Leveraging Auxiliary Information on Marginal Distributions in Nonignorable Models for Item and Unit Nonresponse

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Summary. When handling nonresponse, government agencies and survey organizations typically are forced to make strong, and potentially unrealistic, assumptions about the reasons why values are missing. We present a framework that enables users to reduce reliance on such assumptions by leveraging information from auxiliary data sources, such as administrative databases, on the marginal distributions of the survey variables. We use the framework to impute missing values in voter turnout in the U.S. Current Population Survey (CPS), showing how it allows us to make more reasonable assumptions about the missing values than those used in previous CPS turnout estimates.

Keywords: Auxiliary; Missing; Nonignorable; Survey; Voting

1. Introduction

Many surveys have seen steep declines in response rates (Brick and Williams, 2013; Curtin et al., 2005). Yet, government agencies and survey organizations—henceforth all called agencies—are under increasing budgetary pressures, making fewer resources available for extensive nonresponse follow-up activities. As a result, agencies are forced to account for missing values via statistical methods—for example, survey weight adjustments (Brick and Kalton, 1996) and variants of imputation (Andridge and Little, 2010; Kim, 2011; Rubin, 1987)—that rely on strong assumptions about the reasons why the values are missing, e.g., all values are missing at random (MAR) (Rubin, 1976). Such assumptions could be unrealistic, consequently threatening the validity and usefulness of inferences based on the survey data.

Simultaneously, the digital revolution has seen a proliferation of information that agencies may be able to leverage when accounting for missing data (National Research Council, 2009, 2015). For example, suppose a simple random sample has no unit nonresponse but has item nonresponse on the survey question asking the respondent’s sex. If 70% of participants report female, and the agency knows that the target population includes 50% men and 50% women, the agency likely should impute more men than women for the missing values in the survey question asking the respondent’s sex. In contrast, imputation routines that do not account for the known margin are likely to generate untrustworthy imputations. For example, a MAR model is likely to result in completed data with empirical percentages closer to 70% female than 50% female. Of course, the agency
should not use solely the population margin in imputations; it also should take advantage of observed information in other variables, so as to preserve multivariate relationships.

This example illustrates a broader context. Suppose an agency has accurate estimates of population percentages or counts for some variables in the survey. These can be available from auxiliary data sources, such as censuses, administrative databases, high quality surveys, or private sector data aggregators; see Sadinle and Reiter (2019) for specific examples of auxiliary data sources. The agency seeks to take advantage of this auxiliary information in its methods for handling missing values, which could be due to both item and unit nonresponse. In fact, agencies routinely find themselves in these scenarios; for example, many agencies use population counts as the basis for post-stratification adjustments for unit nonresponse. Usually, however, they do not use such margins in the imputation models for item nonresponse.

In this article, we propose a model-based framework for leveraging auxiliary marginal information when handling both unit and item nonresponse. The margins allow agencies to weaken the assumptions about the reasons for missingness, while also offering flexibility in specification of missing data models. In particular, we show how one can use the margins to specify different missingness mechanisms for unit and item nonresponse; for example, use a nonignorable model for unit nonresponse and an ignorable model for the variables with item nonresponse, or vice versa. We apply and illustrate the framework to handle missing values in a question on voter turnout in the U.S. Current Population Survey. Here, we know the actual number of voters in the election from published statewide totals. We use these auxiliary totals to generate model-based estimates of voter turnout, which we use for substantive empirical analyses examining voter turnout among various population subgroups (age, sex, and state).

The remainder of this article is organized as follows. In Section 2 we present the framework for specifying models for both unit and item nonresponse using information obtained from auxiliary sources, which we refer to as the MD-AM (missing data with auxiliary margins) framework. We base the MD-AM framework on the additive nonignorable (AN) model of Hirano et al. (1998, 2001), which we briefly review in this section. In Section 3 we illustrate an application of the MD-AM framework using the CPS voter turnout data. In Section 4 we conclude and discuss extensions of the MD-AM framework.

2. The MD-AM Framework

The MD-AM framework is based on a two-step process for specifying a joint distribution for the survey variables and indicator variables for nonresponse. Specifically, we characterize the joint distribution using a sequential factorization of conditional models. We use the auxiliary information to guide the specification of the conditional distributions, using models that encode potentially nonignorable nonresponse mechanisms. We require the models to be identifiable as described in Sadinle and Reiter (2019), which corresponds to the usual notion that any set of model parameter values maps to a unique value of the likelihood function (and vice versa). Broadly, the two steps are as follows.

Step 1: Specify model for the observed data. We begin with a model for the survey variables and nonresponse indicators that can be identifiable using the observed data alone, without any auxiliary information. Preferably, the model should allow for
the maximum number of parameters identifiable from the observed data alone. This can be done using either a selection model or a pattern mixture factorization [Glynn et al., 1986; Little, 1993], according to the analyst’s preference. Generally, this step results in ignorable models, such as MAR models, that are often default choices for handling nonresponse in the missing data literature, absent auxiliary data.

**Step 2: Incorporate auxiliary margins.** We next find sets of parameters that can be added to the model in Step 1, so that the model still can be identified because of the auxiliary information. Typically, there are multiple identifiable models—determined by the nature of the auxiliary information—each representing different assumptions about the missingness process. Agencies can choose from among these models according to interpretability and plausibility for the data at hand.

In what follows we demonstrate how to instantiate this framework. We use three scenarios that increase in complication. After introducing notation, in Section 2.2 we start with a scenario involving two binary variables with one subject to item nonresponse. This scenario helps us illustrate the main ideas in a simplistic scenario. In Section 2.3 we extend to two variables with item nonresponse. The extra variable increases the number of identifiable models, and we describe how agencies can consider the practical implications of these choices. In Section 2.4 we add unit nonresponse. We show how the framework allows one to encode different missingness assumptions about unit and item nonresponse. In Section 2.5 we illustrate the framework using simulation studies of the scenario with unit and item nonresponse. In Section 2.6 we discuss issues of model selection and extensions to more variables.

### 2.1. Notation

Let $\mathcal{D}$ comprise data from the survey of $i = 1, \ldots, n$ individuals, and let $\mathcal{A}$ comprise data from the auxiliary database. Let $X = (X_1, \ldots, X_p)$ represent the $p$ variables in both $\mathcal{A}$ and $\mathcal{D}$, where each $X_k = (X_{1k}, \ldots, X_{nk})^T$ for $k = 1, \ldots, p$. Let $Y = (Y_1, \ldots, Y_q)$ represent the $q$ variables in $\mathcal{D}$ but not in $\mathcal{A}$, where each $Y_k = (Y_{1k}, \ldots, Y_{nk})^T$ for $k = 1, \ldots, q$. We disregard variables in $\mathcal{A}$ but not $\mathcal{D}$ as they are not of primary interest. We assume that $\mathcal{A}$ contains sets of marginal probabilities or counts for variables in $X$, summarized from some external database.

