Preliminary results on the suppression of sensing cross-talk in LISA Pathfinder

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Abstract. In the original paper describing the first measurements performed with LISA
Pathfinder, a bulge in the acceleration noise was shown in the 200 mHz - 20 mHz frequency
band. This bulge noise originated from cross-coupling of spacecraft motion into the longitudinal
readout and it was shown that it is possible to subtract this cross-talk noise. We discuss here the
model that was used for subtraction as well as an alternative approach to suppress the cross talk
by realignment of the test masses. Such a realignment was performed after preliminary analysis
of a dedicated cross-talk experiment, and we show the resulting noise suppression. Since then,
the analysis of these experiments is still on-going.

1. Introduction
The well known LISA Pathfinder differential acceleration measurement published in [1] and
displayed for convenience in Fig. 1(a) showed a bulge in the noise curve in the 200 mHz -
20 mHz frequency band. This bulge was caused by cross-coupling of spacecraft motion leaking
into the main differential acceleration measurement and it was shown in [1] that it is possible
to subtract the cross-coupling noise resulting in the blue curve in Fig. 1(a). This subtraction
was performed by fitting the following model to the measured acceleration noise, i.e. to the red
trace in Fig. 1(a) yielding estimates of the coefficients $C_i$ and $\delta_{ifo}$:

$$\Delta g_{\text{cross-talk}} = C_1 \ddot{x}_2[t] + C_2 \ddot{y}_1[t] + C_3 \ddot{y}_2[t] + C_4 \ddot{z}[t] + C_5 \ddot{x}_1[t] + \delta_{ifo} \ddot{x}_1[t].$$

(1)

Each term in this equation defines a cross-coupling of spacecraft motion to the differential
acceleration measurement:

- All but the last summand are cross-coupling terms, because $\phi, \eta, y, z$ are orthogonal
  coordinates to the differential readout along $x$. See Fig. 2(a) for a definition of the coordinate
  system.
- The annotation $\ddot{k}$ with $k \in \{\phi, \eta, y, z\}$, defines a mean motion of both test masses along $k$,
  which is a description of the spacecraft motion along $-k$.
- The last summand, describes a residual coupling of spacecraft motion along $x$ entering
  the differential readout. As the term ‘differential readout’ indicates, $\Delta g$ is defined as
  $\Delta g = \ddot{x}_2 - \ddot{x}_1$ (neglecting here corrections such as for applied or inertial forces). It should
(a) Subtraction method

(b) Suppression method

Figure 1. (a) Result taken from [1], showing that the bulge at 200-20 mHz can be subtracted. (b) In this figure, no subtraction was performed. Instead the reduction was achieved by a realignment of the test masses prior to the second noise run.

therefore be insensitive to any common-mode motion, where \( \ddot{x}_2 = \ddot{x}_1 \). Since spacecraft motion effectively shows as common test mass motion, \( \Delta g \) should not sense spacecraft motion. However, there is a residual leakage, because the interferometer readout response is not identical to TM1 motion as to TM2 motion, such that \( \Delta g = k_2 \ddot{x}_2 - k_1 \ddot{x}_1 \) with \( k_2 \approx k_1 \approx 1 \). The small difference between \( k_1 \) and \( k_2 \) causes the residual cross-coupling.

Fitting eq. (1) to the measurement data and subtracting the resulting model works very well, as visible in Fig. 1(a). However, this technique has several disadvantages: First of all, the model provides no interpretation what the coefficients physically mean, and therefore no explanation how the cross-coupling could be suppressed. Secondly, several parameters show high correlation, such that the given model is not the only one allowing a fit and subtraction with low residuals. Instead, there is a set of models, performing comparably well. Finally, the coefficients vary over mission time, and this variation cannot be explained with the shown model.

For these reasons, there is an ongoing effort, to build a physical model, which can describe the cross-coupling over the entire mission duration with a constant set of parameters, and can be used without adaptation for any set of data to subtract the bulge. Additionally, such a model also allows a direct suppression of the cross-talk. In fact, Fig. 1(a) shows a noise run performed in April 2016 (2016-04-08 to 2016-04-14), which was after a first realignment of the test masses had already reduced the amount of cross-talk. This will be further discussed in the next section.

