The Rarita–Schwinger field: dressing procedure and spin-parity content

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We obtain in analytical form the dressed propagator of the massive Rarita–Schwinger field taking into account all spin components. We found that the nearest analogy for dressing the Rarita–Schwinger field in spin–1/2 sector is dressing the two Dirac fermions of opposite parity with presence of mutual transitions. The calculation of the self-energy contributions confirms that besides the leading spin–3/2 component the Rarita–Schwinger field contains also two spin–1/2 components of different parity.

I. INTRODUCTION

The description of spin–3/2 particles in (effective) quantum field theory is usually based on using of a spin-vector field $\Psi^\mu$, called the Rarita–Schwinger field [1, 2, 3]. However, besides the leading spin–3/2 component, this field contains also two spin–1/2 representations which are usually supposed unphysical. The main difficulties and paradoxes in its description [4, 5, 6] are in fact related with existence of ”extra” components in $\Psi^\mu$ field and with attempts to exclude them in some way. The problem of consistent description of spin–3/2 particles has a long history but we mention here only relatively recent works [7, 8, 9, 10, 11, 12, 13] where the detailed discussion may be found. As for applications of this spin-vector formalism, they are first of all the description of the baryon resonances production (the most investigated one is $\Delta(1232)$) in various processes and theoretical description of gravitino properties.

The most general lagrangian for the free Rarita–Schwinger field has the following form (see e.g. [14, 15, 16, 17]):

$$\mathcal{L} = \bar{\Psi}^\mu \Lambda^{\mu\nu} \Psi^\nu,$$

$$\Lambda^{\mu\nu} = (\hat{\partial} - M)g^{\mu\nu} + A(\gamma^\mu p^\nu + \gamma^\nu p^\mu) + \frac{1}{2}(3A^2 + 2A + 1)\gamma^\mu \hat{\partial} \gamma^\nu + M(3A^2 + 3A + 1)\gamma^\mu \gamma^\nu. \quad (1)$$

Here $M$ is the mass of Rarita–Schwinger field, $A$ is an arbitrary parameter, $p^\mu = i\partial_\mu$.

This lagrangian is invariant under the point transformation:

$$\Psi^\mu \rightarrow \Psi'^\mu = (g^{\mu\nu} + \alpha \gamma^\mu \gamma^\nu)\Psi^\nu, \quad A \rightarrow A' = \frac{A - 2\alpha}{1 + 4\alpha}, \quad (2)$$

with parameter $\alpha \neq -1/4$.

The lagrangian (1) leads to the following equations of motion:

$$\Lambda^{\mu\nu} \Psi^\nu = 0. \quad (3)$$

The free propagator of the Rarita–Schwinger field in a momentum space obeys the equation:

$$\Lambda^{\mu\nu} G^{\nu\rho}_0 = g^{\mu\rho}. \quad (4)$$
The expression for the free propagator $G_{0}^{\mu\nu}$ is well known (see references above), thus we do not present it here.

As concerned for the dressed propagator, its construction is more complicated issue and its total expression is unknown up to now. More exactly, the spin–3/2 contribution may be written out unambiguously (it has the Breit–Wigner form in case of $\Delta(1232)$), while the $s = 1/2$ contributions present some problem. Therefore a practical use of the dressed propagator $G^{\mu\nu}$ needs some approximations in its description. The standard approximation (e.g. [18, 19]) consists in dressing the spin–3/2 components only while the rest components are neglected or considered as bare. Another way to take into account the spin–1/2 components is a numerical solution of appearing system of equations [7, 12]. However, it is rather difficult to understand in this case, how the unphysical degrees of freedom are renormalized and whether they are remained unphysical after the dressing.

The non-leading $s = 1/2$ spin contributions from phenomenological point of view generate the non-resonant background contribution that interfere with resonance. It is not obvious in advance whether these contributions are essential for observables. But at least for $\Delta(1232)$ isobar production in Compton scattering it was noted [18] that the non-leading contributions are necessary for data description.

In this paper we derive an analytical expression for the interacting Rarita–Schwinger field’s propagator with accounting all spin components and discuss its properties. In particular we identify the spin-parity content of different contributions in propagator. The spin–1/2 part has rather compact form and a crucial point for its deriving is the choosing of a suitable basis.

II. DRESSED PROPAGATOR OF THE RARITA–SCHWINGER FIELD

The Dyson–Schwinger equation for the propagator of the Rarita–Schwinger field has the following form

$$G^{\mu\nu} = G_{0}^{\mu\nu} + G^{\mu\alpha} J_{\alpha\beta} G_{0}^{\beta\nu}. \quad (5)$$

Here $G_{0}^{\mu\nu}$ and $G^{\mu\nu}$ are the free and full propagators respectively, $J^{\mu\nu}$ is a self-energy contribution. The equation may be rewritten for inverse propagators as

$$(G^{-1})^{\mu\nu} = (G_{0}^{-1})^{\mu\nu} - J^{\mu\nu}. \quad (6)$$

If to consider the self-energy $J^{\mu\nu}$ as a known value, then the problem is reduced to reversing of the relation (6). For this procedure it is useful to have some basis for both propagators and self-energy.

1. The most natural basis for the spin-tensor $S^{\mu\nu}(p)$ decomposition is the $\gamma$-matrix one:

$$S^{\mu\nu}(p) = g^{\mu\nu} \cdot s_1 + p^\mu p^\nu \cdot s_2 + \tilde{p} p^\mu p^\nu \cdot s_3 + \tilde{p} g^{\mu\nu} \cdot s_4 + p^\mu \gamma^\nu \cdot s_5 + \gamma^\mu p^\nu \cdot s_6 + \sigma^{\mu\nu} \cdot s_7 + \sigma^{\mu\lambda} p^\lambda p^\nu \cdot s_8 + \sigma^{\nu\lambda} p^\lambda p^\mu \cdot s_9 + \gamma^\lambda \gamma^5 \varepsilon^{\lambda\mu\nu\rho} p^\rho \cdot s_{10}. \quad (7)$$

Here $S^{\mu\nu}$ is an arbitrary spin-tensor depending on the momentum $p$, $s_i(p^2)$ are the Lorentz invariant coefficients, and $\sigma^{\mu\nu} = \frac{1}{2}[\gamma^\mu, \gamma^\nu]$. Altogether there are ten independent components in the decomposition of $S^{\mu\nu}(p)$, if parity is conserved.

It is known that the $\gamma$-matrix decomposition is complete, the coefficients $s_i$ are free of kinematic singularities and constraints, and their calculation is rather simple. However this basis is inconvenient at multiplication and reversing of the spin-tensor $S^{\mu\nu}(p)$ because the basis elements

1 The short version of the paper without the spin-parity discussion was published in [20].
2 This is the widely used in the resonant physics “rainbow” approximation, see e.g. recent review [21].
are not orthogonal to each other. As a result the reversing of the spin-tensor $S^\mu\nu(p)$ leads to a system of 10 equations for the coefficients.

