Information transfer via the phase: A local model of Einstein-Podolsky-Rosen experiments.

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Conventionally, one interprets the correlations observed in Einstein-Podolsky-Rosen experiments by Bell’s inequalities and quantum nonlocality. We show, in this paper, that identical correlations arise, if the phase relations of electromagnetic fields are considered. Conceptually, we proceed as follows: First, it is proved that non-factorizability does not mean nonlocality. This is done by a one photon model. Then, it is shown, that the "classical" expression for the correlation sums up photons of different pairs indiscriminately. This feature accounts for the lower correlations in the "classical" model. And finally, an integral is derived, including the properties of both photons while retaining the linearity of fields between the polarizers. This expression describes the measured values correctly. It seems thus that quantum nonlocality can be understood as a combination of linearity of possible electromagnetic fields between the polarizers and a relation of the electromagnetic fields of the two photons via a phase. We expect the same feature to arise in every experiment, where joint probabilities of separate polarization measurements are determined.

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The existence of correlations between particles in space-like separation is arguably the single most important problem in quantum optics. The literature about these Einstein-Podolsky-Rosen (EPR) experiments and their practical realization is impressive. Why is this so? Since Aspect’s first measurements experiments have sought to eliminate all possible loopholes restricting the generality of the results. In the latest set of measurements, performed under strict Einstein locality conditions, a violation of Bell’s inequalities was observed, although the two measurement devices were separated by 400 m. In common terms: the measurement of a property of photon 1 is correlated to the measurement of the same property of photon 2, although the two particles have no possibility to influence each other by any type of conventional field. In his recent review Aspect concludes from the experimental facts that "it is impossible to assign local physical reality to each photon." There is no way, how experimental results can be reconciled with the image of two single photons propagating in opposite directions, where they undergo separate measurements. Consequently, some work has been devoted recently to a careful analysis of the Bell inequalities and an assessment of their validity or invalidity. But even if the Bell inequalities are flawed, the experimental facts remain the same: correlations exist, and we don’t know, why.

In essence, quantum nonlocality derives from the conditional probabilities of two polarization measurements with the polarizer angles set to $\alpha$ and $\beta$, respectively (see Fig. a). The probability of a coincidence at both devices is described by a term $\sin^2(\beta - \alpha)$. Formally this term represents the expectation value of spin-measurements in quantum mechanics. However, there exists no known field which allows for a physical connection between the two measuring devices. For this reason the question of information transfer is still unsolved. We propose, in this report, a solution based on the phase of the photon’s electromagnetic field. And the question, how the two measurements can be related without any physical medium, will be answered approximately by: the photons remember their initial phase at their common origin. It is this memory, which shows up in the ticks of the detectors and coincidence rates.

The measurements are performed using laser pulses, ideally of very short duration, which are converted in an optical converter into two separate pulses of lower frequency and a defined phase relation. The coherence length of the pulses is very high. Due to the short duration of the pulses the electromagnetic fields and their field characteristics are limited in space. But given the high coherence length, their phase along the photon’s path of propagation remains defined and in general equal to the phase of a hypothetical electromagnetic field, covering the whole experimental setup. It is this quality of the laser pulses, which is physically decisive for correlation measurements. In this sense the experiments are of an admirable precision and at the limit of the experimentally feasible today.

In a theoretical treatment reflecting this quality of coherence (also present, e.g. in the representation in quantum mechanics) the electromagnetic field of the single photons and the hypothetical electromagnetic field throughout the system become interchangeable. Their only difference is a scale for the amplitudes. This difference, as will be shown presently, is of no effect on obtained results. In the current treatment we will calculate the field aspects of the correlation measurements by using this hypothetical field. To clarify the issues at hand we initially model two successive polarization mea-
measurements on a single photon (see Fig. 1b). This model will then be generalized to account for coincidence measurements of two photons. The hypothetical field vector of the electric field shall be polarized in x-direction, the amplitude is $E_0$. A straightforward calculation of the electric field after two polarization measurements yields a reduced and rotated amplitude. The amplitude $E_2$ after the measurement with device 2 will be:

$$E_2 = \cos \alpha \cos (\beta - \alpha) |\cos \beta e_x + \sin \beta e_y| \ E_0$$  \hspace{1cm} (1)$$

