CP-violating phases in the CKM matrix in orbifold compactifications

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ABSTRACT

The picture of $CP$-violation in orbifold compactifications in which the $T$-modulus is at a complex fixed point of the modular group is studied. $CP$-violation in the neutral kaon system and in the neutron electric dipole moment are both discussed. The situation where the $T$-modulus takes complex values on the unit circle which are not at a fixed point is also discussed.
String theory may provide new insights into the origin of \( CP \)-violation. It has been argued \[1\] that there is no explicit \( CP \) symmetry breaking in string theory either at the perturbative or the non-perturbative level. However, spontaneous \( CP \)-violation may arise from complex expectation values of moduli or other scalars \[1, 2, 3, 4, 5, 6\]. In any supergravity theory there is the possibility of \( CP \)-violating phases in the soft supersymmetry-breaking \( A \) and \( B \) terms and in the gaugino masses, which are in addition to a possible phase in the CKM matrix and the \( \theta \) parameter of QCD. (For a review with extensive references to the earlier literature see reference \[7\].)

In compactifications of string theory, soft supersymmetry-breaking terms can be functions of moduli such as those associated with the radii and angles characterising the underlying torus of an orbifold compactification. Moreover, in general, the Yukawa couplings also depend upon these moduli. Thus, if the orbifold moduli develop complex expectation values, this can feed through into the corresponding low energy supergravity as potentially \( CP \)-violating phases in the soft supersymmetry-breaking terms and the Yukawa couplings, and, after transforming to the quark mass eigenstates, in the CKM matrix.

The \( CP \)-violating phases in the soft supersymmetry-breaking terms lead to a non-zero neutron electric dipole moment, and, if these phases are larger than about \( 10^{-3} - 10^{-2} \), this electric dipole moment will be above the experimental upper bound. They also contribute to the \( CP \)-violating parameter \( \epsilon_K \) in the neutral kaon system through the squark mass matrix, and phases of order 1 in the soft supersymmetry-breaking terms can easily violate bounds derived from the experimental value of \( \epsilon_K \) by as much as seven orders of magnitude \[7\]. It is therefore important to check that any phases in the soft supersymmetry-breaking terms arising from complex values of the orbifold moduli are not so large as to produce too large a neutron electric dipole moment or too large an \( \epsilon_K \) parameter in the neutral kaon system.
On the other hand, it is an attractive possibility that complex orbifold moduli may induce phases in the Yukawa couplings that emerge as $CP$-violating phases in the CKM matrix. Our knowledge of the weak mixing angles suggests that we require the largest phase in the CKM matrix to be of order $10^{-1} - 10^0$ with the relatively small amount of $CP$-violation in the neutral kaon system being due to small mixing angles. It is a challenge for any model of $CP$-violation to square the large phase required in the CKM matrix, and so in the Yukawas, with the small phases required in the soft supersymmetry-breaking terms, as discussed above. (It might, however, be possible to have only small $CP$-violating phases in the CKM matrix if the soft supersymmetry-breaking terms are non-universal and $\epsilon_K$ is due to squark-gluino box diagrams.)

Elsewhere [4, 5, 6], we studied the expectation values of orbifold $T$-moduli by minimising the modular invariant effective potential. In these calculations, the greatest uncertainty relates to the dilaton dynamics, since, as is well known, a single gaugino condensate superpotential does not stabilize the dilaton expectation value at a realistic value. One approach we adopted, following earlier authors [8], was to simulate the dilaton dynamics by treating the dilaton expectation value $S$ and its corresponding auxiliary field $F_S$ as free parameters, but with $ReS$ fixed at 2. In line with existing calculations of $S$ and its auxiliary field in various contexts, we assumed that $S$ and $F_S$ were both real. Another approach we adopted was to assume stabilisation of the dilaton expectation value by a general multiple gaugino condensate or by a non-perturbative dilaton Kähler potential. The outcome, in any of these approaches, for the case of a single overall modulus $T$, is that there was some region of parameter space for which the minimum of the effective potential gave a real value of $T$ different from 1. There were other regions of parameter space for which the minimum was either at the $PSL(2, Z)$ modular group fixed point at $T = 1$ or at the fixed point
at $T = e^{i\pi/6}$.

