Least Square Estimation Network for Depth Completion

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Abstract—Depth completion is a fundamental task in computer vision and robotics research. Many previous works complete the dense depth map with neural networks directly but most of them are non-interpretable and can not generalize to different situations well. In this paper, we propose an effective image representation method for depth completion tasks. The input of our system is a monocular camera frame and the synchronous sparse depth map. The output of our system is a dense per-pixel depth map of the frame. First we use a neural network to transform each pixel into a feature vector, which we call base functions. Then we pick out the known pixels’ base functions and their depth values. We use a linear least square algorithm to fit the base functions and the depth values. Then we get the weights estimated from the least square algorithm. Finally, we apply the weights to the whole image and predict the final depth map. Our method is interpretable so it can generalize well. Experiments show that our results beat the state-of-the-art on NYU-Depth-V2 dataset [19] both in accuracy and runtime. Moreover, experiments show that our method can generalize well on different numbers of sparse points and different datasets.

I. INTRODUCTION

In many areas such as robotics and autonomous driving, depth estimation is a fundamental task. However, the sensors nowadays are not fully competent for this challenge. RGBD cameras can detect a dense depth map indoor but fails when light changes outdoor. Distance sensors such as LiDAR can detect accurate distance but its measurements are sparse and they are always expensive.

Recently, many depth estimation methods based on deep neural network are proposed and attract much attention. A kind of methods estimate dense depth map only from single camera frame with the help of neural network [6] [16] [7] and achieve promising result on benchmarks. However, predicting dense depth only from a single frame is an ill-posed problem due to that the scale information is missing. With the depth values of some pixels known, the problem changes to a depth completion problem and becomes less ill-posed.

There are mainly three kinds of methods to solve the depth completion problem. In the first one, the sparse depth map is interpolated to a dense one. The interpolation is guided by the hand-crafted features or learned features. Takeda et al. [20] regards the interpolation as a non-parametric estimation and uses a hand-crafted kernel to guide the interpolation. Liu et al. [9] replaces the hand-crafted kernels with the learned kernels. The kernel functions are learned from the guided images. Compared with [20], the interpolated functions are more flexible. However, these kernel regression process is computationally costly and its performance is limited. The second methods use an end-to-end network to predict the dense depth map. The input of the network is the RGB image and the sparse depth map. The output of the network is a dense depth map. Ma et al. [11] predicts the dense depth using convolutional neural network. Bhutani et al. [1] also predict extra information like the per-pixel confidence or object norm map to supervise the results from different aspects. These methods can be trained end-to-end and perform well in some specific datasets. But they have difficulty when generalize in different situations. The third method is an iterative optimization process. Cheng et al. [4] applies the convolutional spatial propagation network to optimize the initial depth map using a learned affinity matrix of the image. The sparse depth map is optional to guide the propagation process. Park et al. Park et al. [14] apply a flexible neighbor pixel choosing strategy for learning the affinity matrix. It can learn the affinity from non-local pixels. These methods can get a satisfactory result due to the iterative process. However, the initial depth map they use comes from other methods. For example, Cheng et al. [4] uses the depth map from the output of Sparse-to-

Fig. 1. Introduction of different methods of predicting depth map using RGB image and sparse depth map. (a) Kernel Regression methods. (b) End-to-End convolutional neural network methods. (c) Our proposed methods. First estimate the base functions of the RGB image and then using sparse points to estimate the weights of the base functions.
Dense [11] network.

In our paper, we propose an image representation method for depth completion task. We estimate a vector to represent each pixel in RGB image, which we call base functions. The depth value of each pixel is the weighted sum of these base functions. We use the iterative re-weighted least square algorithm [8] to estimate the weights of these base functions to fit the depth value of each pixel. Then we apply the weights to the whole image to estimate the dense depth map. In this way, we use a neural network to represent the pixels and build a linear relationship between the pixels’ base functions and the depth values. Experiments show that our method can generalize to different datasets successfully. Moreover, if the depth values of the sampled points have a scale factor to the ground truth values, the whole depth map we predict will have the same scale factor to the ground truth. This feature will benefit a lot in the SLAM applications for if we use the sparse depth values estimated by the SLAM system, the values may have a scale factor different from the ground truth. Experiments show that our method reaches the state of the art result both in accuracy and run time. In conclusion, our contributions are summarized as follows:

1) We propose an image representation method for depth completion task, which can be trained end-to-end. The experiments on NYU-Depth-V2 benchmark show that our method reaches the SOTA results both in accuracy and run time.

