Behaviors of the charge fluctuation in relativistic nucleus-nucleus collisions

Ben-Hao Sa\textsuperscript{1,2,4,5,*}, Xu Cai\textsuperscript{4,2}, Zong-Di Su\textsuperscript{1,4}, An Tai\textsuperscript{3}, and Dai-Mei Zhou\textsuperscript{2}

\textsuperscript{1} China Institute of Atomic Energy, P. O. Box 275 (18), Beijing, 102413 China
\textsuperscript{2} Institute of Particle Physics, Huazhong Normal University, Wuhan, 430079 China
\textsuperscript{3} Department of Physics and Astronomy, University of California, at Los Angeles, Los Angeles, CA 90095 USA
\textsuperscript{4} CCAST (World Lab.), P. O. Box 8730 Beijing, 100080 China
\textsuperscript{5} Institute of Theoretical Physics, Academia Sinica, Beijing, 100080 China

Abstract

Using a hadron and string cascade model, JPCIAE, we investigated the dependence of event-by-event charge fluctuation on the energy, centrality, window size, resonance decay, and the final state interaction for \(Pb + Pb\) collisions at SPS and LHC energies and \(Au + Au\) collisions at RHIC energies. The JPCIAE results of charge fluctuation as a function of the rapidity window size in \(Pb + Pb\) collisions at SPS energies were compared with the preliminary NA49 data. Comparisons with STAR and PHENIX data of \(Au + Au\) collisions at \(\sqrt{s_{nn}} = 130\) GeV were also given. It seems that the final state interaction and resonance decay play a gentle role on the charge fluctuations. The charge fluctuations are slightly decreasing with or nearly independent of the reaction energy and hardly depend on the collision centrality.

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* Email: sabh@iris.ciae.ac.cn
In [1, 2], the energy fluctuation (heat capacity) was first related to the liquid-gas phase transition in intermediate energy heavy-ion collisions. The irregular behavior of heat capacity was then proposed to study the phase transition from hadronic matter to a Quark-Gluon-Plasma (QGP) provided that the event-by-event (E-by-E) fluctuation of temperature is observable in relativistic nucleus-nucleus collisions [3, 4]. Such irregular behavior is also a characteristic of a phase transition: a jump in a first order phase transition and a singularity in a second order one [3]. The E-by-E fluctuation of an observable might supply important information such as the hadronic matter compressibility [3], the position and property of a critical point in the QCD phase diagram of temperature T vs. chemical potential $\mu$ [4], etc. In [4] it was also predicted that the E-by-E fluctuation pattern in average transverse momentum,

![Graph](image)

**FIG. 1:** $\tilde{D}_R$ as a function of the $\Delta y$ in 40, 80 and 158A GeV/c $Pb + Pb$ collisions. The preliminary NA49 data were take from [22].

With the increase of interaction energy a rather high particle multiplicity is accessible and the statistically significant measurements of E-by-E fluctuation became possible for the first time in $Pb + Pb$ collisions at 158A GeV/c [3, 4, 5, 6, 7, 8] and recently in $Au + Au$ collisions at $\sqrt{s_{nn}} = 130$ GeV [11]. Though it was claimed that the non-statistical contributions to E-by-E fluctuation of the average transverse momentum, the $k/\pi$ ratio, and the net charge multiplicity are small [4, 11, 11], the calculations of E-by-E fluctuations based on hadronic transport models [8, 12, 13] and effective models [14, 15, 16] were stimulated.

Since the unit of charge (baryon charge) in the QGP phase is $1/3$ while it is 1 in the hadronic phase, the thermal model predicted that the value of the charged particle ratio
E-by-E fluctuation, $D_R$ (defined below), in the hadronic phase would be a factor of $\sim 2.5 - 4$ larger than that in the QGP phase [17, 18, 19]. The charged particle ratio E-by-E fluctuation was then proposed as a signal of QGP formation if the initial fluctuations survive hadronization and their relaxation time is longer than the collision time [17, 18]. In [20] UrQMD [21] was used to investigate the charged particle ratio fluctuation in $Pb + Pb$ collisions at SPS and $Au + Au$ collisions at the full RHIC energy. However, the UrQMD predictions for the charged particle ratio fluctuation in $Pb + Pb$ collisions at SPS energy were around 3 while the preliminary NA49 data [22] were around 4.

The hadron and string cascade model, JPCIAE, was employed in this paper to further study the charge fluctuations. The model results were compared with the preliminary NA49 data of the charged particle ratio fluctuation as a function of the rapidity window size in $Pb + Pb$ collisions at 40, 80 and 158A GeV/c [22] and with STAR and PHENIX data in $Au + Au$ collisions at $\sqrt{s_{NN}}$=130 GeV [11, 23]. Meanwhile, the dependence of charge fluctuations on reaction energy (from SPS up to LHC), centrality (impact parameter $b$), final state interaction (rescattering), and the resonance decay ($\rho$ and $\omega$) was investigated. This study shows that the charge fluctuations are slightly decreasing with or nearly independent of the reaction energy and hardly depend on the collision centrality. The charge fluctuations are gently affected by the rescattering and the resonance decay ($\rho$ and $\omega$).

