Effective model for particle mass generation

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Abstract

We present an effective model for particle mass generation in which we extract generic features of the Higgs mechanism that do not depend on its interpretation in terms of a Higgs field. In this model the physical vacuum is assumed as a medium at zero temperature which is formed by virtual fermions and antifermions interacting among themselves through the intermediate gauge bosons of the standard model without Higgs sector. As a consequence the fermions acquire their masses from theirs interactions with the vacuum and the gauge bosons from the charge fluctuations of the vacuum. This effective model is completely consistent with the physical mass spectrum, in such a way that the left-handed neutrinos are massive. The masses of the electroweak gauge bosons are properly predicted in terms of the experimental fermions masses and the running coupling constants of the strong, electromagnetic and weak interactions.

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1 Introduction

The Higgs mechanism is the currently accepted procedure to generate the masses of electrically charged fermions and electroweak bosons in particle physics [1]. The implementation of this mechanism requires the existence of a sector of scalar fields which includes, in the Lagrangian density of the model, a Higgs potential and Yukawa terms. In the Minimal Standard Model (MSM), the Higgs field is a doublet in the $SU(2)_L$ space carrying non-zero hypercharge, and a singlet in the $SU(3)_C$ space of color. The Higgs mechanism is based on the fact that the neutral component of the Higgs field doublet spontaneously acquires a non-vanishing vacuum expectation value. Since the vacuum expectation value of the Higgs field is different from zero, the Higgs field vacuum can be interpreted as a medium with a net weak charge. In this way the $SU(3)_C \times SU(2)_L \times U(1)_Y$ symmetry is spontaneously broken into the $SU(3)_C \times U(1)_{em}$ symmetry [2].

As a consequence of the MSM Higgs mechanism, the electroweak gauge bosons acquire their masses, in such a way that the masses depend of the vacuum expectation value of the Higgs field, which is a free parameter in the MSM. This parameter can be fixed by means of the calculation of the muon decay, at tree level, using the Fermi effective coupling constant. Simultaneously, Yukawa couplings between the Higgs field and fermion fields lead to the generation of masses for the electrically charged fermions, that depend on the Yukawa coupling constants, which also are free parameters in the MSM. These constants can be fixed by mean of the experimental values of the fermionic masses. The above mechanism implies the existence of a neutral Higgs boson in the physical spectrum, whose mass is a free parameter in the model. Because there only exist left-handed neutrinos in the Lagrangian density of the MSM, after the spontaneously electroweak symmetry breaking neutrinos remain massless.

In the current picture of the Higgs mechanism [1] the masses of the MSM particles spectrum are generated through the interactions among the electroweak gauge bosons and the electrically charged fermions with the weakly charged Higgs field vacuum. However, there are some physical aspects in this picture of mass generation that are not completely satisfactory and that we summarize in the following questions: What is the possible description of the interactions among the fermions and the electroweak bosons with the Higgs field vacuum? How is it possible to show in a fundamental way that the particle masses are generated by these interactions? Why the origin of the particle masses is only related to the weak interaction? Why the most-intense interactions (strong and electromagnetic) are not related to the mass generation mechanism? Why are there no interactions among the weakly charged left-handed neutrinos and the weakly charged Higgs field vacuum? Why the left-handed neutrinos are massless if they have a weak charge?
All the above questions might have a trivial answer if we only look at things through the current picture of the Higgs mechanism. However, we are interested in to research the possible physics behind the Higgs mechanism. In this way we propose an effective model for particle mass generation in which fermions acquire their masses from theirs interactions with the physical vacuum and gauge bosons from the charge fluctuations of the vacuum.

The physical vacuum is the state of lowest energy of all gauge bosons and fermion fields [3]. As it is well known from the covariant formulation of the quantum field theory [4] the state of lowest energy of the gauge bosons and matter fields has infinite energy. This physical vacuum is a rich medium where there are processes involving virtual massless particles and virtual massless antiparticles with unlimited energy. The physical vacuum is then assumed as a virtual medium at zero temperature which is formed by massless fermions and antifermions interacting among themselves exchanging massless gauge bosons.

The fundamental model describing the dynamics of the physical vacuum is the Standard Model without the Higgs Sector (SMWHS), which is based in the $SU(3)_C \times SU(2)_L \times U(1)_Y$ gauge symmetry group. We assume that each fermion in the physical vacuum has associated a chemical potential which describes the excess of virtual antifermions over virtual fermions. Then there are twelve fermionic chemical potentials $\mu_f$ associated with the six leptons and the six quarks implying an antimatter-matter asymmetry in the physical vacuum. Hence, the physical vacuum is considered as a virtual medium having antimatter finite density. This antimatter-matter asymmetry of the physical vacuum is related with CP violation by the electroweak interactions. Naturally, the mentioned asymmetry has an inverse sign respect to the one of the matter-antimatter asymmetry of the Universe. The existence of fermionic chemical potentials in the physical vacuum does not implies that this vacuum itself carries net charges. This can be understood in a similar way as the existence of the maximal matter-antimatter asymmetry of the Universe does not mean that baryonic matter carries net charges.

The masses of fermions are obtained starting from their self-energies, which give account of the fundamental interactions of massless fermions with the physical vacuum [3]. While quark masses are generated by strong, electromagnetic and weak interactions, the electrically charged lepton masses are only generated by electromagnetic and weak interactions, and the masses of neutrinos are generated from the weak interactions. On the other hand, gauge bosons masses are obtained from the charge fluctuations of the physical vacuum, which are described by the vacuum polarization tensors [3].

We use the following general procedure to calculate the particle masses: Initially we write the one-loop self-energies and the one-loop polarization tensors at finite density and finite temperature; next we calculate the dispersion relations by obtaining
the poles of the fermion and gauge boson propagators; from these dispersion relations we find the fermion and gauge boson effective masses at finite density and finite temperature; finally, we identify these effective masses at zero temperature with the physical masses. This identification can be performed because the virtual medium at zero temperature is representing the physical vacuum.

From a different perspective other works have intended to show that the inertial reaction force, appearing when a macroscopic body is accelerated by an external agent, is originated in a reaction by the physical vacuum that opposes the accelerating action [6]. These works involved only the electromagnetic quantum vacuum and have been able to yield the $F = ma$ expression as well as its relativistic generalization. An expression for the contribution by the electromagnetic quantum vacuum to the inertial mass of a macroscopic object has been found and this has been extended to the gravitational case. Originally these works have used a semiclassical approach [6] which has been easily extended to a quantum version [7].

We find that the fermion and gauge boson masses are functions of the vacuum fermionic chemical potentials $\mu_f$, which are fixed using the experimental fermion masses. Then using the values of the all fermionic chemical potentials obtained we calculate the masses of the electroweak gauge bosons obtaining an agreement with their experimental values. The main result of this effective model for particle mass generation is that the left-handed neutrinos are massive because they have weak charge. The weak interaction among the massless neutrinos and the physical vacuum is the source of the neutrino masses.

Before considering the real case of the physical vacuum described by the SMWHS, in section 2 we first show how to obtain the gauge invariant masses of a fermion and a gauge boson for the case in which the dynamics of the vacuum is described by a non-abelian gauge theory. In section 3 we consider the SMWHS as the model which describes the dynamics of the physical vacuum and we obtain the fermions (quarks and leptons) and the electroweak gauge bosons ($W^\pm$ and $Z^0$) masses. We obtain consistently the masses of the electroweak gauge bosons in terms of the masses of the fermions and the running coupling constants of the three fundamental interactions. In section 4 we focus our interest in to find a restriction about the possible number of families, a prediction of the top quark mass, a highest value for the summing of the square of the neutrino masses. Our conclusions are summarized in section 5.

