Doubly Heavy Baryon $\Xi_{cc}$ Production in $\Upsilon(1S)$ Decay

Shi-Yuan Li$^1$, Zhen-Yang Li$^1$, Zong-Guo Si$^1$, Zhong-Juan Yang$^2$, Xiao Zhang$^1$

1 School of Physics, Shandong University, Jinan, Shandong, 250100, P.R. China and
2 School of Physics and Technology, University of Jinan, Jinan, Shandong, 250022, P.R. China

(Dated: October 6, 2021)

$\Upsilon(1S)$ decay to $\Xi_{cc} + \text{anything}$ is studied. It is shown that the corresponding branching ratio can be as significant as that of $\Upsilon(1S)$ decay to $J/\Psi + \text{anything}$. The non-relativistic heavy quark effective theory framework is employed for the calculation on the decay width.

PACS numbers: Valid PACS appear here

$\Upsilon(1S)$ decay is a good arena to study QCD and hadron physics. Several instructive results have been obtained. For example, recent searches on the exotic XYZ hadrons via the inclusive channel $\Upsilon \to J/\Psi + \text{anything}$ [1] and on light tetraquark hadrons in several channels of $\Upsilon$ decay [2] have been made. Both reported negative results. As a matter of fact, in the energy region above $J/\Psi$ mass at BEPC and that above $\Upsilon$ mass at B factories, many exotic XYZ hadrons have been observed (for a recent review, see [3]). These exotic particles, except those directly couple to the virtual photon in $e^+e^-$ annihilations, are all produced from the decays of either the exited $c\bar{c}$ bound states or the B hadrons. On the other hand, $\Upsilon$ decay is an environment significantly different from those where the exotic particle production is observed. $\Upsilon$ decays via the OZI-suppressed ways, i.e., the annihilation of the $b\bar{b}$ quarks. The dominant mode ($> 80\%$) is the hadronic one generally refered as ’3-gluon’ decay [4], and the subsequent hadronization is a special case of multiproduction. The negative results [1] [2] mentioned above can shed light on property of confinement and the untiarity of the hadronization in multiproduction processes as we have pointed out [5–8].

*Electronic address: lishy@sdu.edu.cn
process prefers to transfer into general hadrons like $J/\Psi$ rather than exotic XYZ’s in this multiproduction process; and that for light hadrons, it is also the similar case, i.e., the above negative experimental results on light exotic hadrons indicate that the dominant decay channels should be $\Upsilon \to h's$, with $h's$ referring to mesons as well as baryons. In one word, $\Upsilon$ generally decays to mesons and baryons, with exotic ones hardly possible to be observed. But the to-date measured decay channels of $\Upsilon$ are much far from exhausting the total decay width. Especially, almost no baryon channel is measured \cite{4}. So measuring the baryon production is an important task for better understanding the dynamics in $\Upsilon$ decay.

Among all the baryons produced in $\Upsilon$ decay, the doubly heavy baryon $\Xi_{cc}$ is the most heavy. SELEX and LHCb have respectively reported the observations of this kind of baryons with different mass \cite{11,12}. One of the possibilities can be that different SU(2) multi-states of $\Xi_{cc}$ are observed by these two Collaborations. To measure these multi-states, and further to explore SU(3) multi-states, can surely help to clarify and deepen our knowledge on the property and production mechanism of $\Xi_{cc}$. $\Upsilon$ decay can provide a clean platform for such measurements.

