Spinors in Quantum Geometrical Theory

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ABSTRACT
Spinors have played an essential but enigmatic role in modern physics since their discovery. Now that quantum-gravitational theories have started to become available, the inclusion of a description of spin in the development is natural and may bring about a profound understanding of the mathematical structure of fundamental physics. A program to attempt this is laid out here. Concepts from a known quantum-geometrical theory are reviewed: (1) Classical physics is replaced by a suitable geometry as a fundamental starting point for quantum mechanics. (2) In this context, a resolution is found for the enigma of wave-particle duality. (3) It is shown how to couple the quantum density to the geometrical density. (4) The mechanical gauge is introduced to allow dimensional reduction. (5) Absolute geometrical equivalence is enforced. The concordant five-dimensional quantum-geometrical theory is summarized to provide an orderly basis for the introduction of spinors. It is supposed that the Pauli–Dirac theory is adaptable. A search is begun for a description that will generate spinors as a natural tangent space. Interactions other than gravity and electrodynamics should then appear intrinsically. These are conjectured to be weak effects for electrons.

Keywords:
Dirac spinors, quantum-gravitational theory, geometry

1 Introduction

A study of Cartan’s book suggests that spinors may function at a fundamental level in the geometry of physics. Interest has continued unabated following discovery of the relativistic equation of the electron by Dirac. If applications are included, the available literature is perhaps the largest of any single topic in physics. Within the context of earlier studies by the author, the contribution of fundamental spin to quantum geometrical theory must be reevaluated.

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FIGURE 1. The fundamental concepts of geometry come from the material properties of physical objects. Practical experience with measuring instruments led to the abstract notions of Euclid.

2 Search for a Fundamental Geometry

It is an ongoing study to find a fundamental geometry that might include all of physics. A very early goal of the author was to discover an elementary but fully geometrical quantum mechanics. This was to be a simple incorporation of essential characteristics. No specific interpretation was presumed among the conflicting beliefs, neither Copenhagen nor hidden variable, nor any other. An elementary understanding by geometry was expected to lead to reconciliation with relativity and gravity. Given the divergent metaphysical issues within the subfields of physics, correctness was to be resolved by recourse to experimental results without the interference of any predisposed phenomenology.

It was soon realized that electrodynamics had to be included. While there are textbook examples of isolated quantum wave functions, practical quantum effects are almost always observed by electromagnetic interaction. (The geometry has been found to support this synthesis.) However, a study involving radiation requires further sophistication. Because radiation is relativistic, special relativity must be incorporated, a fact that is not so obvious in most laboratory experiments. The issues encountered in quantum-electrodynamics become relevant. Because of the usual relativistic constraints, interaction potentials that depend on space-like separated points must be relinquished in favor of covariant forces acting along null lines. Simple instructional examples are hard to find. Both the experimental phenomenology and the concomitant geometrical representation are complicated.

It then became clear that a curvilinear description must be included as well. In general relativity the clock paradox is often used to motivate the analogous development. It is not so obvious that this general type of
difficulty occurs with other interactions. A Lorentz transformation is insufficient to describe any but the simplest motion yet, geometrical equivalence must still be maintained. While an explicit physical “clock” paradox may not be available, neither acceleration nor the complex motion of interference can be modeled by linear coordinate transformations. Any construction suitable for quantum interference will require a curvilinear system, and a Riemannian formalism is necessary. Realizing this is essential to allow for the description of physical interactions in a quantum-relativistic setting.

These studies eventually developed into the aforementioned quantum-geometrical theory. To entertain the question of spinors, a selection of pertinent concepts is reviewed and the current results and conjectures are explained in this context.

3 Epistemology and Geometrization

Attempts to relate quantum systems to gravitation have shown unresolved difficulties. The root causes originate in the metaphysical assumptions of both subfields. New ideas are necessary and some old ones may be untenable.

