Quantum plateau of Andreev reflection induced by spin-orbit coupling

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Abstract – In this work we uncover an interesting quantum plateau behavior for the Andreev reflection between a one-dimensional quantum wire and superconductor. The quantum plateau is achieved by properly tuning the interplay of the spin-orbit coupling within the quantum wire and its tunnel coupling to the superconductor. This plateau behavior is justified to be unique by excluding possible existences in the cases associated with multi-channel quantum wire, the Blonder-Tinkham-Klapwijk continuous model with a barrier, and lattice system with on-site impurity at the interface.

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Introduction. – Andreev reflection (AR) is a remarkable and useful quantum coherent process of two particles in correlation, taking place at the normal metal/superconductor (N/S) interface [1]. In this process, an incident electron in the normal metal picks up another electron below the Fermi level, forming a Cooper pair across the interface in the superconductor and leaving a hole in the normal metal [2]. Owing to the versatile applications in probing material properties, there have been intensive studies on the various AR physics and related phenomena, see, for instance, the review article [3] and some specific examples [4–8]. However, to our knowledge, the role of spin-orbit coupling (SOC) is not fully clarified yet.

Indeed, it is of great interest to incorporate the spin degrees of freedom into the AR process. For instance, in the ferromagnetic/superconducting (F/S) hybrid system, an interplay of the spin degrees of freedom in the ferromagnetic material not only adds new physics to the AR process, but has created a significant technique for measuring the spin polarization of magnetic materials [9]. Another example is the N/S junction with “N” a spin-orbit coupling system [10], which reveals the interesting specular AR phenomenon previously predicted in the graphene-based N/S junction [11,12]. The spin-dependent AR phenomenon has also been studied in the spin-triplet superconductor [13,14], in the superconductor without inversion symmetry [15], and in the superconducting junction on the surface of topological insulator where the spin-orbit coupling is strong [16,17].

In this work we present an AR study on the hybrid system of a quantum wire with Rashba SOC interaction in contact with an $s$-wave superconductor. Instead of the popular Blonder-Tinkham-Klapwijk (BTK) continuous model (approximating the interface as a $\delta$-function potential barrier) [18], we perform a simulation based on a lattice model. Remarkably, our simulation reveals an interesting quantum plateau behavior for this hybrid system in the one-dimensional (1D) case. We justify this unique behavior by excluding its existence in the AR process associated with a multi-channel quantum wire, the BTK continuous model, and the 1D lattice system with on-site impurity at the interface.

Model and methods. – In this work we consider the hybrid system of a quantum wire with SOC interaction and in contact with a superconductor. The quantum wire is modeled as a ribbon in two dimensions, which is semi-infinite along the longitudinal $x$-direction and finite in the lateral $z$-direction. In terms of the tight-binding lattice model, the wire Hamiltonian reads [19]

\[
H_w = \sum_i \epsilon_i a_i^\dagger a_i - t \sum_i [(a_i^\dagger a_{i+\delta x} + a_{i+\delta x}^\dagger a_i) + \text{H.c.}]
+ \sum_i [i\alpha (a_i^\dagger \sigma_x a_{i+\delta z} - a_{i+\delta z}^\dagger \sigma_x a_i) + \text{H.c.}]. \tag{1}
\]

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Here we have abbreviated the electron operators of the $i$-th site with different spin orientation (in the $\sigma$ representation) in compact form as $a_i^\dagger \equiv (a_i^\dagger, a_i^\dagger)$. $\alpha$ is the SOC coefficient under tight-binding lattice description, which is related to its counterpart ($\eta$) in the continuous model as $\alpha = \eta/2a$ ($a$ is the lattice constant). $\epsilon_i$ and $t$ are the tight-binding site energy and hopping amplitude, while the nearest-neighbor hopping implies $\delta_x = \delta_z = 1$.