2) We build a linear relationship between the set of base functions and the depth values. Thanks to the linear relationship, our model has a great generalization ability on different situations such as different sample points and different datasets.

We organize our paper as follows: Section II will review the related works of related area. We will explain our method in details in Section III Section IV will present the experiment results in details. Finally we will conclude the paper in Section V.

II. RELATED WORKS

A. Monocular Depth Estimation

Many previous works have done on predicting the depth map from a monocular RGB image. Early works [7] [13] always rely on hand-crafted features and probabilistic graphical models. Recently many state-of-the-art methods based on supervised models are proposed and make good use of convolutional neural network. These models are always end-to-end. In addition, some other works [21] [22] use multi-task model to synchronously predict the dense depth map and segmentation result, which can benefit the performance for each other. Some methods also predict the uncertainty map at the same time. The uncertainty map is used to finetune the depth map. Bhutani et al. [1] propose an unsupervised methods also demonstrate promising results.

Although many monocular depth estimation methods reach promising results, the task that predicting a dense depth map only from a single RGB image is highly ill-posed. The scale information is unknown. Moreover, many neural network based methods only has a good performance on the dataset it used in training. But they fail when used on other dataset or in real world environment. In our work, apart from RGB image, we make use of the sparse depth map to estimate the dense depth map, which is a less ill-posed problem.

B. Depth Completion

Depth completion task is to predict a dense depth map using a monocular RGB image along with the sparse depth map. In SLAM applications, sparse depth maps can be easily get from distance detectors like LiDAR or from multi-frame tracking such as structure-from-motion algorithm. Many state-of-the-art SLAM methods such as ORB-SLAM [13] and VINS-MONO [15] can predict accurate sparse depth from a set of continuous RGB frames.

Many methods use end-to-end neural network to complete depth from sparse to dense. Ma et al. [11] reached a promising result which only consist of convolutional neural network. With a modern GPU it runs fast which make itself available in many real-time applications such as SLAM. Takenda et al. [20] propose a non-parametric statistics estimation method to interpolate the sparse depth map to a dense one. But the kernel it used is hand-crafted and fixed. Liu et al. [9] promote the algorithm using a learned kernel to replace the fixed kernel. Essentially kernel regression methods are still interpolating methods.

C. Spatial Propagation Methods

Although direct depth completion method have promising performance, spatial propagation with guidance can predict more accurate results. [spatial propagation] use a neural network to predict the affinity matrix of the RGB image and use the affinity matrix to iteratively finetune the coarse dense depth to a final one. [4] promote the work by replacing the traditional spatial propagation process with a convolutional spatial propagation process. Compared with the traditional SPN, it propagates four directions at once which increase the accuracy and fasten the process. [Non local] promote the convolutional spatial propagation process by using a flexible set of neighbor points to replace the fixed set. By doing so it can learn the affinities of non-local points and is more robust. However, these spatial propagation methods needs a coarse depth first. [4] use the [11] to predicts the initial depth and refines it.

D. Multi Frames Optimization

In SLAM applications, we can get a continuous camera frames. [2] use a VAE network in the system and use a latent code to represent each depth map. It optimizes the camera pose and the latent code of each frame simultaneously. [5] integrates learned priors over geometry in a probabilistic factor-graph formulation. [12] at the same time predict a reprojection error image using SLAM reprojection error.
They all can reconstruct the environment from a set of monocular images.

In Spatial propagation methods and multi frames optimization methods, an initial dense depth is needed and plays an important role in the whole system. In our works, we predict the dense depth map from the monocular image and sparse depth points in real time. We test our model on predicting the dense depth map from the monocular image. They all can reconstruct the environment from a set of monocular images.

