Research on the Application of Astrophysics in Magnetohydrodynamics under Big Data

Chuantao Ma
College of Physics and Electronic Engineering Taishan University, Tai’an City, 271000, China
mcht@tsu.edu.cn

Abstract. Based on the background of big data, the paper establishes the evolution equation of electromagnetic fluid state parameters over time called Magnetohydrodynamics (MHD). Compared with kinetic theory, magnetohydrodynamics is much simpler in mathematical processing, and there are many processes in plasma. For some other phenomena in plasma, such as Landau damping, instability in velocity space, etc., the theory of magnetohydrodynamics is unable to describe. The thesis starts from the dynamic equations and establishes the magnetohydrodynamic equations.

Key words: Big data, astrophysics, magnetohydrodynamics, equation establishment.

1. Introduction
Magnetohydrodynamics is a discipline that combines the methods of fluid mechanics and electrodynamics to study the interaction of conductive fluids and electromagnetic fields. It is hard to say that the laws of magnetic and fluid flow are innovations in the 20th century, but magnetohydrodynamics did not become a relatively mature subject until the late 1930s. Magnetohydrodynamics is mainly used in three aspects: astrophysics, controlled thermonuclear reaction and industry [1]. Stars and interstellar gas in the universe are plasmas and have magnetic fields. Therefore, magnetohydrodynamics was first developed and applied in astrophysics, solar physics and geophysics.

Currently, the research topics on the sun include: the nature and origin of the solar magnetic field, and the influence of the magnetic field on the corona, sunspots, and flares. In addition, there are: the possibility of the existence of no force field in interstellar space, the bow shock generated by the interaction between the solar wind and the earth's magnetic field, the explosion of new stars and supernovae, the origin of the earth's magnetic field, and so on. The application of magnetohydrodynamics in controlled thermonuclear reactions may enable humans to obtain huge energy from neon in seawater. The purpose of a controlled thermonuclear reaction is to heat a gas composed of light elements to a high enough temperature for nuclear fusion, and to constrain it for a sufficient time so that the energy produced by the nuclear reaction is greater than the energy consumed. For a mixture of neon and neon, the temperature is required to reach 50 million to 100 million Kelvin and the product of particle density and confinement time is not less than 10 seconds/cm. Tokamak has shown superiority in the research of controlled thermonuclear reaction [2]. The wide application of magnetic fluid mechanics in industry is closely related to magnetic fluid power generation. The basic
equations of magnetohydrodynamics are nonlinear and contain a large number of equations. For this type of equations, even if the initial values are sufficiently smooth, its solutions may appear discontinuous, so it is extremely difficult to solve. However, in practical problems, there is often no need to find the solution of the most general form of equations, but only the solution of the equations of a particular problem.

2. Two-part magnetohydrodynamic equation
Assume that plasma is a two-part electromagnetic fluid composed of an electronic component and an ionic component [3]. First, we introduce the macroscopic state variables of two parts of the magnetic fluid. We know that for a multi-particle system, the macroscopic variables are the statistical averages of the corresponding microscopic variables. In this way, the density \( n_\alpha (\mathbf{r}, t) \), flow velocity, fire \( \mathbf{u}_\alpha (\mathbf{r}, t) \), and temperature \( T_\alpha (\mathbf{r}, t) \) of the category \( \alpha \) component fluid are defined as:

\[
n_\alpha (\mathbf{r}, t) = \int dV f_\alpha (\mathbf{r}, \mathbf{v}, t) \\
n_\alpha (\mathbf{r}, t) \mathbf{u}_\alpha (\mathbf{r}, t) = \int dV \mathbf{v} f_\alpha (\mathbf{r}, \mathbf{v}, t) \\
\frac{3}{2} k_B n_\alpha (\mathbf{r}, t) T_\alpha (\mathbf{r}, t) = \int dV \frac{1}{2} m (\mathbf{v} - \mathbf{u}_\alpha)^2 f_\alpha (\mathbf{r}, \mathbf{v}, t)
\]

Next, we use the plasma kinematics equations to establish the magneto-hydrodynamic equations. The kinetic equation can be written as:

\[
\left[ \frac{\partial}{\partial t} + \mathbf{v} \cdot \nabla + \frac{q_\alpha}{m_\alpha} (\mathbf{E} + \mathbf{v} \times \mathbf{B}) \cdot \nabla \right] f_\alpha (\mathbf{r}, \mathbf{v}, t) = I_\alpha (\mathbf{r}, \mathbf{v}, t) \tag{3}
\]

First define the plasma moment equation:

