String Compactification and Unification of Forces

Jihn E. Kim

Department of Physics and Astronomy and Center for Theoretical Physics,
Seoul National University, Seoul 151-747, Korea

Abstract. I review our recent attempts toward obtaining the MSSM from string orbifold compactification. The required constraints are the existence of three families and R parity, vectorlike exotics, one pair of Higgs doublets, and the SU(5)' hidden sector for dynamical breaking of SUSY toward a GMSB scenario. We also comment on the threshold correction which are influenced by a power law evolution of gauge couplings through the KK radius in non-prime orbifolds and can be used to fit the couplings.

Keywords: orbifold compactification, heterotic string, $Z_{12-I}$ orbifold, Kaluza-Klein masses, coupling unification

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INTRODUCTION

This small workshop is on grand unification theories(GUT) and I will try to obtain GUTs or GUT-like standard models(SM) from string compactification. GUTs introduce the hierarchy problem and supersymmetry(SUSY) has been studied extensively in the last quarter century to understand the hierarchy problem. Now we are finally close to confronting experimental verification/falsification of TeV scale SUSY. If superstring is relevant to low energy physics, it may reveal through an effective supergravity Lagrangian. So, the MSSM phenomenology is the first hurdle to overcome in string phenomenology.

String theory has been studied in many fronts. For obtaining the MSSM group, compactification of the $E_8 \times E_8'$ heterotic string has been most successful, and we follow this route in this talk. Let us start to glimpse the important issues in supergravity related low energy SUSY models:

- In the last 24 years TeV SUSY has been based on supergravity Lagrangian given in [1].
In supergravity, gravitino phenomenology is essential. One unavoidable constraint is the reheating temperature after inflation, $T_{rh} < 10^{-9} - 10^{-7}$ GeV [2, 3].

To verify the existence of gravitino, attempts to detect it at LHC has been proposed via the neutralino decay to gravitino [4].

Most probably, we need an R parity for proton longevity. In this regard, most existing string constructions are ruled out. Especially, the $u' d' d'$ coupling must be forbidden.

One has to solve the so-called $\mu$-problem [6, 7]. More generally, the MSSM problem, “Why only one pair of Higgs doublets at the TeV scale?”, must be understood.

The strong CP problem must be resolved in the string framework, presumably by string axions [8].

One has to resolve the SUSY flavor problem. The gauge mediated SUSY breaking (GMSB) exists in this regard [9], and the recent surge of interest a la ISS [10] reflects the seriousness of the SUSY flavor problem.

There exists the little hierarchy problem. At present the MSSM needs a fine tuning of order 1%, signaling 10-100 TeV SUSY particle masses. In this regard, the negative stop mass possibility has been considered to raise the fine tuning to the level of 5-10% [11]. We hope that this little hierarchy problem will be understood in the end.

One has to understand the moduli stabilization. The KKLT scenario [12] led to the consideration of a phenomenologically interesting mirage mediation [13].

It is required to allow only vectorlike exotics or is better not to have any exotics.

Among these, here we single out the exotic problem which has not been emphasized widely. Most string models accompany exotics. Chiral exotics are dangerous phenomenologically. So, all exotics must be made vectorlike. In string construction, this is a nontrivial condition. Until recently, we did not find exotics-free models. But recently we find exotics-free models [14, 15], where however the weak mixing angle turn out to be not $\frac{3}{8}$. We do not know whether there exist exotics free models with $\sin^2 \theta_W = \frac{3}{8}$. Except this weak mixing angle problem, in the exotics-free models the condition on singlet VEVs is not so strong as in models with exotics, which is a great virtue.

This talk is a top-down approach, and if a specific example is considered then we cite $Z_{12-I}$ orbifold models.

R PARITY AND STRING AXIONS

The R parity or matter parity in the MSSM is basically put in by hand: quarks and leptons are given an odd R parity, Higgs fields are given an even R parity. Note that one of the merits of SO(10) GUTs is that it may have a good and reasonable R parity by assigning

$$16 : R = \text{odd},$$

$$10 : R = \text{even}.$$  

But we can understand this simply as the disparity between spinor ($\mathcal{S}$) and vector ($\mathcal{V}$) representations as shown in Table 1.

For example, $\mathcal{S}\mathcal{S}\mathcal{V}$ coupling is allowed, but $\mathcal{S}\mathcal{S}\mathcal{S}$ coupling is not allowed. This kind of disparity appears in the integer and half-integer angular momenta also in the SO(1,3) Lorentz group. Thus, $u' d' d'$ is of the $\mathcal{S}\mathcal{S}\mathcal{S}$ type and it is forbidden at the cubic level. In this sense, $E_8$ is not good as a GUT because SO(10) matter 16 and SO(10) Higgs 10 are put in the same 27 representation of $E_6$,

$$27 = 16 + 10 + 1.$$  

Usually, in $E_8$ therefore one introduces extra 27 and $\overline{27}$ just for Higgs, which do not mix with matter 27.

However, this attractive feature is not automatically applicable in SO(10) theories with an ultraviolet completion. The reason is that there can exist the SO(10) singlets of the $\mathcal{S}$ type, e.g. (− − − − − − −), and nonrenormalizable interactions allow $u' d' d' (\mathcal{S})/M_P$, and the R parity problem reappears again. In the compactification of heterotic string, we note the $E_8$ adjoint representations appear with the types of

$$\mathcal{S} : (++++++−−−−), \cdots,$$

$$\mathcal{V} : (1 − 1000000), \cdots.$$
TABLE 1. The spinor 16 and the vector 10 of SO(10). For the spinor, we choose odd numbers of minus signs, + and - denote $\frac{1}{2}$ and $-\frac{1}{2}$, respectively, and $B - L = (-2 - 2 - 2 0 0)$. The underline denotes permutations. 16 carries odd numbers of $B - L$ and 10 carries even numbers of $B - L$.

| Weight | SU(3)×SU(2) | Notation | 3(B-L) | Weight | SU(3)×SU(2) | Notation | 3(B-L) |
|--------|-------------|----------|--------|--------|-------------|----------|--------|
| (+ − − + −) | (3, 2) | uL, dL | 1 | (− + + − +) | (3, 1) | uL | −1 |
| (+ + + + −) | (1, 2) | νeL, eL | −3 | (− − + + −) | (1, 1) | N2L | 3 |
| (−1 −1 0 0 0) | (3, 1) | D | 4 | (1 1 0 0 0) | (3, 1) | D’ | −4 |
| (0 0 0 0 −1) | (1, 2) | Hd | 0 | (0 0 0 1 0) | (1, 2) | Hd | 0 |

where the abbreviation $±$ denotes $\pm \frac{1}{2}$. Thus, in the heterotic string compactification the strategy is to put matter representations in $\mathcal{Y}$ type and Higgs representations in $\mathcal{Y}$ type from the original $\mathcal{E}_{8}$, and one must consider nonrenormalizable interactions also.

