Measuring extensive air showers with Cherenkov light detectors of the Yakutsk array: the energy spectrum of cosmic rays

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Abstract. The energy spectrum of cosmic rays in the range \( E \sim 10^{15} \) eV to \( 6 \times 10^{19} \) eV is studied in this paper using air Cherenkov light detectors of the Yakutsk array. The total flux of photons produced by the relativistic electrons (including positrons as well, hereafter) of extensive air showers in the atmosphere is used as an energy estimator of the primary particle initiating a shower. The resultant differential flux of cosmic rays exhibits, in agreement with previous measurements, a knee and ankle feature at energies of \( 3 \times 10^{15} \) and \( \sim 10^{19} \) eV, respectively. A comparison of observational data with simulations is made in the knee and ankle regions in order to choose the models of galactic and extragalactic components of cosmic rays that describe well the energy spectrum measured.

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1. Introduction

The Cherenkov light emitted in the atmosphere by the relativistic electrons of extensive air showers (EASs) of cosmic rays (CRs) carries important information about the shower development and the primary CR particles. A well-known application of the air Cherenkov light technique is γ-ray astronomy. In this paper, we focus on another application of the technique, namely to measure Cherenkov photons from EASs initiated by CRs of energy above $10^{15}$ eV. These giant showers supply plenty of light so that one can detect it with unarmed photomultiplier tubes (PMTs) triggered within the pulse duration.

Since the first observation by Galbraith and Jelley [1] and a systematic measurement of air Cherenkov light properties in the Pamir experiment [2], a number of EAS arrays have been equipped with Cherenkov light detectors. It seems that the most durable and plentiful source of Cherenkov light data is the Yakutsk array experiment [3, 4]. The total flux of light is used to estimate the primary energy in a model-independent manner and the radial distribution of the light intensity at ground level is used to infer the position of the shower maximum, $X_{\text{max}}$, in the atmosphere [3].

In this paper, we focus on the experimental data obtained in Yakutsk with Cherenkov light detectors aiming at the CR energy spectrum. The paper is structured as follows. In section 2 we outline the general characteristics of the Yakutsk array experiment, whereas in sections 3 and 4 the particular properties of Cherenkov light detectors are given: detector design and calibration of the signal. In section 5, the measurement and monitoring of the atmospheric extinction of light are described. The measured lateral distribution of the Cherenkov light intensity at the observation level is given in section 6. The method used to estimate the energy of the particle initiating the EAS is described in section 7. The resultant energy spectrum of CRs is discussed.
in section 8. Our conclusions are set out in section 9. In two appendices, additional information is given that is important for the subjects considered.

2. The Yakutsk array

The Yakutsk array is located at Oktyomtsy near Yakutsk, Russia (61.7°N, 129.4°E), 100 m above sea level (1020 g cm$^{-2}$). At present, it consists of 58 ground-based and six underground scintillation detector stations for measuring charged particles (electrons and muons) and 48 detectors—PMTs in shuttered housing—for observing the atmospheric Cherenkov light. During the more than 30 years of its lifetime, the Yakutsk array has been re-configured several times; the total area covered by the detectors was maximal in about 1990 ($S_{\text{eff}} \sim 17$ km$^2$), now it is $S_{\text{eff}} \sim 10$ km$^2$. In the central part of the array, there is a denser domain with 100–250 m detector spacing. During the whole observation period, approximately $10^6$ showers of primary energy above $10^{15}$ eV were detected; the three highest energy events selected with the axes within the array area and zenith angle $\theta \leq 60^\circ$ have an energy $E > 10^{20}$ eV.

The actual detector arrangement of the array is shown in figure 1. Charged particle detectors having an area of 2 m$^2$ have been built on the stations in couples; the Cherenkov light detectors—PMTs of 176 cm$^2$ and $3 \times 176$ cm$^2$ acceptance area—form the medium, $C_1$ ($\sim 500$ m spacing), and the autonomous, $C_2$ (50–200 m spacing), subsets. The latter were added in 1995 with the aim of studying the air showers in the energy range $10^{15}$–$10^{17}$ eV via Cherenkov light measurements [5].

All detectors/controllers and data processing units of the array are connected to the data handling network shown in figure 2.

3. The design of the air Cherenkov light detector

Charged particle detectors of the array have been described in [3]. Here we will detail a Cherenkov light detector unit. It consists of a vertically mounted PMT (FEU-49B;
Figure 2. Local area network of the Yakutsk array.

Figure 3. The air Cherenkov light detector.

diameter: 15 cm) with an amplifier in a metal container that is blackened inside [6]. An upper hole provides $\theta \leq 55^\circ$ aperture (figure 3). To protect the photocathode from sunlight, a motorized light-proof lid is set up. At night, all the lids of the array can be commanded remotely to open. PMTs and amplifiers are powered round the clock to guarantee stability of performance. When the lid is open, a fan blows out warm air to keep snow and dust from the photocathode surface.

There is also a variant of the detector with three PMTs in a housing and this can operate independently or in summation of the signal in order to increase acceptance area at the shower periphery. In addition, dedicated detectors were used to measure the shape and width of the
Cherenkov signal from the shower. As an example, the pulse shape of the Cherenkov signal at the shower periphery is shown in figure 4, whereas the half-width of the signal as a function of radial distance is given in figure 5.

The spectral sensitivity of the PMT used is shown by the dashed line in figure 6 together with the air Cherenkov light spectrum (solid line) and the atmospheric transmission curve.

Two kinds of triggers were used to select the showers from the background at the Yakutsk array: triggers produced by the scintillators and those produced by the Cherenkov light detectors. In the first case, a coincident signal (if the particle density, $\rho$, is greater than $0.5 \text{ m}^{-2}$ in two scintillators of each station within $2 \mu s$) passes on to the central controller. Trigger-500 is then produced in the case of a coincident signal (in $40 \mu s$) from three or more stations with $\sim 500 \text{ m}$ spacing ($\geq$ sixfold coincidence). Similarly, trigger-1000 is produced by stations with $\sim 1 \text{ km}$ spacing. After the year 1992 when 18 new stations were added, the array area has increased from 2.5 to $7.2 \text{ km}^2$ where trigger-500 operates. That is why we can deal with EAS in the energy range from $3 \times 10^{16}$ to $3 \times 10^{19} \text{ eV}$ by using the same trigger.

The $C_1$ subset of the Cherenkov light detectors has no hardware trigger of its own. Instead, the scintillator trigger controls the detection of Cherenkov signal in this sub-array. The amplitude of the PMT signal is recorded within 5 $\mu s$ after the coincident signal in two scintillators of the host station. In order to exclude detection of an accidental Cherenkov signal.
when there is no trigger-500/1000, a double-trigger condition is used in data analysis:

- Three or more scintillator stations have detected the charged particles ($\rho > 0.5 \text{ m}^{-2}$).
- The Cherenkov light intensities in three or more PMTs are greater than $2.4 \times 10^5 / 4.5 \times 10^5 \text{ m}^{-2}$ depending on the acceptance of PMT.

The thresholds have been chosen to keep the signal/noise ratio greater than 3.

