GASDYNAMICS OF RELATIVISTICALLY EXPANDING GAMMA-RAY BURST SOURCES:
KINEMATICS, ENERGETICS, MAGNETIC FIELDS AND EFFICIENCY

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Abstract: We calculate both analytically and numerically the evolution of a highly relativistic fireball through the stages of free expansion and coasting, and determine the dependence of the thermodynamic and radiation variables in the comoving and laboratory frames. Magnetic fields may have been important in the original impulsive event. We discuss their effect on the fireball dynamics, and consider also their effects on the radiation emitted when the fireball runs into an external medium and is decelerated. The inverse synchro-Compton mechanism can then yield high radiative efficiency in the reverse shock, producing a burst of non-thermal radiation mainly in the MeV to GeV range whose total energy and duration agree with those of typical cosmic gamma ray bursts.

1. Introduction

Gamma-ray burst sources (GRB) have long been suspected to originate from the sudden release of energy in small regions of space, where the initial energy density and characteristic photon energy is so large that an opaque \( e^\pm \) fireball forms (e.g. Cavallo and Rees, 1978, Paczyński, 1986, Goodman, 1986, Shemi and Piran, 1990). If these events were located at distances comparable to (or larger than) typical galactic scales, the \( e^\pm \) fireballs would necessarily remain optically thick out to radii of at least \( \sim 10^9 \) cm (e.g. Zdziarzki, 1982, Imamura and Epstein, 1987), depending on the amount of normal electrons and baryons mixed in with the pairs. If the fireball originated in a region smaller than this, the radiation pressure on the optically thick fireball would cause it to expand; the evolution of this fireball was initially thought to lead, when it became optically thin, to the observed gamma-ray bursts. This model has recently received increased attention because the spatial distribution of GRBs revealed by the BATSE experiment on the Compton Observatory strongly suggested an extended galactic halo or a cosmological origin (Meegan, et al., 1991, Fishman, 1992, Paczyński, 1992).

In its simplest version, the fireball model failed to account for the duration and time-structure of the observed bursts: it predicts a very short burst, emitted when the expanding fireball becomes optically thin (Goodman, 1986), and a quasi-thermal, soft \( \gamma \)-ray spectrum (Goodman, 1986, Paczyński, 1986). Another problem, emphasized by Paczyński, 1990, was the possibility that the \( \gamma \)-rays could easily be degraded to even...
lower energies by adiabatic cooling due to scattering on electrons associated with baryons polluting the pair flow. In fact, in the latter case, most of the fireball energy gets converted into bulk kinetic energy of the polluting baryons, while the photon burst at thinning has much less energy than initially produced. These problems are removed when one considers anisotropic scenarios where high Lorentz factor fireballs can escape the inevitable baryon-polluted slow wind that must accompany the initial energy deposition (e.g. Mészáros and Rees, 1992a, 1992b). In such cases, the main part of the observed gamma-ray radiation occurs when the bulk kinetic energy of the baryons (carrying essentially the full energy of the initial fireball) is re-randomized and radiated away in the blast wave being pushed ahead of the relativistically expanding baryons, and in the reverse shock that propagates inwards into the baryonic fireball gas as the latter is decelerated by the external medium (Rees and Mészáros, 1992, Mészáros and Rees, 1993). This is a very generic mechanism, which operates in almost any scenario for the original energy deposition, as long as it makes a relatively clean, high entropy initial fireball. Moreover, the process depends on the external environment, thereby allowing the possibility that even a standardised type of fireball could create bursts with a variety of complex time-profiles.

The initial development of the fireball determines the final properties of the blast waves, and thus of the bursts. In particular the final bulk Lorentz factor, together with the external density, determines the blast wave burst duration, while the dynamics of the fireball as a function of the baryon loading determines the relative amount of energy in bulk kinetic form (which gets radiated in the blast wave and reverse shock) and in pair and radiation form (which escapes when the fireball thins, usually before the blast wave burst). Since the relativistic dynamics of the fireball expansion determines the entire energetics as well as the temporal characteristics of the burst, a detailed calculation is important.

In the present paper we discuss the dynamics of the relativistic fireball expansion from the acceleration stage through the coasting phase, calculating both numerically and analytically the thermodynamic and radiation variables of the flow through the transition to optical thinness and the saturation of the bulk velocity. We discuss the kinematics both in the comoving and laboratory frames, and indicate the scaling of the behavior with the various parameters of the problem, extending the treatment to the anisotropic (jet) case. We also consider the role of magnetic fields in the dynamics of the evolution, including the case of magnetically dominated fireballs, and investigate the effect of the magnetic field on the radiative efficiency of the reverse shock that arises when the coasting fireball is decelerated in an external medium.

2. Numerical Treatment of a Highly Relativistic Expanding Gas

2.1. Computational Approach

Because of the presence of a natural length in the problem (the initial value of the radius \( r_o \)) the gas dynamic equations do not allow an exact similarity solution valid at all radii. For this reason, a numerical solution is the only way to follow exactly the development of the gas, although approximate analytic solutions are possible in the initial
acceleration stage and in the later coasting phase (see §3). Previous calculations of the free expansion of a relativistic gas in spherical symmetry (e.g. Vitello and Salvati, 1976, using a characteristics method) have followed the evolution over one and a half decades in the expansion factor. However, expansion over such a limited range is usually not sufficient for the bulk velocity to reach its ultimate saturation value, for the cases of large values of the initial radiation to rest mass energy ratio $\eta$ considered in many problems, such as the GRB fireball problem,

$$\eta = \frac{E_o}{M_o c^2} \approx 10^3 E_{51} (M_o/0.6 \times 10^{-6} M_\odot)^{-1}. \quad (2.1)$$

Here $E_o \sim 10^{51} E_{51}$ ergs is the order of magnitude of the initial photon energy expected from the liberation of about a solar mass equivalent of gravitational energy ($GM_\odot^2/R_N \sim \epsilon M_\odot c^2 \sim 10^{54}$ ergs, where $\epsilon \ll 1$, $R_n \sim 10^6$ cm, most of which goes into neutrinos and gravitational waves), while $M_o$ is the total mass of polluting baryons that get mixed in the photon-pair fireball of energy $E_o$.

The numerical treatment of the free expansion of a highly relativistic gas with spherical symmetry calls for the use of Lagrangian hydrodynamics because of the vast range of physical scales present in the problem, of the order of ten or more decades. We have developed a Lagrangian code for relativistic fluid flows that uses a second order Runge-Kutta integrator (Benz 1984) with adaptive time step. This method has the advantage that a Courant (Courant, Friedrichs & Lewy 1928) condition does not limit the size of the time steps, an essential ingredient to achieve short and large time scales. Relativistic artificial viscous stress was implemented in the code and calibrated using relativistic shock tube problems. This was used to verify the accuracy of the same code when artificial viscosity was switched off, which turns to be quite good (and significantly faster) in the cases of free expansion treated here. The reason why this is an excellent approximation is that shocks do not normally occur in such free expansion problems. All the runs described below are therefore for zero artificial viscosity. Typically, simulations consisted of 200 Lagrangian grid points (mass shells). Runs with different number of mass shells were done to check the convergence of our results. We monitored the ability of the code to reproduce the analytic solutions of the plane symmetric rarefaction wave traveling inwards (Vitello & Salvati 1976). We found that for 200 grid points the maximum absolute error ($\leq 3\%$) as well as the maximum cumulative error ($\leq 2\%$) appeared in the velocity.

