On the Advance of Making Language Models Better Reasoners

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Abstract

Large language models such as GPT-3 and PaLM have shown remarkable performance in few-shot learning. However, they still struggle with reasoning tasks such as the arithmetic benchmark GSM8K. Recent advances deliberately guide the language model to generate a chain of reasoning steps before producing the final answer, successfully boosting the GSM8K benchmark from 17.9% to 58.1% in terms of problem solving rate. In this paper, we propose a new approach, DIVERSE (Diverse Verifier on Reasoning Step), to further advance their reasoning capability. DIVERSE first explores different prompts to enhance the diversity in reasoning paths. Second, DIVERSE introduces a verifier to distinguish good answers from bad answers for a better weighted voting. Finally, DIVERSE verifies the correctness of each single step rather than all the steps in a whole. We conduct extensive experiments using the latest language model code-davinci-002 and demonstrate that DIVERSE can achieve new state-of-the-art performance on six out of eight reasoning benchmarks (e.g., GSM8K 74.4% → 83.2%), outperforming the PaLM model with 540B parameters.

1 Introduction

Large-scale pretrained language models have recently exhibit exceptional performance in either few-shot learning with prompt (Radford et al., 2019; Le Scao and Rush, 2021; Jin et al., 2022) or fine-tuning (Houlsby et al., 2019; Hu et al., 2021; He et al., 2022). Gigantic models such as the GPT-3 with 175B parameters (Brown et al., 2020) and the PaLM with 540B parameters (Chowdhery et al., 2022) continue to demonstrate their remarkable performance on a variety of tasks using the few-shot paradigm. However, as illustrated by Cobbe et al. (2021), all these models are still struggling to produce reasonable performance in various reasoning tasks, often requiring multiple steps to produce correct answers.

Recent progresses (Wei et al., 2022; Zhou et al., 2022; Kojima et al., 2022; Lampinen et al., 2022) point out that large pretrained language models have already built in reasoning capabilities although they need design specific prompts to unleash their power. For example, Wei et al. (2022) proposes the insertion of multi-step reasoning paths before generating the final answers, calling their approach chain-of-thought reasoning. Using the GSM8K (Cobbe et al., 2021) arithmetic benchmark as an example, their prompting technique improves the accuracy from 17.9% to 58.1%. As a follow-up, Wang et al. (2022c) continues to introduce a voting mechanism to produce a more accurate answer and boost the performance of GSM8K to 74.4% with the help of the PaLM model containing 540 billion parameters (Chowdhery et al., 2022). Built on top of all these successes, this paper is a continuation of this line of research, with the focus on advancing the reasoning capabilities using gigantic language models in following three areas.

First, motivated by “people think differently, while different thoughts often lead to the same correct answer”, Wang et al. (2022c) proposed self-consistency, i.e., sampling different reasoning paths from the language model, then returning the most consistent final answer by majority voting. We continue to advocate the ideas of diversity and “All Roads Lead to Rome”. However, besides using sampling with a single prompt to produce multiple reasoning paths, we propose to introduce the diversity in the prompt as a complementary solution to produce a higher degree of diversity.

Second, when generating a reasoning path, causal language models have no mechanism to cor-
Prompt & Samples

Q: If there are 3 cars in the parking lot and 2 more cars arrive, how many cars are in the parking lot?
A: There are 3 cars in the parking lot already. 2 more arrive. Now there are 3 + 2 = 5 cars. The answer is 5.

Q: Janet’s ducks lay 16 eggs per day. She eats three for breakfast every morning and bakes muffins for her friends every day with four. She sells the remainder for $2 per egg. How much does she make every day?
A:

[Sample 1] She has 16 - 3 - 4 = 9 eggs left. So she makes 2 * 9 = 18 per day. The answer is 18. (✓)

[Sample 2] This means she uses 3 + 4 = 7 eggs every day. So in total she sells 7 * 2 = 14 per day. The answer is 14. (✗)

[Sample 3] She eats 3 for breakfast, so she has 16 - 3 = 13 left. Then she bakes muffins, so 13 - 4 = 9 eggs left. She makes 9 * $2 = $18. The answer is 18. (✓)

Figure 1: Prompting large language models to generate different reasoning paths, then selecting the final answer via majority voting (Wang et al., 2022c).

