Assembly and transport of nematic colloidal swarms above photo-patterned defects and surfaces

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Abstract

We investigate the dynamic assembly and swarm translocation of anisometric colloidal particles dispersed in a nematic liquid crystal and driven above a photosensitive surface. We use liquid crystal-enabled electrophoresis to propel these particles via an alternating electric field perpendicular to the sample cell. By manipulating the anchoring conditions on one surface of the experimental cell, we obtain a spatially extended spiral pattern of the liquid crystal orientation that induces the dynamic assembly of a rotating colloidal mill. This structure can be transported by translocating the topological defect above the photosensitive surface. We complement our findings with a theoretical model that captures the basic physics of the process, by formulating an analytic equation for the director field above the surface. Our reconfigurable nematic assemblies may be used as a test bed for complex swarming behaviour in biological and artificial microscale systems.

1. Introduction

The formation of coherent structures from a disordered array of interacting, self-propelling elements is currently an active research topic in different fields of science [1–4]. The interest arose from both a fundamental and an applied point of view. On the one hand, monitoring and controlling the transport properties of large populations of artificial micromachines may help understanding the underlying mechanisms that govern spontaneous formation of coherent structures from self-propelled units, as observed in disparate biological systems [5–11]. On the other hand, these investigations have also direct technological applications related with the field of drug delivery in microfluidic and vein networks. In particular, examples include the controlled release of biochemical cargos attached to functionalized colloidal particles [12–15], the miniaturization of simple operations in lab-on-a-chip devices [16–20] or the realization of different functional tasks at the microscale [21–23].

The formation of particle swarms from ensembles of artificial prototypes driven by an external field has been recently reported by different research groups who use electric [24, 25], optic [26] or magnetic fields [27, 28]. In most of the cases, the driven particles are dispersed in an isotropic fluid such as water, and are propelled through lithographic structures or assembled along a circular confinement. An alternative approach that is gaining interest in the community is the use of photosensitive surfaces in a nematic liquid crystal matrix, where the dispersed particles can be driven by an electric field along predefined tracks [29–33]. The use of an anisotropic medium for the particle motion may provide different advantages to self-propelling particle systems. For example the possibility of controlling the mean molecular orientation of the dispersing medium via an external
field, the ability of switching from an anisotropic to an isotropic medium by varying the system temperature, or finally the absence of hard walls or confinement that may alter the particle motion due to steric constraints.

Motivated by these prospects, in this article we investigate the dynamics of an ensemble of anisometric colloidal particles that are dynamically assembled into a rotating mill organized around a single topological defect of a liquid crystal on a surface that imposes planar alignment. In addition, we study the translocation of the colloidal swarm when the topological defect is relocated on the surface. In this case we observe that the condensed pattern disassembles, forming a polarized phase that moves along the director field. To understand the system dynamics, we theoretically determine the director field in contact with the photosensitive surface, and show that a simple simulation scheme describes well the observed phenomena. Our colloidal model system may be used to explore the rich many-body physics of collective organization and swarming in microscale driven systems, and may also inspire the use of similar strategies to control and guide biological entities along pre-designed paths [33, 34].

The paper is outlined as follows. After the description of the experimental system, in section 2, we discuss the propulsion of a single colloidal particle, section 3. In section 4 we describe the formation of the experimentally observed mill pattern and quantify its properties, which is then followed in section 5 by a comparison with a simple numerical model. An analytic expression for spiral pattern is derived in the appendix. The process of translocation is discussed in section 6, in which we compare the results of the measurements with the predictions of the numerical model. We summarize the results and draw the conclusions in section 7.