Multiply both sides of (3) by \( g(\mathbf{v}) \) and integrate \( \mathbf{v} \), then we can get

\[
\int g(\mathbf{v}) \frac{\partial f}{\partial t} d\mathbf{v} = \frac{\partial}{\partial t} \int g(\mathbf{v}) f d\mathbf{v} = \frac{\partial}{\partial t} \langle g \rangle > \\
\int g(\mathbf{v}) \mathbf{v} \cdot \nabla f d\mathbf{v} = \mathbf{v} \cdot \int g(\mathbf{v}) \nabla f d\mathbf{v} = \nabla \cdot \langle g \mathbf{v} \rangle > \\
\frac{q}{m} \int g(\mathbf{v}) \mathbf{E} \cdot \frac{\partial f}{\partial \mathbf{v}} d\mathbf{v} = \frac{q}{m} \int g(\mathbf{v}) \mathbf{E} \cdot \frac{\partial f}{\partial \mathbf{v}} d\mathbf{v} = \frac{q}{m} \mathbf{E} \cdot \int f \frac{\partial g(\mathbf{v})}{\partial \mathbf{v}} d\mathbf{v} = -\frac{q}{m} \mathbf{E} \cdot \langle \frac{\partial g(\mathbf{v})}{\partial \mathbf{v}} \rangle > \\
\frac{q}{m} \int g(\mathbf{v}) (\mathbf{v} \times \mathbf{B}) \cdot \frac{\partial f}{\partial \mathbf{v}} d\mathbf{v} = -\frac{q}{m} \int (\mathbf{v} \times \mathbf{B}) \cdot f \frac{\partial g(\mathbf{v})}{\partial \mathbf{v}} d\mathbf{v} = -\frac{q}{m} \langle (\mathbf{v} \times \mathbf{B}) \cdot \frac{\partial g(\mathbf{v})}{\partial \mathbf{v}} \rangle > \\
\]

Which makes use of the relationship: \( \frac{\partial}{\partial \mathbf{v}} \cdot (\mathbf{v} \times \mathbf{B}) = 0 \)

So, get the moment equation:

\[
\frac{\partial}{\partial t} \langle g \rangle + \nabla \cdot \langle g \mathbf{v} \rangle \mathbf{v} + \frac{q}{m} \mathbf{E} \cdot \left( \langle \frac{\partial g(\mathbf{v})}{\partial \mathbf{v}} \rangle + \langle (\mathbf{v} \times \mathbf{B}) \cdot \frac{\partial g(\mathbf{v})}{\partial \mathbf{v}} \rangle \right) = \int g(\mathbf{v}) \frac{\partial f}{\partial t} d\mathbf{v} \tag{6}
\]

Among them: \( < a > = \int a f d\mathbf{v} \) is the statistical average.

2.1. Momentum balance equation
We set \( g(\mathbf{v}) = m_\alpha \mathbf{v} \) and integrate \( \mathbf{v} \), then we can get
\[
m_t \frac{\partial}{\partial t} (n_t u_a) + m_u \nabla \cdot (n_u u_a u_a) + \nabla \cdot \mathbf{P}_a = \rho_{qa} \mathbf{E} + j_a \times \mathbf{B} + \mathbf{R}_{a\beta}
\]

(7)

Among them
\[
\mathbf{P}(\mathbf{r}, t) = \int d^3v n_m (\mathbf{v} - u_a)(\mathbf{v} - u_a) f_a (\mathbf{r}, \mathbf{v}, t)
\]

(8)

The above formula is the pressure tensor. And
\[
\mathbf{R}_{a\beta} = \int d^3v n_m I_{a\beta}
\]

(9)

We use continuous equation (4), equation (9) can be transformed into
\[
m_t n_a \left( \frac{\partial}{\partial t} + u_a \cdot \nabla \right) u_a = -\nabla \cdot \mathbf{P}_a + \rho_{qa} \mathbf{E} + j_a \times \mathbf{B} + \mathbf{R}_{a\beta}
\]

(10)

\[
m_t n_a \left( \frac{\partial}{\partial t} + u_a \cdot \nabla \right) u_a \]
represents the rate of change of momentum of the fluid element; where
\[
\mathbf{u}_a \cdot \nabla \mathbf{u}_a
\]
is the convection term; \(-\nabla \cdot \mathbf{P}_a\) represents the force generated by the pressure gradient; \(\rho_{qa} \mathbf{E}\) represents the electric field force; \(j_a \times \mathbf{B}\) represents the Lorentz force is the force generated by the current passing through the magnetic field; \(\mathbf{R}_{a\beta}\) represents the \(\alpha\) particle The rate of change of momentum when colliding with the \(\beta\) particle \([4]\). Equation (10) is an unclosed equation, because higher-order moment functions \(\mathbf{P}_a\) and \(\mathbf{R}_{a\beta}\) are involved. Only by solving the dynamic equation can \(\mathbf{P}_a\) and \(\mathbf{R}_{a\beta}\) be calculated strictly. When studying the magnetic fluid state of the plasma, it is usually assumed that the velocity distribution of the charged particles in the plasma is basically isotropic. Therefore:
\[
\mathbf{P} = \mathbf{P}_a \mathbf{j}
\]

(11)

Where \(\mathbf{P}_a = k_b n_a T_a\) is the static pressure. The off-diagonal part of \(\mathbf{P}_a\) is only related to the viscous phenomenon in the plasma. In addition, for collisions between different types of charged particles, the rate of change of momentum can be written in the form of frictional resistance:
\[
\mathbf{R}_{a\beta} \approx -m_a n_a \nu_{a\beta} (\mathbf{u}_a - \mathbf{u}_\beta)
\]

(12)

Where \(\nu_{a\beta}\) is the average collision frequency of momentum transport.

