Avoiding Bias Due to Nonrandom Scheduling
When Modeling Trends in Home-Field Advantage

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Abstract

Existing approaches for estimating home-field advantage (HFA) include modeling the difference between home and away scores as a function of the difference between home and away team ratings that are treated either as fixed or random effects. We uncover an upward bias in the mixed model HFA estimates that is due to the non-random structure of the schedule – and thus the random effect design matrix – and explore why the fixed effects model is not subject to the same bias. Intraconference HFAs and standard errors are calculated for each of 3 college sports and 3 professional sports over 18 seasons and then fitted with conference-specific slopes and intercepts to measure the potential linear population trend in HFA.

Keywords: mixed model, mvglmRank
NOTE

The work in this paper on bias in the mixed model estimators has been superseded by “A Diagnostic for Bias in Linear Mixed Model Estimators Induced by Dependence Between the Random Effects and the Corresponding Model Matrix” https://arxiv.org/abs/2003.08087. In particular, the diagnostics in Equations (7) and (14) were formed with an ad hoc approach, whereas the newly referenced paper takes a thorough theoretical approach. This paper will remain on arXiv per site requirements.

1 Introduction

There has long been observed a “home-field advantage” across a variety of sports (Harville 1977, Harville & Smith 1994, Harville 2003, Pollard & Pollard 2005, Jones 2007, Wang et al. 2011, Goumas 2014). This advantage may be defined as “the net effect of several factors that generally benefit the home team and disadvantage the visiting team” (Harville & Smith 1994). The scope and influence of these factors is debated by sports psychologists. In some sports, such as baseball, there is variation in the structure of the playing field – such as fence distance and height – from stadium to stadium, creating the potential for a team to acclimate to its own field. By contrast, Moskowitz & Wertheim (2012) argue that referee bias due to social influence is a primary factor in home-field advantage.

Recently, sports journalists and researchers have taken interest in a perceived decline in the home-field advantage (HFA) in college football and basketball. Chatelain (2016) notes the decrease in college football HFA by looking at win proportions for the home team within conferences. Davis (2017) makes a similar note of declining proportion of wins for home teams in intraconference NCAA Division I men’s basketball games. However, these analyses assume an underlying binomial model for the number of games in a season won by home teams (Pollard & Pollard 2005) and that game outcomes are independent with an equal probability of the home team winning. Simply tracking the proportion of games won by the home team does not control for the variability in team abilities, nor consider the potential biasing impact of nonrandom scheduling on the estimate, nor provide accurate standard errors for the proportions.

HFA can be approached from two different perspectives:
1. **scoring HFA**: the difference in the expected scores of the home and away teams within a game, after the strengths of each team have been accounted for.

2. **win propensity HFA**: the increased win propensity of a team when playing an opponent at home, beyond the predicted propensity of that team defeating the same opponent at a neutral site.

This investigation studies whether or not there is a linear trend in the scoring HFA of 6 sports over the past 18 seasons: Men’s and Women’s NCAA Division I basketball, the NCAA Football Bowl Subdivision (FBS), the National Football League (NFL), the National Basketball Association (NBA), and the Women’s National Basketball Association (WNBA). The teams of each of these sports are divided into conferences, which are clusters of teams that play each other with higher probability than would be expected if the schedules were randomized across the sport. We are interested in knowing if trends in scoring HFA differ across the conferences.

In the first phase of this investigation, we present a model for estimating HFA (independently) within each conference and in each season. [Pollard & Pollard (2005)](#) acknowledge that schedules may be unbalanced, but do not explore the potential impact on HFA estimates. However, [Harville (1977)](#) Section 5 notes that the estimate for HFA resulting from the mean of the differences between home and away scores is “‘biased’ upwards because, for financial reasons, the better teams are more generally able to arrange schedules with more games at home than away.” However, we will see that a simplified version of the mixed model estimate for HFA proposed by [Harville (1977)](#) is itself subject to upward bias in this situation, by examining a fixed and a random effect model for scoring HFA.

The second phase of the analysis fits a potential linear trend in the yearly conference estimates of HFA that were produced in Phase I. In the college sports, which typically have at least 10 conferences, this phase implements a weighted random coefficient model using the Phase I intraconference HFA estimates as the response and the reciprocal of the squared standard errors from Phase I as weights. The model allows for correlated conference-specific random intercepts and slopes. Due to the presence of only two conferences in each professional sport, the conference-specific intercepts and slopes are instead treated as fixed effects for these sports.

Section 2 presents a fixed team effect model and a random team effect model for esti-
mating the HFA within years – or within conferences within years – (Phase I) and compares the assumptions made by each model regarding the structure of the stochastic schedules. In addition, models for fitting conference-specific linear trends in HFA over time (Phase II) are defined. Section 3 applies the Phase I and Phase II models to 18 seasons of 6 different sports. This section also refits the Phase I model to simulated data sets that are generated by resampling the residuals and team effects in order to explore bias in the HFA estimates. Section 5 closes the paper with a discussion of the results.

2 Methods

2.1 Phase I: Estimating HFA

This section presents both a fixed effects model and a mixed effects model for the home team margins of victory, \( d_i = y_{Hi} - y_{Ai} \), where \( y_{Hi} \) and \( y_{Ai} \) are the home and away team scores, respectively, for game \( i = 1, ..., n \). With \( N \) teams in a data set, the pattern of game opponents and locations (the schedule) is recorded in an \( n \times N \) matrix \( Z \) as follows: if team \( T_H \) hosted team \( T_A \) in game \( i \), then the \( i \)-th row of \( Z \) consists of all zeros except for a 1 in the column corresponding to \( T_H \) and a \( -1 \) in the column corresponding to \( T_A \).

