Unparticle physics effects in $\Lambda_b \rightarrow \Lambda + \text{missing energy}$ processes

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Abstract

We study unparticle physics effects in $\Lambda_b \rightarrow \Lambda + \text{missing energy}$ decay with polarized $\Lambda_b$ and $\Lambda$ baryons. The sensitivity of the branching ratio of this decay and polarizations of $\Lambda_b$ and $\Lambda$ baryons on the scale dimension $d_U$ and effective cut–off parameter $\Lambda_U$ are discussed.

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1 Introduction

Flavor changing neutral current (FCNC) decays induced by the $b \to s$ transition are promising decays for checking predictions of the Standard model (SM) at quantum loop level, since they are forbidden at tree level in the SM. These transitions are also very suitable in looking for new physics beyond the SM.

In the SM the $b \to s \nu \bar{\nu}$ decay receives special attention due to the theoretical advantage that uncertainties in this decay are much smaller compared to other FCNC decays due to the absence of photonic penguin diagrams and hadronic long distance effects. However, in spite of these theoretical advantages experimental measurement of this inclusive channel seems to be very difficult, because it requires a construction of all $X_s$. Therefore, experimentalists focus only on exclusive channels like $B \to K(K^*)\bar{\nu}\nu$. This channel studied extensively on theoretical grounds in many works (see for example [1–4]). Another class of decays, which is described by the $b \to s \bar{\nu}\nu$ transition at inclusive level, is the baryonic $\Lambda_b \to \Lambda \bar{\nu}\nu$ decay.

It should be noted that it is impossible to analyze the helicity structure of the effective Hamiltonian in $B$ meson decays governed by $b \to s$ transition, since the information about chiralities of the quarks is lost in the hadronization process. In contrary to the mesonic decays, baryonic decays could access the helicity structure of the effective Hamiltonian for the $b \to s$ transition [5]. Therefore the heavy baryonic decays can be very rich for studying the polarization effects.

Radiative and semileptonic decays of $\Lambda_b$ baryon, such as $\Lambda_b \to \Lambda \gamma$, $\Lambda_b \to \Lambda_\ell \bar{\nu}_\ell$, $\Lambda_b \to \Lambda_\ell^+ \ell^-$ and $\Lambda_b \to \Lambda \nu \bar{\nu}$, are comprehensively studied in the framework of SM in many works [5–12]. Present status of the experimental investigations of heavy baryons is discussed in [13].

As has already been noted, FCNC transitions are very sensitive to the existence of new physics beyond the SM. One such model is the so–called unparticles is proposed by H. Georgi [14]. It is assumed in this model that, at very high energies the theory contains SM fields and the fields with a non–trivial infrared fixed point so that in the infrared limit it will be an asymptotic conformal theory and be scale–invariant, which are called Banks–Zaks (BZ) fields [15]. In unparticle physics model these two sectors interacted by exchange of particle with a large mass scale $\mu$, below this scale where non–renormalizable couplings between SM and BZ fields will be induced and renormalizable couplings between the BZ fields are then produced by dimensional transmutation, and the scale–invariant unparticle fields emerge below a scale $\Lambda_U$. In the effective theory, below $\Lambda_U$, BZ operators match onto the unparticle operators, and non–renormalizable interactions between SM and unparticle operators can be obtained. An important result in this theory is that unparticle stuff with scale dimension $d_U$ looks like a non–integer number $d_U$ of invisible particles [14], where production might be detectable in missing energy and momentum distributions. Various phenomenological aspects of the unparticle physics have recently been extensively discussed in literature [16]–[56].

In the present work we study the $\Lambda_b \to \Lambda + missing energy$ decay in unparticle physics. The paper is organized as follows: In section 2, we give necessary theoretical framework to describe the differential decay width of $\Lambda_b \to \Lambda + missing energy$ in the SM and in unparticle physics. Section 3 is devoted to numerical analysis and conclusions.
2 Theoretical framework

In the SM $\Lambda_b \rightarrow \Lambda + \text{missing energy}$ channel is described by the $\Lambda_b \rightarrow \Lambda \bar{\nu} \nu$ decay. As has already been noted, unparticles can also contribute to this decay. Therefore, a comparison of the signature of the decay modes $\Lambda_b \rightarrow \Lambda \bar{\nu} \nu$ and $\Lambda_b \rightarrow \Lambda U$ is required.

