Optimal Behaviour of Regulated Firms in Solar Renewable Energy Certificate (SREC) Markets

Arvind Shrivats and Sebastian Jaimungal

Abstract. SREC markets are a relatively novel market-based system to incentivize the production of energy from solar means. A regulator imposes a floor on the amount of energy each regulated firm must generate from solar power in a given period, providing them with certificates for each generated MWh. Firms offset these certificates against the floor, paying a penalty for any lacking certificates. Certificates are tradable assets, allowing firms to purchase / sell them freely. In this work, we formulate a stochastic control problem for generating and trading in SREC markets for a regulated firm’s perspective accounting for generation and trading costs, and the impact both have on prices. We provide a characterisation of the optimal strategy using the stochastic maximum principle and develop a numerical algorithm to solve this control problem, Based on this numerical solution, we provide detail and intuition for the optimal strategy for a regulated firm.

Key words. Commodity Markets, Stochastic Control, SREC, Cap and Trade, Market Design

AMS subject classifications. 37H10, 49L20, 39A14, 91G80

1. Introduction. As the impacts of climate change continue to be felt worldwide, policies to reduce greenhouse gas emissions and promote renewable energy generation will increase in importance. One policy approach which encapsulates many policies is market-based solutions. The most well-known of the policies which fall under this umbrella are carbon cap-and-trade (C&T) markets.

In carbon C&T markets, regulators impose a limit on the amount of carbon dioxide (CO₂) that regulated firms can emit during a certain time period (referred to as a compliance period). They also distribute allowances (credits) to individual firms in the amount of this limit, each allowing for a unit of CO₂ emission, usually one tonne. Firms must offset each of their units of emissions with an allowance, or face a monetary penalty for each allowance they are lacking. These allowances are tradable assets, allowing firms who require more credits than what they were allocated to buy them, and firms who require less to sell them. In this way, C&T markets aim to find an efficient way of allocating the costs of CO₂ abatement across the regulated firms.

In practice, these systems regulate multiple consecutive and disjoint compliance periods, which are linked together through mechanisms such as banking, where unused allowances in period \( n \) can be carried over to period \( n + 1 \). Other linking mechanisms include borrowing from future periods (where a firm may reduce its allotment of allowances in period \( n + 1 \) in order to use them in period \( n \) ) and withdrawal, where non-compliance in period \( n \) reduces period \( n + 1 \) allowances by the amount of non-compliance (in addition to the monetary penalty mentioned above).

A closely related alternative to these cap-and-trade markets are renewable energy cer-
REC markets (REC markets). A regulator sets a floor on the amount of energy generated from renewable sources for each firm (based on a percentage of their total energy generation), and provides certificates for each MWh of energy produced via these means\(^1\). To ensure compliance, they each firm must surrender certificates totaling the floor at the end of each compliance period, with a monetary penalty paid for each lacking certificate. The certificates are traded assets, allowing regulated Load Serving Entities (LSEs) to make a choice about whether to produce electricity from renewable means themselves, or purchase the certificates on the market (or a mix of both). A natural and interesting question is to determine the optimal behaviour for regulated firms.

REC markets can be used to encourage growth of a particular type of renewable energy. The most notable of these systems are Solar REC markets (SREC markets), which have been implemented in many areas of the northeastern United States\(^2\), and are the focus of this work.

The similarities between SREC markets and carbon cap-and-trade markets are clear. However, there are also some notable differences. One key difference between the SREC market and traditional carbon cap-and-trade markets is the uncertainty in the former market is the supply of certificates (driven by some generation process), while in the latter, the uncertainty is in the demand for allowances (driven by an emissions process). In SREC markets, banking is implemented, but borrowing and withdrawal are not. Broadly speaking, SREC markets can be considered the inverse of a cap-and-trade system.

The existing literature on SREC markets focus on certificate price formation. \cite{9} presents a stochastic model for SREC generation, calibrate it to the New Jersey SREC market, and ultimately solves for the certificate price as a function of economy-wide generation capacity and banked SRECs, and investigates the role and impact of regulatory parameters on these markets. \cite{13} studies an alternate design scheme for SREC markets and shows it stabilizes SREC prices. The volatility of REC prices has been noted in other works, such as \cite{3} and \cite{12}. The latter focuses on the Swedish-Norwegian electricity certificate market and develops a stochastic model to analyze price dynamics and policy.

While literature in SREC markets is fairly limited, there is a great deal of work that has been done in carbon cap-and-trade markets, particularly in developing stochastic equilibrium models for emissions markets. \cite{10} present a general stochastic framework for firm behaviour leading to the expression of allowance price as a strip of European binary options written on economy-wide emissions. Agents’ optimal strategies and properties of allowance prices are also studied by \cite{7} and \cite{17} within a single compliance period setup, with the former also making significant contributions through detailed analyses of potential shortcomings of these markets and their alternatives. \cite{6} also proposes a stochastic equilibrium model to explain allowance price formation and develop a model where abatement (switching from less green to more green fuel sources) costs are stochastic. There is also significant work on structural models for financial instruments in emissions markets, such as \cite{11} and \cite{5}.

A natural question in these systems is how should regulated LSEs behave. Here, we use stochastic control techniques to characterize firm specific optimal behaviour through

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\(^1\)Not all generators of renewable energy who participate in REC markets are regulated Load Serving Entities (LSEs), though in this work, we largely focus on the decisions faced by them

\(^2\)The largest and most mature SREC market in North America is the New Jersey SREC Market
generation and trading and discuss potential takeaways from a market design perspective. We believe these results are of interest to both regulators, the designers of REC markets, and the firms regulated by them.

Specifically, we explore a cost minimization problem of a single regulated firm in a single-period SREC market with the goal of understanding their optimal behaviour as a function of their current level of compliance and the market price of SRECs. To this end, we pose the problem as a continuous time stochastic control problem. We provide the optimality conditions, and analyze the form of the optimal controls in feedback form to illuminate features of the solution. In addition, we numerically solve for the optimal controls of the regulated firm as generation and trading costs vary, include a detailed analysis of various scenarios, sample paths, and comparisons to benchmark strategies. We also explore the sensitivity of the optimal controls to the various parameters in the model. We extend these results to a single regulated firm in a multi-period SREC market.

The differences between our work and the extant literature are several. Firstly, we focus on the SREC market, which is a new and burgeoning market and there are few studies (in comparison to carbon C&T markets). Secondly, we focus on the optimal behaviour of firms, rather than what optimal behaviour implies about the price of SRECs. Prior works concern themselves with a stochastic control problem in order to learn about the behaviour of the allowance prices, while we begin with assumed dynamics for the price process of an SREC (which regulated agents exert some control over) and are interested in how the agent should behave given this. The control that agents have over the price process of an SREC is assumed to be similar to the permanent price impacts seen in the literature of optimal execution (see [2], [8]).

The remainder of this paper is organized as follows. Section 2 discusses our model and poses the general optimal behaviour problem in continuous time. Section 3 presents optimality results in a continuous time setting. Section 4 provides a discrete time formulation and numerically solves the dynamic programming equation to characterize the optimal behaviour of a regulated firm. Finally, in Section 5, we present the results including sensitivity analysis and the implications for market design.

2. Model.

2.1. SREC Market Rules. We assume the following set of rules for the SREC market. These rules are exogenous and fixed prior to the market beginning. In an N-period framework, a firm is required to submit \((R_1, \ldots, R_N)\) SRECs corresponding to the compliance periods \([0, T_1], \ldots, [T_{N-1}, T_N]\). For the period \([T_{i-1}, T_i]\), firms pay \(P_i\) for each SREC below \(R_i\) at \(T_i\). Firms receive an SREC for each MWh of electricity they produce through solar means. Further, firms may bank leftover SRECs (not required for compliance in the current period) into the next period, and the banked amount has no expiry. This is a simplifying assumption but can be removed by increasing the state space to include the vintage of the SRECs. Indeed, many SREC markets have limitations on how long an SREC can be banked for (in New Jersey’s SREC market, the largest and most mature in North America, an SREC can be banked for a maximum of four years). The simplifying assumption keeps the dimensionality of the state space low. Finally, an instant after \(T_N\) all SRECs are forfeited and are considered worthless.
2.2. Firm Behaviours. To begin, all processes are defined on the filtered probability space \((\Omega, \mathcal{F}, \mathbb{F} = (\mathcal{F}_t)_{t \in \mathbb{T}}, \mathbb{P})\), where \(\mathbb{F}\) is the natural filtration generated by the SREC price, and \(\mathbb{T} := [0, T]\). The dynamics of SREC prices will be specified below.

We first restrict ourselves to a firm that is optimizing their behaviour in a single compliance period SREC system. A regulated firm can control their generation rate \((\text{SRECs/year})\) at any given time \((g_t)_{t \in \mathbb{T}}\) and their trading rate \((\text{SRECs/year})\) at any given time \((\Gamma_t)_{t \in \mathbb{T}}\). The processes \(g\) and \(\Gamma\) constitute the firm’s controls.

The trading rate may be positive or negative, reflecting that firms can either buy or sell SRECs at the prevailing market rate for SRECs. Firms also incur a trading penalty of \(\frac{1}{2}\gamma \Gamma_t^2\), \(\gamma > 0\), per unit time. This induces a constraint on their trading speed. In general, this could be any convex function of \(\Gamma\).

We assume that a firm has a baseline deterministic generation level, represented by \(h_t\) \((\text{SRECs/year})\), at which there is no cost of generation. Methods similar to [9] may be used to estimate \(h_t\). Deviations from baseline production incurs an instantaneous cost of \(C(g, h) := \frac{1}{2}\xi (g - h)^2\) per unit time, which is similar to [1]. Any differentiable, convex cost function may be used.

The set of admissible controls \(A\) is the set of all \(\mathbb{F}\)-progressively measurable processes \(g, \Gamma\) such that \(\mathbb{E}[\int_0^T g_t^2 dt] < \infty\) and \(\mathbb{E}[\int_0^T \Gamma_t^2 dt] < \infty\). Given an admissible strategy \(g, \Gamma \in A\), the firm holds \(b^{g,\Gamma}_t = (b^{g,\Gamma}_t)_{t \in \mathbb{T}}\) SRECs and the SREC price process is \(S^{g,\Gamma} = (S^{g,\Gamma}_t)_{t \in \mathbb{T}}\).

