A Possible Origin of Dark Matter, Dark Energy, and Particle-Antiparticle Asymmetry

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Abstract
In this paper we present a possible origin of dark matter and dark energy from a solution of the Einstein’s equation to a primordial universe, which was presented in a previous paper. We also analyze the Dirac’s equation in this primordial universe and present the possible origin of the particle-antiparticle asymmetry. We also present ghost primordial particles as candidates to some quantum vacuum constituents.

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1 Introduction

The Einstein’s theory of general relativity is still the best theory to describe problems in astrophysics and cosmology. However, more recent observations in these two areas are apparently difficult to be explained by general relativity. It raises the possibility to consider models involving membranes and parallel universes, dark matter, dark energy, and cosmological constant to explain the behavior of large scale structures like galaxies, clusters of galaxies, and the universe. Some physicists and astronomers believe that the Einstein’s theory needs to be modified while a quantum gravity theory is not developed [1], [2], [3]. Actually, the Einstein’s theory of general relativity offers enormous analytical difficulties for problems involving non-gravitational fields. However, for the primordial universe, nature presents symmetries that make possible a better knowledge. In this paper we present a possible origin of dark matter and dark energy from a solution of the Einstein’s equation, which was presented in a previous paper, [4]. We also analyze the Dirac’s equation for ghost fermions in the primordial universe and conclude that they can be some of the constituents of the quantum vacuum. We also present a justification to particle-antiparticle asymmetry.

This paper is organized as follows. In Sec. 2, we present a primordial and spatially flat solution of the Einstein’s equation with a massive scalar Klein-Gordon field and a cosmological constant. The Jacobi equation is presented for this solution and two primordial forces are identified as dark matter and dark energy, respectively. In Sec. 3, we solve the Dirac’s equation in a vielbein basis for ghost spinors. The PCT theorem is applied and the particle-antiparticle asymmetry is analyzed. We also consider the possibility that the ghost fermions are some constituents of the quantum vacuum. In Sec. 4, we summarize and conclude the results of this paper.
2 Exact Solutions of the Einstein’s Equation

In a previous paper we obtained three solutions of the Einstein’s equation with a Klein-Gordon field and a cosmological constant. In this paper we are interested only in the spatially flat solution. In the following we review part of the referred paper and present new results. The convention used in a local basis was [4]

\[ R_{\mu\nu}^{\alpha} = \partial_{\nu} \Gamma_{\mu\sigma}^{\alpha} - \partial_{\sigma} \Gamma_{\mu\nu}^{\alpha} + \Gamma_{\mu\sigma}^{\eta} \Gamma_{\nu\eta}^{\alpha} - \Gamma_{\nu\sigma}^{\eta} \Gamma_{\mu\eta}^{\alpha} \]  

(2.1)

with Ricci tensor

\[ R_{\mu\nu} = R_{\mu\nu}^{\alpha}. \]  

(2.2)

For this convention we have the following Einstein’s equation, with a cosmological constant \( \Lambda \),

\[ R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R + \Lambda g_{\mu\nu} = -\frac{8\pi G}{c^2} T_{\mu\nu} \]  

(2.3)

where \( T_{\mu\nu} \) is the momentum-energy tensor of a massive scalar field,

\[ T_{\mu\nu} = 2 \nabla_{\mu} \phi \nabla_{\nu} \phi - g_{\mu\nu} \nabla^{\alpha} \phi \nabla_{\alpha} \phi + m^2 g_{\mu\nu} \phi^2 \]  

(2.4)

We have used \((+, -, -, -)\) signature convention and a Friedmann-Robertson-Walker line element given by

\[ ds^2 = dt^2 - \frac{d\sigma^2 e^g}{(1 + Br^2)^2} \]  

(2.5)

where \( d\sigma^2 \) is the three-dimensional Euclidian line element and \( A = 8\pi G/c^2 \), \( B = k/4a^2 \) and with \( k = 0 \), \( k = 1 \) and \( k = -1 \). We also have \( a^2 \) as a constant.

In this paper we pay attention to our spatial flat solution, \( B = 0 \). In this case the field is given by

\[ \phi = \frac{\epsilon mt}{\sqrt{3A}} + b \]  

(2.6)

with \( \epsilon = \pm 1 \) and \( b \) is an arbitrary constant.
The cosmological constant obeys the condition
\[ \Lambda = -\frac{m^2}{3} = -\frac{1}{3} \left( \frac{cM}{\hbar} \right)^2, \tag{2.7} \]
a negative value, associated with the Planck’s constant, the speed of light and a scalar particle of mass \( M \).

