Variations on Supersymmetry Breaking and Neutrino Spectra

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ABSTRACT

The problem of generating light neutrinos within supersymmetric models is discussed. It is shown that the hierarchy of scales induced by supersymmetry breaking can give rise to suppression factors of the correct order of magnitude to produce experimentally allowed neutrino spectra.

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ABSTRACT

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1 Introduction

Small neutrino masses are usually generated through the seesaw mechanism \cite{1}. Three superfields $\tilde{N}_\alpha$ ($\alpha = e, \mu, \tau$), singlets with respect to the Standard Model (SM), are added to the three neutral state components of the three doublets $L_\alpha$. These fields have a large mass $M_R$ that suppresses the electroweak scale naturally obtained from the Yukawa operators $y_S \tilde{N}LH$, for coupling constants $y_S = \mathcal{O}(1)$, when the neutral component of $H$ acquires a vacuum expectation value (vev). Three light neutrino eigenstates $\nu_1, \nu_2, \nu_3$, with mass $\sim v^2/M_R$ are obtained, with very small mixing to the heavy eigenstates $n_1, n_2, n_3$, with mass $\sim M_R$.

This mechanism can be easily implemented in supersymmetric models. In these, however, given the richness with which they are endowed, it is possible to obtain similar suppression factors in different ways, when three SM singlet superfields $\tilde{N}$ are added to the SM particle content. In particular, supersymmetry breaking at some intermediate scale $M_X$, with the typical hierarchy between the Planck mass $M_P$ ($\simeq 10^{18}$ GeV) and the gravitino mass $m_{3/2}$, $m_{3/2} \sim M_X^2/M_P$, or generically between the Planck mass and a typical soft supersymmetry-breaking mass $\tilde{m}$ ($m_{\text{weak}} \lesssim \tilde{m} \ll M_P$, where $m_{\text{weak}}$ is the electroweak scale), may play a crucial role in providing alternative mechanisms for generating small neutrino masses \cite{2, 3, 4}. Moreover, even keeping the seesaw mechanism unaltered, supersymmetry breaking may still be advocated to explain the lightness of an
additional light sterile neutrino, if a fourth SM singlet superfield $S$ is also included. The resulting seventh neutrino eigenstate is denoted by $\nu_s$.

Here, we critically review mechanisms and models recently proposed in Refs. [2] and [5]. Both proposals rely on the same class of models of supersymmetry breaking [6, 7]. They are, however, very different in spirit. The proposal of Ref. [2] replaces or encompasses the conventional seesaw mechanism, accommodating three states $n_i$ of mainly sterile neutrinos that are light or heavy depending on the details of the specific model. The proposal of Ref. [5] makes use of the hierarchy of scales induced by supersymmetry breaking to explain the lightness of the fourth sterile neutrino. The three states $\nu_i$, mainly active ones, are made light by the well-known seesaw suppression generated in the neutrino mass matrix by the three heavy sterile neutrinos of mass $\sim M_R$.

When looking for alternatives to the seesaw mechanism, or when trying to generate a small mass for a fourth sterile neutrino, an excessive tuning of coupling constants can be avoided by forbidding renormalizable interactions such as $y\bar{N}LH$ or $y_SSLH$. Thus, neutrino masses can be generated (in addition to or as a replacement of the seesaw mechanism) in the two following ways.

- **Through non-renormalizable operators**, in which $\bar{N}$ and $S$ are coupled to a spurion field. Small masses are dynamically generated and linked to the large hierarchy between $m_{\text{weak}}$ and $M_P$, or between $m_{\text{weak}}$ and some intermediate scale, as in the seesaw mechanism. Contributions to Dirac masses $m_D$ and $m_{Ds}$ are induced by non-renormalizable interactions of the Yukawa type, i.e. terms of the form $\bar{\nu}_{L\alpha}\nu_{R\beta}$, $\bar{\nu}_{Ra}\nu_{La}$, and $\bar{\nu}_{La}\nu_{Ls}$, where $\nu_{La}$ indicates the fermionic component of the neutral fields in $L_\alpha$ (the $\alpha$-th active neutrino), $\nu_{Ls}$ and $\nu_{Ra}$ are the charge conjugated fermionic components of the superfields $S$ and $\bar{N}_a$, respectively. Majorana masses $m_L$ for active neutrinos $\nu^T_L C \nu_{L\beta}$ can be generated, if chiral-lepton violation is allowed.

- **Through quantum effects.** As in the previous mechanism, loop diagrams give rise to Majorana masses for active neutrinos, if chiral-lepton violation is allowed. Radiative contributions to Dirac masses are possible if an additional gauge group, such as $U(1)_{B-L}$ is included.