To correct previous errors in earlier steps which quickly leads to disorientated results, Cobbe et al. (2021) proposes using a verifier to judge the correctness of each reasoning path, thereby improving the GSM8K accuracy from 33% to 57%. Inspired by their success, we introduce the verifier mechanism to guide the voting mechanism in Wang et al. (2022c). In other words, we believe "not all reasoning paths are equal or good" and use the verifier to score the votes.

Third, as an answer is generated based on the reasoning of multiple steps, when a correct answer is generated, we conjecture all the steps are contributing to the final correctness. However, when a wrong answer is generated, it does not mean all the steps are wrong or contribute to the wrongness. In other words, some steps could still be correct but some follow-up off-course steps result in a final wrong answer. With this motivation, we design a mechanism to assign a fine-grained label to each step and propose a step-aware verifier and attribute the correctness to each single step.

Based on the three key insights, we name this novel method as DiVERSe (diverse verifier on reasoning step). We conduct extensive experiments with analysis using three OpenAI language models (davinci, text-davinci-002, and code-davinci-002) on eight tasks requiring different reasoning skills (arithmetic reasoning, commonsense reasoning, and inductive reasoning). Experimental results clearly demonstrate that DiVERSe can bring significant and consistent improvements on these datasets, comparing to recent state-of-the-art (SOTA) works. Furthermore, DiVERSe also empowers code-davinci-002 to achieve new state-of-the-art results on 6 of the 8 benchmarks1: GSM8K (74.4% → 83.2%), AsDiv (81.9% → 88.7%), MultiArith (99.3% → 99.8%), SVAMP (86.6% → 87.0%), SingleEq (79.5% → 94.9%), and CLUTRR (67.0% → 95.9%). We will make our data publicly available at https://github.com/microsoft/DiVERSe.

2 Preliminaries

Prompting. Prompting means prepending a few exemplars to the task input x and generating the output y from the pretrained language model:

\[ p(y|x) = \prod_{t=1}^{\lfloor y \rfloor} p_{LM}(y_t|x, y_{<t}), \]  

where \( y \) is the concatenation of \( K \) exemplars:

\[ C = (x_1, y_1); (x_2, y_2); \ldots; (x_K, y_K). \]

(2)

We denote prompt as the concatenation of the exemplars \( C \) and the input \( x \).

Reasoning Paths. For reasoning tasks that aim to generate an answer \( y \) for a question \( x \), Wei et al. (2022) proposed the insertion of a reasoning path \( z \) before generating the answer \( y \):

\[ C' = (x_1, z_1, y_1); \ldots; (x_K, z_K, y_K), \]

(3)

Most of the previous SOTA results were achieved by self-consistency on PaLM-540B (Chowdhery et al., 2022).

Table 1: Besides arithmetic reasoning, we also investigate commonsense and inductive reasoning.

| Task                  | Davinci | text-davinci-002 | code-davinci-002 |
|-----------------------|---------|------------------|------------------|
| GSM8K                 | 74.4%   | 83.2%            | 74.4%            |
| AsDiv                 | 81.9%   | 88.7%            | 81.9%            |
| MultiArith            | 99.3%   | 99.8%            | 99.3%            |
| SVAMP                 | 86.6%   | 87.0%            | 86.6%            |
| SingleEq              | 79.5%   | 94.9%            | 79.5%            |
| CLUTRR                | 67.0%   | 95.9%            | 67.0%            |

1 Most of the previous SOTA results were achieved by self-consistency on PaLM-540B (Chowdhery et al., 2022).
where $z_i$ is a text reasoning path of how the answer $y_i$ is reasoned step-by-step for question $x_i$.

