Generation of induced Smith-Purcell radiation in the absence of resonator

Dmitry N. Klochkov$^1$, Alexander I. Artemyev$^2$, Koryun B. Oganesyan$^3$, Yuri V. Rostovtsev$^4$, Marlan O. Scully$^4$, Chin-Kun Hu$^{1,5}$

$^1$Institute of Physics, Academia Sinica, Nankang, Taipei 1152, Taiwan
$^2$General Physics Institute RAS, Vavilova 38, 119991 Moscow, Russia
$^3$Yerevan Physics Institute, 375036, Armenia, Yerevan, 2 Alikhanian Br.
$^4$Department of Physics and Institute for Quantum Studies, Texas A&M University, College Station, Texas 77843-4242, USA
$^5$Center for Nonlinear and Complex Systems and Department of Physics, Chung-Yuang Christian University, Chungli 32023, Taiwan

E-mail: $^1$ dimitri.klochkoff@gmail.com
E-mail: $^5$ huck@phys.sinica.edu.tw

Abstract. The simplest model of the magnetized infinitely thin electron beam is considered. For the grating, which has depth of grooves as a small parameter, the dispersion equation of the Smith-Purcell instability was obtained. It was found that the condition of the Thompson or the Raman regimes of excitation does not depend on beam current but depends on the height of the beam above grating surface. The growth rate of instability in both cases is proportional to the square root of the electron beam current.

The spontaneous Smith-Purcell (SP) radiation is emitted with an electron passes close to the metal surface of a grating [1]. Last years there has been a renewed interest to the induced intense SP radiation produced by relativistic electron beam. The experiment at Dartmouth College [2, 3], where radiation in the SP system was observed, has stimulated new investigations concerning the SP FEL as an open slow wave structure [4, 5, 6, 7, 8, 9].

Several theories have been proposed to describe the operation of a Smith-Purcell FEL. In particular, Schaechter and Ron [10] proposed a theory based on the interaction of an electron-beam with a wave traveling along the grating. The interaction is found to amplify waves incident on the grating and reflected by it, with a gain that depends on the reflection matrix of the grating. They have found that the gain is proportional to the cube root of the electron-beam current, which agrees with the behavior of Cherenkov free-electron lasers and other slow-wave devices. More recently, Kim and Song [4] have proposed a theory in which they assume that the electrons interact with a wave that travels along the surface of the grating. Assuming that at least one Fourier component of the traveling wave is radiative, and they have found that the gain is proportional to the square root of the electron-beam current, $\sim I_b^{1/2}$.

A different result has been obtained in Ref. [5]; in this paper a rectangular grating is considered, assuming that the entire space above the grating is filled by a uniform electron beam. The authors have established the dispersion relation and found the dispersion law $\omega(k)$.  

© 2010 IOP Publishing Ltd
It turns out that the dispersion equation allows only evanescent solutions and the operating point of a SP FEL is fixed by the intersection of the dispersion curve with the beam line. The corresponding small signal gain follows the $\sim I_b^{1/3}$ law.

Hence, there is no general consent on the mechanisms of the SP FEL and the small signal gain obtained by different approaches. Taking also into account that there is currently substantial interest in the implementation of a SP FEL, it is of interest to reexamine the theory of SP FEL.

One can select two theoretical target settings for SP radiation. First of them is a problem of generation of SP-radiation. In this case there is no incoming from infinity wave. SP system generates outgoing waves. Another problem is an amplification or attenuation of incoming waves by SP-system. One can expect that these different problem have different solutions.

In this work we consider only the problem of generation of wave $s$ by SP-system, when the incoming waves are absent. In order that mathematical problems do not cover physical picture, we consider the simplest model of the beam.

For the small signal gain we consider the linear over the field amplitude solutions of system of Maxwell’s and beam’s equations. We take a Cartesian coordinate system in a such manner that the electron beam propagates to the positive $x$ direction in the vacuum over the grating (parallel to the surface of a grating) as it is shown in Fig. 1. For simplicity, we consider a infinitely in $z$ direction sheet of the beam with thickness $\Delta b$ and the non-disturbed density $n_b$ stabilized by external strong magnetic field. We assume the system to be translationally invariant in the $z$ direction. This means that all physical values depend on coordinates $x$ and $y$ only, and $E_z = 0$.

