Game model of three-party shared risk allocation for PPP projects ---based on the perspective of incomplete information

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Abstract. PPP projects involve multi-stakeholders. Rational allocation of risk among project participants is the basis and key to the smooth operation of PPP projects. In the past, most of the studies have only considered the public and private parties, but ignored the bank as the project participant to share the project risk. Therefore, using the bargaining game theory, this paper builds a bargaining game model in which the three parties (government, enterprise and bank) of PPP project deter each other under incomplete information, and works out the corresponding sub-game refined Nash equilibrium, then obtains the proportion of risk sharing among the three participants. The research results provide a scientific basis for project participants to make decision on risk sharing, which is conducive to improving the willingness of project participants to cooperate, and also play a supplementary role in PPP project risk sharing research.

1. Introduction
PPP((Public-Private-Partnership) is a mode in which public sectors cooperate with social capital to provide public goods in the form of franchise. It emphasizes risk-sharing, benefit-sharing and whole-process cooperation. This mode plays an important role in alleviating the pressure of local government debt and activating social capital. At present, the PPP project of new-type urbanization is in full swing, and PPP project has become another major loan investment direction of commercial banks after local financing platforms. The participation of banks is conducive to promoting the participation of private capital, alleviating local debt pressure, revitalizing stock assets, and improving the level and efficiency of government public services. However, PPP project participants are faced with many uncertain risks in the process of project implementation, resulting in low enthusiasm of banks to participate in projects. Therefore, the rational allocation of project risk is important for promoting the development of the project.

At present, the risk sharing of PPP projects is mostly studied by mathematical model, analytic hierarchy process and game theory. Chen [1] established a linear programming model to prove that there exists an rational structure for risk allocation. Gao [2] used the Hall three-dimensional model to model from the three dimensions of risk stages, levels and types, and made a comprehensive score of risk sharing of various risk factors by using this model. Ye [3] used AHP to determine the index weight. Zhou [4] introduced the Harsanyi transformation to construct a bargaining game model under the condition of incomplete information. Based on different bidding sequences, Li [5] constructed a risk sharing model with incomplete information bargaining theory, and obtained the risk sharing ratio of
both parties. Thomas [6] regarded deterrence as a bargaining process and defined mutual deterrence as an ability to inflict harm on the other party. Rubinstein [7] proposed an indefinite bargaining model. Zhu [8] expanded the subject of risk sharing from two parties to three parties and built a three-party static game model. Baj [9] introduced the bank as a participant to build a bargaining game model of mutual deterrence among the three parties under complete information.

By sorting out and analyzing the above literatures, most of the existing studies do not consider the complexity of project participants and only involve the risk sharing between public and private parties. Few studies on tripartite risk allocation are only conducted under the condition of complete information, while in reality, it is difficult for project participants to fully grasp the information of each other. Based on this, this paper introduces the bank as the participant, building a three-party bargaining game model under the condition of incomplete information, more truly and accurately reflects the actual situation of the PPP project, and finds out the rational risk allocation ratio shared by the three parties, thereby improving the enthusiasm and willingness of the project participants to cooperate, ensuring the PPP project running smoothly and promoting its long-term development.

2. Model Building

2.1 Game model of the first stage

Compared with the government, enterprises and banks are in a weak position, so in the first stage, enterprises and banks will form an "alliance" to bargain with the government.

2.1.1 Basic assumptions

The basic assumptions of the model are as follows:

Assumption 1: The government (X) and the "alliance" (Y) are rational and want the negotiation to succeed.

Assumption 2: The information between the government and the"alliance" is asymmetric, and the government is in a dominant position and has the priority to bid.

Assumption 3: Different risks are independent of each other and almost unrelated to each other.

Assumption 4: For a certain risk, the sum of the risk-taking ratio between the government and the alliance is 1. Assuming that the risk-taking ratio of X is h, then the risk-taking ratio of Y is 1-h, both parties will negotiate on the size of h.