### 2.2 Numerical Results

The initial gas configuration adopted here, for simplicity, is that the gas at $t = 0$ is uniformly distributed within $r \leq r_o$, having a total rest mass $M_o$, and that at $t = 0$ an amount of radiation energy $E_o \gg M_o c^2$ is deposited uniformly throughout the spherical gas. The gas is optically thick to its own electrons, and the resulting photon-pair-electron-baryon fluid is essentially an isentropic fluid which can be modeled as a gas of adiabatic index $\gamma_a = 4/3$, as long as the photons are coupled to the matter by radiation drag. The same applies to the case where the energy is mainly in the form of magnetic fields (see §4).
After decoupling, the adiabatic index becomes $\gamma_a = 5/3$. We have carried out calculations for values of $\eta$ ranging from close to unity up to $10^{10}$. As a specific example, we discuss the case of $\eta = 10^4$ below, which is characteristic of the relativistic behavior.

In the course of the expansion most of the mass, as seen by a laboratory observer, is concentrated in a thin shell near the leading edge of the expanding gas (c.f. also Vitello and Salvati, 1976). This is seen in Fig. 1, where the distribution of the lab density is plotted as a function of the (normalized) lab radius for $\eta = 10^4$. The curves are plotted at the times (from top to bottom) when the lab frame expansion factor $r(r_{max})/r_o$ (which is proportional to the lab frame time) is equal to $0.2, 0.4, 0.6, 0.8, 1.0$ times $\eta$.

As the gas expands, the mass shells acquire a bulk velocity which initially increases in time and eventually saturates. The distribution of this bulk Lorentz factor over the various mass shells is shown in Fig. 2, for a fireball of $\eta = 10^4$ at the lab times (increasing from bottom to top) when $r(r_{max})/r_o = \eta^{1/2}, \eta, \eta^{3/2}, \eta^2, \eta^{5/2}, \eta^3$. One sees that the leading edge reaches a value of $\Gamma \sim \eta$, and most of the mass reaches one third of that value, at the laboratory time for which the expansion factor $r(r_{max})/r_o \sim \eta$. After that the average bulk Lorentz factor of most of the matter saturates to the value $\Gamma \sim \eta$, while the leading edge remains within a factor 2 of that value, and the trailing edge of the inner 10% of the matter tapers off fairly steeply to zero.

The average bulk Lorentz factor (or what is nearly the same, $\Gamma(r_{max})$ where the density is maximal) grows initially linearly with radius, and saturates to a value $\Gamma \sim \eta$ after a (lab) expansion factor equal to $\eta$. This is seen in Fig. 3, where $\Gamma(r_{max})$ is plotted against the expansion factor $r(r_{max})/r_o$.

Within the thin shell containing most of the mass, called the mass envelope shell, the bulk of the mass (say the inner 80% away from the outer and inner edges) is distributed quite uniformly, even after the average bulk Lorentz factor has saturated (for $r(r_{max})/r_o \sim \eta$). Only later, particularly after $r(r_{max})/r_o \sim \eta^2$, a slight asymmetry favoring the leading edge of the envelope shell starts to become apparent, but even then the matter within the shell can be considered uniform to a good approximation. This is shown in Fig. 4a, where the lab frame density $\rho_L$ is plotted against the Lagrangian mass coordinate $M/M_t$ (where $M_t \equiv M_o$ is the total rest mass, which is mostly within the shell). In figure 4b we show the same plot for the comoving mass density $\rho_c$. The latter is even more uniform than the laboratory frame density. Thus, a comoving observer would find that the world around it is essentially isotropic and homogeneous, at least far from the edges, but with a density that drops in time, as seen from the decreasing value of the average density. The various curves, from top to bottom, are for (increasing) times when the expansion factor $r(r_{max})/r_o$ is equal to $\eta^{1/2}, \eta, \eta^{3/2}, \eta^2, \eta^{5/2}, \eta^3$.

The width of the mass envelope shell in the lab frame is initially nearly constant an equal to the initial radius, $\Delta r/r_o \sim 1$ (c.f. also Vitello and Salvati, 1976, Goodman, 1986). This remains so, however, only until an expansion factor $r(r_{max})/r_o \sim \eta^2$ is reached. After that, the lab width starts to grow linearly (see Fig. 5a). The comoving frame width, for the shell, grows initially linearly with the expansion factor until $\eta$, remains approximately constant between $\eta$ and $\eta^2$, and then resumes a linear growth with radius (Fig. 5b). The physical reason for this behavior is discussed in §3.
We have also calculated numerically the lab radius at which an expanding fireball becomes Thomson optically thin against its own electrons, as a function of the initial radiation energy to rest mass ratio \( \eta \). This condition is computed in the comoving frame, being the same in the lab frame, as it should, since the optical depth of a constant amount of mass is an invariant between reference frames. This (lab) thinning radius \( r_t \) is found to vary as \( \eta^{-1/2} \), in excellent agreement with the analytic estimate (3.13). Another quantity of interest is the value of average bulk Lorentz factor at which fireballs of varying \( \eta \) become optically thin, as a function of \( \eta \). The numerically computed value of \( \Gamma_t \) grows linearly with \( \eta \) for low values of \( \eta \) where optical thinness is achieved before the bulk velocity \( \Gamma \) saturates to \( \eta \), and \( \Gamma_t \) decreases as \( \eta^{-1/2} \) for larger values of \( \eta \) for which thinness is achieved after the bulk Lorentz factor has saturated (in agreement with Shemi and Piran, 1990). The linear decrease \( \propto \eta^{-1/2} \) changes to \( \eta^0 \) for very large values of \( \eta \) (or initial radiation energies sufficiently larger than \( M_o c^2 \)) at which the opacity of the fireball at optical thinness is still dominated by the pairs (instead of by the polluting baryonic electrons). This occurs for \( \eta \gtrsim 0.6 \times 10^{10} \), above which \( r_t \sim \text{constant}, \Gamma_t \sim 2.4 \times 10^{3} \sim \text{constant} \) (see §3).

3. Analytical Treatment of the Fireball Dynamics

3.1 Accelerated and Saturated Expansion

A fireball expanding into a low density external environment may, in its initial stages, be considered to be expanding in a vacuum. Thus, as long as the inertia of the accumulated external matter can be neglected, shocks will not play an important role in the energetics. Pressure gradients may exist in the expanding fireball gas, particularly if the initial density and pressure distribution is inhomogeneous, but a free (unimpeded) expansion will tend to stretch these out. One may therefore, as a first approximation, neglect internal pressure gradients. Thus, even though we are dealing with a fluid, the dynamics of the expansion would be expected initially to resemble that of a collisionless gas of relativistic particles. The particles initially have an isotropic velocity distribution within a radius \( r_o \), with an initial thermal Lorentz factor \( \eta = E_o/M_o c^2 \). But as they expand, the velocity distribution viewed in the lab frame will become increasingly anisotropic. For the ballistic expansion in the lab frame, when the particles have reached a radius \( r \) their velocity vectors will be confined within an angle \( (r/r_o)^{-1} \) of the radial direction. A transformation to a frame of reference which is outwardly moving with a Lorentz factor

\[
\Gamma \sim \theta^{2/3}(r/r_o),
\]

makes the particle velocity distribution appear again isotropic. This frame is, therefore, the comoving frame of the expanding particles in the initial acceleration phase, which moves outward with a velocity increasing according to \( \Gamma \propto r \). (We have introduced in (3.1) an angular factor \( \theta \) which approximately accounts for jet geometry; for spherical symmetry \( \theta \sim 1 \), see below).

