Acoustic radiation-based optimization of the placement of actuators for active control of noise transmitted through plates

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Abstract

Active Structural Acoustic Control (ASAC) is mostly performed using a passive noise barrier, vibration actuators, sensors and a control system. ASAC reduces or alters the vibration of the barrier structure in a way that blocks the noise propagation through it. However, it is crucial that the actuators are appropriately arranged to be able to effectively control the vibration of the barrier. If the actuators were not optimally arranged, then certain modes of the structure may be uncontrollable, or require a very high control effort. Hence, the locations of the actuators should be determined by a careful optimization process employing a model of the structure. A common approach is to maximize the controllability of the system over a defined frequency range of operation. However, such an optimization procedure often results in a solution that considers numerous vibration modes, only some of which are acoustically-relevant. That is, certain structural modes may vibrate considerably, but their contribution to the noise transmission and radiation would be negligible. Therefore, in the presented research a new acoustic radiation-based approach to the optimisation of the arrangement of actuators is proposed. A model of acoustic radiation is introduced and new cost functions are formulated to focus on modes that strongly contribute to noise transmission or radiation by the noise barrier. For the considered system, this enables an increase in the controllability measure of more than 5 dB for acoustically-relevant modes, which is similar to the level of improvement achieved when the number of actuators is doubled.

1. Introduction

Exposure to excessive acoustic noise is an important problem in modern society and it thus stimulates the development of a variety of noise reduction techniques. One such approach is to separate the recipients from the noise source using noise barriers. However, common passive barriers are often ineffective for low-frequency noise. They also tend to be thick, heavy, and introduce considerable heat insulation that may cause additional problems in certain applications. To overcome this limitation, passive barriers can be complemented with or replaced by actively controlled barriers, which incorporate control sources that may be either acoustic, such as loudspeakers, or structural, such as vibration actuators [1–4]. These active systems are most effective in the low-frequency range, where passive insulation fails. Under certain circumstances, even openings with dedicated active noise control systems can be integrated into barriers to allow air flow [5–7].

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Active noise barriers present many advantages over their passive counterparts, but they have to be carefully implemented in order to operate efficiently and achieve a high level of performance. One of the critical aspects in the design of an active barrier is the arrangement of the actuators, such that they are able to effectively control the vibration of the plate that forms the noise barrier. It is noteworthy that the optimization of the actuator arrangement is also an important step in the design of systems where plates are intentionally designed to emit sound [8].

Different techniques have been proposed over the years to optimize the arrangement of actuators for control applications. One approach primarily focuses on selecting a control strategy and defining a performance index, and then simultaneously optimizing the locations of the actuators and the controller parameters. Liu et al. [9] used a genetic algorithm and the spatial

| Nomenclature                                                                 | Description                                                                 |
|------------------------------------------------------------------------------|------------------------------------------------------------------------------|
| a                                                                            | length of the plate                                                          |
| A                                                                            | state matrix                                                                 |
| b                                                                            | width of the plate                                                           |
| B                                                                            | control matrix                                                               |
| c                                                                            | sound velocity in the air                                                    |
| $D_x, D_y, D_{xy}$                                                          | orthotropic rigidities of the plate                                          |
| $E_x, E_y$                                                                   | Young's moduli of the plate along the x and y directions, respectively       |
| $f_i$                                                                        | force generated by a $i$th actuator                                          |
| G                                                                            | shear modulus                                                                |
| h                                                                            | plate thickness                                                              |
| $k_c$                                                                        | acoustic wavenumber                                                          |
| K                                                                            | stiffness matrix                                                              |
| $m_{ai}$                                                                     | mass of the $i$th actuator                                                   |
| M                                                                            | mass matrix                                                                   |
| $M_a, M_p$                                                                   | mass matrices corresponding to kinetic energy of actuators and the plate, respectively |
| $N$                                                                          | number of employed trial functions                                           |
| $N_a$                                                                        | number of actuators bonded to the plate’s surface                           |
| $p_i(x,y,z)$                                                                 | modal sound pressure amplitude corresponding to $i$th vibration mode of the plate |
| $P_i$                                                                        | modal acoustic power corresponding to $i$th vibration mode of the plate     |
| $q$                                                                          | generalized plate displacement vector                                        |
| $Q$                                                                          | vector of generalized forces                                                 |
| $S_e, S_p$                                                                   | surface which encloses the vibrating plate and surface of the plate, respectively |
| $t$                                                                          | time                                                                         |
| $T$                                                                          | overall kinetic energy of the system                                         |
| $T_a, T_p$                                                                   | kinetic energies of actuators and the plate, respectively                    |
| $u$                                                                          | control vector                                                               |
| $U, U_p$                                                                     | overall potential energy of the system and potential energy of the plate, respectively |
| $v$                                                                          | modal displacement vector                                                    |
| $w(x,y,t)$                                                                   | displacement of the plate to the $z$-direction at time $t > 0$ and position $(x,y)$ |
| $W_c$                                                                        | Gramian matrix of controllability                                             |
| $x$                                                                          | state vector                                                                 |
| $i,j,k$                                                                      | positive integers                                                            |
| $x,y,z$                                                                      | coordinates in the global Cartesian coordinate system                       |
| $x_{ai}, y_{ai}$                                                             | coordinates of the $i$th actuator.                                           |
| $\xi, \eta, \gamma$                                                        | components of the acoustic wavevector                                        |
| $i$                                                                          | imaginary number satisfying equation $i^2 = -1$                              |
| $\lambda_{c,i}$                                                             | $(2i)$th element at the diagonal of the Gramian matrix, corresponding to the $i$th mode |
| $\nu_x, \nu_y$                                                              | Poisson's ratios of the plate corresponding to $x$ and $y$ direction, respectively |
| $\zeta_{d,i}$                                                                | damping coefficient corresponding to $i$th mode                             |
| $\Xi$                                                                        | damping matrix                                                               |
| $\rho_e, \rho_p$                                                            | air density and mass density of the plate material                           |
| $\phi_i(x,y)$                                                                | $i$th time-invariant trial function                                         |
| $\Phi_i$                                                                     | vector containing a set of time-invariant trial functions $\phi_i(x,y)$      |
| $\Phi$                                                                       | $i$th eigenvector ($i$th column in the eigenvector matrix $\Phi$)            |
| $\omega_i, \Omega$                                                          | $i$th eigenfrequency and the eigenfrequencies matrix, respectively           |
In this Section, a model of the vibroacoustic system is presented. The derivation begins with a description of the free vibrations of an orthotropic rectangular plate with inertial actuators attached to its surface. The Kirchhoff–Love theory of thin plates is used for this purpose. The boundary conditions of the plate are assumed to be fully-clamped. Then, the Rayleigh–Ritz method is employed to define an approximate solution, which provides the natural frequencies and mode shapes of the vibrating system. Subsequently, an appropriate Green’s function is used to estimate the acoustic radiation from the optimal placement of piezoelectric actuators for active vibration control of a membrane structure using the controllability Gramian and the particle swarm optimization algorithm was studied by Liu et al. [15].

