Nonlinear Dynamic Model of Machinery

Zhouhong He1 and Xiaowen Liao2*

1Railway Locomotive and Rolling Stock Institute, Wuhan Railway Vocational College of Technology, 430205
2Wuhan Optical Valley Mechanical and Electrical Technology Co., Ltd., 430074

*Corresponding author e-mail: xwliao@fiberhome.com

Abstract. Rotor dynamics theory is the basis for studying the motion of rotating machinery, and rotating machinery such as water turbines, steam turbines, and aero engines are indispensable core equipment for related industries. However, the current research on rotor dynamics is mostly based on linear conditions, which can no longer meet modern requirements. Engineering requirements, so the study of nonlinear rotor dynamics seems significant. The research purpose of this paper is to study the nonlinear dynamic model. Based on the theory of nonlinear rotor dynamics and the theoretical basis of the finite element method, a finite element simulation method is used to study the nonlinear rotor dynamics. In this paper, the finite element model in the discrete mass model is used. After introducing the research method and eddy theory of nonlinear rotor dynamics, the factors that may cause non-linear interference to the rotor motion in the rotor system are analyzed. The problem was simulated and analyzed, and the movement behavior of the eccentric rotor under elastic support was simulated by means of simulation, and the nonlinear effect of eccentricity on the rotor system was proved.

Keywords: Rotating Machinery, Nonlinear Dynamics, Dynamic Model, Finite Element Simulation

1. Introduction

With the completion of the Three Gorges Dam, China's water conservancy and hydropower industry has ushered in another peak after the Gezhouba Hydropower Station. Due to the non-renewable nature of coal resources and its unfriendly environment, people have been trying to find new methods of power generation. Although wind power, nuclear power, and other seemingly good ways have appeared, there are still problems such as low wind power capacity and nuclear power generation with certain safety risks. Hydropower uses renewable energy, which has the advantages of not producing pollutants, large-scale output, and economic security. China is also a country with large hydropower resources, so hydropower will continue to be the current and even the next few decades. The main force of the power industry. Turbine system is the lifeblood of a hydropower station, so its safety and stability must be accurately grasped. When the hydropower plant in our country arranges the maintenance plan of the main power equipment, if they continue to use the traditional periodic maintenance model, it will not only cause unnecessary economic losses to the power plant, but also
have serious hidden safety hazards. Therefore, if an online monitoring system for a large hydropower unit can be developed, it can be monitored at any time, saving maintenance costs and reducing operating risks. Therefore, related research seems urgent.

At present, although the theory and methods of rotor dynamics based on linear vibration systems are very mature, as researchers continue to deepen rotor dynamics, it is found that under linear conditions, it is not enough to describe the behavior of these rotors [1]. At present, the dynamics of rotor-bearing systems are mostly based on linear rotor dynamics theory. For example, traditional rotor dynamics generally uses linear stiffness and damping characteristics when modeling oil film forces. In fact, this is not the case. It is in line with the real situation, but can be approximated under general conditions [2]. With the development of modern technology, due to the increase in rotor speed and mass, the role of non-linear excitation sources such as oil film forces in large rotating machinery has become more and more significant [3]. However, due to the insufficient mathematical model, many complex dynamic behaviors caused by non-linear factors have not been completely understood, and this kind of influence cannot be ignored or even simplified in today's increasingly developed technology, because the analysis results are not comprehensive and accurate. Cannot meet the needs of modern scientific research and development and engineering design accuracy [4]. Therefore, in order to understand the rotor movement of the machine more accurately and more realistically, and to better meet the needs of modern industry, the study of nonlinear rotor dynamics is very important.

This paper combines domestic and foreign related research to briefly introduce and analyze the nonlinear rotor dynamics problem. At the same time, the finite element simulation of the rotor model considering different nonlinear factors is performed using the learned knowledge, and good results are obtained. The object of this article is the nonlinear rotor dynamics problem. The in-depth method is used to explore the ideas and characteristics of the finite element method to study the rotor dynamics problem. Through simulation examples, the reliability of the finite element simulation method to solve rotor dynamics problems is introduced and verified.