2.1.1 Fixed Effects Model for Scoring Advantage

The first plausible model for scoring HFA fits a fixed effect \( \lambda \) for the HFA and vector of fixed team effects \( \beta = (\beta_1, \ldots, \beta_N) \) where the difference between team effects provides an estimate of the score difference in a game between the two teams on a neutral field. For each game \( i \), this model assumes

\[
d_i \sim N(\lambda + \beta_{Hi} - \beta_{Ai}, \sigma^2)
\]

where \( H_i \) and \( A_i \) are the indices for the home and away teams, respectively, in game \( i \). The residuals are assumed to be independent, leading to an overall model

\[
d = \lambda \cdot 1 + Z\beta + \epsilon
\]

where \( 1 \) is a vector of 1’s, \( \epsilon \sim N(0, \sigma^2 I) \) where \( I \) is the identity matrix, and \( d = (d_1, \ldots, d_n) \). This model appears previously as Model 2 of Harville & Smith (1994) and Model 1 of Harville (2003).
The fixed effect model matrix \( Z \) is not full rank and the individual team effects \( \beta_i \) are not estimable. However, \( \lambda \) is estimable, as are the pairwise differences between team effects, provided there is sufficient mixing of the teams (Stroup 2012, Section 5.2.1). For example, in a conference of three teams (A, B, C), \( \lambda \) will be estimable if any two teams have a home and home series, or if, for example, team B plays at team A, team A plays at team C, and team C plays at team B. Provided this is the case and letting \( M^+ \) represent the Moore-Penrose inverse of a matrix \( M \), solutions to model (2) are given by

\[
\begin{bmatrix}
\hat{\lambda} \\
\hat{\beta}
\end{bmatrix} = \begin{bmatrix}
1' \\
Z'
\end{bmatrix}^{-1} \begin{bmatrix}
1'Z \\
(Z'Z)
\end{bmatrix} \begin{bmatrix}
1' \\
Z'
\end{bmatrix} d
\]

An additional concern regarding \( Z \) is that is potentially a stochastic matrix that depends on the team effects, in the sense that better teams may choose to play weaker opponents or more games at home than away. However, since the elements of \( \beta \) are treated as fixed effects, the only variability assumed in their estimates is the variability associated with \( \epsilon \). Taking the conditional expectation of (3) given \([1|Z]\), it appears that two possible sufficient conditions for the model (2) to produce unbiased estimates for \( \lambda \) are either that the model matrix \([1|Z]\) is independent of \( \epsilon \) (which seems reasonable from a sports scheduling perspective) or if \( 1'Z = 0 \). In the later case, each team plays a balanced schedule with the same number of home and away games, and the estimated HFA is simply the mean observed difference, \( \hat{\lambda} = \bar{d} \).

### 2.1.2 Mixed Model for Scoring Advantage

Alternatively, the teams may be modeled with random effects \( \eta = (\eta_1, \ldots, \eta_N) \sim N(0, \sigma^2gI) \) that are assumed to be independent of the error terms. The remaining details are the same as model (2), with difference in home and away scores \( d_i \) being modeled conditional on the random team effects as

\[
d_i|\eta \sim N(\lambda + \eta_{Hi} - \eta_{Ai}, \sigma^2)
\]

producing an overall model

\[
d|\eta \sim N(\lambda * 1 + Z\eta, \sigma^2I)
\]

Harville (1977, Equation 2.1) considers a generalization of this model for ranking sports teams. An advantage of placing a distributional assumption on the team effects is that it provides a form of regularization for the model and avoids the estimability concern for
the team effects. However, the variance imparted to $\eta$ results in a new restriction on the dependence of $Z$ on $\eta$.

As with the fixed effect model (3), the mixed model estimate for $\lambda$ is also equal to $\bar{d}$ when $1'Z = 0$ (Henderson 1975, Equation 5), leading to identical estimates in the presence of a balanced schedule, even if the team assignments were made not-at-random. In the case of an unbalanced but randomly assigned schedule where $1'Z$ and $\eta$ are independent, the mixed model assumptions suggest that we should see similar home field estimates (on average) between the two models: the mixed model assumes $\eta \sim N(0, \sigma^2 g I)$ and thus its product with the off-diagonal quantity of interest from (3) has mean 0 and is distributed as

$$1'Z\eta \sim N(0, \sigma^2 g 1'ZZ'1)$$

(6)

However, if the construction or distribution of $Z$ depends on $\eta$, then the relationship in (6) no longer holds since the factorization $E[Z\eta] = ZE[\eta]$ requires independence. Under a null hypothesis that $\eta \sim N(0, \sigma^2 g I)$ and that $1'Z$ and $\eta$ are independent,

$$\frac{1}{\sigma^2 g} \eta'Z'1(1'ZZ'1)^{-1} 1'Z\eta \sim \chi^2_1$$

(7)

An abnormally large value could indicate dependence between $1'Z$ and $\eta$ that leads to a systematic difference between the estimates of $\lambda$ produced by models (2) and (5). We calculate this value by substituting $\sigma^2 g$ with its REML estimate and $\eta$ with its eBLUP. While these substitutions could alter the null distribution from the expected chi-square, simulations have shown it to be an accurate approximation to observed distributions of this value. NOTE: The previous paragraphs represent an initial ad hoc approach to this problem. Please see https://arxiv.org/abs/2003.08087 for a more developed analysis.

