Parity-violating $\pi NN$ coupling constant in the chiral quark-soliton model

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We investigate the parity-violating $\pi NN$ Yukawa coupling constant $h_{\pi NN}$ within the framework of the SU(2) chiral quark-soliton model, based on the $\Delta S = 0$ effective weak Lagrangian derived within the same framework. We find that the parity-violating $\pi NN$ coupling constant is about $1 \times 10^{-8}$ at the scale of 1 GeV. The results of $h_{\pi NN}$ turn out to be sensitive to the Wilson coefficient. We discuss how the gluonic renormalization suppresses the parity-violating $\pi NN$ coupling constant.

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I. INTRODUCTION

The parity-violating (PV) hadronic processes in low-energy regions have been one of the most fundamental issues in nuclear and hadronic physics for long time (see a recent review [1] for some historical and phenomenological background). However, the weak interactions of hadrons are yet poorly understood because of the strong interaction, compared to lepton-lepton or lepton-hadron weak processes. For example, the long-standing puzzle of the $\Delta I = 1/2$ rule in strangeness-changing weak interactions indicates that the effect of the strong interaction in weak processes raises a non-trivial problem [2,3]. It is even more difficult to study parity-violating nuclear processes because of experimental feasibility and theoretical complication caused by the nonperturbative strong interaction of quarks and gluons. The standard model (SM) asserts that charged weak boson exchange induces flavor-changing weak interactions whereas the neutral current conserves the flavor. The basic ingredient to describe low-energy hadronic weak processes is the quark current-current interaction with $W$ and $Z$ bosons. However, in order to describe low-energy phenomena below 1 GeV, one has to scale down this interaction from the mass scale of the $W$ and $Z$. In the course of this scaling, the quark-gluon interactions are encoded in the Wilson coefficients by the renormalization group equation [3,8], which, however, explains only a perturbative part of the strong interaction.

Desplanques, Donoghue, and Holstein (DDH) [8] suggested that hadronic and nuclear PV processes can be described by one-boson exchange such as $\pi$, $\rho$, and $\omega$-exchanges [8,10] à la the strong nucleon-nucleon ($NN$) potential. The main factors of the $NN$ potential are the seven weak meson-$NN$ coupling constants, i.e. $h_{\pi NN}^I$, $h_{\rho NN}^I$, $h_{\rho NN}^{\rho}$, $h_{\omega NN}^{\omega}$, $h_{\omega NN}^{\omega}$, $h_{\omega NN}^{\omega}$, and $h_{\omega NN}^{\omega}$, where superscripts denote the isospin difference $\Delta I$. Among these coupling constants, it is of utmost importance to understand the PV $\pi NN$ coupling constant, because it governs the long-range part of the PV $NN$ interaction, so that it plays the most significat role in explaining the PV nuclear processes. The PV $\pi NN$ coupling constant can in principle be extracted from various PV reactions $np \rightarrow d\gamma$ [11,13], and $^{18}F^* \rightarrow ^{18}F$ [14,16] but is fraught with large uncertainties. We refer to a recent review [17] for the present status of hadronic PV experiments. It has been also calculated in various theoretical frameworks: the SU(6) quark model [8,15] with the effective weak Hamiltonian, the Skyrme model [17,21], and QCD sum rules [22], and so on. Even though a great amount of efforts was made on understanding $h_{\pi NN}^I$ experimentally as well as theoretically, its quantitative value is still elusive.

In the present work, we investigate the PV $\pi NN$ coupling constant, $h_{\pi NN}^I$, within the framework of the SU(2) chiral quark-soliton model ($\chi$QSM) which is an effective chiral model for QCD in the low-energy region with constituent quarks and the pseudoscalar mesons as the relevant degrees of freedom. The model respects the spontaneous breakdown of chiral symmetry and describes baryons fully relativistically. Moreover, it is deeply related to the QCD
vacuum based on instantons and contains only a few free parameters. These parameters can mostly be fixed to the meson masses and meson decay constants in the mesonic sector. The only remaining free parameter is the constituent quark mass or dynamical quark mass that is also fixed by reproducing the electric properties of the proton. The χQSM was successful in describing lowest-lying baryon properties. Furthermore, the renormalization scale for the χQSM is naturally given by the cut-off parameter for the regularization which is about 0.36 GeV$^2$. Note that it is implicitly related to the inverse of the size of instantons ($\mathcal{O} = 0.35 \text{fm}$). This renormalization scale is very important in general, because the essential feature of the PV hadronic interactions comes from the effective weak Hamiltonian that has a specific scale dependence, as mentioned previously. Thus, the matching of this scale consists of an essential part in investigating any nonleptonic decays and PV hadronic processes.

While the χQSM provides a plausible framework to study the PV $\pi NN$ coupling constant, there are at least two theoretical difficulties. Firstly, the effective weak Hamiltonian has two-body operators and one has to treat the four-point correlation functions in order to compute the PV $\pi NN$ two theoretical difficulties. Firstly, the effective weak Hamiltonian has two-body operators and one has to treat the non-perturbative effects is reflected in the momentum-dependent quark mass, which arises from the zero mode of instantons. However, it is very difficult to handle these problems in the self-consistent approach, we will use the gradient expansion to calculate $h^1_{\pi NN}$, taking the limit of a large soliton size, so that valence quarks in a nucleon plunge into the Dirac sea and the soliton emerges as a topological one, which is quite similar to a skyrmion. Equivalently, we can start directly from the $\Delta S = 0$ effective weak chiral Lagrangian derived in Ref. and quantize the chiral soliton collectively. Then, we introduce a physical pion through quantum fluctuations around the soliton field. This procedure will lead to the results for $h^1_{\pi NN}$ without fitting any parameter. In the present work, we will restrict ourselves the SU(2) case for simplicity and will concentrate on how the low-energy constants (LECs) found in Ref. feature the PV $\pi NN$ coupling constant.

This paper is organized as follows: In Section II we describe briefly a general formalism for the derivation of the PV weak $\pi NN$ coupling constant. In Section III we present the numerical results for $h^1_{\pi NN}$ and discuss the role of the LECs of the $\Delta S = 0$ effective weak chiral Lagrangian. The last section is devoted to the summary and outlook of this work.

