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Effect of vaccination to control COVID-19 with fractal fractional operator

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Abstract Currently, Atangana proposed new fractal-fractional operators that had extensively used to observe the unpredictable elements of an issue. COVID-19 is a pervasive infection today and is hard to fix. In this structure, the novel operators have been used to observing the effect of vaccination in the COVID-19 model with different values of $g_1, g_2$ which are used to show the effect of vaccination. The system will be converted into disease-free according to reproductive number. We used the Atangana-Baleanu fractal-fractional operator to investigate the COVID-19 model qualitatively and quantitatively. By using fixed point theorems we proved the existence and uniqueness of the model with the Atangana-Baleaune fractal-fractional operator. A non-linear assessment helped to find out the stability of the Ulam-Hyers. We simulate the mathematical outcomes, to understand the relationship of operators in several senses, for numerous arrangements of fractional and fractal orders. © 2021 THE AUTHORS. Published by Elsevier BV on behalf of Faculty of Engineering, Alexandria University. This is an open access article under the CC BY-NC-ND license (http://creativecommons.org/licenses/by-nc-nd/4.0/).

1. Introduction

COVID-19 is the disease that took the world by storm. It originated from a wet market in Wuhan, China, where it was transmitted from a bat to a human. The first case was reported in December of 2019 and the numbers grew from there. On March 11, 2020, it was formally proclaimed a pandemic. There are many Symptoms of COVID-19 including fever, dry cough, and tiredness. Among them, uncommon are sore throat, aches, pains, sore throat, diarrhoea, conjunctivitis, headache, and a rash on the skin or discolouration of fingers or toes. The serious signs are difficulty in breathing or shortness of breath, and chest pain or pressure. It spreads from person to person through close contact with an infected person by means of breathing the same air, coughing, sneezing, touching and touching the same surfaces. The most important prevention techniques include, wearing a mask in public at all times, washing your hands and maintaining a 6-foot distance with anyone you don’t live with. Many companies started working on a vaccine as soon as possible and as of now, we have a total of 15 different vaccines for COVID-19 with the most popular...
ones being: Oxford-AstraZeneca, Pfizer-BioNTech, Sputnik V, Moderna, and Johnson. Beside that, mathematicians are also working tirelessly to solve this situation. In order to better understand and forecast disease transmission, mathematical models may be useful and it also aids in the implementation of adequate steps and effective methods for containing the pandemic’s spread and moderate its effects. The aim of this research is to look at one of those models namely an expansion of Kermack and McKendric’s SIR method, which was first implemented in the early twentieth century [1,2]. The authors looked at a homogeneous population affected by an infectious disease in that study. They discovered that the epidemic disease could be analyzed by calculating a simple reproductive number $R_0$, which is dependent on infectiousness, recovery, and death rates: the disease dies out when $R_0$ is, and continues when $R_0 > 1$. A few numerical models have been created to investigate COVID-19 transmission dynamics [3–5]. Nevertheless, these models are subject to a number of uncertainties, including an inadequate explanation of the biological processes regulating disease transmission and a lack of knowledge of certain key parameters. One technique to reduce these uncertainties is to use available data to constrain disease forecasting models. The model performance can be combined with these data to boost prediction and minimize uncertainty [6,7]. When information is visible form evolve into ready for use, this method, famous as information in visible form absorption, following changes the model state and limit to keep the model as very much alike to the genuine in existence path as attainable [8].

Other than ordinary derivatives, fractional order strategies are helpful for exceptional perceptive the interpretation of real-world problems [9]-[10]. Riemann–Liouville established the prime concept of fractional derivatives that built on the law of power. The latest technique fractional derivative has been introduced by Atangana and Alkahtani in [11]. Previous studies [12–15] have suggested a non-singular kernel with an up-to-date fractional derivative that involved trigonometric and exponential functions, and several new techniques for disease models are described in [16–19]. Essential effects have been installed for this new operator, and illustrations are furnished in Khan et al. [20]. To analyze the problem and obtain numerical solutions, a usual approach towards the mode’s method is used in [21].

In this work, some basic notions of fractional calculus are described in segment 2. In segment 3, and we talked about the existence and uniqueness of the model. Moreover, for the COVID-19 model, we discussed the Ulam–Hyres stability. In segment 4, we discussed its simulations. Last segment consist of conclusion.