In the SM, the $\Lambda_b \rightarrow \Lambda \bar{\nu} \nu$ decay is described at quark level by the $b \rightarrow s \bar{\nu} \nu$ transition and receives contributions from $Z$–penguin and box diagrams, where main contributions come from intermediate top quarks. The effective Hamiltonian responsible for $b \rightarrow s \bar{\nu} \nu$ transition is described by only one Wilson coefficient $C_{10}$ and its explicit form is

$$H = \frac{G_F \alpha}{\sqrt{2} \pi} V_{tb} V_{ts}^* C_{10} \bar{s} \gamma_\mu (1 - \gamma_5) b \bar{\nu} \gamma^\mu (1 - \gamma_5) \nu ,$$

where $G_F$ and $\alpha$ are the Fermi constant and structure constants, respectively, $V_{ij}$ are the elements of the Cabibbo–Cobayashi–Maskawa matrix (CKM). The Wilson coefficient $C_{10}$ in Eq. (1), including $O(\alpha_s)$ corrections, has the following form:

$$C_{10} = \frac{X(x_t)}{\sin^2 \theta} ,$$

where

$$X(x_t) = X_0(x_t) + \frac{\alpha_s}{4\pi} X_1(x_t) .$$

$X_0(x_t)$ in Eq. (3) is the usual Inami–Lim function [57] given by:

$$X_0(x_t) = \frac{x_t}{8} \left( \frac{x_t + 2}{x_t - 1} + \frac{3x_t - 6}{(x_t - 1)^2} \ln x_t \right) ,$$

and

$$X_1(x_t) = \frac{4x_t^3 - 5x_t^2 - 23x_t}{3(x_t - 1)^2} - \frac{x_t^4 + x_t^3 - 11x_t^2 + x_t}{(x_t - 1)^3} \ln x_t + \frac{x_t^4 - x_t^3 - 4x_t^2 - 8x_t}{2(x_t - 1)^3} \ln^2 x_t + \frac{x_t^3 - 4x_t}{(x_t - 1)^2} Li_2(1 - x_t) + 8x_t \frac{\partial X_0(x_t)}{\partial x_t} \ln x_\mu .$$

Here,

$$Li_2(1 - x_t) = \int_1^{x_t} \frac{\ln t}{1 - t} dt ;$$

is the spence function, $x_t = m_t^2/m_W^2$, $x_\mu = \mu^2/m_W^2$, and $\mu$ describes the scale dependence when leading QCD corrections are taken into account. The function $X_1(x_t)$ is calculated in [58].

Similarly, at quark level in unparticle physics, $b \rightarrow s + \text{missing energy}$ is described by the $b \rightarrow s U$ transition, where we shall consider two types of unparticle operators:

- Scalar unparticle operators,
• Vector unparticle operators.

For the scalar and vector operators $b \rightarrow sU$ transition is described by the following matrix elements

$$\frac{1}{\Lambda_{dU}^U} \left[ \bar{s} \gamma^\mu (C_S + C_P \gamma_5) b \right] \partial_\mu \mathcal{O}_u,$$

$$\frac{1}{\Lambda_{dU}^d} \left[ \bar{s} \gamma^\mu (C_V + C_A \gamma_5) b \right] \mathcal{O}^\mu_u,$$

where $C_i$ are the dimensionless effective couplings.

Before performing analytical calculations, let us present the forms of the propagators for the scalar and vector unparticle physics [16, 17]

$$D(q^2) = \int d^4xe^{iqx} \langle 0 | \mathcal{T} (\mathcal{O}_u(x)\mathcal{O}_u(0)) | 0 \rangle,$$

$$= \frac{A_{dU}}{2 \sin(d_U \pi)} (-q^2)^{d_U-2},$$

$$D_{\mu\nu} = \frac{A_{dU}}{2 \sin(d_U \pi)} (-q^2)^{d_U-2} \left( -g_{\mu\nu} + \frac{q_{\mu}q_{\nu}}{q^2} \right),$$

where

$$A_{dU} = \frac{16\pi^{5/2}}{(2\pi)^{2dU}} \frac{\Gamma(d_U + 1/2)}{\Gamma(d_U - 1)\Gamma(2d_U)}.$$  

It is found in [14] that, using scale invariance of the unparticle physics, the phase for an unparticle operator with the scale dimension $d_U$ and momentum $q$ is the same as the phase space for $d_U$ invisible massless particles

$$d\Phi_u(q) = A_{dU} \Theta(q^0) \Theta(q^2)(q^2)^{d_U-2} \frac{d^4q}{(2\pi)^2}.$$  