II. GENERAL FORMALISM

In this Section, we will show how to incorporate the $\Delta S = 0$ effective weak Hamiltonian into the effective chiral action. We employ the $\Delta S = 0$ effective weak Hamiltonian derived in Ref. The Hamiltonian reads

$$\mathcal{H}_{\Delta S = 0} = \frac{G_F}{\sqrt{2}} \cos \theta_c \sin \theta_c \left[ \sum_{i=1}^{2} \left( \alpha_i \mathcal{O}(A_i \dagger, A_i) + \beta_i \mathcal{O}(A_i \dagger t_A, A_i t_A) + \text{h.c.} \right) \right] + \sum_{i,j=1}^{2} \left( \gamma_{ij} \mathcal{O}(B_i \dagger, B_j) + \rho_{ij} \mathcal{O}(B_i \dagger t_A, B_j t_A) \right),$$

where $\mathcal{O}(M_i, N_i)$ is defined as a two-body operator $O(M_i, N_i) \equiv -\psi^\dagger \gamma_\mu \gamma_5 M_i \psi \psi^\dagger \gamma_\mu N_i \psi$ in Euclidean space, and $t_A$ denotes the generator of the color SU(3) group, normalized as $\text{tr} t_A t_B = 2 \delta_{AB}$. The definitions of the matrices $A_i$ and $B_i$, and the coefficients $\alpha$, $\beta$, $\gamma$ and $\rho$ can be found in Ref. These coefficients are the functions of the scale-dependent Wilson coefficient $K(\mu)$ defined as

$$K(\mu) = \left( 1 + \frac{g^2(\mu^2)}{16\pi^2} \beta \ln \frac{M_W^2}{\mu^2} \right),$$

where $g(\mu^2)$ denotes the strong running coupling constant, $\mu$ stands for the renormalization point that specifies the energy scale, $b = 11 - 2N_f/3$, and $M_W$ is the mass of the $W$ boson. The coefficient $K$ encodes the effect of the strong interaction from perturbative gluon exchanges.

The four-quark operators are expressed generically by

$$\mathcal{Q}_i(x) = -\psi^\dagger(x) \Gamma_i \psi(x) \psi^\dagger(x) \Gamma_i \psi(x),$$

where $i = 1, \cdots, 12$ labels each four-quark operator in the effective weak Hamiltonian and $\Gamma_{i(1/2)}$ consist of the Dirac
The creation baryon current is written as
\[ J^i \]
with the quantum numbers (\( T T \)). The nucleon state is defined in terms of the Ioffe-type current in Euclidean space (\( s \)). In the chiral limit, where \( M \) stands for the axial-vector current. In the \( \chi \)QSM, the correlation function
\[ H_{\chi NN} = \sum_{i=1}^{12} C_i Q^i(x), \] (4)
where \( C_i \) denotes \( \alpha, \beta, \gamma \) and \( \rho \) according to Eq. (1).

In order to derive \( h_{\pi NN}^i \) in the \( \chi \)QSM, we have to solve the following matrix element:
\[ \langle N | H_{\chi NN} | \pi^a N \rangle = \sum_{i=1}^{12} C_i \langle N | Q^i(z) | \pi^a N \rangle = \sum_{i=1}^{12} C_i \int d^4 \epsilon \epsilon^{ik} \langle k^2 + m_\pi^2 \rangle \langle N | T[Q^i(z)\pi^a(\xi)] | N \rangle. \] (5)

The nucleon current \( J^i_N (J_N) \) plays a role of creating (annihilating) nucleons. The \( N^* (N) \) represents the normalizing factor depending on the initial (final) momentum. The \( J^i_N (J_N) \) consists of \( N_c \) quarks:
\[ J_N(x) = \frac{1}{N_c^2} \sum_{s_1 \cdots s_{N_c}} \sum_{c_1 \cdots c_{N_c}} \Gamma^{s_1 s_2 \cdots s_{N_c}} \langle TT_3 Y \rangle J_{J_{1}Y_{R}}(x) \psi_{s_1 c_1} \cdots \psi_{s_{N_c} c_{N_c}}(x), \] (7)
where \( s_1 \cdots s_{N_c} \) and \( c_1 \cdots c_{N_c} \) denote respectively spin-isospin and color indices. The \( \Gamma^{(i) \langle TT_3 Y \rangle J_{J_{1}Y_{R}}} \) are matrices with the quantum numbers (\( TT_3 Y \)). For the nucleon, \( T = 1/2, Y = 1 \) and \( J = 1/2 \). The right hypercharge will be constrained by the baryon number. The creation baryon current is written as
\[ J^i_N = \frac{1}{N_c^2} \sum_{s_1 \cdots s_{N_c}} \sum_{c_1 \cdots c_{N_c}} \Gamma^{s_1 s_2 \cdots s_{N_c} \cdot \cdot \cdot \cdot \cdot} \langle TT_3 Y \rangle J_{J_{1}Y_{R}}(x) \psi_{s_1 c_1} \cdots \psi_{s_{N_c} c_{N_c}}(x), \] (8)

The partial conservation of the axial-vector current (PCAC) being considered, the matrix elements in Eq. (9) can be related to the following four-point correlation function
\[ \sum_{i=1}^{12} C_i \langle 0 | T[J_N(x)Q^i(z)\partial_\mu A^\mu_N(\xi)J^i_N(y)] | 0 \rangle = \lim_{y_0 \to -\infty} \sum_{x_0 \to +\infty} \mathcal{K}, \] (9)
where \( A^\mu_N \) stands for the axial-vector current. In the \( \chi \)QSM, the correlation function \( \mathcal{K} \) can be expressed as a functional integral
\[ \mathcal{K} = \frac{1}{z} \int D\psi D\psi^\dagger D\xi J_N(x)Q^i(z)\partial_\mu A^\mu_N(\xi)J^i_N(y) \exp \left[ \int d^4x \psi^\dagger \left( i\partial + i\sqrt{M(-\partial^2)}U^\gamma \sqrt{M(-\partial^2)} \right) \psi \right] \] (10)
in the chiral limit, where \( M(-\partial^2) \) denotes the momentum-dependent dynamical quark mass and \( U^\gamma \) represents the chiral field defined as
\[ U^\gamma = \frac{1 + \gamma_5}{2} U + \frac{1 - \gamma_5}{2} U^\dagger \] (11)
with the Goldstone boson field \( U = \exp(i\lambda^a \pi^a/f_\pi) \).

It is, however, extremely complicated to solve Eq. (10) numerically, since the PV \( \pi NN \) coupling constant involves the two-body quark operators \( Q^i \) and the axial-vector one, which will lead to laborious triple sums in quark levels already at the leading order. Moreover, the momentum-dependent quark mass, which is known to be of great significance in describing nonleptonic processes, introduces in addition technical difficulties. One way to avoid these complexities is to use a gradient expansion taking \( \partial U/M \ll 1 \) or equivalently to start from the effective weak chiral Lagrangian already derived in Ref. [29]. Note that though we did not carry out the derivative expansion to order \( p^4 \), it is not difficult to estimate how large the corresponding LECs could be. In Ref. [28], the \( \Delta S = 1 \) effective weak chiral Lagrangian to order \( p^4 \) was investigated in the case of the local chiral quark model. As one can see, all of the LECs are order-of-magnitude smaller than the \( O(p^2) \) LECs. In this sense, even though we go further beyond the leading order, the contribution from higher derivative terms will not enhance or suppress \( h_{\pi NN} \) much. In fact, it will be at most below \((5-10)\% \).

