Use of the Normalized Design Parameters for Designing a Strip Speaker Operated by the Traveling-Wave Control Method

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ABSTRACT A strip speaker is the simplified panel speaker adopting a thin beam as the radiator, which is operated under the traveling-wave control method (TCM) with three actuators. The structural resonances are suppressed fundamentally by nullifying the reflected bending waves near the boundary, thus radiating a smooth sound spectrum in a wide frequency band. This study develops the design guidelines for such TCM-controlled strip speakers by investigating the effect of design variables on acoustic performances, such as the effective frequency range and the radiation efficiency. To this end, the acoustic performances are normalized by the size and physical properties of the beam so that a parametric study in the normalized domain can generalize the conditions for the desired acoustic performances. The result shows that the effective frequency range becomes wider when the beam material has a fast wave speed, and the radiation efficiency is enhanced as the beam density decreases. It is found that the two acoustic performances require a trade-off in selecting the actuator location and beam thickness. The suggested design procedure is validated using acrylic and aluminum beams having the same size. The aluminum beam has a faster wave speed, thus covering the effective bandwidth of 5800 Hz, 2.5 times wider than the acrylic case. On the other hand, the acrylic beam has five times higher radiation efficiency due to its low density. As a result, one can select the dimension and material of the beam appropriately to achieve the desired acoustic performance by using the suggested design method.

INDEX TERMS Active control, bending wave, inverse problem, sound radiation, strip speaker, traveling wave control.

I. INTRODUCTION
A panel speaker, which is thin and occupies a small volume, radiates sound from the bending wave field, either standing wave or traveling wave field, on a thin plate excited by a single actuator or several actuators [1]. However, the multimodal behavior of the plate causes an inherent problem with the quality of the radiated sound. Specifically, the sound spectrum exhibits severe fluctuation at low to medium frequencies, with the resonant responses invoking sharp peaks and troughs. Therefore, the structural resonances need to be suppressed to modulate these irregularities in the radiated sound spectrum.

The simplest way to adjust the vibrational response of a speaker panel to realize desirable characteristics is via passive methods that modify the panel material and dimensions; however, this approach is feasible only in a narrow frequency band [2]–[4]. This vibrational response can also be actively controlled using actuators. The mode control method (MCM) is the most widely applied active method for panel speakers, wherein the vibrational modes are individually controlled for a specific frequency band. In many studies, the MCM has been used to excite only the first mode or create a virtual speaker–baffle system to achieve a flat frequency response [5]–[8]. However, because the number of actuators should be equal to or exceed the number of structural modes to be controlled, the MCM is effective only in the low-frequency range or in a narrow band at mid-frequencies. To overcome this challenge, the actuators can be controlled via the direct inverse identification of the required gain; this approach can be considered a spin-off of the MCM, because it is also based on the concept of structural modes [9]. Yet another active method is the traveling-wave control method (TCM), which eliminates reflected waves from the
panel boundary using phase-controlled actuators [10], [11]. This control concept is similar to those reported in previous works on the active control of structures [12]–[16], focusing on sound radiation. By avoiding the generation of the reverberant, viz., modal, field, the TCM suppresses the emergence of resonances and is more efficient than the MCM for a given number of actuators. The critical problem in the active control methods, which is the need for many actuators, can be resolved by applying TCM and adopting a thin beam, called a ‘strip speaker’ [11]. Using only three actuators, one for the primary excitation and two for wave control, the strip speaker can radiate a smoother sound spectrum in a broader frequency band compared with conventional two-dimensional panel speakers.

A previous study successfully demonstrated the advantages and feasibility of a TCM-controlled strip speaker, but only for test beams with specific geometric and material properties. Therefore, the effects of the location and size of the actuator and the material properties of the radiator on the control performance, which are important parameters in designing a strip speaker, remain to be clarified. For the panel speakers driven by a single exciter, simple design procedures are suggested by modeling the overall speaker system as the electro-mechanical network [1], [17], [18]. The proposed methods can allow the determination of the panel material and size. Still, they possess a fundamental error due to ignoring the complicated modal behavior of the plate in the prediction model. For the panel speakers controlled by the array actuators, the optimal configuration of the actuators is suggested for the plate with a specific size and material. However, the optimization objectives are confined to enhancing the control performance or generating the desired directivity of the strip speaker. Compared with the 2-D panel speaker, the strip speaker possesses high practicality by significantly reducing the control difficulty and the number of actuators. Fig. 1 shows the configuration of a strip speaker adopted in the previous study, which consists of a beam with three actuators to control the vibration field by using the TCM [11].

In the numerical simulation, the boundary condition of both beam edges is assumed as a fixed or simply-supported condition only, which constrain the vibrational behavior. Although the other conditions, such as the free-free or the fixed-free ones, are possible, they have lots of troubles in the practical realization. In particular, the structural robustness is of concern, so the discussion of using them is omitted in this work.

II. CONTROL OF BENDING WAVE FIELD ON A BEAM USING THREE ACTUATORS

The main novelty of the TCM is to remove the reflected actuators near the structural boundary. In the case of the 1-D beam, the structural boundary is simply both ends of the beam, thus requiring a couple of control actuators for the vibration control and the primary actuator acting as the wave generation source. Therefore, without any optimization technique in determining the number of actuators, only three actuators are enough for the TCM to control the strip speaker. Compared with the 2-D panel speaker, the strip speaker possesses high practicality by significantly reducing the control difficulty and the number of actuators.

\[ B \nabla^4 \psi(x) + \omega^2 \mu \psi(x) = j \omega F_0 e^{j \omega t} \delta(x_0). \] (1)