2. Experimental system

We prepare thin cells enclosing a solution of colloidal particles dispersed in a nematic liquid crystal (MLC-7029, Merck) characterized by a negative dielectric anisotropy \( \Delta \varepsilon = -3.6 \) (at 1 kHz). As shown in figure 1(a), these cells are composed by two microscope slides with an area of \( 15 \times 25 \) mm\(^2\), 0.7 mm thickness, and coated with a thin layer of indium-tin oxide (ITO) with a sheet resistance of \( \sim 100 \) \( \Omega \) sq (VisionTek Systems). The plates are cleaned by sonication in a 1% Micro-90 (Sigma-Aldrich) solution, rinsed with ultrapure water (18.2 M\( \Omega \) cm, Millipore Milli-Q), and dried at 80 °C for 30 min. Afterwards, the plates are introduced in a plasma surface treatment equipment (ZEPTO, Diener Electronic). One of the plates is functionalized with a photosensitive surface in a toluene medium (peptide synthesis grade, Scharlau) by mixing two silanes in a ratio 5:1, (3-aminopropyl)triethoxysilane (APTES, Sigma-Aldrich) and an azosilane compound, \( 3 \)-4-(4-octylphenyl)
diazetyl phenoxo)-N-(3-(triethoxysilyl)propyl)-butanamide (GalChimia). Butylamine is added as catalyst. The 
other plate is coated with a thin layer of a polyeimide compound (0626 from Nissan Chemical Industries), 
after plasma surface activation, in order to achieve a strong homeotropic anchoring of the liquid crystal at the surface. Both plates are disposed with the ITO facing inwards and are glued together with a separation of about 23 μm 
using Mylar spacers (Goodfellow). The anisometric particles used in this work have a pear-like shape with two 
connected spherical lobes (Magsphere Inc). The particles are made of polystyrene and have lateral dimensions 
\(d_x = 4 \, \mu m, \, d_y = 3 \, \mu m\), see figure 1(a).

The experimental system is composed of an upright optical microscope (Nikon Eclipse 50iPol) containing two 
collimated epi-illumination LED light sources of wavelength 455 nm (Thorlabs M455L3, 900 mW) and 365 nm (Thorlabs M365L2, 190 mW). Brightfield illumination for microscopy was performed with a red 
longpass filter (Lambda 645 nm) to avoid perturbing the azosilane coating [31, 35–37]. The microscope 
objective is also used to focus light coming from the LEDs onto the sample cell projecting a Gaussian intensity 
profile. Images of the particle dynamics are recorded with a CMOS camera (AVT Marlin F-131B). The external, 
alternating current (AC) is applied with a voltage amplifier (TREK model PZD700) controlled by a function 
generator (Agilent DSOX2002A).

3. Single particle transport

We start by describing the transport properties of an individual anisometric particle. Figure 1(b) illustrates 
different experimental snapshots of a single particle driven in the liquid crystal (LC). To understand the 
mechanism of motion, one has to consider the experimental geometry, as schematically depicted in figure 1(a). In the absence of an external field, the homeotropic alignment at the surface of the two cells forces the nematic 
director \(\vec{n}\) to point along the \(\vec{z}\)-axis. Since the LC has a negative dielectric anisotropy, under the applied field \(\vec{n}\) 
orient perpendicular to the field direction, say along the \(\vec{x}\)-axis. As a consequence, also the main axis of the 
dispersed particle aligns along \(\vec{n}\). Inside the LC, the colloidal particle distorts the nematic matrix creating 
topological defects around its surface [38]. For our pear-shaped colloids we find two point defects located at both 
 apexes of the particle, in the form of a non-symmetric `double-boojum’ [39].

The applied AC electric field induces electroosmotic flows around the particles. The anisometric shape of 
particles breaks the symmetry of these flows leading to net phoretic propulsion at speed \(v\) [29]. Given the 
confined geometry, the motion is quasi-two-dimensional, and thermal fluctuations of the particle are negligible 
due to its relative large size and the high viscosity of the LC medium. Most particles move with their large lobe 
ahead, and exhibit a ballistic-like trajectory as shown by the sequence of images in figure 1(b) (see also Video1 in 
the supporting information). Further, we note that, since the AC field is applied perpendicularly to the plane of 
motion, linear (DC) electrophoretic effects resulting from the attraction toward the two electrodes are also 
negligible.

The particle phoretic speed \(v\) can be controlled by varying the amplitude \(E\) of the applied AC field, as shown 
in figure 1(c). Similarly to [29], we detect a quadratic dependence of \(v\) on \(E\) typical of electroosmotic flows 
developing around the particle, \(v = \beta (E - E_0)^2\). Note that the propulsion velocity is independent of the 
polarity of the field, which enables the application of AC fields. From the fit to the experimental data, we 
determine the threshold field for motion, \(E_0 = 0.16 \pm 0.06 \, \text{V} / \mu \text{m}^2\), figure 1(c). Here \(E_0\) represents the 
minimum field required to generate enough ionic flow around the particle to provide propulsion.