Thus, we will use the \( \Delta S = 0 \) effective weak chiral Lagrangian derived in Ref. [29] as
our starting point, instead of dealing with Eq. (10). Nevertheless, the present approach goes beyond the previous analyses in the Skyrme model [19, 21], because the present scheme incorporates properly the effects of the perturbative quark-gluon strong interaction in the derivation of the PV $\pi NN$ coupling constant.

The leading-order (LO) term of the $\Delta S = 0$ effective weak chiral Lagrangian in the large $N_c$ can be expressed in terms of the vector and axial-vector currents

$$\mathcal{L}_{\text{LO}} = 2\left(\hat{a}_{11} \sum_{i=1}^{2} V_{i}^{\mu} A_{i}^{\mu} + \hat{a}_{22} \sum_{i=4}^{5} V_{i}^{\mu} A_{i}^{\mu}\right) + \left[9\gamma_{11} V_{\mu}^{0} A_{0}^{0} + 3\gamma_{12} \left(-V_{\mu}^{0} + 2V_{\mu}^{3} + \frac{2}{\sqrt{3}} V_{\mu}^{8}\right) A_{0}^{\mu} + 3\gamma_{21} V_{\mu}^{0} \left(-A_{0}^{\mu} + 2A_{3}^{\mu} + \frac{2}{\sqrt{3}} A_{8}^{\mu}\right)\right],$$

where the vector and axial-vector currents are defined as

$$V_{\mu}^{a} = \frac{f_{\pi}^{2}}{2} \text{Tr}[T^{a}(R_{\mu} + L_{\mu})], \quad A_{\mu}^{a} = \frac{f_{\pi}^{2}}{2} \text{Tr}[T^{a}(R_{\mu} - L_{\mu})]$$

in terms of $L_{\mu} = iU^{\dagger} \partial_{\mu} U$, $R_{\mu} = iU \partial_{\mu} U^{\dagger}$, and $T^{a} = \left(\frac{1}{2}, \frac{1}{2}, \cdots, \frac{1}{2}\right)$. The parameter $f_{\pi}$ stands for the pion decay constant $f_{\pi} = 93$ MeV.

The classical soliton field $U_{0}$ is assumed to have a structure of the trivial embedding of the SU(2) hedgehog field as

$$U_{0} = \left(\exp(i\tau \cdot \hat{F}(r)) \quad 0\right) \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

with the profile function of the soliton $F(r)$. This classical soliton field can be fluctuated in such a way that the pion field can be coupled to a weak two-body operator

$$U = \exp(i\tau \cdot \pi/2) U_{0} \exp(i\tau \cdot \pi/2).$$

Similarly, the vector and the axial-vector currents transform as

$$A_{\mu}^{a} = \tilde{A}_{\mu}^{a} + \frac{1}{f_{\pi}} f^{ab}_{\mu} \tilde{V}_{\mu}^{b}, \quad V_{\mu}^{a} = \tilde{V}_{\mu}^{a} + \frac{1}{f_{\pi}} f^{ab}_{\mu} \tilde{A}_{\mu}^{b},$$

where the indices $a, b = 1, \cdots, 8$ and $i = 1, 2, 3$. The current with a tilde indicates that arising from the background soliton field.

Since the PV $\pi NN$ interaction Lagrangian is expressed as

$$\mathcal{L}^{\pi}_{\mu N} = -\frac{1}{\sqrt{2}} h_{\pi NN}^{\dagger} \Psi_{N}(\tau \times \pi)_{3} \Psi_{N},$$

which is linear in the pion field and defined in the SU(2) flavor space (proportional to $(\tau \times \pi)_{3}$), one can easily see that the term $\sum_{i=4}^{5} V_{i}^{\mu} A_{i}^{\mu}$ does not contribute to the PV $\pi NN$ Lagrangian. Moreover, since $f^{ab}_{\mu} = f^{0a}_{\mu} = 0$, the pion fields for the PV $\pi NN$ Lagrangian can survive in the vector and axial-vector currents only when $a = i$. Writing them explicitly, we have

$$A_{\mu}^{i} = \tilde{A}_{\mu}^{i} + \frac{1}{f_{\pi}} \left(\tilde{V}_{\mu}^{i} \times \pi\right)^{i}, \quad V_{\mu}^{i} = \tilde{V}_{\mu}^{i} + \frac{1}{f_{\pi}} (\tilde{A}_{\mu}^{i} \times \pi)^{i}.$$  

Considering the terms contributing to the PV $\pi NN$ vertex, we obtain for the LO Lagrangian

$$\mathcal{L}_{\text{LO}} = \hat{a}_{11} \sum_{i=1}^{2} V_{i}^{\mu} A_{i}^{\mu} + \left(3\gamma_{12} - \gamma_{22}\right) V_{3}^{\mu} A_{0}^{\mu} + \left(3\gamma_{12} - \gamma_{22}\right) V_{\mu}^{0} A^{3} + \frac{2}{\sqrt{3}} \gamma_{22} V_{\mu}^{3} A^{3} + \frac{2}{\sqrt{3}} \gamma_{22} \left(V_{\mu}^{3} A^{8} + V_{\mu}^{8} A^{3}\right) + (V \leftrightarrow A).$$

Extracting the terms linear in the pion field from Eq. (19) and rearranging them, we finally derive the LO PV $\pi NN$ Lagrangian:

$$\mathcal{L}_{\text{LO}} = \frac{1}{f_{\pi}} \left\{ \left(-\hat{a}_{11} + 2\gamma_{22}\right) \left[V_{3}^{\mu}(\mathbf{V}^{\mu} \times \pi)^{3} + A_{3}^{\mu}(\mathbf{A}^{\mu} \times \pi)^{3}\right] + \left(3\gamma_{12} - \gamma_{22}\right) V_{\mu}^{0}(\mathbf{V}^{\mu} \times \pi)^{3} + \left(3\gamma_{12} - \gamma_{22}\right) A_{0}^{\mu}(\mathbf{A}^{\mu} \times \pi)^{3} \right. \right.$$  

$$\left. + \frac{2}{\sqrt{3}} \gamma_{22} \left[A_{\mu}^{8}(\mathbf{A}^{\mu} \times \pi)^{3} + V_{\mu}^{8}(\mathbf{V}^{\mu} \times \pi)^{3}\right]\right\} + \left(\mathbf{O}^{0,3,8} \leftrightarrow \mathbf{O} \times \pi\right).$$
For simplicity, we have omitted the tildes in the currents.

