Published tunneling results of Binnig et al interpreted as related to surface superconductivity in SrTiO$_3$

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Abstract

In 1980 Binnig et al. reported tunneling measurements on Nb-doped SrTiO$_3$, and interpreted their results as indicating two-band superconductivity in the bulk of SrTiO$_3$. However: (1) Effective masses determined from tunneling results in the normal state by Sroubek in 1969 and 1970 are much smaller than those determined by most other methods. The much smaller masses were attributed to properties of a surface layer by the present author in 1971; (2) The only other reports of two-band superconductivity in bulk SrTiO$_3$ can be used to infer much smaller values for the band separations than found by Binnig et al.. In this paper we give an alternative explanation of the results of Binnig et al. in terms of superconductivity in a surface layer. We obtain fair fits to the band gaps versus Fermi energy for the two bands in the three samples where two surface subbands are occupied and to the temperature dependence of the gaps in one crystal, using a model with three adjustable interaction parameters, an adjustable energy for the phonons which dominate the pairing, and an adjustable ratio of the mean-field $T_c$ to the actual $T_c$. We show results for a combined fit to the low-temperature band gaps and to the $T$-dependence in one crystal. The phonon energy which gives the best fit is 21 meV. This is probably an appropriate average over the three longitudinal polar modes and acoustic modes in the material. A large value of about two is found for the ratio $T_{cmf}/T_c$, and we conjecture that this arises because a band with a small Fermi energy, not seen in the tunneling results, plays a part in increasing $T_{cmf}/T_c$.
1. Introduction

In 1980 Binnig et al. [1] claimed that tunneling results into (100) surfaces of SrTiO$_3$ reported by them gave evidence for two-band superconductivity in bulk Nb-doped SrTiO$_3$. However I have had doubts for some time as to whether what they were observing was bulk superconductivity. Two reasons for thinking this are: (1) Effective masses determined from tunneling results of Sroubek in the normal state [2, 3] are much smaller than those determined by most other methods. The much smaller masses were attributed to properties of a surface layer by the present author in 1971 [4]. (2) There are two other reports of two-band superconductivity in bulk SrTiO$_3$ [5, 6], but one of these [5] can be shown to imply much smaller values for the band separation than the 32 meV reported in [1]. This may be seen from the statement in [5] that the Fermi energy when the second band starts to become occupied is 1.8 meV. A third band, which may or may not play a part in the superconductivity, starts to become occupied at a carrier concentration about nineteen times higher, corresponding to a Fermi energy of the order of $19\times 1.8$ meV, i.e. $\sim 13$ meV, still much smaller than the 32 meV mentioned in [1] for the occupation of the second band.

In this paper we give an alternative explanation for the results of [1] where the two gaps observed are associated with the lowest two subbands of a surface layer. At most two subbands of the band which we consider for most of this paper are occupied for the crystals studied. We make use of a two-subband model, and use the type of equations found by Suhl et al. [7] to analyse the data, but we suppose that the mean-field transition temperature $T_{cmf}$ may be larger than the observed $T_c$ where the energy gaps vanish. Although the electron-phonon coupling in the normal state is largest for the highest-energy longitudinal optical phonon [8, 9, 10], this may not be the case for the superconducting state because phonons having a higher energy than the Fermi energy may have reduced effects on the superconducting pairing [11, 12]. Hence we use an adjustable energy for the phonons which dominate the pairing. A large value of about two is found for the ratio $T_{cmf}/T_c$. We conjecture that this arises because a band with a small Fermi energy, not seen in the tunneling results, plays a part in increasing $T_{cmf}/T_c$.

