Non-abelian Self-Dual String and M2-M5 Branes Intersection in Supergravity

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Abstract: We consider the non-abelian theory \cite{1} for an arbitrary number $N_5$ of five-branes and construct self-dual string solution with an arbitrary $N_2$ unit of self-dual charges. This generalizes the previous solution of non-abelian self-dual string \cite{2} of $N_5 = 2, N_2 = 1$. The radius-transverse distance relation describing the M2-branes spike, particularly its dependence on $N_2$ and $N_5$, is obtained and is found to agree precisely with the supergravity description of an intersecting M2-M5 branes system.

Keywords: M-Theory, D-branes, M-branes, Gauge Symmetry.
1. Introduction

The low energy theory of \( N \) coincident M5-branes in a flat spacetime is given by an interacting (2,0) superconformal theory in six dimensions. The understanding of the dynamics of this system is of utmost importance. There exists a number of proposals for the fundamental definition of the quantum theory. For example, in terms of DLCQ instanton quantum mechanics [3,4], deconstruction [5], and more recently in terms of the 5d supersymmetric Yang-Mills theory [6,7]. The 5-dimensional maximally supersymmetric Yang-Mills theory has been shown to be consistent with what we know about M5-branes [8] - [13], as well as on its partition function [14] - [17]. Despite the appearance of a UV divergence [18], it is possible that the theory is non-perturbatively well-defined ¹.

Recently, a six-dimensional action principle for the low energy theory has been proposed [1]. This construction was motivated by the analysis in [19, 20] where a set of 5d Yang-Mills gauge fields was introduced in order to incorporate non-trivial interactions among the 2-form potential. An important feature of this theory is that the self-interaction of the two-form gauge field is mediated by a Yang-Mills gauge field which is constrained to be determined completely in terms of the non-abelian tensor gauge field

\[
F_{\mu\nu} = c \int dx^5 \tilde{H}_{\mu\nu},
\]

and hence is auxiliary. This is necessary since there is no room in the (2,0) tensor multiplet to accommodate additional propagating degrees of freedom.

Although one does not have the full supersymmetric construction of the non-abelian (2,0) theory, by combining knowledge of conformal symmetry and R-symmetry one can argue for the form of the 1/2 BPS equations in the case when only one scalar field is turned on [2]. In the follow up paper [2], the case of two M5-branes was considered and the \( SU(2) \) non-abelian 5-branes equation was solved. It was argued that the solution could be lifted to become a solution of the non-abelian (2,0) theory with self-dual electric and magnetic charges. This is precisely the relation between the pure self-dual string solution in the linear Perry-Schwarz [21] and the half BPS solution of Howe-Lambert-West [22]. It was also shown that the dimensionless constant \( c \), which is a free parameter in the action, is fixed by the charge quantization of the self-dual string solution in the theory. As a result, the solution carries a unit of self-dual charges and describes an M2-brane spike emerging out of the multiple M5-branes worldvolume, with the adjoint scalar representing the transverse dimension.

In the next section, we generalize this construction to the general case of an arbitrary number \( N \) of M5-branes and construct a self-dual string solution with \( N \) unit of self-dual charges. Like the unit charge solution in [2], the new self-dual

¹It is important to understand how this happens, which will deepen our understanding about quantum field theory. It would also have implications on the renormalizability of gravity theory.
string obtains its charge from the auxiliary Yang-Mills field configuration, which is now given by a generalization of the SU(2) Wu-Yang monopole solution to one with arbitrary charge $N_2$. We will show that the value of $c$ is again fixed by the requirement of charge quantization. Moreover we will work out the dependence on $N_2$ and $N_5$ in the radius-transverse distance relation describing the M2-branes spike, and show that it agrees with the supergravity description of an intersecting M2-M5 branes system. Therefore our results provide further support of the model of [1]. The paper is concluded with some further discussions in section 3.

Other related works on multiple M5-branes include: [23] constructed a non-abelian version of (2,0) supersymmetric equation of motion using Lie 3-algebra; twistor space formulation of M5-branes has been proposed [24–26]; [27, 28] constructed a compactified theory of non-abelian 2-form gauge potentials with a self-dual field strength; [29, 30] constructed (1,0) superconformal models with tensor gauge fields as well as Yang-Mills gauge fields, which are further analyzed in [31, 32].