In a similar manner, the next-to-leading order (NLO) effective weak chiral Lagrangian in the large $N_c$ expansion derived in \cite{29} yields the Lagrangian for the PV $\pi NN$ vertex as

\[
\mathcal{L}^\pi_{\text{NLO}} = \frac{1}{N_c f^2} \left\{ - (\bar{\alpha}_{11} + 2 \tilde{\beta}_{11}) (\mathbf{A}_3 \times \pi)_3 + (\bar{\alpha}_{22} + 2 \tilde{\beta}_{22}) \left[ (\mathbf{A}_4 \times \pi)_4 + (\mathbf{A}_5 \times \pi)_5 \right] \\
+ 3 \left( \frac{4\mathcal{I}_1 \mathcal{I}_3}{L_2^2} + 1 \right) (\bar{\gamma}_{12} + 2 \tilde{\rho}_{12}) (\mathbf{A}_0 \times \pi)_3 + 3 \left( \frac{4\mathcal{I}_1 \mathcal{I}_3}{L_2^2} - 1 \right) (\bar{\gamma}_{21} + 2 \tilde{\rho}_{21}) (\mathbf{A}_0 \times \pi)_3 \\
+ 2 (\bar{\gamma}_{22} + 2 \tilde{\rho}_{22}) \left[ (\mathbf{A}_3 \times \pi)_3 + \frac{1}{\sqrt{3}} (\mathbf{A}_8 \times \pi)_3 \right] \right\},
\]  

where

\[
\mathbf{A}_a = \frac{f^2}{4} \text{Tr} \left[ (R_{\mu} \lambda_a R^{\mu} + L_{\mu} \chi_a L^{\mu}) \tau \right] \quad \text{for } a = 0, 3, 8,
\]

\[
(\mathbf{A}_a \times \pi)_a = \frac{f^2}{4} \text{Tr} \left[ (R_{\mu} \lambda_a R^{\mu} + L_{\mu} \chi_a L^{\mu}) f_{a0} \lambda_6 \pi_7 \right] \quad \text{for } a = 4, 5,
\]

and $\lambda_0$ is defined as the unit matrix in SU(3) divided by 3. The integrals $\mathcal{I}_i$ in Eq. (21) were already evaluated in Ref. \cite{29} and are expressed as

\[
\mathcal{I}_1 = - \int \frac{d^4 k}{(2\pi)^4} \frac{M(k)}{k^2 + M^2(k)} = \frac{\langle \bar{\psi} \psi \rangle_M}{4N_c},
\]

\[
\mathcal{I}_2 = \int \frac{d^4 k}{(2\pi)^4} \frac{M^2(k) - \frac{k^2}{2} M(k) M'}{(k^2 + M^2(k))^2} = \frac{f^2}{2N_c},
\]

\[
\mathcal{I}_3 = \int \frac{d^4 k}{(2\pi)^4} \left[ \frac{1}{2} M' k^2 + \frac{1}{2} M' M'' - \frac{M^2}{8M} + \frac{3}{8} \frac{M^2 M''}{(k^2 + M^2(k))^2} \right] = \frac{M + M^2 M' + \frac{3}{2} M' M'' + \frac{3}{8} k^2 M M'' + \frac{3}{8} M M' M + \frac{3}{8} M^2 M''}{(k^2 + M^2(k))^2},
\]

where $\langle \bar{\psi} \psi \rangle_M$ denotes the quark condensate in Minkowski space and $\tilde{M}' = (dM(k)/dk)/2k$.

The next step is to carry out the zero-mode collective quantization of the soliton

\[
U_0(\vec{x}) \to U(\vec{x}, t) = R(t)U_0(\vec{x})R^\dagger(t),
\]

where $R(t)$ stands for the unitary time-dependent SU(3) orientation matrix of the soliton $R(t) = \exp(i\Omega^a(t)\lambda^a/2)$ with its angular velocity $\Omega^a(t)$ that is of order $O(1/N_c)$. Each current is transformed as

\[
V_0^a = \frac{f^2}{2} \text{Tr} \left( i \frac{\lambda^a}{2} R[\{U_0, R^\dagger \dot{R}, U_0^\dagger \} R^\dagger] \right) = \frac{f^2}{2} \left( 1 - \cos P(r) \right) D^{a\alpha} \Omega^\alpha + f^2 \sin^2 P(r) D^{ai} \Omega^i - f^2 \sin^2 P(r) (\hat{r} \cdot \Omega) D^{ai} \hat{r}^i,
\]

\[
V_i^a = \frac{f^2}{2} \text{Tr} \left( i \frac{\lambda^a}{2} R[\{U_0, \partial_i U_0^\dagger \} R^\dagger] \right) = \frac{f^2}{2} \frac{\sin^2 P(r)}{r} \delta_{ijk} \hat{r}^k D^{ai} \hat{r}^j ,
\]

\[
A_0^a = - \frac{f^2}{2} \text{Tr} \left( i \frac{\lambda^a}{2} R[\{U_0, R^\dagger \dot{A}, U_0^\dagger \} R^\dagger] \right) = \frac{f^2}{2} \left( \sin P(r) \cos P(r) \hat{r}^i \epsilon_{ijk} D^{a\alpha} \Omega^j + \sin P(r) \hat{r}^i f_{i\alpha\beta} D^{a\beta} \Omega^\alpha \right) ,
\]

\[
A_i^a = \frac{f^2}{2} \text{Tr} \left( i \frac{\lambda^a}{2} R[\{U_0, \partial_i U_0^\dagger \} R^\dagger] \right) = \frac{f^2}{2} \left[ \frac{\sin 2P(r)}{2r} \delta_{ij} + \frac{(P')^2 - \sin 2P(r)}{2r} \hat{r}_i \hat{r}_j \right] D^{a\alpha} ,
\]

where Italic (Greek) indices run over 1, 2, 3 (4, · · · , 7), respectively, and dot (prime) means the derivative with respect to time (radius), respectively. The Wigner $D$ functions and the angular velocity are defined as

\[
D^{ab}(R) = \frac{1}{2} \text{Tr} (\lambda^a R \lambda^b R^\dagger), \quad R^\dagger \dot{R} = i \frac{1}{2} \lambda^a \Omega^a.
\]
Before we proceed the calculation of $h_{\pi NN}^1$, we want to emphasize that we will investigate $h_{\pi NN}^1$ first in the SU(2) case in this work. Of course, the strange quarks may still play a certain role in describing $h_{\pi NN}^1$. In fact, Ref. [35] showed that the strange quark operator $(\bar{q}\gamma^a q)(\bar{s}\gamma^b s)$ induced by $Z^0$ exchange could contribute significantly to the $NN$ coupling constant. The main argument of Ref. [35] lies in the fact that the $\Delta I = 1$ operator proportional to $h_{\pi NN}^1$ can be related to the $\Delta S = 1$ operator by an SU(3) rotation followed by an isospin rotation. Then, it was found that the linear combination of the strange operators made a large contribution to $h_{\pi NN}^1$, which indicates that it has large matrix elements in the nucleon state. The SU(3) Skyrme model came to the similar conclusion that the four quark operators with the strange quark contributed to $h_{\pi NN}^1$ significantly [21] because of the induced kaon field. Note, however, that Ref. [21] has not used the renormalized effective weak Hamiltonian but started from the bare Hamiltonian. On the other hand, in a recent lattice study [38], the strange quark operators can only contribute to the quark-loop diagrams for which the signal-to-noise ratio remains far too small to bring out any reasonable signal, so that they were neglected. Moreover, recent findings have it that the content of strange quarks in the nucleon in the vector channel is negligible small [39] and that the strangeness in the scalar and axial-vector channels is still hampered by uncertainties [37]. Thus, it is still too early to reach a conclusion on the contribution of strange quark operators to $h_{\pi NN}^1$. In the present work, we will concentrate on the case of SU(2), since it does not vanish even in SU(2). As we will discuss later in detail, this finite result is distinguished from that of the SU(2) Skyrme model [34] in which $h_{\pi NN}^1$ turns out to be equal to zero. The extension of the investigation to SU(3) will be found elsewhere.

