ERBU, Expanding Rubber Band Universe

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Abstract
I put forward a simple unidimensional mechanical analogue of the three-dimensional universe models of modern relativistic cosmology. The main goal of the proposal is the appropriate appreciation of the intrinsic relationship between Hubble’s law and the homogeneity of expanding relativistic models.

Keywords: cosmology, Hubble’s law, relativistic models, mechanical analogy

1 Introduction
The first relativistic models of the universe appeared soon after the publication, in 1915, of the final formulation of the General Relativity Theory (GRT). In 1917, Albert Einstein and Willem de Sitter presented their models and in the beginning of the 1920s Alexander Friedmann presented his [1] [2]. All of these models are solutions of the field equations of GRT for idealized universes, namely, homogeneous universes. Homogeneity simplifies enormously the form of the field equations [3]. The de Sitter and Friedmann solutions represent expanding universes (or, contracting, for one of Friedmann’s models).

The relativistic models of the universe started to have a greater impact in the scientific community with the advent of astronomical observations, which were consistent with the fundamental feature of theoretical models, i.e., their spatial homogeneity. Such observations were the result of the work of many astronomers but were synthesized and presented in a convincing way by one of them, the American astronomer Edwin Hubble, in the form of a relation that became known as “Hubble’s law”. It indicates that galaxies are receding from each other in such a way that the greater the distance between them the greater

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their recession speeds. Mathematically, the Hubble law of expansion is written as $v \propto d$, where $v$ is the galaxy recession speed and $d$ is its distance to a point of reference, namely, the observer location on Earth.

The connection between the homogeneity of relativistic models and Hubble's law of expansion is not always properly appreciated in presentations of modern relativistic cosmology. In general, it is only stressed the existence of the expansion itself and of its more immediate consequence, that is, the necessity of the existence of a beginning for the universe, derived from the extrapolation of the expansion to the past. The initial event is often called Big Bang or initial singularity.

Now, the homogeneity of any expanding body is only preserved if the expansion occurs according to a law of the type velocity of expansion proportional to distance. In other words, all points of the body must have velocities of expansion proportional to their distances to an arbitrary reference point, which is also located in the body. Homogeneity results from the fact that each part of the body expands equally along the expanding body. The uniform expansion of parts does not destroy the homogeneity of the whole. It is the cumulative effect of such small increments along the body that gives rise to a law of expansion of the type velocity proportional to distance.

The body in question here is the universe. If it expands and stays homogeneous, then its expansion must obey a law like the one described above. And that is precisely what Edwin Hubble showed in a clean and objective way at the end of the 1920s.

In the next section I discuss briefly Hubble’s law, presenting the meaning of the terms that appear in its formulation. I show in the third section an unidimensional analogue of the three-dimensional expanding models, which illustrates, in a practical and very simple way, the intrinsic relation between homogeneity and a law of the type $v \propto d$. In the final section I make some considerations about the possible perceptions of the universe, which can be, in general, theoretical and observational. The English cosmologist Edward Milne called these perceptions “world map” and “world picture”, respectively. The unidimensional analogue put forward here is considered in this context and helps the understanding of such generalizations of universe perceptions.

2 Hubble’s law

Hubble’s law is the observational foundation of a theoretical proposition, based in the GRT, known as “the expansion of the universe”. According to this proposition, the universe is expanding in such a way that galaxies are receding faster the further away they are. The discovery of Hubble’s law [4] was one of the greatest scientific achievements of the 20th century. A detailed description of the events and scientists that contributed for its discovery is presented in [5, chap. 14].