III. METHOD

We will describe the whole pipeline of our method in this section. We first describe the architecture of the convolutional neural network. Then, we discuss the position encoding strategy, the least-square algorithm used in our method and the loss function used for training.

A. Neural Network Architecture

The overall target of our system is to estimate a dense depth map using a guided RGB image and a sparse depth map. We divide our system into three main parts. We will describe each part in details next.

Feature Extraction. Traditional CNN detector shows excellent performance in the image regions which have rich object context information. But in some ill-posed regions which have few textures it is difficult to detect the features. We follow the idea in [3] and extract features from the RGB image using a pyramid-like convolutional neural network. We use 4 different size average pooling blocks in the network to extract different level features. Then we concatenate them and output the feature map of the RGB image.

Position Encoding Strategy. Traditionally, we usually focus on the pixel itself of an image but ignore the coordinate information of the pixel. According to [10], adding a coordinate layer into the feature map can improve the performance in many applications including depth prediction. For example, two same objects in one image may have similar features. But in depth prediction they should not be considered as the same region. Therefor we can discriminate them by their coordinates. Moreover, according to [5], mapping the coordinate to high dimension contributes to representing the high frequency features. This enables better fitting of image features. The encoding function we use in the method is as follows:

\[
\gamma(p) = [x, x, y, \sin(2^0 \pi x), \cos(2^0 \pi x),
\sin(2^0 \pi y), \cos(2^0 \pi y), ..., \sin(2^{E-2} \pi x),
\cos(2^{E-2} \pi x), \sin(2^{E-2} \pi y), \cos(2^{E-2} \pi y)]
\]

\(E\) in equation [1] is the number of dimensions of position encoding. In our method, we set \(E = 10\). \(p = (x, y)\) in function denotes pixel position. \(x\) denotes coordinates along the width and \(y\) along the height.

Base Function Estimation. We regard each pixel as a single object, which has \(f\) dimensions of feature and \(p\) dimensions of position. We concatenate the feature map and the position map as the input of a full-connected neural network. The output of the full-connected neural network is a set of base functions of each pixel. We regard each pixel as the weighted sum of its base functions.

\[
D(c, p) = w_0 + \sum_{i=1}^{N} w_i \cdot F_i(\phi(c), \gamma(p)), p \in \Omega.
\]

\(D\) is the depth value of the pixel. \(c = (r, g, b)\) is the color value of each pixel. \(p = (x, y)\) is the coordinate of each pixel. \(N\) is the number of base functions. \(W = [w_i], (i = 0, 1, ..., N)\) is the weights vector to be estimated. \(\Omega\) is the whole image plane.

B. Least Square Algorithm and IRLS Method

We sample the depth points in the ground truth depth map randomly. The total number of valid depth pixels in ground truth depth is \(m\). We randomly sample \(n\) sparse points in the whole depth map. Each valid point in the depth map has the same probability to be sampled. So we should optimize \(W\) to minimize the cost function:
in training. Our training loss consists of three parts. We will

\begin{align}
  \arg\min_{W, b} \sum_{p \in D_0} \|D_{gt} - (w_0 + \sum_{i=1}^{N} w_i \cdot F_i(\phi(c), \gamma(p)))\|. \tag{3}
\end{align}

This is a classical least square question. In real world applications, there might be some noise in the sparse depth map. We can easily apply an iterative re-weighted least square method to reduce the influence of the noise. The algorithm is shown in algorithm 1. But the iterative re-weighted process is optional. If the sparse data we get is reliable, we don’t have to implement the process and only the original least square algorithm is needed.

Algorithm 1 IRLS
Input: sampled depth vector $D$, sampled base functions $F$.
Output: $W = W_{init} = (F \cdot F^T)^{-1} \cdot (F^T \cdot D)$
1. while not meet the stopping criterion do
2. \quad $e = \text{abs}(D - F \cdot W)$
3. \quad $e' = (\max(\text{threshold}, e))^{-1}$
4. \quad $\text{Weight} = \text{diag}(e')$
5. \quad $W = (\text{Weight} \cdot F \cdot F^T)^{-1} \cdot (\text{Weight} \cdot F^T \cdot D)$
6. end while

C. Loss Function
In this part we will explain the loss functions we used in training. Our training loss consists of three parts. We will explain each part in details next.

