The Cosmological Constant and Domain Walls in Orientifold Field Theories and \( \mathcal{N} = 1 \) Gluodynamics

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Abstract

We discuss domain walls and vacuum energy density (cosmological constant) in \( \mathcal{N} = 1 \) gluodynamics and in non-supersymmetric large \( N \) orientifold field theories which have been recently shown to be planar equivalent (in the boson sector) to \( \mathcal{N} = 1 \) gluodynamics. A relation between the vanishing force between two parallel walls and vanishing cosmological constant is pointed out. This relation may explain why the cosmological constant vanishes in the orientifold field theory at leading order although the hadronic spectrum of this theory does not contain fermions in the limit \( N \to \infty \). The cancellation is among even and odd parity bosonic contributions, due to NS-NS and R-R cancellations in the annulus amplitude of the underlying string theory. We use the open-closed string channel duality to describe interaction between the domain walls which is interpreted as the exchange of composite “dilatons” and “axions” coupled to the walls. Finally, we study some planar equivalent pairs in which both theories in the parent-daughter pair are non-supersymmetric.

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1 Introduction

Domain walls are BPS objects which appear in $\mathcal{N} = 1$ supersymmetric (SUSY) gluodynamics [1]. If the gauge group is SU($N$), there are $N$ distinct discrete vacua labeled by the order parameter, the gluino condensate,

$$\langle \lambda \lambda \rangle_k = N \Lambda^3 \exp \left( \frac{i 2 \pi k}{N} \right), \quad k = 0, 1, 2, ..., N - 1. \quad (1)$$

The domain wall $W_{\{k,k+1\}}$ interpolates between the $k$-th and $k+1$ vacua. Moreover, at $N \to \infty$ two parallel domain walls $W_{\{k,k+1\}}$ and $W_{\{k+1,k+2\}}$ are also BPS — there is neither attraction nor repulsion between them.

It is known that the BPS domain walls in $\mathcal{N} = 1$ gluodynamics present a close parallel to D branes in string theory [2]. In particular, a fundamental flux tube can end on the BPS domain wall, similarly to F1 ending on D branes in string theory [2] (see also [3, 4, 5, 6, 7]). The purpose of this paper is three-fold. First, we show that this parallel can be further extended. In string theory the absence of forces between parallel D branes is due to a cancellation between the interactions induced by NS-NS and R-R charges. We show how a similar cancellation works in $\mathcal{N} = 1$ gluodynamics. This parallel yields an important insight revealing a relation between the vanishing of the cosmological constant and cancellation of forces. This observation will be used later.

Second, we will extend this parallel to a non-supersymmetric gauge field theory. Recently, we discussed a non-supersymmetric theory, where exact results on the strong coupling regime could be obtained [8]. The theory, named “orientifold field theory,” is a daughter of $\mathcal{N} = 1$ gluodynamics. The parent-daughter relationship is understood in the sense of [9]. The parent theory is $\mathcal{N} = 1$ gluodynamics with the gauge group U($N$). The daughter theory also has U($N$) gauge group, the same gauge coupling as the parent one, and the fermion sector consisting of one Dirac fermion in the antisymmetric tensor representation.
The advantage of the orientifold daughter compared to orbifold discussed by Strassler \[9\] is the absence of the twisted sector in the former. The nonperturbative planar equivalence between $\mathcal{N} = 1$ gluodynamics and its orbifold, conjectured by Strassler, was questioned in the literature (see e.g. Refs. [10, 11]), with the twisted sector of the orbifold theory being the main suspect. The nonperturbative planar equivalence between $\mathcal{N} = 1$ gluodynamics and its orientifold daughter was shown \[8\] to be on a much more solid theoretical footing. We argued that the orientifold gauge theory, at large $N$, contains $N$ degenerate vacua, has a bifermion condensate which serves as an order parameter, much in the same way as the gluino condensate, Eq. (1). Another finding was the vanishing of the cosmological constant at order $N^2$. These results seem to be surprising since the hadronic spectrum of the orientifold theory is purely bosonic.

The orientifold theory has $N$ discrete degenerate vacua. Hence, one can expect domain walls. Indeed, the daughter inherits domain walls from its supersymmetric parent. Two parallel walls of the type $W_{\{k,k+1\}}$ and $W_{\{k+1,k+2\}}$ are at indefinite equilibrium. In this sense they are “BPS,” although the standard definition of “BPS-ness,” through central charges and supercharges, is certainly not applicable in the non-supersymmetric theory. In this paper we elaborate on physics of the domain walls in the orientifold theory. We will show that these walls carry charges similar to NS-NS and R-R charges. In addition, we will argue that an open-closed string channel duality holds for the analogous field theory annulus amplitude. Moreover, by exploiting the similarity between string theory and field theory we will provide a reason why the cosmological constant of the gauge theory is zero at order $N^2$ despite the fact that the hadronic spectrum of the theory contains only bosons.

