Some remarks regarding finite bounded commutative BCK-algebras

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Abstract. In this chapter, starting from some results obtained in the papers [FV; 19], [FHSV; 19], we provide some examples of finite bounded commutative BCK-algebras, using the Wajsberg algebra associated to a bounded commutative BCK-algebra. This method is an alternative to the Iseki’s construction, since by Iseki’s extension some properties of the obtained algebras are lost.

Keywords: Bounded commutative BCK-algebras, MV-algebras, Wajsberg algebras.

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1. Introduction

BCK-algebras were first introduced in mathematics in 1966 by Y. Imai and K. Iseki, through the paper [II; 66]. These algebras were presented as a generalization of the concept of set-theoretic difference and propositional calculi. The class of BCK-algebras is a proper subclass of the class of BCI-algebras.

Definition 1.1. An algebra $(X, *, \theta)$ of type $(2, 0)$ is called a BCI-algebra if the following conditions are fulfilled:

1) $(x * y) * (x * z) = (z * y) * (z * y) = \theta$, for all $x, y, z \in X$;
2) $(x * (x * y)) * y = \theta$, for all $x, y \in X$;
3) $x * x = \theta$, for all $x \in X$;
4) For all $x, y, z \in X$ such that $x * y = \theta, y * x = \theta$, it results $x = y$.

If a BCI-algebra $X$ satisfies the following identity:

5) $\theta * x = \theta$, for all $x \in X$, then $X$ is called a BCK-algebra.

Definition 1.2. i) A BCK-algebra $(X, *, \theta)$ is called commutative if

$x * (x * y) = y * (y * x)$,

for all $x, y \in X$ and implicative if

$x * (y * x) = x$,
for all \( x, y \in X \).

ii) ([Du; 99]) A BCK-algebra \( (X, \ast, \theta) \) is called positive implicative if and only if

\[
(x \ast y) \ast z = (x \ast z) \ast (y \ast z),
\]

for all \( x, y, z \in X \).

The partial order relation on a BCK-algebra is defined such that \( x \preceq y \) if and only if \( x \ast y = \theta \).

If in the BCK-algebra \( (X, \ast, \theta) \) there is an element 1 such that \( x \preceq 1 \), for all \( x \in X \), then the algebra \( X \) is called a bounded BCK-algebra. In a bounded BCK-algebra, we denote \( 1 \ast x = \overline{x} \).

If in the bounded BCK-algebra \( X \), an element \( x \in X \) satisfies the relation \( x = x \ast x \), then the element \( x \) is called an involution.

If \( (X, \ast, \theta) \) and \( (Y, \circ, \theta) \) are two BCK-algebras, a map \( f: X \to Y \) with the property \( f(x \ast y) = f(x) \circ f(y) \), for all \( x, y \in X \), is called a BCK-algebras morphism. If \( f \) is a bijective map, then \( f \) is an isomorphism of BCK-algebras.

**Definition 1.3.** 1) Let \( (X, \ast, \theta) \) be a BCK algebra and \( Y \) be a nonempty subset of \( X \). Therefore, \( Y \) is called a subalgebra of the algebra \( (X, \ast, \theta) \) if and only if for each \( x, y \in Y \), we have \( x \ast y \in Y \). This implies that \( Y \) is closed to the binary multiplication "\( \ast \)".

It is well known that each BCK-algebra of degree \( n + 1 \) contains a subalgebra of degree \( n \).

2) Let \( (X, \ast, \theta) \) be a BCK algebra and \( I \) be a nonempty subset of \( X \). Therefore, \( I \) is called an ideal of the algebra \( X \) if and only if for each \( x, y \in X \) we have:

i) \( \theta \in I \);

ii) \( x \ast y \in I \) and \( y \in I \), then \( x \in I \).