When the non-perturbative superpotential was generalised to contain the absolute modular invariant function $j(T)$ as well as the Dedekind eta function $\eta(T)$ there were other possibilities in the case of a general multiple gaugino condensate. For some choices of the parameter

$$\rho = \frac{1 - F_S}{y} \quad (1)$$

where

$$y = S + \bar{S} - \tilde{\delta}_{GS} \log(T + \bar{T}) \quad (2)$$

with $\tilde{\delta}_{GS}$ the Green-Schwarz coefficient, there were also minima where $T$ was on the boundary of the fundamental domain at a point different from 1 or $e^{i\pi/6}$. Values of $T$ inside the fundamental domain could also be obtained in the case of a non-perturbative dilaton Kähler potential, but only for choices of the Kähler potential that gave the wrong sign kinetic terms.

The implications for $CP$-violating phases in the soft supersymmetry-breaking terms can be seen by studying the following expressions in terms of the overall modulus $T$. Allowing for the possibility of stringy non-perturbative corrections \[9\] to the dilaton Kähler potential, we write the dilaton and moduli-dependent part of the Kähler potential as

$$K = P(y) - 3 \log(T + \bar{T}) \quad (3)$$

where $y$ is given by (2). Then $\frac{dP}{dy}$ and $\frac{d^2P}{dy^2}$, which occur in the soft supersymmetry-breaking terms, are treated as parameters. Also, allowing for a general multiple gaugino condensate, we write the non-perturbative superpotential in the form

$$W_{np} = \Omega(\Sigma)[\eta(T)]^{-6} \quad (4)$$

where

$$\Sigma = S + 2\tilde{\delta}_{GS} \log \eta(T) \quad (5)$$
and

\[ \Omega(\Sigma) = \Sigma \alpha \beta \gamma e^{\frac{24\pi^2}{\alpha}} \]  

(6)

Then \( \Sigma \) is a parameter to be chosen so that \( y \) is approximately 4. The parameter \( \rho \) of (1) is related to \( \Omega \) by

\[ \rho = \frac{1}{\Omega} \frac{d\Omega}{d\Sigma} \]  

(7)

Then the gaugino masses \( M_a \) are

\[
M_a = m_{3/2} (Re f_a)^{-1} \left[ \frac{\partial \tilde{f}_a}{\partial S} \left( \frac{d^2 P}{dy^2} \right)^{-1} \left( \frac{dP}{dy} + \rho \right) \right. \\
+ \left. \left( \frac{b'_a}{8\pi^2} - \tilde{\delta}_{GS} \right) \left( 1 + \frac{1}{3} \tilde{\delta}_{GS} \frac{dP}{dy} \right)^{-1} \left( \rho \frac{1}{3} \tilde{\delta}_{GS} - 1 \right) (T + \bar{T}) \right] \langle \hat{G} \rangle \]  

(8)

where

\[
\hat{G}(T, \bar{T}) = (T + \bar{T})^{-1} + 2\eta^{-1} \frac{d\eta}{dT} 
\]

(9)

and \( b'_a \) is the usual coefficient occurring in the string loop threshold corrections to the gauge coupling constants \[10, 11, 12\]. Provided the dilaton auxiliary field \( F_S \) is real, as we have discussed earlier, there are no \( CP \)-violating phases in the gaugino masses.

