Leptonic Source of Dark Matter and Radiative Majorana or Dirac Neutrino Mass

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Abstract

The notion of U(1) lepton number (which may only be softly broken) is applied to models of dark matter which interacts with leptons. Previous scotogenic models of Majorana or Dirac neutrino mass are shown to be derivable in this framework without additional symmetries. Only complete renormalizable theories are considered. An explicit class of models with $Z_n$ ($n \geq 5$) lepton and dark symmetry for Dirac neutrinos is derived, as well as an example of $Z_3$ dark symmetry.
Introduction: Two outstanding fundamental issues in particle physics and astroparticle physics are neutrinos and dark matter. They have been shown [1] to be intimately connected in all simple models of dark matter, with dark parity $\pi_D$ derivable from lepton parity $\pi_L$ with the factor $(-1)^{2j}$ for a particle of spin $j$.

To explore further this connection, it is assumed that lepton number may be imposed as a global $U(1)_L$ symmetry in dark-matter extensions of the standard model (SM) of quark and lepton interactions. The extended field theory is required to be renormalizable and the $U(1)_L$ symmetry be respected by all dimension-four terms in the Lagrangian, whereas the soft dimension-three and dimension-two terms are allowed to break $U(1)_L$ to $Z_N$. In particular, the $U(1)_L$ is used to forbid a tree-level Majorana or Dirac neutrino mass, whereas its soft breaking will usher in a radiative Majorana or Dirac neutrino mass through dark matter, i.e. the scotogenic mechanism. Because of the chosen particle content and its original $U(1)_L$ assignments, the resulting theory conserves either lepton parity for Majorana neutrinos, or (redefined) lepton number for Dirac neutrinos. At the same time, a dark symmetry also emerges.

Scotogenic Majorana Neutrino Mass: Consider an extension of the SM with three Higgs doublets: $\Phi = (\phi^+, \phi^0)$, $\eta_1 = (\eta_1^+, \eta_1^0)$, and $\eta_2 = (\eta_2^+, \eta_2^0)$, together with three neutral singlet left-handed $N_L$ and right-handed $N_R$ fermions. They are listed in Table 1. The $U(1)_L$

| fermion/scalar | $SU(2)$ | $U(1)_Y$ | $U(1)_L$ |
|----------------|---------|----------|----------|
| $(\nu, e)_L$   | 2       | $-1/2$   | 1        |
| $e_R$          | 1       | $-1$     | 1        |
| $N_L$          | 1       | 0        | $x_1 \neq -1$ |
| $N_R$          | 1       | 0        | $x_2 \neq 1$ |
| $\Phi = (\phi^+, \phi^0)$ | 2 | $1/2$ | 0 |
| $\eta_1 = (\eta_1^+, \eta_1^0)$ | 2 | $1/2$ | $y \neq 0$ |
| $\eta_2 = (\eta_2^+, \eta_2^0)$ | 2 | $1/2$ | $-y$ |

Table 1: Fermion and scalar content of generic model.
assignments $x_{1,2}$ are chosen to forbid the tree-level couplings $N_L(\nu_L\phi^0 - e_L\phi^+)$ and $\bar{N}_R(\nu_L\phi^0 - e_L\phi^+)$. The choice of $\pm y \neq 0$ is to distinguish $\Phi$ from $\eta_{1,2}$ and to allow the quartic coupling $(\Phi^\dagger\eta_1)(\Phi^\dagger\eta_2)$ as first proposed in Ref. [2]. In the original scotogenic model [3], $\eta_1 = \eta_2$ and is distinguished from $\Phi$ by lepton parity [1]. In the present framework, there are four variations as shown in Fig. 1.

![Figure 1: One-loop diagrams of Majorana neutrino mass.](image)

The fermion line has four possible connections. From top to bottom: $x_1 = y-1, x_2 = y+1$; $x_1 = -y-1, x_2 = -y+1$; $x_2 = y+1 = -y+1$; $x_1 = y-1 = -y-1$. The last two options require $y = 0$ which is ruled out. The first two options requires $x_2 - x_1 = 2$. This means that the soft term $\bar{N}_L N_R$ must break $U(1)_L$ by two units. At the same time, the soft term $N_L N_L$ breaks it by $2y - 2$ or $2y + 2$ units, whereas the soft term $N_R N_R$ does it by $2y + 2$ or $2y - 2$ units. Furthermore, the soft terms $\Phi^\dagger\eta_{1,2}$ would break $U(1)_L$ and allow $\eta_{1,2}^0$ to couple to $\bar{\nu}_L N_R$ through $\phi^0$, so they must be forbidden. To do so, $y$ should be odd, because the $\bar{N}_L N_R$ breaking is even (2 units) and these terms will not be generated if they are assumed to be absent in the beginning.

