Formation of a complex part surface at the micro level during milling

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Abstract. The article discusses formation aspects of micro irregularities (roughness) on a screw surface by a designed cutting tool (disk milling cutter). The cutting process is affected by various factors of a random nature: fluctuations of the removed allowance due to deviations of the surface shape of the workpiece, grain hardness, heterogeneity of the metal structure, microvibration. The amount of asperities in the machined surface depends on the cutting conditions and the geometrical parameters of the cutting tool. The question of predicting the formation of roughness is of great practical importance in the preparation of the technological process of manufacturing parts with a helical surface. To ensure the required amount of microroughness when milling the screw surface of the workpiece, a technique has been developed that allows controlling the roughness due to the discreteness of the cutting process: the main cutting edge of the tool leaves its mark in the form of roughness comb. Modelling is performed on the basis of an invariant method of profiling the cutting tool.

1. Introduction

When processing the workpiece by a cutting tool, it is necessary to predict the roughness (micro irregularities) of a complex surface possibility. A preliminary forecast can be made by simulating the cutting process using computer technology.

After processing of the workpiece by a cutting tool, new surfaces which have a set of physical and geometric characteristics are formed on it. Geometric characteristics are the shape, dimensions of the part, and its deviations \cite{1}. In addition, there are micro geometric characteristics such as all sorts of irregularities, surface roughness.

2. An actual and theoretical roughness

There are several causes of roughness occurrence. Among the roughness occurrence causes the following reasons can be mentioned: the occurrence of surface defects in the chips removal due to the heterogeneous structure of the metal, grains hardness fluctuations, chipping particles due to adhesion phenomena on the tool cutting edges and arising by these reasons, micro vibrations. The probabilistic nature of cutting process leads to the fact that given the quantitative parameters of the cutting mode, given component of the roughness is practically uncontrollable. This component of the roughness can be called an actual roughness.

But there is such a component of roughness that can be controlled. It can be called a theoretical roughness, which occurs directly due to the fact that cutting is a discrete process, and the main cutting edge of the tool leaves its track in the form of roughness scallops.
When milling by disc cutters trochoidal (elongated cycloids) roughness scallops are formed. To reduce the height of the roughness scallops during milling is possible by increasing the number of teeth and reducing the feed value, which reduces the cutting discreteness.

3. The technique of obtaining the required roughness at the given cutting conditions

During machining, the tool and the workpiece carry out the integral motion of the shaping. Then the cut layer can be reproduced as a continuous sum of all intersections of the tool and the workpiece in the time interval \( t_1 - t_2 \). It is possible to merge all positions of a body together so that a certain new body as its spatial trace is formed. If such a space-time body is subtracted from the workpiece, the resulting surface of the part will be obtained.

The theoretical roughness is formed by overlay several traces in a certain order, that is, the roughness is a regular periodic structure. The formation of such a structure occurs at multiple intersections of the trajectory of the main cutting motion and feed motion.

The trace of the main movement can be considered continuous, so there is no need to reproduce each position of the cutting tool tooth, but it is enough to reproduce its continuous trace and further regularly repeated positions of this trace. Thus, when entering a tool and workpiece in the computing environment their intersections can be represented as a single solid teeth trace and then this trace can be repeatedly copied in the feed direction with a certain step.

We determine the value of the micro irregularity left during the processing of a unit helix of the part by a unit milling cutter disc [2]. To do this, we will consider the two traces of a unit disk left at the initial time and after moving the part by the amount of feed to the tooth \( S_z \) (figure 1).

![Figure 1. Scheme of formation of micro-irregularities in milling of complex surface of the product.](image)

The position of a unit disk in the coordinate system of the tool \( OX_uY_uZ_u \) is described by a system of equations [3, 4]:

\[
\begin{align*}
X_u &= R_N \cos \mu
\\
Y_u &= R_N \sin \mu
\\
Z_u &= Z_N
\end{align*}
\]  

(1)
where \( \mu \) is the rotation angle of the point relative to the rotation axis of the tool \( O_u Z_u \).

In the moving coordinate system \( OX_2 Y_2 Z_2 \), with which the coordinate system of the tool is rigidly connected (figure 1), the system (1) is converted to the following form:

\[
\begin{align*}
X_1 &= R \cos \mu + A_w \\
Y_1 &= R \sin \mu \cos \varepsilon + Z_u \sin \varepsilon \\
Z_1 &= R \sin \mu \sin \varepsilon - Z_u \cos \varepsilon
\end{align*}
\]  

(2)

where \( A_w \) is the center distance, \( \varepsilon \) is the center angle.

