Low-Energy Signals from Kinetic Mixing with a Warped Abelian Hidden Sector

Kristian L. McDonald\(^{(a),(b)}\) and David E. Morrissey\(^{(a)}\)

\(^{(a)}\) TRIUMF, 4004 Wesbrook Mall, Vancouver, BC V6T 2A3, Canada.

\(^{(b)}\) Max-Planck-Institut für Kernphysik,
Postfach 10 39 80, 69029 Heidelberg, Germany.

Email: kristian.mcdonald@mpi-hd.mpg.de, dmorri@triumf.ca

Abstract

We investigate the detailed phenomenology of a light Abelian hidden sector in the Randall-Sundrum framework. Relative to other works with light hidden sectors, the main new feature is a tower of hidden Kaluza-Klein vectors that kinetically mix with the Standard Model photon and \(Z\). We investigate the decay properties of the hidden sector fields in some detail, and develop an approach for calculating processes initiated on the ultraviolet brane of a warped space with large injection momentum relative to the infrared scale. Using these results, we determine the detailed bounds on the light warped hidden sector from precision electroweak measurements and low-energy experiments. We find viable regions of parameter space that lead to significant production rates for several of the hidden Kaluza-Klein vectors in meson factories and fixed-target experiments. This offers the possibility of exploring the structure of an extra spacetime dimension with lower-energy probes.
1 Introduction

An interesting possibility for new physics beyond the Standard Model (SM) is a *light hidden sector*, consisting of exotic particles with masses well below the electroweak scale and very weak couplings to the SM [1 2 3]. Even though such states could have been created in a number of previous and current experiments, their rate of production can be consistent with experimental bounds provided their couplings to the SM are sufficiently small. Light hidden sectors can give rise to new and unusual signatures, and their traces might already be present in existing experimental data sets or discoverable in planned upcoming searches [4 5 6 7 8]. It is therefore important to understand the signals of viable low-energy extensions of the SM to ensure that maximal use is made of both existing and forthcoming data sets.

New physics below the electroweak scale arises in a number of scenarios extending the SM, and has been proposed as a central component of several theories of dark matter [9 10 11]. The presence of a sub-electroweak scale introduces another separation of scales beyond the usual weak/Planck hierarchy and one expects the hidden sector to contain some suitable mechanism to ensure radiative stability. The standard solutions to the hierarchy problem can be considered in this context and the sub-electroweak scale of the hidden sector could arise from supersymmetry [10], strong dynamics [12], or a warped extra dimension [13 14]. The details of the stabilization mechanism can significantly modify the resulting experimental signals since they can lead to very different particle spectra below the TeV scale.

In Ref. [14] we investigated a simple hidden sector with an Abelian $U(1)$ hidden gauge symmetry in an extended Randall-Sundrum model [15 16]. Relative to many other realizations of light Abelian hidden sectors, the model predicted an entire Kaluza-Klein (KK) tower of light vector bosons. These modes can have important phenomenological consequences since several modes in the tower can couple significantly to the SM through gauge kinetic mixing. This feature could allow one to study the structure of the warped extra dimension in lower-energy experiments, such as meson factories or fixed-target experiments.

In order to focus specifically on the interesting low-energy physics, in the present work we study a slightly simpler theory than was presented in Ref. [14]. We consider a single warped bulk containing a $U(1)$ gauge theory with the SM confined to the ultraviolet (UV) brane. As before, we couple the SM and the hidden sector through a gauge kinetic mixing operator localized on the UV brane. With an infrared (IR) brane scale of order a GeV this setup reproduces the interesting low-energy phenomenology of Ref. [14]. Beyond the TeV scale one should consider the full structure of Ref. [14] or include some other mechanism to stabilize the electroweak scale, but these details are not important as far as the relevant low-energy phenomenology is concerned. Let us also point out that, via the AdS/CFT correspondence, this model can be considered as a dual description of a purely 4D theory containing fundamental SM fields coupled to a hidden conformal field theory (CFT) with a weakly gauged $U(1)$ subgroup and an order GeV mass gap [17].

In the present work we perform a detailed investigation of the phenomenology of a simple light warped hidden sector with a characteristic mass scale of roughly 10 MeV to 10 GeV. This range is technically natural, but much smaller and much larger hidden mass
scales are also possible (and natural). We concentrate on these particular values primarily because they are phenomenologically interesting. In particular, we will show that a warped hidden sector in this mass range can be consistent with existing bounds from direct searches and astrophysics while also giving rise to potentially observable new signals in lower-energy collider experiments. It is these signals that we investigate in the present work.

The outline of this paper is as follows. In Section 2 we describe the model in more detail. Next we discuss the decays of hidden sector KK modes in Section 3. In Section 4 we address the important issue of reliably computing processes initiated by the SM on the UV brane with energies well above the hidden IR scale but well below the characteristic UV scale. Specifically we discuss the matching of the 4D KK effective theory to the full 5D bulk theory. We apply these results in Sections 5 and 6 where we compute, respectively, the precision electroweak constraints, and the bounds and discovery prospects at fixed-target experiments and meson factories. Section 7 is reserved for our summary and conclusions, and some technical details are collected in a pair of Appendices.

2 A Warped Hidden Sector

In this section we remind the reader of some pertinent features of the warped hidden sector model investigated in Ref. [14] and detail the simplified warped model considered in this work. The basic setup in Ref. [14] consists of the usual RS warped bulk space glued to a second “hidden” warped space at a common UV brane. The IR scale in the visible “throat” is near a TeV in order to solve the electroweak hierarchy problem, and the usual RS picture is assumed, with an IR localized SM Higgs, bulk SM fermions and bulk $SU(3)_c \times SU(2)_L \times U(1)_Y$ gauge bosons. We take the IR scale in the hidden throat to be at or below a GeV. A sketch of the model is given in Fig. 1.

The low-energy phenomenology of such a multithroat construction is quite rich. At energies near the GeV scale the minimal spectrum consists of the SM along with a tower of hidden gauge and gravity KK modes, whose spacing is on the order of a GeV or less. The SM couples to the hidden KK vectors primarily via a localized kinetic mixing operator connecting $U(1)_x$ and $U(1)_Y$ on the shared UV brane. The coupling of hidden KK gravitons to the SM is highly suppressed as the former are strongly localized towards the IR of the hidden throat [14]. Even so, the hidden gravitons still play a phenomenologically important role since their couplings to hidden-sector KK vectors are not suppressed.

As we are primarily interested in the low-energy phenomenology of this setup, it suffices to consider a simplified picture in which the SM is localized on the UV brane of a single hidden warped throat which contains a bulk $U(1)_x$ gauge symmetry and an IR scale near a

---

1 We note that gauge kinetic mixing with a light hidden sector may also be motivated by recent models of dark matter [9] [10]. We do not consider this matter here, but the phenomenology we consider would likely form a central component of warped realizations of these models.

2 For a TeV IR scale in the visible throat to be consistent with precision electroweak bounds, the gauge symmetry in this throat will likely need to be enlarged to include a custodial symmetry as described in Ref. [18]. However, the low energy physics will still match that described here [14].
The geometry of a warped hidden sector. The figure on the left illustrates the two-throat model studied in Ref. [14] while the figure on the right shows the simplified construct we investigate in this work. At energies below the TeV scale the simplified setup with UV localized Standard Model fields provides a good approximation to the full two-throat model. GeV (see Fig. 1). This picture captures all the important low-energy physics and we shall employ it in the present work. At energies above the TeV scale, the phenomenology of this setup differs from that of Ref. [14] and one should include the full two throat structure to accurately determine the high-energy phenomenology. This difference will not significantly modify the low-energy effects in which we are interested.

The metric in the 5D bulk space is given by [15]

\[ ds^2 = \frac{1}{(kz)^2}(\eta_{\mu\nu}dx^\mu dx^\nu - dz^2) = G_{MN}dx^M dx^N, \]

where \( z \in [k^{-1}, R] \) labels the extra dimension and \( \mu, \nu \) (\( M, N \)) are the 4D (5D) Lorentz indices. As per usual, the Planck mass is \( M_{Pl}^2 \sim M_5^2/k \), where \( M_5 \) is the 5D Planck scale. The spectrum of KK gravitons, which we denote as \( h_a \), can be found by perturbing around the background metric of Eq. (1). Their masses are [19]

\[ m_a \simeq \frac{\pi}{R}(a + 1/4), \quad a \geq 1 \]

while the massless zero mode \( h_0 \) is the usual 4D graviton.

The UV-localized SM resides at \( z = k^{-1} \) along with the following gauge kinetic mixing operator [20]

\[ S \supset -\frac{\epsilon_s}{2\sqrt{M_s}} \int_{UV} d^4x \sqrt{-g} g^{\mu\alpha} g^{\nu\beta} B_{\mu\nu} X_{\alpha\beta}, \]

where \( g_{\mu\nu} \) is the induced metric, \( B_{\mu\nu} \) is the SM hypercharge field strength and \( X_{\alpha\beta} \) is the \( U(1)_x \) field strength. Symmetry breaking in the hidden sector can be induced by an explicit IR-localized Higgs (the “Higgsed” case) or by imposing a Dirichlet boundary condition on the IR brane (the “Higgsless” case) [21] 3 We shall focus primarily on the Higgsless case in this work, but will mention the Higgsed case when there are important differences.

3For both sets of boundary conditions we can go to a unitary gauge with \( X_5 = 0 \).
For convenience we remind the reader of some essential features of the KK spectrum in the Higgsless case. Details of the spectrum and kinetic mixing in the Higgsed case can be found in Ref. \[14\]. The KK masses $m_n$ for the hidden vectors can be approximated as

$$m_n \simeq \frac{\pi}{R} (n + 1/4), \quad n > 0,$$

and the mass of the lowest mode, which we label as “0”, is mildly suppressed relative to the hidden IR scale,

$$m_0 \simeq \frac{1}{R} \sqrt{\frac{2}{\log(2kR) - \gamma}},$$

where $\gamma \simeq 0.5772$ is the Euler-Mascheroni constant. In the effective 4D theory, the UV-localized kinetic mixing operator induces mixing between SM hypercharge and the tower of KK vectors:

$$\mathcal{L}_{\text{eff}} \supset -\frac{1}{2} \sum_n \epsilon_n X_n^{\mu\nu} (c_W F_{\mu\nu} - s_W Z_{\mu\nu}),$$

where $s_W$ and $c_W$ refer to the weak mixing angle. The zero–mode kinetic mixing is given by\[3]\n
$$\epsilon_0 = \frac{\epsilon_* f_0(k^{-1})}{M_*^{1/2}} \simeq -\epsilon_* \left( \frac{k}{M_*} \right)^{1/2} \frac{1}{\sqrt{\log(2kR) - \gamma}},$$

where $f_n(k^{-1})$ is the KK mode wavefunction for the $n$-th mode on the UV brane. For the higher modes, one has

$$\epsilon_n = \frac{\epsilon_* f_n(k^{-1})}{M_*^{1/2}} \simeq -\epsilon_* \left( \frac{k}{M_*} \right)^{1/2} \frac{1}{\left[\log(2k/m_n) - \gamma\right]^{1/2}}, \quad n \geq 1.$$  \(8\)

These expressions show that the kinetic mixing parameter $\epsilon_n$ is mildly suppressed for the higher KK modes relative to the lowest mode, with $\epsilon_n \simeq \epsilon_0/6\sqrt{n}$ for $n > 0$.