Here, \( x \) is the coordinate along the beam length, \( \psi \) the velocity, \( \omega \) the angular frequency, \( E \) the Young’s modulus, \( \rho \) the density of the beam, and \( F_0 \) and \( x_0 \) the complex amplitude and position of the excitation force, respectively. Also, \( B \) denotes the bending stiffness of the beam defined as \( EL_hh^3/12 \), and \( m^\prime \) is the mass per unit length defined as \( \rho h L_y \), where \( h \) and \( L_y \) are the beam thickness and width. One can express the vibration response of a thin beam in the steady-state as a linear summation of the modes as

\[ \psi(x, \omega) = \sum_{i=1}^{N} \int_{x_0}^{x_0 + L_x} \psi_i(x) \frac{j \omega F_n \psi_i(x_0)}{\omega_i^2 (1 + j\eta) - \omega^2} m^\prime \psi_i^2(x) dx, \] (2)

\[ \omega_i = \sqrt{\frac{B}{m^\prime}} \left( \frac{\beta_{i,x}}{L_x} \right)^2, \] (3)

where \( N \) is the number of actuators, \( L_x \) the beam length, \( \beta \) the solution of the characteristic equation derived from the boundary condition, and \( \omega_i \) and \( \psi_i \) are the \( i \)th resonance frequency and mode shape function, respectively [22], [23].
Note that (2) is the standing wave representation of the bending wave field. Another way to express the continuous bending wave field is to use the traveling wave representation, in which the summation of propagating waves and non-wave components is used as

$$v(x, \omega) = C_{\text{inc}}e^{-jk_bx} + C_{\text{refl}}e^{jk_bx} + C^n_{\text{inc}}e^{-k_nx} + C^n_{\text{refl}}e^{k_nx}. \quad (4)$$

The first two terms mean propagating waves, and the last two signify non-propagating or evanescent components. Here, $C$ represents the complex coefficient of each component, $k_b$ is the bending wavenumber, $n$ denotes the evanescent component, and the subscripts $\text{inc}$, $\text{refl}$ indicate the incident and reflected waves, respectively.

The TCM can avoid the structural resonance of the beam by removing the reflected waves from the boundary by the control actuators, which is originated from the primary actuator. The target field of the vibration control is the zone between the primary and control actuators. At least four field observation points are needed to decompose the bending wave field into the four propagating and non-propagating wave components as defined in (4). It is assumed that the wave conversion at the boundary is negligible. Hereafter, the large zone between the primary and control actuators to be controlled is called the control area. In contrast, the remaining small zone between the control actuator and the beam edge is called the residual area. The length of the residual area, which represents the distance between the control actuator and fixing boundary, is denoted as $l_y$. At the same time, $l_y$ means the spacing of field points within the control area. After extracting the wave coefficients from the velocity responses at the field points, the transfer function can be determined between the input electric voltage of the control actuator and the coefficient of the reflected wave components. The matrix equation for generating the desired coefficients $C_d$ of the reflected waves from the control actuators can be obtained as [10]

$$K_{ij} \equiv C_{\text{refl},i}/e_j \quad \text{or} \quad [K]_{2 \times 2}[E]_{2 \times 1} = [C_d]_{2 \times 1}. \quad (5)$$

Here, $C_{\text{refl},i}$ denotes the coefficient of the reflected wave in the $i^{th}$ control area, $e_j$ is the input electric voltage of the $j^{th}$ control actuator, $K_{ij}$ is the transfer function, and $[\cdot]$ means the matrix. In this study, the output force of the actuator is assumed to be linear to the input electric voltage, thus maintaining the transfer function $K$ to be constant. The transfer matrix $K$ represents the characteristics of the speaker system, defining the velocity response of the beam caused by the unit input electric voltage of the actuators. Therefore, for controlling the bending wave field in the desired way, it is essential to determine the values of the desired coefficient $C_d$. According to the control objective to nullify the reflected waves from the boundary, the desired coefficients are determined by

$$C_d = -C^n_{\text{refl}}. \quad (7)$$

where the superscript 0 implies that the reflected waves from the boundary actually stem from the incident wave generated by the primary actuator excitation. For deriving the proper input electric voltage of the actuators to generate the desired coefficient $C_d$, the inverse estimation technique is employed, which is widely adopted and described in several studies on the panel speaker control [5]–[11]. The gain of the control actuators can be obtained as follows [24]–[27]:

$$[\hat{E}]_{2 \times 1} = [K]_{2 \times 2}[C_d]_{2 \times 1} = (K^H K)^{-1} K^H C_d. \quad (8)$$

Here, $\dagger$ denotes the pseudo-inverse operator, and the superscript $H$ means the Hermitian transpose. Suppose the boundary condition is ideal, which means that the wave impedance at the structural edges is defined deterministically. In that case, one can determine the transfer function analytically without using any numerical modeling or experimental tuning. However, in practical situations, it is difficult to define the transfer function analytically due to the unknown boundary condition of the structure. Therefore, the empirically measured transfer function is employed for the vibration control in the actual implementation. Fig. 2 shows a schematic of the actual set-up of the strip speaker [11]. At the observation points in the control area, vibration response is measured using a laser vibrometer to avoid the dynamic structural loading, from which the transfer function is calculated compared to the input electric voltage at the actuator. This empirical transfer function determines the input voltage for the desired sound radiation by solving the inverse problem in (8).