The aforementioned studies provide methods for the optimization of actuator locations mainly for the Active Vibration Control (AVC) of plates. Although the same actuator configuration can also be used for Active Structural Acoustic Control (ASAC) as employed in active noise barriers [16–20], it is not necessarily the optimum arrangement for this purpose. The optimization of the actuator arrangement for vibration control entails a search for a solution that generally reaches a trade-off between controlling numerous modes of vibration. Some of these structural modes may radiate sound efficiently, whilst others may vibrate considerably without contributing strongly to the noise transmission or radiation; as a result, these modes do not need to be controlled in the context of a noise barrier. Therefore, in the presented research, a new acoustic radiation-based approach to the optimization of the arrangement of actuators on a plate for the control of noise transmission is proposed. A model of acoustic radiation is introduced into the optimization process and new cost functions are formulated to focus on modes that are truly relevant to the overarching goal of the barrier, which is to block the transmission of noise. The main contribution of this paper is thus providing new insight into the optimization process that should be adopted for the positioning of actuators in active noise barriers.

This paper is organized as follows. Section 2 presents a model of the vibroacoustic system, including both vibration and acoustic radiation phenomena. Section 3 is devoted to the verification of the developed model utilizing a real experimental setup. Then, Section 4 introduces the proposed actuator arrangement optimization process, including formulation of the optimization problem, introduction of new acoustic radiation-based cost functions and a brief description of the employed memetic algorithm. Then, in Section 5 the obtained optimization results are presented and analysed. Finally, advantages and limitations of the proposed approach are pointed out and discussed, and conclusions for future research are drawn.

2. Model of the vibroacoustic system

In this Section, a model of the vibroacoustic system is presented. The derivation begins with a description of the free vibrations of an orthotropic rectangular plate with inertial actuators attached to its surface. The Kirchhoff–Love theory of thin plates is used for this purpose. The boundary conditions of the plate are assumed to be fully-clamped. Then, the Rayleigh–Ritz method is employed to define an approximate solution, which provides the natural frequencies and mode shapes of the vibrating system. Subsequently, an appropriate Green’s function is used to estimate the acoustic radiation from the obtained modes. Finally, a state space form of the model is developed, which facilitates the controllability analysis and formulation of the cost functions used in the optimization of the actuator locations.

2.1. Model of plate vibration

For an orthotropic and homogeneous plate, which occupies the \(x–y\) plane in the reference stress-free state, free vibrations are governed by a differential system [21]

\[
D_x \frac{\partial^4 w}{\partial x^4} + 2(D_x v_y + 2D_{xy}) \frac{\partial^4 w}{\partial x^2 \partial y^2} + D_y \frac{\partial^4 w}{\partial y^4} + \rho h \frac{\partial^2 w}{\partial t^2} = 0,
\]

for \(x \in (0, a), \quad y \in (0, b), \quad t > t_0 > 0.

where

\[
D_x = \frac{E_y h^3}{12(1 - \nu_x \nu_y)}, \quad D_y = \frac{E_x h^3}{12(1 - \nu_x \nu_y)}, \quad D_{xy} = \frac{G h^3}{12}.
\]

The initial conditions are defined by

\[
w(x, y, t_0) = 0, \quad \frac{\partial w(x, y, t)}{\partial t} \bigg|_{t=t_0} = 0.
\]

In Eq. (1)–(4) the function \(w(x, y, t)\) denotes the displacement of the plate from the reference state in the \(z\)-direction at time \(t > 0\) and position \((x, y)\); the lengths of the edges of the rectangular plate are assumed to be equal to \(a\) and \(b\), respec-
tively; $D_x$, $D_y$, and $D_{xy}$ are orthotropic rigidities of the plate; $E_x$ and $E_y$ are the Young’s moduli along the x and y directions, respectively; $G$ is the shear modulus; $\nu_x$ and $\nu_y$ are the Poisson ratios corresponding to the x and y directions, respectively; $\rho_p$ is the mass density of the plate material; and $h$ is the plate thickness.

Considering only the transverse motion and neglecting the effect of rotary inertia, the kinetic and strain energies of the plate, $T_p$ and $U_p$, can be written as

$$T_p = \frac{\rho p h}{2} \int_{S_p} \left( \frac{\partial w}{\partial t} \right)^2 \, dx \, dy,$$

$$U_p = \frac{1}{2} \int_{S_p} \left\{ D_x \left( \frac{\partial^2 w}{\partial x^2} \right)^2 + D_y \left( \frac{\partial^2 w}{\partial y^2} \right)^2 + 2 D_{xy} \frac{\partial w}{\partial x} \frac{\partial w}{\partial y} + 4 D_{yy} \left( \frac{\partial^2 w}{\partial x \partial y} \right)^2 \right\} \, dx \, dy,$$

where $S_p$ is the surface area of the plate. The definition of the kinetic and strain energies of the plate is particularly important, as the Rayleigh–Ritz method is used in this research to find an approximate solution of the differential system (the method is based on the definition of an energy functional).

2.2. Inclusion of the actuators

The plate considered in this paper is assumed to be employed as an actively controlled acoustic barrier. For this purpose, inertial actuators are bonded to the surface of the plate. Their mass is often comparable to the mass of the plate and, therefore, they have a considerable impact on the dynamic response of the plate (both natural frequencies and mode shapes). Hence, in order to develop a model of the system that would remain valid after mounting the actuators, they must also be included in the mathematical modelling.