The soft supersymmetry-breaking \( A \) terms are given by

\[
m^{-1/2}_{3/2} \bar{A}_{\alpha\beta\gamma} = \left( \frac{d^2 P}{dy^2} \right)^{-1} \left( \frac{dP}{dy} + \rho \right) \frac{dP}{dy} \\
+ \left[ 1 + \frac{1}{3} \tilde{\delta}_{GS} \frac{dP}{dy} \right]^{-1} \left( 1 - \rho \frac{1}{3} \tilde{\delta}_{GS} \right) (T + \bar{T}) \hat{G} \\
\times \left( 3 + n_\alpha + n_\beta + n_\gamma - (T + \bar{T}) \frac{\partial \log h_{\alpha\beta\gamma}}{\partial T} \right) 
\]

(10)

where the superpotential term for the Yukawa couplings of \( \phi_\alpha, \phi_\beta \) and \( \phi_\gamma \) is \( h_{\alpha\beta\gamma} \phi_\alpha \phi_\beta \phi_\gamma \), the modular weights of the states are \( n_\alpha, n_\beta \) and \( n_\gamma \), and the usual rescaling by a factor \( \frac{|W_{np}|}{W_{np}} e^{K/2} \) has been carried out to go from the supergravity theory derived from the orbifold compactification to the spontaneously broken globally supersymmetric theory \[3\].
The soft supersymmetry-breaking $B$ term depends upon the mechanism assumed for generating the $\mu$ term of the Higgs scalars $H_1$ and $H_2$, with corresponding superfields $\phi_1$ and $\phi_2$. If we assume that the $\mu$ term is generated non-perturbatively as an explicit superpotential term $\mu_W \phi_1 \phi_2$ then the $B$ term, which in this case we denote by $B_W$, is given by

$$m_{3/2}^{-1} B_W = \left( \frac{d^2 P}{dy^2} \right)^{-1} \left( \frac{dP}{dy} \right) \left( \frac{dP}{dy} + \frac{\partial \log \mu_W}{\partial S} \right)$$

$$+ \left( 1 + \frac{1}{3} \delta_{GS} \frac{dP}{dy} \right)^{-1} (1 - \frac{1}{3} \bar{\rho} \delta_{GS}) (T + \bar{T}) \tilde{G}$$

$$\times \left( 3 + n_1 + n_2 - (T + \bar{T}) \frac{\partial \log \mu_W}{\partial T} - \delta_{GS} \frac{\partial \log \mu_W}{\partial S} \right)$$

(11)

where $n_1$ and $n_2$ are the modular weights of the Higgs scalar superfields, and again the appropriate rescaling has been carried out.

On the other hand, if the $\mu$ term is generated by a term of the form $Z \phi_1 \phi_2 + h.c.$, with $Z$ real, mixing the Higgs superfields in the in the Kähler potential [13], then (before rescaling the Lagrangian) the $B$ term, which in this case we denote by $B_Z$, is given by

$$m_{3/2}^{-1} \mu_{Z}^{eff} B_Z = W_{np} Z \left[ 2 + ((T + \bar{T})(3 + \delta_{GS} \frac{dP}{dy})^{-1}(\rho \delta_{GS} - 3) \hat{G}(T, \bar{T}) + h.c.) \right]$$

$$+ W_{np} Z \left[ -3 + \frac{dP}{dy} + \rho^2 \left( \frac{d^2 P}{dy^2} \right)^{-1} \right.$$  

$$\left. + (3 + \delta_{GS} \frac{dP}{dy})^{-1} |\rho \delta_{GS} - 3|^2 (T + \bar{T})^2 |\hat{G}(T, \bar{T})|^2 \right]$$

(12)

The effective $\mu$ term in the superpotential is $\mu = \mu_{Z}^{eff}$ where

$$\mu_{Z}^{eff} = e^{K/2} |W_{np}| Z \left( 1 - \frac{1}{3} \rho \bar{\delta}_{GS} \right) \left( 1 - (T + \bar{T}) \hat{G}(T, \bar{T}) \right)$$

(13)

To obtain the final form for $B_Z$ in the low energy supersymmetry theory, rescaling of the Lagrangian by $\frac{|W_{np}|}{W_{np}} e^{K/2}$ has to be carried out.

Finally, the soft supersymmetry-breaking scalar masses squared,
which are always real, are given by

\[ m_\alpha^2 = V_0 + m_{3/2}^2 + m_{3/2}^2 n_\alpha \left( 1 + \frac{1}{3} \tilde{\delta}_{GS} \frac{dP}{dy} \right)^{-2} \left| 1 - \frac{1}{3} \tilde{\delta}_{GS} \rho \right|^2 (T + \bar{T})^2 \left| \hat{G}(T, \bar{T}) \right|^2 \]  \hspace{1cm} (14)

where \( V_0 \) is the vacuum energy.

