Steady-state and unsteady-state moisture regime of enclosing structure

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Abstract. This paper describes the moisture transfer problems of enclosing structures which are crucial in modern construction industry. In terms of current condition of science, researchers do not know the moisture transfer law in capillary-porous materials, so they need to work with a variety of mathematical models. We developed the new steady-state and unsteady-state formulation of mathematical models based on moisture potential \( F \), which takes into account water vapour and liquid movements uniformly. It is vital to understand whether we can use steady-state moisture transfer mathematical model in a design engineer work or we must use the unsteady-state one, so we make a comparison between them for a single-layer aerated concrete wall. We compare moisture behaviours of the steady-state and unsteady-state processes in two ways: comparison of the moisture distribution in the thickness of the enclosing structure during maximum moisture accumulation period and comparison of the average moisture of the wall during a year. It was found that the solution of the unsteady-state equation of moisture transfer gives significantly more possibilities than the solution of the steady-state equation of moisture transfer. As a result, we recommend to use the developing unsteady-state mathematical model to predict the moisture regime of enclosing structures.

1. Introduction

Researching of the moisture regime of enclosing structures is one of the modern trends in construction direction [1–5]. Moisture regime is a research complex, which consists of experiments, field investigations and theoretical developments [6]. One of the most important issues is the developing of mathematical models for moisture transfer calculation [7–12].

There are some works which are based on the separate accounting of private moisture transfer potentials. The complexity of this approach is connected with working with a complex system of differential equations and a large number of experiments [13–26].

Another approach, which is based on the moisture potential, allows to replace part of the private transfer potentials with a single moisture potential, which simplifies the mathematical formulation of the problem [27].

One of the famous moisture potential is the potential which was developed by H.M. Künzel:
\[
\frac{dW}{d\phi} \frac{\partial\phi}{\partial \tau} = \nabla (D_v \nabla \phi + \mu \cdot \nabla (\phi E_e)).
\] (1)

where \(D_v\) – liquid moisture transfer coefficient, \(\text{kg/(m} \cdot \text{s})\); \(\phi\) – relative air humidity; \(\mu\) – vapor permeability coefficient, \(\text{kg/(m} \cdot \text{s} \cdot \text{Pa})\); \(E_e\) – saturated water vapor pressure, \(\text{Pa}\); \(W\) – material moisture content, volume percentage; \(\tau\) – time, \(s\).

In this case, the temperature field is determined by the equation:

\[
\frac{dH}{dT} \frac{\partial T}{\partial \tau} = \nabla (\lambda V T) + r \cdot \nabla (\mu \cdot \nabla (\phi E_e)) \cdot \nabla \phi \cdot \nabla (\phi E_e)).
\] (2)

where \(H\) – enthalpy, \(\text{J/m}^3\); \(T\) – absolute temperature, \(\text{K}\); \(r\) – specific heat of the liquid-vapor phase transition, \(\text{J/kg}\); \(\lambda\) – thermal conductivity coefficient, \(\text{W/(m} \cdot \text{C})\).

Another potential is proposed by V.G. Gagarin and V.V. Kozlov [28–30]:

\[
F(w,t) = E_e(t) \cdot \phi(w) + \frac{1}{\mu} \int_{0}^{w} \beta(\zeta) d\zeta.
\] (3)

where \(F\) – moisture potential, \(\text{Pa}\); \(\beta\) – moisture conductivity coefficient, \(\text{kg/(m} \cdot \text{s} \cdot \text{kg/kg})\); \(w\) – material humidity, \(\text{kg/kg}\) (1 kg/kg = 100 % by weight); \(\zeta\) – current material moisture value, \(\text{kg/kg}\).

As we can see from formula (3), the moisture potential \(F\) takes into account the movement of liquid and vaporous moisture [28–30]:

\[
g = -\mu \frac{\partial F}{\partial x}.
\] (4)

where \(g\) – total flow density of vapor and liquid moisture, \(\text{kg/(m}^2 \cdot \text{s})\).

2. Problem

Thus, it becomes possible to develop the mathematical model which is based on the moisture potential \(F\). However, it is crucial to compare the solution of the unsteady-state moisture transfer equation and the solution of the steady-state moisture transfer equation.