### 2.2 Phase II: Fitting a Trend in HFA

Once the estimated intraconference HFAs and associated standard errors have been obtained from either model (2) or model (5), we will check for the presence of a population-wide linear trend in HFA within each sport. In the application in Section 3, separate Phase I models are fit for the different conferences within sports in each year. There are many such conferences for the college sports, but only two for each of the professional sports.
2.2.1 College Sports

For the college sports, we fit a model with a linear trend in time and correlated random slopes and intercepts for conferences. Letting \( \lambda_{ij} \) represent the estimated HFA for conference \( j \) in year \( i \), we fit

\[
\lambda_{ij} = (\alpha_0 + b_{0j}) + (\alpha_1 + b_{1j}) t_i + \epsilon_{ij}
\]

where \( t_i \) is the year (indexed with 2017 as time 0), \( \alpha_0 \) and \( \alpha_1 \) are fixed effects for the population intercept and slope, respectively, and the errors are independent and distributed \( \epsilon_{ij} \sim N(0, \sigma^2_{\lambda}) \) where \( \sigma^2_{\lambda} \) is the standard error of the HFA for conference \( j \) in year \( i \) from Phase I: this weighting incorporates information about the accuracy of the point estimates from Phase I. The conference-level random intercepts and slopes are assumed to be independent across conferences, and independent of \( \epsilon \), distributed as \( (b_{0j}, b_{1j}) \sim N(0, G) \) where \( G \) is a covariance matrix with lower triangle \( \sigma^2_{11}, \sigma^2_{12}, \sigma^2_{22} \) .

Testing for the significance of the contributions of the conference-level random slopes and intercepts is not amenable to the standard likelihood ratio test since the null hypothesis of a zero variance component lies on the boundary of the parameter space (Scheipl et al. 2008): a simulation is used to assess the magnitude of the observed restricted loglikelihood statistic under the null hypothesis \( G = 0 \). However, random effects are retained in the model regardless of their perceived significance. This could lead to reduced power in testing the fixed effects, but guards against bias due to omitted effects since the plots of intraconference HFA in the application of Section 3 suggest heterogeneity in slopes. The resulting fixed effect estimate \( \hat{\alpha}_1 \) represents sport-wide linear trend in HFA, while \( \hat{\alpha}_0 \) estimates the population HFA in 2017.

2.2.2 Professional Sports

Each of the considered professional sports are split into two conferences, which will be denoted A and B. In the NBA and WNBA, these are the Western and Eastern conferences, respectively. In the NFL, they are the NFC and the AFC, respectively. With only two conferences, the HFA estimates from each conference are not able to support the estimation of \( G \) in (8). Instead, the conference intercepts and slopes are fit with fixed effects in model (9) where the errors are weighted and distributed as described in Section 2.2.1. The full model is compared via a likelihood ratio test (using maximum likelihood estimates) to model (10),
which assumes identical slopes and intercepts for both conferences, and to model (11), which drops the linear trend in time from the full model.

\[
\lambda_{ij} = \beta_0 A + \beta_1 A t_i + (\beta_0 B + \beta_1 B t_i) I(j = B) + \epsilon_{ij} \tag{9}
\]

\[
\lambda_{ij} = \beta_0 A + \beta_1 A t_i + \epsilon_{ij} \tag{10}
\]

\[
\lambda_{ij} = \beta_0 A + \beta_0 B I(j = B) + \epsilon_{ij} \tag{11}
\]

Unlike in the random coefficient model, the interpretation of \(\beta_0 A\) and \(\beta_1 A\) changes with the presence of \(\beta_0 B\) and \(\beta_1 B\).

### 2.3 Estimation

The Phase I restricted maximum likelihood (REML) estimates for the linear mixed model (5) are obtained with an EM algorithm (Laird & Ware 1982, Diffey et al. 2013) via the R package `mvglmmRank` (Karl & Broatch 2014), which also uses the resulting model matrices to construct the fixed effect estimates (3). The Phase II models of Sections 2.2.1 and 2.2.2 fit in SAS PROC MIXED. Source code for replicating the Phase I and Phase II analyses is included in the supplementary material (https://github.com/HFAbias18/supplement). The source code can easily be adapted to perform similar analyses on results from other sports using the database provided by Massey (2017).

### 3 Application to College and Professional Basketball and Football

#### 3.1 Data

Intraconference scores from the 2000-2017 seasons for each of the six sports of Men’s and Women’s NCAA Division I basketball, NCAA Football Bowl Subdivision (FBS), NFL, the WNBA, and the NBA are furnished by the website of Massey (2017), with some missing WNBA results obtained from WNBA.com. In all cases, neutral site games are excluded, and college results are limited to intradivision games between Division I teams for basketball and FBS teams for football. Our focus is to examine whether the current generation of athletes is playing the same game as the previous generation, and an 18 year window seems appropriate for this task. The choice of 2000 as a starting year is further explained in the appendix.
Table 1: Median p-values for the test in (7) across the 2000–2017 seasons. One test was performed for each full season of each sport, with results summarized in the first row. Likewise, one test was performed for each conference (including only intraconference games) in each year of each sport, with results summarized in the second row. CFB represents college football, while CBB-M and CBB-W represent men’s and women’s college basketball, respectively.

|                     | NFL | NBA | WNBA | CFB | CBB-M | CBB-W |
|---------------------|-----|-----|------|-----|-------|-------|
| **One Model for Each Season** |     |     |      |     |       |       |
| Expression (7): p-value | 0.40 | 0.25 | 0.52 | 7e-06 | 5e-33 | 2e-19 |
| **One Model for Each Conference in Each Season** |     |     |      |     |       |       |
| Expression (7): p-value | 0.63 | 0.47 | 0.80 | 0.87 | 0.84 | 0.35 |

### 3.2 Phase I: Estimating the HFA

#### 3.2.1 Application to Full Season Data

In an initial application to full seasons of NCAA men’s basketball results (ignoring conference divisions and thus using every game played during each season), Figure [1] shows that the mean $\overline{d}$ is uniformly larger than the estimated HFA from the mixed model (2), which in turn are uniformly larger than the estimates from the fixed effects model (5): this suggests that at least one of the models is producing biased estimates. However, HFA estimates from these three models are identical to each other for the same NBA seasons. The same pattern appears in the other college and professional sports, with three distinct estimates in each of the college sports and three nearly identical estimates in each season of each of the professional sports. These differences between the fixed and random effects estimates in the college sports coincide with unexpectedly large values of the test statistic (7), as shown in the first row of Table (1). A bootstrap simulation helps illustrate this relationship.

