Laser frequency combs and ultracold neutrons to probe braneworlds through induced matter swapping between branes

Michaël Sarrazin\textsuperscript{1} and Fabrice Petit\textsuperscript{2}

\textsuperscript{1}Department of Physics, University of Namur (FUNDP), 61 rue de Bruxelles, B-5000 Namur, Belgium
\textsuperscript{2}Belgian Ceramic Research Centre, 4 avenue du gouverneur Cornez, B-7000 Mons, Belgium

This paper investigates a new experimental framework to test the braneworld hypothesis. Recent theoretical results have shown the possibility of matter exchange between branes under the influence of suitable magnetic vector potentials. It is shown that the required conditions might be achieved with present-day technology. The experiment uses a source of pulsed and coherent electromagnetic radiations and relies on the Hänsch frequency-comb technique well-known in ultra high-precision spectroscopy. A good matter candidate for testing the hypothesis is a polarized ultracold neutron gas for which neutron decay is counted.

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I. INTRODUCTION

Over the last two decades, the concept of braneworld Universe has gained a growing importance in theoretical physics. The braneworld hypothesis assumes that our universe is just a membrane (a brane) embedded in a larger dimensional manifold (a bulk) having \( N > 4 \) dimensions \[1, 2\]. Standard model particles are expected to stay confined in our brane while the other branes are normally considered invisible to us. A wealth of papers has demonstrated that this geometrical approach offers nice explanations to several physical phenomena like the dark matter origin \[3\] or the hierarchy between the electroweak and planck scales \[4\] for instance. As a consequence, finding evidences of the existence of branes or extra dimensions is a major challenge for the 21\textsuperscript{th} century. Experimental results could arise from high energy physics (observation of Kaluza-Klein tower states \[5\] for instance) or low energy physics (deviations from the inverse square law of gravity \[6\] for instance). In the present paper, we are mainly motivated by a low energy approach and we explore how the quantum dynamics of fermions is modified at a non-relativistic energy scale when the higher dimensional bulk contains more than only one brane.

In a previous work \[7\], it has been shown that for a bulk containing at least two branes, matter swapping between these two worlds might be possible (although the effect would remain difficult to observe). In some conditions, this matter exchange could be triggered by using suitable magnetic vector potentials \[8, 9\]. This effect was studied through a quantum description of fermions in a \( M_4 \times Z_2 \) universe, which is a model-independent low energy limit of any two-brane world theory of the Universe \[7\]. However, the situations considered in previous papers were rather simplistic \[7–9\]. In the present work, we investigate further this effect by reconsidering it from a more realistic experimental point of view. To that end, a simple and inexpensive experimental setup is proposed.

In section II, the mathematical and physical assumptions underlying the low energy description of a two-brane world is reminded. The theoretical conditions of matter swapping between branes are given in section III. We discuss the environmental conditions that could preclude the swapping to occur. The role of the magnetic vector potentials, which are required to match the conditions of successful matter exchange between branes is also discussed. Finally, in section IV an experimental setup which might be used to confirm our theoretical prediction is described. The proposed experiment relies on the use of a polarized ultracold neutron gas and coherent electromagnetic radiations thanks to a Hänsch frequency-comb like technique \[10\]. It is demonstrated that for certain experimental parameters, the conditions of a resonant matter exchange between branes may be obtained.

\textsuperscript{*}Electronic address: michael.sarrazin@fundp.ac.be
\textsuperscript{†}Electronic address: f.petit@bcrc.be
II. MODEL OF THE LOW ENERGY LIMIT OF TWO-BRANE WORLDS

In a recent work [11], it has been demonstrated that at low energies any two-brane world model can be described by a simple noncommutative two-sheeted spacetime $M_4 \times Z_2$. For instance, a two-brane world composed of two domain walls on a continuous $M_4 \times R_1$ manifold is well modeled by a discrete product space $M_4 \times Z_2$ at low energies. The continuous real extra dimension $R_1$ is replaced by an effective phenomenological discrete two-point space $Z_2$. At each point along the discrete extra dimension $Z_2$ there is then a four-dimensional spacetime endowed with its own metric field. Both branes/sheets are then separated by a phenomenological distance $\delta$ which is inversely proportional to the overlap integral of the extra-dimensional fermionic wave functions over the fifth dimension $R_1$ [11]. Considering the electromagnetic gauge field, it has been also demonstrated that the five-dimensional $U(1)$ bulk gauge field [11] is substituted by an effective $U(1) \otimes U(1)$ gauge field acting in the $M_4 \times Z_2$ spacetime.

