Hybridization of tensor-optimized and high-momentum antisymmetrized molecular dynamics for light nuclei with bare interaction

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Many-body correlations play an essential role in the ab initio description of nuclei with nuclear bare interactions. We propose a new framework to describe light nuclei by the hybridization of the tensor-optimized antisymmetrized molecular dynamics (TOAMD) and the high-momentum AMD (HM-AMD), which we call “HM-TOAMD.” In this framework, we describe the many-body correlations in terms of not only the correlation functions in TOAMD, but also the high-momentum pairs in the AMD wave function. With the bare nucleon–nucleon interaction AV8', we sufficiently reproduce the energy and radius of the ³H nucleus in HM-TOAMD. The effects of tensor force and short-range repulsion in the bare interaction are nicely described in this new framework. We also discuss the convergence in calculation and flexibility of the model space for this new method.

1. Introduction  In ab initio descriptions of nuclei, nucleon–nucleon (NN) correlations are mainly induced by the strong short-range repulsion and strong tensor force in the bare NN interaction [1,2]. Many-body effects of these NN correlations play essential roles in the accurate description of the nuclear system.

In our recent works, two different approaches have been established for the description of the NN correlations in ab initio calculations of s-shell nuclei with bare interactions [2–6]. These studies are based on the framework of antisymmetrized molecular dynamics (AMD), which has been very successful in investigations of nuclear clustering systems [7,8]. One of our approaches is tensor-optimized antisymmetrized molecular dynamics (TOAMD), which successfully describes s-shell nuclei with bare interaction [2–6]. In the TOAMD wave function, short-range and tensor correlations are described by using the correlation functions condescending to the short-range repulsion and tensor force. By including up to the double products of the correlation functions, the TOAMD results nicely reproduce those obtained from the Green’s function Monte Carlo (GFMC) method. As a next step, we will apply this TOAMD framework to p-shell nuclei, but massive analytical derivations and computation time are necessary to handle this. Other very recent progress in our...
group is the superposition of the AMD basis states, including nucleon pairs with high-momentum (HM) components, which we name “HM-AMD.” In studying s-shell nuclei in HM-AMD, it is found that the superposition of a single nucleon pair with various high-momentum components provides an equivalent effect to two-particle two-hole excitation of the tensor correlation \[9,10\].

In the present work, we propose a new framework for the hybridization of the tensor-optimized and high-momentum AMD, namely “HM-TOAMD.” One important advantage of this combination is that this method can describe more effectively the many-body correlations in light nuclei compared to TOAMD. In particular, for \textit{ab initio} calculation of p-shell and heavier nuclei, many kinds of correlation terms have to be included in the wave function, while the high-momentum pairs in HM-TOAMD are much easier to handle in the formulation part as compared to naive extension of the TOAMD method. In addition, the flexibility of this method provides the possibility of selecting only essential basis states to express the wave function; the important pair correlations can be described by selecting the appropriate high-momentum pairs, and the additional multi-nucleon correlations can be expressed using the correlation functions in the TOAMD part. With this simplicity and flexibility, the new HM-TOAMD is promising for future studies not only of p-shell and cluster nuclei, but also for the treatment of three-nucleon interactions.

In the following paragraphs, the formulation of HM-TOAMD is provided. After that, the effects of short-range and tensor correlations are clearly demonstrated by showing the results of energy curves of \(^3\)H with respect to different imaginary shifts of a high-momentum pair. The energies and radius of the \(^3\)H nucleus are reproduced with HM-TOAMD using the bare AV8\(^\prime\) interaction. In HM-TOAMD, there are two kinds of variational functions; one is a high-momentum pair and the other is correlation functions. Convergence with respect to these variational functions and the flexibility of the model space in this new method are also discussed. The final paragraph contains the conclusion.

