Towards inflation with $n_s = 1$ in light of Hubble tension and primordial gravitational waves

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Abstract

Recently, it has been found that complete resolution of the Hubble tension might point to a scale-invariant Harrison-Zeldovich spectrum of primordial scalar perturbation, i.e. $n_s = 1$ for $H_0 \sim 73 \text{km/s/Mpc}$. We show that for well-known slow-roll models, if inflation ends by a waterfall instability with respect to another field in the field space while inflaton is still at a deep slow-roll region, $n_s$ can be lifted to $n_s = 1$. A surprise of our result is that with pre-recombination early dark energy, chaotic $\phi^2$ inflation, ruled out by Planck+BICEP/Keck in standard $\Lambda$CDM, can be revived, which is now well within testable region of upcoming cosmic microwave background B-mode experiments.

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I. INTRODUCTION

Inflation [1–5] is the current paradigm of early universe, which predicts (nearly) scale-invariant scalar perturbation, as well as primordial gravitational waves (GW). In well-known single field slow-roll inflation models, the spectral index $n_s$ of primordial scalar perturbation follows \cite{6–9}

$$n_s - 1 = -\mathcal{O}(1) / N^*$$ \hspace{1cm} (1)

in large $N^*$ limit, where $N^* = \int H dt$ is the efolds number before the end of inflation. The cosmic microwave background (CMB) perturbation modes exit the horizon at about $N^* \sim 60$ efolds, if inflation ends around $\sim 10^{15}$GeV. Recently, based on standard $\Lambda$CDM model the Planck collaboration obtains $n_s \approx 0.97$ \cite{10}, which is consistent with (1).

However, the current expansion rate of the Universe, Hubble constant $H_0$, inferred by the Planck collaboration \cite{10} assuming $\Lambda$CDM is in $\gtrsim 5\sigma$ tension with that reported recently by the SH0ES collaboration \cite{11} using Cepheid-calibrated supernovas. Currently, it is arriving at a consensus that this so-called Hubble tension likely signals new physics beyond $\Lambda$CDM \cite{12, 13}, see also Refs.\cite{14–17} for reviews and some recent developments.

As a promising resolution of Hubble tension, early dark energy (EDE) \cite{18, 19}, which is non-negligible only for a few decades before recombination\footnote{Actually, “EDE” corresponds to EDE+$\Lambda$CDM, which is a pre-recombination modification to $\Lambda$CDM model, and the evolution after recombination must still be $\Lambda$CDM-like.}, has been extensively studied e.g.\cite{20–30}. In original (axion-like) EDE \cite{19}, the scalar field with $V(\phi) \sim (1 - \cos(\phi/f_a))^3$ is responsible for EDE, which starts to oscillate at critical redshift, and dilutes away rapidly like a fluid with $w > 1/3$ before recombination. In AdS-EDE \cite{25}, since the potential has an anti-de Sitter (AdS) well (temporarily realizing $w > 1$ so EDE dilutes away faster), a larger EDE fraction and so higher $H_0(\approx 73\text{km/s/Mpc})$ can be achieved without spoiling fit to fullPlanck+BAO+Pantheon dataset. Recently, combined analysis of Planck ($\ell_{TT} \lesssim 1000$) with ACT and SPT data for EDE has also been performed, such as Planck+SPTpol \cite{31–33}, Planck+ACT DR4 \cite{34, 35} and Planck+ACT DR4+SPT-3G \cite{36–38}, see also \cite{39–42} for Planck+large scale structure data.

In Ref.\cite{43}, it has been found that in corresponding Hubble-tension-free cosmologies, the bestfit values of cosmological parameters acquired assuming $\Lambda$CDM must shift with $\delta H_0$, \hspace{1cm}
and with fullPlanck+BAO+Pantheon dataset the shift of $n_s$ scales as

$$\delta n_s \simeq 0.4 \frac{\delta H_0}{H_0},$$

which suggests that pre-recombination resolution of Hubble tension is pointing to a scale-invariant Harrison-Zeldovich primordial spectrum, i.e. $n_s = 1$ for $H_0 \sim 73\text{km/s/Mpc}$, see also [44–46] for earlier discussion regarding $N_{ef}$ and $n_T$. The new constraint on tenor-to-scalar ratio $r$ with recent BICEP/Keck data has also been recently considered in Ref.[47]. In Ref.[37, 38], with Planck+ACT+SPT+BAO+Pantheon dataset, similar results have also been found. We outlined the relevant results in Fig.1. Thus it is significant to explore the implication of $n_s = 1$ on primordial Universe.

