Systematics of Gamow-Teller strengths in mid-$fp$-shell nuclei

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(March 31, 2022)

Abstract

We show that the presently available data on the Gamow-Teller (GT) strength in mid-$fp$-shell nuclei are proportional to the product of the numbers of valence protons and neutron holes in the full $fp$-shell. This observation leads to important insights into the mechanism for GT quenching and to a simple parametrization of the Gamow-Teller strengths important for electron capture by $fp$-shell nuclei in the early stage of supernovae.

PACS numbers: 25.40.Kv, 27.50.+e, 97.60.Lf, 23.40.Hc
Weak interactions play an important role in late stellar evolution. In particular, electron capture and nuclear beta decay are essential during the pre-supernova core collapse of a massive star, as these two processes determine, in the early stage of the collapse, the electron-to-baryon ratio. This quantity, in turn, influences both the infall dynamics and the mass of the final homologous core. Bethe et al. [1] and subsequently Fuller et al. [2] recognized that the Gamow-Teller (GT) resonance contributes significantly, and perhaps dominates, both the electron capture and beta decay rates in a collapsing star. GT transition strengths in mid-\(fp\)-shell nuclei (atomic numbers \(Z \approx 22–32\)) are therefore a key ingredient of presupernova studies.

Small \(Q\)-values usually prevent laboratory studies of electron capture and beta decay from sampling the GT strength distribution over a wide range of energies; often, the GT resonance peak cannot be probed. However, the intermediate-energy \((n, p)\) charge-exchange reaction at forward angles is dominated by the \(GT_+\) operator and so provides an alternative and energetically favorable way of determining the required strength function. In recent years the CHARGEX collaboration at TRIUMF has measured \((n, p)\) forward cross sections for several astrophysically important \(fp\)-shell nuclei. The total Gamow-Teller strengths, \(B(GT_+)\), extracted from these measurements are summarized in Table I. These values typically sum the strength to excitation energies of 8 MeV and so span the energies of astrophysical interest; it is generally believed that this low-energy range covers most of the \(GT_+\) strength.

The GT transition is naively viewed as a one-body process in which a proton is changed into a neutron by the \(\vec{\sigma}\tau_+\) operator. The measured \(GT_+\) strengths are thus usually compared to an extreme single particle model [3], in which the nuclear ground state is described by non-interacting nucleons occupying the lowest possible shell-model orbitals. The \(GT_+\) strength is then estimated as

\[
B(GT_+) = \frac{1}{(2j_i + 1)}|\langle \vec{\sigma}\tau_+ \rangle|^2 = \sum_{i,j} \frac{n_i^p n_j^h}{(2j_i + 1)(2j_f + 1)} |\langle i|\vec{\sigma}\tau_+|f \rangle|^2,\]

where \(n_i^p\) is the number of protons in orbital \(i\) of the parent ground state (spin \(j_i\)), \(n_j^h\) is the number of neutron holes in the final daughter orbital (with spin \(j_f\)), and the sum is over all proton orbitals in the initial state and neutron orbitals in the final states; transitions within the inert \(A = 40\) core are blocked by the Pauli principle. As the GT operator conserves orbital angular momentum, an initial proton orbital can be connected to at most two different final neutron orbitals. The single particle matrix elements are given in Ref. [2];
they are roughly equal for the $f_{7/2}$ and $p_{3/2}$ proton orbitals that are important in the following discussion.

It is well known that the observed $GT_+$ strength is “quenched”; i.e., it is significantly smaller than the single particle estimate (1). Aufderheide et al. [3] justifiably criticized pre-supernova studies that use single-particle estimates in calculating electron-capture rates [2]. However, it is also apparent from Table 1 that the quenching factor $q$ (ratio of the single-particle estimate to experiment) varies significantly from one nucleus to another, so that Ref. [3]’s suggestion of a constant quenching, $q = 2$, is also questionable. The purpose of this Letter is to point out a remarkable systematic behavior of the $GT_+$ strengths for mid-$fp$-shell nuclei that offers both insight into the quenching mechanism and an improved estimate of astrophysical rates.

We begin by observing that the $GT_+$ strength should be proportional to the number of valence protons in the $fp$-shell, $Z_{val} = Z - 20$. For two reasons, the $GT_+$ strength also depends upon the number of valence neutrons, $N_{val} = N - 20$. First, the valence neutrons block possible transitions, as is accounted for in Eq. (1). Second, neutron-proton correlations (not included in the single-particle estimate) have been identified as a major source of quenching [4,5]. Motivated by these considerations, we divide the experimental $B(GT_+)$ values in Table 1 by the number of valence protons and plot them as a function of $N_{val}$ in Fig. 1. (We omit the lightest nucleus, $^{45}$Sc, for reasons discussed below.) As expected, the values decrease with increasing $N_{val}$ and do so roughly linearly. We are therefore led to parametrize the measured total $GT_+$ strengths for mid-shell-$fp$-nuclei as

$$B(GT_+) = a \cdot Z_{val} \cdot (b - N_{val}). \quad (2)$$

A closer inspection of the data (see Fig. 1) reveals that $B(GT_+)/Z_{val}$ for the odd-$Z$ nuclei $^{51}$V, $^{55}$Mn, and $^{59}$Co is systematically lower than the values for the neighboring even-$Z$ nuclei, so that the measurements strongly suggest an odd-even dependence. However, we believe that this behavior is caused mainly by kinematics. The $(n,p)$ experiments can reliably determine the $B(GT_+)$ strength only up to daughter excitation energies of about $E_x = 8$ MeV. However, the GT resonance appears in the $(n,p)$ spectra at systematically higher excitation energies for odd-$Z$ targets than for even-$Z$ targets [6,7]. Experiments with odd-$Z$ targets will therefore “miss” a relatively larger fraction of the total strength with $E_x > 8$ MeV.