By convention, quantum mechanics is derived from classical mechanics by quantization. A classical Hamiltonian becomes a quantum wave equa-
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tion by the substitution of derivatives. The process brings the electrodynamical interaction across as well. The geometrical sense of coordinates is unchanged. The partial differential equations of quantum theory come into existence, but unfortunately without proper evolution of concept.

In contrast to this, general relativity has some fundamental geometrical basis outside of classical mechanics. Herein is the conflict. Some physics has a geometrical component while some has not. Resolution of these differences is essential before incorporating the intrinsically geometrical spinors.

As a progenitor of quantum mechanics, geometry may be capable of displacing classical physics. Early studies showed that some of the essential quantum theory is buried in curvilinear geometrical constructions. It was apparent that formal quantization is not always needed and that certain essential parts of quantum theory are found in geometries unconverted by any quantization procedure.

A major change in the epistemological structure of physics is proposed. Classical physics is to be a simple phenomenology and should not be used to generate quantum theory. The quantum is to come from geometry alone.

This is realistic a possibility because there are no experiments that favor classical over quantum. There are no observed classical point particles and, without such particles, there can be no fundamental basis for classical theory. The active quantization of classical mechanics allows inconsistencies to be incorporated. Such intellectual corruption is the root difficulty of a quantum theory of gravity. It is overly optimistic to believe that it is possible to start with a fundamentally incorrect classical theory and arrive, by any rigorous mathematical process, at a correct quantum theory. The referenced studies show that useful unified field theories can be found once these changes are made to the origination of quantum laws.

4 Quantization

A review of the procedure for the conversion of a classical theory to a quantum theory makes the limitations more apparent.

The common strategy is to start with an algebraic relation for the Hamiltonian such as,

$$ H = \frac{p^2}{2m} + V(x) $$

and convert it to

$$ -i\hbar \frac{\partial}{\partial t} = \frac{1}{2m} \left( i\hbar \frac{\partial}{\partial x} \right)^2 + V(x), $$

by the substitution rule.
The resulting equation, under restricted circumstances, is in agreement with experiment. The mathematical process has been studied for some time, but never formally validated [3].

Attempting to quantize in a curvilinear system, makes plain the enigmatic behavior. Execution of the standard method requires a differential substitution for energy and momentum. The difficulty is that there are already derivatives in the theory that lead to intrinsic non-commuting structures. Specifically, the derivatives of the metric tensor that are implicit in the covariance. They persist in the connections

\[
\Gamma^\alpha_{\beta\gamma} = \frac{1}{2}g^{\alpha\delta} \left( \frac{\partial g_{\delta\beta}}{\partial x^\gamma} + \frac{\partial g_{\delta\gamma}}{\partial x^\beta} - \frac{\partial g_{\beta\gamma}}{\partial x^\delta} \right). \tag{1.5}
\]

which are integral to any curvilinear structure. Such terms are essential when there is a gravitational field. A substitutionally introduced derivative (an essential component of methodological quantization) may operate on a scalar, but because the operand is not identified at the time of the substitution, combinations involving tensors or spinors are problematical. No resolution has become available and problems with quantum normal ordering persist. An accepted classical theory must contain the momentum \(p\) and energy \(E\) in such a way as to transform covariantly. Because the Christoffel symbols are required, the theory cannot be quantized. Apparently, any “correct” classical gravitational theory cannot be converted into a correct quantum theory by any formal quantization. The introduction of additional derivatives destroys the covariance if even it can be defined unambiguously.

Moreover, the characteristic gravitational field equations are already second order. A quantization will increase the order, by at least one and probably two. The resulting equations are third or fourth order, even neglecting complications generated by the non-locality of possible expansions of derivatives under the square root. This is an epistemological disaster because this accepted method to establish a quantum-gravitational theory can no longer generate an intellectually sound theory. The alternate solution is preferred here. Use covariant quantities to represent the wave function and then generate field equations by careful selection of a geometrical system.
5 Wave-particle Duality

A deterministic and geometrical quantum theory impels the resolution of wave-particle duality. Conventional wisdom says that real particles behave sometimes like particles and sometimes like waves. Double-talk notwithstanding, the description must be made more precise and the language adjusted to the reality of experiments.

