewith are the eigenfunctions of the operators \( \hat{\xi}_i \) and play the same role as functions \( \exp ik_i Y^{(i)}_0 \) which are the eigenfunctions of the operator \( (\hat{p}_i)_\mu = -\frac{1}{i} \frac{\partial}{\partial Y_{0\mu}} \).

So we give some relation between of momenta (charges) which flow into the basic surface and into edging surfaces. Namely operators \( \hat{\beta}(\hat{\alpha}) \) define fractions...
of i-th and (i+1)-th momenta (charges) for the basic surface. Let us bring definitions and constraints for them:

\[ \hat{\beta}_{in}^{(i+1)} = \tilde{\lambda}_{i+1}^{(+)} \beta_{in}(i+1); \hat{\beta}_{out} = \tilde{\lambda}_{i+1}^{(+)} \beta_{out}(i+1); \]
\[ \hat{\beta}_{in}^{(i)} = \tilde{\lambda}_{i+1}^{(-)} \beta_{in}(i); \hat{\beta}_{out} = \tilde{\lambda}_{i+1}^{(-)} \beta_{out}(i); \]
\[ \beta_{in} = \beta_{out} = \alpha_{in} = \alpha_{out} = 1 \quad (20) \]

Now there are constraints for matrices \( \beta; \alpha \):

\[ \beta_{in}^2 = \beta_{out}^2 = \alpha_{in}^2 = \alpha_{out}^2 = 1 \quad (21) \]
\[ \beta_{out} = \beta_{in}^+; \alpha_{out} = \alpha_{in}^+ \quad (22) \]
\[ [\beta, \alpha] = 0 \quad (23) \]

We can propose some simple choice for \( \beta_{in}, \beta_{out} \):

\[ \beta_{in} = \beta = a \gamma^P + b \gamma^C \gamma^P; \beta_{out} = \beta^+ = a \gamma^P - b \gamma^C \gamma^P; a = \cosh \phi; b = \sinh \phi \quad (24) \]

\[ \gamma^C = \gamma^0 \gamma^2; \gamma^P = \gamma^0 \]

Here \( \gamma^\mu \) are usual Dirac matrices. The product \( \gamma^C \gamma^P \) corresponds to the CP transformation of spinors \( \lambda \). Let us notice that value of \( \phi \) defines the fraction of momentum to be flowing into two-dimensional surface for the closed string sector and therefore \( \phi \sim 10^{-38} \) as it has discussed above (see page 3).

### 4 Extended Virasoro superconformal symmetries for composite superstrings

The spectrum of spurious states which drop out of physical amplitudes and therefore the spectrum of physical composite string states is defined by the symmetries of the vertices \( \hat{V}_{i,i+1}(1) \). Main symmetry of any string model is the superconformal symmetry to be defined by the Virasoro operators \( G_r \). Certainly it requires the operator vertices (11) to have conformal spin \( j \) equal to be one as for all open string models. The operators \( G_r \) satisfy standard superconformal algebra:

\[ \{ G_r, G_s \} = 2 L_{r+s} + \frac{c}{3} \left( r^2 - \frac{1}{4} \right) \delta_{r,-s} \quad (26) \]
\[ [L_n, L_m] = (n - m) L_{n+m} + \frac{c}{12} n(n^2 - 1) \delta_{n,-m} \quad (27) \]
\[ [L_n, G_r] = \left( \frac{n}{2} - r \right) G_{n+r}. \quad (28) \]
In the case of the Neveu-Schwarz model we have the generators $G_{r}^{NS}(8)$ of Virasoro algebra.

For composite superstring model we consider the set of states and of superconformal generators for the i-th section between the $\tilde{V}_{i-1,i}$ vertex and $\tilde{V}_{i,i+1}$ vertex in (10)(see Fig.7).

Namely we have fields on (i-1), i, (i+1) edging surfaces:

$$(Y(i-1), f^{(i-1)}); (J(i-1), \Phi(i-1)); (Y(i), f^{(i)}); (J(i), \Phi(i)); (Y(i+1), f^{(i+1)}); (J(i+1), \Phi(i+1))$$

fields in addition to fields which are on the basic surface:

$$(\partial X, H); (I, \Theta); (I_s, \Theta_s)$$

Now superconformal generators $G_r$ can be defined as the following ones:

$$G_r = G_{r}^{Lor} + G_{r}^{Int}$$

$$G_{r}^{Lor} = \frac{1}{2\pi} \int_{0}^{2\pi} d\tau \left( H\mu \frac{d}{d\tau} X_{\mu} + \hat{P}_{\nu} H^{\nu} + \hat{P}_{1} f^{(1)} + Y_{\mu}^{(i-1)} f^{(i-1)\mu} + Y_{\mu}^{(i)} f^{(i)\mu} + Y_{\mu}^{(i+1)} f^{(i+1)\mu} \right) e^{-ir\tau}$$

$$G_{r}^{Int} = \frac{1}{2\pi} \int_{0}^{2\pi} d\tau \left( (I\Theta) + (I_s\Theta_s) + \xi_{i} \Phi^{(1)} + (J^{(i-1)} \Phi^{(i-1)}) + (J^{(i)} \Phi^{(i)}) + (J^{(i+1)} \Phi^{(i+1)}) \right) e^{-ir\tau}$$

