The Neutrino Emissivity of Strange Stars with Ultra Strong Magnetic Field

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Abstract

The effect of a strong magnetic field on the dominant neutrino emissivity in strange stars is investigated. In an ultra strong magnetic field, there exists an enhanced neutrino emission because the charged particles are confined to the lowest Landau level. The results show that the neutrino emissivity is proportional to $T^5$ instead of conventional $T^6$, which implies more rapid cooling behaviors in magnetars than usual stars

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1 INTRODUCTION

The theoretical possibility that strange quark matter, which made up of roughly equal numbers of up, down, and strange quarks, may be more stable than atomic nuclei (specifically iron, which is the most stable atomic nucleus) constitutes one of the most startling predictions of modern physics [1, 2]. If Witten’s idea is correct, strange quark stars (strange stars) may exist [3, 4]. Theoretical and observational researches on strange stars might provide a scientific basis to test what is the true ground state of the hadron.

Initially a strange star cools mainly via the neutrino emission from its core. The $\beta$ decay of quarks in quark matter is believed to be the important cooling mechanism in strange stars, which was usually investigated in absence of magnetic field in many works[16, 17]. However, Observations of pulsars predict large surface magnetic field, typically of order $10^{12}G$. Some magnetars are observed to have magnetic field of $10^{14} \sim 10^{15}G$. Considering the flux throughout the stars as a simple trapped primordial flux, the internal magnetic field may go up to $10^{18}G$ or even more. In spite of the fact that we do not know yet any appropriate mechanism to produce more intense field, the scalar virial theorem [5] indeed allows the field magnitude to be as large as $10^{20}G$. In the case of super strong magnetic field such that $2eB > p_F(c)^2$ [20], all electrons occupy the Landau ground state with electron spins pointing in the direction opposite

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1For electrons having a Fermi momentum of 50 Mev one obtains $B > 2 \times 10^{17}G$, for quarks such as d quark having a Fermi momentum of 300 Mev one obtains $B < 2 \times 10^{19}G$.  

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to the magnetic field, and the charge neutrality now forces the degenerate quarks also to occupy the lowest Landau level even if the magnetic field is not large enough to break the Fermi surface. We also know that the quantization effect on the charged particles is extremely large when the magnetic field exceeds a critical field strength \( B_c \) defined by equating the cyclotron energy \( \frac{qB}{m_i} \) to \( m_i \) (i denotes electrons and quark flavors) \(^{19}\), where the radius of cyclotron is \( \frac{1}{\sqrt{2\pi eB}} \) compatible to de Broglie wavelength of the particle \(^{20}\). \( B_c \) will respectively reach to \( \sim 10^{13}, 10^{15}, 10^{18} \) gauss for electrons, u(d)-quarks and s-quarks. Obviously, the effect on s-quark is small due to the large quark mass if the internal magnetic field is not over \( 10^{18} \) gauss. In our calculations, we consider the situation that all charged particles involved in the processes confined in the lowest Landau level and it will give some instructions for general treatment regardless of the internal field in stars. In this situation we must use the exact solutions of Dirac equation. Emission processes will be found to be evidently different from the field-free case. Many works recognized the magnetic effect on weak interactions in 90’s of last century \(^{6, 7}\). In neutron stars the magnetic effect on the direct Urca, scattering and absorption processes was studied extensively \(^{8, 20}\). We will show that the neutrino emissivity of ultra magnetized strange star is strongly dependent on the magnetic field.

This paper is organized as follows. The exact solution of the Dirac equation for charged particles in the magnetic field is obtained in Sect. 2. The direct Urca processes dominating the neutrino emissivity in strong magnetized strange stars are calculated in Sect. 3. The related cooling curves are presented in Sect. 4. Finally we summarize our conclusion and discussion in Set. 5.

