Magnetic moments in odd-A Cd isotopes and coupling of particles with the zero-point vibrations

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Background: The coupling of the last nucleon with configurations in the ground state of the even-even core is known to augment the single quasiparticle fragmentation pattern. In a recent experimental study by Yordanov et al. the values of the magnetic dipole and electric quadrupole moments of the $11/2^-$ state in a long chain of Cd isotopes were found to follow a simple trend which we try to explain by means of incorporating long-range correlations in the ground state.

Purpose: Our purpose is to study the influence of the ground-state correlations (GSC) on the magnetic moments and compare our results with the data for the odd-A Cd isotopes.

Method: In order to evaluate if the additional correlations have bearing on the magnetic moments we employ an extension to the quasiparticle-phonon model (QPM) which takes into account quasiparticle-phonon configurations in the ground state of the even-even core to the structure of the odd-A nucleus wave function.

Results: It is shown that the values for the magnetic moments which the applied QPM extension yields deviate further from the Schmidt values. The latter is in agreement with the measured values for the Cd isotopes.

Conclusions: The GSC exert significant influence on the magnetic dipole moments and reveal a potential for reproducing the experimental values for the studied cadmium isotopes.

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I. INTRODUCTION

The important role that the particle-vibration interaction plays in explaining the deviations of nuclear magnetic moments from the Schmidt values was first acknowledged in Refs. [1, 2]. The strength of this interaction depends mainly on the state which the participating nucleon occupies as well as on the distribution of multiparticle–multihole configurations constituting the vibrating core. Intuitively clear, still supported by the experimental data, is the finding that this interaction is weak near the magic nuclei and becomes substantial in the open-shell regions. For example, performing calculations relying on the first and second order perturbation theory, as shown in Ref. [3], this interaction manifests itself as capable of explaining the differences between the magnetic moments in the near magic odd-A Tl isotopes. In the open-shell regions, however, there is a large number of nuclei, referred to as transitional, in which the diversity of configurations contributing to the vibration grows rapidly and the terms which could be neglected near the magic configurations are no longer small. In this respect the data and analysis by Yordanov et al. [4], which we interpret in this work, are important for at least two reasons. In the first place, the long chain of cadmium isotopes, which is explored in Ref. [4], enters the transitional region with respect to the neutron sub-system. Secondly, the measured quantities, namely, the magnetic dipole and electric quadrupole moments, not only triggered the invention of the nuclear shell model and the collective model [5, 6] but still are a major tool to test the validity of modern nuclear theories.

The theoretical interpretations of the data concerning the quadrupole moments, reported in Ref. [4], were previously carried out by using two kinds of pairing models - the seniority scheme [7] and the BCS approximation [8]. Decent agreement with the experimental data is reached by the authors of Ref. [4] using the seniority model at the cost of introducing an effective neutron charge and by neglecting configurations with seniorities greater than 1. A more robust study, which does not compromise the use of effective charges, is performed in Ref. [9] by employing the BCS approximation to account for the pairing of nucleons. The important result of this study is the estimation that the contribution from the polarized core with 40 protons to the quadrupole moment is as large as the contribution from the valence protons in the g9/2 orbital. The authors of Ref. [10] elaborated on the influence of the density dependence of the effective pairing interaction on the quadrupole moments in odd-N Sn and Pb nuclei, which they found to be notable and produce deviations between 10% and 50% in different isotopes. The electric quadrupole moments in odd-A Sn nuclei have been successfully reproduced in Ref. [11] using the nucleon-pair approximation of the shell model, and also the magnetic moments were found to be very...
close to their single-particle estimates.

If the pairing correlations seem sufficient to describe the trends in the behavior of the quadrupole moments, the magnetic moments of the low-lying states require further efforts mainly due to the role of the $1^+$ magnetic excitations and the nucleon correlations in the ground state induced by the long-range part of the residual nucleon-nucleon force. Although the contribution to the wave function coming from configurations owing to the coupling with the magnetic giant dipole resonance are small their influence on the magnetic moments is significant because of the strong M1 transition to the ground state. In Ref. [15] a systematic theoretical analysis of experimental data on magnetic moments in different nuclei is performed utilizing the theory of finite Fermi systems. The effects of the M1 giant resonance on the magnetic moments (the Arima-Horie effect) were the main focus of many research studies while the contribution from other modes seem to be a less explored territory. The configuration mixing generated by such modes can be taken into account in the calculations of the magnetic moments by either introducing effective two-body operators [13, 14] or evaluating the average values of the single-particle operator in multiparticle configurations.

