Acoustic white holes in flowing atomic Bose–Einstein condensates

Carlos Mayoral$^1$, Alessio Recati$^2$, Alessandro Fabbri$^{1,3}$, Renaud Parentani$^4$, Roberto Balbinot$^5$ and Iacopo Carusotto$^{2,6}$

$^1$ Departamento de Física Teórica and IFIC, Universidad de Valencia-CSIC, C Dr Moliner 50, 46100 Burjassot, Spain
$^2$ INO-CNR BEC Center and Dipartimento di Fisica, Universitàdi Trento, via Sommarive 14, I-38123 Povo, Italy
$^3$ APC (Astroparticules et Cosmologie), 10 rue A Domon et L Duquet, 75205 Paris Cedex 13, France
$^4$ Laboratoire de Physique Théorique, CNRS UMR 8627, Bât. 210, Université Paris-Sud 11, 91405 Orsay Cedex, France
$^5$ Dipartimento di Fisica dell’Università Bologna and INFN sezione di Bologna, Via Irnerio 46, 40126 Bologna, Italy
E-mail: carusott@science.unitn.it

New Journal of Physics 13 (2011) 025007 (29pp)
Received 1 October 2010
Published 4 February 2011
Online at http://www.njp.org/
doi:10.1088/1367-2630/13/2/025007

Abstract. We study acoustic white holes in a steadily flowing atomic Bose–Einstein condensate. A white hole configuration is obtained when the flow velocity goes from a super-sonic value in the upstream region to a sub-sonic one in the downstream region. The scattering of phonon wavepackets on a white hole horizon is numerically studied in terms of the Gross–Pitaevskii equation of mean-field theory: dynamical stability of the acoustic white hole is found, as well as a signature of a nonlinear back-action of the incident phonon wavepacket onto the horizon. The correlation pattern of density fluctuations is numerically studied by means of the truncated-Wigner method, which includes quantum fluctuations. Signatures of the white hole radiation of correlated phonon pairs by the horizon are characterized; analogies and differences with Hawking radiation from acoustic black holes are discussed. In particular, a short wavelength feature is identified in the density correlation function, whose amplitude steadily grows in time since the formation of the horizon. The numerical observations

$^6$ Author to whom any correspondence should be addressed.
are quantitatively interpreted by means of an analytical Bogoliubov theory of quantum fluctuations for a white hole configuration within the step-like horizon approximation.

Contents

1. Introduction 2
2. The physical system and the theoretical model 5
3. Dynamical stability of a white hole configuration: mean-field theory 9
   3.1. Scattering of phonon wavepackets on the horizon 9
   3.2. Back-reaction of the wavepacket on the horizon 12
4. The effect of quantum fluctuations 13
   4.1. Signatures of white hole emission in the correlation pattern of density fluctuations 14
   4.2. Comparison with Hawking radiation from black holes 17
5. Analytical theory 19
   5.1. Dynamical stability of white holes 19
   5.2. The $S$-matrix and the white hole radiation 20
   5.3. Density correlations between the internal and the external regions 23
   5.4. Density correlations in the internal region: the growing checkerboard pattern 25
   5.5. Analytic form of correlations in a black hole configuration 26
6. Conclusions 27
Acknowledgments 28
References 28

1. Introduction

Recent experimental advances in the creation and manipulation of atomic Bose–Einstein condensates (BECs) are suggesting these systems as ideal candidates to experimentally address fundamental questions in quantum hydrodynamics [1]. In particular, much attention has been devoted to configurations showing regions of super-sonic flow, where the dynamics are the richest and quantum effects should be easiest to observe. Among the most remarkable recent observations, we may mention the emission of a wake of Bogoliubov phonons by a moving defect via an analogue of the Čerenkov effect [2], the shedding of solitons in a one-dimensional (1D) configuration [3] and the onset of dynamical instabilities in the periodic potential of optical lattices [4]. In the meanwhile, theoretical investigations have addressed general questions about the stability of superflow [5] and have identified the mechanisms underlying the shedding of solitons [6] and vortices [7]. Analytical aspects of the 1D flow have also been investigated in detail [8]. Most of this physics is accurately described within a mean-field approach, where the dynamics of the Bose-condensed atomic cloud is described in terms of the Gross–Pitaevskii equation (GPE) for the condensate wavefunction [1, 9].

In the wake of the formal analogy [10, 11] between the propagation of quantum fields on a curved space–time background [12] and the propagation of sound on inhomogeneously moving fluids, a great deal of effort is at present being devoted to the quantum features of the condensate hydrodynamics. A very intriguing prediction concerns the emission of correlated pairs of phonons by the horizon of an acoustic black hole configuration via a mechanism that is a condensed-matter analogue of the well-celebrated Hawking radiation from gravitational black
holes [13]. Following the proposal of [14], the key signature of this effect was identified in the correlation function of density fluctuations: the correlation between the Hawking phonon and the partner falling into the black hole results in a peculiar, tongue-shaped feature in the correlation pattern. This prediction was confirmed in an ab initio numerical simulation of the dynamics of an atomic BEC [15]. A first experimental attempt at creating an acoustic black hole in an atomic condensate was reported in [16].

As we schematically show in figure 1, an acoustic white hole configuration is obtained by simply reversing the direction of flow of a black hole one: atoms go from an upstream, inner region where the flow is super-sonic to a downstream, external region where the flow is sub-sonic: within a naive hydrodynamic picture, dragging by the moving condensate forbids low-energy phonons from crossing the horizon and penetrating the inner region of the white hole. Questions about the stability of such acoustic white hole configurations has lately attracted much interest, and different answers have been proposed to this problem, ranging from instability [17], to stability [18], to intermediate behaviours depending on boundary conditions [20]. Issues related to the quantum emission of phonons by configurations involving white hole horizons have been addressed in [18, 19, 21, 22].

In this paper, we report a comprehensive theoretical investigation of the physical properties of acoustic white holes in atomic Bose condensates. Focusing our attention on a specific yet realistic configuration that is most suitable for a numerical and analytical study, we address the problem of the dynamical stability of white hole configurations and then the physical properties of the quantum radiation that is emitted by the horizon. To be able to safely isolate the dynamics of the white hole horizon from spurious effects due to the finite size and/or the multiply connected geometry of the system [23], all numerical calculations have been performed using absorbing boundary conditions for the Bogoliubov phonons so as to model the most relevant situation of an infinitely long condensate.

In section 2, we introduce the physical system and discuss the main features of the dispersion of Bogoliubov excitations in, respectively, the super- and sub-sonic regions: this will be the essential ingredient to physically understand the numerical observations presented in the following sections of the paper. Among the different possible choices of the density and flow velocity patterns that result in a white hole horizon, we focus our attention on a configuration that appears most robust against spurious effects.

The dynamical stability of the white hole configuration is assessed in section 3, where the scattering of a phonon wavepacket on the white hole horizon is studied by means of the GPE
of mean-field theory. In addition to the reflected wavepacket in the outward direction, a pair of short-wavelength wavepackets penetrate into the white hole: as a consequence of the strong blue shift experienced while approaching the horizon, low-energy hydrodynamic modes are in fact transformed into high-momentum single-particle excitations that are able to enter the white hole. One of the two wavepackets consists of negative-norm Bogoliubov modes and results from the same mixing of positive and negative norm modes that underlies Hawking radiation. Dynamical stability of the configuration is demonstrated by the fact that no exponentially growing deformation of the horizon develops under the effect of the incident wavepacket.\(^7\) The only visible back-action effect consists of the Bogoliubov–Čerenkov emission of zero-frequency phonons \(^2\) in the upstream direction by the slight deformation of the horizon; a specific discussion of this feature is given in section 3.2. As a function of time, the amplitude of this pattern eventually tends to a finite amplitude value, roughly proportional to the square of the incident wavepacket amplitude.

The effect of quantum fluctuations is studied in section 4 using the same truncated Wigner method as in [15]. In analogy to what was done for acoustic black holes, our key observable is the correlation function of density fluctuations. Several characteristic features are identified and related to the correlated emission of phonon pairs by the white hole horizon into different pairs of modes. The physical mechanism that is responsible for such a white hole radiation is the same as that for Hawking radiation from acoustic black holes, namely the parametric conversion of incident quantum fluctuations into observable radiation. However, the different geometrical configuration introduces substantial differences because of the exchanged role of in-going and out-going modes and the opposite red- versus blue-shift of modes when approaching the horizon. As a result, the physical properties of the white hole emission turn out to significantly differ from the ones of Hawking radiation from black holes. For this reason, we have chosen to refer to it under the name of \textit{white hole radiation} rather than Hawking radiation. More specifically, whereas Hawking radiation from black holes mostly consists of low-\(k\) excitations, the phonons that are emitted by the white hole have a large momentum comparable to the healing length of the condensate; furthermore, the checkerboard pattern that appears in the correlation function for points located inside the white hole keeps growing in time with a characteristic logarithmic or linear law depending on the initial temperature. This is to be contrasted with the late-time stationary state of Hawking radiation from black holes.

An analytical understanding of the white hole radiation is developed in section 5 based on the Bogoliubov theory of dilute BECs within the step-like horizon approximation of [25]. Dynamical stability of the white hole configuration is assessed by checking that complex frequency eigenmodes of the Bogoliubov–de Gennes equations with positive imaginary part are indeed absent. A theory of quantum fluctuations is developed in terms of the \(S\)-matrix describing phonon scattering on the white hole horizon. This provides an analytical expression for the spectral distribution of the white hole emission: in contrast to the thermal character of Hawking radiation from black holes, the phonons that are emitted outside the white hole have a flat spectral distribution. The same approach is then used to evaluate the different contributions to the correlation pattern: the resulting analytical expressions reproduce in a fairly accurate way all of the features that are observed in the numerical calculations. The steady growth in time of

\(^7\) In more complex configurations involving several horizons, the incident wavepacket is able to trigger a dynamical instability leading to an exponentially growing perturbation of the density profile. This phenomenon goes under the name of \textit{black hole laser} [21, 24], and was originally predicted for a pair of adjacent black and white hole horizons. A detailed investigation of this interesting physics will be the subject of a forthcoming publication.
the checkerboard pattern is related to an infrared divergence of the density correlation function at late times: the logarithmic (linear) growth for a zero (finite) initial temperature is recovered by imposing a suitable infrared cut-off to the integral. Conclusions are finally drawn in section 6.

