Normal tau polarisation as a sensitive probe of CP violation in chargino decay

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CP violation in the spin-spin correlations in chargino production and subsequent two-body decay into a tau and a tau-sneutrino is studied at the ILC. From the normal polarisation of the tau, an asymmetry is defined to test the CP-violating phase of the higgsino mass parameter $\mu$. Asymmetries of more than $\pm 70\%$ are obtained, also in scenarios with heavy first and second generation sfermions. Bounds on the statistical significances of the CP asymmetries are estimated. As a result, the normal tau polarisation in the chargino decay is one of the most sensitive probes to constrain or measure the phase $\phi_\mu$ at the ILC, motivating further detailed experimental studies.

I. INTRODUCTION

Within the framework of the standard electro-weak model (SM), the complex Cabbibo-Kobayashi-Maskawa (CKM) matrix is the origin for CP violation \cite{1,2}. It explains all the current laboratory data, but it is not sufficient to generate the matter-antimatter asymmetry of the universe \cite{3}. Thus further theories have to be investigated, that offer new sources of CP violation \cite{4}. The minimal supersymmetric standard model (MSSM) is a promising extension of the SM \cite{5}. To prevent the supersymmetric partners of the known particles from appearing below the LEP and Tevatron energy scale, there has to be supersymmetry (SUSY) break-down, at least at the electro-weak energy scale \cite{6}. Several of the supersymmetric parameters can be complex, including the higgsino mass parameter $\mu$.

Concerning low energy observables, the corresponding SUSY CP phases lead to T-violating electric dipole moments (EDMs), that already would be far beyond the experimental bounds \cite{7,8,9}. This constitutes the SUSY CP problem: The SUSY phases have to be considerably suppressed, unless cancellations appear between different EDM contributions, or the SUSY spectrum is beyond the TeV scale \cite{10,11}. And indeed, attempts to naturally solve the SUSY CP problem, often still require a certain amount of tuning among the SUSY masses, phases and parameters. Recently proposed models are split SUSY \cite{12}, inverted hierarchy models \cite{13}, and also focus point scenarios that attempt to restore naturalness \cite{14}. These models have also been proposed to solve the SUSY flavour problem, to ensure proton stability, and to fulfil cosmological bounds, like constraints on dark matter or primordial nucleosynthesis. It is clear, that CP-sensitive observables outside the low energy sector have to be proposed and measured, in order to tackle the SUSY CP problem, and to reveal the underlying SUSY model.

Concerning future collider experiments at the LHC \cite{15} and ILC \cite{16}, SUSY phases alter SUSY particle masses, cross sections, branching ratios \cite{17,18}, and longitudinal polarisations of final state fermions \cite{19}. Although the SUSY phases can change these CP-even observables by an order of magnitude or more, only CP-odd observables are a direct evidence of CP violation \cite{20}. CP-odd rate asymmetries of cross sections, distributions, or partial decay widths \cite{21}, however, usually do not exceed 10\%, as they require the presence of absorptive phases, unless they are resonantly enhanced \cite{22,23}. At tree level, larger T-odd and CP-odd observables can be defined with triple or epsilon products of particle momenta and/or spins \cite{24,25}.

At the LHC, CP-odd triple product asymmetries have been studied in the decays of third generation squarks \cite{26,27}, and three-body decays of neutralinos which originate from squarks \cite{28,29}. Since triple products are not boost invariant, compared to epsilon products, some of these studies have included boost effects at the LHC \cite{30,31}. For the ILC, triple product asymmetries have been studied in the production and decay of neutralinos \cite{32,33}, and charginos \cite{34,35}, also using transversely polarised beams \cite{36}. The result of these studies is, that the largest asymmetries of the order of 60\% can be obtained if final fermion polarisations are analysed, like the normal tau polarisation in neutralino decay $\tilde{\chi}_i^0 \to \tilde{\tau}_R \tau \bar{\nu}_{\tau}$ \cite{37,38}.

Although the experimental reconstruction of taus is much more involved than those for electrons or muons, tau decays in principle allow for a measurement of their polarisation \cite{39,40}. The fermion polarisation contains additional and unique information on the SUSY couplings \cite{41,42}, which will be lost if only electron or muon momenta are considered for measurements. Taus might also be the dominant final state leptons. In particular in inverted hierarchy models, sleptons of the first and second generations are heavy, such that electron and muon final states will be rare in SUSY particle decay chains. We are thus motivated to analyse the potential of CP-odd effects in chargino production

$$e^+ + e^- \to \tilde{\chi}_i^\pm + \tilde{\chi}_j^\mp, \quad i, j = 1, 2,$$

with longitudinally polarised beams, and the subsequent two-body decay of one chargino into a polarised tau

$$\tilde{\chi}_i^\pm \to \tau^\pm + \bar{\nu}_{\tau}^{(s)}.$$
For the chargino decay, we define a CP-odd asymmetry from the tau polarisation normal to the plane spanned by the $e^-$-beam and the tau momentum. It is highly sensitive to the CP phase $\varphi_\mu$ of the higgsino mass parameter $\mu$, which enters the chargino sector. Since $\varphi_\mu$ is also the most constrained SUSY CP phase from EDM searches, this asymmetry is particularly important. Besides a MSSM scenario with light sleptons, we thus also analyse the CP asymmetries in an inverted hierarchy scenario, which relaxes the strong EDM constraints on the SUSY phases. Scenarios with an inverted hierarchy are attractive for our process of chargino production and decay, since not only is the chargino production cross section enhanced due to destructive sneutrino interference, but the rate of chargino decay into taus is also amplified.

In Section II, we present our formalism. We briefly review chargino mixing in the complex MSSM, and give the relevant parts of the interaction Lagrangian. We calculate the $\tau$ spin density matrix, the normal $\tau$ polarisation, and present the corresponding analytical formulae in the spin-density matrix formalism \cite{52}. In Section III, we present our numerical results. We summarise and conclude in Section IV.

II. FORMALISM

A. Chargino mixing and complex couplings

In the MSSM, the charged winos $\tilde{W}^\pm$ and higgsinos $\tilde{H}^\pm$ mix after electro-weak symmetry breaking, and form the chargino mass eigenstates $\tilde{\chi}_i^\pm$. In the $(W, H)$ basis, their mixing is defined by the complex chargino mass matrix $\tilde{M}_\chi$:

$$M_\chi = \begin{pmatrix} M_2 & m_W \sqrt{2} \sin \beta \\ m_W \sqrt{2} \cos \beta & \mu \end{pmatrix}.$$  \hspace{1cm} (3)

At tree level, the chargino system depends on the SU(2) gaugino mass parameter $M_2$, the higgsino mass parameter $\mu$, and the ratio $\tan \beta = v_2/v_1$ of the vacuum expectation values of the two neutral CP-even Higgs fields. We parametrise the CP violation by the physical phase $\varphi_\mu$ of $\mu = |\mu|e^{i\varphi_\mu}$, taking by convention $M_2$ real and positive, absorbing its possible phase by redefining the fields.

By diagonalising the chargino matrix $\tilde{M}_\chi$, we obtain the chargino masses, $m_{\tilde{\chi}_1^\pm} \geq m_{\tilde{\chi}_2^\pm} \geq 0$, as well as their couplings. In Appendix C, we give the analytic expressions for the two independent, unitary diagonalisation matrices $U$ and $V$. We shall use them for a qualitative understanding of the chargino mixing in the presence of a non-vanishing CP phase $\varphi_\mu \neq 0$.

At the ILC, chargino production $e^+e^- \rightarrow \tilde{\chi}_1^+ \tilde{\chi}_2^-$ proceeds at tree level via $\tilde{\nu}_e$ exchange in the $t$-channel, and $Z, \gamma$, exchange in the $s$-channel, see the Feynman diagrams in Fig. 1. Note that the photon exchange contribution vanishes for non-diagonal chargino production, $i \neq j$. The relevant terms in the MSSM Lagrangian for chargino production are $\tilde{Z}_L$ \cite{53, 54}.

$$L_{\gamma \tilde{\chi}_i^+ \tilde{\chi}_i^-} = -e A_\mu \bar{\chi}_{i\mu} \gamma^\mu \tilde{\chi}_i^+ \delta_{ij}, \quad e > 0, \hspace{1cm} (5)$$

$$L_{\epsilon \tilde{\nu}_e \tilde{\chi}_i^+} = -g V_{1\mu} \bar{\tilde{\nu}}_e \gamma_\mu P_L \tilde{\chi}_i^+ + h.c., \hspace{1cm} (6)$$

$$L_{Z \tilde{\chi}_i^+ \tilde{\chi}_i^-} = \frac{g}{\cos \theta_W} Z_{\mu} \bar{\chi}_{i\mu} \gamma^\mu \left[ O_{1L}^L P_L + O_{1R}^R P_R \right] \tilde{\chi}_i^+ \hspace{1cm}, \hspace{1cm} (7)$$

with $i, j = 1, 2$, and the projectors $P_{iL,R} = \left( 1 \mp \gamma^5 \right)/2$. In Eq. (5), “$\epsilon$” refers to the electron coupling, in Eq. (6) it refers to the electron spinor field. Furthermore

$$O_{ij}^L = -V_{i1} V_{j1}^* - \frac{1}{2} V_{i2} V_{j2}^* + \delta_{ij} \sin^2 \theta_W, \hspace{1cm} (8)$$

$$O_{ij}^R = +U_{i1} U_{j1}^* - \frac{1}{2} U_{i2} U_{j2}^* + \delta_{ij} \sin^2 \theta_W, \hspace{1cm} (9)$$

with the weak mixing angle $\theta_W$, and the weak coupling constant $g = e/\sin \theta_W$. For diagonal chargino production, $i = j$, the $Z$-chargino couplings are real, see Eqs. (5), (6), and the production amplitude has no CP-violating terms at tree level.

