The End of Unified Dark Matter?

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Despite the interest in dark matter and dark energy, it has never been shown that they are in fact two separate substances. We provide the first strong evidence that they are separate by ruling out a broad class of so-called unified dark matter models that have attracted much recent interest. We find that they produce oscillations or exponential blowup of the matter power spectrum inconsistent with observation. For the particular case of generalized Chaplygin gas models, 99.999% of the previously allowed parameter space is excluded, leaving essentially only the standard ΛCDM limit allowed.

I. INTRODUCTION

Despite the broad interest in dark matter and dark energy, their physical properties are still poorly understood. Indeed, it has never even been shown that the two are in fact two separate substances. The goal of this paper is to show that they are.

There is strong evidence from a multitude of observations that there is about six times more cold dark matter (CDM) than baryons in the cosmic matter budget, making up of order 30% of critical density [1,2]. In addition to this clustering dark component, observations of supernovae, the cosmic microwave background fluctuations and galaxy clustering provide mounting evidence of a uniformly distributed dark energy with negative pressure which has come to dominate the universe recently (at redshifts $z \lesssim 1$) and caused its expansion to accelerate. It currently constitutes about two thirds of the critical density [1–3].

Although the dark energy can be explained by introducing the cosmological constant ($\Lambda$) into general relativity (lending the standard model the name ΛCDM), this “solution” has two severe problems, frequently triggering anthropic explanations and general unhappiness. The first problem is explaining its magnitude, since theoretical predictions for $\Lambda$ lie many orders of magnitude above the observed value. The second problem is the so-called cosmic coincidence problem: explaining why the three components of the universe (matter, radiation and $\Lambda$) presently are of similar magnitudes although they all scale differently with the Universe’s expansion.

As a response to these problems, much interest has been devoted to models with dynamical vacuum energy, so-called quintessence [4]. These models typically involve scalar fields with a particular class of potentials, allowing the vacuum energy to become dominant only recently. Although quintessence is the most studied candidate for the dark energy, it generally does not avoid fine tuning in explaining the cosmic coincidence problem. Recently several alternative models have been proposed, such as the condensate models of [5].

An alternative to quintessence which has attracted great interest lately is the so-called generalized Chaplygin gas (hereafter GCG) [6,7,9,10,11] (see also the related earlier work of [12]). Rather than fine tuning some potential, the model explains the acceleration of the Universe via an exotic equation of state causing it to act like dark matter at high density and like dark energy at low density. The model is interesting for phenomenological reasons but can be motivated by a brane-world interpretation [8,7]. An attractive feature of the model is that it can explain both dark energy and dark matter in terms of a single component, and has therefore been referred to as unified dark matter (UDM) or “quartessence” [13]. (See also [17].)

This approach has been thoroughly investigated for its impact on the 0th order cosmology, i.e., the cosmic expansion history (quantified by the Hubble parameter $H(z)$) and corresponding spacetime-geometric observables. An interesting range of models was found to be consistent with SN Ia data [13] and CMB peak locations [15].

Some work has also studied constraints from 1st order cosmology (the growth of linear perturbations), finding an interesting range of models to be consistent with cosmic microwave background (CMB) measurements [16]. There is, however, a fatal flaw in UDM models that manifests itself only at recent times and on smaller (Galactic) scales and has therefore not been revealed by these studies. As we will see, this flaw rules out all GCG models except those that are for all practical purposes identical to the usual ΛCDM model.

The rest of this Letter is organized as follows. In the next section, we review the fundamentals of the GCG model. We then consider in section III the evolution of density inhomogeneities in the model and use the predicted matter power spectrum to constrain it with observational data. Finally, we describe how the basic flaw that rules out the GCG model is indeed a generic feature of a broad class of unified dark matter models.
II. THE CHAPLYGIN GAS

A standard assumption in cosmology is that the pressure of a single substance is, at least in linear perturbation theory, uniquely determined by its density. A generalized Chaplygin gas [6,8,9] is simply a substance where the pressure $p(\rho)$ is a power law

$$p = -A \rho^{-\alpha}$$  \hspace{1cm} (1)

with $A$ a positive constant. The original Chaplygin gas had $\alpha = 1$. The standard $\Lambda$CDM model has two separate dark components, both with $\alpha = -1$, giving a constant equation of state $w = p/\rho$ that equals 0 for dark matter and $-1$ for dark energy.