Of the five parameters we have introduced to describe the hidden sector, the gauge coupling will play no role in this work (in the absence of an IR Higgs). The 5D gravity scale can be fixed by the 4D Planck mass, leaving three free parameters, which we take as $\epsilon_0$, $R$ and the ratio $k/M_*$. As we will detail below, for energies $E \gg M_*/kR$, which can be relevant for hidden $Z$ decays and colliders, the dependence on $R$ drops out of inclusive processes and the dependence on $k$ is very mild (logarithmic). Absent hierarchically small values of $k/M_*$ the phenomenology at such energies is therefore controlled by the single parameter $\epsilon_0$. At lower energies on the order of $R^{-1}$ the theory is sensitive to all three free parameters, but provided $k/M_*$ is non-hierarchical the dependence is primarily on $\epsilon_0$ and $R$.

The application of the AdS/CFT correspondence to RS models \[17\] permits a dual 4D description of the model we have outlined. In the dual picture there is a hidden CFT

---

4These expressions also apply when the SM propagates in its own warped bulk after making the replacement $\epsilon_* \rightarrow \epsilon_* \sqrt{k/M_* \log(kR_1)}$.  

---
possessing certain global symmetries, a $U(1)$ subgroup of which is weakly gauged. The corresponding gauge boson ($\gamma'$) is a fundamental field, external to the CFT, as are the UV localized SM fields (see Ref. [22] for a discussion of the correspondence between KK modes and CFT modes). The conformality of the hidden sector is broken explicitly in the UV and spontaneously in the IR (corresponding respectively to the UV and IR branes in the 5D picture). The 4D model contains a gauge kinetic mixing term between $\gamma'$ and SM hypercharge ($\sim F'_{\mu\nu} B^{\mu\nu}$) and there is further kinetic mixing between $\gamma'$ and the spin-one modes of the CFT ($\sim F'_{\mu\nu} \rho_n^{\mu\nu}$) [18]. The latter mixing is akin to the kinetic mixing of $\rho$ with the photon in the SM, as electromagnetism is a weakly gauged global symmetry of the QCD sector. Note that there is no direct kinetic mixing term between hypercharge and the CFT modes $\rho_n$, but one is induced by their common mixing with $\gamma'$. The SM therefore couples to the CFT modes by its weak coupling (for $E \ll k$) to the fundamental field $\gamma'$.

Before proceeding we note that the hidden sector could also contain additional states with non-zero $U(1)_x$ charge, possibly including a UV-localized dark matter candidate as mentioned in [14], or new matter fields in the bulk. New states on the UV brane will not significantly modify the low-energy phenomenology we discuss provided they are at or above the weak scale. Indeed, the (unspecified) dynamics on the UV brane that ensures the stability of the electroweak sector can also lead to new states in the electroweak mass range. Exotic hidden bulk matter will naturally be much lighter, with zero-mode and low-lying KK mode masses near the characteristic mass scale of the hidden IR brane. This could, for example, lead to a relatively light dark matter candidate with a mass of order MeV–GeV. Such new light states could significantly modify the low-energy phenomenology relative to the minimal model we consider here; if they are lighter (heavier) than the vector zero mode predominantly (partially) invisible final states can arise. We defer the study of this possibility to a future work.

We also note that previous works have considered warped models with sub-weak scales [23, 24], and that much of the phenomenology we consider here would be similar if implemented on the truncated space of Ref. [24]. The gauge kinetic mixing we consider can also find its origin in string models [25] and our phenomenological analysis may be of interest in this regard.

3 Hidden Sector Decays

Kinetic mixing between $U(1)_x$ and hypercharge permits the creation of hidden KK vectors in experiments colliding SM fields together. The expected experimental signal once a hidden vector is produced depends on the decay properties of the vector. It is therefore important to understand these properties to determine whether the vectors decay predominantly into the hidden sector or to the SM, and to determine likely signals. In this section we consider the decay properties of the KK vectors in some detail. Related discussions in a different context can be found in Refs. [26, 27, 28].

The decay of a hidden KK vector $X_n$ to SM fields requires a kinetic mixing insertion and the corresponding widths are suppressed by a factor of $\epsilon_n^2 \ll 1$. For example, the decay
width of the $n$-th KK mode to a pair of SM leptons $\ell \bar{\ell}$ is \[29\]

$$\Gamma(X_n \rightarrow \ell \bar{\ell}) = \frac{1}{12\pi} e^2 e_W^2 c_n^2 m_n \left( 1 + \frac{2m_\ell^2}{m_n^2} \right) \left( 1 - \frac{m_\ell^2}{m_n^2} \right)^{1/2}, \tag{9}$$

for $m_n \ll m_Z$, and similarly the width to hadrons is

$$\Gamma(X_n \rightarrow \text{hadrons}) = \Gamma(X_n \rightarrow \mu^+ \mu^-) R(s = m_n^2), \tag{10}$$

where $R(s)$ is the usual hadronic $R$ parameter, $R = \sigma(e^+ e^- \rightarrow \text{hadrons})/\sigma(e^+ e^- \rightarrow \mu^+ \mu^-)$ \[30\] \[31\]. As the lowest data point in the hadronic cross section data set is at $\sqrt{s} = 0.36$ GeV, well above the pion threshold, we follow \[5\] and use the cross section for $e^+ e^- \rightarrow \pi^+ \pi^-$ in the uncharted region above threshold \[31\] \[32\]. Observe that the total decay width to the SM is roughly independent of mode number, up to a growth in the number of kinematically accessible states, as the KK mass grows like $m_a \sim n$ while $c_n^2$ goes like $\sim 1/n$. These decays are relatively prompt on collider timescales ($c\tau < 1$ mm) for $R^{-1} \sim$ GeV provided $\epsilon_0 \gtrsim 10^{-4}$ \[7\].

Even in the absence of hidden sector matter, heavier KK vectors can still decay within the hidden sector via the creation of a KK graviton and a lighter vector mode, $X_n \rightarrow h_a X_m$. These decays are kinematically allowed for sets of KK numbers satisfying $n > m + a$ in both the Higgsed and Higgsless cases, but decays with $a = 0$ vanish due to wavefunction orthogonality. Therefore all vector modes with $n \geq 2$ can decay via graviton production. We present a computation of the decay width for $X_n \rightarrow h_a X_m$ in Appendix A.1.

For a given KK level $n$, the partial width $\Gamma(X_n \rightarrow h_a X_m)$ can vary substantially as one varies the daughter KK numbers $a$ and $m$. To demonstrate this we plot the branching ratio for the decay $X_n \rightarrow h_a X_m$ against daughter KK number $a$ in Fig. 2 for the mode $n = 45$. The Figure shows points with fixed values of $m + a = (44, 43, 42, 41)$. Decays with values of $a + m < 41$ are subdominant to those plotted. The plot reveals some important features. Firstly, one observes that decays with daughters $h_a$ and $X_m$ satisfying $a + m \sim n$ are dominant. For a fixed value of the graviton mode number $a$ one can see that the branching ratio for decays with $a + m = 44$ is more than a hundred times larger than that with $a + m = 42$. This disparity increases as one decreases the sum $a + m$. Also note that for fixed values of $a + m$ the decays into lighter KK gravitons dominate.

The tendency of KK vectors to decay into daughters satisfying $n \sim a + m$ demonstrates an approximate conservation of KK number present in RS models \[26\]. In the absence of warping, the momentum along the extra dimension would be conserved and KK decays would necessarily conserve KK number with $n = a + m$. Turning on the warping breaks the translational invariance along the extra dimension so that KK number is no longer conserved, however an approximate KK number conservation persists and is encoded in the wavefunction overlap factors $\zeta_{a,mm}$ and $\xi_{a,mm}$ presented in Appendix A.1. As different KK modes are localized at different points along the extra dimension the overlap factors contain oscillatory integrands unless $n \sim a + m$. This results in a suppression of $\zeta_{a,mm}$ and $\xi_{a,mm}$ as one increases the separation between $n$ and $a + m$, explaining the dominance of decays with $n \sim a + m$. 

6
Figure 2: Plot of the Branching Ratio Vs. Daughter KK number $a$ for the hidden sector decay of the $n = 45$-th KK vector. We plot fixed values of $(a+m)$ and from top to bottom the curves satisfy $(a+m) = (44, 43, 42, 41)$. The branching ratio is taken as $\text{BR} \simeq \Gamma(X_{n=45} \rightarrow h_a X_m) / \sum_{m,a} \Gamma(X_{n=45} \rightarrow h_a X_m)$ and we use $R^{-1} = 1$ GeV. The plot is for the Higgsed Case but similar behavior is found in the Higgsless case.

In Fig. 3 we show the total decay width of the $n$-th mode due to two-body decays to graviton and gauge modes for $n \leq 50$. The plot is for the Higgsless case with $k/M_* = 0.1$ and $R^{-1} = 1$ GeV. One observes that a simple parameterization of the total hidden two-body decay width is possible for $n \gg 1$:

$$\Gamma_n \equiv \sum_{m,a} \Gamma(X_n \rightarrow h_a X_m) = \frac{k^2}{M_{pl}^2} \frac{g_*(n)}{8\pi} m_n,$$

where the effective coupling constant $g_*(n)$ can be written as

$$g_*(n) = C n^p,$$

with $C \simeq 0.80$ and $p \simeq 3$ for $R^{-1} = 1$ GeV. Numerically we find that varying $R$ does not appear to change the power $p$ but does induce a small change in the constant $C$. For example, varying $R$ over an order of magnitude produces a shift in $C$ of order a few to ten percent. It will also be helpful to have a parametrization for the hidden width of the lighter modes, $n \sim 1$. Writing these as in Eq. (11) with $C \rightarrow C_n$ we find

$$C_2 \simeq 0.10 \quad , \quad C_3 \simeq 0.40 \quad , \quad C_4 \simeq 0.58 \quad , \quad C_5 \simeq 0.66,$$

for the Higgsless case with $R^{-1} \lesssim 1$ GeV. The parameters $C_n$ vary slowly as one varies the length of the extra dimension $R$, but the above values provide a reasonable approximation for the energies of interest to us.

The parametrization for the hidden two-body decay width $\Gamma_n$ in Eq. (11) gives a good approximation to the total KK mode width when the KK description is perturbatively under control. However, the KK description breaks down at energies of order $(M_*/k)R^{-1}$ as the coupling between KK modes becomes strong [33]. This is borne out in Eq. (11), which shows...
that $\Gamma_n$ is parametrically of the same order as $m_n$ for $n \sim (M_*/k)$, indicating that the modes are bleeding into each other and the KK description is breaking down. At strong coupling higher-order corrections and multi-body decays are expected to become important. Decays to stringy modes, either from a string theoretic UV completion or from strong coupling, may also become significant \[28, 34\]. We will discuss a method for calculating inclusive decay widths at higher energies in Sec. \[4\].

To illustrate the decay channels of the calculable lower KK modes we consider the Higgsless case for $k/M_* = 0.1$ and $R = 1$ GeV$^{-1}$ with the strength of the zero-mode kinetic mixing fixed at $\epsilon_0 = 3 \times 10^{-3}$. Once these parameters are set the kinetic mixing for the higher modes is determined by Eq. (8) as $\epsilon_n \simeq \epsilon_0/6\sqrt{n}$. In this case the zero mode has mass $m_0 \simeq 230$ MeV and can only decay leptonically, $X_0 \rightarrow 2e, 2\mu$, with a width of about a keV. The first KK mode ($n = 1$) also decays to light SM leptons and hadrons with a slight preference for leptons, $BR^{(1)}_{lep} = 0.56$, and a total width of about a keV. The second KK mode ($n = 2$) can decay within the hidden sector in addition to the SM and has a much larger width on the order of 0.2 MeV. Decays to the hidden sector almost completely dominate with a branching ratio $BR(X_2 \rightarrow X_0 h_1) = 0.9998$. Of the decays to the SM, hadronic modes are slightly preferred with $BR^{(2)}_{had} = 2.9 \times 10^{-4}$ and $BR^{(2)}_{lep} = 2.3 \times 10^{-4}$. As the width to the SM does not significantly increase for heavier modes, the $n \geq 2$ modes all decay predominantly within the hidden sector.

