**Q^2 Dependence of Azimuthal Asymmetries**  
in Semi-Inclusive Deep Inelastic Scattering  
and in Drell-Yan

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**Abstract**  
We study several azimuthal asymmetries in semi-inclusive deep inelastic scattering  
and in Drell-Yan, interpreting them within the formalism of the quark correlator,  
with a particular reference to T-odd functions. The correlator contains an  
undetermined energy scale, which we fix on the basis of a simple and rather general  
argument. We find a different value than the one assumed in previous treatments of  
T-odd functions. This implies different predictions on the Q^2 dependence of the  
above mentioned asymmetries. Our theoretical result on unpolarized Drell-Yan is  
compared with available data. Predictions on other azimuthal asymmetries could be  
tested against yields of planned experiments of Drell-Yan and semi-inclusive deep  
inelastic scattering.

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1 Introduction

Azimuthal asymmetries of remarkable size have been observed in various high energy inclusive reactions, especially in unpolarized Drell-Yan[1, 2, 3], in singly polarized semi-inclusive deep inelastic scattering[4-10] (SIDIS) and in inclusive production of (anti-)hyperons[11-15] and pions[16-19] from singly polarized hadronic collisions. The interpretation of such asymmetries from basic principles of QCD is quite challenging and has stimulated the interest of high energy physicists. In particular, in the present paper we focus our attention on the SIDIS and Drell-Yan asymmetries, which are somewhat analogous, since the two reactions are kinematically isomorphic. The theoretical activity about this subject is quite intense and lively, as witnessed by the numerous articles dedicated to the topic[20-70] in the last 15 years.

An important element in the interpretation of such effects is the intrinsic transverse momentum of partons inside a hadron, whose crucial role in high energy reactions has been widely illustrated in the last years[23-25, 71-74]. Indeed, the transverse momentum is connected to the T-odd quark densities[20, 21, 27, 31-34], which provide a quite natural interpretation of the above mentioned asymmetries[27, 31-34, 42]. At the same time, the T-odd functions involve predictions of further azimuthal asymmetries in unpolarized and singly polarized inclusive reactions[29, 26].

These functions explain simultaneously[42] the remarkable $\cos 2\phi$ asymmetry and the negligible $\cos \phi$ Fourier component exhibited by unpolarized Drell-Yan data[1, 2, 3], where $\phi$ is the usual azimuthal angle adopted in the phenomenological fits[1, 2, 3]. The term $\cos 2\phi$ may be just interpreted as a signature[42] of the pair of chiral-odd (and T-odd) functions involved in this picture. However, the current treatment of the T-odd functions does not reproduce the dependence of this asymmetry on the effective mass of the Drell-Yan lepton pair. More generally, some doubts have been cast on the $Q^2$ dependence of the transverse momentum distribution functions[75, 76, 77, 78], where $Q$ is the QCD hard scale. This imposes a revision of the parameterization of the transverse momentum quark correlator, a fundamental theoretical tool for cross section calculations at high energies. This quantity - originally introduced by Ralston and Soper in 1979[79] and successively improved by Mulders and Tangerman[25, 23,
(see also the more recent contributions on the subject[30, 80, 81]) - consists of a \(4 \times 4\) matrix. Therefore it may be parameterized according to the components of the Dirac algebra, taking into account the available vectors and hermiticity and Lorentz and parity invariance. The parameterization - whose coefficients are the quark distribution functions inside the hadron - includes an undetermined energy scale, \(\mu_0\) [73], usually assumed[23, 24, 25] equal to the mass of the hadron related to the active quark. We shall see that this choice is not unique, perhaps not the most appropriate in normalizing some ”leading twist” functions. Alternatively, we propose \(\mu_0 = Q/2\), which explains quite naturally the \(Q^2\) dependence of the unpolarized Drell-Yan asymmetry. Moreover, concerning the SIDIS and other Drell-Yan azimuthal asymmetries, we get predictions which contrast with those given by previous authors, and which could be tested against present[4, 5, 6, 7, 9], forthcoming[10] and future[82, 83, 84, 85] data.

Here we shall not study all azimuthal asymmetries considered in the literature[86, 87, 88, 89], we shall limit ourselves to SIDIS of unpolarized or longitudinally polarized lepton beams off unpolarized or transversely polarized targets, and to unpolarized or singly polarized Drell-Yan, with transverse polarization; moreover, we shall consider just the asymmetries usually classified as leading twist[25, 27, 29].

The paper is organized as follows. In sect. 2 we give the general formulae for the SIDIS and Drell-Yan cross sections, introducing the formalism of the correlator; in particular we illustrate in detail the T-odd functions. Sect. 3 is dedicated to the theoretical formulae for azimuthal asymmetries. In sect. 4 we determine the parameter \(\mu_0\), by comparing the correlator with the quark density matrix in QCD parton model. Such a determination leads to predictions on the \(Q^2\) dependence of the asymmetries, which we illustrate in sect. 5. In sect. 6 we compare our results with experimental data, as regards unpolarized Drell-Yan. Lastly we draw a short conclusion in sect. 7.
2 SIDIS and Drell-Yan Cross Sections

2.1 General formulae

Consider the SIDIS and the Drell-Yan reactions, i.e.,

\[ lh_A \rightarrow l'h_B X \quad \text{and} \quad h_A h_B \rightarrow l^+ l^- X, \]

(1)

where the \( l \)'s are charged leptons and the \( h \)'s are hadrons. Incidentally, these two reactions are topologically equivalent\[22\]. At not too high energies one can adopt one-photon exchange approximation, where the cross sections for such reactions have an expression of the type

\[ \frac{d\sigma}{d\Gamma} = \frac{(4\pi\alpha)^2}{4FQ^4} L^{\mu\nu} W_{\mu\nu}. \]

(2)

Here \( d\Gamma \) is the phase space element, \( \alpha \) the fine structure constant and \( F = \sqrt{(p_1 \cdot p_2)^2 - m_1^2 m_2^2} \) the flux factor, \( p_i \) and \( m_i \) \( (i = 1, 2) \) being the 4-momenta and the masses of the initial particles. Moreover \( L^{\mu\nu} \) and \( W^{\mu\nu} \) are respectively the leptonic and hadronic tensor. In particular, we have

\[ L^{\mu\nu} = \ell^\mu \ell'^\nu + \ell'^\mu \ell^\nu - g^{\mu\nu} \ell \cdot \ell', \]

(3)

where \( \ell \) and \( \ell' \) are the four-momenta of the initial and final lepton (in SIDIS) or of the two final leptons (in Drell-Yan). As regards the hadronic tensor, one often adopts, in the framework of the factorization theorem\[90, 91, 22, 38\], the so-called "handbag" approximation, where all information concerning the "soft" functions of the quark inside the hadrons is encoded in a parameterization of the quark-quark correlator, according to the various Dirac components\[23, 25\]. In this approximation the hadronic tensor reads

\[ W^{\mu\nu} = c \sum_a e_a^2 \int d^2 p_\perp Tr \left[ \Phi_A^a(x_a, p_\perp)\gamma^\mu \Phi_B^b(x_b, q_\perp - p_\perp)\gamma^\nu \right]. \]

(4)

