Research of fracture of materials and structures under shock-wave loadings by means of the program complex EFES

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Abstract. The results of coordinated experimental and numerical studies of fracture of materials and structures under impact are given in the present paper. Numerical simulation is carried out by the author finite element software package EFES, allowing to simulate a three-dimensional setting behavior of complex structures under dynamic loads. Fracture of metallic materials and structures are investigated in the speed range of interaction 50–3000 m/s.

1. Introduction
Currently there are several commercial program software based on parallel computations and used for calculation of stress-strain state and materials and structures fracture under dynamic loading, among them are ANSYS, ABAQUS, LS-DYNA. Software developers try to endow them with multiple-purpose functions and adjust software for solving a wide range of objectives. Against this background it leads to “complication” of the algorithm and loss of efficiency. Loss of efficiency in this case is conditioned by impossibility to make all the processes parallel. In particular this refers to modeling of dynamic fracture of geometrically complex structures with a large number of contacting boundaries [1].

Compared with ANSYS, ABAQUS, and LS-DYNA programs, EFES software developed by the authors has a number of significant advantages:

- it uses original algorithm which is optimized to the maximum extent to calculate the contacting boundaries; this is especially relevant while analyzing the structures of complex geometry;
- it implements the mechanism of “erosion damage” of contacting elements, thus enabling to keep the regularity of finite element grid at the acceptable integration step;
- there are no limitations on the number of processors (cores) and the number of finite elements; as a rule there are such limitations in ANSYS, ABAQUS, LS-DYNA.

2. Mathematical model
Program software is based on three-dimensional approach to behavior of materials and structures. System of equations describing non-stationary adiabatic movements of compressible...
medium in general coordinates \((i = 1, 2, 3)\) includes the following equations \([2,3]\):

\[
\frac{d\rho}{dt} + \rho \nabla_i v^i = 0, \tag{1}
\]

\[
\rho a^k = \nabla_i \sigma^{ik} + F^k, \tag{2}
\]

\[
\frac{dE}{dt} = \frac{1}{\rho} \sigma^{ij} e_{ij}, \tag{3}
\]

Here (1) is continuity equation; (2) is motion equation; (3) is energy equation. In these equations next notation is used:

\[
a^k = \frac{\partial v^k}{\partial t} + v^i \nabla_i v^k, \quad \nabla_i \sigma^{ik} = \sigma^{ik} + \Gamma^k_{im} \sigma^{im} + \Gamma^m_{im} \sigma^{ik},
\]

\[
e_{ij} = \frac{1}{2} (\nabla_i v_j + \nabla_j v_i),
\]

where \(F^k\)—components of mass force vector; \(\Gamma^k_{ij}\)—Christoffel symbols; \(\sigma^{ij}\)—contravariant components of symmetric stress tensor; \(E\)—specific internal energy; \(e_{ij}\)—components of symmetric strain velocity tensor; \(\rho\)—density of medium; \(v^i\)—components of velocity vector; \(a^k\)—components of acceleration vector.

### 2.1. Elasto-plastic model of isotropic materials behavior

Stress tensor is given as a sum of deviatoric \(S^{ij}\) and spherical part \(P\):

\[
\sigma^{ij} = -P g^{ij} + S^{ij}, \tag{4}
\]

where \(g^{ij}\)—metric tensor. Pressure in the material was calculated using Mie–Grüneisen equation as the function of specific internal energy \(E\) and density \(\rho\):

\[
P = \sum_{n=1}^{3} K_n \left( \frac{V_0}{V} - 1 \right)^n + K_0 \rho E, \tag{5}
\]

where \(K_0, K_1, K_2, K_3\)—constants of material. \(V_0\)—initial specific volume, \(V\)—current specific volume.

Suppose that the principle of minimum work of true stresses on the increments of plastic deformations is true for the medium, then the connection of component of strain velocity tensor and stress deviator is as follows:

\[
2G \left( g^{jm} g^{ik} e_{mk} - \frac{1}{3} g^{mk} e_{mk} g^{ij} \right) = \frac{DS^{ij}}{Dt} + \lambda S^{ij}, \quad (\lambda \geq 0). \tag{6}
\]

Here time derivatives of the stress tensor components are accepted according to Jau mann:

\[
\frac{DS^{ij}}{Dt} = \frac{dS^{ij}}{dt} - g^{jm} \omega_{mk} S^{kj} - g^{jm} \omega_{mki} S^{ik},
\]

where \(\omega_{ij} = \frac{1}{2} (\nabla_i v_j - \nabla_j v_i)\), \(G\)—shear modulus.