The expansion occurs at the expense of the comoving frame thermal energy \( E \), which from conservation of energy must therefore drop as \( E \propto r^{-1} \). This agrees with the adiabatic
expansion law $E \propto V^{-1/3} \propto \rho^{1/3}$, where $V$, $\rho$ are comoving volume and comoving baryon density, provided that the comoving volume $V \propto r^3$. The latter follows from the fact that the laboratory frame radial extent of the particles in free expansion is expected to be (initially, at least) $\Delta r \sim r_0$, while the comoving radial width $\Delta R$ is related to the corresponding lab width through $\Delta R = \Delta r \Gamma$ (we denote comoving radii with capital $R$ and laboratory radii with lower case $r$). Since the dimensions transverse to the motion will be the same in the lab and comoving frame, the comoving volume in this (acceleration) phase is $V \propto r^2 \Delta R \propto r^2 r_0 \Gamma \propto r^3$.

The accelerating behavior $\Gamma \propto (r/r_0)$, however, can only go as long as the internal energy of the expanding gas is relativistic. After the comoving energy density drops below the baryon rest mass density $\rho c^2$, the bulk Lorentz factor $\Gamma$ must saturate to the maximum value it can acquire, which is approximately the initial thermal Lorentz factor $\eta$ in the case of significant baryon loading, or more generally

$$\Gamma_s \sim \min \left[ \eta, \Gamma_m \right], \quad (3.2)$$

where the second value is appropriate for $\eta > \Gamma_m$ given in eq.(3.14), as shown in §3.4.

The initial acceleration and the saturation behavior can also be obtained from a phenomenological expression for the average bulk Lorentz factor (e.g., Shemi and Piran, 1990)

$$\Gamma \sim \frac{E_o + M_o c^2}{E + M_o c^2} = \frac{\eta + 1}{\eta(E/E_o) + 1}$$

making the analogy with a section of the expanding universe, i.e. assuming spherical symmetry, homogeneity and isotropy. Here then $E$ is the fluid thermal energy density (initially mainly in photons and leptons, with an admixture of $M_o$ baryons). As long as it is optically thick and $\eta \approx 1$, the pressure is radiation-dominated throughout the expansion, and using the adiabatic behavior of a relativistic fluid one has

$$E/E_o = T/T_o = \left( \rho/\rho_o \right)^{1/3} = (V/V_o)^{-1/3}, \quad (3.4)$$

where $E$, $T$, $V$, $\rho$ are comoving thermal energy, temperature, volume and baryon density. In the linear expansion phase, we can take (from the discussion below equation (3.1)) $V^{1/3} \propto r^{-1}$, where $r$ is lab radius, so From (3.3) and (3.4) one gets for $E_o/E \ll \eta$ the accelerating behavior $\Gamma \sim E_o/E \sim r/r_o$, while for $E_o/E \gg \eta$ one gets the saturated $\Gamma \rightarrow \eta$, c.f. eq.(3.2). The saturation occurs (in spherical symmetry) at $r_s/r_o \sim \eta$. However, for $\eta > \Gamma_m$ given by eq.(3.14), $\eta$ must be replaced by $\Gamma_m$, see §3.4.

3.2 Lab and Comoving Geometry

As the particles move outward with velocity vectors which are increasingly radial, they form a radially expanding shell whose radius is initially $\Delta r \sim r_0$. The radial velocity spread $(v-c)/c \sim \Gamma^{-2}$, and a noticeable departure from the approximately constant width
$r_o$ starts to become appreciable only after $r \gg r_b$ where $\Delta r \sim r\Delta v/c \sim r_b \eta^{-2} \gg r_o$, or $r_b/r_o \gg \eta^2$. The laboratory frame width is therefore

$$\Delta r \sim \max \left[ r_o, r/\Gamma^2 \right] \sim \begin{cases} r_o, & \text{for } r \ll r_b; \\ r/\Gamma^2, & \text{for } r \gg r_b. \end{cases} \quad (3.5)$$

The width in the comoving frame is $\Delta R = \Delta r \Gamma$, which from eqs. (3.1),(3.2) and (3.5) is

$$\Delta R \sim \begin{cases} \theta^{2/3} r, & \text{for } r \ll r_s; \\ r_o \Gamma_s, & \text{for } r_s \ll r \ll r_b; \\ r/\Gamma_s, & \text{for } r \gg r_b, \end{cases} \quad (3.6)$$

where the two characteristic radii for saturation and for the start of the shell linear expansion in the lab frame are

$$r_s/r_o \sim \theta^{-2/3} \Gamma_s, \quad r_b/r_o \sim \Gamma^2_s. \quad (3.7)$$

The comoving volume in the three different stages of free expansion is

$$V = 4\pi \theta^2 r^2 \Delta R \sim \begin{cases} 4\pi \theta^2 r^3, & \text{for } r < r_s; \\ 4\pi \theta^2 \Gamma_s r_o r^2, & \text{for } r_s < r < r_b; \\ 4\pi \theta^2 \Gamma_s^{-1} r^3, & \text{for } r > r_b. \end{cases} \quad (3.8)$$

and the comoving radiation energy and temperature vary according to

$$\left( \frac{E}{E_o} \right) = \left( \frac{T}{T_o} \right) = \begin{cases} \theta^{-2/3}(r_o/r), & \text{for } r < r_s; \\ \theta^{-2/3}(r_o/r_s)(r_s/r)^{2/3} = \theta^{-4/9} \Gamma_s^{-1/3} (r_o/r)^{2/3}, & \text{for } r_s < r < r_b; \\ \theta^{-2/3}(r_o/r_s)(r_b/r)^{2/3} = \theta^{-4/9} \Gamma_s^{-1/3} r_o/r, & \text{for } r > r_b. \end{cases} \quad (3.9)$$

This is therefore the complete $r$ dependence of the adiabatic law (3.4). The factor $\theta$ can be taken to include both an angle dependence and a possible dependence on the statistical weight factor $g$ in the equilibrium energy density $gaT^4$. That is, $\theta^{-2/3} \equiv (g_o/g)^{1/3} \theta'^{-2/3}$, where $(g_o/g)^{1/3} = (11/4)^{1/3}$ accounts for the change in the statistical factor for the equilibrium energy density when the pairs annihilate as the temperature drops below $kT \ll m_e c^2$, and $\theta'^{-2/3}$ accounts for the possibility of expansion along a restricted range of solid angles. If we have radial expansion along two jets of solid angle $\theta_o$, and all of the energy is channeled into this solid angle, then $\theta' \sim \theta_o 2^{-1/2}$ (or $\sim \theta_o/2$ if there is one jet only). If the gas uses some its internal energy to perform work against a medium which prevents escape in the directions outside of the jet angles $\theta_o$, then the effective $\theta' < \theta_o 2^{-1/2}$. In the discussion below (and in the numerical calculations of §2) we have ignored the small shift of the curves associated with the change of the statistical factor, i.e. we simplify to $(g_o/g) \sim 1$, $\theta \sim \theta'$. (Note that this angular factor is only approximately correct, since it does not account for any possible lateral expansion or transverse radiation loss of the jet; it is strictly valid for one-dimensional radial variations within the angle $\theta$). For a jet, the
asymptotic accelerated behavior has the same radial dependence, but displaced to higher radii by \( \theta^{-2/3} \), as is the saturation radius,

\[
\Gamma \sim \theta^{2/3}(r/r_o) \quad , \quad r_s/r_o \sim \theta^{-2/3} \eta \quad , \quad \Gamma_s \sim \eta ,
\]