2.1.2 Setting model parameters

The model parameters are as follows:

1) Negotiation loss coefficient. Negotiation loss coefficient (a>1) refers to the negotiation cost that both parties have to bear and will inevitably occur during the negotiation process, including time cost, information acquisition cost and other transaction costs. According to the actual negotiation of PPP projects, government departments play a dominant role in PPP projects, so the cost that the government needs to bear in the game(a_1) is lower than the negotiation cost of "alliance"(a_2), that is to say: a_1 < a_2.

2) The possibility of the government utilizing its superiority. In the game process, the government will force the other sides to accept its own risk with the probability of k_1, and the probability of k_2 will not force the "alliance" to take more risk.(k_1 + k_2 = 1)

3) Risk transfer quantity. The quantity of government force "alliance" to accept its own risk is “f”, and the risk transfer quantity must be less than the government's own risk-taking ratio h, so 0 ≤ f ≤ h.<1

4) The Harsanyi transformation. Before 1967, Harsanyi proposed to transform the game of incomplete information into a game of complete but imperfect information by introducing a virtual participant -- "Nature", in which Nature takes the lead and decides the characteristics of each participant.
2.1.3 Game model

In the bargaining game, the dominant party has the priority to bid, so the government makes the bid first, and the "alliance" makes the choice according to the allocation plan proposed by the government. If it accepts, it will conclude the negotiation, and if it refuses, it will continue the next round of negotiations.

The first round of negotiations: The government proposes that it can assume the risk of h₁, and the "alliance" needs to share the risk of 1-h₁.

1) if the government forces the "alliance" to accept its own risk quantity f₁ with the probability of k₁, the risk(X'₁) that the government needs to take and the risk(Y'₁)that the "alliance" should take are respectively:

\[ X'_1 = k_1 \times (1 - f_1) \]  
\[ Y'_1 = k_1 \times h_1 + f_1 \]  

(2) if the government does not force the "alliance" to accept its own risk with the probability of k₂, the risk(X''₁) that the government needs to take and the risk(Y''₁) that the "alliance" should take are respectively:

\[ X''_1 = k_2 \times h_1 \]  
\[ Y''_1 = k_2 \times (1 - h_1) \]  

The risks taken by government departments(X₁) and "alliance"(Y₁) in the first round are respectively:

\[ X_1 = k_1 \times (1 - f_1) + k_2 \times h_1 \]  
\[ Y_1 = k_1 \times (1 - h_1 + f_1) + k_2 \times (1 - h_1) \]

If the "alliance" refuses to take the risk proportion, the negotiations automatically enter the second round.

The second round of negotiations: The "alliance" proposes that the risk-taking ratio of the government is h₂, the risk-taking ratio of the "alliance" is 1-h₂, the government's negotiation loss coefficient is a₁, the "alliance" negotiation loss coefficient is a₂. Similarly, the risks taken by government departments(X₂) and "alliance"(Y₂) in the second round are respectively:

\[ X_2 = k_1 \times a_1 \times (1 - f_2) \times h_2 \]  
\[ Y_2 = k_1 \times a_2 \times (1 - h_2) + k_2 \times a_2 \times (1 - h_2) \]  

If the government does not agree with the allocation ratio proposed by the alliance, the second round of negotiations will be concluded and the third round will be held.

The third round of negotiations: Similarly, the risks taken by government departments and "alliance" in the third round X₃ and Y₃ are respectively:

\[ X_3 = k_1 \times a_1^2 \times (1 - f_3) + k_2 \times a_1^2 \times h_3 \]  
\[ Y_3 = k_1 \times a_2^2 \times (1 - h_3 + f_3) + k_2 \times a_2^2 \times (1 - h_3) \]

If neither the government nor the alliance can accept the allocation plan proposed by the other side, the game negotiations in theory will continue indefinitely until both sides reach a consensus.

