Quantum creation and inflationary universes: a critical appraisal

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Abstract

We contrast the possibility of inflation starting a) from the universe’s inception or b) from an earlier non-inflationary state. Neither case is ideal since a) assumes quantum mechanical reasoning is straightforwardly applicable to the early universe; while case b) requires that a singularity still be present. Further, in agreement with Vachaspati and Trodden [1] case b) can only solve the horizon problem if the non-inflationary phase has equation of state $\gamma < 4/3$, so excluding radiation or massless scalar field dominated cases. Other alternative models, such as the smooth branch change in the pre-big bang model, have related problems of requiring homogeneity over non-causally connected large scales.

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1 Introduction

The standard big bang model (SBB) model has a number of puzzles, particularly the so-called horizon and flatness problems, that are believed to require explanation and not just be accepted by fiat, see eg. [2,3]. However, for matter that obeys the strong energy condition, so including radiation or dust sources, there is by necessity an initial singularity, see for example [4]. On such a singular surface one can impose any suitable data one wants to give rise to our universe after it evolves into the future [5,6]. So strictly speaking the presence of a singularity obviates any talk of being unnatural since what is natural at a singular surface. A somewhat related argument has been made that at the singular surface everything can naturally becomes smooth by entropy arguments [7].

The more generally accepted solution to solving the problems of the SBB is to allow a matter source that violates the strong energy condition and which can give rise to a rapidly expanding regime, so called inflation e.g. [2,3]. One imagines that a small homogeneous initial patch can be taken and massively expanded to give our present universe. However, a lower bound on the size of this patch has been made by Vachaspati and Trodden (VT) and which could actually prevent the horizon problem being solved by inflation [1]. We will consider these claims in detail.

Throughout this article we will take so-called chaotic inflation as the prototype for inflation [2,3]. It can be caused by a scalar field with potential $V(\phi)$ sufficiently displaced from its minimum. However, one still needs to know the distribution for the initial conditions of this scalar field. One natural measure for a classical system is the canonical one [8]. But using this measure, and even after assuming an homogeneous scalar field, the probability for inflation is found to be ambiguous: an infinity of both inflationary and non-inflationary solutions [9]. Further, the predicted flatness becomes arbitrary if there is an upper bound in the potential $V(\phi)$, coming from perhaps a conformal anomaly correction or from a higher derivative $R^2$ term in the action [10].

One obvious limitation to these arguments is that the classical equations will fail as the quantum gravity epoch is encountered near the initial singularity. The initial conditions should then be supplied by a suitable quantum measure. One approach using the Wheeler-DeWitt (WDW) equation aims to achieve this, see eg. [2,3]. Although different results are obtained depending upon which boundary conditions are believed correct. The two most common
approaches are the “no boundary” of Hartle-Hawking [11] and Tunnelling ones [12]. There is some disagreement as to whether inflation is actually predicted with the Hartle-Hawking boundary condition [13,14], but the Tunnelling one seems more certain. We just note that the quantum measure can give a more certain prediction compared to a purely classical approach because the initial size of the universe is necessarily “small” when quantum mechanics is relevant. An initial “quantum of energy” can be distributed amongst the kinetic and potential energy of the scalar field. If the boundary conditions allow the potential to dominate then a rapid inflationary expansion ensues. Roughly speaking, large positive energy is produced by producing large volumes with negative gravitational energy, which can subsequently create large amounts of matter. The possibility of inflation is of course dependent on there being present a suitable scalar potential. This is put in by hand and not a prediction of the quantum cosmological theory. The actual source of this potential or more generally a reason for the initial violation of the strong energy condition needs to be understood from the actual quantum gravitational theory once known, perhaps eventually from string theory. Until this time inflation cannot be thought of as a true explanation of the universe’s existence but rather a possible mechanism for amplifying a quantum created universe that is unnaturally small. Since the creation mechanism is not known it could just be the case that this mechanism itself could give a reason for explaining the various puzzles and allowing a larger initial universe. Recall the present visible universe extrapolated back to the Planck time is a size \( \sim 10^{-3} \text{cm} \), a factor \( \sim 10^{30} \) larger than the Planck length \( \sim 10^{-33} \text{cm} \). In other words the creation mechanism is assumed to be driven by quantum effects that are more probable only over small scales, but since notions of time and energy are only properties within our universe it isn’t clear whether quantum mechanics constrains the initial creation in such a way.