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The second case is the $C_2$ subset, which has an independent Cherenkov light trigger formed by three or more PMTs having detected light intensities above a given threshold within 10 $\mu$s. The signal integration time of the individual PMT is 0.5 $\mu$s.

4. Detector calibration

In order to find the absolute response of the array to Cherenkov light we have considered the sensitivity of all the PMTs in the detectors and have measured the conversion factors from light intensity into analog–digital converter (ADC) output. The Cherenkov light emitted by relativistic muons in a transparent medium with known refraction index (optical radiator) was used in the detector calibration, as was suggested in [8]. In this paper we are keeping to the description given in [6, 9]. Distilled water and plexiglas are used to measure the output signal in a geometry shown in figure 7. A telescope composed of two PMTs above and below the radiator,
spaced 1 m from the detector, with the bottom one under the lead shield (10 cm), selects high-energy muons from background CRs. On the photocathode of the detector PMT is placed (above a thin layer of glycerin) a plexiglas disc of 5.5 cm thickness and 15 cm diameter with a polished side surface. As another version of the radiator, distilled water was used without any pan layer. The Cherenkov radiation angle in water is $35^\circ$, whereas it is greater in plexiglas ($48^\circ$) and in glycerin ($47^\circ$).

The number of photons in the wavelength interval ($\lambda_1 = 3000 \, \text{Å}, \lambda_2 = 8000 \, \text{Å}$) emitted in the radiator ($n = 1.49$, $l = 5.5 \, \text{cm}$) is given by the Frank and Tamm formula [10]:

$$\frac{dN_\gamma}{dE_\mu} = 2\pi \alpha l \left( \frac{1}{\lambda_1} - \frac{1}{\lambda_2} \right) \left( 1 - \frac{E_\mu^2}{n^2(E_\mu^2 - m_\mu^2)} \right),$$

where $\alpha = 1/137$; $n$ is the refraction coefficient; $m_\mu$ and $E_\mu$ are the muon mass and energy, respectively. Integrating the above threshold energy ($E_\mu > 0.25 \, \text{GeV}$) with the muon spectrum measured at sea level, we have $N_\gamma = 2850$. Thus, the ratio of the air Cherenkov light intensity $Q(R)$ at distance $R$ from the shower core to $N_\gamma$ is

$$\frac{Q(R) S}{N_\gamma} = \frac{A_s \overline{K}_r}{A_r (1 - R_s)},$$

where $A_s, A_r$ are the detector signal amplitudes from the shower and radiator; $S$ is the photocathode area; $R_s \leq 0.04$ is the reflectivity factor of the air–glass junction.

Optical radiators used in the laboratory for absolute calibration of the detector do not produce a sufficient number of photons for the routine ratio calibration of a multitude of array detectors in the field. Instead, the scintillator disc ($D = 15 \, \text{cm}, h = 5 \, \text{cm}$) is in use as the light source for this case. Intercalibration of different radiators has been performed using the experimental setup (figure 7).

The total calibration error of 21% consists of uncertainties in

- number of photons at the photocathode (10%),
- measurement of signal amplitude from the radiator (9%),
- conversion factor from the radiator to the scintillator (7%) and
- signal variance throughout the photocathode area and acceptance angle (15%).

### 5. Atmospheric extinction of light

The Cherenkov light undergoes extinction in the atmosphere because of absorption on molecules by Rayleigh scattering and Mie scattering by aerosols and absorption. During the observation periods in winter (average temperature $-40^\circ \text{C}$), Cherenkov light absorption in the atmosphere is negligible and only molecular and aerosol scattering of photons is taken into account.
Figure 8. Atmospheric extinction of the Cherenkov light as a function of $X_{\text{max}}$. $\theta = 20^\circ$ (circles). Approximation 3 is shown by the solid line.

Molecular scattering is almost constant, whereas aerosol concentration in the boundary layer above the surrounding terrain is of diurnal and seasonal variability. We are using the event rate of the showers in the energy range $10^{15} - 10^{16}$ eV detected with a PMT subset in one hour and 15 min intervals in order to monitor the atmospheric transparency for the light generated in EAS [3]. The method is based on the fact that the Cherenkov light flux is proportional to the primary energy:

$$\tau_i = \tau_0 \left( \frac{N_i(>Q_{\text{thr}})}{N_0(>Q_{\text{thr}})} \right)^{1/\kappa},$$

where $N(>Q_{\text{thr}})$ is the integral number of events with the light intensity above the threshold detected in the $i$th and basic periods; $\tau$ is the atmospheric extinction coefficient; $\kappa$ is the energy spectrum index.

The resultant extinction coefficient is a product of the variable $\tau_a$ due to the current aerosol concentration and the basic coefficient $\tau_0$ caused by the molecular scattering and minimal aerosol extinction. The Rayleigh scattering parameters evaluated for the Yakutsk array conditions give $\tau_R = 0.9$ [6] and the corresponding attenuation length $\lambda_R = 300$ km, averaged over the wavelength range under consideration.

The integral spectrum index of the Cherenkov light flux around the knee is [6]

$$\kappa = 1.5 \pm 0.03, \quad E_0 < 3 \times 10^{15},$$

$$\kappa = 2.1 \pm 0.04, \quad E_0 > 3 \times 10^{15}.$$

Evaluating the coefficient $\tau_0$ as a function of $X_{\text{max}}$ in the shining point approximation [11] by assuming that the attenuation length due to aerosol scattering is constant above 1.5 km height and decreases below it [12], one can derive the average extinction coefficient shown in figure 8. The approximation is

$$\tau = (0.79 \pm 0.02) - \frac{X_{\text{max}}}{1400 \pm 30} + \frac{X_{\text{max}}^2}{(1.11 \pm 0.02) \times 10^6},$$

where $X_{\text{max}}$ is in g cm$^{-2}$.

Finally, we have found that

$$\tau = 0.765 \pm 0.015$$
at $\bar{E} = 20^0$, $\bar{E} = 7 \times 10^{16}$ eV averaged over all observation periods, and inserting spectrum index in equation (2).

6. Lateral distribution of the air Cherenkov light

During $\sim 15 \times 10^3$ h of observation, $\sim 60 \times 10^3$ showers of energy above $6 \times 10^{16}$ eV were detected by the medium $C_1$ subset. The autonomous $C_2$ array data consisted of $\sim 200 \times 10^3$ showers with $E > 1.2 \times 10^{15}$ eV detected during $\sim 3200$ h of observation. Only the Cherenkov trigger condition is used to select these showers.

The relative dispersion of the light intensity observed with different detectors at $R \sim 500$ m from the shower core has been evaluated by selecting showers with three detectors hit in the interval $400 < R < 500$ m not far from each other ($\leq 75$ m). Intensity dispersion contains, in addition to inherent fluctuations, the following: instrumental errors, core location error, intercalibration error, etc. Supposing chance variation of the signal around an average $Q(R) \propto R^{-2.35}$ [13], we have found that $\delta Q/Q = 0.25 \pm 0.13$. In order to minimize the $Q(R)$ uncertainty due to EAS axis location error, one has to select showers with axes within $R < R_{opt}$ m ($80 < R_{opt} < 250$ if $E_0 > 2 \times 10^{17}$ eV; $80 < R_{opt} < 400$ if $E_0 > 2 \times 10^{18}$ eV) where stations are spaced closely.

