Abstract—The deluge of date rate in today’s networks poses a cost burden on the backhaul network design. Designing cost efficient backhaul solutions becomes an interesting, yet challenging, problem. Traditional technologies for backhaul networks include either radio-frequency backhauls (RF) or optical fibers (OF). While RF is a cost-effective solution as compared to OF, it supports lower data rate requirements. Another promising backhaul solution that may combine both a high data rate and a relatively low cost is the free-space optics (FSO). FSO, however, is sensitive to nature conditions (e.g. rain, fog, line-of-sight, etc.). A more reliable alternative is to combine RF and FSO solutions through a hybrid structure called hybrid RF/FSO. Consider a backhaul network, where the base-stations (BS) can be connected to each other via OF or via hybrid RF/FSO backhaul links. The paper addresses the problem of minimizing the cost of backhaul planning under connectivity and data rates constraints, so as to choose the appropriate cost-effective backhaul type between BSs (i.e., either OF or hybrid RF/FSO). The paper solves the problem using graph theory techniques by introducing the corresponding planning graph. It shows that under a specified realistic assumption about the cost of OF and hybrid RF/FSO links, the problem is equivalent to a maximum weight clique problem, which can be solved with moderate complexity. Simulation results show that our proposed solution shows a close-to-optimal performance, especially for practical prices of the hybrid RF/FSO.

Index Terms—Network planning, optical fiber, free-space optic, backhaul network design, cost minimization.

I. INTRODUCTION

Cellular networks, flooded by a large demand for mobile data services, are expected to undergo fundamental transformation. In order to significantly increase the data capacity, coverage performance, and energy efficiency, the next generation mobile networks (5G) are expected to move from the traditional single, high-powered base-station (BS) to the deployments of multiple overlaying access points of diverse sizes (i.e. microcell, picocell, femtocell, etc.) using different radio access technologies. The resulting network system architecture is referred to as heterogeneous networks (HetNets). Inter and intra-cell interference represents a major cause of the degradation of the HetNets performance. A successful interference mitigation architecture is the so-called heterogeneous cloud radio access network architecture obtained by connecting BSs from different tiers to a cloud (central processor) through backhaul links. The cloud largely improves the performance of HetNets; however, it necessitates a considerable amount of backhaul communications in order to share the data streams between all BSs across the network. Giving that the links are capacity limited, upgrading the backhaul and increasing its capacity to support the tremendous amount of data is a necessity.

Traditional technologies for the backhaul network design include copper, microwave radio links (RF), and optical fibers (OF). The price of copper lines, the most widely used backhaul technology, is growing linearly with the provided capacity. For high data rates, copper lines become expensive. Microwave radio technologies are also used for the backhaul network design. However, they represents only 6% of the total used transport media due to the initial investment in the licensed part of the spectrum. Moreover, low frequency radio is limited in terms of data-rates due to interference problems and high frequency radio are limited in the transmission coverage area.

Optical fiber (OF) backhaul links provide high data-rates over long distances. However, they are expensive to be deployed and require considerable initial investment. Recently, the free-space optics technology (FSO) becomes an interesting substitute for the next generation cellular backhaul networks design. Generally, an FSO link refers to a laser beam between a pair of photo-detector transceivers using the free-space as medium of transportation. Giving that its wavelength is in the micrometer range, which is an unlicensed band, FSO links not only are free to use but they are also immune to electromagnetic interference generated by the radio-frequency (RF) links. The high bandwidth and interference immunity features make an FSO link up to 25 times more effective than a RF link in terms of capacity. In addition, FSO represents a cost effective solution compared to OF.

Unlike OF links that are always reliable, FSO links are less reliable since they are sensitive to weather conditions, such as fog, snow, and rain. Therefore, reliability should be taken into account when designing FSO-based backhaul networks. In order to cope with the varying reliability and combine the advantages of RF (reliability) and FSO (capacity), the hybrid RF/FSO technology has been proposed. Hybrid RF/FSO transmits, when possible, simultaneously on both the RF and FSO links. In harsh weather conditions, in which the FSO link is affected, the data are transmitted solely on the RF link. Moreover, hybrid RF/FSO transceivers can be
quickly deployed over several kilometers [9] and can also be easily combined with OF [10]–[12]. All these benefits make hybrid RF/FSO a suitable complementary option for upgrading the existing backhaul network [7], [13].

In the past few years, hybrid RF/FSO has attracted a large amount of research. Most of the existing work focused on the determination of the factors affecting the FSO link performance and finding solutions to improve the quality [13], [14]. However, fundamental problems of hybrid RF/FSO architecture optimization for the backhaul network topology design are only at their beginning.

The authors in [15] designed an efficient and scalable algorithm to optimize a given physical layer objective for 2 and 3 optical transceivers per node with a minimum number of links. Kashyap et al. [6] designed a routing algorithm for hybrid RF/FSO networks that backs up the traffic to the FSO routes when it could not be carried by the RF links. In [16], the authors considered an hybrid RF/FSO network in which the RF and FSO links operate at different data rates. They derived an upper bound for the capacity per node that is asymptotically achievable for random networks. Rak et al. [8] introduced a linear integer programming model to determine routing in hybrid RF/FSO network in which the FSO link availability is varying with the weather conditions.

Numerous mixed integer programming model have been proposed to formulate the problem of backhaul network design using hybrid RF/FSO technology. In particular, Son et al. [17] presented an algebraic connectivity-based formulation for the design of the backbone of wireless mesh networks with FSO links and solved it using a greedy approach that iteratively inserts nodes to maximize the algebraic connectivity. The authors in [18] proposed to maximize the network throughput by installing as many FSO links as possible under the constraint that the number of FSO links in a node is bounded. In [19], Ahidi et al. introduced a mixed integer programming model to find the optimal placement of FSO links in order to upgrade an existing RF backhaul network. Similarly, reference [20] propose to upgrade an existing RF backhaul network with FSO links using the minimum number of FSO links to guarantee a target network throughput when RF links are non available due to interference. The authors in [11] considered the upgrade of a pre-deployed OF backhaul network using FSO links and mirrors for nodes not in line-of-sight of each other. For two link-disjoint paths networks, they formulated the problem as a mixed integer programming and extended the study in [21] to K link-disjoint paths. In [12], the same group of authors studied the impact of the parameter K on the optimal solution.