Then, during inference, a reasoning path $z$ will be generated before the answer $y$:

$$p(y|C', x) = p(z|C', x) \cdot p(y|C', x, z).$$  \hspace{1cm} (4)

Figure 1 demonstrates this idea in arithmetic reasoning (GSM8K), and Table 1 demonstrates this idea in commonsense reasoning (StrategyQA) and inductive reasoning (CLUTRR).

### 3 Diverse Verifier on Reasoning Step

Figure 2 shows the overview of DIVERSE. The key insights are three-fold: (1) leveraging diverse prompts to induce more diverse reasoning paths from the language models (Section 3.1); (2) training a voting verifier to better derive the final answers from multiple reasoning paths (Section 3.2); (3) leveraging step correctness to further boost the voting verifier (Section 3.3).

#### 3.1 Diverse Prompts

Reasoning can be boosted by exploiting diverse reasoning paths, just as “All Roads lead to Rome”. Wang et al. (2022c) proposed to generate different reasoning paths from language models by sampling decoding. However, it uses a fixed collection of exemplars for all prompts. These fixed exemplars may bias the generation, thus limiting the diversity of the generated reasoning paths.

To mitigate this, we propose to provide $M_1$ different prompts for each question, then sample $M_2$ reasoning paths for each prompt. Therefore, we get $M = M_1 \times M_2$ diverse reasoning paths for each question. Our main experiments use $M_1 = 5$ and $M_2 = 20$.

A critical issue is how to provide diverse prompts\(^2\). Supposing that there is an exemplar base $E$, we can sample $K$ exemplars from it to construct a prompt, and repeat this $M_1$ times independently to construct $M_1$ prompts with diverse exemplars.

For scenarios that do not have sufficient exemplars (i.e., $|E| < K * M_1$), we propose to bootstrap the diversity of prompts by “self-teaching”, i.e., generating pseudo reasoning paths from a few exemplars and some (question, answer) pairs without reasoning paths\(^3\). Suppose that $D$ is a dataset without reasoning paths, consisting of $(x, y^*)$ pairs. Given the small exemplar base $E$, for each $(x, y^*) \in D$, we can use prompting to generate a reasoning path $z$ and the predicted answer $y$. We define the pseudo exemplar base $E'$ as:

$$E' = \{(x, z, y)| (x, y^*) \in D, y = y^*\},$$  \hspace{1cm} (5)

then $E \cup E'$ can be regarded as the new exemplar base for generating diverse prompts.

#### 3.2 Voting Verifier

Wang et al. (2022c) uses majority voting to conclude the agreement $\hat{y}$ of different reasoning paths:

$$\hat{y} = \arg \max_y \sum_{i=1}^3 1_{y_i = y},$$  \hspace{1cm} (6)

where $y_i$ represents the answer of the $i$-th reasoning path, and $1_{y_i = y}$ is an indicator function that returns 1 (or 0) if $y_i = y$ (or not).

This vanilla method has its limitations when reasonable minority solutions are overwhelmed by disorientated majority solutions.

**Verifier.** Sample-then-rank is a widely-used strategy in sequence-to-sequence generation (Shen et al., 2021; Cobbe et al., 2021), i.e., generating i.e., to permute exemplars in the original prompt. However, this strategy does not increase the diversity in terms of exemplars.

\(^2\)This is motivated by Zelikman et al. (2022).

\(^3\)Wang et al. (2022c) tried an ensemble-based approach,
When a reasoning path gives an incorrect answer, we conjecture not all the steps are contributing to the final wrongness. With this motivation, we propose to extend the voting verifier to a step-aware voting verifier by introducing an extended loss function:

\[
\hat{y} = \arg \max_y \sum_{i=1}^{\|D\|} \sum_{j=1}^{\|S_i\|} \text{BCE}(\text{label}_{i,j}, f'(\text{input}_i, j)),
\] (9)

Voting Verifier. We propose voting verifier, which combines the advantages of voting and verifier. Concretely, we combine Equation 6 and Equation 7 to a new version:

\[
\hat{y} = \arg \max_y \sum_{i=1}^{\|D\|} \text{BCE}(\text{label}_i, f(\text{input}_i)),
\] (8)

where BCE means binary cross-entropy loss, \((\text{input}_i, \text{label}_i)\) represents the \(i\)-th item in \(D\).