The JPCIAE model was developed based on PYTHIA [24], which is a well known event generator for hadron-hadron collisions. In the JPCIAE model the radial position of a nucleon
in colliding nucleus A (indicating the atomic number of this nucleus as well) is sampled randomly according to the Woods-Saxon distribution and the solid angle of the nucleon is sampled uniformly in $4\pi$. Each nucleon is given a beam momentum in z direction and zero initial momenta in x and y directions. The collision time of each colliding pair is calculated under the requirement that the least approach distance of the colliding pair along their straight line trajectory (mean field potential is not taken into account in JPCIAE) should be smaller than $\sqrt{\sigma_{\text{tot}}/\pi}$. Here $\sigma_{\text{tot}}$ refers to the total cross section. The nucleon-nucleon collision with the least collision time is then selected from the initial collision list to perform the first collision. Both the particle list and the collision list are then updated such that the new collision list may consist of not only nucleon-nucleon collisions but also collisions between nucleons and produced particles and between produced particles themselves. The next collision is selected from the new collision list and the processes above are repeated until the collision list is empty.

For each executing collision pair, if its CMS energy is above a certain threshold (=4 GeV in program), we assume that strings are formed after the collision and PYTHIA is used to deal with particle production. Otherwise, the collision is treated as a two-body collision [25, 26, 27]. The threshold above is chosen in such a way that JPCIAE correctly reproduces the charged multiplicity distributions in nucleus-nucleus collisions [28]. It should be noted here that the JPCIAE model is not a simple superposition of nucleon-nucleon collisions since the rescatterings of secondary particles are taken into account. We refer to [28] for more details about the JPCIAE model. Note that particle production from strings in JPCIAE is determined by the Lund fragmentation scheme [29], in which only the lowest excitation state of a resonance is included.

If the deviation (i.e. fluctuation [30]) of a physical variable $x$ from its average value per event $<x>$ is defined as

$$\delta x = x - <x>, \quad (1)$$

the variance of $x$ reads [30]

$$< (\delta x)^2 > = < x^2 > - < x >^2. \quad (2)$$

Suppose $x \equiv R = N_+/N_-$ to be the ratio of positively to negatively charged particle multiplicity, the corresponding variance is

$$< (\delta R)^2 > = < R^2 > - < R >^2. \quad (3)$$
FIG. 3: The centrality dependence of charge fluctuations.

Similarly the variance of net charge multiplicity, \( Q = N_+ - N_- \), reads

\[
< (\delta Q)^2 > = < Q^2 > - < Q >^2 .
\] (4)

However, what is interesting is not \( < (\delta R)^2 > \) or \( < (\delta Q)^2 > \) but

\[
D_R \equiv < N_{ch} > < (\delta R)^2 >
\] (5)

or

\[
D_Q \equiv \frac{< (\delta Q)^2 >}{< N_{ch} >},
\] (6)

where \( N_{ch} = N_+ + N_- \) refers to the total charge multiplicity. A relation follows approximately [17]

\[
D_R \simeq 4D_Q .
\] (7)

The thermal (effective) model predictions for \( D_R \) are [17]: \( \sim 4 \) for a pion gas, \( \sim 3 \) for a resonance pion gas (pions from \( \rho \) and \( \omega \) decays), and \( \sim 0.75 \) for massless noninteracting quarks and gluons (that is \( \sim 1 \) from lattice calculations).

As mentioned in [17], one main assumption made in the thermal model predictions is that the studied system can be described as a grand canonical ensemble. However, in experiments or dynamical simulations, the investigated subsystem (e. g. within a rapidity interval \( \Delta y \)) is a finite fraction of the full system (e. g. in the full rapidity region). Therefore, the assumption of a grand canonical ensemble is only valid in the limit of \( N_{ch} >_{\Delta y} / N_{ch} >_{\text{total}} \rightarrow 0 \), such that the rest system plays the role of a thermal resource. In order to compare the
experiments or dynamical simulations with the thermal model predictions it might be better introducing a correction factor \footnote{17}

\[ C_y = 1 - \frac{<N_{ch}>_{\Delta y}}{<N_{ch}>_{total}}. \]  

(8)

Another assumption adopted in the thermal model predictions is the vanishing of net charge \footnote{17}. However, that is actually impossible in experiments or dynamical simulations, the corresponding correction factor \footnote{17} reads

\[ C_\mu = \frac{<N_+>_{\Delta y}}{<N_->_{\Delta y}}. \]  

