Test of Physics beyond the Standard Model in Nuclei

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Abstract

The modern theories of Grand Unification (GUT) and supersymmetric (SUSY) extensions of standard model (SM) suppose that the conservation laws of the SM may be violated to some small degree. The nuclei are well-suited as a laboratory to test fundamental symmetries and fundamental interactions like lepton flavor (LF) and lepton number (LN) conservation. A prominent role between experiments looking for LF and total LN violation play yet not observed processes of neutrinoless double beta decay ($0\nu\beta\beta$-decay). The GUT’s and SUSY models offer a variety of mechanisms which allow $0\nu\beta\beta$-decay to occur. They are based on mixing of Majorana neutrinos and/or R-parity violation hypothesis. Although the $0\nu\beta\beta$-decay has not been seen it is possible to extract from the lower limits of the lifetime upper limits for the effective electron Majorana neutrino mass, effective right handed weak interaction parameters, the effective Majoron coupling constant, R-parity violating SUSY parameters etc.

A condition for obtaining reliable limits for these fundamental quantities is that the nuclear matrix elements governing this process can be calculated correctly. The nuclear structure wave functions can be tested by calculating the two neutrino double beta decay ($2\nu\beta\beta$-decay) for which we have experimental data and not only lower limits as for the $0\nu\beta\beta$-decay. For open shell nuclei the method of choice has been the Quasiparticle Random Phase Approximation, which treats Fermion pairs as bosons. It has been found that by extending the QRPA including Fermion commutation relations better agreement with $2\nu\beta\beta$-decay experiments is achieved. This increases also the reliability of conclusions from the upper limits on the $0\nu\beta\beta$-decay transition probability.

In this work the limits on the LN violating parameters extracted from current $0\nu\beta\beta$-decay experiments are listed. Studies in respect to future $0\nu\beta\beta$-decay experimental projects are also presented.

PACS numbers: 12.60.Jv, 11.30.Er, 11.30.Fs, 23.40.Bw

1 Introduction

The standard model (SM) represents the simplest and most economical theory which describes jointly weak and electromagnetic interactions. It describes well all terrestrial experimental results known today. Nevertheless the SM cannot be considered as the ultimative theory of nature and is likely to describe the effective interaction at low energy of an underlying more fundamental theory. The supersymmetry (SUSY), which is one of the fundamental
new symmetries of nature, is believed to be next step beyond the successful SM. The supersymmetry is the symmetry between fermions and bosons, which has to be broken in order to explain the phenomenology of the elementary particles and their superpartners. It is the only known symmetry which can stabilize the elementary Higgs boson mass with respect to otherwise uncontrollable radiative corrections. The minimal supersymmetric model (MSSM), that leads to the SM at low energies has been the subject of extensive investigations.

Both the SM and the MSSM leads to zero mass for the neutrinos. In view of the observed results on solar (Homestake, Kamiokande, Gallex, and SAGE) and atmospheric neutrinos (IMB, Soudan 2, MACRO and Super-Kamiokande) it is more appropriate to consider the extensions of the SM and the MSSM that can lead to neutrino masses. Neutrino masses either require the existence of right-handed neutrinos or require violation of the lepton number (LN) so that Majorana masses are possible. So, one is forced to go beyond the minimal models again, whereby LF and/or LN violation can be allowed in the theory. A good candidate for such a theory is the left-right symmetric model of Grand Unification (GUT) inaugurated by Salam, Pati, Mohapatra and Senjanović (especially models based on SO(10) which have first been proposed by Fritzsch and Minkowski and its supersymmetric version). The left-right symmetric models, representing generalization of the SU(2)^L ⊗ U(1) SM, predict not only that the neutrino is a Majorana particle, that means it is up to a phase identical with its antiparticle, but automatically predict the neutrino has a mass and a weak right-handed interaction. The basic idea behind grand unified models is an extension of the local gauge invariance from quantum chromodynamics (SU3) involving only the colored quarks also to electrons and neutrinos. We note that the non-supersymmetric left-right models suffer from the hierarchy problem.

The expectations arising from GUT’s and theirs SUSY versions are that the conservation laws of the SM, e.g. LN conservation, may be violated to some small degree. In the left-right symmetric models the LN conservation is broken by the presence of the Majorana neutrino mass. The LN violation is also inbuilt in those SUSY theories where R-parity, defined as \( R_p = (-1)^{3B+L+2S} \) (S, B, and L are the spin, baryon and lepton number, respectively) is not a conserved quantity anymore. The conservation of LN is among the most stringently tested laws of physics nowadays. The nuclei are well-suited as laboratory to test this fundamental symmetry due to the fact that a variety of quantum numbers is available as initial and final states. The neutrinoless double beta decay (0νββ-decay),

\[
(A, Z) \rightarrow (A, Z + 2) + 2e^-, \tag{1}
\]

which involves the emission of two electrons and no neutrinos, has been long recognized as a powerful tool to study the LN conservation. The 0νββ-decay takes place only if the neutrino is a Majorana particle with non-zero mass. The GUT’s and R-parity violating SUSY models offer a plethora of the 0νββ-decay mechanisms triggered by exchange of neutrinos, neutralinos, gluinos, leptoquarks etc. If one assumes that one mechanism at a time dominates, the half-life of the 0νββ-decay can be written as

\[
(T_{1/2}^{0ν})^{-1} = |LNV|^2 \sum_i P_{0i}^{0ν} G_{0i}^{0ν}, \tag{2}
\]

where LNV is some effective LN violating parameter, \( P_{0i}^{0ν} \) is the real part of the product of two nuclear matrix elements governing the 0νββ-decay and \( G_{0i}^{0ν} \) is the integrated kinematical factor. The sum over \( i \) runs over different phase space integrals weighted by a corresponding
product of nuclear matrix elements. There are different LNV parameters, e.g. the effective electron-neutrino mass, effective right handed weak interaction parameters, effective Majoron coupling constant and R-parity violating SUSY parameters, which incorporate elements of the fundamental interaction of Majorana neutrinos and/or R-parity violating interaction of SUSY particles (see e.g. the recent review articles [13, 14]). The value of these parameters can be determined in two ways: i) One can extract upper bound on the LNV parameter from the best presently available experimental lower limit on the half life of the $0\nu\beta\beta$-decay $T_{1/2}^{0\nu-exp}$ after calculating the corresponding nuclear matrix elements. ii) One can use the phenomenological constraints imposed by other experiments, e.g. those looking for the neutrino oscillation, to evaluate the LNV parameter explicitly, which further can be compared with the extracted one. The $0\nu\beta\beta$-decay constraints on LNV parameters must be taken into account by the theoreticians, when they build new theories of grand unification.

At present the searches for $0\nu\beta\beta$-decay are pursued actively for different nuclear isotopes, e.g. $^{76}\text{Ge}$ (Heidelberg-Moscow coll. [15] and IGEX coll. [16]), $^{100}\text{Mo}$ (NEMO [17], and ELEGANTS [21]), $^{116}\text{Cd}$ (the INR exper. [22] and the NEMO exper. [30]), $^{130}\text{Te}$ [24] and $^{136}\text{Xe}$ (the Gotthard Xe exper. [25]). The sensitivity of a given isotope to the different LNV violating signals is determined by the value of the corresponding nuclear matrix element connecting the ground state of the initial and final nuclei with $J = 0^+$ and the value of the kinematical factor determined by the energy release for this process. In order to correctly interpret the results of $0\nu\beta\beta$-decay experiment, i.e. to obtain qualitative answers for the LNV violating parameters, the mechanism of nuclear transitions has to be understood. The $0\nu\beta\beta$-decay nuclear systems of interest are medium and heavy open shell nuclei with complicated nuclear structure. To test our ability to evaluate the nuclear matrix elements that govern the decay rate, it is desirable to describe the two-neutrino double beta decay ($2\nu\beta\beta$-decay) allowed in the SM:

$$ (A, Z) \rightarrow (A, Z + 2) + 2e^- + 2\bar{\nu}. $$

(3)

We note that each mode of the double beta decay requires the construction of the same many-body nuclear structure wave functions.