Toward this objective, the standard practice to obtain a discrete parity is to put it as a subgroup of an anomaly free $U(1)$ gauge group. In string models, we can include the anomalous $U(1)$ gauge group also since the anomaly is cancelled by the Green-Schwarz mechanism [16]. Let us call this $U(1)$ as $U(1)^{\Gamma}$. For example, consider a VEV of a scalar carrying an even $\Gamma$ charge, with $\Gamma = (2 2 2 0 0 0 0 0)$. If $P \in U(1)^{\Gamma}$, then $\Gamma = \text{odd integer for } \mathcal{Y}$ and $\Gamma = \text{even integer for } \mathcal{Y}$. Thus, $P$ is successfully embedded in $U(1)^{\Gamma}$. Yesterday, Mohapatra discussed 126 Higgs of SO(10) and the usefulness of 126 is simply because it belongs to the $\mathcal{Y}$ type.

In the heterotic string, there are four possibilities for $U(1)^{\Gamma}$ by choosing odd number of 2s:

$$
B - L \propto (2 2 2 0 0 0 0) ,
X \propto (2 2 2 2 0 0 0) : X \text{ of the flipped SU(5)}
Q_1 = (0 0 0 0 0 2 0 0),
Q_2 = (0 0 0 0 0 0 2 0).
$$

THE FCNC PROBLEM IN SUPERGRAVITY MODELS AND GMSB

Our prime objective is obtaining the MSSM spectrum with no chiral exotics or even without exotics and at the same time implementing the R parity. Furthermore, requiring a successful hidden sector is very restrictive. There are very few such models if any, since I know only one model presented in this talk. If the hidden sector is introduced toward a dynamical symmetry breaking of SUSY, the best chance for the hidden sector is an SU(5)’ [17, 18].

The orbifold compactification is well known by now [19]. The $\mathcal{E}_{8} \times \mathcal{E}_{8}$ heterotic string gives a good gauge groups and string phenomenology is most successful here. Our experience shows that any orbifold has a same order of complexity. For example, even though $Z_{3}$ orbifold looks the simplest, actually the 27 fixed points makes it very complicated. On the other hand, the $Z_{12-1}$ looks very complicated, but it is simple in Wilson lines with only 3 fixed points and probably it is simpler than others if one knows how to construct models. In Table 2, we list the conditions on Wilson lines. From this table, note that there are four cases of simple Wilson lines, which are underlined. Certainly, $Z_3$ is simpler than $Z_{2}$ on two-torus and hence $Z_{6-1}$ and $Z_{12-1}$ are simplest ones. Among these, only $Z_{12-1}$ are known to have phenomenologically interesting models [20, 18, 15].

In supergravity models, there appear flavor changing neutral currents problems (FCNC) in general. Even if the superpotential is made flavor-conserving, the Kähler potential is restricted by the reality, which is known to break the flavor symmetry. So, the SUSY flavor problem is generic in supergravity models. The SUSY flavor violations are parametrized by squark and slepton mixings $\delta_{LL}, \delta_{RR}, \delta_{LR}$ [21], which are typically of $\mathcal{O}(10^{-2} \sim 10^{-1})$. This SUSY flavor problem led to the GMSB scenario [22]. The well-known examples of dynamical SUSY breaking in simple groups are one family (10 plus $\tilde{5}$) SU(5) model [17] and 16 + 10 of SO(10) [23]. These models are called uncalculable models [24]. In this case, the behavior of vacuum energy is depicted in Fig. 1. In the figure, the runaway vacuum energy from the confining force [25] and the rising vacuum energy from a superpotential give a nonvanishing vacuum energy at a finite value of some fields.
TABLE 2. The string orbifolds and the Wilson line conditions.

| Lattice      | Effective order | Conditions               |
|--------------|-----------------|--------------------------|
| $Z_3$        | SU(3)$^3$       | $3a_1 = 0, 3a_2 = 0, 3a_3 = 0$ | $a_1 = a_2, a_3 = a_4, a_5 = a_6$ |
| $Z_4$        | SU(4)$^2$       | $2a_1 = 0, 2a_4 = 0$     | $a_1 = a_2 = a_3, a_4 = a_5 = a_6$ |
|              | SU(4) x SO(5) x SU(2) | $2a_1 = 0, 2a_5 = 0, 2a_6 = 0$ | $a_1 = a_2 = a_3, a_4 = a_5$ |
|              | SO(5)$^2$ x SU(2)$^2$ | $2a_1 = 0, 2a_4 = 0, 2a_5 = 0, 2a_6 = 0$ | $a_1 = a_3 = 0$ |
| $Z_{6-I}$    | SU(3) x SU(3)$^2$ | $3a_1 = 0$               | $a_1 = a_2, a_3 = a_4 = a_5 = a_6 = 0$ |
| $Z_{6-II}$   | SU(2) x SU(6)   | $2a_1 = 0$               | $a_2 = a_3 = a_4 = a_5 = a_6 = 0$ |
| $Z_7$        | SU(2) x SO(8)   | $3a_1 = 0, 2a_5 = 0$     | $a_1 = a_2, a_3 = a_4 = a_5 = a_6 = 0$ |
|              | SO(5)$^2$ x SU(3)$^2$ | $3a_1 = 0, 2a_3 = 0, 2a_4 = 0$ | $a_1 = a_2, a_3 = a_5 = 0$ |
| $Z_{8-I}$    | SU(2) x SO(5)   | $2a_1 = 0, 2a_5 = 0$     | $a_1 = a_2 = a_3 = a_4, a_5 = 0$ |
| $Z_{8-II}$   | SO(10) x SU(2)  | $2a_4 = 0, 2a_6 = 0$     | $a_1 = a_2 = a_3 = 0, a_4 = a_5$ |
|              | SU(2)$^2$ x SO(8) | $2a_1 = 0, 2a_5 = 0, 2a_6 = 0$ | $a_1 = a_2 = a_3 = a_4$ |
| $Z_{12-I}$   | E6              | no restriction           | $a_1 = a_2 = a_3 = a_4 = a_5 = a_6 = 0$ |
| $Z_{12-II}$  | SU(3) x SO(8)   | $3a_3 = 0$               | $a_3 = a_4, a_1 = a_2 = a_5 = a_6 = 0$ |
|              | SU(2)$^2$ x SO(8) | $2a_1 = 0, 2a_2 = 0$     | $a_3 = a_4 = a_5 = a_6 = 0$ |

Because of the difficulty in obtaining one family SU(5) and 16 + 10 of SO(10) in stable vacua, the recent study of unstable vacua suggested by Intrilligator, Seiberg and Shih (ISS) got a lot of interest [26] because it allows SUSY QCD with vectorlike quarks for dynamical SUSY breaking. Nelson and Seiberg argued for the need of R symmetry to break SUSY dynamically at the ground state [27]. ISS looked for a sufficiently long lived unstable vacuum, where the need of R-symmetry is discarded. In this case, the behavior of the potential is depicted in Fig. 2. Notably, SU(5)$'$ models with six or seven flavors allow SUSY breaking at an unstable vacuum at the origin of some fields [10].

In these GMSB models, messengers (symbolically denoted as $f$) of SUSY breaking to the observable sector are introduced. In the unstable vacuum models, a superpotential of the following form is introduced [28, 26],

$$W_{\text{tree}} = m \overline{Q}Q + \frac{\lambda}{M_{Pl}} \overline{Q} f \tilde{f} + \tilde{f} f, \quad \text{for a local minimum}$$  (2)

where $Q$ and $\overline{Q}$ is the hidden sector quark pair. The R symmetry breaking is introduced by the tree level $W_{\text{tree}}$, including the messengers $f$. Below the confining scale $\Lambda$, this superpotential can be discussed in terms of [28],

$$W_{\text{ISS}} = \overline{S} S^i S_j - \frac{\det S^{ij}}{\Lambda^{N_f-3}} - m_i \Lambda S^{ij}$$  (3)

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where the singlet $S$ develops an F-term and SUSY breaking is mediated by the messenger $f$ sector (generating F term by $W_{\text{tree}}$) to the observable sector.

