Topological Phase Diagram of a Two-Subband Electron System

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We present a phase diagram for a two-dimensional electron system with two populated subbands. Using a gated GaAs/AlGaAs single quantum well, we have mapped out the phases of various quantum Hall states in the density-magnetic field plane. The experimental phase diagram shows a very different topology from the conventional Landau fan diagram. We find regions of negative differential Hall resistance which are interpreted as preliminary evidence of the long sought reentrant quantum Hall transitions. We discuss the origins of the anomalous topology and the negative differential Hall resistance in terms of the Landau level and subband mixing.

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Extensive works has been carried out on modulation-doped GaAs/AlGaAs heterostructures containing a two dimensional electron gas (2DEG) within the framework of quantum Hall effect. In most of these structures, only one subband is populated. Even though studies of heterostructures with two populated electric subbands have a long history, until recently, it has become increasingly apparent that in one-band systems, disorder induced Landau level mixing can play a critical role in the evolution of the quantum Hall effect, especially in the regime of vanishing magnetic fields. Landau level mixing and its effects have been the subjects of numerous recent experimental and theoretical studies. Similarly, in a two-subband system, crossing of Landau levels of the two different subbands can lead to substantial mixing even in relatively strong magnetic fields. The consequences of Landau level mixing on the topology of the phase boundaries between different quantum Hall states in the two-band system are expected to be surprising and possibly profound.

To explore some of these consequences, we have conducted a systematic magneto-transport study on gated, modulation-doped GaAs/AlGaAs single quantum well samples in which there are two populated subbands. We have constructed a topological phase diagram of the two-band system. We found this phase diagram to be considerably more complex than the conventional Landau fan diagram. One of the spectacular consequences of its unusual topology is that there are multiple reentrant quantum Hall transitions. We have observed negative differential Hall resistance in certain regions of the density-magnetic field plane (the $n-B$ plane). The negative differential Hall resistance, in our opinion, is indicative of the reentrant quantum Hall transition.

The sample used in this study is a symmetrical modulation-doped single quantum well with a width of 250 Å. Two Si δ-doped layers ($n_d = 8 \times 10^{11} \text{cm}^{-2}$) are placed on either side of the well. There is a 200 Å spacer between the δ-doped layer and the well on each side. Heavy doping creates a very dense 2DEG, resulting in the filling of both subbands in the well. As determined from the Hall resistance data and Shubnikov-de Hass oscillations, the total density is $n = 1.21 \times 10^{12} \text{cm}^{-2}$. The higher subband has a density $n_1 = 3.3 \times 10^{11} \text{cm}^{-2}$ while the lower subband has a density $n_2 = 8.8 \times 10^{11} \text{cm}^{-2}$ at $B = 0$. The electron mobility at zero gate voltage is about $8 \times 10^4 \text{cm}^2/\text{N-s}$. The samples are patterned into Hall bars with a 3 : 1 aspect ratio using standard lithography techniques. An Al gate was evaporated on top so that by applying a negative gate voltage, the carrier density can be varied continuously. A total of 9 samples with different lengths (varying from 30 μm to 3 mm) were studied systematically. For consistency, we present the data from only one sample here. During the experiment, the sample was thermally connected to the mixing chamber of a dilution refrigerator. Magnetic fields up to 12 T were applied normal to the plane containing the 2DEG. Standard lock-in techniques with an excitation frequency of about 13 Hz and a current of 10 nA were employed to carry out the magnetoresistance measurements.

A typical trace of the diagonal resistivity $\rho_{xx}$ and the Hall resistivity $\rho_{xy}$ as a function of $B$ at a temperature of 70 mK is shown in Fig. 1. The integer numbers in the figure identify each quantum Hall state by its quantized value in the unit of $h/e^2$ (i.e., $S_{xy} = (h/e^2)/R_{xy}$). The peaks in $\rho_{xx}$ represent the positions of the delocalized states and together they mark the phase boundaries between various quantum Hall states. This criterion was used to construct the experimental phase diagram in Fig. 3.

Before presenting the experimental phase diagram, it is useful to discuss what one should expect for the simplest case in the absence of Landau level mixing. Using the energy separation between the two subbands for the present sample, we plot in Fig. 2a the energy $E$ as func-
tion of magnetic field. The corresponding positions of the delocalized states in the n-B plane can be calculated and the resulting phase diagram is displayed in Fig. 2b. From Fig. 2b, one can see that the electrons fill the Landau levels of upper and lower subbands in alternating fashion as the magnetic field is increased. In this case, the phase diagram has an ordinary “fan-like” appearance identical to that for a single band system.

