Tunable terahertz oscillations in superlattices subject to in-plane magnetic field

M. Orlita,1,2 R. Grill,1 L. Smrčka,2 and M. Zvára1

1Charles University, Faculty of Mathematics and Physics, Institute of Physics, Ke Karlovu 5, CZ-121 16 Prague 2, Czech Republic
2Institute of Physics, Academy of Sciences of the Czech Republic, Cukrovarnická 10, CZ-162 53 Prague 6, Czech Republic

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We present a concept of terahertz oscillations in superlattices generated under conditions apparently different from standard Bloch oscillations. The oscillations are induced by crossed magnetic and electric fields both applied to the superlattice in the in-plane direction. The frequency of these oscillations is tunable by the applied fields.

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I. INTRODUCTION

The possibility of Bloch oscillations (BOs), i.e. the periodic motion of an electron in a periodic system induced by a uniform electric field, was mentioned by Zener relatively soon after the basic quantum mechanical theory of the solid state was established.1 Taking a simple picture of electrons, which are not subject to tunneling to other bands or scattering, Zener predicted an oscillatory motion in the real as well as in the reciprocal space with the frequency \( \omega_{BO} = |e|F\Delta/\hbar \), where \( \Delta \) and \( F \) are the system spatial period and the electric field, respectively. However, it took a long time before any experimental evidence of BOs has been found. The key feature for their observation was a pioneering concept of a superlattice (SL) and minibands suggested by Esaki and Tsu that allowed to overcome problems of a strong electron scattering, which made the observation of BOs in bulk semiconductors hardly feasible.

In this paper, we present a simple idea of electron-in-plane-oscillations appearing when crossed in-plane magnetic and electric fields are applied to SL. We start with a general discussion of properties of SL subject to the in-plane magnetic field the effect of which cannot be described within the quasi-classical approximation. The findings are illustrated by simple numerical calculations based on the standard tight-binding (TB) model.

Subsequently, we study influence of an additionally applied electric field and conclude that an oscillatory motion of electrons is induced. We present two models of these oscillations, i) quasi-classical and ii) pure quantum-mechanical. The predicted oscillations are compared to common BOs.

II. SUPERLATTICE SUBJECT TO IN-PLANE MAGNETIC FIELD

Let us consider an infinite SL having its growth axis oriented along the \( z \)-direction, which is described by the periodic potential \( V(z) = V(z + \Delta) \), where \( \Delta \) is the period of SL. The Hamiltonian of an electron in such a system subject to the in-plane magnetic field \( \mathbf{B} = (0, B_{||}, 0) \), with the vector potential gauge \( \mathbf{A} = (B_{||}z, 0, 0) \), reads:

\[
\mathcal{H} = \frac{1}{2m} \left( p_x + |e|B_{||}z \right)^2 + \frac{p_y^2 + p_z^2}{2m} + V(z).
\]  

To find eigenstates of this Hamiltonian, the following ansatz for the wave functions is commonly assumed:

\[
\psi_{k_x,k_y}(x,y,z) = e^{ik_x x + ik_y y} \chi_{k_x}(z). \tag{2}
\]

This way the three-dimensional Hamiltonian (1) is reduced to one-dimensional \( H \) which depends on the parameters \( k_x \) and \( k_y \):

\[
H = \frac{\hbar^2}{2m} \left( k_x + \frac{|e|B_{||}z}{\hbar} \right)^2 + \frac{\hbar^2 k_y^2}{2m} - \frac{\hbar^2}{2m} \frac{d^2}{dz^2} + V(z). \tag{3}
\]

Hence, when the in-plane magnetic field is applied to the SL system, the motion in the \( x \)- and \( z \)-directions become coupled. This coupling can be interpreted as an effect of the Lorentzian force.

To solve the eigenvalue problem defined by the Hamiltonian (3), we can utilize the invariance of this Hamiltonian under transformation \( H(z,k_x) \rightarrow H(z+\Delta,k_x-K_0) \), where \( K_0 = |e|B_{||}\Delta/\hbar \). The whole eigenenergy spectrum then reads:

\[
E_n(k_x,k_y) = E_n(k_x) + \frac{\hbar^2 k_y^2}{2m}, \tag{4}
\]

where \( E_n(k_x) \) are Landau subbands \( n = 1,2,3\ldots \) which are \( K_0 \)-periodic in the momentum \( k_x \).