In the uncalculable models, the effective Lagrangian has a term of the form [29, 15],

$$L = \int d^2 \theta \left( \frac{1}{M^2} W^\alpha W^\alpha_{\alpha} f + M_f \bar{f} f \right), \quad \text{for stable vacuum.} \quad (4)$$

In string models, there appear many heavy charged fields which can act as messengers [15].

To fulfill the condition for the DSB to occur at a relatively low energy scale, later we will introduce different radii for the three complex tori. It is reminiscent of Horava and Witten’s introduction of a distance between two branes in the M-theory [30]. Also extra particles in the desert may be used to fit the data.

One attractive feature of SUSY GUTs is that with the desert hypothesis the coupling constants meet at $\alpha_{\text{GUT}} \simeq \frac{1}{4}$ at the energy scale $(2-3) \times 10^{16}$ GeV. Because of the possibility of populating the desert between the TeV scale and the $(2-3) \times 10^{16}$ GeV scale, we may allow $\alpha_{\text{GUT}} \simeq \frac{1}{4} - \frac{1}{3}$ at the unification point. For the SUSY flavor problem in the GMSB scenario, the gravity mediation to soft parameters must be sub-dominant the GMSB contribution; thus we may require the SUSY breaking scale in the GMSB scenario below $10^{11-12}$ GeV.

The GMSB scenario needs two ingredients:

- SUSY breaking sector in terms of a confining gauge group, e.g. $SU(5)'$, with hidden sector quarks $Q$ confining at the scale $\Lambda_h$.
- Messengers of SUSY breaking at the scale $M_f$.

So, we consider the following scales

On $\Lambda_h$: \[ \frac{\Lambda_h^3}{M_P^2} \leq 10^{-3} \text{ TeV} \Rightarrow \Lambda_h \leq 2 \times 10^{12} \text{ GeV} \quad (5) \]

On a naive estimate of $M_f$: \[ \frac{\xi \Lambda_h^2}{M_f} \approx 10^3 \text{ GeV} \quad (6) \]

where $\xi$ is a model dependent number. Since $M_f < M_P$ is expected, $\Lambda_h$ may be smaller than $10^{12}$ GeV. To estimate the confining scale of the hidden sector, $\Lambda_h$, we consider its one-loop coupling running

$$\frac{1}{\alpha^{h\text{GUT}}} = \frac{1}{\alpha^{h}(\mu)} + \frac{-b_h}{2\pi} \ln \left( \frac{M_{\text{GUT}}^h}{\mu} \right). \quad (7)$$

If $b_h$ is given, $\Lambda_h$ can be calculated in terms of the inverse coupling $A' \equiv 1/\alpha_{\text{GUT}}^h$. The relation between $A'$ and $\Lambda_h$ is

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2 We will comment more on this later.
Figure 3. Constraints on $A'$. The confining scale is defined as the scale $\mu$ where $\alpha_h^b(\mu) = 1$. Using $\xi = 0.1, M_X = 2 \times 10^{16}$ GeV in the upper bound region and $\xi = 0.1, M_X = \frac{1}{2} \times 10^6$ GeV in the lower bound region, we obtain the region bounded by dashed vertical lines. Thick dash curves are for $-b^h_j = 5$ and 9.

shown in Fig. 3. For example, we obtain $A' \simeq 27.4$ in SU(4)$'$ with no matter ($b^h_j = -12$). It may be difficult to find such a model anyway. For SU(5)$'$ with 7 flavors ($b^h_j = -8$) corresponding to an unstable vacuum [10], we obtain $A' \simeq 18.6$ and $\Lambda_h \sim 10^{9-10}$ GeV.

**Orbifolds with Kaluza-Klein Radius Dependence**

An orbifold is a manifold moded out by a discrete action. It was used extensively in the compactification of string models [31], and later adopted in extra-dimensional field theory [32]. The simplest example is 1-dimensional (1D) torus moded out by the $\mathbb{Z}_2$ discrete action as shown in Fig. 4. Because of the identification of two points by $\mathbb{Z}_2$, we can consider only the half set of the manifolds points except at the boundary. The boundary points are called fixed points and the region between the fixed points is called the fundamental region. The area of the fundamental region is the half of the area of the original manifold (circle). The discrete action, say $g$ transforms the point in the manifold $z$ to $gz$. This action $g$ is an operator in quantum mechanics, and the wave functions are acted by this operator. In the $S_1/\mathbb{Z}_2$ orbifold action, the points in the manifolds transform

\[ g : y \rightarrow -y \]  

and a vector potential is acted as

\[ g : \begin{cases} V_\nu(y) \rightarrow -V_\nu(-y) \\ V_\mu(y) \rightarrow +V_\mu(-y). \end{cases} \]  

Another simple orbifold used extensively in string models is a two-dimensional (2D) torus moded out by a $\mathbb{Z}_3$ action as shown in Fig. 5. In this $T_2/\mathbb{Z}_3$ orbifold, there are three fixed points and the area of the fundamental region is $\frac{1}{3}$ (the yellow region of Fig. 5) of the torus area. The coordinate of the 2D torus is customarily represented by a complex number $z$, and the orbifold action is

\[ g : z \rightarrow e^{2\pi i/3}z \]
A 5D SUSY GUT

An interesting field theoretic orbifold is a 5D SUSY GUT with the internal space $S_1/Z_2 \times Z_2'$. For the torus $S_1$, there are only two possibilities of discrete symmetries for moding out, one by $Z_2$ and the other by $Z_2 \times Z_2'$. The $S_1/Z_2 \times Z_2'$ orbifold is shown in Fig. 6. As shown in Fig. 6, there are two fixed points and the area of the fundamental region is 1/4 of the area of the circle. The $\mathcal{N}=1$ 5D SUSY has $\mathcal{N}=2$ in terms of 4D SUSY. An SU(5) GUT group is expected to be broken by the discrete action and also the $\mathcal{N}=2$ SUSY is also expected to be broken directly by the discrete action. Therefore, we need two $Z_2$ as done in Ref. [32]. The 5D wave functions, with coordinate $(x^\mu, y)$ where $\mu = 0, 1, 2, 3$, have mode expansions in $\sum_n \phi(x)e^{iny}$ with mass $n/R$, which in other words has the $\cos ny$ and $\sin ny$ mode expansions. Thus, massless modes ($n = 0$) appear only in the cosine mode or both the $Z_2$ and $Z_2'$ parities being $+$, $(Z_2, Z_2') = (++)$. This method of obtaining massless modes is so simple and therefore attracted a great deal of attention.