**Proposition 1.4.** ([Me-Ju; 94]) Let \( (X, \ast, \theta) \) be a BCK algebra and \( Y \) be a subalgebra of the algebra \( (X, \ast, \theta) \). The following statements are true:

i) \( \theta \in Y \);

ii) \( (Y, \ast, \theta) \) is also a BCK-algebra. □

Let \( X \) be a BCK-algebra, such that \( 1 \notin X \). On the set \( Y = X \cup \{1\} \), we define the following multiplication "\( \circ \)" as follows:

\[
x \circ y = \begin{cases} 
x \ast y, & \text{if } x, y \in X; 
\theta, & \text{if } x \in X, y = 1; 
1, & \text{if } x = 1 \text{ and } y \in X; 
\theta, & \text{if } x = y = 1.
\end{cases}
\]

The obtained algebra \( (Y, \circ, \theta) \) is a bounded BCK-algebra and is obtained by the so-called Iseki’s extension. The algebra \( (Y, \circ, \theta) \) is called the algebra obtained from algebra \( (X, \ast, \theta) \) by Iseki’s extension ([Me-Ju; 94], Theorem 3.6).
Remark 1.5. ([Me-Ju; 94])

i) The Iseki’s extension of a positive implicative BCK-algebra is still a positive implicative BCK-algebra.

ii) The Iseki’s extension of a commutative BCK-algebra, in general, is not a commutative BCK-algebra.

iii) Let $X$ be a BCK-algebra and $Y$ its Iseki’s extension. Therefore $X$ is an ideal in $Y$.

iv) If $I$ is an ideal of the BCK-algebra $X$, $x \in I$ and $y \leq x$, then $y \in I$.

In the following, we will give some examples of finite bounded commutative BCK-algebras. In the finite case, it is very useful to have many examples of such algebras. But, such examples, in general, are not so easy to found. A method for this purpose can be Iseki’s extension. But, from the above, we remark that the Iseki’s extension can’t be always used to obtain examples of finite commutative bounded BCK-algebras with given initial properties, since the commutativity, or other properties, can be lost. From this reason, we use other technique to provide examples of such algebras. We use the connections between finite commutative bounded BCK-algebras and Wajsberg algebras and the algorithm and examples given in the papers [FHSV; 19] and [FV; 19].

2. Connections between finite bounded commutative BCK-algebras and Wajsberg algebras

Definition 2.1. ([CHA; 58]) An abelian monoid $(X, \theta, \oplus)$ is called $MV$-algebra if and only if we have an unary operation $"'$ such that:

i) $(x')' = x$;

ii) $x \oplus \theta' = \theta'$;

iii) $(x' \oplus y)' \oplus y = (y' \oplus x)' \oplus x$, for all $x, y \in X$. ([Mu; 07]). We denote it by $(X, \oplus', \theta)$.

We remark that in an MV-algebra the constant element $\theta'$ is denoted with $1$. This is equivalent with

$$1 = \theta'.$$

With the above definitions, the following multiplications are also defined:

$$x \odot y = (x' \oplus y')',$$

$$x \odot y = x \odot y' = (x' \oplus y)'.'$$

(see ([Mu; 07]))

From [COM; 00], Theorem 1.7.1, for a bounded commutative BCK-algebra $(X, *, \theta, 1)$, if we define

$$x' = 1 \ast x,$$
we obtain that the algebra \((X, \oplus', \theta)\) is an MV-algebra, with
\[
x \oplus y = x \cdot y.
\]

The converse is also true, that means if \((X, \oplus, \theta, 1)\) is a bounded commutative BCK-algebra.

**Definition 2.2.** ([COM; 00], Definition 4.2.1) An algebra \((W, \circ, \neg, 1)\) of type \((2, 1, 0)\) is called a Wajsberg algebra (or W-algebra) if and only if the following conditions are fulfilled:
\begin{enumerate}
  \item[i)] \(1 \circ x = x\);
  \item[ii)] \((x \circ y) \circ [(y \circ z) \circ (x \circ z)] = 1\);
  \item[iii)] \((x \circ y) \circ y = (y \circ x) \circ x\);
  \item[iv)] \((\overline{x} \circ \overline{y}) \circ (y \circ x) = 1\), for every \(x, y, z \in W\).
\end{enumerate}