A straightforward solution of the Schrödinger equation including the Hamiltonian (3) leads to the functions \( \psi^{(n)}_{k_x,k_y}(x,y,z) \) that do not respect the translation symmetry of the SL along the growth axis. This is caused by the additional effective potential parabolic in \( z \), developed at finite \( B_{||} \) in the Hamiltonian (3). Consequently, electron momenta \( k_x \) have to be taken within the interval \( k_x \in (-\infty, +\infty) \) to obtain full set of eigenfunctions (2). Nevertheless, all eigenstates in the \( n \)-th subband having momenta \( k_x + lK_0 \) (\( l \in \mathbb{Z} \)) are degenerate due to
the periodicity of \( E_n(k_x) \) and therefore, the translation periodicity can be restored, when an appropriate linear combination of these states is taken:

\[
\Phi^{(n)}_{k_x,k_y,q}(x,y,z) \propto \sum_{l \in \mathbb{Z}} e^{iql} \psi^{(n)}_{k_x+k_0,k_y}(x,y,z).
\] (5)

The new continuous quantum number \( q \in (-\pi/\Delta, \pi/\Delta) \) thus replaces the discrete index \( l \). Owing to the restored translation periodicity, the eigenstates fulfill the condition:

\[
\Phi^{(n)}_{k_x,k_y,q}(x,y,z-\Delta) = e^{i(K_0 q + \Delta)} \Phi^{(n)}_{k_x,k_y,q}(x,y,z)
\] (6)

and the considered interval of momenta \( k_x \) can now be reduced to the Brillouin zone \( k_x \in (-K_0/2, K_0/2) \). Both reduced and extended schemes are fully equivalent.

Hence, the Hamiltonian describes a system with periodic dispersions \( E_n(k_x) \), whose period \( K_0 \) is tunable by the applied magnetic field \( B_\parallel \). This is in contrast to a fixed period \( 2\pi/\Delta \) of the electron dispersion in the growth direction of SL at zero \( B_\parallel \). The obtained result can be interpreted also as a formation of a 2D lattice in the \( x-z \) plane induced in SL by a finite magnetic field in the \( y \)-direction. This lattice has spatial periods in \( x \)- and \( z \)-directions \( 2\pi/K_0 \) and \( \Delta \), respectively. The magnetic flux trough the unit cell of the lattice is simply \( 2\pi B_\parallel \Delta/K_0 = \hbar/|e| \), i.e. one magnetic flux quantum.

The form of the Hamiltonian implies the appearance of the periodic dispersion even at a negligible small \( B_\parallel \) and thus the standard parabolic dispersion is not attained in the limit \( B_\parallel \to 0^+ \). The problem lies in the infinite size of the considered SL. When a (realistic) superlattice with a finite number of wells is taken into account, we get only a partially periodic dispersion \( E_n(k_x) \) and the number of minima in this dispersion corresponds to the number of wells, see an extreme case of a double quantum well in the Appendix. These minima clearly disappear at low \( B_\parallel \) and the dispersion approaches the expectable parabolic shape.

Before we further utilize the above drawn conclusions, we use a simple TB approximation to get some numerical results, which can illustrate the studied problem. Within the framework of the TB model, the Hamiltonian is transformed into the matrix form and reads:

\[
H_{r,s} = \frac{\hbar^2}{2m} (k_x + r K_0)^2 \delta_{r,s} + t \delta_{r,s+1} \quad (r, s \in \mathbb{Z}),
\] (7)

where the coefficient \( t \) \((t < 0)\) characterizes the tunneling between adjacent quantum wells (QWs) and where the motion in the \( y \)-direction is not included, since it is not affected by \( B_\parallel \). Note that this TB Hamiltonian conserves the periodicity of the original Hamiltonian in momentum \( k_x \). The eigenvalue problem given by the tridiagonal Hamiltonian can be easily solved numerically.

The calculated dispersions for three lowest lying Landau subbands \( E_n(k_x) \) have been plotted in Fig. 1. These results demonstrate both the expected periodicity of subbands in \( k_x \) and the shape of the dispersion curves, which cannot be predicted only from the symmetry of the Hamiltonian. We see that just one minimum per interval \( (k_x, k_x + K_0) \) appears. The dotted lines in Fig. 1 show the limit of very weakly coupled wells, i.e. \( t \to 0 \). The dispersion curves at \( t = 0 \) are purely parabolic and corresponds to dispersions of electrons in isolated QWs. Hence, the widths of individual Landau subbands \( E_n(k_x) \) and the energy gaps between them are given by the strength of \( t \) and can be also tuned by the applied magnetic field \( B_\parallel \).

Henceforth, we will take account of the lowest lying subbands \( E_1(k_x) \equiv E(k_x) \) only. Such approximation is meaningful in strongly coupled SLs, i.e. for high values of \(|t|\) when the separation of this lowest subband from the higher ones is significant. The energy width \( W_0 \) of this subband can be simply estimated at high magnetic fields. We just take \( W_0 \approx \hbar^2 (K_0/2)^2/(2m) = e^2 B_\perp^2 \Delta^2 / 8m \). Obviously, this rough approximation fails if \( W_0 \approx |t| \).

