Exclusive Nonleptonic Decays of Bottom and Charm Baryons in a Relativistic Three-Quark Model: Evaluation of Nonfactorizing Diagrams

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Abstract

Exclusive nonleptonic decays of bottom and charm baryons are studied within a relativistic three-quark model with a Gaussian shape for the momentum dependence of the baryon-three-quark vertex. We include factorizing as well as nonfactorizing contributions to the decay amplitudes. For heavy-to-light transitions $Q \to qud$ the total contribution of the nonfactorizing diagrams amount up to $\sim 60\%$ of the factorizing contributions in amplitude, and up to $\sim 30\%$ for $b \to cud$ transitions. We calculate the rates and the polarization asymmetry parameters for various nonleptonic decays and compare them to existing data and to the results of other model calculations.

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I. INTRODUCTION

During the last years there has been significant progress in the experimental study of nonleptonic decays of heavy baryons \[1\]. New results on the mass spectrum, lifetimes, branching ratios and asymmetry parameters in the decays of the heavy baryons \(\Lambda_c^+, \Sigma_c, \Xi_c, \Lambda_b^0, \ldots\) were reported by various experiments ALEPH, ARGUS, ACCMOR, CLEO, OPAL, etc. The heavy baryon mass spectrum has been determined with good precision (within an accuracy of a few per cent). As to nonleptonic branching ratios, the accuracy of the measurements does not exceed 25-30% even for the better studied Cabibbo-favored decay modes \(\Lambda_c^+ \rightarrow \Lambda^0 + \pi^+\) and \(\Lambda_c^+ \rightarrow p + \bar{K}^0\). For the decay \(\Lambda_c^0 \rightarrow J/\psi \Lambda\) and the Cabibbo-suppressed decay \(\Lambda_c^+ \rightarrow p\phi\) the experimental errors are even larger. The first observation of the \(\Lambda_c^+ \rightarrow p\phi\) decay was reported by the NA32 Collaboration \[2\]. They quoted a branching ratio of \(Br(\Lambda_c^+ \rightarrow p\phi)/Br(\Lambda_c^+ \rightarrow pK^-\pi^+)) = 0.040 \pm 0.027\). A more recent measurement of the \(\Lambda_c^+ \rightarrow p\phi\) decay rate by the CLEO Collaboration resulted in a ratio of branching ratios \(Br(\Lambda_c^+ \rightarrow p\phi)/Br(\Lambda_c^+ \rightarrow pK^-\pi^+)) = 0.024 \pm 0.006 \pm 0.003\) \[3\]. The baryonic decay \(\Lambda_b^0 \rightarrow J/\psi \Lambda\) was first observed by the UA1 Collaboration \[4\]. The measured branching ratio was found to be \(Br(\Lambda_b^0 \rightarrow J/\psi \Lambda) = (1.4 \pm 0.9)\%\) \[1\]. The OPAL Collaboration obtained an upper limit for the branching ratio of \(Br(\Lambda_c^0 \rightarrow J/\psi \Lambda) < 1.1\%\) \[5\]. Recently the CDF Collaboration has reported a much smaller value for the same quantity \(Br(\Lambda_b^0 \rightarrow J/\psi \Lambda) = (0.037 \pm 0.017 \pm 0.004)\%\) from a larger data sample \[6\]. From a theoretical point of view the \(\Lambda_c^+ \rightarrow p\phi\) and \(\Lambda_c^0 \rightarrow J/\psi \Lambda\) decays are simple in as much as they are described by factorizing quark diagrams alone. Their study can shed light on the nature of the nonleptonic interactions and may serve as an additional source for determining the Cabibbo-Kobayashi-Maskawa (CKM) elements and the values of the short-distance Wilson coefficients in the effective nonleptonic Lagrangian \[7\]-\[11\]. In the near future one can expect large quantities of new data on exclusive charm and bottom baryon nonleptonic decays which calls for a comprehensive theoretical analysis of these decays.

There exist a number of theoretical analysis of exclusive nonleptonic heavy baryon decays in the literature (see, e.g. refs. \[12\]-\[27\]) including predictions for their angular decay distributions. The analysis of nonleptonic baryon decays is complicated by the necessity of having to include nonfactorizing contributions. One thus has to go beyond the factorization approximation which had proved quite useful in the analysis of the exclusive nonleptonic decays of heavy mesons. There have been some theoretical attempts to analyse nonleptonic heavy baryon decays using factorizing contributions alone \[25\], the argument being that W-exchange contributions can be neglected in analogy to the power suppressed W-exchange contributions in the inclusive nonleptonic decays of heavy baryons. One might even be tempted to drop the nonfactorizing contributions on account of the fact that they are superficially proportional to \(1/N_c\). However, since \(N_c\)-baryons contain \(N_c\) quarks an extra combinatorial factor proportional to \(N_c\) appears in the amplitudes which cancels the explicit diagrammatic \(1/N_c\) factor \[13\]-\[17\]. There is now ample empirical evidences in the \(c \rightarrow s\) sectors that nonfactorizing diagrams cannot be neglected. For example, in the charm sector the two observed decays \(\Lambda_c^+ \rightarrow \Xi^0 K^+\) and \(\Lambda_c^+ \rightarrow \Sigma \pi\) can only proceed via nonfactorizing diagrams. Their sizeable observed branching ratios may thus serve to obtain a measure of the size of the nonfactorizing contributions.

In the present paper both factorizing and nonfactorizing contributions to exclusive non-
leptonic decays of bottom and charm baryons are taken into account. The decay amplitudes are studied within a relativistic three-quark model with a Gaussian shape for the momentum dependence of the baryon-three-quark vertex. It is shown that the total contribution of the nonfactorizing diagrams can amount up to \( \sim 60\% \) of the factorizing contribution for heavy-to-light transitions and up to \( \sim 30\% \) for \( b \to c \) transition in amplitude. We calculate branching ratios and asymmetry parameters for bottom and charm baryon nonleptonic decays within the Lagrangian Spectator Model approach which generalizes the spectator quark model approach [28,29]. We compare our results with existing data and other theoretical approaches.

The layout of the paper is as follows. In Section 2 we present details of our Lagrangian Spectator Model approach. In Section 3 we discuss the calculation of the matrix elements of nonleptonic decays of bottom and charm baryons. In Section 4 we present the results of our calculations. Section 5 contains our conclusions.

## II. MODEL

A systematic and comprehensive analysis of weak semileptonic and nonleptonic decays of heavy baryons has been carried out within the spectator quark model [14,15,28,29] which is based on the "equal-velocity" approximation [28,29]. Namely, it is assumed that all quarks inside a hadron have equal velocities coinciding with the velocity of the hadron. In other words, the internal relative motion of quarks inside the hadrons is neglected.

The quark-hadron Bethe-Salpeter (BS) wave function satisfies the free-quark Dirac equation in the each quark index, i.e. the quarks are assumed to be noninteracting. With the use of the "equal-velocity" assumption the equations of motion for the wave functions of the individual constituent quarks in the baryon can be rewritten in terms of the hadron velocity, thus imposing restriction on the possible form of the hadronic BS wave function [14,15,28,29].

The explicit form of the BS wave functions for hadron in the initial state is given by

\[
J^P = \frac{1}{2}^+ : \quad \mathcal{B}_{ABC} = \frac{1}{M} \{ \left( (P + M) \gamma_5 C \right)_{\beta \gamma} u_{\alpha}(P) B_{a[bc]} + \text{cycl.}(\alpha,a;\beta,b;\gamma,c) \},
\]

\[
J^P = \frac{3}{2}^+ : \quad \mathcal{B}_{ABC} = \frac{1}{M} \{ \left( (P + M) \gamma_\nu C \right)_{\beta \gamma} u^\nu_{\alpha}(P) B_{(abc)} + \text{cycl.}(\alpha,a;\beta,b;\gamma,c) \},
\]

\[
J^P = 0^{-+} : \quad \mathcal{M}^B_A = \left( (P + M) \gamma_5 \gamma_\nu \right)_{\alpha}^{\beta} \mathcal{M}^b_a
\]

\[
J^P = 1^{--} : \quad \mathcal{M}^B_A = \left( (P + M) \gamma_\nu \right)_{\alpha}^{\beta} \mathcal{M}^b_a
\]

We have suppressed colour indices in (1). \( M \) is the mass of hadron, \( P \) is its total four-momentum, and \( B_{a[bc]}, B_{(abc)}, \mathcal{M}^b_a \) denote the flavour part of the hadronic wave function. Analogous formulae for the final state hadronic wave functions can easily be derived from...
Eq. (1). They can be found in ref. [14]. Note that the BS spin wave functions (1) contain an additional "projector" factor

$$V_+ = \frac{1}{2M}(P + M) = \frac{\gamma + 1}{2}$$

where \(v\) is the "on-shell" four-velocity of hadron, i.e. \(v^2 = 1\). The factor \(V_+\) ensures that in the c.m. frame only the positive-energy components of the full BS wave function survive, as it should indeed be, when the quarks are noninteracting.

Once the explicit form of the hadron wave functions is given, the transition matrix elements for weak decays are parameterized by a few overlap integrals in terms of the spin-independent spatial part of the hadron wave functions. Previously, the overlap integrals have been treated as phenomenological parameters to be determined from a fit to experimental data [14].

In order to go beyond the approach [14] one has to develop a microscopic approach to the overlap integrals appearing in the expressions for the decay amplitudes or, equivalently, one has to specify the form of the hadron-quark transition vertex (hadronic BS wave function) including the explicit momentum dependence of the Lorentz scalar part of this vertex. In the Lagrangian model considered in this paper this dependence is given by the baryon form factor which appears in the nonlocal interaction vertex coupling the baryons to the three quarks. The Lagrangian model has been successfully applied to the description of a wide class of the low and intermediate energy hadron phenomena both in the light [30]- [33] and heavy [34] quark sectors.

In its present form, this model is not immediately applicable to the study of the heavy baryon nonleptonic decays since it does not reproduce the results of the spectator model analysis [14]. The purpose of our present investigation will consist in embedding, step by step, the spectator model spin structure in our Lagrangian approach. Put differently, we attempt to reformulate the spectator model using the Lagrangian language in order to be able to calculate all quantities appearing in the description of the nonleptonic decays of heavy baryons with the use of the Feynman diagram technique.

Let us begin with the formulation of the basic notions of the Lagrangian model taking into account at every step the spin structure imposed by the spectator picture.

The problem of the choice of baryonic currents was discussed in ref. [34] (see also refs. [35]- [39] and [40]- [42]). Let us briefly review the basic notions. Suppose that a baryon is a bound state of three quarks. Let \(y_i\) (i=1,2,3) be the position space four-coordinate of quark \(i\) with
mass \( m_i \). They are expressed through the center of mass coordinate \((x)\) and the relative Jacobi coordinates \((\xi_1, \ldots)\) as

\[
\begin{align*}
y_1 &= x - 3\xi_1 \frac{m_2 + m_3}{\sum_i m_i} \\
y_2 &= x + 3\xi_1 \frac{m_1}{\sum_i m_i} - 2\xi_2 \sqrt{3} \frac{m_3}{m_2 + m_3} \\
y_3 &= x + 3\xi_1 \frac{m_1}{\sum_i m_i} + 2\xi_2 \sqrt{3} \frac{m_2}{m_2 + m_3}
\end{align*}
\]

where

\[
x = \frac{\sum_i m_i y_i}{\sum_i m_i}, \quad \xi_1 = \frac{1}{3} \left( \frac{m_2 y_2 + m_3 y_3}{m_2 + m_3} - y_1 \right), \quad \xi_2 = \frac{y_3 - y_2}{2\sqrt{3}}.
\]

In the case of light baryons we shall work in the limit of SU(3) invariance by assuming that the masses of \( u, d \) and \( s \) quarks are equal to each other in Eq. (3). The breaking of SU(3) symmetry through the position space variables \( y_i \) (via a difference of strange \( m_s \) and nonstrange \( m \) quark masses: \( m_s - m \neq 0 \)) was found to be insignificant [34]. Thus, for light baryons composed of \( u, d \) or \( s \) quarks the coordinates of the quarks may be written as

\[
\begin{align*}
y_1 &= x - 2\xi_1 \quad y_2 = x + \xi_1 - \xi_2 \sqrt{3} \quad y_3 = x + \xi_1 + \xi_2 \sqrt{3}
\end{align*}
\]

For a heavy-light baryon with \( m_1 \gg m_2, m_3 \) one has instead

\[
\begin{align*}
y_1 &= y_Q = x, \quad y_2 = y_{q_1} = x + 3\xi_1 - \xi_2 \sqrt{3}, \quad y_3 = y_{q_2} = x + 3\xi_1 + \xi_2 \sqrt{3}
\end{align*}
\]

We assume that the momentum distribution of the constituents inside a baryon is modelled by an effective relativistic vertex function given by

\[
F \left( \frac{\Lambda_B^2}{18} \sum_{i<j} (y_i - y_j)^2 \right)
\]

which depends only on the sum of the relative coordinates squared in the coordinate space and on a cutoff parameter \( \Lambda_B \). Generally speaking, the shape of this function should be determined from the bound state equation and may depend on the flavours of the quarks involved. In order to reduce the number of free parameters we will use a common Gaussian function for all flavours but we allow for flavour dependent values of the cutoff parameter \( \Lambda_B \). The Gaussian shape guarantees ultraviolet convergence of the matrix elements. The vertex function models the long distance QCD interactions between quarks. For the present application, there are at least three different values for \( \Lambda_B \) corresponding to the \((s, d, u)\),
(c, d, u), and (b, d, u) sectors. However, in order to recover the Isgur-Wise symmetry in the heavy quark limit \((m_Q \rightarrow \infty)\) the cutoff parameter \(\Lambda_B\) has to be the same for charm and bottom baryons.

The Lagrangian describing the interaction of baryons with the three-quark current is written as

\[
\mathcal{L}_B^{\text{int}}(x) = g_B \bar{B}(x) \int dy_1 \int dy_2 \int dy_3 \delta \left( x - \frac{\sum m_i y_i}{\sum m_i} \right) \frac{\Lambda_B^2}{18} \sum_{i<j} (y_i - y_j)^2 F \left( \frac{\Lambda_B^2}{18} \sum_{i<j} (y_i - y_j)^2 \right) 
\]

\[
\times J_B(y_1, y_2, y_3) + \text{h.c.}
\]

(4)

where \(J_B(y_1, y_2, y_3)\) is the three-quark current with quantum numbers of a baryon \(B\):

\[
J_B(y_1, y_2, y_3) = \Gamma_1 q^{a_1}(y_1) q^{a_2}(y_2) C \gamma_5 q^{a_3}(y_3) \varepsilon^{a_1 a_2 a_3}.
\]

(5)

Here \(\Gamma_1,2\) are strings of Dirac matrices, \(C = \gamma^0 \gamma^2\) is the charge conjugation matrix and \(a_i\) are the color indices. The strong coupling constant \(g_B\) in (4) can be calculated from the compositeness condition (see, ref. [34], [37]-[39]), i.e. the renormalization constant of the hadron wave function is set equal to zero, \(Z_H = 1 - g_B^2 \Sigma_B(M_H) = 0\), with \(\Sigma_H\) being the hadron mass operator and \(M_H\) denotes a hadron mass. Note that the latter condition is equivalent to the well-known relativistic normalization condition for the hadronic Bethe-Salpeter (BS) wave function. However, for technical reasons it is more convenient to use the normalization condition for the elastic vector form factor at zero recoil which, of course, is completely equivalent to the compositeness condition (see, discussion about it in ref. [34]).

Possible choices of light and heavy-light baryonic currents have been studied in refs. [35]-[39] and [40]-[42]. For the octet of light baryons, for the \(\Lambda\)-type heavy-light baryons \((\Lambda_Q, \Xi_Q)\) with a light spin zero diquark system, and for the \(\Omega\)-type heavy-light baryons \((\Omega_Q, \Sigma_Q)\) with a light spin one diquark system the currents are written as follows [34].

