SmO thin films: a flexible route to correlated flat bands with nontrivial topology

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Using density functional theory based calculations, we show that the correlated mixed-valent compound SmO is a 3D strongly topological semi-metal as a result of a 4f-5d band inversion at the X point. The [001] surface Bloch spectral density reveals two weakly interacting Dirac cones that are quasi-degenerate at the \( M \)-point and another single Dirac cone at the \( \Gamma \)-point. We also show that the topological non-triviality in SmO is very robust and prevails for a wide range of lattice parameters, making it an ideal candidate to investigate topological nontrivial correlated flat bands in thin-film form. Moreover, the electron filling is tunable by strain. In addition, we find conditions for which the inversion is of the 4f-6s type, making SmO to be a rather unique system. The similarities of the crystal symmetry and the lattice constant of SmO to the well studied ferromagnetic semiconductor EuO, makes SmO/EuO thin film interfaces an excellent contender towards realizing the quantum anomalous Hall effect in a strongly correlated electron system.

Topological insulators (TI) are materials which exhibit a fundamentally new physical phenomena that was first predicted by theorists \cite{1, 2, 3, 4, 5} and subsequently ascertained in experiments \cite{6, 7}. Although most work considers band semiconductors, the concept of topology can be extended to correlated systems for which many exciting new effects can be expected. In analogy to correlation driven fractional quantum Hall states one might expect fractional Chern insulating states to emerge in correlated topological insulators \cite{8, 9, 10, 11}. The experimental realization of such states would open a whole new field of possibilities.

SmB\(_6\) was recently reported to be a correlated mixed valent topological insulator \cite{12, 13, 14}. The highly correlated flat Sm-\( f \) derived bands hybridize with the dispersive and itinerant Sm-\( d \) derived bands to form a mixed valent insulating ground state. Although the insulating state and topology of the bulk of SmB\(_6\) is well defined, [001] surface of SmB\(_6\) is polar and therefore inherently unstable \cite{15}. This hinders the unique determination of topological surface states and in turn inhibits the creation of robust technological devices. Here, using density functional theory (DFT) based calculations, we show that SmO is a mixed valent correlated compound with a band structure similar to the topological nontrivial high-pressure gold phase of SmS \cite{16}. In contrast to SmS, SmO is predicted to have a topological nontrivial ground state at ambient pressure. Additionally, our calculations show that the topological nontrivial ground-state of SmO is stable for a large range of Sm-O distances, including both positive and negative strain. The Sm-\( f \) band filling is thus tunable by strain, which opens up the possibility to create correlated topological nontrivial bands at different filling. Subsequent to the enormous success in the design and fabrication of semiconductors, it is presently possible to grow high-quality oxide thin films and heterostructures \cite{17, 18}. SmO thus seems the ideal candidate for the experimental realization of topological nontrivial correlated flat bands. In addition, experimental realization of the theoretically predicted quantum anomalous Hall effect (QAHE) (quantized Hall conductance in the absence of an external magnetic field) has been challenging due to difficulties in obtaining insulating bulk TI’s concurrent with homogeneous magnetic doping \cite{19}. Recently, notwithstanding the presence of bulk carrier density (i.e. not a nominally insulating bulk), long range ferromagnetism and QAHE was successfully observed for the first time in Cr-doped (Bi,Sb)\(_3\)Te\(_3\) TI thin films \cite{20, 21}. Other proposals towards realizing a QAHE are to either grow a TI on top of a ferromagnetic insulator \cite{22, 23}; transition-metal oxide heterostructures, \cite{24, 25} or to deposit a layer of heavy atoms with large spin-orbit coupling on a magnetic insulator \cite{26}. Fortuitously, a well studied substrate with the same symmetry as SmO is readily available in the form of the ferromagnetic semiconductor EuO \cite{27, 28}. Consequently, we propose growing SmO/EuO thin film interfaces to test the feasibility of obtaining a QAHE in a strongly correlated electron system.