But unlike the Neveu-Schwarz model this composite string model has a new superconformal symmetry which defines by the following generators $\tilde{G}_{r}$:

$$\tilde{G}_r = \tilde{G}_{r}^{Lor} + \tilde{G}_{r}^{Int}$$

$$\tilde{G}_{r}^{Lor} = \frac{1}{2\pi} \int_{0}^{2\pi} d\tau \left( (\partial X_{\mu} \tilde{\beta}^{(i-1)} f^{(i-1)\mu} + \partial X_{\mu} \tilde{\beta}^{(i)} f^{(i)\mu} + \partial X_{\mu} \tilde{\beta}^{(i+1)} f^{(i+1)\mu} + \right)$$
\[
+ \left( Y^{(i-1)} \hat{\beta}^{(i-1)} H^\mu + Y^{(i)} \hat{\beta}^{(i)} H^\mu + Y^{(i+1)} \hat{\beta}^{(i+1)} H^\mu \right) - \\
- \left( Y^{(i-1)} \hat{\beta}^{(i-1)} f^{(i)} \mu + Y^{(i)} \hat{\beta}^{(i)} f^{(i)} \mu + Y^{(i+1)} \hat{\beta}^{(i+1)} f^{(i)} \mu \right) + \\
+ Y^{(i+1)} \hat{\beta}^{(i+1)} f^{(i)} \mu + \hat{p}_1 f^{(1)} \right) e^{-ir\tau} 
\]

\[
\tilde{G}'_{int} = \frac{1}{2\pi} \int_0^{2\pi} d\tau \left( I\hat{\alpha}^{(i-1)} \Phi^{(i-1)} + I\hat{\alpha}^{(i)} \Phi^{(i)} + I\hat{\alpha}^{(i+1)} \Phi^{(i+1)} \right) + \\
+ \left( J^{(i-1)} \hat{\alpha}^{(i-1)} \Theta + J^{(i)} \hat{\alpha}^{(i)} \Theta + J^{(i+1)} \hat{\alpha}^{(i+1)} \Theta \right) - \\
- \left( J^{(i-1)} \hat{\alpha}^{(i-1)} \hat{\alpha}^{(i)} \Phi^{(i)} + J^{(i)} \hat{\alpha}^{(i)} \hat{\alpha}^{(i-1)} \Phi^{(i-1)} + J^{(i+1)} \hat{\alpha}^{(i+1)} \hat{\alpha}^{(i)} \Phi^{(i)} \right) + \left(32\right) \\
+ J^{(i)} \hat{\alpha}^{(i)} \hat{\alpha}^{(i+1)} \Phi^{(i+1)} \right) + I_s \Theta_s + \xi_1 \Phi^{(1)} \right) e^{-ir\tau} 
\]

These operators \( \tilde{G}'_{r} \) have the same commutation relations to the operator vertex \( \hat{V}_{i,i+1} \) as \( G_r \):

\[
[\tilde{G}_{r}, \hat{V}_{i,i+1}(1)] = [G_{r}, \hat{V}_{i,i+1}(1)]; \quad \left(33\right) \\
[\tilde{G}_{r}, \hat{W}_{i,i+1}] = [G_{r}, \hat{W}_{i,i+1}] \quad \left(34\right) 
\]

Just here we have used the definite relations (17)-(23) for momenta and charges in the operator vertex \( \hat{V}_{i,i+1} \). Taking into account the expressions (29)-(32) we can derive the corresponding commutation relations for \( \tilde{G}_{r} \) and \( G_r \):

\[
\{G_{r},G_{s}\} = 2L_{r+s} + \frac{c}{3} \left( r^2 - \frac{1}{4} \right) \delta_{r,-s} 
\]

\[
\{G_{r},\tilde{G}_{s}\} = 2\tilde{G}_{r+s}; \quad \left(35\right) \\
\{G_{r},\tilde{G}_{s}\} = 2\tilde{G}_{r+s}; \quad \left(36\right) 
\]

\[
\{\tilde{G}_{r},\tilde{G}_{s}\} = 4L_{r+s} - 2L_{r+s} + \frac{2c}{3} \left( r^2 - \frac{1}{4} \right) \delta_{r,-s} 
\]

\[
[L_{n},L_{m}] = (n-m)L_{n+m} + \frac{c}{12} n(n^2 - 1)\delta_{n,-m} \quad \left(37\right) 
\]
\[ [\tilde{L}_n, \tilde{L}_m] = (n - m)\tilde{L}_{n+m} \] (39)