For each $k = 1, \ldots, p$, let $R_{ik}^x = 1$ if individual $i$ would not respond to the question on $X_k$ in the survey (i.e., $\mathcal{D}$), and $R_{ik}^x = 0$ otherwise. Similarly, for each $k = 1, \ldots, q$, let $R_{ik}^y = 1$ if individual $i$ would not respond to the question on $Y_k$ in the survey, and $R_{ik}^y = 0$ otherwise. Let $R^x = (R_1^x, \ldots, R_p^x)$ and $R^y = (R_1^y, \ldots, R_q^y)$, where each $R_{1k}^x = (R_{11k}, \ldots, R_{nk}^x)^T$ and $R_{1k}^y = (R_{11k}, \ldots, R_{nk}^y)^T$. Let $R_i^x = (R_{i1}, \ldots, R_{ip})$ and $R_i^y = (R_{i1}^y, \ldots, R_{iq}^y)$. Let $U = (U_1, \ldots, U_n)$, where each $U_i = 1$ if individual $i$ would not respond to the survey at all (unit nonresponse), and $U_i = 0$ otherwise. We note that $(R_{1k}^x, R_{1k}^y)$ is observed for all cases $i$ with $U_i = 0$, whereas $(R_i^x, R_i^y)$ is not observed for all cases $i$ with $U_i = 1$. Let $R_{obs} = \{(R_i^x, R_i^y) : U_i = 0\}$. Finally, we define the observed data as $\mathcal{D}_{obs} = (\mathcal{D}, R_{obs}, U)$.

Following Sadinle and Reiter (2019), for simplicity we use generic notations such as $f$ and $\eta$ for technically different functions and parameters respectively, but their actual meanings within each context should be clear within each context. For example, $f$, $\eta_0$, ...
Table 1. Data structure for example with one variable subject to item nonresponse. Here, “✓” represents observed components and “?” represents missing components.

| X1 | Y1 | Rx1 |
|----|----|------|
| ✓  | ✓  | 0    |
| ?  | ✓  | 1    |
| ✓  | ?  | ?    |

Auxiliary margins →

and $\eta_1$ need not be the same in the conditional probability mass functions $\Pr(X_1 = 1|Y_1) = f(\eta_0 + \eta_1 Y_1)$ and $\Pr(Y_1 = 1|X_1) = f(\eta_0 + \eta_1 X_1)$.

2.2. One variable subject to item nonresponse

We start with the simplest example possible to illustrate the main ideas. Suppose $\mathcal{D}$ comprises two binary variables, $X_1$ and $Y_1$. Further, $\mathcal{A}$ comprises the true marginal probabilities for $X_1$, but it has no margins for $Y_1$. Suppose $X_1$ suffers from item nonresponse and $Y_1$ is fully observed, and there is no unit nonresponse. As a result, we need to specify a model for $R_{x1}$. We assume there is no need to specify models for $U$ and $R_{y1}$. Thus, for convenience, we write $\mathcal{D}_{\text{obs}} = (\mathcal{D}, R_{x1})$. Table 1 represents the relevant information in $\mathcal{D}_{\text{obs}}$ and $\mathcal{A}$ in a graphical format.

To apply the MD-AM framework, we first follow Step 1 to specify a model for $\mathcal{D}_{\text{obs}}$ without using $\mathcal{A}$. Here, and throughout this section, we use a selection model factorization, in which we posit a model for $\mathcal{D}$ and a model for $(R_{x1} | \mathcal{D})$. We present pattern mixture model factorizations in the supplementary material. For a selection model, a reasonable specification for Step 1 includes all parameters except those targeting the direct relationship between $X_1$ and $R_{x1}$. We assume there is no need to specify models for $U$ and $R_{y1}$. Thus, for convenience, we write $\mathcal{D}_{\text{obs}} = (\mathcal{D}, R_{x1})$. Table 1 represents the relevant information in $\mathcal{D}_{\text{obs}}$ and $\mathcal{A}$ in a graphical format.

We next follow Step 2 to incorporate the auxiliary information about $X_1$. The marginal distribution of $X_1$ provides one additional piece of information about the joint distribution of all the variables. Hence, we should be able to identify and estimate one additional term in the models. For the selection model factorization, we can add $\eta_2 X_1$ to (2), resulting in

$$\Pr(R_{x1} = 1|X_1, Y_1) = h_1(\eta_0 + \eta_1 Y_1 + \eta_2 X_1).$$
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Table 2. Data structure for example with two variables subject to item nonresponse. Here, “✓” represents observed components and “?” represents missing components.

|   |   |   |
|---|---|---|
| X | Y | R_x | R_y |
| ✓ | ✓ | 0  | 0  |
| ? | ✓ | 1  |     |
| ✓ | ? | 0  | 1  |
| ? | ? | 1  |     |
| ✓ | ? | ?  | ?  |

If we instead knew all the joint probabilities \( \Pr(X_1, Y_1) \), we would have two additional pieces of information about the joint distribution (from \( \Pr(X_1 | Y_1 = 0) \) and \( \Pr(X_1 | Y_1 = 1) \)). In this case, we could add both \( \eta_2 X_1 \) and \( \eta_3 Y_1 \) to (2).

The model in (1) and (3) corresponds to the additive nonignorable (AN) model of Hirano et al. (2001), which has been used previously to handle attrition in longitudinal studies with refreshment samples (e.g., Nevo, 2003; Bhattacharya, 2008; Das et al., 2013; Deng et al., 2013; Schifeling et al., 2015). The AN model encodes ignorable and nonignorable models as special cases. In particular, \((\eta_1 = 0, \eta_2 = 0)\) results in a missing completely at random (MCAR, Little and Rubin (2002)) mechanism; \((\eta_1 \neq 0, \eta_2 = 0)\) results in a MAR mechanism; and, \(\eta_2 \neq 0\) results in a missing not at random (MNAR, Little and Rubin (2002)) mechanism. Thus, the AN model uses a weaker assumption about the missingness than its special case models. Importantly, while the AN model offers additional flexibility for modeling missingness, it is not assumption free—missing data always forces one to make identifying assumptions. In particular, the AN model posits that the reason for item nonresponse in \(X_1\) depends on \(X_1\) and \(Y_1\) through a function that is additive in \(X_1\) and \(Y_1\). For further discussion of this point, see Deng et al. (2013).

2.3. Two variables with item nonresponse only

We next extend the scenario in Section 2.2 so that both variables suffer from item nonresponse, without unit nonresponse. Thus, we need models for both \(R_x^y\) and \(R_y^x\), so that we add \(R_y^x\) to \(D^{obs}\). Here, we assume that we know the marginal probabilities only for \(X_1\). Table 2 displays the structure of the observed and auxiliary data.