![Figure 1](image)

**Figure 1.** Schematic of an SP-FEL using a sheet electron beam. The $z$-axis is a perpendicular to the list. The sheet electron beam is in the plane $y = b$. Period of the grating is $d$, amplitude of the grating is $h$.

We start from the Maxwell’s equations

$$\nabla \times E = -\frac{1}{c} \frac{\partial B}{\partial t}, \quad \text{div} B = 0, \quad (1)$$

$$\nabla \times B = \frac{1}{c} \frac{\partial E}{\partial t} + \frac{4\pi}{c} j, \quad \text{div} E = 4\pi \rho. \quad (2)$$
and the hydrodynamical equations for the beam, namely, equation of motion

$$\frac{dp}{dt} = \left( \frac{\partial}{\partial t} + v\vec{\nabla} \right) p = eE + e\frac{\vec{v}}{c} \times \vec{B}$$  \hspace{1cm} (3)$$

and continues equation

$$\frac{\partial p}{\partial t} + \text{div} j = 0,$$  \hspace{1cm} (4)

where $j = \rho \vec{v} = \rho \vec{v}$ is a relativistic factor. An initial velocity of electrons is $u = (u, 0, 0)$, and $\gamma_0 = (1 - u^2/c^2)^{1/2}$ is a relativistic factor.

In our case there are only three non-zero components $E_x$, $E_y$ and $B_z$ of electromagnetic field. In order to describe the field in vacuum we introduce the potential $P$ and determine the field components with

$$E_x = \frac{\partial P}{\partial y}, \quad E_y = -\frac{\partial P}{\partial x}, \quad B_z = \frac{1}{c} \frac{\partial P}{\partial t}.$$  \hspace{1cm} (5)

To describe the electromagnetic field in the vacuum space ($\rho = 0$ and $j = 0$ in Eqs. (1),(2)), we take advantage of Floquet's theorem and expand the fields above ($y > b$) and below ($y < b$) beam level in the form:

$$P = \begin{cases} 
\sum_n P_n e^{-i\omega t + iq_n^+ r}, & y > b \\
\sum_n (P_n^+ e^{iq_n^+ r} + P_n^- e^{iq_n^- r}) e^{-i\omega t}, & y < b.
\end{cases}$$  \hspace{1cm} (6)

Here the wave vectors are $q_n^\pm = (q_{nx}^\pm, \pm q_{ny}) = (k_x + n\kappa, \pm q_{ny})$, and $\kappa = 2\pi/d$ is a wave number of the grating, $d$ is a grating period. The dispersion relation for electromagnetic wave in vacuum space is $\omega^2 = (q_n^\pm)^2 c^2$.

Using the boundary conditions on the sheet beam

$$E_x(b + 0) = E_x(b - 0), \quad E_y(b + 0) - E_y(b - 0) = 4\pi \tilde{\rho}.$$  \hspace{1cm} (7)

we can express the partial amplitudes $P_n^+$ and $P_n^-$ in terms of the partial amplitudes $P_n$ of the outgoing waves. Here the surface density of disturbed beam charge $\tilde{\rho}$ is obtained from the linearized equations for the beam.

The solutions, obtained above, define the amplitudes of the field, but do not determine its frequency. In order to get the frequency we have to consider the boundary condition on the surface of the grating. We consider the simplest model of the grating, when the equation of metal surface is defined as

$$y(x) = h \sin(\kappa x) = h \sin \left( \frac{2\pi}{d} x \right).$$  \hspace{1cm} (8)

Tangential vector to the surface is $\mathbf{r}_0 = (1, h \kappa \cos(\kappa x))$. Boundary condition $(\mathbf{r}_0 \mathbf{E}(x, y(x))) = 0$ takes the form

$$E_x(x, y(x)) + h \kappa \cos(\kappa x) E_y(x, y(x)) = 0.$$  \hspace{1cm} (9)

Here $E_x$ and $E_y$ are given by (5). Substituting the solutions of Maxwell equations (5), (6) in boundary condition (9), multiplying by $\exp(-ip xx)$, where $p = 0; \pm 1; \pm 2; \pm 3; ...$, and integrating then over coordinate $x$ within the interval $(0; d)$ we obtain the infinite system of algebraic equation respectively the partial amplitudes $P_n$:

$$\sum_n \left( q_{ny} (n + p) \kappa q_{nx} q_{ny} \right) \left( J_{n-p}(-q_{ny} h) + K(n) \left[ J_{n-p}(-q_{ny} h) - J_{n-p}(q_{ny} h) e^{2i\kappa q_{ny} b} \right] \right) P_n = 0.$$  \hspace{1cm} (10)
Here $J_0(x)$ is a Bessel function.

