Effects of Planetesimal Scattering: Explaining the Observed Offsets from Period Ratios 3:2 and 2:1

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Abstract
The observed deficit and excess of adjacent planet pairs with period ratios narrow and wide of 3:2 and 2:1, the nominal values for the corresponding mean-motion resonances (MMRs), have intrigued many. Previously, using a suite of simulations, Chatterjee & Ford showed that the excess above the 2:1 MMR can be naturally explained if planet pairs, initially trapped in the 2:1 MMR, dynamically interact with nearby planetesimals in a disk. We build on this work by (a) updating the census of discovered planet pairs, (b) extending the study to initially nonresonant as well as resonant planet pairs, (c) using initial planet and orbital properties directly guided by those observed, and (d) extending the initial period ratios to include both 2:1 and 3:2. We find that (1) interactions with planetesimals typically increase the period ratios of both initially resonant and nonresonant planet pairs; (2) starting from an initially flat period ratio distribution for systems across 3:2 and 2:1, these interactions can naturally create the deficits observed narrow of these period ratios; (3) a contribution from initially resonant planet pairs is needed to explain the observed levels of excess wide of 3:2; and (4) a mixture model where about 25% (1%) of planet pairs were initially trapped into 3:2 (2:1) MMRs is favored to explain both the observed deficit and excess of systems across these period ratios. However, up to a few percent of planet pairs are expected to remain in MMR today.

Unified Astronomy Thesaurus concepts: Exoplanet dynamics (490); Exoplanet migration (2205); Exoplanets (498); Orbital resonances (1181)

1. Introduction
The Nobel-winning discovery of 51 Pegasi b, the first planet discovered around a main-sequence star (other than the Sun), by Mayor & Queloz (1995) started the most exciting field in modern astrophysics. Thanks to numerous successful space-based missions, such as NASA’s Kepler (Borucki et al. 2010; Borucki 2016), K2 (Howell et al. 2014; Cleve et al. 2016), and TESS (Ricker et al. 2015); numerous radial velocity surveys, such as HIRES (Vogt et al. 1994), HARPS (Mayor et al. 2003), HARPS-N (Cosentino et al. 2012), SOPHIE (Bouchy & Sophie Team 2006), and ESPRESSO (Pepe et al. 2021); and follow-up missions like the LAMOST-Kepler survey (Dong et al. 2014; Luo et al. 2015) and California-Kepler Survey (Petigura et al. 2017), we now know of more than 5000 exoplanets (NASA Exoplanet Archive, http://exoplanetarchive.ipac.caltech.edu; Akesson et al. 2013) in more than ~3500 planetary systems. The large number of discovered planetary systems not only tells us about the unprecedented diversity of the present-day orbital and structural properties of exoplanetary systems, it also provides clues toward our still-evolving understanding of the formation and dynamical history of these planetary systems (e.g., Ford et al. 2011; Lissauer et al. 2011; Fang & Margot 2012; Rein 2012; Hansen & Murray 2013; Fabrycky et al. 2014; Malhotra 2015; Pu & Wu 2015; Steffen & Hwang 2015; Ballard & Johnson 2016). One of the most curious observed trends among multiplanet systems is the overabundance and deficit of systems of adjacent planet pairs just wide and narrow of two major first-order mean-motion resonances (MMRs), 3:2 and 2:1, with large offsets $\epsilon$ (Lissauer et al. 2011; Fabrycky et al. 2014). Throughout this study, we use $\epsilon = (P_2/P_1)/[(p + 1)/p] - 1$ as a measure of the offset from the nominal position of the $(p + 1)/p$ MMR. Figure 1 shows the distribution of observed period ratios for adjacent planet pairs. The excess of systems wide of 3:2 and deficit narrow of 2:1 are quite prominent. Indeed, Steffen & Hwang (2015) showed that in particular, the deficit narrow of the 2:1 MMR and the excess wide of the 3:2 MMR are statistically significant to an $\approx 99\%$ confidence level, while the excess wide of 2:1 and deficit narrow of 3:2 are not as significant. When first discovered, this was particularly surprising, since overall, the period ratios of adjacent planet pairs seem to show little preference for the nominal resonance positions, in contrast to earlier findings involving Jovian planet pairs that exhibit a clear preference for the 2:1 period commensurability with small offsets (e.g., Butler et al. 2006; Wright et al. 2011). This abundance asymmetry across these period ratios seems to indicate that the near-resonance planet pairs know the existence of these MMRs but somehow manage to avoid the nominal period ratios with large positive offsets. This discovery generated widespread interest, since the existence or absence of MMRs among planet pairs can shed light on the formation of planetary systems and their subsequent evolution. For example, on one hand, the predominance of resonant planet pairs indicates convergent gas disk–driven migration and efficient trapping in stable resonances with little or insufficient subsequent perturbation (e.g., Goldreich & Tremaine 1980; Lee & Peale 2002; Thommes & Lissauer 2003; Kley & Nelson 2012). On the other hand, the absence of two-planet (or more) MMRs can put constraints on either the success rate of resonance trapping due to convergent migration (e.g., Goldreich & Schlichting 2014; Batygin 2015; Deck & Batygin 2015) or the importance of
subsequent dynamical processes that effectively break MMR configurations (e.g., Chatterjee et al. 2008; Matsumura et al. 2010; Izidoro et al. 2017; Ogihara et al. 2018).

Several theoretical investigations have proposed a variety of mechanisms to explain this curious observed trend. However, a general consensus remains illusive. For example, whether most of the near-resonant systems are in resonance or not is yet unknown. It was pointed out that eccentricity damping of planetary orbits while trapped in an MMR may create large offsets from the nominal commensurate period ratios (e.g., Lithwick & Wu 2012). The mechanism for the eccentricity damping, however, created significant debate. Several studies proposed that star–planet tides may be responsible for eccentricity damping at levels necessary for creating the observed trend (e.g., Batygin & Morbidelli 2013; Delisle & Laskar 2014; Xie 2014). In contrast, other studies pointed out that for the star–planet tides to be solely responsible for the observed trend, tides either need to be unusually strong (e.g., Lithwick & Wu 2012), or perhaps there is another source of damping (e.g., Lee et al. 2013; Silburt & Rein 2015). It was also pointed out that the asymmetry does not seem to be particularly sensitive to the distance from the respective host stars, as would be expected if star–planet tidal damping were fully responsible for the observed asymmetry (e.g., Choksi & Chiang 2020). Other studies have suggested that the observed asymmetry can be attributed to the details of the disk–driven migration process itself. For example, Ramos et al. (2017) suggested that the distribution of near-resonant planets in the super-Earth mass range may be dependent on the details of the disk properties, including how flared the disk is. On the other hand, Choksi & Chiang (2020) and Wang et al. (2021) suggested that in the presence of a gas disk, the migrating planets simply get trapped wide of the nominal period ratios due to continued eccentricity damping by the gas disk. Analyzing a sample of three-planet systems, where each pair has a period ratio just wide of the nominal MMR, Pichierri et al. (2019) concluded that some of these triplets could be locked in multiresonant chains trapped via convergent migration, while subsequent gas disk dissipation repels the orbits wide of the exact commensurabilities. On the contrary, Veras & Ford (2012) suggested that a high fraction of the observed near-resonance planet pairs may not truly be in MMR.

While smooth gas disk–driven migration is expected to trap planets in MMRs, stochastic processes such as dynamical scattering have the potential for breaking the MMRs. Motivated by this expectation, several studies combined smooth migration with some source of stochasticity, for example, from turbulent clumps in the gas disk (e.g., Rein 2012; Batygin & Adams 2017) or due to interactions with a residual disk of planetesimals after the gas disk is sufficiently depleted (Chatterjee & Ford 2015, hereafter CF15). On the other hand, Petrovich et al. (2013) suggested that the observed asymmetry may be a result of the planet formation process itself by showing in idealized cases that as planets grow, test particles tend to pile up wide of the first-order MMRs with respect to the growing planet.

Compact multiplanet systems are also susceptible to dynamical instability even without any external perturbations (e.g., Chambers et al. 1996). In nonresonant systems, planet pairs close to MMRs tend to get destabilized on shorter timescales than pairs away from MMRs (e.g., Chatterjee et al. 2008; Funk et al. 2010; Pu & Wu 2015). It was also pointed out that such instabilities preferentially depleted more pairs narrow of the MMR (Pu & Wu 2015; Wu et al. 2019). Compact resonant chains may also get destroyed due to dynamical instabilities (e.g., Izidoro et al. 2017, 2021; Matsumoto & Ogihara 2020; Pichierri & Morbidelli 2020; Goldberg & Batygin 2022).