2. The physical system and the theoretical model

A sketch of the physical system we are considering is shown in the left panel of figure 1. An elongated atomic BEC is steadily flowing along an atomic waveguide in the \( \hat{x} \)-direction. A suitable modulation of the confinement potential and/or of the atom–atom scattering length creates a white hole horizon at \( x = 0 \) separating an upstream \( x < 0 \) region of super-sonic flow from a downstream \( x > 0 \) one of sub-sonic flow. In the gravitational analogy [10, 26, 27], the upstream region corresponds to the interior of the white hole, while the downstream region corresponds to the external space. For comparison, a sketch of the acoustic black hole configuration obtained reversing the flow speed is shown in the right panel [8].

To simplify the theoretical description, the transverse confinement is assumed to be tight enough for the transverse degrees of freedom to be frozen [1] and the system dynamics to be accurately described by a 1D model based on the following second-quantized Hamiltonian,

\[
\mathcal{H} = \int dx \left[ \frac{\hbar^2}{2m} \nabla \hat{\Psi}^\dagger(x) \nabla \hat{\Psi}(x) + V(x, t) \hat{\Psi}^\dagger(x) \hat{\Psi}(x) + \frac{g(x, t)}{2} \hat{\Psi}^\dagger(x) \hat{\Psi}^\dagger(x) \hat{\Psi}(x) \hat{\Psi}(x) \right].
\]

(1)

Here, \( \hat{\Psi}(x) \) and \( \hat{\Psi}^\dagger(x) \) are atomic field operators satisfying Bose commutation rules \( [\hat{\Psi}(x), \hat{\Psi}^\dagger(x')] = \delta(x - x') \), \( m \) is the atomic mass, \( V(x) \) the external potential and \( g(x) \) is the effective 1D atom–atom interaction constant [1]. At the mean-field level, the condensate dynamics is described by the 1D GPE [1]

\[
i \hbar \frac{\partial \psi}{\partial t} = -\frac{\hbar^2}{2m} \frac{\partial^2 \psi}{\partial x^2} + V(x, t) \psi + g(x, t) |\psi|^2 \psi
\]

(2)

for the macroscopic condensate wavefunction \( \psi \). In section 3, this equation will be used to study the propagation of classical perturbations on top of the white hole configurations: full inclusion of the nonlinearity will allow us to study the dynamical stability of the configuration beyond the linearized Bogoliubov theory. In section 4, the so-called truncated Wigner method will be adopted to go beyond the mean-field approximation underlying (2) and include the effect of quantum and thermal fluctuations: these are described in terms of stochastic noise on the initial value of the condensate wavefunction, which then evolves according to the same GPE (2). Expectation values of observables are obtained as averages over the stochastic noise. For a detailed description of the numerical technique, see [15].

For the sake of simplicity, we shall focus our attention on a specific configuration that is most suitable for both analytic and numeric investigations. Initially, the condensate is assumed to have a spatially uniform density \( n_0 \) and a spatially uniform flow speed \( v_0 \) along the positive \( x \)-direction. The external potential and the (repulsive) atom–atom interaction constant are also uniform and equal to respectively \( V(x) = V_0 \) and \( g(x) = g_d > 0 \). Around \( t = t_0 \), a step-like spatial modulation is applied to both the potential and the interaction constant by suitably modifying the transverse confinement potential and/or the atom–atom scattering length via an

Note that this configuration differs from the one considered in [15, 25] by a simple spatial inversion.
external magnetic field tuned in the vicinity of a so-called Feshbach resonance [1]: within a short time $\sigma$, $V$ and $g$ in the upstream $x < 0$ region are brought to their final values $V_u$ and $g_u$, while their values in the $x > 0$ region are kept equal to the initial ones $V_d$ and $g_d$. The transition region around $x = 0$ over which $V$ and $g$ are spatially varying has a characteristic thickness $\sigma_x$.

Away from the interface, both $V$ and $g$ quickly tend to their asymptotic values $V_u$ and $g_u$, $d$.

To avoid the development of spurious instabilities due to the reflection of Bogoliubov excitations at the edges of the integration box, suitable absorbing boundary conditions are implemented for the Bogoliubov modes. In practice, this is done by including a restoring force on the wavefunction that damps out Bogoliubov excitations and brings it back to a pure condensate. For this additional term not to interfere with the physics under investigation, it is spatially restricted to small regions at the edges of the integration box, which are sufficiently far from the white hole horizon.

To reduce the impact of competing processes, such as back-scattering of condensate atoms and soliton shedding from the potential step [6], the external potential $V$ is chosen to exactly compensate for the spatial jump in the Hartree interaction energy $\mu_{u,d} = g_{u,d} n_0$, i.e.

$$V_d + \mu_d = V_u + \mu_u.$$  \hfill (3)

In this way, the plane wave

$$\psi(x, t) = \sqrt{n_0} \exp[i(k_0 x - \omega_0 t)]$$  \hfill (4)

with $\hbar k_0/m = v_0$ and $\omega_0 = \hbar k_0^2/2m$ is for all times a solution of the GPE (2).

The white hole configuration that is the subject of the present paper is characterized by the chain inequality $c_u < v_0 < c_d$ relating the speed of sound in, respectively, the upstream and downstream asymptotic regions $c_{u,d} = \sqrt{\mu_{u,d}/m}$ and the (spatially uniform) flow speed $v_0$. The dispersion of the Bogoliubov modes describing, in the laboratory frame, the propagation of weak disturbances on top of the flowing condensate has the usual form

$$\omega(k) = \Omega_{u,d}(k) + v_0 k = \pm \sqrt{\frac{k^2}{2m} \left( \frac{\hbar^2 k^2}{2m} + 2mc_{u,d}^2 \right) + v_0 k}$$  \hfill (5)

in, respectively, the upstream and downstream regions and is illustrated in panels (a) and (b) of figure 2. For future use, we have defined $\Omega_{u,d}(k)$ as the frequencies measured in the upstream and downstream comoving frames at rest with the condensate.

The main feature of the white hole configuration consists of the fact that long-wavelength phonons are forbidden from penetrating into the inner region of the white hole. As a consequence of dragging by the super-sonic flow, the group velocity turns out to be positive $v_0 \pm c_u > 0$ in the laboratory frame for both $k > 0$ and $k < 0$ phonons. This behaviour has to be compared to the black hole case shown in the lower panels of figure 2: in this case, the direction of the condensate flow is reversed and long-wavelength phonons cannot escape from the inner, super-sonic region. It is, however, crucial to note that the above black and white hole behaviours are restricted to low-$k$ Bogoliubov excitations of hydrodynamic nature. Indeed, large-$k$ Bogoliubov excitations have a single particle character and propagate at a faster group velocity. As a result, they are able to propagate in both directions, that is to penetrate into a white hole and escape from a black hole. The transition from the hydrodynamic to the single-particle regime occurs at a characteristic wavevector given by the inverse of the healing length, $\xi_{u,d} = \hbar/mc_{u,d}$.
In terms of the gravitational analogy, the point separating the upstream region of super-sonic $c_u < v_0$ flow from the downstream one of sub-sonic $v_0 < c_d$ flow then acts as the horizon. The $k < 0$ hydrodynamic phonons that propagate in the sub-sonic region in the leftward direction towards the horizon get slowed down as they approach the horizon and can never penetrate into the super-sonic region. As a result, they pile up in the vicinity of the horizon. Analogously, the group velocity of $k < 0$ hydrodynamic phonons in the super-sonic region tends to zero as the horizon is approached. As a result, they also get piled up on the horizon. The mechanism of this pile-up effect is graphically illustrated in the upper left panel of figure 3.

As propagation on a stationary background conserves the phonon frequency $\omega$ in the laboratory frame, the pile-up effect is in both cases accompanied by an increase of the wavevector $k$ and a corresponding blue shift of the phonon frequency $\Omega(k)$ in the comoving frame. Whereas this blue-shift effect continues for ever in the case of dispersionless (relativistic) fields giving rise to excitations at arbitrarily high $k$ values, in a BEC it is restricted to wavevectors smaller than the inverse healing length by the peculiar super-luminal shape of the Bogoliubov dispersion shown in figure 2.

Note that this convention differs from the one that is usually adopted in relativity, where the modes are labelled according to their left- or right-moving character with respect to the light/sound cone, i.e. their propagation direction in the frame comoving with the condensate [18, 19].

Figure 2. Dispersion of Bogoliubov excitations in the asymptotic regions far away from the horizon. The labels of the modes indicate the up- or down-stream region with respect to the condensate flow and the in- or out-going direction of their propagation direction (as predicted by their group velocity) with respect to the horizon\textsuperscript{9}. Both the frequency $\omega(k)$ and the group velocity of the modes are measured in the laboratory frame. (a, b) White hole configuration with a rightward flow $v_0 > 0$, as sketched in figure 1(a). White hole parameters: $v_0/c_u = 1.5$, $v_0/c_d = 0.75$. (c, d) Black hole configuration with a leftward flow $v_0 < 0$, as sketched in figure 1(b). Black hole parameters: $|v_0|/c_d = 1.5$, $|v_0|/c_u = 0.75$. 

\textsuperscript{9} Note that this convention differs from the one that is usually adopted in relativity, where the modes are labelled according to their left- or right-moving character with respect to the light/sound cone, i.e. their propagation direction in the frame comoving with the condensate [18, 19].