In this paper, we consider the problem of minimizing the network deployment cost under target rate constraints, so as to determine which type of connections exists between each pair of BSs, i.e., OF, hybrid RF/FSO, or none. A major concern is to guarantee network connectivity, which is achieved by connecting, each pair of nodes in the network, possibly via multiple hops. While the deployment cost of hybrid RF/FSO links depends mainly on the cost of the hybrid RF/FSO transceivers, the deployment cost of OF links depends mostly on the distance between the two end nodes. On the other hand, OF links always satisfy the data rate constraint. The performance of hybrid RF/FSO, however, degrades with the distance and the number of installed links. The paper solves the problem using graph theory techniques by introducing the corresponding planning graph. The paper main contribution is to provide a close to optimal explicit solution to the problem. The paper shows that under a specified realistic assumption about the cost of OF and hybrid RF/FSO links, the problem can be reformulated as a maximum weight clique problem, which can be globally solved using efficient algorithms [22].

The rest of this paper is organized as follows: Section II presents the considered system model and the problem formulation. Section III illustrates the proposed solution. Before concluding in Section V, simulation results are presented in Section IV.

II. SYSTEM MODEL AND PROBLEM FORMULATION

A. System Model and Parameters

In this paper, we consider a backhaul network connecting a set \( B = \{b_1, \cdots, b_M\} \) of \( M \) base stations. We assume that all nodes (interchangeably denoting base-stations in this paper) have line-of-sight connections. Note that, with minor modifications, the results claimed in this paper apply to a network with pre-deployed OF links (scenario of [11], [12], [21]). However, due to space limitation, we will only focus on networks with no existing links. Each node can be connected to any other node with either an OF or a hybrid RF/FSO connection, as in Figure 1 which shows a network containing 5 base-stations.

Let \( d(b, b') \) be the distance operator between any two nodes. In other words, \( d(b, b') \) is the distance between the nodes \( b \) and \( b' \), \( \forall (b, b') \in B^2 \). Let \( \pi^{(O)}(x) \) be the cost of an OF link and \( \pi^{(h)}(x) \) the cost of a hybrid RF/FSO link as a function of distance \( x \). We have \( \pi^{(O)}(0) = \pi^{(h)}(0) = 0 \). Both functions \( \pi^{(O)}(x) \) and \( \pi^{(h)}(x) \) are increasing functions of the distance \( x \). Moreover, since hybrid RF/FSO is a cost effective solution, we assume that \( \pi^{(h)}(x) \leq \pi^{(O)}(x), \forall x \geq d^* \), where \( d^* \) is

![Fig. 1. Network containing 5 base-stations connected together with OF and hybrid RF/FSO links.](image-url)
the minimum distance between two base-stations. Figure 2 represents an example of such cost of OF and hybrid RF/FSO links with respect to the distance.

B. Problem Formulation

This paper considers the problems of minimizing the network deployment cost under the following constraints:

1) Connections between nodes can be either OF or hybrid RF/FSO.
2) Each node has a data rate that exceeds the target data rate.
3) Each node can communicate with any other node through single or multiple hop links. In other words, the graph is connected.

Let \( X_{ij}, 1 \leq i, j \leq M \) be a binary variable indicating if base stations \( b_i \) and \( b_j \) are connected with an OF connection. Similarly, let \( Y_{ij}, 1 \leq i, j \leq M \) indicates if they are connected with an hybrid RF/FSO link.

The objective function can be written as:

\[
\sum_{i=1}^{M} \sum_{j=i+1}^{M} X_{ij} \pi^{(O)}(d(b_i, b_j)) + Y_{ij} \pi^{(h)}(d(b_i, b_j))
\]  

(1)

Note that \( \pi^{(O)}(0) = \pi^{(h)}(0) = 0 \) and \( d(b_i, b_j) = 0, 1 \leq i \leq M \). Hence, the objective function can be written as

\[
\sum_{i=1}^{M} \sum_{j=1}^{M} X_{ij} \pi^{(O)}(d(b_i, b_j)) + Y_{ij} \pi^{(h)}(d(b_i, b_j))
\]  

(2)

Moreover, since \( X_{ij} = X_{ji}, Y_{ij} = Y_{ji}, \) and \( d(b_i, b_j) = d(b_j, b_i) \), the objective function can be written as:

\[
\frac{1}{2} \sum_{i=1}^{M} \sum_{j=1}^{M} X_{ij} \pi^{(O)}(d(b_i, b_j)) + Y_{ij} \pi^{(h)}(d(b_i, b_j))
\]  

(3)

Define \( C = [c_{ij}] \) as the adjacency matrix as follows:

\[
c_{ij} = \begin{cases} 
X_{ij} + Y_{ij} & \text{if } 1 \leq i \neq j \leq M \\
0 & \text{otherwise}
\end{cases}
\]  

(4)

Since only one type of connections exists between the same BSs then \( X_{ij}Y_{ij} = 0 \), hence \( c_{ij} \) is a binary variable (i.e. \( c_{ij} \in \{0, 1\} \)).

From a graph theory perspective [23], [24], the graph connectivity constraint is expressed as a function the Laplacian matrix \( L \) defined as \( L = D - C \), where \( D = \text{diag}(d_1, \ldots, d_M) \) be a diagonal matrix with \( d_i = \sum_{j=1}^{M} (X_{ij} + Y_{ij}) \).

\[
Y_{ij} = \sum_{i=1}^{M} c_{ij}
\]

The diagonalization of the Laplacian matrix is given by \( L = QAQ^{-1} \), where \( \Lambda = \text{diag}(\lambda_1, \ldots, \lambda_M) \) with \( \lambda_1 \leq \lambda_2 \leq \cdots \leq \lambda_M \). The connectivity condition of the matrix can be written using the algebraic formulation proposed in [23], [24] as \( \lambda_2 > 0 \).