For the superconductor we adopt a continuous Hamiltonian, in momentum space which reads [18]

$$H_s = \sum_{k,\sigma} \epsilon_k b_{k\sigma}^\dagger b_{k\sigma} + \sum_k \left( \Delta b_{k1}^\dagger b_{-k1}^\dagger + \Delta b_{-k1} b_{k1} \right).$$

(2)

We consider here a two-dimensional (2D) and $s$-wave superconductor. Then the order-parameter $\Delta$ (assuming it real) is independent of the momentum $k = (k_x, k_z)$. The quantum wire and the superconductor are tunnel-coupled, described as [20]

$$H' = \sum_{i,\sigma} [t_{i\sigma} a_{i\sigma} \sigma_i (z_i) + \text{H.c.}].$$

(3)

Here, to reveal the “nearest-neighbor” coupling feature, we have converted the (superconductor) electron operators in momentum space into coordinate representation via

$$b_{\sigma}(z) = \sum_{k_x, k_z} e^{ik_x z} b_{k\sigma}.$$

We attempt to apply the lattice Green’s function technique to compute the Andreev reflection coefficient. Since the hybrid system under study involves mixing of electron and hole, and as well their spins, it will be convenient to implement the lattice Green’s function method in a compact form of the 4-component Nambu representation [21]. In appendix A we present the particular forms in this representation, for the quantum wire Hamiltonian and the superconductor Green’s functions (and self-energies).

Moreover, in order to implement the quantum “transport” approach based on the nonequilibrium Green’s function technique for the interface Andreev reflection problem, we formally split the (semi-infinite) quantum wire into two parts: the finite part is treated as “central device”, and the remaining semi-infinite one as a “transport lead”. Then, the “central device” is subject to self-energy influences from the both (transport) leads. Based on the surface Green’s function technique, the self-energy from the left lead (the SOC quantum wire) is given by [22]

$$\Sigma'_{L}(E) = H_{10} g''(E) H_{01}. $$

(4)

Here, for simplicity, we have dropped the subscript of $H_{10}$. The surface Green’s function $g''(E)$ can be obtained as a self-consistent solution from the Dyson equation [22],

$$g''(E) = \left[ E - H_{00} - H_{10} g''(E) H_{01} \right]^{-1}. $$

In the expressions presented here, we have labeled the first (leftmost) lattice layer of the “central device” by “1”, and the rightmost layer of the left lead by “0”. In general, the Hamiltonian matrix elements between them are still matrices, expanded over the lateral lattice state basis.

Analogously, applying the surface Green’s function method, in appendix A we carry out the self-energy $\Sigma'_{R}$ for the effect of the right lead of superconductor. Then, the full retarded Green’s function of the central device is given by $G''(E) = \left[ E - H_w - (\Sigma'_{L} + \Sigma'_{R}) \right]^{-1}$, and the advanced one is its conjugate $G''(E) = \left[ G''(E) \right]^*$. Following the Keldysh nonequilibrium Green’s function technique, a lengthy algebra gives an expression for the steady-state transport current as [20]

$$I_{ss} = \frac{e}{2\hbar} \int dE \text{Tr} \left[ \left[ \Gamma_{L} G'' R G'' \right]_{ee} (f_L - f_R) \right.$$

$$- [\Gamma_{L} G'' G'' R]_{hh} (f_L - f_R)$$

$$+ \Gamma_{L} G''_{eh} \Gamma_{L} G''_{eh} (f_L - f_R)$$

$$[\Gamma_{L} G''_{el} \Gamma_{L} G''_{eh} (f_L - f_R)] \right.$$

(5)

where $f_{L(R)} = f(E - \mu_{L(R)})$ and $f_{L(R)} = f(E + \mu_{L(R)})$ are, respectively, the occupied and unoccupied Fermi functions, with $\mu_{L(R)}$ the chemical potential. In the above result, “$e$” and “$h$” denote the subspace of electron and hole, which implies the spin and the lateral lattice states unresolved in explicit basis, but remaining in a $2Nc \times 2Nc$ matrix form to be traced after multiplying all the $2Nc \times 2Nc$ matrices. Finally, the rate matrix $\Gamma_{L(R)}$ in the current formula is defined from the self-energy matrix via $\Gamma_{L(R)} = i [\Sigma'_{L(R)} - (\Sigma'_{L(R)})^*]$, while $\Gamma_{L(R)e}$ and $\Gamma_{L(R)h}$ are their electron and hole blocks.