New Journal of Physics 13 (2011) 025007 (http://www.njp.org/)
As the white hole configuration is related to the black hole one by a simple time reversal, the corresponding Bogoliubov dispersions shown in panels (a, b) and (c, d) of figure 2 are connected by a $k \rightarrow -k$ symmetry. From the figure, it is clear to see that under this operation the sign of the group velocity is reversed and the in- and out-going character of the modes are exchanged. Another, related difference involves the zero mode defined by the $\omega(k_Z) = 0$ condition. Such a mode only exists in the super-sonic region and propagates opposite to the condensate flow: whereas in a black hole it propagates towards the horizon, in a white hole it propagates away from the horizon into the inner region. As a result, a perturbation of the white hole horizon from the Hartree condition (3) may result in the Landau–Čerenkov [2] emission of phonons into this zero-energy mode of the wavevector

$$k_Z = -\frac{2m}{\hbar} \sqrt{v_0^2 - c^2_u}$$  \hspace{1cm} (6)

comparable to the inverse healing length $\xi_{u,\omega}^{-1}$ and group velocity

$$v_Z = c^2_u - v_0^2 \quad \frac{v_0}{v_Z}.$$  \hspace{1cm} (7)

In the following sections of the paper, we shall see how this fact plays a crucial role in the physics of the white hole configuration.
3. Dynamical stability of a white hole configuration: mean-field theory

3.1. Scattering of phonon wavepackets on the horizon

In the previous section, we have seen that a condensate wavefunction in the plane wave form (4) is at all times a solution of the GPE (2). Still, one has to assess the dynamical stability of this solution. A simple way to answer this question consists of solving the Bogoliubov–de Gennes equations for the spectrum of linear perturbations on top of the flowing condensate and looking for complex frequency modes. Pioneering calculations in this direction were performed in [20] for different black and white hole configurations and for different boundary conditions. A detailed investigation of black hole laser instabilities in configurations showing a pair of adjacent black and white hole horizons was reported in [21]–[23]. Application of the linearization technique to the white hole case is briefly discussed in section 5.1.

In the present section, we shall follow a different method, which consists in studying the wavefunction evolution under the full GPE (2) for an initial condition slightly perturbed from the plane wave solution (4). This approach takes into account the nonlinear couplings that are neglected in the Bogoliubov theory and that are responsible for interesting back-reaction effects. In all calculations, the white hole is assumed to be already formed well before the experiment is performed so that the condensate background can be safely considered as a stationary one.

In particular, we consider the simple case where a phonon wavepacket modulation has been imprinted on top of the homogeneous solution (4). The wavepacket has a carrier at $k_{wp}$ and a Gaussian envelope of width $\ell_{wp}$ much larger than the wavelength, $k_{wp}\ell_{wp} \gg 1$. Initially, the wavepacket is centred at a position $x_{wp}^0$ far from the horizon, $|x_{wp}^0| \gg \ell_{wp}$. In wavevector space, the phonon wavepacket has a narrow Gaussian shape around $k_{wp}$ of width $\Delta k_{wp} = 1/\ell_{wp} \ll k_{wp}$. The amplitude of the wavepacket is chosen to be weak enough to be in the linear regime; this condition has been checked by verifying that the amplitude of the reflected and transmitted wavepackets scales linearly with the amplitude of the incident wavepacket. A remarkable feature stemming from nonlinear effects beyond this approximation is discussed in section 3.2.

Snapshots of the wavepacket evolution at subsequent times are given in figure 4 for the most illustrative case in which the wavepacket is initially at $x_{wp}^0 > 0$ outside the white hole and propagates towards the horizon on the $d^{in}$ mode with a wavevector $k_{wp} < 0$ well within the hydrodynamic region, $|k_{wp}| \ll \xi_d^{-1}$. Its frequency distribution is concentrated around the carrier frequency $\omega_{wp} \simeq (v_0 - c_d)|k_{wp}|$. As the wavevector distribution is concentrated in the hydrodynamic region $k\xi_d \ll 1$, where the dispersion is almost linear, the wavepacket gets weakly distorted while propagating towards the horizon. Dynamical stability of the white hole configuration is confirmed by the fact that no exponentially growing perturbation of the horizon is triggered by the arrival of the incident wavepacket.

Within a strict hydrodynamic picture where the high-momentum $u_{1,2}^{out}$ modes were completely neglected, one would expect that no excitation can propagate across the horizon into the white hole as no leftward propagating modes are available inside the white hole at $\omega_{wp}$. As a consequence, the incident wavepacket either accumulates at the horizon or is reflected. This picture is oversimplified as the combination of the blue shift at the white hole horizon and the super-luminal character of the Bogoliubov dispersion may lead to a significant transmission into $u_{1,2}^{out}$ modes with large wavevectors in the vicinity of $\pm k_Z$. In [18], it was predicted that the amplitude of these transmitted wavepackets is much larger than the one of the reflected even for very smooth white hole horizons.
Figure 4. Scattering of a phonon wavepacket on a white hole horizon. The different panels show snapshots of the density modulation $\delta n(x) = n(x) - n_0$ at subsequent times. The insets provide a closer look at some very interesting features. White hole parameters are as in figures 2(a) and (b) with $\sigma_x/\xi_d = 0.5$. Wavepacket parameters are $k_{wp}\xi_d = 0.2$, $\sigma_{wp}/\xi_d = 80$. The qualitative behaviour remains unchanged when smoother horizons are considered with $\sigma_x/\xi_d \gg 1$.

These expectations are confirmed in our numerical calculations shown in figure 4. The reflected wavepacket is actually small, but clearly visible in the insets of the two latest time panels. As expected, it propagates away from the horizon at the speed $c_d + v_0$. In agreement with the condition that the carrier frequency $\omega_{wp}$ has to be conserved after scattering, the carrier wavevector of the reflected wavepacket is smaller than the incident one by a factor $(v_0 - c_d)/(v_0 + c_d)$.

At relatively short times after impinging on the horizon (see e.g. the $\mu_d t = 2000$ panel of figure 4), there is a single transmitted wavepacket that propagates into the white hole with a group velocity close to $v_Z$. As one can see in the left inset, its shape is characterized by a carrier at a wavevector $k_Z$ and a slower modulation. The latter can be interpreted as the result of interference between the $u_{1\text{out}}$ and $u_{2\text{out}}$ components of the wavevector approximately $k_{u_{1\text{out}}} \simeq k_Z + \omega_{wp}/v_Z$ and $k_{u_{2\text{out}}} \simeq -k_Z + \omega_{wp}/v_Z$ (see figure 2(a)). From the Bogoliubov expression for the density perturbation \cite{9}, one immediately recognizes oscillations at $\pm k_{u_{1\text{out}}}$ and $\pm k_{u_{2\text{out}}}$. The slow modulation corresponds to the beating of the $\pm k_{u_{1\text{out}}}$ and $\mp k_{u_{2\text{out}}}$ components and has a wavevector $q_{\text{beat}} \approx 2\omega_{wp}/|v_Z| \approx 2(c_d - v_0)|k_{wp}|/|v_Z|$. The quantitative agreement of this Bogoliubov prediction for $q_{\text{beat}}$ with the period of the slow modulation extracted from the $\mu_d t = 2000$ panel of figure 4 confirms our interpretation.
Figure 5. Scattering of a phonon wavepacket on a black hole horizon. The different panels show snapshots of the density modulation \( \delta n(x) = n(x) - n_0 \) at subsequent times. Black hole parameters are as in figures 2(c) and (d) with \( \sigma_x/\xi_u = 0.5 \). Wavepacket parameters are \( k_{wp}\xi_u = 0.05, \sigma_{wp}/\xi_u = 80 \).

At later times (e.g. \( \mu_d t = 3500 \)), the curvature of the Bogoliubov dispersion makes the \( u_{out}^1 \) and \( u_{out}^2 \) wavepackets eventually separate in space. As a result, the slow modulation due to their interference disappears and one is left with two Gaussian wavepackets. The presence of the transmitted wavepacket on the negative norm \( u_{out}^2 \) out-going mode can be interpreted as the result of a stimulated Hawking emission [25]. This classical counterpart of Hawking radiation was recently investigated in experiments using gravity waves in water tanks [28, 29].

Had we considered an incident wavepacket with a smaller wave vector \( k_{wp} \), the time needed for the two transmitted wavepackets to separate would be longer as the group velocities associated with the two roots \( u_{out}^1 \) and \( u_{out}^2 \) would be closer. In the \( |k_{wp}| \rightarrow 0 \) limit, the two wave vectors \( k_{out}^1 \) and \( k_{out}^2 \) tend to \( \pm k_Z \) and the wavepackets never separate.

For the sake of completeness, it is interesting to compare these results with the case of a black hole configuration. As before, we consider a Gaussian wavepacket centred at \( x_{wp}^0 > 0 \) outside the black hole, which propagates towards the horizon on the \( u_{in}^m \) mode with a wavevector \( k_{wp} < 0 \) well within the hydrodynamic region, \( |k_{wp}| \ll \xi_u^{-1} \). As one can see in the snapshots shown in figure 5, the \( u_{in}^m \) incident wavepacket scatters on the horizon and splits into a reflected wavepacket on the \( u_{out}^1 \) mode and a pair of transmitted ones on the modes labelled \( d_{out}^1 \) and \( d_{out}^2 \) in figures 2(c) and (d): while the stronger and faster wavepacket corresponds to the transmitted positive-norm \( d_{out}^1 \) mode, the weaker one corresponds to the negative-norm \( d_{out}^2 \) mode and results from the same mixing of positive and negative modes that underlies Hawking radiation [25]. In contrast to the white hole case, all out-going wavepackets have significantly different group
velocities and separate in space almost immediately; furthermore, the carrier wavevectors of all wavepackets tend to zero in the $|k_{wp}| \to 0$ limit. The reason for these remarkable differences is easily understood in terms of the dispersion curves shown in figure 2.

3.2. Back-reaction of the wavepacket on the horizon

The previous discussion of phonon scattering on the white hole horizon has highlighted quite a similar physics compared to the black hole case: an incident wavepacket splits into several reflected and transmitted wavepackets whose carrier wavevector is fixed by energy conservation arguments. As we are considering weak incident wavepackets, the amplitude of the reflected and transmitted wavepackets scales linearly with the incident one. However, in addition to these expected wavepackets, a striking unexpected feature is visible in the latest time snapshot of figure 4 right inside the white hole horizon: a stationary density modulation with a spatially flat envelope and a fast oscillating carrier. At late times, the amplitude of this density modulation saturates to a finite value. A similar effect was very recently reported for a classical white hole analogue using gravity waves on the surface of a water tank [28].

