Issues in Quarkonium Production

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ABSTRACT

In this talk, I start with a brief introduction to Non-Relativistic QCD (NRQCD) and its applications to quarkonium physics. This theory has provided a consistent framework for the physics of quarkonia, in particular, the colour-octet Fock components predicted by NRQCD have important implications for the phenomenology of charmonium production in experiments. The applications of NRQCD to \(J/\psi\) production at Tevatron and the tests of the theory in other experiments is discussed. In particular, the apparent disagreement of NRQCD with results from HERA on inelastic photoproduction of \(J/\psi\) is discussed and it is shown that the results are rather susceptible to intrinsic transverse momentum smearing. The photoproduction data, therefore, do not provide a good test of NRQCD. It is argued that NRQCD may be tested stringently by looking for the production of other charmonium resonances at the Tevatron, because the production rates for these resonances can be predicted within the NRQCD framework.

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Over the last few years, there has been a considerable advance in the understanding of quarkonium physics due to the development of the non-relativistic effective field theory of QCD, called non-relativistic QCD (NRQCD) \[1\]. The Lagrangian for this effective theory is obtained from the full QCD Lagrangian by neglecting all states with momenta larger than a cutoff of the order of the heavy quark mass, \(m\), and accounting for this exclusion by introducing new interactions in the effective Lagrangian, which are local since the excluded states are relativistic. Beyond the leading order in \(1/m\) the effective theory is non-renormalisable. The scale \(m\) is an ultraviolet cut-off for the physics of the bound state; however the latter is more intimately tied to the scales \(mv\) and \(mv^2\), where \(v\) is the relative velocity of the quarks in the bound state. The physical quarkonium state admits of a Fock expansion in \(v\), and it turns out that the \(Q\bar{Q}\) states appear in either colour-singlet or colour-octet configurations in this series. Of course the physical state must be a colour-singlet, so that a colour-octet \(Q\bar{Q}\) state is connected to the physical state by the emission of one or more soft gluons. In spite of the non-perturbative nature of the soft gluon emissions, the effective theory still gives useful information about the intermediate octet states. This is because the dominant transitions that occur from colour-octet to physical colour-singlet states are via E1 or M1 transitions with higher multipoles being suppressed by powers of \(v\). It then becomes possible to use the usual selection rules for these radiative transitions to keep account of the quantum numbers of the octet states, so that the production of a \(Q\bar{Q}\) pair in a octet state can be calculated and its transition to a physical singlet state can be specified by a non-perturbative matrix element. The cross-section for the production of a meson \(H\) then takes on the following factorised form:

\[
\sigma(H) = \sum_{n=\{\alpha,S,L,J\}} \frac{F_n}{m^{d_n-4}} \langle O^H_{\alpha}(2S+1LJ) \rangle, \tag{1}
\]

where the \(F_n\)'s are the short-distance coefficients and the \(O_n\) are local 4-fermion operators, of naive dimension \(d_n\), describing the long-distance physics. The short-distance coefficients are associated with the production of a \(Q\bar{Q}\) pair with the colour and angular momentum quantum numbers indexed by \(n\). These involve momenta of the order of \(m\) or larger and can be calculated in a perturbation expansion in the QCD coupling \(\alpha_s(m)\). The \(Q\bar{Q}\) pair so produced has a separation of the order of \(1/m\) which is pointlike on the scale of the quarkonium wavefunction, which is of order \(1/(mv)\). The non-perturbative long-distance factor \(\langle O^H_n \rangle\) is proportional to the probability for a pointlike \(Q\bar{Q}\) pair in the state \(n\) to form a bound state \(H\).

The existence of the colour-octet components of the quarkonium wave function is the new feature of the NRQCD approach. Before the development of NRQCD, the production and decay of quarkonia were treated within the framework of the colour-singlet model \[2, 3\]. In this model, it is assumed that the \(Q\bar{Q}\) pair is formed in the short-distance process in a colour-singlet state. The corrections from terms higher order in \(v\) were neglected. While this model gave a reasonable description of low-
energy $J/\psi$ data, it was known that it was incomplete because of an inconsistency in the treatment of the $P$-state quarkonia. This was due to a non-factorising infra-red divergence, noted first in the application of the colour-singlet model to $\chi c$ decays [4], and the proper resolution of this problem was obtained only by including the colour-octet components in the treatment of the $P$-states [5]. The colour-octet components, however, had a more dramatic impact [6] on the phenomenology of $P$-state charmonium production at large $p_T$ at the Tevatron $p\bar{p}$ collider [7] where the colour-singlet model was seen to fail miserably. While the inclusion of the colour-octet components for the $P$-states was necessary from the requirement of theoretical consistency, there was no such problem with the $S$ states because the corresponding amplitude was finite and the colour-octet components were suppressed compared to the colour-singlet component by $O(v^4)$. But the data on direct $J/\psi$ and $\psi'$ production at the Tevatron [7] seem to indicate an important contribution from the colour-octet components for the $S$-states as well [8].

