Particle-in-cell simulations of plasma slabs colliding at a mildly relativistic speed

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Abstract. Plasmas collide at relativistic speeds in many astrophysical and high-energy density laboratory environments. The boundaries that develop between such plasmas and expand at much larger speeds than the ion sound speed $c_s$ are not well understood. Here, we address two identical electron–proton plasma slabs that collide with a relativistic speed and a Mach number $v/c_s$ of over 400. The collision speed, the plasma temperature and magnetic field are such that the growth rate of the two-stream instability exceeds that of all other instabilities. We model a planar turbulent boundary (TB) with one-dimensional (1D) and 2D particle-in-cell (PIC) simulations. We show that the boundary dissipates its energy via electron phase space holes (EPSHs) that accelerate electrons at the boundary to relativistic speeds and increase significantly the speed of some protons. Our results are put into the context of a dynamic accretion disc and the jet of a microquasar. It is shown that the accelerated electrons could contribute to the disc wind and to relativistic leptonic jets, and possibly to the hard radiation component of the accretion disc.
1. Introduction

Microquasars (MQs) are accreting black hole systems which are similar to the accreting galactic central black holes known as active galactic nuclei (AGN). The proximity and comparatively fast dynamics of MQs makes them ideal objects to observe [1]. The central black hole of the MQ is surrounded by a magnetized accretion disc. Typically, it is assumed that the magnetic dynamo effect in accretion discs yields a maximum plasma $\beta \approx 1$, an assumption which is supported by observations of a protostellar accretion disc [2]. The interaction between this magnetized disc and the black hole is thought to be a key element for the global dynamics of the MQ system. Important observations are time-dependent apparent size variations of the accretion disc, variations of the emitted electromagnetic spectrum, as well as the ejection of plasma jets by the MQ [1, 3]. The rapidly varying emission spectra and the associated ejection of plasma jets during the most dynamical state, which is discussed in detail in [1], suggest a non-stationary accretion disc. This state C is associated with a cooling of the thermal component of electrons from 2.2 to 0.6 keV, and with a hardening of the power law distribution in the high-energy tail of the spectrum. During the state $C$, the inner part of the accretion disc becomes unobservable in the x-ray band, suggesting either a cooling of the plasma or a mass loss due to material falling into the black hole, or both.

Numerical simulations show that shocks in the accretion disc [4] may be responsible for the ejection of the jet and the generation of quasi-stationary oscillations [5], and that thus they play an important role in the dynamics of MQ accretion discs. The magnetohydrodynamic (MHD) and hydrodynamic simulations of [4, 5] can capture the global dynamics of the accretion disc but they cannot reveal the microscopic physics of the shocks, e.g. the local particle acceleration. On the other hand, particle-in-cell (PIC) codes [6] that model the full set of the Maxwell equations together with the relativistic Vlasov equation for a collisionless phase space fluid cannot model large macroscopic systems. Many shock simulations with PIC codes have thus modelled only the initial evolution of shocks and only in one or two spatial dimensions [7]–[13]. These shock simulations have addressed non-relativistic flow speeds since these support planar shocks that can be modelled in less than three spatial dimensions. The formation of planar structures on a microscale are due to the dominance of plasma instabilities like the electrostatic two-stream (ETS) instability, due to which waves evolve that have wavevectors that are aligned with the plasma flow velocity vector.
For increasing relativistic flow speeds the shock structure becomes increasingly nonplanar due to the excitation of mixed modes and electromagnetic filamentary (EF) instabilities [14, 15], where wave modes can now grow at many orientations of the wavevector relative to the flow velocity direction. Relativistic shocks that one finds in some jets [16, 17] have thus been modelled in a three-dimensional (3D) geometry with PIC simulations [18]–[20]. These studies have shown the dominance of an EF instability over the ETS instability for such large plasma flow speeds. However, at present the large spatio-temporal scales of such structures can only be resolved for an electron–positron plasma or for a proton–electron plasma having a reduced mass ratio.

The dynamics within the accretion discs of MQs, however, involves plasma flows with speeds that are close to the critical flow speed at which the boundary layer switches from planar to filamentary, if the plasma is unmagnetized. A magnetic field vector that is parallel to the flow direction and a high plasma temperature would, however, reduce the growth rate of the mixed mode and the EF instabilities [21]–[24], while they would leave the growth rate of the ETS instability unchanged, provided the flow speed is large compared to the electron thermal speed. The plasma temperature and the magnetic field occurring during the steady state MQ accretion disc are probably unable to suppress filamentation. While the thermal disc radiation is suggesting a high plasma temperature [1, 3], the maximum plasma $\beta$ is probably limited to unity [2]. Stronger magnetic fields are required to suppress the mixed mode and EF instabilities. Two situations in the non-steady accretion disc may support such a strong relative magnetic field, and therefore the mildly relativistic planar shocks. Firstly, the magnetic field that is built up during the steady state phase of the accretion disc with its high plasma density may reach $\beta \approx 1$, with a high absolute magnetic field strength. During the active phase C, the inner part of the accretion disc may experience a mass loss. If the magnetic field is not convected away with the plasma, the local $\beta$ may decrease below unity, implying a dominant magnetic pressure. Secondly, the magnetic field that transfers energy from the black hole to the plasma jet flow [25] is not likely to drop significantly as we go from the accretion disc to the disc corona, probably in contrast to the plasma density. The internal shocks in MQ jets may thus also develop in a low-$\beta$ plasma.