In order to cut the infinite system we assume that the depth of the grating groove $h$ is a moderate. Since the Bessel function on small variable $x$ drops as $J_0(x) \approx (x/2)^\alpha/\alpha!$ with $\alpha$ increasing, the assumption that $h$ is a small parameter permits us to consider 3-waves approximation. We suppose that there are only three non-zero partial amplitudes, namely, $P_{n-1}$, $P_n$ and $P_{n+1}$. Other amplitudes are assumed to be zero. Then we get three algebraic equations, matrix of which is:

$$
\begin{pmatrix}
q_{n-1y}\alpha_{n-1} & \beta_n^+ & \gamma_{n+1}^- \\
\beta_{n-1}^- & q_{ny}\alpha_n & \beta_{n+1}^- \\
\gamma_{n-1}^- & \beta_{n-1}^- & q_{n+1y}\alpha_{n+1}
\end{pmatrix}.
$$

(11)

Here coefficients are

$$
\alpha_n = 1 + K(n)(1 - e^{2iq_{ny}h}),
$$

$$
\beta_\pm = \frac{h}{2} (q_{ny}^2 \mp \kappa q_n) \left[ 1 + K(n)(1 + e^{2iq_{ny}h}) \right],
$$

$$
\gamma_n = \frac{h^2}{8} q_{ny} (q_{ny}^2 \mp 2\kappa q_n) \alpha_n.
$$

Here we introduce the notation

$$
K(n) = \frac{i}{2} \Delta_0 \omega_b^2 \gamma_0^{-3} \frac{q_{ny}}{(\omega - (q_n^0 u))^2}.
$$

(13)

The nontrivial solution of infinite system of equations (11) takes place if its determinant equals zero. The latter is dispersion equation describing both the spectrum of frequencies and the growth rates of the induced Smith-Purcell radiation. Neglecting the small terms proportional to $h^4$ and $h$ with higher powers, we obtain the simple dispersion equation:

$$
D(\omega, k) = q_{n-1y}q_{ny}q_{n+1y} \alpha_{n-1} \alpha_n \alpha_{n+1} - q_{n-1y} \alpha_{n-1} \beta_n^+ \beta_{n+1}^- - \beta_{n-1}^- \beta_n^+ q_{n+1y} \alpha_{n+1} = 0.
$$

(14)

What is zero-order approximation for dispersion equation that is the question. In the absence of the beam, when $\omega_b = 0$, we find $P_n^- = 0$ and $P_n^+ = P_n$, that means $E_n^- = 0$ and $E_n^+ = E_n$. Substituting this amplitudes in the boundary condition we find that $E_n = 0$. Since all amplitudes equal zero, the determinant of the system is not equal to zero. Indeed, substituting $\omega_b = 0$ in equation (14) we get

$$
D(\omega, k, \omega_b = 0) = q_{n-1y}q_{ny}q_{n+1y} \left. \begin{array}{c} + \frac{h^2}{4} \left\{ q_{n-1y}q_{ny}^2 \mp \kappa q_n + q_{n+1y}(q_{n-1y}^2 + \kappa q_{n-1y} + \kappa q_{n+1y}) + q_{n+1y}(q_{n-1y}^2 - \kappa q_{n-1y})(q_{ny}^2 + \kappa q_n) \right\} \end{array} \right\} \neq 0.
$$

(15)

The latter has a simple physical explanation. In the absence of the beam, the electromagnetic field in the system is absent too, because the grating does not emit the waves. Indeed, the grating plate does not shine in a dark room!

Therefore we can not use condition $\omega_b = 0$ as a zero-order approximation to solve dispersion equation. Instead we use as zero-order approximation condition $h = 0$, which gives

$$
D_0(\omega, k; h = 0) = q_{n-1y}q_{ny}q_{n+1y} \alpha_{n-1} \alpha_n \alpha_{n+1} = 0.
$$

(16)
In general case for an arbitrary angle $\theta$ the product of $q_{n-1y}q_{n+1y}$ does not equal zero, therefore we can write

$$D_0(\omega, k; h = 0) = \alpha_{n-1}\alpha_n\alpha_{n+1} = 0. \quad (17)$$

Without loss generality we assume that the $n$-mode of electromagnetic spectrum excites in the system. The latter means that $\alpha_n = 0$, which gives equation

$$(\omega - (k_x + n\kappa)u)^2 + \frac{i}{2}\Delta \omega_b^2 \gamma_0^{-3}q_{ny}\left(1 - e^{2i\omega_n b}\right) = 0. \quad (18)$$