The actuators can be considered to be small in size compared to the dimensions of the plate, hence, their impact (loading of the plate) can be represented by additional concentrated masses. The influence of the strain caused by these elements bonded to the plate surface is neglected. Assuming also a perfect bonding and neglecting the stiffness of the actuators, the total energy introduced into the system by the actuators can be represented by the kinetic energy expressed as

$$T_a = \sum_{i=1}^{N_a} \frac{1}{2} \int_{S_a} m_{a_i} \left( \frac{\partial w}{\partial t} \right)^2 \, dx \, dy,$$

where $N_a$ is the number of actuators bonded to the surface of the plate; $m_{a_i}$ is the mass of the $i$th actuator; and $x_{a_i}$ and $y_{a_i}$ are the coordinates of the $i$th actuator, respectively. For the sake of brevity, mathematical modelling of actuators bonded to the plate is presented separately from the differential system of the vibrating plate, defining only the kinetic energy related to the actuators (as it is most important for the Rayleigh–Ritz method used to solve the resulting differential system).

2.3. The Rayleigh–Ritz method

The Rayleigh–Ritz method is used to calculate an approximate solution of the presented differential system, obtaining its natural frequencies and mode shapes. To utilize this method, the total energy of the system (derived in the previous part of this section) and carefully selected trial functions need to be defined. More detailed information regarding the Rayleigh–Ritz method itself is provided in [22].

For free vibration of the plate, the solution of $w$ can be expressed in the required form using a predetermined set of admissible trial functions

$$w(x, y, t) = \sum_{i=1}^{N} \phi_i(x, y) q_i(t) = \phi^T q,$$

where $q$ is a generalized plate displacement vector; $\phi$ is a vector, which represents a set of time-invariant trial functions $\phi_i(x, y)$—in this paper, characteristic orthogonal polynomials having the property of Timoshenko beam functions are used; and the superscript T denotes the transpose operator. All of the mentioned vectors are of dimension $(N \times 1)$, where $N$ is the number of employed trial functions. The procedure for forming orthogonal polynomial trial functions for rectangular plates is described in detail in [23].

2.4. Total energy definition

Utilizing Eq. (7), the total kinetic and potential energies, $T$ and $U$, defined by Eqs. (5) and (6), can also be written as functions of the generalized plate displacement vector $q$, mass matrix $M$ of dimensions $(N \times N)$ and stiffness matrix $K$ of dimensions $(N \times N)$ as [24]
\[ T = T_p + T_a = \frac{1}{2} q^T M q, \quad U = U_p = \frac{1}{2} q^T K q. \] (8)

The overall mass matrix \( M \) is calculated as the sum of matrices related to different energy components
\[ M = M_p + M_a, \] (9)
where \( M_p \) and \( M_a \) correspond to the kinetic energies of the plate and the actuators, respectively. The elements of the mass matrices introduced in Eq. (9) are defined as:
\[ M_{p,ij} = \rho_j h \int_x \phi_i \phi_j dx dy, \] (10a)
\[ M_{a,ij} = \sum_k \{m_{a,k} \phi_i \phi_j \} |x = x_{a,k}, y = y_{a,k}. \] (10b)

The elements \( K_{ij} \) of the stiffness matrix \( K \) can be derived as
\[ K_{ij} = \int_x \left\{ D_x \frac{\partial^2 \phi_i}{\partial x^2} \frac{\partial^2 \phi_j}{\partial x^2} + D_y \frac{\partial^2 \phi_i}{\partial y^2} \frac{\partial^2 \phi_j}{\partial y^2} + 2D_{xy} \frac{\partial^2 \phi_i}{\partial x \partial y} \frac{\partial^2 \phi_j}{\partial x \partial y} \right\} dx dy. \] (11)

2.5. Equation of the vibrating structure and a harmonic solution

Using the Lagrange equation of the second kind, the equation of a vibrating structure can be obtained using the stiffness and mass matrices defined above as
\[ Mq + Kq = Q. \] (12)
where \( Q \) is the vector of generalized forces of dimensions \((N \times 1)\). In this paper, inertial actuators are considered and for the purpose of their positioning, their action can be simplified and taken into account as a force acting at a point. Therefore, the control vector \( u \) of dimensions \((N_a \times 1)\) can be defined as
\[ u = [f_1, f_2, \ldots, f_{N_a}]^T, \] (13)
where \( f_i \) is a force generated by the \( i \)th actuator. Then, the vector of generalized forces can be expressed as
\[ Q = \left[ \begin{array}{c} \phi|x = x_{a,1}, \phi|x = x_{a,2}, \ldots, \phi|x = x_{a,N_a} u \end{array} \right] \] (14)

The harmonic solution of Eq. (12) gives the eigenvector matrix \( \Phi \) of dimensions \((N \times N)\) and \( N \) eigenfrequencies \( \omega_i \). Replacing \( q \) by \( \Phi \vec{v} \) and multiplying Eq. (12) on the left by \( \Phi^T \) gives
\[ \Phi^T M \Phi \vec{v} + \Phi^T K \Phi \vec{v} = \Phi^T Q. \] (15)

where \( \vec{v} \) denotes a modal displacement vector of dimensions \((N \times 1)\):
\[ \vec{v} = [v_1, v_2, \ldots, v_N]^T. \] (16)

Taking advantage of the orthonormality of the eigenvectors in the matrix \( \Phi \), the modal mass matrix becomes a unit matrix \( I \), of dimensions \((N \times N)\) and the corresponding modal stiffness matrix becomes a diagonal matrix \( \Omega \) of \( N \) eigenvalues \( \omega_i^2 \) [25], which gives
\[ \Phi^T M \Phi = I_N, \] (17a)
\[ \Phi^T K \Phi = \Omega = \left[ \text{diag}(\omega_1^2, \omega_2^2, \ldots, \omega_N^2) \right]. \] (17b)

Then, by substituting Eq. (17) into Eq. (15), gives
\[ \vec{v} + \Omega \vec{v} = \Phi^T Q. \] (18)

To provide a better representation of the behaviour of a real system, this is extended to
\[ \vec{v} + \Xi \vec{v} + \Omega \vec{v} = \Phi^T Q. \] (19)
where \( \Xi \vec{v} \) is a term introduced to include the damping in the system, and \( \Xi \) is a diagonal matrix of dimensions \((N \times N)\) defined as:
\[ \Xi = \left[ \text{diag}(2\xi_{d,1}\omega_1, 2\xi_{d,2}\omega_2, \ldots, 2\xi_{d,N}\omega_N) \right]. \] (20)
In Eq. (20), the damping ratios, $0 < \xi_d < 1$, are calculated using the thermoelastic damping model for elastic plates described in detail in [26]. The damping mechanism could also be included at the beginning of the modelling in the form of complex bending rigidities. However, this would substantially complicate the derivation. Introducing it instead at this point preserves the brevity of the derivation and leads to an equivalent solution. This approach was also used, e.g., in [13].