SmO crystallizes in the rock-salt structure with \( a = 4.9414-4.943\text{Å} \) \cite{29, 30}. Considering the lattice parameters of trivalent neodymium chalcogenides and interpolating between neodymium and terbium compounds, Leger et. al. obtain a lattice constant of 4.917 Å for Sm\(_{3+}\)O \cite{31}. Similarly, considering other valent lanthanide oxides, a lattice constant of 5.15 Å is expected for Sm\(_{2+}\)O. Ergo, the experimental cell parameter of fcc SmO lies between that of Sm\(_{3+}\)O and Sm\(_{2+}\)O, which lead the authors \cite{32} to conclude that samarium is in an intermediate valence state in SmO. Using Vegard’s law, a valence of 2.92 was assigned \cite{33}. Electrical resistivity measurements as a function of temperature reveals a \( T^2 \) like increase between 4.2 and 20 K, followed by a rapid increase up to 32 K. The resistivity shows a linear temperature dependence above 60 K \cite{34}. Krill and coworkers \cite{35} measured the magnetic susceptibility of SmO.
FIG. 1. (Color online) Phase diagram (d) of SmO as a function of the fcc lattice parameter within LDA+SO+U (U = 6 eV, \(J_H = 0\) eV) along with representative FPLO band structures for the (a) nontrivial \(d - f\) band inversion, (b) trivial insulator and (a) nontrivial \(s - f\) band inversion scenarios. The \(4f\), \(5d\) and \(6s\) orbital character derived bands are represented by red, blue and green colored symbols respectively. The size of the symbols represent the weight of the various orbital contributions to the underlying bands. The Brillouin zone of an fcc lattice is displayed along with the eight time reversal invariant momenta (TRIM) points.

and observed the roughly constant magnetic susceptibility and the non-divergence of the low-temperature susceptibility. They drew parallels to the susceptibilities of other intermediate valence compounds SmB\(_6\) and gold-SmS under pressure, wherein the low temperature behavior is not explainable by crystal field effects alone [44].

The electronic structure calculations are performed using the full-potential non-orthogonal local orbital code (FPLO) [46]. The local density approximation (LDA) with the Perdew and Wang flavor [47] of the exchange and correlation potential was chosen. To account for the strong spin-orbit (SO) coupling of the \(f\) electrons, we employ the full four-component relativistic scheme. Additionally, the strong Coulomb repulsion between the \(4f\) electrons of samarium are included in a mean-field way by applying LDA+SO+U with the "fully localized limit" (FLL) double counting term [48]. This level of theory will not suffice to describe quantitatively the Sm \(4f\) spectral weight and its dispersion, but is sufficient to predict the Sm \(d - f\) inversion. We have tested the robustness of these predictions by varying \(U\) (5 to 7 eV) and \(J_H\) (0 to 0.7 eV) and by using different functionals (LDA and GGA using FPLO and the modified Becke-Johnson approach [49] with \(U=3\) eV and \(J_H=0\) eV using \texttt{wien2k}[50]). Importantly, as pointed out by Martin and Allen [2] for SmB\(_6\) and related systems, the symmetry of the relevant Sm one particle orbitals and the many body electron removal Green’s function is the same. This allows for a possible adiabatic continuation from the LDA results as presented here to the full interacting system without changing the topology of the system. See Supplemental Material [52] for further discussion on the Sm \(4f\) many body state.

Collected in Fig. (S1a) is the FPLO non-spin polarized, full-relativistic band structure of SmO with the inclusion of the strong Coulomb interaction (LDA+SO+U) for the experimental lattice constant. With samarium being in the 2+ configuration, the \(4f\) states are split into lower lying and filled \(4f^{5/2}\) states that can accommodate 6 electrons and higher lying (above 5.5 eV), empty \(4f_{7/2}\) states (see Supplemental Material). We do not observe a direct
energy gap around the Fermi level (E_F). The material is semi-metallic with a “warped gap” such that everywhere in the BZ (at E_F), band number x is always below band number x + 1. There are small electron and hole pockets at X and Γ respectively, but there are no band crossings between the highest occupied 4f_{5/2} bands and the lowest unoccupied bands. Besides, varying U (5 to 7 eV) or J_H (0 to 0.7 eV) does not change the above mentioned features, since the oxygen 2p, samarium 5d and 6s bands experience a constant shift with respect to the localized 4f states. There exists only one report [45] in literature on the transport properties of SmO wherein the resistivity decreases with decreasing temperature and thus could hint to metallic character, consistent with the lack of an energy gap in our calculations. In contrast to the localized and not very dispersive 4f bands, the 5d bands of samarium are dispersive, broad and dip below the Fermi level, retaining a 100% weight at the X point. As a consequence, we observe a 4f-5d band inversion at the X point, resulting in a nontrivial topology, similar in sense to the band inversions reported for SmB_6 and the high pressure gold phase of SmS. This band inversion and the resulting topological indices 1;(000) allows us to classify SmO as a 3D strongly topological semi-metal (more details are provided in Supplemental Material).