2.2. Energy balance equation

We set \(g(\mathbf{v}) = \frac{1}{2} m_v \mathbf{v}^2\) and integrate \(\mathbf{v}\), and use continuous equation (4), momentum balance square product (10), and finally the energy balance equation can be obtained as:
\[
\frac{3}{2} k_b n_a \left( \frac{\partial}{\partial t} + u_a \cdot \nabla \right) T_a = -\mathbf{P} : \nabla \mathbf{u}_a - \nabla \cdot \mathbf{q}_a - u_a \cdot \mathbf{R}_{a\beta} + \mathbf{Q}_{a\beta}
\]

(13)

Among them:
\[
\mathbf{q}_a = \frac{1}{2} m_a n_a \int d\mathbf{v} (\mathbf{v} - u_a)^2 (\mathbf{v} - u_a) f_a (\mathbf{r}, \mathbf{v}, t)
\]

(14)

The above formula is the heat flow vector, and
\[
\mathbf{Q}_{a\beta} = \frac{1}{2} m_a n_a \int d\mathbf{v} \mathbf{v}^2 f_a (\mathbf{r}, \mathbf{v}, t)
\]

(15)

It is the energy change caused by the collision between different kinds of charged particles. In high-temperature plasma systems, people are not too interested in the energy transport of charged particles in
the plasma. When studying the plasma equilibrium, stability and fluctuation process, it can be considered that the distribution of charged particles in the velocity space basically tends to the isotropic Maxwell distribution [5]. Therefore, the equation of state is usually used to determine the pressure of the plasma, which replaces the energy balance equation. For isothermal processes, there are:

\[ p_a = c_1 n_a \]  

(16)

Where \( c_1 \) is a constant. For adiabatic processes, the pressure is

\[ p_a = c_2 n_a^{5/3} \]  

(17)

Where \( c_2 \) is a constant. Thus, for the two-fluid plasma, the magnetohydrodynamic equation is:

\[
\frac{\partial n_a}{\partial t} + \nabla \cdot (n_a \mathbf{v}_a) = 0
\]

\[
m_a n_a \left( \frac{\partial}{\partial t} + \mathbf{v}_a \cdot \nabla \right) \mathbf{v}_a = -\nabla p_a + \rho_{qa} \mathbf{E} + \mathbf{j} \times \mathbf{B} + \mathbf{R}_{a\beta} 
\]

\[
R_{a\beta} = -m_a n_a v_{a\beta} (\mathbf{v}_a - \mathbf{v}_\beta) 
\]

\[
\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} 
\]

\[
\nabla \times \mathbf{B} = \mu_0 \mathbf{j} + \epsilon_0 \frac{\partial \mathbf{E}}{\partial t} 
\]

\[
\nabla \cdot \mathbf{E} = \frac{\rho_{qa}}{\epsilon_0} 
\]

\[
\nabla \cdot \mathbf{B} = 0 
\]

\[
\rho_{qa} = \sum_a q_a n_a 
\]

\[
\mathbf{j} = \sum_a q_a n_a \mathbf{v}_a 
\]

They are coupled with the state equation to form a closed set of equations. In the next few chapters, we will use this set of equations to study the wave process and stability in the plasma.

3. Single magnetohydrodynamic equation

In the previous chapter, we regarded plasma as a two-fluid composed of electronic fluid and ionic fluid. In fact, when studying certain phenomena in plasma, plasma can also be regarded as a single magnetic fluid [6]. Our task in this section is to give the magnetohydrodynamic equation of this single magnetic fluid.

3.1. Continuity equation

We put the mass continuity equation of the electronic component

\[
\frac{\partial \rho_{me}}{\partial t} + \nabla \cdot (\rho_{me} \mathbf{v}_e) = 0 
\]

(19)

Then the mass continuity equation of a single fluid is

\[
\frac{\partial \rho_m}{\partial t} + \nabla \cdot (\rho_m \mathbf{v}) = 0 
\]

(20)

3.2. Momentum balance equation

In order to obtain the momentum balance equation of a single fluid, we assume that the plasma is quasi-neutral, namely \( n_e \approx n_i = n \). According to the momentum balance equation of electrons and ions,
The momentum balance equation of a single fluid is

$$\rho \frac{du}{dt} = -\nabla P + j \times B$$

The following simplified assumptions are used: Since the mass of electrons is much smaller than that of ions, the inertia term of electrons and the neutrality condition of electrons are omitted: $n_e = n_i$, $P = P_i + P_e$, $j = e n_i u_i - e n_e u_e$, $u = u_i$, $R_{ei} = -R_{ie}$.