Once created, a KK graviton can decay to lighter KK vectors. We present the two-body width for graviton decay $h_a \rightarrow X_m X_n$ in Appendix A.2, where we show that graviton decays further demonstrate the approximate conservation of KK number and prefer decays with $a \sim m + n$. These decays are prompt, as the coupling between KK vectors and gravitons is set by $R^{-1}$. For example, in the Higgsless case with $k/M_* = 0.1$ and $R^{-1} = \text{GeV}$ we find $\Gamma(h_1 \rightarrow 2X_0) \simeq 3 \times 10^{-2}$ MeV for the first KK mode. The total width for the heavier vectors increases with KK number $a$ due to the increase in kinematically available final states (see Appendix A.2). Heavier KK gravitons can also decay to lighter KK gravitons. Note,
however, that the lightest KK graviton \((a = 1)\) will decay entirely to pairs of \(X_0\).

Putting these pieces together, there emerges a simple picture of decays within a Higgsless hidden sector. The \(n = 0, 1\) vector KK modes decay directly back to the SM once they are produced. When a higher vector KK mode (or a superposition of them) is created, such as in an energetic collision of SM fields, it decays promptly to lighter KK vector and graviton modes. In turn these decay to even lighter hidden states, producing a cascade down the hidden KK tower. This showering terminates at the \(n = 0, 1\) vectors which decay back to the SM. Therefore the generic final state resulting from the production of heavier hidden sector modes is a high multiplicity of relatively soft and light SM particles, similar to the scenario discussed in Ref. [28].

The presence of an IR-brane hidden Higgs field \(h_x\) can significantly modify the experimental signals of the hidden cascade. With such a Higgs, KK vectors can decay through the process \(X_n \to X_m + h_x\) when kinematically available. The precise way in which the decay spectrum is altered depends on the relationship between the Higgs mass and the vector KK masses (similar to the 4D case [5, 35, 36]). If \(m_h\) lies below twice the lightest KK vector mass \(m_0\), the hidden Higgs will decay only slowly back to the SM through multi-body or loop-induced channels involving \(X_0\). These decays are typically slow relative to experimental timescales for light hidden Higgs masses leading to missing energy in the final state for \(m_h < m_0\), and possibly displaced vertices for \(m_0 < m_h < 2m_0\) [5]. A light Higgs will also allow \(X_1 \to X_0 + h_x\), which almost always dominates over direct decays to the SM and can produce four- and six-lepton final states. As \(m_h\) grows larger than \((m_1 - m_0)\) the \(n = 0, 1\) vector modes again decay exclusively to the SM. The hidden Higgs can still be produced in the decays of heavier KK modes and will decay promptly to pairs of lighter vectors. Hidden decay cascades in this case will be qualitatively very similar to the Higgsless case described above.

Vector and graviton KK modes may also have decays involving a bulk radion \(r_x\), such as \(X_n \to X_m + r_x\). These will be very similar to decays involving an IR-brane hidden Higgs with which it could mix [37]. As for a hidden Higgs, if the radion is very light it can radically alter the decays of the lightest modes, but as its mass increases it mainly affects the heavier KK modes. The precise determination of the radion mass is a somewhat complicated subject that requires a specification of the stabilization dynamics and a determination of their backreaction on the metric [38]. While there is no difficulty of principle in stabilizing the hidden warped extra dimension [39] (even in the full two-throat setup [10]), in practise the precise value of the radion mass will depend on the details of the approach adopted. Provided the radion is heavier than the \(n = 0\) and \(n = 1\) vector KK modes it will not significantly modify the cascade picture presented above. We assume this to be the case in the present work and neglect radion decay channels.

In summary, we find that the dominant two-body decay of the \(n\)-th KK vector is into light KK gravitons \(h_a\) and KK vectors \(X_m\) with \(m \sim n\) and \(n \sim a + m\) for \(n \geq 2\). If kinematically permitted the daughter vector will further decay into lighter gravitons and vectors and the daughter gravitons will themselves decay back into two lighter vectors. The creation of heavier modes therefore results in a cascade decay down the tower in the hidden sector until one ends up with a collection of light KK vectors. The light vectors then decay
4 Matching to Five Dimensions

In the phenomenological analyses to follow, we will be interested in processes initiated by UV-localized SM states at energies both well above and well below the hidden IR scale $R^{-1} \sim \text{GeV}$, but always much less than the UV scale $k$. Since the local cutoff at a given point $z$ along the extra dimension is $M_s/(kz)$, the effective theory description breaks down in the IR for energies approaching $(M_s/k)R^{-1}$ \[17, 33\]. For processes initiated by SM fields on the UV brane, however, where the local cutoff is near the Planck scale, one expects a reliable effective theory description to exist \[17, 33, 41\]. At energies below $(M_s/k)R^{-1}$ only the lightest KK modes are important and the KK effective theory developed above provides a reliable description of the full dynamics. For energies greater than this, the KK modes become increasingly broad and strongly-coupled, and the KK description breaks down. However, the full 5D theory remains weakly-coupled up to the UV scale for processes initiated on the UV brane \[33\]. We make use of this fact by matching the 4D KK theory onto the 5D theory at energies near $R^{-1}$, where both descriptions are sensible. Our matched 5D theory then allows us to compute reliably in the intermediate regime $R^{-1} \lesssim E \ll k$ for processes initiated on the UV brane. In this section we detail this matching. We note that the technical details of this section, though important for our analysis, lie outside of our main phenomenological focus. Readers not concerned with these computational details can proceed directly to Sec. 5.

Consider first the 5D bulk vector propagator at tree-level. We will always contract this propagator with a transverse projector ($-p^2\eta_{\mu\nu} + p_{\mu}p_{\nu}$) arising from the kinetic mixing interaction of Eq. (3), so we only need the gauge-invariant coefficient of the transverse portion of the propagator. With Higgsless (Neumann-Dirichlet) or Higgsed (Neumann-Neumann) boundary conditions it is given by

$$\Delta_p(z, z') = \begin{cases} \pi kzz'/(Y_1,RJ_0,k-J_1,RY_0,k) & (\text{Higgsless}) \\ \pi kzz'/(Y_0,RJ_0,k-Y_1,RJ_1,k) & (\text{Higgsed}) \end{cases}$$

where $p = \sqrt{p^2}$, $z_\gamma = \max\{z, z'\}$, $z_\gamma = \min\{z, z'\}$ and $J_{n,z}$ is shorthand for $J_n(pz)$. The Neumann-Neumann propagator was derived in \[42\]. Taking $z = z' = k^{-1}$ and $R^{-1} \ll |p| \ll k$, both 5D tree-level propagators reduce to

$$\Delta_p(k^{-1}, k^{-1}) \simeq \frac{k}{p^2[\log(2k/p) - \gamma]} \times \left( \frac{1}{2} \tan(pR - N\pi/4)/[\log(2k/p) - \gamma] \right)^{-1}$$

where the upper (lower) term is for $p^2 > 0$ ($p^2 < 0$) and $N = 3$ ($1$) for the Higgsless (Higgsed) case. Both expressions have poles corresponding to the KK masses. In fact it can be shown that the tree-level 5D propagators are equal to a sum over KK modes weighted by KK bulk...
wavefunction factors $f_n(z)$,

$$\Delta_p(z, z') = \sum_n f_n(z)f_n(z'), \quad (16)$$

where the masses $m_n$ are given in Section 2.

The tree-level propagators in Eq. (15) will be modified in an important way by quantum effects \[28, 33, 41, 42, 43, 44\]. In addition to mass and wavefunction corrections, the RS1 propagators will acquire complex self-energies for $p \gtrsim R^{-1}$. These arise from bulk gauge-graviton (or gauge-Higgs) loops, and come with factors of $k/M_*$. Rather than carrying out an explicit 5D calculation of the loop corrections to the gauge boson self-energy, we take a different approach based on matching to RS2 \[16\], where the IR brane is taken to $z \to \infty$, which we argue is a universal IR-independent limit of the RS1 propagators at high momentum \[17\].

For reference, the coefficient of the gauge-invariant transverse part of the RS2 UV-to-UV propagator is given by \[45, 46\]

$$\Delta_{RS2}^{p}(k^{-1}, k^{-1}) = \frac{H_1^{(1)}(p/k)}{pH_0^{(1)}(p/k)} \times \begin{cases} \left(1 - i\frac{\pi}{2}/[\log(2k/p) - \gamma]\right) ; & p^2 > 0 \\ \left(1 - i\frac{\pi}{2}/[\log(2k/p) - \gamma]\right) ; & p^2 < 0 \\ -1 \end{cases} \quad (17)$$

where $H_n^{(1)} = (J_n + iY_n)$. Note the appearance of an imaginary part for $p^2 > 0$, which follows from the imposition of outgoing-wave boundary conditions as $z \to \infty$ \[45, 47\], and represents the escape of vectors into the bulk.

Comparing the tree-level RS1 UV-to-UV propagators in Eq. (15) to the RS2 expression of Eq. (17), we see that they agree at spacelike momentum with $|p| > R^{-1}$. At large timelike momentum, the real parts of the RS1 propagators also match closely with RS2, except near the poles of the log-suppressed tangent term. This agreement can be understood in terms of the UV-to-bulk RS propagators, which become highly oscillatory or exponentially damped for $z > |p|^{-1}$. Thus, the full theory appears to be insensitive to the detailed geometry at $z \gg |p|^{-1}$. In particular, modifying the geometry at large $z$ by adding an IR brane or changing its location should have virtually no effect on the quantum-corrected UV-to-UV propagator at large momentum \[17\].

There is a potential loophole in this argument related to the poles of the tree-level RS1 propagators. These poles correspond to KK masses, and reflect an apparent sensitivity to the IR that persists at large momentum. The tree-level RS1 propagators also lack the imaginary term present in the RS2 propagator for timelike momentum. Both of these features are artifacts of the tree-level approximation wherein the KK modes are treated as being absolutely stable. Consequently, the KK poles at this level may be thought of as standing

---

5 Explicit expressions for the KK wavefunction factors $f_n(z)$ can be found in Ref. \[14\].
waves that reflect off the IR boundary, thereby probing the deep IR and ensuring there is no net dissipation from the UV brane.

Quantum corrections to the propagator will generate an imaginary component in the self-energy. Near the pole of a narrow low-lying KK mode we can identify this imaginary piece with the finite decay width computed in Sec. 3. Its effect is to regulate the KK pole divergences in the tree-level RS1 propagator, and it represents a net dissipation from the UV brane. For lighter modes that are very narrow, some probability for reflection remains and the net dissipation differs to that of RS2. Going to higher timelike momentum will probe heavier KK modes that are increasingly broad. When the widths become as large as the mode spacing \( \pi/R \), the KK resonances begin to overlap significantly and approach a continuum [28]. We expect this continuum to be independent of the IR structure of the theory, and to persist to even higher momenta where the KK theory becomes strongly-coupled. Intuitively, a broad KK mode produced on the UV brane will decay (or shower) well before reaching the IR brane and can no longer probe the deep IR. As the probability of this mode reflecting off the IR brane falls to zero, the dissipation due to decays should agree with RS2 where no reflection is possible.