2.6. State space model

To allow standard control analysis tools to be utilised, Eq. (19) can be written in the usual state-space form

$$\dot{x} = Ax + Bu$$

with the state vector $x$ of dimensions $(2N \times 1)$ given by

$$x = [v_1, \omega_1 v_1, \dot{v}_2, \omega_2 v_2, \ldots, \dot{v}_{3N}, \omega_N v_N]^T.$$  

The state matrix $A = [\text{diag}(A_1, A_2, \ldots, A_N)]$ with dimensions $(2N \times 2N)$, is defined by

$$A_i = \begin{bmatrix} -2\xi_d \omega_i & -\omega_i \\ \omega_i & 0 \end{bmatrix}, \quad i = 1, 2, \ldots, N.$$  

The matrix $B$, of dimensions $(2N \times N_a)$, can be expressed as

$$B = [\text{diag}(b_1, b_2, \ldots, b_{3N})]\Phi^T \begin{bmatrix} \phi|x = x_{a,1} \phi|x = x_{a,2} \cdots \phi|x = x_{a,N_a} \\ y = y_{a,1} y = y_{a,2} \cdots y = y_{a,N_a} \end{bmatrix},$$

where $b_i = [1 \ 0]^T$.

2.7. Acoustic radiation

The aim of this derivation is to determine an estimate of the radiated acoustic power corresponding to the $i$th vibration mode of the considered plate. To describe the acoustic radiation of the plate, it has been assumed that it is placed in an infinite rigid baffle (cf. Fig. 1). Adopting an appropriate Green’s function that has been derived in [27,28], the modal sound pressure amplitude $p_i(x,y,z)$ can be calculated as

$$p_i(x,y,z) = \frac{k_e a b}{4\pi c} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \exp [i(\xi x + \eta y + \gamma z)] M_i(\xi, \eta) \frac{d\xi}{\gamma} d\eta,$$

for
The modal acoustic power $P_i$ can be expressed as

$$P_i = \int_{S_e} |p_i(x,y,z)|^2 \, dS_e.$$  

In theoretical analysis the enclosing surface $S_e$ is often defined as a hemisphere of a sufficient radius. However, to allow for the experimental verification of the model, the surface $S_e$ will be adopted as a limited plane parallel to the plate and at a distance greater than zero. This may affect to some extent the overall estimate of the modal acoustic power, but the alteration is negligible from the point of view of actuator positioning (the absolute values of distance greater than zero. This may affect to some extent the overall estimate of the modal acoustic power, but the alteration is negligible from the point of view of actuator positioning (the absolute values of distance greater than zero). Hence, the values which correspond to the weakly radiating modes to be distinguished from stronger ones).

### 2.8. Controllability of the system

Taking advantage of the fact that the model is expressed in the state-space form, classical methods can be used to describe the controllability of the system [29,30]. The energy-based approach has been employed, and the obtained results are later used in the optimization process for active control purposes.

The control energy required to reach the desired state $x_i$, at time $t = t_1$, assuming the optimal solution, can be expressed as

$$E_c = \int_0^{t_1} \mathbf{u}^T(t) \Phi(t) \mathbf{u}(t) \, dt = (e^{A_{t_1}}x_0 - x_{t_1})^T \Phi^{-1}(t_1)(e^{A_{t_1}}x_0 - x_{t_1}),$$  

where $\Phi(t)$ is the controllability Gramian matrix of dimensions $(2N \times 2N)$. To minimize the required control energy with respect to the locations of the actuators, a measure of the Gramian matrix should be maximized. It has been shown in the literature that instead of using $\Phi(t_1)$, a steady state controllability Gramian matrix $\Phi_e$ can be used for stable systems when time tends to infinity [31]. This controllability Gramian matrix can be calculated by solving the Lyapunov equation, which gives

$$A \Phi_e + \Phi_e A^T + B B^T = 0.$$  

The controllability Gramian matrix is convenient to use, because if the $(2i)$th value on the diagonal of the matrix, $\lambda_{c,i}$, which corresponds to the $i$th eigenmode, is small, the eigenmode is difficult to control (it can be regulated only if a large control energy is available). Hence, the values $\lambda_{c,i}$ should be maximized in order to improve the susceptibility of the system to control inputs. The values $\lambda_{c,i}$ depend on the actuators arrangement. Such information can be an important criterion in the optimization of the actuator placement. Formally, controllability is a dichotomous property, but “controllable” does not indicate the level of control effort that is needed to reach the final state.

The state-space system employed in the controllability evaluation includes only vibration phenomena (the acoustic radiation weighting is not incorporated). This is because expressing the acoustic radiation as a separate term (cf. Section 4.2) provides more flexibility in the cost function design step.

### 2.9. Summary

The derived model of the vibroacoustic system, although based on components available in the literature, combines both vibration and acoustic radiation phenomena in a concise and coherent form, and also takes into account the loading of the plate due to the inertial actuators used for active control. The employment of the Rayleigh–Ritz method facilitates a numerical solution of the system to be found, while the state-space representation enables analysis of the controllability of the system. All of these components provide a complete and reliable model of thin plates used as active noise barriers, which can be applied to optimize the actuator arrangement and is one of the main contributions of this paper.
3. Experimental verification of the model

In this Section, results from an experimental verification of the developed model are presented. For this purpose, an unloaded aluminium plate was used. The plate was attached to a rigid cubic frame. The remaining walls, which were constructed from sound-absorbing materials, were also attached to the frame to form a closed box. The aluminium plate was acoustically excited by a loudspeaker placed inside the box. The loudspeaker was driven by white noise bandlimited up to 1 kHz. Photographs of the laboratory setup are presented in Fig. 2.

The dimensions of the plate area that was free to vibrate (i.e. the area inside the square clamping frame) were equal to 0.420 m × 0.420 m. The plate can be described by the following parameters, which are defined in the model developed in Section 2:

\[ \begin{align*}
    a & = 0.420 \text{ m}, \\
    b & = 0.420 \text{ m}, \\
    h & = 0.001 \text{ m}, \\
    E_x & = 70 \text{ GPa}, \\
    \rho_p & = 2770 \text{ kg/m}^3, \\
    \nu_x & = 0.3, \\
    E_y & = 77 \text{ GPa}, \\
    G & = 26.9 \text{ GPa}.
\end{align*} \]

Firstly, the accuracy of the model of the plate vibration was evaluated. The response of the plate was measured using a laser vibrometer (Polytec PDV-100). The vibrometer was mounted on an automatic positioning system developed by the authors (cf. Fig. 2). Vibration measurements were taken over a uniform grid of 22 × 22 points (the interval between points was equal to 0.02 m).

