The relic density of a cold dark matter (CDM) candidate is calculated in the context of three non-standard cosmological scenarios and its value is compared with the one obtained in the standard regime. In particular, we consider the decoupling of the CDM particle during: (i) A decaying-particle dominated phase or (ii) a kinetic-energy dominated phase or (iii) the decay of a massive particle under the complete or partial domination of kination. We use plausible values (from the viewpoint of supersymmetric models) for the mass and the thermal averaged cross section times the velocity of the cold relic and we investigate scenarios of equilibrium and non-equilibrium production. In the case (i) a low reheat temperature, in the range $1 - 20$ GeV, significantly facilitates the achievement of an acceptable CDM abundance. On the other hand, the presence of kination in the case (ii) can lead to an enhancement of the CDM abundance up to three orders of magnitude. The latter enhancement can be avoided, in the case (iii). In such a situation, the temperature turns out to be frozen to a plateau value which is, mostly, lower than about 40 GeV.

1. Introduction

This review is based on Refs. [1,2,3]. We recall the calculation of the CDM abundance (Sec. 2) in the context of the Standard scenario (SC) (Sec. 3) and we show how this calculation is modified in three different cases: When we consider a Low Reheating Scenario (LRS)\(^1\) (Sec. 4), a Quintessential Scenario (QKS)\(^2\) (Sec. 5) or a Kination-Dominated Reheating (KRS)\(^3\) (Sec. 6). A comparison between the various scenarios is displayed in Table 1 whereas in Table 2 we present some representative combinations of the parameters which produce the central observational value of the CDM abundance. Throughout the subscript or superscript 0 [I] is referred to present-day values (except for the coefficient $V_0$) [to the onset of each scenario] and $\bar{\rho}_i = \rho_i / \rho^0_c$ where $\rho^0_c = 8.099 \times 10^{-47} h^2$ GeV\(^4\) with\(^4\) $h = 0.73$. 

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CDM ABUNDANCE IN NON-STANDARD COSMOLOGIES

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2. The CDM abundance

In light of the recent WMAP3 results\(^4\), the relic density of any CDM candidate \(\chi\), \(\Omega \chi h^2\), is to satisfy the following range of values:

\[
\Omega \chi h^2 = 0.1045^{+0.0075}_{-0.0095} \Rightarrow 0.08 \lesssim \Omega \chi h^2 \lesssim 0.12 \ \text{at 95\% c.l.} \ (1)
\]

We concentrate our presentation mainly on the Lightest SUSY Particle (LSP) which is stable within the SUSY models with \(R\)-parity conservation\(^5\) and consists the most popular, promising and natural CDM candidate\(^6\).

The calculation of \(\Omega \chi h^2\) is based on the formula:

\[
\Omega \chi h^2 = 2.741 \times 10^8 \ Y_0 \ m_\chi/\text{GeV}, \ \text{where} \ Y_0 = n_\chi^0/s_0, \ (2)
\]

\(s \propto T^3\) is the entropy density, \(m_\chi\) is the mass of \(\chi\) and \(n_\chi\) is the number density of \(\chi\)'s, which satisfies a Boltzmann Equation (BE), provided that \(\chi\)'s achieve kinetic equilibrium with plasma. The form of the BE depends on the scenario under consideration. In general, it depends on: (i) The Hubble parameter, \(H = \sqrt{\rho_{\text{BG}}/3m_p}\) \((m_p = M_P/\sqrt{8\pi}, \ \text{with} \ M_P, \ \text{the Planck scale})\) with \(\rho_{\text{BG}}\), the background energy density, (ii) the equilibrium number density of \(\chi\)'s, \(n_{\text{eq}}^\chi\), which obeys the Maxwell-Boltzmann statistics:

\[
n_{\text{eq}}^\chi(x) = \frac{g}{(2\pi)^{3/2}} m_\chi^3 x^{3/2} \ e^{-1/x} P_2(1/x), \ \text{where} \ x = T/m_\chi < 1, \ (3)
\]

with \(g = 2\) the number of degrees of freedom of \(\chi\) and \(P_n(z) = 1 + (4n^2 - 1)/8z\), (iii) the thermal-averaged cross section times velocity of \(\chi\)'s, \(\langle \sigma v \rangle\), which can be mostly expanded as: \(\langle \sigma v \rangle = a + bx\). We focus on the case \(\langle \sigma v \rangle = a\) with \(10^{-15} \ \text{GeV}^{-2} \leq \langle \sigma v \rangle \leq 10^{-7} \ \text{GeV}^{-2}\) which can be naturally produced within SUSY models\(^1,2,3\).