Fig. [4] is a reproduction of the diagram, presented by Hubble in his 1929 article, which is the graphical representation of Hubble’s law. There one sees
two lines that represent different fittings to the data of velocities and distances of galaxies outside the Local Group of galaxies. The continuous line is the fitting to the filled circles, which represent data of 24 galaxies. The dashed line is the fitting to the circles, which represent groupings of the 24 galaxies in 9 groups, according to the galaxy proximity in distances and on the sky plane. The fitting for both lines — with different values of their slopes — is

\[ v = H_0 d, \]  

which is Hubble’s law, where \( H_0 \) is called “Hubble’s constant”. Velocities were obtained from the galaxy spectra. The spectral lines present in the spectra are displaced to wavelengths that are systematically larger than the same spectral lines measured in the laboratory. Such displacement — called “redshift” — may be interpreted as due to the motion of recession of the galaxies with respect to the observer. In the same way, an approaching motion results in a blueshift of the spectral lines (more details in [5, chap. 14]).

![Figure 1: Hubble's law v=H_0d in the way it was presented in 1929 [4, Fig. 1]. The continuous and dashed lines represent fitting to the data according to distinct sampling criteria. Filled circles (continuous line) represent 24 individual galaxies and circles 9 groupings of the same galaxies. Distances, in the abscissa axis, are in parsec (1 pc = 3.26 light-year) and velocities in km/s.](image)

The redshift is usually represented by the letter \( z \) and is defined as \( z \equiv \Delta \lambda / \lambda \), with \( \Delta \lambda = \lambda_0 - \lambda \), where \( \lambda_0 \) is the observed wavelength of a given spectral line and \( \lambda \) is the wavelength of the same line measured in the laboratory on
Earth. Redshifts can be transformed into recession velocities by means of the mathematical expression of the classical Doppler effect $v = cz$, where $z$ is the redshift and $c$ is the speed of light in vacuum.

Distances to the galaxies were determined by various methods, all taking into account the dependence of the observed luminous flux with the inverse of the squared distance to the observed object. In most of the cases, Hubble determined the luminous flux of individual stars present in galaxies and from a comparison with the luminous flux of similar stars in our Milky Way, with known distances, he calculated the distances to the host galaxies of the observed stars. In some cases he used the observed luminous flux of a whole galaxy and compared it with the flux of closer similar galaxies, whose distances were already known from other methods. Distance determinations were precarious, but constituted the best one could do in the end of the 1920s. The Hubble constant $H_0$ determined by Hubble was almost 10 times as large as its value known in present days, mainly because of the uncertainty in distances. Nevertheless, at that time, the important fact was the convincing establishment of the linear relation between $v$ and $d$.

The fundamental importance of Hubble’s law is that a law of this kind represents a necessary feature of the homogeneity of expanding or contracting universe models. The more common cases, therefore the ones we are interested on, are expanding models. These models are only obtained when the assumption of homogeneity of space is made in the solution of GRT’s field equations (see detailed discussion in [3]). If a law of this kind is observationally verified in the real universe, then it means that the assumption that the universe might be spatially homogeneous has an observational foundation and is not a mere theoretical assumption. In the following section I show that a law of the type $v \propto d$ is indeed a consequence of spatial homogeneity by means of the investigation of a simple mechanical analogue of the expanding universe.

## 3 The expanding rubber band

Here I introduce ERBU, the **Expanding Rubber Band Universe**. ERBU is a homogeneous rubber thread shaped in a closed figure, as illustrated in Fig. 2. ERBU is the unidimensional equivalent to ERSU, Edward Harrison’s **Expanding Rubber Sheet Universe** [5, pp. 275 to 280], which is a 2-D analogue to the 3-D expanding universe. As I show below, ERBU is much more practical than ERSU — because it is 1-D and of easy construction — to be used in a demonstration of Hubble’s law in the study of relativistic cosmology. An unidimensional analogue like this has already been discussed in the context of Hubble’s law by Bernard Schutz (cf. [6] p. 349), where it is called **Rubber-Band Model of the Universe**, RBMU. RBMU is used by this author for quantitative applications of Hubble’s law [5] p. 348], and the aspect I discuss here, namely, the homogeneity of relativistic models of the universe, is mentioned but is not appropriately highlighted, specially the straight relation between Hubble’s law and homogeneity. Another difference between ERBU and RBMU is that in the
first one we treat expansion (the stretching of the rubber band) in only one direction, i.e., a linear expansion. In RBMU the rubber band expands from an initial circular shape and must keep this shape. The linear expansion is less rich in detail but is sufficient to the objective of the present work. RBMU’s study is an important complement to the study presented here.