(3.10)

(which is valid only for \( \eta < \Gamma_m \); otherwise \( \eta \) must be replaced by \( \Gamma_m \) given by eq.(3.14) see §3.2). The effect of the approximate angular factor, e.g. in the behavior of the growth of the bulk Lorentz factor (eq.(3.1)), is that, for the same initial energies the jet achieves the same velocities as a corresponding spherical flow, but at larger radii. This is because at the same radii the jet, being confined to narrower angles, has a larger internal entropy than the spherical flow. The saturation Lorentz factor is however the same in both cases.

### 3.3 Optical Depth and Saturation Lorentz Factor

Depending on the value of \( \eta = E_o/M_o c^2 \), the scattering depth of the fireball at optical thinness is dominated by the “polluting” baryonic electrons (if \( \eta < \eta_p \), where the latter is given by eq.(3.19)), or, if the fireball has very low baryon pollution, the scattering opacity previous to reaching optical thinness is dominated by the \( e^\pm \) pairs (Goodman, 1986, Paczyński, 1986, Shemi and Piran, 1990), for \( \eta > \eta_p \).

\(\text{a) Electron-dominated scattering:}\) In the case \( \eta < \eta_p \), when baryonic electrons dominate at thinness, the optical depth is

\[
\tau = \frac{M_o/m_p \sigma \Delta r}{4 \pi r^2 \theta^2 r^2 \Delta r} = \left( \frac{E_o \kappa}{4 \pi r_o^2 c^2} \right) \frac{1}{\theta^2 \eta} = \theta^{-4/3} \frac{\Gamma_m^3}{\eta} \left( \frac{r_o}{r} \right)^2 .
\]

(3.11)

Here \( \sigma \) is scattering cross section, \( \kappa = \sigma/m_p \sim 0.4 \text{ cm}^2 \text{ g}^{-1} \), and \( \Gamma_m \) is defined in eq.(3.14). The initial optical depth is

\[
\tau_o = \Gamma_m^3 \theta^{-4/3} \eta^{-1} = \Sigma_o \kappa = \eta^{-1} \Sigma_{r,o} \kappa ,
\]

where

\[
\Sigma_o \equiv M_o/4 \pi r_o^2 \theta^2 , \quad \Sigma_{r,o} \equiv E_o c^{-2}/4 \pi r_o^2 \theta^2 ,
\]

(3.12)

are the initial baryon mass surface density and the initial radiation equivalent mass surface density. In the course of the expansion, the gas becomes optically thin at a radius \( r_t \) defined by

\[
\frac{r_t}{r_o} = \theta^{-3/2} \Gamma_m^{3/2} \eta^{-1/2} = \left( \Sigma_o \kappa \right)^{1/2} = \tau_o^{1/2} = 1.9 \times 10^8 E_{51} r_6^{-1} \theta^{-1} \eta^{-1/2} ,
\]

(3.13a)

or

\[
r_t = \left( \frac{E_o \kappa}{4 \pi \theta^2 c^2 \eta} \right)^{1/2} = \frac{E_{51} r_6^{-1} \theta^{-1} \eta^{-1/2}}{6 \times 10^5} ,
\]

(3.13b)

where the initial energy \( E_o \) and radius \( r_o \) have been arbitrarily normalized to \( 10^{51} \text{ ergs} \) and \( 10^6 \text{ cm} \), and the second version (3.13b) is independent of \( r_o \). The critical Lorentz factor \( \Gamma_m \) and the critical \( \eta_m \) are defined as

\[
\Gamma_m \equiv \eta_m = \theta^{4/9} (\tau_o \eta)^{1/3} = \theta^{4/9} (\Sigma_o \kappa \eta)^{1/3} = \theta^{4/9} (\Sigma_{r,o} \kappa)^{1/3} = \left( \frac{E_o \kappa}{4 \pi r_o^2 c^2} \right)^{1/3} \theta^{-2/9} = 3.3 \times 10^5 E_{51}^{1/3} r_6^{-2/3} \theta^{-2/9} .
\]

(3.14)
This is the maximum possible bulk Lorentz factor achievable for a given initial radiation energy $E_o$ deposited within a given initial radius $r_o$ (§3.4). It is also, for a given $E_o$ and $r_o$, the “critical” value of $\eta = \eta_m$ (reached at a critical loading mass $M_o = M_m = (E_o/\eta_m c^2)$) for which the thinning radius (3.13) is equal to the saturation radius $(r_s/r_o) \sim \theta^{-2/3}$. For a given $E_o$, as one varies $M_o$ or $\eta$, the critical value $\eta = \eta_m = \Gamma_m$ is reached at a radius $r_m$ given by

$$\frac{r_m}{r_o} = \frac{r_s}{r_o} = \frac{r_t}{r_o} \sim \theta^{-2/3} \Gamma_m.$$  

(3.15)

For $\eta < \Gamma_m$, optical thinness is reached after saturation, whereas for $\eta > \Gamma_m$ optical thinness is reached before saturation, so the bulk Lorentz factor at thinning $\Gamma_t = \theta^{2/3}(r_t/r_o)$ is equal to

$$\Gamma_t = [\eta, \Gamma_m^{3/2} \eta^{-1/2}] \quad \text{for } [\eta < \Gamma_m, \Gamma_m < \eta < \Gamma_p],$$  

(3.16)

where $\Gamma_p$ is given by eq.(3.20).

b) Pair-dominated Scattering: For very large $\eta$, such that $\eta > \eta_p > \Gamma_m$, the $e^\pm$ scattering dominates, instead of baryonic electron scattering, and optical thinness occurs when the pairs fall out of equilibrium, which occurs at a lab radius $r_p$ given by

$$\frac{r_p}{r_o} = \theta^{-2/3} \frac{T_o}{T_p} \sim 2.4 \times 10^{3/4} E_{51}^{1/4} r_6^{-3/4} \theta^{-2/3},$$  

(3.17)

where $T_p \sim 15$ keV is the temperature where the thermal pair density becomes negligible, and $T_o \sim 36.4 E_{51}^{1/4} r_6^{-3/4}$ MeV is the initial temperature. This occurs during the acceleration stage, so at this point, where pairs become optically thin, $\Gamma$ has reached the value

$$\Gamma_t \equiv \Gamma_p = E_o/E_p = T_o/T_p = \theta^{2/3} (r_p/r_o) \sim 2.4 \times 10^{3/4} E_{51}^{1/4} r_6^{-3/4},$$  