Finally, in the third part we explain how the parent-daughter relationship (nonperturbative planar equivalence) between $\mathcal{N} = 1$ gluodynamics and its orientifold can be extended to include pairs of theories none of which is supersymmetric.

2 The Orientifold Field Theory

This “orientifold field theory” was suggested in Refs. [12, 13] in a somewhat different context. The field content of the orientifold gauge field theory differs from the one of its parent theory, $U(N)$ SUSY gluodynamics, in that
the gluinos are replaced by one massless Dirac fermions in the rank-two antisymmetric tensor representation of U(N) (denoted by $\mathbf{3} + \bar{\mathbf{3}}$). The total number of (say) left-handed fermions is thus $N(N-1)$ in the daughter theory and $N^2$ in the parent theory, which agrees to leading order in $1/N$. The realization of the orientifold field theory in string theory is as follows: this theory lives on a brane configuration of type 0A string theory [12] which consists of NS5 branes, D4 branes and an orientifold plane — hence the name “orientifold field theory.”

The massless open strings on the brane correspond to the ultraviolet (UV) degrees of freedom of the field theory: the gauge field and the antisymmetric fermion.

We will assume that our gauge theory has a string theory dual in the spirit of Ref. [14] (yet to be found, though). It is presumably of the type 0B on a curved background, similarly to the orientifold field theory analog of $\mathcal{N} = 4$ SYM which is type 0B on $AdS_5 \times RP^5$ [13]. In this picture the closed strings correspond to the infrared (IR) degrees of freedom, the glueballs and “quarkonia.” Indeed, the type 0 (closed) strings are purely bosonic, in agreement with our expectation from the confining orientifold field theory. Moreover, the bosonic IR spectrum of the gauge theory is even/odd parity degenerate, in accordance with degeneracies between the NS-NS and R-R towers of the type 0 string.

3 Parallel Domain Walls versus D Branes

As was mentioned, the gauge theory fundamental flux tubes can end on a BPS domain wall. Let us assume that for $\mathcal{N} = 1$ gluodynamics/orientifold theory the domain walls have a realization in terms of $Dp$ branes ($p > 1$) of the corresponding type IIB/0B string theory. Their world volume is 012 + ($p-2$) directions transverse to the four-dimensional space-time 0123. Specific AdS/CFT realizations of domain walls in $\mathcal{N} = 1$ theories are given in Refs. [5, 6, 7, 15], mostly in terms of wrapped D5 branes. We deliberately do not specify which particular branes are used to model the BPS walls, since we do not perform actual AdS/CFT calculations. D branes carry the NS-NS charge, as well as the R-R charge [16]. Moreover, interactions induced by these charges exactly cancel guaranteeing that the parallel D branes neither
attract nor repel each other.

Let us see how this is realized in $\mathcal{N} = 1$ SYM. We start from two parallel BPS walls $W_{\{k,k+1\}}$ and $W_{\{k+1,k+2\}}$. Each of them is BPS, with the tension \[ T_{\{k,k+1\}} = T_{\{k+1,k+2\}} = \frac{N}{8\pi^2} |\langle \text{tr} \lambda \lambda \rangle| 2 \sin \frac{\pi}{N}. \] (2)
The tension of the configuration $W_{\{k,k+2\}}$ is
\[ T_{\{k,k+2\}} = \frac{N}{8\pi^2} |\langle \text{tr} \lambda \lambda \rangle| 2 \sin \frac{2\pi}{N}. \] (3)
This means that at leading order in $N$ (i.e. $N^1$) two parallel walls $W_{\{k,k+1\}}$ and $W_{\{k+1,k+2\}}$ do not interact. There is no interaction at the level $N^0$ either. An attraction emerges at the level $N^{-1}$. That the inter-wall interaction potential is $O(N^{-1})$ can be shown on general kinematic grounds (Ref. [17] presents a detailed discussion, see Eq. (37); see also Ref. [18]. This and other aspects of dynamics of inter-wall separation will be considered in Ref. [19].

Our task is to understand dynamics of this phenomenon from the field theory side. Assume that two walls under consideration are separated by a distance $Z \gg m^{-1}$ where $m$ is the mass of the lightest composite meson. What is the origin of the force between these walls?