Subsequently, the model of the acoustic radiation from the plate was also examined. For this purpose, the carriage with the laser vibrometer was complemented with an array of six measurement microphones (Beyerdynamic MM1). The microphones were arranged to measure the sound pressure just above the laser beam at distances from the plate of between 0.1 m and 0.6 m in steps of 0.1 m; this gives a total of 3120 measurement positions.

3.1. Verification of the modelling of plate vibrations

A comparison between the results from the experimental measurements and those calculated using the model is presented in Fig. 3. From these results it can be seen that the consistency between the results is very good, both in terms of the natural frequencies and mode shapes. Some of the modes could not be distinguished with the laboratory setup, which is due to two facts. Firstly, not all of the modes were equally excited with the loudspeaker, which means that weakly excited modes could not be distinguished well enough from the background noise. Secondly, if two modes have similar natural frequencies but significantly different magnitudes, then the more strongly excited mode would dominate and the weakly excited could not be observed. However, it is clear from the presented results that the majority of the first 12 vibrational modes of the plate were captured and are very consistent with the model.

Fig. 2. Photographs of the laboratory setup with an unloaded plate attached to a rigid cubic frame and excited with a loudspeaker placed inside the frame. An automatic positioning system for the laser vibrometer is also shown.
It is worthwhile to provide an additional comment regarding the experimentally measured fifth operational vibration shape. This shape is in fact a superposition of the theoretically calculated modes 5 and 6. This phenomena is visualized in the Fig. 4, which shows the theoretically calculated mode shapes for modes 5 and 6 and their combination, along with the experimentally identified 5th operational vibration shape. This behaviour is typical for square plates, because pairs of natural frequencies are often very close to each other. If the magnitudes of such modes are similar, the coupled shapes can be observed, as in the case of the experimental setup considered here.

It is also noteworthy that the experimentally measured natural frequencies of related pairs of modes, e.g. mode 2 and 3, are not equal (for a square isotropic plate, these frequencies should be equal). this leads to the conclusion that even a mate-

\[ \text{Simulation} \quad \text{Experiment} \]

\( \begin{align*}
\text{Mode 1, 45.0 Hz} & \quad \text{Mode 1, 54.0 Hz} \\
\text{Mode 2, 90.5 Hz} & \quad \text{Mode 2, 88.0 Hz} \\
\text{Mode 3, 93.0 Hz} & \quad \text{Mode 3, 95.5 Hz} \\
\text{Mode 4, 135.4 Hz} & \quad \text{Mode 4, 134.0 Hz} \\
\text{Mode 5, 163.9 Hz} & \quad \text{Mode 5, 166.0 Hz} \\
\text{Mode 6, 170.3 Hz} & \quad \text{Not distinguished} \\
\text{Mode 7, 206.7 Hz} & \quad \text{Mode 7, 207.5 Hz} \\
\text{Mode 8, 210.6 Hz} & \quad \text{Mode 8, 207.5 Hz} \\
\text{Mode 9, 261.1 Hz} & \quad \text{Mode 9, 259.0 Hz} \\
\text{Mode 10, 272.1 Hz} & \quad \text{Not distinguished} \\
\text{Mode 11, 279.4 Hz} & \quad \text{Mode 11, 278.0 Hz} \\
\text{Mode 12, 302.8 Hz} & \quad \text{Not distinguished}
\end{align*} \]
rial that could be expected to be isotropic (a common flat aluminium plate), may in fact be orthotropic to some extent, which may be due to the manufacturing process of the metal sheets. This is important because it justifies the selection of the orthotropic form of the developed model, which could otherwise be replaced by the simpler isotropic model.

3.2. Verification of the modelling of the acoustic radiation from the plate

A comparison between the experimental acoustic measurements and the theoretical calculations is presented in Fig. 5. The visualized modal sound pressure distributions reflect the sound pressure measured and simulated for the first 12 natural frequencies over a measurement grid. The presented grid was 1.00 m wide and 0.64 m high, and located at a distance of 0.1 m from the plate surface, with an interval between the measurement points of 0.04 m; this gives $26 \times 17$ measurement points, which is a total of 442 points. Assuming that the origin of the coordinate system was placed at the lower left corner of the plate, then the covered area can be defined by the following coordinates:

$$x = [-0.29, 0.71], \quad y = [-0.08, 0.56], \quad z = 0.1.$$

From Fig. 5 it can be seen that there is a high level of consistency between the experimental and theoretical results. As discussed in the previous section in relation to the measurements of the vibration response, if a particular mode was sufficiently excited to be accurately measured, then the measurements are consistent with the model. Once again, the fifth mode is worth an additional comment—the observed shape, as in case of the vibration response, is a result of the superposition of the theoretically calculated modes 5 and 6. This phenomena is visualized in the Fig. 6.

For the acoustic measurements, it is also worthwhile to compare the mean mode magnitudes obtained by experimental measurements with the theoretical expectations. This comparison is presented in Fig. 7. Based on the measured vibration magnitudes, the mean sound pressure magnitudes due to the individual modes obtained through the experimental measurements and the model simulations can be compared. The mean values were obtained by averaging over the described measurement grid at a distance of 0.1 m from the plate surface. It follows from the analysis of Fig. 7 that the accuracy of the theoretical prediction is very high, especially taking into account the fact that the utilized acoustic laboratory is not an anechoic chamber and the room acoustics interfere with the measurement to some extent. Hence, the conclusion can be drawn that the model has been successfully verified and that it can be used for the optimization process presented in the following Section.

4. Optimization process

In this section an optimization process is presented that aims to find the optimal placement of a number of actuators mounted to a vibrating plate for the purpose of active control. The objective of the control system is to reduce the noise radiated from the acoustic enclosure via the Active Structural Acoustic Control (ASAC) approach [32,33]. In order to reach this goal, the control system should be able to control the vibration modes of the plate in the frequency range of interest. The ability to control the ith mode can be described by an element on the diagonal of the controllability Gramian matrix, $\mathcal{K}_c$, as derived in Section 2.8. However, some of the vibrational modes are more important as they more strongly transmit or radiate noise when excited; while other modes behave in the exactly opposite manner and can be neglected, since they vibrate without strongly contributing to the radiated acoustic field. In order to reflect this behaviour, the modal acoustic power corresponding to ith vibration mode of the plate, $P_i$, can be used (cf. Section 2.8). Taking this into account, an optimization problem defined by an appropriate cost function will be presented, which will enable an optimal solution to be found for the arrangement of the given actuators.