\[ [\tilde{L}_n, \tilde{L}_m] = 2(n - m)L_{n+m} - (n - m)\tilde{L}_{n+m} + \frac{c}{6} n(n^2 - 1)\delta_{n,-m} \] (40)

\[ [L_n, G_r] = \left(\frac{n}{2} - r\right) G_{n+r} \] (41)

\[ [L_n, \tilde{G}_r] = \left(\frac{n}{2} - r\right) \tilde{G}_{n+r} \] (42)

\[ [\tilde{L}_n, \tilde{G}_r] = (n - 2r) G_{n+r} - \left(\frac{n}{2} - r\right) \tilde{G}_{n+r} \] (43)

Due to this algebra and equations (33), (34) we are able to prove as earlier in classical models that both \( G_r \) and \( \tilde{G}_r \) operators generate spurious states. This commutation agebra allows to extract the independent combinations of operators \( G_r \) and \( \tilde{G}_r \) which define the spectrum of spurious states. We have three sets of this sort:

\[ G^I_r = \frac{1}{3}(\tilde{G}_r + 2G_r) \] (44)

\[ G^{II}_r = (G^Lor_r - \tilde{G}^{Lor}_r) \] (45)

\[ G^{III}_r = (G^{Int}_r - \tilde{G}^{Int}_r) \] (46)

\[ \{G^I_r, G^I_s\} = \{G^{II}_r, G^{II}_s\} = \{G^{III}_r, G^{III}_s\} = 0 \] (47)

\[ [L^I_n, L^I_m] = [L^{II}_n, L^{II}_m] = [L^{III}_n, L^{III}_m] = 0 \] (48)

Now we are able to formulate in i-th section of our amplitude (see Fig.7.) the extended superconformal constraints for physical states:
\[ G^I_r |\text{Phys}\rangle = 0 ; \quad L^I_n |\text{Phys}\rangle = 0 ; \quad r, n > 0 \] (49)

\[ G^{II}_r |\text{Phys}\rangle = 0 ; \quad L^{II}_n |\text{Phys}\rangle = 0 ; \quad r, n > 0 \] (50)

\[ G^{III}_r |\text{Phys}\rangle = 0 ; \quad L^{III}_n |\text{Phys}\rangle = 0 ; \quad r, n > 0 \] (51)

\[ L_0 |\text{Phys}\rangle = \frac{1}{2} |\text{Phys}\rangle ; \] (52)

Let us notice that our construction of vertices (12),(13),(15) according to (17)-(19) contains some definite combinations of fields with Lorentz indices:

\[ k_i \tilde{f}^{(i)} = k_i (f^{(i)} + \hat{\beta}_i H) ; \]
\[ k_i \tilde{Y}^{(i)} = k_i (Y^{(i)} + \hat{\beta}_i \partial X) \] (53)

and with internal quantum numbers (with exception of \( I_s ; \Theta_s \)):

\[ \zeta_i \tilde{\Phi}^{(i)} = \zeta_i (\Phi^{(i)} + \hat{\alpha}_i \Theta) ; \]
\[ \zeta_i \tilde{J}^{(i)} = \zeta_i (J^{(i)} + \hat{\alpha}_i I) \] (54)

So in our analysis of the Fock space of states for the \( i \)-th section of the amplitude (10) (see Fig.7.) we can use these combinations and orthogonal to them components \( p^i \tilde{f}^{(i)} ; p^i \tilde{Y}^{(i)} \) with \( p^i k_i = 0 ; p^i k_{i-1} = 0 \) (on the left side) and \( p^i k_i = 0 ; p^i k_{i+1} = 0 \) (on the right side):

\[ p^i \tilde{f}^{(i)} = p^i (f^{(i)} + \hat{\beta}_i H) ; \]
\[ p^i \tilde{Y}^{(i)} = p^i (Y^{(i)} + \hat{\beta}_i \partial X) \] (55)

Let us notice that all combinations (53),(54)and (55) commute with operators \( G^{II}_r , G^{III}_r \) and \( L^{II}_n , L^{III}_n \).
5 Elimination of states with negative norms and supercurrent constraints

Let us consider the construction of the spectrum generating algebra for this composite superstring by similar way as in classical string models [6]. For the given i-th section (between \(V_{i-1, i}\) and \(V_{i, i+1}\)) we have fields on \(i, i-1, i+1\) edging surfaces and fields on the basic surface. We are able to build the set of transversal states which are similar to DDF states for the Neveu-Schwarz model with help of the operators of type of vertex operators (11) with the conformal spin \(j\) to be equal to one and transversal components of the \(\partial X\) and \(H\)-fields.