2. Formulation

We start writing the AMD basis of \(A\) nucleons to be constructed with the antisymmetrization expressed by the Slater determinant (det) of single nucleon states \[2\]:

\[
|\Phi_{\text{AMD}}\rangle = \text{det}(|\phi_1(\vec{r}_1) \cdots \phi_A(\vec{r}_A))\rangle,
\]

where the single-nucleon states \(\phi(\vec{r})\) with spin \(\sigma\) and isospin \(\tau\) have the form of the Gaussian wave packet with the centroid \(\vec{Z}\) and \(\chi_{\tau,\sigma}\) for spin–isospin components:

\[
\phi(\vec{r}) \propto e^{-\nu(\vec{r}-\vec{Z})^2}\chi_{\tau,\sigma}.
\]

We explain first the AMD basis states with high-momentum components in the HM-AMD method \[9\]. We take the case of \(^3\)H in this paper and consider three kinds of basis states for HM-AMD, which are denoted by the following expressions:

\[
1 : \vec{D}_{p\uparrow,n\uparrow}, \quad 2 : \vec{D}_{p\uparrow,n\downarrow}, \quad 3 : \vec{D}_{n\uparrow,n\downarrow}.
\]

Here, we drop the fourth basis state \(\vec{D}_{n\uparrow,n\uparrow}\) due to the assumption that the spatial wave function is symmetric and the wave function should be antisymmetrized in the spin–isospin space. In the above symbols, \(\vec{D}\) is the vector of the imaginary shift for the Gaussian centroid defined as \(\vec{Z} = \vec{R} + i\vec{D}\), in which the vector \(\vec{R}\) generally represents the spatial position of the centroid. In HM-AMD, each \(\vec{D}\) should be assigned to a nucleon pair with the same magnitude \(D = |\vec{D}|\) and opposite direction \[9,10\]. We take various values for the imaginary shift \(\vec{D}\) for each pair, which are called “high-momentum pairs,” and superpose these AMD basis states in HM-AMD. When we take the basis number \(n_D\) for
The two-body correlation functions are expressed as the superposition of Gaussian functions. This form of TOAMD can be extended by increasing the power of the correlation functions successively. This extension improves the variational accuracy of TOAMD, but increases the computational costs and the formulation and programming costs tremendously as the power of the correlation increases. On the other hand, in HM-AMD, we superpose various directions and magnitudes of the relative momentum components for any nucleon pairs in the AMD basis states. It is much easier to increase the number of correlations in the wave function using the technologies developed for the AMD framework.

We propose here to combine the two methods: TOAMD and HM-AMD. In this framework, named HM-TOAMD, we write the total wave function as:

\[ |\Psi_{\text{HM-TOAMD}}\rangle = \sum_\alpha \left( C_\alpha + \sum_{i=0}^{1} \sum_{s=0}^{1} C_{S,i}^{\ell,s} F_{S,i}^{\ell,s} + \sum_{i=0}^{1} C_{D,i}^{\ell} F_{D,i}^{\ell} \right) \hat{P}_{MK}^I |\Psi_{\text{HM-AMD},\alpha}\rangle. \]  

Equation (8) is given in the linear combination form of the basis states of HM-TOAMD. \( \Psi_{\text{HM-AMD}} \) is the HM-AMD wave function, and the label \( \alpha \) is to distinguish the basis states including both the high-momentum components in \( \Psi_{\text{HM-AMD}} \) and the correlation functions. \( \hat{P}_{MK}^I \) is the projection operator to total angular momentum state \( J \) [11]. The labels \( i \) and \( s \) are the isospin and spin values of the two correlated nucleons; \( C_\alpha, C_{S,i}^{\ell,s}, \) and \( C_{D,i}^{\ell} \) are the superposition coefficients. The tensor part \( F_{D,i}^{\ell} \) contains two-channel tensor operator terms, and the central part \( F_{S,i}^{\ell,s} \) contains four-channel short-range central terms. The radial forms of the correlation function are the Gaussian functions with the same sign of \( D \) as \( \pm D \), the total basis number for high-momentum configurations is \( 6 \times n_D \). We write explicit Case 1 of Eq. (3) as:

\[ \phi_{r_1}^{T}(\vec{r}_1) \phi_{r_2}^{T}(\vec{r}_2) \propto e^{-v(\vec{r}_1 - \vec{R}_1 + i\vec{D}_p^{\uparrow n_1})^2} x_{r_1}^{\uparrow} e^{-v(\vec{r}_2 - \vec{R}_2 + i\vec{D}_p^{\uparrow n_1})^2} x_{r_2}^{\uparrow}. \] (4)