We can factor the joint distribution of all four variables into the product of \(\theta_{y|x, y} = \Pr(X_1 = 1 | Y_1 = y, R_x^y = r^x, R_y^y = r^y)\), \(\pi_{x|y} = \Pr(Y_1 = 1 | R_x^y = r^x, R_y^y = r^y)\), \(q_{y|x} = \Pr(R_x^y = 1 | R_y^y = r)\), and \(p = \Pr(R_y^y = 1)\). This factorization results in a total of 15 parameters, plus the constraint that all joint probabilities sum to one. We can estimate seven of these parameters, \((p, q_0, q_1, \pi_{00}, \pi_{10}, \theta_{000}, \theta_{100})\) from \(D^{obs}\) alone. The auxiliary margin \(\Pr(X_1 = 1)\) adds one more piece of information, which takes the form of a constraint on the inestimable parameters, namely

\[
\Pr(X_1 = 1) - \Pr(X_1 = 1, R_x^y = 0, R_y^y = 0) = pq_1[\theta_{011}(1 - \pi_{11}) + \theta_{111}\pi_{11}] + p(1 - q_1)[\theta_{001}(1 - \pi_{01}) + \theta_{101}\pi_{01}] + (1 - p)q_0[\theta_{010}(1 - \pi_{10}) + \theta_{110}\pi_{10}].
\]
Thus, with this constraint generated by $A$, we are able to estimate a total of eight parameters.

Following Step 1, we specify a model with at most seven parameters, the maximum identifiable from $D^{obs}$ alone. For a selection model factorization, one natural specification couples (1) with

$$\Pr(R_x^1 = 1|X_1, Y_1) = h_1(\eta_0 + \eta_1 Y_1)$$

(5)

$$\Pr(R_y^1 = 1|X_1, Y_1, R_x^1) = k_1(\zeta_0 + \zeta_1 X_1).$$

(6)

We do not use $X_1$ in (5) or $Y_1$ in (6). Doing so would lead to models that are not necessarily identifiable, as we do not observe $X_1$ and $R_x^1$ simultaneously, nor $Y_1$ and $R_y^1$ simultaneously. In contrast, (5) and (6) are special cases of the itemwise conditionally independent (ICIN) mechanism of Sadinle and Reiter (2017).

Alternatively, one could use one of the nonresponse indicators as a predictor. For example, one could replace (6) with

$$\Pr(R_y^1 = 1|X_1, Y_1, R_x^1) = k_1(\zeta_0 + \zeta_1 X_1 + \zeta_2 R_x^1),$$

(7)

resulting in a slightly different variation of the model within the same framework. It is often more relevant or plausible to consider models where item nonresponse for a variable depends on the survey variables rather than the nonresponse indicators for other variables. Thus, we focus on model specification using (6) rather than (7). Nonetheless, analysts can choose (7) should this make more sense for the particular context or as part of sensitivity analyses; the framework facilitates such flexibility.

In Step 2, we add terms involving $X_1$, since we have auxiliary information on this variable. When building off (6), we can add $\eta_2 X_1$ to (5) since (6) already contains $\zeta_1 X_1$, so that the nonresponse models are (5) and

$$\Pr(R_x^1 = 1|X_1, Y_1) = h_1(\eta_0 + \eta_1 Y_1 + \eta_2 X_1).$$

(8)

We note that (8) is an AN model for $X_1$, with the caveat that $Y_1$ also suffers from item nonresponse. Thus, like the AN model in Section 2.2, by using the margin we now have a selection model that includes ignorable and nonignorable mechanisms as special cases.

Should an analyst prefer (7) to (6), we have two ways to implement Step 2. We could add $\zeta_1 X_1$ to (7), so that our nonresponse models are (6) and

$$\Pr(R_y^1 = 1|X_1, Y_1, R_x^1) = k_1(\zeta_0 + \zeta_1 X_1 + \zeta_2 R_x^1);$$

(9)

or, we could add $\eta_2 X_1$ to (5) as before, resulting in (7) and (8) as the nonresponse models. Here, (7) and (8) are also special cases of the ICIN mechanism. Both choices of selection model factorizations are examples of sequentially additive nonignorable models (Sadinle and Reiter, 2019).

2.4. Unit nonresponse and two variables subject to item nonresponse

We now show how to incorporate unit nonresponse within the framework by directly extending our two-variable scenario to include unit nonresponse. Let $X$ contain two binary variables, $X_1$ and $X_2$, and let $Y$ be empty. In other words, we have auxiliary margins for
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Table 3. Data structure for example with unit nonresponse and two variables subject to item nonresponse. Here, “✓” represents observed components and “?” represents missing components.

| $X_1$ | $X_2$ | $R^x_1$ | $R^x_2$ | $U$ |
|------|------|--------|--------|-----|
| ✓    | ✓    | 0      | 0      | ✓   |
| ?    | ✓    | 1      |        |     |
| ✓    | ?    | 0      | 1      | ✓   |
| ?    | ?    | 1      |        |     |
| ?    | ?    | ?      | ?      | 1   |
| ✓    | ?    | ?      | ?      |     |
| ?    | ✓    | ?      | ?      |     |
| ?    | ?    | ?      | ?      |     |

Table 3. Data structure for example with unit nonresponse and two variables subject to item nonresponse. Here, “✓” represents observed components and “?” represents missing components.

We refer to this specification as MCAR+ICIN, as it assumes a MCAR mechanism on the unit nonresponse indicator and ICIN mechanisms on both item nonresponse indicators. Here, $\eta_0$ is identifiable since the marginal probability of $U$ is known from the observed data alone; $\zeta_0$ and $\zeta_1$ are identifiable since the joint relationship between $R^x_1$ and $X_2$ can be estimated from the observed data; and, $\gamma_0$ and $\gamma_1$ are identifiable since the joint relationship between $R^x_2$ and $X_1$ can be estimated from the observed data. Given these assumptions on the nonresponse indicators, $\Theta$ is identifiable.

We can use Step 2 of the MD-AM framework to leverage the two additional constraints from $\mathcal{A}$ and relax some of the assumptions in the MCAR+ICIN model. We can add
two parameters to the models in many ways, reflecting different assumptions about the missingness mechanisms. We present a few of them here.

One option is to use the margins to enhance the model for unit nonresponse. In particular, we couple (12) and (13) with an AN model for $U$,

$$\Pr(U = 1 \mid X_1, X_2) = g(\eta_0 + \eta_1 X_1 + \eta_2 X_2).$$ (14)

This model inherits the flexibility of AN models in the specification for the unit nonresponse model, in that it encompasses conditionally ignorable and nonignorable models as special cases. The model implies that the item nonresponse models are the same for unit respondents and nonrespondents. We have to make this assumption, as we never observe $R_{xi}^t$ for cases with $U_i = 1$.

This specification is particularly appropriate when agencies consider unit nonresponse to be potentially nonignorable and the item nonresponse to be ICIN. It also can be preferred when agencies want to dedicate the auxiliary information to richer modeling of $U$ than $R$. Agencies can do so when unit nonresponse is a greater threat to the quality of inferences than item nonresponse, for example, when the number of unit nonrespondents is much larger than the number of missing item values for each variable. We refer to this specification as AN-U.

