The large-scale structure in a universe dominated by cold plus hot dark matter

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Abstract

Using numerical simulations, we investigate the large-scale gravitational clustering in a flat universe dominated by cold plus hot dark matter (i.e., $\Omega_0 = \Omega_{\text{CDM}} + \Omega_{\text{HDM}} + \Omega_{\text{baryon}} = 1$). Primordial density fluctuation spectrum is taken to have the Zel’ dovich-Harrison form. Three models are studied, with Model I having $\Omega_{\text{CDM}} = 0.69$, $\Omega_{\text{baryon}} = 0.01$, and $\Omega_{\text{HDM}} = 0.30$ in one flavor of neutrinos; Model II having $\Omega_{\text{CDM}} = 0.60$, $\Omega_{\text{baryon}} = 0.10$, and $\Omega_{\text{HDM}} = 0.30$ in one flavor of neutrinos; Model III having $\Omega_{\text{CDM}} = 0.69$, $\Omega_{\text{baryon}} = 0.01$, and $\Omega_{\text{HDM}} = 0.30$ in three flavors of neutrinos. The initial density spectra are normalized by the COBE quadrupole measurement, and galaxies are identified from the peaks of initial density fields above a certain threshold chosen, to match the observed two-point correlation on scales $\lesssim 10 h^{-1}\text{Mpc}$. Thus the clustering properties of both the mass and the galaxies are completely specified. The biasing parameter (for the ‘galaxies’) determined in this way is $b_g \approx 1.2$ for Model I, 1.5 for Model II and 1.6 for Model III.

The clustering and motions of the simulated ‘galaxies’ are compared with recent observations. The spatial distributions of galaxies in the hybrid models are very frothy; filaments, sheets, voids etc. of sizes $10 – 50 h^{-1}\text{Mpc}$ are frequently seen in the simulations. All three models are in good agreement with the observed local bulk motions and with the count-in-cell statistics $\sigma^2(l)$ in redshift surveys of IRAS galaxies. One exception is the $\sigma^2(l)$ of the QDOT survey at $l = 40 h^{-1}\text{Mpc}$: the value is too high to expect in the models. But its statistical significance was recently questioned with an analysis of the 1.2 Jy IRAS survey. Model I does not have sufficient large-scale power to explain the two-point angular correlation function of the APM survey, the two-point correlation function of Abell clusters. Furthermore, its galaxy pairwise velocity dispersion around $1 h^{-1}\text{Mpc}$ is too high to reconcile with the observation. The other two models can be adjusted, within the observational errors, to fit all observations on scales from $\sim 1 h^{-1}\text{Mpc}$ to $\sim 50 h^{-1}\text{Mpc}$, showing that the power spectrum of the initial density fluctuation is close to the predictions of these two models, and indicating that the observational results of galaxy clustering and motions on large scales are consistent under some reasonable theoretical assumptions.

Key words: Clusters: of galaxies –Cosmology –Dark matter –galaxies: formation of –Universe (the): structure of
1. Introduction

Hybrid models, in which the universe is dominated by cold plus hot dark matter (CDM and HDM, respectively), were proposed in the early 1980s as one scenario of structure formation in the universe (Fang et al. 1984; Shafi & Stecker 1984; Valdarnini & Bonometto 1985). These models were not considered very seriously, however, because they depend on at least one more parameter (i.e., the relative fraction of HDM and CDM) than the simplest models in which the universe is dominated by a single kind of dark matter (e.g., CDM or HDM), and there seemed then to be no appealing reasons for studying them in much more detail. The situation has changed recently, because observational evidence has been accumulated to the point where one can conclude that the simplest CDM and HDM models do not work. The hybrid models are one of the simplest revisions of these models.

The standard CDM model (e.g., Davis et al. 1985) had been quite successful in explaining the structure of the universe on scales $\lesssim 10^\mathrm{h^{-1}Mpc}$ (see Davis & Efstathiou 1988 for a review). The success was, however, based on the assumption that galaxies are highly biased tracers (with a bias parameter $b_g \sim 2.5$) of the underlying mass distribution. Since a strong segregation in the clustering amplitude between faint and bright galaxies is not observed (Alimi et al. 1988; Mo & Lahav 1993), a high value of the bias is therefore not favored. Furthermore, the high-bias model fails to provide sufficient power on large scales to match the angular correlation functions of deep galaxy surveys (Maddox et al. 1990, hereafter MESL; Picard 1991; Collins, Nichol, & Lumsden 1992), the large-scale velocity fields (e.g., Lynden-Bell et al. 1988; Bertschinger et al. 1990), the correlation functions of clusters of galaxies (e.g., Bahcall & Soneira 1983; Klypin & Kopylov 1983; Postman et al. 1986; Batuski et al. 1989; Huchra et al. 1990; Postman et al. 1992; Mo et al. 1993; Jing & Valdarnini 1993; Dalton et al. 1992; Nichol et al. 1992), and the COBE measurement of the microwave background radiation (MBR) fluctuations (Smoot et al. 1992). In fact, the COBE result is very close to the prediction of the standard CDM model with $b_g \approx 1$. But the unbiased CDM model has serious problems on small scales; the predicted amplitude of the velocity field on Mpc scales is too large to be compatible with observations, unless a large velocity bias exists between galaxies and dark matter (Couchman & Carlberg 1992).

The problems in pure HDM models are, to some extent, the opposite. The free-streaming motions of neutrinos (of mass $m_\nu$) have erased all fluctuations on scales smaller than $\sim 40(30\text{eV}/m_\nu)^{h^{-1}\text{Mpc}}$, and galaxy formation has to invoke the fragmentations of large pancakes. To have galaxies (quasars) form early enough to be consistent with observations, the amplitude of the primordial density spectrum on large scales would be too large to be compatible with the upper limit of the MBR anisotropy on angles of $\sim 1^\circ$
Furthermore, the potential wells provided by neutrino pancakes are so deep that baryons falling into clusters of galaxies would exceed the observational limits of the x-ray background (e.g. White et al. 1984).

If the formation of structure is mainly due to gravitational instability, one of the simplest cases to study next would be hybrid models, in which the universe is dominated by CDM plus HDM. From the above discussion we can imagine that the difficulties in one model (CDM or HDM) could, to some extent, be overcome in the other, and a hybrid model may do a better job in matching observations than the simplest models do. Indeed, calculations of the linear evolution of the density perturbations have shown that the hybrid models, when normalized on large scales, have less small-scale power than a pure CDM model but much more than a pure HDM model (e.g. Holtzman 1989; Xiang & Kiang 1992; van Dalen & Schaefer 1992; Taylor & Rowan-Robinson 1992), which is in the desirable direction. Davis et al. (1992) and Gelb et al. (1993) recently have performed N-body simulations, with emphasis on the pairwise velocity in the hybrid models. Compared with the observed large-scale structure, the results of all these investigations suggest that the favorable (hybrid) models would have $\Omega_{\text{HDM}} \sim 0.3$.

In order to study these models further, we have carried out numerical simulations for three hybrid models with $\Omega_0 = 1$ and $\Omega_{\text{HDM}} = 0.3$. The three models differ in the relative fraction of baryons and in the number of flavors of massive neutrinos. Since our simulations do not treat CDM and HDM separately, the results depend only on the initial density spectra on scales larger than $\sim 0.5 \, h^{-1}\text{Mpc}$ (the resolution limit of the simulation of box size $60 \, h^{-1}\text{Mpc}$), and they are not expected to be valid on galactic scales. The amplitudes of the initial density spectra are normalized by the COBE measurement of the fluctuation in the MBR (Smoot et al. 1992). Galaxies are identified from the density peaks above certain threshold, to match the observed two-point correlation functions on scales $\lesssim 10 \, h^{-1}\text{Mpc}$. The large-scale clustering and motions of ‘galaxies’ are then determined in the models, which enables us to make direct comparisons with observations.

