A new one-dimensional variable frequency photonic crystal

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In this paper, we have firstly proposed a new one-dimensional variable frequency photonic crystal (VFPCs). We have calculated the transmissivity and the electronic field distribution of VFPCs and compare them with the conventional PCs, and obtained some new results, which should be help to design a new type optical devices, and the two-dimensional and three-dimensional VFPCs can be studied further.

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Keywords: photonic crystal; variable frequency medium; transmissivity; electronic field distribution

1. Introduction

Photonic crystal (PCs) were first introduced theoretically by Yablonovitch [1], and experimentally by John [2]. PCs, constructed with periodic structure of artificial dielectrics or metallic materials, have attracted many researchers in the past two decades for their unique electromagnetic properties and scientific and engineering applications [1-6]. These crystal indicate a range of forbidden frequencies, called photonic band gap, as a result of Bragg scattering of the electromagnetic waves passing through such a periodical structure [7, 8]. As the periodicity of the structure is broken by introducing a layer with different optical properties, a localized defect mode will appear inside the band gap. Enormous potential applications of PCs with defect layers in different areas, such as light emitting diodes, filters and fabrication of lasers have made such structures are interesting research topic in this field.

Such materials are employed for the realization of diverse optical devices, as for example distributed feedback laser [9, 10] and optical switches [11, 12]. The addition of defects in the periodic alternation, or the realization of completely random sequences, results in disordered photonic structures [13-15]. In the case of one-dimensional disordered photonic structures, very interesting physical phenomena have been theoretically predicted or experimentally observed.

In the paper, we have firstly proposed a new one-dimensional variable frequency photonic crystal (VFPCs), which is made up of variable frequency medium. The so-called variable frequency medium is when light passes the medium the light frequency has been changed, which is a new type medium, it changes the passed light frequency. For the conventional medium, it changes the passed light wavelength and unchanged light frequency. When the light passes through the medium, the medium does not absorbs the light energy, the light frequencies should be unchanged, the medium is called conventional medium. When the light passes through the medium, the medium absorbs or releases the light energy, the light frequencies should be changed, the medium is called variable frequency medium. We can make the photonic crystal with the variable frequency medium. We have studied the transmissivity and the electronic field distribution of VFPCs and compare them with the conventional PCs. We obtained some new results: (1) When the variable frequency function \( f(n_b) < 1 \), the band gaps blue shift, and the band gaps width decrease. (2) When the variable frequency function \( f(n_b) > 1 \), the band gaps red shift, and the band gaps width increase, i.e., the variable frequency function is an important factor of effect on transmissivity. (3) When the VFPCs period numbers increase the band gaps width increase. In the conventional PCs, the band gaps are unchanged when the period numbers increase. (4) When the variable frequency function \( f(n_b) < 1 \) the peak value of VFPCs electronic field distribution decreased, the distribution curve left shift. (5) When the variable frequency

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function $f(n_b) > 1$ the peak value of VFPCs electronic field distribution increased, the distribution curve right shift. We think these new results should be help to design a new type optical devices.

2. Transfer matrix, transmissivity and electronic field distribution of PCs

For one-dimensional PCs, the calculations are performed using the transfer matrix method [16], which is the most effective technique to analyze the transmission properties of PCs. For the medium layer $i$, the transfer matrices $M_i$ for $TE$ wave is given by [16]:

$$M_i = \begin{pmatrix} \cos \delta_i & -i \sin \delta_i / \eta_i \\ -i \eta_i \sin \delta_i & \cos \delta_i \end{pmatrix},$$

where $\delta_i = \frac{2\pi}{c} n_i d_i \cos \theta_i$, $c$ is speed of light in vacuum, $\theta_i$ is the ray angle inside the layer $i$ with refractive index $n_i = \sqrt{\varepsilon_i / \mu_i}$, $\cos \theta_i = \sqrt{1 - (n_0^2 \sin^2 \theta_0 / n_i^2)}$, in which $n_0$ is the refractive index of the environment wherein the incidence wave tends to enter the structure, and $\theta_0$ is the incident angle.

The final transfer matrix $M$ for an $N$ period structure is given by:

$$\begin{pmatrix} E_1 \\ H_1 \end{pmatrix} = M_B M_A M_B M_A \cdots M_B M_A \begin{pmatrix} E_{N+1} \\ H_{N+1} \end{pmatrix},$$

$$= M \begin{pmatrix} E_{N+1} \\ H_{N+1} \end{pmatrix} = \begin{pmatrix} A & B \\ C & D \end{pmatrix} \begin{pmatrix} E_{N+1} \\ H_{N+1} \end{pmatrix},$$

where

$$M = \begin{pmatrix} A & B \\ C & D \end{pmatrix},$$

with the total transfer matrix $M$, we can obtain the transmission coefficient $t$, and the transmissivity $T$, they are

$$t = \frac{E_{N+1}}{E_1} = \frac{2\eta_0}{A\eta_0 + B\eta_0 \eta_{N+1} + C + D\eta_{N+1}},$$

$$T = t \cdot t^*.$$

Where $\eta_0 = \eta_{N+1} = \sqrt{\frac{\varepsilon_0}{\mu_0}}$.

