MPC-Based Trajectory Tracking Via Active Front Steering and External Yaw Moment

XIAO WANG, JIBIN HU and JUN NI

ABSTRACT

In this paper, we propose a trajectory tracking method based Model Predictive Control (MPC) by utilizing steering and yaw moment. The main control objective is to track the desired trajectory with the front steering angle and yaw moment. And an external logic computes the torques at the four wheels such that the yaw moment is achieved, at the same time the logic controls the tire slip ratio around 0.15. Finally, the effectiveness of the proposed approach is demonstrated through the simulations at low and high speed.

INTRODUCTION

Automatic vehicle has broad development prospect in the future intelligent transportation system. Its research has gradually entered the stage of rapid development which forms research fields including perception and navigation, autonomous decision-making, path planning and trajectory tracking. And trajectory tracking is closely related to vehicle dynamics, and is one of the key technologies to realize unmanned driving [1].

At first, the trajectory tracking research focuses on using the front steering angle to follow the desired trajectory. Researchers adopt a variety of control methods to compute the front steering angle. For instance, in [2] the authors present controllers design procedure for dynamic trajectory tracking of a highly automated vehicle. Longitudinal and lateral controller are decoupled. The lateral controller is based on adaptive backstepping approach to design a steering control law for purpose of trajectory tracking, at the same time considering the model nonlinearities and parameter uncertainties. In [3-4] the authors adopt $H_{\infty}$, MPC control method to control the front steering angle to achievement trajectory tracking. There are also some examples [5-8] in the “Grand Challenge” race using the front steering angle to following the trajectory.

Current research efforts in trajectory tracking focus on combining the front steering angle and braking to control the vehicle to achieve trajectory tracking more effectively and steadily. For instance, in [9] the added control variable is braking torques in one of the four wheels. The authors use direct Lyapunov method to compute the front steering angle and braking torques, then log the calculated data on vehicle dynamic model to achieve a double lane change maneuver. In [10] the authors use the Nonlinear MPC formulation to track a desired path for obstacle avoidance maneuver, by a combined use of braking and steering. More examples are shown in [11-13].

Xiao Wang, Jibin Hu, Jun Ni, Beijing Institute of Technology, Beijing, China
In this paper, the research on trajectory tracking is achieved by combining front steering angle and direct yaw moment. We assume that the trajectory planning system is available and we focus on the control of the trajectory tracking via front steering angle and direct yaw moment. The reference trajectory is a double lane with a continuously varying curvature. We design a linear time-varying MPC controller in MATLAB/Simulink with S-function module to calculate the front steering angle and direct yaw moment the vehicle needed to track the reference trajectory.

The rest of this paper is organized as follows. In Section 1, we present the vehicle dynamic model and the tire model we used in MPC controller formulated in Section 2. In Section 3, we present the torque calculation module and simulation results obtained with a combined front steering angle and yaw moment MPC controller. At last, the conclusion and future work are followed in Section 4.

MODELING

This section describes the vehicle and tire model used for simulations and controller design. The simplified vehicle dynamic model is shown in Figure 1. In this paper, OXY is the inertial coordinates fixed in ground. \( \text{oxy} \) is the coordinates fixed in vehicle. \( m \) is vehicle mass(950kg). \( I_z \) is vehicle yaw moment of inertia (4426.94kg\( \cdot \)m\(^2\)). \( CG \) is the center of mass. \( a \) and \( b \) are the distance of CG from front axle(1m) and rear axle(1m) respectively. \( \phi \) is the heading angle. \( M \) is external yaw moment. \( \delta \) is the wheel steering angle. \( v_c \) is lateral wheel velocity. \( v_l \) is longitudinal wheel velocity. \( v_x \) is vehicle longitudinal velocity. \( v_y \) is vehicle lateral velocity. \( s \) is slip ratio. \( \alpha \) is tire slip angle. \( \mu \) is road friction coefficient. \( R_e \) is the wheel radius(0.44m). \( \omega_r \) is wheel angular speed. \( F_b \) is longitudinal tire forces \( (i = f, r) \). \( F_{ci} \) is lateral tire forces \( (i = f, r) \). \( F_{xi} \) is the forces along the longitudinal vehicle axes \( (i = f, r) \). \( F_{yi} \) is the forces along the lateral vehicle axes \( (i = f, r) \). \( f \) and \( r \) are subscripts that denote front and rear wheel.

