Analytical determination of the soil strength parameters by the number of impacts of the dynamic instrument falling weight

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Abstract. The analytical connection of the number of impacts with the energy of gravity forces of the falling weight and instrument has been determined. The energy realized when striking the rod has two components: energy of the free falling weight when moving relative to the casing which occurs before the moment of the impact with the collar, and energy of the gravity forces of the weight and casing, which arises additionally while vertical lowering to the rod penetration depth in the transition process. The soil strength is numerically determined as per the normal stress value at the end of the rod being penetrated into the soil. The numerical determination of the normal stresses has been performed by means of the theorem on the mechanical system motion quantity change and theorem on the kinetic energy change. The identity of the numerical figures of the normal stress value at the end of the rod and specific energy of the impact penetration of the cylindrical rod into the test soil has been determined. The analytical connection of the impact energy and soil strength parameters with the dynamic instrument parameters has been obtained – with the falling weight mass, instrument casing mass, weight falling height, rod penetration depth, instrument geometric parameter. For the first time in relation to the impact instrument the loss of the kinetic energy and efficiency of the falling weight impact process in case of the direct inelastic impact of two bodies has been determined according to Carnot theorem. The values of the soil deformation specific energy and of normal stresses under the flat end of the rod for soil classes in the function of the number of the dynamic instrument impacts have been determined.

1. Introduction
In mechanics for the strength determination of soils and materials the method of the impact penetration of a cylindrical rod of a specified length and diameter into soils and materials to be tested is used. The group of authors: Se-Na Lee, Bum-Jae You, Mee-Seub Limb, Sang-Rok-Oh, Song-Soo Han, Sang Heon Lee in work [1] investigate the process of the block foundation creation by means of pile driving into the soil. For the visual observation of the pile driving a high-speed linear-scanning camera is used. The penetration conditions are characterized by the oscillation frequency level equal to 20 kHz, where the striking part rebound is possible at the end of the driving process which is called the “pile driving resistance”.
The authors Charles P. Aubeny, Han Shi in work [2] have investigated the influence of the falling weight speed, soil strength on the process of the cylindrical rod penetration into seabed. The authors
have determined that the penetration depth of a cylindrical body into seabed depends on the striking mass penetration speed. The calculation models of the cylindrical body penetration into seabed take into consideration effective forces, speeds and deformations. Simplified models are able to ensure a close coincidence with the laboratory test results. In work [2] the error value obtained in the result of analytical modeling is not disclosed.

In work [3] U. Dayal has investigated the instrument for the determination of the penetration depth of a cylindrical body with a cone stamp on the end face. According to the instrument test results with a free falling weight, analytical dependences have been proposed characterizing the cylindrical body penetration depth taking into account the resistance at the conical end and bush friction.

In work [4] the authors V. N. Tarasov, S. M. Kuznetsov considered the process of the cylindrical rod penetration into the soil by a free falling weight to a specified depth. The problem of determining stresses on a lateral surface of the soil conical core being formed under the flat end of the penetrated rod has been solved analytically. It has been determined that on the conical surface at the rod end the normal stresses are half as much as the normal stresses under the flat end of the rod.

By means of the dynamic instrument in work [5] I. V. Boyarkina considers the process of the cylindrical rod penetration into the soil to a specified depth by a free falling weight. It has been determined that the total work during the cylindrical rod penetration is equal to the work of the free falling weight with the mass \( m_1 \) during the free falling and work of two masses: of the weight with the mass \( m_1 \) and mass \( m_2 \) of the instrument housing during the rod movement to a specified depth. The relation of the number of impacts of the falling weight with the soil resistance to scooping during its excavation by bucket working units has been determined.

On the basis of the standard design dynamic instrument [6] A.N. Zelenin has developed the classification of the soil strength classes relative to the number of the dynamic instrument impacts. The instrument and proposed classification of soils have been proved to be successful ones and have gained widespread practical application. However, further improvement of the instrument and extension of its application fields is restrained by the lack of the theory of processes occurring during the impact interaction of the instrument rod with the soil in the impact process. In work [6] there are no analytical expressions of the energy balance of the instrument active gravity forces and expressions of the work of the resistance forces during the rod impact penetration into the soil on one impact.