The interaction is due to the meson exchange in the bulk. Consider the lightest mesons, scalar and pseudoscalar. The scalar meson $\sigma$, the “dilaton,” is coupled to the trace of the energy-momentum tensor $\theta^\mu_\mu$,
\[ \frac{\sigma}{f} \theta^\mu_\mu = \frac{3N}{16\pi^2 f} \sigma \text{tr} F^2, \] (4)
(see e.g. Ref. [20]). Here $f$ is a coupling constant scaling as
\[ f \sim N \Lambda. \] (5)
Integrating over the transverse direction and using the fact that $\theta^\mu_\mu$ translates into mass, we find that the “dilaton”-wall coupling (per unit area) is
\[ T \frac{\sigma}{f}, \] (6)
where $T$ is the tension, see Eq. (2). The $\sigma$-wall coupling scales as $N^0$. 

The coupling of the pseudoscalar meson (the “axion” or “η′”; we will denote this field by η — it will have a realization in terms of the RR 0-form of type IIB/0B) to the wall is related to the change of the phase of the gluino condensate across the wall. It can be estimated as

\[ \frac{\eta}{f} \int dz \frac{N}{8\pi^2} \text{tr} F \tilde{F} \rightarrow \frac{\eta}{f} \int dz \frac{N}{8\pi^2} |\text{tr} \lambda \lambda| \frac{\partial \alpha}{\partial z}, \]  

where \( z \) is the coordinate transversal to the wall, and \( \alpha \) is the phase of the order parameter. At large \( N \) the absolute value of the order parameter stays intact across the wall, while \( \int dz (\partial \alpha/\partial z) = 2\pi/N \). Thus, the “axion”-wall coupling scales as \( N^0 \). Note that the sign of the coupling depends on whether we cross the wall from left to right or from right to left. This is why the exchange of the dilaton between two parallel BPS walls leads to the wall attraction while that of the axion leads to repulsion.

Since the “dilaton/axion” coupling to the wall \( \sim N^0 \), we get no force at the level \( N^1 \) for free. The underlying reason is that the BPS domain wall tension scales as \( N^2 \), not as \( N^0 \), as would be natural for solitons, but as \( N^1 \) — the D-brane type of behavior.

Moreover, we want to say that \( \sigma \) and \( \eta \) give contributions which are exactly equal in absolute values (at the level \( N^0 \)) but are opposite in signs. This requires the degeneracy of their masses — which is certainly the case in \( \mathcal{N} = 1 \) SYM — and their couplings to the walls (except for the relative sign). Taking the right-hand side of Eq. (7) literally we get the “axion”-wall coupling in the form,

\[ T \frac{\eta}{f} + O(1/N), \]  

i.e. the same as in Eq. (6).

At finite \( N \) the wall thickness which scales as [4] \((N\Lambda)^{-1}\) is finite too. We have \((2 + 1)\)-dimensional supersymmetry on the wall world volume, which places scalars and pseudoscalars into distinct (non-degenerate) supermultiplets [21]. At \( N = \infty \) the wall thickness vanishes and it is natural that in this limit the wall coupling involves the lowest component of \((3 + 1)\)-dimensional chiral superfield which has the form \( \sigma + i\eta \). This component is coupled to

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1Equation (7) guarantees, automatically, that the “axion”-wall coupling is saturated inside the wall. Outside the wall, in the vacuum, \( \alpha = \text{const.} \), while \( \langle \text{tr} F^2 \rangle = \langle \text{tr} \tilde{F} \tilde{F} \rangle = 0. \)
the $(1/2, 1/2)$ central charge $Z$ as $Z \exp(\sigma + i\eta) + \text{h.c.}$

Thus, the wall “R-R charge” is due to the axion-like nature of $\eta$. The “NS-NS charge” is due to the wall tension. Both charges are indeed equal and scale as $N^0 \Lambda^2$. This guarantees that at order $N^0$ there is no force. The $\sigma$ and $\eta$ coupling to the wall split at order $N^{-1}$.

Needless to say, the very same arguments can be repeated verbatim in the orientifold theories. In the limit $N \to \infty$ the degeneracy of the $\sigma$ and $\eta$ masses holds, and so does the degeneracy of the wall “NS-NS and R-R charges.” The reason why the couplings to the wall (associated with the trace of the energy-momentum tensor for the “dilaton” and the axial charge for the “axion”) are the same is that these charges and couplings are inherited from the parent supersymmetric theory at $N \to \infty$, see [8]. For what follows it will be useful to note that the $\sigma$ exchange alone (before cancellation) generates the interaction potential between the walls (per unit area)

$$V/A \sim N^0 \Lambda^3 e^{-mZ}.$$  \hspace{1cm} (9)

This scaling law is in agreement with the string theory expectations.