A variety of nuclear techniques have been used in attempts to calculate $2\nu\beta\beta$-decay matrix elements, which have been reviewed recently in Refs. [13, 14]. Especially the Quasiparticle Random Phase Approximation (QRPA) and its extensions have been found to be powerful models, considering their simplicity, to describe nuclear matrix elements, which require the summation over a complete set of intermediate nuclear states. The recent $2\nu\beta\beta$-decay calculations [36, 37, 38] including the schematic ones [39, 40] manifest that the inclusion of the Pauli exclusion principle (PEP) in the QRPA improves the predictive power of the theory giving more reliable prediction of the $2\nu\beta\beta$ decay probability.

In this contribution we present the upper limits on some effective LNV violating parameters extracted from the current experimental limits of the $0\nu\beta\beta$-decay lifetime for $A = 76, 82, 96, 100, 116, 128, 130, 136$ and $150$ isotopes, which are quantities of fundamental importance. A discussion in respect to the sensitivity of a given $0\nu\beta\beta$-decaying isotope to the different LNV violating signals is presented. Some related nuclear physical aspects as well as studies in respect to future $0\nu\beta\beta$-decay experiments are addressed.

2 Neutrinoless double beta decay
Figure 1: Mechanisms for $0\nu\beta\beta$-decay associated with the exchange of a Majorana neutrino: (a) the light and the heavy neutrino mass mechanism, (b), (c) right handed current mechanisms, (d) the Majoron mechanism. The following notation is used: $u_L(R)$, $d_L(R)$ and $e_L(R)$ are left- (right-) handed u-quark, d-quark and electron, respectively. $W$ is vector boson (light or hypothetical heavy) and $\nu_i$ (i=1,2,..) is the Majorana neutrino.

2.1 Majorana neutrino mixing mechanisms

The presently favored models of grand unification are left-right symmetric models [9]. They contain left- and hypothetical right-handed vector bosons $W_L^\pm$ and $W_R^\pm$. The vector bosons mediating the left and right-handed interaction are mixed if the mass eigenstates $W_1^\pm$ and $W_2^\pm$ are not identical with the weak eigenstates, which have a definite handedness:

$$
W_1^\pm = \cos \zeta \cdot W_L^\pm + \sin \zeta \cdot W_R^\pm, \\
W_2^\pm = -\sin \zeta \cdot W_L^\pm + \cos \zeta \cdot W_R^\pm.
$$

(4)

$\zeta$ is the mixing angle of the vector bosons. The left-right symmetry is broken since the vector bosons $W_1^\pm$ and $W_2^\pm$ obtain different masses by the Higgs mechanism. Since we have not seen a right-handed weak interaction the mass of the heavy, mainly “right-handed” vector boson must be much larger than the mass of the light (81 GeV) vector boson, which is responsible for the left-handed force.
The weak interaction Hamiltonian must now be generalized.

\[ H_W \approx \frac{G_F}{\sqrt{2}} \left[ (J_L \cdot j_L) + t g_\zeta (J_R \cdot j_L) + t g_\zeta (J_L \cdot j_R) + \left( \frac{M_1^W}{M_2^W} \right)^2 (J_R \cdot j_R) \right] + h.c., \]

where \( g_V = 1 \) and \( g_A = 1.25 \). \( M_1^W \) and \( M_2^W \) are the light and heavy vector boson masses, respectively. The capital \( J_L \) and \( J_R \) indicate the hadronic left- and right-handed currents changing a neutron into a proton, respectively. The lower case \( j_L \) (\( j_R \)) is the left (right) handed leptonic current which create an electron (or annihilate a positron) and annihilate left (right) handed current neutrino \( \nu_{eL} \) (\( \nu_{eR} \)).

The weak interaction Hamiltonian (5) is given for \( \zeta \ll 1 \) and \( M_2^W \gg M_1^W \) keeping only the lower order terms in expansion of \( t g_\zeta \) and \( M_1^W/M_2^W \) parameters.

The left-right symmetric models allow us to explain the smallness of the neutrino mass within the so called see-saw mechanism in the most natural way. It is supposed that the neutrino mixing does take place according to

\[ \nu_{eL} = \sum_{k=\text{light}} U_{ek}^L \chi_k \xi_k + \sum_{k=\text{heavy}} U_{ek}^L N_k \],

\[ \nu_{eR} = \sum_{k=\text{light}} U_{ek}^R \chi_k \hat{\xi}_k + \sum_{k=\text{heavy}} U_{ek}^R N_k \hat{\xi}_k, \]

where, \( \chi_k \) \( (N_k) \) are fields of light (heavy) Majorana neutrinos with masses \( m_k \) \( (m_k << 1 \text{ MeV}) \) and \( M_k \) \( (M_k >> 100 \text{ GeV}) \), respectively, and \( U_{ek}^L, U_{ek}^R \) are unitary mixing matrices.

In the case of the most general lepton mixing originating from a Dirac-Majorana mass term in the Lagrangian the flavor neutrino fields are superposition of three light and three heavy Majorana neutrinos with definite mass. The fields \( \chi_k \) and \( N_k \) satisfy the Majorana condition:

\[ \chi_k \xi_k = C \chi_k^T, \quad N_k \hat{\xi}_k = C N_k^T, \]

where \( C \) denotes the charge conjugation and \( \xi, \hat{\xi} \) are phase factors.

The possible quark level neutrino mixing mechanisms of \( 0\nu\beta\beta \)-decay are displayed in Figs. 1 (a), (b) and (c). If the \( 0\nu\beta\beta \)-decay is triggered by exchange of a light (heavy) left-handed Majorana neutrino [see Fig. 1 (a)] the corresponding amplitude of the process is proportional to the LN violating parameter \( \langle m_\nu \rangle \) \( (\eta_N) \):

\[ \langle m_\nu \rangle = \sum_{k=\text{light}} (U_{ek}^L)^2 \xi_k m_k, \]

\[ \eta_N = \sum_{k=\text{heavy}} (U_{ek}^L)^2 \hat{\xi}_k \frac{m_p}{M_k}, \]

where \( m_p \) is the proton mass. The difference between \( \langle m_\nu \rangle \) and \( \eta_N \) comes from the fact that the neutrino propagator in the first and second case show different dependence on the mass of neutrinos \[ 13 \]. We note that even if the neutrino is a Majorana particle but massless (i.e. there is no mixing of neutrinos), the process in Fig. 1 (a) can not happen since for a pure left-handed weak interaction theory, the emitted neutrino must be right-handed (positive helicity), while the absorbed neutrino must be left-handed (negative helicity). With a finite mass the neutrino has not any more a good helicity and the interference term between the leading helicity and the small admixtures allows a \( 0\nu\beta\beta \)-decay.
The presence of the slight right-handed weak interaction in the GUT's allows the mechanisms drawn in Figs. 1 (b) and (c). In this cases there is no helicity matching problem. The emitted neutrino from the left-handed vertex is right-handed as well as the absorbed neutrino at the right-handed vertex. Assuming only light neutrinos we distinguish two cases. i) The chiralities of the quark hadronic currents are the same as those of the leptonic currents coupled through the W-boson propagator [Fig. 1 (b)]. Thus the $0\nu\beta\beta$-decay amplitude is proportional to

$$<\lambda> = \left(\frac{M_W}{M_1}ight)^2 \sum_{k} U^L_{ek} U^R_{ek} \xi_k.$$  \hspace{1cm} (8)

