Josephson current and $\pi$-state in a ferromagnet with embedded superconducting nanoparticles

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Abstract

On the basis of Usadel equations we investigate superconductor/ferromagnet/superconductor (S/F/S) hybrid systems which consist of superconducting nanostructures (spheres, rods) embedded in ferromagnetic metal. The oscillations of the critical current of the S/F/S Josephson junctions with the thickness of ferromagnetic spacer between superconducting electrodes are studied. We demonstrate that the $\pi$ state can be realized in such structures despite a dispersion of the distances between different parts of the electrodes. The transitions between 0 and $\pi$ states at some thickness of ferromagnetic spacer can be triggered by temperature variation.

(Some figures in this article are in colour only in the electronic version)

1. Introduction

The peculiarity of the proximity effect in superconductor/ferromagnet (S/F) hybrid structures is the damped oscillatory behavior of the Cooper pair wavefunction inside the ferromagnet [1, 2] (for the reviews see [3, 4]). In some ways it is a manifestation of the Larkin–Ovchinnikov–Fulde–Ferrell (LOFF) state induced in a ferromagnet (F) near the interface with the superconductor (S). In contrast with original LOFF, which is possible only in clean superconductors, the damped oscillatory S/F proximity effect is very robust and exists also in the diffusive limit. This special case of the proximity effect is at the origin of the $\pi$-junction [1, 2] which has at the ground state the opposite sign to the superconducting order parameter in the banks. The $\pi$-junction was first observed experimentally in [5] and since then a lot of progress has been made in the physics of $\pi$-junctions and now they are proving to be promising elements of superconducting classical and quantum circuits [6]. Different manifestations of unusual proximity effects and the $\pi$ states have been observed experimentally in various layered S/F hybrids [7–9]. The proximity induced switching between the superconducting states with different vorticities in multiply connected hybrid S/F structures was suggested recently in [10, 11]. Theoretical studies and experiments [12–15] both demonstrated that in the diffusive limit the spin-flip and spin–orbit scattering lead to a decrease of the decay length and an increase of the oscillating period. In addition, the spin-flip scattering may generate the temperature induced transition from the 0 to the $\pi$ state of the junction [14, 15].

Naturally at the first stage of the work on the S/F/S junctions the systems with planar geometry and well controlled F-layer thickness were considered. However, now that the $\pi$ state has proven to be very robust versus different types of impurity scattering [13, 15] (magnetic and non-magnetic) and interface transparency [16, 17] it may be of interest to address the question about how a $\pi$ junction could be realized in S/F/S systems with a poorly defined thickness of F-spacer, in particular for two superconducting particles embedded in a ferromagnet or between a flat superconducting electrode and a small superconducting nanoparticle (such a situation could be of interest for scanning tunnelling microscopy (STM)-like experiments with a superconducting tip). This question is non-trivial because the transition from 0 to the $\pi$ state occurs at a very small characteristic length [3] $\xi_{\ell} = \sqrt{D_{\ell}/R}$, where $D_{\ell}$ is the diffusion constant in ferromagnetic metal and $h$ is the
ferromagnetic exchange field, and the typical values of \(\xi_l\) are in
the nanoscopic range. We could expect that when the variation of
the distance between different parts of S-electrodes is of the
order of \(\xi_l\) the \(\pi\) state would disappear. Our calculations show
that it is not the case and once again the \(\pi\) state appears to be
very robust and the transition between \(0\) and \(\pi\) states is
always present at some distance and also can be triggered by
temperature variation.

In this paper we present the results of a theoretical study
of the peculiarities of the proximity effect and Josephson current
in S/F hybrids which consist of superconducting nanostructures placed in electrical contact with a ferromagnetic metal. The paper is organized as follows. In section 2 we
discuss the basic equations. In section 3 we calculate the Josephson current in two model hybrid S/F/S systems. The first system consists of two superconducting rod-shaped electrodes embedded in a ferromagnet. The second one is a S/F bilayer with a superconducting spherical particle
at the surface of the ferromagnetic layer. We examine the
temperature dependence of the critical current of the S/F/S
junction between the flat superconducting electrode and the S-
particle, taking into account the spin-flip scattering. For both
cases the S/F interface transparency between superconducting
nanoparticles and ferromagnet is assumed to be low to prevent
superconductivity destruction due to proximity. We summarize
our results in section 4.

