Angular Distributions for Multi-body Semileptonic Charmed Baryon Decays

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We perform an analysis of angular distributions in semileptonic decays of charmed baryons $B_i^{(0)} \rightarrow B_j^{(*)}(-B_k^{(*)}B_{4}^{(0)})\ell^+\nu_{\ell}$, where the $B_i = (\Lambda^+_c, \Xi^0(0, +))$ are the SU(3)-antitriplet baryons and $B_j^+ = \Omega^-_c$ is an SU(3) sextet. We will firstly derive analytic expressions for angular distributions using helicity amplitude technique. Based on the lattice QCD results for $\Lambda^+_c \rightarrow \Lambda$ and $\Xi^0 \rightarrow \Xi^-$ form factors and model calculation of the $\Omega^+_c \rightarrow \Omega^-$ transition, we predict branching fractions: $B(\Lambda^+_c \rightarrow p\pi^-e^+\nu_e) = 2.48(15)\%$, $B(\Lambda^+_c \rightarrow p\pi^-\mu^+\nu_{\mu}) = 2.50(14)\%$, $B(\Xi_c \rightarrow \Lambda\pi^-e^+\nu_e) = 2.40(30)\%$, $B(\Xi_c \rightarrow \Lambda\pi^-\mu^+\nu_{\mu}) = 2.41(30)\%$, $B(\Omega_c \rightarrow \Lambda K^-e^+\nu_e) = 0.362(14)\%$, $B(\Omega_c \rightarrow \Lambda K^-\mu^+\nu_{\mu}) = 0.350(14)\%$. Besides, we also predict the $q^2$-dependence and angular distributions of these processes, in particular the coefficients for the $\cos n\theta_\ell \cos n\theta_b \cos n\phi$ ($n = 0, 1, 2, \cdots$) terms. This work can provide a theoretical basis for the ongoing experiments at BESIII, LHCb and BELLE-II.

I. INTRODUCTION

Weak decays of heavy mesons play an important role in testing the standard model (SM), and measuring the Cabibbo-Kobayashi-Maskawa (CKM) matrix elements that describe the quark mixing and also the strength of CP violation. Moreover, any significant deviation from SM predictions for heavy meson decays will provide clues for new physics beyond SM, and recent experimental analyses by Belle and LHCb collaborations [1–5] have revealed notable tensions between the SM predictions of such processes and data. From this viewpoint, the study of semi-leptonic decays of charmed baryons, which can provide an ideal way to determine the $|V_{cd}|$ and $|V_{cs}|$, and examine the CKM unitarity $\sum_{i=d,s,b} |V_{ci}|^2 = 1$, is of great value. In the singly-charmed baryons with $q_1q_2c$, the two light quarks can be decomposed as an antitriplet and a sextet. Focusing on the ground-states with $J^P = 1/2^+$, only four baryonic states, $(\Lambda^+_c, \Xi^0, \Xi^+)$ and $\Omega^0_c$ baryons, can have measurable weak decays, while others such like $\Xi'_c$ and $\Sigma'_c$ have strong and electromagnetic decay modes [6–10].

Among various decay modes, semileptonic decays are simplest [11–13], and in recent years charmed baryon decays have received great interests from both theoretical and experimental sides [14–25]. Semileptonic decays of $\Lambda^+_c$ have been fruitfully studied in quark model and QCD sum rules [26–40], and predictions for branching fractions differ substantially. A precise measurement of branching fractions of $\Lambda^+_c$ weak decays has recently been reported by BESIII collaboration: $B(\Lambda_c \rightarrow \Lambda e^+\nu_e) = 0.0363$ and $B(\Lambda_c \rightarrow \Lambda\mu^+\nu_{\mu}) = 0.0363$ [16, 18]. For $\Xi_c$, the CLEO collaboration has measured the ratio of branching fractions $B(\Xi^0_c \rightarrow \Xi^- e^+\nu_e)/B(\Xi^0_c \rightarrow \Xi^- \pi^+) = 1.72(10 \pm 12 \pm 50)\%$, $B(\Xi^0_c \rightarrow \Xi^- \mu^+\nu_{\mu}) = 1.71(17 \pm 13 \pm 50)\%$. On theoretical side, a variety of models have been developed to analyze $\Xi_c$ weak decays [44–48], including a recent analysis of $\Xi_c \rightarrow \Xi$ transition form factors from lattice QCD [49]. Limited by low production rate and high background levels of current experiments, measurements of $\Omega_c$ decay branching ratios are not available. In theory, branching fractions of $\Omega_c$ weak decays are predicted in light-front quark model: $B(\Omega^0_c \rightarrow \Omega^+ e^+\nu_e) = 5.4(\pm0.2) \times 10^{-3}$ [56]. In this work, we will make an exploration of decay widths for semileptonic decays of charmed baryons with the LQCD results for

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form factors, and in particular we for the first time derive the angular distributions for four-body weak decays of $\Omega_c$. Feynman diagrams for these decay chains are shown in Fig. 1.

The rest of this paper is organized as follows. In Sec. II, we give the theoretical framework for calculating the helicity amplitudes of charmed baryon decays, including the theoretical results of the Lorentz invariant leptonic and hadronic matrix elements. In Sec. III, we list the differential decay widths of the three-body, as well as four-body decay formulas. Integrating out the $q^2$, we obtain numerical results of partial decay width, as well as the illustration of momentum-transfer and angular distributions of the decay width. A brief summary will be presented in the last section.

**II. THEORETICAL FRAMEWORK**

**A. Formalism**

We will focus on semileptonic four-body decays of both SU(3) $\bar{3}$ and $6$ ground states of singly-charmed baryon, denoted as $B_1^{(i)} \rightarrow B_2^{(i)} (\rightarrow B_3^{(i)} B_4^{(i)}) \ell^+ \nu_\ell$, where $\ell = e, \mu$ and $\nu_\ell$ are the charged and neutral leptons. For the antitriplet baryons $B_1 = (\Lambda_c^+, \Xi_c^0, \Xi_c^+) \rightarrow \Lambda^- p \pi^-$, the intermediate states $B_2$ are spin 1/2 baryon with an SU(3) octet, while the sextet $B_3^{(i)} = \Omega_c^0 \rightarrow \Omega^- (\rightarrow \Lambda K^-)$. $B_3^{(i)}$ and $B_4^{(i)}$ are the baryonic and mesonic final states, respectively. Examples of specific processes include:

\begin{align}
\Lambda_c^+ &\rightarrow \Lambda(\rightarrow p\pi^-)\ell^+\nu_\ell, \\
\Xi_c^0 &\rightarrow \Xi^- (\rightarrow \Lambda\pi^-)\ell^+\nu_\ell, \\
\Omega_c^0 &\rightarrow \Omega^- (\rightarrow \Lambda K^-)\ell^+\nu_\ell.
\end{align}

The effective weak Hamiltonian for the semileptonic decays of charmed baryons can be written as

\begin{equation}
\mathcal{H}_{\text{eff}} = \frac{G_F}{\sqrt{2}} V_{cs} \left( \bar{s} \gamma^\mu (1 - \gamma_5) c \right) \left( \bar{\nu}_\ell \gamma_\mu (1 - \gamma_5) \ell \right),
\end{equation}

where $G_F$ is the Fermi constant, and $V_{cs}$ is the CKM matrix element. Based on the above effective Hamiltonian, we can obtain the decay amplitudes of $B_1^{(i)} \rightarrow B_2^{(i)} (\rightarrow B_3^{(i)} B_4^{(i)}) \ell^+ \nu_\ell$

\begin{equation}
\mathcal{M} = \frac{G_F}{\sqrt{2}} V_{cs} \left( B_3^{(i)} B_4^{(i)} \bar{s} \gamma^\mu (1 - \gamma_5) c \right) \left( B_1^{(i)} \ell^+ \nu_\ell \right) \left( \ell^+ \nu_\ell \bar{\nu}_\ell (1 - \gamma_5) \ell^+ |0\rangle.
\end{equation}
With the decomposition of $g_{\mu\nu}$

$$g_{\mu\nu} = -\sum_{\lambda} \epsilon_\mu^*(\lambda)\epsilon_\nu(\lambda) + \frac{q_\mu q_\nu}{q^2},$$

(6)

the above amplitude can be decomposed into the Lorentz invariant hadronic and leptonic matrix elements:

$$\mathcal{M} = \frac{G_F}{\sqrt{2}} V_{cs} \left\langle B_3^{(f)} B_4^{(f)} \right| \bar{s}\gamma^\mu (1 - \gamma_5) c \left| B_1^{(i)} \rightangle \left\langle \ell^+ \nu \ell | \bar{\nu}_\ell \gamma^\nu (1 - \gamma_5) \ell | 0 \right\rangle g_{\mu\nu}
$$

$$= \frac{G_F}{\sqrt{2}} V_{cs} \left( -\sum_{\lambda} H^\mu \epsilon_\mu^*(\lambda) \times L^\nu \epsilon_\nu(\lambda) + H^\mu \epsilon_\mu^*(t) \times L^\nu \epsilon_\nu(t) \right),$$

(7)

where the hadronic part $H^\mu = \left\langle B_3^{(f)} B_4^{(f)} \right| \bar{s}\gamma^\mu (1 - \gamma_5) c \left| B_1^{(i)} \right\rangle$ and leptonic part $L^\nu = \left\langle \ell^+ \nu \ell | \bar{\nu}_\ell \gamma^\nu (1 - \gamma_5) \ell | 0 \right\rangle$, $\epsilon$ is the polarization vector of decomposed $W^+$ and $t$, in which $\epsilon_\mu(t) \equiv g_\mu / \sqrt{q^2}$.

Focusing on the hadronic part, and inserting the 1-particle completeness states of the intermediate baryon

$$1 = \sum_{s_2} \int \frac{d^4 p_2}{(2\pi)^4} \frac{i}{p_2^2 - m_2^2 + im_2 \Gamma_2} \left\langle B_2^{(f)} (p_2, s_2) \right| \left\langle B_2^{(f)} (p_2, s_2) \right| B_2^{(i)} (p_2, s_2) \right| B_2^{(i)} (p_2, s_2) \right|,$$

(8)

we find the $H^\mu$ is given as:

$$H^\mu = \sum_{s_2} \int \frac{d^4 p_2}{(2\pi)^4} \frac{i}{p_2^2 - m_2^2 + im_2 \Gamma_2} \left\langle B_3^{(f)} (p_3, s_3) B_4^{(f)} (p_4) \right| B_2^{(f)} (p_2, s_2) \right| B_2^{(f)} (p_2, s_2) \right| \bar{s}\gamma^\mu (1 - \gamma_5) c \left| B_1^{(i)} (p_1, s_1) \right\rangle.$$

(9)

In the above $p_2$, $m_2$ and $\Gamma_2$ are the four-momentum, mass and decay rate of intermediate state $B_2^{(f)}$ respectively. The $B_1^{(f)} \rightarrow B_2^{(f)}$ transition $H_1$ can be parameterized as hadronic form factors, while the $B_2^{(f)} \rightarrow B_3^{(f)} B_4^{(f)}$ transition, with one final-state $B_1^{(f)}$ is pseudoscalar meson, can be reduced as

$$\left\langle B_3^{(f)} (p_3, s_3) B_4^{(f)} (p_4) \right| B_2^{(f)} (p_2, s_2) \right| B_2^{(f)} (p_2, s_2) \right| \bar{s}\gamma^\mu (1 - \gamma_5) c \left| B_1^{(i)} (p_1, s_1) \right\rangle = \left\langle B_3^{(f)} (p_3, s_3) B_4^{(f)} (p_4) \left| B_2^{(f)} (p_2, s_2) \right| B_2^{(f)} (p_2, s_2) \right| \bar{s}\gamma^\mu (1 - \gamma_5) c \left| B_1^{(i)} (p_1, s_1) \right\rangle - i \int d^4 x H_{\text{int}}(x) \left\langle B_2^{(f)} (p_2, s_2) \right| B_2^{(f)} (p_2, s_2) \right| \bar{s}\gamma^\mu (1 - \gamma_5) c \left| B_1^{(i)} (p_1, s_1) \right\rangle,$$

