How Neutrino and Charged Fermion Masses Are Connected Within Minimal Supersymmetric SO(10)

Borut Bajc
J. Stefan Institute, Ljubljana, Slovenia
E-mail: Borut.Bajc@ijs.si

Goran Senjanović
International Centre for Theoretical Physics, Trieste, Italy
E-mail: goran@ictp.trieste.it

Francesco Vissani
INFN, Laboratori Nazionali del Gran Sasso, Theory Group, Italy
E-mail: vissani@lngs.infn.it

Abstract: Massive neutrinos are a generic prediction of SO(10), and models of unification cry for supersymmetry. Since we have a rather detailed information on neutrino and charged fermion masses, the real question is: how/whether it is possible to build a SO(10) supersymmetric model, that correctly incorporates fermion masses. We show that a simple construction is possible in the context of a minimal theory. We concentrate on the two heaviest generations, discuss the predictions of the model, and briefly comment on open questions.

1. Yukawa Couplings at $M_{GUT}$

In order to avoid unacceptably big Dirac neutrino masses in SO(10) [1], one introduces $\mathbf{126}$-plets scalars. These produce huge Majorana masses for $\nu^c$ [2], and decouple them from the light spectrum:

$$\mathcal{L} = -16i \left[ Y_{ij}^{(10)} \mathbf{10} + Y_{ij}^{(126)} \mathbf{126} \right] \mathbf{16}_j + h.c. \quad (1.1)$$

The $\mathbf{10}$-plet contains two Higgs doublets, that we call $\varphi_u$ and $\varphi_d$, while the $\mathbf{126}$-plet contains one singlet $S$ (needed for $\nu^c$), one triplet $\Delta$ (which may contribute to light neutrino
masses [3] and two doublets $\varphi_u'$ and $\varphi_d'$ (useful to make up for wrong SO(10) mass relations [4]). Indeed, with a self-explanatory notation for the Weyl fermions [5]:

$$
\begin{align*}
16, 10 \ 16_j & \ni \varphi_u (u_i^c u_j + \nu_i^c \nu_j) + \varphi_d \left( d_i^c d_j + e_i^c e_j \right) + (i \leftrightarrow j); \\
16, 126 \ 16_j & \ni \frac{1}{2} (S \nu_i^c \nu_j + \Delta \nu_i \nu_j) + \varphi_u' (u_i^c u_j - 3 \nu_i \nu_j) + \varphi_d' (d_i^c d_j - 3 e_i^c e_j) + (i \leftrightarrow j)
\end{align*}
$$

In this work, we propose a model of the Yukawa couplings, in which all the features of the minimal SO(10) theory are exploited.

2. Beyond the Great (Supersymmetric) Desert

The question of starting up model building is: what does the minimal supersymmetric standard model (MSSM) want from SO(10)? We get an answer by extrapolating the Yukawa couplings from $T = 0$ to $T = \log(M_{\text{GUT}}/M_Z)/2\pi \approx 5.2$ (see appendix A for details). From figure 1, one sees that:

- For 3rd family charged fermions masses: the Hypothesis of leading 10–plet Yukawa coupling [6], that gives $y_t = y_b = y_\tau$ at $M_{\text{GUT}}$ is OK.\(^1\)

- For 2nd family charged fermion masses: the Hypothesis of leading 126–plet Yukawa coupling [7], that gives $y_\mu = -3 \times y_s$ at $M_{\text{GUT}}$ is OK.

This could be an accidental fact, but is suggestive enough to take it seriously.

3. Determining Model and Parameters

Now that we defined the target, the question becomes: how to match MSSM and SO(10) Yukawa couplings? SO(10) can meet the MSSM needs (illustrated in previous figure) after the very simple identification of the MSSM Higgs fields: $H_u \approx \varphi_u$ and $H_d \approx \varphi_d + \varepsilon \varphi_d'$. (Of course, the orthogonal doublets should decouple from the MSSM spectrum, to maintain gauge coupling unification– namely, we need a “doublet-doublet” splitting).