2. Non-Abelian SU($N_5$) Self-Dual String Solution

2.1 Non-abelian theory of multiple M5-branes

In [1], an action for a non-abelian chiral 2-form in 6-dimensions was constructed as a generalization of the abelian theory of Perry-Schwarz [21]. As in Perry-Schwarz, the self-dual tensor gauge field is represented by a $5 \times 5$ antisymmetric field $B_{\mu \nu}$, $\mu, \nu = 0, \ldots, 4$ with $B_{\mu 5} = 0$ and so manifest 6d Lorentz symmetry is given up. Presumably there is a covariant construction generalizing that of PST [33] where our theory will be obtained after a certain gauge fixing.

For $N_5$ coincident M5-branes, the self-duality equation of motion of the theory reads [1]

$$\tilde{H}_{\mu \nu} = \partial_5 B_{\mu \nu},$$

(2.1)

where the gauge field $A_{\mu}$ is constrained by [1] and carries no propagating degrees of freedom. Here

$$H_{\mu \nu \rho} = D_{[\mu} B_{\nu \rho]} = \partial_{[\mu} B_{\nu \rho]} + [A_{[\mu}, B_{\nu \rho]}],$$

(2.2)

$$\tilde{H}_{\mu \nu} = -\frac{1}{6} \epsilon_{\mu \nu \rho \sigma \tau} H^{\rho \sigma \tau}, \quad \epsilon_{01234} = -1,$$

(2.3)

$$F_{\mu \nu} = \partial_{[\mu} A_{\nu]} + [A_{[\mu}, A_{\nu]}]$$

(2.4)

and are in the adjoint representation of SU($N_5$). $c$ is a constant in the theory, which is fixed later by the charge quantization condition of the self-dual string solution [2]. Evidences have been given in [1,2] that this provides a description of the gauge sector of coincident M5 branes. Moreover, by combining knowledge of conformal symmetry and R-symmetry, we have argued that the 1/2 BPS equation in the case when only one scalar field is turned on takes the form

$$H_{ijk} = \epsilon_{ijk} \partial_5 \phi, \quad H_{ij5} = -\epsilon_{ijk} D_k \phi.$$

(2.5)
This generalizes the 1/2 BPS monopole equation in 4-dimensional non-abelian gauge theory. Our convention for the Lie algebra are: \([T^a, T^b] = if^{abc}T^c\), \(F_{\mu\nu} = iF^{a}_{\mu\nu}T^a\), \(A_\mu = iA^a_\mu T^a\) and \(F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu - f^{abc} A^b_\mu A^c_\nu\). In the abelian limit, the equations (2.5) reduce to that of [22].

### 2.2 Generalized non-abelian Wu-Yang monopole

Our construction of the self-dual string solution in the \(SU(N_3)\) theory will be based on a generalization of the \(SU(2)\) charge one Wu-Yang monopole solution to one with arbitrary charge \(n\). Let us start with a review of the generalized Wu-Yang monopole.

Consider an \(SU(2)\) gauge theory, and for reasons which will become clear later, let us denote the Lie algebra generators by \(\alpha^a (a = 1, 2, 3)\) with\
\[
[\alpha^a, \alpha^b] = i\epsilon^{abc}\alpha^c.
\]

The generalized Wu-Yang monopole is given by the following field configuration [34]
\[
A_k = iA_k^a \alpha^a = -\frac{i}{2r} (\tau^{(n)}_\varphi \hat{\theta}_k - n\tau^{(n)}_\theta \hat{\varphi}_k),
\]

i.e.
\[
A_\theta = -\frac{i}{2r} \tau^{(n)}_\varphi, \quad A_\varphi = \frac{in}{2r} \tau^{(n)}_\theta.
\]