The rate constraint can be written as for all BS \( i \) as follows:

\[
\sum_{j=1}^{M} X_{ij} R(O)(d(b_i, b_j)) + Y_{ij} R(h)(d(b_i, b_j)) \geq R_t, \quad (5)
\]

where \( R(O) \) and \( R(h) \) are the rates of the OF and the hybrid RF/FSO links, respectively, and \( R_t \) the target rate. For simplicity, in this paper, we consider the normalized rates. Hence the constraint can be written as:

\[
\sum_{j=1}^{M} X_{ij} \frac{R(O)(d(b_i, b_j))}{R_t} + Y_{ij} \frac{R(h)(d(b_i, b_j))}{R_t} \geq 1, \quad (6)
\]

Since \( X_{ij} \) and \( Y_{ij} \) are binary variable, then the constraint can be rewritten, for all BS \( i \), using the normalized rates as follows:

\[
\sum_{j=1}^{M} X_{ij} R(O)(d(b_i, b_j)) + Y_{ij} R(h)(d(b_i, b_j)) \geq 1 \quad (7)
\]

where \( R(O)(x) \) and \( R(h)(x) \) are the normalized data rates of an OF and hybrid RF/FSO links, respectively, as a function of the distance \( x \). By normalized rates, we refer to the minimum between the actual rate, divided by the target rate, and unity. In other words, if the rate is \( R \), the target rate is \( R_t \) and the normalized rate \( R_n \), then the relationship linking the three quantities can be expressed as \( R_n = \min(R/R_t, 1) \). Since the OF provides high data rates, without loss of generality, we assume that \( R(O)(x) = 1, \forall x > 0 \), and that \( R(h)(x) \) is a decreasing function of the distance. Figure 2 represents an example of such normalized data rates of OF and hybrid RF/FSO links with respect to the distance.
The problem of minimizing the cost of the backhaul network planning can be formulated as:

\[
\begin{align*}
\text{min} & \quad \frac{1}{2} \sum_{i=1}^{M} \sum_{j=1}^{M} X_{ij} \pi^{(O)}(d(b_i, b_j)) \\
& \quad + Y_{ij} \pi^{(h)}(d(b_i, b_j)) \\
\text{s.t.} & \quad X_{ij} = X_{ji}, \quad 1 \leq i, j \leq M \\
& \quad Y_{ij} = Y_{ji}, \quad 1 \leq i, j \leq M \\
& \quad X_{ij}Y_{ij} = 0, \quad 1 \leq i, j \leq M \\
& \quad \sum_{j=1}^{M} X_{ij} R^{(O)}(d(b_i, b_j)) \\
& \quad + Y_{ij} R^{(h)}(d(b_i, b_j)) \geq 1, \quad 1 \leq i \leq M \\
& \quad \lambda_2 > 0 \\
& \quad X_{ij}, Y_{ij} \in \{0, 1\}, \quad 1 \leq i, j \leq M,
\end{align*}
\]

where the optimization is over both binary variables \(X_{ij}\) and \(Y_{ij}\). The optimization problem (8) is equivalent to a weighted Steiner tree problem which is NP-hard \([25], [26]\).

We refer to the solution of this problem as the optimal planning (OPT-PL). In rest of this paper, we propose efficient heuristic to solve the problem under the assumption that the hybrid RF/FSO connection between two nodes that are far away from each other is always more expensive than the OF connections between one of the nodes and its closest neighbor. The heuristic is based on first finding the solution to the problem when only OF links can be used. Afterwards, it solves an approximate of the backhaul network planning problem via relating problem (8) to solution reached by the planning problem when only OF links are allowed.

### A. Optimal Planning Using Optical Fibre Only

For simplicity, in this section we consider that the cost of OF links is linear with the distance. This assumption is not only realistic but also comes without loss of generality. Indeed, the general case be obtained by simply replacing the distance operator \(d(\cdot, \cdot)\) by the corresponding cost of the OF link \(\pi^{(O)}(d(\cdot, \cdot))\) in the problem formulation and the algorithm.

The following lemma introduces the reduced problem when only OF links are allowed.

**Lemma 1.** The problem of backhaul design with minimum cost when only OF links are allowed is the following:

\[
\begin{align*}
\text{min} & \quad \frac{1}{2} \sum_{i=1}^{M} \sum_{j=1}^{M} X_{ij} d(b_i, b_j) \\
\text{s.t.} & \quad X_{ij} = X_{ji}, \quad 1 \leq i, j \leq M \\
& \quad \lambda_2 > 0 \\
& \quad X_{ij} \in \{0, 1\}, \quad 1 \leq i, j \leq M,
\end{align*}
\]
Algorithm 1 Optimal planning using only OF links

Require: $\mathcal{B}, d(., ..)$. 
Initialize $X_{ij} = 0, \ 1 \leq i, j \leq M$. 
Initialize $\mathcal{Z} = \emptyset$. 

for all $b \in \mathcal{B}$ do 
\begin{align*}
\mathcal{Z} &= \{ Z, \{ b \} \}. \\
\end{align*}
end for 

while $|\mathcal{Z}| > 1$ do 
\begin{align*}
(Z_i, Z_j) &= \arg \min_{Z, Z' \in \mathcal{Z} \cap Z \neq Z'} d(Z, Z') = \min_{b \in Z, b' \in Z' \setminus Z} \left[ \min d(b, b') \right]. \\
(b_i, b_j) &= \arg \min_{b \in Z_i, b' \in Z_j} d(b, b'). \\
X_{ij} &= X_{ji} = 1. \\
Z &= Z \setminus \{ Z_i \}. \\
Z &= Z \setminus \{ Z_i, Z_j \}. \\
\end{align*}
end while 

Proof: The proof of this lemma can be found in Appendix \[A\].

To solve the aforementioned problem, we propose to cluster the BSs according to the minimal distance. First, a cluster is created for all BS in the system. Afterwards, the two clusters at minimum distance of each other are merged into a single cluster, and connected. The distance between two clusters is defined as the minimum distance between all BS in each cluster. When two clusters are merged, the two BSs, in each cluster, at minimum distance are connected through an OF link. The process is repeated until only one cluster remains in the system. The steps of the algorithm are summarized in Algorithm \[1\]. The following theorem characterizes the solution produced by Algorithm \[1\] with respect to the problem defined in Lemma \[1\].

\textbf{Theorem 1.} The solution reached by Algorithm \[2\] is the optimal solution to the problem proposed in Lemma \[2\]. We refer to this solution as the optimal OF only planning (OF-OPL).

Proof: The proof of this theorem can be found in Appendix \[B\].

\textbf{B. Problem Approximation}

In this section, we approximate the backhaul network planning problem \[8\] under the assumption that an hybrid RF/FSO connection between two nodes that are not neighbours is more expensive than an OF links between one of the nodes and its closest neighbour. We first define $b_{i*}$ as be the closest node to base-station $b_i$ as follows:

\begin{align*}
b_{i*} &= \arg \min_{b \in \mathcal{B} \setminus \{ b_i \}} d(b_i, b) \quad \text{(19)}
\end{align*}

The set of neighbours $\mathcal{N}_i$ of base-station $b_i$ is defined as the set of base-station that are closest to base-station $b_i$, and that satisfy the connectivity condition. Mathematically, the condition can be written as:

\begin{align*}
\mathcal{N}_i = \{ b \in \mathcal{B} \setminus \{ b_i \} \mid d(b_i, b) \leq \max_{b_j \in \mathcal{B}} \overline{X}_{ij} d(b_i, b_j) \},
\end{align*}

where $\overline{X}_{ij}, \ 1 \leq i \neq j \leq M$ is the optimal solution to the OF only planning problem \[15\].

