Semileptonic Decays of $B_c$ Meson to a $P$-Wave
Charmonium State $\chi_c$ or $h_c$

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Abstract

The semileptonic decays of meson $B_c$ to a $P$-wave charmonium state $\chi_c(3P_J)$ or $h_c(1P_1)$ are computed. The results show that the decays are sizable so they are accessible in Tevatron and in LHC, especially, with the detectors LHCb and BTeV in the foreseeable future, and of them, the one to the $1P_1$ charmonium state potentially offers us a novel window to see the unconfirmed $h_c$ particle. In addition, it is pointed out that since the two charmonium radiative decays $\chi_c(3P_{1,2}) \rightarrow J/\psi + \gamma$ have sizable branching ratios, the cascade decays of the concerned decays and the charmonium radiative decays may affect the result of the observing the $B_c$ meson through the semileptonic decays $B_c \rightarrow J/\psi + l + \nu_l$ substantially.

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The observation of the meson $B_c$ through the semi-leptonic decays $B_c \rightarrow J/\psi + l + \nu_l$ has been reported very recently by CDF group [1]. According to the observation, the mass is $m_{B_c} = 6.40 \pm 0.39 \pm 0.13$ GeV, the lifetime $\tau_{B_c} = 0.46^{+0.18}_{-0.16} \pm 0.03$ ps etc. Nevertheless, all falls in the predicted region, if the theoretical uncertainties and experimental errors are taken into account [4][5]. In this letter, we briefly report the novel results of the estimates on the semileptonic decays of the meson $B_c$ to a $P$-wave chamonium $B_c \rightarrow \chi_c(h_c) + l + \nu_l$ (here $\chi_c$ are the spin triplet states $(c\bar{c})[3P_{0,1,2}]$ and $h_c$ the singlet $(c\bar{c})[1P_1]$). Besides the decays themselves, especially, as reported in the letter being accessible in foreseeable future, are interesting, the cascade decays $B_c \rightarrow \chi_c(3P_{1,2}) + l + \nu_l$ and $\chi_c(3P_{1,2}) \rightarrow J/\psi + \gamma$ followed may affect the result of observing the meson $B_c$ through semileptonic decays as done in CDF substantially, it is because the radiative decays $\chi_c(3P_{1,2}) \rightarrow J/\psi + \gamma$ are sizable.

In order to obtain reliable results, suitable approach to compute the decays should be chosen. It is well known that QCD inspired potential model works very well for non-relativistic

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double-heavy systems. The non-relativistic double heavy systems \((\bar{c}\bar{b})\) and \((\bar{c}b)\), except the reduce mass, are similar to the well-studied systems \((b\bar{b})\) and \((\bar{c}c)\), so it is believed that the static properties of the systems \((\bar{c}\bar{b})\) and \((\bar{c}b)\) can be predicted by potential model so well as the systems \((b\bar{b})\) and \((\bar{c}c)\). To apply the wave functions obtained by the potential model to compute the concerned decays for present purposes is attracting. Whereas, since \(m_{B_c} \sim 6.4\) GeV and \(m_{\chi_c} \sim 3.5\) GeV, to deal with a possible and great, even relativistic, momentum recoil effects properly in the decays, extra special efforts are needed. As done in computing the \(B_c\) meson decays to a S-wave charmonium \((\eta_c, J/\psi\text{ or } \psi')\) in Ref. \[4\], the approach, ‘generalized instantaneous approximation’, are adopted here. The key points of the approach are firstly to set the potential model on the ground of Bethe-Salpeter (B.S.) equation to describe the binding systems instead of Shr¨ odinger equation, then by means of the Mandelstam method \[8\], to establish the weak current matrix-element of the relevant process, i.e. the weak current sandwiched by two bound-state wave functions in B.S. formulation. Thus the current matrix element is written in a fully relativistic formulation. The third step is to make the so-called ‘generalized instantaneous approximation’ on the whole relativistic matrix element. Finally as a result of the approach, the matrix element turns to be formulated by means of the Schr¨ odinger wave functions with proper operators sandwiched, and the Schr¨ odinger wave functions are just the ones obtained by potential model for the systems, which are direct related to the B.S. wave functions by the original instantaneous approximation\[1\]. Note here that since the approach starts with B.S. equation for the binding systems and the Mandelstam formulation for the transition matrix element, the approach has more solid ground on the quantum field theory.