For the subsequent chargino decay into the tau, $\tilde{\chi}_i^+ \rightarrow \tau^+ \tilde{\nu}_\tau^{(*)}$, the contribution to the Lagrangian is $\tilde{Z}_L$.

$$L_{\tilde{\chi}_i^+ \tau \tilde{\nu}_\tau} = -g \bar{\tilde{\chi}_i^+} \left( c_{\ell \tau}^L P_L + c_{\ell \tau}^R P_R \right) \tau \tilde{\nu}_\tau^{*} + h.c., \hspace{1cm} (10)$$

with the left and right $\tau$-$\tilde{\nu}_\tau$-chargino couplings

$$c_{\ell \tau}^L = Y_{\tau} V_{11}, \hspace{1cm} c_{\ell \tau}^R = -Y_{\tau} U_{12}, \hspace{1cm} (11)$$

and the Yukawa coupling

$$Y_{\tau} = \frac{m_\tau}{\sqrt{2} m_W \cos \beta}. \hspace{1cm} (12)$$

In the following, we present the analytical formulae for the normal tau polarisation, which will be a sensitive probe of CP violation in the chargino system.

B. Tau spin density matrix

The unnormalised, $2 \times 2$, hermitean, $\tau$ spin density matrix for chargino production, Eq. (11), and decay, Eq. (12), reads

$$\rho^{\lambda \lambda'} = \int \left( |\mathcal{M}|^2 \right)^{\lambda \lambda'} \mathcal{L} \mathcal{D}s,$$  \hspace{1cm} (13)
with the amplitude $\mathcal{M}$ and the Lorentz invariant phase space element $d\mathcal{Z}$, and $\rho_{\lambda'}$ for decay (D). They can be expanded in terms of the Pauli matrices $\sigma^a$

$$\rho_{\lambda'} = 2 \left[ \sigma^a(\tau) \lambda' \right] (16)$$

$$\rho_{\lambda'} = \lambda' \delta_{\lambda' \lambda} + (\Sigma_0^a)(\tau) (17)$$

with an implicit sum over $a = 1, 2, 3$. For chargino production, the analytical formulae for the coefficients $\mathcal{P}$ and $\Sigma_0^a = \Sigma_0^a \delta_{\lambda' \lambda}$, which depend on the chargino spin vectors $s_{\lambda'}$, are explicitly given in Ref. [53]. In that convention, a sum over the helicities $\lambda'$ of the decaying chargino $\tilde{\chi}_i$, whose decay we do not further consider, gives the factor of 2 in Eq. (10).

For the chargino $\tilde{\chi}_i^\pm$ decay into a tau, Eq. (2), we define a set of tau spin basis vectors $s_j^\tau$, see Appendix A. We then expand the coefficients of the decay matrix $\rho_{\lambda'}$, Eq. (17),

$$D^{\lambda' \lambda} = D_0^{\lambda' \lambda} + D_1^{\lambda' \lambda}$$

$$\Sigma_0^a = \Sigma_0^a + \Sigma_1^a (18)$$

with an implicit sum over $b = 1, 2, 3$. A calculation of

$$\Delta(\bar{\chi}_i) = \frac{1}{p_{\bar{\chi}_i}^2 - m_{\tilde{\chi}_i}^2 + i m_{\tilde{\chi}_i} \Gamma_{\tilde{\chi}_i}}, (15)$$

with the chargino helicities denoted $\lambda_i$, $\lambda'_i$, and an implicit sum over the helicities of chargino $\tilde{\chi}_j$, whose decay is not further considered. The amplitude squared decomposes into the remnant of the chargino propagator,

$$\Delta(\bar{\chi}_i)|^2 = |\Delta(\bar{\chi}_i)|^2 \sum_{\lambda_i, \lambda'_i} (\rho_{\lambda'}^{\lambda \lambda'}) (\rho_{\lambda'}^{\lambda \lambda'}) (14)$$

the expansion coefficients yields

$$D = \frac{g^2}{2} \left| (|V_{\tau\chi}^2| + |V_{\tau\chi}^2|) \right| \left| (\rho_{\lambda'}^{\lambda \lambda'}) \right|$$

$$\Sigma_0^a = \frac{g^2}{2} \left| (|V_{\tau\chi}^2| + |V_{\tau\chi}^2|) \right| \left| \rho_{\lambda'}^{\lambda \lambda'} \right|$$

$$\Sigma_1^a = \frac{g^2}{2} \left| (|V_{\tau\chi}^2| + |V_{\tau\chi}^2|) \right| \left| \rho_{\lambda'}^{\lambda \lambda'} \right|$$

with the weak coupling constant $g$, and the Yukawa coupling $Y_{\tau\chi}$, cf. Eq. (10). The formulae are given for the decay of a positive chargino, $\tilde{\chi}_i \rightarrow \tau^+ \nu_{\tau}$. The signs in parentheses hold for the charge conjugated decay $\tilde{\chi}_i \rightarrow \tau^- \nu_{\tau}$.

The spin-spin correlation term $\Sigma_0^a$ in Eq. (20) explicitly depends on the imaginary part $\im{\mathcal{M}}$ of the chargino matrices $U$ and $V$, cf. Eqs. (19) and (23) in Appendix C. Thus this term is manifestly CP-sensitive, i.e. sensitive to the phase of the chargino sector. The imaginary part is multiplied by the totally antisymmetric (epsilon) product

$$E_{\sigma}^{\lambda \lambda'} = \epsilon_{\mu \nu \rho \sigma} \rho_{\lambda'} \rho_{\lambda'} \rho_{\lambda'} \rho_{\lambda'}$$

We employ the convention for the epsilon tensor $\epsilon_{0123} = 1$. Since each of the spacial components of the four-momenta $p\lambda$, or spin vectors $s\lambda$, changes sign under a naive time transformation, $t \rightarrow -t$, the epsilon product $E_{\sigma}^{\lambda \lambda'}$ is T-odd.

Inserting the density matrices, Eqs. (11) and (17), into Eq. (14), we get for the amplitude squared

$$\left| (\mathcal{M})^{\lambda \lambda'} \right|^2 = 4 |\Delta(\bar{\chi}_i)|^2 \times$$

$$\left[ (PD + \Sigma_0^a + \Sigma_1^a) \delta^{\lambda \lambda'} \right.$$}

$$\left. + (PD + \Sigma_0^a + \Sigma_1^a) \delta^{\lambda \lambda'} \right], (25)$$

with an implicit sum over $a = 1, 2, 3$. Note that the terms proportional to $m_\tau$ in Eqs. (20), (21), and (23), are negligible at high particle energies $E \gg m_\tau$, in particular $D^b$ can be neglected in Eq. (23).

C. Normal tau polarisation and CP asymmetry

The $\tau$ polarisation for the overall event sample is given by the expectation value of the Pauli matrices $\lambda_{\tau}$.
\[ \sigma = (\sigma^1, \sigma^2, \sigma^3) \]

\[ \mathcal{P} = \frac{\text{Tr}\{\sigma\rho\}}{\text{Tr}\{\rho\}}. \]  

(26)

with the \( \tau \) spin density matrix \( \rho \), as given in Eq. \[ 13 \].

In our convention for the polarisation vector \( \mathcal{P} = (P_1, P_2, P_3) \), the components \( P_1 \) and \( P_3 \) are the transverse and longitudinal polarisations in the plane spanned by \( p_+ \) and \( p_- \), respectively, and \( P_2 \) is the polarisation normal to that plane. See our definition of the tau spin basis vectors \( s^b_\tau \) in Appendix A.

The normal \( \tau \) polarisation is equivalently defined as

\[ P_2 \equiv \frac{N(\uparrow) - N(\downarrow)}{N(\uparrow) + N(\downarrow)}, \]  

(27)

with the number of events \( N \) with the \( \tau \) spin up (\( \uparrow \)) or down (\( \downarrow \)), with respect to the quantisation axis \( p_\tau \times p_{e^-} \), cf. Eq. \[ A12 \] in Appendix A. The normal \( \tau \) polarisation can thus also be regarded as an asymmetric

\[ P_2 = \mathcal{A}^T = \frac{\sigma(T > 0) - \sigma(T < 0)}{\sigma(T > 0) + \sigma(T < 0)}, \]  

(28)

of the triple product

\[ T = \xi_\tau \cdot (p_\tau \times p_{e^-}), \]  

(29)

where \( \xi_\tau \) is the direction of the \( \tau \) spin vector for each event. The triple product \( T \) is included in the spin-spin correlation term \( \Sigma_{ab}^\Sigma_{b}^\Sigma_{ab} \) of the amplitude squared \( |\mathcal{M}|^2 \), Eq. \[ 25 \]. The asymmetry thus probes the term which contains the epsilon product \( \mathcal{E}^{ab} \), Eq. \[ 24 \].

Since under naive time reversal, \( t \to -t \), the triple product \( T \) changes sign, the tau polarisation \( P_2 \), Eq. \[ 28 \], is T-odd. Due to CPT invariance \[ 50 \], \( P_2 \) would thus be CP-odd at tree level. In general, \( P_2 \) also has contributions from absorptive phases, e.g. from intermediate s-state resonances or final-state interactions, which do not signal CP violation. Although such absorptive contributions are a higher order effect, and thus expected to be small, they can be eliminated in the true CP asymmetry \[ 10 \]

\[ \mathcal{A}_{\tau}^{CP} = \frac{1}{2}(P_2 - \overline{P}_2), \]  

(30)

where \( \overline{P}_2 \) is the normal tau polarisation for the charged conjugated process \( \tilde{\chi}_i^+ \to \tau^+ \tilde{\nu}_\tau^+ \). For our analysis at tree level, where no absorptive phases are present, we have \( \mathcal{A}_{\tau}^{CP} = P_2 \). Thus we will study \( \mathcal{A}_{\tau}^{CP} \) in the following, which is however equivalent to \( P_2 \) at tree level.