It is important to stress that the equivalence between the continuous two-domain wall approaches and the noncommutative two-sheeted spacetime is rather general and does not rely for instance on the domain walls features or on the bulk dimensionality [7]. It allows to conjecture that at low energy, any multidimensional setup containing two mutative two-sheeted spacetime is rather general and does not rely for instance on the domain walls features or on the bulk dimensionality [7]. It can be noticed that by virtue of the two-sheeted structure of spacetime, the field. Both branes/sheets are then separated by a phenomenological distance $\delta$ which results from the exact and complex brane characteristics, it can be resolute from experiment. One is able to consider in details in a previous work [11], in the following, we are only considering the relevant $M_4 \times Z_2$ limit.

The mathematical description of the noncommutative two-sheeted spacetime is mainly based on the work of Connes, Viet and Wali [12] and relies on the definition of a noncommutative exterior derivative $\mathcal{D}$ acting on $M_4 \times Z_2$. Due to the specific geometrical structure of the bulk, this operator is given by:

$$D_\mu = \left( \frac{\partial_\mu}{0} \ 0 \ \frac{\partial_\mu}{0} \right) \text{ and } D_5 = \left( \begin{array}{cc} 0 & g \\ -g & 0 \end{array} \right)$$

(1)

with $\mu = 0, 1, 2, 3$ and where the term $g = 1/\delta$ acts as a finite difference operator along the discrete dimension. $g$ also appears as a coupling strength, which describes the interaction between the branes. Although $g$ is a parameter, which results from the exact and complex brane characteristics, it can be resolute from experiment. One is able to build the Dirac operator defined as $\mathcal{D} = \Gamma N D_N = \Gamma^\mu D_\mu + \Gamma^5 D_5$ by considering the following extension of the gamma matrices (we are working in the Hilbert space of spinors [12]): $\Gamma^\mu = 1_{2 \times 2} \otimes \gamma^\mu$ and $\Gamma^5 = \sigma_3 \otimes \gamma^5$, where $\gamma^\mu$ and $\gamma^5 = i\gamma^0\gamma^1\gamma^2\gamma^3$ are the usual Dirac matrices and $\sigma_k$ ($k = 1, 2, 3$) the Pauli matrices. By introducing a general mass term $M$, a two-brane Dirac equation is then derived [7]:

$$\mathcal{D}_{\text{dirac}} \Psi = (i\mathcal{D} - M) \Psi = (i\Gamma N D_N - M) \Psi =$$

$$= \left( \begin{array}{cc} i\gamma^\mu \partial_\mu - m & ig\gamma^5 - m_c \\
ig\gamma^5 - m_c & i\gamma^\mu \partial_\mu - m \end{array} \right) \left( \begin{array}{c} \psi_+ \\ \psi_- \end{array} \right) = 0$$

(2)

where "$*$" denotes the complex conjugate. The off-diagonal mass term $m_c$ can be justified from a two-brane(domain wall) structure of the Universe [7]. It can be noticed that by virtue of the two-sheeted structure of spacetime, the wave function $\psi$ of the fermion is split into two components, each component living on a distinct spacetime sheet. If one considers the $(\text{+})$ sheet as our own brane, the $(\text{-})$ sheet can be considered as a hidden brane.

A. Electromagnetic gauge field

As explained in the introduction, we want to show how the electromagnetic field influences the dynamics of fermions. The reason of the electromagnetic force, by contrast to electroweak or strong force, rests on the choice of a simple gauge group and just serves our experimental purpose. Incorporating the electromagnetic field $A$ in the model ($\mathcal{D}_A \rightarrow \mathcal{D} + \mathcal{A}$) [7–9], the usual $U(1)$ gauge field must be substituted by an extended $U(1) \otimes U(1)$ gauge field accounting for the two-brane structure [7]. The group representation is therefore $G = \text{diag}(\exp(-iq\Lambda_+), \exp(-iq\Lambda_-))$. According to the gauge transformation rule: $A' = GAG^\dagger - iG[\mathcal{D}_{\text{dirac}}, G^\dagger]$, the appropriate gauge field is given by (see Ref. [7])

$$A = \left( \begin{array}{cc} iq\gamma^\mu A_\mu^+ & \gamma^\nu \chi \\
\gamma^\nu \chi & iq\gamma^\mu A_\mu^- \end{array} \right)$$

