Monte Carlo simulations of star clusters – III. A million body star cluster.

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1 INTRODUCTION.

Dynamical modeling of real, large stellar systems, like globular clusters or galactic nuclei, and understanding their evolution still is a great challenge both for the theory and hardware/software. Basically, there are two approaches. Direct \( N \)-body method, which requires extremely large hardware requirements and very sophisticated software, and statistical modeling based on the Fokker–Planck and other approximations, which suffers from the poor understanding of the validity of assumptions.

On the side of \( N \)-body simulations recent years brought a significant progress in both hardware and software. First, parallel (even massively parallel) computing has opened a route to gain performance at relatively low cost and little technological advancement. Secondly, the already successful GRAPE special purpose computers (Sugimoto et al. 1990, Makino et al. 1997, Makino & Taiji 1998, Makino 2002, Makino & Hut 2003) have been developed in their present generation (GRAPE-6) and aim at the 100 TlOps speed. NBODY6++ (Spurzem 1999), the successor of Aarseth’s NBODY6 code has been ported on massively parallel, general purpose computers (CRAY T3E and PC clusters) and was used for astrophysical problems related to the dissolution of globular clusters (Baumgardt 2001, Baumgardt, Hut & Heggie 2002) and the decay of massive black hole binaries in galactic nuclei after a merger (Milosavljevic & Merritt 2001, Hensendorf, Sigurdsson & Spurzem 2002). Despite such progresses in hardware and software there is still impossible to model directly evolution of real globular cluster \((N \sim 10^5)\) and galactic nuclei \((N \sim 10^6)\). Recent work by Baumgardt et al (2002) and Baumgardt & Makino (2002) has pushed the limits of present direct modeling to about \(10^5\) using both NBODY6++ and the GRAPE–6 special purpose hardware.

On the side of Fokker–Planck method with finite differences and an anisotropic gaseous model, recent years brought great improvements. Models can be used now to simulate more realistic stellar systems. They can tackle: anisotropy, rotation, a tidal boundary, tidal shocking by galactic disk and bulge, mass spectrum, stellar evolution and dynamical and primordial binaries (Louis & Spurzem 1991, Giersz & Spurzem 1994, Takahashi 1995, 1996, 1997, Spurzem 1996, Takahashi & Portegies Zwart 1998, 2000 hereafter TPZ, Drukier et al 1999, Einsel & Spurzem 1999, Takahashi & Lee 2000, Kim et al. 2002, Kim, Lee & Spurzem 2004, Fiestas, Spurzem & Kim 2005). Unfortunately, the Fokker–Planck approach suffers, among other things, from the uncertainty of differential cross-sections of many processes which are important during cluster evolution. It can not supply detailed information about the formation and movement of all objects present in clus-
ters. Additionally, the anisotropic gaseous model assumes a certain form of heat conductivity and closure relations between the third order moments.

There is an elegant alternative to generate models of star clusters, which can correctly reproduce the stochastic features of real star clusters, but without really integrating all orbits directly as in an $N$–body simulation. They rely on the Fokker–Planck approximation and (hitherto) spherical symmetry, but their data structure is very similar to an $N$–body model. These so–called Monte Carlo models were first presented by Hénon (1971, 1975), Spitzer (1975) and later improved by Stodol`kiewicz (1982, 1985, 1986) and in further work by Giersz (1997, 1998, 2001), and recently by Rasio and his collaborators (Joshi, Rasio & Portegies Zwart 2000, Watters, Joshi & Rasio 2000, Joshi, Nave & Rasio 2001, Fregeau et al. 2003, Gürkan, Freitag & Rasio 2004), and Freitag (Freitag 2000, Freitag & Benz 2001, 2002). The basic idea is to have pseudo–particles, which orbital parameters are given in a smooth, self–consistent potential. However, their orbital motion is not explicitly followed; to model interactions with other particles like two–body relaxation by distant encounters or strong interactions between binaries and other objects, a position of the particle in its orbit, and further free parameters of the individual encounter, are picked from an appropriate distribution by using random numbers. The Monte Carlo scheme takes full advantage of the established physical knowledge about the secular evolution of (spherical) star clusters as inferred from continuum model simulations. Additionally, it describes in a proper way the graininess of the gravitational field and the stochasticity of real $N$–body systems and provides, in a manner as detailed as in direct $N$–body simulations, information about the movement of any objects in the system. This does not include any additional physical approximations or assumptions which are common in Fokker–Planck and gas models (e.g. for conductivity). Because of this, the Monte Carlo scheme can be regarded as a method which lies between direct $N$–body and Fokker–Planck models and combines most of their advantages. A hybrid variant of the Monte Carlo technique combined with a gaseous model has been proposed by Spurzem & Giersz (1996), and applied to systems with a large number of primordial binaries by Giersz & Spurzem (2000) and Giersz & Spurzem (2003) and for tidally limited systems by Spurzem et al. (2005). The hybrid method uses a Monte Carlo model for binaries or any other object for which a statistical description, as used by the gaseous model, is not appropriate due to small numbers of objects or unknown analytic cross sections for interaction processes. The method is particularly useful for investigating the evolution of large stellar systems with a realistic fraction of primordial binaries, but could also be used in the future to include, for example, the build up of massive stars and blue stragglers by stellar collisions.