In eq. (5), the first (second) term describes the electron (hole) transmission from the left to the right leads, while the third (fourth) term is for the incidence of an electron (a hole) accompanied with reflection of a hole (an electron) to the same (left) lead. Therefore, for our present interest, we extract from eq. (5) the AR coefficient as

$$T_A(E) = \text{Tr} \left[ \Gamma_{Lc}(E) G''_{eh}(E) \Gamma_{Lh}(E) G''_{eh}(E) \right] .$$

(6)

Note that this formalism has the advantage of allowing for the incident electron with arbitrary spin orientation and subject to continuous precession in the “central device”. The simulated results in this work correspond to an arbitrary choice for the spin orientation of the incident electron.

Results and discussions. – In our simulations, we use the tight-binding hopping energy $t$ as the units of all energies, including $E$, $t_c$, $\alpha$, and $\Delta$. We commonly set $\Delta = 10^{-3}t$ and assume $\epsilon_i = \epsilon_0$ at the Fermi energy. In fig. 1 we display the central result found in this work for the 1D quantum wire. We summarize in fig. 1(a) the matching condition between the contact coupling $t_c$ and the SOC $\alpha$ for the formation of the AR plateau, while in (b) we illustrate an example for $\alpha = 0.5t$. In connection with this behavior, we mention that in the BTK paper [18], for a 1D wire without SOC, a similar AR plateau can appear only for a vanishing $\delta$-function potential barrier, which is modeled to separate the normal and superconducting parts. In this case, the whole system is a flat 1D wire, thus the result seems not so striking, despite the fact that
and keeping other parameters in (b) unchanged. The inset en-
tact coupling 

(b) Illustrative formation of the AR plateau by tuning the con-
normal transmission in fig. 1(c), by the result of
between the SOC 

is thus more interesting. The proper matching condition 

has no SOC. The “plateau” behavior of the AR coefficient 

part is a 1D wire with SOC; and the superconducting part 

the right part of the wire has suffered the superconducting 

condensation.

In contrast, our system is inhomogeneous: the normal 
part is a 1D wire with SOC; and the superconducting part 
has no SOC. The “plateau” behavior of the AR coefficient 

is thus more interesting. The proper matching condition 

the SOC \( \alpha \) and the contact coupling \( t_c \) for the formation 

the quantum plateau, as summarized in fig. 1(a), (b), is beyond a simple intuition. When satisfying this matching 

condition, one can check that, by closing the superconducting gap \((\text{setting } \Delta = 0)\) and remaining all the 
other parameters unchanged, the normal transmission co-

Efficiency \((\text{setting } \alpha = 0)\) is unity. As an example, we show this ideal 

normal transmission in fig. 1(c), by the result of \( t_c/t_c = 0.6 \) 
in the inset. This result provides a self-consistent support 
to the AR plateau, since the AR is anyhow a coherent 
tunneling process of two electrons, from the normal part into the superconductor. However, as we find in fig. 1(c) 
(inset), there is no such correspondence between the AR 
and the normal transmission coefficients, for the case of 
unmatched SOC \( \alpha \) and coupling \( t_c \) in general.

The quantum plateau behavior is unique, which we 
found exists only for a 1D SOC quantum wire. We jus-
tify this by simulating multichannel quantum wires, with 
results as shown in fig. 2. In fig. 2(a) and (b), for a given 
SOC \( \alpha \), by altering the contact coupling \( t_c \), the quantum 
plateau can no longer be tuned out now. As a comple-
mentary plot, we show in fig. 2(c) the AR coefficient at 
the Fermi energy \((E = 0)\). For comparative purpose, we 
rescale the AR coefficient as \( T_A/N_c \), since for the mul-
tichannel quantum wire the AR coefficient \((\text{the sum of} \ \text{multiple scattering channels})\) can exceed unity. Clearly, 
we see that, only in the 1D case \((N_c = 1)\), can a proper 
tuning of the contact coupling \((t_c)\) and the SOC \( \alpha \) result 
in the quantum plateau behavior. In contrast, for multi-
channel wires \((e.g., N_c = 2 \ \text{and} \ 3)\), the quantum plateau 
cannot be tuned out, as demonstrated in fig. 2(c) by not-
ing that at the edge \((E = \pm \Delta)\), the AR coefficient is “\( N_c \)” 
(the lateral channel numbers).