Spurious states for this basis are defined by products of operators \(G_I^r, G_I^{II}, G_I^{III}\) and \(L_n^I, L_n^{II}, L_n^{III}\). But only these states are not able to get rid of negative norms the spectrum of physical states as it has taken place for usual classical string models since the capacity of those of them which have negative norms is not enough. Powers of \(G_I^{II}, G_I^{III}\) and \(L_n^{II}, L_n^{III}\) do not allowed to do it where as only odd powers of \(G_I^r, L_n^I\) are not enough. For the Fock space under consideration we can obtain states with negative norms not only the powers of time components of the \(\partial X\) and \(H\) fields on the basic surface but as odd powers of components (if they are time-like ones):

\[
\tilde{f}(i), \tilde{Y}(i), \tilde{f}(i+1), \tilde{Y}(i+1)
\]

and

\[
\tilde{f}(i-1), \tilde{Y}(i-1), \tilde{f}(i), \tilde{Y}(i)
\]

Hence additional conditions for the composite string model are required in order to eliminate all negative norms from the spectrum of physical states. There is a simple solution for it. We shall require as gauge conditions the supercurrent conditions generated by \(k_i \tilde{f}(i)\).

Namely we shall take the following constraints for our vertices:

\[
[k_i \tilde{Y}(i), \tilde{W}_n^{i, i+1}] = [\tilde{W}_n^{i, i+1}, k_i \tilde{Y}(i+1)] = 0 \tag{56}
\]

Then we shall have enough states of negative norms generated by all gauge constraints \(G_r^I, G_r^{II}, G_r^{III}, L_n^I, L_n^{II}, L_n^{III}\) and

\[
k_i \tilde{f}(i), k_i \tilde{Y}(i), k_i \tilde{f}(i-1), k_i \tilde{Y}(i-1), k_{i+1} \tilde{f}(i+1), k_{i+1} \tilde{Y}(i+1)
\]

The equations (56) lead to the conditions:

\[
k_i^2 \to 0; k_{i+1}^2 \to 0; (k_i k_{i+1}) \to 0; \tag{57}
\]

Similarly we obtain

\[
[k_i \tilde{Y}(i-1), \tilde{W}_n^{i-1, i}] = [\tilde{W}_n^{i-1, i}, k_i \tilde{Y}(i)] = 0;
\]

\[
k_i^2 \to 0; k_{i-1}^2 \to 0; k_i k_{i-1} \to 0; \tag{58}
\]
So our gauge supercurrents are independent and nilpotent ones:

\[ [k_i \tilde{Y}_n^{(i)}, k_i \tilde{Y}_m^{(i)}] = 0; [k_{i+1} \tilde{Y}_n^{(i+1)}, k_i \tilde{Y}_m^{(i)}] = 0 \]  

(59)

Let us notice that our choice for additional gauge conditions is appropriate for emission of \( \pi \)-mesons (the case of usual quarks). It gives an explanation for massless \( \pi \)-mesons and correct amplitudes for \( \pi \)-mesons interaction [9]. But other quark flavours bring us to gauge supercurrent constrains which contain not only fields with Lorentz indices \( \tilde{Y}^{(i)} \) but and some part of fields \( \tilde{J}^{(i)} \) for internal numbers. So we have to substitute \( k_i \tilde{Y}^{(i)} \) supercurrents to \( k_i^{(LI)} \) instead of \( k_i^{(LI)} \tilde{Y}^{(LI)(i)} \) to be some sum of Lorentz fields and of fields to be carrying internal numbers (their contribution in usual quarks should be vanishing due to the corresponding operator \( \hat{\xi} \) for \( J_n \) in (12),(13),(19)). Then we have the generalized momentum \( k_i^{(LI)} \) instead of \( k_i \) and the conditions \( (k_i^{(LI)})^2 = 0 \) instead of (57) will lead to massive mesons for other flavours (K-mesons and so on). Now we are able to build spectrum generating algebra (SGA) for our set of states in the same manner as for the Neveu- Schwarz string model [6].

We take for the chosen i-th section with the vertex \( V_{i-1,i} \) on the left side and the vertex \( V_{i,i+1} \) on the right side the following way.

We shall use the light-like vectors \( k_i^{(LI)} \) from our vertices \( ((k_i^{(LI)})^2 = 0) \) and consider a state of the generalized momentum \( P^{(gen)} = p_0 + Nk_i^{(LI)} \).

\[
\frac{p_0^2}{2} = -1; \quad (k_i^{(LI)})^2 = 0; \quad (k_i^{(LI)} p_0) = 1 \\
(k_i^{(LI)})^2 = 0; \quad (k_i^{(LI)} k_i^{(LI)}) \rightarrow 0; \quad (k_i^{(LI)} k_i^{(LI)}) = 0. 
\]

(60)

(61)

Transversal components of \( k_i^{(LI)}, p_0 \) are vanishing \( (p_0)_a = (k_i^{(LI)})_a = 0 \). The generalized mass of this state is given by :

\[
\frac{M^{(gen)^2}}{2} = \frac{(p_0 + Nk_i^{(LI)})^2}{2} = -1 + N 
\]