Very detailed observations of globular clusters which are available at present or will become available in future (e.g. the Hubble Space Telescope and the new very large terrestrial telescopes) have extended and will extend our knowledge about their stellar content, internal dynamics and the influence of the environment on cluster evolution (Janes 1991, Djorgovski & Meylan 1993, Smith & Brodie 1993, Hut & Makino 1996, Meylan & Heggie 1997). This data covers luminosity functions and derived mass functions, color–magnitude diagrams, population and kinematical analysis, including binaries and compact stellar evolution remnants, detailed two–dimensional proper motion and radial velocity data, and tidal tails spanning over arcs several degrees wide (Koch et al. 2004). A wealth of information on “peculiar” objects in globular clusters (blue stragglers, X–ray sources (high– and low–luminosity), milisecond pulsars, etc.) suggests a very close interplay between stellar evolution, binary evolution and dynamical interactions. This interplay is far from being understood. Moreover, recent observations suggest that the primordial binary fraction in a globular cluster can be as high as 15%–38% (Rubenstein & Bailyn 1997), and possible existence of intermediate–mass black holes in the centers of some globular clusters (Gehardt et al. 2000). With all these observational data such as King et al. (1998), Piotto & Zoccali (1999), Rubenstein & Bailyn (1999), Ibata et al. (1999), Piotto et al. (1999), Grillmair et al. (1999), Shara et al. (1998), Odenskär et al. (2001), Hansen et al. (2002), Richer et al. (2002) (to mention only a few papers), easily reproducible reliable modelling becomes more important than before. Monte Carlo codes provide all the necessary flexibility to disentangle the mutual interactions between all physical processes which are important during globular cluster evolution and to perform in a reasonable time, detailed simulations of real globular clusters.

The ultimate aim of the project described here is to build a Monte Carlo code (in the framework of the MODEST international collaborations, http://www.manyparticle.com/modest), which will be able to simulate the evolution of real globular clusters, as closely as possible. In this paper (the third in the series) the Monte Carlo code (which was discussed in detail in Papers I and II) is used to simulate the evolution of stellar systems, which consist of comparable number of stars and have mass comparable to real globular clusters. The results of simulations will be compared with those of Chernoff & Weinberg (1990, hereafter CW), Vesperini & Heggie (1997, hereafter VH), Aarseth & Heggie (1998, hereafter AH), Baumgardt & Makino (2002) and Lamers et al. (2005).

The plan of the paper is as follows. In Section 2, a short description of processes implemented into the Monte Carlo code will be presented. In Section 3, the initial conditions will be discussed and results of the simulation will be shown. And finally, in Section 4 the conclusions will be presented.

2 MONTE CARLO METHOD.

The Monte Carlo method can be regarded as a statistical way of solving the Fokker–Planck equation. Its implementation presented in Giersz (1998, hereafter Paper I) and Giersz (2001, hereafter Paper II) is based on the orbit–averaged Monte Carlo method developed in the early seventies by Hénon (1971) and then substantially improved by Stodol`kiewicz (1986, and references therein), and recently by Giersz (Paper I and II), by Freitag (Freitag & Benz 2001, 2002) and by Rasio’s group (Joshi et al. 2000, 2001, Fregeau et al. 2003, 2004, Fregeau, Chatterjee & Rasio 2005, Freitag, Gürkan & Rasio 2005, Freitag, Rasio & Baumgardt 2005). The code is described in detail in paper I, which deals with simulations of isolated, single–mass systems and in Paper II in which additional physical processes were included: multi–mass systems and stellar evolution, mass loss through the tidal boundary, formation of three–body binaries and their subsequent interactions with field stars, and interactions between binaries.

For the sake of completeness, a short description of the main ingredients of the Monte Carlo model will follow. For more detailed discussion of the code readers should refer to Paper I and II.

2.1 Mass Spectrum and Stellar Evolution

To facilitate comparison with results of other numerical simulations of globular cluster evolution (e.g. CW, Fukushige & Heggie 1995,
Giersz & Heggie 1997, VH, AH, TPZ, JNR) and for simplicity a single power–law approximation of IMF was used.

