Restoring Missing Fragments of a Distorted Image Due to Defocusing or Blur of a Known Hardware Function

Alexander V. Kokoshkin, Evgeny P. Novichikhin
Kotelnikov Institute of Radioengineering and Electronics of RAS, Fryazino Branch, http://fire.relarn.ru/
Fryazino 141190, Moscow Region, Russian Federation
E-mail: shvarts65@mail.ru, epnov@mail.ru
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Abstract: In this paper, a comparative analysis of the methods for recovering images distorted by defocusing or blurring from incomplete data is performed using examples. Incomplete data means the absence of any image fragments that were retouched using different types of interpolation - linear, spline and the interpolation method for the sequential calculation of the Fourier spectrum (IMSCS) developed by us. Then, the famous deconvolution method, the Wiener Filter (WF), was applied to the entire image. Analysis of the quality of restoration, carried out on the example of aerospace images, suggests that using IMSCS to fill in missing fragments (gaps) is either preferred or no less competitive than alternative methods. This is a consequence of the fact that IMSCS does not just retouch the gap, but also tries to reconstruct the lost data.

Keywords: distorted images, remote sensing of the earth's surface, blurring, defocusing, missing fragments, interpolation, restoration

1. INTRODUCTION

The development of methods for remote sensing of the Earth's surface and other planets is a topical trend in science and technology. And one of the key tasks of digital image processing is the restoration of missing parts (gaps). In aerospace imagery, data loss can arise due to the peculiarities of the trajectories of the aircraft, cloud shading or technical failures of the recording equipment. In some cases, incompleteness of data in published images may mean deliberate removal of fragments in order to hide information. Thus, the improvement of methods for reconstructing gaps is one of the most important directions.

Defocusing and blurring can be distinguished among the most common image distortions, which can be described on the basis of the convolution equation [1,2] with the corresponding distorting hardware function (HF). In [3-5], the possibility of restoring images distorted by blur and defocusing from incomplete data was considered. The gaps in such images were retouched using linear interpolation. Then the well-known deconvolution methods...
were applied to the entire image. In [6-13], the possibilities of filling in the missing parts of the image using wavelets, various types of interpolation (bilinear, spline, trigonometric, polynomial) are shown in order to reconstruct the distortions caused by a known HF. However, the assumed non-rectangular shape of the gap and the appearance of additional artifacts after deconvolution caused by such retouching limit the possibilities of using these methods.

In [14], we have developed a method for interpolation of a sequentially calculated Fourier spectrum (MIPVS), which is capable of filling in gaps of arbitrary shape. In works [15-19], possible areas of application of MIPVS are shown and a comparative analysis of the methods used to fill gaps in images was carried out.

This work is devoted to a comparative analysis of how the methods of interpolation of missing fragments distorted due to defocusing or blurring of images, with a known instrumental function, affect the final result of reconstruction. MIPVS is compared with linear interpolation and spline interpolation described in [20,21]. The physical meaning of spline interpolation is that for an arbitrary set of reference points (nodes) a system of linear equations is solved that simulates the behavior of a curved elastic plate. At the same time, it is possible to take into account when calculating the spline the border around the gap with a width of one to several nearest pixels. This approach has a certain versatility and can be applied for comparative analysis with the interpolation method of a sequentially calculated Fourier spectrum. Comparing MIPVS with simpler methods of interpolation, for example, linear, will not be entirely correct. Nevertheless, linear interpolation will be used by us in this work, since with the instrumental function of linear blur it exactly corresponds to the distortion. Thus, provided that the orientation of the blur and the interpolation line match exactly, all other methods of filling the gap are in a more difficult position compared to linear interpolation.