(10)

where the subscript “$H$” and “$I$” denote Heisenberg and interaction representation matrix element respectively, $g_{H_2}$ represents the coupling constant of hadronic vertex $B_2^{(f)} \rightarrow B_3^{(f)} B_4^{(f)}$, and factors $A$ and $B$ are constants weighting the contributions from scalar and pseudoscalar density operators [42].
Therefore, the decay amplitude in Eq. (5) can be expressed as a convolution of the Lorentz invariant leptonic part $L(s_\ell, s_\nu, s_W)$ and two hadronic parts $H_1(s_1, s_2, s_W), H_2(s_2, s_3)$:

\[
iM(B_1^{(t)} \rightarrow B_3^{(t)} B_4^{(t)} \ell^+ \nu_\ell) = \sum_{s_2} iM(B_1^{(t)} \rightarrow B_2^{(t)} \ell^+ \nu_\ell) \times iM(B_2^{(t)} \rightarrow B_3^{(t)} B_4^{(t)}) \times \frac{i}{p_2^2 - m_2^2 + im_2 \Gamma_2}
\]

\[
= - \sum_{s_2} \left( \frac{G_F}{\sqrt{2}} V_{cs} \sum_{s_W = \{\pm, 0, t\}} H_1(s_1, s_2, s_W) \times L(s_\ell, s_\nu, s_W) \times H_2(s_2, s_3) \right)
\]

\[
= - \frac{G_F V_{cs}}{\sqrt{2}} \frac{1}{p_2^2 - m_2^2 + im_2 \Gamma_2} \sum_{s_2} \sum_{s_W = \{\pm, 0, t\}} H_1(s_1, s_2, s_W) \times L(s_\ell, s_\nu, s_W) \times H_2(s_2, s_3),
\]

with

\[
L(s_\ell, s_\nu, s_W) \equiv \bar{u}_\nu(p_\nu, s_\nu) \gamma^\nu (1 - \gamma_5) u_\ell(p_\ell, s_\ell) \varepsilon_\nu(s_W),
\]

\[
H_1(s_1, s_2, s_W) \equiv \langle B_2^{(t)}(p_2, s_2) \mid s_\gamma^\mu (1 - \gamma_5) c \mid B_1^{(t)}(p_1, s_1) \rangle \varepsilon_\mu^*(s_W),
\]

\[
H_2(s_2, s_3) \equiv g_{H_2} \bar{u}_{B_3} (p_3, s_3) (A - B \gamma_5) u_{B_2} (p_2, s_2).
\]

Averaging the spin of initial-state and summing over the spins of final-states, we obtain the squared amplitude as

\[
\frac{1}{2} |iM(B_1^{(t)} \rightarrow B_3^{(t)} B_4^{(t)} \ell^+ \nu_\ell)|^2 = \frac{G_F^2 |V_{cs}|^2}{2} \frac{1}{(q_2^2 - m_2^2)^2 + m_2^2 \Gamma_2^2} \sum_{s_1, s_2, s_3} \sum_{s_W = \{\pm, 0, t\}} H_1(s_1, s_2, s_W) \times L(s_\ell, s_\nu, s_W) \times H_2(s_2, s_3)^2.
\]

**B. Kinematics**

With the abbreviations

\[
s_\pm = (m_1 \pm m_2)^2 - q^2, \quad \lambda(m_1, m_2, q) = s_+ s_-,
\]

the momentum of initial- and final-states in the subprocesses can be set as follows.

1. $B_1^{(t)} \rightarrow B_2^{(t)} W^+$ subprocess: in the rest frame of initial-state $B_1^{(t)}$, suppose $B_2^{(t)}$ moves along the positive $z$-direction ($\theta = 0, \phi = 0$) and $W^+$ boson along the negative $z$-direction ($\theta = \pi, \phi = \pi$), so we have

\[
p_1^\mu = (m_1, 0, 0, 0), \quad p_2^\mu = (E_2, 0, 0, |\vec{p}_2|), \quad q^\mu = (E_W, 0, 0, -|\vec{p}_2|),
\]

with

\[
E_1 = \frac{m_1^2 + m_2^2 - q^2}{2m_1}, \quad E_W = \frac{m_1^2 - m_2^2 + q^2}{2m_1}, \quad |\vec{p}_2| = \frac{\sqrt{\lambda(m_1, m_2, q)}}{2m_1}.
\]

2. $B_2^{(t)} \rightarrow B_3^{(t)} B_4^{(t)}$ subprocess: in the rest frame of $B_2^{(t)}$, suppose the final-state baryon $B_3^{(t)}$ moves along $(\theta = \theta_h, \phi = 0)$ direction, thereby the meson $B_4^{(t)}$ moves along $(\theta = \pi - \theta_h, \phi = \pi)$ direction, we have

\[
p_2^\mu = (m_2, 0, 0, 0), \quad p_3^\mu = (E_3, |\vec{p}_3| \sin \theta_h, 0, |\vec{p}_3| \cos \theta_h), \quad p_4^\mu = (E_4, -|\vec{p}_3| \sin \theta_h, 0, -|\vec{p}_3| \cos \theta_h),
\]

with

\[
E_3 = \frac{m_2^2 + m_3^2 - m_4^2}{2m_2}, \quad E_4 = \frac{m_2^2 - m_3^2 + m_4^2}{2m_2}, \quad |\vec{p}_3| = \frac{\sqrt{\lambda(m_2, m_3, m_4)}}{2m_2}.
\]
3. $W^+ \rightarrow \ell^+ \nu_\ell$ subprocess: in the rest frame of $W^+$, suppose the charged lepton moves along $(\theta = \theta_\ell, \phi = \phi)$ direction, thereby the neutrino along $(\theta = \pi - \theta_\ell, \phi = \pi + \phi)$ direction, we have

$$q' = \left( \sqrt{q'^2}, 0, 0, 0 \right), \quad p'_i = (E_i, |\vec{p}_i| \sin \theta_i \cos \phi_i, |\vec{p}_i| \sin \theta_i \sin \phi_i, |\vec{p}_i| \cos \theta_i),$$

$$p'_\ell = (E_\ell, -|\vec{p}_\ell| \sin \theta_\ell \cos \phi_\ell, -|\vec{p}_\ell| \sin \theta_\ell \sin \phi_\ell, -|\vec{p}_\ell| \cos \theta_\ell),$$

with

$$E_i = \frac{q^2 + m_i^2}{2\sqrt{q^2}}, \quad E_\ell = \frac{q^2 - m_\ell^2}{2\sqrt{q^2}}, \quad |\vec{p}_i| = \frac{q^2 - m_i^2}{2\sqrt{q^2}}. \quad (21)$$

For the phase space of $n$-body decays, the four-body one can be generated by the two-body sub-processes recursively. The two-body phase space can be expressed as

$$d\Phi_2(p \rightarrow p_1 p_2) = \frac{1}{(2\pi)^3} \frac{|\vec{p}_1| d\cos \theta}{4\sqrt{s}}, \quad (23)$$

where $\sqrt{s} = \sqrt{p^2}$ is the center-of-mass energy, $\theta$ is the angle between two final-states. Based on this, the three-body phase space of $B_1^{(s)} \rightarrow B_2^{(s)} \ell^+ \nu_\ell$ can be written as

$$d\Phi_3 \left( B_1^{(s)} \rightarrow B_2^{(s)} B_3^{(s)} \ell^+ \nu_\ell \right) = (2\pi)^3 dq_2^2 \times d\Phi_2(W^+ \rightarrow \ell^+ \nu_\ell) \times d\Phi_2 \left( B_1^{(s)} \rightarrow B_2^{(s)} W^+ \right)$$

$$= \frac{(1 - \hat{m}_\ell^2)\sqrt{\lambda(m_1, m_2, q)} d\Phi_2}{(2\pi)^3 32m^3_1} dq^2 d\cos \theta_1, \quad (24)$$

where $\hat{m}_\ell = m_\ell / \sqrt{s}$. Similarly, the total four-body phase space is

$$d\Phi_4 \left( B_1^{(s)} \rightarrow B_3^{(s)} B_4^{(s)} \ell^+ \nu_\ell \right) = (2\pi)^3 dp_2^2 \times d\Phi_3 \left( B_1^{(s)} \rightarrow B_2^{(s)} \ell^+ \nu_\ell \right) \times d\Phi_2 \left( B_2^{(s)} \rightarrow B_3^{(s)} B_4^{(s)} \right)$$

$$= \frac{(1 - \hat{m}_\ell^2)\sqrt{\lambda(m_1, m_2, q)\lambda(m_2, m_3, m_4)} dp_2^2}{(2\pi)^3 256m^5_2 m^3_1} dq^2 dp_2^2 d\cos \theta_1 d\cos \theta_1 d\phi. \quad (25)$$

Note that the integration variable $p_2^2$ is artificially introduced from the insertion of intermediate state $B_2^{(s)}$, and in the narrow-width limit, this integration will be conducted as

$$\int dq_2^2 \frac{m_2 \Gamma_2}{\pi} \frac{1}{(q_2^2 - m_2^2)^2 + m_2^4 \Gamma_2^2} = 1, \quad (26)$$

while

$$\int dq_2^2 \frac{m_2 \Gamma \left( B_2^{(s)} \rightarrow B_3^{(s)} B_4^{(s)} \right)}{\pi} \frac{1}{(q_2^2 - m_2^2)^2 + m_2^4 \Gamma_2^2} = B \left( B_2^{(s)} \rightarrow B_3^{(s)} B_4^{(s)} \right), \quad (27)$$

where $\Gamma \left( B_2^{(s)} \rightarrow B_3^{(s)} B_4^{(s)} \right)$ and $B \left( B_2^{(s)} \rightarrow B_3^{(s)} B_4^{(s)} \right)$ are total width and branching fraction of the subprocess $B_2^{(s)} \rightarrow B_3^{(s)} B_4^{(s)}$ respectively.

Combining the four-body phase space in Eq.(25) and squared amplitude in Eq.(15), we can write down the differential decay width of four-body process $B_1^{(s)} \rightarrow B_3^{(s)} B_4^{(s)} \ell^+ \nu_\ell$:

$$\frac{d\Gamma(B_1^{(s)} \rightarrow B_3^{(s)} B_4^{(s)} \ell^+ \nu_\ell)}{dq^2 d\cos \theta_\ell d\cos \theta_\ell d\phi} = G_F^3 |V_{cs}|^2 \frac{(1 - \hat{m}_\ell^2)\sqrt{\lambda(m_1, m_2, q)\lambda(m_2, m_3, m_4)}}{(2\pi)^3 4096m^5_2 m^3_1} \frac{B \left( B_2^{(s)} \rightarrow B_3^{(s)} B_4^{(s)} \right)}{\Gamma \left( B_2^{(s)} \rightarrow B_3^{(s)} B_4^{(s)} \right)}$$

$$\times \sum_{s_1} \sum_{s_3} \sum_{s_1} \sum_{s_3} H_1(s_1, s_2, s_W) \times L(s_\ell, s_\nu, s_W) \times H_2(s_2, s_3) \bigg| ^2. \quad (28)$$
C. Leptonic part

In this section, we will focus on the Lorentz invariant leptonic part defined in Eq.(12):

\[ L(s_\ell, s_\nu, s_W) \equiv \bar{u}_\nu(p_\nu, s_\nu) \gamma^\nu (1 - \gamma_5) v_\ell(p_\ell, s_\ell) \epsilon_\nu(s_W). \]  

(29)

Combining each components of spinors of leptons and polarization vectors of \( W^+ \) boson, we can obtain the following non-vanishing matrix elements:

\[
L\left( s_\ell = +\frac{1}{2}, s_\nu = -\frac{1}{2}, s_W = 0 \right) = -\sqrt{2} \sin \theta_\ell N_\ell, \\
L\left( s_\ell = +\frac{1}{2}, s_\nu = -\frac{1}{2}, s_W = +1 \right) = -e^{-i\phi} (1 - \cos \theta_\ell) N_\ell, \\
L\left( s_\ell = +\frac{1}{2}, s_\nu = -\frac{1}{2}, s_W = -1 \right) = -e^{i\phi} (1 + \cos \theta_\ell) N_\ell, \\
L\left( s_\ell = -\frac{1}{2}, s_\nu = -\frac{1}{2}, s_W = 0 \right) = -\sqrt{2} \tilde{m}_\ell \cos \theta_\ell N_\ell, \\
L\left( s_\ell = -\frac{1}{2}, s_\nu = -\frac{1}{2}, s_W = +1 \right) = -\tilde{m}_\ell e^{-i\phi} \sin \theta_\ell N_\ell, \\
L\left( s_\ell = -\frac{1}{2}, s_\nu = -\frac{1}{2}, s_W = -1 \right) = \tilde{m}_\ell e^{i\phi} \sin \theta_\ell N_\ell, \\
L\left( s_\ell = -\frac{1}{2}, s_\nu = -\frac{1}{2}, s_W = t \right) = \sqrt{2} \tilde{m}_\ell N_\ell, 
\]

(30)-(36)

where the factor \( N_\ell = i \sqrt{2 (q^2 - m_f^2)} \).