Whenever the value of \( T \) at the minimum of the effective potential is at a zero of \( \hat{G}(T, \bar{T}) \), all \( CP \)-violating phases in the soft supersymmetry-breaking terms are zero, provided \( F_S \) is real, so that the parameter \( \rho \) is also real. Moreover, the \( A \) terms and the soft supersymmetry-breaking scalar masses are universal. This is evident from (10) and (14) because the dependence on the modular weights \( n_\alpha \) and the Yukawas \( h_{\alpha\beta\gamma} \) drops out of the expressions. In addition, the gaugino masses are universal at the unification scale for the coupling constants because \( \text{Re} f_a = g_a^{-2} \) and because the only \( S \) dependence of \( f_a \) is in the tree term \( f_a = S \).

The function \( \hat{G}(T, \bar{T}) \) has zeros at the fixed points \( T = 1 \) and \( T = e^{i\pi/6} \) of the \( \text{PSL}(2, \mathbb{Z}) \) modular group. The fixed point at \( T = e^{i\pi/6} \) is particularly interesting. Despite the large phase of \( T \), the phases \( \phi(B) \) and \( \phi(A) \) of the \( B \) term and the universal \( A \) term are zero. In these circumstances, there is no contribution to the electric dipole moment of the neutron [8] from the complex expectation value of the \( T \) modulus. Moreover, because the soft supersymmetry-breaking terms are universal, there is no contribution to \( \epsilon_K \) from the squark mass matrix [7].

Any minimum obtained from \( T = e^{i\pi/6} \) by a modular transformation is also a zero of \( \hat{G}(T, \bar{T}) \), and the same discussion applies.

The next question that needs to be considered is whether the phase of the \( T \) modulus can induce the phase required in the CKM matrix to account for the size of the \( CP \) violation in the kaon system. In general, the quark mass matrix is given by the Lagrangian terms

\[ \mathcal{L}_M = (h_d)_{fg}(\bar{d}_f)_L (d_g)_R v_1 - (h_u)_{fg}(\bar{u}_f)_L (u_g)_R v_2 \]  \hspace{1cm} (15)

where \( f, g = 1, 2, 3 \) label the quark generations and \( v_1, v_2 \) are Higgs
expectation values. If the transformation to the quark mass eigenstates is

\[ d_L = P_L \tilde{d}_L, u_L = Q_L \tilde{u}_L, d_R = P_R \tilde{d}_R, u_R = Q_R \tilde{u}_R \] (16)

then the charged weak current in terms of the quark mass eigenstates is

\[ J_+^\mu = \bar{\tilde{u}}_L \gamma^\mu V \tilde{d}_L \] (17)

where

\[ V = Q_L^\dagger P_L \] (18)

is the CKM matrix. In terms of the real diagonal matrices $\tilde{M}_u$ and $\tilde{M}_d$ of physical quark masses

\[ V = -v_1 v_2 (\tilde{M}_u)^{-1} Q_R^\dagger h^i_u h_d P_R (\tilde{M}_d)^{-1} \] (19)

where the Higgs expectation values have been chosen real. Generically, the largest phase in the CKM matrix $V$ is of the same order as the largest phase in the Yukawa matrices $h_u$ and $h_d$. (An explicit example of the way in which the phases in the Yukawas feed through into the CKM matrix can be found in [14], where an orbifold-compactification-inspired parameterisation of the Yukawa matrix containing 6 phases is employed.)