If $y = 1$, then $x_1 = 0$ and $x_2 = 2$. The resulting symmetry is just lepton parity, i.e. $\eta_{1,2}$ are odd and $N_{L,R}$ are even. If this symmetry was imposed in the beginning, then $\eta_1 = \eta_2$ and
\[ N_L = \bar{N}_R \text{ may be assumed, and the original scotogenic model } [3] \text{ is recovered. However, with soft breaking } U(1)_L, \eta_1 \neq \eta_2 \text{ and } N_L \neq \bar{N}_R. \text{ Nevertheless, lepton parity } \pi_L \text{ is still conserved, hence also dark parity } \pi_D = \pi_L(\text{e}^{-1})^j \text{ as remarked earlier. Note that this happens for any odd } y, \text{ pointing to the generality of the } U(1)_L \text{ approach.}

In the above example, the dark symmetry is } \pi_D. \text{ If } U(1)_D \text{ is desired, then it has to be imposed as in Ref. [2]. To insist on obtaining } U(1)_D \text{ without imposing it from the beginning, using only } U(1)_L, \text{ the following variation may be considered, as shown in Table 2. The analogous one-loop diagram is shown in Fig. 2. Again } U(1)_L \text{ is broken by the soft } \bar{E}_L E_R \]

![](image)

Table 2: Fermion and scalar content of second example.

| fermion/scalar | \(SU(2)\) | \(U(1)_Y\) | \(U(1)_L\) |
|---------------|---------|---------|---------|
| \((\nu, e)_L\) | 2       | \(-1/2\) | 1       |
| \(e_R\)       | 1       | \(-1\)   | 1       |
| \(E_L\)       | 1       | \(-1\)   | \(x_1 \neq -1\) |
| \(E_R\)       | 1       | \(-1\)   | \(x_2 \neq 1\) |
| \(\Phi = (\phi^+, \phi^0)\) | 2       | \(1/2\)  | 0       |
| \(\eta_1 = (\eta_1^0, \eta_1^-)\) | 2       | \(-1/2\) | \(y \neq 0\) |
| \(\eta_2 = (\eta_2^+, \eta_2^-)\) | 2       | \(3/2\)  | \(-y\)  |
| \(\chi^0\)    | 1       | 0       | \(-y\)  |

Figure 2: Scotogenic Majorana neutrino mass with \(U(1)_D\).

mass term by two units, but now a dark \(U(1)_D\) symmetry remains for \(\eta_1, E, \eta_2^0\). On the other hand, \(\eta_1^0\) is unsuitable as a dark-matter candidate because it couples to the \(Z\) boson.
and thus ruled out by underground direct search experiments. This model (without $\chi^0$) was considered previously in Ref. [4], but dark parity was imposed there and lepton number was said to be broken by the dimension-four term $(\Phi^\dagger \eta_1)(\Phi^\dagger \eta_2)$, without realizing that its correct implementation is softly broken $U(1)_L$ and that a dark $U(1)_D$ symmetry remains. Here, the added complex scalar singlet $\chi^0$ is a viable dark-matter candidate, providing that the $\chi^0(\eta_1^0 \phi^0 - \eta_1^- \phi^+)$ coupling is suitably small. Again, any odd $y$ works, with $x_1 = y - 1, x_2 = y + 1$. In the special case $y = 1$, the additional term $\chi^0 \bar{E}_L E_R$ is allowed, with further possible interesting phenomenology. This second example shows the power of $U(1)_L$ in combination of the chosen particle content in acquiring radiative Majorana neutrino mass together with a dark symmetry.

