Single bubble rising behaviors in Newtonian and non-Newtonian fluids with validation of empirical correlations: A computational fluid dynamics study

Md. Tariqul Islam | Poo Balan Ganesan | Ji Cheng | Mohammad Salah Uddin

1Department of Mechanical Engineering, Faculty of Engineering, University of Malaya, Kuala Lumpur, Malaysia
2School of Mechanical and Mining Engineering, The University of Queensland, Brisbane, Queensland, Australia

Abstract
The bubble rising (BR) dynamic is a common phenomenon in numerous processes of industries. Here, a single air BR behavior is studied using computational fluid dynamics (CFD) modelling in a Newtonian fluid (NF) and non-Newtonian fluid (nNF). The volume of fluid formulation with the continuum surface force equation is used to track the air bubble in a NF, while the viscosity of the nNF fluid is estimated by using the power-law equation. The bubble terminal velocity and its shape deformation, as well as the influence of different dimensionless numbers on BR characterization are investigated. The bubble rise in NF, the bubble terminal velocity increases with decreasing Morton number, and bubble moves up in a zigzag way with shape oscillation for the case of low Morton number of the NF. The bubble rise in nNF, the bubble terminal velocity increases with the increases in the rheological index, and bubble size as well as its shape transforms from an ellipsoidal to ellipsoidal cap types. It is found that the drag coefficient is high in a low rheological index compared with the high rheological index. The CFD results are compared with experimental results and empirical correlations reported in the literature. Good agreements are found between the CFD and the literature data for both fluids.

KEYWORDS
CFD, bubble shape and dimensionless numbers, bubble terminal velocity

1 | INTRODUCTION
The bubble rising (BR) dynamic is a common phenomenon in numerous processes of industries such as chemical, petrochemical and biochemical refinery processes, food process and membrane bioreactors for wastewater treatment plants.1-3 In those cases, the bubble column is widely used as it provides a high interfacial area, good rate of heat and mass transfer between the gas bubble and the liquid phase, and low maintenance cost due to easy construction.1,4,5 The efficient operation of the bubble column is dependent on bubble size and rise velocity, bubble coalescence, and breakup.6 Researchers have carried out a number of experimental investigations of single air BR characterization (eg, bubble formation through
The bubble terminal velocity and its shape deformation in different glycerin solutions were investigated in a wide range of Reynolds numbers and Weber numbers. A correlation was developed by using the Weber number, Morton number, and Reynolds number. A good agreement was found between the experimental and numerical results. Another study of the unsteady motion of a single BR in high viscous quiescent liquid (different concentrations of sucrose solution) was carried out. An empirical correlation was also developed to estimate the drag coefficient by utilizing the Reynolds number, Archimedes number, and Acceleration number. However, BR behaviors were investigated in a viscoelastic fluid. It was found that an intense shear layer formed at the tip of the bubble's rear pole and helps to change the bubble shape. Similarly, a study was performed to investigate single BR characteristics in a viscoelastic polymer solution (Praestol 2500). It was found that the polymer molecules were moved around the bubble from the top to the bottom by aligning its coordinate system with the motion of BR. As a result, the polymer molecules in the bubble lower hemisphere contributed to the BR velocity. Furthermore, the rising behavior of single bubbles was investigated in water and xanthan solution (XGS). The bubbles rose in straight way with tiny oscillations that were convoyed by a steady bubble shape in water and a marginally low rising velocity in XGS. The effect of channel depth, bubble size, superimposed velocity, and rheology of liquid was examined on single BR dynamics in water and XGS. The velocity magnitude, vorticity, strain, and shear stress increased with increasing bubble size and superimposed liquid velocity. Also, the shear stresses acting in XGS were almost two times higher than that of water.

Nowadays, researchers have utilized an advanced computational technique for a single BR study in Newtonian fluid (NF) and non-Newtonian fluid (nNF). There are three types of computational model namely the Eulerian-Eulerian, the Eulerian-Lagrangian, and the direct numerical simulation model. The Eulerian-Eulerian model was used when the gas bubble treats as a pseudo-continuum phase so no need to provide flow specifics for distinct bubbles. The Eulerian-Lagrangian model was used for a nondeformable bubble analysis. The direct numerical simulation model is used to analyze the details feature of a single BR dynamic in liquid. In this model, the motion of the bubble-liquid interface is determined by using the front capturing (Eulerian framework system) approach or front tracking (Lagrangian grid system) approach. However, the computational power and time in the front tracking approach are high in comparison to the computational power and time in the front capturing approach, which tracks the bubble-liquid interface by a scalar-indicator or volume of fluid (VOF) fraction that is calculated by solving an advection equation.

Researchers have used the VOF formulation to investigate single or multiple bubbles rise characterization and bubble formation through an orifice at atmospheric pressure or at elevated pressure conditions in viscous liquid. Also, the VOF model was utilized to investigate the BR analysis in membrane bioreactor processes, ionic liquids, and shear-thinning non-Newtonian fluid (nNF). There are three types of computational model namely the Eulerian-Eulerian, the Eulerian-Lagrangian, and the direct numerical simulation model. The Eulerian-Eulerian model was used when the gas bubble treats as a pseudo-continuum phase so no need to provide flow specifics for distinct bubbles. The Eulerian-Lagrangian model was used for a nondeformable bubble analysis. The direct numerical simulation model is used to analyze the details feature of a single BR dynamic in liquid. In this model, the motion of the bubble-liquid interface is determined by using the front capturing (Eulerian framework system) approach or front tracking (Lagrangian grid system) approach. However, the computational power and time in the front tracking approach are high in comparison to the computational power and time in the front capturing approach, which tracks the bubble-liquid interface by a scalar-indicator or volume of fluid (VOF) fraction that is calculated by solving an advection equation.

