Static detection of equivalent mutants in real-time model-based mutation testing

An Empirical Evaluation

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Abstract
Model-based mutation testing has the potential to effectively drive test generation to reveal faults in software systems. However, it faces a typical efficiency issue since it could produce many mutants that are equivalent to the original system model, making it impossible to generate test cases from them. We consider this problem when model-based mutation testing is applied to real-time system product lines, represented as timed automata. We define novel, time-specific mutation operators and formulate the equivalent mutant problem in the frame of timed refinement relations. Further, we study in which cases a mutation yields an equivalent mutant. Our theoretical results provide guidance to system engineers, allowing them to eliminate mutations from which no test case can be produced. Our empirical evaluation, based on a proof-of-concept implementation and a set of benchmarks from the literature, confirms the validity of our theory and demonstrates that in general our approach can avoid the generation of a significant amount of the equivalent mutants.

Keywords Software product line · mutation-based testing · real-time system

1 Introduction
Testing a real-time system against safety-critical requirements is a difficult problem due to the time-sensitiveness of its behaviour. To help in this task, model-based testing methods...
automate the generation of test cases by using a formal model of the system (Utting et al. 2012). The model drives the generation of test cases according to different criteria, such as classical state or branch coverage (Masri and Zaraket 2016), or feature combination coverage in the specific context of software product lines (Lee et al. 2020). Testing a formal model rather than source code allows to detect, among others, misinterpretations of requirements or systemic issues arising from time-dependent interactions of the system with its environment. Such detections would be harder at source code level.

Mutation testing (Aichernig et al. 2015; Brillout et al. 2009) is a technique commonly used to evaluate the thoroughness of test cases or to support their generation (Andrews et al. 2006; Offutt 2011). It can be applied both to the implementation (source code) and to the specification (model). A set of mutation operators, simulating possible faults in the system, are applied to the model, obtaining a so-called mutant. Thus, given a set of mutants, the effectiveness of a set of test cases can be evaluated according to the number of mutants it detects (i.e., mutants that produce different output than the original system). Test cases generated from a mutant are capable to detect bugs mimicked by that mutation. The fundamental underlying assumption is the existence of a coupling effect, i.e. the fact that “simple faults are coupled to more complex faults in such a way that a test suite that detects simple faults is sensitive enough to likely detect complex faults as well” (Petrovic et al. 2021). It has been shown (Andrews et al. 2006) that mutation-based testing is more effective in finding real faults than other techniques (Offutt 2011; Baker and Habli 2013; Aichernig et al. 2013).

Scalability of this approach is of paramount importance, because a large number of mutations is required in order to build effective test cases. However, many of these mutations are useless because they generate mutants that have no behaviour that the original system had not. In such cases, no test case can be generated to differentiate the mutant from the original system, leading to useless analyses and waste of computational resources. Code-based mutation testing research has worked on methods to detect and avoid equivalent mutants that are semantically equivalent to the original program (Madeyski et al. 2014). In model-based mutation testing, this problem generalizes to that of detecting subsumed mutants, which have less (or equal) behaviour than the original system model.

One viable method is to organise the mutants as a product line of mutations, in the featured mutant model (Devroey et al. 2016). Such a product line enables the effective generation and validation of mutants against given test cases. However, an efficient featured mutant model should be built upon a set of effective mutations (i.e., those producing useful mutants), rather than from random mutations. This constitutes an important contribution to avoiding the equivalent/subsumed mutant problem.

In this paper, we tackle the problem of testing real-time systems effectively and efficiently. We adopt the model-based mutation testing approach for real-time systems presented in Larsen et al. (2017). We augment the set of existing mutations with a few mutation operators that affect the timing of the system behaviour (e.g., one such operator delays the execution of an action by the system), first introduced in Basile et al. (2020a). Then, we address the subsumed mutant problem: we formally prove the conditions under which mutations inherently (i.e., by construction) produce subsumed mutants. We achieve this on the basis of refinement relations, which can be used to show that a model (the system) subsumes another (the mutant). Our endeavour yields clear guidelines for real-time system engineers, which they can follow in order to reduce their testing effort by ignoring equivalent mutants.

This paper builds on results from Basile et al. (2020a); more precisely, we extend it in the following way. We prove novel auxiliary theoretical results concerning non-subsumed mutants (in Section 4.2), while we refer to Basile et al. (2020a) for proofs of earlier
theoretical results reported here for the sake of completeness. Moreover, we add a thorough empirical evaluation (in Section 5), considering more case studies retrieved from the literature, and extending the experiments to consider second-order mutations, for all case studies. Finally, the experiments protocol has been fully automatised (cf. Section 5.2) and the implementation of mutant generation and checking is open source and available online, thus allowing reproducibility of the experiments.

Summarising, our contributions are as follows.

1. We propose novel time-specific mutation operators for real-time models.
2. We study and formally prove under which conditions mutation operators, including the time-specific ones, yield equivalent (or subsumed) mutants, from which no test case can be generated, and we provide guidelines that can be used to prevent the generation of such mutants.
3. We study and formally prove under which conditions mutation operators, including the time-specific ones, yield non-equivalent (or non-subsumed) mutants. These results must also consider when a mutation produces a non-redundant mutant, and auxiliary results not presented before address this issue formally.
4. We formalise our theoretical results using product lines of mutants. We use the featured mutant model (Devroey et al. 2016) to model the variability of the different mutations that can be applied, using a feature-aware extension of timed graphs (the mathematical structure used to check refinement relations), in a similar way that other formalisms have been extended with variability (Cordy et al. 2012b; Cordy et al. 2012a; Classen et al. 2013; Ter Beek et al. 2016; Basile et al. 2020b; Ter Beek et al. 2020; Ter Beek et al. 2021).
5. We implement our approach in a proof-of-concept tool and validate the soundness and effectiveness of the guidelines, based on an industrial system from the automotive domain and several other case studies from the literature, for first-order as well as second-order mutants.
6. The experiments protocol is completely automatised based on software for (i) mutant generation, (ii) mutant checking with the provided tool, and (iii) automatic refinement checking using the off-the-shelf tools Uppaal TIGA (Behrmann et al. 2007) and Ecdar (David et al. 2010b), to validate the proposed approach empirically.

Outline In Section 2, we discuss related work, followed by background material on (featured) timed games and the featured mutant model in Section 3, where we also introduce the novel mutation operators. Our main contributions are presented in Section 4, where we classify mutation operators and present guidelines for selecting effective mutations, and in Section 5, where we report the results of an empirical evaluation of our guidelines for both first- and second-order mutants. In Section 6, finally, we conclude the paper and provide some ideas for future work. Due to their size, the results of the aforementioned empirical evaluation for second-order mutants are reported in Appendix A.

2 Related Work

This paper, as an extension of Basile et al. (2020a), mainly builds upon two recent results on mutation-based testing (Devroey et al. 2016; Larsen et al. 2017). Featured mutant models were introduced in Devroey et al. (2016) for efficiently validating test cases against different possible mutations. Indeed, a single execution on the generated featured transition
system (Classen et al. 2013) suffices to check all mutants at once. However, in contrast to our approach, no guidelines are provided on how to select the mutations to generate the featured mutant model, that is, the mutations are selected randomly.

While Devroey et al. (2016) studies the problem of checking given test cases, Larsen et al. (2017) considers the problem of generating valid test cases for real-time system models. Basically, a test case generated through mutation-based testing is guaranteed by construction to distinguish certain mutants from the system model.

Compared to Devroey et al. (2016), in Larsen et al. (2017) the mutants are not organised as a product line and thus have to be checked one by one to generate the test cases. Moreover, both approaches generate random mutations that may result ineffective for generating/validating the test cases. Our approach improves on this by providing clear guidelines that allow to establish upfront which mutations can safely be ignored since no test case can be produced from them.

In Luthmann et al. (2017) and Luthmann et al. (2019), an approach to the generation of non-subsumed mutants is proposed, using Configurable Parametric Timed Automata (CoPTA) models, which analyses constraints of the generated zone graph. We do not generate zone graphs but instead statically identify mutations to be discarded based on the fact that we know from our theoretical results that they will generate subsumed mutants.

Earlier, in Aichernig et al. (2013), mutation-based testing for timed automata was introduced, extending standard mutation operators presented in Fabbri et al. (1999) with new mutations tailored for timed automata. We use some of those mutations, but also some of the new ones we introduced in Basile et al. (2020a). Compared to Larsen et al. (2017), a $k$-bounded language inclusion test between the mutant and the system model is used rather than refinement checking with Ecdar.

In Aichernig et al. (2013), a Car Alarm System (CAS) model of Ford is used as case study for experiments and evaluation. In Basile et al. (2020a), we used the same case study; in this paper, we consider five further case studies from the literature (Hune et al. 2001; Feo-Arenis et al. 2014; Hoxha et al. 2015; André et al. 2019; Basile et al. 2020c). Similar to Larsen et al. (2017), the approach in Aichernig et al. (2013) comes without a procedure or guidelines for selecting effective mutations, and no product line is used either. In particular, 471 out of a total of 1099 generated mutants are tested and subsequently discarded, because they cannot be used for generating test cases. We present a technique that allows to avoid the generation of ineffective mutants.

Mutation-based test-case generation is also discussed in Aichernig et al. (2015), for the case of UML state machine diagrams. The technique for comparing the mutant with the system model is similar to the one in Aichernig et al. (2013), and the same CAS case study is used for experiments. Mutations are applied randomly and ineffective mutants (i.e., mutants subsumed by the system model) are discarded subsequent to their generation.

Finally, the survey in Jia and Harman (2011) points out that “one barrier to wider application of mutation testing centers on the problems associated with equivalent mutants”. Our paper is an effort in the direction of reducing the generation of ineffective mutants upfront, within the framework proposed by Larsen et al. (2017) and adopting the featured mutant model construction of Devroey et al. (2016).

3 Background

In this section, we provide some background needed for the sequel.
3.1 Timed Games

Timed games (TG) are transition systems which can remain in a certain state or location only a specific amount of time, can execute a transition only within a certain time interval, and distinguish between controllable and uncontrollable actions. TG are based on timed (game) automata (Alur and Dill 1994; Asarin et al. 1998) and form the underlying behavioral structure of featured timed game (automata) (Cordy et al. 2012b; Cordy et al. 2013).

In reactive systems, one usually distinguishes between uncontrollable and controllable actions, that are assigned to inputs and outputs, respectively, if the environment is uncontrollable and vice versa otherwise.

Time is represented by clocks whose values evolve continuously. Clocks can be regarded as chronometers: their value can be inspected and reset, but not modified arbitrarily. Conditions over clock values are called clock constraints.

Definition 1 (Clock constraints) A clock constraint over a set $C$ of clocks is formed according to the grammar $g ::= \top | n \sim c | g \land g$, with $n \in \mathbb{N}, c \in C$, and $\sim \in \{<, \leq, \geq, >\}$.

We denote by $CC(C)$ the set of clock constraints over $C$. In TG, a clock constraint can label either a state or a transition. In case it labels a state, the constraint is a location invariant, which defines the interval of time in which the system can be in the state. In case it labels a transition, it is a transition guard specifying the interval of time during which the system can execute the transition. Note that the domain of the numeric constants in clock constraints is limited to natural numbers. Without loss of generality, we could use real numbers. However, natural numbers facilitate the implementation of clock constraints by allowing efficient data structures.

Definition 2 (Timed games) Let $(Loc, Act, C, Trans, \ell_0, Inv, AP, L)$ be a timed game (TG) where

- $Loc$ is a finite set of locations;
- $Act$ is a finite set of actions, partitioned into controllable actions $Act^c$ and uncontrollable actions $Act^u$;
- $C$ is a finite set of clocks;
- $Trans \subseteq Loc \times CC(C) \times Act \times 2^C \times Loc$ is a transition relation;
- $\ell_0 \in Loc$ is the initial location;
- $Inv : Loc \rightarrow CC(C)$ is a total function associating locations with invariants;
- $AP$ is a set of atomic propositions; and
- $L : Loc \rightarrow 2^{AP}$ is a total function associating locations with atomic propositions satisfied in those locations.

For a transition $t = (\ell, g, \alpha, R, \ell')$, $\ell$ is the starting location, $g$ is the transition guard, $\alpha$ is the action triggering the transition, $R$ is the subset of clocks to reset, and $\ell'$ is the target location. We may also write $t$ as $\ell \xrightarrow{g, \alpha, R} \ell'$ and omit $g$ and/or $R$ when immaterial, and instead of $\{x\}$ for a reset of clock $x$, we may also write $x := 0$.

Example 1 In Fig. 1(left), a TG model of a soda vending machine is depicted. From its initial state $s_0$, the insertion of a euro coin ($\varepsilon$) results in the clock being (re)set to zero and a move to state $s_1$. This input action is modelled as a controllable transition (drawn as a solid
The vending machine can remain in this state for at most 5 time units but only within 2 time units it can deliver a soda bottle (\(\hat{0}\)), returning to its initial state. The latter action is modelled as an uncontrollable transition (drawn as dotted arc). Note that we may speak of (un)controllable transitions when their action labels are (un)controllable. A TG model of a tea vending machine is depicted in Fig. 1(right).

The semantics of a TG is commonly defined as an infinite transition system (TS) whose states consist of a location and a valuation of the clocks. The transitions can be categorised into two types. **Delay transitions** do not change the location of the system, but only represent the passing of time. They may occur only if the invariant of the current location is still satisfied after the delay modelled by the transition. **Discrete transitions** instead occur when the system moves from one location to another. They may occur only if the current clock values satisfy both the guard of the executed transition and the invariant of the target location. After the execution of such transitions, clock values can be reset.

**Definition 3** (TG semantics) We define the semantics of a timed game \(tg = (Loc, Act, C, Trans, \ell_0, Inv, AP, L)\) as the semantics of the TS \((Loc \times Val(C), Act \cup \mathbb{R}_{\geq 0}, Trans', (\ell_0, v_0), AP \cup CC(C), L')\), denoted by \(\llbracket tg \rrbracket_{TG}\), and such that \(Val(C)\) is the set of clock evaluations, i.e., the set of total functions \(v : C \rightarrow \mathbb{R}^+\) that assign a non-negative real value to every clock; \(v_0 = \{ v_0(c) = 0 \mid c \in C \}\); \(L'(\ell, v) = L(\ell) \cup \{ cc \in CC(C) \mid v \models cc \}\); and

\[
\llbracket tg \rrbracket_{TG} = \{ L(\ell_0), L(\ell_1), \ldots \in (2^{AP \cup CC(C)}) \mid \forall i \in \mathbb{N} \cdot \exists \alpha_i \in Act \cup \mathbb{R}_{\geq 0} \cdot ((\ell_i, v) \xrightarrow{\alpha_i} (\ell_{i+1}, v')) \}
\]

### 3.2 Featured Timed Games

Featured timed games (FTG) extend TG with variability in the same way that featured transition systems (FTS) (Classen et al. 2013) extend (labelled) transition systems (LTS). FTS concisely model the behaviour of all products of a product line in a single superimposed LTS through the annotation of transitions with feature expressions, i.e., conditions expressing their existence in products, based on a feature model.

We assume products to be represented by sets of Boolean features and a feature model to be defined as a pair \((F, P \subseteq 2^F)\), where \(F\) is a set of features and \(P\) is the set of valid products. The semantics of a feature model \(\phi\), denoted by \(\llbracket \phi \rrbracket_{FM}\), is then its set of valid products. It can be represented by either a propositional formula or by the usual feature diagram. Let \(\mathbb{B} = \{ \top, \bot \}\) denote the Boolean constants true (\(\top\)) and false (\(\bot\)), and let \(\mathbb{B}(F)\) denote the set of Boolean expressions over \(F\) (i.e., using features as propositional variables). The elements of \(\mathbb{B}(F)\) are also called *feature expressions*. Formally, a feature expression \(\chi\) is a total function \(\{ \top, \bot \}^{|F|} \rightarrow \{ \top, \bot \}\) that associates every combination of features with a truth value. A feature expression can be interpreted as a set of products \(\llbracket \chi \rrbracket \subseteq 2^F\) defined

### Fig. 1 TG models of a soda vending machine (left) and a tea vending machine (right)
as all products \( p \) for which the induced truth assignment (\( \top \) for \( f \in p \), \( \bot \) for \( f \notin p \), for features \( f \in F \)) validates \( \chi \). Feature expressions and clock constraints allow modelling the behaviour of real-time variable-intensive systems.