Here, \( \phi_{r_1}^{T}(\vec{r}_1) \) and \( \phi_{r_2}^{T}(\vec{r}_2) \) are the single-nucleon states of proton and neutron with imaginary shift \( \vec{D}_p^{\uparrow n_1} \). Equation (4) can be rewritten in terms of the relative \( \vec{r} \) and center-of-mass \( \vec{R} \) coordinates as:

\[ \phi_{r_1}^{T}(\vec{r}) \phi_{r_2}^{T}(\vec{r}) \propto e^{-v/2(\vec{r} - \vec{Z}_p)^2} e^{-2v(\vec{R} - \vec{Z}_R)^2} x_{r_1}^{\uparrow} x_{r_2}^{\uparrow}, \] (5)

where \( \vec{r} = \vec{r}_1 - \vec{r}_2, \vec{Z}_p = (\vec{r}_1 + \vec{r}_2)/2, \) and the centroid parameters are

\[ \vec{Z}_p = \vec{R}_1 - \vec{R}_2 \pm 2i\vec{D}_p^{\uparrow n_1}, \quad \vec{Z}_R = (\vec{R}_1 + \vec{R}_2)/2. \] (6)

Clearly, it is found that the imaginary shifts \( \vec{D}_p^{\uparrow n_1} \) only change the relative parameter \( \vec{Z}_p \) between the nucleons of the high-momentum pair. In this way, we introduce correlations among two nucleons in the HM-AMD [9]. For the ground states of \( s \)-shell nuclei, the optimized value for the spatial centroid parameters are \( \vec{R}_i = \vec{0} \).

We now discuss another way to include correlations between a nucleon pair, which is the TOAMD method. The TOAMD wave function in its lowest order (single-\( F \) TOAMD) is given as [2]:

\[ (1 + F_S + F_D) \times |\Psi_{\text{AMD}}\rangle. \] (7)

The two-body correlation functions \( F_S \) for the short-range correlation and \( F_D \) for the tensor correlation are expressed as the superposition of Gaussian functions. This form of TOAMD can be extended by increasing the power of the correlation functions successively. On the other hand, in HM-AMD, we superpose various directions and magnitudes of the relative momentum components for any nucleon pairs in the AMD basis states. It is much easier to increase the number of correlations in the wave function using the technologies developed for the AMD framework.
with spin–isospin operators [3]:

\[ F_{D,\alpha}^t = \sum_{i<j}^{A} \exp(-a_{D,\alpha}r_{ij}^2)O_{ij}^{t}r_{ij}^{2}S_{12}(\vec{r}_{ij}), \]  

(9)

\[ F_{S,\alpha}^{t,s} = \sum_{i<j}^{A} \exp(-a_{S,\alpha}r_{ij}^2)O_{ij}^{t,s}O_{ij}^{s}. \]  

(10)

Here, \( \vec{r}_{ij} = \vec{r}_{i} - \vec{r}_{j}, O_{ij}^{t} = (\vec{r}_{i} \cdot \vec{r}_{j})', \) and \( O_{ij}^{s} = (\vec{r}_{i} \cdot \vec{r}_{j})^{s}. \) In each channel, we superpose the Gaussian functions with the common basis number \( n_{G}. \) Hence, the total number of basis states coming from the bracket term in Eq. (7) as TOAMD is \( 1 + 6 \times n_{G}. \) Thus, the total basis number for HM-TOAMD is \( (1 + 6 \times n_{G}) \times 6 \times n_{D} \) in the GCM-type calculation in Eq. (8).

Finally, we solve the eigenvalue problem in the following equation and determine \( \tilde{C}_{\alpha} = \{C_{\alpha}, C_{\alpha}'^{t}, C_{\alpha}'^{s}\} \) in the linear combination form:

\[ |\Psi_{HM-TOAMD}\rangle = \sum_{\alpha} \tilde{C}_{\alpha} |\Psi_{HM-TOAMD,\alpha}\rangle, \]  

(11)

\[ \sum_{\alpha,\beta} (H_{\alpha\beta} - EN_{\alpha\beta}) \tilde{C}_{\beta} = 0, \]  

(12)

\[ \langle \Psi_{HM-TOAMD,\alpha} |H| \Psi_{HM-TOAMD,\beta}\rangle = H_{\alpha\beta}, \]  

(13)

\[ \langle \Psi_{HM-TOAMD,\alpha} |\Psi_{HM-TOAMD,\beta}\rangle = N_{\alpha\beta}. \]  

(14)