By inserting equation (1) into the energy conservation law, one finds that the GCG density evolves as [13]

$$\rho(t) = \left[ A + \frac{B}{a^{3(1+\alpha)}} \right]^{-\frac{1}{1+\alpha}},$$  \hspace{1cm} (2)

where $a(t)$ is the cosmic scale factor normalized to unity today, i.e., $a = (1 + z)^{-1}$ where $z$ is redshift. Here $B$ is an integration constant. The striking feature here is that although the GCG has $\rho \propto a^{-3}$ when sufficiently compressed, its density will never drop below the value $A^{1/\alpha}$ no matter how much you expand it. Defining

$$\Omega_m^* \equiv \frac{B}{A+B}, \quad \rho_\alpha \equiv (A+B)^{-1/(1+\alpha)},$$  \hspace{1cm} (3)

equation (2) takes the form

$$\rho(a) = \rho_\alpha \left[ (1 - \Omega_m^*) + \Omega_m^* a^{-3(1+\alpha)} \right]^{-1/(1+\alpha)}.$$  \hspace{1cm} (4)

For comparison, a standard flat model with current CDM density parameter $\Omega_m$ as well as dark energy density $(1 - \Omega_m)$ whose equation of state $w_* \equiv \rho/\rho_0$ is constant gives

$$\rho(a) = \rho_* \left[ (1 - \Omega_m) a^{-3(1+w_*)} + \Omega_m a^{-3} \right].$$  \hspace{1cm} (5)

We see that the last two equations bear a striking similarity even though the former involves a single substance and the latter involves two. Both have two free parameters. Both have the current density $\rho(1) = \rho_*$. Making the identification $\Omega_m^* = \Omega_m$, both have $\rho(a) \rightarrow \Omega_m \rho_\alpha a^{-3}$ at early times as $a \rightarrow 0$ (for $w_* < 0$), showing that $\Omega_m^*$ can be interpreted as an effective matter density in the GCG model. Indeed, for the special case $\alpha = 0$ and $w_* = -1$, we see that both models coincide with standard $\Lambda$CDM. For $\alpha = 0$ the GCG model becomes equivalent to $\Lambda$CDM not only to 0th order in perturbation theory as above but to all orders, even in the nonlinear clustering regime.

The 0th order cosmology determined by equation (4) together with the Friedmann equation

$$H \equiv \frac{\dot{a}}{a} = \left[ \frac{8\pi G}{3} \right]^{1/2} \rho_\alpha^{1/2},$$  \hspace{1cm} (6)

(which determines $a(t)$ and the spacetime metric to 0th order) has been thoroughly in previous work [6,8], and by studying constraints from supernovae observations, Makler et al. [13] have placed interesting constraints on the $(\alpha, \Omega_m^*)$ parameter space.

III. GROWTH OF INHOMOGENEITIES

Let us now consider the evolution of density perturbations in this UDM model. Following the standard calculations of [17], we obtain for the relativistic analog of the Newtonian 1st order perturbation equation in Fourier space that a density fluctuation $\delta_k$ with wave vector $k$ evolves as

$$\ddot{\delta}_k + H \dot{\delta}_k [2 - 3(2w - c_s^2)] = \frac{3}{2}H^2 \delta_k \left[ 1 - 6c_s^2 - 3w^2 + 8w \right] = -\left( \frac{kc_s}{a} \right)^2 \delta_k,$$  \hspace{1cm} (7)

where the equation of state $w \equiv p/\rho$ and the squared sound speed $c_s^2 \equiv \partial p/\partial \rho$ are evaluated to 0th order and hence depend only on time, not on position. (We use units where the speed of light $c = 1$ throughout.) This equation is valid on subhorizon scales $|k| \gg H/c$. In other words, the growth of density fluctuations is completely determined by the two functions $w(a)$ and $c_s^2(a)$. Combining equation (1) and equation (4), these two functions are [13]

$$w = -\left[ 1 + \frac{\Omega_m^*}{1 - \Omega_m^*} a^{-3(1+\alpha)} \right]^{-1},$$  \hspace{1cm} (8)

$$c_s^2 = -\alpha w = \alpha \left[ 1 + \frac{\Omega_m^*}{1 - \Omega_m^*} a^{-3(1+\alpha)} \right]^{-1}.$$  \hspace{1cm} (9)