There is a further special reason stands for the observation on $\Xi_{cc}$ in $\Upsilon$ decay. It is noticed that most of the presented data of $\Upsilon$ decay are upper limits \cite{4}. However, the decay channel $\Upsilon \to J/\Psi + X$ is well measured for several times by several collaborations and has attracted wide interests, which is important on the study of PQCD and NRQCD (for the full literature list, please see a recent review \cite{13}). It was pointed out that, based on the soft $J/\Psi$ spectrum by CLEO measurement which was quite rough at that time, and on the calculation of the partial width \cite{9}, the dominant contribution could be $\Upsilon(1S) \to J/\Psi + c\bar{c}g$. Then the spectrum and branching ratio is confirmed by CLEO II \cite{16,17} and later by BELLE \cite{1}, though detailed calculations show that several competing sub-processes contribute \cite{14,15}. This fact strongly implies that the perturbative production of $c\bar{c}c\bar{c}$ in $\Upsilon$ decay is significant. This leads to that the double charm baryon is hence easily produced as argued by the colour connection analysis \cite{18}. For $c_1\bar{c}_2c_3\bar{c}_4g$ system from $\Upsilon$ decay, $c_1\bar{c}_2$ and $c_3\bar{c}_4$ respectively come from a virtual gluon. But $c_1\bar{c}_4$ and $c_3\bar{c}_2$ can respectively be in colour singlet, i.e., the colour space can be reduced as

\[
(3_1 \otimes 3^*_1) \otimes (3_3 \otimes 3^*_2) = (1_{14} + 8_{14}) \otimes (1_{23} + 8_{23}).
\]

This means that such combination of the pair can be colour singlet and easy to translate to
$J/\psi$ for proper invariant mass. One can recognize that the colour space can also be reduced as

$$(3_1 \bigotimes 3_3) \bigotimes (3_2^* \bigotimes 3_4^*) = (3_{13}^* + 6_{13}) \bigotimes (3_{24} + 6_{24}).$$

In such colour states, the two-charm pair can combine with a light quark to become $\Xi_{cc}$ for proper invariant mass. This simple analysis implies that the production rate of $\Xi_{cc} + \bar{c}\bar{c}g$ is expected not small once the $J/\Psi + \bar{c}\bar{c}g$ production rate is not small.

In this paper, we devote to study the production of $\Xi_{cc}$ in $\Upsilon$ decay. We calculate the corresponding partial width and the momentum distribution of $\Xi_{cc}$. Multi-states like $\Xi_{cc}^+$ or $\Xi_{cc}^{++}$ could have different width and lead to quite different feasibility or difficulty in observing them, but their production mechanism is completely the same in $\Upsilon$ decay. Therefore we do not make any distinction for the investigation on the production. In the super B factory, once the center of mass energy is tuned on the $\Upsilon$ resonance, a large sample of $\Upsilon$ decay data can be obtained and could be employed for the measurement. The following calculations show that the branching ratio of $\Xi_{cc}$ production can be order of $10^{-4}$. For the $\Upsilon$ decay, the process with two charm pairs production is easy to be triggered by 3-jet like event shape and strangeness enhancement (e.g., the $K_\pi$ value) \cite{17,20}, of which some of the the charm meson production events can be vetoed by lepton pair or hadron pair mass around $J/\Psi$ mass. In this way, one can get a clean and large sample of events to study the doubly charm baryon multi-states.

In the process $\Upsilon \rightarrow \Xi_{cc} + \bar{c}\bar{c}g$, both bottom and the charm quarks are heavy. For the initial bound state, the colour singlet $b\bar{b}$ pair with $C=-1$, it directly leads to the non-relativistic wave function formulations \cite{21,24}, where the relative momentum between $b$ and $\bar{b}$ is vanishing, namely same as the case of positronium. For the final bound state, a factorization formulation within the heavy quark effective theory framework \cite{25,26} is employed. One subtle point is that, the non-relativistic formulations are investigated in the rest frame of each bound state, respectively; and then a corresponding covariant form of description is obtained, which can be employed in any frame. Here we start from the initial state: The differential width of the process $\Upsilon \rightarrow \Xi_{cc} + \bar{c}\bar{c}g$ can be formulated as

$$\frac{d\Gamma}{dR} = \frac{|B_T \langle \Xi_{cc}\bar{c}\bar{c}g | S | b\bar{b}(3S_1,1)\rangle|^2}{T},$$

where $dR$ is the phase space volume element for $\Xi_{cc}$ and $\bar{c}, \bar{c}, g$ without the constrain of energy momentum conservation; $S$ is the S-Matrix; $B_T$ is related to the wave function of $\Upsilon$
at origin as
\[ B_T = \frac{\Psi_T(0)}{\sqrt{V}2m_b}. \]  

For convenience, we normalize all final state particle states to be $2EV$ (where $E$ is the particle’s energy and $V$ is the volume of the total space). This normalization is also used for all free quarks in bound states. For the initial state, $B_T$ normalizes the state of $\Upsilon$ to be 1, so that the width can be directly written as above. In Eq. (1) the sum over all spin states for final particles and average of the 3 spin states for $\Upsilon$ are not explicitly shown and the ‘time’ $T$ is $2\pi\delta(0)$.