A precise notion of a classical point particle is required and is here taken to be an object that is described by a one dimensional trajectory (geodesic) in space-time. There may be additional parameters, such as mass or charge, that are part of the dynamics. Such is the common notion.

A more sophisticated concept is required for the quantum particle. A one-dimensional trajectory is never sufficient. Contributing to the confusion is the accepted terminology that distinguishes a quantum point objects as being non-composite. Electrons are quantum point like while protons or pions are not, this, even though none can be condensed to a point. The essential idea of a quantum point particle is, in fact, simple cardinality. Electrons, by experiment, come in sets that have a total number associated. Point localization need not occur. The enumeration may be defined by weight, inertia, integrated charge, or other interaction. This notion of number is adequate for quantum mechanics. It allows normalization of the wave function. Experimentally available particles are countable but never point-localized.

The acceptance of quantization can subtly integrate the classical idea of a point particle into the quantum interpretation. The paradox of wave particle duality then comes into play. To illustrate, consider the following diffraction experiment.

As in the diagram, particles travel through a diffraction screen to be counted by a detector. Suppose that the detector consists of a fixed uniform distribution of charged centers, (perhaps protons), to which the electrons are attracted. An individual electron will interact with these charged objects, emit radiation, lose energy and ultimately be captured. Under ideal conditions, it will bind to one of the charged centers. The large initial diffraction pattern will condense into an atomic wave function. Because of the multiple outcomes, this process generates statistics. The particular final center into which the electron cascades is not predictable, but has a probability following the quantum interpretation.

Within the geometrical theory, these statistics are not generated from a fundamental supposition but are the result of the system evolution as follows: (1) The collapse of the diffracted electron wave function proceeds in a finite time according to the limits of relativity theory. It is not instantaneous. (2) The forces which induce the process come from the advanced fields of the particles that absorb the emitted photons. The geometrical theories require an explicit well-defined construction for these forces of radiative reaction, otherwise equivalence cannot be guaranteed. The ra-
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FIGURE 3. Diffracted electrons are captured in a detector. An explicit model for the detector demonstrates the evolution of the wave function from an extended diverging wave to a compact bound state.

Radiative reaction comes from ontological (electromagnetic) fields that are derived from real particles. (3) The radiation is unique to the final electron state. The equations are time reversible and deterministic. The possible final states of the electron are complete and orthogonal. Unitarity demands that the final photon states map to the final electron state. (4) There is no fundamental statistical mechanism. The different results must be taken as due to differences in the configuration of the system of particles that absorb the photons as they are emitted. Experiments with radiation in cavities or those that study correlations make any other conclusion hard to justify.

Suppose that the above experiment is extended by placing an additional

FIGURE 4. Successive diffractions appear to refine the particle position. In practice, this is always accompanied by interactions and radiation.
series of slits between the source and detector. It is often argued that such successive refinement, can identify the point position of a particle. Each slit diffracts the particle further, and by integrating backwards along the probability current, a particular trajectory within the initial emitted wave function can be defined to any limit of precision. It is sometimes argued that this shows that a particle travels on a specific trajectory. No interpretation of this type is demanded by geometrical theory. In fact, the backwards projected refinement of the original trajectory is invalid because radiative interactions are part of the selection process at each slit.

In consequence it is not experimentally possible to identify a geometrical trajectory. To describe particles as individual but enumerable waves is sufficient, and this is entirely possible with curvilinear geometry. The wave function, as a collection of lines of probability flow, rather than any single trajectory, is associated with the geometry.