To study the single overall modulus case, we restrict attention to the $Z_M \times Z_N$ orbifolds, which are the only orbifolds [11] which possess three $N = 2$ moduli $T_i, i = 1, 2, 3$. For these orbifolds, it is known that the contributions $h_i$ to the (unnormalised) Yukawa couplings $h$ from the modulus $T_i$ take one of five forms [15]. (We are assuming that the phases in the CKM matrix derive from renormalisable couplings.) First, $h_i$ may have no moduli dependence if either one or more of the three coupled states is untwisted, or the 3-point function reduces to a 2-point function because the $i$th complex plane is unrotated in one of the three twisted sectors involved. For all other cases we write

\[ h_i \sim \sum_{X_{ci}} \exp(-S_{ci}) \] (20)
where $S_{\alpha i}$ is the classical action continued to Euclidean space, and

$$h = \prod_{i=1}^{3} h_i$$

For the $Z_3 \times Z_3$ orbifold, when all 3 twists in the $i^{th}$ complex plane are $e^{2\pi i/3}$, then

$$h_i \sim \sum_{k_{2i-1},k_{2i}} \exp\left(-\frac{1}{6} \pi T_i g_i(k_{2i-1}, k_{2i})\right)$$

where

$$g_i(k_{2i-1}, k_{2i}) = (2p_{2i-1} + 3k_{2i-1} + 6k_{2i})^2 + 3k_{2i-1}^2$$

$k_{2i-1}, k_{2i}$ are arbitrary integers, and $p_{2i-1} = 0, \pm 1, \pm 2$ depending on the fixed tori involved.

For the $Z_2 \times Z_6, Z_3 \times Z_6, Z_2 \times Z_6'$ and $Z_6 \times Z_6$ orbifolds, in addition to (22), two other forms of Yukawa contribution arise. For one twist of $e^{4\pi i/3}$ and two twists of $e^{2\pi i/6}$

$$h_i \sim \sum_{k_{2i-1},k_{2i}} \exp\left(-\frac{1}{12} \pi T_i e_i(k_{2i-1}, k_{2i})\right)$$

with $p_{2i-1} = 0, \pm 1$. For twists of $e^{2\pi i/6}, e^{2\pi i/3}$ and $-1$

$$h_i \sim \sum_{k_{2i-1},k_{2i}} \exp\left(-\frac{1}{12} \pi T_i e_i(k_{2i-1}, k_{2i})\right)$$

where

$$e_i(k_{2i-1}, k_{2i}) = (q_i + 6k_{2i})^2 + 3(t_i + 6k_{2i} + 4k_{2i-1})^2$$

with

$$q_i = 2n_{2i-1} + 2p_{2i}$$
$$t_i = 2n_{2i-1} + 3p_{2i} - 2p_{2i-1}$$

and the integers $n_{2i-1}$ taking the values 0, ±1, and the integers $p_{2i}$ and $p_{2i-1}$ taking the values 0, 1, depending on the fixed tori involved.

Finally, for the $Z_2 \times Z_4$ and $Z_4 \times Z_4$ orbifolds, the only moduli dependent
contributions to the Yukawa couplings occur for two twists of $e^{2\pi i/4}$ and one twist of $-1$. Then

$$h_i \sim \sum_{k_2i-1, k_2i} \exp \left( -\frac{1}{4} \pi T_i l_i(k_{2i-1}, k_{2i}) \right)$$

where

$$l_i(k_{2i-1}, k_{2i}) = (\tilde{q}_i + 4k_{2i-1} + 2k_{2i})^2 + (\tilde{t}_i - 2k_{2i})^2$$

with

$$\tilde{q}_i = 2\tilde{n}_{2i} + 2\tilde{p}_{2i-1} - \tilde{p}_{2i}$$
$$\tilde{t}_i = \tilde{p}_{2i}$$

and the integers $\tilde{n}_{2i}, \tilde{p}_{2i-1}$ and $\tilde{p}_{2i}$ all taking the values 0, 1, depending on the fixed tori involved.

