Matter-wave interferometry from near-field to far-field diffraction

S Srisuphaphon$^{1,4}$, W Temnuch$^{2,3,4}$, S Buathong$^{1,4}$ and S Deachapunya$^{1,3,4,*}$

$^1$Department of Physics, Faculty of Science, Burapha University, ChonBuri Province, 20131, Thailand
$^2$Department of Physics, Faculty of Science, Kasetsart University, Bangkok Province, 10900, Thailand
$^3$Thailand Center of Excellence in Physics, Commission on Higher Education, Bangkok 10400, Thailand
$^4$Quantum and Nano Optics Research Unit, Burapha University, ChonBuri Province, 20131, Thailand

*E-mail: sarayut@buu.ac.th

Abstract. A simplified explanation of near-field and far-field diffraction with matter-wave is described using the path integral formulation. Our approach is demonstrated with cold Rb atoms in single-, double- and multiple-slit configurations. With the use of an initial wave for the particle beam that corresponds to the slit opening and the number of slits, the transition from near-field to far-field diffraction can also be explored. Identifying the boundary between near-field and far-field diffraction is the first priority for implementing matter-wave diffraction experiments.

1. Introduction

It is well-known that particle diffraction is evidence of the matter-wave duality, which is a cornerstone of modern physics. Previously, most experiments were performed with small particles in the far-field regime, such as the diffraction pattern of electrons through a crystal, as generally found in fundamental physics textbooks. In the present, nevertheless, diffraction in the near-field region has been more widely studied. For example, in the case of coherent light, clear descriptions of the Talbot effect [1] as well as the Talbot-Lau effect can be found [2]. The optical vortex of light can be characterized using the Talbot effect [3]. Regarding matter-wave diffraction, electrical interactions in terms of a random potential were introduced for studying the effects of metal-coated transmission grating on electron diffraction pattern [4]. Even with larger sized particles, the scalar polarizability of the fullerene molecule C60 can be extracted from the diffraction pattern, which has been perturbed with an external electric field [5]. A study of the Casimir-Polder interactions between C70 molecules and grating has been reported [6]. For more complex molecules, such as C$_{30}$H$_{12}$F$_{30}$N$_{2}$O$_{4}$, experimental setups and theoretical descriptions have been reviewed in Ref. [7].

The interference patterns in the far-field regime can be easily described by Fraunhofer diffraction. In the near-field regime, the model used is Fresnel diffraction. The transition between the two regions is an interesting issue. The boundaries of interference pattern for the Talbot effect, for example, have been estimated by using a simple method based on the superposition of waves with different path lengths [8]. For quantum interference, the sum of probability densities, of finding a particle moving along the possible paths through the slit windows, with Feynman path integration, was used to describe the transition region for particles diffracted through one or two slits [9, 10]. The multipath interference
approach has been experimentally tested in triple-slit diffraction of a highly coherent metastable-helium-atom beam [11].

In this work, we propose a simple method to predict the matter-wave interference, which can be applied to both near-field and far-field and the transition regime. With a well-chosen initial wave function, we can derive the exact wave function using Feynman path integration. We provide simulated interference patterns based on this approach.

2. Theory and methods

In this section, we present a theoretical description of matter-wave diffraction through a diffraction element. We start with a particle beam moving along the z-axis through a diffraction element of period \( d \), which is placed along the \( x_0 \)-axis at \( z = 0 \). According to Feynman path integral approach, an initial wave function, corresponding to particle distribution behind the diffraction element, has to be assumed. Therefore, we assume a wave function of the form

\[
\varphi_N(x_0, z_0, t = 0) = C \exp \left\{ \frac{-z_0^2}{\beta_z^2} + ikz_0 \right\} \sum_{n=-\infty}^{\infty} A_n \exp \left\{ i n k_d (x_0 - Jd) \right\} \sum_{j=-\infty}^{\infty} \exp \left\{ -i \frac{(x_0 - jd)^2}{\beta_x^2} \right\},
\]

where \( C \) is the normalizing factor.