(3)

where $\phi$ and $\chi$ are the scalar components of the field $\chi$ and $\gamma = \gamma^0\chi^\dagger\gamma^0$. If $\chi$ is different from zero, each charged particle of each brane becomes sensitive to the electromagnetic fields of both branes irrespective of their localization in the bulk. This kind of exotic interactions has been considered previously in literature within the framework of...
mirror matter paradigm \[13\] and is not covered by the present paper. Moreover, to be consistent with known physics, at least at low energies, \( \chi \) is necessarily tiny (whereas \( qA_\pm \) needs not to be). This is theoretically corroborated by the gauge transformation rule which shows that during each gauge transformation, \(|\varphi|\) (respectively \(|\phi|\)) varies with an amplitude of order \( q \) (respectively \(|m_c|\)) whatever \( \Lambda_+ \) and \( \Lambda_- \). Using the covariant derivative \( \nabla_A \rightarrow \nabla + A \) and according to expression \[3\], the electromagnetic field can be easily introduced in the two-brane Dirac equation (Eq. \[2\]). Then, we get:

\[
(i \nabla_A - M) \Psi = 0
\]

\[
= \left( \begin{array}{cc}
\ii \gamma^\mu (\partial_\mu + \ii q A_+^\mu) - m & \ii q \gamma^5 - \tilde{m}_c \\
\ii \bar{g} \gamma^5 - \bar{m}_c & \ii \gamma^\mu (\partial_\mu + \ii q A_-^\mu) - m
\end{array} \right) \begin{pmatrix}
\psi_+ \\
\psi_-
\end{pmatrix} = 0
\]

with \( \bar{g} = g + \varphi \) and \( \bar{m}_c = m_c - \ii \phi \). It is important to underline that the field \( \chi \) just leads to replace \( g \) and \( m_c \) by the effective parameters \( \tilde{g} \) and \( \bar{m}_c \). Without loss of generality, in the following we fairly assume that \( \tilde{g} \approx g \) and \( \bar{m}_c \approx m_c \) since \(|\varphi|\) (respectively \(|\phi|\)) should not exceed the amplitude of \( g \) (respectively \(|m_c|\)). This choice allows a further simplification of the model. It is somewhat equivalent to set the off-diagonal term \( \chi \) to zero. With such a choice, we simply assume that the electromagnetic field of a brane couples only with the particles belonging to the same brane. Each brane possesses its own current and charge density distribution as sources of the local electromagnetic fields. On the two branes live then the distinct \( A_+^\mu \) and \( A_-^\mu \) electromagnetic fields. The photon fields \( A_\pm^\mu \) behave independently from each other and are totally trapped in their original brane in accordance with observations: photons belonging to a given brane are not able to reach the other brane. As a noticeable consequence, the structures belonging to the branes are mutually invisible by local observers.

### B. Non-relativistic limit and phenomenology

As explained in the introduction, we are concerned by phenomena occurring at non-relativistic energy scale. In Refs. \[8, 9\], it was indeed demonstrated that any relativistic particle is trapped in its own brane. As a consequence, the model predicts that there is no hope to observe exchange of standard model particles between branes for relativistic energies – a conclusion that contrasts with the usual belief on the energy scales at which extra dimensions effects should become noticeable. This is the reason why this paper is restricted to the case of non-relativistic particles only.

Let us derive the non-relativistic limit of the two-brane Dirac equation \[4\]. Following the well-known standard procedure, a two-brane Pauli equation can be derived:

\[
i \hbar \frac{\partial}{\partial t} \begin{pmatrix}
\psi_+ \\
\psi_-
\end{pmatrix} = \{ \mathbf{H}_0 + \mathbf{H}_{cm} + \mathbf{H}_c \} \begin{pmatrix}
\psi_+ \\
\psi_-
\end{pmatrix}
\]

where \( \psi_+ \) and \( \psi_- \) correspond to the wave functions in the \( (+) \) and \( (-) \) branes respectively. \( \psi_+ \) and \( \psi_- \) are here Pauli spinors. The Hamiltonian \( \mathbf{H}_0 \) is a block-diagonal matrix where each block is simply the classical Pauli Hamiltonian expressed in each branes:

\[
\mathbf{H}_\pm = -\frac{\hbar^2}{2m} \left( \nabla - \ii \frac{g}{\hbar} \mathbf{A}_\pm \right)^2 + g_\mu \frac{1}{2} \sigma \cdot \mathbf{B}_\pm + V_\pm
\]

such that \( \mathbf{A}_+ \) and \( \mathbf{A}_- \) correspond to the magnetic vector potentials in the branes \( (+) \) and \( (-) \) respectively. The same convention is applied to the magnetic fields \( \mathbf{B}_\pm \) and to the potentials \( V_\pm \). \( g_\mu \) is the magnetic moment of the particle with \( g_\mu \) the gyromagnetic factor and \( \mu \) the magneton. In addition to these “classical” terms, the two-brane Hamiltonian comprises also new terms specific from the two-brane world:

\[
\mathbf{H}_c = \begin{pmatrix}
0 & m_c c^2 \\
m_c c^2 & 0
\end{pmatrix}
\]

\[
\mathbf{H}_{cm} = -\ii gg_\mu \frac{1}{2} \begin{pmatrix}
0 & \sigma \cdot (\mathbf{A}_+ - \mathbf{A}_-) \\
-\sigma \cdot (\mathbf{A}_+ - \mathbf{A}_-) & 0
\end{pmatrix}
\]