**Light Baryon Currents**

- **vector variant**
  \[
  J_B^V(y_1, y_2, y_3) = \gamma^\mu \gamma^5 q^{a_1}(y_1) q^{a_2}(y_2) C \gamma_\mu q^{a_3}(y_3) \varepsilon^{a_1 a_2 a_3}
  \]

  (6)

- **tensor variant**
  \[
  J_B^T(y_1, y_2, y_3) = \sigma^{\mu \nu} \gamma^5 q^{a_1}(y_1) q^{a_2}(y_2) C \sigma_{\mu \nu} q^{a_3}(y_3) \varepsilon^{a_1 a_2 a_3}
  \]

**Heavy-Light Baryon Currents**

- **pseudoscalar variant**
  \[
  J_{\Lambda_Q}^P = \varepsilon^{abc} q^a s^b C q^c \gamma^5 d^c
  \]

- **axial variant**
  \[
  J_{\Lambda_Q}^A = \varepsilon^{abc} \gamma_\mu q^a s^b C \gamma^\mu \gamma^5 d^c
  \]

- **vector variant**
  \[
  J_{\Omega_Q}^V = \varepsilon^{abc} \gamma_\mu q^a s^b C \gamma_\mu s^c, \quad J_{\Omega_Q}^V;\mu = \varepsilon^{abc} q^a s^b C \gamma_\mu s^c
  \]

- **tensor variant**
  \[
  J_{\Omega_Q}^T = \varepsilon^{abc} \sigma_{\mu \nu} q^a s^b C \sigma_{\mu \nu} s^c, \quad J_{\Omega_Q}^T;\mu = -i \varepsilon^{abc} q^a s^b C \sigma_{\mu \nu} s^c
  \]

(7)
In Table I we give the quark content, the quantum numbers (spin-parity $J^P$, spin $S_{qq}$ and isospin $I_{qq}$ of light diquark) and the experimental (when available) and theoretical mass spectrum of the heavy baryons \[1,15\] analyzed in this paper. Square brackets \[\ldots\] and curly brackets \{\ldots\} denote antisymmetric and symmetric flavour and spin combinations of the light degrees of freedom. The masses of the light baryons are taken from the Review of Particle Properties \[1\].

Next we write down the Lagrangian which describes the interaction of $\Lambda_Q$-baryon with quarks in the heavy quark limit ($m_Q \to \infty$), i.e. to leading order in the $1/m_Q$ expansion

\[
\mathcal{L}_{\Lambda_Q}^{\text{int}}(x) = g_{\Lambda_Q} \bar{\Lambda}_Q(x) \Gamma_1 Q^a(x) \int d\xi_1 \int d\xi_2 \ F(\Lambda_{BQ}^2 \cdot [\xi_1^2 + \xi_2^2]) \\
\times u^b(x + 3\xi_1 - \xi_2\sqrt{3}) C \Gamma_2 d^c(x + 3\xi_1 + \xi_2\sqrt{3}) \epsilon^{abc} + h.c.
\]

where

\[
\Gamma_1 \otimes CT_2 = \begin{cases} 
I \otimes C \gamma^5 & \text{pseudoscalar current} \\
\gamma^\mu \otimes C \gamma^\mu \gamma^5 & \text{axial current}
\end{cases}
\]

One can see that the heavy quark is factorized from the light degrees of freedom in this limit. The vertex form factor $F$ characterizes the distribution of $u$ and $d$ quarks inside the $\Lambda_Q$ baryon. It is readily seen that the Lagrangian (8) exhibits the heavy quark flavour symmetry (symmetry under exchange b with c) if the parameter $\Lambda_{BQ}$ is the same for charm and bottom baryons.

In what follows we shall work with the momentum space representation of the interaction Lagrangians. Performing the requisite Fourier transformation e.g. for the case of the $\Lambda_Q$ baryon we obtain

\[
\mathcal{L}_{\Lambda_Q}^{\text{int}}(p) = g_{\Lambda_{B_Q}} \bar{\Lambda}_Q(p) \int dp_1 \int dp_2 \int dp_3 \int dk_1 \int dk_2 \delta(k_1 - 3(p_2 + p_3)) \delta(k_2 - \sqrt{3}(p_3 - p_2)) \\
\times \delta(p - \sum_i p_i) F\left(\frac{k_1^2 + k_2^2}{\Lambda_{B_Q}^2}\right) \Gamma_1 Q^a(p_1) u^b(p_2) C \Gamma_2 d^c(p_3) \epsilon^{abc} + h.c.
\]

where $p$ and $p_1, p_2, p_3$ are the momenta of the baryon and the constituent quarks, respectively. The relative momenta $k_1$ and $k_2$ may be expressed in terms of the quark momenta $p_i$ in a standard manner \[34\].

For our purposes we also need the effective Lagrangians that describe the coupling of pions, kaons and the vector mesons $\rho$, $\phi$ and $J/\psi$ to their quark constituents. In this paper we also assume that the mesons are point-like objects, i.e. their interaction with the constituent quarks are described by a local nonderivative Lagrangian
\[ L_M(p) = g_M M(p) \int dp_1 \int dp_2 \, \delta(p - p_1 - p_2) \, \bar{q}(p_1) \, \Gamma_M \, q(p_2) + h.c. \]  

(10)

where \( \Gamma_M \) and \( \lambda_M \) are spin and flavour matrices. In other words, we choose the effective meson vertex functions to be constants in momentum space. This is a reliable approximation for the light mesons. For heavy mesons we expect that form factor effects in the meson vertex become important. This prevents us from extending the present approach to cases with heavy mesons in the final states, such as \( \Lambda^0 \to \Lambda^+_c + D^-_s \). In general the form factor effects in the decays involving heavy mesons in the final state are expected to suppress their rates relative to those obtained from a point-like vertex. Exclusive nonleptonic bottom baryon decays involving heavy mesons form the subject of a separate piece of work.

To reproduce the spin amplitude structure of the spectator (or static quark) model analysis \([14,15]\) we assign the projector \( V_+ = (\not{\nu} + 1)/2 \) to each light quark field in the baryon-quark vertex, where \( \nu \) is the "on-shell" four-velocity of hadron as in ref. \([14]\). The conjugate antiquark fields in the mesons are multiplied by the projector \( V_- = (- \not{\nu} + 1)/2 \).

We shall also use the static approximation for \( u, d \) and \( s \) quark propagators

\[ <0|T\{q(x)\bar{q}(y)\}|0> = \frac{1}{\Lambda_q} \delta^{(4)}(x - y) \]  

(11)

where \( \Lambda_q \) is the free parameter having the dimension of mass. We choose this parameter to have the same value \( \Lambda \) for \( u \) and \( d \) quarks and a different value \( \Lambda_s \) for the strange quark.

The model obtained with the use of above prescriptions will be referred to as \textit{Lagrangian Spectator Model} in what follows.

An important property of the Lagrangian Spectator Model is that the structure of the interaction Lagrangians of light and heavy-light baryons with quarks is simplified. Namely, the different options for the choice of baryon currents all become equivalent. For example, the vector and tensor forms of the interaction Lagrangians of \( J^P = 1/2^+ \) light baryons are completely equivalent. For the proton the interaction Lagrangian takes the form

\[ \mathcal{L}^{\text{int}}_P(p) = 4g_p \bar{p}(p) \int dk_1 \int dk_2 F \left( \frac{12 k_1^2 + k_2^2 + k_1 k_2}{\Lambda_{Bq}^2} \right) \]  

(12)

\[ \times V_+ u^{a_1}(k_1 + p) u^{a_2}(k_2) C \gamma_5 V_+ d^{a_3}(-k_1 - k_2) \varepsilon^{a_1 a_2 a_3} + h.c. \]

\[ \equiv 2g_p \bar{p}(p) \int dk_1 \int dk_2 F \left( \frac{12 k_1^2 + k_2^2 + k_1 k_2}{\Lambda_{Bq}^2} \right) \]  

\[ \times V_+ \gamma^\mu \gamma^5 d^{a_1}(k_1 + p) u^{a_2}(k_2) C \gamma_\mu V_+ u^{a_3}(-k_1 - k_2) \varepsilon^{a_1 a_2 a_3} + h.c. \]
In Appendix A we provide a full list of the effective interaction Lagrangians for light baryons in the Lagrangian Spectator Model.

In the Lagrangian Spectator Model the leptonic coupling constants \( f_\pi \) and \( f_K \) are determined by the integrals

\[
    f_\pi = \frac{N_c g_\pi}{4\pi^2} \frac{1}{M_\pi^2 \Lambda^2} \int_{\text{reg}} \frac{d^4k}{\pi^2}, \quad f_K = \frac{N_c g_K}{4\pi^2} \frac{1}{M_K^2 \Lambda_s^2} \int_{\text{reg}} \frac{d^4k}{\pi^2}
\]  

(13)

The meson coupling constants \( g_\pi \) and \( g_K \) in Eq. (13) are determined from the compositeness condition [34] which reads

\[
    1 = \frac{N_c g_\pi^2}{4\pi^2} \frac{1}{M_\pi^2 \Lambda^2} \int_{\text{reg}} \frac{d^4k}{\pi^2}, \quad 1 = \frac{N_c g_K^2}{4\pi^2} \frac{1}{M_K^2 \Lambda_s^2} \int_{\text{reg}} \frac{d^4k}{\pi^2}
\]  

(14)

Equations (13) and (14) contain the ultraviolet divergence since the mesons in our scheme are point-like objects. To regularize these quantities we introduce an ultraviolet cutoff parameter \( \Lambda_{\text{cut}} \). In order to reduce the number of free parameters in the model we relate the cutoff parameter in Eqs. (13) and (14) to the parameters \( \Lambda \) and \( \Lambda_s \) appearing in static light quark propagator (11) via \( \Lambda_{\text{cut}} = \frac{\Lambda}{\Lambda_s^2} \). Here \( q_i \) corresponds to the flavour of the light quark being the constituent. After that we get

\[
    f_\pi = \frac{\sqrt{N_c}}{8\pi} \Lambda, \quad f_K = \frac{\sqrt{N_c}}{2\pi} \frac{(\Lambda \Lambda_s)^{3/2}}{(\Lambda + \Lambda_s)^2}
\]  

(15)

Substituting experimental values for \( f_\pi = 131 \text{ MeV} \) and \( f_K = 160 \text{ MeV} \) in Eqs. (15) we obtain \( \Lambda = 1.90 \text{ GeV} \) and \( \Lambda_s = 3.29 \text{ GeV} \).

For the heavy quark propagator \( S_Q \) we will use the leading term in the inverse mass expansion. Suppose \( p = M_{BQ} v \) is the heavy baryon momentum. We introduce the parameter \( \tilde{\Lambda}_{(q_1q_2)} = M_{(q_1q_2)} - m_Q \) which is the difference between the heavy baryon mass \( M_{(q_1q_2)} \equiv M_{BQ} \) and the heavy quark mass. Keeping in mind that the vertex function falls off sufficiently fast such that the condition \(|k| \ll m_Q\) holds (\( k \) is the virtual momentum of light quarks) one has

\[
    S_Q(p + k) = \frac{1}{m_Q - (\not{p} + \not{k})} = \frac{m_Q + M_{BQ} \not{p} + \not{k}}{m_Q^2 - M_{BQ}^2 - 2M_{BQ}vk - k^2} = S_v(k, \tilde{\Lambda}_{(q_1q_2)}) + O\left(\frac{1}{m_Q}\right)
\]

\[
    S_v(k, \tilde{\Lambda}_{(q_1q_2)}) = -\frac{(1 + \not{p})}{2(v \cdot k + \tilde{\Lambda}_{(q_1q_2)})}
\]  

(16)

In what follows we will assume that \( \tilde{\Lambda} \equiv \tilde{\Lambda}_{uu} = \tilde{\Lambda}_{dd} = \tilde{\Lambda}_{du}, \tilde{\Lambda}_s \equiv \tilde{\Lambda}_{us} = \tilde{\Lambda}_{ds} \). Thus there are altogether three independent parameters: \( \tilde{\Lambda}, \tilde{\Lambda}_s, \) and \( \tilde{\Lambda}_{ss} \).
The vertex function $F$ in the baryon-quark interaction Lagrangians is an arbitrary function except that it should render the Feynman diagrams ultraviolet finite as was mentioned before. In [30]-[34] it was found that the basic physical observables of pion and nucleon low-energy physics depend only weakly on the choice of the vertex functions. In the present paper we choose a Gaussian vertex function for simplicity. In Minkowski space we write

$$F\left(\frac{k^2_1 + k^2_2}{\Lambda^2_B}\right) = \exp\left(\frac{k^2_1 + k^2_2}{\Lambda^2_B}\right)$$

where $\Lambda_B$ is the Gaussian range parameter which is related to the size of a baryon. Note that all calculations are done in the Euclidean region ($k^2_i = -k^2_i_{\text{E}}$) where the above vertex function decreases very rapidly. We consider two different values of the $\Lambda_B$ cutoff parameter: $\Lambda_{B_q}$ for light baryons composed from light ($u, d, s$) quarks and $\Lambda_{B_Q}$ for baryons containing a single heavy quark ($b$ or $c$). The requirement of the unit normalization of the baryonic IW-functions $\zeta(\omega)$ and $\xi_1(\omega)$ at zero recoil $\omega = 1$ ($\zeta(1) = 1$, $\xi_1(1) = 1$) imposes the restriction $\Lambda_{B_b} = \Lambda_{B_c}$. This can be seen by expressing the baryonic IW-functions for arbitrary values of $\Lambda_{B_Q}$ as

$$\zeta(\omega) = \frac{\Phi\left(\sqrt{2\Lambda^2_{B_b}\Lambda^2_{B_c}}/(\Lambda^2_{B_b} + \Lambda^2_{B_c}), \omega\right)}{\sqrt{\Phi(\Lambda_{B_b}, 1) \Phi(\Lambda_{B_c}, 1)}}$$

$$\Phi(\Lambda_{B_Q}, w) = \Lambda^6_{B_Q}(\omega + 1) \int_0^\infty du \int_0^1 dx \exp\left[-18u^2 - 36u^2x(1-x)(\omega - 1) + 36u\frac{\bar{\Lambda}}{\Lambda_{B_Q}}\right]$$

Eq. (17) shows that one recovers $\zeta(1) = 1$ only when $\Lambda_{B_b} = \Lambda_{B_c}$. As was mentioned above, the parameter $\Lambda_{B_Q} = \Lambda_{B_b} = \Lambda_{B_c}$ is one of the adjustable parameters in our calculation.

Thus, there is the following set of adjustable parameters in our model: the cutoff parameters $\Lambda_B$ ($\Lambda_{B_q}$ and $\Lambda_{B_Q}$), and a set of $\bar{\Lambda}_{\{q_1q_2\}}$ binding energy parameters: $\bar{\Lambda}$, $\bar{\Lambda}_s$ and $\bar{\Lambda}_{\{ss\}}$.

