Cellular Multi-User Two-Way MIMO AF Relaying via Signal Space Alignment: Minimum Weighted SINR Maximization

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Abstract

In this paper, we consider linear MIMO transceiver design for a cellular two-way amplify-and-forward relaying system consisting of a single multi-antenna base station, a single multi-antenna relay station, and multiple multi-antenna mobile stations (MSs). Due to the two-way transmission, the MSs could suffer from tremendous multi-user interference. We apply an interference management model exploiting signal space alignment and propose a transceiver design algorithm, which allows for alleviating the loss in spectral efficiency due to half-duplex operation and providing flexible performance optimization accounting for each user’s quality of service priorities. Numerical comparisons to conventional two-way relaying schemes based on bidirectional channel inversion and spatial division multiple access-only processing show that the proposed scheme achieves superior error rate and average data rate performance.

Index Terms

Two-way relaying, amplify-and-forward, signal space alignment, interference management, linear multi-user MIMO transceiver design, quality of service constraints, multigroup multicast, second-order cone programming.

I. INTRODUCTION

The use of relays to improve link reliability and coverage of cellular wireless communication systems has attracted significant research interest since the pioneer works [1]–[4], and various
Relaying protocols are embraced by state-of-the-art and next generation commercial standards [5]–[7]. In practice, most relaying protocols operate in a half-duplex manner and transmission is divided into two phases using orthogonal channel accesses: in the first phase the source node broadcasts its message, and in the second phase the relay station (RS) forwards the source message to the destination node. The deficiency of half-duplex relaying is that when there is no direct link between the source and destination nodes the end-to-end transmission can only achieve half the degrees of freedom (DoF) of the channel.

Two-way relaying is a promising means to alleviate the loss in spectral efficiency due to half-duplex operation. Specifically, given a pair of terminal nodes that are to exchange data, we allow transmissions in both directions to occur concurrently and reduce the total transmission time by half. The bidirectional transmissions will mutually interfere with each other; nonetheless, we can mitigate the impact of interference by employing spatial division multiple access (SDMA) processing at the RS [5], or by applying the principles of analogue network coding (ANC) [9] or physical layer network coding (PNC) [10]. By means of ANC, the RS performs amplify-and-forward (AF) relaying, and the terminal nodes utilize the a priori knowledge of their own transmitted signals to cancel the self-induced backward propagated interference. On the other hand, by means of PNC the RS attempts to decode-and-forward (DF) the network coded version of the terminal node signals, and the terminal nodes utilize the knowledge of their own signals to decode the network code. Note that PNC has strict feasibility requirements for the precoding and modulation and coding (MCS) schemes used at the terminal nodes (cf. [10, Proposition 1]).

It is, however, nontrivial to extend the two-way relaying protocol to cellular multi-user systems. In this case, each node experiences self-induced interference as well as multi-user interference, and thus necessitates more sophisticated interference management techniques. To shed insight on designing an efficient scheme for cellular multi-user two-way relaying, we first review the qualities and limitations of the prominent related works.

**Single-User Two-Way AF Relaying:** In [9], [11]–[15], the authors consider two-way AF relaying between two terminal nodes, and propose linear transceiver designs subject to various performance metrics (e.g., sum rate maximization and error rate minimization). In [16], the authors analyze the random coding error exponent in two-way AF relay networks and investigate rate and power allocation. However, these single-user designs cannot be easily extended to multi-user systems as they do not accommodate for the presence of multi-user interference.
Multi-User Two-Way AF Relaying with Fixed Terminal Node Transceivers: In [8], [17], the authors consider two-way AF relaying between multiple pairs of terminal nodes. These works neither exploit self-interference cancelation nor optimize the terminal node transceivers (i.e., the transceivers are predetermined offline). Instead, they rely solely on conventional SDMA processing at the RS to mitigate the effects of interference. In [18]–[20], the authors consider cellular two-way relaying between a multi-antenna base station (BS) and multiple single-antenna mobile stations (MSs). Since the MSs are only equipped with a single antenna, they cannot apply MIMO interference mitigation techniques. The focus of these works is to jointly design the BS and RS transceivers to heuristically perform UL-DL bidirectional channel inversion, thereby simultaneously zero-force the multi-user interference from the received signals of the BS and the MSs.

Two-Way DF Relaying: In [10], [21], [22], the authors consider single-user two-way DF relaying with PNC. Albeit theoretically promising, the application of PNC is subject to stringent feasibility requirements that greatly restrict the choices of the MCS schemes that could be employed, and the decoding operation at the RS has high computational complexity. In [23], the authors consider the unique scenario of three-user three-way DF relaying using the arguments of interference alignment (IA) [24]–[26]. Yet, this scheme cannot be easily extended to a general number of users as IA may be infeasible.

In this paper, we consider a cellular system consisting of a BS, an RS, and multiple MSs. All nodes are equipped with multiple antennas. We seek to design linear MIMO transceiver for each node to facilitate efficient two-way AF relaying. The contributions and technical challenges of this work are as follows.

- **Two-Way Relaying by Virtue of Signal Space Alignment:** We show that for the cellular multi-user two-way relaying system under study, the MSs could suffer from tremendous multi-user interference. Yet, exploiting the advantage of self-interference cancelation, we can align the signal spaces of the uplink (UL) and downlink (DL) signals to reduce the dimensions occupied by multi-user interference at each MS. Ultimately, this allows us to alleviate the half-duplex loss and achieve the DoF of the channel.

The paradigm of two-way relaying exploiting signal space alignment is also considered in different contexts in [27], [28]. Specifically, in [27] the authors consider a single-user two-way MIMO relaying system, for which they propose a precoding design that align the two-
way signals and deduce an algorithm for optimizing the basis of the aligned signal space. However, this single-user design cannot be easily extended to multi-user systems as it does not accommodate for the presence of multi-user interference. On the other hand, in [28] the authors consider a multi-user two-way multi-carrier relaying system, for which they propose different frequency domain precoding designs based on aligning the two-way signals of each pair of communicating terminal nodes. These frequency domain precoding designs neglect the impact of multi-user interference, and rely on the intrinsic high frequency diversity to mitigate the impact of interference. Note that it is non-trivial to extend these precoding designs to practical two-way MIMO relaying systems whose signal space dimensions are not large.

- **Algorithm for Two-Way Relay Transceiver Design with Quality of Service Constraints:**

  In consideration of the fact that users in cellular systems have different quality of service (QoS) priorities, we formulate the two-way relay transceiver design problem to maximize the minimum weighted per stream signal-to-interference plus noise ratio (SINR) among all UL and DL data streams. This problem does not lead to closed-form solutions and is non-convex, and we propose to solve it using a two-stage algorithm. In the first stage we focus on attaining signal space alignment, and in the second stage we aim at optimizing the weighted per stream SINRs. We show that the second stage subproblem belongs to the class of multigroup multicast problems, which are NP-hard [30, Claim 2]. So we further propose an algorithm to efficiently solve the second stage subproblem using second-order cone programming (SOCP) techniques [31, Section 4.4.2].

  **Outline:** The rest of this paper is organized as follows. In Section II we present the system model. In Section III we discuss the interference management model and formulate the two-way relay transceiver design problem. In Section IV we present the proposed transceiver design algorithm. In Section V we present numerical simulation results. Finally, in Section VI we conclude the paper.

  **Notations:** $\mathbb{C}^{M \times N}$ denotes the set of complex $M \times N$ matrices. Upper and lower case bold letters denote matrices and vectors, respectively. $\text{vec}(\mathbf{X})$ denotes the column-by-column vectorization of $\mathbf{X}$. $[\mathbf{X}_1; \ldots; \mathbf{X}_N]$ and $[\mathbf{X}_1, \ldots, \mathbf{X}_N]$ denote the matrices obtained by vertically concatenating $\mathbf{X}_1, \ldots, \mathbf{X}_N$. $[\mathbf{X}_1; \ldots; \mathbf{X}_N]$ and $[\mathbf{X}_1, \ldots, \mathbf{X}_N]$ denote the matrices obtained by vertically concatenating $\mathbf{X}_1, \ldots, \mathbf{X}_N$.

\footnote{Note that for multi-user DL unicast systems, it is shown in [29] that max-min weighted SINR MIMO transceiver designs can achieve optimal solutions under the special case of rank one channels.}
and horizontally concatenating \( \mathbf{X}_1, \ldots, \mathbf{X}_N \), respectively. \( \text{diag}(\mathbf{X}_1, \ldots, \mathbf{X}_N) \) denotes a block diagonal matrix having \( \mathbf{X}_1, \ldots, \mathbf{X}_N \) in the main diagonal. \( [\mathbf{X}]_{(a:b, c:d)} \) denotes the \( a \)-th to the \( b \)-th row and the \( c \)-th to the \( d \)-th column of \( \mathbf{X} \). \( (\cdot)^T \), \( (\cdot)^H \), and \( (\cdot)^* \) denote transpose, Hermitian transpose, and conjugate, respectively. \( \text{range}(\mathbf{X}) \) denotes the column space of \( \mathbf{X} \). \( \text{null}(\mathbf{X}) \) denotes the orthonormal basis for the null space of \( \mathbf{X} \). \( \text{rank}(\mathbf{X}) \) and \( \text{nullity}(\mathbf{X}) \) denote the rank and the nullity of \( \mathbf{X} \), respectively. \( \text{pinv}(\mathbf{X}) \) denotes the pseudo-inverse of \( \mathbf{X} \). \( \Re(y) \) denotes the real component of \( y \). \( ||\mathbf{X}|| \) denotes the Frobenius norm of \( \mathbf{X} \). \( \succeq_K \) denotes the generalized inequality with respect to the second-order cone, i.e., \( [y; \mathbf{x}] \succeq_K 0 \) means that \( y \geq ||\mathbf{x}|| \). \( \mathbf{X}_1 \otimes \mathbf{X}_2 \) denotes the Kronecker product of \( \mathbf{X}_1 \) and \( \mathbf{X}_2 \). \( \mathbf{x} \sim \mathcal{CN}(\mu, \Xi) \) denotes that \( \mathbf{x} \) is complex Gaussian distributed with mean \( \mu \) and covariance matrix \( \Xi \). \( \mathbb{E}(\cdot) \) denotes expectation. \( \mathcal{K} \) denotes the index set \( \{1, \ldots, K\} \) and \( \mathcal{L}_k \) denotes the index set \( \{1, \ldots, L_k\} \). \( \mathbf{0}_{M \times N} \) denotes an \( M \times N \) matrix of zeros and \( \mathbf{I}_N \) denotes an \( N \times N \) identity matrix.

II. SYSTEM MODEL

We consider a multi-user system where a BS communicates with multiple MSs as illustrated in Fig. 1. Due to the effects of path loss and shadowing, there is no direct link between the BS and the MSs, and a half-duplex RS is deployed to assist data transmission. In conventional relay systems, UL and DL transmissions utilize non-overlapping channel accesses (cf. Fig. 1a and Fig. 1b). We adopt the two-phase two-way relaying protocol whereby UL and DL transmissions share the channel: first in the multi-access (MAC) phase the BS and the MSs concurrently transmit to the RS (cf. Fig. 1c), then in the broadcast (BC) phase the RS forwards the aggregate signals to the BS and the MSs (cf. Fig. 1d). Specifically, we are interested in a time division duplex (TDD) system where conventional one-way relaying requires four time slots to complete the UL and DL transmissions while two-way relaying requires only two time slots as depicted in Fig. 2.

The detailed model of the system under study is shown in Fig. 3. We consider two-way relaying between one BS and \( K \) MSs. For ease of exposition, we focus on the \( k \)-th MS and the same model applies to all the MSs. The BS is equipped with \( N_B \) antennas, the RS is equipped with \( N_R \) antennas, and the \( k \)-th MS is equipped with \( N_k \) antennas. In the DL, the BS transmits \( L_k \) data streams \( \mathbf{s}^{(k)}_D \in \mathbb{C}^{L_k \times 1} \) to the \( k \)-th MS and a total of \( L = \sum_{k=1}^{K} L_k \) data streams \( \mathbf{s}_D \triangleq [\mathbf{s}^{(1)}_D; \ldots; \mathbf{s}^{(K)}_D] \) to all the MSs. In the UL, the \( k \)-th MS transmits \( L_k \) data streams
\( \mathbf{s}_U^{(k)} \in \mathbb{C}^{L_k \times 1} \) to the BS, and altogether the MSs transmit \( L \) data streams \( \mathbf{s}_U \triangleq [\mathbf{s}_U^{(1)}; \ldots; \mathbf{s}_U^{(K)}] \) to the BS. We make the following assumptions about the data model.

