On quantum illumination, quantum reading, and the capacity of quantum computation

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In this brief note, I clarify the crucial differences between three different protocols of quantum channel discrimination, after some confusion has appeared in recent literature.

Reason for this note

In some recent literature, there has been confusion between the protocols of quantum illumination \cite{1, 2}, quantum reading \cite{3}, and a scheme of communication within a discrete-variable quantum computer \cite{4}. All these protocols are based on the idea of quantum channel discrimination (QCD), but they have completely different applications and features, which is the reason why they have different names and should not be confused. Let me provide some clarifications below.

Quantum Illumination

Quantum illumination \cite{1, 2} (see also \cite{5–21}) is the use of input quantum resources (such as entanglement) and output quantum measurements to enhance the detection of a remote low-reflectivity object in a bright thermal-noise environment. It can be represented as a QCD problem where the bit of information associated with the presence or absence of the target is associated with the binary discrimination of two channels, one including a partial reflection from the target and the other one being a completely thermalizing channel (replacing the input with the state of the environment). Here one can show that, despite initial entanglement is lost in the sender-receiver path, the benefits of quantum illumination still survive in the forms of output correlations. These allow one to enhance the sensitivity of detecting the presence of the target-object with respect to the use of classical sources of light (in particular separable states in the DV version of the protocol \cite{1}, and mixtures of coherent states in the CV version \cite{2}). It is called “quantum” illumination because it proves a quantum advantage with respect to classical strategies under the same conditions (e.g., the same mean number of input photons).

The two schemes of quantum illumination and quantum reading have a specific peculiarity (quantum enhancement) that gives them the “quantum name”. At the same time it is clear that they are both schemes of QCD, where classical information is retrieved from a box (target object or memory cell). For instance, see the discussion in Section V.H “Gaussian channel discrimination and applications” of the Gaussian information review \cite{38}. For more details on these protocols, see also the recent review on photonic quantum sensing \cite{39}.

Capacity of quantum computation

The scheme of Ref. \cite{4} is about the communication capacity of quantum computation. Clearly, it is not about target detection or optical storage, but rather communication between registers of a discrete-variable quantum computer. In this scheme, there is a “memory” register \((M)\) where the sender encodes a classical variable \(i\) in \(N\) pure quantum states \(|i\rangle_M\) \(|i\rangle\) with some probability \(p_i\). Then, the receiver has a computation register (\(C\)) prepared in some initial state \(\rho^0_C\). The initial state of the two registers is therefore the tensor-product

\[
\sum_i p_i |i\rangle_M \otimes \rho^0_C. \tag{1}
\]

The two registers are then fed into a quantum computer, which applies the unitary \(U_i\) onto register \(C\) conditionally on the value \(i\) of register \(M\). Here \(U_i\) represents a series of...
quantum gates which describes some quantum algorithm. For instance, \( i \) may be an integer, and the computational output \( \rho_C = \hat{U}_i \rho_C^i \hat{U}_i^\dagger \) may be its factorization according to Shor’s algorithm.

In general, for the input state as in Eq. (1), the quantum computer provides the output
\[
\sum_i p_i |i\rangle_M \langle i| \otimes \rho^i_C.
\] (2)

The receiver measures register \( C \) so as to discriminate between the possible output states \( \rho^i_C \), or equivalently between the possible unitary operations \( \hat{U}_i \). The optimal information accessible to the receiver is the Holevo bound
\[
I(C : i) = S(C) - S(C|i),
\] (3)
where \( S(C) \) is the von Neumann entropy of the reduced state of \( C \), and \( S(C|i) \) is the corresponding conditional von Neumann entropy. This is clearly maximized when \( p_i \) is uniform and the states \( \rho^i_C \) are pure and orthogonal, so that it takes the maximum value \( I(C : i) = \log_2 N \).

By construction, it is clear that \( I(C : i) \) represents the capacity of the quantum computation \( \{\hat{U}_i\} \) because it tells you how good the quantum computer is in providing distinguishable output states (solutions) for different inputs. When the maximum \( \log_2 N \) is achieved, it means that the quantum computation is perfect over the entire input alphabet of \( N \) letters.

Clarifications

Apart from being interpreted as protocol of QCD, the scheme of Ref. [4] is clearly different from both quantum illumination and quantum reading.

- First of all, Ref. [4] is a communication scheme, where sender’s input alphabet is decoded by a receiver. More specifically, it is spatial communication between two registers which is mediated by a quantum computation. It is a two-register description where the unitaries are \( \hat{U}_i \) are not stored in the computational register \( M \) but rather applied in the dynamical process of the quantum computer (they are in fact control-unitaries). In this regard it is clearly different from the static scenario where a classical variable is physically and stably stored into a black box by an ensemble of channels (to describe presence/absence of a target, or the different reflectivities of a memory cell). This means that Ref. [4] is not about readout from storage.

- The input-output process is based on a single signal system processed by a unitary. Today, we know that unitary discrimination can be perfectly solved in a finite number of uses [40]. It is therefore different from what happens in the more general discrimination of quantum channels, where perfect discrimination is not guaranteed (at finite energies) and the optimal states may require the use of idler systems, which are not sent through the box but directly to the output measurement in order to assist the entire process.

- The signal-idler structure, which is missing in Ref. [4], is one of the main features for both quantum illumination and quantum reading. The use of input entanglement and, more generally, quantum correlations, is the main working mechanism of these two protocols under completely general conditions of decoherence. As a matter of fact, as already said above, the “quantum” name in “illumination” and “reading” exactly comes from the comparison of a quantum resource at the input (entanglement) with respect to the use of classical input states (separable states, mixtures of coherent states).

- The communication scheme of Ref. [4] is for a discrete-variable Hilbert space. The main setting for both quantum illumination and quantum reading is bosonic. Quantum illumination provides the possible working mechanism of a lidar (in the optical case) and a radar (in the microwave case). Quantum reading is also working at the optical frequencies, which is the physics in optical storage.

In summary, all the protocols of quantum illumination, quantum reading, and communication of quantum computation can be represented as schemes of QCD. However, they are protocols with different aims and features, which is the reason why they should not be confused one with the other. In particular, the scheme of Ref. [4] is about communication between registers of a quantum computer, clearly not “quantum reading” of a classical memory nor “quantum illumination” of a remote target. With these two protocols, it only shares the basic structure, that of QCD, which is ultimately about the retrieval of classical information from some type of black box.

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