Soliton Random Walk and the Cluster-Stripping Problem in Ultralight Dark Matter

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Simulations of ultralight, $\sim 10^{-22}$ eV, bosonic dark matter exhibit rich wave-like structure, including a soliton core within a surrounding halo that continuously self-interferes on the de Broglie scale. We show here that as an inherent consequence, the soliton undergoes a confined random walk at the base of the halo potential. This is significant for the fate of the ancient central star cluster in Eridanus II, as the agitated soliton gravitationally shakes the star cluster in and out of the soliton on a time scale of $\sim 100$ Myr, so complete tidal disruption of the star cluster can occur within $\sim 1$ Gyr. This destructive effect can be mitigated by tidal stripping of the halo of Eridanus II, thereby reducing the agitation, depending on its orbit around the Milky Way. Our simulations show the Milky Way tide affects the halo much more than the soliton, so the star cluster in Eridanus II can survive for over $5$ Gyr within the soliton if it formed after significant halo stripping.

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Introduction. A Bose-Einstein condensate of ultralight bosons, with mass $m \sim 10^{-22}$ eV and a de Broglie wavelength on an astronomical scale, has emerged as a viable interpretation of dark matter [1–8], which is often termed fuzzy dark matter (FDM) or wave dark matter ($\psi$DM). This has the desirable effect of suppressing dwarf galaxies [9] as the uncertainty principle counters self-gravity on an astronomical scale, has emerged as a viable alternative to the standard cold dark matter model.

Pioneering $\psi$DM simulations have revealed pervasive, wave-like structure, with a soliton core at the base of every halo, surrounded by turbulent density fluctuations that self-interfer [7] [11]. The soliton is a prominent flat-topped overdensity that compares favorably with the density profiles of dwarf galaxies [7] [12] [13]. But this may be in tension with the HI based smoothly rising rotation curves of more massive galaxies [13] [16], which can be better addressed in the future by incorporating hydrodynamics self-consistently [17]. The density fluctuations in the surrounding halo are fully modulated, which may measurably heat stars [13], thicken galactic discs [19], and affect stellar streams [20]. Direct Compton scale oscillations for pulsars residing within dense soliton cores may also be detectable [21].

The existence of the central compact star cluster in the ultra-faint dwarf galaxy Eridanus II (Eri II) provides a unique probe for constraining dark matter models [22]. It has been suggested that the gravitational heating from the oscillations of soliton peak density should destroy the star cluster completely for boson masses in the range $10^{-21}$ eV $\lesssim m \lesssim 10^{-19}$ eV [23]. In this Letter, using self-consistent $\psi$DM simulations, we show that the central soliton exhibits random-walk behavior, which can lead to complete tidal disruption of the star cluster within $\sim 1$ Gyr even for $m \sim 10^{-22}$ eV. Note that this problem is distinctly different and potentially more serious than the gravitational heating problem above.

Simulation setup. We follow the evolution of a compact star cluster embedded in the center of a live $\psi$DM halo that mimics Eri II. For the halo component, we extract it from a cosmological simulation [11] at redshift zero, with a virial mass of $M_{\text{vir}} \sim 6 \times 10^9 M_\odot$ and a radius of $r_{\text{vir}} \sim 50$ kpc. This halo hosts a central soliton with a half-density radius of $r_{\text{sol}} \sim 0.7$ kpc, an enclosed mass within $r_{\text{sol}}$ of $M_{\text{sol}} \sim 1 \times 10^8 M_\odot$, and a peak density of $\rho_{\text{sol}} \sim 3 \times 10^6 \rho_{\text{DM}} \sim 0.1 M_\odot$ pc$^{-3}$ where $\rho_{\text{DM}} \sim 4 \times 10^{-8} M_\odot$ pc$^{-3}$ represents the mean dark matter density. The mass within 280 pc is $\sim 1 \times 10^7 M_\odot$, consistent with Eri II that has a mass of $M_{1/2} = 1.2^{+0.3}_{-0.4} \times 10^7 M_\odot$ within a half-light radius of $r_{1/2} = 277 \pm 14$ pc [24].

The central star cluster in Eri II has a mass of $M_{\text{EII}} \sim 2 \times 10^5 M_\odot$, a half-light radius of $r_{\text{EII}} \sim 13$ pc, an age of $T_{\text{EII}} \gtrsim 3$ Gyr, and a very shallow density profile [24] [25]. We model it by a Plummer sphere with a peak stellar mass density of $\rho_{\text{*,max}} \sim 0.2 M_\odot$ pc$^{-3}$ and a scale radius of $\sim 20$ pc. It corresponds to an enclosed mass within $r_{\text{EII}}$ of $\sim 1 \times 10^5 M_\odot$, in good agreement with observations. The star cluster is self-bound since $\rho_{\text{*,max}} > \rho_{\text{sol}}$. We let the star cluster center coincide with the soliton center initially.