The influence on the magnetic moments coming from the coupling of the last nucleon with the low-lying collective quadrupole and octupole core excitations is studied in Ref. [15]. The generated admixtures from these interactions were found to give important contributions in most of the studied isotopes but are most considerable in odd-Z nuclei.

In the present development we account for the above effects relying on the concepts and instruments of the quasiparticle phonon model (QPM) [16] without making use of two-body operators for the magnetic moment. Of special interest for our research is the evaluation of the role of the quasiparticle and the quasiparticle-phonon configurations residing in the ground state of the even-even core. This problem, which has not been explored so far, is approached by allowing non zero values for the amplitudes in the wave function of the odd-A nucleus corresponding to the above configurations [17-20]. Our previous investigation on this topic showed considerable fragmentation of the low-energy single-quasiparticle states due to such long-range correlations. The latter is a crux in reliably applying the perturbation theory, which otherwise yields rather tractable results [18-21]. In applying this idea using the so-called backward-going amplitudes in the odd-A nucleus wave function to calculate the expectation value of the magnetic operator we registered considerable shifts in the magnetic moments. The subtle effects owing to the account of the ground state correlations is what distinguishes our work from the studies performed in Ref. [15].

The meson exchange currents between the nucleons inside a nucleus, which modify the single-particle nature of the magnetic moment operator [22], are taken into account by introducing effective $g_\text{e}$-factors values.

This paper proceeds as follows. In Sec. II we outline the basics of the approach that we utilize to estimate the magnetic moments. Numerics and physical interpretations are the subject of Sec. III. The results of this work are summarized in Sec. IV.

II. THEORETICAL FRAMEWORK

In the approach that we follow, the properties of the nuclear states are interpreted as a result of the interaction between two types of fictitious particles - quasiparticles and phonons represented in spherical basis by operators denoted by $a_{jm}$ and $Q_{\lambda\mu}$, respectively [16]. In this framework the odd-even nuclei are formed by the interaction of the last quasiparticle with the ground and excited states of the even-even core. The possibilities for the last particle to couple with different states of the even-even core are accounted for by constructing a wave function as a mixture of one quasiparticle and quasiparticle-phonon pure states [17, 18]:

$$\Psi_{\nu}(JM) = O^\dag_{JM\nu}$$  (1)

with

$$O^\dag_{JM\nu} = C_{J\nu} a^\dag_{JM} + \sum_{jM} D_{jM}(J\nu) P^\dag_{JM}(JM) - E_{j\nu} \tilde{a}_{JM} - \sum_{j\lambda} F_{j\lambda}(J\nu) \tilde{P}_{j\lambda}(JM),$$  (2)

where $|\rangle$ denotes the ground state of the even-even core, $P^\dag_{JM}(JM) = [a^\dag_{JM} Q_{JM}]$ is the quasiparticle-phonon creation operator and $\tilde{a}$ stands for time conjugation according to the convention $\tilde{a}_{jm} = (-1)^{j-m} a_{jm}$. The last terms of this equation address the non-zero probabilities for the last quasiparticle to interact with quasiparticle and quasiparticle-phonon configurations residing in the ground state of the even-even core. The importance of these terms to the magnetic and electric moments is discussed in detail in Sec. III.

The dynamics of the physical setting described in this way is governed by the following Hamiltonian:

$$H = H_{MF} + H_{PAIR} + H_{RES},$$  (3)

which includes a part representing the integral effect from the mean-field generating forces of the nucleon-nucleon interaction, monopole pairing field and the residual central long-range interaction between the spatial and spin degrees of freedom:

$$H_{RES} = H_M + H_{SM}.$$  (4)

Assuming this part of the interaction in a separable form we expand it by multipoles and spin-multipoles:
\[ H_M = -\frac{1}{2} \sum_{\rho=\pm 1} (\epsilon_{\rho}^{(\lambda)} + \rho \kappa_{\rho}^{(\lambda)}) \sum_{\tau=\pm 1} M_{\mu}^{\dagger}(\tau) M_{\mu}(\rho), \quad (5) \]

\[ H_{SM} = -\frac{1}{2} \sum_{L=\lambda,\lambda \pm 1, \rho=\pm 1} (\epsilon_{\rho}^{(\lambda L)} + \rho \kappa_{\rho}^{(\lambda L)}) \sum_{M=\pm 1} (S_{LM}^{\lambda})^{\dagger}(\tau) S_{LM}^{\lambda}(\rho), \quad (6) \]

where \( \tau \) enumerates the neutron (\( n \)) and proton (\( p \)) subsystems.