In Fig. 7(a), we show the KK mass spectrum in field theoretic orbifold. To compare, in Fig. 7(b) we also show string theoretic $\mathcal{N}=1$ SUSY spectrum. Without SUSY, we note that one massless mode $(++)$ is not paired by another as shown in Fig. 7(a). On the other hand, string orbifolds with $\mathcal{N}=1$ SUSY, another SUSY partner appears as a massless mode also. Since the splitting of KK masses are $1/R$, we expect the spectra as shown in Fig. 7.
In the SU(5) field theoretic orbifold $S_1/Z_2 \times Z_2'$, the gauge multiplet $24_{SU(5)}$ splits into the SM representations with the following $(Z_2, Z_2')$ parities [32],

$$24_{SU(5)} = \begin{pmatrix} (8,1)^{(++)}_0 & (3,2)^{(-+)}_{-5/6} & (1,3)^{(+)}_0 \end{pmatrix} \oplus \begin{pmatrix} (1)^{(++)}_{1/3} & 0 & (1)^{++}_{-1/2} \end{pmatrix}$$

The $(++)$ states contain massless gauge boson modes, eight gluons plus four SU(2) × U(1) gauge bosons as shown in Fig. 7(a). For the gauge multiplet, the $\mathcal{N}=2$ SUSY is broken down to $\mathcal{N}=1$ SUSY. The sector containing the SM gauge bosons and the sector $G/H$ (the so-called $X,Y$ gauge bosons) split as

$$A_\mu^{SU(5)/SM} \leftrightarrow A_\lambda^{SU(5)/SM}$$

where the vertical transformation is the $\mathcal{N}=1$ SUSY transformation we are interested in and the horizontal transformation is the broken second $\mathcal{N}'=1$ SUSY transformation. In this model, two hypermultiplets $\mathcal{F}_H$ and $\overline{\mathcal{F}}_H$ are introduced. Here, writing only the spin-0 components, the $\mathcal{N}'(\times \mathcal{N})$ relations between bosons are

$$\mathcal{F} = \begin{pmatrix} 3^{(+)}_{H_u^{(++)}} \\ H_u^{(++)} \end{pmatrix} \leftrightarrow \mathcal{F}' = \begin{pmatrix} 3^{(-)}_{H_u^{(-)}} \\ H_u^{(-)} \end{pmatrix}, \quad \overline{\mathcal{F}} = \begin{pmatrix} \overline{3}^{(+)}_{H_d^{(++)}} \\ H_d^{(++)} \end{pmatrix} \leftrightarrow \overline{\mathcal{F}}' = \begin{pmatrix} \overline{3}^{(-)}_{H_d^{(-)}} \\ H_d^{(-)} \end{pmatrix}$$
where the modes containing massless modes are underlined. Note that we obtain the doublet-triplet splitting in this
model since one pair of Higgs doublets can be light while all colored Higgs fields are heavy. In orbifold string models,
this possibility was noted long time ago [33].

With the above KK spectrum Dienes, Dudas and Ghegetta tried to calculate the evolution of gauge couplings above
the TeV scale [34]. A typical form of the running of gauge couplings is

\[
\frac{1}{g^2_\mu} - \frac{1}{g^2_\lambda} + b^0_i \ln \frac{\Lambda}{\mu} - \left(b_i^{(+)} + b_i^{(-)}\right) \ln \frac{\Lambda}{M_R} + \left(b_i^{(+)} + b_i^{(-)} + b_i^{(+)} + b_i^{(-)}\right) \left[\frac{\Lambda}{M_c} - 1\right]
\]

where \(b_i^{(+)} + b_i^{(-)}\) comes from the SU(5) spectrum and \(b_i^{(+)} + b_i^{(-)}\) comes from the SU(5)/SM sector. Here,
\(M_R = 1/R\) and \(M_c\) is the compactification scale. The contribution to \(b_i^{(+)} + b_i^{(-)} + b_i^{(+)} + b_i^{(-)}\) is from the SU(5) spectrum. The power dependence of the coupling constants appears in the last term. However, the field theoretic
orbifold models are not ultraviolet completed models and the unification of coupling constant cannot be predicted. We
will comment on the string threshold correction below. There, the KK spectrum follows the pattern given in Fig. 7(b)
and in the \(Z_{12-1}\) it has the form [35],

\[
\frac{16\pi^2}{g_H^2(\mu)} \simeq \frac{16\pi^2}{g_2^2} + b^0_i \ln \frac{M^2_2}{\mu^2} - \frac{1}{4} b^N_G \ln \frac{M^2_R}{M^2_2} + \frac{1}{4} b^N_G \left[\frac{2\pi M^2_2}{\sqrt{3} M^2_\epsilon} - 2.19\right].
\]

**SU(5)′ HIDDEN SECTOR FROM Z\(_{12-1}\) ORBITFOLD COMPACTIFICATION**

As discussed before, the most promising hidden sector group toward a GMSB is SU(5)′. If we want the hidden sector
gaugino condensation, maybe there are more allowable choices restricted by \(\Lambda_h \approx 10^{13}\) GeV only. In our search of
SU(5)′ hidden sector, we require

- Three chiral families,
- SU(5)′ with \(10 + \overline{5}\) or many pairs of \(\overline{5} + \overline{5}\),
- Vectorlike exotics, or no exotics, which is another strong restriction.

Since we require three chiral families, it restricts very much the possible representations of the remaining gauge groups
since in this orbifold compactifications the total number of chiral fields are not much more than 100. The obvious
question is, “Why \(Z_{12-1}\)?” Probably, \(Z_{12-1}\) is most restrictive in Yukawa couplings, and it has a simple Wilson line
structure as discussed before [20]. The restrictiveness is due to a large integer 12 used, and hence an approximate
R-parity can be easily implemented [36, 14].

The \(Z_{12-1}\) twist is

\[
Z_{12-1} : \quad \phi = \left(\begin{array}{ccc}
\frac{5}{12} & \frac{4}{12} & \frac{1}{12}
\end{array}\right)
\]

where the second twist is 1/3 which appears in the \(Z_3\) orbifolds. The shape of the \(Z_{12-1}\) orbifold is shown in Fig. 8,
where the second orbifold has the shape of the \(Z_3\) orbifold. Here, Wilson lines distinguish three fixed points. Only one
(34)-torus and hence three fixed points of \(Z_{12-1}\) and 27 fixed points (3\(^3\)) in \(Z_3\). In the end, \(Z_3\) is as complicated as \(Z_{12-1}\)
due to the complexity of Wilson lines. But, the geometric discussion is simpler in \(Z_{12-1}\), since we pay attention only to
the (34)-torus. In \(Z_{12-1}\), much of breaking \(E_8 \times E_8′\) is directly done by the shift vector \(V′\) only, which is the reason that
the Wilson line \(a_3\) can be simple.

In string compactification, the modular invariance conditions are to be satisfied. They correspond to choosing the
\((++)\) parities and the anomaly cancelation conditions in field theory orbifold \(S_1/Z_2 \times Z_2\). In \(Z_{12-1}\), the modular
invariance conditions are

\[
12(V^2 - \phi^2) = \text{even integer}
\]
\[
12a_3^2 = \text{even integer}
\]
\[
12V \cdot a_3 = \text{integer}
\]

where \(a_3 = a_4\) and \(a_1 = a_2 = a_5 = a_6 = 0\). The masslessness conditions are

\[
L - \text{mover} : \quad \frac{(P + kV)^2}{2} + \sum_i N_i \phi_i - \bar{c}_k = 0
\]
The SU(4) © 2008 AIP String Compactification and Unification of Forces 2008/2/28 10

Here, SU(2) k of the Pati-Salam type gauge group but not exactly the same is obtained in Ref. [18]. This model is commented briefly. Let us briefly discuss two interesting U T T T 1:

\[
(\bar{r} + k\bar{\phi})^2 - \sum_{\ell} N^R_{\ell} \bar{\phi}_{\ell} - c_k = 0
\]

where \( k = 0(U), 1, \cdots, 11 \). The generalized GSO projection calculates the multiplicity

\[
P_k(f) = \frac{1}{12 \cdot 3} \sum_{i=0}^{N-1} \chi(\theta_k, \theta_i)e^{2\pi i\theta_f}
\]

\[
\Theta_f = \sum_{\ell} (N^L_{\ell} - N^R_{\ell}) \bar{\phi}_{\ell} - \frac{k}{2} (V^2_f - \phi^2) + (P + kV_f) \cdot V_f - (\bar{r} + k\bar{\phi}) \cdot \bar{\phi}, \quad V_f = V + m_f a_3.
\]

Let us briefly discuss two interesting Z12−I models.