Recall that the W-boson propagator can be approximated by $1/M^2$ for $M = M^W_1, M^W_2$ [see Eq. (3)].

ii) The chirality of the right-handed leptonic current is opposite to the coupled left-handed hadronic current [Fig. 1 (c)]. This is possible due to W-boson mixing. The corresponding effective LN violating parameter is

$$<\eta> = t g \sum_{k}^\text{light} U^L_{ek} U^R_{ek} \xi_k.$$  \hspace{1cm} (9)

It is worthwhile to notice that the factor $\sum_k^\text{light} U^L_{ek} U^R_{ek} \xi_k$ in Eqs. (8) and (9) is expected to be small and even in the case there are only light neutrinos vanish due to orthogonality condition \[13\]. It indicates that the values of $<\lambda>$ and $<\eta>$ might be strongly suppressed assuming the see-saw neutrino mixing mechanism.

### 2.2 Majoron mechanism

The spontaneous breaking of the LN in the context of the see-saw model imply the existence of a physical Nambu-Goldstone boson \[13\], called Majoron \[14\], which is a light or massless boson with a very tiny coupling to neutrinos

$$\mathcal{L}_{\phi\nu\nu} = \sum_{i\leq j} \bar{\nu}_i \gamma_5 \nu_j \left( i \text{Im } \phi \right) P_{ij}, \quad P_{i,j} = \sum_{\alpha,\beta=e,\mu,\tau} U^R_{j\beta} g_{\alpha\beta}.$$  \hspace{1cm} (10)

Here, $\nu_i$ denotes both light $\chi_i$ and heavy $N_i$ Majorana neutrinos.

The Majoron $\phi$ might occur in the Majoron mode of the $0\nu\beta\beta$-decay ($0\nu\beta\beta\phi$-decay)

$$(A, Z) \rightarrow (A, Z + 2) + 2e^- + \phi.$$  \hspace{1cm} (11)

and offers a new possibility for looking for a signal of new physics in double beta decay experiments. The $0\nu\beta\beta\phi$-decay mode yields a continuous electron spectrum for the sum of electron energies like the $2\nu\beta\beta$-decay mode but differs from it by the position of the maximum as different numbers of light particles are present in the final state. We remind that in the case of $0\nu\beta\beta$-decay a peak is expected to be at the end of the electron-electron coincidence spectra.

The mechanism leading to a $0\nu\beta\beta\phi$-decay mode is drawn in Fig. 1 (d). The experimental lower limits on the half-life of $0\nu\beta\beta\phi$-decay allow to deduce the upper limit on the effective Majoron coupling constant $<g>$:

$$<g> = \sum_{ij}^\text{light} U^L_{ei} U^L_{ej} P_{ij}.$$  \hspace{1cm} (12)
2.3 R-parity violating SUSY mechanism

Besides the simplest and the best known mechanism of LN violation based on the mixing of massive Majorana neutrinos advocated by different variants of the GUT’s the R-parity violation proposed in the context of the MSSM is becoming the most popular scenario for LN violation (see e.g. reviews [13, 10]). We remind that the R-parity symmetry assigns even R-parity to known particles of the SM and odd R-parity to their superpartners and that the Lagrangian of the MSSM conserves R-parity. The R-parity conservation is not required by gauge invariance or supersymmetry and might be broken at the Planck scale. The R-parity violation ($R_p$) is introduced in the effective Lagrangian (or superpotential) of the MSSM in terms of a certain set of hidden sector fields. The trilinear part of the $R_p$ superpotential takes the form

$$W_R = \lambda_{ijk} L_i L_j E^c_k + \lambda'_{ijk} L_i Q_j D^c_k + \lambda''_{ijk} U^c_i D^c_j D^c_k.$$  \hspace{1cm} (13)

Here $L$ and $Q$ stand for lepton and quark doublet left-handed superfields while $E^c, U^c, D^c$ stand for lepton and up, down quark singlet superfields. $\lambda_{ijk}$, $\lambda'_{ijk}$ and $\lambda''_{ijk}$ are coupling constant and indices $i, j, k$ denote generations. The $\lambda''$-terms are causing baryon number violation and the remaining ones the LN violation. In fact a combination of $\lambda'$ and $\lambda''$ leads to proton decay.

If the $R_p$ SUSY models are correct the 0νββ-decay is feasible. The nuclear 0νββ-decay is triggered by the 0νββ-decay quark transition $dd \rightarrow uu + e^-e^-$. The relevant Feynman diagrams associated with gluino $\tilde{g}$ and neutralino $\chi$ trilinear $R_p$ SUSY contributions to the 0νββ-decay are drawn in Fig. 2. The $R_p$ SUSY vertices are indicated with bold points. We see that the 0νββ-decay amplitude is proportional to the $\lambda'_{111}$ squared.

There are two possibilities of the hadronization, i.e. coming from the quark level to the nucleon level. One can place the four quark into the two initial neutrinos and two final protons, what is just the conventional two-nucleon mechanism of 0νββ-decay ($nn \rightarrow pp + e^-e^-$). This mechanism is strongly suppressed by the nucleon-nucleon repulsion at short distances for the exchange of heavy SUSY particles and heavy Majorana neutrinos. Another possibility is to incorporate quarks involved in pions in flight between nucleons. This possibility have been first pointed out by Pontecorvo [17]. It is the so called pion exchange mechanism ($\pi^- \rightarrow \pi^+ + e^-e^-$). The pion-exchange mode leads to a long-range nuclear interaction, which is significantly less sensitive to short-hand correlations effects. It was found that $R_p$ SUSY pion exchange contribution to the 0νββ-decay absolutely dominates over the conventional two nucleon mode realization [18, 19, 21].

The enhancement of the pion-exchange mode has also an origin in the bosonization of the $\pi^- > \pi^+ + e^-e^-$ vertex and is associated with the pseudoscalar $J_FJ_F$ and tensor $J^T_{\mu\nu}J_{T\mu\nu}$ hadronic current structure of the effective R-parity violating 0νββ-decay Lagrangian on the quark level [18] ($J_F = \pi\gamma_5 d$ and ($J^T_{\mu\nu} = \pi\sigma^{\mu\nu}(1 + \gamma_5)d$). The corresponding hadronic matrix elements are given as follows:

$$< \pi^+(q)|J_FJ_F|\pi^-(q) > \approx \frac{5}{3} < \pi^+(q)|J_F|0 > < 0|J_F|\pi^-(q) >$$

$$= -\frac{10}{9} f_\pi^2 \frac{m_\pi^4}{(m_u + m_d)^2} = -m_\pi^4 c_F,$$

$$< \pi^+(q)|J^T_{\mu\nu}J_{T\mu\nu}|\pi^-(q) > \approx -4 < \pi^+(q)|J_F|0 > < 0|J_F|\pi^-(q) > .$$ \hspace{1cm} (14)

Here $m_\pi$ is the pion mass, $f_\pi = 0.668 \ m_\pi$. Taking the conventional values of the current
Figure 2: Feynman graphs for the supersymmetric contributions to $0\nu\beta\beta$-decay. $u_L$, $d_R$ and $e_L$ have the same meaning as in Fig. 1. $\tilde{u}_L$, $\tilde{d}_R$ and $\tilde{e}_L$ are left-handed u-squark, right-handed d-squark and left-handed selectron, respectively. $\chi$ and $\tilde{g}$ are neutralinos and gauginos, respectively.