**FIGURE 2.** A schematic diagram of the set-up of a strip speaker using the traveling-wave control method (TCM).

One can find that the input gain of the control actuators, or the overall amplitude and phase of the controlled vibration field, is dominated by the primary actuator. Therefore, the operation of the primary actuator properly for the desired sound radiation is crucial. The desired field of the TCM is the pure traveling wave field like an infinite beam excited by the primary actuator because there are no reflected waves in the controlled field in an ideal situation. From the definition of the wave field in an infinite beam, the input gain of the primary actuator can be adjusted for radiating the desired sound.
pressure by using the Rayleigh integral as follows [21, 28]:

\[
p(R, \omega) = \frac{j \omega \rho_0}{2\pi} \int_S v_{\text{inf}}(r, r') e^{-jk_0r'} dS,
\]

\[
v_{\text{inf}}(x) = \frac{\omega F_0}{4Bk_b^3} \left( e^{-jk_b|x|} - je^{-jk_b|x|} \right).
\]

Here, \( R \) is the observation point in the sound field, \( r' \) the distance between each element of the beam and the observation point, \( S \) the radiating area of the beam, \( \rho_0 \) the density of air, and \( k_0 \) the wavenumber of the radiated sound. In this way, one can determine the output force \( F_0 \), or the input electric voltage, of the primary actuator to radiate the desired sound pressure.

**B. NORMALIZATION OF BENDING WAVE FIELD**

The vibrational quantities, such as the resonance frequency, wavelength, and wavenumber, are determined by the physical properties and the geometrical dimension of a finite structure. First, from (3), one can normalize the resonance frequency of the beam as

\[
\omega^* = \frac{L_x^3}{h} c_{l2} \omega_i = \beta_{\omega, x},
\]

\[
c_{l2} = \sqrt{\frac{E}{\rho}}.
\]

where \( c_{l2} \) represents the quasi-longitudinal wave speed of the beam that is determined by the material properties only. The asterisk in the superscript hereafter denotes the normalized vibrational quantities. One can find that the normalized resonance frequency is a nondimensional value and determined by the boundary condition only. Accordingly, the excitation frequency \( \omega \) is also normalized in the same form with (11). The bending wavenumber \( k_b \) and wavelength \( \lambda_b \) have the unit of \( \text{rad/m} \) and \( \text{m} \), respectively. Both variables can be nondimensionalized to the beam length \( L_a \) as follows:

\[
k_b^* (\omega^*) = l_b^* k_b (\omega) = \sqrt{\omega^*},
\]

\[
\lambda_b^* (\omega^*) = \lambda_b (\omega)/L_a = 2\pi/\sqrt{\omega^*}.
\]

One can note that the normalized bending wavenumber is actually the Helmholtz number. Similarly, all parameters related to the length, such as \( l_g, l_f \), and \( x \) - and \( y \)-coordinates, are also normalized as follows:

\[
l_g^* = l_g/L_a,
\]

\[
l_f^* = l_f/L_a,
\]

\[
x^* = x/L_a,
\]

\[
y^* = y/L_x = y y/L_x.
\]

Consequently, the normalized expression of the bending wave field in the beam can be obtained as follows:

\[
v^* (x^*; \omega^*) = \sum_{i=1}^{N} \sum_{n=1}^{N} \frac{j \omega F_n \psi_i (x^*) \psi_i (x^*)}{A_i^* \left( 1 + j \eta \right) \omega_i^*}.
\]

The normalized vibration field is a function of three parameters: loss factor, boundary condition, and excitation force. All parameters in the definition of the normalized vibration response are nondimensional except the excitation force. Therefore, the unit of normalized vibration response is \( N \), from which one can physically interpret the normalized vibration field as the force distributed over the beam. By inspecting the normalized expression in (20), one can quickly understand that the actual vibration amplitude is to be increased with the decrease of the density, thickness, and longitudinal wave speed of the beam.

**III. NORMALIZED ACOUSTIC PERFORMANCES**

The acoustic performance can be evaluated by several figures of merit such as dynamic range, total harmonic distortion (THD), group delay, sensitivity, and directivity. In this study, the strip speaker is assumed to be operated linearly without distortion of the radiated sound. From that point of view, the THD and the group delay need not be considered acoustic performance because they are related to the non-linearity of the actual speaker system. When it comes to directivity, it is determined by the relative relationship between the geometrical dimension of the source and the wavelength of sound freely propagating in the air. However, it is not easy to directly predict the radiated sound field using the normalized bending wave field expression, and the reason will be discussed in section III.C. Therefore, the acoustic performances investigated in this work are confined to the effective frequency range, radiation efficiency, and input gain. The radiation efficiency is defined as the ratio of the radiated sound power to the input electrical power.

**A. EVALUATION INDEX OF WAVE FIELD CONTROL**

The strip speaker controlled by the TCM has the desired field that is the traveling wave field originated by the primary actuator. The desired sound from the control area is also determined by the traveling wave field as defined by (10). Therefore, the acoustic performance of the controlled strip speaker can be evaluated by comparing the resultant wave field with the desired field. When the TCM controls the beam vibration, it is known that the driving-point mobility of the primary actuator is converted into that of an infinite beam [11]. Accordingly, the deviation of driving-point mobility from the infinite beam case is selected as the evaluation index of the control result by the TCM, which is defined by

\[
e_y (\omega^*) = \left| \frac{Y^*_\text{inf} - Y^*}{Y^*_\text{inf}} \right|,
\]

\[
Y^*_\text{inf} = \frac{1 - j}{4k_b^*},
\]
where $Y$ denotes the driving-point mobility of the primary actuator. The small mobility error $\epsilon_Y$ implies that the reflected wave components around the primary actuator are eliminated as intended, ensuring the desired control result.