In the collective particle dynamics we will keep the field parameters fixed to \(E = 0.76 \, \text{V} / \mu \text{m}^2\) and \(f = 10\) 
Hz, which results in an averaged single particle speed of \(\bar{v} = 15.1 \, \mu \text{m} / \text{s}\). Finally, we note that at a fixed 
amplitude, the frequency dependence \(\bar{v} = \nu (f)\) is even more complex, due to the nature of the ionic motion 
within the LC, as reported in previous works [29, 40]. We, therefore, keep the frequency constant through all 
experiments.

4. Dynamic assembly on a spiral pattern

Controlling the orientation of the director, \(\vec{n}\), allows steering the trajectory of the colloidal particles that are 
propelled in the LC by the applied AC field. We investigate the collective dynamics of the anisometric particles 
above a spiral pattern characterized by a central topological defect, which is shown in figure 2(a) under crossed 
polarizer and analyzer. This complex pattern was obtained with the following steps. First we use UV light to 
irradiate a large, circular region of the surface coated with the azosilane (see section 2), which forces the dye 
molecules there to be in the cis configuration, i.e. to orient parallel to the surface. Thus, the LC director \(\vec{n}\) 
acquires a planar anchoring on this area. After that, we irradiate a smaller central spot with blue light, forcing the 
azosilane layer to revert to the trans-configuration. In this situation, the director \(\vec{n}\) acquires a homeotropic 
alignment inside and outside a circular corona of planar molecules. When the AC field is applied, the LC
molecules with planar alignment in this corona extend inwards and outwards, and \( \vec{n} \) forms a spiral pattern with a central topological defect of charge +1, figure 2(a). Indeed, the LC used here is characterized by a bend elastic constant smaller than the splay one, and thus bend distortions are favored during orientation of the LC matrix.

In figure 2(b) we show the dynamic assembly of an ensemble of anisometric particles following this spiral pattern. As shown by the corresponding Video2 in the supporting information, once the external field is applied, particles move towards the topological defect, which appears as a small black spot at the center of the image. Particles follow a spiral trajectory, and once close to the defect they start orbiting around it at a constant distance. As more particles arrive from outside, the density of the rotating cluster increases, until reaching a size of \( N \approx 1000 \) particles after \( t = 113 \) s, figure 2(b). The cluster rotates as a solid body around the topological defect, and is completely dynamic in nature, since its structure can be easily disassembled by switching off the field or reducing its strength below \( E_0 \) (see section 3). The cluster’s rotational motion follows the chirality of the underlying pattern, which is randomly selected upon irradiation, while the particles located in the central denser region keep a constant inter-particle distance larger than the particle diameter. This indicates the presence of a long range repulsive interaction that may arise from the electrostatic dipoles induced by the applied AC field. Another interesting feature is the formation of chains at the periphery of the core. This chaining arises from the LC-mediated quadrupolar elastic interaction induced by the double-boojum defects located at both apexes of the particles. The chains tend to spiral at a certain angle to minimize the elastic energy and ensure the relocation of the incoming particles [38].

In order to analyze the dynamic state, we show in figure 2(c) particle image velocimetry analysis of the colloidal pattern. Indeed, we find that, after a short transient, the cluster of particle organizes into a rotating mill pattern, with an almost constant linear velocity everywhere in the cluster and, thus, with an angular velocity that decays with the distance to the center of the cluster.