In this paper, we revisit the problem and further explore the idea proposed by CF15. They argued that while a significant gas disk is present, disk–driven migration can trap planet pairs into MMRs. After the gas disk is sufficiently depleted, these trapped planet pairs dynamically interact with nearby planetesimals from a residual planetesimal disk. The authors specifically considered planet pairs initially trapped into the 2:1 MMR. They showed for a wide range of disk profiles that these interactions can break the 2:1 MMR, and if so, the planet pairs diverge, creating large offsets. They argued that this process can be responsible for creating the asymmetry in the distribution of near-resonant planet pairs. We improve and expand the scope of this study in several key aspects. Since CF15 only considered planet pairs initially trapped in 2:1 MMR, their results do not explore the effects of planetesimal scattering on planet pairs that are initially narrow of the nominal MMR position. As a result, CF15’s results are unable to explain the deficit of systems narrow of period ratio 2:1. Moreover, CF15 did not explore the effects of planetesimal scattering near 3:2 at all. In our study, we consider planet pairs near 2:1, as well as 3:2; we also consider planet pairs that are initially trapped in MMRs, as well as those that are not. While CF15 considered specific mass ratios between planet pairs in a grid, we consider masses that are directly motivated by the observed planet pairs in the exoplanet database. Furthermore, while CF15 placed the planet pairs in a way such that the inner planet is near 0.5 au, we assign orbital separations guided directly by the observed exoplanet pairs. In order to limit the computational cost, we restrict ourselves to only studying a single planetesimal disk profile that initially follows the profile of the minimum-mass solar nebula (MMSN), since CF15 showed that the results of planet–planetesimal scattering do not qualitatively depend on the

\[ \frac{P_2}{P_1} = 3:2 \]

\[ \frac{P_2}{P_1} = 2:1 \]
adopted disk profile. Note that the main ingredient in CF15, as well as this study, is that during planet formation, a significant fraction of solids may not grow into planets. As a result, after gas dissipates, (almost) fully formed planets coexist with the debris of planet formation (e.g., Schlichting et al. 2012; Mulders et al. 2020). Also note that in the context of planet formation, solids can have a wide variety of names, such as pebbles, debris, rocks, planetesimals, and protoplanets, depending on their sizes, but the boundaries between them are blurry. For simplicity, throughout the paper, we will use the word “planetesimals” rather loosely and mean all leftover solids that did not grow into planets or protoplanets. We will provide a more physically motivated definition of the upper limit of mass for each of the so-called planetesimals later on.

We present our work as follows. In Section 2, we describe the physical picture we are trying to simulate, the initial properties of the planets and planetesimal disks, and the details of our numerical models. In Section 3, we present our key results. We summarize and conclude in Section 4.

2. Numerical Setup

The core-accretion paradigm of planet formation suggests that the newly born planets are embedded in a protoplanetary disk. The disk is initially made of mostly gas and dust. While significant uncertainties remain in how planet cores grow from dust, it is generally assumed that cores grow in the disk, and some of these cores can also accrete significant gas to become giant planets (e.g., Ida & Lin 2004; Armitage 2010; Mordasini 2018; Lee 2019). It is also generally expected that by the time the gas disk dissipates, some of the solids go into the formation of planets or planetary cores, while the rest of the solids remain as leftovers of the planet formation process. This debris of the planet formation process later on either gets accreted by the planets or the star, gets cleared away by dynamical scattering with the planets (e.g., Higuchi et al. 2006), or erodes through collisional grinding (see reviews by Wyatt 2008; Krivov 2010; Hughes et al. 2018). For simplicity, as mentioned before, we will refer to all solids that do not grow into planets (or planetary embryos, or protoplanets, depending on the definition) at the time of gas disk dispersal as planetesimals.

While the gas disk is sufficiently massive, in particular, while the timescale for dynamical instability is long compared to the damping timescale from the gas disk (e.g., Matsumura et al. 2010), dynamical excitations cannot grow, and the possibility of dynamical interactions between planets and planetesimals is reduced. The torques raised by the gas disk, depending on the migration speed and resonance strength, may also trap a fraction of planet pairs into MMRs (Lee & Peale 2002; Papaloizou & Szuszkiewicz 2005; Batygin 2015; Deck & Batygin 2015; Delisle et al. 2015). During this damped dynamics phase, some planet–planetesimal interactions still may occur if the instability timescale is shorter compared to the ever-increasing damping timescale. Eventually, the gas disk depletes completely, leaving behind the planets and a disk of residual solids. Beginning from when the gas disk is sufficiently depleted such that the damping timescale is long compared to the timescale of subsequent dynamical scatterings, the planets and planetesimals can freely interact, and dynamical excitation of orbits can freely grow.

While fully capturing this general scenario is not possible with present-day computational abilities, it is quite likely that at the time of gas dispersal, fully or almost fully formed planets coexist with a sea of planetesimals, which are conceivably the progenitors of what we see, after billions of years of evolution, as the asteroid belt or the Kuiper Belt in our solar system (e.g., Stern 1996; O’Brien et al. 2007, and references therein). The density distribution of planetesimals near the planets is hard to constrain and likely depends on the planet formation process. Since there are few constraints on the details of this stage of the evolution, we make several simplifying assumptions while keeping the physical process we have in mind unaltered to make this problem tractable.

We notice that the interactions that shape the planetary orbits can be easily divided into two clearly separate regimes with some possible interim phase. While the gas disk is sufficiently massive, gas disk–planet interactions are expected to dominate in shaping the orbital properties of the planets; on the other hand, when the gas disk is sufficiently depleted, the planet–planetesimal scattering should dominate the evolution of planetary orbits. There may be some interim phase where some planet–planetesimal scattering is allowed for which the instability timescale is too short compared to the depleting gas disk. Note that, after gas disk depletion, planet–planet scattering is also a possibility (e.g., Rasio & Ford 1996; Lin & Ida 1997; Ford & Chiang 2007; Levison & Morbidelli 2007; Chatterjee et al. 2008). However, planet–planet scattering is expected to perturb orbits too strongly and is not expected to give rise to the subtle observed abundance asymmetry in the period ratio distribution across the MMRs (e.g., Chatterjee et al. 2008; CF15).

Each of our simulated systems consists of two planets orbiting their host star embedded in a dynamically consistent (see Section 2.1.2) planetesimal disk. The same calculations can be repeated with systems with more than two planets too. However, a higher number of planets in the system simply makes the dynamics more complex, making it harder to interpret the results. Nevertheless, our calculations should be applicable to planetary systems with higher planetary multiplicities if it can be assumed that the system is pairwise dynamically stable. We focus on how the planet–planetesimal disk interactions alter the period ratios of planet pairs when they are either trapped in an MMR or in the vicinity of an MMR. Throughout our study, we use the hybrid integrator module MERCURIUS, available in the REBOUND simulation package (Rein & Liu 2012; Rein et al. 2019).

2.1. Creation of Initial Conditions

In this section, we describe the details of how we create and approximate the initial conditions expected to be present at the time of gas disk dispersal. Because of the complex nature of the condition, we create the initial conditions in several steps. These steps are described below.

2.1.1. Stage 1: Planet and Star Properties, Initial Orbits

By design, our setup is agnostic to the formation process. Thus, instead of trying to build the planets from the ground up assuming any specific formation process, we start with two planets that are almost fully formed with minor opportunities for further growth from accretion of planetesimals. In order to preserve the orbital architectures and various scalings present in real systems, we draw the planet pairs and host stars for our models directly from the observed properties of adjacent planet
pairs in multiplanet systems. The observed exoplanet data used are taken from NASA’s Exoplanet Archive (Akeson et al. 2013). Without applying any filters, out of the 837 multiplanet systems, we can find 1272 adjacent planet pairs in total. We exclude systems around giant stars with surface gravity \( \log(g/\text{cm s}^{-2}) \leq 3 \), which reduces the sample size to 820 multiplanet systems with 1254 adjacent planet pairs. We further exclude systems with missing information on critical properties, such as both the mass and radius of the planets or the host star properties are unknown. In addition, we restrict our study to planet pairs with sub-Neptunes \((R_p < 4 R_\oplus)\). While larger and more massive planets may also exist in real systems, changing their orbits requires proportionally higher mass in interacting planetesimals (e.g., CF15). These cuts reduce the number of planet pairs to 905 distributed in a broad range of period ratios. Since the focus of this study is to explain the observed dearth (excess) of systems narrow (wide) of period ratios 3:2 and 2:1, we further select only the planet pairs with period ratios below 2.5, which leaves us with 582 pairs in 377 systems. In this chosen sample, 175, 180, and 219 pairs are in systems with two, three, or more than three known planets, respectively. The asymmetries across period ratios 3:2 and 2:1 for pairs in two-planet systems and systems with higher multiplicities do not show statistically significant differences. Hence, although we simulate two-planet systems only, we keep all observed adjacent pairs to improve the statistics in observed systems.

In a large fraction of our selected sample, precise mass measurements are not available. We follow the following scheme based on the reported planet mass \( (m) \) and the corresponding error \( (\Delta m) \).

1. \( \Delta m/m \leq 0.3 \). We use the nominal mass value reported in the database directly.
2. \( \Delta m/m > 0.3 \) or either mass or radius measurements are available (but not both). We use the \( M-R \) relationship from Chen & Kipping (2017) to estimate the mass (radius) based on the reported radius (mass) in the database.
3. Average planet density \( \rho_p > 60 \text{ g cm}^{-3} \), which roughly corresponds to the density of a 100 \( M_\oplus \) iron planet (Fortney et al. 2007). We use the \( M-R \) relationship (Chen & Kipping 2017) to estimate mass.