(62)

We define the transversal operators of SGA as corresponding vertex operators. So for components corresponding to \( ((Y^{(i)} + \hat{\beta}_i \partial X)_a)_n \equiv (Y_a^{(i)})_n \) we use \( (S_a^{(i)})_n \) operators:

\[
(S_a^{(i)})_n = \frac{1}{2\pi} \int_0^{2\pi} d\tau V_a^{(i)}(nk_i^{(LI)}, \tau) 
\]

(63)
\[ V_a^{(i)}(nk^{(LI)}_i, \tau) = \exp iL_0\tau \left[ G_r, \hat{W}_a^{(i)}(nk^{(LI)}_i, 0) \right] \exp -iL_0\tau \] (64)

\[ \hat{W}_a^{(i)}(k^{(LI)}_i, \tau) = \frac{1}{\sqrt{2}} (f_a^{(i)}(\tau) + \hat{\beta}_i H_a(\tau)) : \exp ik^{(LI)}_i(Y^{(LI)}(i)(\tau) + \hat{\beta}_i \partial X^{(LI)}(\tau)) : = \frac{1}{\sqrt{2}} \tilde{f}_a^{(i)}(\tau) : \exp (ik^{(LI)}_i \tilde{Y}^{(LI)}(i)(\tau)) \] (65)

Similarly for components corresponding to \((\tilde{Y}^{(i-1)})_n\) we have \((S_a^{(i-1)})_n\) operators:

\[ (S_a^{(i-1)})_n = \frac{1}{2\pi} \int_0^{2\pi} d\tau V_a^{(i-1)}(nk^{(LI)}_i, \tau) \] (66)

\[ V_a^{(i-1)}(nk^{(LI)}_i, \tau) = \exp iL_0\tau \left[ G_r, \hat{W}_a^{(i-1)}(nk^{(LI)}_i, 0) \right] \exp -iL_0\tau \] (67)

\[ \hat{W}_a^{(i-1)}(k^{(LI)}_i, \tau) = \frac{1}{\sqrt{2}} \tilde{f}_a^{(i-1)}(\tau) : \exp (ik^{(LI)}_i \tilde{Y}^{(LI)}(i)(\tau)) \] (68)

For components corresponding to \((\tilde{Y}^{(i+1)})_n\) we have \((S_a^{(i+1)})_n\) operators:

\[ (S_a^{(i+1)})_n = \frac{1}{2\pi} \int_0^{2\pi} d\tau V_a^{(i+1)}(nk^{(LI)}_i, \tau) \] (69)

\[ V_a^{(i+1)}(nk^{(LI)}_i, \tau) = \exp iL_0\tau \left[ G_r, \hat{W}_a^{(i+1)}(nk^{(LI)}_i, 0) \right] \exp -iL_0\tau \] (70)

\[ \hat{W}_a^{(i+1)}(k^{(LI)}_i, \tau) = \frac{1}{\sqrt{2}} \tilde{f}_a^{(i+1)}(\tau) : \exp (ik^{(LI)}_i \tilde{Y}^{(LI)}(i)(\tau)) \] (71)

Other transversal SGA operators which are corresponding to transfer of internal quark numbers (flavour, chirality) and do not enter in \(\tilde{Y}^{(LI)}; \tilde{f}^{(LI)}\) (we shall mark them as \(\Phi', \Theta'\) and \(J', I'\) are defined by a similar way:

\[ \hat{W}_a^{(\text{Int})^{(i)}}(k^{(LI)}_i, \tau) = \frac{1}{\sqrt{2}} (\tilde{\Phi}'^{(i)} : \exp (ik^{(LI)}_i \tilde{Y}^{(LI)}(i)(\tau)) \] (72)
\[ \hat{W}_{a}^{\text{Int}(i-1)}(k_1^{(LI)}, \tau) = \frac{1}{\sqrt{2}} (\Phi'_{(i-1)}(\tau) + \hat{\alpha}_{i-1} \Theta'(\tau)) : \exp (ik_1^{(LI)}) \hat{Y}^{(LI)(i)}(\tau) : \] (73)

\[ \hat{W}_{a}^{\text{Int}(i+1)}(k_1^{(LI)}, \tau) = \frac{1}{\sqrt{2}} (\Phi'_{(i+1)} : \exp (ik_1^{(LI)}) \hat{Y}^{(LI)(i)}(\tau) : (74) \]

