Calculation of the resistance forces from the interaction of blade teeth cutting edges with the soil

Aleksander Petrovich Akimov¹, ⁴, Boris Vladimirovich Turovskiy², Yuriy Valentinovich Konstantinov³ and Anton Victorovich Stepanov³

¹Faculty of Engineering, Chuvash State Agricultural Academy, Cheboksary, 428000, Russia
²Faculty of Engineering and Architecture, Kuban State Agrarian University named after I. T. Trubilin, Krasnodar region, Krasnodar, Kalinina St., 13, Russia
³Faculty of Engineering, Chuvash State Agricultural Academy, Cheboksary, 428000, Russia
⁴Email: akimov_mechfak@mail.ru

Abstract. The methods of calculation of the resistance forces from the interaction of blade teeth cutting edges with the soil and their torque about the center of the disk are proposed. The reaction of soil to a blade toothed is replaced by the resultant force applied to its toe. The resultant direction is taken to be opposite to the velocity vector of the blade toe. So soil reactions to a toothed disk depend on kinematic parameter (the ratio of peripheral disk speed to forward speed). It is shown that the total vertical and horizontal reactions of the soil to the blade teeth can be obtained by analytical or graphical addition. The developed mathematical model of the interaction of blade teeth cutting edges with the soil allows to determine the soil reaction to a toothed disk and the moment of soil resistance to cutting by a toothed disk as periodic functions of its rotation angle. It allows to identify ways of reducing the power capacity of a toothed disk. The horizontal component of soil reaction to a toothed disk is the driven force at kinematic parameter values greater than 1.2. Since the maximal driven disk force and the moment of soil resistance to cutting increase with deceleration with increase in kinematic parameter, it is profitable to use operation modes with its less values.

1. Introduction
Construction of mathematical models of interaction of a flat disc with the soil, starting with N. Nerli was studied by different researchers [1-4]. The value of a kinematic parameter of the disc, equal to the ratio of the circumferential speed of the points of the blades to the translational velocity determines the distribution of the soil friction forces on the disk lateral surface and the distribution of elementary forces of soil resistance to cutting to its blade. In work [5] there was developed a generalized mathematical model of interaction of a flat disc with the soil, taking into account the value of this kinematic parameter, which implies as particular cases the known models built earlier. This model allowed to describe theoretically the phenomenon of sliding- skidding of a freely rotating disk in the soil and to determine its essential power characteristics depending on the geometric parameters and operation mode parameters of the disk.
It was shown in experimental studies of different researchers that force characteristics of powered disk fitted at an angle to motion direction also substantially depend on kinematic parameter [6-7].

Toothed disk coulters are widely used to cut residues in active and inactive operation modes [8-9].

The mathematical model of the interaction of toothed rolling coulter with soil was developed by Magalhaes P S G, Bianchini A and Braunbeck O A [10], but the dependences of its force characteristics on coulter rotation angle and kinematic parameter were not studied.

The objective of this study is to develop a mathematical model of interaction of toothed disk with soil, which allows to determine its force characteristics as functions of disk rotation angle and kinematic parameter.

2. Determination of the soil reaction to the toothed disk

When a toothed disk is operating a certain part of power is spent on overcoming the resistance forces on the teeth blade.

Because of the toothed symmetry relative to the direction of incorporation it may be assumed that the resultant reaction of the soil on the cutting toothed blade $P_i$ is submitted to its toe.

In this case it will be determined by the expression:

$$ P_i = KS_0, $$

where $K$ is the resistivity constant of the soil deformation by the toothed; $S_0$ is the cross-section area of the toothed.

We shall accept the direction of soil reaction to be the opposite to the vector of absolute velocity in accordance with figure 1 [11].

Then the components of the soil reaction may be determined by the following equations:

$$ P_{ix} = -P_i \cos (V, x), $$

$$ P_{iy} = -P_i \cos (V, y), $$

where $P_{ix}, P_{iy}$ are the horizontal and vertical components of the soil reaction to cutting to the toothed at the pre-determined angular disk position; $i$ is the consecutive number of the toothed along the toothed disk rotation.