6 Quantum Conformal Coupling

The observation of a probability involves counting particles as they are detected on a test screen. The experimental arrangement is controlled by material objects which serve as measuring rods to a local cartesian system. In this sense the apparatus is fixed and unchanging. A region on the screen is scribed to delineate an area in which particles will be counted as they are detected. The marked area is measured by dimensions \( \delta x \) and \( \delta y \) and the movement by \( \delta z \) and \( \delta t \) according to local metric \( g^\mu_\nu \). A count of particles, as given by quantum theory, is

\[
N = \psi \psi^* \frac{\delta x \delta y \delta z}{\delta t}
\]

Consider now the observation of this effect by an observer using a metric having a different conformal parameter and consequently a different scale, \( g^\prime^\mu_\nu = \lambda g^\mu_\nu \). The new numerical marks are

\[
(\delta x^\prime, \delta y^\prime, \delta z^\prime, \delta t^\prime) = \sqrt{\lambda} \cdot (\delta x, \delta y, \delta z, \delta t)
\] (1.1)

FIGURE 5. Local variations in the conformal parameter of a particle space-time are equivalent to probability density waves.
The count becomes
\[ N = \frac{\psi\psi^* \delta x' \delta y' \delta z'}{\delta t'} \]  
(1.2)
and because \( N \) is an invariant, \( \psi \) must not be taken unchanged. A conformal transformation of the metric changes the measured probability density. The wave function must be compensated. Presumably, \( \sqrt{\lambda} \psi' = \psi \). The conformal structure of the geometry cannot be separated from the quantum mechanical density. It is proposed that as part of the physical-geometrical structure that many of the essential effects of quantum mechanics are manifestations of conformal structure. A quantitative description of particle localization is identified with the relative conformal density between the particle space and the observer.

The conformally invariant theories prevent this coupling and suppress the intrinsic quantum characteristics of the geometry. They cannot be quantized because the geometrical density is isolated from the quantum density. The geometrical interpretation requires a coupling to the conformal factor. To avoid this problem, a single integrated quantum geometrical conformal structure is used throughout this article.

7 The Mechanical Gauge and the Physical Dimensionality

Measurements of physical structure use clocks and rods whose dimensional qualities originate outside of general relativity. Practically, the measuring standards come from quantum mechanics. However, without a consistent quantum-gravitational theory, the inter-compatibility of this construction is uncertain. Quantum mechanics plays an essential role in determining the geometrical size by referencing a fundamental length. This is taken as the Compton wavelength of the electron. Enforcing a length standard removes
the possibility of conformal variations in the observer’s metric. It sets a particular gauge, here called the mechanical gauge. An experimentalist assigns the structure of space-time in this way. Objects rotate and do not change in length, at least in so far as it is possible to compare them with each other. Conformal stability of the observer’s metric is enforced.

\[ \lambda \]

FIGURE 7. The fundamental local scale is referenced to the particles that make up the reference object. The basic quantity is the Compton wavelength of the electron.

In addition, the dimensionality of space-time is inferred by the structure of mechanical objects. These properties come from the constituent particles. In effect then, the three space coordinates and one time coordinate, 3+1, are assigned by the properties of the solutions of the quantum wave equation. For the quantum gravitational theory used here, the enveloping space is five-dimensional. It is anticipated that a higher number of dimensions may work and still provide an effective space-time. Such a larger geometrical construction may allow interactions other than gravity or electromagnetism.

8 Absolute Equivalence

The equivalence of gravitational and inertial mass has a long history. Modern experiments are designed to systematically test the relative contributions of different types of mass-energy. The integrated effects of electrodynamic, quantum, weak, or strong forces are tested against each other with a sensitive balance.

\[ \times \]

FIGURE 8. Equivalence is tested by balancing different materials in combined gravitational and accelerational fields.
For gravity, exact equivalence is expressed by the existence of a local frame. The equivalence of other interactions apparently does not have such an elementary mathematical form of expression. Yet, the experiments to date show that the integrated equivalent of one type of mass is indistinguishable from any other. In the general case, the coordinates that would be needed to display the inter-transformation of forces are not experimentally accessible. Still the mathematical explanation of equivalence for all forces must have a formal structure that goes beyond general relativity. Because current experiments show no discrepancies, some type of equivalent theory is possible.