Fernandes et al. [13] discuss two-band superconductivity in bulk SrTiO$_3$, but use a two-dimensional model for much of their analysis. They also note that Binnig et al.’s tunneling results are sensitive to surface states, and point out that three-dimensional theory does not present a kink in the energy gap which is observed when the second band starts to become occupied. They further find that interband pairing is small compared with intraband pairing, and so their results have some similarity to ours. However, they differ in several ways. First they do not claim as we do, that the results of [1] are entirely associated with surface effects. Secondly, they do not give any detailed fits to results as we do. Thirdly they give a different reason from us for the dominance of intraband over interband pairing. Fourthly they assume an upper cut-off energy for the interaction of 160 meV, higher than the energy of the highest energy phonon, whereas our fitting with an adjustable phonon energy gives a much smaller cut off of about 21 meV.

Another theoretical paper somewhat related to ours is a discussion of multigap structure in electric-field-induced surface superconductivity by Mizohata et al. [14]. Values of surface carrier density and electric field are chosen to be relevant to experiments on SrTiO$_3$ by Ueno et al. [15].

Our use of a model where interaction with longitudinal optical phonons dominates the pairing has some support from a paper by Klimin et al. [16], who, using a dielectric approach to pairing, find that such phonons dominate the pairing. However, from their paper it is not clear to me the relative importance of the three types of longitudinal polar optical phonons in their theory. Transport measurements [17, 18], ARPES in SrTiO$_3$ [19] and on LaAlO$_3$-SrTiO$_3$ bilayers [20], and tunneling [11], all indicate that most interactions in the normal state, at
least at low carrier concentrations, are with the two highest frequency longitudinal optical phonons, as expected from theory. On the other hand, in the superconducting state Swartz et al. \cite{11} find much weaker coupling, which they attribute to the fact that the energy of the high energy phonons is larger than the Fermi energy in superconducting crystals. They conjecture that low energy phonon modes may be most important for the superconductivity, but to me it appears possible that the high energy phonons may still be important, but with interactions giving rise to pairing relatively small because of the low $E_F$. Baratoff and Binnig \cite{21} argue that the highest frequency longitudinal optical phonon is important for superconductivity because of the sharp fall in $T_c$ when the plasmon frequency becomes greater than the frequency of this mode.

In section 2 we analyse the energy gaps at low temperatures as a function of Fermi energies, and in section 3 we analyse the temperature dependence of the energy gaps reported in \cite{1} in one crystal. We have three coupling parameters and the phonon energy relevant for the superconductivity for our low-temperature fitting, and one extra parameter, $T_{cmf}/T_c$, for the fitting of the temperature dependence of the gaps in one crystal. In order not to waste data, in section 3 we find a combined fit for the energy gaps at low temperatures in three crystals showing two gaps, and for the temperature dependence of the energy gaps in one crystal, with appropriate weightings for the two types of data.

The phonon energy which gives our best fit is about 21 meV, but another minimum of our sum of squares between theory and experiment at only a slightly larger value occurs at about at about 28 meV, with a not very high saddle point in between at about 25 meV. Either the 21 meV or 28 meV minimum is probably some average of the three polar longitudinal phonon energies and perhaps acoustic modes, interaction with which was introduced empirically in \cite{16}. Although the strongest coupling to the phonons in the normal state is to the highest frequency longitudinal phonon of energy 99 meV \cite{8}, in the superconducting state there is a reduction of its contribution to the pairing because its energy is greater than the Fermi energy for the crystals being considered (see e.g. \cite{11,12,22}), and also, for pairing, there is a further reduction relative to that with the lower energy phonons because the phonon energy appears in an energy denominator for the phonon-induced pairing. Although the coupling to the lowest energy longitudinal polar phonon is negligible at low carrier concentrations even after a numerical error giving the coupling a factor of the order of ten too small in \cite{8} is corrected (cf \cite{9}), when finite carrier concentrations are considered \cite{23} this coupling may be non-negligible.

In bulk SrTiO$_3$ there is evidence from the isotope effect \cite{24} that the ferroelectric soft mode plays an important part in the pairing, as predicted in \cite{25,26}, but in surface layers, because of the lower static dielectric constant due to large electric fields, this mode is less likely to be of importance.