### III. Semi-Classical Model of Oscillations

Having the periodic band structure \( E(k_x) \) at a given fixed magnetic field \( B_\parallel \), we use a semi-classical consideration to describe the electron motion if an additional constant electric field \( F_x \) is applied in \( x \)-direction. As the influence of \( B_\parallel \) has already been included in the discussed energy spectrum, the semiclassical equation of motion takes a simple form \( \hbar \dot{k}_x = -|e| F_x \) and thus \( k_x \) changes linearly in time. Hence, the electron velocity \( v_x = \hbar^{-1} dE(k_x)/dk_x \) becomes periodic in time and a specific oscillatory motion is generated. This motion is schematically shown in Fig. 2 and can be decomposed.
into a steady shift in the SL growth direction and the oscillations in the $x$-direction. The drift motion in the growth-axis direction becomes apparent especially in the extended scheme of $E(k_x)$. The used equation of motion gives us also possibility to calculate the corresponding oscillatory frequency $\omega_{B_\parallel} = 2\pi F_x/(B_\parallel \Delta)$. Hence, $\omega_{B_\parallel}$ is tunable not only by the electric field, as in the case of BOs but by $B_\parallel$ as well. Both frequencies $\omega_{BO}$ and $\omega_{B_\parallel}$ are functions of $\Delta$ – but whereas the first frequency is linear in $\Delta$, the latter one has the reciprocal dependence. The oscillatory frequency $\omega_{B_\parallel}$ can be rewritten into $\omega_{B_\parallel} = 2\pi v_d/\Delta$, where $v_d = F_x/B_\parallel$ is the drift velocity introduced in 3D for the electron motion perpendicular to the crossed electric and magnetic fields. The ratio $\Delta/v_d$ is then obviously the time needed by an electron to tunnel into the adjacent QW.

The semiclassical model allows us to determine the spatial amplitude of expected oscillations $x_0$ defined in Fig. 2. When we make use of the facts that the electron position is the time integral of the electron velocity and the velocity $v_x$ is derivative of the dispersion curve, we obtain a simple relation $x_0 = W_0/(2|e|F_x)$. As the subband width $W_0$ varies with $B_\parallel$, the amplitude $x_0$ is tunable by the magnetic field as well.

IV. QUANTUM-MECHANICAL MODEL OF OSCILLATIONS

The electron motion in a system with a periodic dispersion $E(k_x) = E(k_x + K_0)$ can be treated in a pure quantum-mechanical way as reviewed e.g. by Hartmann et al. in Ref. 13. The corresponding Hamiltonian is there conveniently written in the momentum representation:

$$H(k_x) = E(k_x) + i|e|F_x \frac{d}{dk_x}$$  \hspace{1cm} (8)

and thus, taking account of the periodicity in $k_x$, the eigenenergies can be easily calculated:

$$E_n = \frac{1}{K_0} \int_0^{K_0} E(k_x) dk_x + n\hbar\omega_{B_\parallel},$$  \hspace{1cm} (9)

Because the first term of this eigenenergy is constant at given $B_\parallel$, we receive an analog of the common Wannier-Stark ladder ($n \in \mathbb{Z}$) discussed in the framework of Bloch oscillations 13.

V. REMARKS ON A POSSIBLE REALIZATION

From a practical point of view, the experiments proving emission of the predicted THz radiation can be the same as in the case of standard BOs. Both coherent and incoherent radiations can be obtained. The coherent THz radiation induced by BOs is achieved e.g. when free electrons are generated in an undoped SL by a femtosecond optical pulse ensuring the same phase of all electrons. The photon laser energy is tuned to an appropriate electron-hole transition in SL. The same technique can be used in our case as well.

The function of the THz generator could be disrupted, if electrons initially localized in the lowest subband tunnel under the effect of $F_x$ to the higher subbands. In such a case, the one-subband model utilized in both semiclassical and quantum-mechanical treatments of oscillations would not be applicable. We illustrate this obstacle on a simple model of a double quantum well in Appendix. This model offers the simplest possibility to check the intersubband tunneling induced by the electric field. It cannot serve as a definite evidence that the tunneling to higher subbands is negligible in superlattices, nevertheless, it illustrates that electrons do not noticeably tunnel to the higher (antibonding) subband in DQW at $B_\parallel$ under conditions typical for the THz oscillations predicted in SLs. Apparently, further investigations in this direction are necessary.