2 Inflation starting from inception

Given, that some quantum process does lay down the initial conditions one can try to justify the likelihood of inflation being present. This is usually done at the Planck epoch where the semiclassical equations are first valid although as mentioned this should only be taken as a rough first guess. For example, in string theory recent developments suggest the Planck scale in higher dimensions could be at a lower energy scale [15]. Although from experimental constraints this cannot account entirely for the factor \( \sim 10^{30} \) that could alone rectify the unnaturalness of the SBB model.
In the case of chaotic inflation the Friedmann equation for a homogeneous
FRW universe is [2,3]

\[ H^2 + \frac{k}{a^2} = \dot{\phi}^2 + V(\phi) \]  

(1)

where we set Newton’s constant \( 8G/3 = 1 \). If the initial Planck energy is
distributed between the kinetic and potential terms of the scalar field then,

\[ \dot{\phi}^2 + V(\phi) \leq M_{pl}^4 \]  

(2)

with \( M_{pl} \) the Planck mass. This constraint is not too severe for the possibility of
obtaining a sufficient potential component since if the kinetic energy
initially dominates the scalar field only decays with a logarithmic dependence
\( \phi \propto \ln(t) \) [16-18].

There is also the question of whether the scalar field is homogeneous
initially. Such a possibility is described by the spatial gradient terms. The
question is not quite the same as the horizon problem which we will elaborate
on later. For inflation to occur the potential should dominate over such
terms, that is \( V(\phi) \gg (\nabla \phi)^2 \). Otherwise, such spatial gradients do not
drive inflationary expansion but rather an expansion \( a \propto t \), which is only on
the verge of power-law inflation see eg.[19]. Provided such gradient terms
are not too large they typically die away faster than the potential energy.
Numerical studies, suggest the inflationary regime will take over provided the
curvature is not too large that the universe could re-collapse before entering
an inflationary phase [19].

The requirement of sub-dominance of the spatial gradient term does at
first hint at a problem with causality. Consider a region over which the initial
\( \phi_i \) varies a unit amount to be \( L \). Then [3,19]

\[ V(\phi) > (\phi/L)^2 \Rightarrow L > \frac{\phi_i}{\sqrt[1/2]{V}} \]  

(3)

Now because in a flat universe \( H^2 = V(\phi) \) this constraint suggests that the
field \( \phi \) is smooth over scales \( L \gg \phi H^{-1} \). Since an initial field \( \phi_i \gg 1 \) is
required for sufficient inflation the smoothing scale is larger than the “causal
horizon” \( \sim H^{-1} \). For a quadratic potential \( V(\phi) = \lambda \phi^4 \) the initial field
can take values up to the Planck epoch values \( \sim \lambda^{-1/4} M_{pl} \). The value of \( \lambda \)
is extremely small \( \sim 10^{-12} \) in order to suppress quantum fluctuations and
give agreement with the COBE measurements [2,3]. This gives that the
corresponding smoothing scale or wavelength of the inhomogeneity should be \( \sim 10^4 H^{-1} \). However, since the initial conditions, of some finite starting value for \( H \), are simply being imposed by some Planck value constraint this requirement has nothing per se to do with causality. The properties of the universe are assumed to be given by some global creation process and the behaviour of light rays, that define the causality constraints, within such a domain are immaterial. The large smoothing scale does still require some justification from a theory of initial conditions. If the initial domain was instead assumed to be chaotic then a smoothing process would need to be postulated in order to make conditions suitable for inflation to occur. Such a mechanism would then, at least classically, be expected to work only over causal scales.

We note in passing that in assisted inflation the smoothing scale might easier be explained \[20\]. In such models a number of small field, which each individually would not produce inflation, might act in concert to do so. Although, in chaotic assisted inflationary models with \( \phi^n \) potentials they seem susceptible to cross coupling terms; they further assume the fundamental Planck scale coming from a higher dimensional theory is at a lower energy scale than the usual 4 dimensional one \[21\]. If the field was naturally smooth over length scales approximately like the higher dimensional Planck length it could naturally appear smooth over length scales much bigger than the 4-dimensional Planck length, when dimensional reduction occurs.

