Effects of threshold resummation

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Abstract

We investigate effects of threshold resummation of logarithmic corrections \( \ln N \) in Mellin space quantitatively. Threshold resummation leads to enhancement of next-to-leading-order QCD predictions for jet production at large jet transverse energy, which is in the trend indicated by experimental data. We show that this enhancement is completely determined by the behavior of threshold resummation at small \( N \), the region where hierarchy among different powers of \( \ln N \) is lost and current next-to-leading-logarithm resummation is not reliable. Our analysis indicates that more accurate threshold resummation formalism should be developed in order to obtain convincing predictions.
1 INTRODUCTION

The formalism of threshold resummation of double logarithmic corrections to QCD processes, which occur in extreme kinematic conditions, has been developed for some time [1, 2, 3]. At the kinematic end points, a special type of corrections $\ln(1-x)/(1-x)$ is produced with $x$ being a parton momentum fraction, which appears as $\ln^2(1/N)$ under the Mellin transformation. There have been abundant formal derivations of threshold resummation for various processes, such as deep inelastic scattering, Drell-Yan, direct photon, and heavy-quark productions [4]. Quantitative studies of threshold resummation effects in heavy quark production [5] and in direct photon production [6] have been performed recently. Such numerical studies are essential in order to justify that threshold resummation indeed collects important dynamics of processes in extreme kinematic conditions.

In this letter we shall analyze effects of threshold resummation from another point of view. For our purpose, it suffices to consider resummation of the logarithmic corrections that can be factorized into parton distribution functions (PDFs). Taking jet production at Tevatron as an example [7], we observe that threshold resummation enhances next-to-leading-order (NLO) QCD predictions for jet production at large jet transverse energy $E_T$. This tendency is qualitatively consistent with experimental data and with the conclusion in [5, 6]. However, the enhanced predictions overshoot data by a factor of 2. A simple investigation reveals that the resummation associated with the quark distribution functions lead to a negligible effect and the overestimation is attributed to the resummation associated with the gluon distribution function. The reason is that the color factor $N_c = 3$ in the latter case is larger than $C_F = 4/3$ in the former case.

We further find that the enhancement is completely determined by the behavior of the resummation associated with the gluon distribution function at low $N$. Unfortunately, this is a region where current next-to-leading-logarithm (NLL) resummation is not reliable, since hierarchy among different powers of $\ln N$ is lost and all nonleading logarithms need to be summed. In other words, the large-$N$ behavior of threshold resummation is reliable, but almost irrelevant to the end-point enhancement, whereas the small-$N$ behavior accounts for the end-point enhancement, but is not reliable in NLL threshold resummation. Moreover, the importance of low-$N$ contributions can not be diminished no matter how extreme kinematic conditions are. Our
analysis indicates that more accurate threshold resummation formalism need to be developed in order to obtain convincing predictions.

## 2 Threshold Resummation

Threshold resummation of the logarithmic corrections that can be factorized into PDFs is written as

$$\tilde{f}(N) = \exp \left[ \int_0^1 \frac{dz}{1-z} \int_{(1-z)^2}^1 \frac{d\lambda}{\lambda} \gamma_K(\alpha_s(\sqrt{\lambda p^+})) \right], \quad (1)$$

where $p^+$ is the longitudinal component of hadron momentum $p$, and the anomalous dimension $\gamma_K$ is given, up to two loops, by

$$\gamma_K = \frac{\alpha_s}{\pi} C_F + \left( \frac{\alpha_s}{\pi} \right)^2 C_F \left[ C_A \left( \frac{67}{36} - \frac{\pi^2}{12} \right) - \frac{5}{18} n_f \right], \quad (2)$$

with $C_A = 3$ being a color factor and $n_f = 4$ the number of quark flavors. In this work we shall adopt its modified version,

$$\tilde{f}(N) = \exp \left[ \int_0^{1-1/N} \frac{dz}{1-z} \int_{(1-z)^2}^1 \frac{d\lambda}{\lambda} \gamma_K(\alpha_s(\sqrt{\lambda p^+})) \right], \quad (3)$$

which is equivalent to Eq. (1) up to $O(1/N)$ corrections. We refer readers to [9] for its detailed derivation. Equation (3) is simpler in analytical manipulation, since the integral in the exponent can be worked out explicitly.

Threshold resummation associated with the gluon distribution function is obtained by substituting $N_c = 3$ for $C_F$ in Eq. (2).