2. Model and basic equations

Since the models of S/F/S junctions we are going to study consist of superconducting particles embedded in a
ferromagnetic matrix or placed on a ferromagnetic substrate,
we start from a description of the damped oscillatory behavior
of the Cooper wavefunction induced by such particles in a
ferromagnet.

We assume the elastic electron-scattering time \(\tau\) to be
rather small, so that the critical temperature \(T_c\) and
our results in section 4.

the order of \(\xi_l\) for the averaged anomalous Green’s
functions \(\tilde{G}(\omega)\) for the normal Green’s function.

The S/F interface between a particle and ferromagnet is
assumed to be characterized by the dimensionless parameter
\(\gamma_s = R_0\sigma_s/\xi_s\) related to the boundary resistance per unit
area \(R_0\). Here \(\xi_s = \sqrt{D_s/2\pi T_c}\) is the superconducting
(normal-metal) coherence length, \(\sigma_s\) and \(\xi_s\) are the normal-
state conductivities of the S- and F-metals, and \(\theta_s\) denotes
a derivative taken in the radial direction. We will assume
that the rigid boundary condition \(\gamma_s \gg \min[\xi_s\sigma_s/\xi_s\sigma_s, 1]\) is
satisfied, when the inverse proximity effect and the suppression
of superconductivity in the S-metal can be neglected [20, 21].

As a result, the pair amplitude \(F_s(r)\) at the S/F interface is equal
to the one far from the boundary:

\[
F_s(R_s) = \frac{\Delta}{\sqrt{\omega^2 + \Delta^2}} = \frac{\Delta}{\omega} G_n, \quad (6)
\]

where \(\Delta\) is the superconducting order parameter and

\[
G_n = \frac{\omega}{\sqrt{\omega^2 + \Delta^2}} \quad (7)
\]
is the normal Green’s function. Using the solution (2) or (3)
we obtain from equations (5) and (6) the value of \(F_s\) at the S/F
boundary \(r = R_s\), and the amplitudes \(A\) for both cases. Finally,
the expressions

\[
F_s^T(r) = \frac{\Delta}{\omega} G_n \frac{K_0(\rho r)}{K_0(\rho R_s)} + \gamma_s \xi_s q K_1(\rho R_s) \quad (8)
\]

describe the damped oscillatory behavior of the anomalous
Green’s function \(F_s^T\) in a ferromagnet surrounding the
superconducting cylinder or sphere, respectively.

The general expression for the supercurrent density is given by

\[
\tilde{J}_s = \frac{i\pi T \sigma_s}{4e} \sum_{\omega,\sigma = \pm} (\tilde{F}_s \nabla F_s - F_s \nabla \tilde{F}_s) \quad (10)
\]

where \(\tilde{F}_s(r, \omega) = F_s^T(r, -\omega)\).
Figure 1. Schematic representation of the F/S hybrid system under consideration: two identical superconducting cylindrical rod-shaped electrodes of radius $R_s$ surrounded by a ferromagnetic metal. The axes of the superconducting cylinders are assumed to be parallel. The figure shows the cross section of the structure by the plane $(x, y)$ perpendicular to the cylinder axis.

3. Critical current of junctions with superconducting particles

Now we proceed with calculations of the Josephson critical current for two examples of mesoscopic hybrid S/F systems. The first one is two identical superconducting cylindrical rod-shaped electrodes surrounded by a ferromagnetic metal (see figure 1). The second one is a S/F bilayer with a superconducting particle at the surface of the ferromagnetic layer (see figure 3).

3.1. S/F/S junction between two superconducting rods

Consider two superconducting cylinders of radius $R_s$ embedded in a ferromagnet, as shown in figure 1. The distance between the cylinder axes is $d > 2R_s$. These rod-shaped electrodes form a Josephson junction in which the weak link between two superconductors is ensured by a ferromagnetic neighborhood. The supercurrent

$$I_c(\varphi) = I_s \sin(\varphi)$$

(11)

flowing across this structure depends on the phase difference $\varphi$ between the order parameters of the rods:

$$\Delta_{1,2} = \Delta e^{i\varphi/2}.$$  

(12)

For large enough distance between the superconducting cylinders ($a = d - 2R_s > 2\xi_f$), the decay of the Cooper pair wavefunction in the ferromagnet in the first approximation occurs independently near either of the electrodes and can be described by the solution (8). Therefore the anomalous Green function $F_\ell(x)$ in the ferromagnet nearby the plane $x = 0$ may be taken as the superposition of the two decaying functions (8), taking into account the phase difference $\varphi$ [22]:

$$F_\ell(x, y) = \frac{\Delta}{\omega} G_n \left[ K_0(qR_s) + |\gamma_bq| K_1(qR_s) \right] e^{i\varphi/2} + K_0(qr_s) e^{-i\varphi/2},$$