The corresponding line element is
\[ ds^2 = dt^2 - d\sigma^2 e^{\left[ -2\epsilon mb(\sqrt{\frac{3}{4}})t - \frac{m^2}{3} t^2 \right]}. \tag{2.8} \]

The universe is in an expansive phase. For \( t > 0 \) we choose \( \epsilon = -1 \) and \( b > 0 \). Then, we rewrite (2.8) as
\[ ds^2 = dt^2 - d\sigma^2 e^{[2mb(\sqrt{\frac{3}{4}})t - \frac{m^2}{3} t^2]}. \tag{2.9} \]

For \( t = -\tau < 0 \) we choose \( \epsilon = +1 \), and \( b > 0 \). Then, we rewrite (2.8) as
\[ ds^2 = d\tau^2 - d\sigma^2 e^{[2mb(\sqrt{\frac{3}{4}})\tau - \frac{m^2}{3} \tau^2]}. \tag{2.10} \]

In this paper we use another convention to the Riemann tensor, as follows,
\[ R^\alpha_{\mu\sigma\nu} = -\partial_\sigma \Gamma^\alpha_{\mu\nu} + \partial_\nu \Gamma^\alpha_{\mu\sigma} - \Gamma^\eta_{\mu\sigma} \Gamma^\alpha_{\nu\eta} + \Gamma^\eta_{\mu\nu} \Gamma^\alpha_{\sigma\eta}. \tag{2.11} \]

which implies
\[ R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R - \Lambda g_{\mu\nu} = \frac{8\pi G}{c^2} T_{\mu\nu}. \tag{2.12} \]

The motion will be simpler in a Fermi-Walker transported tetrad basis.
Let us consider the connection between the tetrad and the local metric tensor
\[ g_{\lambda\pi} = E^\alpha_\lambda (A) E^\beta_\pi (B) \eta(A)(B), \tag{2.13} \]
where \( \eta(A)(B) \) and \( E^\alpha_\lambda (A) \) are the Lorentzian metric and tetrad components, respectively.
From (2.8) we have

\[ E^{(0)}_0 = 1, \quad E^{(1)}_1 = E^{(2)}_2 = E^{(3)}_3 = e^{-\varepsilon m(b(\sqrt{\frac{A}{3}}) - \frac{m^2}{3}t^2)}. \] (2.14)

(2.15)

We now write the 1-form

\[ \theta^{(A)} = dx^\lambda E^{(A)}_\lambda. \] (2.16)

By exterior derivatives of (2.16) and using the Cartan’s second structure equation, we obtain

\[ R^{(1)}_{(0)(0)(1)} = R^{(2)}_{(0)(0)(2)} = R^{(3)}_{(0)(0)(3)} = -\frac{m^2}{3} + \frac{1}{2}[-2 \varepsilon mb(\sqrt{\frac{A}{3}}) - \frac{m^2}{3}t^2]. \] (2.17)

Let us present the Jacobi equation in a Fermi-Walker transported tetrad basis [5],

\[ \frac{d^2 Z^{(A)}}{d\tau^2} + R^{(A)}_{(0)(C)(0)} Z^{(C)} = 0. \] (2.18)

Substituting (2.17) in (2.18) we obtain

\[ \frac{d^2 Z^{(A)}}{d\tau^2} = \left\{-\frac{m^2}{3} + \frac{1}{2}[-2 \varepsilon mb(\sqrt{\frac{A}{3}}) - \frac{m^2}{3}t^2]\right\} Z^{(A)}, \] (2.19)

with \( A = (1, 2, 3) \).

We can rewrite (2.19) as follows

\[ \frac{d^2 Z^{(A)}}{d\tau^2} = \left\{-\frac{m^2}{3} - 2bm^2 \frac{1}{3} \left(\sqrt{\frac{A}{3}}\right) t + \frac{2m^2}{3}b^2 A + \frac{m^4}{18}t^2\right\} Z^{(A)}. \] (2.20)

The Jacobi equation will be appropriate to show the relative acceleration between two particles if we do not have to consider the metric deformation by particles. For the primordial universe (2.8), we have from (2.19) or (2.20) that two massive scalar particles in two geodesics close to each other feel two primordial forces, one attractive (dark matter) and another repulsive (dark energy) both increasing with distance. The same scalar particle will be responsible for the two metric forces which, conveniently, we have identified as dark matter and dark energy. From the gravitational point of view, the
creation of other types of matter by the universe generates three competitive forces, the two primordial forces above presented, and another which, for galaxies, can be expressed by the Newtonian gravity. Inside and outside the galaxies, the resulting force is the sum of these three forces. A correct dynamic description of one or more stars in a galaxy, depends on a set of information about galaxy evolution. Elliptical and spiral galaxies, as well as clusters of galaxies have different dynamics and different evolution processes. The Newtonian gravity is very important to describe the galaxies dynamics but it is not enough. Physicists and astronomers have concluded that the Newtonian gravity only is not sufficient to describe the galaxies dynamics. They believe in the existence of a second attractive force (dark matter) which, in association with the Newtonian gravity, governs the star dynamics. They also believe in the existence of a repulsive force (dark energy) responsible for the expansion on large scale. We believe that the presence of the two primordial forces together with the Newtonian force, can describe the galaxies’s behavior. Inside and outside the galaxies the resulting force is the sum of the three forces. The value of the constant $b$ in (2.6) and (2.8) could be fixed by experimental records of galaxies (dark matter) or cosmological expansion (dark energy) or both. Modifications in the stars motion in galaxies, can be made by appropriate adjustments in the constant $b$. It is possible that $b$ is a new constant of nature, as well as the mass $M$ of the scalar particle. 

