Controlling entanglement sudden death and birth in cavity QED

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We present a scheme to control the entanglement sudden birth and death in cavity quantum electrodynamics system, which consists of two noninteracting atoms each locally interacting with its own vacuum field, by applying and adjusting classical driving fields.

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I. INTRODUCTION

In recent years, entanglement has been considered as a key resource of quantum information processing [1, 2, 3, 4]. A cavity quantum electrodynamics (QED) system is a useful tool to create the entanglement between atoms in cavities and establish quantum communications between different optical cavities. Recently, the manipulation of quantum entanglement for the system of cavity QED has been extensively investigated [5, 6, 7, 8, 9, 10, 11, 12, 13].

Many efforts have been devoted to the study of the evolution of the entanglement under the influence of the environment [14, 15, 16, 17, 18, 19]. It is pointed out by Yu and Eberly [14] that the entanglement of an entangled two-qubit interacting with uncorrelated reservoirs may disappear within a finite time during the dynamics evolution. This phenomenon, called entanglement sudden death (ESD) has been observed in experiment [21, 22]. Recently, the entanglement sudden birth (ESB) in cavity QED has been discussed by Yonac, Yu, and Eberly [23, 24]. More recently, Lopez et al. [25] have studied the entanglement dynamics of a quantum system consisting of two cavities interacting with two independent reservoirs and shown that ESD in a bipartite system independently coupled to reservoirs is related to the ESB. It has been pointed out that the cavity coherent state can be used to control the ESB and ESD in cavity QED [26].

In the present paper, we propose a scheme to control ESB and ESD of a quantum system consisting of two noninteracting atoms each locally interacting with its own vacuum field. The two atoms, which are initially prepared in entangled states, are driven by two classical fields additionally. It is shown that ESB and ESD phenomenon may appear in this system and the time of ESB and ESD can be controlled by classical driving fields. In addition, the amount of the entanglement of the two atoms or cavities can be significantly increased by applying classical fields.

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II. EFFECTIVE HAMILTONIAN

Now, we consider a system consisting of a two-level atom inside a single mode cavity. The atom is driven by a classical field additionally. The Hamiltonian of the system can be described by

\[ H = \omega a^\dagger a + \frac{\omega_0}{2}\sigma_z + g(\sigma_+ a + \sigma_- a^\dagger) + \lambda(e^{-i\omega_c t}\sigma_+ + e^{i\omega_c t}\sigma_-), \] (1)

where \(\omega, \omega_0\) and \(\omega_c\) are the frequency of the cavity, atom and classical field, respectively. The operators \(\sigma_z\) and \(\sigma_\pm\) are defined by

\[ \sigma_\pm = |e\rangle\langle g|, \quad \sigma_\mp = |g\rangle\langle e|, \quad \sigma_z = |e\rangle\langle e| - |g\rangle\langle g|, \]

and \(\sigma_+ = \sigma_z^+\) where \(|e\rangle\) and \(|g\rangle\) are the excited and ground states of the atom. Here, \(a\) and \(a^\dagger\) are the annihilation and creation operators of the cavity; \(g\) and \(\lambda\) are the coupling constants of the interactions of the atom with the cavity and with the classical driving field, respectively. Note that we have set \(\hbar = 1\) throughout this paper.

In the rotating reference frame the Hamiltonian of the system is transformed to the Hamiltonian \(H_1\) under a unitary transformation \(U_1 = \exp(-i\omega_c t\sigma_z/2)\)

\[ H_1 = U_1^\dagger H U_1 - iU_1^\dagger \frac{\partial U_1}{\partial t} = H_1^{(1)} + H_1^{(2)}, \] (2)

with

\[ H_1^{(1)} = \omega a^\dagger a + g(e^{i\omega_c t}\sigma_+ a + e^{-i\omega_c t}\sigma_- a^\dagger) \]
\[ H_1^{(2)} = \frac{\Delta_1}{2}\sigma_z + \lambda(\sigma_+ + \sigma_-), \] (3)

and \(\Delta_1 = \omega_0 - \omega_c\). Using the method similar to that used in Ref.27, diagonalizing the Hamiltonian \(H_1^{(2)}\), and neglecting the terms which do not conserve energies (rotating wave approximation), we can recast the Hamiltonian \(H_1\) as follows:

\[ H_1 = \omega a^\dagger a + \frac{\Omega_1}{2}\sin \theta(\sigma_+ + \sigma_-) + g\cos^2 \frac{\theta}{2}e^{i\omega_c t} \]
\[ \times (-\sin \frac{\theta}{2}\sigma_z + \cos^2 \frac{\theta}{2}\sigma_+ - \sin^2 \frac{\theta}{2}\sigma_-)a + h.c.], \] (4)

with \(\theta = \arctan(\frac{\Delta_1}{\lambda})\). Here \(h.c\) stands for Hermitian conjugation.