Its physical origin can be understood in terms of the zero mode in the Bogoliubov dispersion: the vanishing $\omega$-frequency of this mode explains the stationarity of the pattern and the observed wavelength of the modulation coincides with the value $k_z$ of the zero-mode wavevector in the upstream region. Still, the microscopic rectification process that is responsible for the excitation of a zero-frequency mode from a finite frequency wavepacket requires further investigation as it is not allowed within a linearized Bogoliubov theory on a stationary background.

To assess its origin in terms of some nonlinear effect, we have repeated the same numerical simulation of the GPE with different amplitudes of the incident wavepacket. The results are summarized in figure 6: the late-time amplitude of the zero-mode modulation scales indeed proportionally to the square of the incident wavepacket amplitude. A very simple physical interpretation can be put forward as follows: the incident wavepacket slightly distorts the wavefunction in the horizon region breaking the Hartree condition (3). The resulting perturbation of the Hartree potential is then responsible for the continuous emission of phonons via the Landau–Cerenkov emission discussed in [2, 6] for a uniform super-sonic flow hitting a defect.

This interpretation is confirmed by the numerical observation that no such feature was ever observed for black hole configurations: in this latter case, the super-sonic region is in fact located downstream of the defect so that the zero mode propagates towards the horizon. This fact is clearly visible in figure 2(c). As a result, the zero-mode cannot be excited by a perturbation localized in the horizon region of a black hole.

To summarize, our numerical simulations of the GPE show that the white hole configuration is dynamically stable at mean-field level even beyond the linearized regime considered so far in the literature. Even when it is excited by a sizeable incident wavepacket, the condensate does not develop any exponentially growing perturbation: the amplitude of the scattered wavepackets is roughly proportional to the incident amplitude and the amplitude of the zero-mode perturbation tends at late times to a stationary value proportional to the square of the incident wavepacket amplitude.
4. The effect of quantum fluctuations

After having assessed in the previous section the dynamical stability of a white hole configuration at the level of mean-field theory, we now proceed to investigate the consequences of quantum fluctuations, and in particular the radiation that is continuously emitted by the white hole horizon once this has been formed. Our investigation will make use of the same truncated-Wigner numerical technique used in [15]. For a more detailed discussion of technical details, see [15].

At the initial time $t = 0$, the condensate is assumed to be spatially homogeneous at density $n_0$ and to be flowing at a constant speed $v_0$; both the interaction constant and the external potential are initially flat and equal to $g_d$ and $V_d$, respectively. The white hole horizon is formed at time $t_0$ by ramping $g$ and $V$ in the upstream $x < 0$ region to their final values $g_u$ and $V_u$. This ramp takes place within a time $\sigma_t$. Eventually, $g$ and $V$ tend to an arctan-shaped spatial profile of characteristic thickness $\sigma_x$.

Quantum and thermal fluctuations on top of the initial homogeneous condensate can be treated by means of the standard Bogoliubov theory of dilute Bose gas. Within the Wigner method, this translates [30] into a stochastic initial condition with a suitable Gaussian noise added on top of the plane-wave initial wavefunction (4) of mean-field theory. The variance of the Gaussian noise is determined by the initial temperature $T_0$. Except for figure 9(b), all numerical data shown in the figures refer to the case of a vanishing initial temperature $T_0 = 0$. The time evolution of the stochastic wavefunction starting from its initial state follows the deterministic GPE evolution (2) including the chosen form of the nonlinear interaction constant $g(x, t)$ and of the external potential $V(x, t)$. Expectation values of symmetrically ordered observables at any later time are obtained by taking the corresponding stochastic average over the ensemble of evolved wavefunctions.

New Journal of Physics 13 (2011) 025007 (http://www.njp.org/)
4.1. Signatures of white hole emission in the correlation pattern of density fluctuations

In [15], it was shown that a good deal on information of the physical properties of Hawking radiation can be extracted from the correlation pattern of density fluctuations at equal times, defined as

\[ G^{(2)}(x, x') = \frac{\langle \hat{\Psi}^+(x) \hat{\Psi}^+(x') \hat{\Psi}(x') \hat{\Psi}(x) \rangle}{\langle \hat{\Psi}^+(x) \hat{\Psi}(x) \rangle \langle \hat{\Psi}^+(x') \hat{\Psi}(x') \rangle} \]

\[ = \frac{\langle n(x)n(x') \rangle}{\langle n(x) \rangle \langle n(x') \rangle} - \frac{1}{\langle n(x) \rangle} \delta(x - x'). \]  

(8)

In this section, we follow the same line for the case of a white hole, trying to isolate the peculiar features that characterize the white hole counterpart of the Hawking radiation from black holes. Looking at the Bogoliubov dispersions shown in figures 2(a) and (b), we expect from energy conservation arguments that correlated pairs of quanta can be emitted from the horizon either into \( u_1^{\text{out}} \) and \( u_2^{\text{out}} \) modes or into the \( d^{\text{out}} \) and \( u_2^{\text{out}} \) modes: the negative energy of the \( u_2^{\text{out}} \) partner can in fact compensate for the positive energy of the \( u_1^{\text{out}} \) or \( d^{\text{out}} \) ones, so as to give a zero total energy of the pair. As it happened in black holes, these processes correspond to clearly distinct features in the correlation pattern. An analytical understanding of this white hole radiation will be provided in section 5. Colour plots of a suitably normalized density correlation \( G^{(2)}(x, x') \) are shown in figure 7. For numerical convenience, all density correlation plots have been smoothed out with a Gaussian spatial filter of size \( \ell_a/\xi_d = 1 \). The main features that are visible in the plots can be classified as follows:

(i) The strong and negative correlation strip on the \( x = x' \) main diagonal results from the usual many-body antibunching due to repulsive interactions and has no relation with the presence of a horizon.

(ii) A system of stripes parallel to the main diagonal appears in the \( x < 0 \) region inside the white hole as soon as the horizon is formed. As time goes on, the fringes move away from the main diagonal at an approximately constant speed and eventually disappear from the region of sight. We have checked that their presence and shape do not depend on the presence of the horizon, but rather on the speed of the time modulation: the slower the ramp (i.e. the longer \( \sigma_t \)), the weaker the fringe amplitude. On the basis of [31], these fringes can be interpreted as a consequence of the dynamical Casimir emission of pairs of counterpropagating phonons at all points where the interaction constant is modulated in time, \( g = g_d \rightarrow g_u \). It is interesting to note that in the white hole case the dynamical Casimir phonons are able to travel back to the horizon. As a direct consequence of this fact, feature (ii) extends also into the \( x > 0, x' < 0 \) and \( x < 0, x' > 0 \) quadrants. As the scattering of dynamical Casimir phonons incident off the horizon may interfere with the white hole emission, it is then important to ensure that the intensity of the dynamical Casimir emission is efficiently suppressed by a suitably long formation time \( \sigma_t \).

(iii) Spatially oscillating correlations appear between points right outside and right inside the horizon. The correlation pattern has a sinusoidal dependence as a function of the position of the point located inside the horizon, while the dependence on the outside point is much smoother. The wavevector of the fringes is approximately equal to the zero-mode wavevector \( k_Z \) introduced in the previous section. As time goes on, these fringes extend in a larger area further away from the horizon, but their amplitude at each given point
Figure 7. Colour plots of the rescaled density correlation function \((n_0\xi_d) \times [G^{(2)}(x, x') - 1]\) at two successive times \(\mu_d t = 90\) (a) and 160 (b) after the switch-on of the white hole horizon. Labels (i), (ii), (iii) and (iv) identify the main features discussed in the text. The solid magenta and red lines indicate the directions along which the cuts shown in figures 8(a) and (b) are taken. The white rectangle indicates the region where the checkerboard amplitude shown in figure 9 is measured. The blue dashed line in panel (b) indicates the analytical position of the axis of feature (iii). The horizon is formed within a time \(\mu_d \sigma_t = 10\) around \(\mu_d t_0 = 50\). The thickness of the horizon region over which \(V\) and \(g\) are spatially varying is \(\sigma_x / \xi_d = 0.5\). White holes are parameters: \(v_0/c_u = 1.5\), \(v_0/c_d = 0.75\). The initial temperature is \(T_0 = 0\).
eventually tends to a finite value, constant in time. The last fact is better visible in the cuts taken along the magenta line that are shown in figure 8(a). Remarkably, the envelope of the fringe system shows a slow modulation, with a minimum approximately located along the blue dashed line of figure 7(b), \( x/v_Z = x'/(v_0 + c_d) \), which suggests that this feature is related to the emission of pairs of quanta by the horizon into the \( u_{1,2} \) and \( d \) modes. An analytical discussion of this feature will be provided in section 5.3.

(iv) A checkerboard pattern appears in the correlations between pairs of points located inside the white hole. As time goes on, the checkerboard pattern extends further away from the horizon. The spatial wavevector of the checkerboard is approximately equal to \( k_Z \) in both the \( x \) and \( x' \) directions, which suggests that it is due to the emission of pairs of quanta by the horizon into the \( u_{1,2} \) modes. In contrast to feature (iii) and to what was always observed for black holes in [15], its amplitude at any given point does not tend to a stationary value, but keeps on steadily increasing. This remarkable fact is visible in the cuts of the correlation pattern shown in figure 8(b) and, even more clearly, in the plot of the checkerboard amplitude as a function of time shown in figures 9(a) and (b). For a vanishing initial temperature \( T_0 = 0 \), the growth appears to follow a logarithmic law. For a finite initial temperature \( T_0 > 0 \), the growth appears instead to be linear with a slope proportional to \( T_0 \). An analytical interpretation of all these features will be discussed in section 5.4.
4.2. Comparison with Hawking radiation from black holes

For the sake of completeness, it can be useful to briefly review the main properties of the Hawking emission from a black hole \[15, 25\]. This should facilitate the reader in appreciating the analogies and differences with the white hole radiation. As before, this study is based on the correlation pattern of density fluctuations, some examples of which are shown in figure 10. Several features can be identified:

(i) As expected, the strong and negative correlation strip on the \(x = x'\) main diagonal is almost identical in the white hole and black hole configurations.

