A DIRECT CONSTRAINT ON THE GALACTIC ACCELERATION AND THE OORT LIMIT FROM PULSAR TIMING

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Draft version October 9, 2020

ABSTRACT

We use compiled high-precision pulsar timing measurements to directly measure the Galactic acceleration. We compare the results to static models of the Milky Way, as well as to interacting simulations. Given the accelerations, we use the Poisson equation to derive the Oort limit, which can provide a measure of the dark matter density, given an accounting of the baryon budget. Our best-fitting model gives a mid-plane total density of $0.08^{+0.05}_{-0.02} M_\odot/pc^3$, which is close to, but lower than the estimate from recent Jeans analyses. Given recent accounting of the baryon budget, this also implies a lower value of the local dark matter density. We also find a constraint for the oblateness of the potential that we express in terms of commonly used potentials. The comparison suggests that the pulsars are tracing the oblateness of the disk rather than the halo. We give a fitting function for the vertical acceleration $a_z$: $a_z = -\alpha_1 z; \log_{10}(\alpha_1/Gyr^{-2}) = 3.69^{+0.19}_{-0.12}$. By analyzing interacting simulations of the Milky Way, we find that variations in $da_z/dz$ as a function of vertical height may be a signature of sub-structure. We end by discussing the power of combining constraints from pulsar timing and high-precision radial velocity (RV) measurements towards lines-of-sight near pulsars, to test theories of gravity and constrain dark matter sub-structure.

1. INTRODUCTION

By serving as precise astrophysical clocks, pulsars have been used in many tests of fundamental physics (see e.g., Will [2014]). Among these tests, pulsars can enable the detection of the cosmological gravitational wave background (see e.g., Burke-Spolaor et al. [2019]) and provide constraints on the nature of gravity (e.g., Weisberg & Huang [2016], Zhu et al. [2019]). Here, we explore the idea that pulsars with precisely measured binary orbital periods can serve as effective accelerometers that can be used to directly measure the Galactic acceleration.

It has been proposed that high precision radial velocity (RV) measurements can be used to directly measure the Galactic acceleration (Silverwood & Easther [2019], Ravi et al. [2019], Chakrabarti et al. [2020]). It is essential to quantify the contamination from planets and binaries to the Galactic RV signal, and in Chakrabarti et al. [2020], we showed that even for modest sample sizes, we can reliably expect to extract the Galactic signal, despite the presence of planets and binaries in a realistic Galactic population. Time-dependent potentials as in interacting simulations of the Milky Way lead to differences in the vertical acceleration relative to static models, especially at heights $|z| > 1$ kpc relative to the Galactic midplane (Chakrabarti et al. [2020]). Prior work has focused mainly on kinematical analysis (Kuijken & Gilmore [1989], Holmberg & Flynn [2000], Bovy & Tremaine [2012]) of various stellar tracers to estimate the Galactic acceleration rather than directly measuring it. The analysis of an interacting simulation of the Milky Way by Haines et al. [2019] indicates that there are differences in the true density in the simulation relative to that determined from kinematics (such as in the Jeans approximation, which assumes spherical symmetry and equilibrium), especially for perturbed regions of the disk. In view of the dynamically evolving picture of the Galaxy as manifested by Gaia data (Helmi et al. [2018]), kinematic estimates should be tested against direct measurements of the acceleration.

Here, we analyze line-of-sight accelerations of fourteen pulsar systems in binaries that have precise measurements of their orbital periods ($P_b$) and rate of change in the orbital period ($\dot{P}_b$). We determine the radial and vertical Galactic accelerations by fitting a low-order polynomial to the data. Given these accelerations, we use the Poisson equation to determine the mid-plane density, and accounting for the baryon density from recent work (McKee et al. [2015], Bienaymé et al. [2014]), we then determine the local dark matter density. Our measurement of the local dark matter density can be used to interpret direct detection measurements of dark matter to ultimately understand the nature of the dark matter particle (Read [2014]). We also compare the line-of-sight accelerations to static models, as well as to interacting simulations of the Milky Way.