We seek solution of the equation in the form

$$\omega = \omega_n \pm \Omega_{bn}, \quad (19)$$

where $\omega_n$ satisfies the equation (18) in the limit $\omega_b \to 0$:

$$\omega_n = (k_x + n\kappa)u, \quad (20)$$

which is the dispersion relation for longitudinal beam waves. Taking into account the dispersion relation for electromagnetic wave in vacuum

$$k_x = \frac{\omega_n}{c} \cos \theta, \quad (21)$$

we find the frequency of $n$-mode of spectrum for the limit $\omega_b \to 0$ as the intersection point in the $k_x - \omega$ diagram shown in the Fig. 2:

$$\omega_n = \frac{n\kappa u}{1 - \beta \cos \theta}, \quad (22)$$

where $\beta = u/c$ is a dimensionless velocity of the electron, $\theta$ is an angle between vector $k^+$ and positive direction of $x$-axis, or say another, the angle of observation. Please do not mix up the $n$-mode in spectrum of radiation with the $n$-mode in $q_x$-spectrum of surface wave $E_n$. With help of Eq. (22) we find that

$$q_{ny} = \frac{i\omega_n}{u\gamma_0}, \quad (23)$$

so the $n$-mode of surface wave $E_n$ does not radiate, because it damp in the positive direction of $y$-axis.

Considering the presence of beam as perturbation and substituting the value $q_{ny}$ given by Eq. (23) in Eq. (18) we find from Eq.(18) the beam frequency $\Omega_{bn}$:

$$\Omega_{bn} = \omega_b\gamma_0^{-2} \sqrt[4]{\frac{\gamma_\omega\omega_n}{2u}} \left(1 - e^{-2\frac{\omega_n b}{\omega_n u}}\right). \quad (24)$$

Here we give the physical interpretation of solutions obtained. The condition $h = 0$ means that there is a flat mirror in plane $y = 0$. The fluctuations of charge and current densities of the electron beam produce fluctuations of electromagnetic field, the plane wave of which reflect from the mirror. The resonant condition (22) is a crossing point of dispersion lines for the beam wave and reflect electromagnetic plane wave. The boundary condition gives discrete spectrum, modes of which contain two coupled frequencies: “high” frequency with $\omega = \omega_n + \Omega_{bn}$ and “low” frequency with $\omega = \omega_n - \Omega_{bn}$. The beam frequency $\Omega_{bn}$ depends on the height of beam above surface $y = 0$ and has asymptotes:

$$\Omega_{bn} = \begin{cases} \omega_b\gamma_0^{-2} \sqrt[4]{\frac{\gamma_\omega\omega_n}{2u}}, & b \to \infty \\ \omega_b\gamma_0^{-5/2} \frac{\omega_n}{u} \sqrt{\gamma_\omega^b}, & b \to 0. \end{cases} \quad (25)$$
Now we consider the presence of the grooves of mirror ($h \neq 0$) as perturbation and will seek the solution of dispersion equation near synchronism point (19) in the form:

$$\omega = \omega_n \pm \Omega_{bn} + \delta \omega. \quad (26)$$

Substituting solution (26) in Eq. (14) and neglecting small terms under realistic condition \{\omega_b, \Omega_{bn}, |\delta \omega| \} \ll \kappa u we obtain dispersion equation respectively small shift of frequency:

$$\delta \omega^2 + 2\Omega_{bn} \delta \omega = \frac{h^2}{4} \Delta \omega_n^2 \left( \frac{\omega_n}{u} \right)^3 e^{-\frac{2\omega_n}{\omega_{bn}}} \left( X_n(\theta, u) + iY_n(\theta, u) \right). \quad (27)$$

