Effects of Pimple Height of a Table Tennis Rubber on Ball Rebound Behavior †

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Abstract: The objective of this study was to construct a finite element (FE) model of table tennis rubber (Sandwich rubber) with pimples structure, which can accurately estimate the rebound behavior of the ball at impact, and to investigate effects of its structure on ball rebound behavior. The sandwich rubber is composed of a combination of a rubber and foam layers. The FE model of the sandwich rubber was constructed with non-linearity, strain rate dependency, and energy absorption which were expressed based on the results of material tests. Impact analyses were conducted using the developed model of sandwich rubber and ball with different pimple heights. The simulation results of rebound behavior do not tend to be proportional to the pimple height. The trend of the rebound behavior was mainly affected by the amount of impulse during impact calculated using the horizontal component of the contact force which was varied with changes in pimple height.

Keywords: table tennis; rubber; impact; pimples structure; finite element analysis; coefficient of restitution

1. Introduction

Table tennis rackets consist of a wooden blade covered with a sandwich rubber on single or double surfaces. The sandwich rubber consists of a sheet and foam layers which are made of a rubber material and foamed elastomer, and is used by combining the sheet and foam layers or only the sheet layer. Various types of sandwich rubber can be used by changing types of material properties and combinations of each layer. Especially, the structure with many cylindrical pimples on the sheet layer significantly affects the rebound behavior of the ball after impact, such as the velocity, angle and spin rate, because of the complicated deformation behavior of pimples during impact. Therefore, the performance of the sandwich rubber depends on the pimples structure which is decided by combining shape, array, and number of pimples within the range of the regulations [1]. However, many combinations of the pimples structure make it difficult to evaluate the performance of the sandwich rubber. Currently, evaluations of the sandwich rubber have often been conducted by interviews with players and experimental tests. However, one of the disadvantages of these tests is the subjective assessment data and the need for the production of a prototype sheet for each test, respectively. If the evaluation of the sandwich rubber by these tests could be replaced by computer simulations, it is expected that the cost of preparing prototypes would be reduced, and more efficient evaluation of the sandwich rubber performance would be led by using information on dynamic
behavior, which is neither simply nor easily obtainable by experiment. One of the most effective ways for the requirement is the finite element (FE) method.

There have been many studies in the literature, which have attempted to evaluate rackets both experimentally and computationally. With references to experimental evaluation of the sandwich rubber for examples, tensile property and hardness of rubber materials were reported [2], and effects of the difference of pimples structure on the static friction and force were investigated [3,4]. In a computational study on ball impact, impact and rebound behavior of the ball was observed in different impact conditions [5,6], and the difference of rebound behavior with changing pimple height was investigated [7]. Studies focusing on the components of the rackets were also conducted, but the evaluation of the sandwich rubber using FE analyses could not be sufficiently investigated due to the simple FE model for the ball with elasticity and the inadequate discussion of the deformation behavior of the sandwich rubber during impact. It is, therefore, desirable to construct FE models of the ball and sandwich rubber with high precision, which can express the non-linearity, energy absorption, and strain rate dependency. Modeling is also expected to predict the performance of the sandwich rubber by easily varying the pimples structure in simulation analysis, and lead to more efficient development of equipment, which meets the regulations.

The objective of this study was to construct an impact analysis model between the ball and sandwich rubber, which can express the rebound behavior of the ball, and to investigate the effects of the pimple height on the rebound behavior of the ball.

2. Modeling of Rubber Material

In this study, the commercial FE code LS-DYNA (ver. 971, United States, Livermore Software Technology Corporation) was used for the simulations.

2.1. Material Models of Sheet and Foam Layers

The sheet and foam layers were modeled using *MAT_SIMPLIFIED_RUBBER (MAT_181) and *MAT_FU_CHANG_FOAM (MAT_083) LS-DYNA material model, respectively to represent the non-linearity, strain rate dependency, and energy absorption. The constitutive equations of the material models are defined as shown in Tables 1 and 2 [8]. In general, it is necessary to determine the material constants making up each equation based on material tests, but the material parameters of the selected models are identified by introducing stress-strain (s-s) curves obtained from three kinds of experiment, that is, the uniaxial static tensile and compression tests and the dynamic compression test, into program files of each material.