The outline of the paper is as follows. In §2, we describe in detail the models to be studied and the simulations to be used to trace the structure evolution. The identification procedure of galaxies is also described there. In §3, we present the results of our simulations, and compare them with various observations of galaxy clustering and motions. The clustering properties of rich clusters in the simulations have been analyzed elsewhere (Jing et al. 1993; hereafter JMBF). In §4, we give a brief discussion of our results and summarize our main conclusions.

2. Models and N-body simulation
2.1 Models and simulation method

In this paper, we assume that the universe is flat (i.e. the total mass density parameter $\Omega_0 = 1$), the cosmological constant $\Lambda = 0$, and the Hubble constant $H_0 = 50 \text{km s}^{-1} \text{Mpc}^{-1}$ (written as $H_0 = 100h \text{km s}^{-1} \text{Mpc}^{-1}$). The primordial density fluctuations are assumed to be adiabatic and Gaussian, with a Zel’dovich-Harrison spectrum $P(k) \propto k$. The universe is assumed to consist of CDM, three species of neutrinos (the massive part of which is called HDM), and baryons. In this framework, we study three models with different relative fractions of HDM, CDM and baryons. In Model I, the cosmic mass has 69% in CDM, 30% in one flavor of neutrinos and 1% in baryons. In Model II, 60% of the cosmic mass is in CDM, 30% in one flavor of neutrinos and 10% in baryons. Model III has the same mass composition as Model I, except that it contains three flavors of neutrinos with equal mass. The choices for the relative fractions of CDM and HDM are based on the results of previous studies of the hybrid (CDM+HDM) models (e.g. Holtzman 1989; Xiang & Kiang 1992; van Dalen & Schaefer 1992; Wright et al. 1992; Davis et al. 1992; Efstathiou et al. 1992; Taylor & Rowan-Robinson 1992; Gelb et al. 1993). By comparing models with the observed large-scale structure, these studies suggested that the (most) favored hybrid models would have $\Omega_{\text{HDM}} \sim 0.3$. The inclusion of a baryon component generally suppresses the power on small scales. The two values $\Omega_{\text{baryon}} = 0.01, 0.1$ represent the lower and upper limits given by the standard Big Bang nucleosynthesis calculation (e.g. Olive et al. 1990; Walker et al. 1991). In Model I and Model II, the mass of the neutrinos is about 7eV, which is well below the experimental upper limit on the mass of $\nu_\tau$ (see e.g. Primack 1992 for a recent summary of the experimental limits on neutrinos masses). The neutrino mass in Model III is about 2eV in each flavor. Since the constraints on the masses of $\nu_\mu$ and $\nu_e$ are quite stringent, this model may not be favoured by particle physics. However, since the neutrino mass in this model is smaller, it has less power on small scales.

We will use the transfer functions of the linear density perturbations given by Holtzman (1989) for these models. Holtzman fitted the transfer functions at the present time ($z = 0$) by a parameter-fitting function. For wavenumbers $k \leq k_{\text{max}} = 1.24 \text{Mpc}^{-1} h$ ($h = 0.5$), the fitting function was claimed to have accuracy better than ten percent (i.e. the maximum deviations of the $P(k)$ fit are less than 20%, see Holtzman 1989). As we shall see below, the Nyquist wavenumber in our main set of simulations (in a cube of $240 h^{-1} \text{Mpc}$ in each side) is about $k_N \approx 0.8h \text{Mpc}^{-1}$, well below the value of $k_{\text{max}}$. For the other set where the cube size is $60 h^{-1} \text{Mpc}$, $k_N \approx 3h \text{Mpc}^{-1}$. In this case, we simply extrapolate the fitting functions to $1.24 \leq k \leq 3.0 h \text{Mpc}^{-1}$. van Dalen & Schaefer (1992) and Klypin et al. (1993) have given the transfer functions for Model I and Model II up
to $k = 3 \, h \, \text{Mpc}^{-1}$, which confirmed that the extrapolations are correct within the ten percent accuracy. The linear density power spectra for these models, given by the parameter fitting, are shown in Figure 1.

Our simulations are performed by a particle-mesh code with $128^3$ grid points and $64^3$ particles in cubic comoving volumes (Hockney & Eastwood 1981). The standard Cloud-In-Cell (CIC) scheme is used for mass assignment and force interpolation. The Poisson equation for the gravitational potential is solved by the Fast Fourier Transformation and the gravitational force is calculated from the potential by the staggered-mesh method (Melott 1986). Particle velocities and positions are forwarded by the standard leapfrog integration. The integration variable is the scale factor $a$, and the integration step size $\Delta a$ is $0.1a_i$ ($a_i$ is the scale factor at the initial time). The simulations are started at $z = 8$, so there are 80 steps for the systems to evolve up to the present time $z = 0$. Initial velocities and positions of particles are generated by the Zel’dovich approximation, following the prescription of Efstathiou et al. (1985). The power spectra shown in Figure 1 are used to produce the initial conditions of our simulations. The $P(k)$ are so normalized that the Sachs-Wolfe effect produces the quadrupole $Q = 6 \times 10^{-6}$ detected by the COBE. Because the simulations start at redshift $z = 8$, we simply scale the linear $P(k)$ by $P(k, z) \propto (1 + z)^{-2}$ to get $P(k)$ at $z = 8$ (see discussions of §2.2). For each model, we have done two sets of simulations, using box sizes $L = 60 \, h^{-1}\text{Mpc}$ and $L = 240 \, h^{-1}\text{Mpc}$ respectively. We run five realizations for each simulation. For convenience, we shall call the simulations of the larger $L$ the L240 simulations, and those of the smaller $L$ the L60 simulations. The mass of each particle is about $3 \times 10^{13}\text{M}_\odot$ in the L240 simulations and $5 \times 10^{11}\text{M}_\odot$ in the L60 simulations. The typical spatial resolution of PM codes is one mesh size, so the L240 simulations have a resolution of about $0.5 \, h^{-1}\text{Mpc}$, and the L60 simulations about $0.5 \, h^{-1}\text{Mpc}$. Combining the results of these two sets of simulations, we can obtain sufficient resolution for the purpose of this paper, the study of clustering features on scales $0.5 \lesssim r \lesssim 50 \, h^{-1}\text{Mpc}$.

2.2. The validity of the simulations

The linear power spectra given by Holtzman (1989) are for redshift $z = 0$. The power spectra at $z = 8$ (when we start our simulations) differ from the simple scaling $P(k, z) \propto (1 + z)^{-2}$ for $k > k_J$, where $k_J$ is the (comoving) Jeans wavenumber of neutrinos. At $z = 8$, the Jeans wavenumber is $1.9h\text{Mpc}^{-1}$ for $m_\nu = 7\text{eV}$, and $0.5h\text{Mpc}^{-1}$ for $m_\nu = 2\text{eV}$. Since the Nyquist wavenumbers of the L60 and L240 simulations are 3 and $0.8 \, h \, \text{Mpc}^{-1}$ respectively, the $(1 + z)^{-2}$ scaling suffices for the L240 simulations of Model I and Model II. For the L60 simulations, the use of the above scaling will underestimate the power
of clustering at \( z = 8 \) near the Nyquist wavenumber \( k_N \) because of the neutrinos free-streaming motion between \( z = 8 \) and \( z = 0 \). The influence of the underestimation on our final results was tested when we got the linear power spectra \( P(k) \) of Model II at different \( z \) (Klypin et al. 1993, hereafter K93) after submitting the first version of the paper. We did two simulations of box size 60 \( h^{-1}\text{Mpc} \). The method for doing these simulations is the same as described above. All input parameters (including the random phases in density fields) are kept the same for the two simulations except the initial power spectra are different. In one simulation (NS simulation) we use the spectrum at \( z = 8 \) given by K93, and in the other (S simulation) we use the spectrum obtained by the \( (1 + z)^{-2} \) relation from the spectrum at \( z = 0 \) given by K93. So the method used for the S simulation is exactly the same as that for other simulations presented in this paper. To compare the two simulations, we show their power spectra at several redshifts in Figure 2a. The method for estimating the power spectra will be given in §3.1. For the resolved range of the simulations, the largest difference between the two spectra happens near the Nyquist wavenumber \( k = k_N \). At the initial time \( z = 8 \), the \( P(k) \) of the S simulation is about 40\% lower than that of the NS simulation at \( k_N \). The difference, however, becomes smaller with the development of non-linear clustering, and is only about 15\% at \( k = k_N \) when \( z = 0 \). This is the case because the evolution of large-wavelength fluctuations contributes to the (non-linear) clustering on small scales. As Little et al. (1991) showed, the non-linear clustering on \( k \gtrsim k_c \) is mainly determined by the power on \( k \lesssim k_c \), where \( k_c \) is the transition scale from linear to non-linear clustering and is defined as \( k_c = 2\pi/R_c \) (\( R_c \) is the radius of sphere in which the \( \text{rms} \) density fluctuation is 1). In our simulations, \( R_c \) is about 5 \( h^{-1}\text{Mpc} \). The initial difference in \( P(k) \) is less than 20\% for \( k \lesssim k_c \). This is why the final difference of \( P(k) \) in the two simulations is only 15\% or smaller.