3.3 Step-aware Voting Verifier

Each reasoning path consists of several steps. When a reasoning path gives an incorrect answer, we conjecture not all the steps are contributing to the final wrongness. With this motivation, we propose to extend the voting verifier to a step-aware voting verifier by introducing an extended loss function:

\[
\mathcal{L} = \mathcal{L}_0 + \alpha \cdot \mathcal{L}_1,
\]

\[
\mathcal{L}_1 = \sum_{i=1}^{\|D\|} \sum_{j=1}^{\|S_i\|} \text{BCE}(\text{label}_{i,j}, f'(\text{input}_i, j)),
\] (10)

\[\text{Sample 1}\] She uses 3 + 4 = 7 eggs every day. So she has 16 - 7 = 9 eggs left. So she makes $2 \times 9 = $18 per day. The answer is 18.

\[\text{Sample 2}\] This means 3 + 4 = 7 eggs are used every day. So in total she sells 7 * $2 = $14 per day. The answer is $14.

Figure 3: How we get the step-level labels. Sample 2 is incorrect, but its first step is correct. The reason is: the intermediate result (7) matches with the first step of Sample 1, a correct reasoning path.

We propose deberta-v3-large (He et al., 2021): \(f(x_i, z_i, y_i)\) is the predicted probability of the positive label, given the concatenation of \(x_i, z_i,\) and \(y_i\) as the text input.

\[\mathcal{L}_0 = \sum_{i=1}^{\|D\|} \text{BCE}(\text{label}_i, f(\text{input}_i))\]

\[\text{where BCE means binary cross-entropy loss, } (\text{input}_i, \text{label}_i) \text{ represents the } i\text{-th item in } D.\]

Arithmetic Reasoning. Following Wang et al. (2022c), we use AsDiv (Miao et al., 2020), SingleEq (Koncel-Kedziorski et al., 2015), MultiArith (Royer and Roth, 2015), SVAMP (Patel et al., 2021), and GSM8K (Cobbe et al., 2021).

Commonsense Reasoning. Following Wang et al. (2022c), we use CommonsenseQA (Talmor et al., 2019) and StrategyQA (Geva et al., 2021).

Inductive Reasoning. The definition of the term “inductive learning” could be stated as: given a hyperparameter to combine \(\mathcal{L}_0\) and the step-level auxiliary loss \(\mathcal{L}_1; S_{i,1}, S_{i,2}, ..., S_{i,|S_i|}\) are multiple steps in \(z_i\); \(\text{label}_{i,j}\) represents whether \(S_{i,j}\) is correct or not; \(f'(\text{input}_i, j)\) represents the probability of the positive label for \(S_{i,j}\). 4

A critical issue is how to get the step-level labels (i.e., \(\text{label}_{i,j}\)) for negative training data with wrong answers. We propose to address this issue through a “cross-validation” method. Figure 3 demonstrates an example of this idea. Concretely: \(\text{label}_{i,j} = 1\) if and only if \(\exists 1 \leq k \leq |D|(k \neq i)\) and \(c \geq j\), s.t. \(\text{label}_{k,1}, ..., \text{label}_{k,c}\) “supports” \(S_{i,1}, ..., S_{i,c}\). In different tasks, “supports” can be defined differently: for arithmetic tasks, it means the set of intermediate results of \(S_{k,1}, ..., S_{k,c}\) the same as that of \(S_{i,1}, ..., S_{i,c}\); for other tasks, it means: \(\forall 1 \leq r \leq c, S_{k,r}\) and \(S_{i,r}\) are semantically equivalent. 5
Table 2: The comparison of DiVERSE with previous SOTA results (fine-tuning; non-gigantic pretrained transformers) are: \(a\): Cobbe et al. (2021), \(b\): Miao et al. (2020), \(c\): Roy and Roth (2015), \(d\): Pi et al. (2022), \(e\): Hu et al. (2019a), \(f\): Xu et al. (2021), \(g\): Chowdhery et al. (2022), \(h\): Sinha et al. (2019). The parameter number of each model is hidden to us.