D. Hadronic part \( H_1 \) with spin-1/2 intermediate state

If \( B_1 \) is a singly-charmed baryon antitriplet, the transition matrix elements with weak current \( J^\mu = V^\mu - A^\mu = \bar{s} \gamma^\mu (1 - \gamma_5) c \) in Eq.(13) can be parameterized as [37, 51]

\[
\langle B_2 (p_2, s_2) | V^\mu | B_1 (p_1, s_1) \rangle = \bar{u}_2(p_2, s_2) \left[ \gamma^\mu f_1(q^2) + i \sigma^{\mu\nu} \frac{q_\nu}{m_1} f_2(q^2) + \frac{q^\mu}{m_1} f_3(q^2) \right] u_1(p_1, s_1), \\
\langle B_2 (p_2, s_2) | A^\mu | B_1 (p_1, s_1) \rangle = \bar{u}_2(p_2, s_2) \left[ \gamma^\mu g_1(q^2) + i \sigma^{\mu\nu} \frac{q_\nu}{m_1} g_2(q^2) + \frac{q^\mu}{m_1} g_3(q^2) \right] \gamma_5 u_1(p_1, s_1),
\]

(37)-(38)

with the momentum transfer \( q^\mu = p_1^\mu - p_2^\mu \) and \( \sigma^{\mu\nu} = i[\gamma^\mu, \gamma^\nu]/2 \). The form factors \( f_i \) and \( g_i \) are functions of \( q^2 \), and the relations between these form factors and other parametrizations are collected in appendix.

By combining each spin components of \( B_1, B_2 \) and \( W^+ \), we give the non-zero terms of \( H_1(s_1, s_2, s_W) \) as

\[
H_{1V}\left( s_1 = -\frac{1}{2}, s_2 = \frac{1}{2}, s_W = 1 \right) = H_{1V}\left( s_1 = \frac{1}{2}, s_2 = -\frac{1}{2}, s_W = -1 \right) = \sqrt{2s-} \left[ f_1(q^2) - \frac{m_1 + m_2}{m_1} f_2(q^2) \right], \\
H_{1V}\left( s_1 = \frac{1}{2}, s_2 = -\frac{1}{2}, s_W = 0 \right) = H_{1V}\left( s_1 = -\frac{1}{2}, s_2 = \frac{1}{2}, s_W = 0 \right) = \sqrt{s-} \left[ (m_1 + m_2) f_1(q^2) - \frac{q^2}{m_1} f_2(q^2) \right], \\
H_{1V}\left( s_1 = -\frac{1}{2}, s_2 = -\frac{1}{2}, s_W = t \right) = H_{1V}\left( s_1 = \frac{1}{2}, s_2 = -\frac{1}{2}, s_W = t \right) = \sqrt{s-t} \left[ (m_1 + m_2) f_1(q^2) - \frac{q^2}{m_1} f_2(q^2) \right].
\]

(39)-(40)
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\[ H_{1A} \left( s_1 = -\frac{1}{2}, s_2 = \frac{1}{2}, s_W = 1 \right) = -H_{1A} \left( s_1 = \frac{1}{2}, s_2 = -\frac{1}{2}, s_W = -1 \right) \]
\[ = \sqrt{2s_+} \left[ g_1(q^2) + \frac{m_1 - m_2}{m_1} g_2(q^2) \right], \quad (42) \]
\[ H_{1A} \left( s_1 = \frac{1}{2}, s_2 = 1, s_W = 0 \right) = -H_{1A} \left( s_1 = -\frac{1}{2}, s_2 = -\frac{1}{2}, s_W = 0 \right) \]
\[ = \sqrt{s_+} \left[ (m_1 - m_2) g_1(q^2) + \frac{q^2}{m_1} g_2(q^2) \right], \quad (43) \]
\[ H_{1A} \left( s_1 = \frac{1}{2}, s_2 = \frac{1}{2}, s_W = t \right) = -H_{1A} \left( s_1 = -\frac{1}{2}, s_2 = -\frac{1}{2}, s_W = t \right) \]
\[ = \sqrt{s_-} \left[ (m_1 + m_2) g_1(q^2) - \frac{q^2}{m_1} g_3(q^2) \right]. \quad (44) \]

Reversing the helicities give the same results for the vector current, but an opposite sign exists for the axial-vector current. The total amplitudes are

\[ H_V(s_1, s_2, s_W) = H_{1V}(s_1, s_2, s_W) - H_{1A}(s_1, s_2, s_W). \quad (45) \]

E. Hadronic part \( H_1 \) with spin-3/2 intermediate state

The charmed baryons \( B'_1 = \Omega_c^0 \) will weakly decay into spin-3/2 baryon decuplets \( \Omega^- \), and the transition matrix elements for this process can be parametrized as

\[
\left< B'_2(p_2, s_2) | V^\mu | B'_1(p_1, s_1) \right> = \bar{U}_c(p_2, s_2) \left[ \gamma^\mu p_1^\alpha \frac{f_1(q^2)}{m_1} + \frac{f_2(q^2)}{m_1} p_1^\alpha p_2^\mu \right.
\]
\[ + \frac{f_3(q^2)}{m_1 m_2} p_1^\alpha p_2^\mu + f_4(q^2) g^{\alpha\mu} \left] \gamma^5 u(p_1, s_1), \quad (46) \right.
\]
\[
\left< B'_2(p_2, s_2) | A^\mu | B'_1(p_1, s_1) \right> = \bar{U}_c(p_2, s_2) \left[ \gamma^\mu p_1^\alpha \frac{g_1(q^2)}{m_1} + \frac{g_2(q^2)}{m_1} p_1^\alpha p_2^\mu \right.
\]
\[ + \frac{g_3(q^2)}{s_2 m_1 m_2} p_1^\alpha p_2^\mu + g_4(q^2) g^{\alpha\mu} \left] u(p_1, s_1). \quad (47) \right.
\]

The definition of vectorial spinor \( U_c(p, S_z) \) for spin-3/2 baryon is shown in appendix. Therefore, the non-zero terms of \( H_V(s_1, s_2, s_W) \) are collected as

\[
H_{1V} \left( s_1 = \frac{1}{2}, s_2 = \frac{3}{2}, s_W = 1 \right) = -H_{1V} \left( s_1 = -\frac{1}{2}, s_2 = -\frac{3}{2}, s_W = -1 \right)
\]
\[ = \sqrt{s_-} f_4(q^2), \quad (48) \]
\[
H_{1V} \left( s_1 = -\frac{1}{2}, s_2 = \frac{1}{2}, s_W = 1 \right) = -H_{1V} \left( s_1 = \frac{1}{2}, s_2 = -\frac{1}{2}, s_W = -1 \right)
\]
\[ = \sqrt{s_-} \left( \frac{s_+}{3} \frac{s_+}{m_1 m_2} f_1(q^2) - f_4(q^2) \right), \quad (49) \]
\[
H_{1V} \left( s_1 = \frac{1}{2}, s_2 = 1, s_W = 0 \right) = -H_{1V} \left( s_1 = -\frac{1}{2}, s_2 = -\frac{1}{2}, s_W = 0 \right)
\]
\[ = \sqrt{s_-} \left( \frac{(m_1 - m_2) s_+}{m_1 m_2} f_1(q^2) \right).
\]
\[-\lambda \frac{m_1 m_2 q}{2m_1 m_2} \left( \frac{1}{m_1} f_2 (q^2) + \frac{1}{m_2} f_3 (q^2) \right) \]

\[-m_1^2 - m_2^2 - q^2 f_4 (q^2) \],

\[H_{1V} \left( s_1 = \frac{1}{2}, s_2 = \frac{1}{2}, s_W = t \right) = -H_{1V} \left( s_1 = -\frac{1}{2}, s_2 = -\frac{1}{2}, s_W = t \right) \]

\[= \sqrt{\frac{s_+ - s_-}{2m_1 m_2}} \left[ \frac{m_1 + m_2}{m_1} f_1 (q^2) \right. \]

\[\left. - \frac{m_1^2 - m_2^2 + q^2}{2m_1^2} f_2 (q^2) \right. \]

\[\left. - \frac{m_1^2 - m_2^2 - q^2}{2m_1 m_2} f_3 (q^2) - f_4 (q^2) \right]. \tag{50} \]

\[H_{1A} \left( s_1 = \frac{1}{2}, s_2 = \frac{3}{2}, s_W = 1 \right) = H_{1A} \left( s_1 = -\frac{1}{2}, s_2 = -\frac{3}{2}, s_W = -1 \right) \]

\[= -\sqrt{s_+} g_4 (q^2), \tag{52} \]

\[H_{1A} \left( s_1 = -\frac{1}{2}, s_2 = \frac{1}{2}, s_W = 1 \right) = H_{1A} \left( s_1 = \frac{1}{2}, s_2 = -\frac{1}{2}, s_W = -1 \right) \]

\[= \sqrt{\frac{s_+}{3}} \left( \frac{s_+}{m_1 m_2} g_1 (q^2) - g_4 (q^2) \right), \tag{53} \]

\[H_{1A} \left( s_1 = \frac{1}{2}, s_2 = \frac{1}{2}, s_W = 0 \right) = H_{1A} \left( s_1 = -\frac{1}{2}, s_2 = -\frac{1}{2}, s_W = 0 \right) \]

\[= \sqrt{\frac{s_+}{6q^2}} \left[ (m_1 + m_2) s_- g_1 (q^2) \right. \]

\[\left. + \frac{\lambda (m_1, m_2, q)}{2m_1 m_2} \left( \frac{1}{m_1} g_2 (q^2) + \frac{1}{m_2} g_3 (q^2) \right) \right. \]

\[\left. + \frac{m_1^2 - m_2^2 - q^2}{m_2} g_4 (q^2) \right], \tag{54} \]

\[H_{1A} \left( s_1 = \frac{1}{2}, s_2 = \frac{1}{2}, s_W = t \right) = H_{1A} \left( s_1 = -\frac{1}{2}, s_2 = -\frac{1}{2}, s_W = t \right) \]

\[= \sqrt{\frac{s_+ - s_-}{2m_1 m_2}} \left[ \frac{m_1 - m_2}{m_1} g_1 (q^2) \right. \]

\[\left. + \frac{m_1^2 - m_2^2 + q^2}{2m_1^2} g_2 (q^2) \right. \]

\[\left. + \frac{m_1^2 - m_2^2 - q^2}{2m_1 m_2} g_3 (q^2) + g_4 (q^2) \right], \tag{55} \]

and the total transition amplitudes give

\[H_1 (s_1, s_2, s_W) = H_{1A} (s_1, s_2, s_W) - H_{1V} (s_1, s_2, s_W). \tag{56} \]

\[F. \text{ Hadronic part } H_2 \text{ with spin-1/2 intermediate state} \]

The amplitude for a spin-1/2 charm baryon \( B_2 \) decaying into a spin-1/2 baryon \( B_3 \) and a spin-0 meson \( B_4 \) can be written in the form

\[iM = iG_F m_4^2 \bar{u}_{B_3} (p_1, s_3) (A - B \gamma_5) u_{B_4} (p_2, s_2), \tag{57} \]
which corresponds to the hadronic coupling constant $g_H = G_F m_2^2$ in Eq.(14). $A$ and $B$ are factors weighting the contributions from scalar and pseudoscalar operators [54]. It is convenient to introduce the asymmetry parameter

$$\alpha = \frac{2 \text{Re} |s^* r|}{|s|^2 + |r|^2},$$

with $s = A$ and $r = B \times |p_3|/(E_3 + m_3)$, and follow $(s \pm r)^2 = (|s|^2 + |r|^2)(1 \pm \alpha)$.