We therefore argue that the quantum-corrected 5D RS1 UV-to-UV propagator in the momentum range \( (M_*/k) R^{-1} \ll |p| \ll k \) approaches a universal IR-independent limit that is approximated well by the tree-level RS2 UV-to-UV propagator. These properties are already true at tree-level for spacelike momentum with \( |p| \gg R^{-1} \). At large timelike momentum, perturbatively small quantum corrections smooth out the poles and generate an imaginary component in the propagator, producing a continuum that is insensitive to the IR geometry. Since the 5D RS theory is perturbatively calculable for processes initiated on the UV brane with \( |p| \ll k \), quantum corrections to the the propagator are numerically small. In the case of RS2, the tree-level propagator is smooth at timelike momentum, and can not be modified strongly by perturbative loop effects. The universal high-momentum limit should thus be described well by the tree-level RS2 propagator.

This matching is also consistent with the gauge dual picture of a 4D approximate CFT. The presence of an IR brane corresponds to a spontaneous breaking of conformal invariance at low energies. Going to energies well above the breaking scale, CFT-breaking effects will become increasingly irrelevant, falling off as positive powers of \( R^{-1}/E \), and the theory will approach a pure CFT (with a UV cutoff) that coincides with RS2. Let us also point out that the 5D quantum corrections we have considered correspond to subleading corrections in a \( 1/N \) expansion within the CFT dual.

Our strategy for the rest of the paper is to match the KK theory valid at low energies to the RS2-like 5D limit we argue is valid at higher energies. For this, we use the following empirical form for the quantum-corrected UV-to-UV 5D propagator to interpolate between the two regimes:

\[
\Delta^\text{UV}_p = \sum_n \frac{f_n^2(k^{-1})}{p^2 - m_n^2 + ip\Gamma_n},
\]

(18)
Figure 4: Real and Imaginary parts the 5D UV-to-UV propagator of Eq. (18) together with those for RS2. We use $k/M_* = 0.1$ and $\pi/R = 1$ GeV.

where

$$\bar{\Gamma}_n = \begin{cases} \Gamma_n; & \Gamma_n \leq m_n/4 \\ m_n/4; & \Gamma_n > m_n/4 \end{cases},$$

and $f_n(k^{-1}) = \epsilon_n(M_*)^{1/2}/\epsilon_*$. This form reproduces the correct KK pole structure at low-momenta where the KK theory is calculable and the resonances are narrow, and connects smoothly to the RS2 propagator for $|p| > (M_*/k)R^{-1}$ over the range of momenta of interest.\(^6\)

In Fig. 4 we plot both the real and imaginary parts of Eq. (18), together with those of the RS2 propagator given in Eq. (17). At large momenta the expression of Eq. (18) is relatively insensitive to the precise form of $\bar{\Gamma}_n$ provided it is larger than the KK mode spacing. Fig. 4 also illustrates how the finite KK mode widths smooth out the poles in the tree-level propagator. Although we do not show it here, the form of Eq. (18) also agrees well with the RS2 propagator for $p^2 < 0$ when $|p| \gg R^{-1}$.

5 Precision Electroweak Observables

Having developed the necessary tools we now proceed to the detailed phenomenology. We will consider precision electroweak observables in this section and turn to low-energy signals and constraints in Sec. 6. For simplicity we focus primarily on the Higgsless case with a Dirichlet IR boundary condition in both sections. The Higgsed case will be qualitatively similar, with the main difference being the presence of additional decay channels (like Higgs'-strahlung) that can modify the signals in low-energy experiments.

Mixing between the hidden bulk gauge vector and the SM photon and $Z$ will alter the predictions for electroweak observables. The two classes of observables we study in this

\(^6\)This expression does not fully account for the pole structure in the intermediate regime when the KK modes have widths on the order of the mass spacing $\pi/R$ [15], but we do not expect this to modify our results significantly.
Figure 5: Correction to the $Z$ propagator. The hidden vector is created on the UV brane (represented by the plane) but may propagate into the bulk before producing standard model states.

section are $e^+e^- \rightarrow f\bar{f}$ scattering at $\sqrt{s} \sim m_Z$, where $f$ is a SM fermion, as well as rare $Z$ decay modes. To compute these effects we treat the kinetic mixing operator of Eq. (3) as an interaction that couples the SM to the bulk vector. For $e^+e^- \rightarrow f\bar{f}$ we only need the UV-to-UV bulk propagator, for which we use the matched expression given in Eq. (18). This is illustrated schematically in Fig. 5. By way of a unitarity cut, we can also use this propagator to compute the inclusive rate of rare $Z$ decays. The direct production of hidden sector states at lower energies will be considered in Sec. 6.

5.1 $e^+e^- \rightarrow f\bar{f}$ Processes

The interaction vertices for $X$-SM mixing in unitary gauge are

$$ZX : \quad -i \frac{\epsilon^*}{M_x^{1/2}} s_W p^2 \left( -\eta_{\mu\nu} + p_\mu p_\nu / p^2 \right) \delta(z - k^{-1}) , \quad (20)$$

$$\gamma X : \quad i \frac{\epsilon^*}{M_x^{1/2}} c_W p^2 \left( -\eta_{\mu\nu} + p_\mu p_\nu / p^2 \right) \delta(z - k^{-1}) . \quad (21)$$

Notice that these involve transverse projectors – contracting $p_\mu$ into the vertices gives zero. We can apply these vertices to the process $e^+e^- \rightarrow f\bar{f}$ with $f \neq e$, working to leading non-trivial order in $\epsilon^*_s$ and neglecting fermion masses. With the resulting summed and squared matrix element, which is given in Appendix [13], we have everything we need to compute the full set of precision electroweak observables and compare to experimental observations.

For this, we will take $m_Z$, $G_F$, and $\alpha_{em}(Q^2 \rightarrow 0)$ as input parameters for computing other electroweak observables. The effect of a tower of light hidden KK vectors on $G_F$ and $\alpha_{em}(Q^2 \rightarrow 0)$ is dominated by the lightest mode. The Fermi constant $G_F$ is measured in
muon decay, and is not changed at all at leading order.\(^7\) Currently, the best-measured value of \(\alpha_{em}(Q^2 \to 0)\) comes from \((g-2)\). This can be shifted by a light hidden vector, but the size of the shift cannot be so big as to disagree with the value of \(\alpha_{em}(Q^2 \to 0)\) measured in atomic systems, which are not significantly modified by the new states \([29]\). Both determinations have a much higher precision than the value of \(\alpha\) extrapolated to the weak scale, which has sizeable hadronic uncertainties. Thus, once the low-energy bounds are satisfied, the effective shift in \(\alpha(m_Z)\) relevant for precision electroweak observables is negligible.

Determining the value of \(m_Z\) is more complicated. It is obtained from a parametrized fit (with \(m_Z\) as a parameter) of \(e^+e^- \to f\bar{f}\) cross-sections measured over a range of energies near the \(Z\) pole. Hidden vectors will modify this \(Z\) lineshape, and can thereby lead to a poor fit or a shift in the extracted value of \(m_Z\). To investigate these effects we fix \(\epsilon_0\) such that \(|\epsilon_0| = 1 \times 10^{-2}\), set \(k/M_* = 0.1\) and \(R = \pi/\text{GeV}\), and compute the resulting \(e^+e^- \to f\bar{f}\) cross-sections. Note that for masses of the lightest hidden vector below 10 GeV, \(|\epsilon_0| = 10^{-2}\) is slightly larger than what is consistent with low-energy probes \([8, 29]\).

We show the effect of the hidden bulk vector on the \(e^+e^- \to \mu^+\mu^-\) cross-section around the \(Z\) pole in Fig. 6, where we have used \(\tilde{m}_Z = 91.187\) GeV as a fiducial input value. The peak of the modified cross-section is shifted away from the SM peak by 0.0012 GeV, about half the current uncertainty in the \(Z\) mass of \(\Delta m_Z = 0.0023\) GeV \([30]\). We find that the change in the lineshape can be almost completely eliminated by changing the value of \(m_Z\) by an amount equal to the shift in the peak location when the fiducial value is used, as we also show in Fig. 6. After changing the value of \(m_Z\) in this way, the \(Z\) lineshape (as well as the shifted \(Z\) mass) is consistent with cross-section measurements around the \(Z\)-pole at LEP \([19]\). We use the shifted \(Z\) mass as an input in computing other electroweak observables.

Having fixed the input parameters \(G_F\), \(\alpha_{em}(Q^2 \to 0)\), and \(m_Z\), we find that the modifications to other \(Z\)-pole observables are completely negligible relative to current experimental uncertainties for \(k/M_* = 0.1\), \(|\epsilon_0| = 10^{-2}\), and \(R = \pi/\text{GeV}\). For all the forward-backward and polarization asymmetries, we obtain shifts less than \(2 \times 10^{-4}\) at the \(Z\) pole, well below the current \(10^{-3}\) level of sensitivity \([50]\). This is not entirely unexpected since the dominant mixing of the hidden bulk vector near the \(Z\) pole is with the \(Z\) \([14]\). As a result, the relative couplings of the hidden vector to SM fermions are nearly identical to those of the \(Z\) and the effect on the asymmetries is tiny. Changes to electroweak observables away from the \(Z\) pole, such as \(\sigma(e^+e^- \to \text{hadrons})\) \([51]\) and Bhabha scattering \([52]\), are also much smaller than the precision to which they have been measured for these model parameters \([53]\).

Our results are consistent with Refs. \([53, 54, 55]\) which considered the precision electroweak bounds on a single 4D Abelian hidden vector. For smaller values of \(\epsilon_0\), the effects on electroweak observables will be even less. Stronger bounds can arise when the IR scale is larger, since now the very narrow \(n = 0\), 1 KK modes may be individually resolvable. The precise constraints in this case can be read off from the analysis of Ref. \([53]\).

---

\(^7\)We work implicitly to leading order in \(\epsilon\) effects, and drop any \(\epsilon\) effects appearing at loop order when the main contribution is tree-level.
Figure 6: Fractional correction in $\sigma(e^+e^- \to \mu^+\mu^-)$ relative to the SM for $\epsilon_0 = 10^{-2}$, $k/M_s = 0.1$, and $R = \pi/GeV$. We also show the fractional correction after shifting the input value of $m_Z$ as discussed in the text.

5.2 Rare Z Decays

Another source of constraints, as well as potential new signals, are Z decays into light hidden-sector states. This can occur when a Z mixes into a bulk vector $X$ that escapes into the extra dimension. The bulk vector will subsequently shower into further vector and graviton modes. Showering will cease when the invariant momentum scale approaches the hidden IR scale. At this point we can view the products of the hidden shower as a large multiplicity of light $n = 0, 1$ KK modes. In turn, these will decay back to the SM. The final state of such a decay mode will therefore consist of a large multiplicity of SM mesons and leptons [26, 28].

We can compute the total inclusive rate of such processes by applying a unitarity cut to the matched bulk propagator of Eq. (18) connected to incoming and outgoing Z boson legs. This yields

$$ BR(Z \to \text{hidden}) = -\frac{m_Z^2}{m_Z \Gamma_Z} \frac{\epsilon_0^2}{M_s} s_W^2 \text{Im}(\Delta^{UV}_p) \approx \sum_n \frac{\epsilon_0^2 s_W^2 m_Z^4}{(m_Z^2 - m_n^2)^2 + m_Z^2 \Gamma_n^2} \left( \frac{\hat{\Gamma}_n}{\Gamma_Z} \right). $$

In Fig. 7 we show the resulting branching fraction as a function of the zero-mode mixing $\epsilon_0$ for $\pi/R = 1$ GeV in the left panel, as well as the branching as a function of the KK mass splitting $\pi/R$ for $\epsilon_0 = 10^{-3}$ in the right panel. Both panels also have $(k/M_s) = 0.1$, but the result is insensitive to the precise value for $(M_s/k)R^{-1} \ll m_Z$.