A. Z12−I model without exotics

The Pati-Salam type gauge group but not exactly the same is obtained in Ref. [18]. This model is commented briefly. The SU(4) × SU(2)_W × SU(2)_Y × SU(5) representations are

\[
U_1: (4, 2, 1)_{0}, 2(6, 1, 1)_{0} \quad U_2: 2(4, 1, 2)_{0}, (6, 1, 1)_{0}
\]

\[
(1, 2, 1; 5'; 1)_{-1/10}, 2(1, 2, 1; 5; 1)_{1/10}
\]

\[
U_3: (4, 1, 2)_{0}, 2(1, 2, 1)_{0}, (1, 1; 2, 1; 1)_{0}
\]

\[
T_1_0: (4, 1, 1)_{1/2}, (1, 2, 1)_{1/2}, (1, 1, 2)_{1/2}
\]

\[
T_1_1: (1, 2, 1)_{-1/2}, (1, 1, 2)_{-1/2}
\]

\[
T_2_0: (6, 1, 1)_{0}, 2^0_{60}, 10
\]

\[
T_2_1: 5^{2/5}_2, 3^{0}_0
\]

\[
T_2_2: (1, 2, 2)_{0}, 3^{0}_0, 2^0_{60}, 2 \cdot 10
\]

\[
T_3: (4, 1, 1)_{1/2}, (4, 1, 1)_{-1/2}, (4, 1, 1)_{1/2}, 2(3, 1, 1)_{-1/2}
\]

\[
3(1, 2, 1)_{1/2}, 2(1, 2, 1)_{-1/2}, 2(1, 1, 2; 2; 1)_{1/2}
\]

\[
(1, 1, 2; 2, 1)_{-1/2}
\]

\[
T_{3_0}: 2(1, 1, 2; 2; 1; 3)_0, 2 \cdot 3^{0}_0
\]

\[
T_{3_{12}}: 2(3, 1, 1)_{0}, 2(4, 1, 2)_{0}, 2(6, 1, 1)_{0}, 7 \cdot 2^{0}_0, 9 \cdot 10
\]

\[
T_{4_1}: 2(1, 1, 2; 2; 1; 3)_0, 2 \cdot 3^{0}_0
\]

\[
T_{4_{12}}: (4, 1, 1)_{1/2}, (1, 1, 2)_{1/2}
\]

\[
T_{4_{12}}: (4, 1, 1)_{-1/2}, (1, 1, 2; 2; 1; 1)_{-1/2}, (1, 1, 2)_{-1/2}
\]

\[
T_{6_0}: 6 \cdot 5^{2/5}_2, 5 \cdot 5^{2/5}_2
\]

3 Here, SU(2)_Y is not the same as the SU(2)R of the Pati-Salam model [37].
TABLE 3. Hidden sector SU(5)' representations. We picked up the left-handed chirality only from $T_1$ to $T_{11}$ representations.

| $P + n[V \pm a]$ | Chirality | $\text{No.}\times(\text{Repts.}),_{Y,Q_1,Q_2}$ |
|-------------------|-----------|---------------------------------|
| $(\frac{1}{6} \frac{1}{6} \frac{1}{6} \frac{1}{6} \frac{1}{6} \frac{1}{6} \frac{1}{6} \frac{1}{6} \frac{1}{6} \frac{1}{6} \frac{1}{6} \frac{1}{6} \frac{1}{6} \frac{1}{6})_{T_{13}}$ | $L$ | $(1,1,2;1;5',1)_{L}^{T_{12}}$ |
| $(\frac{1}{6} \frac{1}{6} \frac{1}{6} \frac{1}{6} \frac{1}{6} \frac{1}{6} \frac{1}{6} \frac{1}{6} \frac{1}{6} \frac{1}{6} \frac{1}{6} \frac{1}{6} \frac{1}{6} \frac{1}{6})_{T_{13}}$ | $L$ | $(1,1,1;1;5',1)_{L}^{T_{12}}$ |
| $(0 \ 0 \ 0 \ \frac{1}{2} \ -\frac{1}{2} \ \frac{1}{2} \ 0)_{T_{13}}$ | $L$ | $(1,2,1;1;5',1)_{L}^{T_{12}}$ |
| $(0 \ 0 \ 0 \ \frac{1}{2} \ -\frac{1}{2} \ \frac{1}{2} \ 0)_{T_{13}}$ | $L$ | $(2(1,2,1;1;5',1)_{L}^{T_{12}}$ |
| $(0 \ 0 \ 0 \ 0 \ \frac{1}{2} \ -\frac{1}{2} \ \frac{1}{2} \ 0)_{T_{13}}$ | $L$ | $(4(1,1,1;1;5',1)_{L}^{T_{12}}$ |
| $(0 \ 0 \ 0 \ 0 \ \frac{1}{2} \ -\frac{1}{2} \ \frac{1}{2} \ 0)_{T_{13}}$ | $L$ | $(2(1,1,1;1;5',1)_{L}^{T_{12}}$ |
| $(0 \ 0 \ 0 \ 0 \ \frac{1}{2} \ -\frac{1}{2} \ \frac{1}{2} \ 0)_{T_{13}}$ | $L$ | $(3(1,1,1;1;5',1)_{L}^{T_{12}}$ |

$a$ The 3rd and 4th row have SU(2)$_W$ doublets

So, by making 3 or 4 flavors of SU(5)' heavy by the Higgs mechanism, we obtain the GMSB scenario at the unstable vacuum [10]. However, this model is not attractive in that the hidden sector quarks carry the SM quantum number(s), in particular there are SU(2)$_W$ doublet hidden sector quarks.

So, the $\theta^0$ component VEVs of $\phi - \bar{\phi}$ condensate mesons is almost zero and SU(2)$_W$ is not broken at the SUSY breaking scale by $\theta^0$ component VEVs. But $\theta^2$ components are large and carry SU(2)$_W$ quantum numbers. So, our model, even though very attractive, is breaking SM at the SUSY breaking scale (by the meson F-term and baryon VEVS) and not working as a realistic model.

B. Another exotics free model at stable vacuum

The model presented in [15] is very interesting in realizing

- Three chiral families
- No exotics
- Realization of $R$ parity
- One pair of Higgs doublets
- The GMSB at a stable vacuum.