Therefore, combining each spin components of $B_2$ and $B_3$, we get the following hadronic matrix elements $H_2(s_2, s_3)$:

$$H_2 \left( s_2 = \frac{1}{2}, s_3 = \frac{1}{2} \right) = N_h (s + r) \cos \left( \frac{\theta_h}{2} \right),$$

$$H_2 \left( s_2 = \frac{1}{2}, s_3 = -\frac{1}{2} \right) = -N_h (s - r) \sin \left( \frac{\theta_h}{2} \right),$$

$$H_2 \left( s_2 = -\frac{1}{2}, s_3 = \frac{1}{2} \right) = N_h (s + r) \sin \left( \frac{\theta_h}{2} \right),$$

$$H_2 \left( s_2 = -\frac{1}{2}, s_3 = -\frac{1}{2} \right) = N_h (s - r) \cos \left( \frac{\theta_h}{2} \right),$$

with $N_h = G_F m_2^2 \sqrt{2 m_2 (E_3 + m_3)}$. Using the two-body phase space and squared matrix elements, we obtain the spin-averaged decay width

$$\Gamma (B_2 \rightarrow B_3 B_4) = \frac{1}{4 m_2} \frac{1}{(2\pi)^3} \frac{|p_3| d \cos \theta_h}{4 m_2} \frac{1}{2} |\mathcal{M} (B_2 \rightarrow B_3 B_4)|^2$$

$$= \frac{N_h^2 \sqrt{\lambda (m_2, m_3, m_4)}}{16 \pi m_2^2} (|s|^2 + |r|^2).$$

### G. Hadronic part $H_2$ with spin-3/2 intermediate state

The transition for a spin-3/2 baryon $\Omega^-$ decaying into a spin-1/2 baryon $\Lambda$ and a spin-0 pseudoscalar meson $K^-$ is parametrized as

$$i \mathcal{M} = i \frac{g_h}{m_4} \bar{u}_{B_2^*}^c (p_3, s_3) (A - B \gamma_5) U_{B_2^*}^a (p_2, s_2) p_{4a}.$$

Using the spinors and vectorial spinors, we can obtain the following terms:

$$H_2 \left( s_2 = \frac{3}{2}, s_3 = \frac{1}{2} \right) = N_h' |p_3| (s + r) \sin \theta_h \cos \frac{\theta_h}{2},$$

$$H_2 \left( s_2 = \frac{3}{2}, s_3 = -\frac{1}{2} \right) = -N_h' |p_3| (s - r) \sin \theta_h \sin \frac{\theta_h}{2},$$

$$H_2 \left( S_{12} = -\frac{3}{2}, s_3 = \frac{1}{2} \right) = -N_h' |p_3| (s + r) \sin \theta_h \cos \frac{\theta_h}{2},$$

$$H_2 \left( s_2 = -\frac{3}{2}, s_3 = -\frac{1}{2} \right) = -N_h' |p_3| (s - r) \sin \theta_h \cos \frac{\theta_h}{2},$$

$$H_2 \left( s_2 = \frac{1}{2}, s_3 = \frac{1}{2} \right) = \frac{\sqrt{3}}{3} N_h' |p_3| (s + r) \cos \frac{\theta_h}{2} (1 - 3 \cos \theta_h),$$

$$H_2 \left( s_2 = \frac{1}{2}, s_3 = -\frac{1}{2} \right) = \frac{\sqrt{3}}{3} N_h' |p_3| (s - r) \sin \frac{\theta_h}{2} (1 + 3 \cos \theta_h),$$

$$H_2 \left( s_2 = -\frac{1}{2}, s_3 = \frac{1}{2} \right) = -\frac{\sqrt{3}}{3} N_h' |p_3| (s + r) \sin \frac{\theta_h}{2} (1 + 3 \cos \theta_h),$$

$$H_2 \left( s_2 = -\frac{1}{2}, s_3 = -\frac{1}{2} \right) = \frac{\sqrt{3}}{3} N_h' |p_3| (s - r) \cos \frac{\theta_h}{2} (1 - 3 \cos \theta_h).$$
where \( N'_h = g_h \sqrt{(E_3 + m_3)m_2/m_4} \). The spin-averaged decay width of process is given as

\[
\Gamma (B'_2 \to B'_3 B'_4) = \frac{1}{12\pi m^2_2} \left| \frac{\alpha}{m^2_1} \frac{1}{4} |M (B'_2 \to B'_3 B'_4)|^2 \right|.
\]

III. THEORETICAL RESULTS OF DIFFERENTIAL DECAY WIDTH

A. Differential decay width of \((\Lambda^+_c, \Xi^0_c)\)

Combining the leptonic matrix elements in Eq.(30)-(36) and hadronic ones \( H \) in Eq.(39)-(45), together with the three-body phase space in Eq.(24), we can obtain the three-body decay width of \( B_1 \to B_2 \ell^+ \nu_\ell \) processes, which \( B_1 = (\Lambda^+_c, \Xi^0_c) \) and corresponding \( B_2 = (\Lambda, \Xi^-) \):

\[
\frac{d\Gamma (B_1 \to B_2 \ell^+ \nu_\ell)}{d\cos \theta_\ell dq^2} = \frac{G_F^2 |V_{cs}|^2 q^2 (1 - m_\ell^2)}{128\pi^4 m^2_1} \left( 1 - m_\ell^2 \right)^2 \frac{G_F^2}{2} |V_{cs}|^2 \left[ \begin{array}{c}
\left| H_{\frac{1}{2},1} \right|^2 \left[ (1 - \cos \theta_\ell)^2 + m_\ell^2 \sin^2 \theta_\ell \right]
+ \left| H_{-\frac{1}{2},-1} \right|^2 \left[ (1 + \cos \theta_\ell)^2 + m_\ell^2 \sin^2 \theta_\ell \right]
+ \left( \left| H_{\frac{1}{2},0} \right|^2 + \left| H_{-\frac{1}{2},0} \right|^2 \right) 2 \left( \sin^2 \theta_\ell + m_\ell^2 \cos^2 \theta_\ell \right)
+ \left( \left| H_{\frac{1}{2},-1} \right|^2 + \left| H_{-\frac{1}{2},1} \right|^2 \right) 2m_\ell^2
+ 2 \left( \left| H_{\frac{1}{2},0} H^*_{\frac{1}{2},1} \right| + \left| H_{-\frac{1}{2},0} H^*_{-\frac{1}{2},1} \right| \right) 2m_\ell^2 \cos \theta_\ell \end{array} \right].
\]

Besides, taking the hadronic matrix elements \( H_2 \) from Eq.(59)-(62) into account, we can obtain the four-body differential decay width for \( B_1 \to B_2 B_3 \ell^+ \nu_\ell \) defined in Eq.(28):

\[
\frac{d\Gamma (B_1 \to B_2 B_3 \ell^+ \nu_\ell)}{d\cos \theta_\ell d\cos \theta_\ell d\phi d\theta d\phi dq^2} = \frac{G_F^2 |V_{cs}|^2 q^2 (1 - m_\ell^2)}{(2\pi)^4 256 m^2_1} \left[ \begin{array}{c}
\left| H_{\frac{1}{2},1} \right|^2 \left. (1 + \alpha \cos \theta_\ell) \left[ \tilde{m}_\ell^2 \sin^2 \theta_\ell + (1 - \cos \theta_\ell)^2 \right] \right.
+ \left| H_{-\frac{1}{2},-1} \right|^2 \left. (1 - \alpha \cos \theta_\ell) \left[ \tilde{m}_\ell^2 \sin^2 \theta_\ell + (1 + \cos \theta_\ell)^2 \right] \right.
+ 2 \left. \left| H_{\frac{1}{2},0} \right|^2 \left. (1 + \alpha \cos \theta_\ell) \left[ \tilde{m}_\ell^2 \cos^2 \theta_\ell + \sin^2 \theta_\ell \right] \right.
+ 2 \left. \left| H_{-\frac{1}{2},0} \right|^2 \left. (1 - \alpha \cos \theta_\ell) \left[ \tilde{m}_\ell^2 \cos^2 \theta_\ell + \sin^2 \theta_\ell \right] \right.
+ 2 \left. \left| H_{\frac{1}{2},-1} \right|^2 \left. (1 + \alpha \cos \theta_\ell) \tilde{m}_\ell^2 \right.
+ 2 \left. \left| H_{-\frac{1}{2},1} \right|^2 \left. (1 - \alpha \cos \theta_\ell) \tilde{m}_\ell^2 \right.
+ 2 \sqrt{2} \alpha \sin \theta_\ell \sin \theta_\ell \cos \phi \left. \left| H_{\frac{1}{2},1} H^*_{-\frac{1}{2},0} \right| \left( 1 - \cos \theta_\ell + \tilde{m}_\ell^2 \cos \theta_\ell \right) \right.
+ 2 \sqrt{2} \alpha \sin \theta_\ell \sin \theta_\ell \cos \phi \left. \left| H_{-\frac{1}{2},-1} H^*_{\frac{1}{2},0} \right| \left( 1 + \cos \theta_\ell - \tilde{m}_\ell^2 \cos \theta_\ell \right) \right.
+ 2 \sqrt{2} \alpha \sin \theta_\ell \sin \theta_\ell \cos \phi \left. \left| H_{\frac{1}{2},1} H^*_{-\frac{1}{2},-1} \right| \tilde{m}_\ell^2 \right.
- 2 \sqrt{2} \alpha \sin \theta_\ell \sin \theta_\ell \cos \phi \left. \left| H_{-\frac{1}{2},-1} H^*_{\frac{1}{2},-1} \right| \tilde{m}_\ell^2 \right.
\end{array} \right].
\]
+ 4\tilde{m}_{\ell}^2 \cos \theta_{\ell} \left( H_{-\frac{1}{2},0} H^*_{\frac{1}{2},t} \right) \left( 1 - \alpha \cos \theta_{h} \right) \\
+ 4\tilde{m}_{\ell}^2 \cos \theta_{\ell} \left( H_{\frac{1}{2},0} H^*_{\frac{1}{2},t} \right) \left( 1 + \alpha \cos \theta_{h} \right).
\]  

(75)

By integrating out the parameters step by step, we can get the results of $q^2$, $\theta_{\ell}$-, $\theta_{h}$- and $\phi$-dependence form of total decay width respectively:

- $q^2$-dependence:

\[
d\Gamma \over dq^2 = \frac{\pi G_F^2 |V_{cs}|^2 q^2 \left( 1 - \hat{m}_t^2 \right)}{(2\pi)^4 48 m_m^3} \lambda(m_1, m_2, q) B(B_2 \to B_3 B_4) \\
\times \left[ (2 + \hat{m}_t^2) \left( |H_{-\frac{1}{2},-1}|^2 + |H_{\frac{1}{2},1}|^2 + |H_{-\frac{1}{2},0}|^2 + |H_{\frac{1}{2},0}|^2 \right) + 3\tilde{m}_{\ell}^2 \left( |H_{-\frac{1}{2},t}|^2 + |H_{\frac{1}{2},t}|^2 \right) \right],
\]

(76)

- $\theta_{\ell}$-dependence:

\[
d\Gamma \over d\cos \theta_{\ell} = \int_{q_{\min}^2}^{q_{\max}^2} dq^2 \frac{G_F^2 |V_{cs}|^2 q^2 \left( 1 - \hat{m}_t^2 \right)}{(2\pi)^4 256 m_m^3} \lambda(m_1, m_2, q) B(B_2 \to B_3 B_4) (A_{\ell} + B_{\ell} \cos \theta_{\ell} + C_{\ell} \cos \theta_{\ell}),
\]

with

\[
A_{\ell} = 2\pi \left[ (3 + \hat{m}_t^2) \left( |H_{-\frac{1}{2},-1}|^2 + |H_{\frac{1}{2},1}|^2 \right) + 2 \left( 1 + \hat{m}_t^2 \right) \left( |H_{-\frac{1}{2},0}|^2 + |H_{\frac{1}{2},0}|^2 \right) \\
+ 4\tilde{m}_{\ell}^2 \left( |H_{-\frac{1}{2},t}|^2 + |H_{\frac{1}{2},t}|^2 \right) \right],
\]

(78)

\[
B_{\ell} = 8\pi \left( |H_{-\frac{1}{2},-1}|^2 - |H_{\frac{1}{2},1}|^2 \right) + 2\tilde{m}_{\ell}^2 \left( |H_{-\frac{1}{2},0} H^*_{\frac{1}{2},t}| + |H_{\frac{1}{2},0} H^*_{\frac{1}{2},t}| \right),
\]

(79)

\[
C_{\ell} = 2\pi \left( 1 - \hat{m}_t^2 \right) \left( |H_{-\frac{1}{2},-1}|^2 + |H_{\frac{1}{2},1}|^2 \right) - 2 \left( |H_{-\frac{1}{2},0}|^2 + |H_{\frac{1}{2},0}|^2 \right),
\]

(80)