\[ N(m)dm = Cm^{-\alpha}dm, \quad m_{\text{min}} \leq m \leq m_{\text{max}}, \quad (1) \]

where \( C \) and \( \alpha \) are constants. The \( m_{\text{min}}, \) minimum stellar mass and \( m_{\text{max}}, \) maximum stellar mass were chosen to be \( 0.1M_\odot \) and \( 15.0M_\odot, \) respectively. However, it should be noted that observations give more and more evidence that the mass function in globular/open clusters and for field stars as well is not a simple power–law, but it could be rather approximated by a composite power–law (e.g. Kroupa, Tout & Gilmore 1993, Kroupa 2002). To describe the mass loss due to stellar evolution the same simplified stellar evolution model as adopted by CW was used. More sophisticated models of stellar and binary evolution based on fitting analytical formulae to the evolution tracks for stars and binaries were developed by Portegies Zwart & Verbunt (1996), Hurley, Pols & Tout (2000), Hurley, Tout & Pols (2003) and Belczynski, Kalogera & Bulik (2002). These models, among others, were used in population synthesis codes (e.g. Belczynski & Taam 2004), and simulations of the evolution of real star cluster M67 (e.g. Hurley et al 2005).

The initial masses of stars are generated from the continuous distribution given in equation (1). This ensures a natural spread in their lifetimes. To ensure that the cluster remains close to virial equilibrium during rapid mass loss due to stellar evolution, special care is taken, that no more than 3\% of the total cluster mass is lost during one overall time–step.

### 2.2 Tidal Stripping

For tidally limited star cluster the rate of mass loss is much higher than for isolated systems. This is connected with the tidal stripping of mass across the system boundary by the gravitational field of a parent galaxy. For isolated system mass loss is attributed mainly to rare strong interactions in the dense, inner part of the system. In the present Monte Carlo code a mixed criterion is used to identify escapers: a combination of apocentre and energy–based criteria (see Paper II). This means that no potential escapers are kept in the system. Stars regarded as escapers are instantaneously lost from the system. This is in contrast to \( N \)–body simulations where stars need time proportional to the dynamical time to be removed from the system. Recently, Baumgardt (2000) showed that stars with energy greater than the tidal cutoff energy \( E_t = -GM/r_1, \) in the course of escape, can again become bound to the system, because of distant interactions with field stars. A similar conclusion was reached by Fukushima & Heggie (2000), who showed that the escape time–scale can be very long (much larger than the half–mass relaxation time). Additionally, during the final stages of cluster evolution or for clusters with initially low central concentration, the mass loss across the tidal boundary can become unstable when too many stars are removed from the system at the same time. To properly follow these stages of evolution the time–step has to be decreased to force smaller mass loss.

### 2.3 Three–Body Binaries and their interactions

In the present Monte Carlo code (as it was described in Paper I) all stellar objects, including binaries, are treated as single superstars. This allows to introduce into the code, in a simple and accurate way, processes of stochastic formation of binaries and their subsequent stochastic interactions with field stars and other binaries. The procedure of binary formation in three–body interactions is in great detail described in Paper II. It relies on the observation that the probability that the masses of the three stars involved in the interaction are \( m_1, m_2 \) and \( m_3 \) is \( n_1n_2n_3/n^3 \) (where \( n_1, n_2, n_3 \) and \( n \) are number densities of three interacting stars and the total number density, respectively) and the rate of binary formation is proportional to \( n_1n_2n_3 \). These considerations lead to the formula for the probability of binary formation which depends only on the local total number density instead of local number densities of each mass involved in the interaction (see Equation 7 in Paper II). This procedure substantially reduces fluctuations in the binary formation rate. The determination of the local number density in the Monte Carlo code, particularly for multi–mass systems, is a very delicate and difficult matter (Paper I).

The energy generation in binary – field star interactions is computed according to the (approximately modified for multi–mass case) semi–empirical formula of Spitzer (Spitzer 1987). Unfortunately, for multi–component systems there is no simple semi–empirical formula which can fit all numerical data (Heggie, Hut & McMillan 1996). The total probability of the interaction between a binary of binding energy \( E_b \) consisting of mass \( m_1 \) and \( m_2 \) and a field star of mass \( m_3 \) was deduced from Equations (6–23), (6–11) and (6–12) presented in Spitzer (1987) and results of Heggie (1975) with an appropriately adjusted coefficient (the total probability computed is such a way gives correct value for the equal–mass case).

\[ P_{\text{int}} = \frac{5\sqrt{2\pi}A_{\text{FWHM}}^2m_1^2m_2^2}{8\sqrt{3}\sqrt{m_1^2m_3^2\sigma E_b}} \Delta t, \quad (2) \]

where \( m_{12} = m_1 + m_2, m_{123} = m_{12} + m_3, m_0 \) is the mean stellar mass in a zone, and \( \sigma \) is the one dimensional velocity dispersion. This procedure is, of course, oversimplified and in some situations can not give correct results, for example, when a field star is very light compared to the mass of the binary components.