**B. NORMALIZED EFFECTIVE FREQUENCY RANGE**

The basic assumption of the strip speaker is that the bending wave field is one-dimensional along the beam length. The 1-D assumption requires that the vibration field along the beam width should be uniform and in-phase, for which the bending wavelength is far longer than the width. It signifies that the condition of the 1-D vibration field poses the upper-frequency limit of the vibration control. One can consider that the boundary condition at the side edges of the beam is the free-free condition and assume that the beam is excited symmetrically to the center of the width. The 1st resonance frequency can determine the upper-frequency limit due to 1-D assumption along the beam width, which can be normalized as follows [22], [23]:

\[
k_b L_y < 4.73, \quad (23a)
\]
\[
k_b^* < 4.73 Y. \quad (23b)
\]

Equation (23) implies that the upper-frequency limit increases with the aspect ratio of the beam. Hereafter, the normalized frequency range is expressed as the normalized wavenumber $k_b^*$ because the numerical magnitude of the normalized frequency obtained from the practical structure can increase to a too large value quickly to be handled when the beam becomes more lengthy and thin. Furthermore, the normalized wavenumber can facilitate the understanding of the physical meanings in the controlled wave field. From the definition of the normalized frequency, a thick beam with a fast quasi-longitudinal wave speed bears a high upper-frequency limit for the same normalized wavenumber.

Fig. 3 shows the normalized driving-point mobility of the primary actuator, which is attached to the beam with a fixed boundary condition and a loss factor of 0.01. The control results are obtained by varying $l_g^*$ as 0.02, 0.05, 0.08, 0.10, while maintaining $l_y^*$ = 0.02. One can find that the normalized mobility spectrum becomes smooth by eliminating the cause of resonances, which becomes like an infinite beam. However, the mobility spectrum occasionally exhibits residual sharp peaks and troughs at very high frequencies, and the error compared to the infinite beam increases gradually at very low frequencies. It means that the bending wave field is controlled unsatisfactorily at those frequencies.

As demonstration examples, Fig. 4 shows the normalized bending wave fields controlled at the different frequencies when the actuator array is configured with $l_y^* = 0.02$. The maximum value normalizes the amplitude field for easy understanding. Above all, the bending wave field controlled in the desired way is shown in Fig. 4(b) at $k_{b*} = 48.69$, which corresponds to the 15th resonance frequency of the uncontrolled beam. One can see a typical modal response pattern in the uncontrolled beam case, but the controlled state exhibits the characteristic of the traveling bending wave field with constant amplitude. Meanwhile, the phase varies linearly from the primary actuator position. As an example for very low frequencies, Fig. 4(a) illustrates the controlled field at $k_{b*} = 5$, near the 1st resonance frequency. Although the controlled field seems acceptable with the amplitude and phase fields varying smoothly, it is hard to consider it the traveling wave field because the evanescent waves are dominant due to the slow decaying rate. Accordingly, the mobility error at the primary actuator increases as the excitation frequency decreases. On the other hand, the normalized field controlled at very high frequencies are shown in Fig. 4(c) at $k_{b*} = 157$ and Fig. 4(d) at $k_{b*} = 196.25$, which correspond to the spectral peaks and troughs observed in Fig. 3. Compared with the desired control result in Fig. 4(b), one can find the unexpected fluctuation in the amplitude field of Fig. 4(c). The problem observed in Fig. 4(d) is caused by the standing wave field generated in the small residual area, which amplitude is too large, so controlling the sound radiation is troublesome. In order to avoid the unwanted control results in constructing a strip speaker, the design conditions for achieving the desired control result of vibration and resulting sound radiation are derived in the following sections.

After controlling the bending wave field using the TCM, the wave field in the central part, i.e., the control area, is occupied by the outgoing traveling wave generated from the primary actuator. In contrast, the residual area denoted as $l_g^*$ reverberates by the standing wave with a large amplitude. One should avoid the excitation frequency corresponding to the standing wave field in the residual area in the frequency range of interest because the sharp peaks can appear in the

![Figure 3](image-url)
Here, $A = 1.25$ for a fixed boundary condition and $A = 1$ for a simply-supported boundary condition. If the free boundary condition is considered as the beam edge, one can assign $A = 1.25$ too because the phase jump $\phi_b = -\pi/2$ becomes the same as the fixed boundary condition. Equation (25) permits the prediction of the effective frequency range avoiding the severe standing wave at the residual area near the beam end, which can be observed in Fig. 4(c). If a beam with larger $l_g^*$ is used, more significant spectral peaks would appear at the harmonics of the frequency determined by (25).

When it comes to wave decomposition, it is known that the spacing of field observation points $l_f$ should be shorter than the half of a bending wavelength of interest to avoid the spatial aliasing problem [30]–[32]. Otherwise, the vibration responses at the field points become correlated and dependent on each other, causing poor decomposition performance, resulting in the undesired control result as shown in Fig. 4(d). The condition on the spacing of the field observation points can be expressed in the normalized form as

$$l_f < \lambda_b/2 \quad \text{or} \quad k_b l_f^* < \pi/l_f^*.$$  \hfill (26a)

$$k_b l_f^* < \pi/l_f^*.$$  \hfill (26b)

As a result, one should determine the upper-frequency limit in using the TCM to satisfy the three conditions of (23), (25), and (26).

For investigating the lower-frequency limit to avoid the bending wave field controlled undesirably in Fig. 4(a), the mobility error defined in (21) of a controlled beam with fixed or simply-supported boundary conditions is shown in Fig. 5. In using the TCM to control the beam, one should recall that the bending waves controlled by the control actuators are the propagating ones only. The evanescent waves that decay along the propagating path exponentially dominate the controlled field, particularly at low frequencies. One can find in Fig. 5 that the mobility error is very significant at low frequencies, and it reduces to a value less than 5% when $k_b l_f^*$ is larger than 7.3 regardless of $l_g^*$ and the boundary condition. If the Rayleigh integral in (9) is used to calculate the radiated sound, one can obtain a $\pm 0.5 \text{ dB}$ difference of the sound pressure level from the desired amplitude due to $\pm 5\%$ error in the structural wave field. Therefore, the low-frequency limit for the efficient application of the TCM can be determined as $k_b l_f^* > 7.3$, which allows the mobility error less than 5%. If the tolerance of the mobility error becomes tighter for reducing the deviation of the sound pressure level, the low-frequency limit becomes higher accordingly.