- $\theta_{h}$-dependence:

\[
d\Gamma \over d\cos \theta_{h} = \int_{q_{\min}^2}^{q_{\max}^2} dq^2 \frac{G_F^2 |V_{cs}|^2 q^2 \left( 1 - \hat{m}_t^2 \right)}{(2\pi)^4 256 m_m^3} \lambda(m_1, m_2, q) B(B_2 \to B_3 B_4) (A_{h} + B_{h} \cos \theta_{h}),
\]

with

\[
A_{h} = \frac{8\pi}{3} \left[ (2 + \hat{m}_t^2) \left( |H_{-\frac{1}{2},-1}|^2 + |H_{\frac{1}{2},1}|^2 + |H_{-\frac{1}{2},0}|^2 + |H_{\frac{1}{2},0}|^2 \right) + 3\tilde{m}_{\ell}^2 \left( |H_{-\frac{1}{2},t}|^2 + |H_{\frac{1}{2},t}|^2 \right) \right],
\]

(82)

\[
B_{h} = \frac{8\pi}{3} \alpha \left[ (2 + \hat{m}_t^2) \left( |H_{-\frac{1}{2},-1}|^2 + |H_{\frac{1}{2},1}|^2 - |H_{-\frac{1}{2},0}|^2 + |H_{\frac{1}{2},0}|^2 \right) + 3\tilde{m}_{\ell}^2 \left( - |H_{-\frac{1}{2},t}|^2 + |H_{\frac{1}{2},t}|^2 \right) \right],
\]

(83)

- $\phi$-dependence:

\[
d\Gamma \over d\phi = \int_{q_{\min}^2}^{q_{\max}^2} dq^2 \frac{G_F^2 |V_{cs}|^2 q^2 \left( 1 - \hat{m}_t^2 \right)}{(2\pi)^4 256 m_m^3} \lambda(m_1, m_2, q) B(B_2 \to B_3 B_4) (A_{\phi} + B_{\phi} \cos \phi_{h}),
\]

with

\[
A_{\phi} = \frac{8}{3} \left[ (2 + \hat{m}_t^2) \left( |H_{-\frac{1}{2},-1}|^2 + |H_{\frac{1}{2},1}|^2 + |H_{-\frac{1}{2},0}|^2 + |H_{\frac{1}{2},0}|^2 \right) + 3\tilde{m}_{\ell}^2 \left( |H_{-\frac{1}{2},t}|^2 + |H_{\frac{1}{2},t}|^2 \right) \right],
\]

(85)

\[
B_{\phi} = \frac{\pi^2}{\sqrt{2}} \alpha \left[ |H_{\frac{1}{2},1} H^*_{-\frac{1}{2},0}| + |H_{-\frac{1}{2},-1} H^*_{\frac{1}{2},0}| + \tilde{m}_{\ell}^2 \left( |H_{\frac{1}{2},1} H^*_{-\frac{1}{2},t}| - |H_{-\frac{1}{2},-1} H^*_{\frac{1}{2},t}| \right) \right].
\]

(86)
B. Differential decay width of $\Omega^0_c$

Using the leptonic matrix elements in Eq.(30)-(36) and hadron ones $H_1$ in Eq.(48)-(55), together with the three-body phase space in Eq.(24), we can obtain the three-body decay width from spin-1/2 charmed baryon $\Omega^0_c$ to spin-3/2 baryon $\Omega^-$ and leptons:

$$
\frac{d\Gamma (\Omega^0_c \rightarrow \Omega^- \ell^+ \nu_\ell)}{d\cos \theta_\ell dq^2} = \frac{q^2 \sqrt{\lambda(m_{\Omega^0_c}, m_{\Omega^-}, q)}}{1024\pi^3 m_{\Omega_c}^3} \left(1 - \hat{m}_\ell^2\right)^2 \frac{G_F^2}{2} |V_{cs}|^2 \left\{ \left( |H_{\frac{3}{2},1}|^2 + |H_{\frac{3}{2},-1}|^2 \right) \left( (1 - \cos \theta_\ell)^2 + \hat{m}_\ell^2 \sin^2 \theta_\ell \right) \right.
$$

$$
+ \left( |H_{\frac{3}{2},-1}|^2 + |H_{\frac{3}{2},-1}|^2 \right) \left( (1 + \cos \theta_\ell)^2 + \hat{m}_\ell^2 \sin^2 \theta_\ell \right) 
$$

$$
+ \left( |H_{\frac{3}{2},0}|^2 + |H_{\frac{3}{2},0}|^2 \right) 2 \left( \sin^2 \theta_\ell + \hat{m}_\ell \cos \theta_\ell \right) 
$$

$$
+ \left( |H_{\frac{3}{2},1}^2| + |H_{\frac{3}{2},-1}^2| \right) 2\hat{m}_\ell^2 
$$

$$
+ 2 \left( |H_{\frac{5}{2},0} H_{\frac{3}{2},1}^*| + |H_{\frac{5}{2},0} H_{\frac{3}{2},-1}^*| \right) \left( 2\hat{m}_\ell^2 \cos \theta_\ell \right) \right\}.
$$

(87)

Bring the results of leptonic part in Eq.(30)-(36), hadronic part $H_1$ in Eq.(48)-(55) and $H_2$ in Eq.(65)-(72) together, the differential decay width of four-body process $\Omega^0_c \rightarrow \Lambda K^- \ell^+ \nu_\ell$ can be expressed as

$$
\frac{d\Gamma (\Omega^0_c \rightarrow \Lambda K^- \ell^+ \nu_\ell)}{dq^2 d\cos \theta_\ell d\cos \theta_\ell d\phi} = \frac{3G_F^2 |V_{cs}|^2 q^2 \left(1 - \hat{m}_\ell^2\right)^2 \sqrt{\lambda(m_{\Omega^0_c}, m_{\Omega^-}, q)} B(\Omega^- \rightarrow \Lambda K^-)}{512(2\pi)^3 m_{\Omega_c}^3} \times \left\{ \left( |H_{\frac{3}{2},-1}|^2 \left[ \hat{m}_\ell^2 \sin^2 \theta_\ell + (1 + \cos \theta_\ell)^2 \right] \sin^2 \theta_\ell \left(1 - \alpha \cos \theta_\ell \right) \right. \right.
$$

$$
+ \left| H_{\frac{3}{2},1} \right|^2 \left[ \hat{m}_\ell^2 \sin^2 \theta_\ell + (1 - \cos \theta_\ell)^2 \right] \sin^2 \theta_\ell \left(1 + \alpha \cos \theta_\ell \right) \right.
$$

$$
+ \frac{1}{12} \left| H_{\frac{3}{2},1} \right|^2 \left[ \hat{m}_\ell^2 \sin^2 \theta_\ell + (1 - \cos \theta_\ell)^2 \right] \left[ 2 \left( 5 + 3 \cos 2 \theta_\ell \right) + \alpha \left( 7 \cos \theta_\ell + 9 \cos 3 \theta_\ell \right) \right] \right.
$$

$$
+ \frac{1}{12} \left| H_{\frac{3}{2},-1} \right|^2 \left[ \hat{m}_\ell^2 \sin^2 \theta_\ell + (1 + \cos \theta_\ell)^2 \right] \left[ 2 \left( 5 + 3 \cos 2 \theta_\ell \right) - \alpha \left( 7 \cos \theta_\ell + 9 \cos 3 \theta_\ell \right) \right] \right.
$$

$$
+ \frac{1}{6} \left| H_{\frac{5}{2},0} \right|^2 \left[ \hat{m}_\ell^2 \cos^2 \theta_\ell + \sin^2 \theta_\ell \right] \left[ 2 \left( 5 + 3 \cos 2 \theta_\ell \right) + \alpha \left( 7 \cos \theta_\ell + 9 \cos 3 \theta_\ell \right) \right] \right.
$$

$$
+ \frac{1}{6} \left| H_{\frac{3}{2},1} \right|^2 \left[ \hat{m}_\ell^2 \left[ 2 \left( 5 + 3 \cos 2 \theta_\ell \right) - \alpha \left( 7 \cos \theta_\ell + 9 \cos 3 \theta_\ell \right) \right] \right.
$$

$$
+ \frac{1}{6} \left| H_{\frac{3}{2},-1} \right|^2 \left[ \hat{m}_\ell^2 \left[ 2 \left( 5 + 3 \cos 2 \theta_\ell \right) + \alpha \left( 7 \cos \theta_\ell + 9 \cos 3 \theta_\ell \right) \right] \right.
$$

$$
+ \frac{1}{3} \left| H_{\frac{5}{2},0} H_{\frac{3}{2},1} \right|^2 \left[ \hat{m}_\ell^2 \cos \theta_\ell \left[ 2 \left( 5 + 3 \cos 2 \theta_\ell \right) - \alpha \left( 7 \cos \theta_\ell + 9 \cos 3 \theta_\ell \right) \right] \right.
$$

$$
+ \frac{1}{3} \left| H_{\frac{5}{2},0} H_{\frac{3}{2},-1} \right|^2 \left[ \hat{m}_\ell^2 \cos \theta_\ell \left[ 2 \left( 5 + 3 \cos 2 \theta_\ell \right) + \alpha \left( 7 \cos \theta_\ell + 9 \cos 3 \theta_\ell \right) \right] \right.
$$

$$
+ \frac{2 \sqrt{3}}{3} \left| H_{\frac{5}{2},-1} H_{\frac{3}{2},1} \right| \left( \hat{m}_\ell^2 - 1 \right) \sin^2 \theta_\ell \cos 2 \phi \sin^2 \theta_\ell \left( 1 - 3 \alpha \cos \theta_\ell \right) \right.
$$

$$
+ \frac{2 \sqrt{3}}{3} \left| H_{\frac{5}{2},1} H_{\frac{3}{2},-1} \right| \left( \hat{m}_\ell^2 - 1 \right) \sin^2 \theta_\ell \cos 2 \phi \sin^2 \theta_\ell \left( 1 + 3 \alpha \cos \theta_\ell \right) \right.
$$

$$
+ \frac{\sqrt{6}}{6} \left| H_{\frac{5}{2},-1} H_{\frac{3}{2},0} \right| \sin \theta_\ell \cos \phi \left\{ \left( \sin \theta_\ell - 3 \sin 3 \theta_\ell \right) \left[ \hat{m}_\ell^2 \cos \theta_\ell + \alpha \left( 1 + \cos \theta_\ell \right) \right] \right.
$$

$$
+ 4 \sin 2 \theta_\ell \left( \alpha \hat{m}_\ell^2 \cos \theta_\ell + 1 + \cos \theta_\ell \right) \right\}.$$
\[ -\frac{\sqrt{6}}{6} \left| H_{1/2,0} H_{1/2,0}^* \right| \hat{m}_t^2 \sin \theta_t \cos\phi \left[ (\sin \theta_h - 3 \sin \theta_h) + 4 \sin 2\theta_h \right] \\
+ \frac{\sqrt{6}}{6} \left| H_{1/2,-1} H_{1/2,-1}^* \right| \hat{m}_t^2 \sin \theta_t \cos\phi \left[ (\sin \theta_h - 3 \sin \theta_h) + 4 \sin 2\theta_h \right] \\
- \frac{\sqrt{6}}{6} \left| H_{1/2,0} H_{1/2,0}^* \right| \hat{m}_t^2 \sin \theta_t \cos\phi \left[ 4 \sin 2\theta_h - \alpha (\sin \theta_h - 3 \cos \theta_h) \right] \\
- \frac{\sqrt{6}}{6} \left| H_{1/2,-1} H_{1/2,-1}^* \right| \hat{m}_t^2 \sin \theta_t \cos\phi \left[ 4 \sin 2\theta_h - \alpha (\sin \theta_h - 3 \cos \theta_h) \right] \\
- \frac{\sqrt{2}}{6} \left| H_{1/2,1} H_{1/2,1}^* \right| \cos \theta_t \hat{m}_t^2 \left[ 1 - \cos \theta_t \right] \sin \theta_t \cos\phi \left[ 5 \sin \theta_h + 9 \sin 3\theta_h \right] \\
+ \frac{\sqrt{2}}{6} \left| H_{1/2,-1} H_{1/2,-1}^* \right| \cos \theta_t \hat{m}_t^2 \left[ 1 - \cos \theta_t \right] \sin \theta_t \cos\phi \left[ 5 \sin \theta_h + 9 \sin 3\theta_h \right] \\
- \frac{\sqrt{2}}{6} \left| H_{1/2,1} H_{1/2,1}^* \right| \hat{m}_t^2 \sin \theta_t \cos\phi \left[ 5 \sin \theta_h + 9 \sin 3\theta_h \right] \\
- \frac{\sqrt{2}}{6} \left| H_{1/2,-1} H_{1/2,-1}^* \right| \hat{m}_t^2 \sin \theta_t \cos\phi \left[ 5 \sin \theta_h + 9 \sin 3\theta_h \right]. \tag{88} \]

