Deterministic Differential Search Algorithm for
Distributed Sensor/Relay Networks

Po-Chun Fu, Pei-Rong Li, Li-Ming Wei, Chang-Lin Chen, and Che Lin†
Institute of Communication Engineering & Department of Electrical Engineering
National Tsing Hua University, Hsinchu, Taiwan 30013
E-mail: s9920101@m99.nthu.edu.tw, s9961151@m99.nthu.edu.tw, markwei27990309@yahoo.com.tw
vip410246@gmail.com, clin@ee.nthu.edu.tw

I. INTRODUCTION

For distributed sensor/relay networks, high reliability and power efficiency are often required. However, several implementation issues arise in practice. One such problem is that all the distributed transmitters have limited power supply since the power source of the transmitters cannot be recharged continually. To resolve this, distributed beamforming has been proposed as a viable solution where all distributed transmitters seek to align in phase at the receiver end. However, it is difficult to implement such transmit beamforming in a distributed fashion in practice since perfect channel state information (CSI) need to be made available at all distributed transmitters, requiring tremendous overhead to feed back CSI from the receiver to all distributed transmitters.

In literature, the efforts of designing efficient distributed phase alignment algorithms can be categorized into two classes: deterministic phase adjustment or random phase perturbation algorithms. In [1], [2], Mudumbai et al. proposed and provided initial analysis for an adaptive distributed beamforming which requires only a single bit feedback. In [3], [4], a general set of adaptive distributed beamforming algorithms was reformulated as a local random search algorithm and analyzed. A bio-inspired robust adaptive random search algorithm was proposed and analyzed in [5].

On the other hand, several deterministic distributed beamforming algorithms were also proposed. For example, Thibault et al. introduced a deterministic algorithm with individual power constraint [6]. For amplify-and-forward wireless relay networks, an algorithm using additive deterministic perturbations was presented in [7]. Fertl et al. further investigated a multiplicative deterministic perturbations for distributed beamforming under a total power constraint [8].

In this paper, we propose a novel algorithm that belongs to the category of deterministic phase adjustment algorithm: the Deterministic Differential Search Algorithm (DDSA), where the differences between the measured received signal strength (RSS) are utilized judiciously to help us predict the deterministic phase adjustment done at distributed transmitters. Numerical simulations demonstrate rapid convergence to a predetermined threshold.

†Che Lin is the corresponding author. This work is supported by National Science Council, Taiwan (R.O.C.), under Grant NSC102-2221-E-007-024.
mits for 2 iterations. The $i$-th transmitter alters its phase by iterating through the elements in the set $\Theta_i$ given by:

$$\Theta_i = \{\theta_i[0] + \alpha_j, \quad j = 0, 1, 2\}$$

where $\theta_i[0]$ is the initial phase and $\alpha = 2\pi/3$. The corresponding RSS measured at the receiver can hence be expressed as

$$M_j = \{\text{Mag}(\theta_i[0] + \alpha j), \quad j = 0, 1, 2\}$$

Note that the RSS function in (2) can be rewritten as

$$\text{Mag}(\theta_1[n], \cdots, \theta_N[n]) = \sqrt{P} \left| \sum_{i=1}^{N} a_i e^{j\theta_i[n]} \right|$$

$$= \sqrt{P} \left| \sum_{k=1}^{N} a_k e^{j\theta_k[n]} + a_i e^{j\theta_i[n]} \right|$$

$$= \sqrt{P} |r_i + c_i|$$

$$= \sqrt{P} \sqrt{(|r_i| + |c_i| \cos \beta)^2 + (|c_i| \sin \beta)^2}$$

where $r_i = \sum_{k \neq i} a_k e^{j\theta_k[n]}$, $c_i = a_i e^{j\theta_i[n]}$ and $\beta$ is the phase angle between complex numbers $r_i$ and $c_i$. From this, we have

$$M_j^2 = \left( |r_i| + |c_i| \cos \left( \beta + \frac{2\pi}{3} j \right) \right)^2$$

$$+ \left( |c_i| \sin \left( \beta + \frac{2\pi}{3} j \right) \right)^2$$

$$= |r_i|^2 + |c_i|^2 + 2|r_i||c_i| \cos \left( \beta + \frac{2\pi}{3} j \right), \quad \forall j = 0, 1, 2$$