Having the explicit forms of the effective Hamiltonian at hand for the $b \rightarrow \tilde{\nu}\nu$ transition and effective interaction for the $b \rightarrow sU$ transition, our next problem is computation of the matrix element of (1), (7) and (8), between initial and final state baryons. It follows from Eqs. (1) and (7) that the we need to know the following matrix elements

$$\langle \Lambda | \bar{s} \gamma_\mu b | \Lambda_b \rangle,$$

$$\langle \Lambda | \bar{s} \gamma_\mu \gamma_5 b | \Lambda_b \rangle.$$

These matrix elements can be parametrized in terms of the form factors a follows [11]:

$$\langle \Lambda | \bar{s} \gamma_\mu b | \Lambda_b \rangle = \bar{u}_\Lambda \left[ f_1 \gamma_\mu + f_2 i \sigma_{\mu\nu} q^\nu + f_3 q_\mu \right] u_{\Lambda_b},$$

$$\langle \Lambda | \bar{s} \gamma_\mu \gamma_5 b | \Lambda_b \rangle = \bar{u}_\Lambda \left[ g_1 \gamma_\mu \gamma_5 + g_2 i \sigma_{\mu\nu} q^\nu \gamma_5 + g_3 q_\mu \gamma_5 \right] u_{\Lambda_b},$$

where $q = p_{\Lambda_b} - p_\Lambda$. 

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It follows from these expressions that $\Lambda_b \rightarrow \Lambda + \text{missing energy}$ decay is described in terms of numerous form factors. It is shown in [5] that Heavy Effective Quark Theory (HEQT) reduces the number of independent form factors to two ($F_1$ and $F_2$) irrelevant of the Dirac structure of corresponding operators, i.e.,

$$\langle \Lambda(p_\Lambda) | s\Gamma b | \Lambda_b \rangle = \bar{u}_\Lambda \left[ F_1(q^2) + \not{\bar{q}} F_2(q^2) \right] \Gamma u_{\Lambda_b},$$

where $\Gamma$ is the arbitrary Dirac structure and $\not{\nu} = p_{\Lambda_b}^\mu/m_{\Lambda_b}$ is the four velocity of $\Lambda_b$. Comparing Eqs. (13), (14) and (15) one can easily obtain relations among the form factors (see also [11])

$$f_1 = g_1 = F_1 + \frac{m_\Lambda}{m_{\Lambda_b}} F_2,$$

$$g_2 = f_2 = g_3 = f_3 = \frac{F_2}{m_{\Lambda_b}},$$

which we will use in our numerical analysis.

Using Eqs. (6), (7), (8), (13), (14) and (16) we get the following expression for the matrix elements of the $\Lambda_b \rightarrow \Lambda + \text{missing energy}$ decay:

$$M^{(1)} = \frac{G_F \alpha}{\sqrt{2} \pi} V_{tb} V_{ts}^* C_{10} \bar{u}_\Lambda \left[ \gamma_\mu (f_1 - g_1 \gamma_5) + \not{\bar{q}} \gamma_\mu (f_2 - g_2 \gamma_5) \right] u_{\Lambda_b} \not{\nu} \gamma_\mu (1 - \gamma_5) \nu,$$

$$M^{(2)} = \frac{1}{\Lambda^{d_\Lambda-1}} \bar{u}_\Lambda \left[ A + B \gamma_5 \right] u_{\Lambda_b} O,$$

$$M^{(3)} = \frac{1}{\Lambda^{d_\Lambda-1}} \bar{u}_\Lambda \left[ \gamma_\mu (A^V + B^V \gamma_5) + (C^V + D^V \gamma_5) P_{\Lambda_b \mu} \right] u_{\Lambda_b} O^\mu.$$ 

Here, $i = 1$, $i = 2$ and $i = 3$ correspond to the SM, scalar operator and vector operator contributions, respectively, and,

$$A = C_S \left[ (m_{\Lambda_b} - m_\Lambda) f_1 + q^2 f_3 \right],$$

$$B = C_P \left[ -(m_{\Lambda_b} + m_\Lambda) f_1 + q^2 g_3 \right],$$

$$A^V = C_V \left[ f_1 - (m_{\Lambda_b} + m_\Lambda) f_2 \right],$$

$$B^V = C_A \left[ g_1 - (m_{\Lambda_b} + m_\Lambda) g_2 \right],$$

$$C^V = 2 C_V f_2,$$

$$D^V = 2 C_A g_2.$$

In the derivation of Eqs. (17), (18) and (19), the neutrinos are taken to be massless and we also use $f_3 = f_2$ and $g_3 = g_2$ (see Eq. (16) ).