Vehicle Model

As shown in Figure 1, the longitudinal motion, lateral motion and yaw motion can
be described by

\[ m\ddot{x} = m\ddot{y} + 2F_{yf} + 2F_{yr} \quad (1) \]

\[ m\ddot{y} = -m\ddot{x} + 2F_{xf} + 2F_{yr} \quad (2) \]

\[ I_{z}\ddot{\phi} = 2aF_{xf} - 2bF_{yr} + M \quad (3) \]

The vehicle’s equation of motion in absolute inertial frame are

\[ \dot{Y} = \dot{x} \sin \phi + \dot{y} \cos \phi \]

\[ \dot{X} = \dot{x} \cos \phi - \dot{y} \sin \phi \quad (4) \]

The longitudinal and lateral tire forces lead to the following components along the lateral and longitudinal vehicle axes:

\[ F_{xf} = F_{lf} \cos \delta_f - F_{cf} \sin \delta_f \]
\[ F_{xf} = F_{lf} \sin \delta_f + F_{cf} \cos \delta_f \]
\[ F_{xr} = F_{lr} \cos \delta_r - F_{cr} \sin \delta_r \]
\[ F_{yr} = F_{lr} \sin \delta_r + F_{cr} \cos \delta_r \quad (5) \]

The longitudinal and lateral tire forces is a complicated function of tire slip angle \( \alpha \), slip ratio \( s \), road friction coefficient \( \mu \) and vertical tire force \( F_z \).

\[ F_{l} = f_l(\alpha, s, \mu, F_z) \]
\[ F_{c} = f_c(\alpha, s, \mu, F_z) \quad (6) \]

The slip angle is the angle between the wheel velocity and the direction of the wheel itself:

\[ \alpha = \arctan \frac{v_{c}}{v_{l}} \quad (7) \]

The lateral wheel velocity \( v_c \) and longitudinal wheel velocity \( v_l \) can be expressed as

\[ v_l = v_y \sin \delta + v_x \cos \delta \]
\[ v_c = v_y \cos \delta - v_x \sin \delta \quad (8) \]

And vehicle longitudinal velocity \( v_x \) and vehicle lateral velocity \( v_y \) can be expressed as
\begin{align*}
\dot{v}_y &= \dot{y} + a\dot{\phi} & \dot{v}_r &= \dot{y} - b\dot{\phi} \\
\dot{v}_{sy} &= \dot{x} & \dot{v}_{sr} &= \dot{x}
\end{align*}
\text{(9)}

The slip ratio \( s \) can be described by

\[
s = \begin{cases} 
-1 & (v_i > R_i \omega_i, \omega_i \neq 0) \\
1 - \frac{v_i}{R_i \omega_i} & (v_i < R_i \omega_i, \omega_i \neq 0)
\end{cases}
\text{(10)}
\]

We assume that the vertical tire forces is only effected by the location of the CG.

\[
F_{sf} = -\frac{bmg}{2(a+b)} \\
F_{sr} = -\frac{amg}{2(a+b)}
\text{(11)}
\]

Using the Equations (1.1) ~ (1.11), we can get the nonlinear vehicle dynamic model:

\[
\dot{\xi}(t) = f_{\mu(t)}(\xi(t), u(t))
\text{(12)}
\]

Where the state vector is \( \xi = [\dot{y}, \dot{x}, \phi, \dot{\phi}, Y, X] \), input vector is \( u(t) = [\delta_f, M] \) (we only consider the front steering angle, the rear steering is zero).