In the following, we discuss the contribution from non-renormalizable operators to Dirac and Majorana entries in the neutrino mass matrix, see Sec. 2. The superpotentials for the relevant fields in the different models proposed in Refs. [2] and [5] are listed in Sects. 2.1.1, 2.1.2, 2.2, and 2.3. In Sec. 3, a class of models is outlined in which supersymmetry breaking is induced by the strong dynamics of a gauge group $SU(2)$, $F_Z \neq 0$, where $F_Z$ is the vev of the auxiliary component of a singlet superfield $Z$. The breaking of supersymmetry gives rise to a non-vanishing vev for the scalar component of such a superfield, $A_Z \neq 0$, which is crucial to induce Dirac and Majorana mass terms from
the non-renormalizable operators listed in Sec. 2. The same operators give rise to scalar terms softly breaking supersymmetry, with explicit chiral-lepton violations. These are discussed in Sec. 4. Some of these terms, if not duly suppressed (or altogether forbidden) may give rise to instability problems and/or too large radiative contributions to Majorana neutrino masses. At danger is the model of Ref. [5], for which remedies are anticipated already in Sec. 2.3 and outlined in Secs. 5 and 6. Finally the contributions to Dirac and Majorana masses due to quantum effects, also induced by supersymmetry breaking, are discussed in Sec. 7. The typical neutrino spectra obtained in the models proposed in Refs. [2] and [3] are described in Sec. 8.

2 Contributions from non-renormalizable operators

We list some non-renormalizable operators inducing tree-level neutrino masses. The presentation is arranged in such a way to show the basic structure of the different models proposed in Refs. [2] and [3]. Their classification is according to the type of lepton-number violation in them allowed.

2.1 No gauge group extension; 3 singlets $\bar{N}$

2.1.1 No lepton-number violation allowed

The relevant superpotential operator is:

$$W = \frac{k_{\bar{N}}}{M_P} Z \bar{N} L H ,$$

(1)

where $Z$ is a SM singlet, with a supersymmetry-conserving vacuum expectation value ($vev$) $A_Z$ and a supersymmetry-violating one $F_Z$, with $F_Z \sim M_X^2 \simeq m_{3/2} M_P$. The usual Yukawa coupling operator $y \bar{N} L H$ can be forbidden by a continuous or discrete symmetry $Z_n$:

$$Z_n(\bar{N}) = +1, \quad Z_n(Z) = -1 ,$$

(2)

whereas a Majorana mass for the right-handed neutrinos is forbidden by a discrete lepton-number symmetry $L$:

$$L(+1), \quad \bar{N}(-1), \quad \bar{E}(-1) .$$

(3)

For each generation, the Dirac neutrino mass

$$m_D = k_{\bar{N}} \frac{A_Z v}{M_P}$$

(4)

is generated and one Dirac neutrino eigenstate with mass $m_\nu = m_D$ is induced. Notice that, if $A^2_Z \simeq M^2_X \simeq m_{3/2} M_P$, the neutrino mass is too large, i.e. $m_\nu \simeq 10^3$ eV. Thus, a value $A^2_Z \ll M^2_X \ll m_{3/2} M_P$ is needed.
2.1.2 Explicit lepton-number violation

In this case, the superpotential is:

\[ W = \frac{k_N}{M_P} Z \bar{N} LH + \frac{z}{4 M_P} ZZ \bar{N} + \frac{h}{4 M_P} LHLH. \]  

(5)

The same $Z_n$-symmetry of eq. (2) forbids the usual Yukawa operators and the absence of a large Majorana mass for the fields $\bar{N}$, can be guaranteed by imposing that $Z_n$ is $Z_{n\neq 2}$. An additional $R$-parity symmetry, $R_p$:

\[ R_p(\bar{N}) = - , \quad R_p(Z) = + , \]  

(6)

forbids also dangerous terms such as $M_P Z \bar{N}$. For each generation, the tree-level contribution to the Dirac mass $m_D$, and the Majorana masses for left-handed and right-handed neutrinos, $m_L$ and $m_R$ are, respectively:

\[ m_D = k_N \frac{A_Z v}{M_P} , \quad m_R = z \frac{A_Z^2}{2 M_P} , \quad m_L = \frac{h}{2} \frac{v^2}{M_P}. \]  

(7)

The physical neutrino spectrum consists of two Majorana neutrinos with mass:

\[ m_{\nu_1} \simeq \frac{h}{2} - \frac{2k_N^2}{z} \frac{v^2}{M_P} , \quad m_{\nu_2} \simeq \frac{z}{2} \frac{A_Z^2}{M_P}. \]  

(8)

The only assumption so far is that $v < A_Z$ and that $z$ is unsuppressed. In this case, for a value of $A_Z^2 \simeq M_X^2 \simeq m_3/2 M_P$, $m_{\nu_1}$ is too small, i.e. $m_{\nu_1} \simeq 10^{-5}$ eV, whereas $m_{\nu_2} \simeq m_3/2$. This spectrum may be cured by radiative corrections, which can increase the value of $m_L$.