Having obtained the matrix elements for the $\Lambda_b \rightarrow \Lambda + \text{missing energy},$ the differential decay width can be calculated straightforwardly. As has already been noted, the polarization effects for the $\Lambda_b \rightarrow \Lambda + \text{missing energy},$ are richer than compared to the corresponding mesonic decays, since polarization of $\Lambda_b$ and $\Lambda$ can be measured. In this connection few words about the polarizations of baryons are in order. In the $\Lambda_b$ rest frame, the unit vectors
along the longitudinal, normal and transversal components of Λ polarization are defined in the following way:

\[
\vec{e}_L = \frac{\vec{p}_\Lambda}{|\vec{p}_\Lambda|}, \\
\vec{e}_N = \vec{\xi}_{\Lambda_b} \times \vec{e}_L, \\
\vec{e}_T = \vec{e}_L \times \vec{e}_N,
\]

where \(\vec{p}_\Lambda\) is the momentum of Λ baryon and \(\vec{\xi}_{\Lambda_b}\) is the unit vector along the \(\Lambda_b\) baryon spin in its rest frame. In the rest frame of \(\Lambda_b\) baryon the differential decay width can be written as:

\[
\frac{d\Gamma^{(i)}}{dE_\Lambda} = \left(\frac{d\Gamma^{(i)}}{dE_\Lambda}\right) \frac{1}{4} \left[ 1 + \frac{I_2^{(i)}}{I_1^{(i)}} \vec{e}_L \cdot \vec{\xi}_{\Lambda_b} \right] \left[ 1 + \vec{\mathcal{P}}^{(i)} \cdot \vec{\xi}_{\Lambda_b} \right], \tag{21}
\]

where \(i = 1, 2, 3\), and \(d\Gamma_0^{(i)}/dE_\Lambda\) describes the unpolarized differential decay width.

In Eq. (21) \(\vec{\mathcal{P}}^{(i)}\) is determined as follows:

\[
\vec{\mathcal{P}}^{(i)} = \frac{1}{1 + \frac{I_2^{(i)}}{I_1^{(i)}} \vec{e}_L \cdot \vec{\xi}_{\Lambda_b}} \left[ \left( I_3^{(i)} + \frac{I_2^{(i)}}{I_1^{(i)}} \vec{e}_L \cdot \vec{\xi}_{\Lambda_b} \right) \vec{e}_L + \frac{I_2^{(i)}}{I_1^{(i)}} \vec{e}_T + \frac{I_6^{(i)}}{I_1^{(i)}} \vec{e}_N \right]. \tag{22}
\]

Note that \(\Lambda_b \rightarrow \Lambda \bar{\nu} \nu\) decay, with \(\Lambda_b\) and Λ polarizations is studied in [11]. Explicit expressions of \(d\Gamma_0/dE_\Lambda, \vec{\mathcal{P}}_\Lambda, I_2\) and \(I_1\) in the SM are given in [11], and therefore we do not present them in this work.

After simple calculation, the decay width due to the scalar operator takes the following form:

\[
\frac{d\Gamma_0^{(2)}}{dE_\Lambda} = \frac{1}{2m_{\Lambda_b}} \frac{A_{\Delta t}}{(\Lambda_{\Delta t})^2} \left( q^2 \right)^{d_{\Delta t}-2} |\vec{p}_\Lambda|^2 \frac{dE_\Lambda}{(2\pi)^2} \Gamma^{(2)}, \tag{23}
\]

where \(q = p_{\Lambda_b} - p_{\Lambda}\), and \(|\vec{p}_\Lambda|\) is the magnitude of the \(\Lambda_b\) baryon three momentum, and

\[
I_1^{(2)} = 4m_{\Lambda_b} \left[ |A|^2 (E_\Lambda + m_\Lambda) + |B|^2 (E_\Lambda - m_\Lambda) \right], \\
I_2^{(2)} = -8 \text{Re}[AB^*] m_{\Lambda_b} |\vec{p}_\Lambda|, \\
I_3^{(2)} = -8 \text{Re}[AB^*] m_{\Lambda_b} |\vec{p}_\Lambda| \equiv I_2^{(2)}, \\
I_4^{(2)} = 4 |A|^2 \left[ -|\vec{p}_\Lambda|^2 + m_\Lambda (E_\Lambda + m_\Lambda) \right] m_{\Lambda_b} + 4 |B|^2 \left[ |\vec{p}_\Lambda|^2 - m_\Lambda (E_\Lambda - m_\Lambda) \right] m_{\Lambda_b}, \\
I_5^{(2)} = 4 |A|^2 m_{\Lambda_b} (E_\Lambda + m_\Lambda) - 4 |B|^2 m_{\Lambda_b} (E_\Lambda - m_\Lambda), \\
I_6^{(2)} = 8 |\vec{p}_\Lambda| m_{\Lambda_b} \text{Im}[AB^*]. \tag{24}
\]