In the highest energy domain, measurements of Cherenkov light exist, carried out by the Haverah Park group [14]. In figure 9 our results are given in comparison with [14]. The data are normalized at 200 m core distance and exhibit consistency in the lateral distribution function (LDF) shape. The solid lines are an approximation to our data given in [9].
Figure 10. Results of model calculation and the Yakutsk array data (circles). The lines illustrate the results of: 1: Hara et al [7]; 2: Dyakonov [15]; 3: Ivanenko et al [16]; 4: Lagutin et al [17], and 5: Dedenko et al [18]. The primary particle energy is indicated on the left side of the data/lines.

Model simulations give a variety of LDFs; some of them are close to our data, as illustrated in figure 10. The lateral distribution depends on the attenuation lengths, multiplicity of secondaries in the interactions, primary mass composition, etc, which can be parameterized with $X_{\text{max}}$. Another considerable influence is the angular distribution of electrons in the shower. A combination of these factors results in different LDFs. The results of Ivanenko et al and Dyakonov are in agreement with our measurements.

Our cumulative results on LDF measurements (zenith angle $\theta < 30^\circ$) are given in figure 11. The data of both subsets $C_1$ and $C_2$ are parameterized by the intensity at 150 m from the shower core, $Q_{150}$, the only core distance really present in the shifting range of measurements when the primary energy is rising from $E_0 \sim 10^{15}$ to $10^{19}$ eV. The data are consistent with the previous results of the Yakutsk array concerning the Cherenkov light and can be described by a suitable EAS model simulation [15, 16].

No abrupt change of LDF parameters is seen, so we choose a rather smooth approximation curve to fit the experimental data in the whole energy range [4]:

$$Q(R) = Q_{150} \frac{(R_1 + 150)(R_2 + R)^{1-b}}{(R_1 + R)(R_2 + 150)^{1-b}},$$

where $R_1 = 60$ m; $R_2 = 200$ m; $b = (1.14 \pm 0.06) + (0.30 \pm 0.02) \times \log Q_{150}$.

7. Energy estimation

The total flux of the Cherenkov light emitted, $Q_{\text{tot}}$, is our main estimator of the primary particle energy. In order to derive the relation between $Q_{\text{tot}}$ and the ionization loss of the shower
Figure 11. The radial distribution of the air Cherenkov light. Our experimental data are given by the black and white points alternately, so as not to confuse the adjacent data. The curves are approximations according to equation (4) with $Q_{150}$ fitted to the particular data. An axis distance of $R = 150 \text{ m}$ is indicated by the dotted line.

electrons in the atmosphere, we have used the number of photons induced on the length $dh$ in the atmosphere, $dN_{\gamma}/dh$, in ratio to that induced in the optical radiator, $dN_r/dh$:

$$\frac{dN_{\gamma}}{dh} = \frac{dN_r}{dh} \frac{1}{(1 - 1/n^2_r)} \left(1 - \left(\frac{1}{vn}\right)^2\right),$$

where $v$ is the electron velocity; $n$ and $n_e$ are the refraction indexes in the air and radiator, respectively; $c = 1$. 

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Emission is possible when $vn > 1$ or $E^2 > E_{thr}^2 = m^2/(1 - (1/n^2))$. The number of photons induced in the depth interval $dx$ is

$$\frac{dN_\nu}{dx} = \zeta \left( 1 - \frac{E_{thr}^2}{E^2} \right), \quad \text{(5)}$$

where

$$\zeta = \frac{2 \frac{dN_e}{dx} \frac{n_0 - 1}{\rho_r 1 - (1/n^2)}}{\rho_0}$$

with $\rho_r$ being the radiator density and $\rho_0, n_0$ are the air density and refraction at the observation level.

The total light flux is given using the electron differential energy spectrum in the shower at the depth $x$:

$$Q_{tot}(x) = \int_0^x dx \tau(x_0 - x) \int_{E_{thr}}^{E_0} dE \xi \left( \frac{E_{thr}^2}{E^2} - 1 \right) \frac{dN(x, E, E_0)}{dE}, \quad \text{(6)}$$

where $\tau(x_0 - x)$ is the extinction coefficient of light along the path $x_0 - x$.

The number of electrons in a shower is approximately $N(x, E, E_0) \approx N(x, E_0) \chi(E)$, where $N(x, E_0)$ is the number of electrons with $E > 0$ and $\chi(E)$ is the universal electron spectrum at the shower maximum. Then

$$Q_{tot} = 2\zeta E_{thr}^2 \int_0^{x_0} dx \tau(x_0 - x) N(x, E_0) \int_{E_{thr}}^{E_0} \chi(E) \frac{dE}{E^3}, \quad \text{(7)}$$

leads to the relation to the ionization loss of electrons in the atmosphere $E_i = \varepsilon_0 \int_0^{x_0} N(x, E_0) dx/t_0$:

$$Q_{tot}^{x_0 = 1} \approx 2\zeta t_0 \left( 0.275 - 0.283 \frac{E_{thr}(X_{max})}{\varepsilon_0} \right) \frac{E_i}{\varepsilon_0}, \quad \text{(8)}$$

if we assume that the extinction $\tau = 1$ [19].

In the real case of the Yakutsk array conditions $\bar{T} = -30^\circ \text{C}$, $P = 754$ Torr and the extinction of light described in section 5, it was found that the relation is [19]

$$E_i/Q_{tot} \approx (3.01 \pm 0.36) \times 10^4 (1 - X_{max}/(1700 \pm 270)) \quad \text{(9)}$$

for $X_{max} \in (500, 1000)$ g cm$^{-2}$; the accuracy of the relation is $\sim 5\%$. This demonstrates the advantage of the air Cherenkov light measurement technique: the relation is determined by $X_{max}$ and extinction of light only; interaction model dependence is parameterized by means of $X_{max}$.

In appendix A, we analyze the shower parameters governing the energy fractions transferred to EAS components.

### 7.1. Experimental evaluation of the energy transferred to EAS components

The energy fractions of the main EAS components can be estimated using the Yakutsk data. The ionization loss of electrons is measured when detecting the total flux of the Cherenkov light at ground level. The detector disposition of the Yakutsk array is appropriate for measuring $Q_{tot}$ in the range above about $Q_{150} = 10^7$ m$^{-2}$ as is shown in table 1. In each $Q_{150}$ bin, the LDF extrapolation formula (4) is used to calculate the total flux.