Two additional points support our interpretation of this apparent odd-even effect. First, the data for $^{51}$V and $^{58}$Co show additional $B(GT_+)$ strength at higher excitation energies
between $E_x = 8$ and 12 MeV. Aufderheide et al. have analyzed the data on $^{51}$V, $^{54}$Fe, and $^{58}$Co, and cite $B(GT_+)$ strengths extending to higher excitation energies ($E_x \approx 12$ MeV) for $^{51}$V, $^{54}$Fe, and $^{59}$Co. While the $^{54}$Fe result coincides with the value summed to $E_x = 8$ MeV given in Ref. [16], their values for $^{51}$V and $^{59}$Co are both noticeably larger than those of Ref. [15] (which only give the strengths up to $E_x = 8$ MeV, see Table I) and both agree well with the linear systematics deduced for the even-$Z$ nuclei (Fig. 1 and below). Second, we discuss below recent shell model calculations that agree well with the experimental $B(GT_+)$ values for even-$Z$ nuclei and do not exhibit an odd-even dependence [9]. For the only odd-$Z$ nucleus studied by these methods to date, $^{55}$Mn, the calculated total $B(GT_+)$ per valence proton, $B(GT_+)/Z_{\text{val}} = 0.44$, is in accord with the results for the neighboring even-$Z$ nuclei $^{54}$Cr and $^{56}$Fe. Moreover, the shell model study finds the centroid of the $B(GT_+)$ strength in $^{55}$Mn at higher daughter excitation energies ($E_x \approx 2.4 \pm 1.6$ MeV) than in $^{56}$Fe ($E_x \approx -0.4 \pm 0.2$ MeV), in agreement with both the data and our interpretation.

We therefore suggest that the total $B(GT_+)$ strengths of mid-$fp$-shell nuclei follow the simple parametrization as given in Eq. (2), while the odd-even behavior of the data reflects the experimental excitation energy cut-off at around 8 MeV. As can be seen from the line in Fig. 1, the presently available $B(GT_+)$ data are well-fitted by Eq. (2) if the “total” $B(GT_+)$ values for $^{51}$V and $^{59}$Co given in Ref. [6] are used. We then find the best-fit values $a = (4.55 \pm 0.25) \times 10^{-2}$ and $b = 19.54 \pm 0.32$ with $\chi^2 = 1.0$ per degree of freedom. A similarly good fit is obtained if $b$ is constrained to be 20, where we find $a = (4.29 \pm 0.15) \times 10^{-2}$ with $\chi^2 = 1.1$ per degree of freedom. The parameter $a$ can be interpreted as an average matrix element. Note that fitting the same $B(GT_+)$ data to Eq. (1) multiplied by a constant quenching factor results in $q \approx 3.7$ with $\gamma^2 = 2.4$ per degree of freedom, which is a noticeably worse fit to the data than provided by Eq. (2).

We expect our parametrization (2) to be valid for nuclei between $^{48}$Ti and $^{70}$Ge; i.e., for $Z$ between 22 and 32 and $N$ between approximately 26 and 38. Most of the nuclei whose electron capture and beta-decay rates govern the late stage of stellar evolution are within these ranges. However, it should be noted that our parametrization has been derived solely from experiments with even-$N$ targets, so that its validity for odd-$N$ nuclei remains to be verified. The parametrization is clearly not valid for nuclei with neutron numbers in excess of 40 (which require the inclusion of the $g_{9/2}$ orbital, but will have very small $GT_+$ strengths for $Z = 22–32$) nor for nuclei with only a few valence nucleons, where we do not expect the proton-neutron correlations to be fully developed. For example, for $N_{\text{val}} = 0$
$B(\text{GT}_+)/Z_{\text{val}} = 3$ and there should be no quenching. For $^{45}\text{Sc}$ ($Z_{\text{val}} = 1$) the experimentally observed quenching ($q = 1.1$) is also much less than predicted by Eq. (2) ($q = 3.4$).

Several interesting conclusions follow from the validity of the parametrization (2).

1. The GT$_+$ strength is proportional to $(20-N_{\text{val}})$, the number of neutron holes in the full $fp$-shell. This indicates that the strength is determined by the total number of holes, rather than by the numbers of holes in individual subshells (as is assumed in Eq. (1)), and suggests that proton-neutron correlations are a significant determinant of the GT$_+$ strength. GT$_+$ strengths of mid-$fp$-shell nuclei apparently behave as though there was only one large shell in which all sub-shell structures have been diluted.