For the factorization of the initial bound state, the width is written, based on the above Equation, as
\[ d\Gamma = dR' \frac{1}{3} \frac{1}{\sqrt{M_T^2}} |\Psi_T(0)|^2 |< \Xi_{cc} \bar{c}c\bar{g}|T|b\bar{b} (3S_1, 1) >|^2. \]  

Here $dR' = dR(2\pi)^4\delta^{(4)}(P_i - P_f)$, the factors time $T$ and volume $V$ are cancelled by the $\delta^{(4)}(0)$. $T$ is the T matrix with $S_{fi} = \delta_{fi} + (2\pi)^4\delta^{(4)}(P_i - P_f)T_{fi}$. Sum over all spin states is implicitly indicated.

Employing the project operator formulation (e.g., [21]), and the radial wave function $R_T$ to describe the initial bound state, we get the decay amplitude as,
\[ M_{fi} = \frac{1}{2} \frac{1}{\sqrt{4\pi}} \frac{1}{\sqrt{M_T}} R_T(0) Tr[O_0(\frac{P}{M_T})(-\bar{f})]. \]  

O$_0$ is the amplitude for $b\bar{b} \rightarrow \Xi_{cc} \bar{c}c\bar{g}$, with relative momentum of $b\bar{b}$ vanishing. $P$ and $\epsilon$ are 4-momentum and polarization vector of $\Upsilon$, respectively.

In the final state of the $\Upsilon$ decay, the unobserved part $X$ can be divided into a perturbative part $X_P$ and a non-perturbative part $X_N$. To the lowest-order (tree level) in PQCD,
\[ M_{fi} = \int \frac{d^4q_1}{(2\pi)^4} A_{ij}(k_1, k_2, P_1, P_2, P_3; q_1) \int d^4x_1 e^{-i\epsilon_{x_1}} \times < \Xi_{cc}(k) + X_N|Q_i(x_1)\bar{Q}_j(0)|0 >. \]  

We assign $k_1, k_2, P_1, P_2, P_3, k$ as the momenta of the corresponding particles, $b, \bar{b}, \bar{c}, c, g, \Xi_{cc}$, respectively, $k_1 = k_2 = P/2$. $A_{ij}(k_1, k_2, P_1, P_2, P_3; q_1)$, which includes the initial wave function, can be directly read from FIG.1. Both i and j are Dirac and color indices. In the matrix element, $X_N$ represents the non perturbative effects. $Q(x)$ is the Dirac field for charm quark.
Taking the absolute square of the above amplitude, one gets
\[
d\Gamma = \frac{1}{2M_Y} \sum_{XN} \frac{d^3k}{(2\pi)^3} \int \frac{d^3P_1}{(2\pi)^32E_1} \frac{d^3P_2}{2(2\pi)^32E_2} \frac{d^3P_3}{(2\pi)^32E_3} \times (2\pi)^4\delta^4(Q - P_1 - P_2 - P_3 - k) \\
\times \frac{1}{3} \times \frac{1}{3} \times \frac{1}{2} \times \int \frac{d^4q_1}{(2\pi)^4} \frac{d^4q_3}{(2\pi)^4} A_{ij}(k_1, k_2, P_1, P_2, P_3; q_1) \\
\times [\gamma^0 A^+_t(k_1, k_2, P_1, P_2, P_3; q_3)\gamma^0]_{kl} \int d^4x_1 d^4x_3 e^{-iq_1x_1 + iq_3x_3} \\
\times <0|Q_k(0)Q_t(x_3)|\Xi_{cc} + X_N><\Xi_{cc} + X_N|Q_j(x_1)\bar{Q}_j(0)|0>,
\]

