The Long Distance Contribution to $D \to \pi l^+ l^-$

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Abstract

We calculate the long distance contribution to $D^{+,0} \to \pi^{+,0} l^+ l^-$ decays by the use of a vector meson dominance model, in which the $\phi$-meson plays the central role. The branching ratios obtained are $10^{-6}$ and a few times $10^{-7}$ for the resonance and non-resonance regions respectively. The analysis includes a calculation of $D^+ \to \pi^+ \phi$, consistent with the experimental value.

PACS number(s): 13.20.Fc 12.40.Vv 12.15.Lk
1 INTRODUCTION AND THE MODEL

New experimental limits for transitions involving change of flavour in charm meson decays were obtained recently at FERMILAB [1, 2] and at the Cornell Electron Storage Ring using the CLEO II detector [3]. In particular, upper limits in the range $10^{-4} - 10^{-5}$ were established for exclusive channels of the type $D^{+,0} \rightarrow X^{+,0} l^+ l^-$, where $X^{+,0}$ is a pseudoscalar or vector meson.

The short distance process $c \rightarrow u\gamma$, driven by the magnetic penguin, is known to be of little significance for radiative decays of charm mesons as it leads to a branching ratio of $10^{-12} - 10^{-11}$ only, despite its enhancement by gluonic corrections [4]. In decays to lepton pairs, the short distance $c \rightarrow ul^+ l^-$ transition may be of more relevance, as it contains contributions from both form factors of the electromagnetic penguin, as well as contributions from the $Z^0$-penguin and from the $W$-box diagram. Indeed, the rate for the short distance $c \rightarrow ul^+ l^-$ transition has been calculated [5] to be $1.1 \times 10^{-20}$ GeV, leading to a branching ratio of $1.8 \times 10^{-8}$ for the inclusive process. Accordingly, exclusive process like $D \rightarrow \pi l^+ l^-$, expected to be approximately 10% of the inclusive rate, will have a short distance contribution to the branching ratio of the order of $10^{-9}$. On this basis, it has been argued [3] that decays like $D^{+,0} \rightarrow \pi^{+,0} l^+ l^-$, $D^+_s \rightarrow K^+ l^+ l^-$ constitute “... a large “discovery” window: seeing this decay occur at a branching ratio above $\sim 10^{-7}$ would be strong evidence for new physics”.

It is therefore of obvious interest to obtain reliable estimates for the long distance contributions to these modes. As one suspects [7] that these contributions are the dominant ones in the Standard Model, one should have good control of their estimation, in order to perform a meaningful search for new physics in these decays.

In the present paper, we consider the long distance contribution to the helicity-unsuppressed decays $D \rightarrow \pi l^+ l^-$, which supposedly are the better suited channel for checking the nature of the flavour changing neutral transitions. We note that presently the best experimental upper bounds for the branching ratios are as follows: $1.8 \times 10^{-5}$
for $D^+ \to \pi^+ \mu^+ \mu^-$, $6.6 \times 10^{-5}$ for $D^+ \to \pi^+ e^+ e^-$ [2], and $1.8 \times 10^{-4}$ for $D^0 \to \pi^0 \mu^+ \mu^-$ [1], $4.5 \times 10^{-5}$ for $D^0 \to \pi^0 e^+ e^-$ [3].

Our estimate for the long distance contribution is based on a vector meson dominance mechanism, similar to the approach widely used for calculating the long distance contributions in $b \to s l^+ l^-$ decay [8]. We present the details of our calculation for the $D^+ \to \pi^+ l^+ l^-$ channel, for which the experimental input required by our model is available.

The basic assumption of our model is that the main long distance contribution to $D^+ \to \pi^+ l^+ l^-$ is given by the transition $D^+ \to \pi^+ (V) \to \pi^+ l^+ l^-$, where $V$ is a $\bar{q}q$ vector meson state. It is known that [1]

$$BR(D^+ \to \pi^+ \phi) = (6.1 \pm 0.6) \times 10^{-3},$$
$$BR(D^+ \to \pi^+ \rho) < 1.4 \times 10^{-3},$$
$$BR(D^+ \to \pi^+ \omega) < 7 \times 10^{-3}. \quad (1)$$

Moreover, the branching ratios of $\phi$ for decays to lepton pairs are [4] $BR(\phi \to e^+ e^-) = (3.00 \pm 0.06) \times 10^{-4}$, $BR(\phi \to \mu^+ \mu^-) = (2.48 \pm 0.34) \times 10^{-4}$, while for $\rho$ and $\omega$ the similar branching ratios are nearly one order of magnitude lower. Thus, we may restrict our considerations to the role of the $\phi$-meson only, the contribution of $\rho$ and $\omega$ constituting a relatively small correction.