2. There is another basis used in consideration of the dressed propagator $G^{\mu\nu}$. It is constructed from the following set of operators:\footnote{We changed here, for convenience, the normalization of $P_{21}^{1/2}$, $P_{12}^{1/2}$.}

\begin{align*}
(P_{3/2}^{3/2})^{\mu\nu} &= g^{\mu\nu} - \frac{2}{3} \frac{p^\mu p^\nu}{p^2} - \frac{1}{3} \gamma^{\mu \gamma} \gamma^{\nu} + \frac{1}{3p^2}(\gamma^{\mu \nu} - \gamma^{\nu \mu})\hat{p}, \\
(P_{11}^{1/2})^{\mu\nu} &= \frac{1}{3} \gamma^{\mu \gamma} \gamma^{\nu} - \frac{1}{3} \frac{p^\mu p^\nu}{p^2} - \frac{1}{3p^2}(\gamma^{\mu \nu} - \gamma^{\nu \mu})\hat{p}, \\
(P_{22}^{1/2})^{\mu\nu} &= \frac{p^\mu p^\nu}{p^2}, \\
(P_{21}^{1/2})^{\mu\nu} &= \sqrt{\frac{3}{p^2}} \cdot \frac{1}{3p^2}(-p^\mu + \gamma^\mu \hat{p})\hat{p} p^\nu, \\
(P_{12}^{1/2})^{\mu\nu} &= \sqrt{\frac{3}{p^2}} \cdot \frac{1}{3p^2}p^\mu(-p^\nu + \gamma^\nu \hat{p})\hat{p}.
\end{align*}

The first three operators $P_{3/2}^{3/2}$, $P_{11}^{1/2}$, $P_{22}^{1/2}$ are the projection operators while $P_{21}^{1/2}$, $P_{12}^{1/2}$ are nilpotent ones. As for their physical meaning, it is obvious that $P_{3/2}^{3/2}$ corresponds to spin–3/2. The remaining operators should describe two spin–1/2 representations and transitions between them. Let us rewrite the operators (8) to make their properties more obvious:

\begin{align*}
(P_{3/2}^{3/2})^{\mu\nu} &= g^{\mu\nu} - (P_{11}^{1/2})^{\mu\nu} - (P_{22}^{1/2})^{\mu\nu}, \\
(P_{11}^{1/2})^{\mu\nu} &= 3\pi^\mu \pi^\nu, \\
(P_{22}^{1/2})^{\mu\nu} &= \frac{p^\mu p^\nu}{p^2}, \\
(P_{21}^{1/2})^{\mu\nu} &= \sqrt{\frac{3}{p^2}} \cdot \frac{1}{3p^2} \pi^\mu \pi^\nu, \\
(P_{12}^{1/2})^{\mu\nu} &= \sqrt{\frac{3}{p^2}} \cdot \frac{1}{3p^2}p^\mu \pi^\nu.
\end{align*}

Here we introduced the vector

$$
\pi^\mu = \frac{1}{3p^2}(-p^\mu + \gamma^\mu \hat{p})\hat{p}
$$

with the following properties:

$$
(\pi p) = 0, \quad (\gamma \pi) = (\pi \gamma) = 1, \quad (\pi \pi) = \frac{1}{3}, \quad \hat{p} \pi^\mu = -\pi^\mu \hat{p}.
$$

The set of operators (8) can be used to decompose the considered spin-tensor as follows:\footnote{We changed here, for convenience, the normalization of $P_{21}^{1/2}$, $P_{12}^{1/2}$.}

\begin{equation}
S^{\mu\nu}(p) = (S_1 + S_2 \hat{p})(P_{3/2}^{3/2})^{\mu\nu} + (S_3 + S_4 \hat{p})(P_{11}^{1/2})^{\mu\nu} + (S_5 + S_6 \hat{p})(P_{22}^{1/2})^{\mu\nu} + (S_7 + S_8 \hat{p})(P_{21}^{1/2})^{\mu\nu} + (S_9 + S_{10} \hat{p})(P_{12}^{1/2})^{\mu\nu}.
\end{equation}
Let us call this basis as \( \hat{\rho} \)-basis. It is more convenient at multiplication since the spin–3/2 components \( \mathcal{P}^{3/2} \) have been separated from spin–1/2 ones. However, the spin–1/2 components as before are not orthogonal between themselves and we come to a system of 8 equations when inverting the \( (6) \). Another feature of decomposition \( (12) \) is existence of the poles \( 1/p^2 \) in different terms. So, to avoid this unphysical singularity, we should impose some constraints on the coefficients at zero point.

3. Let us construct the basis which is the most convenient at multiplication of spin-tensors. This basis is built from the operators \( \mathbb{P} \) and the projection operators \( \Lambda^\pm \)

\[
\Lambda^\pm = \frac{\sqrt{p^2} \pm \hat{\rho}}{2\sqrt{p^2}},
\]

where we assume \( \sqrt{p^2} \) to be the first branch of analytical function. Ten elements of this basis look as

\[
\begin{align*}
\mathcal{P}_1 &= \Lambda^+ \mathcal{P}^{3/2}, \\
\mathcal{P}_2 &= \Lambda^- \mathcal{P}^{3/2}, \\
\mathcal{P}_3 &= \Lambda^+ \mathcal{P}_{11}^{1/2}, \\
\mathcal{P}_4 &= \Lambda^- \mathcal{P}_{11}^{1/2}, \\
\mathcal{P}_5 &= \Lambda^+ \mathcal{P}_{22}^{1/2}, \\
\mathcal{P}_6 &= \Lambda^- \mathcal{P}_{22}^{1/2}, \\
\mathcal{P}_7 &= \Lambda^+ \mathcal{P}_{21}^{1/2}, \\
\mathcal{P}_8 &= \Lambda^- \mathcal{P}_{21}^{1/2}, \\
\mathcal{P}_9 &= \Lambda^+ \mathcal{P}_{12}^{1/2}, \\
\mathcal{P}_{10} &= \Lambda^- \mathcal{P}_{12}^{1/2},
\end{align*}
\]

(14)

where tensor indices are omitted. We will call \( (14) \) as the \( \Lambda \)-basis.