1) Loss on predicted depth. We directly estimate the error between the predicted depth and ground truth depth:

\begin{align}
  \ell_{\text{direct}} = \|D_{gt} - D_{\text{predict}}\|'
\end{align}

When $t = 1$ it represents $\ell_1$ loss and $t = 2$ represents $\ell_2$ loss. We use both $\ell_1$ loss in our method in our training.

2) Loss on depth gradient. Previous work has shown that applying depth gradient loss in training can significantly improve the performance. The gradient of the depth map contains information about edges of objects. Therefore this loss supervise the boundaries predictions and makes them sharper.

\begin{align}
  \ell_{\text{grad}} = \|\nabla D_{gt} - \nabla D_{\text{predict}}\|
\end{align}

3) Loss on regularization. We regard the depth value of each pixel as a weighted sum of base functions. To make the most use of each base function and avoid overfitting, we apply a regularization loss function in training.

\begin{align}
  \ell_{\text{regular}} = \|\sum_{i=0}^{N} w_i\|
\end{align}

The total loss is the weighted combination of the above loss functions:

\begin{align}
  \mathcal{L} = \alpha \ell_{\text{direct}} + \beta \ell_{\text{grad}} + \gamma \ell_{\text{regular}}
\end{align}

IV. Experiments
In this section, we will first introduce the details of NYU-Depth-V2 Dataset and then compare our results on benchmark with some state-of-the-art methods. Then we will compare the generalization potential of our methods with others. At last, we will show some novel features of our methods which will benefit follow-up use of the initial dense depth map in many areas such as SLAM.

A. Training on NYU-Depth-V2 Dataset
1) NYU-Depth-V2 Dataset. The NYU-Depth-V2 dataset contains about 407k RGB frames with depth map in 464 different indoor scenes. There are 1449 labeled frames in it whose depth are labeled densely. A small set including 654 images is officially chosen for benchmarking. The resolution of the raw image in NYU-Depth-V2 is $480 \times 640$. We first downsample the image to half the resolution and the center-crop them to $228 \times 304$ in training and evaluating.

2) Experiment Setup. We implement our method using PyTorch 1.8 with CUDA 10.2. We train our model on NYU-Depth-V2 dataset using a NVidia 2080Ti GPU. The weights of the network are initialized randomly. We use a small batchsize 1 and train the model for 100 epoches which spend about 3 days. We set the learning rate to 1e-4 for the first 30 epoches and reduce it to 1e-5 for the last epoches. We set the weight decay to 1e-4 for regularization.

3) Evaluation Metrics. We adopt the standard evaluation metrics. Assume that the ground truth depth value at position $x_i$ is $d_i = D_i(x_i)$. The predicted depth value at position $x_i$ is $\hat{d}_i = \hat{D}_i(x_i)$. The evaluation metrics are specified as follows:

Fig. 3. Compare results of experiments on NYU-Depth-v2 benchmark. In all methods the sampled number in sparse depth map is 1000. (a) RGB Image. (b) Ground Truth. (c) Prediction of Proposed Methods. (d) Regression result of Learned Steering Kernel. (e) Prediction of Sparse-to-Dense Method.
### Table I

**Depth Completion Benchmark on NYUv2**

| Method                      | Error(↓)RMSE | Error(↓)REL | Accuracy(↑)δ_{1.25} | Accuracy(↑)δ_{1.25^2} | Accuracy(↑)δ_{1.25^3} | Runtime(s)(↓)|
|-----------------------------|--------------|-------------|----------------------|-----------------------|-----------------------|----------------|
| Sparse-to-Dense             | 0.230        | 0.044       | 97.12                | 99.37                 | 99.83                 | 0.022          |
| Steering Kernel             | 0.221        | 0.043       | 96.87                | 99.51                 | 99.83                 | -              |
| Learning Steering Kernel    | 0.198        | 0.034       | 97.73                | 99.80                 | 99.89                 | 0.114          |
| Proposed Method             | **0.148**    | **0.031**   | **98.72**            | **99.84**             | **99.95**             | **0.015**      |

1. All methods take 500 sampled points as sparse depth measurements.
2. The result of the interpolated depth of Learning Steering Kernel.