2 EXACT SOLUTION OF DIRAC EQUATION FOR CHARGED PARTICLES IN MAGNETIC FIELD

When an electromagnetic field with 4-potential \( A^\mu \) is included, Dirac’s equation is modified by the minimal coupling procedure of replacing \( \hat{p}^\mu \) by \( \hat{p}^\mu - qA^\mu \), where \( q \) is the charge of the particle. The Dirac Hamiltonian is then replaced by

\[
\hat{H} = \alpha \cdot (\hat{p} - q\hat{A}) + \beta m + q\phi
\]

Here we set \( \phi = 0 \), just considering the charged particles in a constant, uniform magnetic field in the \( z \)-direction. We choose the \textit{Landau gauge}

\[
\hat{A} = (0, Bx, 0)
\]

and assume a trial wave function of the form

\[
\psi(t, \mathbf{x}) = f(x)exp(-i\epsilon t + i\epsilon p_y y + i\epsilon p_z z)
\]

where \( \epsilon = \pm \) is the sign of the energy whose magnitude is \( \epsilon \). On inserting the trial solution (3) into the Dirac’s equation in the form

\[
(i\partial t - \hat{H})\psi(t, \mathbf{x}) = 0
\]
one can obtain the energy and the wave function. The energy is

\[ \varepsilon = \sqrt{p_x^2 + m^2 + 2n|qB|} \]  

(5)

where \( n \) is the Landau level. It is degenerate and can be expressed as other quantum numbers such as the orbital part \( l \) and the spin quantum number \( s \).

\[ n = l + \frac{1}{2}(1 \pm s) \]  

(6)

here the \( \pm \) is on the contrary for the negative and positive charge respectively. One has \( q = |q| \) for positive charged particles and their four solutions are

\[
\psi^e_+(t, \mathbf{x}) = \frac{\exp[-i\varepsilon t + i\varepsilon_q y + i\varepsilon_p z]}{\sqrt{2\varepsilon(\varepsilon + m)L_y L_z}} \begin{pmatrix} (\varepsilon + m)I_{n; p_z}(x) \\ 0 \\ \varepsilon_p I_{n; p_z}(x) \\ -i\sqrt{2n|q|B}I_{n-1; p_z}(x) \end{pmatrix} 
\]  

(7)

for spin up, and

\[
\psi^e_-(t, \mathbf{x}) = \frac{\exp[-i\varepsilon t + i\varepsilon_q y + i\varepsilon_p z]}{\sqrt{2\varepsilon(\varepsilon + m)L_y L_z}} \begin{pmatrix} 0 \\ (\varepsilon + m)I_{n-1; p_z}(x) \\ i\sqrt{2n|q|B}I_{n; p_z}(x) \\ -\varepsilon_p I_{n-1; p_z}(x) \end{pmatrix} 
\]  

(8)

for spin down cases. In Eqs. (7) and (8), \( L_y \) and \( L_z \) are the length scales along the \( y \) and \( z \) directions. For negative particles \( q = -|q| \), the wave functions are somewhat different as

\[
\psi^e_+(t, \mathbf{x}) = \frac{\exp[-i\varepsilon t + i\varepsilon_q y + i\varepsilon_p z]}{\sqrt{2\varepsilon_q(\varepsilon_q + m)L_y L_z}} \begin{pmatrix} (\varepsilon + m)I_{n-1; p_z}(x) \\ 0 \\ \varepsilon_p I_{n-1; p_z}(x) \\ i\sqrt{2n|q|B}I_{n; p_z}(x) \end{pmatrix} 
\]  

(9)

\[
\psi^e_-(t, \mathbf{x}) = \frac{\exp[-i\varepsilon t + i\varepsilon_q y + i\varepsilon_p z]}{\sqrt{2\varepsilon_q(\varepsilon_q + m)L_y L_z}} \begin{pmatrix} 0 \\ (\varepsilon + m)I_{n; p_z}(x) \\ -i\sqrt{2n|q|B}I_{n-1; p_z}(x) \\ -\varepsilon_p I_{n; p_z}(x) \end{pmatrix} 
\]  

(10)

in above four Eqs. (7)-(10)

\[ I_{n; p_z} = \left( \frac{|q|B}{\pi} \right)^{\frac{1}{2}} \exp\left[ -\frac{1}{2}\frac{|q|B}{(x - \frac{p_y}{qB})^2} \right] \times \frac{1}{\sqrt{2^n n!}}H_n\left[ \sqrt{|q|B}(x - \frac{p_y}{qB}) \right] \]  