For a possible realization, we should also check the sample design and experimental conditions to observe the predicted oscillations in the terahertz region. Assuming $\omega_{B_\parallel} \approx \omega_{BO}$ and the same electric field in both cases, we obtain the corresponding magnetic field $B_\parallel \approx h/\Delta^2|e| \cong 16$ T for $\Delta = 16$ nm. Hence, at $B_\parallel < 16$ T the oscillatory frequency is even higher than for BOs, since $\omega_{B_\parallel} \propto B_\parallel^{-1}$. Moreover, having two free parameters $F_x$ and $B_\parallel$ we can independently optimize $\omega_{B_\parallel}$ and $x_0$ to achieve the maximal emitted power. This is impossible for standard BOs, since $\omega_{BO}$ and the corresponding spatial amplitude are governed by the applied electric field only.

The important point in the observation of BOs is the achievement of an oscillation period significantly lower than is the scattering time due to phonons or plasmons. We predict our oscillation for SL systems, where common BOs are observed. Therefore, the same or very similar damping rates as observed in BO experiments could be
expected in our case as well. Hence, the published experimental evidence of BOs suggests that the predicted \( B_\parallel \)-controlled oscillations ought to be experimentally observable.

It is interesting to investigate also the direction characteristics of the expected THz radiation. Since the radiation is generated by the electron oscillatory motion in the \( x \)-direction, the radiation should be emitted mainly in the plane perpendicular to the \( x \)-axis, i.e. in the plane perpendicular to the oscillating dipoles. The predicted device can thus be both edge- or surface-emitting.

An important advantage of presented model in comparison with standard BOs is a fast (in-plane) drain of electrons from the structure after they reach the edge of SL. This fact can be utilized in a significant enhancement of the repetition frequency of the generation of the coherent THz radiation.

VI. CONCLUSIONS

We have investigated behavior of electrons in a superlattice when crossed magnetic and electric fields are applied, both in the in-plane direction. We predict a novel terahertz oscillations in superlattices that are different from Bloch oscillations that appear when the electric field is applied in the growth direction of the superlattice. We have also found a simple expression for the frequency of such oscillations. The suggested realistic design of the structures allows preparation of terahertz emitters controlled by the in-plane magnetic field and described within the TB approximation by the interwell distance \( \Delta \) and the tunnelling coefficient \( t \). The thorough theoretical analysis of such a DQW system can be found elsewhere. The DQW represents an extreme case of a superlattice taken up to now into account. A lateral electric field applied in the \( x \)-direction at \( x > 0 \) forms the potential profile:

\[
\phi(x) = \begin{cases} 
0 & \text{if } x < 0 \\
|e|F_{x}x & \text{if } x > 0
\end{cases}.
\]

The electron described by the bonding subband wave function \( \Psi_B(k_x, x) \) with the momentum \( k_x > 0 \) and energy \( E_B(k_x) \) enters into the system at \( x < 0 \). It is reflected by the potential barrier \( \phi(x) \) and leaves the system again at \( x < 0 \). The reflected electrons may be found both in bonding states \( \Psi_B(-k_x, x) \) and in antibonding states \( \Psi_A(-k'_x, x) \) with \( k'_x > 0 \) being a momentum of the corresponding antibonding state. Due to the elastic reflection the energy of antibonding state \( E_A(-k'_x) \) equals to bonding ones \( E_A(-k'_x) = E_B(\pm k_x) \).

The total wave function \( \Psi(x) \) of the electron at \( x < 0 \) thus reads

\[
\Psi = \frac{\Psi_B(k_x) + c_B \Psi_B(-k_x) + c_A \Psi_A(-k'_x)}{\sqrt{1 + c_B^2 c_B^2 + c_A^2 c_A^2}},
\]

where the complex amplitudes of reflected waves \( c_B \) and \( c_A \) determine the occupancy of respective bonding and antibonding subbands. The wave function \( \Psi \) at \( x > 0 \) is calculated solving the Schrödinger equation numerically and respective \( c_B \) and \( c_A \) are established to accomplish the damping \( \lim_{x\to\infty} \Psi(x) = 0 \). Results of the calculation for different tunnelling coefficients \( t \) are shown in Fig. where model parameters \( m = 0.067 m_0, \Delta = 16 \text{ nm}, B_\parallel = 12 \text{ T} \) and \( F_x = 1920 \text{ V/cm} \) corresponding to the oscillator frequency of 1 THz have been used. The energy of incoming electrons \( E_B(k_x) \) well above the minimum energy of the antibonding subband \( E_A(0) \) was used to enable the intersubband tunneling. We have ascertained that the course of \( |c_B|^2 \) (\( |c_A|^2 \)) is practically insensitive to \( E_B(k_x) \). We find out that increased subband splitting strongly damps the intersubband tunneling and \( |c_A|^2 < 0.01 \) is obtained at \( |t| > 7.5 \text{ meV} \) for chosen parameters. Analogous results are obtained also for other sets of parameters producing oscillations in THz branch.

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