#### 3.2.2 Simulation for Full Seasons

To assess the behavior of the HFA estimates produced by the fixed and mixed effect models given the potentially nonrandom schedules, $Z$, we run two simulations in each full season of each sport (ignoring conferences) using the the fitted team effects $\hat{\eta}$ and error terms $\hat{e}$ from
Figure 1: Fixed effect model estimates (dotted) of the scoring HFA are plotted with corresponding mixed model (dashed) estimates and the mean of $d$ (solid) across seasons of NCAA Men’s Basketball (left) and the NBA (right). The lines in the NBA plot overlap.

The first simulation generates observations with a HFA of 3 points by resampling the residuals with replacement (bootstrapping) as defined in (12). In the second case, presented by (13), the team effects are also resampled (without replacement).

The first simulation represents a scenario in which the season is played repeatedly according to the same schedule, while the second represents a scenario in which the teams are shuffled before assigning them to the same schedule structure, $Z$. Letting $s_1(x)$ represent a function that samples the elements of $x$ with replacement (or without replacement for $s_0$),

$$y_{sim1} = 3 \cdot 1 + Z \hat{\eta} + s_1(\hat{e})$$

$$y_{sim2} = 3 \cdot 1 + Z s_0(\hat{\eta}) + s_1(\hat{e})$$

2000 replicates of $y_{sim1}$ and $y_{sim2}$ are generated for each season of each sport. These simulated score vectors are fit with both the fixed and mixed effect models. For each year $i$, the mean, $m_i$, of the 2000 estimates of $\lambda$, and the coverage probability of the 95% confidence intervals is recorded. The means of these values over all 18 seasons is then reported in Table 2 along with the p-value from the two-sided $t$-test of the null hypothesis that the $m_i$ were drawn from a population with mean 3.

Both of the models reliably recover the simulated HFA of 3 points in all sports when fitting the simulation with randomized team assignments, $y_{sim2}$. Likewise, both models produce unbiased estimates of the HFA in the professional sports, which have more balanced
schedules than the college sports, when the schedule remains fixed in its current configuration in $y_{sim1}$.

However, only the fixed effect model provides unbiased estimates for $y_{sim1}$ in the college sports: the mixed effect model shows upward bias in its estimate of the college HFAs along with poor coverage probabilities and extremely small p-values. The mixed model (5) assumes random assignments of the teams to games and to home location. This is not the case, however, as stronger college teams are often able to schedule more home than away games. The use of random effects to condition on team abilities ameliorates, but does not eliminate, the bias due to these nonrandom assignments, as illustrated in Figure [1]. This behavior in the college sports coincides with abnormal values of the test statistic [7], as reflected in the first row of Table [1].

### 3.2.3 Restriction to Intraconference Games

Games played within conferences of these sports tend to be more balanced (in terms of opponents strength and number of home and away games) than games played between conferences. Instead of fitting the HFA of using full season results, we could also fit each set of intraconference games separately within each season: interconference game results are discarded. As shown in the second row of Table [1], the intraconference games do not show the same evidence of lack of fit with respect to (7) as the full season results.

The simulation [12] was run with 500 iterations for each of the intraconference schedules in each season of each sport (Table [3]). While the restriction to intraconference games largely eliminated the bias in the mixed model estimate of $\lambda$, there is still some remaining evidence of upward bias in these estimates. For example, in women’s college basketball the mixed model for the full season simulations produced a mean estimate of 3.13 with a coverage probability of 0.87. This improved to an estimate of 3.03 and a coverage probability of 0.95 when restricting to intraconference games; however, the p-value of 8e-20 from the two-sided $t$-test indicates that upward bias is still a systematic problem for the random team effect model. As a result, the Phase II models presented in Section [2.2] will make use of the HFA estimates from the fixed team effect model [2].

An additional benefit of restricting to intraconference games is the potential for a comparison of HFA trends within conferences across seasons. Of course, this limits the inference space to intraconference games, requiring either additional assumptions or additional mod-
Table 2: Results for fitting 2000 simulations of $y_{sim1}$ and $y_{sim2}$ in each of the eighteen seasons for each sport, with $\lambda = 3.0$. Coverage probabilities are for 95% confidence intervals. The p-value refers to the two-sided $t$-test of the null hypothesis that the means of the 2000 simulations in each year are drawn from a population with a mean of 3. CFB represents college football, while CBB-M and CBB-W represent men’s and women’s college basketball, respectively.