2.2 Spontaneous lepton-number violation; 3 singlets $\bar{N}$

Spontaneous lepton-number violation is achieved through the spontaneous breaking of an additional gauge group, say a $U(1)_{B-L}$. Due to this new gauge interaction, two additional Higgs bosons, $\Phi$ and $\bar{\Phi}$ (SM singlets) are present. They acquire vev's $<\Phi> = <\bar{\Phi}> = v_{\Phi} = v_{\bar{\Phi}}$. The fields $\bar{N}$ are not neutral with respect to $U(1)_{B-L}$ and have charge $X_{\bar{N}}$, while the charges of $\Phi$ and $\bar{\Phi}$ are respectively $X_{\Phi}$ and $X_{\bar{\Phi}}$. As in the two cases described before, renormalizable neutrino Yukawa interactions are forbidden by the $Z_n$-symmetry in eq. (2), and the relevant superpotential is:

\[ W = \frac{k_{\bar{N}}}{M_P} Z \bar{N} LH + \frac{z}{4 (M_P)^{m+1}} \bar{\Phi}^m ZZ \bar{N} \bar{N} + \frac{h}{4 (M_P)^{m+1}} \Phi^m LHLH , \]  

(9)

if the symmetry that forbids the Yukawa neutrino operator is $Z_{n\neq 2}$. The operator giving rise to the Majorana mass for the fields $\bar{N}$ is however:

\[ W = \frac{1}{2} \frac{z}{(M_P)^{m-1}} \bar{\Phi}^m \bar{N} \bar{N} , \]  

(10)
if $Z_n = Z_2$. The power $m$ in eqs. (9) and (10) is a solution of the equation $2X_N + mX_\Phi = 0$. In the first case, i.e. $Z_n \neq 2$, the tree-level contributions to neutrino masses are:

$$m_D = k_N \frac{A_Z v}{M_P}, \quad M_R = \frac{z}{2} \left( \frac{v_\Phi}{M_P} \right)^m \frac{A^2_Z}{M_P}, \quad m_L = \frac{h}{2} \left( \frac{v_\Phi}{M_P} \right)^m \frac{v^2}{M_P}. \quad (11)$$

If the discrete symmetry is $Z_2$, $M_R$ becomes:

$$M_R = z v_\Phi \left( \frac{v_\Phi}{M_P} \right)^{m-1}, \quad (12)$$

whereas $m_D$ and $m_L$ remain unchanged. The physical spectrum obtained depends on the values of $m$, $v_\Phi$ and $A_Z$. For $Z_n = Z_2$, and $m = 1$, it is $M_R \sim v_\Phi$. In general, however, since $v_\Phi \leq M_P$, the factor $(v_\Phi/M_P)^m$ is a genuine suppression factor and $M_R$ in eq. (12) is $< v_\Phi$. The same is true for $Z_n \neq 2$, i.e. $M_R$ in eq. (11), if $A_Z^2 \lesssim M_X^2 \simeq m^{3/2} M_P$. In particular, for sufficiently large $m$, it is $M_R \ll v_\Phi$. Moreover, if $v_\Phi$ is sufficiently close to $M_P$, the spectrum is similar to that obtained with the superpotential in eq. (9). If $v_\Phi$ is, for example, as in the seesaw case, $\simeq 10^{12}$ GeV, and $m = 1$, then, for $A_Z^2 \simeq M_X^2 \simeq m^{3/2} M_P$, $\nu_2$ would tend to be too heavy, $\sim 10^6$ eV, and $m_{\nu_1} \sim 10$ eV. For larger $m$, the suppression factor becomes increasingly more effective, and the two eigenvalues tend to $\pm k_N A_Z v / M_P$. Again, a solution to the problem may be obtained if $A_Z^2 \ll M_X^2$ is possible. Similar considerations hold in the case of $Z_2$.

### 2.3 Lepton-number violation; 4 singlets: 3 $\bar{N}$’s, 1 $S$

This case is slightly different from the previous ones, in that renormalizable operators that give rise to the usual seesaw mechanism are allowed, whereas it is only the renormalizable Yukawa operator for the fields $S$ that is forbidden. A non-renormalizable operator of the Yukawa type, mediated by a SM singlet $Z$, is allowed. The superpotential is, in this case:

$$W = y \bar{N} LH + \frac{1}{2} M_R \bar{N} \bar{N} + \frac{1}{4} h \frac{LHLH}{M_P} + \frac{k_S}{M_P} \bar{Z} S L H, \quad (13)$$

where, for simplicity, lepton number is assumed to be explicitly broken. The modifications for the case of spontaneous lepton-number violation, which requires an additional gauge group, are obvious. Notice that all bilinear mass terms $SS$ and $\bar{N}S$ are understood to be forbidden as in string scenarios in which $S$ is a moduli field. It is also understood that some string-related dynamical feature forbids the non-renormalizable superpotential operator $(1/M_P)ZZSS$, which could otherwise induce very dangerous scalar interaction terms. Finally, by assuming that the fields relevant for the generation of neutrino masses are charged under a $U(1)_R$ group, with the following $R$-charges:

$$R(Z) = 0, \quad R(H) = 0, \quad R(L) = 1, \quad R(\bar{N}) = 1, \quad R(S) = 1, \quad (14)$$
a similarly dangerous operator in the Kähler potential, \((1/M_P^2)Z^\dagger ZSS\), is also forbidden. The consequences that both operators, \((1/M_P)ZZSS\) and \((1/M_P^2)Z^\dagger ZSS\), could have are discussed in Sec. 4.