But the compactification scale value of the weak mixing angle ($\sin^2 \theta_W$) is not $\frac{3}{8}$, and it remains to be seen whether it renormalizes correctly to the observed one at the electroweak scale. The model is

$$V = \frac{1}{11!}(6 \ 6 \ 6 \ 2 \ 2 \ 2 \ 3 \ 3 \ 3 \ 3 \ 1 \ 1 \ 1)$$

$$a_3 = \frac{1}{11!}(1 \ 1 \ 2 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 1 \ 1 \ 2)$$

Gauge group is SU(3)$_c \times SU(3)_W \times SU(5)' \times SU(3)' \times U(1)_s$. which contains the Lee-Weinberg electroweak model [38]. The model has no exotics. The observable sector fields are shown in Table 4. Note that $U(1)_R$ charges of the SM fermions are odd and those of the Higgs doublets are even. Therefore, by breaking $U(1)_R$ by VEVs of even $\Gamma$ singlets, we break $U(1)_R$ to a discrete matter parity $P$ or $R$ parity. Thus, we achieve realizing a successful $R$ parity [15]. Extra vectorlike doublets are given superheavy masses.

Because of the $R$ parity, the dimension-4 coupling $u' d' c'$ coupling is not present. The dimension-5 coupling of the form $qqql$ is not forbidden by the $R$ parity, but in the present model it is forbidden up to a very high order because of the remaining $U(1)$ gauge symmetries which are listed as subscripts in Table 4.
doublets. Thus, the MSSM problem of Ref. [39] is resolved.

Note that in our model both $H^+$ and $H^-$ appear from $\mathbf{3}$. It is in contrast to the other cases such as in SU(5) SUSY GUT or SO(10) SUSY GUT. Therefore, in our case SU(3)$_W \times U(1)_Y$ we can consider the original SU(3)$_W \times U(1)$ for the discussion of Yukawa couplings.

The $\mu$ problem and one pair of Higgs doublets

Except the three chiral families, the remaining representations form a vector-like one. Generally, if not forbidden by a special symmetry, vector-like representations including Higgs doublets are heavy. Thus, the need for one pair of Higgs doublets is difficult to realize in general, which is the so-called $\mu$ problem. In our model, we present a novel mechanism for allowing one light pair of Higgs doublets. It is achieved because the electroweak gauge group is the Lee-Weinberg SU(3)$_W \times U(1)$. From Table 4, there appear three quark weak triplets, which appear in three colors and hence count in total 9 weak triplets from the quark sector, $3(\mathbf{3}, \mathbf{3}_w)$. From the anomaly cancelation, at low energy we have 9 color singlet weak anti-triplets. This situation is shown in Fig. 9. These color singlet weak triplets are split, according to their quantum numbers, into $\mathbf{3}_w(H^+), \mathbf{3}_w(H^-)$, and $\mathbf{3}_w$ (lepton). Three $\mathbf{3}_w$ (lepton) remain light because of the chirality. The remaining representation $3[\mathbf{3}_w(H^+) + \mathbf{3}_w(H^-)]$ is vectorlike after the breaking of SU(3)$_W \times U(1)$ down to SU(2)$_W \times U(1)_Y$. However, we can consider the original SU(3)$_W \times U(1)$ for the discussion of Yukawa couplings.

Note that in our model both $H^+$ and $H^-$ appear from $\mathbf{3}$. It is in contrast to the other cases such as in SU(5) SUSY GUT or SO(10) SUSY GUT. Therefore, in our case SU(3)$_W \times U(1)_Y$ invariant $H^+$ and $H^-$ coupling must come from $\mathbf{3}_w \times \mathbf{3}_w \times \mathbf{3}_w$. Thus, there appears the Levi-Civita symbol and two $\varepsilon$ symbols must be introduced, one from taking the SU(3)$_W$ singlet and the other from the flavor basis! (What else?) Therefore, in the flavor space the $H^+$ and $H^-$ mass matrix must be antisymmetric and hence its determinant is zero, we conclude there appear one pair of massless Higgs doublets. Thus, the MSSM problem of Ref. [39] is resolved.

It is interesting to compare our result to the old introduction of color:

- In 1960s, it was known that the low-lying baryon and meson multiplets are embedded in $\mathbf{56}$ of the old SU(6) which is completely symmetric. But spin-half quarks are better to be fermions, which led to the introduction of an antisymmetric index. That was the famous SU(3)$_c$ of color, describing strong interactions [40].

### Table 4.

| $P + [kV + ka]$ | No. $\times (\text{Repts.}) \gamma(q, q_2, q_3, q_4)$ | $\Gamma$ | $\text{Label}$ |
|-----------------|-------------------------------------------------|------|-------------|
| $(-1) \frac{1}{3} \frac{1}{3} \frac{1}{3} \frac{1}{3} \frac{1}{3} \frac{1}{3} \frac{1}{3} \frac{1}{3} \frac{1}{3}$ | $3 \cdot (3, 2)_1^L \times (0, 0, 0, 0)$ | 1 | $q_1, q_2, q_3$ |
| $(\frac{1}{3} \frac{1}{3} \frac{1}{3} \frac{1}{3} \frac{1}{3} \frac{1}{3} \frac{1}{3} \frac{1}{3} \frac{1}{3})$ | $2 \cdot (\mathbf{3}, 1)_2^L \times [3, -3, 3, 2, 0, 0]$ | 3 | $u^c, c^c$ |
| $(-1) \frac{1}{3} \frac{1}{3} \frac{1}{3} \frac{1}{3} \frac{1}{3} \frac{1}{3} \frac{1}{3} \frac{1}{3} \frac{1}{3}$ | $(\mathbf{3}, 1)_1^L \times [0.6, -1, 5, 1]$ | 1 | $t^c$ |
| $(\frac{1}{3} \frac{1}{3} \frac{1}{3} \frac{1}{3} \frac{1}{3} \frac{1}{3} \frac{1}{3} \frac{1}{3} \frac{1}{3})$ | $(\mathbf{3}, 1)_1^L \times [-3, -3, 0, 0, 0, 4]$ | 1 | $d^c$ |
| $(-1) \frac{1}{3} \frac{1}{3} \frac{1}{3} \frac{1}{3} \frac{1}{3} \frac{1}{3} \frac{1}{3} \frac{1}{3} \frac{1}{3}$ | $(\mathbf{3}, 1)_1^L \times [-3, 3, -2, 2, 0, 0]$ | 1 | $s^c, b^c$ |
| $(0 0 0 \frac{2}{3} \frac{2}{3} \frac{2}{3} \frac{2}{3} \frac{2}{3} \frac{2}{3} \frac{2}{3} \frac{2}{3} \frac{2}{3})$ | $(1, 2)_2^L \times [-1, 2, -1, 1, 2, 1, 0, 0]$ | 1 | $l_1, l_2, l_3$ |
| $(-1) \frac{1}{3} \frac{1}{3} \frac{1}{3} \frac{1}{3} \frac{1}{3} \frac{1}{3} \frac{1}{3} \frac{1}{3} \frac{1}{3}$ | $(1, 2)_2^L \times [0.6, -1, 5, 1]$ | 0 | $H_u$ |
| $(0 0 0 \frac{2}{3} \frac{2}{3} \frac{2}{3} \frac{2}{3} \frac{2}{3} \frac{2}{3} \frac{2}{3} \frac{2}{3})$ | $(1, 2)_2^L \times [-6, 0, 9, 0, 0]$ | 2 | $H_d$ |
In our supersymmetric theory, superpotential is described by bosonic fields, i.e. by the \( \theta^0 \) components of chiral multiplets. Since the Lee-Weinberg SU(3)\(_W\) introduces an antisymmetric index, we need to introduce another antisymmetric flavor index [15].