Let us set the coordinate system \( OX_2 Y_2 Z_2 \), the axis \( OX_2 \) of which coincides with the axis \( OX_1 \), and the axis \( OZ_2 \) crosses the axis \( OZ_1 \) at an angle equal to the angle of the helix inclination \( \beta \). The centers of these systems also coincide. The position of the first trace in the new coordinate system can be written:

\[
\begin{align*}
X_2 &= X_1 \\
Y_2 &= Y_1 \cos \beta - Z_1 \sin \beta \\
Z_2 &= Y_1 \sin \beta + Z_1 \cos \beta
\end{align*}
\]  

(3)

After substitution of the description of the tool (2) into the system (3) we obtain:

\[
\begin{align*}
X_2 &= R \cos \mu + A_w \\
Y_2 &= R \sin \mu \cos \varepsilon + Z_u \sin \varepsilon - R \sin \mu \sin \varepsilon + Z_u \cos \varepsilon \\
Z_2 &= R \sin \mu \sin \varepsilon + Z_u \sin \varepsilon + R \sin \mu \cos \varepsilon - Z_u \cos \varepsilon
\end{align*}
\]  

(4)

The coordinate system \( OX_1 Y_1 Z_1 \) when moving on the \( S_z \) value will take position \( OX_1' Y_1' Z_1' \). The unit disk in this coordinate system is also described by the system of equations (1). In its turn, the transfer from the \( OX_1 Y_1 Z_1 \) system to the \( OX_1 Y_1' Z_1' \) system is as follows:

\[
\begin{align*}
X_1 &= X_1' \cos \Delta \phi + Y_1' \sin \Delta \phi \\
Y_1 &= X_1' \sin \Delta \phi + Y_1' \cos \Delta \phi, \\
Z_1 &= Z_1' - S_z \cos \beta
\end{align*}
\]  

(5)

where \( \Delta \phi = S_z \cos \beta / P \). After substituting the description of the tool (2) system (5) we will receive:

\[
\begin{align*}
X_1 &= R \cos \mu \cos \Delta \phi + A_w \cos \Delta \phi + R \sin \mu \sin \varepsilon \cos \Delta \phi + Z_u \sin \varepsilon \sin \Delta \phi \\
Y_1 &= -R \cos \mu \sin \Delta \phi - A_w \sin \Delta \phi + R \sin \mu \cos \varepsilon \sin \Delta \phi + Z_u \cos \varepsilon \sin \Delta \phi \\
Z_1 &= R \sin \mu \sin \varepsilon - Z_u \cos \varepsilon - S_z \cos \beta
\end{align*}
\]  

(6)

Having substituted (6) into (4), we obtain:

\[
\begin{align*}
X_2 &= R \cos \mu \cos \Delta \phi + A_w \cos \Delta \phi + R \sin \mu \cos \varepsilon \sin \Delta \phi + Z_u \sin \varepsilon \sin \Delta \phi \\
Y_2 &= -R \cos \mu \sin \Delta \phi \cos \beta - A_w \sin \Delta \phi \cos \beta + R \sin \mu \cos \varepsilon \sin \Delta \phi \cos \beta - Z_u \sin \varepsilon \sin \Delta \phi \cos \beta \\
&\quad - R \sin \mu \sin \varepsilon \sin \Delta \phi - Z_u \cos \varepsilon \sin \Delta \phi - S_z \cos \beta \sin \Delta \phi \\
Z_2 &= -R \cos \mu \sin \Delta \phi \sin \beta - A_w \sin \Delta \phi \sin \beta + R \sin \mu \cos \varepsilon \sin \Delta \phi \sin \beta + Z_u \sin \varepsilon \sin \Delta \phi \sin \beta \\
&\quad + R \sin \mu \sin \varepsilon - Z_u \cos \varepsilon - S_z \cos \beta \sin \Delta \phi - S_z \cos \beta
\end{align*}
\]  

(7)

