The uncertainties in the synthetic indices for stellar populations

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Abstract. The study of line strengths in the spectra of early-type galaxies has proven to be a powerful tool for investigating the age and the metallicity of these systems. When computing models for spectrophotometric narrow-band indices, index calibrations as functions of the relevant stellar parameters (e.g. temperature, gravity and metal content) are used. Thus synthetic indices depend upon these calibrations (called fitting functions), as well as on the stellar evolution ingredients adopted. All these inputs suffer from uncertainties, which impact on the derived value for the indices. In this paper we address this problem quantitatively.

We compute synthetic Mg$_2$, Fe5270, Fe5335 and H$\beta$ indices for Simple Stellar Populations (SSP) of various ages and metallicities, under different prescriptions for the fitting functions. This allows us to estimate the impact of the uncertainties in the fitting functions. By comparing our models to those of other authors computed with the same fitting functions, we estimate the uncertainty associated to the use of different stellar evolution prescriptions. It is found that the modelling of the Horizontal Branch impacts particularly on Fe and H$\beta$. In the range of parameters explored, the uncertainties introduced by the use of different fitting functions can be appreciably larger than the error affecting the observational data. This typically occurs at high $Z$ for the metallic line strengths, at low $Z$ for the H$\beta$ index.

Keywords: stellar populations, spectral indices

1. Introduction

Defined either as equivalent widths or magnitudes, Spectral Indices (SI) trace the strength of particular absorption features and are widely used as age ($t$) and metallicity ($Z$) indicators for stellar systems (e.g. elliptical galaxies, see Faber et al., 1995, Fisher, Franx & Illingworth, 1995, Buzzoni, 1995 and Greggio, 1997). In general, a single SI depends on both parameters $t$ and $Z$ in the same way: metallic lines become stronger with increasing age and $Z$, while Balmer lines become weaker. As a consequence, a given value for the SI corresponds to either relatively young ages and high metallicities, or vice versa, which is known
as the age/metallicity degeneracy (Worthey, 1992). However, since the $H\beta$ line strength is more sensitive to age, while metallic indices are more sensitive to $Z$, these two fundamental parameters are derived by analysing observational data in the 2-dimensional diagram $H\beta$ versus metallic line strengths (e.g. González, 1993; Kuntschner, this volume). Errors in the data are usually taken into account (see Trager, 1998) whereas uncertainties in the models are not considered. The aim of this paper is to estimate these uncertainties.

We examine SI for SSPs (i.e. single age and single metallicity populations of single stars), which are obtained by summing up the contributions from all the stars that compose the population. The $M_{g2}^{\text{SSP}}$, for example, is given by

$$M_{g2}^{\text{SSP}}(t, [\text{Fe}/H]) = -2.5 \cdot \log \frac{\sum_{i} f_{c}^{*} \cdot 10^{-0.4 \cdot (M_{g2}^{*})}}{\sum_{i} f_{c}^{*}}$$

In eq. 1, $f_{c}^{*}$ is the continuum flux (in the relevant wavelength window) and $M_{g2}^{*}$ is the value of the index of a single star of the SSP. The latter is specified by fitting functions, which depend on the stellar parameters effective temperature, surface gravity and metal content ($T_{\text{eff}}$, $g$, $[\text{Fe}/H]$), and are empirically calibrated on stellar samples. Thus synthetic indices depend on: 1) the fitting functions (FF); 2) the stellar evolution input (set of stellar tracks, adopted law for the helium-enrichment parameter $\Delta Y/\Delta Z$, choice for the mass-loss rate, etc); 3) the Evolutionary Population Synthesis (EPS) computational procedure.

In the following we investigate the uncertainties by varying the above input prescriptions.