Taking into account the toothed disk radius $r$ and the toothed angular displacement $\omega t$ the formulae of the directional cosines of the absolute velocity vector of the first toothed toe become the following:

$$ \cos (V, x) = \frac{-\lambda \sin (\alpha_1 + \omega t)}{\sqrt{\lambda^2 - 2 \lambda \sin (\alpha_1 + \omega t) + 1}}, $$

$$ \cos (V, y) = \frac{\lambda \sin (\alpha_1 + \omega t)}{\sqrt{\lambda^2 - 2 \lambda \sin (\alpha_1 + \omega t) + 1}}, $$

where $\lambda$ is the toothed disk radius; $\alpha_1$ is the toothed disk initial angular position; $\omega t$ is the angular displacement of the toothed disk.

\[ \text{Figure 1. Direction of forces resisting to cutting.} \]
\[ \cos (V, y) = \frac{-\lambda \cos (\alpha_1 + \omega t)}{\sqrt{\lambda^2 - 2\lambda \sin(\alpha_1 + \omega t) + 1}}, \quad (5) \]

where \( \lambda = \frac{\omega r}{V_n} \) is the disk kinematic parameter, \( V_n \) is the disk translational velocity.

As each of the previous teeth leads in phase the considered toothed by an angle \( \varphi = (i-1)\frac{2\pi}{n} \), where \( n \) is the total number of disk teeth, in accordance with figure 1 the equations for determining soil reaction of any of the teeth in the soil, may be written down in a general form:

\[ P_{xi} = K S_0 \frac{\lambda \sin(\alpha_1 + \omega t + (i-1)\frac{2\pi}{n})}{\sqrt{\lambda^2 - 2\lambda \sin((i-1)\frac{2\pi}{n}) + 1}}, \]

\[ P_{yi} = K S_0 \frac{\lambda \cos (\alpha_1 + \omega t + (i-1)\frac{2\pi}{n})}{\sqrt{\lambda^2 - 2\lambda \sin((i-1)\frac{2\pi}{n}) + 1}}, \quad (7) \]

The total horizontal and vertical components of the soil reaction to the toothed disk may be found graphically according to figure 2 and analytically, like addition of harmonic oscillations:

\[ P_x = \sum_{i=1}^{n_z} P_{xi} = P_{x1} + P_{x2} + \ldots + P_{xn_3}, \quad (8) \]

\[ P_y = \sum_{i=1}^{n_2} P_{yi} = P_{y1} + P_{y2} + \ldots + P_{yn_3}, \quad (9) \]

where \( n_z \) is the number of teeth that are simultaneously in the soil; \( P_x, P_y \) are the total horizontal and vertical components of the soil reaction to all the immersed teeth at the predetermined angular disk position.

As it may be seen from the diagram, in accordance with figure 2, the total components of the soil reaction to the toothed disk with changing the rotation angle will change, so their ultimate value may be defined as a simple mean:

\[ P_x = \frac{\sum P_{xi}}{m}, \quad (10) \]

\[ P_y = \frac{\sum P_{yi}}{m}, \quad (11) \]

where \( m \) is the number of calculated ordinates within the phase determined by the angular toothed pitch.

Experiments show that a toothed interacts with the soil within the angle \( 3/2(\frac{z}{2} - \alpha_1) \), and the rest of the way in the soil it passes through the crack from the previous toothed [12].
With this consideration in mind, the number of teeth which are simultaneously in the soil may be determined by the formula:

$$n_2 = \left[ \frac{1.5 \left( \frac{r}{n} + a_i \right)}{2\pi r} + 1 \right]$$  \hspace{1cm} (12)

To evaluate the toothed disk power capacity in overcoming the forces resisting to cutting it is necessary to know the moment of resistance of these forces.

3. The determination of the moment of resistance to soil cutting by a toothed disk

The value of the moment of resistance to soil cutting by a toothed disk may be found as multiplication of the resultant force on the application arm, if assuming that the resultant of all the forces acting on the cutting edge toothed side, is attached to the toothed toe:

$$M_{pm} = P_i l_p,$$  \hspace{1cm} (13)

where $M_{pm}$ - is the moment of resistance to soil cutting at its pre-determined angular position; $l_p$ - is the force application arm, the polar distance from the disk center to the resultant of the cutting forces.

To determine the moment of resistance to cutting, let us select the change pattern of the application arm of the soil reaction to the toothed toe.

Arm $l_c$ of cutting reaction $P_i$ or the polar distance to the tangency at point M of the motion trajectory of the toe toothed according to figure 3 may be calculated by the formula:

$$l_p = \frac{|A|}{\sqrt{A^2 + B^2}},$$  \hspace{1cm} (14)

where $A = Y'; B = X'; C = XY' - XY^2$;

$X, Y'$ – are coordinates of point M of the curve;

$X', Y'$ – are values of the derivatives at point M.