By integrating out the parameters step by step, we can get the results of \( q^2 \)-, \( \theta_t \)-, \( \theta_h \)- and \( \phi \)-dependence form of total decay width respectively:

- \( q^2 \)-dependence:

\[
\frac{d\Gamma}{dq^2} = \frac{32\pi}{9} \left[ 2 + \hat{m}_t^2 \left( |H_{-1/2,-1}|^2 + |H_{1/2,1}|^2 + |H_{-1/2,-1}|^2 + |H_{1/2,1}|^2 + |H_{1/2,0}|^2 + |H_{1/2,0}|^2 \right) \\
+ 3\hat{m}_t^2 \left( |H_{1/2,1}|^2 + |H_{1/2,1}|^2 \right) \right] \tag{89} \]

- \( \theta_t \)-dependence:

\[
\frac{d\Gamma}{d\cos \theta_t} = \int_{q_{min}^2}^{q_{max}^2} dq^2 \frac{G_F^2 |V_{cs}|^2 q^2 (1 - \hat{m}_t^2)^2 \sqrt{\lambda(m_{\Omega_1}, m_{\Omega_2}, q)}}{512(2\pi)^4 \hat{m}_t^2} B \left( B_2 \rightarrow B'_2 B'_2 \right) \left( A'_t + B'_t \cos \theta_t + C'_t \cos 2\theta_t \right), \tag{90} \]

with

\[
A'_t = \frac{4\pi}{3} \left[ 3 + \hat{m}_t^2 \right] \left( |H_{1/2,1}|^2 + |H_{-1/2,-1}|^2 + |H_{1/2,1}|^2 + |H_{1/2,-1}|^2 \right) \\
+ 2 \left( 1 + \hat{m}_t^2 \right) \left( |H_{1/2,0}|^2 + |H_{1/2,0}|^2 \right) + 4\hat{m}_t^2 \left( |H_{1/2,1}|^2 + |H_{1/2,1}|^2 \right), \tag{91} \]

\[
B'_t = \frac{16\pi}{3} \left[ \left( |H_{1/2,-1}|^2 - |H_{1/2,1}|^2 + |H_{1/2,-1}|^2 - |H_{1/2,1}|^2 \right) \\
+ 2\hat{m}_t^2 \left( |H_{1/2,0}|^2 + |H_{1/2,0}|^2 \right) \right], \tag{92} \]

\[
C'_t = \frac{4\pi}{3} \left( 1 - \hat{m}_t^2 \right) \left( |H_{1/2,1}|^2 + |H_{-1/2,-1}|^2 + |H_{1/2,1}|^2 + |H_{1/2,-1}|^2 \right) \\
- 2 \left( |H_{1/2,0}|^2 + |H_{1/2,0}|^2 \right), \tag{93} \]

- \( \theta_h \)-dependence:

\[
\frac{d\Gamma}{d\cos \theta_h} = \int_{q_{min}^2}^{q_{max}^2} dq^2 \frac{G_F^2 |V_{cs}|^2 q^2 (1 - \hat{m}_t^2)^2 \sqrt{\lambda(m_{\Omega_1}, m_{\Omega_2}, q)}}{512(2\pi)^4 \hat{m}_t^2} B \left( B_2 \rightarrow B'_2 B'_2 \right) \times \left( A'_h + B'_h \cos \theta_h + C'_h \cos 2\theta_h + D'_h \cos 3\theta_h \right), \tag{94} \]
with
\[
A'_h = \frac{4\pi}{9} \left\{ (2 + \hat{m}_\ell^2) \left[ 3 \left( |H_{\frac{1}{2},1}|^2 + |H_{-\frac{1}{2},-1}|^2 \right) + 5 \left( |H_{\frac{1}{2},1}|^2 + |H_{-\frac{1}{2},-1}|^2 + |H_{\frac{1}{2},0}|^2 + |H_{-\frac{1}{2},0}|^2 \right) \right] \\
+ 15\hat{m}_\ell^2 \left( \left| H_{\frac{1}{2},t} \right|^2 + \left| H_{-\frac{1}{2},t} \right|^2 \right) \right\},
\]
(95)
\[
B'_h = \frac{2\pi\alpha}{9} \left\{ (2 + \hat{m}_\ell^2) \left[ 3 \left( |H_{\frac{1}{2},1}|^2 - |H_{-\frac{1}{2},-1}|^2 \right) + 7 \left( |H_{\frac{1}{2},1}|^2 - |H_{-\frac{1}{2},-1}|^2 + |H_{\frac{1}{2},0}|^2 - |H_{-\frac{1}{2},0}|^2 \right) \right] \\
+ 21\hat{m}_\ell^2 \left( \left| H_{\frac{1}{2},t} \right|^2 - \left| H_{-\frac{1}{2},t} \right|^2 \right) \right\},
\]
(96)
\[
C'_h = \frac{4\pi}{3} \left\{ (2 + \hat{m}_\ell^2) \left[ \left( |H_{-\frac{1}{2},-1}|^2 - |H_{\frac{1}{2},1}|^2 \right) + 3 \left( |H_{\frac{1}{2},1}|^2 - |H_{-\frac{1}{2},-1}|^2 + |H_{\frac{1}{2},0}|^2 - |H_{-\frac{1}{2},0}|^2 \right) \right] \\
+ 3\hat{m}_\ell^2 \left( \left| H_{\frac{1}{2},t} \right|^2 + \left| H_{-\frac{1}{2},t} \right|^2 \right) \right\},
\]
(97)
\[
D'_h = \frac{2\pi\alpha}{3} \left\{ (2 + \hat{m}_\ell^2) \left[ \left( |H_{\frac{1}{2},-1}|^2 - |H_{-\frac{1}{2},1}|^2 \right) + 3 \left( |H_{\frac{1}{2},1}|^2 - |H_{-\frac{1}{2},-1}|^2 + |H_{\frac{1}{2},0}|^2 - |H_{-\frac{1}{2},0}|^2 \right) \right] \\
- 9\hat{m}_\ell^2 \left( \left| H_{-\frac{1}{2},t} \right|^2 - \left| H_{\frac{1}{2},t} \right|^2 \right) \right\},
\]
(98)

- $\phi$-dependence:
\[
\frac{d\Gamma}{d\phi} = \int_{\sigma_{\text{min}}}^{\sigma_{\text{max}}} dq \frac{3G_F^2 |V_{CS}|^2 q^2 (1 - \hat{m}_\ell^2)^2 \sqrt{\lambda (m_H, m_{Q}, q)}}{512(2\pi)^4 m_{1c}^2} B(B_2' \rightarrow B_3 B_4') (A'_\phi + B'_\phi \cos \phi + C'_\phi \cos 2\phi),
\]
(99)

with
\[
A'_\phi = \frac{16}{9} \left\{ (2 + \hat{m}_\ell^2) \left[ |H_{-\frac{1}{2},-1}|^2 + |H_{\frac{1}{2},1}|^2 + |H_{-\frac{1}{2},-1}|^2 + |H_{\frac{1}{2},0}|^2 + |H_{-\frac{1}{2},0}|^2 \right] \\
+ 3\hat{m}_\ell^2 \left( \left| H_{-\frac{1}{2},t} \right|^2 + \left| H_{\frac{1}{2},t} \right|^2 \right) \right\},
\]
(100)
\[
B'_\phi = \frac{\pi^2}{12\sqrt{2}} \left[ \sqrt{3} \alpha \left( |H_{\frac{1}{2},-1}H_{\frac{1}{2},0}| + |H_{\frac{1}{2},1}H_{\frac{1}{2},0}| \right) + \sqrt{3}\hat{m}_\ell^2 \left( \left| H_{-\frac{1}{2},-1}H_{\frac{1}{2},0}^* \right| + \alpha \left| H_{\frac{1}{2},1}H_{\frac{1}{2},0}^* \right| \right) \\
+ 5 \left( |H_{\frac{1}{2},-1}H_{\frac{1}{2},0}^*| + \alpha \left| H_{\frac{1}{2},1}H_{\frac{1}{2},0}^* \right| \right) - 5\hat{m}_\ell^2 \left( \alpha \left| H_{-\frac{1}{2},-1}H_{\frac{1}{2},0}^* \right| + \left| H_{\frac{1}{2},1}H_{\frac{1}{2},0}^* \right| \right) \right],
\]
(101)
\[
C'_\phi = -\frac{32}{9\sqrt{3}} (1 - \hat{m}_\ell^2)^2 \left( \left| H_{\frac{1}{2},-1}H_{\frac{1}{2},0}^* \right| + \left| H_{\frac{1}{2},1}H_{\frac{1}{2},-1}^* \right| \right).
\]
(102)

### IV. NUMERICAL ANALYSIS

#### A. Input

| Baryons   | $\Lambda_\pi$ | $\Xi_0$ | $\Omega_0$ | $\Lambda$ | $\Xi$ | $\Omega$ |
|-----------|---------------|--------|------------|------------|------|--------|
| masses(GeV)| 2.2860        | 2.4709 | 2.6952     | 1.1156     | 1.3217| 1.6724 |
| lifetimes($10^{-13}$s) | 2.024 | 1.530 | 2.680 | 2632 | 1639 | 821 |

**TABLE I: The masses of initial- and final-state baryons**

In this section, all parameters used in calculation will be collected, including the baryon masses, and CKM matrix. In addition, the lepton mass $m_e = 0.005$ GeV, $m_\mu = 0.1134$ GeV, and Fermi constant $G_F = 1.166 \times 10^{-5}$ GeV$^{-2}$. The
CKM matrix element $V_{cs} = 0.973$. In the calculation of heavy baryons four-body decays, the asymmetry parameters and branch ratios are collected as [10]

$$
\alpha(\Lambda^0 \to p\pi^-) = 0.732, \quad \alpha(\Xi^- \to \Lambda^0\pi^-) = -0.401, \quad \alpha(\Omega^- \to \Lambda^0 k^-) = 0.0157, \quad
$$

$$
\mathcal{B}(\Lambda^0 \to p\pi^-) = 63.900\%, \quad \mathcal{B}(\Xi^- \to \Lambda^0\pi^-) = 99.887\%, \quad \mathcal{B}(\Omega^- \to \Lambda^0K^-) = 67.800\%. \quad (103)
$$

## B. Numerical Results and Discussions

In hadronic part with spin-1/2 intermediate state, we use the lattice QCD calculation of the $\Lambda_c^+ \to \Lambda$ [55] and $\Xi_c^0 \to \Xi^-$ [49] for the decay process of $\Lambda_c^+ \to \Lambda\ell^+\nu_\ell$ and $\Xi_c^0 \to \Xi^\ell^+\nu_\ell$ respectively. In order to access the $q^2$-distribution, we use the modified z-expansions in the physical limits [53], and the fit functions are shown as

$$
f(q^2) = \frac{1}{1 - q^2/m^2_{\text{pole}}} \sum_{n=0}^{n_{\text{max}}} a_n^f [z(q^2)]^n. \quad (104)
$$

The expansion variable is defined as

$$
z(q^2) = \frac{\sqrt{t_+ - q^2} - \sqrt{t_+ - t_0}}{\sqrt{t_+ - t_0} - \sqrt{t_+ - t_0}}, \quad (105)
$$

with $t_0 = q^2_{\text{max}} = (m_{\Xi_c} - m_{\Xi})^2$ for $\Xi_c \to \Xi$, $t_0 = q^2_{\text{max}} = (m_{\Lambda_c} - m_{\Lambda})^2$ for $\Lambda_c \to \Lambda$, and $t_+ = (m_D + m_K)^2$. The pole masses in the form factors are used as $m_{\text{pole}}^{f+, f_\perp} = 2.12$ GeV, $m_0^{f_\perp} = 2.318$ GeV, $m_0^{g_+, g_\perp} = 2.460$ GeV, and $m_0^{\text{pole}} = 1.968$ GeV.