### III. MATRIX ELEMENTS OF WEAK DECAYS OF HEAVY BARYONS

The weak nonleptonic decays of bottom and charm baryons are described by the diagrams I, IIa, IIb and III in Fig. 1. [1]

\[\text{[1] In the terminology of [30] diagram I corresponds to factorizable external and internal W-emission, IIa to nonfactorizable internal W-emission and IIb and III to nonfactorizable W-exchange.}\]
Diagram I corresponds to the so-called factorizing contribution. Diagrams IIa, IIb and III correspond to the nonfactorizing contributions. The vertices $O_\mu \bullet \bullet O_\mu$ correspond to the nonleptonic interaction described by a standard effective four-fermion Lagrangian [7]-[11]. For $b \to c\bar ud$ and $c \to s\bar ud$ transitions the effective four-fermion vertices read

$$\mathcal{L}_{eff} = \frac{G_F}{\sqrt{2}} V_{cb} V^*_{ud} \left[ c_1 (\bar{c}^{a_1} O_\mu b^{a_2}) (\bar{b}^{a_2} O_\mu u^{a_1}) + c_2 (\bar{c}^{a_1} O_\mu b^{a_2}) (\bar{b}^{a_2} O_\mu u^{a_1}) \right]$$

(18)

$$+ \frac{G_F}{\sqrt{2}} V_{cs} V^*_{ud} \left[ c^* \bar{s}^{a_1} O_\mu c^{a_2} (\bar{u}^{a_2} O_\mu d^{a_1}) + c^* \bar{s}^{a_1} O_\mu c^{a_2} (\bar{u}^{a_2} O_\mu d^{a_1}) \right] + h.c.,$$

$$O_\mu = \gamma_\mu (1 + \gamma_5)$$

Here $c_1, c_2$ are short distance Wilson coefficients for $b \to c\bar ud$ transitions and $c^*_1, c^*_2$ are the Wilson coefficients for $c \to s\bar ud$ transitions. It is well-known that the factorizing contributions are proportional to the following two linear combinations

$$a_1 = c_1 + \frac{c_2}{N_c} = c_1 + \xi c_2$$

(19)

$$a_2 = c_2 + \frac{c_1}{N_c} = c_2 + \xi c_1$$

(20)

and the same for $a^*_1$ and $a^*_2$. Here $N_c$ is the number of colors and $\xi = 1/N_c$ is the color singlet projection factor. Phenomenological considerations of the nonleptonic decays of $D$ and $B$ mesons give the following values for the Wilson coefficients

$$a^*_1 \approx 1.2 \pm 0.10 \approx c_1, \quad a^*_2 \approx -0.5 \pm 0.10 \approx c_2 \quad [11]$$

$$a_1 \approx 1.05 \pm 0.10, \quad a_2 \approx 0.25 \pm 0.05 \quad \text{(see e.g. refs. in [10])}$$

The phenomenological results for the coefficients $a^*_{1,2}$ can be seen to correspond to a suppression of the $1/N_c$ term in Eq. (19). A straightforward calculation of these coefficients in the leading logarithmic approximation has been performed in refs. [7,8,10]. For $D$-meson decays it was shown that the coefficient $a^*_1$ is weakly dependent on the choice of the renormalization scheme for fixed values of the renormalization scale and the QCD cutoff parameter:

\[ \gamma_5 = \begin{pmatrix} 0 & -I \\ -I & 0 \end{pmatrix} \]

\[ \text{We employ the notation} \]

\[ \text{\[11\]} \]
\(a_1^* = 1.31 \pm 0.19\) (in accordance with phenomenology). In contrast to this the value of \(a_2^*\) strongly depends on the renormalization scheme, ranging from \(-0.47\pm0.15\) to \(-0.60\pm0.22\). A detailed discussion can be found in ref. [10]. A first calculation of the Wilson coefficients \(a_i\) for bottom hadron decays was done in ref. [7,8]. A more refined analysis of the renormalization coefficients within various renormalization schemes can be found in ref. [10] where was shown that the value of the coefficient \(a_1\) depends weakly on details of calculations: \(a_1=1.01\pm0.02\) (in accordance with phenomenological analysis). As for the case of charm decays the coefficient \(a_2\) is more sensitive to the choice of the renormalization scheme and ranges from \(0.15\pm0.05\) to \(0.20\pm0.05\).

The matrix elements describing heavy-to-heavy \((b \to c)\) and heavy-to-light \((Q \to q)\) transitions can be written as

- heavy-to-heavy transition

**Factorizing contribution**

Diagram I

\[
T_{B_b \to B_c + M}^{fac} = \frac{G_F}{\sqrt{2}} V_{cb} V_{q_1 q_2}^\dagger \chi_\pm < B_c | J^{V+A}_\mu | B_b > \cdot < M | J^{V+A}_\mu | 0 >,
\]

\[
< B_c | J^{V+A}_\mu | B_b > = \frac{N_c g_{BQ}^2}{(4\pi)^4 \Lambda_{q_1} \Lambda_{q_2}} \int \frac{d^4 k}{\pi^2 i} \int \frac{d^4 k'}{\pi^2 i} \exp \left[ \frac{18k^2 + 6(2k' + k)^2}{\Lambda_{BQ}^2} \right] \]

\[
\times \bar{u}(v_2) \Gamma_2 S_{v_2}(k, \Lambda) O_{A_\mu} S_{v_1}(k, \Lambda) \Gamma_1 u(v_1) \ Tr \left[ \Gamma_2'(1+\gamma_2)(1+\gamma_1) \Gamma_1' \right]
\]

(21)

For the matrix elements of the current operator \(J^{V+A}_\mu\) sandwiched between one-meson state \(< M |\) and the vacuum \(|0 >\) we use the standard definitions

\[
< M^P(P_3) | A^\mu | 0 > = f_P P_3^\mu \quad \text{for the pseudoscalar mesons}
\]

\[
< M^V(P_3) | V^\mu | 0 > = f_V M_3 \varepsilon^\mu \quad \text{for the vector mesons}
\]

Here \(\chi_+ = a_1\) for transition with a charged meson in the final state and \(\chi_- = a_2\) for transition with a neutral meson in the final state. \(P_3\) and \(M_3\) are the four-momentum and the mass of the meson, respectively, \(f_P\) is the leptonic decay constant of pseudoscalar meson and \(f_V\) is
the decay constant of vector meson into $e^+e^-$ pair. For $f_P$ and $f_V$ we use the experimental values [1]: $f_\pi = 131$ MeV, $f_K = 160$ MeV, $f_\phi = 237$ MeV, $f_{J/\psi} = 405$ MeV.

Nonfactorizing contributions

Diagram IIa

$$T^{IIa}_{B_b\rightarrow B_c+M} = \frac{G_F}{\sqrt{2}} V_{cb} V_{q_1q_2}^\dagger \frac{N_c! g_{BQ}^2 g_M}{(4\pi)^6 \Lambda_{q_1} \cdots \Lambda_{q_4}} \int \frac{d^4k}{\pi^2i} \int \frac{d^4k'}{\pi^2i} \int \frac{d^4k''}{\pi^2i}$$

$$\times \exp \left[ \frac{9k^2 + 9k'^2 + 3(2k'' + 2k' - k - p_3)^2 + 3(2k'' + k' - p_3)^2}{\Lambda_{BQ}^2} \right]$$

$$\times \bar{u}(v_2)\Gamma_2 S_{v_2}(k, \bar{\Lambda}) O^\mu S_{v_1}(k, \bar{\Lambda}) \Gamma_1 u(v_1) \text{ Tr} \left[ \Gamma'_2 (1+ \not{\psi}_2) \Gamma_M (1+ \not{\psi}_3) O_\mu (1+ \not{\psi}_1) \Gamma'_1 \right]$$

Here $g_M$ is the meson-quark coupling constant which is calculated with the use of the compositeness condition. The Dirac structure $\Gamma_M$ specifies the mesonic final state, i.e. $\Gamma_M = i\gamma_5$ for pseudoscalar mesons and $\Gamma_M = \gamma_\mu$ for vector mesons.

Diagram IIb

$$T^{IIb}_{B_b\rightarrow B_c+M} = \frac{G_F}{\sqrt{2}} V_{cb} V_{q_1q_2}^\dagger \frac{N_c! g_{BQ}^2 g_M}{(4\pi)^6 \Lambda_{q_1} \cdots \Lambda_{q_4}} \int \frac{d^4k}{\pi^2i} \int \frac{d^4k'}{\pi^2i} \int \frac{d^4k''}{\pi^2i}$$

$$\times \exp \left[ \frac{9k^2 + 9k'^2 + 3(2k'' + k + p_3)^2 + 3(2k'' + 2k - k' + p_3)^2}{\Lambda_{BQ}^2} \right]$$

$$\times \bar{u}(v_2)\Gamma_2 S_{v_2}(k, \bar{\Lambda}) O^\mu S_{v_1}(k, \bar{\Lambda}) \Gamma_1 u(v_1) \text{ Tr} \left[ \Gamma'_2 (1+ \not{\psi}_2) O_\mu \Gamma_M (1+ \not{\psi}_3) (1+ \not{\psi}_1) \Gamma'_1 \right]$$
Diagram III

\[ T_{B_0 \rightarrow B_c + M}^{III} = \frac{G_F}{\sqrt{2}} V_{cb} V_{q_1 q_2}^\dagger \frac{N_c! g_{B_q}^2 g_M}{(4\pi)^6 \Lambda_{q_1} \cdots \Lambda_{q_4}} \int \frac{d^4 k}{\pi^2 i} \int \frac{d^4 k'}{\pi^2 i} \int \frac{d^4 k''}{\pi^2 i} \]

\[ \times \exp \left[ \frac{9k^2 + 9k'^2 + 3(2k'' - k - p_3)^2 + 3(2k'' - k' + p_3)^2}{\Lambda^2_{B_q}} \right] \]

\[ \times \bar{u}(v_2)\Gamma_2 O^\mu S_{v_1}(k, \bar{\Lambda})\Gamma_1 u(v_1) \text{ Tr} \left[ \Gamma_2'(1+ \gamma_2)(1+ \gamma_1)\Gamma_1' \right] \]

- heavy-to-light transition

Factorizing contribution

Diagram I

\[ T_{B_Q \rightarrow B_q + M}^{I} = \frac{G_F}{\sqrt{2}} V_{q_1 q_2} \chi_+ \cdot \frac{N_c! g_{B_Q} g_{B_q}}{(4\pi)^6 \Lambda_{q_1} \Lambda_{q_4}} \int \frac{d^4 k}{\pi^2 i} \int \frac{d^4 k'}{\pi^2 i} \exp \left[ \frac{9k^2 + 3(2k' + k)^2}{\Lambda^2_{B_Q}} \right] \]

\[ \times \exp \left[ \frac{(3k + 2p_2)^2 + 3(2k' + k)^2}{\Lambda^2_{B_q}} \right] \]

\[ \times \bar{u}(p_2)\Gamma_2 O^\mu S_{v_1}(k, \bar{\Lambda})\Gamma_1 u(v_1) \text{ Tr} \left[ \Gamma_2'(1+ \gamma_2)(1+ \gamma_1)\Gamma_1' \right] \]

Nonfactorizing contributions

Diagram IIa

\[ T_{B_Q \rightarrow B_q + M}^{IIa} = \frac{G_F}{\sqrt{2}} V_{q_1 q_2} \frac{N_c! g_{B_Q} g_{B_q} g_M}{(4\pi)^6 \Lambda_{q_1} \cdots \Lambda_{q_4}} \int \frac{d^4 k}{\pi^2 i} \int \frac{d^4 k'}{\pi^2 i} \int \frac{d^4 k''}{\pi^2 i} \]

\[ \times \exp \left[ \frac{9k^2 + 3(2k'' + 2k' - k - p_3)^2 + 3(2k'' + k' - p_3)^2}{\Lambda^2_{B_Q}} \right] \]

\[ \times \bar{u}(p_2)\Gamma_2 O^\mu S_{v_1}(k, \bar{\Lambda})\Gamma_1 u(v_1) \text{ Tr} \left[ \Gamma_2'(1+ \gamma_2)\Gamma_M(1+ \gamma_3)O_\mu(1+ \gamma_1)\Gamma_1' \right] \]
Diagram IIb

\[
T_{B_Q \rightarrow B_q+M}^{IIb} = \frac{G_F}{\sqrt{2}} V_{Qq} V_{q_1q_2}^+ \frac{N_c! \, g_{B_Q} \, g_{B_q} \, g_M}{(4\pi)^6 \Lambda q_1 \cdots \Lambda q_4} \int \frac{d^4k}{\pi^2 i} \int \frac{d^4k'}{\pi^2 i} \int \frac{d^4k''}{\pi^2 i}  
\]

\[
\times \exp \left[ \frac{9k^2 + 3(2k'' + k + p_3)^2}{\Lambda_{BQ}^2} + \frac{(3k' + 2p_2)^2 + 3(2k'' + 2k - k' + p_3)^2}{\Lambda_{Bq}^2} \right] 
\]

\[
\times \bar{u}(p_2) \Gamma_2 O^\mu S_{v_1}(k, \bar{\Lambda}) \Gamma_1 u(v_1) \, \text{Tr} \left[ \Gamma_2' (1 + \gamma_2) O_{\mu} \Gamma_M (1 + \gamma_3) (1 + \gamma_1') \Gamma_1' \right] 
\]

Diagram III

\[
T_{B_Q \rightarrow B_q+M}^{III} = \frac{G_F}{\sqrt{2}} V_{Qq} V_{q_1q_2}^+ \frac{N_c! \, g_{B_Q} \, g_{B_q} \, g_M}{(4\pi)^6 \Lambda q_1 \cdots \Lambda q_4} \int \frac{d^4k}{\pi^2 i} \int \frac{d^4k'}{\pi^2 i} \int \frac{d^4k''}{\pi^2 i}  
\]

\[
\times \exp \left[ \frac{9k^2 + 3(2k'' - k - p_3)^2}{\Lambda_{BQ}^2} + \frac{(3k' + 2p_2)^2 + 3(2k'' - k' + p_3)^2}{\Lambda_{Bq}^2} \right] 
\]

\[
\times \bar{u}(p_2) \Gamma_2 O^\mu S_{v_1}(k, \bar{\Lambda}) \Gamma_1 u(v_1) \, \text{Tr} \left[ \Gamma_2' (1 + \gamma_2) O_{\mu} (1 + \gamma_1') \Gamma_1' \Gamma_M (1 + \gamma_3) \right] 
\]

Details of the calculation of the matrix elements (21)-(28) can be found in Appendix B.

Below we list the Lorentz-spinor parts \( \bar{u}(p_2) ... u(p_1) \text{Tr} [...] \) of the individual diagrams where one has to differentiate between the various possible light diquark transitions. (\( M_1, M_2 \) and \( M_3 \) denote the masses of the initial and final baryons, and the meson, respectively)

- **scalar-to-scalar diquark transitions**

Factorizing Diagram (I)

\[
(M_1 M_2 M_3) v_3^\mu [\bar{u}(\gamma_2 + 1) O_{\mu} (\gamma_1 + 1) u] \text{Tr} [\gamma_5 (\gamma_2 + 1) (\gamma_1 + 1) \gamma_5] 
\]

\[
= 8Q_+ \bar{u}(M_- - M_+ \gamma_5) u \bigg|_{M_2/M_1 \rightarrow 0} = 8M_1^3 \bar{u}(1 - \gamma_5) u 
\]
Diagram IIa

\[(M_1 M_2 M_3) \bar{u}(\gamma_2 + 1) O_{\mu}(\gamma_1 + 1) u \text{Tr}[\gamma_5(\gamma_2 + 1) \gamma_5(\gamma_3 + 1) O_{\mu}(\gamma_1 + 1) \gamma_5]\]

\[= 16M_1 \bar{u}[-P_+ - \gamma_5 Q_+] \Bigg|_{M_2/M_1 \to 0} \implies 16M_1^2 \bar{u}(1 - \gamma_5)u \tag{30}\]

Diagram IIb

\[-(M_1 M_2 M_3) \bar{u}(\gamma_2 + 1) O_{\mu}(\gamma_1 + 1) u \text{Tr}[\gamma_5(\gamma_2 + 1) O_{\mu} \gamma_5(\gamma_3 + 1)(\gamma_1 + 1) \gamma_5]\]

\[= 16M_2 \bar{u}[D_+ - \gamma_5 Q_+] \Bigg|_{M_2/M_1 \to 0} \implies 16M_1^2 M_2 \bar{u}(1 - \gamma_5)u \tag{31}\]

Diagram III

\[(M_1 M_2 M_3) \bar{u}(\gamma_2 + 1) O_{\mu}(\gamma_1 + 1) u \text{Tr}[\gamma_5(\gamma_2 + 1) O_{\mu} \gamma_5(\gamma_3 + 1) \gamma_5 \gamma_5(\gamma_3 + 1)]\]

\[= 32 (M_1 M_2) \sum_{i=1}^{3} M_i \bar{u} \gamma_5 u \Bigg|_{M_2/M_1 \to 0} \implies 32M_1^2 M_2 \bar{u} \gamma_5 u \tag{32}\]

- **vector-to-scalar diquark transitions**

Diagram IIa

\[(M_1 M_2 M_3) [\bar{u} \gamma^\beta \gamma^5(\gamma_2 + 1) O_{\mu}(\gamma_1 + 1) u \text{Tr}[\gamma_5(\gamma_2 + 1) \gamma_5(\gamma_3 + 1) O_{\mu}(\gamma_1 + 1) \gamma_5]\]

\[= 16M_1 \bar{u}[3P_+ - \gamma_5 Q_+] \bigg|_{M_2/M_1 \to 0} \implies -16M_1^2 \bar{u}(3 + \gamma_5)u \tag{33}\]