**Remark 2.3.** ([COM; 00], Lemma 4.2.2 and Theorem 4.2.5)
\begin{enumerate}
  \item[i)] For the Wajsberg algebra \((W, \circ, \neg, 1)\), if we define the following multiplications
    \[
    x \odot y = (x \circ y)
    \]
    and
    \[
    x \oplus y = \overline{x} \circ y,
    \]
    for all \(x, y \in W\), we obtain that \((W, \oplus, \circ, \neg, \overline{1})\) is an MV-algebra.
  \item[ii)] Conversely, if \((X, \oplus', \circ, 1)\) is an MV-algebra, defining on \(X\) the operation
    \[
    x \circ y = x' \oplus y,
    \]
    we obtain that \((X, \circ', 1)\) is a Wajsberg algebra.
\end{enumerate}

**Remark 2.4.** From the above, if \((W, \circ, \neg, 1)\) is a Wajsberg algebra, then
\[
(W, \oplus, \circ, \neg, 0, 1)
\]

\text{is an MV-algebra, with}
\[
x \odot y = \overline{x \oplus y} = \overline{x \circ y}.
\]
\text{Defining}

\[
x \cdot y = \overline{x \circ y},
\]
we have that \((W, \cdot, \circ, \theta, 1)\) is a bounded commutative BCK-algebra.

Using the above remark, starting from some known finite examples of Wajsberg algebras given in the papers [FHSV; 19] and [FV; 19], we can obtain examples of finite commutative bounded BCK-algebras, using the following algorithm.

**The Algorithm**
\begin{enumerate}
  \item[1)] Let \(n\) be a natural number, \(n \neq 0\) and
  \[
n = r_1r_2...r_t, r_i \in \mathbb{N}, 1 < r_i < n, i \in \{1, 2, ..., t\},
  \]
  \[
  \]
be the decomposition of the number \( n \) in factors. The decompositions with the same terms, but with other order of them in the product, will be counted one time. The number of all such decompositions will be denoted with \( \pi_n \).

2) There are only \( \pi_n \) nonisomorphic, as ordered sets, Wajsberg algebras with \( n \) elements. We obtain these algebras as a finite product of totally ordered Wajsberg algebras (see [BV; 10] and [FV; 19], Theorem 4.8).

3) Using Remark 2.4 from above, a commutative bounded BCK-algebra can be associated to each Wajsberg algebra.

## 3. Examples of finite commutative bounded BCK-algebras

In the following, we use some examples of Wajsberg algebras given in the paper [FHSV; 19]. To these algebras, we will associate the corresponding commutative bounded BCK-algebras and we give their subalgebras and ideals.

**Example 3.1.** Let \( W = \{O \leq A \leq B \leq E\} \) be a totally ordered set. On \( W \) we define the multiplication \( \circ_1 \) as in the below table, such that \((W, \circ_1, E)\) is a Wajsberg algebra. We have \( A = B \) and \( B = A \).

| \( \circ_1 \) | \( O \) | \( A \) | \( B \) | \( E \) |
|---|---|---|---|---|
| \( O \) | \( E \) | \( E \) | \( E \) | \( E \) |
| \( A \) | \( B \) | \( E \) | \( E \) | \( E \) |
| \( B \) | \( A \) | \( B \) | \( E \) | \( E \) |
| \( E \) | \( O \) | \( A \) | \( B \) | \( E \) |

(see [FHSV; 19], Example 4.1.1)

Therefore, the associated commutative bounded BCK-algebras \((W, \ast_1, O)\) has multiplication given in the below table:

| \( \ast_1 \) | \( O \) | \( A \) | \( B \) | \( E \) |
|---|---|---|---|---|
| \( O \) | \( O \) | \( O \) | \( O \) | \( O \) |
| \( A \) | \( A \) | \( O \) | \( O \) | \( O \) |
| \( B \) | \( B \) | \( A \) | \( O \) | \( O \) |
| \( E \) | \( E \) | \( B \) | \( A \) | \( O \) |

The proper subalgebras of this algebra are: \( \{O, A\} \), \( \{O, B\} \), \( \{O, E\} \), \( \{O, A, B\} \). There are no proper ideals in the algebra \((W, \ast_1, O)\).