- $\Lambda_c^+ \to p\pi^- \ell^+\nu_\ell$

We collect the fitted form factors parameters from [55] in Table II. The resulting SM predictions for the $\Lambda_c^+ \to p\pi^- \ell^+\nu_\ell$ decay widths and branching fractions with corresponding error estimates are listed as

$$
\Lambda_c^+ \to p\pi^- e^+\nu_e: \quad \Gamma = 8.061(482) \times 10^{-14} \text{s}^{-1}, \quad \mathcal{B} = 2.48\%(15), \quad (106)
$$

$$
\Lambda_c^+ \to p\pi^- \mu^+\nu_\mu: \quad \Gamma = 8.126(462) \times 10^{-14} \text{s}^{-1}, \quad \mathcal{B} = 2.50\%(14). \quad (107)
$$

| $a_0^{f_\perp}$ | $a_1^{f_\perp}$ | $a_2^{f_\perp}$ | $a_0^{f_+}$ | $a_1^{f_+}$ | $a_2^{f_+}$ | $a_0^g$ | $a_1^g$ | $a_2^g$ |
|----------------|----------------|----------------|-------------|-------------|-------------|---------|---------|---------|
| Nominal 1.30 ± 0.06 | -3.27 ± 1.18 | 7.16 ± 11.6 | 0.81 ± 0.03 | -2.89 ± 0.52 | 7.82 ± 4.53 | 0.77 ± 0.02 | -2.24 ± 0.51 | 5.38 ± 4.80 |
| $\alpha_0^{g_\perp}$ | $\alpha_1^{g_\perp}$ | $\alpha_2^{g_\perp}$ | $\alpha_0^{g_+}$ | $\alpha_1^{g_+}$ | $\alpha_2^{g_+}$ | $\alpha_0^g$ | $\alpha_1^g$ | $\alpha_2^g$ |
| Nominal 0.68 ± 0.02 | -1.19 ± 0.35 | 6.24 ± 4.89 | 0.68 ± 0.02 | -2.44 ± 0.25 | 13.7 ± 2.15 | 0.71 ± 0.03 | -2.86 ± 0.44 | 11.8 ± 2.47 |

TABLE II: The $\Lambda_c \to \Lambda$ form factors calculated on lattice [55].

Based on the form factors, we predict the differential decay widths for $\Lambda_c^+ \to p\pi^- \ell^+\nu_\ell$ as function of $q^2$ in Fig. 3(a). Note that the increasing errors in small $q^2$ region come from the uncertainties of form factors at large momentum transfer in lattice calculations. In Eq.(76-84), we show the theoretical results of $\theta_l^-$, $\theta_h^-$ and $\phi$-dependence of total four-body decay width with different final state leptons, in which the coefficients are functions of $q^2$ only. After we integrate out $q^2$, the angular distribution with $\cos \theta_l$, $\cos \theta_h$ and $\phi$ are shown as

1. $\Lambda_c^+ \to p\pi^- e^+\nu_e$

$$
\frac{d\Gamma}{\Gamma d\cos \theta_l} = 0.4448(254) + 0.1992(172) \cos \theta_l - 0.1657(204) \cos 2\theta_l, \quad (108)
$$
\[
\frac{d\Gamma}{d\cos \theta_h} = 0.5000(299) - 0.3188(210) \cos \theta_h, \\
\frac{d\Gamma}{d\phi} = 0.1592(95) - 0.0276(35) \cos \theta_h.
\]

2. \( \Lambda_c^+ \rightarrow p\pi^-\mu^+\nu_\mu \)

\[
\frac{d\Gamma}{d\cos \theta_l} = 0.455(57) + 0.265(28) \cos \theta_l - 0.135(24) \cos 2\theta_l, \\
\frac{d\Gamma}{d\cos \theta_h} = 0.500(63) + 0.162(19) \cos \theta_h, \\
\frac{d\Gamma}{d\phi} = 0.159(20) + 0.012(3) \cos \theta_h.
\]

\begin{itemize}
  \item \( \Xi_c^0 \rightarrow \Lambda\pi^-\ell^+\nu_\ell \)
  Based on the recently announced \( \Xi_c \rightarrow \Xi \) form factors calculated by lattice QCD [49], where the \( z \)-expansion parameters of helicity-based form factors collected in Table III, we perform the predictions for \( \Xi_c^0 \rightarrow \Lambda\pi^-\ell^+\nu_\ell \) decay widths and branching fractions with corresponding uncertainties
  \[
  \Xi_c \rightarrow \Lambda\pi^-e^+\nu_e : \quad \Gamma = 1.031(129) \times 10^{-13} \text{s}^{-1}, \quad B = 2.40(30)\%, \\
  \Xi_c \rightarrow \Lambda\pi^-\mu^+\nu_\mu : \quad \Gamma = 1.037(131) \times 10^{-13} \text{s}^{-1}, \quad B = 2.41(30)\%.
  \]
\end{itemize}
We also predict the differential decay widths for $\Xi_c^0 \to \Lambda \pi^- \ell^+ \nu_\ell$ as function of $q^2$ in Fig. 5, the errors in plots mainly come from the uncertainties of form factors extracted from lattice QCD, and its behaviors is same as $\Lambda_c^+ \to p \pi^- \ell^+ \nu_\ell$. The results of $\theta_l$, $\theta_h$- and $\phi$-dependence of differential decay widths with different leptonic final-states, are listed as follows

1. $\Xi_c^0 \to \Lambda \pi^- e^+ \nu_e$

$$\frac{d\Gamma}{d\cos\theta_l} = 0.4478(557) + 0.2363(249) \cos \theta_l - 0.1565(273) \cos 2\theta_l,$$

$$\frac{d\Gamma}{d\cos\theta_h} = 0.5000(626) + 0.1616(191) \cos \theta_h,$$

$$\frac{d\Gamma}{d\phi} = 0.1592(199) + 0.0123(30) \cos \theta_\phi .$$

and corresponding plots of angular distributions are collected in Fig. 5.

2. $\Xi_c^0 \to \Lambda \pi^- \mu^+ \nu_\mu$

$$\frac{d\Gamma}{d\cos\theta_l} = 0.4549(569) + 0.2650(280) \cos \theta_l - 0.1354(240) \cos 2\theta_l,$$

$$\frac{d\Gamma}{d\cos\theta_h} = 0.5000(630) + 0.1617(193) \cos \theta_h,$$

$$\frac{d\Gamma}{d\phi} = 0.1592(200) + 0.0122(29) \cos \theta_\phi ,$$

and corresponding plots of angular distributions are collected in Fig. 5.

- $\Omega_c^0 \to AK^- \ell^+ \nu_\ell$

Due to the absence of lattice QCD calculation, we use $\Omega_c^0 \to \Omega^-$ transition form factors calculated by light-front quark model [56]. The form factors can be expressed as the following double-pole form:

$$F(q^2) = \frac{F(0)}{1-a(q^2/m_F^2) + b(q^2/m_F^2)} ,$$

where $F(0)$ is the value of the form factors at $q^2 = 0$ with $m_F = 1.86$ GeV, $\delta \equiv \delta m_c/m_c = \pm 0.04$. The numerical results from light-front quark model are collected in Table IV.

|      | $f_1$     | $f_2$    | $f_3$      | $f_4$      | $g_1$      | $g_2$      | $g_3$    | $g_4$    |
|------|-----------|----------|------------|------------|-----------|-----------|---------|---------|
| $F(0)$ | 0.54 + 0.13$\delta$ | 0.35 - 0.36$\delta$ | 0.33 + 0.59$\delta$ | 0.97 + 0.22$\delta$ | 2.05 + 1.38$\delta$ | -0.06 + 0.33$\delta$ | -1.32 - 0.32$\delta$ | -0.44 + 0.11$\delta$ |
| a    | -0.27     | -30.0    | 0.96       | -0.53      | -3.66     | -1.15     | -4.01   | -1.29   |
| b    | 1.65      | 96.82    | 9.25       | 1.41       | 1.41      | 71.66     | 5.68    | -0.58   |

TABLE IV: $\Omega_c^0 \to \Omega^-$ transition form factors with $F(0)$ at $q^2 = 0$. 

|      | $f_1$     | $f_2$ | $f_3$ | $f_4$ | $g_1$ | $g_2$ | $g_3$ | $g_4$ |
|------|-----------|-------|-------|-------|-------|-------|-------|-------|
|      | $a_0$     | $a_1$ | $a_2$ |       |       |       |       |       |
|      | 1.51 ± 0.09 | -1.88 ± 1.21 | 1.71 ± 0.49 | 0.64 ± 0.09 | 0.77 ± 0.07 | 0.56 ± 0.07 | 0.63 ± 0.07 | 0.56 ± 0.08 |
|      | $g_0$     | $g_1$ |       |       |       |       |       |       |
|      | -0.27     | -1.83 ± 1.22 | 0.56 ± 0.51 | -4.09 ± 1.18 | -0.35 ± 1.26 | -1.63 ± 1.36 | -0.00 ± 1.38 |       |
|      | $g_3$     | $g_4$ |       |       |       |       |       |       |
|      | 1.41      | 1.41  |       |       |       |       |       |       |
|      | 71.66     | 5.68  |       |       |       |       |       |       |
|      | 0.96      | -0.53 | 0.15 ± 0.29 |       |       |       |       |       |
|      | -3.66     |       | 0.15 ± 0.29 |       |       |       |       |       |
|      | -1.15     |       | 0.14 ± 0.29 |       |       |       |       |       |

TABLE III: Results for the $z$-expansion parameters describing $\Xi_c \to \Xi$ form factors with statistical errors.
FIG. 4: Differential decay width for $\Xi^0 \rightarrow \Lambda \pi^- e^+ \nu_e$ as function of $q^2$ (a), $\cos \theta_l$ (b), $\cos \theta_h$ (c) and $\phi$ (d). Note that in sub-fig.(b) and (c), the errors increasing with $\theta_l$ form -1 to 1 and $\theta_h$ from 1 to -1, as a results of the remarkable uncertainties of form factors at small $q^2$ region.

Through the decay width of $\Omega_c$ in Eq.(88) and integrating out all variables, we can obtain the numerical results of decay widths and branching fractions with errors:

$\Omega_c \rightarrow \Lambda K^- e^+ \nu_e$ : $\Gamma = 8.889(344) \times 10^{-15} s^{-1}$, $B = 0.362(14)\%$, (119)

$\Omega_c \rightarrow \Lambda K^- \mu^+ \nu_\mu$ : $\Gamma = 8.771(343) \times 10^{-15} s^{-1}$, $B = 0.350(14)\%$. (120)

Fig. 5(a) shows the differential decay widths of $\Omega_c^0 \rightarrow \Lambda K^- e^+ \nu_e$ as function of $q^2$. We also give the results of angular distributions of total decay width, the numerical results are listed in Eq.(121,122), and in Fig.5 the illustration of the angular distributions with $\phi$, $\cos \theta_l$ and $\cos \theta_h$ are displayed.