2. Basic Definition

In this segment, We describe some primary notions which are helpful to analysis the system.

Definition 2.1. Let $0 \leq v$, $\phi \leq 1$ then $U(t)\text{in the Riemann–Liouville for fractal fractional operator}$ is defined as [22]:

$$\frac{d}{dt}U(t) = \lim_{\phi \to 0} \frac{U(t) - U(t)}{\phi}.$$

Thus, the fractal-fractional integral of $U(t)$ with order $(v, \phi)$ and having power law type kernel is defined as:

$$\text{FFPI}_0^\phi U(t) = \int_0^t (t - \psi)^{-\phi-1} U(\psi)d\psi.$$

Definition 2.2. Let $0 \leq v$, $\phi \leq 1$ then $U(t)\text{in the Riemann–Liouville for fractal fractional operator having exponentially decaying kernel is given as}$ [22]:

$$\text{FFD}_0^\phi U(t) = \frac{M(v)}{m - v} \frac{d}{dt} U(t) + \frac{\phi}{M(v)} \int_0^t \psi^{-\phi} U(\psi)d\psi.$$

Definition 2.3. Let $0 \leq v$, $\phi \leq 1$ then $U(t)\text{in the Riemann–Liouville for fractal fractional operator with generalized Mittag–Leffler kernel (FFM) is defined as}$ [22]:

$$\text{FFM}_0^\phi U(t) = \frac{A_B(v) (1 - v)}{1 - v} \frac{d}{dt} U(t) + \frac{\phi}{A_B(v)} \int_0^t \psi^{-\phi} U(\psi)d\psi,$$

where $0 < v$, $\phi \leq 1$ and $A_B(v) = 1 - \frac{v}{\Gamma_\phi}$.

Thus, $U(t)$ with order $(v, \phi)$ and having exponentially decaying type kernel is defined as:

$$\text{ffm}_0^\phi U(t) = \frac{A_B(v) (1 - v)}{1 - v} \frac{d}{dt} U(t) + \frac{\phi}{A_B(v)} \int_0^t \psi^{-\phi} U(\psi)d\psi.$$

3. Fractional order COVID-19 model

To simulate the epidemic of COVID-19, we expand the SEIR model into seven parts. These variables are $S(t)$ susceptible, $U(t)$ exposed (uncovered), $C(t)$ infectious (contagious), $A(t)$ quarantined (alone), $R(t)$ recovered, $P(t)$ dead (passed) and $B(t)$ vaccinated cases. The model has non-negative initial conditions that is $S(0) = S_0$, $U(0) = U_0$, $C(0) = C_0$, $A(0) = A_0$, $R(0) = R_0$, $P(0) = P_0$, $B(0) = B_0$ and the coefficients $H$ new hosts per unit of time, $\gamma$ shows the transmission rate divided by N, $\alpha$ denotes the vaccination rate, $\upsilon$ express natural passing rate, $\delta$ reveals average latent time, $\nu$ indicate average quarantine time, $\gamma$ convey average days of infection, $\xi$ display mortality rate, $\lambda$ demonstrate duration of recovery and $\rho$ specify average days until death. The system of governing equations for the model is given in [23], under fractal-fractional operators in Atangana-Baleanu sense.
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\[ FFM D_{0,t}^{\phi} S(t) = H - \eta S(t) C(t) - \alpha S(t) - \nu S(t), \]

\[ FFM D_{0,t}^{\Phi} U(t) = \eta S(t) C(t) - \delta U(t) + \nu \eta B(t) C(t) - \nu U(t), \]

\[ FFM D_{0,t}^{\phi} C(t) = \delta U(t) - \gamma C(t) - \nu C(t), \]

\[ FFM D_{0,t}^{\phi} A(t) = \gamma C(t) - (1 - \xi) \lambda A(t) - \xi \rho A(t) - \nu A(t), \]

\[ (1) \]

\[ FFM D_{0,t}^{\phi} R(t) = (1 - \xi) \lambda A(t) - \nu R(t), \]

\[ FFM D_{0,t}^{\phi} P(t) = \xi \rho A(t), \]

\[ FFM D_{0,t}^{\phi} B(t) = S(t) - \eta B(t) C(t) - \nu B(t). \]

