Mathematical modeling of steel fiber concrete under dynamic impact

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Abstract. This paper introduces a continuum mechanics mathematical model that describes the processes of deformation and destruction of steel-fiber-concrete under a shock wave impact. A computer modeling method was applied to study the processes of shock wave impact of a steel cylindrical rod and concrete and steel fiber concrete plates. The impact speeds were within 100-500 m/s.

1. Introduction
This paper presents a study that investigates the behavior of steel fiber concrete (SFC) under a shock wave impact. Physical mechanical properties of fiber concrete under static and dynamic loading are studied by a number of scientists [1, 2]. Except for results obtained for regular fiber concretes [3] also was considered operation of high-performance powder concrete reinforced with steel fiber under static and dynamic loading. In the present work SFC is modeled as a homogeneous two-phase steel-concrete mixture with an initial density of \( \rho_0 = \rho_{lc} + \rho_{ls} \). Where \( \rho_{lc}, \rho_{ls}, \rho_{ls} \) are initial volume concentrations and densities of steel and concrete. A dynamic destruction of SFC is considered as a process of growth and fusion of micro defects under a stress that emerges during the impact process.

A special volume of a porous media \( v \) is considered a sum of a special volume of material matrix \( v_{ml} \), a special volume of the pores \( v_p \) and a special volume \( v_t \) that emerges due to the opening of cracks: \( v = v_m + v_p + v_t \). The porosity of material can be characterized as relative volume of hollows \( \xi = \xi_p + \xi_t \), or a parameter \( \alpha = v/v_{ml} \), that are dependent as \( \alpha = 1/(1 - \xi) \). Where \( \xi_p = v_p/v \), \( \xi_t = v_t/v \) – relative volumes of pores and cracks.

2. Mathematical modeling of a collision between a cylindrical rod and a steel-fiber-concrete plate
The equation system that describes the move of a porous elastic-plastic medium is presented below [4-6]:

\[
\begin{align*}
\frac{d}{dt} \int_\Omega \rho dV &= 0; & \frac{d}{dt} \int_\Omega \rho u dV &= \int_s \eta \cdot \sigma dS; & \frac{d}{dt} \int_\Omega \rho EdV &= \int_s \eta \cdot \sigma u dS; & e &= \frac{s}{2\mu} + \lambda s; \\
\end{align*}
\]

\[
\begin{align*}
\sigma : s &= \frac{2}{3} \sigma_0^2; & p &= \frac{1}{\alpha} \left[ \frac{c_0^2 \rho_0 (1 - \gamma \eta / 2 \eta)^2}{(1 - s \eta)^2} + \rho_0 \gamma \delta \right].
\end{align*}
\]
where \( t \) – time; \( V \) – integration volume; \( S \) – surface; \( n \) – outward normal; \( \rho \) – density; \( \sigma = -p\mathbf{g} + s \) – stress tensor; \( s \) – its deviator; \( p \) – pressure; \( \mathbf{g} \) – metric tensor; \( \mathbf{u} \) – velocity vector; \( E = \varepsilon + \mathbf{u} \cdot \mathbf{u}/2 \) – specific total energy; \( \varepsilon \) – specific internal energy; \( \varepsilon = \mathbf{d} \cdot (\mathbf{d} \cdot \mathbf{g}) \mathbf{g}/3 \) – deformation strain deviator; \( \mathbf{d} = (\nabla \mathbf{u} + \nabla \mathbf{u}^T)/2 \) – strain-rate tensor; \( \mathbf{s}^\prime = \mathbf{s} + \mathbf{w} \cdot \mathbf{w} - \mathbf{w} \cdot \mathbf{w} \) – derivative of deviatoric stress (Jaumann-Noll); \( \mu = \mu_0 (1 - \varepsilon_0)/[1 - (6\rho_0 c_0^2 + 12\mu_0 c_0^4)/(9\rho_0 c_0^2 + 8\mu_0)] \), \( \sigma_T = Y_0/\alpha \) – effective shear modulus and yield strength; \( \omega = (\nabla \mathbf{u} - \nabla \mathbf{u})/2 \) – vortex tensor; \( \rho_0, c_0, \mu_0, Y_0, s_0 \) – material matrix constants; \( \eta = 1 - \rho_0 \nu / \alpha \). The parameter \( \lambda \) is excluded by yield condition.