\( \mathbf{H}_c \) is responsible for free spontaneous oscillations of fermions between each brane. Nevertheless, as mentioned in Refs. \[7, 9\], these oscillations are expected hardly observable due to environmental effects which freeze the oscillations (see also section \[III \A\]). By contrast \( \mathbf{H}_{cm} \) is a geometrical coupling involving the gauge fields of the two branes. It is worth noticing that this coupling term depends on the magnetic moment and on the difference between the local (i.e.
on a brane) values of the magnetic vector potentials (see Refs. [7–9] for more details and discussions). The coupling term $H_{cm}$ implies that matter exchange should be possible between the branes. Its form suggests that particles could experience Rabi like oscillations between the branes that might be triggered by suitable magnetic vector potentials (see Fig. 1).

### III. MATTER OSCILLATIONS BETWEEN BRANES

We consider the possibility of a matter exchange (or swapping) between the two branes through an idealized case and from equations (5) to (8). Let us consider a neutral particle endowed with a magnetic moment (a neutron for instance), initially ($t = 0$) localized in our brane in a region of curlless rotating magnetic vector potential such that $A_+ = A_\mu = A_\mu(t)$ and $A_- = 0$, with $e(t) = (\cos \omega t, \sin \omega t, 0)$. $\omega$ is the angular frequency of the field $A_\mu$, which can be null ($\omega = 0$) in the static field case. Assuming that the conventional part of the Pauli Hamiltonian $H_{\pm}$ can be written as $H_{\pm} = V_{\pm}$, any particle initially in a spin-down state for instance (according to $e_3 = (0, 0, 1)$) and localized in our brane at $t = 0$ can be detected in the second brane at time $t$ according to the probability [9]:

$$P(t) = \frac{4\Omega_p^2}{\eta^2 - \omega^2} \sin^2 \left(\frac{1}{2} \sqrt{\frac{\eta^2}{\omega^2} + 4\Omega_p^2 t}\right)$$

where $\Omega_p = gg_s \mu A_p/(2\hbar)$ and $\eta = (V_+ - V_-)/\hbar$. In addition, in the second brane, the particle is then in a spin-up state. $\eta\hbar$ is an effective potential that sums up all the interactions between the particle and its environment described by $H_{\pm}$. Those environmental effects are discussed in section IIIA. Eq. (9) shows how the particle is transferred in the other brane through a process involving Rabi like oscillations. Eq. (9) shows that a resonant exchange occurs whenever the magnetic vector potential rotates with an angular frequency $\omega = \eta$. A similar expression is obtained by applying the substitution $\omega \to -\omega$ if the particle is in a spin-up state at $t = 0$ (the resonance is then achieved with a counterrotative vector potential). In this case, the particle is in a spin-down state in the second brane after exchange. Note that Eq. (9) corresponds to a resonant process with an half-height width $\Delta \omega = 4\Omega_p$. The weaker the coupling constant $g$ is, the narrower the resonance is. By contrast, the greater $A_\mu$ is, the broader the resonance is.

Of course when $\omega \neq \eta$, or even in the static field case ($\omega = 0$), matter oscillations between branes occurs as well. Nevertheless, due to environmental effects discussed in section IIIA the amplitude of these oscillations must probably be strongly damped and hardly observable.

It is clear that the situation described by Eq. (9) remains rather simplistic. In the suggested experiment (see section IV) we will propose a more realistic situation to achieve matter exchange through the use of coherent electromagnetic radiations.

#### A. Freezing oscillations

As detailed in previous papers [8, 9], common environmental interactions are strong enough to suppress matter oscillations between adjacent branes. This can be easily checked by considering Eq. (9) showing that if $\omega \neq \eta$ then $P(t)$ decreases as $\eta$ increases in comparison to $\Omega_p$. Based on the fact that no such oscillations has been observed so
far, we can expect that the ratio $\Omega_p/\eta$ is usually very small. Therefore, any experiment seeking for such oscillations should focus on the quest of resonant responses (i.e. when $\omega = \eta$) in order to avoid the environmental confinement.

Next, if one considers an experiment involving a set of particles with strong collisional dynamics, collisions are likely to occur so frequently that coherent oscillatory behavior cannot develop. From the point of view of a single particle, each collision resets the probability of transfer according to a quantum Zeno like effect. This oscillation damping is also expected to increase dramatically with temperature and quickly fluctuating ambient fields (radiations and Earth’s magnetic field for instance).