Another problem is associated with the neutrino free-streaming motion, because we follow the evolution of structures by only one type of particles (i.e. cold particles). This procedure should be a good approximation for wavenumbers less than \( k_J \), so it is valid for the L240 simulations of Model I and Model II. But it may fail for the L60 simulations. As a check, we have run a two-component simulation with box size 60 \( h^{-1}\text{Mpc} \) for Model II. Because of the free-streaming motion of the hot component, the usual way to set the initial conditions for simulations by using the Zel’’dovich approximation is no longer valid. In Davis et al. (1992), the initial condition is generated by simply spreading the hot particles uniformly in the simulation box and giving each of them a randomly-oriented velocity drawn from a Fermi-Dirac distribution. This is obviously an approximation, though the approximation may not be too bad because of their small simulation box (7 \( h^{-1}\text{Mpc} \) on each side). In K93, the authors claimed that the Zel’dovich approximation was used to set
the initial conditions for their simulations, but did not give any details. We believe that the use of the Zel'dovich approximation in this context is not justified.

To test the influence of the hot component on our results of the L60 simulations, we adopt an ‘approximate’ method to generate the initial conditions for the two-component simulations. The method is still based on the Zel’dovich approximation but requires that 1.) the initial position displacement of cold particles corresponds to a random realization of the power spectrum $P_c(k)$ of cold particles; 2.) the initial position displacement of hot particles corresponds to a random realization of the power spectrum $P_h(k)$ of hot particles; 3.) the initial velocities of both cold and hot particles, contributed by gravitational clustering, are assigned by the usual Zel’dovich approximation, assuming a power spectrum $P(k) = (0.3\sqrt{P_h(k)} + 0.7\sqrt{P_c(k)})^2$ and a flat universe dominated by cold dark matter; 4.) in realizing the above three steps, the random phases of both CDM and HDM perturbations are kept the same, so that they correspond to a single random process; and 5.) each hot particle is given a thermal motion randomly oriented and drawn from the Fermi-Dirac distribution. We believe that all of the above requirements are correct except the third one. The third is incorrect in general sense because of the free-streaming motion of neutrinos, but may be not a bad approximation for the L60 simulation, for a large fraction of the peculiar velocity induced by gravitational instability in the linear regime is from long-wavelength fluctuations and the free-streaming motion affects only short-wavelength fluctuations on a few Mpc after $z = 8$. Although we are unclear how accurate the initial conditions generated in this way can be, the test presented here may give some idea about the importance of the free-streaming motion in the simulations.

In the simulation, we use three sets of $64^3$ particles: $64^3$ cold particles and $2 \times 64^3$ hot particles. The mass of each cold particle is $3.5 \times 10^{11} M_\odot$ and each hot particle has mass of $7.5 \times 10^{10} M_\odot$. The initial $P(k)$ at $z = 8$ of both cold and hot particles are from K93. The hot particles are grouped into pairs at the initial time. The two particles of each pair have the same initial position, the same initial velocity induced by the gravitational clustering, but oppositely directed thermal velocities of equal magnitude. As suggested by K93, this may prevent the simulated thermal motions from generating spurious fluctuations. The random phases of the density fluctuations are designed to be the same as in the NS simulation, so that these two sets of simulations can be compared directly. The simulation is evolved to the present time by the way described earlier.

In Figure 2b, we present the power spectra $P(k)$ for both cold and hot particles at several redshifts. For comparison, the power spectra of the NS simulation are also plotted. By design, the power spectra of the cold particles in both simulations are identical initially. The hot particles have much less power on small scales owing to the free-streaming motion.
Because of the hot component, the clustering of cold particles becomes subsequently weaker in the two-component simulation than in the NS simulation, as is clearly shown by the power spectra at redshift $z \geq 1$. At redshift $z = 1$, the $P_c(k)$ of the two-component simulation is lower, near the Nyquist wavenumber, than that of the NS simulation by 25%, an amount comparable to the result based on the linear calculation. At $z \approx 1$, the Jeans wavenumber of neutrinos becomes $4 h \text{Mpc}^{-1}$, similar to the force resolution size of the simulation. The free-streaming motion is no longer important later on, and the hot component catches up with the clustering of the cold component in the two-component simulation. In the meantime, non-linear clustering becomes more and more important. As discussed in the last paragraph, long wavelength perturbation can significantly influence the clustering at short wavelengths. As a result, the clustering difference between the distributions of the cold particles in the two sets of simulations becomes smaller at $z < 1$. At the end of the simulations, the $P_c(k)$ in the two-component simulation is only $\sim 5\%$ lower than that in the NS simulation.

The power spectrum of hot particles, as shown in Figure 2b, appears to grow much faster near $k_N$ than that of cold particles, especially at large redshift $z$. This unphysical behavior is of common nature of N-body simulations. Because the simulations consist of limited number of particles, shot noise must play role to some extent. At the beginning of simulation, particles are usually placed regularly, thus minimizing the white noise. If the particles are cold, when clustering is weak (linear), particles are moving coherently and the shot noise is still largely suppressed (because particles are not random in space); with clustering increasing, the shot noise become even less important. But in the simulation of hot particles, the behavior appears much more significant for their thermal motions. Because they have a component of random motion, they become more or less randomly distributed in space on scale of the random motion. We should point out that the power spectra shown in the figures are superpositions of physical clustering and white noise. This is why the power spectrum of neutrinos appears to grow much faster than that of cold particles, especially at large $z$. But this does not mean that neutrinos really cluster faster than cold dark matter.

The shot noise effect has little influence on our final conclusions made in the paper. In fact, we did another simulation which has two times more hot particles. We found that the shot noise effect of hot particles is much less significant at $z = 4$, 2, and 1, as expected. But anyway, this effect is still showing up. However, the most interesting point is that the distributions of cold particles in the two simulations are identical at all redshifts (difference in $P(k)$ never exceeds 1%) and that the distributions of hot particles are nearly identical at the final stage of the simulations (difference in $P(k)$ is only $2\%$ at $k = k_N$). By the way,
the fact that shot noise does not amplify clustering in the N-body simulations was noticed by Efstathiou et al. (1985) and Davis et al. (1985).

From the above tests, we expect that, in the L60 simulations of Model I and Model II, our neglect of a hot component overestimates $P_c(k)$ by $\sim 5\%$. And since the scaling $P(k, z) \propto (1 + z)^{-2}$ for getting $P(k)$ at $z = 8$ underestimates $P_c(k)$ by $\sim 15\%$, the net systematic errors arising from the two effects amount to $10\%$ in the power spectrum measurement. This error is smaller than the fitting error of $P(k)$ in Holtzman (1989). At the present, we are unclear how large an error these two effects will lead to in the simulations of Model III. The error should be larger, because of the smaller neutrino mass. As pointed out before, this model is not favored by particle physics. We will consider this model as a phenomenological model described by the power spectrum used in this paper.