The above contributions to the Yukaws may be cast in terms of Jacobi $\theta$ functions $\theta_3$ and $\theta_2$ where

$$\theta_3(\nu, i\gamma T) = \sum_{n=-\infty}^{\infty} e^{-\pi \gamma T n^2} e^{2\pi i n \nu}$$

$$\theta_2(\nu, i\gamma T) = \sum_{n=-\infty}^{\infty} e^{-\pi \gamma T (n-1/2)^2} e^{2\pi i (n-1/2) \nu}$$

Then, corresponding to equations (22), (24), (25) and (28), respectively we have

$$h_i \sim e^{-2\pi p_{2i-1}^2 T_{i}/3} \left[ \theta_3(ip_{2i-1} T_i, 2iT_i) \theta_3(ip_{2i-1} T_i, 6iT_i) + \theta_2(ip_{2i-1} T_i, 2iT_i) \theta_2(ip_{2i-1} T_i, 6iT_i) \right]$$

$$h_i \sim e^{-\pi p_{2i-1}^2 T_{i}/3} \left[ \theta_3(ip_{2i-1} T_i/2, iT_i) \theta_3(ip_{2i-1} T_i/2, 3iT_i) + \theta_2(ip_{2i-1} T_i/2, iT_i) \theta_2(ip_{2i-1} T_i/2, 3iT_i) \right]$$
\[ h_i \sim e^{-\pi(q_i^2 + \tilde{q}_i^2)T_i/12} \left[ \theta_3(it_iT_i, 4iT_i)\theta_3(i\tilde{q}_iT_i, 12iT_i) \right. \\
+ \left. \theta_2(it_iT_i, 4iT_i)\theta_2(i\tilde{q}_iT_i, 12iT_i) \right] \]  

(35)

and

\[ h_i \sim e^{-\pi(\tilde{q}_i^2 + \tilde{t}_i^2)T_i/12} \left[ \theta_3(i\tilde{q}_iT_i, 4iT_i)\theta_3(-i\tilde{t}_iT_i, 4iT_i) \right. \\
+ \left. \theta_2(i\tilde{q}_iT_i, 4iT_i)\theta_2(-i\tilde{t}_iT_i, 4iT_i) \right] \]  

(36)

The Yukawas \( h = \prod_{i=1}^{3} h_i \) correspond to the superpotential terms. However, the normalisation factors deriving from the Kähler potential are functions of \( T_i + \bar{T_i} \) and are not relevant for the present discussion, because they do not contribute to any phases. As discussed above, it is of interest to consider the case \( T_1 = T_2 = T_3 = T = e^{i\pi/6} \) corresponding to one of the fixed points of the \( PSL(2, \mathbb{Z}) \) modular group. Then, the possible phases of \( h_i \) in (35) . . . (38) for the allowed values of the integers \( p_{2i-1}, q_i, t_i, \tilde{q}_i \) and \( \tilde{t}_i \) are given in tables 1-4. For a particular orbifold, the possible phases of \( h \) are obtained by combining the phases of \( h_i, i = 1, 2, 3 \). The values obtained are of the right order of magnitude to yield phases of order \( 10^{-1} - 10^0 \) in the CKM matrix.

When, as discussed earlier, the non-perturbative superpotential \( W_{np} \) is generalised to involve the absolute modular invariant \( j(T) \) as well as the Dedekind eta function \( \eta(T) \), minimisation of the effective potential can lead to values of \( T \) on the unit circle that differ from 1 or \( e^{i\pi/6} \). Then, \( W_{np} \) contains an extra factor \( H(T) \), where the most general form of \( H(T) \) to avoid singularities in the fundamental domain is

\[ H(T) = (j - 1728)^{m/2}j^{n/3}P(j) \]  

(37)