Pulsar timing has previously been used to infer the potential in globular clusters (Prager et al. [2017]), and very recently for the Galaxy (Phillips et al. [2020]). We draw attention to the work by Phillips et al. [2020], which is contemporaneous with ours. A key difference in our work arises from our analysis of orbital periods (rather than spin periods), as well as our inclusion of both the vertical and radial components of the acceleration. Phillips et al. [2020]’s value of the acceleration corresponds to a veloc-
ity for the local standard of rest $V_{\mathrm{LSR}} \sim 350 \text{ km/s}$. This value is at odds with the value determined by Quillen et al. (2020) using the Galactocentric radius of the Sun measured by the GRAVITY collaboration et al. (2018), the proper motion of the radio source associated with Sgr A*, and the tangential component of the solar peculiar motion by Schönrich et al. (2010), which gives $233.3 \pm 1.4 \text{ km/s}$. The value in Quillen et al. (2020) is consistent with the measurement using trigonometric parallaxes of high-mass star formation regions from Reid et al. (2019). The discrepancy may be due to their statistical analysis of spin periods rather than the direct analysis that can be done for orbital periods. The current distribution of pulsars with precisely measured $P_0$ corresponds to approximately a square kpc in area. A small area coverage like this provides significantly more leverage in measuring gradients in vertical accelerations than radial accelerations. Thus, while we solve for both components of the acceleration simultaneously, we will focus here on vertical accelerations.

This paper is organized as follows. In §2.1, we review the properties of the pulsars we have selected here, and our method for determining Galactic accelerations from pulsar timing data. We compare the line-of-sight accelerations of the pulsars to various static models and give the best-fit values in §2.2. Here, we also present our values for the Oort limit, the local dark matter density, and a parameter that is sensitive to the oblateness of the potential. In §2.3, we compare the results to interacting simulations. We discuss some additional implications of our work and conclude in §3.

2. ANALYSIS AND RESULTS

2.1. Pulsar Timing Measurements

We select binary pulsars from the ATNF pulsar catalogue (Manchester et al. 2005) that have precisely measured $P_0$ (within 2-sigma), distances, and proper motions (either from pulsar timing or very-long-baseline interferometry, VLBI). We do not include pulsars (i) in globular clusters where the additional accelerations induce a change to the observed $P_0$, (ii) in systems undergoing ablation or mass transfer that changes the orbital parameters, or (iii) without parameter uncertainties. Our sources along with their parameters are provided in Table 1.

For some sources, there are multiple measurements of the observed binary period $P_{0,\text{obs}}$ reported. In that case, we choose the data set with lowest uncertainty on $P_{b,\text{obs}}$ and use the other timing model parameters from that data set required for our analysis. Additionally, for some sources, there are multiple measurements of the parallax, e.g., timing parallax and VLBI measurements, and we adopt the parallax value with the lowest uncertainty. In the case of PSRs J0737–3039A/B and J2222–0137, where insufficient astrometric information was measured, we used the parallaxes and proper motions derived from VLBI for the purpose of improving gravitational tests with these systems (Deller et al. 2009, 2013). Since all of our sources are within ~ kpc of the Sun, we cannot obtain constraints on multi-component potentials. The Hulse-Taylor system (Weisberg & Huang 2016) is at present the only source that is at a larger radial distance. We do not include it currently in our analysis as a single source does not help in constraining global potentials, and therefore we focus on the simple potentials we outline below.

Acceleration due to the Galactic potential will impact observed pulsar parameters, namely the spin period and orbital period for those in binaries. Measurements of the Galactic acceleration by use of observed spin periods are statistical in nature since they require knowledge of the intrinsic distribution of spin periods and spin-downs whereas the use of binary orbital periods do not.

For a binary system in the Galaxy not undergoing mass transfer, we may write the observed orbital period drift rate $\dot{P}_{b,\text{obs}}$ as:

$$\dot{P}_{b,\text{obs}} = \dot{P}_{b,\text{Gal}} - \dot{P}_{b,\text{Shk}} - \dot{P}_{b,\text{GR}},$$