2.3 A simple biasing prescription

Since the amplitude of the initial density power spectrum is normalized by the COBE quadrupole observation, the clustering strength of the underlying mass at the present time is uniquely determined. Using the two-point correlation function $\xi$ as a clustering measure, we found that the mass particles in all three models are less clustered than the galaxies in the universe (see §3.2). We therefore introduce a biasing mechanism to select galaxies in our simulation, so that the two-point correlation functions $\xi(r)$ of the simulated galaxies have amplitudes $r_0$ similar to those of the observed galaxies (Groth & Peebles 1977; Davis & Peebles 1983). It should be pointed out that the biasing mechanisms in galaxy formation theories are not well established. They may depend on many complicated physical processes (e.g., Dekel & Rees 1987). To identify galaxies in our simulation, we follow the simple but plausible prescription of White et al. (1987, hereafter WFDE). The key idea of this prescription is that only peaks above a certain threshold $\nu_s$ of the density field smoothed on galactic scales ($r_s$) will eventually evolve into galaxies observed today (e.g., Kaiser 1984; Bardeen et al. 1986; the smoothed field will be called $F_s$ below). Because the particle distribution is not resolvable on the scale $r_s$ in the L240 simulations, we cannot identify the density peaks (or ‘galaxies’) directly. Instead, we first smooth the density field [given by $P(k)$] on a much larger scale $r_b$ ($\gg r_s$) to produce a background field $F_b$ which is able to be resolved by the simulation, and then use the analytical formula of Bardeen et al. (1986) to calculate the expected peak density $n(\geq \nu_s)$ of $F_s$ in the vicinity of a point where the density contrast of $F_b$ is known. In implementing this method, we obtain first the density contrasts of $F_b$ on the $128^3$ grid points and then the expected number of peaks on each grid point. Because particles are uniformly placed on centers of meshes before being perturbed, the expected peak number $N_p$ associated with each particle is just the
sum of the expected peak numbers of eight neighbouring grid points.

In our simulations, we have used a Gaussian window of width $r_s = 0.54 \, h^{-1}\text{Mpc}$ to produce the smoothed field $F_s$. The background field $F_b$ is generated by smoothing the original density field using a top-hat window of radius $k_b = 0.72 \, h \, \text{Mpc}^{-1}$ in $k$-space [i.e., $W(k) = 1$ for $k \leq k_b$; $W(k) = 0$ for $k > k_b$]. As emphasized in WFDE, using the top-hat window ensures that the clustering properties of the peaks identified above are asymptotically the same as those of the real peaks of $F_s$. The parameters $r_s$ and $k_b$ are the same as in WFDE. The $\nu_s$ values are adjusted such that the correlation length $r_0$ of peaks (i.e. $\xi(r_0) = 1$) is about $6.0 \, h^{-1}\text{Mpc}$ (Davis & Peebles 1983; de Lapparent, Geller & Huchra 1989). The $\nu_s$ values thus determined are 0.0 for Model I, 0.6 for Model II, and 0.8 for Model III. The mean number densities of peaks are 0.019 $h^3\text{Mpc}^{-3}$ (Model I), 0.015 $h^3\text{Mpc}^{-3}$ (Model II), and 0.016 $h^3\text{Mpc}^{-3}$ (Model III), all comparable to the observed density of galaxies brighter than $M_{b,j} \approx -17.7$ (e.g., Loveday et al. 1992). Our choices for the amount of biasing (i.e. $\nu_s$) are only for fitting the observed correlation function of galaxies. No attempt has been made to find any physical argument to support these choices, because physics on possible biasing processes is far unclear. The mean densities of peaks are about five times larger than that of the standard CDM simulation (WFDE), and may correspond to the density of fainter galaxies.

3. Clustering properties and comparison with observations

3.1 Power spectrum and evolution

As an example, Figure 3 shows the evolution of the density power spectrum $P(k)$ of Model I in one realization of the L240 simulation. The choice of this realization is arbitrary, and the features shown by this example are typical. We calculate the power spectrum from the particle distribution at redshift $z = 8$, 4, 2, 1, and 0 respectively. The particle distribution is first converted by the CIC to a density field on 128$^3$ grid points, then the $P(k)$ is obtained by Fourier transformation of the density field. Beyond the Nyquist wavenumber, the $P(k)$ is meaningless because it is seriously affected by the ‘alias’ effect and the discreteness of particles. The results are shown in the figure by different types of symbols for different redshifts. For comparison, we also plot the power spectrum predicted by the linear perturbation theory (solid lines). The power spectrum calculated from the particle distribution at $z = 8$ is in good agreement with the input spectrum up to $k \approx k_N = 32k_0$ (where $k_0$ is the fundamental wavenumber of the simulation; $k_N$ is the particle Nyquist wavenumber in the simulation), indicating that the initial simulation conditions
have been properly generated. At $z \gtrsim 2$, the density perturbations follow the linear perturbation theory for $k \lesssim k_N$. Thereafter, non-linear effects become more and more important on scales $\gtrsim k_N$. At the present epoch ($z = 0$), the density perturbations depart from the linear prediction for $k \gtrsim 6k_0$, showing the importance of non-linear evolution on these scales.

The power spectrum at the present time can be related to many observational quantities, as we shall see below. The evolved power spectrum obtained from each ensemble of the simulations is presented in Figure 4, with the error bars showing the $1\sigma$ scatters between the five realizations. The open and filled circles show the results of the power spectra of the mass-density field, $P_m(k)$, of the L240 and of the L60 simulations respectively. Below the Nyquist wavenumber ($k_{N2} = 0.84 h \text{Mpc}^{-1}$) of the L240 simulations, the spectra of the L240 simulations agree quite well with those of the L60. Above $k_{N2}$, the power spectra $P_m(k)$ of the L240 simulations are smaller than those of the L60 simulations, which is partly due to the force softening in the N-body code and partly due to the regular placement of particles on the grid points in generating the initial conditions. Combining these two sets of simulations, we can therefore study the clustering properties of the models on a scale $k \lesssim k_{N1} = 3.3 h \text{Mpc}^{-1}$ (where $k_{N1}$ is the Nyquist wavenumber of the L60 simulations).

We have also applied the power spectrum analysis to the spatial peak distributions. In the following calculations, we shall mainly consider the density field of peaks. The peak-density fields are generated by the CIC assignment of peaks to $128^3$ grid points. Their power spectra, $P_p(k)$, are depicted in Figure 4 by triangles. The results of the L240 simulations are shown by open symbols, and those of the L60 by filled ones. For clarity, the peak power spectra of $P_p(k)$ have been shifted by a factor of 10 in the figure. Compared with the power spectra $P_m(k)$ of the mass distributions, the $P_p(k)$ are amplified by roughly a constant factor $b^2 > 1$ ($b$ is usually called “the biasing factor”). This means that the linear biasing assumption is approximately valid in the peak scheme adopted here. For the peak height thresholds we specified, the values of $b$ are about 1.2 in Model I, 1.5 in Model II, and 1.6 in Model III. It is interesting to note that these values are consistent with recent observational results (Yahil 1988; Kaiser et al. 1991; Lahav & Kaiser 1989).

Comparing the evolved spectra with those predicted by the linear perturbation theory (the dotted lines in Fig. 4), one sees that the effect of the nonlinear evolution is obvious for $k > 0.15 h \text{Mpc}^{-1}$. We then fit the evolved spectra by simple analytical formulae up to wavenumber $k_{N1}$, the resolution limit of the L60 simulations. The formulae we use have asymptotically the same functional form as the linear power spectra for $k < 0.1 h \text{Mpc}^{-1}$, and are described by a power-law $k^{-\alpha}$ with $\alpha \approx 1.3$ for $k \gtrsim 0.5 h \text{Mpc}^{-1}$. The results of
the fit are shown by the solid lines in Figure 4 (valid for $k \lesssim k_{N1}$). These fit curves will be used in the following subsections.