The last condition is enforced because in some observed systems, the reported masses are from dynamical constraints. They are generally overestimated and may create unphysical models. We further make sure that all planet masses are lower than the mass of Neptune. At last, we have all necessary physical properties, including the mass and radius of 574 planet pairs around 374 host stars.

We now proceed with initializing their orbital properties. Note that we cannot directly import the observed orbital periods \( (P_{\text{orb}}) \) for our planets, since these would potentially change due to planet–planetesimal scattering. On the other hand, from CF15, we know that the amount of migration via planet–planetesimal scattering is limited. Hence, to ensure that the planet pairs are roughly at \( P_{\text{orb}} \) similar to the observed, we directly import the observed \( P_{\text{ orb}} \) of the outer planet \( (P_2) \). We divide our models into two major subsets and assign the rest of the orbital properties in the following way.

1. \( \text{Initially nonresonant pairs} \). In this subset, all planet pairs are initially nonresonant but are near period ratios 3:2 or 2:1. We assign \( P_1 \) such that \( \epsilon \) is distributed uniformly in the range \(-0.14 \leq \epsilon \leq +0.12 \). The eccentricities (inclinations) of the planetary orbits are drawn according to the Rayleigh distribution with mean 0.04 (0.024; Xie et al. 2016). Orbital phase angles are chosen uniformly in their full ranges. Once the orbits are created, we ascertain that the resulting orbits are stable against planet–planet scattering and orbit crossing using the generalized stability criteria (Gladman 1993; Petit et al. 2018). If we find that a planet pair is unstable, we redraw the orbital properties until we find a stable orbital configuration. This configuration is further integrated for 100 \( P_2 \) to ensure orbital stability. We collect all relevant properties for these planet pairs at the end of these integrations.
2. \( \text{Initially resonant pairs} \). In this subset, we create initially resonant planet pairs in the following way. We first assign \( P_1 \) such that the planet pairs lie just outside the nominal position of the MMR, e.g., \( P_2/P_1 = 1.52 \) for the 3:2 MMR and \( P_2/P_1 = 2.05 \) for the 2:1 MMR. Initially, both planets’ orbits are circular and coplanar.

We apply a slow inward migration on the outer planet with migration timescale \( T_{\text{mig}} = 10^6 P_2 \) and eccentricity damping timescale \( T_e = 10^2 P_2 \) for a duration of \( T_{\text{mig}} = 2 \times 10^4 P_2 \) to trap the planets in MMR. We ensure trapping into MMR via libration of the resonant angles for \( (p + 1)/p \) MMR,

\[
\begin{align*}
\theta_1 &= \left( p + 1 \right) \lambda_o - p \lambda_i - \varpi_i \\
\theta_2 &= \left( p + 1 \right) \lambda_o - p \lambda_i - \varpi_o \\
\theta_3 &= \varpi_i - \varpi_o, 
\end{align*}
\]

where \( \lambda \) (\( \varpi \)) is the mean longitude (longitude of pericenter), and the subscript \( i \) (\( o \)) refers to the inner (outer) planet. If the adopted timescales, \( T_{\text{mig}}, T_e, \) and \( T_{\text{mig}} \) result in overshooting the resonance instead of trapping, we adjust \( T_{\text{mig}} \) to ensure trapping (needed in 43 systems out of 2600 across both of the MMRs). Note that the exact choice of these timescales is not directly relevant for this study. We are simply interested in creating a large set of initially resonant planet pairs with properties that are observationally motivated. We collect all relevant properties for these planet pairs at the end of the resonance trapping integrations.

2.1.2. Stage 2: Introduction of Planetesimal Disk

We adopt an initial planetesimal disk where the surface density \( \Sigma(r) \propto r^{-3/2} \), similar to the profile of the MMSN (e.g., Weidenschilling 1977; Hayashi 1981), where \( r \) denotes the distance from the star. Note that we do not claim that the density profiles of planetesimals are always expected to be continuous or given by a power law. In fact, it is expected that the solid delivery may be significantly dependent on the planet formation scenario. For example, in the case of significant migration, the planetesimal profile may not be continuous and may instead have overdensities at resonance locations (Wyatt 2003). On the other hand, in situ formation models envision that there may have been a wide diversity of profiles for solids in both the exponent and the normalization (e.g., Hansen & Murray 2012, 2013; Chiang & Laughlin 2013; Moriarty & Ballard 2016; MacDonald et al. 2020). In contrast,
the inside-out formation model predicts that there is a continuous supply of solids streaming in from the outer disk (Chatterjee & Tan 2014, 2015; Hu et al. 2016). Authors CF15 studied the effects of different disk profiles and showed that a change in the disk profile simply changes the fraction of planetesimals close enough to interact with the planets and does not change the qualitative outcome of these interactions. Hence, to keep this already complex problem numerically tractable, we start with the MMSN-like disk profile and do not explore other disk profiles. Nevertheless, it can potentially be interesting to investigate different initial disk profiles in a future work.

To avoid any unpredictable edge effects, we set the inner (outer) edge of the disk at $P_3/3$ ($3P_3$), sufficiently far away from the positions of the planet pair. The orbital elements of the planetesimals (e.g., eccentricity, inclination, and other phase angles) are assigned in the same way as the nonresonant planet pairs (Section 2.1.1). We treat the planetesimals as pseudo-test particles to limit computational cost; while planetesimal–planet interactions are taken into account, planetesimal–planetesimal interactions are ignored. This is an approximation often adopted by similar studies of dynamical systems (Tsiganis et al. 2005; Raymond et al. 2012; Bonson et al. 2014; Izidoro et al. 2014; CF15; Mulders et al. 2020) and ignores the possibility of collisional growth and fragmentation of the planetesimals themselves. Since we are interested in the effects of a sea of small solids on the planets, this approximation is acceptable for our study. Furthermore, we expect that the leftover solids will not quickly grow themselves in the presence of the planets; rather, they will either get scattered or be accreted by the planets.

The sizes of the planetesimals are found using density $\rho_{pl} = 4 \text{ g cm}^{-3}$ and equal mass given by $m_{pl} = m_d/N_{pl}$, where $m_d$ ($N_{pl}$) denotes the total initial mass (number) of planetesimals in the disk. Although, in reality, the solid disk may have a variety of sizes (e.g., Morbidelli et al. 2009; Weidenschilling 2011; Schlichting et al. 2013), we do not expect that this simplification changes the nature of the physical process we are interested in. We parameterize the total disk mass to be a function of the total disk mass $m_p = m_1 + m_2$, $m_d = k \times m_p$, where $k$ is a constant. We explore $k = 0.1, 0.5, 1$, and $2$. It is important to note that the total disk mass $m_d$ is actually not an interesting parameter in this problem and only relevant for our bookkeeping. What really matters is the total mass of planetesimals that are close enough to the planets to dynamically interact with them (see discussion in Appendix A).

The choice of $N_{pl}$ in our simulations is somewhat arbitrary. We intend to investigate how the orbital properties of the planets evolve as a result of the cumulative effects of a large number of weak interactions from a sea of low-mass objects. Thus, on one hand, the higher the $N_{pl}$, the better. On the other hand, the computational cost scales as $N_{pl}$, which forces us to make a pragmatic choice. We choose $N_{pl}$ by demanding that the fractional change in the semimajor axis caused by a single planet–planetesimal interaction should always be small, $\Delta a/a \leq 10^{-3}$. Since $|\Delta a|a_{\text{max}} \sim m_{pl}/m$ for a single planet–planetesimal encounter, this requirement ensures that the mass of a planetesimal never exceeds 1/1000 of the mass of the lower-mass planet in the simulated system. As a fiducial value, we choose $N_{pl} = 5000$. In a handful of systems with disparate planet masses, our fiducial $N_{pl} = 5000$ does not satisfy the condition that $m_{pl} < 10^{-3}$ times the lower-mass planet. In these cases, we increase $N_{pl}$ as required, up to $N_{pl} = 10^4$. Our tests indicate that as long as $m_{pl}$ is sufficiently small compared to $m_p$, the results we are interested in do not change, and we capture the cumulative effects of a large number of weak encounters, as expected from leftover solids interacting with the planets (see a more detailed discussion in Appendix B). This consideration indirectly imposes a constraint on the maximum planet–planet mass ratio we can explore for a given $m_d/m_p$. For example, for $m_d/m_p = 1.0$, the maximum mass ratio between the planet pairs is $\approx 9:1$; hence, we restrict the maximum mass ratio to this value. Using this additional constraint, we are left with 548 planet pairs for $m_d/m_p = 0.1, 0.5$, and 1.0 and 529 pairs for $m_d/m_p = 2.0$ models. Discarding extreme mass ratio systems should not affect our overall results significantly, since this is a small fraction of all systems. It is also expected that adjacent planet pairs are more similar to each other than two randomly drawn planets (e.g., Weiss et al. 2018); we suspect that many of the apparently high mass ratios may not be real and are a result of using the nominal $M–R$ relationship, which, in reality, has large spreads (e.g., CF15; Wolfgang et al. 2016; Chen & Kipping 2017).