\textbf{Remark 1.} The results presented in this paper do not depend on the definition of the set of neighbours $\overline{N}_i$ of node $b_i$ as long as $\overline{N}_i \subseteq \mathcal{N}_i$. Intuitively, as the set $\overline{N}_i$ gets bigger and bigger, the approximation of the solution is more and more tight. For $\overline{N}_i = \mathcal{B} \setminus \{ b_i \}$, the proposed algorithm reduces to an exhaustive search.

The assumption that two nodes that are far away from each others (i.e. not neighbours) connected with hybrid RF/FSO link generate a cost greater that the cost of the same nodes connected with OF links with their closest neighbors can be written as:

\begin{align*}
\pi^{(O)}(d(b_i, b_{i*})) + \pi^{(O)}(d(b_j, b_{j*})) \leq \pi^{(h)}(d(b_i, b_j)) \quad \forall (b_i, b_j) \notin \overline{N}_i \times \overline{N}_i \quad \text{(21)}
\end{align*}

Based on the above assumption, the following lemma approximates the optimization problem \[8\] under the assumption \[21\].

\textbf{Lemma 2.} The problem of backhaul network cost minimization design using OF and hybrid RF/FSO connections can be approximated by the following problem:

\begin{align*}
\min & \quad \frac{1}{2} \sum_{i=1}^{M} \sum_{j=1}^{M} X_{ij} \pi^{(O)}(d(b_i, b_j)) \quad \text{+} \quad Y_{ij} \pi^{(h)}(d(b_i, b_j)) \quad \text{(22)}
\end{align*}

s.t. \quad X_{ij} = X_{ji}, \ 1 \leq i, j \leq M \quad \text{(23)}

\begin{align*}
Y_{ij} &= Y_{ji}, \ 1 \leq i, j \leq M \quad \text{(24)}
X_{ij} Y_{ij} &= 0, \ 1 \leq i, j \leq M \quad \text{(25)}
\end{align*}

\begin{align*}
\sum_{j=1}^{M} X_{ij} \overline{R}^{(O)}(d(b_i, b_j)) \quad \text{+} \quad Y_{ij} \overline{R}^{(h)}(d(b_i, b_j)) \quad \geq \frac{1}{2}, \ 1 \leq i \leq M \quad \text{(26)}
\quad (X_{ij} + Y_{ij}) \overline{X}_{ij} = \overline{X}_{ij}, \ 1 \leq i \neq j \leq M \quad \text{(27)}
\quad X_{ij}, Y_{ij} \in \{ 0, 1 \}, \ 1 \leq i, j \leq M \quad \text{(28)}
\end{align*}

Proof: To prove this lemma, we first prove that any solution to \[22\], is a feasible solution to \[8\]. To show that a feasible solution to \[22\], is a feasible solution to \[8\], we just show that constraint \[27\] is included in constraint \[13\] since it is the only constraint changing from one formulation to the other. Constraint \[27\] ensures that, for all connections $\overline{X}_{ij} = 1$ that are generated by Algorithm \[1\] then a similar
connections (OF or hybrid RF/FSO link) between nodes \( b_i \) and \( b_j \) should exist. For connections \( X_{ij} = 0 \), the constraint is always satisfied and connection may or may not exist. From Theorem 1 Algorithm 1 produces a connected graph. In other words, \( \lambda_2 > 0 \). Therefore, constraint (27) is included in constraint (13) which conclude that a feasible solution to (22), is a feasible solution to (8). In Theorem 2 we show that the optimal solution to (22) is the optimal solution to (8) in many scenarios (but not all). Therefore, the approximation of problem (8) by the problem (22) is tight.

C. Proposed Solution

This section proposes the solution for the approximate problem (22). The solution is based first on constructing the network planning graph, and then on formulating the problem as a graph theory problem that can be optimally solved with moderate complexity.

1) Planning Graph: In this section, we introduce the undirected planning graph \( G(V,E) \), where \( V \) is the set of vertices and \( E \) the set of edges. Before stating the vertices construction and the edge connection, we first introduce the clustering \( C_i \) for each node \( b_i, 1 \leq i \leq M \) in the network as follows:

\[
C_i = \{ (X_{ij}, Y_{ij}) , \ldots , (X_{ij,N_i}, Y_{ij,N_i}) \} \text{ such that } \sum_{k=1}^{|N_i|} b_{ijk} = N_i
\]

\[
X_{ij},Y_{ij} = 0,1 \leq k \leq |N_i|
\]

\[
(X_{ij} + Y_{ijk})X_{ijk} = X_{ijk}, 1 \leq k \leq |N_i|
\]

\[
\sum_{k=1}^{|N_i|} X_{ijk} R^{(O)}(d(b_i,b_j)) + Y_{ijk} R^{(h)}(d(b_i,b_j)) \geq 1 \}
\]

(29)

Let the weight of cluster \( \alpha_i = \{ (X_{ij}, Y_{ij}) , \ldots , (X_{ij,N_i}, Y_{ij,N_i}) \} \in C_i \) be defined as:

\[
w(\alpha_i) = -\frac{1}{2} \sum_{k=1}^{|B_i|} X_{ijk} \pi^{(O)}(d(b_i,b_j)) + Y_{ijk} \pi^{(h)}(d(b_i,b_j))
\]

(30)

Note that the size of a clustering \( C_i \) is always strictly bounded by \( 2^{|N_i|+1} \). For each cluster \( \alpha_i \in C_i \), a vertex \( v_{ij}, 1 \leq j < 2^{|N_i|+1} \) is generated. Two distinct vertices \( v_{ij} \) and \( v_{kl} \) are connected with an edge in \( E \) if the two following conditions are satisfied:

1) C1: \( i \neq k \): The vertices represents different nodes in the network.

2) C2: \( (X_{ik}, Y_{ik}) = (X_{ki}, Y_{ki}) \) if \( (b_i, b_k) \in (N_k, N_i) \): The vertices are non conflicting.