If the initial domain is not created uniformly then the homogeneous patch that is to inflate must be of sufficient size. Recall that the scalar field behaves as a negative pressure and any outside positive or zero pressure will wish to equalize the situation by rushing in. Assuming this equalization can proceed at the speed of light one finds that the homogeneous domain must be of a size greater than \( \sim 3H^{-1} \) \[19\]. One can speculate that other ways of isolating the interior from higher pressure and weakening this requirement are possible cf.\[22\] where large negative pressure of water in various natural systems can occur. In the inflationary case this could correspond to a highly non-trivially topological structure where a negative pressure might be isolated from its surroundings. Once such suitable domains occur then inflation will proceed. Because of a combination of the fluctuations that are generated during inflation and the finite horizon size \( \sim H^{-1} \) in De Sitter space inflation never stops entirely once started, provided a requirement on the size of the scalar field \( \phi > \phi_* \) is achieved \[2\]. If the initial domain does not have a sufficiently large
scalar field value one can hope that a quantum “instanton” effect can produce a domain with sufficiently large scalar field provided there is sufficient starting volume that the probability is not infinitesimally small [2,23]. One can try and make inflation eternal into the past but geodesic completeness is not possible without violating the more extreme weak energy condition [24]. A singularity is therefore present if one tries to extend an inflationary phase backwards indefinitely. Without some creation mechanism to give the initial inflationary conditions the model would still suffer the same initial singularity problem as in the usual SBB model. The quantum fluctuations also drive the average field into Planck energy densities where the theory is unknown, although one can argue that inflation is still future eternal even if an absorbing boundary is imposed at the Planck scale to remove such singular states[23]. One worrying aspect of inflation is that a non-minimal coupling i.e. $\xi R \phi^2$, can, apart from preventing inflation entirely for larger $\xi$, change the renormalization schemes that are required to produce a linear growth in $<\phi^2>$ that is required to drive eternal inflation cf. [25]. The large fluctuations might also cause black hole or other defect formation, which will reduce the surface gravity and so possibly regulate the eternal inflationary mechanism cf.[26,27] .

### 3 Quantum creation of initial state

How realistic is the concept of quantum creation? We would just like to mention a few aspects of this problem that often seem lost among all the technicalities in the literature.

In usual quantum mechanics one is given the relevant potential, or action in the path integral formulism, prior to doing a calculation. The situation is more confused in quantum cosmology since the potential, determined partly by the matter components, is supposedly also being predicted. One might wonder if this is at all sensible, or at least that the notion of “creation” is overstated since the relevant quantities are assumed to exist prior to starting the calculation. The formulism more accurately tries to obtain which quantities are most dominant amongst all the allowed possibilities: “quantum determination of the cosmological state” would be a more apt description.

With this more limited aim let us consider further the “quantum tunneling from nothing” and Hartle-Hawking (HH) schemes. They both depend crucially on the WDW potential, given, in the simplest models by [2,3]

$$U = ka^2 - V(\phi)a^4$$  \hspace{1cm} (4)
being positive to produce a forbidden or Euclidean region. But this requires a number of assumptions:

i) The curvature $k$ must be closed ($k = 1$) to produce a forbidden region ($U > 0$) for a small. This region can then be tunnelling through, or oppositely “anti-tunnelled” in the HH case. But this naive treatment of the curvature is precisely the quantity that is likely to be modified as general relativity is superceded by quantum gravitational effects. Because with $V(\phi)$ roughly constant, the strong energy condition is being violated curvature dominates at small scales unlike for usual matter (eg. radiation) where the curvature only becomes relevant at late times when the scale factor is large. One might expect curvature to wildly vary on small scales and the crucial $ka^2$ term to be inadequate for its full description.