The banked amount and SREC price processes satisfy the stochastic differential equations (SDEs)

\[
\begin{align*}
(2.1a) & \quad S^{g,\Gamma}_t = S_0 + \int_0^t (\mu_u + \eta \Gamma_u - \psi g_u) \, du + \int_0^t \sigma_u \, dB_u, \\
(2.1b) & \quad b^{g,\Gamma}_t = b_0 + \int_0^t (g_u + \Gamma_u) \, du.
\end{align*}
\]

Here, \(B = (B_t)_{t \in \mathbb{T}}\) is a \(\mathbb{P}\)-Brownian motion, and \(\mu, \sigma\) are deterministic functions of time. We assume \(\int_0^T \sigma_u^2 du < \infty\) and \(\int_0^T \mu_u du < \infty\). In this model, trading and generation impact the price in opposite directions, and we assume their impacts are linear. This is similar to the price impact commonly studied in optimal execution problems (see e.g., [8]). These impacts can be viewed as being due to excess buying (selling) of SRECs pushing the price up (down) and due to excess (shortage of) supply caused by excess (decreased) generation pushing the price downwards (upwards). In this way, a firm’s behaviour impacts market prices.

Putting the modeling pieces together, a regulated firm’s performance criterion for the single-period problem is

\[
(2.2) \quad J^{g,\Gamma}(t, b, S) = \mathbb{E}_{t,b,S} \left[ - \int_t^T C(g_u, h_u) \, du - \int_t^T \Gamma_u S^{g,\Gamma}_u \, du - \frac{1}{2} \int_t^T \Gamma_u^2 \, du - P(R - b^{g,\Gamma}_t) + \right],
\]

where \(\mathbb{E}_{t,b,S} [\cdot]\) represents taking expectation conditioned on \(b^{g,\Gamma}_t = b\), and \(S^{g,\Gamma}_t = S\). The firm’s value function is

\[
(2.3) \quad V(t, b, S) = \sup_{(g, \Gamma) \in A} J^{g,\Gamma}(t, b, S),
\]

and they seek the cost minimization strategy that attains the sup (if it exists).
In the next section, we characterize the optimal trading strategy and the relationship to SREC price using the stochastic maximum principle as well as the dynamic programming principle approach.

3. Continuous time approach.

3.1. Stochastic Maximum Approach. Here, we apply the Stochastic (Pontryagin) Maximum Principle (see, the seminal works of [14] and [15]) to solve for the optimal strategy in (2.3). In this approach, the optimal controls are characterised as a system of coupled forward backward stochastic differential equations (FBSDEs) and our key result in this section is contained in the following proposition.

Proposition 3.1 (Optimality Conditions). The processes \((g, \Gamma)\) satisfying the FBSDE

\[
\begin{align*}
    d\Gamma_t &= \frac{1}{\gamma} (dM_t - \mu_t \ dt + \psi g \ dt), \quad \Gamma_T = \frac{1}{\gamma} (P \mathbb{1}_{b_T < R} - S_T), \quad \text{and} \\
    dg_t &= \frac{1}{\zeta} (dZ_t + (\zeta h'(t) - \psi \Gamma_t) \ dt), \quad g_T = \frac{1}{\zeta} (P \mathbb{1}_{b_T < R} + \zeta h(T)),
\end{align*}
\]

for all \(t \in [0, T]\), where the processes \((M, Z) = (M_t, Z_t)_{t \in \mathbb{T}}\) are martingales, are the optimal controls for problem (2.3).

Proof. The Hamiltonian corresponding to the performance criterion (2.2) and dynamics (2.1) is

\[
H(t, b, S, g, \Gamma, y, z) = -\frac{\zeta^2}{2} (g - h_t)^2 - S \Gamma - \frac{\gamma^2}{2} \Gamma^2 + y_b (g + \Gamma) + y_S (\mu_t + \eta \Gamma - \psi g) + \sigma_t z_S,
\]

where \(y = (y_b, y_s)\) and \(z = (z_b, z_S)\). This is concave in the controls \(g, \Gamma\) and state variables \(b, S\). Moreover, the adjoint processes \((y_{b,t}, y_{S,t})_{t \in \mathbb{T}}\) satisfy the BSDEs

\[
\begin{align*}
    dy_{b,t} &= z_{b,t} dB_t, \quad y_{b,T} = P \mathbb{1}_{b_T < R}, \\
    dy_{S,t} &= \Gamma_t \ dt + z_{S,t} dB_t, \quad \text{and} \quad y_{S,T} = 0.
\end{align*}
\]

The stochastic maximum principle implies that if there exists a solution \((\hat{y}, \hat{z})\) to the BSDE system (3.4), then the strategy \((g_t, \Gamma_t)\) that maximizes \(H(t, b_t, S_t, g_t, \Gamma_t, \hat{y}_t, \hat{z}_t)\) maximizes the performance criterion, and is the optimal control we seek.

As both BSDEs have linear drivers, their solution is straightforward (see [16], Chapter 6) and given by

\[
\begin{align*}
    y_{b,t} &= P \mathbb{P}_t (b_T < R) \quad \text{and} \quad y_{S,t} = -\mathbb{E}_t \left[ \int_t^T \Gamma_u du \right].
\end{align*}
\]

The strategy, in feedback form, that maximizes the Hamiltonian is given by the first order conditions

\[
\begin{align*}
    \frac{\partial H}{\partial \Gamma} = 0 & \iff y_b + \eta y_S - S - \gamma \Gamma = 0, \quad \text{and} \\
    \frac{\partial H}{\partial g} = 0 & \iff y_b - \psi y_S - \zeta (g - h) = 0,
\end{align*}
\]
and substituting the solutions to the adjoint processes (3.5), we obtain the optimality conditions

\begin{align}
(3.7a) & \quad P \mathbb{P}_t(b_T < R) - \eta \mathbb{E}_t \left[ \int_t^T \Gamma_u du \right] - S_t - \Gamma_t \gamma = 0, \\
(3.7b) & \quad P \mathbb{P}_t(b_T < R) + \psi \mathbb{E}_t \left[ \int_t^T \Gamma_u du \right] - \zeta (g_t - h_t) = 0.
\end{align}

By the Stochastic Maximum Principle, solutions to (3.7a) and (3.7b) are the optimal controls we seek. We next rewrite the optimality conditions as the solution to FBSDEs. From (3.7a), we have

\begin{equation}
(3.8) \quad Y_t + \eta \int_0^t \Gamma_u du - S_t = \Gamma_t \gamma,
\end{equation}

where \( Y = (Y_t)_{t \in \mathbb{T}} \) is the Doob-martingale defined by

\begin{equation}
(3.9) \quad Y_t = P \mathbb{P}_t(b_T < R) - \eta \mathbb{E}_t \left[ \int_0^T \Gamma_u du \right].
\end{equation}

Rearranging (3.8), we arrive at (3.1) where the terminal condition follows immediately from (3.7a) and \( M = (M_t)_{t \in [0,T]} \) is the martingale defined by

\begin{equation}
(3.10) \quad M_t = Y_t - \int_0^t \sigma_u dB_u.
\end{equation}

From (3.7b), we apply a similar argument leading to (3.2) where \( Z = (Z_t)_{t \in \mathbb{T}} \) is the Doob-martingale defined by

\begin{equation}
(3.11) \quad Z_t = P \mathbb{P}_t(b_T < R) + \psi \mathbb{E}_t \left[ \int_0^T \Gamma_u du \right].
\end{equation}

We end this subsection with a few comments and interpretations of the optimality conditions. In [10] the authors develop optimality conditions in a carbon C&T system along similar lines but without trading speed penalty or price impact. Here, including the trading speed penalty and the impact of trading and generation on SREC prices modify the optimality conditions. When \( \eta = \psi = 0 \), (3.7b) reduces to \( P \mathbb{P}_t(b_T < R) = \zeta (g_t - h_t) \). This is similar to the result that the marginal cost of generation is equal to the product of the penalty and probability of non-compliance found in [10]. Moreover, when \( \eta = \psi = 0 \), (3.7a) reduces to \( P \mathbb{P}_t(b_T < R) - \gamma \Gamma_t = S_t \). Hence, in this case, the SREC price equals the penalty scaled by the probability of non-compliance but modified by the optimal trading of the firm.

Similar behavior persists in the general case when \( \eta > 0, \psi > 0 \). From (3.7a), the SREC price equals the penalty scaled by the probability of non-compliance, but modified by the time-\( t \) marginal cost of the firm’s trading and their expectations of future trading. That is, low prices are associated with high rate of trading and high expected future rate of trading.

From (3.7b), the penalty scaled by the probability of non-compliance equals the difference between the marginal cost of generation and re-scaled (by \( \psi \)) expected future trading.

The FBSDEs (3.1)-(3.2) can be solved numerically using Least Square Monte Carlo techniques, however, we will consider a dynamic programming approach to solving the original problem (2.3).
3.2. HJB Approach. The dynamic programming principle provides additional insight into the solution of the problem. Here, for simplicity, we assume \( \mu_t, \sigma_t \) are constants denoted by \( \mu, \sigma \). Using standard techniques (see e.g., [16]), the dynamic programming principle (DPP) applied to (2.3) implies that the value function is the unique viscosity solution to the dynamic programming equation (DPE) or Hamilton-Jacobi-Bellman (HJB) equation

\[
\begin{align*}
\partial_t V(t,b,S) + \sup_{g,\Gamma} \left\{ \mathcal{L}^{b,S} V(t,b,S) + F(t,b,S,g,\Gamma) \right\} &= 0, \\
V(T,b,S) &= G(b),
\end{align*}
\]

where \( G(b) = \left( R - b \right)_+ \), \( F(t,b,S,g,\Gamma) = -\frac{1}{2} \zeta(g - h_t)^2 - S \Gamma - \frac{\eta}{2} \Gamma^2 \), and the operator \( \mathcal{L}^{b,S} \) acts on functions as follows

\[
\mathcal{L}^{b,S} V = (\mu + \eta \Gamma - \psi g) \partial_S V + (g + \Gamma) \partial_b V + \frac{1}{2} \sigma^2 \partial_{SS} V.
\]

The first order conditions provides the optimal controls in feedback form

\[
\begin{align*}
g^*(t,b,S) &= h_t + \frac{1}{\zeta} (\partial_b V(t,b,S) - \psi \partial_S V(t,b,S)) , \\
\Gamma^*(t,b,S) &= \frac{1}{\gamma} (\partial_b V(t,b,S) + \eta \partial_S V(t,b,S) - S).
\end{align*}
\]

From the above, the optimal level of trading has a negative linear relationship with respect to the SREC market price \( S \). Hence, as \( S \) increases, the optimal level of trading decreases. That is, the firm buys less (or equivalently, sell more) as SREC prices increase.