2) Proposed Algorithm: The following theorem characterize the solution of the approximated backhaul network planning problem (22).

\[\text{Theorem 2. Let } (X^{*}_{ij}, Y^{*}_{ij}), 1 \leq i,j \leq M \text{ be the optimal solution to the planning problem (22) then we have } X^{*}_{ij} + Y^{*}_{ij} = 1 \text{ only if } (i,j) \in N_j \times N_i\]

\[\text{Proof: The proof of this theorem can be found in Appendix C}\]

The following theorem links the solution of problem (22) to the planning graph.

\[\text{Theorem 3. The solution of the approximation of the backhaul network design using hybrid RF/FSO can be formulated as a maximum weight clique, among the cliques of size } M, \text{ in the planning graph, in which the weight of each vertex } v_{ij} \text{ is the weight of the corresponding cluster } \alpha_i \text{ defined in (30).}\]

\[\text{Proof: The proof of this theorem can be found in Appendix D}\]

IV. Simulation Results

This section shows the performance of the proposed solution to the backhaul network planning problem using hybrid RF/FSO technology. The base stations are randomly placed on a 5 Km long square. The cost of an OF link is taken to be 13.2$ per meter and the cost of an hybrid RF/FSO link is taken to be independent of the distance. Given the prices offered by the different constructors (FSONA, LightPointe, and RedLine), 2 type of price are identified \( \pi^{(F)} = 10000 \) and 20000. The price \( \pi^{(F)} = 40000 \) is proposed as a cut of price for which hybrid RF/FSO do not represent any advantage. The normalized data rate of an hybrid RF/FSO links is taken to be constant over a distance \( x \) after which it decays exponentially. For illustration purposes, the distance \( x \) is assumed to be 3Km unless indicated otherwise. The numbers of base-stations, price of the hybrid RF/FSO transceivers, and the distance \( x \) vary in the simulations so as to study the methods performance for various scenarios.

The optimal planning solution (solution of (8)) denoted by “OPT-PL”, the planning solution using only OF links (solution of (15)) denoted by “OF-O-PL” and our proposed solution (solution of (22)) denoted by “FSO-PL” are simulated in this section.

Figure 4 plots the cost of the network versus the number of BSs, for different costs of the hybrid RF/FSO transceivers. We clearly see that the degradation of our proposed solution against the optimal solution becomes less severe when first the number of base stations increases, and secondly when the hybrid RF/FSO transceivers becomes more expensive. The increase in performance in the first case can be explained by the fact that the connectivity opportunities of nodes increase as the number of base-stations increases, due to the increase in the neighbours sets \( N_i \). The gain in performance when the price of the hybrid RF/FSO transceivers increases can be explained by the fact that our assumption (21) becomes more valid as the price of the hybrid RF/FSO transceivers increases.

Figure 5 and Figure 6 illustrate the cost of the network and the ratio of the OF link, respectively, against the cost of the hybrid RF/FSO transceivers. As shown in Figure 5, the performance of our proposed algorithm is more and more close to the one of the optimal planning as the cost of the hybrid RF/FSO transceivers increases. From Figure 6, we clearly see that if the hybrid RF/FSO transceivers are expensive enough,
both the optimal and our proposed solution contain only OF links. In fact for expensive hybrid RF/FSO transceivers, the OF links offers a noticeable rate advantage which explain their use. It is worthy mentioning that for a cost $\pi(h) \geq 30000$ in Figure 6, even though the link nature used in the optimal solution and our proposed solution are different, the total cost of the network is almost the same (Figure 5 for $M = 7$).

Finally, to quantify the performance of the proposed algorithms with respect to the distance $x$, Figure 7 and Figure 8 plot, respectively, the cost of the network and the ratio of the OF link used against the distance $x$ for different prices of the hybrid RF/FSO transceivers. Again, we clearly see that for expensive hybrid RF/FSO transceivers both the optimal solution and our proposed solution use exclusively OF links. Figure 7 shows that, even for cheap hybrid RF/FSO transceivers, our proposed solution performs as good as the optimal solution for a distance $x \geq 4$. Figure 8 depicts that for a small $x$, our proposed solution uses more OF links than the optimal solution. Whereas for a $x \geq 4$ the ratio is almost the same. This can be explained by our choice of neighbours $N_i$. The connectivity opportunities of our proposed solution are less than the one of the optimal solution. Hence, for small $x$, to
V. CONCLUSION

In this paper, we consider the problem of backhaul network design using the OF and hybrid RF/FSO technologies. We first formulate the planning problem under connectivity and rate constraints. We, then, solve the problem optimally when only OF links are allowed. Using the solution of the OF deployment, we formulate an approximation of the general planning problem and show that under a realistic assumption about the relative cost of the OF links and the hybrid RF/FSO transceivers, the solution can be expressed as a maximum weight clique in the planning graph. Simulation results show that our approach shows a close-to-optimal performance, especially for practical prices of the hybrid RF/FSO.

APPENDIX A
PROOF OF LEMMA 1

The problem formulation when only OF links are allowed can be simply obtained by setting \( Y_{ij} = 0 \), \( 1 \leq i, j \leq M \) in (5). Constraints (10) and (11) become redundant and the problem can be written as:

\[
\min \frac{1}{2} \sum_{i=1}^{M} \sum_{j=1}^{M} X_{ij} \pi(O)(d(b_i, b_j))
\]

subject to

\[
X_{ij} = X_{ji}, \quad 1 \leq i, j \leq M
\]

\[
\sum_{j=1}^{M} X_{ij} R(O)(d(b_i, b_j)) \geq 1, \quad 1 \leq i \leq M
\]

\[
\lambda_2 > 0
\]

\[
X_{ij} \in \{0, 1\}, \quad 1 \leq i, j \leq M.
\]  \hspace{1cm} (A.1)

Note that since \( \pi(O)(x) \) is a linear function with respect to the distance, then the objective function can reduce to:

\[
\min \frac{1}{2} \sum_{i=1}^{M} \sum_{j=1}^{M} X_{ij} d(b_i, b_j)
\]

subject to

\[
X_{ij} = X_{ji}, \quad 1 \leq i, j \leq M
\]

\[
\lambda_2 > 0
\]

\[
X_{ij} \in \{0, 1\}, \quad 1 \leq i, j \leq M.
\]  \hspace{1cm} (A.2)

APPENDIX B
PROOF OF THEOREM 1

To prove this theorem, we first proof that Algorithm 1 produces a feasible solution to the problem. Afterwards, we proof that any graph that can be reduced, using Algorithm 2 to a single cluster includes the graph outputted by Algorithm 1. Finally, we show that any solution that cannot be reduced to a single cluster is not optimal.

Algorithm 2 can be seen as a complement of Algorithm 1. As for Algorithm 1 in Algorithm 2 begins by generating a cluster of each BS in the system. Afterwards, two clusters at minimum distance of each other and whose BSs ta minimum distance are connected are merged in a single cluster. The process is repeated until no further connection can be found. The steps of the algorithm are summarized in Algorithm 2.