4.1. Optimization problem

The optimization variables defined for the considered problem are the coordinates of a predefined number of actuators, $N_a$. A flat rectangular plate is considered, hence two coordinates per ith actuator, $x_a$ and $y_a$, are sufficient to unambiguously describe its location. Hence, the optimization algorithm is required to find a solution in an $2N_a$–dimensional space.
Due to physical dimensions of the actuators, certain constraints have to be defined in order to maintain the practicability of the solution. Namely, margins from the plate edges and between the actuators should be maintained, with the assumption that the actuators can be attached only from one side of the plate. Inertial actuators are considered in this paper, which are most commonly manufactured with a round footprint, although the method could be extended to more complex geometries as required. The first resulting constraint ensures that the actuators are placed within the boundaries of the plate; the dimensions of the considered rectangular plate are $a \times b$, hence, the coordinates of $i$th actuator $x_{a,i} \in (\frac{1}{2}d_{a,i}, a - \frac{1}{2}d_{a,i})$ and $y_{a,i} \in (\frac{1}{2}d_{a,i}, b - \frac{1}{2}d_{a,i})$, where $d_{a,i}$ is the diameter of the $i$th actuator. The second constraint ensures that actuators do not over-

Fig. 5. A comparison of initial 12 modal sound pressure distributions in the near field of the rigid casing wall, calculated with the mathematical model and experimentally measured—1 mm thick aluminium unloaded plate. Size of the measurement grid is in [m], and the z-axis depicts normalized amplitude.
For $i \neq j$, the $i$th and $j$th actuators should not be closer than a distance of $\frac{1}{2} d_{a_i} + \frac{1}{2} d_{a_j}$, which is represented by the following constraint: $(x_{a_i} - x_{a_j})^2 + (y_{a_i} - y_{a_j})^2 \geq \left(\frac{1}{2} d_{a_i} + \frac{1}{2} d_{a_j}\right)^2$.

### 4.2. Cost functions

The cost functions for the described problem can be formulated in a number of ways. In this research, six cost functions will be evaluated and analysed. Firstly, three cost functions that do not take into account the acoustic radiation, $J_1$-$J_3$, are formulated as follows,

$$J_1 = \min_i \lambda_{c,i} \quad (31a)$$

$$J_2 = N_j^{-1} \left( \sum_i \lambda_{c,i} \right) \quad (31b)$$

$$J_3 = \left( \prod_i \lambda_{c,i} \right)^{N_j^{-1}} \quad (31c)$$

for $i \in \{1, 2, \ldots, N_j\}$, where $N_j$ is the number of modes considered in the cost function. The same range of $i$ is also considered for the other cost functions. All three cost functions $J_1$-$J_3$ focus on maximizing the controllability of the system, however, they result in a different balance between the $N_j$ controllability measures, $\lambda_{c,i}$, corresponding to the $N_j$ considered modes.

Cost function $J_1$ represents only the least controllable mode, and thus ensures that there are no uncontrollable resonances within the frequency range of interest. Cost function $J_2$, which represents the mean controllability of the modes within the...
frequency range of interest, may increase the controllability of certain modes, even if this happens at the expense of reducing the controllability of other modes. Finally, cost function $J_3$ should lead to solutions that provide a trade-off between $J_1$ and $J_2$, making sure that the smallest of the factors is maximized, whilst also benefiting to some extent an increase in the controllability of the other modes in the frequency range of interest.

Subsequently, three additional cost functions are defined, $J_4$-$J_6$, which are analogous to the initial three cost functions, but take into account the acoustic radiation. These cost functions are defined as

$$J_4 = \min_i \left( \frac{\lambda_{c,i}}{P_i} \right), \quad J_5 = N_i^{-1} \left( \sum_i \frac{\lambda_{c,i}}{P_i} \right), \quad J_6 = \left( \prod_i \frac{\lambda_{c,i}}{P_i} \right)^{N_i^{-1}}.$$  

In each case, the division of $\lambda_{c,i}$ by $P_i$ forces the optimization algorithm to seek solutions with better controllability (more energy efficient) for the $i$th mode, if the $i$th mode acoustic radiation measure $P_i$ is higher. That is, the cost functions are weighted to focus the effort into the controllability of the strongly radiating structural modes.

| Table 1 | Results of the optimization for cost functions $J_1$-$J_6$ with $N_a = 3$ and $N_I = 12$. The natural frequencies $\omega_i$ are given in [Hz], while values of the cost functions $J_1$-$J_6$, $\lambda_{c,i}$, $P_i$ and $\lambda_{c,i}/P_i$ are given in [dB]. Resulting values of the cost functions used as the optimization index are marked with bold font. Individual modes of high acoustic radiation ($P_i > 30$ dB) are highlighted with a grey background. The actuators placement is also given.

|                | Cost functions used in the optimization | Taking into account the acoustic radiation |
|----------------|-----------------------------------------|------------------------------------------|
| Neglecting the acoustic radiation | $J_1$ | $J_2$ | $J_3$ | $J_4$ | $J_5$ | $J_6$ |
| Obtained values | $\omega_i$ | $\lambda_{c,i}$ | $P_i$ | $\frac{\lambda_{c,i}}{P_i}$ | $\omega_i$ | $\lambda_{c,i}$ | $P_i$ | $\frac{\lambda_{c,i}}{P_i}$ | $\omega_i$ | $\lambda_{c,i}$ | $P_i$ | $\frac{\lambda_{c,i}}{P_i}$ | $\omega_i$ | $\lambda_{c,i}$ | $P_i$ | $\frac{\lambda_{c,i}}{P_i}$ |
| 1              | 43 55 28 27 | 20 73 21 52 | 41 60 28 32 | 29 68 24 44 | 26 69 23 46 | 44 54 28 26 |
| 2              | 75 58 22 36 | 69 41 22 19 | 65 60 26 34 | 82 47 23 24 | 45 65 18 47 | 72 61 21 39 |
| 3              | 77 59 21 38 | 72 36 20 16 | 81 60 21 39 | 85 43 21 22 | 67 63 11 52 | 76 59 21 38 |
| 4              | 102 61 23 31 | 109 49 37 13 | 100 54 22 32 | 113 56 35 21 | 83 36 20 16 | 103 51 21 30 |
| 5              | 119 54 22 33 | 133 44 24 11 | 113 54 20 33 | 116 55 34 21 | 117 53 34 19 | 110 56 22 33 |
| 6              | 136 56 19 37 | 142 23 25 26 | 129 56 23 33 | 132 56 35 21 | 129 29 22 8 | 130 58 20 37 |
| 7              | 146 50 22 28 | 155 37 28 9 | 151 51 29 22 | 149 52 17 35 | 137 46 34 12 | 155 51 16 35 |
| 8              | 178 50 32 19 | 175 22 21 1 | 176 49 28 21 | 166 40 19 21 | 160 33 23 10 | 168 48 19 29 |
| 9              | 184 50 33 17 | 220 13 24 12 | 183 51 22 29 | 186 48 27 21 | 167 43 31 12 | 179 48 34 13 |
| 10             | 194 50 19 31 | 225 11 15 -5 | 184 51 33 17 | 189 47 25 22 | 194 33 26 8 | 197 47 14 33 |
| 11             | 203 50 30 20 | 247 38 30 8 | 201 50 27 23 | 217 51 23 27 | 212 42 27 15 | 199 53 14 38 |
| 12             | 212 52 22 30 | 301 14 22 -8 | 212 50 27 22 | 223 48 22 26 | 236 42 23 20 | 203 50 17 34 |