For $\epsilon_0 \leq 10^{-2}$ the branching fraction is too small to modify the total Z width in an appreciable way. Even so, branching fractions greater than a few times $10^{-6}$, corresponding
Figure 7: Total branching fraction for $Z \rightarrow \text{hidden}$ as a function of $\epsilon_0$ (left) and $\pi/R$ (right). Both panels have $k/M_*=0.1$, while the left panel also has $\pi/R = 1$ GeV and the right panel has $\epsilon_0 = 10^{-3}$.

Note that this is potentially much more sensitive than indirect bounds from $e^+e^-\rightarrow f\bar{f}$ processes. However, the high-multiplicity final states predicted by this scenario were not explicitly searched for, and thus we do not quote a precise limit on $\epsilon_0$. When the KK mode spacing is significantly larger than a GeV, mixing of individual KK modes with the $Z$ can be resonantly enhanced.

Observe that the branching fraction in Fig. 7 becomes independent of $R$ for $R^{-1} \lesssim 10$ GeV. This feature is readily understood; for $m_Z \gg (M_*/k)R^{-1}$ the bulk propagator does not probe the deep IR and is well approximated by the $R$-independent RS2 form. Consequently a simple expression for the hidden $Z$-width can be obtained. Using Eq. (17) in Eq. (22) the branching fraction in this regime can be approximated as

$$BR(Z \rightarrow \text{hidden}) \simeq \frac{\pi}{2} \frac{k}{M_*} \frac{s_W^2 \epsilon_*^2}{[\log(2k/m_Z) - \gamma]^2} \left( \frac{m_Z}{\Gamma_Z} \right)$$

$$\simeq \frac{\pi}{2} \frac{s_W^2 \epsilon_0^2}{[\log(2k/m_Z) - \gamma]} \left( \frac{m_Z}{\Gamma_Z} \right) \sim 0.4 \epsilon_0^2.$$  \hspace{1cm} (23)

We note that among the parameters describing the hidden sector, this result is sensitive to $\epsilon_0$ alone (up to a very weak logarithmic $k$ dependence). Since this parameter also controls the direct production of the lightest vector mode in low-energy experiments, the model predicts an important correlation between these different observables.
6 Low-Energy Measurements

Low-energy experiments with a very high precision or luminosity can in many cases provide a much more sensitive test of the present scenario than higher-energy collider experiments. Specific examples include measurements of the anomalous magnetic moment of the muon, fixed-target and beam dump experiments, and meson factories. In this section we investigate the power of such lower-energy probes to test a light warped hidden sector.

6.1 Sub-GeV Constraints and Searches

Experimental constraints on a single Abelian hidden vector have been discussed extensively in Refs. [5, 8, 29]. For vector masses below a GeV the strongest model-independent bounds come from the anomalous $\epsilon$ and $\mu$ magnetic moments for larger kinetic mixing $\epsilon$ [29], together with null beam dump searches for smaller kinetic mixing [8]. Vector masses as small as 10 MeV are allowed for $\epsilon \lesssim 10^{-3}$. Much lighter vectors can also be consistent with existing bounds for extremely small values of $\epsilon$ [57], but we do not consider this possibility here.

In the present scenario, the single vector is replaced by a tower of KK resonances. Even so, with the exception of meson factories, the corresponding low-energy experimental bounds on the tower are almost completely dominated by the lightest zero mode. We find that these bounds are nearly identical to the case of a single Abelian hidden vector with the same mass and kinetic mixing as the zero mode. This occurs for two reasons. First, the constraints due to leptonic magnetic moments and beam dump experiments generally become weaker as the vector mass increases. Second, and usually more importantly, the zero mode has a significantly larger kinetic mixing coupling to the SM than the heavier KK modes, as can be seen by comparing Eqs. (7) and (8).

Summarizing these limits, for $m_0$ below 10 GeV the corresponding bounds can be obtained by applying the constraints of Refs. [8, 29] to the zero mode vector. For such light masses $\epsilon_0 \lesssim 3 \times 10^{-3}$ is needed to satisfy the constraints from leptonic magnetic moments [29] and searches for exotic events within the BABAR $T(3S)$ data set [8]. Masses below about 0.5 GeV are further constrained by beam dump searches and supernova cooling, leading to bounds on smaller values of $\epsilon_0$. Together, there remains a pocket of allowed $m_0$-$\epsilon_0$ values in the range of $m_0 = 10$ MeV-10 GeV as exhibited in Ref. [8]. Masses below $m_0 \simeq 10$ MeV are only consistent with very small values of the kinetic mixing, below $\epsilon_0 \lesssim 5 \times 10^{-8}$ [8, 57]. Once these low-energy constraints are satisfied (for $m_0 \lesssim 10$ GeV) the constraints from precision electroweak data are also met.

Current and planned fixed-target experiments will provide even more stringent bounds on light hidden vectors in the near future. Hidden vector production in such experiments occurs dominantly in the forward direction as it is enhanced by a collinear singularity cut off by the vector mass. Thus the lightest KK modes will be produced most abundantly. The fixed-target search techniques proposed in Refs. [6, 7, 8] are suited for hidden vectors that decay directly to a pair of SM fermions. These searches can therefore also be sensitive to the $n = 0, 1$ KK modes of a bulk hidden vector. Modified search techniques will likely be
Figure 8: Hidden sector production in the s-channel. This only requires one kinetic mixing insertion (represented by the cross) and therefore scales as $\epsilon^2$, improving the prospects for production. Resonant enhancement can occur when $\sqrt{s} \simeq m_n$ for one of the small-$n$ modes.

needed to detect the higher KK modes, which become progressively broader and decay to complicated multi-body final states.

### 6.2 New Searches at Meson Factories

Meson factories like DAΦNE, BABAR and Belle provide some of the most promising means by which to probe the warped hidden sectors we consider. The specific signals depend on the value of the hidden IR scale relative to the center-of-mass (CoM) energy $\sqrt{s}$ of the given experiment. If $\sqrt{s} \simeq m_n$ for one of the narrow small-$n$ KK modes, hidden vector production can be resonantly enhanced and specific exclusive signatures can be searched for. However at higher CoM energies $\sqrt{s} \gtrsim (M_*/k) R^{-1}$, well above the mass of the narrow lighter KK modes, one instead probes the continuum part of the spectrum and there is no resonant enhancement. In this regime the typical signal consists of a large multiplicity of soft SM particles.

The approaches discussed for discovering a single Abelian hidden vector in Refs. [5, 6, 7, 58] also apply to the narrow $n = 0, 1$ KK modes in the present scenario when they decay primarily to the SM. These searches focus on single vector production via $e^+e^- \rightarrow \gamma X_n$. The hidden vector decays to a pair of SM leptons or pions, giving signals like $\ell^+\ell^-\gamma$. The tiny vector width leads to a distinctive dilepton invariant mass peak that can potentially be distinguished from the smooth SM background. A detailed search for this signal within the BABAR and Belle $\Upsilon(3s)$ and $\Upsilon(4s)$ datasets could potentially probe hidden vectors with kinetic mixing as low as $10^{-3}$. [6, 7, 8]. These searches will be most sensitive to the $n = 0$ mode as the single vector production rate scales like $\epsilon^2_n \simeq \epsilon^2_0/(36n^2)$, (see (7) and (8)). More complicated multi-lepton or pion final states can also arise for the $n = 0, 1$ modes if there is a light hidden Higgs [5, 35, 58].

The same production channel can also be used to probe narrow $n > 1$ vector modes that decay mainly into the hidden sector before cascading back to the SM. For example, production of an $n = 2$ vector mode would give

$$e^+e^- \rightarrow \gamma X_2 \rightarrow \gamma X_0 h_1.$$  \hfill (24)
Figure 9: Inclusive cross section for hidden sector production at the center-of-mass energy of DAΦNE ($\sqrt{s} = 1.02$ GeV) as a function of the hidden IR scale $R^{-1}$. The plot is for the Higgsless case with the kinetic mixing parameter for the zero mode fixed at $\epsilon_0 = 10^{-3}$.

The KK graviton decays via $h_1 \to 2X_0$ so the final state consists of $\gamma + 6\ell$ (for leptonic $n = 0$ KK decays). Similar statements hold for single vector production in Kaon decays [29, 7]. Again, even higher multiplicities in the final state can arise if there is an explicit light IR Higgs.

Hidden sector production can be resonantly enhanced when $\sqrt{s} \sim m_n$. For two reasons, this enhancement is not significant for hidden vectors that decay entirely to the SM, such as the $n = 0, 1$ KK vectors. First, the relevant process in this case is $e^+e^- \to X \to f\bar{f}$, and its rate is proportional to $\epsilon^4$. Second, the resonance is typically much narrower than the beam energy spread and gets strongly smeared out. In the present scenario, however, the KK gravitons in the spectrum permit more efficient $s$-channel production processes, such as that of Fig. 8, which only require one kinetic mixing insertion and thus go like $\epsilon^2$. The graviton decay modes of a hidden vector also broaden its resonance. This is analogous to multi-vector production in non-Abelian hidden gauge sectors [6], as one might expect from gauge-gravity duality.

We estimate the prospects for observing resonant production by calculating the inclusive hidden-sector cross section at meson factories. At the relatively low energies (1–10 GeV) of these machines we may neglect diagrams involving the $Z$ boson and consider the insertion of a kinetic mixing operator between a hidden vector $X_n$ and the SM photon. Applying a unitarity cut gives the inclusive hidden sector cross section as

$$\sigma(e^+e^- \to \text{Hidden}) \simeq \sum_n \frac{e^2 c_w^2 \epsilon_n^2 \Gamma_n \sqrt{s}}{(s - m_n^2)^2 + s \Gamma_n^2}, \quad (s \ll m_Z^2) \quad (25)$$

where $s$ is the usual Mandelstam variable. We plot this inclusive cross section as a function
of $1/R$ at the CoM energy of DAΦNE ($\sqrt{s} = 1.02$ GeV) in Fig. 9 and for the B-factories ($\sqrt{s} = 10.58$ GeV) in Fig. 10. In both figures we fix $\epsilon_0 = 10^{-3}$.

The peaks in Figs. 9 and 10 occur at $s$-channel KK resonances, when $1/R$ is such that $\sqrt{s} \simeq m_n$ for one of the narrow, small $n$ KK modes. For example, the peak at $R^{-1} \simeq 5$ GeV in Fig. 9 occurs because DAΦNE would be sitting right on the zero mode mass of $m_0 \simeq 1$ GeV. In practice, this very narrow peak will be significantly smeared out by the spread in beam energy; about 0.2 MeV at DAΦNE [59] and a few MeV at the B-factories [60]. As a result, it is unlikely to be visible as a direct $s$-channel resonance [7, 29]. The same also applies to the $n = 1$ mode when it decays entirely to the SM via kinetic mixing. Higher vector KK modes have decays to the hidden sector, making them much broader and relatively insensitive to the energy spread of the meson factory beams. As $1/R$ decreases, the peaks become more and more closely-spaced and less sharp. This corresponds to resonances occurring at ever larger KK mode numbers, which eventually begin to overlap with each other.