2. BASIC ASSUMPTIONS

As test we use aerospace images taken from publicly available Internet resources. There are a large number of methods for determining the distorting hardware function from the original image and methods of "blind" deconvolution [1,22-26], but any inaccuracy in determining the HF parameters will inevitably lead to a deterioration in the quality of reconstruction. Therefore, in order to eliminate additional interfering factors, this work assumes that the hardware functions are accurately known. The gap is filled using linear interpolation, spline and MIPVS. The missing fragment is square and varies in size from experience to experience. Its shape was chosen to be square in order to put all the considered interpolation methods into equal conditions, since the MIPVS (in contrast to alternative methods) has an arbitrary lacuna shape that does not cause difficulties. Reconstruction of the image with the reconstructed lacuna is performed using the Wiener Filter (WF) [1]. No correction of artifacts after deconvolution is performed. An objective criterion for the quality of restoration will be the relative values of the standard deviation of the result of restoration with an interpolated gap from the result of restoration of an image without a gap taken as "ideal".

In [4] it was shown that the irreplaceable area is determined by the relative size of the gap (in [4] of the shading object) and the
amount of defocusing of the image. Thus, in our work, the size of the defocusing spot is chosen large enough for reasons of the fact that with a small size of the HF, or as they sometimes say - the point blur function (PBF), the shading effects are hardly noticeable. According to the results of [4], it is obvious that inside the gap one can only hope for partial recovery of the lost data, and only within the radius of the blur spot from the edge of the gap. **Fig. 1** shows the gap in black, gray circles denote the HF coverage area at different sizes of the gap and HF relative to each other.

Thus, with a known HF with a radius of \( S = 10 \) pixels, the forecast for the reconstruction of a lacuna with a size of no more than 20 by 20 pixels is positive. In the proposed work, we vary the size of the gap from 10 by 10 pixels to 30 by 30 pixels. Those, we begin the study of the reconstruction quality from the zone of sufficiently confident restoration (a gap of minimum size – **Fig. 1a**) and end in the zone of obviously problematic restoration of the center of a gap of maximum size – **Fig. 1c**. In what follows, all figures in this work show the results of image reconstruction for the most difficult of the selected cases – a gap size of 30 by 30 pixels. This is done so that readers can visually (expertly) assess the quality of the restoration.

Similar reasoning was used to select the relative sizes of the gap and the length of the HF of rectilinear blur. When modeling the instrumental blur function, we assume that in the process of shooting, distortion occurs due to the steady progressive motion of the recording system (presumably an aircraft) relative to the image scene. Thus, we take a horizontal line of length \( SL = 21 \) pixels for the hardware blur function.

3. HARDWARE DEFOCUS FUNCTION

The sequence of actions required to conduct a comparative analysis of the effect of interpolation methods for missing fragments on the final result of restoration is as follows:

**Fig. 2a** shows an aerospace image of Rostov Veliky, 512 by 512 pixels. In **Fig. 2b** image of **Fig. 2a** is shown after defocusing with the known HF with a radius of \( S = 10 \) pixels. **Fig. 2c** is figure 2b with a missing fragment measuring 30 by 30 pixels (black). **Fig. 2d** – restoration using the Wiener filter of **Fig. 2c** with a gap filled using linear interpolation. **Fig. 2e** – restoration using the Wiener filter of **Fig. 2c** with a gap filled using spline interpolation. **Fig. 2f** – restoration using the Wiener filter of **Fig. 2c** with a gap filled with MIPVS.

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**Fig. 1.** The coverage area of the HF with different sizes of the gap (black square) and HF "round spot" (gray circles): (a) – HF is much wider than the gap; (b) – the HF radius is equal to the half-width of the square lacuna; (c) HF is much smaller than the gap.
Fig. 3 is a fragment of Fig. 2. It shows a 90 by 90 pixel patch with a 30 by 30 pixel gap inside. Each of Fig. 3 (from \(a\) to \(e\)) is a fragment of the corresponding Fig. 2 (from \(a\) to \(e\)).

**Fig. 2.** (a) - original test aerospace image; (b) - defocused image in Fig. 2a \((S = 10\) pixels\); (c) - image of Fig. 2b with the missing fragment 30 by 30 pixels (black); (d) - restoration using the Wiener filter of Fig. 2c with a gap filled using linear interpolation; (e) - restoration using the Wiener filter of Fig. 2c with the gap filled with the help of spline interpolation; (f) - restoration using the Wiener filter of Fig. 2c with a gap filled with MIPVS.