|                | NFL | NBA | WNBA | CFB | CBB-M | CBB-W |
|----------------|-----|-----|------|-----|-------|-------|
| **Fixed team effects for $y_{sim1}$** |     |     |      |     |       |       |
| mean($\hat{\lambda}$) | 3.00 | 3.00 | 3.00 | 3.00 | 3.00  | 3.00  |
| Coverage Prob.  | 0.95 | 0.95 | 0.95 | 0.95 | 0.95  | 0.95  |
| p-value         | 0.40 | 0.77 | 0.63 | 0.66 | 5e-03 | 0.87  |
| **Random team effects for $y_{sim1}$** |     |     |      |     |       |       |
| mean($\hat{\lambda}$) | 3.02 | 3.00 | 3.00 | 3.37 | 3.26  | 3.13  |
| Coverage Prob.  | 0.95 | 0.95 | 0.95 | 0.89 | 0.62  | 0.87  |
| p-value         | 0.02 | 0.37 | 0.12 | 4e-15| 1e-19 | 4e-18 |
| **Fixed team effects for $y_{sim2}$** |     |     |      |     |       |       |
| mean($\hat{\lambda}$) | 3.00 | 3.00 | 3.01 | 2.99 | 3.00  | 3.00  |
| Coverage Prob.  | 0.95 | 0.95 | 0.95 | 0.95 | 0.95  | 0.95  |
| p-value         | 0.65 | 0.56 | 0.03 | 0.02 | 0.21  | 0.47  |
| **Random team effects for $y_{sim2}$** |     |     |      |     |       |       |
| mean($\hat{\lambda}$) | 3.00 | 3.00 | 3.01 | 2.99 | 3.00  | 3.00  |
| Coverage Prob.  | 0.95 | 0.95 | 0.95 | 0.95 | 0.95  | 0.95  |
| p-value         | 0.66 | 0.57 | 0.03 | 0.02 | 0.29  | 0.41  |
Table 3: Results for fitting 500 simulations of $y_{sim1}$ in each conference in each of the eighteen seasons, for each sport, with $\lambda = 3.0$. Coverage probabilities are for 95% confidence intervals. The p-value refers to the two-sided t-test of the null hypothesis that the population of simulated conference means (averages over the 500 simulations in each conference in each season) has mean equal to 3. CFB represents college football, while CBB-M and CBB-W represent men’s and women’s college basketball, respectively.

| Fixed team effects for $y_{sim1}$ | NFL | NBA | WNBA | CFB | CBB-M | CBB-W |
|-----------------------------------|-----|-----|------|-----|-------|-------|
| mean($\hat{\lambda}$)            | 2.99| 3.00| 3.00 | 3.00| 3.00  | 3.00  |
| Coverage Prob.                    | 0.95| 0.95| 0.95 | 0.94| 0.95  | 0.95  |
| p-value                           | 0.38| 0.76| 0.58 | 0.51| 0.99  | 0.35  |

| Random team effects for $y_{sim1}$| NFL | NBA | WNBA | CFB | CBB-M | CBB-W |
|----------------------------------|-----|-----|------|-----|-------|-------|
| mean($\hat{\lambda}$)           | 3.02| 3.00| 3.01 | 2.99| 3.01  | 3.03  |
| Coverage Prob.                   | 0.95| 0.95| 0.95 | 0.94| 0.94  | 0.95  |
| p-value                          | 0.09| 0.39| 0.12 | 0.48| 6e-08 | 8e-20 |

eling structure to generalize conclusions to interconference games.

### 3.3 Phase II: Trends in Home-Field Advantage

After fitting separate HFA estimates for each conference in each season of each sport, the resulting point estimates and standard errors are used to fit the Phase II models of Sections 2.2.1 and 2.2.2. The results for the college sports appear in Table 4 and those for the professional sports appear in Table 5. The conference-specific trends in scoring HFA are plotted in Figure 2.

Most estimates of $\sigma^2_{\lambda}$ in Tables 4 and 5 are near 1 due to the use of the squared reciprocals of the standard errors of $\hat{\lambda}$ as weights in the linear models. Estimates of error variance away from 1 indicate either over- or under-dispersion in the Phase II model from the estimated Phase I variation.
Figure 2: Each point represents the estimated scoring HFA for a conference within each season and sport. Graphs have common limits for the y-axis. In the college sports, the lines correspond to different conferences. In the professional sports, the triangles (circles) and solid (dashed) lines represent the NFC (AFC) conference in the NFL and the Western (Eastern) Conference in the NBA and WNBA.
Table 4: Phase II results of fitting the random coefficient model (8) to the Phase I estimated HFAs and associated standard errors for college sports. CFB represents college football, while CBB-M and CBB-W represent men’s and women’s college basketball, respectively.

|                | CFB     | CBB-M    | CBB-W    |
|----------------|---------|----------|----------|
| **Estimate**   |         |          |          |
| $\hat{\alpha}_0$ | 2.352   | 2.969    | 2.817    |
| $\hat{\alpha}_1$ | -0.102  | -0.074   | -0.049   |
| $\hat{\sigma}^2_\lambda$ | 1.020   | 1.100    | 1.048    |
| $\hat{\sigma}^2_1$ | 0.946   | 0.503    | 0.350    |
| $\hat{\sigma}_{12}$ | 0.106   | 0.001    | -0.003   |
| $\hat{\sigma}^2_2$ | 0.013   | 0.001    | 1e-20    |
| mean($w_{2017}^2$) | 3.970   | 1.129    | 1.272    |
| **95% Confidence Intervals** |         |          |          |
| $\alpha_0$ lower | 1.402   | 2.661    | 2.534    |
| $\alpha_0$ upper | 3.302   | 3.276    | 3.099    |
| $\alpha_1$ lower | -0.205  | -0.097   | -0.068   |
| $\alpha_1$ upper | 0.002   | -0.051   | -0.029   |
| **p-values for null hypothesis** |         |          |          |
| $\alpha_0 = 0$ | 3e-4    | 5e-19    | 1e-18    |
| $\alpha_1 = 0$ | 0.053   | 3e-7     | 9e-7     |
| $G = 0$        | 0.169   | <1e-5    | <1e-5    |
Table 5: Phase II results of fitting the linear model (9) to the Phase I estimated HFAs and associated standard errors for professional sports. p-values at bottom are generated by likelihood ratio tests of model (9) against reduced models (10) and (11), respectively