Quark masses $m_u = 4.2$ MeV, $m_d = 7.4$ MeV one gets $c_P \approx 214$. In the case of the exchange of a heavy Majorana neutrino there is a vector and axial-vector hadronic current structure $J_{AV}^\mu J_{AV,\mu}$ of the effective Lagrangian ($J_{AV}^\mu = \bar{u}_\alpha \gamma^\mu (1 - \gamma_5) d_\alpha$). We have

$$< \pi^+(q)|J_{AV}^\mu J_{AV,\mu}|\pi^-(q)> \approx \frac{8}{3} < \pi^+(q)|J_P|0> <0|J_P|\pi^-(q)> = \frac{-8}{3} f_\pi q^2 = -m_\pi^4 c_A (q^2).$$

(15)

Assuming the average momentum of the exchanged pion to be about 100 MeV we find $c_A \approx 0.61$. We note that $c_P \gg c_A$.

In order to derive a limit on the R-parity violating first generation Yukawa coupling $\lambda'_{111}$ from the observed absence of the $0\nu\beta\beta$-decay it is necessary to use viable phenomenological assumptions about some of the fundamental parameters of the $R_p$ MSSM. In Ref. [48] the ansatz of universal sparticle masses was assumed and that the lightest neutralino is bino-like. Within such phenomenological scenarios it was found that the gluino and neutralino exchange mechanisms are of comparable importance [13, 48]. Another possibility is to implement
relations among the weak scale values of all parameters entering the superpotential and the soft SUSY breaking Lagrangian and their values at the GUT scale. This scenario have been outlined in Refs. [49, 50]. It was shown that there is no unique answer to the problem of the dominance of neutralino and gluino contribution to 0νββ-decay. The dominance of any of them is bound with a different choice of the SUSY parameters $m_0$ and $m_{1/2}$. It is worthwhile to notice that in the extraction of λ′ the main uncertainty comes from the parameters of supersymmetry and not from the nuclear physics side [49, 51].

3 2νββ-decay and nuclear structure

Since there are measurements available for the 2νββ-decay with the geochemical method ($^{82}Se$ [31], $^{96}Zr$ [22], $^{128}Te$ and $^{130}Te$ [23]) and with the radiochemical method ($^{238}U$ [35]) and for seven nuclei even laboratory measurements ($^{48}Ca$ [33], $^{76}Ge$ [28], $^{82}Se$ [29], $^{96}Zr$ [19, 74, 20], $^{100}Mo$ [26, 34], $^{116}Cd$ [22, 30] and $^{150}Nd$ [26]), one could try to calculate for a test of the theory the double beta-decay with two neutrinos and compare them with the data. We note that a positive evidence for a 2νββ-decay transition to the 0+1 excited state of final nucleus was observed for $^{100}Mo$ [27].

The inverse half-life of the 2νββ-decay is free of unknown parameters on the particle physics side and can be expressed as a product of a phase-space factor $G_{2ν}$ and the Gamow-Teller transition matrix element $M_{2ν}^{GT}$ in second order:

$$ [T_{1/2}^{2ν}(0^+_{g.s.} \rightarrow 0^+_{g.s.})]^{-1} = G_{2ν} \left| M_{2ν}^{GT} \right|^2, \quad (16) $$

where

$$ M_{2ν}^{GT} = \sum_n < 0_f^+ | A_k(0) | 1_n^+ > < 1_n^+ | A_k(0) | 0_i^+ > \frac{1}{E_n - E_i + \Delta}. \quad (17) $$

$|0_i^+ >$, $|0_f^+ >$ and $|1_n^+ >$ are respectively the wave functions of the initial, final and intermediate nuclei with corresponding energies $E_i$, $E_f$ and $E_n$. $\Delta$ denotes the average energy $\Delta = (E_i - E_f)/2$. $A_k$ is the Gamow-Teller transition operator $A_k = \sum_i \tau_i^+ (\vec{\sigma}_i)_k$, k=1,2,3.

The calculation of $M_{2ν}^{GT}$ remains to be challenging and attracts the specialists of different nuclear models. The computational complexity of $M_{2ν}^{GT}$ consists in the reliable description of the complete set of the intermediate nuclear states. Recently, it has been shown the summation over the intermediate nuclear states in the present 2νββ-decay studies corresponds to a summation over a class of meson exchange diagrams within the S-matrix approach [52].

The nuclear shell model gives a satisfactory description only of the low lying excited states of nuclei. In the heavier nuclei there is a large number of basis states in the shell model which does not allow to perform realistic calculation without severe truncations. It is supposed to be the reason that the shell model predictions of 2νββ-decay rate for heavier nuclei, especially those for the Te region, show deviations from the experimental data [53]. We note that the feasibility of shell model calculations is growing with increasing computer facilities allowing to handle much larger configuration spaces.

Many different nuclear structure aspects of the many-body Green function $M_{2ν}^{GT}$ have been discussed. It was suggested by Abad et al. [44] that $M_{2ν}^{GT}$ could be dominated by the transitions through the lowest intermediate 1+ state (so called Single-State-Dominance-Hypothesis (SSDH)). The SSDH could be realized in two ways:i) There is the true dominance
\[ \nu = l^2 \]
\[ s^2 = 1 + 2 - 0 + 0 + 0 + \beta - /0/0/0/0 /1/1/1/1 (n p) \beta + /0/0/0/0 /1/1/1/1 (p n) \]

Figure 3: The upper part shows the way how in the Random Phase Approximation (RPA) the \(2\nu\beta\beta\) decay is calculated. For the Fermi transitions the \(\beta^- (n \rightarrow p)\) amplitude moves just a neutron into the same proton level and the \(\beta^+ (p \rightarrow n)\) amplitude moves a proton into the same neutron level. For the Gamow-Teller transitions it can also involve a spin flip, but the orbital part remains the same. One immediately realizes that the occupation and non-occupation amplitudes favor the \(\beta^-\) amplitude, but disfavor the \(\beta^+\) amplitude. There one has a transition from an unoccupied to an occupied single particle state, which is two-fold small \((s^2)\) first by the fact that the occupation amplitude for the proton \(v_p\) and secondly that the unoccupation amplitude for the neutron state \(u_n\) are both small. Therefore the \(2\nu\beta\beta\) is drastically reduced.

of the first \(1^+\) state, i.e. the contribution from higher lying \(1^+\) states to \(M_{GT}\) is negligible. ii) There is a cancelations among the contributions of higher lying \(1^+\) states of the intermediate nucleus. The idea of SSDH have been outlined in Ref. [55] showing that some experimental and theoretical evidence supports it for a few \(2\nu\beta\beta\)-decay systems.