\[ M_{\mu}^{\dagger} = \sum_{jj',mm'} \langle jm | i^\lambda R_\lambda(r) Y_{\lambda\mu} | j'm' \rangle a_{j'm'}^{\dagger} a_{jm}, \quad (7) \]

and

\[ (S_{LM}^{\lambda})^{\dagger} = \sum_{jj',mm'} \langle jm | i^\lambda R_\lambda(r) |j'm' \rangle a_{j'm'}^{\dagger} a_{jm}, \quad (8) \]

are the single-particle multipole and spin-multipole operators [16]. From the sum over \( L \) in Eq. (6) we include only the terms with \( L = \lambda - 1 \). In the phonon space, the eigenstates of this part of the interaction are of unnatural parity \((-1)^L\). Of particular interest to our present research are the \( 1^\pm \) states which in QPM account for the dipole core spin polarization [15], induced by the \( \sigma \sigma \) forces. The reduced matrix elements related to equations (7) and (8) are denoted by \( f^{(\lambda)} \) and \( f^{(\lambda L)} \) respectively.

We obtain the eigenstates of the system, defined by the Hamiltonian (4), using an approximate step-by-step diagonalization procedure in which initially the first two terms are diagonalized using the canonical Bogoliubov transformation

\[ a_{jm} = u_j a_{jm} + (-)^{j-m} v_j a_{j'-m}^{\dagger}. \quad (9) \]

The term of the Hamiltonian (\( H_{RES} \)), which contains non-diagonal elements in the quasiparticle basis after the first step of this procedure, couples different quasiparticles to form mixed states which in the QPM are understood using the concept of phonons

\[ Q^{\lambda}_{\mu i} = \frac{1}{2} \sum_{jj'} | \psi_{jj'}^{\lambda} A^{\dagger}(jj'; \lambda \mu) - (-1)^{\lambda-\mu} \phi_{jj'}^{\lambda} A(jj'; \lambda - \mu) \rangle, \quad (10) \]

where \( A^{\dagger} \) (and its inverse) stand for the bifermion quasiparticle operator:

\[ A^{\dagger}(jj'; \lambda \mu) = \sum_{mm'} \langle jmj'm' | \lambda \mu \rangle a_{j'm'}^{\dagger} a_{jm}, \quad (11) \]

In order to determine the structure of even-even nuclei, this part of the interaction is diagonalized in a space spanned by one-phonon wave functions where it is assumed that the ground state of the even-even core is a vacuum for the phonon operators, i.e., \( Q_{\lambda\mu i} = 0 \). Of special attention is the fact that the core’s ground state from Eq. (1) contains additional correlations that are not included in the phonon vacuum state. The latter are incorporated by equating numbers rather than wave functions as theorized in the equation-of-motion method [23] which we apply in the following form:

\[ \langle \{ \delta O_{JM\nu}, H, O_{J'M\nu}^{\dagger} \} \rangle = \eta_{J\nu} \langle \{ \delta O_{JM}, O_{J'M}^{\dagger} \} \rangle. \quad (12) \]

This method allows to harness the already obtained phonon vacuum state for calculating the average values of the Hamiltonian. In Eq. (12) \( \{ \cdot, \cdot \} \) stands for the double commutator and \( \eta_{J\nu} \) is the energy of the \( \nu \)-th eigenstate with angular momentum \( J \). Despite lowering of the particle rank, the double commutator yields two-body operators whose average values still depend on the ground-state correlations. In this work, we evaluate the operators’ average values using the random phase approximation (RPA) with corrections for certain three-quasiparticle configurations affected by the Paulli exclusion principle [24].