Now let us see how this picture is implemented in string theory. From the open string standpoint the result of the zero force is natural. This is the Casimir force between the walls. Since the UV degrees of freedom are bose-fermi degenerate the vacuum energy and, hence, the Casimir force is zero.

\textsuperscript{2}If the BPS wall in $\mathcal{N} = 1$ gluodynamics becomes an ordinary D-brane at $N = \infty$, then it must support a massless U(1) gauge field. The gauge field in 1+2 dimensions is equivalent to a (pseudo)scalar field with the $S_1$ target space [22]. The above argument suggests an answer to a question formulated in Ref. [21]. Namely, the U(1) gauge field on the world volume of the domain wall was shown to be described by the Lagrangian

$$\mathcal{L}_{1+2} = -\frac{1}{4e^2} F_{mn}F^{mn} + \frac{N}{16\pi} F_{mn} A_k \varepsilon^{mnk} + \text{ferm. terms.}$$

The Chern-Simons term makes the $A$ field massive, $m_A = N e^2/(4\pi)$, non-degenerate with the translational modulus. The scaling law of $m_A$ depends on that of $e^2$. We suggest that at $N \to \infty$ the degeneracy is restored, i.e. $e^2 \sim \Lambda N^{-2}$, so that $m_A \sim \Lambda N^{-1} \to 0$ at $N \to \infty$. 

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Figure 1: The annulus diagram for the orientifold field theory. The D branes are domain walls. Closed strings are bosonic glueballs and open strings are UV degrees of freedom.

In the gauge theory the closed strings are the glueballs of the field theory [23, 24]. Let us consider the large separation case, first. At the lowest level we have a massive scalar and a pseudoscalar (we assume a mass gap). These two exactly degenerate states correspond to the “dilaton” and “axion” (RR 0-form of type IIB/0B). They are expected to become massive when the theory is defined on curved background [23]. The type IIB/0B action contains the couplings

\[ e^{-\Phi} \text{tr} F^2 + C \text{tr} F \tilde{F}, \]  

where \( \Phi \) denotes the dilaton and \( C \) the R-R zero-form. In addition we have a coupling of the graviton and the R-R four-form

\[ \eta^{\mu \rho} h^{\nu \lambda} \text{tr} F_{\mu \nu} F_{\rho \lambda} + C^{\mu \nu \rho \lambda} \text{tr} F_{\mu \nu} F_{\rho \lambda}. \]  

In the gauge theory the “graviton” (tensor meson) and the four-form are heavier glueballs since they carry higher spins than the dilaton and the zero-form. Similarly, the whole tower of degenerate bosonic hadrons of the “orientifold field theory” should correspond to NS-NS and R-R fields of type II/0 string theory. This gives us a new picture of why the force between domain walls is zero in terms of the glueball exchanges: even-parity glueballs lead to an attractive force between the walls whereas odd-parity glueballs lead
to repulsion. The sum of the two is exactly zero at the leading $N^0$ order. The $1/N$ force between the walls is related to a possible non-vanishing force between parallel D branes in curved space at the order $O(g_s^2)$.

We can also exploit the above picture to estimate the potential between a wall and a anti-wall. This configuration is not BPS and a non-zero force is expected at the leading $N^0$ order. At large separations, the force is controlled by an exchange of the lowest massive closed strings. These are the dilaton and the 0-form. Now, their contributions add up. We get an attractive potential as indicated in Eq. (9).

4 Vanishing of the Vacuum Energy in $\mathcal{N} = 1$ SYM and Orientifold Theory

One of the surprising results of our previous work [8] is that the $N^2$ part of the vacuum energy density vanishes in the “orientifold field theory.” While this result makes sense from the UV point of view, where we have bose-fermi degeneracy, it looks rather “mysterious” from the IR point of view, since at the level of the composite color-singlet states we have only bosonic degrees of freedom. Indeed, since at large $N$ we have free bosons, it is legitimate to sum the bosonic contributions to the vacuum energy density as follows:

$$\sum_n \sum_k \frac{1}{2} \sqrt{k^2 + M_n^2},$$

where $M_n$ are the hadron masses. The paradox arise since the sum runs over positive contributions. How can positive contributions sum to zero?