We further justify the quantum plateau behavior by con-
sidering the BTK 1D continuous model \([18]\). The BTK 
model assumes a normal quantum wire connecting with a 
superconductor through a \( \delta \)-potential barrier \((\text{with height} \ V_0)\). In our case, we further consider the quantum wire 
with the Rashba SOC interaction \((\text{with strength} \ \eta)\). In 
appendix B, we present a detailed solution for this system and obtain the AR coefficient as

\[
T_A = \frac{(1 + x^2)(4Z_2^2 + 1)}{(4Z_2^2 + 1) + x^2(2Z_1^2 + 2Z_2^2 + 1)^2},
\]

where \( Z_1 = \frac{mV_0}{\hbar^2 k_F} \) and \( Z_2 = \frac{m\eta}{\hbar^2 k_F} \), with \( m \) the electron mass and \( \hbar k_F \) the Fermi momentum. Also, for \( E \leq \Delta \), we 
have introduced the dimensionless parameter \( x = (\Delta^2 - E^2)^{1/2}/E \). From eq. (7), one can check that \( T_A = 1 \) only 
at \( E = \Delta \), and \( T_A(E) < 1 \) for other \( E \). So we conclude 
that the quantum plateau behavior does not appear in the 
BTK model for a nonzero height of barrier.

To understand the above result, which seems in con-
trast with the result observed earlier in fig. 1, let us re-
turn to the 1D lattice model. The \( \delta \)-potential barrier, 
in a certain sense, is analogous to an “impurity” at 
the end of the 1D lattice chain, through which the quantum 
wire is coupled to the superconductor. Based on this sort 
of “impurity” model, we perform further simulations and 
report the results in fig. 3. In fig. 3(a) we show that, 
for a given SOC \( \alpha \) and altering the contact coupling \((t_c)\), 
one can no longer tune out the quantum plateau for the
AR coefficient. Indeed, this differs from what we observed in fig. 1, but is consistent with the BTK model discussed above. In fig. 3(b) we present a more complete plot for the absence of the quantum plateau. For several impurity site energies ($\epsilon_{im}$), we display how the quantum plateau behavior disappears. In this plot, we employ the maximum value ($T_A^{\text{max}}$) of the AR coefficient for each $t_c$. No quantum plateau appears for nonzero $\delta$.

**Concluding remarks.** – We thus arrive at the conclusion that the quantum plateau of AR can be formed for a homogeneous 1D wire in contact with a superconductor, as a result of the participation of the SOC interaction in the quantum wire. For this behavior, the SOC effect is essential and not obvious. First, the incident electron can be initially in arbitrary spin orientation and experiences continuous spin precession during its propagation. Second, at the interface, two electrons with opposite spins coherently tunneling into the superconductor and even a quantum plateau can be induced. The AR plateau also implies a SOC-induced “transparency” for the interface, which does not cause normal reflections.

To summarize, in this work we predict a quantum plateau behavior for the Andreev reflection in a 1D quantum wire system, associated with spin-orbit coupling. It would be of interest to verify this behavior by experiment in possible engineered 1D systems.

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**Appendix A: particulars in Nambu representation.** – The hybrid system under the present study involves mixing of electron and hole, together with their spins. Let us introduce a generalized Nambu representation [21], $\psi_i = (a_{i\uparrow}, a_{i\downarrow}, a_{i\uparrow}^\dagger, a_{i\downarrow}^\dagger)^T$, for the electron operators of the $i$-th layer lattice sites along the lateral ($z$) direction. The quantum wire Hamiltonian can be re-expressed in a compact form as

$$H_w = \frac{1}{2} \sum_i \left[ \psi_i^\dagger H_i \psi_i + (\psi_i^\dagger H_{i,i+1} \psi_{i+1} + \text{H.c.}) \right]. \quad (A.1)$$

