"Möbius" Microring Resonator

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A new category of optical microring resonator, which is analogous to the Möbius strip, is proposed. The "Möbius" microring resonator allows the conversion between modes with different polarizations in the ring and light must circulate two cycles to be converted back to the original polarization status, which is similar to a Möbius strip. This topology structure of polarization leads to the free spectral range be half of that corresponds to the cavity round trip. The eigenmodes of this microring are hybridizations of different polarizations and the breaking of the rotation invariance of the ring makes the transmission be related with the polarization of input light. Our work opens the door towards the photonic devices with nontrivial mode topology and provides a novel way to engineering photonic structures for fundamental studies.

Over the past decades, photonic integrated circuits (PICs) have been extensively studied [1–3]. The high refractive index contrast between the waveguides and their cladding allows for the realization of compact optical devices. Besides, because of the CMOS-compatible materials, the photonic circuits can be fabricated by the well-developed processes for integrated electronic circuits. All of these advantages make PICs be of great potential for scalable classical and quantum information processing [4–7]. Among the various components of PICs, integrated optical microcavities are crucial [8] and different kinds of microcavities have been demonstrated over the past years [9–11]. Owing to the advantages of high quality factor, small mode volume and reconfigurability, microring resonators made by a closed uniform waveguides are used in many realms, including filter [12], ultrasensitive sensor [13, 14], nonlinear optics [15–17], and optomechanics [18, 19].

By twisting one end of an ordinary waveguide through 180° and making the ends join together, a new resonator similar to the Möbius strip will be created [20]. The Möbius strip is famous for its nontrivial topological structure. As is shown in Figure 1(a), a Möbius strip has only one surface, an ant traveling along the strip can come back to the same point after two cycles, and thus the length of roundtrip path is double of that of the original strip. In recent years, a series of studies about Möbius strips emerge in optics [21–23], but it’s very challenging to fabricate a photonic structure with such a complex geometric topology [24–26].

In this paper, we propose a new category of microring resonator with non-trivial mode polarization properties. By properly engineering the microring with a polarization rotator (PR) [27–31], light must circulate two cycles to be converted back to the original status. Therefore, the free spectral range (FSR) of the resonator is approximately 1/2 of that for a traditional microring without PR, which makes the resonator be analogous to a Möbius strip [32]. Furthermore, the effective optical path of the resonator can be N times of the optical circumference for a generalized N-fold “Möbius” microring resonator (MMR). Such a new kind of resonator promises the applications for sensing and polarization analysis.

The microring resonator proposed to realize the “Möbius” topology in the virtual dimension of modes instead of real geometry space is shown in Fig.1(b), where the two polarization states of the light in the ring are analogous to the two surfaces of the original strip. As we know, due to the structural characteristics of the integrated waveguide, rectangular waveguides exhibit strong polarization-maintaining properties with well-defined TE (H-polarized) and TM (V-polarized) polarized modes supported. When etched corner is introduced at a section of nonuniform waveguide, the H- and V-polarization

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modes in rectangular waveguide are not the eigenmodes of the etched waveguide anymore, which are shown in the inset of Fig. 1(c), and the H- and V-polarization modes can convert to each other in the etched waveguide, so it serves as a PR. When the conversion rate between the two polarization modes approaches unit, the H-polarization mode traveling in the microring will become V-polarization mode after one cycle of propagation and the V-polarization mode will be converted back to the H-polarization mode after another cycle of propagation. So the resonator is similar to a Möbius strip and the two polarization modes corresponding to the two surfaces of the original strip. In this letter, we choose the Aluminum Nitride (AlN) microring for analyses, since AlN possesses excellent linear and nonlinear optical properties [15, 33, 34]. The thickness and width of microring are $t = 650\,\text{nm}$ and $w$, respectively. The working wavelength is around $\lambda = 1550\,\text{nm}$ with the refractive index about 2.1. Instead of directly solving the modes in the MMR, we start with the analyses of the propagating light in it.

For the light propagating along the PR, the corresponding mode effective refractive index $n_{\text{eff}}$ for different waveguide width of the PR is shown in Fig. 2(a). Due to the etched corner induced mode coupling, there are several avoid crossing regions, i.e. A, B, and C. In these regions, the coupled-mode equations for H- and V-polarized modes read

$$\frac{d}{dz} \begin{bmatrix} E_H \\ E_V \end{bmatrix} = \begin{bmatrix} -ik_0 n_H & -tg \\ -tg & -ik_0 n_V \end{bmatrix} \begin{bmatrix} E_H \\ E_V \end{bmatrix},$$