(3.18)

valid for $\eta > \eta_p$. The latter is the value of $\eta$ above which the pair-dominated regime occurs,

$$\eta_p = \frac{\Gamma_m^3}{\Gamma_p^2} \sim 0.63 \times 10^{10} E_{51}^{1/2} r_6^{-1/2} \theta^{-2/3},$$  

(3.19)

in terms of which one can write the Lorentz factor and the radius at which pairs become optically thin as

$$\Gamma_p = \frac{\Gamma_m^{3/2}}{\eta_p^{1/2}}, \quad \frac{r_p}{r_o} = \theta^{-2/3} \Gamma_p = \theta^{-2/3} \frac{\Gamma_m^{3/2}}{\eta_p^{1/2}}.$$  

(3.20)

3.4 Photon Drag and Final Baryon Lorentz Factor

Photons are obviously coupled to the baryons when $\tau > 1$, which ensures a radiation-like equation of state with adiabatic index $4/3$. However, even after $\tau < 1$, baryons may remain coupled to the photons if the density of the latter is so large that the Compton drag time is shorter than the expansion time in the comoving frame. The comoving Compton drag time is the time during which an electron sweeps up an amount of photons whose
mass equivalent equals one proton plus one electron’s mass, \( t_D \sim m_p c^2/\sigma_T c u_{\gamma} \), where \( u_{\gamma} \) is the comoving photon energy density. This is longer than the Compton cooling time by a factor \( m_p/m_e \). The ratio of the comoving baryon rest mass energy density \( u_m \) and the comoving photon energy density \( u_{\gamma} \) is

\[
\varepsilon \equiv \frac{u_{\gamma}}{u_m} = \frac{E}{M_o c^2} = \eta \frac{E}{E_o},
\]

which is equal to \( \theta^{-2/3} \eta r_o/r \) if \( r < r_s \). Dividing the Compton drag time \( t_D \) by the comoving expansion time \( \Delta R/c \) and multiplying and dividing by the comoving baryon density \( n \) (comoving baryon rest energy density \( u_m \)), the ratio of the Compton drag time to the expansion time is

\[
\zeta \equiv \frac{t_D}{t_E} = \frac{u_m}{u_{\gamma}} \tau = \frac{1}{\varepsilon \tau} = \left( \frac{E_o}{E} \right) \frac{\theta^{4/3}}{\Gamma_m^3} \left( \frac{r}{r_o} \right)^2 = \theta^2 \Gamma_m^{-3} \left( \frac{r}{r_o} \right)^3,
\]

where \( \tau \) is the optical depth (3.11), and the last equality is valid for \( r \approx r_s \), with different dependences for \( r \approx r_s \). The final decoupling of photons and baryons occurs at a radius \( r_f \) where \( \zeta = 1 \), or at the thinning radius \( r_t \), whichever is largest.

The final bulk Lorentz factor of the baryons is the value that it has at decoupling from the photons, either when the fireball becomes optically thin or when the Compton drag has become longer than the expansion time, \( \zeta > 1 \), whichever occurs last. For \( \eta < \Gamma_m \), the fireball saturates to a value \( \Gamma = \eta \), and thinning occurs at a radius larger by a factor \( \left( r_t/r_o \right) = (\Gamma_m/\eta)^{3/2} > 1 \) where \( \zeta_t = (\Gamma_m/\eta) > 1 \), so the final baryon Lorentz factor is \( \Gamma_f = \eta \) in this case. On the other hand, for \( \eta > \Gamma_m \), at optical thinness \( \varepsilon_t = (\eta/\Gamma_m)^{3/2} > 1 \) and \( \zeta_t = (\Gamma_m/\eta)^{3/2} < 1 \), so the Compton drag keeps photons and baryons coupled beyond \( r_t \). This means that, even though most photons do not collide, most electrons do keep colliding with a small fraction of the photons, and keep being accelerated (with their baryons). They finally decouple when \( \zeta_f = 1 \) at \( (r_f/r_o) = \theta^{-2/3} \Gamma_m \), where \( \Gamma_f \sim \Gamma_m \), \( \varepsilon_f = (\eta/\Gamma_m) > 1 \) and \( \tau_f = (\Gamma_m/\eta) < 1 \). The terminal baryon bulk Lorentz factor is therefore

\[
\Gamma_f = \min \left[ \eta, \Gamma_m \right], \quad \text{for} \quad \left[ \eta \leq \Gamma_m, \eta \geq \Gamma_m \right].
\]

This is equal to the Lorentz factor at thinning \( \Gamma_t \) in the case \( \eta < \Gamma_m \), when \( \Gamma_t = \Gamma_f = \eta \), but \( \Gamma_t \neq \Gamma_f \) for \( \eta > \Gamma_m \).

3.5 Baryon Loading and Final Radiation to Kinetic Energy Ratio

One can distinguish four baryon-loading regimes, characterized by the value of \( \eta \):
H) High-load fireballs, for \( 1 < \eta < \Gamma_m \), or \( 0.6 \times 10^{-3} M_\odot \approx M_o \approx 1.8 \times 10^{-9} M_\odot \). In this regime \( \Gamma \) grows linearly with \( r/r_o \) until reaching \( r_s/r_o = \theta^{-2/3} \eta \), where it saturates to \( \eta \), and becomes optically thin at \( r_t/r_o = \Gamma_m (\Gamma_m/\eta)^{1/2} > \Gamma_m > r_s/r_o \). The observer-frame
(Doppler blueshifted) energy of the radiation escaping under the assumption of isotropy (even if in reality it is beamed) and the kinetic energy of the baryons is

\[ E_{r,ob} = E_o \theta^{-4/9} \eta^{-1/3} \left( \frac{r_o}{r_t} \right)^{2/3} = E_o \left( \frac{\eta}{\Gamma_m} \right)^{1/3} < E_o, \quad E_k = M_o c^2 \eta \simeq E_o. \]

That is, the thinning radiation burst is a small fraction of \( E_o \) (unless \( \eta \to \Gamma_m \)), most of the energy having gone into the kinetic energy of the baryons.

M) Critical-load Fireballs, for \( \eta = \Gamma_m \) or \( M_o \sim 1.8 \times 10^{-9} M_\odot \). In this case \( \Gamma_s = \Gamma_m \) at \( r_s = r_t = r_f \), and both the observer-frame radiation energy and the baryon kinetic energy assume their maximal value, \( E_{r,ob} \sim E_o/2, E_k \sim E_o/2 \).

