Neutrino-Electron Scattering: Charge Radius and Effective Couplings

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Abstract. In this work the neutral-current scattering cross-section for neutrinos on electrons is calculated assuming that a massive Dirac neutrino is characterized by a phenomenological parameter, a charge radius \( \langle r^2 \rangle \) and the right-handed currents are present in the framework of a Left-Right symmetric model (LR). Using the CHARM II result for the charge radius of the muon-neutrino \( |\langle r^2 \rangle| < 6.0 \times 10^{-33} \text{ cm}^2 \), we place a bound on \( -7.9 \times 10^{-33} \text{ cm}^2 \leq \langle r^2 \rangle_{LR} \leq 7.9 \times 10^{-33} \text{ cm}^2 \). We discuss the relationship between the electron neutral couplings \( g_{eV} \) and \( g_{eA} \) and the LR model parameters.

1. Introduction
In general, a photon may couple to charged leptons through its electric charge, magnetic dipole moment (MM), electric dipole moment (EDM) and the anapole moment (AM). This coupling may be parametrized using a matrix element in which the usual \( \gamma^a \) is replaced by a more general Lorentz-invariant form [1]:

\[
\Gamma^\mu = F_Q(q^2)\gamma^\mu + F_M(q^2)i\sigma^{\mu\nu}q_\nu + F_E(q^2)\sigma^{\mu\nu}q_\nu\gamma_5 + F_A(q^2)(q^2\gamma^\mu - q^\mu q^5)\gamma_5,
\]

where \( F_{Q,M,E,A}(q^2) \) are the electromagnetic form factors of the neutrino, corresponding to the charge radius, MM, EDM and AM, respectively.

Electromagnetic properties of neutrinos are of fundamental importance and serve as a probe of physics beyond the SM. Several authors have shown that the charge radius of the neutrino is not a physical quantity [2], as demonstrated by the fact that it is gauge-dependent [3]. However, other authors claim that they can extract a gauge-independent neutrino charge radius which is, therefore, a physical observable. A definition of the neutrino charge radius that satisfies physical requirements, i.e. it is a physical observable, has recently been provided in the framework of the Pinch Technique formalism [4].

In this paper, we start from a Left-Right symmetric model (LR) [5] and assuming that a massive Dirac neutrino is characterized by a phenomenological parameter, a charge radius \( \langle r^2 \rangle_\nu \) we calculate the cross-section of the process \( \nu_\mu e^- \to \nu_\mu e^- \). We also estimate bounds on the charge radius of the muon-neutrino in the framework of the LR model.
2. The Muon-neutrino Electron Cross-Section

In this section we obtain the corresponding amplitude for the process

$$\nu_\mu(k_1) + e^-(p_1) \rightarrow \nu_\mu(k_2) + e^-(p_2),$$

mediated by the photon $\gamma$ and the neutral gauge bosons $Z_L$ and $Z_R$. We assume that a massive Dirac neutrino is characterized by a phenomenological parameters, a charge radius $\langle r^2_\nu \rangle$. Therefore, the expression for the cross-section of the process $\nu_\mu e^- \rightarrow \nu_\mu e^-$ is given by

$$\sigma_{LR}^{T} = \frac{G_F^2 m_e E_\nu}{2 \pi} \left\{ 2 \delta^2 + 2 \delta(P + S) + \frac{(P + S)^2 + (Q + R)^2}{2} \right\},$$

where $P, Q, R, S$ are given by

$$P = (A + 2B + C)g_V, \quad Q = (-A + C)g_A, \quad R = (-A + C)g_V, \quad S = (A - 2B + C)g_A.$$ (4)

The constants $A, B$ and $C$ depend only on the parameters of the LR model

$$A = (c_\phi - \frac{s^2_W}{r_W} s_\phi)^2 + \gamma \frac{s^2_W}{r_W} c_\phi + s_\phi)^2,$$

$$B = (c_\phi - \frac{s^2_W}{r_W} s_\phi)(-\frac{c^2_W}{r_W} s_\phi) + \gamma (\frac{s^2_W}{r_W} c_\phi + s_\phi)(\frac{c^2_W}{r_W} c_\phi),$$

$$C = (\frac{c^2_W}{r_W} s_\phi)^2 + \gamma (\frac{c^2_W}{r_W} c_\phi)^2,$$

$$\gamma = \frac{(M_{Z_L})^2}{M_{Z_R}},$$

$$\delta = \frac{\sqrt{2} \pi \alpha}{3G_F} \langle r^2 \rangle,$$

where $c_\phi = \cos \phi$, $s_\phi = \sin \phi$, $c_W = \cos \theta_W$, $s_W = \sin \theta_W$, $r_W = \sqrt{\cos 2\theta_W}$ and $M_{Z_L}$, $M_{Z_R}$ are the masses of the light and heavy massive neutral vector bosons that participate in the interaction. $\gamma$ together with $\phi$ are the two new parameters that are introduced in the LR model, while $\langle r^2 \rangle$ is the charge radius of the muon-neutrino. Bounds on this quantity are reported in the literature [6]. We can in addition take the limit $\phi = 0$ and $M_{Z_2} \rightarrow \infty$ and get the standard model cross-section.