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Table 1. The fraction of the air Cherenkov light, $\Delta Q$, charged particles, $\Delta S$, and muons, $\Delta \mu$, actually measured in EAS events in the Yakutsk array experiment.

| $Q_{150}$ (m$^{-2}$) | $10^6$ | $10^7$ | $10^8$ | $10^9$ |
|---------------------|------|------|------|------|
| $\Delta Q$ (%)     | 50  | 70  | 90  | 85  |
| $\Delta S$ (%)     | –   | –   | 14  | 13  |
| $\Delta \mu$ (%)   | –   | –   | 65  | 65  |

Table 2. The primary energy fractions of EAS components: ionization loss of electrons in the atmosphere, $E_i$, and in the ground, $E_g$; energy of muons at ground level, $E_\mu$; energy of EAS components unobservable at the Yakutsk array, $E_{\text{unobs}}$. $E_0 = E_i + E_g + E_\mu + E_{\text{unobs}}$, $\theta = 0^\circ$.

| Energy deposit channel | $E_0$ fraction (%) | Experimental uncertainty (%) |
|-----------------------|---------------------|-----------------------------|
| $E_i$                 | 80                  | 77                           | 30                           |
| $E_g$                 | 9                   | 15                           | 60                           |
| $E_\mu$               | 6                   | 4                            | 10                           |
| $E_{\text{unobs}}$    | 5                   | 4                            | 20                           |

Conversion of the measured $Q_{\text{tot}}$ to $E_i$ is carried out with equation (9) with parameters relevant to the particular observation period, taking into account detector calibration and atmospheric extinction of light.

Other portions of the energy carried out by electromagnetic and muonic components beyond sea level are evaluated via the total number of electrons:

$$E_g = \epsilon_0 N_e \lambda_e / t_0,$$

where $\epsilon_0$ and $t_0$ are the critical energy and radiation length of electrons in air; attenuation length $\lambda_e$ is derived from the zenith angle dependence of $N_e$ [20]; and muons are measured at the ground level:

$$E_\mu = N_\mu (E > 1 \text{ GeV}) \overline{E_\mu},$$

where the average energy of muons $\overline{E_\mu}$ is taken from the MSU array data [21].

Residual energy fractions transferred to neutrinos $E_\nu$, nucleons $E_h$, etc, unmeasurable with this array, are estimated using computational modeling [19]. The resulting apportioning of the primary energy $E_0$ is given in table 2.

The energy fraction carried by the electromagnetic component ($E_{\text{em}} = E_i + E_g$) appears to be the basic contribution to the total energy of the shower, and its energy dependence (measured with Cherenkov light detectors + scintillators of the Yakutsk array [22]) is illustrated in figure 12 in comparison with CORSIKA/QGSJET estimation [23].

Due to the air Cherenkov total light flux and the electron and muon number, which are experimental values measured at the ground level, only about 5% of the primary energy in the interval $E_0 \in (10^{18}, 10^{19})$ eV is calculated by using model assumptions. So we consider the energy estimation used to be model independent within these bounds.
Figure 12. The electromagnetic component energy estimation from the Yakutsk array data: $C_1$ subset (squares) and $C_2$ subset (rhombuses). The solid curve is the result of a CORSIKA/QGSJET-based estimation.

Figure 13. The correlation between charged particle density at 600 m from the core, $S_{600}$, and air Cherenkov light intensity at 150 m, $Q_{150}$, measured in the same showers with $\theta < 15^\circ$.

Moonless nights, when air Cherenkov light measurements are possible, constitute $\sim 10\%$ of the observation period. In order to evaluate the primary energy of the bulk of showers, the correlation

$$S_{600} = 1.56 \times 10^{-8} Q_{150}^{1.01}$$

is used between the charged particle density at 600 m from the shower core, $S_{600}$, and the light intensity at 150 m from the core (figure 13), which, in turn, is related to the total flux of the Cherenkov light in the atmosphere $Q_{\text{tot}} = a_1 Q_{150}^{b_1}$; the parameters are given in the first three columns of table 3. Using the derived relation between the primary energy and $Q_{\text{tot}}$ (equation (9)), the final formula was found to link $Q_{150}$ with $E_0$ in the interval $\theta < 15^\circ$ [19]:

$$E_0 = a_2 Q_{150}^{b_2},$$

(10)
Table 3. Parameters of the relation between $Q_{150}$ and the total flux of air Cherenkov light and the energy of the primary particle initiating EAS.

| $Q_{150}$ (m$^{-2}$) | $a_1$   | $b_1$     | $a_2$ | $b_2$  |
|---------------------|---------|-----------|-------|--------|
| $<10^6$             | $2.39 \times 10^6$ | 0.90     | $(6.87 \pm 1.44) \times 10^{10}$ | 0.87 $\pm$ 0.02 |
| $10^6-10^8$         | $1.33 \times 10^6$ | 0.94     | $(3.78 \pm 0.72) \times 10^{10}$ | 0.91 $\pm$ 0.02 |
| $10^8-10^9$         | $7.91 \times 10^5$ | 0.97     | $(2.90 \pm 0.29) \times 10^{10}$ | 0.93 $\pm$ 0.02 |
| $>10^9$             | $5.75 \times 10^5$ | 0.98     | $(5.59 \pm 0.56) \times 10^9$   | 1.01 $\pm$ 0.02 |

Figure 14. $S_{300}$ as a function of $x = 1020/\cos \theta$ for different CR intensities. Open symbols ($S_i$) are the equi-intensity method results and the filled ones ($Q_i$) are derived by fixing $Q_{400}$.

where the numerical values of $a_2$, $b_2$ are summarized in the last two columns of table 3 for different $Q_{150}$ intervals.

The observed densities $S_{300}/S_{600}$ at various zenith angles are connected to the ‘vertical’ one ($\theta = 0^\circ$) along the attenuation curve [24]. In order to measure the attenuation length of these densities for fixed energy, we have used two different methods—the well-known equi-intensity cut method, and fixing the Cherenkov light intensity at 400 m from the core as the equivalent of the primary energy, taking into account the light absorption in the atmosphere. In figure 14 the results are given. Experimental points are consistent with each other for the two methods used and can be described by the sum of two components—a soft component (electrons, attenuation length $\lambda_e = 200$ g cm$^{-2}$) and a hard component (muons, $\lambda_\mu = 1000$ g cm$^{-2}$) [20]:

$$S_{300}(\theta) = S_{300}(0^\circ) \left( (1 - \beta_{300}) \exp \left( \frac{x_0 - x}{\lambda_e} \right) + \beta_{300} \exp \left( \frac{x_0 - x}{\lambda_\mu} \right) \right),$$

(11)
Table 4. The highest energy EAS events detected with the Yakutsk array

| Date        | θ (deg) | log E₀ | δE₀ (%) | b (deg) | l (deg) |
|-------------|---------|--------|---------|---------|---------|
| 18.02.04    | 47.7    | 20.16  | 42      | 16.3    | 140.2   |
| 07.05.89    | 58.7    | 20.14  | 46      | 2.7     | 161.6   |
| 21.12.77    | 46.0    | 20.01  | 40      | 50.0    | 220.6   |
| 15.02.78    | 9.6     | 19.99  | 32      | 15.5    | 102.0   |

where β₃₀₀ is the hard-component fraction. The attenuation curve for S₆₀₀ is the same but β is different:

\[
\beta_{300} = (0.563 \pm 0.032) S_{300}(0°)^{-0.185 \pm 0.02}, \\
\beta_{600} = (0.62 \pm 0.006) S_{600}(0°)^{-0.076 \pm 0.03}. \tag{12}
\]

Experimental uncertainties in estimating the EAS component energies are summarized in the last column of table 2. The main contribution arises from δEᵣ, which is governed by uncertainties in the atmospheric transparency (15%), detector calibration (21%) and the total light flux measurement (15%). Errors in estimation of Nₑ, λₑ, Nₑ determine the next two items (for ionization loss in the ground and δEₑ). The resultant uncertainty in energy estimation is the sum of all errors weighed with the second/third columns of table 2: δE₀ ~ 32% [19]. An extra 20% was added due to a Q₁₅₀–S₆₀₀ conversion uncertainty.

