About the equilibrium of airship in the parking lot

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Abstract. The paper discusses the questions of constructing a mathematical model of the equilibrium of an airship in a parking lot with a two-cable parking scheme and also the problem of calculating forces acting on the elements of the cable system that holds the airship.

1. Introduction

Nowadays, many countries are working on the design of airships, analysis of their flight characteristics and research of their behavior in various parts of the trajectory [1–27]. One of the most important problems in this case is the problem of modeling the behavior of the airship in the parking lot, especially in the case of parking using a cable system, since the reliable equilibrium of the airship in the parking lot determines the safety of its operation as a whole. One may say, that the cable parking of the airship in the parking lot might be the most stressful and dangerous in all the airship flight operation.

In this regard, it is necessary to consider the construction of a mathematical model of the equilibrium of the airship in the parking lot first. This question is also important because in the airship equilibrium position it is possible to measure the aerodynamic forces and moments acting on it, just as it is done when conducting experiments in a wind tunnel. For this purpose, it is enough to provide each stretch with a dynamometer that can record the tension force of the cable, and then calculate the total force acting on the airship in one direction or another. In this case it is important that the experiment is carried out not on reduced models, as it is done in a wind tunnel, but on a real airship, and therefore it is not necessary to recalculate the aerodynamic characteristics from the model to nature. The results obtained in this paper are sufficient to solve this problem. In addition, they can be used to estimate the required strength of cables.

2. Basic assumption

Let's assume that the airship, while parked, is held in a stationary position by cables attached to some devices located on the ground (winches, etc.). The dynamic diagram of this position for a two-cable scheme is shown in Figure 1.

In Figure 1, $A, B, a, b$ — the mounting point of the cables on ground and on airship body, respectively; $l_A, l_a$ — cable lengths; $N_A, N_a$ — the reaction forces of the bonds; $\vec{F}$ — aerostatic force and the weight force; $O$ — center of volume; $O_1$ — center of mass; $O_x, y, z$ — earth coordinate system fixed in space; $Oxy$ — the coordinate system associated with the airship, the origin of which is located in the center of the volume. The axis $Ox$ is directed along the longitudinal axis of symmetry.
of the hull towards its bow tip; \( \vec{W} \) — wind speed vector at the earth’s surface. We assume that 
\( \vec{W} = \text{const}, \vec{W} \parallel O_x x, \) and that all forces acting on the airship are located in a plane (a flat problem). The elasticity of the airship hull and its deformations will be ignored. Accounting for these deformations is an important task that requires separate consideration. In particular, the matrix of attached masses of a deformable airship will have a significantly more complex appearance than in the case of a rigid hull.

3. **Nonlinear mathematical model of equilibrium for two-cable mounting scheme**

In this case, the main problems are the equilibrium position of the airship, the cable mounting scheme, the estimation of forces arising in the mounting points, and the previously mentioned problem of experimental obtaining of the aerodynamic characteristics of the airship. The task of determining the mounting scheme is mainly understood as the choice of the position of points A, B and the length of the cables. Everywhere below, it is assumed that in this statement, it is possible to ignore the slackness of the cables and imagine them located along a straight line.

To determine the equilibrium conditions of a non-free rigid body, it will be considered that this body as free, replacing the bonds imposed on it with unknown reaction forces, the action of which on the body is equivalent to the action of bonds.

In the plane case under consideration, the forces and moments acting on the airship when the bonds are released are as follows: \( \vec{N}^A, \vec{N}^B \) — bond reaction forces; \( \vec{G} \) — weight applied at the center of mass, which generally does not coincide with the center of volume; \( \vec{A} \) — aerostatic force applied at the center of the volume; \( \vec{R} = (R_x, R_y, R_z) \) — the aerodynamic force given by projections in the associated coordinate system; \( \vec{M} = (M_x, M_y, M_z) \) — aerodynamic moment. Unknown reaction forces \( \vec{N}^A, \vec{N}^B \) will be determined in the earth's coordinate system.

The equilibrium conditions of the airship are expressed in vector form as follow:

\[
\vec{N}^A + \vec{N}^B + \vec{G} + \vec{A} + \vec{R} = 0,
\]

\[
\vec{O}a \times \vec{N}^A + \vec{Ob} \times \vec{N}^B + \vec{Oo} \times \vec{G} + \vec{M} = 0.
\]

The equation of moments is written with respect to the point O.

The scalar form of writing equations (1) makes it possible to obtain six conditions connecting the following twelve unknown quantities: \( x, y, z \) — parameters that determine the position of the pole of the airship in a fixed coordinate system; \( \varphi, \psi, \gamma \) — parameters that determine its angular position; \( N_{x}^A, N_{y}^A, N_{z}^A, N_{x}^B, N_{y}^B, N_{z}^B \) — projections of reaction forces on the axes of the earth's coordinate system. The missing six equations can be obtained if we take into account the action of connections.