By integrating out the heavy fields \(\bar{N}\), the usual effective superpotential

\[
W = -\frac{y^2}{2M_R}LHLH + \frac{k_S}{M_P}ZSLH,
\]

is obtained, where the subleading operator, with coupling \(h\), is neglected. Four light states are now present, three with Majorana masses \(m_L\), one with Dirac mass \(m_{Ds}\):

\[
m_{Ds} = k_S \frac{A_Z v}{M_P}, \quad m_L = -\frac{y^2 v^2}{M_R}.
\]

Values of \(M_R \sim 10^{13}–10^{14}\) give rise to \(m_L\)'s in the range 0.1–1 eV, whereas \(m_{Ds}\) is again too large if \(A_Z^2 \simeq M_X^2 \simeq m_{3/2} M_P\).

### 3 An interesting class of models

An interesting class of models was introduced in the past years, in which the dynamics of a strongly interacting \(SU(2)\) gauge group induces the breaking of supersymmetry through non-perturbative effects \([6, 7]\). The scale of supersymmetry breaking coincides with the dynamical scale \(\Lambda\) of this gauge interaction. A fundamental role is played by a superfield \(Z\), singlet of \(SU(2)\) and of the SM gauge group, which is the field that acquires a supersymmetry-breaking vev \(F_Z \sim \Lambda^2\). (For the definition of the field \(Z\), see Ref. \([8]\).) This field is identified with the singlet mediating the non-renormalizable neutrino operators in eqs. \((1), (5), (9),\) and \((13)\).

It is known that in these models the \(Z\) direction is flat at the tree level, but it is lifted by loop corrections. Corrections to the Kähler potential from the strong \(SU(2)\) interaction are non-calculable and can only be estimated. They give rise to a quadratic term in \(Z\) in the scalar potential, with coupling \(k\). Making a specific dynamical assumption on the sign of this term, i.e. \(k > 0\), a tiny vev for \(Z\) is induced \([9]\):

\[
A_Z \simeq \frac{m_{3/2}}{\lambda k},
\]

where \(\lambda\) is the coupling of the effective low-energy superpotential obtained after integrating out heavy fields active under the strong \(SU(2)\). It was shown in \([10]\), however, that a quadratic term in the scalar potential is obtained from loops of light particles. These corrections are calculable and larger than the non-calculable one as long as \(\lambda\) is in a perturbative regime \([10]\). This quadratic term induces a vev:

\[
A_Z \simeq \frac{16\pi^2}{\lambda^3} m_{3/2}.
\]
In the following, $\lambda$ is assumed to be in the perturbative regime and only the value in eq. (18) is used for the vev $A_Z$. (The numerical result obtained in Ref. [5] for the mass of the sterile neutrino, makes use of the value of $A_Z$ in eq. (17), corresponding to a non-perturbative coupling $\lambda$.)

The ratio $A_Z/M_P$ in eqs. (4), (7), and (11) may be sufficiently small to induce suitable values of neutrino masses. Indeed, when the breaking of supersymmetry is transmitted to the SM visible sector via gravitational interactions ($m_3/2 \simeq 1$ TeV), the ratio $A_Z/M_P$ is of order $\sim 10^{-13}$. It is needless to say that, if the transmission of supersymmetry breaking is via gauge interactions (in general, $m_3/2 \ll m_{\text{weak}}$), this ratio may give rise to contributions to neutrino masses that are too small. The radiative mechanism may then be the only one to generate sizable values of $m_L$ and $m_D$ in this class of models.

Notice that in the case of the superpotentials in eqs. (1), (5), and (9), the singlet $Z$ is charged under the discrete group $Z_n$, while it is neutral in the physics picture described by the superpotential in eq. (13). In this picture, $Z$ can couple to vector multiplets, giving rise to gaugino masses and can therefore be the main supersymmetry-breaking agent, when the information of supersymmetry breaking is transmitted to the SM visible sector via gravitational interactions. It is assumed in Ref. [5] that $F_Z \sim \Lambda^2 \sim M_X^2 \sim m_{3/2} M_P$. In the scenarios described by the superpotential in eqs. (1), (5), and (9), $Z$ cannot induce gaugino masses and it does not need to be the singlet with the largest supersymmetry-breaking vev. Thus, it can be $F_Z \sim \Lambda^2 < M_X^2 \sim m_{3/2} M_P$. The situation is obviously more free when the breaking of supersymmetry is transmitted to the visible sector by gauge interactions, which induce gaugino masses at the quantum level. In this case, $F_Z$ can be smaller (and even considerably smaller) than $M_X^2$.

### 4 Scalar interactions

In addition to the mass terms $\tilde{m}_N^2 |\tilde{N}|^2$ and $\tilde{m}_S^2 |\tilde{S}|^2$ induced by Kähler potential operators, trilinear as well as bilinear terms holomorphic in the scalar components of $\tilde{N}$ and $S$ are also generated.