Considering that the first resonance frequency of the fixed and the simply-supported beams are $k_b^* = 4.73$ and 3.14, respectively, one can find that the effect of the evanescent waves becomes negligible when the excitation frequency is far higher than the first resonance frequency. Furthermore, the mobility error is obtained with almost the same value regardless of the boundary condition. It is because the TCM-controlled bending wave field is dominated by the propagating waves not affected by the structural boundary. As a result,
the two parameters only: volume velocity \( Q \) and angular 

**FIGURE 5.** Effect of boundary condition on the error of the normalized mobility at the primary actuator position varying the gap length \( l_g^* \):

- \( l_g^* = 0.02; \) dashed line
- \( l_g^* = 0.05; \) dashed-dotted line
- \( l_g^* = 0.08; \) dashed-dotted-dotted line
- \( l_g^* = 0.10. \) (a) Fixed, (b) simply-supported.

one can summarize the normalized effective frequency of the strip speaker controlled by the TCM as follows:

\[
\begin{align}
7.3 < k^*_b < 4.73 \gamma & \quad \text{or} \quad (27a) \\
7.3 < k^*_b < \pi / l^*_g & \quad \text{or} \quad (27b) \\
7.3 < k^*_b < \alpha \pi / l^*_g & \quad \text{(27c)}
\end{align}
\]

Here, \( \alpha = 1.25 \) for fixed boundary conditions, and \( \alpha = 1 \) for simply-supported boundary conditions. For widening the normalized effective frequency range, it is recommended to use a beam with a high aspect ratio, locate the control actuators close to the beam edges, and select the field observation points with narrow spacing. The actual effective frequency range can be converted from the normalized form in (11) using the material properties and dimension of the beam.

**C. NORMALIZED RADIATION EFFICIENCY**

The Rayleigh integral in (9) cannot be normalized directly by the vibrational quantities because of the wavenumber of sound propagating in the air. For connecting the radiated sound pressure with the normalized wave amplitude on the beam assuming the far-field condition \( (R \approx \infty) \), the Rayleigh integral can be approximated as

\[
\begin{align}
p(R, \omega) & \approx \frac{j \rho_0}{2 \pi R} e^{-j k_R R} \omega Q(\omega), \quad (28) \\
Q(\omega) & = \int_S |p(\omega)|^2 dS, \quad (29)
\end{align}
\]

where \( Q \) denotes the volume velocity generated from the bending wave field on the beam surface. One can find that the sound pressure amplitude at a far-field position depends on the two parameters only: volume velocity \( Q(\omega) \) and angular frequency \( \omega \). Therefore, \( \omega Q(\omega) \) in (28) can be interpreted as the major contributor to the overall bending wave field. The far-field assumption simplifies the beam wave field as a simple source, thus avoiding the need for detailed characteristics in sound radiation such as directivity. However, it is still useful for describing the approximated acoustic contribution by the bending wave field.

The radiation efficiency of the strip speaker is defined as the mechno-acoustic power efficiency, which describes the conversion ratio between the input electric power of actuators and the radiated sound power in this work. The sound power \( W_{\text{rad}} \) radiated from the bending wave field on a beam, and the total electric input power \( W_{\text{in}} \) spent by the actuators are given by

\[
\begin{align}
W_{\text{rad}} & = \frac{\pi R^2}{\rho_0 c_0} \frac{1}{N} \sum_{i=1}^{N} |p|^2 \approx \frac{\rho_0}{4 \pi c_0} \frac{\omega Q(\omega)}{\omega^2}, \quad (30) \\
W_{\text{in}} & = \frac{3}{\pi^2} \sum_{n=1}^{3} |F_n(\omega)|^2. \quad (31)
\end{align}
\]

Here, \( \kappa \) represents the sensitivity of the actuator, which is assumed as a linear constant relating the output force and the input voltage. Consequently, the radiation efficiency can be defined and expressed in the normalized form as

\[
\sigma(\omega) \equiv \frac{W_{\text{rad}}}{W_{\text{in}}} = \frac{\kappa^2}{\rho^2 \pi^2} \sigma^*(\omega^*). \quad (32)
\]

One should note that the normalized radiation efficiency \( \sigma^* \) in (32) is an approximated value, which is adopted only to investigate the acoustic performance of the strip speaker using the normalized bending wave field. One can find that the radiation efficiency is determined by the density and thickness of the beam and the sensitivity of the actuator in transforming the electric input voltage to the output force. As mentioned before, the sensitivity \( \kappa \) is assumed as constant within the frequency range of interest, which is not satisfied in a practical situation.