For $\omega = \omega_n$ the wave number $q_{n+1y}$ is always an imaginary value. We rewrite $q_{n+1y}$ as $q_{n+1y} = ig_{n+1}$, where $g_{n+1} = \frac{\omega_n}{\omega_{bn}} \sqrt{(n + 1 - \beta \cos \theta)^2 - n^2 \beta^2}$. The wave number $q_{n-1y}$ can be as a real as an imaginary for $\omega = \omega_n$. Under condition:

$$1 - \beta \cos \theta < n < \frac{1 - \beta \cos \theta}{1 - \beta} \quad (28)$$

the wave number $q_{n-1y}$ is a real and then the coefficients $X_n$ and $Y_n$ are equal to

$$X_n = \left(1 + \frac{\kappa u \gamma_0^2}{\omega_n}\right) \left(\frac{\kappa}{g_{n+1}} \left(1 + \frac{\kappa u}{\omega_n} - \frac{g_{n+1}u}{\omega_n}\right)\right),$$

$$Y_n = \left(1 - \frac{\kappa u \gamma_0^2}{\omega_n}\right) \left(1 - \frac{\kappa u}{\omega_n}\right) \left(1 + \frac{\kappa u}{\omega_n}\right) \left(1 - \frac{\kappa u}{\omega_n}\right) \frac{\kappa}{i g_{n-1}}. \quad (29)$$

Another case realizes under condition

$$n \notin \left(\frac{1 - \beta \cos \theta}{1 + \beta}, \frac{1 - \beta \cos \theta}{1 - \beta}\right), \quad (30)$$

when the wave number $q_{n-1y} = ig_{n-1}$ is an imaginary and then the coefficients $X_n$ and $Y_n$ are equal to

$$X_n = \left(1 + \frac{\kappa u \gamma_0^2}{\omega_n}\right) \left(\frac{\kappa}{g_{n+1}} \left(1 + \frac{\kappa u}{\omega_n} - \frac{g_{n+1}u}{\omega_n}\right)\right) \left(\frac{\kappa}{\omega_n} \right) \left(1 - \frac{\kappa u \gamma_0^2}{\omega_n}\right) \left(\frac{\kappa}{g_{n-1}}\right);$$

$$Y_n = 0. \quad (31)$$

Here $g_{n-1} = \frac{\omega_n}{\omega_{bn}} \sqrt{(n + 1 - \beta \cos \theta)^2 - n^2 \beta^2}$.

The square equation (27) has a simple solutions. Here we consider two interesting cases. At first, we consider the Thompson (single-particle approximation) type of excitation, when the electron beam radiates as a single particle or a bunch. In this case the beam waves can be neglected and condition of generation is $|\delta \omega| > \Omega_{bn}$. Under the condition we can omit the second term in left-hand side of dispersion equation and obtain the solution:

$$\delta \omega_n = \frac{h \omega_n}{2u} \sqrt{\frac{\Delta \omega_n}{u} \frac{\omega_b}{\gamma_0^5/2} e^{-\frac{2\omega_n}{\omega_{bn}}} \left( X_n + iY_n \right)^{1/2}. \quad (32)$$

The growth rate of instability of n-mode (22) ($\delta \omega = \delta \omega''_n + i \delta \omega''_n$) in this case is

$$\delta \omega''_n = \pm \frac{\omega_n}{2u} \frac{\omega_b}{\gamma_0^5/2} \sqrt{\frac{\Delta \omega_n}{u} e^{-\frac{2\omega_n}{\omega_{bn}}} \left( X_n + Y_n \right)^{1/4} \sin \left| \frac{\psi}{2} \right|}, \quad (33)$$
where angle $\psi$ is defined with $\psi = \arccos(\frac{X_n}{\sqrt{X_n^2 + Y_n^2}})$. For sufficiently large value of the relativistic factor $\gamma_0$, when the inequality $\gamma_0 \gg \frac{\omega_n b}{u}$ holds, the growth rate of instability is proportional to square root of inverse $\gamma_0$: $\delta \omega'' \sim \gamma_0^{-1/2}$. The condition of the Thompson regime excitation does not depend on beam current and has form:

$$\frac{\hbar \omega_n}{u} \left| \frac{X_n + iY_n}{\sqrt{\gamma_0^2 \left( \frac{2e\gamma_0}{\omega_{n0}} - 1 \right)}} \right| \gg 1. \quad (34)$$

This inequality holds for $b \to 0$ and for the big value of grooves depth $h$ of grating.