Before we present our solution, we would like to make two comments. First, the sum (12) is not well defined since the expected Regge trajectory is not bounded from above and therefore a regularization is needed. Second, in the above sum (12) the $N$ dependence of each individual mode is $N^0$. The expected UV $N^2$ dependence of the vacuum energy is hidden in the sum over hadronic modes. Thus, though formally (12) represents the vacuum energy density of the theory, it is not the most efficient way to calculate it. Below, we present an alternative way of calculation of the vacuum energy density — which explains naturally the vanishing result.
Let us consider the contribution to the cosmological constant from the open string sector. At large $N$ it is dominated by the annulus diagram where each boundary consists of $N$ D branes and a summation over the various D branes is assumed. The Möbius and Klein-bottle as well as higher-genus amplitudes are suppressed at large $N$. It is not surprising that the cosmological constant vanishes, as we have $N^2$ bosons (NS open strings) and $N^2$ fermions (Ramond fermionic open strings).

The annulus diagram has another interpretation. It represents the force between the D branes. The force is mediated by bosonic closed strings. In a SUSY setup (the type II string), D branes are BPS objects — hence, the zero force. As has been discussed above, the balance, at large separation, is achieved in this case due to cancellation between the dilaton, the graviton and the massless R-R forms.

It is interesting that the force between parallel selfdual D branes is zero also in type-0 string theory [25, 13]

\[ A = N^2(V_8 - S_8) \equiv 0. \]  

This is due to the underlying SUSY on the world sheet. The mechanism is exactly as in the type II case: the R-R modes cancel the contributions of the NS-NS modes. Note that since we are interested only in the planar gauge theory, we can restrict ourselves to $g_{st} = 0$ on the string theory side. Therefore, higher-genus amplitudes are irrelevant for our discussion. At this level the relevant type-0 amplitudes, as well as the bosonic spectrum, are identical to the type II ones, in a not too surprising similarity with the situation in the large $N$ dual gauge theories (the type-0 string becomes, in a sense, supersymmetric at the tree level). In particular, the induced dilaton tadpole and cosmological constant are irrelevant, and, thus, the background inherited from the supersymmetric theory remains intact.

The vanishing of the annulus diagram leads to an explanation of the “mysterious” vanishing of the cosmological constant in the orientifold field theory: if one views the hadrons (in the spirit of the AdS/CFT) as closed strings, the degenerate bosonic spectrum is the reason behind the vanishing result for both wall-wall interaction and the cosmological constant.

We hasten to add that though the mechanism is similar, there is a difference between the two cases: the wall-wall interaction involves the force
between “D2” branes (wrapped D5 branes), whereas the vanishing cosmological constant involves the force between “D3” branes. The two sorts of branes are not necessarily the same — it depends on the specific realization. However, from the bulk point of view, the mechanism is identical. The only requirement is the degeneracy of the NS-NS and the R-R tower and their couplings to the branes.

The difference between string theory and field theory is that in string theory the force between D branes, from the closed string standpoint, is related to the contribution to the cosmological constant from the open string sector. However, closed strings and open strings are independent degrees of freedom, and so one has to add the contribution of the closed strings to the cosmological constant. In the gauge theory string picture, the closed strings are simply hadrons made out of the constituent open strings — the gluons and quarks. Therefore, the value of the cosmological constant can be determined by either ultraviolet (UV) or infrared (IR) degrees of freedom. It is the same quantity.

Below we will present a purely field-theoretic consideration which will, hopefully, make transparent the issue of the vanishing of the vacuum energy density $E$ (at level $N^2$) in the orientifold theory. Usually it is believed that one needs full supersymmetry to guarantee that $E = 0$. It turns out that a milder requirement — the degeneracy between scalar and pseudoscalar glueballs/mesons — does the same job. Of course, in supersymmetric theories this degeneracy is automatic. The orientifold field theory is the first example where it takes place (to leading order in $1/N$) without full supersymmetry.

To begin with, we will outline some general relations relevant to $E$ which are valid in any gauge theory with no mass scale other than the dynamically generated $\Lambda$. The vacuum energy density $E$ is defined through the trace of the energy-momentum tensor,

$$E = \frac{1}{4} \langle \theta^\mu \rangle = \frac{1}{4} \int DA D\Psi \theta^\mu \exp(iS),$$

$$\theta^\mu = -\frac{3N}{32\pi^2} F^2,$$  \hspace{1cm} (14)

where

$$F^2 \equiv F_{\mu\nu}^a F^{\mu\nu,a}, \quad F \tilde{F} \equiv F_{\mu\nu}^a \tilde{F}^{\mu\nu,a}. \hspace{1cm} (15)$$
The second line in Eq. (14) is exact in $\mathcal{N} = 1$ gluodynamics and is valid up to $1/N$ corrections in the orientifold theory.

