Is the dynamics of scaling dark energy detectable?

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Abstract. We highlight the unexpected impact of nucleosynthesis and other early universe constraints on the detectability of scaling quintessence dynamics at late times, showing that such dynamics may well be invisible until the unveiling of the Stage-IV dark energy experiments (DUNE, JDEM, LSST, SKA). Nucleosynthesis strongly limits potential deviations from $\Lambda$CDM. Surprisingly, the standard Chevallier–Polarski–Linder parametrization, $w(z) = w_0 + \frac{w_a}{1 + z}$, cannot match the nucleosynthesis bound for minimally coupled scaling fields. Given that such models are arguably the best-motivated alternatives to a cosmological constant these results may significantly impact future cosmological survey design and imply that dark energy may well be dynamical even if we do not detect any dynamics in the next decade.

Keywords: dark energy theory, big bang nucleosynthesis

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1. Introduction

The quintessential enigma of modern cosmology is that of dark energy—the shadowy source of the accelerated expansion of the cosmos. Einstein’s ‘greatest blunder’—the cosmological constant, \( \Lambda \), remains the most economical explanation for this observed acceleration, but more exciting alternatives could imply the existence of new types of matter, modifications of the Einstein equations or even violation of the Copernican principle.

Of these however, one can argue that only scalar fields are consistent with current constraints while simultaneously being theoretically well founded. The more exotic alternatives either are claimed to have theoretical pathologies or are likely to be indistinguishable from the vanilla \( \Lambda \)CDM (CDM: cold dark matter) model which, given all current data, remains flavour of the month [1]. Models in the former category include K-essence [2], which may exhibit an unbounded adiabatic speed of sound [3], and the DGP model [4], which has some theoretical problems with ghosts (see [5] and references therein) while those in the latter class include \( f(R) \) modifications of gravity [6] and unified dark energy [7]. Of the scalar fields, arguably the best-motivated and most compelling are the scaling quintessence models [8]–[10]. We consider such scaling models with a redshift-dependent equation of state parameter \( w(z) = p/\rho \) that tracks the dominant energy density component of the cosmos \( (w = \frac{1}{3}, 0) \) until a redshift \( z = z_t \) is reached at which point it undergoes a transition to \( w < 0 \) which triggers the onset of acceleration.

Big bang nucleosynthesis (BBN) provides strong constraints on the energy density of dark energy during the radiation dominated era at a temperature of \( T \sim 1 \) MeV, implying that \( \Omega_{\text{DE}}(T \sim 1 \) MeV) < \( \epsilon = 0.045 \) at 2\( \sigma \) [11]. If the scalar field has reached the scaling attractor solution at the time of BBN, this early universe constraint on the energy density of the dark energy will be carried forward until the field exists the scaling regime (at \( z_t \)). The time at which the attractor for the scalar field is approached depends on the details of the physics before the radiation dominated epoch. But in [10] it was shown that, for a typical inflationary model with the usual method of reheating, the field will approach the
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attractor long before nucleosynthesis. Therefore we assume that the field is already scaling with the background fluid at the time of BBN. In fact, our results are unchanged even if scaling only occurs by decoupling, since similar or better constraints on the dark energy density exist from the Cosmic Microwave Background (CMB), \( \Omega_{\text{DE}}(T \sim 1\text{eV}) < 0.04 \) [12]. Constraints on \( w(z) \) from BBN taken together with recent data require that \( z_t > 5 \) [13].

In the case of scaling dark energy models these constraints at early times are preserved by the subsequent evolution of the cosmos, implying that the limit \( \Omega_{\text{DE}}(z_t) < \epsilon = 0.045 \) since \( \Omega_{\text{DE}}(z) \) is constant in perfectly scaling models. It is standard to write \( \Omega_{\text{DE}}(z) = H_0^2 \Omega_{\text{DE}}(z = 0) f(z) \) where \( f(z) \) determines the redshift dependence of the dark energy density. The evolution of \( \Omega_{\text{DE}} \) as a function of redshift is illustrated for various models in figure 1. From the above discussion, plus the scaling requirement that \( w(z_n = z_t) = 0 \), one can show that

\[
f(z_n = z_t) = \frac{\epsilon (1 + z)^3}{r(1 - \epsilon)} = \frac{0.047}{r} (1 + z)^3,
\]

where \( r = \Omega_{\text{DE}}/\Omega_m \) is evaluated today.