First, the Hamiltonian matrix $H_{i,i+1}$ reads

$$H_{i,i+1} = \begin{bmatrix} -\tilde{t}_+ & 0 & 0 & 0 \\ 0 & -\tilde{t}_- & 0 & 0 \\ 0 & 0 & \tilde{t}_+ & 0 \\ 0 & 0 & 0 & \tilde{t}_- \end{bmatrix} \otimes I_{Nc \times Nc}, \quad (A.2)$$

where $\tilde{t}_ \pm = t \pm i\alpha$. The second Hamiltonian matrix, $H_{i,i}$, has three parts: $H_{i,i} = H_0 + H_1 + H_2$. Each is given by, respectively,

$$H_0 = \begin{bmatrix} \epsilon_i & 0 & 0 & 0 \\ 0 & \epsilon_i & 0 & 0 \\ 0 & 0 & -\epsilon_i & 0 \\ 0 & 0 & 0 & -\epsilon_i \end{bmatrix} \otimes I_{Nc \times Nc}, \quad (A.3)$$

$$H_1 = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 1 & 0 & \cdots & 0 & \cdots \\ \vdots & \vdots & \ddots & \ddots & \vdots \\ 0 & \cdots & 0 & 1 & 0 \end{bmatrix}_{Nc \times Nc}, \quad (A.4)$$

$$H_2 = \begin{bmatrix} 0 & \alpha & 0 & 0 \\ -i & 0 & \cdots & 0 \\ \alpha & 0 & 0 & 0 \\ 0 & \cdots & 0 & i \end{bmatrix}_{Nc \times Nc} \otimes I_{Nc \times Nc}, \quad (A.5)$$

$$H = \begin{bmatrix} 0 & \alpha & 0 & 0 \\ -i & 0 & \cdots & 0 \\ \alpha & 0 & 0 & 0 \\ 0 & \cdots & 0 & i \end{bmatrix}_{Nc \times Nc} \otimes I_{Nc \times Nc}, \quad (A.5)$$
Similarly, for the superconductor (Hamiltonian and Green’s functions), we introduce the 4-component Nambu representation $\psi_1^\dagger = (b_1^\dagger, b_1^\dagger, b_1, b_1^\dagger)$). Originally, the electron operators in the superconductor Hamiltonian, eq. (2), are defined in momentum space. For the purpose of applying the surface Green’s function technique, we introduce the “surface” electron operator via $b_0(z) = \sum_{k_x, k_y} e^{i (k_x x + k_y y)}$. In this representation, the (retarded) surface Green’s function of the superconductor reads [21]

$$g^{R}_S (z, z', t) = -i \theta(t) \langle \{ \psi_1 (z, t), \psi_1^\dagger (z', 0) \} \rangle \times \left\{ \begin{array}{c} \sigma_x \frac{\Delta}{E} \sigma_z \\ \frac{\Delta}{E} \sigma_x \sigma_z \end{array} \right\} .$$

(A.6)

Applying the equation-of-motion method, in the frequency domain one obtains [20]

$$g^{R}_S (z, z', E) = -i \pi \rho_{ph} \int [k_F (z - z')] \beta(E)$$

$\times \left\{ \begin{array}{c} \sigma_x \frac{\Delta}{E} \sigma_z \\ \frac{\Delta}{E} \sigma_x \sigma_z \end{array} \right\} .$$

(A.7)

In this result, $\sigma_x$ is the Pauli matrix (the third one), and $\sigma_y$ an identity matrix. Other notations used here are the density of states $\rho$, the Fermi momentum $k_F$, and the first-Bessel function $J_0$. We also introduced: $\beta(E) = |E|/\sqrt{E^2 - \Delta^2}$ for $|E| > \Delta$; and $\beta(E) = -i E/\sqrt{\Delta^2 - E^2}$ for $|E| < \Delta$.

Knowing $g^{R}_S (z, z', E)$, the self-energy contribution of the superconductor to the “central device” is accordingly obtained via $\Sigma^{R}_{ij} = \int [\omega] g^{R}_S (z_i, z_j, \omega)$, where $z_i(j)$ corresponds to the “site” at the superconductor surface coupled to the $i$-th ($j$-th) site of the quantum wire.