Inserting now the explicit form of the density matrix \( \rho \), Eq. \[ 14 \], into Eq. \[ 24 \], together with Eq. \[ 26 \], we obtain the CP asymmetry

\[ \mathcal{A}_{\tau}^{CP} = P_2 = \frac{\int \Sigma_{ab}^\Sigma_{b}^\Sigma_{ab} dLips}{\int PD dLips}, \]  

(31)

where we have used the narrow width approximation for the propagators in the phase space element \( dLips \), see Eq. \[ 14 \]. Note that in the denominator of \( \mathcal{A}_{\tau}^{CP} \), Eq. \[ 31 \], all spin correlation terms vanish, \( \int \Sigma_{ab}^\Sigma_{b}^\Sigma_{ab} dLips = 0 \), when integrated over phase space. In the numerator only the spin-spin correlation term \( \Sigma_{ab}^\Sigma_{b}^\Sigma_{ab} = 0 \) contributes, since only \( \Sigma_{ab}^b \), Eq. \[ 29 \], contains the T-odd epsilon product \( \mathcal{E}^{ab} \), see Eq. \[ 24 \].

D. Parameter dependence of the CP asymmetry

To qualitatively understand the dependence of the asymmetry \( \mathcal{A}_{\tau}^{CP} \), Eq. \[ 31 \], on the parameters of the chargino sector, we study in some detail its dependence on the \( \tau \tilde{\nu}_\tau \)-chargino couplings, \( c_{i\tau}^L \) and \( c_{i\tau}^R \), cf. Eq. \[ 19 \]. From the explicit form of the decay terms D, Eq. \[ 20 \], and \( \Sigma_{ab}^b \), Eqs. \[ 29 \], \[ 31 \], we find that the asymmetry

\[ \mathcal{A}_{\tau}^{CP} = \eta_1 \frac{\int \Sigma_{ab}^\Sigma_{b}^\Sigma_{ab} dLips}{\int P \sigma \cdot P_\tau \cdot dLips}, \]  

(32)

with \( (p_{\chi_i} \cdot p_\tau) = (m_{\chi_i}^2 - m_{\tilde{\nu}_\tau}^2) / 2 \), is proportional to the decay coupling factor

\[ \eta_1 = \frac{\text{Im}\{c_{i\tau}^L c_{i\tau}^R \ast\}}{\frac{1}{2}[(c_{i\tau}^L)^2 + (c_{i\tau}^R)^2}] = \frac{Y_{\tau} \text{Im}\{V_{i1} U_{i2}\}}{\frac{1}{2}(|V_{i1}|^2 + |V_{i2}|^2)}, \]  

(33)

with \( \eta_1 \in [-1, 1] \). Using the explicit forms of the matrix elements \( U \) and \( V \), see Eqs. \[ C1 \] and \[ C2 \], we can rewrite the factor \( \eta_1 \) for \( \tilde{\chi}_i^+ \to \tau^+ \tilde{\nu}_\tau^{(*)} \), or \( \tilde{\chi}_i^- \to \tau^- \tilde{\nu}_\tau^{(*)} \) decay, respectively

\[ \eta_1 = \frac{-Y_{\tau} \sin(\theta_1) \cos(\theta_2)}{\frac{1}{2} \cos^2(\theta_2) + Y_{\tau}^2 \sin^2(\theta_1)} \sin(\gamma_1 + \phi_1). \]  

(34)

\[ \eta_2 = \frac{Y_{\tau} \cos(\theta_1) \sin(\theta_2)}{\frac{1}{2} \sin^2(\theta_2) + Y_{\tau}^2 \cos^2(\theta_1)} \sin(\gamma_2 + \phi_2). \]  

(35)

with the angles \( \theta_{1,2} \), which describe chargino mixing, and \( \gamma_{1,2} \) and \( \phi_{1,2} \) which describe their CP properties, cf. Eqs. \[ C3 \] and \[ C4 \] in App. C.

Since \( \eta_1 \) is proportional to the Yukawa coupling \( Y_\tau \), Eq. \[ 12 \], the asymmetry will be enhanced for increasing \( \tan \beta \). Then the phase dependence of the asymmetries will be \( \mathcal{A}_{\tau}^{CP} \propto \eta_1 \sin(\gamma_1 + \phi_1) \approx \sin(\psi_1) \), since we find \( \phi_1, \gamma_2 \to \phi_1 \), \( \phi_2, \gamma_1 \to 0 \) for \( \tan \beta \gg 1 \). Note that the asymmetry will be additionally suppressed if

\[ \text{Note that for } \tilde{\chi}_i^+ \tilde{\chi}_j^- \text{ production, with } i \neq j, \text{ there is, in principle, also a contribution from the CP-violating normal chargino polarisation } \Sigma_{ab}^{ab=2} \text{ to the asymmetry } \mathcal{A}_{\tau}^{CP}, \text{ which projects out the CP-even parts of } \Sigma_{ab}^{ab=2}. \text{ However, } \Sigma_{ab}^{ab=2} \text{ is numerically small in our scenarios with large } \tan \beta, \text{ so that we do not discuss its impact here; see Refs. } \[ 34 \] \text{ for CP asymmetries in chargino production}. \]
\[ \eta_1 = \frac{2 Y_\tau \text{Im}(V_{11}U_{12})}{|V_{11}|^2 + Y_\tau^2 |U_{12}|^2} \]

\[ \eta_2 = \frac{2 Y_\tau \text{Im}(V_{21}U_{22})}{|V_{21}|^2 + Y_\tau^2 |U_{22}|^2} \]

**FIG. 2:** Contour lines in the $M_2-|\mu|$ plane for (a) the proportionality factor $\eta_1$, Eq. (34), of the asymmetry $A_{\text{CP}}^\tau$ for $e^+e^- \rightarrow \tilde{\chi}_1^\pm \tilde{\chi}_1^\mp$, $\tilde{\chi}_1^\pm \rightarrow \tau^\pm \nu_\tau$ and the scenario as defined in Table I, (b) the proportionality factor $\eta_2$, Eq. (35), of the asymmetry $A_{\text{CP}}^\tau$ for $e^+e^- \rightarrow \tilde{\chi}_2^\pm \tilde{\chi}_1^\mp$, $\tilde{\chi}_2^\pm \rightarrow \tau^\pm \nu_\tau$ and the scenario as given in Table I. In each case with a centre-of-mass energy $\sqrt{s} = 500$ GeV. Above the dashed line the lightest neutralino is no longer the LSP since $m_{\tilde{\chi}_1^1} < m_{\tilde{\chi}_1^0}$. In the grey-shaded area $m_{\tilde{\chi}_1^\pm} < 104$ GeV. In (a), the band between the dashed-dotted lines, and in (b) the triangle below the dashed-dotted line mark the kinematically allowed regions, see Figs. 3(c) and 6(c), respectively.
we also give its theoretical statistical significance discuss in detail. Finally, to get a lower bound on the cross sections and decay branching ratios, which we beam polarisations. The feasibility of measuring the for \( \tilde{\nu} \), \( \gamma \rightarrow \tilde{\nu} \mu \), which means that the phase dependent part of \( \eta_i \) vanishes, \( \sin(\gamma_i + \phi_i) \rightarrow 0 \).

Second, we expect maximal asymmetries for equal left and right chargino couplings, \( |\epsilon^{\mu}_{\nu L}| \approx |\epsilon^{\mu}_{\nu R}| \), where the coupling factor can be maximal \( \eta \equiv \pm 1 \), see Eq. (33). Concerning the mixing of the charginos, parametrised by the angles \( \theta_{1,2} \), we expect maximal asymmetries in a mixed gaugino-higgsino scenario, \( |\mu| \approx M_2 \), however “corrected” by the Yukawa coupling, such that \( \cos(\theta_2) \approx Y_\tau \sin(\theta_1) \) for \( \tilde{\chi}^+_1 \) decay, and \( \sin(\theta_2) \approx Y_\tau \cos(\theta_1) \) for \( \tilde{\chi}^+_2 \) decay, see Eqs. (33) and (35), respectively.

In Figs. (2a) and (b), we show the \( \eta \) factors \( \eta_1 \) and \( \eta_2 \) in the \( M_2-\mu \) plane for the scenarios as given in Table I and II accordingly. 

## III. NUMERICAL RESULTS

We present numerical results for the CP asymmetry \( A_{\text{CP}} \), Eq. (31), for chargino production \( e^+e^- \rightarrow \tilde{\chi}^+_1 \tilde{\chi}^0_1 \) and decay \( \tilde{\chi}^+_1 \rightarrow \tau^+\tilde{\nu}_\tau \). We choose a centre-of-mass energy of \( \sqrt{s} = 500 \) GeV and longitudinally polarised beams \((P_e^-|P_e^+| = (-0.8|0.6))\). We include all spin correlations between chargino production and decay, since only they include CP-violating terms at tree level.

We study the dependence of the CP asymmetry on the MSSM parameters \( \mu = |\mu|e^{i\varphi_\mu} \), \( M_2 \), and \( \tan\beta \). For \( \tilde{\chi}_1^- \tilde{\chi}_1^0 \) production, we study the dependence on the beam polarisations. The feasibility of measuring the asymmetry also depends on the chargino production cross sections and decay branching ratios, which we discuss in detail. Finally, to get a lower bound on the event rates necessary to observe the CP asymmetry, we also give its theoretical statistical significance \( S_\tau \), Eq. (D3).