In flat or open geometries there is no forbidden sector, but the model can still be quantized although the relevant boundary conditions to impose are less well motivated [28]. Assuming open compact geometries one can argue that inflation is likely by imposing analogous “outgoing like” boundary conditions[28]. There is a further possibility of removing forbidden regions by use of signature change [29]. Because the lapse function is arbitrary and not determined by the Einstein field equations it can change sign to prevent the forbidden region being classically disallowed. With such a signature change variable present, one can further quantize the model, although there seems further scope for different boundary conditions [30-32]. These extra possibilities seems symptomatic of what could occur as the Planck scale is approached and curvature and time become less structured cf.[33].

ii) Matter that satisfies the strong energy condition also has to be removed to create a Euclidean region at small size. This is done indirectly by imposing boundary conditions that disallow such matter to dominate as $a \rightarrow 0$. As emphasized by ref.[34] quantum uncertainty should at least introduce such a matter source of “zero point” size. But any such matter eg. radiation, stiff matter, now dominates over the curvature or $V(\phi)$ term. For stiff matter or equivalently the kinetic term of a scalar field a singularity represented by a wildly oscillating wave function is produced. Even if the singularity is absent or suitably regularized (possibly by using a separation constant-see eg.[28]) the model represents a Lorentzian model similar again to the case with non-closed curvature present. The constants that determine the matter content of this universe are arbitrary, just as the size of an hy-
drogen atom is determined by the potential and not the quantization per se. In these cases the universe can have sufficient size that inflation would be unnecessary and depending upon the initial perturbation spectrum could give a suitable model for our universe.

iii) When strong energy satisfying matter is present eg. radiation, one typically needs to impose the boundary condition \( \Psi(a = 0) = 0 \) [34]. But this certainly does not explain why the universe comes into existence and indeed is not consistent if one considers zero scale factor to be the relevant starting value. Likewise the inclusion of a forbidden region isolating \( a = 0 \) from the rest of the universe does not explain why the zero scale factor should have special status and not itself require explanation. Quantum tunneling is from one well defined state to another: it is not a magical process that explains something from nothing. The actual cosmological problem is further compounded by the fact the potential itself is being predicted.

iv) One can even question whether the aim of obtaining the initial value of \( V(\phi) \), to determine the ensuing amount of inflation, is achievable. Consider the analogous problem of field emission or enhancing the alpha particle decay of a nucleus by applying an electric field \( E \) - see eg.[36]. The relevant potential is

\[
U = W - Ea
\]  

where \( W \) is the work function of the metal in the field emission problem, but which for our purpose is simply a constant. Note how in the quantum cosmology case the potential \( V(\phi) \) is analogous to the electric field (ignoring some unimportant discrepancy in the factors of \( a \)). Now in the field emission problem, one asks: given an applied electric field what is the probability of electron emission? The answer is proportional to \( \sim \exp(-1/E) \), so a larger electric field enhances the emission. But in the quantum cosmology problem we are trying to ask: given tunneling occurred (a universe exists) what is the value of the potential \( V(\phi) \)? In the electron case this would be indeterminate for a single event, but given there is a reservoir of electrons that all eventually tunnel this problem is still solvable, by repeated observation, given that every other variable of the problem is known. The longer the observation the more certain the applied field can known. But for the alpha decay which is also a single one off event, there is an ambiguity in trying to
know the applied field if you don’t know how long the system has already remained undecayed. And the quantum cosmology case is, at least as complex, and analogous to this problem since the time being absent means there is no time limit on when the decay (or initial creation) should proceed. One might try to argue that if $V(\phi)$ is initially too small the Euclidean region would extend over large scale factors, and similarly for negative $V(\phi)$ the universe is entirely Euclidean. Indeed, the HH scheme apparently favours the Euclidean region extending to large scales, while the “tunneling from nothing” prefers it confined to small scales [37]. Without use of anthropic arguments one might have to postulate a “source of universes” that can come into existence, analogous to the electrons in the metal, so that “on average” $V(\phi)$ can be assumed to be large. But because time is not even existing exterior to the universes even this sort of modification is interpretatively suspect, since how would the “reservoir” be defined. Requiring a surfeit of universes is also rather extravagant. See also ref.[38] in this regard, who try to suggest why universes are not still being continuously created around us, but this rapidly starts getting into the quagmire of different interpretations of quantum mechanics. The points I have made are rather general and mostly independent of more abstruse arguments about which interpretation of quantum mechanics is valid, although ultimately this could have a bearing. Working with the path integral formulism would raise equivalent problems and concerns.