Moreover, two fundamental cases of \(\chi\)-production can be singled out\(^7\): \(\chi\)'s do or do not maintain chemical equilibrium with plasma. In the first case (EP) the current value of \(n_\chi/s\) follows \(n_{\text{eq}}^\chi/s\) and at some \(T = T_P\), \(n_\chi/s\) becomes larger than \(n_{\text{eq}}^\chi/s\). On the other hand, in the case of non-EP, \(n_\chi/s \gg n_{\text{eq}}^\chi/s\) (non-EPI) or \(n_\chi/s \ll n_{\text{eq}}^\chi/s\) (non-EPII) at least at the point of the maximal \(\chi\) production\(^3,7\).

| SC | LRS | QKS | KRS |
|----|-----|-----|-----|
| \(\rho_\eta = \rho_\phi = 0\) | \(\rho_{\phi_1} \gg \rho_{\phi_{11}}, \ \rho_\eta = 0\) | \(\rho_{\phi_1} \gg \rho_{\phi_{11}}, \ \rho_\eta = 0\) | \(\rho_{\phi_1} \gg \rho_{\phi_{11}} \gg \rho_{\phi_{111}}\) |
| \(H \propto T^2\) | \(H \propto T^4\) | \(H \propto T^3\) | \(H \propto T^4\) |
| \(T \propto R^{-1}\) | \(T \propto R^{-3/8}\) | \(T \propto R^{-1}\) | \(T = \text{cst}\) |
| \(sR^3 = \text{cst}\) | \(sR^3 \neq \text{cst}\) | \(sR^3 = \text{cst}\) | \(sR^3 \neq \text{cst}\) |
| \(N_\chi = 0\) | \(N_\chi \neq 0\) | \(N_\chi = 0\) | \(N_\chi \neq 0\) |

Table 1. Standard vs non-Standard Scenarios
3. The Standard Cosmological Scenario (SC)

According to the SC\(^5\), \(\chi\)'s (i) are produced through thermal scatterings, (ii) reach chemical equilibrium with plasma and (iii) decouple from the cosmic fluid at a temperature \(T = T_F \sim 15\) GeV during the radiation-dominated (RD) era. The consequences of the assumptions above are: (i) The form of the relevant BE is (dot denotes derivative w.r.t the cosmic time),

\[
\dot{n}_\chi + 3Hn_\chi + \langle \sigma v \rangle (n^2_\chi - n^{eq2}_\chi) = 0 \tag{4}
\]

which can be solved numerically (or semi-analytically using the freeze-out procedure\(^1,\text{2,3}\)) with initial condition \(n_\chi(x = 1) = n^{eq}_\chi(x = 1)\) or \(n_\chi(x = 1) = 0\), (ii) the required \(\langle \sigma v \rangle\) is \(\langle \sigma v \rangle \gtrsim 10^{-20}\) GeV\(^{-2}\) (note that with \(\langle \sigma v \rangle \approx 2.9 \times 10^{-29}\) GeV\(^{-2}\) and \(n_\chi(x = 1) = 0\), we can obtain \(\Omega_\chi h^2 = 0.1\) if we allow for non-EPII), (iii) the cosmological evolution during the \(\chi\) decoupling is RD and so, \(\rho_{BG} \propto \rho_R \propto T^4\). Therefore, \(H \propto T^2\) and \(T \propto R^{-1}\), where \(R\) is the scale factor of the universe (see Table 1).

In this context, the \(\Omega_\chi h^2\) calculation depends only on two parameters: \(m_\chi\) and \(\langle \sigma v \rangle\). As shown in Table 2 (1st column), \(\langle \sigma v \rangle \sim 10^{-9}\) GeV\(^{-2}\) can ensure acceptable \(\Omega_\chi h^2\)'s. Such a requirement strongly restricts the parameter space of many particle models. However, this picture can drastically change if one or more assumptions of the SC are lifted.