Diagram IIb

\[(M_1 M_2 M_3) [\bar{u} \gamma^\beta \gamma^5(\gamma_2 + 1) O_{\mu}(\gamma_1 + 1) u \text{Tr}[\gamma_5(\gamma_2 + 1) O_{\mu} \gamma_5(\gamma_3 + 1)(\gamma_1 + 1) \gamma_5]\]

\[= -48M_2 \bar{u}[D_+ - \gamma_5 Q_+] \bigg|_{M_2/M_1 \to 0} \implies -48M_1^2 M_2 \bar{u}(1 - \gamma_5)u \tag{34}\]

Diagram III
\[(M_1M_2M_3)[\bar{u}\gamma^\alpha\gamma^5(\not{p}_2 + 1)O_\mu(\not{p}_1 + 1)u]Tr[\gamma_\beta(\not{p}_2 + 1)O_\mu(\not{p}_1 + 1)\gamma_\gamma_5(\not{p}_3 + 1)]\]

\[= -96(M_1M_2)\sum_{i=1}^{3} M_i \bar{u}\gamma_5 u \bigg|_{M_2/M_1 \to 0} \implies -96M_1^2M_2\bar{u}\gamma_5 u \quad (35)\]

- vector-to-vector diquark transitions

Factorizing Diagram (I)

\[(M_1M_2M_3)v_3^\mu[\bar{u}\gamma^\alpha\alpha^5(\not{p}_2 + 1)O_\mu(\not{p}_1 + 1)\gamma^\beta\gamma^5 u]Tr[\gamma_\alpha(\not{p}_2 + 1)(\not{p}_1 + 1)\gamma_\beta]\]

\[= -8Q_+\bar{u}(3M_\gamma + M_+\gamma_5)u \bigg|_{M_2/M_1 \to 0} \implies -8M_1^3\bar{u}(3 + \gamma_5)u \quad (36)\]

Diagram IIa

\[(M_1M_2M_3)[\bar{u}\gamma^\alpha\gamma^5(\not{p}_2 + 1)O_\mu(\not{p}_1 + 1)\gamma^\beta\gamma^5 u]Tr[\gamma_\alpha(\not{p}_2 + 1)\gamma^5(\not{p}_3 + 1)O_\mu(\not{p}_1 + 1)\gamma_\beta]\]

\[= 48M_1\bar{u}[3P_+ - \gamma_5Q_+]u \bigg|_{M_2/M_1 \to 0} \implies -48M_1^3\bar{u}(3 + \gamma_5)u \quad (37)\]

Diagram IIb

\[(M_1M_2M_3)[\bar{u}\gamma^\alpha\gamma^5(\not{p}_2 + 1)O_\mu(\not{p}_1 + 1)\gamma^\beta\gamma^5 u]Tr[\gamma_\alpha(\not{p}_2 + 1)O_\mu\gamma_5(\not{p}_3 + 1)(\not{p}_1 + 1)\gamma_\beta]\]

\[= -48M_2\bar{u}[3D_+ + \gamma_5Q_+]u \bigg|_{M_2/M_1 \to 0} \implies -48M_1^2M_2\bar{u}(3 + \gamma_5)u \quad (38)\]

Diagram III

\[(M_1M_2M_3)[\bar{u}\gamma^\alpha\gamma^5(\not{p}_2 + 1)O_\mu(\not{p}_1 + 1)\gamma^\beta\gamma^5 u][\gamma_\alpha(\not{p}_2 + 1)O_\mu(\not{p}_1 + 1)\gamma_\gamma_5(\not{p}_3 + 1)]\]

\[= -288 (M_1M_2)\sum_{i=1}^{3} M_i\bar{u}\gamma_5 u \bigg|_{M_2/M_1 \to 0} \implies -288M_1^2M_2\bar{u}\gamma_5 u \quad (39)\]

where

\[Q_+ = (M_1 + M_2)^2 - M_3^2, \quad P_+ = (M_2 + M_3)^2 - M_1^2, \quad D_+ = (M_1 + M_3)^2 - M_2^2\]
The relations Eqs. (29)-(39) are in a complete agreement with the result of spectator model analysis [14]. Note also that the contributions arising from the diagrams IIb and III can be seen to be down by the helicity flip factor \( (M_2/M_1) \) in agreement with the result of [14].

The general invariant matrix element describing exclusive weak nonleptonic decays of heavy baryons \( 1/2^+ \rightarrow 1/2^+ + 0^- \) is given by one

\[
M = M_I + M_{IIa} + M_{IIb} + M_{III} \equiv A - \gamma_5 B
\]  

(40)

where the amplitudes \( M_I, M_{IIa}, M_{IIb}, \) and \( M_{III} \) are determined from the diagrams I, IIa, IIb, and III, respectively. Our results are given in the form

**Factorizing contribution:**

Diagram I :  
\[
M_I = c_W \chi_{\pm} f_P \frac{Q^+}{4 M_1 M_2} \left( M_- \epsilon_{FD} - M_+ \epsilon_+ \cdot \gamma^5 \right) f(M_1, M_2, M_3) 
\]

(41)

**Nonfactorizing contributions**

Diagram IIa :  
\[
M_{IIa} = c_W c_- \frac{H_2(M_1, M_2, M_3)}{4 M_1 M_2} \left( P_+ \epsilon_{IIa}^P - Q_+ \epsilon_{IIa}^Q \cdot \gamma^5 \right) M_1 
\]

(42)

Diagram IIb :  
\[
M_{IIb} = c_W c_- \frac{H_2(M_1, M_2, M_3)}{4 M_1 M_2} \left( D_+ \epsilon_{IIb}^D - Q_+ \epsilon_{IIb}^Q \cdot \gamma^5 \right) M_2 
\]

(43)

Diagram III :  
\[
M_{III} = c_W c_- \frac{H_3(M_1, M_2, M_3)}{4 M_1 M_2} \sum_{i=1}^{3} M_i(M_1 M_2) \epsilon_{III} \cdot \gamma^5
\]

(44)

Here, \( c_W = G_F/\sqrt{2} V_{QQ}'(q) V_{QQ}^* \), \( f_P \) \( (P = \pi, K) \) are meson leptonic decay constants; \( c_- = c_1 - c_2 \) and \( \epsilon_{FD}^\pm, \epsilon_{IIa}^P, \epsilon_{IIa}^Q, \epsilon_{IIb}^P, \epsilon_{IIb}^Q, \epsilon_{III} \) are flavor coefficients whose values are listed in Tables IIa and IIb. The full list of expressions for the form factors \( f(M_1, M_2, M_3), H_2(M_1, M_2, M_3) \) and \( H_3(M_1, M_2, M_3) \) appearing in Eqs. (41)-(44) is given below. At the present stage we only give a complete analysis of the Cabibbo-favored nonleptonic decays only for \( 1/2^+ \rightarrow 1/2^+ + 0^- \) transitions. In addition to these decays we shall also consider the factorizing processes with vector mesons \( \Lambda_c^+ \rightarrow p\phi \) and \( \Lambda_b^0 \rightarrow J/\psi\Lambda \) which were recently measured by the CLEO [3] and CDF [6] Collaborations.
\[ b \to c \text{ transitions} \]

\[
f(\omega) = \frac{R(\omega, \bar{\Lambda})}{R(1, \bar{\Lambda})}, \quad \omega = \frac{M_1^2 + M_2^2 - M_3^2}{2M_1M_2}
\]

\[
H_i(\omega) = d_i t_i(r) \frac{R_H(\omega, \bar{\Lambda}_i, \bar{\Lambda}_f)}{\sqrt{R(1, \bar{\Lambda})R(1, \bar{\Lambda})}} \frac{8}{9\pi\sqrt{3}} \frac{\Lambda_{BQ}^4}{\lambda^3}, \quad (i = 2, 3)
\]

where

\[
R(\omega, \bar{\Lambda}) = \int_0^\infty du \int_0^1 d\alpha \exp \left\{ -18u^2 [1 + 2\alpha (1 - \alpha)(\omega - 1)] + 36u\bar{\Lambda}/\Lambda_{BQ} \right\}
\]

\[
R_H(\omega, \bar{\Lambda}_i, \bar{\Lambda}_f) = \int_0^\infty du \int_0^1 d\alpha \exp \left\{ -72u^2 [1 + 2\alpha (1 - \alpha)(\omega - 1)] \right\}
\]

\[ + \exp \left\{ 144u(\bar{\Lambda}_i \alpha + \bar{\Lambda}_f (1 - \alpha))/\Lambda_{BQ} - 432u^2(\alpha^2 + (1 - \alpha)^2) \right\} \]

Here \(d_2 = 1\) and \(d_3 = 0.5\exp[9M_3^2/2\Lambda_{BQ}^2]\). The parameters \(\bar{\Lambda}_i\) and \(\bar{\Lambda}_f\) correspond to initial and final baryons, respectively. The parameters \(t_i(r)\), where \(r = \Lambda/\Lambda_s\), are given in Table IIIa.

It is well-known that there are altogether three IW functions \(\zeta(\omega), \xi_1(\omega)\) and \(\xi_2(\omega)\) describing current induced ground state to ground state transitions. Here \(\zeta(\omega)\) describes \(\Lambda_Q\)-type baryon transitions and \(\Omega_Q\)-type baryon transitions \[43,44\]. In our approach they are expressed via a single universal function \(f(\omega)\)

\[
\zeta(\omega) = \xi_1(\omega) = \xi_2(\omega)(1 + \omega) = f(\omega) \frac{Q_+}{M_1M_2} = f(\omega) \frac{\omega + 1}{2}, \quad f(1) = 1
\]

This result coincides with the prediction of large-\(N_c\) QCD \[45\] and reproduces the result of the spectator quark model \[43\].

**Heavy-light transitions**

\[
f(M_1, M_2, M_3) = \frac{R_{FD}(M_1, M_2, M_3, \bar{\Lambda})}{\sqrt{R(1, \bar{\Lambda})}} \frac{8R^2}{(1 + R)^3} \frac{1}{\sqrt{\chi(r)}}, \quad R = \frac{\Lambda_{BQ}^2}{\Lambda_{Bq}^2}
\]

\[
R_{FD}(M_1, M_2, M_3, \bar{\Lambda}) = \int_0^\infty d\alpha \exp \left[ -9\alpha^2 (1 + R) + 18\alpha \frac{\bar{\Lambda}}{\Lambda_{BQ}} (1 + R) \right]
\]

\[ \times \exp \left[ -12\alpha R \omega \frac{M_2}{\Lambda_{BQ}} + \frac{4R}{R + 1} \frac{M_2^2}{\Lambda_{BQ}^2} \right] \]

\[
H_i(M_1, M_2, M_3, \bar{\Lambda}) = \frac{t_i(r)}{\sqrt{\chi(r)}} \frac{R_{H_i}(M_1, M_2, M_3, \bar{\Lambda})}{(1 + R)\sqrt{R(1, \bar{\Lambda})}} \frac{4}{9\pi\sqrt{3}} \frac{\Lambda_{BQ}^4}{\lambda^3}, \quad (i = 2, 3)
\]
where

\[ R_{H_2} = \int_0^\infty d\alpha \exp \left[ 36\alpha^2(1+R)(3R+4) + 72\alpha \frac{\bar{\Lambda}}{\Lambda_{BQ}}(1+R) - 12\alpha R \omega \frac{M_2}{\Lambda_{BQ}} + \frac{R}{1+R} \frac{M_2^2}{\Lambda_{BQ}^2} \right] \]

\[ R_{H_3} = \int_0^\infty d\alpha \exp \left[ 36\alpha^2(1+R)(3R+4) + 72\alpha \frac{\bar{\Lambda}}{\Lambda_{BQ}}(1+R) - 12\alpha R \frac{M_2^2 + M_3^2}{\Lambda_{BQ} M_1} \right] \]

\[ \times \exp \left[ \frac{R}{1+R} \frac{M_2^2 + 6M_3^2}{\Lambda_{BQ}^2} \right] \]

The parameters \( \chi(r) \), \( t_2(r) \) and \( t_3(r) \) are given in Table IIIb. The terms proportional to \( (M_1 - M_2)/\Lambda_{BQ} \) in the exponents in Eqs. (45) and (47) have been dropped for physical reasons.

**IV. RESULTS**

In this section we give our numerical results for the decay rates and the asymmetry parameters in the nonleptonic decays of \( \Lambda_Q \), \( \Xi_Q \) and \( \Omega_Q \) baryons. Let us specify the model parameters. Our model contains the following set of parameters: the cutoff parameters \( \Lambda_{B_q} \) and \( \Lambda_{B_Q} \) and the binding energy parameters (\( \bar{\Lambda}, \bar{\Lambda}_s \) and \( \bar{\Lambda}_{ss} \)). Three of the parameters (\( \Lambda_{B_q}, \Lambda_{B_Q} \) and \( \bar{\Lambda} \)) are used to fit known branching ratios of five nonleptonic decays \( \Lambda_c^+ \rightarrow \Lambda^0 \pi^+, \Lambda_c^+ \rightarrow \Sigma^0 \pi^+, \Lambda_c^+ \rightarrow \Sigma^+ \pi^0, \Lambda_c^+ \rightarrow pK^0 \) and \( \Lambda_c^+ \rightarrow \Xi^0 K^+ \). Moreover, in the fit we impose the condition \( \rho^2 = 1 \) on the slope of baryonic Isgur-Wise function. The fit yields the following values for these model parameters: \( \Lambda_{B_q} = 3.037 \) GeV, \( \Lambda_{B_Q} = 2.408 \) GeV, \( \bar{\Lambda} = 0.9 \) GeV. One has to remark that the values \( \Lambda_{B_q} \) and \( \Lambda_{B_Q} \) are the phenomenological parameters which in principle are related to the size of a baryon. However their magnitude is not strongly constrained by the experimental values of baryon observables and allows for the variation in a rather wide range. Note that the obtained value \( \Lambda_{B_Q} = 2.408 \) GeV is close to \( \Lambda_{B_Q} = 2.5 \) GeV coming from analysis of semileptonic heavy baryon decays in relativistic three-quark model which uses the constituent quark masses [34]. As to the cutoff parameter in the light-baryon vertex, in Ref. [33] it was demonstrated that the experimental data both for the dimensionless (nucleon magnetic moments) as well as dimensionful (nucleon charge radii) observables can be described successfully, using the value of the parameter \( \Lambda_{B_q} \) from the interval \( \sim (1-3) \) GeV provided the constituent quark mass is properly fitted. In particular, for the value \( \Lambda_{B_q} = 3.037 \) GeV, with the constituent quark mass \( m_q = 315 \) MeV, we obtain for the nucleon magnetic moments and charge radii: \( \mu_p = 2.62 \) (experiment 2.79), \( \mu_n = - \)

\[ \]
1.61 (experiment -1.91), $r^E_p = 0.82$ fm (experiment $0.86\pm0.01$ fm), $< r^2 >^E_n = -0.188$ fm$^2$ (experiment $-0.119\pm0.004$ fm$^2$), $r^M_p = 0.74$ fm (experiment $0.86\pm0.06$ fm), $r^M_n = 0.76$ fm (experiment $0.88\pm0.07$ fm). The parameters $\bar{\Lambda}_s$ and $\bar{\Lambda}_{ss}$ cannot be determined at present due to the lack of experimental information on the decays of heavy-light baryons containing one or two strange quarks. For the time being we fix them at the values $\bar{\Lambda}_s=1$ GeV and $\bar{\Lambda}_{ss}=1.1$ GeV. The masses of hadrons are taken from [1,15]. In what follows we will use the following values for the Cabibbo-Kobayashi-Maskawa matrix elements $V_{qq'}$ [1]:

$$|V_{cb}| = 0.04, \quad |V_{ud}| \approx |V_{cs}| = 0.975, \quad |V_{us}| \approx |V_{cd}| = 0.22, \quad |V_{ub}| = 0.0035 \quad (48)$$

The Wilson coefficients are taken to be $a_1 = 1.03$, $a_2 = 0.10$, $a_1^* = 1.3$, $a_2^* = -0.65$.

In order to check on the consistency of our approach, we shall prove that the Isgur-Wise functions $\xi_1$ and $\xi_2$ satisfy the model-independent Bjorken-Xu inequalities [47]. As was mentioned in Sec. 3 the baryonic IW functions $\zeta(\omega)$, $\xi_1(\omega)$ and $\xi_2(\omega)$, corresponding to $\Lambda_Q$-type and $\Omega_Q$-type heavy-heavy weak baryon transitions, are expressed via a single universal function $f(\omega)$ (see, Eqs. (45) and 46)).