**Definition 4** (Featured timed games) Given a timed game \((\text{Loc}, \text{Act}, C, \text{Trans}, \text{Loc}_0, \text{Inv}, \text{AP}, L)\), the decuple \((\text{Loc}, \text{Act}, C, \text{Trans}, \text{Loc}_0, \text{Inv}, \text{AP}, L, \varphi, \gamma)\) is a featured timed game (FTG) where

- \( \varphi \) is a feature model over a finite set \( F \) of features; and
- \( \gamma : (\text{Trans} \cup (\text{Loc} \rightarrow \text{CC}(C))) \rightarrow 2^F \) is a total function associating feature expressions to transitions and invariants.

As for FTS, the function \( \gamma \) associates a feature expression \( \chi \) to some transition \( t = (\ell, g, \alpha, R, \ell') \) such that \( \gamma(t) = \chi \) encodes the set of products able to execute \( t \). We may also write \( t \) as \( \ell \xrightarrow{[\chi]g, \alpha, R} \ell' \) and omit \( g \) and/or \( R \) when immaterial. The function \( \gamma \) moreover associates a feature expression \( \chi \) to a location invariant \( \text{Inv}(\ell) = g \), for some \( \ell \in \text{Loc} \), such that \( \gamma(g) = \chi \), which we may also write as \([\chi]g\), encodes the set of products with the invariant \( g \) in location \( \ell \). Note that \( [\top] \) stands for a feature expression that is always satisfied (by any product).

**Example 2** In Fig. 2(left), an FTG \( ftg \) of a product line of vending machines is depicted. The feature model is \( s \lor t \), with features \( s \) for soda and \( t \) for tea. From the initial state \( s_0 \), the insertion of a euro coin (\( \epsilon \)), which is always possible (the feature expression is always true) and which results in the clock being (re)set to zero, leads to state \( s_1 \). This is a controllable (input) action. A vending machine can remain in this state for at most 5 time units. Vending machines with feature \( s \) can deliver a soda bottle (\( \downarrow \)) before 2 time units have passed. Vending machines with feature \( t \) can deliver a cup of tea (\( \triangledown \)) after at least 2 time units have passed (producing tea takes more time). Note that in the presence of both features, after precisely 2 time units have passed, a choice occurs. Both (output) actions are uncontrollable.

FTG model real-time behaviour of a product line. Moreover, from an FTG we can derive TG modelling behaviour of specific products. This is achieved by projection of an FTG onto

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**Fig. 2** FTG model of a product line of vending machines (left) and TG models of two of its products (right) reproduced from Fig. 1
a product \( p \) obtained in much the same way as an LTS is obtained from an FTS (Classen et al. 2013): all transitions and invariants unavailable in product \( p \) are removed.

**Definition 5** (FTG projections) The projection of an FTG \( \text{ftg} = (\text{Loc}, \text{Act}, C, \text{Trans}, \text{Loc}_0, \text{Inv}, \text{AP}, L, \varphi, \gamma) \) onto a valid product \( p \in [\varphi]_{FM} \) is the TG \( \text{ftg}|_p = (\text{Loc}, \text{Act}, C, \text{Trans}', \text{Loc}_0, \text{Inv}', \text{AP}, L) \) where

\[
\text{Trans}' = \{ t = (\ell, g, \alpha, R, \ell') | t \in \text{Trans} \land p \models \gamma(t) \}; \quad \text{and} \quad \text{Inv}'(\ell) = \text{Inv}(\ell)|_p, \ \forall \ell \in \text{Loc} \text{ and the projection of an invariant } g \text{ onto a product } p \text{ is recursively defined as}
\]

\[
g|_p = \begin{cases} (g_1)|_p \land (g_2)|_p & \text{if } g = g_1 \land g_2 \\ g' & \text{if } (g = [\chi]g') \land p \in [\chi] \\ \top & \text{if } (g = [\chi]g') \land p \not\in [\chi] \end{cases}
\]

**Example 3** In Fig. 2(right), products \( \text{ftg}|_{\{s\}} \) and \( \text{ftg}|_{\{t\}} \) of the FTG \( \text{ftg} \) are depicted. The TG \( \text{ftg}|_{\{s\}} \) in Fig. 2(bottom-right) is a model of the vending machine that can only deliver soda bottles, whereas the TG \( \text{ftg}|_{\{t\}} \) in Fig. 2(top-right) is a model of the vending machine that can only deliver tea. Product \( \text{ftg}|_{\{s,t\}} \) is not shown.

The semantics of an FTG model of a product line is defined as a function that associates every valid product with the semantics of its projection.

**Definition 6** (FTG semantics) The semantics of an FTG \( \text{ftg} = (\text{Loc}, \text{Act}, C, \text{Trans}, \text{Loc}_0, \text{Inv}, \text{AP}, L, \varphi, \gamma) \) is defined as the function \([\text{ftg}]_{FTG}\) such that

\[
\forall p \in [\varphi]_{FM} \cdot [\text{ftg}]_{FTG}(p) = [\text{ftg}|_p]_{TG}
\]

### 3.3 Featured Mutant Model

The idea underlying model-based mutation testing is to guide the test-case generation by mutants, which are typically obtained through random mutations of the original model. Organising the mutants as a product line of mutations, a family of variations of the system under test (SUT), coined the featured mutant model (FMM) in Devroey et al. (2016), enables the efficient generation, configuration, and execution of mutants. Each feature in the FMM corresponds to a single application of one mutant operator on the original model.

Like Devroey et al. (2016), we use a selection of the operators proposed by Fabbri et al. (1999), based on Chow (1978) and Weyuker et al. (1994), to generate mutants from a TS:

- **TMI** Transition MIssing operator removes a transition;
- **TAD** Transition ADd operator adds a transition between two states;
- **SMI** State MIssing operator removes a state (other than the initial state) and all its incoming/outgoing transitions.

Additionally, we introduce the following operators specific to timed models, which change the constant in clock constraints, which we recall to be either a transition guard or a location invariant:

- **CXL** Constant eXchange L operator increases the constant of a clock constraint;
- **CXS** Constant eXchange S operator decreases the constant of a clock constraint;
- **CCN** Clock Constraint Negation operator negates a clock constraint.
The CCN operator is inspired by the \( \mu_{ng} \) operator from Aichernig et al. (2013), where only clock constraints appearing as transition guards are negated.

Each operator can be used to generate mutants using either the enumerative approach or the FMM approach. In the enumerative approach, each mutation transforms an FTG model \( \text{ftg} \), representing the SUT behaviour, into a mutant \( \text{ftg}_m \).

**Example 4** The FTG in Fig. 3(right) has been obtained from the FTG in Fig. 3(left) by applying the mutation operators TMI, CXL, and CXS. The transition labelled with a soda bottle was removed (TMI). Moreover, constant 2 in the clock constraint that acts as transition guard was increased to 4 to model that producing a tea takes more time (CXL). Instead, constant 5 in the clock constraint that acts as location invariant was decreased to 4 to model that the vending machine takes less time to produce a drink (CXS). Thus, the transition from \( s_1 \) to \( s_0 \) that models the delivery of a cup of tea now occurs (instantaneously) precisely when \( x = 4 \). The feature model was not changed.

In the FMM approach, each mutation operator is added as a feature to the existing feature model. When considering first-order mutation (only one mutation can be applied to the original system), the features/mutations are mutually exclusive. For higher order mutations, disjunction is used instead.

**Example 5** Adding the TMI, CXL, and CXS operators to the FTG in Fig. 4(left), results in the FTG \( \text{ftg}_{fmm} \) depicted in Fig. 4(right) with feature model \( \varphi_{fmm} \) depicted in Fig. 5. We now explain this.

To begin with, the TMI operator removes the transition \( t_1 = s_1 \xrightarrow{[s]x \leq 2} s_0 \) of the base model in the following way:

1. The feature expression \( \neg tmi \) is added to the feature expression of \( t_1 \), resulting in transition \( s_1 \xrightarrow{[s \wedge \neg tmi]x \leq 2} s_0 \), meaning that this transition may be fired only if the \( tmi \) mutation is deactivated (and if \( s \) is true);
2. The feature \( tmi \) is added to the feature model \( \varphi_{fmm} \) representing the application of the mutation operator (cf. Figure 5).

Moreover, the CXL operator increases the constant 2 to 4 in the clock constraint that acts as guard on the transition \( t_2 = s_1 \xrightarrow{\lceil x \geq 2 \rceil} s_0 \) of the base model, in the following way:

\[
\begin{align*}
[s]x &\leq 2, \lceil \rceil \\
[s] &\leq 2, \lceil \rceil \\
\end{align*}
\]

\[
\begin{align*}
[t]x &\geq 2, \lceil \rceil \\
[\top] &\leq 5 \\
\end{align*}
\]

**Fig. 3** FTG (right) resulting from the application of mutation operators TMI, CXL, and CXS to the FTG (left) reproduced from Fig. 2(left)
The feature expression $\neg\text{cxl}$ is added to the feature expression of $t_2$, resulting in transition $s_1 \xrightarrow{[t \wedge \neg\text{cxl}]x \geq 2} s_0$, meaning that this transition may be fired only if the $\text{cxl}$ mutation is deactivated (and if $t$ is true);

2. The transition $s_1 \xrightarrow{[t \wedge \text{cxl}]x \geq 4} s_0$ is added, meaning that this transition with feature expression $t \wedge \text{cxl}$ and clock constraint $x \geq 4$ may be fired only if the $\text{cxl}$ mutation is activated (and if $t$ is true);

3. The feature $\text{cxl}$ is added to the feature model $\varphi_{fmm}$ representing the application of the mutation operator (cf. Figure 5).

Finally, the $\text{CXS}$ operator decreases the constant 5 to 4 in the featured clock constraint $[\top]x \leq 5$, which acts as invariant of the state $s_1$ of the base model, in the following way:

1. The feature expression $\neg\text{cxs}$ is added to the featured clock constraint of state $s_1$, meaning that the updated featured clock constraint $[\neg\text{cxs}]x \leq 5$ acts as invariant $x \leq 5$ of $s_1$ only if the $\text{cxs}$ mutation is deactivated;

2. The feature expression $\text{cxs}$ is added to the featured clock constraint of state $s_1$, meaning that the updated featured clock constraint $[\text{cxs}]x \leq 4$ acts as invariant $x \leq 4$ of $s_1$ only if the $\text{cxs}$ mutation is activated;

3. The feature $\text{cxs}$ is added to the feature model $\varphi_{fmm}$ representing the application of the mutation operator (cf. Figure 5).
Hence, mutation operators are added to the FMM under construction.

4 Classifying Mutations

Our main theoretical contribution is a classification of mutations to identify those that are effective (i.e., can be used to generate test cases). Our key idea from Basile et al. (2020a) is that, by construction, some mutations produce mutants that have the same (or a subset of the) behaviour of the SUT. Discarding them will speed-up the mutation testing process, as we would avoid fruitless attempts to generate test cases. Thus, we aim to characterise these mutations by formally proving under which conditions (i.e., mutation operator and the elements of the model to which it is applied) the produced mutant is subsumed by the SUT.

Recall that a test case generated from a mutant provides a sequence of inputs that makes the mutant behave differently than the SUT (in terms of accepted inputs, produced outputs, or execution time). Thus, the goal of the test case is to distinguish whether the system on which it is executed is the original one or the mutant. For a mutant to remain “live” (as it is named in the jargon), there must be no test case that can distinguish it from the SUT. This is equivalent to proving that the mutant is a refinement of the SUT (Larsen et al. 2017). Refinement checking is solved as a two-player timed game, where one player (playing the “whenever” transitions of the forthcoming Definition 7) wins if the mutant is not a refinement of the system (the mutant is killed) and the other player (playing the “then” transitions of Definition 7) wins if the mutant is a refinement (the mutant is alive). If the mutant is not a refinement, then the counterexample represents the test case that distinguishes the mutant from the SUT.

In what follows, we consider the mutation operators mentioned in Section 3.3 and state under which conditions their application results in a refinement of the original model. First-order mutations were shown to offer a higher fault-revealing ability (Papadakis and Malevris 2010). Our theoretical results hold for first-order and also higher order mutations. As such, when proving refinement relations, we consider the general case where mutations are applied to mutants of the SUT (either subsumed or not). Similarly, our work generalises to the case where the original model represents the behaviour of not only one system, but of a whole product line of systems. Thus, our theoretical developments are defined over FTG rather than single TG. To summarise, all results described hereafter apply to (1) any-order mutations and (2) families of systems.

4.1 Subsumed Mutants

To begin with, we formalise the notion of refinement between TG, adapted from David et al. (2010a) and Larsen et al. (2017). In Larsen et al. (2017), real-time systems are modelled as timed I/O automata, in which input actions are defined controllable and output actions are defined uncontrollable. The main idea is to perform a refinement check between the mutant and the system model, using Eedar (David et al. 2010b), which is a tool built on top of Uppaal TIGA (Behrmann et al. 2007) that implements the timed interface theory from David et al. (2010a). Basically, a refinement model (i.e., a live mutant) must be able to mimic all controllable transitions of the original system model, while the original model must be able to mimic all uncontrollable transitions of the refinement. In our case, controllable transitions correspond to inputs (since a live mutant must accept all inputs that the original system accepts), whereas uncontrollable transitions correspond to outputs and delays (since a live mutant should not exhibit any behaviour that does not belong to the system). Note that this
is the opposite of the standard notion of modal refinement, where the inputs are seen as sent by an uncontrolled environment (Larsen et al. 2007). In other words, here the viewpoint is switched to the environment (David et al. 2010a; Larsen et al. 2017).

**Definition 7 (Refinement)** A TG $tg_1 = (Loc_1, Act_1, C_1, Trans_1, \ell_01, Inv_1, AP_1, L_1)$ is a refinement of a TG $tg_2 = (Loc_2, Act_2, C_2, Trans_2, \ell_02, Inv_2, AP_2, L_2)$, denoted as $tg_1 \preceq tg_2$, if there exists a binary relation $R \subseteq (Loc_1, Val(C_1)) \times (Loc_2, Val(C_2))$ that contains $s = ((\ell_01, v_01), (\ell_02, v_02))$ and is such that for each pair of locations and clocks values $((\ell_1, v_1), (\ell_2, v_2)) \in R$, it holds:

- whenever $(\ell_2, v_2) \xrightarrow{\alpha} (\ell'_2, v_2)$ for some $\ell'_2$ and $\alpha \in Act_2$, then $(\ell_1, v_1) \xrightarrow{\alpha} (\ell'_1, v_1)$ for some $\ell'_1$, $\alpha \in Act_1$ and $((\ell_1, v_1), (\ell'_2, v_2)) \in R$
- whenever $(\ell_1, v_1) \xrightarrow{\alpha} (\ell'_1, v_1)$ for some $\ell'_1$ and $\alpha \in Act_1$, then $(\ell_2, v_2) \xrightarrow{\alpha} (\ell'_2, v_2)$ for some $\ell'_2$, $\alpha \in Act_2$ and $((\ell'_1, v_1), (\ell'_2, v_2)) \in R$
- whenever $(\ell_1, v_1) \xrightarrow{\delta} (\ell_1, v'_1)$ for some $v'$ and $\delta \in \mathbb{R}_{\geq 0}$, then $(\ell_2, v_2) \xrightarrow{\delta} (\ell_2, v'_2)$ for some $v'_2$ and $((\ell_1, v'_1), (\ell_2, v'_2)) \in R$

We now provide a definition of subsumed mutant, where $Op_{fmm}$ is the set of mutations. Basically, after applying an additional mutation the resulting mutant is a refinement of the former one on which the additional mutation was not applied.

**Definition 8 (Subsumed mutant)** Let $ftg$ be an FTG and let $\llbracket \phi \rrbracket$ be the set of mutants with $m, m' \in \llbracket \phi \rrbracket$. We say that $m$ differs from $m'$ by $op$ iff $m = m' \cup op$ for some $op \in Op_{fmm}$. Moreover, we say that $m$ is subsumed by $m'$ iff $ftg_{m'} \succeq ftg_{m'}$, and we say that it is non-subsumed otherwise.

**Example 6** Consider the FTG $ftg_{fmm}$ of Example 5, reproduced in Fig. 6(left), and its mutants $m_1 = \{s, tmi_1\}$ and $m'_1 = \{s\}$, depicted in Figs. 6(top-right) and 6(bottom-right), respectively, i.e., with $t_1 = s_1 \xrightarrow{\pi < 2, \Delta} s_0$.