The actual phase diagram is, however, very different from this simple picture. We present in Fig. 3 an experimental phase diagram. The density on the right axis is related to the gate voltage on the left axis by a linear relation determined by the sample geometry. To construct the phase diagram, we have swept both the gate voltage, i.e., the carrier density, at a fixed magnetic field (a “Vg-scan”), and the field at a fixed gate-voltage (a “B-scan”). Each peak in \( \rho_{xx} \) corresponds to a single data point in the phase diagram. The data points represent the phase boundaries between various quantum Hall liquid states. Limited by the base temperature of our cryostat, the plateaus in regions very near to the intersections of phase boundary lines are normally not well resolved. In such cases, we follow the evolutionary development of the plateaus away from these places to assign sensible values of \( S_{xy} \). Moreover, we could not determine the phase boundaries reliably at low magnetic fields (\( B \leq 2 \) T) as the peak becomes progressively more difficult to resolve in a decreasing magnetic field.

In the low-density regime of the phase diagram (i.e., \( n \leq 8.5 \times 10^{11} \text{ cm}^{-2} \)), with the upper subband depopulated, the experiment phase diagram is identical to that of a one-subband system. Note the transition of spin-resolved quantum Hall states to spin-degenerate quantum Hall state at around 8 T. This type of level “pinch off” has been studied recently in detail both theoretically and experimentally. With the upper subband populated, the phase diagram becomes very rich in topology. The most pronounced feature is the sawtooth-like pattern for densities in the range between \( n = 9 \times 10^{11} \text{ cm}^{-2} \) and \( 11 \times 10^{11} \text{ cm}^{-2} \). A similar pattern can also be seen for higher densities between \( n = 10.5 \times 10^{11} \text{ cm}^{-2} \) and \( 12 \times 10^{11} \text{ cm}^{-2} \). There are also apparently “triple” and “quadruple” points which separate the different quantum Hall phases.

Despite the complexity of this phase diagram, we find that the “selection rules” for the quantum Hall phase transitions are never violated. According to the selection rules, the Fermi level must cross one delocalized level at a time. For the level counting, “one” is used for the spin resolved case and “two” is used for the spin unresolved case. Therefore for either a “Vg-scan” or a “B-scan”, the number for \( S_{xy} \) changes either by one for crossing a spin-resolved subband level, or by two for crossing a spin-degenerate subband level, or even conceivably by four for crossing a spin-degenerate and subband-mixed level.

One of the striking consequences of this unusual topology is that the differential Hall resistance (NDHR) \( d\rho_{xy}/dB \) can be negative during the B-scan in certain regions of the n-B diagram. For example a NDHR can be seen in the Fig. 1 around \( B = 7 \) T (in the circled area). This NDHR is certainly unusual. In a one-band system, only positive differential Hall resistance (i.e. the classical Hall resistance or in the region between two plateaus) and zero differential Hall resistance (i.e., in the plateau region) have been observed. For this two-band system we found, in fact, that NDHR can be seen during a B-scan...
along a trajectory cutting through the top portion of any sawtooth.

We present, in the left shown of Fig. 4, the evolution of the NDHR for various $V_g$ at a fixed temperature of 70 mK for the sawtooth between 6 to 8 T. For the convenience of tracking $S_{xy}$, we have plotted $1/R_{xy}$ (in unit of $e^2/h$) as the vertical axis. At $V_g = -0.34$ V (at the tip of the tooth), a slight dip is seen at the middle of the well developed $h/7e^2$ Hall plateau. As the $V_g$ gets more negative, the dip shows more deviation from $h/7e^2$. The dip gets progressively deeper and wider. At $V_g = -0.38$ V, the deviation is the greatest while the high field portion of the $h/7e^2$ plateau is still visible. The high-field side of this dip leads to the unusual NDHR. As the gate voltage gets even more negative, the dip develops into the $h/6e^2$ Hall plateau (see for example $-0.54$ V) giving, eventually, the normal $S_{xy} = 7$ to $S_{xy} = 6$ (“6-7” in short) quantum Hall transition.