**Tire Model**

Except for aerodynamic force and gravity, all of the forces affecting vehicle handing stability are from the tires. Therefore it is important to use a practical and user-friendly tire model. In this paper, we choose Pacejka tire model to describe the tire’s nonlinear dynamic character. This is a complex, semi-empirical nonlinear model that fits the concrete expression based the experiment data. It uses formula of trigonometric functions to describe the longitudinal tire force and lateral tire force and also consider the interaction between the longitudinal and lateral tire force in combined braking and steering. Moreover the parameters in Pacejka tire model have definite physical meanings, so it is convenient to observe the tire character in real time and analyze the vehicle dynamic performance.

**MODEL PREDICTIVE CONTROL PROBLEM**

The real-time performance of the control algorithm is important for unmanned vehicle motion control. Compared with nonlinear MPC, the linear time-varying MPC
is real time and relatively simple to calculate. Therefore we choose linear time-varying model as the prediction model.

In order to obtain a linear time-varying model, we linearize and discretize the model (12) with a fixed sampling time $T_s$

$$
\xi(t+1) = f(\xi(t), u(t)) \\
u(t) = u(t-1) + \Delta u(t)
$$

(13)

Where $u(t)=[\delta f, M]$ and $\Delta u(t)=[\Delta \delta f, \Delta M]$.

We define the following output map:

$$
\eta(t) = h(\xi(t)) = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0
\end{bmatrix} \xi(t)
$$

(14)

And consider the following cost function:

$$
J(\xi(t), \Delta U_t) = \sum_{i=0}^{N_p} \left\| \eta_{t+i,t} - \eta_{ref,i,t} \right\|_Q^2 + \sum_{i=0}^{N_c-1} \left\| \Delta u_{t+i,t} \right\|_R^2 + \rho \epsilon^2
$$

(15)

Where, as in standard MPC notation ([4]), $\Delta U_t = [\Delta u_{t,t}, \Delta u_{t+1,t}, \ldots, \Delta u_{t+N_c-1,t}]$ is the optimization vector at time $t$, $\eta_{t+i,t}$ is the output vector predicted at time $t+i$ obtained by state $\xi_{t,t}$ and applying to the system (2.1) and (2.2) the input sequence $\Delta u_{t,t}$, $\Delta u_{t+1,t}, \ldots, \Delta u_{t+N_c-1,t}$, $\eta_{ref}$ denote the corresponding reference signal. $N_p$ is the output prediction horizon and $N_c$ is the control horizon. Like standard MPC scheme, we use $N_p > N_c$ and when $N_c < i <= N_p$, $\Delta u_{t+i,t}=0$. In (2.3), the first summand reflects the requirement to track the reference trajectory, the second summand reflects the requirement of control quantity’s steady change and the third summand is relaxation factor which is to ensure every time has a feasible solution. $Q$, $R$ and $\rho$ are weighting matrices. The whole function is ensure the vehicle system to track the desired trajectory quickly and steadily.

At each time step $t$, we solve the following finite horizon optimal control problem:

$$
\min_{\Delta U_t} J(\xi(t), \Delta U_t)
$$

(16a)

Subj. to
\[\ddot{\xi}_{k+1,t} = f^d_{\mu_{k,t}}(\dot{\xi}_{k,t}, u_{k,t}), k = t, \ldots, t + N_p - 1\]
\[u_{k,t} = u_{k-1,t} + \Delta u_{k,t}, k = t, \ldots, t + N_c - 1\]
\[\eta_{k,t} = h(\xi_{k,t}), k = t+1, \ldots, t + N_p\]
\[\xi_{t,t} = \xi(t)\]
\[u_{t-1,t} = u(t-1)\]
\[\mu_{k,t} = \mu_{t,t}, k = t, \ldots, t + N_p\]
\[u_{f,\min} \leq u_{k,t} \leq u_{f,\max}, k = t, t+1, \ldots, t + N_c - 1\]
\[\Delta u_{f,\min} \leq \Delta u_{k,t} \leq \Delta u_{f,\max}, k = t, t+1, \ldots, t + N_c - 1\]