**Fig. 3.** Fragments of 90 by 90 pixels: (a) - the original test aerospace image; (b) - defocused image of Fig. 2a \((S = 10\) pixels\); (c) - images 2b with a missing fragment of 30 by 30 pixels (black); (d) - Fig. 2d, reconstructed using the Wiener Filter, with a gap filled with linear interpolation; (e) - Fig. 2c reconstructed using the Wiener Filter with a gap filled with spline interpolation; (f) - Fig. 2c, reconstructed using the Wiener Filter, with a gap filled with MIPVS.
A close examination of the images in Fig. 2d,e,f it can be seen that on the reconstructed images the intensity of artifacts over the entire area gradually decreases from filling the gap using linear interpolation to filling the gap using MIPVS. This is confirmed by the data in Fig. 4a, which shows the dependence of the standard deviation over the entire image of the result of reconstruction with an interpolated gap on the reconstruction of an image without a gap, taken as "ideal", along the horizontal gap size (DL). The standard deviation for the corresponding interpolation is indicated as follows: SKOL - linear, SKOS - spline, SKOF - MIPVS. In Fig. 4b shows similar dependences of the RMS on the size of the gap, calculated only inside the gap itself. The standard deviation for the corresponding interpolation is indicated as follows: SKOLm - linear, SKOSm - spline, SKOFm - MIPVS. Figs 4a and 4b show a

Fig. 4. Case of AF defocusing. The standard deviation of the result of reconstruction with an interpolated lacuna from the reconstruction of an image without a lacuna taken as "ideal": (a) - over the entire image (interpolation designations: SKOL - linear, SKOS - spline, SKOF - MIPVS); (b) - only by the area of the gap (notations of interpolations: SKOLm - linear, SKOSm - spline, SKOFm - MIPVS); (c) - relations OtnS = SKOS/SKOF, OtnL = SKOL/SKOF; (d) - ratios OtnSm = SKOSm/SKOFm, OtnLm = SKOLm/SKOFm.
significant advantage of MIPVS (solid line) compared to linear (dashed line) and spline (dashed line) interpolations. Fig. 4c depicts the dependences $OtnS = SKOS/SKOF$, $OtnL = SKOL/SKOF$ on the size of the gap $DL$. They show how many times the RMSD of restoration over the entire image with linear ($OtnL$) or spline ($OtnS$) interpolations is greater than the RMSD of restoration with MIPVS. Similar to Fig. 4d illustrates dependencies $OtnSm = SKOSm/ SKOFm$, $OtnLm = SKOLm/ SKOFm$ referring to RMS only inside the reconstructed lacuna. All dependencies indicate significantly better performance of MIPVS compared to competing methods.

4. HARDWARE BLUR FUNCTION
We assume that linear interpolation occurs exactly along the blur line (length $SL = 21$ pixels). In this case, other types of filling the gap are in a more difficult position. The initial test is the image in Fig. 2a. In order to save space, here (in Fig. 5) we present only fragments of the test image measuring 90 by 90 pixels with a gap of 30 by 30 pixels in the middle. In fact, as with HF defocus, restoration was performed over the entire 512 by 512 pixel image. That is, all actions for HF blur were carried out as for HF defocusing.

Fig. 6 is analogous to Fig. 4, for HF blur only. The designations are the same. The behavior of the curves in Fig. 6 indicates that, according to the objective criterion of standard deviation from the "ideal", none of the investigated interpolations has an advantage over the alternative ones. This is especially evident in Fig. 6c and Fig. 6d, which, respectively demonstrate the behavior of the dependence $OtnS = SKOS/ SKOF$, $OtnL = SKOL/ SKOF$ (for RMS over the entire image) on the size of the gap $DL$ and the dependence $OtnSm = SKOSm/ SKOFm$. 