(1)

where $\dot{P}_{b,\text{Gal}} = P_b a_{\text{Gal}}/c$ is the rate induced by the Galactic potential, $a_{\text{Gal}}$ is the relative line-of-sight Galactic acceleration between the solar system and the pulsar, $c$ is the speed of light, and $\dot{P}_{b,\text{Shk}}$ is the apparent drift rate caused by the binary’s transverse motion (known as the Shklovskii effect; Shklovskii 1970; Damour & Taylor 1991), which is given by:

$$\dot{P}_{b,\text{Shk}} = \mu^2 d P_b / c,$$

(2)

for a system at distance $d$ with a proper motion $\mu$. The term $\dot{P}_{b,\text{GR}}$ describes the rate at which the system is losing energy due to gravitational radiation (Weisberg & Huang 2016), and can be computed given the orbital period, eccentricity $e$, and the masses of the pulsar $m_p$ and its companion $m_c$ (determined from Shapiro delay; Shapiro 1964) as

$$\dot{P}_{b,\text{GR}} = -\frac{192\pi G m_p^5}{5c^5} \left( \frac{P_b}{2\pi} \right)^{-5/3} \left( 1 - e^2 \right)^{-7/2} \times \left( 1 + \frac{73}{24} e^2 + \frac{37}{96} e^4 \right) \frac{m_p m_c}{(m_p + m_c)^{7/3}}.$$

(3)

Given these terms, we can then calculate the line-of-sight Galactic acceleration, $a_{\text{Gal}}$ as:

$$a_{\text{Gal}} = \dot{P}_{b,\text{Gal}} - \dot{P}_{b,\text{Mod}}.$$

(4)

We define the observed line-of-sight acceleration, $a_{b,\text{LOS}}$, as

$$a_{b,\text{LOS}} = \frac{c\dot{P}_{b,\text{obs}}}{P_b}.$$

(5)

This is simply a redefinition of the observed binary period drift rate $\dot{P}_{b,\text{obs}}$. As a result, it cannot be compared to a true acceleration as it includes both the Shklovskii effect and secular GR effects, $\dot{P}_{b,\text{GR}}$. Likewise, we also compute a model line-of-sight acceleration, $a_{b,\text{LOS}}$, that includes these additional effects, which we compare to the observed values.

2.2. Comparison of pulsar timing data with static models of the Milky Way

Figure 1 shows the fractional difference between the model line-of-sight acceleration $a_{b,\text{LOS}}$ for various static potentials and the observed values ($a_{b,\text{LOS}}$) for all the pulsars in our sample. As these pulsars currently cover a
The radial component may be written: 

$$ \Phi(R,z) = \frac{1}{2} \alpha_1 z^2 + \frac{1}{3} \alpha_2 |z|^3 $$

and the components of the acceleration as:

$$ a_R = \frac{\partial}{\partial R} \Phi(R,z), \quad a_z = -\frac{\partial}{\partial z} \Phi(R,z) $$

for an axisymmetric potential. For this and all other potentials, we fit for the vertical and radial accelerations simultaneously. We refer to the $\beta = 0$, $\alpha_2 = 0$ case as the $\alpha_1$ model, the $\beta = 0$, $\alpha_2 \neq 0$ as the $(\alpha_1, \alpha_2)$ model, and the $\beta \neq 0$, $\alpha_2 = 0$ case as the $(\alpha_1, \beta)$ model in Table 2.

We also consider an exponential disk model of the form $\Phi = \rho_0 \exp(-|z|/\zeta_0)$, as well as the Hernquist potential (Hernquist 1990), where $M_h$ and $a_H$ are the mass normalization and scale length respectively for the Hernquist potential. We also compare to the MWPotential2014 model that was presented by Bovy (2015), which is denoted “MWP” in Table 1. Finally, we consider a variant of the potential given in Eqs. [6] and [7] and introduce a cross-term:

$$ \Phi(R,z) = V_{\text{LSR}}^2 \ln(R/R_\odot) + (R/R_\odot)\gamma z^2 + \frac{1}{2} \alpha_1 z^2. $$

This model assumes that the potential is symmetric about the Galactic plane and expands to second order in $z$. To first order in $R - R_\odot$ we can write $\ln(R/R_\odot) \sim (R - R_\odot)/R_\odot$. We discuss below the sensitivity of $\gamma$ to the oblateness of the potential. We refer to this model as the “cross”-term model.