The mass of the scalar particle can be estimated by astronomic measurements of cosmological constant. From (2.7) and (2.19) we conclude that $\Lambda$ is associated with an attractive force, for us conveniently identified as dark matter, and a repulsive force, identified as dark energy. The second term in the second member of (2.19) is positive and, therefore, identified as dark energy. Notice that the dark energy term is a function of time, of the constant $b$, and also of the term identified as the dark matter. We noticed in (2.20) that the constants $m$ and $b$ are present in the terms associated with the dark matter (attractive force) and to the terms associated with the dark energy (repulsive force). Then, $\Lambda$ is present in the dark matter and in the dark energy. In other words, it is not possible to separate them, because dark matter and dark energy are scalar particle effects. Originally, the cosmological constant was associated with a repulsive force so that the estimate given in the following is associated with a large scale expansion. It can be slightly different from a realistic and definitive value, but our objective is to point that we cannot detect the scalar particle, or, at best, we have a very
low probability of doing so.

Using the experimental limit for the constant $\Lambda$ in (2.7) [6], it is possible to obtain a superior limit for the mass of the scalar particle. The cosmological constant was estimated as

$$\Lambda < 10^{-54} cm^{-2}. \quad (2.21)$$

Using it and (2.7) we obtain

$$M < (6).10^{-65} g. \quad (2.22)$$

where (2.35) was used. There is another limit for the cosmological constant [3] given by

$$\Lambda Lp^2 < 10^{-123}, \quad (2.23)$$

or

$$\Lambda < 10^{-57} cm^{-2}. \quad (2.24)$$

Using it and (2.7) we obtain

$$M < (1.9).10^{-66} g. \quad (2.25)$$

The relationship between the electron rest mass and the mass of the scalar particle is approximately given by

$$m_e \sim (4.79).10^{38} M. \quad (2.26)$$

The universe expansion can be calculated. In other words, it is possible to calculate the starting point of the universe contraction. Using (2.33) in (2.8) we obtain

$$t \sim \frac{1}{\sqrt{-\Lambda}}. \quad (2.27)$$

Notice that we have assumed $c = 1$ in (2.8). Therefore, for numeric results involving time, we regain $ct$, so that

$$t \sim 10^{27} \frac{cm}{c} \sim (3.33)10^9 years, \quad (2.28)$$

From (2.24) and (2.27) we have

$$t \sim (3.162)10^{28} \frac{cm}{c} \sim (33.4)10^9 years. \quad (2.29)$$
Note that (2.28) is incompatible with the geological data of the Earth. If (2.29) is a good estimate, it will be almost impossible to detect the scalar particle. Its influence will be predominantly gravitational and it is given by (2.8). Consequently, for many classical situations as, for instance, the solar system dynamics, the effect on the ordinary matter would be insignificant. For this condition we consider only the ordinary matter in the Einstein’s equation. The scalar particle can be very important for galaxies, clusters of galaxies, and large structures. It is necessary an investigation to evaluate the influence of an intense gravitational field generated by a classical black hole geometry in the primordial scalar particle.

The primordial universe (2.8) starts with scalar particles and is non-singular at $t = 0$. It is an expansible universe if the curvature $R^{(3)}(A)_{(A)}$ obeys a simple inequality. With the time evolution, other types of matter were created and complex interactions among particles are checked every day. Analytical solutions of (2.10) with the inclusion of other fields are very difficult. However, as the influence of the primordial universe (2.8) could have been very important in the past and can be very important in the present, it is reasonable to suppose that other primordial particles are ghosts, so that (2.8) is a consequence of the scalar particles only. In other words, the momentum-energy tensors of other primordial fields have not contributed to the curvature of the primordial universe in the past nor in the present, although such particles interact with all that, playing an important part in the evolution of the universe as well as in the creation of the ordinary matter. We recall that in the cosmological models, metrics as the Friedmann-Robertson-Walker are important to the initial large structure formation as well as to the universe evolution. But, gradually each local distribution of matter will be more and more important and the effect of all distributions of matter in the universe is represented by a moment-energy tensor of a fluid in the Einstein’s equation for a Friedmann-Robertson-Walker metric. However, if (2.8) is responsible for the dark matter and the dark energy, we will have a different situation. In this case, (2.8) would determine the evolution of the universe in the past and in the present, and due to the mass estimate of the scalar particle, its interaction with other particles would be predominantly gravitational.