A third example uses a scalar triplet and a singlet, so that the soft bilinear (and trilinear) scalar terms are absent to begin with. Hence there is no constraint on $y$ at this stage. However, the dark fermions must now be doublets and they can form bilinear terms with the SM lepton doublets. To prevent their existence, $y$ must again be odd as shown below. This

| fermion/scalar | $SU(2)$ | $U(1)_Y$ | $U(1)_L$ |
|---------------|---------|---------|---------|
| $(\nu, e)_L$  | 2       | $-1/2$  | 1       |
| $e_R$         | 1       | $-1$    | 1       |
| $(N, E)_L$    | 2       | $-1/2$  | $x_1 \neq 1$ |
| $(N, E)_R$    | 2       | $-1/2$  | $x_2 \neq 1$ |
| $\Phi = (\phi^+, \phi^0)$ | 2 | $1/2$ | 0 |
| $\rho = (\rho^+, \rho^0, \rho^-)$ | 3 | 0 | $y$ |
| $\chi^+$      | 1       | 1       | $-y$    |

Table 3: Fermion and scalar content of third example.

model was one of the compilations considered in Refs. [5, 6]. Again their basic assumption was to have dark parity to begin with, in which case the scalar triplet $\rho$ may be chosen real. Here only $U(1)_L$ is used, which requires $x_1 = y - 1, x_2 = y + 1$ as in the previous two examples. The soft term $\bar{E}_L E_R$ again breaks $U(1)_L$ by two units, whereas the soft term $\bar{e}_L E_R$
would break it by $y$, so again $y$ must be odd to allow the latter to be absent. Since $\rho^0$ has no coupling to $Z$, it is a viable dark-matter candidate in this case and the dark symmetry $U(1)_D$ emerges as a consequence of softly broken $U(1)_L$ together with the chosen particle content.

**Scotogenic Dirac Neutrino Mass**: The same idea of using $U(1)_L$ may be applied to Dirac neutrinos. Assuming that $U(1)_L$ comes from gauged $B - L$, there have been three recent studies \[7, 8, 9\]. The following analysis shares many of their methods and results, but with an important difference. Whereas they consider only dimension-five operators for obtaining radiative Dirac neutrino masses, the adopted procedure here is to use dimension-four operators with softly broken $U(1)_L$, i.e. the imposition of $U(1)_L$ to forbid the tree-level mass, but to allow a radiative mass to appear from dimension-two and/or dimension-three terms which break $U(1)_L$, so that a dark symmetry emerges as well.

To have a Dirac neutrino mass, the right-handed singlet neutrino $\nu_R$ must exist. It should pair up with $\nu_L$ through the SM Higgs boson $\phi^0$. Hence it should have $L = 1$ under $U(1)_L$. In that case, a tree-level Yukawa coupling is allowed which must however be very small to account for the observed neutrino mass limit of 1.1 eV \[10\]. To forbid this tree-level coupling, a symmetry is routinely applied to distinguish $\nu_R$ from the other SM particles, but $L = 1$ is retained. For a short review, see Ref. \[11\]. A generic one-loop diagram is depicted in Fig. 3, with its particle content shown in Table 4.

![Figure 3: Scotogenic Dirac neutrino mass.](image-url)
Table 4: Fermion and scalar content for scotogenic Dirac neutrino mass.

| fermion/scalar | $SU(2)$ | $U(1)_Y$ | $U(1)_L$ | * | $Z^L_n$ | $Z^D_n$ |
|---------------|---------|----------|----------|---|---------|---------|
| $(\nu, e)_L$ | 2       | $-1/2$   | 1        | 1 | $\omega$ | 1       |
| $e_R$        | 1       | $-1$     | 1        | 1 | $\omega$ | 1       |
| $\nu_R$      | 1       | 0        | $x$      | $-n+1$ | $\omega$ | 1       |
| $N_L$        | 1       | 0        | $y$      | $2-n$ | $\omega^2$ | $\omega$ |
| $N_R$        | 1       | 0        | $y$      | $2-n$ | $\omega^2$ | $\omega$ |
| $\Phi = (\phi^\pm, \phi^0)$ | 2 | $1/2$ | 0 | 0 | 1 | 1 |
| $\eta = (\eta^+, \eta^0)$ | 2 | $1/2$ | $y-1$ | $1-n$ | $\omega$ | $\omega$ |
| $\chi^0$    | 1       | 0        | $y-x$   | 1 | $\omega$ | $\omega$ |