In a real system emerging out of a gas disk, the planetesimal disk profile is not expected to remain a pure power law. Even in the presence of damping from the gas disk, significant dynamical clearing is expected for planetesimals that are in orbits that are unstable on short timescales. To model this, we introduce the planetesimals as zero-mass particles and gradually increase their mass to the desired value over a timescale $t_{\text{ramp}} = 10^4 P_2$. Thus, at the beginning, the planets perturb the orbits of the nearby planetesimals without any effect on their own orbits. As a result, only planetesimal orbits that are dynamically consistent with the orbits of the planets remain. The gradual increase of $m_{pl}$ also ensures that the systems do not get shocked by a sudden introduction of a massive planetesimal disk. The choice of $t_{\text{ramp}}$ is somewhat arbitrary, but it is easy to understand how $t_{\text{ramp}}$ affects the evolution. A small $t_{\text{ramp}}$ may shock the system and potentially create instabilities early. A large $t_{\text{ramp}}$ can lead to a lot of planet–planetesimal interactions before the planetesimals reach the full predetermined mass. In our tests, we find that as long as $t_{\text{ramp}} \gg P_2$, the choice of $t_{\text{ramp}}$ does not alter the planets’ orbits significantly, and the exact value has little effect on the final results. At the end of this clearing stage in each system, we obtain two planets embedded in a dynamically consistent disk of planetesimals. Figure 2 shows an example of a planetesimal disk and the embedded planet pairs at the time of the introduction of the planetesimal disk (gray) and when the planetesimals have attained their full mass (blue). The planet positions are given by the vertical lines. At the end of stage 2, we are left with two planets embedded in a disk of planetesimals with structures in the density profile that are dynamically consistent with the presence of the planets. We envision that as long as the planet formation process is wasteful in use of the solid reservoir, planetary systems emerging out of a gas disk may have conditions that are dynamically similar to our setup at the end of stage 2.

We create 1300 systems of planet pairs embedded in a dynamically consistent planetesimal disk, each around initial period ratios of 3:2 and 2:1 for the initially nonresonant set. For the set of initially resonant planet pairs, we again create 1300 systems each for the 3:2 and 2:1 MMRs. Thus, in total, we have 5200 planet pairs initialized either in initially resonant or in
initially nonresonant orbits. This is done for each \( m_d/m_p \), which means that we have simulated a total of 5200 \( \times 4 \) systems.

### 2.2. Stage 3: Evolution of Embedded Planet Pairs

We simulate the planet pairs embedded in a dynamically consistent disk for \( 10^6 P_p \) to study how the planet pairs evolve due to the cumulative effect of many stochastic small interactions with the planetesimals. The frequency of interactions between the planets and planetesimals is initially high and gradually decreases with time, as expected. The majority of these interactions happen earlier than our chosen stopping time. Moreover, we verify that longer integrations do not alter the results in a statistically significant way.

### 3. Results

In this section, we first describe the typical evolution of initially resonant and nonresonant planet pairs under the influence of a planetesimal disk using example systems. Afterward, we describe how these interactions shape the final \( \epsilon \) distributions of the whole ensemble of planet pairs we simulate.

#### 3.1. Typical Evolution of Planet Pairs

In a typical system, the planetesimals interact with the planets stochastically. Similar to the findings of CF15, a wide range of outcomes of individual planet–planetesimal interactions are possible. No single interaction produces any significant perturbations to the planetary orbits. However, the cumulative effect of a large number of encounters typically increases the ratio of the planetary orbital periods.\(^5\)

##### 3.1.1. Initially Nonresonant Pairs

Figure 3 shows the evolution of the orbital properties for an initially nonresonant planet pair under the influence of planet–planetesimal interactions as an example. Note that throughout the paper, \( t = 0 \) indicates the start of stage 1 in our initial setup. As a result, the effects of the planet–planetesimal scattering are fully underway starting from \( 1.1 \times 10^3 P_p \). In this example system, the planets are initially wide of the nominal 3:2 MMR with an initial offset \( \epsilon_{\text{ini}} = 0.115 \). Throughout the evolution, the resonant angles corresponding to 3:2 commensurability circulate (first panel). While at the beginning, the period ratio remains roughly unchanged, it monotonically increases after \( t \sim 10^3 P_p \) and reaches \( \epsilon = 0.129 \) at the integration stopping time (second panel). In this example, both planets migrate inward; however, the inner planet migrates more than the outer (third panel). In our simulations, we find that several combinations are possible, and the typical migration direction leading to an increase in \( \epsilon \) varies depending on the initial resonance status and period ratio. For example, in the case of initially nonresonant planets near 3:2, the inner planet typically migrates inward, while the outer planet migrates outward. In contrast, near 2:1, both planets tend to migrate inward, while the inner planet migrates more than the outer. The orbital eccentricities for both planets are damped due to planet–planetesimal interactions.\(^6\)

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\(^5\) For initially nonresonant planets, \( \epsilon \) increases in \( \sim 97\% \) (\( \sim 65\% \)) of our simulated systems near 3:2 (2:1). The corresponding fraction for systems initially in the 3:2 (2:1) MMR is \( \sim 96\% \) (\( \sim 92\% \)).
planet migrates outward. Both eccentricities are damped. In this particular example, the inner planet migrates inward, and the outer planet continues the inward migration, while the outer planet migrates outward. This jump in $\epsilon$ is likely a result of resonant repulsion (Lithwick & Wu 2012). After this jump, $\epsilon$ continues to monotonically increase as long as the planets have access to planetesimals to interact with. In general, systems that are close enough to an MMR on the narrow side in the presence of a sufficiently massive planetesimal disk jump across the resonance, and this jump is primarily responsible for the observed paucity of systems narrow of the 3:2 and 2:1 MMRs. The evolution of systems narrow of the 2:1 MMR is very similar, with some differences showing up at the population level depending on the relative strengths of the 3:2 and 2:1 MMRs. There will be more discussion of this later.

\subsection{Initially Resonant Pairs}

The evolution of planet pairs initially in 2:1 MMR and embedded in a planetesimal disk was discussed in detail in CF15. In this study, we consider planet pairs initially trapped in 2:1 as well as 3:2 MMR. The behavior of planet pairs initially in resonance under the influence of a planetesimal disk is similar to nonresonant planet pairs except that the interactions first need to break the resonance before affecting the essentially monotonic increase in $\epsilon$. Figure 5 shows the evolution of orbital properties for a planet pair initially trapped in the 3:2 MMR embedded in a planetesimal disk with an initial $m_d/m_p = 0.5$ as an example. The resonant angles librate until $t \sim 2 \times 10^3 P_2$ (top panel), indicating that the planet pair is initially trapped in the 3:2 MMR. While the planets are trapped in the resonance, the evolution of $\epsilon$ is random and fluctuates with a small ($\lesssim 10^{-5}$) amplitude. Once the resonance is broken at $t \sim 2 \times 10^3 P_2$, easily identified by the circulation of the resonant angles, the planet pair practically becomes a nonresonant planet pair wide of the MMRs, as discussed in Section 3.1 (Figure 3). Then the planets undergo divergent migration as long as enough planetesimals are available in the vicinity to interact. In this particular example, the two planets migrate inward together while they are trapped in the resonance. However, once the resonance breaks, the inner planet continues the inward migration, while the outer planet migrates outward. In general, for the case of initially resonant planet pairs, similar to the case of initially nonresonant pairs, $\epsilon$ increases because either both planets migrate inward (inner planet migrates more than the outer) or the inner planet migrates inward and the outer planet migrates outward.

In the example of Figure 5, the eccentricities of both planets decrease to very low values due to planet–planetesimal interactions, similar to the initially nonresonant planet pairs (bottom panels in Figures 3–5). In general, a higher total mass in interacting planetesimals ($m_{\text{tot}}$) leads to more damping. For example, while the mean eccentricity for initially nonresonant planets is $\bar{\epsilon} = 0.04$ at the beginning of stage I (Section 2.1.1), it reduces to $\bar{\epsilon} = 0.034, 0.018, 0.009$, and 0.004 in the final snapshot for our models with $m_d/m_p = 0.1, 0.05, 1.0$, and 2.0, respectively. Similarly, in the case of initially resonant planets, $\bar{\epsilon}$ reduces from $\approx 0.02$ at the end of stage I to $\approx 0.01$ and $< 0.004$ in models with initial $m_d/m_p = 0.1$ and $> 0.5$, respectively. Similar to eccentricities, the mutual inclinations ($i_m$) also decrease. For example, initially, $i_m = 0.024$ for planetesimal scattering (fourth panel). At the integration stopping time, the planets open large enough cavities in the planetesimal disk (Figure 2), and planet–planetesimal scattering becomes infrequent. All planet pairs initially wide or narrow of the 3:2 or 2:1 period ratios show very similar evolution if the increase in period ratios does not push them very close to the resonance.