It is important to notice the presence of two different time scales, one associated with a local distribution, as well as with a large scale structure of ordinary matter, and another associated with the cosmological time of (2.8). The embedding of (2.8) and of a classical metric in an $n$ dimensional flat
space is a possible strategy to consider the gravitational interaction between
the scalar particles and the ordinary matter.

The primordial universe (2.8) is non-singular at $t = 0$. It is cyclical and etern-
al, and could have different cycles. Although it is not the only possibility, a
negative curvature is the simplest mechanism for an expansive universe and
it will be considered.

For (2.8) the curvature is given by

$$R^{(A)}(A) = -2\left(\frac{cM}{\hbar}\right)^2 + 6\left[-2 \in \frac{cM}{\hbar}b\left(\sqrt{\frac{A}{3}}\right) - \left(\frac{cM}{\hbar}\right)^2 \frac{t}{3}\right]^2.$$  

(2.30)

We have at $t = 0$

$$R^{(A)}(A)(t = 0) =$$

$$= 2\left(\frac{cM}{\hbar}\right)^2[-1 + 4b^2.A],$$

(2.31)

which is a finite curvature. We consider an initial negative curvature $R^{(A)}(A)$
as the simplest condition for the primordial expansive universe

$$R^{(A)}(A)(t = 0) < 0,$$

(2.32)

so that

$$\|b\| < \frac{1}{2\sqrt{A}},$$

(2.33)

or

$$\|b\| < \frac{c}{2\sqrt{8\pi G}},$$

(2.34)

or

$$\|b\| < 10^{15}\sqrt{\frac{g}{cm}},$$

(2.35)
where (2.35) is a superior limit for \( b \). We have obtained two superior limits for the mass of the scalar particle and for the constant \( b \) given by (2.22) and (2.35), respectively. Note that \( b \) and \( M \) can be two new constants of nature. For (2.24), \( M \) will be given by (2.25), smaller than (2.22), reinforcing the previous conclusion that it is very difficult to detect this scalar particle. Note that our choice of an initial negative curvature, as the expansion mechanism, imposed a superior limit for the constant \( b \). However, other mechanisms are possible, so that the constant \( b \) can assume another limit, compatible with experimental results.

In section 3 we use the first quantization to interactions among primordial scalar particles, primordial ghost fermions, and ordinary fermions.

### 3 The Dirac’s Equation

In this section we consider the Dirac’s equation and show, using the PCT theorem, the particle-antiparticle asymmetry in a primordial universe as (2.8). To preserve the solution (2.8) and consequently the Einstein’s equation, it is reasonable to suppose that other primordial fields are ghosts, so that (2.8) is a consequence of the scalar particles only. In other words, the momentum-energy tensors of other primordial fields have not contributed and still do not contribute today to the curvature of the primordial universe, although such particles interact with all that, playing an important part in the evolution of the universe as well as in the creation of the ordinary matter. These other primordial fields can be identified as some quantum vacuum constituents.

In the following, we consider the Dirac momentum-energy tensor [7], [8], [9],

\[
T_{(A)(B)} = i\{\bar{\psi}\gamma_{(A)}\nabla_{(B)}\psi + \bar{\psi}\gamma_{(B)}\nabla_{(A)}\psi - \nabla_{(A)}\bar{\psi}\gamma_{(B)}\psi - \nabla_{(B)}\bar{\psi}\gamma_{(A)}\psi\}. \tag{3.1}
\]

For ghost fields, the momentum-energy tensor obeys the following condition [10],

\[
T_{(A)(B)} = 0. \tag{3.2}
\]