The ratio of intersubband to intrasubband interactions depends on the range of interactions giving rise to pairing in comparison with the width of the surface layer. To calculate this ratio for a realistic form of interactions would be quite difficult. However, in section 4, we consider a one-dimensional square-well model for the interaction potential, and show how, in this model, the ratio of intersubband to intrasubband interactions depends on the ratio of the width $w$ of the square well to the width $d$ of the surface layer. We also find the value of $w/d \approx 0.47$ which gives a value for this ratio in agreement with the value of about 0.14 determined from our data fitting. This value of 0.14 is smaller than the value of 2/3 found for local interactions \cite{27}. Paskin and Singh’s result that the intrasubband interaction is increased by a factor of 3/2 for local interactions for very thin films was argued by us to be increased by a factor 9/4 in very thin whiskers, and was used speculatively in combination with enhanced densities of states at low carrier concentrations in such films or whiskers to estimate high transition temperatures in (111) films and
[111] whiskers of 0.7, 1.4 and 2.1 nm thicknesses in [28].

2. Energy gaps as a function of Fermi energy

Tunneling into eight Nb-doped crystals was reported in [1]. The Fermi energies for these crystals varied from about 18 to 60 meV. From results on three of the crystals it was found that the second band commenced in energy at about 32 meV above the bottom of the lowest band. In our interpretation we regard the two bands as surface subbands. If we were to assume wave functions which vanish at the edges of a surface layer of given thickness, then the third subband would commence at an energy of about \((8/3) \times 32\) meV \(\approx 85\) meV. However, by studying the type of model for the surface layer used in [4] we see that the barrier at the edge of the surface layer does not extend above the Fermi energy, and so, for a maximum Fermi energy of about 60 meV in the crystals studied in [1], there will be no third surface subband. We also ignore any possible pairing interaction between bulk and surface subbands. Although the fact that the superconducting \(T_c\) drops sharply when the plasmon energy exceeds the energy of 99 meV of the highest energy longitudinal optical phonon has been argued to imply that that phonon dominates the pairing [21], because of the generally held view that pairing mediated by phonons of energy larger than or of the order of the Fermi energy is smaller than it would be for large Fermi energies (see e.g. [11, 12, 22]), we allow the phonon energy involved in the pairing to be adjustable, and assume that any value we find is an appropriate average over several phonon energies. The limits to the integrals involved in the BCS-type of theory have to be altered if the Fermi energy measured from the bottom of any band is smaller than the phonon energy.

The densities of states of the subbands would all be equal within the subbands if the surface layer were of a constant thickness. However, since the layer thickness will increase as the energy increases, we allow for differing densities of states in the two subbands. An important parameter appearing in our theory is the ratio \(r\) of the pairing potential between subbands to that within the lowest subband. Although, for local pairing interactions this ratio is \(2/3\) [27], we shall allow this ratio to be adjustable to be determined from our data fitting. In section 4 we shall show that, for a square-well type of pairing potential \(r\) becomes smaller when the size of the square well becomes comparable with the width of the surface layer, and that an empirical value of about 0.14 determined by our data fitting to two types of data in section 3 will occur for a square-well type of pairing potential if the ratio \(w/d\) of the size \(w\) of the square well to the layer thickness \(d\) is 0.47.

For a two-band superconductor with gap parameters \(\Delta_i, i = 1, 2\), with constant densities of states in each subband between the bottoms of the bands and \(E_{Fi} + \hbar\omega\), at \(T = 0\) there are two quantities \(F_i(\Delta_i) (i=1,2)\) analogous to \(F(A)\) on page 553 of [7], given by

\[
F_i(\Delta_i) = \frac{1}{2} \int_{-p_i}^{\hbar\omega} \left[1/(c^2 + \Delta_i^2)\right]^{1/2} \mathrm{d}p_i = (1/2)[\arcsinh(p_i/\Delta_i) + \arcsinh(\hbar\omega/\Delta_i)] \tag{1}
\]

where \(p_i = \min(|E_F|, \hbar\omega)\), and \(p_2 = \min(|E_F - E_c|, \hbar\omega)\), where \(E_F\) is the Fermi energy measured from the bottom of the lower band, and \(E_c\) is the energy of the bottom of the upper band above that of the lower band. We have made use of the fact that the \(\tanh\) in the expression for \(F(A)\) in [7] is replaced by unity at \(T = 0\).