Fig. 5. Fragments of 90 by 90 pixels: (a) - the original test aerospace image; (b) - image distorted by blur in Fig. 2a ($SL = 21$ pixels); (c) - images 5b with a missing fragment of 30 by 30 pixels (black); (d) - Fig. 5c reconstructed using the Wiener Filter with a gap filled using linear interpolation; (e) - Fig. 5c reconstructed using the Wiener Filter with a gap filled with spline interpolation; (f) - Fig. 5c reconstructed using the Wiener Filter with a gap filled with MIPVS.
SKOFm, OtnLm = SKOLm/SKOFm (for standard deviation, only inside the restored gap). In Fig. 6c and Fig. 6d standard deviation of restoration over the entire image (or standard deviation only in the gap) with linear or spline interpolation is practically equal to standard deviation of restoration with MIPVS, i.e. their ratio is close to unity for all sizes of the gap. At the same time, for HF defocusing (Fig. 4c and Fig. 4d), the reconstruction after MIPVS showed much better results than the reconstruction after retouching the gap using competing interpolation methods. Note that the expert assessment of the restoration results in Fig. 5d, Fig. 5e, 5f, due to the visually lesser artifacts, indicates a preference for using MIPVS. However, according to the objective criterion of standard deviation from "ideal", none of the interpolation methods has any advantages.
5. FINAL EXAMPLE

In conclusion, we will give an example for the well-known aerospace image "the position of the Egyptian air defense S-75 in the desert"

**Fig. 7.** Original photo: https://picryl.com/media/an-aerial-view-of-soviet-built-sa-2b-guideline-surface-to-air-missiles-positioned-016058.

For this image, a similar sequence of actions for modeling, processing and analysis was carried out as for the previous photo (in the case of HF defocusing). The lacuna was created in the area of the lower left rocket launcher. **Fig. 8a** is a fragment of the image in Fig. 7. The sequence of Figures 8 is analogous to Fig. 3 90 by 90 pixels with a gap of 30 by 30 pixels inside the resulting sequence of actions: defocusing - Fig. 8b, creating a gap in Fig. 8c, reconstruction using the WF with linear interpolation of the gap - Fig. 8d, restoration with the help of WF with spline interpolation of the gap - Fig. 8e, restoration with the help of WF during lacuna interpolation using MIPVS - Fig. 8f.

The behavior of the curved lines in **Fig. 9** in general terms corresponds to similar graphs presented in Fig. 4. I.e. indicates an advantage when using MIPVS for retouching missing fragments over alternative interpolation methods. We believe that this happens because MIPVS, working in the frequency domain, uses all available data (original image), while spline or linear
interpolation, performing actions in the spatial domain, operates with information about the edges of the gap. Thus, MIPVS not only retouches, but also tries to reconstruct the lost data [14,15,16,19]. Some fluctuations in the curves in Fig. 9 take place due to the specifics of the selected image and, among other things, depend on the location of the lacuna. Obviously, if the gap is located on a "flat" place (clear sky or smooth desert), then there is not much difference which of the interpolation methods fill the place of the lost data. It is quite another matter if there is no fragment of a highly informative image (saturated with various elements).

6. CONCLUSIONS
Comparative analysis, both by expert judgment and by the objective standard deviation criterion from the "ideal", shows that when solving the problems of restoring missing fragments of images distorted due
to defocusing with a known instrumental function, the use of MIPVS is preferable compared to linear and spline interpolations. At the same time, despite the seemingly favorable for MIPVS expert assessment with HF blur, standard deviation does not show any advantages for any of the methods of lacuna retouching considered here. Thus, the MIPVS proposed by us in each of the considered HF options is either preferable or no less competitive than alternative methods. We believe that this happens because MIPVS, working in the frequency domain, uses all available data (the entire original image), while spline or linear interpolation, performing actions in the spatial domain, operates with information only about the edges of the gap. Thus, MIPVS not only retouches, but also tries to reconstruct the lost data [14,15,16,19].

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