Researchers have used the VOF formulation to investigate single or multiple bubbles rise characterization and bubble formation through an orifice at atmospheric pressure or at elevated pressure conditions in viscous liquid. Also, the VOF model was utilized to investigate the BR analysis in membrane bioreactor processes, ionic liquids, and shear-thinning non-Newtonian fluid (nNF). There are three types of computational model namely the Eulerian-Eulerian, the Eulerian-Lagrangian, and the direct numerical simulation model. The Eulerian-Eulerian model was used when the gas bubble treats as a pseudo-continuum phase so no need to provide flow specifics for distinct bubbles. The Eulerian-Lagrangian model was used for a nondeformable bubble analysis. The direct numerical simulation model is used to analyze the details feature of a single BR dynamic in liquid. In this model, the motion of the bubble-liquid interface is determined by using the front capturing (Eulerian framework system) approach or front tracking (Lagrangian grid system) approach. However, the computational power and time in the front tracking approach are high in comparison to the computational power and time in the front capturing approach, which tracks the bubble-liquid interface by a scalar-indicator or volume of fluid (VOF) fraction that is calculated by solving an advection equation.

Researchers have used the VOF formulation to investigate single or multiple bubbles rise characterization and bubble formation through an orifice at atmospheric pressure or at elevated pressure conditions in viscous liquid. Also, the VOF model was utilized to investigate the BR analysis in membrane bioreactor processes, ionic liquids, and shear-thinning non-Newtonian fluid (nNF). There are three types of computational model namely the Eulerian-Eulerian, the Eulerian-Lagrangian, and the direct numerical simulation model. The Eulerian-Eulerian model was used when the gas bubble treats as a pseudo-continuum phase so no need to provide flow specifics for distinct bubbles. The Eulerian-Lagrangian model was used for a nondeformable bubble analysis. The direct numerical simulation model is used to analyze the details feature of a single BR dynamic in liquid. In this model, the motion of the bubble-liquid interface is determined by using the front capturing (Eulerian framework system) approach or front tracking (Lagrangian grid system) approach. However, the computational power and time in the front tracking approach are high in comparison to the computational power and time in the front capturing approach, which tracks the bubble-liquid interface by a scalar-indicator or volume of fluid (VOF) fraction that is calculated by solving an advection equation.

Researchers have used the VOF formulation to investigate single or multiple bubbles rise characterization and bubble formation through an orifice at atmospheric pressure or at elevated pressure conditions in viscous liquid. Also, the VOF model was utilized to investigate the BR analysis in membrane bioreactor processes, ionic liquids, and shear-thinning non-Newtonian fluid (nNF). There are three types of computational model namely the Eulerian-Eulerian, the Eulerian-Lagrangian, and the direct numerical simulation model. The Eulerian-Eulerian model was used when the gas bubble treats as a pseudo-continuum phase so no need to provide flow specifics for distinct bubbles. The Eulerian-Lagrangian model was used for a nondeformable bubble analysis. The direct numerical simulation model is used to analyze the details feature of a single BR dynamic in liquid. In this model, the motion of the bubble-liquid interface is determined by using the front capturing (Eulerian framework system) approach or front tracking (Lagrangian grid system) approach. However, the computational power and time in the front tracking approach are high in comparison to the computational power and time in the front capturing approach, which tracks the bubble-liquid interface by a scalar-indicator or volume of fluid (VOF) fraction that is calculated by solving an advection equation.

Researchers have used the VOF formulation to investigate single or multiple bubbles rise characterization and bubble formation through an orifice at atmospheric pressure or at elevated pressure conditions in viscous liquid. Also, the VOF model was utilized to investigate the BR analysis in membrane bioreactor processes, ionic liquids, and shear-thinning non-Newtonian fluid (nNF). There are three types of computational model namely the Eulerian-Eulerian, the Eulerian-Lagrangian, and the direct numerical simulation model. The Eulerian-Eulerian model was used when the gas bubble treats as a pseudo-continuum phase so no need to provide flow specifics for distinct bubbles. The Eulerian-Lagrangian model was used for a nondeformable bubble analysis. The direct numerical simulation model is used to analyze the details feature of a single BR dynamic in liquid. In this model, the motion of the bubble-liquid interface is determined by using the front capturing (Eulerian framework system) approach or front tracking (Lagrangian grid system) approach. However, the computational power and time in the front tracking approach are high in comparison to the computational power and time in the front capturing approach, which tracks the bubble-liquid interface by a scalar-indicator or volume of fluid (VOF) fraction that is calculated by solving an advection equation.

Researchers have used the VOF formulation to investigate single or multiple bubbles rise characterization and bubble formation through an orifice at atmospheric pressure or at elevated pressure conditions in viscous liquid. Also, the VOF model was utilized to investigate the BR analysis in membrane bioreactor processes, ionic liquids, and shear-thinning non-Newtonian fluid (nNF). There are three types of computational model namely the Eulerian-Eulerian, the Eulerian-Lagrangian, and the direct numerical simulation model. The Eulerian-Eulerian model was used when the gas bubble treats as a pseudo-continuum phase so no need to provide flow specifics for distinct bubbles. The Eulerian-Lagrangian model was used for a nondeformable bubble analysis. The direct numerical simulation model is used to analyze the details feature of a single BR dynamic in liquid. In this model, the motion of the bubble-liquid interface is determined by using the front capturing (Eulerian framework system) approach or front tracking (Lagrangian grid system) approach. However, the computational power and time in the front tracking approach are high in comparison to the computational power and time in the front capturing approach, which tracks the bubble-liquid interface by a scalar-indicator or volume of fluid (VOF) fraction that is calculated by solving an advection equation.

