Social Trust Prediction via Max-norm Constrained 1-bit Matrix Completion

Jing Wang\textsuperscript{1}, Jie Shen\textsuperscript{2}, Huan Xu\textsuperscript{3}
\textsuperscript{1} Hefei University of Technology, China
wangjing@mail.hfut.edu.cn
\textsuperscript{2} Rutgers University, USA
js2007@rutgers.edu
\textsuperscript{3} National University of Singapore, Singapore
mpexuh@nus.edu.sg

Abstract

Social trust prediction addresses the significant problem of exploring interactions among users in social networks. Naturally, this problem can be formulated in the matrix completion framework, with each entry indicating the trustness or distrustfulness. However, there are two challenges: 1) the observed data are with sign (1-bit) measurements; 2) they are typically sampled non-uniformly. Most of the previous matrix completion methods do not well handle the two issues. Motivated by the recent progress of max-norm, we propose to solve the problem with a 1-bit max-norm constrained formulation. Since max-norm is not easy to optimize, we utilize a reformulation of max-norm which facilitates an efficient projected gradient decent algorithm. We demonstrate the superiority of our formulation on two benchmark datasets.

1 Introduction

With the increasing popularity of social networks, there exist many interesting and difficult problems, such as friends recommendation, information propagation, etc. In this paper, we study the problem of social trust prediction, which aims to estimate the positive or negative relationship among the users based on the existing trustness information associated with them. This problem plays an important role in social networks as the system can block the invitation from some users based on the existing trustness information.

Social trust prediction methods fall into three categories: 1) the observed entry is either 1 or −1, which is analogous to the binary classification problem. But in our problem, we are handling much more complex matrix data. Fortunately, Srebro\textit{et al}. presented a maximum margin matrix factorization framework that unifies the binary problem for vector case and matrix case. The key idea in their work is a low-norm matrix factorization, which will also be utilized in this paper. Second, the locations of the entries are sampled non-uniformly, which gaps the theory and practice. To tackle this challenge, we suggest using the max-norm as a convex surrogate for a low-rank matrix. The third one models the problem as a binary classification one and utilizes techniques such as logistic regression.

Challenges. However, there are two issues emerging in social trust prediction which are not well characterized by the algorithms in previous works. First, the value of the observed entry is either 1 or −1, which is analogous to the binary classification problem. But in our problem, we are handling much more complex matrix data. Fortunately, Srebro\textit{et al}. presented a maximum margin matrix factorization framework that unifies the binary problem for vector case and matrix case. The key idea in their work is a low-norm matrix factorization, which will also be utilized in this paper. Second, the locations of the entries are sampled non-uniformly, which gaps the theory and practice.

Our contributions are two-folds: 1) To the best of our knowledge, we are the first to address the social trust prediction problem by utilizing a max-norm constrained formulation. 2) Although a max-norm constrained problem can be solved by SDP solvers and an accurate enough solution can be achieved, we here utilize a projected gradient algorithm that is scalable to large scale datasets. We empirically show the improvement of our formulation for the non-uniform 1-bit benchmarks.

2 Related Work

Social interaction is investigated intensively in the last decades. The social interaction indicates the friendship, support, enemy or disapproval as shown in Figure 1. Online users
user-item matrix, such as transitivity. Meanwhile, the social trust in reality is extremely sparse. For instance, Facebook has hundreds of millions of users, but most of them have less than 1,000 friends. Besides, the people with similar personality tend to behave similarly. To sum up, the data matrix of social trust has both sparse and low-rank structure. Thus, the social trust prediction problem is especially suitable for the matrix completion model. That is the focus of our paper.

The problem of matrix completion is to recover a low-rank matrix from a subset of entries \([Candès and Recht, 2009]\), which is given by:

\[
\begin{align*}
\min_{X} & \quad \text{rank}(X) \\
\text{s.t.} & \quad P_Z(X) = P_Z(Z)
\end{align*}
\]  

(2.1)

where \(Z \in \mathbb{R}^{p \times n}\) is the data matrix, \(X\) is the recovered matrix, and \(\Omega\) is the index set of observed entries. The optimization problem (2.1) is not only NP-hard, but requires double exponential time complexity with the number of samples \([Recht et al., 2010]\). To solve the above problem, one alternative is to use nuclear norm as a relaxation to the rank function:

\[
\begin{align*}
\min_{X} & \quad ||X||_* \\
\text{s.t.} & \quad P_Z(X) = P_Z(M)
\end{align*}
\]  

