Hydraulic Project Development and Corresponding Ecological Factor Analysis

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Abstract
Currently, hydraulic project development requires assessment of environment implications. Understanding fish shape, swimming ability and energy cost of the fish during swimming will help to design fish-friendly hydraulic structure. In this paper, a self-propulsion of 3D-fish model has been developed. The interaction of the fish and fluid are taken into account. The fish swimming velocity, torques and energy expending during swimming with different shapes are calculated. The fish swimming ability with different beat frequency (1.43, 0.94 and 0.64 Hz) are also analysed. The study has following objectives: to determine the mechanisms of fish swimming, fish swimming velocity and analyses the energy of the fish expended consider the fish length, body shapes, tail beat frequency.

Keywords: Fish swimming velocity; Fish locomotion; Energy expending; Fish swimming simulation

Introduction
Understanding fish swimming ability and energy cost could help for better design fish-friendly hydraulic project. The fish swimming capacity is depends on a variety of factors such as the metabolic cost [1], oxygen requirement [2], turbulence in the water [3], types of muscles used [1] and fatigue times [4]. The fish swimming mechanism is also researched by many biologist, engineers and scientist [5]. For example, Heisey et al. [6,7] studies the survivability of fish passing through obstructs. Videler [8] and Webb [9] studied the relationship between fish swimming speed and the fish tail beat frequency and fish tail beat amplitude respectively. The length and speed of the body wave and body undulates are have infects on fish swimming [10-13]. The shape of the fish also has been considerable discussion in the literature, with various authors considering the fish shapes [14-18]. Modelling of swimming fish is a both river engineering and biological model system for understanding the mechanics activity during locomotion. In the last decade, a huge amount of efforts have been done by mathematicians to understand the dynamics of swimming in a fluid [19,20]. These models help to better understand the impressive swimming capabilities of fish. In order to gain insight into fish-locomotion, mathematic models are the desirable approach to use for testing fish locomotion to study fish swimming velocity, torques and energy cost during swimming. Experiments have shown that most fish swim using body undulations to generate thrust and maneuver in three dimensions [18]. So the patterns of body bending play a crucial role to understand their locomotion. According to Lauder et. al [18], the pattern of body bending during steady rectilinear locomotion has similar general characteristics in many fishes and involves a wave of increasing amplitude passing from the head region toward the tail. Real life fish are bodies of complexity shape and a complex surface roughness distribution due to the scales. For that reason, most researchers choose a prolate ellipsoid instead of fish. However, prolate ellipsoid could be faced with the huge error of biological criteria. Hence, in our research, we use the actual fish body shape instead of prolate ellipsoid to study the fish biological characteristics. To the best of authors’ knowledge, there are no published results on simulating the fish swimming ability and energy cost with the consideration of fish shapes, length and tail beat frequency. This paper a mathematic model is used to simulate the fish undulatory locomotion in the water. After a brief introduction of the theory of this model, we discuss the design of a robotic fish that allows measurement of self-
propelled velocity, torques and the energy cost with two types of fish. We also address an important technical issue: to check the fish swimming ability in an ideal fluid. Our overall aim is to understand the swimming mechanics of different shape and different types of fish and calculate the energy of the fish expended which could be used as guidance criteria to design low fish mortality turbine and fish way.

Mathematical Formulations and Evaluation

The constrain-based formulation for self-propulsion is described as follow. The fish is consisted by fish bone and the ellipse shape attached to the fish bone. The fish and its deformations are modelled according to a beam model. In other words, after the deformations of the fishbone are prescribed, each orthogonal ellipse of the fish is supposed to move rigidly, remaining orthogonal to the fishbone. Each ellipse shape is treated as a rigid body and the fish swimming paths are oriented by the first one. In the study, we assume the arc length of the fishbone to 1dm with 7 rigid bodies (Figure 2). In order to ensure the smooth of the fish, we need make the first and second derivatives for those parameters. When the second order discretization is made, the fish shape smoothly can be guaranteed (Figure 3A, 3B). After the salmon shape is defined, the sequential coupling of the salmon shape into the surface triangular mesh was done with accuracy of 10−3 and zero-gradient condition at the mesh on the computational domain is set. At every moment, the position and orientation of the fish are given by a pair (r, R) where r is a vector giving the position of the centre of mass of the fish and R is a rotation matrix giving its orientation. Every rigid fishbone (3D) has six degrees of freedom corresponding to its rigid component of motion and it is carried out in a frames(r, ε) related to the fish. Where ε is the Euler angle (α, β, γ) of fish bone. We set the rotation angle (γ) of fish to 5 degrees which is to insure the stability of fish is during fish swimming [21]. The angle of α and β is change with time, in this simulation we define α and β are the tangent angle of sint in x and y axis. The fish tail beat frequency is also determined by sinusoidal motion. The first fish bone is defined by the real fish swimming path which is included in fish swimming control file (Figure 1). The fish swimming control file can be used to define the fish movement function. After get the hydrodynamics position of the first rigid bones of the fish, the following rigid bones can be defined through linear movement and rotation matrix. In i part of the rigid bones, the position of Pᵢ is defining as follows:

\[ Pᵢ = Pᵢ₋₁ + Aᵢ₋₁ Pᵢ₋₁ \]

Where \( Pᵢ₋₁ \) is linear distance between the i part and the i-1 part of the fish, \( Pᵢ₋₁ \) is the relative position of i−1 part. The rotation matrix A is defined as follow:

\[
A = \begin{bmatrix}
\cos \alpha \cos \gamma & -\cos \beta \sin \alpha \sin \gamma & \sin \alpha \cos \gamma + \cos \beta \sin \alpha \sin \gamma & \sin \beta \sin \gamma \\
\cos \alpha \sin \gamma & \cos \beta \cos \alpha \cos \gamma & -\sin \alpha \sin \gamma + \cos \beta \cos \alpha \cos \gamma & \sin \beta \cos \gamma \\
\sin \beta \sin \alpha & \sin \beta \cos \alpha & 0 & 0
\end{bmatrix}
\]

And the inverse rotation matrix of A is as below:

\[
A^{-1} = \begin{bmatrix}
\cos \alpha \cos \gamma & \cos \beta \sin \alpha \sin \gamma & -\sin \alpha \cos \gamma + \cos \beta \sin \alpha \sin \gamma & \sin \beta \sin \gamma \\
\cos \alpha \sin \gamma & \cos \beta \cos \alpha \sin \gamma & -\sin \alpha \sin \gamma + \cos \beta \cos \alpha \sin \gamma & \sin \beta \cos \gamma \\
\sin \beta \sin \alpha & \sin \beta \cos \alpha & 0 & 0
\end{bmatrix}
\]
The total computational domain $\Phi_f$ occupied by the fish is $\Phi_s$ and the fluid domain $\Phi_f/\Phi_s$. $\Omega$ is the surface of the fish and $n$ (Eq. 4) means the unitary normal vector to $\Omega$ directed toward the exterior of the fluid. The fluid is assumed to be irrotational, inviscid and incompressible and governed by Euler equations. The potential acting on the fish from fluid flow is calculated by classical Kirchhoff’s equations. The forces and torques acting on the body are due to a combination of added-mass effects, skin friction and free vorticity [22]. These have been described by Chambrion et al [23,24], Kanso et al [25], Munnier [20,26] and Kirchhoff’s equations has proved to be a very efficient for determining the hydrodynamic forces [27] and it has been a standard method when dealing with a problem of interactions between a perfect fluid and rigid immersed structures. The classical Kirchhoff’s equations are derived under the assumption that the rigid body is submerged in an infinitely large volume of an ideal fluid. Assume $v$ and $\omega$ to be the linear and angular velocity vectors of the rigid body. The key issue in this model is to calculate the linear and angular velocity in fluid and through linear and angular velocity, the torques, the forces and the energy can be gotten. In order to calculate eulerian velocity exerted by the fluid on the fish’s surface, at every time step, the Laplace equation which is refer to as Kirchhoff’s equations set on $\Omega$ will be solved according to the equations in fluid dynamics [24]. The overall potential is gotten as follow:

$$\frac{d \sigma T}{dt} = \nabla \times v + q_h + q$$

$$\frac{d \sigma T}{dt} = \nabla \times \omega + F_h + F$$

$$T = \frac{1}{2}(\omega^T I \omega + m\omega^2)$$

$$Q_h = -\int p x \times n d\sigma$$

$$H_k = -\int p n d\sigma$$

Where $\omega$ and $\upsilon$ are the angular and linear velocity vectors of the fish body; respectively; $t$ is simulation time, $\sigma$ is the density of the fish surface, $x$ is the vertical distance between the surface mesh and the fish bones. I is the moment of inertia tensor; $m$ is the fish body’s mass which is the integral of mesh; $n$ is a unit normal to the surface of the body; $p$ is a pressure at the fish body; $T$, $Q$ and $H$ are the Kinetic energy, hydrodynamic torque and force acting on the body, respectively. The integration is performed over the fluid-exposed portion of the fish’s surface and the fish surface satisfies Neumann boundary condition. To solve the equations 1 to 5, the PDEs (governing the fluid flow) and the ODEs (driving the rigid motion of the immersed body) was used which are the toolboxes on Matlab [23,24]. In the algorithm above, the fish can be rigid and deforming, which is depends on the nature of the fish bending angles $\alpha$, $\beta$, $\gamma$ in the rotation matrix $A$ and $A^{-1}$. Additionally, the fish is three-dimensional and it is immersed surface and curve can be rigorously derived. The spatial order of accuracy will depend on the discretization of the spatial derivatives, and the interpolation scheme used in the Eulerian- Lagrangian coupling between the fluid mesh and the marker particles representing the solid body. The flow chart of our model is shown in Figure 1 (Figure 1).

Convergence criteria are based on the maximum errors in calculation. Convergence is ensured when the maximum errors become less than 10-9. The current numerical technique has been very successfully applied and validated in a series of recent papers [24]. Tests were conducted for the application to determine the dependency of solutions upon mesh size. Solutions were obtained with successive refinement of mesh size, i.e., reduction in mesh sizes. When solutions were observed to be unaffected by further refinements, they were presumed to be mesh independent.

**Fish Definition**

In this research we choose salmon as research object (Figure 4) to simulate. A general description of the fish calculation is presented as follow: First we set the parameters of the salmons (Table 1) and the shapes of salmons are described in Figure 2. The flow chart of our model is shown in Figure 1 (Figure 1). The trajectories of virtual fish were determined by the fish swimming control file (Figure 1). The fish swimming path was defined as moving on straight line in fluid. We get the velocity of the fish swimming (Figure 6), calculated the torques of the fish, and energy (T) of the fish during swimming (Figure 7). The simulation was carried out for a total time of 20 second. The time step was 0.01 s for most of this period. During the first second, a dynamic time...
step was used (starting with a step of 0.001 s) in order to achieve convergence at each time step during the initial fast transient period starting from the prescribed initial condition. All programs were implemented in Mat lab codes and simulations were run on AMD Athlon II PCs. Data analysis and visualization were done with Mat lab. Simulation time for a fish that of a fish with 681 finite volumes in 20 s, took approximately 1 hour (Table 1) (Figures 2-4).

Table 1: Fish shape parameters of salmon one and salmon two (mm).

| Number | Semi-minor axes of the ellipses of fish one (mm) | Semi-major axes of the ellipses of fish one (mm) | Semi-minor axes of the ellipses of fish two (mm) | Semi-major axes of the ellipses of fish two (mm) |
|--------|-----------------------------------------------|-----------------------------------------------|-----------------------------------------------|-----------------------------------------------|
| 1      | 0.12                                          | 0.2                                           | 0.24                                          | 0.28                                          |
| 2      | 0.4                                           | 0.4                                           | 0.48                                          | 0.44                                          |
| 3      | 1.7                                           | 1.8                                           | 1.8                                           | 1.9                                           |
| 4      | 0.1                                           | 0.2                                           | 0.12                                          | 0.23                                          |
| 5      | 10                                            | 10                                            | 12                                            | 12                                            |
| 6      | 0.4                                           | 10                                            | 0.38                                          | 0.38                                          |
| 7      | 0.4                                           | 0.4                                           | 0.38                                          | 0.38                                          |

Results and Discussion

In this section we validate the model by focus on two different shapes of fishes: I) salmon fish one, II) salmon fish two. These cases have been chosen to demonstrate the capability of our approach at fish swimming velocity, torques and energy expending simulation, to demonstrate easy application of the method to different fish, and to quantitatively compare the effect of shape and tail beat frequency on swimming performance of two fish.

The effect of shape on swimming performance

The two salmon shapes are indicated in Figure 2(b) and Figure 3 (b) and the comparison of salmon fish one and salmon fish two swimming distance is shown in Figure 5. The swimming distance, velocity, torques and energy expends of salmon fish one is represented by green grids, while salmon fish two is shown using blue grids to represent it. In order to ensure the consistency of the simulation, for the first and second fish, simulation time is set the same (20 s) and the tail beat frequency of two fish is also set the same (0.95 Hz). The velocity, torques and energy that fish spend during swimming are calculated and given in Figure 6, 7 and 8. The figure 8 is the animation image of the fish swimming. It is clear from the swimming distance shown in Figure 5(a) and Figure 5(b) that the first fish swimming distances is much bigger than the second one. The values of the swimming distance of fish one and fish two are 35.02 and 14.91 cm respectively. The average swimming velocity of fish one is also bigger than the fish two with 1.750 and 0.746 cm/s respectively. The energy expending of the two fishes are $1.49 \times 10^{-4}$ and $1.73 \times 10^{-4}$ watt respectively. A comparison of the effect of shapes on swimming performance are shown that the shape of fish have big influence in swimming ability which is similar to the conclusion of Reid [5]. Although the second fish spend more energy than the first one, but the velocity of the second fish are lower than the first one (Figures 5-9).