Provided \(p_i\) and \(\hbar\omega\) are significantly greater than \(|\Delta_i|\), then the \(\arcsinh\)'s in equations (1) can be approximated by the logarithms of \(2p_i/\Delta_i\).

We have four parameters to consider in this section. These may be taken to be \(\lambda_1\) and \(\lambda_2\), the densities-of-states interaction products within the two subbands, the ratio \(r\) of inter-subband interactions to intrasubband interactions, and the phonon energy \(\hbar\omega\).

When two bands are occupied, equation (4) of [7] gives, at \(T = 0\), using our notation introduced above, the following two simultane-
ous equations to solve:

\[ \Delta_1 = \Delta_1 \lambda_1 F_1(\Delta_1) + r \Delta_2 \lambda_2 F_2(\Delta_2), \]  
\[ \Delta_2 = \Delta_2 \lambda_2 F_2(\Delta_2) + r \Delta_1 \lambda_1 F_1(\Delta_1). \]  

(2) (3)

If only one band is occupied then equation (3) does not occur, and the second term on the right-hand side of equation (2) is omitted.

For the crystals with two subbands partially occupied, we treat the three coupling parameters as adjustable. We solve equations (2) and (3) numerically for crystals 6, 7 and 8, and then, we find the values of \( \lambda_1, \lambda_2, r \) and \( h\omega \) which give the best root-mean-square difference between calculated and six observed energy gaps for crystals 6, 7 and 8, after making small estimated corrections due to the non-zero value of temperature at which the energy gaps were found. However, with four parameters and only six data points, uncertainties in parameters will be too large for their values to have much meaning, and so we do not present results for parameters until a combined fit to two types of data for a total of 29 points has been performed in the next section.

3. Temperature dependence of gaps in one crystal

In the work mentioned in the last section, we were able to find a fair fit to the energy gaps as a function of Fermi energy determined by Binnig at al. [1] using a model where the superconductivity is associated with two surface subbands, although we postponed showing results there because of insufficient data to determine parameters well. In this section we examine the temperature dependence of the energy gap in one crystal using the same type of model. The transition temperature for the crystal considered may be estimated to satisfy \( k_B T_c \approx 54 \mu\text{eV} \) from figure 2 of [1].

The equations we use are:

\[ \Delta_2 \lambda_1 s_2 = \Delta_1 (1 - \lambda_1 s_1) \]  
\[ \Delta_1 \lambda_2 s_1 = \Delta_2 (1 - \lambda_2 s_2). \]  

Here \( \Delta_1 \) and \( \Delta_2 \) are the two energy-gap parameters, \( \lambda_i, i = 1, 2 \), are the intrasubband values of the density-of-states interaction products, \( \lambda_{21} = \lambda_1 r, \lambda_{12} = \lambda_2 r \), where \( r \) is the ratio of intersubband to intrasubband interactions. Also

\[ s_1 = 0.5s_{11} + 0.5s_{12}, \]  
\[ s_2 = 0.5s_{21} + 0.5s_{22}, \]  

(6) (7)

where

\[ s_{11} = \int_0^{q_1} f_1(x)dx, \]  
\[ s_{12} = \int_0^{h\omega} f_1(x)dx, \]  
\[ s_{21} = \int_0^{q_2} f_2(x)dx, \]  
\[ s_{22} = \int_0^{h\omega} f_2(x)dx. \]  

(8) (9) (10) (11)

Here \( q_1 = \min(59.5\text{meV}, h\omega) \), \( q_2 = \min(59.5\text{meV} - E_c, h\omega) \) and

\[ f_1 = \tanh[(x^2 + \Delta_1^2)^{\frac{1}{2}}/2t]/(x^2 + \Delta_1^2)^{\frac{1}{2}}, \]

\[ f_2 = \tanh[(x^2 + \Delta_2^2)^{\frac{1}{2}}/2t]/(x^2 + \Delta_2^2)^{\frac{1}{2}}, \]

with \( t = k_B T_{\text{cmf}}(T/T_c) \), where \( T_{\text{cmf}} \) is the mean-field transition temperature, \( T_c \) is the temperature where the observed gap vanishes, and \( E_c \) is the energy of the bottom of the second subband above the bottom of the first.