(2.2)

where \(||X||_*\) denotes the sum of singular values of matrix \(X\). \([Cai et al., 2010]\) developed a first-order procedure to solve the convex problem (2.2), namely singular value projection (SVP) algorithm. \([Keshavan et al., 2010]\) solved the problem by first trimming each row and column with too few entries, then compute the truncated SVD of the trimmed matrix. Under certain conditions, it showed accurate recovery on the order of \(nd\log n\) samples \((n\) is the number of samples, \(d\) is the rank of recovered matrix). With the rapid development of matrix completion problem, some more efficient methods have been proposed \([Candès and Tao, 2010], [Gross, 2011], [Wang and Xu, 2012], [Huang et al., 2014]\).

However, all the methods mentioned above use the nuclear norm as the surrogate to the rank, whose exact recovery can be guaranteed only when the data are sampled uniformly, which is not practical in real world applications. On the other hand, recent empirical on max-norm \([Srebro et al., 2004]\) shows promising results for non-uniform data if one utilize the max-norm as a surrogate \([Salakhutdinov and Srebro, 2010]\). Notably, for some specific problems, such as collaborative filtering, \([Srebro and Shraibman, 2005]\) proved that the generalization error bound for max-norm is better than the nuclear norm. More recently, \([Shen et al., 2014]\) reported encouraging results on the subspace recovery task (which is closely relevant to matrix completion). Since the social trust data is non-uniformly sampled, we believe that a max-norm regularized formulation can better handle the challenge than the nuclear norm. Our formulation is also inspired by a recent theoretical study on matrix completion with 1-bit measurement \([Cai and Zhou, 2013]\), which established a minimax
lower bound on the general sampling model and derived the optimal convergence rate in terms of Frobenius norm loss. Furthermore, there are several practical algorithms to solve max-norm regularized or max-norm constrained problems, see [Lee et al., 2010] and [Shen et al., 2014] for example.

2.1 Overview
After review of related work in Section 2, we introduce the notations and formulate the problem in Section 3. Then we give algorithm to solve the max-norm constrained 1-bit matrix completion (MMC) problem in Section 4. Meanwhile, we also provide an equivalent SDP formulation for the MMC, which can be accurately solved at the expense of efficiency. Then we report the empirical study on two benchmark datasets in Section 5. Section 6 concludes this paper and discusses possible future work.

3 Notations and Problem Setup
In this section, we introduce the notations that will be used in this paper. Capital letters such as \( M \) are used for matrices and lowercase bold letters such as \( v \) denotes vectors. The \( i \)-th row and \( j \)-th column of a matrix \( M \) is denoted by \( m(i) \) and \( m(j) \) respectively, and the \( (i,j) \)-th entry is denoted by \( M_{ij} \). For a vector \( v \), we use \( v_i \) to denote its \( i \)-th element. We denote the \( \ell_2 \) norm of a vector \( v \) by \( \| v \|_2 \). For a matrix \( M \in \mathbb{R}^{p \times n} \), we use the Frobenius norm by \( \| M \|_F \) and \( \| M \|_{2,\infty} \) denotes the maximum \( \ell_2 \) row norm of \( M \), i.e.,

\[
\| M \|_{2,\infty} := \max_{i} \| m(i) \|_2.
\]

We further define the max-norm of \( M \) [Littel et al., 2007],

\[
\| M \|_{\max} = \min_{U,V,M=UV^T} \max \{ \| U \|_{2,\infty}^2, \| V \|_{2,\infty}^2 \}, \quad (3.1)
\]

where we enumerate all possible factorizations to obtain the minimum.

Intuition on max-norm. At a first sight, the max-norm is hard to understand. We simply explain why it is a tighter approximation to the rank function than the nuclear norm. Again, we write the nuclear norm of \( M \) as a factorization form [Recht et al., 2010]:

\[
\| M \|_\nu := \min_{U,V,M=UV^T} \frac{1}{2} \left( \| U \|_F^2 + \| V \|_F^2 \right).
\]

Note that the Frobenius norm is the sum of the square of the \( \ell_2 \) row norm. Thus, a nuclear norm regularizer actually constrains the average of the \( \ell_2 \) row norm, while the max-norm constrains the maximum of the \( \ell_2 \) row norm!