A well known issue with LDA when dealing with semiconductors, is the underestimation of band gaps. Since nontrivial topology depends on band inversions, under-estimation of band gaps could sometimes lead to wrong sequence of the underlying band structure. For example, in HgTe, a s-p TI, LDA correctly predicts the band inversion, but results in an incorrect band sequence at the Γ point. The correct band sequence is obtained by using the MBJLDA exchange potential [53]. Literature on the efficiency of MBJLDA over the traditional LDA+U for f electron systems is still scarce. In a recent work on SmS, the authors have employed MBJLDA+SO+U (U = 3 eV) to open a band gap of 0.2 eV in the ambient pressure black phase such that the gap is consistent with the available experiments [25]. Using the same parameters, the authors conclude that the high pressure gold phase of SmS is a topological metal. We compared the LDA+SO+U results with that of MBJLDA+SO and obtain a consistent picture for the band inversion between the two approaches (see Supplemental Material). No direct band gap is opened with MBJLDA+SO+U (U = 3 eV), in accordance with our LDA+SO+U results and as well as with the experimental report [45].

To provide additional confirmation of the topological non-triviality in SmO and to explicitly identify the protected surface states, we have calculated the Bloch spectral density (A_B(k)) of the 12 topmost surface layers of a semi infinite solid [51] with [001] surface. We obtain two weakly interacting quasi-degenerate Dirac-cones at M, and a single Dirac-cone at Γ which is hidden in the bulk and becomes a surface resonance due to the semimetallic nature of SmO. To clearly identify the Dirac-cone at Γ, we have repeated the slab calculations such that the hole pockets at Γ for the bulk are pushed down.

FIG. 2. (Color online) Left: Surface Bloch spectral density (A_B(k)) of the first 12 SmO layers of a semi-infinite solid with [001]-surface. Note, that there are two Dirac cones at the X-point, while the one at the Γ-point falls into the bulk projected band structure and hence forms a surface resonance (see right panel). Right: A_B(k) with downwards shifted 4f-electron pocket around the Γ-point to reveal the third Dirac cone.

FIG. 3. (Color online) Maximum possible f valence due to f → d or f → s promotion in SmO while retaining the nontrivial topology as a function of lattice parameter. The color map follows that of Fig. S1(d): light blue = nontrivial topology from d − f inversion, light green = nontrivial topology from s − f inversion, light pink = trivial topology. The equilibrium lattice parameters for a high-spin 3+ and 2+ state are computed using L(S)DA+SO+U scheme (dashed red line). The estimates from an empirical extrapolation for the limiting cases are obtained from Ref. [45] (dashed green line). The filled green circle denotes the experimental bulk SmO. SmO remains a topological semi-metal for a wide range of tensile strain (≥ 1%), since the calculated maximum valence, is above both the empirical extrapolation and L(S)DA+U+SO estimates.
This in turn, allows one to unambiguously identify all three Dirac-cones in a [001] terminated SmO. More details are provided in the Supplemental Material.