Here
\[
\hat{\theta}_k dx^k := \cos \theta \cos \varphi dx^1 + \cos \theta \sin \varphi dx^2 - \sin \theta dx^3,
\]
\[
\hat{\varphi}_k dx^k := -\sin \varphi dx^1 + \cos \varphi dx^2,
\]
\[
x^1 = r \sin \theta \cos \varphi, \quad x^2 = r \sin \theta \sin \varphi, \quad x^3 = r \cos \theta,
\]

\[
\tau^{(n)}_r := (\sin \theta \cos(n\varphi), \sin \theta \sin(n\varphi), \cos \theta),
\]
\[
\tau^{(n)}_\theta := (\cos \theta \cos(n\varphi), \cos \theta \sin(n\varphi), -\sin \theta) = \frac{\partial \tau^{(n)}_r}{\partial \theta},
\]
\[
\tau^{(n)}_\varphi := (-\sin(n\varphi), \cos(n\varphi), 0) = \frac{1}{n \sin \theta} \frac{\partial \tau^{(n)}_r}{\partial \varphi}
\]

and
\[
\tau^{(n)}_r = \tau^{(n)}_r \cdot \vec{\alpha}, \quad \tau^{(n)}_\theta = \tau^{(n)}_\theta \cdot \vec{\alpha}, \quad \tau^{(n)}_\varphi = \tau^{(n)}_\varphi \cdot \vec{\alpha},
\]

The Wu-Yang monopole configuration considered in [2] is given by the case \(n = 1\).

It is useful to note that
\[
A_k = U \bar{A}_k U^{-1} + U \partial_k U^{-1},
\]

with
\[
U = e^{-i \vec{\alpha} \cdot \vec{\tau}^{(n)}}
\]
\[ \bar{A}_k = \frac{n}{r} \frac{1 - \cos \theta}{\sin \theta} \hat{\phi}_k \times i \alpha^3, \quad (2.15) \]

As a result,
\[ F_{ij} = U \bar{F}_{ij} U^{-1}, \quad (2.16) \]

where
\[ \bar{F}_{ij} = \partial_i \bar{A}_j - \partial_j \bar{A}_i = n \epsilon_{ijk} \frac{x^k}{r^3} \times i \alpha_3. \quad (2.17) \]

Since
\[ U \alpha^3 U^{-1} = \hat{r}^{(n)}_a \alpha^a, \quad (2.18) \]

we obtain
\[ F_{ij} = n \epsilon_{ijk} \frac{x^k}{r^3} \hat{r}^{(n)}_a \alpha^a. \quad (2.19) \]

Therefore (2.7), (2.19) provide a generalization of the Wu-Yang monopole solution to “charge” \( n \).

We also note that
\[ D_i (\hat{r}^{(n)}_a \alpha^a) = 0 \quad (2.20) \]

holds for the generalized Wu-Yang monopole. This is a generalization of \( D_i (x^a \alpha^a / r) = 0 \) for the charge one case.

### 2.3 Non-abelian self-dual string and the distance-radius relation

In this subsection we construct a self-dual string solution where the tensor gauge field is embedded in an \( SU(2) \) sub-algebra of \( SU(N_5) \). Let us denote the generators of the \( SU(2) \) factor by \( \alpha^a \):
\[ [\alpha^a, \alpha^b] = i \epsilon^{abc} \alpha^c. \quad (2.21) \]

As an \( N_5 \times N_5 \) irreducible representation \( \mathcal{R} \) of \( SU(2) \), the Casimir operator is given by
\[ \alpha_1^2 + \alpha_2^2 + \alpha_3^2 = \frac{1}{4} (N_5^2 - 1) := R^2. \quad (2.22) \]

Therefore
\[ \text{tr}(\alpha^a \alpha^b) = \mu^2 \delta^{ab}, \quad (2.23) \]

where
\[ \mu^2 := \frac{1}{12} N_5 (N_5^2 - 1). \quad (2.24) \]

Consider an ansatz of the field strength
\[ H_{\mu \nu \lambda} = H_{\mu \nu \lambda}^{(PS)} \hat{r}^{(n)}_a \alpha^a, \quad (2.25) \]

where \( H_{\mu \nu \lambda}^{(PS)} \) is given by the abelian Perry-Schwarz solution [21]
\[ H_{045}^{(PS)} = \frac{\beta x^5}{\rho^4}, \quad H_{ij5}^{(PS)} = -\frac{\epsilon_{ijk} \beta x^k}{\rho^4} \quad (2.26) \]
with $\beta$ being a constant to be fixed. Obviously the field strength is self-dual. Since $\hat{r}_a^{(n)}$ is independent of $x_5$, we can immediately integrate $H_{045}$, $H_{ij5}$ over $x_5$ and obtain