The transformation $T(\alpha, \beta)$ with $E_2 = T(\alpha, \beta)E_0$ describes the events in three discrete steps: (i) A reduction of the field at the first device ($\cos \alpha$). (ii) A conditional reduction of the amplitude at the second device ($\cos(\beta - \alpha)$). (iii) A rotation of the amplitude vector ($\cos \beta e_x + \sin \beta e_y$). The decisive step is step (ii): the conditional reduction of the amplitude at device 2 given the measurement at 1. The term combines two operations: the rotation of the field vector at device 1, and the reduction of the amplitude at device 2.

$$\begin{align*}
E_2 & = \cos \alpha \cos (\beta - \alpha) |\cos \beta e_x + \sin \beta e_y| \ E_0 \\
& = \cos \alpha \cos (\beta - \alpha) |\cos \beta e_x + \sin \beta e_y| \\
& = \cos \alpha \cos \beta e_x + \sin \alpha \sin \beta e_y, \\
& \text{where} \\
& |\cos \beta e_x + \sin \beta e_y| = \sqrt{\cos^2 \beta + \sin^2 \beta} = 1,
\end{align*}$$

expression can be visualized. Two polarizers with perpendicular planes of polarization block out all light. But a third, in a diagonal plane of polarization between the former two, is sufficient to let a part of the original light pass all three polarizers. Although the effect seems puzzling to the layman, it is nonetheless completely understandable. It shows, what could be called the memory of the photon’s electromagnetic field: for the outcome of a polarization measurement the previous treatment is generally relevant. The conditional probability equally must be expressed in terms of settings at different locations.

If the measurement is repeated with polarizers set to the angles $\gamma, \delta, \epsilon$ etc, then the representation contains terms of the form $\cos(\gamma - \beta) \cos(\delta - \gamma) \cos(\epsilon - \delta)$ etc. If the probability of a measurement is proportional to the intensity of the electromagnetic field, which it commonly is in measurements using cascade detectors, then the reduction of the probability in two consecutive measurements is equal to $\cos^2(\beta - \alpha)$. But this means, that we can calculate the conditional probability of the two measurements. The result is:

$$P(\alpha, \beta) = 1 - \cos^2(\beta - \alpha) = \sin^2(\beta - \alpha)$$  \hspace{1cm} (2)$$

Since the reduction is a relative measure, the actual numerical value of the amplitude is not decisive. Therefore the conditional probability calculated from this (hypothetical) electromagnetic field and the one computed from the actual photon field are strictly equivalent. The result is also equal to the one quoted above and which is the standard result in EPR-type experiments. Although the mathematical expression suggests nonlocality, the actual physical process can be completely local. It is only required that the photon has a finite volume and that the two measuring devices are sufficiently far apart.

These conditions are met in most state-of-the-art experiments. Note that the correlation probability, in this one photon model, cannot be factorized into events at device 1 and device 2. But that does not mean an involvement of nonlocality: the photon is perfectly localized at every given moment along its path.

That this feature is also inherent in two photon model is indicated by the fact that the one-photon model (determine the probability of measurement 2 after 1 has been performed) is an exact representation of the probability calculus of the two photon case (determine the probability of 2 under the condition that 1 has been performed). The only changes in a two photon model should therefore be the initial phase between the two photons and a factor of two for the detector counts and probabilities. Even with a space-like separation of the two devices, the correlation probability then contains a connection between settings at device 1 and 2.