The first term of $\gamma_K$ leads to the leading (double)-logarithm summation, and the second term leads to the NLL summation. Note that Eq. (3) is not complete at the NLL level. To obtain a full NLL summation, contributions from soft gluon exchanges among initial- and final-state partons should be taken into account, which are process-dependent. While Eq. (3) is process-independent, since it sums factorizable corrections. In this work we shall comment on applications of threshold resummation to various QCD processes, such as deep inelastic scattering, direct photon production, and jet production. Furthermore, our goal is to demonstrate that the end-point enhancement is determined by the small-$N$ behavior of threshold resummation. Hence, Eq. (3) serves the purpose.
We propose an expansion of the threshold resummation $\tilde{f}(N)$ in Eq. (3) in terms of polynomials in $N$:

$$\tilde{f}(N) = \sum_{i=0}^{n} a_i C(N - 1, i), \quad C(N, i) \equiv \frac{N!}{i!(N - i)!}.$$  (4)

The above series corresponds to a simple form in momentum fraction space,

$$f(x) = \sum_{i=0}^{n} \frac{a_i}{i!} \delta^{(i)}(1 - x),$$  (5)

which can be easily verified by performing the Mellin transformation

$$\tilde{f}(N) = \int_{0}^{1} dx x^{N-1} f(x).$$  (6)

The first coefficient $a_0 = \tilde{f}(1) = 1$ gives the initial condition of threshold resummation. The other coefficients $a_i$ are determined by best fit to Eq. (3). On the other hand, Eq. (3) is not appropriate for large $N$, the region in which the integration variable $\lambda$ may be as small as $1/N^2$, the running coupling constant $\alpha_s(p^+/N)$ diverges, and perturbation theory is not applicable. In this region Eq. (3) should be replaced by a nonperturbative function, which is of course model-dependent. For example, a minimal prescription which takes into account only the Landau singularity was introduced in [10]. It is straightforward to extrapolate Eq. (4) to the $N \to \infty$ limit, and this extrapolation can be regarded as a nonperturbative model.

It turns out that an expansion up to $n = 3$ in Eq. (4) describes the growth of Eq. (3) with $N$ very precisely. The parameters $a_i$ for the quark and gluon distribution functions from best fit to Eq. (3) for $\Lambda_{QCD} = 0.2$ GeV in $\alpha_s$ and for center-of-mass energy $\sqrt{s} = \sqrt{2p^+} = 1800$ GeV at Tevatron are listed below:

| parton  | $a_1$               | $a_2$               | $a_3$                |
|---------|---------------------|---------------------|----------------------|
| quark   | $1.6198 \times 10^{-2}$ | $-8.5872 \times 10^{-6}$ | $2.2515 \times 10^{-8}$ |
| gluon   | $1.1248 \times 10^{-1}$ | $7.2253 \times 10^{-5}$  | $2.5192 \times 10^{-6}$  |

For smaller $\sqrt{s}$, such as $\sqrt{s} = 38.7$ GeV for direct photon production in E706 [11], we obtain the parameters
The parameters $a_i$ increase as $\sqrt{s}$ decreases, implying stronger resummation effects, because the running $\alpha_s$ is larger at lower energies. For even lower $\sqrt{s}$, such as those for deep inelastic scattering, $a_i$ are even larger.

The modified PDF $\bar{\phi}$ is written as the convolution of threshold resummation with the original distribution function $\phi$ [4]:

$$
\bar{\phi}(x) = \int_x^1 \frac{d\xi}{\xi} f(\xi) \phi(x/\xi),
$$

$$
\quad = (1 - a_1 + a_2 - a_3)\phi(x) - (a_1 - 2a_2 + 3a_3)x \frac{d}{dx} \phi(x) \\
\quad + \frac{1}{2}(a_2 - 3a_3)x^2 \frac{d^2}{dx^2} \phi(x) - \frac{1}{6}a_3x^3 \frac{d^3}{dx^3} \phi(x).
$$

(7)

Briefly speaking, threshold resummation effectively modifies a PDF, and the modification is energy- and process-dependent. Hence, before performing a global determination of PDFs, one should clarify threshold resummation effects.