(1)

where light propagates along the z-direction, $k_0 = 2\pi/\lambda$, $n_{H(V)}$ is the $n_{\text{eff}}$ of H- (V-) polarized mode, $g$ is the coupling strength. The coupling leads to the minimum propagation constant difference between normal modes is 2$g$. To realize a broadband and robust PR, the waveguide width $w$ along the z-direction of the PR is varying slowly. According to the Landau-Zener tunneling theory [35, 36], the maximum conversion rate can be estimated by

$$\eta \approx 1 - e^{-2\pi g^2/|\kappa dw/dz|},$$

(2)

where $dw/dz$ is the varying rate of the waveguide width and $\kappa = k_0 d(n_H - n_V)/dw$. Therefore, the evolution of the H- and V-polarized light after one round-trip can be described by

$$\begin{bmatrix} E_H' \\ E_V' \end{bmatrix} = T \begin{bmatrix} E_H \\ E_V \end{bmatrix},$$

(3)

with the transfer matrix

$$T = \begin{bmatrix} e^{-i\varphi_H} & 0 \\ 0 & e^{-i\varphi_V} \end{bmatrix} \begin{bmatrix} \sqrt{1-\eta} & \sqrt{\eta} \\ -\sqrt{\eta} & \sqrt{1-\eta} \end{bmatrix}.$$  

(4)

We can further solve the eigenvalues by the $T$, and obtain the spectrum of an MMR with different $dw/dz$. In Fig. 2(c), we focus on the coupling of fundamental H- and V-polarized modes with $w$ around 740 nm (avoiding region A) and the effective radius of the ring is 200 $\mu\text{m}$. The slot size is fixed to 50nm $\times$ 50nm, which induces the coupling strength $g = 10^{-3}k_0$ and $\kappa = 0.31k_0$ $\mu\text{m}^{-1}$. In the following analyses, the PR is treated as a point and we neglect the change of phase in the coupling region when calculating the roundtrip phase for H- or V-polarized light.

When $dw/dz$ is large, $\eta$ is small as indicated by Eq. (2). In Fig. 2(c), for $dw/dz \rightarrow 0.1$, the conversion is nonadiabatic, the propagation of H- and V-polarized modes are independent and thus there are two independent families of spectra for two polarization modes, just corresponds to that in traditional microrings. However, by reducing the geometry parameter $dw/dz \rightarrow 10^{-5}$, the conversion will be adiabatic and $\eta$ approaches 1. In this case, the two separated optical paths of two polarization modes combine to one path and therefore the effective length of
the path is double. As a result, the eigenmodes of MMR are hybridizations of H- and V-polarized light and the FSR becomes half of that when the conversion is non-adiabatic. The mode topology when conversion changes from non-adiabatic to adiabatic is shown in Fig. 2(b).

In Fig. 2(d), we generalize the MMR to three mode families by including two PRs based on the avoid-crossing regions B and C, respectively. Similarly, we found that when the conversion in PRs are non-adiabatic, the propagation of three modes are independent and there are three sets of independent spectra. When the conversion is adiabatic, the eigenmodes of the ring will be a hybridization of these three modes and the length of light path approximatively becomes 3 times of that when the conversion is non-adiabatic. The concept of “Möbius” mode topology can be further developed by introducing higher order modes in the ring. When there are N adiabatic PRs in the ring with N cascaded avoid crossings as shown in Fig. 2(a), the light path will become N times of the circumference for the N-fold MMR.

![Diagram of MMR]

FIG. 3. (a) The schematic of add-drop MMR. The light is input at port 1. (b) and (c) are the transmission spectra of two polarization modes monitored at port 2 and 3 respectively with different phase φ when the amplitude ratio of the EV/EH is fixed at 1. (d) The schematic of coupled MMRs. (e) and (f) are the transmission spectra of two polarization light with different θ1 (θ2 = π) and θ2 (θ1 = 0.5π) respectively and the amplitude ratio of the EV/EH is 0. The dash lines are the results of H-polarized light and solid lines are the results of V-polarized light.

In practical applications, we use adjacent waveguides to couple with the cavity evanescently and the waveguide width is around the avoid-crossing region A, as depicted in Fig. 3(a). Because the eigenmodes of the MMR are hybridizations of two polarization modes, the cavity field will couple with the H- and V-polarized modes in the waveguide simultaneously. The Hamiltonian of the system can be written as

\[ H = -\Delta a^\dagger a + \sum_{j=1,2} \left( \sqrt{\kappa_{H,j}} E_{H,j} a^\dagger + \sqrt{\kappa_{V,j}} E_{V,j} e^{i\varphi_j} a^\dagger + h.c. \right) \]

where a is the bosonic operator of the MMR cavity mode, Δ is the external probe laser frequency detuning, \( \kappa_{H(V),j} \) are external coupling strength between the cavity mode and j-th input waveguide of H(V)-polarized mode, \( E_{H(V),j} \) are the corresponding input fields, and \( \varphi_j \) is the phase difference between two polarization modes. Due to the PR in the ring breaks the rotation invariance of the ring, the phase φ also depends on the relative position of the PR. If we rotate the ring by an angle θ, because of the birefringence effect, we will have \( \varphi = \varphi + k_0 R \Theta(n_V - n_H) \), where R is the radius of MMR. At steady state, the cavity field is