| Modes | $x_{a,i}$ (m) | $y_{a,i}$ (m) | $x_{a,j}$ (m) | $y_{a,j}$ (m) | $x_{a,i}$ (m) | $y_{a,i}$ (m) | $x_{a,j}$ (m) | $y_{a,j}$ (m) | $x_{a,i}$ (m) | $y_{a,i}$ (m) | $x_{a,j}$ (m) | $y_{a,j}$ (m) |
|-------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|
| 1     | 0.379 0.317    | 0.194 0.254    | 0.380 0.384    | 0.377 0.385    | 0.187 0.239    | 0.097 0.373    | 0.382 0.372    |
| 2     | 0.382 0.383    | 0.255 0.223    | 0.358 0.311    | 0.044 0.390    | 0.120 0.309    | 0.308 0.376    |
| 3     | 0.174 0.381    | 0.194 0.194    | 0.244 0.376    | 0.202 0.200    | 0.293 0.134    | 0.382 0.372    |

| Actuators placement | An overview: | An overview: | An overview: | An overview: | An overview: | An overview: |
|---------------------|--------------|--------------|--------------|--------------|--------------|--------------|
4.3. Optimization algorithm

The search space that follows from the optimization problem described in the previous subsections is very complicated and contains numerous local maxima. Therefore, an efficient algorithm must be employed in order to find a solution that satisfies the defined requirements. A Memetic Algorithm (MA) can be utilised for such a task, which is a hybrid form of a population-based approach coupled with separate individual learning [34]. The MA combines advantages of a global search, as offered by evolutionary algorithms, and local refinement procedures, which enhance convergence to the local maxima [34,35]. Due to these complementary properties, MA are particularly suitable for solving complex multi-parameter optimization problems, such as the placement of sensors and actuators [36,37].

### Table 2

Results of the optimization for the cost functions $J_1$ and $J_4$ with $N_a = 12$ and $N_a$ equal to 3, 6 or 9. The natural frequencies $\omega_i$ are given in [Hz], while values of the cost functions $J_1$ and $J_4$, $P_i$, and $\lambda_i$, are given in [dB]. Resulting values of the cost functions used as the optimization index are marked with bold font. Individual modes of high acoustic radiation ($P_i > 30$ dB) are highlighted with a grey background. The placement of the actuators is also given.

| Cost functions used in the optimization | J1 | J4 |
|----------------------------------------|----|----|
| $N_a = 3$ | $N_a = 6$ | $N_a = 9$ | $N_a = 3$ |
| **J1** | | | |
| 50 | 53 | 56 | 40 |
| 55 | 59 | 63 | 58 |
| 53 | 56 | 59 | 51 |
| 17 | 19 | 21 | 21 |
| 33 | 38 | 44 | 34 |
| 29 | 31 | 37 | 25 |
| **J6** | | | |
| **Obtained values** | | | |
| \(\omega_i, \lambda_i, P_i, \frac{\lambda_i}{P_i}\) | | | |
| **Modes** | | | |
| 1 | 43 55 28 27 | 32 66 25 41 | 22 71 22 48 | 29 68 24 44 |
| 2 | 75 58 22 36 | 46 65 19 47 | 40 66 14 52 | 82 47 23 24 |
| 3 | 77 59 21 38 | 79 58 29 29 | 61 60 26 34 | 85 43 21 22 |
| 4 | 102 52 21 31 | 86 53 23 30 | 71 60 21 39 | 113 56 35 21 |
| 5 | 119 54 22 33 | 104 57 24 33 | 82 60 14 47 | 116 55 34 21 |
| 6 | 136 56 19 37 | 114 56 28 28 | 90 57 18 38 | 132 56 35 21 |
| 7 | 146 50 22 28 | 124 55 31 25 | 104 58 17 41 | 149 52 17 35 |
| 8 | 178 50 32 19 | 132 55 24 31 | 112 56 31 25 | 166 40 19 21 |
| 9 | 184 50 33 17 | 144 53 15 38 | 121 56 27 29 | 186 48 27 21 |
| 10 | 194 50 19 31 | 152 53 34 19 | 127 56 29 27 | 189 47 25 22 |
| 11 | 203 50 30 20 | 174 53 29 24 | 141 56 20 37 | 217 51 23 27 |
| 12 | 212 52 22 30 | 186 53 25 28 | 147 56 36 21 | 223 48 22 26 |
| **Actuators placement** | | | |
| An overview: | An overview: | An overview: | An overview: |
5. Analysis of optimization results

In this Section, an analysis of the optimization results obtained for the arrangement of real actuators is presented. Dayton Audio DAEX32EP-4 are considered as actuators in this paper. They have a circular form factor, with a mass \( m_{a,i} = 0.115 \) kg and a diameter \( d_{a,i} = 0.060 \) m. The dimensions of the considered plate are \( a = 0.420 \) m and \( b = 0.420 \) m, hence, based on the constraints defined in SubSection 4.1, the coordinates of the \( i \)th actuator are given as \( x_{a,i} \in (0.030, 0.390), y_{a,i} \in (0.030, 0.390) \) and \((x_{a,i} - x_{a,j})^2 + (y_{a,i} - y_{a,j})^2 \geq (0.060)^2\) for \( i \neq j \).