In summary, the arguments of the “no boundary” or “tunneling from nothing” proposal, which respectively are claimed to give distributions $\sim \exp(1/V(\phi))$ and $\sim \exp(-1/V(\phi))$ [2,3] for the initial $V(\phi)$, are at best provisional, with the usual quantum analogies and definitions of probability applied in a cavalier manner. One at least hopes that quantum mechanics can still be reasonably applied and is actually valid during the early epoch.

Returning, again to some general notions of inflation. It is often supposed that the scale invariant fluctuations that are generated during inflation and the subsequent Doppler peaks in the CMB spectrum are a validation of inflation. But as emphasized in ref.[6] such a signature does not validate inflation since other models with fluctuations created early can give such behaviour [6], and of sufficient size [39] . An example of an alternate mechanism for fluctuations is if a state of “self-organized criticality” could occur during the early universe. So giving fluctuations over scales of many magnitudes [40]. Such processes, which are believed a general feature of nature [41], could occur during phase transitions at the Planck scale, hopefully without requiring
Because of our limited knowledge of quantum creation it is difficult to know whether a subsequent inflationary stage will be necessary. Instead of treating inflation as an additional phase to correct the failings of the initial creation process one might just as well, with our present understanding, include it within the initial creation scheme. Such a scheme has the advantage of regularizing singularities, although it still depends on the type of matter present and factor ordering terms. But where the singularities are present is precisely when the quantum gravity corrections are most important and the WDW equation as presently formulated is unlikely to be valid. In summary, the initial state resulting from a creation scheme is rather provisional. Whether it requires a subsequent inflationary stage is actually unknown until this state is better defined.

4 Inflation from previously non-inflationary conditions

What about the naturalness of inflation in models that don’t inflate initially from their inception. Singularities seem a general consequence of producing conditions that give inflationary behaviour. The idea of producing a “universe in the lab” was required to expand so rapidly to avoid re-collapse that a singularity would be present [42]. This can be seen clearer by noting that in a FRW universe regions of size bigger than the so-called apparent horizon $\sim \rho^{-1/2}$ have a necessary singularity -see page 353 in ref.[4] where such a quantity is called the Schwarzschild length of matter density $\rho$. But this size, $\sim H^{-1}$ for the flat $k = 0$ case, is the minimum required to isolate an inflationary patch from its surroundings for sufficient time to start inflating. In fact requiring the initial patch size to be larger than the apparent horizon size has recently been claimed by VT to be a major problem in setting up inflationary conditions within a patch. However, depending on the matter source it need not strictly violate causality which is rather determined by the particle horizon: the distance light travels from the beginning of the universe. The large initial patch size means rather that a singularity is present.

Let us first recall the nature of the horizon problem. It occurs because the the particle horizon size, defined as

$$ r = c \int_0^t \frac{dt}{a(t)} $$

(6)

is finite, see eg.[2,3]. The horizon proper distance $R$ is this quantity $r$ multiplied by the scale factor i.e. $R = a \ast r$. For any strong-energy satisfying
matter source this quantity $R$ grows linearly with time. But in SSB cosmology the rate of change of the scale factor, given by $a \sim t^p$ and $1/3 < p < 1$, grows increasingly rapidly as $t \to 0$. The horizon cannot keep pace with the scale factor ‘velocity’ $\dot{a} \sim 1/t^{1-p}$. But note that this is only impossible for times below unity $0 < t < 1$. If the horizon problem was solved, by some (quantum) process, at the Planck time $t_{pl} = t = 1$ it would remain permanently solved during the ensuing evolution [43]. Note also that in models that inflate from their inception the usual space-like singularity of the FRW universe becomes null like when $p > 1$ - see eg.[44]. The idea of inflation is to take an initial domain of size less than the corresponding particle horizon size and allow it to expand greatly to encompass our universe. Let us see how this requirement can be constrained. Fortunately, most of the relevant quantities have already been obtained in work on the holography conjecture and can readily be applied in this context [45,46]. For this purpose a useful form of the FRW metric is

$$ds^2 = a^2(\eta) \left(-d\eta^2 + d\chi^2 + f^2(\chi)d\Omega^2\right)$$

(7)

where $f(\chi) = \sinh \chi \ , \chi , \sin \chi \ ,\text{ corresponding to open, flat and closed universes respectively.}$ We can define a number of important quantities. The \textit{Hubble horizon} is defined by

$$r_H = H^{-1}$$

(8)

