Exciton condensation in semiconductor quantum wells in nonuniform electric field

A.A. Chernyuk ¹, V.S. Kopp ², V.I. Sugakov ¹

¹Institute for Nuclear Research, Nat. Acad. of Sci. of Ukraine, pr. Nauky, 47, Kyiv-03680, Ukraine
²Taras Shevchenko Kyiv National University, pr. Glushkova, 2, Kyiv-03127, Ukraine
(e-mail: sugakov@kinr.kiev.ua)

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Abstract

The structure appearance in exciton density distribution in semiconductor double quantum well with transverse electric field applied is studied, in the case when the metal electrode contains a round window. It is suggested that there is exciton condensed phase, free energy of which can be described by phenomenological Landau model. For the exciton density determination the traditional theory of phase transitions was used generated for the case of the finite exciton lifetime, a presence of the pumping and nonhomogenity of the system. It is shown that at high exciton density the different types of structures appear: periodic distribution of exciton condensed phase islands or condensed phase rings. The behavior of the structures depending on the pumping, the window size and temperature is analyzed. The obtained results are agreed with experimental data.

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1 Introduction

Interexciton interaction in semiconductor double quantum wells is intensively studied, particularly in connection with a problem of revealing Bose-Einstein condensation of excitons [1]. Important results were obtained under investigation of so-called “indirect excitons” in double quantum wells. In the case of applying the electric field, recombination of indirect exciton is hampered and the lifetime becomes long, therefore great concentration of excitons can be created. In semiconductor alloys based on GaAs/AlGaAs double quantum wells, a narrow luminescence line was revealed at low temperatures, which had some specific features: the line appears at pumping, larger than some threshold value, which depends on temperature; the emission intensity of the line depends super-linearly on pumping etc. [2, 3]. In exciton luminescence spectrum in double quantum well the
luminescence from the laser spot region was revealed to be surrounded by a concentric bright ring of emission, separated from the central spot by a dark region [4, 5]; the distance between the ring and the laser spot enlarges with pumping growth, and at low temperatures the ring was observed to be broken down into periodically sited fragments [4].

There are different interpretations of the structure formation. A series of explanations of the phenomena was connected with Bose statistics of excitons. According to [6], the mechanism, based on stimulated Bose character of exciton statistics in relaxation process, gives rise to instability in the exciton subsystem. In papers [7, 8] it was shown the possibility of appearance of periodic structures in solutions of nonlinear quantum equations like nonlinear Schrodinger equation. But the investigation of observed peculiarities, changes in the structure depending on temperature, pumping and the parameters of exciton system in [7, 8] was not considered.

Another alternative approach for the explanation of the peculiarities in the system with great exciton density was developed in [9–13]. The explanation is not connected with Bose statistics of excitons. In these papers a traditional phase transition theory, generalized for the case of unstable particles and a presence of the pumping, is applied. The appearance and the properties of the forming structures are related with: 1) the presence of the attraction between excitons, which leads to condensed phase formation (the possibility of condensed phase formation of indirect excitons was shown in [14]), and 2) non-equilibrium of the system, caused by finite exciton lifetime.

As it was shown in [9], the system of attracting excitons is unstable with respect to density super-lattice formation. The properties of exciton condensation in 2D systems in a statistical approach, generalized for the case particles with finite lifetime, were studied in [11]. Ring fragmentation outside of the laser irradiation region in quantum wells observed in [4] was explained in a statistical approach [10] and in spinodal decay model, generalized for the system of unstable particles [12, 13]. The emerging structure in case of interaction of two laser spots, was simulated in [14]. The method employed in [12, 13], applied for the investigation of a nonuniform system, allowed to explain the transition between fragmented luminescence ring and a continuous one with temperature or other parameters changing, the formation of localized spots in the emission, caused by defects in the structure and etc. So, the appearance of a structure in high-density exciton systems was explained in the papers [9–13] with the processes of self-organization in non-equilibrium systems caused by both the finite value of the exciton lifetime and the presence of pumping. Such explanations do not require the Bose-Einstein condensation of excitons.

Recently the interesting effects were observed in the papers [16–18]. The excitation and emission of indirect excitons was performed through “a window” in metallic gate with the diameter of order of several micrometers. In the case of excitation intensity growth, a regular ring pattern of equidistant bright spots along the perimeter of the window was observed. At higher pumping or temperatures the structure washed out and transforms to the emission from a ring. With expanding window size, the structure became complicated. These experiments are explained in this paper, continuing the viewpoint of the papers [9–13].