Assumption 1 (Data Model): All data streams are independent and have unit power. The covariance matrix of the DL data streams is given by \( \mathbb{E}(\mathbf{s}_D(\mathbf{s}_D)^\dagger) = \mathbf{I}_L \), and the covariance matrix of the UL data streams is given by \( \mathbb{E}(\mathbf{s}_U^{(k)}(\mathbf{s}_U^{(k)})^\dagger) = \mathbf{I}_{L_k} \).

A. Two-Way Relaying MAC Phase

In the MAC phase, the BS precodes the DL data streams \( \mathbf{s}_D \) using the precoder matrix \( \mathbf{W}_B \triangleq [\mathbf{W}_{B,1}, \ldots, \mathbf{W}_{B,K}] \), where \( \mathbf{W}_{B,k} \in \mathbb{C}^{N_B \times L_k} \) is the precoder matrix for data streams \( \mathbf{s}_D^{(k)} \). Thus, the transmitted signals of the BS are given by \( \mathbf{x}_B = \mathbf{W}_B \mathbf{s}_D \). Similarly, the \( k \)-th MS precodes the data streams \( \mathbf{s}_U^{(k)} \) using the precoder matrix \( \mathbf{W}_k \in \mathbb{C}^{N_k \times L_k} \), and the transmitted signals of the \( k \)-th MS are given by \( \mathbf{x}_k = \mathbf{W}_k \mathbf{s}_U^{(k)} \). We make the following assumptions about the transmit power constraints of the BS and the MSs.

Assumption 2 (BS and MS Transmit Power Constraints): The maximum transmit power of the BS is given by \( \mathbb{E}(||\mathbf{x}_B||^2) = ||\mathbf{W}_B||^2 \leq P_B \). The maximum transmit power of the \( k \)-th MS is given by \( \mathbb{E}(||\mathbf{x}_k||^2) = ||\mathbf{W}_k||^2 \leq P_k \).

Let \( \mathbf{H}_{R,B} \in \mathbb{C}^{N_R \times N_B} \) denote the channel matrix from the BS to the RS, and let \( \mathbf{H}_{R,k} \in \mathbb{C}^{N_R \times N_k} \) denote the channel matrix from the \( k \)-th MS to the RS. It follows that the received signals of the RS can be expressed as

\[
\mathbf{y}_R = \sum_{m=1}^{K} \mathbf{H}_{R,m} \mathbf{x}_m + \mathbf{H}_{R,B} \mathbf{x}_B + \mathbf{n}_R = \mathbf{U} \mathbf{s}_U + \mathbf{D} \mathbf{s}_D + \mathbf{n}_R, \tag{1}
\]

where \( \mathbf{U} \triangleq [\mathbf{H}_{R,1} \mathbf{W}_1, \ldots, \mathbf{H}_{R,K} \mathbf{W}_K] \) and \( \mathbf{D} \triangleq \mathbf{H}_{R,B} \mathbf{W}_B \) represent the MAC phase effective channel matrices of the UL and DL data streams, respectively, and \( \mathbf{n}_R \sim \mathcal{CN}(0_{N_R \times 1}, N_0 \mathbf{I}_{N_R}) \) is the AWGN. We make the following assumptions about the channel model.

Assumption 3 (Channel Model): All channels are independent and exhibit quasi-static fading such that the channel matrices \( \{\mathbf{H}_{R,1}, \ldots, \mathbf{H}_{R,K}, \mathbf{H}_{R,B}\} \) remain unchanged during a fading block of two time slots spanning the MAC and BC phases. Moreover, the forward and reverse channels are reciprocal: the channel matrix from the RS to the \( k \)-th MS is given by \( (\mathbf{H}_{R,k})^T \), and the channel matrix from the RS to the BS is given by \( (\mathbf{H}_{R,B})^T \). Without loss of generality, we assume \( \text{rank}(\mathbf{H}_{R,k}) \geq L_k \) and \( \text{rank}(\mathbf{H}_{R,B}) \geq L \).
B. Two-Way Relaying BC Phase

In the BC phase, the RS amplifies the received signals $y_R$ using the transformation matrix $W_R \in \mathbb{C}^{N_R \times N_R}$, and the forwarded signals are given by

$$x_R = W_R y_R = W_R U_s U + W_R D_s D + W_R n_R. \quad (2)$$

We make the following assumption about the transmit power constraint of the RS.

**Assumption 4 (RS Transmit Power Constraint):** The maximum transmit power of the RS is given by

$$\mathbb{E}(|x_R|^2) = \sum_{m=1}^{K} \| W_R H_{R,m} W_m \|^2 + \| W_R H_{R,B} W_B \|^2 + N_0 \| W_R \|^2 \leq P_R. \quad \Box$$

Accordingly, the received signals of the BS are given by

$$y_B = (H_{R,B})^T x_R + n_B = (H_{R,B})^T W_R U_s U + (H_{R,B})^T W_R n_R + n_B + i_B, \quad (3)$$

where $n_B \sim \mathcal{CN}(0_{N_B \times 1}, N_0 I_{N_B})$ is the AWGN and $i_B \triangleq (H_{R,B})^T W_R D_s D$ is the backward propagated self-interference. Likewise, the received signals of the $k$-th MS are given by

$$y_k = (H_{R,k})^T x_R + n_k = (H_{R,k})^T W_R H_{R,B} W_{B,k} s_D^{(k)} + (H_{R,k})^T W_R \tilde{D}_k s_D + \tilde{U}_k U_s U + (H_{R,k})^T W_R n_R + n_k + i_k, \quad (4)$$

where $n_k \sim \mathcal{CN}(0_{N_k \times 1}, N_0 I_{N_k})$ is the AWGN, $i_k \triangleq (H_{R,k})^T W_R H_{R,k} W_k s_U^{(k)}$ is the self-interference, and $\tilde{U}_k \triangleq [H_{R,1}W_1, \ldots, H_{R,k-1}W_{k-1}, 0_{N_R \times L_k}, H_{R,k+1}W_{k+1}, \ldots, H_{R,K}W_K]$ and $\tilde{D}_k \triangleq H_{R,B} [W_{B,1}, \ldots, W_{B,k-1}, 0_{N_B \times L_k}, W_{B,k+1}, \ldots, W_{B,K}]$ are the MAC phase effective channel matrices of the UL and DL interference streams, respectively.

C. Receive Processing

The BS and MS receive processing consists of two steps. Inherent to the two-way relaying protocol, the BS and the MSs can exploit the a priori knowledge of their own transmitted signals to cancel the backward propagated self-interference\(^2\). After that, the BS and the MSs process the resultant signals using linear equalizers to produce data stream estimates. Specifically, the BS cancels the self-interference $i_B$ from the received signals $y_B$ and processes them using the

\(^2\)We shall elaborate in Section [IV-D] the assumptions on the side information available at each node to facilitate transceiver design and self-interference cancelation.
equalizer matrix $\mathbf{V}_B \triangleq [\mathbf{V}_B^{(1)}; \ldots ; \mathbf{V}_B^{(K)}]$, where $\mathbf{V}_B^{(k)} \in \mathbb{C}^{L_k \times N_B}$ is the equalizer matrix for data streams $s^{(k)}_U$. The UL data stream estimates are given by

$$\tilde{s}_U = \mathbf{V}_B (\mathbf{y}_B - \mathbf{i}_B) = [\tilde{s}^{(1)}_U; \ldots ; \tilde{s}^{(K)}_U], \quad (5a)$$

$$\tilde{s}^{(k)}_U = \mathbf{V}_B^{(k)} (\mathbf{H}_{R,B})^T \mathbf{W}_R \mathbf{U} \tilde{s}_U + \mathbf{V}_B^{(k)} (\mathbf{H}_{R,B})^T \mathbf{W}_R \mathbf{n}_R + \mathbf{V}_B^{(k)} \mathbf{n}_B. \quad (5b)$$

In the same way, the $k$-th MS cancels the self-interference $\mathbf{i}_k$ from the received signals $\mathbf{y}_k$ and processes them using the equalizer matrix $\mathbf{V}_k \in \mathbb{C}^{L_k \times N_k}$ to produce the DL data stream estimates

$$\tilde{s}^{(k)}_D = \mathbf{V}_k (\mathbf{y}_k - \mathbf{i}_k) = \mathbf{V}_k (\mathbf{H}_{R,k})^T \mathbf{W}_R \mathbf{H}_{R,B} \mathbf{W}_{B,k} s^{(k)}_D + \mathbf{V}_k (\mathbf{H}_{R,k})^T \mathbf{W}_R (\tilde{\mathbf{D}}_k \mathbf{s}_D + \tilde{\mathbf{U}}_k \mathbf{s}_U) + \mathbf{V}_k (\mathbf{H}_{R,k})^T \mathbf{W}_R \mathbf{n}_R + \mathbf{V}_k \mathbf{n}_k. \quad (6)$$

In the UL the SINR of the data stream estimate $[\tilde{s}^{(k)}_U]_{(l)}$ is given by

$$\gamma^{(k,l)}_U \triangleq \frac{\left| [\mathbf{V}_B^{(k)}]_{(l,:)}(\mathbf{H}_{R,B})^T \mathbf{W}_R \mathbf{H}_{R,k} [\mathbf{W}_k]_{:,l} \right|^2}{\left( ||[\mathbf{V}_B^{(k)}]_{(l,:)}(\mathbf{H}_{R,B})^T \mathbf{W}_R \mathbf{U}||^2 - ||[\mathbf{V}_B^{(k)}]_{(l,:)}(\mathbf{H}_{R,B})^T \mathbf{W}_R \mathbf{H}_{R,k} [\mathbf{W}_k]_{:,l} ||^2 \right) + N_0 (||[\mathbf{V}_B^{(k)}]_{(l,:)}(\mathbf{H}_{R,B})^T \mathbf{W}_R||^2 + ||[\mathbf{V}_B^{(k)}]_{(l,:)}||^2)}, \quad (7)$$

and in the DL the SINR of the data stream estimate $[\tilde{s}^{(k)}_D]_{(l)}$ is given by

$$\gamma^{(k,l)}_D \triangleq \frac{\left| [\mathbf{V}_k]_{(l,:)}(\mathbf{H}_{R,k})^T \mathbf{W}_R \mathbf{H}_{R,B} [\mathbf{W}_{B,k}]_{:,l} \right|^2}{\left( ||[\mathbf{V}_k]_{(l,:)}(\mathbf{H}_{R,k})^T \mathbf{W}_R [\mathbf{D}_k, \tilde{\mathbf{U}}_k]||^2 - ||[\mathbf{V}_k]_{(l,:)}(\mathbf{H}_{R,k})^T \mathbf{W}_R \mathbf{H}_{R,B} [\mathbf{W}_{B,k}]_{:,l} ||^2 \right) + N_0 (||[\mathbf{V}_k]_{(l,:)}(\mathbf{H}_{R,k})^T \mathbf{W}_R||^2 + ||[\mathbf{V}_k]_{(l,:)}||^2)}. \quad (8)$$

Furthermore, the achievable data rate for each data stream can be expressed as

$$\text{UL: } C^{(k,l)}_U = \frac{1}{2} \log_2(1 + \gamma^{(k,l)}_U), \quad \text{DL: } C^{(k,l)}_D = \frac{1}{2} \log_2(1 + \gamma^{(k,l)}_D), \quad (9)$$

where the factor of 1/2 accounts for the half-duplex loss.