|                  | NFL   | NBA   | WNBA  |
|------------------|-------|-------|-------|
| **Estimate**     |       |       |       |
| $\hat{\beta}_0$A | 2.753 | 3.333 | 3.705 |
| $\hat{\beta}_0$B | -0.276| -0.775| -1.702|
| $\hat{\beta}_1$A | 0.035 | -0.021| -0.013|
| $\hat{\beta}_1$B | -0.036| -0.023| -0.075|
| $\sigma^2_\lambda$ | 0.865 | 1.096 | 0.715 |
| mean($w_{2017}^2$) | 1.583 | 0.399 | 2.875 |

|                  |       |       |       |
| **95% Confidence Intervals** |       |       |       |
| $\hat{\beta}_0$A lower | 1.673 | 2.776 | 2.641 |
| $\hat{\beta}_0$A upper | 3.832 | 3.890 | 4.770 |
| $\hat{\beta}_0$B lower | -1.796| -1.566| -3.277|
| $\hat{\beta}_0$B upper | 1.243 | 0.016 | -0.128|
| $\hat{\beta}_1$A lower | -0.074| -0.077| -0.113|
| $\hat{\beta}_1$A upper | 0.144 | 0.035 | 0.087 |
| $\hat{\beta}_1$B lower | -0.188| -0.101| -0.223|
| $\hat{\beta}_1$B upper | 0.116 | 0.056 | 0.073 |

|                  |       |       |       |
| **p-value for null hypothesis** |       |       |       |
| $\beta_0 = \beta_1 = 0$ | 0.874 | 0.011 | 0.013 |
| $\beta_1 = \beta_1 = 0$ | 0.784 | 0.179 | 0.221 |
3.3.1 College Results

The estimates $\hat{\alpha}_1$ and p-values for the test $\alpha_1 = 0$ in Table 4 indicate the strength of the trend of the population of conference-level HFAs over time. While college football produces the steepest downward slope of the three college sports, there is not strong evidence that this slope is not an artifact of conference and error variability over the observed period ($p=0.053$). Individually, the SEC and PAC10/12 conferences show an increase in estimated HFA over this time period, while the steepest decreases occur in the MAC, Sun Belt, and WAC conferences (see the supplementary data).

By contrast, there is a strong indication of downward linear trends in the men’s and women’s college basketball HFA. With estimates of $-0.074$ ($p = 3e-07$) and $-0.049$ ($p = 9e-07$), respectively, these translate into a decrease of 1.332 points in the HFA of men’s college basketball teams over the 18 year period considered, and a decrease of 0.882 points for the women’s HFA. The magnitudes of these decreases correspond to 45% and 31% of the estimated 2017 HFA ($\hat{\alpha}_0$) in each sport, making them appear practically as well as statistically significant. We can conclude that there has been a significant and noticeable decrease in the HFA of men’s and women’s college basketball intraconference college games from 2000 to 2017. As such, it seems that this would be a worthwhile topic for further study by sports journalists.

There is evidence of conference-level heterogeneity in the HFA trends. Variability in conference-level intercepts in all three sports is evident from Figure 2 which also reveals variability in conference-level slopes in college football. Even though the p-value does not provide evidence against the hypothesis that $G = 0$ for the college football results, an examination of the HFA by time plots for each of the conferences reveals too much variability to be overlooked. This variability is also reflected in the magnitudes of the $\hat{\sigma}_1^2$ and $\hat{\sigma}_2^2$ estimates in Table 4.

While Figure 2 does not include conference labels, the source file (available in the supplementary material) reveals similarity in the conference-level HFA estimates from men’s and women’s college basketball: in both cases, the Big 12 consistently had among the largest scoring HFAs while the Ivy League had among the smallest (Figure 3). Positive correlation between the conference-level HFA for men’s and women’s sports suggests the possible existence of important conference-level factors that affect HFA in a similar way for both men’s and women’s sports within the same conference. For sports journalists, investigating
Figure 3: Scoring HFA for intraconference games for the Ivy League and Big 12 conferences of men’s and women’s college basketball.

and understanding why there is a greater HFA observed in intraconference Big 12 basketball games than in intraconference Ivy League games could lead to a better understanding of factors that contribute to HFA in general.

### 3.3.2 Professional Results

We test for a linear trend in intraconference HFA in the professional sports with a likelihood ratio test of the full model (9) against the reduced model (11) with $\beta_1A = \beta_1B = 0$. The resulting p-values from this test appear in the last row of Table 5: there is no indication of a sustained trend in any of the three professional sports. This is consistent with the plots of the HFAs in the second column of Figure 2.

The NBA and WNBA plots suggest a difference between the trends of the Eastern and Western conference HFAs. In the WNBA, the fitted 2017 Eastern conference HFA is 1.7 points ($1.702/3.705 = 46\%$) lower than the Western conference HFA, while in the NBA the fitted 2017 Eastern conference HFA is 0.8 points ($0.775/3.333 = 23\%$) lower than the Western conference HFA. This is formally tested in a likelihood ratio test comparing the full model (9) against the reduced model (10) where $\beta_0B = \beta_1B = 0$: the resulting p-values appear in the penultimate row of Table 5. With p-values near 0.01, the tests offer slight (keeping in mind the large number of p-values being reported in this paper) but inconclusive support for the difference in conference slopes and intercepts. Given the size of the estimated differences between conference HFAs and the relative ease with which these HFAs can be
fit, this appears to be a worthwhile trend to monitor over the next few seasons and then to investigate for root causes if the trend continues. One possible explanation is that the teams are farther apart in the Western conferences of the WNBA and the NBA. We explored this by adding distance between teams (or quantiles thereof) as a covariate to model (2) and fitting full season NBA results, but did not find any significant association.