Given the observed data \( Z \in \mathbb{R}^{p \times n} \), we are interested in approximating \( Z \) with a low-rank matrix \( X \), which can be formulated by,

\[
\begin{align*}
\text{minimize} & \quad \frac{1}{2} \| P_{\Omega}(Z - X) \|_F^2, \\
\text{s.t.} & \quad \text{rank}(X) \leq d,
\end{align*}
\]

where \( \Omega \) is an index set of observed entries and \( d \) is some expected rank. \( P_{\Omega}(M) \) is a projection operator on a matrix \( M \) such that \( P_{\Omega}(m_{ij}) = m_{ij} \) if \((i,j) \in \Omega \) and zero otherwise. However, it is usually intractable to optimize the above program as the rank function is non-convex and non-continuous [Recht et al., 2010]. One common approach is to use the nuclear norm as a convex surrogate to the rank function. However, it is well known that the nuclear norm cannot well handle the non-uniform data. Motivated by the recent progress in max-norm [Salakhutdinov and Srebro, 2010; Cai and Zhou, 2013; Shen et al., 2014], we use the max-norm as an alternative convex relaxation, which gives the following formulation:

\[
\begin{align*}
\min_{X} & \quad \frac{1}{2} \| P_{\Omega}(Z - X) \|_F^2, \\
\text{s.t.} & \quad \| X \|_{\max} \leq \lambda^2,
\end{align*}
\]

(3.2)

where \( \lambda \) is some tunable parameter.

4 Algorithm
The max-norm is convex and moreover, it can be solved by any SDP solver. Formally, we have the following lemma:

**Lemma 4.1** ([Srebro, 2004]). For any matrix \( X \in \mathbb{R}^{p \times n} \) and \( \lambda \in \mathbb{R} \), \( \| X \|_{\max} \leq \lambda \) if and only if there exist \( A \in \mathbb{R}^{p\times p} \) and \( B \in \mathbb{R}^{n\times n} \), such that \( \left[ \begin{array}{c} A \\ X \end{array} \right] \) is semi-definite positive and each diagonal element in \( A \) and \( B \) is upper bounded by \( \lambda \).

With Lemma 4.1 on hand, one can formulate Problem 3.2 as an SDP:

\[
\begin{align*}
\min_{X,A,B} & \quad \frac{1}{2} \| P_{\Omega}(Z - X) \|_F^2, \\
\text{s.t.} & \quad A_{ii} \leq \lambda^2, \quad B_{jj} \leq \lambda^2, \quad \forall i \in [p], \quad j \in [n],
\end{align*}
\]

(4.1)

And this program can be solved by any SDP solver to obtain accurate enough solution.

However, SDP solvers are not scalable to large matrices. Thus, in this paper, we apply a projected gradient method to solve Problem 3.2, which is due to [Lee et al., 2010]. A key technique is the reformulation of the max-norm (3.1). Assume that the rank of the optimal solution \( X^* \) produced by the SDP (4.1) is at most \( d \). Then we can safely factorize \( X = UV^T \), with \( U \in \mathbb{R}^{p \times d} \) and \( V \in \mathbb{R}^{n \times d} \). Combining the factorization and the definition, we obtain the following equivalent program :

\[
\begin{align*}
\min_{U,V} & \quad \frac{1}{2} \| P_{\Omega}(Z - UV^T) \|_F^2, \\
\text{s.t.} & \quad \| U \|_{2,\infty} \leq \lambda, \quad \| V \|_{2,\infty} \leq \lambda.
\end{align*}
\]

(4.2)

Note that the gradient of the objective function w.r.t. \( U \) and \( V \) can be easily computed. That is,

\[
\nabla_U f(Z, U, V) = P_{\Omega}((UV^T - Z)V),
\]

\[
\nabla_V f(Z, U, V) = P_{\Omega}((VU^T - Z^TU)).
\]

(4.3)

Here, for simplicity we define

\[
f(Z, U, V) = \frac{1}{2} \| P_{\Omega}(Z - UV^T) \|_F^2.
\]
If we further pick the step size \( |\cdot|\) from Algorithm 1, we pick a sufficiently large value for \( \lambda \) to make the sequence of \( (U_t, V_t) \) at each iteration converge if they violate the constraints. If not, we can proceed to the next iteration. Otherwise, we can scale the rows of \( U \) and/or \( V \) by \( \frac{\lambda}{\|X - \lambda \|_2,\infty} \) and/or \( \frac{\lambda}{\|Y - \lambda \|_2,\infty} \) respectively. In this way, we have the process:

\[
\Pi(M) = \begin{cases} \frac{\lambda}{\|M\|_2,\infty} M, & \text{if } \|M\|_2,\infty > \lambda, \\ M, & \text{otherwise.} \end{cases}
\]

If we further pick the step size \( \alpha_t \) via the Armijo rule [Armijo and others, 1966], it can be shown that the sequence of \( (U_t, V_t) \) will converge to a stationary point [Bertsekas, 1999]. The algorithm is summarized in Algorithm 1.