2. Results

The SI presented here are computed with the EPS procedure described in Maraston (1998), which makes use of the Fuel Consumption Theorem (Renzini and Buzzoni, 1986) to evaluate the energetics of Post Main Sequence (PMS) stars. The input stellar tracks are taken from Bono et al. (1997) and from Cassisi (1998, private communication). In order to estimate the impact of uncertainties in the FF, we explore the two independent sets from Worthey et al. (1994, hereafter WFF) and from Buzzoni et al. (1992; 1994, hereafter BFF). We selected the indices common to the two sets, namely $M_{g2}$, $H\beta$, $Fe5270$ and $Fe5335$. In addition, the comparison of our models with those of Worthey (1994, hereafter WSSP) and of Buzzoni et al. (1992; 1994, hereafter BSSP) for the same FF, gives us clues on the uncertainties due to the use of different EPS.
The uncertainties in the synthetic indices for stellar populations

Figure 1. Left-hand panel: the effect of different EPS. The upper part of the diagram shows the comparison with WSSP at 12 Gyr, the lower part shows the comparison with BSSP at 15 Gyr. Right-hand panel: the uncertainty due to the FF. The curves are labelled with the SSP ages in Gyr.

procedures and stellar evolution input. We emphasize that the results obtained in this way provide lower limits for the uncertainties affecting the model indices.

In this contribution we discuss part of the results obtained in a comprehensive study that will be presented in a forthcoming paper (Greggio and Maraston, in preparation, hereafter GM99). We concentrate here on old ages, that are more relevant for elliptical galaxies.

2.1. The Mg$_2$ index

Figure 1 shows the effect of different EPS prescriptions (left-hand panel) and of different FF (right-hand panel), on the synthetic Mg$_2$. The largest discrepancy due to the EPS is $\sim 0.02$ mag for [Fe/H] $\sim -0.5$ for the 15 Gyr old SSPs compared to Buzzoni’s models. When using the Mg$_2$ index to estimate the metallicity [Fe/H] of a stellar system, the EPS procedure introduces an appreciable uncertainty. In the cases explored here, the largest values are 0.2 dex, for BSSP at Mg$_2 \sim 0.2$, and $\sim 0.15$ dex for WSSP at Mg$_2 \sim 0.3$. The Mg$_2$ indices in BSSP models are systematically stronger than ours. This is likely due to the use of a different $\Delta Y/\Delta Z$ of the evolutionary tracks used in the two EPS, namely $\Delta Y/\Delta Z \sim 1$ in BSSP and $\Delta Y/\Delta Z \sim 2.5$ in this work. At fixed stellar mass, a lower $Y$ implies longer lifetimes on the Red Giant Branch (RGB) and shorter lifetimes on the Horizontal Branch (HB) (see Renzini, 1994). Therefore, the indices of RGB stars receive relatively more weight in eq. (1). Since the RGB is an important contributor to the total Mg$_2$ (see GM99), and since Mg$_2^*$ becomes stronger
with decreasing $T_{\text{eff}}$, an SSP with a lower $Y$-content exhibits a stronger Mg$_2$.

The discrepancies introduced by the different FF (Fig. 1, right-hand panel) are relatively small at old ages (see also Table 1). For the 3 Gyr models, instead, WFF lead to Mg$_2$ values which are systematically higher by $\sim 0.015$ mag with respect to BFF. This comes from the higher intrinsic Mg$_2$ given by the WFF for the $T_{\text{eff}}$ typical of a 3 Gyr turn-off ($\sim 6300$ K). The largest uncertainty in the [Fe/H] determination hence appears for the 3 Gyr models, with $\Delta$[Fe/H] $\sim 0.15$.

2.2. THE IRON INDICES

Figure 2 shows the same comparisons as in Figure 1 for the average iron index $\langle$Fe$\rangle = (\text{Fe}5335 + \text{Fe}5270)/2$. Concerning the EPS procedure (left-hand panel), the discrepancy to BSSP is systematically increasing with metallicity up to 0.4 Å, with our models predicting $\langle$Fe$\rangle$ stronger than Buzzoni’s. Again, at least part of this inconsistency likely comes from the different $\Delta Y/\Delta Z$ adopted. At [Fe/H] $> -0.5$, the HB phase is spent at $T_{\text{eff}} \sim 5300 - 4600$ K, where the Fe fitting functions yield the strongest values for the indices (Buzzoni et al., 1994). It follows that the Fe indices are very sensitive to the lifetime of this phase. As mentioned in the previous section, at fixed $Z$, a larger value of $Y$ implies a longer HB lifetime, which in turn determines stronger Fe indices. Thus for a given value of $\langle$Fe$\rangle$ our models with BFF yield a lower [Fe/H], compared to BSSP, with a discrepancy reaching 0.2 dex at [Fe/H] $> 0$. 