Another option is to use a MCAR model for unit nonresponse and AN models for the item nonresponse indicators. That is, we use (11) and

$$\Pr(R_{xi}^1 = 1 \mid X_1, X_2, U) = h_1(\zeta_0 + \zeta_1 X_2 + \zeta_2 X_1)$$ (15)

$$\Pr(R_{xi}^2 = 1 \mid X_1, X_2, U, R_{xi}^1) = h_2(\gamma_0 + \gamma_1 X_1 + \gamma_2 X_2).$$ (16)

This specification is appealing for situations where agencies feel it important to use richer models for $R$ than $U$, for example, when the amount of item nonresponse is much larger than the amount of unit nonresponse. We refer to this specification as AN-R.

Finally, a compromise model is an ICIN model for unit nonresponse and one of the item nonresponse indicators, plus an AN model for the other item nonresponse indicator. Specifically, we can use

$$\Pr(U = 1 \mid X_1, X_2) = g(\eta_0 + \eta_2 X_2),$$ (17)

(13), and (15), which we refer to as AN-R$^1_2$ in our simulations. Similarly, we can use

$$\Pr(U = 1 \mid X_1, X_2) = g(\eta_0 + \eta_1 X_1),$$ (18)

(12), and (16), which we refer to as AN-R$^2_1$ in our simulations. Such models can be useful when $D$ has a large amount of unit nonresponse, and only one of the variables has a large amount of item nonresponse (and the other does not). In this way, we utilize the information from $A$ to enrichen the models for both unit and item nonresponse.

2.5. Illustrative simulations

As the previous example shows, the MD-AM framework offers agencies the flexibility to specify different mechanisms for item and unit nonresponse, and the ability to enrich the modeling for the nonresponse deemed most critical to the quality of survey inferences.
Table 4. Parameter values for generating nonresponse under each of the five nonresponse mechanisms. “M+I” stands for MCAR+ICIN.

| Nonresponse Mechanism | Parameter | M+I | AN-U | AN-R | AN-R\textsuperscript{x1} | AN-R\textsuperscript{x2} |
|-----------------------|-----------|-----|------|------|----------------|----------------|
| η\textsubscript{0}     |           | -1.71 | -1.00 | -1.71 | -1.21 | -1.92 |
| η\textsubscript{1}     |           | -1.10 | -1.00 | -1.10 | -1.39 |      |
| η\textsubscript{2}     |           | -1.45 | -1.00 | -1.45 | -1.50 |      |
| ζ\textsubscript{0}     |           | -1.34 | -1.00 | -1.34 | -1.40 | -1.34 |
| ζ\textsubscript{1}     |           | -1.00 | -1.00 | -1.00 | -2.00 | -1.00 |
| ζ\textsubscript{2}     |           |       |       |       | -1.05 | -1.05 |
| γ\textsubscript{0}     |           | -1.00 | -1.00 | -1.58 | -1.00 | -1.58 |
| γ\textsubscript{1}     |           | -1.26 | -1.26 | -1.85 | -1.26 | -1.85 |

We now illustrate these models empirically using simulation studies for the scenario in Section 2.4. The supplementary material includes simulation results for other scenarios.

We simulate five datasets, each comprising \( n = 10,000 \) individuals, from the following sequence of logistic regressions.

\[
X_{i1} \sim \text{Bern}(\pi_{X_{i1}}); \quad \text{logit}(\pi_{X_{i1}}) = \alpha_0
\]

\[
X_{i2} | X_{i1} \sim \text{Bern}(\pi_{X_{i2}}); \quad \text{logit}(\pi_{X_{i2}}) = \beta_0 + \beta_1 X_{i1},
\]

where \( \alpha_0 = 0.65 \) and \( (\beta_0, \beta_1) = (0.5, -1) \). This creates a moderate amount of dependence between \( X_1 \) and \( X_2 \) in each dataset, which we look to preserve in the inferences resulting from the MD-AM framework. For each simulated dataset, we generate missing data from one of the five different nonresponse mechanisms discussed in Section 2.4, that is, MCAR+ICIN, AN-U, AN-R, AN-R\textsuperscript{x1} and AN-R\textsuperscript{x2}. This way, each dataset suffers from nonresponse generated according to exactly one of the five nonresponse mechanisms. We set the values for the parameters in the models for \( U, R_x, R_y \) and \( R_z \), in the five mechanisms, to result in approximately 15% missing cases subject to each form of nonresponse; the parameter values are shown in Table 4. We do so to mimic the average nonresponse rates in the variables that suffer most from nonresponse in the CPS application described in Section 3. This results in approximately 40% of sampled observations subject to some form of missing data, in each dataset.

We fit models based on (19) and (20), and logistic regressions corresponding to the five nonresponse mechanisms, to each simulated dataset. In each case, we re-estimate all parameters to examine how accurately each model estimates the joint distribution of \( X_1 \) and \( X_2 \), as well as other parameters. We fit all models using Bayesian Markov chain Monte Carlo (MCMC) samplers with non-informative priors for all parameters. We run each MCMC sampler for 10,000 iterations, discarding the first 5,000 as burn-in, resulting in 5,000 posterior samples; we base inferences on all 5,000 post burn-in samples. Although we use posterior inference throughout this article, one also could use approximately independent draws from the posterior samples to perform multiple imputation (MI) [Carpenter and Kenward, 2013] [Reiter and Raghunathan, 2007] [Rubin].
Table 5. Results for simulation study with unit nonresponse and two variables subject to item nonresponse. Entries are posterior means with standard errors in parenthesis. Here, “Par” stands for parameter, and “M+I” stands for MCAR+ICIN.