L) Low-load or underloaded fireballs, for \( \Gamma_m < \eta < \eta_p \), or \( 1.8 \times 10^{-9} M_\odot \lesssim M_o \lesssim 0.95 \times 10^{-13} M_\odot \). For these (rather low) values of the baryon pollutant mass, \( \Gamma \) grows linearly with \( r/r_o \) until becoming optically thin at \( r_t/r_o < r_s/r_o \sim r_m/r_o \), where it has the value \( \Gamma_t \sim \theta^{2/3} r_t/r_o = \Gamma_m (\Gamma_m/\eta)^{1/2} < \eta \). Most of the radiation energy escapes from here without further scattering, while the baryons continue coupled by Compton drag to a small fraction of the photons until the radius \( r_f \) where \( \Gamma_f = \Gamma_m \). The observer-frame radiation energy observed (assumed over \( 4\pi \)) and the final kinetic energy of the protons are

\[ E_{r,ob} \sim E_o \theta^{-2/3} \left( \frac{r_o}{r_t} \right) \Gamma \sim E_o, \quad E_k = M_o c^2 \Gamma \sim E_o \left( \frac{\Gamma_m}{\eta} \right) < E_o. \]

P) Pair fireballs, for \( \eta > \eta_p \) or \( M_o \lesssim 0.95 \times 10^{-13} M_\odot \). In this extremely underloaded regime, the pairs dominate the Thomson opacity and the value of \( \Gamma \) grows linearly with \( r/r_o \) but it always becomes optically thin at the same value of the radius where \( \Gamma \sim \Gamma_p \), \( r_t/r_o = r_p/r_o \sim \theta^{-2/3} \Gamma_p \), given by eqs.(3.18)-(3.20), independent of \( \eta \) as long as the latter is greater than \( \eta_p \). The observer-frame radiation energy at thinness under the assumption of isotropy and the final kinetic energy of the baryons is again given by eq.(3.25).

The significance of the ratio of observed radiation energy to kinetic energy, in our “standard” model (Mészáros and Rees, 1993), is that this represents the ratio of the energy in a short precursor burst to that in the main burst, the latter coming from the recovery of the kinetic energy upon interaction with the external medium (§4).

4. Magnetic Fields, Efficiency and Photon Energies

4.1 Magnetic Fireballs

In almost any model of the initial energy release, the initial mass motions are expected to be extremely violent \( (v \sim c) \), and this could magnify any pre-existing magnetic fields, via compression, shearing, turbulent dynamo mechanisms, Parker-type instabilities, etc. (e.g. Usov, 1992, Narayan, Paczyński and Piran, 1992, Thompson and Duncan, 1993). If the initial total “disposable” energy (i.e., that portion of the available gravitational energy that is not lost in the form of neutrinos) is \( E_o = 10^{51} E_{51} \) ergs, this might be distributed between radiation and magnetic components as

\[ E_{B_o} = \xi E_o, \quad E_{r,o} = (1 - \xi) E_o, \]
where $\xi \leq 1$. One may define separate magnetic and radiation $\eta$ parameters

$$\eta_B = E_{Bo}/M_o c^2 = \xi \eta , \quad \eta_r = E_{ro}/M_o c^2 = (1 - \xi) \eta$$

(4.2)

in terms of the usual total $\eta = E_o/M_o c^2$, with $\eta = \eta_B + \eta_r = \xi \eta + (1 - \xi) \eta$. Even if the magnetic fields have a large scale structure, the expansion is essentially isotropic in the comoving frame, and therefore the magnetic energy density will evolve $\propto V^{-4/3}$, so the total magnetic energy and radiation energy in the comoving frame vary in a similar manner, $E_B/E_{Bo} = B^2/B_o^2 = E_r/E_{ro} = (V/V_o)^{-1/3}$, where the volume factors are given by eqs. (3.9). The bulk Lorentz factor, similarly to (3.3), is again

$$\Gamma = E_o + M_o \over E_r + E_B + M_o = \frac{\eta + 1}{\eta_r(E_r/E_{ro}) + \eta_B(E_B/E_{Bo}) + 1} = \frac{\eta + 1}{\theta^{-2/3}(r_o/r) \eta + 1},$$

(4.3)

so that, as in the case of pure radiation, $\Gamma \sim \theta^{2/3}(r/r_o)$ for $r \ll r_s$ while $\Gamma \sim \eta$ for $r \gg r_s$, with $r_s/r_o = \theta^{-2/3}\Gamma_s$ and $\Gamma_s = \min[\eta, \Gamma_m]$, where $\Gamma_m$ is given by eqs(3.14). The behavior is therefore the same as in §3, even if all the disposable energy $E_o$ goes into magnetic fields ($\xi \to 1$), i.e. a purely magnetic fireball. The maximal initial magnetic field will be

$$B_o = (8\pi \xi E_o/V_o)^{1/2} = 10^{17} \xi^{1/2} E_{51}^{1/2} r_6^{-3/2} \text{ G},$$

(4.4)

and the comoving field strength evolves according to

$$\frac{B}{B_o} = \begin{cases} \frac{\theta^{-4/3}(r_o/r)^2}{\theta^{-8/9}\Gamma_s^{-2/3}(r_o/r)^4/3} & \text{for } r < r_s; \\ \frac{\theta^{-8/9}\Gamma_s^{2/3}(r_o/r)^2}{\theta^{-8/9}\Gamma_s^{-2/3}(r_o/r)^2} & \text{for } r_s < r < r_b; \\ \theta^{-4/3}(r_o/r)^2 & \text{for } r > r_b; \end{cases}$$

(4.5)

where $r_s$, $r_b$ are given in (3.7). Even after most of the pairs annihilate, the exponentially small fraction of frozen-in surviving pairs is enough to provide the currents needed to support large-scale fields, so that the behavior (4.5) is uninterrupted beyond the radius at which annihilation, optical thinness, photon drag decoupling, etc. occur.

We may also consider briefly the most extreme scenario of magnetic field dominance, that where the initial field $B_0$ is in equipartition not with the disposable energy $E_o \sim 10^{51} E_{51}$ ergs (which, as in supernovae, is of order $10^{-3}$ of the total liberated binding energy from the gravitational collapse of about a solar mass) but rather is in equipartition with the binding energy itself, $E_b \simeq GM_\odot^2/R_N \sim 10^{54} E_{54}$ ergs. This might occur in a rapidly-spinning object, where rotational and shearing motions would involve a large fraction of the total binding energy. In this case

$$B_o = B_m \sim 3 \times 10^{18}(\xi/30) \text{ G},$$

(4.6)

where, in terms of the first of equations (4.1), $\xi \sim 30$. Now, however, $\eta_r$ is no longer defined as $(1 - \xi) \eta$ but rather just as $\eta_r = E_{ro}/M_o c^2$. The dynamic considerations are similar to those just described, but the dynamics is entirely described by using $\eta_B$ instead of $\eta$ everywhere in §3, and increasing $\Gamma_m$ in eq.(3.14) by a factor 10 (since $E_{51} \sim 10^5$).
4.2 Standard Shock Deceleration Model

The magnetic fields are carried on in the comoving frame of whatever polluting baryons \( M_o \) were present originally in the fireball, and continue expanding with the latter and the corresponding electrons plus surviving pairs until the fireball matter is decelerated by the external medium (Rees and Mészáros, 1992). For a fireball which has saturated its bulk Lorentz factor to a value \( \Gamma_s = \min[\eta, \Gamma_m] \) the deceleration occurs when the mass of external matter swept up by the fireball equals \( \Gamma_s^{-1} M_o \); the swept-up external matter is shock heated to a comoving average thermal Lorentz factor \( \gamma_s \sim \Gamma_s \sim \eta >> 1 \), while a reverse shock starts to propagate into the adiabatically cooled fireball material, eventually reheating it to a marginally relativistic average thermal Lorentz factor \( \gamma_r \sim \Gamma_r \approx 2 \). If the forward and reverse shocks are able to create a field which approaches equipartition with the post-shock thermal energies of the particles (similarly to what appears to occur in supernova remnant shocks), both the forward and reverse shock regions can radiate away the entire thermalized bulk kinetic energy of the shock, i.e. the entire total initial disposable energy \( E_o \) of the event. However, because of the dependence of the radiative efficiency on the magnetic field strength, it is worthwhile to consider departures from shock equipartition, as well as the effect of frozen-in primordial fields discussed in the previous subsection, whose initial strength \( B_o \) may have been amplified by shear or turbulent dynamo effects during the cataclysmic event of the initial energy release, parametrized through \( \xi \) defined in the first of equations (4.1).