Unfortunately, there is no way to actually apply the equivalence. The reason is simple: polarizations with arbitrary angles follow, classically, a $\cos^2 \phi$ characteristic. In this context it should be noted that we employ, in the following, the term phase in both of its meanings: it can,
e.g. for circular polarization, be the actual angle of the field vector at a given moment, or it describes the propagation of the photons along their path from the source to the polarizers. Since these two variables are connected by the wave features of the photons, a separation seems unnecessary. On the other hand, using a common symbol simplifies the notation considerably. Returning to single polarizations, it is clear that treating each measurement separately destroys the vital phase relations. Why are these relations important? Consider the two photon model where the electromagnetic fields of the photons are related by a phase at their origin. Given an experimental setup, the hypothetical electromagnetic field, accounting for the coherence of the photon beams, is exactly defined by the boundary conditions at the two polarizers. This derives from the time inversion symmetry of the wave equation. Now consider the way, correlation probabilities are treated e.g. in Furry’s model \[11\]. The integral over all measurements is:

\[
P(\alpha, \beta) = \int dx \cos^2(x) \cos^2(x - (\alpha - \beta))
\]  

(3)

The integral clearly describes the impacts, but does so by completely separating the two measurements. There is no way to guarantee, in this expression, that the electromagnetic fields are indeed linear throughout the space between the polarizers. To the point: the integral combines not only measurements, which belong to the same photon pair, but also measurements, where one measurement belongs to a different pair than the other. And in the latter case the phase between events is, of course, arbitrary. However, the integration does not completely destroy the correlations one expects if only photons of the same pair are considered. The remainder of an interference pattern is still obtained. It has been said, in this respect, that quantum correlations are stronger than classical ones. This seems not quite right. Only in quantum mechanics the correlations are computed by an expression conserving the linearity and thus guaranteeing, that photon 1 and photon 2 really belong to the same pair.

A physical approach to this problem, which could be called the analysis of the EPR problem from a field theoretical and statistical angle, can be based on a non standard photon model \[14\]. In this model transversal properties are described by electromagnetic fields, while longitudinal properties are related to the kinetic energy of the photons. To account for both features the fields must be described by complex vectors. In the same spirit the polarization measurement is performed on a complex function, where the electromagnetic component is \(\cos \phi\) and the kinetic component \(i \sin \phi\). In electrodynamics complex fields are standard practise, as is the computation of intensity from the real part of the fields alone. In a two photon measurement, where the fields between polarizers preserve their full linear features, only possible relations between the two measurements must be included. Which means, that the square of the fields cannot be computed for each measurement alone. It has to be calculated including interference terms. An integral along these lines would be the following:

\[
I(\alpha, \beta) = \int_0^{2\pi} dx \left\{ \left[ \cos(x - \alpha) + i \sin(x - \alpha) \right] \left[ \sin(x - \beta - \phi_0) + i \cos(x - \beta - \phi_0) \right] \right\}^2
\]

(4)

The exchange of \(\sin\) and \(\cos\) in the field of the second photon signals a phase shift of \(\pi/2\). We set \(\phi_0\) to zero for convenience. Now if the correlation probability, or the correlated change of intensity in the measurements, is taken from the real part of this integral, we arrive for the normalized integral at an expression already deduced by Kracklauer, and which is equivalent to the measured correlations \[13\]:

\[
P(\phi) = \frac{I(\alpha, \beta) - I}\{\cos x \sin(x - \phi) - \sin x \cos(x - \phi)\}^2}{2 \int_0^{2\pi} dx \left\{ \cos^2 x + \sin^2 x \right\}
\]

(5)

In the general case this leads to the following expression for the average correlation probability:

\[
P(\alpha, \beta) = \frac{1}{2} \sin^2(\beta - \alpha - \phi_0)
\]

(6)

Here \(\phi_0\) denotes the setup and the type of measurement (either transmission or adsorption as the relevant events). Can we justify the two operations? (i) Taking only the real part of the complex valued function. (ii) Linking it to intensity. The manipulations are strictly valid only in the hypothetical field, because only in this case the boundary conditions apply. In terms of classical expressions, where the intensity usually is computed in this way, there seems no problem. The expression again is a relative one, so it should also apply to the single photons and their measurements.