Similarly, the optimal generation amount can be interpreted as the baseline amount of SRECs \( (h) \) plus the marginal value gained by the generation of an SREC (as generating an SREC increases \( b \) and negatively impacts \( S \)), modulated by the cost of generation parameter \( \zeta \).

Generation and trading have opposite dependence in their sensitivity to asset price; that is, the coefficients of the \( \partial_S V \) terms in (3.14) have opposite signs. If an incremental change in SREC price increases (decreases) the value function, then it increases (reduces) trading and simultaneously reduces (increases) generation. One reason is that as SRECs are purchased (sold), trading impacts prices and pushes them upwards (downwards).

Substituting the feedback form into the HJB equation leads to the semi-linear parabolic PDE

\[
\begin{align*}
\partial_t V + \mathcal{L}^S V - \frac{1}{2} \zeta h^2 + \frac{1}{\gamma} (\partial_b V - \psi \partial_S V + \zeta h)^2 + \frac{1}{\gamma} (\partial_b V + \eta \partial_S V - S)^2 &= 0, \\
V(T,b,S) &= G(b),
\end{align*}
\]

where \( \mathcal{L}^S = \mu \partial_s + \frac{1}{2} \sigma^2 \partial_{ss} \) is the generator of the no-impact SREC price. This PDE is difficult to solve analytically. One can solve it numerically using finite differences methods and then apply (3.14) to obtain the optimal controls. However, due to the lack of a convexity term in \( b \), numerical instabilities occur and require large number of grid point methods, or more sophisticated finite-difference schemes. Instead, we formulate a discrete time version of the problem directly and solve it numerically.
4. Discrete time version of problem. Thus far, we formulated the cost minimization problem of a single regulated firm using continuous time optimal control techniques to characterize the solution and tease out some essential features of the optimal strategy. To obtain numerical solutions, however, we solve a discrete time version of the problem which we find has better numerical stability. Indeed, a discrete time formulation more closely approximates practice, as regulated firms typically take actions only at discrete time points within a compliance period.

To this end, let \( n \) be the number of decision points within a single compliance period (we are still in the single firm, single-period case), which occur at \( 0 = t_1 < t_2 < \ldots < t_n < T = t_{n+1} \). For simplicity, we assume these are equally spaced such that \( t_k = k\Delta t \).

The processes \( g_t, \Gamma_t \) are now piecewise constant within \([t_i, t_{i+1})\), and the firm controls \( \{g_{t_i}, \Gamma_{t_i}\}_{i \in \mathcal{N}} \) where \( \mathcal{N} := \{0, \ldots, n\} \). Intuitively, at each time point, the regulated firm chooses their trading and generating behaviour over the next interval of length \( \Delta t \). In this section, \( g, \Gamma \) represent vectors whose elements are these controls.

Under the same assumptions as earlier, the performance criterion (corresponding to the total cost) for an arbitrary admissible control is

\[
J^{g,\Gamma}(m, b, S) = \mathbb{E}_{t_m, b, S}\left\{ \sum_{i=m}^{n} \left( \frac{\zeta}{2}(g_{t_i} - h_{t_i})^2 + \Gamma_{t_i}S_{t_i}^{g,\Gamma} + \frac{\gamma}{2} \Gamma_{t_i}^2 \right) \Delta t + P(R - b^{g,\Gamma}_T) \right\},
\]

In the above, the dynamics of the state variables \((b, S)\) are modified for discrete time to

\[
S_{t_i}^{g,\Gamma} = \min \left( S_{t_{i-1}}^{g,\Gamma} + (\mu + \eta \Gamma_{t_{i-1}} - \psi g_{t_{i-1}}) \Delta t + \sigma \sqrt{\Delta t} Z_{t_i}, P \right)
\]

\[
b_{t_i}^{g,\Gamma} = b_{t_{i-1}}^{g,\Gamma} + (g_{t_{i-1}} + \Gamma_{t_{i-1}}) \Delta t
\]

where \( Z_{t_i} \sim N(0, 1), \text{ iid, for all } i \in \mathcal{N} \).

Note that (4.2a) is the discrete time analogue of (2.1a), which we further cap at \( P \) and floor at 0. The cap and floor ensures that SREC prices remain in the closed interval \([0, P]\) as prices outside this interval cannot occur in real markets.

We aim to optimize (4.1) with respect to \( g, \Gamma \) and determine the value of the position the regulated firm, as well as their optimal behaviour. Hence, we seek

\[
V(t, b, S) = \sup_{g, \Gamma \in A} J^{g,\Gamma}(t, b, S),
\]

and the strategy that attains the sup, if it exists. Applying the Bellman Principle to (4.3) implies

\[
V(t_i, b, S) = \inf_{g_{t_i}, \Gamma_{t_i}} \left\{ \left( \frac{\zeta}{2}(g_{t_i} - h_{t_i})^2 + \Gamma_{t_i}S_{t_i}^{g,\Gamma} + \frac{\gamma}{2} \Gamma_{t_i}^2 \right) \Delta t + \mathbb{E}_{t_i} \left[ V(t_{i+1}, b_{t_{i+1}}^{g,\Gamma}, S_{t_{i+1}}^{g,\Gamma}) \right] \right\},
\]

and

\[
V(T, b, S) = P(R - b)_+. \]

In the next section, we provide a numerical scheme for solving this optimization problem.
5. Solution Algorithm and Results.

5.1. Parameter Choice and Optimal Behaviour. We use the following numerical algorithm for solving (4.4) with state variable dynamics in (4.2):

1. Choose a grid of \( b \) and \( S \) values denoted by \( \mathcal{G} \). We use a uniform grid of 101 points in \( b \) from 0 to \( 2R \), so that \( R \) is on the grid, and a uniform grid of \( S \) with \( \Delta S = \sqrt{3} \Delta t \sigma \) and lower and upper bounds of 0 and \( P \) respectively. In this manner, the number of grid points in \( S \) is tuned to the volatility over a time-step\(^3\).

2. Minimize (4.4a) at \( i = n \) (corresponding to \( t = T - \Delta t \)) with respect to \( (g_{t_n}, \Gamma_{t_n}) \) for every point in \( \mathcal{G} \). As the terminal condition in (4.4b) is independent of \( S_{t_{n+1}}^{g_{t_n}, \Gamma_{t_n}} \) and \( b_{t_{n+1}}^{g_{t_n}, \Gamma_{t_n}} \) is deterministic given \( (g_{t_n}, \Gamma_{t_n}) \), minimizing (4.4a) is a deterministic convex optimization problem.

3. Step backwards from \( i + 1 \) to \( i \), by minimizing (4.4a) with respect to \( (g_{t_i}, \Gamma_{t_i}) \) at time \( t_i \) for all points in \( \mathcal{G} \).

To so this, we require an estimate of \( \mathbb{E}_{t_i} \left[ V \left( t_{i+1}, b_{t_{i+1}}^{g_{t_i}, \Gamma_{t_i}}, S_{t_{i+1}}^{g_{t_i}, \Gamma_{t_i}} \right) \right] \) for each \( (S_{t_i}, b_{t_i}) \in \mathcal{G} \). This is achieved by simulation as follows:

A. Select a pair \( (S_{t_i}, b_{t_i}) \in \mathcal{G} \)
   i. Select a candidate pair \( (g_{t_i}, \Gamma_{t_i}) \)
      (a) Simulate \( b_{t_{i+1}}^{g_{t_i}, \Gamma_{t_i}} \) using (4.2b), which results in a unique value.
      (b) Simulate 100 scenarios of \( S_{t_{i+1}}^{g_{t_i}, \Gamma_{t_i}} \) by applying (4.2a) – use the same set of random numbers for all points in \( \mathcal{G} \).
      (c) For each simulated pair of \( (b_{t_{i+1}}^{g_{t_i}, \Gamma_{t_i}}, S_{t_{i+1}}^{g_{t_i}, \Gamma_{t_i}}) \), estimate the one-step-ahead value function \( V \left( t_{i+1}, b_{t_{i+1}}^{g_{t_i}, \Gamma_{t_i}}, S_{t_{i+1}}^{g_{t_i}, \Gamma_{t_i}} \right) \) by interpolation.
      (d) Use the empirical mean of the result of (c) as an estimate of the true mean at \( (b_{t_i}, S_{t_i}) \).
   ii. Use Matlab’s ‘fminsearch’ function to determine next candidate pair \( (g_{t_i}, \Gamma_{t_i}) \) and repeat from (i) until converged, store optimal pair and value function.

B. Go to next grid point in \( \mathcal{G} \) repeat from A.

This procedure provides an estimate of the value function at all grid points \( \mathcal{G} \) and at all times \( \mathcal{T} := \{ t_i \}_{i \in \mathbb{N}} \), as well as the optimal generation and trading rates on \( \mathcal{G} \times \mathcal{T} \).

For the first set of numerical experiments, we use the parameters reported in Tables 1 and 2.

These parameters are chosen for illustrative purposes, as calibration to a particular firm is itself a non-trivial problem and requires proprietary knowledge of a firm’s cost function and baseline production (which also varies significantly from firm to firm). Instead, we provide broad-level intuition regarding the optimal behaviour of a firm in a single-period SREC market with reasonable parameters. The penalty of \( P = \$300 \) is motivated by the New Jersey SREC market, where the non-compliance penalty in compliance period ending

\(^3\)As with any numerical solution, there is a trade-off between grid size (accuracy of the dynamic program solution) and run-time. The grid we use provides an acceptable trade-off between these two, and we observed no further increase in accuracy by increasing the grid size.
Table 1
Compliance parameters.

| n  | T  | P ($/ lacking SRECs) | R (SRECs) | h_t (SREC/y) |
|----|----|----------------------|-----------|--------------|
| 50 | 1  | 300                  | 500       | 500          |

Table 2
Model Parameters.

| µ  | σ  | ψ  | η  | ζ  | γ  |
|----|----|----|----|----|----|
| 0  | 10 | 0  | 0  | 0.6| 0.6|

May 2018 is $308. The choice $h_t = \frac{1}{T}R$ implies the regulated firm has the capability to produce the required SRECs by generating at their baseline. We set $\mu = \psi = \eta = 0$ so that, in this baseline case, $S$ is a martingale. This choice results in the firm’s generation and trading having no price impact on prices.