The implementation of interactions between binaries in the code is based on the method described by Stodolskiewicz (1986) and then improved in Paper II. Only strong interactions are considered, and only two types of outcomes are permitted: heavier binary and two single stars, or two binaries in a hyperbolic relative orbit. Stable three–body configurations are not allowed. In the case when binaries are only formed in dynamical processes, and only a few binaries are present, at any time, in the system, it is very difficult to use the binary density (Giersz & Spurzem 2000) to calculate the probability of a binary–binary interaction. Given binary can hit only binaries (regarded as targets) whose actual distance from the cluster center lies between its pericenter and apocenter. This leads to the following formula for the binary–binary interaction probability.

\[ P_{\text{int}} = \frac{4\pi\rho^2}{|v_r|^2T} \Delta t, \quad (3) \]

where \( |v_r| \) is the relative velocity between interacting binaries, \( |v_r|, \) \( r, \) \( T \) and \( p \) are the radial velocity, radial position, orbital period of a binary in the system and impact parameter (equal to 2.5 times the semi–major axis of the softer binary), respectively. The outcome of the interaction is as follows (see details in Stod{\l}kiewicz 1986, Paper II and Mikkola 1983, 1984):

- Two binaries in a hyperbolic relative orbit (12\% of all interactions). Energy generation is equal to 0.4 times the binding energy of the softer binary.
- One binary and two escapers (88\% of all interactions). The softer binary is destroyed and the harder binary increases its bind-
Table 1. Models $^a$

| Model | $W_0$ | $\alpha$ | $M_T$ | $R_G$ | $r_{t,M}$ | $t_{\text{scale}}$ | $F$ |
|-------|-------|--------|-------|-------|--------|--------|-----|
| W3235 | 3     | 2.35   | 319305| 2.00  | 3.1311 | 79797   | 4.62|
| W335  | 3     | 3.5    | 166577| 2.77  | 3.1311 | 61418  | 3.34|
| W5235a| 5     | 2.35   | 319305| 2.00  | 4.3576 | 48602  | 4.62|
| W7235b| 7     | 2.35   | 319305| 2.00  | 6.9752 | 23999  | 4.62|

$^a$ $M_T$ is the total cluster mass in solar mass, $r_{t,M}$ is the tidal radius in Monte Carlo units, $t_{\text{scale}}$ is the time scaling factor to scale simulation time to physical time in $10^6$ yrs, $R_G$ is the distance from the Galactic Center in kpc, $F$ in $10^4$ (Chernoff & Weinberg 1990). The first entry after $W$ describes the King model and the following numbers the mass function power-law index.

$^b$ two models with different initial random seed were simulated.

In order to scale the Monte Carlo time to physical units the following formula is used

$$ t_{\text{scale}} = 14.91 \left( \frac{r_t}{r_{t,M}} \right)^{1.5} \sqrt{\frac{N_T}{M_T \ln(\gamma N_T)}}, $$

where $r_{t,M}$ is the tidal radius in Monte Carlo units, $M$ is the total cluster mass and $r_t$ is the tidal radius, both are in solar mass and pc, respectively and $\gamma$ is equal to 0.11 as for single–mass systems (Giersz & Heggie 1994). However, for multi–mass systems it could be much smaller (Giersz & Heggie 1996, 1997). The initial model is not in virial equilibrium, because of statistical noise and because of masses, which are assigned independently from the positions and velocities. Therefore, the model has to be initially rescaled to fulfill the virial equilibrium condition. During all the simulations the virial ratio is kept within $<2\%$ of the equilibrium value (for the worst case within $<5\%$). Standard $N$–body units (Heggie & Mathieu 1986), in which the total mass $M = 1$, $G = 1$ and the initial total energy of the cluster is equal to $-1/4$, have been adopted for all runs. Monte Carlo time is equal to $N$–body time divided by $N_T/\ln(\gamma N_T)$. In the course of evolution, when the cluster looses mass, the tidal radius changes according to $r_t \propto M^{1/3}$.

Finally, a few words about the efficiency of the Monte Carlo code presented here. The simulation of a million body systems took about two/three weeks (depending on an initial model) on a Pentium 4, 2 GHz PC. This is still much faster than the biggest direct $N$–body simulations performed on GRAPE–4 (Teraflop special–purpose hardware). Nevertheless, the speed of the code is not high enough to perform large scale survey simulations in a reasonable time. It is clear, that to simulate the evolution of real globular clusters with all physical processes in operation, a substantial speed–up of the code is needed. This can be done, either by parallelizing the code (in a similar way to JNR), introducing a more efficient way of determining the new positions of superstars, introducing superstars which contain a different number of stars, or by using a hybrid code (as was done by Giersz & Spurzem 2000, 2003). However, the Monte Carlo code has presently a great relative advantage over the $N$–body code for simulations with a large number of primordial binaries. Primordial binaries substantially downgrade the performance of $N$–body codes on supercomputers or on special–purpose hardware. They can be introduced into the Monte Carlo code in a natural way, practically without a substantial loss of performance (Giersz & Spurzem 2000, 2003).