5. Theoretical description of the colloidal mill

To model the dynamics of particles assembled in the colloidal mill, we apply the minimal scheme able to capture all the general trends of the dynamic assembly. We consider particles to be spherical with a prescribed phoretic speed and restrict our consideration to the two-dimensional \((x, y)\) plane, which is parallel to the electrodes. The overdamped motion of an ensemble of \( N \) colloidal particles with positions \( \vec{r}_i \) \((i = 1, ..., N)\) in a LC matrix is described by the following equations of motion

Figure 2. (a) Polarized microscope image taken under crossed polarizer and analyzer, showing the spiral pattern induced by light on the nematic LC due to the irradiated azosilane layer. The small inset at the top shows a photoaligned corona before the application of the AC field. (b) Sequence of experimental images of a mill pattern composed by anisometric colloidal particles assembled around a topological defect (black spots). The applied field has a frequency \( f = 10 \) Hz and an amplitude \( E = 0.76 V_{\mu m}^{-1} \). Scale bars for all images are 25 \( \mu m \), and all are separated by 37 s. The corresponding video (Video2) can be found in the supporting information, which is available online. (c) Particle image velocimetry of the largest nematic colloidal mill shown in panel (b). The color bar on the right side shows the amplitude of the velocity field.
The director of the liquid crystal, and which is given in equation (A.7). In polar coordinates, the director field spirals out of the origin according to \( \mathbf{n} = (n_x, n_y) = (\cos \psi, \sin \psi) \). Since we are interested in patterns that spiral in, we flip the sign of the radial component of the director, \( n_x \rightarrow -n_x \), and then pass to Cartesian coordinates \((x, y)\). As a result, the components of the director field, \( \mathbf{n}(r) = (n_x, n_y) \), can be presented as \( n_x(r) = -\cos \theta n_x - \sin \theta n_y = -\cos(\theta(r) - \psi(r)), \quad n_y(r) = -\sin \theta n_x + \cos \theta n_y = -\sin(\theta(r) - \psi(r)) \).

The corresponding pattern generated by equation (3) in the domain \( 0 \leq x \leq L_x, 0 \leq y \leq L_y \) with \( L_x = 300 \, \mu m \) and \( L_y = 300 \, \mu m \) and placed at the position \( q = (150 \, \mu m, 150 \, \mu m) \) is shown in figure 3(a). The spiral pattern is characterized by the parameters \( \alpha_1 = -\pi/5, \alpha_2 = -\pi/4 \) (see the appendix), and the inner and outer radii are set to \( r_1 = 2 \, \mu m \) and \( r_2 = 1000 \, \mu m \), respectively. We note that although the pattern is derived for the elastic constants \( K_1 = 16.1 \) pN and \( K_2 = 15.0 \) pN, which are close in value, equation (A.7), valid for \( K_1 = K_2 \), gives a slightly different pattern with respect to the one-constant approximation, equation (A.8), but it represents a more accurate solution.

In figure 3(b) we show the result from a numerical simulation, where \( N = 800 \) particles are assembled above the spiral pattern (see also the corresponding Video3 in the supporting information). In order to reproduce the dynamic features observed in the experimental system, we introduce long-range repulsive interaction between the colloidal particles. We have checked and found that different long-range repulsion potentials seem to work fine to describe the dynamics of the colloidal assembly process. To be specific, in the control simulations we stick to the repulsive potential of the form \( U(r_i) \propto 1/r_i^d \), which leads to the repulsive force exerted on particle \( i \) by
particle $j$

$$F_{ij}^{(1)} = -\frac{\partial U_i(r)}{\partial r_i} = C_i r_{ij},$$

(4)

where $r_{ij} = r_i - r_j$, $r_{ij} = |r_{ij}|$. The constant $C_i / \zeta_0$ is taken as 2000 $\mu$m$^3$/s$^{-1}$ in the simulations. We note that this law of repulsion is a justified assumption because the particles are polarized in the applied AC field and acquire an electric dipole moment [41, 42]. This dipole is always perpendicular to the electrodes and to the plane of particles, and therefore all the dipole moments stay parallel to each other. As a result, they are expected to repel according to the above dipolar interaction force.