The coefficients \(\vec{e}_L, \vec{e}_T\) and \(\vec{e}_N\), in Eq. (22) corresponding to the longitudinal, transversal and normal polarization asymmetries of Λ can also be defined as:

\[
\mathcal{P}_\Lambda^{(i)} = \frac{d\Gamma^{(i)}(\vec{\xi}_{\Lambda} \cdot \vec{e}_j = 1) - d\Gamma^{(i)}(\vec{\xi}_{\Lambda} \cdot \vec{e}_j = -1)}{d\Gamma^{(i)}(\vec{\xi}_{\Lambda} \cdot \vec{e}_j = 1) + d\Gamma^{(i)}(\vec{\xi}_{\Lambda} \cdot \vec{e}_j = -1)},
\]

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where \( j = L, T, N \).

Performing similar calculations for unpolarized decay due to the vector operator, we get:

\[
\frac{d\Gamma^{(3)}_0}{dE_\Lambda} = \frac{1}{2m_{\Lambda_b}} \left( \frac{A_{d\ell}}{(A_{d\ell}^{\mu-1})^2} \right) (q^2)^{d\ell-2} \frac{\widetilde{\rho}_\Lambda}{(2\pi)^2} I_1^{(3)},
\]

and the expressions of the functions entering into Eq. (22) for the vector operator case are as follows:

\[
I_1^{(3)} = 4 \left( \frac{|D_V|^2}{q^2} \right) m_{\Lambda_b}^3 (E_\Lambda - m_\Lambda) [(E_\Lambda - m_{\Lambda_b})^2 - q^2]
+ 4 \left( \frac{|C_V|^2}{q^2} \right) m_{\Lambda_b}^3 (E_\Lambda + m_\Lambda) [(E_\Lambda - m_{\Lambda_b})^2 - q^2]
+ 4 \left( \frac{|A_V|^2}{q^2} \right) m_{\Lambda_b} \left[ -2m_{\Lambda_b} E_\Lambda + E_\Lambda (2m_\Lambda^2 + 2m_{\Lambda_b}^2 + q^2) - m_\Lambda (2m_\Lambda m_{\Lambda_b} + 3q^2) \right]
+ 4 \left( \frac{|B_V|^2}{q^2} \right) m_{\Lambda_b} \left[ -2m_{\Lambda_b} E_\Lambda + E_\Lambda (2m_\Lambda^2 + 2m_{\Lambda_b}^2 + q^2) - m_\Lambda (2m_\Lambda m_{\Lambda_b} - 3q^2) \right]
+ 8 \left( \frac{\text{Re}[A_V C_V^*]}{q^2} \right) m_{\Lambda_b}^2 (E_\Lambda + m_\Lambda) [(E_\Lambda - m_{\Lambda_b}) (m_\Lambda - m_{\Lambda_b}) - q^2]
+ 8 \left( \frac{\text{Re}[B_V D_V^*]}{q^2} \right) m_{\Lambda_b}^2 (E_\Lambda - m_\Lambda) [(E_\Lambda - m_{\Lambda_b}) (m_\Lambda + m_{\Lambda_b}) + q^2]
\]

\[
I_2^{(3)} = -8 \left( \frac{\tilde{\rho}_\Lambda}{q^2} \right) \left\{ \text{Re}[C_V D_V^*] m_{\Lambda_b}^3 [(E_\Lambda - m_{\Lambda_b})^2 - q^2] \right. \\
+ \text{Re}[A_V B_V^*] m_{\Lambda_b} [2m_\Lambda^2 - 2m_{\Lambda_b} E_\Lambda + q^2] + \text{Re}[B_V C_V^*] m_{\Lambda_b}^2 [(E_\Lambda - m_{\Lambda_b}) (m_\Lambda + m_{\Lambda_b})] \\
+ \text{Re}[A_V D_V^*] m_{\Lambda_b}^2 [(E_\Lambda - m_{\Lambda_b}) (m_\Lambda - m_{\Lambda_b}) - q^2] \right\},
\]

\[
I_3^{(3)} = \frac{8}{q^2} \left\{ -\text{Re}[C_V D_V^*] m_{\Lambda_b}^3 [(E_\Lambda - m_{\Lambda_b})^2 - q^2] \\
- \text{Re}[A_V D_V^*] m_{\Lambda_b}^2 [(E_\Lambda - m_{\Lambda_b}) (m_\Lambda - m_{\Lambda_b}) - q^2] \\
- \text{Re}[B_V C_V^*] m_{\Lambda_b}^2 [(E_\Lambda - m_{\Lambda_b}) (m_\Lambda + m_{\Lambda_b}) - q^2] \\
+ \text{Re}[A_V B_V^*] m_{\Lambda_b} [2m_{\Lambda_b} (m_\Lambda - E_\Lambda) + q^2] \right\},
\]