(11)

where \( H_n \) is the well-know Hermite polynomial. It is convenient to write

\[ \xi = (|q|B)^{1/2}(x - \frac{p_y}{qB}) \]  

(12)

then Eq. (11) can be rewritten as

\[ I_{n; p_z} = \left( \frac{|q|B}{\pi} \right)^{\frac{1}{2}} \exp\left[ -\frac{\xi^2}{2} \right] \times \frac{1}{\sqrt{2^n n!}}H_n(\xi) \]  

(13)
3 NEUTRINO EMISSIVITY

The following weak processes are important in quark matter consisting of up, down and strange quarks,

\[ d \to u + e^- + \bar{\nu}_e, \quad u + e^- \to d + \nu_e. \]  

\[ s \to u + e^- + \bar{\nu}_e, \quad u + e^- \to s + \nu_e. \]

We will employ the Weinberg-Salam theory to calculate the neutrino emissivity of these reactions in strong magnetic field. The interaction Lagrangian density for the charged current reaction (14) and (15) may be expressed as \( \mathcal{L}_{\text{int}} = (G_f/\sqrt{2})\cos \theta_c J_{\mu}^{\text{eff}} \), where \( G_f \approx 1.355 \times 10^{-49} \text{ erg cm}^{-3} \) is the Fermi weak coupling constant and \( \theta_c \), the Cabibbo angle. The lepton and nucleon charged weak currents are \( l_\mu = \psi_e \gamma_\mu (1 - \gamma_5) \psi_u \) and \( j_{\mu}^{\text{eff}} = \bar{\psi}_u \gamma^\mu (1 - \gamma_5) \psi_d \) respectively.

when the u, d quarks and electrons are Landau quantized. The emissivity due to the antineutrino emission process in the presence of a uniform magnetic field \( B \) along \( z \)-axis is given by

\[
\varepsilon_\nu(B) = \frac{3}{\sqrt{2}} \int_{|q|BL/2}^{-|q|BL/2} L_x dp_x \int_{-\infty}^{\infty} L_y dp_y \int_{|q|BL/2}^{-|q|BL/2} L_z dp_z \frac{L_y dp_y}{2\pi} \frac{L_x dp_x}{2\pi} \frac{L_z dp_z}{2\pi} \int_{-\infty}^{\infty} V d^3 \vec{p}_\nu \times \varepsilon_\nu W_{fi}(\varepsilon_\nu) (1 - f(\varepsilon_\nu))(1 - f(\varepsilon_u)) 
\]

(16)

where we consider all charged particles are in the lowest Landau level. The prefactor 3 takes into account the d quark’s flavour degenerate. The function \( f(\varepsilon_i) \) denotes the Fermi-Dirac functions for the \( i \)-th particle. The transition rate per unit volume due to the antineutrino process may be derived from Fermi’s golden rule and is given by \( W_{fi} = (|S_{fi}|^2)/tV \). Here \( t \) represents time and \( V = L_x L_y L_z \) the normalization volume. The \( S \) matrix element is given by

\[
S_{fi} = \frac{G_F}{\sqrt{2}} \int d^4X \bar{\psi}_e \gamma_\mu (1 - \gamma_5) \psi_u \bar{\psi}_u \gamma^\mu (1 - \gamma_5) \psi_d 
\]

(17)

when all the charged particles in the lowest Landau level \( (n = 0) \), their wave function can be expressed very simply as

\[
\psi_e(x) = \left( \frac{eB}{\pi} \right)^{1/4} \frac{1}{\sqrt{L_y L_z}} \exp[-ie\varepsilon_e t + iep_eyy + iep_ezz] \exp(-\xi_e^2) U_{e,-1} 
\]

(18)

\[
\psi_d(x) = \left( \frac{eB}{3\pi} \right)^{1/4} \frac{1}{\sqrt{L_y L_z}} \exp[-ie\varepsilon_d t + iep_dgy + iep_dzz] \exp(-\xi_d^2) U_{d,-1} 
\]