All these SGA operators \( (S_a^{(i)})_n \) have correct gauge and commutation properties:

\[ [\tilde{G}_r, (S_a^{(i)})_n] = [G_r, (S_a^{(i)})_n] = 0; \] (75)

\[ [k_1^{(LI)} \hat{Y}^{(LI)(i)}, (S_a^{(i)})_n] = 0; [k_{i-1}^{(LI)} \hat{Y}^{(LI)(i-1)}, (S_a^{(i)})_n] = 0 \] (76)

\[ [k_1^{(LI)} \hat{f}^{(LI)(i)}, (S_a^{(i)})_n] = 0; [k_{i-1}^{(LI)} \hat{f}^{(LI)(i-1)}, (S_a^{(i)})_n] = 0; \] (77)

As for the Neveu-Schwarz model more complicated constructions are used for independent transversal SGA operators \( (B_a^{(i)})_r \) corresponding to \( (f_r^{(i)} + \tilde{\beta}_i H_r) \equiv \hat{f}_r^{(i)} \) components:

\[ (B_a^{(i)})_r = \frac{1}{2\pi} \int_0^{2\pi} d\tau U_a^{(i)}(rk_1^{(LI)}, \tau) \] (78)

\[ U_a^{(i)}(k_1^{(LI)}, \tau) = \exp L_0 \tau \left[ G_r, \hat{Z}_a^{(i)}(k_1^{(LI)}, 0) \right] \exp -iL_0 \tau \] (79)

\[ Z_a^{(i)}(k_1^{(LI)}, \tau) = -\frac{1}{\sqrt{2}} \hat{f}_a^{(i)}(\tau) (k_1^{(LI)} \hat{f}^{(i)}(\tau)) \] (80)

These SGA operators satisfy necessary constraints:
\{G_r, (B_a^{(i)})_s\} = \{\bar{G}_r, (B_a^{(i)})_s\} = 0; \quad (81)

\[ [k_i^{(LL)} Y^{(LL)(i)}, (B_a^{(i)})_r] = 0; [k_{i-1}^{(LL)} Y^{(LL)(i-1)}, (B_a^{(i)})_r] = 0 \quad (82) \]

\{k_i^{(LL)} f^{(LL)(i)}, (B_a^{(i)})_r\} = 0; \{k_{i-1}^{(LL)} f^{(LL)(i-1)}, (B_a^{(i)})_r\} = 0; \quad (83)

Similarly we build transversal SGA operators \((\Theta^{(i)})_r\) which are carrying internal numbers and correspond to the combinations which do not enter in \(Y^{(LL)}; f^{(LL)}\) i.e. correspond to the \(\Phi', \Theta'\) and \(J', I'\) supercurrents:

\[(\Theta^{(i)})_r = \frac{1}{2\pi} \int_0^{2\pi} d\tau U^{Int(i)}(r k_i^{(LL)}, \tau) \quad (84)\]

\[U^{Int(i)}(k_i^{(LL)}, \tau) = \exp iL_0 \tau [G_r, \hat{Z}^{Int(i)}(k_i^{(LL)}, 0)] \exp -iL_0 \tau \quad (85)\]

\[Z^{Int(i)}(k_i^{(LL)}, \tau) = -\frac{1}{\sqrt{2}} \bar{\Phi}^{(i)}(\tau)(k_i^{(LL)} f^{(i)}(\tau)) \quad (86)\]

Again we have similar expressions for \((\Theta^{(i-1)})_r\) and \((\Theta^{(i+1)})_r\).

All transversal SGA operators satisfy simple commutation algebra:

\[ [S_a^{(i)} m, (S_b^{(j)})_m] = m \delta_{ij} \delta_{a,b} \delta_{m+n,0} \quad (87) \]

\[ [(S_a^{(i)})_m, (B_b^{(j)})_r] = 0; \{(B_a^{(i)})_r, (B_b^{(j)})_s\} = \delta_{ij} \delta_{a,b} \delta_{m+n,0} \]

So we can construct similarly to the DDF states transversal states \(|Phys\rangle\) from powers of the transversal SGA operators:

\[|Phys\rangle = \prod ((S_a^{(i)}_{-n})^{m_{a,n}} |\Psi_0\rangle \quad (88)\]

These states will satisfy the following conditions:
\[
G_r |\text{Phys} \rangle = 0; L_n |\text{Phys} \rangle = 0; n > 0; r > 0 \quad (89)
\]
\[
\tilde{G}_r = 0; \tilde{L}_n |\text{Phys} \rangle = 0; n > 0; r > 0 \quad (89)
\]
\[
(G_{Lr}^{\text{Lor}} - G_r^{\text{Lor}}) |\text{Phys} \rangle = 0; (L_{n}^{\text{Lor}} - L_n^{\text{Lor}}) |\text{Phys} \rangle = 0; \quad (90)
\]
\[
(k_i^{(LL)} \tilde{Y}^{(LL)(i)}|n\rangle \rangle |\text{Phys} \rangle = 0; (k_i^{(LL)} \tilde{Y}^{(LL)(i-1)}|n\rangle \rangle |\text{Phys} \rangle = 0; \quad (90)
\]
\[
(k_r^{(LL)} \tilde{f}(LL)(i)|r\rangle \rangle |\text{Phys} \rangle = 0; (k_r^{(LL)} \tilde{f}(LL)(i-1)|r\rangle \rangle |\text{Phys} \rangle = 0; \quad (90)
\]