III. INTERFERENCE MANAGEMENT AND TRANSCEIVER DESIGN PROBLEM FORMULATION

In this section, we first discuss the motivations behind the interference management model to exploit signal space alignment. We then proceed to formulate the transceiver design problem.
A. Interference Management via Signal Space Alignment

As shown in (5) and (6), the UL and DL data stream estimates are given by

\[
\tilde{s}^{(k)}_U = V_B^{(k)} (H_{R,B})^T W_R U s_U + V_B^{(k)} (H_{R,B})^T W_R n_R + V_B^{(k)} n_B,
\]

\[
\tilde{s}^{(k)}_D = V_k (H_{R,k})^T W_R H_{R,B} W_{B,k} s^{(k)}_D + V_k (H_{R,k})^T W_R \tilde{D}_k s_D + \tilde{U}_k s_U + V_k (H_{R,k})^T W_R n_R + V_k n_k,
\]

where the DL data stream estimates \( \tilde{s}^{(k)}_D \) are prone to multi-user interference and it is nontrivial to mitigate its effects. On the one hand, it is detrimental to performance if we naively treat the multi-user interference as noise since its strength could be comparable to the desired signals. On the other hand, it is not always spectrally efficient if we were to mitigate interference by solely using conventional SDMA processing at the RS [8]. Under this approach, all the signal streams that constitute the RS forwarded signals (2) must be linearly independent, which implies that

\[
\text{rank}(\begin{bmatrix} W_R U, W_R D \end{bmatrix}) = 2L \quad \text{and the channel matrices must satisfy}
\]

\[
L_k \leq \text{rank}(H_{R,k}) \leq N_k, \quad \forall k \in K, \quad L \leq \text{rank}(H_{R,B}) \leq N_B, \quad (10a)
\]

\[
2L \leq \text{rank}(H_{R,k}) + \text{nullity}(H_{R,k}), \quad \text{rank}(H_{R,B}) + \text{nullity}(H_{R,B}) = N_R, \quad (10b)
\]

so the MSs (BS) can only transmit \( L \leq N_R / 2 \) data streams in the UL (DL).

Taking into consideration that each node is capable of canceling the backward propagated self-interference in the received signals, we can allow the self-interference to overlap with the desired signals since ultimately it does not affect the decoding of the desired signals. As such, we can facilitate interference management by perfectly aligning the UL and DL signal spaces to reduce the dimension of the multi-user interference space at each node as exemplified in Fig. 4. Mathematically, aligning the UL and DL signal spaces can be represented as

\[
\text{BS: } \text{range}(H_{R,B}^T W_R H_{R,m} W_m) = \text{range}(H_{R,B}^T W_R H_{R,B} W_{B,m}), \quad \forall m \in K, \quad (11a)
\]

\[
\text{k-th MS: } \text{range}(H_{R,k}^T W_R H_{R,B} W_{B,m}) = \text{range}(H_{R,k}^T W_R H_{R,m} W_m), \quad \forall m \in K, \quad (11b)
\]

and this can be manifested by constructing the RS forwarded signals (2) such that

\[
\begin{align*}
\text{UL signals} & = \text{range}(W_R H_{R,m} W_m) \\
\text{DL signals} & = \text{range}(W_R H_{R,B} W_{B,m}).
\end{align*}
\]
In order for all the UL and DL signal streams to be linearly independent, the rank of the RS forwarded signals should be

\[
\text{UL: } \text{rank}([W_{R,H}W_{R,1}, \ldots, W_{R,H}W_{R,K}]) = L,
\]

(13a)

\[
\text{DL: } \text{rank}([W_{R,H}W_{R,B}, W_{R,H}W_{R,1}, \ldots, W_{R,H}W_{R,B}W_{B,K}]) = L,
\]

(13b)

\[
\text{rank}([W_{R,H}W_{R,1}, \ldots, W_{R,H}W_{R,K}, W_{R,H}W_{R,B}, W_{R,H}W_{R,1}, \ldots, W_{R,H}W_{R,B}W_{B,K}]) = L,
\]

(13c)

and from (13) it suffices that the channel matrices satisfy

\[
L_k \leq \text{rank}(H_{R,k}) \leq N_k, \quad \forall k \in K, \quad L \leq \text{rank}(H_{R,B}) \leq N_B,
\]

(14a)

\[
L \leq \text{rank}(H_{R,k}) + \text{nullity}(H_{R,k}), \quad \text{rank}(H_{R,B}) + \text{nullity}(H_{R,B}) = N_R,
\]

(14b)

so the MSs (BS) can transmit \( L \leq N_R \) data streams in the UL (DL). Comparing (14b) and (10b) shows that we can achieve superior multiplexing gain by exploiting signal space alignment than by performing conventional SDMA processing.

Consider again the UL and DL data stream estimates in (5) and (6). Exploiting signal space alignment, in the DL data stream estimates \( \tilde{s}_D^{(k)} \) the UL and DL multi-user interference streams span the same signal space and appear as if they were one set of streams. In effect, UL and DL transmissions perform similarly to separated one-way relaying transmissions.

**Remark 1 (Feasibility of Signal Space Alignment):** Aligning the UL and DL signal spaces as per (12) requires that the two-way signals between the BS and the \( k \)-th MS be aligned when received by the RS (i.e., a single node), which then broadcasts the aligned signals back to the BS and the \( k \)-th MS. Using the arguments of coordinated transmission and reception [32], it can be shown that the alignment operation is feasible if the number of antennas and data streams for each node and the rank of the channel matrices satisfy (14). Note that this is unlike conventional IA for interference channels, which is subject to stringent feasibility conditions [24]–[26], due to the requirement to simultaneously align interference at multiple nodes.

**Remark 2 (DoF of One- and Two-Way AF Relaying):** As per [32], [33], the achievable DoF of multi-user one-way half-duplex AF relaying is \( 1/2 \min\{\text{rank}(H_{R,B}), \text{rank}([H_{R,1}, \ldots, H_{R,K}])\} \), where the factor of \( 1/2 \) accounts for the half-duplex loss. Exploiting signal space alignment, the achievable DoF of two-way relaying is \( \min\{\text{rank}(H_{R,B}), \text{rank}([H_{R,1}, \ldots, H_{R,K}])\} \).
Corollary 1: For independent and identically distributed (i.i.d.) Rayleigh fading channels, with probability 1, \( \text{rank}(H_{R,k}) = \min\{N_k, N_R\} \), \( \text{rank}([H_{R,1}, \ldots, H_{R,K}]) = \min\{N_R, \sum_{k=1}^K N_k\} \), and \( \text{rank}(H_{R,B}) = \min\{N_B, N_R\} \). Thus, the achievable DoF of two-way relaying is given by
\[
\min\{\min\{N_B, N_R\}, \min\{N_R, \sum_{k=1}^K N_k\}\} = \min\{N_B, N_R, \sum_{k=1}^K N_k\}.
\]

B. Transceiver Design Optimization

In the preceding discussion, we have focused on interference management without regard for QoS considerations. In practice, the users might have different service priorities, and their data streams might have heterogeneous requirements (for example, in terms of throughput and reliability). However, as shown in (7)–(9), the achievable data rate for each data stream is intricately related to the channel qualities of all links and the transceiver matrices of all nodes. One issue is that typically the transmit power of the BS is substantially higher than the MSs, and the transceiver design should accommodate for the unequal transmit powers to ensure that both UL and DL transmissions are of satisfactory performance. Toward this end, we seek to optimize the transceiver design to maximize the minimum weighted SINR among all data streams.

Specifically, we associate with each UL and DL data stream a weight factor that corresponds to its priority, where the higher the priority of a data stream the larger its weight factor. In the UL let \( \omega_U^{(k)}(l) \geq 1 \) denote the weight factor for data stream \( s_U^{(k)}(l) \), and in the DL let \( \omega_D^{(k)}(l) \geq 1 \) denote the weight factor for data stream \( s_D^{(k)}(l) \). We define the weighted per stream SINRs as
\[
\begin{align*}
\text{UL:} & \quad (\omega_U^{(k)}(l))^{-1} \gamma_U^{(k,l)}, \\
\text{DL:} & \quad (\omega_D^{(k)}(l))^{-1} \gamma_D^{(k,l)}.
\end{align*}
\]

By maximizing the minimum weighted per stream SINR, we can simultaneously enhance the performance of all data streams while accounting for their relative priorities. Altogether, we formulate the two-way relaying transceiver design problem as follows.

\text{Problem 1 (Two-Way Relaying Transceiver Design with QoS Constraints):} Given the transmit power constraint of each node and the priority weight factor of each data stream, we design the two-way relaying transceiver processing – exploiting signal space alignment – to maximize the minimum weighted per stream SINR.
we can simplify the freedoms of optimization to obtain effective and pragmatic transceiver designs.

with the signal streams of the given by RS precoder matrix

we define a signal model as follows. We divide the RS transformation matrix into two components

in order to take advantage of its structures and solve it more easily.

Note that Problem $Q$ is difficult to solve since it does not lead to closed-form solutions and is non-convex. As we show in the next section, it is also nontrivial to reformulate Problem $Q$ in order to take advantage of its structures and solve it more easily.

IV. PROPOSED SOLUTION

A. Two-Stage Transceiver Design Paradigm

The transceiver design problem, Problem $Q$, does not lead to closed-form solutions and is non-convex (since the objective function and constraints are not jointly convex in all the optimization variables); thus, we cannot efficiently solve for all the transceiver matrices in a single-shot manner. We propose to solve the transceiver design problem using a two-stage paradigm: in the first stage we focus on attaining alignment between the UL and DL signal streams, and in the second stage we aim at optimizing the weighted per stream SINRs.

To facilitate decomposing the transceiver design problem into two stages, we first extend the signal model as follows. We divide the RS transformation matrix into two components

$$W_R \triangleq F_R A_R;$$

we define $A_R \triangleq [A_R^{(1)}; \ldots; A_R^{(K)}]$ as the RS equalizer matrix and $F_R \triangleq [F_R^{(1)}; \ldots; F_R^{(K)}]$ as the RS precoder matrix, where the submatrices $A_R^{(k)} \in \mathbb{C}^{L_k \times N_R}$ and $F_R^{(k)} \in \mathbb{C}^{N_R \times L_k}$ are associated with the signal streams of the $k$-th MS. Substituting (17) into (2), the RS forwarded signals are given by $x_R = F_R (A_R [H_{R,1} W_1, \ldots, H_{R,K} W_K] s_U + A_R [H_{R,B} W_{B,1}, \ldots, H_{R,B} W_{B,K}] s_D + n_R)$.

3 The general two-way transceiver optimization problem is extremely complex. By imposing the signal alignment structure, we can simplify the freedoms of optimization to obtain effective and pragmatic transceiver designs.
Analogous to interference alignment (cf. [26] and references therein), aligning the UL and DL signal streams [16d] can be encompassed by the following conditions:

\[
\text{UL: } \mathbf{A}_R^{(k)} \mathbf{H}_{R,k} \mathbf{W}_k = \text{diag}(\phi^{(k)}), \quad \mathbf{A}_R^{(k)} \mathbf{H}_{R,m} \mathbf{W}_m = \mathbf{0}_{L_k \times L_m}, \quad k \neq m, \quad (18a)
\]

\[
\text{DL: } \mathbf{A}_R^{(k)} \mathbf{H}_{R,B} \mathbf{W}_{B,k} = \text{diag}(\psi^{(k)}), \quad \mathbf{A}_R^{(k)} \mathbf{H}_{R,B} \mathbf{W}_{B,m} = \mathbf{0}_{L_k \times L_m}, \quad k \neq m. \quad (18b)
\]

**Remark 3 (Interpretation of (18))**: We design the precoder matrices \(\mathbf{W}_{B,k}\) and \(\mathbf{W}_k\) such that the two-way signals between the BS and the \(k\)-th MS are perfectly aligned at the RS and linearly independent to other users’ signals.