To illustrate the energy estimation method used, four showers detected at the Yakutsk array in the range \(E₀ \geq 10^{20}\) eV, \(θ < 60°\), and the axes within the array area, are given in table 4 (one event is added slightly below the threshold, because the energy estimation error is larger than the tiny difference).

8. The energy spectrum of CRs derived from air Cherenkov light measurements

The Cherenkov light detector subsets of the Yakutsk array give us the opportunity to reconstruct the energy spectrum of CRs in the energy range from \(E₀ \sim 10^{15}\) eV to \(6 \times 10^{19}\) eV. The total numbers of electrons and muons measured at the ground level are used to estimate the additional energy fractions carried by EAS components but the final relation (10) comprises the light intensity alone.

The intensity of CRs has been evaluated using the number of EAS events derived from the showers detected with PMTs, and aperture \(S_{eff} TΩ\) of the sub-arrays in a particular energy interval, where the array area bounded by the perimeter, \(S_{eff}\), depends on the primary energy and the zenith angle; Ω is the acceptance solid angle; and \(T\) is the sum of observation periods. Under the condition of the array configuration changing due to rearrangements and non-active detectors, \(S_{eff}\) has to be calculated for a given period using the Monte Carlo technique.

The acceptance area has been simulated as a function of \(Q_{150}\) averaged in the zenith angle interval (0°, 30°) for the two Cherenkov light detector subsets shown in figure 1. A lateral distribution fit according to equation (4) is used together with instrumental and statistical errors (Gaussian with 25% relative deviation) to model the trigger of each detector subset with 100 000 fake showers. In the case of the autonomous sub-array, the Cherenkov trigger is simulated, whereas in the \(C₁\) case the double trigger for scintillator and PMT signals has been modeled.
The average \( S_{\text{eff}} = S_0 n_{\text{triggered}}/10^4 \), where \( S_0 \) is the array area inside the perimeter; \( n_{\text{triggered}} \) is the triggered number of events) is plotted against the parameter \( Q_{150} \) in figure 15. Corrections for inoperative detectors are not shown here.

The shower data gathered after the latest array re-configuration were used to work out the spectrum: namely, 1993–2007 for the medium \( C_1 \) subset, and 1995–2007 for autonomous subset. In order to evaluate the intensity of the primary flux, we have collected data during observation periods with a light extinction better than 0.65 and shower axes within the area of the corresponding sub-array.

The resulting differential all-particle spectrum of CRs is shown in figure 16 in comparison with the data from other Cherenkov detector arrays, namely BLANKA \([25]\) and Tunka \([26]\).

Hereinafter, two energy regions below and above \( E = 10^{18} \) eV are analyzed separately in order to consider the two irregularities in the spectrum.

The region below \( 10^{18} \) eV is shown in detail in figure 17 where CR intensity is multiplied by \( E^{2.75} \) in order to emphasize the spectrum irregularity. In addition to spectra around the knee measured by the Cherenkov detector arrays (BLANKA, Tunka and Yakutsk taken from the previous figure), the data of scintillation detector arrays (Akeno \([29]\), KASCADE \([30]\) and Tibet \([31]\)) are given for comparison. Due to the air Cherenkov light signal being proportional to the number of electrons in the shower, no difference is expected in the spectrum shape measured with scintillator or PMT arrays. Measured CR intensities are corrected in the case of the Cherenkov detector arrays with the algorithm in appendix B. Integral energy spectrum indices below and above the knee are assumed to be 1.67 and 2.1, respectively, as measured by the Tibet-III array, while energy evaluation errors are estimated as 0.12 (Blanka), 0.2 (Tunka) and 0.25 (Yakutsk). Original intensities from Akeno, KASCADE and Tibet arrays are not changed because the energy/intensity reconstruction procedures from \( N_e / N_\mu \) measurements include the conversion factor \( V.1 \) needed.
Figure 16. CR flux measured by arrays equipped with air Cherenkov light detectors. Vertical bars indicate statistical errors.

Figure 17. The energy spectrum of CRs around the knee region. Observational data are from arrays equipped with Cherenkov light and scintillation detectors. Two panels show the spectra before (left) and after (right) energy corrections applied to the data, as described in the text.
Figure 18. Our energy spectrum and the results of galactic CR simulations. Curves: BV, the SNR acceleration model [32]; KP, the diffusion model [33]; EW, the single source model [34] and LNU, the anomalous diffusion model [35].

For the reconstructed energy and intensity there is an interaction model/primary composition dependence in all the data from arrays. For instance, the Tibet-III results indicate a $\sim 20\%$ systematic error due to chemical composition and $\sim 10\%$ discrepancy between QGSJET01c and SIBYLL2.1 interaction models below $E = 10^{16}$ eV. The uncertainty may be much worse if we use interaction models that are not carefully tuned to the measured EAS observables.

We have used energy correction factors to compare the measured differential spectra around the knee. The intensities ($J \times E^{2.75}$) shown in the left panel of figure 17 were shifted by factors 1.2, 1.05 and 1.05 in the Blanka, Tunka and Yakutsk cases, respectively; the other three results were shifted along energy correction factors 0.95 (Akeno), 0.97 (KASCADE) and 1.02 (Tibet). The resultant spectra are plotted in the right panel of figure 17. All data exhibit the knee approximately in the same energy interval, while the intensity/energy estimates are different, especially in the Akeno and BLANCA flux data.

As examples of model calculations, four predicted spectra are shown together with the Cherenkov light data of the Yakutsk array in figure 18. Only the shape of the all-particle spectra can be compared with the data because of the free parameters in models—the intensity of CRs and, to a lesser degree, a knee position due to the magnetic field uncertainty in the sources and in the interstellar medium.

All the model spectra are in agreement with experiment, especially in view of the dispersion in the results of arrays (e.g. figure 17) which is greater than the shape variation due to models. At energy $E > 0.1$ EeV, the anomalous diffusion model spectrum is harder than that observed in Yakutsk, but the points are within experimental errors. The lack of CRs in supernova remnant (SNR) acceleration and diffusion models above an energy of 0.1 EeV can be filled up by the extragalactic component.

A hint of the fine structure is seen in the energy spectrum measured with the Yakutsk array. Although it is possible that the undulations around the knee are caused by the instrumental
Table 5. Energy estimation errors, $\sigma$, and intensity conversion factors, $R_J$, for EAS arrays.

| Array        | AGASA | HiRes | PAO   | Yakutsk |
|--------------|-------|-------|-------|---------|
| $\sigma$ (%) | 25    | 17    | 22    | 32      |
| $R_J (\lg E < 18.5)$ | 1.18 | 1.08 | 1.14 | 1.31    |
| $R_J (18.5 < \lg E < 19.8)$ | 1.12 | 1.05 | 1.09 | 1.20    |
| $R_J (\lg E > 19.8)$ | 1.65 | 1.26 | 1.47 | 2.27    |

errors\(^2\), there are 40 size spectra and five Cherenkov light spectra measured before 2005, which demonstrate the second excess (‘peak’) at $\lg E = \lg E_{\text{knee}} + 0.6$ besides the knee itself \[^{[36]}\]. The second peak in our data is approximately at this energy. However, more Cherenkov light data are needed to scrutinize the subject. Our autonomous sub-array is capable of supplying a sufficient sample of EAS events within the next few years.