The matrix coefficients \( c_0 \) and \( s_0 \) of a linear dependency of a shock wave velocity \( D \) form a mass velocity \( u \) (\( D = c_0 + s_0 u \)) are defined by the shock adiabats of the mixture’s components \( D_i = c_{0i} + s_{0i} u \) (\( i = 1, 2 \)). A shock adiabat for the variable \( v_m \), \( p_m \) is presented below.

\[
v_m(p_m) = \sum_{i=1}^{2} m_i \left[ v_{0i} - \frac{1}{p_m} \left( \frac{c_0}{s_0} \left( \frac{s_{0i} p_m + 1}{\rho_0 c_{0i}} - \frac{1}{2} \right) \right) \right]^{\gamma^2}
\]

(2)

If the following relationship to the shock wave is applied:

\[
D = \sqrt{\frac{p_m}{\nu_0 - \nu_m(p_m)}}; \quad u = \sqrt{p_m(\nu_0 - \nu_m(p_m))},
\]

(3)

one can build a dependency of shock wave velocity from the mass velocity to derive the coefficients \( c_0 \) and \( s_0 \).

The Gruneisen coefficient \( \gamma_0 \) for the mixture is defined as its components’ Gruneisen coefficients \( \gamma_{0i} \):

\[
\frac{\nu_0}{\gamma_0} = \sum_{i=1}^{2} m_i \frac{v_{0i}}{\gamma_{0i}}
\]

(4)

The shear modulus \( \mu_0 \) and the yield strength \( Y_0 \) are defined as:

\[
\mu_0 = 1/\left( \frac{V_1}{\mu_{01}} + \frac{V_2}{\mu_{02}} \right); \quad Y_0 = m_1 Y_1 + m_2 Y_2,
\]

(5)

where \( m_i = V_i P_{0i} \) - mass concentrations of steel (\( i = 1 \)) and concrete (\( i = 2 \)) in SFC; \( \mu_{0i}, Y_i (i = 1, 2) \) – shear modules and yield strengths for the mixture components.

Equations that describe the change of the parameter \( \alpha \) under tension and compression are required to lock the system (1). The equation that defines the parameter \( \alpha \) under an elastic deformation of SFC is presented below:

\[
\rho_0 \gamma_0 \varepsilon + \frac{c_0^2 \rho_0 (1 - \gamma_0 \eta)/2 \eta}{(1 - s_0 \eta)^2} + \frac{3 \mu_0 (\alpha - \alpha_0)}{8(1 - \nu) N_0 R \alpha \alpha_0} = 0
\]

(6)

In deriving the equation (6) it was considered that no new cracks appeared during the loading process. The deformation was caused only by the growth of initial crack with a characteristic dimension \( R \):

\[
\dot{R}/R = F_1 + F_2,
\]

(7)
where: \( F_1 = (\alpha s_i - s_0) / \eta_i \) at \( \alpha s_i > s_0 \) and \( F_1 = 0 \) at \( \alpha s_i \leq s_0 \),

\[
F_2 = (\eta s - p_i) / \eta_i \text{ at } p < 0, \| \eta s \| > p_i \text{ and } F_2 = 0 \text{ and } p \geq 0, \| \eta s \| \leq p_i ;
\]

\[
p_s = p_0 (1 - R / R_s) ; \ s_s = s_0 (1 - R / R_s) ; \ R_s = \beta / \sqrt[3]{N_0} ; \ s_0, p_0, \eta_1, \eta_2, \beta \ - \text{material constants} ; \ N_0 \ - \text{number of crack is the volume unite} ; \ \nu \ - \text{Poisson coefficient} ; \ \alpha_0 \ - \text{initial porosity of SFC}.
\]

It is considered that a fusion of micro cracks in SFC starts when the specific size reaches the critical value \( R_s = \beta / \sqrt[3]{N_0} \) at a constant concentration \( N_0 \).

The process of fragmentation of the damaged by cracks material is described within a porous elastic-plastic medium model. The system (1) is locked by the equations that relate pressure \( p \) and porosity \( \alpha \) under compression.

\[
(p \geq \frac{2}{3} \sigma_T \ln(\frac{\alpha}{\alpha - 1}))
\]

\[
\rho_0 \varepsilon_0 + \frac{c_s^2}{2} \rho_0 \left( \frac{1 - \gamma_0 \eta / 2}{1 - s_0 \eta} \right)^2 - \frac{2}{3} \sigma_T \ln(\frac{\alpha}{\alpha - 1}) = 0 \tag{8}
\]

and at decompression \( (p \leq \alpha_s \ln(\frac{\alpha}{\alpha - 1})) \)

\[
\rho_0 \varepsilon_0 + \frac{c_s^2}{2} \rho_0 \left( \frac{1 - \gamma_0 \eta / 2}{1 - s_0 \eta} \right)^2 + \alpha_s \ln(\frac{\alpha}{\alpha - 1}) = 0 \tag{9}
\]

The fragmentation of the damaged by cracks material under tension load happens the relative volume of the hollows reaches the critical value \( \xi_s = \frac{\alpha_s - 1}{\alpha_s} \). If the damaged material is being loaded by a compression impact then the fragmentation criterion will be the intensity value of the plastic deformations \( \varepsilon_s \) :

\[
\varepsilon_s = \frac{\sqrt{3}}{3} \sqrt[6]{\sqrt{T_2} - T_1^2}, \tag{10}
\]

where \( \alpha_s \) – model parameter; \( T_1 \) and \( T_2 \) – the first and the second invariants of the deformation tensor.