2. Proposed Method

2.1 Rotor Dynamics

2.1.1 Rotor dynamics analysis method. In the analysis of rotor dynamics, there are two kinds of mechanical models often used: continuous mass model and discrete mass model. Both models have their own characteristics, which are applicable to different problems and different actual needs [5]. The continuous mass model basically considers the rotor as an elastic body with continuous mass distribution according to the actual structure of the rotor. The continuous mass model is closer to reality in mathematical modeling, so the error caused by simplification is not large [6]. However, due to the complexity of the actual rotor geometry, it is difficult to mathematically list the boundary conditions of the partial differential equations. The solution is more difficult, so it is very limited in practical applications. Discrete mass model discretizes the actual structure and turns continuous infinite-degree-of-freedom models into discrete finite-degree-of-freedom models. Ordinary differential equations are often used to describe its motion. The mathematical modeling of the discrete mass model and its solution are relatively easy, and when the degree of freedom of the discrete model is sufficient, the calculation results are sufficient to meet the requirements of engineering accuracy, so it is widely used [7]. Discrete mass models can be subdivided into finite element models and centralized mass models. A finite element model is a combination of discrete continuous bodies separated into a limited number of elements connected to each other in a certain way. Because the number of elements in the finite element model can be large, and the simplified approximation to the actual structure is very small, more accurate calculation results can be obtained. Compared with the continuous mass model and finite element model, the centralized mass model has a simple structure and a small amount of calculation. If it is simplified and reasonable, it can also obtain more accurate results, which is more suitable for theoretical derivation and basic analysis.
2.1.2 Transfer matrix method. The lumped mass method has great advantages in dealing with simple models and is favored by theoretical researchers. However, when dealing with complex structures, it can better handle the dynamics of multi-disk rotor systems. Recursive matrix method, but it seems inadequate [8]. When using the transfer matrix method to solve the dynamic behavior of the rotor system, due to the characteristics of the transfer matrix method itself: the increase of the degree of freedom of the system does not affect the order of the calculation matrix [9-10]. The basic idea of the transfer matrix method is: first establish a centralized mass model of the system, and then divide it into several typical unit components including disks, shaft segments, and supports. These physical quantities of shear force are used as the state vector of the end face of each shaft segment. According to the continuous conditions, the basic equations of the transfer matrix shown in formula (1) are obtained, and then the expressions of the state variables on the two ends of the rotor are obtained. Finally, it is solved according to the boundary conditions to obtain the rotor dynamics information.

\[
\{S_n\} = [T_n \ldots T_1] \{S_0\}
\]

Where \(\{S_n\}\) is the state vector of the end cross section, \(\{S_0\}\) is the state vector of the initial cross section, and \([T_n]\) is the transfer matrix.

2.2 Finite Element Simulation Method of Rotor Dynamics

2.2.1 Advantages and disadvantages of finite element method for solving rotor dynamics problems. The main disadvantage of the finite element method for analyzing rotor dynamics is that the rotor model is simplified very little, which leads to a large amount of calculation, especially if the many nonlinear characteristics of the rotor system are considered, the establishment of the entire rotor system including surrounding structures is limited Meta-model, it will be very difficult to solve [11-12]. However, in recent years, with the continuous improvement of computer performance and the continuous optimization and enhancement of finite element simulation software functions, if some reasonable simplification methods are adopted for the finite element model, then for general rotor dynamics engineering problems, use finite element The method for analysis can get very valuable results [13].

Among several commonly used mechanical models, the advantage of the finite element method compared to the concentrated mass method is that the concentrated mass method cannot accurately calculate the size and position of mass and inertia, resulting in inaccurate calculation of system parameters. Simple rotor system [14-15], compared with the transfer matrix method, on the one hand, it is difficult to analyze the surrounding structure of the rotor system, such as bearings, which limits the space for future improvement; on the other hand, the transfer matrix method does not have general-purpose calculation software. All calculations need to rewrite the program, which is far less mature than the finite element method. Therefore, if the amount of calculation is kept within an acceptable range, the finite element method is undoubtedly the optimal method to solve the rotor dynamics problem. The ANSYS software to be used in this paper is a set of mature FEM commercial software. It is based on the basic principle of the finite element method and provides an auxiliary method for the research and analysis of rotor dynamics problems.