Due to the restriction on the lengthy for a letter, beyond a brief report we cannot describe the details of the estimates and on more decay modes here, but will put them in another achieved paper \[7\].

For the exclusive semileptonic decays \(B_c \rightarrow X_c + \ell^{\pm} + \nu_\ell\), the \(T-\)matrix element:

\[
T = \frac{G_F}{\sqrt{2}} V_{ij} \bar{u}_\nu \gamma_\mu (1 - \gamma_5) \nu_\ell < X_c(p', \epsilon) | J^\mu | B_c(p) >,
\]

where \(X_c\) indicates \(\chi_c\) and \(h_c\), \(\bar{u}_\nu\) and \(\nu_\ell\) are the spinors of the leptons, \(V_{ij}\) is the Cabibbo-Kobayashi-Maskawa (CKM) matrix element and \(J^\mu\) is the charged weak interaction current responsible for the decay, \(p, p'\) are the momenta of initial state \(B_c\) and final state \(X_c\). Thus we have:

\[
\sum |T|^2 = \frac{G_F^2}{2} |V_{ij}|^2 l^{\mu\nu} h_{\mu\nu},
\]

where \(l^{\mu\nu}\), being the leptonic tensor, can be calculated straightforwardly, and \(h_{\mu\nu}\), being the hadronic tensor, can be written as:

\[
h_{\mu\nu} = -\alpha g_{\mu\nu} + \beta_{++}(p + p')_\mu (p + p')_\nu + \beta_{+-}(p + p')_\mu (p - p')_\nu + \beta_{-+}(p - p')_\mu (p + p')_\nu + \beta_{--}(p - p')_\mu (p - p')_\nu + i\gamma_5 \epsilon_{\mu\nu\rho\sigma}(p + p')^\rho (p - p')^\sigma,
\]

and with the leptonic and hadronic tensors the differential decay rate may be obtained:

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1It is shown by Salpeter first in early 50’s.
\[
\frac{d^3 \Gamma}{dxdy} = |V_{ij}|^2 G_F^2 M^5 \left\{ \alpha \left( \frac{y - m_i^2}{M^2} \right) + 2\beta_+ + \left[ 2x \left( 1 - \frac{M'^2}{M^2} + y \right) - 4x^2 - y \right] \right. \\
+ \left. \frac{m_i^2}{4M^2} \left( 8x + \frac{4M'^2 - m_i^2}{M^2} - 3y \right) \right\} + 4(\beta_+ + \beta_-) \frac{m_i^2}{M^2} \left( 2 - 4x + y - \frac{2M'^2 - m_i^2}{M^2} \right) + 4\beta_- \frac{m_i^2}{M^2} \left( y - \frac{m_i^2}{M^2} \right) - \gamma \left[ y \left( 1 - \frac{M'^2}{M^2} - 4x + y \right) + \frac{m_i^2}{M^2} \left( 1 - \frac{M'^2}{M^2} + y \right) \right],
\]

where \( x \equiv E_\ell / M \) and \( y \equiv (p - p')^2 / M^2 \), \( M, m_i \) are the masses of \( B_c \) meson and the charged lepton respectively, \( M' \) is the mass of final state \( X_c \). The coefficient functions \( \alpha, \beta_+, \gamma \) are related to the form factors which are defined in different cases respectively:

1. In the case \( X_c = h_c([1P_1]) \) state, the vector current matrix element can be written as:
\[
\langle X_c(p', \epsilon)|V_\mu|B_c(p)\rangle \equiv r \epsilon^*_\mu + s_+(\epsilon^* \cdot p)(p + p')_\mu + s_-(\epsilon^* \cdot p)(p - p')_\mu,
\]

and the axial vector current matrix element:
\[
\langle X(p', \epsilon)|A_\mu|B_c(p)\rangle \equiv i\mu_\nu \rho_\sigma \epsilon^* \mu \nu (p + p')_\rho (p - p')_\sigma,
\]

where \( p \) and \( p' \) are the momenta of \( B_c \) and \( h_c \) respectively, \( \epsilon \) is the polarization vector of \( h_c \).