![Figure 1. Dynamic diagram of airship arrangement in two-cable mounting scheme.](image)
that restrict the motion of the device. If neither of the two bonds is released in the equilibrium position (all the cables are taut), the following two geometric relations must be fulfilled:

\[
\overrightarrow{O_oA} = \overrightarrow{O_o} + \overrightarrow{Oa} + \overrightarrow{aA}, \\
\overrightarrow{O_oB} = \overrightarrow{O_o} + \overrightarrow{Ob} + \overrightarrow{bB}.
\]

(2)

Ignoring the slackness of the cables in the equilibrium position, i.e. assuming that their configuration in this position differs a little from a straight line and, consequently, the direction of the reaction forces coincides with the direction of the vectors and, it is obtained:

\[
\overrightarrow{aA} = \frac{\overrightarrow{N^A}}{\overrightarrow{N^A}} (1 + k |\overrightarrow{N^A}|) \overrightarrow{l_A}, \\
\overrightarrow{bB} = \frac{\overrightarrow{N^B}}{\overrightarrow{N^B}} (1 + k |\overrightarrow{N^B}|) \overrightarrow{l_B},
\]

(3)

where, \(k\) — the coefficient of linear cable tension.

To loopback the resulting system of equations (1–3) it is necessary to define dependencies \(R(\alpha, \beta), M(\alpha, \beta)\), to express the angle of attack \(\alpha\) and sideslip angle \(\beta\) using the desired unknowns and note that the vector quantities appearing in (1, 2) are defined in different coordinate systems.

Let’s design the vectors \(\overrightarrow{R}, \overrightarrow{M}, \overrightarrow{Oa}, \overrightarrow{Ob}, \overrightarrow{OO}\), defined in the associated coordinate system on the axis of the earth's coordinate system using the corresponding transformation matrix. Then

\[
\begin{pmatrix}
R_x \\
R_y \\
R_z \\
M_x \\
M_y \\
M_z
\end{pmatrix} = Q
\begin{pmatrix}
R_x \\
R_y \\
R_z \\
M_x \\
M_y \\
M_z
\end{pmatrix},
\]

(4)

where \(R_x = -c_x q U^2\), \(R_y = c_y q U^2\), \(R_z = c_z q U^2\), \(M_x = m_x q U\), \(M_y = m_y q U\), \(M_z = m_z q U\),

\[
c_x = c_{x0} + c_{x1}\alpha^2 + c_{x2}\beta^2, \\
c_y = c_{y0} + c_{y1}\alpha^2 + c_{y2}\alpha|\alpha|, \\
c_z = c_{z0} + c_{z1}\beta^2, \\
m_x = m_{x0} + m_{x1}\alpha + m_{x2}\alpha|\alpha|, \\
m_y = m_{y0} + m_{y1}\alpha + m_{y2}\alpha|\alpha|, \\
m_z = m_{z0} + m_{z1}\beta + m_{z2}\beta|\beta|. 
\]

\(W\) — modulus of the wind speed vector; \(U\) — the volume of the device.

Nonlinear terms in the expressions of aerodynamic coefficients can be taken in a different form, for example, in the form of a cubic dependence from \(\alpha, \beta\) which is not of fundamental importance for constructing the final mathematical model.

Wind speed vector can be written as follow;

\[
\begin{pmatrix}
W_x \\
W_y \\
W_z
\end{pmatrix} = Q^T
\begin{pmatrix}
W_{x0} \\
W_{y0} \\
W_{z0}
\end{pmatrix},
\]

(6)

where \(W_{x0}, W_{y0}, W_{z0}\) = 0 since the problem is flat due to the above assumption.

For the angles of attack \(\alpha\) and sideslip angle \(\beta\), let’s write the following equations
\[
\alpha = -\arctg \left( \frac{W_y}{W_x} \right), \quad \beta = \arcsin \left( \frac{W_z}{W} \right).
\]

When designing equations (1) on the axis of the earth's coordinate system, it is necessary to additionally take into account the following relations:

\[
\bar{\overline{a}} = Q \begin{pmatrix} x_a \\ y_a \\ z_a \end{pmatrix}, \quad \bar{\overline{b}} = Q \begin{pmatrix} x_b \\ y_b \\ z_b \end{pmatrix}, \quad \bar{\overline{O}} = Q \begin{pmatrix} x_c \\ y_c \\ z_c \end{pmatrix},
\]

where \((x_a, y_a, z_a), (x_b, y_b, z_b), (x_c, y_c, z_c)\) are elements of vectors \(\bar{\overline{a}}, \bar{\overline{b}}, \bar{\overline{O}}\) in associated axes.

The system of nonlinear equations (1–9) is the desired mathematical model of the equilibrium of the airship in the parking lot. This is a closed system of equations with respect to the twelve unknowns:

\[x_g, y_g, z_g, \theta, \psi, \gamma, N_A^g, N_A^e, N_B^g, N_B^e, N_C^g, N_C^e.\]

The vector \(\vec{W}\) is considered as known here.

4. Conclusion
The constructed nonlinear mathematical model is quite difficult to use, since it requires knowledge of the equilibrium position, which allows to correctly set the lengths of both cables. This follows from the fact that when setting the problem, the equilibrium position was supposed to be found among all possible positions of the system, at which both cables are taut. In real conditions, this requirement can be achieved by adjusting the length of the cables, and these adjustments will depend on the size and direction of the wind speed vector \(\vec{W}\).

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