The values of $\zeta$ and $\gamma$ are motivated by the upper bounds they imply for $g_t, \Gamma_t$. Specifically, consider the case of a firm that cannot generate enough solar energy to meet the requirements, and hence will fail to comply. The benefit of generating SRECs is to reduce their non-compliance obligation, and with each generated SREC their obligation is reduced by $P$. Therefore, the costs and benefits of generation over a time-step are (independent of trading activity)

\begin{equation}
K_1(g_t) = \frac{1}{2}\zeta(g_t - h_t)^2\Delta t, \quad \text{and} \quad B_1(g_t) = Pg_t\Delta t,
\end{equation}

respectively. The firm generates energy in order to minimize $N_1(g_t) := K_1(g_t) - B_1(g_t)$ which occurs at $g_t^* = \frac{P}{\zeta} + h_t$. For the chosen parameters, $g^* = 1,000$ which is exactly twice the baseline rate $h_t$. In other words, this choice of $\zeta$ ensures the firm’s maximum generation rate is bounded by twice their baseline.

We conduct a similar exercise for $\Gamma_t$. Once again, consider a firm that will fail to comply. In this scenario, a rational firm will purchase SRECs. As before, the benefit of a firm purchasing SRECs is to reduce their non-compliance obligation, and with each generated SREC their obligation is reduced by $P$. As such, the costs and benefits to purchase over the next time-step are (independent of generation activity):

\begin{equation}
K_2(\Gamma_t) = \frac{1}{2}\gamma\Gamma_t^2\Delta t + S_t\Gamma_t\Delta t, \quad \text{and} \quad B_2(\Gamma_t) = Pg_t\Delta t,
\end{equation}

respectively. The firm purchases in order to minimize $N_2(\Gamma_t) := K_2(\Gamma_t) - B_2(\Gamma_t)$ which occurs at $\Gamma_t^* = \frac{P-S}{\gamma}$. For the chosen parameters, this is maximized when $S = 0$ and results in $\Gamma^* = 500$. The significance of this computation is to show we have chosen parameters that result in a reasonable upper bound on the amount of trading a firm will partake in.

Repeating the same exercise for a firm that is guaranteed to comply (and thus is motivated to sell), we obtain $g_t^* = h_t = 500$ and $\Gamma_t = -\frac{S}{\gamma}$ which is maximized (in absolute value) at $-500$ for the chosen parameters.
Figure 1. Optimal firm behaviour (top panel: generation rate, bottom panel: trading rate) as a function of banked SRECs for various time-steps and SREC market prices. Parameters in Tables 1 and 2.

For the parameters in Tables 1 and 2, this simple analysis shows that generation and trading rates are restricted to the range $g_t \in [500, 1,000]$ and $\Gamma_t \in [-500, 500]$, which is a reasonable range of possible values given our choices of $h_t$ and $R$.

In Section 5.4, we consider other parameters. In particular, we explore how various levels of $\zeta, \gamma$ impact firm behaviour, as well as the effect of price impact ($\psi \neq 0, \eta \neq 0$). We restrict to $h_t$ being constant but study alternative baselines in Section 5.4. As total generated SRECs is important from the perspective of a market designer, we investigate how the various parameters affect it.

A regulated firm’s optimal behaviour is one of the key outputs from solving the Bellman equation. Figure 1 shows the dependence of the optimal trading and generation rate on banked SRECs for three SREC prices at six points in time.

The most notable feature is the distinct regimes of generation/trading. For low levels of banked SRECs and near the terminal date, the firm generates/purchases until the marginal cost of producing/purchasing another SREC exceeds $P$, as the firm is almost assured to fail to comply. This follows the classic microeconomic adage of conducting an activity until the marginal benefit from the activity equals the marginal cost. In this regime, the marginal benefit of an additional SREC to the firm is $P$, as each additional SREC lowers their non-compliance obligation by $P$.

As the banked amount increases, the firm reaches a point where the marginal benefit from an additional SREC decreases from $P$. This occurs as the probability of compliance becomes non-negligible, as additional SRECs in excess of $R$ provide smaller marginal benefit than $P$. This is a result of the sale price of an SREC being bounded above by $P$ and leads to a decrease in optimal generation and optimal trading. The firm adjusts its behaviour
so that its marginal costs are in line with this marginal benefit. This eventually leads to the firm selling as opposed to purchasing SRECs, as the net proceeds from the sale exceed the marginal value of retaining those certificates. This decrease continues until the firm no longer benefits from additional SRECs. That is, at a certain level of \( b \), the marginal benefit of an additional SREC is zero. Specifically, having an additional SREC does not increase the firm’s likelihood of compliance, nor can they sell the additional SREC to make a profit. This results in optimal generation plateauing at the baseline \( h_t \) as the firm can produce at the baseline with zero marginal cost. Similarly, optimal trading plateaus at the level where the marginal revenue from trading equals the marginal cost.

If we hold \( b, S \) constant, generation and purchasing are increasing in \( t \). In the case where \( b \) and \( S \) are such that compliance is non-guaranteed, this is natural, as with less time until the end of a compliance period, the firm needs to accumulate more SRECs in order to comply. For values of \( b \) and \( S \) for which compliance is guaranteed, we note that this property will not always hold, and is dependent on the value of \( \gamma \). This is covered in more detail in Section 5.4.3.

Trading is influenced by a change in SREC price, which is in accordance with our intuition and coincides with the theoretical results from Section 3. As SREC prices increase, the regulated firm chooses to purchase less, regardless of banked SRECs. We also see that higher SREC prices generally imply higher generation, as the firm chooses to generate their own SRECs, either to avoid paying high prices for them in the market, or to sell in the market and capitalize on the high prices (which of these two factors is the larger contributor depends on how much is banked).

5.2. Sample Paths. In Figure 2, we show the dynamics of optimal firm behaviour through the compliance period. Here, \( S_0 = 150, b_0 = 0 \) and we simulate a path for \( S \) and at each time-step. Along this path, we adopt the optimal firm strategy in accordance with their banked SRECs and the SREC price.

From Figure 2, the regulated firm banks SRECs at a steady rate. However, the generation and trading processes exhibit notable variation over time. In particular, the inverse relationship between trading rate and SREC price is notable, as is the positive relationship between generation rate and SREC price. Instantaneous incurred costs denote the cost incurred to the firm at each time-step. That is for \( i = 1, 2, ..., n \), we have

\[
\text{Instantaneous Incurred Cost} = \left( \frac{1}{2} \left( g_{t_i} - h_{t_i} \right)^2 + \Gamma_{t_i} S_{t_i}^{\text{g,T}} + \frac{\gamma}{2} \Gamma_{t_i}^2 \right) \Delta t.
\]

In this case, instantaneous incurred costs are negative at all time-steps, which signifies that the firm is making a profit in the system, due to their sale of SRECs.

Next we investigate the distribution of various quantities of interest, such as final SREC total \( (b_T) \), total generated amount \( (\int_0^T g_u du) \), total traded amount \( (\int_0^T \Gamma_u du) \), and total profit (negative of costs), by performing multiple simulations. For the base-line parameter choice, and with \( b_0 = 0, S_0 = 150 \), we present summary statistics using 1,000 simulated paths of \( S \) in Table 3.

In this one-period setup, the firm’s optimal behaviour results in \( b_T = 500 \) SRECs in almost every path. As there is no advantage to additional SRECs above the requirement
in a single-period framework, any strategy resulting in $b_T < 500$ exposes the firm to the non-compliance penalty, and any strategy resulting in $b_T > 500$ leaves money on the table as remaining SRECs are worthless.

5.3. Comparison With Other Strategies. A natural question is how the resulting optimal strategy compares with other simple strategies. In particular, we compare the optimal strategy (for the parameter choice in Section 5.1) with the following strategies:

1. A constant generation strategy with $g_t = 500$, and $\Gamma_t = 0$ for $t \in [0, T]$ (recall $R = 500, h_t = 500$). We denote this as the ‘No-Trade’ strategy.

2. A constant strategy where the firm follows the mean behaviour analysed in Section 5.2. That is, they produce and trade at a constant s.t. $\int_0^T g_u du = 625$ and $\int_0^T \Gamma_u du = -125$. We denote this as the ‘Mean Behaviour’ strategy.
The Mean-Behaviour strategy is in fact optimal (for this parameter set) if we restrict the firm to constant behaviours for their controls (i.e. $g_t = g, \Gamma_t = \Gamma$ for every $t \in [0, T]$) and require that they comply with the requirement $(g + \Gamma = R)^4$. See Appendix A for details.

Table 4 reports the profit statistics using 1,000 simulations with $S_0 = 150$. The strategy suggested as the output to the dynamic program is referred to as the 'Optimal' strategy.

| Strategy      | Mean Profit | Std. Dev. of Profit | Q1 Profit | Q3 Profit |
|---------------|-------------|---------------------|-----------|-----------|
| No-Trade      | 0           | 0                   | 0         | 0         |
| Optimal       | 9,380       | 724                 | 8,920     | 9,850     |
| Mean-Behaviour| 9,360       | 724                 | 8,910     | 9,830     |

Table 4
Summary statistics of the three strategies: No-Trade, Optimal, and Mean-Behaviour. Initial condition $S_0 = 150, b_0 = 0$ and all remaining parameters in Tables 1 and 2.

The No-Trade strategy is trivial. As there is no randomness associated with generation, the profit from the No-Trade strategy is deterministic. In particular, the profit is 0 in this case due to the cost function and the parameters we chose (in particular, that $h_t = 500, R = 500$). Thus, the firm can generate SRECs ‘cost-free’ (at their baseline production) and produce enough to comply with the requirement. It is worth noting that as $\gamma$ tends to $\infty$, the No-Trade strategy becomes optimal.