|        | Par | Truth | M+I      | AN-U     | AN-R     | AN-R₁⁺ | AN-R₂⁺ |
|--------|-----|-------|----------|----------|----------|--------|--------|
|        | α₀  | .65   | .66 (.02)| .65 (.01)| .65 (.01)| .65 (.01)| .65 (.01) |
|        | β₀  | .50   | .50 (.05)| .47 (.04)| .45 (.04)| .46 (.04)| .46 (.04) |
| M+I    | β₁  | -1.00 | -0.97 (.06)| -0.97 (.06)| -0.94 (.06)| -0.96 (.06)| -0.95 (.06) |
|        | α₀  | .65   | .78 (.03)| .65 (.01)| .67 (.01)| .66 (.01)| .65 (.01) |
| AN-U   | β₀  | .50   | .82 (.05)| .44 (.04)| .43 (.05)| .50 (.04)| .42 (.05) |
|        | β₁  | -1.00 | -1.16 (.06)| -0.92 (.06)| -0.89 (.07)| -1.00 (.06)| -0.88 (.07) |
|        | α₀  | .65   | .80 (.02)| .66 (.01)| .65 (.01)| .67 (.01)| .66 (.01) |
| AN-R   | β₀  | .50   | .32 (.05)| .44 (.04)| .44 (.05)| .40 (.04)| .52 (.05) |
|        | β₁  | -1.00 | -1.09 (.06)| -0.92 (.06)| -0.92 (.07)| -0.87 (.06)| -1.05 (.06) |
| AN-R₁⁺ | α₀  | .65   | .75 (.02)| .66 (.01)| .66 (.01)| .65 (.01)| .66 (.01) |
|        | β₀  | .50   | .84 (.05)| .49 (.05)| .40 (.05)| .44 (.05)| .40 (.04) |
|        | β₁  | -1.00 | -1.16 (.06)| -0.99 (.07)| -0.84 (.06)| -0.92 (.07)| -0.85 (.06) |
| AN-R₂⁺ | α₀  | .65   | .87 (.03)| .65 (.01)| .66 (.01)| .68 (.01)| .65 (.01) |
|        | β₀  | .50   | .37 (.05)| .37 (.04)| .39 (.04)| .29 (.04)| .43 (.04) |
|        | β₁  | -1.00 | -0.68 (.06)| -0.83 (.06)| -0.82 (.06)| -0.69 (.06)| -0.90 (.06) |

1987 [Schafer, 1997], which can be appealing for data dissemination [Alanya et al., 2015; Little and Vartivarian, 2005; Peytchev, 2012].

To incorporate the auxiliary marginal information in our simulations, we follow the approach of Schifeling and Reiter (2016) by augmenting the observed data with a large number of synthetic observations with empirical distributions that match the marginal probabilities in \( \mathcal{A} \). Specifically, for each simulated dataset, we generate \( n^* = 3n \) synthetic observations for each of the two variables with available auxiliary margins, resulting in a total of 60,000 synthetic observations added to \( D_{obs} \). We leave values of \((U, R₁, R₂)\) completely missing for the synthetic observations. We leave \( X₂ \) missing for the 30,000 synthetic observations corresponding to the auxiliary margin of \( X₁ \), and leave \( X₁ \) missing for the 30,000 synthetic observations corresponding to the auxiliary margin of \( X₂ \). We use this approach for estimating models that utilize \( \mathcal{A} \) (AN-U, AN-R, AN-R₁⁺ and AN-R₂⁺), but do not for baseline models that do not utilize \( \mathcal{A} \) (MCAR+ICIN). By using a large \( n^* \), we treat the auxiliary margins as having negligible standard errors. When \( \mathcal{A} \) has non-negligible uncertainty, one can make \( n^* \) smaller to correspond to the desired standard error, following the approach in Schifeling and Reiter (2016).

We repeat the simulation procedure five times; that is, we generate five simulated datasets each time. Results are qualitatively similar across all five repeated runs, so for ease of illustration we present results for one simulation run here. Table 5 displays the estimated posterior means and standard errors of the parameters of the joint distribution of \((X₁, X₂)\). The supplementary material includes estimates of the parameters of the nonresponse models. As expected, we are able to estimate true values of parameters accurately when using the correct models. This validates that the models are identifiable,
and that the augmented margins approach works effectively. When the data are generated according to the MCAR+ICIN mechanism, we are able to estimate the joint distribution of \((X_1, X_2)\) accurately when using any of the MD-AM models, illustrating the additional flexibility of using the framework. In contrast, when the true nonresponse model is not the MCAR+ICIN, fitting the MCAR+ICIN model results in less accurate estimates than all four other MD-AM models. In other words, any of the MD-AM options are an improvement on MCAR+ICIN.

For the most part, the results are quite similar when we either assume AN-U and the true nonresponse model is AN-R, or vice versa. This shows the flexibility the MD-AM framework offers as we are able to account for nonignorable unit nonresponse by allowing for nonignorable item nonresponse in all variables and vice versa. This is especially plausible when the unit nonresponse rate is very similar to the item nonresponse rate for each variable, as is the case in this simulation setup. AN-U and AN-R also perform just as good or better than AN-R_x^1 (AN-R_x^2) when the true mechanism is AN-R_x^2 (AN-R_x^1). Both AN-R_x^1 and AN-R_x^2 perform relatively well as compromise methods when the true mechanism follow either AN-U or AN-R. However, when the true mechanism is AN-R_x^2, AN-R_x^1 results in the least accurate estimates of all methods. This underlines the importance of examining sensitivity to different specifications. When the missingness mechanism for one variable is nonignorable but we assume that it is ignorable, whereas the true mechanism for another variable is ignorable but we assume that it is nonignorable, we could potentially end up with biased or inaccurate estimates. Analysts must keep this in mind when applying the MD-AM framework.

2.6. Implementation considerations

These examples illustrate how the MD-AM framework enables agencies to tailor how they use information in the auxiliary marginal distributions. For example, agencies can use \(A\) to specify AN models for unit (item) nonresponse indicator when they want to dedicate model flexibility for unit (item) nonresponse models. However, agencies need not select only one model in the MD-AM framework. They can examine sensitivity of inferences to different specifications, as we do implicitly in the simulations of Section 2.5. When using the MD-AM framework to release multiple imputations, we conjecture that agencies can use the approach of Siddique et al. (2012), to incorporate uncertainty regarding the missing data mechanism.

When specifying the sequence of conditional models, one needs to decide the ordering of the variables. For the survey variables, the order is somewhat arbitrary, in that we seek to characterize their joint distribution. For practicality, we recommend following the advice in typical missing data imputation routines (Burgette and Reiter, 2010; van Buuren et al., 2006; van Buuren and Groothuis-Oudshoorn, 2011; van Buuren, 2012) and ordering from least to most missing values. For the nonresponse indicators, we find it convenient to put variables with auxiliary margins early in the sequence and variables without auxiliary margins later in the sequence. It can be easier to interpret the nonresponse mechanisms, and thus decide how to use the information in \(A\), in models with fewer terms, which is the case for the models early in the sequence. In our simulations, the ordering of the nonresponse indicators does not seem to affect the results noticeably, especially when we do not use the nonresponse indicators as predictors. Nonetheless,
agencies can assess sensitivity of results to orderings of the variables.

Theoretically, it might be impossible to distinguish between unit nonresponse and item nonresponse for some cases, e.g., when an individual does not provide information on any of the questions used in a particular analysis. This is not a problem in all our simulation scenarios by design. However, when agencies cannot distinguish the two forms of nonresponse, they may need to incorporate assumptions about the nonresponse indicators into the modeling in order to have identifiable models. For example, agencies can treat individuals who do not respond to any of the questions being analyzed as equivalent to unit nonrespondents. In this case, we add the constraint of zero probability to the chance that all item nonresponse indicators equal one; for example, set $Pr(R_x^2 = 1)$ to zero whenever $R_x^1 = 1$ and vice-versa. In this way, the model for $U$ completely captures all unit nonrespondents plus item nonrespondents who do not respond to any questions.