Let \( \mathcal{Z} = \{Z_1, \cdots, Z_{|\mathcal{Z}|} \} \) be the clustering at any step of Algorithm 1. First note that \( \mathcal{Z} \) is a partition of \( B \). We proof by induction that nodes inside any cluster \( Z_i, 1 \leq i \leq |\mathcal{Z}| \) are connected. Clearly, for a cluster \( Z_i \) with \( |Z_i| = 1 \) (the cluster contains a single node), all nodes inside the cluster are connected. Assume that all clusters \( Z_i \) of size \( |Z_i| \leq n \) are connected. From the last step of Algorithm 1 clusters of size \( n + 1 \) can be generated only by merging two clusters \( Z_j \) and \( Z_k \) with \( |Z_j|, |Z_k| < n \) and \( |Z_j| + |Z_k| = n + 1 \).
Algorithm 2 Clustering Algorithm

Require: $B, d(., .), X_{ij}, 1 \leq i, j \leq M$. Initialize $Z = \emptyset$.

for all $b \in B$ do
   $Z = \{Z, \{b\}\}$.
end for

Initialize $t =$ TRUE.

while $t =$ TRUE do
   $t =$ FALSE.
   
   for all $Z \neq Z' \in Z$ do
      if $\sum_{b_i, b_j \in Z} X_{ij} = 1$ then
         $Z^* = \min_{X \in Z, X \neq Z'} d(X, Z^*), \min_{b_i, b_j \in Z} d(b_i, b_j)$.
         if $Z = Z^*$ and $Z' = Z'^*$ then
            \begin{align*}
            (b_i, b_j) &= \min_{b_i, b_j \in Z} d(b_i, b_j). \\
            \text{if } X_{ij} = 1 \text{ then } \\
            Z &= Z \setminus \{Z\}, \\
            Z &= Z \setminus \{Z'\}, \\
            Z &= \{Z, Z_1, Z_2\}, \\
            t &= $TRUE
            \end{align*}
         end if
      end if
   end for
end while

Since by construction such clusters are connected ($X_{ij} = X_{kj} = 1$ with $b_j \in Z_j$ and $b_k \in Z_k$), then the resulting cluster $Z_i$ from merging $Z_j$ and $Z_k$ is also connected. Therefore, all nodes within any arbitrary cluster $Z_i$, $1 \leq i \leq |Z|$ are connected. Finally, since $Z$ contains a single cluster at the end of Algorithm 1 (i.e. $|Z| = 1$) and it is a partition of $B$ (contains all the nodes in the network), then all the nodes are connected. Hence the outputted solution satisfy constraint (17). By construction, we can easily see that the connections are binary and symmetric. In other words, the outputted solution satisfy constraints (16) and (18) which conclude that it is a feasible solution.

Let $X_{ij}, 1 \leq i, j \leq M$ be a feasible solution and let $Z$ be the outputted clustering by Algorithm 2 when inputted $X_{ij}$. We can clearly see that $Z$ is a partition of $B$. Therefore, if $|Z| = 1$, then $Z = \overline{Z}$ since there exist only a unique partition containing a single element (the set $B$ itself). This concludes that the any graph that can be reduced, using Algorithm 2 to a single cluster includes all the connections that are created in the graph outputted by Algorithm 1. Since $\pi(O)$ is a strictly positive function and that the graph outputted by Algorithm 1 have the minimum number of connections among all the graphs that can be reduced to a single cluster using Algorithm 2, then the solution of Algorithm 1 is the best solution among the solutions that can be reduced to a single cluster using Algorithm 2.

Now assume that $|Z| \neq 1$. We can clearly see that $|Z| \geq 3$. Otherwise, if there exist only two clusters (i.e. $|Z| = 2$) and since the solution is feasible, then they are connected. Due to the fact they are only two, then two cases can be distinguished:

- The BSs at minimum distance of each others are connected and hence they can be reduced to a single cluster. Therefore, clustering $|Z| = 2$ cannot be outputted by Algorithm 2.
- The BSs at minimum distance of each others are not connected. Then the solution having the same connections except for the connection between the two cluster being replaced with the connection of BSs at minimum distance of each others produces a feasible solution at a lower cost.

Therefore, the initial solution is not optimal.

Let $Z = \{Z_1, \cdots, Z_{|Z|}\}, |Z| \geq 3$ be the outputted clustering by Algorithm 2. Define the reduced clustering as $\overline{Z} = \{Z_1, Z_2, \{Z_3 \cup \cdots \cup Z_{|Z|}\}\} = \{\overline{Z}_1, \overline{Z}_2, \overline{Z}_3\}$. We can clearly see that $|\overline{Z}| = 3$ with none of the clusters connected and at minimum distance of each other.

**Lemma 3.** For any three points in the plane, there must exist two points at minimum distance of each others.

**Proof:** Let $a, b, c$ be the three points in the plane and assume that there do not exist two points at minimum distance of each other. The only possible configuration (up to a permutation of the points) is that $a$ at minimum distance from $b$ and $b$ from $c$. Hence $b$ is at minimum distance from $c$ which is at its turn at minimum distance from $a$. These conditions yield $d(a, b) < d(a, c), d(b, c) < d(b, a)$ and $d(a, c) < d(a, b)$. Since the distance operator is symmetric then, $d(a, b) < d(c, b) = d(b, c) < d(b, a) = d(a, b)$. In other words, $d(a, b) < d(a, c)$, which is impossible. Therefore for any three points in the plane, there must exist two points at minimum distance of each others. $\blacksquare$

From Lemma 3 there must exist two clusters at minimum distance of each other. Since they have not been reduced to a single cluster, then they are not connected. For simplicity, assume $\overline{Z}_1$ and $\overline{Z}_2$ are at minimum distance of each other and since the graph is connected then $\overline{Z}_1$ and $\overline{Z}_3$ are connected and similarly for $\overline{Z}_2$ and $\overline{Z}_3$. Moreover, it can be easily concluded that all nodes inside the clusters are connected. Otherwise, assume $\overline{Z}_3$ can be split into two non connected clusters $\overline{Z}$ and $\overline{Z}'$ with $\overline{Z}_1$ connected only to $\overline{Z}$ and $\overline{Z}_2$ connected only to $\overline{Z}'$. Then, since $\overline{Z}_1$ and $\overline{Z}_2$ are not connected, the whole graph is not connected and the solution is not feasible.