Fig. 6 shows the radiation efficiency of the beam controlled by the TCM, of which the loss factor is \( \eta = 0.01 \). The TCM exhibits its advantageous feature in radiating sound at mid to high frequencies for both fixed and simply-supported boundary conditions. The radiation efficiency increases with the actuator location \( l_g^* \) because the mobility of the actuators becomes higher when the excitation point is far from the beam edge, which constrains the wave interaction within a narrow space of \( l_g^* \). However, one can recall that the high-frequency limit for achieving the desired sound radiation is reduced with the \( l_g^* \). The simply-supported condition bears a better radiation efficiency than the fixed condition when it comes to the boundary condition. Similar to the discussion on the actuator position, the reason is that the simply-supported beam has higher mobility than the fixed condition that constrains the dynamic motion more strictly. It should be noted that, in a practical situation, the actual boundary condition of the strip speaker cannot be precisely defined ideally like
the speaker system and the smoothness of the radiated sound spectrum of the radiation efficiency ensures the stability of improved with the slight increase of the loss factor. The flat throughs caused by the minimum volume velocity are much is a bit reduced by the damping effect, the sharp spectral dips in the target frequency range in designing the strip some frequencies, which causes a small amount of acoustic effect or modifying the design variables to avoid the spectral troughs in the control area is multiples of the bending wavelength. The input gain of actuators for the structural vibration control is described as a force in using the normalized bending wave field. When the TCM controls the beam, the ratio of the output force between the primary and the control actuators is determined by the desired control result, as shown in Fig. 8. One can observe the spectral peaks caused by the standing wave in the residual area as defined by (25). As discussed in the radiation efficiency, the small mobility magnitude reduces the output force required for controlling the beam. Such a phenomenon can be observed in the simply-supported beam case when the actuator is located far from the beam edge. In particular, at low frequencies, the output force of the control actuator becomes more significant than that of the primary actuator when $l_g^* < 5/8b_1^* \lambda_b$ for the fixed beam, and $l_g^* < 1/2b_1^* \lambda_b$ for the simply-supported beam. Depending on the desired effective frequency range, the result can be used to determine the actuator satisfying the dynamic range in controlling the beam. For predicting the actual input gain of actuators, the sensitivity $\kappa$ between the output force and the input electric voltage should be known. If the actuator is a moving-coil type, the sensitivity can be assumed as a constant within the operating frequency range by definition. On the other hand, it is usually non-linear and a function of excitation spectrum as well. As a result, it is recommendable that some amount of proper additional damping is needed to design a strip speaker. If one wants to compensate for the loss of the acoustic response due to the increase of the loss factor, the best way would be to select the beam material with a low density which can enhance the radiation efficiency and widen the effective frequency range simultaneously.

D. INPUT GAIN OF ACTUATORS

FIGURE 6. Normalized radiation efficiency of the beam controlled by the TCM varying the actuator location $l_g^*$ for two different boundary conditions ($\eta = 0.01$): (a) fixed, (b) simply-supported. Here, $l_g^* = 0.03$, $l_g^* = 0.05$, $l_g^* = 0.08$, $l_g^* = 0.10$.

FIGURE 7. Effect of loss factor $\eta$ on the radiation efficiency of the beam controlled by the TCM ($l_g^* = 0.02$): $\eta = 0.01$; $\eta = 0.05$; $\eta = 0.10$; $\eta = 0.20$. (a) Fixed, (b) simply-supported.
IV. ACOUSTIC PERFORMANCE TEST

The sound radiated from the test strip speaker is measured by varying the beam material to verify the acoustic performance predicted by the normalized bending wave field. The beam size is 315 mm x 40 mm x 2 mm, fixed at both edges. The observation point in the sound field is an on-axis point, 1-m apart from the beam center, and the bending wave field on a beam is controlled by the TCM to satisfy the desired sound level of 45 dB at all frequencies. The identical three actuators are arranged with \( l_g = 20 \text{ mm} \), while the spacing of field observation points is chosen as \( l_f = 20 \text{ mm} \). The moving-coil type actuator with a sensitivity of \( \kappa = 0.26 \text{ N/V} \) is selected because it generates the output force linear to the input electric voltage within the design operating range [11]. Compared with the other types, such as the piezoelectric or the magnetostriuctive actuators, the linearity between the output force and the input voltage assures the constant transfer function, which is advantageous for the vibration field. Aluminum and acrylic are selected as the beam material, and their physical properties are shown in Table 1. One can see the effective frequency range determined by the actuator layout and physical properties too. As discussed in the preceding sections, the effective frequency range of the aluminum beam is much broader than the acrylic beam due to the faster quasi-longitudinal wave speed \( c_{12} \).

The on-axis sound pressure level at a 1-m apart from the beam center is shown in Fig. 9(a), which is displayed in the normalized bending wave number \( k^*_{b} = k_b L_x \) for comparing different beam materials. One can find that the sound spectrum radiated from the traveling wave field on the controlled beam is different from the uncontrolled beam having the typical modal behavior. The sharp peaks caused by the structural resonances of the uncontrolled beam are reduced by the active control using the TCM. Although several spectral dips caused by the small volume velocity of the controlled beam are observed in both cases, the acrylic beam exhibits a smoother spectral shape than the aluminum beam due to the difference in damping. Fig. 9(b) shows the radiation efficiency of the controlled aluminum and acrylic strip speakers. In this case, the sound power required to calculate the radiation efficiency is obtained by using the Rayleigh integral, not using the approximated value by the volume velocity. One can find that the radiation efficiency becomes much higher in using the acrylic beam because of its high mobility magnitude determined by the quasi-longitudinal wave speed. According to (32), the acrylic beam has at least five times higher radiation efficiency than the aluminum beam under

| TABLE 1. Physical properties and the effective frequency range of the test beams. |
|---------------------|---------------------|---------------------|
|                      | Aluminum           | Acrylic             |
| \( E \), GPa         | 60                 | 3.2                 |
| \( \rho \), kg/m³     | 2700               | 1180                |
| \( \eta \)           | 0.01               | 0.1                 |
| \( c_{12} \), m/s    | 4174               | 1647                |
| Effective frequency  | 250-6050           | 82-2110             |

FIGURE 9. The sound radiated from the test strip speaker controlled by the TCM. The dashed line denotes the uncontrolled and the solid line the controlled beam: \( - - - - - - - - \), \( L_p = 45 \text{ dB} \); \( \bigcirc \), aluminum beam; \( \times \), acrylic beam. (a) On-axis sound level at the 1-m position. (b) Radiation efficiency.
which is determined to increase the radiation efficiency in the frequency range of interest. One can find a more detailed description of the experimental set-up and the other results in the corresponding previous work.