The difficulty of the calculation of \(M_{GT}\) consists in the fact that the \(2\nu\beta\beta\)-decay matrix elements are strongly suppressed. Its value is only small fraction of the double Gamow-Teller sum rule that scales roughly like the number of pairs of unpaired neutrons [56],

\[ \sum_f |<f| \vec{A} \cdot \vec{A} |i>|^2 \approx (N - Z)(N - Z - 1). \] (18)

The proton-neutron QRPA (pn-QRPA) has been found successful in describing the suppression mechanism for the \(2\nu\beta\beta\)-decay [57, 58]. Fig. 3 explains why the \(2\nu\beta\beta\)-decay amplitude is so drastically reduced. Therefore the small effects which normally do not play a major role
can affect the $2\nu\beta\beta$-decay transition probability. If one looks to the second leg of the double beta-decay which is calculated backwards as a $\beta^+ (p \rightarrow n)$ decay from the final nucleus to the intermediate nucleus one finds that the matrix elements involved in these diagrams are Pauli suppressed by a factor $(u_n v_p)^2 = (small)^4$. The neutron-particle proton-hole force in the isovector channel, which is usually included is repulsive while the particle-particle force usually neglected is attractive. Therefore both excitations tend to cancel each other and therefore the amplitude $\beta^+$ is drastically reduced. This cancelation for the second leg could be even complete, i.e. the backgoing amplitudes and thus groundstate correlations cancel the leading forward going terms.

Although one can obtain agreement within pn-QRPA with the measured $2\nu\beta\beta$ data multiplying the particle-particle G-matrix elements of the nuclear Hamiltonian with a factor $g_{pp}$ in a range of $0.8 \leq g_{pp} \leq 1.2$ ( $g_{pp}$ in principle should be equal to unity), two leaps of faith are usually quoted: i) The extreme sensitivity of $M_{GT}$ to the strength of particle-particle interaction which does not allow a reliable prediction of the $2\nu\beta\beta$-decay probability. We note that $M_{GT}$ as a function of $g_{pp}$ crosses zero. ii) The collapse of the pn-QRPA solution within the physical range of $g_{pp}$, what is supposed to be a phase transition. The collapse is caused by generation of too many ground state correlations with increasing strength of the attractive proton-neutron interaction.

The study of the QRPA approximation scheme for different model spaces manifest that the problems i) and ii) are related with each other: The undesirable behavior of the pn-QRPA has its origin in the quasiboson approximation (QBA) violating the Pauli exclusion principle (PEP) and causing the QRPA excitation operators behave like bosons. The renormalized QRPA, which considers the PEP in an approximate way, shifts the collapse of the QRPA outside the physical range of $g_{pp}$ and shows a less sensitive dependence of $M_{GT}^{2\nu}$ on $g_{pp}$ [20, 21]. It allows us to predict more reliable values of the double beta decay matrix elements. The importance of the PEP for solving the problem of the QRPA collapse has been shown clearly within the schematic models, which are trying to simulate the realistic cases either by analytical solutions or by a minimal computational effort [19, 20]. In Ref. [21], to our knowledge for the first time, the solution of the QRPA equation with full consideration of the PEP was presented. It was found that restoring the PEP, the QRPA solutions are considerably stabilized and a better agreement with the exact solution is obtained. A new extension of the standard pn-QRPA “QRPA with PEP” was proposed, which consider the PEP in more appropriate way as the RQRPA and might work well also in the case of realistic calculations.

We note that the calculation of the $2\nu\beta\beta$-decay nuclear transition continues to be subject of interest, which stimulates the rapid development of the nuclear theory [20, 21, 22, 23, 24, 25].

4 Limits on LN violating parameters

The limits deduced for LN violating parameters depend on the values of nuclear matrix element $M E_i^{0\nu}$, of the kinematical factor $G_i^{0\nu}$ and of the current experimental limit for a given isotope [see Eq. (2)]. Thus there is useful to introduce sensitivity parameters for a given isotope to the different LN violating parameters, which depend only on the characteristics
of a given nuclear system. There are the following:

\[
\begin{align*}
\zeta_{<m_\nu>} (Y) & = 10^7 |M_{<m_\nu>}^{\text{light}}| \sqrt{G_{01} \text{ year}}, & \zeta_{\eta_N} (Y) & = 10^6 |M_{<m_\nu>}^{\text{heavy}}| \sqrt{G_{01} \text{ year}}, \\
\zeta_{<\lambda>} (Y) & = 10^7 |M_{\nu \beta \beta 1}^{\nu \beta \beta}| \sqrt{G_{\lambda} \text{ year}}, & \zeta_{<\eta>} (Y) & = 10^5 |M_{\nu \beta \beta 1}^{\nu \beta \beta}| \sqrt{C_{\eta} \text{ year}}, \\
\zeta_{<g>} (Y) & = 10^8 |M_{<m_\nu>}^{\text{light}}| \sqrt{G_B \text{ year}}, & \zeta_{\lambda_{111}} (Y) & = 10^5 |M_{\pi N}^{\pi N}| \sqrt{G_{01} \text{ year}}.
\end{align*}
\]

The explicit form of \( M_{<m_\nu>}^{\text{light}}, M_{<m_\nu>}^{\text{heavy}}, M_{\nu \beta \beta 1}^{\nu \beta \beta}, C_{\lambda}, C_{\eta}, C_{\lambda}, \text{ and } M_{\pi N}^{\pi N} \) can be found e.g. in Refs. \cite{66,13,49}.

Admittedly there is a rather large spread between the calculated values of nuclear matrix elements within different nuclear theories (for \(^{76}\text{Ge}\) the calculated rates differ by a factor of 3 \cite{13,14}). In principle there are no exact criteria to decide which of them are correct. Nevertheless one can argue the RQRPA method offers more reliable results than the QRPA as the ground state correlation are better under control due to the consideration of PEP.

The present limits on LN violating parameters \(< m_\nu >, \eta_N, < \lambda >, < \eta >, < g >\) are associated with the two-nucleon mechanism for which the correct treatment of the weak nucleon current \( J^{\mu 1} \) is crucial. We have:

\[
J^{\mu 1}_L = \Psi \tau^+ \left[ g_V(q^2)\gamma^\mu - i g_M(q^2) \frac{\sigma^{\mu \nu}}{2m_p} q^\nu - g_A(q^2)\gamma^\mu \gamma_5 + g_P(q^2)q^\mu \gamma_5 \right] \Psi,
\]

where \( q^\mu = (p - p')^\mu \) is the momentum transferred from hadrons to leptons (\( p \) and \( p' \) are four momenta of neutron and proton, respectively) and \( \sigma^{\mu \nu} = (i/2)\left[ \gamma^\mu, \gamma^\nu \right] \). \( g_V(q^2), g_M(q^2), g_A(q^2) \) and \( g_P(q^2) \) are the vector, weak-magnetism, axial-vector and induced pseudoscalar formfactors, respectively, which are real functions of a Lorentz scalar \( q^2 \).

The matrix elements \( M_{<m_\nu>}^{\text{light}} \) and \( M_{<m_\nu>}^{\text{heavy}} \) have been calculated by neglecting the role of induced nucleon currents (weak magnetism and induced pseudoscalar terms). Recently, it has been shown that they contribute significantly to the \( 0\nu\beta\beta \)-decay amplitude \cite{66}. They modify the Gamow-Teller contribution and create a new tensor contribution. Their contribution is as important as that of unchanged Fermi matrix element. It was found that indeed such corrections cause a more or less uniform reduction of the \( M_{<m_\nu>}^{\text{light}} \) by approximately 30\% throughout the periodic table. In the case of heavy neutrino exchange (\( M_{<m_\nu>}^{\text{heavy}} \)) the effect is much larger (a factor of 3) \cite{66}. The nucleon finite size has been taken into account through the phenomenological formfactors and the PCAC hypothesis. We note that in calculating the matrix elements involving the exchange of heavy neutrinos, the treatment of the short-range repulsion and nucleon finite size is crucial \cite{67}. It is expected that the correct treatment of the induced pseudoscalar term (which is equivalent to a modification of the axial-vector current due to PCAC) might influence significantly also the amplitude of \( 0\nu\beta\beta \)-decay in the case of the right-handed current mechanisms. It goes without saying that the validity of this argument can be ultimatively assessed by numerical calculations.