## 2 Non-abelian gauge theory case

In this section we first consider a more simple case in which the dynamics of the vacuum is described by a non-abelian gauge theory, and in this context we calculate the fermion and gauge boson masses. The vacuum is assumed to be a quantum medium
at zero temperature constituted by virtual massless fermions and antifermions interacting among themselves through massless non-abelian gauge bosons. We also assume that there exist an excess of virtual antifermions over virtual fermions in the vacuum. This antimatter-matter asymmetry of the vacuum is described by non-vanishing fermionic chemical potentials \( \mu_{f_i} \), where \( f_i \) represents the different fermion species. In this section, for simplicity we will take \( \mu_{f_1} = \mu_{f_2} = \ldots = \mu_f \).

The non-abelian gauge theory describing the dynamics of the vacuum is given by the following Lagrangian density \([8]\):

\[
L = -\frac{1}{4} F^\mu_\nu F^{\nu A}_\mu + \bar{\psi}_m \gamma^\mu \left( \delta_{mn} i \partial_\mu + g L^A_{mn} A^A_\mu \right) \psi_n,
\]

(2.1)

where \( A \) runs over the generators of the group and \( m, n \) over the states of the fermion representation. The covariant derivative is \( D_\mu = \delta_\mu + ig T_A A^A_\mu \), being \( T_A \) the generators of the \( SU(N) \) gauge group and \( g \) the gauge coupling constant. The representation matrices \( L^A_{mn} \) are normalized by \( Tr(L^A L^B) = T(R) \delta^{AB} \) where \( T(R) \) is the index of the representation. In the calculation of the fermionic self-energy appears \( (L^A L^A)_{mn} = C(R) \delta_{mn} \), where \( C(R) \) is the quadratic Casimir invariant of the representation \([8]\).

At finite temperature and density, Feynman rules for vertices are the same as those at \( T = 0 \) and \( \mu_f = 0 \), while the propagators in the Feynman gauge for massless gauge bosons \( D_{\mu\nu}(p) \), massless scalars \( D(p) \) and massless fermions \( S(p) \) are \([9]\):

\[
D_{\mu\nu}(p) = -g_{\mu\nu} \left[ \frac{1}{p^2 + i\epsilon} - i \Gamma_b(p) \right],
\]

(2.2)

\[
D(p) = \frac{1}{p^2 + i\epsilon} - i \Gamma_b(p),
\]

(2.3)

\[
S(p) = \frac{\not{p}}{p^2 + i\epsilon} + i \not{\phi} \Gamma_f(p),
\]

(2.4)

where \( p \) is the particle four-momentum and the plasma temperature \( T \) is introduced through the functions \( \Gamma_b(p) \) and \( \Gamma_f(p) \), which are given by

\[
\Gamma_b(p) = 2\pi \delta(p^2) n_b(p),
\]

(2.5)

\[
\Gamma_f(p) = 2\pi \delta(p^2) n_f(p),
\]

(2.6)

with

\[
n_b(p) = \frac{1}{e^{\mu_f u/T} - 1},
\]

(2.7)

\[
n_f(p) = \theta(p \cdot u) n_{f_1}(p) + \theta(-p \cdot u) n_{f_f}(p),
\]

(2.8)
being \( n_b(p) \) the Bose-Einstein distribution function. The Fermi-Dirac distribution functions for fermions \( n_\bar{f}(p) \) and for anti-fermions \( n_f^+(p) \) are:

\[
n_f^+(p) = \frac{1}{e^{(p-u+\mu_f)/T} + 1}.
\] (2.9)

In the distribution functions (2.7) and (2.8), \( u^\alpha \) is the four-velocity of the center-of-mass frame of the dense plasma, with \( u^\alpha u_\alpha = 1 \).

### 2.1 Self-energy and fermion mass

We first consider the propagation of a massless fermion in a medium at finite density and finite temperature. The finite density of the medium is associated with the fact that it has more antifermions than fermions. The fermion mass is calculated by following the general procedure described in the introduction.

For a non-abelian gauge theory with parity and chirality conservation, the real part of the self-energy for a massless fermion is written as:

\[
\text{Re } \Sigma'(K) = -aK - b\eta,
\] (2.10)

\( a \) and \( b \) being the Lorentz-invariant functions and \( K^\alpha \) the fermion momentum. These functions depend on the Lorentz scalars \( \omega \) and \( k \) defined as \( \omega \equiv (K \cdot u) \) and \( k \equiv [(K \cdot u)^2 - K^2]^{1/2} \). Taking for convenience \( u^\alpha = (1, 0, 0, 0) \) we have \( K^2 = \omega^2 - k^2 \) and then, \( \omega \) and \( k \) can be interpreted as the energy and three-momentum, respectively. Beginning with (2.10) it is possible to write:

\[
a(\omega, k) = \frac{1}{4k^2} \left[ Tr(K \text{Re } \Sigma') - \omega Tr(\eta \text{Re } \Sigma') \right],
\] (2.11)

\[
b(\omega, k) = \frac{1}{4k^2} \left[ (\omega^2 - k^2) Tr(\eta \text{Re } \Sigma') - \omega Tr(K \text{Re } \Sigma') \right].
\] (2.12)

The fermion propagator, including only mass corrections, is given by [11]

\[
S(p) = \frac{1}{K - \text{Re } \Sigma'(K)} = \frac{1}{r} \frac{\gamma^0 \omega n - \gamma_i k^i}{n^2 \omega^2 - k^2},
\] (2.13)

where \( n = 1 + b(\omega, k)/r\omega \) and \( r = 1 + a(\omega, k) \). The propagator poles can be found when:

\[
\left[ 1 + \frac{b(w, k)}{w(1 + a(w, k))} \right]^2 w^2 - k^2 = 0.
\] (2.14)

We observe in (2.14) that \( n \) plays a role similar to that of the index of refraction in optics. To solve the equation (2.14), \( a(\omega, k) \) and \( b(\omega, k) \) are first calculated from the
relations (2.11) and (2.12) in terms of the real part of the fermionic self-energy. The contribution to the fermionic self-energy from the one-loop diagram, which can be constructed in this theory, is given by

\[ \Sigma(K) = ig^2 C(R) \int \frac{d^4p}{(2\pi)^4} D_{\mu\nu}(p) \gamma^\mu S(p + K) \gamma^\nu, \]  

(2.15)

where \( g \) is the interaction coupling constant and \( C(R) \) is the quadratic Casimir invariant of the representation. For the fundamental representation of SU(N), \( C(R) = (N^2 - 1)/2N \) [10]. We have that \( C(R) = 1 \) for the U(1) gauge symmetry group, \( C(R) = 1/4 \) for SU(2) and \( C(R) = 4/3 \) for SU(3).