In this paper, we examine the balance of plasmas that collide at a Mach number $v/c_s \sim 440$ in low-$\beta$ plasma, with $c_s$ being the ion sound speed, in which the magnetic field direction is aligned with the flow direction, which we discuss in detail in section 2. We observe the evolution of a turbulent boundary (TB) self-consistently out of two colliding warm and equally dense plasma slabs. This is necessary because the high Mach number related to the relative flow speed of the plasma slabs, with respect to the upstream ion sound speed, and their similar density does not lead to a full shock involving the ions [10]–[12], [26]–[29], which is confirmed in this paper. We set the relative flow speed of the colliding plasmas close to that found in [5]. We thus expand the early PIC simulations of electrostatic (ES) shocks [11, 12] into the mildly relativistic regime, while limiting the flow speed to a value at which the TB can still be considered planar for the chosen plasma temperature, magnetic field strength and direction. We verify this in section 3 by means of a 2D simulation of the initial development of the TB. The 2D simulation reveals, however, that a 1D simulation could not represent correctly the evolution of the most intense nonlinear structure far downstream that develops in response to our specific choice for the initial conditions. The TB itself can be modelled in a 1D high-resolution simulation, which we discuss in section 4. The statistically significant plasma representation minimizes effects like a reduced stability of electron phase space holes (EPSHs) [30], and an insufficient representation of the energetic particles [31] which are important for this work. In what follows, we will refer to nonlinear ES phase space structures as the EPSHs, even if they develop into partially
electromagnetic structures [32]–[34] and fill up with electrons during the later stages of their evolution. The simulation further reveals the nature of the TB, in which the dynamics is regulated by the EPSHs [35]–[38]. Our simulation shows that the developing boundary is a sequence of relativistic EPSHs, which are convected downstream. Such EPSHs are also observed close to solar system shocks [39]. The electrons are also ejected into the upstream region by the TB and reach relativistic speeds in this frame. The downstream electrons achieve a thermal energy of about 15 keV, which is comparable to the energy of the hard electromagnetic radiation of the MQ accretion disc. In section 5, we discuss the relevance of our simulations and our interpretations with regard to a better understanding of the electron acceleration to relativistic speeds within the accretion discs of MQs, and the generation of energetic electron beams by colliding plasmas that have relativistic speeds.

2. Simulation method and initial conditions

Our PIC simulation code solves the Vlasov–Maxwell equations by the method of characteristics. The phase space fluid is approximated by an ensemble of computational particles (CPs), which are then followed through space and time under the influence of the ES and electromagnetic fields that are calculated self-consistently from all CPs. These equations can be normalized, which enhances their applicability.

Let \( j_p, E_p \) and \( B_p \) be the current, the electric and magnetic fields in physical units, which we denote by the index \( p \). The electron mass, the proton mass and the magnitude of the electron charge are \( m_e, m_p \) and \( e \), respectively. We model the full proton mass \( m_p = 1836 m_e \). With the electron number density \( n_e \) and the vacuum permittivity \( \epsilon_0 \), we introduce the electron plasma frequency \( \omega_{pe} = (n_e e^2 / \epsilon_0 m_e)^{1/2} \). The electron cyclotron frequency is \( \omega_{ce} = e B_0 / m_e \), where \( B_0 \) is the magnitude of the background magnetic field. The electron skin depth is \( \lambda_e = c / \omega_{pe} \), where \( c \) is the vacuum speed of light. The physical mass and charge of a particle with index \( j \) are \( m_{j,p} = m_e m_j \) and \( q_{j,p} = eq_j \). We introduce \( E_p = cm_e \omega_{pe} E / e \) for the electric field, \( B_p = m_e \omega_{pe} B / e \) for the magnetic field and \( j_p = n_e c e j \) for the current. The time is \( t_p = t / \omega_{pe} \), the space \( (x_p, y_p, z_p) = (x, y, z) \lambda_e \) and therefrom \( \nabla_p = \lambda_e^{-1} \nabla \) and \( dt_p = dt / \omega_{pe} \). The velocity \( v_p = cv \). The equations that are solved by a relativistic and electromagnetic PIC code are