Since we will calculate $h_{\pi NN}^1$ in the process of $n\pi^+ \to p$, we can rewrite the LO and NLO Lagrangians in SU(2) as follows:

$$\mathcal{L}_{\pi NN}^{\text{LO}} = \frac{i\sqrt{2}}{f_\pi} \left\{ (-\bar{\alpha}_{11} + 2\bar{\gamma}_{22}) [V^3_\mu V^{\mu +} + A^3_\mu A^{\mu +}] + 2(3\bar{\gamma}_{21} - \bar{\gamma}_{22})V^3_\mu V^{\mu +} \right.\nonumber$$

$$+ 2(3\bar{\gamma}_{12} - \bar{\gamma}_{22})A^0_\mu A^{\mu +} + 2\bar{\gamma}_{22} [A^0_\mu A^{\mu +} + V^3_\mu V^{\mu +}] \right\} \pi^- + \left( O^{0, 3} \leftrightarrow O^+ \right) \right.\nonumber$$

$$\mathcal{L}_{\pi NN}^{\text{NLO}} = \frac{i\sqrt{2}}{N_c f_\pi} \left\{ (-\bar{\alpha}_{11} + 2\bar{\gamma}_{11})\Lambda^3_\alpha + 3 \left( \frac{4\bar{I}_3 I_3}{I_3^2} + 1 \right) (\bar{\gamma}_{11} + 2\bar{\rho}_{12})\Lambda^3_\alpha \right.\nonumber$$

$$+ 3 \left( \frac{4\bar{I}_3 I_3}{I_3^2} - 1 \right) (\bar{\gamma}_{21} + 2\bar{\rho}_{21})\Lambda^3_\alpha + 2(\bar{\gamma}_{22} + 2\bar{\rho}_{22})(\Lambda^3_\alpha + \Lambda^3_\alpha) \right\} \pi^-,$$  \hspace{1cm} (33)

where we have used the identity

$$\left( O \times \pi \right)^3 = \sqrt{2}i(\pi^- O^- - \pi^+ O^+)$$  \hspace{1cm} (35)

with the definitions $O^b = \frac{1}{2}(O^1 \pm iO^2)$ and $\pi^\pm = \frac{1}{\sqrt{2}}(\pi_1 \pm i\pi_2)$. The eighth component of the Gell-Mann matrices becomes the unity matrix with factor $1/\sqrt{3}$ in going from SU(3) to SU(2). The PV $\pi NN$ coupling constant, $h_{\pi NN}^1$, can be directly read from the matrix element

$$h_{\pi NN}^1 = i(p \uparrow |\mathcal{L}_{\pi NN}^{\text{LO}}|n \uparrow, \pi^+) \nonumber$$  \hspace{1cm} (36)

where

$$\mathcal{L}_{\pi NN}^{\text{LO}} = -h_{\pi NN}^1 \Psi_N i(\pi^- \tau^+ - \pi^+ \tau^-) \Psi_N \nonumber$$  \hspace{1cm} (37)

Let us first compute $h_{\pi NN}^1$ with the LO Lagrangian. Note that the iso-scalar current vanishes identically in the present model. By using the results in the previous Section, we can see that the temporal component can contribute to $h_{\pi NN}^1$ because of the orthogonality of $D^{ab}$. This produces the following expression:

$$V_0^3 V_0^+ = \frac{4f_\pi^4}{15\sqrt{2}} \sin^4 P(r) \times \left[ 6D^{3i} \Omega^i (D^{3j} + iD^{2j})\Omega^j + D^{3i} \Omega^i (D^{1j} + iD^{2j})\Omega^j + D^{3i} \Omega^j (D^{1j} + iD^{2j})\Omega^i \right], \nonumber$$

$$A_0^3 A_0^+ = \frac{4f_\pi^4}{3\sqrt{2}} \sin^2 P(r) \cos^2 P(r) \left[ D^{3i} \Omega^i (D^{1j} + iD^{2j})\Omega^j - D^{3i} \Omega^j (D^{1j} + iD^{2j})\Omega^i \right]. \nonumber$$  \hspace{1cm} (38)

Because of the zero-mode quantization, the angular velocity is expressed in terms of the spin operator $S^i \Omega^i = S^i I^i$, where $I$ is the moment of inertia of the soliton. The spin operator and Wigner $D$ function satisfy the commutation relation $[S^i, D^{0j}] = i\epsilon^{ijk} D^{ak}$. Then, the matrix elements of Eq. (38) are written as

$$\langle p \uparrow |V_0^3 V_0^+|n \uparrow \rangle = \frac{f_\pi^4}{15\sqrt{2}} \frac{5}{2I} \sin^4 P(r) = -\langle p \uparrow |V_0^3 V_0^+|n \uparrow \rangle \nonumber$$

$$\langle p \uparrow |A_0^3 A_0^+|n \uparrow \rangle = -\frac{f_\pi^4}{3\sqrt{2}} \frac{3}{2I^2} \sin^2 P(r) \cos^2 P(r) = -\langle p \uparrow |A_0^3 A_0^+|n \uparrow \rangle \nonumber$$  \hspace{1cm} (39)
Since the LO Lagrangian is symmetric under the exchange of the indices 3 and +, it turns out that
\[ h^1_{\pi NN}(\text{LO}) = 0. \] (40)

This null result of the LO \( h^1_{\pi NN} \) was also obtained in the minimal Skyrme model [34].

The NLO Lagrangian has a rather complicated structure, so that it is convenient to analyze first \( \Lambda^i_{0,3} \). Introducing \( r^i_\mu \) and \( l^i_\mu \) as
\[ R_\mu = -\tau^i r^i_\mu, \quad L_\mu = -\tau^i l^i_\mu, \] (41)
we rewrite the expressions for \( \Lambda^i_\mu \) as
\[
\begin{align*}
\Lambda^i_a &= \frac{f_4}{4} \text{Tr} \left[ (R_\alpha R^\alpha + L_\alpha L^\alpha) r^i \right] = \frac{f_4}{4} \left( r^m_\mu r^{\mu m} + l^m_\mu l^{\mu m} \right) \text{Tr} (r^m_\mu r^{\mu n} r^n \tau^i) \\
&= \frac{f_4}{2} \left( r^m_\mu r^{\mu m} - \delta^i_a r^m_\mu r^{\mu m} + r^{\mu n} r_\mu + (r \leftrightarrow l) \right), \text{ for } a \neq 0, \\
\Lambda^i_0 &= \frac{f_4}{12} \text{Tr} \left[ (R_\mu R^\mu + L_\mu L^\mu) r^i \right] = \frac{f_4}{12} \left( r^m_\mu r^{\mu m} + l^m_\mu l^{\mu m} \right) \text{Tr} (r^m_\mu r^{\mu n} r^n \tau^i) \\
&= \frac{f_4}{6} \left( r^m_\mu r^{\mu m} + l^m_\mu l^{\mu m} \right). 
\end{align*}
\] (42)