In this case, $m_1$ differs from $m'_1$ by $tmi_1$. Moreover, let $tg_1 = ftg_{fmm|m_1}$ and $tg_2 = ftg_{fmm|m'_1}$. It holds that $tg_1 \preceq tg_2$, i.e., $tg_1$ is subsumed by $tg_2$. Indeed, for all values $v$ of $x$ in the interval $[0, 5]$, the three points of Definition 7 hold for $((s_1tg_1, v_{tg_1}), (s_1tg_2, v_{tg_2}))$, and there is no configuration $(s_1, v)$ with $v > 5$ because it would violate the invariant of $s_1$.

We only consider deterministic TG, as usual (Larsen et al. 2017; Aichernig et al. 2015; Aichernig et al. 2013). The following proposition identifies conditions under which a mutant is subsumed.

**Proposition 1** (Basile et al. 2020a) Let $ftg$ be an FTG, let $\llbracket \phi \rrbracket$ be the set of mutants with $m, m' \in \llbracket \phi \rrbracket$, and let $m$ differ from $m'$ by $op$. Then $m$ is a subsumed mutant of $m'$ iff $op$ has introduced either less uncontrollable or more controllable behaviour (or trivially if the behaviour is unchanged).

In the remainder of this section, we present several results for identifying mutations that generate subsumed mutants by construction. Proof (sketches) can be found in Basile et al. (2020a). We start with those operations that were proposed by Fabbri et al. (1999), followed by the novel ones introduced in this paper.
The TMI mutation is used to remove a transition from the system. The following lemma shows that removing an uncontrollable transition from a mutant, by construction the resulting mutant is subsumed by the original one.

**Lemma 1 (TMI Subsumed)**  Let \( ftg \) be an FTG and let \( \varphi \) be the set of mutants with \( m, m' \in \varphi \) and \( m = \{tmi_t\} \cup m' \) for some \( t \in Trans_{ftg,m'} \) with action in \( Act^c \). Then \( ftg\mid_m \) is subsumed by \( ftg\mid_{m'} \).

**Example 7** We illustrate the usefulness of this result. Recall that test-case generation is more effective if the number of subsumed mutants is minimised. Continuing the previous example, since \( t_1 \) is an uncontrollable transition, Lemma 1 implies that \( ftg_{fmm|m_1} \) is subsumed by \( ftg_{fmm|m'_1} \), i.e., this is not a good candidate mutation for the configuration \( m'_1 \).

The TAD mutation is used to add a transition to the system. The next lemma shows that by adding a controllable transition to a mutant, the obtained mutant is subsumed by the original one.

**Lemma 2 (TAD Subsumed)**  Let \( ftg \) be an FTG and let \( \varphi \) be the set of mutants with \( m, m' \in \varphi \) and \( m = \{tad_t\} \cup m' \) for some \( t \) with action in \( Act^c \). Then \( ftg\mid_m \) is subsumed by \( ftg\mid_{m'} \).

The state missing SMI mutation removes a location from the system (not the initial location however). This is equivalent to making the location unreachable, i.e., removing all its incoming transitions. Hence, the results on TMI can be applied. The following lemma shows when this mutation produces a subsumed mutant.

**Lemma 3 (SMI Subsumed)**  Let \( ftg \) be an FTG and let \( \varphi \) be the set of mutants with \( m, m' \in \varphi \) and \( m = \{smi_\ell\} \cup m' \) for some \( \ell \in Loc_{ftg,m'} \). Then \( ftg\mid_m \) is subsumed by \( ftg\mid_{m'} \) if there exists no transition \( t \) with target location \( \ell \) and action \( \alpha \in Act^c \).

We continue with the mutation operators that were firstly introduced in Basile et al. (2020a).

The CXL mutation increases the constant of a clock constraint. The next lemma shows when the mutation operator CXL applied on a transition produces a mutant that is subsumed.
Lemma 4 (CXL Subsumed Transitions) Let \( \mathcal{fg} \) be an FTG and let \( \left[ \varphi \right] \) be the set of mutants with \( m, m' \in \left[ \varphi \right] \) and \( m = \{ cxl_t \} \cup m' \) for some \( t \in \text{Trans}_\mathcal{ftg} \) with source \( \ell \) and either (i) action in \( \text{Act}^\ell \) and guard \( g = x \leq k \) or (ii) action in \( \text{Act}^\ell \), guard \( g == k \) and \( \text{Inv}(\ell) = x \leq k \) or (iii) action in \( \text{Act}^\ell \) and guard \( g = x \geq k \). Then \( \mathcal{ftg}|_m \) is subsumed by \( \mathcal{ftg}|_{m'} \).

Example 8 Recall from Example 5 the mutation operator \( \text{CXL} \) applied to the FTG \( \mathcal{ftg}_{fmm} \) that is reproduced in Fig. 7(left), and now consider its mutants \( m_1 = \{ t, cxl_{t_2}, cxss_1 \} \) and \( m'_1 = \{ t, cxss_1 \} \), depicted in Figs. 7(top-right) and 7(bottom-right), respectively, i.e., with \( t_2 = s_1 \overset{x \geq 2, \not\downarrow}{\rightarrow} s_0 \). Since \( t_2 \) is an uncontrollable transition, Lemma 4(iii) implies that \( \mathcal{ftg}_{fmm|m_1} \) is subsumed by \( \mathcal{ftg}_{fmm|m'_1} \), i.e., this is not a good candidate mutation for the configuration \( m'_1 \).

Finally, the next lemma identifies the conditions under which applying \( \text{CXL} \) on an invariant yields a subsumed mutant.

Lemma 5 (CXL Subsumed Invariants) Let \( \mathcal{fg} \) be an FTG and let \( \left[ \varphi \right] \) be the set of mutants with \( m, m' \in \left[ \varphi \right] \) and \( m = \{ cxl_\ell \} \cup m' \) for some location \( \ell \in \text{Loc}_\mathcal{ftg} \) with \( \text{Inv}(\ell) = x \geq k \) and for all valuations \( \nu \) of clock \( x \) such that \( k < \nu \leq k' \) for \( k' \) mutation, \( (\ell, \nu) \) can only be reached through a transition with action in \( \text{Act}^\ell \) and target \( \ell \). Then \( \mathcal{ftg}|_m \) is subsumed by \( \mathcal{ftg}|_{m'} \).

Example 9 Consider Fig. 8, assume that \( \mathcal{tg}_1 = \mathcal{ftg}|_m \) and \( \mathcal{tg}_2 = \mathcal{ftg}|_{m'} \), for some \( \mathcal{ftg} \) and \( m = \{ cxls_1 \} \cup m' \), where \( cxl \) increases by one unit the clock constraint of location \( s_1 \). By Lemma 5, it holds that \( \mathcal{tg}_1 \) is subsumed by \( \mathcal{tg}_2 \).

CXS mutation We now turn our attention to the mutation \( \text{CXS} \) that decreases the constant of a clock constraint. The next lemma shows that the mutation operator \( \text{CXS} \) applied to a guard of the form \( x \leq k \) of an uncontrollable transition or to a guard of the form \( x \geq k \) of a controllable transition produces a mutant that is subsumed.

![Fig. 7 FTG \( \mathcal{ftg}_{fmm} \) (left) reproduced from Fig. 4 (right) and its mutants \( \mathcal{ftg}_{fmm|m_1} \) (top-right) and \( \mathcal{ftg}_{fmm|m'_1} \) (bottom-right) ](image-url)
**Lemma 6** (CXS Subsumed Transitions) *Let $fg$ be an FTG and let $\mathcal{M}$ be the set of mutants with $m, m' \in \mathcal{M}$ and $m = \{cxs_t\} \cup m'$ for some $t \in Trans_{fg|_m}$ with either (i) action in $Act^u$ and guard $g = x \leq k$; or (ii) action in $Act^c$ and guard $g = x \geq k$. Then $fg|_m$ is subsumed by $fg|_{m'}$.***

Finally, the next lemma identifies the conditions under which the application of CXS on an invariant yields a subsumed mutant.

**Lemma 7** (CXS Subsumed Invariants) *Let $fg$ be an FTG and let $\mathcal{M}$ be the set of mutants with $m, m' \in \mathcal{M}$ and $m = \{cxs_{\ell}\} \cup m'$ for some location $\ell \in Loc_{fg|_{m'}}$ and either (i) $Inv(\ell) = x \leq k$ and for all valuations $v$ of clock $x$ such that $k' \leq v < k$ for $k'$ mutation, $(\ell, v)$ can only be reached through a transition with action in $Act^u$ and target $\ell$ or (ii) $Inv(\ell) = x \geq k$ and for all valuations $v$ of clock $x$ such that $k' \leq v < k$ for $k'$ mutation, $(\ell, v)$ can only be reached through a transition with action in $Act^c$ and target $\ell$. Then $fg|_m$ is subsumed by $fg|_{m'}$.***

**Example 10** Consider again Fig. 8, now assuming that $tg_3 = ftg|_{m'}$ and $tg_4 = ftg|_{m}$, for some $ftg$ and $m = \{cxs_{s_1}\} \cup m'$, where $cxs$ decreases by one unit the constant of the clock constraint of location $s_1$. By Lemma 7, it holds that $tg_3$ is subsumed by $tg_4$.

### 4.2 Auxiliary Results on Non-subsumed Mutants

Although our focus is on detecting subsumed mutants, the developed theory is also helpful in spotting when a specific mutation yields by construction a non-subsumed mutant. The following auxiliary results target non-subsumed mutants and complement the results stated so far (from Basile et al. 2020a). At this point, it is important to note that the experiments in Section 5 will only exploit results on subsumed mutants to discard ineffective mutations. However, the auxiliary results presented in this section could be exploited to perform a different evaluation: instead of discarding subsumed mutants, only generate (statically known) non-subsumed ones. This evaluation is harder, because only in specific cases it is possible to statically detect when a mutation is non-redundant. This is left for future work. A TG is
said to be non-redundant if every location \( \ell \) is reachable in at least one trace, it is not time-locked (i.e., delay is possible), and every transition is executable in at least one trace. For the non-subsumed lemmata, we will only consider non-redundant TG. Indeed, mutating a redundant element may produce a subsumed mutant. Note that redundant specifications are ill-defined and should be amended prior to any application, be it testing, model checking or any other.

We introduce the auxiliary definition of non-redundancy preserving mutation. This will be useful for the forthcoming results about non-redundant higher order mutations, whose hypothesis is that the mutated mutant is non-redundant.

Proof (sketches) of the results already reported earlier can be found in Basile et al. (2020a).

**Definition 9** (Non-redundancy preserving mutation) Let \( ftg \) be an FTG and let \([\varphi]\) be the set of mutants with \( m, m' \in [\varphi] \) and with \( m \) that differs from \( m' \) by the application of some mutation \( op \in Op_{fmm} \). Then, \( m \) is a non-redundancy preserving mutation of \( m' \) iff \( ftg|_{m'} \) is non-redundant implies \( ftg|_{m} \) is non-redundant.

Below follow two generic results on non-redundancy preserving on location or transition for any mutation. Note that the results below are operating at the syntactic level (i.e., statically). This means that the information on the specific values of clocks is missing. This information is only known at the semantics level (i.e., during the execution), where states are pairs of locations and clock evaluations. However, under specific hypothesis, it is possible to infer (statically) the values of clocks. Intuitively, if all clocks are reset when entering a location, the clocks evaluation when entering the location is statically known to be zero, and it is possible to predict whether guards and invariants will be satisfied. This allows to provide a result on non-redundancy preserving of a generic mutation involving a location that can be checked statically.

**Proposition 2** (Non-redundancy preserving mutation on location) Let \( ftg \) be an FTG and let \([\varphi]\) be the set of mutants with \( m, m' \in [\varphi] \) and with \( m \) that differs from \( m' \) by the application of some mutation \( op \in Op_{fmm} \). Then, \( m \) is a non-redundancy preserving mutation of \( m' \) iff \( ftg|_{m'} \) is non-redundant implies \( ftg|_{m} \) is non-redundant.

**Proof** (sketch) By assumption \( t' \) is non-redundant, so \( \ell \) is reachable through \( t' \). By assumption, when reaching \( \ell \) through \( t' \), \( Inv(\ell) \) is satisfied. Moreover, all guards of outgoing transitions of \( \ell \) at some point are satisfied by hypothesis. By the fact that all variables are reset in each transition, it holds that the behaviour of the underlying transition system is unchanged, thus \( ftg|_{m} \) is non-redundant. \( \square \)

We also provide a result on non-redundancy preserving of a generic mutation involving a transition that can be checked statically.

**Proposition 3** (Non-redundancy preserving mutation on transition) Let \( ftg \) be an FTG and let \([\varphi]\) be the set of mutants with \( m, m' \in [\varphi] \) and with \( m \) that differs from \( m' \) by the application of some mutation \( op \in Op_{fmm} \). Then, \( m \) is a non-redundancy preserving mutation of \( m' \) iff \( ftg|_{m'} \) is non-redundant implies \( ftg|_{m} \) is non-redundant.
Proof (sketch) By assumption $\ell$ is reachable, and at some point transition $t$ can be fired. By the fact that all variables are reset in $t$, it holds that the behaviour of the underlying transition system is unchanged, thus $\mathit{ftg}|_m$ is non-redundant.

**TMI mutation** In case the location target of the removed transition is target of another transition, the mutation is non redundancy-preserving. This result (and the others) builds on the hypothesis of non-redundancy.

**Proposition 4** (Non redundancy-preserving TMI mutation) Let $\mathit{ftg}$ be an FTG and let $[\mathit{\varphi}]$ be the set of mutants with $m, m' \in [\mathit{\varphi}]$ and with $m = \{tmi_t\} \cup m'$ for some $t \in \mathit{Trans}_{\mathit{ftg}|_m'}$ and let $t' \in \mathit{Trans}_{\mathit{ftg}|_m'}$ such that $t \neq t'$ and $t, t'$ have the same target location. Then, $\mathit{ftg}|_{m'}$ is non-redundant implies $\mathit{ftg}|_m$ is non-redundant.

Proof Since $\ell$ is reachable by a transition $t' \neq t$ and by the fact that the projection removes all syntactically redundant elements, it follows that $\ell$ and $t'$ are non-redundant and thus $\mathit{ftg}|_m$ is non-redundant.

The following lemma shows that the application of a mutation TMI on a transition $t$ ($tmi_t$ in the following) of a mutant $m'$, i.e., removing such a transition from $m'$, produces by construction a mutant $m$ that is non-subsumed by $m'$, in case $t$ is controllable.

**Lemma 8** (TMI Non-subsumed) Let $\mathit{ftg}$ be an FTG and let $[\mathit{\varphi}]$ be the set of mutants with $m, m' \in [\mathit{\varphi}]$ and with $m = \{tmi_t\} \cup m'$ for some $t \in \mathit{Trans}_{\mathit{ftg}|_m'}$ with action in $\mathit{Act}^e$. Then $\mathit{ftg}|_m$ is non-subsumed by $\mathit{ftg}|_{m'}$.

**Example 11** We illustrate the usefulness of this result by means of an example. Consider the FTG $\mathit{ftg}^e_{fmm}$, depicted in Fig. 9(left), and its mutants $m_2 = \{tmi_{t_2}\}$ and $m_2' = \emptyset$, depicted in Figs. 9(middle) and 9(right), respectively, i.e., $t_2 = s_0 \xrightarrow{e, x:=0} s_1$. Since $t_2$ is a controllable transition, Lemma 8 implies that $\mathit{ftg}^e_{fmm|m_2}$ is non-subsumed by $\mathit{ftg}^e_{fmm|m_2'}$, i.e., this is a good candidate mutation for the configuration $m_2'$.

**TAD mutation** The non-redundancy condition for transition adding mutation is trivial.

**Proposition 5** (Non redundancy-preserving TAD mutation) Let $\mathit{ftg}$ be an FTG and let $[\mathit{\varphi}]$ be the set of mutants with $m, m' \in [\mathit{\varphi}]$ and with $m = \{tad_t\} \cup m'$ for some $t$ non-redundant in $\mathit{ftg}|_m$ Then, $\mathit{ftg}|_{m'}$ is non-redundant implies $\mathit{ftg}|_m$ is non-redundant.
We note that a non-redundant transition to add could simply have a guard trivially true and an empty set of reset clocks (this will not be the case for the experiments in Section 5).