We find a qualitatively different evolution if $\epsilon_{\text{ini}} < 0$ and the planet pairs cross the nearest first-order MMR (3:2 or 2:1) as the period ratio increases. Figure 4 shows the evolution of two planets with initial period ratios narrow of the 3:2 MMR as an example. As the planets come sufficiently close to the nominal period ratio for the 3:2 resonance, $\epsilon$ suddenly jumps across zero, the nominal position of the resonance. As a result, the planets overshoot the 3:2 resonance and get deposited significantly wide ($\epsilon \gtrsim 0.005$) of 3:2. The resonant angles circulate almost throughout the evolution except for a short time during the resonance crossing, when they librate. This jump in $\epsilon$ is likely a result of resonant repulsion (Lithwick & Wu 2012). After this jump, $\epsilon$ continues to monotonically increase as long as the planets have access to planetesimals to interact with. In general, systems that are close enough to an MMR on the narrow side in the presence of a sufficiently massive planetesimal disk jump across the resonance, and this jump is primarily responsible for the observed paucity of systems narrow of the 3:2 and 2:1 MMRs. The evolution of systems narrow of the 2:1 MMR is very similar, with some differences showing up at the population level depending on the relative strengths of the 3:2 and 2:1 MMRs. There will be more discussion of this later.
changes, and planetesimal interactions break the resonance, and the resonant angles start damping. Overall, planetesimal interactions make the two-planet system more stable by reducing eccentricities and increasing period ratios.

3.2. Distribution of \( \epsilon \)

The change in offset, \( \Delta \epsilon = \epsilon_{\text{fin}} - \epsilon_{\text{ini}} \), where \( \epsilon_{\text{fin}} \) (\( \epsilon_{\text{ini}} \)) denotes the final (initial) \( \epsilon \), depends on the number of strong planet–planetesimal interactions in a particular system. As mentioned earlier, the overall \( m_d \) is not interesting for our problem; instead, \( m_{\text{ini}} \) is the key quantity. For the example system shown in Figure 2, \( m_{\text{ini}}/m_p \approx 0.09 \), whereas \( m_d/m_p = 0.5 \). Indeed, we find that for various different \( m_d/m_p \), if \( m_{\text{ini}}/m_p \) is roughly similar, the \( \epsilon \) evolution too remains unchanged (see Appendix A for more details).

It is impractical to track every strong scattering between the planets and planetesimals for all of our simulations because of the unreasonably large data size. Instead, we use the growth of the total planet mass (\( \Delta m_p \)) via planet–planetesimal collisions as a proxy for the number of strong planet–planetesimal encounters. In Figure 6, we show \( \Delta \epsilon \) as a function of \( \Delta m_p/m_p \) for all planet pairs in our simulations. Different colors denote models using different initial \( m_d/m_p \). The top and bottom panels show systems near the 3:2 and 2:1, respectively. The right and left panels show models with initially nonresonant and resonant systems. Several trends become apparent. The higher the level of interactions (and as a result, growth of planet mass via planet–planetesimal collisions), the larger the \( \Delta \epsilon \). Moreover, for any particular \( \Delta m_p/m_p \), there is a large spread in \( \Delta \epsilon \), which illustrates the stochastic nature of the evolution. Clearly, it is not possible to draw a one-to-one correspondence between \( \Delta \epsilon \) in a particular system and the total number of strong scattering interactions.\(^5\) There is significant overlap between systems with different initial \( m_d/m_p \). Moreover, we find that for the same \( m_d/m_p \), systems near \( P_2/P_1 = 3:2 \) exhibit a statistically higher \( \Delta \epsilon \) compared to those near \( P_2/P_1 = 2:1 \). Near both 2:1 and 3:2, the initially resonant planet pairs show a smaller \( \Delta \epsilon \) compared to those that are initially not in any resonance. This is because in the initially resonant systems, the interactions first need to break the resonance before \( \epsilon \) can grow freely. Interestingly, if negative, \( \Delta \epsilon \) remains small, \( |\Delta \epsilon| \lesssim 10^{-2} \). Thus, while some systems may exhibit \( \Delta \epsilon < 0 \), they do not move significantly away from their initial offset. In contrast, in the case of \( \Delta \epsilon > 0 \), the planetary orbits can diverge much more significantly.

3.2.1. Initially Nonresonant Pairs

Figures 7 and 8 show the \( \epsilon_{\text{fin}} \) distribution of initially nonresonant planet pairs near the 3:2 and 2:1 MMRs for models with different initial \( m_d/m_p \). The blue solid (dashed) histograms show the final (initial) \( \epsilon \) distributions. We show the observed distribution in gray for reference. Interactions with

\(^5\) For a longer discussion of this aspect, see CF15.
For any given $m_d/m_p$, the deficit near 3:2 is wider than that near 2:1. In contrast, for any given $m_d/m_p$, the deficit narrow of 2:1 is more prominent compared to that narrow of 3:2. These differences can be understood by noticing the difference in $\Delta \epsilon$ magnitudes near 3:2 and 2:1 for any given $m_d/m_p$ (Figure 6). Narrow of a resonance, there is a competition between how efficiently planet pairs can cross $\epsilon = 0$, creating a deficit, and replenish this deficit from lower period ratios via a smooth $\epsilon$ increase. Since $\Delta \epsilon$ is statistically larger for planet pairs near 3:2 (compared to that for planet pairs near 2:1) for any given $m_d/m_p$, replenishment of the deficit is more efficient narrow of 3:2. Interestingly, in the observed systems too, the deficit narrow of 2:1 is statistically more significant compared to that narrow of 3:2 (Steffen & Hwang 2015).

For the same reason, the deficit wide of 3:2 extends to larger positive $\epsilon$ compared to 2:1. For example, for $m_d/m_p \geq 0.5$, the distribution of model systems with $\epsilon > 0$ near 3:2 peaks at a higher $\epsilon$ compared to the $\epsilon$ corresponding to the peak in the observed systems. This suggests that planetesimal interactions with only initially nonresonant planet pairs may not fully explain the deficit as well as the excess of systems observed narrow and wide of 3:2. In contrast, while the bins narrow of 2:1 show a clear deficit of model systems, those wide of 2:1 do not exhibit any significant deficits.

### 3.2.2. Initially Resonant Pairs

Figure 9 shows the initial and final distributions for $\epsilon$ for models where planet pairs are initially trapped in the 3:2 MMR. Initially, all planet pairs are concentrated in the bin just wide of $\epsilon = 0$. As a result of planet–planetesimal interactions, $\epsilon$ fluctuates and grows on an average while the planet pairs are still in resonance. If the resonance breaks, $\epsilon$ can freely increase, and a monotonic increase in $\epsilon$ follows (Figure 5). Hence, planet pairs start populating the bins wide of the resonance.

As expected, higher-mass disks lead to higher levels of perturbations by the planetesimals, and as a result, resonance breaks in a higher fraction of systems, and the planet pairs are pushed to higher $\epsilon$ values. In a significant fraction of our simulated initially resonant systems, especially those with the lower $m_d/m_p$, the planet–planetesimal interactions are not sufficient to break the resonance. For example, $\approx 99\%$ ($\approx 2\%$) of our models initially trapped in the 3:2 MMR embedded in an initial disk with $m_d/m_p = 0.1$ (2) remain trapped in resonance at the integration stopping time (see Table 1).

Figure 10 shows the $\epsilon$ distributions for systems that are initially trapped in the 2:1 MMR. While the qualitative nature in this case is very similar to that for systems initially trapped in the 3:2 MMR, a notable difference is that it is harder to break the 2:1 MMR (Table 1). For example, even in our models with the highest $m_d/m_p$, 23% of the systems remain trapped in the 2:1 MMR (in contrast, only 2% remain trapped in 3:2). As a result, wide of the 2:1 MMR, the $\epsilon$ distribution does not exhibit a peak but rather a steady decrease from the peak corresponding to the systems still trapped in resonance. The range in final $\epsilon$ for systems initially in the 3:2 MMR is much larger compared to those initially in 2:1.

Interestingly, we find excellent agreement between the locations of the peaks in the observed and model $\epsilon$ distributions for systems initially in the 3:2 MMR, especially for models using $m_d/m_p = 0.5$ and 1.

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**Figure 7.** Distribution of $\epsilon$ for initially nonresonant systems near 3:2. Blue dashed and solid lines denote the initial ($\epsilon_{ini}$) and final ($\epsilon_{fin}$) distributions, respectively. Different panels show results from different initial $m_d/m_p$ (see legend). The gray line denotes the $\epsilon$ distribution of observed systems as a reference. Starting from a flat distribution, planetesimal interactions naturally produce a deficit of planet pairs near $\epsilon = 0$ for a sufficiently massive planetesimal disk ($m_d/m_p \geq 0.5$). The peak wide of $\epsilon = 0$ in the observed distribution is not reproduced.