For $\sqrt{s} \gg M_* / kR$ the inclusive cross-section becomes roughly independent of $1/R$, as can be seen in Fig. 10 (similar behaviour would be seen in Fig. 9 at smaller values of $1/R$). This reflects the IR insensitivity of the theory for large injection energies on the UV brane, as discussed in Section 4. In this region the inclusive cross section can be written in a simple form by using the RS2 propagator of Eq. (17) in Eq. (25):

$$
\sigma(e^+ e^- \rightarrow \text{Hidden}) \simeq \frac{\pi}{2} \frac{k}{M_*} \frac{e^2 c_W^2 c_s^2}{\log(2k/\sqrt{s}) - \gamma} \frac{1}{s} \frac{1}{s}
$$

$$
\simeq \frac{\pi}{2} \frac{e^2 c_W^2 c_s^2}{\log(2k/\sqrt{s}) - \gamma} \frac{1}{s}.
$$
Figure 11: Production of six-lepton final states in the $s$-channel. Hidden sector production occurs via $e^+e^- \to h_1X_0$ and the KK graviton promptly decays to $2X_0$ (collectively represented by the blob). The $3X_0$'s in turn decay to six light SM fields. Production is only suppressed by $\epsilon^2$ and can be resonantly enhanced for $\sqrt{s} \simeq m_2$. Similar processes lead to eight-lepton final states.

We note that, up to a mild logarithmic sensitivity to $k$, this depends on the single unknown parameter $\epsilon_0$ and can be written as:

$$\sigma(e^+e^- \to \text{Hidden}) \simeq \left( \frac{\epsilon_0}{10^{-3}} \right)^2 \times \left( \frac{10.58 \text{ GeV}}{\sqrt{s}} \right)^2 \times 10 \text{ fb} .$$

For $\epsilon_0 = 10^{-3}$ one obtains an asymptotic small-$1/R$ inclusive cross section of $\sigma(e^+e^- \to \text{Hidden}) \simeq 10 \text{ fb}$ for the $B$-factories, as seen in Fig. 10.

Once created, hidden states will cascade down to the lightest vector modes, $n = 0, 1$. Provided the kinetic mixing is not too small, $\epsilon_{0,1} \gtrsim 10^{-4}$, these lightest hidden modes will decay relatively promptly to the SM. Thus a typical final state will consist of a number of SM fields with pairwise invariant masses equal to one of the KK vector masses. A light hidden Higgs or smaller values of $\epsilon_{0,1}$ can also give rise to displaced vertices.

The optimal detection strategy at meson factories depends on the quantity $R\sqrt{s}$. For $R\sqrt{s} \lesssim 1$, only the lightest modes will be produced in association with a photon, with $n = 0$ modes dominating. The resulting final state will consist of a pair of SM particles reconstructing the KK vector mass. For larger $R\sqrt{s}$ there can be resonant production of hidden vectors in the $s$-channel. The final states in this case will consist of a number of SM fields with pairwise invariant masses equal to the mass of the mode $n = 0$ or $n = 1$. The precise number of final-state SM particles increases with $R\sqrt{s}$, but as the multiplicity increases so too will the combinatoric problem of reconstructing pairwise invariant mass peaks. On the other hand, a high multiplicity of charged pions and leptons should make these events very distinctive. Scanning the inclusive multiparticle cross section in energy may allow one to probe heavier KK vector resonances that are not overly broad.

More stringent bounds on warped Abelian hidden sectors (or possible discovery signals) could potentially be obtained with existing data. New searches for narrow resonances in four-lepton final states have recently been undertaken by the BABAR Collaboration [61]. Their analysis demands that the four leptons reconstruct to (nearly) the full beam energy, resulting
in very strong bounds of $\sigma(e^+e^- \to 2\ell 2\ell') < (25-60) \text{ab}$ for hidden vector masses in the range $[0.24, 5.3] \text{GeV}$. This final state can result from the $s$-channel process $e^+e^- \to W' W' \to 2\ell 2\ell'$ in theories with a non-Abelian hidden sector, but does not occur through the $s$-channel in our warped model. Four-lepton final states can occur in the present model via light vector production in the $t$-channel, $e^+e^- \to 2X_{0,1}$, with the final state vectors decaying to lepton pairs. However, due to the $e^4$ suppression the BABAR bound is not severe.

On the other hand, improved analysis of six-lepton final states like

$$e^+e^- \to 2\mu 4e, \ 4\mu 2e, \ 6\mu,$$

(26)
could improve the bounds on the warped model. These occur via hidden sector production in the $s$-channel with only $e^2$ suppression (see Fig. 11), permitting more efficient production. The same is true of eight-lepton final states from processes like $e^+e^- \to h_2 X_0 \to 4X_0$. Given the strong bounds from the four-lepton BABAR analysis (61) relative to the typical cross sections shown in Fig. 10 similar experimental studies for $N > 4$ final-state leptons could greatly improve the prospects for detection. One hopes that such analysis will be forthcoming. Note that with a total integrated luminosity of $\sim 1 \text{ab}$, a production cross section of order $a \text{fb}$ corresponds to a few hundred to a few thousand hidden sector production events.

Together these features suggest an interesting multi-faceted approach to experimentally studying the model for $m_Z R \gg 1$. One may be able to probe individual resonances directly at low-energy colliders and fixed target experiments operating at order GeV energies, and extract information on the kinetic mixing parameter $\epsilon_0$. Via Eq. (26) one could then predict the expected inclusive cross section for hidden sector production at the $B$-factories. Furthermore these two probes can be combined with an expected signal from hidden $Z$ decays as given in Eq. (23). These correlations provide important means by which to discriminate the present framework from alternative light hidden sectors.

7 Conclusion

In this work we have investigated the detailed phenomenology of an Abelian warped hidden sector, focusing on hidden symmetry breaking scales much less than the weak scale. Such light hidden sectors may be of interest in connection with recent models of dark matter, but more generally comprise an interesting scenario for beyond-the-SM physics. Despite the relative simplicity of our model the low-energy phenomenology was seen to be quite rich. The main new feature of our construct, relative to other works with light hidden sectors, is the existence of a tower of hidden KK vectors that kinetically mix with SM hypercharge.

We have considered the decay properties of the hidden sector fields in some detail, and developed a useful approach for calculating processes initiated on the UV brane with large injection momentum relative to the hidden IR scale. Using these results, we have considered the detailed bounds on the model from precision electroweak observables and low-energy experiments, and found that viable parameter sets permitting significant numbers of hidden sector production events exist.
For a hidden IR scale in the range 10 MeV to 10 GeV, we find that the constraints on a tower of hidden vectors are nearly identical to those on a single hidden vector whose mass and kinetic mixing equals that of the zero mode. This is primarily due to the properties of the KK vectors: the higher modes are heavier and, more importantly, their kinetic mixing strength decreases (relative to the zero mode). The most stringent bounds are obtained by applying existing constraints from beam dumps, $\Upsilon$ decays, and anomalous lepton magnetic moments directly to the zero mode. The precise bounds can be read off the figures in Refs. [8, 29], and generally require $\epsilon_0 \lesssim 3 \times 10^{-3}$ for $m_0 \sim \mathcal{O}(\text{GeV})$. Once the zero mode is made to satisfy these low-energy constraints, compatibility with precision electroweak data is ensured.

Relative to models with a single hidden vector there are, however, some important differences that can have implications for future searches. The hidden sector contains multiple light vectors that can potentially be probed as individual resonances at low-energy experiments like the $B$-factories and fixed-target experiments for values of the kinetic mixing parameter that are consistent with existing direct search constraints. The model also predicts hidden sector decays of the $Z$-boson with rates essentially dependent on the single parameter $\epsilon_0$ for the interesting range $m_Z R \gg 1$. As this parameter controls the production and decay rate of the lightest vectors, important correlations exist between the low-energy signals and the hidden $Z$ width. Future experimental studies of six- and eight-lepton final states could also improve the bounds on the model, or potentially discover evidence for a hidden warped extra dimension.

Acknowledgements

We thank Kaustubh Agashe, Brian Batell, Robert McPherson, John Ng, Lisa Randall, Steve Robertson, Matt Schwartz, Jessie Shelton, Andy Spray, Matt Strassler, Jay Wacker, and Lian-Tao Wang for helpful discussions. KM thanks the theory group at the University of Melbourne for hospitality while parts of this work were completed. DM thanks the Aspen Center for Physics for their hospitality. This work was supported by NSERC.

Appendix

A Hidden Sector Decays

In this appendix we present the decay widths for KK vectors and KK gravitons into the hidden sector.
A.1 Vector Decays

To determine the coupling between KK vectors and gravitons one expands the metric as

$$G_{\mu\nu} \to (kz)^{-2} \left[ \eta_{\mu\nu} + \frac{2}{M^2} h_{\mu\nu}(x, z) \right],$$

where we work in the gauge $\partial^\mu h_{\mu\nu} = 0 = h_{\mu\nu}$. KK expanding the graviton fluctuation as

$$h_{\mu\nu}(x, z) = \sum_a h^{(a)}_{\mu\nu}(x) f^{(a)}(z),$$

the effective 4D Lagrangian contains the following coupling between the KK gravitons and vectors:

$$\mathcal{L}_{\text{eff}} \supset \frac{k}{M_{Pl}} \sum_{a,m,n} \eta^{\rho\sigma} \eta^{\nu\beta} \left[ \zeta_{a,mn} X_{\mu\nu} X_{\alpha\beta} - \xi_{a,mn} X^m X^n \right].$$

Here the factors $\zeta_{a,mn}$ and $\xi_{a,mn}$ encode the wavefunction overlap along the extra dimension,

$$\zeta_{a,mn} = \frac{1}{k^{3/2}} \frac{dz}{(kz)} f^{(a)} f^{(m)} f^{(n)},$$

$$\xi_{a,mn} = \frac{1}{k^{3/2}} \frac{dz}{(kz)} f^{(a)} \partial_z f^{(m)} \partial_z f^{(n)},$$

and $f^{(m)}_X, f^{(n)}_X$ are the KK vector profiles. The vertex derived from Eq. (29) is

$$h^{(a)}_{\mu\nu}(p) X^m_{\nu}(k_1) X^n_{\beta}(k_2) :$$

$$-2i \frac{k}{M_{Pl}} \left\{ \eta^{\rho\sigma} \eta^{\nu\beta} \left[ \zeta_{a,mn} + k_1 \cdot k_2 \zeta_{a,mn} \right] + \zeta_{a,mn} \left[ \eta^{\rho\sigma} k_1^\rho k_2^\sigma - \eta^{\nu\beta} k_1^\nu k_2^\beta - \eta^{\rho\sigma} k_1^\nu k_2^\beta \right] \right\},$$

where the graviton momentum $p$ is defined as incoming and the vector momenta $k_{1,2}$ are outgoing. This vertex can be used to calculate the decay width for $X_n \to X_m h_a$. We find the width to be

$$\Gamma(X_n \to X_m h_a) = \frac{1}{144\pi} \frac{k^2 m^7}{M_{Pl}^2 m_a^4} \times [1 - (r_m + r_a)^2]^{1/2} [1 - (r_m - r_a)^2]^{1/2} \times \left\{ 2 \xi_{a,mn} G_{a,mn}^\xi + 40 \zeta_{a,mn} \left[ \frac{\xi_{a,mn}}{m_m m_a} \right] G_{a,mn}^\zeta \left[ \frac{\xi_{a,mn}}{m_m m_a} \right] + \left[ \frac{\xi_{a,mn}}{m_m m_a} \right]^2 G_{a,mn}^\xi \right\},$$

where we write the mass ratios as $r_{m,a} = m_{m,a}/m_a$ and define the following set of constants

$$G_{a,mn}^\zeta = 1 + 2(7r_a^2 - 2r_m^2) + (6r_m^4 - r_m^2 r_a^4 + r_a^4) - (4r_m^4 + r_a^2 r_m^2 - 34r_a^4 r_m^2 + 9r_a^6) + (r_m^8 + r_m^2 r_a^4 + r_a^2 r_m^4 - 9r_a^6 r_m^2 + 6r_a^8),$$

$$G_{a,mn}^\xi = r_m r_a^2 \left\{ 1 + (r_a^2 - 2r_m^2) + (r_m^4 + r_a^2 r_m^2 - 2r_a^4) \right\},$$

$$G_{a,mn}^\zeta = 1 + 2(3r_a^2 - 2r_m^2) - 2(7r_a^4 + 3r_a^2 r_m^2 - 3r_m^4) + 2(3r_a^6 + 42r_a^2 r_m^4 - 3r_m^4 r_a^2 - 2r_a^6) + (r_a^2 - r_m^2)^2 (r_m^4 + 8r_a^2 r_m^2 + r_a^4).$$