(19)

\[
\psi_u(x) = \left( \frac{2eB}{3\pi} \right)^{1/4} \frac{1}{\sqrt{L_y L_z}} \exp[-ie\varepsilon_u t + iep_uyy + iep_uzz] \exp(-\xi_u^2) U_{u,1} 
\]

(20)

\[
\psi_{\nu,1}(x) = \frac{1}{\sqrt{L_x L_y L_z}} \exp[-ie\varepsilon_\nu t + iep_\nu x + iep_\nu y + iep_\nu z] U_{\nu,1} 
\]

(21)
\[ \psi_{\nu,-1}(x) = \frac{1}{\sqrt{L_x L_y L_z}} \exp[-i\varepsilon_{\nu}t + i\varepsilon_{\nu}x + i\varepsilon_{\nu}y + i\varepsilon_{\nu}z]U_{\nu,-1} \]  

(22)

where \( \varepsilon_{\nu} = (eB)^{1/2}(x + \frac{p_{\nu u}}{eB}) \), \( \varepsilon_{d} = (eB)^{1/2}(x + \frac{3p_{\nu u}}{eB}) \) and \( \varepsilon_{u} = (\frac{2eB}{3})^{1/2}(x - \frac{3p_{\nu u}}{2eB}) \). The negative charged particles in the lowest Landau level spin down, while the positive charged particles spin up. So from Eqs. (7)-(10), we know that

\[
\begin{align*}
U_{e,-1} &= \frac{1}{\sqrt{2\varepsilon_{e}(\varepsilon_{e} + m_{e})}} \begin{pmatrix} 0 \\ \varepsilon_{e} + m_{e} \\ -\varepsilon_{e} \end{pmatrix} \\
U_{d,-1} &= \frac{1}{\sqrt{2\varepsilon_{d}(\varepsilon_{d} + m_{d})}} \begin{pmatrix} 0 \\ \varepsilon_{d} + m_{d} \\ -\varepsilon_{d} \end{pmatrix} \\
U_{u,+1} &= \frac{1}{\sqrt{2\varepsilon_{u}(\varepsilon_{u} + m_{u})}} \begin{pmatrix} \varepsilon_{u} + m_{u} \\ 0 \\ p_{uz} \end{pmatrix} \\
U_{\nu,+1} &= \frac{1}{\sqrt{2\varepsilon_{\nu}(\varepsilon_{\nu} + m_{\nu})}} \begin{pmatrix} \varepsilon_{\nu} + m_{\nu} \\ 0 \\ p_{\nu z} \end{pmatrix} \\
U_{\nu,-1} &= \frac{1}{\sqrt{2\varepsilon_{\nu}(\varepsilon_{\nu} + m_{\nu})}} \begin{pmatrix} \varepsilon_{\nu} + m_{\nu} \\ -p_{\nu z} \\ -\varepsilon_{\nu} \end{pmatrix}
\end{align*}
\]

(23) - (27)

Because only positive energy solution will contribute (\( \varepsilon = +1 \)), using these wave functions it is straightforward to calculate the transition rate per unit volume which is given by (for convenience, we take \( p_{\nu x} \approx p_{\nu y} \approx p_{\nu z} \approx 0 \))

\[
W_{fi} = \frac{G_{F}^{2}\cos^{2}\theta_{W}\sqrt{\pi}L_{x}L_{y}L_{z}}{6\sqrt{\pi}L_{x}L_{y}L_{z}}|M|_{2}^{2}\delta(\varepsilon_{d} - \varepsilon_{u} - \varepsilon_{e} - \varepsilon_{\nu}) \\
\delta(p_{d\nu} - p_{u\nu} - p_{e\nu} - p_{\nu z})\delta(p_{dz} - p_{uz} - p_{ez} - p_{\nu z}) \\
e^{\exp[-\frac{p_{\nu y}^{2}}{eB} - \frac{5p_{\nu u}^{2}}{2eB} - \frac{p_{\nu u}^{2}}{2eB} - \frac{p_{\nu y}p_{du}}{eB} - \frac{p_{\nu y}p_{du}}{eB} + \frac{p_{du}p_{cy}}{eB} + \frac{p_{du}p_{cy}}{eB}]} \quad (28)
\]