(13)

where $r_s = \sqrt{(x \pm d/2)^2 + y^2}$. Using the expression (10), we obtain the sinusoidal current–phase relation (11) in the S/F/S Josephson junction between two superconducting rod-shaped electrodes for the case of low transparent S/F interfaces. For the critical current of such a Josephson structure, we have

$$I_c = \frac{2\pi T_c}{\sigma} e\sum_{a\neq 0} \frac{\Delta^2 G_n^2 R e}{m^2} \left[ K_0(qR_s) + (\gamma_bq)e^{i\varphi/2} K_1(qR_s) \right]^2 \int_0^\infty dy K_0(qr_s) K_1(qr_s) \left( \frac{r_0}{r_s} \right)^2,$$

(14)

where $r_0 = \sqrt{y^2 + d^2/4}$. In the limit of large $R_s \gg \xi_f$ a curvature of the electrodes is not essential and the formula (14) coincides with the corresponding expressions for the critical current of S/F/S layered structures with a large interface transparency parameter $\gamma_b$ previously obtained in [15]. The critical current equation (14) can be simplified for $h \gg \pi T_c$ and $R_s, r_0 \gg \xi_f$ and may be written as

$$I_c = \frac{I_0}{\xi_f} \int_{-\infty}^{\infty} dy \frac{e^{-2i(\sqrt{y^2 + d^2} - R_s)/\xi_f}}{y^2 + d^2/4} \left( \frac{\sqrt{y^2 + d^2/4} - R_s}{\xi_f} + \pi \right),$$

(15)

$$I_0 = \frac{\pi \sigma_0 \Delta^2 e F_c}{2\sqrt{\pi} \gamma_b^2 \xi_f^2} \tanh \left( \frac{\Delta}{2T_c} \right).$$

(16)

Note that our approach is valid for large enough distance between the superconducting cylinders $a > 2\xi_f$ and the first transition into the $\varphi$ state at $a_0$ is described only qualitatively. It has been demonstrated [23] that for the planar S/F/S junction with low interface transparency the first transition into the $\varphi$ state occurs at F-layer thickness smaller than $\xi_f$ (its actual value depends on the exchange field $h$ and transparency parameter $\gamma_b$). A similar situation is expected for the embedded superconducting particles and to find the corresponding interparticle distance we need to solve our problem exactly.

The dependence of the critical current $I_c$ as a function of the distance $a$ between the superconducting cylindrical electrodes calculated from equation (15) is presented in figure 2 for several values of the radius $R_s$. From the figure,
Note that equation (17) transforms into the linear equation (1) in the limit of small \( \Theta_i \ll 1 \). For simplicity we restrict ourselves to the case of thick F-layer \( (d \gg \xi) \) then the decay of superconducting order parameter occurs independently near each S/F interface. In that case, the behavior of the anomalous Green’s function near each interface can be treated separately, assuming that the F-layer thickness is infinite. Following [14, 15], the analytical solution of equation (17) for a flat transparent interface at \( z = d \) can be written as

\[
\frac{\sqrt{1 - \epsilon^2 \sin^2(\Theta_i/2)} - \cos(\Theta_i/2)}{\sqrt{1 - \epsilon^2 \sin^2(\Theta_i/2)} + \cos(\Theta_i/2)} = f_0 e^{2\eta(z-d)},
\]

where

\[
\epsilon^2 = (1/\tau_a)(|\omega| + i\hbar \text{sgn} \omega + 1/\tau_a)^{-1}.
\]

The integration constant \( f_0 \) should be determined from the boundary condition at the surface \( z = d \). As before, the rigid boundary conditions are assumed to be valid at \( z = d \):

\[
\Theta_f(d) = \arctan \frac{\Delta}{\omega}.
\]

From equations (18) and (19) we get

\[
f_0 = \frac{(1 - \epsilon^2) F_n^2}{\sqrt{(1 - \epsilon^2) F_n^2 + 1} + 1}.
\]

\[
F_n = \frac{|\Delta|}{\omega + \sqrt{\omega^2 + |\Delta|^2}}.
\]