Note that the same environmental effects are responsible for the negligible role of $H_{ce}$, which cannot be artificially enhanced by contrast to $H_{ce}$.

Therefore a prerequisite for observing the oscillatory behavior of the particles between branes is to keep each particle isolated from the environment as much as possible and to apply very specific magnetic vector potentials, i.e. a context never met in any kind of experiments so far.

As a consequence, it seems natural to work preferentially with ultracold neutrons, which are insensitive to electric fields. In addition, neutron magnetic sensibility is the result of its magnetic moment only. Therefore, convenient Helmholtz coils should be used to cancel any ambient magnetic field (as the Earth’s one for instance). Other magnetic contributions from ambient electromagnetic waves can be canceled too in a large spectral frequency domain by working with a cooled setup (to avoid black body emissions) and using a combination of convenient shields, such as Faraday cage and lead brick walls. Moreover, one may assume that by using a low-density neutron gas the collisions between particles should be prevented. Of course, an experimental setup emptied from atmospheric gases must be used to avoid undesirable collisions between neutrons and atmospheric molecules.

At last, note that for a neutron shielded from magnetic fields, $\eta = (V_{grav,+} - V_{grav,-})/\hbar$ (i.e. only gravitational contributions are considered). However, it is difficult to assess the value of $\eta\hbar$. For instance, the estimations given in Ref. [9] suggest that $V_{grav,+}$ could be of the order of $\sim 500$ eV owing to the milky way core gravitational influence exerted on neutrons. Nevertheless, since the gravitational contribution of the other brane ($V_{grav,-}$) remains unknown, $\eta$ appears therefore as an effective unknown parameter of the model.

### B. Magnetic vector potential. Ambient and artificial contributions

Before detailing the experimental setup, we first need to address the issue of a hypothetical ambient magnetic vector potential. The existence of such a potential was recently debated in literature in a context of photon mass measurement [14]. For instance, in Refs. [14], ambient magnetic vector potential is estimated by integrating $B_{amb} = \nabla \times A_{amb}$ and considering ambient magnetic fields $B_{amb}$. It was demonstrating that $A_{amb} = RB_{amb}$, where $R$ is the typical distance from sources. From earth magnetic field measurements, it was therefore assessed an ambient contribution $A_{amb}$ of about $200$ T$m$ whereas from the Coma galactic cluster magnetic field, a value of $A_{amb} \sim 10^{12}$ T$m$ was calculated. Although these assessments are not really constraining the present model, they raise a number of questions. Indeed, several authors have pointed out that any assessment of the ambient magnetic vector potential from such simple calculations was unreliable (see Luo et al [14]). The magnetic vector potential derived that way is the transversal contribution only though any vector potential comprises three parts: $A_{amb} = A_\perp + A_\parallel + A_{cte}$. Here $A_\perp$ is the transverse part such that $B_{amb} = \nabla \times A_\perp$, $A_\parallel$ is the longitudinal component such that $\nabla \times A_\parallel = \mathbf{0}$, and $A_{cte}$ is a constant vector. It is obvious that a rigorous assessment of the ambient magnetic vector potential has to take into account all these components. Unfortunately they cannot be determined since several boundary conditions are still unknown due to their astrophysical origins. In addition, Eq. (8) shows that it is the net difference $\delta A = A_+ - A_-$ between the vector potentials of the two branes that is relevant, not the local values on the branes. From all these reasons, the effective ambient magnetic vector potential $\delta A_{amb}$ "could be very large... or null!". To simplify, we consider hereafter $\delta A_{amb}$ as an unknown parameter of the model. Nevertheless, to be consistent with observations, Eq. (9) shows that any ambient magnetic vector potential $\delta A_{amb}$ should be balanced by an ambient confining potential $\eta\hbar$, such that $\Omega_p/\eta \ll 1$ (in order to avoid unseen oscillations of "free" fermions).

Finally, since in the following one is seeking for a resonant oscillatory mechanism, a first requirement is to consider a rotating magnetic vector potential as explained before. Such a potential can obviously be obtained from a coherent electromagnetic wave with a circular polarization. Indeed, we will just need to consider the magnetic vector potential $\mathbf{A}$ related to the electric and magnetic field $\mathbf{E} = -\partial \mathbf{A}/\partial t$ and $\mathbf{B} = \nabla \times \mathbf{A}$ of the electromagnetic wave.