The performed review shows that the number of impacts of the falling weight in the instrument characterize the strength and are proportional to the soil strength, at the same time the soil parameters are normalized as per the State Standard GOST 30067-93 [7]. The lack of analytical expressions characterizing the impact process is explained by the lack of data in GOST 30067-93 relative to the instrument casing mass for measuring the soil strength and dimensions of the instrument elements which characterize this mass. It’s necessary to normalize the falling weight mass \( m_1 \) and mass \( m_2 \) of the instrument casing by definite values, as they are important parameters of the dynamic instrument.

The analytical expressions characterizing the instrument workflow are necessary for further extension of the instrument application fields, for example, for the determination of the soil strength characteristics during the compaction by different static and vibration methods.

2. Task definition
To obtain analytical equations connecting the energy of active forces with the energy of resistance forces at the end of the rod being penetrated into the soil on the basis of the energy balance of the dynamic instrument operating forces. To determine the analytical dependence of the energy of active forces on the dynamic instrument parameters, justify the strength characteristics of the soils to be tested and specific energy of the soil deformation on impact. To define the soil strength characteristics in the form of numerical values of the normal stresses at the rod end, determine dependences of the dynamic instrument parameters with the deformation energy during the impact penetration of the rod into the soil to be tested.

3. Theory
The construction soil strength is generally assessed by the number of impacts of the dynamic instrument during the cylindrical rod penetration into the soil with the diameter \( d=0.0113 \) m, length
$S = 0.1 \text{ m}$, rod end sectional area is equal to $A = 1 \text{ cm}^2$. The reference weight of the instrument with the mass $m_1 = 2.5 \text{ kg}$ freely falls from the height $H = 0.4 \text{ m}$, delivering an impact and performing work on each impact.

The soil strength determination instrument consists of the falling weight with the mass $m_1$ and casing with the mass $m_2$, ensuring the vertical free falling of the weight from the height $H$ (Figure 1).

![Figure 1. Design diagram of the instrument parameters intended for the soil strength determination](image)

Soils at a positive temperature can have five strength classes (table 1) [8, 9].

| Soil class | I    | II   | III  | IV   | V    |
|------------|------|------|------|------|------|
| Number of impacts, $C_{imp}$ | 1-4  | 5-8  | 9-16 | 17-34| 35-70|
| Relative change of impact number | 4    | 1.6  | 1.78 | 2    | 2    |

Table 1 is supplemented by the line of the relative change in the number of impacts for each class of soils.

The number of impacts $C_{imp}$ characterizes objectively enough the soil ability to resist the impact effects during the cylindrical rod penetration into the soil. The number of impacts is directly included in the formula for the determination of the specific resistance of the soil to cutting and digging, as it is not just some quantitative measure, but it has the specific energy content and is a definite reference of the impact effect on the soil [5]. The reference weight during the free falling from the height $H = 0.4 \text{ m}$ acquires the speed $V = \sqrt{2gH}$.

The weight free falling process is complete at the time of the weight contact with the collar on the instrument casing. At this moment the direct inelastic impact and immediate speed transformation of the falling weight into the joint movement speed of two masses [8] occur

$$U = V - \frac{m_1}{m_1 + m_2},$$

(1)

where $V, U$ – respectively the falling weight speed at the end of the free falling and initial speed of the joint movement of two masses after the impact during the cylindrical rod penetration into the soil.

After the impact process completion the transition process of the rod penetration into the soil takes place. At the same time the kinetic energy of two masses acquired on impact is consumed for overcoming resistance forces during the cylindrical rod penetration into the soil.