Researchers have used the VOF formulation to investigate single or multiple bubbles rise characterization and bubble formation through an orifice at atmospheric pressure or at elevated pressure conditions in viscous liquid. Also, the VOF model was utilized to investigate the BR analysis in membrane bioreactor processes, ionic liquids, and shear-thinning non-Newtonian fluid (nNF). There are three types of computational model namely the Eulerian-Eulerian, the Eulerian-Lagrangian, and the direct numerical simulation model. The Eulerian-Eulerian model was used when the gas bubble treats as a pseudo-continuum phase so no need to provide flow specifics for distinct bubbles. The Eulerian-Lagrangian model was used for a nondeformable bubble analysis. The direct numerical simulation model is used to analyze the details feature of a single BR dynamic in liquid. In this model, the motion of the bubble-liquid interface is determined by using the front capturing (Eulerian framework system) approach or front tracking (Lagrangian grid system) approach. However, the computational power and time in the front tracking approach are high in comparison to the computational power and time in the front capturing approach, which tracks the bubble-liquid interface by a scalar-indicator or volume of fluid (VOF) fraction that is calculated by solving an advection equation.

Researchers have used the VOF formulation to investigate single or multiple bubbles rise characterization and bubble formation through an orifice at atmospheric pressure or at elevated pressure conditions in viscous liquid. Also, the VOF model was utilized to investigate the BR analysis in membrane bioreactor processes, ionic liquids, and shear-thinning non-Newtonian fluid (nNF). There are three types of computational model namely the Eulerian-Eulerian, the Eulerian-Lagrangian, and the direct numerical simulation model. The Eulerian-Eulerian model was used when the gas bubble treats as a pseudo-continuum phase so no need to provide flow specifics for distinct bubbles. The Eulerian-Lagrangian model was used for a nondeformable bubble analysis. The direct numerical simulation model is used to analyze the details feature of a single BR dynamic in liquid. In this model, the motion of the bubble-liquid interface is determined by using the front capturing (Eulerian framework system) approach or front tracking (Lagrangian grid system) approach. However, the computational power and time in the front tracking approach are high in comparison to the computational power and time in the front capturing approach, which tracks the bubble-liquid interface by a scalar-indicator or volume of fluid (VOF) fraction that is calculated by solving an advection equation.

In literature, most of the single BR studies were analyzed by using dimensionless numbers namely Reynolds number, drag coefficient, Weber number, Eotvos number, Galilei number, and dimensionless liquid properties. Very few studies focused on the BR characterization by other dimensionless numbers such as Velocity number, Flow number, Archimedes number, and Capillary number. These dimensionless numbers are also useful for single BR analysis. In this study, single BR characterization is investigated by using those dimensionless numbers in
NF and shear-thinning nNF. The VOF formulation with the continuum surface force (CSF) equation is used to track the bubble-liquid interface in the NF, while the power-law equation is used to calculate the viscosity of the shear-thinning nNF. The bubble terminal velocity and its deformation, and the influence of dimensionless number on BR behaviors are examined and compared with experimental data and empirical correlations reported in the literature.

2 | COMPUTATIONAL FLUID DYNAMICS (CFD) MODEL

2.1 | Governing equations

2.1.1 | VOF method

The VOF tracks the interface on fixed meshes and is used to track the bubble interface by using the volume fraction function. The transport equation is described as follows:

\[
\frac{\partial \varepsilon_L}{\partial t} + \vec{W} \cdot \nabla \varepsilon_L = 0,
\]

(1)

where \( \vec{W} \) and \( \varepsilon_L \) are the liquid velocity and the liquid volume fraction, respectively. The fully liquid and gas phase are defined by using the value of \( \varepsilon_L = 1 \) and 0, respectively. The interface boundary between the bubble and the liquid is known as \( 0 < \varepsilon_L < 1 \). Here, the piecewise linear interface construction algorithm is used to solve Equation (1) since it can provide a high precision to reconstruct the bubble-liquid interface. The local-averaged density (\( \rho \)) and viscosity (\( \mu \)) of liquid in the interface are calculated as follows:

\[
\rho = \rho_L \varepsilon_L + \rho_G (1 - \varepsilon_L),
\]

(2)

\[
\mu = \mu_L \varepsilon_L + \mu_G (1 - \varepsilon_L),
\]

(3)

where the subscripts \( L \) and \( G \) denote the liquid and the gas phase, respectively.

2.1.2 | Continuity and momentum equations

The continuity and the momentum equations for incompressible flow are written as follows:

\[
\nabla \cdot \vec{W} = 0,
\]

(4)

\[
\frac{\partial}{\partial t} (\rho \vec{W}) + \nabla \cdot (\rho \vec{W} \vec{W}) = -\nabla P + \nabla \cdot [\mu \{\nabla \vec{W} + (\nabla \vec{W})^T\}] + \rho \vec{g} + \vec{F},
\]

(5)

where \( P \), \( t \), and \( g \) represent the liquid pressure, reference time, and gravitational acceleration, respectively. \( \vec{F} \) is the volume force that is computed by using the CSF method developed by Brackbill et al. The equation is expressed as follows:

\[
\vec{F} = \frac{\sigma k (\nabla \cdot \varepsilon_L)}{0.5(\rho_L + \rho_G)},
\]

(6)

where \( \sigma \) is the liquid surface tension. \( k \) represents the curvature of the bubble-liquid interface and can be described as follows:

\[
k = (\nabla \cdot \hat{n}) = \frac{1}{\hat{n}} \left[ \left( \frac{\hat{n}}{\hat{n} \cdot \nabla} \nabla \right) \hat{n} - (\nabla \cdot \hat{n}) \right].
\]

(7)
Constitutive equation of nNF

In an nNF, the power-law equation is used to calculate the liquid viscosity and is written as follows:\(^{17}\):

\[
\mu = K \dot{\gamma}^{f-1},
\]