Figure 2. Left-hand panel: the effect of different EPS. The upper part of the diagram shows the comparison with WSSP at 12 Gyr, the lower part shows the comparison with BSSP at 15 Gyr. Right-hand panel: the uncertainty due to the FF. The curves are labelled with the SSP ages in Gyr.
The uncertainties in the synthetic indices for stellar populations

Figure 3. Left-hand panel: the effect of different EPS. In the top diagram we show our models for 8, 12 and 15 Gyr (solid lines), and Worthey’s models for 8, 12 and 17 Gyr (dotted lines). In the bottom diagram ours and Buzzoni’s models are shown for ages of 10 and 15 Gyr. In both diagrams the triangles represent a model in which the mass-loss during the RGB (see the text) is halved. Right-hand panel: Hβ indices for two different choices for the fitting functions.

The different FF (right-hand panel) imply systematic differences in the slopes of the relations index/metallicity at old ages. At large values of ⟨Fe⟩, BFF indices are stronger than those computed with WFF (particularly for the Fe5270, see Table 1). The difference increases with metallicity up to ∆⟨Fe⟩ ∼ 0.2 Å and mainly results from the different values of the FF. We finally notice that within the explored range, the largest uncertainty in the metallicity determination due to the use of the two sets of FF amounts to ∆[Fe/H] ∼ 0.15 dex at ⟨Fe⟩ ∼ 3.

2.3. The Hβ index

The comparison of the Hβ indices from our models with BSSP and WSSP is shown in the left-hand panel of Figure 3. At low metallicities ([Fe/H] ≤ −0.5 dex) and old ages (t ≥ 12 Gyr), the Hβ index is much stronger in our computations. This large discrepancy comes from the typical temperature of the HB phase in the various EPSs, which is controlled by the assumptions on the mass-loss on the RGB. At a given age (i.e. at fixed evolutionary mass at the turn-off), the larger the mass-loss, the lower the mass of the HB star, which implies larger T_{eff} for the HB phase. In our models, the mass-loss is computed using the scheme of Reimers (1977) with the canonical efficiency η = 0.33, a value calibrated on Galactic globular clusters (see e.g. Fusi-Pecci & Renzini, 1976). At 15 Gyr and [Fe/H] = −1.35, the resulting evolutionary mass on the HB is ∼ 0.65 M⊙ in our models, while Buzzoni (∼ 0.71 M⊙) and
Worthey (~ 0.83 $M_\odot$) use higher values. We checked our computations comparing the $M_{\text{HB}}$ values with those obtained by Greggio and Renzini (1990) with the same $\eta$ values, and found no appreciable difference.

Thus, in our models the HBs are hotter, which leads to the strong H$\beta$ enhancement, since the relative FF are very sensitive to $T_{\text{eff}}$. Old and metal-poor ([Fe/H] $\lesssim$ −0.5) SSPs predominantly show this effect, while for higher metallicity, and younger ages the evolutionary mass on the HB is relatively large, and the HB remains at low temperatures ($T_{\text{eff}} \sim 4600 - 4900$ K). However, the $T_{\text{eff}}$ of the HB depends on the prescriptions for the mass-loss, and a larger value for $\eta$ would make the HB hotter also at higher $Z$ (see e.g., Greggio & Renzini, 1990). Note that Worthey & Ottaviani (1997) show an analogous effect of the RGB mass-loss on the H$\gamma$ and H$\delta$ indices, as a consequence of a change in the HB morphology with respect to Worthey 1994 (see the caption of their Fig. 6). Conversely, reducing the mass-loss on the RGB leads to weaker H$\beta$ indices at old ages/high metallicities, as shown in Figure 3 by the model represented as a triangle. Therefore, H$\beta$ is not a reliable age indicator, its value also depending on the HB morphology. Since the observed HB in Globular Clusters very often show intermediate morphologies (see, e.g., Dickens, 1972), in GM99 the results for mixed HB will be discussed.