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The origin of the deficit is not hard to understand. Planet pairs, initially narrow or wide of a resonance, that do not cross $\epsilon = 0$ exhibit a smooth increase in $\epsilon$ over time due to interactions with the planetesimals (e.g., Figure 3). This smooth increase in $\epsilon$ gradually shifts planet pairs from lower to higher $\epsilon$. In contrast, when planet pairs cross $\epsilon = 0$, $\epsilon$ increases rapidly (e.g., Figure 4). This sudden jump in $\epsilon$ creates the deficit just narrow of the resonance, while the width of the deficit is dependent on the magnitude of this jump.
3.3. Comparison with Observed Systems

So far, we have focused on describing the effects of planetesimal scattering for ensembles that completely consist of either initially nonresonant or initially resonant systems. However, reality almost certainly is not as simple, and it is expected that a fraction of the observed systems near 3:2 or 2:1 may have been trapped in MMR in the past. In addition, the amount of perturbation from a planetesimal disk a planetary system experiences also likely varies from system to system. Hence, to compare our simulated systems with those observed, we need to combine initially resonant and nonresonant systems that are allowed to go through varying degrees of planetesimal perturbations. In what follows, we first describe how we combine all of our simulated models; then, we compare the simulated and observed $\epsilon$ distributions near the 3:2 and 2:1 MMRs.

3.3.1. Combining Models

We have already seen that $\Delta \epsilon$ is correlated with $\Delta m_p/m_p$, our adopted proxy for the level of perturbation provided by the planetesimal disk (Section 3.2, Figure 6). Furthermore, a priori, we do not know the relative contributions of initially resonant and nonresonant systems, $\beta \equiv N_{\text{res}}/(N_{\text{res}} + N_{\text{non-res}})$, for the observed planet pairs. Here $N_{\text{res}}$ ($N_{\text{non-res}}$) denotes the number of initially resonant (nonresonant) systems, and $N_{\text{res}} + N_{\text{non-res}} = N_{\text{total}}$, the total number of planet pairs. We treat $\Delta m_p/m_p$ and $\beta$ as parameters, and, using our models, we estimate the posterior distributions of $\Delta m_p/m_p$ and $\beta$ given the...
observed $\epsilon$ distributions using Markov Chain Monte Carlo (MCMC).\footnote{Note that instead of $\Delta m_p/m_p$, in principle, one can use other measures of planet–planetesimal interactions, including $m_{\text{init}}/m_p$. However, it is simply easier to track $\Delta m_p/m_p$ in our simulations.}

We use emcee (Foreman-Mackey et al. 2013) to run our MCMC for a two-dimensional parameter estimation. We find the posteriors separately for systems near 2:1 and 3:2. For systems near each resonance, we combine all models for all initial $m_{\text{init}}/m_p$ into two separate sets, initially resonant and initially nonresonant. For any given value of $\Delta m_p/m_p$, we select systems that are within $\Delta m_p/m_p = \pm 0.02$ separately from the initially resonant and nonresonant sets. Then, for any given value of $\beta$, we randomly select (with replacement) $N_{\text{res}}$ ($N_{\text{non-res}}$) systems from these subsets (selected based on $\Delta m_p/m_p$) from the initially resonant (nonresonant) systems such that $N_{\text{res}}/N_{\text{total}} = \beta$ and $N_{\text{total}} = N_{\text{res}} + N_{\text{non-res}} = 10^3$. We create a probability density function (PDF) for $\epsilon$ based on this randomly selected mixture of planet pairs with specific ($\beta$, $\Delta m_p/m_p$) using kernel densities with Gaussian kernels. Note that the observed number of pairs we want to compare with is fewer in number by a factor of $\sim 10$. We intentionally choose a high sample size from our models to reduce statistical noise in the model PDF for $\epsilon$. We define the log-likelihood of a particular value $\{\beta, \Delta m_p/m_p\}$ for systems near any given resonance as

$$\ln L \left( \beta, \Delta m_p \right) = \ln \mathcal{P} \left( \epsilon_{\text{obs}}, \beta \right),$$

where the index $i$ denotes the observed systems, and $\mathcal{P}$ denotes the conditional probability of the occurrence of $\epsilon_{\text{obs},i}$ given $\{\beta, \Delta m_p/m_p\}$. We then proceed to find the posterior distribution,

$$\mathcal{P} \left( \beta, \Delta m_p/m_p \right),$$

where $\epsilon_{\text{obs}}$ is the observed distribution of $\epsilon$ for near-resonant systems corresponding to either $P_2/P_1 = 3:2$ or 2:1. For each value of $P_2/P_1$, we use 256 walkers and 20,000 steps for each walker. We use the default Stretch-Move method in the emcee package for the jumps. We discard the first 5000 steps as burn-in. Furthermore, we prune the walker positions by choosing one position in 10. We use flat priors; i.e., $\beta$ is flat between zero and 1, and $\Delta m_p/m_p$ is flat between 0% and 30% based on the range we find in our simulations (e.g., Figure 6).

\begin{table}

| Fraction of Unbroken MMRs | $m_{\text{init}}/m_p$ | 3:2 MMR | 2:1 MMR |
|---------------------------|-----------------------|---------|---------|
| $\epsilon_{\text{i}} \times 10^3$ | $\epsilon_{\text{m}} \times 10^3$ | $\epsilon_{\text{i}} \times 10^3$ | $\epsilon_{\text{m}} \times 10^3$ |
| 0.1 | 0.99 | 1$\pm$2$^4$ | 6$\pm$9$^8$ | 0.97 | 1$\pm$2$^3$ | 10$\pm$7$^6$ |
| 0.5 | 0.27 | 6$\pm$4$^4$ | 26$\pm$16$^6$ | 0.48 | 2$\pm$2$^2$ | 5$^5$ |
| 1.0 | 0.04 | 5$\pm$4$^4$ | 45$\pm$30$^8$ | 0.26 | 2$\pm$2$^2$ | 6$\pm$1$^3$ |
| 2.0 | 0.02 | 2$\pm$2$^4$ | 96$\pm$36$^6$ | 0.23 | 2$\pm$2$^2$ | 8$\pm$3$^0$ |

Note. Fraction of systems ($F_i$) where the initial MMR is not broken by the integration stopping time for the two resonances we consider for different initial $m_{\text{init}}/m_p$. The median value and $1\sigma$ range of $\epsilon_{\text{init}}$ are given for systems that are still in resonance ($\epsilon_{\text{i}}$) and those that broke out of resonance ($\epsilon_{\text{m}}$).

Figure 10. Same as Figure 9 but for systems initially in the 2:1 MMR. A higher fraction of systems remain in resonance until the simulation stopping time compared to systems initially in the 3:2 MMR for any given initial $m_{\text{init}}/m_p$ (also see Table 1).

3.3.2. Estimated Parameters

Figure 11 shows the posterior distributions for $\Delta m_p/m_p$ and $\beta$, as well as the $\epsilon$ distributions created using the parameter values drawn from the posteriors for simulated systems near 3:2. The corresponding observed distributions are also shown for comparison. We use bootstrap by randomly selecting (with replacement) half the number of observed planet pairs $10^4$ times and create distributions for all of them. The histogram heights and error bars for the observed distribution in the figure show the median and the span between the 16th and 84th percentiles for each bin from our bootstrap exercise. We generate the simulated histogram in the following way. We randomly draw $10^4$ samples of $\{\beta, \Delta m_p/m_p\}$ from the posterior distribution. For each of these draws, we create a mixed set of planet pairs and collect the values of $\epsilon$. Using these $10^4$ synthetic distributions, we create the $\epsilon$ distribution,
where the histogram heights and the shaded regions denote the median and the 16th and 84th percentiles in each bin.

Overall, the model and observed distributions of $\epsilon$ for planet pairs near 3:2 agree within their respective $1\sigma$ equivalent errors. The bin immediately narrow of 3:2 is the only one where the simulated and observed distributions are apparently not within $1\sigma$ equivalent error bars of each other. The error bar for this bin in the observed distribution is likely grossly underestimated. In bootstrap, any bin that contains zero systems would always contain zero systems in all bootstrap draws and hence would show a zero error bar, which is not realistic. In addition, as discussed earlier, the prominence of the deficit narrow of a resonance is dependent on the competition between the efficiency of jumping across $\epsilon = 0$ and that of replenishment due to a smooth increase in $\epsilon$ (Section 3.2.1). The lack of prominence of the deficit in the bin immediately narrow of 3:2 for the simulated systems may be an artifact of our assumed flat initial $\epsilon$ distribution. A close inspection of the observed systems indicates that in reality, the $\epsilon$ distribution narrow of 3:2 has a decreasing trend as we move away from 3:2. Due to this difference, we may be replenishing the deficit in the bin immediately narrow of 3:2 more in our simulations compared to the observed systems.