The \textit{particle horizon}, or the distance travelled by light from the initial moment of the universe, is simply,

$$\chi_{PH} = \eta$$

(9)

for this metric. The \textit{apparent horizon} is given by [45]

$$\chi_{AH} = \frac{1}{\sqrt{H^2 + k/a^2}} \Rightarrow \frac{1}{\sqrt{p}}$$

(10)

Roughly speaking light rays beyond the apparent horizon are seen to move away from the origin, a so-called anti-trapped behaviour. Note that in the flat case $k = 0$ the apparent horizon and Hubble horizon coincide.

Vachaspati and Trodden [1] have argued that the initial inflationary patch must have sufficient size $x$ that it reaches the anti-trapped surface i.e. $x > r\chi_{AH}$. Otherwise the weak energy condition is violated for light rays that
could otherwise enter the inflating region from normal or trapped regions. For a perfect fluid with equation of state \( p = (\gamma - 1)\rho \) the apparent horizon has the following time dependence \([45, 46]\)

\[
\chi_{AH} = \frac{d\gamma - 2}{2}\eta
\]  

with \( d \) the number of space dimension i.e. 3 in the usual 4 dimensional space-time case. However the causal particle horizon has a different time dependence simply \( \chi_{PH} = \eta \) so the condition

\[
\chi_{AH} < x < \chi_{PH}
\]  

can be satisfied for

\[
\frac{d\gamma - 2}{2} < 1 \quad d=3 \quad \gamma < 4/3
\]  

This does exclude the case of radiation (\( \gamma = 4/3 \)) or stiffer equations of state. But if \( \gamma \) was gradually reducing before inflation occurred, recall the strong energy condition is violated as \( \gamma \) falls below 2/3, this causal constraint can be satisfied. The condition can be thought of as saying the effective value of \( \gamma \) cannot switch suddenly but rather must fall below 4/3 for sufficient time to allow the causal or particle horizon to be larger than the apparent horizon. This result is independent of whether curvature is present. Although in the open and flat cases one might argue that the presence of an infinite spacial section gives a more likely chance that inflationary conditions would occur to give an inflationary patch. In the closed case only during the expansion phase is an anti-trapped surface present- see Fig.(4) in ref.\([46]\). This means that producing inflation to avoid an impending “big crunch” singularity during a collapsing phase will violate not only the strong energy condition. Now it is true that needing \( x > \chi_{AH} \) is difficult to justify in terms of particle physics processes, but if this patch could be smaller than \( \chi_{AH} \) one could avoid the singularity in a FRW universe since the matter would be insufficient to converge the light rays into the past. See chapter 10 in ref.\([4]\) for a proof of this argument. So allowing an initial domain of size \( x < \chi_{AH} \) to inflate, would have allowed singularities to be expunged from this cosmology: the result that this cannot be done without violating the weak energy condition is therefore consistent with the studies of eternal inflation that singularities have to be present when the model is continued into the past \([24]\).
There are some alternative metrics with non-singular solutions, but like Minkowski space they don’t have anti-trapped regions [44,47]. Achieving inflation is such spaces would likewise require the violation of the weak-energy condition.

5 Higher dimensions and Brane inflation

We can make some remarks about specific inflationary models. For example in higher dimensional theories it will be more difficult to satisfy the constraint. In 5 dimensions the constraint becomes $\gamma < 1$ so now excluding the pressureless dust equation of state. In 7 dimensions or more the strong-energy condition has to be violated from the start to ensure the constraint is met.

In 5 dimensional Brane models the Hubble parameter is further modified, typically such that $H = \rho$, on the 4 dimensional Brane corresponding to our universe [48]. Matter is effectively stiffened and the particle horizon can extend beyond the apparent horizon now only for $\gamma < 2/3$: which would violates the strong energy condition in usual 4-dimensional general relativity. Indeed it is known that the modified Friedmann equation has a more stringent requirement, that $\gamma < 1/3$, for inflationary behaviour [48]. But it also appears impossible to solve the horizon problem without matter that is initially inflationary anyway. Unless such matter is present in string theory, the inflationary conditions would need to be understood by non-causal initial conditions or weak energy violating quantum mechanisms. Otherwise one again would have to postulate inflation from its initial inception.