According to researches [1] the striking part rebound from the pile takes place at the striking part speed of more than $5 \text{ m/s}$, that’s why in the instrument to be considered the direct inelastic impact is implemented without the rebound of the mass $m_1$ at the speed of the weigh movement equal to $U = 2.801 \text{ m/s}$. 
According to the momentum theorem the momentum before the impact is equal to the momentum after the impact

\[ m_1V = (m_1 + m_2)V' \quad \text{(2)} \]

During the direct inelastic impact of two bodies the kinetic energy loss takes place

\[ \Delta T = \frac{m_1V^2}{2} - \frac{(m_1 + m_2)V'^2}{2} \quad \text{(3)} \]

The energy relative loss value on impact is equal to

\[ \delta_{\text{imp}} = \frac{m_1V^2 - (m_1 + m_2)V'^2}{m_1V^2} = \frac{V^2 - (1 + m_2/m_1)U^2}{V'} \quad \text{(4)} \]

According to formula (4) for the impact measuring instrument with the parameters \(m_1=2.5\, \text{kg}, m_2=0.565\, \text{kg}, H=0.4\, \text{m}, V=2.801\, \text{m/s}, U=2.285\, \text{m/s}\) the energy relative loss is determined by the formula

\[ \delta_{\text{imp}} = 1 - (1 + m_2/m_1) \frac{U^2}{V'} = 0.1141 \text{ or } 11.41\% \quad \text{(5)} \]

As it is shown in works [5, 8] the energy of the rod penetration into the soil is determined by two components: during the weight free falling from the height \(H=0.4\, \text{m}\) and additional energy of the lowering into the soil of two masses during the process of the cylindrical rod penetration

\[ A = m_1gHC_{\text{imp}} + (m_1 + m_2)gS_z, \quad \text{(6)} \]

where \(S_z\) – length of the cylindrical rod to be penetrated into the soil \(S_z=0.1\, \text{m}\).

The work of the active forces determined by formula (6) is consumed for overcoming the resistance forces. Under the flat end of the rod the normal stress distribution diagram \(\sigma_z\) appears (see Figure 1). In the impact effect process the contact stresses \(\sigma_z\) instantly acquire the limit values \(\sigma_z = \sigma_{\text{max}}\), whereby the soil starts to move laterally releasing the space under the cylindrical rod end.

In Figure 2 the stress change characteristic \(\sigma_z\) under the rod end is shown.

![Figure 2. Stress change characteristic under the rod end during the penetration into the soil](image)

The preliminary deformation value \(\Delta z\) during the process of impact is a small value, after which realization the transition process occurs of the material flow in the transverse direction relative to the rod movement. As it is known, the process of the material flowability is performed during the constant acting force and constant stress [8].

In the transition process of the cylindrical rod penetration into the soil the energy transformation of the active forces into work takes place which is consumed for overcoming the resistance forces. As shown above, the work of the active forces of gravity during the rod penetration process is determined by formula (6).

The work of the resistance forces during the rod penetration is determined by the formula

\[ A^* = \frac{\pi d^2}{4} \sigma_z S_z \quad \text{(7)} \]

According to the theorem of kinetic energy change the considered works are always equal. Comparing expressions (6) and (7) one can determine the normal stresses under the rod end.
Formula (8) can be given the form of the linear function

$$\sigma_z = \frac{m_i H C_{\text{imp}} + (m_1 + m_2)g S_z}{\pi d^2 S_z},$$

(8)

Formula (9) shows that the normal stresses $\sigma_z$ represent the linear analytical function. At the same time, the numerical value of the normal stresses $\sigma_z$ is nothing other than the ratio of the deformation energy to the deformation volume by formula (8), which has been named as the specific energy [5].

4. Research results

Figure 3 shows the dependence of the normal stresses $\sigma_z$ under the rod end on the number of impacts $C_{\text{imp}}$ for soils of the 1st - 5th classes.

In this case the impact instrument for the soil strength determination represents a technical example wherein the volume of the soil being deformed is equal to the volume of the cylindrical rod to be penetrated into the soil with the diameter $d$ and height $S_z$ (see Figure 1).