(34)
We note that the sum over graviton polarizations necessary for calculating the KK decays is

\[
\sum_{\text{pol.}} e_{\alpha,\rho \sigma}(p) e^{*}_{\alpha,\mu \nu}(p) = B_{\rho \sigma, \mu \nu}(p),
\]

(35)

where

\[
B_{\rho \sigma, \mu \nu}(p) = \left( \eta_{\rho \mu} - \frac{p_{\rho} p_{\mu}}{m_a^2} \right) \left( \eta_{\sigma \nu} - \frac{p_{\sigma} p_{\nu}}{m_a^2} \right) + \left( \eta_{\rho \nu} - \frac{p_{\rho} p_{\nu}}{m_a^2} \right) \left( \eta_{\sigma \mu} - \frac{p_{\sigma} p_{\mu}}{m_a^2} \right) \\
- \frac{2}{3} \left( \eta_{\rho \sigma} - \frac{p_{\rho} p_{\sigma}}{m_a^2} \right) \left( \eta_{\mu \nu} - \frac{p_{\mu} p_{\nu}}{m_a^2} \right).
\]

(36)

A.2 Hidden KK Graviton Decays

The KK graviton decay \( h_a \rightarrow X_m X_n \) is kinematically available for

\[
a \geq m + n \quad \text{Higgsed}, \\
a > m + n \quad \text{Higgsless},
\]

(37)

where the Higgsed case assumes that the Higgs VEV is less than the IR scale so that the zero mode vector acquires a mass \( m_0 \sim (5R)^{-1} \). For \( a > 0 \) the KK graviton masses are approximately \( m_a R \approx \left( a + 1/4 \right) \pi \) and for a given value of \( n > 0 \) there are less available channels in the Higgsless case. The width for the decay of a KK graviton into two vectors is

\[
\Gamma(h_a \rightarrow X_m X_n) = \frac{S_{mn} k^2}{240 \pi M_{Pl}^2} m_a^3 [1 - (\bar{r}_m + \bar{r}_n)^2]^{1/2} [1 - (\bar{r}_m - \bar{r}_n)^2]^{1/2} \\
\times \left\{ 12 \zeta_{a,mn}^2 F_{a,mn}^\xi + 80 \zeta_{a,mn} \left[ \frac{\zeta_{a,mn}}{m_m m_n} \right] F_{a,mn}^{\xi \zeta} + \left[ \frac{\zeta_{a,mn}}{m_m m_n} \right]^2 F_{a,mn}^\xi \right\},
\]

(38)

where we define the symmetry factor and mass ratios respectively as

\[
S_{mn} = [1 - (1/2) \delta_{mn}], \quad \bar{r}_{m,n} = \frac{m_{m,n}}{m_a},
\]

(39)

and also define the following constants:

\[
F_{a,mn}^\xi = 1 - \frac{3}{2} (\bar{r}_m + \bar{r}_n)^4 + \frac{1}{6} (\bar{r}_m^4 + 34 \bar{r}_m^2 \bar{r}_n^2 + \bar{r}_n^4) + \frac{1}{6} (\bar{r}_m^2 - \bar{r}_n^2)^2 (\bar{r}_m^2 + \bar{r}_n^2) + \frac{1}{6} (\bar{r}_m^2 - \bar{r}_n^2)^4 \\
F_{a,mn}^{\zeta \xi} = \bar{r}_m \bar{r}_n \left\{ 1 - \frac{1}{2} (\bar{r}_m^2 + \bar{r}_n^2) - \frac{1}{2} (\bar{r}_m^2 - \bar{r}_n^2)^2 \right\} \\
F_{a,mn}^{\zeta} = 1 + 6(\bar{r}_m^2 + \bar{r}_n^2) - 14(\bar{r}_m^4 - 6\bar{r}_m^2 \bar{r}_n^2 + \bar{r}_n^4) + 6(\bar{r}_m^2 - \bar{r}_n^2)^2 (\bar{r}_m^2 + \bar{r}_n^2) + (\bar{r}_m^2 - \bar{r}_n^2)^4.
\]

With an order GeV IR scale the KK graviton decays are prompt, as can be seen in Fig. 12 where we plot the two-body decay width \( \sum_{m,n} \Gamma(h_a \rightarrow X_m X_n) \). Heavier modes are even
broader due to the increase in available final states. Similar to the hidden vector decays the presence of an approximate KK number conservation means that decays with \( a \sim m + n \) are dominant, as can be seen in Fig. 13 where we plot \( \Gamma(h_a \to X_m X_n) \) against daughter KK number for the Higgsless case with fixed \( m + n \).

Note that crossing symmetry requires the matrix element for the KK graviton decay and the KK vector decay to be related and one can show that this requires the amplitudes for decay to be related via

\[
\sum_{\text{pol.}} |\mathcal{M}(X_n \to h_a X_m)|^2_{\xi \to -\xi} = \sum_{\text{pol.}} |\mathcal{M}(h_a \to X_m X_n)|^2.
\]  

We have checked that this relation holds for the matrix elements we have employed. We also note that, in general, stabilization of the extra dimension will involve some additional bulk field like a Goldberger-Wise scalar \cite{39} or some flux. This will couple to KK gravitons and one expects additional decay channels as a result.

\section*{B Amplitude for \( e^+e^- \to f \bar{f} \)}

Using the vertices from Sec. 5.1 we can calculate the process \( e^+e^- \to f \bar{f} \) with \( f \neq e \). We work to leading non-trivial order in \( \epsilon_* \) and neglect the fermion masses. The resulting summed and squared matrix element is:

\[
\frac{1}{4} \sum_{s,s'} |\mathcal{M}|^2 = \left( \frac{s^2}{4} \right) \left[ (|a_{LL}|^2 + |a_{RR}|^2 + |a_{LR}|^2 + |a_{RL}|^2) (1 + \cos^2 \theta) \right. \\
+ \left. (|a_{LL}|^2 + |a_{RR}|^2 - |a_{LR}|^2 - |a_{RL}|^2) (2 \cos \theta) \right],
\]

where the factors \( a_{AB} \) (\( A B = L, R \)) are given by

\[
a_{AB} = (a_Z + a_{ZXZ}) g_{A_2}^f g_{B_2}^f + (a_\gamma + a_{\gamma X}) g_{A_2}^f g_{B_2}^f + a_{ZX\gamma} g_{A_2}^f g_{B_2}^f + a_{\gamma X Z} g_{A_2}^f g_{B_2}^f.
\]
Figure 13: Higgsless Case: Plot of $\Gamma(h_{a=45} \to X_n X_m)$ Vs. Daughter KK number $n$ for fixed values of $(m+n)$. From top to bottom the curves satisfy $(m+n) = (44, 43, 42, 41)$.

Here, $g_{LZ}^f = g_{RZ}^f = e Q = (\bar{g}/c_W s_W) Q$, $g_{LZ}^f = \bar{g}(t^3_L - Q s^2_W)$, and $g_{RZ}^f = \bar{g}(-Q s^2_W)$, $\bar{g} = \sqrt{g^2 + g'^2}$, while the $a_V$ terms are given by

\[
a_Z = \frac{1}{p^2 - m^2_W + i\Gamma_Z m_Z} := \Delta^Z_p \quad (43)\\
\]
\[
a_{ZXZ} = \frac{\epsilon^2}{M^2} s^2_W p^4 (\Delta^Z_p)^2 \Delta^{UV}_p \quad (44)\\
\]
\[
a_{\gamma} = \frac{1}{p^2} \quad (45)\\
\]
\[
a_{\gamma X\gamma} = \frac{\epsilon^2}{M^2} c^2_W \Delta^{UV}_p \quad (46)\\
\]
\[
a_{ZX\gamma} = \frac{\epsilon^2}{M^2} (-c_W s_W) p^2 \Delta^Z_p \Delta^{UV}_p = a_{\gamma X\gamma}. \quad (47)\\
\]

Here, $\Delta^{UV}_p$ is the 5D bulk-to-bulk propagator for which we use the matched expression of Eq. (18). When the final state is $e^+e^-$, there is an additional $t$-channel contribution to the amplitude.

References

[1] M. J. Strassler, K. M. Zurek, Phys. Lett. B651, 374-379 (2007) [hep-ph/0604261].

[2] B. Patt and F. Wilczek, [hep-ph/0605188]

[3] K. M. Zurek, 1001.2563 [hep-ph].

[4] N. Borodatchenkova, D. Choudhury and M. Drees, Phys. Rev. Lett. 96, 141802 (2006) [hep-ph/0510147]; S. Heinemeyer, Y. Kahn, M. Schmitt and M. Velasco, 0705.4056 [hep-ex]; M. Ahlers, H. Gies, J. Jaeckel, J. Redondo and A. Ringwald, Phys. Rev. D 76 (2007) 115005 [0706.2836 [hep-ph]]; J. Jaeckel and A. Ringwald, Phys. Lett. B 659
(2008) 509 [0707.2063 [hep-ph]]; M. Freytsis, G. Ovanesyan and J. Thaler, JHEP 1001
(2010) 111 [0909.2862 [hep-ph]]; J. Jaeckel, A. Ringwald, Ann. Rev. Nucl. Part. Sci.
60, 405-437 (2010). [arXiv:1002.0329 [hep-ph]]. L. Barze, G. Balossini, C. Bignamini,
C. M. C. Calame, G. Montagna, O. Nicrosini and F. Piccinini, 1007.4984 [hep-ph].

[5] B. Batell, M. Pospelov and A. Ritz, Phys. Rev. D 79, 115008 (2009) [0903.0363 [hep-ph]];
B. Batell, M. Pospelov and A. Ritz, 0911.4938 [hep-ph].

[6] R. Essig, P. Schuster and N. Toro, Phys. Rev. D 80, 015003 (2009) [0903.3941 [hep-ph]].

[7] M. Reece and L. T. Wang, JHEP 0907, 051 (2009) [0904.1743 [hep-ph]].

[8] J. D. Bjorken, R. Essig, P. Schuster and N. Toro, Phys. Rev. D 80, 075018 (2009)
[0906.0580 [hep-ph]]. R. Essig, P. Schuster, N. Toro and B. Wojtsekhowski, [1001.2557
[hep-ph]].

