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Study of the Uncertainty of the Gluon Distribution

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The uncertainty in the calculation of many important new processes at the Tevatron and LHC is dominated by that concerning the gluon distribution function. We investigate the uncertainty in the gluon distribution of the proton by systematically varying the gluon parameters in the global QCD analysis of parton distributions. The results depend critically on the parton momentum fraction $x$ and the QCD scale $Q^2$. The uncertainties are presented for integrated gluon-gluon and gluon-quark luminosities for both the Tevatron and LHC as a function of $\sqrt{s} = \sqrt{x_1 x_2} = \sqrt{s}/s$, the most relevant quantity for new particle production. The uncertainties are reasonably small, except for large $x$.

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Many hadron-collider signatures of physics beyond the Standard Model have a gluon in the initial state, in either the signal process or the important background processes. Other new signatures within the Standard Model can also have gluons in the initial state. One example of this is the production of a light Higgs boson at the LHC via the process $gg \to H \to \gamma\gamma$. Another example is the measurement of $V_{tb}$ using single-top production at the Tevatron via the process $gW \to tb$. It is important to estimate the theoretical uncertainty in the Quantum Chromodynamics (QCD) calculations of these new processes. Since the quark distributions of the nucleon are relatively well-determined, the dominant uncertainty in these cases is due to that of the gluon distribution. The conventional method of estimating parton distribution uncertainties is to compare different published parton parameterizations. This is a completely unreliable approach since the authors of most published sets of parton distributions adopt similar assumptions and use similar data sets. The differences between these sets have little to do with the range of possible variations of the parton distributions as constrained by current theory and available data.

In this paper we focus on the uncertainty of the gluon distribution within the framework of the CTEQ global QCD analysis\(^1\)\(^2\), and present a more complete estimate of the uncertainties. Not surprisingly, we will find that the uncertainty is a function of the gluon $x$ and $Q^2$.

The approach we adopt in this paper is to systematically vary the gluon distribution parameters in the global analysis framework. We then conservatively delineate the range of admissible distributions as that bounded by fits which show clear disagreements with more than one data set. For this purpose, we adopt the CTEQ4M parton distribution set\(^1\) as the standard and explore the range of possible variations of the gluon distribution around it. The conclusions of this study should apply to all modern parton distribution sets since they are in rather good agreement with each other\(^1,2\).

**Constraints Based on the Momentum Sum Rule**  
The momentum fraction of the proton carried by quarks is determined by deep-inelastic scattering data to be 58% in the CTEQ4M analysis ($Q=1.6$ GeV)\(^1\). The uncertainty in this number is mainly due to normalization uncertainties of the experimental data sets, which is typically ±2%. Therefore the total gluon momentum fraction in the CTEQ4M fit is 42% with an uncertainty of about 2%. This is an extremely important constraint that is not fully appreciated. If the flux of gluons in a certain $x$ range is increased, the flux must be reduced by almost the same amount somewhere else.

The largest component of the gluon momentum is carried at medium values of $x$ ($0.01 < x < 0.3$), since this has the largest product of the number of gluons

\(1\)\(^2\)
and the momentum fraction of each gluon. At small $x$, typical uncertainties from HERA in the gluon distribution are shown to be 30-40\%. At larger $x$ the fixed target Drell-Yan data is sensitive to the gluon through the coupling of gluons and anti-quarks.

These results, combined with the momentum sum rule, restrict the amount by which the gluon in the medium $x$ range can be compensated by changes at large and small $x$. This important constraint also serves to explain the quantitative results on parton distribution uncertainties discussed in the following sections.

**Scanning the Gluon Parameters** We now perform a detailed study of the range of possible variation of the gluon distribution by systematically varying the gluon parameters in a global analysis. The CTEQ4 gluon parametrization at $Q_0$ is: $A_0 x A_1 (1 - x)^2 (1 + A_3 x A_4)$, with $A_0 = 1.1229$, $A_1 = -0.206$, $A_2 = 4.673$, $A_3 = 4.269$, $A_4 = 1.508$ for the standard CTEQ4M parton distribution set. We have then systematically varied the values of $A_1$, $A_2$, and $A_3$, each time refitting the other quark and gluon parameters using the CTEQ procedure described in Ref. 1. The range of variation of each parameter was expanded until clear disagreements with more than one data set were observed. In order to be conservative in this study, we performed these scans using only the

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1We found the variation of $A_3$ was easily compensated by changes in $A_4$, and vice versa. Therefore in the $A_3$ parameter scan described below $A_4$ was fixed to the CTEQ4M value.
well-established DIS and Drell-Yan data sets. This also allows us to establish a baseline uncertainty estimate with the processes that are best understood theoretically.