The clustering connecting $\overline{Z}_1$ with $\overline{Z}_2$ and $\overline{Z}_1$ with $\overline{Z}_3$ (or $\overline{Z}_2$ with $\overline{Z}_3$) produces also a feasible solution at a lower cost since the sum of the distances is minimized ($d(\overline{Z}_1, \overline{Z}_2) \leq \min(d(\overline{Z}_1, \overline{Z}_3), d(\overline{Z}_2, \overline{Z}_3))$). Therefore, $\overline{Z}$ is not optimal and by extension $Z$ is also not optimal. Finally, we can conclude
that the optimal solution is the solution containing a single cluster. Therefore, the solution outputted by Algorithm 1 is the optimal solution to the problem proposed in Lemma 1.

APPENDIX C
PROOF OF THEOREM 2
In this theorem, we show that the optimal solution \( X_{ij}, Y_{ij} \), 1 \( \leq i, j \leq M \) to (22) should satisfy \( Y_{ij}^{*} = 1 \) only if \( (i, j) \in \mathcal{N}_i \times \mathcal{N}_j \).

Remark 2. Note that if any feasible solution \( X_{ij}, Y_{ij} \) to the general problem (9) that verify Theorem 2 then the solution is feasible to (22). In other words, if \( X_{ij}, Y_{ij} \) feasible to (9) and \( X_{ij} + Y_{ij} = 1 \) only if \( (i, j) \in \mathcal{N}_i \times \mathcal{N}_j \), then \( X_{ij} + Y_{ij} = X_{ij} \). This can be easily concluded given the construction of \( \mathcal{N}_i \), \( \leq i \leq M \) as the minimum set of nodes that can generate a connected graph. Since all \((x, y)\) such that \( X_{xy} \) is 1 are at the edge of at least one of the \( \mathcal{N}_i \), then the only connected solution that satisfies Theorem 2 is feasible to (22). In that scenario, the optimal solution of (9) and (22) are the same.

Assume \( \exists (x, y) \) such that \( X_{xy}^{*} + Y_{xy}^{*} = 1 \) and \((x, y) \notin \mathcal{N}_x \times \mathcal{N}_y \). Two scenarios can be distinguished:

- \( \mathcal{B} \setminus \{b_x, b_y\} \) represents a connected subgraph.
- \( \mathcal{B} \setminus \{b_x, b_y\} \) is not a connected subgraph.

For the first scenario, consider the reduced network \( \{b_x, b_y, \bar{b}_k\} \). Clearly, we have \( b_x^{*}, b_y^{*} \subset \bar{b}_k \). Define the following planning:

\[
\tilde{X}_{ij} = \begin{cases} 
1 & \text{if } i = x \text{ and } j = x^* \\
1 & \text{if } j = y \text{ and } j = y^* \\
0 & \text{if } i = x \text{ and } y = y \\
X_{ij}^* & \text{otherwise} \\
\end{cases}
\]

\[
\tilde{Y}_{ij} = \begin{cases} 
0 & \text{if } i = x \text{ and } j = x^* \\
0 & \text{if } j = y \text{ and } j = y^* \\
0 & \text{if } i = x \text{ and } j = y \\
Y_{ij}^* & \text{otherwise} \\
\end{cases}
\]

(C.1)

in which the connection between \( b_x \) and \( b_y \) is replaced by two connections between \( b_x \) and \( b_x^* \), between \( b_y \) and \( b_y^* \).

We can clearly see that the network is connected. Moreover, since \( b_x \) and \( b_y \) are connected with an OF link, then the data rate constraint is satisfied for both nodes. Therefore, \( \tilde{X}_{ij}, \tilde{Y}_{ij}, 1 \leq i, j \leq M \) represents a feasible solution. Moreover, the difference in cost between the optimal planning \( X_{ij}^*, Y_{ij}^* \) and the planning \( \tilde{X}_{ij}, \tilde{Y}_{ij} \) is lower bounded by:

\[
\pi(d(b_x, b_y)) - \pi(O)(d(b_x, b_x^*))) + \pi(O)(d(b_y, b_y^*))) \\
\geq \pi(h)(d(b_x, b_y)) - \pi(O)(d(b_x, b_x^*))) + \pi(O)(d(b_y, b_y^*))) \\
\geq 0
\]

(C.2)

From assumption (21), the difference is positive. This concludes that \( X_{ij}^*, Y_{ij}^*, 1 \leq i, j \leq M \) is not the optimal solution.

Remark 3. For scenario 1, \( X_{ij}^*, Y_{ij}^*, 1 \leq i, j \leq M \) can be the optimal solution to the original problem (9). Hence, for this configuration, the optimal solution of (9) and (22) are the same.

For scenario 2, let the network be reduced to \( \{b_x, b_y, \bar{b}_k\} \) with \( b_x \) connected to \( b_k \), which is a connected subgraph, \( b_y \) connected \( b_k \), which is a connected subgraph, and \( b_k \) and \( b_1 \) are not connected. Since \( X_{ij}^*, Y_{ij}^*, 1 \leq i, j \leq M \) is a feasible solution to (22), then it satisfies constraint (27). In other words, \( X_{kl} + Y_{kl} = 1, \forall k, l \) such that \( X_{kl} = 1 \). Note that \( X_{xy} = 0 \). Otherwise, by construction of the neighbours set, we have \((x, y) \notin \mathcal{N}_y \times \mathcal{N}_x \). Define the planning \( \tilde{X}_{ij}, \tilde{Y}_{ij} \) such that

\[
\tilde{X}_{ij} = \begin{cases} 
0 & \text{if } i = x \text{ and } j = y \\
X_{ij}^* & \text{otherwise} \\
\end{cases}
\]

\[
\tilde{Y}_{ij} = \begin{cases} 
0 & \text{if } i = x \text{ and } j = y \\
Y_{ij}^* & \text{otherwise} \\
\end{cases}
\]

(C.3)

The planning \( \tilde{X}_{ij}, \tilde{Y}_{ij} \) satisfy (27). However, we can clearly see that the graph is not connected. Therefore, \( X_{ij}^*, Y_{ij}^*, 1 \leq i, j \leq M \) is not a feasible solution. This concludes that scenario 2 is not feasible. Finally, we conclude that the optimal solution \( X_{ij}^*, Y_{ij}^*, 1 \leq i, j \leq M \) should satisfy \( X_{xy}^* + Y_{xy}^* = 1 \) only if \((x, y) \notin \mathcal{N}_y \times \mathcal{N}_x \).