With initial condition

\[ S(0) \geq 0, U(0) \geq 0, C(0) \geq 0, A(0) \geq 0, R(0) \geq 0, \]

\[ P(0) \geq 0, B(0) \geq 0 \]

3.1. Positivity Analysis and Equilibria

Put Left hand side of the system (1) equal to zero, we get following endemc point

\[ Q^*(S, U, C, A, R, T) = \frac{H}{\nu + \infty}, 0, 0, 0, 0, 0, 0, 0, 0, 0 \]

and reproductive number \( R_0 \) details given in [24], we have

\[ R_0 = \frac{\eta \delta H}{\nu (\nu + \delta + \gamma) (\nu + \infty)} \]

Sensitivity of \( R_0 \) can be analyzed by taking the partial derivatives of reproductive number for the involved parameters as follows

\[ \frac{\partial R_0}{\partial \eta} = \frac{\eta \delta H}{(\nu + \delta + \gamma) (\nu + \infty)} > 0, \]

\[ \frac{\partial R_0}{\partial \nu} = \frac{\eta \delta H}{(\nu + \delta + \gamma) (\nu + \infty)} > 0, \]

\[ \frac{\partial R_0}{\partial \delta} = \frac{\eta \delta H}{(\nu + \delta + \gamma) (\nu + \infty)} > 0, \]

\[ \frac{\partial R_0}{\partial \gamma} = \frac{\eta \delta H}{(\nu + \delta + \gamma) (\nu + \infty)} > 0, \]

\[ \frac{\partial R_0}{\partial \alpha} = \frac{\eta \delta H}{(\nu + \delta + \gamma) (\nu + \infty)} > 0. \]

Clearly, in case of change in parameter \( R_0 \) is very sensitive. In this manuscript, \( H, \eta, \delta, \nu, \alpha \) are growing while \( \gamma \) is reducing. Thus, based on sensitivity analysis, we can say that prevention is better to control the disease.

**Theorem 3.1.** The solution of the proposed fractional-order model (1) along initial conditions is unique and bounded in \( R^+_0 \).

**Proof.** In system (1), we can get its existence and uniqueness on the time interval \((0, \infty)\). Afterwards, we need to show that the non-negative region \( R^+_0 \) is a positively invariant region.

From model (1), we find

\[ r_{st} D_{0,t}^{\phi} S(t)_{|_{t=0}} = H > 0, \]

\[ r_{st} D_{0,t}^{\phi} U(t)_{|_{t=0}} = \eta S(t) C(t) > 0, \]

\[ r_{st} D_{0,t}^{\phi} C(t)_{|_{t=0}} = \delta U(t) - \gamma C(t) - \nu C(t), \]

\[ r_{st} D_{0,t}^{\phi} A(t)_{|_{t=0}} = \gamma C(t) \geq 0, \]

\[ r_{st} D_{0,t}^{\phi} R(t)_{|_{t=0}} = (1 - \xi) \lambda A(t) \geq 0, \]

\[ r_{st} D_{0,t}^{\phi} P(t)_{|_{t=0}} = \xi \rho A(t), \]

\[ r_{st} D_{0,t}^{\phi} B(t)_{|_{t=0}} = S(t) \geq 0. \]

If \((S(0), U(0), C(0), A(0), R(0), P(0), B(0)) \in R^+_0\), then according to Eqs. (2), the solution cannot escape from the hyperplanes. Also on each hyperplane bounding the non-negative orthant, the vector field points into \( R^+_0 \), i.e., the domain \( R^+_0 \) is a positively invariant set.

3.2. Existence and uniqueness of the Model

We use the Atangana-Baleanu strategy of fractal fractional to derive the existence and uniqueness of the model. Consider

\[ AB_{D_{0,t}^{\phi}} S(t) = H - \eta S(t) C(t) - \alpha S(t) - \nu S(t), \]

\[ ABC D_{0,t}^{\phi} U(t) = \eta S(t) C(t) - \delta U(t) + \nu \eta B(t) C(t) - \nu U(t), \]

\[ ABC D_{0,t}^{\phi} C(t) = \delta U(t) - \gamma C(t) - \nu C(t), \]

\[ ABC D_{0,t}^{\phi} A(t) = \gamma C(t) - (1 - \xi) \lambda A(t) - \xi \rho A(t) - \nu A(t), \]

\[ ABC D_{0,t}^{\phi} R(t) = (1 - \xi) \lambda A(t) - \nu R(t), \]

\[ ABC D_{0,t}^{\phi} P(t) = \xi \rho A(t), \]

\[ ABC D_{0,t}^{\phi} B(t) = S(t) - \eta B(t) C(t) - \nu B(t). \]