The bases \( |\Psi_{HM-TOAMD,\alpha}\rangle \) are carefully chosen to reflect the physical properties of NN correlations. For the Gaussian range parameters \( a \) in Eqs. (9) and (10), a series of bases ranging from 0.01 fm\(^{-2}\) to 5 fm\(^{-2}\) are adopted and optimized for each channel by single-\( F \) TOAMD calculation. For the imaginary shifts, absolute values \( \vec{D} \) ranging from 1 fm to 12 fm are adopted for each kind of high-momentum pair, with corresponding momentum \( \langle k_{z} \rangle = 2\nu \cdot \text{Im}(D_{z}) \) ranging from 0.28 fm\(^{-1}\) to 2.8 fm\(^{-1}\) using \( \nu = 0.14 \) fm\(^{-2}\).

We discuss the difference between the new HM-TOAMD approach and our previous TOAMD and HM-AMD by showing the three-body correlations in the case of \(^3\)H in Fig. 1. In this figure, panel (a) shows the “double-\( F \) TOAMD” approach [3–6], which includes the double products of two kinds of NN correlations described by functions \( F_{D} \) and \( F_{S} \) as in Eqs. (9) and (10). Panel (b) shows the pure HM-AMD approach (T. Myo, in preparation; M. Isaka et al., in preparation), in which all the correlations are described by using the double high-momentum pairs as in Eqs. (3) and (4). In our new hybrid description HM-TOAMD, one of the correlations is described by the correlation functions \( F_{D} \) and \( F_{S} \), and the other is described by the high-momentum pairs, as shown in panel (c).

3. Results We discuss first the effect of the imaginary shift \( \vec{D} \) on the total energy for \(^3\)H. In our previous study of HM-AMD, we showed that the imaginary shift \( |\vec{D}| = 5 \) fm \( (k_{z} = 2.5 \) fm\(^{-1}\)) is energetically favored by the tensor interaction [9]. In our new hybrid approach, we can discuss the effects of \( \vec{D} \) in a more accurate situation, where we include the single correlation function in terms of TOAMD and the second correlation in terms of the high-momentum pair as shown in Fig. 1(c). Figure 2 shows the energy curves of \(^3\)H for three kinds of high-momentum pairs as functions of \( D_{z} = |\vec{D}| \) in the \( z \)-direction, which is parallel to the spin direction of the nucleons, with \( n_{D} = 1, \) where we employ the number of Gaussian expansions \( n_{G} = 3 \) in \( F_{D} \) and \( F_{S} \) of the TOAMD part.
Fig. 1. Diagrams of the three-body correlations in double products of correlation functions for $^3$H: (a) TOAMD, (b) HM-AMD, (c) HM-TOAMD. “$F_S, F_D$” denote the NN correlations described by the correlation functions in Eqs. (7) and (8), and “HM” denotes the NN correlations described by high-momentum pairs in Eqs. (3) and (4).

Fig. 2. Energy curves of the $^3$H nucleus calculated with HM-TOAMD and the bare interaction AV8′ with respect to the shift $D_z$ in the $z$-direction when only a single high-momentum pair is included as $n_D = 1$. The symbols $D_{p↑,n↑}$, $D_{p↑,n↓}$, and $D_{n↑,n↓}$ denote different kinds of high-momentum pairs with positive shift $D$ as in Eq. (4). We use $n_G = 3$ and $\nu = 0.14$ fm$^{-2}$.

We use the AV8′ bare interaction. In this figure, we see clearly two minima around $D_z = 4$ fm ($k_z = 1.12$ fm$^{-1}$) and $D_z = 10$ fm ($k_z = 2.8$ fm$^{-1}$) for the ($p \uparrow, n \downarrow$) high-momentum pair, which are caused by the effects of the tensor interaction and the short-range repulsion in the AV8′ interaction, respectively. For the ($p \uparrow, n \uparrow$) high-momentum pair, we obtain the minimum around $D_z = 8$ fm corresponding to the short-range effect. For these results, we should keep in mind that both the strong short-range effect and the tensor correlation have already been included in the correlation function $F_S$ and $F_D$ of TOAMD, respectively. For the ($n \uparrow, n \downarrow$) high-momentum pair the tensor correlation is weak, and then we can confirm only the effect of short-range correlation with the energy minimum around $D_z = 10$ fm, which should be caused by the short-range repulsion.