2.1.4 Optimal solution

This model is a bargaining game model under the condition of incomplete information. According to the Harsanyi transformation theory, this model should be solved by setting up an inverse basis point, which is the same in any round, that is, h₁=h₂. This paper chooses the third round of negotiations with deadline as the reverse basis point. If X₃ is greater than X₃, the government will reject the result of the second round of negotiations, and then the third round of negotiations is required. However, for each negotiation, the alliance will bear the negotiation loss, and a₂ is greater than a₁. Therefore, in order to avoid increasing the negotiation loss, the alliance will minimize its risk expectation in the second round of allocation proposal and make X₃ is not greater than X₃, that is X₂ = X₁:

\[ h_2 = a_1 h_1 - k_1 a_2 f_2 + k_2 f_2 \]

Then the risk expectations of the second and third rounds of "alliance " are:

\[ Y_2 = a_1^2 (1 - h_2) + k_1 a_2 f_2 \]  
\[ Y_3 = a_2^2 (1 - h_3) + k_1 a_2^2 f_3 \]
\[ Y_2/Y_1 = (a_2/a_1) + [(1-a_1)a_2(f_1k_1 + 1-h_3)] \]  

(14)

Because of \( a_2 > a_1 > 1 \), obviously, \( 0 < Y_2/Y_1 < 1 \), so the risk sharing ratio of the alliance in the second round is less than that in the third round, so the negotiation will not enter the third stage. Similarly, in the first round of negotiations, the rational strategy is \( Y_1 = Y_2 \):

\[ h_1 = 1 + k_1/(1-a_1h_1 + k_1a_2f_1) \]  

(15)

For an infinite round of negotiations, game no matter from which round, the risk proportion of bargaining should be the same, so \( h_1 = h_3 \):

\[ h_3 = [k_1(a_1a_2f_1 + a_2 - 1)] / (a_1a_2) \]  

(16)

\[ 1 - h_3 = [k_1(a_1 - 1) - k_1(a_1a_2f_1)] / (a_1a_2) \]  

(17)

If \( f = f_0 \) is a constant, the equilibrium solution of risk sharing between government and alliance is as follows:

\[ h = (a_2 - 1)/(a_1a_2 - 1) + k_1f \]  

(18)

\[ 1 - h = (a_1a_2 - a_2)/(a_1a_2 - 1) - k_1f \]  

(19)

\( k_1f \) refers to the risk that the government transfers to the alliance by taking advantage of its strong position, so the actual risk-taking ratio of the government departments is \((a_2 - 1)/(a_1a_2 - 1)\), the actual risk-taking ratio of the alliance is \((a_1a_2 - a_2)/(a_1a_2 - 1)\).

2.2 Game model of the second stage

Therefore, in this stage, a game model is constructed to carry out risk redistribution under the condition of incomplete information with asymmetric status of both parties (the bank and enterprise).

2.2.1 Basic assumptions

The basic assumptions of the model are as follows:

Assumption 1: Bank (A) and enterprise (B) are rational people, and neither of them wants the negotiation to break down.

Assumption 2: Bank is in a dominant position and the bank makes the first bid.

Assumption 3: All risks are independent of each other, and there is little correlation between different risks, the initial values are set at 1.

Assumption 4: The sum of the risk-taking ratio for a certain risk assumed by the bank and the enterprise is \( M \), the proportion of risk assumed by the bank is \( r_1 \), then the proportion of risk assumed by the enterprise is \( M - r_1 \), and the two parties negotiate on the size of \( r_1 \).

2.2.2 Setting model parameters

The model parameters are as follows:

1) Negotiation loss coefficient \( \eta > 1 \). According to the actual situation of PPP project negotiation, banks play a dominant role in PPP projects for enterprises, so the cost that banks need to bear in the game is lower than the negotiation cost of enterprises, that is \( 1 < \eta_1 < \eta_2 \).

2) The possibility of the bank utilizing its superiority. In the process of game, the bank transfers the risk to the other parties with the probability of \( k' \).