4 Application to Value-Added Models

Some value-added models (VAMs) for education evaluation (McCaffrey et al. 2003) also utilize a stochastic random effects model matrix, $Z$, to track students’ classroom assignments. Previous work has used a variety of methods to explore the possibility that nonrandom assignment of students to classrooms could bias model estimates (Ballou et al. 2004). As with the HFA example of Section 3, it seems that the test of Equation 7 could be useful for flagging data and model combinations with potentially biased fixed effect estimates. The VAM applications use more general mixed models for student test scores (Mariano et al. 2010) with $Y \sim N(X\beta + Z\eta, R)$ with $\eta \sim N(0, G)$, where classroom-level random effects are modeled with $\eta$, student-level correlation is modeled in $R$, and $\beta$ includes student- or classroom-level fixed effects. Provided that the inverse exists,

$$\eta'Z'R^{-1}X(X'R^{-1}ZGZ'R^{-1}X)^{-1}X'R^{-1}Z\eta \sim \chi^2_{\text{rank}(X'R^{-1}Z)}$$

(14)

We have applied this test (14) to the complete persistence value-added model (Ballou et al. 2004, Mariano et al. 2010) for three years of data from a cohort of students from a large urban school district (Karl et al. 2013). The test does not reveal anything unexpected when only the three yearly means are included in $X$ as fixed effects (p-value 0.98), but when Race/Ethnicity is added as a fixed effect, the p-value drops to 0.0005. It is possible that this resonance between $X'R^{-1}$ and $Z\eta$ signals that estimates of the fixed effects in these models will be biased in the same way as the mixed model HFA estimates for college basketball results (Figure 1). Indeed, 200 simulations of the three year data set using the fitted values $\hat{G}, \hat{\eta}, \hat{\beta}$ and error vectors sampled from $N(0, \hat{R})$ reveals that the estimates of the race/ethnicity fixed effects are biased upward for white (0.3% bias, p=1e-33) and Asian (0.2% bias, p=1e-05) students, and downward for Hispanic (-0.4% bias, p=1e-44) students. No bias is detected for the yearly means. The magnitude of the biases in this application...
are so small that they may not be practically relevant, but they do demonstrate the ability
of the statistic (14) to flag data sets with biased fixed effect estimates due to the structure
of $Z$, without having to run lengthy simulations.

5 Concluding Remarks

Division I college men’s and women’s basketball intraconference games have shown strong
downward linear trends in scoring HFA over the last two decades. While college football
shows an even steeper decrease on intraconference HFA over the same period, there is also
much more variability in the Phase I within-season estimates in college football than in college
basketball. There is also greater conference-level variability in the Phase II estimates of slopes
and intercepts in the HFA linear trends for football. Due to the magnitude of the variability,
it is difficult to say if the observed downward trend in college football intraconference HFA
represents a systematic shift in population behavior or is merely an artifact of a sample from
a noisy process with constant mean.

This paper focused on the scoring HFA instead of the win propensity HFA in order to
explore the biasing effects of the nonrandom schedules without also considering the potential
bias due to the requisite integral approximations in the case of binary game results (Breslow
& Lin 1995). The source code provided in the supplementary material (https://github.
com/HFAbias18/supplement) can be modified to fit the win propensity HFA in a generalized
linear mixed model with a probit link and a fully exponential Laplace approximation (Karl
2012, Broatch & Karl 2017a,b) by changing the method argument of the mvglmmRank function
call. No changes would be required for the Phase II models, since they would simply continue
to make use of the point estimates and standard errors that are produced by mvglmmRank.
In fact, we did fit these models, but chose not to include them, for the sake of brevity, as
we did not see any noticeable differences in the Phase II results from those presented for the
scoring HFA (outside of slightly larger p-values for the Phase II population trends, possibly
due to the information loss in the discretization of the game results).

This analysis has raised two interesting questions about men’s and women’s college bas-
ketball for further investigation by sports researchers. What factors contribute to the differ-
ence in intraconference HFAs, such as those shown between the Big 12 and the Ivy League
(Figure 3)? Second, what factors are driving the practically and statistically significant
downward trends in the HFAs? Davis (2017) asked then-head basketball coach at Louisville, Rick Pitino, about a potential decline in HFA. Pitino responded that he believes television has improved refereeing due to increased visibility of bias and errors, noting that the advances in video technology over the past seven years have increased the use of video review for referee evaluation, and that this may make referees less susceptible to home-court crowd pressure. Certainly there will be other theories about contributing factors.

**Appendix: p-hacking Disclosure**

The American Statistical Association (ASA) statement on p-values (Wasserstein & Lazar 2016) and the associated commentaries call for a reform in the approach and presentation of statistical analyses. Among its recommendations are that journal authors document additional tests and analysis that were performed but not included in the text of the article.

- Before committing to the analysis of linear trends in HFA, we examined a plot of the estimates to see that a line provided a plausible explanation for the trend. Had there been evidence of curvature or autocorrelation, the analysis would have likely taken a different approach. We also originally allowed for correlation in the Phase II professional results of model (9) by adding Toeplitz bands to the error covariance matrix, but chose not to include these effects in the model presented in the paper after finding no significant correlation. Note that the random coefficient model (8) for the college results naturally incorporates correlation at the conference level.