Fig. 2 shows ERBU. The material used in its confection is constituted by a rubber band and two pieces of string.

![Figure 2: The string loops tied to the rubber band represent “galaxies” A and B. For the purpose of discussion, the observer O is located at the left end of the band. He may however be on any point of the ERBU.](image)

The ERBU of Fig. 2 may be used in three different ways. (i) Fixing the left end O and stretching the right end one notes that the left loop A moves much less than the right loop B. This is a behavior that follows from the ERBU Hubble’s law: the larger the distance to the “observer”, in this case the left end O, the larger the galaxy’s displacement and therefore the larger its speed. In this way, v is proportional to the distance d, as explained in the previous section. (ii) Fixing the right end and stretching the left end, the loops behavior is reversed; the speed of galaxy B is now lesser than the speed of galaxy A. (iii) Simultaneously moving both ends, the previous scenarios happen simultaneously too; in other words, there is no privileged observer.

All that occurs in order to preserve the band homogeneity. Fig. 3 shows the ERBU of Fig. 2 before and after the expansion — or stretching — of the rubber band.

Let us assume that the band is stretched during a time interval $t_e$. The displacement of galaxy A in this interval is $\mathbf{AA}'$. One may imagine that such displacement is the sum of small displacements which occur along $\mathbf{OA}$. Since the band is homogeneous, all these small displacements have the same magnitude $\delta d$. The total displacement $\mathbf{AA}'$ is equal to $N \times \delta d$, where $N$ is the number of displacements $\delta d$. Hence, the larger is $\mathbf{OA}$, the larger is the number $N$ and, consequently, the larger is $\mathbf{AA}'$, that is to say, $\mathbf{AA}'$ is proportional to $\mathbf{OA}$. Likewise, the displacement $\mathbf{BB}'$, of galaxy B, is proportional to $\mathbf{OB}$. Such behavior patterns result in a “Hubble’s law” for the ERBU, as shown next.
Figure 3: The lines represent the rubber band of Fig. 2. The distance from the observer to galaxy B is, in this example, equal to 7 times the distance to galaxy A. The bottom line shows the expanded rubber band. The band is homogeneous and remains homogeneous after the expansion and hence one has OB’= 7×OA’. The fine traces in the bottom line denote the initial positions of the galaxies.

Let γ be the constant of the proportionality described above. One has then d’=γd, where d’ represents the generic displacement of a galaxy from its initial distance to the reference point O. Note that γ is a dimensionless constant. For the galaxy A, for example, d’=AA’ and d=OA. Since the displacement d’ occurred during the time interval t₀, one can write then d’=vt₀, where v is the galaxy velocity in the displacement. In Fig. 3 one can see that the displacement velocity of galaxy B will be larger than the displacement velocity of galaxy A, because in the same time interval t₀ its displacement was BB’=7×AA’. The velocity of B will be therefore 7 times as large as the velocity of A.

The relation d’=γd becomes hence vt₀ = γd, or v=(γ/t₀)d. Making γ/t₀ ≡ H_G, the Hubble’s constant of the ERBU, we have the expression of Hubble’s law for the ERBU as:

\[ v = H_G d, \]

where v is the velocity of displacement of any point of the rubber band when the band is stretched, H_G ≡ γ/t₀ is the stretching — or expansion — constant of the rubber band and d is the distance of the point to the reference O. Eq. 2 is entirely analogous to eq. 1 of Hubble’s cosmological expansion, and one may note in both equations that H₀ and H_G have physical dimensions of 1/time.

In the expanding universe models, similarly to what occurs in the rubber band, Hubble’s law describes an expansion that preserves the universe homogeneity and, as in the band, there is no privileged observer or point of reference as well.

