Seismic interferometry and ambient noise tomography: theoretical background and application in south India

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Abstract. Seismic interferometry can be used to extract useful information about Earth’s subsurface from the ambient noise wave field. It is an important new tool for exploring seismically quiescent areas. The method involves extraction of empirical Green’s function from the background ambient vibrations of the Earth, followed by computation of group or phase velocity and tomographic imaging. Here we provide a review of seismic interferometry and ambient noise tomography (ANT) and present an example of the method in south India.

1. Introduction
The traditional methods in seismology for imaging the earth’s interior use time and amplitude of earthquake waves recorded over seismographs. The methods therefore could be best used only in seismologically active areas. Elsewhere, it has limited resolution. Over the last decades, a new method known as seismic or wave field interferometry has revolutionized passive seismology imaging. The method is based on ambient seismic noise caused mainly by wind, ocean waves, rock fracturing and anthropogenic activity that constantly travel through the Earth. This complex wave field contains information about the Earth’s subsurface. The feasibility of the method has been established experimentally (Weaver and Lobkis, 2001; Loarose et al., 2005) and theoretically (Snieder, 2004; Wapenaar, 2004). The methodology allow us to decode the information contained in the ambient noise wave field to create an artificial seismogram that can then be used to image the subsurface of the Earth following traditional seismological tomographic or imaging methods.

Surface wave empirical Green functions (EGFs) can be determined from cross-correlations between long time sequences of ambient seismic noise observed at different stations. Shapiro and Campillo (2004) demonstrated that EGFs estimated from ambient noise possess dispersion characteristics similar to earthquake derived measurements. The dispersion measured from surface wave EGFs have been inverted to produce the corresponding tomography maps in several regions (Shapiro et al., 2005; Moschetti et al., 2007; Kao et al., 2013, Borah et al., 2014, Guo et al., 2015). Most of these studies obtained Rayleigh wave group velocity maps in the period range from 5 s to 70 s.

In this paper we review the theory of seismic interferometry as applied to ambient noise data. We then construct the virtual seismogram using noise propagating across south India and 3-D tomographic images are generated thereof.

2. Seismic Interferometry: Theoretical Background
Interferometry is a general interference phenomenon between pairs of signals and is used to gain useful information about the medium. Compared to many other tomographic imaging tools with long
history, application of ambient noise to Earth imaging is recent (Wapenaar, 2004; Campilo and Paul, 2003; Wapenaar et al, 2011). The basic theory behind seismic interferometry is that Green’s function between two seismic stations can be estimated by cross correlating long time series of ambient noise recoded at those stations. This Green’s function may be thought of as the seismogram recorded at one location due to an impulsive or instantaneous source of energy at the other.

Suppose two receivers at positions r1 and r2 are surrounded by energy sources located on an arbitrary surroundings boundary S (Figure 1). The wave field emanating from each source propagates into medium in the interior of S and is recorded at both receivers. The signals recorded at the two receivers are then cross-correlated. When the cross-correlation of all the sources are added together, the energy that travels along the path will add constructively and the energy that doesn’t travel along the path will add destructively. Thus the Green’s function obtained between r1 and r2 will be as if one of the receivers had actually been a source (Figure 1) (Wapenaar, 2004). This case is actually observed where each source is fired sequentially and impulsively. For the case of random noise, a surface S exists such that it joins all the noise sources and since noise sources may all fire at the same or at overlapping time, their recorded signals at two receivers are already summed together which actually takes place naturally. Snieder (2004) showed that the seismic sources located around the extensions of the inter-receiver path contribute most to the interferometric Green’s function construction and thus whole boundary of source is not necessary in order to approximate the inter-receiver Green’s function.

![Figure 1](image)

**Figure 1:** (a) Two receivers (triangles) are surrounded by a boundary S of sources each of which sends a wavefield into the interior and exterior of S. (b) the seismic interferometry method turns one of the receivers (r1) into virtual source from which a real seismogram is obtained. (c) sources located within the grey regions contribute the most to the Green’s function computation. (Nicolson et al., 2012).

2.1. **Conceptual Framework for Correlation and Green’s Function**

Recent works by Wapenaar & Fokkema (2005), and a host of others have confirmed both theoretically and practically the 3D elastodynamic generalization of Claerbout’s (1967) 1-D postulate that the autocorrelation of noise generated at depth at a surface station yields the reflection profile of the earth under that station and using two different surface points yields the impulse response function between these points, as is indicated by the cartoon in Figure 2.