3.2 Two-point correlation functions

The two-point correlation functions $\xi(r)$ of the simulation particles and of the peaks are plotted in Figure 5 for the three models. The squares show the results for the peaks, and the circles represent the results for the mass particles. The functions $\xi(r)$ of the L60 simulations are shown by the filled symbols, and those of the L240 by the open ones. In calculating the peak-peak correlation function, we use the peak number to weight the pair counts. Similar to the power spectra (§3.1), the correlation functions of the peaks are roughly a factor $b^2$ higher than those of the underlying mass. The functions $\xi(r)$ of the L60 and L240 simulations are in reasonable agreement on intermediate scales. Because of the resolution limitation in the simulations, clustering is suppressed on small scales, as shown by the flattening of $\xi(r)$ on $r \lesssim 1 \, h^{-1}\text{Mpc}$ in the L240 simulations and on $r \lesssim 0.3 \, h^{-1}\text{Mpc}$ in the L60 simulations. $\xi(r)$ of the L60 simulations are smaller than those of the L240 simulations on scales $r \approx 10 \, h^{-1}\text{Mpc}$, which is probably caused by the suppression of the non-linear clustering development on scales close to the simulation box size and by the truncation of the power on scales larger than the size of the box.

In all three models, the correlation functions have approximately a power-law form $r^{-\gamma}$ with $\gamma \approx 1.7$ for $0.5 \lesssim r \lesssim 10 \, h^{-1}\text{Mpc}$ (see the dashed line which shows the power law $r^{-1.8}$). This is true for both the underlying mass and the peaks. The correlation lengths $r_0$ [defined as the scale where $\xi(r)$ is 1] of the mass particles are about 4.8, 4.0, and 3.0 $h^{-1}\text{Mpc}$ for Model I, Model II and Model III respectively. The corresponding values for the peaks are 6.3, 6.4, and 5.9 $h^{-1}\text{Mpc}$. The results for the peaks, by our design, are in agreement with the observed correlation function of galaxies (Davis & Peebles 1983; de Lapparent et al. 1989). The data of MESL also suggest that the correlation length of galaxies is around 6 $h^{-1}\text{Mpc}$ (see below).

The two-point correlation function and the power spectrum form a Fourier transform pair. Given a power spectrum, one can easily calculate its correlation function. The solid and the dotted curves in Figure 5 show $\xi(r)$ for the peaks and for the underlying mass calculated from the power spectra shown in Figure 4 (the solid curves). In these calculations, we have included the softened part of the power spectrum $P(k)$ on scales $k \geq k_{N1}$, because the correlation functions $\xi(r)$ estimated from the pair counts have also been affected by the same amount of softening. $\xi(r)$ obtained from $P(k)$ remains positive up to about $44 \, h^{-1}\text{Mpc}$ in Model I, $51 \, h^{-1}\text{Mpc}$ in Model II and $60 \, h^{-1}\text{Mpc}$ in Model III. However, $\xi(r)$ is difficult to detect below 0.01 from direct pair counting, because of
the integral constraint \( \int_0^\infty \xi(r)r^2dr = 0 \) and the lack of clustering power outside of the simulation box.

The angular two-point correlation function \( \omega(\theta) \), determined from large deep surveys of galaxies, challenges strongly the standard CDM model (MESL; Picard 1991; Collins et al. 1992). Here we calculate \( \omega(\theta) \) for the hybrid models. We convert the peak correlation functions \( \xi(r) \) (the solid curves in Figure 5) to \( \omega(\theta) \) by using the relativistic version of the Limber equation (Peebles, 1980 §56). An extrapolation by the power law \( \propto r^{-1.7} \) is made for \( \xi(r) \) on scales \( r \lesssim 0.4 h^{-1}\)Mpc, to compensate for the softening effect in the simulations. The extrapolation is important only on very small angular scales \( \theta \lesssim 0.07^\circ \). In order to compare the theoretical \( \omega(\theta) \) with the APM observation, the luminosity function and its evolution model given by MESL are used here. Furthermore, the clustering evolution is assumed to be stable, i.e., \( \xi(r, z) \propto (1 + z)^{-3} \), as in MESL. In Figure 6, we show the angular correlation function \( w(\theta) \) of the hybrid models scaled to the Lick catalog depth (the solid curves). The APM observational data are also plotted for comparison. Of the three models, Model III has the strongest and Model I the weakest power on large angular scales, as expected because the correlation functions \( \xi(r) \) of peaks are essentially normalized on scales \( r \lesssim 10 h^{-1}\)Mpc. Model III is clearly in good agreement with the \( \omega(\theta) \) of the APM survey; Model II is acceptable, considering that the data for \( \omega(\theta) < 0.01 \) may be not reliable due to the possible plate-to-plate sensitivity variation in the survey (MESL). Model I does not have sufficient power on large scales to explain the APM data.

### 3.3 Count-in-cell statistic of galaxies

Another important observational result of the large scale structure comes from the count-in-cell statistic. Assuming that the window function for counting is \( W(\vec{r}) \), the variance \( \sigma^2 \) of the counts \( N \) in cells can be easily related to the two-point correlation function \( \xi(r) \):

\[
\sigma^2 = \frac{1}{[\int_{V_\mu} W(\vec{r})d\vec{r}]^2} \int \xi(|\vec{r}_1 - \vec{r}_2|)W(\vec{r}_1)W(\vec{r}_2)d\vec{r}_1d\vec{r}_2, \tag{1}
\]

or equivalently to the power spectrum \( P(k) \):

\[
\sigma^2 = \frac{V_\mu}{(2\pi)^2} \int P(k)|W(\vec{k})|^2d\vec{k}, \tag{2}
\]

where \( W(\vec{k}) = \int_{V_\mu} W(\vec{r})e^{-i\vec{k} \cdot \vec{r}}d\vec{r}/[\int W(\vec{r})d\vec{r}]^2 \) and \( V_\mu \) is a sufficiently large rectangular volume in which the \( P(k) \) is measured.
Efstathiou et al. (1990, hereafter E90) and Saunders et al. (1991) have applied this statistic to the QDOT redshift survey of IRAS galaxies. E90 used cubic cells, i.e.,
\[ W(\vec{k}) = \frac{\sin(\frac{1}{2}k_x) \sin(\frac{1}{2}k_y) \sin(\frac{1}{2}k_z)}{(\frac{1}{2}k_x)(\frac{1}{2}k_y)(\frac{1}{2}k_z)}, \]
where \( l \) is the side length of a cell. Saunders et al. (1991) used Gaussian windows for their measurement. Since the two observations are closely related and are based on the same database, their constraints on theoretical models are quite similar. Therefore we test the hybrid models only against the measurement of E90.