2. The $N_{\text{val}}$-dependence of our parametrization suggests that correlations introduced by higher shells (e.g., the $g_{9/2}$ orbital) do not significantly change the $B(\text{GT}_+)$ values for the range of nuclei covered by the empirical formula (2).

3. The apparent sensitivity of $B(\text{GT}_+)$ to the number of neutron holes in the full $fp$-shell suggests that shell model calculations attempting to reproduce the GT strength must include the full $fp$-shell model space. This observation is in accord with recent complete $0h\omega$ $fp$-shell Monte Carlo calculations [9], which show quenching significantly greater than that calculated in restricted ($2p-2h$) model spaces due to proton-neutron correlations [11]. In fact, these calculations, employing the Brown-Richter interaction [11], yield $B(\text{GT}_+)$ values for $^{54}\text{Cr}$, $^{54}\text{Fe}$, $^{55}\text{Mn}$, and $^{56}\text{Fe}$ that agree well with experiment and with the empirical parametrization (see Fig. 1). Only the shell model value for $^{58}\text{Ni}$ is significantly larger than experiment, a result that is quite sensitive to the Hamiltonian assumed. A recent complete $0h\omega$ direct diagonalization of $^{48}\text{Ti}$ yields a $B(\text{GT}_+)$ value of $1.26 = 0.045Z_{\text{val}}(20 - N_{\text{val}})$ [12], again in agreement with the data and Eq. (2).

4. Our attempt at a similar parametrization for $sd$-shell nuclei failed, indicating that the GT strengths in these nuclei are more sensitive to the sub-shell structure.

In summary, we have observed that presently available ($n,p$) data for mid-$fp$-shell nuclei show that the total GT$_+$ strength is proportional to the product of the number of valence protons and the number of neutron holes in the full $fp$-shell. This observation suggests the importance of neutron-proton correlations throughout all of the $fp$ orbitals in reproducing the observed quenching. It also leads to a simple empirical parametrization of the GT$_+$.
strength valid for all nuclei between $^{48}$Ti and $^{70}$Ge. As the electron capture and beta-decay rates for nuclei in this range determine the early stages of a supernova collapse, our parametrization will help to reduce the uncertainties in pre-supernova studies. We also note that the odd-even dependence of the excitation energy of the GT resonance should be quite important in astrophysical applications, but has apparently been neglected in supernova studies to date.

ACKNOWLEDGMENTS

We thank the CHARGEX collaboration for providing us with their $(n, p)$ data prior to publication and are grateful to Petr Vogel for helpful discussions. This work was supported in part by the National Science Foundation, Grants No. PHY91-15574 and PHY90-13248.
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FIGURES

FIG. 1. Plot of the experimental Gamow-Teller strength, $B_{\text{GT}^+}$, per valence proton as a function of the number of valence neutrons. Open symbols with error bars denote data summed to $E_x \approx 8$ MeV, while the full symbols represent the Gamow-Teller strengths summed to $E_x \approx 12$ MeV for the odd-$Z$ nuclei $^{51}$V and $^{59}$Co [3]. Symbols without error bars denote recent full-space shell model results, as discussed in the text. The line shows the best fit to Eq. (2) when $b$ is constrained to be 20 and the adjusted data for $^{51}$V and $^{59}$Co are used.
TABLES

TABLE I. Comparison of measured Gamow-Teller strengths for $E_x < 8$ MeV extracted from $(n, p)$ data on midshell $fp$-nuclei with the prediction of the single particle model (1). The quenching factor $q$ is defined as the ratio of these two quantities. For $^{51}$V and $^{59}$Co the “total” Gamow-Teller strength for $E_x < 12$ MeV as given in Ref. 6 is also listed.

| Target | $(Z, N)$ | $B(GT^+)$ | Single particle estimate | $q$ |
|--------|----------|-----------|-------------------------|-----|
| $^{45}$Sc | (21,24) | 2.1 | 2.36 | 1.1 |
| $^{48}$Ti | (22,26) | 1.31±0.2 | 4.07 | 3.1±0.5 |
| $^{51}$V | (23,28) | 1.2±0.15 | 5.14 | 4.3±0.4 |
| | | 1.48±0.15 | | 3.5±0.4 |
| $^{54}$Fe | (26,28) | 3.1±0.6 | 10.29 | 3.3±0.6 |
| $^{55}$Mn | (25,30) | 1.72±0.2 | 8.57 | 5.0±0.5 |
| $^{56}$Fe | (26,30) | 2.85±0.3 | 10.29 | 3.6±0.4 |
| $^{58}$Ni | (28,30) | 3.76±0.40 | 13.71 | 3.6±0.2 |
| $^{59}$Co | (27,32) | 1.9±0.2 | 12.02 | 6.3±0.4 |
| | | 2.39±0.25 | | 5.0±0.5 |
| $^{70}$Ge | (32,38) | 0.84±0.13 | 2.67 | 3.2±0.5 |
This figure "fig1-1.png" is available in "png" format from:

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