The non-renormalizable operators $(1/M_P) Z \tilde{N} \tilde{L} H$ and $(1/M_P) Z \tilde{S} \tilde{L} H$ give rise to the terms:

$$ A_{\tilde{N}} \bar{\tilde{N}} \tilde{L} H \,, \quad A_{\tilde{S}} \bar{\tilde{S}} \tilde{L} H \,, \quad (19) $$

with dimensionful couplings given by:

$$ A_{\tilde{N}} = k_{\tilde{N}} \left( \frac{F_Z}{M_P} \right) \,, \quad A_{\tilde{S}} = k_{\tilde{S}} \left( \frac{F_Z}{M_P} \right) \,. \quad (20) $$

These are of order of the gravitino mass if $F_Z$ is the maximal supersymmetry-breaking vev, and therefore $\gtrsim m_{\text{weak}}$ in gravity-mediated scenarios of transmission of supersymmetry.
breaking, for couplings $k_{\tilde{N}}$ and $k_{\tilde{S}}$ of $\mathcal{O}(1)$. $A_{\tilde{N}}$ can be much smaller than $m_{\text{weak}}$ when this assumption is dropped, i.e. when $\mathcal{F}_Z < M_X^2$, or in gauge-mediated scenarios of supersymmetry breaking.

Bilinear terms, such as:

$$B_{\tilde{N}}^2 \tilde{N} \tilde{N}, \quad B_{\tilde{S}}^2 \tilde{S} \tilde{S},$$

are also generated, the first in the models of Secs. 2.1.2, and 2.2 (no such term is possible in the model of Sec. 2.1.1 in which lepton number is conserved), the second in the model described in Sec. 2.3.

In the models of Secs. 2.1.2, and 2.2, in which lepton number is violated explicitly or spontaneously, due to the presence of additional symmetries, a supersymmetric mass term $(1/2)M_R N N$ is induced by non-renormalizable operators, Thus, a bilinear term

$$B_{\tilde{N}}^2 = m_{3/2} M_R$$

is always generated via supergravity effects. Moreover, in the model of Secs. 2.1.2, and that of Sec. 2.2 with $Z_\eta \neq 2$, a term

$$B_{\tilde{N}}^2 = \frac{\mathcal{F}_Z Z S}{A_Z} M_R$$

is directly induced by the relevant non-renormalizable terms in the superpotentials of eqs. (5) and (9). In gravity-mediated scenarios of transmission of supersymmetry breaking, no instabilities or tachyonic eigenvalues for the scalar components of $\tilde{N}$ arise, as far as $B_{\tilde{N}}^2 < M_R^2$, if $M_R \gg \tilde{m}_{\tilde{N}}$, or as far as $B_{\tilde{N}}^2 < \tilde{m}_{\tilde{N}}^2$, if $M_R < \tilde{m}_{\tilde{N}}$. All potentially dangerous situations can be avoided by requiring $\mathcal{F}_Z < \tilde{M}_X^2 \sim 3/2 M_P$. (See discussion in Ref. [2] and in the previous Section.) No problems in general arise in scenarios in which the transmission of supersymmetry breaking is via gauge interactions.

In the model of Sec. 2.3, chiral-lepton number is violated through the field $\tilde{N}$, but a renormalizable mass term for the field $S$ is forbidden by “stringy” reasons. However, if no charges are assigned to any of the relevant fields, it is difficult to forbid the non-renormalizable operator $(1/M_P)Z Z S S$ already mentioned in Sec. 2.3. Indeed, the combination of fields $Z S$ has, in principle, the same quantum numbers of the field $\tilde{N}$, unless some higher scale string dynamics is advocated to distinguish between $Z S$ and $\tilde{N}$. Thus, forbidding $(1/M_P)Z Z S S$ would be equivalent to forbidding the see-saw operator $(1/M_R) L H L H$. Such an operator would give rise to an acceptable tree-level Majorana mass for the sterile neutrino, of order $A_Z^2/M_P \sim (16\pi^2/\lambda^3)2 m_{3/2}^2/M_P$, with mixing $(v/m_{3/2})$ to the active neutrinos. It would, however, also give rise to a tachyonic scalar component of the field $S$, and to a very large radiative contributions to the Majorana mass of active neutrinos, through the large bilinear term

$$B_{\tilde{S}}^2 = \frac{16\pi^2}{\lambda^3} m_{3/2}^2.$$
Similarly, the Kähler potential operator \( (1/M^2) Z^\dagger Z SS \), would induce the dangerous value of \( B_S^2 \):

\[
B_S^2 = m_{3/2}^2.
\]

The simple assignment of \( R \)-charges in eq. (14) can, however, forbid this operator.