The IW-functions $\xi_1$ and $\xi_2$ must satisfy to the two model-independent Bjorken-Xu inequalities in [47]. The first inequality reads

$$1 \geq \frac{2 + \omega^2}{3} \xi_1^2(\omega) + \frac{(\omega^2 - 1)^2}{3} \xi_2^2(\omega) + \frac{2}{3}(\omega - \omega^3)\xi_1(\omega)\xi_2(\omega) \quad (49)$$

The inequality (49) implies a second inequality, namely a model-independent restriction on the slope (radius) of the form factor $\xi_1(\omega)$

$$\rho^2_{\xi_1} \equiv -\frac{d\xi_1(\omega)}{d\omega} \bigg|_{\omega=1} \geq \frac{1}{3} - \frac{2}{3} \xi_2(1) \quad (50)$$

From the inequality (49) we find an upper limit for the universal function $f(\omega)$

$$\xi_1(\omega) \leq 1 \text{ or } f(\omega) \leq \sqrt{\frac{2}{1+\omega}} \quad (51)$$

which we impose as a condition.

From the inequality (49) for the slope of the function $\xi_1(\omega)$ we see that $\rho^2_{\xi_1} \geq 0$. For the choice of model parameters corresponding to the best fit the universal function $f(\omega)$ and the slope of the $\xi_1$ satisfy to the Bjorken-Xu inequalities [47] and (50). In this case the charge radii of the $\zeta$ and $\xi$ functions are equal to 0.84. Our form factor function $f(\omega)$ is well approximated by the formula
\[
f(\omega) \approx \left[ \frac{2}{1 + \omega} \right]^{1+0.68/\omega}
\]

In Table IV we present the branching ratios of the decays \( \Lambda_c^+ \to \Lambda^0 \pi^+ \), \( \Lambda_c^+ \to \Sigma^0 \pi^+ \), \( \Lambda_c^+ \to \Sigma^+ \pi^0 \), and \( \Lambda_c^+ \to \Xi^0 K^+ \) which are described nicely using a three-parameter fit. Our predictions for the other heavy-to-light decay modes are listed in Table IV. In Table V we give the calculated values for the asymmetry parameters in the nonleptonic decays of \( 1/2^+ \) charm and bottom baryons into octet of light baryons and pseudoscalar mesons (pions and kaons). The relevant formulae for the decay rates and the asymmetry parameters in terms of the invariant amplitudes \( A \) and \( B \) are listed in ref. [14]. For comparison in Tables IV and V we quote the results predicted by other phenomenological approaches. It is seen that rates of decays which proceed only via the nonfactorizing diagrams are not suppressed.

In Table VI we list our predictions for the parity-violating \((A)\) and parity-conserving \((B)\) amplitudes in the decays \( \Lambda_c^+ \to \Lambda \pi^+ \) and \( \Lambda_c^+ \to \Sigma^+ \pi^0 \) in units of \( G_F V_{cs} V_{ud} \times 10^{-2} \text{GeV}^2 \).

In Table VII we give the predictions for the rates and the asymmetry parameters in the nonleptonic decays of bottom baryons into charm baryons with the use of the same model parameters. A clear pattern emerges. The dominant rates are into channels with factorizing contributions. Rates which proceed only via nonfactorizing diagrams are small but not negligibly small. The total contribution of the nonfactorizing diagrams can be seen to be destructive. The sum of nonfactorizing contributions amount up to 30% of the factorizing contribution in amplitude. Using \( \tau(\Lambda_b) = (1.14 \pm 0.08) \times 10^{-12} \text{s} \) [1] we predict a branching ratio of the mode \( \Lambda_b \to \Lambda_c \pi \) to be \((0.44 \pm 0.003)\% \). If one neglects the nonfactorizing contributions for this mode as was done in [26] one would obtain an enhanced rate of \( \Gamma = 0.665 \times 10^{10} \text{s}^{-1} \). The prediction for the asymmetry parameter remains at \( \alpha \approx -1 \) and is thus not affected by such an omission.

In Tables VIII and IX we analyze the nonfactorizing contributions to the decay amplitudes for the transitions \( \Lambda_c^+ \to \Lambda \pi^+ \) and \( \Lambda_b^0 \to \Lambda_c^+ \pi^- \). It is seen that the total contribution of the nonfactorizing diagrams are destructive. They can amount up to \( \sim 60 \% \) of the factorizing contribution in amplitude of heavy-to-light transition and up to \( \sim 30 \% \) of the factorizing contribution in amplitude of \( b \to c \) transition. Also we calculate the values for overlap integrals \( f, H_2 \) and \( H_3 \) for these modes. They turn out to be equal to \( f = 0.51, H_2 = 43 \text{ MeV} \) and \( H_3 = 14 \text{ MeV} \) for \( \Lambda_c^+ \to \Lambda \pi^+ \) and \( f = 0.61, H_2 = 24 \text{ MeV} \) and \( H_3 = 12 \text{ MeV} \) for \( \Lambda_b^0 \to \Lambda_c^+ \pi^- \). For comparison we quote the results for overlap integrals evaluated for the decay \( \Lambda_c^+ \to \Lambda \pi^+ \) in ref. [14]: \( f = 0.34, H_2 = 40 \text{ MeV} \) and \( H_3 = -4 \text{ MeV} \).
In Tables X and XI we present the predictions for the $\Lambda_c^+ \to p\phi$ and $\Lambda_b^0 \to J/\psi\Lambda$ decays for various values of the $a_2$ and $a_2^*$ parameters. As mentioned before these processes are described by the factorizing diagram alone. The corresponding weak hadronic matrix elements in the spectator approximation have a trivial spin structure given by the matrix $O_\mu$. For this reason the asymmetry parameter for these transitions does not depend on the model parameters and can be expressed through the hadron masses

$$\alpha \left( \frac{1}{2}^+ \to \frac{1}{2}^+ + 1^- \right) = -\frac{M_1^2 - M_2^2 - 2M_3^2}{\sqrt{Q^+Q^-} + \frac{3}{2}M_3^2(Q^+ + Q^-)} \quad (53)$$

In particular, the asymmetry parameter in the decay $\Lambda_c^+ \to p\phi$ is equal to $-0.26$ and $\alpha = 0.21$ for $\Lambda_b^0 \to J/\psi\Lambda$ transition. It is seen that for the accepted value of the Wilson coefficient $a_2 = 0.10$ our approach gives the prediction for the branching $Br(\Lambda_b^0 \to J/\psi\Lambda) = 0.027$ which is consistent with the recent CDF data $Br(\Lambda_b^0 \to J/\psi\Lambda) = 0.037 \pm 0.017 \pm 0.004$ [6]. For the rare decay $\Lambda_c^+ \to p\phi$ our approach for the accepted value of the corresponding Wilson coefficient $a_2^* = -0.65$ yields the branching ratio $Br(p\phi)/Br(pK^-\pi^+) = 0.105$ which overestimates the known experimental data from CLEO [3] and NA32 [2] measurements.

V. CONCLUSION

We have studied the exclusive nonleptonic decays of heavy-light baryons into charm and light baryons. The decay rates and the asymmetry parameters have been calculated. It would be interesting to test our predictions in $b \to c$ transitions in future experiments.

We have shown that rates of decays which proceed only via the nonfactorizing diagrams are suppressed but not completely suppressed for both cases of heavy-to-light and heavy-to-heavy transitions. We have analyzed in detail the nonfactorizing contributions to the decay amplitudes for the transitions $\Lambda_c^+ \to \Lambda\pi^+$ and $\Lambda_b^0 \to \Lambda_c^+\pi^-$. It was shown that the total contribution of the nonfactorizing diagrams are destructive. They amount up to $\sim 60\%$ of the factorizing contribution in amplitude of heavy-to-light transition and up to $\sim 30\%$ of the factorizing contribution in amplitude of $b \to c$ transition. Finally, we give the predictions for the $\Lambda_c^+ \to p\phi$ and $\Lambda_b^0 \to J/\psi\Lambda$ decays for various values of the $a_2$ and $a_2^*$ parameters.

The generalization to the channels $\frac{1}{2}^+ \to \frac{1}{2}^+ + 1^-$, $\frac{3}{2}^+ \to \frac{3}{2}^+ + 0^-$ and $\frac{1}{2}^+ \to \frac{1}{2}^+ + 1^-$ involving the ground state partners of the mesons and baryons in the final state is straightforward and will be treated in a subsequent paper. In this paper we have only discussed the Cabibbo favoured decays induced by the transitions $b \to c\bar{u}d$ with a light pseudoscalar
meson in the final state. There are also a number of Cabibbo favoured decays with heavy mesons in the final state which include the decays induced by the quark transitions \( b \to c \bar{c}s \). The treatment of heavy mesons in the final state requires some refinements in our simple Lagrangian spectator model. Again, exclusive nonleptonic heavy baryon decays involving heavy mesons in the final state are the subject of a future publication.

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APPENDIX A: HADRON-QUARK INTERACTION LAGRANGIANS

Below we present a complete list of hadronic interaction Lagrangians used in the calculations. We start from the consideration of various possible couplings of three quarks in the light baryons. It is well known that there are five possible nonderivative forms of such coupling for octet baryons. We start from the consideration of various possible couplings of three quarks in

\[\bar{B}^{km} q_{m_1} q_{m_2} C\gamma_5 q_{m_3} \varepsilon^{a_1 a_2 a_3} \varepsilon^{km2m3}\]

Pseudoscalar variant

\[\bar{B}^{km} \gamma_5 q_{m_1} q_{m_2} C q_{m_3} \varepsilon^{a_1 a_2 a_3} \varepsilon^{km2m3}\]

Scalar variant

\[\bar{B}^{km} \gamma_\mu q_{m_1} q_{m_2} C\gamma_\mu \gamma_5 q_{m_3} \varepsilon^{a_1 a_2 a_3} \varepsilon^{km2m3}\]

Axial variant

\[\bar{B}^{km} \lambda_{m_1}^{m_2} \gamma_\mu \gamma_5 q_{m_1} q_{m_2} \lambda^{m_3} C\gamma_\mu q_{m_3} \varepsilon^{a_1 a_2 a_3} \varepsilon^{km2m3}\]

Vector variant

\[\bar{B}^{km} \lambda_{m_1}^{m_2} \gamma_\mu \gamma_5 q_{m_1} q_{m_2} \lambda^{m_3} C\gamma_\mu q_{m_3} \varepsilon^{a_1 a_2 a_3} \varepsilon^{km2m3}\]

Tensor variant

\[\bar{B}^{km} \lambda_{m_1}^{m_2} \sigma_{\mu \nu} \gamma_5 q_{m_1} q_{m_2} \lambda^{m_3} C\sigma_{\mu \nu} q_{m_3} \varepsilon^{a_1 a_2 a_3} \varepsilon^{km2m3}\]

where \(\bar{B}^{km}\) is the baryonic octet matrix

\[
\bar{B}^{km} = \begin{pmatrix}
\bar{\Sigma}^0 / \sqrt{2} + \bar{\Lambda}^0 / \sqrt{6} & \bar{\Sigma}^- & -\bar{\Xi}^- \\
-\bar{\Sigma}^0 / \sqrt{2} + \bar{\Lambda}^0 / \sqrt{6} & \bar{\Sigma}^- & -\bar{\Xi}^- \\
\bar{\rho} & \bar{\eta} & -2\bar{\Lambda}^0 / \sqrt{6}
\end{pmatrix}
\]

(A2)

It is well known that these five forms can be combined in the two linearly independent SU(3) invariant combinations called vector variant and tensor variant (see, Eqs. (3)).

In order to reproduce the results of the spectator model in the Lagrangian formulation, one has to modify the baryonic currents writing them in terms of the "projected" quark fields, replacing \(q \rightarrow V_+ q\), where \(V_+ = 1/2 (\bar{\rho} + 1)\) is the projector introduced is Sec. 2. With the use of the "on-shell" conditions \(BV_+ = B\) and \(v^2 = 1\) it is easy to verify that there exist simple relations between various interaction Lagrangians obtained from Eq. (A1) via the substitution \(q \rightarrow V_+ q\):

\[
\bar{B}^{km} V_+ q_{m_1} q_{m_2} C\gamma_5 V_+ q_{m_3} \varepsilon^{a_1 a_2 a_3} \varepsilon^{km2m3} = -\bar{B}^{km} \gamma_\mu V_+ q_{m_1} q_{m_2} C\gamma_\mu \gamma_5 V_+ q_{m_3} \varepsilon^{a_1 a_2 a_3} \varepsilon^{km2m3}
\]

\[
\bar{B}^{km} \gamma_\mu V_+ q_{m_1} q_{m_2} C V_+ q_{m_3} \varepsilon^{a_1 a_2 a_3} \varepsilon^{km2m3} = 0
\]

\[
\bar{B}^{km} \lambda_{m_1}^{m_2} \gamma_\mu \gamma_5 V_+ q_{m_1} q_{m_2} \lambda^{m_3} C\gamma_\mu V_+ q_{m_3} \varepsilon^{a_1 a_2 a_3} \varepsilon^{km2m3} = \frac{1}{2} \bar{B}^{km} \lambda_{m_1}^{m_2} \sigma_{\mu \nu} \gamma_5 V_+ q_{m_1} q_{m_2} \lambda^{m_3} C\sigma_{\mu \nu} V_+ q_{m_3} \varepsilon^{a_1 a_2 a_3} \varepsilon^{km2m3}
\]

(A3)

Since the vector and tensor Lagrangians (3) are completely equivalent to each other on the baryon mass shell one can start with either of them. Note that the vector and pseudoscalar...
forms of interaction Lagrangians transform into each other under Fierz transformations (on baryon mass shell)

\[
[B^{\alpha_1 V^{\alpha_1, \alpha_2}} q^{\alpha_2}] \otimes [q^{\alpha_3} (C\gamma_5 V)_{\alpha_3, \alpha_4} q^{\alpha_4}] = \frac{1}{2} \left\{ [B^{\alpha_1 V^{\alpha_1, \alpha_4}} q^{\alpha_4}] \otimes [q^{\alpha_3} (C\gamma_5 V)_{\alpha_3, \alpha_2} q^{\alpha_2}] + [B^{\alpha_1 (\gamma^\mu \gamma_5 V)_{\alpha_1, \alpha_4}} q^{\alpha_4}] \otimes [q^{\alpha_3} (C\gamma^\mu V)_{\alpha_3, \alpha_2} q^{\alpha_2}] \right\}
\]  

(A4)

Here \( (\alpha_i) \) denote the spinor indices.

For \( SU(3) \) octet of light baryons the interaction Lagrangians are listed in Table XII. The interaction Lagrangians for heavy-light baryons are given in Table XIII. The meson-quark-antiquark interaction Lagrangians are listed in Table XIV.

The baryon-quark couplings \( g_{BQ} \) are determined from the normalization condition for vector current. For heavy-light baryons they are given by

\[
g_{BQ}^{-2} = \frac{N_c!}{(4\pi)^4} \frac{\Lambda_{BQ}^6}{18\Lambda_q \Lambda_{q_2}} \cdot R_Q
\]

(A5)

where \( \Lambda_q \) and \( \Lambda_{q_2} \) are light quark cutoff parameters and \( R_Q \) is the structure integral which depends on the ratio \( \bar{\Lambda}/\Lambda_{BQ} \)

\[
R_Q = \int_0^\infty du \exp \left[ -18u^2 + 36u \frac{\bar{\Lambda}}{\Lambda_{BQ}} \right]
\]

In the case of light baryons the couplings are given by

\[
g_{Bq}^{-2} = \frac{N_c!}{(4\pi)^4} \frac{\Lambda_{Bq}^8}{27\Lambda_q^4} \cdot \kappa
\]

(A6)

where \( \kappa = \begin{cases} 1 & \text{for nucleons} \\ r(2/3 + r/3) & \text{for } \Lambda^0 \text{ and the triplet of } \Sigma \text{ hyperons} \\ r^2(2r/3 + 1/3) & \text{for the doublet of } \Xi \text{ hyperons} \end{cases} \)
APPENDIX B: THE CALCULATION TECHNIQUE

To elucidate the calculation of the matrix elements (21)-(28) we consider the four relevant integrals in Euclidean space corresponding to the factorizing contributions from $b \rightarrow c$ and heavy-light transitions and the typical nonfactorizing ones coming from the diagram II$_a$. The calculations of the nonfactorizing contributions from diagrams II$_b$ and III can be carried out analogously.