$$B_{\mu\nu} = B_{\mu\nu}^{(PS)} \hat{r}_a^{(n)} \alpha^a, \quad \mu\nu = ij \text{ or } 04,$$ \hspace{1cm} (2.27)

where the Perry-Schwarz 2-form potential is:

$$B_{ij}^{(PS)} = -\frac{1}{2} \frac{\beta x_k x_m}{r^3} \left( \frac{x^5 r}{\rho^2} + \tan^{-1} \left( \frac{x^5}{r} \right) \right), \quad B_{04}^{(PS)} = -\frac{\beta}{2\rho^2}. \hspace{1cm} (2.28)$$

One still need to check that this $B$-field reproduces correctly the other components $H_{ijk}$. To check this, we note that the constraint (1.1) gives

$$F_{ij} = -c \beta \pi \frac{1}{2} \epsilon_{ijm} x_m \hat{r}_a^{(n)} \alpha^a \quad F_{04} = 0. \hspace{1cm} (2.29)$$

Therefore if we take

$$\beta = -\frac{2n}{c\pi}, \hspace{1cm} (2.30)$$

for an integer $n$, then the auxiliary gauge field is given by the generalized Wu-Yang monopole. As a result of (2.20), we have

$$D[\lambda B_{\mu\nu}] = \partial[\lambda B_{\mu\nu}^{(PS)}] \hat{r}_a^{(n)} \alpha^a \hspace{1cm} (2.31)$$

and the result agrees with (2.25).

To obtain a self-dual string solution, we observe that the BPS equation (2.5) can be solved with

$$\hat{\phi} = -\left( u + \frac{\beta}{2\rho^2} \right) \hat{r}_a^{(n)} \alpha^a. \hspace{1cm} (2.32)$$

This leaves an unbroken $U(1)$ generated by

$$\hat{\phi} := \phi/|\phi| = \mp i\hat{r}_a^{(n)} \alpha^a/\mu \hspace{1cm} (2.33)$$

at $\rho \to \infty$. Here $|\phi|^2 := \text{tr}(\phi^2)$ and the $- (+)$ sign is chosen for $u > 0 (u < 0)$. Without loss of generality, we take $u > 0$ below. The asymptotic $U(1)$ fields are identified by a projection

$$H_{\mu\nu\lambda}^{U(1)} := \text{tr}(H_{\mu\nu\lambda} \hat{\phi}), \quad B_{\mu\nu}^{U(1)} := \text{tr}(B_{\mu\nu} \hat{\phi}), \hspace{1cm} (2.34)$$

then

$$H_{\mu\nu\lambda}^{U(1)} = \mu H_{\mu\nu\lambda}^{(PS)}, \quad B_{\mu\nu}^{U(1)} = \mu B_{\mu\nu}^{(PS)}. \hspace{1cm} (2.35)$$

The $U(1)$ magnetic and electric charges (per unit length) are defined by

$$2\pi^2 P = \int_{S^3} H_{\mu\nu\lambda}^{U(1)}, \quad 2\pi^2 Q = \int_{S^3} * H_{\mu\nu\lambda}^{U(1)}. \hspace{1cm} (2.36)$$
This gives the $U(1)$ charges,
\begin{equation}
P = Q = \mu \beta \tag{2.37}
\end{equation}

Note that our normalization for the charges differs from that of [21] by a factor of $2\pi^2$ of the volume of unit $S^3$. Our definition is consistent with the Gauss law of the form:
\begin{equation}
\nabla \cdot \vec{B} = 2\pi^2 P \delta^{(4)}(x), \quad \nabla \cdot \vec{E} = 2\pi^2 Q \delta^{(4)}(x) \tag{2.38}
\end{equation}
for a string lying in the $x^1$ direction, and $B_p := \epsilon_{pqrs}H^{qrs}/3!, \quad E_p := H_{01p}, \quad (p = 2, 3, 4, 5)$. With this normalization of charges, the charge quantization condition for dyonic strings in an abelian theory of 2-form in six-dimensions reads [35, 36]
\begin{equation}
PQ' + QP' = 2\pi \frac{Z}{(2\pi^2)^2}. \tag{2.39}
\end{equation}