### IV. SETUP FOR RESONANT NEUTRON SWAPPING BETWEEN BRANES

In the present section, we will give a sketch of an experimental setup that might be suitable to investigate the resonant mechanism roughly described in section III. As said previously, this experiment involves coherent electro-
magnetic radiations. In a Coulomb gauge, a rotative magnetic vector potential can be obtained from a circularly polarized electromagnetic wave. To achieve the conditions of a successful matter exchange, the applied electromagnetic radiations. In a Coulomb gauge, a rotative magnetic vector potential can be obtained from a circularly polarized electromagnetic wave. To achieve the conditions of a successful matter exchange, the applied electromagnetic radiations.

FIG. 2: Naive view of the experimental device for the study of particle swapping between branes. a: Inside the cavity, mirror 1 (m1) (respectively mirror 2 (m2)) has a reflection coefficient $R_1$ (respectively $R_2$). L as well as $\omega$ can be tunable. The ultracold neutron gas is stored in the vessel $V$ surrounded by a radiation detector $D$. Neutrons spins ($s$) are polarized perpendicularly to the plane of rotation of $A_p$. The pulsed electromagnetic beam $B$ comes from the laser source $L_a$ and the circular polarization is ensured by the polarizer $P$. b: Sketch of the amplitude of the magnetic vector potential against time $t$. The field is a set of $N$ coherent pulses (dashed line illustrates the coherence between each pulse). $T$ is the period time between each pulse while $\tau$ is the pulse duration.

Let us now study the applicability of this technique to force the swapping of neutrons between the branes. The neutrons energy is assumed to be low enough to make the non-relativistic equations valid. Under a suitable field $A$ constraint, a neutron $n$ may disappear from our brane to the other one, i.e. $n \rightarrow n'$ (where the "prime" denotes the neutron in the second brane), but for an observer in our brane we get $n \rightarrow \text{nothing}$ (Fig.1). Ultracold neutrons are stored in a convenient vessel (emptied from atmospheric gases) which is put in a Fabry-Pérot cavity made of two mirrors (Fig.2.a). The cavity is supplied with a coherent electromagnetic wave having a magnetic field amplitude $B_0$. The magnetic vector potential in the cavity can be expressed as [10]:

$$A_p(t) = \frac{B_0 c}{\omega} \sum_{n=0}^{\infty} (R_1 R_2)^{n/2} \Upsilon_r(t-nT)e(t-nT)$$

where $e(t) = (\cos \omega t, \sin \omega t, 0)$, and $\omega$ is the angular frequency of the wave. $R_1$ and $R_2$ are the reflection coefficients of the two mirrors limiting the cavity such that $(R_1 R_2)^{n/2}$ allows to define the amplitude of the $n$th pulse. $\Upsilon_r(t)$ is the envelope function of a pulse, with $\tau$ the temporal pulse-width (see Fig.2.b). For the sake of simplicity we set $\Upsilon_r(t) = \text{rect}(t/\tau)$. $T = 2L/c$ is the pulse roundtrip time in the cavity (see Fig.2.b).

In the experiment, neutrons are submitted to the ambient gravitational fields ($V_{\text{amb}}$) and the magnetic field of the electromagnetic wave. For completeness, we are also considering an ambient magnetic vector potential $\delta A_{\text{amb}} = \delta A_{\text{amb}} u$, where $u = (\sin \theta \cos \varphi, \sin \theta \sin \varphi, \cos \theta)$ in the system of reference OxTyz (see Fig.2.a). Therefore, Eq. (5) can be expressed as

$$i\hbar \frac{\partial}{\partial t} \langle \Psi \rangle = \left\{ \begin{array}{ccc} V_+ & -iE_0 \sigma \cdot u \\ iE_0 \sigma \cdot u & V_- \end{array} \right\} + E_0 \sum_{n=0}^{\infty} f_n(t-nT) \times \left\{ \begin{array}{ccc} \sigma \cdot e(t-nT) & -i\kappa \sigma \cdot e(t-nT) \\ i\kappa \sigma \cdot e(t-nT) & 0 \end{array} \right\} \langle \Psi \rangle$$

with $E_0 = (1/2)g_m B_0$, $\kappa = gc/\omega$, $f_n(t-nT) = (R_1 R_2)^{n/2} \Upsilon_r(t-nT)$ and $E_a = (1/2)g_{mB} \delta A_{\text{amb}}$. 