(9)

The \( D_R \) with corrections above is denoted as

\[ \tilde{D}_R = \frac{D_R}{C_y C_\mu}. \]  

(10)

The fluctuation is usually composed of statistical fluctuation and dynamical fluctuation. There are many sources to be considered as dynamical fluctuations, such as string fragmentation (or QCD color fluctuations), centrality (impact parameter or participants), rescattering, resonance decay, etc. On the contrary, the statistical fluctuation is no dynamical origin and could be described in a stochastic scenario by probability distribution functions \footnote{11}, \footnote{30}. Only a finite number of events could be generated in experiments or dynamical simulations causes also the statistical fluctuation. Though it is necessary to study the influences of reaction energy, centrality, rescattering, and resonance decay individually, an alternative way to investigate the non-statistical contribution is to compare E-by-E fluctuation distribution extracted from real events with ones from mixed events \footnote{10}. The mixed events here are
constructed from the real events so that in principle only statistical fluctuation survives in
the mixed events.

FIG. 5: The $\hat{D}_R$, $D_R$, and $4D_Q$ as a function of $\Delta \eta$: (a) comparison of the results from JPCIAE
and UrQMD for $Au + Au$ collisions at $\sqrt{s_{nn}}=200$ GeV; (b) comparison of the JPCIAE results for
$Au + Au$ collisions at $\sqrt{s_{nn}}=130$ and 200 GeV.

In Fig. 1 the JPCIAE results of $\hat{D}_R$ as a function of $\Delta y$ in 40, 80 and 158A GeV/c $Pb + Pb$
collisions (full circles) are compared to the NA49 preliminary data (full triangles) 22. Corresponding
to the centrality cut of 7.2% at 40 and 80A Gev/c and 10% at 158A GeV/c in the NA49 experiments the impact parameters in the JPCIAE calculations were set to be
$b \leq 3.57$ fm and $b \leq 4.20$ fm, respectively. $\Delta y$ was set around 2.9, 3.2, and 3.6, respectively,
for 40, 80, and 158A GeV/c energies, and the $p_t$ window was set to be 0.005 < $p_t$ < 2.5
GeV/c for all the three beam momenta as in the NA49 experiments. The dashed and solid
lines in this figure are the thermal model predictions for a resonance pion gas and the lattice
Monte Carlo result for a quark-gluon gas, respectively 20. One sees from this figure that
the JPCIAE results are generally compatible with the preliminary NA49 data for 40 and
80A GeV/c $Pb + Pb$ collisions. However, for 158A GeV/c $Pb + Pb$ collisions there exists
discrepancies in the $\Delta y$ dependence between preliminary NA49 data and JPCIAE results.
Such differences are not due to statistics and require further studies.

The effects of rescattering and resonance decay ($\rho$ and $\omega$ primarily 17) on the distribution
of $\hat{D}_R$ vs. $\Delta y$ are shown in Fig. 2 for 158A GeV/c $Pb + Pb$ collisions. In this figure,
the circles, the triangles, and the squares are, respectively, the results of default JPCIAE,
JPCIAE without rescattering, and JPCIAE without $\rho$ and $\omega$ resonance decays. In the JPCIAE calculations the impact parameter was $b \leq 3.5$ fm, $\Delta y$ was set around 3 and the $p_t$ window was $0 < p_t < 5$ GeV/c. Globally speaking, the rescattering effect is gentle, that is consistent with the conclusion from the RQMD model [31]. The effect of resonance decay seems weak, too and the shift in $\tilde{D}_R$ is smaller than 1 over all the $\Delta y$ region unlike what is expected based on the thermal model predictions for a pion gas and resonance pion gas. However, in the default JPCIAE calculations no all the mesons are from $\rho$ and $\omega$ resonance decays, which may explain in part why the shift is smaller than 1.

Fig. 3 (a) compares the JPCIAE results (circles) of centrality dependence of $\tilde{D}_R$ in $Pb + Pb$ collisions at 158A GeV/c with UrQMD results (squares, taken from [20]). In both calculations the rapidity window was $2.5 < y < 4.5$. The discrepancies between JPCIAE and UrQMD results might attribute in part to the higher resonance states included in the UrQMD model. In Fig. 3 (b) the centrality dependences of $\tilde{D}_R$, $D_R$, and $4D_Q$ in Au + Au collisions at $\sqrt{s_{nn}}=200$ GeV from JPCIAE (-0.5 < $\eta$ < 0.5) are given by full circles, open circles, and full squares, respectively. One sees from Fig. 3 that the charge fluctuation measures (i. e. the $\tilde{D}_R$, $D_R$, and $4D_Q$) are not so sensitive to the impact parameter. That is consistent with the

![FIG. 6: The energy dependence of $\tilde{D}_R$, $D_R$, and $4D_Q$.](image)