Extending the MD-AM approach to more variables, as well as categorical variables with more than two levels, is conceptually straightforward. When the data include $Y$ variables with missing values, as is usually the case, we simply add $Y$ in the models for $f(X,Y)$ and add conditional models for each $R_y$. We recommend putting the conditional models for $R_y$ at the end of the sequence. Because we do not have marginal distributions for these $Y$ variables, we are forced to make stronger assumptions about them, such as ICIN or MAR. When the data include multiple variables with margins in $A$, we recommend treating them as we do $X_1$ and $X_2$ in the illustrative simulations. For example, if we have the marginal distribution for some variable $X_k$, we can add terms involving $X_k$ to the item nonresponse model for $R_x^1$ or to the unit nonresponse model for $U$. As before, with the MD-AM framework, the agency can choose where to dedicate the extra modeling flexibility. We demonstrate this with an analysis of voter turnout in CPS data, as we now describe.

3. Application to CPS Voter Turnout Data

Scholars and policymakers often lament over low voter turnout in the United States. Turnout rates typically fall just below 60% in presidential elections, 40% in midterm elections, and are lower still in off-year local elections—among the worst participation rates in advanced democracies (Leighley and Nagler, 2013). Turnout is also unequal among demographic subgroups, and especially low among young Americans, who consistently vote at rates 20 to 30 percentage points lower than older citizens (Holbein and Hillygus, 2016). Although there is widespread recognition of low and unequal electoral participation, it turns out that estimating turnout rates, especially for subgroups, can be difficult.

Given that we have official government counts of ballots cast in an election, it might seem puzzling that calculating turnout rates is at all complicated. There are two problems. First, estimates of ballots cast are often not available by population subgroups of interest. The ability to calculate votes by demographic subgroups depends on the information available in voter registration records, which varies across states. Second, to calculate a turnout rate, we need an estimate of the denominator, i.e., the voting eligible population. Although there are official estimates of the voting age population from Census Bureau data, not all adults living in a locale are in fact eligible to vote. The
discrepancy between voting age population and vote eligible population is mostly due to disenfranchised felons, non-citizens, and citizens living abroad. This discrepancy has grown over time, and varies considerably across states and subgroups of the population [McDonald and Popkin, 2001]. Given these issues with official election records, many researchers rely instead on survey-based estimates of turnout.

Among surveys, the CPS is considered the gold standard for estimating voter turnout. Every Congressional election year the CPS November Supplement asks a variety of questions about voter registration and turnout. Response rates for the survey far exceed those of other surveys; in 2008, for instance, the RR6 rate was 88 percent (U.S. Census Bureau 2010, 16-4). Additionally, the CPS is one of the few surveys with sufficient sample size to make turnout estimates by state, as the sample size exceeds 75,000 voting-age citizens, stratified by state.

Nonetheless, the CPS voter turnout measure is plagued by high levels of missing data, as we document in Section 3.1. This may be due in part to proxy responses, where one individual in a household answers on behalf of all members of the household. In its official reports, the CPS treats “Don’t Know,” “Refused,” and “No Response” as indicating that the respondent did not vote: “Nonrespondents and people who reported that they did not know if they voted were included in the ‘did not vote’ class because of the general overreporting by [other] respondents in the sample” (U.S. Census Bureau [2010a], at “Voter, Reported Participation”). This means the official Census Bureau estimates impute item nonrespondents in the November Supplement as nonvoters. Biased estimates occur from this practice. For example, in the 2008 U.S. Presidential Election between Barack Obama and John McCain, despite a historic number of ballots cast, the official CPS estimate reported a turnout rate that was slightly lower than their 2004 estimates (Hur and Achen, 2013). Although imputing all item nonrespondents as nonvoters helps to reduce bias in the overall turnout estimates, it is unclear if it might increase bias in estimates for particular subgroups. For example, young people are both more likely to be survey nonrespondents and to be non-voters, raising the possibility that CPS estimates of youth turnout could be biased without corrections for nonresponse.

Alternatively, one could drop all persons who do not respond to all of the variables of interest, and re-weight the remaining available cases (Hur and Achen, 2013). While this may work in some scenarios, it has drawbacks as a general strategy. It forces unit nonrespondents’ and item nonrespondents’ values to be from the same distribution; it sacrifices the information in partially complete records; it can make the subset of records and values of weights used in the analysis model dependent on the variables used in that model; and, it complicates estimation of standard errors. We instead apply the MD-AM framework, as we now describe.

3.1. Data
We restrict our analysis to 2012 CPS data from Florida (FL), Georgia (GA), North Carolina (NC) and South Carolina (SC). All are southern states that vary in demographic composition, as well as their battleground election status in 2012. We use the four variables described in Table 6. We discuss extensions to additional variables in Section 4. The resulting dataset comprises $n = 10,800$ individuals with data missing according to the nonresponse rates in Table 7. Item nonresponse is trivial for sex and fairly low for
Table 6. Description of variables used in illustration.

| Variable Categories                                                                 |
|-------------------------------------------------------------------------------------|
| State 1 = Florida, 2 = Georgia, 3 = North Carolina, 4 = South Carolina               |
| Sex 0 = Male, 1 = Female                                                             |
| Age 1 = 18 - 29, 2 = 30 - 49, 3 = 50 - 69, 4 = 70+                                |
| Vote 0 = Did not vote; 1 = Voted                                                    |

Table 7. Unit and item nonresponse rates by state. Seven individuals are missing sex.

| Unit | Item    | Vote | Sex | Age |
|------|---------|------|-----|-----|
| FL   | Vote .18| .18  | .00 | .07 |
| GA   | Vote .11| .16  | .00 | .05 |
| NC   | Vote .14| .11  | .00 | .03 |
| SC   | Vote .16| .10  | .00 | .03 |

age, but it is substantial in vote. Unit nonresponse is also substantial in all four states.

We use the voter-eligible population (VEP) for highest office as marginal information for voter turnout by state. These percentages are as follows: FL = 62.8%, GA = 59.0%, NC = 64.8% and SC = 56.3%. We obtain this aggregate-level information for 2012 from The United States Elections Project (USEP) [http://www.electproject.org/2012g](http://www.electproject.org/2012g), which compiles government data to create election year estimates of the voting eligible population from the American Community Survey and Department of Justice felon estimates (McDonald, 2008). We also use marginal information for the age groups by state from the 2010 census. Although this auxiliary information is two years behind the 2012 CPS, we use it nonetheless for the purposes of this illustration because: (i) the decennial census provides the most accurate data on demographics across states, in comparison to other data sources, and (ii) we expect the demographic information to be fairly consistent between 2010 and 2012, and fairly similar both for eligible and non-eligible voters, at least up to small standard errors.