The Optimal and Mean-Behaviour strategies appear to have results that more similar. To investigate this further, we show the histogram of the difference of the profits of the two strategies in Figure 3 on a sample-path basis: i.e., for a same sample path of $S$, we compute the resulting profit from both the Mean-Behaviour and Optimal strategy, compute their difference, and then generate a histogram of the results. The results show that the Optimal strategy outperforms the Mean-Behaviour strategy in every sample path of $S$. Hence, adjusting behaviour in response to SREC prices provides value over a static optimal control regardless of the path of $S$.

\footnote{It is clear that any strategy with $g + \Gamma \neq R$ is not optimal for these parameters. In a one period model with $h_t = R$, a strategy with $g + \Gamma > R$ implies the firm spends money to generate additional SRECs, some of which expire valueless. If $g + \Gamma < R$, the firm incurs non-compliance penalties that cannot be made up for by sales, as $S \leq P.$}
5.4. Parameter Sensitivity. In this section we investigate how varying parameters affect the optimal behaviour and resulting summary statistics, and explore the intuition behind the resulting effects.

5.4.1. Sensitivity to Price Impact. In the previous subsections we set price impact to zero ($\eta = \psi = 0$). Here, we analyze the firm’s behaviour when these are non-zero. In particular, we now set $\eta = 0.01, \psi = 0.005$ to demonstrate the effect of price impact. We justify these choices in a similar manner to Section 5.1. Recall that previously, we discussed that under the parameter choices of Section 5.1, $g_0 \leq 1,000, |\Gamma_t| \leq 500$. Under these choices of generation and trading, price impact parameters $\eta = 0.01, \psi = 0.005$ results in net price impacts of $500 \times 0.01 \times \frac{1}{50} = 0.1$ per time-step for trading and $1,000 \times 0.005 \times \frac{1}{50} = 0.1$ per time-step for generation. These are sizeable impacts, but not so large that the model seems implausible.

In Figure 4, we plot the regulated firm’s optimal behaviour as a function of banked SRECs, across three different prices of S and at six points in time. Comparing Figure 4 with Figure 1, we see that the optimal controls have the same shape with or without price impact but there is some additional structure. The optimal controls at each of the ‘plateaus’ now vary slightly with time-step. This can be seen more clearly in Table 5, where we record the values of generation and trading for each of the six time-steps detailed in Figure 4 when $S = 251$ and $b = 1000$, as an illustration of this property.

As Table 5 illustrates, at high levels of banking, firms generate less and purchase more (equivalently, sell less) at earlier time-steps than they do at later time-steps. This is to lower the impact of their behaviour on $S$, and in particular, to push prices upwards in order
to capitalize on future sales. The inverse behaviour occurs for low banking levels. Firms generate more and purchase more in order to push the prices down and make compliance more attainable. These effects are proportional to the magnitude of $\eta$ and $\psi$, and increase with $S$.

We continue by considering the sample path of an optimally behaving firm through a compliance period, in the same manner as Section 5.2. Figure 5 shows a sample path of the optimal behaviour under three price impact scenarios using the same random seed for the path of $S$, with $S_0 = 150$ and $b_0 = 0$. Specifically, these scenarios are

1. No price impact: $(\eta, \psi) = (0, 0)$
2. Standard price impact: $(\eta, \psi) = (0.01, 0.005)$
3. Strong price impact: $(\eta, \psi) = (0.02, 0.01)$

The optimal controls appear similar across each price impact scenario, and reflect the behaviour already discussed earlier for Figure 2. In particular, in all cases, the firm accumulates banked SRECs at identical rates. However, we see that in the presence of price impacts the firm generates and sells less in order to mitigate their price impact. This is more pronounced in the strong price impact scenario. The resulting impact of the firm’s behaviour on the SREC price path itself is evident. The SREC price in the no price impact scenario (blue) dominates the SREC price in the price impact scenario (red), which itself dominates the SREC price in the strong price impact scenario (yellow). Larger price impacts result in a lower path of $S$, as the firm is generating and selling SRECs. Finally, we see the firm generates less revenue when price impacts are active, as the market will always move against their behaviour. We also observe that as time progresses, the difference in generated revenue between the two parameter sets increases.

To study this further, we compare an optimally behaving firm in a single-period model that is subject to various price impact scenarios to an identical optimally behaving firm that is operating in the baseline scenario of $\eta = \psi = 0$. We consider all $(\eta, \psi)$ pairs where $\eta \in \{0, 0.01, 0.02\}$ and $\psi \in \{0, 0.005, 0.01\}$. To do this, we simulate 1,000 paths of $S$ in each price impact scenario, using the same random numbers in each scenario. In each path of $S$, we calculate total generation, total trading, and profit for the firm, and the difference between each quantity and their analogous amount under the baseline scenario. We calculate the mean and standard deviation of these differences across all paths, for

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**Table 5**

Optimal generation and trading values at each time-step for a regulated firm with $S = 251, b = 1000$, and price impact parameters $\eta = 0.01, \psi = 0.005$. All remaining parameters in Tables 1 and 2.

| Time Step | $g_t$   | $\Gamma_t$ |
|-----------|---------|------------|
| 50        | 500.0   | -418.0     |
| 40        | 499.3   | -416.6     |
| 30        | 498.6   | -415.3     |
| 20        | 497.9   | -413.9     |
| 10        | 497.3   | -412.5     |
| 1         | 496.7   | -411.4     |

---

Note that the price impacts mean the paths of $S$ itself are different in each scenario.
Figure 5. Sample path of optimal firm behaviour with price impacts $\eta = 0.02, \psi = 0.01$ (yellow), $\eta = 0.01, \psi = 0.005$ (red), and $\eta = \psi = 0$ (blue). Initial condition $S_0 = 150, b_0 = 0$ and remaining parameters as in Tables 1 and 2.

Firstly, we note that banked SRECs (the sum of total generation and total trading) is equal to $R$ in each price impact scenario, with changes in generation and trading perfectly offsetting. For fixed $\psi$, increasing $\eta$ results in lower generation, less selling, and consequently, lower profit. This is the result of the firm attempting to mitigate their price impact through sales, resulting in lower generation as a consequence (as it is optimal to have $b_T = R$).

Similarly, for fixed $\eta$, increasing $\phi$ results in lower generation, less selling, and lower profit. This is the result of the firm mitigating their price impact through generation, and results in lower sales as a consequence. These are consistent with the results we’ve seen throughout this subsection. The impact on total generation and total trading as a result of changing $\phi$ is more significant than the impact of changing $\eta$ because for these parameters (specifically for the chosen $S_0$), $|g_t|$ is generally larger than $|\Gamma_t|$.

Lastly, we observe that all the differences in the means of the quantities are significant at the 5% level.

5.4.2. Sensitivity to Baseline Generation. Thus far, we investigated the optimal firm behaviour when the baseline generation rate $h_t = \frac{R}{T}$, which results in perfect compliance. More generally, a wide array of LSEs are regulated by SREC systems, with varying levels of investment into solar and thus, capability to successfully meet compliance requirements while generating SRECs at their baseline generation rate. As such, it is important to
consider the optimal behaviours of firms with various levels of production capability.

In particular, this has important ramifications from the perspective of a regulator in charge of market design. Ultimately, the goal of SREC systems is to promote investment into solar energy generation. Consequently, the amount of SRECs generated by a regulated firm is a quantity that would be monitored carefully by regulators and market designers. To briefly study the impact of $h_t$ on total SRECs generated by a firm, we simulate 1,000 paths of $S$ starting from $S_0$, with the regulated firm behaving optimally at each time-step. We examine how the distribution of $\int_0^T g_u \, du$ changes across varying levels of $h_t$ for the firm, with all other parameters at their benchmark reported in Tables 1 and 2. Specifically, we look at (constant) values of $h_t$ from $0R$ to $1.25R$, in increments of $0.25R$. We also consider various initialization points of the path of $S$: $S_0 = 49, 150, 251$ (which represent low, medium, and high prices of $S$ respectively). The results are reported in Table 7.

| $\eta$ | $\psi$ | $\int_0^T g_u \, du$ | $\int_0^T \Gamma_u \, du$ | Profit |
|--------|--------|-----------------|-----------------|--------|
|        |        | mean | std.dev. | mean | std.dev. | mean | std.dev. |
| 0      | 0      | 624.92 | 4.67 | -124.92 | 4.67 | 9,380 | 697 |
| 0      | 0.005  | -1.52 | 0.02 | 1.52 | 0.02 | -190 | 10 |
| 0      | 0.01   | -3.03 | 0.03 | 3.03 | 0.03 | -380 | 20 |
| 0.01   | 0      | -1.00 | 0.03 | 1.00 | 0.03 | -70  | 6 |
| 0.01   | 0.005  | -2.51 | 0.05 | 2.51 | 0.05 | -260 | 16 |
| 0.01   | 0.01   | -3.99 | 0.07 | 3.99 | 0.07 | -440 | 26 |
| 0.02   | 0      | -1.99 | 0.06 | 1.99 | 0.06 | -150 | 11 |
| 0.02   | 0.005  | -3.48 | 0.08 | 3.48 | 0.08 | -330 | 21 |
| 0.02   | 0.01   | -4.95 | 0.10 | 4.95 | 0.10 | -520 | 31 |

**Table 6**

Mean and standard deviation of differences in quantities of interest between an optimally behaving firm under various price impact scenarios and an optimally behaving firm subject to the baseline scenario of $\eta = \psi = 0$. We use 1,000 sample paths of $S$, with initial condition $S_0 = 150$ and remaining parameters as in Tables 1 and 2.

| $h_t$ | $S_0 = 49$ | $S_0 = 150$ | $S_0 = 251$ |
|-------|----------------|----------------|----------------|
|       | mean | std.dev | mean | std.dev | mean | std.dev |
| 0     | 290.77 | 4.60 | 375.94 | 4.63 | 458.95 | 4.75 |
| 0.25R | 353.54 | 4.55 | 437.45 | 4.76 | 521.42 | 4.69 |
| 0.50R | 416.00 | 4.56 | 500.15 | 4.65 | 583.97 | 4.73 |
| 0.75R | 478.60 | 4.55 | 562.64 | 4.74 | 646.69 | 4.64 |
| $R$   | 541.06 | 4.84 | 625.00 | 4.63 | 709.08 | 4.72 |
| 1.25R | 625.00 | 0.00 | 687.47 | 4.68 | 771.61 | 4.76 |