In Fig. 4 the JPCIAE results of $D_R$ and $4D_Q$ as a function of $\langle N_{ch} \rangle$ in peripheral $Au + Au$ collisions at $\sqrt{s_{nn}}=130$ GeV (-0.35 < $\eta$ < 0.35, $p_t > 0.2$) were compared with PHENIX data [11]. For simplicity only part of $D_R$ data points were copied (full squares with error bar) and compared with JPCIAE results (open squares with error bar). Most
$D_R$ results from JPCIAE were lower than PHENIX data, at peak region especially. That might attribute in part to the PHENIX spectrometer has an acceptance of $\pi/2$ radians in azimuthal angle, since a linear extrapolation to full azimuthal coverage leads to the decreasing of fluctuation measures $^{[11]}$. In Fig. 4 the PHENIX data of $4D_Q$ with error bar were denoted simply by shaded region and compared with JPCIAE results of $4D_Q$ (open circles). One sees that the JPCIAE results of $4D_Q$ are compatible with PHENIX data.

Fig. 5 (a) compared the JPCIAE results of $\tilde{D}_R$ and $D_R$ as a function of $\Delta \eta$ ($b \leq 2$ fm) in $Au + Au$ collisions at $\sqrt{s_{nn}}=200$ GeV with the UrQMD results ($b \leq 2$ fm, taken from $^{[20]}$ where $\Delta y$ was used). The full and open squares and the full and open triangles in this panel are, respectively, the results of $\tilde{D}_R$ and $D_R$ from JPCIAE and UrQMD. Generally speaking, the results of JPCIAE are systematically higher than those of UrQMD, similar to Fig. 3 (a). In Fig. 5 (b) the JPCIAE results of $\tilde{D}_R$, $D_R$ and $4D_Q$ as a function of $\Delta \eta$ in $Au + Au$ collisions at $\sqrt{s_{nn}}=130$ (open squares, circles, and triangles, respectively) and at 200 GeV (full squares, circles, and triangles, respectively) are compared with each other. In those calculations the centrality and $p_t$ cuts were, respectively, 10% most central collisions and $p_t > 0.2$. The thick stick at $\Delta \eta=0.7$ is the PHENIX datum $^{[11]}$ of $4D_Q$ in $Au + Au$ collisions at $\sqrt{s_{nn}}=130$ GeV, which is about 10% lower than the corresponding JPCIAE result (open triangle). It is also interesting to note that the JPCIAE result of $D_Q \sim 0.9$ at $\Delta \eta=1$ in $Au + Au$ collisions at $\sqrt{s_{nn}}=130$ GeV is about 10% higher than the corresponding STAR datum 0.8 extracted under the assumption of zero net charge $^{[23]}$. From Fig. 5 (b) one sees that globally speaking the charge fluctuation measures are not sensitive to the change of energy from 130 to 200 GeV, which is consistent with the conclusions in $^{[18, 20, 31]}$.

Finally, the JPCIAE results of energy dependence of $\tilde{D}_R$ (full circles), $D_R$ (open circles), and $4D_Q$ (full triangles) from SPS to RHIC and then to LHC energy were given in Fig. 6. In those calculations the centrality cuts were all set to be 10% most central collisions and rapidity windows are $2.5 < y < 3.5$ for $Pb + Pb$ at SPS and $-0.5 < \eta < 0.5$ for $Au + Au$ at RHIC and $Pb + Pb$ at LHC energy, respectively. One sees from Fig. 6 that the $D_R$ might be decreasing slightly with energy. However, the $\tilde{D}_R$ and $4D_Q$ show almost no energy dependence within error bar.

In summary, a hadron and string cascade model, JPCIAE, has been employed in this paper to investigate the energy, centrality, rescattering, and resonance decay dependences of the charge fluctuation measures. Within the framework of this model the calculated results
seem compatible with the preliminary NA49 data for $Pb + Pb$ collisions at 40 and 80\,A GeV/c. For 158\,A GeV/c $Pb + Pb$ collisions there exists discrepancies in the $\Delta y$ dependence between JPCIAE results and preliminary NA49 data. The JPCIAE results for $Au + Au$ collisions at $\sqrt{s_{nn}}=56$, 130 and 200 GeV and for $Pb + Pb$ collisions at $\sqrt{s_{nn}}=5500$ GeV were given as well. Comparisons between JPCIAE results and experimental data from STAR and PHENIX were also made for $Au + Au$ collisions at $\sqrt{s_{nn}}=130$ GeV. It seems that the charge fluctuation measures are nearly independent of the collision centrality. Their dependence on the reaction energy is weak. It is also found that the effect of resonance decay ($\rho$ and $\omega$) on the charge fluctuation measures is gentle. However, the rescattering effect might be somewhat stronger than resonance decay.

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