3.2. Modeling

Let $S_i$, $G_i$, $A_i$ and $V_i$ represent the state, sex, age and vote of the $i = 1, \ldots, n$ individuals in the data. All four variables are in $D$. Population margins for all variables but $G_i$ are in $A$. We do not rely on auxiliary margins for sex from the 2010 census because item nonresponse is trivial for sex—only seven cases are missing a value for sex—and to also illustrate how the absence of auxiliary information for at least one variable affects model specification in this application. As before, let $U_i$ represent the unit nonresponse indicator for individual $i$. Also, let $R^G_i$, $R^A_i$ and $R^V_i$ be item nonresponse indicators for individual $i$, for sex, age and vote respectively, where each equals one if the corresponding variable is missing and equals zero otherwise.

The item nonresponse in all states has a monotone pattern: sex is always missing...
when age and vote are missing, and age is always missing when vote is missing. We therefore include constraints in the item nonresponse models to respect the monotone patterns. Without these constraints, the models would fail to capture the nonresponse process adequately. Thus, we generally recommend that one investigates the empirical distributions of the missing data indicators to help with model specification.

The relationships between \( S_i \) and the other variables, as well as the nonresponse indicators, are always observed. Therefore, we use \( S_i \) as a covariate in all the models. Following Step 1 of the MD-AM framework, we specify the following models for the distribution of \((G_i, A_i, V_i)\).

\[
G_i | S_i \sim \text{Bern}(\pi_i^G); \\
\text{logit}(\pi_i^G) = \beta_1 + \beta_{2j} \mathbb{I}[S_i = j] \\
A_i | G_i, S_i \sim \text{Cat}(\text{Pr}[A_i \leq k] - \text{Pr}[A_i \leq k - 1]); \\
\text{logit}(\text{Pr}[A_i \leq k]) = \phi_{1,k} + \phi_{2j} \mathbb{I}[S_i = j] + \phi_3 G_i \\
V_i | A_i, G_i, S_i \sim \text{Bern}(\pi_i^V); \\
\text{logit}(\pi_i^V) = \nu_1 + \nu_{2j} \mathbb{I}[S_i = j] + \nu_{3j} \mathbb{I}[A_i = j] + \nu_4 G_i + \nu_{3jk} \mathbb{I}[S_i = j, A_i = k].
\]

Here, the model for \( A_i \) is a proportional odds regression, and the models for \( G_i \) and \( V_i \) are logistic regressions. For parsimony, we exclude all interaction terms except \( \mathbb{I}[S_i = j, A_i = k] \). We do not see evidence that the excluded interaction terms have important predictive power based on exploratory data analysis. In scenarios where the interaction terms are potentially important or of inferential interest, they can be included since they can be estimated based on our discussions in Section 2.

Following Step 2 of the MD-AM framework, we next specify models for the nonresponse indicators. Since we are most interested in estimating turnout, and \( V \) has a high item nonresponse rate, we first use the margin for \( V \) to estimate an AN model for \( R^V \). Since we have a high unit nonresponse rate and a low item nonresponse rate for age, we use the margin for \( A \) to enrich the model for \( U \) and sacrifice the use of an AN model for \( R^A \). We therefore use an ICIN model for \( R^A \), as a default option for estimating the maximum number of parameters possible based on the remaining information. Since we do not have a margin for sex, we cannot include \( G \) in the models for \( R^G \) or \( U \). Since the nonresponse rate for sex is so low, we adopt a MAR model for \( R_i^G \) for computational convenience, even though we can identify parameters for \( A \) and \( V \) in the model for \( R^G \).

We write our specification formally as

\[
U_i | \ldots \sim \text{Bern}(\pi_i^U); \\
\text{logit}(\pi_i^U) = \gamma_1 + \gamma_{2j} \mathbb{I}[S_i = j] + \gamma_{3j} \mathbb{I}[A_i = j] \\
R_i^G | U_i, \ldots \sim \text{Bern}(\pi_i^{RG}); \\
\text{logit}(\pi_i^{RG}) = \eta_1 + \eta_{2j} \mathbb{I}[S_i = j] \\
R_i^A | R_i^G, U_i, \ldots \sim \text{Bern}(\pi_i^{RA}(1-R_i^G)); \\
\text{logit}(\pi_i^{RA}) = \alpha_1 + \alpha_{2j} \mathbb{I}[S_i = j] + \alpha_3 G_i + \alpha_4 V_i
\]
In each model, "..." represents conditioning on \((V_i, A_i, G_i, S_i)\). We cannot add \(\gamma_4 V_i\) to (24), because this information can only come from the margin for vote \((V_i \text{ and } U_i \text{ are never observed together})\) and we have chosen to use the margin to estimate \(\psi_5 V_i\) in (27).

We incorporate marginal information by augmenting the observed data with \(n^* = 3n\) synthetic observations for each of the two variables with available auxiliary margins, resulting in a total of 64,800 synthetic observations added to the observed data. For each margin, we augment with three times the size of the observed data so that the empirical margins match the auxiliary information with negligible standard error. We fit all models using Bayesian MCMC, with non-informative priors for all parameters. We run the MCMC sampler for 10,000 iterations, discarding the first 5,000 as burn-in, resulting in 5,000 posterior samples. We base inferences on all 5,000 post burn-in posterior samples.

### 3.3 Results

Figure 1 displays one of the key advantages of the MD-AM framework: it generates posterior predictive distributions for turnout for those who did not respond to the survey. The MD-AM models in (21) to (27) predict more than 48% of unit nonrespondents in each state in the data to be voters. Again, these individuals are omitted from the calculation of turnout for the official CPS estimates. As a predictive check, we also construct posterior predictive intervals for all four-way joint probabilities from the contingency table for
Table 8. Turnout estimates of sub-populations by state for MD-AM compared to the complete cases. "M" stands for male and "F" stands for female. MD is our MD-AM framework and CC uses complete cases.

|        | FL MD | FL CC | GA MD | GA CC | NC MD | NC CC | SC MD | SC CC |
|--------|-------|-------|-------|-------|-------|-------|-------|-------|
| Full   | .62   | .75   | .60   | .73   | .65   | .77   | .58   | .73   |
| M      | .60   | .73   | .57   | .71   | .63   | .76   | .55   | .68   |
| F      | .65   | .77   | .62   | .75   | .67   | .79   | .60   | .77   |
| <30    | .47   | .55   | .44   | .56   | .50   | .64   | .47   | .64   |
| 30-49  | .60   | .73   | .61   | .74   | .65   | .76   | .55   | .70   |
| 50-69  | .69   | .82   | .70   | .83   | .72   | .82   | .68   | .78   |
| 70+    | .72   | .84   | .62   | .76   | .76   | .84   | .62   | .75   |
| <30(M) | .45   | .52   | .42   | .49   | .48   | .58   | .44   | .57   |
| 30-49(M)| .58  | .58   | .59   | .61   | .63   | .68   | .53   | .72   |
| 50-69(M)| .67  | .69   | .68   | .70   | .70   | .72   | .66   | .67   |
| 70+(M) | .70   | .77   | .60   | .77   | .74   | .81   | .60   | .73   |
| <30(F) | .50   | .80   | .47   | .83   | .53   | .83   | .49   | .71   |
| 30-49(F)| .62  | .84   | .63   | .83   | .67   | .82   | .57   | .83   |
| 50-69(F)| .71  | .86   | .72   | .81   | .74   | .88   | .70   | .76   |
| 70+(F) | .74   | .81   | .64   | .73   | .78   | .82   | .64   | .74   |

\( \mathcal{D} \), based on all 5,000 posterior samples. For approximately 94% of the four-way joint probabilities, the predictive intervals contained the observed data point estimates.