Here \( M \) is the invariant matrix element. Squaring it and summing over neutrino states, we get

\[
|M|^{2} = \frac{(\varepsilon_{d} + m_{d} + p_{dz})(\varepsilon_{e} + m_{e} - p_{ez} + p_{ez})^{2}(\varepsilon_{u} + m_{u} + p_{uz})^{2}}{4\varepsilon_{d}\varepsilon_{e}\varepsilon_{u}(\varepsilon_{e} + m_{e})(\varepsilon_{u} + m_{u})} \times [\frac{(\varepsilon_{\nu} + m_{\nu} - p_{\nu z})^{2} + p_{\nu z}^{2}}{\varepsilon_{\nu}(\varepsilon_{\nu} + m_{\nu})}]^{2} \quad (29)
\]
when the charged particle confined in the lowest Landau level, its energy is \( \varepsilon = \sqrt{p_z^2 + m^2} \). So if the particle’s mass is zero, we have \( \varepsilon = \pm p_z \). Here we consider electron, u and d quarks’ mass are zero and neglect the neutrino’s momentum, therefore only if

\[
p_{dz} > 0, \ p_{ez} < 0 \ p_{uz} > 0
\]

the \(|M|^2\) is not zero, it is

\[
|M|^2 = 16
\]

and note that the range \( L_x \) of \( x \) can be fixed in a natural way \([11]\) corresponding to the magnetic field \( L_x = 1/\sqrt{eB} \). Considering all these factors, the neutrino emissivity can be written as

\[
\dot{\varepsilon}_\nu(B) = \frac{8G^2F^2\cos^2\theta_e eB}{(2\pi)^6 \sqrt{\pi}} \int_{-\sqrt{1/3eB/2}}^{\sqrt{1/3eB/2}} dp_{dy} \int_{-\sqrt{2/3eB/2}}^{\sqrt{2/3eB/2}} dp_{uy} \int_{-\sqrt{1/eB/2}}^{\sqrt{1/eB/2}} dp_{cy}
\]

\[
\delta(p_{dy} - p_{cy} - p_{uy}) \exp[-\frac{p_{cy}^2}{2eB} - \frac{5p_{cy}^2}{2eB} - \frac{p_{uy}^2}{2eB} - \frac{p_{uy}p_{dy}}{eB} - \frac{p_{uy}p_{cy}}{eB} + \frac{p_{dy}p_{uy}}{eB}]
\]

\[
\delta(\varepsilon_d - \varepsilon_u - \varepsilon_e - \varepsilon_{\nu e}) \delta(p_{dz} - p_{uz} - p_{ez} - p_{\nu e z})
\]

Integrals over \( p_{dy}, p_{uy} \) and \( p_{cy} \) can be performed numerically. The integrals over \( p_{ez}, p_{dz} \) and \( p_{uz} \) are converted into integrals over \( d\varepsilon_{e z}, d\varepsilon_{d z} \) and \( d\varepsilon_{u z} \) respectively. The integral over neutrino solid angle is performed by using the \( z \)-component momentum conserving delta function. Finally using the standard procedure \([12]\), the neutrino emissivity can be cast into

\[
\dot{\varepsilon}_\nu(B) = \frac{3\sqrt{2\pi}G^2\cos^2\theta_e eB}{16(2\pi)^6} \int_{-\sqrt{2eB/3/2}}^{\sqrt{2eB/3/2}} d\varepsilon_{e z} \int_{-\sqrt{1/eB/2}}^{\sqrt{1/eB/2}} d\varepsilon_{d z} \int_{-\sqrt{1/eB/2}}^{\sqrt{1/eB/2}} d\varepsilon_{u z}
\]

\[
\delta(\varepsilon_d - \varepsilon_u - \varepsilon_e - \varepsilon_{\nu e}) \delta(p_{dz} - p_{uz} - p_{ez} - p_{\nu e z})
\]

