Improved cosmological parameter constraints from CMB and $H(z)$ data

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Received 10 July 2008
Accepted 5 October 2008
Published 22 October 2008

Abstract. We discuss the cosmological degeneracies among the Hubble parameter $H(z)$, the age of the Universe and cosmological parameters describing simple variations from the minimal $\Lambda$CDM (CDM: cold dark matter) model. We show that independent determinations of the Hubble parameter $H(z)$ such as those recently obtained from ages of passively evolving galaxies, combined with cosmic microwave background data (Wilkinson Microwave Anisotropy Probe five-year data), provide stringent constraints on possible deviations from the $\Lambda$CDM model. In particular we find that this data combination constrains at the 68% (95%) confidence limit the following parameters: the sum of the neutrino masses $\sum m_\nu < 0.5$ (1.0) eV, the number of relativistic neutrino species $N_{\text{rel}} = 4.1^{+0.4}_{-0.3} \pm 0.15$, the dark energy equation of state parameter $w = -0.95 \pm 0.17$ (±0.32), and the curvature $\Omega_k = 0.002 \pm 0.006$ (±0.014), in excellent agreement with data set combinations involving the cosmic microwave background, supernovae and baryon acoustic oscillations. This offers a valuable consistency check for systematic errors.

Keywords: CMBR experiments, galaxy evolution

ArXiv ePrint: 0807.0039 [astro-ph]
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1. Introduction

The recent measurements of cosmic microwave background (CMB) anisotropies and polarization [2, 3], alone or in combination with other cosmological data sets, have provided confirmation of the standard cosmological model and an accurate determination of some of its key parameters.

In particular, the new determination of the age of the Universe, \( t_0 = 13.68 \pm 0.13 \) Gyr, improves by an order of magnitude on previous determinations from, e.g., cosmochronology of long-lived radioactive nuclei [4] and population synthesis of the oldest stellar populations [5]–[7] and by a factor of 2 on previous determinations from CMB data.

With cosmological parameters so tightly constrained within the framework of the standard flat \( \Lambda \)CDM model, it is important however to constrain possible deviations from the standard cosmological model. Beyond the primordial parameters describing the shape of the primordial power spectrum and late-time parameters such as the optical depth to the last scattering surface, CMB observations already constrain directly parameters such as [8] the angular size distance to the last scattering combined with the sound horizon at decoupling, the baryon-to-photon ratio and the redshift of matter–radiation equality.

This implies that, for models beyond the standard flat \( \Lambda \)CDM, CMB data alone still show large degeneracies among ‘derived’ cosmological parameters such as the matter density parameter \( \Omega_m \), the curvature \( \Omega_k \), the dark energy equation of state parameter \( w \), the effective number of relativistic neutrino species \( N_{\text{eff}} \), the sum of neutrino masses \( \sum m_{\nu} \) and the Hubble parameter \( H_0 \). For example (see e.g., [9]–[11]), departures from the standard model described by a deviation from three neutrino species can arise from the decay of dark matter particles [12]–[15], quintessence [16], exotic models [17], and additional hypothetical relativistic particles. This affects the matter–radiation equality yielding, even for a flat, cosmological constant-dominated model, degeneracies among \( N_{\text{eff}}, H_0 \) and \( \Omega_m \). A departure from dark energy being described by a cosmological constant (i.e. a component with equation of state \( w \neq -1 \)) yields a different angular size distance to the last scattering, and thus degeneracies among \( w, H_0 \) and \( \Omega_m \) even for

\[^4\] Note that the name ‘derived parameters’ has sometimes been used in the literature with a slightly different emphasis, denoting parameters such as the bias \( b \), \( \sigma_8 \), etc.
a flat Universe. Finally, relaxing the flatness assumption yields the so-called ‘geometric degeneracies’ (among age or $H_0$ and $\Omega_m$ and $\Omega_\Lambda$).