## 5 Stability of the scalar potential

We start by discussing the scalar potential relevant for the model described in Sec. 2.3. It is easy to see that this potential has a \( D \)-flat direction when:

\[
\tilde{L} = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ \phi \end{pmatrix}, \quad H = \frac{1}{\sqrt{2}} \begin{pmatrix} \phi \\ 0 \end{pmatrix},
\]

where \( H \) is here understood as the scalar component of the superfield indicated by the same symbol. For a choice of phases such that \( \phi^2 \tilde{S} = -|\phi|^2 |\tilde{S}| \), the scalar potential for \( \tilde{S} \) and \( \phi \) around the origin, is given by:

\[
V = \tilde{m}_\phi^2 |\phi|^2 + \tilde{m}_S^2 |\tilde{S}|^2 - 2k_S m_{3/2} |\phi|^2 |\tilde{S}| + \cdots.
\]

In this expression, the ellipsis denotes higher order terms, suppressed by small factors of \( (m_{\text{weak}}/M_R) \), \( (m_{\text{weak}}/M_P) \), or \( (A_Z/M_P) \), as long as the field values \( |\phi| \) and \( |\tilde{S}| \) are not too large compared to the electroweak scale. It is clear that such a potential has a minimum deeper than the origin for values of the field \( |\phi| \) and \( |\tilde{S}| \) larger than the electroweak scale. In fact, the potential in eq. (27) crosses the plane \( V = 0 \) at the following field values:

\[
|\tilde{S}_0| \approx \frac{1}{2k_S} \left( \frac{\tilde{m}_\phi^2}{m_{3/2}} \right) (1 + a),
\]

\[
|\phi_0| \approx \frac{1}{2k_S} \left( \frac{\tilde{m}_\phi \tilde{m}_S}{m_{3/2}} \right) \left( \frac{1 + a}{\sqrt{a}} \right),
\]

where \( a \) is an arbitrary real number. These values are of the order of weak scale and tunneling to the unwanted minimum may occur.

However, an estimate of the tunneling rate to the false vacuum shows that a value of the coupling \( k_S < 1 \) by about one order of magnitude, is sufficient to guarantee that the origin is a metastable vacuum. The tunnelling rate can be estimated from the four dimensional Euclidean action \( S_4 \) evaluated with the bounce solution for the potential in eq. (27). (See also discussion in [12].) The tunneling rate per volume is roughly given by \( \Gamma_4 \sim \tilde{m}^4 \exp(-S_4) \), where \( \tilde{m} \) is a typical scale of the potential, \( \tilde{m} \sim \tilde{m}_\phi \sim \tilde{m}_S \sim m_{3/2} \). The requirement that the false vacuum has not decayed in our past light-cone gives a constraint \( \Gamma_4 L^4 \ll 1 \), where \( L \) is the present size and age of the visible universe. Thus,
$S_4$ must satisfy $S_4 > 400 + 4 \ln (m/\text{TeV})$. On the other hand, from eq. (27), by redefining $\phi$ and $\tilde{S}$, one can find $S_4 \sim (1/k_S^2) \times \tilde{S}_4$, where $\tilde{S}_4$ is a dimensionless numerical factor of $O(10)$ [13]. Therefore, a value $k_S \approx 0.1$ is sufficient to avoid tunneling to the dangerous vacuum. Even if $k_s \approx 0.01$, we may reproduce the result in Ref. [4] adopting the value $A_Z$ in eq. (18).

Notice that a similar problem may arise in the models described in Secs. 2.1 and 2.2, except for the case in which an unsuppressed mass term for the superfield $N$ is present (see the superpotential in eq. (10) with $m = 1$). With a choice of phases for $\phi$ and $\tilde{N}$ similar to that made earlier for $\phi$ and $\tilde{S}$, the scalar potential is in these cases:

$$V = \tilde{m}_\phi^2 \phi^2 + \tilde{m}_N^2 |\tilde{N}|^2 - 2k_N \left( \frac{F_Z}{M_P} \right) |\phi|^2 |\tilde{N}| + \cdots .$$

(30)

As mentioned earlier, however, in these models the singlet $Z$ does not need to have the maximal value of supersymmetry-breaking $vev F_Z$, as it is assumed in the model of Sec. 2.3. Therefore, a suppression of the trilinear term $|\phi|^2 |\tilde{N}|$ may be achieved through a smaller value of $F_Z$, while keeping the coefficient $k_N \sim 1$. (This and other types of solutions were advocated for the analogous problem present in scenarios in which tree-level Yukawa couplings for quarks and leptons are forbidden by symmetries [14].)

6 Another cosmological problem

It was shown in the previous Section that the scalar potential in eq. (27), as well as that in eq. (30), if $F_Z/M_P \sim m_3/2$, have minima deeper than the origin. The vacuum tunneling from the origin to these minima can be easily avoided. However, another cosmological problem may, in principle, arise. If the values of the scalar fields $\tilde{S}$ or $\tilde{N}$ are larger than the weak scale at the end of inflation, they do not roll toward the vacuum in which we live, but towards the unwanted minimum, which is far from the origin, and the desired vacuum is never realized. However, if a scalar field is charged, it is naturally expected that at the end of inflation such a field is localized at the origin, which is an enhanced symmetry point [15]. The fields $\tilde{N}$ are charged in all the models outlined above (see Ref. [2]) and this problem is naturally avoided. The $R$-charges introduced in eq. (14) to forbid a dangerous Kähler potential operator, turns out to be very important to prevent this possible cosmological problem, by forcing the field $S$ at the origin.