Substituting (2.2) and (2.4) into (2.15), the fermionic self-energy can be written as \( \Sigma(K) = \Sigma(0) + \Sigma'(K) \), where \( \Sigma(0) \) is the zero-density and zero-temperature contribution and \( \Sigma'(K) \) is the finite-density and finite-temperature contribution. It is easy to see that:

\[ \Sigma(0) = -ig^2 C(R) \int \frac{d^4p}{(2\pi)^4} \frac{g_{\mu\nu} \gamma^\mu (p + K)}{(p + K)^2} \]  

and

\[ \Sigma'(K) = 2g^2 C(R) \int \frac{d^4p}{(2\pi)^4} (\phi + \mathbf{K}) \left\{ \frac{\Gamma_b(p)}{(p + K)^2} - \frac{\Gamma_f(p + K)}{p^2} + i\Gamma_b(p)\Gamma_f(p) \right\}. \]  

(2.17)

Keeping only the real part (\( \text{Re} \Sigma'(K) \)) of the finite-density and finite-temperature contribution, we obtain:

\[ \text{Re} \Sigma'(K) = 2g^2 C(R) \int \frac{d^4p}{(2\pi)^4} \left[ (\phi + \mathbf{K}) \Gamma_b(p) + \phi \Gamma_f(p) \right] \frac{1}{(p + K)^2}. \]  

(2.18)

If we multiply (2.18) by either \( \mathbf{K} \) or \( \phi \), take the trace and perform the integrations over \( p_0 \) and the two angular variables, the functions (2.11) and (2.12) can be written in the notation given in [12] as:

\[ a(\omega, k) = g^2 C(R) A(w, k, \mu_f), \]  

(2.19)

\[ b(\omega, k) = g^2 C(R) B(w, k, \mu_f), \]  

(2.20)

where the integrals over the modulus of the three-momentum \( p = |\mathbf{p}| \), \( A(\omega, k, \mu_f) \) and \( B(\omega, k, \mu_f) \), are:

\[ A(\omega, k, \mu_f) = \frac{1}{k^2} \int_0^\infty \frac{dp}{8\pi^2} \left[ 2p - \frac{\omega p}{k} \log \left( \frac{\omega + k}{\omega - k} \right) \right] \left[ 2n_b(p) + n^-_f(p) + n^+_f(p) \right]. \]
\[ B(\omega, k, \mu_f) = \frac{1}{k^2} \times \int_0^\infty \frac{dp}{8\pi^2} \left[ \frac{p(\omega^2 - k^2)}{k} \log \left( \frac{\omega + k}{\omega - k} \right) - 2\omega p \right] \left[ 2n_e(p) + n_f^-(p) + n_f^+(p) \right]. \] (2. 21)

The integrals (2.21) and (2.22) have been obtained in the high density approximation, i.e., \( \mu_f \gg k \) and \( \mu_f \gg \omega \), and keeping the leading terms in temperature and chemical potential [13]. Evaluating these integrals we obtain that \( a(\omega, k) \) and \( b(\omega, k) \) are given by:

\[
a(\omega, k) = \frac{M^2_F}{k^2} \left[ 1 - \omega \frac{2k}{\omega - k} \log \frac{\omega + k}{\omega - k} \right], \] (2. 23)

\[
b(\omega, k) = \frac{M^2_F}{k^2} \left[ \frac{\omega^2 - k^2}{2k} \log \frac{\omega + k}{\omega - k} - \omega \right], \] (2. 24)

where the fermion effective mass \( M_F \) is:

\[
M^2_F(T, \mu_f) = \frac{g^2C(R)}{8} \left( T^2 + \frac{\mu_f^2}{\pi^2} \right). \] (2. 25)

The value of \( M_F \) given by (2.25) is in agreement with [14]-[17]. We are interested in the effective mass at \( T = 0 \), which corresponds precisely to the case in which the vacuum is described by a virtual medium at zero temperature. For this case:

\[
M^2_F(0, \mu_f) = M^2_f = \frac{g^2C(R)}{8} \frac{\mu_f^2}{\pi^2}. \] (2. 26)

Substituting (2.23) and (2.24) into (2.14), we obtain for the limit \( k \ll M_F \) that:

\[
\omega^2(k) = M^2_F \left[ 1 + \frac{2}{3} \frac{k}{M_F} + \frac{5}{9} \frac{k^2}{M^2_F} + \ldots \right] \] (2. 27)

This dispersion relation is gauge invariant as the calculation has been done at leading order in temperature and chemical potential [13].

It is well known that the relativistic energy in the vacuum for a massive fermion at rest is \( \omega^2(0) = m_f^2 \). It is clear from (2.27) that if \( k = 0 \) then \( \omega^2(0) = M^2_f \) and thereby we can identify the fermion effective mass at zero temperature as the rest mass of the fermion, i.e. \( m_f^2 = M^2_f \). So the gauge invariant fermion mass, which is generated by the SU(N) gauge interaction of the massless fermion with the vacuum, is:

\[
m_f^2 = \frac{g^2C(R)}{8} \frac{\mu_f^2}{\pi^2}, \] (2. 28)

being \( \mu_f \) a free parameter.
2.2 Polarization tensor and gauge boson mass

The gauge boson mass is due to the charge fluctuations of the vacuum. This mass is calculated following the general procedure presented in the introduction. The most general form of the polarization tensor which preserves invariance under rotations, translations and gauge transformations is [18]:

\[ \Pi_{\mu\nu}(K) = P_{\mu\nu} \Pi_T(K) + Q_{\mu\nu} \Pi_L(K), \]  

(2.29)

where the Lorentz-invariant functions \( \Pi_L \) and \( \Pi_T \), which characterize the longitudinal and transverse modes respectively, are obtained by contraction:

\[ \Pi_L(K) = -\frac{K^2}{k^2} u^\mu u^\nu \Pi_{\mu\nu}, \]  

(2.30)

\[ \Pi_T(K) = -\frac{1}{2} \Pi_L + \frac{1}{2} g^{\mu\nu} \Pi_{\mu\nu}. \]  

(2.31)

The bosonic dispersion relations are obtained by looking at the poles of the full propagator, which results of adding all vacuum polarization insertions. The full bosonic propagator is [18]:

\[ D_{\mu\nu}(K) = \frac{Q_{\mu\nu}}{K^2 - \Pi_L(K)} + \frac{P_{\mu\nu}}{K^2 - \Pi_T(K)} - (\xi - 1) \frac{K_\mu K_\nu}{K^4}, \]  

(2.32)

where \( \xi \) is a gauge parameter. The gauge invariant dispersion relations, describing the two propagation modes, are found for:

\[ K^2 - \Pi_L(K) = 0, \]  

(2.33)

\[ K^2 - \Pi_T(K) = 0. \]  

(2.34)

The one-loop contribution to the vacuum polarization from the diagram, which can be constructed in this theory, is given by

\[ \Pi_{\mu\nu}(K) = ig^2 C(R) \int \frac{d^4 p}{(2\pi)^4} Tr [\gamma_\mu S(p) \gamma_\nu S(p + K)], \]  

(2.35)

where \( S \) is the fermion propagator (2.4). Substituting (2.4) into (2.35) the polarization tensor can be written as \( \Pi_{\mu\nu}(K) = \Pi_{\mu\nu}(0) + \Pi'_{\mu\nu}(K) \), where \( \Pi_{\mu\nu}(0) \) is the zero-density and zero-temperature contribution and \( \Pi'_{\mu\nu}(K) \) is the finite-density and finite-temperature contribution.

It is easy to see that the real part of the finite-density and finite-temperature contribution \( \text{Re} \Pi'_{\mu\nu}(K) \) is given by

\[ \text{Re} \Pi'_{\mu\nu}(K) = \frac{g^2 C(R)}{2} \int \frac{d^4 p (p^2 + p \cdot K) g^{\mu\nu} - 2p^\mu p^\nu - p^\mu K^\nu - p^\nu K^\mu}{(p + K)^2} \Gamma_f(p). \]  