The governing equation of $\psi$DM is the Schrödinger-Poisson equation [2], which in physical coordinates reads

$$\left[ \frac{i}{m} \frac{\partial}{\partial t} + \frac{\hbar^2}{2m} \nabla^2 - V_\psi \right] \psi = 0,$$

(1)

$$\nabla^2 V_\psi = 4\pi G m |\psi|^2,$$

(2)

where $\psi$ is the wave function, $V_\psi$ is the gravitational potential, $m$ is the boson mass, $\hbar$ is the reduced Planck
constant, and $G$ is the gravitational constant. $\rho_{\text{DM}} = m|\psi|^2$ gives the dark matter mass density. We adopt $m = 8 \times 10^{-23}$ eV throughout this work to be consistent with [7] [11].

To evolve $\psi$DM and stars, we use the code GAMER [20], which supports adaptive mesh refinement with hybrid MPI/OpenMP/GPU parallelization. It employs an explicit finite-difference method to solve Eqs. (1) and (2) [7] and further incorporates an auxiliary continuity equation to ensure mass conservation to the machine precision. The code has been extensively applied to $\psi$DM simulations (e.g., [7] [11] [27]).

We simulate a volume of size $L = 250 \text{kpc}$, with a root grid $N = 128^3$ and up to nine refinement levels. Grid refinement is performed on patches of $8^3$ cells. Two refinement criteria are adopted. First, to resolve the $\psi$DM halo and soliton, cells with $\rho_{\text{DM}} > 10^4 \rho_{\text{DM}}$ are refined to level $l + 1$ for $0 \leq l \leq 3$, leading to a resolution of $0.12 \text{kpc} \sim (1/6) r_{\text{sol}}$. Second, to resolve the central star cluster, patches containing more than $10^3$ particles are refined, giving a maximum resolution of $3.8 \text{pc} \lesssim (1/3) r_{\text{FEH}}$. This extremely high resolution results in simulation time-steps as small as $\Delta t \sim 150 \text{yr}$ due to the stringent Courant-Friedrichs-Lewy stability condition imposed by Eq. (1). The total number of collisionless particles for sampling the star cluster is $2 \times 10^5$, corresponding to a particle mass resolution of $3.6 \times 10^{-2} M_\odot$. We adopt isolated boundary conditions for gravity and sponge boundary conditions for wave function. We have validated the numerical convergence of all the presented results by adjusting the spatial resolution by up to a factor of 4 and the particle mass resolution by a factor of 10. We have also confirmed that an isolated star cluster without a $\psi$DM halo is stable over a Hubble time in our simulation setup.

Isolated $\psi$DM halo. We start by simulating an isolated system by ignoring the tidal field of the Milky Way. Fig. 1 shows the motion of the central soliton, revealing for the first time a confined Brownian (random-walk-like) motion at the base of the halo potential. This random motion exhibits a characteristic length scale similar to the soliton radius $r_{\text{sol}}$ and a time scale comparable to the oscillation period of the soliton wave function, $t_{\text{sol}} \sim 120(\rho_{\text{sol}}/0.1 M_\odot \text{pc}^{-3})^{-1/2} \sim 120 \text{Myr}$. We adopt the halo center of mass as the halo center and the soliton peak density position as the soliton center. The drift of the halo center of mass caused by numerical errors is found to be of order $10^{-2} r_{\text{sol}}$, indicating that this soliton random motion is not a numerical artifact.

The star cluster can be treated as a tracer as the soliton is five orders of magnitude more massive. The characteristic free-fall time of a star cluster located just outside the soliton is $t_{\text{ff}} \sim (G \rho_{\text{sol}})^{-1/2} \sim 50 \text{Myr}$, smaller but comparable to $t_{\text{sol}}$. It suggests that the star cluster can only loosely trace the random motion of soliton, resulting in a maximum separation between them, denoted as $R_{\text{cs}}$, of order $r_{\text{sol}}$. Fig. 2(a)–(d) illustrates this feature.

This large separation can have a great impact on the survival of the star cluster due to tidal stripping, which can be seen as follows. The soliton density profile features
FIG. 2: Tidal stripping of a star cluster caused by soliton random walk. (a)–(d) Projected stellar mass density in a 4 kpc thick slab centered on the initial center of the star cluster. Contours indicate the soliton boundaries (≈ r_{sol}) where the dark matter density drops by half. Although the star cluster and soliton centers coincide at the beginning by design, the soliton random motion quickly results in a separation between them of order r_{sol}, leading to a large tidal field that disrupts the star cluster (see text for details). (e) Stellar density profiles. The dash-double-dot line shows a soliton profile for comparison. The vertical (dash-dash-dot) line indicates the half-light radius (r_{EII}) of the central star cluster in Eri II. (f) Enclosed stellar mass within r_{EII} as a function of time, normalized to its initial value. The star cluster loses ∼ 99% of its original mass after ∼ 1 Gyr.