The benefits of applying the factorization on the max-norm are two-folds: 1) the memory cost can be significantly reduced from \( O(nm) \) of SDP to \( O(d(p + n)) \). 2) it facilitates the projected gradient algorithm, which is computationally efficient when working on large matrices (see Section 5). However, note that Problem (4.2) is non-convex. Fortunately, [Burer and Monteiro, 2005] proved that as long as we pick a sufficiently large value for \( \lambda \), then any local minimum of Eq. (4.2) is a global optimum. In Section 5 we will report the influence of \( \lambda \) on the performance. Actually, in Algorithm 1 the stopping criteria is set to be a maximum iteration. One may also check if it reaches a local minima as the stopping criteria, as discussed in [Cai and Zhou, 2013].

### 4.1 Heuristic on \( \lambda \)

The \( \lambda \) is the only tunable parameter in our algorithm. For our problem, note that the data is of 1-bit measurements, i.e., \( |Z_{ij}| = 1 \) for \( (i,j) \in \Omega \). Also note that \( Z_{ij} = u(i)v(j)^T \). Thus, \( |u(i)v(j)^T| = 1 \). So we have \( \lambda \geq 1 \). However, if we choose a large \( \lambda \), the estimation \( |X_{ij}| \) may deviate away from 1. We find that \( \lambda = 1.2 \) lead to satisfactory improvement.

### 5 Experiments

In this section, we empirically evaluate our method for the matrix completion performance. We will first introduce the used datasets. In the experimental settings, we present the comparative methods and evaluation metrics. Then we report encouraging results on two benchmark datasets. We also examine the influence of matrix rank \( d \).

#### 5.1 Datasets

We conduct the experiments on two benchmark datasets: Epinions and Slashdot. In these two datasets, the users are connected by explicit positive (trust) or negative (distrust) links (i.e. the 1-bit measurements in \( Z \)). The first dataset contains 119,217 nodes (users) and 841,000 edges (links), 85.0% of which are positive. The Slashdot dataset contains 82,144 users and 549,202 links, and 77.4% of the edges are labled as positive. Table 1 gives a summary description about the subset used in our experiment.

It is clear that the distribution of links are not uniform since each user has his/her individual preference and own friendship network. Following [Huang et al., 2013], we select 2,000 users with the highest degrees from each dataset to form the observation matrix \( Z \).

#### 5.2 Experimental Settings

**Baselines.** We choose four state-of-the-art methods as baselines, including SVP [Jain et al., 2010], SVT [Cai et al., 2010], OPTSpace [Keshavan et al., 2010] and RRMC [Huang et al., 2013]. Since SVT and RRMC need a specifed rank, we tune the rank for these methods and choose the best performance as the final result.

**Evaluation Metrics.** Let \( T \) be the index set of all observed entries. We use two evaluation metrics to measure the performance, mean average error (MAE) and root mean square error (RMSE), computed as follows:

\[
MAE = \frac{\sum_{(i,j)\in T} |X_{ij} - M_{ij}|/(|T| - |\Omega|)}{\sqrt{\sum_{(i,j)\in T} (X_{ij} - M_{ij})^2/(|T| - |\Omega|)}},
\]

where \(|T|\) denotes the cardinality of \( T \).