The numerical values of the sensitivity parameters \( \zeta_X (Y) (X = < m_\nu >, \eta_N, < \lambda >, \eta >, < g > \text{ and } \lambda_{111}) \) are listed in Table \range{1}. In their calculation we used values of \( M_{<m_\nu>}^{\text{light}}, M_{<m_\nu>}^{\text{heavy}}, \) and \( M_{\pi N}^{\pi N} \) calculated within the pn-RQRPA \cite{66,49}. As there are no available pn-RQRPA results for \( C_{\lambda}, C_{\eta} \) we used those of Ref. \cite{11}. The quantity \( \zeta_X (Y) \) is an intrinsic characteristic of an isotope \( Y \). The large numerical values of the sensitivity \( \zeta_X (Y) \) correspond to those isotopes within the group of \( \beta\beta \) decaying nuclei which are the most promising candidates for searching for the LN violating parameter \( X \). However, we remind
that there are also other microscopic and macroscopic properties of the isotope, which are important for building a $0\nu\beta\beta$-decay detector. By glancing the Table I we see that the most sensitive isotope is $^{150}Nd$. It is mostly due to the large phase space integral and partial due to the larger nuclear matrix element [88, 49]. We remark that the nucleus $^{150}Nd$ is deformed and that in the calculation of the corresponding nuclear matrix element the effects of nuclear deformation, which might be important, were not taken into account.

It is expected that the experimental constraints on the half-life of the $0\nu\beta\beta$-decay are expected to be more stringent in future. Knowing the values of $\zeta_X(Y)$ there is a straightforward way to deduce limits on LN violating parameters $X$ from the experimental half-lives $T_{1/2}^{0\nu-exp}$:

$$\frac{<m_\nu>}{m_e} \leq \frac{10^{-5}}{\zeta_{m_\nu}} \sqrt{\frac{10^{24} \text{ years}}{T_{1/2}^{0\nu-exp}}}, \quad \frac{\eta_N}{\zeta_N} \leq \frac{10^{-6}}{\zeta_{\eta_N}} \sqrt{\frac{10^{24} \text{ years}}{T_{1/2}^{0\nu-exp}}},$$

$$\frac{<\lambda>}{\zeta_\lambda} \leq \frac{10^{-5}}{\zeta_\lambda} \sqrt{\frac{10^{24} \text{ years}}{T_{1/2}^{0\nu-exp}}}, \quad \frac{<\eta>}{\zeta_\eta} \leq \frac{10^{-7}}{\zeta_\eta} \sqrt{\frac{10^{24} \text{ years}}{T_{1/2}^{0\nu-exp}}},$$

$$(\lambda'_{111})^2 \leq \kappa^2 \left(\frac{m_q}{100 \text{ GeV}}\right)^4 \left(\frac{m_q}{100 \text{ GeV}}\right) \frac{10^{-7}}{\zeta_{\lambda'_{111}}} \sqrt{\frac{10^{24} \text{ years}}{T_{1/2}^{0\nu-exp}}}.$$  \tag{21}

$\kappa$ is equal to 1.8 (gluino phenomenological scenario [13]). The normalization of $10^{24}$ years was chosen so that the $\zeta$’s are of order unity.

The current experimental upper bounds on the $0\nu\beta\beta$-decay effective LN violating parameters of interest for different isotopes are shown in Table I. We see that the Heidelberg-Moscow experiment [13] (In Table I we are giving the sensitivity of the experiment for $^{76}Ge$ being $T_{1/2}^{exp-0\nu} \geq 1.6 \times 10^{25}$ as we want to compare this value with those from other experiments.) offers the most restrictive limit for $<m_\nu>$, $\eta_N$, $<\lambda>$, $<\eta>$ and the $^{128}Te$ experiment [23] for $<g>$. We note that if the experimental data from an other geochemical experiment on double beta decay of $^{128}Te$ would be considered ($T_{1/2}^{exp} = 1.5 \times 10^{24}$, see Refs. cited in [23]), one would get less stringent limit on $<g>$ ($<g> \leq 1.5 \times 10^{-4}$), which is comparable with the upper bound offered by the $^{100}Mo$ experiment [24] (see Table I).

At present the largest attention is paid to the effective electron Majorana neutrino mass parameter $<m_\nu>$ in light of the positive signals in favor of oscillations of solar, atmospheric and terrestrial neutrinos. The masses and mixing angles can be determined from the available experimental data on neutrino oscillations and from astrophysical arguments by using some viable assumptions (hierarchical and non-hierarchical neutrino spectra etc.). At present the three and four neutrino mixing patterns are the most favorable ones [58, 69]. However, there is a discussion whether we really need the fourth sterile neutrino to fit the current experimental data. Knowing the elements of the neutrino mixing matrix one can draw conclusions about the $0\nu\beta\beta$-decay, in particular on $<m_\nu>$, assuming neutrinos to be Majorana particles. From the study of different neutrino mixing schemes it follows that the upper bound on effective Majorana neutrino mass $<m_\nu>$ could be ranging from $10^{-2}$ to 1 eV [58, 70, 71]. From the Table I we see that the Heidelberg-Moscow $0\nu\beta\beta$-decay
The present neutrino experiments do not provide us with useful information about the neutrino mixing. ii) The lepton number violation is in reach of near future $\nu_{\beta\beta}$-decay experiments, if the neutrino is a Majorana particle. An issue which only $\nu_{\beta\beta}$-decay plays an important role in deciding among the alternative possibilities of neutrino mixing. We remind that the discovery of the $\nu_{\beta\beta}$-decay for a given isotope, respectively. $\zeta(X)$ denotes according to Eq. (19) the sensitivity of a given nucleus $Y$ to the LN violating parameter $X$. The upper limits on $< m_{\nu} >, \eta_{N}, < \lambda >, < \eta >, < g >$ and $\lambda_{111}'$ are presented. gch.-geochemical data.