The first part represents the Gaussian wave packet with full width at half maximum \( 2\beta_z \) propagating along the z-axis. The wave vector \( k = 2\pi / \lambda_{db} \) where \( \lambda_{db} \) denotes the de Broglie wavelength for a particle of mass \( m \) moving along the z-axis with average speed \( v \). The second part is the transmission function with the Fourier components \( A_n = \sin \left( n\pi f \right)/n\pi \) for periodicity along the \( x_0 \)-axis, with the wave vector \( k_d = 2\pi / d \) and open fraction \( f \) [2]. The third part represents the Gaussian distribution around the center at the \( j^{th} \) slit window with \( \beta_x = wfd \). Here \( w \) is defined as a size parameter for modulating the probability pattern \( |\varphi_N|^2 \) according to the characteristics of the slit. The phase shift \( x_0 - Jd \), where \( J = (N - 1)/2 \), has to be added for adjusting by the number \( N \) of slits. For example, for the slit counts \( N = 1, 2, 5, 8 \) with open fraction \( f = 0.5 \) and any period \( d \), the proper probability distributions can be found by substituting \( w = 0.715 \), as shown in figure 1.

Consequently, the wave function \( \psi_N(x, z, t \geq 0) \) in \( xz \) plane behind the slit can be obtained by using the path integration

\[
\psi_N(x, z, t) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} dx_0 dz_0 K(x, z, t; x_0, z_0, 0) \varphi_N(x_0, z_0, 0),
\]

where

\[
K(x, z, t; x_0, z_0, 0) = \frac{m}{2\pi i \hbar t} \exp \left\{ \frac{-im}{2\hbar t} \left[ (x - x_0)^2 + (z - z_0)^2 \right] \right\},
\]

is the propagator for a free particle of mass \( m \) moving in the \( xz \) plane. Since the initial wave \( \varphi_N \) (equation (1)) is written in terms of a quadratic Gaussian, the integration in equation (2) can be carried out explicitly. On evaluating the Gaussian integral, we obtain the exact wave function
Figure 1. The probability densities of finding a particle behind the slits at \( t = 0 \) for the cases \( N = 1 \) (single slit) (a), 2 (double slit) (b), 5 (c), and 8 (d), respectively. The simulations computed modulus squared of equation (1) with \( f = 0.5 \) with the size parameter \( w = 0.715 \).

\[
\psi_N(x, z, t) = \frac{C}{\sqrt{\gamma_x \gamma_z}} \exp\left\{Z_{re}(t) + Z_{im}(t)\right\} \sum_{n} \sum_{j} A_n \exp\left\{X_{re}(n, j, t) + X_{im}(n, j, t)\right\},
\]

where \( \gamma_{x,z} = 1 + \left(2i\theta / m\beta_z^2\right) \). The exponential functions are as follows:

\[
Z_{re}(t) = \frac{-1}{\beta_z^2 \gamma_x} \left(z - \frac{kht}{m}\right)^2,
\]

\[
Z_{im}(t) = \frac{ik}{\gamma_x} \left(z - \frac{kht}{2m}\right) - \frac{ihtz^2}{m\gamma_x^2},
\]

\[
X_{re}(n, j, t) = \frac{-1}{\gamma_z} \left[(x - jd)^2 + 4\pi n(j - J)t\right] / m,
\]

\[
X_{im}(n, j, t) = \frac{i}{\gamma_z} \left[ nk_d (x - Jd) - \frac{n^2 \pi^2 h\theta}{md^2}\right].
\]
Figure 2. Interference patterns both in near-field and far-field diffraction with $N = 1$ (a), 2 (b), 4 (c), 8 (d), 12 (e), and 20 (f). The insets in (a) and (b) show diffraction in the near-field regime. Higher Talbot orders can be observed with the larger numbers of slits. The triangle area in (f) exhibits the internal area of the near-field interference pattern, with a tenth of Talbot length indicated by the vertical dashed line.