Now, we use an old trick [26] to express the trace of the energy-momentum tensor in terms of a two-point function. The idea is to vary both sides of Eq. (14) with respect to $1/g^2$ invoking the fact that the only dimensional parameter of the theory, $\Lambda$, exponentially depends on $1/g^2$. In this way one obtains [26]

$$\mathcal{E} = -i \int d^4x \left\langle \text{vac} \left| T \left\{ \frac{1}{4} \theta^\mu(x), \frac{1}{4} \theta^\nu(0) \right\} \right| \text{vac} \right\rangle_{\text{conn}} \equiv -i \int d^4x \left\langle \frac{3N}{128\pi^2} F^2(x), \frac{3N}{128\pi^2} F^2(0) \right\rangle_{\text{conn}}. \quad (16)$$

The above expression (16) is formal — it is not well-defined in the ultraviolet. This is the same divergence that plagues any calculation of $\mathcal{E}$ (remember, Eq. (16) is general, it is not related to supersymmetry). In order to give sense to this relation one needs a particular regularization. For instance, one can always think of a non-SUSY theory as a SUSY theory with mass terms for unwanted superpartners (soft SUSY breaking).

In supersymmetric theories SUSY prompts us a natural regularization. Indeed, let us consider the two-point function of the lowest components of two chiral superfields $D^2W^2$,

$$\langle \text{tr} D^2W^2(x), \text{tr} D^2W^2(0) \rangle. \quad (17)$$

Supersymmetry Ward identity tells us that this two-point function vanishes identically. There are two remarkable facts encoded in Eq. (17). First, since $D^2W^2 \propto \left( F^2 + iF\tilde{F} \right)$, the vanishing of (17) is not due to the boson-fermion cancellation but, rather, due to the cancellation between even/odd parity mesons (glueballs).

Second, Eq. (17) generalizes Eq. (16), so that the expression for the vacuum energy density takes the form

$$\mathcal{E} = -i \left( \frac{3N}{128\pi^2} \right)^2 \int d^4x \left( \langle F^2(x), F^2(0) \rangle - \langle F\tilde{F}(x), F\tilde{F}(0) \rangle \right), \quad (18)$$
where the connected correlators are understood on the right-hand side. To see that Eq. (18) is a heir of Eq. (16), please, observe that

$$0 = -i \int d^4x \left\langle \frac{3N}{128\pi^2} F\tilde{F}(x), \frac{3N}{128\pi^2} F\tilde{F}(0) \right\rangle.$$  \hspace{1cm} (19)

The vanishing in Eq. (19) is due to the fact that $F\tilde{F}$ is proportional to the divergence of the axial current $a_\mu$ both in $\mathcal{N} = 1$ gluodynamics and in the orientifold theory.

It is absolutely clear that this regularization works perfectly in the orientifold theories (at level $N^2$). Indeed, the part of the two-point function that involves $F^2$ is saturated at $N \to \infty$ by the propagators of the glueballs with the even parity. Similarly, the part that involves $F\tilde{F}$ is saturated by the odd parity glueballs. We then get

$$E = \sum_{\text{even parity}} \frac{\lambda_n^2}{M_n^2} - \sum_{\text{odd parity}} \frac{\lambda_n^2}{M_n^2}, \quad \lambda_n^2 \sim N^2 \quad \text{for all} \ n, \hspace{1cm} (20)$$

where $\lambda_n$ are the couplings to $T^\mu_\mu$ and $\partial^\mu a_\mu$, respectively, and $M_n$ are the glueballs masses. Clearly, if the masses and the couplings of the glueballs are even/odd-parity degenerate, as is the case in $\mathcal{N} = 1$ gluodynamics and in the large-$N$ orientifold field theory, $E$ vanishes.

In summary, in the ultraviolet calculation the fermi-bose degeneracy was responsible for the vanishing of the cosmological constant both in supersymmetric gluodynamics and in orientifold theory (where the cancellation was at level $N^2$). In dealing with $E$ a certain regularization procedure is needed. In SUSY it is implicit. In passing from the UV language to the IR one, we make it explicit through Eq. (18). The range of the applicability of the latter is wider than just SUSY. It is perfectly applicable in the orientifold theories too.

The expression (20) is in a remarkable agreement with our string theory picture. It shows that only bosonic glueballs are involved and also that the even and odd parity glueballs contribute with the opposite signs.