The largest magnetic fields achievable with current technology are obtained by ultrashort laser pulses and can be of the order of 10^7 T for instance (for a laser intensity about 10^{18} W/cm^2 [13]). For such intense fields, \( E_0 \) is about 10^{-3} eV. As explained previously, the term \( \eta = (V_+ - V_-)/\hbar \) accounts for the environmental contributions acting on the particle. Its value is presently unknown as it depends on the gravitational field in the "hidden" brane. Nevertheless, Ref. [9] and above suggest that it might be larger than 1 eV (absolute value) in such a way that \( \hbar |\eta| \gg E_0 \). It is worth reminding that Eq. (9) also implies \( \hbar |\eta|/E_a \gg 1 \) for consistency (to avoid undetected "free" oscillations of fermions). According to Eq. (11), if \( E_0 = E_a = 0 \), in our brane, a neutron can be described by the eigenstates \( |\psi_{\pm1/2}\rangle \) with energy \( V_+ \) (the subscript "\( \pm1/2 \)" denotes both spin states along the \( Oz \) axis), whereas in the second brane, it will be described by the eigenstates \( |\psi_{\pm1/2}\rangle \) with energy \( V_- \). In Eq. (11), it is then possible to write

\[
\Psi = a(t)e^{-i\hbar^{-1}V_+t}|\psi_{+1/2}\rangle + b(t)e^{-i\hbar^{-1}V_-t}|\psi_{-1/2}\rangle + c(t)e^{-i\hbar^{-1}V_+t}|\psi_{-1/2}\rangle + d(t)e^{-i\hbar^{-1}V_-t}|\psi_{+1/2}\rangle
\]  

Substituting (12) into (11), the Pauli equation (11) reads:

\[
\begin{cases}
\bar{\hbar}\partial_t a(t) = E_0 b(t)e^{-i\omega t}\Lambda(t) - \bar{\hbar}E_a d(t)e^{-i\eta t}\Lambda(t) - \bar{\hbar}E_a c(t) \cos\theta e^{i\eta t} - \bar{\hbar}E_a d(t) e^{-i\varphi}\sin\theta e^{i\eta t} \\
\bar{\hbar}\partial_t b(t) = E_0 a(t)e^{i\omega t}\Lambda^*(t) - \bar{\hbar}E_a c(t)e^{i\eta t}\Lambda^*(t) + \bar{\hbar}E_a d(t) \cos\theta e^{-i\eta t} - \bar{\hbar}E_a c(t) e^{i\varphi}\sin\theta e^{-i\eta t} \\
\bar{\hbar}\partial_t c(t) = \bar{\hbar}E_a b(t)e^{-i\eta t}\Lambda(t) + \bar{\hbar}E_a c(t) \cos\theta e^{-i\eta t} + \bar{\hbar}E_a b(t) e^{-i\varphi}\sin\theta e^{-i\eta t} \\
\bar{\hbar}\partial_t d(t) = \bar{\hbar}E_a b(t)e^{-i\eta t}\Lambda^*(t) - \bar{\hbar}E_a c(t) e^{-i\varphi}\sin\theta e^{-i\eta t}
\end{cases}
\]  

with \( \Lambda(t) = \sum_{n=0}^{\infty} f_n(t - nT)e^{i\omega nT} \), and where the star denotes the complex conjugate. The arguments of the exponential terms in this system suggest that a resonant process occurs at \( \omega \approx \eta \). Considering the expected values of \( g \) and \( \eta \) mentioned above, we get \( \kappa \ll 1 \). Since then \( \kappa E_0 \ll E_0 \ll \hbar |\eta| \) and \( \hbar |\eta| \gg E_a \), the secular approximation can be applied: the terms exhibiting rapid variations when \( \omega \approx \eta \) can be neglected, i.e. terms with \( \pm i\eta t \), \( \pm i\omega t \) and \( \pm (i\eta + \omega)t \) (It can be checked that the exponential terms with \( \pm i\eta t \), \( \pm i\omega t \) and \( \pm (i\eta + \omega)t \) arguments lead to amplitude contributions \( E_a/(\hbar \eta) \), \( E_0/(\hbar \eta) \) and \( \kappa E_0/(2\hbar \eta) \) respectively, which are then very smaller than 1). These simplifications allow to write the system (13) in a very compact form:

\[
\begin{cases}
\partial_t a(t) = -\Omega_p e^{i(\eta - \omega)t}\Lambda(t)d(t) \\
\partial_t d(t) = \Omega_p e^{-i(\eta - \omega)t}\Lambda^*(t)a(t)
\end{cases}
\]  

with \( \Omega_p = \kappa E_0/\hbar \). Since the influence of the electromagnetic wave can be treated as a simple perturbation, the system (14) can be solved analytically by using first order perturbation theory. For a neutron initially localized in our brane at time \( t = 0 \), the probability \( P = |d(t)|^2 \) to find the particle in the second brane is finally:

\[
P \approx \tau^2 \Omega_p^2 \frac{\sin^2((\eta - \omega)\tau/2)}{((\eta - \omega)\tau/2)^2} S
\]  