\[
I_4^{(3)} = \frac{4}{q^2 m_\Lambda} \left\{ |D_V|^2 m_{\Lambda_b}^3 \left[ |\tilde{\rho}_\Lambda|^2 + m_\Lambda (m_\Lambda - E_\Lambda) \right] [(E_\Lambda - m_{\Lambda_b})^2 - q^2] \\
+ |C_V|^2 m_{\Lambda_b}^3 \left[ -|\tilde{\rho}_\Lambda|^2 + m_\Lambda (m_\Lambda + E_\Lambda) \right] [(E_\Lambda - m_{\Lambda_b})^2 - q^2] \\
+ 2 \text{Re}[A_V C_V^*] m_{\Lambda_b}^2 \left[ -|\tilde{\rho}_\Lambda|^2 + m_\Lambda (m_\Lambda + E_\Lambda) \right] [(E_\Lambda - m_{\Lambda_b}) (m_\Lambda - m_{\Lambda_b}) - q^2] \\
+ 2 \text{Re}[B_V D_V^*] m_{\Lambda_b}^2 \left[ |\tilde{\rho}_\Lambda|^2 + m_\Lambda (m_\Lambda - E_\Lambda) \right] [(E_\Lambda - m_{\Lambda_b}) (m_\Lambda + m_{\Lambda_b}) + q^2] \\
+ |B_V|^2 m_{\Lambda_b} [2m_\Lambda (E_\Lambda - m_{\Lambda_b}) (m_{\Lambda_b} E_\Lambda - m_\Lambda^2) + 2 |\tilde{\rho}_\Lambda|^2 (m_\Lambda^2 + m_\Lambda m_{\Lambda_b} + m_{\Lambda_b} (m_{\Lambda_b} - E_\Lambda))] \right\}.
\]
\[- q^2 \left( |\vec{p}_\Lambda|^2 + m_\Lambda (m_\Lambda - E_\Lambda) \right) \]
\[- |A_V|^2 m_{\Lambda_b} \left( |\vec{p}_\Lambda|^2 \left( 2m_\Lambda^2 - 2m_{\Lambda_b}(E_\Lambda + m_\Lambda) + 2m_{\Lambda_b}^2 - q^2 \right) \right. \]
\[+ m_\Lambda \left( 2E_\Lambda^2 m_{\Lambda_b} + 2m_\Lambda^2 m_{\Lambda_b} + m_\Lambda q^2 - 2E_\Lambda (m_\Lambda^2 + m_{\Lambda_b}^2) + E_\Lambda q^2 \right) \bigg) \bigg], \]

\[I_5^{(3)} = -4 |B_V|^2 m_{\Lambda_b} (E_\Lambda - m_\Lambda) + 4 |A_V|^2 m_{\Lambda_b} (E_\Lambda + m_\Lambda). \]

For the case when \( \Lambda_b \) is unpolarized, we get from Eq. (22) that

\[\vec{P}_\Lambda^{(i)} = \alpha^{(i)}_\Lambda \vec{e}_L,\]

with

\[\alpha^{(i)}_\Lambda = \frac{I_5^{(i)}}{I_4^{(i)}},\]

which means that, in this case \( \Lambda \) polarization is purely longitudinal.

For \( \Lambda \) unpolarized, by performing summation over \( \Lambda \) spin in Eq. (22), we get

\[\frac{d\Gamma^{(i)}}{dE_\Lambda} = \left( \frac{d\Gamma^{(i)}_0}{dE_\Lambda} \right) \frac{1}{2} \left[ 1 + \alpha^{(i)}_\Lambda \vec{e}_{\Lambda_b} \cdot \vec{e}_L \right],\]

where

\[\alpha^{(i)}_{\Lambda_b} = \frac{I_5^{(i)}}{I_4^{(i)}}.\]

Note that the normal component \( P_N^{(i)} \) of \( \Lambda \) polarization is a T–odd quantity and its non–zero value indicates CP violation. In the SM and considered version of unparticle physics, there is no CP violating phase (in the SM case \( V_{tb}V_{ts}^* \)), therefore, it cannot induce \( P_N \) in \( \Lambda_b \to \Lambda + missing\ energy \) decay when both baryons are polarized. Obviously, if \( P_N \) is measured in experiments, it clearly is an indication of the fact that there do not exist new CP violating sources.

### 3 Numerical analysis

In this section we calculate the numerical values of the differential branching ratio and polarizations of \( \Lambda_b \to \Lambda \bar{\nu}\nu \) decay in unparticle physics.