Let us determine the equations of motion of point M in the system of fixed coordinates XOY. The coordinates of point $M_1$ in accordance with figure 3, in the system are defined by the equations:

$$\begin{cases} X = V_n t + r \cos (\omega t + \omega \lambda) \\ Y = -r \sin (\omega t + \omega \lambda) \end{cases}$$  \hspace{1cm} (15)

Substituting the values of forward velocity $V_n$ expressed through $\omega, r, \lambda$ to the equations (15), we shall get:
\[ X = r \cos (a_1 + \omega t) + \frac{r}{\lambda} \omega t \]
\[ Y = -r \sin (a_1 + \omega t) \]

and their derivatives or components of the absolute velocity of point M:
\[ X' = -r \cos (a_1 + \omega t) + \frac{r}{\lambda} \omega \]
\[ Y' = -r \sin (a_1 + \omega t) \]

After defining the expressions and corresponding mathematical transformations, the expression (14) will take the following form:
\[ l_c = r \frac{\lambda \sin (a_1 + \omega t)}{\sqrt{\lambda^2 - 2 \lambda \sin (a_1 + \omega t) + 1}} \]

Taking into account the obtained expressions of the force application arm (18) the moment of resistance to cutting by one (the first) toothed will be equal to:
\[ M_{c1} = P_1 r \frac{\lambda \sin (a_1 + \omega t)}{\sqrt{\lambda^2 - 2 \lambda \sin (a_1 + \omega t) + 1}} \]

The resisting moment of the cutting reaction for every previous toothed in the soil in the pre-determined angular position is calculated by the formula:
\[ M_{c2} = K S_0 r \frac{\lambda \sin (a_1 + \omega t + (i - j) \frac{2 \pi}{m})}{\sqrt{\lambda^2 - 2 \lambda \sin (a_1 + \omega t + (i - j) \frac{2 \pi}{m}) + 1}} \]

The average resisting moment to cutting by a toothed disk is found similar to the method of determining the components of the cutting soil reactions:
\[ M_c = \frac{\sum M_{ci}}{m} \]

where \( M_{pi} \) – is the moment of resistance to soil cutting by a toothed disk at a pre-determined angular position of the disk.

Basing on the obtained dependencies (10), (11), (21) we can make a theoretical calculation of the vertical and horizontal components of the soil reactions and the moment of resistance to cutting of the toothed disk and the analysis of the regularities of their changes from the kinematic mode, the geometric parameters of the toothed disk and physical-mechanical properties of the soil [13].

It is found out that the power capacity of a flat disc gets predominant influence of cutting resistance forces and friction forces on its surfaces in interaction with the soil [14].

Basing on this, we may write down:
\[ M_s = M_f + M_c, \]

where \( M_s \) – is the moment of resistance of a toothed disk from friction forces on lateral surfaces \( M_f \) and the moment of cutting resistance forces.

The algebraic sum of the horizontal components of friction forces and cutting resistance forces is the horizontal component of the soil reaction to a toothed disk:
\[ R_x = F_x + P_x \]

Thus, by the obtained formulas (10), (11), (21), (22) and (23) one can to determine the soil reaction to a toothed disk and the moment of soil resistance to cutting by a toothed disk.

4. Results

Let us to study the disk with six teeth \((n=6)\) with the help of the developed mathematical model. The value of \( \alpha_1 = \pi/6 \) corresponds to the disk depth \( h = 0.5 \), at which two teeth will be in the soil at once.

Graphs of the dimensionless horizontal component of soil reaction to cutting \( P_x \) \((K S_0)\) versus the disk rotation angle for different \( \lambda \) values drawn by formulae (6) and (8) are shown in the figure 4a. As it follows from it, the horizontal component of soil reaction to cutting, which is a driven force \((\lambda \geq 1.2)\), is a periodic function with the period \( T = \pi/3 \). Its graph is symmetric one with regard to the straight line \( \varphi = \pi/6 \). For \( \lambda \geq 2 \) the largest value of driven force achieves at the point \( \varphi = \pi/6 \).
Graph of the dimensionless maximal disk driven force versus the parameter $\lambda$ is shown in the figure 5. It follows from it, that the maximal driven force monotonically increases with the increase in $\lambda$, at first rapidly and then more and more slowly. As calculations show, at increasing $\lambda$ from 2 to 4 the maximal driven force increases by 31.4%, and at increasing from 4 to 6 – only by 4.8%. Since energy consumptions monotonically increase with increase in $\lambda$, it is profitable to use operation modes with less values of $\lambda$.