(ii) The moving pattern of fringes due to the dynamical Casimir emission of phonons by the time-modulated interaction constant is also very similar in the black and white hole configurations. The main difference is purely geometrical: as a result of the different relative positions of the super- and sub-sonic regions, the low-\(k\) dynamical Casimir phonons are not able to travel back to the horizon and are fully dragged into the black hole.

(iii) A symmetric pair of negative correlation tongues extends from the horizon point for points \(x\) and \(x'\) located on opposite sides from the horizon. Their axis is located along the \(x/(v_0 + c_d) = x'/(v_0 + c_u)\) straight line. While their length linearly grows with time since the horizon formation time, their maximum height remains almost constant in time. Their

---

**Figure 9.** Time evolution of the peak-to-peak amplitude of the checkerboard pattern measured within the region indicated by the white rectangle in figure 7. The upper panel (a) shows a semilog plot for a zero initial temperature \(T_0 = 0\). The lower panel (b) shows a linear plot for different temperatures \(k_B T_0/\mu_d = 0\) (black), 0.1 (red) and 0.2 (blue). The cyan straight lines are guides to the eye, which emphasize that the growth is approximately logarithmic (linear) in the zero (finite) temperature cases.
Figure 10. Colour plots of the rescaled density correlation function $(n_0 \xi_d) \times [G^{(2)}(x, x') - 1]$ at two successive times $\mu_u t = 40$ (a) and 160 (b) after the switch-on of a black hole horizon. Labels (i), (ii), (iii) and (iv) identify the main features discussed in the text. The horizon is formed within a time $\mu_u \sigma_t = 0.5$ around $\mu_u t_0 = 2.5$. The thickness of the horizon region over which $V$ and $g$ are spatially varying is $\sigma_x/\xi_u = 0.5$. The calculation has been performed using the truncated-Wigner method. The same system parameters as in figures 2(c) and (d). The initial temperature is $T_0 = 0$.

Physical origin lies in the stationary emission of correlated pairs of quanta by the horizon into the modes labelled as $u_{\text{out}}$ and $d_{\text{out}}^{\text{out}}$ in figures 2(c) and (d): while the $d_{\text{out}}^2$ partner falls into the black hole, the $u$ partner flies away giving rise to the standard Hawking emission.

(iv) Another pair of positive correlation tongues extends from the horizon point for both $x, x' < 0$ inside the horizon. Their axis is located along the $x/(v_0 + c_d) = x'/(v_0 - c_d)$ line, which suggests an interpretation in terms of the stationary emission of correlated pairs of $d_{\text{out}}^1$ and $d_{\text{out}}^2$ quanta by the horizon.

(v) Another pair of weaker tongues appears in the $x < 0, x' > 0$ region along the $x/(v_0 - c_d) = x'/(v_0 + c_u)$ line, which suggests an interpretation in terms of correlations between the $d_{\text{out}}^1$ and $u_{\text{out}}$ modes.

A general fact of features (iii)–(v) is that none of them shows any significant modulation: this is due to the fact that the wavevector of all out-going phonon modes tends to zero in a black hole configuration for low frequencies. This is to be contrasted with the white hole case where the wavevector of $u_{\text{out}}^{1,2}$ modes tends to $\pm k_Z$, which is of the order of the inverse healing length $1/\xi_u$ in the super-sonic region.

To summarize, in this section we have described the properties of the quantum emission of phonons from the white hole horizon. As in the black hole case, the peculiar quantum correlations between the constituents of each pair result in long distance correlations of the density fluctuations. Several novel features have been highlighted: as the quantum emission into the white hole consists of short-wavelength excitations comparable to the healing length, the correlation pattern shows a fast sinusoidal modulation. Furthermore, the amplitude of the checkerboard pattern for pairs of points located inside the white hole keeps on growing in time. This is in stark contrast with the stationary character of Hawking radiation from black holes. The consequence of this steady growth of density fluctuations for the stability of the white hole...
configuration requires a detailed study of the back-action effect of quantum fluctuations onto the flow and will be the subject of further work.

5. Analytical theory

In this section, we sketch an analytical framework offering a physical interpretation of the numerical observations discussed in the previous sections. More details will be given in future publications, in particular in Carlos Mayoral’s PhD thesis [32]. Our approach is based on the step-like horizon model first introduced at the level of mean-field theory in [20] and then developed into a full theory of quantum fluctuations and Hawking radiation in [25]. Here, we will show that this theoretical model is able to explain most of the numerically observed physics and to provide quantitatively accurate predictions for the most relevant observables.

5.1. Dynamical stability of white holes

In this subsection, we address the issue of the dynamical stability of the white hole configuration. The numerical observations reported in section 3 indicate that the chosen configuration is dynamically stable. Here we confirm this conclusion by analytical means. Our analytical approach is inspired by [20] and comes to similar conclusions. For analytical simplicity, we restrict our attention to the same configuration as that used in the numerical calculations, that is, a BEC in the plane wave state (4) with a spatially constant density $n_0$ and flow speed $v_0$; the step-like white hole horizon is created by means of a sudden jump at $x = 0$ of both the nonlinear interaction constant $g(x)$ and the external potential $V(x)$.

The dynamical stability of this configuration against perturbations of the horizon region is assessed by looking for eigenmodes of the Bogoliubov–Gennes equations [1, 9] with complex frequency: the system is unstable if at least an eigenmode $\alpha$ exists with a frequency $\tilde{\omega}_\alpha$ such that $\text{Im}[\tilde{\omega}_\alpha] > 0$. As we are interested in instabilities that start in the horizon region, boundary conditions have to imposed that the mode wavefunction of the perturbation decays to zero at infinity on both the sides of the horizon. At the horizon, the mode wavefunction has to be continuous along with its first spatial derivative.

More precisely, we are looking for eigenmodes of the Bogoliubov–Gennes equations,

$$\hbar \tilde{\omega} u(x) = -\frac{\hbar^2}{2m} \frac{\partial^2}{\partial x^2} u(x) + V(x) u(x) + 2 g(x) |\phi_0|^2 u(x) + g(x) \phi_0^2(x) v(x) - \mu u(x), \quad (9)$$

$$\hbar \tilde{\omega} v(x) = \frac{\hbar^2}{2m} \frac{\partial^2}{\partial x^2} v(x) - V(x) v(x) - 2 g(x) |\phi_0|^2 v(x) - g(x) \phi_0^2(x) u(x) + \mu v(x). \quad (10)$$

Within the spatially homogeneous upstream $x < 0$ region, the eigenmodes can be expanded into plane waves, as follows,

$$u(x) = \sum_i \alpha_i \tilde{u}_i^{(i)} e^{i k_i^{(i)} x}, \quad (11)$$

$$v(x) = \sum_i \alpha_i \tilde{v}_i^{(i)} e^{i k_i^{(i)} x}, \quad (12)$$

New Journal of Physics 13 (2011) 025007 (http://www.njp.org/)
where the sum over \( i \) runs over the Bogoliubov modes at frequency \( \tilde{\omega} \), whose complex wavevector \( \tilde{k}_u^{(i)} \) satisfies the (complex) dispersion relation

\[
\hbar^2 (\tilde{\omega} - \tilde{k}_u^{(i)} v_0)^2 = \frac{\hbar^2 \tilde{k}_u^{(i)}^2}{2m} \left( \frac{\hbar^2 \tilde{k}_u^{(i)}^2}{2m} + 2g_u |\phi_0|^2 \right)
\]

(13)

describing Bogoliubov modes in a moving fluid at \( v_0 \). As usual, the amplitudes \( \tilde{u}_u \) and \( \tilde{v}_u \) are determined by inserting the plane wave ansatz (11)–(12) into the Bogoliubov–de Gennes equations (9)–(10). In contrast to the standard cases [1, 9], the complex value of \( \tilde{\omega} \) makes the \( \tilde{u}, \tilde{v} \) coefficients complex as well.

The boundary condition that the mode wavefunction has to decay to zero for \( x \to -\infty \) restricts the sum to those Bogoliubov modes whose wavevector \( \text{Im}[\tilde{k}_u^{(i)}] < 0 \): out of the four solutions of (13), only two can then be retained\(^10\). Analogous conditions hold for the downstream, \( x > 0 \) region. The matching condition at the horizon insists that both \( u(x) \) and \( v(x) \) have to be continuous, as well as their first spatial derivatives.

The boundary conditions at infinity plus the matching conditions at the horizon reduce our problem to the determination of four amplitude coefficients (two modes per \( u, d \) side) that have to satisfy four matching conditions. Non-trivial solutions (the so-called asymptotically bound modes (ABMs) [21, 22]) exist only for special values of the complex frequency \( \tilde{\omega} \) that make the determinant of the corresponding linear system vanish. As one can see in figure 11, this never happens for the white hole system under investigation here. As dynamical instabilities are signalled by the existence of ABMs with \( \text{Im}[\tilde{\omega}] > 0 \), the complete absence of ABMs confirms the numerical observation of section 3 that white holes are dynamically stable objects. Of course, the linearized Bogoliubov theory that is used in this section is not able to capture the stationary Bogoliubov–Čerenkov density modulation pattern discussed in section 3.2, which appears as a consequence of nonlinear back-action effects.

In a forthcoming paper, this theory will be extended to more complex configurations involving a pair of adjacent black and white hole horizons: in agreement with [21, 22], the black hole laser instability shows up in this case as a series of discrete eigenmodes with a positive imaginary part of the frequency. Each of them corresponds to a zero of the determinant at the corresponding complex frequency \( \tilde{\omega} \). In a figure analogous to figure 11, such modes would appear as narrow blue spots corresponding to sharp minima of the logarithm.

5.2. The S-matrix and the white hole radiation

In the previous subsection, we have shown that no complex frequency eigenmodes appear in a white hole configuration. All the physics of quantum fluctuations can then be studied in terms of the input–output formalism of quantum optics [33, 34]: the central object of this approach is the \( S(\omega) \) matrix connecting the amplitude operators for the out-going Bogoliubov modes to those of the in-going modes at the same real frequency \( \omega \). This is a direct extension to the white hole configuration of the Bogoliubov theory of acoustic black holes first introduced in [35] and fully developed in [18, 19, 25].