We have also investigated the temperature dependence of $d\rho_{xy}/dB$ at $V_g = -0.41$ V. $B$-scan traces, in the right shown of Fig. 4, were taken at $T = 4.2$ K, 1.2 K, and 70 mK. At the highest temperature, the $h/7e^2$ plateau is not resolved and there was no sign of the NDHR. At $T = 1.2$ K, the plateau starts to form and a small dip becomes visible near the expected positions of the $h/7e^2$ and $h/6e^2$ plateau. As the temperature goes further down, both $h/7e^2$ and $h/6e^2$ plateaus are well resolved and the dip becomes deeper. The deviation has reached a value of about $h/6.5e^2$ at 70 mK. It is apparent that the NDHR is associated closely with the formation of the quantum Hall states of $S_{xy} = 6$ and $S_{xy} = 7$ in this case.

In an attempt to understand the topological anomalies of the phase diagram, we have performed a simple numerical calculation to account for the effect of Landau level mixing of the two bands. In this calculation, we have made the simple assumption that the density of states can be modeled as two sequences of Gaussian functions centered around the Landau levels for the lower and upper subband respectively. The width of the Gaussian functions is determined from the conductivity of the sample. For each and every maximum in the density of states, the electron density is calculated at a given magnetic field. We assume that the delocalized states lie at the local maxima in the resulting density of states. In this way, we can obtain a theoretical phase diagram. Of course, in reality, the Landau level mixing due to both the level repulsion and disorder broadening is far more complicated than this simple assumption. Our simple calculation is nevertheless able to produce the sawtooth-like structure qualitatively as seen in the experimental data. Therefore, we believe the sawtooth pattern is a result of the mixing of the Landau levels of the two bands. Every time the two levels move towards a crossing, the position of the delocalized states deviate from the normal fan lines and “float up” in density or equivalently in energy. At the same time, when two levels move away from the crossing, the position of the delocalized states “sinks down” back to the normal fan lines. We think this effect has the same origin as the “floating” observed in the one-band system in a vanishing $B$ [3]. In our numerical calculations, the above criterion results in a general floating up in energy of individual delocalized states with decreasing $B$. Therefore, the unusual sawtooth patterns are caused by delocalized states rising above their corresponding host Landau levels. The sawtooth structure at 7.5 T can be identified as due to the mixing of the spin-degenerate third Landau level ($N = 2$) of the first subband with the spin-up state of the lowest Landau level of the second subband ($N' = 0 \uparrow$). The features at around 5.5 T, 4.4 T, and 3.5 T, are due to the mixing of the
(N′ = 0) level with (N = 3, 4, 5) levels respectively. Another portion of the sawtooth pattern at high densities is due to the mixing of (N′ = 1) level with the N = 5, 6 levels.

It is important to point out here that the sawtooth patterns indicate a sequence of reentrant quantum Hall transitions (i.e., 7-6-7, 10-8-10, and 12-10-12 etc.). This type of reentrant quantum Hall transitions have been proposed theoretically for single band quantum Hall systems. Experimentally, it has not been seen to date. An analogous transition which has been observed is the 0-2-0 transition or the 0-1-0 transition. For the present experiment, we believe the NDHR observed in certain regions shows preliminary evidence of the long sought reentrant quantum Hall transitions. For example, in the region between 6 T and 8 T, a B-scan at the appropriate Vg at the top portion of the sawtooth is equivalent to traversing the phase diagram horizontally, thus cutting through two sides of a sawtooth. To the left of the sawtooth, we can identify the quantum Hall state as $S_{xy} = 7$ state. Inside the sawtooth it is a $S_{xy} = 6$ state. To the right, within a very narrow range of B, it is again a $S_{xy} = 7$ state. For the 7-6-7 transition, the Hall resistance should vary from $h/7e^2$ to $h/6e^2$ and back to $h/7e^2$ with increasing B. However, the $S_{xy} = 6$ and the "re-entrant" $S_{xy} = 7$ plateaus in our experiment cannot be well resolved simultaneously at a given $V_g$. As a result, the values of the dip and the peak in $R_{xy}$ only reach, at best, about $h/6.5e^2$ rather than $h/6e^2$ and $h/7e^2$ respectively. The NDHR can be considered as a precursor of the reentrant $S_{xy} = 7$ state. We believe one should be able to observe the true 7-6-7 transition at lower temperatures. We however cannot eliminate the possibility of that a true quantum Hall state (i.e., with zero diagonal resistance and a quantized Hall plateau) would be intrinsically prohibited by the Landau level mixing.

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