### Hidden sector

The hidden sector gauge group for breaking SUSY is SU(5)\(^{\prime}\) and in Table 5 we list the SU(5)\(^{\prime}\) representations. After removing the vectorlike representation there remain 10\(^\prime\) and \( \bar{5}^{\prime} \) which are starred in Table 5. This set of chiral representations is the source of dynamical SUSY breaking in SU(5)\(^{\prime}\) [41], and an F term appears for the chiral gauge multiplet, \( \mathcal{W}^{a\alpha} \mathcal{W}^{b}_d \) for example [29]. This F term splits the SUSY partner masses of messengers \( f \),

\[
\mathcal{L} = \int d^2 \theta \left( \frac{1}{M^2} \bar{f} f^{\alpha\beta} \mathcal{W}^{a\alpha} \mathcal{W}^{b}_d + M_f \bar{f} f \right) + \text{h.c.}
\]

where \( M \) is the parameter, presumably above 10\(^{12}\) GeV. For example, vectorlike \( Q_{em} = -\frac{1}{2} \) D-type quarks can be colored messengers. The superpartner mass splittings of the messenger sector transmit the information to the observable sector via gauge interactions and hence the soft masses of squarks and sleptons appear as flavor independent [22].

[Noted added:]

One can see that the \( \mathcal{W}^{c\alpha} \mathcal{W}^{d}_b \) in Eq. (25) develops an F term. We can consider the following operators below the confinement scale of SU(5)\(^{\prime}\),

\[
Z \sim \mathcal{W}^{a\alpha} \mathcal{W}^{b}_d \quad \text{(26)}
\]

\[
Z' \sim \epsilon_{abc} f g h \mathcal{W}^{a\alpha} \mathcal{W}^{d}_d \bar{10}^{id} \bar{5}^{j} \bar{5}^{k} \bar{10}^{sh} \quad \text{(27)}
\]

where the contraction of spinor indices of the chiral gauge multiplet \( \mathcal{W} \) is implied. Under the global symmetry \( U(1)_A \times U(1)_B \times U(1)_R \), \( 10, \bar{5} \) and \( \mathcal{W}^{c\alpha} \) transform as

\[
\begin{align*}
\begin{array}{ccc}
U(1)_A & U(1)_B & U(1)_R \\
10 & p & 1 \\
\bar{5} & q & -3 \\
\mathcal{W} & 0 & 0 \\
Z & 0 & 0 \\
Z' & 3p + q & 3r + s + 2
\end{array}
\end{align*}
\]

\[\begin{array}{c}
\text{(28)}
\end{array}\]
where $U(1)_R$ charges are given as anomaly free. $U(1)_R$ is also chosen as anomaly free, which gives the relation $3r + s = -6$. Thus, $Z'$ carries $R = -4$. On the other hand $U(1)_A$ is anomalous. The fermionic zero modes contributes to the instanton amplitudes as $e^{-8\pi^2/8^2}(µ + iθ) = (Λ / µ)^{3(Nc - Σ_f)} = (Λ / µ)^{15 - 2}$ where Λ is the dynamically generated mass scale. The so-called 't Hooft’s determinental instanton amplitude carries flavors (e.g. represented as $2N_c$ gluino lines plus $2N_q$ quark lines in SUSY QCD) and hence after integrating out the one-loop beta function we assign $2\sum_f \ell(f)$ charge for the $U(1)_A$ quantum number to the scale $Λ^{(3Nc - Σ_f)}$. Thus, $Λ^{13}$ carries the $U(1)_A$ charge $3p + q$. Including this instanton amplitude, we try to include all possible terms allowed by $U(1)_A \times U(1)_B \times U(1)_R$. Namely, $U(1)_A$ for the instanton interaction is respected if we consider the combination $Λ^{13}$ divided by $10 \cdot 10 \cdot \overline{5}^5$ or $Λ^{10}$ by $Z'$. In this way, we can write all possible terms. $U(1)_A \times U(1)_R$ symmetries dictate the following effective superpotential, after redefining $Z$ and $Z'$ as dimension-1 fields,

$$W_{\text{eff}} = \sum a c_{m}^{Z_{1}^{2}a} Z_{1}^{2a} Z_{a}^{2a} \tag{29}$$

where $c_{a}$ are dimensionless constants. The determinental interaction corresponds to $a = -1$. Strong dynamics may also allow the $a = 0$ term. In (29) we included all terms allowed just from the symmetry argument. Considering only the two terms with $a = -1$ and $a = 0$ the SUSY conditions cannot be satisfied simultaneously, but a runaway solution results. So, we consider at least three terms, for which we choose $a = -1, 0$ and $1$, for an illustration. Then, the SUSY conditions are

$$\frac{\partial W}{\partial Z} = -c_{-1}m^{3}Z^{-2}Z' - 1 + c_{0}m^{2} + 3c_{1}m^{-1}Z^{2}Z' = 0 \tag{30}$$

$$\frac{\partial W}{\partial Z'} = -c_{-1}m^{3}Z^{-1}Z'^{-2} + c_{1}m^{-1}Z^{3} = 0 \tag{31}$$

which cannot be satisfied simultaneously unless $c_{0} + 2\sqrt{c_{1}c_{-1}} = 0$. The symmetry principle allows many terms including the determinental interaction, and in general SUSY is broken.

**Discrete symmetries**

In the nonprime orbifolds, there are invariant torii in which case there exist some discrete symmetries. These discrete symmetries can be used for obtaining fermion mass spectrum. In the $Z_{12 - I}$ orbifold, $T_3$ and $T_6$ sectors have the following twists

$$Z_{12 - I} : \quad 3\phi = \left( \begin{array}{ccc} 1 & 0 & 1 \\ 1 & 0 & 4 \\ 0 & 1 & 4 \end{array} \right) , \quad 6\phi = \left( \begin{array}{ccc} 1 & 2 & 0 \\ 0 & 1 & 2 \\ 2 & 0 & 1 \end{array} \right) \tag{32}$$

which are $Z_4$ and $Z_2$, respectively. Thus, they have the fixed points as shown in Fig. 10. For example, in $T_6$ we may consider an $S_3$ symmetry because the four fixed points cannot be distinguished by Wilson lines. The Yukawa couplings must respect this kind of discrete symmetry, which can be used to obtain nonabelian discrete symmetries by a further manipulation [43].

**THRESHOLD CORRECTION**

The threshold correction via one loop is the torus topology, and the orbifolds on torus is the natural place to consider one loop corrections of closed strings. In string compactification, the threshold correction comes from non-prime orbifolds. The reason is that they contain invariant torus, and a large radius $R$ can be introduced. In $R \rightarrow \infty$, we have a 6D model, i.e. we obtain an $\mathcal{N} = 2$ SUSY. The $\mathcal{N} = 2$ models are vectorlike in 4D, and have masses of the form $1/R$ times integer. The simplest invariant torus is the $Z_2$ substructure.