Accordingly, the RS forwarded signals can be expressed as

\[
\mathbf{x}_R = \mathbf{F}_R (\Phi \mathbf{s}_U + \Psi \mathbf{s}_D + \mathbf{A}_R \mathbf{n}_R), \quad (19)
\]

where \(\Phi \triangleq \text{diag}(\phi^{(1)}, \ldots, \phi^{(K)})\) and \(\Psi \triangleq \text{diag}(\psi^{(1)}, \ldots, \psi^{(K)})\) represent the effective gains of the UL and DL data streams, respectively. The transmit power constraint of the RS forwarded signals is given by \(\mathbb{E} (||\mathbf{x}_R||^2) = ||\mathbf{F}_R \Phi||^2 + ||\mathbf{F}_R \Psi||^2 + N_0 ||\mathbf{F}_R \mathbf{A}_R||^2 \leq P_R\). We define the SINRs of the RS forwarded signals as

\[
\text{UL: } \gamma_{U}^{(k,l)} = \frac{|[\phi^{(k)}]_{(l)}|^2}{N_0 ||[\mathbf{A}_R^{(k)}]_{(l,:),l,.}||^2}, \quad (20a)
\]

\[
\text{DL: } \gamma_{D}^{(k,l)} = \frac{|[\psi^{(k)}]_{(l)}|^2}{N_0 ||[\mathbf{A}_R^{(k)}]_{(l,:),l,.}||^2}. \quad (20b)
\]

From (19), (3) and (4), the end-to-end received signals of the BS can be expressed as

\[
\mathbf{y}_B = (\mathbf{H}_{R,B})^T \mathbf{F}_R \Phi \mathbf{s}_U + (\mathbf{H}_{R,B})^T \mathbf{F}_R \mathbf{A}_R \mathbf{n}_R + \mathbf{n}_B + \mathbf{i}_B, \quad (21)
\]

and the end-to-end received signals of the \(k\)-th MS can be expressed as

\[
\mathbf{y}_k = (\mathbf{H}_{R,k})^T \mathbf{F}_R \psi^{(k)} \mathbf{s}_D + (\mathbf{H}_{R,k})^T \mathbf{F}_R (\tilde{\Phi}_k \mathbf{s}_D + \tilde{\Phi}_k \mathbf{s}_U) + (\mathbf{H}_{R,k})^T \mathbf{F}_R \mathbf{A}_R \mathbf{n}_R + \mathbf{n}_k + \mathbf{i}_k, \quad (22)
\]

where \(\tilde{\Phi}_k \triangleq \text{diag}(\phi^{(1)}, \ldots, \phi^{(k-1)}, 0_{L_k \times 1}, \phi^{(k+1)}, \ldots, \phi^{(K)})\) represent the effective gains of the UL interference streams, and \(\tilde{\Psi}_k \triangleq \text{diag}(\psi^{(1)}, \ldots, \psi^{(k-1)}, 0_{L_k \times 1}, \psi^{(k+1)}, \ldots, \psi^{(K)})\) represent the effective gains of the DL interference streams. Therefore, the UL and DL data stream estimates [5], [6] can be expressed as

\[
\tilde{\mathbf{s}}_U^{(k)} = \mathbf{V}_B^{(k)} (\mathbf{H}_{R,B})^T \mathbf{F}_R \Phi \mathbf{s}_U + \mathbf{V}_B^{(k)} (\mathbf{H}_{R,B})^T \mathbf{F}_R \mathbf{A}_R \mathbf{n}_R + \mathbf{V}_R^{(k)} \mathbf{n}_B, \quad (23)
\]

\[
\tilde{\mathbf{s}}_D^{(k)} = \mathbf{V}_k (\mathbf{H}_{R,k})^T \mathbf{F}_R \psi^{(k)} \mathbf{s}_D + \mathbf{V}_k (\mathbf{H}_{R,k})^T \mathbf{F}_R (\tilde{\Phi}_k \mathbf{s}_D + \tilde{\Phi}_k \mathbf{s}_U) + \mathbf{V}_k (\mathbf{H}_{R,k})^T \mathbf{F}_R \mathbf{A}_R \mathbf{n}_R + \mathbf{V}_k \mathbf{n}_k. \quad (24)
\]
and the end-to-end SINRs of the data stream estimates (7), (8) are equivalently given by

\[ \gamma_{U}^{(k,l)} = \frac{\| [V_{B}^{(k)}]_{(l,:)} (H_{R,B})^{T} [F_{R}^{(k)}]_{(:,l)} [\Phi^{(k)}]_{(l,:)} \|_{2}^{2}}{\left( \| [V_{B}^{(k)}]_{(l,:)} (H_{R,B}) F_{R} \Phi \|_{2}^{2} - \| [V_{B}^{(k)}]_{(l,:)} (H_{R,B})^{T} [F_{R}^{(k)}]_{(:,l)} [\Phi^{(k)}]_{(l,:)} \|_{2}^{2} \right) + N_0 \left( \| [V_{B}^{(k)}]_{(l,:)} (H_{R,B})^{T} F_{R} A_{R} \|_{2}^{2} + \| [V_{B}^{(k)}]_{(l,:)} \|_{2}^{2} \right) } \], 

(25)

\[ \gamma_{D}^{(k,l)} = \frac{\| [V_{k}]_{(l,:)} (H_{R,k})^{T} [F_{R}^{(k)}]_{(:,l)} [\Psi^{(k)}]_{(l,:)} \|_{2}^{2}}{\left( \| [V_{k}]_{(l,:)} (H_{R,k}) F_{R} [\tilde{\Phi}, \tilde{\Psi}] \|_{2}^{2} - \| [V_{k}]_{(l,:)} (H_{R,k})^{T} [F_{R}^{(k)}]_{(:,l)} [\Psi^{(k)}]_{(l,:)} \|_{2}^{2} \right) + N_0 \left( \| [V_{k}]_{(l,:)} (H_{R,k})^{T} F_{R} A_{R} \|_{2}^{2} + \| [V_{k}]_{(l,:)} \|_{2}^{2} \right) } \]. 

(26)

Lemma 1 (Decomposition of Transceiver Design): The transceiver design problem, Problem \( Q \), can be equivalently decomposed into two stages. The first stage processing finds the BS and MS precoder matrices and the RS equalizer matrix \( \{ W_{B}^{*}, \{ W_{k}^{*} \}_{k=1}^{K}, A_{R}^{*} \} \), subject to the alignment conditions (18), to maximize the minimum weighted SINR of the RS forwarded signals.

First Stage Processing

\[ \begin{align*}
\{ W_{B}^{*}, \{ W_{k}^{*} \}_{k=1}^{K}, A_{R}^{*} \} & := M \{ P_{B}, \{ P_{k} \}_{k=1}^{K}, \{ \omega_{U}^{(k)}, \omega_{D}^{(k)} \}_{k=1}^{K} \} \\
& \arg \max_{W_{B}, \{ W_{k} \}_{k=1}^{K}, A_{R}} \min_{\forall k \in K} \left\{ \left( [\omega_{U}^{(k)}]_{(l,:)} \right)^{-1} \gamma_{U}^{(k,l)} \left( [\omega_{D}^{(k)}]_{(l,:)} \right)^{-1} \gamma_{D}^{(k,l)} \right\} \\
& \text{s.t.} \quad \| W_{B} \|_{2}^{2} \leq P_{B}, \quad \| W_{k} \|_{2}^{2} \leq P_{k}, \quad \forall k \in K, \\
& \quad A_{R}^{(k)} H_{R,k} W_{k} = \text{diag}(\phi^{(k)}), \quad A_{R}^{(k)} H_{R,m} W_{m} = 0_{L_{k} \times L_{m}}, \quad k \neq m, \\
& \quad A_{R}^{(k)} H_{R,B} W_{B,k} = \text{diag}(\psi^{(k)}), \quad A_{R}^{(k)} H_{R,B} W_{B,m} = 0_{L_{k} \times L_{m}}, \quad k \neq m.
\end{align*} \]

(27)

The second stage processing finds the RS precoder matrix and the BS and MS equalizer matrices \( \{ F_{R}^{*}, V_{B}^{*}, \{ V_{k}^{*} \}_{k=1}^{K} \} \) to maximize the minimum weighted end-to-end SINR of the data stream estimates.

Second Stage Processing

\[ \begin{align*}
\{ F_{R}^{*}, V_{B}^{*}, \{ V_{k}^{*} \}_{k=1}^{K} \} & := B \{ W_{B}^{*}, \{ W_{k}^{*} \}_{k=1}^{K}, A_{R}^{*}, P_{R}, \{ \omega_{U}^{(k)}, \omega_{D}^{(k)} \}_{k=1}^{K} \} \\
& \arg \max_{F_{R}, V_{B}, \{ V_{k} \}_{k=1}^{K}} \min_{\forall k \in K} \left\{ \left( [\omega_{U}^{(k)}]_{(l,:)} \right)^{-1} \gamma_{U}^{(k,l)} \left( [\omega_{D}^{(k)}]_{(l,:)} \right)^{-1} \gamma_{D}^{(k,l)} \right\} \\
& \text{s.t.} \quad \| F_{R} \Phi \|_{2}^{2} + \| F_{R} \Psi \|_{2}^{2} + N_0 \| F_{R} A_{R} \|_{2}^{2} \leq P_{R}.
\end{align*} \]

(28)

Proof: Refer to Appendix A.

The top-level steps of the proposed two-stage transceiver design are summarized in Algorithm 1 and illustrated in Fig. 5. We shall elaborate the details of the first and second stage processing in the following subsections.
B. First Stage Processing

To solve the first stage processing, Problem $\mathcal{M}$, we focus our attention on coordinate eigen-mode transmission at the MSs, zero-forcing equalization at the RS, and zero-forcing transmission at the BS [34].

- **MS Precoder Matrices**: Let $\mathcal{G}_k$ denote the set of the right singular vectors of the channel matrix $\mathbf{H}_{R,k}$. The $k$-th MS precoder matrix $\mathbf{W}_k$ is given as $[\mathbf{W}_k]_{(:,l)} = \sqrt{\lambda_k^{(l)}} \mathbf{g}_k^{(l)}$, where $\mathbf{g}_k^{(l)} \in \mathcal{G}_k$ is the beam direction, and $\lambda_k^{(l)}$ is the allocated power satisfying the transmit power constraint $\|\mathbf{W}_k\|^2 = \sum_{l=1}^{L_k} \lambda_k^{(l)} = P_k$.

- **RS Equalizer Matrix**: The zero-forcing equalizer matrix is given as

$$\mathbf{A}_R = \text{pinv}(\left[\mathbf{H}_{R,1}[\mathbf{g}_1^{(1)}, \ldots, \mathbf{g}_1^{(L_1)}], \ldots, \mathbf{H}_{R,K}[\mathbf{g}_K^{(1)}, \ldots, \mathbf{g}_K^{(L_K)}]\right]).$$

- **BS Precoder Matrix**: Let $\mathbf{A}_R^{(k,l)}$ denote the matrix obtained by removing $[\mathbf{A}_R^{(k,l)}]_{(:,l)}$ from $\mathbf{A}_R$ and let $\mathbf{G}_B^{(k,l)} = \text{null}(\mathbf{A}_R^{(k,l)} \mathbf{H}_{R,B})$. The BS precoder matrix $\mathbf{W}_B$ is given as $[\mathbf{W}_B]_{(:,l)} = \sqrt{\lambda_B^{(k,l)}} \mathbf{g}_B^{(k,l)}$, where $\mathbf{g}_B^{(k,l)} \in \mathcal{G}_B^{(k,l)}$ is the beam direction, and $\lambda_B^{(k,l)}$ is the allocated power satisfying the transmit power constraint $\|\mathbf{W}_B\|^2 = \sum_{k=1}^{K} \sum_{l=1}^{L_k} \lambda_B^{(k,l)} = P_B$.

As such, the SINRs of the RS forwarded signals (20) can be equivalently expressed as

$$\tilde{\gamma}_U^{(k,l)} = \kappa_U^{(k,l)} \lambda_k^{(l)} \gamma_U^{(k,l)} = \kappa_D^{(l)} \lambda_{(k,l)},$$

where $\kappa_U^{(k,l)} = \frac{||[\mathbf{A}_R^{(k,l)}]_{(:,l)} \mathbf{H}_{R,k} \mathbf{g}_k^{(l)}||^2}{N_0} ||[\mathbf{A}_R^{(k,l)}]_{(:,l)} ||^2$ and $\kappa_D^{(l)} = \frac{||[\mathbf{A}_R^{(k,l)}]_{(:,l)} \mathbf{H}_{R,B} \mathbf{g}_B^{(k,l)}||^2}{N_0} ||[\mathbf{A}_R^{(k,l)}]_{(:,l)} ||^2$.