Having determined the structural composition of the odd-even nucleus, estimates for the observable quantities of interest are obtained by evaluating the average values of the corresponding operators. For the magnetic dipole and electric quadrupole moments they are defined as

\[ \mu_1(J\nu) = \sqrt{\frac{4\pi}{3}} \langle JJ\nu | M(M; 10) | JJ\nu \rangle, \quad (13) \]

\[ Q_2(J\nu) = \sqrt{\frac{16\pi}{5}} \langle JJ\nu | E(10) | JJ\nu \rangle, \quad (14) \]

where the electric and magnetic multipole operators are expressed as

\[ M(X; \lambda \mu) = \frac{1}{\pi \lambda} \sum_{j_1 j_2} \sum_{m_1 m_2} (-1)^{j_2 - m_2} \mathcal{F}^{(\lambda)}_{j_1 j_2} (j_1 m_1, j_2 - m_2 | \lambda \mu \rangle a_{j_1 m_1}^{\dagger} a_{j_2 m_2}. \quad (15) \]

Hereafter \( \pi \lambda = \sqrt{2\lambda + 1} \). \( \mathcal{F}^{(\lambda)}_{j_1 j_2} \) are the reduced single particle matrix elements:
\[ \mathcal{F}^{(\lambda)}_{j_1j_2} = \begin{cases} e \langle j_2 | r^{\lambda} Y_{\lambda \mu} | j_1 \rangle, & \text{for electric transitions}, \\ \mu_0 \left( g_s \langle j_2 | s \nabla (r^{\lambda} Y_{\lambda \mu}) | j_1 \rangle + g \gamma^{2/3} \langle j_2 | \nabla (r^{\lambda} Y_{\lambda \mu}) | j_1 \rangle \right), & \text{for magnetic transitions}. \end{cases} \] (16)

Here \( e \) and \( \mu_0 \) are the electron charge and nuclear magneton, respectively.

The matrix elements of the electromagnetic operator \( [15] \) in nuclear wave functions derived by using the independent-particle approximation can be decomposed into two parts \([20]\) - one evaluating its expectation value in the even-even core and the other representing the matrix element of this operator between the corresponding single-particle states. For the magnetic moments, only the second term gives a contribution. Depending on the degree of correlation of the core in its ground state this simple picture gets modified by correcting these terms and also by considering additional terms which vanish in the single-particle model. To take such corrections into account, we represent all respective quantities in terms of quasiparticles \([9]\) and phonons \([10]\). In that way the matrix elements in Eqs. \([13]\) and \([14]\) are obtained in a form which can be derived from the following formulae:

\[
\langle J_2 M_2 \nu_2 | \mathcal{M} (X; \lambda \mu) | J_1 M_1 \nu_1 \rangle = \frac{\langle J_1 - M_1 \lambda - \mu | J_2 - M_2 \rangle}{\pi_{J_2}} (x_{q-p-q} + x_{q-p-h} + x_{p-h-p}) \tag{17}
\]

where \( x_{q-p-q} \) gives the transition amplitude between two quasiparticle states

\[
x_{q-p-q} = \langle \left\{ \left( C_{j_2 \nu_2} \alpha_{j_2 M_2} - E_{j_2 \nu_2} F_{j_2}^\dagger (J_2 \nu_2) \right) \mathcal{M} (X; \lambda \mu) C_{j_1 \nu_1} \alpha_{j_1 M_1} - E_{j_1 \nu_1} F_{j_1}^\dagger (J_2 \nu_2) \right\} \rangle = (-C_{j_1 \nu_1} C_{j_2 \nu_2} + E_{j_1 \nu_1} E_{j_2 \nu_2}) \mathcal{F}^{(\lambda)}_{j_1 j_2} \psi_{j_1 j_2} \tag{18}
\]

\[ x_{q-p-h} \] evaluates the transition amplitudes between quasiparticle and quasiparticle-phonon states:

\[
x_{q-p-h} = \frac{1}{2 \pi \lambda} \sum_{i} \pi_{j_1} \left\{ C_{j_2 \nu_2} D_{j_2}^{\dagger} (J_2 \nu_1) + E_{j_2 \nu_2} F_{j_2}^\dagger (J_2 \nu_1) \right\} - (-1)^{J_1 + J_2 + \lambda} \pi_{j_2} \left\{ C_{j_1 \nu_1} D_{j_1}^{\dagger} (J_2 \nu_2) - E_{j_1 \nu_1} F_{j_1}^\dagger (J_2 \nu_2) \right\} \]

\[
\sum_{12} \mathcal{F}^{(\lambda)}_{j_1 j_2} \psi_{j_1 j_2} \psi_{j_1 j_2} \tag{19}
\]

and \( x_{p-h-p} \) corresponds to the transition amplitudes between two quasiparticle-phonon states

\[
x_{p-h-p} = \pi_{j_1} \pi_{j_2} \sum_{j_1 j_2 \nu, \nu'} (-)^{J_1 + J_2} \left\{ D_{j_1}^{\nu \nu'} (J_2 \nu_2) D_{j_2}^{\nu \nu'} (J_1 \nu_1) - F_{j_1}^{\nu \nu'} (J_2 \nu_2) F_{j_2}^{\nu \nu'} (J_1 \nu_1) \right\} \mathcal{F}^{(\lambda)}_{j_1 j_2} \psi_{j_1 j_2} \tag{20}
\]

\[ \psi_{j_1 j_2} \]