In order to go beyond the concordance $\Lambda$CDM model parameter determination, one needs extra data sets that probe different physics and are affected by different systematics. In this work we concentrate on the measurements of $H(z)$ using passively evolving red-envelope galaxies and how using them helps to constrain cosmological parameters, dropping the assumption of the concordance model. In particular we show that the recent determinations of $H_0$ from the HST Key Project [18] and $H(z)$ provided by [1] (SVJ; based on [19] and references therein) can provide, when combined with CMB and other cosmological data, new and tighter constraints on deviations from the standard $\Lambda$CDM model, as first shown in [11,19]. This approach of combining different data sets to constrain parameters that are otherwise poorly constrained is called a ‘concordance approach’. While it is a very powerful approach, the same ‘concordance’ approach is used to test data sets for systematic errors. It is therefore important to consider enough data sets to have an overconstrained problem and as diverse data sets as possible, relying on different physics and affected by different systematics. Only in this case, if all data sets agree, can one be confident that the systematic errors are safely below the statistical errors and that the cosmological constraints are robust.

After obtaining constraints on deviations from the simple $\Lambda$CDM model obtained with WMAP five-year data and $H(z)$ measurements, we compare them with those obtained from the combination of WMAP five-year data with baryon acoustic oscillation and supernovae data. We find good agreement between the two approaches. We conclude that any possible systematic effect in the non-CMB data sets is below the statistical errors, and that there is no evidence for a deviation from the flat $\Lambda$CDM model, thus offering support to the standard cosmological model.

2. Data analysis: method

The method for extracting cosmological parameters from the different data sets that we adopt is based on the publicly available Markov chain Monte Carlo package cosmomc [20] and the sampling of the posterior distribution given by Monte Carlo Markov chains released with the WMAP five-year data [3]. The standard $\Lambda$CDM model is described by the following set of cosmological parameters: the physical baryon and CDM densities, $\omega_b = \Omega_b h^2$ and $\omega_c = \Omega_c h^2$, the density parameter of dark energy, $\Omega_\Lambda$, the scalar spectral index, $n_s$, and amplitude, $A_s$, and the optical depth to reionization, $\tau$.\footnote{We marginalize over the SZ amplitude parameter as was done by the WMAP team [3].} For all these parameters the chosen boundaries of the priors do not affect the cosmological constraints. We consider deviations from this model described by the addition of a single extra parameter. The models which show significant degeneracies among $H(z)$ and the additional parameters are: models where we add the possibility of having an extra background of relativistic particles (parametrized by $N_{\text{eff}} \neq 3.04$), or where we fix the effective number of neutrinos to $N_{\text{eff}} = 3.04$ but allow them to have significant non-zero mass $\sum m_\nu \neq 0$; models where we consider the possibility of a (constant) dark energy equation of state $w \neq -1$; and finally models with non-flat geometry $\Omega_k \neq 0$.\footnote{We marginalize over the SZ amplitude parameter as was done by the WMAP team [3].}
We then study how determinations of the rate of expansion $H(z)$ can constrain these deviations. We consider the Hubble Key Project determination of the Hubble constant [18] (HST) and the determination of the redshift dependence of the Hubble parameter $H(z)$ from observations of passively evolving galaxies [1] (SVJ). This combination (WMAP5 + HST + ages) is referred to as ‘WMAP5 + H’.

Finally, we also consider a model which deviates from the standard ΛCDM by the addition of two parameters: $\Omega_k$ and $w$, and investigate how the $H(z)$ data set helps break the CMB only degeneracy.

To conclude, we compare these constraints to those obtained with the combination of WMAP five-year data with supernovae and baryon acoustic oscillations [21, 22]. This combination is referred to as ‘WMAP5 + SN + BAO’.

### 2.1. $H(z)$ determination

An important observable for constraining cosmological parameters is a direct measurement of the Hubble parameter $H(z) = (\dot{a}/a)$, as this measures directly the expansion rate of the Universe at a given redshift. For example, $H(z)$ gives a more direct measurement of the equation of state of dark energy than the angular diameter distance $d_A(z)$ or the luminosity distance $d_L(z)$. This is easy to see by recalling that, adopting a FRW metric, using Einstein’s equations and considering a flat Universe composed of matter and dark energy with equation of state $p_Q = w_Q(z)\rho_Q$, $H^2 = H_0^2[\rho_T(z)/\rho_T(0)]$ and thus

$$\frac{H(z)}{H_0} = (1 + z)^{3/2} \left[ \Omega_M(0) + \Omega_Q(0) \exp \left[ 3 \int_0^z \frac{dz'}{1 + z'} w_Q \right] \right]^{1/2}, \quad (1)$$

where the subscripts $Q$, $M$ and $T$ refer respectively to the dark energy, the matter, and the total contents. The quantities $d_A(z)$ and $d_L(z)$ are related to $H(z)$ via $d_A(z)(1 + z) = d_L(z)/(1 + z) = \int_0^z d z'/H(z)$.