The Dirac’s equation in a curved space is given by

\[
i\gamma^{(D)}(\partial(D) - \Gamma_{(D)})\psi - \bar{m}\psi = 0, \tag{3.3}
\]
where
\[
\Gamma(D) = -ieA(D)I - \frac{1}{4}\gamma_{(A)(B)(C)}\gamma^{(A)}\gamma^{(B)},
\] (3.4)
and the \(\gamma_{(A)(B)(C)}\) are the Ricci’s rotation coefficients.
For two spaces, one flat and the other curved, the Dirac’s matrices are related by
\[
\gamma^{(A)} = E^{(A)}(x)\gamma^{\lambda}(x).
\] (3.5)
In this paper we use the following representation
\[
\gamma^{(0)} = \begin{pmatrix}
I & 0 \\
0 & -I
\end{pmatrix}
\] (3.6)
\[
\gamma^{(k)} = \begin{pmatrix}
0 & \sigma^{k} \\
-\sigma^{k} & 0
\end{pmatrix}
\] (3.7)
\[
\gamma^{k} = \frac{1}{R}\gamma^{(k)},
\] (3.8)
where from (2.15)
\[
R = e^{-2(\sqrt{\frac{T}{3}})t + \frac{m^2 + e^2}{2}}.
\] (3.9)
For (2.8) we have
\[
\Gamma_{(0)} = -ieA_{(0)}I,
\] (3.10)
\[
\Gamma_{(k)} = -ieA_{(k)} + \frac{R}{2R}\gamma^{(k)}\gamma^{(0)}.
\] (3.11)
Substituting (3.3) in (3.2) we obtain
\[
\gamma^{(D)}[i\partial(D) - eA(D) + i\frac{1}{4}\gamma_{(A)(B)(C)}\gamma^{(A)}\gamma^{(B)}]\psi - \bar{m}\psi = 0,
\] (3.12)
where \(\bar{m}\) is a fermion mass.
By a charge conjugation we obtain
\[
\gamma^{(D)}[i\partial(D) + eA(D) + i\frac{1}{4}\gamma_{(A)(B)(D)}\gamma^{(A)}\gamma^{(B)}]\psi^{(c)} - \bar{m}\psi^{(c)} = 0.
\] (3.13)
Using (3.12) into (3.1), we verify the condition (3.2) if
\[
A_{(k)} = 0,
\] (3.14)
A_{(0)} = 0, \quad (3.15)

and

ψ = ψ(t), \quad (3.16)

where ψ(t) is a function of time only. Explicitly

\partial_t \psi + \frac{3 \dot{R}}{2R} \psi + i \bar{m} \gamma^{(0)} \psi = 0, \quad (3.17)

\partial_t \psi^{(c)} + \frac{3 \dot{R}}{2R} \psi^{(c)} + i \bar{m} \gamma^{(0)} \psi^{(c)} = 0, \quad (3.18)

In the following we present the difference between ψ_l and ψ^{(r)}. ψ^{(r)} is one of the four linearly independent solutions of the Dirac’s equation in the form of column matrices, and ψ_l is an element of one of these four matrices. We use similar considerations for the charge conjugate solutions. Substituting (3.16) and (3.9) in (3.12)

\partial_t \psi_j + \{i \bar{m} c^2 + \frac{3 \dot{R}}{2R}\} \psi_j = 0, \quad (3.19)

\partial_t \psi_l + \{-i \bar{m} c^2 + \frac{3 \dot{R}}{2R}\} \psi_l = 0. \quad (3.20)

Explicitly

\partial_t \psi_j + \{i \bar{m} c^2 - 3 \in mb \sqrt{\frac{A}{3} - m^2 t}\} \psi_j = 0, \quad (3.21)

\partial_t \psi_l + \{-i \bar{m} c^2 - 3 \in mb \sqrt{\frac{A}{3} - m^2 t}\} \psi_l = 0, \quad (3.22)

where \( j = 1,2 \) and \( l = 3,4 \).

By integration of (3.19) and (3.20) we have

\psi_j = e^{-i \bar{m} c^2 t} \exp \left(-\frac{3 \dot{R}}{2R} c_j \right), \quad (3.23)

\psi_k = e^{i \bar{m} c^2 t} \exp \left(-\frac{3 \dot{R}}{2R} c_k \right), \quad (3.24)
where \( c_j \) and \( c_k \) are constants.

The four linearly independent solutions are given by

\[
\psi^{(r)} = e^{-i\bar{m}c^2t} \exp\left(-\frac{3\bar{R}}{2R}\right)u^{(r)} \quad (3.25)
\]

\[
\psi^{(s)} = e^{i\bar{m}c^2t} \exp\left(-\frac{3\bar{R}}{2R}\right)u^{(s)} \quad (3.26)
\]

Explicitly we have

\[
\psi^{(r)} = e^{-i\bar{m}c^2t}e^{i\bar{m}b\sqrt{\frac{T}{2}}t_{\pm}^{2}}u^{(r)}, \quad (3.27)
\]

\[
\psi^{(s)} = e^{i\bar{m}c^2t}e^{i\bar{m}b\sqrt{\frac{T}{2}}t_{\pm}^{2}}u^{(s)}, \quad (3.28)
\]

where

\[
u^{(r)} = \begin{pmatrix} \delta_{1r} \\ \delta_{2r} \\ 0 \\ 0 \end{pmatrix} \quad (3.29)
\]

\[
u^{(s)} = \begin{pmatrix} 0 \\ 0 \\ \delta_{3s} \\ \delta_{4s} \end{pmatrix} \quad (3.30)
\]

where \( r = 1, 2 \) and \( s = 3, 4 \).