3) Risk transfer quantity. The amount of bank force others to accept its own risk is \( q \), and there is \( 0 \leq q \leq r_1 < 1 \).

2.2.3 Game model

The first round of negotiations: The bank first proposes the risk ratio \( r_1 \), and the enterprise is \( M - r_1 \). At the end of the first round, the risk-taking ratios of bank and enterprise are:

\[ A_1 = k'(r_1 - q_1 + k'r_1) \]  

(20)

\[ B_1 = k'(M - r_1 + q_1 + k'(M - r_1)) \]  

(21)

Similarly, after the second round of negotiations, the risk allocation ratios of both parties are:

\[ A_2 = k_1\eta_1(r_2 - q_2 + k'\eta_1r_2) \]  

(22)

\[ B_2 = k_1\eta_2(M - r_2 + q_2 + k'\eta_2(M - r_2)) \]  

(23)
Similarly, after the third round of negotiations, the risk distribution proportions of both parties are:

\[ A_3 = k_1' \eta_1^2 (r_1-q_1) + k_2' \eta_1^2 r_1 \]  
\[ B_3 = k_1' \eta_2^2 (M-r_2+q_3) + k_2' \eta_2^2 (M-r_3) \]  

(24)  

(25)

2.2.4 Optimal solution

Same as the first stage, using the same solution idea, letting \( A_2 = A_1 \), \( B_1 = B_2 \), we can get that:

\[ r = [(\eta_1^2-1) (a_1 a_2-a_2)]/[(\eta_1^2-1) (a_1 a_2-a_2)] \]  
\[ M-r=[(\eta_1^2-\eta_2)(a_1 a_2-a_2)]/[(\eta_1^2-1) (a_1 a_2-a_2)] \]  

(26)  

(27)

To sum up, the actual risk proportions shared by the government, bank, and enterprise are as follows:

Government: \( X = (a_2-1) / (a_1 a_2-1) \)  

Bank: \( A = [(\eta_1^2-1) (a_1 a_2-a_2)]/[(\eta_1^2-1) (a_1 a_2-a_2)] \)  

Enterprise: \( B = [(\eta_1^2-\eta_2) (a_1 a_2-a_2)]/[(\eta_1^2-1) (a_1 a_2-a_2)] \)  

(28)  

(29)  

(30)

3. Numerical Results

This paper takes a subway line project as an example for empirical analysis. The total length of the line is nearly 25 kilometers, and 18 stations are set on the whole line. The total investment of the project is nearly 8.2 billion yuan. The pre-project negotiation is a typical game negotiation under incomplete information.

3.1 Risk indicator system

In this paper, through questionnaire survey, literature analysis and interviews with 25 personnel familiar with the project, combining with the practical situation of the project, the risk index system of the project is obtained, and the collected data are processed to obtain the corresponding parameter values under each risk category, as shown in Table 1. The corresponding risk-sharing ratios of three parties are obtained by using formulas (18), (28), (29) and (30), respectively, as shown in Table 2.

| Types of risk                      | Impact factors | \( a_1 \) | \( a_2 \) | \( \eta_1 \) | \( \eta_2 \) | \( \eta_3 \) | \( k_1 \) | \( f \) |
|-----------------------------------|----------------|----------|----------|-------------|-------------|-------------|----------|------|
| Market risk                       |                | 1.14     | 1.21     | 1.16        | 1.18        | 0.53        | 0.15     |
| Risk of cost overruns             |                | 1.32     | 1.35     | 1.34        | 1.37        | 0.62        | 0.14     |
| Social natural risk               |                | 1.03     | 1.07     | 1.05        | 1.09        | 0.71        | 0.08     |
| Project construction risk         |                | 1.18     | 1.24     | 1.22        | 1.29        | 0.65        | 0.06     |
| Profitable risk                   |                | 1.21     | 1.25     | 1.26        | 1.31        | 0.78        | 0.09     |
| Completion risk                   |                | 1.06     | 1.11     | 1.07        | 1.15        | 0.64        | 0.12     |
| Policy risk                       |                | 1.33     | 1.39     | 1.34        | 1.37        | 0.39        | 0.03     |