Here $x \neq 1$ is imposed so that $\nu_R$ does not couple to $\nu_L$ at tree level. To connect them in one loop, the trilinear $\bar{\eta}^0 \phi^0 \chi^0$ term must break $U(1)_L$ softly by $x - 1$. Now $N_L$ and $N_R$ are assumed to have the same $U(1)_L$ charge, i.e. $y$, so that $y \neq \pm 1$ and $y \neq \pm x$ are required. Furthermore, $N_L N_L$ or $N_R N_R$ would break $U(1)_L$ by $2y$, $\nu_R \nu_R$ by $2x$, $N_R \nu_R$ by $x + y$, $\bar{N}_L \nu_R$ by $x - y$, and $\chi^0 \chi^0$ by $2(y-x)$. To allow them to be absent in a complete theory, their $U(1)_L$ charges must not be zero, or divisible by the required $U(1)_L$ breaking, i.e. $x - 1$. To have a solution, $x$ and $y$ must be chosen so that the residual symmetry after $U(1)_L$ breaking maintains an effective lepton symmetry together with a dark symmetry.

Let $x = -n + 1$, then $U(1)_L$ breaks to $Z_n$. If for example $n = 3$ [2], it would be impossible for neutrinos to be Majorana, i.e. they must remain Dirac as shown in Ref. [12].

The structure of Fig. 3 is well-known [13, 14]. It is realized conventionally by 3 symmetries: (A) conventional lepton number, where $\nu_{L,R}, N_{L,R}$ have $L = 1$, and $\Phi, \eta, \chi$ have $L = 0$, which is strictly conserved; (B) dark $Z_2$ symmetry, under which $N_{L,R}, \eta, \chi$ are odd and others even, which is strictly conserved; and (C) an ad hoc $Z_2$ symmetry under which $\nu_R, \chi$ are odd and all others even, which is softly broken by the $\bar{\eta}^T \Phi \chi$ term. In previous applications, $\chi$ is assumed to be a real neutral scalar singlet for simplicity. Here it is crucial that it is complex to carry the nonzero $U(1)_L$ charge $y - x$. 

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In Table 4, in the column denoted by $\ast$, the $U(1)_L$ charges are chosen explicitly. The terms $\bar{N}_L\nu_R$ and $\chi^0\chi^0$ have $U(1)_L$ charges $-1$ and $2$. They are not zero or divisible by $n \geq 3$. The terms $\nu_R\nu_R$, $N_R\nu_R$ and $N_RN_R$ should also be absent, their $U(1)_L$ charges divided by $n$ are $(2/n) - 2$, $(3/n) - 2$ and $(4/n) - 2$, hence $n = 3, 4$ are ruled out. All higher values of $n$ are acceptable. The resulting theory allows two related symmetries: (I) $Z_n^L$ lepton symmetry under which $\nu_{L,R}, e_{L,R}, \eta, \chi \sim \omega$ and $N_{L,R} \sim \omega^2$, where $\omega^n = 1$; (II) $Z_n^D$ dark symmetry, derivable from lepton symmetry by multiplying the latter by $\omega^{-2j}$ where $j$ is the particle’s spin. As a result, $\nu_{L,R}, e_{L,R} \sim 1$ and $N_{L,R}, \eta, \chi \sim \omega$. This is the Dirac generalization of the Majorana case of the derivation of dark parity $\pi_D$ from lepton parity $\pi_L$ first pointed out in Ref. [1].

In a renormalizable theory, $Z_n$ symmetry is not simply realizable for large $n$ because the Lagrangian admits only terms of dimension four or less. In the above example for $n \geq 5$, the Lagrangian cannot admit the term $(\chi^0)^n$, hence the true symmetry of the theory is a redefined $U(1)_L$, where $\nu_{L,R}, e_{L,R}, \eta, \chi \sim 1$ and $N_{L,R} \sim 2$. The dark symmetry is then $U(1)_D$ where it is derived from $U(1)_L$ by subtracting from the latter $2j$ where $j$ is the particle’s spin, i.e. $\nu_{L,R}, e_{L,R} \sim 0$ and $N_{L,R}, \eta, \chi \sim 1$. This shows that scotogenic Dirac neutrino mass is derivable from softly broken $U(1)_L$ alone with emergent $U(1)$ lepton and dark symmetries.