2.2.2 Rotor dynamic equation of motion. As with using the finite element method to analyze other problems, the rotor structure needs to be divided into units, then the motion equations of all micro-element bodies are combined into the macro-state equation of motion of the rotor system, and the boundary conditions are finally solved. The general dynamic equation expression is:

\[
[M]\{U\} + ([C] + [G])\{U\} + ([K] + [B])\{U\} = \{F\}
\]

In the formula, [M], [C], and [K] represent mass, damping, and stiffness matrices respectively; \(\{U\}\)
is the displacement matrix, \([B]\) represents the rotation damping matrix, \([F]\) is the external force matrix.

The basic equations and ideas of the finite element method are simple, and the expression is simple. It can be used to analyze the rotor and the complex rotor system structure considering the surrounding structure. It has great advantages in engineering applications.

2.2.3 The main points of ANSYS dealing with rotor dynamics: For ANSYS software, anyone who has done finite element simulation will not be unfamiliar. As the most mature finite element simulation at present, one of the real software, its function is very powerful whether it is pre-processing or post-processing.

1) Element selection in the rotor dynamics simulation, the elements in the rotating structure must consider the gyro effect in the rotation angle, so the commonly used elements are: MASS21, BEAM188, BEAM189, SHELL181, SHELL281, SOLID185, SOLID186, SOLID272, SOLID273, PIPE288 and PIPE289. Etc. MASS21 particle unit is generally used to simulate the rigid turntable, that is, the degeneration of the turntable is not considered during the simulation. This is because under normal motion conditions, the degeneration of the rotor shaft is much larger than that of the turntable, and usually the state of the rotor and the shaft we study is the key to the entire rotor system. For non-axisymmetric components, it can be equivalent to an axisymmetric mass. A new mass element is defined at the center of gravity, and the element constant and the moment of inertia are defined using real constants. The shaft generally uses BEAM188 element, which is regarded as a beam element for simulation. This element is a three-dimensional finite stress beam element. In extreme cases, such as large disturbances, its nonlinearity needs to be considered. The other structural units are selected and modeled according to the actual model of the shaft. For example, if you are analyzing the rotation of the turntable plane, you can use a shell unit to simulate the turntable and so on. The rotor is supported by bearings, but in fact this support is not rigid but elastic. In fact, the support elasticity has a great influence in engineering analysis. When doing simulation, we must consider it. Bearings are generally simulated with spring-damped COMBIN14 or COMBIN214 units. Tension and compression can be considered, but bending and torsion are not considered. The COMBIN14 element can be expressed as a non-linear relationship. More complex non-linear situations will require the use of the MATRIX27 element for simulation, which may be the development direction of oil film force simulation in the future.

2) Points for modeling and solving process When building a rotor model, be sure to separate the rotating parts from the non-rotating parts, and ensure that the rotating parts are axisymmetric. If not, you can make an equivalent mass force as described in the previous section. For more complex models, it is recommended to import from external CAD software. During the loading process, the rotor dynamics simulation generally uses transient analysis and harmonious response analysis. The rotor dynamics analysis must apply the speed, and pay attention to activating and setting the gyro response option. As for the operation of applying force and constraints, it is basically the same as the general operation. The same, it will not be explained. It should be noted that, because it is a rotational force, when performing harmonic response analysis, this force will be expressed in the form of a complex number, so the real and imaginary components need to be used to define the rotational load.