2. General results

One can derive model-independent constraints on cosmological observables (we consider the Hubble rate, \( H(z) \), and distance modulus, \( \mu(z) \)) for scaling models. The BBN constraint implies that \( \Omega_{\text{DE}}(z \geq z_t) < \epsilon = 0.045 \) since \( \Omega_{\text{DE}}(z \geq z_t) \) is constant in perfectly scaling models. It is standard to write \( \rho_{\text{DE}}(z) = H_0^2 \Omega_{\text{DE}}(z = 0) f(z) \) where

\[
f(z) = \exp \left[ 3 \int_0^z \frac{1 + w(z')}{1 + z'} \, dz' \right]
\]

determines the redshift dependence of the dark energy density. The evolution of \( \Omega_{\text{DE}} \) as a function of redshift is illustrated for various models in figure 1. From the above discussion, plus the scaling requirement that \( w(z \geq z_t) = 0 \), one can show that

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Figure 1. Evolution of $\Omega_{DE}(z)$ for the models that we consider, showing their approach to the BBN limits of $\epsilon = \Omega_{DE}(z = z_{BBN}) = 0.045$ for the polynomial parametrization of $w(z)$ and $\frac{4}{3}\epsilon$ for the double-exponential potential. For comparison we also show the curves for $\Lambda$ and the CPL [15] $w(z)$ with the lowest asymptotic value of $\Omega_{DE}$ in this model while still assuming $w \geq -1$, showing its inability to match the BBN constraint and describe a canonical scalar field. Figure 2 shows the corresponding observational quantities for the scaling quintessence models.

Using this result implies the following general but stringent constraint:

$$\frac{H_{DE}(z \geq z_t)}{H_{\Lambda}(z \geq z_t)} = \sqrt{\frac{(1 + z_t)^3}{(1 - \epsilon)((1 + z_t)^3 + r)}} \leq \sqrt{\frac{1}{1 - \epsilon}} = 1.023,$$

where the last equality arises from imposing $\epsilon = 0.045$ and the upper bound from setting $r = 0$ as $z_t \to \infty$. This limit can clearly be seen in the top middle panel of figure 2. This robust result implies that detecting deviations from $\Lambda$ at high redshift will be difficult using Hubble rate measurements alone since they are bound to be less than 2.3%. This does not, however, strongly constrain the behaviour of $H_{DE}/H_{\Lambda}$ for $z < z_t$, once the field has left the scaling regime. We will show in two classes of models that its maximum value is less than 5% and occurs around $z \approx 1$. The latter result is good news for Baryon Acoustic Oscillation (BAO) surveys such as WiggleZ, BOSS and WFMOS [16] which will probe this range of redshifts.

Similarly one can also place robust bounds on the deviation of the distance modulus, $\mu(z)$, from the $\Lambda$ prediction. The quantity $\Delta \mu \equiv \mu_{DE}(z) - \mu_{\Lambda}(z)$ is given by

$$\Delta \mu = 5 \log_{10}\left(\frac{d_L_{DE}(z)}{d_L_{\Lambda}(z)}\right),$$

where $d_L(z)$ is the corresponding model luminosity distance. If we assume that there exists a number $\alpha$ such that $H(z)/H_{\Lambda}(z) \leq 1 + \alpha^2$ for all $z$ (for example in the constraint
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Figure 2. BBN compatible scaling dark energy models: allowed observables for the polynomial $w(z)$ for $z_t$ from 4.02 (light brown) to 10.02 (dark brown; top panel) and double-exponential potential with $-30 \leq \mu \leq 1$ (bottom) showing the small deviations from the $\Lambda$ predictions. The bottom light brown curves are from $\mu = 1$ (for which $w(0) = -0.8$) to $\mu = 0$, $w(0) = -1$ and the dark brown curves are for $\mu = -30, -25, -20, -15, -10, -5, -4, -3, -2, -1, -0.8, -0.6, -0.4, -0.2$. All models have $|d w / d z(0)| < 0.2$. In the model with $w(0) < -0.9$, $H_{DE}(z)$ deviates from $H_{\Lambda}(z)$ by at most 2.7% (marked by the horizontal line in the bottom middle panel) and implies a deviation in distance modulus of less than 0.05 mag. The error bars in the right panels correspond to the Stage-III (large boxed errors) and Stage-IV (small triangular errors) supernova surveys respectively. These are produced for the bottom curve in each case except for the Stage-IV (SNAP-like) errors in the double-exponential case which correspond to the $w(0) = -0.9$ model (thicker line asymptoting to $\sim 0.04$ mag). Note that the ratio $H(z)/H_{\Lambda}$ for the double-exponential potential does not converge to 1.023 since the matter dominated value of $\Omega_{DE}(z)$ is constrained to be $3/4$ of the radiation dominated value. Hence the Hubble rate is forced to $\sim 1 + (3/8) \epsilon \sim 1.017$.