**Table 7**

Total generation for various levels of $h_t$ and $S_0$. Initial condition $b_0 = 0$, and remaining parameters as in Tables 1 and 2.
We also produce the analogous table for how the distribution of $\int_0^T \Gamma_u du$ changes across $h_t$, and report the results in Table 8.

| $h_t$ | $S_0 = 49$ | $S_0 = 150$ | $S_0 = 251$ |
|-------|------------|-------------|-------------|
|       | mean       | std.dev     | mean        | std.dev     | mean        | std.dev     |
| 0     | 209.23     | 4.60        | 125.06      | 4.63        | 41.05       | 4.75        |
| 0.25R | 146.46     | 4.56        | 62.55       | 4.76        | -21.42      | 4.69        |
| 0.50R | 84.00      | 4.56        | -0.15       | 4.65        | -83.97      | 4.73        |
| 0.75R | 21.40      | 4.55        | -62.64      | 4.74        | -146.69     | 4.64        |
| R     | -41.06     | 4.84        | -125.00     | 4.63        | -209.08     | 4.72        |
| 1.25R | -82.29     | 9.20        | -187.47     | 4.68        | -271.61     | 4.76        |

Table 8
Total trading for various levels of $h_t$ and $S_0$. Initial condition $b_0 = 0$, and remaining parameters as in Tables 1 and 2.

From Table 7, we see that total generation is increasing in both the initial SREC price and baseline generation rate. Both results are intuitive, as high SREC prices implies that generating more will be profitable, while at a fixed price, deviating from the baseline adds additional cost. In all but one scenario, the firm produces more than their baseline production. This scenario occurs when $h = 1.25R$ and prices are low. Here, the firm produces at its baseline and sells the excess SRECs to profit rather than incur a cost to slow down.

Conversely, from Table 8, we see that purchasing is decreasing in the initial SREC price and baseline generation rate. A low SREC price means purchasing is more viable, while the opposite occurs if the SREC price is high. Similarly, as the firm’s baseline production increases, the likelihood of the firm possessing excess SRECs does as well, allowing them to sell more freely.

Table 7 also shows that lower producing firms are incentivized to increase production over their baseline during the course of the compliance period relative to higher producing firms. For example at $S_0 = 49$, a firm with baseline $h_t = 0$ produces an additional 291.06 MWh over its baseline, while a firm with baseline $h_t = 500$ produces only an additional 41.01 MWh above its baseline. That is, lower producing firms respond to requirements by investing in generation. This is the ultimate goal of SREC markets and our setup provides evidence that setting $R$ to be significantly above economy-wide baseline generation can incite higher degrees of investment into solar generation. Of course, we have a simple setting here and the analysis does not consider the impact to the firm’s profit, political and lobbying pressures against high requirements, and other important factors.

Lastly, we note that the sum of total generation and total trading is above $R$ in every scenario, meaning that the regulated firm complies with the requirement and faces no additional monetary penalty for failing to do so. It is possible to choose parameters such that this does not occur. For example, a very high cost of trading ($\gamma$) and penalty for deviating from $h_t$ ($\zeta$) would make compliance prohibitively expensive for a firm with $h_t = 0$ that they would instead choose to bear the non-compliance penalty. The compliance parameters set by the regulatory body are also of importance in this regard. In particular, the choice of $R$ and $P$ is hugely important for the regulatory body, as it will greatly impact the degree
to which firms will be able to and be incentivized to comply. Calibrating the compliance parameters of SREC systems from a regulators perspective remains an interesting area for future work and we aim to continue building towards it. Our model is the first significant first step in that direction.

5.4.3. Sensitivity to Trading and Generation Costs. To conclude our analysis of the single period model, we explore sensitivity to generation and trading speed costs ($\zeta$ and $\gamma$). Figure 6 shows how the optimal behaviour changes for various values of $\zeta$ and $\gamma$, across six time-steps, for fixed SREC price level ($S_t = 150$).

![Graph showing optimal generation and trading rates for differing levels of $\zeta$ (generation cost parameter) and $\gamma$ (trading speed penalty parameter) when $S_t = 150$. ($\zeta, \gamma$) = (0.2, 0.2) (left), (0.6, 0.6) (centre), (1, 1) (right). Remaining parameters as in Tables 1 and 2.](image)

From Figure 6, we can make some observations about precisely how perturbing $\zeta, \gamma$ simultaneously impacts the optimal behaviour of the firm. The central subplots contain the firm’s optimal behaviour in the default setting of $\zeta = 0.6, \gamma = 0.6$ as in Table 2. If we increase $\zeta, \gamma$ to 1, as in the rightmost figure, we see that the range of optimal behaviours is compressed. Optimal generation is in the range $[500, 800]$ if $(\zeta, \gamma) = (1, 1)$, while it is in the range $[500, 1000]$ if $(\zeta, \gamma) = (0.6, 0.6)$. Similarly, the range of optimal trading is $[-150, 150]$ if $(\zeta, \gamma) = (1, 1)$, while it is in the range $[-250, 250]$ if $(\zeta, \gamma) = (0.6, 0.6)$. Looking to the leftmost figure, where $(\zeta, \gamma) = (0.2, 0.2)$, we see a similar story told. The range of optimal generation here is $[500, 2000]$ and the range of optimal trading is $[-750, 750]$. This is the result of lower parameters corresponding to lower costs and increased capacity of the firm to invest in generation (if $\zeta$ is low) and to trade (if $\gamma$ is low). These ranges are consistent
with the optimal controls in feedback form (3.14a) and (3.14b)\textsuperscript{6}.

Additionally, it is clear that lowering both $\zeta$ and $\gamma$ widens the range over which the firm’s optimal behaviour has non-zero derivative in $b$. Recall from Section 5.1 that the regimes observed in the optimal behaviours correspond to the marginal benefit of the firm holding an additional SREC. In particular, the optimal behaviour having non-zero derivative corresponds to the values of $b, S$ where the marginal benefit of an additional SREC is between 0 and $P$. The expansion of this range is the result of the increased generation and trading capacity that the firm experiences due to lower costs.

Finally, we remark that the leftmost plots in Figure 6 have the property that for values of $b$ above $R$, optimal generation and purchasing are larger at earlier time-steps than later time-steps. This is the result of $\gamma$ being decreased, specifically. With lower $\gamma$, the firm can more aggressively sell excess SRECs and rid itself of them before time $T$. This means that at earlier time-steps, the firm continues to generate above their baseline in order to acquire more SRECs to sell later in the period. Later in the period, the firm prefers to liquidate their excess SRECs in order to ensure they do not have excess inventories at time $T$, resulting in the observed behaviour. This does not happen in the cases where $\gamma = 0.6$ or 1 as the firm is limited in how quickly it can viably liquidate excess SRECs by its trading speed penalty.

We do not include the plots of $(\zeta, \gamma)$ combinations where $\zeta \neq \gamma$ to avoid repetition. The results and interpretation are identical to those discussed above, with changes in $\zeta$ impacting optimal generation and changes in $\gamma$ impacting optimal trading.

5.5. Multi-period model. Thus far, we have considered a single period compliance framework. In practice, SREC markets consist of multiple periods. In this section, we present the results from numerically solving for the optimal behaviour of a regulated firm in an $N$-period SREC market, which is described in Section 2. Much of the behaviour and intuition discussed in the earlier parts of this section also applies to the multi-period model. We assume that there are $n$ decision points within each compliance period. Specifically, we assume

\[ (4.1) \quad 0 = t_1 < \cdots < t_n < T_1 = t_{n+1} < \cdots t_{2n} < T_2 = t_{2n+1} < \cdots < t_{Nn} < T_N = t_{Nn+1}. \]

We assume these are equally spaced such that $t_k = k\Delta t$. $t_{Nn+1}$ is not a decision point. Therefore, there are $Nn$ decision points, from $t_1, \ldots, t_{Nn}$.

As before, we continue to assume that $P$ and $R$ are constant across each of the $N$ periods, and that the processes $g_t, \Gamma_t$ are piecewise constant within $[t_i, t_{i+1})$, with the firm controlling $\{g_{t_i}, \Gamma_{t_i}\}_{i \in \mathbb{N}}$, where $\mathbb{N} = \{0, \ldots, N \times n\}$. Like Section 4, the intuition remains that the regulated firm chooses their trading and generating behaviour over the next interval of length $\Delta t$.

We denote the end points of the $i$-th period by $T_i$, $i = 1, \ldots, N$ and assume firms may bank unused certificates in one period to future periods with no expiry. As previously mentioned, in SREC markets, certificates generally cannot be banked indefinitely, but for a finite amount of periods. Assuming firms can bank unused certificates indefinitely reduces the dimensionality of the problem significantly, making it more computationally tractable.

\footnote{To check this, we make use of the fact that $0 \leq \partial_b V \leq P$.}
Under the assumptions detailed above, the performance criterion (corresponding to the total cost) for an arbitrary admissible control is

\[
J^g,\Gamma(k, b, S) = \mathbb{E}_{t_k, b, S} \left[ \sum_{i=k}^{N_n} \left\{ \frac{\zeta}{2} (g_{t_i} - h_{t_i})^2 + \Gamma_{t_i} S_{g,\Gamma}^t + \frac{\gamma}{2} \Gamma_{t_i}^2 \right\} \Delta t \right. \\
\left. + P(R - b_{g,\Gamma}^t - \Delta t(g_{t_i} + \Gamma_{t_i})) + \mathbb{I}_{\cup_{j=1}^{N_n} t_{i+1} = T_j} \right].
\]

(5.5)

The dynamics of the state variables \((b, S)\) are modified as follows

\[
S_{g,\Gamma}^t = \min \left( \left( S_{g,\Gamma}^{t-1} + \left( \mu + \eta \Gamma_{t-1} - \psi g_{t-1} \right) \Delta t + \sigma \sqrt{\Delta t} Z_{t_i} \right)_+, P \right) \\
b_{g,\Gamma}^t = \left( b_{g,\Gamma}^{t-1} + \left( g_{t-1} + \Gamma_{t-1} \right) \Delta t - R_{\cup_{j=1}^{N_n} t_{i+1} = T_j} \right)_+
\]

(5.6a, 5.6b)

where \(Z_{t_i} \sim N(0, 1), \text{iid, for all } i \in \mathfrak{N}\).