Table 1: The present state of the Majorana neutrino mass (light and heavy), right-handed current, Majoron and $R_{p}$ SUSY searches in $0\nu\beta\beta$-decay experiments. $T_{1/2}^{exp-0\nu}$ and $T_{1/2}^{exp-0\nu\phi}$ are the best presently available lower limit on the half-life of the $0\nu\beta\beta$-decay and $0\nu\beta\beta\phi$-decay for a given isotope, respectively. $\zeta (X)$ denotes according to Eq. (19) the sensitivity of a given nucleus $Y$ to the LN violating parameter $X$. The upper limits on $< m_{\nu} >, \eta_{N}, < \lambda >, < \eta >, < g >$ and $\lambda_{111}'$ are presented. gch.-geochemical data.

| Nucleus | $^{76}Ge$ | $^{82}Se$ | $^{96}Zr$ | $^{100}Mo$ | $^{106}Cd$ | $^{128}Te$ | $^{130}Te$ | $^{136}Xe$ | $^{150}Nd$ |
|---------|---------|---------|---------|---------|---------|---------|---------|---------|---------|
| $T_{1/2}^{exp-0\nu}$ [years] | $1.6\times 10^{25}$ | $2.8\times 10^{22}$ | $7.7\times 10^{24}$ | $5.6\times 10^{22}$ | $4.4\times 10^{23}$ | $1.2\times 10^{21}$ |
| C.L. [%] | 90 | 90 | 90 | 90 | 90 | 90 | 90 | 90 | 90 |
| Ref. | [15] | [16] | [20] | [21] | [22] | [23] | [24] | [25] | [26] |
| $T_{1/2}^{exp-0\nu\phi}$ [years] | $7.9\times 10^{21}$ | $3.1\times 10^{21}$ | $1.2\times 10^{21}$ | $7.7\times 10^{24}$ | $2.7\times 10^{21}$ | $7.2\times 10^{21}$ | $2.8\times 10^{21}$ |
| C.L. [%] | 90 | 90 | 90 | 90 | 90 | gch. | gch. | 90 | 90 |
| Ref. | [23] | [24] | [24] | [25] | [26] | [25] | [26] |
| $\zeta_{\eta_{N}}$ | 2.49 | 4.95 | 4.04 | 7.69 | 5.11 | 1.02 | 4.24 | 1.60 | 17.3 |
| $\zeta_{\eta_{N}}$ | 2.90 | 5.64 | 3.98 | 7.10 | 5.36 | 1.25 | 5.45 | 3.43 | 18.5 |
| $\zeta_{\lambda_{111}}$ | 3.35 | 6.92 | 10.3 | 1.81 | 1.60 | 0.66 | 8.62 | 6.06 | 27.6 |
| $\zeta_{\eta_{N}}$ | 5.67 | 3.91 | 8.20 | 5.39 | 2.28 | 2.86 | 12.7 | 9.00 | 40.9 |
| $\zeta_{\eta_{N}}$ | 2.41 | 6.59 | 5.93 | 10.5 | 6.60 | 0.53 | 5.08 | 1.90 | 26.7 |
| $\zeta_{\lambda_{111}}$ | 5.57 | 10.9 | 11.6 | 17.9 | 10.9 | 3.25 | 14.7 | 8.92 | 54.7 |
| $< m_{\nu} >$ [eV] | 0.51 | 8.7 | 40. | 4.0 | 5.9 | 1.8 | 5.1 | 4.8 | 8.5 |
| $\eta_{N}$ [10^{-7}] | 0.86 | 15. | 79. | 8.4 | 11. | 2.9 | 7.7 | 4.4 | 16. |
| $< \lambda >$ [10^{-6}] | 0.75 | 12. | 31. | 33. | 36. | 5.5 | 4.9 | 2.5 | 10.4 |
| $< \eta >$ [10^{-8}] | 0.44 | 22. | 38. | 11. | 26. | 1.3 | 3.3 | 1.7 | 7.1 |
| $< g >$ [10^{-4}] | 4.7 | 3.1 | 8.5 | 1.7 | 4.4 | 0.69 | 3.8 | 6.2 | 2.2 |
| $\lambda_{111}'$ (100 GeV) [10^{-4}] | 1.2 | 5.0 | 9.4 | 3.3 | 4.2 | 1.9 | 3.0 | 2.3 | 4.1 |
| $\lambda_{111}'$ (1 TeV) [10^{-2}] | 3.8 | 16. | 30. | 10. | 13. | 6.0 | 9.6 | 7.4 | 13. |

experiment implies $< m_{\nu} >$ to be less 0.5 eV. This fact allows us to make two important conclusions: i) The $0\nu\beta\beta$-decay plays an important role in deciding among the alternative possibilities of neutrino mixing. ii) The lepton number violation is in reach of near future $0\nu\beta\beta$-decay experiments, if the neutrino is a Majorana particle. An issue which only $0\nu\beta\beta$-decay can decide. We remind that the discovery of the $0\nu\beta\beta$-decay, what would be a major achievement for particle physics and cosmology, will implies only the upper bound on $< m_{\nu} >$ as a plethora of other $0\nu\beta\beta$-decay mechanisms is in the game. It is supposed that only further measurements of $0\nu\beta\beta$-decay transitions to the excited states of daughter nucleus together with inclusion of nuclear theory and study of different differential characteristics will allow us to decide which mechanism is the dominant one.

The present neutrino experiments does not provide us with useful information about the mixing of heavy Majorana neutrinos ($M_{k} >> 100$ GeV). Therefore, it is difficult to extract
the mass of heavy neutrinos from the current limit \( \eta_N \leq 8.6 \times 10^{-8} \) (see Table 1). If one assumes the corresponding \( U_{ei} \) to be of the order of unity for the lightest heavy neutrino mass \( M_i \) one gets: \( M_i \geq 1.1 \times 10^4 \) TeV. However, this element of the neutrino mixing matrix is expected to be rather small due to large differences in masses of light and heavy neutrinos within the see-saw mechanism. Therefore the real limit on \( M_k \) is supposed to be much weaker. It is worthwhile to notice that the limit on \( \eta_N \) is extremely sensitive to the nucleon finite size and the short–range correlation effects [61]. The heavy neutrino exchange nuclear matrix elements evaluated without inclusion of the induced nucleon currents [72] are considerably overestimated [66] and should not be used in deducing the limit on \( \eta_N \).

The present particle physics phenomenology does not allow us to deduce the magnitude of \( \sum_{k}^{\text{light}} U_{ek}^{L} U_{ek}^{R} \xi_k \) entering the expressions for the effective right-handed current parameters \(< \lambda >\) and \(< \eta >\). If we assume its value is about unity, then we get from the current limits on \(|< \lambda >| \leq 7.5 \times 10^{-7}\) and \(|< \eta >| \leq 4.4 \times 10^{-9}\) (see Table [1]) the corresponding bounds on the mass \( M_{W}^{*} \) of the heavy vector boson \( W_{R}^{*} \) and the mixing angle \( \zeta \) as follows: \( M_{W}^{*} \geq 93 \) TeV and \(|tg\zeta| \leq 4.4 \times 10^{-9}\). Mohapatra has shown that in the left-right symmetric model with spontaneous R-parity violation there is an upper limit on \( M_{W}^{*} \) (in the limit \( \tan \zeta \to 0 \) \( M_{W}^{*} = M_{W}^{*} \)) of at most 10 TeV [73]. By using this value for upper bound on \( M_{W}^{*} \) (for the lower limit on \( M_{W}^{*} \) we consider the value 100 GeV) we find

\[
|\sum_{k}^{\text{light}} U_{ek}^{L} U_{ek}^{R} \xi_k| \leq 1.1 \times 10^{-6} - 1.1 \times 10^{-2},
\]

\[
|tg\zeta| \leq 3.8 \times 10^{-7} - 3.8 \times 10^{-3}.
\]

We remark that these limits could be modified after the pseudoscalar term of the nuclear current is properly taken into account.