The decomposition of a spin-tensor in this basis has the following form:

\[
S^{\mu\nu}(p) = \sum_{i=1}^{10} \mathcal{P}_i^{\mu\nu} \bar{S}_i(p^2).
\]

(15)

The coefficients \( \bar{S}_i \) are calculated in analogy with \( \gamma \)-matrix decomposition. Besides, we found (with computer analytical calculations) the matrix relating the \( \Lambda \)-basis with the \( \gamma \)-matrix basis and convinced ourselves that this matrix is not singular. Therefore the elements of this basis \( (14) \) are independent and the basis is complete. It is easy to connect the expansion coefficients \( (12) \) and \( (15) \) between themselves.

\[
\begin{align*}
\bar{S}_1 &= S_1 + \sqrt{p^2} S_2, \\
\bar{S}_3 &= S_3 + \sqrt{p^2} S_4, \\
\bar{S}_5 &= S_5 + \sqrt{p^2} S_6, \\
\bar{S}_7 &= S_7 + \sqrt{p^2} S_8, \\
\bar{S}_9 &= S_9 + \sqrt{p^2} S_{10}, \\
\bar{S}_2 &= S_1 - \sqrt{p^2} S_2, \\
\bar{S}_4 &= S_3 - \sqrt{p^2} S_4, \\
\bar{S}_6 &= S_5 - \sqrt{p^2} S_6, \\
\bar{S}_8 &= S_7 - \sqrt{p^2} S_8, \\
\bar{S}_{10} &= S_9 - \sqrt{p^2} S_{10}.
\end{align*}
\]

(16)

The \( \Lambda \)-basis has very simple multiplicative properties which are represented in the Table [1].

The first six basis elements are projection operators, while the remaining four elements are nilpotent. We are convinced by direct calculations that there are no other projection operators besides indicated.

Now we can return to the Dyson–Schwinger equation \( (6) \). Let us denote the inverse dressed propagator \( (G^{-1})^{\mu\nu} \) and free one \( (G_0^{-1})^{\mu\nu} \) by \( S^{\mu\nu} \) and \( S_0^{\mu\nu} \) respectively. Decomposing the \( S^{\mu\nu} \), \( S_0^{\mu\nu} \) and \( J^{\mu\nu} \) in \( \Lambda \)-basis according to \( (14) \) we reduce the equation \( (6) \) to set of equations for the scalar coefficients

\[
\bar{S}_i(p^2) = \bar{S}_0i(p^2) + J_i(p^2)
\]
Table I: Properties of the Λ-basis at multiplication.

| $\mathcal{P}_1$ | $\mathcal{P}_2$ | $\mathcal{P}_3$ | $\mathcal{P}_4$ | $\mathcal{P}_5$ | $\mathcal{P}_6$ | $\mathcal{P}_7$ | $\mathcal{P}_8$ | $\mathcal{P}_9$ | $\mathcal{P}_{10}$ |
|----------------|----------------|----------------|--------------|--------------|--------------|--------------|--------------|--------------|--------------|
| $\mathcal{P}_1$ | 1              | 0              | 0            | 0            | 0            | 0            | 0            | 0            | 0            |
| $\mathcal{P}_2$ | 0              | $\mathcal{P}_2$ | 0            | 0            | 0            | 0            | 0            | 0            | 0            |
| $\mathcal{P}_3$ | 0              | 0              | $\mathcal{P}_3$ | 0            | 0            | $\mathcal{P}_7$ | 0            | 0            | 0            |
| $\mathcal{P}_4$ | 0              | 0              | 0            | $\mathcal{P}_4$ | 0            | 0            | 0            | $\mathcal{P}_8$ | 0            |
| $\mathcal{P}_5$ | 0              | 0              | 0            | 0            | $\mathcal{P}_5$ | 0            | 0            | 0            | $\mathcal{P}_9$ |
| $\mathcal{P}_6$ | 0              | 0              | 0            | 0            | 0            | 0            | $\mathcal{P}_6$ | 0            | 0            | $\mathcal{P}_{10}$ |
| $\mathcal{P}_7$ | 0              | 0              | 0            | 0            | 0            | 0            | 0            | 0            | $\mathcal{P}_7$ |
| $\mathcal{P}_8$ | 0              | 0              | 0            | $\mathcal{P}_8$ | 0            | 0            | 0            | 0            | 0            | $\mathcal{P}_4$ |
| $\mathcal{P}_9$ | 0              | 0              | 0            | 0            | 0            | 0            | 0            | $\mathcal{P}_5$ | 0            | 0            |
| $\mathcal{P}_{10}$ | 0              | 0              | 0            | 0            | $\mathcal{P}_{10}$ | 0            | 0            | 0            | 0            | $\mathcal{P}_6$ |

The values $\tilde{S}_i$ are defined by the bare propagator and the self-energy and may be considered as known.

The dressed propagator also can be found in such form

$$G^{\mu\nu} = \sum_{i=1}^{10} \mathcal{P}_i^{\mu\nu} \cdot \tilde{G}_i(p^2)$$

The existing 6 projection operators take part in the decomposition of $g^{\mu\nu}$:

$$g^{\mu\nu} = \sum_{i=1}^{6} \mathcal{P}_i^{\mu\nu}.$$ (18)

Now solving the equation

$$G^{\mu\nu} S^{\nu\lambda} = g^{\mu\lambda}$$

in Λ-basis, we obtain a set of equations for the scalar coefficients $\tilde{G}_i$.

$$\tilde{G}_1 \tilde{S}_1 = 1,$$
$$\tilde{G}_2 \tilde{S}_2 = 1,$$
$$\tilde{G}_3 \tilde{S}_3 + \tilde{G}_7 \tilde{S}_{10} = 1,$$
$$\tilde{G}_4 \tilde{S}_4 + \tilde{G}_8 \tilde{S}_9 = 1,$$
$$\tilde{G}_3 \tilde{S}_7 + \tilde{G}_7 \tilde{S}_6 = 0,$$
$$\tilde{G}_4 \tilde{S}_8 + \tilde{G}_8 \tilde{S}_5 = 0,$$
$$\tilde{G}_5 \tilde{S}_5 + \tilde{G}_9 \tilde{S}_8 = 1,$$
$$\tilde{G}_6 \tilde{S}_6 + \tilde{G}_{10} \tilde{S}_7 = 1,$$
$$\tilde{G}_5 \tilde{S}_9 + \tilde{G}_9 \tilde{S}_4 = 0,$$
$$\tilde{G}_6 \tilde{S}_{10} + \tilde{G}_{10} \tilde{S}_3 = 0.$$ (19)

The equations are easy to solve:

$$\tilde{G}_1 = \frac{1}{\tilde{S}_1}, \quad \tilde{G}_2 = \frac{1}{\tilde{S}_2},$$
$$\tilde{G}_3 = \frac{\tilde{S}_6}{\Delta_1}, \quad \tilde{G}_4 = \frac{\tilde{S}_5}{\Delta_2}, \quad \tilde{G}_5 = \frac{\tilde{S}_4}{\Delta_2}, \quad \tilde{G}_6 = \frac{\tilde{S}_3}{\Delta_1},$$
$$\tilde{G}_7 = \frac{-\tilde{S}_7}{\Delta_1}, \quad \tilde{G}_8 = \frac{-\tilde{S}_8}{\Delta_2}, \quad \tilde{G}_9 = \frac{-\tilde{S}_9}{\Delta_2}, \quad \tilde{G}_{10} = \frac{-\tilde{S}_{10}}{\Delta_1}.$$ (20)
where
\[ \Delta_1 = \bar{S}_3 S_6 - \bar{S}_7 S_{10}, \quad \Delta_2 = \bar{S}_4 S_5 - \bar{S}_8 S_9. \] (21)