We define that \(\Delta U_t^* = [\Delta u_{t,t}^*, \Delta u_{t+1,t}^*, \ldots, \Delta u_{t+N_c-1,t}^*]\) is the sequence of optimal steering angle and yaw moment increments computed at time \(t\) by solving the problem (2.4) for the current observed state \(\xi(t)\) and the previous input \(u(t-1)\). According to the basic principle of the MPC, the first sample of \(\Delta U_t^*\) apply to the vehicle system as the actual control increment:

\[u(t) = u(t-1) + \Delta u_{t,t}^*\] (17)

The system executes the controlled variable until the next moment. At the next moment, the system predicts the output of the next time based on the state information and obtain a new control increment sequence through the optimization process. Cycle until the system completes the control process.

**SIMULATION**

In this paper, we use a double lane [10] as the desired trajectory and the simulation environment is Simulink. And the simulation model is 7 degrees of freedom (longitudinal, lateral, yaw and the rotational of four wheels) vehicle dynamic model. The tire model uses the aforementioned Pacejka tire model and the model predictive controller built with S function computes the front steering angle and the yaw moment needed for vehicle to track the reference trajectory. Then we introduce the driving force distribution method of four wheels.

\[\sum F_x = (F_{x11} + F_{x12}) \cos \delta_f + F_{x21} + F_{x22}\] (18)

\[M = (F_{x11} + F_{x12}) \sin \delta_f * a + (-F_{x11} \cos \delta_f + F_{x12} \cos \delta_f - F_{x21} + F_{x22}) \frac{B}{2}\] (19)

\[F_{x11} : F_{x21} = F_{z11} : F_{z21}\] (20)

\[F_{x12} : F_{x22} = F_{z12} : F_{z22}\] (21)
Where $F_{x11}$, $F_{x12}$, $F_{x21}$, $F_{x22}$ represent the longitudinal tire forces of the front left tire, the front right tire, the rear left tire, and the rear right tire, respectively. As the same way, $F_{z11}$, $F_{z12}$, $F_{z21}$, $F_{z22}$ represent the vertical tire forces of the front left tire, the front right tire, the rear left tire, and the rear right tire, respectively. And B is the wheel-track. $\sum F_x$ is the resultant of the longitudinal force computed by the PID to track the desired speed. In addition, the torques at the four wheels are controlled by sliding mode control.

$$T = F_x R - K_s sat\left(\frac{s - s_d}{\Psi}\right)$$  \hspace{1cm} (22)

Where $T$ is torques at wheel, $F_x$ is longitudinal force of the tire, $K_s$ is a coefficient, $s$ is the tire slip ratio, $s_d$ is desired tire slip ratio which we set 0.15, $\Psi$ is the thickness coefficient namely a margin condition, $sat$ is a saturation function whose output is 0-1.

Next we present the simulation result of MPC controller at 15m/s and 20m/s in Figure 2 and Figure 3 respectively. The yaw angle(upper left), the yaw rate(upper middle), the lateral vehicle position(upper right), the front steering angle(lower left) and the yaw moment(low right) is show below. And the red dotted line represents the desired values and the blue solid line represents the simulation results.

We can see the very good tracking performance from the simulation results in Figure 2 at 15m/s. From the Figure 2d and 2e we can see the front steering angle and the yaw moment cooperates well to follow the desired trajectory. At approximatively 2s, the vehicle turn left. In the same time, the controller outputs a positive steering angle and a positive yaw moment. After 6s, the vehicle begins to turn right, in the same time the controller outputs a decreasing yaw moment and a oscillating but leveling off steering angle to track the double lane.