Another case is the Raman (or collective) regime of generation, when influence of beam waves is appreciable and $|\delta \omega| \ll \Omega_{bn}$. For the Raman regime we obtain

$$\delta \omega = \pm \frac{\hbar^2 \omega_n^2 \omega_0 b}{4u^2 \gamma_0^2} \left[ \frac{\Delta \delta \omega_n}{2u} \frac{e^{-\frac{2e\gamma_0}{\omega_{n0}}}}{\sqrt{1 - e^{-\frac{2e\gamma_0}{\omega_{n0}}}}} (X_n + iY_n) \right]. \quad (35)$$

The growth rate of instability is

$$\delta \omega'' = \pm \frac{\hbar^2 \omega_n^2 \omega_0 b}{4u^2 \gamma_0^2} \left[ \frac{\Delta \delta \omega_n}{2u} \frac{e^{-\frac{2e\gamma_0}{\omega_{n0}}}}{\sqrt{1 - e^{-\frac{2e\gamma_0}{\omega_{n0}}}}} Y_n(\theta, u) \right]. \quad (36)$$

If $Y_n(\theta, u) > 0$, then the high-frequency branch with $\omega = \omega_n + \Omega_{bn}$ excites. In the opposite case the low-frequency waves with $\omega = \omega_n - \Omega_{bn}$ generates. For sufficiently large value of the relativistic factor $\gamma_0$, when the inequality $\gamma_0 \gg \frac{\omega_n b}{u}$ holds, the growth rate of instability is proportional to inverse $\gamma_0$: $\delta \omega'' \sim \gamma_0^{-1}$. Condition for the Raman type of excitation is

$$\frac{\hbar^2 \omega_n^2}{4u^2 \gamma_0^2} \left| \frac{Y_n}{e^{-\frac{2e\gamma_0}{\omega_{n0}}} - 1} \right| \ll 1. \quad (37)$$

It does not depend on Langmuir beam frequency (or beam current), but it depends on the beam height $b$ above the grating.

The mechanism of light radiation is following. The surface wave with mode number $n$ acting on the beam electrons, perturbs the trajectories of latter. Electrons, oscillating near equilibrium point $r = r_0 + ut$, emit electromagnetic wave. Because, the amplitude of oscillation is proportional to the magnitude of surface wave on the height $h$ of beam, the value of growth rate depends on the beam height above the grating surface.

When $q_{n-1y} = 0$ the 3-waves approximation does not work. It can be when $n \approx (1 - \beta \cos \theta)(1 \pm \beta)$, for example, for $n = 1$ and $\theta = 0$. In this case we have to include additional evanescent modes and consider 5-waves approximation, namely, we assume that there are only five non-zero amplitudes: $E_{n-2x}$, $E_{n-1x}$, $E_{nx}$, $E_{n+1x}$ and $E_{n+2x}$. Other amplitudes are assumed to equal zero. Taking into account that $h$ is a small parameter and neglecting small terms as above, we obtain the following dispersion equation:

$$D(\omega, k) = \left( q_{n-2y}\alpha_{n-2q_{n-1y}\alpha_{n-1}q_{n-1}q_{n+2}} + q_{n-2y}\alpha_{n-2q_{n-1y}\alpha_{n-1}q_{n+2}} - q_{n-2y}\alpha_{n-2q_{n-1y}\alpha_{n-1}q_{n+2}} + \beta_{n-2y}\alpha_{n-2q_{n-1y}\alpha_{n-1}q_{n+2}} \right) q_{n-2y} \frac{q_{n-1y}q_{n-1}q_{n+2}q_{n+2}q_{n+2}q_{n+2}}{q_{n-2y}\alpha_{n-2q_{n-1y}\alpha_{n-1}q_{n+2}}} + q_{n-2y}\alpha_{n-2q_{n-1y}\alpha_{n-1}q_{n+2}} + \beta_{n-2y}\alpha_{n-2q_{n-1y}\alpha_{n-1}q_{n+2}} \beta_{n-1y}\alpha_{n-1}q_{n+2}q_{n+2}q_{n+2}q_{n+2}q_{n+2}q_{n+2} = 0. \quad (38)$$
Considering the solution of Eq. (38) near the synchronism point (26), we can rewrite the equation respectively $\delta \omega$:

$$\delta \omega^2 \pm 2 \Omega_{bn} \delta \omega + \frac{i}{2} \frac{\Delta \omega_{b}^2 \omega_n^2}{u^2 \gamma_0^3} A - \frac{2 \omega b}{u \gamma_0} = 0,$$

(39)

where

$$A = q_{n-2y} q_{n+2y} \left\{ \left( 1 + \frac{\kappa \omega_n^2}{\omega_n} \right) q_{n-1y} \beta_{n+1}^+ - \left( 1 - \frac{\kappa \omega_n^2}{\omega_n} \right) q_{n+1y} \beta_{n-1}^- \right\},$$