Remark 4. Scenario 2 can be a feasible scenario if \( X_{ij}^*, Y_{ij}^*, 1 \leq i, j \leq M \) is the optimal solution to the original problem (9). In that case, two scenarios can be distinguished:

- \( b_x^* \notin \mathcal{N}_y \) and \( b_y^* \notin \mathcal{N}_x \). In that case \( \tilde{X}_{ij}, \tilde{Y}_{ij} \) presented for scenario 1 produces a feasible solution with lower cost. Therefore, the optimal solution of (9) and (22) are the same.
- \( b_x^* \notin \mathcal{N}_y \) or \( b_y^* \notin \mathcal{N}_x \). In this configuration, no conclusion can be reached about the optimal solution of (9) and (22) is an upper bound of the minimum of (9).

Therefore, the approximation of the problem (9) by the problem (22) is a tight approximation.

APPENDIX D
PROOF OF THEOREM 3
To prove this theorem, we first prove that there is a one to one mapping between the set of feasible solution of a modified version of problem (22) and the set of cliques of degree \( M \) in the planning graph \( G(V, E) \). To conclude the proof, we show that the weight of the clique is equivalent to the merit function of the optimization problem (22).

From Theorem 2 we have \( X_{ij} = 0 \) and \( Y_{ij} = 0, \forall (b_i, b_j) \notin \mathcal{N}_i \times \mathcal{N}_j \). Hence the objective function (22) and constraint (26) can be replaced by:

\[
\max - \frac{1}{2} \sum_{i=1}^{M} \sum_{b_j \in \mathcal{N}_i} X_{ij} \pi(O)(d(b_i, b_j)) + Y_{ij} \pi(h)(d(b_i, b_j)) \\
+ \sum_{b_j \in \mathcal{N}_i} X_{ij} R(O)(d(b_i, b_j)) \\
\geq 1, 1 \leq i \leq M
\]

(D.1)
Similarly, since \( X_{ij} = 0 \) and \( Y_{ij} = 0 \), \( \forall (b_i, b_j) \not\in \mathcal{N}_j \times \mathcal{N}_i \), then \((X_{ij}, Y_{ij}) = (X_{ji}, Y_{ji})\) is always verified for \((b_i, b_j) \not\in \mathcal{N}_j \times \mathcal{N}_i\). Hence the constraints (23), (24), and (25) can be replaced by:

\[
(X_{ij}, Y_{ij}) = (X_{ji}, Y_{ji}), \quad \forall (b_i, b_j) \in \mathcal{N}_j \times \mathcal{N}_i
\]

\[
X_{ij}Y_{ij} = 0, \quad \forall (b_i, b_j) \in \mathcal{N}_j \times \mathcal{N}_i
\]  

(D.2)

By definition of the set \( \mathcal{N}_i \), we have \( \bar{X}_{ij} = 0, \forall j \not\in \mathcal{N}_i \). Therefore, constraint (27) may be written as:

\[
(X_{ij} + Y_{ij})\bar{X}_{ij} = \bar{X}_{ij}, \forall j \in \mathcal{N}_i, 1 \leq i \leq M
\]  

(D.3)

The problem (22) can be reformulated as:

\[
\max -\frac{1}{2} \sum_{i=1}^{M} \sum_{b_j \in \mathcal{N}_i} X_{ij} \pi^{(O)}(d(b_i, b_j)) + Y_{ij} \pi^{(H)}(d(b_i, b_j))
\]

subject to

\[
(X_{ij}, Y_{ij}) = (X_{ji}, Y_{ji}), \quad \forall (b_i, b_j) \in \mathcal{N}_j \times \mathcal{N}_i
\]

\[
X_{ij}Y_{ij} = 0, \quad \forall (b_i, b_j) \in \mathcal{N}_j \times \mathcal{N}_i
\]

\[
\sum_{b_j \in \mathcal{N}_i} X_{ij} R^{(O)}(d(b_i, b_j)) + Y_{ij} R^{(H)}(d(b_i, b_j)) \geq \Gamma_i, \quad 1 \leq i \leq M
\]

\[
(X_{ij} + Y_{ij})\bar{X}_{ij} = \bar{X}_{ij}, \forall j \in \mathcal{N}_i, 1 \leq i \leq M
\]

\[
X_{ij}, Y_{ij} \in \{0,1\}, \quad 1 \leq i, j \leq M
\]  

(D.4)

Let \( \alpha_i = \{X_{ij}, Y_{ij}\}, \ldots, (X_{ij}, Y_{ij}) \} \) be the new variable. Using the variables \( \alpha_i \) and the definition of the sets \( \mathcal{C}_i \), the problem can be written as:

\[
\max \sum_{i=1}^{M} w(\alpha_i)
\]

subject to

\[
\alpha_i \in \mathcal{C}_i, 1 \leq i \leq M
\]  

(D.5a)

\[
(X_{ij}, Y_{ij}) = (X_{ji}, Y_{ji}), \quad \forall 1 \leq i \neq j \leq M.
\]  

(D.5b)

Let \( \mathcal{C} \) be the set of cliques of degree \( M \) in the planning graph and let \( \mathcal{F} \) be the set of feasible solutions to the optimization problem (D.5). We prove that any clique \( \mathcal{C} = \{v_1, \ldots, v_M\} \in \mathcal{C} \) satisfy constraints (D.5a), and (D.5b). Then, we prove the converse. In other words, for element in \( \mathcal{F} \), there exists a clique in \( \mathcal{C} \).

Let \( \mathcal{C} = \{v_1, \ldots, v_M\} \in \mathcal{C} \). Assume \( \exists k, i, j \) such that \( v_i, v_j \in \mathcal{C}_k \). Since all the vertices in a clique are connected, then from the connectivity condition C1, vertices \( v_i \) and \( v_j \) are not connected. Hence \( \not\exists k, i, j \) such that \( v_i, v_j \in \mathcal{C}_k \). Given that the clique contain \( M \) elements, then constraint (D.5a) is satisfied. The connectivity condition C2 ensures that \((X_{ij}, Y_{ij}) = (X_{ji}, Y_{ji})\) for all vertices. Therefore, \( \mathcal{C} \) is a feasible solution to (D.5). Similarly, let \( \{c_1, \ldots, c_M\} \) be a feasible solution to (D.5), then clearly the vertices corresponding to each cluster are connected. Finally, there a one to one mapping between \( \mathcal{C} \) and \( \mathcal{F} \). Moreover, the weight of the clique is \( w(C) = \sum_{i=1}^{M} w(\alpha_i) \) which conclude that the solution of (D.5) is the maximum weight clique, among the clique of size \( M \), in the planning graph.

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