Here, index \( a \) runs over \( a = 1, 2, 3 \). Then, \( r^i_\mu \) and \( l^i_\mu \) become
\[
\begin{align*}
r^a_\mu &= -D^{ab} (-\sin P(r) \cos P(r)) e^{blm} \Omega^l \hat{r}^m + \sin^2 P(r) \delta^b_1 \Omega^l, \\
r^i_\mu &= -D^{ab} \left( i \hat{r}^b \partial_\mu P(r) + \delta^b_1 \frac{\sin 2 P(r)}{2r} + \frac{\sin^2 P(r)}{r} e^{ikb} r^k \right), \\
l^0_\mu &= -D^{ab} (\sin P(r) \cos P(r)) e^{blm} \Omega^l \hat{r}^m + \sin^2 P(r) \delta^b_1 \Omega^l, \\
l^i_\mu &= -D^{ab} \left( -i \hat{r}^b \partial_\mu P(r) - \delta^b_1 \frac{\sin 2 P(r)}{2r} + \frac{\sin^2 P(r)}{r} e^{ikb} r^k \right),
\end{align*}
\] (44)
(45)
(46)
(47)
where the transverse Kronecker delta is expressed as \( \delta^a_1 = \delta^{ab} - \hat{r}^a \hat{r}^b \). Putting these results together, we arrive at the expressions for \( \Lambda^i_3 \) and \( \Lambda^i_0 \):
\[
\begin{align*}
\Lambda^+_3 &= \frac{f_4}{4} \text{Tr} \left[ (R_\mu \tau_3 R^\mu + L_\mu \tau_3 L^\mu) r^+ \right] = \frac{f_4}{4} \left( (r^1_\mu + i r^2_\mu) r^3_\mu + r^3_\mu (r^1_\mu + i r^2_\mu) + (r \leftrightarrow l) \right), \\
\Lambda^+_0 &= \frac{f_4}{4} \text{Tr} \left[ (R_\mu \lambda_3 R^\mu + L_\mu \lambda_3 L^\mu) r^+ \right] = \frac{f_4}{12} \left( (r^1_\mu + i r^2_\mu) r^3_\mu - r^3_\mu (r^1_\mu + i r^2_\mu) + (r \leftrightarrow l) \right).
\end{align*}
\] (48)

Since
\[
\int d^3x \langle p \uparrow | r^3_\mu (r^1_\mu + i r^2_\mu) | n \uparrow \rangle = - \int d^3x \langle p \uparrow | (r^{12}_\mu + i r^{23}_\mu) r^0_\mu | n \uparrow \rangle = \frac{2\pi}{3 f^2} \int dr r^2 \sin^2 P(r) (\sin^2 P(r) - 3 \cos^2 P(r)),
\] (49)
one can easily see that only \( \Lambda^+_0 \) contributes to \( h^1_{\pi NN} \). As a result, \( h^1_{\pi NN} \) from the NLO Lagrangian turns out to be
\[
h^1_{\pi NN}(\text{NLO}) = \frac{8\sqrt{2}\pi}{3 f^2} \left( N_9 + \frac{2}{3} N_{10} \right) \int dr r^2 \sin^2 P(r) (\sin^2 P(r) - 3 \cos^2 P(r)),
\] (50)
where the LECs \( N_9 \) and \( N_{10} \) are given as [29]
\[
\begin{align*}
N_9 &= 4 N_c \left[ 4 \bar{L}_3 \gamma_2 (\gamma_1 + \gamma_2 + 2 \rho_12 + 2 \rho_21) + T_2 \gamma_2 (\gamma_1 - \gamma_2 + 2 \rho_12 - 2 \rho_21) \right] \\
&= 4 \langle \bar{\psi} \psi \rangle M L_3 (\gamma_1 + \gamma_2 + 2 \rho_12 + 2 \rho_21) + \frac{f_4}{4 N_c} (\gamma_1 - \gamma_2 + 2 \rho_12 - 2 \rho_21), \\
N_{10} &= 4 N_c T_2^2 (\gamma_22 + 2 \rho_22) = \frac{f_4}{4 N_c} (\gamma_22 + \rho_22).
\end{align*}
\] (51)
(52)
As we will discuss later, the LECs \( N_9 \) and \( N_{10} \) are essential to describe the PV \( \pi N N \) coupling constant.
III. RESULTS AND DISCUSSION

We are now in a position to calculate Eq. (50) numerically. In doing so, we make use of the momentum dependent quark mass derived from the instanton vacuum [26] and the corresponding results of the LECs obtained in Ref. [29]. The value of $M_0 = M(k = 0)$ is taken to be 350 MeV as in Ref. [29], which was fixed by the saddle-point equation from the instanton vacuum [26]. Moreover, we employ three different types of the solitonic profile function to examine the dependence of $h_{\pi NN}^1$ on them. The first one is the arctangent profile function $P(r)$ [32]

$$P(r) = 2 \arctan \left( \frac{r_0}{r} \right)^2,$$

where $r_0$ is given by $r_0 = \sqrt{\frac{3g_A}{m_\pi^2}}$. Employing $g_A = 1.26$ and $f_\pi = 93$ MeV, we obtain $r_0 = 0.582$ fm. We use a physical profile function as a second one, which associates with the proper pion tail of the nucleon

$$P(r) = \begin{cases} 
2 \arctan \left( \frac{r_0}{r} \right)^2 & (r \leq r_x), \\
A e^{-m_\pi r} (1 + m_\pi r)^2/r^2 & (r > r_x), 
\end{cases}$$

where $m_\pi$ denotes the pion mass and $A = 2r_0^2$. $r_x$ is determined by the intersection of the arctangent function ($r \leq r_x$) and pion tail ($r > r_x$). If one takes the limit $m_\pi \to 0$ for the pion tail, the physical profile function becomes identical with the arctangent one at large $r$. With the physical pion mass considered, we have $r_x = 0.749$ fm. The final one is the linear profile function initially proposed by Skyrme [33]

$$P(r) = \begin{cases} 
\pi(1 - u/\lambda) & (u \leq \lambda), \\
0 & (u > \lambda)
\end{cases}$$

where $u = 2ef_\pi r$ with $e = 4.84$ and $\lambda = 3.342$.