**Lemma 2 (Beam Directions and Power Allocation at the BS and MSs)**: To maximize the minimum weighted SINR of the RS forwarded signals, the power allocation at the BS and MSs are, respectively, given by

$$\begin{align*}
\lambda_B^{(k,l)} &= \frac{[\mathbf{\omega}_D^{(k,l)}]_{(:,l)} (\kappa_B^{(k,l)})^{-1} P_B}{\sum_{m=1}^{K} \sum_{q=1}^{L_m} [\mathbf{\omega}_D^{(m)}]_{(:,q)} (\kappa_B^{(m,q)})^{-1}}, \\
\lambda_k^{(l)} &= \frac{[\mathbf{\omega}_U^{(k,l)}]_{(:,l)} (\kappa_U^{(k,l)})^{-1} P_k}{\sum_{q=1}^{K} \sum_{m=1}^{L_m} [\mathbf{\omega}_U^{(m)}]_{(:,q)} (\kappa_U^{(m,q)})^{-1}}.
\end{align*}$$

It follows that the weighted SINRs of the RS forwarded signals can be expressed as

$$\begin{align*}
\text{UL:} & \quad (\mathbf{\omega}_U^{(k,l)})^{-1} (\gamma_U^{(k,l)}) = \frac{P_k}{\sum_{q=1}^{K} \sum_{m=1}^{L_m} [\mathbf{\omega}_U^{(m)}]_{(:,q)} (\kappa_U^{(m,q)})^{-1}}, \\
\text{DL:} & \quad (\mathbf{\omega}_D^{(k,l)})^{-1} (\gamma_D^{(k,l)}) = \frac{P_B}{\sum_{m=1}^{K} \sum_{q=1}^{L_m} [\mathbf{\omega}_D^{(m)}]_{(:,q)} (\kappa_D^{(m,q)})^{-1}},
\end{align*}$$

and selection of the beam directions to maximize the minimum weighted SINR can be performed using combinatorial search.

**Proof**: Refer to Appendix B.
C. Second Stage Processing

We now proceed to describe the algorithm for solving the second stage processing, Problem B. As per (25)-(26), the SINRs of the data stream estimates are not jointly convex in the RS precoder matrix and the BS and MS equalizer matrices \( \{ \mathbf{F}_R, \mathbf{V}_B, \{ \mathbf{V}_k \}_{k=1}^K \} \). However, for a fixed precoder matrix \( \mathbf{F}_R \) there are closed-form solutions for the equalizer matrices \( \mathbf{V}_B \) and \( \mathbf{V}_k \); conversely, for fixed \( \mathbf{V}_B \) and \( \mathbf{V}_k \) we can cast the problem of solving for \( \mathbf{F}_R \) as a quasi-convex problem. This motivates the approach to progressively refine the transceiver matrices by iteratively alternate between solving for the BS and MS equalizer matrices and the RS precoder matrix. In this regard, we alternatingly optimize each one of the RS precoder matrix and the BS and MS equalizer matrices in the form of the following subproblems, and the convergence proof is provided in Appendix C.

\[
\text{RS Precoder Matrix } \begin{cases} \mathbf{F}_R^*, \gamma_0 \end{cases} := \mathcal{B}_R \{ \mathbf{V}_B, \{ \mathbf{V}_k \}_{k=1}^K, \mathbf{W}_B^*, \{ \mathbf{W}_k^* \}_{k=1}^K, \mathbf{A}_R^*, P_R, \{ \omega^{(k)}_U, \omega^{(k)}_D \}_{k=1}^K \}
\end{cases}
\]

\[
\max_{\mathbf{F}_R, \gamma_0} \gamma_0 \quad \begin{cases} \text{s.t.} \left( [\omega^{(k)}_U(j)]_0 \right)^{-1} \gamma^{(k,j)}_U, (\omega^{(k)}_D(j)_0)^{-1} \gamma^{(k,j)}_D \geq \gamma_0, \forall k \in \mathcal{K}, \forall l \in \mathcal{L}_k, \right. \right.
\right.
\]  
\[
\left. \left| |\mathbf{F}_R \Phi| |^2 + \left| |\mathbf{F}_R \Psi| |^2 + N_0| |\mathbf{F}_R \mathbf{A}_R| |^2 \leq P_R. \right. \right. \]
\]

\[
\text{BS Equalizer } \begin{cases} \mathbf{V}_B^* := \mathcal{B}_B \{ \mathbf{F}_R, \mathbf{W}_B^*, \{ \mathbf{W}_k^* \}_{k=1}^K, \mathbf{A}_R^*, \{ \omega^{(k)}_U \}_{k=1}^K \} \end{cases}
\]

\[
\arg \max_{\mathbf{V}_B} \min_{\forall k \in \mathcal{K}} \left\{ \left( [\omega^{(k)}_U(j)]_0 \right)^{-1} \gamma^{(k,j)}_U \right\}. \tag{34}
\]

\[
\text{k-th MS Equalizer } \begin{cases} \mathbf{V}_k^* := \mathcal{B}_k \{ \mathbf{F}_R, \mathbf{W}_B^*, \{ \mathbf{W}_k^* \}_{k=1}^K, \mathbf{A}_R^*, \omega^{(k)}_D \} \end{cases}
\]

\[
\arg \max_{\mathbf{V}_k} \min_{\forall l \in \mathcal{L}_k} \left\{ \left( [\omega^{(k)}_D(j)]_0 \right)^{-1} \gamma^{(k,j)}_D \right\}. \tag{35}
\]

First, for Problem \( \mathcal{B}_B \) and Problem \( \mathcal{B}_k \), the per stream SINRs are maximized with a minimum mean squared error (MMSE) equalizer matrix \[35\]. Hence, the BS equalizer matrix is given by

\[
\mathbf{V}_B^* = \left( (\mathbf{H}_{R,B})^T \mathbf{F}_R \Phi \right)^\dagger \left( (\mathbf{H}_{R,B})^T \mathbf{F}_R \Phi \left( (\mathbf{H}_{R,B})^T \mathbf{F}_R \Phi \right)^\dagger + \Omega_B \right)^{-1}, \tag{36}
\]

where \( \Omega_B \equal N_0(\mathbf{H}_{R,B})^T \mathbf{F}_R \mathbf{A}_R ((\mathbf{H}_{R,B})^T \mathbf{F}_R \mathbf{A}_R)^\dagger + N_0 \mathbf{I}_{N_B} \) is the covariance matrix of the aggregate noise at the BS. Likewise, the \( k \)-th MS equalizer matrix is given by

\[
\mathbf{V}_k^* = \left( (\mathbf{H}_{R,k})^T \mathbf{F}_R^{(k)} \Psi^{(k)} \right)^\dagger \left( (\mathbf{H}_{R,k})^T \mathbf{F}_R \left[ \Psi, \tilde{\Phi}_k \right] \left( (\mathbf{H}_{R,k})^T \mathbf{F}_R \left[ \Psi, \tilde{\Phi}_k \right] \right)^\dagger + \Omega_k \right)^{-1}, \tag{37}
\]

where \( \Omega_k \equal N_0(\mathbf{H}_{R,k})^T \mathbf{F}_R \mathbf{A}_R ((\mathbf{H}_{R,k})^T \mathbf{F}_R \mathbf{A}_R)^\dagger + N_0 \mathbf{I}_{N_k} \) is the covariance matrix of the aggregate noise at the \( k \)-th MS.
Second, note that Problem $B_R$ corresponds to designing precoders for $L$ two-user multicast groups, where the precoder $[F_R^{(k)}]_{(l)}$ is used for multicasting the signal stream that encapsulates the UL data stream $[s_U^{(k)}]_{(l)}$ and the DL data stream $[s_D^{(k)}]_{(l)}$. As per [30, Claim 2], this multigroup multicast problem is NP-hard. We propose to solve for the RS precoder matrix $F_R$ using Algorithm 2 as derived in Appendix D. In a nutshell, we cast Problem $B_R$ as a quasi-convex problem and solve it using the bisection method [31, Section 4.2.5]. To do so, we define the SOCP feasibility problem of designing $F_R$ that achieves a target value of the minimum weighted per stream SINR $\gamma_0$ as:

$$F_R^* := \tilde{B}_R \{\gamma_0, V_B, \{V_k\}_{k=1}^K, W_B^*, \{W_k^*\}_{k=1}^K, A_R^*, P_R, \{\omega_U^{(k)}\}, \{\omega_D^{(k)}\}_{k=1}^K\}$$

$$\begin{align*}
\text{find } F_R & \quad \text{s.t.} \\
& \quad \left[\tilde{\alpha}_U^{(k,l)} ; \beta_U^{(k,l)} ; \delta_U^{(k,l)} \right] \succeq 0, \\
& \quad \left[\tilde{\alpha}_D^{(k,l)} ; \beta_D^{(k,l)} ; \delta_D^{(k,l)} \right] \succeq 0, \\
& \quad \sqrt{P_R} ; \rho \succeq 0.
\end{align*}$$

Starting with an interval that is expected to contain the optimum value of $\gamma_0$, we repeatedly bisect the interval and select the subinterval in which Problem $\tilde{B}_R$ is feasible until $\gamma_0$ converges.

### D. Implementation Considerations

First, we make the following assumptions about the synchronization requirement on the UL and DL signals.

**Assumption 5 (Synchronization Requirement):** The transmitted signals of the BS and the MSs are frequency and time synchronous [12, Section 2.2]. For instance, in a practical system such as IEEE 802.16m, the BS and MSs would be scheduled to transmit and receive over the same frequency-time resource units [5, Section 16.3.4.1].

Second, we make the following assumptions on the side information available at each node to facilitate transceiver design and self-interference cancelation. Under these assumptions, the proposed transceiver design problem can be solved in a distributed fashion.

**Assumption 6 (Side Information at the RS):** The RS has knowledge of global channel state information (CSI) $\mathcal{H}_R = \{H_{R,1}, \ldots, H_{R,K}, H_{R,B}\}$. For instance, the RS can accurately estimate the channel matrices of all links by observing the reciprocal reverse channels.

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4 Please refer to [36], [37] for discussion on the unicast precoder design problem, which is not NP-hard.

5 Due to page limit, please refer to Appendix D for the expressions for (38b) and (38c).
As per Assumption 6, the RS can locally solve the transceiver design problem (using Algorithm 1) and broadcast the RS transformation matrix to the BS and MSs.

Assumption 7 (Side Information at the BS and MSs): The BS and each MS has knowledge of the channel matrix between itself and the RS, the two-hop effective channel matrix, and the RS transformation matrix. Thus, the side information at the BS and the \( k \)-th MS include

\[
\mathcal{H}_B = \{ H_{R,B}, (H_{R,B})^T F_R \Phi, F_R, A_R \}, \quad \mathcal{H}_k = \{ H_{R,k}, (H_{R,k})^T F_R [\Psi, \tilde{\Phi}_k], F_R, A_R \}.
\]

For instance, the BS and each MS can estimate the channel matrix between itself and the RS by observing the reciprocal reverse channel, and can estimate the two-hop effective channel matrix using pilot-assisted techniques.

As per Assumption 7, the BS and MSs can locally determine their precoder and equalizer matrices (using Lemma 2, (36), and (37)), and have sufficient information to deduce and cancel self-interference.

Remark 4 (Channel Estimation and Feedback): In a practical system such as IEEE 802.16m, pilot symbols are embedded in frequency-time resource units to facilitate channel estimation [5, Section 16.3.4.4], and each node can perform channel estimation using techniques such as those defined in [38] and references therein. On the other hand, the RS can broadcast the RS transformation matrix to the BS and MSs by means of high fidelity unquantized feedback [5, Section 16.3.6.2.5.6].

V. SIMULATION RESULTS AND DISCUSSIONS

In this section, we provide numerical simulation results to assess the performance of the proposed transceiver design. For illustration, we consider the following simulation settings.

A. Simulation Settings

We consider a system with \( K = 3 \) MSs. In particular, we focus on MIMO configurations similar to those defined in the IEEE 802.16m standard [5]: the BS is equipped with up to \( N_B = 8 \) antennas and the MSs are equipped with \( N_k = \{2, 4\} \) antennas. As an example, we investigate the scenario in which the BS exchanges \( L_1 = 2, L_2 = 1, \) and \( L_3 = 1 \) data streams with the MSs.
We evaluate the performance of the proposed scheme using the packet error rate (PER) and the average sum rate\(^6\) as performance metrics. In the PER simulations, we employ the convolutional turbo code (CTC) defined in the IEEE 802.16m standard [5, Section 16.3.10.1.5]: each packet contains eight information bytes coded at rate 1/3 and modulated using QPSK. We compare the performance of the proposed scheme against the following prominent baseline schemes. Since these schemes were originally designed for single-antenna MSs, they do not consider MS precoder and equalizer designs. We extend these schemes to generate the \(k\)-th MS precoder matrix \(W_k\) from the principal right singular vectors of the channel matrix \(H_{R,k}\) with equal power allocation across the data streams, and we obtain the \(k\)-th MS equalizer matrix as \(V_k = (W_k)^T\).