Under the (assumed) hypothesis that the added transition is executable in at least one trace, such a mutation produces a non-subsumed mutant if an uncontrollable transition is added and a subsumed mutant if a controllable transition is added.

The next lemma shows that the application of a mutation TAD on a transition \( t \) (\( tad_t \) in the following) of a mutant \( m' \), i.e., adding such a transition to \( m' \), produces by construction a mutant \( m \) that is non-subsumed by \( m' \), in case \( t \) is uncontrollable. Note that the added transition is non-redundant in the mutant.

**Lemma 9** (TAD Non-subsumed) Let \( ftg \) be an FTG and let \( \{\varphi\} \) be the set of mutants with \( m, m' \in \{\varphi\} \) and \( m = \{tad_t\} \cup m' \) for some \( t \) with action in Act. Then \( ftg|_m \) is non-subsumed by \( ftg|_{m'} \).

SMI mutation The next lemma is about non-redundancy preserving state-missing mutation. Basically, the hypothesis of this lemma checks that no redundant locations are created by the mutation.

**Proposition 6** (Non redundancy-preserving SMI mutation) Let \( ftg \) be an FTG and let \( \{\varphi\} \) be the set of mutants with \( m, m' \in \{\varphi\} \) and with \( m = \{smi_\ell\} \cup m' \) for some \( \ell \in Loc_{ftg|_{m'}} \) where \( \forall \ell' \neq \ell \) such that \( \ell' \) is target of \( t \) and \( \ell \) is source of \( t \), \( \exists \ell' \in Trans_{ftg|_{m'}} \) with target \( \ell' \) and source \( \ell'' \neq \ell, \ell' \). Then, \( ftg|_{m'} \) is non-redundant implies \( ftg|_{m} \) is non-redundant.

**Proof** The condition \( \forall \ell' \neq \ell \) such that \( \ell' \) is target of \( t \) and \( \ell \) is source of \( t \), \( \exists \ell' \in Trans_{ftg|_{m'}} \) with target \( \ell' \) and source \( \ell'' \neq \ell, \ell' \) ensures that the removal of such transitions \( t \) do not cause any redundant location, using the same argument used in Proposition 4.

CXL mutation The condition for non-redundancy preserving CXL mutation is based on a relaxation of the clock constraint.

**Proposition 7** (Non redundancy-preserving CXL mutation) Let \( ftg \) be an FTG and let \( \{\varphi\} \) be the set of mutants with \( m, m' \in \{\varphi\} \) and with \( m = \{cxl_x\} \cup m' \) for some \( x \in Trans_{ftg|_{m'}} \cup Loc_{ftg|_{m'}} \) with clock constraint \( x \leq k \). Then, \( ftg|_{m'} \) is non-redundant implies \( ftg|_{m} \) is non-redundant.

**Proof** This mutation introduces a relaxation of the clock constraint of transition \( t \). As such, the previous behaviour is still available and by the hypothesis of non-redundancy of \( ftg|_{m'} \) it follows that \( ftg|_{m} \) is non-redundant.

Let \( c \) be a clock and let \( k \) be some constant. Then such a mutation does not generate a subsumed mutant when applied to a guard of the form \( x \leq k \) of an uncontrollable transition or to a guard of the form \( x \geq k \) of a controllable transition, under conditions discussed in the next lemma.

**Lemma 10** (CXL Non-subsumed Transitions) Let \( ftg \) be an FTG and let \( \{\varphi\} \) be the set of mutants with \( m, m' \in \{\varphi\} \) and \( m = \{cxl_x\} \cup m' \) for some \( x \in Trans_{ftg|_{m'}} \cup Loc_{ftg|_{m'}} \) with clock constraint \( x \leq k \). Then, \( ftg|_{m'} \) is non-redundant.
either (i) action $\alpha \in \text{Act}^\mu$, guard $g = x \leq k$ and there exists a valuation of clock $k < v \leq k'$ for $k'$ mutation such that $(\ell, v) \xrightarrow{\alpha} f_{\text{ftg}|m}$ or (ii) action $\alpha \in \text{Act}^\mu$, guard $g = x \geq k$ and there exists a valuation of clock $k \leq v < k'$ for $k'$ mutation such that $(\ell, v) \xrightarrow{\alpha} f_{\text{ftg}|m'}$. Then $f_{\text{ftg}|m}$ is non-subsumed by $f_{\text{ftg}|m'}$.

The next lemma provides the conditions under which the application of the mutation operator $\text{CXL}$ on an invariant of a location $\ell$ (written as $\text{cxl}_\ell$), yields a non-subsumed mutant.

**Lemma 11** (CXL Non-subsumed Invariants) Let $f_{\text{ftg}}$ be an FTG and let $\llbracket \varphi \rrbracket$ be the set of mutants with $m, m' \in \llbracket \varphi \rrbracket$ and $m = \{\text{cxl}_\ell\} \cup m'$ for some location $\ell \in \text{Loc}_{\text{ftg}|m'}$ and either (i) $\text{Inv}(\ell) = x \geq k$ and there exists a valuation $v$ of clock $x$ such that $k \leq v < k'$ for $k'$ mutation such that $(\ell, v) \xrightarrow{\alpha} f_{\text{ftg}|m}$ or (ii) $\text{Inv}(\ell) = x \leq k$. Then $f_{\text{ftg}|m}$ is non-subsumed by $f_{\text{ftg}|m'}$.

**CXS mutation** Also the condition for non-redundancy preserving CXS mutation is based on a relaxation of the clock constraint.

**Proposition 8** (Non redundancy-preserving CXS mutation) Let $f_{\text{ftg}}$ be an FTG and let $\llbracket \varphi \rrbracket$ be the set of mutants with $m, m' \in \llbracket \varphi \rrbracket$ and $m = \{\text{cxs}_x\} \cup m'$ for some $x \in \text{Trans}_{f_{\text{ftg}|m'}} \cup \text{Loc}_{f_{\text{ftg}|m'}}$ with clock constraint $x \geq k$. Then, $f_{\text{ftg}|m'}$ is non-redundant implies $f_{\text{ftg}|m}$ is non-redundant.

*Proof* This mutation introduces a relaxation of the clock constraint. Thus, the previous behaviour is still available and by the hypothesis of non-redundancy of $f_{\text{ftg}|m'}$ it follows that $f_{\text{ftg}|m}$ is non-redundant. \qed

Again, let $c$ be a clock and let $k$ be some constant. Then such a mutation produces a non-subsumed mutant when applied to a guard of the form $x \leq k$ of a controllable transition or to a guard of the form $x \geq k$ of an uncontrollable transition, as the next lemma shows.

**Lemma 12** (CXS Non-subsumed Transitions) Let $f_{\text{ftg}}$ be an FTG and let $\llbracket \varphi \rrbracket$ be the set of mutants with $m, m' \in \llbracket \varphi \rrbracket$ and $m = \{\text{cxs}_x\} \cup m'$ for some $x \in \text{Trans}_{f_{\text{ftg}|m'}} \cup \text{Loc}_{f_{\text{ftg}|m'}}$ with source $\ell$ and either (i) action in $\text{Act}^\mu$, $g = x \leq k$ and there exists a clock valuation $k' < v \leq k$ with $k'$ mutation such that $(\ell, v) \xrightarrow{\alpha} f_{\text{ftg}|m'}$ or (ii) action in $\text{Act}^\mu$, $g = x \geq k$ and there exists a clock valuation $k' \leq v < k$ with $k'$ mutation such that $(\ell, v) \xrightarrow{\alpha} f_{\text{ftg}|m'}$. Then $f_{\text{ftg}|m}$ is non-subsumed by $f_{\text{ftg}|m'}$.

The next lemma provides the conditions under which the application of the mutation operator $\text{CXS}$ on an invariant of a location $\ell$ (written as $\text{cxs}_\ell$) produces a non-subsumed mutant.

**Lemma 13** (CXS Non-subsumed Invariants) Let $f_{\text{ftg}}$ be an FTG and let $\llbracket \varphi \rrbracket$ be the set of mutants with $m, m' \in \llbracket \varphi \rrbracket$ and $m = \{\text{cxs}_\ell\} \cup m'$ for some location $\ell \in \text{Loc}_{f_{\text{ftg}|m'}}$ and either (i) $\text{Inv}(\ell) = x \geq k$ and there exists a valuation $v$ of clock $x$ such that $k' \leq v < k$ for $k'$ mutation such that $(\ell, v) \xrightarrow{\alpha} f_{\text{ftg}|m}$ or (ii) $\text{Inv}(\ell) = x \leq k$ and there exists a valuation $v$ of clock $x$ such that $k' \leq v < k$ for
k’ mutation such that (ℓ, v) is reached through a transition with action in Act and target ℓ. Then \( ftg_{m} \) is non-subsumed by \( ftg_{m'} \).

**Example 12** Recall from Example 5 the mutation operator \( \text{CXS} \) applied to the FTG \( ftgf_{\text{fmm}} \) reproduced in Fig. 10(left), and consider its mutants \( m_1 = \{t, \text{cxs}, s_1\} \) and \( m'_1 = \{t\} \), depicted in Figs. 10(top-right) and 10(bottom-right), respectively. Since the clock constraint \( x \leq 5 \) acting as an invariant of \( s_1 \) is reached through the controllable transition \( s_0 \xrightarrow{\epsilon, x:=0} s_1 \), Lemma 13(ii) implies that \( ftgf_{\text{fmm}|m_1} \) is non-subsumed by \( ftgf_{\text{fmm}|m'_1} \), i.e., this is a good candidate mutation for the configuration \( m'_1 \).

**CCN Mutation** We turn our attention to the CCN operator that negates a clock constraint of a transition. For this mutation, no non-redundancy preserving properties have been identified. Indeed, the negation of a clock constraint requires to explore the zone graph to check if the corresponding transition is non-redundant, as well as other locations and transitions only reachable by that transition. For all non-redundant TG, this mutation always generates a non-subsumed mutant.

**Lemma 14 (CCN Non-subsumed)** Let \( ftg \) be an FTG and let \( \varphi \) be the set of mutants with \( m, m' \in \varphi \) and \( m = \{\text{ccn}_t\} \cup m' \) for some \( t \in \text{Trans}^{ftg}_{\text{fmm}} \). Then \( ftg_{m} \) is non-subsumed by \( ftg_{m'} \).

### 4.3 Classifying Mutations

The following main theorem sums up all the results presented in this section. The specific additional conditions that need to hold for each mutation operator can be found in the corresponding lemmata.

**Theorem 1 (Classifying mutations)** Let \( ftg \) be an FTG and let \( \varphi \) be the set of mutants with \( m, m' \in \varphi \) and with \( m \) that differs from \( m' \) by the application of some mutation \( op \in \text{Op}_{\text{fmm}} \). Table 1 summarises when \( ftg_{m} \) is (non-)subsumed by \( ftg_{m'} \) based on the applied mutation.

![Fig. 10](image-url) FTG \( ftgf_{\text{fmm}} \) (left) reproduced from Fig. 4 (right) and its mutants \( ftgf_{\text{fmm}|m_1} \) (top-right) and \( ftgf_{\text{fmm}|m'_1} \) (bottom-right) reproduced from Fig. 2(top-right)
Table 1 Subsumption relation for mutation operators, where ✗ stands for subsumed and ✓ for non-subsumed, while their subscripts x refer to the corresponding Lemmata x for the specific additional conditions that must hold for each mutation operator.

| subject                  | operator                  | TMI | TAD | SMI | CXL | CXS | CCN |
|--------------------------|---------------------------|-----|-----|-----|-----|-----|-----|
| controllable transition  | ✓₈                         | ✓₂  |     | ✓₄  | ✓₁₄ | ✓₁₄ |     |
| + clock constraint ≤     | ✓₄                         |     | ✓₁₃ | ✓₁₄ |     |     |     |
| + clock constraint ≥     | ✓₁₀                        |     |     | ✓₆  | ✓₁₄ |     |     |
| uncontrollable transition| ✓₁                         | ✓₉  |     |     |     |     |     |
| + clock constraint ≤     | ✓₁₀                        |     |     | ✓₆  | ✓₁₄ |     |     |
| + clock constraint ==    | ✓₄                         |     |     | ✓₄  | ✓₁₄ |     |     |
| + clock constraint ≥     | ✓₄                         |     |     | ✓₁₃ |     |     |     |

Proof The proof is obtained by cases, applying the lemmata discussed so far.

4.4 Generating Effective Mutations

Based on our results for detecting subsumed mutants (i.e., Lemmata 1–7), we provide guidelines for generating effective FMM and their corresponding FTG. The FTG in Fig. 3 (right) is an example of a “bad” model. This is because two out of three mutants of the model are subsumed, and a subsumed mutant cannot be used to generate effective test cases (Larsen et al. 2017). Hence, while building the FMM and the corresponding FTG (cf. Section 3.3), ideally one wants to minimise the number of subsumed mutants, thus maximising the effectiveness of the test-case generation phase. To do so, we select from Table 1 those results for detecting subsumed mutants that are applicable by only checking the syntax of the original model, rather than those based on the semantics.

Note that in Basile et al. (2020a), we reported two more commandments. The seventh commandment in Basile et al. (2020a) was distilled from the current Lemma 11, and it is a condition ensuring that the generated mutant is non-subsumed. As stated in Section 4.2, Lemma 11 also requires the original model to be non-redundant. This was the first commandment in (Basile et al. 2020a), in fact a pre-requisite on the used models. Since our focus is on subsumption detection and to improve the separation of concerns, all results about non-subsumed detection have now been rendered as auxiliary and are thus no longer part of the guidelines (and neither is the non-redundant requirement).
Finally, we note that such guidelines could be implemented directly as constraints in the feature model of the FMM (cf. Figure 5), such that subsumed mutants are prevented from being generated. In the next section, we provide an empirical evaluation of the results presented in this section, i.e., certain subsets of mutations are guaranteed to produce mutants that are a refinement of the SUT and thus there is no need to use them.

5 Evaluation

To further validate our theoretical results and their benefits, we conduct a completely automatised empirical evaluation based on a proof-of-concept tool we developed, extending the preliminary experiments performed in Basile et al. (2020a).

5.1 Research Questions and Methodology

The objective of our empirical evaluation is to identify the mutation operators and the conditions under which a non-effective (i.e., subsumed) mutant is generated. We already addressed this by formally proving that mutants resulting from specific operators are subsumed under specific conditions, as reported in Theorem 1 and Table 1. To raise confidence in our results, we confront our theory with a practical implementation. Thus, we ask:

RQ1: Are our guidelines sound, i.e., are all mutants rejected by the guidelines indeed subsumed mutants?

Our next question concerns the benefits of avoiding the generation of mutants that are subsumed by construction. In practice, the saved computation time is dependent on the concrete test-case generation and execution platform. Instead, we measure these benefits in
a relative way, as the percentage of subsumed mutants that our guidelines can detect. Thus, we ask:

**RQ2:** How complete are our guidelines in detecting subsumed mutants?

To answer these questions, we apply the mutation operators to a given original model to produce first-order mutants. To produce second-order mutants we apply second mutations on first-order mutants. Then, we check whether those mutants violate guidelines and whether they are subsumed by the original system model, using the refinement check implemented in the Uppaal or Ecdar tools (David et al. 2010b).

### 5.2 Implementation

A proof-of-concept software has been implemented to perform the evaluation. The development of this software has been made necessary due to the high number of mutants to check for each case study. Indeed, in Basile et al. (2020a) the experiments were only considering first-order mutations of a single case study and were only partially automatised, requiring manual intervention to collect the data, parse the logs, and to produce the output tables. As a side-product, the software can be used as a tool to automatically check the guidelines on given mutants of Uppaal/Ecdar models, as well as for mutant generation with the mutation operators discussed in Section 3.3.

It is important to state at this point that the organisation of TG into the framework of FTG and FMM has been used to obtain a clean formalisation of the theoretical results discussed in Section 4. However, the guidelines for detecting subsumed mutants are independent from the way in which mutants are generated, i.e., they are not constrained to use FMM or FTG.

The experiments protocol is modular and has been divided into two main activities, depicted in Fig. 11, which led to the (parallel) production of two applications called AppMutant (developed by the third, fourth, and first authors) and AppEcdar (developed by the first author).