How $n_s = 1$ would affect our understanding about inflation? At first thought, it seems that (1) is not compatible with the result in Hubble-tension-free cosmologies, since for $N_\ast \approx 60$ it is hardly possible to achieve $n_s = 1$ in the slow-roll models satisfying (1). Actually, $n_s \geq 0.99$ puts a lower bound $N_\ast > \mathcal{O}(10^2)$. Such perturbation modes are still far larger than our observable Universe today. This seems to pose a serious challenge to slow-roll inflation, implying that corresponding models might need to be reconsidered e.g.[48–50], see also recent [51–53].

However, inspired by recent Ref.[54], we might have a different story. In (1), $N_\ast$ is the “distance” between $\phi_\ast(\epsilon \ll 1)$ and $\phi_e(\epsilon = 1)$ at which inflation ends, $\phi$ being the inflaton. Typically $N_\ast \approx 60$. However, inflation can also be terminated at $\phi_e$ when $\epsilon \ll 1$ by waterfall instability with respect to another field $\sigma$, like in the hybrid inflation models [55, 56], which suggests that $\Delta N \approx 60$ dose not necessarily require $N_\ast \approx 60$, see Fig.2. Thus we can actually have $n_s$ arbitrarily close to 1 by pushing $N_\ast$ to a sufficiently large value $N_\ast \gg \Delta N \approx 60$ while ending inflation by certain mechanism at $N_\ast - 60$.

We will present this possibility. In our (hybrid) uplift of $n_s \approx 0.97$ to $n_s = 1$, the potential of inflaton $\phi$ still preserves the shape of well-known single field slow-roll inflation models, see section-II, but inflation ends by the waterfall instability while $\phi$ is still in the deep slow-roll region. A surprise of our result is that certain models originally thought to be ruled out by Planck+BICEP/Keck based on $\Lambda$CDM [57], specially chaotic $\phi^2$ inflation [5], can be revived by this hybrid uplift to $n_s = 1$, which is now well within the testable region of upcoming CMB B-mode experiments, such as BICEP Array [58] and CMB-S4 [59].
FIG. 1: $n_s$ vs. $H_0$. In upper panel with the fullPlanck+BAO+Pantheon dataset, we adopt the result in Ref.[19] for original axion-like EDE (the SH0ES result as a Gaussian prior on $H_0$), and Ref.[33] for AdS-EDE. In lower panel with the Planck+ACT+SPT+BAO+Pantheon dataset, Ref.[37] (Planck $\ell_{TT} < 650$) for axion-like EDE and Ref.[38] (Planck $\ell_{TT} < 1000$) for AdS-EDE. Grey band represents the recent SH0ES result $H_0 = 73.04 \pm 1.04\, \text{km/s/Mpc}$ [11], and black solid line marks $n_s = 1$. 
II. HYBRID UPLIFT TO $n_s = 1$

The scenario we consider is sketched in Fig.2, in which

$$V(\phi, \sigma) = V_{\text{inf}}(\phi) + \frac{1}{4\lambda} \left[ (\lambda \sigma^2 - M^2)^2 - M^4 \right] + \frac{g^2}{2} \sigma^2 \phi^2, \quad (3)$$

and $V_{\text{inf}}$ is the well-known inflation potentials satisfying (1). Initially, $\sigma = 0$ and $\partial^2_\sigma V > 0$, the inflaton $\phi$ slow rolls along $V_{\text{inf}}$ and $\epsilon \ll 0.01$. At $\phi = \phi_c \simeq M/g$, we still have $\epsilon(\phi_c) \ll 0.01$, but $\partial^2_\sigma V \lesssim 0$ so that the inflation will rapidly end by a waterfall instability along $\sigma^2$, see also [60, 61] for the effective energy momentum tensor approach of multi-fields perturbations. In original hybrid inflation [55, 56], when $\sigma = 0$, $V = V_{\text{inf}} + M^4/4\lambda$ is lifted by $V_{\text{up}} = M^4/4\lambda$. Generally, for $V_{\text{inf}} \sim \phi^2$, when $V_{\text{inf}} \ll V_{\text{up}}$, one has $n_s - 1 > 0$ [62]. However, here we subtract out the uplift $V_{\text{up}}$. We will see that for $V_{\text{inf}} \sim \phi^2$, we have $n_s - 1 \approx 0$ but $< 0$.

