Error Analysis of Directional Coupler for Lower Hybrid Wave Power Measurement

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Abstract As one of the important parameters for lower hybrid wave (LHW) systems with 24 4.6 GHz/250 kW continuous-wave klystron amplifiers, the microwave power needs to be accurately acquired in real time for the experimental advanced superconducting tokamak (EAST) experiment. A new 4.6 GHz/250 kW directional coupler was developed for power measurements in an LHW system. The design principle and characteristic parameters of the coupler are discussed based on transmission line theory and the system requirements. The calculations and simulations indicate that the measurement error of the incident power is less than 5% when the isolation is greater than 25 dB. The directional coupler was also applied to the 4.6 GHz LHW system. The results show that the device can meet the requirements of LHW system operations. Comprehensive Research Facility for Fusion Technology (CRAFT) is being built. This work is beneficial to the design of power measurement system for LHW system.

1. Introduction

The lower hybrid current drive (LHCD) is an important method to drive non-inductive current and achieve long pulse operations on the Experimental Advanced Superconducting Tokamak (EAST) [1-4] and the Comprehensive Research Facility for Fusion Technology (CRAFT). The EAST 4.6 GHz/6 MW lower hybrid wave (LHW) system contains 24 VKC7849A klystron amplifiers with a capacity of 250 kW under continuous operation. [5,6] As one of the key parameters for the LHW, microwave power must be accurately obtained in real time and during the various experiments on the EAST. [7,8] Two directional couplers are installed on each branch for real-time measurements of the branch power data while operating the LHW system.

One branch of 4.6GHz LHW system shown in Fig. 1 mainly includes a low power microwave source, a klystron amplifier, transmission line and antenna. The transmission line is composed of five arc detectors, two directional couplers, one circulator, one DC break and two ceramic windows. The LHW system includes arc detection system and high reflected power protection system, which are used to protect the safety of ceramic windows. The accuracy of power measurement is very important for high reflected power protection system.

This article discusses the errors in the microwave power as measured using directional couplers under different load reflection coefficients and distances to the load. Signal analysis and program calculations indicate that the difference between the measured and true values is due primarily to the insufficient directivity of the directional coupler and the mismatched load. Finally, a new 4.6 GHz/250 kW directional coupler is designed and installed in the LHW system.
2. Microwave signal analysis

The directional coupler method is a common, high-power, microwave, real-time, measurement technique. The simplified microwave circuit model for the LHW system shown in Fig. 2 illustrates the normalized arbitrary complex load impedance \( Z_l \) to represent the plasma at the terminal. The left side of dashed line 1 is the microwave source (klystron) and the right side of dotted line 2 is the load (plasma). The transmission line is between dashed lines 1 and 2 with an electrical length \( l \) and normalized characteristic impedance \( Z_0 = 1 \).

On a lossless transmission line terminated at an arbitrary load impedance, the ratio of the reflected voltage to the incident voltage is:

\[
\Gamma_l = \Gamma_L e^{-j2\beta l}
\]

with \( \Gamma_L \) is the reflection coefficient at the terminal, \( \beta \) is the Spatial Frequency. The spatial frequency is:

\[
\beta = \frac{2\pi}{\lambda_g}
\]

with \( \lambda_g \) is the waveguide wavelength.

The voltage can be written as

\[
V_g + V_r \Gamma_g = V_i
\]
\[
V_r = V_i \Gamma_l e^{-j4\beta l}
\]

So that

\[
V_i = \frac{V_g}{1 - \Gamma_g \Gamma_l e^{-j4\beta l}}
\]
\[
V_r = \frac{V_g \Gamma_l e^{-j4\beta l}}{1 - \Gamma_g \Gamma_l e^{-j4\beta l}}
\]

The input power of coupling port is

\[
P = |V_i + V_r|^2
\]
Substituting Eqs. (5) and (6) in (7) gives

\[ P = K |V_g|^2 \]  

where

\[ K = \frac{|1 + r_ie^{-j\beta l}|^2}{|1 - r_ie^{-j\beta l}|^2} \]  

(8)

(9)

The K factor is related to the plasma load and cannot be controlled. The signal to the coupling port of the directional coupler can also be obtained using a similar analysis method.

\[ P' = K' |V_g'|^2 \]  

where

\[ K' = \frac{|1 + r_I e^{-j\beta l'}|^2}{|1 - r_I e^{-j\beta l'}|^2} \]  

(10)

(11)

Assuming there is no reflection at the source (\( \Gamma_g = 0 \)) and the incident voltage is one (V_g = 1), from Eq. (11),

\[ K' = |1 + r_I e^{-j\beta l'}|^2 \]  

\[ P' = \left( 1 + r_I^2 + 2r_I \cos \left( -\frac{4\pi l'}{\lambda} \right) \right) \]  

(12)

(13)

From (13), the power measurement value is related to the reflection coefficient (\( \Gamma_I \)) and the electrical length (\( l' \)).