**B. Benchmark Evaluation**

We evaluate our model on NYU-Depth-V2. Table I shows the comparison on NYU-Depth-V2 benchmark of accuracy and runtime. Our model outperforms Sparse-to-Dense and the interpolated result of Learning Steering Kernels both in accuracy and runtime. Learning Steering Kernel contains a kernel regression process which cost much time. Sparse-to-Dense is an end-to-end convolutional neural network so it is faster. However its structure is more complex so the time consuming is more than ours. Our method mainly contains feature extraction and least square algorithm. The feature extraction part is consist of convolutional neural network which process on GPU and run fast. The least square algorithm is a linear calculation which also run fast. In depth completion task runtime is an important indicator because many applications should run in real time, such as SLAM application.

**C. Ablation Study**

**Position Encoding:** To prove that the position encoding part contributes to the performance, we conduct ablation study on position encoding. We set the number of base functions to 128 from beginning to end. The number of sampled depth points is 1000. We found that without position encoding the network will converge to a condition which performs worse than before. The results are shown in Table II.

**Generalization wrt. Numer of Sampled Points:** Sparse points are only used in the least square calculation part in our methods. The more points we have, the better fitting performance we will show. Due to the reason that our fitting process is a linear process, the relationship between performance increasing and the points number increasing can be basically regarded as positive linear correlation, compared with the End-to-end depth completion method. We compare the different methods with different sampled points number. Shown as figure 4 and figure 5, our methods and Learning Steering Kernel have a obviously increasing of...
performance while sampled points number increases. However, the Sparse-to-Dense method show little improvement with sampled points number increasing.

**Generalization wrt. Different Scales:** In SLAM applications, predicted 3D coordinates of sparse feature points may have a scale factor estimation error. In our method, the relationship between base functions and the depth value is linear. Therefore if the sparse depth values have a multiple error with the ground truth data, the final predicted depth of our method will have the same multiple error with the ground truth data. This feature is good for use in SLAM applications. We test our method on NYU-Depth-V2 dataset. We manually multiply the sparse depth by a factor and predict the dense depth. Then we calculate the evaluation error between the predicted dense depth and the ground truth depth multiplied by the same factor. In figure 7, we can see that the performance of the Sparse-to-Dense method fluctuates greatly, while the Learning-Steering-Kernel Method and the proposed method basically don’t change.

**Generalization wrt. Different Datasets:** We use the model trained on NYU-Depth-V2 dataset and directly test it in the KITTI dataset which is an outdoor driving dataset to test the generalization. We use a selected subset of KITTI which contains 1000 RGB images along with depth maps. The depth maps are not dense but they have more valid points than we need. In three methods, we all sample 1000 points. For comparison, we all center-crop the image to the size $228 \times 304$. The results are listed in table III. The predict depth image is shown in figure 6.

**TABLE II**

| Method                | RMSE   | REL   |
|-----------------------|--------|-------|
| With Position Encoding| 0.1285 | 0.02707 |
| Without Position Encoding | 0.1705 | 0.0375 |

**TABLE III**

| Method                | RMSE $\delta$ | REL $\delta$ | Accuracy $\%$ |
|-----------------------|---------------|---------------|---------------|
| Sparse-to-Dense       | 1029.95       | 0.099         | 92.9          |
| Proposed Method       | 600.74        | 0.051         | 97.9          |

1 All methods take 1000 sampled points as sparse depth measurements.
2 The unit is millimeter.

**V. CONCLUSIONS**

In this paper, we proposed a least square estimation network to complete the sparse depth map to a dense one. With experiments on standard dataset we show that our performance is promising both in accuracy and runtime. Besides, we show that our estimation has a great generalization ability wrt. different number of sampled points. And our method is more robust facing the scale factor change of sparse depth points. There features are essential and advantageous for usage in SLAM.
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