For the calculation of the chargino decay widths and branching ratios, we consider the two-body decays (44)

\[
\begin{align*}
\tilde{\chi}_i^+ & \rightarrow W^+\tilde{\chi}_k^0, \\
\tilde{\chi}_i^+ & \rightarrow \mu^+\tilde{\nu}_\mu, \\
\tilde{\chi}_i^+ & \rightarrow \tau^+\tilde{\nu}_\tau, \\
\tilde{\chi}_2^+ & \rightarrow \tilde{\chi}_1^+\tilde{\chi}_1^0, \\
\tilde{\chi}_2^+ & \rightarrow \tilde{\chi}_1^+\tilde{\chi}_1^0, \\
\tilde{\chi}_2^+ & \rightarrow \tilde{\chi}_1^+\tilde{\chi}_1^0, \\
\end{align*}
\]

for \( i = 1, 2 \) and \( k = 1, \ldots, 4 \). We neglect three-body decays, which are suppressed by phase-space. In order to reduce the number of parameters, we use the GUT inspired relation \( |M_1| = 5/3 M_2 \tan^2 \theta_w \). This significantly constrains the neutralino sector (57). We take stau mixing into account, and set the mass of the trilinear scalar coupling parameter to \( A_\tau = 250 \) GeV. Since its phase does not contribute to the CP asymmetry, we fix it to \( \varphi_A = 0 \), as well as \( \varphi_1 = 0 \), which is the phase of the gaugino mass parameter \( M_1 \). We fix the soft-breaking parameters \( M_{\tilde{E}} = M_{\tilde{L}} \) in the slepton sector. When varying \( \mu \) and \( M_2 \), this can lead to excluded regions in the plots where the lightest stau \( \tilde{\tau}_1 \) is the LSP, \( m_{\tilde{\tau}_1} < m_{\tilde{\chi}_1^0} \), which we indicate by a dashed line, cf. Fig. 2. The Higgs mass parameter is fixed to \( M_A = 1000 \) GeV. Our results are insensitive to this choice, as long as we stay in the decoupling limit.

In order to show the full phase dependence on \( \varphi_\mu \), we relax the constraints from the electric dipole moments (EDMs). Our purpose is to demonstrate that a non-vanishing CP phase in the chargino sector would lead to large asymmetries. Their measurement in chargino decays will be a sensitive probe to constrain the phase \( \varphi_\mu \) independently from EDM measurements. There can be cancellations between different contributions to the EDMs, which can in principle be achieved by tuning parameters and CP phases outside the chargino sector (14–16).

On the other hand, large phases can be in agreement with the EDM bounds for scenarios with heavy first and second generation sfermion masses, of the order of 10 TeV. The third generation can stay light with masses of the order of 100 GeV (10). Such scenarios are particularly interesting also for our process. Heavy electron sneutrinos enhance chargino production cross sections, while at the same time the decay channels into taus, \( \tilde{\chi}_1^+ \rightarrow \tau^+\tilde{\nu}_\tau \), will be dominating. We will study such a scenario at the end of the numerical section.

### A. Chargino pair production

\[ e^+e^- \rightarrow \tilde{\chi}_1^+\tilde{\chi}_1^- \text{ and decay } \tilde{\chi}_1^+ \rightarrow \tau^+\tilde{\nu}_\tau \]

We first study the production of the lightest chargino pair \( e^+e^- \rightarrow \tilde{\chi}_1^+\tilde{\chi}_1^- \). Note that the production amplitude

| \( \tan\beta \) | \( \varphi_\mu \) | \( M_2 \) | \( |\mu| \) | \( M_{\tilde{E}} = M_{\tilde{L}} \) |
|---|---|---|---|---|
| 25 | 0.5\pi | 380 | 240 | 200 |

#### TABLE I: Scenario for chargino production \( e^+e^- \rightarrow \tilde{\chi}_1^+\tilde{\chi}_1^- \) and decay \( \tilde{\chi}_1^+ \rightarrow \tau^+\tilde{\nu}_\tau \). The mass parameters \( M_2, |\mu|, M_{\tilde{E}} \) and \( M_{\tilde{L}} \) are given in GeV.
\[ \sigma_P(e^+e^- \rightarrow \tilde{\chi}_1^+\tilde{\chi}_1^-) \] in fb

\[ \text{BR}(\tilde{\chi}_1^+ \rightarrow \tau^+\tilde{\nu}_\tau) \] in %

\[ A_{CP}^\tau \] in %

\[ S_\tau = \frac{|A_{CP}^\tau|\sqrt{2E\sigma}}{\sqrt{1-(A_{CP}^\tau)^2}} \]

FIG. 3: Contour lines in the \( M_2-|\mu| \) plane of (a) the production cross section, (b) branching ratio, (c) CP asymmetry of the normal tau polarisation, and (d) its significance for \( e^+e^- \rightarrow \tilde{\chi}_1^+\tilde{\chi}_1^- \rightarrow \tau^+\tilde{\nu}_\tau \), with a centre-of-mass energy \( \sqrt{s} = 500 \text{ GeV} \), longitudinally polarised beams \( (P_e^-|P_e^+|) = (-0.8|0.6) \), and an integrated luminosity \( \mathcal{L} = 500 \text{ fb}^{-1} \). The other SUSY parameters are defined in Table I. The area (a) above the zero contour line of the production cross section is kinematically forbidden by \( \sqrt{s} < 2m_{\tilde{\chi}_1^\pm} \), and the area (b) below the zero contour line of the branching ratio by \( m_{\tilde{\chi}_1^\pm} < m_{\tilde{\nu}_\tau} \). Above the dashed line the lightest neutralino is no longer the LSP since \( m_{\chi_1^0} < m_{\chi_1^\pm} \). In the grey-shaded area \( m_{\tilde{\chi}_1^\pm} < 104 \text{ GeV} \).
\( A_{\tau}^{CP} \) in %

\( S_{\tau} = \frac{|A_{\tau}^{CP}|\sqrt{2L\sigma}}{\sqrt{1-(A_{\tau}^{CP})^2}} \)

FIG. 4: Phase dependence of (a) the CP asymmetry of the normal tau polarisation and (b) its significance for \( e^+e^- \rightarrow \tilde{\chi}_1^+\tilde{\chi}_1^-; \tilde{\chi}_1^\pm \rightarrow \tau^\pm\tilde{\nu}_\tau \), for various values of \( \tan\beta \) with \( (P_{e^-}|P_{e^+}) = (-0.8|0.6) \) at \( \sqrt{s} = 500 \) GeV, and \( L = 500 \) fb\(^{-1}\). The other SUSY parameters are defined in Table I.

\( P_{e^+} \)

\( A_{\tau}^{CP} \) in %

\( S_{\tau} = \frac{|A_{\tau}^{CP}|\sqrt{2L\sigma}}{\sqrt{1-(A_{\tau}^{CP})^2}} \)

FIG. 5: Contour lines in the \( P_{e^-}P_{e^+} \) plane of (a) the CP asymmetry of the normal tau polarisation and (b) its significance for \( e^+e^- \rightarrow \tilde{\chi}_1^+\tilde{\chi}_1^-; \tilde{\chi}_1^\pm \rightarrow \tau^\pm\tilde{\nu}_\tau \), with \( \sqrt{s} = 500 \) GeV and \( L = 500 \) fb\(^{-1}\), for the scenario in Table I.
is purely real for equal chargino pair production, $\tilde{\chi}_1^+\tilde{\chi}_1^-$. Then the Z-chargino couplings are real, see Eqs. (3), (4), and the t-channel sneutrino amplitude depends only on the modulus of the sneutrino couplings, $|V_{i1}|^2$, see Eq. (5). Thus, at treelevel, a CP asymmetry in general can only receive CP-odd contributions from the chargino decay. We centre our numerical discussion around a reference scenario, see Table I which is in some sense optimised to give large significances. We choose beam polarisations of $(P_{e^-}|P_{e^+}) = (-0.8|0.6)$, which enhance the $\gamma$ exchange and $\gamma Z$ interference contributions in the production, with respect to the destructive contributions from $\gamma\tilde{\nu}_e$ and $Z\tilde{\nu}_e$ interference. This favours higher production cross sections and asymmetries compared to the reversed polarisations $(P_{e^-}|P_{e^+}) = (0.8|-0.6)$, since $\gamma Z$ interference then becomes destructive, too.

1. $M_2-|\mu|$ dependence

In Fig. (3a), we show contour lines of the chargino pair production cross section $\sigma_P(e^+e^- \rightarrow \tilde{\chi}_1^+\tilde{\chi}_1^-)$ in the $M_2-|\mu|$ plane for the scenario given in Table I. For $\mu \lesssim 200$ GeV, the cross section $\sigma_P$ mainly receives $\gamma$ exchange, and $\gamma Z$ interference contributions, which add to the contributions from pure $Z$, and $\tilde{\nu}_e$ exchange, to give more than $\sigma_P = 1000$ fb. These contributions are about twice as large as the destructive contributions from $\gamma\tilde{\nu}_e$ and $Z\tilde{\nu}_e$ interference. For $|\mu|, M_2 \gtrsim 200$ GeV, the cross section is reduced by the growing contributions from $Z\tilde{\nu}_e$ and $\gamma\tilde{\nu}_e$ interference. For $|\mu| \gtrsim 200$ GeV and $M_2 \lesssim 200$ GeV, the $Z\tilde{\nu}_e$ and $\gamma\tilde{\nu}_e$ interference contributions again become weaker, so that the production cross section is dominated by pure $\tilde{\nu}_e$ exchange.