The sample fits shown in Fig. 1a display clear disagreement with more than one data set. With $A_2 = 1.0$ for example, both H1 and ZEUS $\chi^2$ increased by 50 (for 170-180 data points); with $A_2 = 9.0$, all of the fixed target DIS data sets (with 60-170 data points each) had increased $\chi^2$ of 15 units or more over those of the CTEQ4M fit. Similar criteria were applied for extreme fits to the $A_1, A_3$ parameter scans. The effect of the momentum sum rule constraint is dramatically demonstrated at $Q=5$ GeV. The relatively small changes at moderate values of $x$ are compensated by large changes at small and large $x$.

A reasonable estimate of the current uncertainties on the gluon distribution can be obtained by examining the range of variations spanned by those fits which do not clearly contradict any of the data sets used. The general conclusions are that the range of gluon distributions is within 10% of CTEQ4M below $x < 0.15$ and $Q > 100$ GeV, and the uncertainties grow significantly at larger $x$.

**Uncertainty on the luminosity functions**  For assessing the range of predictions on cross-sections for Standard Model and new physics processes, it is more important to know the uncertainties on the gluon-gluon and gluon-quark luminosity functions in the appropriate kinematic region (in $\tau = x_1 x_2 = \delta/s$), rather than on the parton distributions themselves. Therefore, we turn to the relevant integrated parton-parton luminosity functions. The gluon-gluon luminosity function is defined to be: $\tau dL/d\tau = G \otimes G$. This quantity is directly proportional to the cross-section for the s-channel production of a single particle; it also gives a good estimate for more complicated production mechanisms. It is most appropriate when the experimental acceptances do not play a major role in the cross section calculation.

In analogy with the discussion of gluon-gluon luminosities, we have also studied the variations of gluon-quark luminosities: $\tau dL/d\tau = G \otimes Q$. A complete analysis can be found in ref. 5; here we will simply present the results using the parton uncertainties determined in the previous section.

We summarize in Fig. 2 and Table 1 the uncertainties for the gluon-gluon and gluon-quark luminosity function. Since the sizes of the bands were almost identical for $\sqrt{s} = 2$ or 14 TeV, we only give one set of numbers for each. Above $\sqrt{s} > 0.4$ the uncertainties are increasing rapidly and should simply be considered as being unconstrained by these data.

These are not meant to be precise uncertainties obtained by rigorous statistical analysis, but reasonable error estimates in the same spirit as the estimate
Figure 2: The ratio of luminosities compared to CTEQ4M is shown as a function of $\sqrt{s}$.

for the uncertainty in the theory that one obtains by varying the $\mu^2$ scale in the calculation.

| $\sqrt{s}$ Range | Gluon-Gluon | Gluon-Quark |
|------------------|-------------|-------------|
| $< 0.1$          | $\pm 10\%$ | $\pm 10\%$ |
| $0.1$ to $2$     | $\pm 20\%$ | $\pm 10\%$ |
| $2$ to $3$       | $\pm 30\%$ | $\pm 15\%$ |
| $3$ to $4$       | $\pm 60\%$ | $\pm 20\%$ |

Table 1: Recommended uncertainties on gluon-gluon and gluon-quark luminosities for both the Tevatron and LHC, as a function of $\sqrt{s}$. This is compared to CTEQ4M as the default parton distribution set.

Our baseline variations of the gluon distribution can be used as the benchmark for comparison with present and future measurements which have the potential to narrow the uncertainties. For this purpose the various parton distribution sets which typify the variations shown in this paper will be made available to interested users.

**Conclusions** We have studied the uncertainty in the gluon distribution of the nucleon by systematically varying the relevant parameters in the QCD global analysis. This uncertainty dominates the current uncertainty in the calculation of many important new processes at the Tevatron and LHC. The
uncertainty depends critically on parton $x$ and $Q^2$. We present a table of estimated uncertainties for integrated gluon-gluon and gluon-quark luminosities for both the Tevatron and LHC as a function of $\sqrt{T} = \sqrt{x_1 x_2} = \sqrt{\hat{s}}/\hat{s}$. The uncertainties are reasonably small, except for large $x$, where future emphasis should be placed.

1. H.L. Lai et al., Phys. Rev. D55:1280, (1997).
2. A.D. Martin et al., Phys. Lett. B387:419, (1996).
3. Proceedings of the 5th International Workshop on Deep Inelastic Scattering and QCD, DIS97, AIP Conference Proceedings No. 407.
4. All of the eight parton distribution variations described in this paper are available from the CTEQ web site, http://www.phys.psu.edu/~cteq/.
5. J. Huston et al., hep-ph/9801444; and references therein.