### 4. Magnetohydrodynamic balance and stability

For a magnetically confined plasma system, the first question that people care about is whether the forces on the system are balanced? If the force can reach equilibrium, the next question is whether this equilibrium state is stable, that is, after a small disturbance, is the deviation from the equilibrium state large? Below we use the language of mathematics to describe the problem of balance and stability.

Suppose $x_0$ is a certain physical quantity in the equilibrium state of the system, which satisfies the following equation:

$$f(x_0) = 0.$$ 

If a disturbance is applied to the system, the physical quantity $x_0$ becomes $x(t) = x_0 + x_i$ and satisfies the following equation of motion:

$$\frac{dx(t)}{dt} = f(x(t)).$$

We suppose that the perturbation is perturbation, then equation (24) can be linearized:

$$\frac{dx(t)}{dt} = f'(x_0) x_i.$$

We suppose that the disturbance $x_i$ can be expressed in the form of a mountain $x_i(t) \sim \exp(-i \omega t)$, from which the disturbance frequency can be obtained as:

$$\omega = if'(x_0).$$

If $\text{Im}(\omega) = f'(x_0) < 0$, it indicates that the disturbance will be quickly attenuated, and the corresponding equilibrium state of the system is stable. On the contrary, $\text{Im}(\omega) = f'(x_0) > 0$ indicates that the equilibrium state $f(x_0) = 0$ is unstable. We can also use Figure 1 below to illustrate the problem of balance and stability [7]. Figure 1a corresponds to a stable balance, because after the ball deviates from the equilibrium position, it can still return to its original state; while Figure 1b corresponds to an unstable balance. Once the ball is disturbed, it will gradually deviate from its equilibrium position.
The direction of the magnetic pressure points from the strong magnetic field to the weak magnetic field (see Figure 2). Like thermal pressure, it can cause the density of the magnetic field (that is, the strength of the magnetic field) to propagate outward, and the waves generated at this time are the same as sound waves. It is a longitudinal wave, called a (slow) magnetoacoustics wave. If the thermal pressure and the magnetic pressure simultaneously drive the disturbance of the magnetic fluid to propagate outward, a faster magnetoacoustics wave will be produced.

Figure 2. Magnetic pressure

The other term $\nabla \cdot (\frac{B^2}{\mu})$ is called magnetic tension, which can be further rewritten into a form with clearer physical meaning

$$\nabla \cdot \frac{B^2}{\mu} = \frac{1}{\mu} \left[ (\nabla \cdot \vec{B}) \vec{B} + \vec{B} \cdot \nabla \vec{B} \right] = \frac{1}{\mu} B(b \cdot \nabla) B b$$

$$= \frac{1}{\mu} \left( B(b \cdot \nabla) b + B^2 (b \cdot \nabla b) \right) = \frac{1}{\mu} \left[ \frac{1}{2} b b \cdot \nabla B^2 + B^2 \kappa \right]$$

(27)

Where $\vec{b} = \vec{B} / B$ is $\vec{B}$ the unit vector in the direction, $\kappa = b \cdot \nabla b$ is the curvature of a point on the magnetic force line, the absolute value of which is $\kappa = \left| \frac{d\vec{b}}{dl} \right|$, and the direction points to the centre of curvature of this point on the magnetic force line (see Figure 3). When the magnetic field line is a straight line, the curvature $\kappa$ is zero everywhere on it.

Figure 3. Curvature of magnetic field lines

5. Conclusion

For this system of magnetohydrodynamic equations, the Roe and HLLC formats are implemented to solve one-dimensional and two types of equations. For the magneto gas dynamics equations, according to the three conditions that the Roe matrix satisfies, a Roe matrix without any special requirements for
the adiabatic coefficient is constructed, and when it degenerates to Euler's equation, the new Roe average of Euler's equation is obtained. For the non-ideal isentropic compressible flow transverse magnetic field dynamic equations, two different methods are adopted to construct the Roe matrix. For the HLLC format, according to the idea of constructing the HLLC format of the gas dynamics equations and the characteristics of the two types of equations, the HLLC format of the two types of equations is constructed. The paper compares and analyses the numerical results and verifies the validity of the format.

Acknowledgements
Ma C.-T. Also acknowledges support from the Scientific Research Projects for Universities in Shandong Province, China (J18KB101)

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