\[
\nabla \times E = -\frac{\partial B}{\partial t}, \quad (1)
\]

\[
\nabla \times B = j + \frac{\partial E}{\partial t}, \quad (2)
\]

\[
\frac{dp_j}{dt} = \frac{q_j}{m_j}(E + v \times B), \quad (3)
\]

where \( \mathbf{p}_j = v_j \Gamma(v_j) \) is the normalized relativistic momentum of the particle species \( j \). We will refer to the evolving momentum components \( p_x, p_y, p_z \) in physical units, which are normalized explicitly by \( m_{p,c} \) or \( m_{e,c} \). The Poisson equation and \( \nabla \cdot \mathbf{B} = 0 \) are fulfilled by the virtual particle PIC scheme [6]. The particles and the fields are connected through the currents, which are calculated from the ensemble of the CPs.

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As an initial condition, we consider the case of two colliding plasma slabs, as sketched in figure 1, each occupying one half of the simulation box. The slabs are spatially homogenous along the $y$- and $z$-directions, and we employ periodic boundary conditions in both directions. The normal of the collision boundary, the magnetic field direction $\mathbf{B}$ and the beam flow directions $\pm v_b$ are aligned with the $x$-axis. The speed modulus of the colliding plasma slabs $v_b$ is chosen such that the relative collision speed in the beam frame is $v_r = 2v_b/(1 + v_b^2) = 0.35$. This value $v_r$ is close to the maximum plasma speed jump found in the hydrodynamic simulations of [5]. Each cell in the simulations has the equal side lengths $\Delta x = \Delta y = \Delta z = 1/33.75$. In section 3, we resolve in the 2D simulation the $x$-direction by $N_x = 1600$ cells and the $y$-direction by $N_y = 200$ cells. The centre of the box along the $x$-direction is at $x_0 = 23.7$. The 1D simulation in section 4 resolves the $x$-direction by $N_x = 6 \times 10^4$ cells and $x_0 = 888.6$. The unresolved directions are represented by one interior cell and two boundary cells. In the following, the term TB refers to the boundaries that develop by physical processes in front of the respective plasma slabs. Both are located at $x_0$ at the simulation’s start (figure 1) but then the two TBs expand in opposite directions. Note that the TB is not a shock, since the protons are not thermalized by it. The full shock will develop once the protons are slowed down by energy dissipation [11] or possibly by collisions, to a Mach number of a few, provided the colliding plasma is not highly asymmetric [10]. The simulation is advanced in time with the step duration $\Delta t = 1/70.3$. We advance the 2D system in section 3 by $T_1 = 4000 \Delta t \approx 56.9$ and the 1D system in section 4 by $T_2 = 8.4 \times 10^4 \Delta t \approx 1200$. Each of the slabs is represented by $N_1 \approx 6.1 \times 10^7$ CPs for the electrons and by $N_2 \approx 2.3 \times 10^7$ CPs for the protons. All plasma species have the same number density $N$ and the same temperature $T_0 = 6.9 \times 10^6$ K, which equals the temperature ($\sim 0.6$ keV) of the thermal emissions during stage C [1]. The Mach number is $M_s/c_s \approx 440$, where $c_s = (K_BT_0/m_p)^{1/2}$ is the ion acoustic speed and $K_B$ is the Boltzmann constant.

The ETS instability that initially develops in an unmagnetized plasma between the two equally dense colliding electron beams, which move at a speed modulus $v_b$, leads to the growth of ES waves at the exponential rate $\gamma_T \approx \omega_{pe}/2\Gamma(v_b)$, as discussed, for example, in [40]. By solving the linear dispersion relation of a fast electron beam that is moving through an unmagnetized background plasma [14, 41], one finds that the ETS instability is not the fastest growing instability. Without an external magnetic field, the boundary would wiggle, as in previous shock simulations [13]. We thus impose a magnetic field so that the electron gyrofrequency
Figure 2. The panel (a) shows the normalized $E_x$ field amplitude at the simulation’s end $t = T_1$. The panel (b) shows the corresponding normalized $E_y$ field amplitude. The plasma slabs have moved the distance modulus $|x - x_0| = 10.3$ and thus overlap over an interval with the width 20.6.

is $\omega_{ce} = \omega_{pe}$. With the permeability $\mu_0$ of free space, we obtain for an isothermal electron–proton plasma a value $\beta = 4N\mu_0 K_B T_0 / B_0^2 \approx 4.7 \times 10^{-3}$.