In order to estimate the radiative efficiency at the deceleration shock, we use the parameters of our “standard” shock model, e.g. Mészáros and Rees, (1993). For typical parameters, the shock deceleration occurs after the fireball has saturated (and after it has become optically thin, producing a brief and weak precursor burst). The deceleration radius \( r_d \) at which the fireball gas starts to feel the inertia of an external medium of uniform density \( n_{ext} = n_1 \) \( \text{cm}^{-3} \) is

\[
r_d/r_o = (\eta_{ext}/\eta^2)^{1/3}, \quad \text{where} \quad \eta_{ext} = (E_o/V_o n_{ext} m_p c^2),
\]

or

\[
r_d = (3E_o/4\pi n_{ext} m_p c^2 \eta^2)^{1/3},
\]

which is

\[
r_d \sim 10^{16} n_1^{1/3} E_{51}^{-1/3} \eta_3^{-2/3} \text{ cm}.
\]

(For maximal magnetic dominance, \( E_{51} \sim 10^3 \), \( \eta \equiv \eta_B \), \( \xi \sim 30 \), but for the purposes of this example we consider the more conservative case of \( E_{51} \sim 1 \), \( \xi \approx 1 \), \( n_1 \sim 1 \). The comoving width of the forward and reverse shocked shells is \( \eta \) times larger than their laboratory width \( \sim r_d/\eta^2 \), or \( \Delta R \sim r_d/\eta \sim 10^{13} \eta_3^{-5/3} \) cm. The comoving expansion time is consequently

\[
t_{ex} = \Delta t_c = \Delta R/c \sim 10^3 \eta_3^{-5/3} \text{ s},
\]

while the laboratory expansion time (the observed GRB burst time, if the shock is radiatively efficient) is \( \Delta t = r_d/\eta^2 \sim 1 \eta_3^{-8/3} \) s. The total number of baryons (protons) involved in the fireball is \( N_{p,o} = E_o/\eta m_p c^2 \sim 10^{54} \eta^{-1} \), and the comoving density in the preshocked fireball gas (for \( r_d > r_b \) in eqs.(3.8) with (3.4)) is

\[
n = n_d = N_{p,o} \eta/4\pi \theta^2 r_d^3 \sim 10^6 \eta_3^2 \text{ cm}^{-3}.
\]
If the reverse shock, whose bulk Lorentz factor achieves at most a value $\Gamma_r \sim 2$, produces a shock equipartition field, this is $B_{d,e} = (8\pi n m_p c^2 \Gamma_s 4)^{1/2} \sim 10^2 \eta_3$ G, while if the magnetic field is mainly the original, possibly amplified and adiabatically expanded fireball magnetic field (4.4) or (4.6), this is

$$B_d = 4B_o \eta^{2/3} (r_d/r_o)^2 = 4 \times 10^{-1} \xi \eta_3^2 \text{ G}, \quad (4.10)$$

where in both cases we have included a factor 4 to take into account the shock compression factor. Aside from the different $\eta$ dependence, we may consider both the shock-equipartition and frozen-in fields to be given by (4.10), where the shock-equipartition field case has $\xi \sim 250 \eta_3^{-1}$ while the frozen-in field has $\xi \approx 1$ (or $\xi \approx 30$ in the extreme magnetic dominance case).

### 4.3 Radiative Efficiency and Photon Energies

The efficiency of the blast wave moving ahead of the contact discontinuity can only be affected by the field $B_{d,e}$ developed in the shock, as discussed in our previous papers. Here we focus on the radiative efficiency of the reverse-shocked fireball material, which can be affected by the original, possibly amplified, fireball field $B_d$ of eq.(4.10). The “fireball” at the stage just before the reverse shock moves into it contains only a small fraction of the cooled original pairs and the original polluting baryons and electrons. With a field of order (4.10), relativistic electrons of Lorentz factor $\gamma \sim 10^6$ produced by the reverse shock are sufficient to ensure a synchrotron cooling time comparable to the (comoving) expansion time (4.6), i.e. near unit radiative efficiency, and photons of characteristic energy $\sim 1\eta_3$ MeV in the laboratory frame. If shock acceleration is considered, limited by synchrotron losses, the maximum possible $\gamma \lesssim 2 \times 10^7 B^{-1/2}$ and the maximum energy of the synchrotron photons is $\sim 1\eta_3$ GeV in the laboratory frame, independent of the field strength.

Synchrotron losses, however, will almost certainly be surpassed in importance by the inverse Compton (IC) losses of the electrons, in the radiation field of the synchrotron photons. Without diffusive Fermi-type acceleration, the electrons passing through the shock may energetically attain a maximum average Lorentz factor

$$\gamma \sim (m_p/m_e) \Gamma_r \sim 4 \times 10^3 \zeta. \quad (4.11)$$

With this average Lorentz factor, the ratio of synchrotron cooling to expansion time $t_{sy}/t_{ex} \sim 5 \times 10^8 \gamma^{-1} B^{-2} \sim 2.5 \times 10^3 \zeta^{-1} \xi^{-2} \eta_3^{-7/3} \gg 1$, i.e. the synchrotron efficiency is low. The energy density in synchrotron photons is $u_{sy} \sim n P_{sy} \Delta R/c \sim 2 \times 10^2 \zeta^2 \eta_3^{1/3} u_B$, where $u_B = B^2/8\pi$ is the magnetic energy density. The ratio of inverse Compton cooling time to expansion time is therefore

$$\frac{t_{IC}}{t_{ex}} \sim 10^1 \eta^{-3} \zeta^{-2} \eta_3^{-8/3}, \quad (4.12)$$
giving a first-order inverse Compton efficiency of 10% even for this modest average Lorentz factor $\gamma \sim 4 \times 10^3 \zeta$ and $\xi \sim 1$. The synchrotron photons (IR in the comoving frame, optical in the lab frame for this $\gamma$ and $\eta_3 \sim 1$) are boosted up by the single inverse Compton scattering to MeV and GeV energies in the comoving and lab frames.