Here, exact absolute equivalence is assumed. It is supposed that any type of force can be transformed into any other type of force, at least internally. This assumption is easy to use, but is stronger than experiments can establish directly. The mechanical expression of space-time, resulting in the reduced dimensionality, may conceal the additional symmetries. Verification must depend on the development of a specific geometrical thesis.

FIGURE 9. A quantum particle must be represented by all of the lines of its probability current. A local inertial frame only applies to the local motion of a single quantum particle. Each particle must have its own set of local frames.

The principle of equivalence expressed as the existence of a local frame can be adapted to a quantum particle. Rather than using an individual classical trajectory, the entire probability current of a wave-particle solution is taken at one time. Only the local part of the quantum motion can be associated with a Lorentz frame.

Quantum-gravitational equivalence is more subtle than the equivalent construction in classical physics. A convergent particle wave could be from either a diffraction event or alternatively from the converging force of a gravitational field. The equivalence of the wave equation derivatives and
the Christoffel derivatives is essential. Any identification of the difference requires knowledge of the cause of the convergence. It cannot be elucidated by study of the wave alone.

9 Essential Results in Five Dimensions

For the conceptual development in hand, a mathematical summary of the five-dimensional theory is to the point. Five coordinates \( (x^0 \cdots x^4) \equiv x^m \sim (t, x, y, z, \tau) \) are chosen where \( (x^0 \cdots x^3) \equiv x^h \sim (t, x, y, z, \tau) \) can be taken to coincide with the observers space-time. The five metric in standard form is

\[
\gamma_{mn} = \left( g_{\mu\nu} - A_\mu A_\nu A_\mu A_\nu - 1 \right)
\]  
(1.1)

and can be rewritten with adjustable conformal factors as

\[
\gamma_{mn} = \omega \left( \chi g_{\mu\nu} - \chi^2 A_\mu A_\nu \chi A_\mu A_\nu - 1 \right).
\]  
(1.2)

There is a preferred set of geodesics

\[
\frac{dx^\mu}{ds} = \omega g^{\mu\nu} \chi A_\nu
\]  
(1.3)

which are tangent to the probability current and which remain invariant under changes in \( \lambda, \chi, \) and \( \omega \). It can be shown that these factors relate to the causative explanation of the geodesic curvature in concordance with absolute equivalence.

A discussion of the use of \( \chi \) and \( \lambda \) is beyond this talk, but \( \omega \) is relevant. With \( \lambda = 1 \) and \( \chi = 1 \), the curvature scalar \( \Theta \equiv \Theta(\gamma_{\mu\nu}) \equiv \Theta(\gamma_{\mu\nu}(\omega)) \) becomes, upon being set equal to zero in a field free region,

\[
(\partial^2)^2 x^m \Psi \equiv \left( \frac{\partial^2}{\partial t^2} - \frac{\partial^2}{\partial x^2} - \frac{\partial^2}{\partial y^2} - \frac{\partial^2}{\partial z^2} - \frac{\partial^2}{\partial \tau^2} \right) \Psi = 0
\]  
(1.4)

Where the linear wave function \( \Psi \) is equal to \( \omega^{3/4} \). Adding the condition

\[
\frac{\partial \Psi}{\partial \tau} = im
\]  
(1.5)

produces a Klein-Gordon equation for mass \( m \). The quantum theory appears by the choice of geometry.