Another kind of possible long distance contributions comes from the W-annihilation (or W-exchange) diagram, which is found for example to be large in $B \to \rho \gamma$ [10], and needs therefore to be discussed further. In the present case of $D \to \pi e^+ e^-$, the mechanism for the W-annihilation (or W-exchange) diagram is through $D \to \pi \gamma^*$ and $\gamma^* \to e^+ e^-$, where the virtual photon comes from one of the four quark lines. Because naive (constituent) quark model does not work for the light pseudo-Goldstone meson $\pi$, we need to use a hadronic model to estimate these contributions. At the hadronic level, the process $D \to \pi \gamma^*$ corresponds to the electromagnetic transition $D \to D \gamma^*$ followed by the weak transition $D \to \pi$, or $D \to \pi$ followed by $\pi \to \pi \gamma^*$. These
electromagnetic transitions are described by the electromagnetic formfactors of the $D$ and the $\pi$ mesons, which can be taken in the VMD model as dominated by vector mesons as $\psi$, $\rho$ and $\omega$. The contribution from $\psi$ is smaller than those from the other two since this $\psi$ is highly off-shell. The remaining contributions from $\rho$ and $\omega$ in the $W$-annihilation (or $W$-exchange) mechanism correspond to the full processes $D \to \pi \rho$ and $D \to \pi \omega$. These effects have been included in our VMD model and found to be small.

In the second section we present our approach to the calculation of $D^+ \to \pi^+ \phi$ and in the third section we calculate its contribution to the $D \to \pi l^+ l^-$ modes.

## 2 THE $D^+ \to \pi^+ \phi$ TRANSITION

We begin with a treatment of the process $D^+ \to \pi^+ \phi$, which is the main contribution to $D^+ \to \pi^+ l^+ l^-$ in our approach. The effective Hamiltonian for this transition is

$$H_{eff} = \frac{G_F}{\sqrt{2}} V_{us}^* V_{cs} a_2 \bar{s} \gamma^\mu (1 - \gamma_5) s \bar{u} \gamma_\mu (1 - \gamma_5) c,$$

(2)

where $a_2$ is QCD-coefficient which is taken as $|a_2| = 0.55 \pm 0.1$ from the overall fit to nonleptonic $D$ decays\[11\]. We shall assume the usefulness of the factorization hypothesis\[11\], thus

$$< \pi^+ \phi | H_{eff} | D^+ > = \frac{G_F}{\sqrt{2}} V_{us}^* V_{cs} a_2 < \phi | \bar{s} \gamma^\mu (1 - \gamma_5) s | 0 > < \pi^+ | \bar{u} \gamma_\mu (1 - \gamma_5) c | D^+ >,$$

(3)

and we define $g_\phi$ by

$$< \phi | \bar{s} \gamma^\mu s | 0 > = ig_\phi \epsilon^*_\mu.$$

(4)

$g_\phi$ will be determined from the observed lepton pair decay $\phi \to e^+ e^-$. The hadronic matrix element is parameterized by two independent form factors $f_+$ and $f_-$ as
\[ \langle \pi^+ | \bar{u} \gamma_\mu (1 - \gamma_5) c | D^+ \rangle = f_+(p_D + p_\pi)_\mu + f_-(p_D - p_\pi)_\mu. \] (5)

The \( f_- \) form factor does not contribute in the decay to a lepton pair; \( f_+ \) is related by isospin symmetry to the form factor of the semileptonic decay \( D^+ \rightarrow \pi^0 l^+ \bar{\nu}_l \). Accordingly, we turn to the latter for learning the \( f_+ \) form factor.

Using the heavy meson chiral perturbation theory, Wise\[12\] has calculated the \( f_+ \) form factor near the zero recoil point \( q_m^2 = (p_D - p_\pi)_m = (m_{D^+} - m_{\pi^0})^2 \) in \( D^+ \rightarrow \pi^0 l^+ \bar{\nu}_l \) to be:

\[
\begin{align*}
\langle q_m^2 \rangle &= \frac{-f_D}{2\sqrt{2}f_\pi} \left( 1 - g \frac{p_D \cdot p_\pi - m_D^2}{p_D \cdot p_\pi + m_D \Delta} \right) \\
&\sim \frac{-f_D}{2\sqrt{2}f_\pi} \left( 1 + g \frac{m_D - m_\pi}{m_\pi + \Delta} \right), \\
\Delta &\equiv m_{D^*} - m_D.
\end{align*}
\] (6)