This shows a second reason why the GCG has been considered promising for cosmology it starts out behaving like pressureless CDM ($w \approx 0$, $c_s \approx 0$) early on (for $a \ll 1$) and gradually approaches cosmological constant behavior ($w \approx -1$) at late times. There is also an intermediate state where the effective equation of state is $p = \alpha \rho$ [6]. (Going beyond 1st order perturbation theory, the GCG that gets gravitationally bound in galactic halos maintains its density high enough to keep acting like CDM forever.)

To solve equation (7) numerically, we change the independent variable from $t$ to $\ln a$. Using the properties

$$\frac{d}{dt} = H \frac{d}{d\ln a}, \quad \delta_k = H^2 \delta'' + \frac{1}{2}(H^2)' \delta',$$  \hspace{1cm} (10)

where $' \equiv d/d\ln a$ and

$$\xi \equiv \frac{(H^2)'}{2H^2} = -\frac{3}{2} \left( 1 + (1/\Omega_m^* - 1)a^{3(1+\alpha)} \right)^{-1},$$  \hspace{1cm} (11)

equation (7) takes the form
Defining a critical wavelength it the Horizon scale divided by the perturbation scale. \( k/aH \) by the prefactor speed is tiny (13) are of order unity or smaller, the sound speed is exponentially (18). This oscillation is confirmed by the numerical solutions, and is analogous to the acoustic oscillations in the photon-baryon fluid in the pre-decoupling epoch. If \( c_s^2 < 0 \), corresponding to negative \( \alpha \), fluctuations below this wavelength will be violently unstable and grow exponentially [18].

A key point which has apparently been overlooked in prior work is that whereas all the other terms in equation (13) are of order unity or smaller, the sound speed term \((kc_s/aH)^2\) can be much larger even if the sound speed is tiny \( |c_s| \ll 1 \). This is because \( c_s \) is multiplied by the prefactor \( k/aH \) which can be enormous, since it it the Horizon scale divided by the perturbation scale. Defining a critical wavelength \( \lambda_c \) by

\[
\lambda_c^2 = \frac{c_s^2}{(aH)^2} = -\frac{\alpha w}{(aH)^2},
\]

the pressure term in equation (13) becomes simply \((\lambda_c k)^2\), so we expect oscillations or exponential blowup in the power spectrum on scales \( k \gtrsim \lambda_c^{-1} \). These are created mainly during the recent transition period when both \( a \) and \( -w \) are or order unity (growing from 0 to 1), and since neither effect is seen in observed data, we therefore expect to obtain constraints of order \( |\alpha| \lesssim (H/k)^2 \), the squared ratio of the perturbation scale to the horizon scale. This heuristic argument thus suggests that Galaxy clustering constraints on scales down to \( 10^{-1} \text{Mpc} \) would give the constraint \( |\alpha| \lesssim (10^{-1}\text{Mpc}/3000\text{h}^{-1}\text{Mpc})^2 \approx 10^{-5} \) — we will see that this approximation is in fact fairly accurate.

For our numerical calculations, we evolved a scale invariant Harrison-Zeldovich spectrum up to redshift \( z = 100 \) (before which the GCG is indistinguishable from \( \Lambda \text{CDM} \)) with CMBfast [19] to correctly include all the relevant effects (early super-horizon evolution, pre-recombination acoustic oscillations, Silk-damping, etc.), with cosmological parameters given by the concordance model of [2]. We then used equation (13) to evolve the fluctuations from \( z = 100 \) until today. Results for a sample of \( \alpha \)-values are plotted in Figure 1, and show how tiny non-zero values of \( \alpha \) result in large changes on small scales as expected.