2 Model of the system

Let us deal with qualitative picture of structure appearance of exciton condensed phase in quantum well in the case of a window in the electrode. Increasing the electric field, the position of the indirect
exciton level shifts towards low energies. Under the window the electric field is smaller, than in the region remoted from the window, therefore, a hump for exciton potential energy arises in the well. Excitons, created by the light through the window, roll down in nonuniform field from the middle of the well towards the region under the window. Due to a finite lifetime they can not move far away from the region under the window. As a result, in the well under the perimeter of the window the maximum of the exciton density appears. Thus, at the pumping growth the condensed phase forms in the region of maximum exciton density, i.e., on the ring along the perimeter of window, like it was observed in experiments [16–18].

Let us find the exciton potential energy, caused by the window presence in metallic electrode above the quantum well plane (Fig. 1).

In external electric field an exciton gets additional energy \( V = -p_z E_z \), where \( p_z \) is the dipole momentum of an exciton (the axis \( Oz \) is perpendicular to the quantum well plane). In experiments [16–18] semiconductor with double quantum well is covered by a metallic mask, in which a round window is located. The uniform electric field of the intensity \( E_0 \) is applied perpendicular to the quantum well plane. Due to the presence of the window in the mask, the electric field is distorted at immediate proximity of the window. Let us calculate nonuniform addition to external uniform electric field into the exciton potential energy. Let the upper electrode with the coordinate \( z = 0 \) has a round window of the radius \( r_0 \). For the determination of the field we have to solve Laplace equation for the potential with the following boundary conditions: the potential of the electrostatic filed is constant at both electrodes; the difference of the potentials of electrodes ought to be equal to the value, at which the field between electrodes is equal to \( E_0 \) far away from the window. For that we shall use the solution of the task given in [19] about the field, created by the ground plane with a window, located in external electric field of the intensity \( E_0 \). As a matter of fact, this solution does not satisfy the condition of the constant potential on the lower electrode. But a change of the potential introduced by the window falls down with moving off from the window as a dipole potential [19], and thus, it is small in the region of the lower electrode under conditions that \( r_0 \ll L \), where \( L \) is the distance between electrodes. Let us consider that \( L \gg r_0, z \), i.e., the plane of the well is located significantly
nearer to the upper electrode, than to lower one. Besides this, for using the solution of the task [19], in which the environment for both sides of the window is identical, we consider, that the upper electrode (the electrode with the window) is inner and is located inside semiconductor environment. So, the considered system slightly differs from the investigated in the experiments [16–18], but it gives the same qualitative behavior of the potential at the change of the both the window radius and the location of the well relatively the electrode.

Thus, the additional potential, created by the window is

\[ \varphi = \frac{E_0 z}{\pi} \left( \arctg \frac{r_0}{\sqrt{\xi}} - \frac{r_0}{\sqrt{\xi}} \right), \]  

where \( \xi \equiv \frac{1}{2} \left[ \rho^2 + z^2 - r_0^2 + \sqrt{(\rho^2 + z^2 - r_0^2)^2 + 4z^2r_0^2} \right] \) is flattened spheroidal coordinate, \( \rho \) is the radial coordinate in the quantum well plane. At great fields, when the electrons and holes are distributed to different wells and dipole momentum does not depend on \( E_0 \), the exciton potential energy \( V(\rho, z) = p_z \cdot \partial \varphi / \partial z \). We shall characterize the value of the potential energy by “pulling force” \( \lambda = p_z E_0 / (kT_c) \). A radial profile of the exciton energy \( V(\rho) \) at different values of \( z/r_0 \) is presented in Fig. 2.

Fig. 2. A radial profile of the exciton potential energy \( V \) in nonuniform electric field as a function of the ratio \( \rho/r_0 \) at different values of the parameter \( z/r_0 \). The pulling force \( \lambda = 30 \) is for all curves.

Coordinate \( z \) characterizes the distance from the quantum well to the plane of the electrode with the window. With decreasing this distance, the potential hump for an exciton, created by the window, enlarges, and the slope becomes more steep. In the region below the window perimeter inside the well round potential well for excitons arises (it is showed by the arrow in Fig. 2). The exciton energy in this well is lower, than the energy in regions, remote from the window. The depth of the well increases, if the plane of the quantum well approaches to the window. The appearance of the well is related with charge origination nearby edges of the window. In the work [18] the shift in the emission spectrum of indirect excitons from the regions below the window perimeter was observed, and this is probably related with the formation of the aforementioned round potential well.
3 Exciton density equation

We assume the time of post-excitation establishment of quasilocal equilibrium of electrons and holes and binding into excitons to be much smaller than the time of the exciton lifetime and the time of equilibrium establishment between different regions. In this case, the free energy of quasilocal state can be regarded as a function of the exciton density. A phenomenological equation for the exciton density \( n \) can be written down as

\[
\frac{\partial n}{\partial t} = -\text{div} j + G - \frac{n}{\tau},
\]

(2)

where \( G(r) \) is the pumping (the number of excitons, created in a unit area in a unit time), \( \tau \) is the exciton lifetime, \( j = -M\nabla \mu \) is the exciton current density, where \( \mu \) is the chemical potential, \( M \) is the exciton mobility. We shall use for \( M \) the Einstein condition \( M = nD/kT \). The corrections for the condition caused by Bose statistics of excitons were studied in [20] for quantum wells. But for the temperature and the exciton density under investigations these corrections are not essential.