- The flexibility to choose a starting and ending year for the study comes with a potential danger of data dredging by selecting these time points based on the results of the analysis. The decision to exclude any game results beyond the end of the 2017 season for each sport was made in advance and based on the manuscript timeline, while the decision to begin with the 2000 season was determined by data set availability and a desire to compare all of the sports over the same seasons. Early drafts did consider additional prior seasons for some sports: no major changes to the conclusions would have resulted from inclusion of these seasons, although there would have been a need to address curvature in the football and men’s basketball HFA prior to 2001.
• Besides the statistical tests reported here, many others were performed during research for this paper. For example, an early draft only considered full season results from the Phase I mixed model, and also explored trends in the estimated home and away mean scores and conditional home and away error variances from a joint model for home and away scores \cite{Broatch2017b}. These were abandoned for the sake of providing a tighter narrative; however, these fitted values remain in the full season Phase I results of the supplementary data and may be further explored by the reader. Even though the original intention of the research was to examine the strength of linear trends in HFA, the number of such other p-values considered during manuscript preparation must be taken into account when considering the strength of the p-values reported in the paper.

References

Ballou, D., Sanders, W. & Wright, P. (2004), ‘Controlling for student background in value-added assessment of teachers’, *Journal of Educational and Behavioral Statistics* **29**, 37–65.

Breslow, N. E. & Lin, X. (1995), ‘Bias correction in generalised linear mixed models with a single component of dispersion’, *Biometrika* **82**, 81–91.

Broatch, J. E. & Karl, A. T. (2017a), ‘Multivariate generalized linear mixed models for joint estimation of sporting outcomes’, *Italian Journal of Applied Statistics* p. To Appear.

Broatch, J. E. & Karl, A. T. (2017b), ‘Multivariate generalized linear mixed models for joint estimation of sporting outcomes’, *arXiv:1710.05284* .

Chatelain, D. (2016), ‘College football’s greatest myth’, *Omaha World-Herald*. 
URL: http://dataomaha.com/bigstory/sports/college-footballs-greatest-myth

Davis, S. (2017), ‘Hoop thoughts: What is happening to home court advantage?’, *Sports Illustrated*. 
URL: https://www.si.com/college-basketball/2017/01/09/hoop-thoughts-home-court-advantage-baylor-kansas-villanova
Diffey, S., Welsh, A., Smith, A. & Cullis, B. R. (2013), A faster and computationally more efficient REML (PX)EM algorithm for linear mixed models, Technical Report 2-13, Centre for Statistical and Survey Methodology, University of Wollongong.

URL: http://ro.uow.edu.au/cssmwp/108

Goumas, C. (2014), ‘Home advantage and referee bias in European football’, European Journal of Sport Science 14(sup1), S243–S249. PMID: 24444213.

Harville, D. (1977), ‘The use of linear-model methodology to rate high school or college football teams’, Journal of the American Statistical Association 72(358), 278–289.

Harville, D. A. (2003), ‘The selection or seeding of college basketball or football teams for postseason competition’, Journal of the American Statistical Association 98(461), 17–27.

Harville, D. A. & Smith, M. H. (1994), ‘The home-court advantage: How large is it, and does it vary from team to team?’, The American Statistician 48(1), 22–28.

Henderson, C. R. (1975), ‘Best linear unbiased estimation and prediction under a selection model’, Biometrics 31, 423.

Jones, M. B. (2007), ‘Home advantage in the NBA as a game-long process’, Journal of Quantitative Analysis in Sports 3(4).

Karl, A. T. (2012), ‘The sensitivity of college football rankings to several modeling choices’, Journal of Quantitative Analysis in Sports 8(3), doi:10.1515/1559-0410.1471.

Karl, A. T. & Broatch, J. (2014), mvglmmRank: Multivariate Generalized Linear Mixed Models for Ranking Sports Teams. R package version 1.0-2.

Karl, A. T., Yang, Y. & Lohr, S. L. (2013), ‘A correlated random effects model for nonignorable missing data in value-added assessment of teacher effects’, Journal of Educational and Behavioral Statistics 38, 577–603.

Laird, N. M. & Ware, J. H. (1982), ‘Random-effects models for longitudinal data’, Biometrics 38, 963–974.

Mariano, L. T., McCaffrey, D. F. & Lockwood, J. (2010), ‘A model for teacher effects from longitudinal data without assuming vertical scaling’, Journal of Educational and Behavioral Statistics 35, 253–279.
Massey, K. (2017), ‘Massey Ratings’.

**URL:** [https://www.masseyratings.com/data.php](https://www.masseyratings.com/data.php)

McCaffrey, D. F., Lockwood, J., Koretz, D. M. & Hamilton, L. S. (2003), *Evaluating Value-Added Models for Teacher Accountability*, Pittsburgh: The RAND Corporation.

Moskowitz, T. & Wertheim, L. J. (2012), *Scorecasting: The Hidden Influences Behind How Sports Are Played and Games Are Won*, New York: Random House.

Pollard, R. & Pollard, G. (2005), ‘Long-term trends in home advantage in professional team sports in North America and England’, *Journal of Sports Sciences* 23(4), 337–350.

Scheipl, F., Greven, S. & Kuchenhoff, H. (2008), ‘Size and power of tests for a zero random effect variance or polynomial regression in additive and linear mixed models’, *Computational Statistics & Data Analysis* 52(7), 3283 – 3299.

Stroup, W. W. (2012), *Generalized Linear Mixed Models: Modern Concepts, Methods and Applications*, Boca Raton: CRC Press.

Wang, W., Johnston, R. & Jones, K. (2011), ‘Home advantage in american college football games: A multilevel modelling approach’, *Journal of Quantitative Analysis in Sports* pp. 1559–0410.

Wasserstein, R. L. & Lazar, N. A. (2016), ‘The ASA’s statement on p-values: Context, process, and purpose’, *The American Statistician* 70(2), 129–133.