A sketch of the in-going and out-going modes for the white hole configuration was shown in figures 2(a) and (b): in the low-energy sector below the cut-off frequency \( \omega_{\text{max}} \) (defined as

\(^{10}\) It can be shown that the number of such solutions is always two. More details of this issue as well as the relation between this boundary condition and the in-going versus out-going character of the modes will be given in future publications.

New Journal of Physics 13 (2011) 025007 (http://www.njp.org/)
Figure 11. A log-scale colour plot of the determinant whose zeros define the ABMs of a white hole configuration with a step-like horizon. The x- and y-axis are the real and imaginary parts of the complex frequency $\tilde{\omega}$. As the determinant has no zeros in the complex $\tilde{\omega}$ plane, no ABMs exist. This proves a fortiori that the white hole is dynamically stable. The white hole parameters are the same as in previous figures.

The maximum of the dispersion of the $u_2$ mode, there are three in-going modes, labelled $d^{in}$, $u_1^{in}$ and $u_2^{in}$, and three out-going modes, labelled $d^{out}$, $u_1^{out}$ and $u_2^{out}$. As was discussed in previous works, the $d$ and $u_1$ out-going (in-going) modes have a positive Bogoliubov norm and enter the input–output formalism as annihilation operators $\hat{b}_{d,u_1}(\omega)$ $(\hat{a}_{d,u_1}(\omega))$. On the other hand, the negative norm $u_2$ modes enter the formalism as creation operators $\hat{b}_{u_2}^{\dagger}(\omega)$ $(\hat{a}_{u_2}^{\dagger}(\omega))$. The input–output relations have the form

$$
\begin{pmatrix}
\hat{b}_{d}(\omega) \\
\hat{b}_{u_1}(\omega) \\
\hat{b}_{u_2}^{\dagger}(\omega)
\end{pmatrix} = S(\omega)
\begin{pmatrix}
\hat{a}_{d}(\omega) \\
\hat{a}_{u_1}(\omega) \\
\hat{a}_{u_2}^{\dagger}(\omega)
\end{pmatrix}.
$$

(14)

The white hole radiation consists of a finite occupation of the out-going modes, even for a vacuum state of the in-going ones. In this framework, it appears as a direct consequence of the mixing of creation and destruction operators by the $S$-matrix (14). As usual in quantum optics [33, 34], such processes lead to a squeezed vacuum state as the output state of the modes after having scattered on the horizon. Except for some remarks at the end of section 5.4, our investigation is focused on the case of a $T_0 = 0$ initial state where the in-going modes are assumed to be in their vacuum state defined by $\hat{a}_{d,u_1,u_2}^{\dagger}|\text{vac}\rangle = 0$. The population of the out-going $d$ and $u_1$ modes is proportional to $|S_{du_2}|^2$ and $|S_{u_1u_2}|^2$, respectively. Thanks to the unitarity of the $S$-matrix, the population of the negative frequency partners is proportional to $|S_{u_2u_2}|^2 = |S_{du_2}|^2 + |S_{u_1u_2}|^2$, which confirms that phonons are created in pairs that carry no net energy.
A simple way of estimating the matrix elements of $S(\omega)$ is to look at the time evolution of incident wavepackets as was done in section 3 and to extract the reflection and transmission amplitudes from the intensity and phase of the out-going wavepackets. Alternatively, one can obtain the Bogoliubov modes at a given $\omega$ by solving the corresponding Bogoliubov–de Gennes differential equations: a numerical solution of this problem was performed in [18] for smooth configurations of the flow velocity and the speed of sound. Closed analytical formulae for the transmission and reflection amplitudes of a black hole in the opposite limit of a step-like horizon were obtained in [25] and then in [36]: imposing appropriate boundary conditions at the horizon $x = 0$, one is able to match the Bogoliubov plane-wave solutions within each homogeneous region for $x \gtrless 0$ and then build global solutions of the Bogoliubov problem.

Here, the last strategy is applied to a white hole configuration to obtain closed expressions for the matrix elements of $S(\omega)$ in the low-frequency limit within the step-like horizon approximation. In particular, we shall focus our attention on the matrix elements for the negative frequency $u_2$ in-going mode that are involved in the white hole radiation,$$
|S_{u_1u_2}|^2 \simeq |S_{u_2u_2}|^2 \simeq \frac{(v_0 - c_d)(v_0^2 - c_u^2)^{3/2}(c_d + c_u)}{2c_u(c_d + v_0)(c_u - c_d)} \frac{1}{\omega}, \quad (15)
$$

$$
|S_{d_1u_2}|^2 \simeq \frac{c_d}{c_u} \frac{(v_0 - c_u)^2}{(v_0 + c_d)^2}. \quad (16)
$$

This behaviour of the low-energy $S$-matrix elements is common to all in-going modes: all of the matrix elements for out-going $u_1$ and $u_2$ modes diverge as $1/\sqrt{\omega}$, whereas the ones for out-going $d$ mode tend to a constant value. As a consequence, the spectral distribution of the quanta that are emitted into the $u_1\omega_\text{max}$ modes into the white hole has the usual $1/\omega$ thermal form of Hawking emission, whereas the radiation leaving the horizon has a flat spectrum. This remarkable fact was first noted in [18] as a result of a full Bogoliubov calculation with a smooth horizon, and constitutes a crucial difference from the black hole case where the Hawking emission outside the horizon has a thermal $1/\omega$ spectrum.

For higher frequencies (but still below $\omega_\text{max}$), the $S_{u_1u_2}$ matrix element quickly decreases to small values, whereas the $S_{u_2u_2}$ one keeps a sizeable value as it ends up describing simple reflection processes. For a smooth horizon of thickness $\sigma \gg \xi$, these coefficients are expected to tend to, respectively, 0 and 1 with an exponential law of the canonical Hawking form,$$
|S_{u_1u_2}^{(H)}| = [1 - \exp(-\omega/\omega_H)]^{-1/2}, \quad (17)
$$

$$
|S_{u_2u_2}^{(H)}| = \exp(-\omega/2\omega_H) |S_{u_2u_2}|, \quad (18)
$$

where the specific value of the cut-off $\omega_H = k_B T_H/h$ is determined by the analogue of the surface gravity of the horizon configuration under examination [10].

Before proceeding with the calculation of the density correlations, it is interesting to point out a remarkable relation connecting the $S$-matrix of black hole and white hole configurations related by a time-reversal symmetry [18, 19],

$$
[S^{WH}]^\dagger = [S^{BH}]^{-1}, \quad (19)
$$

with the convention that the $d$, $u_1$ and $u_2$ modes of the white hole correspond after time reversal (that exchanges the upstream and downstream regions) to the $u$, $d_1$ and $d_2$ ones of the black hole as defined in [25]. Making use of the $\eta$-unitarity of $S$, that is, $S^\dagger \eta S = \eta$ with $\eta = \text{diag}[1, 1, -1]$, this leads to

$$
S^{BH} = \eta [S^{WH}]^\dagger \eta. \quad (20)
$$
5.3. Density correlations between the internal and the external regions

The input–output approach reviewed in the previous section is the starting point for a calculation of the correlation function of density fluctuations. Such a calculation was first performed in [25] for the black hole case and is briefly reviewed in section 5.5. The results agreed with the numerical observations of [15]. Here, we shall extend this theory to the case of white holes.

Following the same lines as in [25], one can show that the density correlation between the internal and the external regions of the white hole far away from the horizon can be written in the form

\[ G^{(2)}(x, x') = 1 + \int_0^\infty d\omega \delta G_0^{(2)}(\omega, x, x'), \]  

with

\[ n_0 \delta G_0^{(2)}(\omega, x, x') = \frac{1}{4\pi} \sum_{l=u_1, u_2} \left\{ \frac{R_{l\leftrightarrow l'}(\omega) R_{l\leftrightarrow l'}(\omega)}{\sqrt{|v^\text{out}_{l,g,d} v^\text{out}_{l,g,d}|}} e^{i k_\omega x' - i k_{l\leftrightarrow l'}(\omega) x} S_{l_1}^* (\omega) S_{l_2} (\omega) + \text{c.c.} \right\}. \]  

(21)

(22)

Here, the point x is taken in the upstream region inside the white hole, while the point x' is taken in the downstream region outside the white hole. Of course, \( \delta G_0^{(2)}(\omega, x, x') \) is non-zero only within the \( \omega < \omega_{\text{max}} \) region where the \( u_2^\text{in} \) mode exists. As already mentioned, we are here restricting our attention to the zero temperature case, \( T_0 = 0 \). Formula (22) clearly indicates that the process that is responsible for the correlation signal can be interpreted as the conversion of quantum fluctuations from the \( u_2^\text{in} \) mode into out-going pairs of \( d^\text{out} \) and \( u_1^\text{out} \) or \( d^\text{out} \) and \( u_2^\text{out} \) real Bogoliubov excitations: the \( d^\text{out} \) quantum is emitted away from the white hole in the rightward direction, while the \( u_1^\text{out} \) and \( u_2^\text{out} \) quanta propagate upstream into the white hole.

We have defined \( k_{l\leftrightarrow l'}^\text{out}(\omega) \) as the value of the wavevector of the \( d, u_1 \) and \( u_2 \) outgoing modes at the frequency \( \omega \). The corresponding group velocities in the laboratory frame are defined as \( v^\text{out}_{l,g,d}(\omega) \). The density components \( R_{l,d,u_1,u_2}^\text{out}(\omega) \) of the Bogoliubov modes are written in terms of the Bogoliubov \( U_k \) and \( V_k \) coefficients as \( R_k = U_k + V_k = [\hbar k^2/2m\Omega(k)]^{1/2} \) [1, 9]. In particular, note how \( R_{k,d}^\text{out}(\omega) \) goes as \( \sqrt{k_d^\text{out}(\omega)} \sim \sqrt{\omega} \) in the low-frequency limit, while \( R_{k,u_1}^\text{out}(\omega) \) tends to finite values simply because both \( k_{u_1,2}^\text{out}(\omega \to 0) = \pm k_Z \): this simple fact will play a crucial role in the following.