Here \( c \) is due to color degree of freedom, \( c = 1 \) for SIDIS and \( c = 1/3 \) for Drell-Yan. \( \Phi_A \) and \( \Phi_B \) are correlators, relating the active (anti-)quarks to the (initial or final) hadrons \( h_A \) and \( h_B \). \( a \) and \( b \) are the flavors of the active partons, with \( a = u, d, s, \bar{u}, \bar{d}, \bar{s} \) and \( b = a \) in SIDIS, \( b = \bar{a} \) in Drell-Yan; \( e_a \) is the fractional charge.
of flavor $a$. In Drell-Yan $\Phi_A$ and $\Phi_B$ encode information on (anti-)quark distributions inside the initial hadrons: the $x'$s are the longitudinal fractional momenta of the active quark and antiquark, $p_\perp$ is the transverse momentum of the active parton of $h_A$ and $q_\perp$ is the transverse momentum of the lepton pair. In SIDIS $\Phi_B$ is replaced by the fragmentation correlator $\Delta[z, z(q_\perp - p_\perp)]$, describing the fragmentation of the struck quark into the final hadron $h_B$ (see subsect. 2.4). Here $z$ is the longitudinal fractional momentum of $h_B$ with respect to the fragmenting quark and $z q_\perp$ is the transverse momentum of $h_B$ with respect to the virtual photon momentum. Approximation (4) holds for the hadronic tensor under the condition\cite{42, 35, 36, 37}

$$q_\perp << Q,$$

where $q_\perp = |q_\perp|$. Moreover we neglect the Sudakov suppression\cite{92, 93}, as usually assumed at moderate $Q^2$\cite{42, 54-62}.

### 2.2 Parameterization of the Correlator

The correlator for a nucleon may be parameterized according to the Dirac algebra, taking into account hermiticity and Lorentz and parity invariance. It is conveniently split into a T-even and a T-odd term, i.e.,

$$\Phi = \Phi_e + \Phi_o,$$

where $\Phi_e$ is even under time reversal and $\Phi_o$ is odd under the same transformation. At leading twist one has\cite{25, 94, 80, 81}

$$\Phi_e \simeq \frac{P}{\sqrt{2}} \left\{ f_1 \delta_+ + (\lambda g_{1L} + \lambda_{1T}) \gamma_5 \delta_+ + \frac{1}{2} h_{1T} \gamma_5 [\theta_\perp, \delta_+] \right\}$$

$$+ \frac{P}{2 \sqrt{2}} (\lambda h_{1L} + \lambda_{1T} h_{1T}) \gamma_5 [\theta_\perp, \delta_+] ,$$

$$\Phi_o \simeq \frac{P}{\sqrt{2}} \left\{ f_{1T} \epsilon_{\mu \nu \rho \sigma} \gamma^\mu n_+^\nu S_\perp^\rho \eta_\perp + ih_{1L} \frac{1}{2} [\theta_\perp, \delta_+] \right\} .$$

In formulae (7) and (8) we have used the notations of refs.\cite{25, 23} for the "soft" functions*. $n_\pm$ are lightlike, dimensionless vectors, such that $n_+ \cdot n_- = 1$ and whose

*The correlator (6) has a different normalization than in ref.\cite{25}, in accord with the definition (4) of the hadronic tensor.
space components are along (+) or opposite to (-) the nucleon momentum. Moreover the Pauli-Lubanski vector of the nucleon, denoted as $S$ and such that $S^2 = -1$, may be decomposed as

$$S = \frac{P}{M} + S_\perp. \quad (9)$$

Here $P$ is the nucleon four-momentum, with $P^2 = M^2$, $\lambda = -S \cdot n_0$ and $n_0 \equiv (0, 0, 0, 1)$ in the nucleon rest frame, taking the $z$-axis along the nucleon momentum. Thirdly,

$$\mathcal{P} = \frac{1}{\sqrt{2}} p \cdot n_-, \quad \lambda_\perp = -S \cdot \eta_\perp, \quad \eta_\perp = p_\perp / \mu_0, \quad p_\perp = p - (p \cdot n_-) n_+ - (p \cdot n_+) n_- \quad (10)$$

and $p$ is the quark four-momentum. Notice that the parameter $\mathcal{P}$ is similar to the one introduced by Jaffe and Ji\cite{95, 96} with the same notation: denoting that parameter by $\mathcal{P}_{JJ}$, one has $\mathcal{P} = x\mathcal{P}_{JJ}/\sqrt{2}$. Lastly, the energy scale $\mu_0$, encoded in the dimensionless vector $\eta_\perp$, has been introduced in such a way that all functions involved in the parameterization of $\Phi$ have the dimensions of a probability density. This scale - defined for the first time in ref.\cite{73}, where it was denoted by $m_D$ - determines the normalization of the functions which depend on $\eta_\perp$; therefore $\mu_0$ has to be chosen in such a way that these functions may be interpreted just as probability densities. We shall see in sect. 4 that taking $\mu_0$ equal to the rest mass of the hadron, as usually done\cite{23, 24, 25}, is not, perhaps, the most appropriate in this sense. Two observations are in order about $\mu_0$. First of all, it is washed out by integration over $p_\perp$ of the correlator, therefore it does not influence the common\cite{95, 96} distribution functions. Secondly, we can reasonably assume that, for sufficiently large $Q^2$, this parameter is independent of the perturbative interactions among partons: as we shall see, this conjecture can be proved rigorously.

### 2.3 T-odd functions

As explained in the introduction, the T-odd functions deserve especial attention. In particular, the two functions introduced in formula (8) may be interpreted as quark densities: $h_1^\perp$ corresponds to the quark transversity in an unpolarized (or spinless)
hadron, while $f_{1T}$ is the density of unpolarized quarks inside a transversely polarized spinning hadron[27, 77].

A possible mechanism for generating these effects has been analyzed in detail, from different points of view, by various authors[35-41, 44-53]. In particular, the function $f_{1T}$, known as the Sivers function, may give rise to a single spin asymmetry, as predicted for the first time many years ago by Sivers[20, 21] as a consequence of coherence among partons. Essential ingredients for producing the effect are[35, 36, 37]

a) two amplitudes with different quark helicities and different components ($\Delta L_z = 1$) of the orbital angular momentum;

b) a phase difference between such amplitudes, caused, for example, by one gluon exchange between the spectator partons and the active quark, either before or after the hard scattering: owing to the different orbital angular momenta, the gluon interaction causes a different phase shift in the two amplitudes.

Incidentally, a $\Delta L_z = 1$ is connected to the anomalous magnetic moment of the nucleon[35-37, 48-53]; however, the difference in quark helicities could be attributed, in part, also to spontaneous chiral symmetry breaking[38]. In this connection, we think that the correct origin and the basic mechanisms for producing the Sivers asymmetry should be investigated more deeply.

The initial and final state interactions may be described by the so-called link operator, introduced in the definition of bilocal functions in order to assure gauge invariance[25, 38, 39, 40, 41]. Moreover they cause also a nonvanishing $h_{1T}$ [44, 45, 46, 47]: in a scalar diquark model, this function is equal to $f_{1T}$ [42] (see also ref.[47]). In the mechanism which generates quark transversity in an unpolarized nucleon, angular momentum conservation implies a change by one or two units of orbital angular momentum of the quark; this change can be connected to a pseudovector particle exchange, while the above mentioned initial or final state interactions are interpreted as Regge (or absorptive) cuts[44, 45, 46, 47].

From the above discussion it follows that quark-gluon interactions are essential for producing T-odd functions. Indeed, if such interactions are turned off, T-odd functions are forbidden by time reversal invariance[22] in transverse momentum space. On the contrary, they are allowed in the impact parameter space[48-53]: the Sivers
asymmetry can be viewed as a left-right asymmetry with respect to the nucleon spin in that space, where final state interactions produce a chromodynamic lensing for the struck quark[48-53].