The pioneering work on the threshold correction in string models has been calculated in Ref. [44, 45, 46], and we have recently implemented the method to add Wilson lines [35]. The invariant sublattices are under $G' \subseteq G$. Here,

---

4 See, for example, Ref. [42].
5 With one $10$ and one $\overline{5}$, the combination $10 \cdot 10 \cdot \overline{5}^5$ is not possible, but $Z'$ is possible as shown in (27). However, other combinations of $\overline{Y} \cdot \overline{Y}'$ with matter fields are not possible.
the $\mathcal{N}=2$ SUSY KK masses are described by a large radius ($R$), encoded in modulus of the metric. The simplest substructure $\mathbb{Z}_3$ appears in $\mathbb{Z}_{6-I}$ and $\mathbb{Z}_{12-I}$ orbifolds. But, there has not appeared a phenomenologically interesting $\mathbb{Z}_{6-I}$ model, and we restrict the discussion to the $\mathbb{Z}_{12-I}$ models. Specifically, we work with the model presented in Ref. [14]:

\[
\phi = \left( \frac{5}{12}, \frac{4}{12}, \frac{1}{12}, \frac{5}{12}, \frac{4}{12}, \frac{1}{12} \right)
\]

\[
V = \left( \frac{1}{4}, \frac{1}{4}, \frac{1}{4}, \frac{1}{4}, \frac{5}{12}, \frac{5}{12} \right) \left( \frac{1}{4}, \frac{3}{4}, 0, 0, 0, 0 \right)
\]

\[
a_3 = \left( \frac{2}{3}, \frac{2}{3}, \frac{2}{3}, \frac{2}{3}, \frac{2}{3}, \frac{2}{3}, 0, 0 \right)
\]

(33)

The invariant torus is the second one, i.e. the $(34)$-torus which obeys the $\mathbb{Z}_3$ identification. The radius $R$ of the $(34)$-torus is large compared to the compactification radii of $(12)$- and $(56)$-torii. Introduction of the Wilson line $a_3$ in the $(34)$-torus breaks $G$ down to the SM gauge group. The anticipated evolution of gauge couplings are shown in Fig. 11.

\[
\Delta_i = \left( \frac{Z'}{Z} \right) \frac{b_i^{N=2}}{2} \int \frac{d^2 \tau}{\tau_2} \left( \mathbb{Z}_{\text{torus}}(\tau, \bar{\tau}) - 1 \right)
\]

(34)

where $Z' = 3$ (from the $(34)$-torus) and $Z = 12$ in our case. Eq. (34) gives the compactification size ($R$) dependence through the modular parameter with the following metric,

\[
\{ \hat{e}_1 = (\sqrt{2}, 0), \hat{e}_2 = (-\sqrt{1/2}, \sqrt{3/2}) \}; \quad g_{ab} = \left( \begin{array}{cc} 2 & -1 \\ -1 & 2 \end{array} \right)
\]

(35)

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Thus, we obtain the following $R$ dependence of the gauge couplings,

$$
\begin{align*}
\frac{4\pi}{\alpha_{H_0}(\mu)} &= \frac{4\pi}{\alpha_s} + b^0 H_0 \log \frac{M^2}{\mu^2} - \frac{b_H}{4} \left[ \log \frac{R^2}{\alpha} + 1.89 \right] + \frac{(b_H + b_{G/H})}{4} \left[ \frac{2\pi R^2}{\sqrt{3} \alpha'} - 0.30 \right] \\
\end{align*}
$$

where $H_0$ is the SM gauge group. Between $R$ and the string scale, the contribution to the $\beta$-function coefficient is given by $b_H$: the corresponding group may not be the SM group. The $\beta$-function coefficient $b_H + b_{G/H}$ is the coefficient for the full group $G$ which is the gauge group obtained by $V$. Introduction of the Wilson line $a_4$ in the (34)-torus introduces the $R$ dependence. Because of the string calculation in our scheme, the resultant power behavior is reliable. It is a reliable calculation, not like the expressions written in extra dimensional field theory [34]. In particular, we point out that the $R$-squared and constant terms are also reliable, and predicts how gauge couplings behave above the so-called GUT scale.

Between $R$ and the string scale, the contribution to beta function coefficient is given by $b_H$. We note that the corresponding group may not be the SM group. Actually, we need singlet Higgs VEVs to give large masses for exotic particles [14]. Since SU(4) above the scale $1/R$ gives a complicated form for its U(1) subgroup, we break the SU(4) by VEVs of these singlets. So, we consider only the subgroup SU(2)$_W$ of the broken SU(4) and consider the N=2 $b_1$ (the $b_H$ term in Eq. (37)) in terms of another parameter $h_i$,

$$
\begin{align*}
b_i &= h_i \left( \log \frac{M^2}{M^2_R} + 1.89 \right).
\end{align*}
$$

The hypercharge definition must be made judiciously to avoid chiral exotics or even to remove all exotics, as discussed in [14]: Model E with vectorlike exotics and Model S without exotics,

$$
\begin{align*}
\text{Model E : } Y_E &= \left( \begin{array}{ccc} 1 & 1 & 1 \\ -1 & -1 & 2 \\ 0 & 0 & 0 \end{array} \right)(0^8)', \quad \sin^2 \theta_W = \frac{3}{8} \\
\text{Model S : } Y_S &= \left( \begin{array}{ccc} 1 & 1 & 1 \\ -1 & -1 & 2 \\ 0 & 0 & 0 \end{array} \right)(0 0 1 0^5)', \quad \sin^2 \theta_W = \frac{3}{14}.
\end{align*}
$$

In Model E, $\sin^2 \theta_W$ is the standard one and we obtain the usual result. Here, however, there exists another parameter $R$ which can be used to fit the strong coupling constant $\alpha_s(M_Z)$ to the observed value [14]. On the other hand, Model S has a much smaller $\sin^2 \theta_W$ and the parameters $R$ and the string scale $M_s$ can be used to fit to the observed values of the mixing angle and the strong coupling, $\sin^2 \theta_W(M_Z) = 0.22306 \pm 0.00033$ and $\alpha_s(M_Z) = 0.1216 \pm 0.0017$ [47]. The allowed regions of $R = M^2_R$ and $M_s$ are

$$
\begin{align*}
\text{Model S : } & \frac{M_s}{M_Z} \approx 1.70 \times 10^{15}, \quad \frac{M_s}{M_R} \approx 3.68.
\end{align*}
$$

CONCLUSION

We showed some interesting explanations of the SUGRA problems by the orbifold compactification of the $E_8 \times E'_8$ heterotic string. We also considered the GMSB possibility in the orbifold compactification with a desirable MSSM spectrum. We observed that a 6D SUSY GUT is realized with the KK mass dependent threshold corrections. These corrections are reliable unlike in extra-dimensional field theory. In some models, three families appear with no exotics. The GMSB at a stable vacuum in SU(5)' with $10'$ plus $\tilde{5}$ is shown to be possible. In this model, we obtained just the MSSM spectrum, i.e. with one pair of Higgs doublets. The R parity embedding is shown to be successful. We also discussed the gauge coupling unification in nonprime orbifolds with the KK mode contribution to the evolution equation. The KK mass parameter $1/R$ is used to obtain the coupling unification even with a GUT scale value $\sin^2 \theta_W^{GUT} \neq \frac{3}{8}$.

The orbifold compactification of the $E_8 \times E'_8$ heterotic string gives enough good phenomenologies, which is not competed in other superstrings. Yet, we have to resolve the moduli stabilization problem in this kind of heterotic string models.
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