3 RESULTS

In Paper II a survey of the evolution of $N$–body systems influenced by the tidal field of a parent galaxy, by stellar evolution and by energy generation in interactions between binaries and field stars and binaries themselves was discussed. Here, the first Monte Carlo simulations of million body systems are presented. The results will be discussed from the point of view of system parameters which can be verified by observational data. Among others, evolution of LMF and GMF, anisotropy, total mass, mean mass, mass segregation and density profiles will be presented.

3.1 Initial Models

The initial conditions were chosen in a similar way as in the “collaborative experiment” (Heggie et al 1999) and Paper II. The positions and velocities of all stars were drawn from the King model (King 1966). All models have the same total number of stars $N_T = 1,000,000$ and the same tidal radius $r_t = 33.57$ pc. Masses of stars are drawn from the power–law mass function according to Equation (1). The minimum mass was chosen to be $0.1 M_\odot$ and the maximum mass $15 M_\odot$. Two different values of the power–law index were chosen: $\alpha = 2.35$ and 3.5. The sets of initial King models were characterized by $W_0 = 3, 5$ and 7. All models are listed in Table 1.

$F$ is the parameter (introduced by CW), which is proportional to the half–mass relaxation time.

$$ F \equiv \frac{M_T}{M_\odot} \frac{R_G}{\text{kpc}} \frac{220 \text{ km s}^{-1}}{v_g} \frac{1}{\ln N}, $$

where $R_G$ is the distance of the globular cluster to the Galactic Center, and $v_g$ is the circular speed of the cluster around the Galaxy.
nally, there is the phase connected with the tidal disruption of a cluster. The last phase is well visible only in model W7235, which evolution is followed long enough. In model W3235 on can see a phase when the total cluster mass is nearly constant and very small (about a few thousands of the initial mass). This is an artificial effect connected with the fact that the Monte Carlo code can not properly deal with systems with the total binding energy close to zero and consist of only a few dozen particles. It is interesting to note, that the second phase (gradual tidal stripping) can be divided into two subphases, clearly separated by the core collapse time. For the post collapse phase the rate of mass loss is slightly steeper than in the collapse phase. This can be connected with the fact that hard interactions, which involve binaries, start to operate introducing an additional mechanism of the mass loss (escaping stars and binaries). Models with the same mass–function ($\alpha \leq -2.35$) and with different $W_0$ show very similar evolution during the phase of tidal stripping (except W3235). It seems that the initial mass loss due to stellar evolution is not sufficiently strong to substantially change the initial cluster structure. Model W3235 is not initially concentrated strongly enough to survive without substantial structure changes, the violent mass loss event. Figure 1 presents qualitatively very similar features as figures shown by VH, Paper II, Baumgardt & Makino (2002) and Lamers et al (2005), Figure 1, Figures 1-3, Figure 1 and Figure 2, respectively.

The total mass loss consists of mass loss due to stellar evolution ($evo$) and mass loss connected with tidal stripping ($esc$). Results presented in Figure 2 are in a good qualitative agreement with the $N$–body results of VH (Figure 2), taking into account that $\alpha$ adopted by VH is $-2.5$ instead $-2.35$. According to Equation 11 of VH, the evolution of the total mass of the cluster can be fitted by a simple analytical expression, which consists of two terms: $evo$ and $esc$. The amount of mass loss because of $esc$ can be fitted (as it was shown in VH) by a straight line with a coefficient $\beta$. The values of $\beta$ listed by VH are 0.828 and 0.790 for models $W_0 = 7$, and $W_0 = 5$, respectively. The equivalent values for the Monte Carlo models are 0.999 and 0.943. The bigger values of $\beta$ for the Monte Carlo models can be attributed mainly to the different: escape criterion, power–low index of the initial mass function and slight increase of the rate of mass loss due to binary interactions (see Figure 1). As it was stated in Section 2.2, the escape criterion adopted in this paper (together with the instantaneous removal of escapers) leads to a higher escape rate than in $N$–body simulations. Additionally, less steep initial mass function causes stronger mass loss due to stellar evolution and consequently leads to more escapers than the steeper mass function adopted by VH. Mass loss connected with stellar evolution dominates the initial phase of cluster evolution. Then the rate of stellar evolution substantially slows down and escape due to tidal stripping takes over. During this phase of evolution the rate of mass loss due to stellar evolution is nearly constant, and higher for shallower mass functions (as expected). Energy carried away by stellar evolution events dominates the energy loss due to tidal stripping, even though the tidal mass loss is higher.

It is worth to note, that all models except W3235 show clear gravothermal oscillations in the post collapse phase. The oscillations become less and less visible when the cluster is approaching the final stages of dissolution.

Generally, agreement throughout the evolution between the results presented here and those by AH, VH, Baumgardt & Makino (2002) and Lamers et al (2005) for $N$–body models, TPZ for 2–D Fokker–Planck models and by JNR and Paper II for Monte Carlo models is rather good. In all cases, the qualitative behaviour is very similar. Nevertheless, the treatment of escapers for a tidally limited system proposed by Spurzem et al (2005) can be implemented to properly and more accurately follow the mass loss due to tidal stripping and the final stages of the cluster evolution, just before the dissolution.