(8)

where \(K\) and \(f\) are the consistency coefficient and the power-law index or rheological index, respectively. The local shear rate (\(\dot{\gamma}\)) can be expressed as follows:

\[
\dot{\gamma} = \sqrt{2(\overrightarrow{\tau} : \overrightarrow{\tau})},
\]

(9)

where \(\overrightarrow{\tau}\) is the strain rate tensor and can be described as follows:

\[
\overrightarrow{\tau} = \frac{1}{2} (\nabla \overrightarrow{W} + \nabla \overrightarrow{W}^T).
\]

(10)

In Equation (5), the viscosity of the nNF can be estimated by combining Equations (8) to (10). For nondimensional analysis of a single rising bubble, the Morton number, Reynolds number, Capillary number, Eotvos number, Weber number, drag coefficient, and Archimedes number are described as follows:

For NF,

\[
Mo = \frac{g(\rho_L - \rho_G)\mu_L^4}{\sigma^3 \rho_L^2}. \quad (11)
\]

For nNF,\(^{36}\)

\[
Mo_{non} = \frac{g(\rho_L - \rho_G)[K(U_T/d_{eq})^{f-1}]^4}{\sigma^3 \rho_L^2}, \quad (12)
\]

\[
Re = \frac{\rho_L U_T d_{eq}}{\mu_L}, \quad (13)
\]

\[
Ca = \frac{\mu_L U_T}{\sigma}, \quad (14)
\]

\[
Eo = \frac{g(\rho_L - \rho_G)d_{eq}}{\sigma}, \quad (15)
\]

\[
We = \frac{\rho_L d_{eq} U_T^2}{\sigma}, \quad (16)
\]

\[
C_d = \frac{4g d_{eq}^3 (\rho_L - \rho_G)}{3 \rho_L d_w^2 U_T^2}, \quad (17)
\]

\[
Ar = \frac{g d_{eq}^3 (\rho_L - \rho_G)^2}{\mu_L^2}, \quad (18)
\]

where \(U_T\) is the bubble terminal velocity. \(d_{eq}\) is the bubble equivalent diameter which is calculated as follows:\(^{37}\):

\[
d_{eq} = (d_h \times d_w^2)^{1/3}; \quad \text{here,} \quad d_h \text{ and } d_w \text{ are the shortest distance and longest distance of the bubble, respectively.}
FIGURE 1  A schematic view of the computational domain for bubble rising characterization

TABLE 1  Properties of the NF and nNF

| Cases | Sucrose/water (vol%) | $\rho_L$ (kg/m³) | $\mu_L$ (Pas) | $\sigma$ (N/m) | Mo (Equation (11)) (—) |
|-------|---------------------|-----------------|--------------|---------------|----------------------|
| 1     | 41/59               | 1183.8          | 0.0060       | 0.0745        | $2.67 \times 10^{-8}$ |
| 2     | 49/51               | 1222.6          | 0.0114       | 0.0786        | $2.82 \times 10^{-7}$ |
| 3     | 63/37               | 1308.4          | 0.1066       | 0.0812        | $1.80 \times 10^{-3}$ |

| nNF   |                     |                 |              |               | Mo (Equation (12)) (—) |
|-------|---------------------|-----------------|--------------|---------------|----------------------|
| 4     | 0.025               | 996.0           | 0.00612      | 0.8248        | $4.36-6.1 \times 10^{-9}$ |
| 5     | 0.10                | 997.0           | 0.09503      | 0.5481        | $1.04-1.98 \times 10^{-5}$ |

Abbreviations: NF, Newtonian fluid; nNF, non-Newtonian fluid.

2.2  Physical model

Literature review shows that several researchers investigated the BR behaviors in a full two-dimensional (2D) computational domain.28,38,39 Here, a 2D domain is selected since the present study is mainly focused on the BR characterization in a high Mo case. Figure 1 shows a schematic view of the 2D computational domain, which is used to investigate single air BR characterization in NF and shear-thinning nNF. In the initial condition, a sphere bubble with a diameter ($d_b$) is placed at $10 + 0.5d_b$ mm height from the bottom of the domain in centerline direction. The domain width, $D = d_b/0.07$ mm is used to minimize the wall effect13,40 and $H = 3 \times D$ mm is used as the domain height. The air bubble diameter of 2.5 mm and 10, 12, and 17 mm are used in sucrose solution as an NF9 and XGS as an nNF,37 respectively. The gravitational acceleration ($g = 9.81$ m/s²) acts along the negative y-direction. The fluid properties are shown in Table 1. In the NF, the ranges of Mo for cases 1 to 3 are $2.67 \times 10^{-8}$ to $1.80 \times 10^{-3}$, respectively (Table 1). In the nNF, $Mo_{non}$ varies with the shear rate which is a function of bubble size and terminal velocity. Values of $Mo_{non}$ for case 4 and 5 are $4.36 \times 10^{-9}$-$6.1 \times 10^{-9}$ and $1.04 \times 10^{-5}$-$1.98 \times 10^{-5}$, respectively. The wall and top of the domain are defined as no-slip and pressure outlet boundary conditions, respectively.

2.3  Numerical procedure

The governing equations were solved by using Ansys-Fluent 14.0 that is based on the finite volume method. The QUICK upwind and PRESTO schemes were used to solve the momentum equation and the liquid pressure, respectively. The pressure-velocity coupling is solved by using the PISO scheme which can able to give a fast convergence rate with minimum loss of the solution stability. Here, the fixed Courant number of 0.25 is used for solving the governing equations. The time step for every simulation was $10^{-4}$ seconds and the total simulation time was 0.5 second. Values of 0.3 and 0.7
were considered as a relaxation factor of the pressure and momentum, respectively. The value of $10^{-6}$ was used as the residual scale for the convergence criteria for all equations.