1. $\Omega_c^0 \rightarrow \Lambda K^- e^+ \nu_e$

$$\frac{d\Gamma}{\Gamma d \cos \theta_l} = 0.472(2) + 0.244(3) \cos \theta_l - 0.085(14) \cos 2\theta_l,$$

$$\frac{d\Gamma}{\Gamma d \cos \theta_h} = 0.548(24) - 0.002(0) \cos \theta_h + 0.143(13) \cos 2\theta_h - 0.001(0) \cos 3\theta_h,$$

$$\frac{d\Gamma}{\Gamma d \phi} = 0.159(6) - 0.0233(8) \cos \phi - 0.0173(4) \cos 2\phi.$$ (121)

2. $\Omega_c^0 \rightarrow \Lambda K^- \mu^+ \nu_\mu$

$$\frac{d\Gamma}{\Gamma d \cos \theta_l} = 0.479(16) + 0.273(5) \cos \theta_l - 0.064(11) \cos 2\theta_l,$$
\[
\frac{d\Gamma}{\Gamma \, d \cos \theta_h} = 0.5479(238) - 0.0021(6) \cos \theta_h + 0.1436(131) \cos 2 \theta_h - 0.0014(0) \cos 3 \theta_h ,
\]
\[
\frac{d\Gamma}{\Gamma \, d \phi} = 0.1596(62) - 0.0224(8) \cos \theta_l - 0.0120(2) \cos 2 \theta_l .
\]

(122)

In summary, we have studied charmed baryon four-body semileptonic decays including the antitriplet charmed baryons (\(\Lambda_c^+\), \(\Xi_c^0\)) and sextet \(\Omega_c^0\). With the form factors of the \(\Lambda_c^+ \to \Lambda \) and \(\Xi_c^0 \to \Xi^-\) calculated by lattice QCD and the light-front quark model calculation of \(\Omega_c^0 \to \Omega^-\) form factors, we have predicted \(\Gamma(\Lambda_c^+ \to p \pi^- e^+ \nu_e) = 8.061(482) \times 10^{-14} \text{s}^{-1}\), \(\Gamma(\Lambda_c^+ \to p \pi^- \mu^+ \nu_\mu) = 8.126(462) \times 10^{-14} \text{s}^{-1}\), \(\Gamma(\Xi_c^0 \to \Lambda \pi^- e^+ \nu_e) = 1.031(129) \times 10^{-13} \text{s}^{-1}\), \(\Gamma(\Xi_c^0 \to \Lambda \pi^- \mu^+ \nu_\mu) = 1.037(131) \times 10^{-13} \text{s}^{-1}\), \(\Gamma(\Omega_c^0 \to \Lambda K^- e^+ \nu_e) = 8.889(344) \times 10^{-15} \text{s}^{-1}\), \(\Gamma(\Omega_c^0 \to \Lambda K^- \mu^+ \nu_\mu) = 8.771(343) \times 10^{-15} \text{s}^{-1}\), and \(B(\Lambda_c^+ \to p \pi^- e^+ \nu_e) = 2.48(15)\%\), \(B(\Lambda_c^+ \to p \pi^- \mu^+ \nu_\mu) = 2.50(14)\%\), \(B(\Xi_c^0 \to \Lambda \pi^- e^+ \nu_e) = 2.40(30)\%\), \(B(\Xi_c^0 \to \Lambda \pi^- \mu^+ \nu_\mu) = 2.41(30)\%\), \(B(\Omega_c^0 \to \Lambda K^- e^+ \nu_e) = 0.362(14)\%\), \(B(\Omega_c^0 \to \Lambda K^- \mu^+ \nu_\mu) = 0.350(14)\%\), using helicity amplitude technique. In addition, we give the angular distributions of all the process with different angulars \(\cos \theta_l, \cos \theta_h\) and \(\phi\). In the future, we expect to study the more general cases of semileptonic charmed baryon decays by calculating the form factors such as \(\Xi_c \to \Lambda\) and so on. This work can provide a theoretical basis for the ongoing experiments at BESIII, LHCb and BELLE-II.
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APPENDIX

A. Definitions of spinors, polarization vectors and vectorial spinors

In this part, we will introduce the spinors and polarization vectors of fermions and vector boson [57]. The spinors for spin-1/2 fermions are

\[ u (\vec{p}, + \frac{1}{2}) = N_\rho \begin{pmatrix} e^{-i\phi/2} \cos(\theta/2) \\ e^{i\phi/2} \sin(\theta/2) \\ \frac{|\vec{p}|}{E+m} e^{-i\phi/2} \cos(\theta/2) \\ \frac{|\vec{p}|}{E+m} e^{i\phi/2} \sin(\theta/2) \end{pmatrix}, \]

(123)

\[ u (\vec{p}, - \frac{1}{2}) = N_\rho \begin{pmatrix} -e^{-i\phi/2} \sin(\theta/2) \\ e^{i\phi/2} \cos(\theta/2) \\ \frac{|\vec{p}|}{E+m} e^{-i\phi/2} \sin(\theta/2) \\ -\frac{|\vec{p}|}{E+m} e^{i\phi/2} \cos(\theta/2) \end{pmatrix}, \]

(124)

\[ (\vec{p}, - \frac{1}{2}) = 3 \begin{pmatrix} 1/2 \\ 0 \\ 0 \\ 1 \end{pmatrix}, \]

(125)

where \( N_\rho = \sqrt{E + m} \) and \( p^\mu = (p^0, \vec{p} \sin \theta \cos \phi, \vec{p} \sin \theta \sin \phi, \vec{p} \cos \theta) \). The spinors for anti-fermions can be obtained by \( \bar{\psi}(\vec{p}, s) = -i\gamma^2 \bar{u}^* (\vec{p}, s) \). The polarization vectors can be expressed as

\[ e^\mu (1) = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ -\cos \theta \cos \phi + i \sin \phi \\ -\cos \theta \sin \phi - i \cos \phi \sin \theta \\ -\sin \phi \end{pmatrix}, \]

\[ e^\mu (-1) = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ \cos \theta \cos \phi + i \sin \phi \\ \cos \theta \sin \phi - i \cos \phi \sin \theta \\ i \sin \phi \cos \phi \end{pmatrix}, \]

\[ e^\mu (0) = \frac{1}{\sqrt{m}} \begin{pmatrix} |\vec{p}| \cos \phi \sin \phi, p^0 \sin \theta \cos \phi, p^0 \sin \theta \sin \phi, p^0 \cos \theta \end{pmatrix}. \]

(126)

The vectorial spinor \( U_\alpha (\vec{p}, S_z) \) for spin-3/2 baryon is given as

\[ U_\alpha (\vec{p}, S_z) = \sum_{s_z, s'_z} \left< s = 1, s_z; s'_z = \frac{1}{2}, s' = \frac{1}{2}; s = \frac{1}{2}, S = \frac{3}{2}, S_z \right| \epsilon_\alpha (\vec{p}, s_z) \bar{u} (\vec{p}, s'_z), \]

(127)

where \( \bar{u}(\vec{p}, s'_z) \) is wave function for spin-1/2 and \( \epsilon_\alpha (\vec{p}, s_z) \) for spin-1. And the Clebsch-Gordan coefficients

\[ S_z = \frac{3}{2} : \left< s_z = 1, s'_z = \frac{1}{2}; S_z = \frac{3}{2} \right| = 1, \]

(128)

\[ S_z = \frac{1}{2} : \left< s_z = 1, s'_z = -\frac{1}{2}; S_z = \frac{1}{2} \right| = \frac{1}{\sqrt{3}}, \]

(129)

\[ S_z = -\frac{1}{2} : \left< s_z = 0, s'_z = -\frac{1}{2}; S_z = -\frac{1}{2} \right| = \frac{1}{\sqrt{3}}, \]

(130)

\[ S_z = -\frac{3}{2} : \left< s_z = -1, s'_z = -\frac{1}{2}; S_z = -\frac{3}{2} \right| = 1. \]

(131)

B. Transformations of various-based form factors

Apart from the generalized parametrization in Eq.(37,38) for spin-1/2-to-spin-1/2 case, another commonly treatment named helicity-based form factors,

\[ \langle B_2 (p_2, s_2) | V^\mu | B_1 (p_1, s_1) \rangle = \bar{u} (p_2, s_2) \left( m_1 - m_2 \right) q^\mu q^2 f_0 (q^2) \]

[57] P. R. Auvil and J. J. Brehm, Phys. Rev. 145, no. 4, 1152 (1966). doi:10.1103/PhysRev.145.1152
Similarly, we can obtain the helicity-based form factors from the generalized ones in Eq.\(46,47\) under the following transformation:

\[
\begin{pmatrix}
    f_{\perp} (q^2) \\
    f_0 (q^2) \\
    f_{+} (q^2)
\end{pmatrix}
= \begin{pmatrix}
    1 & -m_1 + m_2 & m_1 \\
    1 & 0 & q^2 \\
    1 & -m_1 (m_1 + m_2) & q^2
\end{pmatrix}
\begin{pmatrix}
    f_1 (q^2) \\
    f_2 (q^2) \\
    f_3 (q^2)
\end{pmatrix},
\]

\[
\begin{pmatrix}
    g_{\perp} (q^2) \\
    g_0 (q^2) \\
    g_{+} (q^2)
\end{pmatrix}
= \begin{pmatrix}
    1 & m_1 - m_2 & m_1 \\
    1 & 0 & q^2 \\
    1 & -m_1 (m_1 + m_2) & q^2
\end{pmatrix}
\begin{pmatrix}
    g_1 (q^2) \\
    g_2 (q^2) \\
    g_3 (q^2)
\end{pmatrix}.
\]

The helicity-based parametrized form factors of \(\Omega^0 \rightarrow \Omega^-\) are given as

\[
\begin{align*}
\langle B_2(p_2, s_2) | A^{\mu} | B_1(p_1, s_1) \rangle &= -\bar{u}(p_2, s_2) \gamma_5 \left[ m_2 \left( m_2 + m_1 \right) p_1^\mu q^\mu \right. \\
&+ f_{+} \frac{m_1}{s_+} \left( m_1 - m_2 \right) p_1^\mu \left( q^2 (p_1^\mu + p_2^\mu) - \left( m_2 - m_1 \right) q^\mu \right) \\
&+ f_{\perp} \frac{m_1}{s_+} \left( p_1^\mu \gamma^\mu - 2 p_2^\mu (m_2 p_1^\mu - m_1 p_2^\mu) \right) s_+ \\
&+ f_{+} \frac{m_1}{s_-} \left( p_1^\mu \gamma^\mu + 2 p_2^\mu (m_2 p_1^\mu + m_1 p_2^\mu) \right) s_- \\
&\left. \right] u(p_1, s_1),
\end{align*}
\]

\[
\begin{align*}
\langle B_2(p_2, s_2) | \bar{s} \gamma^\mu \gamma_5 \gamma_5 | B_1(p_1, s_1) \rangle &= \bar{u}(p_2, s_2) \left[ m_2 \left( m_2 - m_1 \right) p_1^\mu q^\mu \right. \\
&+ g_{+} \frac{m_2}{s_+} \left( m_2 + m_1 \right) p_1^\mu \left( q^2 (p_1^\mu + p_2^\mu) - \left( m_2 - m_1 \right) q^\mu \right) \\
&+ g_{\perp} \frac{m_2}{s_+} \left( p_1^\mu \gamma^\mu - 2 p_2^\mu (m_2 p_1^\mu + m_1 p_2^\mu) \right) s_+ \\
&+ g_{+} \frac{m_2}{s_-} \left( p_1^\mu \gamma^\mu + 2 p_2^\mu (m_2 p_1^\mu - m_1 p_2^\mu) \right) s_- \\
&\left. \right] u(p_1, s_1).
\end{align*}
\]

Similarly, we can obtain the helicity-based form factors from the generalized ones in Eq.\(46,47\) under the following transformation

\[
\begin{pmatrix}
    f_{\perp} (q^2) \\
    f_{+} (q^2) \\
    f_{0} (q^2) \\
    f_{\perp} (q^2)
\end{pmatrix}
= \begin{pmatrix}
    \frac{s_+}{m_1 m_2} & 0 & 0 & -1 \\
    0 & 0 & 0 & 1 \\
    \frac{m_1 - m_2}{m_1 m_2} & 0 & 0 & 0 \\
    \frac{m_1 - m_2}{m_1 m_2} & \frac{s_+}{m_1 m_2} & -1 & 0
\end{pmatrix}
\begin{pmatrix}
    f_1 (q^2) \\
    f_2 (q^2) \\
    f_3 (q^2) \\
    f_4 (q^2)
\end{pmatrix},
\]

\[
(137)
\]
\[
\begin{pmatrix}
g_\perp(q^2) \\
g_\perp'(q^2) \\
g_0(q^2) \\
g_+(q^2)
\end{pmatrix}
= \begin{pmatrix}
\frac{s_-}{m_1 m_2} & 0 & 0 & -1 \\
0 & \frac{(m_1^2 - m_2^2 + q^2) s_+}{2 m_1^2 m_2 (m_1 - m_2)} & \frac{(m_1^2 - m_2^2 - q^2) s_-}{2 m_1 m_2^2 (m_1 - m_2)} & -2 (m_1 + m_2)^2 + 2 q^2 + \frac{(m_1^2 - m_2^2 - q^2) s_-}{2 m_1 m_2^2 (m_1 - m_2)} \\
\frac{s_-}{m_1 m_2} & \frac{2 m_1^2 m_2 (m_1 - m_2)}{2 m_1^2 m_2 + 2 m_1^2 m_2} & \frac{2 m_1 m_2^2 (m_1 - m_2)}{2 m_1^2 m_2 + 2 m_1 m_2^2} & -2 + \frac{s_-}{s_+ \frac{(m_1^2 + m_2^2) m_2}{(m_1 + m_2) m_2}} \\
\end{pmatrix}
\begin{pmatrix}
g_1(q^2) \\
g_2(q^2) \\
g_3(q^2) \\
g_4(q^2)
\end{pmatrix}.
\]

(138)