[9] M. Pospelov, A. Ritz and M. B. Voloshin, Phys. Lett. B 662, 53 (2008) [0711.4866
[hep-ph]]; M. Pospelov and A. Ritz, Phys. Lett. B 671, 391 (2009) [0810.1502 [hep-ph]].

[10] N. Arkani-Hamed, D. P. Finkbeiner, T. R. Slatyer and N. Weiner, Phys. Rev. D 79,
015014 (2009) [0810.0713 [hep-ph]]; N. Arkani-Hamed and N. Weiner, JHEP 0812, 104
(2008) [0810.0714 [hep-ph]].

[11] D. Hooper and K. M. Zurek, Phys. Rev. D 77, 087302 (2008) [0801.3686 [hep-ph]];
D. E. Morrissey, D. Poland and K. M. Zurek, JHEP 0907, 050 (2009) [0904.2567 [hep-
ph]]; M. Baumgart, C. Cheung, J. T. Ruderman, L. T. Wang and I. Yavin, JHEP 0904,
014 (2009) [0901.0283 [hep-ph]].

[12] D. S. M. Alves, S. R. Behbahani, P. Schuster et al., Phys. Lett. B692, 323-326 (2010)
[0903.3945 [hep-ph]]; T. Hambye and M. H. G. Tytgat, Phys. Lett. B 683, 39 (2010)
[0907.1007 [hep-ph]]; D. S. M. Alves, S. R. Behbahani, P. Schuster and J. G. Wacker,
JHEP 1006, 113 (2010) [1003.4729 [hep-ph]]; R. Foot, Phys. Lett. B 692, 65 (2010)
[1004.1424 [hep-ph]].

[13] T. Gherghetta, B. Harling, JHEP 1004, 039 (2010). [1002.2967 [hep-ph]]; D. Bunk,
J. Hubisz, Phys. Rev. D81, 125009 (2010). [1002.3160 [hep-ph]].

[14] K. L. McDonald and D. E. Morrissey, JHEP 1005, 056 (2010) [1002.3361 [hep-ph]].

[15] L. Randall and R. Sundrum, Phys. Rev. Lett. 83, 3370 (1999) [hep-ph/9905221].

[16] L. Randall and R. Sundrum, Phys. Rev. Lett. 83, 4690 (1999) [hep-th/9906064].

[17] N. Arkani-Hamed, M. Porrati and L. Randall, JHEP 0108, 017 (2001) [hep-th/0012148];
R. Rattazzi and A. Zaffaroni, JHEP 0104, 021 (2001) [hep-th/0012248]; M. Perez-
Victoria, JHEP 0105, 064 (2001) [hep-th/0105048].

[18] K. Agashe, A. Delgado, M. J. May and R. Sundrum, JHEP 0308, 050 (2003)
[hep-ph/0308036]; K. Agashe, R. Contino, L. Da Rold and A. Pomarol, Phys. Lett.
B 641, 62 (2006) [hep-ph/0605341].
[19] H. Davoudiasl, J. L. Hewett and T. G. Rizzo, Phys. Rev. Lett. 84, 2080 (2000) [hep-ph/9909255].

[20] B. Holdom, Phys. Lett. B 166, 196 (1986); R. Foot and X. G. He, Phys. Lett. B 267, 509 (1991).

[21] C. Csaki, C. Grojean, H. Murayama, L. Pilo and J. Terning, Phys. Rev. D 69, 055006 (2004) [hep-ph/0305237]; C. Csaki, C. Grojean, L. Pilo and J. Terning, Phys. Rev. Lett. 92, 101802 (2004) [hep-ph/0308038].

[22] B. Batell and T. Gherghetta, Phys. Rev. D 76 (2007) 045017 [0706.0890 [hep-th]].

[23] B. Gripaios, Nucl. Phys. B 768, 157 (2007) [Erratum-ibid. 830, 390 (2010)] [hep-ph/0612118]; T. Flacke and D. Maybury, JHEP 0703 (2007) 007 [hep-ph/0612126].

[24] K. L. McDonald, Phys. Lett. B 696, 266 (2011) [arXiv:1010.2659 [hep-ph]].

[25] S. A. Abel, M. D. Goodsell, J. Jaeckel, V. V. Khoze and A. Ringwald, JHEP 0807 (2008) 124 [0803.1449 [hep-ph]]; M. Goodsell, J. Jaeckel, J. Redondo and A. Ringwald, JHEP 0911, 027 (2009) [0909.0515 [hep-ph]]; M. Bullimore, J. P. Conlon and L. T. Witkowski, [1009.2380 [hep-th]].

[26] C. Csaki, M. Reece and J. Terning, JHEP 0905, 067 (2009) [0811.3001 [hep-ph]].

[27] M. A. Stephanov, Phys. Rev. D76, 035008 (2007). [0705.3049 [hep-ph]].

[28] M. J. Strassler, [0801.0629 [hep-ph]].

[29] M. Pospelov, Phys. Rev. D 80, 095002 (2009) [0811.1030 [hep-ph]].

[30] C. Amsler et al. [Particle Data Group], Phys. Lett. B 667, 1 (2008).

[31] V. V. Ezhela, S. B. Lugovsky and O. V. Zenin, [hep-ph/0312114].

[32] M. Davier, S. Eidelman, A. Hocker and Z. Zhang, Eur. Phys. J. C 27, 497 (2003) [hep-ph/0208177].

[33] W. D. Goldberger and I. Z. Rothstein, Phys. Rev. Lett. 89, 131601 (2002) [hep-th/0204160]; W. D. Goldberger and I. Z. Rothstein, Phys. Rev. D 68, 125011 (2003) [hep-th/0208060]; W. D. Goldberger and I. Z. Rothstein, Phys. Rev. D 68, 125012 (2003) [hep-ph/0303158].

[34] M. Reece and L. T. Wang, JHEP 1007, 040 (2010) [1003.5669 [hep-ph]].

[35] S. Gopalakrishna, S. Jung and J. D. Wells, Phys. Rev. D 78, 055002 (2008) [0801.3456 [hep-ph]].

[36] P. Schuster, N. Toro and I. Yavin, Phys. Rev. D 81, 016002 (2010) [0910.1602 [hep-ph]].

[37] C. Csaki, M. L. Graesser and G. D. Kribs, Phys. Rev. D 63, 065002 (2001) [hep-th/0008151]; K. m. Cheung, Phys. Rev. D 63, 056007 (2001) [hep-ph/0009232].

30
[38] O. DeWolfe, D. Z. Freedman, S. S. Gubser and A. Karch, Phys. Rev. D 62, 046008 (2000) [hep-th/9909134]; C. Csaki, M. L. Graesser and G. D. Kribs, Phys. Rev. D 63, 065002 (2001) [hep-th/0008151]; T. Konstandin, G. Nardini and M. Quiros, 1007.1468 [hep-ph].

[39] W. D. Goldberger and M. B. Wise, Phys. Rev. Lett. 83, 4922 (1999) [hep-ph/9907447].

[40] S. S. C. Law, K. L. McDonald, Phys. Rev. D82, 104032 (2010) arXiv:1008.4336 [hep-ph].

[41] A. Pomarol, Phys. Rev. Lett. 85, 4004 (2000) [hep-ph/0005293].

[42] L. Randall and M. D. Schwartz, JHEP 0111, 003 (2001) [hep-th/0108114].

[43] K. w. Choi, H. D. Kim and I. W. Kim, JHEP 0211, 033 (2002) [hep-ph/0202257].

[44] K. Agashe, A. Delgado and R. Sundrum, Nucl. Phys. B 643, 172 (2002) [hep-ph/0206099]; K. Agashe and A. Delgado, Phys. Rev. D 67, 046003 (2003) [hep-th/0209212]; K. Agashe, A. Delgado and R. Sundrum, Annals Phys. 304, 145 (2003) [hep-ph/0212028].

[45] S. L. Dubovsky, V. A. Rubakov and P. G. Tinyakov, Phys. Rev. D 62, 105011 (2000) [hep-th/0006046]; S. L. Dubovsky and V. A. Rubakov, Int. J. Mod. Phys. A 16, 4331 (2001) [hep-th/0105243].

[46] A. Friedland, M. Giannotti and M. Graesser, Phys. Lett. B 678, 149 (2009) [0902.3676 [hep-th]]; A. Friedland, M. Giannotti and M. L. Graesser, JHEP 0909, 033 (2009) [0905.2607 [hep-th]].

[47] S. B. Giddings, E. Katz and L. Randall, JHEP 0003, 023 (2000) [hep-th/0002091].

[48] G. Cacciapaglia, A. Deandrea and S. De Curtis, Phys. Lett. B 682, 43 (2009) [0906.3417 [hep-ph]].

[49] G. Abbiendi et al. [OPAL Collaboration], Eur. Phys. J. C 19, 587 (2001) [hep-ex/0012018]; [ALEPH Collaboration and DELPHI Collaboration and L3 Collaboration and ], Phys. Rept. 427, 257 (2006) [hep-ex/0509008].

[50] J. Alcaraz et al. [ALEPH Collaboration and DELPHI Collaboration and L3 Collaboration and ], [hep-ex/0612034]. LEP Electroweak Working Group, [0911.2604 [hep-ex]].

[51] P. Janot, Phys. Lett. B 594, 23 (2004) [hep-ph/0403157].

[52] D. Karlen and H. Burkhardt, Eur. Phys. J. C 22, 39 (2001) [hep-ex/0105065]; G. Abbiendi et al. [OPAL Collaboration], Eur. Phys. J. C 45, 1 (2006) [hep-ex/0505072].

[53] A. Hook, E. Izaguirre, J. G. Wacker, [1006.0973 [hep-ph]].

[54] W. F. Chang, J. N. Ng and J. M. S. Wu, Phys. Rev. D 74, 095005 (2006) [Erratum-ibid. D 79, 039902 (2009)] [hep-ph/0608068].
[55] D. Feldman, Z. Liu and P. Nath, Phys. Rev. D 75, 115001 (2007) [hep-ph/0702123].

[56] P. D. Acton et al. [OPAL Collaboration], Phys. Lett. B 311, 391 (1993); M. Acciarri et al. [L3 Collaboration], Phys. Lett. B 353, 136 (1995).

[57] M. Ahlers, J. Jaeckel, J. Redondo and A. Ringwald, Phys. Rev. D 78, 075005 (2008) [0807.4143 [hep-ph]].

[58] P. Fayet, Phys. Rev. D 75, 115017 (2007) [hep-ph/0702176]. P. Fayet, Phys. Rev. D 74, 054034 (2006) [hep-ph/0607318].

[59] F. Bossi, E. De Lucia, J. Lee-Franzini, S. Miscetti and M. Palutan [KLOE Collaboration], Riv. Nuovo Cim. 31, 531 (2008) [0811.1929 [hep-ex]]; G. Amelino-Camelia et al., Eur. Phys. J. C 68, 619 (2010) [1003.3868 [hep-ex]].

[60] B. Aubert et al. [BABAR Collaboration], Nucl. Instrum. Meth. A 479, 1 (2002) [hep-ex/0105044]; B. Aubert et al. [BABAR Collaboration], Phys. Rev. D 72, 032005 (2005) [hep-ex/0405025].

[61] B. Aubert et al. [BABAR Collaboration], [0908.2821 [hep-ex]].

[62] T. Han, J. D. Lykken and R. J. Zhang, Phys. Rev. D 59, 105006 (1999) [hep-ph/9811350]; G. F. Giudice, R. Rattazzi and J. D. Wells, Nucl. Phys. B 544, 3 (1999) [hep-ph/9811291].