3. Error analysis of power measurement

The incident (reflected) port of a bi-directional coupler is assumed to have a coupling coefficient \( C_i \) (\( C_r \)) and directivity coefficient \( D_i \) (\( D_r \)). The incident and reflected microwave signals in the transmission line are given by Eqs. (5) and (6), giving a voltage for the incident port of a bi-directional coupler as

\[ V_g' = \frac{\sqrt{C_i}V_g + \sqrt{C_r}D_r e^{-j\beta l}V_g e^{-j\beta l}}{1 - r_ie^{-j\beta l}} \]  

(14)

From (14) the measured power of microwave detector is

\[ P'_i = K'|V_g'|^2 = \frac{K'C_i |V_g|^2}{|1 - r_ie^{-j\beta l}|^2} \]  

(15)

When the isolation of the coupler is ideal \((D_i = 0)\), Eq. (15) reduces to

\[ P'_i |_{D_i = 0} = K'|V_g|^2 \]  

(16)

which is the real incident power in the waveguide. The \( K' \) factor is related to the load of the isolation port of the directional coupler.

The measured incident power by the detector \( P'_i \) is normalized to the true incident power in the waveguide \( P'_i |_{waveguide} \) and is defined as the normalized incident power \( P'_i |_{waveguide} \) as

\[ P_Ni = P'_i / P'_i |_{waveguide} = |1 + \sqrt{D_r} \Gamma_I e^{-j\beta l} |^2 \]  

(17)

The error of the incident power measurement is

\[ P_{ei} = |1 + \sqrt{D_r} \Gamma_I e^{-j\beta l} |^2 - 1 \]  

(18)

It is easily shown that the errors in the reflected power measurement are

\[ P_{er} = |r_I e^{-j\beta l} \sqrt{D_r} |^2 - 1 \]  

(19)

From Eqs. (18) and (19), the errors in the power measurements between the measured and true values is related to the directivity coefficient \( D \), the reflection coefficient \( \Gamma_I \), and the electrical length \( l \) from the load to the measuring port. Consider the simpler case where the load is purely resistance, the imaginary part of \( \Gamma_I \) is 0, and Eqs. (18) and (19) can be written as
4. Simulation and discussion

To obtain accurate power measurements, a program was written to perform the calculations in Eqs. (18) and (19). The curve (surface) of power with respect to electrical length and load reflection coefficient are given below.

\[
P_{EI} = D_l \Gamma_L^2 + 2\sqrt{D_l} \Gamma_L \cos \left( \frac{8\pi l}{\lambda_g} \right) \tag{20}
\]

\[
P_{ER} = \frac{D_l + 2\sqrt{D_l} \Gamma_L \cos \left( \frac{8\pi l}{\lambda_g} \right)}{\Gamma_L^2} \tag{21}
\]

Fig. 3 The three-dimensional surface plot of the power measurement error PE, when load reflection coefficient ΓL and electrical length λ change at the same time.

From formula (18) and (19), the power measurement error shown in Fig. 3 changes with different electrical lengths and load reflection coefficient. It is seen that the power measurement error changes periodically with the electrical length with a period of \( \lambda_g/4 \). There are always two points ( \( \lambda_g/16 \) and \( 3\lambda_g/16 \)) that intersect within a \( \lambda_g/4 \) period. For these points, the measured value is equal to the true value, the measurement error is zero. In the case of full reflection (\( \Gamma_L = -1 \)), if the incident and reflective ports have the same directivity, the relative errors for the incident and reflected powers are the same. In other cases, the incident power is greater than the reflected power. This causes the relative errors of incident power to be smaller than the one of reflected power. Generally, the relative error of the reflection power is greater than the one of the incident power. It can be seen from the figure that if the electrical length is given, the relative error value can be reduced only by increasing the directionality of the coupler. If the microwave power total is totally reflected (\( \Gamma_L = -1 \)) and the port directivity is consistent, the relative error between incident power and reflected power is the same. When the load is completely matched (\( \Gamma_L = 0 \)), the measured power value is the same as the real power value for incident part. At the same time, the relative error is infinite because the true reflected power is zero.