E90 measured the variance \( \sigma^2 \) using cubic cells of \( l = 10, 20, 30, 40, \) and \( 60 \, h^{-1}\text{Mpc} \). Their results are shown in Figure 7. To see the predictions of the hybrid models for the variance, we first calculate \( \sigma^2(l) \) analytically by Eqs. (2) and (3) with the power spectra \( P(k) \) given by the fits in Figure 4 for galaxies. The \( \sigma^2(l) \) predicted are shown in Figure 7 by dotted lines. The models have higher \( \sigma^2(l) \) at \( l \approx 10 \, h^{-1}\text{Mpc} \) and faster decrease of \( \sigma^2(l) \) with \( l \) than the observation. However E90 measured \( \sigma^2 \) in redshift space. Redshift distortion can reduce the clustering on small scales and enhance that on large scales (Peebles 1980; Kaiser 1987), thus making \( \sigma(l) \) look more flat. Therefore we apply the statistic
\[ \sigma^2 = \frac{\langle N^2 \rangle - \langle N \rangle^2 - \langle N \rangle \langle N \rangle}{\langle N \rangle^2}, \]
directly to our simulations, to measure \( \sigma(l) \) for the distributions of peaks in redshift space, where \( N \) is the number of galaxies in a cell. When we transform the particle positions from real space to redshift space, we simply assume that the ‘observer’ is at the center of the simulation box and is at rest in the comoving frame. We have not attempted to choose an ‘observer’ having the same environs as ours (e.g., the same peculiar velocity relative to the comoving frame, Virgo cluster being around, etc.), though the statistic may be influenced by the choice of the observer. The results are given in Figure 7 as the solid lines. Indeed, the redshift distortion amplifies \( \sigma^2(l) \) by a constant factor on scales \( l \geq 20 \, h^{-1}\text{Mpc} \), with an amplification factor consistent with the prediction of Kaiser (1987). For \( l \) around \( 10 \, h^{-1}\text{Mpc} \), the scales where the clustering transits from the nonlinear to the linear regime, the values of \( \sigma^2(l) \) are not affected strongly by the redshift distortion. The values of \( \sigma^2(l) \) predicted by the models are higher than the observational results. However since the peaks in the simulations are selected to match the clustering strength of optical galaxies, and IRAS galaxies may be less strongly clustered (e.g., Lahav et al. 1990; Saunders et al. 1992), the discrepancy can be explained. Since it is not quite certain how much IRAS galaxies are biased with respect to optical galaxies, we simply shift each
solid line in Fig.7 downward by a constant factor \( (b_{OI}^2) \) to fit the observational upper limit of \( \sigma^2(l) \) at \( l = 10 \, h^{-1}\text{Mpc} \). These are the dashed lines in the figure. The value of \( b_{OI} \) is 1.3 for Model I, 1.5 for Model II and 1.4 for Model III. It is interesting to note that the relative biasing parameter for optical and IRAS galaxies obtained by Lahav et al. (1990) is about 1.7. Compared with the standard CDM model (the open squares), the hybrid models have much more power on large scales. Furthermore, Model II and Model III give better fits to the observation than Model I. The results of these three models are consistent with all of the observed data points except the one at \( l = 40 \, h^{-1}\text{Mpc} \), where the models still do not have sufficient power to reproduce the high \( \sigma^2 \) value. Should this observed value not be a statistical fluke, we would need more excess power on large scales than the hybrid models have. Furthermore the power spectrum required would be quite different from the predictions of the CDM and the hybrid models, because these models produce smooth curves of \( \sigma^2(l) \) without the type of bump at \( l = 40 \, h^{-1}\text{Mpc} \) seen in the data. As Fisher et al. (1993) recently pointed out, the value is significantly in excess of the variance determined from their 1.2 Jy IRAS redshift survey.

3.4 The bulk motions on large scales

Our local universe within a distance of \( < 50 \, h^{-1}\text{Mpc} \) was found to move coherently at a velocity of \( \sim 600 \, \text{kms}^{-1} \) in the direction of Centaurus (Lynden-Bell et al. 1988; Faber & Burstein 1989). This is one piece of evidence for the existence of more large-scale clustering power in the universe than given by the standard CDM model (e.g., Vittorio, Juszkiewicz, & Davis 1986). A recent reanalysis of the Lynden-Bell et al.’s data, using a reconstruction method, gives a bulk motion velocity of \( 388 \pm 67 \, \text{kms}^{-1} \) toward \( L = 177^\circ \), \( B = -15^\circ \) for a sphere of radius \( 40 \, h^{-1}\text{Mpc} \) around the Local group (LG), and a bulk motion of \( 327 \pm 87 \, \text{kms}^{-1} \) toward \( L = 194^\circ \), \( B = 5^\circ \), for a sphere of radius \( 60 \, h^{-1}\text{Mpc} \) (Bertschinger et al. 1990).

The relation between the bulk motion \( V(R) \) and the density power spectrum \( P(k) \) has been extensively discussed in Juszkiewicz et al. (1990, JVW) and Lahav et al. (1990, LKH). Because the observational region is centered on the LG, it seems reasonable to study relevant statistics under the condition that the central parts in the theoretical models have the same peculiar velocity as the LG†. The peculiar velocity of the LG is \( 622 \, \text{kms}^{-1} \) according to the COBE observation (Smoot et al. 1991). We model the LG by a top-hat

\[† \text{ The condition may still be insufficient for an ‘appropriate’ observer, since our location in the Universe is so specific. Several rich clusters and large voids around us may significantly influence measurement of the bulk motions.} \]
window of radius $5\,h^{-1}\text{Mpc}$. All information about bulk motion $\vec{V}(R)$ is thus contained in the conditional probability distribution function of $\vec{V}(R)$, which depends only on $P(k)$ (JWV; LKH). Using the formulae derived by JWV and LKH, and using the density power spectra given in §3.1, we show in Figure 8 the predictions of the hybrid models for the bulk motion. The solid lines are the rms bulk velocities of the models. For these three models, the predictions for the bulk velocity are indistinguishable and all agree well with the statistical results of Bertschinger et al. (1990). This result, plus the so-called ‘cosmic variance’ which is given by the theoretical upper and lower limits of the 95% confidence level (dashed lines), means that the constraint on models given by the present observations of bulk motion is very weak.

3.5 The pairwise velocity dispersion on small scales

The pairwise velocity dispersion around $1\,h^{-1}\text{Mpc}$ has been considered to be a strong test of galaxy formation theories. As in the observations, we calculate the one dimensional velocity dispersion, i.e., the rms relative velocity $\langle v^2_|| \rangle^{1/2}$ along the connection lines of pairs. We would like to point out that the observational value depends to some degree on the model assumed for the pairwise velocity distribution (cf. Peebles 1980). To have a precise comparison between the model and the observation, we should use exactly the same procedure to estimate the dispersion in the simulation as in the observation. Here we ignore the error possibly caused by the model of the pairwise velocity distribution. Only the L60 simulations are used for this analysis. In the simulations, each particle carries less than one peak, that is, one ‘galaxy’ consists of more than one particle. The pairwise velocity dispersion of ‘galaxies’ could be very sensitive to the procedures used to identify them (Couchman & Carlberg 1992; K93). Since we are unclear which identification procedure is really correct, here we have measured the velocity dispersion of the mass and assume that it is equal to the velocity dispersion of galaxies (cf. Couchman & Carlberg 1992). In Figure 9, we present $\langle v^2_|| \rangle^{1/2}$ for the three models. Again, error bars are the 1σ scatters between the five realizations. $\langle v^2_|| \rangle^{1/2}$ is between 800 and 1100 kms$^{-1}$ in Model I, between 700 and 950 kms$^{-1}$ in Model II, and between 600 and 800 kms$^{-1}$ in Model III. All of these values are significantly higher than the observational values: $340 \pm 40$ kms$^{-1}$ for the CfA survey (Davis & Peebles 1983); $250 \pm 50$ kms$^{-1}$ for the AAT pencil-beam samples (Bean et al. 1983); $\sim 400$ kms$^{-1}$ for the CfA slice survey around the Coma cluster (de Lapparent, Geller & Huchra 1988); and $300 \pm 100$ kms$^{-1}$ for the KOS pencil-beam survey (Efostathiou & Jedredjewski 1984). However there were two statistical studies which gave a much larger velocity dispersion for galaxies. Efostathiou & Jedredjewski (1984) got $540 \pm 100$ kms$^{-1}$ for the KOSS pencil beam survey, and Hale-Sutton et al. (1989) obtained $600 \pm 140$ kms$^{-1}$
for the AAT sparse pencil-beam survey. The last two values are in agreement with the predictions of Model II and Model III.

We point out here that almost all N-body simulations, carried out to date, give high values of the pairwise velocity dispersion. The standard CDM model without bias predicts \( \langle v^2 \rangle^{1/2} \geq 1000 \text{ km s}^{-1} \) (e.g., Davis et al. 1985; Couchman & Carlberg 1992). Even the standard CDM model with a high bias \( b_g = 2 \sim 2.5 \) (e.g., Davis et al. 1985; Park 1991) or a low density (\( \Omega = 0.2 \)) CDM model (Kauffmann & White 1992), predicts a \( \langle v^2 \rangle^{1/2} \) value between 400 and 600 km s\(^{-1}\). As Gelb et al. (1993) showed recently, the pairwise velocity dispersion is very sensitive to the bias parameter; \( \langle v^2 \rangle^{1/2} \approx 500 \text{ km s}^{-1} \) requires the \( \text{rms} \) density contrast \( \sigma_8 \) in a sphere of radius \( 8h^{-1}\text{Mpc} \) to be 0.5 in both the flat CDM and the hybrid (with a contribution of neutrinos of less than 30% in mass) models. For the three models studied here, the values of \( \sigma_8 \) are 0.86 for Model I, 0.73 for Model II and 0.63 for Model III. All are larger than 0.5. The high values of \( \langle v^2 \rangle^{1/2} \) in these models are therefore expected.