(40)

$$B = q_{n-2y} q_{n-1y} q_{n+1y} q_{n+2y} - q_{n-2y} q_{n-1y} \beta_{n+1}^- \beta_{n+2}^+ - \beta_{n-2}^- \beta_{n-1}^- q_{n+1y} q_{n+2y}.$$

For the case $q_{n-1y} = 0$ the dispersion equation reduces to the form:

$$\delta \omega^2 \pm 2 \Omega_{bn} \delta \omega + \frac{i}{2} \frac{\Delta \omega_{b}^2 \omega_n^2}{u^2 \gamma_0^3} q_{n-2y} \frac{2 \omega b}{u \gamma_0} = 0.$$

(41)

Here $q_{n-2y} = 2 \kappa^2 \left( \frac{n}{1 - \beta \cos \theta} - 1.5 \right)$ is a positive value. For the Thompson regime of excitation we find the growth rate:

$$\delta \omega'' = \pm \frac{\omega_b \omega_n}{u \gamma_0^{5/2}} \sqrt{\frac{\Delta \omega_{b}^2 \omega_n^2}{2 q_{n-2y} + \kappa \omega_n^{2x}}},$$

(42)

which does not depend on small parameter $h$.

For the Raman regime of generation we find the growth rate:

$$\delta \omega'' = \pm \sqrt{\frac{\Delta \omega_{b}^2 \omega_n^2}{2 \frac{\kappa \omega_n^2}{u \gamma_0^{3/2}} q_{n-2y} + \kappa \omega_n^{2x}}} e^{-\frac{2 \omega b}{u \gamma_0}},$$

(43)

In this case the growth rate does not depend on small groove depth $h$ too. Therefore, we have a peak of the growth rate under condition $q_{n-1y} = 0$.

We have found that zero-order approximation for solution of dispersion equation of SP-generation corresponds to the mirror boundary case, when the electron beam propagates above plane metal surface (mirror). The growth rates for both the Thompson and the Raman types of waves excitation are proportional to the Langmuir frequency $\omega_b$ or square root of beam current $I_b^{1/2}$. The conditions of excitations for both Thompson and Raman types do not depend on Langmuir beam frequency (or beam current), but depend on the beam height $h$ above the grating. It is essential distinction between the Cherenkov and SP instabilities.

This work was supported by the National Science Council of the Republic of China (Taiwan) under Grant Nos. NSC 96-2911-M-001-003-MY3 & 98-2911-I-001-028 (for Chin-Kun Hu), and National Center for Theoretical Sciences in Taiwan and supported by the International Science and Technology Center, Moscow (project A-1602). We gratefully acknowledge the support from the Defense Advanced Research Projects, the Office of Naval Research, the NSF grant EEC-0540832 (MIRTHE ERC), and the Robert A. Welch Foundation (Grant #A1261).

[1] S.J. Smith and E.M. Purcell, Phys. Rev. 92, 1069 (1953).
[2] J. Urata, M. Goldstein, M. F. Klimm, A. Naumov, C. Platt, and J. E. Walsh, Phys. Rev. Lett. 80, 516 (1998).
[3] A. Bakhtyari, J. E. Walsh, and J. H. Brownell, Phys. Rev. E 65, 066503 (2002).
[4] K.-J. Kim and S.-B. Song, Nucl. Instrum. Methods Phys. Res., Sect. A 475, 158 (2001).
[5] H. L. Andrews and C. A. Brau, Phys. Rev. ST Accel. Beams 7, 070701 (2004).
[6] H.L. Andrews, C. H. Boulware, C. A. Brau, and J. D. Jarvis, Phys. Rev. ST Accel. Beams 8, 050703 (2005).
[7] H.L. Andrews, C. H. Boulware, C. A. Brau, J. T. Donohue, J. Gardelle, and J. D. Jarvis, New J. Phys. 8, 289 (2006).
[8] Vinit Kumar and Kwange-Je Kim, Phys. Rev. E 73, 026501 (2006).
[9] G. F. Mkrtchian, Phys. Rev. ST Accel. Beams 10, 080701 (2007).
[10] L. Schaechter and A. Ron, Phys. Rev. A 40, 876 (1989).
[11] P. M. Van den berg, Appl. Sci. Res. 24, 261 (1971).