The Hamiltonian \(H_1\) can be diagonalized by a final unitary transformation \(U_2 = \exp[\frac{i\omega_c t}{2}(\sigma_+ + \sigma_-)]\). Then, we can rewrite the Hamiltonian of the system

\[ H_2 = \omega a^\dagger a + \frac{\omega'}{2}\sin \theta(\sigma_+ + \sigma_-) + g'(\sin \frac{\theta}{2}\sigma_z \]
\[ + \cos^2 \frac{\theta}{2}\sigma_+ - \sin^2 \frac{\theta}{2}\sigma_-)a + h.c.], \] (5)

where \(\omega' = \sqrt{\Delta_1^2 + 4\lambda^2} + \omega_c\) and \(g' = g\cos^2 \frac{\theta}{2}\). It is worth noting that the unitary transformations \(U_1\) and \(U_2\) are both local unitary transformations. As we known the entanglement of a quantum system does not change under local unitary transformations. Thus, the entanglement of the system considered here will not be changed by applying the local unitary transformations \(U_1\) and \(U_2\).
In this section, we investigate ESD and ESB of a quantum system consisting of two noninteracting atoms each locally interacting with its own vacuum field. Each atom interacts with its own vacuum field where the interaction of the system is described by $H_2$. We show how to control entanglement sudden death and birth of a quantum system formed by two two-level atoms and two cavities via classical driving fields. Assume the two-level atoms are prepared in entangled states and the cavities are prepared in vacuum states, i.e., the whole system is initially prepared in the state

$$|\psi(0)\rangle = (\alpha|a_1\rangle|a_2\rangle + \beta|a_1\rangle|a_2\rangle)|c_1\rangle|c_2\rangle,$$

(6)

where the subscripts $a_1$, $a_2$, $c_1$, and $c_2$ refer to atom 1, atom 2, cavity 1, and cavity 2, respectively. Here, $|\pm\rangle$ can be interpreted as the dressed states of the two-level atom. They are defined as follows:

$$|+\rangle = \cos \frac{\theta}{2}|e\rangle + \sin \frac{\theta}{2}|g\rangle,$$

$$|\rangle = -\sin \frac{\theta}{2}|e\rangle + \cos \frac{\theta}{2}|g\rangle.$$

(7)

After some algebra, we find the state of the whole system at time $t$ is

$$|\psi(t)\rangle = \alpha|a_1\rangle|a_2\rangle|c_1\rangle|c_2\rangle + \beta f_1(t)|a_1\rangle|a_2\rangle|c_1\rangle|c_2\rangle + \beta f_2(t)|a_1\rangle|a_2\rangle|c_1\rangle|c_2\rangle + \beta f_1(t)f_2(t)|a_1\rangle|a_2\rangle|c_1\rangle|c_2\rangle + \beta f_1(t)f_2(t)|a_1\rangle|a_2\rangle|c_1\rangle|c_2\rangle,$$

(8)

with

$$f_1(t) = e^{i\Delta_2 t/2[\cos (\Omega t) - \frac{i\Delta_2}{2\Omega} \sin (\Omega t)]},$$

$$f_2(t) = -ig\cos \frac{\theta}{2}e^{-i\Delta_2 t/2 \sin (\Omega t)/\Omega},$$

$$\Delta_2 = \sqrt{(\omega_0 - \omega_c)^2 + 4\lambda^2 + \omega_c - \omega},$$

$$\Omega = \frac{\Delta_2^2}{4} + (g\cos^2 \frac{\theta}{2})^2.$$ (9)

Tracing over the degrees of the freedom of cavities, we obtain the reduced density matrix of two atoms

$$\rho_{a_1a_2}(t) = ||\alpha|^2 + |\beta f_2(t)|^2||a_1\rangle|a_2\rangle - (a_1\rangle|a_2\rangle|a_1\rangle|a_2\rangle$$

$$+ |\beta f_1(t)f_2(t)|^2|a_1\rangle|a_2\rangle + h.c.||a_1\rangle|a_2\rangle.$$ (10)
Similarly, the reduced density matrix of two cavities is

\[
\rho_{c_1c_2}(t) = [|\alpha|^2 + |\beta f_1(t)|^2]|-a_1\rangle|-a_2\rangle\langle-a_1|-a_2| + |\beta f_1(t)|^2|-a_1\rangle|a_1\rangle\langle a_1|\langle a_1|-a_2| + |\beta f_1(t)f_2(t)|^2(-a_1\rangle|a_1\rangle\langle a_1|-a_2| + |\beta f_1(t)f_2(t)|^2|-a_1\rangle|a_1\rangle\langle a_1|-a_2| + \alpha^2 f_2^2(t)|-a_1\rangle|-a_2\rangle\langle a_1|a_2\rangle + h.c].
\]  

(11)

In order to show this more clearly, we plot the two-qubit concurrence for different partitions \(C_{a_1a_2}\) (solid line) and \(C_{c_1c_2}\) (dotted line) with \(\alpha = \sqrt{3}/\sqrt{10}, \beta = \sqrt{7}/\sqrt{10}, \omega = 3, \omega_0 = 2, g = 1\) in Fig.2. Comparing Fig.1 and Fig.2, one can see that the time for which ESD(\(t_{ESD}\)) and ESB(\(t_{ESB}\)) occur could be adjusted by controlling the frequency \(\omega_c\) and strength \(\lambda\) of classical driving fields. In addition, the amount of entanglement between two cavities can also be controlled by classical driving fields.