The \( G_1, G_2 \) terms which describe the spin–3/2 have the usual resonant form and could be obtained from (12) as well. As for \( G_3 - G_{10} \) coefficients which describe the spin–1/2 contributions, they have a more complicated structure. Let us consider the denominators of (20) in more details.

\[ \Delta_1 = \bar{S}_3 S_6 - \bar{S}_7 S_{10} = (S_3 + \sqrt{p^2} S_4)(S_5 - \sqrt{p^2} S_6) - (S_7 + \sqrt{p^2} S_8)(S_9 - \sqrt{p^2} S_{10}), \]
\[ \Delta_2 = \Delta_1(\sqrt{p^2} \rightarrow -\sqrt{p^2}). \] (22)

The appearance of \( \sqrt{p^2} \) factor is typical for fermions — see below and this apparent branch point \( \sqrt{p^2} \) is canceled in total expression for the dressed Rarita–Schwinger propagator (20).

Thus we obtained the simple analytical expression (20) for the interacting Rarita-Schwinger field propagator which accounts all spin components. The new moment here is a closed expression for spin–1/2 sector where we can expect the dressing of two spin–1/2 components with mutual transitions. To derive it we introduced the spin-tensor basis (14) with very simple multiplicative properties. This basis is singular and it seems unavoidable (recall the vector field case). Nevertheless the singularity of a basis is not so big obstacle in its use though it needs additional constrains on coefficients. We did not suppose here any symmetry properties of the self-energy \( J^{\mu\nu} \) restricting ourselves by general case. Of course the concrete form of interaction will lead to some symmetry of \( J^{\mu\nu} \) and it will be important at renormalization.

### III. DRESSING OF DIRAC FERMIONS

The obtained answer for the interacting Rarita-Schwinger field’s propagator has rather unusual structure, so before renormalization it would be useful to clarify the physical meaning of the formulae. In search for nearest analogy for dressing of Rarita–Schwinger field we consider below few examples for dressing of Dirac fermions. We will use the projection operators \( \Lambda^\pm \) (13) which has been appeared in consideration of Rarita–Schwinger field. We found them very convenient in case of Dirac fermions also.

#### A. Dressed fermion propagator

The dressed fermion propagator \( G(p) \) is the solution of the Dyson–Schwinger equation
\[ G(p) = G_0 + G \Sigma G_0, \] (23)
where \( G_0 \) is a bare propagator and \( \Sigma \) is a self-energy contribution.

Let us introduce here new notations to emphasize the analogy with Rarita–Schwinger field.

\[ P_1 = \Lambda^+ = \frac{\sqrt{p^2} + \hat{p}}{2 \sqrt{p^2}}, \quad P_2 = \Lambda^- = \frac{\sqrt{p^2} - \hat{p}}{2 \sqrt{p^2}}. \] (24)

Decomposition of any \( 4 \times 4 \) matrix depending on one momentum \( p \) has the form
\[ S(p) = \sum_{M=1}^{2} P_M \tilde{S}^M. \] (25)
The Dyson–Schwinger equation in this basis takes form
\[ G^M = G_0^M + G^M \Sigma G_0^M, \quad M = 1, 2, \] (26)
or, equivalently,
\[ (G^M)^{-1} = (G_0^M)^{-1} - \Sigma^M. \] (27)

More detailed answer is
\[
\begin{align*}
(G_{M=1}^M)^{-1} &= (G_{0}^M)^{-1} - \Sigma^{M=1} = -m_0 - A(p^2) + \sqrt{p^2}(1 - B(p^2)), \\
(G_{M=2}^M)^{-1} &= (G_{0}^M)^{-1} - \Sigma^{M=2} = -m_0 - A(p^2) - \sqrt{p^2}(1 - B(p^2)),
\end{align*}
\]
where \(A,B\) are usually used components of the self-energy contribution
\[
\Sigma(p) = A(p^2) + \hat{p}B(p^2) = \Lambda^+\Sigma^1 + \Lambda^\pm\Sigma^2,
\]
\[ \Sigma^1 = A + \sqrt{p^2}B, \quad \Sigma^2 = A - \sqrt{p^2}B. \]

The renormalization has some features because of using of \(\Lambda^\pm\) operators, however the final answer coincides with standard (see Appendix).

Let us look at the self-energy contribution \(\Sigma(p)\). As an example we shall consider the dressing of baryon resonance \(N' (J^P = 1/2\pm)\) due to interaction with \(\pi N\) system. Interaction lagrangian is of the form
\[
L_{int} = g\bar{\Psi} \Gamma(x)\gamma^5\Psi(x) \cdot \phi(x) + h.c. \quad \text{for} \quad N' = 1/2^+ \quad \text{(30)}
\]
and
\[
L_{int} = g\bar{\Psi} \Gamma(x)\Psi(x) \cdot \phi(x) + h.c. \quad \text{for} \quad N' = 1/2^- \quad \text{(31)}
\]
Let us write down the one-loop self-energy contribution.

\[
\Sigma(p) = -ig^2 \int \frac{d^4k}{(2\pi)^4} \gamma^5 \frac{1}{\hat{p} + k - m_N} \gamma^5 \frac{1}{k^2 - m_{\pi}^2} = I \cdot A(p^2) + \hat{p}B(p^2) \quad \text{(32)}
\]
The loop discontinuity is calculated according to Landau–Cutkosky rule\(^5\)
\[
\Delta A = \frac{ig^2 m_N}{(2\pi)^2} I_0, \quad \Delta B = -\frac{ig^2}{(2\pi)^2} I_0 \frac{p^2 + m_N^2 - m_{\pi}^2}{2p^2}. \quad \text{(33)}
\]
Here \(I_0\) is the basic integral without indices.

\[
I_0 = \int d^4k \delta(k^2 - m_{\pi}^2) \delta((p+k)^2 - m_N^2) = \theta(p^2 - (m_N + m_{\pi})^2) \frac{\pi}{2} \sqrt{\frac{\lambda(p^2, m_N^2, m_{\pi}^2)}{(p^2)^2}}, \quad \text{(34)}
\]

\(^4\) The isotopic is not essential for our purpose, so we do not write these indices.
\(^5\) This is a way to avoid unphysical singularities: to renormalize the \(A, B\) components and then to calculate \(\Sigma^\pm\).
where \( \lambda(a, b, c) = (a - b - c)^2 - 4bc \).