Having established the nontrivial topology in SmO for the experimental bulk lattice parameter, we consider the scenario of growing thin films of SmO. In general, the lattice parameter of the substrate plays a decisive role in determining the lattice parameter of the thin film. Then, the relevant question to answer is the robustness of the topological semi-metal state as a function of lattice parameter variation. To this end, we have investigated the topological indices for various lattice parameters. We have retained the cubic symmetry of the unit cell (i.e. isotropic volume change) in SmO, based on the experimental studies on isostructural samarium systems, which evidences a negative value of the Poisson ratio $\nu$ (elastic constant $C_{12} < 0$), characteristic of an isotropic volume change$^{[55, 56]}$. Since the nontrivial topology is obtained due to $d-f$ band inversion at the $\Gamma$ point, SmO is topological as long as the $5d$ band bottom is below the $E_F$. The nontrivial topology is suppressed when the $5d$ band bottom moves above $E_F$ or when other bands dip below $E_F$ (Fig. S1(b)). In the case of SmO, it is the $6s$ band bottom at the $\Gamma$ point that shifts to lower values as a function of increasing lattice parameter. The shift of the $5d$ and $6s$ band bottoms (at $\Gamma$ and $\Gamma$ respectively) with respect to the $E_F$ as a function of lattice parameter are collected in Fig. S1(d) for LDA+$U$ approach. Treating the strong $4f$ correlations on a mean-field level, the topological semi-metal state is quite robust and remains so, up to 5.38 Å, a 9% increase in lattice parameter. On an experimental level, this result is very promising, since it provides a large range of substrate lattice constants for which SmO thin films reveal topological non-triviality. During the evaluation of the topological indices using LDA+$U$, we became aware of another interesting feature, a nontrivial topology due to $s-f$ band inversion at $\Gamma$ for 11% and larger lattice constants ($\geq 5.5$ Å). In LDA+$U$, the $5d$ band bottom shifts above the $E_F$ and becomes trivial before the $6s$ band bottom dips below $E_F$. Further increase in the lattice parameter then results in a $s-f$ band inversion at $\Gamma$ which is again topological (Fig. S1(c)). Although, the probability for a successful growth of SmO on a substrate with a 11% and larger lattice constant may be low, it nonetheless offers another exciting route towards realizing a topologically nontrivial state, only this time with a $s-f$ band inversion. We verified the validity of our results for a range of $U$ values ($5$ to $7$ eV) and as well as for the MBJLDA exchange potential.

In the calculations so far, we have allowed Sm to be in the intermediate valence state and have treated this on a DFT level. The mixed valence situation can be viewed as a $4f$ to $5d$ promotion of a certain fractional amount of electrons due to the hybridization of the localized $4f$ states with the band like $5d$ states. Based on the handful of experimental reports available in literature, the valence assigned to Sm in SmO is $2.92$, i.e. a promotion of $0.92$ electrons. To actually calculate the amount of electron promotion is a difficult task, since DFT in the Kohn-Sham scheme is based on a single Slater determinant approximation and prevents a correct description of a many body intermediate valence state. In other words, the position of the $4f$ bands relative to the $5d/6s$ as calculated in DFT is not a reliable quantity (see Supplemental Material). Yet, we can address the effect of mixed valency on the topological properties of SmO in a rigorous quantitative way, by calculating the amount of electrons (integration of the density of states) that can be accommodated in the dispersive $5d$ or $6s$ band, thereby mimicking various amounts of $4f \rightarrow 5d$ or $4f \rightarrow 6s$ promotion. For the lattice constants which has the band inversion at the $X$ point, we calculate the amount of electrons that can be contained in the $5d$ band before beginning to populate the $6s$ band. Note that the nontrivial topology is maintained as long as only the $5d$ is occupied and the $6s$ remains unoccupied. Fig. S2 displays this electron amount as a function of the lattice constant. Analogously, on the other side of the phase diagram, wherein non-triviality is manifested due to a $s-f$ inversion, we calculate the amount of electrons that can be contained in the $6s$ band before beginning to populate the $5d$ band. Equilibrium lattice constants for the $3+$ and $2+$ limiting cases are estimated using spin-polarized L(S)DA+$U$ schemes (see Supplemental Material). We observe that the theoretical estimated valence and lattice constants always yield a topological nontrivial ground state. Though the estimate from an empirical extrapolation for the SmO valence places the bulk in the trivial region of the phase diagram, nontrivial topology is quickly reinstated for lattice parameters with a small tensile strain ($\geq 1\%$). Spectroscopy experiments are necessary to confirm the samarium valency, and positions of the samarium $s$ and $d$ bands with respect to the $f$ states in bulk SmO and SmO thin films.