Assuming \( E_c = 32 \text{ meV} \) as found in [1], for given \( \lambda_1, \lambda_2, r, h\omega \) and \( T_{\text{cmf}} \), we have two simultaneous integral equations to solve to obtain the energy gaps for a given temperature. We treat this problem as a subroutine, and then try minimising the sum of squares of differences between theoretical and experimental reduced energy gaps as a function of \( T \), where experimental points for both energy gaps are shown in the inset of figure 2 of [1]. If we take \( k_B T_c \) of this crystal to have the value which can be inferred from figure 1 of [1], i.e. \( k_B T_c \approx 0.66 \Delta_8(0) \), where \( \Delta_8(0) \) is the larger energy gap at \( T = 0 \) for this crystal inferred from the observed gap at \( T \approx 0.2T_c \), we find that we cannot obtain a good fit for all temperatures. So we adopt a different strategy, supposing that the \( T_c \) inferred from figure 2 of [1] is
smaller than the mean-field transition temperature $T_{cmf}$ because in general mean-field theory overestimates $T_c$. So as not to waste data, we perform a combined fit to the results for the energy gaps at low temperatures for the crystals with two energy gaps, as discussed in section 2, and to the temperature dependence of the gaps in one crystal, with weightings for the two types of points inversely proportional to the squares of the average values for the two types of points in the units used.

The best fit to a total of 29 points, six for low-$T$ energy gaps in three crystals, and 23 for the $T$-dependence of the energy gap in one crystal, gives a root-mean-square error of 5.5% of the mean values of the experimental points. The values of parameters for the fit are $\lambda_1 = 0.1478$, $\lambda_2 = 0.1181$, $r = 0.143$, $\hbar\omega = 21$ meV and $T_{cmf}/T_c = 2.055$. The theoretical and experimental data for the above parameters are shown in figures 1 and 2. We note that $\lambda_2$ from our fitting is slightly smaller than $\lambda_1$, as expected, since the effective width of the surface layer will be larger for a larger mean energy of a subband (see figure 1 of [4]). The value of $T_{cmf}/T_c$ is required to be greater than unity but with $T_{cmf}$ being of the same order of magnitude as $T_c$. Values found from our data fitting are consistent with these requirements. Further section 4 gives some restrictions on the value of the ratio of inter to intrasubband interactions if we have some information about the range of the pairing potential to the width of the surface layer.

By taking fixed values for the phonon energy and varying the other parameters we find that there is quite a large uncertainty in the phonon energy on the higher energy side, and we estimate that $\bar{\hbar}\omega = 21^{+8}_{-1}$ meV, with a small region around $\bar{\hbar}\omega \approx 25$ meV where the sum of squares of differences between theory and experiment is larger than the standard error (68% confidence level).

For the five crystals with only one subband occupied, the interaction parameter $\lambda_1$ could have a different value from what it has in the crystals with two subbands occupied because of different screening, and so fitting these points with a different value of $\lambda_1$ will not help in determination of parameters. However, if we assume that $\hbar\omega = 21$ meV, we find that $\lambda_1$ varies from 0.142 and 0.148 for the five crystals, with an average of 0.144. These values are not
much different from the value of $\lambda_1 = 0.1478$
found by fitting 29 points in this section.