At the end of this section the evolution of the density profiles for model W7235 will be discussed (see Figure 3). The very large number of particles in models allows substantial reduction in statistical fluctuations and computation (with high precision) of the density profiles in the course of cluster evolution. From the theoretical considerations it follows that, during the collapse phase the nearly isothermal core is building up. At the time of core bounce (about 3.5 Gyr) the density profile reaches the theoretically predicted power–low index equal to $-2.2$. Then this index is preserved during the further post–collapse evolution. For models W7235, W5235 and W335 the evolution of density profiles is practically the same. Only for model W3235, which undergoes strong mass loss due to stellar evolution and tidal stripping the density profiles do not show any signs of power–low core development. The initial shape of the profile is practically preserved (taking into account the reduction of central density and tidal radius).
3.3 Anisotropy evolution.

The degree of velocity anisotropy is measured by a quantity

\[ A \equiv 2 - \frac{2\sigma_r^2}{\sigma_t^2} \]  \hspace{1cm} (6)

where \( \sigma_r \) and \( \sigma_t \) are the radial and tangential one–dimensional velocity dispersions, respectively. For isotropic systems \( A \) is equal to zero. Systems preferentially populated by radial orbits have \( A \) positive and systems preferentially populated by tangential orbits have \( A \) negative. The evolution of anisotropy is presented in Figures 4 to 6 for models W3235, W5235 and W7235, respectively. As can be seen in Figure 4, for systems which can not survive the violent initial mass loss without strong structural changes (see discussion in the previous section), anisotropy stays close to zero with very large fluctuations. The system does not live long enough and the tidal stripping is strong enough to prevent the developing of substantial positive or negative anisotropy. For model W5235 the cluster is not initially very strongly concentrated, so the mass loss due to stellar evolution and consequently by tidal stripping prevents the development of positive anisotropy in the outer and middle parts of the system (see Figure 5). From the very beginning anisotropy for these regions becomes negative. Stars, which are preferentially on radial orbits, escape leaving mainly stars on tangential orbits. Finally, just before cluster disruption, the anisotropy in the whole system becomes slightly positive. At that time of cluster evolution most stars have radial orbits. Clusters concentrated stronger (W7235), develop from the very beginning a small positive anisotropy in the outer and middle parts of the system (see Figure 6). In the course of evolution the amount of anisotropy in the outer parts of the system is reduced substantially by tidal stripping, as stars on radial orbits escape preferentially. As tidal stripping exposes deeper and deeper parts of the system, the anisotropy (for large Lagrangian radii) gradually decreases and eventually becomes negative. At the same time the anisotropy in the middle and inner parts of the system stays close to zero.

It is interesting to note that for clusters which undergo core collapse (W5235 and W7235) the anisotropy in the inner parts of the system becomes slightly positive just around the collapse time. This behaviour can be attributed to the binary activities, which are mainly concentrated in the cluster core. Interactions between binaries and field stars (relaxation and energy generation) put stars on more radial orbits.

The anisotropy of main–sequence stars shows very similar behaviour to that discussed above. Mass segregation forces white dwarfs and neutron stars (most massive stars) to preferentially occupy the inner parts of the system. Therefore the anisotropy for them is much more modest than for the main–sequence stars, and stays close to zero. Anisotropy evolution agrees very well with the results of Paper II and qualitatively with the results obtained by Takahashi (1997) and Takahashi & Lee (2000).

3.4 Mass segregation.

Three main effects may be expected to govern the evolution of the mass function in models studied in this paper. First, there is a period of violent mass loss due to stellar evolution of the most massive stars. It takes place mainly during the first few hundred million years and its amplitude strongly depends on the slope of the mass function. Secondly, there is the process of rapid mass segregation, caused by two–body distant encounters (relaxation). It takes place...
mainly during core collapse and is relatively unaffected by the presence of a tidal field. Thirdly, there is the effect of the tidal field itself, which becomes more important after core collapse or even earlier (for models with a shallow mass function and with low concentration). Because the tides preferentially remove stars from the outer parts of the system, which are mainly populated by low-mass stars (mass segregation), one can expect that the mean mass should increase as evolution proceeds, making allowance for stellar evolution.