Using fixed point results, we show that the model under consideration has at least one and unique solution. As integral satisfies the condition of differentiability, so we can express the model as

\[ o_{ABR} D_{0,t}^{\phi} S(t) = \phi \delta^{\rho - 1} J(t, S, U, C, A, R, P, B), \]

\[ o_{ABR} D_{0,t}^{\phi} U(t) = \phi \delta^{\rho - 1} K(t, S, U, C, A, R, P, B), \]

\[ o_{ABR} D_{0,t}^{\phi} C(t) = \phi \delta^{\rho - 1} L(t, S, U, C, A, R, P, B), \]

\[ o_{ABR} D_{0,t}^{\phi} A(t) = \phi \delta^{\rho - 1} M(t, S, U, C, A, R, P, B), \]

\[ o_{ABR} D_{0,t}^{\phi} R(t) = \phi \delta^{\rho - 1} N(t, S, U, C, A, R, P, B), \]

\[ o_{ABR} D_{0,t}^{\phi} P(t) = \phi \delta^{\rho - 1} O(t, S, U, C, A, R, P, B), \]

\[ o_{ABR} D_{0,t}^{\phi} B(t) = \phi \delta^{\rho - 1} P(t, S, U, C, A, R, P, B), \]

where

\[ J(t, S, U, C, A, R, P, B) = H - \eta S(t) C(t) - \alpha S(t) - \nu S(t), \]

\[ K(t, S, U, C, A, R, P, B) = \eta S(t) C(t) - \delta U(t) + \nu \eta B(t) C(t) - \nu U(t), \]

\[ L(t, S, U, C, A, R, P, B) = \delta U(t) - \gamma C(t) - \nu C(t), \]

\[ M(t, S, U, C, A, R, P, B) = \gamma C(t) - (1 - \xi) \lambda A(t) - \xi \rho A(t) - \nu A(t), \]

\[ N(t, S, U, C, A, R, P, B) = (1 - \xi) \lambda A(t) - \nu R(t), \]

\[ O(t, S, U, C, A, R, P, B) = \xi \rho A(t), \]

\[ V(t, S, U, C, A, R, P, B) = S(t) - \eta B(t) C(t) - \nu B(t). \]

We can rewrite the system (4) as:

\[ o_{ABR} D_{0,t}^{\phi} S(t) = \phi \delta^{\rho - 1} \Omega(t, \psi(t)), \]

\[ \psi(0) = \psi_0. \]

By replacing \( o_{ABR} D_{0,t}^{\phi} \) by \( o_{ABR} D_{0,t}^{\phi} \) and applying fractional integral, we have

\[ \psi(t) = \psi(0) + \frac{\phi \delta^{\rho - 1} (1 - \Theta)}{AB(\Theta)} + \frac{\Theta \phi}{AB(\Theta) T(\Theta)} \]

\[ \times \int_0^t \lambda^{\rho - 1} (t - \lambda)^{\Theta - 1} \Omega(t, \psi(t)) d\lambda, \]

where
\[ \psi(t) = \begin{cases} J(t) & \text{if } \psi(0) = J(0) \\ K(t) & \text{if } \psi(0) = K(0) \\ L(t) & \text{if } \psi(0) = L(0) \\ M(t, \phi(0)) = M(0, \Omega(\psi(i))) = \begin{cases} M(t, \psi(0)) & \text{if } \psi(0) = M(t, \psi(0)) \\ N(t) & \text{if } \psi(0) = N(0) \\ O(t) & \text{if } \psi(0) = O(0) \\ V(t) & \text{if } \psi(0) = V(0) \end{cases} \end{cases} \]

If \( Y \subseteq [0, T] \), we use a Banach space \( Z = Y \times Y \times Y \times Y \times Y \times Y \) then its norm is

\[ \|\psi(t)\| = \text{max}_{x \in [0, r]} |J(t)| + K(t) + L(t) + M(t) + N(t) + O(t) + V(t) |. \]