3. Initial evolution of the instability: 2D simulation

Here we verify that the magnetic pressure and the plasma temperature are high enough to suppress, at least initially, the mixed mode and filamentation instabilities to yield a planar TB. The pure ETS instability leads to planar ES structures, since our slabs are spatially homogeneous in the plane orthogonal to $v_b$. Since the ES field vector is aligned with $v_b$, we expect a growth of the $E_z$ field component. The $E_z$ component of the electric field does not grow in the simulation (not shown). The electromagnetic fields, at least during the linear phase of the instability, are given by the $E_y$ field component.

We show the spatial distribution over a suitable box interval of the $E_x$ component in figure 2(a) and that of the $E_y$ component in figure 2(b) at $T_1 = 56.9$. Both slabs have moved a distance modulus $|x - x_0| \approx 10.3$ along the $x$-direction. The figure 2 reveals a planar TB. The two energetic structures at a distance modulus $|x - x_0| \approx 2$ are accompanied by an oscillating magnetic field in the $B_z$ direction (not shown) and they are thus partially electromagnetic. At the time $t = T_1$, they are predominantly ES and quasi-planar and are connected to an electron
Figure 3. The electron density for $x > x_0$, normalized by the total electron number density $2N$, at the simulation’s end at $t = T_1$. A value of unity corresponds thus to twice the electron density of a single slab. The density drops to 0.5 for $x - x_0 > 10.3$, where the slabs do not yet overlap.

density depletion of about 30%, which is displayed in figure 3 for a subsection of the simulation box at the time $t = T_1$. The density depletion is bounded by a thin high-density layer, which shows the electron charge density modulations along the $y$-direction.

To obtain further information about the time evolution of the charge density modulations, we examine the growth of the electric field. We perform a Fourier transform of the electric field $E_y$ component over the full box length along the $y$-direction and for each cell in the $x$-direction. This is done for each time step $t_s$ with available field data. We square the result and obtain the spatial power spectrum

$$E_y^2(x, k_y, t_s) = N_y^{-2} \left| \sum_{j=1}^{N_y} E_y(x, j\Delta y, t_s) \exp(-ik_y j\Delta y) \right|^2,$$

(4)

with $N_y$ being the number of simulation cells in the $y$-direction, $\Delta y$ being their respective length, $l$ taking integer values and $k_y = 2\pi l/N_y \Delta y$.

We integrate this spatial power spectrum over the $x$-interval $x_0 < x < 26.7$, which gives the averaged power spectrum $\hat{E}_y^2(k_y, t_s)$. Since the density depletion is the only structure with a significant $E_y$ component in figure 2, it will be the main contributor to $\hat{E}_y^2$. Figure 2(b) reveals that the depletion supports a dominant electric field modulation with the wavenumber $\hat{k}_y = 6\pi/N_y \Delta y$. The associated spatial power spectrum $\hat{E}_y^2(\hat{k}_y, t)$ over the full simulation time is plotted in figure 4. The power spectrum shows a clear modulation with the period $\Delta T \approx 10$ and the oscillation period of the electric field amplitude will thus be $2\Delta T$, giving the frequency $\omega \approx \omega_{pe}/3$. The oscillations of the density depletion can thus not be due to the non-oscillatory filamentation instability. The mixed polarization $|E_x| > |E_y| \neq 0$ would suggest mixed modes [24] but their wavevectors would not point along the symmetry axis of the density depletion. The oscillation is thus likely to be an eigenmode of the plasma structure.
The strong electric fields found in figure 2 will yield a rapid electron flow. The TB shows planar electric fields and the partially electromagnetic structure at \( x - x_0 \approx 2 \) is still quasiplanar. To visualize the electron flow, we can therefore integrate the electron phase space distribution over the \( y \)-direction. The strong \( B \) field implies a link between the \( p_y \) and \( p_z \) momentum components and we introduce the perpendicular momentum \( p_{\perp}^2 = p_y^2 + p_z^2 \). We show the phase space distribution at \( t = T_1 \) in the \( x, p_{\perp} \) plane in figure 5(a) and in the \( x, p_x \) plane in figure 5(b).

The strong electron density depletion in figure 2 is caused by an EPSH or, in 2D, a phase space tube. The exponential growth of \( \hat{E}_{\gamma}^2(\hat{k}_y, t) \) in figure 4 shows that it is unstable in 2D [34], whereas it would be stable in 1D [35]. Such phase space tubes have been examined elsewhere [32, 33] for non-relativistic plasma flows, which have shown that the tubes kink. This tendency is also revealed by the figure 3 and it is this kink deformation that is leading to the growing \( E_y \) field.

This kink motion has been explained in [32, 33] in terms of the interaction between the tube and an ES whistler. For other plasma parameters, the large EPSH may thus radiate; strong whistlers are actually observed close to non-relativistic shocks and they may play an important role in the energy dissipation and electron acceleration at such structures [13, 42]. Figure 5(b) shows a second strong EPSH at the leading edge of the slab. This EPSH shows no modulation in the \( p_{\perp} \) direction and it is thus ES. The difference between both EPSHs is that the ES one is time-evolving, which is probably suppressing the kink instability.