Given the magnetic field, the integration in Eq. (33) can be calculated numerically and it varies about linearly according to the magnetic field. So the neutrino emissivity can be approximately written as

\[
\dot{\varepsilon}_\nu(B) \approx 6.03 \times 10^{26} eB^2 T^5
\]

the neutrino emissivity of reaction (15) is similar to that if the magnetic field strength is higher than \( 10^{18} \) gauss.
4 COOLING CURVES OF STRONG MAGNETIZED STRANGE STAR

Because of the coulomb barrier, strange stars may have a similar thin crust to neutron stars. We postulate that the crust of strange stars has the same structure with the same mass neutron stars. In our calculations, we take $M = 1.4M_\odot$ and $R = 10km$, so the corresponding central density of the strange stars is $\rho_c \simeq 6.65 \times 10^{14} g cm^{-3}$. With regard to the relationship between the interior temperature $T$ and the surface temperature $T_s$, we use the isothermal core approximation which assumes that the internal temperature $T$ is constant within the stellar core with $\rho_c \geq \rho_b = 10^{10} g cm^{-3}$ (Glen & Suthland 1983) and apply the result of Potekhin et al. (1997). We need discuss the heat capacity besides neutrino emissivity for the calculation of the cooling curves. We adopt the standard thermodynamic formula (Lifshitz and Pitaevskii, 1980).

$$c_j = \frac{g_j}{(2\pi)^3} \int d^3\vec{p}_j \frac{1}{\varepsilon_j^2} df_j$$

where $g_j$, $\varepsilon_j$ and $\vec{p}_j$ stand for the particles’ degeneracy, energy and momentum respectively, $\mu_j$ is the chemical potential, and $f_j$ is the Fermi-Dirac distribution. In field-free case, we have for an almost ideal, strongly degenerate ultra-relativistic gas

$$c_j = \frac{g_j\rho_c^2 T}{6}$$

In magnetic field, it reads

$$c_j = \frac{g_j eB}{(2\pi)^2} \int dp_{zj} \frac{1}{\varepsilon_j^2} df_j$$

If we assume the charged particles to be in the lowest Landau level in the strong magnetic field. We obtain

$$c_{\text{quark}} = \frac{g_j eBT}{6}$$

We can now make cooling simulation which is reduced to solve the equation of thermal balance

$$C \frac{dT}{dt} = -L_\gamma - L_\nu$$

where $C$, $L_\gamma$, and $L_\nu$ denote respectively total heat capacity, neutrino and surface luminosity, $C = \sum c_j dV$, $L_\nu = \int \dot{\varepsilon}_\nu dV$, $L_\gamma = 4\pi R^2 \sigma T^4$ with Stefan-Boltzmann constant $\sigma$ and surface temperature $T_s$. Here we will compare two cases of the s-quark polarized and without polarization. which are showed in figure 1 and 2. In both the cases studied, the magnetic field results in more faster cooling compared to the the field-free case.

5 SUMMARY

We have calculated the neutrino emissivity of strange stars with Ultra strong magnetic field in some approximation and obtained an analytic formula. Our results show
that the neutrino emissivity is proportional to $T^5$ instead of $T^6$ as in the field-free case\cite{16}. Bandyopadhyay\cite{18} had already calculated the neutrino emissivity of magnetized strange stars, but they treated d and s quarks as free particles and they found that the relation between the neutrino emissivity and the temperature is the same as in the field-free case, with only a different coefficient. Furthermore, we find that the emissivity is strongly dependent on the magnetic field but independent of the electron fraction due to the extremely polarization. In the cases we studied, we show that the magnetic field accelerates the cooling of the strange stars. Finally, we emphasize that a general treatment must consider the contribution of different Landau levels of all involved charged particles because of the finite field strength. But we expect the neutrino emissivity is still proportional to $T^5$, with only corrections to the coefficient. course, the analytical calculation is difficult and the numerical method should be applied. The future works will be necessary.

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Figure 1: Cooling curves with the s-quark not polarized
Figure 2: Cooling curves with the s-quark polarized