We present an operator \( \mathcal{R} : L \rightarrow L \) as:

\[ \mathcal{R}(\psi)(t) = \psi(0) + \frac{\phi T^{1-\theta}(1 - \Theta)}{AB(\Theta)} \Omega(t, \psi(t)) + \frac{\Theta \phi}{AB(\Theta) \Gamma(\Theta)} \int_0^t \xi^{1-\theta}(t - \xi)^{1-\theta} \Omega(t, \psi(t))d\xi \]

(6)

If a function \( \Omega(t, \psi(t)) \) full fills the extension and the condition of Lipschitz then for \( \psi \in B \) there exists some positive constants \( L_0, M_0 \) such that

\[ \Omega(t, \psi(t)) \leq L_0|\psi(t)| + M_0. \]

(7)

and for \( \psi, \bar{\psi} \in B \) there exists a constant \( U_0 > 0 \) such that

\[ \|\Omega(t, \psi(t)) - \Omega(t, \bar{\psi}(t))\| \leq U_0|\psi(t) - \bar{\psi}(t)|. \]

(8)

**Theorem 3.2.** For the set of continuous function \( \Omega : [0, T] \times Y \rightarrow R \) there exists at least single outcome for system (1) if the Eq. (7) is true.

**Proof.** Initially, our purpose is to represent that the operator \( \mathcal{R} \) defined in (6) is completely continuous. As \( \Omega \) is a continuous function, so this implies \( \mathcal{R} \) is a continuous function too. Consider \( H = \{ \psi \in Y : \|\psi\| \leq R, R > 0 \} \). For any \( \psi \in Y \), we have

\[ \mathcal{R}(\psi)(t) = \text{max}_{x \in [0, r]} |\psi(t)| + \frac{\phi T^{1-\theta}(1 - \Theta)}{AB(\Theta)} \Omega(t, \psi(t)) + \frac{\Theta \phi}{AB(\Theta) \Gamma(\Theta)} \int_0^t \xi^{1-\theta}(t - \xi)^{1-\theta} \Omega(t, \psi(t))d\xi \]

\[ \leq \psi(0) + \frac{\phi T^{1-\theta}(1 - \Theta)}{AB(\Theta)} (L_0|\psi| + M_0) + \text{max}_{x \in [0, r]} \]

\[ \times \frac{\Theta \phi}{AB(\Theta) \Gamma(\Theta)} \int_0^t \xi^{1-\theta}(t - \xi)^{1-\theta} \Omega(t, \Pi(t))d\xi \]

\[ \leq \psi(0) + \frac{\phi T^{1-\theta}(1 - \Theta)}{AB(\Theta)} (L_0|\psi| + M_0) + \frac{\Theta \phi}{AB(\Theta) \Gamma(\Theta)} (L_0|\psi| + M_0) T^{1-\theta} H(t + \phi) \]

\[ \leq R. \]

As a result, \( \mathcal{R} \) is uniformly bounded, and \( H(\Theta + \phi) \) representing the beta function.

For equicontinuity of \( \mathcal{R} \), we take \( t_1 < t_2 \leq T \). Then consider

\[ \mathcal{R}(\psi)(t_2) - \mathcal{R}(\psi)(t_1) = \left( \frac{\phi T^{1-\theta}(1 - \Theta)}{AB(\Theta)} \Omega(t_2, \psi(t_2)) + \Theta \phi \right) \int_0^{t_2} \xi^{1-\theta}(t_2 - \xi)^{1-\theta} \Omega(t_2, \psi(t_2))d\xi \]

\[ - \frac{\phi T^{1-\theta}(1 - \Theta)}{AB(\Theta)} \Omega(t_1, \psi(t_1)) + \frac{\Theta \phi}{AB(\Theta) \Gamma(\Theta)} \int_0^{t_1} \xi^{1-\theta}(t_1 - \xi)^{1-\theta} \Omega(t_1, \psi(t_1))d\xi \]

\[ \leq \left[ \frac{\phi T^{1-\theta}(1 - \Theta)}{AB(\Theta)} + \frac{\Theta \phi}{AB(\Theta) \Gamma(\Theta)} \right] \left( \theta^{1-\phi} H(t + \phi) - \theta^{1-\phi} H(t_1 + \phi) \right) \]

\[ \leq \rho \|\mathcal{R}(\psi) - \mathcal{R}(\psi)\|. \]

Hence, \( \mathcal{R} \) has contraction. Thus, by using the Banach contraction principle, it has a special outcome.