If $Z_n$ symmetry is desired, the scalar sector must be extended. For example, if $n = 5$, in addition to $\chi \sim \omega$, another scalar $\sigma \sim \omega^3$ may be added. The coexisting terms $\chi^3\sigma^* + \chi^2\sigma$ would then enforce $Z_5^D$ as a dark symmetry, but the lepton symmetry would become global $U(1)$ as conventionally defined, i.e. $L = 1$ for $\nu_{L,R}$ and $N_{L,R}$. To enforce $Z_5^L$ as a lepton symmetry, a third scalar $\kappa$ may be added with $U(1)_L$ charge $= 7$. In that case, the terms $\chi\sigma^2\kappa^* + \kappa N_R\nu_R$ would allow $\nu_{L,R}, \chi, \eta \sim \omega$ and $N_{L,R}, \kappa \sim \omega^2$ under $Z_5^L$ with $\omega^5 = 1$.

Since $\chi^0$ mixes with $\eta^0$, the neutral scalar of the dark sector of this model requires this mixing to be very small and the lighter eigenstate to be mostly $\chi^0$ for it to be a viable
dark-matter candidate. Alternatively the lightest $N$ may also be chosen as dark matter. For details, see Ref. [14].

A possible variation of the model is to add a neutral scalar singlet $\zeta$ with $U(1)_L$ charge $n$, and require that $U(1)_L$ be spontaneously broken only. In that case, the term $\eta^i\Phi\chi$ is replaced with $\zeta^*\eta^i\Phi\chi$. Neutrinos obtain radiative Dirac masses as before, with new emergent $U(1)$ lepton and dark symmetries, but now a massless Goldstone boson appears. It is the analog of the majoron which comes from breaking $U(1)_L$ spontaneously to $Z_2$ and is applicable to Majorana neutrinos, whereas here it is the massless diracon [15] which comes from breaking $U(1)_L$ spontaneously to $Z_n$ ($n \geq 5$) and is applicable to Dirac neutrinos.

There is a further use of $\zeta$, if it is allowed to couple anomalously to a pair of exotic quarks (color fermion triplets) or a color fermion octet [16, 17]. Then this diracon becomes a QCD (quantum chromodynamics) axion and $U(1)_L$ is an extended version of Peccei-Quinn symmetry, as proposed many years ago [18, 19] for Majorana neutrinos, and very recently also for Dirac neutrinos [20, 21]. In these scenarios, dark matter consists of both the axion and a WIMP (weakly interacting massive particle) [22].

In Fig. 3, the fermion singlets $N_{L,R}$ may be replaced with the doublets $(E^0, E^-)_{L,R}$ as shown in Table 5. This construction eliminates the existence of many fermion bilinears

| fermion/scalar | $SU(2)$ | $U(1)_Y$ | $U(1)_L$ | $**$ | $L$ | $Z_3^D$ |
|----------------|---------|----------|---------|------|-----|--------|
| $(\nu, e)_L$  | 2       | $-1/2$   | 1       | 1    | 1   | 1      |
| $e_R$          | 1       | $-1$     | 1       | 1    | 1   | 1      |
| $\nu_R$        | 1       | 0        | $x$     | $-2$ | 1   | 1      |
| $(E^0, E^-)_L$ | 2       | $-1/2$   | $y$     | 2    | 1   | $\omega$ |
| $(E^0, E^-)_R$ | 2       | $-1/2$   | $y$     | 2    | 1   | $\omega$ |
| $\Phi = (\phi^+, \phi^0)$ | 2 | $1/2$ | 0 | 0 | 0 | 1 |
| $\eta = (\eta^+, \eta^0)$ | 2 | $1/2$ | $x-y$ | $-4$ | 0 | $\omega^{-1}$ |
| $\chi^0$       | 1       | 0        | $y-1$   | 1    | 0   | $\omega$ |