In Figure 3 the simulation results at 20m/s are shown. We can see the tracking errors are larger than the errors at 15m/s. The reason is that the higher the speed is, the harder the vehicle is to control.
CONCLUSION

In this paper, we presented a combined steering and yaw moment MPC controller based on vehicle dynamic model. The controller computes the front steering angle and the yaw moment the vehicle needed to track the desired trajectory. And an external logic computes the torques at the four wheels such that the desired yaw moment is achieved. Finally, we showed the controller has good tracking performance at both low and high speed by simulation results. From the simulation results we can see the tracking errors become larger as the vehicle speed higher, improving the control effect in high speed is the following work in the future.

REFERENCES

1. TANG. 2011. “Kinematics Analysis of Motion Simulation Subsystem for Unmanned Vehicle,” J. Chinese Journal of Mechanical Engineering, 24(6):923-934.
2. Hima S. 2011. “Controller design for trajectory tracking of autonomous passenger vehicles,” in *International IEEE Conference on Intelligent Transportation Systems*, Glaser S, Chaibet A, et al, eds. Washington, DC: IEEE, pp. 1459-1464.

3. Wang R, Jing H, Hu C, et al. 2016. “Robust $H_{\infty}$ Path Following Control for Autonomous Ground Vehicles With Delay and Data Dropout,” *J. IEEE Transactions on Intelligent Transportation Systems*, 17(7):1-9.

4. Paolo Falcone, Francesco Borrelli, Jahan Asgari, et al. 2007. “Predictive Active Steering Control for Autonomous Vehicle Systems,” *J. IEEE Transactions on Control Systems Technology*, 15(3): 566-580.

5. Werling M, Gröll L, Brethauer G. 2010. “Invariant Trajectory Tracking With a Full-Size Autonomous Road Vehicle,” *J. IEEE Transactions on Robotics*, 26(4):758-765.

6. T. B. Foote, L. B. Cremean, J. H. Gillula, G. H. Hines, D. Kogan, K. L. Kriechbaum, J. C. Lamb, J. Leibs, L. Lindzey, C. E. Rasmussen, A. D. Stewart, J. W. Burdick, and R. M. Murray. 2006. "Alice: An information-rich autonomous vehicle for high-speed desert navigation," *J. Journal of Field Robot.*, 23(9): 777-810.

7. Montemerlo M. 2006. “Winning the DARPA Grand Challenge with an AI Robot,” in *National Conference on Artificial Intelligence and the Eighteenth Innovative Applications of Artificial Intelligence Conference*, Thrun S, Dahlkamp H, et al, eds. Boston: Aai’, pp. 1-6.

8. S. Thrun, M. Montemerlo, H. Dahlkamp et al. 2006. “Stanley: The Robot that Won the DARPA Grand Challenge,” *J. Journal of Field Robotics*, 23(9): 661-692.

9. Nenggen Ding and Saied Taheri. 2010. “An adaptive integrated algorithm for active front steering and direct yaw moment control based on direct Lyapunov method,” *J. Vehicle System Dynamics*, 48(10):1193-1213.

10. Paolo Falcone, H. Eric Tseng, Francesco Borrelli, Jahan Asgari and Davor Hrovat. 2008. “MPC-based yaw and lateral stabilisation via active front steering and braking,” *J. Vehicle System Dynamics*, 46(S1): 611-628.

11. E. Ono, Y. Hattori, Y. Muragishi and K. Koibuchi. 2006. “Vehicle dynamics integrated control for four-wheel-distributed steering and four-wheel-distributed traction/braking systems,” *J. Vehicle System Dynamics*, 44(2): 139-151.

12. Huang X, Zhang H, Zhang G, et al. 2014. “Robust Weighted Gain-Scheduling ($H_{\infty}$) Vehicle Lateral Motion Control With Considerations of Steering System Backlash-Type Hysteresis,” *J. IEEE Transactions on Control Systems Technology*, 22(5):1740-1753.

13. Hui Zhang, Xinjie Zhang and Junmin Wang. 2014. “Robust gain-scheduling energy to-peak control of vehicle lateral dynamics stabilisation,” *J. Vehicle System Dynamics*, 52(3): 309-340.