New Group Chain Acceptance Sampling Plans (NGChSP-1) using Minimum Angle Method for Generalized Exponential Distribution
(Pelan Persampelan Baharu Penerimaan Kumpulan Berantai (NGChSP-1) menggunakan Kaedah Sudut Minimum untuk Taburan Eksponen Teritlak)

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ABSTRACT
The established group chain acceptance sampling plans (GCSP-1) functions with five acceptance criteria, while the modified group of chain acceptance sampling plans (MGChSP-1) operates with three acceptance criteria. Since the acceptance criteria affect the performances of the sampling plans, therefore, this article suggests a balanced approach by introducing a new group of chain acceptance sampling plans (NGChSP-1), where it functions with four acceptance criteria. The NGChSP-1 is developed by using minimum angle method which caters for producer’s and consumer’s risks. The generalized exponential distribution is selected as the lifetime distribution and the simulation for the NGChSP-1 is conducted at various values of design parameters using the Scilab programming. The finding shows that the optimal number of groups and the corresponding smallest theta for NGChSP-1 are smaller compared to those for the GCSP-1. For illustration purposes, the NGChSP-1 is then applied to real data of air conditioning equipment.

Keywords: Generalized exponential distribution; minimum angle method; new group chain acceptance sampling plans (NGChSP-1)

INTRODUCTION
In acceptance sampling, different sampling plans have different acceptance criteria. Acceptance criteria are the conditions imposed on a sampling plan in order to decide whether a lot is accepted or in worst-case scenario, the lot is rejected. A sampling plan with many acceptance criteria is ideally the choice for the producers while the consumers may prefer to have a sampling plan with less acceptance criteria. Mughal (2018) discussed two group chain acceptance sampling plans for Pareto distribution of the 2nd kind, which are: group chain acceptance sampling plans (GCSP-1), and modified group chain acceptance sampling plans (MGChSP-1). The GCSP-1 has five acceptance criteria while the MGChSP-1 only has three acceptance criteria.
Based on the acceptance criteria, readers can observe that the GChSP-1 is a loose sampling plan (five acceptance criteria) while the MGChSP-1 is a really tight sampling plan (three acceptance criteria). The difference in acceptance criteria leads to a conflict for the two main stakeholders in acceptance sampling, which are producers and consumers. The producers are pleased with GChSP-1 as it reduces the probability of rejecting a good lot (producer’s risk) but the consumers are unhappy since it increases the probability of accepting a bad lot (consumer’s risk). For MGChSP-1, the producers are unhappy with the plan as it increases the producer’s risk but the consumers are happy with the plan as it decreases the consumer’s risk.

In order to overcome the conflict, new group chain acceptance sampling plans (NGChSP-1) are introduced. The NGChSP-1 manages to solve the conflict as it has four acceptance criteria, where the four acceptance criteria stand between the GChSP-1 (five acceptance criteria) and MGChSP-1 (three acceptance criteria). With the introduction of NGChSP-1, it provides win-win situations for both stakeholders now as it has moderate risks for them. The NGChSP-1 is a better plan compared to the GChSP-1 as the former has tighter acceptance criteria compared to the latter which allows too many defectives (more than one defective) to be found in the previous lots. The NGChSP-1 only allows one defective in the previous lots while the GChSP-1 does not bother on how many defectives in the previous lots. In other words, the NGChSP-1 imposes strict conditions regarding the previous lots while the GChSP-1 does not. This condition eventually makes the NGChSP-1 a better plan compared to the GChSP-1.

This article proposes the NGChSP-1 for generalized exponential distribution using minimum angle method. The generalized exponential distribution is chosen as previous researchers have shown that it may closely represent lifetime of electronic products. For example, Nelson (1982) proved that the failure rate for diesel fans followed exponential distribution. For capacitors and integrated circuit, O’Connor et al. (2016) showed that the two electronic products exhibited exponential distribution.

Motivated by this scenario, the generalized exponential distribution has been used rigorously by previous researchers. These include Epstein (1954) for single acceptance sampling plans (SSP), Rao (2010) for group acceptance sampling plans (GSP), Aslam et al. (2011) also for GSP and Ramaswamy and Jayasri (2012) for chain acceptance sampling plans (ChSP-1). The lifetime distribution used is not limited to generalized exponential distribution as other researchers also applied different lifetime distributions to different acceptance sampling plans. For instance, Aslam (2008) proposed economic reliability for the SSP using generalized Rayleigh distribution, Srinivasa Rao (2011) for double acceptance sampling plans (DSP) using Marshall-Olkin extended Lomax distribution and Mughal et al. (2016) used Pareto distribution of the 2nd kind for GSP.