| Method                  | GSM8K | AtoY | MultiArith | SVAMP | SingleEq | CommonsenseQA | StrategyQA | CLUTRR |
|-------------------------|-------|------|------------|-------|----------|----------------|------------|--------|
| Previous SOTA (Fine-tuning) | 37.7  | 75.3%| 60.3%  | 57.4% | 32.5%  | 51.2% | 73.9% | 67.0% |
| 9–12 year olds (Cobbe et al., 2021) | 60    | -    | -       | -     | -       | -    | -    | -     |
| LAMA (I)                |       |      |          |       |         |      |       |       |
| Greedy Decode           | 17.1  | 49.0 | 51.8     | 38.9  | 56.6    | 57.9 | 59.4 | -     |
| Self-Consistency        | 27.7  | 58.2 | 75.7     | 53.3  | -       | 63.1 | 67.8 | -     |
| PaL (S)                 |       |      |          |       |         |      |       |       |
| Greedy Decode           | 56.5  | 74.0 | 94.7     | 79.0  | 79.5    | 79.0 | 57.3 | -     |
| Self-Consistency        | 74.4  | 81.9 | 99.3     | 86.6  | -       | 80.7 | 81.6 | -     |
| GPT-3 (4B)              | 8.7   | 31.4 | 34.1     | 21.2  | 38.2    | 48.2 | 59.3 | 33.6 |
| Greedy Decode           | 18.9  | 52.8 | 68.6     | 44.6  | 59.6    | 57.4 | 65.9 | 42.5 |
| Self-Consistency        | 30.9 (±12.0) | 57.6 (±18.9) | 87.4 (±19.0) | 46.9 (±2.3) | 65.1 (±5.5) | 75.0 (±17.6) | 67.9 (±20.0) | 92.5 (±98.0) |
| text-davinci-002        | 37.1  | 60.8 | 70.7     | 60.0  | 73.3    | 65.5 | 60.3 | 18.4 |
| Greedy Decode           | 37.1  | 60.8 | 70.7     | 60.0  | 73.3    | 65.5 | 60.3 | 18.4 |
| Self-Consistency        | 78.2  | 76.9 | 88.6     | 78.2  | 87.2    | 79.2 | 72.9 | 15.8 |
| DiVerSE                | 70.2 (±12.0) | 83.5 (±16.6) | 96.4 (±16.8) | 82.7 (±14.5) | 86.5 (±6.7) | 79.2 (±4.3) | 73.1 (±2.4) | 68.5 (±53.7) |
| text-davinci-002        | 55.3  | 75.5 | 88.8     | 70.5  | 87.5    | 73.4 | 73.8 | 32.9 |
| Greedy Decode           | 76.7  | 86.2 | 98.6     | 85.8  | 93.7    | 77.3 | 78.3 | 55.6 |
| Self-Consistency        | 82.3 (±5.6) | 98.7 (±1.5) | 99.8 (±1.2) | 87.5 (±1.2) | 94.9 (±1.2) | 79.9 (±2.6) | 77.7 (±0.6) | 95.9 (±68.3) |

Table 2: The comparison of DiVERSE, Greedy Decode and Self-Consistency. The previous SOTA results (fine-tuning; non-gigantic pretrained transformers) are: \(a\): Cobbe et al. (2021), \(b\): Miao et al. (2020), \(c\): Roy and Roth (2015), \(d\): Pi et al. (2022), \(e\): Hu et al. (2019a), \(f\): Xu et al. (2021), \(g\): Chowdhery et al. (2022), \(h\): Sinha et al. (2019). The parameter number of each model is hidden to us.