**Training and Testing.** We randomly split the dataset for training and testing. In particular, the number of observation measurements \( \Omega \) for training ranges from 10% to 60%, with step size 10%. For each split, we run all the algorithms for 20 trials, with the training data in each trail being randomly sampled. Then, we report the mean and standard deviation of MAE and RMSE over all 20 trails.

| Dataset  | Epinions | Slashdot |
|----------|----------|----------|
| #of Users | 2,000    | 2,000    |
| #of Trust | 171,731  | 68,932   |
| #of Distrust | 18,916  | 20,032   |
Table 2: MAE Results on Epinions Dataset

| Observed entries (%) | SVT       | OPTSpace  | SVP       | RRMC      | MMC       |
|----------------------|-----------|-----------|-----------|-----------|-----------|
| 10                   | 0.359±0.004 | 0.289±0.019 | 0.450±0.008 | 0.576±0.001 | 0.254±0.003 |
| 20                   | 0.394±0.022 | 0.236±0.005 | 0.294±0.002 | 0.518±0.002 | 0.212±0.003 |
| 30                   | 0.360±0.057 | 0.219±0.009 | 0.248±0.001 | 0.460±0.002 | 0.201±0.002 |
| 40                   | 0.410±0.099 | 0.205±0.008 | 0.224±0.001 | 0.418±0.002 | 0.193±0.001 |
| 50                   | 0.471±0.129 | 0.197±0.007 | 0.210±0.001 | 0.386±0.001 | 0.190±0.002 |
| 60                   | 0.476±0.146 | 0.197±0.003 | 0.199±0.001 | 0.362±0.002 | 0.206±0.003 |

Table 3: RMSE Results on Epinions Dataset

| Observed entries (%) | SVT       | OPTSpace  | SVP       | RRMC      | MMC       |
|----------------------|-----------|-----------|-----------|-----------|-----------|
| 10                   | 0.513±0.010 | 0.530±0.021 | 0.610±0.010 | 0.650±0.001 | 0.466±0.004 |
| 20                   | 0.559±0.031 | 0.456±0.005 | 0.459±0.002 | 0.606±0.002 | 0.406±0.004 |
| 30                   | 0.532±0.082 | 0.422±0.011 | 0.415±0.002 | 0.563±0.002 | 0.383±0.002 |
| 40                   | 0.620±0.171 | 0.406±0.015 | 0.394±0.002 | 0.533±0.002 | 0.371±0.002 |
| 50                   | 0.719±0.225 | 0.398±0.016 | 0.381±0.001 | 0.509±0.001 | 0.364±0.001 |
| 60                   | 0.728±0.288 | 0.403±0.009 | 0.371±0.002 | 0.491±0.002 | 0.365±0.002 |

5.3 Experimental Results

We report detailed results from Table 2 to Table 5. From the results in Tables 2 and 3, we observe that MMC outperforms the other methods in terms of both evaluation metrics on the Epinions dataset most of the time. In particular, when there are few observations 10% (which indicates a hard task), MMC obtains the RMSE of 0.466, much better than OPTSpace (0.530), SVP (0.610) and RRMC (0.650). Except on the case with 60% observed entries, OPTSpace obtains the smallest MSE with 0.197, but our algorithm is comparative with 0.206. In a nutshell, the gap between MMC and the baselines becomes larger as the fraction of observed entries decreases.

Similarly, our method achieves the least MAE and RMSE on the Slashdot dataset (see Table 4 and 5). For instance with 30% observed entries, MMC obtains the MAE with less than 0.4, much better than the comparative methods, such as SVT (0.513), OPTSpace (0.427), SVP (0.501) and RRMC (0.686). In terms of RMSE, in the case of 20% observed entries, the RMSE values of other methods are all above 0.7 while our method reaches 0.679. In sum, our method is superior than the comparative methods on two real-life datasets in terms of MSE and RMSE most of the time.

Since we have studied the effectiveness of our method, here we examine the computational efficiency in Table 6 which is important for practical applications. To test the time complexity of the methods, we report the averaged time cost on the Epinions dataset with 10% observed entries and Slashdot with 20% observed entries. To illustrate the trade-off between accuracy and efficiency, we also report the MAE and RMSE. As we see, SVP is the most efficient method, whose running time is 0.84 seconds on Epinions while ours is 1.92 seconds. On Slashdot, it also achieves the best performance in terms of efficiency. However, our method enjoys a significant improvement of MAE and RMSE compared to all base-