Note that the wave function (3.16), for the metric (2.8), is given by

\[
\psi = \psi(t) = e^{i\bar{m}b\sqrt{\frac{T}{2}}t_{\pm}^{2}}\left[\sum_r c_r e^{-i\bar{m}c^2t}u^{(r)} + \sum_s d_s e^{i\bar{m}c^2t}u^{(s)}\right],
\]

\[
(3.31)
\]

with

\[
||c_1||^2 + ||c_2||^2 = ||d_3||^2 + ||d_4||^2,
\]

\[
(3.32)
\]

obtained from the condition

\[
\bar{\psi}\psi = 0,
\]

\[
(3.33)
\]
for ghost fermions in (2.8).

Note that (3.33) is also true if (3.16) is given by one of the four solutions (3.27) and (3.28), or a linear combination of two solutions, one from (3.27) and the other from (3.28). The above is also true for ghost neutrinos. For ghost neutrinos we have

$$\partial_t \psi_j - \{3 \in mb \sqrt{\frac{A}{3}} + m^2 t\} \psi_j = 0, \quad (3.34)$$

$$\partial_t \psi_l - \{3 \in mb \sqrt{\frac{A}{3}} + m^2 t\} \psi_l = 0, \quad (3.35)$$

where \(j = 1, 2\) and \(l = 3, 4\). The solutions are given by

$$\psi^{(r)} = e^{\frac{3}{2} \sqrt{\frac{A}{3}} t + \frac{m^2}{2} t^2} u^{(r)} \quad (3.36)$$

$$\psi^{(s)} = e^{\frac{3}{2} \sqrt{\frac{A}{3}} t + \frac{m^2}{2} t^2} u^{(s)} \quad (3.37)$$

where \(r = 1, 2\) and \(s = 3, 4\).

Now we will define the following wave functions

$$\Phi^{(s)} = \exp \left( \frac{3}{2R} \right) \psi^{(s)}, \quad (3.38)$$

$$\Phi^{(r)} = \exp \left( \frac{3}{2R} \right) \psi^{(r)}, \quad (3.39)$$

so that

$$\Phi^{(r)} = e^{-imc^2 t} u^{(r)}, \quad (3.40)$$

$$\Phi^{(s)} = e^{imc^2 t} u^{(s)}. \quad (3.41)$$

The wave functions (3.40) and (3.41) are the four linearly independent solutions of the Dirac’s equation for a rest particle in the Minkowski space-time

$$\partial_t \Phi + i\bar{m}c^2 \gamma^{(0)} \Phi = 0. \quad (3.42)$$

Substituting (3.38) and (3.39) in (3.42) we obtain (3.17). For a Lorentz’s boost applied to (3.40) and (3.41) we obtain plane wave functions for a free particle. This does not imply, as a consequence of (3.38) and (3.39), that
the boost is a correct procedure for $\psi^{(r)}$ and $\psi^{(s)}$. In other words, the new functions, obtained from $\psi^{(r)}$ and $\psi^{(s)}$ by a boost, are not solutions of the Dirac’s equation.

Although the calculus was based on (2.8), the expansible initial phase of (2.8), with $t > 0$ and $\epsilon = -1$, is given by (2.9). It is interesting to rewrite the wave functions as follows

$$
\psi^{(r)} = e^{-i mc^2 t \epsilon} \frac{1}{\sqrt{2i}} \left[ -3mb\sqrt{\frac{4t}{s^2} - \frac{m^2}{s^2}} + \frac{m^2}{s^2} \right] u^{(s)}
$$

(3.43)

$$
\psi^{(s)} = e^{i mc^2 t \epsilon} \frac{1}{\sqrt{2i}} \left[ -3mb\sqrt{\frac{4t}{s^2} - \frac{m^2}{s^2}} + \frac{m^2}{s^2} \right] u^{(s)}
$$

(3.44)

for a massive fermion, and

$$
\psi^{(r)} = e^{-i mc^2 t \epsilon} \frac{1}{\sqrt{2i}} \left[ -3mb\sqrt{\frac{4t}{s^2} - \frac{m^2}{s^2}} + \frac{m^2}{s^2} \right] u^{(s)}
$$

(3.45)

$$
\psi^{(s)} = e^{i mc^2 t \epsilon} \frac{1}{\sqrt{2i}} \left[ -3mb\sqrt{\frac{4t}{s^2} - \frac{m^2}{s^2}} + \frac{m^2}{s^2} \right] u^{(s)}
$$

(3.46)

for a neutrino.

The simplified form of the wave functions is a consequence of the symmetries in the primordial universe, as well as the ghost particles hypothesis for other primordial particles.

Notice that the cosmological time $t$ should be used in the Dirac’s equation for the primordial phase of the universe. In the present phase we should use it in the Dirac’s equation when we consider the evolution of ghost fermions only. For interactions involving ordinary matter, the time in the Dirac’s equation will be another time associated with the ordinary matter.

Strong reflection ($x'^{(D)} \rightarrow -x^{(D)}$) followed by Hermitian conjugation is equivalent to time reflection, charge conjugation, and parity (PCT), if the product of the three phases is chosen to be one [11].

