Resonant intrinsic spin Hall effect in p-type GaAs quantum well structure

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We study intrinsic spin Hall effect in p-type GaAs quantum well structure described by Luttinger Hamiltonian and a Rashba spin-orbit coupling arising from the structural inversion symmetry breaking. The Rashba term induces an energy level crossing in the lowest heavy hole subband, which gives rise to a resonant spin Hall conductance. The resonance may be used to identify the intrinsic spin Hall effect in experiments.

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The study of spin Hall effect (SHE), in which an electric field induces a transverse spin current, has recently evolved into a subject of intense research for its potential application to the information processing. The intrinsic SHE was proposed by Murakami et al.\textsuperscript{1} in p-type semiconductor of a Luttinger Hamiltonian and by Sinova et al.\textsuperscript{2} in 2-dimensional (2D) electron systems with Rashba spin-orbit coupling. Their works have generated a lot of theoretical activities.\textsuperscript{3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19} Current theoretical understanding is that the intrinsic SHE does not survive in the diffusive transport in the thermodynamic limit for the 2D Rashba electron system\textsuperscript{20} in the absence of strong magnetic fields\textsuperscript{21, 22}, but the effect appears to be robust in the 2D hole gases\textsuperscript{15, 16}, p-doped bulk semiconductors and the modified Rashba coupling case\textsuperscript{17}. The earlier theoretical work on the extrinsic SHE is associated with the impurity scattering, such as the skew scattering and the side jump processes\textsuperscript{20, 21, 22}. On the experimental side, there have been two groups reporting the observation of spin Hall effect. Kato et al.\textsuperscript{23} used Kerr rotation microscopy to detect and image electrically induced electron-spin polarization near the edge of a n-type semiconductor channel. The effect was suggested to be extrinsic based on the weak dependence on crystal orientation for the strained samples. Wunderlich et al.\textsuperscript{24} observed the SHE in 2D hole system with spin-orbit coupling, and interpreted the effect to be intrinsic. In view of the unfamiliarity of the spin Hall transport, it will be desirable and important to experimentally identify if the observed SHE is intrinsic. Such an identification requires careful study of properties of the intrinsic SHE.

In this Letter we study the intrinsic SHE in p-type GaAs quantum well structure described by a Luttinger Hamiltonian with a Rashba spin-orbit coupling arising from the translational symmetry breaking. The Rashba term hybridizes the electronic sub-bands of the Luttinger Hamiltonian in a quantum well and induces energy level crossings in the both heavy and light hole subbands. The level crossing, if it occurs at the Fermi level, gives rise to a resonant intrinsic SHE characterized by a sharp peak and a sign change in the spin Hall conductance. By tuning the Rashba coupling strength and/or carrier density, this type of resonance may be observed in experiment to distinguish the intrinsic from the extrinsic SHE. In the latter case, one does not expect such drastic changes. The sign of the extrinsic SHE induced in the skew scattering, which dominates over the side jump process in the weak disorder limit of our interest here, depends on the sign of the impurity potential, and does not change with changing carrier density or the Rashba coupling strength.

We consider an effective Hamiltonian for the hole doped quantum well with the structural inversion symmetry breaking, described by the Luttinger Hamiltonian with a confinement potential along the z direction and an additional Rashba coupling term,

\[ H = H_L - \lambda (\hat{z} \times \vec{p}) \cdot \vec{S} + V(z) \]

where \( \lambda \) is the Rashba spin-orbit coupling, \( \vec{p} \) is the momentum, and \( \vec{S} = (S_x, S_y, S_z) \) are the spin-3/2 operators. \( V(z) \) is a confinement potential along the z-direction. For simplicity, we choose \( V(z) = +\infty \) for \( |z| > L \) and \( V(z) = 0 \) otherwise. Note that we have assumed that the only effect of the structural inversion symmetry breaking is to induce a Rashba coupling term in Eq. (1), and any asymmetry in \( V(z) \) has been neglected. We expect that the simplification of \( V(z) \) will not alter the qualitative physics we discuss below. \( H_L \) is the Luttinger effective Hamiltonian describing the hole motion in the valence band,

\[ H_L = -\frac{\hbar^2}{2m} \left( \gamma_1 + \frac{5\gamma_2}{2} \right) \nabla^2 + 2\gamma_2 (\vec{S} \cdot \nabla)^2 \]

Since the translational symmetry is broken only along the z direction, the momentum \( \hbar \hat{k} \) in the \( x-y \) plane remains to be a good quantum number. For a given \( \hat{k} \), \( H \) can be reduced to a 1D effective Hamiltonian

\[ H_{\hat{k}} = \frac{\hbar^2}{2m} (k^2 - \partial_z^2 ) (\gamma_1 + \frac{5\gamma_2}{2}) + V(z) \]

\[-\frac{\hbar^2 \gamma_2}{m} (S_x k_x + S_y k_y - S_z \partial_z)^2 - \lambda h(k_y S_x - k_x S_y), \]