4. 1D simulation of the TB layer

4.1. Initial development

In the previous section, we have demonstrated that with suitably chosen values of the magnetic field and plasma temperature [24], the boundary between the two colliding plasma slabs can be
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Figure 5. The panel (a) shows the 10-logarithm of the electron phase space density in the $x, p_{\perp}$ plane. The panel (b) shows the electron phase space density in the $x, p_x$ plane. Both distributions are sampled at the simulation’s end $t = T_1$.

considered planar for collision speeds as high as $v_r = 0.35$, at least over some spatio-temporal intervals. We exploit this planar behaviour and examine the plasma collision with a 1D simulation. More specifically, we discuss the TB that develops at the leading edge of the plasma slab 1 in figure 1, which expands from the initial position $x = x_0$ towards increasing $x$-values. The term TB will henceforth refer to this boundary. The system is symmetric against the initial contact coordinate $x_0$, as demonstrated by figure 6. Two dominant groups of waves move symmetrically from $x_0$ towards positive and negative $x$-directions. The fast ES waves are following the slab front. These are associated with a purely ES-EPSH, as shown in figure 5. Two slower EPSHs, the partially electromagnetic ones from figure 5, are formed during the initial development phase of the ETS instability and they are thus closely related to our initial plasma conditions. The phase space distributions of the electrons in the 1D and 2D simulations agree up to this time, as found from a comparison of figure 5(b) and the electron distribution at the time $t = 63$ shown in figure 7. Both EPSHs trap electrons of both plasma slabs and they are thus a key mechanism for the electron phase space mixing and thermalization.

The solitary EPSH at $x - x_0 \approx 2$ in figure 7 moves at a speed $v \approx 2.5 \times 10^{-2}c$, which is less than the initial electron thermal speed. This is a characteristic of solitary EPSHs [35]. The slow EPSHs in our simulation are well defined and stable because they developed out of initially cooler plasma. Figure 6 also reveals that these have been the first EPSHs to appear. The limited

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Figure 6. The $E_x$ field amplitude close to $x_0$. At $t \approx 10$ strong ES fields develop. The broad field distributions move with the slab front. Two symmetric bipolar pulses are generated during the initial instability. They are slower than $v_b$ until $t \approx 170$ when they accelerate. A multitude of weaker bipolar pulses can also be observed.

Figure 7. The EPSHs at the simulation time $t = 63$. The colour shows the 10-logarithm of the number of CPs. Panel (a) shows the electrons of the plasma slab 1 and panel (b) shows the electrons of the plasma slab 2.

overlap between both plasma slabs at this early time caused the formation of two strong solitary EPSHs. Solitary EPSHs cannot collapse through the sideband- or the coalescence instability and are thus stable in one space dimension. The solitary EPSHs are thus a consequence of our initial conditions that imposed a discontinuity in velocity across $x_0$ and cooler plasmas on either side of $x_0$. The solitary EPSHs eventually accelerate to $v \approx v_b$ (figure 6), probably in response to the
increasing electron temperature or due to their interaction with the proton beams [43]. We further find weaker bi-polar structures in the downstream region between the two leading ES-EPSHs.

To visualize the TB, we show the $E_x(x, t)$ field in figure 8, in the reference frame of the plasma slab 1. The TB has developed between the upstream region ($x_0 - v_b t > 0$) that is occupied only by the plasma slab 2 and the downstream region ($x_0 - v_b t < 0$). This boundary is not time-stationary. Field structures emerge from this TB and they move into the upstream and downstream regions. We perform a Fourier transform of the $E_x(x, t)$ data in figure 8 over time and square the result. We show the resulting power spectrum in figure 9. This power spectrum reveals the turbulent nature of the boundary. The broadband oscillations are Doppler shifted to lower frequencies owing to the measurement of $E_x(x, t)$ in the reference frame of the plasma slab 1.