The NGChSP-1 is developed by using minimum angle method, where the method considers producer’s and consumer’s risks simultaneously. This method has been used by several researchers including Ramaswamy and Sutharani (2013) for the DSP, Suresh and Vinitha (2014) for generalized two plan system and Teh et al. (2019) for the MGChSP-1. It was proven that the sample size (for DSP) and the optimal number of groups (for MGChSP-1) satisfied both stakeholders, producers and consumers, compared to the previous method, where it only satisfied the consumers’ side (Ramaswamy & Sutharani, 2013; Teh et al. 2019).

**Materials and Methods**

In this study, the minimum angle method is applied to the NGChSP-1. The method calculates the angle between line A and line B, as portrayed in Figure 1.

![Figure 1](https://example.com/figure1.png)

**FIGURE 1.** The minimum angle method as illustrated by Ramaswamy and Sutharani (2013)
Based on Figure 1, the point A is coordinated as 
\((p_1, 1 - \alpha)\) where \(p_1\) represent the fraction defective at the acceptable quality level (AQL) and \(\alpha\) is the producer’s risk. Point B is located at \((p_2, \beta)\) where \(p_2\) is the fraction defective at the rejectable quality level (RQL) and \(\beta\) is the consumer’s risk. In order to calculate the angle, the following formula is used

\[
\tan \theta = \frac{BC}{AC} = \frac{(p_2 - p_1)}{L(p_1) - L(p_2)}
\]  

(1)

For the NGChSP-1, the operating steps are as follows: First, find the optimal number of groups \((g)\). Next, allocate the number of products \((r)\) to each group. The sample size \((n = g \times r)\). After that, count the number of defectives \((d)\) during test termination time \((t)\). Lastly, accept the current lot if \(d = 0\) given that the preceding lots have at most one defective. The current lot is also accepted if \(d = 1\), given that there is no defective recorded in the preceding \(i\) lots. Reject the current lot if \(d > 1\). Based on the operating steps, the probability of lot acceptance, for the NGChSP-1 is

\[
L(p) = (P_0)^g[(i + 1)P_1 + P_0]
\]

(2)

where \(P_0\) and \(P_1\) stand for the probability of zero defective and probability of one defective, respectively.

In this study, \(P_0\) and \(P_1\) are derived by using binomial distribution. Upon simplification, (2) can be written as

\[
L(p) = (1 - p)^{gr(i+1)}\left[\frac{(i + 1)gr^p}{1 - p} + 1\right].
\]

(3)

where \(p\) is the fraction defective.

For the fraction defective, \(p\), it is acquired from the cumulative distribution function (CDF) of generalized exponential distribution. Teh et al. (2019) has shown that the fraction defective, \(p\) for the generalized exponential distribution is given by

\[
p = \left[1 - \exp \left(-a \left(\frac{1}{\mu_p}\right)\right)\right]^{1/\lambda}.
\]

(4)

Based on (4), \(a\) is the specified constant representing how long the inspection will be conducted. In this study, \(a\) is set from 0.25 to 2.00, with an increment of 0.25. For instance, if a product has lifetime of 1000 h and the specified constant, \(a\) for the inspection is 0.25, then the inspection is conducted and truncated when it reaches 250 h. The inspection is longer if higher \(a\) is selected. For example, if \(a\) is 2, then the inspection time is stopped at 2000 h.

The mean ratio is written as the true mean life, \(\mu_0\) over the specified mean life, \(\mu_p\). The true mean life is the actual lifetime of a product while the specified mean life is usually printed by the manufacturer on the product. Basically, the mean ratio represents the quality (usually lifetime) of a product. If a product has higher mean ratio, then the lifetime of a product is longer, and vice versa.

Since this study accommodates two main stakeholders in acceptance sampling, there are two ways to calculate the fraction defective, \(p\). For the first stakeholder (the producers), the fraction defective is calculated by using (4) and the mean ratio is 1. On the other hand, for the consumers (second stakeholder), the fraction defective is also determined using (4), but the mean ratio varies from 2 to 12.

There are other design parameters such as shape parameters, \(\lambda\), number of preceding lots, \(i\) and number of products, \(r\). The shape parameters, \(\lambda\), is set from 1 to 3 while the number of preceding lots, \(i\), varies from 1 to 4. The number of products, \(r\) starts from 2 to 5, and 1 cannot be the value as it would turn the NGChSP-1 to the ordinary chain acceptance sampling plans. The optimal number of groups, \(g\) is obtained by solving (5) and (6) given as

\[
L(p_i) \geq 1 - \alpha
\]

(5)

and

\[
L(p_i) \leq \beta.
\]

(6)

Both risks are set at 0.10, and the corresponding smallest \(\theta\), is calculated by using (1). The optimal number of groups, \(g\) and the corresponding smallest \(\theta\) at the different values of design parameters are shown in Tables 1 to 3 in the Result and Discussion section.