3.3. Ulam-Hyers stability

Here, we are aiming to illustrate the Ulam-Hyers stability of the suggested demonstration.

**Definition 3.4.** If for any positive \( \epsilon \) and for all \( \psi \in [W[0, T], R] \) there exists a positive operator \( \mathcal{R}_{\theta, \phi} \), then system (1) is Ulam-Hyers stable.
\[ 0 \text{FVM} D^\phi_{\theta,\phi} \psi(t) - \Omega(t, \psi(t)) \leq \varepsilon, \quad t \in [0, T], \]

so we get a special outcome \( \varepsilon \in (W[0, T], R) \) such that

\[ |\psi(t) - \sigma(t)| \leq \mathcal{R}_{\varepsilon, \phi, \rho}, \quad t \in [0, T]. \]

If we suppose a perturbation \( \gamma \in W[0, T] \) then \( \gamma(0) = 0. \) Consider

\( \gamma(t) > 0, \) we have \( \gamma(t) \leq \varepsilon \)

\( \text{(ii)} \) \( 0 \text{FVM} D^\phi_{\theta,\phi} \psi(t) = \Omega(\psi(t)) + \gamma(t), \)

**Lemma 3.5.** A perturbed model has outcome

\[ 0 \text{FVM} D^\phi_{\theta,\phi} \psi(t) = \Omega(\psi(t)) + \gamma(t), \quad \psi(0) = \psi_0, \]

fulfills the following relation

\[ |\mathcal{R}(t) - \{\psi(0) + \frac{\phi^{\phi-1}(1 - \Theta)}{AB(v)} \Omega(t, \psi(t)) + \frac{\Theta \phi}{AB(\theta)\Gamma(\theta)} \int_0^t \lambda^{\phi-1}(t - \lambda)^{\phi-1} \Omega(\lambda, \psi(\lambda))d\lambda\}| \leq \mathcal{R}_{\theta,\phi,\rho}, \]

\[ v_{0,\phi} = \frac{\Theta \phi}{AB(\theta)\Gamma(\theta)} T^{\phi-1} \mathcal{H}(\Theta, \phi). \]

**Lemma 3.6.** If we consider the condition of Eq. (8) with lemma (3.5) then it has Ulam-Hyers stable solution under the condition \( \rho < 1. \)

**Proof.** Consider \( \theta \in A \) be a special result, and \( \psi \in A \) be any result of the model, then

\[ |\psi(t) - \theta(t)| = |\psi(t) - \{\theta(0) + \frac{\phi^{\phi-1}(1 - v)}{AB(v)} \Omega(t, \theta(t)) + \frac{\Theta \phi}{AB(\theta)\Gamma(\theta)} \int_0^t \lambda^{\phi-1}(t - \lambda)^{\phi-1} \Omega(\lambda, \theta(\lambda))d\lambda\}|
\]

\[ \leq |\psi(t) - \{\theta(0) + \frac{\phi^{\phi-1}(1 - v)}{AB(v)} \Omega(t, \psi(t)) + \frac{\Theta \phi}{AB(\theta)\Gamma(\theta)} \int_0^t \lambda^{\phi-1}(t - \lambda)^{\phi-1} \Omega(\lambda, \psi(\lambda))d\lambda\}|
\]

\[ + |\psi(0) + \frac{\phi^{\phi-1}(1 - v)}{AB(v)} \Omega(t, \psi(t)) + \frac{\Theta \phi}{AB(\theta)\Gamma(\theta)} \int_0^t \lambda^{\phi-1}(t - \lambda)^{\phi-1} \Omega(\lambda, \psi(\lambda))d\lambda|
\]

\[ - |\theta(0) + \frac{\phi^{\phi-1}(1 - v)}{AB(v)} \Omega(t, \theta(t)) + \frac{\Theta \phi}{AB(\theta)\Gamma(\theta)} \int_0^t \lambda^{\phi-1}(t - \lambda)^{\phi-1} \Omega(\lambda, \theta(\lambda))d\lambda|
\]

\[ \leq v_{0,\phi} + \rho |\psi(t) - \theta(t)|.
\]

Consequently,

\[ ||\psi - \theta|| \leq v_{0,\phi} + \rho ||\psi - \theta||. \]

Moreover, we can write the above expression as

\[ ||\psi - \theta|| \leq \mathcal{R}_{\varepsilon, \phi, \rho} \]

where \( \mathcal{R}_{\varepsilon, \phi, \rho} = \frac{\varepsilon}{1 - \rho}. \) Hence it is Ulam-Hyers stable.