3. Materials and methods

3.1. The mathematical model of steady-state moisture transfer

In the case of steady-state process, moisture transfer is described by the equation:

\[
\frac{\partial}{\partial x} (\mu \frac{\partial F(w,t)}{\partial x}) = 0.
\] (5)

Third-order boundary conditions on enclosing structure surfaces for the moisture potential distribution:
where \(F_N\) – material moisture potential next to enclosing structure surface, which contacts with inside air, \(Pa\); \(F_i\) – moisture potential of enclosing structure section, which contacts with outside air, \(Pa\); \(F_{in}\) – inside air moisture potential, \(Pa\); \(F_{ext}\) – outside air moisture potential, \(Pa\); \(R_{m,ext}\) – resistance to moisture exchange between outside air and the surface of the enclosing structure, \((m^2 s Pa)/kg\); \(R_{m,in}\) – resistance to moisture exchange between inside air and the surface of the enclosing structure, \((m^2 s Pa)/kg\).

Boundary condition between enclosing structure layers:

\[
-\mu_1 \frac{\partial F}{\partial x} \bigg|_{x=0} = -\mu_2 \frac{\partial F}{\partial x} \bigg|_{x=v}.
\]  

where \(v\) – section of multi-layer enclosing structure, in which there is a material joint: \(\mu_1\) – vapor permeability coefficient of enclosing structure layer which is the nearest to the building outside surface, \(kg/(m s Pa)\); \(\mu_2\) – vapor permeability coefficient of enclosing structure layer which is the nearest to the enclosing structure inner surface, \(kg/(m s Pa)\).

Similarly, the steady-state heat conduction equation is considered:

\[
\frac{\partial}{\partial x}(\lambda \cdot \frac{\partial t}{\partial x}) = 0.
\]  

The third-order boundary condition on an enclosing structure outside surface for the temperature distribution:

\[
-\lambda \frac{\partial t}{\partial x} \bigg|_{x=0} = \alpha_{ext} \left( t_{ext} - t_i \right).
\]  

where \(t_{ext}\) – outside air temperature, °C; \(t_i\) – temperature of enclosing structure section, which contacts with outside air, °C; \(\alpha_{ext}\) – heat transfer coefficient between outside air and the surface of the enclosing structure, \(W/(m^2 °C)\).

The third-order boundary conditions on an enclosing structure inside surface for the temperature distribution:

\[
\lambda \frac{\partial t}{\partial x} \bigg|_{x=v} = \alpha_{in} \left( t_{in} - t_N \right).
\]
where \( t_a \) – inside air temperature, \( ^\circ \text{C} \); \( t_n \) – temperature of enclosing structure section, which contacts with inside air, \( ^\circ \text{C} \); \( \alpha \) – heat transfer coefficient between inside air and the surface of the enclosing structure, \( W/(m^2 \cdot ^\circ \text{C}) \), \( l \) – enclosing structure thickness, \( m \).

Boundary condition between enclosing structure layers:

\[
-\lambda \frac{\partial t}{\partial x} \bigg|_{x=\pm \theta_e} = -\lambda \frac{\partial t}{\partial x} \bigg|_{x=\pm \theta_e} .
\]  

(12)

Saturated water vapor pressure is described by analytical expression:

\[
E_t = 1.84 \cdot 10^{11} \cdot \exp(-5330/(273+t)).
\]  

(13)

3.2. The mathematical model of steady-state moisture transfer

In unsteady-state moisture regime moisture transfer is described by the differential equation:

\[
\frac{\partial F(w,t)}{\partial t} = \kappa(w,t) \cdot E_t(t) \frac{\partial^2 F(w,t)}{\partial x^2}.
\]  

(14)

where \( \kappa \) – material heat-humidity characteristic coefficient, \( m^2/(s \cdot \text{Pa}) \).

The solution of the moisture transfer equation (14) is possible to obtain by the finite difference method using an explicit difference scheme:

\[
\begin{aligned}
F_{1}^{k+1} &= F_{1}^{k} + \Delta t \frac{h}{h^2} E_{1}^{k} \left( F_{i+1}^{k} - (1 + \frac{h}{\mu R_{m,ext}}) F_{i}^{k} + F_{i-1}^{k} \right) + \Delta t \frac{h}{h^2} E_{1}^{k} \left( \frac{h}{\mu R_{m,ext}} F_{ext}^{k} \right), \\
F_{i}^{k+1} &= F_{i}^{k} + \Delta t \frac{h}{h^2} E_{i}^{k} \left( F_{i+1}^{k} - 2 \cdot F_{i}^{k} + F_{i-1}^{k} \right), \quad i = 2, \ldots, N - 1, \quad k = 0,1, \ldots \\
F_{N}^{k+1} &= F_{N}^{k} + \Delta t \frac{h}{h^2} E_{N}^{k} \left( F_{N-1}^{k} - (1 + \frac{h}{\mu R_{m,in}}) F_{N}^{k} \right) + \Delta t \frac{h}{h^2} E_{N}^{k} \left( \frac{h}{\mu R_{m,in}} F_{int}^{k} \right),
\end{aligned}
\]  