For systems near 3:2, although the $\beta$ posterior distribution is broad, there is a clear preference for a nonzero contribution from systems initially trapped in the 3:2 MMR. Our models suggest that planetesimal interactions can naturally create the deficit narrow of 3:2 from an ensemble of nonresonant planet pairs with an initially flat $\epsilon$ distribution (Section 3.2.1), while a nonzero contribution of systems initially trapped in the 3:2 MMR via planetesimal interactions helps create the excess of systems just wide of 3:2, as observed. For the systems near 3:2, the dominant peak in the posterior distribution is approximately at $\{\beta, \Delta m_p/m_p\} = \{0.25, 0.04\}$. Although contributions from initially resonant systems are needed to explain the excess of observed systems wide of 3:2, most of these initially resonant systems ($\sim84\%$) are not in resonance in the end. We find that only $4.9\%$ of systems in the range $-0.1 < \epsilon < 0.1$ are finally in resonance. These resonant systems predominantly occupy the bin right next to the nominal MMR at $\epsilon = 0$. In contrast, the second bin wide of the MMR is the most populated in the observed $\epsilon$ distribution (Figure 11).

The situation around 2:1 is different. For systems near 2:1, the $\beta$ posterior distribution monotonically increases all the way to $\beta = 0$, indicating that the observed $\epsilon$ distribution prefers little contribution from systems initially trapped in the 2:1 MMR (Figure 12). Our results suggest that this is because breaking the 2:1 MMR is considerably harder compared to 3:2. Any significant contribution from planet pairs initially trapped in the 2:1 MMR creates a $\epsilon_{\text{fin}}$ distribution exhibiting an excess too close to 2:1 compared to what is observed. However, the deficit narrow of 2:1 can be naturally explained as a result of planet–planetesimal interactions of initially nonresonant planet pairs.

The relative ease in breaking the resonance between 3:2 and 2:1 also creates a significant difference between where the posteriors for $\Delta m_p/m_p$ peak for 3:2 and 2:1; while the primary peak in the posterior distribution for 3:2 is near $\Delta m_p/m_p \approx 0.04$, for 2:1, it is near $\Delta m_p/m_p \approx 0.25$. Clearly, a higher level of planetesimal interaction is necessary to significantly perturb the planet pairs near or initially trapped in the 2:1 MMR.

### 4. Summary and Discussion

In this study, we have investigated the evolution of planet pairs in or in the vicinity of the 3:2 and 2:1 MRs as a result of interactions with nearby planetesimals in a residual disk after gas dispersal. Using a large number of $N$-body simulations involving two planets and thousands of planetesimals, we have investigated whether such interactions, expected to be common immediately after gas dispersal, can explain the observed deficit and excess of planet pairs narrow and wide of these period ratios.

We find that interactions with planetesimals typically increase the period ratios of planet pairs, trapped in an MMR or not (Figure 3). For a small fraction of systems, the period ratios may decrease; however, in these cases, the fractional change is much smaller compared to that when the period ratios
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increase (e.g., Figure 6). If the increase in period ratios for planet pairs that are initially narrow of an MMR brings them sufficiently close to the MMR ($\epsilon \sim -0.005$), the planet pairs overshoot the resonance and get deposited wide of $\epsilon = 0$ (Figure 4). Following the jump, the offset keeps on increasing as long as the planets have access to planetesimals to interact with.

Using an initially flat distribution of $\epsilon$ across 3:2 and 2:1, we find that planetesimal interactions naturally create a deficit of systems narrow of the nominal resonance positions at $\epsilon = 0$. For systems near the 2:1 MMR, the final $\epsilon$ distribution generated from initially nonresonant planet pairs closely resembles that for the observed adjacent planet pairs for any disk with $m_d/m_p \geq 0.5$ (Figure 8). However, for model systems near the 3:2 MMR, the deficit spans a wider range of $\epsilon$ and can extend beyond where an excess is actually observed (Figure 7). This indicates that while the observed deficit of systems narrow of 3:2 can be explained by the evolution of initially nonresonant planet pairs embedded in a disk of residual solids, the observed excess wide of 3:2 may not be explained using initially nonresonant planets only.

Our models with initially resonant planet pairs show that the offset typically remains limited to $\epsilon \sim 10^{-3}$ while the planet pairs are still in resonance. However, once the random fluctuations break the resonance, $\epsilon$ monotonically increases (Figure 5). For any given $m_d/m_p$, it is significantly easier to break the 3:2 MMR compared to breaking the 2:1 MMR (e.g., Figures 9 and 10, Table 1).

Using the simulated ensembles of initially resonant and nonresonant planet pairs and the observed $\epsilon$ distribution, we constrain the level of planetesimal interactions ($\Delta m_p/m_p$) and the relative contribution of initially resonant systems ($\beta$) to explain the observed asymmetric abundances across 3:2 and 2:1. We find that a significant fraction ($\beta \approx 0.25$) of initially resonant planet pairs is needed to explain the observed $\epsilon$ distribution across 3:2. While the deficit narrow of 3:2 can be explained by planetesimal perturbations on initially nonresonant planet pairs, the observed excess wide of 3:2 requires a contribution from initially resonant pairs. Interestingly, the posterior distribution clearly excludes $\beta = 0$ for systems near 3:2. For these systems, $\Delta m_p/m_p \approx 4\%$ is most preferred (Figure 11). Higher $\Delta m_p/m_p$ would create larger $\epsilon$, and the excess wide of 3:2 would move further away compared to the location where it is observed. Although a significant fraction of initially resonant planet pairs are needed to explain the observed $\epsilon$ distribution across 3:2, only about 4% of all final systems are expected to be in resonance within $-0.1 < \epsilon < 0.1$.

This finding is broadly consistent with the findings of Izidoro et al. (2017), where they tried to explain the observed period ratio distribution from super-Earth systems in compact resonant chains that undergo dynamical instability once the gas dissipates. Their simulations match the observed period ratio distribution if less than 25% of the resonant chains remain stable while the rest get destabilized. Although the physical process and initial simulation setup are quite different, interestingly, we also find that a similar fraction of initially resonant systems is needed to explain the observed asymmetry across 3:2. From the abundance of planet pairs wide of the MMRs, they also suggested that ~5% of planet pairs may be in resonance at present, very similar to the fraction of planet pairs that are still in resonance in our best-fit model.

In contrast to our findings for systems near 3:2, the observed $\epsilon$ distribution across 2:1 supports little contribution from initially resonant planet pairs (Figure 12). Interestingly, this is in contrast to the implied expectations of CF15 in the details. They considered only planet pairs initially in the 2:1 MMR and showed that perturbations from a disk of solids can break the resonance and preferentially increase $\epsilon$. While the evolution of embedded planet pairs found in this study is in qualitative agreement with the findings of CF15, our results suggest that a significant contribution from planet pairs initially in 2:1 MMR would be inconsistent with the observations. Our simulations also indicate that a higher level of planetesimal interactions ($\Delta m_p/m_p \approx 25\%$) is needed to explain the observed $\epsilon$ distribution across 2:1.

The difference in the relative contributions from initially resonant planet pairs in the 3:2 and 2:1 MMRs found in this study is interesting and likely related to the types of planets we study here. For example, Deck & Batygin (2015) suggested that due to type I migration, relevant for the planets we study.

Figure 12. Same as Figure 11 but for systems near 2:1. We find excellent agreement between the $\epsilon$ distributions of simulated and observed systems, including the deficit narrow of 2:1. The posterior distribution of $\beta$ monotonically increases all the way down to $\beta = 0$, indicating that there is little contribution from systems initially in the 2:1 MMR. The median of the posterior distribution for $\Delta m_p/m_p \approx 0.25$, which is higher than the same found for systems near 3:2 MMR. This is consistent with our finding that a higher level of planetesimal interactions is needed to significantly alter planet pairs near the 2:1 MMR (e.g., Figure 6).

For systems near 2:1 MMR, the deficit narrow of the nominal resonance positions at $\epsilon = 0$. For systems near the 2:1 MMR, the final $\epsilon$ distribution generated from initially nonresonant planet pairs closely resembles that for the observed adjacent planet pairs for any disk with $m_d/m_p \geq 0.5$ (Figure 8). However, for model systems near the 3:2 MMR, the deficit spans a wider range of $\epsilon$ and can extend beyond where an excess is actually observed (Figure 7). This indicates that while the observed deficit of systems narrow of 3:2 can be explained by the evolution of initially nonresonant planet pairs embedded in a disk of residual solids, the observed excess wide of 3:2 may not be explained using initially nonresonant planets only.

Our models with initially resonant planet pairs show that the offset typically remains limited to $\epsilon \sim 10^{-3}$ while the planet pairs are still in resonance. However, once the random fluctuations break the resonance, $\epsilon$ monotonically increases (Figure 5). For any given $m_d/m_p$, it is significantly easier to break the 3:2 MMR compared to breaking the 2:1 MMR (e.g., Figures 9 and 10, Table 1).