The Green’s function of a medium between two points A and B represents the record obtained at A if an impulsive source is applied at B. In a completely random wavefield, the cross-correlation of signals recorded between two points converges to the complete Green’s function of the medium, including all reflection, scattering and propagation modes (Weaver & Lobkis, 2001). If the sources of the ambient noise are evenly distributed and recorded by sensors at two points A(φ, A), B(φ, B), then time derivative of the correlation is the exact Green’s function of the medium, as if the source was at A or B (Weaver and Lobkis, 2001; Wapenaar, 2004):

\[
\frac{d}{dt} C_{AB}(\tau) = \frac{d}{dt} \int \varphi_A(t) \varphi_B(t + \tau) dt \\
\propto G^+(A, B, \tau) - G^-(A, B, -\tau)
\]

(1.0)
Figure 2: Schematic diagram showing the cross-correlation of noise at two surface stations.

It has been established that normalized cross-spectral density $C_{A,B}(\omega)$ at frequency $\omega$ between two receivers $A$ and $B$ separated by a distance $r$ is

$$C_{A,B}(\omega) = \frac{\sin(\omega r/c)}{\omega r/c}$$

(1.1)

In time domain (taking Fourier transform) the normalized correlation function is

$$C_{A,B}(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} C_{A,B}(\omega) e^{i\omega t} d\omega$$

Therefore

$$\frac{d}{dt} C_{A,B}(t) = \frac{1}{4\pi r/c}[\delta(t + r/c) - \delta(t - r/c)]$$

(1.2)

The above equation (1.2) is our required exact Green’s function.

The surface wave parts of inter-receiver Green’s functions appear particularly clearly in seismograms constructed from seismic interferometry, because strong sources of seismic noise are in general restricted to locations within or on the Earth's crust. Surface wave travel along the interfaces between different layers; within the Earth, they propagate particularly strongly within the crust and upper-mantle. Seismic surface waves can be divided into Rayleigh waves which have longitudinal and vertical motion, and Love waves which have transverse horizontal vibration. Both these types of surface waves are observable on cross-correlations of ambient seismic noise.

3. Ambient Noise Tomography

The ambient noise data processing is best described by Bensen et al. (2007). The procedure is divided into four main phases (1) single station data preparation, (2) cross-correlation and temporal stacking, (3) measurement of dispersion curves and (4) quality control, including error analysis and selection of the acceptable measurements. Single station data preparation includes removal of instrument response, mean and trend; band-pass filter and cut to a length of one day for each individual station. Earthquake signals and instrument irregularities are then to remove by temporal normalization. After temporal normalization, the signals are whitened in frequency. Before whitening, ambient noise is most energetic in the microseism bends below 20 sec period. Frequency whitening is carried out to broaden the period band of the dispersion measurement. Next is to perform cross-correlation between each pair of station for each available day. In order to enhance the signal to noise ratio (SNR), stacking of available daily cross-correlation for each component and station pair is done. This stacked cross-correlation can be converted to Empirical Green’s function by introducing an additive phase factor (Lin et al., 2008). Next step is to calculate the dispersions. One particularly useful property of surface wave is that they are dispersive: the longer period waves within a packet of surface wave energy have a longer wavelength and hence penetrate deeper into Earth. Since different frequencies are sensitive to properties at different depths, study of surface wave dispersion allows us to infer information about how seismic velocity varies with depth in the Earth (e.g. Dziewonski et al., 1969). Typically, periods
below about 20 s are mainly sensitive to crustal structure and properties, and above 20 s are also sensitive to properties of the upper mantle. Inverting surface wave velocities at different periods, measured for many paths within a given region, to obtain models of the Earth’s velocity structure with depth is known as surface wave tomography. Since seismic interferometry does not depend on the location of impulsive sources such as earthquakes, rather only the location of the receivers, the resolution of ambient noise tomography in relatively aseismic regions can be greater than that achieved by local surface wave tomography using earthquakes.

3.1. Ambient noise tomography in South India

We applied ambient noise tomography to image the subsurface of southern India. Seismograms recorded at 57 broadband sites (Figure 3a) have been analyzed for the study. These stations operated for approximately three years data (from February 2009 to April 2012).

Figure 3: (a) Station location map of south India, (b) Cross-Correlation gather for the station SUP. All waveform are bandpass between 5 and 10-s period.