We use the Markov-chain Monte Carlo (MCMC) code emcee (Foreman-Mackey et al. 2013) to explore the likelihood distribution of the data. The log likelihood function is given by:

$$ \log(L) = \log(P(\theta)) - \sum_i \frac{(a_{\text{Obs}} - a_{\text{Mod}})^2}{2\sigma_i^2}. $$
where \( \log(P(\theta)) \) is the log prior on the parameters, \( \theta \), \( N \) is the number of pulsars, and \( \sigma_i \) are the uncertainties. The number of parameters used are \( k + 3N \), where \( k \) is the number of parameters used in the various galactic models. The three parameters that we use per pulsar are the parallax, e.g., distance, proper motion, \( \mu \), and the secular GR effect, \( P_{GR} \). As these parameters have constraints on them, we use a log prior of the form 
\[
-\frac{(\theta_i - \theta_i, \text{Obs})^2}{\sigma_i, \text{Obs}^2}
\]
where \( \sigma_i, \text{Obs} \) is the published 1-\( \sigma \) error on these measurements. For the \( k \) parameters used in galactic models, we choose a flat distribution, but test its effects on our results. Thus, in the MCMC calculation of the posterior distribution, we incorporate uncertainties in the measured \( \dot{P}_{\text{Obs}} \) as well as uncertainties in terms that affect the calculation of the Shklovskii term (the distance and proper motion uncertainties) and the uncertainties in the calculation of \( \dot{P}_{GR} \) (i.e., the uncertainties on the mass of the pulsar and its companion and the eccentricity).

As shown in Figure 1, the agreement between models and the observations are mostly within the errors of the measured uncertainties; those outside the measured uncertainties are within 2\( \sigma \). Table 1 lists the best-fit parameters for the range of models we have considered here, along with the Akaike Information Criteria (AIC; Akaike 1974), which is given by:
\[
\text{AIC} = -2 \ln L + 2k \tag{11}
\]
where \( L \) is the likelihood and \( k \) is the number of parameters in the model. The model with the lowest AIC is considered better at describing the data. A \( \Delta \text{AIC} \) of 2 is considered positive evidence in favor of the model with the lower AIC, while a \( \Delta \text{AIC} \) of 6 indicates strong evidence (Kass & Raftery 1995). The \( \alpha_1, (\alpha_1, \beta), \) and \( (\alpha_1, \gamma) \) models all have a best fit value of \( \log_{10}(\alpha_1/\text{Gyr}^{-2}) \approx 3.6 - 3.8 \). Our best-fit value for \( \alpha_1 \) (which describes the frequency of low-amplitude vertical oscillations) is close to a recent estimate by Quillen et al. (2020) to match the data presented from the Jeans analysis by Holmberg & Flynn (2000). We do not obtain constraints on \( \beta \), the slope of the rotation curve, though the best-fit values are comparable to recent works (Li et al. 2019; Mróz et al. 2019). It is not surprising that we do not obtain a constraint for \( \beta \) as our radial range is restricted to \( \sim 1 \) kpc.

We may express a log-oblate (LO) potential with a core as:
\[
\Phi_{\text{LO}}(R, z) = \frac{V_{\text{LSR}}^2}{2} \ln \left( \frac{R^2}{R_\odot^2} + \frac{z^2}{q^2 R_\odot^2} + \frac{d^2}{R_\odot^2} \right) \tag{12}
\]
where \( d \) is the core size and \( q < 1 \) gives an oblate potential. A second-order expansion in \( z \) and first-order expansion in \( R \) about \( R_\odot \) gives:
\[
\gamma_{\text{LO}} = -\frac{V_{\text{LSR}}^2 R_\odot^2}{R_\odot^2 + d^2} \tag{13}
\]
Evaluating this term for a log-spherical potential with
\[ d = 0 \text{ gives } \log_{10} (\gamma_{\text{LO}} / \text{Gyr}^{-2}) = -2.93, \text{ for } V_{\text{LSR}} = 233.3 \text{ km/s}. \] 

For the Miyamoto-Nagai (MN) disk:

\[ \Phi_{\text{MN}}(R, z) = \frac{-GM_d}{\sqrt{R^2 + (a + \sqrt{z^2 + b^2})^2}} \]  