We constrain \( \alpha \) by making a \( \chi^2 \) fit of the theoretically predicted power spectrum against that observed with the 2dF 100k Galaxy Redshift Survey [20] as analyzed by [21]. For each \( \alpha \), we use the best fitting power normalization to ensure that our constraints come only from the shape of the power spectrum, not from the overall amplitude which involves mass-to-light bias. To be conservative and stay firmly in the linear regime, we discard data with \( k > 0.3h/\text{Mpc} \). We run our code for a fine grid of models with \(-1 < \alpha < 1\) to find the corresponding \( \chi^2 \) values. The likelihood function \( e^{-\Delta \chi^2/2} \) is plotted in Figure 2. It predictably peaks around \( \alpha \approx 0 \), and the observed skewness is simply due to the fact that the oscillating solution (\( \alpha > 0 \)) is easier to fit than the exponentially unstable solution (\( \alpha < 0 \)). Setting \( \Delta \chi^2 = 1 \) cut-off as in a crude Bayesian analysis gives the constraints \(-0.00000081 < \alpha < 0.0000079 \).

To place this result in context, Figure 3 shows the 0th order constraints from Makler et al. [13] with our new strict constraints superimposed.

### IV. CONCLUSIONS

Above we showed that GCG models with \( |\alpha| \gg 10^{-5} \) are ruled out by observation, since they cause fluctuations or blowup in the matter power spectrum that are not observed. Let us now examine the assumptions that went into this and the broader implications.

First of all, our extremely tight constraints imply that the narrow range of allowed GCG models will be completely indistinguishable from \( \Lambda \text{CDM} \) to both to
FIG. 2. The likelihood function $e^{-\Delta \chi^2/2}$ as a function of the GCG parameter $\alpha$. It is sharply peaked around $\alpha = 0$ which is equivalent to the $\Lambda$CDM model. From top to bottom, the horizontal dashed lines correspond to $\Delta \chi^2 = 1$ and 4, respectively.

FIG. 3. The graph is showing constraints from previous work by Makler et al. Our new constraints from first order perturbation theory are superimposed on the plot as shown above. 0th order and at the early times when primary CMB anisotropies are produced. This means that the corresponding standard constraints on cosmological parameters from CMB, SN Ia etc. apply also to the GCG models making the identification $\Omega^*_m = \Omega_m$, so that there are no interesting degeneracies between $\alpha$ and other parameters that can significantly widen the allowed $\alpha$-range. We have therefore used standard constraints $0.2 < \Omega_m < 0.4$ for the allowed region in Figure 3.

Second, our limiting our constraints to linear scales $k < 0.3h$/Mpc was probably overly conservative. As reviewed in [22,23], there are quite strong constraints on the linear power spectrum on much smaller scales from weak lensing, from the Lyman $\alpha$ forest and perhaps even from lensing of halo substructure [24] which if used would tighten our upper limit on $\alpha$ to $10^{-6}$, $10^{-7}$ and $10^{-10}$, respectively.

Third, we saw that all that really mattered in equation (13) as far as the constraints were concerned was the pressure term $(\lambda c k)^2$. This means that our results apply more generally than merely to the GCG case: any unified dark matter model where $p$ is a unique function of $\rho$ is ruled out if the sound speed is non-negligible, i.e., if the function $p(\rho)$ departs substantially from a constant over the range where pressure has an effect — quantitatively, if $|d\ln p/d\ln \rho| \gtrsim 10^{-5}$, again rendering it indistinguishable from standard $\Lambda$CDM.

In contrast, standard quintessence models have no such problems. Although they typically have high sound speeds causing oscillations as above, this does not prevent the dark matter from clustering since it is a separate dynamic component. Quintessence models would fail as above if there the two components were tightly coupled, and this is effectively what happens with UDM since the two are one and the same substance.

To salvage the UDM idea in some form, its pressure must not be uniquely determined by its density — not even on subhorizon scales. As worked out in detail by Hu [18], the effective sound speed can under some circumstances differ from the adiabatic sound speed, and it is only the former that must approximately vanish to satisfy our constraints. Although it may be possible to concoct such models, say by introducing another physical field upon which $p$ depends and making it fluctuate in such a way as to cancel the problematic pressure gradients, this would be giving up much of the elegance and simplicity that gave the unified dark matter idea its appeal, essentially substituting one extra field for another.

In conclusion, precision data is gradually allowing us to test rather than assume the physics underlying modern cosmology. We have taken a step in this direction by ruling out a broad class of so-called unified dark matter models. Our results indicate that dark energy is either indistinguishable from a pure cosmological constant or a separate component from the dark matter with a life of its own.

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