Initial conditions for QCD evolution of double parton distributions

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Double parton distribution functions (DPDFs) are used in the QCD description of double parton scattering. The DPDFs evolve with hard scales through relatively new QCD evolution equations which obey nontrivial momentum and valence quark number sum rules. Based on the constructed numerical program, we present results on the QCD evolution of the DPDFs. In particular, we discuss the problem how to specify initial conditions for the evolution equations which exactly fulfill the sum rules.

XXI International Workshop on Deep-Inelastic Scattering and Related Subjects
22-26 April, 2013
Marseilles, France

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http://pos.sissa.it/
1. Introduction

Double parton distribution functions are used in the description of double hard scattering \[1\]. Their QCD evolution equations are known in the leading logarithmic approximation (LLA) \[2, 3, 4, 5, 6, 7, 8, 9\]. The DPDFs obey nontrivial sum rules which are conserved by the evolution equations. In this presentation we address the problem how to specify initial conditions for the evolution equations which exactly obey these sum rules.

2. Parton distribution functions

In the single parton scattering (SPS), the final state of the hadron-hadron collision has been produced from only one hard interaction while in the double parton scattering, two hard subprocesses occur. For the description of the SPS we use the single parton distribution functions, \(D_f(x, Q)\), while for the double parton scattering - the double parton distribution functions denoted by \(D_{f_1f_2}(x_1, x_2, Q_1, Q_2)\). The DPDF depend on parton flavours \(f_1, f_2\) (including gluon), longitudinal momentum fractions \(x_1, x_2\) and two hard scales \(Q_1, Q_2\). The parton momentum fractions obey the condition,

\[x_1 + x_2 \leq 1, \quad (2.1)\]

which says that the sum of partons’ momenta cannot exceed the total nucleon momentum.

3. Evolution equations

The general form of QCD evolution equations for single PDFs is given by

\[\partial_t D_f(x, t) = \sum_{f'} \int_0^1 du \mathcal{K}_{ff'}(x, u, t) D_{f'}(u, t), \quad (3.1)\]

with the evolution parameter \(t = \ln(\frac{Q^2}{Q_0^2})\). The integral kernels \(\mathcal{K}_{ff'}\) presented in above equation describe real and virtual parton emission. The real emission kernels take the following form

\[\mathcal{K}^R_{ff'}(x, u, t) = \frac{1}{u} P_{ff'}(\frac{x}{u}, t) \theta(u - x) \quad (3.2)\]

in which \(P_{ff'}\) are splitting functions computed perturbatively in QCD in powers of the strong coupling constant \(\alpha_s\):

\[P_{ff'}(z, t) = \frac{\alpha_s(t)}{2\pi} P_{ff'}^{(0)}(z) + \frac{\alpha_s^2(t)}{(2\pi)^2} P_{ff'}^{(1)}(z) + ... \quad (3.3)\]

After including the splitting functions, we find the well known DGLAP evolution equations for the single PDFs:

\[\partial_t D_f(x, t) = \sum_{f'} \int_x^1 \frac{dz}{z} P_{ff'}(z, t) D_{f'}(\frac{x}{z}, t) - D_f(x, t) \sum_{f'} \int_0^1 dz z P_{ff'}(z, t). \quad (3.4)\]
In the leading logarithmic approximation, the evolution equations of the DPDFs (for equal two hard scales, $Q_1 = Q_2, \equiv Q$) have the following form, see [3] for more details,

$$\frac{\partial}{\partial t} D_{f_1 f_2} (x_1, x_2, t) = \sum_{f'} \int_0^{1-x_2} du \mathcal{K}_{f' f}(x_1, u, t) D_{f_1 f_2} (u, x_2, t)$$

$$+ \sum_{f'} \int_0^{1-x_1} du \mathcal{K}_{f' f'}(x_2, u, t) D_{f_1 f'} (x_1, u, t)$$

$$+ \sum_{f'} \mathcal{K}^R_{f' \rightarrow f_1 f_2} (x_1, x_1 + x_2, t) D_{f'} (x_1 + x_2, t). \quad (3.5)$$

The upper integration limits reflect condition (2.1). The third term contains the single PDFs and because of that eqs. (3.5) and (3.1) have to be solved together. That is why the initial conditions for both the single and double PDFs have to be specified at some initial scale $Q_0$.

4. Sum rules

The DGLAP evolution equations (3.4) preserve the momentum sum rule for the single PDFs:

$$\sum_i \int_0^1 dx x D_f (x, Q) = 1 \quad (4.1)$$

while the evolution equations (3.5) preserve the momentum sum rule for the DPDFs

$$\sum_{f_i} \int_0^{1-x_2} dx_1 x_1 D_{f_1 f_2} (x_1, x_2, Q) \frac{D_{f_1 f_2} (x_1, x_2, Q)}{D_{f_2} (x_2, Q)} = (1 - x_2). \quad (4.2)$$

The ratio of the double and single PDFs in the above relation looks like a conditional probability to find a parton with the momentum fraction $x_1$, while the second parton is fixed. Thus, it is clearly seen that the new momentum sum rule relates the double and single PDFs

$$\sum_{f_i} \int_0^{1-x_2} dx_1 x_1 D_{f_1 f_2} (x_1, x_2, Q) = (1 - x_2) D_{f_2} (x_2, Q). \quad (4.3)$$

