Stabilization of nonlinear plasma waves by radiation

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Abstract

It is shown that the interaction of a plasma with thermal radiation leads to the stabilization of both periodic and solitary nonlinear plasma waves. The stabilized periodic nonlinear plasma waves were indeed observed in the experiment carried out by Looney and Brown. There are many branches of radiation-induced solitary waves in plasmas, corresponding in particular to the density waves in hot plasmas of accretion disks.

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It is known that longitudinal plasma waves are subject to the linear and nonlinear Landau damping [1,2]. On the other hand, electron plasma waves are unstable with respect to the modulation instability and formation of envelope solitons. However, in a plasma without a magnetic field, plane solitons should be also unstable due to the transverse modulation, and therefore they cannot exist, according to the theoretical predictions [3]. The theories of Langmuir wave collapse and Landau damping do not take into account the interaction of a plasma with thermal radiation. Here we show that the interaction of a plasma with radiation leads to the stabilization of both periodic and solitary electron plasma waves. Note that there is no direct experimental evidence of the existing of Langmuir wave collapse [3].

In the experiment carried out by Looney and Brown [4], a beam of several hundred $eV$ electrons, injected into the plasma of a dc discharge from an auxiliary electron gun, excited oscillations in the plasma at the plasma electron oscillation frequency. However, Looney and Brown have observed that the frequency of oscillation varies in a discontinuous manner with the electron density $n_e$ which is proportional to the gun discharge current. The frequency remained nearly constant over small variations of the electron density and then jumped abruptly to a new frequency. Along one of these frequency plateaus, the intensity of the oscillation rose to a maximum and then fell to a much lower value before a jump to a new frequency occurred. The frequency corresponding to the strongest signal for each frequency range was consistent with the Tonks - Langmuir relation

$$\omega_{pe} = \left(4\pi n_e e^2/m_e\right)^{1/2},$$

where $m_e$ is the mass of an electron.

A movable probe showed the existence of standing - wave patterns of the oscillatory energy in the region of the plasma. Nodes of the patterns coincided with the electrodes which limited the region of the plasma traversed by the beam. Each of the observed frequency plateau corresponded to the definite number of waves in a standing - wave pattern.

We argue that Looney and Brown have observed nonlinear plasma oscillations stabilized by the plasma emission. It was shown recently ([5] and Refs therein) that thermal emission has a stimulated character. According to this conception thermal emission from non-uniform gas is produced by an ensemble of individual emitters. Each of these emitters is an elementary resonator the size of which has an order of magnitude of mean free path $l$.
of photons

\[ l = \frac{1}{n\sigma} \]  \hspace{1cm} (2)

where \( n \) is the number density of particles and \( \sigma \) is the absorption cross-section.

The emission of each elementary resonator is coherent, with the wavelength

\[ \lambda = al, \]  \hspace{1cm} (3)

where \( a \) is a dimensionless constant, and thermal emission of gaseous layer is incoherent sum of radiation produced by individual emitters.

An elementary resonator emits in the direction opposite to the direction of the density gradient. The wall of the resonator corresponding to the lower density is half-transparent due to the decrease of absorption with the decreasing gas density.

The relation (3) implies that the radiation with the wavelength \( \lambda \) is produced by the gaseous layer with the definite number density of particles \( n \).

In the case of the experiment carried out by Looney and Brown, \( n = n_i = n_e \), due to the neutrality of the plasma, and the absorption cross-section we assume to be equal to those for the fully ionized plasma, \( \sigma = \sigma_0 = 10^{-9} \text{cm}^2 \). The wavelength of nonlinear plasma wave coincides with the wavelength of plasma emission as determined by the relation (3).

The wavelength, \( \lambda \), of standing wave can be calculated as \( \lambda = d/N \), where \( d = 1.5 \text{cm} \) is the plasma length, the thickness of the ion sheaths at each of two electrodes which limited the plasma region traversed by the electron beam being neglected, and \( N \) is the number of waves in a standing - wave pattern. The electron density corresponding to the strongest signal for each frequency plateau can be taken from Fig.2 in [4].