In Fig. 3, we show the tensor matrix elements as functions of the shift parameter $D_z$, similar to the case of Fig. 2. It is found that the $p-n$ pairs commonly show the two minima; one minimum is located at 4 fm, which comes from the tensor correlation in the high-momentum pair. Another minimum around 8–10 fm comes from the short-range correlation in the high-momentum pair, while the correlation functions in the TOAMD part contribute to describe the tensor correlation.

To obtain an accurate solution of the $^3$H wave function, we superpose the basis states having different $\vec{D_S}$ with the number of $n_D$ for three kinds of pairs in Eq. (3). First, we consider only the imaginary shift $\vec{D_S}$ along the $z$-axis and show the results by adding successively the different
Fig. 3. Tensor matrix elements of $^3$H as functions of the shift $D$. Notations are the same as used in Figure 2.

Fig. 4. Energy of the $^3$H nucleus calculated with HM-TOAMD using the bare interaction $AV8'$ and successively adding the various kinds of high-momentum pairs. All the imaginary shifts $\vec{D}$ are adopted to be parallel with the $z$-axis. $\pm D$ denote different high-momentum pair shifts in Eq. (3). The label of each line denotes the values of $|\vec{D}|$ superposed. We set the range parameter $\nu = 0.14$ fm$^{-2}$.

combination of high-momentum pairs in Fig. 4. When we include no high-momentum pairs, which corresponds to the horizontal coordinate 0 in Fig. 4, the HM-TOAMD wave function in Eq. (8) reduces to the single-$F$ TOAMD wave function as $(1 + F_S + F_D)|\Psi_{AMD}\rangle$, and the energy of $^3$H is $-4.67$ MeV. After the successive addition of high-momentum pairs into the wave function of HM-TOAMD, the total energy variationally decreases and gets closer to the value given by the double-$F$ TOAMD. The addition of the first negative shift $-D_{p\uparrow,n\uparrow}$ restores the parity symmetry of the total wave function combined with the positive shift $D$, which contributes more than 1 MeV, as shown in the green line with $n_G = 6$ and $n_D = 7$. This result shows the importance of the parity symmetry of the HM-AMD basis. It is also true for rotational symmetry, for which the angular momentum projection could also contribute more than 1 MeV in the final calculation. In Fig. 4, it is also shown clearly that the first five kinds of pairs contribute almost all the energy gains compared to the single-$F$ TOAMD result. The addition of the last pair shift $-D_{n\uparrow,n\downarrow}$ provides a tiny improvement of the total energy, which is only 8 keV in the green line. This convergence shows that the first five kinds of pairs are enough to provide accurate descriptions of the NN correlations. It is also found that the energy of $^3$H is not improved by adding more high-momentum basis states having $\vec{D}s$ along the
Table 1. Energies of $^3$H (ground state) from HM-TOAMD, double-$F$ TOAMD ($F^2$-TOAMD; Ref. [5]), and few-body calculations [12,13] in units of MeV. The radius is in units of fm.

|                     | HM-TOAMD | $F^2$-TOAMD (Refs. [3,5]) | Few-body (Refs. [12,13]) |
|---------------------|----------|---------------------------|--------------------------|
| Energy              | $-7.64$  | $-7.68$                   | $-7.76$                  |
| Kinetic             | $47.29$  | $47.21$                   | $47.57$                  |
| Central             | $-22.47$ | $-22.44$                  | $-22.49$                 |
| Tensor              | $-30.60$ | $-30.60$                  | $-30.84$                 |
| LS                  | $-1.87$  | $-1.86$                   | $-2.00$                  |
| Radius              | $1.73$   | $1.75$                    | $1.75$                   |

$x$- or $y$-axis. Hence, it can be concluded that the model space of high-momentum pairs along the $z$-direction, parallel to the nucleon spin, is already enough in HM-TOAMD. This property is different from the previous analysis of $^4$He with the pure HM-AMD for tensor correlation [9], which shows the importance of both directions of spin-parallel ($z$) and spin-perpendicular ($x$ or $y$) cases.