(2.36)
Substituting (2.36) in (2.30) and (2.31) and keeping the leading terms in temperature and chemical potential, we obtain for the high density approximation \( \mu_f \gg k \) and \( \mu_f \gg \omega \) that:

\[
\text{Re} \Pi'_L(K) = 3M_B^2 \left[ 1 - \frac{\omega}{2k} \log \frac{\omega + k}{\omega - k} \right], \tag{2.37}
\]

\[
\text{Re} \Pi'_T(K) = \frac{3}{2}M_B^2 \left[ \frac{\omega^2}{k^2} + \left( 1 - \frac{\omega^2}{k^2} \right) \frac{\omega}{2k} \log \frac{\omega + k}{\omega - k} \right], \tag{2.38}
\]

where the gauge boson effective mass \( M_B \) is:

\[
M_B^2(T, \mu_f) = \frac{1}{6}Ng^2T^2 + \frac{1}{2}g^2C(R) \left[ \frac{T^2}{6} + \frac{\mu_f^2}{2\pi^2} \right], \tag{2.39}
\]

being \( N \) the gauge group dimension. The non-abelian effective mass (2.39) is in agreement with [17]. The abelian gauge boson, associated with a U(1) gauge invariant theory, acquires an effective mass \( M_{B(a)} \) defined by

\[
M_{B(a)}^2(T, \mu_f) = e^2 \left[ \frac{T^2}{6} + \frac{\mu_f^2}{2\pi^2} \right], \tag{2.40}
\]

being \( e \) the interaction coupling constant associated with the U(1) abelian gauge group. The abelian effective mass (2.40) is in agreement with [19]. Because the vacuum is described by a virtual medium at \( T = 0 \), then the non-abelian gauge boson effective mass generated by the quantum fluctuations of the vacuum is:

\[
M_{B(na)}^2(0, \mu_f) = M_{B(na)}^2 = g^2C(R)\frac{\mu_f^2}{4\pi^2}, \tag{2.41}
\]

and the abelian gauge boson effective mass generated by the quantum fluctuations of the vacuum is:

\[
M_{B(a)}^2(0, \mu_f) = M_{B(a)}^2 = e^2\frac{\mu_f^2}{2\pi^2}, \tag{2.42}
\]

in agreement with the result obtained at finite density and zero temperature [20]. For the limit \( k \ll M_{B(a)} \), it is possible to obtain the dispersion relations for the transverse and longitudinal propagation modes [18]:

\[
\omega_L^2 = M_B^2 + \frac{3}{5}k_L^2 + \ldots \tag{2.43}
\]

\[
\omega_T^2 = M_B^2 + \frac{6}{5}k_T^2 + \ldots \tag{2.44}
\]
We note that (2.43) and (2.44) have the same value when the three-momentum goes to zero. It is clear from (2.43) and (2.44) that for $k = 0$ then $\omega^2(0) = M_B^2$ and it is possible to recognize the gauge boson effective mass as the true gauge boson mass. The non-abelian gauge boson mass is:

$$m_{b(na)}^2 = M_B^2 = g^2 C(R) \frac{\mu_f^2}{4\pi^2}, \quad (2.45)$$

and the abelian gauge boson mass is:

$$m_{b(a)}^2 = M_B^2 = e^2 \frac{\mu_f^2}{2\pi^2}. \quad (2.46)$$

We observe that the gauge boson mass is a function on the chemical potential that is a free parameter in this effective model. It is important to note that if the fermionic chemical potential has an imaginary value, then the gauge boson effective mass, given by (2.45) or (2.46), would be negative [21].

3 SMWHS case

In this section, following the same procedure as in the previous one, we calculate the fermions and electroweak gauge bosons masses for the case in which the dynamics of the physical vacuum is described by the SMWHS. The physical vacuum is assumed to be a virtual medium at zero temperature constituted by virtual massless quarks, antiquarks, leptons and antileptons interacting among themselves through massless $G$ gluons (for the case of the quarks and antiquarks), massless $W^\pm$ electroweak gauge bosons, massless $W^3$ gauge bosons and massless $B$ bosons. In this quantum medium there exist an excess of virtual antifermions over virtual fermions. This fact is described by non-vanishing chemical potentials associated with the different fermion flavors. The chemical potentials are represented for the six quarks by the symbols $\mu_u, \mu_d, \mu_c, \mu_s, \mu_t, \mu_b$. For the chemical potentials of the charged leptons we use $\mu_e, \mu_\mu, \mu_\tau$ and for neutrinos $\mu_\nu_e, \mu_\nu_\mu, \mu_\nu_\tau$. These non-vanishing chemical potentials are input parameters in the effective model of particle mass generation.

The dynamics of the vacuum associated with the strong interaction is described by Quantum Chromodynamics (QCD), while the electroweak dynamics of the physical vacuum is described by the $SU(2)_L \times U(1)_Y$ electroweak standard model without Higgs sector. This model is defined by the following Lagrangian density:

$$\mathcal{L}_{ew} = \mathcal{L}_{YM} + \mathcal{L}_{FB} + \mathcal{L}_{GF} + \mathcal{L}_{FP}, \quad (3.1)$$
where $\mathcal{L}_{YM}$ is the Yang-Mills Lagrangian density, $\mathcal{L}_{FB}$ is the fermionic-bosonic Lagrangian density, $\mathcal{L}_{GF}$ is the gauge fixing Lagrangian density and $\mathcal{L}_{FP}$ is the Fadeev-Popov Lagrangian density. The $\mathcal{L}_{YM}$ is given by
\begin{equation}
\mathcal{L}_{YM} = -\frac{1}{4} W^A_{\mu\nu} W^A_{\mu\nu} - \frac{1}{4} F_{\mu\nu} F_{\mu\nu} \tag{3.2}
\end{equation}
where $W^{A}_{\mu\nu} = \partial_{\mu} W_{\nu}^{A} - \partial_{\nu} W_{\mu}^{A} + g_{w} F^{ABC} W_{\mu}^{B} W_{\nu}^{C}$ is the energy-momentum tensor associated with the $SU(2)_L$ group and $F_{\mu\nu} = \partial_{\mu} B_{\nu} - \partial_{\nu} B_{\mu}$ is the one associated with the $U(1)_Y$ group. The $\mathcal{L}_{FB}$ is written as:
\begin{equation}
\mathcal{L}_{FB} = i \bar{L} \gamma^\mu D_\mu L + i \bar{\psi}^I \gamma^\mu D_\mu \psi^I + i \bar{\psi}^L \gamma^\mu D_\mu \psi^L, \tag{3.3}
\end{equation}
where $D_\mu L = (\partial_\mu + ig_{e} Y_{L} B_{\mu}/2 + ig_{w} T_{i} W_{\mu}^{i}) L$ and $D_\mu R = (\partial_\mu + ig_{e} Y_{R} B_{\mu}/2) R$, being $g_{w}$ the gauge coupling constant associated with the $SU(2)_L$ group, $g_{e}$ the one associated to the $U(1)_Y$ group, $Y_{L} = -1$, $Y_{R} = -2$ and $T_{i} = \sigma_{i}/2$. The $SU(2)_L$ left-handed doublet $(L)$ is given by
\begin{equation}
L = \begin{pmatrix} \psi^L \\ \psi^R \end{pmatrix} \tag{3.4}
\end{equation}

### 3.1 Masses of the fermions

Initially we consider the propagation of massless fermions in a medium at finite density and finite temperature. The fermion masses are calculated following the same procedure as in the previous section. For a non-abelian gauge theory with parity violation and quirality conservation like the SMWHS, the real part of the self-energy for a massless fermion is [12]:
\begin{equation}
\text{Re} \Sigma'(K) = -\mathcal{K}(a_{L} P_{L} + a_{R} P_{R}) - \bar{\psi}(b_{L} P_{L} + b_{R} P_{R}), \tag{3.5}
\end{equation}
where $P_{L} \equiv \frac{1}{2}(1 - \gamma_{5})$ and $P_{R} \equiv \frac{1}{2}(1 + \gamma_{5})$ are respectively the left- and right-handed chiral projectors. The functions $a_{L}$, $a_{R}$, $b_{L}$ and $b_{R}$ are the chiral projections of the Lorentz-invariant functions $a$, $b$ and they are defined in the following way:
\begin{align}
a &= a_{L} P_{L} + a_{R} P_{R}, \tag{3.6} \\
b &= b_{L} P_{L} + b_{R} P_{R}. \tag{3.7}
\end{align}