The motivation to expand the threshold resummation into a series of $C(N-1,i)$ up to $i = 3$ is as follows. The $C(N-1,1)$ term and the $C(N-1,3)$ term determine the behavior of the resummation at small $N$ and at large $N$, respectively. If the series terminates at $i < 3$, the role of each $C(N-1,i)$ in determining the behavior of the resummation in different regions of $N$ is not significant. If the series contains terms with $i > 3$, numerical handling of higher derivatives of modified PDFs will be difficult. Since the resummation associated with the gluon distribution function dominates, we take it as an example to demonstrate the above idea. For $N \sim 10$, the $i = 1$ term $a_1C(N-1,1)$ in the case with $\sqrt{s} = 1800$ GeV is of order unity, while the $i = 3$ term $a_3C(N-1,3)$ is only of order $10^{-3}$. As $N$ increases up to $10^3$, the edge for perturbation theory to be applicable, the $i = 1$ term, being of order $10^2$, becomes smaller than the $i = 3$ term, which is of order $10^3$. Hence, a variation of $a_1$ implies a variation of the small-$N$ behavior of threshold resummation, and a variation of $a_3$ implies a variation of the large-$N$ behavior.

For $N \sim 10^3$, a double logarithm $\ln^2 N$ is larger than a single logarithm $\ln N$ by a factor of 7, indicating that hierarchy among different powers of $\ln N$
exist and current NLL resummation is reliable. For \( N \) as small as 10, \( \ln^2 N \) and \( \ln N \) are in fact of the same order, and NLL resummation is not reliable. Therefore, by varying the coefficients \( a_1 \) and \( a_3 \), we can investigate how sensitive the end-point enhancement of NLO predictions is to the small-\( N \) portion of the NLL resummation, which is not reliable, and to the large-\( N \) portion, which is reliable. It will be satisfactory, if the enhancement is insensitive to the small-\( N \) behavior of the NLL resummation. However, we shall demonstrate that this is not the case. The reason is obvious from Eq. (7): the coefficient of each term on the right-hand side of Eq. (7) is dominated by \( a_i \) with smaller \( i \). The above reasoning applies to cases like E706 with \( \sqrt{s} = 38.7 \) GeV or lower, for which the corresponding parameters \( a_i \) have the same relation \( a_1 \gg a_2 \gg a_3 \). In these cases the end-point enhancement is determined by the behavior of threshold resummation at smaller \( N \), and the controversy stated above is more serious.

3 NUMERICAL ANALYSIS

Consider jet production with transverse energy \( E_T \) in \( p\bar{p} \) collision,

\[
p(p_1) + \bar{p}(p_2) \rightarrow J(E_T) + X .
\]  

The hadron momenta are assigned as \( p_1 = (p_1^+, 0, 0, T) \) and \( p_2 = (0, p_2^-, 0, T) \) with \( p_1^+ = p_2^- = \sqrt{s}/2 \). Partons carry the momenta \( \xi_i p_i \) with \( \xi_i, i = 1, 2, \) being the momentum fractions. In fact, the transverse momenta \( k_T \) of partons should be taken into account, when transverse degrees of freedom of final states are measured \[12\]. The corresponding \( k_T \) resummation, if included, is expected to further enhance the cross section at high \( E_T \). In the present work \( k_T \) resummation will not be considered, since we concentrate on effects of threshold resummation.

The factorization of jet production is basically similar to that of direct photon production in \[12\]. The self-energy correction to a parton and the loop correction with a real gluon connecting the two partons from the same hadron, contain both collinear divergences from the loop momentum \( l \) parallel to \( p_i \) and soft divergences from small \( l \). Since soft divergences cancel between the above corrections \[13\], the remaining collinear divergences are absorbed into a PDF associated with the hadron \( i \). They are the corrections which produce the logarithms \( \ln(1/N) \) that have been summed into Eq. (3) in
axial gauge. In the considered kinematic region for jet production, the other radiative corrections, because of the soft cancellation, are absorbed into a hard scattering amplitude $H$, which corresponds to a parton-level differential cross section.

The factorization formula for jet production with the threshold resummation for PDFs included are written as

$$\frac{d\sigma(E_T)}{dE_T} = \int d\xi_1 d\xi_2 \tilde{\phi}(\xi_1, E_T/2)\tilde{\phi}(\xi_2, E_T/2)H(\xi_1, \xi_2, s, E_T/2).$$

We have set the renormalization (factorization) scale $\mu$ of $\tilde{\phi}$ and $H$ to the characteristic scale $E_T/2$. The original PDFs $\phi$ evolve to $E_T/2$ according to the Dokshitzer-Gribov-Lipatov-Altarelli-Parisi equation [14], which sums another type of single logarithms $\ln E_T$. In the following numerical analysis we employ the NLO QCD calculations for jet production derived in [15] and the CTEQ4M set [16] for the original PDFs. We have also checked the CTEQ3M set [17] and found that our conclusion does not depend on the choice of PDF sets.