Training set of objects whose classes are known, find a rule for predicting the class of an unseen object as a function of its attribute values (Quinlan, 1990). We use CLUTRR (Sinha et al., 2019), a diagnostic benchmark for inductive reasoning from text, requiring natural language understanding (NLU) systems to infer kinship relations between characters in short stories.

4.2 Details

Language Models. We use three OpenAI language models: davinci, text-davinci-002 and code-davinci-002. We use the default parameters except a temperature of 0.5 for sampling decoding.

Exemplars. For arithmetic/commonsense/inductive reasoning, each prompt contains 5/7/7 exemplars. For DiVerSE, each question has 5 different prompts, and 20 reasoning paths are sampled from the language model for each prompt. For arithmetic reasoning, the exemplars are randomly sampled from the training dataset of GSM8K; for CLUTRR, the exemplars are sampled from its training dataset, with reasoning paths synthesized by handcraft rules; for StrategyQA and CommonsenseQA, we construct 1,000 pseudo exemplars by “self-teaching” (Section 3.1) from the 7 “seed” exemplars provided by Wei et al. (2022).

Datasets without Reasoning Paths. For each task, we sample 1,000 (question, answer) pairs from the training dataset to train the verifier.

Verifier. We fine-tune deberta-v3-large (He et al., 2021) with learning rate $1 \times 10^{-5}$ and batch size 128. For the step-aware verifier, we select the best $\alpha$ among 0.0/0.1/0.2/0.3.
guage model with 540 billion parameters.

On all the five arithmetic reasoning tasks, DiVERSE (with code-davinci-002) achieves new SOTA results, with an average improvement of 6.2%. On the two commonsense reasoning tasks, the performance of DiVERSE is slightly lower (−2.2%) than that of PaLM-based self-consistency. We speculate that the reason might be: these two commonsense reasoning tasks are multiple choice tasks rather than open-ended generation tasks, resulting in more false-positive exemplars in the pseudo exemplar base (Section 3.1). Details will be discussed in Section 5.3. Regarding inductive reasoning, DiVERSE achieves a surprisingly good performance of 95.9% on the CLUTRR task, outperforming (+28.9%) the previous SOTA result with fine-tuning (Sinha et al., 2019).  

5.3 Noises in Multiple Choice Tasks

In our experimental setup, StrategyQA and CommonsenseQA are a bit more challenging than other tasks, as they use pseudo exemplars generated through “self-teaching” (Section 3.1).

“Self-teaching” may lead to bad exemplars, whose reasoning paths are invalid but happen to yield answers coinciding with the ground truth. Questions in StrategyQA and CommonsenseQA are two-choice and four-choice questions, respectively. Therefore, such noise would be more serious in StrategyQA than in CommonsenseQA. This somehow explains why DiVERSE can achieve comparable performance (−0.8%) as the PaLM-based SOTA on CommonsenseQA, while it sees a 3.9% performance decline to PaLM on StrategyQA.

6 Ablations

We also conduct ablation experiments and analysis to investigate the keys to the success of DiVERSE.

6.1 Diverse Prompts vs. Sampling Decoding

To evaluate the effectiveness of diverse prompts, we compare ⟨M₁ = 5, M₂ = 20⟩ with ⟨M₁ = 1, M₂ = 100⟩ (i.e., sampling decoding used by self-consistency), both of which use majority voting.

| Method         | GSM8K | CQA   | CLUTRR |
|----------------|-------|-------|--------|
| davinci:       |       |       |        |
| M₁ = 1, M₂ = 100 | 18.9  | 57.4  | 42.5   |
| M₁ = 5, M₂ = 20  | 21.3  | 57.5  | 45.9   |
| text-davinci-002: |       |       |        |
| M₁ = 1, M₂ = 100 | 58.2  | 72.9  | 15.8   |
| M₁ = 5, M₂ = 20  | 61.3  | 77.3  | 21.2   |
| code-davinci-002: |       |       |        |
| M₁ = 1, M₂ = 100 | 76.7  | 77.3  | 35.6   |
| M₁ = 5, M₂ = 20  | 80.0  | 78.8  | 43.8   |

Table 3: The effectiveness of diverse prompts ⟨(5, 20)⟩ compared to pure sampling decoding (Wang et al., 2022c), under majority voting.