In Fig. (3b), we show contours for the chargino branching ratio into the tau, $BR(\tilde{\chi}_1^+ \rightarrow \tau^+\tilde{\nu}_e)$, as a function of $|\mu|$ and $M_2$. We indicate the thresholds of the rivaling decay channels by coloured lines. The branching ratios $BR(\tilde{\chi}_1^+ \rightarrow \ell^+\tilde{\nu}_\ell)$, $\ell = e, \mu$ are typically of the same order as that for the decay into the tau. Branching ratios into left sleptons are of the order of $BR(\chi_1^+ \rightarrow \tilde{\ell}_1^\pm\nu_\ell) < 3\%$, $\ell = e, \mu$. The competitive chargino decays into staus can reach up to $BR(\tilde{\chi}_1^+ \rightarrow \tilde{\tau}_1^\pm\nu_\tau) = 54\%$ and $BR(\tilde{\chi}_1^+ \rightarrow \tilde{\tau}_2^\pm\nu_\tau) = 15\%$. Above the cyan-coloured contour in Fig. (3b), the decay into the $W$ boson opens, with $BR(\chi_1^+ \rightarrow W^+\tilde{\chi}_1^0) < 5\%$. The other chargino decays $\chi_1^+ \rightarrow W^+\chi_1^0$, $n = 2, 3, 4$, are kinematically excluded, since already $m_{\chi_2^+} \approx m_{\tilde{\nu}_2}$.

In Fig. (3c), we show the CP asymmetry $A_{\chi\chi}^{CP}$, Eq. (51), within its kinematically allowed range in the $M_2-|\mu|$ plane for chargino production. $\sqrt{\mathcal{S}} \geq 2m_{\chi_2^+}$, and decay, $m_{\chi_2^+} \gtrsim m_{\tilde{\nu}_2}$. For increasing $M_2$, the asymmetry receives increasing contributions from pure $\tilde{\nu}_e$ exchange, whereas the $\gamma\tilde{\nu}_e$ and $Z\tilde{\nu}_e$ interference terms, which enter with opposite sign, get reduced. Although the cross section $\sigma = \sigma_P(e^+e^- \rightarrow \tilde{\chi}_1^+\tilde{\chi}_1^-) \times BR(\tilde{\chi}_1^+ \rightarrow \tau^+\tilde{\nu}_e)$, which enters the denominator of Eq. (61), also increases with increasing $M_2$, the asymmetry attains its maximum of more than $A_{\chi\chi}^{CP} = -70\%$ for large $M_2 \gtrsim 350$ GeV. The reason is that the coupling factor $\eta_1$, Eq. (53), to which the asymmetry is proportional, is here also maximal for large $M_2$. As discussed in Subsect. III D, $\eta_1$ and thus the asymmetry is largest for $|V_{i1}| \sim |Y_eU_{12}|$. For tan $\beta = 25$, we have $Y_e \sim 1/3$, resulting in a maximum of $\eta_1$ and the asymmetry for $|\mu| \sim M_2/2$ see Fig. (3a), which is in good agreement with the location of the maximum of the asymmetry in Fig. (3c).

In Fig. (3d), we show the corresponding theoretical significance $S_\tau$, which is defined in Eq. (D3), Appendix I for $\mathcal{L} = 500$ fb$^{-1}$.

2. $\varphi_\mu$ and tan$\beta$ dependence

In Figs. (4a) and (b), we show the $\varphi_\mu$ dependence of $A_{\chi\chi}^{CP}$ and $S_\tau$, respectively, for different values of tan $\beta$. First, the asymmetry is increasing for increasing tan $\beta$, since $A_{\chi\chi}^{CP} \sim Y_e$, as discussed in Subsection III D. Second, concerning the phase dependence of the asymmetry, we find for large tan $\beta$ that $A_{\chi\chi}^{CP} \sim \sin(\varphi_\mu)$, as also discussed in Subsection III D. We observe from Fig. (4a) the almost perfect sinusoidal behaviour of $A_{\chi\chi}^{CP}$ for large tan $\beta = 20$. For smaller values of tan $\beta$, the sine-shape of the asymmetry gets less pronounced such that its maxima are not necessarily obtained for maximal CP phases $\varphi_\mu = \pm \pi/2$, but are slightly shifted away. It is remarkable that in our scenario, see Table I the asymmetry can still be $A_{\chi\chi}^{CP} = \pm 22\%$ even for $\varphi_\mu = \pm 0.1\pi$. Small phases $\varphi_\mu$ are suggested by the experimental upper bounds on the EDMs, and the asymmetry will be a sensitive probe even for small CP phases.

3. Beam polarisation dependence

In Fig. (5a), we show the beam polarisation dependence of the asymmetry $A_{\chi\chi}^{CP}$ for our benchmark scenario, see Table I. For unpolarised beams the asymmetry is $A_{\chi\chi}^{CP} = -43\%$, and varies between $A_{\chi\chi}^{CP} = -74\%$ for $(P_{e^-}|P_{e^+}) = (-0.8|0.6)$, and $A_{\chi\chi}^{CP} = +60\%$ for $(P_{e^-}|P_{e^+}) = (0.8|-0.6)$. The strong dependence of the asymmetry on the beam polarisations is due to the enhancement of the chargino production channels with $\tilde{\nu}_e$ exchange for negative electron beam polarisation, $P_{e^-} < 0$, and positive positron beam polarisation, $P_{e^+} > 0$. For oppositely polarised beams, $P_{e^-} > 0$, $P_{e^+} < 0$, the $Z$ exchange contributions are enhanced, and those of $\tilde{\nu}_e$ are suppressed. Since the $Z$ contributions enter with opposite sign, also the sign of $A_{\chi\chi}^{CP}$ changes, see Fig. (5a).

The corresponding theoretical statistical significance $S_\tau$ is shown in Fig. (5b). The production cross section
\[ \sigma_P(e^+e^- \rightarrow \tilde{\chi}_1^\pm \tilde{\chi}_1^\mp) \] varies from 332 fb for unpolarised beams to 418 fb for \( (P_{e^-}:P_{e^+}) = (-0.8:0.6) \), and 121 fb for \( (P_{e^-}:P_{e^+}) = (0.8:0.6) \). Thus the largest values of \( \sigma_P \) are obtained for polarised beams, where \( \nu_e \) exchange contributions are enhanced. The significance then reaches up to \( S_\tau = 450 \).

### B. Production of an unequal pair of charginos

\[ e^+ + e^- \rightarrow \tilde{\chi}_i^+ + \tilde{\chi}_j^- \] and decay \( \tilde{\chi}_i^+ \rightarrow \tau^+ + \nu_\tau \)

For \( \tilde{\chi}_1^+ \tilde{\chi}_2^- \) production, the CP asymmetry \( A_{\chi}^{\text{CP}} \) in principle also receives non-vanishing CP-odd contributions from the production. However, in our benchmark scenario with large \( \tan \beta = 25 \), see Table II, those contributions are smaller than 1%. The dominant contributions will still be from the decay, and we discuss the asymmetries for the decay of \( \tilde{\chi}_1^\pm \) and \( \tilde{\chi}_2^\pm \) separately.

#### 1. Decay of \( \tilde{\chi}_2^+ \rightarrow \tau^+ + \nu_\tau \)

In Fig. 5(a), we show the \( M_2-|\mu| \) dependence of the production cross section \( \sigma_P(e^+e^- \rightarrow \tilde{\chi}_2^+ \tilde{\chi}_1^-) \) which can attain values of several hundred fb. In contrast to the production of an equal pair of charginos, \( e^+e^- \rightarrow \tilde{\chi}_i^+ \tilde{\chi}_i^- \), the cross section receives destructive contributions from \( Z\nu_\tau \) interference, only. The dominant contribution is from pure \( \nu_\tau \) exchange. With increasing \( |\mu| \), that contribution decreases faster than the \( Z\nu_\tau \) interference term and the production cross section is reduced.

**TABLE II: Scenario for \( e^+e^- \rightarrow \tilde{\chi}_1^\pm \tilde{\chi}_2^\pm \) production and decay \( \tilde{\chi}_{1,2} \rightarrow \tau^\pm \nu_\tau^{(\tau)} \).** The mass parameters \( M_2, |\mu|, M_R \) and \( M_L \) are given in GeV.

| \( \tan \beta \) | \( \varphi_\mu \) | \( M_2 \) | \( |\mu| \) | \( M_R = M_L \) |
|---|---|---|---|---|
| 25 | 0.5\( \pi \) | 250 | 200 | 150 |

#### Calculated mass spectrum.

| \( \ell \) | \( m \) [GeV] | \( \tilde{\chi} \) | \( m \) [GeV] |
|---|---|---|---|
| \( \tilde{e}_R, \tilde{\mu}_R \) | 156 | \( \tilde{\chi}_1^0 \) | 115 |
| \( \tilde{e}_L, \tilde{\mu}_L \) | 157 | \( \tilde{\chi}_2^0 \) | 177 |
| \( \tilde{\nu}_e, \tilde{\nu}_\mu \) | 136 | \( \tilde{\chi}_3^0 \) | 210 |
| \( \tilde{\tau}_1 \) | 125 | \( \tilde{\chi}_4^0 \) | 294 |
| \( \tilde{\tau}_2 \) | 183 | \( \tilde{\chi}_1^{\pm} \) | 170 |
| \( \tilde{\nu}_\tau \) | 136 | \( \tilde{\chi}_2^{\pm} \) | 294 |
| \( \text{BR}(\tilde{\chi}_1^+ \rightarrow \tau^+ \nu_\tau) \% \) | 28 |
| \( \text{BR}(\tilde{\chi}_2^+ \rightarrow \tau^+ \nu_\tau) \% \) | 14 |
| \( \sigma_P(e^+e^- \rightarrow \tilde{\chi}_1^+ \tilde{\chi}_2^-) \) [fb] | 444 |

In contrast to the lightest chargino \( \tilde{\chi}_1^+ \), the decay of the heavy chargino \( \tilde{\chi}_2^+ \rightarrow \tau^+ \nu_\tau \) is kinematically allowed in the entire \( M_2-|\mu| \) plane, see Fig. 5(b). However, its branching ratio is small, \( \text{BR}(\tilde{\chi}_2^+ \rightarrow \tau^+ \nu_\tau) < 14\% \), since all other decay channels can open, except \( \text{BR}(\tilde{\chi}_2^+ \rightarrow \chi_0^0 W^+) \), see Eq. (33). Within the parameter range of Fig. 5(b), we find \( \text{BR}(\tilde{\chi}_2^+ \rightarrow \ell \nu_\ell) < 28\% \), and \( \text{BR}(\tilde{\chi}_2^+ \rightarrow \ell_L \nu_\ell) < 15\% \) for \( \ell = e, \mu \), as well as \( \text{BR}(\tilde{\chi}_2^+ \rightarrow \chi_0^0 W^+) < 41\% \). Other decays can reach up to \( \text{BR}(\tilde{\chi}_2^+ \rightarrow \chi_0^0 W^+) = 7\% \), for \( i = 1, 3 \), and \( \text{BR}(\tilde{\chi}_2^+ \rightarrow \chi_i^0 H_i^+) = 21\% \).