Using these three profile functions, we can immediately compute the PV $\pi NN$ coupling constant $h_{\pi NN}^1$. Figure 1 draws the results of $h_{\pi NN}^1$ as a function of the Wilson coefficient $K$. The solid curve depicts that with the physical profile function, whereas the dashed and short-dashed ones correspond to those with the arctangent and linear profile functions respectively. One can regard the difference between the results with the physical profile function and those

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{figure1.png}
\caption{(Color online) PV $\pi NN$ coupling constant $h_{\pi NN}^1$ as a function of the Wilson coefficient $K$ in units of $10^{-8}$. The solid curve draws the result with the physical profile function, the dashed one depicts that with the arctangent profile, and the short-dashed one does that with the linear one.}
\end{figure}
with the arctangent one as effects of the finite pion mass, which contribute to \( h_{\pi NN}^1 \) approximately by 10%. Moreover, the type of the profile function does not change much the general features of \( h_{\pi NN}^1 \), though we preferably take the results with the physical one as our final values.

We find out from Fig. 1 that \( h_{\pi NN}^1 \) is rather sensitive to the Wilson coefficient \( K \) and it decreases monotonically, as \( K \) increases. We notice that its sign is even changed around \( K = 6 \). This can be easily understood. The LECs \( N_9 \) and \( N_{10} \) in Eq. (50) play essential roles in determining the \( K \) dependence of \( h_{\pi NN}^1 \). Figure 2 draws the results of the LECs \( N_9 \) and \( N_{10} \) as functions of \( K \). While \( N_9 \) depends rather strongly on \( K \), \( N_{10} \) does mildly on \( K \). Moreover, \( N_9 \) is dominant over \( N_{10} \), so that the PV \( \pi NN \) coupling constant is mainly governed by \( N_9 \). Since \( N_9 \) is the main contribution to \( h_{\pi NN}^1 \), we want to examine it in detail. We can easily see that the first term of Eq. (51) containing the quark condensate is much larger than the second one. Moreover, since \( \approx 10 \) for \( \mu \approx 6 \) GeV as done for the \( \Delta S = 1 \) case \([3]\). Thus, we obtain \( h_{\pi NN}^1 \approx 1 \times 10^{-8} \) for \( K = 4 \). However, if one neglects

\[ N_9 \approx 4(\bar{\psi}\psi)_M I_3 (\check{\gamma}_{21} + 2\check{\rho}_{12}) . \]  

As shown in Eq. (1), the coefficient \( \check{\gamma}_{21} \) comes from the original effective weak Hamiltonian at the mass scale of the \( W \) boson \( \mu = M_W = 80.4 \) GeV corresponding to \( K = 1 \). In this case, only \( \check{\gamma}_{21} \) survives in \( N_9 \). However, when we start to scale the Hamiltonian down to \( \mu \approx 1 \) GeV that corresponds to \( K \approx 4 \), the gluonic renormalization arising from gluon exchange parallel to Z-boson exchange is turned on. As a result, the \( 2\check{\rho}_{12} \) term becomes as large as a half of the \( \check{\gamma}_{21} \) one \([2, 22]\) at this scale. If one goes further down to the scale at which \( K \approx 6^{-1} \), the correction of \( \check{\rho}_{12} \) cancels out the contribution of \( \check{\gamma}_{21} \), so that \( h_{\pi NN}^1 \) almost vanishes, as already shown in Figs. 1 and 2. This cancellation implies that the effects of the gluon renormalization leads to the suppression of the PV \( \pi NN \) coupling constant in the present approach of the SU(2) \( \chi \) QSM.

Since Ref. \([23]\) derived the effective weak chiral Lagrangian based on the \( \Delta S = 0 \) effective weak Hamiltonian, it is plausible to take the value \( K = 4 \) for \( h_{\pi NN}^1 \), which corresponds to the renormalization scale of the charm quark mass \( \mu \approx 1 \) GeV as done for the \( \Delta S = 1 \) case \([1]\). Thus, we obtain \( h_{\pi NN}^1 \approx 1 \times 10^{-8} \) for \( K = 4 \). However, if one neglects

\[ 1 \text{ There is a caveat in scaling further down below 1 GeV, because the matching problem becomes non-trivial below the charm quark mass. Furthermore, we still do not know how to incorporate all possible nonperturbative effects consistently below 1 GeV.} \]
all the renormalization effects, i.e. if one takes $K = 1$, we have $h_{\pi NN}^1 \approx 4 \times 10^{-8}$, which is similar to that of the SU(2) Skyrme model with vector mesons ($h_{\pi NN}^1 = (2 - 3) \times 10^{-8}$) [19] in which the effective weak Hamiltonian at \( \mu = M_W \) or with $K = 1$ was used. Note that the present result is almost 40 times smaller than the “best” value of DDH ($h_{\pi NN}^1 = 4.5 \times 10^{-7}$) [8].

![Graph](image-url)

**FIG. 3:** (Color online) PV $\pi NN$ coupling constant as a function of the pion mass $m_{\pi}$ in units of $10^{-8}$. The Wilson coefficient $K = 4$ is used, which corresponds to $\mu \approx 1$ GeV.

Very recently, Ref. [38] has reported the first result of lattice QCD: $h_{\pi NN}^1 = (1.099 \pm 0.505^{+0.058}_{-0.064}) \times 10^{-7}$ with the pion mass $m_{\pi} = 389$ MeV. Thus, it is interesting to compare the present result with the lattice one. In order to do that, we need to examine the dependence of $h_{\pi NN}^1$ on the pion mass $m_{\pi}$. Figure 3 depicts the PV $\pi NN$ coupling constant as a function of $m_{\pi}$. We employ here the physical profile function with $K = 4$. Interestingly, $h_{\pi NN}^1$ starts to increase, as $m_{\pi}$ does. As a result, $h_{\pi NN}^1$ turns out to be around $1.8 \times 10^{-8}$ for $m_{\pi} = 400$ MeV. Though it is still around five times less than that of the lattice calculation, we can infer from Fig. 3 that lattice results with the physical pion mass might be quite smaller than that of Ref. [38]. Moreover, if one takes $K = 1$, $h_{\pi NN}^1$ with $m_{\pi} = 389$ MeV would become $h_{\pi NN}^1 \approx 6.77 \times 10^{-8}$ that is comparable to the lattice one, though the value $K = 1$ does not seem tenable for $h_{\pi NN}^1$ as discussed previously. However, one has to keep in mind that Ref. [38] has not performed the calculation of the full matrix element, since the quark-loop diagrams were omitted because of technical difficulties. A quantitative comparison with full lattice calculations is still being awaited.

**IV. SUMMARY AND OUTLOOK**

We have investigated the parity-violating pion-nucleon coupling constant $h_{\pi NN}^1$ within the framework of the chiral quark-soliton model with the gradient expansion used. Starting from the $\Delta S = 0$ effective weak chiral Lagrangian derived in the same framework, we have calculated the parity-violating $\pi NN$ coupling constant. It was found that it vanished at the leading order in the large $N_c$, i.e. $h_{\pi NN}^1(\text{LO}) = 0$, which is of order $O(N_c^{-1/2})$, but it was finite to the next-to-leading order, i.e. $O(N_c^{-3/2})$.