In Fig. 1 we show the modification from the threshold resummation on NLO QCD predictions for jet production at Tevatron. It has been observed that there seems to be an excess of the CDF data at high $E_T$ compared to the NLO predictions with usual PDFs [16, 17, 18, 19], whereas the D0 data are in good agreement with the predictions. Note that the CDF and D0 data do not conflict each other if considering the large systematic uncertainties ($8 \sim 30\%$) in addition to the statistical ones. Hence, it is expected that the threshold resummation causes a small amount of enhancement of the NLO predictions at high $E_T$. However, it is found that the enhancement is a factor of 2. We explore the source that is responsible for this huge overestimation. If turning off the threshold resummation associated with the gluon distribution function, the end-point enhancement falls dramatically. If turning off the threshold resummation associated with the quark distribution function, the enhancement almost remains invariant. That is, the resummation associated with the quark distribution function contributes only few percent of the full enhancement, and the overestimation is attributed to the resummation associated with the gluon distribution function as shown in Fig. 1.

We then investigate which portion in $N$ of the resummation associated with the gluon distribution function accounts for the end-point enhancement. As stated before, the parameters $a_1$ and $a_3$ control the small-$N$ and large-$N$
behaviors of threshold resummation, respectively. If setting \( a_1 \) to zero, which changes the small-\( N \) behavior of the resummation but leaves almost invariant the large-\( N \) behavior, the resultant predictions coincide with those from the NLO calculations without including resummation effects as shown in Fig. 2. If setting \( a_3 \) to zero, which changes the large-\( N \) behavior but leaves invariant the small-\( N \) behavior, the enhancement remains the same. This investigation indicates that the behavior of the resummation at small-\( N \) determines the end-point enhancement. As argued before, this is the region where hierarchy among different powers of \( \ln N \) is lost and more accurate formulas including summation of all nonleading logarithms are required. Before ex-
Figure 2: Enhancements from the threshold resummation for gluon, from the resummation with $a_1$ set to zero, and from the resummation with $a_3$ set to zero (which appears to coincide with dotted line for gluon). The NLO predictions and experimental data (with statistical errors only) are also shown.

stracting reliable predictions from threshold resummation, this point must be taken into account. At last, we observe that the end-point enhancement can be adjusted by tuning $a_1$, i.e., by varying the small-$N$ behavior of threshold resummation. Choosing $a_1 = 0.02$ in the resummation associated with the gluon distribution function, the resultant predictions are similar to those from applying the CTEQ4HJ PDFs [16, 20], and well describe the CDF and D0 data simultaneously.

We emphasize that the above controversy always exists no matter how large $E_T$ is reached. With higher $E_T$, behaviors of PDFs at larger momentum
fraction are probed. In this region we have $x \to 1$ in the inverse Mellin transformation,

$$f(x) \equiv \frac{1}{2\pi i} \int_{c-i\infty}^{c+\infty} dN x^{-N} \tilde{f}(N),$$

with $c$ being an arbitrary constant, and contributions from $\tilde{f}(N)$ in the whole range of $N$ are equally important. In cases with intermediate $E_T$, for which $x$ may not approach unity, contributions from the small-$N$ region dominate. That is, no matter how large $E_T$ is reached, the small-$N$ region always contributes. We have also analyzed the data of direct photon production in E706 [11] and of deep inelastic scattering in BCDMS [21] and NMC [22] with lower $\sqrt{s}$ using the above method, and arrived at the same conclusion: in the region where perturbation theory is applicable, small-$N$ contributions always determine the end-point enhancement.

4 CONCLUSION

In this letter we have shown that the behavior of the threshold resummation associated with the gluon distribution function at small $N$ determines the end-point enhancement of NLO predictions for jet production completely. A variation of the large-$N$ behavior of the NLL resummation, which is reliable, does not affect the enhancement. However, in the small-$N$ region hierarchy among different powers of $\ln N$ disappears and current NLL resummation is not reliable. This controversy also exists in processes with higher $E_T$ or lower center-of-mass energies. Hence, to obtain convincing predictions, more accurate formalism for threshold resummation which sums all nonleading logarithms need to be developed. We emphasize that it is not the goal of this work to explain experimental data. Even if data can be explained by complete NLL threshold resummation, the controversy we have found remains. Before attempting to understand data using threshold resummation, our conclusion should be taken into account.

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