By virtue of the large surface to volume ratio, thin films of systems with nontrivial topology with dominant surface states are highly sought-after. Owing to the simple $fcc$ symmetry and the flexibility in observing the topological ground state in SmO for a wide range of lattice parameters, we anticipate plenty of options for suitable substrates. One particular substrate that invokes special interest is the $fcc$ ferromagnetic semiconductor EuO$^{[35, 37]}$, which has a lattice constant of $5.14$ Å, only a $4\%$ lattice mismatch with that of bulk SmO. EuO by itself is an attractive material with many functionalities, including metal-insulator transition, magnetic phase transition, colossal magneto-resistance, etc$^{[39, 42]}$. These functionalities have produced an abundance of research on growing high quality thin films of EuO. So, a quick progress in the engineering of SmO/EuO thin film interfaces can be expected. Additionally, this would open up the possi-
bility for another experimental realization of the QAHE, this time in a strongly correlated electron system with topologically nontrivial flat bands. Despite the lack of a bulk band gap in Cr-doped (Bi,Sb)$_2$Te$_3$ thin films, QAHE was observed at 30 mK [29][30]. In a similar manner, the presence of a semi-metallic bulk in SmO will give rise to bulk carriers, but given the tiny electron and hole pockets around $X$ and $\Gamma$ respectively, and by making use of thin films, the amount of bulk charge carriers can be kept quite small and should not completely overshadow the QAHE.

In summary, we have investigated in detail the electronic structure of SmO and established the nontrivial topology of the band structure such that SmO can be classified as a 3D strongly topological semi-metal. The topological state in SmO is very robust and prevails for a wide range of lattice parameters, which leads us to propose SmO as an ideal candidate for further investigations of topologically nontrivial correlated flat bands in bulk and thin-film form.

The authors thank Prof. J. Allen, S. Wirth and P. Thalmeier for discussions. D.K. acknowledges funding by the DFG within FOR 1346.
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TABLE S1. Calculated topological indices of selected bands in SmO within LDA+SO+U approximation (a = 4.941 Å). \( \varepsilon_n \) refers to the band energy at \( \Gamma \); \( n \) refers to the band indices, shown in Fig. S1, \( \Gamma, X \) and \( L \) are the TRIM points. \( \nu_0; (\nu_1 \nu_2 \nu_3) \) are the four topological indices. The parity of the Kramers doublet at the various TRIM points are denoted by \( O \) (odd) and \( E \) (even). Additionally, the dominating orbital character of the bands at the TRIM points are listed.

| \( \varepsilon_n (\Gamma) \) eV | (n) | \( \nu_0; (\nu_1 \nu_2 \nu_3) \) |
|----------------|-----|----------------------|
| -3.650         | O   (2p) | O (2p) E (2p) 0; (000) |
| -0.493 (1.2)   | O (4f/2) | E (5d) O (4f/2) 1; (000) |
| +0.030 (3.4)   | O (4f/2) | O (4f/2) O (4f/2) 1; (000) |
| +0.030 (5.6)   | O (4f/2) | O (4f/2) O (4f/2) 1; (000) |
| +1.457 (7.8)   | E (6s) | O (5d) E (5d) 0; (000) |

O2p and Sm 4f must be trivial due to the fact that SO coupling is too small to create band inversions bridging this gap. Hence the highest O2p band forms the trivial base line for discussing the topological properties of the remaining bands. In the table, we list the energy of the bands at the \( \Gamma \) point, the band indices (as indicated in Fig. S1), the parity of the Kramers doublets (O: odd, E: even) together with the dominating orbital character at the three symmetry distinguished classes of TRIM points as well as the resulting topological indices obtained from all bands including the ones mentioned in the first column. Note that, bands numbered 3...6 are trivial bands, and do not change the topology.