2. Cross-coupling suppression

Of all terms given in eq. (1), we will focus now on the first four summands. For these four terms exists a multitude of mechanisms contributing to the parameters \( C_1 \) to \( C_4 \). Some of these are geometric, others result from the interference of Gaussian beams and their phase detection (see e.g. [2, 3] for more information on some non-geometric cross-talk mechanisms). Unfortunately, the amount of cross-coupling originating from non-geometric effects needs to be regarded as unknown. The geometric effects however, are known very well and can be used to counteract any unknown effects.

In Fig. 2 we illustrate two related geometrical effects. If a test mass is tilted with respect to...
the optical bench and performs a lateral motion, the reflection point on the test mass moves longitudinally, i.e. in x-direction. This causes a direct coupling of lateral motion into the longitudinal readout signal, since the optical pathlength of the laser beam changes.

A second geometric coupling effect occurs, if the test mass reflection point does not coincide with the center of rotation of the test mass - as shown in Fig. 2(c). In this case, a test mass tilt causes a longitudinal displacement of the reflection point and thereby a change in the optical pathlength. In both cases, there was no motion of the test mass along x present, but since the reflection point on the test mass was displaced in x-direction, the interferometer senses an apparent test mass x-motion.

In fact, these two coupling mechanisms are closely related since in both cases, the coupling occurs due to a combination of rotation and lateral displacement, and both can be described by the following simple equation:

\[ x_{rp} = y \phi. \]  

Here, \( x_{rp} \) annotates the displacement of the reflection point along \( x \), and thereby the apparent longitudinal test mass displacement. The lateral shift between the center of rotation and the reflection point is labeled \( y \), and \( \phi \) is the test mass angle with respect to the optical bench. The same equation holds for the orthogonal plane:

\[ x_{rp} = z \eta \]  

where \( z \) is a vertical displacement of a reflection point with respect to the center of rotation, and \( \eta \) a test mass tilt around the y-axis, measured relative to the optical bench (see Fig. 2(a) for an illustration of the coordinate system). Since eq. (2) and (3) are linear in each degree of freedom, they can be used to counter act all other cross-coupling mechanisms - at least to first order.

For example, assume a cross-coupling of TM1 \( \phi \)-jitter was measured to be \( c_1 \cdot \phi_1 + O(\phi_1^2) \). In that case, a lateral displacement of TM1 by \( y_1 = -c_1 \) would suppress this measured cross talk to first order. This means, by a well chosen set of rotations and shifts of the test masses, the amount of cross-coupling leaking into \( \Delta g \) can be reduced. In order to find the required offsets, we injected sinusoidal motion in each degree of freedom and extracted the parameters via fitting a dedicated model to the measured data. Fig. 3 shows an excerpt of the injections performed.
Figure 3. Example of injections performed during the ‘engineering-day’ cross-talk experiment performed on 2016-March-16. The labels ‘GRS $z_1, z_2$’ indicate, that the shown vertical test mass displacement ($z_1, z_2$) is taken from the ‘Gravitational Reference Sensor’ readout.

on 2016-March-16: the common-mode and counter-mode injections into the vertical test mass motion. To the right-hand axis, also the measured differential acceleration $\Delta g$ is plotted, which shows a clear correlation to the injections and thereby the cross-coupling. A preliminary data analysis resulted in a first estimate of the parameters which were then used to realign the test masses. The resulting reduction of cross-coupling noise is given in Fig. 1(b). Please note, that eq. 2 and eq. 3 are not the physical model yet, which is needed to describe the cross-talk with constant parameters over the entire mission duration. Instead, these are toy-models describing two geometric cross-talk effects. Furthermore, the parameters in this model are written here not in terms of readout signals, which is necessary for applying the model to the data. The full model is still work in progress, the analysis of the cross-talk behaviour is therefore still ongoing and further information will follow in a separate paper.

3. Acknowledgments

This work has been made possible by the LISA Pathfinder mission, which is part of the space-science program of the European Space Agency. We gratefully acknowledge support by the European Space Agency (ESA) (22331/09/NL/HB, 16238/10/NL/HB), by the Deutsches Zentrum für Luft- und Raumfahrt (DLR) with funding of the Bundesministerium für Wirtschaft und Energie with a decision of the Deutschen Bundestag (DLR project reference numbers FKZ OQ 0501 and FKZ 50 Q O 1601), and thank the German Research Foundation for funding the Cluster of Excellence QUEST (Centre for Quantum Engineering and Space-Time Research).

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