The experiment protocol is organised into a control part and an experimental part. The control part uses the run-time refinement checking offered by Uppaal (originally introduced...
in Ecdar) to check, for each mutant, whether or not it is a refinement. The experimental part statically analyses the mutants to check if they are violating the guidelines presented in this paper.

We note that whilst the theory presented in Section 4 only distinguishes between subsumed and non-subsumed mutants, the refinement checking performed by Uppaal may also return inconsistent as output. An inconsistent model is such that it cannot be refined to one satisfying two additional requirements, called output urgency and independent progress (cf. David et al. 2015, Def. 4). Output urgency states that an automaton cannot delay an enabled output. Independent progress is not satisfied by an automaton if it cannot internally (i.e., by only delaying or firing uncontrollable (output) actions) prevent an invariant from being violated in a location, against all possible inputs that may or may not arrive from the environment.

When answering a refinement query $S_m \leq S$, Uppaal firstly refines $S_m$ to be consistent. This means that the uncontrollable behaviour of $S_m$ violating the requirements of consistency is pruned before performing the refinement checking (after pruning, the model can still be inconsistent because the controllable (input) behaviour is not pruned). Since, as for a subsumed mutant, an inconsistent model is not usable, we consider both subsumed and inconsistent mutants as “bad” mutants. More specifically, an inconsistent model is subsumed by any other model (similarly to how from false premises any conclusion can be reached), thus an inconsistent model is also a subsumed model.

As we will see in Section 5.4, this additional pruning operation of Uppaal that introduces further redundant mutated behaviour may cause subsumed or inconsistent mutants to be undetected by the commandments (for which this behaviour is not considered redundant).

Initially, the original models have been collected with only few adjustments (cf. Section 5.3) to make them consistent and usable by Uppaal. Indeed, we assume the competent programmer hypothesis (DeMillo et al. 1978) (i.e., developers produce initial models close to being correct). Thus, we assume that developers make only small mistakes (e.g., we will only mutate the clock constants by one time unit, see below). These simple mistakes (simulated by small mutations) can be put in cascade or coupled to form other emergent faults using higher-order mutants, according to the coupling effect (DeMillo et al. 1978).

The first activity is the generation of mutants which uses these original models to generate their mutants, in particular first-order and second-order (i.e., two mutations are applied) mutants. Each mutant operator is systematically applied to each relevant element (location or transition) of each model. The second-order mutants generation can be performed exhaustively or on a sample of each mutation ($\leq 10\%$, see below). Sampling can be useful when the number of second-order mutants is huge, due to a combinatorial explosion. We generated all mutants for all case studies, and sampling had been used earlier to quickly validate the code and models. Moreover, the first activity also performs the experimental part on the generated mutants: for each mutant it is checked whether or not it violates some guideline.

AppMutant is the software that has been developed (in Java) to perform both the generation of mutants and the guidelines checking. The class diagrams of AppMutant are displayed in Fig. 12.

Each mutation is represented as a class instantiating the interface Mutation, offering facilities to compute the number of possible applications of the mutation to a specific model, as well as applying the mutation to a model (given as a parameter). Which of the identified possible mutants is to be generated (and stored) is a second parameter to the method apply. The guideline checking is also performed in the method apply: a Boolean flag is returned by this method with value $true$ in case the generated mutant is violating a guideline, and value $false$ otherwise.
Fig. 12  Class diagrams of AppMutant (for readability, dependencies are not drawn as arrows)
For example, the abstract class `ConstraintSmallerMutation` implements `Mutation` and represents the mutation operator `CXS`. Two implementations of this abstract class are available, depending on whether the mutation is applied to a transition or to a location. In both cases, a method `reduceBound` is available to reduce the constant of the clock constraint. In the performed experiments, the bound is always reduced (resp., incremented) by one unit in `CXS` (resp., `CXL`). Note that `CCN` is only applied to transitions. The results presented in Section 4 are applicable to constraints with no conjunctions nor disjunctions. Accordingly, only these constraints can be mutated by the mutation operators. The TAD mutation operator is implemented by the abstract class `TransitionAddedMutation`. This abstract class is implemented by `TransitionAddedControllable` and `TransitionAddedUncontrollable`. The implementation of this mutation operator simply clones a transition (the first one encountered in the model), then changes the cloned transition source and target states and its action, and adds it to the model. In particular, a unique dummy action is used to avoid non-determinism.

The `Mutator` class is used to apply a `Mutation` to a `Model`. The class `Model` offers facilities to manage a `Uppaal` model (stored in `.xml` format). `AppMutant` is the executable class. It features methods for running first-order and second-order experiments.

Sampling is implemented in the `runTwoMutationsSampling` utility method. More in detail, this method takes as parameters the two mutation operators to apply, and the original model is retrieved with the method `original`. The method `getPossibleMutations` of the retrieved model returns the number of possible mutations that can be applied to that model for a given mutation operator (as parameter). By calling this method for the two mutation operators and multiplying the returned values (say, `n1` and `n2`) the total number of possible mutations to apply is computed. The sampled number of mutants to generate is computed as \( \max(n_1 \times n_2, 1000) \). For each mutant, the selected mutations to apply are chosen uniformly in the interval \([0, n_1]\) for the first mutation and \([0, n_2]\) for the second mutation.

The executable `AppMutant` has options for generating either first-order or second-order mutants, with or without sampling. The outputs of the execution are the created mutants (stored as `Uppaal` models) and a comma separated values file containing for each mutation operator the number of violations. Moreover, for each mutation a file listing the mutants that are violating guidelines is provided. Each generated model comes equipped with two or three automata: the original model (`Spec`), the first-order mutant (`Spec_mutant`), and the second-order mutant (`Spec_mutant_mutant`).

After computing both mutants and the violations log, the second activity is carried out using the `AppEcdar` application. `AppEcdar` has two concerns: execute for each mutant model the refinement checking and parse the logs.

During the refinement checking the queries evaluated by `Uppaal` are as follows (for first- and second-order mutants, respectively):

\[
\text{refinement} : \ Spec\_mutant \leq \ Spec \\
\text{refinement} : \ Spec\_mutant\_mutant \leq \ Spec
\]

The evaluations of these queries for each mutant model produce logs that are grouped by their mutation operators and stored.

Concerning log parsing, the logs of violations and refinement checking are processed to provide as output the \LaTeX\ tables showed in this paper. Moreover, for validation purposes,
a text file (called false negatives) is used to store all mutants violating a guideline whose refinement checking output is non-subsumed. The class diagrams of AppEcdar are displayed in Fig. 13. The two main concerns of this activity are separated into two classes extending the Consumer functional interface of Java. The RefinementChecker class overrides the method apply(Integer) of Consumer<Integer> to execute the refinement checking, where the parameter indicates whether first-order or second-order mutants are processed. This class basically launches the verifytga Uppaal process, and can be executed on either Windows, Linux or Mac OS X. Similarly, LogParser extends the same interface and is used to perform the parsing: the mutants and violations logs parsing are decomposed into two separate methods. The class AppEcdar is the executable class. It offers options to select which case study to analyse, whether to perform refinement checking (indicating whether for first or second order), and whether to perform logs parsing (indicating whether for first or second order).

To enable the replication of our experiments, the source code, models, binaries and a video tutorial are publicly available.¹ Note that, for refinement checking, a distribution of Ecdar or Uppaal TIGA is required.² The experiments have been executed on a Windows machine with Java version 9.0.4 and a Linux Machine with OpenJDK 1.8, both using Uppaal TIGA 4.1.4 (rev. 5535), March 2014 academic version.

¹Available at https://doi.org/10.5281/zenodo.5749732. The sources are also available at https://bitbucket.org/davidebasile84/timed_mutation/src/master/
²Available at https://uppaal.org/downloads/other/
5.3 Subject Systems

In this section, we describe the six case studies we analysed and provide references to the literature. While some case studies (i.e., the Car Alarm System) were already provided as TG in the literature, others required minor adaptations to be usable by Uppaal. Examples of such minor adaptations are: declarations of inputs as controllable and outputs as uncontrollable, and declaration of internal transitions as uncontrollable.

We also faced three issues. The first is that some case studies were parametric, whilst Uppaal TIGA cannot process parametric timed automata (i.e., it is not possible to have a transition using variables in the guard and channel synchronisation). Parametric constraints may also be mutually exclusive such that each set-up of parameters makes some transitions redundant. We solved this by instantiating each parameter to a specific value, taken from the literature when possible, and removing redundant behaviour.

The second issue is that Uppaal TIGA and timed interfaces do not support shared memory, i.e., no global variables are allowed in refinement checking. Since we analysed automata in isolation, all variables were made local.

The third issue is that Uppaal TIGA requires that each model is consistent (cf. Section 5). In some of the case studies the automata had originally been designed to be consistent when composed with the other automata of the same case study (with specific parameter assignments), but not for any possible environment, as instead is required by the consistency checking of Uppaal TIGA. We solved this third issue by fixing the automata not satisfying these properties for the specific set-up of parameters. Indeed, performing refinement checking of inconsistent models would be useless.

We specify below the changes we applied to the models in order to deal with these three issues. We evaluated the guidelines on six case studies retrieved from the literature that are detailed next.

5.3.1 CAS

The Car Alarm System (CAS) automaton we used in Basile et al. (2020a) stems from Ford’s automotive demonstrator in the MOGENTES project, and is depicted in Fig. 14 (for readability, we omitted the sink state with incoming transitions from all states receiving unexpected inputs). The original model accounts six automata that are composed synchronously. We applied the mutations on the main automaton of the model, called system. This automaton has already been used for experiments in Aichernig et al. (2015), Larsen et al. (2017), and Aichernig et al. (2013). The system model allows as inputs the unlocking, locking, closing, and opening of a car’s door. The outputs are the signals for arming, unarming, and turning the sound and flash alarms on and off. We mainly used the adaptation of the automaton to Uppaal in Larsen et al. (2017), in which all input transitions are marked as controllable and all output transitions as uncontrollable.

5.3.2 Coffee

The coffee machine (COFFEE) model stems from an introduction to parametric timed automata (André et al. 2019), for didactic purposes. The model is composed of only one automaton, depicted in Fig. 15, and allows a user to press a button for coffee and possibly add sugar, upon which a cup of coffee is prepared and eventually delivered.
5.3.3 Accel

This is another parametric timed automata model from the automotive domain, stemming from Waga et al. (2017) and André et al. (2018), where it was presented as generated from a Simulink model of a scenario of monitoring the gear change of an automatic transmission system (Hoxha et al. 2015). The automaton that has been used for the experiments is depicted in Fig. 16. It has been slightly modified from its original version. In particular, we abstracted away the clock variable \( x_{abs} \). It models a basic automatic transmission requirement from Hoxha et al. (2015) (through a pattern that matches a part of a timed word that violates it): when shifting into any gear, there should be no shift from that gear to any other gear within a given time threshold.

5.3.4 Hotel

This is a timed service contract automata model from Basile et al. (2020c), depicted in Fig. 17. It models a hotel booking system that offers different types of rooms, requests payment either in cash or by card (of different duration) from clients, and possibly emits a receipt.
5.3.5 WFAS

The Wireless Fire Alarm System (WFAS) model is a parametric timed automaton model from Benes et al. (2015). It stems from Feo-Arenis et al. (2014), where it was presented as a formalisation of the requirements specified by a company specialised in radio technology, with the aim of verifying that the WFAS design passes the conformance tests put forward by European standards. It models two wireless sensors communicating with an alarm controller over a communication channel. The controller synchronises the two sensors and uses a clock $x$.

As for ACCEL, the automaton that has been used for the experiments, depicted in Fig. 18, has been slightly modified from its original version. We retrieved the values of the

![Diagram](image_url)
parameters from Benes et al. (2015). We abstracted away the clock variable $y$ from the clock invariant and removed redundant transitions.

We added two additional uncontrollable transitions from states $\text{cont}_1$ and $\text{cont}_3$ to the $\text{timeout}$ state to satisfy independent progress and make the model consistent.

5.3.6 Mutex

This a parametric timed automaton model from Hune et al. (2001) of Fischer’s mutual exclusion protocol (Lamport 1987) (cf. Ter Beek and Kleijn 2012 for an untimed token-based solution). Its purpose is to guarantee mutually exclusive access to a critical section among competing processes. Also this automaton, depicted in Fig. 19, has been slightly modified. The original model used a global variable $\text{lock}$ shared with the other (replicated) automata. We pruned two transitions that were never enabled (and redundant) since this automaton is analysed in isolation.
5.4 Validation Results

Here we interpret the results of our experiments for RQ1 and RQ2 for all the case studies. We applied all mutation operators discussed in the paper, namely TMI, TAD, SMI, CXL, CXS, and CCN. Mutations are applied singularly for first-order mutants or in pairs for second-order mutants. For each case study, we provide a table for first-order mutations and a table for second-order mutations (for reasons of readability, the latter are reported in Appendix A). Each table reports the total number of mutants, that of non-subsumed mutants, of subsumed mutants, of inconsistent mutants, and of mutants violating the guidelines, as well as—in the last column—the ratio of mutants that are statically detected to be subsumed by our guidelines, computed as follows:

$$\frac{\text{(models) violating (guidelines)}}{\text{subsumed + inconsistent (models)}}$$

5.4.1 First Order

We start by discussing the results for first-order experiments. Firstly, no false negatives are detected: none of the mutants violating guidelines is non-subsumed, thus validating the theoretical results presented in the initial part of Section 4 (i.e., Lemmata 1–7).

CAS Table 2 contains the results for first-order mutants. We note that we refined the mutant generation phase, to reduce the number of inconsistent mutants generated. Thus, the results in Table 2 are not based on the same mutants as those in Basile et al. (2020a). A total of 721 first-order mutants were generated. A total of 77% of the first-order subsumed mutants and inconsistent mutants generated for this case study are detected by both Uppaal and the guidelines.

As showed in Fig. 14, many locations of this automaton have upper-bound invariants (i.e., $\leq$) and only one outgoing transition. This means that whenever one of these transitions is removed, the mutant may become inconsistent. For example, from the initial location openUnlocked, the sequence close? lock? of inputs leads to the location closedLocked. A mutation that removes the only uncontrollable outgoing transition of closedLocked produces an inconsistent mutant. Indeed, if no input arrives the system can only delay for a finite amount of time before the invariant $c \leq 20$ is violated. This is the case for TMI and SMI mutations, the two mutation operators that remove transitions.

We also note two locations with invariant $e \leq 300$ and incoming uncontrollable transition with guard $e = 300$. In this case, increasing the constant of the clock guard or
Table 2  CAS first-order mutants results

| operator | measure | mutants | non-subsumed | subsumed | inconsistent | violating guidelines | violating subsumed + inconsistent |
|----------|---------|---------|--------------|----------|--------------|----------------------|----------------------------------|
| TMI      |         | 88      | 71           | 6        | 11           | 17                   | 100%                             |
| TAD      |         | 578     | 197          | 381      | 0            | 289                  | 76%                              |
| SMI      |         | 16      | 7            | 0        | 9            | 8                    | 89%                              |
| CXL      |         | 20      | 8            | 9        | 3            | 6                    | 50%                              |
| CXS      |         | 13      | 6            | 3        | 4            | 6                    | 86%                              |
| COX      |         | 6       | 6            | 0        | 0            | 0                    | -                                |
| totals   |         | 721     | 295          | 399      | 27           | 326                  | 77%                              |

decreasing the constant of the clock invariant makes the uncontrollable transition redundant. This in turn makes the source state of the (only outgoing uncontrollable) redundant transition not satisfying the independent progress property: the corresponding mutant is inconsistent. This is the case for mutations CXL and CXS. Moreover, the worst performance of the guidelines are for mutation CXL. This is due to the fact that enlarging a clock constraint constant in this specific automaton is likely to make an element redundant (as in the previous example), and thus make the corresponding mutant subsumed, even if no guideline is violated. Since the original model is consistent, TMI applied to a controllable transition (and thus not violating a guideline) never produces a redundant element. Indeed, in this automaton, 100% of subsumed or inconsistent mutants produced with TMI are detected to be violating a guideline.

**COFFEE**  Table 3 contains the results for first-order mutants. A total of 55 first-order mutants were generated. A total of 91% of the first-order subsumed mutants and inconsistent mutants generated for this case study could have been avoided, because they are statically detected to be violating the guidelines.