As in the single-period case, we seek

\[
V(t, b, S) = \sup_{g, \Gamma \in \mathcal{A}} J^g,\Gamma(t, b, S),
\]

(5.7)

and the strategy that attains the sup, if it exists. Applying the Bellman Principle to (5.7) implies

\[
V(t_i, b, S) = \inf_{g_i, \Gamma_i} \left\{ \left\{ \frac{\zeta}{2} (g_{t_i} - h_{t_i})^2 + \Gamma_{t_i} S_{g,\Gamma}^t + \frac{\gamma}{2} \Gamma_{t_i}^2 \right\} \Delta t \right. \\
\left. + P(R - b_{g,\Gamma}^t - \Delta t(g_{t_i} + \Gamma_{t_i})) + \mathbb{I}_{\cup_{j=1}^{N_n} t_{i+1} = T_j} \right] \\
+ \mathbb{E}_{t_i} \left[ V \left( t_{i+1}, b_{g,\Gamma}^{t_{i+1}}, S_{g,\Gamma}^{t_{i+1}} \right) \right], \quad \text{and}
\]

(5.8a, 5.8b)

\[
V(T, b, S) = P(R - b)_+.
\]

We note that the dynamics of \(b\) in the multi-period framework are such that \(b_{T_j}\) represents the firm’s SRECs after submitting the compliance requirement for the compliance period ending at \(T_j\). We adjust our solution algorithm described in Section 5.1 to account for the assumptions stated above, using the same model parameters, and choosing \(N = 5\). We denote the current period by \(m\). As the algorithm we use to solve for the optimal controls numerically in the multi-period problem is very similar to the algorithm detailed in Section 5.1, we do not explain it in detail here. We proceed to analyzing the results.

5.5.1. Multi-period models without price impacts. In Figure 7, we plot the dependence of the optimal generation and trading rate of the firm in the first period \((m = 1)\) of the 5-period model against banked SRECs, for three SREC prices, at six points in time, with all remaining parameters as in Tables 1 and 2. Much of the intuition surrounding Figure 1 applies here. There are, however, obvious differences between Figures 1 and 7. As before, for low levels of banked SRECs, across all values of \(S\), and near the end of the
compliance period, the firm generates until the marginal cost of producing another SREC exceeds $P$, and purchases until the marginal cost of purchasing another SREC exceeds $P$, as the firm is almost assured to fail to comply. In this regime, the marginal benefit of an additional SREC is $P$, as each additional SREC lowers their non-compliance obligation by $P$.

As the banked amount increases, the firm reaches a point where the marginal benefit from an additional SREC decreases from $P$. This occurs as the probability of compliance becomes non-negligible, as additional SRECs in excess of $R$ provide smaller marginal benefit than $P$. This leads to a decrease in optimal generation and optimal trading, as the firm adjusts its behaviour so that its marginal costs are in line with this marginal benefit. Thus far, this is the same interpretation as the single-period setting. As $b$ continues to increase, the firm holds sufficient banked SRECs such that they will be able to acquire surplus certificates above $R$. These surplus SRECs have no value in the current period to the firm. They may, however, bank SRECs putting the firm in a better position for future compliance periods. In the single-period case, at the end of the compliance period, holding additional SRECs lack utility. Here, however, we see an abrupt change in the slope of the optimal controls, and a slower decay in generation and purchasing rate when compared to Figure 1.

This decrease continues until the firm no longer benefits from additional SRECs. That is, at a certain level of $b$, the marginal benefit of an additional SREC is zero. Specifically, having an additional SREC does not increase the firm’s likelihood of compliance in current or future periods, nor can the firm sell the additional SREC for a profit (taking into account their trading costs and $S$). As in Figure 1, this results in optimal generation plateauing.
Figure 8. Paths of three optimally behaving firms in a 5-period compliance system with $S_0 = 150$, $b_0 = 0$ (blue), $b_0 = 250$ (red), $b_0 = 500$ (yellow). Remaining parameters as in Tables 1 and 2.

at the baseline amount $h_t$ and optimal trading plateauing at the level where the marginal revenue from trading equals the marginal cost. This plateau is not visible in every subplot in Figure 7 due to axis limits and the fact that $m = 1$. As $m$ increases, this plateau occurs for lower values of $b$, as there are fewer future periods to position oneself for. See Appendix B Figures 14–17 for the analogous figures for $m = 2, 3, 4, \text{and } 5$. The impact of SREC price on generation and trading is similar to the single period case.

The optimal controls for $m = 1, 2, 3$ and $4$ are similar to one another, while $m = 5$ is identical to the single-period case as it must be since the performance criterion is time-consistent. As $m$ increases, however, the firm has fewer future periods to position themselves for. Consequently, the firm’s optimal generation and purchasing behaviour decays more quickly for larger $m$, once they are in the regime where they may acquire more than $R$ SRECs. See Appendix B, Figures 14–17.

Figure 8 shows a sample path of the optimal strategy for three firms (with the same cost functions) throughout the course of the 5-period SREC market, with each period lasting 1 year. The firms differing in their initial banking: Firm 1 has $b_0 = 0$, Firm 2 has $b_0 = 250$ and Firm 3 has $b_0 = 500$. We set $S_0 = 150$ and simulate a path for $S$. At each time-step, each firm behaves optimally given their values of banked SRECs and the SREC price.

There are clear similarities between Figure 8 and Figure 2. Specifically, we see the same positive relationship between SREC price and generation rate, and the same inverse relationship between SREC price and purchasing. In this sample path, the SREC price generally decreases over the compliance period, and accordingly, we observe each firm generating less and selling less as this occurs. We also see the banked SRECs for all three firms
converge to $R = 500$ as $t \to 5$. As in the single period model, there is no advantage to banking SRECs in the terminal period, so an optimally behaving firm will, if compliance is possible, end with exactly $R$ SRECs in the terminal period. Consequently, Firm 3 accumulates SRECs at a slower rate than Firm 2, who accumulates SRECs at a slower rate than Firm 1. The rate at which each firm accumulates SRECs is almost constant within each compliance period, but drops in an amount equal to the requirement. This results in the *converging saw-tooth* pattern in the first subplot of Figure 8.

The optimal behaviours of each firm differ by almost a constant, suggesting that they react similarly to changes in $S$. The primary difference is due to their initial banked SRECs $b_0$. Firm 1 has no spare SRECs at $t = 0$, and generates the most and sell the least. Firm 3 has 500 spare SRECs at $t = 0$ – enough for an entire period of compliance. As such, they produce the least and sell the most. Firm 2 operates between Firm 1 and Firm 3. Naturally, Firm 3 profits the most from this system, due to their initial position.

As in Section 5.2, we simulate many paths similar to the method above in order to obtain summary statistics and learn about the distribution of various quantities for each firm. In Figures 9 we plot the histograms of total generated SRECs and total traded SRECs for a regulated firm in each period, based on 1,000 sample paths of $S$, with $S_0 = 150, b_0 = 250$.

![Figure 9](image-url)

*Figure 9. Histogram of firm generation and trading across each compliance period with $S_0 = 150, b_0 = 250$. Remaining parameters as in Tables 1 and 2.*

In Figure 9 we observe that each histogram exhibits symmetry. The most notable aspect of these plots is that the mean of total generated SRECs and total traded SRECs does not change significantly as the period changes. Consequently, given the initial value of the SREC price $S_0$, we expect the firm to have similar aggregate behaviour across all periods. This is a consequence of our choice of parameters which results in $S_t$ being a martingale. The variance, however, of each firm’s aggregate behaviour increases as the periods progress. This is the result of simulating forward paths of $S_t$ conditioning on $\mathcal{F}_0$, as $\text{Var}(S_t|S_0)$ is increasing in $t$. The increased variance in the paths of $S$ as time passes corresponds to increased variance in total generation and trading. These patterns persists across various choices of $S_0$ and $b_0$. To avoid repetition, plots for other initial conditions are not included.
in this work.

As in the single-period case, the correlation coefficient between total generation and total trading in each period is $-1$, indicating that the two quantities exist on a line in $\mathbb{R}^2$. In particular, they satisfy the linear relation $\int_0^{T_i} g_u du + \int_0^{T_i} \Gamma_u du = \frac{nR-b_0}{n}$ for all $i \in [n]$. This further implies that for all $i \in [n]$, the distribution of banked SRECs at time $T_i$ (before compliance) is a point mass.

To conclude this subsection, we briefly compare the behaviour of an optimally behaving firm in the multi-period SREC system and a firm in the same system who always behaves as if they are in a single-period SREC system across a sample path of $S$. For simplicity, we refer to the firm that behaves optimally as the ‘optimal’ firm, and the firm that behaves as if they are in the single-period system as the ‘naive’ firm.

In Figure 10, we observe that the naive firm starts the period by generating its baseline and selling a large amount. As the naive firm behaves as though they are in the single-period case, their objective here is to liquidate all excess SRECs, so they do not expire worthless (as they would in a single-period setting). However, in this case, it means that they have prevented themselves from banking any SRECs into future periods. From period 2 onward, their production jumps and they sell relatively less through the rest of the periods, as they strive for compliance without the advantage of any banked SRECs. From periods 2-5, the naive firm behaves in a manner similar to Firm 1 in Figure 8.

On the other hand, the optimal firm produces more and sells less in the first period than the naive firm, as they recognize that additional SRECs above $R$ can be banked and
used in future periods. As such, we see their behaviour remain similar across all periods, which is consistent with the results seen in Figures 8 and 9. This behaviour also results in the naive firm earning more revenue than the optimal firm in period 1, but earning less in periods 2-5, and underperforming the optimal strategy on an aggregate basis, as can be seen by the cumulative incurred costs.

5.5.2. Multi-period model with price impacts. The previous multi-period analysis assumed no price impact ($\psi = \eta = 0$). As in Section 5.4.1, we now consider the firm’s optimal behaviour with price impact. To avoid repetition, we only present results for a single price impact scenario; specifically, we choose $\eta = 0.01$, $\psi = 0.005$.