As shown in appendix, T-odd functions can be related[97, 78] to the Qiu-Sterman [98, 99, 100] effect, which takes into account quark-gluon-quark correlations (see also ref.[101, 102]). This relation will be illustrated especially at the end of sect. 5, in connection with singly polarized Drell-Yan: T-odd functions turn out to produce an asymmetry whose $Q^2$ dependence coincides with the one obtained by assuming one of the above mentioned correlations[103, 104]. T-odd functions can be approximately factorized[38] - up to a sign, according as to whether the functions are involved in SIDIS or in Drell-Yan[38, 97] - if condition (5) is fulfilled[42]; otherwise one is faced with serious difficulties as regards universality of the effect[105].

### 2.4 Fragmentation Correlator

The fragmentation correlator can be parameterized analogously to $\Phi$, see subsect. 2.2. We have, in the case of quark fragmentation into a pion,

$$\Delta = \Delta_e + \Delta_o,$$

where, at leading twist, the T-even part is given by

$$\Delta_e \simeq \frac{1}{2} k \cdot n'_+ D\hat{p}'_-$$

and the T-odd part reads

$$\Delta_o \simeq \frac{1}{4\mu_0^5} H_1^\perp [k_\perp, \hat{p}'_\perp].$$

Here $k$ is the four-momentum of the quark, $k_\perp = k - (k \cdot n'_\perp)n'_+ - (k \cdot n'_\perp)n'_-$ and $n'_\pm$ are a pair of lightlike vectors, defined analogously to $n_\pm$, but such that the space component of $n'_-$ is along the pion momentum. $\mu_0^5$ is an energy scale analogous to $\mu_0$. Lastly $D$ and $H_1^\perp$ are fragmentation functions, $D$ is the usual one, chiral even, while $H_1^\perp$ - the Collins function[22] - is chiral odd. It is important to notice that the latter function is interaction dependent, as well as the T-odd distribution functions: indeed, it has been shown[106] that this function would vanish in absence of interactions among partons.
3 SIDIS and Drell-Yan Asymmetries

Now we deduce the expressions of the asymmetries involved in the two reactions considered, according to the formalism introduced in the previous section (see also, e. g., refs.[25, 29, 107, 108] for SIDIS and ref.[109] for Drell-Yan). As regards SIDIS, we treat the cases where the initial lepton is either unpolarized or longitudinally polarized, while the nucleon target is either unpolarized or tranversely polarized. On the other hand, concerning Drell-Yan, we consider the situations where at most one of the two initial hadrons (typically a proton) is transversely polarized. For our aims, the most relevant kinematic variables are two azimuthal angles, denoted as $\phi$ and $\phi_S$. In the case of Drell-Yan they are the azimuthal angles, respectively, of the momentum of the positive lepton and of the spin of the initial polarized hadron, in a frame at rest in the center of mass of the final lepton pair. Different frames, related to one another by rotations, have been defined: while the $x$-axis is usually taken along $q_\perp$, the $z$-axis may be taken along the beam momentum - Gottfried-Jackson(GJ) frame[110] -, antiparallel to the target momentum - U-channel (UC) frame[1, 2, 3] -, or along the bisector of the beam momentum and of the direction opposite to the target momentum - Collins-Soper (CS) frame[111]. We shall discuss the frame dependence of the asymmetry parameters later on. As far as SIDIS is concerned, $\phi$ and $\phi_S$ are respectively the azimuthal angles - defined in the Breit frame where the proton momentum is opposite to the photon momentum - of the final hadron momentum and of the target spin vector with respect to the production plane.

3.1 SIDIS Asymmetries

The doubly polarized SIDIS cross section, with a longitudinally polarized lepton and a tranversely polarized nucleon, may be written as a sum of 4 terms, i. e.,

$$\left( \frac{d\sigma}{d\Gamma} \right) = \left( \frac{d\sigma}{d\Gamma} \right)_{UU} + \left( \frac{d\sigma}{d\Gamma} \right)_{UT} + \left( \frac{d\sigma}{d\Gamma} \right)_{LU} + \left( \frac{d\sigma}{d\Gamma} \right)_{LT}. \tag{15}$$

Here we have singled out the unpolarized ($UU$), the singly polarized - either with a transversely polarized target, ($UT$), or with a longitudinally polarized beam, ($LU$) -
and the doubly polarized \((LT)\) contributions. According to the formalism introduced in sect. 2, we get, at leading twist approximation,

\[
\left(\frac{d\sigma}{d\Gamma}\right)_{UU} \simeq \sum_a e_a^2[U_0^a + U_1^a\cos 2\phi], \tag{16}
\]

\[
\left(\frac{d\sigma}{d\Gamma}\right)_{UT} \simeq \sum_a e_a^2[S_1^a\sin(\phi + \phi_S) + S_2^a\sin(\phi - \phi_S) + S_3^a\sin(3\phi - \phi_S) + S_4^a\sin 2\phi], \tag{17}
\]

\[
\left(\frac{d\sigma}{d\Gamma}\right)_{LU} \simeq 0, \tag{18}
\]

\[
\left(\frac{d\sigma}{d\Gamma}\right)_{LT} \simeq \sum_a e_a^2[D_1^a + D_2^a\cos(\phi - \phi_S)]. \tag{19}
\]

Here we have expressed the cross section in units \(\alpha^2xz^2s/Q^4\), where \(s\) is the overall c.m. energy squared. Moreover, omitting the flavor indices of the functions involved, we have

\[
U_0 = A(y)\mathcal{F}[w_{U_0}, f_1, D], \tag{20}
\]

\[
U_1 = -C(y)\frac{q_1^2}{\mu_0\mu_0^0}\mathcal{F}[w_{U_1}, h_{1T}^+, H_{1T}^+], \tag{21}
\]

\[
S_1 = C(y)|S_\perp|\frac{q_1^2}{\mu_0^0}\mathcal{F}[w_{S_1}, h_{1T}^+], \tag{22}
\]

\[
S_2 = A(y)|S_\perp|\frac{q_1^2}{\mu_0^0}\mathcal{F}[w_{S_2}, f_{1T}^+, D], \tag{23}
\]

\[
S_3 = C(y)|S_\perp|\frac{q_3^2}{\mu_0^0\mu_0^0}\mathcal{F}[w_{S_3}, h_{1T}^+, H_{1T}^+], \tag{24}
\]

\[
S_4 = \lambda C(y)\frac{q_1^2}{\mu_0^0\mu_0^0}\mathcal{F}[w_{S_4}, h_{1T}^+], \tag{25}
\]

\[
D_1 = \lambda\lambda_i^0\frac{1}{2}E(y)\mathcal{F}[w_{D_1}, g_{1L}, D], \tag{26}
\]

\[
D_2 = \lambda\lambda_i^0\frac{1}{2}E(y)\frac{q_1^2}{\mu_0^0}\mathcal{F}[w_{D_2}, g_{1T}, D]. \tag{27}
\]

We have denoted by \(\lambda_\ell\) and \(S_\perp\) respectively the helicity of the initial lepton and the transverse component of the nucleon spin vector, with \(|S_\perp| = \sin \phi_S\) and \(\lambda = \cos \phi_S\). Moreover

\[
A(y) = 1 - y + 1/2y^2, \quad C(y) = 1 - y, \quad E(y) = y(2 - y), \tag{28}
\]
where $y \simeq q^-/\ell^-$ and $q$ is the four-momentum of the virtual photon, such that $|q^2| = Q^2$. Lastly, $F$ is a functional \cite{29},

$$F[w, f, D] = \int d^2 p_\perp w(p_\perp, q_\perp) f(p_\perp) D[z, z(q_\perp - p_\perp)], \quad (29)$$