Using the simulated ensembles of initially resonant and nonresonant planet pairs and the observed $\epsilon$ distribution, we constrain the level of planetesimal interactions ($\Delta m_p/m_p$) and the relative contribution of initially resonant systems ($\beta$) to explain the observed asymmetric abundances across 3:2 and 2:1. We find that a significant fraction ($\beta \approx 0.25$) of initially resonant planet pairs is needed to explain the observed $\epsilon$ distribution across 3:2. While the deficit narrow of 3:2 can be explained by planetesimal perturbations on initially nonresonant planet pairs, the observed excess wide of 3:2 requires a contribution from initially resonant pairs. Interestingly, the posterior distribution clearly excludes $\beta = 0$ for systems near 3:2. For these systems, $\Delta m_p/m_p \approx 4\%$ is most preferred (Figure 11). Higher $\Delta m_p/m_p$ would create larger $\epsilon$, and the excess wide of 3:2 would move further away compared to the location where it is observed. Although a significant fraction of initially resonant planet pairs are needed to explain the observed $\epsilon$ distribution across 3:2, only about 4% of all final systems are expected to be in resonance within $-0.1 < \epsilon < 0.1$.

This finding is broadly consistent with the findings of Izidoro et al. (2017), where they tried to explain the observed period ratio distribution from super-Earth systems in compact resonant chains that undergo dynamical instability once the gas dissipates. Their simulations match the observed period ratio distribution if less than 25% of the resonant chains remain stable while the rest get destabilized. Although the physical process and initial simulation setup are quite different, interestingly, we also find that a similar fraction of initially resonant systems is needed to explain the observed asymmetry across 3:2. From the abundance of planet pairs wide of the MMRs, they also suggested that ~5% of planet pairs may be in resonance at present, very similar to the fraction of planet pairs that are still in resonance in our best-fit model.

In contrast to our findings for systems near 3:2, the observed $\epsilon$ distribution across 2:1 supports little contribution from initially resonant planet pairs (Figure 12). Interestingly, this is in contrast to the implied expectations of CF15 in the details. They considered only planet pairs initially in the 2:1 MMR and showed that perturbations from a disk of solids can break the resonance and preferentially increase $\epsilon$. While the evolution of embedded planet pairs found in this study is in qualitative agreement with the findings of CF15, our results suggest that a significant contribution from planet pairs initially in 2:1 MMR would be inconsistent with the observations. Our simulations also indicate that a higher level of planetesimal interactions ($\Delta m_p/m_p \approx 25\%$) is needed to explain the observed $\epsilon$ distribution across 2:1.

The difference in the relative contributions from initially resonant planet pairs in the 3:2 and 2:1 MMRs found in this study is interesting and likely related to the types of planets we study here. For example, Deck & Batygin (2015) suggested that due to type I migration, relevant for the planets we study.
here, the stability of the 2:1 resonance requires a more demanding planet–planet mass ratio, with the inner planet 12× more massive than the outer planet, compared to 3:2, where the required mass ratio is ~1. This is consistent with our requirement of a higher fraction of systems trapped into the 3:2 MMR compared to that trapped into the 2:1 MMR at the onset of planetesimal-driven evolution.

Overall, we find excellent agreement between the $\epsilon$ distributions of our simulated and observed systems across both 3:2 and 2:1. The excellent agreement between models and observations with little need for fine-tuning indicates that planetesimal interactions can explain the $\epsilon$ distribution across 3:2 and 2:1. Our models also indicate that up to a few percent of all near-resonant systems may be currently in resonance. While our finding of a low fraction of true resonances at present is consistent with some past studies (e.g., Veras & Ford 2012; Deck & Batygin 2015; Izidoro et al. 2017, 2021), it is in contradiction with proposed scenarios that invoke damping while still in resonance (e.g., Choksi & Chiang 2020). Future measurements of the relative abundance of truly resonant pairs among the near-resonant can ultimately shed light on the allowed mechanisms out of the many proposed.

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Figure A1. Left: initial planetesimal disk profiles for three different disks (solid histograms). The blue solid histogram shows a disk that spans from $P_1/3$ to $3P_2$ and has an initial mass of $m_{\text{int}}/m_p = 0.5$. The distribution of this disk after stage 3 is shown by the blue dotted histogram indicating the region around the planets where interacting planetesimals reside. The positions of the planets are marked by the vertical dashed lines. This example system consists of $m_1 = 2.1$ and $m_2 = 5.48 \, M_\oplus$, planets initially not in resonance around a 0.84 $M_\odot$ star. The orange (green) histogram shows a disk that spans $P_1/2 < P_0 < 2P_2$ ($P_1/1.3 < P_0 < 1.3P_2$) and has an initial mass of $m_{\text{int}}/m_p = 0.34$ (0.19). However, each disk has roughly the same $m_{\text{int}}/m_p \sim 0.1$ and $\Delta m_p/m_p \sim 0.09$. Right: $\epsilon$ evolution of the planet pair embedded in the three disks shown in the left panel. The colors denote disks with the same colors in the left panel. Each line of the same color shows a different evolutionary track corresponding to a different random realization within the same disk model. The $\epsilon$ evolution is very similar between different disks of different $m_{\text{int}}/m_p$, but the same $m_{\text{int}}/m_p$. 

**Appendix A**

**Total Disk Mass versus Total Mass of Interacting Planetesimals**

The planetesimal disk we consider throughout this study is distributed in the region $P_1/3 < P_{pl} < 3P_2$, where $P_{pl}$ is the orbital period of the planetesimals. We consider this large range simply to avoid any edge effects. Although we describe the disks via the total disk mass $m_{\text{disk}}$, what really matters is the total mass of planetesimals close enough to the planets to take part in dynamical interactions, $m_{\text{int}}$. This can be illustrated via the following experiment. Figure A1 shows the distribution of planetesimals in the same example system as shown in Figure 2. In this same system, in addition to our fiducial disk spanning $P_1/3 < P_{pl} < 3P_2$ (blue solid), we model the effects of two other disks truncated at closer distances from the planet pair, $P_1/2 < P_{pl} < 2P_2$ (orange) and $P_1/1.3 < P_{pl} < 1.3P_2$ (green). The total disk mass $m_{\text{int}}/m_p = 0.5, 0.34$, and 0.19, depending on where we truncate. In each case, however, $m_{\text{int}}/m_p = 0.09–0.1$ and $\Delta m_p/m_p = 0.087–0.092$ remain roughly the same. We estimate $m_{\text{int}}$ by taking the difference between the planetesimal density profiles at the end of stages 2 and 3 within the gap carved by the planets, and $\Delta m_p/m_p$ is simply the fractional change in the total planet mass. We simulate four realizations for each of the three disks simply by changing the random seed. The $\epsilon$ evolution is very similar in all three cases, resulting in very similar $\epsilon_{\text{int}}$. The differences in $\epsilon$ between models of different disks are well within the statistical fluctuations in $\epsilon$ between different realizations of the same disk. This indicates that the evolution of the planet pair is determined by $m_{\text{int}}/m_p$ (or, equivalently, $\Delta m_p/m_p$), rather than $m_{\text{int}}/m_p$. 

**Software:** Rebound (Rein & Liu 2012), emcee (Foreman-Mackey et al. 2013), corner (Foreman-Mackey 2016), matplotlib (Hunter 2007), numpy (Harris et al. 2020), pandas (McKinney 2010; Reback et al. 2020), scikit-learn (Pedregosa et al. 2011).
Appendix B

Individual Planetesimal Mass versus Total Mass of Interacting Planetesimals

In Figure B1, we show the evolution of the offset ($\epsilon$) for the system shown in Figure 3 for two different values of $m_{pl}$, keeping the total disk mass ($m_d/m_p = 0.5$) fixed. In the first case, we run four different realizations with $N_{pl} = 5000$ according to the criteria discussed in Section 2.1.2 (blue; each curve represents an independent realization with a different random seed). In the other set of simulations, we again run four different realizations with $N_{pl} = 10,000$ (orange). The only difference between these two sets is that the individual planetesimal masses $m_{pl}$ are different by a factor of 2 from each other. The $\epsilon$ evolutions in the two sets are very similar to each other. The differences between the simulations between the sets is well within the differences coming from statistical fluctuations within a set. This test indicates that as long as $m_{pl}$ is sufficiently small compared to $m_p$ (see discussion in Section 2.1.2), the exact value of $m_{pl}$ does not affect the outcome of the simulations in a statistically significant manner. Hence, each planetesimal in our study can also be considered as a swarm of smaller bodies interacting with the planets.

Figure B1. The $\epsilon$ evolution of a planet pair embedded in planetesimal disks of equal $m_d/m_p = 0.5$ but with different $N_{pl} = 5000$ (blue) and 10,000 (orange), resulting in different individual planetesimal masses $m_{pl} = m_d/N_{pl}$. The system parameters are the same as in Figure 2. Different tracks of the same color denote the evolution of different realizations, created using different random seeds, of the same model system. In each of these cases, the total mass of interacting planetesimals is roughly the same, $m_{pl}/m_p \sim 0.1$. The overall evolution of planets does not depend on $N_{pl}$ (or $m_{pl}$) as long as $N_{pl}$ is sufficiently large or, equivalently, $m_{pl}/m_p$ is sufficiently small.
