Calculation of electromagnetic torque for synchronous motor with modulated magnetic flux and smooth harmonic rotor

A F Shevchenko and L G Shevchenko

Novosibirsk State Technical University, 20 Karl Marks Av., Novosibirsk, 630073, Russia

Abstract. Results of the electromagnetic torque calculation for the synchronous motor with modulated magnetic flux and a smooth harmonic rotor are presented in this paper. The value of the torque is determined from the electromagnetic forces, which appear due to interaction of magnetic field in the gap with the rotor surface elements. The obtained analytical expression makes it possible to determine easily the electromagnetic torque for the considered motor in the MathCAD environment.

1. Introduction

Motors with high and ultra-high rotational speeds (over 100 000 rpm) are necessary for a number of technological processes. The maximum rotational speed of electrical machines is limited by the mechanical strength of the rotor and a valid power source frequency. Therefore, such machines are performed with a small number of poles and special design of the rotor. In this paper the authors consider one of the motors of this type – synchronous reluctance motor with a harmonic rotor [1, 2].

The motor considered in this paper combines individual design elements of such electrical machines as rolling rotor motors (RRM), SR – motors, as well as classical synchronous reluctance motors (SRM) [3,4,5]. The cross section of this motor is shown in figure 1.

The stator winding is made of individual coils placed on the stator poles (teeth), i.e. in the same way as in the RRM with pulsating flow and in SR – motors [6]. The rotor is a salient pole with number of poles, usually, equal to two. The air gap between the stator and the rotor should change so that the law of the magnetic conductivity variation of the air gap approaches a sinusoidal one (with a constant component). It is clear that the number of rotor poles can be more than two but it also can be equal to one. However, the last option cannot be used because of single sided pressure forces.

A smaller number of rotor poles (teeth) allows obtaining higher speeds of the motor compared to the SR-motor with the same power supply frequency. Therefore, it can operate on high and ultra-high speeds. Exceptional simplicity of the rotor - laminated, without teeth, oval shape also contributes to this. The rotor shape will significantly reduce the aerodynamic noise.

Magnetic field distribution shown in figure 1 can explain the motor principle of operation. The ferromagnetic body placed in magnetic field is exposed to electromagnetic force which is perpendicular to the body surface. Since the rotor surface is curved, this force has a tangential component which forms the electromagnetic torque. It is easy to see that the electromagnetic torque appears due to magnetic conductivity difference along the d and q axes. Moreover, the driving torque is produced by electromagnetic forces acting in the region of the air gap where the derivative of the air gap conductivity along the angular coordinate is positive:

\[ f = F_m \frac{d\lambda}{d\alpha} > 0 \]
Figure 1. The motor design and the winding diagram with q=2/5

The rotating speed of reluctance motors is determined by the power supply frequency $f_1$:

$$\omega_r = \frac{2\omega_1}{z_2} = \frac{2 \cdot 2\pi f_1}{z_2},$$

and it does not depend on the number of the stator teeth and, consequently, on the fractionality of the used winding [1].

The considered reluctance motor does not have a starter winding; therefore, it can operate only from the semiconductor frequency convertor or in valve mode. However, it must be equipped with a rotor position sensor for this.

Either sinusoidal current or the bipolar current pulses flow through the reluctance motor windings. That is why one can use conventional commutation switcher circuit for such controlled motor. The voltage of a rectangular shape is applied to the winding in the valve and step modes.

One of the important problems in the study and design of any electric machine is the determination of its electromagnetic torque. The electromagnetic torque of electric motor can be found both from the variation of the magnetic field energy stored in the air gap [2] and through the electromagnetic forces. The determination of the motor electromagnetic torque through the electromagnetic forces is considered in the paper. To find the total torque, it is necessary to sum the elementary torques appearing from the interaction of the magnetic field in the gap with the surface elements of the rotor.

2. Electromagnetic torque

Let us assume that the magnetic field in the air gap is plane-parallel. In this case, the torques from the electromagnetic forces applied to the elements of the rotor curve $dl$ can be summed.