To account for a finite size of particles, we also introduce a steep short-range repulsive potential of the form

$$U_3(r_{ij}) \propto \left(\frac{\sigma}{r_{ij}}\right)^{48} - \left(\frac{\sigma}{r_{ij}}\right)^{24} + 1/4,$$

which is applied at distances $r_{ij} < \eta_0 = 2^{1/24}\sigma$ and is vanishing otherwise. Here, $\sigma$ is an effective diameter of the particle. At short distances, $r_{ij} < \eta_0$, this potential leads to the strictly repulsive force

$$F_{ij}^{(2)} = -\frac{\partial U_3(r_{ij})}{\partial r_i} = 24C_i \left[\left(\frac{\sigma}{r_{ij}}\right)^{48} - \left(\frac{\sigma}{r_{ij}}\right)^{24}\right] \frac{r_{ij}}{r_{ij}^3},$$

(5)

In our study, we set $C_3 / \zeta_0 \approx 1 \mu$m$^3$/s$^{-1}$ and $\sigma = 5 \mu$m. Note that $\sigma$ is slightly larger than the actual size of particles, $d_a, d_c$. To better replicate the experimental observations, we assume that defects in the liquid crystal in the vicinity of particles prevent the direct contact of particles, leading to a larger effective hard core distance.

Finally, we also account for quadrupolar interactions, which cause a weak chaining of particles depending on the underlying pattern. The corresponding force can be represented as [43]

$$F_{ij}^{(3)} = -\frac{\partial U_3(r_{ij})}{\partial r_i}, \quad U_3(r_{ij}) = C_3 \left(3 - 30 \cos^2 \theta + 35 \cos^4 \theta\right),$$

(6)

where $\theta$ is the angle between the far-field orientation of the nematic director $n$ and the vector connecting the center of particles, $r_{ij}$. In the simulations, we set $C_3 / \zeta_0 = 500 \mu$m$^3$/s$^{-1}$. Note that figure 3(b) reflects the consequence of these interactions leading to spiral chaining at the periphery of the hard core.

The comparison between the experimental and numerical data is shown in figure 4, where we plot different observables related with the dynamics of the colloidal mill as obtained in experiments (figures 4(a), (b)) and simulations (figures 4(c) and (d)). In the former case, we use video microscopy and tracking routines [44] to extract the positions ($x_i, y_i$) of each particle $i$, and analyze the radial and azimuthal particle velocities averaged over the ensemble, ($v_{r0}, v_{\phi0}$) with $v_{r,\phi} = \frac{1}{\sum_i |\mathbf{v}_i|} \sum_i |\mathbf{v}_i|$, as measured in polar coordinates from the central defect. Both quantities are shown in figure 4(a) for the rotating mill, and compared with the numerical case in figure 4(c). Both types of data present the same trend, with a vanishing radial velocity when the mill is formed and an azimuthal component that approaches $-1$. Both quantities are negative since the particles are moving toward the center thus against the radial direction, outwards by definition ($v_{r0} < 0$), and in a clockwise sense ($v_{\phi0} < 0$). The main difference between experiments and simulations refer to transient states determined by initial conditions, which are clearly different in both cases. As expected, the convergence to the final state shows similar trends in the evolution of $v_{r0}$ and $v_{\phi0}$.

Apart from the instantaneous velocity, we characterize the dynamics in terms of two other order parameters that describe, respectively, the degree of particle alignment and collective rotation. The first one is the normalized polarity $P$ of the pattern, defined as:

$$P = \frac{\sum_i \mathbf{v}_i}{\sum_i |\mathbf{v}_i|},$$

(7)

with $\mathbf{v}_i$ being the velocity of particle $i$ at position $r_i$ from the center of mass of the ensemble, and the summation is performed over all particles in the ensemble. The second parameter is the normalized angular momentum of the particle ensemble, defined as

$$M = \frac{\sum_i r_i \times \mathbf{v}_i}{\sum_i |r_i||\mathbf{v}_i|}.$$

(8)

As described in [45], both order parameters are necessary to quantitatively determine whether the collection of propelling particles is in a coherent flock state (high value of $P$, low $M$) or in a single-mill state (low $P$, high $M$). As shown in figures 4(b) and (d), we find that in the experiments and simulations both assemblies show a negligible polarity $P$, while the angular momentum $M$ increases toward unity.
6. Colloidal translocation: experimental and numerical results

While we just described the formation of a colloidal mill, the photosensitive surface also allows to transport these dynamic states everywhere within the cell. For a given assembly, particle speed due to liquid crystal-enabled electrophoresis (LCEEP) drops down to zero at electric field frequencies of about 50 Hz. For greater stability, we choose 1 kHz as the frequency to freeze the assembly in place, while we pull the topological defect from the initial position (center of the formed colloidal mill) for a certain distance. This translocation is performed through a disclination line that forms as the UV light spot is dragged between the initial and the final destination on the photosensitive surface. We demonstrate this feature in the sequence of images in figure 5(a), where a mill pattern is translocated a distance of 175 μm (see also Video4 in the supporting information).