Recently, Lagutin et al \[^{[37]}\] examined the contribution of a nearby SNR-type source to the energy spectrum of CRs produced in the anomalous diffusion model. They found several sequential peaks caused by H, He and CNO nuclei around the knee in the all-particle spectrum, confirming the results of the single source model \[^{[34]}\] in the case of the ‘background’ anomalous diffusion model.

Our conclusion concerning the part of the energy spectrum below $10^{18}$ eV is that while all four models considered are in agreement with our measurements in the knee region, e.g. the intensities below and above the knee, the only model able to describe the fine structure in the spectrum is the single source model. Hence, this model combining a recent nearby SNR with the background (anomalous) diffusion of CRs in the Galaxy is the best fit for the Yakutsk array data.

When comparing the upper half of the spectrum measured for $E > 10^{18}$ eV with the energy spectra observed by giant EAS arrays, including that of the Yakutsk array, a somewhat different energy estimation has to be taken into account \[^{[3, 20, 38]}\]. Due to the comparatively small acceptance area of the Cherenkov detector subsets (figure 15), our data are reliable up to $\sim 10^{19}$ eV; this includes the ankle region. Another purpose is to compare ultra-high energy cosmic ray (UHECR) intensities measured with different techniques.

In table 5, we give the energy estimation errors of the Akeno Giant Air Shower Array (AGASA) \[^{[39]}\], High-Resolution Fly’s Eye (HiRes) \[^{[40]}\], PAO \[^{[41]}\] and Yakutsk \[^{[19]}\] experiments and also the intensity conversion factors calculated as described in appendix B for energy bins where the index is constant and the variation of instrumental errors is negligible. The integral energy spectrum index of CRs is assumed to be 2.3 and 1.9 below and above the ankle in the spectrum, and $\kappa = 4$ at $\lg E > 19.8$, in accordance with the results of HiRes \[^{[40]}\].

The observed UHECR spectra are given in the left panel of figure 19 with the intensity correction factors applied: the observed intensities are decreased by $R_J$ and spectra are shifted in $\lg E$ by $0.434\sigma$.\(^3\) This is a preliminary, crude procedure for reconstructing the spectrum breaks, but for the purpose of the intensity comparison it may be sufficient.

\(^2\) Vertical bars show statistical errors only.

\(^3\) This may be unwarranted in the PAO case due to \[^{[42]}\].
Table 6. Correction factors to energy scales for the pairs of EAS arrays/detectors, $R_E$, averaged in the region $E > 10^{18}$ eV.

|       | AGASA | HiRes | PAO | $Y_{Sc}$ | $Y_{Ch}$ |
|-------|-------|-------|-----|----------|----------|
| AGASA | 1     | 0.75  | 0.63| 1.05     | 0.82     |
| HiRes | 1.33  | 1     | 0.85| 1.40     | 1.08     |
| PAO   | 1.6   | 1.2   | 1   | 1.70     | 1.30     |
| $Y_{Sc}$ | 0.91 | 0.71  | 0.6 | 1        | 0.75     |
| $Y_{Ch}$ | 1.22 | 0.93  | 0.80| 1.33     | 1        |

Figure 19. UHECR energy spectra measured by AGASA (open rhombuses) [39], HiRes I and II (open and filled triangles) [40], PAO (filled rhombuses) [41], the Yakutsk array scintillation detectors (open, crossed and filled circles) [20] and the air Cherenkov light detectors (squares) [43]. Statistical errors are shown by the vertical bars, whereas horizontal bars indicate energy bins. The spectra are given before (left panel) and after (right panel) the energy corrections applied to the data.

Two spectra of the Yakutsk array are in agreement with each other within errors above $10^{19}$ eV but diverge at lower energies. The possible reason for a discrepancy can be systematic errors in primary energy and CR intensity estimation near the scintillator threshold of trigger-500. Work is in progress to surmount the divergence.

Comparison of the UHECR spectra measured using different detectors and energy reconstruction methods reveals the existence of systematic differences between the resultant energies obtained at the arrays. The average values, $\overline{E}$, should be corrected, too\(^4\). While the true primary energy is unknown, cross calibration of energy estimation methods can be carried out by adjusting correction factors, $R_E$, for the pairs of observed spectra, converging it together. Then the resulting spread of factors elucidates the confidence interval for the CR energy estimated.

Table 6 demonstrates the variety of correction factors for the energy estimation methods of the different groups.

\(^4\) For the given primary energy $E$. 

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Figure 20. The Yakutsk array data from the previous figure (with energy correction) in comparison with the results of UHECR propagation models given by the curves: Berezinsky, Gazizov and Grigorieva (BGG); Bahcall and Waxman (BW); Wibig and Wolfendale (WgW); and De Marco and Stanev (MS).

To illustrate the result of corrections $R_J \times R_E$ applied to the estimated energies and intensities, the measured spectra are replotted in the right panel of figure 19. Energy scale factors are used here (arbitrarily, but preferably close to the median) from the last column of table 6, although any other column of the table may be used as well.

The shapes of the UHECR spectra measured by all the arrays are in agreement with each other within errors if the energy estimations are calibrated. Namely, the observed positions of the ankle and energy thresholds of GZK suppression are in satisfactory agreement.

A difference in the energy of EAS primary particles estimated based on the data from two subsets of the Yakutsk array detectors—namely scintillators, $Y_{Sc}$, and air Cherenkov light detectors, $Y_{Ch}$, originates, presumably, from the systematic error of the relation used between the charged particle density at 600 m from the shower core and the light intensity at 150 m from the core.

The UHECR propagation modeling results are illustrated using four examples in figure 20 in comparison with the Yakutsk array data.

Bahcall and Waxman [44] gave a two-component (galactic+extragalactic) model with the shape insensitive to the choice of the absolute energy scale.

A different approach was used by Berezinsky et al [45] assuming UHECRs as extragalactic protons from uniformly distributed sources. The electron–positron pair production in collisions of protons with relic photons results in the energy spectrum of extragalactic CRs with the ‘dip’ feature in this model.

De Marco and Stanev’s model fits the UHECR spectra measured by AGASA and HiRes with different injection spectra at CR sources that are uniformly and homogeneously distributed in the Universe [46]. The best fit they found assuming that CRs ($E > 10^{19}$ eV) are protons, varying the index, emissivity and cosmological evolution parameters of the injection spectrum, is given in figure 20.

5 Except AGASA data.
Wibig and Wolfendale [47] focus on the ankle in the primary energy spectrum, attributing it to the rapid transition from the galactic to the extragalactic component of CRs. The sum of a smoothly falling galactic spectrum and a power-law extragalactic spectrum fitted to the Yakutsk array scintillator data is shown.