Factorizing Contribution ($b \rightarrow c$ transition)

\[
I_{F}^{b\rightarrow c}(\omega_E) = \int \frac{d^4k_E}{\pi^2} \int \frac{d^4k'_E}{\pi^2} \exp \left[ -\frac{18k_E^2 + 6(2k'_E + k)^2}{\Lambda_{BQ}^2} \right] \frac{1}{k_{E}v_{1E} - \Lambda} \frac{1}{k_{E}v_{2E} - \Lambda}
\]  

(B1)

Factorizing Contribution ($Q \rightarrow q$ transition)

\[
I_{F}^{Q \rightarrow q}(\omega_E, M_2) = \int \frac{d^4k_E}{\pi^2} \int \frac{d^4k'_E}{\pi^2} \exp \left[ -\frac{9k_E^2 + 3(2k'_E + k)^2}{\Lambda_{BQ}^2} \right]
\]

\[
\times \exp \left[ -\frac{(3k_E + 2p_{2E})^2 + 3(2k'_E + k)^2}{\Lambda_{BQ}^2} \right] \frac{1}{k_{E}v_{1E} - \Lambda} \frac{1}{k_{E}v_{2E} - \Lambda}
\]  

(B2)

Nonfactorizing Contribution ($b \rightarrow c$ transition)

\[
I_{NF}^{b\rightarrow c}(\omega_E, M_2) = \int \frac{d^4k_E}{\pi^2} \int \frac{d^4k'_E}{\pi^2} \int \frac{d^4k''_E}{\pi^2} \exp \left[ -\frac{9k_E^2 + 3(2k''_E + k'_E - p_{3E})^2}{\Lambda_{BQ}^2} \right]
\]

\[
\times \exp \left[ \frac{9k'^2_E + 3(2k''_E + k'_E - p_{3E})^2}{\Lambda_{BQ}^2} \right] \frac{1}{k_{E}v_{1E} - \Lambda} \frac{1}{k'_{E}v_{2E} - \Lambda}
\]  

(B3)

Nonfactorizing Contribution ($Q \rightarrow q$ transition)

\[
I_{NF}^{Q \rightarrow q}(\omega_E, M_2) = \int \frac{d^4k_E}{\pi^2} \int \frac{d^4k'_E}{\pi^2} \int \frac{d^4k''_E}{\pi^2} \exp \left[ -\frac{9k_E^2 + 3(2k''_E + k'_E - p_{3E})^2}{\Lambda_{BQ}^2} \right]
\]

\[
\times \exp \left[ -\frac{(3k'_E + 2p_{2E})^2 + 3(2k''_E + k_E - p_{3E})^2}{\Lambda_{BQ}^2} \right] \frac{1}{k_{E}v_{1E} - \Lambda}
\]  

(B4)

The final light baryon state carries the Euclidean momenta $p_{2E}$ with the mass-shell condition: $p_{2E}^2 = -M_2^2$. The dimensionless variable $\omega_E$ is defined as $\omega_E = v_{1E} \cdot p_{2E}/M_2 = -\omega$.  

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Scaling all momentum variables in the above integrals by $\Lambda_{BQ}$ and using the Feynman parametrization
\[
\frac{1}{A} = \int_{0}^{\infty} d\alpha \exp(-\alpha A)
\] (B5)
we have
\[
I_{F}^{b\rightarrow c}(\omega_{E}) = 4\Lambda_{BQ}^{6} \int_{0}^{\infty} d\alpha \int_{0}^{\infty} d\beta \int \frac{d^{4}k_{E}}{\pi^{2}} \int \frac{d^{4}k'_{E}}{\pi^{2}} \exp[-18k_{E}^{2} - 24k'_{E}^{2}]
\]
\[
\times \exp\left[-\frac{(\alpha + \beta)^{2}}{18} + \frac{\alpha\beta}{9}(\omega_{E} + 1) + 2(\alpha + \beta)\frac{\bar{\Lambda}}{\Lambda_{BQ}}\right]
\]
\[
I_{F}^{Q\rightarrow q}(\omega_{E}, M_{2}) = 2\Lambda_{BQ}^{7} \int_{0}^{\infty} d\alpha \int \frac{d^{4}k_{E}}{\pi^{2}} \int \frac{d^{4}k'_{E}}{\pi^{2}} \exp[-9(1 + R)k_{E}^{2} - 12(1 + R)k'_{E}^{2}]
\]
\[
\times \exp\left[-\frac{\alpha^{2} - 12R\alpha\omega_{E} - 36RM_{2}^{2}}{9(1 + R)} + 2\alpha\frac{\bar{\Lambda}}{\Lambda_{BQ}}\right]
\]
\[
I_{NF}^{b\rightarrow c}(\omega_{E}) = 4\Lambda_{BQ}^{10} \int_{0}^{\infty} d\alpha \int_{0}^{\infty} d\beta \int \frac{d^{4}k_{E}}{\pi^{2}} \int \frac{d^{4}k'_{E}}{\pi^{2}} \int \frac{d^{4}k''_{E}}{\pi^{2}} \exp[-12k_{E}^{2} - 21k'_{E}^{2}]
\]
\[
\times \exp\left[-\frac{72k''_{E}^{2}}{7} - \frac{(\alpha + \beta)^{2}}{72} + \frac{\alpha\beta}{72}(\omega_{E} + 1) + 2(\alpha + \beta)\frac{\bar{\Lambda}}{\Lambda_{BQ}} - \frac{\alpha^{2} + \beta^{2}}{12}\right]
\]
\[
I_{NF}^{Q\rightarrow q}(\omega_{E}, M_{2}) = 2\Lambda_{BQ}^{11} \int_{0}^{\infty} d\alpha \int \frac{d^{4}k_{E}}{\pi^{2}} \int \frac{d^{4}k'_{E}}{\pi^{2}} \int \frac{d^{4}k''_{E}}{\pi^{2}} \exp[-12k_{E}^{2} - 3R(3 + 4R)k'_{E}^{2}]
\]
\[
\times \exp\left[-\frac{36R(1 + R)k''_{E}^{2}}{3 + 4R} - \frac{\alpha^{2} - 12R\alpha\omega_{E} - 9M_{2}^{2}}{9(1 + R)} + 2\alpha\frac{\bar{\Lambda}}{\Lambda_{BQ}}\right]
\]
After integration over $k_{E}$, $k'_{E}$ and $k''_{E}$ we arrive at
\[
I_{F}^{b\rightarrow c}(-w) = \frac{\Lambda_{BQ}^{6}}{12^{2}} \int_{0}^{\infty} du \int_{0}^{1} dx \exp[-18u^{2} - 36u^{2}x(1 - x)(\omega - 1) + 36u\frac{\bar{\Lambda}}{\Lambda_{BQ}}]
\]
\[
I_{F}^{Q\rightarrow q}(-M_{2}, -w) = \frac{2\Lambda_{BQ}^{7}}{36^{2}(1 + R)^{3}} \int_{0}^{\infty} du \exp[-9(1 + R)u^{2} + 18(1 + R)u\frac{\bar{\Lambda}}{\Lambda_{BQ}}]
\]
\[
\times \exp[-12Ru\omega\frac{M_{2}}{\Lambda_{BQ}} + \frac{4R}{R + 1}\frac{M_{2}^{2}}{\Lambda_{BQ}^{2}}]
\]
\[ I_{NF}^{b \rightarrow c}(-w) = \frac{\Lambda_{BQ}^{10}}{36^2} \int_{0}^{\infty} du \int_{0}^{1} dx \exp \left[ -72u^2 - 144u^2x(1-x)(\omega-1) \right] \]
\[ \times \exp \left[ 144u \frac{\bar{\Lambda}}{\Lambda_{BQ}} - 432u^2(x^2 + (1-x)^2) \right] \]
\[ I_{NF}^{Q \rightarrow q}(-M_2^2, -w) = \frac{2\Lambda_{BQ}^{11}}{216^2R^2(1+R)} \int_{0}^{\infty} du \exp \left[ -36(1+R)(3R+4)u^2 \right] \]
\[ \times \exp \left[ 72(1+R)u \frac{\bar{\Lambda}}{\Lambda_{BQ}} - 12Ru\omega \frac{M_2}{\Lambda_{BQ}} + \frac{R}{R+1} \frac{M_2^2}{\Lambda_{BQ}^2} \right] \]
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TABLE I Quantum numbers of heavy-light baryons.

TABLE IIa Flavor coefficients for heavy-heavy decays (\(C \equiv \cos \delta_p, S \equiv \sin \delta_p, \delta_p = \theta_p - \theta_I\), where \(\theta_p = -11^\circ\) is the \(\eta - \eta'\) mixing angle).

TABLE IIb Flavor coefficients for heavy-light decays (\(C \equiv \cos \delta_p, S \equiv \sin \delta_p, \tan \pm = 1 \pm \tan \delta_p \cdot r \sqrt{2}, \delta_p = \theta_p - \theta_I\), where \(\theta_p = -11^\circ\) is the \(\eta - \eta'\) mixing angle, \(\theta_I = 35^\circ\)).

TABLE IIIa Factors \(t_i(r)\) for heavy-heavy decays (\(C \equiv \cos \delta_p, S \equiv \sin \delta_p, \delta_p = \theta_p - \theta_I\), where \(\theta_p = -11^\circ\) is the \(\eta - \eta'\) mixing angle, \(\theta_I = 35^\circ\)).

TABLE IIIb Factors \(\chi(r)\) and \(t_i(r)\) for heavy-light decays.

TABLE IV Branching ratios (in \%) in nonleptonic decays \(1/2^+ \rightarrow 1/2^+ + 0^-\) of heavy baryons (heavy-light transitions). Numerical values of CKM elements and Wilson coefficients: \(|V_{cs}| = 0.975, |V_{ud}| = 0.975, |V_{ub}| = 0.0035, a_1^* = 1.3, a_2^* = -0.65\).

TABLE V Asymmetry parameters \(\alpha\) in the nonleptonic decays \(1/2^+ \rightarrow 1/2^+ + 0^-\) of heavy baryons (heavy-light transitions). Numerical values of CKM elements and Wilson coefficients: \(|V_{cs}| = 0.975, |V_{ud}| = 0.975, |V_{ub}| = 0.0035, a_1^* = 1.3, a_2^* = -0.65\).

TABLE VI Invariant amplitudes in the decay \(\Lambda_c^+ \rightarrow \Lambda \pi^+\) and \(\Lambda_c^+ \rightarrow \Sigma^+ \pi^0\) (in units of \(G_F V_{cs} V_{ud} \times 10^{-2} \text{ GeV}^2\))

TABLE VII Decay rates and asymmetry parameters in heavy-heavy transitions. Numerical values of CKM elements and Wilson coefficients: \(|V_{cb}| = 0.04, |V_{ud}| = 0.975, a_1 = 1.03, a_2 = 0.10\).

TABLE VIII Decay \(\Lambda_c^+ \rightarrow \Lambda \pi^+:\) contribution of nonfactorizing diagrams (in \% relative to the factorizing contribution)

TABLE IX Decay \(\Lambda_b^0 \rightarrow \Lambda_c^+ \pi^-\): contribution of nonfactorizing diagrams (in \% relative to the factorizing contribution)

TABLE X Predictions for \(\Lambda_c^+ \rightarrow p \phi\) decay rate for different values of the Wilson coefficient \(a_2^*\).

TABLE XI Predictions for \(\Lambda_b^0 \rightarrow J/\psi \Lambda\) decay rate for different values of the Wilson
coefficient $a_2$.

**TABLE XII** Light baryon Lagrangians

**TABLE XIII** Heavy-light baryon Lagrangians

**TABLE XIV** Meson Lagrangians.

**List of Figures**

**FIG. 1** Diagrams contributing to the matrix element of heavy baryon nonleptonic decay: factorizing diagram (I), nonfactorizing diagrams (IIa), (IIb) and (III).
| Baryon | Quark Content | $J^P$ | $(S_{qq}, I_{qq})$ | Mass (GeV) |
|--------|---------------|-------|-----------------|------------|
| $\Lambda_c^+$ | c[ud] | $\frac{1}{2}^+$ | (0,0) | 2.285 |
| $\Xi_c^+$ | c[us] | $\frac{1}{2}^+$ | (0,1/2) | 2.470 |
| $\Xi_c^0$ | c[ds] | $\frac{1}{2}^+$ | (0,1/2) | 2.466 |
| $\Xi^{*+}_c$ | c{us} | $\frac{1}{2}^+$ | (1,1/2) | 2.470 |
| $\Xi^{*0}_c$ | c{ds} | $\frac{1}{2}^+$ | (1,1/2) | 2.466 |
| $\Sigma^0_c$ | c{dd} | $\frac{1}{2}^+$ | (1,1) | 2.453 |
| $\Omega^0_c$ | c{ss} | $\frac{1}{2}^+$ | (1,0) | 2.704 |
| $\Lambda^0_b$ | b[ud] | $\frac{1}{2}^+$ | (0,0) | 5.640 |
| $\Xi^0_b$ | b[us] | $\frac{1}{2}^+$ | (0,1/2) | 5.800 |
| $\Omega^-_b$ | b{ss} | $\frac{1}{2}^+$ | (1,0) | 6.040 |
| Decay                             | \( \ell_{FD}^- \) | \( \ell_{FD}^+ \) | \( \ell_{II}^- \) | \( \ell_{II}^+ \) | \( \ell_{III}^- \) | \( \ell_{III}^+ \) |
|----------------------------------|-------------------|-------------------|-------------------|-------------------|-------------------|-------------------|
| \( \Lambda_b^0 \to \Lambda_c^+ \pi^- \) | -1                | -1                | - \frac{\sqrt{3}}{2} | \frac{1}{2}       | \frac{\sqrt{3}}{2} | \frac{1}{2}       |
| \( \Lambda_b^0 \to \Sigma_c^+ \pi^- \) | 0                  | 0                  | \frac{\sqrt{2}}{2} | - \frac{1}{\sqrt{3}} | \frac{\sqrt{3}}{2} | - \frac{2}{\sqrt{3}} |
| \( \Lambda_b^0 \to \Sigma_c^0 \pi^0 \) | 0                  | 0                  | - \frac{\sqrt{3}}{2} | \frac{1}{\sqrt{3}} | - \frac{\sqrt{3}}{2} | 2 \sqrt{3}       |
| \( \Lambda_b^0 \to \Sigma_c^0 \eta \) | 0                  | 0                  | - \frac{\sqrt{3}}{2} | \frac{1}{\sqrt{3}} | \frac{\sqrt{2}}{2} | 2 \sqrt{6}       |
| \( \Lambda_b^0 \to \Sigma_c^0 \eta' \) | 0                  | 0                  | \frac{\sqrt{3}}{2} | - \frac{1}{\sqrt{3}} | \frac{\sqrt{3}}{2} | - \frac{2}{\sqrt{3}} |
| \( \Lambda_b^0 \to \Xi_c^0 K^0 \) | 0                  | 0                  | - \frac{\sqrt{3}}{2} | \frac{1}{\sqrt{3}} | \frac{\sqrt{3}}{2} | 0                 |
| \( \Xi_b^0 \to \Xi_c^+ \pi^- \) | -1                | -1                | - \frac{\sqrt{3}}{2} | \frac{1}{\sqrt{3}} | \frac{\sqrt{3}}{2} | 0                 |
| \( \Xi_b^0 \to \Xi_c^+ \pi^- \) | 0                  | 0                  | - \frac{\sqrt{3}}{2} | \frac{1}{\sqrt{3}} | \frac{\sqrt{3}}{2} | 0                 |
| \( \Xi_b^0 \to \Xi_c^0 \pi^0 \) | 0                  | 0                  | \frac{1}{\sqrt{2}}  | - \frac{1}{\sqrt{2}} | \frac{1}{\sqrt{2}} | 0                 |
| \( \Xi_b^0 \to \Xi_c^0 \eta \) | 0                  | 0                  | \frac{\sqrt{3}}{2} | - \frac{1}{\sqrt{2}} | \frac{\sqrt{3}}{2} | - \frac{2}{\sqrt{2}} |
| \( \Xi_b^0 \to \Xi_c^0 \eta' \) | 0                  | 0                  | \frac{\sqrt{3}}{2} | - \frac{1}{\sqrt{2}} | \frac{\sqrt{3}}{2} | - \frac{2}{\sqrt{2}} |
| \( \Xi_b^0 \to \Xi_c^0 K^0 \) | 0                  | 0                  | - \frac{\sqrt{3}}{2} | \frac{1}{\sqrt{2}} | \frac{\sqrt{3}}{2} | 0                 |
| \( \Xi_b^0 \to \Omega_c^0 K^0 \) | 0                  | 0                  | - \frac{\sqrt{3}}{2} | \frac{1}{\sqrt{2}} | \frac{\sqrt{3}}{2} | 2 \sqrt{6}       |
| \( \Omega_c^0 \to \Xi_c^0 \pi^- \) | -1                | -1                | - \frac{\sqrt{3}}{2} | \frac{1}{\sqrt{2}} | \frac{\sqrt{3}}{2} | 0                 |
| \( \Omega_c^0 \to \Xi_c^0 \pi^- \) | 0                  | 0                  | 0                  | 0                  | 0                  | 0                 |
| \( \Omega_c^0 \to \Xi_c^0 K^- \) | 0                  | 0                  | 0                  | 0                  | 0                  | 0                 |
| \( \Omega_c^0 \to \Omega_c^0 K^- \) | 0                  | 0                  | 0                  | 0                  | 0                  | 0                 |