The inverse fermion propagator is given by
\begin{equation}
S^{-1}(K) = \mathcal{L} P_{L} + \mathcal{R} P_{R} \tag{3.8}
\end{equation}
where:
\begin{align}
\mathcal{L}^\mu &= (1 + a_{L}) K^\mu + b_{L} u^\mu \tag{3.9} \\
\mathcal{R}^\mu &= (1 + a_{R}) K^\mu + b_{R} u^\mu. \tag{3.10}
\end{align}
The fermion propagator follows from the inversion of (3.8):

\[ S = \frac{1}{D} \left[ (\mathcal{L}^2 \Re) P_L + (\Re^2 \mathcal{L}) P_R \right]. \tag{3.11} \]

being \( D(\omega, k) = \mathcal{L}^2 \Re^2 \). The poles of the propagator correspond to values \( \omega \) and \( k \) for which the determinant \( D \) in (3.11) vanishes:

\[ \mathcal{L}^2 \Re^2 = 0. \tag{3.12} \]

In the rest frame of the dense plasma \( u = (1, \vec{0}) \), Eq.(3.12) leads to the fermion dispersion relations for a chirally invariant gauge theory with parity violation, as the case of the SMWHS. Thus, the fermion dispersion relations are given by [12]

\[ [\omega(1 + a_L) + b_L]^2 - k^2 [1 + a_L]^2 = 0, \tag{3.13} \]

\[ [\omega(1 + a_R) + b_R]^2 - k^2 [1 + a_R]^2 = 0. \tag{3.14} \]

Left- and right-handed components of the fermion dispersion relations obey decoupled relations. The Lorentz invariant functions \( a(\omega, k) \) and \( b(\omega, k) \) are calculated from expressions (2.11) and (2.12) through the real part of the fermion self-energy. This self-energy is obtained adding all the possible gauge boson contributions admitted by the Feynman rules of the SMWHS.

We work in the basis of gauge bosons given by \( B_\mu, W_3^\mu, W_\pm^\mu \), where the charged electroweak gauge bosons are \( W_\pm^\mu = (W_1^\mu \pm i W_2^\mu)/\sqrt{2} \). The diagrams with an exchange of \( W_\pm^\mu \) gauge bosons induce a flavor change in the incoming fermion \( i \) to a different outgoing fermion \( j \).

### 3.1.1 Quark masses

For the quark sector, in the case of the flavor change contributions mentioned, the flavor \( i \) (\( I \)) of the internal quark (inside the loop) runs over the up (\( i \)) or down (\( I \)) quarks flavors according to the type of the external quark (outside the loop). As each contribution to the quark self-energy is proportional to (2.21)-(2.22), the functions \( a_L, a_R, b_L \) and \( b_R \) are given by

\[ a_L(\omega, k)_{ij} = [f_S + f_{W^3} + f_B]A(\omega, k, \mu_i) + \sum_I f_{W^\pm} A(\omega, k, \mu_I), \tag{3.15} \]

\[ b_L(\omega, k)_{ij} = [f_S + f_{W^3} + f_B]B(\omega, k, \mu_i) + \sum_I f_{W^\pm} B(\omega, k, \mu_I), \tag{3.16} \]

\[ a_R(\omega, k)_{ij} = [f_S + f_B]A(\omega, k, \mu_i), \tag{3.17} \]

\[ b_R(\omega, k)_{ij} = [f_S + f_B]B(\omega, k, \mu_i). \tag{3.18} \]
The coefficients $f$ are:

$$f_s = \frac{4}{3} g_s^2 \delta_{ij}, \quad (3.19)$$

$$f_{W^3} = \frac{1}{4} g_w^2 \delta_{ij}, \quad (3.20)$$

$$f_B = \frac{1}{4} g_e^2 \delta_{ij}, \quad (3.21)$$

$$f_{W^\pm} = \frac{1}{2} g_w^2 K_1^+ K_{ij}, \quad (3.22)$$

where $K$ represents the CKM matrix and $g_s$ is the strong running coupling constant associated with the $SU(3)_C$ group. The integrals $A(\omega, k, \mu_f)$ and $B(\omega, k, \mu_f)$ are obtained in the high density approximation ($\mu_f \gg k$ and $\mu_f \gg \omega$) and, keeping the leading terms in temperature and chemical potential, they are given by

$$A(\omega, k, \mu_f) = \frac{1}{8k^2} \left( T^2 + \frac{\mu_f^2}{\pi^2} \right) \left[ 1 - \frac{\omega}{2k} \log \frac{\omega + k}{\omega - k} \right], \quad (3.23)$$

$$B(\omega, k, \mu_f) = \frac{1}{8k^2} \left( T^2 + \frac{\mu_f^2}{\pi^2} \right) \left[ \frac{\omega^2 - k^2}{2k} \log \frac{\omega + k}{\omega - k} - \omega \right]. \quad (3.24)$$

The chiral projections of the Lorentz-invariant functions are:

$$a_{L}(\omega, k)_{ij} = \frac{1}{8k^2} \left[ 1 - F(\frac{\omega}{k}) \right] \left[ l_{ij}(T^2 + \frac{\mu_f^2}{\pi^2}) + h_{ij}(T^2 + \frac{\mu_f^2}{\pi^2}) \right], \quad (3.25)$$

$$b_{L}(\omega, k)_{ij} = -\frac{1}{8k^2} \left[ \omega \frac{k}{\omega} + \left( \frac{k}{\omega} - \frac{\omega}{k} \right) F(\frac{\omega}{k}) \right] \left[ r_{ij}(T^2 + \frac{\mu_f^2}{\pi^2}) + h_{ij}(T^2 + \frac{\mu_f^2}{\pi^2}) \right], \quad (3.26)$$

$$a_{R}(\omega, k)_{ij} = \frac{1}{8k^2} \left[ 1 - F(\frac{\omega}{k}) \right] \left[ r_{ij}(T^2 + \frac{\mu_f^2}{\pi^2}) \right], \quad (3.27)$$

$$b_{R}(\omega, k)_{ij} = -\frac{1}{8k^2} \left[ \omega \frac{k}{\omega} + \left( \frac{k}{\omega} - \frac{\omega}{k} \right) F(\frac{\omega}{k}) \right] \left[ r_{ij}(T^2 + \frac{\mu_f^2}{\pi^2}) \right], \quad (3.28)$$

where $F(x)$ is

$$F(x) = \frac{x}{2} \log \left( \frac{x + 1}{x - 1} \right), \quad (3.29)$$

and the coefficients $l_{ij}, h_{ij}$ and $r_{ij}$ are given by

$$l_{ij} = \left( \frac{4}{3} g_s^2 + \frac{1}{4} g_w^2 + \frac{1}{4} g_e^2 \right) \delta_{ij}, \quad (3.30)$$
\[ h_{ij} = \sum_l \left( \frac{g_w^2}{2} \right) K_i^l K_j^l, \quad (3.31) \]
\[ r_{ij} = \left( \frac{4}{3} g_s^2 + \frac{1}{4} g_e^2 \right) \delta_{ij}. \quad (3.32) \]

Substituting (3.25)-(3.26) into (3.13), and (3.27)-(3.28) into (3.14), we obtain for the limit \( k \ll M_{(i,I)_{L,R}} \) that:

\[ \omega^2(k) = M^2_{(i,I)_{L,R}} \left[ 1 + \frac{2}{3} \frac{k}{M_{(i,I)_{L,R}}} + \frac{5}{9} \frac{k^2}{M^2_{(i,I)_{L,R}}} + \ldots \right], \quad (3.33) \]

where

\[ M^2_{(i,I)_L}(T,\mu_f) = (l_{ij} + h_{ij}) \frac{T^2}{8} + l_{ij} \frac{\mu^2_{(i,I)_L}}{8\pi^2} + h_{ij} \frac{\mu^2_{(i,I)_L}}{8\pi^2}, \quad (3.34) \]
\[ M^2_{(i,I)_R}(T,\mu_f) = r_{ij} \frac{T^2}{8} + r_{ij} \frac{\mu^2_{(i,I)_R}}{8\pi^2}. \quad (3.35) \]