We next turn to explicit comparisons with other approaches of turnout estimation. For starters, we compare MD-AM to a complete case (or uncorrected) turnout estimate in Table 8. The complete case method suggests higher turnout rates across every subgroup, particularly among the youth respondent subgroups. The large differences between estimates illustrates the differences in unit and item nonresponse. It also could reflect an upward bias from using self-reported turnout data (DeBell et al., 2018), which has been found to be around 6% in other election surveys (Enamorado and Imai, 2019; Jackman and Spahn, 2014). We discuss the interaction of the MD-AM framework and the potential measurement error in Section 4.

Next, we turn to turnout estimate comparisons between MD-AM and the CPS method. Table 9 allows us to make a variety of comparisons between these methods’ estimates of turnout. At the state-level, MD-AM is comparable to the CPS estimates in Florida and Georgia, but differs from the CPS estimates in North and South Carolina. Turning to subgroup turnout estimates, we see that MD-AM estimates of turnout differ from the CPS method for the age categories in North Carolina and South Carolina. Estimates of turnout for those under the age of 30 are around 6 to 9 percentage points lower than CPS estimates in North Carolina and South Carolina. These trends continue when we further break down voters under 30 by sex (see the supplementary material for full marginal results). As expected, these subgroup estimates are the most dramatically different across the methods, likely because youth voters are known to experience both low levels of turnout and high levels of survey non-response compared to other age groups.
Table 9. Turnout estimates of sub-populations by state for different methods. MD is our MD-AM framework and CPS is the current weighted CPS estimates using the U.S. Census Bureau’s method.

|       | FL  | GA  | NC  | SC  |
|-------|-----|-----|-----|-----|
|       | MD  | CPS | MD  | CPS | MD  | CPS | MD  | CPS |
| Full  | .62 | .61 | .60 | .62 | .65 | .69 | .58 | .65 |
| <30   | .47 | .46 | .44 | .47 | .50 | .56 | .47 | .56 |
| 30-49 | .60 | .58 | .61 | .61 | .65 | .69 | .55 | .62 |
| 50-69 | .69 | .67 | .70 | .72 | .72 | .73 | .68 | .71 |
| 70+   | .72 | .70 | .62 | .66 | .76 | .77 | .62 | .70 |

Fig. 2. Distributions of deviations in MD-AM estimates for voter composition from Catalist estimates.

While benchmark comparisons do not exist for turnout among subgroups, we are able to provide a validation of our turnout estimates by comparing the demographic composition of voters in our sample compared to voter files in those states. To do so, we rely on voter files by the data firm Catalist. The Catalist data offers fully observed estimates (with negligible standard error) of the joint probabilities of turnout rates for each subgroup variable we included in our analysis. However, reliable auxiliary marginal information in the Catalist data is available only for those registered to vote, and not for the entire voting eligible population. Figure 2 displays the comparisons between the composition of voters in the Catalist data and the MD-AM estimates in each of the four states. MD-AM produces a composition of voters nearly identical to estimates produced using Catalist’s verified voters in these states. Overall, we find that using more reasonable assumptions about missingness creates different turnout estimates from the CPS method in two of the four states, for subgroups where we would expect both low turnout and survey non-response to be common (young voters).

As a sensitivity analysis, we also consider including $V_i$ in the unit nonresponse model.
instead of the item nonresponse model for vote. We modify (24) and (27) as follows.

\[
U_i | \ldots \sim \text{Bern}(\pi_{U_i}^U);
\]
\[
\logit(\pi_{U_i}^U) = \gamma_1 + \gamma_2 j \mathbb{I}[S_i = j] + \gamma_3 j \mathbb{I}[A_i = j] + \gamma_4 V_i \tag{28}
\]
\[
R_i^V | R_i^A, R_i^G, U_i, \ldots \sim \text{Bern}(\pi_{R_i}^V | (1 - R_i^A));
\]
\[
\logit(\pi_{R_i}^V) = \psi_1 + \psi_2 j \mathbb{I}[S_i = j] + \psi_3 j \mathbb{I}[A_i = j] + \psi_4 G_i \tag{29}
\]

When we replace (24) and (27) with (28) and (29), and leave the other models unchanged, the overall conclusions remain the same. However, now most unit nonrespondents are predicted to be non-voters, and instead, more than half of the item nonrespondents for vote are predicted to be voters. This phenomenon is due to the large differences between the turnout rates from the complete cases and those from the VEP. Essentially, for the model to make up for the large differences, a lot of nonrespondents must be predicted as nonvoters, and whether those would be unit or item nonrespondents depends on whether the margin for vote is used in the unit or item nonresponse models. In fact, our results in Table 9 are mostly similar to the CPS estimates especially in Florida and Georgia even though we make different assumptions. There is so much of an upward bias in the observed data estimates of vote turnout that, one way or another, a large number of missing values in the vote variable must be imputed as nonvoters to match the margin. This would not be the case if the rates in the complete cases were closer to the true margins. Both MD-AM models, however, predict that a sizeable proportion of nonrespondents are in fact voters, which clearly challenges the assumption of the CPS method.

4. Discussion

The MD-AM framework provides a flexible framework for handling unit and item nonresponse when auxiliary data are available. Specifically, the framework allows analysts to leverage information on marginal distributions to identify extra parameters in model-based approaches for nonresponse. Analysts can dedicate these extra parameters to the models where flexibility is most beneficial, thereby allowing for nonignorable missing data models and enriching nonresponse modeling more broadly.

With respect to the CPS application, one area for future improvement is to deal more carefully with the reporting error in voter turnout. Some respondents are inclined to say they voted even though they did not, because it is socially desirable to vote. The MD-AM framework could be extended to handle reporting error through a hierarchical specification, as we can add a reporting model to explain how reported values are generated from the true unobserved values. This is a subject for future work.

We presented MD-AM via specifying a sequence of parametric regressions. This can be challenging in practice with large numbers of variables, as the number of possible model specifications can be very large, especially when one considers interaction terms. Model selection is further complicated by identification constraints. Thus, an important future research topic is to incorporate nonparametric methods, such as classification and regression trees and Bayesian additive regression trees, in the MD-AM framework.
Finally, the framework presented here does not incorporate survey design variables or survey weights. In complex surveys, using such design information can improve multiple imputation inferences (Reiter et al., 2006). Finding practical methods for general complex designs is another important topic for future research.

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