Perhaps the most interesting lesson from this picture is that the cosmological constant can vanish even though only bosonic IR degrees of freedom are present in the given gauge theory (at least, to the leading order in $N$). In addition, the “correct” calculation of the cosmological constant in $\mathcal{N} = 1$ SYM,
using the IR degrees of freedom, involves a cancellation among the degenerate bosons and omission of the fermions!

5 Non-supersymmetric Parent-Daughter Pairs

A simple proliferation of the fermion fields in the form of “flavors” leads to non-supersymmetric parent-daughter pairs. This was first mentioned in Ref. [10] in the context of $\mathbb{Z}_2$ orbifolds. The planar equivalence here holds perturbatively [27, 28] but most likely fails at the nonperturbative level. If we use, as a starting point, $\mathcal{N} = 1$ gluodynamics and its orientifold, the planar equivalence has solid chances to hold nonperturbatively.

Non-supersymmetric planar-equivalent pairs were mentioned in passing in Ref. [8]. For instance, gauge theories with one and the same number of Dirac fermions either in the antisymmetric two-index or symmetric two-index representations are planar equivalent. Now we would like to discuss in more detail parent-daughter pairs which are obtained from $\mathcal{N} = 1$ gluodynamics and its orientifold by introducing fermion replicas. Thus, as previously, the parent and daughter theories share one and the same gauge group, $U(N)$, and one and the same gauge coupling. The two respective fermion sectors are:

(i) $k$ species of the Weyl fermions in the adjoint, to be denoted as $(\lambda^A)^i_j$;
and

(ii) $k$ species of the Dirac fermions $(\Psi^A)_{[ij]}$.

Here $i, j$ are (anti)fundamental indices running $i, j = 1, 2, ..., N$ while $A$ is the flavor index running $A = 1, 2, ..., k$. Note that $k \leq 5$. Otherwise we lose asymptotic freedom. Each Dirac fermion is equivalent to two Weyl fermions,

$$\Psi_{[ij]} \rightarrow \left\{ \eta_{[ij]}, \; \xi^{[ij]} \right\}.$$ 

Of course, since both theories are non-supersymmetric, predictive power is significantly reduced compared to the case of a SUSY parent. Still, one can benefit from the comparison of both theories, in particular, the Goldstone meson sectors. Let us start with the case (ii), $k$ species of $\Psi_{[ij]}$. Since the fermion fields are Dirac and in the complex representation of the gauge group,
the theory has the same (non-anomalous) chiral symmetry as QCD with \( k \) flavors, namely, \( \text{SU}(k)_L \times \text{SU}(k)_R \). Various arguments tell us \([29, 30, 31]\) that the pattern of the chiral symmetry breaking is the same as in QCD too, namely

\[
\text{SU}(k)_L \times \text{SU}(k)_R \to \text{SU}(k)_V .
\] (21)

The only distinction is that in QCD the constant \( f \) scales as \( \sqrt{N} \Lambda \) while in our case its scaling law is \( N \Lambda \). Moreover, the coefficient \( n \) in front of the Wess-Zumino-Novikov-Witten term \( \Gamma \) (see e.g. Ref. [32]) equals \( N \) in QCD and \((1/2)N(N-1)\) in the case at hand (see below).

All axial (non-anomalous) currents are spontaneously broken, giving rise to \( k^2 - 1 \) Goldstone mesons, “pions.” Some of them — those coupled to the axial currents that can be elevated from the daughter theory (ii) to the parent theory (i) — persist in the parent theory (i), where the fermion fields belong to the real representation, with the same coupling to the corresponding axial currents. This is because of the planar equivalence of two theories. (Remember, currents with the structure \( \bar{\xi}\xi - \bar{\eta}\eta \) cannot be elevated from (ii) to (i).)

It is not difficult to count the number of the axial currents that are elevated from (ii) to (i): there are \((1/2)k(k-1)\) off-diagonal currents of the type \( \bar{\xi}^A\xi^B + \bar{\eta}^A\eta^B (A \neq B) \) plus all \( k-1 \) diagonal axial currents of the type \( \bar{\xi}^A\xi^A + \bar{\eta}^A\eta^A \) (no summation over \( A \)). Altogether we get

\[
\frac{k(k+1)}{2} - 1
\]

Goldstone mesons. This obviously corresponds to the following pattern of the chiral symmetry breaking:

\[
\text{SU}(k) \to \text{SO}(k) ,
\] (22)

with the Goldstone mesons in the symmetric two-index representation of \( \text{SO}(k) \). The pattern of the chiral symmetry breaking for the quarks belonging to a real representation of the gauge group indicated in Eq. (22) was advocated many times in the literature \([29, 30, 31, 33]\), but no complete proof was ever given.