Fig. 5(c) shows the CP asymmetry \( A_{\chi}^{\text{CP}} \). It reaches more than 70\%, and is enhanced kinematically by the rising destructive interference \( Z\nu_\tau \) in the production process for \( |\mu| \gtrsim 200 \text{ GeV} \). These lead to lower contributions to \( P \) and hence to larger asymmetries, cf. Eq. (31). In addition, the coupling factor \( \eta_\ell \), Eq. (33), is maximal for \( |\mu| \approx 300 \text{ GeV} \) and \( M_2 \approx 200 \text{ GeV} \), the condition for maximal interference of the gaugino-higgsino components. See the discussion in Subsection IID and Fig. 2(b).

The corresponding significance, \( S_{\tau} \), Eq. (24), which is shown in Fig. 1(d), is smaller than for the production of an equal pair of charginos, as the cross section \( \sigma_P(e^+e^- \rightarrow \tilde{\chi}_2^+ \tilde{\chi}_1^-) \times \text{BR}(\tilde{\chi}_2^+ \rightarrow \tau^+ \nu_\tau) \) is smaller by a factor of about 2. However for \( L = 500 \text{ fb}^{-1} \), it can still attain values of \( S_{\tau} = 150 \).

The \( \varphi_\mu \) dependence of the asymmetry and its significance is shown for various values of \( \tan \beta \) in Fig. 7. Again, we can clearly observe the two striking features, \( A_{\chi}^{\text{CP}} \propto Y_\tau \), and \( A_{\chi}^{\text{CP}} \propto \sin(\varphi_\mu) \). These are due to the special dependence of the asymmetry on the \( \tau-\tilde{\nu}_\tau \)-chargino couplings, and can be qualitatively understood, see the discussion in Subsection IID.

#### 2. Decay of \( \tilde{\chi}_1^+ \rightarrow \tau^+ + \nu_\tau \)

In Fig. 5(a), we show the \( M_2-|\mu| \) dependence of the CP asymmetry. Large values, up to \( A_{\chi}^{\text{CP}} = -74\% \), are obtained towards \( M_2 \sim 2|\mu| \), where also \( \eta_1 \), Eq. (33), is maximal; compare with the asymmetry in Fig. 5(c). The corresponding branching ratio \( \text{BR}(\tilde{\chi}_1^+ \rightarrow \tau^+ \nu_\tau) \) does not exceed 30\%. The decay channels into the light leptons \( \text{BR}(\tilde{\chi}_1^+ \rightarrow \ell^+ \nu_\ell) \), \( \ell = e, \mu \) and into the lightest stau \( \text{BR}(\tilde{\chi}_1^+ \rightarrow \tilde{\tau}_1^0 \nu_\tau) \) are the most competitive ones with up to 20\% and 30\%, respectively. Towards the production threshold, \( \text{BR}(\tilde{\chi}_1^+ \rightarrow \ell_L \nu_\ell) \) is of the order of 5\%. Together with the production cross section \( \sigma_P(e^+e^- \rightarrow \tilde{\chi}_1^+ \tilde{\chi}_1^-) \), as shown in Fig. 5(a), the product of production and decay branching ratio, \( \sigma = \sigma_P \times \text{BR}(\tilde{\chi}_1^+ \rightarrow \tau^+ \nu_\tau) \), can be as large as 140 fb. The statistical significance, shown in Fig. 5(b), reaches \( S_{\tau} = 200 \), for \( L = 500 \text{ fb}^{-1} \).
FIG. 6: Contour lines in the $M_2$-$|\mu|$ plane of (a) the production cross section, (b) branching ratio, (c) CP asymmetry of the normal tau polarisation and (d) its significance for $e^+e^- \rightarrow \tilde{\chi}_2^\pm \tilde{\chi}_1^\mp$, with a centre-of-mass energy $\sqrt{s} = 500$ GeV, longitudinally polarised beams $(P_+|P_-) = (-0.8|0.6)$, and an integrated luminosity $L = 500$ fb$^{-1}$. The other SUSY parameters are defined in Table II. The area $A$ above the zero contour line of the cross section is kinematically forbidden by $\sqrt{s} < m_{\tilde{\chi}_2^\pm} + m_{\chi_1^-}$. Above the dashed line the lightest neutralino is no longer the LSP since $m_{\tilde{\chi}_1^0} < m_{\tilde{\tau}_1^\pm}$. In the grey-shaded area $m_{\tilde{\chi}_1^+} < 104$ GeV.
FIG. 7: Phase dependence of (a) the CP asymmetry of the normal tau polarisation and (b) its significance for $e^+ e^- \rightarrow \tilde{\chi}_1^\pm \tilde{\chi}_1^\mp$, for various values of tan $\beta$ with $(P_e^+ |P_e^-) = (-0.8|0.6)$ at $\sqrt{s} = 500$ GeV, and $L = 500$ fb$^{-1}$. The other SUSY parameters are defined in Table II.

FIG. 8: Contour lines in the $M_2$-$|\mu|$ plane of (a) the CP asymmetry of the normal tau polarisation and (b) its significance for $e^+ e^- \rightarrow \tilde{\chi}_1^\pm \tilde{\chi}_2^0$; $\tilde{\chi}_1^\pm \rightarrow \tau^\pm \tilde{\nu}_\tau$, with a centre-of-mass energy $\sqrt{s} = 500$ GeV, longitudinally polarised beams $(P_e^+ |P_e^-) = (-0.8|0.6)$, and an integrated luminosity $L = 500$ fb$^{-1}$. The other SUSY parameters are defined in Table III. The area 1 is kinematically forbidden by $\sqrt{s} < m_{\tilde{\chi}_1^+} + m_{\tilde{\chi}_2^0}$, and the area 2 is kinematically forbidden by $m_{\tilde{\chi}_1^+} < m_{\tilde{\nu}_\tau}$. Above the dashed line the lightest neutralino is no longer the LSP since $m_{\tilde{\chi}_1^+} < \tilde{\chi}_1^0$. In the grey-shaded area $m_{\tilde{\chi}_1^\pm} < 104$ GeV.
C. Inverted hierarchy scenario

The phase $\varphi_\mu$ of the higgsino mass parameter $\mu$ contributes already at the one-loop level to the electric dipole moments (EDMs) of the electron [8], the neutron [9], and the mercury atom [10,11]. The dominant contribution to the electron EDM from chargino exchange, for example, is proportional to $\sin \varphi_\mu$ [8,14]. The phase $\varphi_\mu$ is thus strongly constrained by the experimental upper limits on the EDMs with $|\varphi_\mu| \lesssim 0.1 \pi$ in general [13]. However, the bounds from the EDMs are highly model dependent. For instance, cancellations between different SUSY contributions to the EDMs can resolve the restrictions on the phases [14,15].

Another way to fulfil the EDM bounds is to assume sufficiently heavy sleptons and/or squarks. Since sparticle masses of the order of 10 TeV are required [14], such solutions are unnatural. Models with heavy sparticles have been discussed in the literature, like split-SUSY [13] or focus-point scenarios [18]. If only the first and second generation squarks are heavy, naturalness can be reconciled while experimental constraints can still be fulfilled, see e.g. Ref. [13] for such an inverted hierarchy approach.

Heavy sfermions of the first and second generations are particularly interesting for our process of chargino production $e^+e^- \rightarrow \tilde{\chi}^\pm_1 \tilde{\chi}^\mp_1$ and decay into the tau, $\tilde{\chi}^\pm_1 \rightarrow \tau^\pm \tilde{\nu}_\tau(\gamma)$. First, the negative contributions from sneutrino $\tilde{\nu}_\tau$ interference to the production cross sections are considerably reduced. Second, the chargino branching ratio into the tau is enhanced, since the chargino decay channels $\tilde{\ell}_R \nu_\ell$ and $\tilde{\ell}_L \bar{\nu}_\ell$ are closed due to the heavy sleptons of the first and second generations, $\tilde{\ell} = e, \mu$. In order to compare with our previous results, we take the parameters as in Table I but now choose heavy soft breaking masses for the selectrons and smuons $M_{\tilde{e}} = M_{\tilde{\mu}} = 15$ TeV. See new reference scenario and the resulting mass spectrum in Table III.