A bound of $\alpha^2 = 0.025$ gives $|\Delta \mu(z)| \leq 0.054$ mag—see the top row of figure 2. This is a conservative upper bound since we have shown that for $z \geq z_t$, $\alpha^2 \leq \epsilon/2$. For the oscillating double-exponential $w(z)$ models (with $-30 \leq \mu < 0$) considered later, one has $\alpha^2 < 0.015$ which yields the constraint $|\Delta \mu(z)| \leq 0.032$ mag. These general results do
not constrain $H(z)$ for $z < z_t$, which instead requires a specific model for $w(z)$. We now consider two classes of models which describe a wide range of scalar field dynamics.

3. Specific forms of the scaling potential

3.1. Polynomial $w(z)$ parametrization

First we consider a quadratic parametrization of the dark energy equation of state, $w(z)$ [17]:

$$w(z) = \begin{cases} w_0 + w_1 z + w_2 z^2 & \text{for } z < z_t, \\ 0 & \text{for } z \geq z_t. \end{cases}$$  \hspace{1cm} (6)

We apply the constraint $w(z) \geq -1$ since we want to describe minimally coupled scalar fields with canonical kinetic terms. The linear case with $w_2 = 0$ requires $z_t \approx 6.2$ to match BBN (for $\epsilon = 0.045$) if $w_0 = -1$ and if we allow $w_0$ to be free the BBN constraint implies the correlation $w_1 \approx -0.4w_0 - 0.2$ for the interesting region $-1 < w_0 < -0.8$. The other case that we consider is $w_0 = -1, w_2 \neq 0$. Continuity at $z = z_t$ then implies

$$w_2 = \frac{1}{z_t^2} - \frac{w_1}{z_t}. \hspace{1cm} (7)$$

The BBN constraint provides $w_1$ in terms of $z_t$. The resulting family of curves and observables are shown in the top row of figure 2. We note that for $z \leq 1$ the BBN constraint is so strong that all models have $w(z) < -0.8$, as shown in figure 2, and the largest deviation of $\dot{H}(z)$ from the $\Lambda$ comparison model is about 2.7% with the largest deviation in the distance modulus only about 0.03 magnitudes (occurring at $z = 2$). This shows that if $w \approx -1$ today we cannot expect significant deviations from $\Lambda$ at any redshift and only Stage-IV experiments [14] are likely to detect dark energy dynamics with any real significance. However while current data favour $w(0) \approx -1$ they are consistent with larger values. To study how this zero point affects our results we now consider simulations of a scalar field with a popular family of scalar field potentials, $V(\phi)$, which also allows us to examine the impact of oscillations in $w(z)$.

3.2. Double-exponential potential

A single-exponential potential is well known to give early scaling [9] but cannot also lead to late-time acceleration. One well-studied way to combine the two is via the double-exponential potential [18]:

$$V(\phi) = M_1^4 e^{-\lambda \phi} + M_2^4 e^{-\mu \phi}, \hspace{1cm} (8)$$

where late-time acceleration is induced if $\mu \leq \sqrt{2}$. The dynamics of $\phi$ is sensitive to the sign of $\mu$, in that negative values of $\mu$ yield a potential that oscillates around the minimum of the field. This can clearly be seen in figure 2, for the potentials with $-30 \leq \mu \leq 0$. We assume that the scalar field is in the scaling regime during radiation domination, and hence satisfying the BBN constraint requires $\lambda \geq 2/\sqrt{\epsilon} \geq 9.43$ [9]. We choose $\lambda = 9.43$ to maximize deviations from $\Lambda$. Unlike in a perfectly scaling model of the form that we assumed in the previous section, $\Omega_\phi$ actually decreases in the transition to matter domination and we have $\Omega_\phi < 3/4 \times \epsilon = 0.034$ during matter domination, leading to
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The extra $3/4$ factor is specific to the double-exponential potential and is responsible for the reduction of the asymptotic values of $H_{\text{DE}}(z)/H_{\Lambda\text{CDM}}$, seen in the bottom middle panel of figure 2, relative to the predictions of equation (3). We choose $M_1 = 10^{-14} m_{\text{pl}}$ and use Planck units where $\kappa = 1$.

We numerically solve the evolution equation

$$\ddot{\phi} + 3H\dot{\phi} + V_{,\phi} = 0$$

for $\phi$ with $V(\phi)$ given by equation (8), together with the radiation and matter fluids. These are all coupled to gravity through the Friedmann equation

$$H^2 = \frac{\kappa^2}{3} \left[ \frac{1}{2} \dot{\phi}^2 + V(\phi) + \rho_r + \rho_m \right].$$

For each value of $\mu$ we find $M_2$ such that $\Omega_{\text{DE}} = 0.7$ today which implies $M_2 \sim 10^{-31} m_{\text{pl}}$ for $\mu \sim 1$. The resulting $w(z)$ curves for these models, together with the predicted observables $(H(z), \Delta \mu(z))$, are shown in the bottom row of figure 2. For negative $\mu$—we study $-30 \leq \mu < 0$—the potential has a global minimum and for $z \lesssim 0.2$ the equation of state satisfies $w_{,\phi} \leq -0.98$. As a result all the negative $\mu$ models show tiny deviations from $\Lambda$: less than 1.5% for $H(z)$ and less than 0.015 mag for $\Delta \mu$ (see figure 2). This will make detection extremely difficult even with the Stage-IV dark energy experiments such as DUNE, JDEM, LSST and SKA [14].