(15)

where \( \Delta t \) – time step, \( s \); \( h \) – coordinate step, \( m \); \( F_{ext}^{k} \) – moisture potential of outer air at the \( k \)-th time step, \( Pa \); \( F_{1}^{k} \) – moisture potential in the first section of the enclosing structure at the \( k \)-th time step, \( Pa \); \( F_{2}^{k} \) – moisture potential in the second section of the enclosing structure at the \( k \)-th time step, \( Pa \); \( F_{i+1}^{k} \) – moisture potential in the first section of the enclosing structure at the \( (k+1) \)-th time step, \( Pa \); \( F_{i}^{k} \) – moisture potential in the \( (i-1) \)-th section of the enclosing structure at the \( k \)-th time step, \( Pa \); \( F_{i+1}^{k} \) – moisture potential in the \( i \)-th section of the enclosing structure at the \( k \)-th time step, \( Pa \); \( F_{i}^{k} \) – moisture potential in the \( i \)-th section of the enclosing structure at the \( (k+1) \)-th time step, \( Pa \); \( F_{N}^{k} \) – moisture potential in the \( (N-1) \)-th section of the enclosing structure at the \( k \)-th time step, \( Pa \); \( F_{N}^{k} \) –
moisture potential in the \( N \)-th section of the enclosing structure at the \((k+1)\)-th time step, \( P_a \); \( F^k_w \) — moisture potential of inner air at the \( k \)-th time step, \( P_a \); \( \kappa^k_F \) — material heat-humidity characteristic coefficient in the first section of the enclosing structure at the \( k \)-th time step, \( m^2/(s\cdot Pa) \); \( E^k_i \) — saturated water vapor pressure in the first section of the enclosing structure at the \( k \)-th time step, \( P_a \); \( E^k_{1i} \) — saturated water vapor pressure in the \( i \)-th section of the enclosing structure at the \( k \)-th time step, \( P_a \); \( E^k_{iN} \) — saturated water vapor pressure in the \( N \)-th section of the enclosing structure at the \( k \)-th time step, \( P_a \); \( \kappa^k_{F,i} \) — material heat-humidity characteristic coefficient in the \( i \)-th section of the enclosing structure at the \( k \)-th time step, \( m^2/(s\cdot Pa) \); \( \kappa^k_{F,N} \) — material heat-humidity characteristic coefficient in the \( N \)-th section of the enclosing structure at the \( k \)-th time step, \( m^2/(s\cdot Pa) \).

4. Results and discussion

The results of calculating the moisture regime using the steady-state and the unsteady-state methods, which are based on the theory of the moisture potential \( F \), are compared. A single-layer aerated concrete wall in Moscow (Russian Federation) was investigated. Inside the building the temperature of 20 °C and the relative humidity of 55 % are constantly maintained. A comparison of the moisture distribution of the enclosing structure, which is determined by separate methods of the theory of moisture potential \( F \), during maximum moisture accumulation period and during a year is presented at (Fig. 1, Fig. 2).

![Figure 1. Moisture distribution along the thickness of the enclosing structure during maximum moisture accumulation period (1 — distribution, which is obtained according to the solution of the steady-state moisture transfer equation; 2 — distribution, which is obtained according to the solution of the unsteady-state moisture transfer equation).](image-url)
Figure 2. Moisture distribution during a year (1 – distribution, which is obtained according to the solution of the steady-state moisture transfer equation; 2 – distribution, which is obtained according to the solution of the unsteady-state moisture transfer equation).

As it can be seen from the graph (Fig. 1), the highest moisture is achieved using the steady-state method for assessing the humidity regime. This is because the steady-state regime does not take into account the kinetics of humidification. Also, the unsteady-state method allows to more accurately determine the moment of maximum moisture accumulation.

5. Conclusion

Thus, we formulated the mathematical models of steady-state and unsteady-state moisture transfer processes. The solution of the steady-state moisture transfer equation can be found as an analytical expression, whereas the solution of the unsteady-state moisture transfer equation can be obtained by the finite difference method using an explicit difference scheme.

We used these two methods for a the single-layer enclosing structure of aerated concrete blocks, analyzed the moisture behaviour in the thickness of the enclosing structure during maximum moisture accumulation period, and the average moisture of the wall during a year. As a result, the steady-state method of calculation is simpler to use in engineer work, however, it gives less opportunities. The unsteady-state method has higher accuracy than steady-state one, because the unsteady-state formulation of mathematical model takes into account the inertia of humidification processes.

Authors recommend using the developing unsteady-state mathematical model to predict the moisture regime of enclosing structures.

The prospect of developing the topic is the creation of analytical calculation methods for assessing the unsteady-state moisture regime.

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