Table 4 shows the results. We show neither pure diverse prompts nor pure sampling decoding is the best setup. In other words, we need to combine diverse prompts with sampling decoding as a complementary solution.

6.2 The Power of Voting Verifier

We compare three algorithms to conclude the agreement from diverse reasoning paths: majority voting, verifier, and voting verifier. Table 5 shows the results. Comparing to majority voting, our voting verifier can significantly and consistently boost the reasoning performance across different tasks and different language models. Verifier without voting often outperforms majority voting, but extending it to voting verifier can further boost the performance.
Table 5: The effectiveness of voting verifier. All experiments in this table use $\langle M_1, M_2 \rangle = (5, 20)$.

| Method         | GSM8K | CQA | CLUTRR |
|----------------|-------|-----|--------|
| davinci:       |       |     |        |
| Voting         | 21.3  | 57.4| 45.9   |
| Verifier       | 27.0  | 74.1| 93.2   |
| Voting Verifier| **30.6** | **75.0** | **92.5** |
| text-davinci-002: |       |     |        |
| Voting         | 61.3  | 77.3| 35.6   |
| Verifier       | 62.7  | 77.9| 93.8   |
| Voting Verifier| **68.9** | **79.2** | **93.8** |
| code-davinci-002: |       |     |        |
| Voting         | 80.0  | 75.4| 43.8   |
| Verifier       | 65.9  | **78.8** | **95.9** |
| Voting Verifier| **82.3** | **78.8** | **95.9** |

Table 6: The effectiveness of step-aware voting verifier, with $\langle M_1, M_2 \rangle = (5, 20)$.

| Method         | GSM8K | CommonsenseQA |
|----------------|-------|----------------|
| davinci:       |       |                |
| DiVERSe (without step) | 30.6  | 75.0           |
| DiVERSe (with step)    | **30.9** | **76.0**      |
| text-davinci-002: |       |                |
| DiVERSe (without step) | 68.9  | 79.2           |
| DiVERSe (with step)    | **70.2** | **79.8**      |
| code-davinci-002: |       |                |
| DiVERSe (without step) | 82.3  | 78.8           |
| DiVERSe (with step)    | 81.5  | **79.9**       |

6.3 The Impact of Step-aware Voting Verifier

Table 6 compares the performance of DiVERSe with / without step-level information in its voting verifier. We observe that, extending the voting verifier to a step-aware version can bring performance gains in most of the experiments. For code-davinci-002 on GSM8K, the step-aware version leads to slightly performance drop. We conjecture code-davinci-002 is more powerful and can generate higher quality reasoning paths for GSM8K, thus reducing the necessity of step-level information.

6.4 How Many Diverse Outputs Do We Need?

Figure 4 compares the accuracy at different $M$. For each question, we generate 100 reasoning paths, then sample $M$ reasoning paths from them to generate the final answer. We observe that: (1) more reasoning paths lead to better performance, but marginal effects start to appear at $M \geq 50$; (2) DiVERSe performs significantly and consistently better than self-consistency at different $M$ values.

6.5 How Many Training Data Do We Need?

DiVERSe requires a dataset without reasoning paths for training the verifier. Figure 5 shows how the size of this dataset affects the performance. We observe that: the performance is only reduced by about 2%, even if the size of training data is cut by 75% (from 1,000 to 250). With the same reasoning paths, voting verifier performs better than majority voting, while verifier without voting causes significant performance drops.