The analysis of various existing experiments gives the following bounds on the parameters in Eq. (6)\[9, 13\]:

\[
\begin{align*}
f_D &\leq 0.31 \text{GeV}, \\
|g| &\leq 0.63.
\end{align*}
\] (7)

On the other hand, two measurements\[14\] of the formfactor \( f_+(q^2) \) in the semileptonic decay have been analyzed under the assumption of a monopole behaviour for it,

\[
f_+(q^2) = f_+(q_m^2) \frac{1 - q_m^2/m_{D^*}^2}{1 - q^2/m_{D^*}^2},
\] (8)

and the data may be summarized as\[1\]

\[
f_+(0) = (1.0 \pm 0.3) \times f^K_+(0) = (1.0 \pm 0.3) \times (0.75 \pm 0.03),
\] (9)

which shows the consistency of Eqs. (6) - (8), within the accuracy obtained so far. A new recent measurement by E687 gave \(|f_+(0)/f^K_+(0)| = 1.03 \pm 0.16 \pm 0.02\[15\] \), which is consistent with the value cited in Eq. (9).
Using now, for example, the reasonable set of values \( f_D = 0.20 \text{GeV} \) and \( g = 0.6 \), as well as Eq. (8), we find \( \Gamma(D^+ \to \pi^+ \phi) = (3.7 \pm 1.4) \times 10^{-15} \text{GeV} \), which is in remarkable agreement with the experimental average \( \Gamma(D^+ \to \pi^+ \phi) = (3.8 \pm 0.4) \times 10^{-15} \text{GeV} \).

This approach leads to \( f_+(0) = 0.92 \) which agrees with the results of Refs. [14, 15].

In the calculation of the leptonic decay \( D^+ \to \pi^+ l^+ l^- \) we shall normalize to the observed \( D^+ \to \pi^+ \phi \) rate. Nevertheless, it is appropriate to emphasize at this point that the theoretical treatment described here accounts well for this mode; this provides the needed confidence in its use as the major contribution to the \( D \to \pi l^+ l^- \) decays.

3 LONG DISTANCE CONTRIBUTION TO \( D^+ \to \pi^+ l^+ l^- \)

The effective coupling between \( \phi \) and \( l^+ l^- \), via a photon propagator is

\[
g_{\phi l} = \frac{g_{\nu \alpha}}{(p_+ + p_-)^2} \left[ \frac{1}{3} e g_{\phi} \epsilon^\nu \right] = \frac{e^2}{3} \bar{u}_l \gamma^\nu v_l \frac{g_{\phi}}{(p_+ + p_-)^2} \epsilon^\nu.
\]

For the \( \phi \)-meson decaying onshell, we replace \( 1/(p_+ + p_-)^2 \to 1/m_\phi^2 \).

We shall assume that \( g_{\phi}(q^2) \) defined in Eq. (10) does not vary appreciably with \( q^2 \) in the region of interest for our calculation, which is taken as \( m_\eta \leq \sqrt{m_\phi^2 - (m_D - m_\pi)} \), and we use for it the value determined from \( \phi \to e^+ e^- \) decay

\[
g_{\phi} = (492 \text{MeV})^2.
\]
The amplitude for the decay is then

$$A(D^+ \rightarrow \pi^+ l^+ l^-) = \frac{G_F}{\sqrt{2}} a_2 V^*_{us} V_{cs} < \pi^+ | \bar{u} \gamma_{\mu}(1 - \gamma_5) c | D^+ > g_{\phi} \left[ \frac{g_{\mu -} - (p_+ + p_-)_{\mu} (p_+ + p_-)_{\nu}}{(p_+ + p_-)^2 - m_{\phi}^2 + i \Gamma_{\phi} m_{\phi}} \right] \frac{e^2}{3} \frac{1}{m_{\phi}^2 - m_{\mu}^2 - m_{\pi}^2 + i \Gamma_{\phi} m_{\phi}} f_+(m_{\phi}^2) \bar{u}_l \gamma_{\nu} v_{l+}, \tag{12}$$

where we used for the $\phi$ propagator a Breit-Wigner form to account for the behaviour throughout the region of decay.