That’s why in formula (8) the normal stresses $\sigma_z$ at the same time represent the characteristic of the soil strength and physical parameter, i.e. the work assigned to the deformation volume,

$$A_{\text{imp}} = \frac{A}{V},$$

(10)

where $A_{\text{imp}}$ – specific work per the volume unit, J/m³; $A$ – deformation work; $V$ – volume of the soil deformation equal to the rod volume to be penetrated into the soil.

The normal stresses under the rod end $\sigma_z = \frac{4P}{\pi d^2}$.

In this case in Figure 3 the numerical values of the normal stresses $\sigma_z$ and specific work are identical

$$A_{\text{imp}} = \sigma_z.$$  

(11)

The dimensionality of the normal stresses $\sigma_z$ [Pa] is transformed into the dimensionality of the specific energy [J/m³] with the retention of numerical values.

For the 1st-5th class soils table 2 gives data for the total work $A$ to be performed by the striker; mean force of impact $P=A/S_z$; normal stresses $\sigma_z$ on the flat end of the rod; specific deformation work $A_{\text{imp}} = A/V$; rod movement per one impact $\Delta S_z$; instrument weight acceleration $\ddot{z}_{\text{max}} = \frac{P}{m_1 + m_2}$.

| Soil class | Number | Total | Mean force | Stress under | Specific | Rod movement | Instrument |
|------------|--------|-------|------------|--------------|----------|--------------|------------|

Table 2. Parameters characterizing the soil strength of different classes
Table 2 shows that the dynamic instrument allows you to determine the set of the most important parameters characterizing the soil strength.

The impact process efficiency of the dynamic instrument can be determined as the ratio of useful work to consumed work during the rod penetration in one impact

$$\eta_{imp} = \frac{m_1 V^2 - (m_1 + m_2)(V - U)^2}{m_1 V^2} = 1 - \left(1 + \frac{m_2}{m_1}\right)(1 - U/V)^2. \quad (12)$$

For the instrument parameters the impact process efficiency is equal to: $$\eta_{imp} = 1 - \delta_{imp} = 0.8859.$$

In table 2 the striker’s peculiarity has been found out lying in the fact that its basic parameters have been determined in such a manner that the number of impacts of the dynamic instrument $C_{imp}$ approximately corresponds to the value of stresses $\sigma_z [MPa]$ and specific energy $A_{imp} [MJ/m^3]$.

In the result of the determination of the number of impacts the averaging of soil properties with respect to the depth $S_z = 0.1 m$ takes place.

The most effective application field of the dynamic instrument is connected with the determination of the strength of soil foundations during the compaction with static and vibration rollers.

The obtained results allow to describe the transition process of the striker rod penetration into the soil massive in one impact. The initial conditions of the weight movement process with the rod during the impact penetration into the soil appear as follows: at $t=0$ $z_0 = 0$; $\dot{z}_0=U = 2.285 \ m/s$.

After the impact of the weight of the mass $m_1$ with the instrument casing of the mass $m_2$ (see Figure 1) both masses move down according to the differential equation

$$\ddot{z} = g - \frac{\sigma_z}{m_1 + m_2} \frac{zd^2}{4}, \quad (13)$$

where $g$ – free fall acceleration of bodies.

In equation (13) the force on the rod end to be created by the normal stress $\sigma_z$ is considered as the resistance force $P$ (see Figure 1).

As the normal stresses $\sigma_z$ under the rod end for different soil classes have been determined, then equation (13) allows to describe the transition process of the rod movement $z$ in one impact for all soil classes.

For the 1st class soil with the number of impacts $C_{imp} = 1$ the first and second integrals of differential equation (13) appear as follows

$$\dot{z} = -32.0065t + U; \quad z = -32.0065\frac{t^2}{2} + Ut. \quad (14)$$

For the 5th class soil with the number of impacts $C_{imp} = 70$ the first and second integrals appear as follows

$$\dot{z} = -2240.46t + U; \quad z = -2240.46\frac{t^2}{2} + Ut. \quad (15)$$
Figure 4 shows the transition processes of the speed $\dot{z}$ and movement $z$ during the cylindrical rod penetration into the soil. Equations (14), (15) allow to determine the time $\tau$ of the transition process of the rod penetration into the soil and movement of the rod $\Delta z_{\text{max}}$, corresponding to one impact.