where \( m \) and \( n \) are positive integers, and \( P(j) \) is a polynomial in \( j \). This results in modification of \( (10) . . . (14) \) by the replacement of \( (\delta_{GS} \rho -...
3) $\dot{G}(T, T)$ by $(\tilde{\delta}_{GS\rho} - 3)\dot{G}(T, T) + \frac{d\log H}{dT}$. At least for the case where stabilisation of the dilaton expectation value is due to a stringy non-perturbative dilaton Kähler potential, there is a region of parameter space for which the value of $T$ at the minimum of the effective potential is such that $(\tilde{\delta}_{GS\rho} - 3)\dot{G}(T, T) + \frac{d\log H}{dT}$ is zero. Then, the $CP$-violating phases in the supersymmetry-breaking terms are again zero and the soft supersymmetry-breaking terms remain universal. However, the minimum is no longer at a zero of $\dot{G}(T, \bar{T})$, and consequently no longer at a fixed point of the modular group. We therefore have to evaluate the phases of the Yukawas at a complex value of $T$ different from $e^{i\pi/6}$. For example, for $P(J) = 1, m = n = 1, \tilde{\delta}_{GS} = -30, \frac{dP}{dy} = 2.45, \frac{d^2P}{dy^2} = 0.45$, there is a minimum with $T$ on the unit circle at

$$T = 0.971143367 \pm 0.238496458 \, i$$

The range of the phases of $h_i$ are listed in tables 5-8. Again the phases are predominantly of order $10^{-1} - 10^0$. Of course there are also minima with $T$ obtained by performing a modular transformation on $e^{i\pi/6}$ or (38). We have checked that the order of magnitude of the largest phase for each variety of Yukawa does not change when modular transformations are carried out. In a full calculation of the CKM matrix from a detailed 3 generation orbifold model, the phases of more than one Yukawa coupling will enter and the result for the observable $CP$-violating phases must be modular invariant.

In conclusion, we have studied orbifold compactifications where the value of the orbifold modulus is at a fixed point $e^{i\pi/6}$ of the modular group [8, 4, 5, 6]. (This happens for quite a wide range of choices of the dilaton auxiliary field $F_S$ or the derivatives $\frac{dP}{dy}$ and $\frac{d^2P}{dy^2}$ of the dilaton Kähler potential.) The $CP$-violating phases in the soft supersymmetry-breaking terms are zero and the soft supersymmetry-breaking terms are universal [4, 5, 6]. As a result, the contribution to the electric dipole moment of the neutron from the phase of the $T$ modulus is zero.
Moreover, large contributions to the $\epsilon_K$ parameter in the neutral kaon system arising from the squark mass matrix are avoided [7]. On the other hand, the largest phase in the CKM matrix is expected to be of order $10^{-1} - 10^0$ generically. The picture of $CP$-violation in the neutral kaon system is thus essentially the same as in the standard model [7] requiring small weak mixing angles to explain the smallness of the $CP$-violation in this system and implying larger $CP$-violation in neutral $B$ decays. However, unlike the generic situation in low energy supergravity, the smallness of the neutron electric dipole moment is rather natural. Similar conclusions can be drawn when the non-perturbative superpotential is generalised to include the absolute modular invariant $j(T)$ when $T$ has complex values on the unit circle that result in the generalisation of $\hat{G}(T, \bar{T})$ to this case being zero.

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| $p_{2i-1}$ | $\text{Arg}(h_i)$ |
|-----------|-----------------|
| 0         | 0               |
| $\pm 1$   | $-\pi/3$        |
| $\pm 2$   | $-\pi/3$        |

Table 1: Phases of Yukawa coupling Eq.33 at the fixed point $e^{i\pi/6}$ of the modular group

| $p_{2i-1}$ | $\text{Arg}(h_i)$ |
|-----------|-----------------|
| 0         | -0.3746         |
| $\pm 1$   | -0.5899         |

Table 2: Phases of Yukawa coupling Eq.34 at the fixed point $e^{i\pi/6}$ of the modular group

| $n_{2i-1}$ | $p_{2i}$ | $p_{2i-1}$ | $\text{Arg}(h_i)$ |
|-----------|---------|-----------|-----------------|
| -1        | 0       | 0         | $\pi/3$        |
| -1        | 1       | 0         | -0.392         |
| 1         | 0       | 0         | $\pi/3$        |
| 1         | 1       | 1         | -0.982         |
| 0         | 1       | 0         | -0.982         |
| 1         | 1       | 0         | -0.982         |
| 0         | 1       | 1         | -0.982         |
| -1        | 0       | 1         | $-\pi/6$       |
| -1        | 1       | 1         | -0.392         |
| 0         | 0       | 0         | 0.0            |
| 1         | 0       | 1         | $-\pi/6$       |
| 0         | 0       | 1         | $-\pi/2$       |