Linearizing the solution (18) for \( \Theta_f \ll 1 \) we obtain the anomalous Green’s function in a ferromagnet \( (0 \leq z \leq d) \) induced by a flat superconducting electrode:

\[
\Theta_f \simeq \frac{4 F_n}{(1 - \epsilon^2) F_n^2 + 1} e^{\eta(z-d)}.
\]

The total anomalous Green’s function in the F-layer far from both the S/F interfaces may be taken as the superposition of the two decaying functions (9) and (22), taking into account the phase difference in each superconducting electrode

\[
F_f = \frac{\Delta}{\omega} G_n \frac{R_s e^{-q(r - R_s) + i\eta/2}}{r(1 + 2\gamma \xi_s(q + 1/R_s))} + \frac{4 F_n e^{(z-d)+i\eta/2}}{(1 - \epsilon^2) F_n^2 + 1}.
\]

where \( r = \sqrt{x^2 + y^2 + z^2} \).

To derive the general expression for the critical current \( I_c \) we have to calculate the total Josephson current flowing through a virtual surface \( \Gamma \) : the points of the surface \( \Gamma \) are equidistant from both electrodes of the junction. The surface \( \Gamma \) is a paraboloid the form of which is described by the equations:

\[
z = z_c - (x^2 + y^2)/4z_c, \quad z_c = (R_s + d)/2.
\]

Using the solution (23) and equation (10), one can arrive at a sinusoidal current–phase relation (11) with the critical current (see appendix for details):

\[
I_c = I_0 \frac{z_c R_s T}{\xi T_c} \times 
\sum_{n=0}^{\infty} \left\{ \frac{F_n(\Delta G_n/\omega) e^{-q(d-R_s)}}{\sqrt{(1 - \epsilon^2) F_n^2 + 1} + 1} (1 + 2\gamma \xi_s(q + 1/R_s)) \times \left( q \int_0^{z_c} du e^{-2\eta u}/u + z_c + 1/2 \int_0^{z_c} du e^{-2\eta u}/(u + z_c)^2 \right) \right\},
\]

where \( I_0 = 64\pi^2 T_c \sigma \xi / e \).
The dependence of the critical current $I_c$ (25) as a function of the thickness $a = d - R_s$ of the ferromagnetic spacer separating the superconducting plate and the particle is presented in figure 4 for several values of the particle radius $R_s$ and the magnetic scattering time $\tau_s$. It is clearly seen from figure 4 that with a decrease of the particle radius $R_s$, the position $a_0$ of the first zero of the critical current is shifted towards larger values of the distance $a$ between superconducting electrodes. Figure 4(b) demonstrates the influence of magnetic scattering on the proximity effect and the critical current in the S/F bilayer with the particle: decrease of the magnetic scattering time $\tau_s$ leads to decrease of decay length and increase of the oscillation period of the anomalous Green’s function $F_f$ [14]. This results in a much stronger decrease of the critical current in the S/F/S junction with increase of the thickness $a$, if the magnetic scattering time $\tau_s$ becomes relatively small $\tau_s^{-1} \gg h$.

Figure 5 shows the temperature dependence of the S/F/S junction critical current $I_c$ (25) at several values of the thickness of the ferromagnetic spacer between the superconducting electrodes. The F-spacer thickness $a$ is chosen close to the first transition from 0 to the $\pi$ state: $a \sim a_0 \approx 3\xi_f$. The nonmonotonic dependences $I_c(T)$ demonstrate the $0-\pi$ transition due to a change of the temperature. The transition temperature $T^*$ ($I_c(T^*) = 0$) seems to be very sensitive to the size of the superconducting particle. It should be noted, however, that the temperature $T^*$ is determined rather by the scale $a = d - R_s$ than by the scales $d$ or $R_s$, separately.

4. Conclusion

To sum up, we have analyzed the Josephson effect in S/F/S hybrid structures with a poorly defined thickness of F-spacer. As an example, we have calculated the Josephson current between two rod-shaped superconducting electrodes embedded in a ferromagnet or between flat superconducting electrode and the small superconducting nanoparticle at the surface of the F-layer. For both cases we have demonstrated the possibility of the realization of $\pi$ junctions in such hybrid systems. We have studied the dependence of the transitions between 0 and $\pi$ states both on the size of superconducting particles and the temperature. The $\pi$ state has been proven.
to be very robust with respect to a geometry of the S/F/S junction. In the dirty limit the transition into the π state is determined rather by the thickness of the F-spacer between superconducting electrodes than by the shape of the electrodes. Naturally our calculations can be easily generalized to the different shape of the S particles (for example spherical) with similar conclusions.