- **Baseline 1 (Bidirectional Channel Inversion Naive Algorithm [19])**: The BS precoder and equalizer matrices and the RS transformation matrix are determined using pseudo-inverse methods.
- **Baseline 2 (Bidirectional Channel Inversion Greedy Algorithm [20])**: The BS precoder and equalizer matrices are determined using pseudo-inverse methods. A greedy iterative algorithm is employed to determine the RS transformation matrix that maximizes the asymptotic per stream SINRs.
- **Baseline 3 (Two-Way Relaying using Conventional SDMA Processing [8])**: The RS transformation matrix is devised to spatially multiplex all data streams. Since this scheme does not provide BS precoder and equalizer designs, we generate the BS precoder matrix \(W_B\) from the principal right singular vectors of the channel matrix \(H_{R,B}\) with equal power allocation across the data streams, and we obtain the BS equalizer matrix as \(V_B = (W_B)^T\).

In the simulation results we define the signal-to-noise ratio (SNR) as \(P_B/N_0\). We set the RS and MS transmit powers such that \(P_B/L = P_R/L = P_k/L_k\), so the transmit power per data stream is the same for all nodes. We assume i.i.d. Rayleigh fading, so the channel matrices are given by\(^7\) \(\text{vec}(H_{R,B}) \sim \mathcal{CN}(0_{N_B N_R \times 1}, I_{N_B N_R})\) and \(\text{vec}(H_{R,k}) \sim \mathcal{CN}(0_{N_k N_R \times 1}, I_{N_k N_R})\).

### B. Performance Comparisons

In Fig. 6 and Fig. 8, we present the performance results when the RS is equipped with \(N_R = 4\) antennas. Note that in this setting the number of spatial dimensions at the RS does not suffice

\(^6\)The average sum rate is defined as \(E \left[ \sum_{k=1}^{K} \sum_{l=1}^{L_k} (C_U^{(k,l)} + C_D^{(k,l)}) \right]\), where \(C_U^{(k,l)}\) and \(C_D^{(k,l)}\) are the UL and DL per stream achievable data rates, respectively, as given in (9).

\(^7\)Note that \(\text{rank}(H_{R,B}) = \min\{N_B, N_R\}\) and \(\text{rank}(H_{R,k}) = \min\{N_k, N_R\}\) with probability 1.
for performing two-way relaying using conventional SDMA processing (i.e., $N_R < 2L$), and so Baseline 3 is not feasible. Moreover, we assume that User 2 has higher service priority than the other users; as an example, we set the priority weight factors to $[\omega_U^{(2)}]_{(1)} = [\omega_D^{(2)}]_{(1)} = 2$ and $[\omega_U^{(k)}]_{(l)} = [\omega_D^{(k)}]_{(l)} = 1$ otherwise.

First, in Fig. 6 we show the PER performance results when the BS is equipped with $N_B = 4$ antennas and the MSs are equipped with $N_k = 2$ antennas. It can be seen that the proposed scheme exhibits better error performance than the baseline schemes. For instance, the proposed scheme achieves in excess of 10 dB SNR gain over the baseline schemes at $10^{-2}$ PER. This is attributed to the fact that the proposed scheme efficiently exploits the multiple spatial dimensions at the MSs, whereas the baseline schemes were originally designed for single antenna MSs and cannot efficiently exploit the available spatial dimensions. On the other hand, reflecting the QoS priority settings, for the proposed scheme User 2 has approximately 3 dB SNR gain over the other users for all PER values smaller than $10^{-1}$.

Second, in Fig. 8 we show the average sum rate performance results. In Fig. 8a, we show the average data rate versus SNR when the BS is equipped with $N_B = 4$ antennas. It can be seen that the proposed scheme achieves significant data rate gain over the baseline schemes. Moreover, the proposed scheme alleviates the half-duplex loss (cf. Remark 2 and Corollary 1) and achieves the DoF equal to $\min\{N_B, N_R, \sum_{k=1}^{K} N_k\} = 4$. In Fig. 8b, we show the average sum rate versus the number of BS antennas at 25 dB SNR. It can be seen that the data rate of the proposed scheme improves monotonically with the number of antennas at the BS and MSs. Note that the inferior performance of Baseline 3 is due to the fact this scheme requires more spatial dimensions at the RS to be feasible.

Finally, in Fig. 7 we show the PER performance results when the BS is equipped with $N_B = 4$ antennas, the MSs are equipped with $N_k = 2$ antennas, and the RS is equipped with $N_R = 8$ antennas. In this setting, there are enough spatial dimensions at the RS (i.e., $N_R \geq 2L$) for Baseline 3 to be feasible. For simplicity of comparison, we assume all the users have the same service priority and we set all priority weight factors to $[\omega_U^{(k)}]_{(l)} = [\omega_D^{(k)}]_{(l)} = 1$. It can be seen that the proposed scheme substantially outperforms Baseline 3 (e.g., up to 19 dB SNR gain at $10^{-2}$ PER). This is because the proposed scheme can efficiently exploit the spatial dimensions

---

Note that Baseline 1 is infeasible as it requires $N_B \geq N_R$. Therefore, we have excluded it from the comparison.
at the RS to mitigate interference as well as to achieve beamforming gain, whereas Baseline 3 uses all the spatial dimensions to null interference.

VI. CONCLUSIONS

In cellular multi-user two-way AF relaying systems, each node experiences self-induced backward propagated interference as well as multi-user interference. As a result, conventional self-interference cancelation approaches for single-user two-way relay systems do not suffice to mitigate the impact of interference. We applied an interference management model exploiting signal space alignment and proposed a linear MIMO transceiver design algorithm, which allows for alleviating the half-duplex loss and providing flexible performance optimization accounting for each user’s QoS priorities. Numerical comparisons to two-way relaying schemes based on bidirectional channel inversion and SDMA-only processing show that the proposed scheme achieves superior error rate and average data rate performance.

APPENDIX A: PROOF OF LEMMA I

We first show that the end-to-end per stream SINRs are predicated by the first hop. Specifically, the end-to-end SINRs of the data stream estimates (25)-(26) can be expressed as

\[ \gamma_{UL}^{(k,l)} = \xi_U^{(k,l)} \gamma_0^{(k,l)} \]

\[ \gamma_{DL}^{(k,l)} = \xi_D^{(k,l)} \gamma_0^{(k,l)} \]

where \( \gamma_0^{(k,l)} \) and \( \gamma_0^{(k,l)} \) are the SINRs of the RS forwarded signals (20) and

\[ \xi_U^{(k,l)} = \frac{N_0 \left[ V_k^{(k)} \right] (l,:) \left( H_{R,B} \right)^T \left[ F_R^{(k)} \right] (l,:) \left[ \phi^{(m)} \right] (q,:) \left[ \phi^{(m)} \right] (q,:) \left[ \phi^{(m)} \right] (q,:) \left[ \phi^{(m)} \right] (q,:) \right] + N_0 \left[ V_B^{(k)} \right] (l,:) \left( H_{R,B} \right)^T \left[ F_R^{(k)} \right] (l,:) \left[ \phi^{(m)} \right] (q,:) \left[ \phi^{(m)} \right] (q,:) \left[ \phi^{(m)} \right] (q,:) \left[ \phi^{(m)} \right] (q,:) \right] }{ \sum_{m=1}^{K} \sum_{q=1}^{L_m} \left[ V_k^{(k)} \right] (l,:) \left( H_{R,B} \right)^T \left[ F_R^{(k)} \right] (l,:) \left[ \phi^{(m)} \right] (q,:) \left[ \phi^{(m)} \right] (q,:) \left[ \phi^{(m)} \right] (q,:) \left[ \phi^{(m)} \right] (q,:) \right] + \sum_{m=1}^{K} \sum_{q=1}^{L_m} \left[ V_B^{(k)} \right] (l,:) \left( H_{R,B} \right)^T \left[ F_R^{(k)} \right] (l,:) \left[ \phi^{(m)} \right] (q,:) \left[ \phi^{(m)} \right] (q,:) \left[ \phi^{(m)} \right] (q,:) \left[ \phi^{(m)} \right] (q,:) \right] } \]

\[ \xi_D^{(k,l)} = \frac{N_0 \left[ V_k^{(k)} \right] (l,:) \left( H_{R,B} \right)^T \left[ F_R^{(k)} \right] (l,:) \left[ \phi^{(m)} \right] (q,:) \left[ \phi^{(m)} \right] (q,:) \left[ \phi^{(m)} \right] (q,:) \left[ \phi^{(m)} \right] (q,:) \right] }{ \sum_{m=1}^{K} \sum_{q=1}^{L_m} \left[ V_k^{(k)} \right] (l,:) \left( H_{R,B} \right)^T \left[ F_R^{(k)} \right] (l,:) \left[ \phi^{(m)} \right] (q,:) \left[ \phi^{(m)} \right] (q,:) \left[ \phi^{(m)} \right] (q,:) \left[ \phi^{(m)} \right] (q,:) \right] + \sum_{m=1}^{K} \sum_{q=1}^{L_m} \left[ V_B^{(k)} \right] (l,:) \left( H_{R,B} \right)^T \left[ F_R^{(k)} \right] (l,:) \left[ \phi^{(m)} \right] (q,:) \left[ \phi^{(m)} \right] (q,:) \left[ \phi^{(m)} \right] (q,:) \left[ \phi^{(m)} \right] (q,:) \right] } \]
Consider $\xi_{U}^{(k,l)}$ for example. Note that

$$|| [V_{B}^{(k)}]_{l,(i)} (H_{R,B})^{T} F_{R} A_{R} ||^2$$

$$= || \sum_{m=1}^{K} \sum_{q=1}^{L_{m}} [V_{B}^{(k)}]_{l,(i)} (H_{R,B})^{T} [F_{R}^{(m)}]_{l,(q)} [A_{R}^{(m)}]_{l,(q)} + [V_{B}^{(k)}]_{l,(i)} (H_{R,B})^{T} [F_{R}^{(k)}]_{l,(i)} [A_{R}^{(k)}]_{l,(i)} ||^2$$

$$\approx || [V_{B}^{(k)}]_{l,(i)} (H_{R,B})^{T} [F_{R}^{(k)}]_{l,(i)} [A_{R}^{(k)}]_{l,(i)} ||^2,$$

where (a) follows from the fact that the terms $\{ [V_{B}^{(k)}]_{l,(i)} (H_{R,B})^{T} [F_{R}^{(m)}]_{l,(q)} \}$ should be negligible to suppress interference. It can be deduced that $\xi_{U}^{(k,l)} \leq 1$ and similarly $\xi_{D}^{(k,l)} \leq 1$, so the end-to-end per stream SINRs are limited by the SINRs of the RS forwarded signals, i.e.,

$$UL: \gamma_{U}^{(k,l)} \leq \tilde{\gamma}_{U}^{(k,l)}, \quad DL: \gamma_{D}^{(k,l)} \leq \tilde{\gamma}_{D}^{(k,l)}.$$  \hfill (41)

As per (41), the minimum weighted end-to-end SINR of the data stream estimates is limited by the minimum weighted SINR of the RS forwarded signals, i.e.,

$$\min_{\forall k \in K, l \in \mathcal{L}_{k}} \{ (\omega_{U}^{(k)})^{-1} (\gamma_{U}^{(k,l)})^{-1}, (\omega_{D}^{(k)})^{-1} (\gamma_{D}^{(k,l)})^{-1} \} \leq \min_{\forall k \in K, l \in \mathcal{L}_{k}} \{ (\omega_{U}^{(k)})^{-1} (\tilde{\gamma}_{U}^{(k,l)})^{-1}, (\omega_{D}^{(k)})^{-1} (\tilde{\gamma}_{D}^{(k,l)})^{-1} \}.$$  \hfill (42)