where the spin summation of the baryon $\Xi_{cc}$, and the polarization and color summation of two anti-charm quarks are implied. Here we take nonrelativistic normalization for the baryon $\Xi_{cc}$. We can eliminate the sum over $X_N$ by using translational covariance. Defining
the creation operator $a^\dagger(k)$ for $\Xi_{cc}$ with the three momentum $k$, we obtain

$$d\Gamma = \frac{1}{2M} \frac{1}{18 (2\pi)^3} \int \frac{d^3P_1}{(2\pi)^3} \frac{d^3P_2}{(2\pi)^3} \frac{d^3P_3}{(2\pi)^3} \int \frac{d^4q_1}{(2\pi)^4} \frac{d^4q_3}{(2\pi)^4} A_{ij}(k_1, k_2, P_1, P_2, P_3; q_1)$$

$$\times [\gamma^0 A^\dagger(k_1, k_2, P_1, P_2, P_3; q_3) \gamma^0]_{kl}$$

$$\times \int d^4x_1 d^4x_2 d^4x_3 e^{-i q_1 x_1 + i q_3 x_3 - i q_2 x_2}$$

$$\times < 0 | Q_k(0) Q_l(x_3) a^\dagger_k a^\dagger_k \overline{q}_i(x_1) \overline{q}_j(x_2) | 0 > ,$$

(7)

with $q_2 = k - q_1$.

We use heavy quark effective field theory to deal with the $\Xi_{cc}$ state. In $\Xi_{cc}$ rest frame, the heavy quarks move with a small velocity $v_c$. Hence, the Fourier transformed matrix element can be expanded in $v_Q$ with fields of NRQCD. The relation between NRQCD fields and Dirac fields $Q(x)$ in the rest frame is

$$Q(x) = e^{-imc t} \left\{ \begin{array}{c} \psi(x) \\ 0 \end{array} \right\} + O(v_c) + ...,$$

(8)

where $\psi(x)$ is NRQCD field. We will work at the leading order of $v_c$. We denote $v$ as the velocity of $\Xi_{cc}$ with $v^\mu = k^\mu / M_{\Xi_{cc}}$ to express our result of Fourier transformed matrix element in a covariance way. Hence, the Fourier transformed matrix element in the rest frame is

$$v^0 \int d^4q_1 d^4q_2 d^4q_3 e^{-i q_1 x_1 - i q_2 x_2 + i q_3 x_3}$$

$$< 0 | Q_k(0) Q_l(x_3) a^\dagger_k a^\dagger_k \overline{q}_i(x_1) \overline{q}_j(x_2) | 0 >$$

$$= \int d^4q_1 d^4q_2 d^4q_3 e^{-i q_1 x_1 - i q_2 x_2 + i q_3 x_3}$$

$$< 0 | Q_k(0) Q_l(x_3) a^\dagger(k = 0) a^\dagger(k = 0) \overline{q}_i(x_1) \overline{q}_j(x_2) | 0 > .$$

(9)

Using Eq.(8), the matrix element in Eq.(9) can be expanded with $\psi(x)$ and $\psi^\dagger(x)$. The spacetime dependence of the matrix element with NRQCD field is controlled by the scale $m_c v_c$. At the leading order of $v_c$ one can neglect the spacetime dependence and the mass of the baryon $M_{\Xi_{cc}}$ is approximated by $2m_c$. With the approximation the matrix element in Eq.(9) is

$$< 0 | \phi_{\lambda_3}^{a_3}(0) \phi_{\lambda_4}^{a_4}(0) a^\dagger a^\dagger \psi_{\lambda_1}^{a_1}(0) \psi_{\lambda_2}^{a_2}(0) | 0 >$$