$w$, $f$ and $D$ being, respectively, a weight function, a distribution function and a fragmentation function. As to the weight functions, we have

$$w_{U_0} = w_{D_1} = 1, \quad (30)$$

$$w_{U_1} = w_{S_4} = 2\hat{u} \cdot \hat{p}_\perp \hat{u} \cdot \hat{k}_\perp - \hat{k}_\perp \cdot \hat{p}_\perp, \quad (31)$$

$$w_{S_1} = \hat{u} \cdot \hat{k}_\perp, \quad w_{S_2} = w_{D_2} = \hat{u} \cdot \hat{p}_\perp, \quad (32)$$

$$w_{S_3} = 4(\hat{u} \cdot \hat{p}_\perp)^2 \hat{u} \cdot \hat{k}_\perp - 2\hat{u} \cdot \hat{p}_\perp \hat{k}_\perp \cdot \hat{p}_\perp - \hat{u} \cdot \hat{k}_\perp \hat{p}_\perp^2. \quad (33)$$

Here we have set $\hat{u} = q_\perp/q_\perp$, $\hat{p}_\perp = p_\perp/q_\perp$ and $\hat{k}_\perp = (q_\perp - p_\perp)/q_\perp$. Notice that the first two terms of the cross section (17) correspond respectively to the Collins and Sivers asymmetry \cite{22, 20, 21}.

### 3.2 Weighted Asymmetries in SIDIS

The weighted asymmetries are defined as

$$A_W = \frac{\langle W \rangle}{\langle 1 \rangle}. \quad (34)$$

Here brackets denote integration of the weighted cross section over $q_\perp$ and over the azimuthal angles defined above. $W$ is the weight function, consisting of the Fourier component we want to pick up [see eqs. (16) to (19)], times $(q_\perp/\mu_0)^{n_a}(q_\perp/\mu_0)^{n_b}$, where $n_a$ and $n_b$ are respectively the powers with which $\hat{p}_\perp$ and $\hat{k}_\perp$ appear in the functions $w$ [see eqs. (30) to (33)]. For instance, the weight function corresponding to the Collins asymmetry is $W_{S_1} = (q_\perp/\mu_0)\sin(\phi + \phi_S)$.

### 3.3 Drell-Yan Asymmetries

In the case of singly polarized Drell-Yan with a transversely polarized proton, we have (see also ref.\cite{109})

$$\left(\frac{d\sigma}{d\Gamma'}\right)_{UU} = \sum_a e_a^2 [U_0^a + U_1^a \cos 2\phi], \quad (35)$$
\[
\left( \frac{d\sigma}{d\Gamma} \right)_{UT} = \sum_a e_a^2 [S'_1 a \sin(\phi + \phi_S) + S'_2 a \sin(\phi - \phi_S) \\
+ S'_3 a \sin(3\phi - \phi_S)].
\] (36)

Here we have adopted the same approximation as before and have expressed the cross section in units \(\alpha^2/3Q^2\). Moreover

\[
U_0' = A'(y)F[w_{U_0}, f_1, \overline{f}_1],
\]

(37)

\[
U_1' = C'(y) \frac{q^2}{\mu_0} F[w_{U_1}, h_{1T}^+, \overline{h}_1^+],
\]

(38)

\[
S'_1 = -C'(y) \frac{q^2}{\mu_0} F[w_{S_1}, h_{1T}^+, \overline{h}_1^+],
\]

(39)

\[
S'_2 = A'(y) \frac{q^2}{\mu_0} F[w_{S_2}, f_{1T}^+, \overline{f}_1],
\]

(40)

\[
S'_3 = -C'(y) \frac{q^2}{\mu_0 \mu'_0} F[w_{S_3}, h_{1T}^+, \overline{h}_1^+],
\]

(41)

\[
A'(y) = 1/2 - y + y^2, \quad C'(y) = y(1 - y)
\]

(42)

and

\[
y = 1/2(1 + \cos \theta),
\]

(43)

\(\theta\) being the polar angle of the positive lepton in one of the frames (GJ, UC, CS) defined at the beginning of this section. \(\mu_0\) and \(\mu'_0\) are energy scales relative to the two initial hadrons in the Drell-Yan process. The change of sign of the T-odd functions with respect to SIDIS has been taken into account in the coefficients \(S'_1\), \(S'_2\) and \(S'_3\), as already discussed at the end of subsect. 2.3.

### 4 Determining \(\mu_0\)

Here we derive the appropriate value of the parameter \(\mu_0\) for sufficiently large \(Q^2\). To this end we expand (see appendix) the correlator in powers of the coupling and, by exploiting the Politzer\[112\] theorem on equations of motion (see also ref.[113]), we compare the zero order term and the first order correction respectively with the T-even and with the T-odd parameterizations, eqs. (7) and (8). We shall show that the two procedures lead to consistent results.
Figure 1: Behavior of the asymmetry parameter $\nu$ vs the dimensionless parameter $\rho = q_\perp/Q$. Data are taken from refs.[1, 2]: circles correspond to $\sqrt{s} = 16.2$ GeV, squares to $\sqrt{s} = 19.1$ GeV and triangles to $\sqrt{s} = 23.2$ GeV. The best fit is made with formula (61), $A_0 = 1.177$.

### 4.1 Spin Density Matrix

In appendix we show that, in the case of a transversely polarized nucleon, one has[114, 115], in the limit of $g \to 0$,

$$\Phi \to \rho = \frac{1}{2}(\not{p} + m)[f_1(x, p_\perp^2) + \gamma_5 S_q h_{1T}(x, p_\perp^2)].$$

(44)

Here $m$ is the rest mass of the quark, such that $p^2 = m^2$, and $S_q$ is (up to a sign) the quark Pauli-Lubanski vector, defined so as to coincide, in the quark rest frame[116], with the Pauli-Lubanski vector $S$ of the nucleon in its rest frame. Now we compare the various Dirac components of the density matrix (44) with those of the T-even correlator (7), taking into account relation (A. 21) between $S$ and $S_q$, and

$$p = \sqrt{2} p n_+ + p_\perp + O\left(p^{-1}\right).$$

(45)
As a result we get the following relations for a free, on-shell quark:

\[ \lambda_\perp h_{1T}^\perp = (1 - \epsilon_1)\overline{\lambda}_\perp h_{1T}, \quad (46) \]

\[ \lambda_\perp g_{1T} = (1 - \epsilon_2)\overline{\lambda}_\perp h_{1T}. \quad (47) \]

Here

\[ \overline{\lambda}_\perp = -p_\perp \cdot S/P, \quad (48) \]

moreover \( \epsilon_1 \simeq m/P \) and \( \epsilon_2 \simeq m/2P \) are the correction terms due to the quark mass, which is small for light flavors. The terms of order \( O \left( \left( m^2 + p_\perp^2 \right)/P^2 \right) \) have been neglected.