This position leads us to identify the MSSM Yukawa couplings in the following manner:

$$
Y_u \approx Y(10)^{\text{diagonal by definition}} \\
Y_d \approx Y(10) + \epsilon Y(126) \\
Y_e \approx Y(10) - 3 \epsilon Y(126)
$$

Since we know the Yukawa couplings (after extrapolation at $M_{\text{GUT}}$), we can deduce the size of several elements of the SO(10) Yukawa matrices. The chain of deduction we follow and the numerical values we obtain at $M_{\text{GUT}}$ are shown in this table:

| Couplings | Value |
|-----------|-------|
| $y_t, y_b, y_\tau$ | $Y_{33}^{(10)} \simeq 0.94 \gg \epsilon Y_{33}^{(126)}$ |
| $y_\mu, y_s$ | $\epsilon Y_{22}^{(126)} \simeq 1.4 \times 10^{-2} > Y_{22}^{(10)}$ |
| $y_c$ | $Y_{22}^{(10)} \simeq 1.8 \times 10^{-3}$ |
| $V_{cb}$ | $\epsilon Y_{23}^{(126)} \simeq 2.7 \times 10^{-2}$ |

\(^1\)We tuned the vev ratio $\tan \beta = \langle H_u \rangle / \langle H_d \rangle \sim 55.4$ to get this. We use 1 loop “running” and $\alpha_3 = 0.118$. 

International Europhysics Conference on HEP Francesco Vissani
Figure 1: Upper panel: Running of MSSM Yukawa couplings of third generation from $M_Z$ till $M_{\text{GUT}}$ ($y_t$ is the largest at $M_Z$, $y_\tau$ the smallest). Lower panel: same for second generation ($y_\mu$ is the largest, $y_c$ is the smallest). (We denote by $y_x$ the Yukawa coupling of the particle $x$, e.g.: $y_t$ for top, $y_c$ for charm, $y_\mu$ for muon. For a given $\tan\beta$, $y_x$ is computed from the mass of $x$ at $T = 0$.)

Two remarks are in order:

- We kept the deduction as simple as possible e.g. we did not perform detailed diagonalizations to get these numbers, which saves us from considering their phases. (However, we feel that it is fair to say that higher order effects, threshold and non-log corrections etc. could make a much more accurate treatment meaningless.)

- The only unknown element of the 2nd–3rd family blocks is $\epsilon Y^{(126)}_{33}$ (though one may reasonably guess that it is not too far from $\epsilon Y^{(126)}_{22}$ or $\epsilon Y^{(126)}_{23}$).

Till here, we showed that the model is not contradicting known things...

4. Neutrino Features

Now we come to the fermion of the day: the neutrino. In order to formulate our proposal, we will base our discussion on this provocative question: what do these neutrinos want?

We recapitulate the experimental situation by means of the following table:

| $\Delta m^2_{31}$ | $[1.5, 5] \times 10^{-3} \text{ eV}^2$ | atmospheric neutrinos |
|-------------------|---------------------------------|-----------------------|
| $\Delta m^2_{21}$ | $[2, 50] \times 10^{-5} \text{ eV}^2$ | solar LMA ($\sigma < 2 \times 10^{-7} \text{ eV}^2$) |
| $\theta_{23}$     | $[35^\circ, 55^\circ]$             | atmospheric neutrinos |
| $\theta_{13}$     | $< 10^\circ$                           | CHOOZ+atm.+K2K (depends on $\Delta m^2_{21}$) |
| $\theta_{12}$     | $[25^\circ, 43^\circ]$             | solar neutrinos (99 % CL) |
We will be mostly concerned with the first three items. As remarked by several people (see e.g. [8]) a neutrino mass matrix with a “dominant block” is strongly suggested:

\[ \frac{M_\nu}{\Delta m^2_{31}} = \frac{1}{2} \times \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{pmatrix} + O \left( \theta_{13}, \theta_{23} - \frac{\pi}{4}, \sqrt{\frac{\Delta m^2_{21}}{\Delta m^2_{31}}} \right) \]

But, due to hierarchical Yukawa couplings, the seesaw does not yield this pattern \textit{generically} (however, see also [10]). Often, small values of \( \theta_{23} \) are found, as pointed out in [4, 9] and as illustrated here:

\[ M_D M^{-1}_R M_D = \begin{pmatrix} e & 0 \\ 0 & a \\ 0 & b \end{pmatrix} \cdot \begin{pmatrix} a & b & c \\ b & c & \eta \end{pmatrix} \cdot \begin{pmatrix} e & 0 \\ 0 & 1 \end{pmatrix} \]

Thus, we are led to try another mass mechanism, and we welcome the fact that we have the triplet \( \Delta \) at our disposal [3] (by the way, we arrived at a common sense answer to the question on “neutrino wishes”: neutrinos want to be different from the other fermions).