It may also be possible that some of the statistical results for the galaxy pairwise velocity dispersion listed above represent only a lower limit of the true value. As pointed out by Mo, Jing & Börner (1993; hereafter MJB), the statistical result of the galaxy pairwise velocity dispersion is very sensitive to sampling effects, such as the correction of the Virgocentric infall in the CfA sample and the exclusion of Coma region from the CfA slice. Without these corrections, MJB found a value 450 km s\(^{-1}\) for the CfA survey and \( \sim 1000 \text{ km s}^{-1} \) for the CfA slice sample. MJB have also analyzed the Southern Sky Redshift Survey (da Costa et al. 1991) and the 2Jy IRAS galaxy survey (Strauss et al. 1992). The pairwise velocity dispersions they found are \( \sim 400 \text{ km s}^{-1} \) for the SSRS sample and \( \sim 350 \text{ km s}^{-1} \) for the IRAS sample. From these results, they obtained \( \langle v^2 \rangle^{1/2} = 500 \pm 50 \text{ km s}^{-1} \). Therefore the discrepancy between the predictions of our models (especially Models II & III) and the observation is alleviated.

The density power spectra in the simulations are normalized by the COBE quadrupole result. Taking the lower limit allowed by the uncertainty (\( \sim 25\% \)) in the quadrupole measurement, one can expect to find a pairwise velocity dispersion of \( \sim 500 \text{ km s}^{-1} \) in Model II and Model III, in good agreement with the observation. The modest reduction of the amplitude of the power spectrum \( P(k) \) has, however, little influence on the results of the previous sections. As discussed in JMBF, this reduction can also improve the mass function of rich clusters in Model II and Model III.

### 3.6 Filaments, bubbles and voids

Recent observations have revealed a variety of structures in the universe (Geller &
Huchra 1988; references therein). Although the morphology of these structures has potential importance for the theories of cosmogony, it is difficult to assess the statistical significance from just a few individual structures (e.g., voids, filaments, superclusters, bubbles etc.). In this section, we do not intend to set any constraints on the models from such observations, because we think only well defined and objective statistics are useful to constrain models. Instead, we just show a few simulation slices of Model II, to give a qualitative impression on the frothiness of the hybrid models.

In Figure 10, for example, we show the spatial distributions of galaxies and the underlying mass in two square slices of dimension $60 \times 60 \ (h^{-1}\text{Mpc})^2$ and thickness $18 h^{-1}\text{Mpc}$ in the L60 simulation. There are about 900 galaxies and 20,000 mass particles in each slice. Since in the L60 simulation the peak number carried by each mass particle is less than 1, the galaxies can be selected by a Monte Carlo experiment, with the selection probability equal to the peak number of each particle (see, e.g., Weinberg & Gunn 1989). The number of galaxies thus selected is the same as the number of peaks, and the correlation function of the galaxies was found to be the same as that given by particle pairs weighted by peak numbers (§3.2). There are originally $\sim 70,000$ mass particles in each of the slices. For clarity we have only plotted 30% of them. We use thick dots to denote galaxies and use thin points to denote mass particles. From the plots, it is obvious that the galaxies delineate the structures of the mass distribution, and that the distribution of galaxies is more clumpy than that of the underlying mass (the biasing factor is $\sim 1.5$, see §3.1). In the left panel one finds prominent filaments with lengths $\sim 50 h^{-1}\text{Mpc}$. In between the filaments one finds significant underdensity regions (voids) of similar sizes. The spatial distribution shown in the right panel looks much smoother. Here one finds many small voids with sizes of about $10 - 20 h^{-1}\text{Mpc}$, surrounded by filaments or/walls.

In Figure 11, we show a redshift slice of the galaxy distribution, which is from one realization of the L240 simulation. For this plot, we assume that the ‘observer’ is sitting at the center, and the richest cluster in the simulation has equatorial coordinates $\alpha = 13^h$, $\delta = 29^\circ$, and redshift $cz = 8,500\ \text{kms}^{-1}$, the position of the Coma cluster in the real universe. The slice has its boundary specified by the declination range $26.5^\circ \leq \delta \leq 38.5^\circ$ and by the depth $cz = 12,000\ \text{kms}^{-1}$. The sample of galaxies is constructed by using a radial selection function which corresponds to an apparent- magnitude limit $m_B = 15.5$. These selection criteria mimic the CfA slice survey of de Lapparent, Geller & Huchra (1988). However unlike the observation, the sample does neither have a boundary in right-ascension nor take into account the galactic extinction effect. There are pronounced large empty regions around the rich cluster, The galaxy distribution in this slice looks quite similar to that in the CfA sample, having rich clusters surrounded by large voids.
4 Discussion and Conclusions

The N-body simulations carried out are for the purpose of testing the models in more detail. They are not designed to probe the model-parameter space allowed by present observations. One can perhaps find many hybrid models compatible with the observations, by tuning model parameters such as the cosmological density parameter $\Omega_0$, the Hubble constant $H_0$, the relative fractions of CDM, HDM and baryonic components, the shape of the primordial density spectrum, and the statistic (Gaussian or non-Gaussian) of the primordial density fluctuations. The models studied in this paper are favored by an inflationary universe, and by previous studies based on the linear and non-linear calculations.

The success of Model II and Model III in matching the observed large-scale clustering and motions of galaxies demonstrates that the shape of the initial density spectrum for structure formation in the universe is very close to those given by these models. However, it is very difficult to assess how serious one should take these specific models. Model III is not favored by the current experimental data of neutrino masses. However, the assumptions made on the species and masses of the dark matter particles in Model II are not in conflict with currently popular theories of particle physics, but there is still no compelling evidence for this model. But since the constraints from particle physics are not stringent, the possibility for the dark matter to have the properties assumed may not be very small. Although the two models have very different assumptions on the properties of the dark matter, their transfer functions are very similar. This means that the initial density spectrum given by these specific models may represent those of a class of (dark-matter) models, and therefore may have more validity than the models themselves.

Galaxy formation may be a potential problem for the hybrid models. Since these models have less power on small scales than the standard CDM model, galaxy formation at high redshift is less efficient. Indeed, our simulation show that significant nonlinear evolution occurs only after $z \approx 2$. It is not known if galactic-scale perturbations can collapse early enough to account for the existence of quasars and radio galaxies observed at $z > 2$. As discussed by Melott (1990; see also Buchert & Blanchard 1993), if quasars (and radio galaxies) are objects as rare as they are observed to be, the constraint imposed by the galaxy-formation time is not very stringent even in a pure HDM model. To study galaxy formation in the hybrid model in more detail, it is necessary to have simulations which incorporate the streaming motion of neutrinos and the hydrodynamical processes of baryons on small scales. Such a simulation is inevitably much more difficult to carry out than that for the CDM model. Until now only few pioneer studies have been done (e.g. Davis et al. 1992; K93). It is fair to say that galaxy formation in hybrid models is still an
open question.