a flat core within ≈ r_{sol} and a steep outer gradient. So, for R_{cs} > r_{sol}, a star at a distance r_{s} with respect to the star cluster center will be subject to a tidal field f_{tidal} ≈ 2G M_{sol}r_{s}/R_{cs}^{3} ≈ r_{cs}^{3}, assuming r_{s} ≪ R_{cs}. On the other hand, for R_{cs} < r_{sol}, the star cluster will be compressed instead of tidally stripped. As a result, tidal stripping is most effective when R_{cs} ∼ r_{sol}, which is exactly what happens here. We can assess the significance of the tidal field at r_{s} by computing the ratio of f_{tidal} to the star cluster self-gravity f_{s},

$$\frac{f_{tidal}}{f_{s}} \approx \frac{3}{2\pi} \frac{M_{sol}}{\rho_{s,\text{max}} R_{cs}^{3}} \approx 2 \frac{\rho_{sol}}{\rho_{s,\text{max}}} \approx O(1), \quad (3)$$

where \(\rho_{sol} \approx 0.1 M_{\odot} \text{ pc}^{-3}\) and \(\rho_{s,\text{max}} \approx 0.2 M_{\odot} \text{ pc}^{-3}\) as described earlier. The fact that this ratio is of order unity and independent of r_{s}, suggests that the entire star cluster is marginally stable and vulnerable to tidal disruption after \(t \gg t_{\text{eff}}\).

This expectation is confirmed in Fig. 2(e) and (f), showing that the star cluster loses ∼ 90% and 99% of its original mass within r_{EII} after ∼ 0.7 and 1 Gyr, respectively. This disruption time scale is noticeably shorter than T_{EII} and can potentially present a serious challenge for \(\psi\)-DM, which we refer to as the cluster-stripping problem. Nevertheless, in the following, we shall further show how the Milky Way tides may alleviate this problem by reducing the halo agitation of the soliton.

Tidally disrupted \(\psi\)-DM halo. Satellite galaxies of the Milky Way are subject to its tidal field. For example, the Fornax spheroidal galaxy is estimated to have a tidal radius of only ∼ 1.8 − 2.8 kpc. Eri II, although currently at a Galactocentric distance of ∼ 370 kpc, may be on its second or third orbit around the Milky Way with an infall time of ∼ 4 − 10 Gyr ago and a periapsis of ∼ 100 − 200 kpc. The question naturally arises as to whether the Milky Way tides can affect the soliton random motion, and thus the survival of the central star cluster in Eri II.

Modeling the exact tidal stripping process of Eri II requires detailed information about its orbital parameters, which unfortunately still suffers from large uncertainties. As a proof-of-concept study, we adopt a circular orbit of radius R_{MW} = 100 kpc for the satellite galaxy and a point mass of M_{MW} = 1 × 10^{12} M_{\odot} to approximate the tidal field of the Milky Way. To speed up the simulations, we choose a moving non-rotating coordinate system, with the Milky Way center orbiting the coordinate origin that coincides with the center of mass of the satellite halo, in order to get rid of the small wavelength in \(\psi\) associated with the high orbital velocity. Assuming \(r \ll R_{MW}\), where \(r\) and \(R_{MW}(t)\) are the position vectors of a simulation cell and the Milky Way center, respectively, the tidal potential can be approximated as

$$V_{\text{tidal}}(r, t) \approx \frac{GM_{MW}}{2R_{MW}^{2}} \left[ r^{2} - 3 \left( \frac{r \cdot R_{MW}(t)}{R_{MW}} \right)^{2} \right]. \quad (4)$$
The total potential is given by $V_{\text{tot}} = V_\psi + V_{\text{tidal}}$. We evolve the system for $9\,\text{Gyr}$.

Fig. 3 shows the tidal stripping process of a satellite halo. The halo surrounding the central soliton is found to be vulnerable to tidal disruption; the density at $r \gtrsim 3\,r_{\text{sol}}$ decreases by more than an order of magnitude after $\sim 2\,\text{Gyr}$. In comparison, the soliton, which is strongly gravitationally bound, is resilient to tidal disruption and stays intact during the entire simulation period, which ensures that the average dark matter density remains consistent with the observational constraint of Eri II (i.e., $M_{1/2} \sim 1 \times 10^7\,M_\odot$).

During the tidal stripping of the halo, the soliton random motion is significantly diminished. Fig. 3(a) shows the separation between the halo and soliton centers as a function of time for a system subject to the Milky Way tides. We define the halo center as the center of mass of high-density regions ($\rho_{\text{DM}} > 10^4\bar{\rho}_{\text{DM}}$) to exclude the stripped material. The separation, normalized to $r_{\text{sol}}$, reduces from $\mathcal{O}(1)$ to $\mathcal{O}(10^{-2})$ after $\sim 5\,\text{Gyr}$. This result is not surprising given that the random motion originates from the interaction between soliton and its halo.