The chemical potential can be expressed by free energy: \( \mu = \delta F/\delta n \). We chose the free energy in the Landau model:

\[
F[n] = \int d\mathbf{r} \left[ \frac{K}{2} (\nabla n)^2 + f(n) + nV \right].
\]

(3)

The term \( \frac{K}{2} (\nabla n)^2 \) characterizes the energy of the non-homogeneity. Exciton additional energy in a nonuniform electric field is taken into account with the term \( nV \). According to Landau method, we expand the free energy in a power series of \( (n - n_c) \) in the vicinity of its minimum:

\[
f(n) = f(n_c) + \frac{a}{2} (n - n_c)^2 + \frac{b}{4} (n - n_c)^4.
\]

(4)

The parameters \( a, b \) and \( n_c \) in (4) are phenomenological and may be obtained from quantum-mechanics calculations of the free energy of the exciton system at infinite exciton lifetime approximation or comparing the theory with the experiment. After the substitution (3) and (4) in Eq. (2), the latter is reduced to

\[
\frac{\partial n}{\partial t} = \frac{D}{kT} \nabla \left[ n \nabla \left( \frac{df}{dn} - K\Delta n \right) + n \nabla V \right] + G - \frac{n}{\tau}.
\]

(5)

We must remark that for small densities in the free energy (3) the term \( kTn (\ln n - 1) \) has to be added, which leads in the Eq. (5) to the well-known term \( D\Delta n \). But its contribution is inessential at great exciton densities.

Eq. (5) is a non-linear 2D phenomenological equation, which describes density distribution of interacting excitons of high concentration taking into account the pumping and finite lifetime. We shall solve it instead of Gross-Pitayevsky equation [21], because the wave function loses coherence at distances of order or smaller than the distance between excitons. The finite lifetime and the presence of constant exciton generation lead to new qualitative peculiarities in the process of phase formation in comparison with phase formation for stable particles. We assume, that Eq. (5) is also applied, if a condensed phase is electron-hole liquid. Then, \( n \) in the free energy (3) is the density of electron-hole pairs.

For numeric calculations let the units of length, concentration and time be

\[
l_u = \sqrt{\frac{K}{-a}}, \quad n_u = \sqrt{\frac{-a}{b}}, \quad t_u = \frac{kTK}{Da^2n_u},
\]

(6)
correspondingly. Then, the generation rate and the energy are measured in the units of \( G_u = n_u/t_u \)
and \( V_u = -an_u \), correspondingly. In dimensionless variables the exciton density equation \( (5) \) becomes
\[
\frac{\partial n}{\partial t} = \nabla \left[ n \nabla \left( -\Delta n + (3n_c^2 - 1)n - 3n_c n^2 + n^3 \right) + n \nabla V \right] + G - \frac{n}{\tau}.
\] (7)

4 Calculations and results discussion

Eq. (7) was solved numerically in a 2D system in the shape of a rectangular plate, the sizes of which exceeds much the window radius, so, at the further increasing of window sizes, the quasiparticle density distribution does not almost change. The following values of the parameters of the system were chosen: \( \tau = 10 \text{ ns} \), \( T_c = 5.7 \text{ K} \), \( n_c = 1.2n_u = 3.33 \cdot 10^{10} \text{ cm}^{-2} \), \( Kn_c^2 = 15.7 \text{ meV} \), \( -an_c = 0.826 \text{ meV} \), \( bn_c^3 = 1.70 \text{ meV} \). The free path length is \( l = \sqrt{D\tau} = 1.41 \mu\text{m} \). The pumping \( G \) is constant on the disc with the radius \( r_0 \) and equals to zero outside the disc. An example of the stationary solution of Eq. (7) for the exciton density is shown in Fig. 3.