Table 4: MAE Results on Slashdot Dataset

| Observed entries (%) | SVT       | OPTSpace   | SVP       | RRMC     | MMC       |
|----------------------|-----------|------------|-----------|----------|-----------|
| 10                   | 0.679±0.008 | 0.554±0.017 | 0.755±0.005 | 0.715±0.001 | 0.546±0.008 |
| 20                   | 0.562±0.004 | 0.458±0.008 | 0.582±0.007 | 0.704±0.001 | 0.437±0.008 |
| 30                   | 0.513±0.030 | 0.427±0.009 | 0.501±0.003 | 0.686±0.003 | 0.395±0.006 |
| 40                   | 0.506±0.041 | 0.395±0.009 | 0.460±0.002 | 0.647±0.006 | 0.380±0.009 |
| 50                   | 0.495±0.060 | 0.376±0.004 | 0.432±0.002 | 0.609±0.002 | 0.366±0.003 |
| 60                   | 0.520±0.065 | 0.362±0.011 | 0.413±0.002 | 0.585±0.002 | 0.350±0.002 |

Table 5: RMSE Results on Slashdot Dataset

| Observed entries (%) | SVT       | OPTSpace   | SVP       | RRMC     | MMC       |
|----------------------|-----------|------------|-----------|----------|-----------|
| 10                   | 0.788±0.008 | 0.826±0.021 | 0.873±0.004 | 0.829±0.001 | 0.774±0.008 |
| 20                   | 0.718±0.020 | 0.704±0.013 | 0.746±0.007 | 0.821±0.001 | 0.679±0.009 |
| 30                   | 0.680±0.043 | 0.652±0.006 | 0.680±0.003 | 0.807±0.002 | 0.633±0.008 |
| 40                   | 0.670±0.056 | 0.620±0.009 | 0.647±0.003 | 0.778±0.005 | 0.615±0.011 |
| 50                   | 0.642±0.055 | 0.596±0.009 | 0.624±0.003 | 0.749±0.002 | 0.581±0.006 |
| 60                   | 0.699±0.084 | 0.577±0.011 | 0.609±0.002 | 0.730±0.002 | 0.566±0.002 |

Table 6: The trade-off between accuracy and efficiency.

| Dataset    | SVT MAE | SVT RMSE | SVT Time | OPTSpace MAE | OPTSpace RMSE | OPTSpace Time | SVP MAE | SVP RMSE | SVP Time | RRMC MAE | RRMC RMSE | RRMC Time | MMC MAE | MMC RMSE | MMC Time |
|------------|---------|----------|----------|-------------|---------------|---------------|---------|----------|----------|----------|-----------|-----------|---------|----------|-----------|
| Epinions   | 0.466   | 0.531    | 0.285    | 0.531       | 15.26         | 0.450         | 0.610   | 0.84     | 0.576    | 0.83     | 43.11     | 0.262     | 0.679   | 4.11     |
| Slashdot   | 0.618   | 0.705    | 9.23     | 0.705       | 50.73         | 0.582         | 0.746   | 0.03     | 0.715    | 0.83     | 44.12     | 0.437     | 0.679   | 2.41     |

lines. Also, our algorithm is orders of magnitude faster than other three baselines. This implies that MMC favors a good trade-off between the accuracy and efficiency.

5.4 Examine The Influence of d

The non-convex reformulation (4.2) requires an explicit rank estimation d on the true matrix. In this section, we investigate the influence of d on the Epinions dataset as an example. The rank d is chosen from [1, 5, 50, 100, 300, 500] and the results are plot in Figure 2. We observe that the rank has little influence on the performance. This is possibly because that the actual data has a low-rank structure (close to rank one). And from [Burer and Monteiro, 2005], we know that if d is larger than the actual rank, any local minimum of Eq. (4.2) is also a global optima.

6 Conclusion and Future Work

In this paper, we formulated the social trust prediction in the matrix completion framework. In particular, due to the special structure of the social trust problem, i.e., the measurements are 1-bit and the observed entries are non-uniformly sampled, we presented a max-norm constrained 1-bit matrix completion (MMC) algorithm. Since SDP solvers are not scalable to large scale matrices, we utilized a non-convex reformulation of the max-norm, which facilitates an efficient projected gradient decent algorithm. We empirically examined our algorithm on two benchmark datasets. Compared to other state-of-the-art matrix completion formulations, MMC consistently outperformed them, which meets with recently developed theories on max-norm. We also studied the trade-off between the accuracy and efficiency and observed that MMC achieved superior accuracy while keeping comparable computational efficiency.

The max-norm has been studied for several years and in many applications, such as collaborative filtering, clustering, subspace recovery. It is empirically and theoretically shown to be superior than the popular nuclear norm. This work investigates the power of max-norm for social trust problem and demonstrates encouraging results. It is interesting and promising to apply max-norm as a convex surrogate to other practical problems such as face recognition, subspace clustering etc.
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