4. The Low Reheating Scenario (LRS)

The modern cosmo-particles theories are abundant in scalar massive particles (e.g. moduli, PQ-flatons, dilatons) which can decay when \(H\) becomes equal to their mass creating episodes of reheating. In the LRS, we assume that such a scalar particle \(\phi\), with mass \(m_\phi\), decays with a rate \(\Gamma_\phi\) into radiation, producing an average number \(N_\chi\) of \(\chi\)'s, rapidly thermalized. The energy densities of \(\phi\) and the produced radiation and \(\rho_R\) and \(\rho_\phi\) and \(n_\tilde{\chi}\), satisfy the following BEs \((\Delta_\phi = (m_\phi - N_\chi m_\chi)/m_\phi)\):

\[
\dot{\rho}_\phi + 3H\rho_\phi + \Gamma_\phi \rho_\phi = 0, \tag{5}
\]

\[
\dot{\rho}_R + 4H\rho_R - \Gamma_\phi \Delta_\phi \rho_\phi - 2m_\chi \langle \sigma v \rangle (n^2_\chi - n^{eq2}_\chi) = 0, \tag{6}
\]

\[
\dot{n}_\chi + 3Hn_\chi + \langle \sigma v \rangle (n^2_\chi - n^{eq2}_\chi) - \Gamma_\phi N_\chi n_\phi = 0, \tag{7}
\]

The system above can be solved, imposing the following initial conditions:

\[
H_I = \frac{m_\phi}{H_0} \Rightarrow \rho_\phi I = \frac{m_\phi^2}{H_0^2} \text{ and } \rho_{\tilde{\chi}} I = \rho_\chi I = 0. \tag{8}
\]

Our investigation verifies that the reheating process is not instantaneous\(^8\). Until its completion – i.e., \(\rho_\phi(T_{RH}) = \rho_R(T_{RH})\) –, the maximal temperature, \(T_{max} = f(\Gamma_\phi, \rho_\chi I)\), can become much larger than the so-called reheat
Table 2. Combinations of parameters leading to $\Omega_\chi h^2 = 0.1$ in the Standard and non-Standard Scenarios.

| $\chi$-Production: | SC | LRS | QKS | KRS |
|-------------------|----|-----|-----|-----|
| $\langle \sigma v \rangle$ (GeV$^{-2}$) | $2 \times 10^{-9}$ | $10^{-10}$ | $10^{-10}$ | $10^{-8}$ | $3 \times 10^{-8}$ | $10^{-10}$ | $10^{-10}$ | $10^{-10}$ |
| $T_\phi$ (GeV) | $-$ | 5.5 | 0.001 | 5.5 | $-$ | $-$ | 30 | 30 | 5.5 |
| $N_\chi$ | $-$ | 0 | $10^{-3}$ | $5 \times 10^{-5}$ | 0 | 0 | $10^{-6}$ | 0 | 0 |
| $\Omega_{\chi}^{NS}$ | $-$ | $-$ | $-$ | $-$ | 0.1 | 0.001 | 0.02 | $3 \times 10^{-9}$ | $2.4 \times 10^{-15}$ |
| $\log \rho_\phi_1$ | $-$ | 95.62 | 95.62 | 95.62 | $-$ | $-$ | 69.3 | 73.815 | 76.5 |
| $T_{PL}$ (GeV) | $-$ | $-$ | $-$ | $-$ | $-$ | $-$ | 1.2 | 16 | 32.5 |
| $\Omega_\chi h^2|_{SC}$ | 0.1 | 1.9 | 1.9 | 0.023 | $9 \times 10^{-5}$ | $7 \times 10^{-4}$ | 1.9 | 1.9 | 1.9 |

Note: We fix $m_\chi = 350$ GeV for the results presented in this table. We also use $m_\phi = 10^6$ GeV in the LRS and KRS and $\lambda = 0.5$ in the QKS and KRS (note, however, that the results on $\Omega_\chi h^2$ are $\lambda$-independent). Recall that in the LRS, $T_\phi = T_{RH}$ and the results on $\Omega_\chi h^2$ are $\rho_\phi_1$-independent and remain invariant for fixed $N_\chi m_{\phi}^{-1}$ (and, obviously, $T_\phi$, $m_\chi$, $\langle \sigma v \rangle$) – however, the type of the $\chi$-production does depend separately on $N_\chi$ and $m_\phi$. In each case, shown is the type of the $\chi$-production and the obtained value of $\Omega_\chi h^2$ in the SC, $\Omega_\chi h^2|_{SC}$. 
temperature, $T_{RH}$. Also, for $T > T_{RH}$, $\rho_{bc} \simeq \rho_\phi \Rightarrow H \propto T^4$ with $T \propto R^{-3/8}$ and an entropy production occurs (see Table 1).