(10)
where we suppress the notation $k = 0$ and it is always implied that NRQCD matrix elements are defined in the rest frame of $\Xi_{cc}$. The superscripts $a_i (i = 1, 2, 3, 4)$ are used to label the color of quark fields, while the subscripts $\lambda_i (i = 1, 2, 3, 4)$ for the quark spin indices. We obtain the matrix element by two parameters, $h_1, h_3$ as following:

\[
< 0 | \psi_{\lambda_3}^{a_3} (0) \psi_{\lambda_4}^{a_4} (0) a^+ a \psi_{\lambda_1}^{a_1} (0) \psi_{\lambda_2}^{a_2} (0) | 0 >
= (\varepsilon)_{\lambda_3 \lambda_4} (\varepsilon)_{\lambda_2 \lambda_1} \cdot (\delta_{a_1 a_4} \delta_{a_2 a_3} + \delta_{a_1 a_3} \delta_{a_2 a_4}) \cdot h_1
+ (\sigma^n \varepsilon)_{\lambda_3 \lambda_4} (\varepsilon \sigma^n)_{\lambda_2 \lambda_1} \cdot (\delta_{a_1 a_4} \delta_{a_2 a_3} \delta_{a_1 a_3} - \delta_{a_2 a_4}) \cdot h_3,
\]

\(11\)

where $\sigma^i (i = 1, 2, 3)$ are Pauli matrices. $\varepsilon = i \sigma^2$ is totally anti-symmetric. The parameters $h_1$ and $h_3$ are defined as:

\[
h_1 = \frac{1}{48} < 0 | [\psi_{\epsilon}^{a_1} \psi_{\epsilon}^{a_2} + \psi_{\bar{\epsilon}}^{a_2} \psi_{\bar{\epsilon}}^{a_1}] a^+ a \psi_{\epsilon}^{a_2} \varepsilon \psi_{\epsilon}^{a_1} | 0 >,
\]

\[
h_3 = \frac{1}{72} < 0 | [\psi_{\epsilon}^{a_1} \psi_{\epsilon}^{a_2} - \psi_{\bar{\epsilon}}^{a_2} \psi_{\bar{\epsilon}}^{a_1}] a^+ a \psi_{\epsilon}^{a_2} \sigma^n \varepsilon \psi_{\epsilon}^{a_1} | 0 >.
\]

\(12\)

$h_1 (h_3)$ represents the probability for a cc pair in a $^1S_0(^3S_1)$ state and in the color state of $6(\bar{3}^*)$ to transform into the baryon. It is the Pauli exclusion principle determines that only these two kinds of combination of colour and spin states, which are asymmetric, are possible \[26\]. With these results the Fourier transformed matrix element in Eq.\(9\) can be expressed as:

\[
v^0 \int d^4 x_1 d^4 x_2 d^4 x_3 e^{-i q_1 x_1 - i q_2 x_2 + i q_3 x_3} < 0 | Q_{\epsilon}^{a_3} (0) Q_{\bar{\epsilon}}^{a_4} (x_3) a^1 (k) a(k) \overline{Q}_{\epsilon}^{a_1} (x_1) \overline{Q}_{\bar{\epsilon}}^{a_2} (x_2) | 0 >
= (2\pi)^4 \delta^4 (q_1 - m_c v) (2\pi)^4 \delta^4 (q_2 - m_c v) (2\pi)^4 \delta^4 (q_3 - m_c v) \\
\times \left[ - (\delta_{a_1 a_4} \delta_{a_2 a_3} + \delta_{a_1 a_3} \delta_{a_2 a_4}) \right] \\
(\bar{P}_v C \gamma_5 P_v)_{ji} (P_v \gamma_5 C \bar{P}_v)_{ik} h_1 \\
+ (\delta_{a_1 a_4} \delta_{a_2 a_3} - \delta_{a_1 a_3} \delta_{a_2 a_4}) (\bar{P}_v C \gamma^\mu P_v)_{ji} \\
(P_v \gamma^\mu C \bar{P}_v)_{ik} (v_\mu v_\nu - g_\mu_\nu) h_3 + ...
\]