We consider that material behaves elastic \((\lambda = 0)\), if von Mises condition is followed:

\[
S^{ij} S_{ij} \leq \frac{2}{3} \sigma_d^2, \tag{7}
\]
material behavior is considered plastic ($\lambda > 0$), when von Mises criterion is not followed [4]. In this case $\sigma_d$—dynamic tensile yield stress, that can be in the general case the function of rate of deformations, pressure and temperature. In case the condition (7) is violated, we apply the procedure of correction of stresses considering the material plasticity for calculation of component of stress deviator. For this purpose components $S_{ij}$ are multiplied by normalizing factor. It is actually the same as the description of medium behavior in plastic zone as proved by equations of Prandtl–Reiss theory.

The limiting value of plastic deformations intensity is accepted as a local criterion of shear fracture:

$$e_u < \frac{\sqrt{2}}{3} \sqrt{3T_2 - T_1^2},$$

where $T_1, T_2$—first and second invariants of deformation tensor.

Numerical simulation is conducted by finite element method [5] using author’s algorithm [6]. The contact surfaces are calculated according to [7]. The final elemental mesh is created by the open source mesh generator–NETGEN [8]. Effective application of parallel computing is achieved by using an OpenMP technology and by vectorization using modern processor instructions SSE and AVX.

3. Problem statement

We consider the interaction of $n$ bodies in general, three-dimensional case in Cartesian reference system $XYZ$ (figure 1). Each body is of a given shape and occupies the zone $D_k$ ($k = 1, 2, 3, \ldots, n$), limited by the surface $\Sigma_k$ correspondingly. Surfaces $\Sigma_k$ are divided into subsurfaces of free $\Sigma_k^\text{free}$ and contact $\Sigma_k^\text{cont}$ surfaces. Velocity vector of projectile equals $\overline{u}$. Cosines of angles between velocity vector and coordinates axes equal correspondingly $l, m, n$.

3.1. Initial conditions

Following initial ($t = 0$) conditions are used:

$$\sigma^{ij} = P = E = 0,$$

$$u = u_0 l, \quad v = v_0 m, \quad w = v_0 n,$$

$$u = v = w = 0,$$

$$\rho = \rho_k,$$

if $(x, y, z) \in D_k$, (9)

if $(x, y, z) \in D_l$, (10)

if $(x, y, z) \in D_{i=2,3,\ldots,n}$, (11)

if $(x, y, z) \in D_k$. (12)

Here $u, v, w$ are components of velocity vector along axes $X, Y, Z$ correspondingly.
3.2. Boundary conditions

Free surfaces are characterized by the conditions

$$T_{nn} = T_{ns} = T_{n\tau} = 0, \quad \text{if } (x, y, z) \in \Sigma_k^{\text{free}}. \quad (13)$$

Contact surfaces are characterized by slipping conditions, friction-free:

$$v_n^+ = v_n^-, \quad T_{nn}^+ = T_{nn}^-,$$

$$T_{n\tau_1}^+ = T_{n\tau_1}^-, \quad T_{n\tau_2}^+ = T_{n\tau_2}^-,$$

$$\text{if } (x, y, z) \in \Sigma_k^{\text{cont}}. \quad (14)$$

Here $\pi$—unit vector that is normal to the surface at the considered point, $\tau$ and $\tau$—unit vectors, tangent to surface in this point, $T_n$—force vector on site with normal $\pi$, $\tau$—velocity vector. Suffix numbers of vectors $T_n$ and $\tau$ indicate projections on the corresponding basis vectors; plus sign “+” characterizes the value of parameters in material at the upper boundary of contact surface, minus sign “−” characterizes at the lower boundary.

Therefore, the system of equations (1)–(8) together with initial and boundary conditions (9)–(14) solves boundary-value problem completely.

4. Calculation results and their comparison with experimental data

Experimental studies of the interaction of the projectiles with spaced barriers were carried out on the basis of the ballistic complex of the Second Central Research Institute of the Russian Ministry of Defense. The stand for tests contains ballistic unit, a complex of the measuring, registering and synchronizing equipment. The projectiles velocity before first barrier was registered by an electrocontact method, further by using piezoelectric sensors.