The rubber-band “Hubble’s constant” is related to its elasticity because a “hard” rubber band expands (or stretches) with more difficulty than a “soft” one. By analogy, one may say that the cosmological Hubble’s constant is related to the elasticity of the spatial tissue. Space and space-time in GRT are physical entities. Rubber analogies of the universe, like the one presented here, show that
It is conceptually appropriate attributing an elastic property to space (space elasticity is discussed, for example, in [2] pp. 286-287). It must be pointed out that Harrison’s ERSU, being a rubber sheet, i.e., a 2-D object, has an important advantage over ERBU, namely, to enable the discussion of the behavior of two-dimensional patterns in expanding models (e.g., [5] p. 276). Now, Schutz’s RBMU expands keeping a circular shape, while ERBU expands linearly, a feature that is sufficient for the discussion of homogeneity, and it is also, because of this same feature, more easily handled than RBMU is. The latter, as said before, needs to keep a circular shape while being stretched.

The ERBU allows, therefore, the experimental verification of one of the most important consequences of the homogeneity of a physical system, namely, the validity of a law of the kind $v \propto d$, in other words, the validity of a “Hubble’s law”.

4 Final remarks

As we saw, ERBU is a simple mechanical analogue of the expanding universe, which allows a better conceptual understanding of Hubble’s law. It is suitable also for the discussion of two interesting cosmological concepts introduced by the English physicist, mathematician and cosmologist Edward Milne (1896-1950). These are the “world map” and the “world picture”. They are general concepts and may be applied to any cosmological models, either in expansion or not. The map and the picture of the universe are two possible ways of perception of the universe. Such concepts are explored more extensively in [5] cap. 14; I make next a brief presentation of their meanings.

The world map is what is perceived by cosmic observers external to the universe, i.e., by godlike spectators (cf. [5] p. 279). Such a spectator sees all the cosmos as it is in a given instant of time. In our analogue, the external spectator sees the whole ERBU, that is, the rubber band. The external spectator is, generally speaking, everyone that handles the ERBU.

As to the putative observer located in the ERBU’s point O, he sees, according to Milne, the “world picture”, and has observational limitations that do not exist for the godlike observer. The observer located in O is a wormlike denizen of the ERBU. For the real universe, such limitations are more obvious. There, the wormlike denizen sees bodies that are distant in space and remote in time and is unable of perceiving the whole cosmos as it is in a given instant of time, i.e., the world map, because of the finiteness of the speed of light. In the real universe this is, however, the only way of observing the universe. In other words, we are wormlike denizens of the real universe.

The world map is perceived by someone from the outside, being necessarily a theoretical view. The world picture is perceived by someone from the inside, being then an observational view. The perception of the expansion, in this case, has two fundamental limitations, one arising from its observational nature, namely, the finiteness of the speed of light, and another of a theoretical nature,
i.e., the necessity of choosing a cosmological model that describes the expansion. This choice determines the way redshifts are transformed into velocities, as well as how distances to observed galaxies are calculated. In this way, the velocity-distance relation is only linear for small redshifts (and consequently small distances), because then the influence of the finiteness of the speed of light and of the adopted theoretical model for the expansion are negligible. For large distances the velocity-distance relation depends on the adopted model of expansion, because primary observational data are not velocities but redshifts. And these must be transformed into velocities in accordance with the model. The function \( v = cz \), equivalent to the classical Doppler effect, is only valid for small redshifts \( z \) (like those used by Hubble in 1929); for large values of \( z \), the function \( v(z) \) depends on the adopted model for the expanding universe. For example, for the critical Friedmann model \(^2\) this function is not linear and is illustrated in figure 2 of \(^8\) and in figure 15.8 of \(^5\).

In conclusion, the law \( v=H_0d \) holds for any distance in the world map, as long as it is homogeneous, because as we saw in the previous section, it is a law of this sort that describes the homogeneity of expanding universes. However, for large distances in the world picture, the velocity-distance law is not, in general, linear. Non linearity starts to be observed for redshifts larger than 0.1 \(^5\), that is, for distances larger than about 1 billion light-years, for Hubble’s constant of 72 (km/s)/Mpc \(^9\) (1 Mpc = 3, 26 × 10\(^6\) light-year).

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