While some of the current constraints on the dark energy equation of state parameter $w_Q(z)$ are based on integrated measurements of $H(z)$ (like the angular diameter distance), other observables have already provided direct measurements of $H(z)$, like the determination of the star populations of luminous red galaxies [1]. Other techniques that can provide a direct measurement of $H(z)$ are using the power spectrum of the peculiar velocities, as measured, for example, from the KSZ effect [23, 24], or the baryonic acoustic oscillation (BAO) scale in the radial direction [25, 26]. The BAO technique has recently received renewed attention because of its potential for providing a standard ruler at different redshifts, and because of its robustness to systematic effects; it is thus considered a powerful method for determining the nature of dark energy.

In this paper we constrain cosmological parameters for models beyond the standard ΛCDM, using the $H(z)$ determinations provided by [1], obtained from the study of the evolution of the star populations in massive ($>2.2 L_\ast$) luminous red galaxies (LRG). Recent studies [27, 28] have clearly established that massive LRG have formed more than 95% of their stars at redshifts higher than 4. These galaxies, therefore, form a very uniform population, whose stars are evolving passively after the very first short episode of active star formation [29, 30]. Because the stars evolve passively, these massive LRG are excellent cosmic clocks, i.e. they provide a direct measurement of $dt/dz$; the observational evidence discards further star formation activity in these galaxies. Dating of the stellar population
can be achieved by modeling the integrated light of the stellar population using synthetic stellar population models, in a similar way to what is done for open and globular clusters in the Milky Way. The dating of the stellar population needs to be done on the integrated spectrum because individual stars are not resolved and therefore the requirements on the observed spectrum are stringent, as one needs a very wide wavelength coverage, spectral resolution and very high signal-to-noise ratio, \( S/N \). In [28], it has been shown that the spectra of these massive LRG at a redshift \( z \sim 0.15 \) are extremely similar, with differences of only 0.02 mag, which is further evidence of the uniformity of the stellar populations in these galaxies. There have already been examples of accurate dating of the stellar populations in LRGs [1, 6, 19, 31] where it has been shown that galaxy spectra with sufficient wavelength coverage (the UV region is crucial), wavelength resolution (about 3 Å) and high enough \( S/N \) (at least 10 per resolution element of 3 Å) can provide sensible constraints on cosmological parameters. The resulting \( H(z) \) data obtained from these studies, and used in this paper, are the points with error bars shown in figure 1. The interested reader can find more details in [1, 19].

3. Results and conclusions

Measurements of \( H(z) \) constrain the age of the Universe at different redshifts and thus break the CMB only degeneracies among the age of the Universe today \( (t_0) \) and the parameters describing deviations from the \( \Lambda \)CDM model. As shown in table 1, the age of the Universe as constrained by WMAP5 only data is very sensitive to the presence of some of these parameters, especially to the possibility of having a background of \( N_{\text{eff}} \) relativistic
Figure 2. Left: constraints on the total mass of relativistic neutrinos from WMAP five-year data alone (dotted line), WMAP5 + HST (dashed line) and WMAP5 + H (solid line). The total sum of the neutrino masses, $\Sigma m_\nu$, is constrained to be below 0.48 (0.93) eV at 68% (95%) confidence level, by the combination WMAP5 + H. Right: constraints on the effective number of relativistic neutrino species from WMAP five-year data alone (dotted line), WMAP5 + HST (dashed line) and WMAP5 + H (solid line). The effective number of neutrino species is constrained to be $N_{\text{rel}} = 4.1^{+0.4}_{-0.9} (^{+1.1}_{-1.5})$ at the 68% (95%) confidence level, by the combination WMAP5 + H. The WMAP five-year data only constraint has a hard prior $N_{\text{eff}} < 10$ imposed. Adding HST or H constraints makes the determination insensitive to the prior.