\(13\)

where $P_v = \frac{1 + \gamma_v}{2}, \bar{P}_v = \frac{1 - \gamma_v}{2}$; $C = i \gamma^2 \gamma^0$, the charge conjugation operator.
With the above formula, we obtain the decay width as following:

$$d\Gamma = \frac{16\pi^4\alpha_s^3|R_\Upsilon(0)|^2M_\Xi}{9M_\Upsilon^2}\frac{1}{[(P_1+k/2)(P_2+k/2)]^2}$$

$$\sum_{c=1}^{8}(\sum_{\xi=1}^{6}\overline{A}_{\xi}^{abc})(\sum_{\zeta=1}^{6}A_{\zeta}^{a'b'c})H^{aba'b'}$$

$$\frac{d^3k}{(2\pi)^32E_k}\prod_{i=1}^{3}\frac{d^3P_i}{(2\pi)^32E_i}(2\pi)^4\delta^4(Q - P_1 - P_2 - P_3 - k),$$

where

$$H^{aba'b'} = -(Tr[T^a T^{a'} T^b T^{b'}] + Tr[T^a T^{a'}] Tr[T^b T^{b'}]) \times h_1 \times B_1$$

$$+ (Tr[T^a T^{a'} T^b T^{b'}] - Tr[T^a T^{a'}] Tr[T^b T^{b'}]) \times h_3 \times B_2,$$

$$B_1 = Tr[\gamma^\alpha (P_2 - m_c)\gamma^a P_v \gamma_\alpha (-P_1 - m_c)\gamma^\beta \gamma^\beta P_v \gamma_\beta P_v],$$

$$B_2 = Tr[\gamma^\alpha (P_2 - m_c)\gamma^a P_v \gamma_\alpha (-P_1 - m_c)\gamma^\beta \gamma^\beta P_v \gamma_\beta P_v](v_\mu v_\mu - g_{\mu\nu}).$$

The function $A_\xi(\xi = 1, 2, 3, 4, 5, 6)$ are given in Appendix A.

The radial wave function for $\Upsilon$ at origin can be obtained, e.g., by fitting its leptonic decay width. On the other hand, the value of $h_1$ and $h_3$ is difficult to be obtained. There are no experiment results now. Here we employ a potential model with the radial wave function $R_{cc}(r)$ at origin [10] to get the numerical value of $h_3$

$$h_3 = \frac{|R_{cc}(0)|^2}{4\pi},$$

with its value to be 0.0287 GeV$^3$. There is no practical model for $h_1$, which can be taken as a free parameter, the reason is explained later. In the numerical calculations, we take $\Psi_\Upsilon(0) = 2.194 GeV^3/2$, $M_\Upsilon = 9.46 GeV, M_\Xi = 3.621 GeV, m_b = 4.73 GeV, \alpha_s(m_c) = 0.253$. $m_c/m_b$ is taken to be parameter, and the dependence of branching ratio on $m_c/m_b$ is studied as shown in FIG.2.

With $m_c/m_b = 0.25$ the partial width is $\Gamma = (0.0126h_1 + 0.240h_3) \text{ KeV}$. Here we see that the perturbative part corresponding to $h_1$ is much smaller than that of $h_3$. So if there is
no special enhancement on $h_1$, this part of contribution can not be significant. Here for simplicity we take $h_1 = h_3$, and the decay width is 7.256eV, leading to the branching ratio as $1.34 \times 10^{-4}$. The $\Xi_{cc}$ momentum distributions are shown in FIG.3 and FIG. 4. The momentum distributions of $\bar{c}$ are shown in FIG.5 and FIG 6.

![Graph](image1)

FIG. 2: Dependence of branching ratio on $m_c/m_b$.

![Graph](image2)

FIG. 3: The momentum distribution of $\Xi_{cc}, h_3 = 0$

![Graph](image3)

FIG. 4: The momentum distribution of $\Xi_{cc}, h_1 = 0$

The experiment of BELLE in 2016 has collected $102 \times 10^6 \Upsilon$ events [1, 13]. So it is possible to make a scan on the $\Xi_{cc}$ production. In the future, further precise measurement
on the production of $\Xi_{cc}$ can even be made with more large luminosity at BELLE2. Similar productions characteristic of the partonic state with four charm (anti)quarks can also be studied in $\Upsilon$ decay.