The configurations for three, six and nine actuators have been optimized using the six cost functions, \( J_1, J_2, \ldots, J_6 \), defined in the previous Section. The objective was to maximize the controllability of the plate used as an active acoustic barrier. The low frequency range was considered, hence the first \( N_f = 12 \) vibration modes of the plate were considered in the optimization process. The obtained results are summarized in Table 1.

It follows from an analysis of the results presented in Table 1 that the introduction of an acoustic radiation estimate into the cost function \( J_1 \), obtaining \( J_4 \), enables an increase in the controllability measure \( \lambda_{c,i} \) of more than 5 dB for acoustically-relevant modes (where \( P_i \geq 30 \) dB; in Table 1 they are highlighted with a grey background). An increase in the controllability measure \( \lambda_{c,i} \) means that the \( i \)th mode is more excited with the same control effort (e.g. an increase of \( \lambda_{c,i} \) by 5 dB means that the modal velocity of the \( i \)th mode is by 5 dB greater with the same control effort). The price that is paid for this increase is a smaller controllability for modes that are less responsible for acoustic radiation or transmission. The minimal controllability measure \( \lambda_{c,i} \) for \( J_1 \) is 50 dB, while for \( J_4 \) it is 40 dB. However, the least controllable mode for the solution obtained with \( J_4 \) has \( P_i = 19 \) dB, which means that its role in acoustic radiation or transmission will be minor compared to the other modes that have a \( P_i \) that is more than 15 dB higher.

It is also interesting to highlight that modifying cost function \( J_1 \) to give \( J_4 \) provides a similar increase in the controllability of the acoustically-relevant modes to that achieved by employing additional actuators, as shown by the results presented in Table 2. By optimizing the actuator locations using \( J_4 \), a similar controllability can be reached as achieved when the number of actuators are doubled and optimized using \( J_1 \). In other words, by using \( J_4 \) the number of actuators, \( N_a \), could be reduced, e.g. from 6 to 3 per plate, whilst maintaining a similar level of controllability in terms of the acoustically-relevant modes. In practical noise control applications, this reduction in the required number of actuators offers a significant reduction in the cost and control system complexity, which is a considerable advantage.

Referring again to the results presented in Table 1, it can be seen that the results obtained for both \( J_2 \) and \( J_3 \) are in general inferior to the results obtained with \( J_1 \) and \( J_4 \). The reason for this is that if a sum is employed in the cost function \( J_2 \) and \( J_3 \), it can be beneficial to maximize only one component of the sum and neglect the others. In the considered optimization problem, the controllability of the first mode was maximized, but the remaining modes were neglected and, as a result, these cost functions do not meet the objective.

It is interesting to note from the results presented in Table 1 that both \( J_1 \) and \( J_4 \) result in similar cost function values. The controllability of all considered modes has been maximized using these cost functions. However, introduction of acoustic radiation measure into cost function \( J_3 \), which gives \( J_6 \), provides unsatisfactory results. It turns out that it is beneficial for \( J_6 \) to have a single mode of high acoustic radiation and lower controllability, while maximizing controllability of the other less acoustically-relevant modes. This is, therefore, an unacceptable solution for the considered application.

Acoustic and structural vibration responses of the plate, obtained for the different optimization indices \( J_1, J_6 \), are presented in Fig. 8. These responses are calculated for the solutions summarized in Table 1, hence in all cases the number of actuators \( N_a = 3 \). Both responses, driven with the primary uniform excitation and by the optimally arranged actuators, are presented. The responses of the plate due to primary uniform excitation are obtained by applying an equal excitation to all structural modes, instead of simulating an external acoustic excitation. These responses correspond to the result of a uniform wide-band external excitation that can be produced by many types of common noise sources. The responses due to excitation by the optimally arranged actuators are obtained by simulating actuator action as forces acting at the optimized actuator locations \( (x_{a,i}, y_{a,i}) \). A wideband signal again was used as the input to the actuators. The magnitude of the input signals to the actuators was arbitrarily chosen and was the same in all evaluated cases and for all actuators. The larger the response due to the actuators (shown by the red line) compared to the response when driven by the primary uniform excitation (shown by the black line), the easier it will be for the control system to reduce the noise transmission or radiation in the considered frequency range.

It follows from analysis of Fig. 8 that in the case of the structural responses obtained for \( J_1 \) and \( J_4 \), the responses due to optimally arranged actuators nearly match for all considered peaks in the responses due to primary uniform excitation. However, in the case of the acoustic responses obtained for \( J_1 \) and \( J_6 \), the highest peaks in the responses due to primary uniform excitation are higher than the corresponding responses due to the actuators, while the response due to the optimally arranged actuators obtained for \( J_6 \) dominates the highest peaks in the acoustic response of the plate due to primary uniform excitation. Still, an enhanced performance in acoustic response obtained for \( J_6 \) is traded for a weaker structural response for the modes that are less responsible for acoustic radiation or transmission. These remarks are consistent with the previous conclusions drawn from the analysis of Table 1.
6. Conclusions

Active noise control methods are gaining growing attention as a practical method for controlling noise in the increasingly noise-polluted world. This paper develops a new acoustic radiation-based method to optimize the arrangement of actuators for active noise barriers. A model of acoustic radiation was introduced into the optimization process and new cost functions were formulated in order to focus on modes that are truly relevant to the noise transmission and radiation. This optimization process constitutes the main novelty of the presented research. The study involved mathematical modelling, laboratory experiments and numerical simulations in order to evaluate the proposed optimization method.

The employed model was validated experimentally and employed in the optimization of the actuators for a real laboratory setup. It follows from an analysis that introduction of the acoustic radiation measure into the cost function in the form

![Fig. 8. Acoustic and structural vibration responses of the plate, obtained for different optimization indices $J_1$-$J_6$ as summarized in Table 1. Responses are shown for both the primary uniform excitation and when excited using the optimally arranged actuators.](image-url)
of $J_a$ offers best performance and enables an increase in the controllability measure of more than 5 dB for acoustically-relevant modes. The increase in controllability for these modes is comparable to that achieved by employing additional actuators (at least doubling the number of actuators for the considered system). From a different point of view, this method could also be used to rearrange the actuators in order to try to reduce their number, while maintaining the same level of controllability. Such reduction in practical noise control applications entails a significant reduction in the cost and the control system complexity.

These advantages are traded for a reduction in the controllability of the modes that are less responsible for acoustic radiation or transmission. However, such modes have been shown to have a modal acoustic power that is at least 15 dB lower than that due to the dominant modes, which means that their contribution to the noise transmission and radiation is negligible.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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