We have followed the data preparation steps as discussed in the previous section. For each station the data has been resampled to 10 samples per second and split into one day segments, followed by the removal of mean, trend and instrument response. The resulting waveform is then tapered, bandpass filtered between periods 1-60 s followed by time domain normalization and spectral whitening. After all of these preprocessing steps were applied, cross-correlation between each of the resulting day segment was computed. The resulting cross-correlations are two sided functions, where the signal at positive and negative lag times (i.e. causal and acausal components) represents energy travelling in opposite directions between pair of stations. The cross-correlation functions (CCFs) were calculated for the daily waveforms of each pair of station and stacked to increase the signal to noise ratio (SNR). Cross-correlation gather for a station SUP is shown in Figure 3b. We have applied frequency time analysis (FTAN) multiple-filter technique (Dziewonski et al. 1969) with phase match filtering (Levshin and Ritzwoller, 2001) to measure the group velocity dispersion curves. After calculating the dispersion for all EGF’s, we selected data with the signal to noise ratio (SNR) greater than 10 and the interstation spacing more than three wavelengths at a given period (Bensen et al 2008). SNR is defined as the ratio of peak amplitude in the signal window to root mean square (rms) noise in the trailing noise window (Bensen et al., 2007). We used 2514 interstation paths with a distance ranging
from 44.7 km to 1359 km. The maximum number of dispersion curve measurement is 1350 for 10 s period. For the shortest periods (T < 4 s) the number of measurements decreases significantly as waveforms with poor signal to noise ratio have been discarded.

Before performing a tomographic inversion with surface wave travel time data it is important to understand how well the subsurface structure could be resolved by the geometry of stations and virtual sources. This is achieved by generating synthetic data for each of a series of checkerboard models which represent an imaginary Earth’s group velocity structures, calculating a solution model using the synthetic data and comparing the solution velocity models with the synthetic Earth models. The resolution of the surface wave tomography depends primarily on path coverage (Figure 4) and their azimuth distribution, has been studied using checkerboard test where, input models are a square cells of side 0.7°. Velocity perturbations of ±8% are assigned to each cell with average velocity shown at the bottom left corner at each period in Figure 4.

After computing the group velocity dispersion measurement a tomographic inversion is performed for periods between 2 and 10 s (Figure 5) using the non-linear 2-D tomographic inversion technique developed by Rawlinson and Sambridge (2003). The method combines the Fast Marching Method (FMM) (Rawlinson and Sambridge, 2004a, b) for calculation of forward problem and gradient method based on subspace technique (Kennett et al., 1998) for inversion, where the minimization is carried out simultaneously along several search directions that together span a subspace of the model space. The method has been successfully used for ambient noise tomography of several regions (Young 2011, Saygin & Kennet 2012, Ren et al. 2013 etc). A detailed explanation of the methodology is presented in Rawlinson and Sambridge (2004a) and Saygin and Kennett (2012).

![Figure 4: Ray path coverage and checkerboard resolution test at 2 s and 10 s period.](image-url)
4. Discussions and Conclusions
In this paper we described the background of seismic interferometry and the ambient noise tomography method. We have shown that seismic interferometry can be used to compute inter-receiver surface wave using ambient noise data recorded in southern India. We have presented the first surface wave group velocity map of south India from ambient seismic noise and they contain useful information of surface geology. The group velocity can be converted to shear velocity for 3-D variations in subsurface structures with the depth. Further work will aim to use Love as well as Rayleigh waves to achieve better lithological constraints along south India.

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References
[1] Bensen, G D, Ritzwoller M H, Barmin M P, Levshin A L, Lin F, Moschetti M P, Shapiro N M, Yang Y, 2007, Processing seismic ambient noise data to obtain reliable broad band surface wave dispersion measurements, Geophys. J. Int., 169(3), 1239–1260.
[2] Bensen, G D, Ritzwoller M H and Shapiro N M, 2008 Broadband ambient noise surface wave tomography across the United States, J. Geophys. Res., 113(B5).
[3] Borah, K, Rai S S, Prakasam K S, Gupta S, Priestley K and Gaur V K, 2014 Seismic imaging of crust beneath the Dharwar Craton, India, from ambient noise and teleseismic receiver function modelling, Geophys. J. Int., 197(2), 748–767.
[4] Campillo, M, Paul A, 2003 Long-range correlations in the diffuse seismic coda. Science 299, 547–549.
[5] Claerbout, J, F, 1967, Synthesis of a layered medium from its acoustic transmission response, Geophysics, 33, 264.
[6] Dziewonski, A, Bloch S, and Landisman M, 1969 A technique for the analysis of

Figure 5: Group velocity perturbation (%) maps for selected periods of 2 and 10 s. Velocity perturbations are relative to the average group velocity plotted at the bottom left corner of each plot.
transient seismic signals, *Bull. Seismol. Soc. Am.*, 59(1), 427–444.