Employing three different profile functions, that is, the arctangent, physical, and linear ones, we calculated the parity-violating $\pi NN$ coupling constant to the next to the leading order. It turns out that the values of $h_{\pi NN}^1$ depend sensitively on the values of the Wilson coefficient $K$ and vanishes around $K \approx 6$. The reason can be found in the fact that the contribution of the gluonic renormalization constant $\tilde{\rho}_{12}$ cancels out that of the $\tilde{\gamma}_{21}$, which is the leading one. It indicates that the perturbative gluonic contribution suppresses the parity-violating $\pi NN$ coupling constant. Taking the scale of the charm quark mass, i.e. $\mu \approx 1$ GeV, we found $h_{\pi NN}^1 \approx 1 \times 10^{-8}$, which is almost 40 times smaller
than the “best value” of Ref. [8]. If the $\mu = M_W$ is selected, the value of $h_{\pi NN}^1$ turns out to be similar to that from the Skyrme model with vector mesons [19]. We also compared the present result with that of the lattice calculation. Thus, we examined the dependence of the parity-violating $\pi NN$ coupling constant on the pion mass and found that $h_{\pi NN}^1$ increased as $m_\pi$ did. If one uses $m_\pi = 400$ MeV, the result turns out to be almost two times larger than that with the physical value $m_\pi = 140$ MeV but is still about five times smaller than the lattice one. However, we want to emphasize that neither the present result nor the lattice one is the final one.

In order to understand the parity-violating $\pi NN$ coupling constant $h_{\pi NN}^1$ more completely and quantitatively, we have to consider the following important physics: Since we have considered the SU(2) case in the present work, the effects of strangeness were left out. As already mentioned in Section II, however, the strange quark operators may play a certain role in describing the parity-violating $\pi NN$ coupling constant. Extending from SU(2) to SU(3) is lengthy but straightforward in the present framework. Starting from Eqs. (20, 21), we employ the quantization with the embedding (14). In particular, the singlet current is distinguishable from the octet one in SU(3), so that this would make difference in predicting the parity-violating $\pi NN$ coupling constant. Moreover, the fourth and fifth flavor components of the vector currents enter the next-to-leading order Lagrangian, this would also contribute to the embedding (1). In particular, the singlet current is distinguishable from the octet one in SU(3), so that this would make difference in predicting the parity-violating $\pi NN$ coupling constant. Moreover, the fourth and fifth flavor components of the vector currents enter the next-to-leading order Lagrangian, this would also contribute to $h_{\pi NN}^1$.

As was seen in Section III, the gluon renormalization plays a role of suppressing the parity-violating $\pi NN$ coupling constant. However, the effective weak Hamiltonian at two-loop order was derived very recently in Ref. [39], where the QCD penguin diagrams were also considered. This Hamiltonian is more complete than that from Ref. [8]. Thus, it is of great significance to investigate the $\Delta I = 1/2$ rule in nonleptonic decays [6, 7]. The corresponding investigations are under way.

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References

[1] B. R. Holstein, J. Phys. G 36 (2009) 104003.
[2] J. Kambor, J. H. Missimer and D. Wyler, Nucl. Phys. B 346 (1990) 17.
[3] M. Artuso et al., Eur. Phys. J. C 57 (2008) 309.
[4] L. Lellouch, arXiv:1104.5484 [hep-lat].
[5] M. K. Gaillard and B. W. Lee, Phys. Rev. Lett. 33 (1974) 108.
[6] M. A. Shifman, A. I. Vainshtein and V. I. Zakharov, Nucl. Phys. B 120 (1977) 316.
[7] G. Buchalla, A. J. Buras and M. E. Lautenbacher, Rev. Mod. Phys. 68 (1996) 1125.
[8] B. Desplanques, J. F. Donoghue and B. R. Holstein, Annals Phys. 124 (1980) 449.
[9] R. D. C. Miller and B. H. J. McKellar, Phys. Rept. 106 (1984) 169.
[10] V. M. Dubovik and S. V. Zenkin, Annals Phys. 172 (1986) 100.
[11] J. F. Cavaignac, B. Vignon and R. Wilson, Phys. Lett. B 67 (1977) 148.
[12] M. T. Gericke et al., Phys. Rev. C 83 (2011) 015505.
[13] W. M. Snow et al., Nucl. Instrum. Meth. A 440 (2000) 729.
[14] C. A. Barnes et al., Phys. Rev. Lett. 40 (1978) 840.
[15] P. G. Bizzeti et al., Lett. Nuovo Cim. 29 (1980) 167.
[16] S. A. Page et al., Phys. Rev. C 35 (1987) 1119.
[17] W. C. Haxton, arXiv:0802.2984 [nucl-th].
[18] G. B. Feldman, S. A. Page et al., Nucl. Phys. C 43 (1991) 863.
[19] N. Kaiser and U. G. Meissner, Nucl. Phys. A 499 (1989) 699.
[20] N. Kaiser and U. G. Meissner, Nucl. Phys. A 510 (1990) 759.
[21] U. G. Meissner and H. Weigel, Phys. Lett. B 447 (1999) 1.
[22] E. M. Henley, W. Y. P. Hwang and L. S. Kisslinger, Phys. Lett. B 367 (1996) 21. nucl-th/9511002.
[23] M. Franz, H. -Ch. Kim and K. Goeke, Nucl. Phys. A 699 (2002) 541.
[28] M. Franz, H. -Ch. Kim, K. Goeke, Nucl. Phys. B 562 (1999) 213.
[29] H. -J. Lee, C. H. Hyun, C. -H. Lee and H. -Ch. Kim, Eur. Phys. J. C 45 (2006) 451.
[30] W. Broniowski, B. Golli and G. Ripka, Nucl. Phys. A 703 (2002) 667 [hep-ph/0107139].
[31] D. Diakonov, V. Y. .Petrov and P. V. Pobylitsa, Nucl. Phys. B 306 (1988) 809.
[32] D. Diakonov, hep-ph/9802298.
[33] T. H. R. Skyrme, Proc. R. Soc. London A 260 (1961) 127.
[34] M. Z. Shmatikov, Sov. J. Nucl. Phys. 49 (1989) 565 [Yad. Fiz. 49 (1989) 910].
[35] D. B. Kaplan and M. J. Savage, Nucl. Phys. A 556 (1993) 653 [Erratum-ibid. A 570 (1994) 833] [Erratum-ibid. A 580 (1994) 679].
[36] Z. Ahmed et al. [HAPPEX Collaboration], Phys. Rev. Lett. 108 (2012) 102001 [arXiv:1107.0913 [nucl-ex]].
[37] J. R. Ellis, K. A. Olive and C. Savage, Phys. Rev. D 77 (2008) 065026 [arXiv:0801.3656 [hep-ph]].
[38] J. Wasem, Phys. Rev. C 85 (2012) 022501.
[39] B. C. Tiburzi, arXiv:1201.4852 [hep-ph].