Fig. 11 compares the measured and the predicted sound level spectra, illustrated in the linear frequency range, not the normalized bending wave number, to show the acoustic performance practically. One can find that the measured sound pressure level radiated from the test beams are similar to the predicted results. In the case of the acrylic beam, the deviation between the predicted and the measured results is higher than the aluminum beam because the acrylic is more vulnerable to the external mass-loading by the actuators. As expected from the normalized acoustic performances, the frequency range for the effective sound radiation becomes narrower in using the acrylic beam than the aluminum beam. On the other hand, the acrylic beam has higher radiation efficiency than the aluminum, thus radiating the desired sound level with less input power, as shown in Fig. 9(b). Accordingly, one can choose the acrylic beam for the woofer due to the high radiation efficiency covering the low frequencies. In contrast, the aluminum beam would be appropriate for the midrange speaker covering a wide frequency range. As a result, it is crucial to select the proper material and size of the beam depending on the desired acoustic performance.

V. CONCLUSION

The acoustic performance of a strip speaker actively controlled using three actuators is generalized using the bending wave field normalized to the dimensions and physical properties of the beam. The traveling wave control method (TCM) is applied to radiate sound with the desired quality by actively suppressing the structural resonances. The acoustic performance is defined in terms of the effective frequency range for achieving the desired vibration control result and the radiation efficiency, related to the power conversion between the input electric voltage and the radiated sound. The findings of this work indicate that for widening the effective frequency range, the beam thickness and the quasi-longitudinal wave speed should be increased, and the beam length should be decreased. For securing the effective frequency range at higher frequencies, the control actuator should be located close to the beam edge to avoid the unwanted standing wave field in the residual area. Similarly, the field observation points in the control area should be arranged with a spacing narrow enough to remove the spatial aliasing problem for widening the effective frequency range. Due to the traveling wave field in the TCM, the radiation efficiency exhibits spectral dips caused by the minimum volume velocity. One can exclude the spectral dips at high frequencies by locating the control actuators far from the beam edges to lower the upper-frequency limit. It is thought that the most effective way to resolve the spectral fluctuation is to increase the structural loss factor moderately. The beam density and thickness should be decreased for improving the radiation efficiency, whereas the beam thickness should be increased to widen the same condition of beam thickness and actuators. Within the effective frequency range, the mean value of the radiation efficiency is 0.0135 % for the aluminum beam and 0.0675 % for the acrylic beam. Even though the acrylic beam radiates the sound more efficiently than the aluminum beam, one should also recall that the effective frequency range is much broader in using the aluminum beam. On the other hand, the aluminum beam can increase the radiation efficiency by locating the control actuators further from the beam edges as discussed in Fig. 6, while sacrificing the upper limit of the effective frequency range. Consequently, the trade-off between the effective frequency bandwidth and radiation efficiency is needed, considering the constraints in designing the overall speaker system and the desired acoustic performance.

For the validation of the suggested method to predict the acoustic performance of the strip speaker, experimental data is employed from the previous work, which is conducted using the test beams as shown in Fig. 10 [11]. In this case, the control actuators are configured with \( l_g = 34 \) mm for aluminum beam and \( l_g = 20 \) mm for acrylic beam,
the effective frequency range. Therefore, the effective frequency range and radiation efficiency need to be traded off appropriately depending on the desired acoustic performance and design constraints of the speaker system. One can readily compare and evaluate the acoustic performances from the normalized parameters and conditions for the desired acoustic performances. For example, under the same beam size, the effective frequency range of an aluminum beam is 2.5 times wider than an acrylic beam because its quasi-longitudinal wave speed is 2.5 times faster. On the other hand, the density of the acrylic beam is 2.3 times lighter than the aluminum beam, which results in the radiation efficiency of an acrylic beam being 5 times higher than an aluminum beam. Therefore, one can conclude that light and flexible material is advantageous in designing a woofer for very low-frequency sound radiation. In contrast, the stiff beams made of metallic materials are proper for the midrange speakers being 5 times higher than an aluminum beam. Thus, the density of the acrylic beam being 5 times higher than an aluminum beam, which results in the radiation efficiency of an acrylic beam being 5 times higher than an aluminum beam. Therefore, one can conclude that light and flexible material is advantageous in designing a woofer for very low-frequency sound radiation. In contrast, the stiff beams made of metallic materials are proper for the midrange speakers covering a wide frequency range. In summary, the findings of this work can serve as design guidelines for strip speakers. In future work, the generalization method could be expanded to improve the acoustic performance of two-dimensional panel speakers actively controlled by an actuator array or excited by a single actuator.