The radiative efficiency will be larger in the presence of a diffusive (or similar) shock acceleration mechanism that produces an electron power-law distribution. Considering the electron shock acceleration to be limited by first-order inverse Compton losses (using the Thomson limit in the electron rest frame and balancing $t_{IC}$ with the acceleration time $t_a \sim \gamma/\omega_B$, where $\omega_B = eB/m_e c$) one gets a maximum $\gamma$ significantly higher than (4.11) by a factor $\zeta_m \sim 4.5 \times 10^1 \xi^{-1/4} \eta_3^{-7/12}$. The synchrotron photon energy in the electron rest frame becomes of order 1 MeV at a slightly lower energy, $\zeta_m \sim 2 \times 10^1 \xi^{-1/3} \eta_3^{-2/3}$. Since the transition to the Klein-Nishina cross section at this energy will effectively limit the maximum Lorentz factor to this value, we will use this second value of $\zeta_m \sim 20$, i.e. $\gamma_m \sim 10^5$. With this, the synchrotron and first-order IC photons have a characteristic maximum energy of

$$E_{sy} \sim 10^{1.1/3} \zeta_3^{5/3} \text{ keV}, \quad E_{IC} \sim 10^{14.1} \zeta^{-1/3} \eta_3^{1/3} \text{ eV}, \quad (4.13)$$

in the lab frame, and a radiation efficiency

$$\epsilon_{sy} \sim 10^{-2} \zeta^{5/3} \eta_3^{5/3}, \quad \epsilon_{IC} \sim \min[1, 10^{3} \zeta \eta_3^{2/3}]. \quad (4.14)$$

Another effect which could increase the efficiency is the higher order IC scattering (e.g. Rees, 1967). The Thomson optical depth of the fireball shell material is

$$\tau_T \sim n \sigma_T f \Delta R \sim 0.6 \times 10^{-5} f \eta_3^{1/3}, \quad (4.15)$$

where $f \leq 1$ is the fraction of the fireball material that has been heated by the shock. When $\tau_T$ is larger than $\gamma^{-2}$ (as long as the scattering is not Klein-Nishina dominated) the conditions are fulfilled for both first order and higher order inverse Compton scattering to dominate over synchrotron losses. This is because the synchrotron, first-order IC, second-order IC, etc. photon energy densities are

$$u_{sy} \sim n P_{sy} f \Delta R/c \sim n \sigma_T \gamma^2 u_B f \Delta R \sim \tau_T \gamma^2 u_B, \quad (4.16a)$$

$$u_{IC} \sim n P_{IC} f \Delta R/c \sim n \sigma_T \gamma^2 u_{sy} f \Delta R \sim \tau_T^2 \gamma^4 u_B, \quad (4.16b)$$

$$u_{IC'} \sim n P_{IC'} f \Delta R/c \sim n \sigma_T \gamma^2 u_{IC} f \Delta R \sim \tau_T^2 \gamma^6 u_B, \quad (4.16c)$$

and for $\tau_T \gg \gamma^{-2}$ the ratio of the successive energy densities in eq(4.16) are larger by increasing factors of $\tau_T \gamma^2 \gtrsim 1$. Even for $\gamma \sim 10^3$ the IC losses exceed synchrotron losses by a factor 10, and the second order IC' losses would exceed first order IC losses by another factor of 10, were it not for the fact that for second order IC and the maximal $\gamma$ the photons in the electron rest-frame are now subject to the Klein-Nishina decrease in the cross section. (For $\gamma \sim 10^5$ the respective factors are $10^5$, $10^{10}$, etc., and the importance of the IC losses
make themselves felt already when the shock has heated a small fraction \( f \gtrsim 10^{-5} \) of the fireball material). Notice that for both our simplest model (using the average Lorentz factor \( \gamma \sim 10^{9} \)) and for the non-thermal (diffusive acceleration) model with \( \gamma_m \sim 10^5 \), the first order IC is in the Thomson limit and exceeds synchrotron losses, but the Klein-Nishina losses diminish the importance (at least as far as the efficiency is concerned) of the second and higher order IC scattering, although they will affect the spectrum. In the power-law spectrum model, the first-order IC efficiency is unity for \( \xi \eta^2 \gtrsim 10^{-3} \), i.e. for initial fields \( B_0 \gtrsim 10^{14} \eta^{-2/3} \) G in the fireball, or for fields developed by turbulent instabilities in the reverse shock which are a factor \( 10^{-5} \eta^{-2/3} \) below equipartition with the thermal (reverse) post-shock particles, a fairly undemanding assumption.

5. Discussion

We have presented both analytical and numerical calculations of the evolution of a freely expanding ultrarelativistic gas produced by an impulsive energy release. The expansion and cooling from an initially optically thick and extremely hot fireball is followed through the stage where the rest-frame energy density becomes comparable to the rest mass density, where the expansion bulk Lorentz factor, which until then grew proportionally to the expansion factor, saturates to a value which is equal to the smallest of the ratio of the initial thermal energy to rest mass energy in the fireball or the bulk Lorentz factor at which Compton drag becomes negligible. The laboratory and comoving frame geometry as well as the radiation and gasdynamic variables were investigated for a large range of values of \( \eta = E_o/M_o c^2 \), and we derived analytic scaling laws showing the dependence of the variables as a function of the various initial parameters, including an approximate angular scaling for the case of jet-like expansion within a limited range of solid angles.

We have also devoted particular attention to the likely magnetic field content of fire-balls. An initial magnetic field, possibly amplified by the violent mass motions in the initial impulsive event, may contribute a dynamically-significant fraction of the total fireball energy; there is also the possibility that a field develops in the shock heated fireball after the ram pressure of an external medium decelerates the expansion. When the fireball runs into an external medium and is decelerated, the efficiency with which the re-randomized energy is radiated depends on the magnetic field strength, and on how the electrons are accelerated by the resultant shocks. (The radiation processes during the deceleration phase which give rise to the typical burst profiles are, fortunately, rather less sensitive to precisely how the fireball was originally formed – the formation mechanism may have resembled the impulsive model discussed in sections 2 and 3, or could alternatively have been spread over as long as a second, as in the scenarios of Usov (1992) or Narayan et al (1992) ). We find, for a range of assumptions, that inverse synchro-Compton cooling should be extremely efficient in radiating away the kinetic energy of the cooled fireball in a time scale short compared to the expansion time, which gives the right order of magnitude duration and total energy for a gamma ray burst source, and produces a nonthermal spectrum with the bulk of the energy in the the MeV to GeV range.
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Figure Captions

Fig. 1: Mass distribution in the laboratory frame for a fireball of $\eta = 10^4$ at the times when the lab expansion factor $r(\rho_{max})/r_o$ (proportional to the lab time) is equal to 0.2, 0.4, 0.6, 0.8, 1.0 times $\eta$, from top to bottom.

Fig. 2: Distribution of the bulk Lorentz factor $\Gamma$ over the various mass shells (i.e. against the Lagrangian mass coordinate), for $\eta = 10^4$, at the times when the lab expansion factor (which is proportional to the lab time) is $r(\rho_{max})/r_o = \eta^{1/2}, \eta, \eta^{3/2}, \eta^2, \eta^{5/2}, \eta^3$, from bottom to top.

Fig. 3: The average bulk Lorentz factor $\Gamma$ as a function of the expansion factor $r(\rho_{max})/r_o$, for $\eta = 10^4$.

Fig. 4: a) The lab frame mass density against the Lagrangian mass coordinate, at the times (from top to bottom) when the expansion factor is $r(\rho_{max})/r_o = \eta^{1/2}, \eta, \eta^{3/2}, \eta^2, \eta^{5/2}, \eta^3$.
   b) The comoving frame mass density against the Lagrangian mass coordinate, for the same instants as a)

Fig. 5: a) The lab frame width of the shell where most (80%) of the mass is located, as a function of the expansion factor $r(\rho_{max})/r_o$, for $\eta = 10^4$.
   b) The comoving frame width of the shell containing most (80%) of the matter, as a function of the expansion factor $r(\rho_{max})/r_o$, for $\eta = 10^4$. 