(14)

where \( M_d, a, b \) are the mass of the disk and the scale lengths respectively. By expanding this potential to second order in \( z \) near \( z = 0 \) and to first order in \( R \) near \( R_0 \), one can show that the oblateness parameter for the Miyamoto-Nagai disk can be written as:

\[ \gamma_{\text{MN}} = \frac{GM_d}{b} \frac{a + b}{(R_0^2 + (a + b)^2)^{5/2}} \frac{3R_0^2}{2} \]  

(15)

Evaluating this quantity using the values listed in [4] and [5], i.e., \( M_d = 10^{11} M_\odot \), \( b = 0.26 \text{ kpc} \), \( a = 6.5 \text{ kpc} \), gives \( \log_{10} (\gamma_{\text{MN}} / \text{Gyr}^{-2}) = -3.94 \), which is closer to our best-fit value for \( \gamma \), suggesting that the pulsars trace the oblateness of the disk rather than that of the halo.

We do not find clear trends in the residuals as a function of radius and vertical height for the potentials considered here. Models that are not symmetrical about the galactic plane (due to a warp or a lopsided mass distribution) or are non-axisymmetric may be constrained in future studies. While our focus here has been in measuring the acceleration with a small sample of pulsars, direct acceleration measurements have the potential to provide a clear view of dark matter sub-structure for a larger sample of pulsars, with higher precision measurements.

2.3. The Oort limit from pulsar timing

The Oort limit, or the volume mass density at the Galactic mid-plane, has traditionally been determined using kinematical tracers of the gravitational field ([8], Holmberg & Flynn 2000, which assume spherical symmetry and equilibrium. Using Eq. 7 and Poisson’s equation applied in the mid-plane at \( R_0 \), we can determine the frequency of low-amplitude vertical oscillations:

\[ \nu^2 = \left. \frac{d^2 \Phi}{dz^2} \right|_{z=0, R=R_0} = 4\pi G \rho_0 - 2\beta \Omega^2_0 \]  

(16)

where \( \rho_0 \) is the mid-plane mass density and we have used the potential of equation 6 for the radial derivative terms. Using the values of \( \alpha_1 \) and \( \beta \) from Table 1, we obtain an Oort limit of 0.080 \pm 0.003 M_\odot /pc^3, which value of the Oort limit is close to, but somewhat lower relative to recent estimates using the Jeans equation ([9], Holmberg & Flynn 2000). Considering the baryon budget found by [9], or 0.084 \pm 0.012 M_\odot /pc^3, we obtain a local dark matter density \( \rho_{\text{DM}} = -0.004 \pm 0.003 M_\odot /pc^3 \), which is lower than, but within the range of prior work by [9]. It is close to but lower than the work by [9], who found \( \rho_{\text{DM}} = 0.013 \pm 0.003 M_\odot /pc^3 \). It is also consistent with having no dark matter in the mid-plane. Using the values of the baryon density from [9], of 0.077 \pm 0.007 M_\odot /pc^3 gives \( \rho_{\text{DM}} = 0.003 \pm 0.003 M_\odot /pc^3 \). While the uncertainties on these values are large, our analysis does suggest that \( \rho_{\text{DM}} \) from the Jeans estimate may be an overestimate. Improving the uncertainties on the Oort limit would allow us to directly determine the viability of disk dark matter models ([10], 2014).

2.4. Comparison of pulsar timing data with interacting simulations of the Milky Way

Figure 3 depicts a comparison of the quantity \( \alpha_1 / \alpha_1 \) from simulations of the Antlia 2 dwarf interacting with the Milky Way and the Sgr dwarf from [9], compared to our fit for \( \alpha_1 \), the value for \( \alpha_1 / \alpha_1 \) for the static Hernquist potential with \( M_b = 2 \times 10^{12} M_\odot \) and \( a_M = 30 \text{ kpc} \), and for the MWPotential2014 model from [11]. As is clear, \( \alpha_1 / \alpha_1 \) is not a constant for interacting simulations and varies relative to the Galactic mid-plane. A larger and more densely populated sample of pulsars should be able to trace the deviation of \( \alpha_1 / \alpha_1 \) from a constant value, which is a signature of sub-structure, either due to interactions with dwarf galaxies, or dark matter sub-structure.