In Fig. 4, we start our calculation with $n_D = 1$ using the imaginary shift $D_z = 10$ fm ($k_z = 2.8$ fm$^{-1}$), which is favored to express the short-range correlation and corresponds to the red line in the figure. After superposing all six kinds of high-momentum pairs commonly keeping this shift value, the energy of $^3$H is $-6.3$ MeV, improved by 1.6 MeV compared with the single-$F$ TOAMD. The further addition of the high-momentum basis state with $D_z = 4$ fm ($k_z = 1.12$ fm$^{-1}$), in which the tensor correlation is favored, provides additional improvement by 0.8 MeV for the total energy. Also, we see a contribution of about 0.15 MeV from the $D_z = 1$ fm ($k_z = 0.28$ fm$^{-1}$) pairs; this small shift in the momentum space can express the low-momentum components of pairs in the $^3$H nucleus. After superposition of $n_D = 7$ with different $D_z$s for all kinds of high-momentum pairs, as shown by the green line in Fig. 4, we get the final result for the energy of $^3$H as $-7.64$ MeV, which is 40 keV higher than the double-$F$ TOAMD result $-7.68$ MeV [3] and 120 keV higher than the GFMC result $-7.76$ MeV [12]. We list in detail our present results in Table 1, with other calculations for the Hamiltonian components and root-mean-square of the matter radius. Each component is found to nicely reproduce the corresponding values of double-$F$ TOAMD and few-body calculations. In particular, the present HM-TOAMD provides almost equivalent solutions to those of double-$F$ TOAMD, which indicates the reliability of HM-TOAMD.

We further discuss the relation between the correlation functions $F_D, F_S$ and the high-momentum pairs. In the above calculations, as shown in Fig. 4, the correlation functions $F_D$ and $F_S$ are expanded by using the Gaussian functions with the number $n_G = 3$–6, which is smaller than the choice of 7–13 bases in our previous TOAMD calculations [3–6]. In Fig. 5, we show the results with different Gaussian numbers $n_G$ for the correlation functions while keeping the number of high-momentum pairs as $n_D = 6$. When we include no high-momentum pairs at point “0” in the horizontal axis, we can see clear differences between the energies obtained with different $n_G$. After we include all kinds of high-momentum pairs, these differences in the total energy become very small and the resulting energies are almost the same. This result indicates that the small model space of the correlation functions to express the NN correlations, due to the small number $n_G = 3$ in $F_D$ and $F_S$, can be compensated by using high-momentum pairs with larger $n_D$. Thus, essentially the $F_D, F_S$ and the high-momentum pairs play a role in describing the same NN correlations and compensate each other. Hence, we can choose flexibly the basis numbers $n_G$ for $F_D$ and $F_S$ and $n_D$ for high-momentum pairs to set a reasonable model space that can describe the NN correlations of nuclei. In future study of
Fig. 5. Energy of $^3$H nucleus calculated with HM-TOAMD using the bare AV8' interaction with respect to successively adding the various kinds of high-momentum pairs. All the imaginary shifts $\vec{D}$ are adopted to be parallel with the $z$-axis. $\pm D$ denote the different kinds of high-momentum pair shifts defined in Eq. (3). Six shift parameters $D_z$ are superposed for each high-momentum pair. The label of each line denotes the number of Gaussian expansions $n_G$ in $F_D$ and $F_S$. We set the range parameter $\nu = 0.14$ fm$^{-2}$.

$p$-shell nuclei including the clustering states, we can benefit from this flexibility of model space in HM-TOAMD. By utilizing the high-momentum pair, we also simplify the calculations with multiple correlation functions, such as the triple product case and beyond.

4. Conclusion In conclusion, we have formulated a new ab initio framework with the hybridization of tensor-optimized and high-momentum AMD, namely “HM-TOAMD.” The NN correlations in the wave function are described by using both the correlation functions in the TOAMD approach and the high-momentum pairs in the HM-AMD approach. In the present hybrid description, the energy curves of the $^3$H nucleus as functions of high-momentum components show clearly the effects of tensor interaction and strong short-range repulsion. We also find that the convergence of the solutions can be obtained by including only the high-momentum pairs in the $z$-direction, parallel to the spin direction of the nucleons. With the AV8' interaction, the total energy, Hamiltonian components, and matter radius obtained in HM-TOAMD agree very well with other ab initio frameworks such as double-$F$ TOAMD, GFMC, and few-body calculations. We also show the flexibility of the model space in this new hybrid method to reduce the total basis number for the numerical calculation. This is essential for future ab initio studies of $p$-shell nuclei and nuclei with clustering structures in which the triple and further products of correlation functions must be included.

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