TABLE IIa
| Decay          | $\ell_{FD}$ | $\ell_{FD}^*$ | $\ell_{II}^+$ | $\ell_{II}^+$ | $\ell_{II}^+$ | $\ell_{II}^+$ | $\ell_{II}^+$ |
|---------------|-------------|---------------|---------------|---------------|---------------|---------------|---------------|
| $\Lambda_c^+ \to \Lambda^0 \pi^+$ | $-1$        | $-1$          | $-\frac{1}{2}$ | $\frac{1}{2}$ | $\frac{1}{2}$ | $\frac{1}{2}$ | $-2$          |
| $\Lambda_c^+ \to \Sigma^0 \pi^+$ | $0$         | $0$           | $-\frac{\sqrt{3}}{2}$ | $\frac{1}{2\sqrt{3}}$ | $-\frac{\sqrt{3}}{2}$ | $-\frac{\sqrt{3}}{2}$ | $2\sqrt{3}$ |
| $\Lambda_c^+ \to \Sigma^+ \pi^0$ | $0$         | $0$           | $\frac{\sqrt{3}}{2}$ | $\frac{1}{2\sqrt{3}}$ | $\frac{\sqrt{3}}{2}$ | $\frac{\sqrt{3}}{2}$ | $-2\sqrt{3}$ |
| $\Lambda_c^+ \to \Sigma^+ \eta$  | $0$         | $0$           | $-\frac{\sqrt{3}}{2} \cdot S \cdot \operatorname{ctg}_+$ | $-\frac{1}{2\sqrt{3}} \cdot S \cdot \operatorname{ctg}_-$ | $\frac{\sqrt{3}}{2} \cdot S$ | $\frac{\sqrt{3}}{2} \cdot S$ | $\sqrt{3} \cdot S$ |
| $\Lambda_c^+ \to \Sigma^+ \eta'$ | $0$         | $0$           | $\frac{\sqrt{3}}{2} \cdot \operatorname{ctg}_-$ | $\frac{1}{2\sqrt{3}} \cdot \operatorname{ctg}_+$ | $-\frac{3}{2} \cdot C$ | $-\frac{\sqrt{3}}{2} \cdot C$ | $-\sqrt{3} \cdot C$ |
| $\Lambda_c^+ \to p K^0$ | $\frac{3}{\sqrt{6}}$ | $\frac{1}{\sqrt{6}}$ | $-\frac{3}{\sqrt{6}}$ | $\frac{1}{\sqrt{6}}$ | $0$            | $0$            | $0$            |
| $\Lambda_c^+ \to \Xi^0 K^+$ | $0$         | $0$           | $0$            | $\frac{1}{\sqrt{6}}$ | $0$            | $0$            | $-2\sqrt{6}$ |
| $\Xi^+ \to \Sigma^+ K^0$ | $\frac{3}{\sqrt{6}}$ | $\frac{3}{\sqrt{6}}$ | $0$            | $0$            | $-\frac{3}{\sqrt{6}}$ | $-\frac{3}{\sqrt{6}}$ | $0$            |
| $\Xi^+ \to \Xi^0 K^0$ | $\frac{3}{\sqrt{6}}$ | $\frac{3}{\sqrt{6}}$ | $0$            | $0$            | $-\frac{3}{\sqrt{6}}$ | $-\frac{3}{\sqrt{6}}$ | $0$            |
| $\Xi^0 \to \Lambda^0 K^0$ | $\frac{1}{2}$ | $-\frac{1}{2}$ | $1$            | $0$            | $\frac{1}{2}$ | $\frac{1}{2}$ | $-2$          |
| $\Xi^0 \to \Sigma^0 K^0$ | $\frac{1}{2}$ | $-\frac{1}{2}$ | $0$            | $-\frac{1}{2}$ | $\frac{1}{2}$ | $\frac{1}{2}$ | $2\sqrt{3}$ |
| $\Xi^0 \to \Sigma^+ K^-$ | $0$         | $0$           | $0$            | $\frac{2}{\sqrt{6}}$ | $0$            | $0$            | $-2\sqrt{6}$ |
| $\Xi^0 \to \Xi^0 \pi^0$ | $0$         | $0$           | $\frac{\sqrt{3}}{2}$ | $-\frac{1}{2\sqrt{3}}$ | $\frac{3}{2}$ | $\frac{3}{2}$ | $0$            |
| $\Xi^0 \to \Xi^0 \eta$ | $0$         | $0$           | $-\frac{\sqrt{3}}{2} \cdot S \cdot \operatorname{ctg}_+$ | $\frac{1}{2\sqrt{3}} \cdot S \cdot \operatorname{ctg}_-$ | $\frac{3}{2} \cdot S$ | $\frac{3}{2} \cdot S$ | $\sqrt{6} \cdot C$ |
| $\Xi^0 \to \Xi^0 \eta'$ | $0$         | $0$           | $\frac{\sqrt{3}}{2} \cdot \operatorname{ctg}_-$ | $\frac{1}{2\sqrt{3}} \cdot \operatorname{ctg}_+$ | $\frac{3}{2} \cdot C$ | $\frac{3}{2} \cdot C$ | $\sqrt{6} \cdot S$ |
| $\Xi^0 \to \Xi^- \pi^+$ | $-\frac{3}{\sqrt{6}}$ | $-\frac{3}{\sqrt{6}}$ | $-\frac{3}{\sqrt{6}}$ | $\frac{1}{\sqrt{6}}$ | $0$            | $0$            | $0$            |
| $\Omega^0 \to \Xi^0 K^0$ | $-1$        | $1$           | $0$            | $0$            | $-3$           | $1$           | $0$            |
| $\Lambda_b^0 \to \Lambda^0 \pi^0$ | $\frac{1}{\sqrt{2}}$ | $\frac{1}{\sqrt{2}}$ | $-\frac{1}{2\sqrt{2}}$ | $\frac{1}{2\sqrt{2}}$ | $\frac{1}{2\sqrt{2}}$ | $\frac{1}{2\sqrt{2}}$ | $\sqrt{2}$ |
| $\Lambda_b^0 \to p K^-$ | $\frac{1}{\sqrt{6}}$ | $\frac{1}{\sqrt{6}}$ | $\frac{1}{\sqrt{6}}$ | $\frac{1}{\sqrt{6}}$ | $0$            | $0$            | $0$            |
| Decay | $t_2(r)$ | $t_3(r)$ |
|-------|----------|----------|
| $\Lambda_b^0 \to \Lambda_c^+ \pi^-$ | 1 | 1 |
| $\Lambda_b^0 \to \Sigma_c^+ \pi^-$ | 1 | 1 |
| $\Lambda_b^0 \to \Sigma_c^0 \pi^0$ | 1 | 1 |
| $\Lambda_b^0 \to \Sigma_c^0 \eta$ | $\frac{r^2}{\sqrt{C^2 \cdot r^2 + S^2}}$ | $\frac{r^2}{\sqrt{C^2 \cdot r^2 + S^2}}$ |
| $\Lambda_b^0 \to \Sigma_c^0 \eta'$ | $\frac{r^2}{\sqrt{S^2 \cdot r^2 + C^2}}$ | $\frac{r^2}{\sqrt{S^2 \cdot r^2 + C^2}}$ |
| $\Xi_b^- \to \Xi_c^0 K^0$ | $(1 + r)^2/4$ | $(1 + r)^2/4$ |
| $\Xi_b^- \to \Xi_c^0 K^0$ | $(1 + r)^2/4$ | $(1 + r)^2/4$ |
| $\Xi_c^- \to \Xi_c^0 \pi^-$ | 1 | 1 |
| $\Xi_c^- \to \Xi_c^0 \pi^-$ | 1 | 1 |
| $\Xi_b^- \to \Xi_b^0 \pi^0$ | 1 | 1 |
| $\Xi_b^- \to \Xi_b^0 \eta$ | $\frac{r^2}{\sqrt{C^2 \cdot r^2 + S^2}}$ | $\frac{r^2}{\sqrt{C^2 \cdot r^2 + S^2}}$ |
| $\Xi_b^- \to \Xi_b^0 \eta'$ | $\frac{r^2}{\sqrt{S^2 \cdot r^2 + C^2}}$ | $\frac{r^2}{\sqrt{S^2 \cdot r^2 + C^2}}$ |
| $\Xi_b^- \to \Xi_b^0 \pi^0$ | 1 | 1 |
| $\Xi_b^- \to \Xi_b^0 \eta'$ | $\frac{r^2}{\sqrt{S^2 \cdot r^2 + C^2}}$ | $\frac{r^2}{\sqrt{S^2 \cdot r^2 + C^2}}$ |
| $\Xi_b^- \to \Lambda_c^0 K^-$ | $(1 + r)^2/4$ | $(1 + r)^2/4$ |
| $\Xi_b^- \to \Sigma_c^* K^-$ | $(1 + r)^2/4$ | $(1 + r)^2/4$ |
| $\Xi_b^- \to \Sigma_c^0 K^0$ | 0 | $(1 + r)^2/4$ |
| $\Xi_b^- \to \Omega_c^0 K^0$ | $(1 + r)^2/4$ | 0 |
| $\Xi_b^- \to \Xi_b^0 \pi^-$ | 1 | 1 |
| $\Xi_b^- \to \Xi_b^0 \pi^-$ | 1 | 1 |
| $\Xi_b^- \to \Xi_b^0 \pi^-$ | 1 | 1 |
| $\Omega_b^- \to \Omega_b^0 K^-$ | $(1 + r)^2/4$ | 0 |
| $\Omega_b^- \to \Omega_b^0 \pi^-$ | 1 | 1 |
### TABLE IIIb

| Decay                   | $\chi_f$ | $t_2(r)$ | $t_3(r)$ |
|-------------------------|----------|----------|----------|
| $\Lambda_c^+ \to \Lambda^0 \pi^+$                      | $\frac{2}{3\sqrt{r}} + \frac{1}{3\sqrt{r}}$ | 1         | 1         |
| $\Lambda_c^+ \to \Sigma^0 \pi^+$                         | $\frac{2}{3\sqrt{r}} + \frac{1}{3\sqrt{r}}$ | 1         | 1         |
| $\Lambda_c^+ \to \Sigma^+ \pi^0$                         | $\frac{2}{3\sqrt{r}} + \frac{1}{3\sqrt{r}}$ | 1         | 1         |
| $\Lambda_c^+ \to \Sigma^+ \eta$                          | $\frac{2}{3\sqrt{r}} + \frac{1}{3\sqrt{r}}$ | 1         | 1         |
| $\Lambda_c^+ \to \Sigma^+ \eta'$                         | $\frac{2}{3\sqrt{r}} + \frac{1}{3\sqrt{r}}$ | 1         | 1         |
| $\Lambda_c^+ \to pK^0$                                    | 1        | $(1 + r)^2 \sqrt{r/4}$ | $(1 + r)^2 \sqrt{r/4}$ |
| $\Lambda_c^+ \to \Xi^+ K^+$                               | $\frac{2}{3\sqrt{r}} + \frac{1}{3\sqrt{r}}$ | $(1 + r)^2 \sqrt{r/4}$ | $(1 + r)^2 \sqrt{r/4}$ |
| $\Xi_c^+ \to \Sigma^+ \pi^0$                             | $\frac{2}{3\sqrt{r}} + \frac{1}{3\sqrt{r}}$ | $(1 + r)^2 \sqrt{r/4}$ | $(1 + r)^2 \sqrt{r/4}$ |
| $\Xi_c^+ \to \Xi^+ K^0$                                   | $\frac{2}{3\sqrt{r}} + \frac{1}{3\sqrt{r}}$ | $(1 + r)^2 \sqrt{r/4}$ | $(1 + r)^2 \sqrt{r/4}$ |
| $\Xi_c^+ \to \Xi^0 K^0$                                   | $\frac{2}{3\sqrt{r}} + \frac{1}{3\sqrt{r}}$ | $(1 + r)^2 \sqrt{r/4}$ | $(1 + r)^2 \sqrt{r/4}$ |
| $\Xi_c^+ \to \Sigma^+ K^-$                                | $\frac{2}{3\sqrt{r}} + \frac{1}{3\sqrt{r}}$ | $(1 + r)^2 \sqrt{r/4}$ | $(1 + r)^2 \sqrt{r/4}$ |
| $\Xi_c^+ \to \Xi^0 \pi^0$                                 | $\frac{2}{3\sqrt{r}} + \frac{1}{3\sqrt{r}}$ | 1         | 1         |
| $\Xi_c^+ \to \Xi^0 \eta$                                  | $\frac{2}{3\sqrt{r}} + \frac{1}{3\sqrt{r}}$ | 1         | $r$       |
| $\Xi_c^+ \to \Xi^0 \eta'$                                 | $\frac{2}{3\sqrt{r}} + \frac{1}{3\sqrt{r}}$ | 1         | $r$       |
| $\Xi_c^+ \to \Xi^0 \pi^+$                                 | $\frac{2}{3\sqrt{r}} + \frac{1}{3\sqrt{r}}$ | 1         | 1         |
| $\Omega_c^+ \to \Xi^0 K^0$                                | $\frac{2}{3\sqrt{r}} + \frac{1}{3\sqrt{r}}$ | $(1 + r)^2 \sqrt{r/4}$ | $(1 + r)^2 \sqrt{r/4}$ |
| $\Lambda_b^+ \to \Lambda^0 \pi^0$                        | $\frac{2}{3\sqrt{r}} + \frac{1}{3\sqrt{r}}$ | 1         | 1         |
| $\Lambda_b^+ \to pK^-$                                    | 1        | $(1 + r)^2 \sqrt{r/4}$ | $(1 + r)^2 \sqrt{r/4}$ |
| Process                                      | Körner, Krämer [14] | Xu, Kamal [22] | Cheng, Tseng [24] | Our     | Experiment [1] |
|---------------------------------------------|----------------------|----------------|-------------------|---------|----------------|
| $\Lambda^+_c \rightarrow \Lambda \pi^+$     | 0.76                 | 1.67           | 0.91              | 0.79    | 0.79±0.18      |
| $\Lambda^+_c \rightarrow \Sigma^0 \pi^+$   | 0.33                 | 0.35           | 0.74              | 0.88    | 0.88±0.20      |
| $\Lambda^+_c \rightarrow \Sigma^+ \pi^0$  | 0.33                 | 0.35           | 0.74              | 0.88    | 0.88±0.22      |
| $\Lambda^+_c \rightarrow \Sigma^+ \eta$   | 0.16                 |                | 0.11              | 0.48    | 0.48±0.17      |
| $\Lambda^+_c \rightarrow \Sigma^+ \eta'$  | 1.28                 |                |                   | 0.12    |                |
| $\Lambda^+_c \rightarrow pK^0$             | 2.16                 | 1.24           | 1.30              | 2.06    | 2.2±0.4        |
| $\Lambda^+_c \rightarrow \Xi^0 K^+$        | 0.27                 | 0.10           |                   | 0.31    | 0.34±0.09      |
| $\Xi^+_c \rightarrow \Sigma^+ K^0$         | 5.11                 | 0.35           | 0.67              | 3.08    |                |
| $\Xi^+_c \rightarrow \Xi^0 \pi^+$          | 2.80                 | 2.66           | 3.12              | 4.40    | 1.2±0.5±0.3   |
| $\Xi^0_c \rightarrow \Lambda K^0$          | 0.11                 | 0.32           | 0.24              | 0.42    |                |
| $\Xi^0_c \rightarrow \Sigma^0 K^0$         | 1.03                 | 0.08           | 0.12              | 0.20    |                |
| $\Xi^0_c \rightarrow \Sigma^+ K^-$         | 0.11                 | 0.11           |                   | 0.27    |                |
| $\Xi^0_c \rightarrow \Xi^0 \pi^0$          | 0.03                 | 0.49           | 0.25              | 0.04    |                |
| $\Xi^0_c \rightarrow \Xi^0 \eta$          | 0.21                 |                |                   | 0.28    |                |
| $\Xi^0_c \rightarrow \Xi^0 \eta'$          | 0.74                 |                |                   | 0.31    |                |
| $\Xi^0_c \rightarrow \Xi^- \pi^+$          | 0.91                 | 1.52           | 1.10              | 1.22    |                |
| $\Omega^0_c \rightarrow \Xi^0 K^0$         | 1.10                 |                | 0.08              | 0.02    |                |
| $\Lambda^0_b \rightarrow \Lambda \pi^0$   | 4.92×10^{-8}         |                |                   |        |                |
| $\Lambda^0_b \rightarrow pK^-$             | 2.11×10^{-7}         |                |                   |        |                |
| Process                              | Körner, Krämer [13] | Xu, Kamal [22] | Cheng, Tseng [24] | Our     | Experiment [1] |
|--------------------------------------|----------------------|----------------|-------------------|---------|----------------|
| \( \Lambda_c^+ \to \Lambda \pi^+ \) | -0.70                | -0.67          | -0.95             | -0.95   | -0.98 ± 0.19   |
| \( \Lambda_c^+ \to \Sigma^0 \pi^+ \) | 0.70                 | 0.92           | 0.78              | 0.43    |                |
| \( \Lambda_c^+ \to \Sigma^+ \pi^0 \) | 0.71                 | 0.92           | 0.78              | 0.43    | -0.45 ± 0.31 ± 0.06 |
| \( \Lambda_c^+ \to \Sigma^+ \eta \) | 0.33                 |                | 0.55              |         |                |
| \( \Lambda_c^+ \to \Sigma^+ \eta' \) | -0.45                |                | -0.05             |         |                |
| \( \Lambda_c^+ \to pK^0 \)          | -1.0                 | 0.51           | -0.49             | -0.97   |                |
| \( \Lambda_c^+ \to \Xi^0 K^+ \)     | 0                    | 0              | 0                 | 0       |                |
| \( \Xi_c^0 \to \Sigma^+ K^0 \)      | -1.0                 | 0.24           | -0.09             | -0.99   |                |
| \( \Xi_c^+ \to \Xi^0 \pi^+ \)       | -0.78                | -0.81          | -0.77             | -1.0    |                |
| \( \Xi_c^0 \to \Lambda K^0 \)       | -0.76                | 1.0            | -0.73             | -0.75   |                |
| \( \Xi_c^0 \to \Sigma^0 K^0 \)      | -0.96                | -0.99          | -0.59             | -0.55   |                |
| \( \Xi_c^0 \to \Sigma^+ K^- \)      | 0                    | 0              | 0                 | 0       |                |
| \( \Xi_c^0 \to \Xi^0 \pi^0 \)       | 0.92                 | 0.92           | -0.54             | 0.94    |                |
| \( \Xi_c^0 \to \Xi^0 \eta \)        | -0.92                |                | -1.0              |         |                |
| \( \Xi_c^0 \to \Xi^0 \eta' \)       | -0.38                |                | -0.32             |         |                |
| \( \Xi_c^0 \to \Xi^- \pi^+ \)       | -0.38                | -0.38          | -0.99             | -0.84   |                |
| \( \Omega_c^0 \to \Xi^0 K^0 \)      | 0.51                 |                | -0.93             | -0.81   |                |
| \( \Lambda_b^0 \to \Lambda \pi^0 \) |                     |                |                   | -1.0    |                |
| \( \Lambda_b^0 \to pK^- \)          |                     |                |                   | -0.88   |                |
### TABLE VI