Figure 2 shows computational configuration of projectile in section, which is sphere-shaped made from steel ShKh15, of diameter 12.7 mm and barrier made from alloy D16T, 4.9 mm thick at sequential points in time. Initial velocity of projectile is 1001 m/s, velocity vector makes angle of 60° with normal to the barrier. Calculations demonstrate the dynamics of barrier fracture and formation of fragment field at the back surface of the barrier.
Figure 3 demonstrates experimental and calculated hole in the barrier after interaction with projectile. Experimental and numerical results find good qualitative and quantitative correspondence. The largest diameter of the hole during the experiment comprises 26.6 mm, the calculated value comprises 26.9 mm (relative discrepancy $\delta = 1.1\%$). The value of projectile velocity after barrier penetration makes 843 m/s, the calculated value makes 846 m/s ($\delta = 0.4\%$).

Figure 4 shows calculated configurations of sphere-shaped projectile under normal interaction with spaced barrier from eight plates. Projectile is made from steel ShKh15; material of spaced barrier is alloy D16T. Plates in the barrier are 2.8 mm thick, projectile diameter is 9.5 mm. Initial velocity of projectile is 1010 m/s. In this case the projectile penetrates seven barriers and stops at the eighth one, forming crater inside it. Crater diameter comprises 8 mm in the experiment and 7.2 mm in the calculation ($\delta = 10\%$). Experimental and computational results of projectile velocity after penetration of the first barrier have been also compared: the experimental value is 925 m/s, the calculated one is 912 m/s ($\delta = 10\%$). Diameter of the hole in the first barrier in the experiment makes 9.4 mm, in calculation—9.5 mm ($\delta = 1\%$).

Figure 5 shows calculated dependency of center-of-mass velocity of projectile in time. It is characterized by row echelon form; areas of heavy breaking correspond to the period of projectile interaction with plates of spaced barrier.

Experimental and numerical studies have been conducted on interaction of projectile with spaced barrier made from steel. Figure 6 shows in section the calculated configurations of interaction of projectile made from steel ShKh15, diameter 12.7 mm with spaced barrier, comprising four plates made from steel, each plate is 3.9 mm thick. The impact occurs along the normal, initial velocity is 1600 m/s. Through-penetration of all the plates is not observed both in the experiment and in the computation, projectile decelerates at the fourth plate, forming a crater inside it. Final picture of barrier fracture is given in figure 7: calculated section of the spaced barrier (figure 7a), pictures of the first plate (figure 7b) and forth plate (figure 7c) after interaction with projectile. Diameter of the hole in the first plate in the experiment made 17.7 mm, in the calculation—18.6 mm ($\delta = 5.1\%$), the depth of crater in the fourth plate in the experiment is 2 mm, as calculated—2.3 mm ($\delta = 15\%$).

Figure 8 shows the dependency of the center-of-mass velocity on time and enables to evaluate the deceleration dynamics of projectile during interaction with spaced barrier.

We further consider application of program software to calculation of real structures on the example of shell structure that simulates ballistic missile. Shell structure has several chambers divided by partitions, shell is made of steel. Shell and partitions are 2 mm thick. Figure 9 shows calculated configuration of the shell and stress field $\sigma_{xx}$ (in Pa) at its falling flatwise on rigid base with velocity of 200 m/s at the time period 300 ms in a volume and longitudinal cross section. Calculations enable to evaluate response to load, both of the structure on the whole and work of different structural elements. Interaction of structure with elongated steel projectile is given in figure 10. Initial velocity of projectile is 1 km/s, elongation 15 calibres, angle of interaction 60°.
Figure 4. Calculated configurations of projectile and spaced barrier.

Figure 5. Time variation of center-of-mass velocity of projectile.
Figure 6. Calculated configurations of projectile and spaced barrier.

Figure 7. Final picture of barrier fracture: (a)—calculation; (b)—experiment, first plate; (c)—experiment, fourth plate.

Figure 8. Time variation of the center-of-mass velocity of projectile.
Figure 9. Configuration of shell and stress distribution $\sigma_{xx}$ (in Pa) $t = 300$ ms.

Figure 10. Interaction of elongated projectile with the shell. Longitudinal cross section.
5. Conclusion
Conducted research proved correspondence of the suggested models and calculation algorithm. Program software EFES can be used to carry out wide-parametric numerical experiments to study materials properties and design of structures under dynamic loads.

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