Figure 4. Dependence of the speed $\dot{z}$ and depth $z$ of the cylindrical rod penetration into the soil with the parameters $m_1=2.5$ kg, $m_2=0.565$ kg, $H=0.4$ m on the time $t$:

- a) 1$^\text{st}$ class soil, $C_{\text{imp}}=1$;
- b) 5$^\text{th}$ class soil, $C_{\text{imp}}=70$

In Figure 4.a for the 1$^\text{st}$ class soil the movement on one impact $C_{\text{imp}}=1$ relative to analytical dependencies has been equal to $z_{\text{max}}=0.08154$ m, transition process time $\tau=0.071$ s. In Figure 4,b for the 5$^\text{th}$ class soil at $C_{\text{imp}}=70$ the movement $z_{\text{max}}=0.001165$ m, transition process time $\tau=0.00101$ s.

5. Discussion of results

Such results for the dynamic instrument has been received for the first time and they show that under given conditions the rod movement relative to the analytical dependencies differs from the values in Table 3.

The indicated deviations of the rod movements within the limits $\delta_c=18.46\%$ and $\delta_c=18.54\%$ for the 1$^\text{st}$ and 5$^\text{th}$ class soil are explained by losses in kinetic energy on a direct central inelastic impact equal to $\delta_{\text{imp}}=11.41\%$. That’s why the actual deviation of the obtained analytical results is equal to $\delta\%=\delta_c-\delta_{\text{imp}}=7.1\%$.

The number of impacts of the dynamic instrument represents an integer value, that’s why the absolute probability of an error for all soil classes is similar $\Delta C_{\text{imp}}=1$, at the same time the actual error of measurement stipulated by the integrality of the parameter $C_{\text{imp}}$ represents a variable value (see Table 3).

Table 3. Relative measurement error of the number of impacts $C_{\text{imp}}$ for all soil classes at $\Delta C_{\text{imp}}=1$

| Soil classes | I | II | III | IV | V |
|-------------|---|----|-----|----|---|
| Number of impacts, $C_{\text{imp}}$ | 1-4 | 5-8 | 9-16 | 17-34 | 35-70 |
| Error stipulated by integrality $C_{\text{imp}}$, $\delta\%$ | 50-25 | 20-12.5 | 11.1-6.25 | 5.88-2.94 | 1.8-1.42 |
The measurement error of the number of impacts connected with the measurement integrity $C_{imp}$ is changed insignificantly. The average value of this error is commensurately with the analytical calculation error, which is practically constant for all soil classes. Despite the indicated peculiarities of the instrument the number of impacts required for the rod penetration into the soil is a sufficiently reliable comparative assessment of the soil strength.

Analytic expressions have been obtained which connect the normal stresses $\sigma_z$ at the end of the rod being penetrated into the soil with the masses of the falling weight and instrument casing, weight falling height and instrument parameters. The identity of the mean numerical values of the stresses $\sigma_z$ and specific energy $A_{imp}$ of the rod penetration into the soil has been shown. As for the direct inelastic impact of two masses the kinetic energy loss on impact and efficiency of the impact process have been obtained. The transition process of the rod penetration into the ground after the impact is described by the differential equation with the constant right-hand side. The first integral of the equation is the speed linear function $z\dot{z}$, second integral $z$ of the rod movement is a second-degree equation. The results to be obtained, i.e. rod movement in one impact and time $\tau$ of the transition process of the instrument represent important characteristics of the dynamic instrument.

6. Summary and conclusion
The analytical method of the calculation of the cylindrical rod penetration process into the soil allows to improve the instrument parameters aimed at determining characteristics of the soil strength and expand the number of parameters of the instrument impact process characterizing the strength of construction soils.

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