The boundary fluctuations evolve on electron and proton timescales. Figure 10 displays the electron phase space distribution close to the TB at $t = 626$. At this time, the slab has propagated a distance $x - x_0 \approx 113$ into the upstream region. The boundary is constituted by a well defined but non-stationary EPSH. Its dynamical evolution, e.g. its deformation, collapse and re-structuring, gives rise to a TB. The EPSH does grow to a size that allows the trapping of the electrons of both plasma slabs. This is confirmed unambiguously with the help of a separation of the electrons of the plasma slab 1 in figure 10(a), and those of the plasma slab 2 in figure 10(b). The downstream EPSHs are comparable in size and we therefore obtain electric fields of similar strength in the downstream region $x - x_0 - v_b t < 0$ displayed in figure 8 and, for the time $t = 626$, in figure 11. Only the $E_x$ component is shown since it exceeds the oscillations of all other field components by over two orders of magnitude. Figure 11(b) shows electric field pulses close to the TB. The oscillatory fields in figure 11(c) evidence plasma structures that outrun the TB, which is confirmed by figure 12 that shows the electron distribution at the same time. The electrons which are transported by these structures originate from both plasma slabs and are thus downstream electrons that have leaked through the boundary, and reflected upstream electrons. These EPSHs give rise to the ES structures outrunning the TB in figure 8.
Figure 9. The power spectrum $\log_{10}|E_2^2|$ as a function of $\Omega = \omega/\omega_{pe}$ in the reference frame of slab 1. The leading edge of the plasma slab 1 is located at the zero position. The boundary is not time stationary. Its reformation involves oscillations with $\omega < \omega_{pe}$. The horizontal line shows upstream waves with $\omega \approx \omega_{pe}$.

Figure 10. The EPSH that constitutes the TB at the time $t = 626$. The leading edge of the slab is located at the position $x - x_0 = 113$. Panel (a) shows the electrons that originate from the slab 1. Panel (b) shows the electrons of slab 2. Both slabs contribute to the trapped electron population at the boundary.
Figure 11. The electric field $E_x$ components at the time $t = 626$. Panel (a) shows the large scale field. Panel (b) shows the electric field close to the TB and panel (c) shows the upstream electric field.

Figure 12. A train of EPSHs is emerging from the boundary and perturbing the upstream electrons, as shown here at $t = 626$. The colour scale is defined by the number of CPs. Panel (a) shows the density of reflected electrons. We do not display the electrons of both slabs separately due to their low number density. Panel (b) shows the 10-logarithmic density of the incoming upstream electrons.
Figure 13. EPSHs close to \( x_0 \) at \( t = 626 \). We find a highly structured electron phase space distribution at \( x_0 \), which is 113 \( \lambda_e \) or 2.6 proton skin depths behind the boundary. Panel (a) shows the electrons of the plasma slab 1. Panel (b) shows the electrons of the plasma slab 2. The colour shows the 10-logarithm of the number of CPs.

From figure 6, we find stable bipolar ES field structures that are convected downstream. These are the EPSHs and are present even at the position \( x_0 \). The EPSH distribution at the time \( t = 626 \) is displayed in figure 13 and they involve practically the whole electron bulk population. The strong EPSH at \( x - x_0 \approx 16 \) in figure 13(a) is constituted primarily of the electrons of the plasma slab 1. Its large ES potential, however, picks up electrons also of the plasma slab 2, as figure 13(b) is showing.

The strong EPSH at the leading edge of slab 1 in figure 6 interacts also with the protons, which is revealed at \( t = 626 \) by figure 14. The EPSHs close to the slab front yield a negative charge layer that drags the protons with them. The protons of the leading edge of the plasma slab 1 in figure 14(a) are thus starting to form phase space holes and we observe the acceleration of protons. The slab front is also spreading out. The strong potential that is supported by the plasma slab 1 also accelerates the protons of the plasma slab 2. The formation of phase space holes in the proton distribution is a key mechanism for the proton thermalization and the resulting formation of a full shock [11, 12]. This will require, however, a significantly longer time than \( t = 626 \).

4.2. Late times

Our simulation evolves the TB until \( T_2 = 1200 \). During the time interval \( 626 < t < T_2 \) the TB evolves as a dynamic structure. The qualitative TB structure and phase space distribution of the downstream electrons remains, however, unchanged. This is demonstrated by the supplementary movie 1, which shows the electron phase space distribution close to the TB for \( 626 < t < T_2 \). The proton distribution, on the other hand, is evolving under the influence of strong ES fields associated with the EPSHs at the TB. It develops filaments, as displayed by the supplementary movie 2, which shows the proton distribution close to the TB during the times \( 626 < t < T_2 \).
Figure 14. The proton distribution at the time $t = 626$. The colour scale corresponds to the 10-logarithm of the number of CPs. The panel (a) shows the proton distribution of the plasma slab 1 and the panel (b) shows that of the plasma slab 2.

Figure 15. The electron phase space distribution at $t = T_2$. The colour scale shows the 10-logarithm of the number of computational electrons. The leading edge of the slab is located at $x - x_0 \approx 216$.

The global electron phase space distribution at $t = T_2$ is displayed by figure 15 that shows a system, in which a hot downstream electron population is separated from a cooler upstream electron population by a turbulent ES layer. We will now consider the downstream electron plasma, the boundary layer and the upstream region separately.