A set of the superconducting particles embedded in a ferromagnetic matrix realize a Josephson network. Depending on the geometry of this network and its state (0 or π) it may reveal a spontaneous current similar to that observed in superconducting arrays of π junctions [24]. For example, the equilibrium phase difference for a triangular two-dimensional (2D) π junction network is equal to 2π/3 which corresponds to the current state. For typical parameters of a Nb/CuNi hybrid system ($T_c = 9K$, $\xi_t = 2 nm$, $\rho_n = 1/\sigma_n \approx 60 \mu\Omega \text{cm}$ [14]) one can get from (25) the following estimate of the Josephson energy $E_J = \phi_0 I_c/2\pi c$ of the S/F/S junction: $E_J/T_c \sim 10^3 (I_c/I_0) < 1$, i.e. an observation of spontaneous currents near π is expected to be masked by strong temperature fluctuations. Despite this restriction we believe that intrinsically frustrated superconducting networks induced by the proximity effect can be experimentally observable in such S/F/S composites. In particular, a 2D Josephson network of π junctions may serve as a laboratory to study the phase transitions with continuous degeneracy [25].

The possibility of fabricating regular 2D and 3D arrays of Josephson π junctions may open interesting perspectives in the study of the very rich physics of Josephson systems. In particular, a 2D Josephson network of π junctions may serve as a laboratory to study the phase transitions with continuous degeneracy [25].

### Appendix. Josephson current in an S/F bilayer with a superconducting particle

The general expression for the supercurrent is given by equation (10), where the anomalous Green’s function $F_l$ near the surface $\Gamma$ (see figure 3) may be written as

$$F_l = A_1 e^{i q \frac{z}{2}} + A_2 e^{-i q \frac{z}{2}} \sqrt{\rho^2 + 4 z^2},$$  

(A.1)

$$A_1 = \frac{4 F_c e^{-q d}}{(1 - e^{-q d})^2 + 1},$$

(A.2)

$$A_2 = \frac{\left( A G_{\alpha}(\omega) \right)^2 R_s}{1 + \gamma_n \xi_n (q + 1/R_s)}.$$  

where $r^2 = \rho^2 + z^2$ and the functions $G_{\alpha}$, $F_c$ are determined by expressions (7) and (21), respectively. At the surface $\Gamma$ the function $F_l$ and the projection of the vector $\nabla F_l$ along the normal

$$\nabla F_l = \frac{\rho}{\sqrt{\rho^2 + 4 z^2}} \xi_0 + \frac{2 z}{\sqrt{\rho^2 + 4 z^2}} \xi_0$$  

to the surface are

$$F_l |\Gamma = \left( A_1 e^{i q \frac{z}{2}} + A_2 e^{-i q \frac{z}{2}} \sqrt{\rho^2 + 4 z^2} \right) e^{-i \frac{q z}{2}},$$  

(A.3)

$$\nabla F_l |\Gamma = \frac{2 z e^{-i q \frac{z}{2}}}{\sqrt{\rho^2 + 4 z^2}} A_2 e^{-i \frac{q z}{2}} e^{i \frac{q z}{2}}.$$  

(A.4)

Substitution of equations (A.3) and (A.4) into the expression for the supercurrent (10) and taking into account the symmetry relations $q(-\omega) = q^*(\omega), A_{1,2}(\omega) = A_{1,2}^*(\omega)$ leads to the following formula

$$\langle J_s, \mathbf{n} \rangle |\Gamma = J_c(\rho) \sin \psi,$$

(A.5)

$$J_c(\rho) = \frac{32 \pi T_\sigma}{e} \sum_{\omega > 0} \frac{\zeta^2 R_s A_1 A_2}{(\rho^2 + 4 z^2)^{3/2}} \left( q + \frac{4 z}{\rho^2 + 4 z^2} e^{-i q \frac{z}{2}} \right).$$

(A.6)

Further integration of supercurrent density (A.6) over the surface $\Gamma$

$$I_c = \int_\Gamma dS J_c = \frac{\pi}{z_c} \int_0^{2\pi} d\rho \rho \sqrt{\rho^2 + 4 z_c^2} J_c(\rho)$$

(7.7)

results in expressions (25) for the critical current $I_c$ of the S/F/S Josephson junction between superconducting plate and particle.

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