**BAND STRUCTURE USING MODIFIED BECKE JOHNSON APPROACH**

To make sure that the topological characteristics are not dependent on the choice of exchange and correlation functional, we calculated the band structure of SmO using the recently proposed modified Becke-Johnson functional (MBJLDA) as implemented in WIEN2K. Fig. S2 shows the band structure obtained using MBJLDA+SO. The \( f - d \) band inversion at \( X \) point is consistent to our LDA+SO+U calculations, though the hybridization gap and the warped band-gap in MBJLDA+SO are smaller (but, of the same order).

**SURFACE STATE CALCULATIONS**

To assess the nature of the surface band structure we calculated the Bloch spectral density \( (A_{Bl}(k)) \) of the 12 topmost surface layers of a semi infinite solid with \([001]\) surface. For this purpose the DFT band structure was fitted with atom centered maximally projected Wannier functions of Sm 5d, 6s and 4f and O 2p character. A soft energy cutoff was employed to achieve band disen-tanglement at \(+6eV\). The resulting fit models all relevant bands around the Fermi level within a few meV accuracy. Afterwards, the obtained bulk hopping parameters were mapped onto a semi infinite solid and a Greens function technique was used to calculate \( A_{Bl}(k) \) for the surface layers. The resulting surface bands clearly show two weakly interacting Dirac cones around the surface projected \( M \)-point. In order to be of topology induced nature, there must be an odd number of Dirac cones. The surface Brillouin zone for a \([001]\) surface ends up with two bulk \( X \)-points being projected onto the surface \( \bar{M} \)-point and one bulk \( X \)-point being projected onto the surface \( \Gamma \)-point (see Fig S3). Hence, one expects two Dirac cones at \( \bar{M} \) and one at \( \bar{\Gamma} \). The back-folding of the bulk band structure due to projection onto \([001]\) leads to the appearance of electron pocket states from the bulk \( X \)-point at \( \bar{\Gamma} \), which overlap with the bulk projected states of the bulk \( \Gamma \) hole pocket at \( \bar{\Gamma} \). Hence, the third Dirac cone will be immersed into projected bulk states and can only form a surface resonance. In order to prove the existence of this third Dirac cone, we performed an additional calculation.
where the 4f electron pocket around the bulk Γ-point are lowered in energy by application of a k-dependent potential. This removes bulk projected states from the low energy region at the surface Γ point and reveals the third Dirac cone. Finally, to rule out drastic effects of the relaxation of the surface electronic structure we performed a full DFT slab calculation for 21 SmO layers. The resulting slab band structure shows the same two weakly interacting Dirac cones around the ¯M-point as the semi-infinite calculation. Slight shifts of bands occur, but the overall structure equals that of the mapped model calculation.

NOTES ON THE MANY BODY STATE OF SMO

The Sm 4f states are known to be strongly correlated. Nonetheless one can obtain useful information about this system using DFT within the local density approximation. The large onsite Coulomb repulsion between the f electrons restricts the local valence occupations to fluctuate between the f5 and f6 configurations. The lattice constants for the limiting cases of a pure 2+ (f6) and pure 3+ (f5) SmO compound are determined using spin polarized L(S)DA+SO+U. The local atomic f6 (f5) configuration has a lowest Hunds-rule multiplet state which belongs to the 7F0 term with J = 0, belonging to the A1g representation) we use non-spin polarized DFT calculations. In this case the f6 configuration is represented by a state with six electrons in the j = 5/2 bands. Although this state has an incorrect expectation value of L (⟨L2⟩ = 24/7) and S (⟨S2⟩ = 24/7) compared to the many body state (and thus a to high local Coulomb energy), it does have the right symmetry (A1g) and total momentum (J = 0). The f5 configuration is represented by single hole excitations starting from the f6 configuration. A peculiarity of starting from a many body f6 state with A1g symmetry is that the many body one electron excitations have the same symmetry as the independent electron bands found in LDA, as shown by Martin and Allen[2].

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FIG. S3. (Color online) Bulk and surface Brillouin zone of a fcc lattice.