In figure 16, the front end of the plasma slab 1 has expanded by about 216 $\lambda_e$ or five proton skin depths into the upstream region at $t = T_2$. We see relativistic EPSHs at this position.
Figure 16. The electron phase space distribution close to the front of slab 1 at $t = T_2$. The relative position is $x' = x - x_0 - v_b t$. The colour denotes the 10-logarithm of the number of computational electrons. Dense relativistic downstream EPSHs at $x' < 10$ are separated from tenuous upstream EPSHs at $x' > 10$.

The expected slab boundary separates dense relativistic EPSHs from tenuous ones. The dense EPSHs show up to the left of the boundary and may be associated with the downstream plasma. The tenuous EPSHs propagate freely to the right, well ahead of the slab boundary, and they belong to the upstream region. The dense EPSHs are convected downstream, as we find from figure 6, and they eventually thermalize. The spatial interval that is resolved by our simulation appears to be large enough to facilitate this process, since figure 17 shows a quasi-thermal electron distribution close to $x_0$ at $t = T_2$, with only a few remaining EPSHs.

The electron momentum distribution in the far downstream region can be compared with that in the far upstream region by integrating the phase space distributions over space at the time $t = T_2$. The result is shown in figure 18. The downstream electron distribution is a flat-top distribution, as found also for the final distribution of relativistic ETS instabilities [31, 44], close to solar system shocks [45] and in early simulations of ES shocks [11]. The plateau expands to $p_x/m_e c \approx 0.25$ and the downstream temperature is thus $\approx 15$ keV. The flanks of the distribution decrease slower than exponential and can be approximated by the power law $\sim p_x^{-12}$. The upstream distribution is a superposition of the unperturbed upstream electron population of the slab 2 and a tenuous electron beam. The beam electrons reach a $p_x/m_e c \approx 1$. The boundary moves with $v_t = 0.35 c$ in the upstream frame and a specularly reflected electron beam would move at the speed $2v_t/(1 + v_t^2) \approx 0.66$. In the reference frame of slab 2 that is the upstream plasma, however, the peak speed of the electron beam in figure 18 is $|v| \approx 0.8$. The supplementary movie 1 shows that EPSHs form ahead of the TB and some of the fastest upstream electrons are due to beam relaxation processes. The varying sizes of the EPSHs close to the TB can accelerate further electrons to high energies.

The proton distribution close to the leading edge of the plasma slab 1 at $t = T_2$ has further expanded, as depicted in figure 19. The charge distribution associated with the EPSHs has accelerated the protons by about $20\%$, and the fastest protons have outrun the theoretical boundary.
Figure 17. The electron distribution far from the front end of slab 1 at \( t = T_2 \). The relative position is \( x' - x_0 - v_b t \) and \( x_0 \) has the location \( x' \approx -216 \). The colour denotes the 10-logarithm of the number of computational electrons. The EPSHs have filled up with electrons, and the electron plasma has thus thermalized.

Figure 18. The downstream and the upstream electron distributions in units of computational electrons at the time \( t = T_2 \). We integrate the phase space distribution of the downstream electrons from \( x_0 \) to \( x_0 + 40 \) and display the result by the dashed curve. A power law \( N_e(p_x) \sim p_x^{-12} \) is overplotted at large \( p_x \). The upstream electrons are integrated from \( x_0 + 345 \) to \( x_0 + 385 \) (solid curve).
Figure 19. The 10-logarithmic proton distribution of slab 1 close to the boundary at \( t = T_2 \). The relative position is \( x' = x - x_0 - v_b t \) and \( p_0 = m_p v_b c \Gamma(v_b) \), where \( v_b \) is in normalized units. The protons outrun the expected location of the boundary at \( x' = 0 \) due to their acceleration by the electron charge layer and start to form phase space holes.

location by about 15 \( \lambda_e \). As a consequence, the initial sharp drop of the charge density at the plasma boundary has smeared out. The low density \( \text{[10]} \) of this proton beam precursor and the charge density modulations of the proton beam \( \text{[46, 47]} \) may also give rise to a shock.

5. Discussion and conclusions

The present simulation studies have addressed the microphysics of the equally dense plasma slabs that are colliding at mildly relativistic speeds. For the plasma flow speeds that exceed the ion acoustic speed by less than a few times, ES shocks are formed. This has been demonstrated by the early PIC simulations in the \( \text{[11, 12]} \). These shocks dissipate their energy by trapping electrons \( \text{[26]} \) or by reflecting ions \( \text{[27, 28]} \). Our aim here has been to assess whether ES shocks can form between identical plasma slabs that are colliding at a Mach number 440 and without the involvement of the EF instability \( \text{[18, 19]} \). Such initial plasma conditions have been achieved by selecting a plasma flow speed that is limited to mildly relativistic values and a strong magnetic field that is aligned with the plasma flow velocity vector \( \text{[24]} \).