In order to determine \( \mu_0 \), we observe that the functions \( g_{1T}, h_{1T} \) and \( h_{1T}^\perp \), involved in formulae (46) and (47), are twist 2, therefore they may be interpreted as quark densities. For example, \( g_{1T} \) is the helicity density of a quark in a transversely polarized nucleon. Therefore it is natural to fix \( \mu_0 \) in such a way that \( g_{1T} \) and \( h_{1T}^\perp \) are normalized like \( h_{1T} \). This implies, neglecting the quark mass,

\[ \lambda_\perp = \overline{\lambda}_\perp, \quad (49) \]

and, according to eqs. (10) and (48),

\[ \mu_0 = P = \frac{1}{\sqrt{2}} p \cdot n_-. \quad (50) \]

4.2 First Order Correction in the Coupling

We show in appendix that the first order correction in \( g \) of the correlator - denoted as \( \Phi_1 \) in the following - is T-odd and corresponds to a quark-gluon-quark correlation. This confirms that T-odd functions vanish if we neglect quark-gluon interactions inside the hadron. Moreover the result binds us to compare \( \Phi_1 \) to the parameterization (8) of the T-odd correlator. As proven in appendix, the comparison yields

\[ \mu_0 \propto P, \quad (51) \]

consistent with eq. (50). Result (51) is a consequence of the Politzer theorem, of four-momentum conservation and of kinematics of one-gluon exchange. But the T-odd functions \( h_{1T}^\perp \) and \( f_{1T}^\perp \) may be interpreted as quark densities (see subsect. 2.3),
provided they are properly normalized. Therefore we adopt for them the same normalization as for the T-even density functions - e. g., $g_{1T}$ - which involve the vector $\eta_{\perp}$. This leads again to result (50), analogously to the case of noninteracting partons. Therefore we conclude that, for sufficiently large $Q^2$, the energy scale $\mu_0$ has to be identified with $\mathcal{P}$. This is true both for some (T-even) functions of the parameterization (7) - which survive also in absence of quark-gluon interactions - and for the T-odd correlator (8), which, on the contrary, depends on such interactions. Thus the conjecture proposed at the end of subsect. 2.2 is confirmed. Incidentally, eqs. (46) and (47) involve just T-even functions, therefore, according to the results deduced in appendix, these equations are valid up to terms of $O(g^2)$ and are expected to hold down to reasonably small $Q^2$.

4.3 Remarks

At this point some important remarks are in order.

a) Of course, we expect $\mu_0$ to be modified by nonperturbative interactions: for example, in the case of the already cited quark-diquark model[35-37, 44-47] (see subsect. 2.3), the interference term scales with $Q^2$, in agreement with the assumption $\mu_0 = M [79, 25]$. However, the virtuality of the exchanged gluon increases proportionally to $Q^2 [117, 118]$, so that, at increasing $Q^2$, the gluon ”sees” the single partons rather than the diquark as a whole and eq. (50) appears more appropriate. To summarize, if we take into account the intrinsic transverse momentum of quarks, we are faced with the normalization scale $\mu_0$, which, for large $Q^2$, is equal to $\mathcal{P}$, while for smaller $Q^2$ (such that nonperturbative interactions are not negligible) it is of the order of the hadron mass, in accord with a phase space restriction.

b) Since, as already observed, the parameter $\mu_0$ is encoded in the four-vector $\eta_{\perp} = p_{\perp}/\mu_0$, result (50) implies that, for sufficiently large $Q^2$, the transverse momentum has to be treated as an effective higher twist. This agrees with the observation by Qiu and Sterman[98, 99, 100] that, owing to gauge invariance, transverse momentum has to be paired with a transversely polarized gluon, which, through quark-gluon-quark correlations, gives rise to a higher twist contribution. But, as already stressed, and
As shown in appendix, T-odd functions may be viewed as correlations of this kind. On the other hand, also the function \( g_{1T} \), T-even and classified as twist 2, results to be suppressed for large \( P \). All that casts some doubts on the correlation between the twist of an operator and the \( P \) dependence of the corresponding coefficient[75, 76], when transverse momentum dependent functions are involved. Indeed, in this case, although the Dirac operator commutes with the Hamiltonian of a free quark[95, 96], its mean value over the nucleon state may be suppressed, if the density function describes interference between two quark states with different orbital angular momenta. This occurs for those "soft" functions which are washed out by \( p_\perp \)-integration.

c) It is worth comparing our approach to Kotzinian’s[73], who starts from the approximate expression of the density matrix for a free ultra-relativistic fermion and adapts it to the case of a quark in the nucleon. He parameterizes the density matrix with the 6 twist-2, T-even functions that appear in the parameterization (7). Similar results are obtained by Ralston and Soper[79] and by Tangerman and Mulders[23]. The difference with our approach is that those authors do not take into account the Politzer theorem[112], which implies relations among the "soft" functions.

4.4 Determining \( \mu_0 \) in the Fragmentation Correlator

As regards the fragmentation correlator, we adapt our previous line of reasoning to the case of a quark fragmenting into a transversely polarized spin-1/2 particle, say a \( \Lambda \). For \( g \to 0 \) one has

\[
\Delta \to \rho' = \frac{1}{2}(\slashed{k} + M')(D + H_1 \gamma_5 S').
\] (52)

Here \( M' \) and \( S' \) are, respectively, the mass and the Pauli-Lubanski vector of the final hadron, whereas \( H_1 \) is the transversely polarized fragmentation function; the other symbols are those introduced in subsect. 2.4. By comparing this limiting expression with a parameterization of \( \Delta \) - analogous to eq. (7) as regards twist-2 terms - , we get

\[
\mu_0^\pi = k \cdot n'_\perp / \sqrt{2}.
\]
Figure 2: Behavior of the asymmetry parameter $\nu$ vs the effective mass $Q$ of the final lepton pair at fixed $q_\perp$. $\sqrt{s} = 23.2$ GeV Data from ref.[2], CS frame, and fitted with formula (61), $A_0 \cdot q_\perp^2 = 2.52$ GeV$^2$.

5 $Q^2$ Dependence of Asymmetries

Now we apply the result of the previous section to the processes considered in the present paper. It is convenient to take the space component of $n_-$ along the direction of one of the two initial hadrons (Drell-Yan) or along the direction of the virtual photon (SIDIS). In both cases we get $P \simeq Q/2$. Therefore we assume

$$\mu_0 = \frac{Q}{2},$$

the result being trivially extended to $\mu'_0$ and to $\mu_0^\pi$. As a consequence, we conclude that the azimuthal asymmetries illustrated in sect. 3 decrease with $Q^2$. In particular, as regards SIDIS, we predict

$$S_1, S_2, D_2 \propto \rho, \quad U_1 \propto \rho^2, \quad S_3 \propto \rho^3$$

and

$$D_2 \propto \frac{M}{Q}, \quad S_4 \propto \rho^2 \frac{M}{Q},$$

(54)
where

$$\rho = q_\perp / Q.$$  \hfill (56)

Results (55) are consequences of the fact that $\lambda$ [see eq. (9)] is proportional to $Q^{-1}$ for a transversely polarized nucleon. Such predictions might be checked by comparing data of experiments which have been realized (HERMES[4, 5, 6, 7] and COMPASS[9]) with those planned (CLAS[10]), which operate in different ranges of $Q^2$. A strategy could be, for instance, to isolate the various Fourier components in the cross section [see eqs. (20) to (27)] by means of the weighted asymmetries and to study their $Q^2$ dependence. A particular remark is in order as regards the unpolarized SIDIS asymmetry, which we predict to decrease as $1/Q^2$, just like the twist-4 $\cos 2\phi$ asymmetry arising as a consequence of the quark transverse momentum[119, 120, 73]. This makes the two asymmetries hardly distinguishable, but the last asymmetry can be parameterized, as well as the $\cos \phi$ asymmetry (the Cahn effect[34]), by means of the unpolarized quark density.

Concerning Drell-Yan, the predictions are

$$S'_1, S'_2 \propto \rho, \quad U'_1 \propto \rho^2, \quad S'_3 \propto \rho^3.$$  \hfill (57)
As regards $U'_1$, the result will be checked against unpolarized Drell-Yan data in the next section; the other three predictions could be verified, in principle, by comparison with data from experiments planned at various facilities, like RHIC[82], GSI[83, 84] and FNAL[85]. It is important to observe that, according to the approximation assumed in the present article, the asymmetry terms $S'_1$, $S'_2$, $S'_3$ and $U'_1$ are invariant under rotations (see eqs. (38) to (41)) and therefore independent of the frame chosen (CS, GJ or UC).