Acknowledgments

This work is supported by National Natural Science Foundation of China (grant Nos. 11635009, 11775130) and the Natural Science Foundation of Shandong Province (grant ZR2017MA002).
Appendix A

The functions $\bar{A}_\xi(\xi = 1, ..., 6)$ in the decay width are:

$$
\bar{A}_1 = Tr^a[T^c T^b T^a] \frac{1}{[(q - P_3)^2 - m^2][(q - P_1 - k/2)^2 - m^2]}
\times Tr[\gamma^* (P_3)(m + P_3 - q)\gamma_\alpha(q - P_1 - \frac{k}{2} + m)\gamma_\beta(M + \not{P})\not{\epsilon}]
$$

$$
\bar{A}_2 = Tr^a[T^b T^c T^a] \frac{1}{[(P_2 + k/2 - q)^2 - m^2][(q - P_1 - k/2)^2 - m^2]}
\times Tr[\gamma_\alpha(m + P_2 + \frac{k}{2} - q)\gamma^* (P_3)(q - P_1 - \frac{k}{2} + m)\gamma_\beta(M + \not{P})\not{\epsilon}]
$$

$$
\bar{A}_3 = Tr^a[T^c T^a T^b] \frac{1}{[(P_3 - q)^2 - m^2][(q - P_2 - k/2)^2 - m^2]}
\times Tr[\gamma^* (P_3)(m + P_3 - q)\gamma_\beta(q - P_2 - \frac{k}{2} + m)\gamma_\alpha(M + \not{P})\not{\epsilon}]
$$

$$
\bar{A}_4 = Tr^a[T^a T^b T^c] \frac{1}{[(P_1 + k/2 - q)^2 - m^2][(q - P_2 - k/2)^2 - m^2]}
\times Tr[\gamma_\beta(m + P_1 + \frac{k}{2} - q)\gamma^* (P_3)(q - P_2 - \frac{k}{2} + m)\gamma_\alpha(M + \not{P})\not{\epsilon}]
$$

$$
\bar{A}_5 = Tr^a[T^b T^a T^c] \frac{1}{[(P_2 + k/2 - q)^2 - m^2][(q - P_3)^2 - m^2]}
\times Tr[\gamma_\alpha(m + P_2 + \frac{k}{2} - q)\gamma_\beta(q - P_3 + m)\gamma^* (P_3)(M + \not{P})\not{\epsilon}]
$$

$$
\bar{A}_6 = Tr^a[T^a T^b T^c] \frac{1}{[(P_1 + k/2 - q)^2 - m^2][(q - P_3)^2 - m^2]}
\times Tr[\gamma_\beta(m + P_1 + \frac{k}{2} - q)\gamma_\alpha(q - P_3 + m)\gamma^* (P_3)(M + \not{P})\not{\epsilon}]
$$

(A1)

Here $Tr^a[...]$ means only keeping the symmetric part; $m = m_b$, $M = M_T$, $q = P/2$. $\varepsilon(P_3)$ is the polarization vector of the gluon with momentum $P_3$.

[1] C. P. Shen et al. [Belle Collaboration], Phys. Rev. D 93 (2016) no.11, 112013 doi:10.1103/PhysRevD.93.112013 arXiv:1605.00990 [hep-ex].

[2] S. Jia et al. [Belle Collaboration], Phys. Rev. D 96 (2017) no.11, 112002 doi:10.1103/PhysRevD.96.112002 arXiv:1711.01690 [hep-ex].

[3] N. Brambilla, S. Eidelman, C. Hanhart, A. Nefediev, C. P. Shen, C. E. Thomas, A. Vairo and C. Z. Yuan, arXiv:1907.07583 [hep-ex].

[4] M. Tanabashi et al. (Particle Data Group), Phys. Rev. D 98, 030001 (2018).
[5] W. Han, S. Y. Li, Y. H. Shang, F. L. Shao and T. Yao, Phys. Rev. C 80 (2009), 035202 doi:10.1103/PhysRevC.80.035202 [arXiv:0906.2473 [hep-ph]].

[6] Y. Jin, S. Y. Li and S. Q. Li, Phys. Rev. D 94 (2016) no.1, 014023 doi:10.1103/PhysRevD.94.014023 [arXiv:1603.03250 [hep-ph]].