Fig. 3. Exciton density distribution \( n(x, y) \) in the quantum well plane at the parameters: \( r_0 = 2.5 \mu\text{m}, \)
\( z/r_0 = 1.44 \), \( T = 1.71 \text{ K} \), \( G\tau = 9.71 \cdot 10^{10} \text{ cm}^{-2} \), \( \lambda = 30 \)

The points of high density can be attributed to a condensed phase, and the points of low density are assumed to be a gas phase. Thus, the formed fragments are periodically sited islands of the exciton condensed phase, that corresponds to the structure, observed in [16–18]. One can see from Fig. 3, that the islands of the condensed phase appear at the boundary of the ring, as it is follows from the qualitative analysis of the influence of nonuniform potential distribution. A part of an island is under the metal mask and may not be observed in the luminescence. The size of the region, “hidden” below the mask, depends on the distance of the quantum well with respect to the upper electrode. With moving off the electrode, the islands shift towards the window center and “stretch” from the electrode. With approaching the well plane to the electrode islands “hide” below the electrode.

Hereinafter, we present the results of the investigation of the structure behavior at external parameters changing. With broadening the window, the number of islands increases (Fig. 4), as the length of the circle, restricting the window, increases. At the same time, the shape of the fragments
does almost not change. At some small window radius only the spot in the center forms, but the
critical value of the pumping, necessary for its appearance, is greater, than the critical value for
islands formation in the systems with bigger window.

Fig. 4. Exciton density distribution with growth of the window radius. The radius equals to: a) \( r_0 = 1.4 \, \mu m \), b) \( r_0 = 2.5 \, \mu m \), c) \( r_0 = 3.6 \, \mu m \). Other parameters are the same as in Fig. 3, except for the case ‘a’, where
the pumping \( G \) is larger (see the text). The case ‘b’ is a density plot of 3D image in Fig. 3.

Islands are formed only at the pumping exceeding some threshold, because the condensed phase
forms at large exciton density. At small pumping the structure does not appear, and at large
irradiation intensity distinct islands merge into a continuous condensed phase ring (Fig. 5).

Fig. 5. Exciton density distribution with the pumping growth. The pumping \( G \) equals to: a) \( 0.86G_0 \), b) \( G_0 \), c) \( 1.14G_0 \); \( G_0 \tau = 9.71 \cdot 10^{10} \, \text{cm}^{-2} \). Other parameters \( (r_0, z, T) \) are the same as in Fig. 3.

Let us examine temperature dependence in the Landau model. The phase transition occurs, if
the coefficient \( a \) in the free energy density (4) changes the sign. Temperature dependence of other
coefficients can be neglected. In Landau approximation, the parameter \( a \) depends on temperature
like
\[
a(T) = \alpha \left( 1 - \frac{T}{T_c} \right), \tag{8}
\]
where \( T_c \) is the critical temperature, \( \alpha < 0 \). Linear dependence (8) is real in the framework of
self-consistent field approximation and may not be fulfilled, if fluctuations play the essential role. At
derivation of the equation in dimensionless variables, we shall use the units (6), but substituting
\( a \) and \( T \) for \( \alpha \) and \( T_c \), correspondingly. If temperature is measures in \( T_c \), then in new variables the
exciton density equation (7) is modified to:
\[
\frac{\partial n}{\partial t} = \frac{1}{T} \nabla \left[ n \nabla \left( -\Delta n + (3n_c^2 - 1 + T)n - 3n_c n^2 + n^3 \right) + n \nabla V \right] + G - \frac{n}{\tau}.
\]
As numeric calculations show, with increasing the temperature distinct islands of the condensed phase merge into a continuous ring, and at higher temperatures the emission from the window is homogeneous (Fig. 6).

![Fig. 6. Exciton density distribution with temperature increasing. Temperature equals to: a) 1.71 K, b) 1.94 K, c) 3.14 K. Other parameters \((G, r_0, z)\) are the same as in Fig. 3](image)

### 5 Conclusions

In the paper the investigation of exciton condensation in the framework of traditional phase transition theory performed for the system in nonuniform electric field taking into account the effects of a finite value of the exciton lifetime and the presence of the pumping. The main results are the following. 1) At the pumping, exceeding some threshold value, the exciton condensed phase appears in the form of the islands localized nearby the edge of the window or continuous rings. 2) The number of the islands increases with the rise of window radius. A continuous ring can break out into distinct periodically sited condensed phase islands. 3) At small size of the window, only the spot in the window center appears. 4) With the rise of pumping and temperature the distinct islands merge into continuous ring.

All above-mentioned peculiarities were observed in the works [16-18]. We have to remark, that Bose-Einstein condensation was not drawn for results obtained, but quantum statistics of excitons may play some role in the formation of parameters, which were used in the phenomenological model. We consider, that the formed structure (the formation of islands, their location, dynamics at parameters changing) is the consequence of non-equilibrium of the system, i.e., the structure is an example of self-organization processes in non-equilibrium systems [22, 23]. In the considered case the origin of a non-equilibrium state is the finite value of the exciton lifetime. The general theory of spatial structure formation for unstable particles at phase transitions is presented in [24, 25].

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