with

\[
S = \left| \sum_{n=0}^{\infty} \left( \sqrt{R_1 R_2 e^{-i\eta T}} \right)^n \right|^2 = S_{\text{max}} \left( 1 + F \sin^2(\eta T/2) \right)^{-1}
\]  

where \( S_{\text{max}} = (1 - \sqrt{R_1 R_2})^{-2} \), and \( F = 4\sqrt{R_1 R_2}/(1 - \sqrt{R_1 R_2})^2 \). According to Eqs. (14), a neutron initially in a spin-up state in our brane reaches the other brane in a spin-down state (and reciprocally). Therefore, in order to achieve a successful transfer, neutron gas in the vessel must be polarized prior to the experiment with a direction of polarization normal to the plane of rotation of the magnetic vector potential of the incident electromagnetic wave (Fig.2a). It must be pointed out that the influence of a hypothetic ambient magnetic vector potential vanishes in that case.

The excitation time \( t_{\text{cav}} \) during which neutrons feel a set of coherent pulses is tuned by the cavity properties defined by \( R_1 \) and \( R_2 \). \( t_{\text{cav}} \) is related to the quality factor \( Q \) of the cavity such that \( Q = \omega t_{\text{cav}} \). From the description of a Fabry-Pérot cavity, it is well known that

\[
t_{\text{cav}} = \frac{1}{2} \frac{1}{R_1 - R_2} \tau
\]
In the present experiment, the rate $\Gamma$ of neutron exchange between branes is simply (at $\omega = \eta$):

$$\Gamma = P/t_c = \tau^2 \Omega_p^2 S/t_c$$

(18)

It appears that the collisional dynamics between neutrons or in relation with the storage chamber walls is negligible provided that collisional time $t_c$ is greater than $t_s$ which is assumed to be satisfied in the experimental conditions. Since the neutron gas must be strongly polarized prior to the experiment, the Pauli exclusion principle will favor an increase of the collisional time $t_c$ thus promoting the experimental success.

Let $N_0$ be the initial number of neutrons in the storage chamber, $\Gamma_n$ the usual decay rate of neutron and $t_s$ the storage time (i.e. the duration of an experiment). In a first calibration experiment, $N_n(t_s) = N_0(1 - \exp(-\Gamma_n t_s))$ will be the number of detected usual neutron decays without applied electromagnetic field. In a second experiment where the wave is switched on, the number of recorded events will be then given by: $N_{c,f}(t_s) = N_0 \exp(-\Gamma_f t_s)(1 - \exp(-\Gamma_n t_s))$. Then, the neutron transfer between branes could be simply detected by measuring the ratio: $N_{c,f}(t_s)/N_n(t_s) = \exp(-\Gamma_f t_s)$. For instance, let us use the previous limit parameters (i.e. $h/\eta = 1$ eV, $g = 10^3$ m$^{-1}$, $E_0 = 10^{-5}$ eV). We consider a cavity with a quality factor $Q = 5 \cdot 10^8$ and a length $L = 5$ m. The source is endowed with a pulse-width $\tau = 1$ fs. At resonance $\omega = \eta$ which corresponds to a wavelength of the source $\lambda = 1.24 \mu$m. One then obtains $\Gamma/\Gamma_n \sim 0.1$. It is an attractive result that shows that the effect could be constructively investigated with present day technology. In this example, the value of the environmental contributions $\eta$ suggests to use an infrared electromagnetic source ($\lambda = 1.24 \mu$m). Of course, as previously suggested, $\eta$ can reach many possible values very different from each other. As a consequence, in order to increase the chance of success of such an experiment it could be highly beneficial to consider different electromagnetic sources such as terahertz lasers, coherent synchrotron radiation sources in X-rays domain or free-electron lasers in a wide wavelength domain.

V. CONCLUSION

Recent theoretical results have suggested the possibility of matter swapping between branes. In the present work, a new experimental framework has been proposed to demonstrate this mechanism at a laboratory scale. The experiment, involving a resonant mechanism, uses an ultracold and polarized neutron gas interacting with a fine tuned electromagnetic pulsed radiation. This experiment relies on the Hänsch frequency comb technique, commonly used in spectroscopy to overcome the limitations of the large bandwidth of electromagnetic sources. The recorded experimental data are simply the rates at which particle exchange occurs between branes. Any success in this experiment would imply major consequences extending far beyond the demonstration of the existence of other branes. Moreover, the relative simplicity of the suggested setup is of considerable interest: existing devices used in optical spectroscopy and neutron physics investigations could possibly match the requirements of a successful experiment. Thus the present work takes place in a relevant current trend in experimental works for new physics searches at a low energy scale.

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