5. The Triplet Option

We are assuming that neutrinos take mass \textit{mostly} from the triplet \( \Delta : M_\nu \propto Y^{(126)} \). Running back to \( M_Z \) the MSSM Yukawa couplings, we get a simple expression for the \( \nu_\mu - \nu_\tau \) block of the neutrino mass matrix:

\[ M_\nu \propto \begin{pmatrix} 1 & 1.7 \\ 1.7 & x \end{pmatrix} \]

(We have “\( x \)”, for \( Y^{(126)}_{33} \) is unknown, and also because seesaw might contribute to 33-entry—see e.g. [11]). Clearly, eq. (5.1) can underlie a “dominant block”, thus:

\[ \theta_{23} \text{ can be large} \]

\[ \text{we expect a weak mass hierarchy (not } m_3 \gg m_2) \]

These two properties correlate, as can be seen in figure 2. To further illustrate this result (assuming \( m_3^2 \simeq \Delta m^2_{31} = 3 \times 10^{-3} \text{ eV}^2 \) and \( m_2^2 \simeq \Delta m^2_{21} \)) we note that:

- If \( \theta_{23} = 45^\circ \), then \( \Delta m^2_{21} > 2 \times 10^{-4} \text{ eV}^2 \);
- If \( \Delta m^2_{21} = 5 \times 10^{-5} \text{ eV}^2 \), then\(^2 \theta_{23} < 40^\circ \).

We conclude that the minimal SO(10) model for Yukawa coupling we propose is predictive, despite (thanks to?) its simplicity.

\(^2\text{Quite tough to test experimentally, since it is equivalent to } \sin^2 2\theta_{23} < 0.97...\)
Figure 2: Possible values of the mass hierarchy parameter $m_2^2/m_3^2$ and of the atmospheric mixing angle $\theta_{23}$, obtained varying the complex input parameter $x$ (eq. 5.1). A rectangle encloses the range of permitted values, estimated assuming that the lightest neutrino mass $m_1$ is negligible.

6. Summary and Discussion

* We discussed an “economical embedding” of MSSM into SO(10), in a sense that all features of 126-plet have been exploited, namely: we use singlet, doublets and triplet vev’s.

* The most important step in the construction: how the masses of the charged fermions of the 2nd and 3rd generations are explained (Sects. 2 and 3). 3rd family unification suggests the large $\tan \beta = \langle H_u \rangle / \langle H_d \rangle$ regime; this is not an appealing case, but perhaps it is still viable (incidentally, it permits us to accommodate a “heavy” Higgs field, $m_h < 135$ GeV).

* The triplet mechanism for neutrino mass generation is at least likely (discussion in Sect. 4). The correlations among $(M_\nu)_{22} \leftrightarrow m_\mu, m_s$, and $(M_\nu)_{23} \leftrightarrow V_{cb}$ imply (eq. (5.1)):

$$\theta_{23} \in [35^\circ, 55^\circ] \iff \frac{m_2^2}{m_3^2} \in \left[ \frac{1}{250}, \frac{1}{3} \right]$$

Solar $\nu$ solutions with big hierarchy are disfavored, while LMA fits well the scheme. After the $\Delta m_{21}^2$ measurement–at KamLAND?–we will get an upper bound on $\theta_{23}$ (fig. 2 and Sect. 5).

* A pending question is: masses of 1st family fermions (also $m_1$); proton decay rate; feasibility of baryogenesis-through-leptogenesis mechanism. These features are strictly tied among them, and require further study.

To conclude, we stress the main goals achieved: We showed that it is possible to build a simple model for fermion masses based on supersymmetric SO(10), with renormalizable couplings only. This model accounts for the masses of second and third generation fermions. It has large $\theta_{23}$, and prefers the solar neutrino solutions with weak mass hierarchy.
Acknowledgments

F.V. thanks the Organizers of the “International Europhysics Conference on HEP” for the beautiful conference, and the Conveners for kind invitation and discussions. The work of B.B. is supported by the Ministry of Education, Science and Sport of the Republic of Slovenia. The work of G.S. is partially supported by EEC, under the TMR contracts ERBFMRX-CT960090 and HPRN-CT-2000-00152. We express our gratitude to INFN, which permitted the development of the present study by supporting an exchange program with the International Centre for Theoretical Physics.