All the models used demonstrate GZK suppression and ankle features in agreement with the data from arrays. This is not surprising in view of the fact that the source emissivity and the ankle position in the energy scale are free parameters, and the GZK effect is embedded in the models.

The lack of EAS events above $10^{19}$ eV observed with the Cherenkov light detectors hinders us from choosing a model of better fit. The index of the energy spectrum observed in the interval $(10^{19}, 3 \times 10^{19})$ eV is $d \log J/d \log E = -2.1$ for Cherenkov light detector data and is $-2.7$ for charged particle detector data. If we use these values as the confidence bounds, then model indices are within the interval. The model of Bahcall and Waxman is closest to our Cherenkov light detector data and can be considered as preferable among equally matching models.

9. Conclusions

The total flux of the air Cherenkov light with subsidiary data on the electron and muon sizes at the ground level is used to estimate the energy of the primary CR particle initiating EAS. The relation between the total flux and ionization loss of electrons in the atmosphere is derived, which depends on the extinction of light and $X_{\text{max}}$ parameters; the latter is the only parameter to accumulate the interaction model dependence of EAS development in this case.

The independent measurement technique based on the Cherenkov light detectors of the Yakutsk array enabled us to observe the CR energy spectrum in the range from $E \sim 10^{15}$ eV to $6 \times 10^{19}$ eV. Two spectra measured with different detectors of the Yakutsk array—scintillators and Cherenkov light detectors—exhibit an ankle feature below $E = 10^{19}$ eV. The autonomous sub-array data confirm the previous observations of the knee at $E \sim 3 \times 10^{15}$ eV. A comparison of our results with the data from other EAS arrays shows the agreement among spectra if the energy estimations are corrected.

The energy spectra predicted for several models of galactic CRs demonstrate agreement within experimental errors with our data in the knee region. However, only the single source model describes the fine structure of the spectrum observed, attributing it to the contribution of nuclei from a recent nearby supernova. We have chosen this model as the best fit for our data below $10^{18}$ eV.

At the highest energies, extragalactic CRs forming a dip or transition between galactic and extragalactic components above $10^{18}$ eV are thought to be responsible for an ankle detected in the energy spectrum. A comparison of our data with models of these types shows satisfactory agreement with all of them (the best fit with Bahcall and Waxman’s model), so we cannot distinguish between different scenarios of the ankle formation based on the energy spectrum measurement alone. There is a need for additional data, presumably, concerning the mass composition of UHECRs, in order to elucidate the origin of an ankle in the energy spectrum.

6. Index $-2.37$.

7. Except [47], which is not intended by this sentence.
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Appendix A. Energy balance of EAS components

The energy fractions of the EAS primary particle transferred to the shower components can be described on the basis of hadron transport equations. If \( E_k \) \((k = N, \pi, \mu, \nu, e, \gamma)\) is the energy transferred to nucleons, charged pions, muons + neutrinos and electrons + photons, a few cascade parameters determine the ratios between \( E_k \) values—i.e. the energy balance in the shower. For instance, the transport equation for the charged pion density \( \pi(x, E) \) at a depth \( x \) is

\[
\frac{\partial \pi(x, E)}{\partial x} = -\left( \frac{1}{\lambda_\pi} + \frac{B_\pi}{xE} \right) \pi(x, E) + \frac{2}{3\lambda_\pi} \int_{E}^{E_0} \pi(x, U) w_{\pi\pi}(E, U) dU \\
+ \frac{2}{3\lambda_N} \int_{E}^{E_0} N(x, U) w_{\pi N}(E, U) dU,
\]

where the interaction mean free paths \( \lambda_\pi \) and \( \lambda_N \) are assumed to be constant; \( w_{\pi\pi}(E, U) \) and \( w_{\pi N}(E, U) \) are the spectra of charged pions produced in pion–air and nucleon–air interactions. Equation (A.1) can be transformed (integrating over \( E^2 \)) into.

\[
\frac{dE_\pi}{dx} = -\frac{E_\pi}{\lambda_\pi} \frac{B_\pi}{x} \pi(x, E > 0) E dE + \frac{2E_\pi}{3\lambda_\pi} + \frac{2K_N E_N}{3\lambda_N},
\]

where \( E_\pi(x) = \int_{0}^{E_0} \pi(x, E) E dE; \) \( \pi(x, E > 0) = \int_{0}^{E_0} \pi(x, E) E dE; \) \( K_N \) is the nucleon inelasticity assumed to be constant; \( E_N = E_0 \exp(-K_Nx/\lambda_N) \).

In the energy range \( E \gg B_\pi \), the only parameters for defining the solution are \( K_N/\lambda_N \) and \( \lambda_\pi \). This means that the energy transferred to charged pions is independent of the spectra of pions produced in nuclear interactions. Hence, in the general case, we can use the simple \( \delta \)-model with the production spectrum \( w_{ik}(E, U) = n_s \delta(E - U/n_s) \), where \( n_s \) is the multiplicity of secondaries, to balance the components energy in a shower. Because of the net value of \( E_\mu + \nu/E_0 \leq 0.1 \), the uncertainty due to the simplified model should be of the second order of magnitude.

To summarize, the main model parameters governing the energy balance in the shower are the average inelasticity coefficients, mean free paths, multiplicity of secondaries and the fragmentation rate of the primary nucleus. The influence of other model characteristics such as ‘the form of the rapidity distribution of constituent quarks’ in collisions is weak.

The exact analytic solution of equations in the case of constant \( \lambda_\pi, \lambda_N, K_N \) and \( B_\pi = 0 \) is shown in figure A.1 together with the \( \delta \)-model results. The figure demonstrates the influence of rising cross sections and the real decay rate of charged pions. An asymptotic \((x = \infty)\) estimation of \( E_{\pi\pi} \) with the CORSIKA/QGSJET code at \( E_0 = 10^{18} \text{ eV} \) [23] is shown in the figure as well.
Figure A.1. The energy carried in a cascade by nucleons (N), charged pions ($\pi$), muons and neutrinos ($\mu + \nu$), electrons and photons (e + $\gamma$). Dashed curves are analytic expressions with $\lambda_i$, $K_N = \text{const}$, $B_\pi = 0$: $E_N = E_0 \exp(-K_N x/\lambda_N)$; $E_\pi = 2/3 E_0 (1 - \exp(-K_N x/\lambda_N)) \exp(-x/\lambda_\pi/3)$; $E_{e\gamma} = E_0 - E_N - E_\pi$. Solid curves are the $\delta$-model results: cross sections are supposed to increase $\propto 0.08 \ln E$; $B_\pi = 120 \text{ GeV}$; $K_N = 0.5$; $n_e \propto E^{1/4}$; $E_0 = 10^{18} \text{ eV}$. The open circle represents the asymptotic estimation for $E_{e\gamma}$ of the CORSIKA/QGSJET code.

Appendix B. Conversion of the measured CR intensity into the spectrum of the primary beam

There is a correction to be applied to the measured intensity of CRs before any comparison of the energy spectra observed by different EAS arrays.