The discrepancy in the synthetic H$\beta$ due to the treatment of the HB phase can be as large as $\sim$ 1.5 Å (see, e.g., the BSSP case at [Fe/H] $\sim$ −1.3, $t = 15$ Gyr). At low $Z$, this effect implies an age/metallicity degeneracy, with old SSPs ($t \sim 12 - 14$ Gyr) having the same H$\beta$ index of intermediate-age populations (see also Greggio, 1997). This effect is shown in the right-hand panel of Fig. 3, with the models at [Fe/H] $\sim$ −1.35 showing a minimum around 12 Gyr. Concerning the uncertainties introduced by the FF, BFF systematically lead to higher index values for all the explored values of age and [Fe/H]. For solar and supersolar metallicities, at H$\beta$ $\sim$ 1.5 − 1.6, models computed with WFF predict ages younger by $\sim$ 3 Gyr, compared to models computed with BFF.

3. Summary

We have shown that current synthetic SI are affected by uncertainties that cannot be neglected when using these models to derive the age and the metallicity of stellar systems. These uncertainties originate from the EPS procedure/stellar input and from the use of different Fitting Functions.
The uncertainties in the synthetic indices for stellar populations

The Mg$_2$ index seems to be the least affected by the different modelling procedures. The EPS ingredient that mostly influence its values is the $\Delta Y/\Delta Z$ parameter, in the sense that at fixed $Z$ a higher $Y$ leads to a lower Mg$_2$. The dependence of the Mg$_2$ index on the adopted FF is found small at old ages, while for intermediate age SSPs WFF predict systematically stronger values.

Also the $\langle$Fe$\rangle$ index depends on $\Delta Y/\Delta Z$, a higher $Y$ yielding a stronger $\langle$Fe$\rangle$. Models of the Fe indices adopting different FF show major discrepancies at high [Fe/H] and old ages.

The H$\beta$ index is highly sensitive to the treatment of Horizontal Branch stars. In metal-poor SSPs ([Fe/H] $\lesssim$ −0.5), an age/metallicity degeneracy is found, since at $t \gtrsim 12$ Gyr H$\beta$ becomes stronger with increasing age. The sensitivity of this index on the FF is found to be large in all the age and metallicity ranges explored, but especially in the metal-poor regime.

Table 1 collects the estimated uncertainties due to different fitting functions, expressed as SI(WFF)−SI(BFF), for 3,10 and 15 Gyr old SSPs and ([Fe/H] = −1.35, 0.0, 0.35). The 2$\sigma$ observational errors (from González, 1993) are reported. Values in bold-face are larger than the quoted observational errors. No value means a negligible uncertainty ($\sim 10^{-3}$). There are regimes in which the uncertainty related to the FF adopted in the modelling is appreciably larger than the errors affecting the data. This happens at high $Z$ for Fe5270, and at low $Z$ for the H$\beta$ index. Thus, it appears that uncertainties in the theoretical models should be taken into account when deriving ages and metallicities for stellar systems.

To conclude, we stress that the estimates reported here are only indicative of the real uncertainties affecting these indices. For example, a subset of FF and SSP models existing in the literature is considered here. Besides, variations in the parameters controlling the stellar evolution input of EPS (e.g. $\eta$, $\Delta Y/\Delta Z$, IMF slope, etc.) have not been discussed explicitly. A more thorough investigation will be the subject of a forthcoming paper (GM99).

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Table I. Uncertainties on SI due to the use of different fitting functions. Observational errors are from González 1993 (first line). For the various ages and metallicities SI(WFF) – SI(BFF) is shown.

| [Fe/H] | age   | ∆Mg₂ (mag) | ∆ Fe5270 (Å) | ∆ Fe5335 (Å) | ∆ Hβ (Å) |
|-------|-------|-------------|--------------|--------------|----------|
| −1.35 | 3 Gyr | +0.010      | −0.10        | +0.06        | −0.50    |
|       | 10 Gyr| −          | −0.12        | +0.22        | −0.25    |
|       | 15 Gyr| −0.014      | −0.04        | +0.13        | −0.47    |
| 0.00  | 3 Gyr | +0.016      | −0.02        | +0.18        | −0.11    |
|       | 10 Gyr| −          | −0.23        | -            | −0.09    |
|       | 15 Gyr| −0.27       | −0.05        | −0.13        |          |
| 0.35  | 3 Gyr | +0.016      | −0.20        | +0.13        | −0.07    |
|       | 10 Gyr| −0.39       | −0.08        | −0.10        |          |
|       | 15 Gyr| −0.39       | −0.12        | −0.15        |          |

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