The transition form factors \( f_i \) and \( g_i \), as well as \( d_\ell \) and \( \Lambda^d_\ell \), are the main input parameters in the numerical analysis. For calculation of the form factors we use the results obtained from QCD sum rules method in corporation with HQET. As has already been noted, HQET reduces the number of independent form factors to two. Moreover, the \( q^2 \) dependence of \( F_i \) which is obtained in terms of three–parameter fit has the form [11]

\[F_i(q^2) = \frac{F_i(0)}{1 - a_F^i(q^2/m_{\Lambda_b}^2) + b_F^i(q^2/m_{\Lambda_b}^2)^2},\]
Table 1: Form factors for $\Lambda_b \to \Lambda + \text{missing energy}$ decay in a three parameter fit.

|   | $F(0)$ | $a_F$   | $b_F$   |
|---|-------|--------|--------|
| $F_1(0)$ | 0.462 | -0.0182 | -0.000176 |
| $F_2(0)$ | -0.077 | -0.0685 | 0.001460 |

The values of $F_i(0)$, $a_F^i$, and $b_F^i$ are given in Table–1.

It is emphasized in [14] that unparticles behave as a non–integer number of particles and it is shown there that the very peculiar shape of $u$–quark energy distribution in the $t \to dU$ decay and it can serve as a good test in discovering unparticles experimentally. Along the same lines, the energy distribution of $K$ and $K^*$ mesons in the $B \to K(K^*) + \text{missing energy}$ decay is analyzed [33] and it is seen that this decay, especially in the presence of vector unparticle operators, is very distinctive compared to that of the SM prediction. Similar situation can take place for the $\Lambda_b \to \Lambda + \text{missing energy}$ decay. In what follows, we try to answer the intriguing question whether the polarization observables can be useful for the experimental observation of unparticles.

It is shown in [31] that if vector operators couple to the flavor non–diagonal current, $d_U$ should be larger than $d_U > 2$. On the other hand, the bound for the scalar operators turns out to be $d_U > 1$, and these are the bounds we will use in our numerical analysis. The values of other parameters are chosen as $C_P = C_S = 2.0 \times 10^{-3}$ for scalar operators; and $C_V = C_A = 10^{-3}$ for vector operator, and $\Lambda_U = 1 \text{ TeV}$ for both cases.

In Figs. (1) and (2), we present the dependence of the differential decay width as a function of the $\Lambda$ baryon energy $E_\Lambda$ for scalar and vector operators, for various choices of $d_U$, respectively. From these figures we see that the distribution for the final $\Lambda$ baryon energy for both operators are similar. Note that, the behavior of the dependence of the differential decay width on the energy distribution $E_\Lambda$ for these operators and SM case are very similar to each other.

In Figs. (3) and (4) we present the dependence of the branching ratios of the $\Lambda_b \to \Lambda + \text{missing energy}$ decay on $d_U$ at fixed values of the effective coupling constants $C_S$, $C_P$, $C_V$ and $C_A$, at different values of the cut–off scale $\Lambda_U$, for scalar and vector operators. For completeness, in these figures we also present the SM result for the branching ratio of the $\Lambda_b \to \Lambda \bar{\nu} \nu$ decay. From these figures we see that, for $d_U > 2$, the value of the branching ratio is smaller compared to that of the SM case in the presence of the vector operator; while the branching ratio can exceed the SM prediction in the presence of the scalar operator when $d_U < 1.7$, whose behavior is determined by $\Lambda_U$. Therefore, determination of the value of the branching ratio can put stringent restrictions to the values of $d_U$ and $\Lambda_U$.

In Figs. (5) and (6), we present the dependence of the branching ratio of the $\Lambda_b \to \Lambda + \text{missing energy}$ decay on cut–off scale $\Lambda_U$ at fixed values of $C_S$, $C_P$, $C_V$ and $C_A$, respectively. We observe from these figures that, the branching ratios are very sensitive to the values of these effective couplings. It follows from Fig. (5) that, for the scalar operator case and up to $\Lambda_U = 8 \text{ TeV}$, the value of the branching ratio exceeds that of the SM prediction, for all choices of the fixed values of $d_U$, which can give useful information about unparticle physics.
Now let us discuss the numerical values of the Λ and Λₖ baryon polarizations. As we proceed in analyzing the Λₖ polarizations we have assumed that Λ is not polarized, and when we analyze Λ polarizations we have assumed that \((l_2^{(k)}/l_1^{(k)})\vec{e}_L \cdot \vec{ξ}_{Λₖ}\) is small which can be neglected in numerical calculations.