[7] S. Y. Li, “Production of multi-quark hadrons in high energy multi-production processes,” Proceedings of 2017 Moriond QCD.

[8] Y. Jin, S. Y. Li, Y. R. Liu, L. Meng, Z. G. Si and X. F. Zhang, Chin. Phys. C 41 (2017) no.8, 083106 doi:10.1088/1674-1137/41/8/083106 [arXiv:1610.04411 [hep-ph]].

[9] S. Y. Li, Q. b. Xie and Q. Wang, Phys. Lett. B 482 (2000) 65 doi:10.1016/S0370-2693(00)00439-1 [hep-ph/9912328].

[10] A. V. Berezhnoy, V. V. Kiselev, A. K. Likhoded and A. I. Onishchenko, Phys. Rev. D 57 (1998) 4385 doi:10.1103/PhysRevD.57.4385 [hep-ph/9710339].

[11] A. Ocherashvili et al. [SELEX], Phys. Lett. B 628 (2005), 18-24 doi:10.1016/j.physletb.2005.09.043 [arXiv:hep-ex/0406033 [hep-ex]].

[12] R. Aaij et al. [LHCb], Phys. Rev. Lett. 119 (2017) no.11, 112001 doi:10.1103/PhysRevLett.119.112001 [arXiv:1707.01621 [hep-ex]].

[13] S. Jia, X. Zhou and C. Shen, [arXiv:2005.05892 [hep-ex]].

[14] Z. G. He, B. A. Kniehl and X. P. Wang, Phys. Rev. D 101 (2020) no.7, 074002 doi:10.1103/PhysRevD.101.074002 [arXiv:1912.10232 [hep-ph]].

[15] Z. G. He and J. X. Wang, Phys. Rev. D 82 (2010), 094033 doi:10.1103/PhysRevD.82.094033 [arXiv:1009.1563 [hep-ph]].

[16] R. A. Briere et al. [CLEO], Phys. Rev. D 70 (2004), 072001 doi:10.1103/PhysRevD.70.072001 [arXiv:hep-ex/0407030 [hep-ex]].

[17] W. Han and S. Y. Li, Phys. Rev. D 74 (2006), 117502 doi:10.1103/PhysRevD.74.117502 [arXiv:hep-ph/0607251 [hep-ph]].

[18] W. Han, S. Y. Li, Z. G. Si and Z. J. Yang, Phys. Lett. B 642 (2006), 62-67 doi:10.1016/j.physletb.2006.08.067 [arXiv:hep-ph/0601195 [hep-ph]].

[19] Y. Jin, S. Y. Li, Z. G. Si, Z. J. Yang and T. Yao, Phys. Lett. B 727 (2013), 468-473 doi:10.1016/j.physletb.2013.10.070 [arXiv:1309.5849 [hep-ph]].

[20] Y. Jin, S. Y. Li, Y. R. Liu, Z. G. Si and T. Yao, Phys. Rev. D 89 (2014) no.9, 094006 doi:10.1103/PhysRevD.89.094006 [arXiv:1401.6652 [hep-ph]].
[21] J. H. Kühn, J. Kaplan and E. G. O. Safiani, Nucl. Phys. B 157, 125 (1979).

[22] C. H. Chang, Nucl. Phys. B 172, 425 (1980).

[23] R. Baier and R. Rückl, Phys. Lett. B 102, 364 (1981); and Z. Phys. C 19, 251 (1983).

[24] O. Nachtmann, 'Elementary particle Physics Concept and Phenomena', Springer-Verlag, 1990.

[25] S. Y. Li, Y. R. Liu, Y. N. Liu, Z. G. Si and X. F. Zhang, Commun. Theor. Phys. 69 (2018) no.3, 291 doi:10.1088/0253-6102/69/3/291 [arXiv:1706.04765 [hep-ph]].

[26] J. P. Ma and Z. G. Si, Phys. Lett. B 568, 135 (2003) [hep-ph/0305079].

[27] In the following we use $\Upsilon$ to represent this 1S state.