2. In the case \( X_c = \chi_c([3P_0]) \) state, the axial current
\[
\langle X_c(p')|A_\mu|B_c(p)\rangle \equiv u_+(p + p')_\mu + u_-(p - p')_\mu.
\]
The vector matrix element vanishes.

3. In the case \( X_c = \chi_c([3P_1]) \) state,
\[
\langle X_c(p', \epsilon)|V_\mu|B_c(p)\rangle \equiv le^*_\mu + c_+(\epsilon^* \cdot p)(p + p')_\mu + c_-(\epsilon^* \cdot p)(p - p')_\mu,
\]

and
\[
\langle X_c(p', \epsilon)|A_\mu|B_c(p)\rangle \equiv i\rho_\mu \nu \sigma \epsilon^* \mu \nu (p + p')_\rho (p - p')_\sigma.
\]

4. In the case \( X_c = \chi_c([3P_2]) \) state,
\[
\langle X_c(p', \epsilon)|V_\mu|B_c(p)\rangle \equiv ke^*_\mu \nu (p + p')_\nu + b_+(\epsilon^* \rho p^\rho p^\sigma)(p + p')_\mu + b_-(\epsilon^* \rho p^\rho p^\sigma)(p - p')_\mu
\]

and
\[
\langle X_c(p', \epsilon)|V_\mu|B_c(p)\rangle \equiv i\rho_\mu \nu \sigma \epsilon^* \mu \nu (p + p')_\rho (p - p')_\sigma.
\]
FIG. 1. The universal function of $\xi_1$ and $\xi_2$: overlap-integrations of $\chi_c(h_c)$ and $B_c$ wave functions, where the solid line is of $\xi_1$, the dashed line is of $\xi_2$.

The form factors $r, s_+, s_-, v, u_+, u_-, l, c_+, c_-, k, b_+, b_-$ and $h$ are functions of the momentum transfer $(p - p')^2$ and can be calculated precisely with the generalized instantaneous approximation approach [4]. Similar to the heavy mesons where all factors depend on only one function (Isgur-Wise function), here all the form factors will depend on two independent functions $\xi_1$, $\xi_2$ with certain kinematics factors (see the example for $h_c$ below) instead, and the functions $\xi_1$ and $\xi_2$ are just two different overlap-integrations of the wave functions of the initial and final bound states:

$$
\epsilon^\lambda(L) \cdot \epsilon_0 \xi_1 = \int \frac{d^3q_{p'\perp}}{(2\pi)^3} \psi_{n_{1M_z}}^* (q'T) \psi_{n_{00}} (qT),
$$

$$
\epsilon^\alpha_\lambda(L) \xi_2 = \int \frac{d^3q_{p'\perp}}{(2\pi)^3} \psi_{n_{1M_z}}^* (q'T) \psi_{n_{00}} (qT) q_{p'\perp},
$$

where

$$
\epsilon_0 = \frac{p - \frac{p \cdot p'}{M'z} p'}{\sqrt{\frac{(p \cdot p')^2}{M'^2} - M^2}}
$$

describes the polarization vector along recoil momentum $p'$, and $\epsilon^\lambda(L)$ is polarization vector for orbital angular momentum.

The two functions $\xi_1$ and $\xi_2$ take into account the recoil effects properly indeed. Note especially that $\xi_1$ cannot be obtained by boosting the final state wave function as done in the cases when recoil is small in atomic and nuclear processes ($\xi_1$ approaches to zero when the recoil momentum vanishes).