Note that (4) describes 3 linearly independent equations with 3 independent variables $|r_i|$, $|c_i|$ and $\beta$. Therefore, we can obtain $|r_i|$, $|c_i|$ and $\beta$ by solving (4). Once $\beta$ is obtained, we can predict and adjust the phase at the $i$th distributed transmitter accordingly. Ideally, one can feed back $\beta$ directly to the $i$th distributed transmitter to achieve perfect phase alignment from the viewpoint of the $i$th transmitter. However, if there is only limited bandwidth for the reverse feedback link, proper quantization is necessary. For example, if 3 bits are available for the reverse feedback link, $\beta$ can be quantized as $\{0, \pi/4, \pi/2, 3\pi/4, \pi, 5\pi/4, 3\pi/2, 7\pi/4\}$. With this feedback information, the $i$th transmitter can decide whether to subtract either one of $\{0, \pi/4, \pi/2, 3\pi/4, \pi, 5\pi/4, 3\pi/2, 7\pi/4\}$ to achieve a higher RSS function.

As a numerical example to demonstrate the rapid convergence of our proposed DDSA, we assume that there are $N_s = 500$ distributed transmitters. In Figure 1 we compare the convergence behavior between DDSA and the following schemes: a) one-bit scheme proposed in [11], b) BioRASA proposed in [5], and c) DBSA proposed in [9]. We clearly observe that our proposed DDSA exhibit superior convergence behavior and is the first to reach $\text{RSS}=0.95$. Therefore, our proposed DDSA presents an efficient and simple alternative to existing adaptive distributed beamforming algorithms. It is important to note that DDSA can also be easily extended to the case where more limited feedback information is available. This is one of our future extensions.

**REFERENCES**

[1] R. Mudumbai, B. Wild, U. Madhow, and K. Raman, “Distributed beamforming using 1 bit feedback: From concept to realization,” in Allerton Conference on Communication, Control, and Computing, 2006.

[2] R. Mudumbai, J. P. Hespanha, U. Madhow, and G. Barriac, “Distributed transmit beamforming using feedback control,” IEEE Trans. Info. Theory, vol. 56, no. 1, pp. 411–426, Jan 2010.

[3] C. Lin, V. V. Veeravalli, and S. P. Meyn, “A random search framework for convergence analysis of distributed beamforming with feedback,” IEEE Transactions on Information Theory, vol. 56, no. 12, Dec. 2010.

[4] C.-C. Chen, C.-S. Tseng, and C. Lin, “A general proof of convergence for adaptive distributed beamforming schemes,” in IEEE International Conference on Acoustics, Speech and Signal Processing (ICASSP), May 2011.

[5] C.-S. Tseng, C.-C. Chen, and C. Lin, “A bio-inspired robust adaptive random search algorithm for distributed beamforming,” in IEEE International Conference on Communications (ICC), Jun. 2011.

[6] I. Thibault, G. Corazza, and L. Deambrogio, “Random, deterministic, and hybrid algorithms for distributed beamforming,” in Advanced satellite multimedia systems conference and the 11th signal processing for space communications workshop, 2010.

[7] P. Fertl, A. Hottinen, and G. Matz, “Perturbation-based distributed beamforming for wireless relay networks,” in Global Telecommunications Conference, 2008. IEEE GLOBECOM 2008. IEEE, Nov 2008, pp. 1–5.

[8] —, “A multiplicative weight perturbation scheme for distributed beamforming in wireless relay networks with 1-bit feedback,” in Acoustics, Speech and Signal Processing, 2009. ICASSP 2009. IEEE International Conference on, April 2009, pp. 2625–2628.

[9] J. Denis, C.-S. Tseng, C.-W. Lee, C.-Y. Tsai, and C. Lin, “Deterministic bisection search algorithm for distributed sensor/relay networks,” in Global Communications Conference (GLOBECOM), 2012 IEEE, Dec 2012, pp. 4851–4855.