Since at the $\eta$-mass the sizable $D^+ \rightarrow \pi^+ (\eta) \rightarrow \pi^+ \mu^+ \mu^-$ opens $[ BR(D^+ \rightarrow \pi^+ \eta) = (7.5 \pm 2.5) \times 10^{-3} ]$, we shall impose a lower cut on the $m_{\eta\mu}$ spectrum of the muon pairs above the $\eta$ mass. We have also checked that changing this cut up to 700MeV does not affect practically our results. We shall make for convenience the same cut in the $e^+ e^-$ channel. Then, the spectra for $e^+ e^-$ and $\mu^+ \mu^-$ are essentially identical in the chosen region for $m_{\eta\mu}$. By restricting our considerations to this region, we also avoid a possible ambiguity concerning the $q^2$ dependence in the region close to $q^2 = 0$.

The contribution of $\phi$ to the long distance $D^+ \rightarrow \pi^+ l^+ l^-$ decay in the resonance region is given by

$$\Gamma_R = \int_{(m_{\phi} - \Gamma_{\phi}/2)^2}^{(m_{\phi} + \Gamma_{\phi}/2)^2} dm_{\eta\mu}^2 \frac{d \Gamma(D^+ \rightarrow \pi^+ l^+ l^-)}{dm_{\eta\mu}^2}, \tag{13}$$

and we find it to contribute to the branching ratio

$$(\Gamma_R) BR(D^+ \rightarrow \pi^+ l^+ l^-) = 0.82 \times 10^{-6}. \tag{14}$$

If we extend the limits in (13) to $m_{\phi} \pm \Gamma_{\phi}$, this branching ratio becomes $1.22 \times 10^{-6}$.

Now we turn to the region outside the resonance, which is the main object of the present investigation. We denote this contribution as $\Gamma_{NR}$, and we calculate it from the amplitude of Eq. (12) for the region $m_{\eta} \leq m_{\eta\mu} \leq (m_{\phi} - \Delta)$ and $(m_{\phi} + \Delta) \leq m_{\eta\mu} \leq (m_D - m_\pi)$. The branching ratios for the long distance contribution outside the resonance region, for several values of $\Delta$, are given in Table 1.
TABLE 1. The long distance contribution $\Gamma_{NR}$ to the $D^+ \rightarrow \pi^+\mu^+\mu^-$ or $D^+ \rightarrow \pi^+e^+e^-$ decay rates, outside the $\phi$-resonance (beyond $m_\phi \pm \Delta$).

| $\Delta$ (in MeV) | $\Gamma_{NR}$ (in $10^{-19}$GeV) | Branching ratio |
|-------------------|----------------------------------|-----------------|
| 5                 | 3.66                             | $5.9 \times 10^{-7}$ |
| 10                | 1.92                             | $3.1 \times 10^{-7}$ |
| 20                | 0.95                             | $1.5 \times 10^{-7}$ |
| 40                | 0.45                             | $0.73 \times 10^{-7}$ |

We remind the reader now that the short distance contribution to these decays is about two orders of magnitude smaller than the values of Tables 1. Hence, the typical interference distribution in $m_{\pm}$ expected in $b \rightarrow s l^+l^-$ should not appear in $D \rightarrow \pi l^+l^-$, the spectrum of the lepton pair in our case being given by the matrix element of Eq. (12) and phase space only.

In the present paper, we concentrated on the $D^+ \rightarrow \pi^+l^+l^-$ modes. For the parallel decays $D^0 \rightarrow \pi^0l^+l^-$, we expect in our model a branching ratio smaller by approximately 5, due to a factor of 2 from the $\Delta I = \frac{1}{2}$ weak $D \rightarrow \pi$ transition and a factor of 2.5 from the $D^+ - D^0$ lifetime difference.

We remark that a previous long-distance calculation of $D \rightarrow \pi l^+l^-$ (first Ref. of [7]) has considered it as evolving from the $D \rightarrow D^*\pi$ process, with the virtual $D^*$ decaying to a lepton pair. A rate of $10^{-8}$ has been found for this contribution; this is consistent with our result as this specific diagram is obviously only part of the form factor we considered here for the transition (5).

We conclude by stressing that our results, presented in Eq. (13) and in Table 1, invalidate the earlier expectations that branching ratios above $\sim 10^{-7}$ would constitute a signal for physics beyond the Standard Model.

This research was supported in part by Grant 5421-3-96 from the Ministry of Science and the Arts of Israel. The work of P.S. has been supported in part also by the Fund for Promotion of Research at the Technion. One of us (P.S.) acknowledges a helpful
discussion with Professor Lalit Sehgal. We also thank our colleagues Prof. Gad Eilam, Dr. Dan Pirjol and Prof. Daniel Wyler for helpful remarks.

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