This automaton shows the best performances of the guidelines when compared to the other case studies. Indeed, 100% of subsumed or inconsistent mutants for mutation operators TMI, SMI, and CXS are violating guidelines, whilst TAD has also a higher percentage of mutants detected by the guidelines. Operator CXL has six subsumed mutants, of which four are detected by the guidelines. These two subsumed mutants not detected by the guidelines result from the application of CXL to either location `add_sugar` or `preparing_coffee` (cf. Figure 15). In both cases, the constant is incremented by one.

We note that both models (original and mutant) can reach a configuration where \( \ell = \text{add\_sugar} \) and \( y = 5 \). From this configuration, if the mutant delays by one time unit then the original model cannot also delay. According to Definition 7 this proves that the mutant is not a refinement. However, in Uppaal this additional delay of the mutant is not allowed. The reason is that the reached configuration (\( \ell = \text{add\_sugar} \) and \( y = 6 \)) is “bad”: it violates the independent progress property (neither delays nor uncontrollable moves are allowed), which is required by Uppaal to hold for the mutant. Such transitions leading to “bad” configurations are pruned by Uppaal before performing the refinement checking (David et al. 2015). Accordingly, Uppaal detects this mutant to be subsumed. Basically, the increment of the constant adds redundant uncontrollable behaviour that does not modify the original specification.
The same reasoning applies to the case where CXL is applied to the location `preparing_coffee`. On the converse, this is not the case when applying CXL to the location `cdone`: the added uncontrollable behaviour is not redundant because of a further outgoing uncontrollable transition. Indeed, this mutant is detected by Uppaal to be non-subsumed.

**ACCEL** Table 4 contains the results for first-order mutants. A total of 451 first-order mutants were generated. A total of 86% of the first-order subsumed mutants and inconsistent mutants generated for this case study are statically detected by the guidelines. In this case, the percentage of detected mutants is 100% for the mutation operators TMI, SMI, CXS, and CXL.

The only inconsistent mutant is generated by TAD when adding a controllable transition from the initial state `pre_s0` to state `s_end`. Indeed, the system cannot block inputs, and from the target state `s_end` no delay is allowed (the location is urgent) and no outgoing uncontrollable transition is present. This makes the mutant inconsistent (and correctly spotted as violating the second commandment). We note that the ACCEL model has no invariant on locations, and thus all locations are satisfying independent progress (except for the urgent one).

There are 16% TAD mutants detected by Uppaal as subsumed but not detected by the guidelines. These added uncontrollable transitions have no guard but only a dummy output.

| Table 3   | COFFEE first-order mutants results |
|-----------|-----------------------------------|
| operator  | measure                           | mutants | non-subsumed | subsumed | inconsistent | violating guidelines | violating subsumed + inconsistent |
|          |                                   |         |              |          |             |                      |                                      |
| TMI      |                                   | 6       | 0            | 6        | 0           | 6                    | 100%                                  |
| TAD      |                                   | 32      | 15           | 17       | 0           | 16                   | 94%                                   |
| SMI      |                                   | 3       | 0            | 3        | 0           | 3                    | 100%                                  |
| CXS      |                                   | 6       | 3            | 3        | 0           | 3                    | 100%                                  |
| CXL      |                                   | 7       | 1            | 6        | 0           | 4                    | 67%                                   |
| CCN      |                                   | 1       | 1            | 0        | 0           | 0                    | -                                     |
| totals   |                                   | 55      | 20           | 35       | 0           | 32                   | 91%                                   |

| Table 4   | ACCEL first-order mutants results |
|-----------|-----------------------------------|
| operator  | measure                           | mutants | non-subsumed | subsumed | inconsistent | violating guidelines | violating subsumed + inconsistent |
|          |                                   |         |              |          |             |                      |                                      |
| TMI      |                                   | 33      | 0            | 33       | 0           | 33                   | 100%                                  |
| TAD      |                                   | 392     | 158          | 233      | 1           | 196                  | 84%                                   |
| SMI      |                                   | 13      | 0            | 13       | 0           | 13                   | 100%                                  |
| CXS      |                                   | 3       | 1            | 2        | 0           | 2                    | 100%                                  |
| CXL      |                                   | 5       | 2            | 3        | 0           | 3                    | 100%                                  |
| CCN      |                                   | 5       | 4            | 1        | 0           | 0                    | 0%                                     |
| totals   |                                   | 451     | 165          | 285      | 1           | 247                  | 86%                                   |
For example, the mutation adding an additional uncontrollable transition from the initial state to itself is detected as being subsumed, although it is not according to Definition 7. Since all locations are reachable, satisfy independent progress, and no constraints on locations are present, we argue that these 16% TAD subsumed mutants may be false positives (although we have no details on the internal implementation of Uppaal). However, due the high number of such subsumed mutants (i.e., 233), it is not possible to manually inspect each one of them. Notwithstanding false positives (which deteriorate the percentage of subsumed mutants detected by the guidelines), the guidelines in this case study are still efficient, by statically detecting 86% of the subsumed mutants.

Finally, in this case study we also note the presence of a subsumed mutant for the mutation operator CCN. This mutation is negating the clock guard of the transition with source s_soon_end and target s_end. This would apparently be a counterexample to Lemma 14. However, this is not the case because the generated mutant is redundant, thus violating the hypothesis of Section 4.2. Indeed, this specific mutation is not preserving non-redundancy of the mutant. This is because the negated clock guard $x \leq 0$ can never be enabled, since the target state is only reached when $x > 25$. This last example emphasises the hardness of statically detecting non-redundancy preserving mutations.

**HOTEL** Table 5 contains the results for first-order mutants. A total of 39 first-order mutants were generated.

A total of 51% of the first-order subsumed mutants and inconsistent mutants generated for this case study are statically detected by the guidelines, this being the worst performances when compared to the other case studies. This is because Hotel represents a “bad” model for the guidelines. We remark that the guidelines do not consider additional constraints imposed by Uppaal on the mutants, causing the increment of redundancy in the mutants. Indeed, since location $q_{H1}$ (cf. Figure 17) does not satisfy independent progress, Uppaal prunes the only two outgoing transitions from the initial state, prior to start refinement checking. Thus, all elements (apart from the initial location) are redundant in the model when checking for refinement. Indeed, all mutants generated by TMI, SMI, CXL, CXS, and CCN are mutating redundant elements and thus are all subsumed.

For example, the four subsumed mutants of CCN (negating the 4 clock guards) are all cases of mutations adding redundant uncontrollable behaviour. Now consider the mutation negating the guards of the transition with source state $q_{H0}$ and target state $q_{H1}$. By Definition 7, from the initial configuration ($q_{H0}, y = 0$) the mutant could fire this mutated uncontrollable transition (and indeed this mutation is not violating any guideline). However,

| operator | mutants | non-subsumed | subsumed | inconsistent | violating guidelines | violating subsumed + inconsistent |
|----------|---------|--------------|----------|--------------|----------------------|----------------------------------|
| TMI      | 5       | 0            | 5        | 0            | 3                    | 60%                              |
| TAD      | 18      | 4            | 13       | 1            | 9                    | 64%                              |
| SMI      | 2       | 0            | 2        | 0            | 1                    | 50%                              |
| CXL      | 5       | 0            | 5        | 0            | 2                    | 40%                              |
| CXS      | 5       | 0            | 5        | 0            | 3                    | 60%                              |
| CCN      | 4       | 0            | 4        | 0            | 0                    | 0%                               |
| totals   | 39      | 4            | 34       | 1            | 18                   | 51%                              |
since the reached location $q_{H1}$ does not satisfy independent progress, this uncontrollable behaviour is pruned by Uppaal prior to the refinement checking (i.e., the added behaviour is redundant). Therefore, the mutant is detected as subsumed.

The only mutant detected as inconsistent by Uppaal is a TAD mutation adding a controllable transition $(q_{H0}, y \geq 4, \text{dummy}, y = 0, q_{H1})$. Indeed, as previously stated, location $q_{H1}$ does not satisfy independent progress, and the system cannot prevent to reach this “bad” location due to the presence of the added dummy input transition. Note that this mutant is also violating a guideline: if this mutant were not inconsistent, it would anyway be subsumed.

Finally, concerning TAD applied to uncontrollable transitions, there are four non-subsumed mutants and five subsumed mutants (not detected by the guidelines). The four non-subsumed mutants are obtained by adding transitions from location $q_{H1}$ to either of the locations of the model (3 in total). These mutants are non-subsumed because the mutations make location $q_{H1}$ satisfy independent progress (and thus non-redundant). The last case is adding a transition from the initial state to $q_{H2}$. This transition is not redundant because it does not involve state $q_{H2}$, and the corresponding mutant is non-subsumed.

**WFAS** Table 6 contains the results for first-order mutants. A total of 121 first-order mutants were generated. A total of 74% of the first-order subsumed mutants and inconsistent mutants generated for this case study are detected by the guidelines.

Mutation TMI produces four inconsistent mutants that are all detected by the guidelines. Indeed, removing one of such four uncontrollable transitions makes the source state inconsistent (i.e., not satisfying independent progress).

One application of SMI to the state timeout causes the violation of the third commandment. The corresponding mutant is inconsistent. Its mutation operator corresponds to removing two uncontrollable transitions (similar to TMI). Removing state $cont_3$ (and all its incident transitions) makes state $cont_2$ not satisfying independent progress, and thus inconsistent. Since $cont_3$ has one incoming controllable transition, it does not violate the third commandment. The application of SMI to locations fail, $cont_4$, and $cont_2$ produces non-subsumed mutants.

Mutation TAD adds 36 controllable transitions (producing 36 mutants), all detected as violating guidelines and all subsumed. Of the 36 uncontrollable transitions added, 26 produce non-subsumed mutants, while ten produce subsumed mutants (not detected by the guidelines). These ten subsumed mutants all have an additional uncontrollable transition

| operator | measure | mutants | non-subsumed | subsumed | inconsistent | violating guidelines | violating subsumed + inconsistent |
|----------|---------|---------|--------------|----------|-------------|----------------------|----------------------------------|
| TMI      |         | 14      | 10           | 0        | 4           | 4                    | 100%                             |
| TAD      |         | 72      | 26           | 46       | 0           | 36                   | 78%                              |
| SMI      |         | 5       | 3            | 0        | 2           | 1                    | 50%                              |
| COS      |         | 12      | 8            | 0        | 4           | 0                    | 0%                               |
| COL      |         | 12      | 2            | 6        | 4           | 8                    | 80%                              |
| CCN      |         | 6       | 6            | 0        | 0           | 0                    | -                                |
| totals   |         | 121     | 55           | 52       | 14          | 49                   | 74%                              |
with guard \( x \geq 2 \) and a dummy output. Two with source \( \text{cont}_1 \) and target \( \text{cont}_1 \) (resp., \( \text{cont}_3 \)) and two with source \( \text{timeout} \) and target \( \text{fail} \) (resp., \( \text{timeout} \)) are not redundant, and their target state is not violating independent progress. These four mutants are false positives. The remaining added transitions are one with source \( \text{cont}_2 \) and target \( \text{cont}_2 \), two with source \( \text{cont}_3 \) and target \( \text{cont}_1 \) (resp., \( \text{cont}_3 \)), one with source \( \text{cont}_4 \) and target \( \text{cont}_4 \), two with source \( \text{fail} \) and target \( \text{fail} \) (resp., \( \text{timeout} \)). From these source states the added transitions are never enabled, and thus redundant.

The application of \( \text{CXS} \) to any clock guard produces a non-subsumed mutants (for a total of 8 mutants), whereas its application to a clock invariant produces an inconsistent mutant. Of these four inconsistent mutants none is detected to be violating a guideline. Indeed, these are all cases of mutations that deactivate an uncontrollable transition, and similar to \( \text{TMI} \) this in turns causes the mutated state to not satisfy independent progress.

There are only two non-subsumed \( \text{CXL} \) mutants, namely the mutations applied to either location \( \text{cont}_1 \) or \( \text{cont}_3 \). For example, the mutation on \( \text{cont}_1 \) allows to delay by three time units before firing the transition to \( \text{timeout} \). This behaviour is not allowed in the original model and thus according to Definition 7 the mutant is non-subsumed. The same happens with the mutation on \( \text{cont}_3 \). The other two applications of \( \text{CXL} \) to locations \( \text{cont}_2 \) and \( \text{cont}_4 \) are subsumed and are not detected by the guidelines. This is because \( \text{Uppaal} \) prunes the transitions delaying of one further time unit in both locations prior to refinement, since this (uncontrollable) behaviour leads to a configuration not satisfying independent progress. The application of \( \text{CXL} \) to transitions produces four inconsistent and four subsumed mutants. These are all detected to be violating the third commandment and point (i) of the fifth commandment. The four inconsistent mutants are mutating the guards of uncontrollable transitions, making them redundant and the source location violating independent progress.

Finally, all \( \text{CCN} \) mutations are producing non-subsumed mutants. We conclude that this case study is susceptible to violate independent progress when mutated, this being the main reason for the alternating performances of the mutation operators, whereas for \( \text{TAD} \) this is due to the possibility of adding redundant uncontrollable transitions.

**MUTEX** Table 7 contains the results for first-order mutants. A total of 47 first-order mutants were generated. A total of 73% of the first-order subsumed mutants and inconsistent mutants generated for this case study are detected to be violating some guideline.

In this case study, mutation operators \( \text{TMI} \), \( \text{SMI} \), \( \text{CXL} \), and \( \text{CXS} \) detect 100% of subsumed or inconsistent mutants. The only \( \text{CCN} \) mutant is non-subsumed. Mutation \( \text{TAD} \) applied to controllable transitions produces 16 mutants, of which two are inconsistent and the others are subsumed. All 16 mutants are violating the second commandment. One of the two inconsistent mutants is generated by adding a controllable transition from \( \text{start}_1 \) to \( \text{tryenter}_1 \). In this case, the target state does not satisfy independent progress, because the only outgoing uncontrollable transition becomes disabled. The other inconsistent mutant is similar, in this case however the source state of the added transition is \( \text{set}_1 \).

Finally, the only subsumed mutants not detected by the guidelines are due to mutation \( \text{TAD} \) applied to uncontrollable transitions. This produces six non-subsumed mutants and ten subsumed mutants that are not detected by the guidelines. This is because the ten subsumed mutants are adding uncontrollable transitions whose guard is never enabled (due to the implementation of the \( \text{TAD} \) operator), and are thus redundant. Hence, in this specific case study the ten subsumed mutants are missed by the guidelines because of the way the \( \text{TAD} \) operator is implemented, i.e., the chosen guard for the added transition may render the new transition redundant.
5.4.2 Second Order

We now interpret the outcomes of the second-order experiments. Due to the high number of mutants (hundreds of thousands), it is not feasible to manually inspect each of the generated mutants (as we did for the first-order mutants). However, since second-order mutants are generated by mutating first-order mutants, the insights on the first-order experiments can be extended to the case of second-order mutants. The results of the second-order experiments for the six case studies are reported in Tables 8, 9, 10, 11, 12 and 13 and summarised in Fig. 22.

We observe a deterioration in the percentage of subsumed/inconsistent mutants detected by the guidelines. The average deterioration is $\approx 25\%$, with a minimum of 15% deterioration for ACCEL and a maximum of 34% for WFAS. By looking at the first-order experiments, this variation can be explained by the fact that ACCEL’s undetected subsumed mutants are false positives, and none of them is violating independent progress. The converse happens for WFAS, whose mutants are highly susceptible to violate independent progress, and this is amplified in the second-order experiments.

We note that in the second-order experiments it is required that both mutations are violating the guidelines. If only one mutation is violating a guideline but the other is not, then the corresponding second-order mutant is detected as not violating the guidelines. Indeed, although the second mutation is relative to the corresponding first-order mutant (i.e., second-order mutants are obtained by performing an additional mutation on first-order mutants), the “bad” mutants are those that are refinements of the original model, and not of their relative first-order mutant. Indeed, we are interested in detecting all mutants (no matter of what order) that are subsumed by the original model.