3) Post-processing means can use ANSYS general function to complete the analysis of many results, such as displaying motion trajectory, studying the force change of each node, etc. After multiple modal solutions, the user can draw a Campbell diagram, which is a special post-processing method specialized in rotor dynamics. Using the Campbell chart, users can easily get the most important physical quantity when studying the rotor system-critical speed. In the Campbell diagram, the intersection of the frequency curve and the straight line of the extracted speed is the critical speed. Therefore, making a good Campbell diagram can greatly help the rotor dynamics analysis.
3. Experiments

3.1 Modeling

3.1.1 Unit selection principles. Here BEAM188 unit is used to simulate the bearing, and MASS21 particle unit is used to simulate the rigid turntable. Because it is necessary to perform transient analysis, it is more suitable to use COMBIN14 unit to simulate the bearing. Because COMBIN14 unit can only set parameters in one direction, it is necessary to define the unit for the x and y directions respectively.

3.1.2 Transient analysis. By using array parameters to define the rotational force of each time step and the magnitude of the force in each direction. In this example, the rigid turntable is simulated with a particle unit. For eccentric processing, it can be equivalent to a centrifugal force that changes with the rotation of the axis, that is, an external force that changes with time. It is appropriate to use the function definition function in ANSYS to define the Eccentric force. The expression of eccentric force is:

$$ F_p = m_p \omega^2 = m_p l_p \left(\frac{10000}{5*t}\right)^2 $$

Where $m_p$ is the eccentric mass, $l_p$ is the eccentricity, and $t$ is the time of axis rotation, $(10000/5*t)$ is the instant speed.

When loading on the model, since it is a rotational force, the component forces in the vertical and horizontal directions are constantly changing, so the force needs to be disassembled into real and imaginary components for definition and loading.

The established model is shown in Figure 1. At this point, the pre-processing stage of the finite element simulation of the model is basically completed.

![Figure 1. Schematic diagram of eccentric rotor model](image)

The model sizes used in this experiment are as follows:

| Component | Size |
|-----------|------|
| Shaft length L1 | 0.3m |
| L2 | 0.7m |
| L3 | 0.8m |
| L4 | 0.35m |
| Radius R | 0.02m |
| Turntable mass | 30kg |
| R1 | 0.5m |

4. Discussion
4.1 Analysis of Critical Speed of Rotor Model Without Eccentricity

According to the experimental data in this article, before the transient analysis, a modal analysis can be performed on the model to determine its critical speed when eccentricity is not considered. The first three stages of critical speed values are shown in Table 1.

Table 1. Analysis of critical speed of rotor model without considering eccentricity

| Critical speed (rps) | First order | Second order | Third order |
|----------------------|-------------|--------------|-------------|
|                      | 15.799      | 15.851       | 101.56      |
|                      | 102.96      | 145.53       | 146.91      |

It can be seen from Table 1 that there are two critical speeds for each order frequency. This is because the elastic speed of the bearing is taken into account, which causes the critical speeds in the x direction and y direction of the rotor to be no longer the same. It will also become an ellipse, which is consistent with the conclusion derived from the basic theory of rotor dynamics. This axis trajectory diagram is very useful for analyzing rotor dynamics. Because of its clear and intuitive expression, it has high reference value for engineering applications. On the one hand, you can understand the dynamic behavior of any point on the rotor, and on the other hand, you can find the point. The corresponding maximum amplitude value or interval in the plane.

4.2 Amplitude Analysis of Disk Center Vibration Under Different Eccentricity

In order to study the effect of eccentricity on the rotor system, the change of the maximum amplitude of the disk center was studied by changing the eccentricity. The results are shown in Figure 2.

Figure 2. Vibration amplitude of disc center under different eccentricities

From Figure 2, it can be calculated that the effect of the eccentricity on the rotor is a non-linear relationship, and the effect of the eccentricity on the rotor system is demonstrated to be non-linear. However, it can also be seen from the data that when the amount of eccentricity is small, the effect of eccentricity on the rotor is an approximately linear relationship. Only in the case of large masses and large eccentricities, it is necessary to consider the non-linear effect of eccentricity. Generally, it can be treated as a linear simplification in the case of small eccentricity.