It is known that the elementary force acting on the element of the rotor curve $dl$ (figure 2) is [7]:

$$\vec{F} = \frac{\mu_0 H^2}{2} \, d\vec{l}$$  \hspace{1cm} (1)

where $H$ is the magnetic field strength in the air gap region.
Figure 2. Elementary force, acting on the element of the rotor surface

The torque vector, arising from this force is:
\[ \bar{M} = \bar{F} \times \bar{r}, \quad (2) \]

and its module is:
\[ |M| = |F| \cdot |r| \cdot \sin \gamma. \quad (3) \]

The torque from all electromagnetic forces along the contour of the rotor is determined by integrating the torque (3) along this contour.
\[ M = \oint |F| \cdot |r| \cdot \sin \gamma \, dl. \quad (4) \]

It is difficult to calculate the integral in the form mentioned above, because the radius \( r \) (and consequently \( \sin \gamma \)) varies along the contour of the rotor.

To solve this problem, it is convenient to use the conformal mapping of the initial air gap region on a simpler annular domain. Due to the symmetry of the original region, it is sufficient to map only half of it to the ring. It is convenient to use the intermediate mapping on the region between two eccentric circles. At first, half of the initial region on the plane \( z \) with coordinates \( x \) and \( y \) is conformally transformed into the region between two eccentric circles on the plane \( \mathbb{z} \) with \( xx \) and \( yy \) coordinates, as shown in figure 3.

Figure 3. The conformal transformation of the half of the initial region into the domain between two eccentric circles

The mapping is carried out by known function [8,9]:

\[ \mathbb{z}(z) = z^2 \]
Then the region between two eccentric circles on the \(zz\) plane with the \(xx\) and \(yy\) coordinates is conformally transformed to the annular domain on the \(w\) plane with \(u\) and \(v\) coordinates (as shown in figure 4) also by the known mapping function. \[w(zz) = \frac{zz - a}{1 - a \cdot zz}\] (6)

Figure 4. The conformal transformation of the domain between two eccentric circles into the annular region

In this case, the inverse conformal transformation formulas have the form:

\[zz(w) = \frac{a + w}{1 + a \cdot w}\] (7)

\[z(w) = \sqrt[1 + a \cdot w]{\frac{a + w}{1 + a \cdot w}}.\] (8)

After accepting the stator radius in the initial region equal to one and performing some mathematical transformations, one obtains relation between the constant \(a\) and the initial region parameters, such as \(\delta\) (the minimum air gap) and \(\Delta\) (the maximum air gap).

\[a = k - \sqrt{k^2 - 1},\] (9)

where

\[k = \frac{1 - (1 - \Delta)^2(1 - \delta)^2}{(1 - \delta)^2 - (1 - \Delta)^2}.\]

And the stator radius in the annular region is:

\[\rho_0 = \frac{(1 - \Delta)^2 + a}{1 + a(1 - \Delta)^2}.\] (10)

These conformal transformations make it possible to change from the coordinates of the original domain to the coordinates of the annular domain:

\[r \cdot e^{i\theta} \rightarrow \rho_0 \cdot e^{i\alpha},\]

where \(r, \theta\) - the radius (which is changing along the contour of the rotor) and rotor rotation angle in the initial air gap region, \(\rho_0, \alpha\) - the radius (which is constant along the contour of the rotor) and the rotor rotation angle in the annular region.

The initial contour of the rotor is represented on the \(z\) plane as shown in figure 5. In this figure, the curve element \(dl\) is denoted by \(dz\) and the radius \(r\) - by the complex variable \(z\). Taking into account, that \(\sin \gamma = \cos(90 - \gamma)\), one can write the integral (4) as:
Let us take into account the definition of the scalar product. On the one hand, the scalar product $\bar{z} \cdot dz$ is equal to:

$$\bar{z} \cdot dz = |z| \cdot |dz| \cdot \cos(90 - \gamma);$$

on the other hand:

$$\bar{z} \cdot dz = x \cdot dx + y \cdot dy = \frac{1}{2}(z \cdot d\bar{z} + \bar{z} \cdot dz),$$

where $z$, $\bar{z}$ and $d\bar{z}, dz$ are complex conjugate values.