By this property, the transceiver design problem, Problem $Q$, can be decomposed into two stages. In the first stage processing, we find the BS and MS precoder matrices and the RS equalizer matrix that maximize the minimum weighted SINR of the RS forwarded signals, i.e.,

$$\arg \max_{W_{B}, W_{R}^{(k)}, A_{R}} \min_{\forall k \in K, l \in \mathcal{L}_{k}} \{ (\omega_{U}^{(k)})^{-1} (\gamma_{U}^{(k,l)})^{-1}, (\omega_{D}^{(k)})^{-1} (\gamma_{D}^{(k,l)})^{-1} \},$$

and thereby implicitly maximize the achievable minimum weighted end-to-end SINR of the data streams estimates. Then, in the second stage processing, we find the RS precoder matrix and the BS and MS equalizer matrices to holistically maximize the minimum weighted end-to-end SINR of the data streams estimates, i.e.,

$$\arg \max_{F_{R}, V_{B}, W_{B}} \min_{\forall k \in K, l \in \mathcal{L}_{k}} \{ (\omega_{U}^{(k)})^{-1} (\gamma_{U}^{(k,l)})^{-1}, (\omega_{D}^{(k)})^{-1} (\gamma_{D}^{(k,l)})^{-1} \}.$$  \hfill (43)

**APPENDIX B: PROOF OF LEMMA 2**

Substituting (30) into (27a), the problem of maximizing the minimum weighted SINR of the RS forwarded signals can be reformulated as

$$\arg \max_{\{\lambda_{k}^{(l)}\} \forall k \in K, l \in \mathcal{L}_{k}} \min_{\forall k \in K, l \in \mathcal{L}_{k}} \{ (\omega_{U}^{(k)})^{-1} (\gamma_{U}^{(k,l)})^{-1}, (\omega_{D}^{(k)})^{-1} (\gamma_{D}^{(k,l)})^{-1} \} \leq \arg \max_{\{\lambda_{k}^{(l)}\} \forall k \in K, l \in \mathcal{L}_{k}} \min_{\forall k \in K, l \in \mathcal{L}_{k}} \{ (\omega_{U}^{(k)})^{-1} (\gamma_{U}^{(k,l)})^{-1}, (\omega_{D}^{(k)})^{-1} (\gamma_{D}^{(k,l)})^{-1} \}.$$
Therefore, for fixed beam directions \( \{ g_{k}^{(l)} : g_{B}^{(k,l)} \} \), the power allocation at each node can be separately determined. For instance, the power allocation at the \( k \)-th MS can be determined according to \( (\lambda_{k}^{(l)})^{*} = \arg \max_{\lambda_{k}^{(l)}} \min_{\gamma \in D} \left\{ \left[ \left( \omega_{U}^{(k)} \right)_{(l)} \right]^{-1} \gamma \right\} \), whereby

\[
\left( \left[ \omega_{U}^{(k)} \right]_{(l)} \right)^{-1} \gamma_{U}^{(k,l)} = \ldots = \left( \left[ \omega_{U}^{(k)} \right]_{(L_{k})} \right)^{-1} \gamma_{U}^{(k,L_{k})} \iff \left( \left[ \omega_{U}^{(k)} \right]_{(l)} \right)^{-1} \gamma_{U}^{(k,l)} = \ldots = \left( \left[ \omega_{U}^{(k)} \right]_{(L_{k})} \right)^{-1} \gamma_{U}^{(k,L_{k})} \left( \lambda_{k}^{(L_{k})} \right)^{*}.
\]

(44)

It can be shown that (44) is satisfied with \( (\lambda_{k}^{(l)})^{*} = \frac{\left[ \omega_{U}^{(k)} \right]_{(l)} \left( \kappa_{U}^{(k,l)} \right)^{-1} P_{k}}{\sum_{q=1}^{L_{k}} \left[ \omega_{U}^{(k)} \right]_{(q)} \left( \kappa_{U}^{(k,q)} \right)^{-1} - P_{k}} \), which in turn yields weighted UL SINRs of \( \left( \left[ \omega_{U}^{(k)} \right]_{(l)} \right)^{-1} \gamma_{U}^{(k,l)} = \frac{\left[ \omega_{U}^{(k)} \right]_{(l)} \left( \kappa_{U}^{(k,l)} \right)^{-1} P_{k}}{\sum_{q=1}^{L_{k}} \left[ \omega_{U}^{(k)} \right]_{(q)} \left( \kappa_{U}^{(k,q)} \right)^{-1} - P_{k}} \). Analogously, the power allocation at the BS is given by \( (\lambda_{B}^{(l)})^{*} = \frac{\left[ \omega_{D}^{(k)} \right]_{(l)} \left( \kappa_{D}^{(k,l)} \right)^{-1} P_{B}}{\sum_{m=1}^{K_{m}} \sum_{q=1}^{L_{m}} \left[ \omega_{D}^{(m)} \right]_{(q)} \left( \kappa_{D}^{(m,q)} \right)^{-1} - P_{B}} \), which yields weighted DL SINRs of \( \left( \left[ \omega_{D}^{(k)} \right]_{(l)} \right)^{-1} \gamma_{D}^{(k,l)} = \frac{\left[ \omega_{D}^{(k)} \right]_{(l)} \left( \kappa_{D}^{(k,l)} \right)^{-1} P_{B}}{\sum_{m=1}^{K_{m}} \sum_{q=1}^{L_{m}} \left[ \omega_{D}^{(m)} \right]_{(q)} \left( \kappa_{D}^{(m,q)} \right)^{-1} - P_{B}} \).

APPENDIX C: CONVERGENCE OF THE SECOND STAGE PROCESSING

At the \( q \)-th iteration of the second stage processing, we denote the RS precoder matrix as \( F_{R}^{(q)} \), the BS equalizer matrix as \( V_{B}^{(q)} \), the \( k \)-th MS equalizer matrix as \( V_{k}^{(q)} \), and the minimum weighted per stream SINR as \( \gamma_{-}^{(q)} \). Moreover, we denote as \( \gamma_{U}^{(k,l)} \{ F_{R}^{(a)}, V_{B}^{(b)} \} \) and \( \gamma_{D}^{(k,l)} \{ F_{R}^{(a)}, V_{k}^{(b)} \} \) the UL and DL per stream SINR given \( F_{R}^{(a)} \), \( V_{B}^{(b)} \), and \( V_{k}^{(b)} \). We show that each iteration of the second stage processing monotonically increases the minimum weighted per stream SINR.

In Step 2.1, given the BS and MS equalizer matrices \( \{ V_{B}^{(q-1)}, V_{k}^{(q-1)} \}_{k=1}^{K} \), we solve for the RS precoder matrix \( F_{R}^{(q)} \) to improve the minimum weighted per stream SINR, i.e.,

\[
\gamma_{0} = \min_{\forall k \in K} \left\{ \left( \left[ \omega_{U}^{(k)} \right]_{(l)} \right)^{-1} \gamma_{U}^{(k,l)} \{ F_{R}^{(q)}, V_{B}^{(q-1)} \}, \left( \left[ \omega_{D}^{(k)} \right]_{(l)} \right)^{-1} \gamma_{D}^{(k,l)} \{ F_{R}^{(q)}, V_{k}^{(q-1)} \} \right\}
\]

(45)

\[
\geq \min_{\forall k \in K} \left\{ \left( \left[ \omega_{U}^{(k)} \right]_{(l)} \right)^{-1} \gamma_{U}^{(k,l)} \{ F_{R}^{(q-1)}, V_{B}^{(q-1)} \}, \left( \left[ \omega_{D}^{(k)} \right]_{(l)} \right)^{-1} \gamma_{D}^{(k,l)} \{ F_{R}^{(q-1)}, V_{k}^{(q-1)} \} \right\}
= \gamma_{-}^{(q-1)}.
\]

In Step 2.2 and Step 2.3, given the RS precoder matrix \( F_{R}^{(q)} \), we solve for the BS and MS equalizer matrices \( \{ V_{B}^{(q)}, V_{k}^{(q)} \}_{k=1}^{K} \) to improve the per stream SINR, i.e.,

\[
\gamma_{U}^{(k,l)} \{ F_{R}^{(q)}, V_{B}^{(q)} \} \geq \gamma_{U}^{(k,l)} \{ F_{R}^{(q-1)}, V_{B}^{(q-1)} \}, \gamma_{D}^{(k,l)} \{ F_{R}^{(q)}, V_{k}^{(q)} \} \geq \gamma_{D}^{(k,l)} \{ F_{R}^{(q-1)}, V_{k}^{(q-1)} \};
\]
hence the minimum weighted per stream SINR satisfies
\[
\gamma[q] = \min_{\gamma, \forall k \in K} \{ (\omega_U)_{(l)}^{(k)} \}^{-1} \gamma_U^{(k,l)} \{ F_R[q], V_B[q] \}; (\omega_D)_{(l)}^{(k)} \}^{-1} \gamma_D^{(k,l)} \{ F_R[q], V_B[q] \} \geq \gamma_0 . \tag{46}
\]

It follows from (45) and (46) that the weighted minimum per stream SINR increases with each iteration, i.e., \( \gamma[q] \geq \gamma_0 \geq \gamma[q-1] \), and the second stage processing must converge.

**APPENDIX D: DERIVATION OF ALGORITHM 2**

For ease of exposition, we express the constraints in Problem \( B_R \) in vector form. The transmit power constraint (35c) can be expressed as
\[
\sqrt{P_R} \geq \sqrt{|| F_R \Phi ||^2 + || F_R \Psi ||^2 + N_0|| F_R A_R ||^2 } \tag{a} = || \rho ||, \tag{47}
\]
\[
\rho \triangleq \left( \left( ( \Phi(\Phi) + \Psi(\Psi) + N_0 A_R (A_R)^\dagger \right)^{1/2} \right)^T \otimes I_{N_R} \right) vec(F_R) \tag{48}
\]
and equality (a) follows from the Kronecker product property \( vec(XYZ) = (Z)^T \otimes X vec(Y) \).

With some algebraic manipulations and using the aforementioned Kronecker product property, the UL SINR constraints (35b) can be expressed as
\[
(\omega_U)_{(l)}^{(k)} \gamma_U^{(k,l)} \geq \gamma_0 \iff \alpha_U^{(k,l)} \geq \left| \left| [\beta_U^{(k,l)}; \delta_U^{(k,l)}] \right| \right|, \tag{49}
\]
\[
\alpha_U^{(k,l)} \triangleq \sqrt{1 + (\omega_U)_{(l)}^{(k)} \gamma_0} \left| \left| \phi_U^{(k,l)} \right| \right| \left| \left| \phi_V^{(l)} \right| \right| \left| \left| \phi_B^{(k)} \right| \right| \left| \left| \phi_{\delta}^{(k,l)} \right| \right| \right| \right| \right| \right| \right| \right| \right| \right| \right| \right| \right| \right| \right| \right| \right| \right| \right| \right| \right| \right| \right| \right| \right| \right| \right| \right| \right| \right| \right| \right| \right| \right| \right| \right| \right| \right| \right| \right| \right| \right| \right| \right| \right| \right| \right| \right| \right| \right| \right| \right| \right| \right| \right| \right| \right| \right| \right| \right| \right| \right| \right| \right| \right| \right| \right| \right| \right| \right| \right| \right| \right| \right| \right| \right| \right| \right| \right| \right| \right| \right| \right| \right| \right| \right| \right| \right| \right| \right| \right| \right| \right| \right| \right| \right| \right| \right| \right| \right| \right| \right| \right| \right| \right| \right| \right| \right| \right| \right| \right| \right| \right| \right| \right| \right| \right| \right| \right| \right| \right| \right| \right| \right| \right| \right| \right| \right| \right| \right| \right| \right| \right| \right| \right| \right| \right| \right| \right| \right| \right| \right| \right| \right| \right| \right| \right| \right| \right| \right| \right| \right| \right| \right| \right| \right| \right| \right| \right| \right| \right| \right| \right| \right| \right| \right| \right| \right| \right| \right| \right| \right| \right| \right| \right| \right| \right| \right| \right| \right| \right| \right| \right| \right| \right| \right| \right| \right| \right| \right| \right| \right| \right| \right| \right| \right| \right| \right| \right| \right| \right| \right| \right| \right| \right| \right| \right| \right| \right| \right| \right| \right| \right| \right| \right| \right| \right| \right| \right| \right| \right| \right| \right| \right| \right| \right| \right| \right| \right| \right| \right| \right| \right| \right| \right| \right| \right| \right| \right| \right| \right| \right| \right| \right| \right| \right| \right| \right| \right| \right| \right| \right| \right| \right| \right| \right| \right| \right| \right| \right| \right| \right| \right| \right| \right| \right| \right| \right| \right| \right| \right| \right| \right| \right| \right| \right| \right| \right| \right| \right| \right| \right| \right| \right| \right| \right| \right| \right| \right| \right| \right| \right| \right| \right| \right| \right| \right| \right| \right| \right| \right| \right| \right| \right| \right| \right| \right| \right| \right| \right| \right| \right| \right| \right| \right| \right| \right| \right| \right| \right| \right| \right| \right| \right| \right| \right| \right| \right| \right| \right| \right| \right| \right| \right| \right| \right| \right| \right| \right| \right| \right| \right| \right| \right| \right| \right| \right| \right| \right| \right| \right| \right| \right| \right| \right| \right| \right| \right| \right| \right| \right| \right| \right| \right| \right| \right| \right| \right| \right| \right| \rf
Note that the transmit power constraint \((51\text{c})\) is convex in the RS precoder matrix \(\mathbf{F}_R\), but the SINR constraints \((51\text{b})\) are non-convex in \(\mathbf{F}_R\) and the minimum weighted per stream SINR slack variable \(\gamma_0\) since \(\alpha_{(l)}^{(k),l}\) and \(\alpha_{(l)}^{(k),l}\) are not affine in \(\mathbf{F}_R\) and \(\gamma_0\).