**Appendix B: 1D continuous model.** In this appendix we present a detailed solution for the AR coefficient based on the BTK 1D continuous model [18] for tunneling through a $\delta$-function potential barrier (with height $V_0$), in the presence of Rashba SOC in the quantum wire (with strength $\eta$). For simplicity but not affecting the conclusion, in the following analysis we consider only the incident electron with spin-up orientation. This can reduce the Nambu representation from four to two dimensions. Accordingly, the Bogoliubov-de Gennes Hamiltonian for the total system is expressed in a compact form as

$$H_{up} = \left[ \begin{array}{c} H_1 - \mu & \Delta \Theta(x) \\ \Delta \Theta(x) & H_1^\dagger + \mu \end{array} \right] ,$$

(B.1)

where $\Theta(x)$ is the “step” function, and

$$H_1 = - \frac{\hbar^2}{2m} \frac{d^2}{dx^2} + \eta \Theta(-x) \frac{d}{dx} + \left( V_0 - \frac{\eta}{2} \right) \delta(x) ,$$

$$H_1^\dagger = - \frac{\hbar^2}{2m} \frac{d^2}{dx^2} - \eta \Theta(x) \frac{d}{dx} + \left( V_0 + \frac{\eta}{2} \right) \delta(x) .$$

(B.2)

In this two-component Nambu representation, the wave function is a spinor:

$$\Psi(x, t) = \begin{bmatrix} f(x, t) \\ g(x, t) \end{bmatrix} .$$

(B.3)

For the SOC quantum wire (normal part), substituting the spinor wave function into the Schrödinger equation $i\hbar \frac{df(x, t)}{dt} = H_{up} \Psi$, and considering the stationary solution of $f(x, t) = u e^{i q x - i \omega t/\hbar}$ and $g(x, t) = v e^{i q x - i \omega t/\hbar}$, we have

$$E u = \left[ \frac{\hbar^2 q^2}{2m} - \hbar q - \mu \right] u ,$$

$$E v = - \left[ \frac{\hbar^2 q^2}{2m} - \hbar q - \mu \right] v .$$

(B.4)

Simply, we obtain four spinor wave functions:

$$\Psi^{(c)}_{q^\pm} = \begin{bmatrix} 1 \\ 0 \end{bmatrix} e^{i q^\pm x} ,$$

(B.5)

and

$$\Psi^{(h)}_{q^\pm} = \begin{bmatrix} 0 \\ 1 \end{bmatrix} e^{i q^\pm x} .$$

(B.6)

$q^\pm_j (j = 1, 2)$ are given by $q^\pm_j = q_{so} + (-1)^{j-1} \hat{q}^\pm$, with $q_{so} = m \eta \hbar$ and $\hat{q}^\pm = \sqrt{2m (\mu \pm E + E_{so})}/\hbar$, where $E_{so} = m \eta^2/(2 \hbar^2)$.

Similarly, for the superconductor, the stationary Schrödinger equation reads

$$E \tilde{u} = \left[ \frac{\hbar^2 k^2}{2m} - \mu \right] \tilde{u} + \Delta \tilde{v} ,$$

$$E \tilde{v} = - \left[ \frac{\hbar^2 k^2}{2m} - \mu \right] \tilde{v} + \Delta \tilde{u} .$$

(B.7)

Here we have assumed $f(x, t) = \tilde{u} e^{i k x - i \omega t/\hbar}$ and $g(x, t) = \tilde{v} e^{i k x - i \omega t/\hbar}$. Accordingly, we obtain the quasiparticle wave functions as

$$\Psi^{(c)}_{\pm k^\pm} = \begin{bmatrix} u_0 \\ v_0 \end{bmatrix} e^{i k^\pm x} ,$$

(B.8)

and

$$\Psi^{(h)}_{\pm k^\pm} = \begin{bmatrix} v_0 \\ u_0 \end{bmatrix} e^{i k^\pm x} .$$

(B.9)

where $u_0 = 1 - v_0^2 = 1/2 [1 + (E^2 - \Delta^2)^{1/2} / E]$, and $E = \sqrt{(\hbar^2 k^2/2m - \mu)^2 + \Delta^2}$ (here taking only the positive root). In this context, we also applied the following considerations: for $\hbar^2 k^2/2m - \mu > 0$, $\tilde{u} = u_0$ and $\tilde{v} = v_0$; while for $\hbar^2 k^2/2m - \mu < 0$, $\tilde{u} = v_0$ and $\tilde{v} = u_0$. The wave vector numbers read $k^\pm = \sqrt{2m |\mu \pm (E^2 - \Delta^2)^{1/2}|}/\hbar$. 67013-p5
As mentioned earlier, we consider the incidence of a spin-up electron with a sub-gap energy. In this regime, the dominant process is AR. The associated incident, reflecting, and transmitting waves are described as