More in detail, given a model $\text{Spec}$, a first-order mutant $\text{Spec}_{\text{mutant}}$, and a second-order mutant $\text{Spec}_{\text{mutant\_mutant}}$, $\text{AppEcdar}$ checks whether $\text{Spec}_{\text{mutant\_mutant}} \leq \text{Spec}$. If the first mutation on the original model violates any guideline, by the theoretical results presented in Section 4, it holds that $\text{Spec}_{\text{mutant}} \leq \text{Spec}$. If also the second mutation of the first-order mutant violates any guideline, we know that $\text{Spec}_{\text{mutant\_mutant}} \leq \text{Spec}_{\text{mutant}}$. At this point, by the transitivity of $\leq$, it is possible to conclude that $\text{Spec}_{\text{mutant\_mutant}} \leq \text{Spec}$. However, if one of the two violations is not violating any guideline, this conclusion cannot be reached. This also entails that first-order subsumed mutants cannot be discarded when performing second-order mutations, since otherwise it would not be possible to conclude that $\text{Spec}_{\text{mutant\_mutant}} \leq \text{Spec}$ using the theoretical results of Section 4.
One of the causes of this deterioration of the guidelines’ performances in the second-order mutants is the conjunction of two violation requirements.

Consider the models in Fig. 20. The first-order mutant MUTANTtad introduces controllable behaviour (with TAD, by adding a controllable transition), thus being subsumed by the original model SUT and violating the second commandment. Another first-order mutant MUTANTcxs, mutated with CXS, is subsumed by SUT due to the ninth commandment. Even though the two mutations are separately producing first-order mutants that are subsumed and separately violating guidelines, the application of CXS to MUTANTtad produces a mutant MUTANTtadcxs that is not violating the guidelines, even if MUTANTtadcxs <= SUT.

Indeed, applying the CXS mutation to MUTANTtad reduces the added controllable behaviour (i.e., it restricts the clock invariant target of the added transition), and thus MUTANTtadcxs \( \not\leq \) MUTANTtad. Correspondingly, the second mutation is not violating any guideline and the experiments cannot conclude that the second-order mutant is violating the guidelines. However, since this reduced controllable behaviour was not present in SUT, it holds that MUTANTtadcxs <= SUT.

Finally, whilst applying a mutation on the original model is less prone to mutate a redundant element, this is not the case for second-order mutants. Indeed, if one mutation produces redundant behaviour (possibly also due to independent progress), then applying the second mutation on one such redundant element could produce a subsumed mutant not detected by our guidelines. Also, in case of an inconsistent first-order mutant, it is unlikely that the second mutation will fix the inconsistency.

We note that the presented theoretical results only account for a single mutation of a model (being it the original model or a mutant), and the second-order results are obtained by transitivity of <=. Further results about applying more than one single mutation on a model could help to improve the performances of higher-order mutants (e.g., syntactic conditions under which second-order mutants are subsumed by the SUT). However, such results are hard to obtain due to the possible presence of redundant behaviour and interactions between
mutations. For example, it is not always true that if two mutations produce two separate subsumed first-order mutants, then their composition produces a second-order subsumed mutant (as occurs in Fig. 20).

To show this, consider Fig. 21. This example shows that the two first-order mutants MUTANT1 (applying CXS on a transition) and MUTANT2 (applying CXL on a location) are both subsumed by the original model SUT, because they are mutating redundant elements. However, the application of the two mutations (no matter in what order) produces a second-order mutant that is not subsumed by the SUT. This is because an interaction occurs between the two mutations, causing the redundant (uncontrollable) behaviour of the original model to become non-redundant and the second-order mutant to be non-subsumed.

Regardless of the obstacles when applying the guidelines to higher-order mutants, in our experiments we were able to statically detect 71% of the subsumed/inconsistent second-order mutants for the ACCEL case study, and 65% for the COFFEE case study.

5.5 Final Considerations

The experiments confirm that all mutants violating the guidelines were subsumed or inconsistent, as expected, thus providing further confidence in our results and an answer to RQ1. This has been theoretically proved in Section 4, and the correctness of the proofs was confirmed during the experiments. Indeed, AppEcdar performed a false negative checking, to check for the presence of non-subsumed mutants violating the guidelines. No such mutants have been identified in any of the case studies. The correctness of the theoretical results can also be observed by the fact that, for each case study, the number of mutants violating the guidelines is never greater than the number of mutants that are either subsumed or inconsistent. We remark that all subsumed and inconsistent mutants that are violating the guidelines are those that would not be generated if one were to apply the guidelines presented in Section 4.4.

![Image](image.png)

**Fig. 21** An example, using Uppaal TIGA, showing two subsumed first-order mutants whose composition of mutations is not subsumed
To answer RQ2, we report a histogram summarising the outcome of the experiments in Fig. 22. The results confirm the gain of our approach: more than half of all subsumed first-order mutants are detected by our guidelines for each case study, detecting 91%, 86%, and 77% subsumed mutants for, respectively, the COFFEE, ACCEL, and CAS case studies. As discussed in Section 5.4.2, this holds also for second-order mutants. Indeed, the guidelines were capable of detecting more than half of the second-order subsumed or inconsistent mutants for the case studies COFFEE, ACCEL, and CAS, and fewer for the remaining three case studies.

We also report a histogram in Fig. 23 showing what is the likelihood of generating a subsumed mutant when applying random mutations, with or without using the guidelines, for both first-order and second-order experiments. We assume that all mutations have the same probability of being applied. For each case study, the probability of generating a subsumed mutant is computed as

\[
1 - \frac{\text{non-subsumed mutants}}{\text{total mutants} - \text{mutants violating guidelines}}
\]

(the parameter of \text{mutants violating guidelines} is equal to zero when generating mutants without using the guidelines). Notably, the guidelines can drastically diminish the probability of generating subsumed mutants in some case study. The histogram shows how this probability gets lower when applying two mutations (independently of the usage of the guidelines). Indeed, by augmenting the number of mutations, it is less likely that they will have no effect on the mutated model. We note that HOTEL has the highest probability of generating subsumed mutants. This also explains the worst performances of the guidelines for this particular case study.

Several directions of improvement of the static analysis performed by the guidelines are possible. Different implementations of the mutation operators can be investigated. For example, our implementation of the TAD mutation operator adds uncontrollable transition that may be guarded. This is likely to introduce subsumed mutants not detected by our

![Figure 22](image-url) Percentage of subsumed mutants that are detected by the guidelines
guidelines when such guards are never satisfied. By changing the implementation of TAD such that no guarded transition is added, the generated mutants are less likely to be subsumed. We note that TAD mutation is responsible for many undetected subsumed mutants in all the analysed case studies.

Parametric timed games (Luthmann et al. 2019; Luthmann et al. 2017) can be investigated to perform exhaustive experiments for the mutations CXL and CXS for all possible constant value updates. In the presented experiments, CXL and CXS updated the constants of clock constraints by increasing or decreasing by one unit.

More research is needed for the static detection of redundant elements (e.g., dangling locations). Our experiments have exploited the refinement checking provided by Uppaal, which however performs a pre-processing step to prune uncontrollable behaviour not consistent. For experiments performed with Uppaal, syntactically detecting locations not satisfying independent progress is possible. For example, if a location has a clock invariant of the form $x \leq k$ and no outgoing uncontrollable transition, then this location is not satisfying independent progress. This is the case for location $q_{H1}$ of the HOTEL case study, which is indeed generating the worst performances for our guidelines. We argue that by statically pre-processing models similar to the HOTEL model to remove such “bad” locations, the performances can be improved.

In Fig. 21, an example of two mutations producing subsumed mutants whose composition is producing a non-subsumed mutant is displayed. However, these two mutations are not violating the guidelines. Indeed, the set of mutants violating the guidelines is a strict subset of the subsumed mutants. Further research is needed to prove or disprove whether the composition of two mutations violating guidelines always yields a subsumed mutant.

Finally, we note that the variation of percentages of detected mutants in each case study
also depends on the number of controllable and uncontrollable transitions, as well as the presence of different types of constraints, which are specific to each case study.

5.6 Threats toValidity

Our empirical results essentially rely on the refinement relation as implemented in Uppaal/Ecdar. Should this implementation deviate (even slightly) from the definition we employ (itself based on the paper introducing Ecdar (David et al. 2010b)), we might witness the occurrence of false positives (mutants erroneously labelled as subsumed). To mitigate this risk, we conducted a non-exhaustive manual analysis of the sampled mutants.

Nevertheless, we did not remove these few false positives, which would actually improve our results since it would increase the percentage of subsumed mutants detected by our guidelines.

6 Conclusion and Future Work

We presented a methodology for discarding ineffective mutations for testing real-time systems. An effective mutant can be used to generate test cases that distinguish the mutant from the original system model. The framework of TG and Ecdar refinement checking of Larsen et al. (2017) was adopted, and mutants were organised as a product line of mutations using the approach of Devroey et al. (2016). Our guidelines to the construction of such a featured mutant model can be encoded as constraints in the feature model, to guarantee that effective mutants will be generated. Our experiments confirmed the soundness of our approach and demonstrated that our actionable guidelines can significantly reduce the number of subsumed mutants.

In future work, we plan to exploit the auxiliary results from Section 4.2 to perform a different evaluation than the one presented in this paper, viz., instead of discarding subsumed mutants, only generate (statically known) non-subsumed ones. As mentioned before, this evaluation is harder because only in specific cases it is possible to detect whether a mutation is non-redundant.

In the future, we also plan to investigate a family-based technique for checking refinements all-at-once directly on the FTG, in order to take further advantage of the product-line approach and of our technique for building effective featured mutant models. This would allow the generation of the smallest set of test cases that can distinguish all killable mutants. One way to do so is to design a feature-aware extension of the refinement checking procedure of David et al. (2010a, 2010b). By associating a feature to each mutant (Devroey et al. 2016), one could then collect the feature expressions identifying all mutants for which the refinement holds, and those for which it does not, in a single play. This problem was studied in the non-timed case (Cordy et al. 2012a), but it remains unaddressed for real-time systems. The addition of time makes this problem challenging, as there is no known efficient way to encode time and variability in a single data structure (Cordy et al. 2012b).

Our work also provides the foundations to evaluate real-time test cases. To this aim, one could apply the approach of Devroey et al. (2016) on a featured timed model to identify which mutants are killed. Again, this would require data structures combining time with variability.
## Appendix A: Results for Second-Order Mutations

Table 8 CAS second-order mutants results

| operator | measure | mutants | non-subsumed | subsumed | inconsistent | violating guidelines | violating subsumed + inconsistent |
|----------|---------|---------|--------------|----------|-------------|----------------------|----------------------------------|
| TMI-TMI  |         | 7657    | 5844         | 30       | 1783        | 272                  | 15%                              |
| TMI-TAD  |         | 50866   | 42576        | 2335     | 5955        | 4913                 | 59%                              |
| TMI-SMI  |         | 1409    | 573          | 0        | 836         | 136                  | 16%                              |
| TMI-CXS  |         | 1137    | 672          | 18       | 447         | 102                  | 22%                              |
| TMI-CXL  |         | 1753    | 1248         | 57       | 448         | 99                   | 20%                              |
| TMI-CCN  |         | 523     | 456          | 0        | 67          | 0                    | 0%                               |
| TAD-TMI  |         | 51444   | 42576        | 2913     | 5955        | 4913                 | 55%                              |
| TAD-TAD  |         | 334084  | 188798       | 145286   | 0           | 83521                | 57%                              |
| TAD-SMI  |         | 9250    | 4285         | 33       | 4932        | 2176                 | 44%                              |
| TAD-CXS  |         | 7517    | 4168         | 1161     | 2188        | 1683                 | 50%                              |
| TAD-CXL  |         | 11563   | 6497         | 3383     | 1683        | 1734                 | 34%                              |
| TAD-CCN  |         | 3468    | 3468         | 0        | 0           | 0                    | -                                |
| SMI-TMI  |         | 1251    | 470          | 0        | 781         | 121                  | 15%                              |
| SMI-TAD  |         | 8194    | 3823         | 33       | 4338        | 2048                 | 47%                              |
| SMI-SMI  |         | 241     | 80           | 0        | 161         | 59                   | 37%                              |
| SMI-CXS  |         | 188     | 59           | 0        | 129         | 44                   | 34%                              |
| SMI-CXL  |         | 293     | 109          | 0        | 184         | 42                   | 23%                              |
| SMI-CCN  |         | 85      | 33           | 0        | 52          | 0                    | 0%                               |
| CXS-TMI  |         | 1146    | 678          | 18       | 450         | 102                  | 22%                              |
| CXS-TAD  |         | 7517    | 4168         | 1161     | 2188        | 1734                 | 52%                              |
| CXS-SMI  |         | 210     | 70           | 0        | 140         | 48                   | 34%                              |
| CXS-CXS  |         | 172     | 80           | 9        | 83          | 36                   | 39%                              |
| CXS-CXL  |         | 263     | 120          | 40       | 103         | 36                   | 25%                              |
| CXS-CCN  |         | 79      | 54           | 0        | 25          | 0                    | 0%                               |
| CXL-TMI  |         | 1762    | 1254         | 57       | 451         | 102                  | 20%                              |
| CXL-TAD  |         | 11563   | 6497         | 3383     | 1683        | 1734                 | 34%                              |
| CXL-SMI  |         | 322     | 124          | 0        | 198         | 48                   | 24%                              |
| CXL-CXS  |         | 270     | 120          | 47       | 103         | 36                   | 24%                              |
| CXL-CXL  |         | 403     | 206          | 87       | 110         | 36                   | 18%                              |
| CXL-CCN  |         | 121     | 96           | 6        | 19          | 0                    | 0%                               |
| CCN-TMI  |         | 529     | 462          | 0        | 67          | 0                    | 0%                               |
| CCN-TAD  |         | 3468    | 3468         | 0        | 0           | 0                    | -                                |
| CCN-SMI  |         | 97      | 42           | 0        | 55          | 0                    | 0%                               |
| CCN-CXS  |         | 79      | 54           | 0        | 25          | 0                    | 0%                               |
| CCN-CXL  |         | 121     | 96           | 6        | 19          | 0                    | 0%                               |
| CCN-CCN  |         | 36      | 30           | 6        | 0           | 0                    | 0%                               |
| totals   |         | 519081  | 323354       | 160069   | 35658       | 105775               | 54%                              |
### Table 9  COFFEE second-order mutants results

| operator | measure | mutants | non-subsumed | subsumed | inconsistent | violating guidelines | violating subsumed + inconsistent |
|----------|---------|---------|--------------|----------|-------------|----------------------|-----------------------------------|
| TMI—TMI |         | 30      | 0            | 30       | 0           | 30                   | 100%                              |
| TMI—TAD |         | 193     | 61           | 128      | 4           | 96                   | 73%                               |
| TMI—SMI |         | 18      | 0            | 18       | 0           | 18                   | 100%                              |
| TMI—CXS |         | 33      | 8            | 25       | 0           | 18                   | 72%                               |
| TMI—CXL |         | 38      | 2            | 36       | 0           | 20                   | 56%                               |
| TMI—CCN |         | 5       | 4            | 1        | 0           | 0                    | 0%                                |
| TAD—TMI |         | 225     | 61           | 160      | 4           | 96                   | 59%                               |
| TAD—TAD |         | 1024    | 735          | 289      | 0           | 256                  | 89%                               |
| TAD—SMI |         | 97      | 11           | 82       | 4           | 36                   | 42%                               |
| TAD—CXS |         | 193     | 124          | 65       | 4           | 36                   | 60%                               |
| TAD—CXL |         | 225     | 105          | 116      | 4           | 64                   | 53%                               |
| TAD—CCN |         | 32      | 32           | 0        | 0           | 0                    | -                                 |
| SMI—TMI |         | 9       | 0            | 9        | 0           | 9                    | 100%                              |
| SMI—TAD |         | 55      | 11           | 40       | 4           | 27                   | 61%                               |
| SMI—SMI |         | 6       | 0            | 6        | 0           | 6                    | 100%                              |
| SMI—CXS |         | 10      | 0            | 10       | 0           | 6                    | 60%                               |
| SMI—CXL |         | 12      | 0            | 12       | 0           | 6                    | 50%                               |
| SMI—CCN |         | 2       | 2            | 0        | 0           | 0                    | -                                 |
| CXS—TMI |         | 36      | 8            | 28       | 0           | 18                   | 64%                               |
| CXS—TAD |         | 193     | 124          | 65       | 4           | 48                   | 70%                               |
| CXS—SMI |         | 18      | 0            | 18       | 0           | 9                    | 50%                               |
| CXS—CXS |         | 36      | 19           | 17       | 0           | 9                    | 53%                               |
| CXS—CXL |         | 42      | 14           | 28       | 0           | 12                   | 43%                               |
| CXS—CCN |         | 6       | 6            | 0        | 0           | 0                    | -                                 |
| CXL—TMI |         | 42      | 2            | 40       | 0           | 24                   | 60%                               |
| CXL—TAD |         | 225     | 105          | 116      | 4           | 64                   | 53%                               |
| CXL—SMI |         | 21      | 0            | 21       | 0           | 12                   | 57%                               |
| CXL—CXS |         | 43      | 14           | 29       | 0           | 12                   | 41%                               |
| CXL—CXL |         | 49      | 13           | 36       | 0           | 16                   | 44%                               |
| CXL—CCN |         | 7       | 7            | 0        | 0           | 0                    | -                                 |
| CCN—TMI |         | 6       | 4            | 2        | 0           | 0                    | 0%                                |
| CCN—TAD |         | 32      | 32           | 0        | 0           | 0                    | -                                 |
| CCN—SMI |         | 3       | 2            | 1        | 0           | 0                    | 0%                                |
| CCN—CXS |         | 6       | 6            | 0        | 0           | 0                    | -                                 |
| CCN—CXL |         | 7       | 7            | 0        | 0           | 0                    | -                                 |
| CCN—CCN |         | 1       | 0            | 1        | 0           | 0                    | 0%                                |
| totals   |         | 2980    | 1519         | 1429     | 32          | 948                  | 65%                               |
### Table 10  ACCEL second-order mutants results