Graphs of the dimensionless horizontal component of soil reaction to cutting $P_x/(K_S0)$ versus the disk rotation angle for different $\lambda$ values drawn by formulae (6) and (8) are shown in the figure 4a. As it follows from them, the horizontal component of soil reaction to cutting, which is a driven force ($\lambda\geq1.2$), is a periodic function with the period $T = \pi/3$. Its graph is symmetric one with regard to the straight line $\varphi=\pi/6$. For $\lambda \geq 2$ the largest value of driven force achieves at the point $\varphi=\pi/6$.

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Graphs of the dimensionless moment of soil resistance to cutting by a disk $M_c/(K_S0)$ versus its rotation angle for different $\lambda$ values were drawn by formulae (20) and (21) (figure 4b). This moment is also a periodic ($T=\pi/3$) function of disk rotation angle with the graph symmetric with regard to the straight line $\varphi=\pi/6$. The moment is practically a constant at $\lambda \geq 2$. The calculation show, that at $\lambda=2$ the maximal deviation of moment from its average value does not exceed one percent and it decreases with increase in $\lambda$.
5. Conclusions
The developed mathematical model of the interaction of blade teeth cutting edges with the soil allows to determine the soil reaction to a toothed disk, the moment of soil resistance to cutting by a toothed disk, and thus to identify theoretically the ways of reducing the toothed disk energy consumptions in different operation modes.

Horizontal component of soil reaction to cutting and moment of soil resistance to cutting are periodic functions of the teeth disk rotation angle.

Horizontal component of soil reaction to cutting is a driven force at the kinematic parameter $\lambda \geq 1.2$. The maximal disk driven force increases with deceleration with increase in kinematic parameter. So it is profitable to use operation modes with less values of $\lambda$.

Moment of soil resistance to cutting does not practically depend on disk rotation angle at $\lambda \geq 2$.

References
[1] Nerli N 1930 Sul Problema dinamico dell’ aratro a disco. (Instratto del Bolletino del R. Instituto. Supereir Agrogro di Pisa)
[2] Nerli N 1930 Sul vantaggio dinamico del coltro rotante (Pisa: Tip. ed. Mariotti Pacini)
[3] Sineokov G N 1949 Disk tillage tools (Moscow)
[4] Kanarev F M 1983 Rotary tillage machines and tools. (Moscow)
[5] Medvedev V I, Konstantinov Y V and Akimov A P 2001 Generalized mathematical model of disk blade interaction with the soil Tractors and agricultural machines 2 34 (in Russian)
[6] Hann M J and Giessibl J 1998 Force measurements on driven discs J. agric. Engng Res. 69 149
[7] Singh S P, Singh B and Vatsa D K 1995 Design and development of powered one-way plough Agricultural mechanization in Asia, Africa and Latin America 26 (3) 9
[8] Bianchini A and Magalhaes P S G 2008 Evaluation of coulters for cutting sugar cane residue in a soil bin Biosystems Engineering 100 370
[9] Ahmad F, Weimin D, Qishou D, Rehim A and Jabran K 2017 Comparative performance of various disc-type furrow openers in no-till paddy field conditions Sustainability 9 1143
[10] Magalhaes P S G, Bianchini A and Braunbeck O A 2007 Simulated and experimental analyses of a toothed rolling coulter for cutting crop residues Biosystems Engineering 96 (2) 193
[11] Turovskiy B V, Efremova V N and Sidorenko S M 2016 Mathematical model of toothed geometry of a flat working body The collection: Modern state of the applied science in the field of mechanics and energy, the materials of the all-Russian research-practice conference held as part of the events dedicated to the 85th anniversary of the Chuvash State Agricultural Academy p 231 (in Russian)
[12] Turovskiy B V, Efremova V N, Sidorenko S M and Trifonov I K 2016 Substantiation of form of vehicle flat disk stirring operating elements Proceedings of Kuban State Agrarian University 61 p 194 (in Russian)
[13] Akimov A P, Konstantinov Y V and Fedorov D I 2013 The method of calculating the resistance and moment of soil cutting resistance Tractors and agricultural machines 3 32 (in Russian)
[14] Koshchaev A G 2016 Assessment of potential hazard of technological processes The collection: Scientific provision of agricultural and industrial complex. Collection of articles of the proceedings of the 71st research-to-practice conference of teachers following the results of R&D for the year 2015 Responsible for the release is Koshchaev A G p 253 (in Russian)