In slow-roll approximation, one has ($M_p = 1$)

$$N_* \approx \Delta N + \int_{\phi_c}^{\phi^*} \frac{d\phi}{\sqrt{2\epsilon}} = \left( \int_{\phi_c}^{\phi^*_c} + \int_{\phi^*_c}^{\phi^*} \right) \frac{d\phi}{\sqrt{2\epsilon}} \approx N(\phi_*). \quad (4)$$

The results of both $n_s$ and $r$ are determined by $\phi_*$, value of the inflaton field $\phi$ when the corresponding perturbation mode exits horizon during inflation, thus they are related to $N_*$ rather than $\Delta N$. This indicates that we can have $N_* > \mathcal{O}(10^2)$ in (1) and $n_s \simeq 1$, while still having $\Delta N \approx 60$. In certain sense, with (3), what we do corresponds to push the inflaton $\phi$ deeply into slow-roll region at which $N_* \gg \Delta N \approx 60$. Inflation will end at $N_* - 60$ so that the modes exiting horizon near $N_*$ can be just at CMB window.

Here, we show that the addition of a potential uplift $V_{\text{up}}$ in Ref.[54] is actually equivalent to pushing the inflaton $\phi$ deeply into the slow-roll region without $V_{\text{up}}$, i.e. eq.(3). By lifting $V_{\text{inf}} = V_0 \left( 1 - e^{-\gamma \phi} \right)$ to $V_{\text{up}} + V_{\text{inf}}$, Ref.[54] found

$$n_s - 1 \approx -\frac{2}{(V_{\text{up}} + V_0) e^{\gamma \phi_*}} \approx -\frac{2}{\gamma^{-2} e^{\gamma \phi_*}} = -\frac{2}{\Delta N + \gamma^{-2} e^{\gamma \phi_*}}, \quad \tilde{\phi} \equiv \phi + \frac{1}{\gamma} \ln(V_{\text{up}}/V_0), \quad (5)$$

where the second approximate equality is obtained in the large $V_{\text{up}} \gg V_0$ limit in the uplifted potential $V_{\text{up}} + V_{\text{inf}}$. This is simply equivalent to the large $N_*$ (or equivalently large $\phi_c$ and

\footnote{At the minima of $\sigma$ when $\phi < \phi_c$, the corresponding potential $V = V_{\text{inf}} - (M^2 - g^2 \phi^2)^2/4\lambda$ might be negative for $|\phi| \ll M/g$ and certain $V_{\text{inf}}$. In this case (3) should be thought of as an effective potential only captures the shape of field space for $\phi > \phi_c$ and $\phi \sim \phi_c$.}
FIG. 2: Left panel: The slow-roll potential $V_{\text{inf}}$. Right panel: $V(\phi, \sigma)$ obtained by hybrid lifting $V_{\text{inf}}$ according to (3). $\Delta N$ is the efolds number in the original slow-roll model. In well-known slow-roll models, inflation ends at $\phi_e$ where $\epsilon \simeq 1$. The perturbation modes exiting horizon near $N_s \gg \Delta N \approx 60$ is still far outside of our current Hubble horizon. However, after the hybrid uplift of $V_{\text{inf}}$ to the $\phi - \sigma$ space (initially $\partial^2_{\sigma} V > 0$ at $\sigma = 0$), inflation can end at $\phi = \phi_c$, at which we still have $\epsilon(\phi_c) \ll 0.01$ but $\partial^2_{\sigma} V < 0$, by a waterfall instability along $\sigma$, see the right panel, so that the modes exiting horizon near $N_s \gg 60$ can be just at CMB window, thus $n_s = 1$ in light of (1).