Table 1. This table shows the constitutive equations for MAT_181 [8]. The constants of $g(\lambda)$ are decided based on s-s curves [9,10]. The value of $K$ needs to be identified by a user.

$$
\tau_i = f(\lambda_i) + K (\frac{1}{2} + \frac{1}{2} \Sigma_{k=1}^n f(\lambda_k) 
$$

$$
f(\lambda) = \lambda g(\lambda) + \lambda^{-1/2} g(\lambda^{-1/2}) + \lambda^{1/2} g(\lambda^{1/2}) + \cdots + f(\lambda^{1/2})
$$

$J (=\lambda^2 \lambda^2 \lambda^2)$: Relative volume $K$: Bulk moduli
Table 2. This table shows the constitutive equations for MAT_083 [8]. Material constants in Table 2 are identified based on s-s curves [11,12].

\[ \sigma(t) = \sigma\left[ E^N(t), E^N(t), S(t) \right] \]  

\[ E^N_i = \frac{\sigma_i}{\|\varepsilon_i\|^2} D_0 \exp \left( -c_0 \left[ \frac{\text{tr}(\varepsilon_i)}{\|\varepsilon_i\|^2} \right]^{2n_0} \right) \]  

\[ S_{ij} = \left[ c_1 (a_{ij} R - c_2 S_{ij}) P + c_3 W^{n_0} \left( \|E^N\| \right)^{n_1} I_{ij} \right]^R \]

\[ R = 1 + c_4 \left( \frac{\|E^N\|}{c_5} - 1 \right)^{n_3} \]

\[ P = \text{tr} (\sigma E^N) \]

\[ W = \int \text{tr} (\sigma E^N) \]

\( \text{S}(t) \): the state variable  \( E^N(t) \): nonlinear part of the strain  \( E^N(t) \): expression for the past history of  \( E^N \)  \( D_0, c_0, c_1, c_2, c_3, c_4, n_0, n_1, n_2, n_3, a_{ij} \): material constants

2.2 Experimental Tests for Material Modeling

Static tensile and compression tests were conducted using a material testing machine. In the tensile test, the cross-section size of the specimen was 10 mm × 7 mm, and the distance between chucks was 100 mm. In the compression test, the specimen size was 30 mm × 30 mm × 10 mm. And the test speed of both tests was conducted at a velocity of 50 mm/min.

A drop weight impact compression test apparatus was designed and manufactured to conduct the dynamic compression test for the specimen (30 mm × 30 mm × 10 mm). A free-falling drop weight along guide bars perpendicular to the horizon was collided with the specimen. The impact load and the deformation of the specimen were synchronously measured using a piezoelectric force sensor and a laser displacement transducer, respectively, at a sampling rate of 10 kHz. The impact velocities were controlled at 1.0, 2.0 and 3.0 m/s by changing the fall length.

2.3 Accuracy Validation of Material Models

To confirm the accuracy of the material models, FE analyses were conducted with the constructed models under the same conditions as those of the drop impact test as shown in Figure 1. The simulation results of both materials for both the loading and unloading processes almost agreed with the experimental results. The simulation results have ranges of the history which do not clearly match with the experimented results. This minor error seems to be cause by the possibility that the contact condition between the weight and specimen is not completely represented in FE analyses. Moreover, the high quantitative accuracy of the simulation results can be obtained at impact velocities of 1.0 and 2.0 m/s.

![Figure 1](image-url). These figures show the results of the s-s curves for the experiments and the FE analyses at a velocity of 3.0 m/s: (a) the sheet; (b) the foam.
3. Construction of Impact Simulation Model for Ball and Rubber Model

3.1 Impact Tests for Accuracy Validation of Impact Simulation Model

Impact experiments between the ball and a rubber plate were conducted to obtain the dynamic behavior of the ball using a high-speed video camera. The ball was fired from an air gun and collided with the plate (without pimples structure) affixed on the surface of a fixed flat steel plate. The ball behavior of incidence and rebound, such as the velocities \( v_{in} \) and \( v_{out} \), angles \( \theta_{in} \) and \( \theta_{out} \), and spin rate \( \omega_{out} \) of the ball were ascertained from images which were tracked at a frame rate of 20,000 fps and a shutter speed of 1/100,000 s. The velocity and angles of incidence were 60 km/h, and 0 (normal impact) and 70 degrees.

3.2 Modeling of Impact between Ball and Rubber Plate

An impact simulation was conducted using a ball model with viscosity developed in our previous study [13], and the rubber plate model which was defined by MAT_181 and was composed of hexahedral solid elements (element size: 0.2 mm) as shown in Figure 2. The ball model collided with the rubber plate, which was constrained on the opposite side of the impact surface, under the same impact conditions as those of the experiment.

Table 3 shows the results of the ball rebound behavior for the experiment and the FE analysis. The simulation results tend to fall within the range of the experimental results for both normal and oblique impact conditions. The constructed impact model can express the rebound behavior for a range of impact conditions in this study.

![Figure 2.](image)

**Figure 2.** This figure shows the constructed impact simulation model between the ball and the rubber plate without pimples structure. To confirm the accuracy of the model, the simulation results of the ball rebound behavior were compared to those from the experiment.