From equations (2) and (3), we find the wavelength of standing wave in the form

\[ \lambda = a/ (n_e\sigma_0). \]  \hspace{1cm} (4)

Since \( \lambda = d/N \), where \( N \) is a half-integer number, oscillations would be observed only at definite values of the electron density \( n_e \). However, an elementary resonator has a finite bandwidth [6]

\[ \Delta \nu \approx v_{Te}/al, \]  \hspace{1cm} (5)
where $v_{Te}$ is the thermal velocity of electrons. Due to the finite bandwidth of an elementary resonator, oscillations can be observed also at other values of the electron density, the intensity of the oscillation at the “wings” of bandwidth much lower than the resonant intensity, as it was indeed observed by Looney and Brown.

According to the relation (4), the product of the wavelength, $\lambda$, by the electron density, $n_e$, corresponding to the maximum intensity of oscillations, should be a constant for all frequency plateaus observed by Looney and Brown. In Table 1 we give the calculated values of $a = \lambda n_e \sigma_0$ for each frequency plateau.

### Table 1

An elementary resonator in the Looney - Brown experiment.

| Number of waves $N$ | Wavelength $\lambda$ | Electron density $n_e \left(10^9 cm^{-3}\right)$ | Product $a = \lambda n_e \sigma_0$ |
|---------------------|---------------------|-----------------------------------------------|----------------------------------|
| 3/2                 | 1.0                 | 4.0                                           | 4.0                              |
| 2                   | 0.75                | 6.0                                           | 4.5                              |
| 5/2                 | 0.6                 | 7.5                                           | 4.5                              |
| 3                   | 0.5                 | 10.0                                          | 5.0                              |
| 7/2                 | 0.4                 | 12.5                                          | 5.0                              |

The wavelength, $\lambda$, of standing wave is calculated as $\lambda = d/N$, where $d = 1.5 cm$ is the plasma length, and $N$ is the number of waves in a standing - wave pattern. The electron density, $n_e$, corresponding to the maximum intensity of oscillations, is taken from Fig.2 in [4]. The absorption cross-section is assumed to be equal $\sigma_0 = 10^{-9} cm^2$.

The calculated value of $a$ is slightly varying with the electron density $n_e$. A possible explanation is that the relation (4) has a statistical origin and suggests a sufficiently large number of waves, $N$. However, in the experiment by Looney and Brown, only the lowest modes with small values of $N$ were observed. For small numbers of waves in a standing - wave pattern, the edge effects, such as the ion sheaths at each of two electrodes which
limited the plasma region traversed by the electron beam, are essential and can disturb
the experimental results. An accurate estimation of such edge effects with account for the
interaction of a plasma with radiation seems to be a difficult problem.

There seem to be different types of envelope solitary waves stabilized by radiation. One of
these types of radiation-induced solitary waves in a plasma may be called a Looney-Brown
soliton. It is an ordinary Langmuir solitary wave [3,7] in which the radiation pressure is
small compared to the high-frequency pressure, but the wavelength of a carrying mode
is determined by the relation (4), similar to the wavelength of the periodic wave in the
experiment by Looney and Brown.

The periodic wave is unstable with respect to the formation of local regions of high-
frequency electric field. Plasma is pushed out from these regions by high-frequency pressure,
so that the cavities of lower plasma density are formed, an electric field being locked in these
cavities. The Looney-Brown envelope soliton represents such a localized region of lower
plasma density travelling at a group velocity of electron plasma waves

\[ u = 3T_e k_0 / (m_e \omega_{p0}) \]

where \( T_e \) is the electron temperature, the electron plasma frequency \( \omega_{p0} \) corresponds to the
undisturbed plasma density \( n_0 \), and \( k_0 \) is the wave number of the carrying mode,

\[ k_0 = 2\pi / \lambda = 2\pi n_0 \sigma_0 / a. \]

Here we make use of the equation (4) for the wavelength \( \lambda \). Substituting relations (4) and (7) in equation (6), we obtain

\[ u = (9T_e^2 n_0 \sigma_0^2 / am_e e^2)^{1/2}. \]

For the values of the electron temperature \( T_e \approx 1eV \) and the plasma density \( n_0 \approx 10^{12} cm^{-3} \) which are characteristic for low-temperature high-density plasma, the last equation
gives \( u \approx 3 \times 10^8 cms^{-1} \).

Another type of radiation-induced solitary wave is the density waves propagating in hot
plasmas of accretion disks surrounding the compact astrophysical objects such as active
galactic nuclei and pulsars [6]. In this case, the radiation pressure is not small and cannot
be neglected.
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