Thus, we have described both traces on the plane I-I. Now by equating the coordinates \( X_2 \) and \( Z_2 \) from the systems (4) and (7) we obtain a system of equations:

\[
\begin{align*}
R \cos \mu + A_w &= R \cos \mu \cos \Delta \phi + A_w \cos \Delta \phi + R \sin \mu \cos \varepsilon \sin \Delta \phi + Z_u \sin \varepsilon \sin \Delta \phi \\
R \sin \mu \cos \varepsilon \sin \Delta \phi + Z_u \sin \varepsilon \sin \Delta \phi &= R \sin \mu \cos \varepsilon \sin \Delta \phi - Z_u \cos \varepsilon \sin \Delta \phi \\
&\quad - R \sin \mu \sin \varepsilon \sin \Delta \phi - Z_u \cos \varepsilon \sin \Delta \phi - S_z \cos \beta \sin \Delta \phi \\
&\quad + R \sin \mu \sin \varepsilon - Z_u \cos \varepsilon - S_z \cos \beta
\end{align*}
\]  

(8)

From the first equation of this system we obtain:

\[
\cos \mu = -A_w R + \frac{Z_u \sin \varepsilon \sin \Delta \phi}{R(1 - \cos \Delta \phi)} + \sin \mu \left( \frac{\cos \varepsilon \sin \Delta \phi}{1 - \cos \Delta \phi} \right).
\]  

(9)

Let us substitute the result into the second equation of the system (8) and express \( \sin \mu \):
\[
\sin \mu = \frac{C_4}{C_3}
\]
(10)

where
\[
C_1 = \frac{A_w}{R} + \frac{Z_u \sin \Delta \varphi}{R(1-\cos \Delta \varphi)},
\]
\[
C_2 = \frac{\cos \varphi}{1-\cos \Delta \varphi},
\]
\[
C_3 = R \sin \beta \cos \Delta \varphi - R \sin \beta \sin \Delta \varphi C_2;
\]
\[
C_4 = R \sin \beta \sin \Delta \varphi C_1 + Z_u \sin \beta \sin \varphi - Z_u \sin \beta \sin \Delta \varphi \cos \varphi + S_z \cos^2 \beta.
\]

From equation (10) we will get:
\[
\mu = \arcsin \frac{C_4}{C_3}.
\]
(11)

Having substituted the found value \(\mu\) into the system (4), we obtain the coordinates \((x_{2N}; z_{2N})\) of the intersection point (N) of two traces of the unit disk on the plane I-I.

4. Results and conclusions

The initial height of the micro-irregularities \(R_{ZO}\) is defined as the distance between the point N and the helix line trace on the plane I-I. The helix is located on the cylinder of radius r. In the section of the cylinder by the plane II there will be an ellipse, with which helix trace on this plane coincides within the step \(S_z\). The equation of the ellipse is written as follows:
\[
\frac{x^2}{r^2} + \frac{z^2 \sin^2 \beta}{r^2} = 1.
\]
(12)

Since the value \(S_z\) is small, the normal line to the ellipse will approximately pass through the point N perpendicular to the segment connecting the points 1 and 2. Equation of this line would be:
\[
x_2 = kZ_2 - b,
\]
(13)

where
\[
k = -\frac{x_{21} - x_{22}}{x_{21} - x_{22}},
\]
\[
b = k z_{2N} + x_{2N},
\]

\(x_{21}, z_{21}, x_{22}, z_{22}\) are coordinates of points 1 and 2 respectively.

Let us find the intersection point (P) of the normal line (13) with the ellipse (12). To do this, we will transform a system (12) and write down a system of equations:
\[
\begin{cases}
  x_2^2 + \frac{z_2^2 \sin^2 \beta}{r^2} - r^2 = 0 \\
  x_2 = kZ_2 - b.
\end{cases}
\]
(14)

Let us substitute the second equation of the system (14) into the system (1) and after some transformations we will obtain a quadratic equation
\[
z_2^2 (k^2 + \sin^2 \beta) - 2kbZ_2 + (b^2 - r^2) = 0,
\]
(15)

the solution of which is
\[
z_{2P} = \frac{2kb \pm \sqrt{4k^2 b^2 - 4(k^2 + \sin^2 \beta)(b^2 - r^2)}}{2(k^2 + \sin^2 \beta)}.
\]
(16)

Having substituted the found value \(z_{2P}\) in the second equation of the system (1), we will obtain the coordinate \(x_{2P}\) after some transformations.

The original height of micro irregularity \(R_{ZO}\) is defined as:
\[
R_{ZO} = \sqrt{(x_{2N} - x_{2P})^2 + (z_{2N} - z_{2P})^2}.
\]
(17)

The scallop height is [2]:
The presented technique allows us to estimate the number of cutter teeth required to obtain the required roughness under specified cutting conditions.

References
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