2.4 Mesh

A uniform structured with different sizes of the grid such as 0.25, 0.20, and 0.15 mm (which generates total elements number of $6.93 \times 10^4$, $1.08 \times 10^5$ and $1.93 \times 10^5$, respectively) are examined for the mesh dependency study. Figure 2 shows the 2.5 mm BR distance with time for the different mesh sizes. For all cases, the CFD results are almost the same up to $t = 0.10$ second and after that, there is some difference in results until 0.5 second. Very less difference can be seen between the results in the size of the mesh 0.20 and 0.15 mm. While 0.25 mm mesh size produces a low estimation of the BR distance. Considering all, mesh size of 0.20 mm is selected.

3 Results and Discussion

3.1 BR characteristics in NF

3.1.1 Terminal velocity

The bubble terminal velocity is one of the key parameters for BR characterization. Figure 3 shows the instantaneous BR velocity with simulation time for different Morton numbers, $Mo$. For all cases, the BR velocity increases until 0.2 second and then there is no variation on its velocity up to 0.5 second. It means that the instantaneous rise velocities are in a steady condition. Values of 0.196, 0.162 and 0.063 m/s are found as the terminal velocity for low, medium and high $Mo$, respectively. The experimental data of 2.5 mm bubble in low $Mo$ (Case 1) is also included. Up to 2.36% relative error (($\text{Experimental} - \text{CFD}) / \text{Experimental} \times 100$) is found by comparing between the CFD and the experimental data. This low error represents a good benchmark for the present CFD methods.

The CFD results are also compared with the empirical correlation developed by Rodrigue, to predict single BR velocity based on velocity number ($V_N$) and flow number ($F_N$). The correlation was developed by using a large range of physical properties and hydrodynamic regimes, and can be described as follows:

$$ V_N = \frac{1000F_N}{12000 + 588F_N^{2/3}}. $$

The velocity number ($V_N$) and flow number ($F_N$) are calculated as follows:

$$ V_N = (Re^2Ca)^{1/3}, $$

(Figure 2 Instantaneous rise distance of bubble, $d_b = 2.5$ mm vs time for different sizes of mesh)
Figure 3  The instantaneous rising velocity of a bubble with time for different Morton number (Mo). Experimental data ($d_b = 2.5$ mm) is taken from Zhang et al. 9

Figure 4  Velocity number vs Flow number

$$F_N = E_0 \left( \frac{Re}{Ca} \right)^{2/3}.$$  \hspace{1cm} (21)

Figure 4 shows a comparison between the CFD results and the correlation (Equation (19)) results. The velocity number increases with increasing flow number and the relative error is less than 10% which is also a reasonable agreement for the present CFD method.

3.1.2  Rise trajectory and shape

Figure 5 shows the BR trajectory for different Morton number, Mo. In a low Mo, the bubble moves up and follows a zigzag path from the left to right. During zigzag motion, the bubble changes its axis (or shape oscillation) from 150° to 45° until the top of the domain (Figure 5B). In this case, a strong vortex of liquid jet acts behind the bubble that rises faster with shape oscillation. The bubble shape oscillation was also observed by other researchers. 7,11 In medium Mo (Figure 5A), the bubble moves up from the left to right with less oscillation. However, there is no trajectory of the bubble for the case of high Mo because the surface tension force maintains the shape with minimum surface energy. As a result, the bubble aspect ratio is high, and a low terminal velocity is observed.

In addition, the bubble shapes are examined with a well-known shape regime chart which was developed by using a vast amount of experimental data and theoretical analysis. 43 Figure 6 shows the bubble shape regime chart with Reynolds number ($Re$) vs Eotvos number ($Eo$) for a wide range of Mo. The computed bubble shapes agree well with the shape regime chart for all cases.
FIGURE 5  (A) 2.5 mm Bubble rising trajectory for different Morton number (Mo), and (B) zoom view of bubble rising trajectory in low Mo condition.

FIGURE 6  Bubble shape regime chart for different Morton number taken from Clift\textsuperscript{43} and the contour shows the computed bubble shape at $t = 0.4$ second. Note: s, oe, oed, oec, sc, and sk denote the spherical, oblate ellipsoidal, oblate ellipsoidal-disk, oblate ellipsoid-cap, spherical-cap, and skirted type shape of a bubble, respectively.

Also, the bubble shape can be examined with the Weber number ($We$) which is the ratio of dynamic pressure to the surface tension. When $We < 1$, the bubble maintains a spherical shape that alters to an oblate ellipsoidal or oblate ellipsoidal disk shape due to the velocity pressure at $We > 1$\textsuperscript{44}. For example, $We$ of 0.123 and 1.51 represent a spherical and oblate ellipsoidal shape for low and high $Mo$, respectively (refer to Figure 6). The calculation of the $We$ is compared with available empirical correlation developed by Raymond & Rosant.\textsuperscript{8} The authors developed the correlation with the range of $Mo = O (10^{-8}) - O (7)$ and can be expressed as follows:

$$We = 0.42Mo^{0.35}Re^{5/3}.$$  \hfill (22)
Figure 7 shows the Weber number ($We$) as a function of Reynolds number ($Re$) for different $Mo$. It is found that $We$ increases with increasing $Re$. Results from CFD match well with correlation Equation (22) and less than 5% relative error is obtained. Similarly, Cai et al.\textsuperscript{45} investigated a single BR behavior in high viscous liquid and a wide range of $Mo = O(10^{-11})$—$O(163)$. The authors developed a relationship between the bubble aspect ratio (the ratio of shortest distance to longest distance of the bubble, $d_h/d_w$) and dimensionless numbers such as $Re$, $Eo$, and $Mo$. The equation is described as follows:

$$\frac{d_h}{d_w} = \frac{4.67 Re^{0.625} Mo^{0.291}}{Eo}. \quad (23)$$

Figure 8 shows the bubble aspect ratio vs the Reynolds number for different $Mo$. The bubble aspect ratio decreases with increasing $Re$. At low $Re < 3$, the bubble contains its original shape which becomes an ellipsoidal shape due to the surface tension force and the buoyancy when $Re > 3$. The computed results are well fit with correlation (Equation (23)) and up to 5% relative error is found.