**4. Simulation and discussion**

For initial condition \( C(0) = U(0) = 3, R(0) = P(0) = 0 \) and for the values of parameters, \( H \) denotes the new birth rate has value 2300 persons/days [25], we supposed transmission rate before and after vaccine is \( \eta_1 = 5.85 \times 10^{-9} \text{day}^{-1} \) and \( \eta_2 = 3.43 \times 10^{-7} \text{day}^{-1} \) respectively, \( \alpha = 3.5 \times 10^{-4} \text{day}^{-1} \) represents vaccine rate [26], Natural death rate shows \( v = 3 \times 10^{-7} \text{persons/days} \) [26], duration of process is \( \delta^{-1} = 5.5 \text{days} \) [27], rate of ineffective of vaccine is \( v = 0.05 \) [28], \( \gamma^{-1} = 3.8 \text{days} \) is duration of spreading disease time [27], the rate of disaster is \( \xi = 0.014 \) [28], \( \lambda^{-1} = 10 \text{days} \) shows revivial time [27] and \( \rho^{-1} = 15 \text{days} \) express the time until death[27]. We take into account the subsequent 2 amounts: the essential time frame gives the measure of the beginning of the pandemic once the intervention had not been applied utilizing a fundamental duplicate assortment bigger than one. \( R_0 = 2.5, \) which provides \( \eta_1 = 5.85 \times 10^{-9}, \)though the time of play compares to the sun once the mediation was applied. Throughout this era, the numbers of latest cases square measure comparatively consistent. Hence, we will in general utilize a fundamental duplicate variety \( R_0 = 1, \) which gives \( \eta_2 = 3.43 \times 10^{-9}. \) Simulation of the data will present in Figs. 1–7 with the effect of value \( \eta_1 \) and \( \eta_2. \) We also observe that in Fig. 3 infected population should decrease gradually after vaccination and the recovered population also increase shown in Fig. 5. If we continue the process of vaccination in society, this will be possible we have a disease-free endemic point. For putting \( \eta_1 \) values, we have a reproductive number greater than one. But by putting \( \eta_2 \) getting the reproductive number equal to 1 which shows that infection will not be carried out in society with one or more than one way of infection. Simulation results in Figs. 1–7 support our justification and the result are helpful to overcome the risk of COVID-19 in society.

**5. Conclusion**

In this formation, results also show that growing the vaccination movement meaningfully decreases the number of definite cases and deaths. We used Atagan-Baleanu fractal-fractional derivative technique to reformulate the COVID-19 model. We explored it in both cases qualitatively and quantitatively. The existence and uniqueness and the stability covered under qualitative while the numerical simulations can be considered under quantitative. We proved the Ulam-Hyers stability of the COVID-19 model. In the simulations, we can see a change in the protection of the immune and COVID-19 cells is due to the difference in fractal order. Memory effects of fractal and fractional orders will be observed in figures. The proposed scheme is very helpful for planning, decision making and different control strategies like using mask, social distance and vaccination. Others parameters, we used in future modeling process to control the infection COVID-19 in society.
Fig. 1 Simulation of $S(t)$ with Fractal Fractional Operator for $\eta_1, \eta_2$.

Fig. 2 Simulation of $U(t)$ with Fractal Fractional Operator for $\eta_1, \eta_2$.

Fig. 3 Simulation of $C(t)$ with Fractal Fractional Operator for $\eta_1, \eta_2$.

Fig. 4 Simulation of $A(t)$ with Fractal Fractional Operator for $\eta_1, \eta_2$.

Fig. 5 Simulation of $R(t)$ with Fractal Fractional Operator for $\eta_1, \eta_2$.

Fig. 6 Simulation of $P(t)$ with Fractal Fractional Operator for $\eta_1, \eta_2$. 
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Fig. 7 Simulation of $B(t)$ with Fractal Fractional Operator for $\eta_1, \eta_2$.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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