There is another approach where the presence of extra branes might allow a “short circuit” of space-like separations compared to when only a single brane is present[49]. This is rather similar to earlier suggestions that wormholes could alleviate the horizon problem during an early quantum gravitational phase [50]. For such a scheme to work the branes have to be curved and aligned in just the right fortuitous manner.

6 Pre-Big Bang models

The VT requirement is also relevant for the pre-big bang scenario [51]. In this model the universe starts at time minus infinity and expands towards a singularity at time zero, which will corresponds to the usual Big bang beginning of the universe. I have earlier criticized whether this is a true inflationary behaviour but will ignore this aspect in the present discussion [52].

The expansion is driven by the dilaton present in string theory. In the
Einstein frame this dilaton is simply a massless scalar field or equivalently a stiff fluid driven model. A mechanism is required to end the contraction phase and branch now into a suitably expanding behaviour. One assumes that higher order corrections become increasingly important as the universe becomes increasingly hotter and turbulent. Because there are no anti-trapped regions in a collapsing universe one cannot enter an inflationary phase without violating the VT constraint.

Even for a patch just locally trying to avoid the singularity and enter a non-inflationary expansion there is a similar constraint. As the universe collapses one needs to ensure that the particle horizon (starting from the time new string states are being produced) keeps the universe causally connected. In a collapsing model the usual particle horizon of an expanding stiff fluid is converted to an event horizon as the singularity is approached. But because the matter is constantly being modified by new processes becoming important an effective particle horizon is also present. To avoid the singularity and create a smooth post big bang phase one needs to set up uniform conditions over a length scale larger than the trapped surface. Only for perfect fluids with $\gamma < 4/3$ can the event horizon remain larger than the size of the trapped surface and one might justify an homogeneous distribution of strong energy violating material to avoid the impending singularity. In the dilaton driven case it seem that the VT constraint will mean that the weak energy condition is necessarily violated. Similar conclusions have been obtained in specific models [53] but the requirement of requiring weak energy condition violation is more transparent when using the VT constraint. One might hope that the stiff equation of state is modified to that below $\gamma = 4/3$ before one tries to implement the branch change in order to require only the strong energy condition to be violated. There is another potential problem even if this matter can be distributed. The measure for such branch changes or bounces is likely to depend on any upper bound in some effective scalar potential and singular solutions are still likely to be more probable cf.[10]. Such criticism could also be levelled at the “evolution of universes” scheme of Smolin [54], where every collapsing black hole is envisioned as bouncing into to another universe.

To summarize the difficulty, in the pre-big bang phase one is sailing too close to the singularity where processes occur increasingly rapidly. Near such a surface, horizon problems again result since one wishes to influence a sufficiently large patch to avoid the singularity and keep the resulting post
big bang universe smooth and coherent. The situation required is rather opposite to the usual ending of inflation where parts of the universe often remain in the inflationary state. In this case none of the universe can be allowed to remain pre-big bang or inflationary as a space like singularity would result. Whether, one can justify such a simultaneous exit therefore depends on knowing what quantum gravitational effects will come into play and what energy conditions can be justifiably violated. Otherwise the pre-big bang model simply brushes its problems under the branch change carpet.

7 Variable constant models

Another possible solution to the horizon problem is to postulate that the various constants particularly $c$ could take different values during the early universe [55-57]. This alone is not too helpful since a space-like singularity cannot be crossed by any finite value of $c$ and a higher $c$ just means one has to go further back in time to see an equivalent horizon problem [58]. There is also a causality problem, of sorts, as to why $c$ can change simultaneously over the whole universe and constantly stay equal throughout, once the value of $c$ has started to reduce and causal contact lost. This constraint is similar to the VT one for inflation in that the behaviour for $c$ really has to be pre-programed in the universe from its conception. Changing such constants also tends to suppress any quantum gravitational epoch at the beginning of the universe [59]. However, this quantum epoch can surface at a later time, which must be pushed sufficiently far into the future [58]. In this regard these model have some similarity with the pre-big bang phase which also start in a classical state and tend towards a quantum gravitational singular region. This makes such models more difficult to conceive of by quantum creation schemes, but it must be admitted that quantization alone does not explain why creation occurs. Neither, does quantum cosmology explain why the various constants take their actual values, or why even the various (quantum) laws of nature are applicable to the event.