| Reference | \( \Lambda_c^+ \rightarrow \Lambda \pi^+ \) | \( \Lambda_c^+ \rightarrow \Sigma^+ \pi^0 \) |
|-----------|------------------------------------------|------------------------------------------|
| \( \Lambda_c^+ \rightarrow \Lambda \pi^+ \) | \(-3.0^{+0.8}_{-1.2} \) | \(-12.7^{+2.4}_{-2.6} \) |
| \( \Lambda_c^+ \rightarrow \Sigma^+ \pi^0 \) | \(-1.3^{+0.9}_{-1.1} \) | \(-17.3^{+2.3}_{-2.9} \) |
| Xu and Kamal [22] | \(-2.7 \) | \(20.8 \) |
| Cheng and Tseng [23] | \(-3.5 \) | \(-6.0 \) |
| Körner and Krämer [14] | \(-1.9 \) | \(-14.6 \) |
| Our | \(-4.2 \) | \(-17.2 \) |

### TABLE VII

| Process | \( \Gamma \) (in \( 10^{10} \text{ s}^{-1} \)) | \( \alpha \) | Process | \( \Gamma \) (in \( 10^{10} \text{ s}^{-1} \)) | \( \alpha \) |
|---------|---------------------------------|-----|---------|---------------------------------|-----|
| \( \Lambda_b^0 \rightarrow \Lambda_b^+ \pi^- \) | 0.382 | -0.99 | \( \Xi_b^0 \rightarrow \Xi_b^{0'} \pi^0 \) | 0.014 | -0.94 |
| \( \Lambda_b^0 \rightarrow \Sigma_b^+ \pi^- \) | 0.039 | 0.65 | \( \Xi_b^0 \rightarrow \Xi_b^{0'} \eta \) | 0.015 | -0.98 |
| \( \Lambda_b^0 \rightarrow \Sigma_b^0 \pi^0 \) | 0.039 | 0.65 | \( \Xi_b^0 \rightarrow \Xi_b^{0'} \eta' \) | 0.021 | 0.97 |
| \( \Lambda_b^0 \rightarrow \Sigma_b^0 \eta \) | 0.023 | 0.79 | \( \Xi_b^0 \rightarrow \Lambda_b^+ K^- \) | 0.010 | -0.73 |
| \( \Lambda_b^0 \rightarrow \Sigma_b^0 \eta' \) | 0.029 | 0.99 | \( \Xi_b^0 \rightarrow \Sigma_b^+ K^- \) | 0.030 | -0.74 |
| \( \Lambda_b^0 \rightarrow \Xi_b^0 K^0 \) | 0.021 | -0.81 | \( \Xi_b^0 \rightarrow \Sigma_b^0 K^0 \) | 0.021 | 0 |
| \( \Xi_b^0 \rightarrow \Xi_b^{0'} K^0 \) | 0.032 | 0.98 | \( \Xi_b^0 \rightarrow \Omega_b^0 K^0 \) | 0.023 | 0.65 |
| \( \Xi_b^0 \rightarrow \Xi_b^{0'} \pi^- \) | 0.479 | -1.00 | \( \Xi_b^- \rightarrow \Xi_b^{0'} \pi^- \) | 0.645 | -0.97 |
| \( \Xi_b^0 \rightarrow \Xi_b^{0'} \pi^- \) | 0.018 | 0.61 | \( \Xi_b^- \rightarrow \Xi_b^{0'} \pi^- \) | 0.007 | -1.00 |
| \( \Xi_b^0 \rightarrow \Xi_b^{0'} \pi^0 \) | 0.002 | -0.99 | \( \Xi_b^- \rightarrow \Sigma_b^0 K^- \) | 0.016 | -0.98 |
| \( \Xi_b^- \rightarrow \Xi_b^{0'} \eta \) | 0.012 | -0.86 | \( \Omega_b^- \rightarrow \Omega_b^{0'} \pi^- \) | 0.352 | 0.60 |
| \( \Xi_b^- \rightarrow \Xi_b^{0'} \eta' \) | 0.003 | 0.71 | | | |
### TABLE VIII

| Amplitude | Diagram |\( II_a \) |\( II_b \) |\( II_a + II_b \) |\( III \) |
|-----------|---------|------------|------------|----------------|---------|
| A         | -29.8%  | -18.5%     | -48.3%     |                |         |
| B         | -32.4%  | -15.9%     | -48.3%     | -13.9%         |         |

### TABLE IX

| Amplitude | Diagram |\( II_a \) |\( II_b \) |\( II_a + II_b \) |\( III \) |
|-----------|---------|------------|------------|----------------|---------|
| A         | -13.9%  | -6.2%      | -20.1%     |                |         |
| B         | -14.3%  | -5.8%      | -20.1%     | -8.5%          |         |
| Ratio of interest | \( \text{Br}(p\phi)/\text{Br}(pK^-\pi^+) \) (in %) |
|-------------------|---------------------------------|
| CLEO [3]          | 0.024±0.006±0.003               |
| NA32 [4]          | 0.04±0.03                       |
| Körner & Krämer [14] | 0.05                   |
| Cheng & Tseng [25] | 0.016                           |
| Datta [27]        | 0.01                            |
| Our               | 0.022 \((a_2^*=0.30)\)          |
|                   | 0.030 \((a_2^*=0.35)\)          |
|                   | 0.040 \((a_2^*=0.40)\)          |
|                   | 0.050 \((a_2^*=0.45)\)          |
|                   | 0.062 \((a_2^*=0.50)\)          |
|                   | 0.075 \((a_2^*=0.55)\)          |
|                   | 0.090 \((a_2^*=0.60)\)          |
|                   | 0.105 \((a_2^*=0.65)\)          |

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\begin{align*}
\text{TABLE XI} \\
\begin{array}{|c|c|}
\hline 
\text{Ratio of interest} & \text{Br}(\Lambda \rightarrow J/\psi \Lambda) \text{ (in \%) } \\
\hline 
\text{UA1} & 1.4 \pm 0.9 \\
\text{OPAL} & < 1.1 \\
\text{CDF} & 0.037 \pm 0.017 \pm 0.004 \\
\text{Cheng \& Tseng} & 0.011 \\
\text{Cheng} & 0.016 \\
\text{Our} & 0.027 (a_2 = 0.10) \\
& 0.061 (a_2 = 0.15) \\
& 0.108 (a_2 = 0.20) \\
& 0.169 (a_2 = 0.25) \\
& 0.243 (a_2 = 0.25) \\
\hline
\end{array}
\end{align*}

\begin{align*}
\text{TABLE XII} \\
\begin{array}{|c|l|}
\hline 
\text{Baryon} & \text{Lagrangian} \\
\hline 
p & g_\mu \gamma_\mu \gamma_5 V_+ d_{a_1} u_{a_2} C_{\gamma_\mu V_+ d_{a_3} e_{a_1 a_2 a_3}} = 2 g_\mu \bar{V}_+ u_{a_1} u_{a_2} C_{\gamma_5 V_+ d_{a_3} e_{a_1 a_2 a_3}} \\
& - g_{\lambda_0} \Lambda_0 \gamma_\mu \gamma_5 V_+ u_{a_1} d_{a_2} C_{\gamma_{\lambda_0} \bar{V}_+ V_+ u_{a_3} e_{a_1 a_2 a_3}} = -2 g_{\lambda_0} \Lambda_0 d_{a_1} d_{a_2} C_{\gamma_5 V_+ u_{a_3} e_{a_1 a_2 a_3}} \\
& \Lambda^0 & \sqrt{2} g_{\lambda_0} \Lambda_0 \gamma_\mu \gamma_5 V_+ u_{a_1} d_{a_2} C_{\gamma_{\lambda_0} \bar{V}_+ V_+ u_{a_3} e_{a_1 a_2 a_3}} = - \sqrt{2} g_{\lambda_0} \Lambda_0 d_{a_1} d_{a_2} C_{\gamma_5 V_+ u_{a_3} e_{a_1 a_2 a_3}} \\
& \Xi^0 & g_{\Xi^0} \Xi^0 \gamma_\mu \gamma_5 V_+ u_{a_1} s_{a_2} C_{\gamma_{\Xi^0} \bar{V}_+ V_+ s_{a_3} e_{a_1 a_2 a_3}} = 2 g_{\Xi^0} \Xi^0 d_{a_1} s_{a_2} C_{\gamma_5 V_+ s_{a_3} e_{a_1 a_2 a_3}} \\
& \Xi^- & g_{\Xi^-} \Xi^- \gamma_\mu \gamma_5 V_+ s_{a_1} d_{a_2} C_{\gamma_{\Xi^-} \bar{V}_+ s_{a_3} e_{a_1 a_2 a_3}} = 2 g_{\Xi^-} \Xi^- s_{a_1} d_{a_2} C_{\gamma_5 V_+ s_{a_3} e_{a_1 a_2 a_3}} \\
\hline
\end{array}
\end{align*}
### TABLE XIII

| Baryon | Lagrangian |
|--------|------------|
| $\Lambda_Q$ | $- g_{\Lambda_Q} \Lambda_Q Q^{a_1} u^{a_2} C \gamma_5 V_+ d^{a_3} \varepsilon^{a_2 a_3}$ |
| $\Xi_Q$ | $g_{\Xi_Q} \Xi_Q Q^{a_1} u^{a_2} C \gamma_5 V_+ s^{a_3} \varepsilon^{a_1 a_2 a_3}$ |
| $\Xi_Q'$ | $\frac{1}{\sqrt{6}} g_{\Xi_Q} \Xi_Q' Q^{a_1} u^{a_2} C \gamma_5 V_+ d^{a_3} \varepsilon^{a_1 a_2 a_3}$ |
| $\Sigma_Q$ | $\frac{1}{\sqrt{6}} g_{\Sigma_Q} \Sigma_Q Q^{a_1} u^{a_2} C \gamma_5 V_+ d^{a_3} \varepsilon^{a_1 a_2 a_3}$ |
| $\Xi_Q''$ | $g_{\Xi_Q''} \Xi_Q'' Q^{a_1} u^{a_2} C \gamma_5 V_+ s^{a_3} \varepsilon^{a_1 a_2 a_3}$ |
| $\Omega_Q$ | $\frac{1}{\sqrt{3}} g_{\Omega_Q} \Omega Q^{a_1} s^{a_2} C \gamma_5 V_+ s^{a_3} \varepsilon^{a_1 a_2 a_3}$ |

### TABLE XIV

| Meson | Lagrangian |
|-------|------------|
| $\pi$ | $g_{\pi} \pi^+ \bar{u} i \gamma^5 d + h.c.$ |
| $K$ | $g_K K^+ \bar{u} i \gamma^5 V_+ s + K^0 \bar{d} i \gamma^5 V_+ s + h.c.$ |
| $\phi$ | $g_{\phi} \phi \bar{s} (\gamma^\mu - v^\mu) V_+ s$ |
| $J/\psi$ | $g_{J/\psi} \psi (J/\psi) \bar{c} (\gamma^\mu - v^\mu) V_+ c$ |
Diagram I
Diagram IIa:

\[ k_1 = k' + p_2 \]
\[ k_3 = -(k' + k'') + p_3/2 \]
\[ k_5 = k'' + p_3/2 \]
\[ k_2 = k + p_1 \]
\[ k_4 = k'' - p_3/2 \]
\[ k_6 = k'' + k' - k - p_3/2 \]
Diagram II:

\[
\begin{align*}
k_1 &= k' + p_2 \\
k_3 &= -(k + k'') - p_3/2 \\
k_5 &= k'' - p_3/2 \\
k_2 &= k + p_1 \\
k_4 &= k'' - k' + k + p_3/2 \\
k_6 &= k'' + p_3/2
\end{align*}
\]
Diagram III:

\[ k_1 = k' + p_2 \]
\[ k_3 = - (k' + k'') + p_3/2 \]
\[ k_5 = k'' - p_3/2 \]
\[ k_2 = k + p_1 \]
\[ k_4 = - (k + k'') - p_3/2 \]
\[ k_6 = k'' + p_3/2 \]