| Types of risk                      | Risk-taking ratio | The government's actual share | Government risk transfer ratio | Nominal government share | The bank's actual share | The actual share of the enterprise |
|-----------------------------------|-------------------|------------------------------|-------------------------------|-------------------------|------------------------|-------------------------------|
| Market risk                       |                   | 55.35%                       | 7.95%                         | 63.30%                  | 21.79%                 | 22.86%                        |
| Risk of cost overruns             |                   | 44.76%                       | 8.68%                         | 53.44%                  | 24.45%                 | 30.79%                        |
| Social natural risk               |                   | 68.56%                       | 5.68%                         | 74.24%                  | 19.58%                 | 11.86%                        |
| Project construction risk         |                   | 51.81%                       | 3.90%                         | 55.71%                  | 24.35%                 | 23.84%                        |
| Profitable risk                   |                   | 48.78%                       | 7.02%                         | 55.80%                  | 24.41%                 | 26.81%                        |
| Completion risk                   |                   | 62.29%                       | 7.68%                         | 69.97%                  | 24.54%                 | 13.17%                        |
| Policy risk                       |                   | 45.95%                       | 1.17%                         | 47.12%                  | 23.93%                 | 30.12%                        |
3.2 Game analysis of risk sharing

The Table 2 shows that under incomplete information, the government uses its dominant position to exert strong pressure on other project participants to transfer risks, so the risk proportion that the government actually undertakes is always lower than the risk proportion that it nominally undertakes. Among the listed risks, when the project encounters the risk of cost overruns and market risks, the risk proportion of government transfer is the highest, which indicates that under the circumstances of insufficient funds and large market competition, the government hopes that banks and enterprises will seek solutions by themselves, not willing to take action.

4. Conclusions

Rationality of risk sharing is the key factor to determine the success of PPP projects. Previous risk studies mostly focused on both public and private parties, and did not fully consider the diversity of participants and their preferences and responses to risk. On the basis of previous research on PPP risk sharing, this paper constructs a bargaining game model of mutual deterrence among government, bank and enterprise under incomplete information, and works out the corresponding Nash equilibrium of sub-game, and obtains the optimal risk sharing proportion. The results show that the proportion of risk sharing is related to the negotiation loss coefficient of the negotiating parties. If one party wants to reduce its risk bearing value, it needs to increase the cost to understand the information and strategies of the negotiating parties and reduce the number of negotiation as much as possible. The size of negotiation loss coefficient in PPP project needs to be obtained through questionnaires and interviews with the relevant persons in charge. This paper will supplement the risk study of PPP model, provide a more scientific and reasonable basis for decision-making of risk sharing for tripartite participants in PPP projects, reduce the randomness and blindness of risk allocation, and help to improve the enthusiasm of banks to enter the PPP field and promote the healthy development of PPP projects.

References

[1] B.Chen,C.G.Xu, Technology & Economy in Areas of Communications, 13 (1): 126-128(2011)
[2] D.M.Gao,Y.J. Luo, Project Management Technology, 15,8(2017)
[3] X.D.Ye, Journal of Engineering Management, 5:44-47(2012)
[4] S.Zhou,M.S.Li, Engineering Economy, 27,4(2017)
[5] Y.Li, Public Finance Research, 10: 50-57(2015)
[6] T. C. Schelling ,New Haven: Yale University Press, (1966)
[7] A.Rubinstein, Econometrica: Journal of the Econometric Society, 50 (1): 97(1982)
[8] X.D.Zhu, X.Xiao, N.Zheng, Journal of Hebei University of Technology, 2: 97-101(2013)
[9] X.Y.Bai, H.Xu, Y.Xu, Journal of Weinan Normal University, 33,6(2018)