8 Conclusions

We have seen how the constraint of VT for the initial patch size of an inflationary domain is actually the requirement that a singularity be present. The horizon problem can still be solved by inflation for matter sources that initially have $\gamma < 4/3$ before inflation occurs. For stiffer equations of state the requirement that the singularity be present means the particle horizon size is too small to keep the created inflationary domain within causal contact. Although, in practice the particle horizon size is likely to be modified by
absorption and other quantum effects at high energies making the universe possibly opaque to light rays. The horizon problem could therefore be more difficult to allay than suggested by this classical analysis. Even having causal contact does not alone explain why the scalar field is homogeneous across the required domain size. This is, in some sense, a well ordered state, with low entropy that requires further explanation. The fact that the radiation case is discounted means the quantum vacuum state is necessarily unstable near the initial singularity [60]. The so-called Scalar Ricci curvature hypothesis (SRCH) cannot be implemented. It would be interesting to see if inflationary conditions are still compatible with the Weyl-Curvature hypothesis cf.[5,60].

The fact that inflation goes hand in hand with singularities is rather worrying. If singularities are anyway present how seriously should one take the inflationary paradigm. It would appear no better than the usual SBB model which also suffers from initial singularities so always emerging from unknown conditions. Even the violation of the strong energy condition is insufficient to remove the singularity. One can postulate that the weak energy condition be violated to try and obviate this limitation, but this is rather drastic. It is likely to be very unstable, for example to perturbations and anisotropy increasing. Quantum gravitational effects might play a role since the proof that inflation has initial singularities depends on inflationary domains that finish inflating never again undergoing inflationary behaviour. If space could be recycled then the proofs of no past-eternal inflation might be obviated cf.[61]. If the Supernova data suggesting an accelerating universe[62]is confirmed then some possibility of inflation being re-entered could be argued. It would certainly prove that strong energy condition violating matter exists and that gravity can be repulsive over large scales. Of course, even having an eternal model into the past does not entirely explain its existence. There is also the related suggestion that the presence of closed timelike curves can give an explanation of the universe [34]. But time is an internal property of the universe and doesn’t help explain the coming into existence of the cosmological model.

The other alternative to achieve inflation is to postulate that the inflationary stage is not preceded by an earlier phase but is started suddenly by some “quantum creation event”. As explained earlier, we are rather trying to use quantum reasoning to determine conditions assuming certain cosmological models are existing in the first place. The event is rather analogous to a “scanning tunneling microscope (STM)”-see eg.[36], with the scalar
potential $V(\phi)$ playing the role of the external potential applied to the “tip of the probe”. This is rather uncertain extrapolation of quantum mechanics to a conceptually different type of problem. On a more practical level, even the notion that quantum tunneling takes place is dependent on the curvature dominating over matter at small scale factors. Firstly we don’t know how curvature should be treated at quantum gravitational scales and secondly we have no reason to expect that matter should not be the dominant contribution. Although boundary conditions impose such restrictions on what terms in the Wheeler DeWitt equation can dominate they arbitrarily remove generality from the approach. Unlike the (STM) example, in the cosmology case one is trying to determine the “potential on the probe” in order to predict the amount of inflation and it appears a “one shot” event. To take the analogy further: we are trying to find the potential while the probe itself is coming into existence. This, at the very least, suffers from interpretational problems for making predictions.

Other related approaches to inflation such as the ending of the pre-big bang phase or the variation of the speed of light have related problems of causality. They require coherence lengths that are larger than particle horizon lengths and so are not possible to justify.

In conclusion, and in agreement with VT, inflation as a means of resolving the horizon problem is difficult to justify: although the possibility is not entirely excluded for $\gamma < 4/3$. The consequence of having inflationary behaviour is that singularities are present. A complete justification of having inflationary cosmological models will therefore require such singularities to be removed or regularized in some way. Either by quantum gravitational processes that violate the weak energy condition, or circumscribed by quantum reasoning bypassing such singular behaviour. This still would not explain creation but would allow us to take one step closer to formulating the properties of the event that initially sparked the universe.

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