While it is clear that the correct description of the Tevatron large-$p_T$ data requires that the colour-octet components of the quarkonium wave function have to be taken into account, the major problem is that the corresponding long-distance matrix elements are a priori unknown and can be obtained only by fitting to the Tevatron data [3, 10]. The direct $J/\psi$ production cross section in the NRQCD approach receives contributions from the colour-singlet $3S_1^{[1]}$ channel and the colour-octet $3P_J^{[8]}$, $1S_0^{[8]}$, and $3S_1^{[8]}$ channels. The non-perturbative parameter for the colour-singlet channel is known from $J/\psi$ leptonic decay. Given this input, the three non-perturbative parameters $\langle O(3P_J^{[8]}) \rangle$, $\langle O(1S_0^{[8]}) \rangle$, $\langle O(3S_1^{[8]}) \rangle$ (which we call matrix elements $M_1$, $M_2$, and $M_3$ respectively) are extracted from a fit to the CDF data. It turns out that for $p_T > 4$ GeV, the $p_T$ dependence of the short-distance coefficients corresponding to the $3P_J^{[8]}$ and the $1S_0^{[8]}$ channels are identical. The $3S_1^{[8]}$ channel on the other hand has a different $p_T$ distribution, because fragmentation-type contributions are present only for this channel. Consequently, the shape of the experimental $p_T$ distributions can be used to determine $M_3$ separately, but only a linear combination of $M_1$ and $M_2$ (i.e. $M_1/m_c^2 + M_2/3$) can be fitted.

Clearly it is important to have other tests of NRQCD, and much effort has been made recently to understand the implications of these colour-octet channels for $J/\psi$ production in other experiments. We discuss some of these below.

1. The prediction [11, 12, 13] for prompt $J/\psi$ production at LEP in the colour-singlet model is of the order of $3 \times 10^{-5}$, which is almost an order of magnitude below the experimental number for the branching fraction obtained from LEP [14]. Recently, the colour-octet contributions to $J/\psi$ production in this channel have been studied [15, 16] and it is found that the inclusion of the colour-octet contributions in the fragmentation functions results in a predictions for the branching ratio which is $1.4 \times 10^{-4}$ which is compatible with the measured values
of the branching fraction from LEP \cite{14}. A more accurate analysis, resumming large logarithms in $E_{J/\psi}/M_Z$ ignored in Ref. \cite{16} has been recently performed \cite{17}.

2. The production of $J/\psi$ in low energy $e^+e^-$ machines can also provide a stringent test of the colour-octet mechanism \cite{18}. In this case, the colour-octet contributions dominate near the upper endpoint of the $J/\psi$ energy spectrum, and the signature for the colour-octet process is a dramatic change in the angular distribution of the $J/\psi$ near the endpoint.

3. One striking prediction of the colour-octet fragmentation process both for $p\bar{p}$ colliders and for $J/\psi$ production at the Z-peak, is that the $J/\psi$ coming from the process $g \rightarrow J/\psi X$ is produced in a transversely polarised state \cite{19}. For the colour-octet $c\bar{c}$ production, this is predicted to be a 100% transverse polarisation, and heavy-quark spin symmetry will then ensure that non-perturbative effects which convert the $c\bar{c}$ to a $J/\psi$ will change this polarisation only very mildly. This spin-alignment can, therefore, be used as a test of colour-octet fragmentation.

4. The colour-octet components are found \cite{20} to dominate the production processes in fixed-target $pp$ and $\pi p$ experiments. Using the colour-octet matrix elements extracted from elastic photoproduction data it is possible to get a very good description of the $\sqrt{s}$-dependence and also the $x_F$ and rapidity distributions. More recently, NLO corrections to the fixed-target cross-sections have been calculated \cite{21}.