The valence quark number sum rule for the single PDFs has the well known form

$$\int_0^1 dx \{ D_{q_i} (x, Q) - D_{\bar{q}_i} (x, Q) \} = N_i \quad (4.4)$$

where $N_i$ is the number of valence quarks. For the DPDFs, the following relation holds, depending on the second parton flavour, [4]

$$\int_0^{1-x_2} dx_1 \{ D_{q_i f_2} (x_1, x_2, Q) - D_{\bar{q}_i f_2} (x_1, x_2, Q) \} = \begin{cases} N_i D_{f_2} (x_2, Q) & \text{for } f_2 \neq q_i, \bar{q}_i \\ (N_i - 1) D_{f_2} (x_2, Q) & \text{for } f_2 = q_i \\ (N_i + 1) D_{f_2} (x_2, Q) & \text{for } f_2 = \bar{q}_i \end{cases} \quad (4.5)$$

It is important to emphasize again that the momentum and valence quark number sum rules are conserved by the evolution equations (3.4) and (3.5) once they are imposed at an initial scale $Q_0$. 

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5. Initial conditions

In order to solve eqs. (3.4) and (3.5) we need to specify initial conditions for the DPDFs. For practical reason, their form is to built using the existing single PDFs. For example, in [3, 4] a symmetric input with respect to the parton interchange was proposed,

\[ D_{f_1 f_2}(x_1, x_2, Q_0) = D_{f_1}(x_1, Q_0) D_{f_2}(x_2, Q_0) \frac{(1 - x_1 - x_2)^2}{(1 - x_1)^2 + n_1 (1 - x_2)^2 + n_2}, \]

which is also positive definite (provided the single PDFs are positive). In the below, we show how the momentum and valence quark number sum rules are fulfilled by this input by showing the ratios of the r.h.s to l.h.s of eqs. (4.3) and (4.5) for \( q_i = u \) (and \( n_1 = n_2 = 0 \) in (5.1), for simplicity). We see that the valence quark number sum rule is significantly violated.

| Momentum rule | Valence number rule |
|---------------|---------------------|
| Ratio         |                     |
| 2             | 1.75                |
| 1.5           | 1.25                |
| 1             | 0.75                |
| 0.5           | 0.25                |
| 0             | 0                   |

\begin{align*}
\text{Ratio} & = \frac{f_2 = g, u, \bar{u}}{f_2 = g, u, \bar{u}} \\
\text{Ratio} & = \frac{f_2 = u}{f_2 = \bar{u}} \\
\text{Ratio} & = \frac{f_2 = \bar{u}}{f_2 = g} \\
\end{align*}

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\text{Ratio} & = \frac{f_2 = u}{f_2 = \bar{u}} \\
\text{Ratio} & = \frac{f_2 = \bar{u}}{f_2 = g} \\
\end{align*}

Is it possible to construct an input which exactly fulfill the sum rules (4.3) and (4.5)? In order to obey the momentum sum rule we could use an asymmetric ansatz (we skip \( Q_0 \) in the notation):

\[ D_{f_1 f_2}(x_1, x_2) = \frac{1}{1 - x_2} \left[ D_{f_1} \left( \frac{x_1}{1 - x_2} \right) \right] D_{f_2}(x_2). \]

To fulfill the valence number sum rule we need to introduce corrections for identical quark flavours and antiflavours,

\[ D_{f_i f_i}(x_1, x_2) = \frac{1}{1 - x_2} \left\{ D_{f_i} \left( \frac{x_1}{1 - x_2} \right) - \frac{1}{2} \right\} D_{f_i}(x_2) \]

\[ D_{f_i f_i}(x_1, x_2) = \frac{1}{1 - x_2} \left\{ D_{f_i} \left( \frac{x_1}{1 - x_2} \right) + \frac{1}{2} \right\} D_{f_i}(x_2), \]

which do not spoil the already fulfilled momentum sum rule. However, we pay the price that the DPDFs for identical flavours are not positive definite because of the factor \(-1/2\). We cannot avoid such a situation in the contruction which uses single PDFs.
The graphical comparison of the symmetric and asymmetric inputs is shown below.

For the distributions $D_{uu}$ both inputs give similar results in the small $x_1$ region, while for large $x_1$ there are differences between them because of the lack of positive definiteness of the asymmetric input. The same results are found for the evolved distribution.

For $D_{u\bar{u}}$, both distributions are positive but there are differences between them at the large $x_1$: 
And finally, for the $D_{gu}$ distributions both inputs give the same results in the whole $x_1$ domain.

![Graph showing $D_{gu}(x_1, x_2=10^{-3})$](image)

6. Summary

The specification of the initial conditions for evolution of the DPDFs is not a simple task. The symmetric input obeys parton symmetry and positivity but does not exactly fulfill sum rules. On the other hand, the asymmetric input obeys the sum rules exactly but it is not positive definite. There also exist an alternative solution: one could specify positive initial double distributions and then generate the single PDFs using the sum rules. However, a little is known about the DPDFs from experiments. For small values of parton momentum fractions, the factorized form, $D_{f_1f_2}(x_1, x_2, Q) \approx D_{f_1}(x_1, Q) D_{f_2}(x_2, Q)$, is a good approximation. However, the problem occurs for large values of $x$ ($> 10^{-2}$).

Acknowledgement

This work was supported by the Polish NCN grants DEC-2011/01/B/ST2/03915 and DEC-2012/05/N/ST2/02678.

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