Table 5: Fermion and scalar content for scotogenic $Z_3$ Dirac neutrino mass.
except $\nu_R\nu_R$ and $\bar{\nu}_L E^0_R + e^+_L E^-_R$. Hence only $2x$ and $y - 1$ must not be zero or divisible by $x - 1$. Also $y \neq x$ is required. As a result, it is possible to have $Z_3$ dark symmetry, i.e. $x = -2$ and $y = 2$, as shown in the column denoted by **. The analogous one-loop diagram for scotogenic Dirac neutrino mass is shown in Fig. 4. The $U(1)_L$ symmetry is broken to $Z_3$

\begin{equation}
\phi^0 \\
\chi^0 \\
\eta^0 \\
\nu_L \\
E^0_R \\
E^0_L \\
\nu_R
\end{equation}

Figure 4: Scotogenic Dirac neutrino mass with $Z_3$ dark symmetry.

by the soft trilinear scalar terms $\Phi^\dagger \eta \chi$ and $\chi^3$. However, the dimension-four term $\chi^0 \nu_R \nu_R$ which is allowed by $Z_3$ is not allowed by the original $U(1)_L$. Hence the breaking does not affect the fermions of this model and the conventional assignment of $L = 1$ may be applied to $\nu_{L,R}, E^0_{L,R}$ with $L = 0$ for all the scalars. The $Z_3^D$ dark symmetry emerges as before.

In this example, the $U(1)_L$ global symmetry is anomalous. To make it anomaly-free so that it can be promoted to a gauge symmetry, the three copies of $\nu_R$ with charge $-2$ should be changed to 1, as in the conventional assignments for gauge $B - L$. The difference is then $3(1) - 3(-2) = 9$ for the sum of $U(1)_L$ charges and $3(1) - 3(-8) = 27$ for the sum of the cubes of the charges. A complete renormalizable anomaly-free gauge $U(1)_L$ theory is then possible with the following additional particle content.

Singlet right-handed fermions $\psi_{2,3,4}$ are added with $U(1)_L$ charges $-2, 3, -4$ respectively. Let there be 3 copies each of $\psi_{2,4}$ and 9 copies of $\psi_3$. Then $3(-2 - 4) + 9(3) = 9$ and $3(-8 - 64) + 9(27) = 27$, satisfying the requirement for the theory to be anomaly-free. To break the gauge $U(1)_L$ symmetry to $Z_3$, the scalar singlets $\zeta_{3,6}$ with charges $3, 6$ are used, so that the terms $\chi^3 \zeta^*_3, \zeta^2_3 \zeta^*_6, \psi_3 \psi_3 \zeta^*_6$, and $\psi_2 \psi_4 \zeta_6$ are allowed in the complete Lagrangian, ensuring
that all new fermions acquire nonzero masses. The new fermions $\psi_{2,3,4}$ have $L = 1, 0, -1$ and transform trivially under $Z_3^D$. In addition $\psi_3$ has its own accidental (or predestined) $Z_2$ symmetry from the chosen particle content of the theory and the imposed $U(1)_L$ symmetry. The Dirac neutrino mass matrix is now $6 \times 6$ with 3 tree-level masses and 3 one-loop masses, the latter linking only to the left-handed SM neutrinos. However, there could be mixing between the two sectors which may be a source of nonunitarity of the observed $3 \times 3$ neutrino mixing matrix.

**Conclusion**: The intrinsic connection between lepton number and dark symmetry has been demonstrated with three examples in the case of Majorana neutrinos where $U(1)_L$ is broken softly to $Z_2$ lepton parity $\pi_L$. In the first example, $Z_2$ dark parity $\pi_D = \pi_L(-1)^{2j}$ emerges and scotogenic Majorana neutrino mass is obtained. In the second and third examples, by choosing different particles in the dark sector, a dark $U(1)_D$ symmetry is maintained.

Using the same connection, two examples of scotogenic Dirac neutrino mass have also been described, one with emergent $Z_n$ lepton and dark symmetry for $n \geq 5$. However, without enlarging the necessary scalar sector, the requirement of renormalizability of these models implies that the true symmetry of the Lagrangian is a redefined $U(1)_L$ such that a dark $U(1)_D$ is obtained by subtracting the assigned lepton number of a particle by $2j$ where $j$ is the particle’s spin.

The other example chooses a different set of dark fermions so that $Z_3$ dark symmetry emerges which is maintained explicitly by the renormalizable Lagrangian of the model. It also sustains a conserved lepton number in the conventional way.

These two examples generalize the case of $Z_2$ lepton and dark parity [1] for Majorana neutrinos to Dirac neutrinos.

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