During translocation, the colloidal swarm moves as a polarized pattern developing a leading edge that is followed by all other particles. These dynamic features are reflected in figure 5(b), where we evaluate the corresponding two order parameters $M$ and $P$. A colloidal mill is initially assembled under $f = 10$ Hz. It is subsequently held in place by switching to $f = 1$ kHz and a UV light spot is dragged from the center of the mill to a destination spot (figure 5(a)). When the frequency is switched back to 10 Hz, LCEEP becomes active again, dismantling the assembly as particles are driven towards the destination spot. Both order parameters rapidly exchange values raising the value of $P$ and reducing the rotational motion (thus $M$). The particles move at a constant speed towards the translocated topological defect, raising the value of $P$ to unity, while $M$ vanishes. After 150 s, enough particles have gathered around the destination spot for the mill pattern to form again, thus increasing the corresponding value of $M$ and reducing the degree of alignment of the propelled particles, thus lowering $P$.

In order to reproduce numerically the experimental situation illustrated in figure 5, we perform a procedure similar to the experimental one. In particular, the operation of transferring one defect from one initial position to a new one at a certain distance is obtained by switching from one spiral pattern to another one with a shifted position after a time $t_1$

$$ n(r, t) = \begin{cases} n(\psi_1(r - q_1)), & 0 \leq t < t_1, \\ n(\psi_2(r - q_2)), & t \geq t_1, \end{cases} \quad (9) $$

where the two patterns are determined by the functions $\psi_1$ and $\psi_2$ and $q_1$ and $q_2$ specify their locations. The two patterns in equation (9) are generated using equation (A.7) as shown in figure 6(a).
We then simulate the translocation process using equation (1) for \( N = 1000 \) particles in a simulation domain corresponding to a rectangular box, \( 0 \leq x \leq L_x, 0 \leq y \leq L_y \) with \( L_x = 500 \mu m \) and \( L_y = 675 \mu m \). Spiral patterns 1 and 2 in equation (9) are characterized by the functions \( \psi_1 \) with \( \alpha_1 = \pi/6, \alpha_2 = \pi/4 \) located at \( q_1 = (250 \mu m, 250 \mu m) \) and \( \psi_2 \) with \( \alpha_1 = \pi/6, \alpha_2 = -\pi/10 \) located at \( q_2 = (415 \mu m, 250 \mu m) \), respectively. Other parameters are kept the same, as for the case of mill.

Figure 6(b) shows the results of the numerical simulations (see also Video5 in the supporting information) with the corresponding calculated order parameters \( P \) and \( M \) shown in figure 6(c). At time \( t = 0 \), the particles are uniformly distributed within a circle of radius 200 \( \mu m \) centered at (200 \( \mu m, 200 \mu m \)). They start to move along the spiral orbits towards the center of attraction, \( q_1 \), forming a rotating mill. During this process, both parameters, \( P \) and \( M \), are non-vanishing and their evolution have opposite tendencies following the previously identified trends.

The switch from pattern 1 to pattern 2 occurs at \( t = t_1 = 50 \) s, when the behavior of \( P \) and \( M \) displays an abrupt jump caused by the instant shift of the ‘center of attraction’ and particle propulsion towards position 2. In contrast to this instantaneous switch, reconstruction of the destination mill is gradual, in analogy with the experiments, as can be seen from the smooth decrease in \( P \) and increase in \( M \). Eventually, the center of mass of particles nearly reaches the position of the spiral pattern, \( q_2 \), and we observe another rotating mill. This state approximately corresponds to \( P \approx 0 \) and \( M \approx 1 \).