3. DISCUSSION & CONCLUSION

We discuss here briefly additional implications of our work. Pulsar timing measurements have been analyzed to constrain general relativity and alternate theories of gravity, most notably in the consistency of gravitational radiation (e.g., [12], [13], [14] but also in tests of the strong equivalence principle (e.g., [15], [16], and the time-variability of the gravitational constant, \( G \) ([17], [18], [19] while assuming a Galactic potential that is derived from kinematical analysis (for example, [20]). Obtaining high-precision RV measurements towards lines-of-sight with pulsars can enable us to determine the Galactic potential via this complementary approach, which can thereby provide significantly more precise constraints on the measurements described above and constrain theories of gravity. Although the uncertainties in fits for the time rate of change of the orbital period for binary pulsars due to gravitational radiation...
have improved (for the Hulse-Taylor system they are now within $\sim 1$-sigma of the value predicted by relativity), they are currently dominated by the assumed values for the Galactic potential (Weisberg & Huang 2016). Direct measurement of the potential would provide more robust constraints in these tests of gravity.

Our focus here and in prior work (Chakrabarti et al. 2020) is on relative accelerations, i.e., accelerations measured relative to the Sun. The solar acceleration has been measured by VLBI observations (Xu et al. 2012; Titov & Lambert 2013; Titov & Krausn 2018). Zakamska & Tremaine (2005) have discussed the intriguing possibility of obtaining constraints on undiscovered planets or distant stellar companions from the acceleration of the solar system barycenter using pulsar timing observations. The effect of a distant giant planet as in the work by Batygin & Brown (2016) or that of the nearest stars is given current measurement uncertainties.

We summarize our main findings below:

- By fitting a low-order polynomial for the Galactic potential to line-of-sight accelerations of fourteen binary pulsar systems, we infer an Oort limit of $0.08_{-0.02}^{+0.05} M_\odot/pc^2$. Given the baryon budget from McKee, Parravano, & Hollenbach (2015), this gives $\rho_{DM} = 0.003_{-0.02}^{+0.05} M_\odot/pc^2$; for the baryon budget from Bienaymé et al. (2014), $\rho_{DM} = 0.0034_{-0.02}^{+0.05} M_\odot/pc^2$. The uncertainties in the local dark matter density are mainly due to the current uncertainty in the Oort limit from pulsar timing. A larger sample of pulsars, with higher precision measurements (not only for $P_{\text{orb}}$ but also for the distances and proper motions) would serve to improve the precision of this measurement.

  - The vertical acceleration profile can be described by $a_z = -\alpha_1 z$; our best-fit value for $\alpha_1$ is $\log_{10}(\alpha_1/\text{Gyr}^{-2}) = 3.60_{-0.12}^{+0.19}$.
  - The data imply an additional constraint on an oblateness parameter, $\log_{10}(\gamma/\text{Gyr}^{-2}) = -4.9_{-1.1}^{+0.9}$. This value of $\gamma$ is closer to that for disk models (which have larger $\gamma$) than halo models, which suggests that the pulsars trace the oblateness of the disk rather than the halo.
  - Our analysis of dynamical simulations suggests that dark matter sub-structure or interactions with dwarf galaxies may manifest as deviations in $da_z/dz$ from a pure polynomial fit (such as our $\alpha_1$) or static models. Nevertheless, the average value of $da_z/dz$ in the simulations we have considered here is close to our fit for $\alpha_1$.
  - The measurement of the Galactic acceleration using high precision RV observations near pulsars can provide significantly more precise constraints on $\mathcal{F}_{GR}$, $G$, and other post-Newtonian parameters than has been obtained thus far (for which prior work has assumed pre-formulated potentials that employ kinematic estimates).

SC gratefully acknowledges support from the RCSA Time Domain Astrophysics Scialog award, NASA ATP NNX17AK90G, NSF AAG 2009574, and from the Institute for Advanced Study. PC is supported by the NASA ATP program through NASA grant NNH17ZDA001N-ATP. MLT and SJV are members of the NANOGrav project, which receives support from NSF Physics Frontiers Center award number 1430284. MLT also acknowledges support from NSF AAG 2009468. SC thanks S. Tremaine, J. Goodman, and J. Wright for helpful discussions on the solar acceleration and R. Rafikov on pulsars.

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