7 Radiative contributions to neutrino masses

Radiative contributions to Dirac masses $m_D$ and Majorana masses $m_L$ arise from the two diagrams shown in Fig. [1] and/or the diagram in Fig. [4], depending on the model
Figure 1: Diagrams contributing to the Dirac and Majorana neutrino masses $m_D$ and $m_L$ discussed. The first diagram in Fig. 1 requires a gauge interaction for the field $\bar{N}$ and therefore is present only when the SM gauge group is extended (see Sec. 2.2). The two diagrams giving rise to the Majorana masses $m_L$ are possible when chiral-lepton number is violated, specifically if bilinear terms $B^2_{\bar{N}}\bar{N}\bar{N}$ and $B^2_{\bar{S}}\bar{S}\bar{S}$ exist.

An explicit evaluation of these loop diagrams yields the following results. The radiative contribution to Dirac masses $m_D$ is:

$$m_D \sim \frac{1}{8\pi^2} (g_X X_N) (g_Y Y_L) \left( \frac{A_{\bar{N}} v}{\tilde{m}} \right) \left( \frac{\tilde{m}}{M_G} \right)^2$$

(31)

where $g_X$ and $g_Y$ are respectively the coupling constants of the additional gauge group $U(1)_{B-L}$ and of $U(1)_Y$; $X_N$ is the $U(1)_{B-L}$ charge of the superfields $\bar{N}$, $Y_L$ is the $U(1)_Y$ charge of the doublet superfield $L$. Finally, $M_G$ is the scale of $U(1)_{B-L}$ violation: $M_G \propto v_\Phi = v_\Phi$ and $\tilde{m}$ is the typical soft mass for scalar fields and light gauginos.

The radiative contribution to Majorana masses $m_L$ shown in the second diagram of Fig. 1 i.e. due to the exchange of $\bar{N}$, is:

$$m_L \sim \frac{1}{8\pi^2} (g_Y Y_L)^2 \left( \frac{A_{\bar{N}} v}{\tilde{m}} \right)^2 \left( \frac{B M_R}{\tilde{m}} \right) \left( \frac{\tilde{m}}{M_R} \right)^4$$

(32)

if the Majorana mass for the fields $\bar{N}$ is $M_R \gg m_{\text{weak}}$, whereas it is:

$$m_L \sim \frac{1}{8\pi^2} (g_Y Y_L)^2 \left( \frac{A_{\bar{N}} v}{\tilde{m}} \right)^2 \left( \frac{B M_R}{\tilde{m}} \right),$$

(33)

for $M_R \ll m_{\text{weak}}$. Notice that the symbol $B^2_{\bar{N}}$ in the Figure is here replaced by $B M_R$.

It is interesting to see that in a seesaw type of setting, i.e. when $M_G$ and $M_R$ are very large compared to $\tilde{m}$, $m_L$ and $m_D$, and therefore the final mass eigenvalues, are more suppressed by the large scales than in the typical seesaw mechanism. A part for a loop suppression factor $(1/8\pi^2)$, they are reduced away from the electroweak scale respectively by factors $(\tilde{m}/M_G)^2$ and $(\tilde{m}/M_R)^3$. If the tree-level contributions to $m_D$ and $m_L$ are
forbidden as in the model of Sec. 2.2, the scale of $U(1)_{B-L}$ can be much smaller than in the typical seesaw mechanism. Notice that in this model $M_G$ and $M_R$ are in general disentangled and fixing the value of one ($M_G$) to get reasonable values of $m_D$, does not necessarily imply that $m_L$ is very small.

If $M_R$ is very small, and $B^2_N = BM_R$ is also small, the suppression to $m_L$ comes from $(BM_R/\tilde{m}^2)$ and $A_S/\tilde{m}$. This is particularly true of the model of Sec. 2.1.2 for which the options $F_Z < M^2_X$, or $m_{3/2} < m_{\text{weak}}$ as when the transmission of supersymmetry breaking is mediated by gauge interactions, turns out to be of crucial importance.

In the model of Sec. 2.3, since $M_R$ and possibly $M_G$ (if the model is realized in such a way to have spontaneous violation of lepton number) are large, i.e. $\sim 10^{13}$ GeV, the radiative contributions to $m_D$ and $m_L$ from the two diagrams of Fig. 1 are very small. A radiative contribution to $m_L$, however, arises from from the exchange of $\tilde{S}$ as shown in Fig. 2. The natural scale for $\tilde{S}$ is the soft mass $\tilde{m}$. Therefore, the expression for the radiative contribution to $m_L$ is similar to that in eq. (33):

$$m_L \sim \frac{1}{8\pi^2} \left( g_Y Y_L \right)^2 \left( \frac{A_S v}{\tilde{m}^2} \right)^2 \left( \frac{B^2_S}{\tilde{m}} \right)$$

(34)

Since in this model $F_Z$ is assumed to be the maximal supersymmetry breaking vev, $F_Z \sim M^2_X$, it is $A_S \lesssim \tilde{m}$ when the suppression of order of magnitude for the coupling $k_S$ discussed in Sec. 3 is kept into account. It becomes clear, then, why the values of $B^2_S$ given in eq. (24) and (25) are far too dangerous: indeed, they tend to move the value of $m_L$ towards the electroweak scale! It is therefore mandatory to have all operators involving the bilinear combination $SS$ and giving rise to the soft term $B^2_S$ sufficiently suppressed in order to avoid too large contributions to $m_L$. 