| operator  | measure | mutants | non-subsumed | subsumed | inconsistent | violating guidelines | violating subsumed + inconsistent |
|-----------|---------|---------|--------------|----------|--------------|----------------------|-----------------------------------|
| TMI–TMI   |         | 1056    | 0            | 1056     | 0            | 1056                 | 100%                              |
| TMI–TAD   |         | 12937   | 4944         | 7959     | 34           | 6468                 | 81%                               |
| TMI–SMI   |         | 429     | 0            | 429      | 0            | 429                  | 100%                              |
| TMI–CXS   |         | 96      | 32           | 64       | 0            | 64                   | 100%                              |
| TMI–CXL   |         | 160     | 61           | 99       | 0            | 96                   | 97%                               |
| TMI–CCN   |         | 160     | 125          | 35       | 0            | 0                    | 0%                                |
| TAD–TMI   |         | 13329   | 4936         | 8359     | 34           | 6468                 | 77%                               |
| TAD–TAD   |         | 153667  | 97510        | 55375    | 782          | 38416                | 68%                               |
| TAD–SMI   |         | 5097    | 1434         | 3650     | 13           | 2366                 | 65%                               |
| TAD–CXS   |         | 1177    | 704          | 469      | 4            | 392                  | 83%                               |
| TAD–CXL   |         | 1961    | 1248         | 707      | 6            | 588                  | 82%                               |
| TAD–CCN   |         | 1961    | 1709         | 246      | 6            | 0                    | 0%                                |
| SMI–TMI   |         | 380     | 0            | 380      | 0            | 380                  | 100%                              |
| SMI–TAD   |         | 4395    | 1434         | 2948     | 13           | 2197                 | 74%                               |
| SMI–SMI   |         | 156     | 0            | 156      | 0            | 156                  | 100%                              |
| SMI–CXS   |         | 33      | 10           | 23       | 0            | 22                   | 96%                               |
| SMI–CXL   |         | 55      | 18           | 37       | 0            | 33                   | 89%                               |
| SMI–CCN   |         | 55      | 39           | 16       | 0            | 0                    | 0%                                |
| CXS–TMI   |         | 99      | 32           | 67       | 0            | 66                   | 99%                               |
| CXS–TAD   |         | 1177    | 704          | 469      | 4            | 392                  | 83%                               |
| CXS–SMI   |         | 39      | 10           | 29       | 0            | 26                   | 90%                               |
| CXS–CXS   |         | 9       | 5            | 4        | 0            | 4                    | 100%                              |
| CXS–CXL   |         | 15      | 6            | 9        | 0            | 6                    | 67%                               |
| CXS–CCN   |         | 15      | 13           | 2        | 0            | 0                    | 0%                                |
| CXL–TMI   |         | 165     | 61           | 104      | 0            | 99                   | 95%                               |
| CXL–TAD   |         | 1961    | 1248         | 707      | 6            | 588                  | 82%                               |
| CXL–SMI   |         | 65      | 18           | 47       | 0            | 39                   | 83%                               |
| CXL–CXS   |         | 15      | 6            | 9        | 0            | 6                    | 67%                               |
| CXL–CXL   |         | 25      | 16           | 9        | 0            | 9                    | 100%                              |
| CXL–CCN   |         | 25      | 22           | 3        | 0            | 0                    | 0%                                |
| CCN–TMI   |         | 165     | 125          | 40       | 0            | 0                    | 0%                                |
| CCN–TAD   |         | 1961    | 1709         | 246      | 6            | 0                    | 0%                                |
| CCN–SMI   |         | 65      | 39           | 26       | 0            | 0                    | 0%                                |
| CCN–CXS   |         | 15      | 13           | 2        | 0            | 0                    | 0%                                |
| CCN–CXL   |         | 25      | 22           | 3        | 0            | 0                    | 0%                                |
| CCN–CCN   |         | 25      | 20           | 5        | 0            | 0                    | 0%                                |
| totals    |         | 202970  | 118273       | 83789    | 908          | 60366                | 71%                               |
| operator     | mutants | non-subsumed | subsumed | inconsistent | violating guidelines | violating subsumed + inconsistent |
|--------------|---------|--------------|----------|--------------|-----------------------|-----------------------------------|
| TMI−TMI      | 20      | 0            | 20       | 0            | 6                     | 30%                               |
| TMI−TAD      | 91      | 19           | 66       | 6            | 27                    | 38%                               |
| TMI−SMI      | 10      | 0            | 10       | 0            | 3                     | 30%                               |
| TMI−CXS      | 21      | 0            | 21       | 0            | 9                     | 43%                               |
| TMI−CXL      | 21      | 0            | 21       | 0            | 4                     | 19%                               |
| TMI−CCN      | 16      | 0            | 16       | 0            | 0                     | 0%                                |
| TAD−TMI      | 109     | 19           | 84       | 6            | 27                    | 30%                               |
| TAD−TAD      | 327     | 124          | 169      | 34           | 81                    | 40%                               |
| TAD−SMI      | 37      | 3            | 32       | 2            | 6                     | 18%                               |
| TAD−CXS      | 110     | 24           | 78       | 8            | 33                    | 38%                               |
| TAD−CXL      | 110     | 24           | 78       | 8            | 18                    | 21%                               |
| TAD−CCN      | 91      | 20           | 65       | 6            | 0                     | 0%                                |
| SMI−TMI      | 4       | 0            | 4        | 0            | 1                     | 25%                               |
| SMI−TAD      | 17      | 3            | 12       | 2            | 4                     | 29%                               |
| SMI−SMI      | 2       | 0            | 2        | 0            | 1                     | 50%                               |
| SMI−CXS      | 4       | 0            | 4        | 0            | 0                     | 0%                                |
| SMI−CXL      | 4       | 0            | 4        | 0            | 1                     | 25%                               |
| SMI−CCN      | 3       | 0            | 3        | 0            | 0                     | 0%                                |
| CXS−TMI      | 25      | 0            | 25       | 0            | 9                     | 36%                               |
| CXS−TAD      | 92      | 20           | 65       | 7            | 27                    | 38%                               |
| CXS−SMI      | 10      | 0            | 10       | 0            | 3                     | 30%                               |
| CXS−CXS      | 25      | 0            | 25       | 0            | 9                     | 36%                               |
| CXS−CXL      | 25      | 0            | 25       | 0            | 6                     | 24%                               |
| CXS−CCN      | 20      | 0            | 20       | 0            | 0                     | 0%                                |
| CXL−TMI      | 25      | 0            | 25       | 0            | 6                     | 24%                               |
| CXL−TAD      | 92      | 20           | 65       | 7            | 18                    | 25%                               |
| CXL−SMI      | 10      | 0            | 10       | 0            | 2                     | 20%                               |
| CXL−CXS      | 25      | 0            | 25       | 0            | 6                     | 24%                               |
| CXL−CXL      | 25      | 0            | 25       | 0            | 4                     | 16%                               |
| CXL−CCN      | 20      | 0            | 20       | 0            | 0                     | 0%                                |
| CCN−TMI      | 20      | 0            | 20       | 0            | 0                     | 0%                                |
| CCN−TAD      | 73      | 16           | 52       | 5            | 0                     | 0%                                |
| CCN−SMI      | 8       | 0            | 8        | 0            | 0                     | 0%                                |
| CCN−CXS      | 20      | 0            | 20       | 0            | 0                     | 0%                                |
| CCN−CXL      | 20      | 0            | 20       | 0            | 0                     | 0%                                |
| CCN−CCN      | 16      | 0            | 16       | 0            | 0                     | 0%                                |
| totals       | 1548    | 292          | 1165     | 91           | 311                   | 25%                               |
### Table 12  WFAS second-order mutants results

| operator | measure | mutants | non-subsumed | subsumed | inconsistent | violating guidelines | violating subsumed + inconsistent |
|----------|---------|---------|--------------|----------|-------------|----------------------|-----------------------------------|
| TMI−TMI | 183     | 98      | 0            | 85       | 12          | 14%                  |                                   |
| TMI−TAD | 1010    | 738     | 6            | 266      | 144         | 53%                  |                                   |
| TMI−SMI | 71      | 35      | 0            | 36       | 4           | 11%                  |                                   |
| TMI−CXS | 162     | 80      | 0            | 82       | 0           | 0%                   |                                   |
| TMI−CXL | 162     | 80      | 0            | 82       | 28          | 34%                  |                                   |
| TMI−CCN | 79      | 56      | 0            | 23       | 0           | 0%                   |                                   |
| TAD−TMI | 1082    | 738     | 78           | 266      | 144         | 42%                  |                                   |
| TAD−TAD | 5184    | 3068    | 2116         | 0        | 1296        | 61%                  |                                   |
| TAD−SMI | 362     | 221     | 0            | 141      | 30          | 21%                  |                                   |
| TAD−CXS | 939     | 609     | 46           | 284      | 36          | 11%                  |                                   |
| TAD−CXL | 939     | 330     | 337          | 272      | 288         | 47%                  |                                   |
| TAD−CCN | 505     | 446     | 48           | 11       | 0           | 0%                   |                                   |
| SMI−TMI | 49      | 21      | 0            | 28       | 2           | 7%                   |                                   |
| SMI−TAD | 277     | 180     | 0            | 97       | 25          | 26%                  |                                   |
| SMI−SMI | 21      | 8       | 0            | 13       | 0           | 0%                   |                                   |
| SMI−CXS | 47      | 20      | 0            | 27       | 0           | 0%                   |                                   |
| SMI−CXL | 47      | 20      | 0            | 27       | 6           | 22%                  |                                   |
| SMI−CCN | 22      | 14      | 0            | 8        | 0           | 0%                   |                                   |
| CXS−TMI | 170     | 84      | 0            | 86       | 0           | 0%                   |                                   |
| CXS−TAD | 867     | 581     | 2            | 284      | 0           | 0%                   |                                   |
| CXS−SMI | 62      | 28      | 0            | 34       | 0           | 0%                   |                                   |
| CXS−CXS | 143     | 68      | 0            | 75       | 0           | 0%                   |                                   |
| CXS−CXL | 147     | 60      | 12           | 75       | 0           | 0%                   |                                   |
| CXS−CCN | 73      | 50      | 0            | 23       | 0           | 0%                   |                                   |
| CXL−TMI | 170     | 84      | 0            | 86       | 32          | 37%                  |                                   |
| CXL−TAD | 867     | 316     | 273          | 278      | 288         | 52%                  |                                   |
| CXL−SMI | 62      | 28      | 0            | 34       | 8           | 24%                  |                                   |
| CXL−CXS | 147     | 60      | 12           | 75       | 0           | 0%                   |                                   |
| CXL−CXL | 147     | 36      | 36           | 75       | 64          | 58%                  |                                   |
| CXL−CCN | 73      | 50      | 0            | 23       | 0           | 0%                   |                                   |
| CCN−TMI | 85      | 60      | 0            | 25       | 0           | 0%                   |                                   |
| CCN−TAD | 433     | 427     | 0            | 6        | 0           | 0%                   |                                   |
| CCN−SMI | 31      | 18      | 0            | 13       | 0           | 0%                   |                                   |
| CCN−CXS | 73      | 50      | 0            | 23       | 0           | 0%                   |                                   |
| CCN−CXL | 73      | 50      | 0            | 23       | 0           | 0%                   |                                   |
| CCN−CCN | 36      | 30      | 6            | 0        | 0           | 0%                   |                                   |
| totals  | 14800   | 8842    | 2972         | 2986     | 2407        | 40%                  |                                   |
Table 13  MUTEX second-order mutants results

| operator | mutants | non-subsumed | subsumed | inconsistent | violating guidelines | violating subsumed + inconsistent |
|----------|---------|--------------|----------|--------------|----------------------|-----------------------------------|
| TMI-TMI  | 13      | 0            | 0        | 13           | 12                   | 92%                               |
| TMI-TAD  | 130     | 20           | 24       | 86           | 64                   | 58%                               |
| TMI-SMI  | 13      | 0            | 0        | 13           | 12                   | 92%                               |
| TMI-CXS  | 13      | 0            | 0        | 13           | 12                   | 92%                               |
| TMI-CXL  | 17      | 3            | 0        | 14           | 3                    | 21%                               |
| TMI-CCN  | 4       | 1            | 0        | 3            | 0                    | 0%                                |
| TAD-TMI  | 162     | 19           | 56       | 87           | 64                   | 45%                               |
| TAD-TAD  | 1027    | 344          | 576      | 107          | 256                  | 37%                               |
| TAD-SMI  | 98      | 10           | 0        | 88           | 36                   | 41%                               |
| TAD-CXS  | 98      | 7            | 0        | 91           | 36                   | 40%                               |
| TAD-CXL  | 131     | 95           | 0        | 36           | 16                   | 44%                               |
| TAD-CCN  | 33      | 31           | 0        | 2            | 0                    | 0%                                |
| SMI-TMI  | 7       | 0            | 0        | 7            | 6                    | 86%                               |
| SMI-TAD  | 56      | 8            | 0        | 48           | 27                   | 56%                               |
| SMI-SMI  | 7       | 0            | 0        | 7            | 6                    | 86%                               |
| SMI-CXS  | 8       | 0            | 0        | 8            | 7                    | 88%                               |
| SMI-CXL  | 10      | 0            | 0        | 10           | 1                    | 10%                               |
| SMI-CCN  | 2       | 0            | 0        | 2            | 0                    | 0%                                |
| CXS-TMI  | 13      | 0            | 0        | 13           | 12                   | 92%                               |
| CXS-TAD  | 98      | 7            | 0        | 91           | 48                   | 53%                               |
| CXS-SMI  | 10      | 0            | 0        | 10           | 9                    | 90%                               |
| CXS-CXS  | 10      | 0            | 0        | 10           | 9                    | 90%                               |
| CXS-CXL  | 14      | 0            | 3        | 11           | 3                    | 21%                               |
| CXS-CCN  | 4       | 1            | 0        | 3            | 0                    | 0%                                |
| CXL-TMI  | 18      | 3            | 0        | 15           | 4                    | 27%                               |
| CXL-TAD  | 131     | 95           | 0        | 36           | 16                   | 44%                               |
| CXL-SMI  | 14      | 0            | 0        | 14           | 3                    | 21%                               |
| CXL-CXS  | 15      | 0            | 4        | 11           | 3                    | 20%                               |
| CXL-CXL  | 19      | 11           | 0        | 8            | 1                    | 12%                               |
| CXL-CCN  | 4       | 4            | 0        | 0            | 0                    | -                                 |
| CCN-TMI  | 5       | 1            | 0        | 4            | 0                    | 0%                                |
| CCN-TAD  | 33      | 31           | 0        | 2            | 0                    | 0%                                |
| CCN-SMI  | 4       | 0            | 0        | 4            | 0                    | 0%                                |
| CCN-CXS  | 4       | 1            | 0        | 3            | 0                    | 0%                                |
| CCN-CXL  | 4       | 4            | 0        | 0            | 0                    | -                                 |
| CCN-CCN  | 1       | 0            | 1        | 0            | 0                    | 0%                                |
| totals   | 2230    | 696          | 664      | 870          | 666                  | 43%                               |
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Declarations

Conflict of Interests  The authors have no conflict of interest.

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