### 3.1.3 Drag coefficient

The drag force is another important parameter for BR characterization. The computed results are examined with Tomiyama et al. correlation\textsuperscript{46} which is expressed as follows:

$$C_d = \frac{16}{Re} (1 + 0.15 Re^{0.687}). \quad (24)$$

---

**FIGURE 7** Weber number vs Reynolds number

**FIGURE 8** Bubble aspect ratio vs Reynolds number
Figure 9 illustrates the drag coefficient vs Reynolds number. The drag coefficient decreases with increasing \( Re \) and good agreement can be seen between the computed results and the correlation results (Equation (24)). Also, Zhang et al.\(^9\) experimental data are included and up to 8% relative error is found by comparing the correlation and the experimental data. Similarly, Zhang et al.\(^9\) developed a correlation to estimate the drag coefficient based on bubble Archimedes number \( (Ar) \) and \( Re \). The authors reported that if the bubble velocity reached into a steady condition which represented the Acceleration number is equal to zero, that is same for this present work. The correlation for the steady condition is expressed as follows:

\[
C_d = 2.275 \left( \frac{Ar^{0.801}}{Re^{1.79}} \right).
\]  

Figure 10 shows the drag coefficient vs Archimedes number. The drag coefficient decreases with increasing Archimedes number. An overprediction (up to 5%) can be seen by comparing with Zhang et al.’s estimations.

### 3.2 BR characteristics in nNF

#### 3.2.1 Terminal velocity

In this section, single BR dynamics are examined in the shear-thinning nNF. Three distinct sizes of bubble, \( d_b = 10, 12 \) and 17 mm are considered in XGS of 0.025% and 0.10% concentration solution, corresponding to rheological index \( (f) \) of 0.8248 and 0.5481, respectively. Figure 11 shows the bubble terminal velocity vs bubble diameter for \( f = 0.8248 \) and 0.5481. The
bubble terminal velocity increases with increasing bubble diameter. In high rheological index, $f = 0.8248$, the terminal velocity of $d_b = 10$ and $12$ mm gives an overprediction from the experimental data reported by Hassan et al.,$^{37}$ while an under-prediction is obtained in low rheological index, $f = 0.5481$ due to the shear-thinning effect. In both $f = 0.8248$ and $0.5481$, the terminal velocity of the large bubble ($d_b = 17$ mm) is in under-prediction compared to the Hassan et al.’s results.

### 3.2.2 Rise trajectory and shape

Figure 12 illustrates a BR trajectory with shape deformation (like as dimpled ellipsoidal cap shape) which happens at the beginning of the rising stage at $t = 0.05$ second. The changes in shape occur due to the unbalanced pressure difference, buoyancy force, and drag force. However, the bubble deformation and swing increase with increasing bubble size and the large bubble experiences more resistance by the nNF as well as a large deformation due to the effect of strong wake at bubble lower surface. In Figure 12A ($d_b = 10$ mm), the bubble shapes are close to a spherical-cap which changes to wobbling shape when $d_b$ is increased to 12 mm (Figure 12B) at $t = 0.2-0.5$ second. In Figure 12C ($d_b = 17$ mm), the dimpled ellipsoidal cap shape ($t = 0.05$ second) changes to skirted type shape ($t = 0.1$ second) and then the edge of the bubble breakup into small bubbles and form ellipsoidal cap shape bubble ($t = 0.2-0.5$ second).

### 3.2.3 Drag coefficient

In a non-Newtonian power-law fluid, the Reynolds number based on the aspherical bubble with a vertical symmetry axis can be calculated as follows$^{17}$:

$$Re_{non} = \frac{d_w U_T^{2-f} \rho_L}{K}.$$  \hspace{1cm} (26)

Figure 13 shows the drag coefficient vs Reynolds number for two different rheological indexes of 0.8248 and 0.5481. The drag coefficient decreases with increasing Reynolds number. The lower drag coefficient can be seen for a high rheological index ($f = 0.8248$) than that of the low rheological index, $f = 0.5481$. The CFD results are also compared with the available correlation:

For high Reynolds number,$^{47}$

$$C_d = \frac{16}{Re_{non}} (1 + 0.173 Re_{non}^{0.657}) + \frac{0.413}{1 + 16300 Re_{non}^{1.09}}.$$  \hspace{1cm} (27)

For low Reynolds number,$^{47}$

$$C_d = \frac{24}{Re_{non}} (1 + 0.173 Re_{non}^{0.657}) + \frac{0.413}{1 + 16300 Re_{non}^{1.09}}.$$  \hspace{1cm} (28)

Up to 12% relative error is found by comparing between the CFD results and these correlation results.
**FIGURE 12** Shape of bubble in two different concentrations of xanthan solution in the liquid for different time. (A) \(d_0 = 10\, \text{mm}\), (B) \(d_0 = 12\, \text{mm}\) and (C) \(d_0 = 17\, \text{mm}\)

**FIGURE 13** Drag coefficient vs Reynolds number for two different rheological indexes \((f)\)
4 | CONCLUSION

In this study, single air BR behaviors in an NF and nNF are investigated by using the VOF with the continuum surface force formulations. The power-law equation is used to estimate the viscosity of the nNF. The bubble terminal velocity increases with decreasing Morton number, and bubble moves up in a zigzag way with shape oscillation for the case of low Morton number of NF. In the nNF, the bubble terminal velocity increases with the increases of the rheological index and the bubble size as well as its shape changes from an ellipsoidal to ellipsoidal-cap. Also, the drag coefficient is high in a low rheological index compared to the high rheological index. Finally, the computed results show good agreements with experimental results and empirical correlations reported in the literature.