In fact, it turns out that the interactions can also be represented for another functional representation of the parameter \( \omega \). Setting to zero the Ricci tensor of the conformally multiplied metric

\[
0 = \Theta^{ij}(\gamma_{mn}(\omega)),
\]  
(1.6)
it can be shown that this conformal contribution to the Ricci curvature of the original metric can be calculated as

$$\Theta^{ij}(\gamma_{mn}(0)) = T^{ij}(\omega).$$  (1.7)

with $T^{ij}$ known as a function of $\omega$. And furthermore, for a suitable choice of $\omega$, this becomes, as expressed in four dimensional notation,

$$R^{\alpha\beta} = 8\pi\kappa[F^\alpha_\mu F^{\mu\beta} + m|\psi|^2 \frac{e^2}{m^2} A^\alpha A^\beta + m|\psi|^2 \frac{1 - (e^2/m^2) A^2}{2 - (e^2/m^2) A^2} g^{\alpha\beta}].$$  (1.8)

and at the same time

$$F^{\beta\mu}|_\mu = 4\pi\alpha|\psi|^2 A^\beta.$$  (1.9)

For spinors, the point is that the standard quantum field equation and the effects of external interactions are mediated by the overall conformal factor $\omega$. In addition, the condition of inferred dimensionality is satisfied. The resulting system predicts quantum-dynamical interactions that are in appearance four-dimensional. The mechanical gauge is correct and depends on the separation constant, $m$, of the geometrical field equation. In fact, a universal mechanical gauge is realized by choosing the same type of particle at each point and relating the scale of that mass to the fifth coordinate. (Scale changes in $\tau$ cause proportional changes in the local mass of all particles and are unobservable.)

This construction allows for a theory of mass that comes from the geometrical structure. It is presumed that more complicated systems of separation constants could be found in higher dimensional spaces and so might give a mass spectrum.

10 The Need for Spinors

The motivation for the study of spinors is simply this: There are, at present, no known stable massive bosons. All decay into other particles which, in the end, have spin $1/2$. The internal mechanism of the five theory does not directly accommodate spin. None of the fields nor the coordinates have spinor transformation properties. Still group theory predicts a deeper irreducible spinor representation of the local coordinate transformations. Apparently, spin is built into even the simplest notion of geometry. A full understanding remains elusive.

It is known that the source current from a particular electron does not interact with itself, but always with another. Consider a pair production event after which the positron anihilates with an outside electron. While each electron must not be affected by its own source currents, each must interact with the currents of the other even though they eventually connect through the positron. The enigma has not been resolved in the context of a
FIGURE 10. The positron from an e-p pair annihilates smoothly with another electron. The electrons must be antisymmetric with respect to each other.

geometrical theory. Antisymmetrization and the associated indistinguishability seem important. This and the ubiquitous presence of spin 1/2 are taken as symptoms of a deeper geometrical structure.

11 Pauli-Dirac Theory

The most elementary approach is to reconsider the discussion of Pauli and others [4] in which the five anti-commuting Dirac matrices are each associated with one of the five coordinate directions. Let

\[ \dot{\gamma}^0, \dot{\gamma}^1, \ldots, \dot{\gamma}^4 \equiv \dot{\gamma}_{AB} \]  

for \( m = 0, \ldots, 4 \) and \( A, B = 1, \ldots, 4 \) such that

\[ 1_{\dot{\gamma}^{mn}} = 1 \begin{pmatrix} \delta^{\mu\nu} & 0 \\ 0 & -1 \end{pmatrix} = \frac{1}{2} \{ \dot{\gamma}^m, \dot{\gamma}^n \} = \frac{1}{2} (\gamma^m \gamma^n + \gamma^n \gamma^m) \]  

It is convenient to use the standard basis for \( \gamma^\mu, \mu = 0 \ldots 3 \) and to let \( \dot{\gamma}^4 = i \gamma^5 \). In addition, define \( \sigma^{mn} = [\gamma^m, \gamma^n] \equiv \gamma^m \gamma^n - \gamma^n \gamma^m \) and let 1
be the unit matrix. For the particle metric (as used to represent the fields of an individual particle, say the electron.)