Mildly relativistic plasma flows in a moderately hot plasma are thought to exist in the accretion discs of MQs \( \text{[3, 5]} \). Accordingly, we have considered plasma slabs that collide at the maximum flow speed of \( v_r = 0.35c \), which the hydrodynamic simulations of \( \text{[5]} \) have predicted. Our ambition has been to model the plasma slabs colliding at the maximum Mach number that is thought to be realistic, and we have thus chosen a plasma temperature of 600 eV for all plasma species. This temperature is similar to that of the thermal emissions observed from MQs during the active phase C of the accretion disc of GRS 1915 + 105 \( \text{[1, 3]} \). We have demonstrated with the help of a 2D PIC simulation that the speed, temperature and magnetic field aid the ETS instability to dominate. The only exception has been a slowly moving solitary EPSH
that originated from the selection of our initial conditions, and that showed signs of instability in 2D. Since the ETS instability can develop planar TBs, as demonstrated by the 2D simulation, we could represent the colliding plasma slabs in one spatial dimension at a high simulation resolution.

Our simulations have shown the development of a TB layer that dissipates the plasma flow energy by processes involving electron phase space structures. The EPSHs have not been confined to the boundary, but they have expanded into the downstream and upstream for distances of a few proton skin depths. The downstream electron distribution far from the TB has approached a flat-top distribution \([11, 31, 44, 45]\) with a temperature of about 15 keV that is comparable to the energy of the hard electromagnetic MQ emissions. The flanks of the flat-top distributions decrease as the power law \(p_x^{-12}\). An important finding of the present study has been the formation of a relativistically fast electron flow, reaching a peak momentum \(p_x \approx 1.3m_ec\), or \(\Gamma \approx 1.6\) in the reference frame of the plasma slab 2. These electrons with energies of a few 100 keV originate from both the downstream and upstream regions, and they have been accelerated by the EPSHs located at the front end of the plasma slab 1. We can understand this process as a reflection of the upstream electrons and an electron leaking from the hot downstream population into the upstream, similar to the ion leaking at astrophysical plasma shocks \([48]\). It is likely that an ambient perpendicular magnetic field component will further enhance the electron energies, e.g. by the electron surfing acceleration mechanism \([46, 49, 50]\), by a shock surfing acceleration \([51]\), or by other acceleration processes, e.g. plasma wave accelerators, wave breaking or secondary waves \([31, 47, 52, 53]\). We may thus obtain an energetic electron population that can precipitate into the disc wind, into a hot halo population and possibly into a disc jet. The simulation results presented here may also be representative for internal shocks of MQ jets and, in this case, the reflected electrons may form superluminal jet structures.

Our PIC simulations confirm that the upstream ions cannot be trapped or reflected by the electric fields of the boundary at the considered high Mach number and shocks involving also the protons do not form during the simulation time. The presence of the EPSHs has therefore, during the considered simulation time, not changed the global dynamics of the high Mach number shock sketched out in \([12]\). The proton slabs can interpenetrate and move along ballistic paths. A phase space mixing of the upstream and the downstream protons may eventually be accomplished by the formation of phase space holes in the proton distribution \([11, 12]\). An interaction between protons, electrons and incoherent ES fields \([40]\), instabilities involving oblique modes \([53]\), or the formation of shocks out of plasma flow speed gradients \([47, 54]\) may further thermalize the plasma. The protons of the plasma slab are also accelerated due to the appearance of negative charge layers ahead of the shock, as it is well-known for the interaction of lasers with solids \([55]\). These fast tenuous proton precursors may give rise to asymmetric shocks \([10]\). It is thus evident that the microphysics of the colliding plasma slabs is important on the spatio-temporal scales considered here.

The accretion disc of the GRB 1915 + 105 probably has a typical hydrogen number density and, by charge neutrality, also an electron number density of \(N_e \approx 10^{12} \text{ cm}^{-3}\) \([3]\). If the plasma density in the inner accretion disc drops by a factor 100 during the state C, the density and the corresponding plasma frequency would be \(N_i \approx 10^{10} \text{ cm}^{-3}\) and \(\omega_{pe} \approx 6 \times 10^9 \text{ s}^{-1}\). An important observation of this study is that the protons of both plasma slabs maintain mildly relativistic relative flow speeds during the physical time \(1200/\omega_{pe} \approx 2 \times 10^{-7}\). During this time, a slab moving with the speed \(v_r = 0.35c\) would move a distance approximately 20 m. The visible formation of proton structures may indicate that the full shock will form on a spatio-temporal
range that is exceeding our simulation time by a factor 10. The inner accretion disc has the much larger radial extent of tens of kilometres [1]. The hydrodynamic and MHD simulations by [4, 5], which assume a full shock, may thus be valid on the macroscopic scale of the accretion disc.

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