From the parity conservation one can see that in the transition \( N'(1/2^+) \rightarrow N(1/2^+) + \pi(0^-) \) the \( \pi N \) pair has the orbital momentum \( l = 1 \). But according to threshold quantum-mechanical theorems \(^{23}\), the imaginary part of a loop should behave as \( q^{2l+1} \) at \( q \rightarrow 0 \), which does not correspond to \(^{33}\). \(^6\) Let us calculate the imaginary part of \( \Sigma^M \) component according to \(^{33}\)

\[
\text{Im } \Sigma^1 = \text{Im } (A + \sqrt{p^2} B) = \frac{g^2 I_0}{4\sqrt{p^2} (2\pi)^2} \left( \sqrt{p^2} - m_N - m_\pi \right) \left( \sqrt{p^2} - m_N + m_\pi \right) \sim q^3, \\
\text{Im } \Sigma^2 = \text{Im } (A - \sqrt{p^2} B) = -\frac{g^2 I_0}{4\sqrt{p^2} (2\pi)^2} \left[ (\sqrt{p^2} + m_N)^2 - m_\pi^2 \right] \sim q^1.
\]

One can see that the \( \Sigma^1 \) component (accompanied by the \( \Lambda^+ \) projector) demonstrates the proper threshold behavior.

\( \frac{1}{2} \leftrightarrow \frac{1}{2} \)

\[
\Sigma(p) = i g^2 \int \frac{d^4k}{(2\pi)^4} \frac{1}{k + p - m_N} \cdot \frac{1}{k^2 - m_\pi^2} = IA(p^2) + \hat{p}B(p^2), \\
\Delta A = -ig^2 \frac{m_N}{(2\pi)^2} I_0, \quad \Delta B = \frac{-ig^2}{(2\pi)^2} \frac{p^2 + m_N^2 - m_\pi^2}{2p^2}.
\]

Again the \( \Sigma^1 \) component demonstrates the proper orbital momentum (\( l = 0 \) in this case)

\[
\text{Im } \Sigma^1 = -\frac{g^2 I_0}{4\sqrt{p^2} (2\pi)^2} \left[ (\sqrt{p^2} + m_N)^2 - m_\pi^2 \right] \sim q^1, \\
\text{Im } \Sigma^2 = \frac{g^2 I_0}{4\sqrt{p^2} (2\pi)^2} \left( \sqrt{p^2} - m_N - m_\pi \right) \left( \sqrt{p^2} - m_N + m_\pi \right) \sim q^3.
\]

So the considered examples indicate that the correct threshold behavior (\( i.e. \) correct parity) demonstrates only \( \Sigma^1 \) component which has the \( 1/\left(\sqrt{p^2} - m\right) \) pole. As for another component \( \Sigma^2 \), which has pole of the form \( 1/\left(-\sqrt{p^2} - m\right) \), it has the opposite parity. The appearance of opposite parity contributions in a propagator is quite natural since the fermion and anti-fermion parities are opposite.

**B. Fermion dressing with parity violation**

Let us consider a dressing of the fermion state with parity violation. Such situation, arises, in particular for dressing of the \( t \)-quark propagator. Dyson–Schwinger equation has the same form but

\[ q^2 = \lambda(p^2, m_N^2, m_\pi^2) = \left[ p^2 - (m_N + m_\pi)^2 \right] \left[ p^2 - (m_N - m_\pi)^2 \right] / 4p^2 \]
the self-energy contribution $\Sigma$ contains the parity violating terms
\[
\Sigma(p) = A(p^2) + \hat{p}B(p^2) + \gamma^5C(p^2) + \hat{p}\gamma^5D(p^2).
\] (38)

Now the basis will contain four operators:
\[
P_1 = \Lambda^+, \quad P_2 = \Lambda^-, \quad P_3 = \Lambda^+\gamma^5, \quad P_4 = \Lambda^-\gamma^5,
\] (39)
where $P_{1,2}$ are projection operators and $P_{3,4}$ are nilpotent ones. The expansion of any $\gamma$-matrix depending on $p$ now has the form (compare with (23))
\[
S(p) = \sum_{M=1}^{4} P_M S^M.
\] (40)

This set of operators has simple multiplication properties (see Table II).

| $P_1$ | $P_2$ | $P_3$ | $P_4$ |
|-------|-------|-------|-------|
| $P_1$ | $P_1$ | $P_3$ | $P_1$ |
| $P_2$ | $P_2$ | $P_4$ | $P_2$ |
| $P_3$ | $P_3$ | $P_1$ | $P_3$ |
| $P_4$ | $P_4$ | $P_2$ | $P_4$ |

Table II: Multiplication properties of the basis (39) operators

Let us denote the inverse dressed and bare propagators as $S(p)$ and $S_0(p)$ respectively. With using of the basis (40) the Dyson–Schwinger equation is reduced to
\[
\bar{S}^M = (\bar{S}_0)^M - \bar{\Sigma}^M, \quad M = 1, \ldots, 4.
\]

So the problem is reduced to reversing of the known $S(p)$ matrix
\[
\left( \sum_{M=1}^{4} P_M \bar{G}^M \right) \left( \sum_{L=1}^{4} P_L \bar{S}^L \right) = P_1 + P_2.
\] (41)

We obtain the set of equations on the unknown coefficients $G^M$
\[
\bar{G}^1 \bar{S}^1 + \bar{G}^3 \bar{S}^4 = 1 \quad \bar{G}^1 \bar{S}^3 + \bar{G}^3 \bar{S}^2 = 0
\]
\[
\bar{G}^2 \bar{S}^2 + \bar{G}^4 \bar{S}^3 = 1 \quad \bar{G}^4 \bar{S}^3 + \bar{G}^2 \bar{S}^4 = 0,
\] (42)

which are easy to solve. The answer is
\[
\bar{G}_1 = \frac{S_2}{\Delta}, \quad \bar{G}_2 = \frac{S_1}{\Delta}, \quad \bar{G}_3 = -\frac{S_3}{\Delta}, \quad \bar{G}_4 = -\frac{S_4}{\Delta},
\] (43)
where $\Delta = \bar{S}_1 \bar{S}_2 - \bar{S}_3 \bar{S}_4$.

This example resembles the dressing of the Rarita–Schwinger field by its algebraic structure (compare Tables 1,2) but it contains only few degrees of freedom.
C. Joint dressing of two fermions of the same parities

Let we have two bare fermion states \( N', N'' \) which are dressed in presence of mutual transitions. Suppose that we have two fermions of the same parity and there is no parity violation in lagrangian. The Dyson–Schwinger equation acquires matrix indices

\[
G_{ij} = (G_0)_{ij} + G_{ik} \Sigma_{kl} (G_0)_{lj}, \quad i, j, k, l = 1, 2. \tag{44}
\]

Each element of this equation has also the \( \gamma \)-matrix indices which are not shown.