As it was explained in the previous section, we are interested in the effective masses at \( T = 0 \). For this case:

\[ M^2_{(i,I)_L}(0,\mu_f) = l_{ij} \frac{\mu^2_{(i,I)_L}}{8\pi^2} + h_{ij} \frac{\mu^2_{(i,I)_L}}{8\pi^2}, \quad (3.36) \]
\[ M^2_{(i,I)_R}(0,\mu_f) = r_{ij} \frac{\mu^2_{(i,I)_R}}{8\pi^2}. \quad (3.37) \]

Following the same argument as in section 2, we can identify the quark effective masses at zero temperature with the rest masses of quarks. Coming from the left-handed and right-handed representations, we find that the masses of the left-handed quarks are:

\[ m_i^2 = \left[ \frac{4}{3} g_s^2 + \frac{1}{4} g_w^2 + \frac{1}{4} g_e^2 \right] \frac{\mu^2_{i,L}}{8\pi^2} + \left[ \frac{1}{2} g_w^2 \right] \frac{\mu^2_{i,L}}{8\pi^2}, \quad (3.38) \]
\[ m_I^2 = \left[ \frac{4}{3} g_s^2 + \frac{1}{4} g_w^2 + \frac{1}{4} g_e^2 \right] \frac{\mu^2_{I,L}}{8\pi^2} + \left[ \frac{1}{2} g_w^2 \right] \frac{\mu^2_{I,L}}{8\pi^2}, \quad (3.39) \]

and the masses of the right-handed quarks are:

\[ m_i^2 = \left[ \frac{4}{3} g_s^2 + \frac{1}{4} g_e^2 \right] \frac{\mu^2_{i,R}}{8\pi^2}, \quad (3.40) \]
\[ m_I^2 = \left[ \frac{4}{3} g_s^2 + \frac{1}{4} g_e^2 \right] \frac{\mu^2_{I,R}}{8\pi^2}. \quad (3.41) \]
where the couple of indexes \((i, I)\) run over quarks \((u, d)\), \((c, s)\) and \((t, b)\). It is known that the masses of the left-handed quarks are the same that the ones of right-handed quarks. This means that the left-handed quark chemical potentials \(\mu_{q_L}\) are different than the right-handed quark chemical potentials \(\mu_{f_R}\).

If we call

\[
a_q = \frac{1}{8\pi^2} \left[ \frac{4}{3}g_s^2 + \frac{1}{4}g_w^2 + \frac{1}{4}g_e^2 \right],
\]

(3.42)

\[
b_q = \frac{1}{8\pi^2} \left[ \frac{1}{2}g_w^2 \right],
\]

(3.43)

it is easy to see that the expressions (3.38) and (3.39) lead to

\[
\mu_{u_L}^2 = \frac{a_q m_u^2 - b_q m_d^2}{a_q^2 - b_q^2},
\]

(3.44)

\[
\mu_{d_L}^2 = \frac{-b_q m_u^2 + a_q m_d^2}{a_q^2 - b_q^2},
\]

(3.45)

and if we call

\[
c_q = \frac{1}{8\pi^2} \left[ \frac{4}{3}g_s^2 + \frac{1}{4}g_e^2 \right],
\]

(3.46)

the expressions (3.40) and (3.41) can be written as

\[
\mu_{u_R}^2 = \frac{m_u^2}{c_q},
\]

(3.47)

\[
\mu_{d_R}^2 = \frac{m_d^2}{c_q},
\]

(3.48)

and similar expressions for the other two quark doublets \((c, s)\) and \((t, b)\).

If we take the experimental central values for the strong constant \(\alpha_s = 0.1176\), the fine-structure constant as \(\alpha_e = 7.2973525376 \times 10^{-3}\) and the cosine of the electroweak mixing angle as \(\cos \theta_w = M_W/M_Z = 0.8043/0.1876 = 0.881732\) [5], then we have that \(g_s = 1.21565\), \(g_w = 0.641911\) and \(g_e = 0.34344\). Putting the central values for the masses of the quarks, given by [5] \(m_u = 0.00225\) GeV, \(m_d = 0.005\) GeV, \(m_c = 1.25\) GeV, \(m_s = 0.095\) GeV, \(m_t = 172.371\) GeV and \(m_b = 4.20\) GeV, into the expressions (3.44), (3.45), (3.47) and (3.48), we obtain that the squares of the left-handed quark chemical potentials are given by

\[
\mu_{u_L}^2 = 9.9068 \times 10^{-5},
\]

(3.49)

\[
\mu_{d_L}^2 = 9.2896 \times 10^{-4},
\]

(3.50)

\[
\mu_{c_L}^2 = 59.2015,
\]

(3.51)
\[ \mu_{s_L}^2 = -5.4612, \quad \mu_{t_L}^2 = 1.1263 \times 10^6, \quad \mu_{b_L}^2 = -1.0969 \times 10^5, \quad (3.52) \]
\[ \mu_{u_R}^2 = 1.9987 \times 10^{-4}, \quad \mu_{d_R}^2 = 9.8701 \times 10^{-4}, \quad \mu_{c_R}^2 = 61.6883, \quad \mu_{s_R}^2 = 0.3563, \quad \mu_{t_R}^2 = 1.1730 \times 10^6, \quad \mu_{b_R}^2 = 696.436, \quad (3.55) \]

and the squares of the right-handed quark chemical potentials are

\[ \mu_{u_R}^2 = \frac{1}{4} g_w^2 + \frac{1}{4} g_e^2, \quad (3.57) \]

where the left- and right-handed chemical potentials are given in GeV^2 units.

### 3.1.2 Lepton masses

For the lepton sector, the contributions to the fermion self-energy are proportional to (2.21)-(2.22) and the functions \(a_L, a_R, b_L, b_R\) are given by

\[ a_L(\omega, k)_{ij} = [f_{W^3} + f_B]A(\omega, k, \mu_i) + \sum_l f_{W^\pm} A(\omega, k, \mu_l), \quad (3.61) \]
\[ b_L(\omega, k)_{ij} = [f_{W^3} + f_B]B(\omega, k, \mu_i) + \sum_l f_{W^\pm} B(\omega, k, \mu_l), \quad (3.62) \]
\[ a_R(\omega, k)_{ij} = [f_B]A(\omega, k, \mu_i), \quad (3.63) \]
\[ b_R(\omega, k)_{ij} = [f_B]B(\omega, k, \mu_i), \quad (3.64) \]

being for this case \(f_{W^\pm} = g_w^2/2\) and the other coefficients \(f_{W^3}\) and \(f_B\) are given by (3.20) and (3.21), respectively.