Let us notice that all transversal SGA operators on the left side with (i-1)- and (i)- operators ( (i+1)-operators are vanishing there) can be defined in (63)-(68),(72),(73),(78)-(80) and in (84)-(86) with replacement of all (i)-fields to (i-1)-fields and vice versa of all (i-1)-fields to (i)-fields. It is true and for all transversal SGA operators on the right side with (i+1)- and (i)- operators ( (i-1)- operators are vanishing there ). They can be defined with replacement of all (i)-fields to (i+1)-fields and vice versa of all (i+1)-fields to (i)-fields. This possibility to reformulate these sets of states allows to move from states of i-th section under consideration to states in (i-1)-th section and so on.

Moving from these DDF type states to arbitrary states we can obtain them as usually with help of ordered powers of the conformal generators \( G_{r}^{I}, L_{n}^{I}; G_{r}^{II}, L_{n}^{II}; G_{r}^{III}, L_{n}^{III} \) and of powers of the supercurrent operators \( k_i^{(LI)} \tilde{Y}^{(LI)(i)} \), \( k_i^{(LI)} \tilde{f}(LI)(i) \) acting on |\text{Phys} \rangle states:

\[
(G_{r}^{I})^{\lambda(1)}(G_{r}^{I})^{\lambda(3)}...(L_{n}^{I})^{\mu(1)}(L_{n}^{I})^{\mu(2)}... \prod_r (k_i^{(LL)} \tilde{f}(LL)(i-1)) ^{\gamma(i-1,r)} \]
\[
\prod_n (k_i^{(LL)} \tilde{Y}^{(LL)(i-1)}) ^{\delta(i-1,n)} \prod_r (k_i^{(LL)} \tilde{f}(LL)(i)) ^{\gamma(i,r)} \prod_n (k_i^{(LL)} \tilde{Y}^{(LL)(i)}) ^{\delta(i,n)} |\text{Phys} \rangle \quad (90)
\]

Then we can repeat considerations in the Neveu-Schwarz model [6] for the theorem about absence of ghosts in the spectrum of physical states in our case for the critical value of the number of effective dimension in relation to \( G_{I}^{I} \) and \( G_{I}^{II} \) operators and taking into account the conditions (60), (61).

In critical case the operators \( G_{I}^{I} \) and \( G_{I}^{I} + 2(G_{I}^{I})^3 = G_{I}^{III} \) define null states:

\[
\{ G_{I}^{I}, G_{I}^{I} \} = 0 \quad (91)
\]
\[ |S_{-\frac{3}{2}}\rangle = G^{I}_{-\frac{3}{2}} |\text{Phys}\rangle; \langle S_{-\frac{3}{2}} | S_{-\frac{3}{2}} \rangle = 0 \] (92)

\[ |S_{-\frac{3}{2}}\rangle = G^{I}_{-\frac{3}{2}} |\text{Phys}\rangle; \langle S_{-\frac{3}{2}} | S_{-\frac{3}{2}} \rangle = 0 \] (93)

\[ \langle |S_{\frac{3}{2}} \rangle | S_{-\frac{3}{2}} \rangle = 0 \] (94)

The critical case corresponds to the condition (93) It requires definite values of numbers of fields:

\[ d_{\text{crit}} = d_1 + \frac{3}{2} d_s = 15 \] (95)

with the condition:

\[ L_0 = \bar{L}_0 = \frac{1}{2} \] (96)

Here \( d_s \) is the number of isotopic scalar two-dimensional fields \( I_s \) on the basic surface which have no partners on edging surfaces. And \( d_1 \) is the number of other fields. For the critical case we can prove by the same way as in the Neveu-Schwarz model that the powers indices \( \lambda \) and \( \mu \) in (90) are vanishing if all constraints for physical states are fulfilled and hence the norms of all physical states are nonnegative.