In order to study the entanglement of above system described by density matrix \(\rho\), we adopt the measure concurrence which is defined as

\[
C = \max \{ 0, \lambda_1 - \lambda_2 - \lambda_3 - \lambda_4 \},
\]  

(12)

where the \(\lambda_i (i=1,2,3,4)\) are the square roots of the eigenvalues in decreasing order of the magnitude of the “spin-flipped” density matrix operator \(R = \rho(\sigma_y \otimes \sigma_y)\rho^*(\sigma_y \otimes \sigma_y)\) and \(\sigma_y\) is the Pauli Y matrix, i.e., \(\sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}\).

Particularly, for a density matrix of the form

\[
\rho = \begin{pmatrix} a & 0 & 0 & 0 \\ 0 & b & z & 0 \\ 0 & z^* & c & 0 \\ 0 & 0 & 0 & d \end{pmatrix},
\]  

(13)

the concurrence is

\[
C = 2 \max \{ 0, |z| - \sqrt{ad} \}.
\]  

(14)

Combining the above equation with the reduced density matrix, we find that the concurrence of two atoms is

\[
C_{a_1a_2}(t) = 2f_1(t)\max \{ 0, |\alpha\beta| - |\beta f_2(t)|^2 \},
\]  

(15)

and the concurrence of two cavities is

\[
C_{c_1c_2}(t) = 2f_2(t)^2\max \{ 0, |\alpha\beta| - |\beta f_1(t)|^2 \}.
\]  

(16)
see time of ESD($t_{ESD}$) and ESB($t_{ESB}$) depend on the parameters $\alpha$ and $\beta$. In the case of $\alpha = 1/\sqrt{10}$ and $\beta = 3/\sqrt{10}$, $t_{ESD} < t_{ESB}$, that is, ESB appears after ESD. However, when $\alpha = \sqrt{3}/\sqrt{10}$ and $\beta = \sqrt{7}/\sqrt{10}$, $t_{ESD} > t_{ESB}$, that is, ESB appears before ESD. Again, the time of ESD and ESB and the amount of entanglement between two cavities can be controlled by adjusting classical driving fields.

We now turn to show the influence of classical driving fields on the distribution of entanglement in the present system. The bipartite entanglement of $a_1 \otimes a_2, c_1 \otimes c_2, a_1 \otimes c_2$, and $c_1 \otimes a_2$ are displayed in Fig.3. It is not difficult to see that the concurrence $C_{a_1 a_2}, C_{c_1 c_2}, C_{a_1 c_2}$, and $C_{c_1 a_2}$ are periodic functions of time $t$. The periods of them depend on the strength and the frequencies of classical driving fields. Comparing the right panel and the left panel of Fig.3, we find that the time of ESB and ESD and the amount of the entanglement of two qubits can be controlled by classical driving fields. For example, $t_{ESD}$ and the amount of $C_{c_1 c_2}$ (dashed line) of the right panel are larger than that of the left panel.

IV. CONCLUSIONS

In summary, we have considered a quantum system consisting of two noninteracting atoms each locally interacting with its own vacuum field. The two atoms, which are driven by two classical fields, are initially prepared in entangled states. We find that classical driving fields can increase the amount of entanglement of the two-atom system. It is worth noting that the time of ESB and ESD can be controlled by the classical driving fields. The approach presented in the present Letter may have potential applications in quantum information processing.

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FIG. 1 The concurrence of two atoms (solid line) and two cavities (dotted line) are plotted as a function of \( t \) with \( \alpha = 1/\sqrt{10}, \beta = 3/\sqrt{10}, \omega = 3, \omega_0 = 2, g = 1 \). Right panel: \( \omega_c = \lambda = 0 \). Left panel: \( \omega_c = \lambda = 1 \).

FIG. 2 The concurrence of two atoms (solid line) and two cavities (dotted line) are plotted as a function of \( t \) with \( \alpha = \sqrt{3}/\sqrt{10}, \beta = \sqrt{7}/\sqrt{10}, \omega = 3, \omega_0 = 2, g = 1 \). Right panel: \( \omega_c = \lambda = 0 \). Left panel: \( \omega_c = \lambda = 1 \).

FIG. 3 The concurrence of two qubits for different partitions are plotted as a function of \( t \) with \( \alpha = \sqrt{3}/\sqrt{10}, \beta = \sqrt{7}/\sqrt{10}, \omega = 3, \omega_0 = 2, g = 1 \). Right panel: \( \omega_c = \lambda = 0 \). Left panel: \( \omega_c = \lambda = 1 \).
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