The 0\( \nu \beta \beta \)-decay offer the most stringent limit on the R-parity violating first generation Yukawa coupling \( \lambda'_{111} \) [49]. Its value depends on the masses of SUSY particles (see Eq. (21)). If the masses of squark \( m_{\tilde{q}} \) and gluino \( m_{\tilde{g}} \) would be at their present experimental lower bounds of 100 GeV we deduce from the observed absence of the 0\( \nu \beta \beta \)-decay \( \lambda'_{111} \leq 1.2 \times 10^{-4} \) (phenomenological scenario). A conservative upper bound is obtained using the SUSY ”naturalness” upper bound \( m_{\tilde{q}}, m_{\tilde{g}} \approx 1 \) TeV. It gives \( \lambda'_{111} \leq 3.8 \times 10^{-2} \) (see Table [1]). We mention that the limits on \( \lambda'_{111} \) depend only weakly on the details of the nuclear structure as \( \lambda'_{111} \) is proportional to the inverse square root of the nuclear matrix element. In addition, the corresponding nuclear matrix elements are changing only slightly within the physical range of parameters of the nuclear Hamiltonian [19]. However, \( \lambda'_{111} \) depends quadratic on the masses of SUSY particles. In the GUT’s constrained SUSY scenario there is a rather large SUSY parameter space. By using different sample of relevant SUSY parameters one ends up with significantly different limits on \( \lambda'_{111} \) [71]. Finally, we stress that the above limits are very strong and lie beyond the reach of near future accelerator experiments (HERA, TEVATRON) [18]. However, we note that the collider experiments are potentially sensitive to other couplings \( \lambda'_{ijk}, \lambda''_{ijk} \) etc.

There are new experimental proposals for measurements of the 0\( \nu \beta \beta \)-decay for different isotopes. A new NEMO 3 experiment is under construction, which is expected to reach a lower limit on the 0\( \nu \beta \beta \)-decay half-life of the order of 10\(^{25}\) years for \(^{82}\)Se, \(^{96}\)Zr, \(^{100}\)Mo, \(^{116}\)Cd, \(^{130}\)Te and \(^{150}\)Nd nuclei in a period of about six years [74]. The KAMLAND [75], CUORE [76] and GENIUS [73] experiments are under consideration at moment. The KAMLAND
Table 2: The expected limits on LN violating parameters of interest from the future $0\nu\beta\beta$-decay experiments. The same notations as in Table I is used.

| exper. | nucl.  | $T_{1/2}^{exp-0\nu}$ | $T_{1/2}^{exp-0\nu\phi}$ | $<m_\nu>$ | $\eta_N$ | $<\lambda>$ | $<\eta>$ | $<g>$ | $\lambda'_{111}$ |
|-------|--------|----------------------|---------------------------|----------|---------|----------|---------|------|-----------------|
| current | $^{76}Ge$ | 1.6 | 10$^{25}$ | | | 0.51 | 8.6 | 7.5 | 4.4 | 12. |
| | $^{128}Te$ | 77. | 10$^{23}$ | | | | | | | |
| NEMO-3 | $^{82}Se$ | 1. | 1. | 0.33 | 5.6 | 4.6 | 8.1 | 4.8 | 9.7 | |
| | $^{96}Zr$ | 1. | 1. | 0.40 | 7.9 | 3.1 | 3.8 | 5.3 | 9.4 | |
| | $^{100}Mo$ | 1. | 1. | 0.21 | 4.4 | 17. | 5.9 | 3.0 | 7.6 | |
| | $^{116}Cd$ | 1. | 1. | 0.32 | 5.9 | 20. | 14. | 4.8 | 9.7 | |
| | $^{130}Te$ | 1. | 1. | 0.38 | 5.8 | 3.7 | 2.5 | 6.2 | 8.3 | |
| | $^{150}Nd$ | 1. | 1. | 0.093 | 1.7 | 1.1 | 0.77 | 1.2 | 4.3 | |
| KAMLAND | $^{136}Xe$ | 5. | | 0.45 | 4.1 | 2.3 | 1.6 | | 7.2 | |
| CUORE | $^{130}Te$ | 10. | | 0.12 | 1.8 | 1.2 | 0.79 | | 4.7 | |
| GENIUS | $^{76}Ge$ | 580. | | 0.027 | 0.45 | 0.39 | 0.23 | | 2.8 | |

The present limit on the effective Majorana neutrino mass $<m_\nu>$ ≤ 0.5 eV deduced from the $0\nu\beta\beta$-decay experiment have been discussed in connection with the different neutrino

5 Summary

In this contribution we have discussed the problem of lepton number violation in the context of rare nuclear processes, in particular of the $0\nu\beta\beta$-decay, which has a broad potential for providing important information on modern particle physics. We have shown that the $0\nu\beta\beta$-decay has strong impact on physics beyond the Standard model in the way it constrains the parameters of other theories. The mechanism of LN violation within the $0\nu\beta\beta$-decay has been discussed in the context of the problem of neutrino mixing and the R-parity violating SUSY extensions of the Standard model. The relevant LN violating parameters have been the effective Majorana neutrino mass $<m_\nu>$, effective heavy neutrino mass parameter $\eta_N$, effective right-handed weak interaction parameters $<\lambda>$ and $<\eta>$, effective Majoron coupling constant $<g>$ and the first generation trilinear R-parity violating SUSY coupling $\lambda'_{111}$. The restrictions on the lepton number violating parameters have be deduced from the current experimental constraints on $0\nu\beta\beta$-decay half-life time for several isotopes of interest.
mixing scenarios advocated by current data of neutrino experiments. We conclude that the $0\nu\beta\beta$-decay experiment plays an important role in the determination of the character of the neutrino mass spectrum. Some analysis in respect to the heavy Majorana neutrino have been also presented. By using the upper and the lower limit on the mass of the heavy vector boson constrained by the left-right symmetric model with spontaneous R-parity violation we have determined the allowed range for the mixing angle of vector bosons. It has been found that the current upper bound for the R-parity violating SUSY interaction constant $\lambda'_{111}$ is $\leq 1.2 \times 10^{-4}$ ($\leq 3.8 \times 10^{-2}$) assuming the masses of SUSY particle to be on the scale of 100 GeV (1 TeV). A discussion of the dominance of the pion-exchange R-parity violating mode for the $0\nu\beta\beta$-decay process was also presented.

The interpretation of the LN violating parameters involves some nuclear physics. It is necessary to explore the nuclear part of the $0\nu\beta\beta$-decay probability. The predictive power of different nuclear wave functions can be tested in the $2\nu\beta\beta$-decay. One needs a good description of the experimental data for the $2\nu\beta\beta$ probability. We have discussed the recent progress in the field of the calculation of double beta decay matrix elements associated with the inclusion of the Pauli exclusion principle in the QRPA. The reliability of the calculated $0\nu\beta\beta$-decay matrix elements was addressed.

We found it useful to introduce sensitivity parameters $\zeta_X(Y)$ for a given isotope Y associated with different LN violating signals, which are free of influences from particle physics and relate simply the experimental half-lives with LN violating parameters. The largest value of $\zeta_X(Y)$ determines that isotope, which is the most sensitive to a given lepton number violating parameter X. It is an important information for planning new $0\nu\beta\beta$-decay experiments.

The $0\nu\beta\beta$-decay offers with both theoretical and experimental investigations a view to physics beyond the SM. New $0\nu\beta\beta$-decay experiments are in preparation or under consideration (NEMO 3, KAMLAND, CUORE, GENIUS). They could verify the validity of different mixing scheme of neutrinos. The expected limits on the LN violating parameters which could be reached in these experiments are presented in Table 2. However, there is a possibility that the $0\nu\beta\beta$-decay could be detected in the forthcoming experiments. This would establish that the neutrino is a massive Majorana particle. The recent development in neutrino physics has triggered the hope that we could be close to this achievement.

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