Approximating the \( S \)-matrix products with their low-energy form for a step-like horizon configuration,

\[ S_{u_1,u_2}^* S_{d,u_2} \approx \frac{1}{\sqrt{\omega}} \frac{(v_0^2 - c_u^2)^3/2}{c_u (c_u - c_d)(c_d + v_0)} \left[ \frac{1}{\sqrt{|v^2_0 - c_u^2 - \sqrt{c_d^2 - v_0^2}} \right] \frac{c_d (c_d - v_0)}{2(c_d + v_0)}, \]  

\[ S_{u_2,u_2}^* S_{d,u_2} = - \left[ S_{u_1,u_2}^* S_{d,u_2} \right]^*, \]  

and summing up the two terms, one is led to the final expression,

\[ n_0 \delta G_0^{(2)}(x, x') \simeq \frac{(v_0 - c_d) \sqrt{(v_0^2 - c_u^2)(c_d - v_0)}}{2\pi (c_d + v_0)^{3/2} c_u (c_u - c_d)} \left[ \sqrt{v_0^2 - c_u^2} \cos(k_Z x) + \sqrt{c_d^2 - v_0^2} \sin(k_Z x) \right] \]  

\[ \times \frac{1 - \cos \left[ \frac{\omega_{\text{max}}}{v_Z} \left( \frac{x}{v_Z} - \frac{x'}{v_{g,d}} \right) \right]}{\frac{x}{v_Z} - \frac{x'}{v_{g,d}}}, \]  

(23)

(24)

(25)
Figure 12. The upper (a) and (b) panels: separated plots of the contribution of the $d-u_1$ and $d-u_2$ pairs to the correlation function of density fluctuations. The lower (c) plot: full analytical prediction (25) (red solid line). Numerical prediction evaluated at a late time $\mu_d t = 180$ and rescaled to compensate for the spatial smoothening procedure (green dashed line). All of the curves are cuts of the correlation function along the $x' = 63.5 \xi_d$ line. The vertical dotted line indicates the position of the intersection of the cut line with the destructive interference line (26).
5.4. Density correlations in the internal region: the growing checkerboard pattern

The same framework can be applied to the correlations between points located in the internal region of the white hole far away from the horizon. In this case, auto-correlations in the $u_1$ and $u_2$ outgoing modes are important, as well as cross-correlations between the two modes. Hence, the density correlation function is given by three contributions,

\[ n_0 \delta G^{(2)}_0(\omega, x, x') = \frac{1}{4\pi} \left[ \frac{|R^{\text{out}}_{u_1}|^2}{|\nu_{u_1}|} e^{i k_{\text{out}}^{u_1}(x'-x)} |S_{u_1u_2}^{\ast}|^2 + \frac{|R^{\text{out}}_{u_2}|^2}{|\nu_{u_2}|} e^{i k_{\text{out}}^{u_2}(x'-x)} |S_{u_2u_2}^{\ast}|^2 \right. \]
\[ + \left. \frac{R^{\text{out}}_{u_1} R^{\text{out}}_{u_2}}{|\nu_{u_1}| |\nu_{u_2}|} e^{i[k_{\text{out}}^{u_1}(x'-x')]} S_{u_1u_2}^{\ast} S_{u_2u_2} + c.c. \right] (x \leftrightarrow x'). \] (27)

The expressions for the low-frequency limit of the $S$-matrix elements within the step-like horizon approximation are

\[ S_{u_1u_2}^{\ast} S_{u_2u_2} \approx \frac{(v_0^2 - c_u^2)^{3/2}(c_d - v_0)}{2\omega c_u(c_d + v_0)(c_u - c_d)^2} \left[ 2v_0^2 - c_d^2 - c_u^2 + 2i\sqrt{(c_d^2 - v_0^2)(v_0^2 - c_u^2)} \right], \] (28)

\[ |S_{u_1u_2}|^2 \approx |S_{u_2u_2}|^2 \approx \frac{(v_0 - c_d)(v_0^2 - c_u^2)^{3/2}(c_u + c_d)}{2c_u(c_d + v_0)(c_u - c_d)\omega}. \] (29)

Inserting these expressions into (21), the integral over $\omega$ can be trivially performed. For the contribution of the $u_1^{\text{out}} - u_2^{\text{out}}$ cross-correlations and the sum of the $u_1^{\ast}$ and $u_2^{\ast}$ self-correlations, one obtains respectively

\[ n_0 \delta G^{(2)}_0(x, x') \bigg|_{u_1u_2} \approx \frac{(v_0^2 - c_u^2)(c_d - v_0)}{2\pi c_u(c_d + v_0)(c_u - c_d)^2} \left\{ (2v_0^2 - c_u^2 - c_d^2) \cos[k_z(x + x')] \right. \]
\[ + \left. 2\sqrt{(c_d^2 - v_0^2)(v_0^2 - c_u^2)} \sin[k_z(x + x')] \right\} I_\epsilon, \] (30)

\[ n_0 \delta G^{(2)}_0(x, x') \bigg|_{u_1u_2} \approx \frac{(v_0^2 - c_u^2)(v_0 - c_d)(c_u + c_d)}{2\pi c_d(c_d + v_0)(c_u - c_d)} \cos[k_z(x - x')] I_\epsilon. \] (31)

The trigonometric factors in (30) and (31) are responsible for the checkerboard pattern that was observed in figure 7: the former gives fringes parallel to the anti-diagonal $x + x' = 0$, while the latter gives the fringes parallel to the main diagonal $x = x'$. The most interesting factor is, however, the integral,

\[ I_\epsilon = \int_\epsilon^{\omega_{\text{max}}} \frac{d\omega}{\omega} \cos \left[ \omega \left( \frac{x - x'}{v_Z} \right) \right], \] (32)

which diverges when the infrared cut-off $\epsilon \rightarrow 0$.

A way of coping with this divergence is to assume a scaling $\epsilon \propto 1/\tau$ for the infrared cut-off, $\tau$ being the time elapsed since the formation of the horizon. In this way, $I_\epsilon$ becomes time dependent: in particular, its value for $x = x'$ follows a logarithmic growth with $\tau$. This fact is in agreement with the growth of the checkerboard pattern amplitude with time that was observed in the numerical simulations. In figure 9(a), we have tried to fit the numerical data with a logarithmic law (cyan line): the agreement is indeed quite good.

As a further check of our interpretation, we have repeated the same calculation for the finite temperature $T_0 > 0$ case, which introduces an additional $1/\omega$ factor into (32). The same
strategy of imposing an infrared cut-off at $1/\tau$ then correctly explains the linear growth of the checkerboard pattern in time that was observed in the numerical calculation of figure 9(b). All of these features will be discussed thoroughly in [32].

5.5. Analytic form of correlations in a black hole configuration

For the sake of completeness and to better understand the peculiarities of the white hole emission, it is useful to briefly review the analytical form of the different features that appear in the correlation pattern of black holes [25]. The main ingredient of the calculation is again the $S$-matrix that connects the amplitude operators for the out-going $u^{\text{out}}$, $d_1^{\text{out}}$ and $d_2^{\text{out}}$ Bogoliubov modes to those of the in-going $u^{\text{in}}$, $d_1^{\text{in}}$ and $d_2^{\text{in}}$ ones. From the relation (20), it is immediately clear that for all $l = \{u, d_1, d_2\}$, $S_{ld_1}$ and $S_{ld_2}$ scale as $1/\sqrt{\omega}$, whereas $S_{lu}$ tends to a finite value in the low-frequency limit.

5.5.1. Density correlation between the internal and the external regions. The density correlation between the inside and outside regions of the black hole have the form

$$n_0 \delta G_0^{(2)}(\omega, x, x') = \frac{1}{4\pi} \sum_{l=d_1, d_2} \left\{ \frac{R_{u}^{\text{out}}(\omega)R_{c}^{\text{out}}(\omega)}{|v_{g,u}^{\text{out}}v_{g,l}^{\text{out}}|} e^{i(k_{u}^{\text{out}}(\omega)x - k_{l}^{\text{out}}(\omega)x')} S_{ld_1}(\omega) S_{ld_2}(\omega) + \text{c.c.} \right\}.$$ (33)

Here, the point $x > 0$ is located in the upstream region outside the black hole, while the point $x' < 0$ is located in the downstream region inside the black hole.

As in the white hole case, expression (33) clearly indicates that the process responsible for the correlation signal consists of quantum fluctuations incident on the horizon from the $d_2$ mode, which scatter into a correlated pair of real out-going Bogoliubov excitations either into the $u$ and $d_1$ modes or into $u$ and $d_2$ modes. As already mentioned, in the black hole configuration the wavevector $k_{l}^{\text{out}}$ of all out-going $l = \{u, d_1, d_2\}$ modes tends linearly to zero in the zero frequency limit, which implies that all $R_{l}^{\text{out}}(\omega) \propto \sqrt{\omega}$.