Let us consider the critical value of the effective dimension \( d_{\text{crit}} = d_1 + \frac{3}{2} d_s = 15 \). Since \( d_1 \) is a number of two sets of fields \((\bar{Y}^{(i-1)}, J^{(i-1)})\) and \((\bar{Y}^{(i)}, J^{(i)})\) on the left or \((\bar{Y}^{(i+1)}, J^{(i+1)})\) and \((\bar{Y}^{(i)}, J^{(i)})\) on the right) and hence \( d_1 \) is an even number. It gives only \( d_s = 2, 6, 10 \) but the values 6,10 give only three or zero for the number of \( Y \)-fields that is not enough for our Minkovsky space-time. So we have only one possibility: \( d_s = 2; d_1 = 12 \). That means four fields for all \( Y \)-fields i.e. \( Y^{(i)}_\mu, \mu = 0, 1, 2, 3 \) and two \( J^{(i)} \)-fields: \( d_1 = 12 = 2(4 + 2) \). A natural choice for two supercurrents \( \tilde{J}^{(i)} \) are two chiral nonabelian isotopic currents that corresponds two terms in \( G_r \), namely \((-i)(\Phi^L_1 \Phi^L_2 \Phi^L_3 + \Phi^R_1 \Phi^R_2 \Phi^R_3) \equiv J^L_3 \Phi^L_3 + J^R_3 \Phi^R_3 \) for each of i,i-1,i+1-th edging surfaces. It means in our above consideration \( J^{L(i)}_n \) and \( J^{R(i)}_n \) components of currents with \((J^{L(i)}_L)_0 = \bar{\lambda}^{(i)}_1 (1 + \gamma_5) \xi^L \bar{\lambda}^{(i)}_1 \) and \((J^{R(i)}_R)_0 = \bar{\lambda}^{(i)}_1 (1 - \gamma_5) \xi^R \bar{\lambda}^{(i)}_1 \).

So our currents satisfy a nonabelian Kac-Moody algebra for L and R currents:

\[ [(J^{L(i)}_a)_n, (J^{L(i)}_b)_m] = i\epsilon_{abc} (J^{L(i)}_c)_m+n + n\delta_{ab}\delta_{n,-m} \] (97)
\[
[(J^R_a)_n, (J^R_b)_m] = i\epsilon_{abc}(J^R_c)_{m+n} + n\delta_{ab}\delta_{n,-m} \tag{97}
\]
\[
[(J^L_a)_n, (J^R_b)_m] = 0 \tag{98}
\]

The rest two \(I_s\)-fields are abelian isotopic scalar fields for left chiral and right chiral currents on the basic surface.

Just nonabelian Kac-Moody currents lead us to the fermion two-dimensional fields dominance. Namely the number of fermion fields exceeds the number of boson fields in our composite superstring model. (Let us notice that we have a supersymmetry only on the world two-dimensional surface.) This fermion dominance gives superconvergence for one-loop planar string diagrams (Fig.8.).

6 Conclusion

So we have the consistent composite string model for hadron interactions. It can give the realistic spectrum of physical hadron states with the leading meson trajectory \(\alpha_\rho(t) = \frac{1}{2} + \alpha't\). This spectrum is free from states of negative norms. This model possesses an extended Virasoro superconformal symmetry on world surfaces (basic and two edging) and an additional gauge symmetry generated by nilpotent supercurrents. Just these supercurrent symmetries lead to the zeroth mass of pions and to the intercept of \(\rho\)-trajectory \(\alpha_\rho(0)\) to be equal to one half. Due to all symmetries it is possible to build the spectrum generating
algebra of operators for critical case and to prove absence of negative norms in the spectrum of physical states in this composite string model.

We have in this approach instead of point-like quarks at ends of string two-dimensional fields on two edging surfaces which are carrying quark quantum numbers.

It is worth to notice that these quark nontrivial quantum numbers lead to an interesting possibility of a consistent description of closed string sector to be arising for nonplanar loop diagrams in this model.

In order to describe fermions (baryons) we should move to the Ramond picture on basic two-dimensional surface. In so doing we shall hold the relations between edging and basic fields for this three quark configuration (quantum numbers for two quarks on two edging surfaces and an Ramond quark is on the basic surface). In the following publications we shall consider amplitudes with different flavours of hadrons, lightest states of a closed string sector and one-loop corrections for this composite superstring model.

I would like to thank participants of theoretical seminar of PNPI and HSQCD-2008 participants for attention to this work and useful discussions of results.

References

[1] Y.Nambu, Lectures at the Copenhagen Symposium "Symmetries and quark models" (1970);

[2] G.Veneziano, Nuovo cim.57A (1968) 190;

[3] C.Lovelace, Phys Lett. B28 (1968) 264; D.V.Shirkov, Sov.Phys.Usp. 102 (1970)87;

[4] A.V.Anisovich, V.V.Anisovich and A.V.Sarantsev, Phys.Rev.D62 (2000) 051502; V.V.Anisovich, Phys.Usp. 47 (2004) 45;

[5] C.Lovelace, Nucl.Phys. B148 (1979) 253;
[6] M.B.Greene, J.H.Schwarz and E.Witten, Superstring theory (Cambridge Univ. Press, N.Y. 1987);

[7] C.Lovelace, Phys.Lett B32 (1970) 490; V.Alessandrini, D.Amati, M.Lebellac and D.Olive, Phys.Rep.1 (1971) 269;

[8] V.A.Kudryavtsev, JETP Lett.58 (1993) 321;

[9] V.A.Kudryavtsev, Phys.At.Nucl.58 (1995) 131