In the special case \( k = 0 \), \( S_z \) is a good quantum number
and the eigen wave-functions of $H_k$ are found to be,

$$\Psi_{an}(z) = \begin{cases} \cos(q_n z)\chi_a, & n = \text{odd} \\ \sin(q_n z)\chi_a, & n = \text{even.} \end{cases} \tag{4}$$

with $q_n = n\pi/2L$, and $n$ being positive integers. $\chi_a$ is the eigenstate of $S_z$ corresponding to the eigenvalue of $S_z^2 = a = 3/2, -3/2, 1/2, -1/2$. The eigenstates are two-fold degenerate corresponding to the eigenvalues

$$E_{\pm 3/2, n} = \hbar^2 q_n^2/2m_{hh}$$

$$E_{\pm 1/2, n} = \hbar^2 q_n^2/2m_{lh}, \tag{5}$$

with $m_{lh} = m/(\gamma_1 + 2\gamma_2)$ and $m_{hh} = m/(\gamma_1 - 2\gamma_2)$ the effective masses for light and heavy hole subbands, respectively. The splitting of the heavy and light hole subbands at $k = 0$ is due to the $\gamma_2$ term in $H_L$. Note that the Rashba coupling term vanishes at $k = 0$.

For $\vec{k} \neq 0$, $S_z$ is no longer a good quantum number, and the two-fold degeneracy splits and all the heavy and light hole subbands mix to each other in general. Two limiting cases were considered previously. One is the limit $2kL/\pi \ll 1$, while the Rashba coupling $\lambda$ is finite. This case was studied by Schliemann and Loss, who used a lowest order perturbation theory to derive a simplified effective theory by approximately projecting the full Hamiltonian to the lowest heavy hole subband. The splitting of the lowest heavy hole subband due to the Rashba coupling was found to be proportional to $k^2$, which gives the value of the spin Hall conductance of the order of $9e/8\pi$. In this limit, the spin Hall effect is purely contributed from the Rashba term. The other limiting case is $\lambda = 0$, which was considered by Bernevig and Zhang, who calculated the spin Hall conductance by including both the lowest heavy hole and light hole subbands. The spin Hall effect in this case is purely caused by the Luttinger type spin-orbit coupling.

Below we shall study the electronic structure of Eq. (1) at a finite $\vec{k}$ and $\lambda$. We use basis wavefunctions of the $k = 0$ eigenstates, given by Eqs. (4), and apply a truncated method, in which only $N$ basis states of the lowest energies of Eqs. (5) are kept. We then diagonalize $\hat{H}_k^\perp$ within this truncated Hilbert space by numerical means. As $N$ increases, the eigen energies of the lowest subbands converge quickly. In Fig. 1(a) we plot the lowest four subbands in the Rashba free case, namely HH1, LH1, HH2 and HH3 from the bottom to top, with a double degeneracy for each subband. Here HH and LH denote heavy hole and light hole respectively. In our calculations, we use $m = \gamma_1 = 7.0, \gamma_2 = 1.9$. With this choice of the parameters, the correct band structure of the sub-bands are reproduced, and the results are in good agreement with the previous calculations using the evolope function method. The Rashba term lifts the double degeneracy of each sub-band at finite $k$, as shown in Fig. 1(b) and (c). For $k \ll \pi/2L$, the energy splitting is found to be proportional to $k^2$ for the HH1 band and to $k$ for the LH1 subband, consistent with the previous study based on the leading order perturbation around the $\Gamma$ point. With the increment of $k$, the interplay between the Rashba and Luttinger type spin-orbit couplings leads to a level crossing within the sub-bands. For relatively small Rashba coupling ($\lambda = \hbar^2/mL$), the level crossing occurs only in the LH1 subband. While for large Rashba coupling ($\lambda = 3\hbar^2/mL$), level crossings are found in both LH1 and HH1 subbands. A careful analysis reveals that HH2 subband is important to the level crossings in both HH1 and LH1 subbands.

We now discuss the spin Hall conductance. We consider a linear response of the spin current tensor component $j_{s, x}$ to a transverse electric field along the $y$-direction, where we define $j_{s, x} = (\nu_x S_x + S_x \nu_x)/2$, and $\nu_x = \partial H/\partial p_x$ is the $x-$ component of the velocity operator. The spin Hall conductance can be calculated by using Kubo formula,

$$\sigma^z_{s} = -\frac{2e}{\hbar} \int \frac{d^2 \vec{k}}{(2\pi)^2} \sum_{i' < n} f(E_{i', \vec{k}}) - f(E_{i, \vec{k}}) \frac{(E_{i', \vec{k}} - E_{i, \vec{k}})^2}{(E_{i', \vec{k}} - E_{i, \vec{k}})^2} \times Im \langle i | j_{s, x}^{i'} | i \rangle \langle i' | v_y | i \rangle \tag{6}$$