The dispersion relation for the leptons are similar to the relations (3.33), but in this case the effective masses (3.34) and (3.35) are given by

\[ M^2_{(i,l)}(T, \mu_f) = (l + h) \frac{T^2}{8} + l \frac{\mu_{[i,l]L}^2}{8\pi^2} + h \frac{\mu_{[i,l]R}^2}{8\pi^2}, \quad (3.65) \]
\[ M^2_{(i,r)}(T, \mu_f) = r \frac{T^2}{8} + r \frac{\mu_{[i,r]R}^2}{8\pi^2}, \quad (3.66) \]

where the coefficients \(l, h\) and \(r\), for the charged leptons are given by

\[ l = \left( \frac{1}{4} g_w^2 + \frac{1}{4} g_e^2 \right), \quad (3.67) \]
\[ h = \left( \frac{1}{2} g_w^2 \right), \quad (3.68) \]
\[ r = \left( \frac{1}{4} g_e^2 \right), \quad (3.69) \]
and for the neutrinos these coefficients are:
\[ l = \left( \frac{1}{4} g_w^2 \right), \quad (3.70) \]
\[ h = \left( \frac{1}{2} g_w^2 \right), \quad (3.71) \]
\[ r = 0. \quad (3.72) \]

The leptonic effective masses (3.65) and (3.66) at zero temperature can be interpreted as the masses of the leptons. Coming from the left-handed and right-handed representations, respectively, we find that the masses of the left-handed leptons are given by:
\[ m_i^2 = \left[ \frac{1}{4} g_w^2 \right] \frac{\mu_i^2}{8\pi^2} + \left[ \frac{1}{2} g_w^2 \right] \frac{\mu_{L_i}^2}{8\pi^2}, \quad (3.73) \]
\[ m_I^2 = \left[ \frac{1}{4} g_w^2 + \frac{1}{4} g_e^2 \right] \frac{\mu_{L_i}^2}{8\pi^2} + \left[ \frac{1}{2} g_w^2 \right] \frac{\mu_{L_i}^2}{8\pi^2}, \quad (3.74) \]
and the masses of the right-handed charged leptons are
\[ m_I^2 = \left[ \frac{1}{4} g_e^2 \right] \frac{\mu_{L_i}^2}{8\pi^2}, \quad (3.75) \]
where the couple of indexes \((i, I)\) run over leptons \((\nu_e, e), (\nu_\mu, \mu)\) and \((\nu_\tau, \tau)\). We note that our effective model predicts that neutrinos are massive. The \(W^3\) and \(W^\pm\) interactions among the massless neutrinos with the physical vacuum are the origin of the left-handed neutrinos masses, as we can observe from (3.73).

If we call
\[ a_l = \frac{1}{8\pi^2} \left[ \frac{1}{4} g_w^2 \right], \quad (3.76) \]
\[ b_l = \frac{1}{8\pi^2} \left[ \frac{1}{2} g_w^2 \right], \quad (3.77) \]
\[ c_l = \frac{1}{8\pi^2} \left[ \frac{1}{4} g_w^2 + \frac{1}{4} g_e^2 \right], \quad (3.78) \]
then the expressions (3.73) and (3.74) lead to
\[ \mu_{\nu_L}^2 = \frac{c_l m_\nu^2 - b_l m_e^2}{a_l c_l - b_l^2}, \quad (3.79) \]
\[ \mu_{e_L}^2 = \frac{-b_l m_\nu^2 + a_l m_e^2}{a_l c_l - b_l^2}, \quad (3.80) \]
and if we call
\[ d_t = \frac{1}{8\pi^2} \left[ \frac{1}{4} g_e^2 \right], \]  

the expression (3.75) can be written as
\[ \mu_{\tau R}^2 = \frac{m_\tau^2}{d_t}, \]  

and similar expressions for the other two lepton doublets \((\nu_\mu, \mu)\) and \((\nu_\tau, \tau)\). Assuming the neutrinos to be massless, \(m_{\nu_e} = m_{\nu_\mu} = m_{\nu_\tau} = 0\), and putting the experimental central values for the masses of the charged leptons, given by \([5]\) \(m_e = 0.51099892 \times 10^{-3}\) GeV, \(m_\mu = 0.105658369\) GeV, \(m_\tau = 1.77699\) GeV, in the expressions (3.79), (3.80) and (3.82), we obtain that the squares of the left-handed lepton chemical potentials are
\[
\begin{align*}
\mu_{\nu_e L}^2 &= 3.9182 \times 10^{-4}, \\
\mu_{e L}^2 &= -3.0462 \times 10^{-4}, \\
\mu_{\mu L}^2 &= 16.8179, \\
\mu_{\nu_\mu L}^2 &= -13.0751, \\
\mu_{\nu_\tau L}^2 &= 4.7569 \times 10^3, \\
\mu_{\tau L}^2 &= -3.6982 \times 10^3,
\end{align*}
\]

and the the squares of the right-handed charged lepton chemical potentials are
\[
\begin{align*}
\mu_{\nu_e R}^2 &= 6.9645 \times 10^{-4}, \\
\mu_{\mu R}^2 &= 29.8929, \\
\mu_{\tau R}^2 &= 8.4551 \times 10^3,
\end{align*}
\]

where the left- and right-handed chemical potentials are given in GeV\(^2\) units. Neutrino masses are not known but direct experimental results shown that neutrino masses are of order 1 eV \([5]\), and cosmological interpretations from five-year WMAP observations find a limit on the total mass of massive neutrinos of \(\Sigma m_\nu < 0.6\) eV (95% CL) \([22]\). These results assure that the values of the left-handed lepton chemical potentials obtained taking neutrinos to be massless will change little if we take the true small neutrinos masses.

We observe that for five of the six fermion doublets the square of the chemical potential associated to the down fermion of the doublet has a negative value. This behavior is observed if there is a large difference between the masses of the two fermions of the doublet. This means that it behavior is not observed for the quark
doublet which is formed by the up and down quarks, because these two quarks have masses very close. In this case, the chemical potentials associated to these two quarks are positives.

From expressions (3.38), (3.39), (3.73) and (3.74) it can be seen that our effective model does not predict the fermions masses values because the fermionic chemical potentials \( \mu_{f_i} \) are free parameters. However, we have fixed the values of these \( \mu_{f_i} \) starting from the known experimental values for the fermions masses. This limitation of our effective model is similar to what happens in the MSM with Higgs mechanism, in the sense that the masses of the fermions depend on the Yukawa coupling constants which are free parameters. In a similar way to what happens here when we find the values of the vacuum chemical potentials, the Yukawa coupling constants can be fixed by means of the experimental values of the fermion masses.

3.2 Masses of the electroweak gauge bosons

The masses of the electroweak gauge bosons are originated from the charge fluctuations of the vacuum. These masses are calculated following a sequence of steps that we present now: In the first step we write the one-loop polarization tensor at finite density and finite temperature; then we calculate the one-loop bosonic dispersion relations in the high density approximation by obtention of the poles of the gauge boson propagators; starting from these dispersion relations we obtain the electroweak gauge boson effective masses at finite density and finite temperature; finally we identify these effective masses at zero temperature with the masses of the electroweak gauge bosons.

To evaluate the bosonic polarization tensor associated with the \( W^\pm, W^3, B^\mu \) gauge boson propagators, we follow the same procedure as in the section 2.2. On the other hand, applying the expressions (2.45) and (2.46) in the SMWHS, we obtain that the masses of the gauge bosons are:

\[
M^2_{W^\pm} = \frac{g^2_w}{2} \frac{\mu^2_{u_L} + \mu^2_{d_L} + \mu^2_{e_L} - \mu^2_{s_L} - \mu^2_{t_L} + \mu^2_{c_L} + \mu^2_{b_L} + \sum_{i=1}^{3} (\mu^2_{v_{iL}} - \mu^2_{e_{iL}})}{4\pi^2}, \quad (3.92)
\]

\[
M^2_{W^3} = \frac{g^2_w}{4} \frac{\mu^2_{u_L} + \mu^2_{d_L} + \mu^2_{e_L} - \mu^2_{s_L} + \mu^2_{t_L} - \mu^2_{b_L} + \sum_{i=1}^{3} (\mu^2_{v_{iL}} - \mu^2_{e_{iL}})}{2\pi^2}, \quad (3.93)
\]

\[
M^2_B = \frac{g^2_e}{4} \frac{\mu^2_{u_L} + \mu^2_{d_L} + \mu^2_{e_L} - \mu^2_{s_L} + \mu^2_{t_L} - \mu^2_{b_L} + \sum_{i=1}^{3} (\mu^2_{v_{iL}} - \mu^2_{e_{iL}})}{2\pi^2}, \quad (3.94)
\]

where the sum runs over the three lepton families. It is important to remember that if the left-handed fermionic chemical potential has an imaginary value, then its contribution to the gauge boson effective mass, as in the case (2.45) or (2.46), would be negative. This fact means that finally the contribution from each fermionic chemical potential to the masses of the gauge bosons is always positive.
Substituting the obtained left-handed fermionic chemicals potentials values (3.55)-(3.60) and (3.83)-(3.91) into the expressions (3.92)-(3.94), we obtain

\[ M_{W^\pm} = M_{W^3} = 80.403 \text{ GeV}, \quad (3. \ 95) \]
\[ M_B = 42.9954 \text{ GeV}, \quad (3. \ 96) \]

where the \( M_W \) value computed is in agreement with its experimental value given by \( M_W^{exp} = 80.403 \pm 0.029 \text{ GeV} \) [5].