$$\Psi_i = \begin{bmatrix} 1 \\ 0 \end{bmatrix} e^{iq_0 x},$$

$$\Psi_r = a \begin{bmatrix} 0 \\ 1 \end{bmatrix} e^{iq_1 x} + b \begin{bmatrix} 1 \\ 0 \end{bmatrix} e^{iq_2 x}, \quad (B.10)$$

$$\Psi_t = c \begin{bmatrix} v_0 \\ v_0 \end{bmatrix} e^{ik^+ x} + d \begin{bmatrix} v_0 \\ u_0 \end{bmatrix} e^{-ik^- x}.$$ 

Following the standard procedures for solving this sort of tunneling problems, we apply the boundary conditions at the interface for the wave functions and their derivatives. The first boundary condition reads

$$\Psi_S(0) = \Psi_N(0) \equiv \Psi(0). \quad (B.11)$$

Here we have denoted $\Psi_S = \Psi_i$ and $\Psi_N = \Psi_i + \Psi_r$. Crossing the $\delta$-function barrier, the second boundary condition is given by

$$-\frac{\hbar^2}{2m} (\Psi_S - \Psi_N) = (V_0 - \frac{\eta}{2}) \Psi(0). \quad (B.12)$$

Noting that $E \leq \Delta \ll \mu$, we can approximate $k^+ \simeq k^- \simeq k_F = \sqrt{2m\mu}/\hbar$ and $q_j^\pm \simeq q_0 + (-1)^{j-1} \sqrt{2m(\mu + E_{so})}/\hbar \equiv q_j$. We further introduce $\bar{q}_j = |q_j|/k_F$, for the sake of brevity in expressions. More explicitly, the boundary conditions read

$$1 + b = cu_0 + dv_0,$$

$$a = cv_0 + du_0, \quad (B.13)$$

and

$$\frac{i\hbar^2 k_F}{2m}(cu_0 - dv_0 - \bar{q}_1 + \bar{b}_2) = (1 + b) \left( V_0 - \frac{\eta}{2} \right),$$

$$\frac{i\hbar^2 k_F}{2m}(cv_0 - du_0 - a \bar{q}_1) = a \left( V_0 - \frac{\eta}{2} \right). \quad (B.14)$$

Solving this set of linear equations yields

$$a = \frac{2u_0 v_0}{\gamma} (\bar{q}_1 + \bar{q}_2),$$

$$b = \frac{1}{\gamma} (u_0^2 - v_0^2)(4Z_1^2 + 4iZ_1 \bar{q}_1 + (\bar{q}_2 - \bar{q}_1)\bar{q}_1),$$

$$c = \frac{u_0}{\gamma} (\bar{q}_1 + \bar{q}_2)(1 + \bar{q}_1 - 2iZ_1),$$

$$d = \frac{v_0}{\gamma} (\bar{q}_1 + \bar{q}_2)(1 - \bar{q}_1 + 2iZ_1), \quad (B.15)$$

where

$$Z = \frac{m(V_0 - i\frac{\eta}{2})}{k_F^2} = Z_1 - iZ_2,$$

$$Z_1 = \frac{mV_0}{k_F^2} = V_0/\hbar v_F, \quad (B.16)$$

$$Z_2 = \frac{m\eta}{2\hbar^2 k_F} = \frac{\bar{q}_1 - \bar{q}_2}{4}.$$ 

and

$$\gamma = (\bar{q}_1 + \bar{q}_2) + (u_0^2 - v_0^2)(4|Z|^2 + 2). \quad (B.17)$$

Since $E \leq \Delta$, we introduce a real and dimensionless factor $x \equiv (\Delta^2 - E^2)^{1/2}/E$. Then, $u_0^2 = \frac{1}{2}(1 + ix)$ and $v_0^2 = \frac{1}{2}(1 - ix)$. We finally obtain the AR coefficient as

$$T_A(E) = |a|^2 = \frac{(1 + x^2)(\bar{q}_1 + \bar{q}_2)^2}{(1 + x^2)(4Z_1^2 + 1) + (4Z_2^2 + 1)^2}. \quad (B.18)$$

We find that at the excitation edge $T_A(E = \Delta) = 1$, otherwise $T_A(E) < 1$.

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