$\phi_c$ limit in $V_{\text{inf}}$ (without $V_{\text{up}}$) because in light of (4) we have

$$N_s \approx \int_{\phi_e}^{\phi_c} \frac{d\phi}{\sqrt{2\epsilon}} = \frac{\gamma}{2} \left| e^{\gamma \phi} \right|^*_{\phi_e},$$

where $\epsilon = \frac{V''_{\text{inf}}}{2V_{\text{inf}}} = \frac{\gamma^2}{2} e^{-2\gamma \phi}$ is used. Thus combining (6) with (5), we obtain (1), which indicates that large $V_{\text{up}}$ or $\phi_c$ limit in Ref.[54] actually is equivalent to the large $N_s$ limit. However, here we straightly push $N_s \approx 60$ to a sufficiently large value $N_s > \mathcal{O}(10^2)$, and do not make the uplift of $V_{\text{inf}}$ to $V_{\text{up}} + V_{\text{inf}}$, so fully preserve the shape of single field slow-roll potentials satisfying (1), thus (1) can be directly applied. The advantage of our inflation model will be seen in the $\phi^p$ inflation.

(hybrid) Starobinski inflation In Starobinsky ($R^2$) model [4], the effective potential is $V_{\text{inf}}(\phi) \sim \left(1 - e^{-\sqrt{2/3} \phi}\right)^2$, and

$$n_s - 1 \approx -\frac{2}{N_s}, \quad r \approx \frac{12}{N_s^2},$$

which corresponds to $\alpha = 1$ in $\alpha$-attractor inflation [7, 63–65]. (7) is compatible with the $\Lambda$CDM constraint $n_s \approx 0.967$ for $N_s = \Delta N \approx 60$ in standard slow-roll inflation. However,
in Hubble-tension-free cosmologies, e.g. AdS-EDE, \( n_s = 0.998 \pm 0.005 \), see Fig.1 for the fullPlanck+BAO+Pantheon result, this would require that in the hybrid lifted Starobinski model, we need \( N_s \gtrsim 300 \gg \Delta N \) and inflation ending at \( N \approx N_s - 60 \), thus \( n_s \approx 1 - 2/N_s \gtrsim 0.993 \) and \( r \approx \frac{12}{N_s^2} \lesssim 1.3 \times 10^{-4} \). This tensor-to-scalar ratio is far smaller than that in the standard Starobinski model.

**(hybrid) \( \phi^p \) inflation** In corresponding models, \( V_{inf}(\phi) \sim \phi^p \), and

\[
n_s - 1 \approx -\frac{p/2 + 1}{N_s}, \quad r \approx \frac{4p}{N_s},
\]

Here, \( p = 2 \) is the chaotic inflation [5], \( p = 2/3, 1 \) correspond to the monodromy inflation [66, 67]. We have \( n_s \approx 1 - 2/60 = 0.97 \) for \( p = 2 \) and \( N_s = 60 \), which seems compatible with the ΛCDM constraint. However, since \( r \approx 8/N_s = 0.13 \), \( \phi^2 \) model has been ruled out by Planck+BICEP/Keck data in ΛCDM [57].

In Fig.3, we plot the \( r - n_s \) posterior in ΛCDM and AdS-EDE (as an example of EDE) models, respectively. Following Ref.[47], the AdS-EDE results are obtained with the fullPlanck+BK18+BAO+Pantheon dataset using the modified versions\(^3\) of CLASS cosmology code [68, 69] and MontePython-3.4 Monte Carlo Markov Chian (MCMC) sampler [70, 71]. The ΛCDM results are directly produced from the public available BK18 chains\(^4\) (fullPlanck+BK18+BAO) [57]. AdS-EDE fits the full Planck CMB and BICEP/Keck18 B-mode data slightly better than ΛCDM with bestfit \( \chi^2 \) improvements \( \Delta \chi^2_{\text{Planck}} = -5.49 \) and \( \Delta \chi^2_{\text{BK18}} = -1.32 \).

In Hubble-tension-free AdS-EDE model, \( n_s = 0.998 \pm 0.005 \), which requires that \( N_s \gtrsim 300 \gg \Delta N \). It is interesting to note that the hybrid uplift to \( n_s = 1 \) generally lowers \( r \), which has the potential to revive inflation models killed by Planck+BICEP/Keck based on ΛCDM due to too large \( r \). In Fig.3, when \( N_s > 300 \), we have \( 0.993 \lesssim n_s \leq 1 \) and \( r \approx 8/N_s \lesssim 0.03 \) for chaotic \( \phi^2 \) inflation, perfectly consistent with current constraints. Since here \( r \approx |n_s - 1| \) \( (r \approx (n_s - 1)^2 \) in Starobinski model), so in certain sense, \( n_s = 1 \) might also explain the non-detection of \( r \) in current observations.