Using the expansion (25) (there is no parity violation so the basis contains only two elements) we reduce equation (44) to independent equations on \( \bar{G}_M \)

\[
(\bar{G}^M)_{ij} = (\bar{G}_0^M)_{ij} + (\bar{G}_0^M)_{ik} (\bar{\Sigma}^M)_{kl} (\bar{G}_0^M)_{lj}, \quad M = 1, 2. \tag{45}
\]

Let us rewrite (45) in the matrix form

\[
\bar{G}^M = \bar{G}_0^M + \bar{G}^M \bar{\Sigma}^M \bar{G}_0^M, \quad M = 1, 2, \tag{46}
\]

and write down its solution

\[
G^M = \left[ (\bar{G}_0^M)^{-1} - \bar{\Sigma}^M \right]^{-1} = \left[ \left( \bar{G}_0^M \right)^{-1}_{11} - \bar{\Sigma}^M_{11} \left( \bar{G}_0^M \right)^{-1}_{22} - \bar{\Sigma}^M_{22} \right]^{-1} =
\]

\[
= \frac{1}{\Delta^M} \left( \left( \bar{G}_0^M \right)^{-1}_{22} - \bar{\Sigma}^M_{22} \right) \left( \left( \bar{G}_0^M \right)^{-1}_{11} - \bar{\Sigma}^M_{11} \right), \tag{47}
\]

\[
\Delta^M = \left[ (\bar{G}_0^M)_{11}^{22} - \bar{\Sigma}^M_{11} \left( \bar{G}_0^M \right)_{12}^{22} - \bar{\Sigma}^M_{12}, \right] - \bar{\Sigma}^M_{12} \bar{\Sigma}^M_{21}.
\]

Now we will calculate the loop contributions \( \Sigma_{ij} \) for above considered example of baryon dressing by \( \pi N \) intermediate state. For case of the dressing of two states \( N', N'' \) of the same parity the self-energy contribution coincides with above written out (32), (35) besides the coupling constants.

\[
\Sigma_{ij} = ig_i g_j \int \frac{d^4k}{(2\pi)^4} \gamma^5 \frac{1}{\hat{p} + k - m} \gamma^5 \frac{1}{k^2 - m^2} \quad \text{for} \quad N', N'' = 1/2^+, \tag{48}
\]

\[
\Sigma_{ij} = ig_i g_j \int \frac{d^4k}{(2\pi)^4} \frac{1}{\hat{p} + k - m} \frac{1}{k^2 - m^2} \quad \text{for} \quad N', N'' = 1/2^-.
\]

So the above conclusions about the threshold behavior of imaginary parts (35), (37) are kept in this case also.

D. Joint dressing of two fermions of opposite parities

Let us consider the nearest analogy to the Rarita–Schwinger filed: the joint dressing of two fermions of the different parity \( 1/2^\pm \). We shall suppose that interaction conserves the parity.

As in previous example the Dyson–Schwinger equation has the matrix form (44) and again it is reduced (cp. (41)) to equation

\[
\left( \sum_{M=1}^{4} \mathcal{P}_M \bar{G}^M \right) \left( \sum_{L=1}^{4} \mathcal{P}_L \bar{S}^L \right) = \mathcal{P}_1 + \mathcal{P}_2, \tag{49}
\]
where $\tilde{G}_M$, $\tilde{S}_L$ are now 2-dimensional matrixes.

This yields the matrix analogies of equations (42).

$$
\begin{align*}
\tilde{G}_1 \tilde{S}_1 + \tilde{G}_3 \tilde{S}_4 &= E_2, \\
\tilde{G}_2 \tilde{S}_2 + \tilde{G}_4 \tilde{S}_3 &= E_2,
\end{align*}
$$

(50)

where $E_2$ is a unit matrix $2 \times 2$. It is easy to write down the solution of system (that is the matrix analogy of (43))

$$
\begin{align*}
\tilde{G}_1 &= \left[ \tilde{S}_1 - \tilde{S}_3(\tilde{S}_2)^{-1} \tilde{S}_4 \right]^{-1}, \\
\tilde{G}_3 &= -\left[ \tilde{S}_1 - \tilde{S}_3(\tilde{S}_2)^{-1} \tilde{S}_4 \right]^{-1} \tilde{S}_3(\tilde{S}_2)^{-1}, \\
\tilde{G}_2 &= \left[ \tilde{S}_2 - \tilde{S}_4(\tilde{S}_1)^{-1} \tilde{S}_3 \right]^{-1}, \\
\tilde{G}_4 &= -\left[ \tilde{S}_2 - \tilde{S}_4(\tilde{S}_1)^{-1} \tilde{S}_3 \right]^{-1} \tilde{S}_4(\tilde{S}_1)^{-1}.
\end{align*}
$$

(51)

Now let us concretize these general formulae. Suppose that we have two fermions of different parity but there is no parity violation in lagrangian. It means that the diagonal loops contain only the $I$ and $\hat{p}$ matrixes.

$$
\begin{array}{c}
\begin{array}{c}
1 \quad 2 \\
\bigcirc \\
1 \quad 2
\end{array}
\end{array} \quad \Sigma_{ii} \sim I, \hat{p},
$$

while the non-diagonal ones have $\gamma^5$

$$
\begin{array}{c}
\begin{array}{c}
1 \quad 2 \\
\bigcirc \\
1 \quad 2
\end{array}
\end{array} \quad \Sigma_{ij} \sim \gamma^5, \hat{p}\gamma^5 \text{ for } i \neq j
$$

Therefore the decomposition of the inverse propagator in this basis looks as follows

$$
S(p) = \mathcal{P}_1 \left( \begin{array}{cc}
-m_1 + E - \Sigma_{11}^{(1)} & 0 \\
0 & -m_2 + E - \Sigma_{22}^{(1)}
\end{array} \right) + \\
\mathcal{P}_2 \left( \begin{array}{cc}
-m_1 - E - \Sigma_{11}^{(2)} & 0 \\
0 & -m_2 - E - \Sigma_{22}^{(2)}
\end{array} \right) + \\
\mathcal{P}_3 \left( \begin{array}{cc}
0 & -\Sigma_{12}^{(3)} \\
-\Sigma_{21}^{(3)} & 0
\end{array} \right) + \mathcal{P}_4 \left( \begin{array}{cc}
0 & -\Sigma_{12}^{(4)} \\
-\Sigma_{21}^{(4)} & 0
\end{array} \right),
$$

where $E = \sqrt{\hat{p}^2}$, and $i, j = 1, 2$ numerate the dressing fermion states.