Table 3: Phases of Yukawa coupling Eq.35 at the fixed point $e^{i\pi/6}$ of the modular group
\[ n_2^i \tilde{p}_{2i-1} \tilde{p}_{2i} \arg(h_i) \]

| \( n_2^i \) | \( \tilde{p}_{2i-1} \) | \( \tilde{p}_{2i} \) | \( \arg(h_i) \) |
|-----------|-----------|-----------|----------|
| 1         | 0         | 0         | \(-\pi/2\) |
| 1         | 1         | 1         | \(-\pi/4\) |
| 0         | 0         | 0         | 0        |
| 1         | 0         | 1         | \(-\pi/4\) |
| 0         | 1         | 1         | \(-\pi/4\) |
| 1         | 1         | 0         | 0        |
| 0         | 1         | 0         | \(-\pi/2\) |
| 0         | 0         | 1         | \(-\pi/4\) |

Table 4: Phases of Yukawa coupling Eq.36 at the fixed point \( e^{i\pi^6} \) of the modular group

\[ p_{2i-1} \arg(h_i) \]

| \( p_{2i-1} \) | \( \arg(h_i) \) |
|----------------|----------------|
| 0              | -0.013383      |
| \( \pm 1 \)    | -0.50174       |
| \( \pm 2 \)    | -0.50174       |

Table 5: Phases of Yukawa coupling Eq.33 for minimum of \( T \) on the unit circle (Eq.38)

\[ p_{2i-1} \arg(h_i) \]

| \( p_{2i-1} \) | \( \arg(h_i) \) |
|----------------|----------------|
| 0              | -0.159         |
| \( \pm 1 \)    | -0.285         |

Table 6: Phases of Yukawa coupling Eq.34 for minimum of \( T \) on the unit circle (Eq.38)
\[ n_{2i-1} \quad p_{2i} \quad p_{2i-1} \quad \text{Arg}(h_i) \]

| \( n_{2i-1} \) | \( p_{2i} \) | \( p_{2i-1} \) | \( \text{Arg}(h_i) \) |
|----------------|------|-------------|----------------|
| -1             | 0    | 0           | -0.999         |
| -1             | 1    | 0           | -0.1897        |
| 1              | 0    | 0           | -0.999         |
| 1              | 1    | 1           | -0.4704        |
| 0              | 1    | 0           | -0.4704        |
| 1              | 1    | 0           | -0.4704        |
| 0              | 1    | 1           | -0.4704        |
| -1             | 0    | 1           | -0.25422       |
| -1             | 1    | 1           | -0.1897        |
| 0              | 0    | 0           | 0.0            |
| 1              | 0    | 1           | -0.25422       |
| 0              | 0    | 1           | -0.75149       |

Table 7: Phases of Yukawa coupling Eq.35 for minimum of \( T \) on the unit circle (Eq.38)

\[ \tilde{n}_{2i} \quad \tilde{p}_{2i-1} \quad \tilde{p}_{2i} \quad \text{Arg}(h_i) \]

| \( \tilde{n}_{2i} \) | \( \tilde{p}_{2i-1} \) | \( \tilde{p}_{2i} \) | \( \text{Arg}(h_i) \) |
|----------------------|----------------------|----------------------|----------------------|
| 1                    | 0                    | 0                     | -0.74926             |
| 1                    | 1                    | 1                     | -0.37909             |
| 0                    | 0                    | 0                     | -0.0089              |
| 1                    | 0                    | 1                     | -0.37909             |
| 0                    | 1                    | 1                     | -0.37909             |
| 1                    | 1                    | 0                     | -0.00892             |
| 0                    | 1                    | 0                     | -0.74926             |
| 0                    | 0                    | 1                     | -0.37909             |

Table 8: Phases of Yukawa coupling Eq.36 for minimum of \( T \) on the unit circle (Eq.38)