In formulae \([18]-[20]\), the plus and minus signs in \( \pm \) and \( \mp \) apply to the magnetic and the electric moment, respectively. If the residual interaction is switched off, then the terms \([19]\) and \([20]\) disappear and the term \([18]\) gives the well-known Schmidt values. The terms \([19]\) and \([20]\) have the following important difference: while the term \([19]\) involves properties of the phonons whose angular momenta coincide with the multipolarity of the transition operator, the summation in Eq. \([20]\), in contrast, runs over all angular momenta \( \lambda = 1, 2, 3 \ldots \)

\[ \text{III. RESULTS} \]

For the evaluation of the electromagnetic moments it is of great importance to know the values of the structural coefficients entering into Eqs. \([18]\), \([19]\) and \([20]\). In performing calculations for determining these coefficients we retain only the quadrupole term from the multipole expansion of the residual interaction between the nucleons’ spatial degrees of freedom and only the dipole term from the part describing the residual interaction between the spacial \([3]\) and spin \([4]\) degrees of freedom. In our investigation the strengths of these interactions are free parameters that are fixed by fitting on the experi-
TABLE I. Major components of the 11/2− state in 121Cd, 123Cd, 125Cd and 127Cd calculated using both forward (third column) and forward+backward (second column) amplitudes in the wave functions.

| Isotope | FRW+BCW | FRW |
|---------|---------|-----|
|         | C(D)    | E(F) | Component | C(D) | Component |
| 121Cd   | 0.88    | 0.02 | ν1h11/2   | 0.98 | ν1h11/2   |
|         | -0.01   | -0.42 | ν1h11/2 ⊗ 2+ | 0.08 | ν2f7/2 ⊗ 2+ |
|         | 0.14    | -0.11 | ν2f7/2 ⊗ 2+ | 0.08 | ν1h11/2 ⊗ 1− |
|         | 0.05    | 0.00  | ν1h11/2 ⊗ 1− | 0.07 | ν1h11/2 ⊗ 1− |
| 123Cd   | 0.87    | -0.01 | ν1h11/2   | 0.98 | ν1h11/2   |
|         | -0.08   | -0.41 | ν1h11/2 ⊗ 2+ | 0.10 | ν1h11/2 ⊗ 1− |
|         | 0.14    | -0.11 | ν2f7/2 ⊗ 2+ | 0.07 | ν2f7/2 ⊗ 2+ |
|         | 0.1     | 0.00  | ν1h11/2 ⊗ 1− | -0.06 | νh9/2 ⊗ 1+ |
| 125Cd   | 0.91    | -0.03 | ν1h11/2   | 0.98 | ν1h11/2   |
|         | -0.15   | -0.30 | ν1h11/2 ⊗ 2+ | -0.13 | ν1h11/2 ⊗ 1− |
|         | 0.09    | -0.10 | ν2f7/2 ⊗ 2+ | 0.10 | ν1h11/2 ⊗ 1− |
|         | 0.09    | 0.00  | ν1h11/2 ⊗ 1− | 0.07 | ν2f7/2 ⊗ 2+ |
| 127Cd   | 0.92    | -0.05 | ν1h11/2   | 0.97 | ν1h11/2   |
|         | -0.22   | -0.23 | ν1h11/2 ⊗ 2+ | -0.20 | ν1h11/2 ⊗ 2+ |
|         | 0.10    | 0.00  | ν1h11/2 ⊗ 1− | 0.10 | ν1h11/2 ⊗ 1− |
|         | 0.06    | -0.08 | ν2f7/2 ⊗ 2+ | 0.05 | ν2f7/2 ⊗ 2+ |