Figure 2: Radiative contribution to $m_L$ from the sterile neutrino field $S$. 

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8 Neutrino spectra and Conclusions

The neutrino spectrum obtained in the model of Sec. 2.3 is that obtained from the conventional seesaw mechanism with the addition of one light state, mainly sterile. There are, therefore, three heavy states $n_i$ with mass $\sim M_R$, and four light states $\nu_i$ and $\nu_s$. (We assume here that radiative contributions to $m_L$ are negligible.) In the basis in which charged leptons have a diagonal mass matrix, for $k_s \sim 0.1\sim 0.01$, as argued in Sec. 4 and the value of $A_Z$ in eq. (18), it is $m_{Ds} \sim 10^{-4}$ eV. The values of $m_L$ are $\sim y^2 \times 1$ eV. Therefore, if $\nu_e$ has a Majorana mass $m_L \sim 10^{-4}$ eV, the solar $\nu_e-\nu_{Ls}$ oscillation can be explained as a quasi-vacuum oscillation. With suitable choices of the couplings $y$, the other two active neutrinos together with $\nu_e$ can explain the atmospheric and LSND oscillations. The overall picture is that of the so-called 2+2 neutrino spectrum [16]. (A model for the recently advocated “1+3” picture of neutrino masses [17, 16, 18, in which all neutrino masses are also generated at the tree level, is proposed in Ref. [19].)

The model in Sec. 2.1.1 give rise to only three light neutrinos with Dirac mass given in eq. (8). The oscillation patterns of solar and atmospheric neutrino experiments have to rely on a specific texture of the couplings $k_N$ and no explanation is possible for the LSND experiment.

The models of Secs. 2.1.2 and 2.2 are more complex and allow a large variety of neutrino spectra. They have an interesting interplay among tree-level and radiative contributions to neutrino masses and replace completely the conventional seesaw mechanism.

The model (or class of models) in Sec. 2.2 may nevertheless retain some of the qualitative feature of the seesaw mechanism spectrum if the Majorana mass $M_R$ for the fields $\tilde{N}$ is large. Three light states $\nu_i$ are present, of mainly active neutrinos, and three heavy states $n_i$ with mass $\sim M_R$. In this case, however, since no tree-level Yukawa couplings exist, the small mass of the light states $\nu_i$ is not due to the pattern of light-heavy scales in the neutrino mass matrix. What moves away from the electroweak scale ($m_{\text{weak}} \sim v$) the Dirac and Majorana entries in the neutrino mass matrix are: i) suppression factors coming from non-renormalizable operators, i.e. $A_Z/M_P$ and $A_Z^2/(vM_P)$ where $A_Z$ is a vacuum expectation value induced by supersymmetry breaking; ii) the factors $1/(16\pi^2)(\tilde{m}/M_R)^3$ and $1/(16\pi^2)(\tilde{m}/M_G)^2$, originated by loop diagrams, where $\tilde{m}$ is a typical mass softly breaking supersymmetry and $M_G$ is the scale of the additional gauge group present in the model.

Both classes of models in Secs. 2.1.2 and 2.2 however, may have rather small Majorana masses $M_R$ at the tree-level and can easily realize the possibility of light sterile neutrinos. Exact predictions depend on the particular realization of these models. The six states $\nu_i$ and $n_i$ are all light. The three $n_i$-neutrinos, however, can still be heavier than the $\nu_i$'s, in which case, the mixing angles between $n_i$'s and $\nu_i$'s are very small. There is also the other interesting case in which the $\nu_i$ and $n_i$ states are roughly at the same
scale or nearly degenerate, with mixing angles ranging from small to maximal. Flavour oscillations, therefore, can be accommodated in this class of scenarios, relying on specific textures of the tree-level couplings of non-renormalizable operators, or of the trilinear and bilinear neutrino soft parameters. Oscillations among active and sterile neutrinos are also possible.

Since neutrino masses are much smaller than all other masses, it is plausible to assume that their origin is different from that of the other lepton and quark masses. They may be induced, after electroweak symmetry breaking, by operators of higher dimensionality than the usual Yukawa operators or by quantum effects. We have shown that both possibilities are easily implemented in supersymmetric models and that the generation of neutrino masses is intimately linked to the breaking of supersymmetry. In this sense, neutrinos may be fundamentally different from all other fermions. The recently gathered evidence pointing to the fact that they are massive and the pattern of oscillation among different neutrinos, may therefore be the first evidence for supersymmetry.

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