1. Parameter dependence

In Fig. 2(a), we show the $M_2-|\mu|$ dependence of the cross section for chargino production $e^+e^- \rightarrow \tilde{\chi}^+_1 \tilde{\chi}^-_1$ for our new reference scenario with heavy sneutrinos. Due to the heavy electron sneutrino, $m_{\tilde{\nu}_e} = 15$ TeV, the negative interference contributions from $Z\tilde{\nu}_e$ and $\gamma\tilde{\nu}_e$ are thoroughly suppressed, which enhances the cross section. In the scenario with light sneutrinos, in particular for large values of $|\mu|$, the pure $\tilde{\nu}_e$ exchange is the largest contribution to the cross section, see the discussion concerning Fig. 3(a) in Subsection III A 1. Although the constructive $\tilde{\nu}_e$ exchange contributions are also lost for heavy sneutrinos, there is still a net surplus in the production cross section, compare Fig. 3(a) and Fig. 2(a), since they are of the same order as one of the destructive channels, $Z\tilde{\nu}_e$ or $\gamma\tilde{\nu}_e$.

The branching ratio $BR(\tilde{\chi}^+_1 \rightarrow \tau^+ \tilde{\nu}_\tau)$ is only reduced by the rivaling decay into the lightest stau, which is at least $BR(\tilde{\chi}^+_1 \rightarrow \tilde{\tau}\nu_\tau) = 50\%$ in Fig. 2(b). The product of production and decay $\sigma = \sigma_P(e^+e^- \rightarrow \tilde{\chi}^+_1 \tilde{\chi}^-_1) \times BR(\tilde{\chi}^+_1 \rightarrow \tau^+ \tilde{\nu}_\tau)$ for heavy sneutrinos is thus of the order of several hundred fb, see Table III. In contrast to the strong impact of a heavy sneutrino on the cross section, the CP asymmetry $A_{\chi_1-\chi_1}$ is only slightly enhanced, compare Fig. 2(c) with Fig. 2(c). The asymmetry is mainly determined by the coupling factor $\eta_1$, see Eq. (32), which still allows for asymmetries of more than 70%. Together with the enhanced cross section, this leads to sizable significances of the order of several hundred standard deviations over statistical fluctuations, which we show in Fig. 2(d). Also the phase $\varphi_\mu$ and $\tan \beta$ dependence of the asymmetries is still governed by the coupling factor $\eta_1$, Eq. (32). In Fig. 3(a), we observe the same sinuss-like dependence of $A_{\chi_1-\chi_1}$, which increases with increasing $\tan \beta$, cf. Fig. 2(a), and see the discussion in Subsection III D.

To summarise, the CP asymmetries in the decay of a chargino into a polarised tau are a powerful tool to probe $\varphi_\mu$, which might be large in particular in scenarios with flavour violation [16], or heavy sfermions of the first and second generations [19].

IV. SUMMARY AND CONCLUSIONS

We have studied CP violation in chargino production with longitudinally polarised beams, $e^+e^- \rightarrow \tilde{\chi}^+_1 \tilde{\chi}^-_1$, and the subsequent two-body decay of one of the charginos

| $\tan \beta$ | $\varphi_\mu$ | $M_2$ | $|\mu|$ | $M_{\tilde{e}} = M_{\tilde{\mu}}$ | $M_{\tilde{\phi}} = M_{\tilde{\phi}_L}$ |
| 25 | 0.5$\pi$ | 380 | 240 | $15\times 10^3$ | 200 |

Calculated mass spectrum.

| $\ell$ | $m$ [GeV] | $\tilde{\chi}$ | $m$ [GeV] |
| --- | --- | --- | --- |
| $\tilde{e}_R, \tilde{\mu}_R$ | $15 \times 10^3$ | $\tilde{\chi}_1^0$ | 175 |
| $\tilde{e}_L, \tilde{\mu}_L$ | $15 \times 10^3$ | $\tilde{\chi}_2^0$ | 238 |
| $\tilde{\nu}_e, \tilde{\nu}_\mu$ | $15 \times 10^3$ | $\tilde{\chi}_3^0$ | 247 |
| $\tilde{\tau}_1$ | 177 | $\tilde{\chi}_3^0$ | 405 |
| $\tilde{\tau}_2$ | 230 | $\tilde{\chi}_3^\pm$ | 225 |
| $\tilde{\nu}_\tau$ | 189 | $\tilde{\chi}_3^\mp$ | 405 |

| $BR(\tilde{\chi}^+_1 \rightarrow \tau^+ \tilde{\nu}_\tau)$ [%] | $\sigma_P(e^+e^- \rightarrow \tilde{\chi}^+_1 \tilde{\chi}^-_1)$ [fb] |
| --- | --- |
| 49 | 805 |

TABLE III: Scenario for chargino pair production $e^+e^- \rightarrow \tilde{\chi}^+_1 \tilde{\chi}^-_1$ and decay $\tilde{\chi}^+_1 \rightarrow \tau^+ \tilde{\nu}_\tau$ with heavy first and second slepton generations. The mass parameters $M_2, |\mu|, M_{\tilde{e}}, M_{\tilde{\mu}}, M_{\tilde{\phi}}$ are given in GeV.
\[ \sigma_P(e^+e^- \rightarrow \tilde{\chi}^+_1 \tilde{\chi}^-_1) \text{ in fb} \]

\[ \text{BR}(\tilde{\chi}^+_1 \rightarrow \tau^+ \tilde{\nu}_\tau) \text{ in \%} \]

\[ \Delta A^\text{CP} \text{ in \%} \]

\[ S_\tau = \frac{|A^\text{CP}| \sqrt{2E_{\text{cm}}}}{\sqrt{1-(A^\text{CP})^2}} \]

**FIG. 9:** Contour lines in the \(M_2-|\mu|\) plane of (a) the production cross section, (b) branching ratio, (c) CP asymmetry of the normal tau polarisation, and (d) its significance, for \(e^+e^- \rightarrow \tilde{\chi}^+_1 \tilde{\chi}^-_1, \tilde{\chi}^0_1 \rightarrow \tau^+ \tilde{\nu}_\tau(\ast)\), for a spectrum of heavy 1st and 2nd slepton generations as given in Table II with a centre-of-mass energy \(\sqrt{s} = 500\,\text{GeV}\), longitudinally polarised beams (\(P_{e^-} |P_{e^+}| = (-0.8,0.6)\), and an integrated luminosity \(L = 500\,\text{fb}^{-1}\). The area (\(\mathcal{A}\)) above the zero contour line of the production cross section is kinematically forbidden by \(\sqrt{s} < 2m_{\tilde{\chi}^\pm_1}\), and the area (\(\mathcal{B}\)) below the zero contour line of the branching ratio by \(m_{\tilde{\chi}^+_1} < m_{\tilde{\nu}_\tau}\). Above the dashed line the lightest neutralino is no longer the LSP since \(m_{\tilde{\chi}_1^0} < \tilde{\chi}_1^0\). In the grey-shaded area \(m_{\chi^+_1} < 104\,\text{GeV}\).
In a numerical discussion we have considered equal chargino pair production $e^+e^- \rightarrow \tilde{\chi}_1^+ \tilde{\chi}_1^-$, and unequal chargino pair production, $e^+e^- \rightarrow \tilde{\chi}_1^+ \tilde{\chi}_2^-$, with the ensuing decay of either $\tilde{\chi}_1^+$ or $\tilde{\chi}_2^+$ respectively. We have studied the dependence of the CP asymmetries on the MSSM parameters of the chargino sector $M_2, |\mu|, \varphi_\mu$, and $\tan \beta$. The asymmetries are considerably enhanced for large $\tan \beta$, where the tau Yukawa coupling is enhanced. The size of the asymmetries also strongly depends on the gaugino-higgsino composition of the charginos, and can be maximal for equally sized left and right $\tau-\tilde{\nu}_\tau$-chargino couplings.

We have found that $A_\text{CP}$ can attain values of more than 70%. The asymmetry is already present at tree level and can be sizable even for small phases of $\mu$, as suggested by the experimental limits on EDMs. Moreover, by choosing different beam polarisations the $Z$, $\gamma$ and $\tilde{\nu}_\tau$ contributions can be enhanced or suppressed. A proper choice of beam polarisations can thus considerably enhance both, the asymmetry, and the production cross sections. An analysis of statistical errors shows that the asymmetries are well accessible in future $e^+e^-$ collider experiments in the 500 GeV range with high luminosity and longitudinally polarised beams.

Since the phase $\varphi_\mu$ of the higgsino mass parameter $\mu$ is the most constrained SUSY CP phase, as suggested by EDM bounds, a measurement of the normal tau polarisation will be a powerful tool to constrain $\varphi_\mu$ independently from the low energy measurements. Moreover, we have shown that the asymmetry can be sizable in inverted hierarchy scenarios, with heavy sfermions of the first and second generations, where the EDM constraints on the SUSY phases are less severe.

To summarise, CP asymmetries in the decay of a chargino into a polarised tau are one of the most sensitive probes to measure or constrain $\varphi_\mu$ at the ILC. Since the feasibility of measuring the tau polarisation can only be addressed in a detailed experimental study, we want to motivate such thorough analyses, to explore the potential of measuring SUSY CP phases at high energy colliders.