The PCT theorem can be written as follows,

$$
\psi' (x'^{(D)}) = i \gamma^{(5)} \psi (x^{(D)}),
$$

(3.47)

with

$$
x'^{(D)} = -x^{(D)}.
$$

(3.48)

Let us consider the Dirac’s equation as follows

$$
\gamma^{(D)} \partial^{(D)} \psi + i mc^2 \psi + \frac{1}{4} \gamma^{(A)(B)(D)} \gamma^{(A)} \gamma^{(B)} \psi = 0.
$$

(3.49)
Multiplying it by $\gamma^{(5)}$ from the left and using the $\gamma$’s properties

$$-\gamma^{(D)} \partial_{(D)} \gamma^{(5)} \psi(x^{(D)}) + i\bar{m}c^2 \gamma^{(5)} \psi - \frac{1}{4} \gamma_{(A)(B)(D)} \gamma^{(D)} \gamma^{(A)} \gamma^{(B)} \gamma^{(5)} \psi = 0.$$ \hspace{1cm} (3.50)

But

$$-\partial_{(D)} \gamma^{(5)} \psi(x^{(D)}) = \partial_{(D')} \gamma^{(5)} \psi(x^{(D')}).$$ \hspace{1cm} (3.51)

Substituting in (3.50) we obtain

$$-\gamma^{(D)} \partial_{(D)} \gamma^{(5)} \psi(x^{(D)}) + i\bar{m}c^2 \gamma^{(5)} \psi - \frac{1}{4} \gamma_{(A)(B)(D)} \gamma^{(D)} \gamma^{(A)} \gamma^{(B)} \gamma^{(5)} \psi = 0.$$ \hspace{1cm} (3.52)

For the coordinate $x^{(D)}$ the covariance of the Dirac’s equation implies

$$\gamma^{(D)} \partial_{(D')} \gamma^{(5)} \psi(x^{(C)}) + i\bar{m}c^2 \gamma^{(5)} \psi(x^{(C)}) + \frac{1}{4} \gamma_{(A)(B)(D)} \gamma^{(D)} \gamma^{(A)} \gamma^{(B)} \gamma^{(5)} \psi = 0.$$ \hspace{1cm} (3.53)

For (2.8) the equation (3.53) will be

$$\partial_{t'} \psi(t') + \frac{3R'}{2R} \psi(t') + i\bar{m}c(0) \psi(t') = 0,$$ \hspace{1cm} (3.54)

or

$$\partial_{t'} \psi(t') + \{-3 \in mb \sqrt{\frac{A}{3} - m^2 t'}\} \psi(t') + i\bar{m}c(0) \psi(t') = 0,$$ \hspace{1cm} (3.55)

with solutions

$$\psi^{(r)}(t') = e^{-i\bar{m}c2t'} e^{\frac{3mb}{4t'} + \frac{m^2}{t'} |t'|} u^{(r)},$$ \hspace{1cm} (3.56)

$$\psi^{(s)}(t') = e^{i\bar{m}c2t'} e^{\frac{3mb}{4t'} + \frac{m^2}{t'} |t'|} u^{(s)}.$$ \hspace{1cm} (3.57)

Substituting (3.48) in (3.56) and in (3.57) we obtain

$$\psi^{(r)}(t') = \psi^{(r)}(-t) = e^{i\bar{m}c2t} e^{-3mb\sqrt{\frac{A}{3} + \frac{m^2}{t} t^2}} u^{(r)},$$ \hspace{1cm} (3.58)

$$\psi^{(s)}(t') = \psi^{(s)}(-t) = e^{-i\bar{m}c2t} e^{-3mb\sqrt{\frac{A}{3} + \frac{m^2}{t} t^2}} u^{(s)}.$$ \hspace{1cm} (3.59)

For (2.8) the equation (3.50) (or (3.52) ) will be

$$-\partial_t \gamma^{(5)} \psi(t) + \{3 \in mb \sqrt{\frac{A}{3} + m^2 t}\} \gamma^{(5)} \psi(t) + i\bar{m}c^2 \gamma^{(0)} \gamma^{(5)} \psi(t) = 0,$$ \hspace{1cm} (3.60)
with solutions
\[
\gamma(5)\psi^{(r)}(t) = e^{imc^2t}e^{\left[3\in \sqrt{\frac{4}{3}t + \frac{m^2}{2}}\right]}u^{(r)}, 
\]
(3.61)
\[
\gamma(5)\psi^{(s)}(t) = e^{-imc^2t}e^{\left[3\in \sqrt{\frac{4}{3}t + \frac{m^2}{2}}\right]}u^{(s)}. 
\]
(3.62)

We conclude that (3.58), (3.59), (3.61) and (3.62) do not obey the PCT theorem (3.47) and (3.48). The PCT theorem is violated in the primordial phase of the universe.