Substituting all into solution (51), we obtain the matrix of dressed propagator

$$
G = \Lambda^+ \left( \begin{array}{cc}
-m_2 - E - \Sigma_{22}^{2} & 0 \\
\Delta_1 & -m_1 - E - \Sigma_{11}^{2}
\end{array} \right) \Lambda^- \left( \begin{array}{cc}
-m_2 + E - \Sigma_{22}^{1} & 0 \\
\Delta_2 & -m_1 + E - \Sigma_{11}^{1}
\end{array} \right) + \\
+ \Lambda^+ \gamma^5 \left( \begin{array}{cc}
0 & -\Sigma_{12}^{3} \\
\Sigma_{21}^{3} & 0
\end{array} \right) \Lambda^- \gamma^5 \left( \begin{array}{cc}
0 & -\Sigma_{12}^{4} \\
\Sigma_{21}^{4} & 0
\end{array} \right).
$$

(52)

Here

$$
\begin{align*}
\Delta_1 &= (-m_1 + E - \Sigma_{11}^{1})(-m_2 - E - \Sigma_{22}^{2}) - \Sigma_{12}^{3}\Sigma_{21}^{4}, \\
\Delta_2 &= (-m_1 - E - \Sigma_{11}^{1})(-m_2 + E - \Sigma_{22}^{1}) - \Sigma_{12}^{3}\Sigma_{21}^{4} = \Delta_1(E \to -E).
\end{align*}
$$
Let us compare (52) with the dressing of two fermions of the same parity. For this purpose we will rewrite the above solution (47) in similar form

\[
G = \Lambda^+ \frac{1}{\Delta_1} \left( -m_2 + E - \Sigma_{22}^1 \right) - \frac{1}{\Delta_1} \left( -m_1 + E - \Sigma_{11}^1 \right) + \Lambda^- \frac{1}{\Delta_2} \left( -m_2 - E - \Sigma_{22}^2 \right) - \frac{1}{\Delta_2} \left( -m_1 - E - \Sigma_{11}^2 \right)
\]

\[
\Delta_1 = ( - m_1 + E - \Sigma_{11}^1 ) ( - m_2 + E - \Sigma_{22}^1 ) - \Sigma_{12}^1 \Sigma_{21}^1
\]

\[
\Delta_2 = ( - m_1 - E - \Sigma_{11}^2 ) ( - m_2 - E - \Sigma_{22}^2 ) - \Sigma_{12}^2 \Sigma_{21}^2 = \Delta_1 (E \rightarrow -E).
\]

(53)

Let us remind that the both cases (52), (53) correspond to conservation of parity in lagrangian. The appearance of nilpotent operators in decomposition (52) is an indication for transitions between states of different parity. They are absent in case of mixing of the same parity states. Besides the denominators have different structure.

Let us summarize our consideration of the dressing of Dirac fermions.

1) We found very convenient the using of the projection operators \( \Lambda^\pm = (\sqrt{p^2} \pm \hat{p})/2\sqrt{p^2} \) for solving of Dyson-Schwinger equation especially in case of few dressing states.

2) The projection operators \( \Lambda^\pm \) are very useful in another aspect: its coefficients have the definite parity. But as one can see from the loop calculations (35), (36) the components \( \Lambda^\pm \) have different parity. There is such correspondence: the parity of the field \( \Psi \) is the parity of the component \( \Lambda^+ \), which has the pole \( 1/(E - m) \). Another component \( \Lambda^- \) which has the pole \( 1/(-E - m) \) demonstrates the opposite parity.

3) In contrast to boson case, even if the interactions conserve the parity, the loop transitions between different parity states are not zero: they are proportional to nilpotent operator \( \mathcal{P}\mathcal{P} = 0 \).

4) The joint dressing of two fermions without parity violation in vertex has different picture in dependence of parities of dressing states. One can illustrate it in the following scheme:

\[
\begin{align*}
J^P = 1/2^\pm & \iff J^P = 1/2^\pm \\
\Lambda^+ & \iff \Lambda^+ \\
\Lambda^- & \iff \Lambda^-
\end{align*}
\]

Another difference is the appearance of nilpotent operators in the second case.

IV. SPIN-PARITY OF THE RARITA–SCHWINGER FIELD COMPONENTS

Comparing the Tables I and II one can conclude that the presence of nilpotent operators \( \mathcal{P}_7 - \mathcal{P}_{10} \) in the decomposition (17) of the Rarita–Schwinger propagator is an indication for transitions between components of different parity 1/2\(^\pm\). To make sure in this conclusion we can calculate the loop contributions in propagator. As an example we take the standard \( \pi N \Delta \) interaction lagrangian

\[
L_{\text{int}} = g_{\pi N \Delta} \bar{\Psi}^\mu(x) \left( g^\mu{}^\nu + a\gamma^\mu \gamma^\nu \right) \Psi(x) \cdot \partial_{\nu} \phi(x) + \text{h.c.}.
\]

(54)
where the vertex contains an additional parameter \( a \).\(^7\)

The one-loop self-energy contribution is

\[
J^{\mu
u}(p) = -ig^2\pi N\Delta \int \frac{d^4k}{(2\pi)^4} (g^{\mu p} + a\gamma^\mu \gamma^p)k^p \frac{1}{p + k - m_N} k^\lambda (g^{\lambda \nu} + a\gamma^\lambda \gamma^\nu) \frac{1}{k^2 - m_\pi^2}.
\] (55)

Let us calculate the discontinuity of loop contribution in \( \hat{p} \) basis \(^{[12]} \).

\[
\Delta J_1 = -ig^2I_0 \frac{m_N}{12s} \lambda(s, m_N^2, m_\pi^2),
\]

\[
\Delta J_2 = -ig^2I_0 \frac{1}{24s^2}(s + m_N^2 - m_\pi^2)\lambda,
\]

\[
\Delta J_3 = -ig^2I_0 \frac{m_N}{12s} (\lambda + 6a\lambda - 36a^2m_\pi^2s),
\]

\[
\Delta J_4 = -ig^2I_0 \frac{1}{24s^2}[(s + m_N^2 - m_\pi^2)\lambda + 12as\lambda + 36a^2s(s^2 - m_\pi^2 s - 2m_\pi^2 s - m_\pi^2 m_N^2 + m_N^4)],
\]

\[
\Delta J_5 = ig^2I_0 \frac{m_N}{4s}[(s - m_N^2 + m_\pi^2)^2 + 2a(s - m_N^2 + m_\pi^2)^2 + 4a^2m_\pi^2s],
\]

\[
\Delta J_6 = ig^2I_0 \frac{1}{8s^2}[(s + m_N^2 - m_\pi^2)(s - m_N^2 + m_\pi^2)^2 + 4as(s - m_N^2 + m_\pi^2)(s - m_N^2 - m_\pi^2) + 4a^2(s^2 - m_\pi^2 s - 2m_N^2 s - m_\pi^2 m_N^2 + m_N^4)],
\]

\[
\Delta J_7 = ig^2I_0 \sqrt{\frac{3}{s}} \cdot \frac{1}{24s} [(s - m_N^2 + m_\pi^2)\lambda + 4as(2s^2 - m_\pi^2 s - 4m_\pi^2 s + 2m_N^4 - m_\pi^2 m_N^2 - m_N^4) + 12a^2s(s^2 - m_\pi^2 s - 2m_N^2 s - m_\pi^2 m_N^2 + m_N^4)],
\]

\[
\Delta J_8 = -ig^2I_0 \sqrt{\frac{3}{s}} \cdot \frac{am_N}{6s} [(s^2 + 4m_\pi^2 s - 2m_N^2 s + m_N^4 - 2m_N^2 m_\pi^2 + m_\pi^4) + 6asm_\pi^2],
\]

\[
\Delta J_9 = \Delta J_7
\]

\[
\Delta J_{10} = -\Delta J_8.
\]