**RESULTS AND DISCUSSION**

Performances of the NGChSP-1 are measured based on the optimal number of groups, \(g\) and the corresponding smallest \(\theta\). For group-related acceptance sampling plans, the plan with low optimal number of groups has better performance since it reduces the inspection time. The reduction is contributed by the fact that there is less products to be inspected. Meanwhile, the smaller \(\theta\) is better as it approaches the ideal OC curve, as discussed by previous researchers (Ramaseswamy & Sutharani 2013; Suresh & Vinitha 2014; Teh et al. 2019).
Tables 1 to 3 display the optimal number of groups, \( q \) and the corresponding smallest theta, \( \theta \) for generalized exponential distribution at the different values of design parameters.

The information from Tables 1 to 3 act as a guidance for the inspection activity. Let’s say a product is expected to be inspected using the NGChSP-1 at the risk of 10%. If the product exhibits generalized exponential distribution as its lifetime with 1 as the shape parameter, the lifetime of a product is 12 times higher than the specified lifetime, the inspection activity is scheduled to stop at 25% of the specified lifetime, and the inspection platform only allows 2 products, therefore, the suggested groups is 6. Thus, it means the inspection is conducted for 12 samples in total, but it is divided into 6 groups with 2 products in each group.

For these design parameters \( (\lambda, \mu, i, r, a) = (1, 12, 1, 2, 0.25) \) the corresponding smallest theta, \( \theta \) is 12.65115 which indicates that the stated design parameters indeed approaches the ideal OC curve better compared to the other design parameters. Bear in mind that the stated design parameters are only applicable to a product when the product has generalized exponential distribution with one as the shape parameter. If a product still follows generalized exponential distribution but with different shape parameters, then Tables 2 and 3 will guide the inspection personnel to select the best optimal number of groups and the smallest theta at the required design parameters.

In order to demonstrate the application of NGChSP-1, a real data set is used. Teh (2018) has shown that 15 air conditioning equipment in Boeing 720 fleet followed generalized exponential distribution with one as the shape parameter. If the NGChSP-1 is used to inspect the 15 air conditioning equipment with design parameters of \( (\lambda, \mu, i, r, a) = (1, 12, 1, 2, 0.25) \), then the suggested groups is 6, with 2 air conditioning equipment in each group. The inspection is done simultaneously on the 6 groups and the number of defective is recorded during the 25% inspection time. The air conditioning equipment is accepted if there is no defective found in the current lot provided that there is at most one defective in the 1 preceding lot. Besides that, the air conditioning equipment is also accepted if one defective is found, given that there is no defective in the 1 preceding lot. Otherwise, the air conditioning equipment lot is rejected.

One set of design parameters is selected to compare the performances between GChSP-1 and NGChSP-1. Table 4 shows the optimal number of groups and the corresponding smallest theta at different values of specified constant when the other design parameters are \( (\lambda, \mu, i, r, a) = (3, 10, 1, 2) \). For all values of the specified constant, the NGChSP-1 records smaller number of optimal groups and the corresponding smallest theta compared to the GChSP-1. For instance, the NGChSP-1 suggests 303 as the optimal number of groups while GChSP-1 requires 451 when the specified constant is 0.25. For the corresponding smallest theta, NGChSP-1 creates \( \theta = 0.61935 \) while the GChSP-1 documents the theta as \( \theta = 0.61943 \). This pattern also holds when the values of specified constant change from 0.25 to 2, as shown in Table 4.