4. Estimate of the ratio of intrasubband
to intersubband pairing strengths for a
square-well potential

In this section, assuming a square-well pairing
potential, we calculate the ratio $r$ of intersubband
to intrasubband pairing strengths as a
function of the ratio of the size of the square
well in the direction normal to the surface to
the surface thickness, and find the size of the
square well which gives agreement with our em-
pirical value of $r$ determined in the last section.

We suppose that we have a surface layer
of thickness $d$ with vanishing wave functions at
$z = 0$ and $z = d$, where $z$ is the coordinate
perpendicular to the surface. We also suppose
that, after integrating over coordinates parallel
to the surface, we have a potential well $V(z-z')$
of constant magnitude between electrons at $r$
and $r'$ for $|z-z'| < z_e$ and zero otherwise. To
find the pairing strength between subbands $n$
and $n'$ we numerically integrate

$$
(2/d^2) \int_0^d \int_0^d \sin(\pi nz/d)\sin(\pi nz'/d) \times
$$

$$
V(z-z')\sin(\pi n'z/d)\sin(\pi n'z')dzdz',
$$

(14)

and find the ratio of the pairing strength be-
tween subbands 1 and 2 to the intrasubband
pairing strength within subband 1 as a function
of $w/d$, where $w$ is the range of the pair-
ing interaction. For a square-well type of pair-
ing potential some results are shown in figure
3. We see that when $w$ becomes comparable
with $d$ the ratio $r$ can become small. When
$w/d > 0.46$, the intrasubband interaction is
decreased below what it would be for the bulk if
we ignore other changes such as increased den-
sities of states for thin layers.

We see from the figure that $r = V_{21}/V_{11} = 0.14$
when the ratio $w/d \approx 0.47$.

We suppose that the bare effective mass
in the $z$-direction is equal to a heavy bare mass
of perhaps about $6m_e$ (this mass will depend
on the carrier concentration, as can be inferred
from figure 3 of [18]), and in [29] we estimated
that it was about $7.3m_e$ for a carrier concentra-
tion of $9.4 \times 10^{20}$ cm$^{-3}$). With a mass of
$6m_e$ in the $z$-direction, the energy of the first
excited state of about 32 meV from [11] implies
that the surface state thickness $d$ is given by

$$
3(\pi/d)^2h^2/(2 \times 6m_e) = 32 \text{ meV}.
$$

(15)

This gives $d \approx 2.4$ nm.

Probably a characteristic length for the interac-
tion between two large polarons will be the
radius of a polaron with bare mass equal to
the reduced bare mass of two polarons, i.e. half
the bare mass of a polaron. If we assume a bare
mass of in the $z$-direction of $6m_e$ as discussed
above, then the radius for a particle of half this
mass will be $(h/6m_e \omega)^{1/2}$. If $h\omega = 0.021$ eV,
then this radius is 0.78 nm, giving a diameter
of 1.56 nm. Thus the ratio of this to the width
of the surface region is 1.56/3.4 = 0.46. This is
very close to the of 0.47 found empirically by us
for a square-well potential and a surface barrier
also of square-well shape. In view of the fact
that neither a square well for the inter-particle
potential nor the surface potential are realis-
tic, the good agreement of the estimated ratio
of $w/d$ with the empirically determined ratio
must probably be regarded as a lucky coinci-
dence.

For the interaction between two large po-
larons at $r_1$ and $r_2$, judging by the potential
due to a single polaron [30], the interaction

\begin{figure}
\centering
\includegraphics[width=\textwidth]{figure3.png}
\caption{Ratio of interband to intraband pairing strengths $V_{21}$ and $V_1$ and ratio of $V_1$ to the
pairing strength $V$ in the bulk as a function of $w/d$.}
\end{figure}
starts flat at zero separation, and then changes to an interaction of the form proportional to \(1/(r_1 - r_2)\exp\left[-(r_1 - r_2)/r_{pr}\right]\), where \(r_{pr}\) is the polaron radius for a particle with bare mass of half the single-particle mass, as discussed above. Thus, apart from the tail to the potential, the interaction is not far from a square-well form. There may also be a deepening of the interaction potential with a much shorter range due to deformation-potential interaction with acoustic phonons.