For us, the charge quantization condition is modified due to the existence of a non-trivial center in the residual gauge group of the non-abelian theory. In fact, in a non-abelian Yang-Mills gauge theory with an unbroken gauge group of the form
\begin{equation}
H = U(1) \times K, \tag{2.40}
\end{equation}
where the $U(1)$ factor allows one to define the electric and magnetic charges, and $K$ is any residual gauge group, Corrigan and Olive have shown [37] that the charge quantization takes the form
\begin{equation}
e^{4\pi i qg} = k, \tag{2.41}
\end{equation}
where $k$ is an element in $C(K)$, the center of $K$. For example, if $K = SU(N)$, then $C(K) = Z_N,$
\begin{equation}
k = e^{2\pi in/N} \mathbf{1}_N, \quad n = \text{integers} \tag{2.42}
\end{equation}
and the charge quantization condition for the monopoles reads
\begin{equation}
qg = \frac{n}{2N}. \tag{2.43}
\end{equation}
For us, since the symmetry is broken down by the scalar field as $SU(N_5) \to U(1) \times SU(N_5 - 1)$ and since (2,0) supersymmetry demands that all fields in theory to be in the adjoint representation, this means the center of the residual gauge symmetry is given by $Z_{N_5-1}$. The same argument as Corrigan and Olive then gives
\begin{equation}
PQ' + QP' = 2\pi \frac{Z}{N_5 - 1 \cdot (2\pi^2)^2}. \tag{2.44}
\end{equation}
We can now use the charge quantization condition (2.44) to fix the value of $\beta$. Let us consider the situation where the self-dual string configuration arises as the intersection of a number $N_2$ of coincident M2-branes ending perpendicularly on our

\footnote{The normalization of the magnetic charge was taken as $\nabla \cdot \vec{B} = 4\pi g$ in [37].}
system of M5-branes. In this case, the charges $P,Q$ should be proportional to $N_2$. Substituting (2.37), we obtain

$$P = Q = \mu \beta = \frac{N_2}{\sqrt{N_5 - 1}} Q_0,$$

where $Q_0 = \sqrt{\pi/(2\pi^2)}$ is the minimal unit of charge in the abelian theory [21, 22].

From the field theory point of view, the geometrical shape of the M2-branes spike is described by the scalar field $X = 4\phi$. Here the normalization factor of 4 was obtained from the analysis of [22] where the geometrical relations between target space coordinates and worldvolume scalar field is the clearest in the superembedding formalism. It is convenient to introduce the root-mean-square distance (setting $l_p = 1$)

$$D := \sqrt{\frac{1}{N_5} |\text{tr}(4\phi)^2|}$$

as a measure of the transverse distance of the M2-branes spike from the system of M5-branes. The cross section of the M2-branes spike is an $S^3$ and the radius $\rho$ is governed by the transverse distance-radius relation

$$D = D_0 + \frac{2N_2 Q_0}{\sqrt{N_5(N_5 - 1)} \rho^2}$$

where

$$D_0 := \frac{4\mu \mu}{\sqrt{N_5}}$$

is a constant.

In addition to the worldvolume description, the system of intersecting M2-M5 branes also admits a supergravity description from which one can extract the transverse distance-radius relation and compare with our field theory result. However, the supergravity solution, beyond the brane probe approximation, for a system of M2-branes intersecting on a system of separated M5-branes where $(N_5 - 1)$ of them are in coincidence and another single M5-brane is separated at a finite distance away from the main group is not known. The closest system which admits a supergravity solution is the system of $N_2$ M2-branes intersecting a system of $N_5$ coincident M5-branes [38]. In this paper the technique of blackfold is applied and the transverse distance-radius relation

$$D = \frac{2\pi N_2}{N_5} \frac{1}{\rho^2}$$

is obtained. At distance $D$ large enough compared with the separation so that one cannot resolve the details of the separation, one can expect the supergravity solution for our system can be approximated by the supergravity solution of this system. In this case, one can ignore the first term in our field theory result (2.46) and our transverse distance-radius relation

$$D = \frac{2N_2 Q_0}{N_5 \rho^2}$$
agrees precisely with that of supergravity on their $N_2$ and $N_5$ dependence. Note that, however, (2.48) and (2.49) differs by an overall scale factor of $2\pi^2\sqrt{\pi}$. This is presumably due to a different unit is employed in supergravity.