Based on this, the integrand takes the following form:

$$|z| \cdot |dz| \cdot \cos(90 - \gamma) = \frac{1}{2}(z \cdot d\bar{z} + \bar{z} \cdot dz)$$

(12)

Further, if one considers a complex variable $z$ as a function of the angle $\alpha$ and uses the inverse conformal transformation $z(w)$, one can write for the contour of the rotor that:

$$z(\alpha) = z(w) = z(\rho_0 e^{ia\alpha}),$$

where

$$z(w) = \sqrt{-\frac{a + w}{1 + \bar{a} \cdot w}}.$$

Then it is possible to substitute the following expressions in (12):

$$z = z(\rho_0 \cdot e^{ia\alpha}),$$

$$dz = z'(\rho_0 \cdot e^{ia\alpha}) \cdot \rho_0 \cdot i \cdot e^{ia\alpha} \cdot d\alpha.$$  

(13)

The derivative in (13) is:

$$z'(w) = \frac{1}{2} \frac{1 - (|k|)^2}{1 + \bar{a}w} \frac{1 + \bar{a}w}{a + w}.$$

The final form of the integral for the torque is:

$$M = \int_0^{2\pi} H_0^2 \cdot \mu_0 \cdot i \cdot \rho_0 \cdot \left( z(\rho_0 e^{ia\alpha}) \cdot z'(\rho_0 e^{ia\alpha}) \cdot z'(\rho_0 e^{ia\alpha}) \cdot z(\rho_0 e^{ia\alpha}) \right) d\alpha$$

(14)
where \( l \) is the length of the motor and
\[
H = \left[ \sum_{n=1}^{\infty} n \left( c_n e^{i\alpha} + \overline{c_n} e^{-i\alpha} \right) \right] \frac{2 \cdot \rho_0^{n+1}}{1 - \rho_0^{2n}} \frac{1}{z'} \left( \rho_0 e^{i\alpha} \right) 
\]

strength of the magnetic field in the initial region of the air gap.

The formula for the strength of the magnetic field in the air gap is obtained from the analytical solution of the Laplace equation in the annular region in the form of series, using inverse conformal transformation formulas. The coefficient \( c_n \) in the formula is the coefficient of the Fourier series, which represents the boundary condition on the stator surface in the initial region of the air gap.

3. Conclusion

Given the known engine parameters and magnetic potential distribution on the stator surface, the resulting formula for the integral (14) makes it possible to calculate the torque of the studied motor through the electromagnetic forces appearing due to interaction of magnetic field in the gap with the rotor surface elements. This definite integral can be easily calculated in the MathCad system. As an example, some calculation results of the electromagnetic torque dependence on the torque angle for the reluctance type motor are shown in figure 6.

![Figure 6](image)

**Figure 6.** Angular characteristic of the motor with modulated magnetic flux and smooth rotor

In this case the direct-sequence currents flow through the motor windings. It is obvious that using the obtained expressions, it is possible to calculate the electromagnetic torque for any laws of phase current variation.

4. References

[1] Shevchenko A F and Shevchenko L G 2000 New electrical motor with variable air gap conductivity (SR- motor) for high speed electric drives _Elektrotehnika_ 11 20-3
[2] Shevchenko A F and Shevchenko L G 2002 Magnetic field and electromagnetic torque of synchronous motor with modulated magnetic flux and smooth harmonic rotor _Russian Electrical Engineering_ 73(7) 57
[3] Bertinov A I and Varley V V 1969 _Rolling Rotor Motors_ (Moscow: Energiya)
[4] Ray W F, Lawrenson P J, Davis R M, Stephenson J M, Fulton N N and Blake R J 1986 High-performance switched reluctance brushless drives _IEEE Transactions on Industry Applications_ IA-22(4) 722-730
[5] Lawrenson P 1992 A brief status review of switched reluctance drives _EPE Journal_ 2(3) 133-134
[6] Shevchenko A F and Chestyunina T V 2009 Analysis of magnetomotive forces of fractional-slot windings of electrical machines Russian Electrical Engineering 80(12) 641

[7] Ivanov–Smolensky A V 1989 Electromagnetic Forces and Energy Transformation in Electrical Machines (Moscow: Vysshaya Shkola)

[8] Lavrentiev M A and Shabat B V 1965 Methods of the Theory of Functions of a Complex Variable (Moscow: Nauka)

[9] Bornshtein I N and Semendyaev K A 1986 Mathematics Handbook (Moscow: Nauka)