**Example 3.2.** Let \( W = \{O \leq A \leq B \leq E\} \) be a totally ordered set. On \( W \) we define the multiplication \( \circ_2 \) as in the below table, such that \((W, \circ_2, E)\) is a Wajsberg algebra.

| \( \circ_2 \) | \( O \) | \( A \) | \( B \) | \( E \) |
|---|---|---|---|---|
| \( O \) | \( E \) | \( E \) | \( E \) | \( E \) |
| \( A \) | \( B \) | \( E \) | \( E \) | \( E \) |
| \( B \) | \( A \) | \( A \) | \( E \) | \( E \) |
| \( E \) | \( O \) | \( A \) | \( B \) | \( E \) |
Therefore, the associated commutative bounded BCK-algebras \((W, *, O)\) has multiplication given below:

\[
\begin{array}{c|cccc}
* & O & A & B & E \\
\hline
O & O & O & O & O \\
A & A & O & A & O \\
B & B & B & O & O \\
E & E & B & A & O \\
\end{array}
\]

The proper subalgebras of this algebra are: \{O, A\}, \{O, B\}, \{O, E\}, \{O, A, B\}.  
The proper ideals are: \{O, A\}, \{O, B\}.

**Example 3.3.** Let \(W = \{O \leq A \leq B \leq C \leq D \leq E\}\) be a totally ordered set.  
On \(W\) we define a multiplication \(\circ_3\) given in the below table, such that \((W, \circ_3, E)\) is a Wajsberg algebra.  
We have \(\overline{A} = D, \overline{B} = C, \overline{C} = B, \overline{D} = A\).

\[
\begin{array}{c|cccccc}
\circ_3 & O & A & B & C & D & E \\
\hline
O & E & E & E & E & E & E \\
A & D & E & E & E & E & E \\
B & C & D & E & E & E & E \\
C & B & C & D & E & E & E \\
D & A & B & C & D & E & E \\
E & O & A & B & C & D & E \\
\end{array}
\]

Therefore, the associated commutative bounded BCK-algebras \((W, *, O)\) has multiplication defined below:

\[
\begin{array}{c|cccccc}
* & O & A & B & C & D & E \\
\hline
O & O & O & O & O & O & O \\
A & A & O & O & O & O & O \\
B & B & A & O & O & O & O \\
C & C & B & A & O & O & O \\
D & D & C & B & A & O & O \\
E & E & D & C & B & A & O \\
\end{array}
\]

The proper subalgebras of this algebra are: \{O, A\}, \{O, B\}, \{O, C\}, \{O, D\}, \{O, E\}, \{O, A, B\}, \{O, B, D\}, \{O, A, B, C\}, \{O, A, B, C, D\}.  
This algebra has no proper ideals.

**Example 3.4.** Let \(W = \{O \leq A \leq B \leq C \leq D \leq E\}\) be a totally ordered set.  
On \(W\) we define a multiplication \(\circ_4\) given in the below table, such that \((W, \circ_4, E)\) is a Wajsberg algebra.  
We have

\[
\begin{array}{c|cccccc}
\circ_4 & O & A & B & C & D & E \\
\hline
O & E & E & E & E & E & E \\
A & D & E & D & E & E & E \\
B & C & D & C & D & E & E \\
C & B & B & B & E & E & E \\
D & A & B & D & E & E & E \\
E & O & A & B & C & D & E \\
\end{array}
\]
Therefore, the associated commutative bounded BCK-algebras \((W, \ast_4, O)\) has multiplication given in the following table:

\[
\begin{array}{c|cccccc}
\ast_4 & O & A & B & C & D & E \\
\hline
O & O & O & O & O & O & O \\
A & A & O & O & A & O & O \\
B & B & A & O & B & A & O \\
C & C & C & C & O & O & O \\
D & D & C & C & A & O & O \\
E & E & D & C & B & A & O \\
\end{array}
\]

The proper subalgebras of this algebra are: \(\{O, A\}, \{O, B\}, \{O, C\}, \{O, D\}, \{O, E\}, \{O, A, B\}, \{O, A, B, C\}, \{O, A, B, C, D\}, \{O, A, C\}, \{O, A, C, D\}\).