Without a proper understanding of the physical processes of galaxy formation, any biasing procedure for identifying ‘galaxies’ in simulations is not well justified. The procedure we used here is based on the biasing formalism of Bardeen et al. (1986), and is very similar to that of WFDE for the simulation of the standard CDM model in large boxes. For the standard CDM model, it is known that the ‘galaxies’ selected in this way correspond well to the real density peaks in high-resolution simulations (Park 1991). Presumably this result is also true for the hybrid models, for the structure formation here is still a bottom-up gravitational hierarchy. But to justify this needs simulations of high resolution. However since the biasing factor is determined by the mass correlation function (which is, in turn, uniquely determined by normalizing the initial density spectrum to the COBE measurement of the MBR fluctuation) and the observed two-point correlation function of (bright) galaxies, the value of the biasing parameter obtained for these models should be reliable, unless the COBE result is seriously in error. Our results suggest that a modest bias of galaxy formation is required for these models to match both the COBE result and the galaxy clustering on small scales. An interesting question is whether or not such a bias can be naturally achieved by the galaxy formation process in these models.

Keeping the above discussed questions in mind, we now summarize our main conclusions as follows.

1. Hybrid models of the universe which contain (in mass) \( \sim 30\% \) HDM, \( \sim 70\% \) CDM and \( \geq 1\% \) baryons, when normalized to the COBE quadrupole result of the MBR, provide a reasonable fit to the observed galaxy clustering and motion of the universe on large and intermediate scales.

2. For the hybrid models to match both the COBE results and the observed two-point correlation function of galaxies, a modest bias, with a biasing parameter \( b_g \sim 1.5 \), is needed. This value is in good agreement with observations.

3. The constraints from different observations of galaxy clustering and motions are consistent with each other, supporting that the large-scale structure of the universe is from the gravitational instability of the initial density fluctuations which have a power spectrum very close to those predicted by the hybrid models.

4. The most stringent constraint on the models comes from the angular-correlation function \([w(\theta)]\) of galaxies of the APM survey (Maddox et al. 1990), and from the count-in-cell function \([\sigma^2(l)]\) of IRAS galaxies of the QDOT survey (e.g. Efstathiou et al. 1990). Of the three models we studied (see §2 for definition), Model III is in good agreement with the results of the APM survey; Model II is acceptable, considering the possible selection effects in the survey; Model I does not have sufficient power of galaxy clustering on large...
scales to match the APM data. The three models are consistent with the statistical results of the count-in-cell variance $\sigma^2(l)$, except for the observed data of $\sigma^2(l)$ at $l = 40 \, h^{-1}\text{Mpc}$ which are too high.

5. All of the three models are consistent with the observations of the bulk motions of galaxies, with Model III giving the best fit. The constraint given by this observation is weak.

6. If the velocity bias is negligible, the pairwise velocity dispersion $\langle v^2 \rangle^{1/2}$ in the models is uniquely determined by the normalization to the COBE quadrupole measurement. The values of $\langle v^2 \rangle^{1/2}$ are between 800 and 1100 kms$^{-1}$ in Model I, between 700 and 950 kms$^{-1}$ in Model II, and between 600 and 800 kms$^{-1}$ in Model III. These values are still larger than the recent statistical result $\sim 500$ kms$^{-1}$ obtained by MJB based on various redshift surveys of galaxies. However, the discrepancy is not severe, especially for Model II and Model III. Using a modestly reduced (by $\sim 15\%$) amplitude of the MBR quadrupole (which is within the COBE detection uncertainty) to normalize models, one can pull the model prediction down to $\sim 500$ kms$^{-1}$. This slight change in normalization does not influence other conclusions of the paper.

7. Voids of the size of Bootes, filaments of the size of the Great Wall are easily produced in the hybrid models. The morphology of the large-scale structure resembles well that revealed by observations.

8. To have a complete discussion on our simulations, we also summarize the results of JMBF on the clustering properties of rich clusters in the models. They found that both the coherence length and the correlation length of the cluster-cluster correlation function in the hybrid models are larger than in the standard CDM model, and increase systematically with the power of the initial density spectrum on large scales. The correlation length is $r_0 = (15.5 \pm 0.8), (20.0 \pm 0.8)$ and $(23.0 \pm 0.9) \, h^{-1}\text{Mpc}$ for Model I, Model II and Model III, respectively. The corresponding coherence lengths are about 40, 50, 70 $h^{-1}\text{Mpc}$. Model III matches the observed correlation function in both amplitude and coherent length. Model II is also consistent with the observations within the theoretical and observational uncertainties. Model I gives a correlation function which is too small in both correlation amplitude and in coherent length.

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Figure captions

Fig.1. The density power spectra of the hybrid models at redshift $z = 8$, normalized by the COBE quadrupole result of the microwave background radiation.

Fig.2. The influence of the free-streaming motion on the power spectrum evolution. (a) The power spectra $P(k)$ estimated from the particle distributions of the NS simulation (long-dash curves) and the S simulation (dotted curves) at $z = 8, 4, 2, 1,$ and 0 (from the bottom to the top); (b) The power spectra $P(k)$ of cold (solid curves) and hot particles (dashed curves) in the two-component simulation at $z = 8, 4, 2, 1,$ and 0 (from the bottom to the top). The power spectra of the NS simulation are plotted (long-dash curves) for comparison. The wavenumber is in units of the fundamental wavenumber $k_0 = 2\pi/60\, h\, \text{Mpc}^{-1}$.

Fig.3. The evolution of density power spectrum $P(k)$ in one realization of the L240 simulation of Model I. Different symbols show the power spectra at redshift $z = 8, 4, 2, 1,$ and 0 [$P(k)$ is a decreasing function of $z$]. The solid lines are the predictions for linear perturbation evolution. $k$ is in units of the fundamental wavenumber $k_0 = 2\pi/240\, h\, \text{Mpc}^{-1}$.

Fig.4. The evolved power spectra of peaks (triangles) and underlying mass (circles) at redshift $z = 0$. For clarity, the spectra of the peaks are shifted vertically by a factor of 10. The open and filled symbols show the results of the L240 and of the L60 simulations respectively. The dotted lines are the linear predictions for the density power spectra. The solid lines are our fits of the evolved spectra. The fit results are sufficiently accurate for $k_{N1} < 3.3\, h^{-1}\text{Mpc}$, the resolution limit of the L60 simulation.

Fig.5. The spatial two-point correlation functions of mass (circles) and of peaks (squares). The open and filled symbols show the results of the L240 and of the L60 simulations respectively. The solid and dotted curves are the Fourier transforms of the fitted spectra.

Fig.6. The angular two-point correlation functions of galaxies in the hybrid models, scaled to the Lick catalogue depth, are compared with the observational data of the APM survey (Maddox et al. 1990; the APM data were kindly provided by S. Maddox).

Fig.7. The count-in-cell variances $\sigma^2(l)$ in cubic volumes of sides $l$. The observational data, from Efstathiou et al. (1990), are plotted as squares, together with error bars representing the 95% confidence level. The dotted curves show the variances of peaks in
real space; the solid curves show the $\sigma^2(l)$ of peaks in redshift space; the dashed curves are the solid ones shifted down so that the $\sigma^2(l)$ equals the observational upper limit at $l = 10 \, h^{-1}\text{Mpc}$. The open squares are the prediction of the standard CDM model.

**Fig.8.** The bulk motion of a sphere as a function of its radius $R$. The solid lines show the theoretical \textit{rms} values, and the dashed ones show the upper or lower limits at the 95\% confidence level. The filled squares and their error bars are from the statistical analysis of Bertschinger et al. (1990).

**Fig.9.** The pairwise velocity dispersion along the connection lines of pairs of particles, as a function of their separation.

**Fig.10.** Two examples of spatial galaxy (thick dots) and underlying mass (thin points) distributions in Model II. The distributions are projected on a rectangular slice of $60 \times 60 (\, h^{-1}\text{Mpc})^2$. The thickness of the slice is $18 \, h^{-1}\text{Mpc}$.

**Fig.11.** The galaxy distribution in a redshift slice of $26.5^\circ \leq \delta \leq 38.5^\circ$ in declination, constructed from one realization of the L240 simulation of Model II. The slice contains the richest cluster of the simulation, which is set to be at the position of Coma cluster in the universe. The distribution is subjected to the radial selection function of the CfA slice survey. The outer boundary of the slice is $12,000 \, \text{kms}^{-1}$.