\[ 1 \gamma^{mn} = 1 \left( \dot{g}^{\mu\nu} A^\mu A^\nu A^\tau A^\rho - 1 \right) = \frac{1}{2} \{ \gamma^m, \gamma^n \} \quad (1.3) \]

where direct calculation gives \( \gamma^\mu = \dot{\gamma}^\mu \) and \( \gamma^4 = \dot{\gamma}^4 - A_\mu \dot{\gamma}^\mu \)

The standard Dirac equation

\[ i \hbar \dot{\gamma}^\mu \frac{\partial}{\partial x^\mu} \psi - mc\psi = 0 \quad (1.4) \]

can be rewritten with the five dimensional notation. Define a similarity transformation by \( S = \frac{1}{\sqrt{2}}(1 + \gamma^4) \), then left multiply by \( S\dot{\gamma}^4 \), right multiply by \( e^{i mc \hbar x^4} \), and insert the pair \( S^{-1} S \).

\[ i \hbar (S\gamma^4 \gamma^\mu S^{-1} \frac{\partial}{\partial x^\mu} S\psi e^{i mc \hbar x^4} - mc S\gamma^4 S^{-1} \psi e^{i mc \hbar x^4} = 0 \quad (1.5) \]

Setting \( \Psi = S\psi e^{i mc \hbar x^4} \) and noting that \( S\gamma^4 S^{-1} = \gamma_4 \) and \( S\gamma^4 \gamma^\mu S^{-1} = \gamma^\mu \) gives

\[ \gamma^m \frac{\partial}{\partial x^m} \Psi = 0. \quad (1.6) \]

An interesting question is how to connect the conformal part of \( \Psi \) with the five conformal factor \( \omega \). That is, if \( \gamma^i \rightarrow \omega \gamma^i \), is this equivalent to \( \gamma^i \rightarrow \omega^{1/2} \gamma^i \) or \( \Psi \rightarrow \omega^{1/2} \Psi \)? Certainly, there are conformal contributions to the curvature and other complexities as well, but because of the conformal flatness of the five theory, a construction of some kind should be possible. The conformal substructure of the Dirac theory is at issue. A possible immediate application is a fundamental understanding of the interaction of spin one-half particles with the gravitational field.

The most interesting question is whether and in what way weak interactions can be included. Because experimental tests of equivalence include the integrated rest-mass-energy of the weak forces, a theory that can even describe weak interactions must show a detailed equivalence if it is to be exact. The internal gauge transformation must exist. Perturbative descriptions of neutrino-electron interactions should be the limit of exact geometrical effects. It is not yet entirely clear how to do this. The five dimensional spinor approach is a direct avenue of investigation.

12 Spinor Coordinate Systems

More radical is to search for a geometrical system that has spinors as a natural tangent space. For a classical point particle, the choice of local frame is not unique. Space rotations about the particle center are degenerate. For
real particles, the spin breaks the symmetry. It is conventional to assign a spinor basis space in place of a local Lorenz frame. But, to develop a fundamental geometry of spin, some sort of coordinates should themselves provide the local spinor orientation.

**FIGURE 11.** Electrons have spinor type local tangent frames. These are not naturally derivable from the usual coordinate space. A coordinate sub-manifold is sought.

Suppose that there is such a base space. Choose four complex valued coordinates \((\xi^1 \cdots \xi^4) \equiv \xi^A \in \mathbb{C}\) which are to be related to the physical coordinates \((x^0 \cdots x^4) \equiv x^m\). Some of the usual difficulties with holonomy may not apply because of the conformal flatness.