From the cross-section expression (3), we obtain the interference cross-section which is given by

$$\sigma_{LR}^{I} = \frac{2G_F^2 m_e E_\nu \delta}{3 \pi} (2P + S).$$ (6)

We rewrite the interference cross-section in order to identify the neutrino charge radius in the LR model.
\begin{equation}
\langle r^2 \rangle_{LR} = \langle r^2 \rangle [1 + \left( \frac{2g_V a_W + g_A b_W}{2g_V + g_A}\right) \gamma + \left( \gamma - 1 \right)\left( \frac{2g_V + g_A b_W}{2g_V + g_A}\right) s_\phi c_\phi],
\end{equation}

where \( \langle r^2 \rangle_{LR} \) is the charge radius of the muon-neutrino in the LR model and \( \langle r^2 \rangle \) is the charge radius in the minimal extension of the standard model.

The total cross-section Eq. (3) is written in such a way that we can express the theoretical predictions for the electron couplings constant \( (g^e_V)_{LR} \) and \( (g^e_A)_{LR} \) such that give the SM couplings in the limit \( \phi = 0 \) and \( M_{Z_2} \to \infty \):

\begin{equation}
(g^e_A)_{LR}[M_{Z_L}, M_{Z_R}, \phi, \delta, \sin \theta_W, g^e_A] = [\delta f_3 + \frac{1}{2}(f_1 f_3 + f_2^2)](g^e_A)_{SM},
\end{equation}

\begin{equation}
(g^e_V)_{LR}[M_{Z_L}, M_{Z_R}, \phi, \delta, \sin \theta_W, g^e_V] = \delta f_3 + \frac{1}{2}(f_1 f_3 + f_2^2)(g^e_V)_{SM}.
\end{equation}

In these expressions, with the limits \( \delta = 0, \phi = 0 \) and \( M_{Z_R} \to \infty \), the SM couplings are recovered.

3. Results and Conclusions

In this section, we present the numerical results obtained for the charge radius of the muon-neutrino in the framework of a Left-Right symmetric model \( \langle r^2 \rangle_{LR} \) and the electron couplings constants \( (g^e_V)_{LR} \) and \( (g^e_A)_{LR} \). For the mixing angle \( \phi \) between \( Z_L \) and \( Z_R \), we use the reported data in Ref. [7]: \(-1.66 \times 10^{-3} \leq \phi \leq 1.22 \times 10^{-3} \), with a 90% C.L.

In order to estimate a limit on the charge radius of the muon-neutrino \( \langle r^2 \rangle_{LR} \) in the framework of the Left-Right symmetric model, we plot the expression (7) in Fig. 1. In this figure, we show the allowed region for \( \langle r^2 \rangle_{LR} \) as a function of \( \phi \) with 90% C.L. The allowed region is the rectangle band that is a result of both factors in Eq. (7). In this figure, the second factor gives the rectangle band while \( \langle r^2 \rangle \) gives the band width. This analysis was done using the experimental value for \( \langle r^2 \rangle \) reported by CHARM II [6] with a 90% C.L. In the same figure, we show the \( \langle r^2 \rangle \) (\( \phi = 0 \)) result at 90% C.L. with the dashed horizontal lines. The allowed region in the LR model (dotted line) for \( \langle r^2 \rangle_{LR} \) is wider than the one for the \( \langle r^2 \rangle \), and is given by

\begin{equation}
-7.9 \times 10^{-33} \text{ cm}^2 \leq \langle r^2 \rangle_{LR} \leq 7.9 \times 10^{-33} \text{ cm}^2, \ 90\% \ C.L.,
\end{equation}

whose value is quite close to that reported by other authors [6].

Fig. 2 shows the charge radius \( \langle r^2 \rangle_{LR} \) as a function of the LR parameters \( \phi \) and \( M_{Z_R} \). This figure shows a strong dependence of the charge radius with respect to the model parameters.

In Fig. 3 we have plotted \( (g^e_A)_{LR} \) from Eq. (9) as a function of the LR parameters \( \phi \) and \( M_{Z_R} \). We have chosen the range \(-1.66 \times 10^{-3} \leq \phi \leq 1.22 \times 10^{-3} \) and \( 300 \leq M_{Z_R} \leq 800 \text{ GeV} \) where \( \phi \) is measured in radians.

In Fig. 4 we have plotted \( (g^e_V)_{LR} \) from Eq. (8) as a function of \( \phi \) and \( M_{Z_R} \). The range of variation for the LR parameters is the same as in Fig. 3. In this case the experimental value, \( (g^e_V)^{exp}_{LR} = -0.035 \) is reached for small values of \( M_{Z_R} \) and \( \phi \). The effect of \( \phi \) and \( M_{Z_R} \) on \( (g^e_A)_{LR} \) and \( (g^e_V)_{LR} \) are similar.

In summary, we have estimated bounds that can be derived from the muon-neutrino electron scattering. Our bounds on the neutrino charge radius \( \langle r^2 \rangle_{LR} \) and the electron couplings constants \( (g^e_V)_{LR} \) and \( (g^e_A)_{LR} \) are consistent with those reported in the literature and in some cases, improve the existing bounds. However, new experiments dedicated to the detailed study of electron (anti) neutrino interactions with matter, for example the reactor MUNU, as well as radioactive sources of neutrinos such as the BOREXINO detector, should be able to improve existing limits on the neutrino charge radius, magnetic moment and other parameters.
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Figure 1. Allowed region for $\langle r^2 \rangle_{LR}$ as a function of the mixing angle $\phi$ with the value $\langle r^2 \rangle$.

Figure 2. Plot of $\langle r^2 \rangle_{LR}$ as a function of the LR parameters $\phi$ and $M_{Z_R}$.

Figure 3. Plot of $(g^\nu e)_A^{LR}$ as a function of the LR parameters $\phi$ and $M_{Z_R}$.

Figure 4. Plot of $(g_A^{\nu\nu})_{LR}$ as a function of the LR parameters $\phi$ and $M_{Z_R}$.

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