CONFLICT OF INTEREST
The authors declare no conflict of interest.

AUTHOR CONTRIBUTIONS
Md. Tariqul Islam contributed to the investigation, methodology, validation, and writing of the original draft. Poo B. Ganesan contributed to the investigation, methodology, validation, and writing of the review and editing. Ji Cheng contributed to the investigation. Mohammad S. Uddin contributed to the writing of the review and editing.

NOMENCLATURE

ALPHABETIC

| Symbol | Description | Units |
|--------|-------------|-------|
| Ar     | Archimedes number | (—)   |
| d_b    | initial bubble diameter | (mm) |
| d_h    | bubble height | (mm) |
| d_w    | bubble width | (mm) |
| d_eq   | bubble equivalent diameter | (mm) |
| Ca     | capillary number | (—) |
| C_d    | drag coefficient | (—) |
| D      | column width | (mm) |
| Eo     | Eotvos number | (—) |
| f      | flow index | (—) |
| F      | volume force | (N/m³) |
| F_N    | flownumber | (—) |
| g      | gravitational acceleration | (m/s²) |
| H      | column height | (mm) |
| k      | surface curvature of the interface | (—) |
| K      | consistency coefficient | (Pa s') |
| Mo     | Morton number | (—) |
| Mo_non | Morton number of non-Newtonian fluid | (—) |
| P      | liquid pressure | (N/m²) |
| Re     | Reynolds number in Newtonian fluid | (—) |
| Re_non | Reynolds number in non-Newtonian fluid | (—) |
| t      | time | (s) |
| U_T    | bubble terminal velocity | (m/s) |
| V_N    | velocity number | (—) |
| W      | liquid velocity | (m/s) |
| We     | Weber number | (—) |

GREEK SYMBOLS

| Symbol | Description | Units |
|--------|-------------|-------|
| ε_L    | volume fraction function | (—) |
| ρ      | density | (kg/m³) |
| μ      | viscosity | (Pa s) |
| σ      | surface tension coefficient | (N/m) |
REFERENCES