The free parameters of the LRS are: $m_{\chi}$, $\langle \sigma v \rangle$, $\Gamma_\phi$, $m_\phi$, $N_\chi$, $\rho_{\phi I}$. However, the results on $\Omega_\chi h^2$ do not depend on the explicit value of $\rho_{\phi I}$ as long as $T_{RH} < T_T < T_{max}$, and are invariant\textsuperscript{1,10} for fixed $N_\chi m_\phi^{-1}$ (and $T_\phi$, $m_{\chi}$, $\langle \sigma v \rangle$). Moreover, $\Gamma_\phi$ can be replaced by $T_\phi \simeq T_{RH}$ through the relation\textsuperscript{3}:

$$\Gamma_\phi = 5 \sqrt{\frac{\pi^3 g_{\ast e}(T_\phi)}{45}} \frac{T^2}{M_P} \simeq \sqrt{\frac{5\pi^3 g_{\ast e}(T_\phi)}{72}} \frac{T^2}{m_v}.$$ \hfill (9)

As shown in Table 2, we can obtain acceptable $\Omega_\chi h^2$'s (i) for relatively low $\langle \sigma v \rangle$'s (2nd, 3rd columns) with low $T_{RH} = 5.5$ GeV and $N_\chi = 0\textsuperscript{1,9}$ (EP) or much lower $T_{RH} = 1$ MeV and $N_\chi \neq 0$ (non-EP/II) and (ii) for larger $\langle \sigma v \rangle$'s (4th column) with larger $N_\chi$'s (EP).

5. The Quintessential Scenario (QKS)

Another role that a scalar field could play when it does not couple to matter (contrary to $\phi$) is this of quintessence. Such a field, $q$, satisfies the equation:

$$\ddot{q} + 3H\dot{q} + dV/dq = 0, \quad \text{where} \quad V = V_0 e^{-\lambda q/m_v},$$ \hfill (10)

is the adopted potential\textsuperscript{11} and undergoes three phases during its cosmological evolution:\textsuperscript{2} (i) The kination\textsuperscript{12} dominated (KD) phase, where the energy density of $q$ is $\rho_q = \dot{q}^2/2 + V \simeq \dot{q}^2/2$, (ii) the frozen-field dominated (FD) phase, where $\rho_q$ is constant and (iii) the late-time attractor dominated (AD) phase, with $w_q = \lambda^2/3 - 1$ for $\lambda < \sqrt{3}$. Today we obtain a transition from the FD to the AD phase\textsuperscript{13}. Although this does not provide a satisfactory resolution of the coincidence problem, the observational data\textsuperscript{4}:

$$\Omega_q^0 = \Omega_{DE}^0 = 0.74 \pm 0.12 \quad \text{and} \quad w_q < -0.83,$$ \hfill (11)

can be reproduced with $\lambda \leq 1.1$ and by conveniently adjusting $V_0\textsuperscript{2,13}$.

For a reasonable region of initial conditions ($q_1 = 0$ and $\Omega_q^1 = 1$), $\rho_q \simeq \dot{q}^2/2$ can dominate over radiation, creating a totally KD era in conjunction with the satisfaction of the nucleosynthesis (NS) constraint\textsuperscript{15}, $\Omega_q^{NS} \lesssim 0.21$. As a consequence, during the KD era we obtain: $\rho_{bc} \simeq \rho_q \Rightarrow H \propto T^3$ with $T \propto R^{-1}$ (see Table 1). Combining eqs. (4) and (10) we can show that if the $\chi$-decoupling occurs during the KD era, $\Omega_\chi h^2$ increases\textsuperscript{14} w.r.t $\Omega_\chi h^2_{|SC}$ (see 5th and 6th column in Table 2). This enhancement of $\Omega_\chi h^2$ turns out to be a single-valued function of the quintessential parameter at the eve of NS$^2$, $\Omega_q^{NS}$, for fixed $m_{\chi}$ and $\langle \sigma v \rangle$. Therefore, the $\Omega_\chi h^2$ calculation depends only on the parameters: $m_{\chi}$, $\langle \sigma v \rangle$, $\Omega_q^{NS}$ ($\lambda \leq 1.1$).
6. The Kination-Dominated Reheating (KRS)