The electronic field distribution at position $x$ is [16]

$$\begin{pmatrix} E(x) \\ H(x) \end{pmatrix} = M_A (a - x) M_B (b) (M_A (a) M_B (b))^{N-1} \begin{pmatrix} E_{N+1} \\ H_{N+1} \end{pmatrix},$$

$$= \begin{pmatrix} A'(x) & B'(x) \\ C'(x) & D'(x) \end{pmatrix} \begin{pmatrix} E_{N+1} \\ H_{N+1} \end{pmatrix},$$

with $E_{N+1} = E_1 \cdot t$ and $H_{N+1} = \sqrt{\varepsilon_0 / \mu_0} \cdot E_{N+1}$, we have

$$E(x) = (A'(x) + B'(x) \sqrt{\varepsilon_0 / \mu_0}) E_1 \cdot t,$$

and

$$\frac{|E(x)|^2}{E_1^2} = |A'(x) + B'(x) \sqrt{\varepsilon_0 / \mu_0}|^2 \cdot |t|^2.$$

3. Transform matrix and transmissivity of VFPCs
In the following, we have firstly proposed a new one-dimensional photonic crystal of variable frequency medium (VFPCs). The so-called variable frequency medium is when light passes the medium the light frequency has been changed, it is

\[ \omega = f(n)\omega_0, \]

where \( \omega_0 \) is the incident light frequency, \( \omega \) is the frequency of light in the medium and \( n \) is the refractive index of variable frequency medium, and \( f(n) \) is called variable frequency function. The structure of one-dimensional VFPCs is \((AB)^N\), where the medium \( B \) is the variable frequency medium, the medium \( A \) is not the variable frequency medium (conventional medium), and \( N \) is the period numbers.

For the VFPCs, its transfer matrices are similar to the conventional transfer matrices (1), they are

\[
M_{A_i} = \begin{pmatrix}
\cos \delta_{a_i} & -i \sin \delta_{a_i} / \eta_a \\
-i \eta_a \sin \delta_{a_i} & \cos \delta_{a_i}
\end{pmatrix} (i = 1, 2, \cdots, N),
\]

\[
M_{B_i} = \begin{pmatrix}
\cos \delta_{b_i} & -i \sin \delta_{b_i} / \eta_b \\
-i \eta_b \sin \delta_{b_i} & \cos \delta_{b_i}
\end{pmatrix} (i = 1, 2, \cdots, N),
\]

but the phases should be modified as:

\[
\delta_{a_1} = \frac{\omega_0}{c} \cdot n_a \cdot a,
\]

\[
\delta_{b_1} = \frac{f(n_b) \cdot \omega_0}{c} \cdot n_b \cdot b,
\]

\[
\delta_{a_2} = \frac{f(n_b) \cdot \omega_0}{c} \cdot n_a \cdot a,
\]

\[
\delta_{b_2} = \frac{f^2(n_b) \cdot \omega_0}{c} \cdot n_b \cdot b,
\]

and the \( i \)-th period phase is

\[
\delta_{a_i} = \frac{f^{i-1}(n_b) \cdot \omega_0}{c} \cdot n_a \cdot a,
\]

\[
\delta_{b_i} = \frac{f^i(n_b) \cdot \omega_0}{c} \cdot n_b \cdot b,
\]

We consider the structure of VFPCs is \((AB)^{12}\), and the variable frequency function is \( f(n_b) \). When the \( f(n_b) > 1 \), the light frequency should be increased in medium \( B \), when the \( f(n_b) < 1 \), the light frequency should be decreased in medium \( B \). When \( f(n_b) = 1 \) it is called conventional PCs, and \( f(n_b) \neq 1 \) it is called variable frequency PCs (VFPCs).