We remark that the field theory description and the supergravity description are good only in their respective regime of validity. To confirm the validity of the agreement we found above, one need to check that there indeed exists an overlapping regime where one can trust both descriptions and hence compare the results sensibly. To check this, we note that our description of the M2-branes spike as a worldvolume soliton of the M5-branes is good provided that higher derivative corrections to our non-abelian action is small:

$$l_p|\partial^2 \Phi| \ll |\partial \Phi|.$$  \hspace{1cm} (2.50)

On the other hand, the validity of the blackfold description [38] was discussed in [39]. It was found there that for zero angular velocity which is our case, one needs

$$\rho \gg \max(l_p, \rho_c),$$  \hspace{1cm} (2.51)

where $\rho_c := N_5^{1/3}(1 + \sqrt{1 + 64N_2^2/N_5^3})^{1/6}l_p$. Therefore in the region (2.51), both the supergravity description and the M5-branes worldvolume description are valid. Now in order for the field theory result (2.46) to reduce to the form (2.48), it is required that

$$\rho \ll \left(\frac{3}{4\pi^3}\right)^{1/4}\left(\frac{N_2}{uN_5^2}\right)^{1/2}.$$  \hspace{1cm} (2.52)

Thus, by arranging the parameters $N_2$, $N_5$ and $u$ (for example a scaling limit involving large $N_2$, $N_5$ and small $u l_p^2$), a non-empty region of $\rho$ satisfying both (2.51) and (2.52) can always be achieved, and so the agreement we found is justified.

Summarizing, we can take the constant $c$ of the action to be $c = -2\mu\sqrt{N_5 - 1}/(Q_0\pi)$. Then (2.27), (2.32) provide a self-dual string solution to the theory if we take

$$\beta = \frac{N_2Q_0}{\mu\sqrt{N_5 - 1}}.$$  \hspace{1cm} (2.53)

3. Discussions

In this paper, we have constructed a self-dual string solution in the $SU(N_5)$ non-abelian theory of five-branes. Our solution carries an arbitrary $N_2$ unit of self-dual charge and obtains its charge through the generalized non-abelian Wu-Yang monopole configuration carried by the auxiliary one-form gauge field. We have also shown that the radius-transverse distance relation describing the M2-branes spike agrees precisely with the supergravity description of the intersecting M2-M5 branes system. Our results in this paper therefore provide further evidence that the non-abelian theory [1] does give a description for a system of multiple M5-branes.
Our self-dual string solution was obtained for a special symmetry breaking where there is a residual $SU(N_5 - 1)$ gauge symmetry. For a more general configurations of the Higgs field, the residual symmetry at infinity could be smaller and the self-dual string would be characterized by more number of charges. The discussion is similar to that of non-abelian monopole [40]. It will be interesting to construct these other kinds of self-dual strings and understand their dual description in supergravity and M-theory.

Our results were derived by solving the self-dual string equation (2.3). As argued in [2], this could be obtained from the 1/2 BPS condition of a supersymmetry transformation law of the form

$$
\delta \psi = (\Gamma^M \Gamma^I D_M \phi^I + \frac{1}{3!} \Gamma^{MNP} H_{MNP}) \epsilon. \tag{3.1}
$$

Our results thus provide support that (3.1) is indeed the correct supersymmetry transformation law for the (2,0) supersymmetric theory. This additional information should be helpful for tackling the so far mysterious nature [1] of the (2,0) supersymmetry. It would also be interesting to consider other BPS solutions and check them against the supergravity description.

With the solution at hand, one may perform a small fluctuation analysis of the solution and use it to learn more about the dynamics of non-abelian self-dual string. It would also be interesting to include couplings to a background $C$-field. In [41,42], the quantum Nambu geometry has been obtained as the quantum geometry for M5-branes in a large constant background $C$-field. One should be able to construct the star-product for this geometry and use it to derive the “Seiberg-Witten” map for the non-abelian M5-branes theory in a background $C$-field.

Finally we note that our field theory description is not good near the spike region ($\rho \simeq 0$). Apparently, unlike the non-abelian BPS monopole, the non-abelian interaction here is not sufficient to remove the spike singularity. Nevertheless one can expect it to be smoothen out only in the complete description of M-theory with all higher derivative corrections included [21].

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