In view of the two previous situations, the obvious question would be: What happens if we have both quintessence and low reheating? Or, a low $T_{RH}$ could assist us to the reduction of $\Omega_\chi h^2$, in the presence of a KD phase? This novel cosmological set-up can be analyzed by solving the system of Eqs. (5)-(7) and (10) with constraint $\Omega_I^q = 1$ and initial conditions:

$$ q_i = 0, \quad H_i = m_\phi \Rightarrow \dot{q}_i = \sqrt{2\rho_0^q(m_\phi/H_0)} \quad \text{and} \quad \bar{\rho}_i = \bar{\rho}_{qI} = 0. \quad (12) $$

We can distinguish two types of $q$-domination, depending whether $\phi$ decays before or after it becomes the dominant component of the universe. In both cases, $\rho_{BG} \simeq \rho_q \Rightarrow H \propto T^3$, entropy production occurs and a prominent period of constant maximal temperature, $T_{PL} = f(\bar{\rho}_{qI}, \Gamma_\phi/m_\phi)$, arises\(^3\) (see Table 1). The free parameters of this scenario are: $\lambda, \bar{\rho}_{qI}, m_\phi, T_\phi, N_\chi, m_\chi, \langle \sigma v \rangle$. As in the QKS the $\Omega_\chi h^2$ calculation is $\lambda$-independent for $\lambda \leq 1.1$ but unlike the LRS it severely depends on $\bar{\rho}_{qI}$, and $T_\phi$ does not coincide with the maximal temperature of the RD era.

The crucial difference between the KRS and the QKS is that $\Omega_\chi h^2$ does not exclusively increase with $\Omega_I^NS$. Indeed, when an increase of $\Omega_I^NS$ is generated by an increase of $T_\phi$ (which results to an increase of $T_{PL}$), $\Omega_\chi h^2$ increases with $\Omega_I^NS$ – see Fig. 1-(a). On the contrary, when the increase of $\Omega_I^NS$ is due to the decrease of $\bar{\rho}_{qI}$ (which results to a decrease of $T_{PL}$), $\Omega_\chi h^2$ decreases as $\Omega_I^NS$ increases – see Fig. 1-(b). This is, because $n_\chi$ decreases rapidly with $T_{PL}$ due to the exponential suppression of $n_{eq}^q$ in Eq. (3).

As shown in Table 2 (7th and 8th columns), $\Omega_\chi h^2$ reaches the range
of Eq. (1) with $N_\chi \sim (10^{-7} - 10^{-5})$ when $T_{PL} \ll T_F$ (non-EPI) and with $N_\chi \sim 0$ when $T_{PL} \sim T_F$ (non-EPII). As $\Omega_{NS}^q$ decreases, $(T_{PL} - T_F)$ increases and $\Omega_\chi h^2$ approaches its value in the LRS (9th column in Table 2).

7. Conclusions

We considered three deviations from the SC and we showed\textsuperscript{1,2,3} that: (i) In the LRS with $T_{RH} < 20$ GeV, $\Omega_\chi h^2$ decreases w.r.t its value in the SC for low $N_\chi$’s and increases for larger $N_\chi$’s. Both EP and non-EP are possible for commonly obtainable $\langle \sigma v \rangle$’s, (ii) in the QKS, $\Omega_\chi h^2$ increases drastically (almost 3 orders of magnitude for $\Omega_{NS}^q$ close to its upper bound) (iii) in the KRS, $\Omega_\chi h^2$ becomes cosmologically interesting for $N_\chi \sim (10^{-7} - 10^{-5})$ when $T_{PL} \ll T_F$ (non-EPI), and for $N_\chi \sim 0$ when $T_{PL} \sim T_F$ (non-EPII); EP is activated for $T_{PL} > T_F$ and the results on $\Omega_\chi h^2$ approach their values in the LRS or SC as $T_{PL}$ increases well beyond $T_F$.

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