For the surface potential we expect a sharp rise at the boundary between the surface region and the bulk, where the effective mass changes, but a more gradual change of potential near the surface whose detail will depend on the electric-field dependence of the dielectric constant, with a form qualitatively similar to that shown in figure 1 of [4].

5. Remarks on the large ratio of \(T_{cmf}\) to \(T_c\) found by our data fitting

Differences between mean-field transition temperatures and observed transition temperatures are usually thought to arise because phase coherence occurs at a lower temperature than the pairing temperature. In the present problem we are taking the observed transition temperature as the temperature where the tunneling gap vanishes. When there is pairing but no phase coherence of the pairs, there will still be energy gaps, but these can be expected to spread over a range of values due to the thermal spread of the center-of-mass energy, and so no sharp peaks in the tunneling spectrum can be expected except at low temperatures.

The difference between the pairing temperature and the temperature at which there is condensation of the pairs can arise because of being close to or in the Bose-Einstein condensation (BEC) region [31], and can be enhanced by low dimensionality. For the crystal for which the temperature dependence of the gap has been reported, the gap is several orders of magnitude smaller than the Fermi energy, and so we are not close to the BEC region. However, with only two subbands partially occupied, we are close to two-dimensionality. We have not made an exhaustive literature search for differences between the mean-field \(T_c\) and the actual \(T_c\), but note that L. Miu [32] analyses results on resistivity of \(\text{YBa}_2\text{O}_4\), and finds that the ratio of the mean-field \(T_c\) to the Berezinsky-Kosterlitz-Thouless temperature is \((73/47)=1.55\). This is not quite as large as our estimated value of just over two. As a first guess we might conjecture that the lower carrier concentration in \(\text{SrTiO}_3\) could contribute to the larger value. However, for a one-band model, Kagan finds theoretically [33] that

\[
|T_c^{\text{BCS}} - T_c^{\text{BKT}}| \sim T_c^{\text{BCS}}/E_F,
\]

which is small compared to one in our system. Also, Chubukov et al. [34] find that, for a two-band model with interband pairing dominating, if \(E_F >> E_0\), where \(2E_0\) is a pair binding energy, then one also gets \(T_c \approx T_{\text{ins}}\), where \(T_{\text{ins}}\) is the onset temperature of the pairing, presumably to be identified with \(T_{cmf}\). Thus, it seems that we need to bring in further postulates to explain the large ratio of \(T_{cmf}/T_c\). One possibility is that, beside the two subbands discussed in the main part of the paper, there is a bulk or surface band which has a larger gap, and a small carrier concentration and Fermi energy, for which the transition to the BEC regions is reached or close to being reached. Bands with small occupations and Fermi energies have been discussed by several authors in connection with underdoped cuprates [35], some of the Fe-based superconductors [36], and in connection with shape resonances associated with quantum confinement effects [37, 38].

6. Conclusions

It has been argued that Binnig et al.’s tunneling data [1] in Nb-doped \(\text{SrTiO}_3\) are associated with superconductivity in a surface layer. Fair fits to the energy gaps at low temperature in three crystals and to the temperature dependence of the energy gap in one crystal have been obtained using a model with three adjustable interaction parameters, an adjustable
phonon energy for the phonons which dominate the pairing, and one other partially adjustable parameter, viz. the ratio of the mean-field transition temperature to the observed $T_c$. The phonon energy found by data fitting is approximately 21 meV, which probably represents a weighted energy of the phonons which contribute to the pairing. A large value found for $T_{cmf}/T_c$ may imply that a band with small occupation and Fermi energy plays a part in increasing $T_{cmf}$. The ratio $r$ of intersubband to intrasubband interactions from our data fitting is about 0.14, smaller than the value of 2/3 for local interactions. We show that, for a square-well pairing potential, a value $r = 0.14$ is obtained when the ratio of the width of the square well to the width of the surface layer is 0.47.

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