1. Yang G, Du B, Fan L. Bubble formation and dynamics in gas–liquid–solid fluidization—a review. Chem Eng Sci. 2007;62:2-27.
2. Böhm L, Brehmer M, Kraume M. Comparison of the single bubble ascent in a Newtonian and a non-Newtonian liquid: a phenomenological PIV study. Chem Ing Tech. 2016;88:93-106.
3. Böhm L, Kraume M. Fluid dynamics of bubble swarms rising in Newtonian and non-Newtonian liquids in flat sheet membrane systems. J Membr Sci. 2015;475:533-544.
4. Pourtousi M, Sahu J, Ganesan P. Effect of interfacial forces and turbulence models on predicting flow pattern inside the bubble column. Chem Eng Process Process Intens. 2014;75:38-47.
5. Kantarci N, Borak F, Ulgen KO. Bubble column reactors. Process Biochem. 2005;40:2263-2283.
6. Shah Y, Kelkar BG, Godbole S, Deckwer WD. Design parameters estimations for bubble column reactors. AIChE J. 1982;28:353-379.
7. Tomiyama A, Celata G, Hosokawa S, Yoshida S. Terminal velocity of single bubbles in surface tension force dominant regime. Int J Multiphase Flow. 2002;28:1497-1519.
8. Raymond F, Rosant J-M. A numerical and experimental study of the terminal velocity and shape of bubbles in viscous liquids. Chem Eng Sci. 2000;55:943-955.
9. Zhang L, Yang C, Mao Z-S. Unsteady motion of a single bubble in highly viscous liquid and empirical correlation of drag coefficient. Chem Eng Sci. 2008;63:2099-2106.
10. Fragedakis D, Pavlidis M, Dimakopoulos Y, Tsamopoulos J. On the velocity discontinuity at a critical volume of a bubble rising in a viscoelastic fluid. J Fluid Mech. 2016;789:310-346.
11. Böhm L, Kurita T, Kimura K, Kraume M. Rising behaviour of single bubbles in narrow rectangular channels in Newtonian and non-Newtonian liquids. Int J Multiph Flow. 2014;65:11-23.
12. Niethammer M, Brenn G, Marschall H, Bothe D. An extended version of fluid method and its application to single bubbles rising in a viscoelastic liquid. J Comput Phys. 2019;387:326-355.
13. Ganesan PB, Islam MT, Sahu JN, Sandaran SC. Effect of column angles to rise velocity of a single bubble: a CFD study. Prog Comput Fluid Dyn. 2016;16:288-299.
14. Ganesan PB, Tariqul Islam M, Pokrajac D, Hamad F, Sandaran SC. Coalescence and rising behavior of co-axial and lateral bubbles in viscous fluid: a CFD study. Asia-Pac J Chem Eng. 2015;10:670-680.
15. Islam MT, Ganesan P, Cheng J. A pair of bubbles' rising dynamics in a xanthan gum solution: a CFD study. RSC Adv. 2015;5:7819-7831.
16. Pang M, Lu M. Numerical study on dynamics of single bubble rising in shear-thinning power-law fluid in different gravity environment. Vacuum. 2018;153:101-111.
17. Premlata A, Tripathi MK, Karri B, Sahu KC. Numerical and experimental investigations of an air bubble rising in a Carreau-Yasuda shearthinning liquid. Phys Fluids. 2017;29:033103.
18. Salomons EM, Blumrich R, Heimann D. Eulerian time-domain model for sound propagation over a finite-impedance ground surface. Comparison with frequency-domain models. Acta Acust United Acust. 2002;88:483-492.
19. Soonti SG, Pallewar PG, Ghosh AB, Atta A. Understanding the influence of rheological properties of shear-thinning liquids on segmented flow in microchannel using CLSVOF based CFD model. Can J Chem Eng. 2019;97:1208-1220.
20. Islam M, Ganesan P, Sahu JN, Hamad FA. Numerical study to investigate the effect of inlet gas velocity and Reynolds number on bubble formation in a viscous liquid. Therm Sci. 2015;19:2127-2138.
21. Islam MT, Ganesan PB, Sahu JN, Sandaran SC. Effect of orifice size and bond number on bubble formation characteristics: a CFD study. Can J Chem Eng. 2015;93:1869-1879.
22. Islam MT, Ganesan PB, Billah MM, Uddin MN. A numerical study of single air bubble formation comparison between in viscous liquid and in water. Asia-Pac J Chem Eng. 2019;14(6):e2367.
23. Li Y, Yang G, Zhang J, Fan L-S. Numerical simulations of bubble formation dynamics in gas–liquid–solid fluidization at high pressures. Powder Technol. 2001;116:246-260.
24. Ratkovich N, Chan C, Berube PR, Nopens I. Experimental study and CFD modelling of a two-phase slug flow for an airlift tubular membrane. Chem Eng Sci. 2009;64:3576-3584.
25. Buetehorn S, Volmering D, Vossenkaul K, Wintgens T, Wessling M, Melin T. CFD simulation of single-and multi-phase flows through submerged membrane units with irregular fiber arrangement. J Membr Sci. 2011;384:184-197.
26. Bao D, Zhang X, Dong H, Ouyang Z, Zhang X, Zhang S. Numerical simulations of bubble behavior and mass transfer in CO2 capture system with ionic liquids. Chem Eng Sci. 2015;135:76-88.
29. Zhang L, Yang C, Mao Z-S. Numerical simulation of a bubble rising in shear-thinning fluids. J Nonnewton Fluid Mech. 2010;165:555-567.
30. Aguirre VA, Castillo BA, Narvaez CP. Numerical simulation of a bubble rising in an environment consisting of xanthan gum. Paper presented at: AIP Conference Proceedings; 2017:020022.
31. Farhangi M, Passandideh-Fard M, Moin H. Numerical study of bubble rise and interaction in a viscous liquid. Int J Comput Fluid Dynam. 2010;24:13-28.
32. Tripathi MK, Sahu KC, Govindarajan R. Dynamics of an initially spherical bubble rising in quiescent liquid. Nat Commun. 2015;6:6268.
33. Li S-b, Yan Z, Li R-d, Wang L. Numerical simulation of single bubble rising in shear-thinning fluids by level set method. J Central South Univ. 2016;23:1000-1006.
34. Youngs DL. Time-dependent multi-material flow with large fluid distortion. Numer Methods Fluid Dynam. 1982;24:273-285.
35. Brackbill JU, Kothe DB, Zemach C. A continuum method for modeling surface tension. J Comput Phys. 1992;100:335-354.
36. Wenyuan F, Youguang M, Shaokun J, Ke Y, Huaizhi L. An experimental investigation for bubble rising in non-Newtonian fluids and empirical correlation of drag coefficient. J Fluids Eng. 2010;132:021305.
37. Hassan N, Khan MMK, Rasul M. A study of bubble trajectory and drag co-efficient in water and non-Newtonian fluids. WSEAS Trans Fluid Mech. 2008;3:270.
38. Ma D, Liu M, Zu Y, Tang C. Two-dimensional volume of fluid simulation studies on single bubble formation and dynamics in bubble columns. Chem Eng Sci. 2012;72:61-77.
39. Roshdi S, Kasiri N, Rahbar-Kelishami A. VOF simulation of single rising drops in three liquid-liquid extraction systems using CSF and CSS interfacial force models. Braz J Chem Eng. 2018;35:1315-1331.
40. Krishna R, Urseanu M v, Van Baten J, Ellenberger J. Wall effects on the rise of single gas bubbles in liquids. Int Commun Heat Mass Transf. 1999;26:781-790.
41. A. Fluent. 12.0 Theory Guide. Vol 5. Canonsburg, Pennsylvania: Ansys Inc; 2009.
42. Rodrigue D. Generalized correlation for bubble motion. AIChE J. 2001;47:39-44.
43. Clift R, Grace JR, Weber ME. Bubbles, Drops, and Particles. New York: Academic Press; 1978.
44. Gupta A, Kumar R. Lattice Boltzmann simulation to study multiple bubble dynamics. Int J Heat Mass Transf. 2008;51:5192-5203.
45. Ziqi C, Yuyun B, Zhengming G. Hydrodynamic behavior of a single bubble rising in viscous liquids. Chin J Chem Eng. 2010;18:923-930.
46. Tomiyama A, Kataoka I, Zun I, Sakaguchi T. Drag coefficients of single bubbles under normal and micro gravity conditions. JSME Int J Ser B Fluids Therm Eng. 1998;41:472-479.
47. Hassan N, Khan MMK, Rasul M, Rackemann DW. An experimental study of bubble rise characteristics in non-Newtonian (power-law) fluids; 2007.

How to cite this article: Islam MT, Ganesan PB, Cheng J, Uddin MS. Single bubble rising behaviors in Newtonian and non-Newtonian fluids with validation of empirical correlations: A computational fluid dynamics study. Engineering Reports. 2020;2:e12100. https://doi.org/10.1002/eng2.12100