The basic results are illustrated in Figure 7 for the total mean mass inside the 10% and 50% Lagrangian radii and white dwarfs inside 10% Lagrangian radius for model W5235. The violent and strong mass loss due to stellar evolution is characterized by an initial decrease of the mean mass. The evolution of the most massive stars will remove a substantial amount of mass from the system and, consequently, greatly lower the mean mass in the whole system. This behaviour is visible also in the other models, but with one exception. The mean mass inside the 10% Lagrangian radius for model W7235 is increasing instead of decreasing. For this model, the initial concentration is high enough to force very quick mass segregation in the central part of the system. The most massive main sequence stars (not evolved yet): white dwarfs and neutron stars, sink into the center. For model W3235 for most of the time the mean mass stays nearly constant for the whole system. Just before the cluster dissolution it substantially increases. For all models tidal stripping connected mainly with removal of less massive stars forces the mean mass in the middle and outer parts of the system to steadily increase. The much faster increase of the mean mass inside the 10% Lagrangian radius (mass segregation) than in the outer Lagrangian radii (50% and 99%) during the collapse phase nearly stops at the core bounce time. The subsequent evolution is characterized by the slower rate of increase of the mean mass (close to the time of the core bounce the mean mass is practically constant). This is in good agreement with results obtained by Giersz & Heggie (1997) for small N-body simulations and for Monte Carlo simulations presented in Paper II. The reason for the nearly constant mean mass is unclear. The slow increase of the mean mass can be attributed to the binary activity, which leads to removal of some low mass stars from the core. For all models the effect of tides manifests itself by a gradual increase of the mean mass in the halo.

The evolution of the mean mass for main–sequence stars and white dwarfs for the inner 10% Lagrangian radii is shown on Figure 8. When less and less massive main–sequence stars finish their evolution as less and less massive white dwarfs, the mean mass of white dwarfs decreases with time. The newly created less massive white dwarfs affect the mean mass of white dwarfs more strongly for the steeper mass function than for the shallower one. For the steeper mass function a smaller number of massive white dwarfs is created comparable to the shallower mass function. At the time around the core bounce the average mass of white dwarfs increases. This is connected with the decreasing core size and continuing mass segregation. Deeper in the cluster center a higher fraction of massive white dwarfs is present. In the post collapse phase the white dwarfs average mass is decreasing. Binaries start to be created mainly from the most massive stars (neutron stars and white dwarfs) deep in the core. They dynamically interact with other stars and in the course of evolution are removed from the system together with some massive white dwarfs. This leads to the decrease of the mean mass of white dwarfs.

The mean mass of main–sequence stars initially slightly de-
Stellar evolution will change a stellar content of the cluster. The evolution of the total mean stellar mass in the different zones: $R(0-0.01)$, $R(0.01-0.05)$, $R(0.05-0.1)$, $R(0.1-0.3)$, $R(0.3-0.6)$ increases, which is connected with stellar evolution of the most massive stars. Then, there is a period of the gradual increase of the mean mass due to mass segregation. At late phases of cluster evolution the rate of increase of the mean mass speeds up. Around the core bounce time one can observe the break of the rate of increase of the main-sequence average mass. This can be again attributed to binary activities.

The evolution of the total mean stellar mass in the different places of the system (Figure 7) and the evolution of the average mass inside 10% Lagrangian radius for main–sequence stars and white dwarfs (Figure 8) agree very well with the results obtained in Paper II.

Stellar evolution will change a stellar content of the cluster. In the course of time main–sequence stars will evolve creating first black holes, neutron stars and then white dwarfs with decreasing mass. In Figure 9, evolution of the mass ratio of main–sequence stars ($ms$), white dwarfs ($wd$) and neutron stars/black holes ($ns$) to the actual total cluster mass and the evolution of the ratio of the actual total mass ($ms + wd + ns$) to the initial total mass, are shown for model W5235. The fraction of evolved stars present in the system at any time results from two processes which act in opposite directions: stellar evolution (formation) and evaporation across the tidal boundary (loss). At the beginning of cluster evolution the sharp increase of the mass ratio of evolved stars and decrease of the mass ratio of main–sequence stars and the total mass ratio are connected with the stellar evolution of the most massive stars in the system. Then, the steady increase of the mass ratio for $wd$ and $ns$ is observed. After the time of core bounce the ratio for $ns$ is slowly decreasing because of binary activities (the most massive stars (neutron stars) are with the highest probability involved in binary formation and finally removed from the system by interactions with the field stars and other binaries). The ratios for $ms + wd + ns$ and $ms$ are decreasing, as expected. At the time close to the dissolution time evolved stars consists of up to 75% of the mass of the all stars and up to 63% of the number of all stars. The fraction of mass/number of $wd$ is larger for systems with shorter relaxation time and with stronger influence of the tidal field of a parent galaxy. This is in a very good agreement with the results obtained by VH.

Figure 10 shows the evolution of the mass ratio of evolved stars to the actual total mass for different zones: $R(0-0.01)$, $R(0.01-0.05)$, $R(0.05-0.1)$, $R(0.1-0.3)$, $R(0.3-0.6)$ and $R(0.6-r_1)$ for model W5235.
The Monte Carlo method describes in a proper way the graininess of the gravitational field and the stochasticity of real $N$–body systems. It provides, in almost as much detail as $N$–body simulations, information about the movement of any object in the system. In that respect the Monte Carlo scheme can be regarded as a method which lies between direct $N$–body and Fokker–Planck models and combines most advantages of both these methods. This is the first important step in the direction of simulating the evolution of a real globular cluster.