Physically speaking, one has to make sure that the integration contains only photons belonging to the same pair. A way to achieve this, is the linearity requirement. Its ultimate mathematical foundation is the linearity of both theories: quantum mechanics and electrodynamics. In quantum mechanics this is done by describing the measurements locally, and by retaining the phase between the two points of measurement \[14\]. A method to the same effect in electrodynamics is to retain the linearity in the integral, and to compute the square of the real part. The chosen method seems to be rather a matter of taste, once the physics of the problem is clear. And the physics behind both treatments is the same: a phase connection between the two separate photons due to the high coherence of the beam.

The conceptual steps leading to this result can be rephrased as follows. First it was proved that non-factorizability does not mean nonlocality. This was done by the one photon model. Then it was shown, that the "classical" formulation sums up photons of different pairs indiscriminately. And finally, using a non standard model
of photons, we provided a formulation for the integral including the features of both photons while retaining linearity. The last step can probably be solved in a different way. And it is, in fact, in quantum mechanics. This is not decisive. Decisive is the shift of focus. While previously the main question was:

- How can there be a nonlocal connection between events?

- How can we calculate the correlation including only photons of the same pair?

The former question leads, and did in fact lead, to somewhat ridiculous speculations about the metaphysical implications of a physical theory. In our view something, the next generation will laugh about. The latter question is only a technicality. It can be solved by methods strictly within the limits of theoretical physics. And it has already been solved by the methods used in quantum mechanics. One only has to reinterpret these methods as such: a technicality to guarantee that the linearity of the fields between the polarizers is conserved. Once this point is understood in quantum mechanics, a great deal, maybe all, of the paradox will simply disappear.

In a sense, EPR experiments are similar to interference measurements: because the result depends on the phase between the two locations. Therefore, the main feature of the presented model, the individual photon being in phase with the extended field, may be worth a more general application. Then the wave-particle duality could eventually be comprehended as an individual (the particle, the photon) participating via its phase in some larger, possibly infinite structure (the wave). What this would mean, historically, seems clear: a recovery of Louis de Broglie’s conception of a harmony of phases on a different basis.

Incidentally, this expression for the correlation probability is equal, but for a factor of 2, to the probability in the single photon case. It seems that a time ordered representation of measurements retains the linearity of fields, while it accounts for correlations in a straightforward manner. The equivalence, implied by semantics, has its counterpart in physics. The one photon model is also the only case, where a single event can be traced in a precise spacetime picture. A clear distinction between single events and their statistical treatment, which is a feature of this model, can contribute much to remove the paradox in quantum mechanics. This has been already been emphasized by Barut [13]. Unfortunately, the lesson seems forgotten. And in standard quantum mechanics, the distinction is not made in principle.

Whether the imaginary part of the integral has a significance of its own, e.g. related to the kinetic energy and thus momenta of the photons (which are in principle measurable), cannot be said with certainty at the moment. This point will be addressed in future.

This result is in accordance with all experimental findings. Physically speaking, the model is a local model of EPR-type measurements, regardless of the actual separation of the two measuring devices. And the only physical condition for the derivation are a relation of the phase (or the angle of rotation) of the electromagnetic field of photon 1 and photon 2, and a coherence length high enough to guarantee the right phase relation between the finite fields of the photons and the hypothetical field extending throughout the system.

Does the current model violate the Bell inequalities? It is easy to see, that it does, because the joint probability \( P(\alpha, \beta) = \frac{1}{2} \sin^2(\alpha - \beta) \) contains the interference term. Like in the conventional model truncating the phase information by a locality condition yields disagreement between calculated and measured values. Although, in the present case, the model is perfectly local. The main point, from a physical perspective, is the difference between conceptual nonlocality - which exists, without doubt, in quantum theory - and physical nonlocality - which requires a connection explicitly contradicted by special relativity. Saying that, we also wish to make it clear, that we do not enter into the more subtle points of the debate, how information is transferred instantaneously from one device to the other without violating special relativity.