Note that \(t'\) is a negative time. As in the obtention of (2.10), make \(t' = -\tau\), where \(\tau\) is a positive time. It is convenient to avoid a confusing notation. Then, for \(e = +1\), and \(b > 0\), as in (2.10), the solutions (3.56) and (3.57) assume the following form
\[
\psi^{(r)}(t') = e^{imc^2\tau}e^{\left[-3\in \sqrt{\frac{4}{3}\tau + \frac{m^2}{2}}\right]}u^{(r)}, 
\]
(3.63)
\[
\psi^{(s)}(t') = e^{-imc^2\tau}e^{\left[-3\in \sqrt{\frac{4}{3}\tau + \frac{m^2}{2}}\right]}u^{(s)}. 
\]
(3.64)

We conclude that (3.63) and (3.64) are calculated in the universe (2.10). The solutions (3.61) and (3.62) live in our universe (2.9), where \(t > 0\), \(e = -1\), and \(b > 0\). In (2.9) they assume the following forms
\[
\gamma(5)\psi^{(r)}(t) = e^{imc^2t}e^{\left[-3\in \sqrt{\frac{4}{3}t + \frac{m^2}{2}}\right]}u^{(r)}, 
\]
(3.65)
\[
\gamma(5)\psi^{(s)}(t) = e^{-imc^2t}e^{\left[-3\in \sqrt{\frac{4}{3}t + \frac{m^2}{2}}\right]}u^{(s)}. 
\]
(3.66)

The solutions (3.63), (3.64), (3.65) and (3.66) satisfy the Dirac’s equation and obey the PCT theorem.

### 4 Concluding Remarks

The embedding in an n-dimensional flat space of one metric only is well known. The embedding of (2.8) and of a classical metric in an n-dimensional flat space is a possible strategy to consider the gravitational interaction between the scalar particle and the ordinary matter.

The Jacobi equation (2.20) eliminates fictitious forces and identifies attractive (dark matter) and repulsive (dark energy) forces as a scalar particle effect.
Strong reflection \((x^{(D)} \rightarrow -x^{(D)})\) followed by Hermitian conjugation is equivalent to time reflection, charge conjugation, and parity \((PCT)\), \([11]\).

The solutions (3.40) and (3.41) of the Dirac’s equation (3.42) for massive fermions obey the \(PCT\) theorem. The same is not true for the solutions (3.25) and (3.26) of the Dirac’s equation (3.17). Then, the \(PCT\) theorem is violated in the primordial phase of the universe. In this phase the particle-antiparticle symmetry is incompatible with (2.8). The solutions of the Dirac’s equation present a real and initial expansible factor. If we use the \(PCT\) theorem in the anti-fermion (or fermion) wave functions, we will have an asymmetry between the initial expansible factors due to \(t \rightarrow -t\). Consequently, for creation and annihilation processes, fermions and anti-fermions have different probabilities. This could give the necessary and quantitative initial difference between particles and antiparticles to explain the observed asymmetry.

If we consider that (2.8) represents two expansible parallel universes, one of particles, with \(t > 0, \epsilon = -1, b > 0\), described by (2.9), and the other of anti-particles, with \(t' = -t = -\tau < 0, \epsilon = +1\), and \(b > 0\), described by (2.10), then the \(PCT\) theorem will be obeyed if \(\gamma^{(5)}\psi(t)\) and \(\psi'(t')\) are calculated in (2.9) and (2.10), respectively. For the particle parallel universe (2.9), the presence of anti-particles will be possible by processes involving energy-momentum conservation, as a pair creation. We interpret the presence of anti-particles in the parallel universe (2.9) as a capture from the parallel universe (2.10). Processes involving energy-momentum conservation, as a pair creation, can be a bridge between the two primordial universes. We believe that future attempts at anti-hydrogen trapping will be successful \([12]\). From our above hypotheses, anti-hydrogen trapping can be thought as a capture from the antiparticle parallel universe (2.10). This allows us to consider the possibility of a temporary hydrogen trapping in the parallel universe (2.10) as a symmetric process. It is a conjecture.

If (2.8) correctly describes the dark matter and the dark energy as an effect of the primordial scalar particle, the Einstein’s equation needs to be preserved. This implies in considering other primordial particles as being ghosts. If we preserve the Dirac’s equation and the \(PCT\) theorem in the primordial phase, we will need to consider the existence of the two parallel and expansible primordial universes (2.9) and (2.10).

Notice that the cosmological time \(t\) should be used in the Dirac’s equation for the primordial phase of the universe. In the present phase we should use it in the Dirac’s equation when we consider the evolution of ghost fermions only.
For interactions involving ordinary matter, the time in the Dirac’s equation will be another time, associated with the ordinary matter. In this case the metric (2.8) can be considered as a small background field, and the PCT theorem can be obeyed, but embedding will be necessary for a more realistic approach.

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