where $f$ is the Fermi distribution function, and $E_{i, \vec{k}}$ is the energy of the $i^{th}$ subband with the in-plane momentum $\vec{k}$. The calculated spin Hall conductance at zero temperature as a function of $\lambda$ and hole density for GaAs quantum well is shown in Fig. 2 and Fig. 4, where we have assumed a carrier lifetime $\tau = 2.0 \times 10^{-11}s$. At a lower hole density and a large $\lambda$, there is a resonance associated with the level crossing of the HH1 subbands. At a higher hole density, the resonance is associated with the level crossing of the LH1 subbands, insensitive to the value of $\lambda$. In Fig. 2 we show $\sigma^z_{s}$ as a function of $\lambda$ for a lower hole density case. A resonance is clearly seen at $\lambda \approx 3.15\hbar^2/mL$, associated with the level crossing of the HH1 subbands at the Fermi energy. The resonance becomes a singularity in $\sigma^z_{s}$ if we use $\tau \rightarrow \infty$ and is
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The parameters corresponding to the functions of hole density in GaAs quantum well for various well thickness.

A few remarks are in order. The first is on the possible cancellation of the SHE due to the vortex correction. Recent theoretical works show that the vertex correction is severe in the 2D Rashba electron system, but is dis-

FIG. 3: The resonant Rashba coupling for the HH1 subbands as functions of hole density for various thickness $2L$ in GaAs quantum well.

smoother if $\tau$ is 10 times smaller. The resonance may be used to identify the intrinsic SHE by tuning the Rashba coupling in experiments.

In Fig. 3 we plot the resonant Rashba coupling associated with the level crossing of the HH1 subbands as functions of hole density in GaAs quantum well for various well thickness. The parameters corresponding to the resonance appears to be within the experimental region.

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FIG. 2: Spin Hall conductance of GaAs quantum well as a function of dimensionless Rashba coupling $\lambda/\lambda_0$, with $\lambda_0 = \hbar^2/mL$. The hole density $n_p = 5.0 \times 10^{11}/cm^2$ and half-thickness $2L = 83\,\AA$. A finite life time $\tau = 2.0 \times 10^{-11}$, equivalent to a mobility of $10^4 cm^2/sV$, is assumed.

In the experiment of Wunderlich et al. [25], the hole density is $2.0 \times 10^{12} cm^{-2}$ and the effective width of the quantum well can be estimated to be $2L = 83\,\AA$ by fitting the Fermi level subband splitting of the LH1 and HH1 subbands. The Rashba coupling constant can also be extracted by fitting the splitting of the HH1 subband at the Fermi level, which is approximately $\lambda = 1.5 \times 10^{-11} eV/m$. From figure 3, the required Rashba coupling for the resonance is around $8.5 \times 10^{-11} eV/m$, which is several times larger than the parameter in Wunderlich’s experiment.

In order to observe the resonance in the HH1 subband, one will need to either increase the Rashba coupling by about 6 times or to increase the thickness of the quantum well to around 200\,\AA while keeping the 2D carrier density unchanged. Note that as shown in figure 3, the resonance requires quite high mobility (around $10^4 cm^2/sV$). To observe the possible resonance associated with the level crossing in the LH1 subband, one would need to tune the

$\bar{\mu}$

$A^2$

$V$

$\sigma^x$

$\tau$

$\lambda$

$\tau$

$\lambda$

$\lambda$

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carrier density around $1.2 \times 10^{13} \text{cm}^{-2}$ or the well thickness. The value of the Rashba coupling does not play much role in this case.

![FIG. 4: Spin Hall conductance as a function of hole density for GaAs quantum well of half-thickness $2L = 83\AA$ and Rashba coupling $\lambda = \hbar^2/mL$. Dashed line is the spin Hall conductance at $\lambda = 0$.

In summary, we have studied the electronic structure and the intrinsic transverse spin transport properties of the p-type GaAs quantum well. The Rashba spin-orbit coupling arising from the structure inversion symmetry breaking splits the subbands of the Luttinger Hamiltonian, and induces level crossings within the lowest heavy hole subbands and the lowest light hole subbands. These level crossings, if occurring at the Fermi level, give rise to resonant spin Hall conductance. Our calculations show that the parameters (the hole density, the well thickness, and the Rashba coupling strength) for the resonance are likely reachable in experiments. This phenomenon can be used to distinguish the intrinsic spin Hall effect from the extrinsic one. We expect the resonant spin Hall effect associated with the heavy hole subbands be robust since the vertex correction to the heavy hole subband at small momentum has been shown to be non-severe. The robustness of the resonant effect associated with the light hole sub-bands requires careful examination of the vertex corrections. We have been benefited from many useful and stimulating discussions with Bradley Foreman, Shun-Qing Shen, Zidan Wang, Shou-Cheng Zhang, to whom we would like to thank. This work was supported in part by Hong Kong’s RGC grant and NSFC in China.

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