Given the result of this calculation, we are faced by a considerable problem. Bell states, in his paper, that in a theory, in which parameters are added to quantum mechanics to determine the results of individual measurements without changing the statistical predictions, there must be a mechanism, whereby the setting of one measuring device can influence the reading of another instrument, however remote.” In this model we found that the conditional probability of the measurement 2 given 1 necessarily involves the setting of 1, which expresses the phase of the photon’s electromagnetic fields. Our principal question concerning the Bell inequalities can then be formulated as follows. If (i) the result of our calculation is equal to the result of measurements, and (ii) the calculation involves, necessarily, settings at two separate locations, while (iii) the whole model is nevertheless strictly local, then: What is the logical flaw in Bell’s acclaimed inequalities? Because the Bell inequalities, it should be remembered, are taken as a proof that local and realistic models of EPR-like measurements do not exist. Whereas this model, based on the phase of the electromagnetic fields, is a local and realistic model.

Bell assumed the correlation function \( P(a, b) \), to be given by (in the general case the plane of polarization is described by a vector in space) [12]:

\[
P(a, b) = \int d\lambda \rho(\lambda) A(a, \lambda) B(b, \lambda)
\]  

(7)

Here \( \lambda \) is the hidden variable, \( a, b \) the setting of device 1, 2, and \( A, B \) the expectation value of the spin of photon 1 or 2. The only other assumption, used in the derivation is the limitation of the expectation values \( A, B \). The main problem, with Bell’s definition, is...
the separation of the two photons. He assumed that the two systems can be described independently; this is the conceptual meaning of the product $\hat{A}(a, \lambda)\hat{B}(b, \lambda)$. The assumption is incorrect, considering the intrinsic electromagnetic fields, if these fields are related by a phase. So that the logical separation, Bell assumed, would only be given if we measure two separate particles with random phases. In this case the Bell inequalities are not violated. The experiments with down-converted photons measure, in effect, the phase relation between the two photons \[1\]. While Bell, simply said, described a measurement with two independent photons. That the phase relation lies at the bottom of the problem, can also be demonstrated in a different, more abstract approach \[14\].

One point seems to deserve special emphasis. Bell's statement is frequently interpreted in terms of nonlocal connections between the two measuring devices. Within the current model, this interpretation is incorrect. Because the interaction between a polarizer and a photon is always strictly local. It depends only on two local variables (i) The setting of the polarizer, and (ii) the orientation of the photon's electromagnetic field. Apart from the phase due to the emission at a common source, there is no connection between the measurement events. This can clearly be seen, if two different outcomes are analyzed: correlations exist, or do not exist between the two measurements. The only difference between these two cases is the quality of the photons: in one case their phase is related, in the other case it is not. Therefore, the connection can be changed simply by changing the quality of the photons. Nonlocality requires that the two events are connected not only via the photons. Since the correlations change by changing the photons, the claim of any nonlocal connection must be rejected. The model is therefore local.

The current understanding of this important experiment can be used to shed new light on the old questions, connected to Einstein's original paper \[1\]. Because the insight into the physics of the actual process behind a measurement of polarization (or photon spin, in the terminology of quantum mechanics), may be used to ask, whether Einstein was right, when he suspected quantum theory to be incomplete \[2\]? On the basis of this result, we may conclude that he was. Because the measurement of the electromagnetic field of a single photon and the phase relations between two photons is well beyond a purely statistical interpretation of quantum theory. But so was Bohr \[14\]: because there is no defined quality of any single photon, which exists after the emission from a common source. All that exists, is a periodic electromagnetic field in some angle of polarization. The problem at their time seemed to be the assumption of "objective" properties of photons. And the rather simple concept of a particle. A periodic field is no "objective" property, neither is the superposition of two discrete states of polarization.

In summary, we have presented, what we consider the first step towards a fully local model of EPR measurements. Locality was regained by an analysis of the measurements within a field theoretical approach, and using a non standard model of photons. It was essential, for the derivation, that the linearity of electromagnetic fields between the polarizers was preserved. The same requirement, in quantum mechanics, goes under the name of entanglement. It was shown that only a combined field theoretical and statistical treatment provides sufficient constraints for mathematical models to construct a representation of these measurements on a local basis. We expect the same feature, correlations of polarization measurements in space like separation, to arise in every experiment, where joint probabilities of separate measurements are determined.

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