It was shown that the results obtained in this paper are in qualitative agreement with these presented by CW, VH, AH, TPZ, JNR, Paper II, Baumgardt & Makino (2002) and Lamers et al 2005. Particularly good agreement is obtained with VH’s $N$–body simulations and what is not surprising, with results of Monte Carlo simulations presented in Paper II. All models survive the phase of rapid mass loss and then undergo core collapse and then subsequent post–collapse expansion (except model W3235) in a manner similar to isolated models. The expansion phase is eventually reversed when tidal limitation becomes important. As in isolated models, mass segregation substantially slows down by the end of core collapse. Mass loss connected with stellar evolution dominates the initial phase of cluster evolution. Then the rate of stellar evolution substantially slows down and escape due to tidal stripping takes over. During this phase of evolution the rate of mass loss is nearly constant, and higher for shallower mass functions. Energy carried away by stellar evolution events dominates the energy loss due to tidal stripping, even though the tidal mass loss is higher. For the first time, because of the large number of particles in simulations (1,000,000) which results in substantial reduction of statistical fluctuations, it was possible to compute in an accurate way the evolution of density profiles. The development of power–low density profile is clearly visible and agrees very well with theoretical predictions. The observed power–low index is equal to about $-2.2$.

The strongly concentrated model (W7235), shows a modest initial build up of anisotropy in the outer parts of the system. As tidal stripping exposes the inner parts of the system, the anisotropy gradually decreases and eventually becomes slightly negative. Model W5235 from the very beginning develops in the outer parts of the system negative anisotropy. The cluster is not concentrated enough to prevent removal of stars which are preferentially on radial orbits. The negative anisotropy stays negative until the time of cluster disruption, when it becomes slightly positive (during cluster disruption most stars are on radial orbits). Because of mass segregation, and due to evaporation across the tidal boundary, which preferentially removes low mass stars, the mean mass in the cluster increases with time. During the core collapse the rate of increase of the mean mass is highest in the central parts of the system (mass segregation). After the core bounce there is a substantial increase in the mean mass in the middle and outer parts of the system (tidal stripping), and more modest increase in the inner parts of the system, which is mainly connected with binary activity. The fraction of evolved stars is increasing during the cluster evolution. This fraction is larger for systems with shorter relaxation time and stronger influence of the tidal field of a parent galaxy. The mass ratio of evolved stars can be as high as nearly 65% at the outer parts of the system up to nearly 98% in the core. At the time of 15 Gyr these ratios are 90% and 10%, respectively. The presence of a substantial number of practically invisible stars has very important consequences for the interpretation of observational data and it has to be taken into account when the global globular cluster parameters are drawn from the observations. Because of stellar evolution, mass segregation and evaporation of stars the IMF is quickly forgotten and impossible
to recover from the observational data. The actual GMF closely resembles the LMF for the middle parts of the system (close to half–mass radius). These results are in an excellent agreement with $N$–body simulations presented by VH.

In order to perform simulations of real globular clusters the description of some processes, already included in the code has to be improved, and several additional physical processes have to be added to the code. Stellar and binary evolution, more accurate treatment for energy generation by binaries, particularly in binary–binary interaction, and proper treatment of the escape process in the presence of a tidal field are still waiting for improvement. One of the population synthesis codes developed by Hurley (Hurley et al 2000, 2002), Portegies Zwart (Portegies Zwart & Verbunt 1966) and Belczynski (Belczynski, Kalogera & Bulik 2002) can be used to follow more accurately single star and binary evolution. The implementation of techniques used in Aarseth’s $N$BODY codes for direct integrations of a few body subsystems can cure the problem of energy generation in 3–body and 4–body interactions (Giersz & Spurzem 2003). The treatment of escapers proposed by Spurzem et al (2005) can be implemented to solve the problem of escapers in the tidal field (problem which is inherently connected with gaseous, Fokker–Planck and Monte Carlo codes). The tidal shock heating of the cluster due to passages through the Galactic disk, interaction with the bulge, shock–induced relaxation, primordial binaries, physical collisions between single stars and binaries are some of the processes, which are waiting for implementation into the code. The inclusion of all these processes does not pose a fundamental theoretical challenge, but is rather complicated from the technical point of view. The international collaboration called MODEST was setup a few years ago to solve problems with merging all available codes (hydrodynamical, stellar evolution and dynamical) into a code which can deal in detail with the evolution of real globular clusters (see http://www.manybody.org/modest). Inclusion into the Monte Carlo code of as much as possible physical processes will allow to perform detailed comparison between simulations and observed properties of globular clusters, and will also help to understand the conditions of globular cluster formation and explain how peculiar objects observed in clusters can be formed. These types of simulations will also help us to introduce, in a proper way, into future $N$–body simulations all the necessary processes required to simulate the evolution of real globular clusters on a star–by–star basis from their birth to their death.

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