A. 1 loop renormalization group equations

We assume supersymmetry, in order to comply with one-step unification of gauge couplings. The renormalization group equations relevant to our analysis are:

\[
\begin{align*}
\alpha'_t &= \alpha_t [6\alpha_t + \alpha_b - 16/3\alpha_3 - 3\alpha_2 - 13/9\alpha_1]
\alpha'_b &= \alpha_b [6\alpha_b + \alpha_t + \alpha_\tau - 16/3\alpha_3 - 3\alpha_2 - 7/9\alpha_1]
\alpha'_\tau &= \alpha_\tau [4\alpha_\tau + 3\alpha_b - 3\alpha_2 - 3\alpha_1]
\alpha'_c &= \alpha_c [3\alpha_\tau - 16/3\alpha_3 - 3\alpha_2 - 13/9\alpha_1]
\alpha'_s &= \alpha_s [3\alpha_b + \alpha_\tau - 16/3\alpha_3 - 3\alpha_2 - 7/9\alpha_1]
\alpha'_\mu &= \alpha_\mu [3\alpha_b + \alpha_\tau - 3\alpha_2 - 3\alpha_1]
A' &= -A(\alpha_t + \alpha_b)/2
\lambda' &= 0
\rho' &= 0
\eta' &= 0
M'_{ij} &= M_{ij} [\alpha_\tau (k_i + k_j)/2 + 3\alpha_\tau - 3\alpha_2 - \alpha_1]
\end{align*}
\]

The symbol ’ (=prime) denotes derivative with respect to \(T = \log(Q/M_Z)/2\pi\). We define \(\alpha_x = y_x^2/4\pi\) for \(x = t, b, \tau, c, s, \mu\), analogously to gauge \(\alpha_i\)’s. \(A, \lambda, \eta, \rho\) are the Wolfenstein parameters. \(M_{ij}\) are the entries of neutrino mass matrix; \(k_3 = 1\), and \(k_2 = 0\). \(\alpha_1\) is normalized in standard model fashion—not SU(5)’s.
References

[1] H. Georgi, in Coral Gables 1975, “Theories and experiments in high energy physics”, New York 1975, 329-339; H. Fritzsch and P. Minkowski, Annals Phys. 93, 193 (1975).

[2] M. Gell-Mann et al. in “Supergravity”, Stony Brook 1979; T. Yanagida in “Proceeding of the workshop on unified theory and baryon number in the Universe”, KEK 1979; R.N. Mohapatra and G. Senjanović, Phys. Rev. Lett. 44 (1980) 912.

[3] R.N. Mohapatra and G. Senjanović, Phys. Rev. D23, 165 (1981).

[4] K.S. Babu and R.N. Mohapatra, Phys. Rev. Lett. 70 (1993) 2845.

[5] R. Barbieri, D.V. Nanopoulos, G. Morehio and F. Strocchi, Phys. Lett. B 90 (1980) 91.

[6] H. Georgi and D.V. Nanopoulos, Nucl. Phys. B 159 (1979) 16.

[7] H. Georgi and C. Jarlskog, Phys. Lett. B 86 (1979) 297.

[8] This remark was first done in Z.G. Berezhiani and A. Rossi, Phys. Lett. B 367 (1996) 219. A recent discussion in F. Vissani, 9th M. Grossmann meeting, Rome 2000, hep-ph/0102235.

[9] B. Brahmachari and R.N. Mohapatra, Phys. Rev. D 58 (1998) 015001.

[10] There is not a “no-go theorem” for seesaw mechanism in SO(10). See K.T. Mahanthappa et al., hep-ph/0110037, talk given in the session on CP violation of HEP2001, and K. Matsuda et al., hep-ph/0108202, appeared after the conference. However, note that they find solutions with pronounced hierarchy as LOW and QVO, that are not expected in our model.

[11] As formalized in eq. (17) of A.S. Joshipura and E.A. Paschos, hep-ph/9906498.