### TABLE 1. The optimal number of groups for generalized exponential distribution (\( \lambda = 1 \))

| mean ratio | \( i \) | \( r \) | 0.25 | 0.5 | 0.75 | 1 | 1.25 | 1.5 | 1.75 | 2 |
|------------|--------|--------|------|-----|------|---|------|-----|-------|---|
| 2          | 1      | 2      | -    | -   | -    | - | -    | -   | -     | - |
|            | 2      | 3      | -    | -   | -    | - | -    | -   | -     | - |
|            | 3      | 4      | -    | -   | -    | - | -    | -   | -     | - |
|            | 4      | 5      | -    | -   | -    | - | -    | -   | -     | - |
| 4          | 1      | 2      | -    | -   | -    | - | -    | -   | -     | - |
|            | 2      | 3      | -    | -   | -    | - | -    | -   | -     | - |
|            | 3      | 4      | -    | -   | -    | - | -    | -   | -     | - |
|            | 4      | 5      | -    | -   | -    | - | -    | -   | -     | - |
| 6          | 1      | 2      | -    | -   | -    | - | -    | -   | -     | - |
|            | 2      | 3      | -    | -   | -    | - | -    | -   | -     | - |
|            | 3      | 4      | -    | -   | -    | - | -    | -   | -     | - |
|            | 4      | 5      | -    | -   | -    | - | -    | -   | -     | - |
| mean ratio | $i$ | $r$ | 0.25 | 0.5  | 0.75 | 1    | 1.25 | 1.5   | 1.75 | 2    |
|------------|-----|-----|------|------|------|------|------|-------|------|------|
|            | 1   | 2   | -    | -    | -    | -    | -    | -     | -    | -    |
|            | 2   | 3   | -    | -    | -    | -    | -    | -     | -    | -    |
|            | 3   | 4   | -    | -    | -    | -    | -    | -     | -    | -    |
|            | 4   | 5   | -    | -    | -    | -    | -    | -     | -    | -    |
|            | 8   | -   | 5    | -    | -    | -    | -    | 1     | 1    | -    |
|            | 10  | 2   | 3    | 2    | -    | 1    | 1    | 1     | 1    | -    |
|            | 12  | 2   | 3    | 2    | 3    | -    | 1    | 1     | 1    | 1    |
|            |     |     |      |      |      |      |      |       |      |      |
|            | 2   | 3   | 3    | 2    | -    | -    | -    | -     | -    | -    |
|            | 3   | 4   | 4    | 2    | -    | -    | -    | -     | -    | -    |
|            | 4   | 5   | 5    | 1    | -    | -    | -    | -     | -    | -    |
|            | 1   | 2   | 35   | 10   | 5    | 3    | 2    | 2     | 2    | 2    |
|            | 2   | 3   | 16   | 5    | 2    | 2    | 1    | 1     | -    | -    |
|            | 3   | 4   | 9    | 3    | 1    | 1    | -    | -     | -    | -    |
|            | 4   | 5   | 6    | 2    | 1    | -    | -    | -     | -    | -    |

TABLE 2. The optimal number of groups for generalized exponential distribution ($\lambda = 2$)
| i  | r  | 0.25 | 0.5 | 0.75 | 1   | 1.25 | 1.5 | 1.75 | 2   |
|----|----|------|-----|------|-----|------|-----|------|-----|
| 2  | 1  | -    | -   | -    | -   | -    | -   | -    | -   |
| 2  | 2  | -    | -   | -    | -   | -    | -   | -    | -   |
| 3  | 3  | -    | -   | -    | -   | -    | -   | -    | -   |
| 4  | 4  | -    | -   | -    | -   | -    | -   | -    | -   |
| 4  | 5  | -    | -   | -    | -   | -    | -   | -    | -   |
| 12 | 1  | 183  | 30  | 11   | 6   | 4    | 3   | 2    | 2   |
| 2  | 2  | (0.61661°) | (3.46669°) | (8.30933°) | (14.07898°) | (19.80127°) | (24.90914°) | (29.95912°) | (32.61997°) |
| 2  | 3  | (0.61661°) | (3.46694°) | (8.30816°) | (14.11066°) | (19.89826°) | (24.98058°) | (29.06020°) |
| 3  | 4  | (0.61661°) | (3.46830°) | (8.31151°) | (14.24823°) | (19.80127°) | (25.59747°) |
| 4  | 5  | (0.61661°) | (3.46735°) | (8.31828°) | (14.08314°) | (20.57428°) |

**TABLE 3.** The optimal number of groups for generalized exponential distribution ($\lambda = 3$)

**Generalized exponential distribution, $\lambda = 3$**

**Specified constant, $a$**
| Sampling plans | a | GChSP-1 | NGChSP-1 |
|---------------|---|---------|---------|
|               | 0.25 | 451 | 303 |
|               | 0.61943° | 0.61935° |
|               | 0.5 | 74 | 50 |
|               | 3.48113° | 3.48039° |
|               | 0.75 | 28 | 19 |
|               | 8.34181° | 8.33928° |
|               | 1 | 15 | 10 |
|               | 14.14413° | 14.1383° |

**TABLE 4.** Performance comparison between GChSP-1 and NGChSP-1
CONCLUSION

This paper proposes a new sampling plan named new group chain acceptance sampling plans (NGChSP-1). The NGChSP-1 operates with four acceptance criteria, where it improves the established group chain acceptance sampling plans (GChSP-1) which functions with five acceptance criteria. The NGChSP-1 is developed using the minimum angle method, where the method caters for the producer’s and consumer’s risks.

The performances of the NGChSP-1 are then measured based on the optimal number of groups and the corresponding smallest theta, where the two performance indicators are calculated at different values of design parameters. The design parameters involved are the shape parameter, the mean ratio, the number of preceding lots, the number of products and the specified constant.

Performance comparison shows that the NGChSP-1 is better than the GChSP-1. For all values of the specified constant, the NGChSP-1 shows smaller optimal number of groups and the corresponding smallest theta compared to the GChSP-1.

For future research, the NGChSP-1 can be further explored by using different quality parameters such as percentile or median, different underlying distribution such as weighted binomial and different lifetime distribution such as generalized Rayleigh distribution.

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