For this reason, none of the black hole correlations show any modulation and all features have a sharp tongue-like shape; within the low-frequency approximation of the $S$-matrix mentioned above, one indeed obtains

$$n_0 \delta G_0^{(2)}(x, x') \simeq \sum_{l=d_1, d_2} A_{l} c_{u}^2 \omega_{\text{max}} \text{sinc} \left( \frac{\omega_{\text{max}}(\sqrt{x_{\text{out}}^2-v_{l}^{\text{out}}^2})}{\sqrt{x_{\text{out}}^2-v_{u}^{\text{out}}^2}} \right),$$ (34)

where \text{sinc}(x) = \sin x / x and the $A_{l}$ coefficient has the following explicit form,

$$A_{d_1} (d_2) = \frac{c_{u} (c_{u} + v_{l})}{c_{d} (c_{u} - v_{l})} \left( \frac{v_{l}^2}{c_{u}^2} - \frac{c_{d}^2}{c_{u}^2} \right)^{3/2} \frac{c_{u}}{c_{d} + (-)c_{u}},$$ (35)

in terms of the microscopic parameters of the system. $v_{g,u}^{\text{out}} = v_{0} + c_{u} > 0$ is the group velocity of the $u^{\text{out}}$ mode escaping from the black hole. The group velocities $v_{g,d_1,2}^{\text{out}} < 0$ of the $d_1,2^{\text{out}}$ outgoing modes are very different from each other, $v_{g,d_1,2}^{\text{out}} = v_{0} \mp c_{d}$. This simple fact has the important consequence that the corresponding $u-d_2^{\text{out}}$ and $u-d_1^{\text{out}}$ features (iii) and (v) in the correlation pattern of figure 10(b) are spatially separated and do not interfere, as it was instead the case for a white hole in section 5.3. Feature (iii) may be considered the main signature on the density correlation function of the Hawking emission from acoustic black holes.
5.5.2. Density correlations in the internal regions. The correlation function for points both located inside the black hole, \( x, x' < 0 \), contains a single contribution that originates from the \( d_1, d_2 \) modes, namely

\[
n_0 \delta G^{(2)}_0(\omega, x, x') = \frac{1}{4\pi} \left( \frac{R^{\text{out}}_{d_1}(\omega) R^{\text{out}}_{d_2}(\omega)}{\sqrt{|v^{\text{out}}_{g.d_1} v^{\text{out}}_{g.d_2}|}} \right) e^{-i(k^{\text{out}}_{d_1}(\omega) x - k^{\text{out}}_{d_2}(\omega) x')} S^*_d(\omega) S_d(\omega) + \text{c.c.} \right) + (x \leftrightarrow x').
\]

(36)

Within the same low-frequency expansion approximation, one obtains a tongue-shaped contribution of the form

\[
n_0 \delta G^{(2)}_0(\omega, x, x') \simeq - \frac{B \Omega c^2 v}{4\pi v^{\text{out}}_{g.d_1} v^{\text{out}}_{g.d_2} c_d} \frac{\text{sinc} \left( \Omega \left( \frac{x}{v^{\text{out}}_{d_1}} - \frac{x'}{v^{\text{out}}_{d_2}} \right) \right)} + (x \leftrightarrow x').
\]

(37)

with

\[
B = - \frac{c_d}{2c_d} \left( \frac{c_v}{c_d} + \frac{v_0}{c_d} \right) \left( \frac{v^2_0}{c^2_v} - \frac{c^2_d}{c^2_u} \right)^{3/2}
\]

(38)

that correctly reproduces feature (iv) of figure 10(b). In particular, in contrast to the white hole case, no infrared divergence appears in the integral, which confirms the stationarity in time of the Hawking radiation pattern.

In summary, in this section we have developed a Bogoliubov theory of quantum fluctuations in a white hole configuration within the step-like approximation. This theory provides analytical formulae for the physical observables that were considered in the numerical simulations reported in the previous section. In contrast to the thermal character of the Hawking radiation from a black hole, the spectral distribution of the emitted radiation outside the white hole turns out to be flat. The steady growth of the density fluctuations in time is signalled by infrared divergences in the Bogoliubov calculation. By imposing a suitable cut-off on the integrals, one is able to reproduce the numerically observed trend. Inclusion of the terms describing the back-action of fluctuations onto the condensate is at present in progress and is expected to throw light on the effect of quantum fluctuations on the stability of the acoustic white hole configuration.

6. Conclusions

In this paper, we have reported a comprehensive investigation of the physics of acoustic white hole configurations. We have focused our attention on a very simple model that is amenable to analytical and numerical works. Using the Gross–Pitaevskii equation for the condensate dynamics, we have established the dynamical stability of the white hole at the mean-field level: no exponentially growing perturbation of the horizon appears when this is hit by an incident wavepacket of Bogoliubov phonons. A nonlinear back-action effect due to the perturbation of the horizon by the incident wavepacket is visible as a stationary pattern of Bogoliubov–Čerenkov waves by the perturbed horizon; the amplitude of this pattern scales as the square of the incident wavepacket one.

Quantum fluctuations are included within a truncated Wigner numerical approach; the physics of an infinitely long condensate is reproduced by imposing absorbing boundary conditions.
conditions on fluctuations. Signatures of the white hole emission by the horizon are identified in the correlation function of density fluctuations. In contrast to the Hawking emission from black holes, the white hole emission does not consist of low-energy excitations only, but involves high wavevector modes. As a result, the density correlation pattern shows fast oscillating features on the scale of the healing length. Furthermore, some of these features are not stationary in time, but indefinitely grow with time with a logarithmic (linear) law for a zero (finite) initial temperature. All of the features found in the numerical observations are fully recovered and physically interpreted by an analytical calculation within a step-like approximation for the horizon. Interestingly, this theory predicts a flat spectral distribution of the radiation emitted outside the acoustic white hole: this is in stark contrast to the thermal character of Hawking radiation from black holes.

In addition to its intrinsic interest from the point of view of quantum fluctuations in condensed matter systems, our results may open up important avenues in gravitational physics and quantum field theory in curved space–times. Even though black and white hole configurations are related to each other by a simple time reversal symmetry and the quantum radiation originates from the same parametric emission process, the observable properties of the radiation turn out to be significantly different in the two cases, starting from the thermal versus flat frequency distribution of the emission in the two cases. On the one hand, Hawking radiation from black holes is concentrated in the low- \( k \) hydrodynamical modes whose dispersion has a linear, relativistic-like behaviour; for this reason, Hawking radiation has turned out to be very robust against modifications of the ultraviolet behaviour of the dispersion relation. On the other hand, white hole radiation is intrinsically not a hydrodynamical process: because of the large associated blue shift, it is in fact highly sensitive to the ultraviolet behaviour of the Bogoliubov dispersion relation. On the basis of this, we expect that an experimental observation of the white hole radiation would not only provide a strong indication of the actual existence of Hawking radiation from black holes, but also show promising potential as a much better magnifying glass to test the intimate microscopic structure of the quantum vacuum.

**Acknowledgments**

We are indebted to N Pavloff and M Rinaldi for stimulating discussions. AF thanks the Generalitat Valenciana for financial support. AF and CM are supported in part by MICINN grant no. FIS2008-06078-C03-02. CM is indebted to the Spanish MEC for an FPU fellowship. CM thanks the Università di Bologna and the BEC Center in Trento for their charming hospitality during the elaboration of this work.

**References**

[1] Pitaevskii L and Stringari S 2003 *Bose–Einstein condensation* (Oxford: Clarendon)
[2] Carusotto I, Hu S X, Collins L A and Smerzi A 2007 *Phys. Rev. Lett.* 97 260403
[3] Engels P and Atherton C 2007 *Phys. Rev. Lett.* 99 160405
[4] Fallani L, De Sarlo L, Lye J E, Modugno M, Saers R, Fort C and Inguscio M 2001 *Phys. Rev. Lett.* 93 140406
[5] Wu B and Niu Q 2001 *Phys. Rev. A* 64 061603
Smerzi A, Trombettoni A, Kevrekidis P G and Bishop A R 2002 *Phys. Rev. Lett.* 89 170402
Modugno M, Tozzo C and Dalfovo F 2004 *Phys. Rev. A* 70 043625

*New Journal of Physics* 13 (2011) 025007 (http://www.njp.org/)
[6] Hakim V 1997 *Phys. Rev. E* **55** 2835
Pavloff N 2002 *Phys. Rev. A* **66** 013610

[7] Frisch T, Pomeau Y and Rica S 1992 *Phys. Rev. Lett.* **69** 1644
Winiecki T, Jackson B, McCann J F and Adams C S 2000 *J. Phys. B: At. Mol. Opt. Phys.* **33** 4069
Kamchatnov A and Pitaevskii L 2008 *Phys. Rev. Lett.* **100** 160402

[8] Leszczyszyn A M, El G A, Gladush Yu G and Kamchatnov A M 2009 *Phys. Rev. A* **79** 063608

[9] Castin Y 2001 Coherent atomic matter waves *Lecture Notes of Les Houches Summer School* ed R Kaiser, C Westbrook and F David (Berlin: Springer) p 1–136

[10] Unruh W G 1981 *Phys. Rev. Lett.* **46** 1351

[11] Novello M, Visser M and Volovik G (ed) 2002 *Artificial Black Holes* (Singapore: World Scientific)

[12] Birrell N D and Davies P C W 1982 *Quantum Fields in Curved Space* (Cambridge: Cambridge University Press)

[13] Hawking S W 1974 *Nature* **248** 30
Hawking S W 1975 *Commun. Math. Phys.* **43** 199

[14] Balbinot R, Fabbri A, Fagnocchi S, Recati A and Carusotto I 2008 *Phys. Rev. A* **78** 021603
Carusotto I, Fagnocchi S, Recati A, Balbinot R and Fabbri A 2008 *New J. Phys.* **10** 103001

[15] Lahav O, Itah A, Blumkin A, Gordon C and Steinhauer J 2010 *Phys. Rev. Lett.* **105** 240401

[16] Leonhardt U, Kiss T and Öhberg P 2003 *Phys. Rev. A* **67** 033602

[17] Macher J and Parentani R 2009 *Phys. Rev. A* **80** 043601
Macher J and Parentani R 2009 *Phys. Rev. D* **79** 124008

[18] Barceló C, Cano A, Garay L J and Jannes G 2006 *Phys. Rev. D* **74** 024008

[19] Coutant A and Parentani R 2010 *Phys. Rev. D* **81** 084042

[20] Finazzi S and Parentani R 2010 *New J. Phys.* **12** 095015

[21] Garay L J, Anglin J R, Cirac J I and Zoller P 2001 *Phys. Rev. A* **63** 023611
Jain P, Bradley A S and Gardiner C W 2007 *Phys. Rev. A* **76** 023617

[22] Corley S and Jacobson T 1999 *Phys. Rev. D* **59** 124011

[23] Mayoral Saenz C 2011 *PhD Thesis* Universidad de Valencia

[24] Walls D F and Milburn G J 1994 *Quantum Optics* (Berlin: Springer)

[25] Ciuti C and Carusotto I 2006 *Phys. Rev. A* **74** 033811

[26] Leonhardt U, Kiss T and Ohberg P 2003 *J. Opt. B: Quantum Semiclassical. Opt.* **5** S42

[27] Fabbri A and Mayoral C 2010 arXiv:1004.4876 unpublished
Mayoral C, Fabbri A and Rinaldi M 2010 arXiv:1008.2125 unpublished

New Journal of Physics 13 (2011) 025007 (http://www.njp.org/)