Due to well known physical reasons \( W^3_\mu \) and \( B_\mu \) gauge bosons are mixed. After diagonalization of the mass matrix, we get the physical fields \( A_\mu \) and \( Z_\mu \) corresponding to the massless photon and the neutral \( Z^0 \) boson of mass \( M_Z \) respectively, with the relations [23, 24]:

\[ M_Z^2 = M_W^2 + M_B^2, \quad (3. \ 97) \]
\[ \cos \theta_w = \frac{M_W}{M_Z}, \quad \sin \theta_w = \frac{M_B}{M_Z}. \quad (3. \ 98) \]

where \( \theta_w \) is the weak mixing angle:

\[ W^3_\mu = B_\mu \sin \theta_w - W^3_\mu \cos \theta_w, \quad (3. \ 99) \]
\[ A_\mu = B_\mu \cos \theta_w + W^3_\mu \sin \theta_w. \quad (3. \ 100) \]

Substituting (3.112) and (3.96) into (3.97) we obtain that:

\[ M_Z = 91.1876 \text{ GeV}, \quad (3. \ 101) \]

also in agreement with its experimental value given by \( M_Z^{exp} = 91.1876 \pm 0.0021 \text{ GeV} \) [5].

Substituting the expressions for the fermionic chemical potentials given by (3.44), (3.45), (3.79), (3.80) into the expressions (3.92), (3.93), (3.94), we then obtain that the masses of the electroweak gauge bosons \( W \) and \( Z \) in terms of the masses of the fermions and the running coupling constants of the strong, weak and electromagnetic interactions are given by

\[ M_W^2 = g_w^2 (A_1 + A_2 + A_3 - A_4), \quad (3. \ 102) \]
\[ M_Z^2 = (g_e^2 + g_w^2) (A_1 + A_2 + A_3 - A_4), \quad (3. \ 103) \]

where the parameters \( A_1, A_2, A_3 \) and \( A_4 \) are

\[ A_1 = \frac{m_u^2 + m_d^2}{B_1}, \quad (3. \ 104) \]
\[ A_2 = \frac{m_e^2 - m_t^2 + m_i^2 - m_b^2}{B_2}, \quad (3.105) \]
\[ A_3 = \frac{3(m_e^2 + m_{\mu}^2 + m_{\tau}^2)}{B_3}, \quad (3.106) \]
\[ A_4 = \frac{(3 + g_s^2/g_w^2)(m_{\nu_e}^2 + m_{\nu_{\mu}}^2 + m_{\nu_{\tau}}^2)}{B_3}, \quad (3.107) \]

being
\[ B_1 = \frac{4}{3} g_s^2 + \frac{3}{4} g_w^2 + \frac{1}{4} g_e^2, \quad (3.108) \]
\[ B_2 = \frac{4}{3} g_s^2 - \frac{1}{4} g_w^2 + \frac{1}{4} g_e^2, \quad (3.109) \]
\[ B_3 = \frac{3}{4} g_w^2 - \frac{1}{4} g_e^2. \quad (3.110) \]

If we take the experimental central values for the strong constant at the \( M_Z \) scale as \( \alpha_s(M_Z) = 0.1176 \), the fine-structure constant as \( \alpha_e = 7.2973525376 \times 10^{-3} \) and the cosine of the electroweak mixing angle as \( \cos \theta_w = M_W/M_Z = 0.881732 \) [5], then \( g_s = 1.21565 \), \( g_w = 0.641911 \) and \( g_e = 0.34344 \). Putting these values for \( g_s \), \( g_w \) and \( g_e \) and the central values for the masses of the electrically charged fermions, given by [5] \( m_u = 0.00225 \) GeV, \( m_d = 0.005 \) GeV, \( m_c = 1.25 \) GeV, \( m_s = 0.095 \) GeV, \( m_t = 172.371 \) GeV, \( m_b = 4.20 \) GeV, \( m_e = 0.51099892 \times 10^{-3} \) GeV, \( m_{\mu} = 0.105658369 \) GeV, \( m_{\tau} = 1.77699 \) GeV, into the expressions (3.102) and (3.103), and assuming the neutrinos to be massless, \( m_{\nu_e} = m_{\nu_{\mu}} = m_{\nu_{\tau}} = 0 \), we obtain that the theoretical masses of the \( W \) and \( Z \) electroweak gauge bosons are
\[ M_{W^\pm} = 80.403 \text{ GeV} \quad (3.111) \]
\[ M_Z = 91.1876 \text{ GeV}. \quad (3.112) \]

These theoretical masses are in agreement with their experimental values given by \( M_W^{exp} = 80.403 \pm 0.029 \) GeV and \( M_Z^{exp} = 91.1876 \pm 0.0021 \) GeV [5]. The parameters \( A_1 \), \( A_2 \), \( A_3 \) and \( A_4 \) in the expressions (3.102) and (3.103) have the following values \( A_1 = 1.30201 \times 10^{-5}, A_2 = 15655, A_3 = 34.0068 \) and \( A_4 = 0 \). We observe that \( A_2 \) is very large respect to \( A_3 \) and \( A_1 \). Through the definition of the parameter \( A_2 \), given by (3.105), it is possible to conclude that the masses of the electroweak gauge bosons come mainly from the top quark chemical potential \( \mu_t \) and the strong running coupling constant \( g_s \).

### 4 Conclusions
We presented an effective model for particle mass generation in which we extracted some generic features of the Higgs mechanism that do not depend on its interpretation in terms of a Higgs field. The physical vacuum has been assumed to be a medium at zero temperature which is formed by virtual massless fermions and antifermions interacting among themselves by means of massless gauge bosons. The fundamental effective model describing the dynamics of this physical vacuum is the SMWHS. We have assumed that each fermion flavor in the physical vacuum has associated with it a chemical potential $\mu_f$ in such a way that there is an excess of virtual antifermions over virtual fermions. This fact implies that the vacuum is thought to be a virtual medium having a net antimatter finite density.

Fermion masses are calculated starting from the fermion self-energy, which represents the fundamental interactions of a massless fermion with the physical vacuum. The gauge boson masses are calculated from the charge fluctuations of the physical vacuum, which are described by the vacuum polarization tensor. We have used the following general procedure to calculate these particle masses: Initially we write the one-loop self-energies and polarization tensors at finite density and finite temperature; next we calculate the dispersion relations by obtention of the poles of the fermion and gauge boson propagators; starting from these dispersion relations we obtain the fermion and gauge boson effective masses at finite density and finite temperature; finally we identify these particle effective masses at zero temperature with the physical particle masses. This identification can be performed because in our effective model the virtual medium, at finite density and zero temperature, represents the physical vacuum.

Using this effective model for particle mass generation, we obtain the masses of the electroweak gauge bosons in agreement with their experimental values. A further result of our effective model is that the left-handed neutrinos are massive because they have weak charge.

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