Figure 5: a) Fish swimming distance of salmon fish one in 20 seconds; B) Fish swimming distance of salmon fish two in 20 seconds.

Figure 6: a) Fish swimming velocity of salmon fish one; B) Fish swimming velocity of salmon fish two.
The effect of tail beat frequency on swimming performance

The tail beat frequency is often used for the analysis of swimming speed and many researchers have study the relationship between fish swimming and tail beat frequency (Table 2). Here, we use the same fish shapes with salmon fish one (Figure 2b) and same swimming time (20 s) with difference frequency to check the correlation between fish swimming velocity and tail beat frequency in our model. The tail beat frequency used in our simulation is defined as 1.43 and 0.64 Hz respectively. The swimming distance, the swimming velocity, the energy expending during swimming and the fish swimming images are shown in Figure 10, 11, 12 and 13.

Figure 10 shows salmon fish one swimming distance for different tail beat frequency. From Figure 10 (a) and 10 (b), it is apparent that the fish with high frequency can swim longer than low frequency and the swim velocity are also increased with the beat frequency increase. The energy cost of the two fishes is also difference in two beat frequency types. The values of the swimming distance of fish one and fish two are 49.12 and 25.84 cm respectively. The value of the mean velocity of fish with high F and low F are 2.456 and 1.292 cm/s respectively. The energy expending of the two fishes are $1.49 \times 10^{-4}$ and $0.61 \times 10^{-4}$ watt respectively. This result has the same trend with empirical equation made by He et al [28], Sepulveda et al [29], Ohlberger et al [30] and Li et al [31]. Comparing the results...
of Table 2 and the results determined by our model, it appears that until both the fish shapes and swimming types are taken into consideration, the equation can be more accuracy. At this point, our model is more reasonable and described more specifically. Basically, even we don’t have the real fish data to evaluate our model, the agreement between the results in Table 2 and model calculation gives confidence to accept the model predictions. It is important that our model is matched to the actual situation of fish swimming, so that the fish swimming velocity and energy cost can be quantify with high accuracy (Table 2) (Figures 10-13).

| Fish type          | Equation of F and V | Developed by            |
|--------------------|---------------------|-------------------------|
| Salmon             | V = 0.95F-0.46      | He & Wardle [28]        |
| Isurus oxyrinchus  | F = 0.94+0.14V      | Sepulveda et al [29]    |
| Lemon shark,      | F = 1.5V0.48        | Sepulveda et al [29]    |
| Negaprion brevirostris | F = 1.31V+0.35    | Ohlberger et [30]       |
| Cyprinoid          | V = 20.65F-34.80    | Li et al [31]           |

Figure 10: a) Fish swimming distance of salmon fish one in salmon 20 seconds (F=1.43); B) Fish swimming salmon 20 fish one in 20 seconds (F=0.64).

Figure 11: a) Fish swimming velocity of salmon fish one (F=1.43); B) Fish swimming velocity of salmon fish one (F=0.64).

Figure 12: a) Energy expended by salmon fish one during one swimming of salmon fish one (F=1.43); B) Energy expended by one salmon fish during swimming of salmon fish one.
Conclusion

We presented a computational approach for fully resolved simulation of self-propulsion of 3d fish through an ideal fluid. This algorithm is developed that solves for the velocity of the fish for prescribed deformation kinematics. A solution for the fish swimming ability, torques energy expending and the swimming path are developed. The codes of fish swimming velocity, swimming cost of the fish is also obtained. The effect of the shapes and the effects of fish beat frequency on swimming performance are also analysed. So the purpose of this model would be used to assist in the design of fish-friendliness of hydraulic structure, the resulting computer tool had to be very efficient and relatively easy to use. The fish design concepts provided here can be used for both re-evaluating existing hydraulic structure as well as new structures design in order to improve their compliance with the new age of environmental awareness and fish safety. These concepts would also benefit the hydropower plant in many ways. We believe that understand the effects of fish shapes and tail beat frequency on fish swimming ability would result in a more efficient hydraulic operation and reduced operation and maintenance costs.

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Conflict of Interest

No conflict of interest.

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