**Table 3.** This table shows the results of the ball rebound behavior for the experiment and the FE analysis. The red and black markers represent the results of the FE analysis and the range of the experimental results, respectively. \( COR = \frac{v_{out}}{v_{in}} \)

|                  | Normal impact   | Oblique impact |
|------------------|-----------------|----------------|
| \( COR \)        | 0.65, 0.66, 0.66 | 0.47, 0.475, 0.48, 0.485, 0.49 |
| \( \theta_{out} \) | No data         | 59, 60, 61, 62, 63, 64 (degree) |
| \( \omega_{out} \) | No data         | 6000, 6200, 6400, 6600, 6800 (rpm) |

4. Effects of Pimple Height on Rebound Behavior

Impact simulations were conducted using the models of the ball and sandwich rubber with changes in the height of pimples, in order to investigate its effects on ball rebound behavior, as shown in Table 4. Three rubber models (IDs: R01, R02, R03) were prepared by varying the pimple height from 0.50 to 1.00 mm. The layers of sheet and foam were expressed by hexahedral solid elements.
(element size: about 0.2 mm), and the surface of the pimples joined to the contact surface of the foam. The impact conditions were under the same conditions as those in Chapter 3.

**Table 4.** This table shows the geometry and design parameters of pimples structure.

| ID | Sheet layer | Foam layer | Foam layer |
|---|---|---|---|
|  | $t_0$ [mm] | $\phi$ [mm] | $t_1$ [mm] | $d$ [mm] | $t_2$ [mm] |
| R01 | 1.00 | 1.50 | 0.50 | 2.50 | 2.0 |
| R02 | 1.00 | 1.50 | 0.75 | 2.50 | 2.0 |
| R03 | 1.00 | 1.50 | 1.00 | 2.50 | 2.0 |

From the simulation results of the rebound behavior, as shown in Figure 3, the simulation results tend to change depending on the pimple height in a range of changes in this study, although the rate of change for the rebound behavior tends to decrease as the height increases. The trends of $\theta_{\text{out}}$ and $\omega_{\text{out}}$ with different height have the inverse correlation, and the change of $\theta_{\text{out}}$ and $\omega_{\text{out}}$ tends to correlate with that of the $y$ component of $v_{\text{out}}$. In contrast, the pimple height has a much smaller effect on the results of the $z$ component of $v_{\text{out}}$ than that of the $y$ component. Therefore, the pimple height strongly affects the horizontal component of $v_{\text{out}}$ rather than the vertical one. As a result, the magnitude of the horizontal component of $v_{\text{out}}$ is thought to contribute to the trends of $\theta_{\text{out}}$ and $\omega_{\text{out}}$.

In order to investigate the causes of the tendency depending on the velocity components, the contact force during impact was analyzed, as shown in Figure 4. The maximum value of the force for both components tends to decrease as the height of the pimples increases. If it is assumed that the magnitude of $\omega_{\text{out}}$ increases with the maximum value of the $y$ component force, that is, the horizontal component which is estimated to contribute to an occurrence of the spin, the relationship of magnitude between $\omega_{\text{out}}$ and the $y$ component force is not based on this assumption. This indicates that the trend of $\omega_{\text{out}}$ cannot be explained only by the contact force. On the other hand, focusing on the impulse, the simulation results of the $y$ component impulse tend to be R02, R03, and R01 in the order of magnitude, which depends on the contact time and the positive and negative components of the force, as well as the maximum value of the force. The order of the $y$ component impulse tends to correspond to that of $\omega_{\text{out}}$. Although the order of the $z$ component impulse also tends to correspond to the $z$ component of $v_{\text{out}}$, the height of the pimples has a smaller effect on the behavior of the vertical component than that of the horizontal component. These trends suggest that the change in stiffness of the pimples structure with the pimple height mainly affects the horizontal component impulse, and the component tends to affect the $\theta_{\text{out}}$ and $\omega_{\text{out}}$.

![Figure 3](image-url) **Figure 3.** These figures show the simulation results of the rebound behavior of the ball with different pimple heights at incidence velocity and angle of 60 km/h and 70 degrees: (a) the $y$ and $z$ components of $v_{\text{out}}$; (b) $\theta_{\text{out}}$; (c) $\omega_{\text{out}}$. The $y$ and $z$ axes are defined as shown in Table 4.
Figure 4. These figures show the simulation results of the contact force and impulse during impact with different pimple heights: (a) the $y$ component force; (b) the $z$ component force; (c) the $y$ component impulse; (d) the $z$ component impulse.

5. Conclusions

An FE model of the sandwich rubber was constructed by introducing the experimental data obtained from static and dynamic tests on the materials of sheet and foam into the material models. High accuracy of the material models was confirmed by comparing the results of the ball rebound behavior between the experiment and the simulation under the same conditions. Impact simulation analyses were conducted using constructed models with varying pimple heights.

The results of the rebound angle and spin rate tend to be affected not by the order of the pimple height, but by the horizontal component impulse which is varied with changes in pimple height, especially the trend of the spin rate strongly depending on the trend of the horizontal component impulse.

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