The quantity $\hat{E} = \text{‘primary particle energy’}$ that has been estimated after a shower detection, and the actual energy of the particle, $E$, which has initiated the EAS, are different values, connected with each other by a relation to be found. Estimated energy has distribution around the given mean value $g(\hat{E}, E), formed by the instrumental errors and fluctuations of the shower parameters with an rms deviation, $\sigma$. Energy fluctuation is small in comparison with instrumental errors. Our aim here is to calculate the exact difference between the observed intensity of CRs, $J(\hat{E}) \, d\hat{E}$, and the original one $J(E) \, dE$ in the case of a rapidly falling power law spectrum.

The problem was solved, in general, by Zatsepin and Kalmykov in the previous century. The measured number of EAS particles, the so-called shower size, $N_e$, is connected with the primary energy. The function $g(N_e, E)$ depending on $E/N_e$ has been found by Zatsepin [48]. Then Kalmykov has calculated the measured intensity in the case of a lognormal distribution of $N_e$ [49]:

$$J(N_e) = J_0(N_e) \exp\left(\frac{\sigma_N^2 \kappa (\kappa - a_N)}{2a_N^2}\right),$$

where $\sigma_N$ is the rms deviation of $\ln N_e$; $\kappa$ is the spectrum index; $a_N = (E/N_e) dN_e/dE$.

Here, we also assume the lognormal distribution of $y = \ln \hat{E}$ with the average value equal to $\ln E$. The observed intensity of CRs is then given by the convolution of the primary spectrum,
Figure B.1. Simulation results of the spectrum measurement with ankle and knee features.

\[ J(z) = J_0 \exp(-\kappa z), \] and the distribution of instrumental errors and fluctuations

\[ \hat{J}(z) = \int_{-\infty}^{\infty} J(z - y) g(y) \, dy = J_0 \int_{-\infty}^{\infty} \exp(-\kappa z + \kappa y) \frac{\exp(-y^2/2\sigma^2)}{\sqrt{2\pi\sigma}} \, dy. \] (B.2)

The resultant initial-to-observed intensity conversion factor is

\[ R_J = \frac{\hat{J}(z)}{J(z)} = \exp\left(\frac{\sigma^2\kappa^2}{2}\right). \]

The necessary conditions are a constant index and rms error, or at least both changing only slowly with energy.

A distinct feature of the break in the spectrum is its shift along the energy scale. Due to the smeared transition between the conversion factors below and above the break, its position moves upward in energy.

To illustrate this we have simulated the primary energy spectrum with ankle and knee: random primary energies have been generated according to a broken power law with the index \( \gamma = 3.3 \) and 2.9 below and above \( 10^{18} \text{ eV} \) in the case of the ankle, and \( \gamma = 2.9 \) and 5 for the knee. The energy estimation is modelled by adding randomly a Gaussian error to \( \ln E \) with \( \sigma = 0.32 \). Figure B.1 shows the ‘measured’ and primary spectra. The different shifts in the logarithm of intensity on both sides of the break are clearly seen, and it is also seen that the position of the observed break moves to the right.
Appendix C. Data tables

Table C.1. CR flux measured with the $C_2$ sub-array detectors. Statistical errors are given where appreciable.

| Energy (eV) | $J(E)$ (m$^{-2}$ sr$^{-1}$ s$^{-1}$ eV$^{-1}$) |
|-------------|-----------------------------------------------|
| 1.60 × 10$^{15}$ | 8.55 × 10$^{-22}$ |
| 2.00 × 10$^{15}$ | 4.63 × 10$^{-22}$ |
| 2.50 × 10$^{15}$ | 2.56 × 10$^{-22}$ |
| 2.90 × 10$^{15}$ | 1.80 × 10$^{-22}$ |
| 3.50 × 10$^{15}$ | 1.05 × 10$^{-22}$ |
| 4.60 × 10$^{15}$ | 4.83 × 10$^{-23}$ |
| 5.70 × 10$^{15}$ | 2.32 × 10$^{-23}$ |
| 6.30 × 10$^{15}$ | 1.68 × 10$^{-23}$ |
| 7.80 × 10$^{15}$ | 9.06 × 10$^{-24}$ |
| 9.50 × 10$^{15}$ | 5.25 × 10$^{-24}$ |
| 1.15 × 10$^{16}$ | 2.83 × 10$^{-24}$ |
| 1.40 × 10$^{16}$ | 1.60 × 10$^{-24}$ |
| 1.78 × 10$^{16}$ | 7.27 × 10$^{-25}$ |
| 2.19 × 10$^{16}$ | 3.33 × 10$^{-25}$ |
| 2.80 × 10$^{16}$ | 1.73 × 10$^{-25}$ |
| 3.50 × 10$^{16}$ | (8.77 ± 0.42) × 10$^{-26}$ |
| 4.20 × 10$^{16}$ | (4.95 ± 0.23) × 10$^{-26}$ |
| 5.10 × 10$^{16}$ | (2.71 ± 0.14) × 10$^{-26}$ |
| 6.60 × 10$^{16}$ | (1.20 ± 0.07) × 10$^{-26}$ |
| 7.90 × 10$^{16}$ | (7.10 ± 0.43) × 10$^{-27}$ |
| 1.00 × 10$^{17}$ | (3.80 ± 0.25) × 10$^{-27}$ |
| 1.66 × 10$^{17}$ | (7.43 ± 0.66) × 10$^{-28}$ |
| 2.00 × 10$^{17}$ | (4.25 ± 0.63) × 10$^{-28}$ |
| 3.10 × 10$^{17}$ | (1.10 ± 0.24) × 10$^{-28}$ |

Table C.2. CR flux measured with the $C_1$ sub-array detectors.

| Energy (eV) | $J(E)$ (m$^{-2}$ sr$^{-1}$ s$^{-1}$ eV$^{-1}$) |
|-------------|-----------------------------------------------|
| 2.60 × 10$^{17}$ | 2.16 × 10$^{-28}$ |
| 4.10 × 10$^{17}$ | 5.91 × 10$^{-29}$ |
| 6.50 × 10$^{17}$ | 1.60 × 10$^{-29}$ |
| 1.00 × 10$^{18}$ | 4.20 × 10$^{-30}$ |
| 1.27 × 10$^{18}$ | (1.35 ± 0.07) × 10$^{-30}$ |
| 2.02 × 10$^{18}$ | (3.34 ± 0.23) × 10$^{-31}$ |
| 3.22 × 10$^{18}$ | (7.95 ± 0.87) × 10$^{-32}$ |
| 5.08 × 10$^{18}$ | (1.69 ± 0.37) × 10$^{-32}$ |
| 8.06 × 10$^{18}$ | (3.35 ± 1.07) × 10$^{-33}$ |
| 1.27 × 10$^{19}$ | (1.52 ± 0.75) × 10$^{-33}$ |
| 2.02 × 10$^{19}$ | (4.39 ± 2.54) × 10$^{-34}$ |
| 3.22 × 10$^{19}$ | (1.69 ± 0.99) × 10$^{-34}$ |
| 5.08 × 10$^{19}$ | (1.41 ± 1.01) × 10$^{-34}$ |
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