In order to obtain a mathematically tractable solution to Problem \(B_R\), we cast the SINR constraints as convex functions in \(\mathbf{F}_R\) by tightening these constraints as follows. We define

\[
\tilde{\alpha}_{(l)}^{(k),l} \triangleq \sqrt{1 + \left(\left[\vec{\omega}_{(k),l}^{(k)}\right]_l\right)^{-1} \left[\left[\vec{\phi}_{(k),l}^{(k)}\right]_l \mid \Re\left(\left[\mathbf{v}_{B}^{(k)}\right]_l \left(\mathbf{H}_{R,B}\right)^{T} \left[\mathbf{F}_{R}^{(k)}\right]_l\right)\right]},
\]

\[
\tilde{\alpha}_{(l)}^{(k),l} \triangleq \sqrt{1 + \left(\left[\vec{\omega}_{(l),l}^{(l)}\right]_l\right)^{-1} \left[\left[\vec{\psi}_{(l),l}^{(l)}\right]_l \mid \Re\left(\left[\mathbf{v}_{k}^{(l)}\right]_l \left(\mathbf{H}_{R,k}\right)^{T} \left[\mathbf{F}_{R}^{(k)}\right]_l\right)\right]},
\]

which are affine in \(\mathbf{F}_R\). Since \(\left[\left[\mathbf{v}_{B}^{(k)}\right]_l \left(\mathbf{H}_{R,B}\right)^{T} \left[\mathbf{F}_{R}^{(k)}\right]_l\right] \mid \Re\left(\left[\mathbf{v}_{B}^{(k)}\right]_l \left(\mathbf{H}_{R,B}\right)^{T} \left[\mathbf{F}_{R}^{(k)}\right]_l\right)\) and \(\left[\left[\mathbf{v}_{k}^{(l)}\right]_l \left(\mathbf{H}_{R,k}\right)^{T} \left[\mathbf{F}_{R}^{(k)}\right]_l\right] \mid \Re\left(\left[\mathbf{v}_{k}^{(l)}\right]_l \left(\mathbf{H}_{R,k}\right)^{T} \left[\mathbf{F}_{R}^{(k)}\right]_l\right)\), \(\alpha_{(l)}^{(k),l}\) is lower bounded by \(\tilde{\alpha}_{U}^{(k),l}\) and \(\alpha_{(l)}^{(k),l}\) is lower bounded by \(\tilde{\alpha}_{D}^{(k),l}\). If we tighten the SINR constraints as

\[
\tilde{\alpha}_{U}^{(k),l} \geq \left\| \left[\beta_{U}^{(k),l} ; \delta_{U}^{(k),l} \right]\right\|, \quad \tilde{\alpha}_{D}^{(k),l} \geq \left\| \left[\beta_{D}^{(k),l} ; \delta_{D}^{(k),l} \right]\right\|,
\]

then the SINR constraints degenerate into convex functions in \(\mathbf{F}_R\). Yet, the SINR constraints are still non-convex in \(\gamma_0\). Altogether, the precoder design problem is quasi-convex and it can be solved using the bisection method \([31, \text{Section 4.2.5}]\). Specifically, we define the SOCP feasibility problem of designing \(\mathbf{F}_R\) that achieves a target value of \(\gamma_0\) as

\[
\mathbf{F}_R^* := \bar{B}_R \left\{ \gamma_0, \mathbf{V}_B, \{\mathbf{v}_{k}\}_{k=1}^{K}, \mathbf{W}_B, \{\mathbf{w}_{k}\}_{k=1}^{K}, \mathbf{A}_R, P, \{\omega_{U}^{(k)}, \omega_{D}^{(k)}\}_{k=1}^{K} \right\}
\]

\[
\begin{cases}
\text{find} \; \mathbf{F}_R \\
\text{s.t.} \; \left[\alpha_{U}^{(k),l} ; \beta_{U}^{(k),l} ; \delta_{U}^{(k),l}\right] \succeq 0, \left[\alpha_{D}^{(k),l} ; \beta_{D}^{(k),l} ; \delta_{D}^{(k),l}\right] \succeq 0, \forall k \in \mathcal{K}, \forall l \in \mathcal{L}_k,
\end{cases}
\]

where \((55\text{b})\) and \((55\text{c})\) correspond to \((54)\) and \((47)\) expressed as SOC constraints, respectively. Starting with an interval that is expected to contain the optimum value of \(\gamma_0\), we repeatedly bisect the interval and select the subinterval in which Problem \(\bar{B}_R\) is feasible until \(\gamma_0\) converges.

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Algorithm 1 Top-Level Algorithm

\textbf{Outputs:} \(W_B^*, \{W_k^*\}_{k=1}^K, W_R^*, \{V_k^*\}_{k=1}^K\) \textbf{Inputs:} \(P_B, \{P_k\}_{k=1}^K, P_R, \{\omega_U^{(k)}\}_{k=1}^K, \omega_D^{(k)}\) \(\forall k = 1, \ldots, K\)

\textit{First Stage Processing}

\textbf{Step 1:} Solve \(\{W_B^*, \{W_k^*\}_{k=1}^K, A_R^*\} := \mathcal{M}\{P_B, \{P_k\}_{k=1}^K, \{\omega_U^{(k)}, \omega_D^{(k)}\}_{k=1}^K\}\) using Lemma 2.

\textit{Second Stage Processing}

\textbf{Step 2.0:} Initialize \(V_B = (W_B^*)^T\) and \(V_k = (W_k^*)^T, \forall k \in \mathcal{K}\).

Repeat

\textbf{Step 2.1:} Solve \(\{F_{R}^*, \gamma_0\} := \mathcal{B}_R\{V_B, \{V_k\}_{k=1}^K, W_B^*, \{W_k^*\}_{k=1}^K, A_R^*, P_R, \{\omega_U^{(k)}\}_{k=1}^K, \omega_D^{(k)}\}_{k=1}^K\}\) using Algorithm 2.

\textbf{Step 2.2:} Solve \(V_B^* := \mathcal{B}_B\{F_{R}^*, W_B^*, \{W_k^*\}_{k=1}^K, A_R^*, \{\omega_U^{(k)}\}_{k=1}^K\}\) using (36).

\textbf{Step 2.3:} Solve \(V_k^* := \mathcal{B}_k\{F_{R}^*, W_B^*, \{W_k^*\}_{k=1}^K, A_R^*, \omega_D^{(k)}\}\), \(\forall k \in \mathcal{K}\), using (37).

Until the minimum weighted per stream SINR converges.

\textbf{Step 3:} Set \(W_R^* = F_{R}^* A_R^*\).

Algorithm 2 RS Precoder Matrix Optimization

\textbf{Outputs:} \(F_{R}^*\) \textbf{Inputs:} \(V_B, \{V_k\}_{k=1}^K, W_B^*, \{W_k^*\}_{k=1}^K, \{A_R^*\}_{k=1}^K, P_R, \{\omega_U^{(k)}\}_{k=1}^K, \omega_D^{(k)}\) \(\forall k = 1, \ldots, K\)

\textbf{Step 0:} Initialize \(\gamma_{\text{min}} = \min_{\forall k \in \mathcal{K}, \forall l \in \mathcal{L}_k} \left\{ \left(\omega_U^{(k)}\right)_{(l)}^{-1} \gamma_U^{(k, l)}, \left(\omega_D^{(k)}\right)_{(l)}^{-1} \gamma_D^{(k, l)} \right\}\)

and \(\gamma_{\text{max}} = \max_{\forall k \in \mathcal{K}, \forall l \in \mathcal{L}_k} \left\{ \left(\omega_U^{(k)}\right)_{(l)}^{-1} \gamma_U^{(k, l)}, \left(\omega_D^{(k)}\right)_{(l)}^{-1} \gamma_D^{(k, l)} \right\}\).

Repeat \(\triangleright\) Bisection Method

\textbf{Step 1:} Set the target minimum weighted per stream SINR \(\gamma_0 = (\gamma_{\text{min}} + \gamma_{\text{max}})/2\).

Solve \(F_{R}^* := \mathcal{B}_R\{\gamma_0, V_B, \{V_k\}_{k=1}^K, W_B^*, \{W_k^*\}_{k=1}^K, \{A_R^*\}_{k=1}^K, P_R, \{\omega_U^{(k)}\}_{k=1}^K, \omega_D^{(k)}\}_{k=1}^K\}\).

\textbf{Step 2:} If Problem \(\mathcal{B}_R\) is feasible, set \(\gamma_{\text{min}} = \gamma_0\); else set \(\gamma_{\text{max}} = \gamma_0\).

Until \(\gamma_0\) converges.
Fig. 1. Multi-user relay system. (a) One-way transmission UL. (b) One-way transmission DL. (c) Two-way transmission MAC phase. (d) Two-way transmission BC phase.

Fig. 2. Timing diagram for multi-user one- and two-way relaying in TDD systems.

Fig. 3. System model of two-way relaying between one BS and $K$ MSs. The BS is equipped with $N_B$ antennas, the RS is equipped with $N_R$ antennas, and the $k$-th MS is equipped with $N_k$ antennas. The BS and the $k$-th MS exchange $L_k$ data streams.
Fig. 4. Illustration of signal space alignment. The BS exchanges two data streams with each of two MSs. At each node, desired signals are aligned with the backward propagated self-interference.

Fig. 5. Illustration of the two-stage transceiver design algorithm.
Fig. 6. Packet error rate versus SNR. The BS, the RS, and the MSs are equipped with \( N_B = 4 \), \( N_R = 4 \), and \( N_k = 2 \) antennas, respectively. The BS exchanges \( L_1 = 2 \), \( L_2 = 1 \), and \( L_3 = 1 \) data streams with the MSs. User 2 has higher service priority than the other users: \([\omega_U^{(2)}](1) = [\omega_D^{(2)}](1) = 2 \) and \([\omega_U^{(k)}](l) = [\omega_D^{(k)}](l) = 1 \) otherwise.

Fig. 7. Packet error rate versus SNR. The BS, the RS, and the MSs are equipped with \( N_B = 4 \), \( N_R = 8 \), and \( N_k = 2 \) antennas, respectively. The BS exchanges \( L_1 = 2 \), \( L_2 = 1 \), and \( L_3 = 1 \) data streams with the MSs. All users have the same service priority: \([\omega_U^{(k)}](l) = [\omega_D^{(k)}](l) = 1 \), \( \forall k, l \).
Fig. 8. Average sum rate comparison. The RS and the MSs are equipped with $N_R = 4$ and $N_k = \{2, 4\}$ antennas, respectively. The BS exchanges $L_1 = 2$, $L_2 = 1$, and $L_3 = 1$ data streams with the MSs. User 2 has higher service priority than the other users: $[\omega^{(2)}_U]_l(1) = [\omega^{(2)}_D]_l(1) = 2$ and $[\omega^{(k)}_U]_l(i) = [\omega^{(k)}_D]_l(i) = 1$ otherwise. (a) Average sum rate versus SNR when the BS is equipped with $N_B = 4$ antennas. (b) Average sum rate versus the number of BS antennas $N_B$ at 25 dB SNR.