5. The associated production of a $J/\psi + \gamma$ is also a crucial test of the colour-octet components \cite{22} and also of the fragmentation picture \cite{23}. Similar tests can be conceived of with double $J/\psi$ production at the Tevatron \cite{24}.

6. $J/\psi$ and $\psi'$ production in $pp$ collisions at centre-of-mass energies of 14 TeV at the LHC also provides a crucial test of colour-octet fragmentation \cite{25}. Recently, $J/\psi + \gamma$ production at the LHC has also been studied \cite{26} at the LHC.

One important cross-check is the inelastic photoproduction of $J/\psi$ at the HERA $ep$ collider \cite{27}. The inelasticity of the events is ensured by choosing $z \equiv p_T / p_{J/\psi} > 1$ GeV. The surprising feature of the comparisons \cite{28, 29} of the NRQCD results with the data from HERA is that the colour-singlet model prediction is in agreement with the data while including the colour-octet component leads to violent disagreement with the data at large $z$. While the colour-singlet cross section dominates in most of the low-$z$ region, the colour-octet contribution increases steeply in the large-$z$ ($0.8 < z < 0.9$) region and this rise is not seen in the data. In these comparisons, the values of the non-perturbative matrix elements are taken to be those determined from a fit to the Tevatron large-$p_T$ data.
Naively, one would think that this points to a failure of NRQCD. But this conclusion is premature. The reason is that while at the Tevatron the measured $p_T$ of the $J/\psi$ is greater than about 5 GeV, at HERA the $p_T$ can be as small as $\mathcal{O}(1)$ GeV. At such small values of $p_T$ (and also for $z$ very close to unity), there could be significant perturbative and non-perturbative soft physics effects. One way to explore the effect of such contributions is to include transverse momentum smearing of the partons inside the proton, and study the effects of the parton transverse momentum, $k_T$, on the $J/\psi$ distributions both at the Tevatron and at HERA. It has been demonstrated \[30\] that the $z$ distribution measured at HERA is particularly sensitive to the effects of $k_T$ smearing, and that inelastic photoproduction at HERA, with the present kinematic cuts, is not a clean test of NRQCD \[1\].

In Fig. 1, the results of the fits to the Tevatron data are shown, for three different values of average $k_T$, $\langle k_T \rangle$, viz. $\langle k_T \rangle = 0, 0.7, 1.0$ GeV. It is observed that the effect of the $k_T$-smearing on the parameters extracted from the data is very modest. Fig. 1 shows that the fits to the data when $k_T$-smearing is included are very good and comparable in quality to the case $\langle k_T \rangle = 0$. Taking these fitted values of the parameters, inelastic $J/\psi$ photoproduction at HERA is considered, for the same choice of parton distributions, scales etc. as used in the Tevatron fits. The $z$ distribution, for $\sqrt{s_{\gamma p}} = 100$ GeV and $p_T > 1$ GeV, is compared with the data from HERA in Fig. 2. Again the theoretical curves in Fig. 2 are for $\langle k_T \rangle = 0, 0.7, 1.0$ GeV.

In the absence of smearing, $\langle k_T \rangle = 0$, we see that the colour-octet component makes a large contribution at $z$ close to 1 which is not supported by the data. However the introduction of $k_T$ makes a substantial change to the octet contribution. Whereas the effect of $k_T$-smearing is very small for large-$p_T$ production at the Tevatron, these effects are found to be very important for $J/\psi$ production at HERA. In particular, smearing significantly reduces the size of the cross section and the $z$ distribution also becomes flatter, in better agreement with the HERA data. It is safe to conclude that while a direct comparison of the NRQCD predictions with the $z$-dependence of the inelastic photoproduction cross section for $J/\psi$ at HERA show a marked disagreement between the two, we argue that such a comparison is misleading. The inelastic photoproduction process does not provide a clean test of NRQCD because of the very low $p_T$-cut ($\sim 1$ GeV) used in the HERA experiments making the data very susceptible to effects like $k_T$ smearing.