It is required that the effective field theory responsible for the evolution of our Universe must be UV-complete, otherwise it belongs to the swampland. According to (4), we have \( \Delta N \sim \frac{\Delta \phi}{\sqrt{2} \epsilon} \), where \( \Delta \phi = \phi_s - \phi_c \). In \( \phi^2 \) inflation without hybrid uplift, we have \( N_s = \Delta N \)

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\(^3\) The corresponding cosmological code is available at: https://github.com/genye00/class_multiscf.

\(^4\) Available at http://bicepkeck.org/bk18_2021_release.html.
FIG. 3: Predictions of Starobinsky and $\phi^p$ ($p = 2, 2/3$) models with respect to 68% and 95% C.L. contour of $r - n_s$. Here, we adopt the result in recent BICEP/Keck Ref.\[57\] for $\Lambda$CDM and that in Ref.\[47\] (fullPlanck+BICEP/Keck+BAO+Pantheon) for AdS-EDE.

and $\epsilon \sim 1/N_*$, so the field excursion of inflaton is $\Delta \phi \approx \sqrt{2N_*} > 1$, contradicting the swampland conjecture $\Delta \phi < 1$. However, for hybrid $\phi^2$ inflation, we have

$$\Delta \phi \approx \sqrt{\frac{2(\Delta N)^2}{N_*}},$$

where $\Delta N \approx 60 \ll N_*$. Thus the swampland conjecture $\Delta \phi < 1$ requires $N_* \approx 10^4$. In this case, $r \approx 8/N_* \sim 10^{-3}$, also consistent with the Lyth bound \[72\].

**(hybrid) polynomial attractors** In corresponding models, $V_{\text{inf}}(\phi) \sim 1 - \left(\frac{\mu}{\phi}\right)^p$, see recent Ref.\[73\], which was invented in D-brane inflation \[74–76\], and

$$n_s - 1 \approx -\frac{2}{N_*} \left(\frac{p + 1}{p + 2}\right), \quad r \approx \frac{8p^2}{[p(p + 2)N_*]^{2p+2}} \frac{\mu^{2p+2}}{2^{p+1}},$$

for $\mu \ll 1$. Thus for e.g. $p = 2$, we have $n_s - 1 = -3/2N_*$ and $r \approx N_*^{-3/2}/\mu$, which is also compatible with the $\Lambda$CDM constraint $n_s \approx 0.97$ for $N_* = \Delta N \approx 60$. However, in Hubble-tension-free cosmologies, $n_s = 0.998 \pm 0.005$, this would require that $N_* \gtrsim 200 \gg \Delta N$ and inflation ends at $N \gtrsim 140$ (60 efolds after the CMB modes exit horizon at $N_* \approx 200$). Thus
we have $n_s \simeq 1 - 3/2N_* = 0.993$, and $r \simeq 5 \times 10^{-4}\mu$, which is still consistent with current constraint but even smaller than that in hybrid Starobinsky model.

III. CONCLUSION

The complete resolution of Hubble tension might be pointing to a scale-invariant Harrison-Zeldovich spectrum of primordial scalar perturbation, i.e. $n_s = 1$ for $H_0 \sim 73$km/s/Mpc. We propose a scheme to lift $n_s$ predicted by well-known slow-roll inflation models to $n_s = 1$. In corresponding models satisfying (1), if inflation ends by a waterfall instability when inflaton is still at a deep slow-roll region, $n_s$ can be lifted to $n_s = 1$. Particularly, it is found that chaotic $\phi^2$ inflation ruled out by Planck+BICEP/Keck [57] can be revived by this hybrid uplift, which is testable with upcoming CMB B-mode experiments.

The inflation might continue after the waterfall instability. It is possible that the waterfall instability is caused by the nucleations and collisions of vacuum bubbles [56]. This will yield a sub-horizon stochastic GW background, which after being reddened by subsequent inflation can explain the recently observed NANOGrav signal [77, 78]. It is also possible that EDE is the remnant after the waterfall instability along $\sigma$, so that inflaton, EDE and current dark energy could live harmoniously together in a landscape. Relevant models might imprint lots of observable signals to be explored.

Though our discussion is slightly simplified, it highlights a significant point that until the Hubble tension is solved completely, it seems premature to claim which model of inflation is favored or ruled out by current data, since new physics beyond $\Lambda$CDM might bring unforeseen impact on primordial Universe.

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