Table 1. Determination of the age of the Universe (68.3% c.l.) for several different cosmological models for WMAP five-year data alone and WMAP5 + H data.

| Age (Gyr) | $\Lambda$CDM | $\Lambda$CDM + $N_{\text{eff}}$ | $\Lambda$CDM + $\sum m_\nu$ | $\Lambda$CDM + $\Omega_k$ wCDM |
|----------|---------------|----------------------|-----------------------------|-------------------------------|
| WMAP five-year data | 13.69 ± 0.13 | 12.08 ± 1.29 | 14.06 ± 0.27 | 16.32 ± 1.76 |
| WMAP5 + H | 13.65$^{+0.14}_{-0.10}$ | 12.87$^{+0.61}_{-0.31}$ | 13.81$^{+0.24}_{-0.14}$ | 13.61$^{+0.29}_{-0.44}$ |

particles (with $N_{\text{eff}}$ not fixed to 3.04) or allowing a non-zero curvature (see the first row, third and fifth columns of table 1). This is because many models which deviate from the standard $\Lambda$CDM but are consistent with CMB data are not a good fit to the $H(z)$ data. Some illustrative examples are shown in figure 1. The combination WMAP5 + H significantly reduces the degeneracies among $t_0$ and some of the ‘extra’ parameters, thus improving the constraints on the age of the Universe in those models by, for example, almost a factor of 3 for the case with $N_{\text{eff}}$ not fixed to 3.04, and almost a factor of 5 for the case of non-zero curvature (see the same columns but the second row in table 1).

In figure 2 we explore the resulting constraints on the neutrino properties. In all cases the dotted line shows the WMAP five-year data only result, the dashed line is for WMAP5 + HST and the solid line is for WMAP5 + H. We find that the combination WMAP5 + H constrains the sum of neutrino masses to be $\sum m_\nu < 0.48$ eV and <0.93 eV at the 68% and 95% confidence levels, respectively, thus improving the WMAP only constraints by 50%. The constraint on the effective number of neutrino species is $N_{\text{rel}} = 4.1^{+0.4}_{-0.9} (^{+1.1}_{-1.5})$ at the 68% (95%) confidence level and $N_{\text{rel}} > 2.2$ at better than
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Figure 3. Left: constraints on the curvature of a ΛCDM model from WMAP five-year data alone (dotted line), WMAP5 + HST (dashed line) and WMAP5 + H (solid line). With the $H(z)$ measurements, the curvature is constrained to $0.002 \pm 0.006$ (±0.014) at the 68% (95%) confidence level. The WMAP five-year data only line shows the well known geometric degeneracy. Right: constraints on the dark energy equation of state parameter from WMAP five-year data alone (dotted line), WMAP5 + HST (dashed line) and WMAP5 + H (solid line). With the $H(z)$ measurements we obtain $w = -0.95 \pm 0.17$ (±0.32) at the 68% (95%) confidence level.

The left panel of figure 3 shows the constraints on the geometry of the Universe. The WMAP5 + H combination yields $\Omega_k = 0.002 \pm 0.006$ (±0.014) at the 68% (95%) confidence levels, thus breaking the geometric degeneracy.

In the right panel of figure 3, we report the constraints on the dark energy equation of state parameter (assumed constant). The WMAP5 + H combination yields $w = -0.95 \pm 0.17$ (±0.32) at the 68% (95%) confidence level, which improves the WMAP five-year data only constraints by a factor $\sim 70\%$. While the WMAP five-year data constraint has a hard prior on the Hubble constant $H_0 < 100$ km s$^{-1}$ Mpc$^{-1}$ which imposes a lower limit on $w$, the WMAP5 + HST and WMAP5 + H combinations are insensitive to this prior.

In table 2 we compare the WMAP5 + H constraints on deviations from the ΛCDM model, with those obtained by the combination of WMAP five-year data with baryon acoustic oscillation data (BAO) [22] and with supernovae data as obtained by [21].