Better tests of NRQCD may be obtained by studying other observables at the Tevatron itself. The study of the polarisation of the produced $J/\psi$ mentioned earlier is one example; in the following, we will discuss the production of other charmonium resonances \[33, 34\] whose cross-section can be predicted in NRQCD. One important feature of the NRQCD Lagrangian is that it shows an approximate heavy-quark symmetry, which is valid to $O(v^2) \sim 0.3$. The implication of this symmetry is that the

\[1\] Other effects such as soft-gluon resummation \[31\] and the breakdown of NRQCD factorisation near $z = 1$ \[22\] have been discussed in the context of this discrepancy.
nonperturbative parameters have a weak dependence on the magnetic quantum number. Using this symmetry some non-perturbative matrix elements can be expressed in terms of others already determined from the Tevatron data. In particular, the $^1P_1$ matrix elements can be inferred from the Tevatron data on $\chi$ production and, therefore, the production of the $^1P_1$ charmonium state, $h_c$, can be predicted in NRQCD [33].

The production of the $h_c$ is interesting in its own right: charmonium spectroscopy [35] predicts this state to exist at the centre-of-gravity of the $\chi_c(3P_J)$ states. While the E760 collaboration at the Fermilab has reported [36] the first observation of this resonance its existence needs further confirmation.

The cross-section for $h_c$ production at the Tevatron energy ($\sqrt{s} = 1.8$ TeV) has been presented in Ref. [33]. For 20 pb$^{-1}$ total luminosity, for $p_T$ integrated between 5 and 20 GeV we expect of the order of 650 events in the $J/\psi + \pi$ channel. Of these, the
Figure 2: The HERA data [27] for $d\sigma/dz$ (in nb) for $J/\psi$ production at $\sqrt{s_{\gamma p}} = 100$ GeV with $p_T > 1$ GeV, compared with model predictions for three choices of the intrinsic transverse momentum distribution, namely $\langle k_T \rangle = 0$, 0.7, 1.0 GeV.

contribution from the colour-singlet channel is a little more than 40, while the octet channel gives more than 600 events. The colour-octet dominance is more pronounced at large-$p_T$. Recent results on $J/\psi$ production from CDF are based on a total luminosity of 110 pb$^{-1}$. For this sample, more than 3000 events can be expected to come from the decay of the $h_c$ into a $J/\psi$ and a $\pi$. With this large event rate, the $h_c$ should certainly be observable if the $p\pi^0$ coming from its decay can be reconstructed efficiently.

A similar prediction for the absolute production rate can be made for $\eta_c$ production [34], where the two-photon decay mode of the $\eta_c$ has been considered. Heavy quark symmetry allows the $\eta_c$ cross-section to be determined in terms of the non-perturbative parameters $M_1$ and $M_2$ obtained from $J/\psi$ data. But as explained before, the $J/\psi$ data do not allow for a separate determination of these parameters, but only a linear combination of these parameters. We can saturate the linear combination with either
$M_1$ or $M_2$ and we obtain the $\eta_c$ event rate in both these cases. For the integrated event rate, with a $p_T$-cut of 5 GeV and assuming an integrated luminosity of 110 pb$^{-1}$, we find that the number of $\eta_c \rightarrow \gamma\gamma$ lies between 425 and 7700, depending on whether $M_1$ or $M_2$ saturates the linear combination. The sensitivity of the event rate to $M_1$ and $M_2$ shows that the experimental measurement of the $\eta_c$ cross-section will allow for an accurate determination of these non-perturbative parameters. We reiterate that the rates for $h_c$ and $\eta_c$ are predictions of NRQCD, and it is not possible to have similar predictions in alternative approaches to quarkonium production like colour-evaporation [37].

In conclusion, NRQCD provides a predictive theoretical framework for quarkonium physics. In particular, the anomalies in the $J/\psi$ production at the Tevatron are properly understood using NRQCD. Several other tests of the theory, proposed in the literature, have been discussed. In particular the inelastic photoproduction of $J/\psi$ at HERA is discussed and it shown that the apparent disagreement of the experimental results with the predictions of NRQCD is misleading. Because of the low values of $p_T$ in the photoproduction case, we find that the effect of $k_T$-smearing is important and that, indeed, for $\langle k_T \rangle \sim 0.7$ GeV, the discrepancy between theory and experiment is no longer observed. On the other hand, the inclusion of $k_T$ smearing has a very modest effect on the large-$p_T$ $J/\psi$ data from the Tevatron. Better tests of NRQCD may be obtained by studying other observables at the Tevatron itself, such as the study of the polarisation of the produced $J/\psi$ [19] or the production of other charmonium resonances [33, 34] whose cross-section can be predicted in NRQCD.

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