We conclude this section by a brief comment on the topological properties of the corresponding chiral Lagrangian and how they match the underlying gauge field theory expectations. The theory (i) is expected to be confining
and support flux tubes — fundamental color charges cannot be screened. On the other hand, we do not expect stable baryons with mass growing with $N$. Composite color-singlet states of gluon and $(\lambda^A)_i^j$ form baryons with $M \sim N^0$.

At the same time, the theory (ii) does not have baryons with $M \sim N^0$. Here the baryon masses grow with $N$. The theory is expected to be confining too, but two flux tubes (each attached to a color source in the fundamental representation) can be screened by $(\Psi^A)^{ij}$.

As was suggested in Refs. [32, 33], at large $N$ one can try to identify baryons with the Skyrmions supported by the corresponding chiral Lagrangians. Since

$$\pi_3\{\text{SU}(k)/\text{O}(k)\} = Z_4 \quad \text{at} \quad k = 3, \quad \pi_3\{\text{SU}(k)/\text{O}(k)\} = Z_2 \quad \text{at} \quad k \geq 4;$$

$$\pi_3\{\text{SU}(k)\} = Z \quad \text{at all} \quad k$$

both theories, (i) and (ii), yield Skyrmions with $M_{\text{Skyrme}} \sim N^2$, albeit the theory (ii) has a richer spectrum. The above scaling law, $M_{\text{Skyrme}} \sim N^2$, is due to the fact that $f$ scales as $N$ in the theories under consideration.

Skyrmion statistics is determined by the (quantized) factor in front of the Wess-Zumino-Novikov-Witten term,

$$(-1)^{N(N-1)/2}. \quad (24)$$

It has half-integer spin provided that $N(N-1)/2$ is odd, i.e. $N = 4p + 2$ or $N = 4p + 3$ where $p$ is an integer. In both cases one can construct, in the microscopic theory, interpolating baryon currents with an odd number of constituents scaling as $N$. Why then the Skyrmion mass scales as $N^2$? A possible explanation is as follows. For quarks in the the fundamental representation of SU($N$) the color wave function is antisymmetric, which allows them all to be in the $S$ wave in the coordinate space. With antisymmetric two-index spinor fields the color wave function is symmetric, which would require them to occupy orbits with angular momentum up to $\sim N$. Then the scaling law $M_{\text{Skyrme}} \sim N^2$ seems natural.

Since $\pi_2\{\text{SU}(k)\} = 0$ the chiral sector of the theory (ii) does not support flux tubes. Albeit disappointing, such a situation was anticipated by Witten [33] who noted that topology of the full space of the large $N$ theory need not coincide with topology of its Goldstone sector.
In light of this remark we can understand the complete failure of the Skyrmion description of the theory (i). In particular, since \( \pi_2 \{ \text{SU}(k)/\text{O}(k) \} = \mathbb{Z}_2 \) at \( k \geq 3 \) we get flux tubes in the chiral theory, while we do not expect them in the microscopic theory. Moreover, stable Skyrmions of the chiral sector should become unstable in the full theory.

6 Conclusions

In this work we tried to further develop a parallel between \( \mathcal{N} = 1 \) gluodynamics and its non-supersymmetric orientifold daughter on the one hand, and string/D brane paradigm, on the other. We discussed forces between two BPS domain walls in field theory terms and established contact with the string theory description. In the latter, there is a well-known cancellation between NS-NS and R-R interactions. In field theory terms this cancellation manifest itself as follows: the exchange of a composite dilaton coupled to the domain wall is canceled (at leading order) by that of a composite axion. The string-theory interpretation allows us to establish a one-to-one relation between vanishing force and vanishing cosmological constant. Thus, the question “who is responsible for the vanishing cosmological constant in non-supersymmetric orientifold field theory?” gets a rather unexpected answer — the degeneracy of the even-odd parity composite mesons.

In the last part of the paper we discuss “flavor proliferation” as a device allowing one to get planar equivalent pairs in which both theories in the parent-daughter pair are non-supersymmetric, starting from the original pair — \( \mathcal{N} = 1 \) gluodynamics and its orientifold daughter. Although we loose predictive power based on supersymmetry of the parent, some predictions survive. In particular, we compare Goldstone meson sectors, and obtain consequences for the patterns of the chiral symmetry breaking.

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