\[ a = \sum_{j=1,2} \sqrt{\kappa_{H,j}} E_{H,j} + e^{i\varphi_j} \sqrt{\kappa_{V,j}} E_{V,j} / \Delta + j/2 \]

where \( \kappa_{tot} = \sum_{j=1,2} (\kappa_{H,j} + \kappa_{V,j}) + \kappa_i \) is the total loss of the cavity and \( \kappa_i \) is the intrinsic loss. When light is incident only from port 1, the transmission at port 2 is \( E_{out2} = (E_{H1} - j\sqrt{\kappa_{H1} a} e^{i\varphi_1}) e^{i\varphi} \) and the transmission at port 3 is \( E_{out3} = -j\sqrt{\kappa_{H2} a} e^{i\varphi} \) and \( \kappa_{V3} a e^{i\varphi} \). These equations indicate that the transmission not only depends on the detuning but also the power ratio of two input polarization modes and the phase difference \( \varphi \). In Fig. 3(b-c), the transmission spectra of the H- and V-polarized modes are plotted for different \( \varphi \) with \( E_o = E_H = 1 \) and \( \kappa_{H1} = \kappa_{V1} = 1.5\kappa_{H2} = 1.5\kappa_{V2} = 1.5\kappa_i \). When \( \varphi = 0 \), the transmission spectra show coincident Lorentz type and it is in critical coupling condition with \( \kappa_{H1} + \kappa_{V1} = \kappa_{H2} + \kappa_{V2} + \kappa_i \). There is a special situation that when \( \varphi = \pi \), the transmission of both the H- and V-polarized light is 1. It seems that the MMR is absent. The transmission spectra of the H- and V-polarized light become asymmetrical Fano-lineshapes when \( \varphi \) is neither 0 nor \( \pi \). As two polarization modes can convert to each other in the MMR, which may induce the transmission to exceed 1. All of these Fano-like lineshapes can be described by the equation of \( T(\Delta) = (q(\Delta^2 + b^2)^{-1} + 1)^{-1} \).

The asymmetry parameters \( q \) and \( b \) are shape parameters describing the Fano-like interference. By fitting the lineshapes with the equation, we can get the parameters \( q \) of each curve in Fig. 3(b). Since changing the phase difference \( \varphi \) affects the transmission spectra, it can be used for sensing. In addition, by solving the equations of \( E_{out2} \) and \( E_{out3} \), we can get the power and the phase of two polarized light at the input port, which means the device may also be used as a polarization analyzer.

In fact, the MMR coupled with waveguides is a linear interference structure, the independent H- and V-
polarized light are two interference channels. For a certain wavelength, when two polarization modes are incident, rotating the MMR is equivalent to changing the relative phase difference of two interference channels at input ports, which will change the output power of two channels. However, when only one polarization mode is input, changing the initial phase of the incident light do not affect the time-averaged power of the output light.

For coupled MMRs, which can be treated as a more complex interference structure, as shown in Fig. 3(d). The radius of two cavities are 100 μm and the conversion rate of the PRs in MMRs is 1. The loss of the MMR1 is larger than that of the MMR2. The transmission matrix method is used to analyze the influence of different θ1 and θ2 on the transmission spectrum with only H-polarized input. We found that the transmission spectra for a specific polarization mode are the Lorentz line-shapes of MMR1 with a coupled-cavities-induced splitting as shown in Figs. 3(e-f), which is similar to that in traditional coupled microrings. Due to the coupling between two polarization modes in MMRs, whatever the input light is, there are always two polarization modes in MMRs. According to Fig. 3(a-c), rotating MMR will change the power coupling between MMRs, which means the intrinsic parameters of the interference structure are changed, and thus the degree of the splitting is affected. Meanwhile, rotating the MMR1 also equivalently changes the initial phase of the input H-polarized light, but according to the previous analyses, it will not affect the output power. So the results of changing θ1 are similar to that of changing θ2, as illustrated in Figs. 3(e-f).

Additionally, such a coupled MMRs can be used for the polarization dependent sensing. When there is an external perturbation to the MRR2, it will induce round-trip phase shift to both polarization modes, i.e. φh and φv. The summation of the two phases φh + φv will induce a resonance frequency shift of MMR2, while the difference of the two phases φh − φv is equivalent to a geometric rotation, i.e. the changing of θ2, thus the line shape of the spectrum will also be changed.

In conclusion, a new type of microring resonator, which is analogous to the Möbius strip is proposed. The theoretical analyses reveal that the “Möbius” microring resonator holds many distinguishable properties, including the nontrivial mode polarization properties and the fractional mode FSR compared to traditional microring cavity of the same size. Besides, the spectral properties of single MMR and coupled MMRs are studied. The spectra show Fano-like features, which are related to the relative phase and the power ratio of the input light with orthogonal polarizations. Such features might be used in sensing and polarization analysis. Our work opens the door towards the topological photonics by exploiting virtual dimension of mode family and provides a novel way to engineering photonic structures for functional devices and fundamental studies.

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