The interplay between the neutron subshell 1h11/2 and the quasiparticle in the ground state, one obtains an increased fragmentation of the 1h11/2 quasiparticle strength (conf. Refs. 17, 20) which causes a reduction in the contribution $x_{qp}$ to the quantities of interest. In the FRW-BCW model version the largest part of this strength is transferred to the $\nu1h_{11/2} \otimes 2^+_1$ admixture in the ground state of the even-even core which interacts most intensely with the last quasiparticle. The main part of the strength of this interaction is given by

$$V(Jj\lambda i) = \left\langle \nu1h_{11/2}^{1}\nu1h_{11/2}^{1} \otimes 2^+_1 \right\rangle = -\frac{1}{4} \frac{\pi}{\lambda} \sum_{\nu} \varphi_j^{\nu} (\lambda i \nu) \varphi_j^{\nu}$$

where $\varphi_j^{\nu}$ are the phonons’ backward amplitudes and $\varphi_j^{\nu} (\lambda i \nu)$ is defined in 17.

As seen from Eq. (22), the only two-quasiparticle state which influences this interaction strength is the one which bears nucleons from shells designated by the quantum numbers of the participating non-collectivized quasiparticles, namely $J$ and $j$. The high amplitude $\varphi_j^{\nu}$ (between 0.3 and 0.5 in different isotopes) of annihilating the two-quasineutron state $[\nu1h_{11/2} \otimes \nu1h_{11/2}]_2$ in the ground state for the formation of the $2^+_1$ phonon, explains the enhanced magnitude of the interaction between the neutron subshell 1h11/2 and the quasiparticle in the ground state of the even-even core which interacts most intensely with the last quasiparticle.
The magnetic moment, \( M \), of a nucleus is given by its charge and spin distribution, and the interaction between charged particles, i.e., protons and neutrons in the nucleus, and the magnetic field. The interaction is described by the vertex function, \( V(J,j) \), which is given by:

\[
V(J,j) = -\left<\alpha_{JM} H P_{JM}^+ (JM)\right> = \begin{cases} 
\frac{1}{\sqrt{2}} \sum_{l,j} f_{l,j}^{(1/2+)} v_{l,j}^{(1/2+)} \sqrt{\lambda^*} & \text{for } \lambda^* = 2^+, \\
\frac{1}{\sqrt{2}} \sum_{l,j} f_{l,j}^{(1/2+)} v_{l,j}^{(1/2+)} \sqrt{\lambda^*} & \text{for } \lambda^* = 1^+, 
\end{cases}
\]

where \( v_{l,j}^{(\pm)} = u_j u_j \pm v_f v_j \) with \( u_j \) and \( v_j \) being the pairing occupation numbers for the level \( j \).

The trends for these interaction vertices, imposed by the relationship in Eq. (23), when changing the neutron number are complex because of their dependence on both the pairing properties of the non-collectivized particles and the degree of collectivity of the participating phonon. For instance, the key to understanding the trends in \( V(\nu 1h_{11/2}; \nu 1h_{11/2} \otimes 2^+_1) \), plotted in Fig. 1, are the changes in the values of \( v_{\nu 1h_{11/2}; \nu 1h_{11/2}}^{(-)} \) which drop down from 0.4619 in \(^{117}\text{Cd}\) passing through 0.05 in \(^{121}\text{Cd}\) and reaching -0.530 in \(^{127}\text{Cd}\). Analogously, the line for the vertex \( V(\nu 1h_{11/2}; \nu 2f_{7/2} \otimes 2^+_1) \) is explained by the attenuation of the quantity \( v_{\nu 1h_{11/2}; \nu 2f_{7/2}}^{(-)} \) from 0.61 in \(^{121}\text{Cd}\) to 0.36 in \(^{127}\text{Cd}\). In contrast, the vertex \( V(\nu 1h_{11/2}; \nu 1h_{11/2} \otimes 1^+_2) \) does not depend on the pairing directly since \( v_{\nu 1h_{11/2}; \nu 1h_{11/2}}^{(+)} = 1 \) and its evolution over the isotope chain is driven by the collective properties of the \( 1^+_2 \) state in the respective even-even core.