As a conclusion of this section it is worth recalling that the Drell-Yan single spin asymmetry, integrated over the transverse momentum of the final muon pair, was studied some years ago, in terms of a quark-gluon-quark correlation function, and it was found to decrease as $Q^{-1}$ [103, 104, 121, 122, 123] (see also refs.[98-102, 124]), consistent with our result (57).
Figure 5: Same as fig 2. Data from ref.[1], √s = 19.1 GeV. A₀ · q₂⊥ = 2.71 GeV².

6 Azimuthal Asymmetry in Unpolarized Drell-Yan

As is well-known, unpolarized Drell-Yan presents an azimuthal asymmetry. This has been seen, for example, in reactions of the type[1, 2, 3]

\[ \pi^- N \rightarrow \mu^+ \mu^- X, \]  

where \( N \) is an unpolarized tungsten or deuterium target, which scatters off a negative pion beam. The Drell-Yan angular differential cross section is conventionally expressed as

\[ \frac{1}{\sigma} \frac{d\sigma}{d\Omega} = \frac{3}{4\pi} \frac{1}{\lambda + 3} \left( 1 + \lambda \cos^2 \theta + \mu \sin 2\theta \cos \phi + \frac{1}{2} \nu \sin^2 \theta \cos 2\phi \right). \]  

Here \( \Omega = (\theta, \phi) \), \( \theta \) and \( \phi \) being respectively the polar and azimuthal angle of the \( \mu^+ \) momentum in the one of the frames defined in sect. 3. Moreover \( \lambda, \mu \) and \( \nu \) are parameters, which are functions of the overall center-of-mass energy squared \( s \), of \( q^2 \), of \( Q \) and of the Feynman longitudinal fractional momentum \( x_F \) of the muon pair with respect to the initial beam.
In the naive Drell-Yan model, where the parton transverse momentum and QCD corrections are neglected, one has \(\lambda = 1, \mu = \nu = 0\). Therefore deviations of such parameters from the above predictions - observed experimentally both for \(\lambda\) and \(\nu\), while \(\mu\) is consistent with 0 [1, 2, 3] - can be attributed to transverse momentum or gluon effects, as illustrated in ref.[1]. The main contribution to the Drell-Yan cross section derives from QCD first order effects, typically from the partonic reactions \(qq \rightarrow g\gamma^*\) and \(qq \rightarrow q\gamma^*\) [1, 2, 3, 125], which also could account for the asymmetry parameter \(\nu\) [1, 2, 3, 126]. However, such effects fulfill the Lam-Tung[127] relation, which, instead, turns out to be rather strongly violated (see refs.[42, 1, 2, 3] and refs. therein). This fact has led people to propose alternative mechanisms[42, 43] for explaining the asymmetry. Furthermore we notice that data[1, 2, 3] exhibit for \(\nu\) a substantial independence of the frame chosen (CS, GJ, UC), just as predicted by T-odd functions (see the previous section), while first order perturbative QCD corrections would imply[126] a considerable frame dependence for that parameter.

The behavior of Drell-Yan data may be understood by observing that the cross section is very sensitive also to power corrections[128, 129, 130] (see also refs.[131, 132]). In particular, a \(\lambda \neq 1\) is obtained by assuming for reaction (58) a simple model of initial state interactions[129, 130], somewhat similar to the quark-gluon-quark correlations[98, 99, 100]. Here the Drell-Yan unpolarized cross section is of the type

\[
d\sigma \propto |f_0 + f_1|^2,
\]

where \(f_0\) is the naive Drell-Yan amplitude and \(f_1\) consists of two terms (due to gauge invariance), describing one gluon exchange between the spectator quark of the meson and each active parton. It results[129, 130] \(|f_0|^2 \propto (1 - x)^2(1 + \cos^2\theta)|f_1|^2 \propto \rho^2\cos^2\theta\) and \(2\Re f_0f_1^* \propto (1 - x)\cos^2\theta\cos\psi_0\), where \(\psi_0\) is the relative phase of the two amplitudes. This implies \(\lambda - 1 \propto \rho^2/(1 - x)^2\), in good agreement with data[1, 2, 3]. However, \(\mu\) depends crucially on \(\psi_0\), moreover the third term of eq. (59) is absent. This could be recovered by inserting in eq. (60) a third amplitude, say \(f_1'\), describing one gluon exchange between each active parton and the spectator partons of the nucleon: the missing asymmetry is reproduced by the interference term \(2\Re f_1'f_1^*\), as

\[
\]

21
sketched at the end of subsect. 2.3, which amounts to recovering T-odd functions.[42]

Indeed, the features of the parameters $\mu$ and $\nu$ are suitably interpreted in terms of the correlator: comparison of eq. (35) with eq. (59) yields $\mu = 0$ and

$$\nu = A_0 \frac{q^2}{Q^2} = A_0 \rho^2,$$

with

$$A_0 = \frac{\mathcal{F}[w_{L1}^0, h_{T1}]}{\mathcal{F}[w_{L1}^0, f_1, f_1]}.$$  \hfill (61)

Here eqs. (37), (38), (42) and the second eq. (57) have been taken into account. We make some approximations concerning $A_0$. First of all, we neglect its $q_\perp$ dependence: for example, if we assume a gaussian behavior as regards the $p_\perp$-dependence of the density functions, the $q_\perp$ dependence disappears in the ratio (62). Secondly we neglect the $Q^2$ evolution of the ”soft” functions involved; such a dependence is expected to be quite smooth, as follows by assuming factorization and demanding factorization scale independence for the hadronic tensor[133]. Lastly, as told in subsect. 2.1, we neglect the Sudakov suppression: indeed, this effect, as well as the previous one, would imply a weak $Q^2$ dependence[92, 93], more complicated than the (approximate) $\rho$-dependence[92, 93] exhibited by data (see fig. 1 of the present paper and tables 1 to 3 in ref.[1]). Therefore we approximate $A_0$ by a constant.

We fit formula (61) to the experimental results of $\nu$ at different energies, both as a function of $\rho$ (fig. 1) and as a function of $Q$ at fixed $q_\perp$ (figs. 2 to 6), treating $A_0$ as a free parameter. We stress that the $\rho$-dependence of $\nu$ cannot be reproduced by the assumption $\mu_0 = M$ [23, 24, 25], not even taking into account the effects, just discussed, of QCD evolution and Sudakov suppression. This assumption would provide also a poor approximation to data of $\nu$ versus $Q$ at fixed $q_\perp$.

7 Conclusion

We have studied the parameterization of the transverse momentum dependent quark correlator, both for distributions inside the hadron and for fragmentation processes. We are faced with the energy scale $\mu_0$, introduced in the parameterization for dimensional reasons, and determining the normalization of some of the quark densities...
(or fragmentation functions) involved. Comparison of the parameterization with the limiting expression of the correlator for noninteracting quarks yields \( \mu_0 = p \cdot n_\perp \), contrary to the usual\cite{23, 24, 25} assumption, \( \mu_0 = M \), which appears more appropriate for situations where nonperturbative interactions among partons are present. The two different assumptions lead to different predictions on the \( Q^2 \) dependence of azimuthal asymmetries in SIDIS and Drell-Yan. Our result agrees with previous approaches to azimuthal asymmetries, in particular with the \( Q^2 \) dependence predicted by quark-quark-gluon correlations\cite{98, 99, 100} for Drell-Yan single spin asymmetry, and also with data of azimuthal asymmetry in unpolarized Drell-Yan. In particular, our interpretation of this asymmetry is considerably simpler than the one which could be obtained with the usual assumption about \( \mu_0 \). Further challenges to the two different theoretical predictions could come from future Drell-Yan experiments\cite{82, 83, 84, 85} and from comparison between present\cite{4, 5, 6, 7, 9} and incoming\cite{10} SIDIS data.