4. Numerical result

In this section, we report our numerical results of VFPCs. The VFPCs main parameters are: The medium \( A \) refractive indexes \( n_a = 1.40 \), and thickness \( a = 320nm \), the medium \( B \) refractive indexes \( n_b = 2.58 \), thickness \( b = 138nm \). The structure is \((AB)^{12}\). In FIG. 1, we study the effect of the variable frequency function \( f(n_b) \) on VFPCs transmissivity, the variable frequency function \( f(n_b) \) in the FIG. 1 (a), (b) and (c) are 0.98, 1 and 1.02, respectively. The FIG. 1 (b) is the transmissivity of conventional PCs because
of \( f(n_b) = 1 \), and FIG. 1 (a) and (c) are the transmissivity of VFPCs because of \( f \neq 1 \). Comparing the transmissivity of VFPCs with conventional PCs, we can obtain the new results: (1) When the variable frequency function \( f(n_b) < 1 \) (FIG. 1 (a)), the band gaps blue shift, and the band gaps width decrease. (2) When the variable frequency function \( f(n_b) > 1 \) (FIG. 1 (c)), the band gaps red shift, and the band gaps width increase. From the results (1) and (2), we find that the variable frequency function is an important factor of effect on transmissivity. In FIG. 2, we study the effect of the variable frequency medium thickness on the VFPCs transmissivity, wherein variable frequency function \( f(n_b) = 1.02 \). The FIG. 2 (a), (b) and (c) thickness \( b \) are 98 nm, 138 nm and 178 nm, respectively. From FIG. 2 (a), (b) and (c), we can find when the variable frequency medium thickness \( b \) increase the band gaps red shift, and the band gaps width increase. In FIG. 3, we study the effect of the variable frequency medium refractive index on the VFPCs transmissivity, wherein variable frequency function \( f(n_b) = 1.02 \). The FIG. 3 (a), (b) and (c) thickness \( n_b \) are 2.18, 2.58 and 2.98, respectively. From FIG. 3 (a), (b) and (c), we can find when the variable frequency medium refractive index \( n_b \) increase the band gaps red shift, and the band gaps width increase. In FIG. 4, we study the effect of the variable frequency medium thickness \( b \) increase the band gaps red shift, and the band gaps width increase. In FIG. 5, we calculate the electronic field distribution of the VFPCs, and consider the influence of variable frequency function \( f(n_b) \) on VFPCs electronic field distribution. The variable frequency function \( f(n_b) = 0.97, 1 \) and 1.03 are according to the dash dot line, solid line and dot line of electronic field distribution, respectively. comparing with the conventional PCs (\( f(n_b) = 1 \)), we can obtain new results: (1) When the variable frequency function \( f(n_b) < 1 \) the peak value of VFPCs electronic field distribution decreased, the distribution curve left shift. (2) When the variable frequency function \( f(n_b) > 1 \) the peak value of VFPCs electronic field distribution increased, the distribution curve right shift.

5. Conclusion

In summary, we have studied the transmissivity and the electronic field distribution of one-dimensional VFPCs and compare them with the conventional PCs. We obtained some new results: (1) When the variable frequency function \( f(n_b) < 1 \), the band gaps blue shift, and the band gaps width decrease. (2) When the variable frequency function \( f(n_b) > 1 \), the band gaps red shift, and the band gaps width increase, i.e., the variable frequency function is an important factor of effect on transmissivity. (3) When the VFPCs period numbers increase the band gaps width increase. In the conventional PCs, the band gaps are unchanged when the period numbers increase. (4) When the variable frequency function \( f(n_b) < 1 \) the peak value of VFPCs electronic field distribution decreased, the distribution curve left shift. (5) When the variable frequency function \( f(n_b) > 1 \) the peak value of VFPCs electronic field distribution increased, the distribution curve right shift. We think these new results should be help to design a new type optical devices and we can further study the two-dimensional and three-dimensional VFPCs.

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FIG. 1: Comparing the transmissivity of VFPCs with conventional PCs. (a) VFPCs $f(n_b) = 0.98$, (b) conventional PCs $f(n_b) = 1$, (c) VFPCs $f(n_b) = 1.02$.

FIG. 2: The effect of the variable frequency medium thickness $b$ on the VFPCs transmissivity. (a) $b = 98nm$, (b) $b = 138nm$, (c) $b = 178nm$. 
FIG. 3: The effect of the variable frequency medium refractive index $n_b$ on the VFPCs transmissivity. (a) $n_b = 2.18$, (b) $n_b = 2.58$, (c) $n_b = 2.98$.

FIG. 4: The effect of the period number $N$ on the VFPCs transmissivity. (a) $N = 8$, (b) $N = 10$, (c) $N = 12$. 
FIG. 5: The effect of the variable frequency function $f(n_b)$ on the VFPCs electronic field distribution $|E/E_i|^2$. (a) $f(n_b) = 0.97$ dash dot line, (b) $f(n_b) = 1$ solid line, (c) $f(n_b) = 1.03$ dot line.