Finally, we consider a model which deviates from the standard ΛCDM in two parameters: curvature is allowed to vary and the equation of state of dark energy (allowed
Figure 4. Left: constraints in the $\Omega_k$–$w$ plane from WMAP five-year data alone (purple), WMAP5 + H (blue 68% and 95% c.l.). For comparison we show (right) the WMAP team’s WMAP five-year data only constraints (black), WMAP5 +BAO (yellow) and WMAP5 + supernovae (red; see [21]). The differences in the WMAP five-year data only constraints are due to different choices of priors. Most notably, different boundaries on the $H_0$ prior are used: $0.4 < h < 1$ (left) versus $h < 1$ (right), and on $w$ (on the left panel there is an additional prior $w < -0.3$).

Table 2. Cosmological constraints at 68 % (95%) c.l. on the extra parameters characterizing deviations of the standard $\Lambda$CDM model, comparing their values as extracted from WMAP five-year data only, WMAP5 + BAO + SN and WMAP5 + H.

| Parameter | WMAP five-year data only | WMAP5 + BAO + SN | WMAP5 + H |
|-----------|--------------------------|------------------|-----------|
| $N_{\text{eff}}$ | $>2.3$ (95%) | $4.4^{+1.5}_{-1.5}$ a | $4.10^{+0.37}_{-0.94}^{+1.12}_{-1.50}$ |
| $\sum m_{\nu}$ | $>1.3$ eV (95%) | $<0.61$ eV (95%) | $<0.93$ eV (95%) |
| $w$ | $>-2.37$ (95%) | $-0.972^{+0.061}_{-0.060}^{+0.112}_{-0.138}$ | $-0.945^{+0.194}_{-0.155}^{+0.311}_{-0.350}$ |
| $\Omega_k$ | $>-0.68$ (95%) | $-0.0052^{+0.0064}_{-0.0064}^{+0.0137}_{-0.0123}$ | $0.002^{+0.0059}_{-0.0059}^{+0.012}^{+0.018}_{-0.018}$ |

a With HST prior.

to cluster) is assumed to be constant but not fixed to $w = -1$. When running the WMAP five-year data only Markov chain we use priors different to those used in [3], the most important differences being as regards $h$ and $w$: we use $0.4 < h < 100$ and $-2.5 < w < -0.3$, and a flat prior on the angular size distance to the last scattering surface rather than a flat prior on $\Omega_\Lambda$. In figure 4 we show how the addition of $H(z)$ data helps break the degeneracies. For comparison, in the right hand panel of figure 4 we show the constraints obtained by [21] from the combination WMAP5 + BAO and WMAP5 + SN. As already noted by e.g. [33]–[35], measurements of $H(z)$ are crucial for breaking degeneracies among the curvature and dark energy properties.
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We conclude that the addition of Hubble parameter determinations at different redshifts breaks the CMB only degeneracies arising in models that allow deviations from the simple flat $\Lambda$CDM model. We find constraints on the number of effective neutrino species, the sum of neutrino masses, the curvature of the Universe and the equation of state parameter for dark energy. These constraints are comparable to those obtained from the combination of WMAP five-year data with supernovae and baryon acoustic oscillation data [21]. This ‘concordance’ approach shows that systematic errors in non-CMB data sets are smaller than the statistical errors and offers further support to the simple flat $\Lambda$CDM model. Finally note that future BAO surveys will have the potential to constrain $H(z)$ with % accuracy in several $z$ bins, i.e. they will constrain not only dark energy models but other deviations from the standard $\Lambda$CDM model, as explored in this paper.

Acknowledgments

We would like to thank Troels Haugboelle and Hiranya Peiris for useful comments on this work and help with MCMCs, and A Melchiorri for comments on the manuscript. We also thank the anonymous referee for comments that greatly improved the presentation of the material. We acknowledge the use of the Legacy Archive for Microwave Background Data Analysis (LAMBDA). Support for LAMBDA is provided by the NASA Office of Space Science. DGF is supported by a FPU contract, with ref. AP2005-1092. LV acknowledges the support of FP7-PEOPLE-2007-4-3-IRGn202182 and CSIC I3 grant 2007501034. RJ is supported by a CSIC I3 grant and FP7-PEOPLE-2007-IRG. This work was supported in part by the Spanish Ministry of Education and Science (MEC) through the Consolider Ingenio-2010 program, under project CSD2007-00060 Physics of the Accelerating Universe (PAU).

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