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Appendix A: Momenta and spin vectors

We choose a coordinate frame for the centre-of-mass system such that the momentum $p_{\hat{f}}$ of the chargino $\hat{f}$ points in the z-direction [53]. The scattering angle $\hat{s}(p_{\hat{f}}, p_{\hat{f}'} )$ is denoted by $\theta$, and the azimuthal angle is set to zero. Explicitly the momenta are then [53]

\[
\begin{align*}
  p_{\hat{f}'} &= E_{\hat{f}}(1,-\sin \theta,0,0,\cos \theta), \\
  p_{\hat{f}} &= (E_{\hat{f}},0,0,0), \\
  p_{\hat{f}} &= E_{\hat{f}}(1,\sin \theta,0,0,\cos \theta), \\
  p_{\hat{f}} &= (E_{\hat{f}},0,0,0),
\end{align*}
\]  

with the beam energy $E_b = \sqrt{s}/2$, and [53]

\[
\begin{align*}
  E_{\hat{f}} &= \frac{s + m_{\hat{f}}^2 - m_{\hat{f}}^2}{2s}, \\
  E_{\hat{f}'} &= \frac{s + m_{\hat{f}}^2 - m_{\hat{f}}^2}{2s}, \\
  q &= \frac{\sqrt{\lambda(s, m_{\hat{f}}^2, m_{\hat{f}}^2)}}{2s},
\end{align*}
\]

with

\[
\lambda(x,y,z) = x^2 + y^2 + z^2 - 2(xy + xz + yz).
\]  

For the chargino decay, $\hat{f} \rightarrow \tau^+\nu_\tau$, we parametrise the tau momentum in terms of the decay angle $\theta_D = \hat{s}(p_\tau, p_{\hat{f}})$, and its azimuth $\phi_D$.

\[
(p_{\tau})^T = \begin{pmatrix} E_\tau & -|p_\tau| \sin \theta_D \cos \phi_D \\ |p_\tau| \sin \theta_D \sin \phi_D & -|p_\tau| \cos \theta_D \end{pmatrix}, \quad \text{A9}
\]

\[
|p_\tau| = \frac{m_{\hat{f}}^2 - m_{\hat{f}'}^2}{2(E_{\hat{f}} - q \cos \theta_D)}. \quad \text{A10}
\]

The $\tau$ spin vectors are defined as

\[
\begin{align*}
  s_{1,\mu} &= \left( 0, \frac{s_2 \times s_3}{|s_2 \times s_3|} \right), \\
  s_{2,\mu} &= \left( 0, \frac{p_\tau \times p_{\hat{f}}}{|p_\tau \times p_{\hat{f}}|} \right), \\
  s_{3,\mu} &= \frac{1}{m_\tau} \left( |p_\tau| E_\tau p_\tau \right).
\end{align*}
\]

with

\[
s_{1,\mu} \cdot s_{1,\nu} = -\delta_{\mu\nu}, \quad s_{2,\mu} \cdot p_\tau = 0. \quad \text{A14}
\]

The chargino and tau spin vectors fulfil the completeness relation [52,53]

\[
\sum c_{\nu}^* s_{1,\nu} \cdot s_{1,\mu} = -g_{\mu\nu} + \frac{p_{\hat{f}} \cdot p_{\hat{f}'}}{m_{\hat{f}'}^2}. \quad \text{A15}
\]

Appendix B: Phase space

For chargino production $e^+ e^- \rightarrow \chi_{\pm}^0 \chi_\mp^\pm$, and subsequent decay of one of the charginos, $\chi_\pm^\pm \rightarrow \tau^\pm \nu_\tau$, the Lorentz invariant phase-space element decomposes into two-body phase-space elements [58]

\[
d\mathcal{L}ip(s; p_{\hat{f}}, p_{\hat{f}'} \nu) = \frac{1}{2\pi} d\mathcal{L}ip(s; p_{\hat{f}}, p_{\hat{f}'} \nu) \times d\mathcal{L}ip(s_{\hat{f}'}; \nu, \nu). \quad \text{B1}
\]

The decay of the other chargino $\chi_\mp^\pm$ is not considered further. The constituent parts are

\[
d\mathcal{L}ip(s; p_{\hat{f}}, p_{\hat{f}'} \nu) = \frac{g}{8\pi s} \sin \theta \, \cos \gamma, \quad \text{B2}
\]

\[
d\mathcal{L}ip(s_{\hat{f}'}; \nu, \nu) = \frac{1}{2(2\pi)^2} \frac{|p_{\tau}|^2}{m_{\hat{f}}^2 - m_{\hat{f}'}^2}, \quad \text{B3}
\]

with $d\Omega_D = \sin \theta_D d\theta_D d\phi_D$, and $s_{\hat{f}} = p_{\hat{f}}^2$.

We use the narrow width approximation for the chargino propagator, $\Delta(\chi_{\pm})$, Eq. (15),

\[
\int |\Delta(\chi_{\pm})|^2 \, ds_{\hat{f}} = \frac{\pi}{m_{\hat{f}} \Gamma_{\hat{f}}}. \quad \text{B4}
\]

This approximation should be justified for $(\Gamma_{\hat{f}}/m_{\hat{f}})^2 \ll 1$, which holds in our case for chargino widths $\Gamma_{\hat{f}} \lesssim 1$ GeV and masses $m_{\hat{f}} \approx 100$ GeV. However, the naive $O(\Gamma/m)$-expectation of the error can easily receive large off-shell corrections of an order of magnitude and more, in particular at threshold, or due to interferences with other resonant or non-resonant processes. For a recent discussion of these issues, see, for example, Ref. [57].

Appendix C: Chargino diagonalisation matrices

The matrices $U$ and $V$, which diagonalise the chargino matrix $M_{\chi}$, see Eq. (3), can be parametrised by [23]

\[
U = \begin{pmatrix} e^{i\gamma_1} & 0 \\ 0 & e^{i\gamma_2} \end{pmatrix} \begin{pmatrix} \cos \theta_1 & e^{i\phi_1} \sin \theta_1 \\ -e^{-i\phi_1} \sin \theta_1 & \cos \theta_1 \end{pmatrix}, \quad \text{C1}
\]

\[
V = \begin{pmatrix} \cos \theta_2 & e^{-i\phi_2} \sin \theta_2 \\ -e^{i\phi_2} \sin \theta_2 & \cos \theta_2 \end{pmatrix}. \quad \text{C2}
\]

The mixing angles, $-\pi/2 \leq \theta_{1,2} \leq 0$, are

\[
t(2\theta_1) = \sqrt{M_{\chi}^2 c^2(\beta) + |\mu|^2 s^2(\beta) + M_{\chi} |\mu| s(2\beta) c(\varphi_\mu)} \left/ \frac{2\sqrt{2} M_{\chi}^2 - |\mu|^2 - 2M_{\chi}^2 c(2\beta)}{M_{\chi}^2 - |\mu|^2 - 2M_{\chi}^2 c(2\beta)} \right., \quad \text{C3a}
\]

\[
t(2\theta_2) = \sqrt{M_{\chi}^2 s^2(\beta) + |\mu|^2 c^2(\beta) + M_{\chi} |\mu| s(2\beta) c(\varphi_\mu)} \left/ \frac{2\sqrt{2} M_{\chi}^2 - |\mu|^2 - 2M_{\chi}^2 c(2\beta)}{M_{\chi}^2 - |\mu|^2 - 2M_{\chi}^2 c(2\beta)} \right., \quad \text{C3b}
\]
with the short hand notations \( s(\alpha) = \sin(\alpha), c(\alpha) = \cos(\alpha), \) and \( t(\alpha) = \tan(\alpha). \) For a CP-violating chargino system, \( \varphi_\mu \neq 0, \) the following CP phases enter

\[
\begin{align*}
t(\phi_1) &= s(\varphi_\mu) \left[ c(\varphi_\mu) + \frac{M_2}{|\mu| t(\beta)} \right]^{-1}, \\
t(\phi_2) &= -s(\varphi_\mu) \left[ c(\varphi_\mu) + \frac{M_2 t(\beta)}{|\mu|} \right]^{-1}, \\
t(\gamma_1) &= -\frac{s(\varphi_\mu)}{c(\varphi_\mu) + \frac{M_2 (m_{\chi^+_1}^2 - |\mu|^2)}{|\mu|m_{\chi^+_1}^2 s(2\beta)}}, \\
t(\gamma_2) &= \frac{s(\varphi_\mu)}{c(\varphi_\mu) + \frac{M_2 m_{\chi^+_1}^2 s(2\beta)}{|\mu| (m_{\chi^+_2}^2 - M_2^2)}}.
\end{align*}
\]

The chargino masses are

\[
m_{\chi^+_1,2}^2 = \frac{1}{2} \left( M_2^2 + |\mu|^2 + 2m_W^2 \mp \kappa \right),
\]

with

\[
\kappa^2 = \left( M_2^2 - |\mu|^2 \right)^2 + 4m_W^4 c^2(2\beta) + 4m_W^2 \left[ M_2^2 - |\mu|^2 + 2M_2 s(2\beta) c(\varphi_\mu) \right].
\]

### Appendix D: Theoretical statistical significance

To measure a non-zero value of the CP asymmetry \( \mathcal{A}_{\text{CP}}^\tau, \) Eq. (30), over statistical fluctuations, we define its theoretical statistical significance by

\[
S_\tau = \frac{|\mathcal{A}_{\text{CP}}^\tau|}{\sigma_A},
\]

such that \( S_\tau \) is the expected number of standard deviations \( \sigma_A \) to which the asymmetry \( \mathcal{A}_{\text{CP}}^\tau \) can be determined to differ from zero. Since the variance is given by

\[
\sigma_A^2 = \frac{1 - |\mathcal{A}_{\text{CP}}^\tau|^2}{2N},
\]

with the total number of events \( N = \mathcal{L}_\sigma, \) we find

\[
S_\tau = \frac{|\mathcal{A}_{\text{CP}}^\tau| \sqrt{2\mathcal{L}_\sigma}}{\sqrt{1 - |\mathcal{A}_{\text{CP}}^\tau|^2}}.
\]

Note that our definition of the statistical significance \( S_\tau \) is purely based on the theoretical signal rate and its asymmetry. Detector efficiencies, event reconstruction efficiencies, and contributions from CP-even backgrounds are neglected, which would reduce the significance. The definition has thus to be regarded as an absolute upper bound only. In order to give realistic values of the statistical significances to observe a CP signal, a detailed experimental study is necessary.

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