A number of transformations are possible, but not all have suitable properties. The following construction is under study. Let \(\xi^A = \xi^A_r + i\xi^A_i\) with \(\xi^A_r, \xi^A_i\) real. Define a coordinate relation by

\[
x^m = \frac{1}{2} \gamma_{AB} \xi^A \xi^B.
\]

where \(\overline{\xi}^A\) is the complex conjugate A five dimensional real space is generated by a restriction from the four dimensional complex space. The important question is the existence of differential equations and their relationship to more general coordinate transformations.

Further characteristics are useful. (1) Translations at least are allowed in \(\mathbb{C}^4\) and also in \(\mathbb{R}^5\). (2) Apparently, expansion of the equation \((\partial^2)_{x} \Psi = 0\) from five dimensions to the complex four dimensional space gives something which may be written in the form \((\partial^2)_{\xi} (\partial^2)_{\xi} \Psi = 0\) where \((\partial^2)_{\xi}\) represents an eight dimensional d’Alembertian in the \(\xi^A_r\) and \(\xi^A_i\). The essential point is that the Klein Gordon equation in five space can be written in terms of a suitable differential invariant in spinor space. Apparently, there are characteristic solutions on \(\mathbb{C}^4\) that map to standard quantum wave functions on \(\mathbb{R}^5\). (3) The conformal structure of interactions in 5-space is transferred to the complex 4-space and induces a quantum structure with the required gravito-electromagnetic interactions in place. (4) Also, it seems that the relevant interactions, as induced by the second order derivatives of the
FIGURE 12. The complex plane can be mapped stereographically to the surface of a sphere in two different ways. The projection line from a point on the plane can go through either the center or the antipode. Both cases are shown together. Two spheres of radius one and two are tangent at the origin, point $O$. The line projection goes through the point $P$ which is at the center of the larger sphere and at the antipode of the smaller one. The angle $\theta$ from the vertical axis to the projected point on the smaller sphere is exactly twice the angle $\phi$ for the larger sphere. There is a quadratic mapping from one to the other. The double valued relation of $\xi$ to $x$ is analogous to the required spinor map of $\mathbb{R}^5$ to $\mathbb{C}^4$ suggesting a spinor coordinate transformation.

A number of conjectures remain indeterminate. (1) Is it possible to represent the Dirac wave function as a unit vector in $\mathbb{C}^4$ multiplied by a conformal factor? (2) What, in detail, is the differential geometry of this scheme? (3) Is the phenomenology correct? (4) Can all known properties of the electron be described in this way? (5) What addition structures may be needed? (6) Is it correct to use general complex nonsingular transformations for the $\xi$'s? (7) Are there other analogous coordinate spaces and transformation systems that might apply to higher particles?
FIGURE 13. Geometry provides a basis for physics. The spinors may provide a physical-geometrical map between different dimensionalities.

13 Summary

More than the presentation of any new result, this discussion is intended to define an ongoing approach to an unsolved problem. The ultimate question, a geometry for all of physics, may find a solution as new types of geometrical systems become associated with the known experimental facts. Certain ideas from the five dimensional description of quantum mechanics and gravity enable the introduction of spinors. Spin has always been geometrical and its association with other fundamental geometrical constructions should be fruitful. Certain ideas are important. Relinquishment of quantization abolishes the need for a classical theory. Exclusion of constructions that depend on discreet classical particles allows a fully wave oriented development. A dynamics based on conformal effects permits a mathematically simple scheme that is compatible with spinors. Equivalence should be implied for all interactions. The concept of inferred dimensionality permits the use of spaces having more than four dimensions to describe the physical-mechanical world.

Specific suggestions may bring the Dirac system into a geometrical foundation. The classical five-dimensional theory that associates each one of the five anti-commuting Dirac matrices with one of the five coordinates may be adaptable. Explicit introduction of spinor coordinates, particle by particle, may be possible. In any case, a fully geometrical structure with interactions is expected.

The immediate goal, as a problem in physics, is to understand electrons and their interactions. The mathematical goal is to understand how to handle these types of geometrical-particle structures so that more complicated systems might be accessible in a fully equivalent theory.
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