Here \( I_0 \) is the base integral \(^{[31]} \), \( \lambda(a, b, c) = (a - b - c)^2 - 4bc \), arguments of \( \lambda \) are the same in all expressions, but are indicated only in first one.

We observed in case of Dirac fermions that the propagator decomposition in basis of projection operators demonstrates the definite parity. We can expect the similar property for Rarita–Schwinger field in \( \Lambda \)-basis. Let us verify it by calculating the threshold behavior of imaginary part. Using \(^{[50]} \), one can convince yourself that

\[
\Delta \bar{J}_1 = \Delta J_1 + E\Delta J_2 \sim q^3
\]

\[
\Delta \bar{J}_2 = \Delta J_2 - E\Delta J_2 \sim q^5
\]

\[
\Delta \bar{J}_3 = \Delta J_3 + E\Delta J_4 \sim q^3
\]

\[
\Delta \bar{J}_4 = \Delta J_3 - E\Delta J_4 \sim q
\]

\[
\Delta \bar{J}_5 = \Delta J_5 + E\Delta J_6 \sim q
\]

\[
\Delta \bar{J}_6 = \Delta J_5 - E\Delta J_6 \sim q^3.
\]

Such a behavior indicates that the components \( J_1, J_2 \) exhibit the spin-parity \(^8 \) 3/2\(^+\), while the pairs of coefficient \( J_3, J_4 \) and \( J_5, J_6 \) correspond to 1/2\(^+\), 1/2\(^-\) contributions respectively.

---

\(^7\) To maintain the symmetry of free lagrangian \(^2\) parameter \( a \) in vertex must be related with parameter \( A \) in the free lagrangian \( a = A(1 + 4x)/2 + z \) (see, \(^{[14]} \)). Here \( z \) is so called "off-shell" parameter the meaning of which is controversial.

\(^8\) More exactly \( J_1 \sim 3/2^+, \ J_2 \sim 3/2^- \), we found in above that the field parity is in fact the parity of \( \Lambda^+ \) component in propagator.
V. CONCLUSION

We obtained in closed analytic form the dressed propagator of Rarita–Schwinger field taking into account all spin components. To derive it we introduced the spin-tensor basis \((14)\) with very simple multiplicative properties. It is constructed by combining the standard tensor operators \((8)\) and \(\Lambda^\pm\) projection operators \((13)\). This basis consists of 6 projection operators and 4 nilpotent ones.

In search of analogy for dressing of \(s = 1/2\) sector of the Rarita–Schwinger field we have considered the dressing of Dirac fermions in different variants. As a result we have found that the nearest analogy is the joint dressing of two Dirac fermions of different parity. Using of projection operators \(\Lambda^\pm\) is turned out to be very convenient in this consideration. Besides the simple multiplicative properties, this basis is convenient also in another aspect: its coefficients exhibit the definite parity. The presence of nilpotent operators in propagator decomposition yields indication for transitions between different parity states. Calculation of the self-energy contributions in case of \(\Delta\) isobar confirms it: in the Rarita–Schwinger field besides the leading \(s = 3/2\) contribution there are also two \(s = 1/2\) components of opposite parity. Similar conclusion was obtained in [24] on the base of algebraic methods and construction of the corresponding space of the Lorentz group representations.

The obtained dressed propagator \((20)\) solves an algebraic part of the problem, the following step is renormalization. Note that the investigation of the dressed propagator is an alternative for more conventional methods based on equations of motion (see, i.e. Ref. [8, 25] and references therein). Instead of analysis of motion equations and constrains it leads to investigation of poles in complex energy plane. However the renormalization problem of the Rarita–Schwinger field needs a more careful consideration.

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APPENDIX

RENNORMALIZATION OF DIRAC FERMION PROPAGATOR IN \(\Lambda^\pm\) BASIS

The dressed inverse propagator has the form

\[
S(p) = \hat{p} - m - A(p^2) - \hat{p}B(p^2) = -m - A(p^2) + \hat{p}\left(1 - B(p^2)\right),
\]

where \(A(p^2)\) and \(B(p^2)\) are components of self-energy \(\Sigma(p)\)

\[
\Sigma(p) = A(p^2) + \hat{p}B(p^2),
\]

The standard procedure of renormalization makes use of decomposition of the inverse propagator in terms of \(\hat{p} - m\). If we use the mass shell subtraction scheme then we have the following condition

\[
S(p) = \hat{p} - m + o(\hat{p} - m) \quad \text{when} \quad (\hat{p} - m) \to 0
\]

where \(m\) is a renormalized fermion mass. It leads to conditions for coefficients \(A\) and \(B\).

\[
A(m^2) + mB(m^2) = 0,
2mA'(m^2) + B(m^2) + 2m^2B'(m^2) = 0
\]
To derive this conditions it is necessary to use the relation $p^2 = \hat{p}^2$. The conditions (59) define the subtraction constants in the loops.

With using of $\Lambda^\pm$ projection operators basis the procedure of the renormalization becomes slightly different. The inverse propagator has form

$$S(p) = \Lambda^+ S^1 + \Lambda^- S^2,$$

where

$$
S^1 = -m - A(p^2) + \sqrt{p^2}\left(1 - B(p^2)\right),
$$
$$
S^2 = -m - A(p^2) - \sqrt{p^2}\left(1 - B(p^2)\right).
$$

One should renormalize the scalar functions $S^{1,2}$ depending on argument $E = \sqrt{p^2}$. Let us require $S^1$ to have zero at the point $E = m$ with unit slope.

$$S^1 = E - m + o(E - m) \quad \text{when} \quad E \to m$$

It is easy to see that this condition coincides with (58) after substitution $\hat{p} \to \sqrt{p^2}$ and therefore we get the same conditions (59) on the subtraction constants.

This condition defines completely the subtraction constants of $A(p^2), B(p^2)$ so the another component $S^2$ is fixed also. As a result we obtain the dressed renormalized propagator $G(p)$, which coincides with the standard expression.

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