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**Appendix**

We illustrate some features of the correlator; in particular, we expand this quantity in powers of the coupling and study in detail the zero and first order term of the expansion. The correlator is defined as

\[\Phi = N \int \Phi'(p; P, S)dp^- \].

(A. 1)

Here \(N\) is a normalization constant, to be determined below, and \(\Phi'(p; P, S)\) is defined in such a way that

\[\Phi'_{ij}(p; P, S) = \int \frac{d^4x}{(2\pi)^4} e^{ipx} \langle P, S| \overline{\psi}_j(0) \mathcal{L}(x) \psi_i(x) | P, S \rangle, \]

(A. 2)

\(\psi\) being the quark (or antiquark) field of a given flavor and \(|P, S\rangle\) a state of a nucleon with a given four-momentum \(P\) and Pauli-Lubanski four-vector \(S\), while \(p\) is the quark four-momentum. The color index has been omitted in \(\psi\) for the sake of simplicity. Moreover

\[\mathcal{L}(x) = \text{Pexp}[-igA_I(x)], \quad \text{with} \quad A_I(x) = \int_{0(I)}^x \lambda_a A^a_\mu(z)dz^\mu, \]

(A. 3)

is the gauge link operator. Here \(g\) is the coupling and "P" denotes the path-ordered product along a given integration contour \(I\), \(\lambda_a\) and \(A^a_\mu\) being respectively the Gell-Mann matrices and the gluon fields. The link operator depends on the choice of \(I\), which has to be fixed so as to make a physical sense. According to previous treatments[38, 25], we define two different contours, \(I_\pm\), as sets of three pieces of straight lines, from the origin to \(x_{1\infty} \equiv (\pm \infty, 0, 0)\), from \(x_{1\infty}\) to \(x_{2\infty} \equiv (\pm \infty, \pm 1, 0)\) and from \(x_{2\infty}\) to \(x \equiv (\pm 1, \pm 1, 0)\); here the \(\pm\) sign has to be chosen, according as to whether final or initial state interactions[38, 25] are involved in the reaction. We have adopted a frame - to be used throughout this appendix - whose \(z\)-axis is taken along the nucleon momentum, with \(x^\pm = 1/\sqrt{2}(t \pm z)\).

**T-even and T-odd correlator**

We set

\[\Phi'_{E(O)} = \frac{1}{2}[\Phi'_+ \pm \Phi'_-]. \]

(A. 4)
where $\Phi'_\pm$ corresponds to the contour $I_\pm$ in eqs. (A.3), while $\Phi'_E$ and $\Phi'_O$ select respectively the T-even and the T-odd "soft" functions. These two correlators contain respectively the link operators $L_E(x)$ and $L_O(x)$, where

$$L_{E(O)}(x) = \frac{1}{2} \mathcal{P} \left\{ \exp \left[ -ig\Lambda_{I_+}(x) \right] \pm \exp \left[ -ig\Lambda_{I_-}(x) \right] \right\} \quad (A.5)$$

and $\Lambda_{I_\pm}(x)$ are defined by the second eq. (A.3). It is convenient to consider an axial gauge with antisymmetric boundary conditions[25], to be named $G$-gauge in the following. This yields

$$\Lambda_{I_-}(x) = -\Lambda_{I_+}(x) \quad (A.6)$$

and therefore

$$L_E(x) = \mathcal{P} \cos \left[ g\Lambda_{I_+}(x) \right], \quad L_O(x) = -\mathcal{P} \sin \left[ g\Lambda_{I_+}(x) \right]. \quad (A.7)$$

Then the T-even (T-odd) part of the correlator consists of a series of even (odd) powers of $g$, each term being endowed with an even (odd) number of gluon legs. Moreover eq. (A.6) implies that the T-even functions are independent of the contour ($I_+$ or $I_-$), while T-odd ones change sign according as to whether they are generated by initial or final state interactions[38]. In this sense, such functions are not strictly universal[38].

The two conclusions above, as well as the power expansion in the coupling, turn out to be gauge independent, since the the correlator is by definition gauge independent for any value of $g$ and the same is true for any term in the expansion. As a consequence, the zero order term is T-even, while the first order correction is T-odd. This confirms that no T-odd terms occur without interactions among partons, as claimed also by other authors[35, 36, 37, 38].

Let us consider the expansion of $\Phi'$ in powers of $g$, i. e.,

$$\Phi' = \Phi'_0 - ig\Phi'_1 + ... \quad (A.8)$$

with

$$(\Phi'_0)_{ij} = \int \frac{d^4x}{(2\pi)^4} e^{ipx} \langle P, S| \bar{\psi}_j(0)\psi_i(x) |P, S\rangle \quad (A.9)$$
and
\[(\Phi')_{ij} = \int \frac{d^4x}{(2\pi)^4} e^{ipx} \int_{0(x)}^x dz'^\mu (P, S | \bar{j}(0) \lambda_a A^a_\mu(z) \psi_i(x) | P, S). \tag{A. 10}\]
We stress that the term (A. 10) consists of a quark-gluon-quark correlation, analogous to the one introduced by Efremov and Teryaev \cite{101, 102} and by Qiu and Sterman \cite{98, 99, 100}. Inserting expansion (A. 8) into eq. (A. 1), we get
\[\Phi = \Phi_0 + \Phi_1 + ..., \tag{A. 11}\]
where
\[\Phi_0 = N \int \Phi'_0(p; P, S) dp^- \quad \text{and} \quad \Phi_1 = -igN \int \Phi'_1(p; P, S) dp^- \tag{A. 12}\]
From now on we shall consider a transversely polarized nucleon. Then, according to our previous considerations, we may identify, at the two lowest orders in $g$, $\Phi_0$ with $\Phi_e$ and $\Phi_1$ with $\Phi_o$, where $\Phi_e$ and $\Phi_o$ are given, respectively, by eqs. (7) and (8) in the text. Now we study in detail these two terms of the expansion.

**Zero order term**

We apply the Politzer theorem on equations of motion \cite{112}, i.e.,
\[\langle P, S | \bar{\psi}(0) \mathcal{L}(x)(i \not\!{D} - m) \psi(x) | P, S \rangle = 0. \tag{A. 13}\]
Here $D_\mu = \partial_\mu - ig\lambda_a A^a_\mu$ is the covariant derivative. The result (A. 13) survives renormalization and applies also to off-shell quarks. Expanding $\mathcal{L}(x)$ in powers of $g$, at zero order the theorem implies that the quark can be treated as if it were on shell (see also ref. \cite{113}). Then we consider the Fourier expansion of the unrenormalized field of an on-shell quark, i.e.,
\[\psi(x) = \int \frac{d^4p}{(2\pi)^{3/2}} \sqrt{m \mathcal{P}} \delta \left( p^- - \sqrt{m^2 + \mathbf{p}_1^2} \right) e^{-ipx} \sum_s u_s(p)c_s(p), \tag{A. 14}\]
where $m$ is the rest mass of the quark, $s = \pm 1/2$ its spin component along the nucleon polarization in the quark rest frame, $u$ its four-spinor, $c$ the destruction operator for the flavor considered and $\mathcal{P}$ is defined by the first eq. (10) in the text: in our frame $\mathcal{P} = p^+/\sqrt{2}$. As regards the normalization of $u_s$ and $c_s$, we assume
\[\bar{u}_s u_s = 2m, \quad \langle P, S | c^\dagger_{s'}(\bar{p}') c_{s'}(\bar{p}) | P, S \rangle = (2\pi)^3 \delta^3(\bar{p}' - \bar{p}) \delta_{ss'} q_s(\bar{p}), \tag{A. 15}\]
where \( \tilde{p} \equiv (p^+, p_\perp) \) and \( q_s(\tilde{p}) \) is the probability density to find a quark with spin component \( s \) along the nucleon (transverse) polarization and four-momentum \( p \equiv (p^-, \tilde{p}) \), with \( p^- = \sqrt{m^2 + p_\perp^2/2p^+} \). For an antiquark the definition is quite analogous.