The first order core polarization is manifested by the significant contribution to the magnetic moment of the quasiparticle\( \otimes 1^+ \)-phonon states. The structure of the \( 1^+ \) states has a simple form and is represented by a mixture of coupled quasiparticle states belonging to spin-orbit doublets. Although the contribution to the odd-even nucleus wave function coming from the configurations including \( 1^+ \) states is small, its importance to describing the magnetic moments is vital, as it will be discussed in the following section. The \( 1^+ \) state which mostly contributes to the structure of the first \( 11/2^- \) states in the odd-even cadmium isotopes is the one acquiring the highest degree of collectivity and having an energy in the region of 13 MeV. Its structure is dominated by the configuration \([\nu 1h_{11/2} \otimes \nu 1h_{9/2}]_1 \otimes \nu 1h_{11/2}]_{11/2} \). If the higher order correction, given by Eq. (20), could be neglected, then the coupling of the last nucleon with the \( 1^+ \) states might be treated by following the classical approaches by Blin-Stoyle and Arima and Horie, which are based on the smallness of the mixing coefficients. In our calculations we confirm that the perturbing configuration from Ref. [1], which applied to the \( 11/2^- \) state in the cadmium isotopes is \([\nu 1h_{11/2} \otimes \nu 1h_{9/2}]_1 \otimes \nu 1h_{11/2}]_{11/2} \), indeed con-
tributes the most to the magnetic moment if the backward amplitudes are not taken into account. However, the inclusion of a single complex configuration to the wave function is not adequate for describing the magnetic moments because the coupling with excitation modes of the even-even core having higher angular momenta is crucial to gain a more realistic picture of the wave function.

A. Magnetic moment

The particularities in the structure of the odd-A isotopes discussed in the previous section determine the deviations of the magnetic moment of the 11/2\textsuperscript{+}, 3/2\textsuperscript{+} and 1/2\textsuperscript{+} states from their single-particle estimates. The results from the calculations performed using the featured QPM versions are plotted in Fig. 2 and are compared to the experimental values. The dotted and dashed lines in this figure depict the most important achievement of this work - the finding that the interaction between the last quasiparticle and the ground-state phonon admixtures produces configurations which contribute significantly to the magnetic moment of odd-A nuclei and reveal a potential for reproducing their experimental values, which proves impossible if this interaction is neglected. The importance of each contribution from Eqs. (18)-(20) to the magnetic dipole moment is visualized in Fig. 3. The enhanced fragmentation due to the quasiparticle-phonon interaction in the ground state leads to systematically shrunked values of the single quasiparticle contribution \( \mu_{qp-qp} \) and to an increase in the quasiparticle-phonon contribution \( \mu_{qp-ph} \) leading to an overall decrease in the magnitude of the magnetic moment. The escalation of the magnetic transitions between different quasiparticle\( \otimes \)phonon configurations, given by \( \mu_{ph-ph} \), is due to configurations involving a quadrupole phonon, of which \( \nu \hbar_{11/2} \otimes 2_1^+ \) plays the most important role. It is worth noting that because of the weakened coupling between the quasiparticles and the quadrupole phonons in the core’s ground state near the neutron shell closure, the quantity \( \mu_{ph-ph} \) tends to diminish while \( \mu_{qp-qp} \) remain almost unchanged along the isotope chain. This interaction, however, leaves the first order core polarization term \( \mu_{qp-ph} \), describing the magnetic transitions between quasiparticle and quasiparticle\( \otimes 1^+ \)-phonon states, virtually unaffected because the latter configurations represent a negligible part in the 11/2\textsuperscript{+} state mixture.

However, despite its capacity of reaching the experimental values, this theoretical development suffers from the shortcoming (conf. [17, 20]) that the residual interaction strength, which yields results that are of sound agreement with the odd-A experimental data, generates substantially less collective 2\textsuperscript{+} \textsubscript{1} states in the even-even cores than the ones implied from the data for the neighboring even-even nuclei. The origin of this inconsistency is the set of approximation techniques embedded in the considered QPM versions, namely, the BCS and RPA, which tend to overestimate the degree of correlations in the ground state for open-shell nuclei. One path to healing this problem is to apply the more consistent Extended RPA [25], or use a tractable method based on the variational principle for the ground state, as in [20].

IV. CONCLUSIONS

The magnetic dipole moments of the low-lying states in odd-A Cd nuclei are found to be significantly affected by the correlations in the ground state. The obtained corrections allow one to reproduce the experimental values in open-shell nuclei, which proves impossible if the existence of the quasiparticle\( \otimes \)phonon configurations in the ground states of even-even is ignored. Despite the reported improvements, the calculations based on this version of the model exhibit a very high sensitivity on interaction parameters which limit its predictive power, and pertinent work in this direction is ongoing.

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FIG. 3. Contribution to the magnetic moment of the 11/2- state in the series of isotopes $^{117-127}$Cd

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