7. Conclusions

In this article we have demonstrated a method to trap and transport a large ensemble of anisometric colloids dispersed in a nematic liquid crystal matrix and driven by nonlinear electrophoresis. The particles are propelled above a photosensitive surface where a topological defect generating a spiral pattern is created or erased by light. The experiments are combined with a theoretical model that calculates the pattern topology and explains the observed dynamic states and how they depend on the underlying pattern orientation. While our colloidal swarm was transported along a simple track, namely a line connecting two topological defects, in principle more complex patterns may be easily designed to steer the colloidal motion. The advantages of this method over other strategies to propel microscale matter in viscous fluids are the possibility to control the mean molecular orientation of the dispersing medium using an external field, and the independent control of the surface topology through optical means. These two external fields are uncoupled and can be easily interchanged during the experiments, a feature that gives further functionality to the transport in colloidal systems.

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Appendix. Plane spiral patterns of the director field in a nematic liquid crystal

To model mill patterns, here we are interested in finding spiral solutions of the director field that are consistent with the continuum theory of liquid crystals and at the same time can be adjusted to render the experimental conditions. A simple 'magic spiral' solution for a nematic liquid crystal confined by two concentric cylinders of radii $r_1 < r_2$ was obtained by Parodi, see [46]:

Figure 6. (a) Top (Bottom) image shows the vector field of the spiral pattern obtained for $\alpha_1 = \pi/6, \alpha_2 = \pi/4 (\alpha_1 = \pi/6, \alpha_2 = -\pi/10)$. Both vector fields cover the same area ($500 \times 675 \mu m^2$) and are exchanged in the simulation after $t_1 = 50$ s. (b) Sequence of images from numerical simulation showing the translocation process for $N = 1000$ particles. First ($t < t_1$) a colloidal mill is assembled above the pattern shown in the top figure 6(a), and later ($t > t_1$) is translated by switching the underlying pattern, bottom figure 6(a). Scale bars for all the images are 20 $\mu$m. See corresponding Video5 in the supporting information. (c) Corresponding normalized order parameters $M$ and $P$ versus time.

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\[
\psi(\rho) = \frac{\pi}{2} \ln\left(\frac{\rho / n}{\rho / n}\right) 
\]

(A.1)

showing a monotonic change of the angle made by the director with the radial direction. It is based on three basic assumptions: (i) the director field is restricted to the plane normal to the axes of the cylinders; (ii) the anchoring is homeotropic (normal director) at the inner cylinder and planar (tangential director) at the outer one; (iii) the one-constant approximation, implying that the splay, \( K_S \), and bend, \( K_B \), constants are equal.

Later, Williams went beyond the one-constant approximation and also extended the analysis to an arbitrary alignment at the outer boundary but kept it homeotropic at the inner one [47]. However, to achieve a reasonable agreement with the experiment, it is crucial for us to be able to arbitrarily adjust the alignment of the director field also at the inner border. Furthermore, the radial dependence of the angle \( \psi(\rho) \) can be non-monotonic even at slightly different constants, which may become important for describing spiral patterns at large distances. Therefore, for the sake of generality, we will stick to the case \( K_S = K_B \) and adapt the discussed solutions.

We start by formulating the Frank–Oseen free energy of the nematic liquid crystal. For the two-dimensional case as in assumption (i) above \((n_z = 0)\), the corresponding energy expressed as a function of the director field \( \mathbf{n} = \mathbf{n}(r) \) and written per unit length along the cylinder is

\[
F = \frac{1}{2} \int_0^{r_0} \mathbf{d}r \{K_S(\nabla \cdot \mathbf{n})^2 + K_B[\mathbf{n} \times (\nabla \times \mathbf{n})]^2\}. 
\]

(A.2)

We now proceed to the polar coordinates \((\rho, \theta)\) and represent the director field as \((n_{\rho}, n_{\theta}) = (\cos \psi, \sin \psi)\) with \(\psi = \psi(\rho)\). Using this ansatz in equation (A.2), integrating it over the polar angle \(\theta\) and introducing the combinations of elastic constants, \(K_\pm = (K_S \pm K_B)/2\), we obtain

\[
F = \pi \int_0^{r_0} \rho d\rho \left[K_+ \left(\psi_{\rho}^2 + \frac{1}{\rho^2}\right) - K_- \left(\psi_{\rho}^2 \cos 2\psi - \frac{2\psi_{\rho} \sin 2\psi}{\rho} - \cos 2\psi\right)\right]. 
\]