To see the behavior, let us present the functions $\xi_1$ and $\xi_2$ versus $t_m - t$ in Fig.1, where $t_m = (M - M')^2$ and $t = (P - P')^2$. 
The precise formula of the form factors on the functions $\xi_1, \xi_2$ in various cases, being lengthy for the letter, cannot be written down here, we put them elsewhere in Ref. [7], but to see the general features, only the form factors $r, s_+, s_-$ and $v$ for $h_c$ are given here:

$$r = \frac{(m_1' - m_2')(m_1 + \omega_{10} - m_2 + \omega_{20})\xi_2}{8m'_1\omega_{10}\omega_{20}} - \frac{(m_1' + m_2)(m_1 + \omega_{10} + m_2 + \omega_{20})\xi_2(p\cdot p')}{8M'Mm'_1\omega_{10}\omega_{20}}, \quad (13)$$

$$s_+ = \frac{m_2[M(m_2 + \omega_{20} - m_1 - \omega_{10}) - M'(m_1 + \omega_{10} + m_2 + \omega_{20})]\xi_2}{8M'M^2\omega_{10}\omega_{20}^2} + \frac{m_2[M(m_2 + \omega_{20} - m_1 - \omega_{10}) - M'(m_1 + \omega_{10} + m_2 + \omega_{20})]\xi_1}{8M'M\omega_{10}\omega_{20}nep} + \frac{m_2[M(m_2\omega_{20} + \omega_{20}^2 - m_1\omega_{20} - \omega_{10}^2) - M'(m_1\omega_{20} - \omega_{10}^2 + m_2\omega_{20} + \omega_{20}^2)]\xi_2}{8M'M^2\omega_{10}^3\omega_{20}} + \frac{(m_1' + m_2)(m_1 + \omega_{10} + m_2 + \omega_{20})\xi_2}{16M'Mm'_1\omega_{10}\omega_{20}} - \frac{(M' - M)\xi_1}{8M'M^2\omega_{10}}, \quad (14)$$

$$s_- = \frac{m_2[-M(m_2 + \omega_{20} - m_1 - \omega_{10}) - M'(m_1 + \omega_{10} + m_2 + \omega_{20})]\xi_2}{8M'M^2\omega_{10}\omega_{20}^2} + \frac{m_2[-M(m_2 + \omega_{20} - m_1 - \omega_{10}) - M'(m_1 + \omega_{10} + m_2 + \omega_{20})]\xi_1}{8M'M\omega_{10}\omega_{20}nep} + \frac{m_2[-M(m_2\omega_{20} + \omega_{20}^2 - m_1\omega_{20} - \omega_{10}^2) - M'(m_1\omega_{20} - \omega_{10}^2 + m_2\omega_{20} + \omega_{20}^2)]\xi_2}{8M'M^2\omega_{10}^3\omega_{20}} - \frac{(m_1' + m_2)(m_1 + \omega_{10} + m_2 + \omega_{20})\xi_2}{16M'Mm'_1\omega_{10}\omega_{20}} + \frac{(M' - M)\xi_1}{8M'M^2\omega_{10}}, \quad (15)$$

$$v = -\frac{(m_1' + m_2)(m_1 + m_2 + \omega_{10} + \omega_{20})\xi_2}{16M'Mm'_1\omega_{10}\omega_{20}}, \quad (16)$$

Here $m_1, m_2$ are ‘effective masses’ of $\bar{b}$-quark and $c$-quark in the meson $B_c$ and $m'_1, m_2$ are those of $\bar{c}$-quark and $c$-quark in the $p$-wave charmonium state respectively.

For convenience, here we have also introduced the auxiliary parameters:

$$\omega_{20} \equiv \omega_2'\frac{p\cdot p'}{MM'},$$

$$\omega_{10} \equiv \sqrt{\omega_{20}^2 - m_2^2 + m_1^2},$$

$$nep = \sqrt{(p\cdot p')^2 - M^2}.$$
\[ \beta_{++} = \frac{r^2}{4M'^2} - M'^2yv^2 + \frac{1}{2} \left[ \frac{M^2}{M'^2}(1 - y) - 1 \right] rs_+ + \frac{1}{2}\left[ \frac{M^2}{M'^2}(1 - y) - 3 \right] rs_- + \frac{M^2}{M'^2}s_+s_- , \]

(18)

\[ \beta_{+-} = \frac{r^2}{4M'^2} + (M^2 - M'^2)v^2 + \frac{1}{4} \left[ -\frac{M^2}{M'^2}(1 - y) - 3 \right] rs_+ 
+ \frac{1}{4} \left[ \frac{M^2}{M'^2}(1 - y) - 1 \right] rs_- + \frac{M^2}{M'^2}s_+s_- , \]