In contrast, positive values of $\mu$ ($0 \leq \mu \leq 1$) simply modify the slope of the potential. They can yield values of $w_{,\phi}$ significantly different from $-1$ today; e.g. for $\mu \sim 1$ one finds $w_{,\phi}(0) \sim -0.8$ which is consistent (at about the $2\sigma$ level [1,13]) with current observations and which we therefore take as the upper bound for $\mu$. The bottom row of figure 2 shows that the maximum allowed deviation for $H(z)$ from $H_{\Lambda}(z)$ in this case is about 5%, peaking at $z \sim 1$ with a maximum value of $\Delta \mu \sim 0.9$ mag. Such a model will be detectable with Stage-III supernova surveys (at the 99.97% confidence level) and with the upcoming BAO experiments, since the maximum deviation in $H(z)$ coincides with the redshift ranges in which they will operate, i.e. $z \sim 0.7$–1.1. However, for values more consistent with the current best fits, $w_{,\phi}(0) < -0.9$, one finds much smaller deviations of 2.7% and 0.045 mag respectively for $H(z)$ and $\Delta \mu$ which again will require Stage-IV experiments for conclusive detection as can be seen in the right-hand panels of figure 2. Similar results will apply to other modifications of the exponential potential; e.g. [19].

4. Performance of standard parametrizations

The most widely used parametrization for dark energy, the Chevallier–Polarski–Linder (CPL) parametrization [15],

$$w(z) = w_0 + w_a \frac{z}{1 + z},$$

is also the basis for the DETF figure of merit [14]. Surprisingly this parametrization fails dramatically to meet the BBN constraint if we demand $w(z) \geq -1$, $\Omega_{\text{DE}} \sim 0.7$ today and $w(z \geq z_t) = 0$ as before (for some $z_t$). This is clearly visible in the top curve of figure 1, which shows the lowest attainable value of $\Omega_{\text{DE}}$ with $w_0 = -1$. In retrospect this is understandable since to reach the scaling value $w = 0$ for some $z_t$.
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requires $w_\phi > -w_0$. However in this case $w(z)$ does not spend enough time at sufficiently negative values to force $\Omega_{DE}(z)$ down to the BBN value. The least phantom value of $w_0$ that is able to satisfy the BBN constraint is $w_0 = -1.3$. In contrast the logarithmic expansion $w(z) = w_0 + w_1 \ln(1 + z)$ is able to match the BBN constraint with $w(z) \geq -1$, but only for $z_t > 12.4$.

5. Conclusions

Scaling field models are arguably the best-motivated alternatives to the cosmological constant. We have shown that the constraints on the energy density of the scalar field at the time of big bang nucleosynthesis and decoupling strongly limit their allowed dynamics. If $w$ today is not close to the maximum value allowed by current data then detection of dynamics will likely have to wait a decade for the Stage-IV DETF experiments. Of course, these strong conclusions are only true for scaling models and if one allows exotic phantom behaviour ($w < -1$) the conclusion is much more rosy. We discussed two specific families of these scaling models—imposing the BBN and CMB constraints on general scaling models is left for future work.

One might ask how the constraints discussed here will affect growth of structure. Since the growth in minimally coupled scalar fields smoothly approaches that of $\Lambda$CDM when $w(z) \to -1$ (see e.g. figure 2 in [20]) our results show that the growth in models allowed by BBN and CMB constraints will be close to those in $\Lambda$CDM at $z < 1$, making detection difficult. To what extent there are deviations at higher redshift, and whether they will be detectable by future cluster or weak lensing surveys, is again left as a subject for future study.

Finally we have shown that the standard CPL parametrization, $w(z) = w_0 + w_a z/(1 + z)$, fails dramatically to match the BBN constraint when describing scaling fields which satisfy $w \geq -1$. This is particularly important given that the CPL parametrization is the basis of the DETF figure of merit [14] which is now the de facto standard for the optimization of future cosmological surveys; e.g. [21]. A concern therefore is that optimizations may be unwittingly biased away from scaling dark energy models. More work in this area is clearly needed to assess the implications for cosmological survey design, but it is clear that the current non-detection of dark energy dynamics should neither come as a surprise, nor discourage us from the hunt.

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