Substituting eq. (A.14) and the second eq. (A.15) into eq. (A.9) and into the first eq. (A.12), we get

\[
(\Phi_0)_{ij} = \frac{N}{2P} \sum_s q_s(\tilde{p}) [u_s(\tilde{p})]_i [\overline{u}_s(\tilde{p})]_j. \tag{A.16}
\]

But \([u_s(\tilde{p})]_i [\overline{u}_s(\tilde{p})]_j\) is nothing but the matrix element \( \rho_{ij} \) of the spin density matrix of the quark. Therefore, taking into account the first eq. (A.15), eq. (A.16) yields

\[
\Phi_0 = \frac{N}{2P} \sum_s q_s(\tilde{p}) \frac{1}{2}(\not{\! p} + m)(1 + 2s\gamma^5 S_q). \tag{A.17}
\]

where \( 2sS_q \) is the Pauli-Lubanski vector of the quark in a transversely polarized nucleon. We normalize the correlator according to the definition (4) of the hadronic tensor, that is, demanding that it reduce to the spin density matrix in the limit of \( g \to 0 \). Therefore

\[
N = 2P. \tag{A.18}
\]

Eq. (A.17) can be conveniently rewritten as

\[
\Phi_0 = \frac{1}{2}(\not{\! p} + m)[f_1(x, p^2_\perp) + \gamma^5 S_q h_{1T}(x, p^2_\perp)], \tag{A.19}
\]

where we have set, according to the definitions of the density functions [in the scaling limit, \( q_s(\tilde{p}) \to q_s(x, p^2_\perp) \)],

\[
f_1 = \sum_{s=\pm 1/2} q_s, \quad h_{1T} = \sum_{s=\pm 1/2} 2sg_s. \tag{A.20}
\]

Eq. (A.19) corresponds to formula (44) in the text. According to the Politzer theorem, renormalization modifies the functions \( f_1 \) and \( h_{1T} \), but not the structure of this expression.

Now we express \( S_q \) as a function of \( S \). The two vectors do not coincide, since the spin operator for a massive particle has to be defined in the particle rest frame[116]. Taking into account the proper Lorentz boosts, we get, for a transversely polarized nucleon,

\[
S^q = S + \overline{\lambda}_\perp \frac{\not{\! p}}{m} + O(p^2_\perp), \tag{A.21}
\]
with \( \eta_\perp = \frac{p_\perp}{P} \) and \( \overline{\lambda}_\perp = -S \cdot \eta_\perp \).

**First order correction**

Now we consider the term (A. 10), which, as shown before, is T-odd. This term gives rise to a final (or initial) state interaction\[103, 30, 97, 121, 35, 36, 37\], in the sense that the spectator partons of a given hadron may exchange a gluon either with the final active quark (in SIDIS) or with the initial active quark of another hadron (in Drell-Yan). This kind of interaction selects the direction of the momentum of the gluon in the triple correlation (A. 10). Indeed, if the gluon is emitted by the spectator partons, it must have the same direction as the active quarks in that correlation; if absorbed, it must have opposite direction. This observation is quite important, as we shall see.

We apply again the Politzer theorem, eq. (A. 13), now considering the first order correction of \( \mathcal{L}(x) \). We get, adopting the G-gauge,

\[(\not{p} - m)\Phi_1' = \mathcal{M}, \quad (A. 22)\]

where

\[
\mathcal{M}_{ij} = \int \frac{d^4x}{(2\pi)^4} e^{ipx} \langle P, S | \bar{\psi}_j(0) [\mathcal{A}_2 - \mathcal{A}]_{ik} \psi_k(x) | P, S \rangle \quad (A. 23)
\]

and \( \mathcal{A} = \lambda_a \mathcal{A}^a(x), \mathcal{A}_2 = \lambda_a \mathcal{A}^a(x_{2\infty}) \). The matrix \( \mathcal{M} \) is a quark-gluon-quark correlator, with the dimensions of a momentum; this matrix is a “soft” quantity, therefore independent of the ”hard” scale \( p^+ \). Eq. (A. 22) implies

\[\Phi'_1 = \frac{\not{p} + m}{p^2 - m^2} \mathcal{M}, \quad (A. 24)\]

with \( p^2 \neq m^2 \). On the other hand, by considering the Fourier expansion of the gluon field in eq. (A. 23), and by applying again Politzer’s theorem, we conclude that the other quark in the triple correlation is on shell. Now we show that

\[ p^2 \propto (p^+)^2 \quad (A. 25) \]

for \( p^+ \to \infty \), so that \( \Phi'_1 \) decreases like \( (p^+)^{-1} \) in that limit.
Our previous considerations and four-momentum conservation imply that the quarks in the triple correlation have four-momenta $p$ and $p \pm k$ respectively, such that

$$(p \pm k)^2 = m^2.$$  (A. 26)

Here $k$ is the four-momentum of the gluon in the triple correlation, the $\pm$ sign referring respectively to gluon emission and absorption. Since the gluon is exchanged between two color charges, it is space-like, moreover

$$k \equiv (k_0, \mathbf{k}), \quad \text{with} \quad 0 < k_0 \sim |k_x| \sim |k_y| << |k_z| = O(p^+).$$  (A. 27)

But, according to the previous discussion, the kinematics of one gluon exchange demands $k_z$ to be positive for emission and negative for absorption. Then condition (A. 25) follows from eqs. (A. 27) and (A. 26).

However, in order to justify the parameterization (8) for $\Phi_0(p)$ in the text, one has to make an approximation. Indeed, the hadronic tensor in SIDIS and in Drell-Yan is not rigorously factorizable into two terms, if we include the mechanism of one gluon exchange illustrated before. As discussed by Collins in ref.[38] and papers therein, an approximate factorization may be assumed, provided the transverse momentum of the final hadron (in SIDIS) or of the final pair (in Drell-Yan) is much smaller than the "hard" scale, see condition (5). Under such a condition, taking into account results (A. 24) and (A. 25) and the T-odd character of $\Phi'_1$, this term may be parameterized in the following way:

$$-ig\Phi'_1 = \frac{K}{p^+} \left\{ \tilde{f}^+_{1T} \epsilon_{\mu \nu \rho \sigma} \gamma^\mu n^\nu p^\rho S^\sigma_{\perp} + i\tilde{h}_{1T}^+ \frac{1}{2} \{ \hat{b}_{\perp}, \hat{n}_{\perp} \} \right\}.$$  (A. 28)

Here $K$ is a numerical constant and $\tilde{f}^+_{1T}$ and $\tilde{h}_{1T}^+$ are two "soft" functions. Inserting eq. (A. 28) into the second eq. (A. 12) yields a parameterization for $\Phi_1$, which, by comparison to eq. (8), leads us to conclude that $\mu_0 \propto \mathcal{P}$. As discussed in subsect. 4.2, a suitable choice of the normalization of the two T-odd "soft" functions (which uniquely fixes the constant $K$) leads to eq. (50), as in the noninteracting case.
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