(19)

\[ \beta_{-+} = \beta_{+-} \]

(20)

\[ \beta_{--} = \frac{r^2}{4M'^2} + \left[ M^2y - 2(M^2 + M'^2) \right] v^2 + \frac{1}{2} \left[ -\frac{M^2}{M'^2}(1 - y) - 3 \right] rs_- + \frac{M^2}{M'^2}s_+s_- , \]

(21)

\[ \gamma = 2rv. \]

(22)

In the other cases for \( \chi_c[3P_{0,1,2}] \), the situation is similar. Now with Eq.(4) and the functions \( \alpha, \beta_{++}, \beta_{+-}, \beta_{-+}, \beta_{--} \) and \( \gamma \) in each case for \( h_c \) and \( \chi_c[3P_{0,1,2}] \), the differential decay rates may be obtained respectively for all the semileptonic decays.

In our numerical calculations, the values of the relevant parameters are taken from ref. [9]: \( V_{bc} = 0.04, m_c = 1.846 \text{ GeV}, m_b = 5.243 \text{ GeV}, M' = 3.497 \text{ GeV} \) (the average value of the mass of \( \chi_c \) and \( h_c \)) and \( M = 6.213 \text{ GeV} \) (the mass of \( B_c \)). The wave functions of \( B_c \) and \( \chi_c, h_c \) are taken from potential model with suitable parameters [9].

The integrated widths for the semileptonic decays \( B_c \rightarrow h_c(\chi_c) + e(\mu, \tau) + \nu \), as final results, are shown in Table I.

**Table I. Decay widths with the unit \( 10^{-15} \text{ GeV} \)**

| \( l \) | \( \Gamma(B_c \rightarrow h_c[3P_1] \ell \nu) \) | \( \Gamma(B_c \rightarrow \chi_c[3P_0] \ell \nu) \) | \( \Gamma(B_c \rightarrow \chi_c[3P_1] \ell \nu) \) | \( \Gamma(B_c \rightarrow \chi_c[3P_2] \ell \nu) \) |
|---|---|---|---|---|
| \( e(\mu) \) | 2.509 | 1.686 | 2.206 | 2.732 |
| \( \tau \) | 0.356 | 0.249 | 0.346 | 0.422 |

In the table, the first line means those of \( l = e, \mu \) and the second one means that of \( l = \tau \).

To see the further feature of the decays, we present the lepton energy spectra for \( e, \mu \) only in Fig.2, where \( |p_\ell| \) is the momentum of lepton.
FIG. 2. The lepton energy spectrum for $B_c \to \chi_c + e(\mu) + \nu$, where the solid line is the result of $h_c[1P_1]$ state, the dotted-blank-dashed line is of $\chi_c[3P_0]$, the dashed line is of $\chi_c[3P_1]$, the dotted-dashed line is of $\chi_c[3P_2]$.

If comparing the results in Table 1 with the decays of $B_c$ to $S$-wave charmonium $J/\psi$ and $\eta_c$ ($\Gamma(B_c \to J/\psi + l + \nu) \sim 25 \cdot 10^{-15}\text{GeV}$, ref. [4,5]) and considering the fact about the observation of $B_c$ by CDF group, one can conclude that the decays concerned here are accessible in Tevatron and in LHC, especially, in the detectors LHCB and BTeV in the foreseeable future. If considering the fact that the $P$-wave charmonium $h_c[1P_1]$ has not been confirmed experimentally yet, it is quite interesting that the $B_c$ decay to the $P$-wave state $h_c$, in fact, potentially offers us a novel window to see it. Furthermore, since the two charmonium radiative decays $\chi_c[3P_{1,2}] \to J/\psi + \gamma$ have sizable branching, the cascade decays i.e. the decays $B_c \to \chi_c[3P_{1,2}] + l + \nu_l$ with an accordingly radiative one $\chi_c[3P_{1,2}] \to J/\psi + \gamma$, being a substantial background, may make certain affects on the result of the observation of the $B_c$-meson through its semileptonic decays.

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