This expression is simplified in terms of a rescaled radial coordinate, \(s(\rho) = \ln(\rho/r_1)\), resulting in

\[
F = \pi \int_0^{s_1} ds \left[K_+(s^2 + 1) - K_- (\psi_{s}^2 \cos 2\psi - 2\psi_{s} \sin 2\psi - \cos 2\psi)\right], 
\]

(A.3)

where \(s(r_1) = 0\) and \(s(r_2) = \ln(r_2/r_1)\). Here, the subscripts ‘\(\rho\)’ and ‘\(s\)’ denote the corresponding derivatives.

Applying a standard procedure to minimize the energy functional (A.3), we arrive at an ordinary differential equation for \(\psi(s)\)

\[
(K_+ - K_- \cos 2\psi)\psi_{ss} + K_+ \psi_s^2 \sin 2\psi + K_- \sin 2\psi = 0, 
\]

(A.4)

which is supplemented by the boundary conditions

\[
\psi(0) = \alpha_1, \quad \psi(s_2) = \alpha_2. 
\]

(A.5)

The values \(\alpha_1\) and \(\alpha_2\) determine the arbitrary alignments of the director field at the inner \((\rho = r_1)\) and outer \((\rho = r_2)\) borders, respectively.

The boundary value problem described by equations (A.4) and (A.5) admits two simple fundamental solutions. One is given by \(\psi(s) = \alpha_1 = \alpha_2 = 0, \pi\). The sink-like pattern with \(\psi(s) = \pi\) corresponds to an aster, \(\mathbf{n} = (n_{\rho}, n_{\theta}) = (-1, 0)\), while the source-like pattern \(\psi(s) = 0\) describes an antiaster, \(\mathbf{n} = (n_{\rho}, n_{\theta}) = (1, 0)\). Another important solution is given by \(\psi(s) = \alpha_1 = \alpha_2 = \pm\pi/2\), which corresponds to a vortex, \(\mathbf{n} = (n_{\rho}, n_{\theta}) = (0, \pm 1)\), where the upper and lower signs are for counter- and clockwise directions of rotation, respectively. Because of nonlinearity of equation (A.4), it admits no exact analytic solution for a spiral pattern and should be obtained numerically.

To make an analytical progress, we resort to the approximation of weak distortion, \(\psi \ll 1\). By retaining the leading terms only, we end up with the linear ordinary differential equation

\[
\psi_{ss} + k^2 \psi = 0, \quad k^2 = \frac{2K_-}{K_+ - K_-} = \frac{K_S - K_B}{K_B}. 
\]

(A.6)

Solving equation (A.6) for the case \(K_S > K_B\), as in the experiment, and satisfying boundary conditions, equation (A.5), yields

\[
\psi(\rho) = \alpha_1 \cos(ks(\rho)) + \frac{\alpha_2 - \alpha_1 \cos k_2}{\sin k_2} \sin(ks(\rho)) \quad (K_S > K_B). 
\]

(A.7)

Note that the solution in the opposite case of \(K_S < K_B\) is given by equation (A.7) with trigonometric functions replaced by their corresponding hyperbolic counterparts. The special case of \(\alpha_1 = 0\) is in agreement with the results of Williams [47].
The partial case of one-constant approximation, $K_S = K_B$, follows from equation (A.7) in the limit $k \to 0$, 
\[
\psi(\rho) = \alpha_1 + (\alpha_2 - \alpha_1) \frac{\ln(\rho/\eta)}{\ln(r_2/\eta)} \quad (K_S = K_B),
\]  
which can alternatively be figured out directly from equation (A.4) for $K_S = 0$. Again, the special case of $\alpha_2 = 0$ and $\alpha_1 = \pi/2$ is in agreement with the results by Parodi [46], see equation (A.1). We finally stress that in contrast to one-constant approximation solution (A.8) with a strictly monotonic dependence of $\psi(\rho)$ for $0 \leq \alpha_1$, $\alpha_2 \leq \pi/2$, $r_1 \leq \rho \leq r_2$, the corresponding solution (A.7) is more general, leading to periodic radial undulations of the director field for $K_S > K_B$.

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