Analogous to Figure 7, Figure 11 shows the optimal behaviour of a regulated firm as a function of banked SRECs, across three different prices of $S$ and at six points in time during the first compliance period when there is price impact.

![Figure 11](image)

Figure 11. Optimal firm behaviour as a function of banked SRECs across various time-steps (during the first of five compliance periods) and SREC market prices with price impact $\eta = 0.01$, $\psi = 0.005$ and remaining parameters as in Tables 1 and 2.

Figures 11 and 7 are very similar. Specifically, the optimal controls at each of the ‘plateaus’ now vary slightly with time-step. Furthermore, with price impacts activated firms generate more (less) and purchase less (more) when far from compliance (near compliance) relative to their optimal behaviour without price impacts activated. This behaviour ensures that they influence the price of SRECs to their advantage (relative to their no-price-impact behaviour). That is, when far from compliance, their behaviours will keep $S$ relatively lower than if they reprised their no-price-impact behaviour, and when on track to comply, their behaviours will keep $S$ relatively higher.

We next explore the sample path behaviour of a regulated firm in a 5-period market with price impact. To do so, we replicate Figure 5 in the multi-period setting. Here, we
choose $S_0 = 150, b_0 = 0$ and the results are shown in Figure 12.

Regardless of the activation of price impacts, the optimally behaving firm accumulates SRECs at nearly identical rates, which are consistent across periods, as in Figure 8, resulting in the same saw-tooth pattern in the first subplot of Figure 12. As expected, the regulated firm generates less and sells less when price impacts are active, in order to mitigate their price impact. Nonetheless, we see that the price paths diverge over time, with the path of $S$ when price impacts are active being dominated by the path of $S$ when price impacts are inactive. Lastly, we note that activating price impacts results in lower profit for the firm. Each of these properties were exhibited in the single-period setting, in Figure 5.

Finally, we repeat Figure 9 when price impacts are activated. We simulate many paths of $S$ with $S_0 = 150, b_0 = 250$ in order to obtain summary statistics and learn about the distribution of various quantities for each firm. In Figures 13 we plot the histograms of total generated SRECs and total traded SRECs for a regulated firm in each period, based on 1,000 such sample paths.

From the figure, we see that the mean of total generation and total trading is no longer invariant across periods, like it was when $\psi = \eta = 0$. This is the result of the price impacts removing the martingale property of $S_t$. Like before, we see that the variance of each firm’s aggregate behaviour increases as the periods progress. This is the result of simulating forward paths of $S_t$ conditioning on $\mathcal{F}_0$, as $\text{Var}(S_t|S_0)$ is increasing in $t$. As before, these patterns persist across various choices of $S_0$ and $b_0$. To avoid repetition, plots for other initial conditions are not included in this work.

6. Conclusion. In this work, we characterize the optimal behaviour of a single regulated LSE in a single-period SREC market. In particular, we characterize their optimal generation
Many further extensions are possible. Interactions between agents are a critical component of real SREC markets that are largely ignored in this single-firm setup. In particular, incorporating partial information of firms would be a very challenging but mathematically interesting problem that would more closely mimic the realities of SREC markets. This could potentially necessitate the use of a mean field games approach. Improved calibration to real world parameters would also increase the applicability of this work for use by regulated firms and regulators.

However, even our simple model reveals salient facts about the nature of these systems and how firms should behave when regulated by them. Our single-period model reveals that the optimal generation and trading of regulated firms broadly exists in three regimes, depending on the marginal benefit received from holding an additional SREC. We observe that a firm’s trading behaviour is more sensitive to changes in $S$ than its generation behaviour, and that higher SREC prices imply greater generation and lower purchasing (more selling). We also show that the optimal behaviour outperforms the optimal constant behaviour strategy for our parameter choice, and study the effects of including price impacts in our model, as well as sensitivity to other parameters.

When extending to the multiple-period framework, we observe many similarities, but also the key difference that a fourth regime exists in the optimal generation and trading of regulated firms; that is, the regime where a marginal SREC does not provide value in the current period, but may be banked to provide value in the future. Additionally, we compare and contrast the optimal behaviours of firms throughout the multiple-period

Figure 13. Histogram of firm generation and trading across each compliance period with $S_0 = 150, b_0 = 250$. Price impacts activated with $\eta = 0.01, \psi = 0.005$. Remaining parameters as in Tables 1 and 2.
framework based on different initialization points. Lastly, we discover that conditional on $S_0$ and $b_0$, the mean of a firm's total generation and total trading does not change across periods, but the variance of said quantities increase.

In providing these results, we have produced a framework and numerical solution that would be of use for both regulated firms and regulatory bodies who both have immense interest in understanding the optimal behaviour of regulated LSEs in these systems.
Appendix A. Optimal Constant Behaviour Strategy.

A sub-problem to the one considered in prior subsections is how a firm should behave if it is restricted to behaving in a constant manner over the course of the compliance period. This provides insight into how their controls will change with respect to parameter changes. As such, consider a regulated firm that is optimizing their behaviour at $t = 0$, with $b_0 = 0$ and $T = 1$. For simplicity, further assume that $h_t, \mu_t, \sigma_t$ are constants. That is, $h_t = h, \mu_t = \mu, \sigma_t = \sigma$. In particular, we consider the case where we constrain $g + \Gamma = R$

Proposition A.1 (Optimal Constant Behaviours).

Consider a single firm that is regulated in a single-period SREC market, with the following additional assumptions:

- $h_t = h, \mu_t = \mu, \sigma_t = \sigma$
- $b_0 = 0, T = 1$
The controls $g_t, \Gamma_t$ must be constant across the period (i.e. $g_t = g, \Gamma_t = \Gamma$ for all $t \in [0, T]$).

Therefore, the firm aims to maximize the following:

$$J(g, \Gamma) = E \left[ -\int_0^T \frac{\zeta}{2} (g - h)^2 du - \int_0^T \Gamma S_u^g \Gamma du - \int_0^T \frac{\zeta}{2} \Gamma^2 du \right]$$

The optimal control is given by $(g^*, \Gamma^*) = \left( \frac{2S_0 + \mu + R(\psi + 2\eta + 2\gamma) + 2\zeta h}{2(\eta + \gamma + \psi + \zeta)}, -\frac{2S_0 + \mu - R(\psi + 2\zeta) + 2\zeta h}{2(\eta + \gamma + \psi + \zeta)} \right)$.

**Proof.** The RHS of (A.1) is identical to the RHS of (2.2) updated for the additional assumptions made (note that we can remove the non-compliance penalty as the constraint $g + \Gamma = R$ ensures compliance). This is a constrained optimization problem, which we solve through the use of Lagrange multipliers.

$$J(g, \Gamma) = E \left[ -\int_0^T \frac{\zeta}{2} (g - h)^2 du - \int_0^T \Gamma S_u^g \Gamma du - \int_0^T \frac{\zeta}{2} \Gamma^2 du \right]$$

$$= -\frac{\zeta}{2} (g - h)^2 - \Gamma E \left[ \int_0^T S_u^g \Gamma du \right] - \frac{\zeta}{2} \Gamma^2$$

$$= -\frac{\zeta}{2} (g - h)^2 - \Gamma \int_0^T (S_0 + (\mu + \eta \Gamma - \psi g)u) du - \frac{\zeta}{2} \Gamma^2$$

$$= -\frac{\zeta}{2} (g - h)^2 - \Gamma (S_0 + \frac{\mu}{2} - \frac{1}{2} (\eta + \gamma) \Gamma^2 + \frac{\psi}{2} g \Gamma).$$

We introduce $\lambda$ as an auxiliary variable, and define:

$$\mathcal{L}(g, \Gamma, \lambda) := -\frac{\zeta}{2} (g - h)^2 - \Gamma (S_0 + \frac{\mu}{2} - \frac{1}{2} (\eta + \gamma) \Gamma^2 + \frac{\psi}{2} g \Gamma) - \lambda (g + \Gamma - R).$$

Set $\nabla \mathcal{L} = 0$ to obtain the following system of equations, which is a necessary condition for a candidate optimizer to satisfy:

$$-\zeta (g - h) + \frac{\psi}{2} \Gamma - \lambda = 0,$$

$$-S_0 - \frac{\mu}{2} - (\eta + \gamma) \Gamma + \frac{\psi}{2} g - \lambda = 0,$$

$$-g - \Gamma + R = 0.$$

This is a system of three equations and three unknowns. The solution to this system is given by (with $\lambda$ omitted):

$$g = \frac{2S_0 + \mu + R(\psi + 2\eta + 2\gamma) + 2\zeta h}{2(\eta + \gamma + \psi + \zeta)}, \quad \text{and} \quad \Gamma = -\frac{2S_0 + \mu - R(\psi + 2\zeta) + 2\zeta h}{2(\eta + \gamma + \psi + \zeta)}.$$

Since $J$ is concave and the equality constraint is linear, a point satisfying the necessary conditions for an optimizer is in fact the global optimizer (see [4]). Therefore, $g^*, \Gamma^*$ take the forms seen in (A.7)
In Section 5.3, we consider the Mean-Behaviour strategy given by $g = 625, \Gamma = -125$. By substituting the applicable parameters from Section 5.1 into (A.7), we see that these values coincide with the optimal behaviours given by A.1. For these parameters, it can also be shown that the Mean-Behaviour strategy is optimal among all constant behaviour strategies.

Appendix B. Additional Figures.

Included below are plots of the regulated firm’s optimal behaviour in the context of Section 5.5, for periods 2-5 of a 5-period model.

$m = 2$

![Figure 14. Optimal firm behaviour as a function of banked SRECs across various time-steps (during the second of five compliance periods) and SREC market prices. Remaining parameters as in Tables 1 and 2.](image-url)
Figure 15. Optimal firm behaviour as a function of banked SRECs across various time-steps (during the third of five compliance periods) and SREC market prices. Remaining parameters as in Tables 1 and 2.

Figure 16. Optimal firm behaviour as a function of banked SRECs across various time-steps (during the fourth of five compliance periods) and SREC market prices. Remaining parameters as in Tables 1 and 2.
Figure 17. Optimal firm behaviour as a function of banked SRECs across various time-steps (during the fifth of five compliance periods) and SREC market prices. Remaining parameters as in Tables 1 and 2.