The destructed material is modeled as a granulated medium that is tolerant to compression and not tolerant to a stretching load. In [3] design-experiment method was used to investigate the process of the impact interaction of steel cylindrical hitters (diameter \( d_0 = 7.65 \text{ mm} \), length = 23 mm) with a concrete plate (width = 200 mm) at 130÷700 m/s. The concrete consisted of one part of cement 400, 2 parts of fine send. The production time = 30 days. The diameter (D) of the facial break-away in the concrete plate and the depth of the crate (L) were measured during the experiment. The experimental results have matched the results of mathematical modelling. The presented study focuses on the modeling of the impact interaction of the above mentioned hitter and a SFC plate. The results of the mathematical modeling of the impact interaction are shown in Table 1 and Figure 1.

**Table 1.** Calculated penetration depth.

| \( V, \text{ m/c} \) | \( L_1/d_0 \) | \( L_2/d_0 \) | \( R, \% \) | \( V, \text{ m/c} \) | \( L_1/d_0 \) | \( L_2/d_0 \) | \( R, \% \) |
|---|---|---|---|---|---|---|---|
| 100 | 0,15 | - | - | 150 | 0,89 | 0,16 | -82 |
| 200 | 1,65 | 0,67 | -59 | 244 | 2,25 | 1,31 | -42 |
| 300 | 2,76 | 2,06 | -25 | 350 | 3,2 | 2,54 | -21 |
Where \( V \) is the velocity of the impact; \( L_1/d_0, L_2/d_0 \) – relative crater depth in concrete and SFC plates; \( R \) – difference between craters’ depth. Figure 1 presents the impact interaction of the cylindrical hitter with concrete and SFC plates at 350 m/s.

![Figure 1. Impact interaction of the cylindrical hitter with concrete (a) and SFC (b) plates at 350 m/s.](image)

As shown in Table 1 the penetration depth of the steel cylinder in the concrete plate is deeper than in the SFC plate. The process of the crater origination in the concrete plate starts at about 90 m/s. Meanwhile the same process in the SFC plate starts at 135 m/s. The crater’s depth in the concrete plate is 0.15 \( d_0 \) at 100 m/s. Meanwhile the impact at the same velocity the SFC plate causes 0.16 \( d_0 \) crater. The difference between two craters is 82%. The difference decreases with an impact’s velocity growth. At 350 m/s the difference is 21%. In [6] a design-experiment method was used to investigate the process of the impact interaction of the above mentioned hitter with concrete and ferroconcrete plates at 300÷750 m/s. A hitter perforates 24 mm concrete plate at 495 m/s. Facial and rear break-away originate during the experiment. Their diameters are \( d_f = 3.0 \cdot d_0 \); \( d_r = 5.2 \cdot d_0 \). The hitter’s velocity is \( U_k = 327 \) m/s after it has left the plate.

Figure 2 presents the impact perforation of concrete and SFC plates by a cylindrical hitter at 495 m/s. A facial break-away \( d_f/d_0 = 3.06 \) and a rear break-away \( d_r/d_0 = 2.53 \) have originated in the plate. The velocity of the hitter after it has left the plate is 310 m/s. The difference between the modeling and experimental velocity measurements is 5%. One can observe the difference between rear break-away (30%). A relatively large difference can be explained by the limitations of the model where the maximum value of the barrier is 5.8 \( d_0 \) meanwhile it could reach up to 7.8 \( d_0 \) during the experiment. A facial break-away (\( d_f = 1.8 \cdot d_0 \)) originates when a SFC plate is being perforated. A rear break-away diameter is \( d_r = 1.47 \cdot d_0 \). The velocity of the hitter after it has left a SFC plate is 285 m/s. This is 8 % lower in comparison to a concrete plate.
Figure 2. Configuration of a steel hitter and a concrete (a) and SFC (b) plates at 495 m/s in t= 130 micro seconds.

Conclusions
The results of the study demonstrate the capabilities of the mathematical model to calculate stress-strain state and destruction of SFC structures under shock wave load. Comparison of the results for concrete and SFC showed that the application of steel fiber allows increasing a dynamic strength of a plate up to 500 m/c (10%).

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