[7] Guo, Z, Chen Y J, Ning J, Feng Y, Grand S P, Niu F, Kawaktsu H, Tanaka S, Obayashi M, Ni J, 2015 High resolution 3-D crustal structure beneath NE China from joint inversion of ambient noise and receiver functions using NECESSArray data, *Earth Planet. Sci. Lett.*, 416, 1-11.

[8] Kao, H, Behr Y, Currie, C A, Hyndman R, Townend J, Lin F C, Ritzwoller M H, Shan S, He J, 2013 Ambient seismic noise tomography of Canada and adjacent regions: part I. crustal structures, *J. Geophys. Res.*, 118, 5865-5887.

[9] Kennett, B, L, N, Seismic wave propagation and seismic tomography, Research School Of Earth Sciences, Institute of Advanced Studies, The Australian National University, Canberra, 1998.

[10] Larose E., Derode A, Clorennec D, Margerin L, and Campillo M, 2005 Passive retrieval of Rayleigh waves in disordered elastic media, *Phys. Rev. E*, 72(4) 46,607.

[11] Levshin, A L, and Ritzwoller M H 2001 Automated detection, extraction, and Measurement of Regional Surface Waves, *Pure Appl. Geophys.*, 158(8), 1531–1545.

[12] Lin, F., M.P. Moschetti, and M.H. Ritzwoller (2008). Surface wave tomography of the western United States from ambient seismic noise: Rayleigh and Love wave phase velocity maps, *Geophys. J. Int.*, 173(1), 281–298.

[13] Moschetti M P, Ritzwoller M H and Shapiro N M, 2007 Surface wave tomography of the western United States from ambient seismic noise: Rayleigh wave group velocity maps, *Geochem. Geophys. Geosys.*, 8, Q08010.

[14] Nicolson, H, Curtis A, Baptie B, Galetti E, 2012 Seismic interferometry and ambient noise tomography in British Isles, *Proceeding of the Geologists Association*, 123, 74-86.

[15] Rawlinson, N, Sambridge M, 2003 Seismic traveltime tomography of the crust and lithosphere, *Advances in Geophysics*, pp. 81–198, Elsevier {BV}.

[16] Rawlinson, N, Sambridge M, 2004a Multiple reflection and transmission phases in complex layered media using a multistage fast marching method, *Geophysics*, 69(5), 1338.

[17] Rawlinson, N, Sambridge M, 2004b Wave front evolution in strongly heterogeneous layered media using the fast marching method, *Geophys. J. Int.*, 156(3), 631–647.

[18] Ren, Y, Grecu B, Stuart G, Houseman G, Hegedus E, 2013 Crustal structure of the Carpathian Pannonian region from ambient noise tomography, *Geophys. J. Int.*, 195(2), 1351-1369

[19] Shapiro N, Campillo M, 2004 Emergence of broadband Rayleigh waves from correlations of the ambient seismic noise. *Physical Review Letters* 31 (7), 1615-1619.

[20] Shapiro N, Campillo M, Stehly L, Ritzwoller M H, 2005 High-resolution surfacewave tomography from ambient seismic noise. *Science* 307, 1615.

[21] Snieder R, 2004 Extracting the Green’s function from the correlation of coda waves: a derivation based on stationary phase. *Physical Review* E69 046610.1- 046610.8.

[22] Wapenaar K, 2004 Retrieving the elastodynamic Green’s function of an arbitrary homogeneous medium by cross correlation. *Physical Review* E 69, 046610.

[23] Wapenaar, K, Fokkema J, 2005 Seismic interferometry, time-reversal and reciprocity, 67th EAGE Conference & Exhibition.

[24] Wapenaar, K, Ruigrok E, Neut V D, Draganov J D, 2011 Improved surfacewave retrieval from ambient seismic noise by multi-dimensional deconvolution. *Geophysical Research Letters* 38, L01313.

[25] Weaver R, Lobkis O, 2001 Ultrasonics without a source: thermal fluctuation correlations at MHz frequencies. *Physical Review Letters* 87 (13), 134301.

[26] Wessel, P. & Smith, W.H.F., 1998. New, improved version of the Generic Mapping Tools released, EOS, *Trans. Am. geophys. Un.*, 79, 579.

[27] Young, M K, Rawlinson N, Arroucau P, Reading A M, and Tkalčič H, 2011 High frequency ambient noise tomography of southeast Australia: New constraints on Tasmania’s tectonic past, *Geophys. Res. Lett.*, 38(13).