The proper ideals of this algebra are: \(\{O, A, B\}, \{O, C\}\).

**Example 3.5.** Let \(W = \{O \leq A \leq B \leq C \leq D \leq E\}\) be a totally ordered set. On \(W\) we define a multiplication \(\circ_5\) given in the below table, such that \((W, \circ_5, E)\) is a Wajsberg algebra. We have

\[
\begin{array}{c|cccccc}
\circ_5 & O & A & B & C & D & E \\
\hline
O & E & E & E & E & E & E \\
A & C & E & A & D & D & E \\
B & D & E & E & D & D & E \\
C & A & E & A & E & E & E \\
D & B & A & B & A & E & E \\
E & O & A & B & C & D & E \\
\end{array}
\]

Therefore, the associated commutative bounded BCK-algebras \((W, \ast_5, O)\) has multiplication defined in the following table:

\[
\begin{array}{c|cccccc}
\ast_5 & O & A & B & C & D & E \\
\hline
O & O & O & O & O & O & O \\
A & A & O & C & B & B & O \\
B & B & O & O & B & B & O \\
C & C & O & C & O & O & O \\
D & D & C & D & C & O & O \\
E & E & C & D & A & B & O \\
\end{array}
\]

The proper subalgebras of this algebra are: \(\{O, A\}, \{O, B\}, \{O, C\}, \{O, D\}, \{O, E\}, \{O, B, C\}, \{O, C, D\}, \{O, A, B, C\}, \{O, A, B, C, D\}\).

All proper ideals are: \(\{O, C, D\}, \{O, B\}\).

**Example 3.6.** Let \(W = \{O \leq X \leq Y \leq Z \leq T \leq U \leq V \leq E\}\) be a totally ordered set. On \(W\) we define a multiplication \(\circ_6\) which can be found in the below table. We obtain that \((W, \circ_6, E)\) is a Wajsberg algebra. We have \(\overline{X} = V, \overline{Y} = U, \overline{Z} = T, \overline{T} = U, \overline{U} = V, \overline{V} = E\).
\( \overline{Z} = T \). Therefore the algebra \( W \) has the following multiplication table:

| \( O \) | \( X \) | \( Y \) | \( Z \) | \( T \) | \( U \) | \( V \) | \( E \) |
|-----|-----|-----|-----|-----|-----|-----|-----|
| \( O \) | \( E \) | \( E \) | \( E \) | \( E \) | \( E \) | \( E \) | \( E \) |
| \( X \) | \( V \) | \( E \) | \( E \) | \( E \) | \( E \) | \( E \) | \( E \) |
| \( Y \) | \( U \) | \( V \) | \( E \) | \( E \) | \( E \) | \( E \) | \( E \) |
| \( Z \) | \( T \) | \( U \) | \( V \) | \( E \) | \( E \) | \( E \) | \( E \) |
| \( T \) | \( Z \) | \( T \) | \( U \) | \( V \) | \( E \) | \( E \) | \( E \) |
| \( U \) | \( Y \) | \( Z \) | \( T \) | \( U \) | \( V \) | \( E \) | \( E \) |
| \( V \) | \( X \) | \( Y \) | \( Z \) | \( T \) | \( U \) | \( V \) | \( E \) |
| \( E \) | \( O \) | \( X \) | \( Y \) | \( Z \) | \( T \) | \( U \) | \( V \) |

From here, we get that the associated commutative bounded BCK-algebras \( (W, \ast_6, O) \) has multiplication given in the below table:

| \( O \) | \( X \) | \( Y \) | \( Z \) | \( T \) | \( U \) | \( V \) | \( E \) |
|-----|-----|-----|-----|-----|-----|-----|-----|
| \( O \) | \( O \) | \( O \) | \( O \) | \( O \) | \( O \) | \( O \) | \( O \) |
| \( X \) | \( X \) | \( O \) | \( O \) | \( O \) | \( O \) | \( O \) | \( O \) |
| \( Y \) | \( Y \) | \( X \) | \( O \) | \( O \) | \( O \) | \( O \) | \( O \) |
| \( Z \) | \( Z \) | \( Y \) | \( X \) | \( O \) | \( O \) | \( O \) | \( O \) |
| \( T \) | \( T \) | \( Z \) | \( Y \) | \( X \) | \( O \) | \( O \) | \( O \) |
| \( U \) | \( U \) | \( T \) | \( Z \) | \( Y \) | \( X \) | \( O \) | \( O \) |
| \( V \) | \( V \) | \( U \) | \( T \) | \( Z \) | \( Y \) | \( X \) | \( O \) |
| \( E \) | \( E \) | \( V \) | \( U \) | \( T \) | \( Z \) | \( T \) | \( X \) |

The proper subalgebras of this algebra are: \( \{O, J \}, J \in \{X, Y, Z, T, U, V, E\} \), \( \{O, X, Y\}, \{O, X, Y, Z\}, \{O, X, Y, Z, T\}, \{O, X, Y, Z, T, U\}, \{O, X, Y, Z, T, U, V\} \). There are no proper ideals.

**Example 3.7.** Let \( W = \{O \leq X \leq Y \leq Z \leq T \leq U \leq V \leq E\} \) be a totally ordered set. On \( W \) we define a multiplication \( \ast_7 \) given in the below table, such that \((W, \ast_7, E)\) is a Wajsberg algebra.

| \( O \) | \( X \) | \( Y \) | \( Z \) | \( T \) | \( U \) | \( V \) | \( E \) |
|-----|-----|-----|-----|-----|-----|-----|-----|
| \( O \) | \( E \) | \( E \) | \( E \) | \( E \) | \( E \) | \( E \) | \( E \) |
| \( X \) | \( V \) | \( E \) | \( V \) | \( E \) | \( V \) | \( E \) | \( V \) |
| \( Y \) | \( U \) | \( U \) | \( E \) | \( E \) | \( E \) | \( E \) | \( E \) |
| \( Z \) | \( T \) | \( U \) | \( V \) | \( E \) | \( V \) | \( E \) | \( V \) |
| \( T \) | \( Z \) | \( Z \) | \( U \) | \( U \) | \( E \) | \( E \) | \( E \) |
| \( U \) | \( Y \) | \( Z \) | \( T \) | \( U \) | \( T \) | \( E \) | \( V \) |
| \( V \) | \( X \) | \( X \) | \( Z \) | \( Z \) | \( U \) | \( U \) | \( E \) |
| \( E \) | \( O \) | \( X \) | \( Y \) | \( Z \) | \( T \) | \( U \) | \( V \) |

Therefore, the associated commutative bounded BCK-algebras \((W, \ast_7, O)\) has
multiplication defined in the below table:

| 7 | O | X | Y | Z | T | U | V | E |
|---|---|---|---|---|---|---|---|---|
| O | O | O | O | O | O | O | O | O |
| X | X | O | X | O | X | O | X | O |
| Y | Y | Y | O | O | O | O | O | O |
| Z | Z | Y | X | O | X | O | X | O |
| T | T | T | Y | Y | O | O | O | O |
| U | U | T | Z | Y | Z | O | X | O |
| V | V | V | T | T | Y | Y | O | O |
| E | E | V | U | T | Z | Y | X | O |

The proper subalgebras of this algebra are: \(\{O, J\}, J \in \{X, Y, Z, T, U, V, E\}\), \(\{O, X, Y\}\), \(\{O, X, Y, Z\}\), \(\{O, T, Y\}\), \(\{O, T, Y, V\}\), \(\{O, X, Y, Z, T\}\), \(\{O, X, Y, Z, T, U\}\), \(\{O, X, Y, Z, T, U, V\}\).

All proper ideals are: \(\{O, Y, T, V\}\), \(\{O, X\}\).

**Conclusions.** In this chapter, we provided an algorithm for finding examples of finite commutative bounded BCK-algebras, using their connections with Wajsberg algebras. This algorithm allows us to find such examples no matter the order of the algebra. This thing is very useful, since examples of such algebras are very rarely encountered in the specialty books.

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