REFERENCES

[1] M. R. Bai and T. Huang, “Development of panel loudspeaker systems: Design, evaluation and enhancement,” J. Acoust. Soc. Amer., vol. 109, no. 6, pp. 2751–2761, Jun. 2001.
[2] E. Y. Prokofieva, K. V. Horoshenkov, and N. Harris, “The acoustic emission of a distributed mode loudspeaker near a porous layer,” J. Acoust. Soc. Amer., vol. 111, no. 6, pp. 2665–2670, Jun. 2002.
[3] G. Lu and Y. Shen, “Model optimization of orthotropic distributed-mode loudspeaker using attached masses,” J. Acoust. Soc. Amer., vol. 126, no. 5, pp. 2294–2300, Nov. 2009.
[4] S. Zhang, Y. Shen, X. Shen, and J. Zhou, “Model optimization of distributed-mode loudspeaker using attached masses,” J. Audio Eng. Soc., vol. 54, pp. 295–305, Apr. 2006.
[5] J.-H. Woo and J.-G. Ih, “Vibration rendering on a thin plate with actuator array at the periphery,” J. Sound Vibrat., vol. 349, pp. 150–162, Aug. 2015.
[6] D. Anderson and M. Bocko, “Modal crossover networks for flat-panel loudspeakers,” J. Audio Eng. Soc., vol. 64, no. 4, pp. 229–240, Apr. 2016.
[7] J.-H. Woo and J.-G. Ih, “Generation of a virtual speaker and baffle on a thin plate controlled by an actuator array at the boundary,” IEEE/ASME Trans. Mechatronics, vol. 24, no. 3, pp. 1197–1207, Jun. 2019.
[8] N. Benbara, M. Rebillat, and N. Mechbal, “Bending waves focusing in arbitrary shaped plate-like structures: Application to spatial audio in cars,” J. Sound Vibrat., vol. 487, Nov. 2020, Art. no. 115587.
[9] J.-H. Woo, J.-G. Ih, and Y. Park, “Comparison of two vibro-acoustic inverse methods to radiate a uniform sound field from a plate,” J. Sound Vibrat., vol. 458, pp. 445–457, Oct. 2019.
[10] K.-H. Lee and J.-G. Ih, “A simulation study on the array control of a rectangular panel speaker for improving the sound radiation performance,” J. Sound Vibrat., vol. 488, Dec. 2020, Art. no. 115631.
[11] K.-H. Lee, J.-G. Ih, and D. Jung, “A strip speaker using the traveling bending wave on a beam controlled by three actuators,” J. Sound Vibrat., vol. 504, Jul. 2021, Art. no. 116136.
[12] A. H. von Flotow and J. B. Schafer, “Wave absorbing controllers for a flexible beam,” J. Guid., Control Dyn., vol. 9, pp. 673–680, Nov. 1986.
[13] N. Tanaka and Y. Kikushima, “Optimal vibration feedback control of an Euler–Bernoulli beam: Toward realization of the active sink method,” J. Vibrat. Acoust., vol. 121, no. 2, pp. 174–182, Apr. 1999.
[14] B. R. Mace, E. Rustighi, N. S. Ferguson, and D. Doherty, “Active control of flexural vibration: An adaptive anechoic termination,” in Motion and Vibration Control. Dordrecht, The Netherlands: Springer, 2009, pp. 231–240.
[15] H. Iwamoto, N. Tanaka, and S. G. Hill, “Adaptive feedforward control of a rectangular panel using a wave filter constructed with point sensors,” J. Sound Vibrat., vol. 330, no. 11, pp. 2401–2418, May 2011.
[16] H. Iwamoto, N. Tanaka, and S. G. Hill, “Feedback control of wave propagation in a rectangular panel—Part 1: Theoretical investigation of fundamental characteristics,” Mech. Syst. Signal Process., vol. 59, nos. 1–2, pp. 3–19, Aug. 2013.
[17] N. Harris and M. O. Hawskford, “The distributed-mode loudspeaker (DML) as a broad-band acoustic radiator,” in Proc. 103rd Conf. Audio Eng. Soc., New York, NY, USA, Sep. 1997, p. 4526.
[18] B. Zénker, M. Heinl, S. Merchel, and M. E. Altmüny, “Low-frequency performance of a woofer-driven flat-panel loudspeaker (Part 2: Numerical system optimization and large signal analysis),” in Proc. 149th Conf. Audio Eng. Soc., Oct. 2020, p. 10443.
[19] D. A. Anderson, M. C. Heilemann, and M. F. Bocko, “Optimized driver placement for array-driven flat-panel loudspeakers,” Arch. Acoust., vol. 42, no. 1, pp. 93–104, Mar. 2017.
[20] O. Jeon, H. Ryu, H.-G. Kim, and S. Wang, “Vibration localization prediction and optimal exciter placement for improving the sound field optimization performance of multi-channel distributed mode loudspeakers,” J. Sound Vibrat., vol. 481, Sep. 2020, Art. no. 115424.
[21] L. Cremer, M. Heckl, and E. E. Ungar, Structural-Borne Sound. Berlin, Germany: Springer, 1973, ch. 3.
[22] D. J. Inman, Engineering Vibration. London, U.K.: Pearson, 2014, ch. 6.
[23] A. W. Leissa, Vibration of Plates. Washington, DC, USA: NASA, 1973, ch. 4.
[24] M. R. Bai, J.-G. Ih, and J. Benesty, Acoustic Array Systems: Theory, Implementation, and Application. Singapore: Wiley, 2013, ch. 6.
[25] B. Kim and J. Ih, “On the reconstruction of the vibro-acoustic field over the surface enclosing an interior space using the boundary element method,” J. Acoust. Soc. Amer., vol. 100, no. 5, pp. 3003–3016, Nov. 1996.
[26] A. N. Tikhonov and V. Y. Arsenin, Solutions of Ill-Posed Problems. New York, NY, USA: Wiley, 1977, ch. 2.
[27] P. C. Hansen, Rank-Deficient and Discrete Ill-Posed Problems. Philadelphia, PA, USA: SIAM, 1998, ch. 7.
[28] D. T. Blackstock, Fundamentals of Physical Acoustics. New York, NY, USA: Wiley, 2000, ch. 13.
[29] P. M. Morse and K. U. Ingard, Theoretical Acoustics. New York, NY, USA: McGraw-Hill, 1968, ch. 5.
[30] C. R. Halkyard and B. R. Mace, “Structural intensity in beams—Waves, transducer systems and the conditioning problem,” J. Sound Vibrat., vol. 185, no. 2, pp. 279–298, Aug. 1995.
[31] S.-H. Jang and J.-G. Ih, “On the multiple microphone method for measuring in-duct acoustic properties in the presence of mean flow,” J. Acoust. Soc. Amer., vol. 103, no. 3, pp. 1520–1526, Jun. 1998.
[32] B. R. Mace and C. R. Halkyard, “Time domain estimation of response and intensity in beams using wave decomposition and reconstruction,” J. Sound Vibrat., vol. 230, no. 3, pp. 561–589, Feb. 2000.

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