The probability density distribution of particles in $xz$-plane behind the multiple slits is given by $|\psi_N|^2$. The first exponential function $\exp\{Z_{re}(t)\}$ is the Gaussian wave packet moving along the $z$-axis with group velocity $v = k h / m = h / m \lambda_{db}$. The phase function $Z_{im}(t)$ involves the phase velocity $k h / 2 m$ and the constant phase ($\propto z^2$) that can be omitted from calculation of the probability density $|\psi_N|^2$. The phase function $X_{re}(n, j, t)$ gives the contribution of the number of slits $N$ from the initial wave $\varphi_N(x_0, z_0, 0)$. Periodicity of the interference pattern can be found in $X_{im}(n, j, t)$ by substituting of the time $t = z / v$. The second term $n^2 \pi^2 ht / md^2$ can be reduced to $n^2 \pi^2 z / L_T$ where $L_T = d^2 / \lambda_{db}$ is the so-called Talbot length [12].

3. Results and discussion

Here we demonstrate our approach with cold Rubidium 85 (Rb) atoms in single-, double-, and multiple-slit diffraction cases. Our simulations were done with equation (4). The cold atomic temperature was about 2.5 $\mu$K [13, 14], the grating period 1 $\mu$m, the open fraction 0.3, and the sum truncated at $n = \pm 25$ was employed. Figure 2 represents our simulations of the cases with numbers of slits $N = 1$ (a), 2 (b), 4 (c), 8 (d), 12 (e), and 20 (f). The numbers of slit $N = 1$ (a), and 2 (b) are the single-slit and double-slit configurations in far-field diffraction. The insets show both with unclear patterns in the near-field regime. In order to produce sharp patterns in the near-field by diffraction, higher numbers of slits are necessary, as shown in figure 2 (f). The triangle area exhibits the internal area of the near-field interference pattern, with a tenth of Talbot length. For $N = 50$, the simulation (figure 3) shows a fine pattern of the optical carpet in the range of one Talbot length.
Figure 3. Optical carpet of cold Rubidium 85 (Rb) atoms with the grating period of 1 µm, and open fraction of 0.3. The dashed line indicates position of the first Talbot length.

4. Conclusions

We show that the path integral calculation can describe matter-wave optics both in near-field and far-field diffraction. This corresponds to diffraction phenomena in wave optics [8]. Our demonstration used cold Rb atoms in single-, double-, and multiple-slit configurations. The transition from near-field to far-field diffraction can also be explored. Here our study can provide useful information for quantum experiments with matter-wave interferometry. Further studies including velocity distribution and various grating parameters shall be pursued in the future.

Acknowledgments

We gratefully acknowledge the support from Thailand Center of Excellence in Physics (ThEP-60-PET-BUU8) and the grant from Faculty of Science and Burapha University (1/2562).

References

[1] Talbot H F 1836 Philos. Mag. 9 401–7
[2] Case W B, Tomandl M, Deachapunya S and Arndt M 2009 Opt. Express 17 20966–74
[3] Panthong P, Srisuphaphon S, Chiangga S and Deachapunya S 2018 Appl. Opt. 57 1657–61
[4] Gronniger G, Barwick B and Batelaan H 2005 Appl. Phys. Lett. 87 124104
[5] Berninger M, Stefanov A, Deachapunya S and Arndt M 2007 Phys. Rev. A 76 013607
[6] Brezger B, Hackermueller L, Uttenthaler S, Petschinka J, Arndt M and Zeilinger A 2002 Phys. Rev. Lett. 88 100404
[7] Arndt M and Hornberger K 2009 Preprint 0903.1614v1
[8] Temnuch W, Deachapunya S, Panthong P, Chiangga S and Srisuphaphon S 2018 Wave Motion 78 60–7
[9] Beau M 2012 Eur. J. Phys. 33 1023–39
[10] Jones E R, Bach R A and Batelaan H 2015 Eur. J. Phys. 36 065048
[11] Barnea A R, Cheshnovsky O and Even U 2018 Phys. Rev. A 97 023601
[12] Artyotha C, Deachapunya S, Kunavakarn B, Kheaomaingam N and Srisuphaphon S 2018 J. Phys.: Conf. Ser. 1144 012053
[13] Mellish A S and Wilson A C 2002 Am. J. Phys. 70 965–74
[14] Bodart Q, Merlet S, Malossi N, Pereira Dos Santos F, Bouyer P and Landragin A 2010 Appl. Phys. Lett. 96 134101