5. Design and experiment

From discuss above, it can be seen that for incident power measurement, the smaller the reflection coefficient, the more accurate the measured data. The measurement of reflected power is just the opposite. For the measurement error of incident power and reflected power, its accuracy varies with the period of electrical length.

To reduce the errors in the power measurements, a new directional coupler is developed and applied to the proposed 4.6 GHz LHW system. The maximum input power of the self-made microwave detector is 27 dBm. According to the power measurement requirements and the characteristics of the detector, the coupling degree of the directional coupler needs to reach 58 dB. Fig. 3 indicates that the power measurement error is less than 5% when the load reflection coefficient is less than 0.44. The high
reflection protection action is usually triggered when the reflection is greater than 30 kW in typical experiments. Based on the above analysis, a directionality of 25 dB could ensure that the measurement error of the incident power is less than 5%. The new directional coupler integrates the matching loads with a microstrip structure to improve the measurement accuracy.

The incident and reflected coupling ports of the 4.6 GHz/250 kW continuous-wave directional coupler have the same parameters. Compared with previous designs [11], the proposed directional coupler has a microstrip load and water cooling, which further reduces the power measurement error. At the same time, the directional coupler changes its coupling degree (50 to 58 dB) and the direction of the coupling port. A vector network analyzer (Agilent E5071B) is used to measure the characters with typical coupling values of 58 dBm with a directionality of 25 dBm.

All directional couplers have been tested on 250kW continuous wave test bench. [12] Here are two power measurement methods on the high-power test bench shown in Fig. 4, one is the water load method, and the other is the directional coupler method. They are confirmed each other to ensure the accuracy of the power measurement. Table 1 shows the comparison of measured power with water load and two directional couplers. Two kinds of power meters are used in the system, one is Agilent U200b connected to new coupler, the other is a self-made detector connected to old one. It can be seen from Table 1 that the power measurement error of directional coupler and water load at high power is less than 5%. The measured data with the new coupler is more accurately than the data with the old one. The new coupler has good heat dissipation and a high degree of coupling, which can reduce the microwave measurement error due to thermal effects.

### Table 1 Comparison of measured power between water load and directional couplers

| Number | Water load (KW) | Old coupler1 (KW) (self-made detector) | New coupler 2 (KW) (Agilent U200b) |
|---|---|---|---|
| 1 | 43 | 44 | 46.6 |
| 2 | 92 | 91 | 94 |
| 3 | 142 | 140 | 145 |
| 4 | 192 | 188 | 194 |
| 5 | 242 | 235 | 243 |

### Table 2. The LHW power data in EAST experiments when high reflection protection occurs

| Number | Measured incident power (kW) | Reflected power (kW) | Incidence power measurement error(%) |
|---|---|---|---|
| Zero-reflected power | High-reflected power | |
| 1 | 48 | 49 | 25 | 2.08 |
| 2 | 52.5 | 54 | 23 | 2.85 |
| 3 | 68 | 66 | 20 | 2.94 |
| 4 | 61 | 60 | 25 | 1.63 |
| 5 | 82 | 83 | 35 | 1.21 |
| 6 | 103 | 104 | 30 | 0.97 |
A high reflection protection is triggered when the plasma is mismatched in the EAST experiment, which alters the incident power data. A selection of power data from several EAST experiments in 2018 and 2019 are shown in Table II. The measurement error for the incident power caused by high reflections is less than 5%.

6. CONCLUSION
The directional coupler method is used to measure microwave power in real time based on the EAST experiment. The 4.6 GHz/250 kW continuous-wave directional coupler is an indispensable part of the LHW system. New design parameters for directional couplers are proposed based on transmission line theory and the system requirements. Simulations show that the measurement error of the incident power is less than 5% when the isolation is greater than 25 dB. The newly designed directional coupler reduces the power measurement error and has a more compact structure. The operational results in the EAST experiments indicate that the design can meet the needs of LHW systems. Comprehensive Research Facility for Fusion Technology (CRAFT) is being built in HEFEI now. This work will guide the design of power measurement system for LHW system in further.

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