In Fig. 3(b), we assess how the above findings affect the cluster-stripping problem by conducting four simulations and in each of them, we add the central star cluster at a different epoch, $t_0 = 0, 2, 3, 4\,\text{Gyr}$. We find that the later the star cluster is added, the more stable it is, because the tidal field induced by soliton random motion becomes weaker over time. For $t_0 = 0$, the result is almost identical to the case without considering the Milky Way tides (see Fig. 2) since the tidal disruption time scale of the star cluster ($\lesssim 1\,\text{Gyr}$) is shorter than that of the halo ($\sim 2 - 4\,\text{Gyr}$). In comparison, for $t_0 = 4\,\text{Gyr}$, at which the ambient medium of soliton has been largely stripped away (see Fig. 3), the star cluster can stay intact throughout the remaining simulation time in $5\,\text{Gyr}$. This period is longer than $T_{\text{EII}}$ and thus free of the cluster-stripping problem.

**Concluding remarks.** We have reported a cluster-stripping problem specific to dwarf galaxy-sized \(\psi\)DM halos hosting a central compact star cluster, such as Eri II. Our unprecedentedly high-resolution simulations reveal, for the first time, random-walk-like behavior in the soliton core. This can displace the star cluster slightly outside the soliton radius ($r_{\text{sol}}$), leading to efficient tidal disruption of the star cluster.

As a possible solution, we have further demonstrated that if Eri II is bound to the Milky Way, the tidal field of the Milky Way may readily disrupt the outer part of a \(\psi\)DM subhalo, leaving the relatively dense soliton intact. In this case, we find that the degree of soliton random motion declines significantly, and so a star cluster forming centrally after substantial halo removal can survive much longer within the soliton. More accurate proper motion measurements of Eri II in the future will help clarify this possibility.

We emphasize that the \(\psi\)DM halo adopted here, which is consistent with the observed average dark matter density of Eri II (i.e., $M_{1/2} \sim 1 \times 10^7\,M_\odot$), is directly extracted from a cosmological simulation with $m \sim 1 \times$
$10^{-22}$ eV \cite{11}. Therefore, it appears to undermine the claim that Eri II cannot form for $m \lesssim 8 \times 10^{-22}$ eV based on the subhalo mass function \cite{22}, which likely underestimates the total halo mass by one to two orders of magnitude compared to our simulations. Our study also confirms that the star cluster heating due to the oscillations of soliton peak density is irrelevant for $m \sim 1 \times 10^{-22}$ eV, in agreement with \cite{22}.

The soliton peak density scales as $\rho_{\text{sol}} \propto m^{-2} r_{\text{sol}}^{-4}$ \cite{11}. For $m \lesssim 5 \times 10^{-22}$ eV, a soliton of $\rho_{\text{sol}} \sim \rho_\ast \max$ can still satisfy the constraint $M_{1/2} \sim 1 \times 10^{7}$ M$_\odot$. So the tidal field remains roughly the same (Eq. \ref{eq:3}). For even larger $m$, $\rho_{\text{sol}}$ must increase, resulting in more efficient tidal stripping under the assumption that the characteristic length scale of soliton random motion remains to be of order $r_{\text{sol}}$.

For studies using the soliton properties to test $\psi$DM, it would be important to take into account its random motion reported here. Examples include the dynamical friction and core stalling in dwarf galaxies \cite{8, 31, 32, 10}, the galactic rotation curves \cite{14, 16}, and the excess mass in the Milky Way center \cite{23, 33}. But it remains to be investigated how soliton random walk changes with halo mass.

A possible physical interpretation of the soliton random motion is that while a halo comprises a wide range of wave function frequencies, only the beating frequencies of low-lying modes near the halo center are comparable to $t^{-1}_{\text{sol}}$ \cite{27}. The gradient of the dominant dipole potential perturbation near the soliton, $\delta V_{\psi, l=1}(t) = 8\pi G \nabla^2 \text{Re}(\psi_{\text{sol}}^* (t) \sum_{m} \psi_{\text{halo}, n=1, m}(t))$, may result in the observed soliton random motion. Here $\psi_{\text{halo}}$ and $\psi_{\text{sol}}$ represent the halo and soliton components of the total wave function $\psi$, respectively, $n, l, m$ are the well-known quantum numbers, and the summation over $n$ is for low $n$’s. If $|\psi_{\text{halo}}|^2 / |\psi_{\text{sol}}|^2 = \epsilon$, the soliton density fluctuations are enhanced to order of $\epsilon^{1/2}$ and so are the potential fluctuations near the soliton. Thus the soliton random motion can be substantial.

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