5. Conclusions

After analyzing the research significance and research status of nonlinear rotor dynamics, this paper first introduced the finite element simulation method to a certain length, and focused on using the finite element software ANSYS to simulate the eccentric rotor model. On the one hand, based on
the combination of simulation results and theory, the feasibility of the finite element simulation method to solve the nonlinear rotor dynamics and the operability in engineering applications are verified. Broad platform and method demonstration.

References
[1] Simon Bäuerle, Hartmut Hetzler. Rotor-systems with compliant seals: A comparison of the rotordynamics using the Muszynska model and Hirs' lubrication equations[J]. Pamm, 2017, 17(1):359-360.
[2] N. A. Saeed, M. Kamel. Active magnetic bearing-based tuned controller to suppress lateral vibrations of a nonlinear Jeffcott rotor system[J]. Nonlinear Dynamics, 2017, 90(8):457-478.
[3] Jiyong Ma, Dongmei Xiao. Nonlinear dynamics of a mathematical model on action potential duration and calcium transient in paced cardiac cells[J]. Mathematical Biosciences, 2017, 18(9):2377-2396.
[4] Dongxiao Wang, Xiaodan Gao, Ke Meng. Utilization of kinetic energy from wind turbine for grid connections: A review paper[J]. Iet Renewable Power Generation, 2018, 12(6):615-624.
[5] Wanhui Liu, Kai Feng, Peng Lyu. Bifurcation and nonlinear dynamic behaviours of a metal mesh damped flexible pivot tilting pad gas bearing system[J]. Nonlinear Dynamics, 2017, 91(4):655-677.
[6] A. Grządziela, M. Kluczyk. The impact estimation of damping foundations in dynamics of the rotor system[J]. Diagnostyka, 2017, 18(3):55-61.
[7] Lee, Seongkyu, Brentner, Kenneth S. Morris, Philip J. Long-Range and Nonlinear Propagation of Helicopter High-Speed Impulsive Noise[J]. Journal of the American Helicopter Society, 2017, 2(2):1633-1643.
[8] JIANG Ke-jian, ZHU Chang-sheng. Parameter identification for stiffness and damping of active magnetic bearing in flexible rotor system[J]. Journal of Vibration Engineering, 2017, 30(6):883-892.
[9] Yevych, Medulych, Vysochanskii. Nonlinear dynamics of ferroelectrics with three-well local potential[J]. Condensed Matter Physics, 2018, 21(2):23001.
[10] Phuong Le Quang, Ian L. Braly, John K. Katahara. Nonlinear photocarrier recombination dynamics in mixed-halide CH3NH3Pb (11− x Br x )3 perovskite thin films[J]. Applied Physics Express, 2017, 10(10):102401.
[11] Yen-Ju Chen, Yen-I Lee, Wen-Cheng Chang. Modelling of the material flow of Nd-Fe-B magnets under high temperature deformation via finite element simulation method[J]. Science & Technology of Advanced Materials, 2017, 18(1):611-619.
[12] Chenguoy Yao, Di Hu, Zhongyong Zhao. Simulation Modeling of Lumped Parameter Circuit for Power Transformer Winding Based on the Finite Element Method[J]. Gaodianya Jishu/High Voltage Engineering, 2018, 44(11):3517-3523.
[13] Qi Tao, Gerald Pinter, Thomas Anttreter. Model free kinetics coupled with finite element method for curing simulation of thermosetting epoxy resins[J]. Journal of Applied Polymer Science, 2018, 135(27):46408.
[14] Yan-qiu Zhang, Shu-yong Jiang, Ya-nan Zhao. Simulation of isothermal precision extrusion of NiTi shape memory alloy pipe coupling by combining finite element method with cellular automaton[J]. Journal of Central South University, 2017, 24(3):506-514.
[15] Shengyong Ding, Guojian Shao, Ang Li. Numerical simulation of holes and inclusions using adaptive polygonal finite element method[J]. Journal of Mechanical Science & Technology, 2017, 31(9):4305-4317.