The results we get for \(α_{Λₖ} = I_2/I_1\) can be summarized as follows:

- In the presence of the scalar operator the magnitude of \(α_{Λₖ}\) starts from zero and approaches sharply to the value one; and from \(E_Λ \simeq 1.25 \text{ GeV}\) on, its value continues to be very close to one. It is further observed that in the whole physical region of \(E_Λ\), its value is independent of the values of the parameters \(C_P\) and \(C_S\). Averaged value of \(α_{Λₖ}\) in unparticle physics model is equal to 0.98.

- In the vector operator case the situation is drastically different from that above-mentioned scalar operator case. In other words, in the region \((E_Λ)_{min} \leq E_Λ \leq 1.70 \text{ GeV}\), \(α_{Λₖ}\) gets negative values and at \(E_Λ = 1.70 \text{ GeV}\), \(α_{Λₖ}\) becomes zero. Starting from \(E_Λ = 1.70 \text{ GeV}\) on, \(α_{Λₖ}\) increases with the increasing values of \(E_Λ\). Similar situation occurs for the SM case as well. (see [11]). More essential than that, similar to the scalar operator case, \(α_{Λₖ}\) is insensitive to the values of the parameters \(C_P\) and \(C_A\). Note that in the SM \(\langle α_{Λₖ} \rangle = -0.33\), while this model predicts \(\langle α_{Λₖ} \rangle = 0.86\).

From the analysis of Λ polarizations we get the following results:

- In the presence of scalar and vector operators, longitudinal polarization of Λ exhibits practically the same behavior, namely, up to \(E_Λ = 1.25 \text{ GeV}\), \(P_L\) increases and from that point on it remains constant for all kinematical region, and its value is independent of the parameters \(C_S\), \(C_P\), \(C_V\) and \(C_A\).

- The averaged value of the longitudinal polarization is \(\langle P_L \rangle = 0.98\) in the scalar and \(\langle P_L \rangle = 0.99\) in the vector operator cases, respectively. Note that \(\langle P_L \rangle \approx -0.33\) in the SM case.

- Transversal polarization of Λ decreases with increasing values of \(E_Λ\) and from \(E_Λ = 1.75 \text{ GeV}\) on it approaches to zero in the presence of the scalar operator, being insensitive to the parameters \(C_S\) and \(C_P\). This behavior is very different in the vector operator case. While the sign of \(P_T\) is negative up to \(E_Λ = 1.25 \text{ GeV}\), it starts increasing from this point on, and attains at constant value 10% after \(E_Λ = 1.5 \text{ GeV}\). Similar to the scalar operator case, \(P_T\) is insensitive to the numerical values of \(C_V\) and \(C_A\).

- The averaged value of the transversal polarization is \(\langle P_T \rangle = 7\%\) in the vector and \(\langle P_T \rangle = 4.6\%\) in the scalar operator case, respectively, and \(\langle P_T \rangle = 5.4\%\) in the SM case.

In conclusion, we have studied the possible manifestation of unparticles on the missing energy signatures of the rare Λₖ decays. The branching ratio, Λ and Λₖ baryon polarizations are studied in unparticle physics in the presence of scalar and vector operators. The value of the branching ratio is very sensitive to the choice of the model.
parameter $d_H$. The averaged values of $\Lambda$ and $\Lambda_b$ baryon polarizations significantly differ from the corresponding SM values. Therefore experimental measurements of the branching ratio of the process $\Lambda_b \rightarrow \Lambda + \text{missing energy}$ decay, as well as $\Lambda$ and $\Lambda_b$ baryon polarizations can give useful information on unparticle physics.
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Figure captions

Fig. (1) The dependence of the unpolarized differential decay width on $\Lambda$ baryon energy at $\Lambda_U = 1 \, TeV$, and at fixed values of $d_U$ for the scalar operator.

Fig. (2) The same as in Fig. (1), but for the vector operator.

Fig. (3) The dependence of the branching ratio of $\Lambda_b \to \Lambda + missing \, energy$ decay on $d_U$ at fixed values of $\Lambda_U$ for the scalar operator.

Fig. (4) The same as in Fig. (3), but for the vector operator.

Fig. (5) The dependence of the branching ratio of $\Lambda_b \to \Lambda + missing \, energy$ decay on $\Lambda_U$ at fixed values of $d_U$ for the scalar operator.

Fig. (6) The same as in Fig. (5), but for the vector operator.
Figure 1:

Figure 2:
Figure 3:

Figure 4:
Figure 5:

\[ \Lambda_U \ (TeV) \]

Figure 6:

\[ \Lambda_U \ (TeV) \]

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