6.6 The Impact of the Number of Exemplars

We conduct experiments for $k = 3/5/8$ ($k$ means how many exemplars are used in each prompt) on GSM8K. Figure 6 shows the results. We observe that: using 8 exemplars in each prompt can further boost the accuracy of GSM8K to 83.2%.

7 Related Work

Reasoning Skills. Researchers in the literature have proposed many benchmarks requiring various reasoning skills, including commonsense reasoning (Zellers et al., 2018; Talmor et al., 2019; Bhagavatula et al., 2019; Geva et al., 2021) numerical reasoning (Dua et al., 2019), multi-hop reasoning (Yang et al., 2018), arithmetic reasoning (Koncel-Kedziorski et al., 2015; Roy and Roth, 2015; Miao et al., 2020; Patel et al., 2021; Cobbe et al., 2021), logical reasoning (Liu et al., 2020; Yu et al., 2020), inductive reasoning (Sinha et al., 2019) and tabular reasoning (Chen et al., 2020; Zhu et al., 2021).
Reasoning with Symbolic Systems. Much research in the literature enhances the reasoning capabilities of machine learning systems by exploiting symbolic systems, including knowledge graphs (Mihaylov and Frank, 2018; Bauer et al., 2018; Kundu et al., 2019; Wang et al., 2019; Lin et al., 2019; Ding et al., 2019; Feng et al., 2020; Wang et al., 2022b), or question taxonomies (Dua et al., 2019; Andor et al., 2019; Hu et al., 2019b; Wang et al., 2022a). Although these methods work well on specific benchmarks, they usually require domain-specific designs and human efforts, thus limiting the generalizability.

Reasoning via Language Models. This line of work aims to address reasoning tasks in a general sequence-to-sequence manner, empowered by reasoning-aware pre-training or fine-tuning of language models. For example, Deng et al. (2021) proposed to train the language model with crawled data from the internet; Asai and Hajishirzi (2020) proposed a logic-guided data augmentation method to pre-train the language model; Shen et al. (2021); Cobbe et al. (2021) proposed to train a verifier to rank solutions sampled from fine-tuned language models; Geva et al. (2020); Yoran et al. (2022); Campagna et al. (2020); Wang et al. (2022a) proposed to equip language models with reasoning abilities by generating training examples with human-designed templates; Pi et al. (2022) proposed to inject reasoning capabilities into language models by continual pre-training on program execution data. 

Reasoning via Prompting Gigantic Language Models. Gigantic language models like GPT-3 (Brown et al., 2020) have demonstrated impressive few-shot learning capabilities in many tasks and have attracted many research interests on making gigantic language models better few-shot learners (Zhao et al., 2021; Holtzman et al., 2021; Min et al., 2021; Liu et al., 2022; Lu et al., 2021; Rubin et al., 2021; Min et al., 2022). However, these methods struggle to address tasks requiring reasoning skills. To mitigate this, recently there is a line of research that focuses on unleashing the reasoning capabilities of gigantic language models via better prompting strategies. Wei et al. (2022) proposed chain-of-thought reasoning, of which the key insight is the insertion of multi-step reasoning paths before generating the final answers; Wang et al. (2022c) proposed to improve chain-of-thought reasoning via self-consistency, of which the key insight is to conclude the most consistent answer from different reasoning paths sampled from the language model; Zhou et al. (2022); Creswell et al. (2022) proposed to leverage gigantic language models to decompose questions into sub-questions, thereby addressing them in an iterative manner; Kojima et al. (2022) proposed that gigantic language models can even be good zero-shot reasoners, by designing prompts that can induce language models to do reasoning step-by-step; Lampinen et al. (2022) proposed building a prompt by selecting examples and explanations together, thus substantially improving performance over selecting examples alone. Despite their great successes, these works come with
their limitations. This paper is a continuation of this line of research, focusing on diverse verifier on reasoning steps.

8 Conclusion and Future Work

We introduce DiVerSe, an effective and general method to make large language models better reasoners. As a continuation of the line of research that prompting language models using multi-step reasoning paths, the key insights of DiVerSe are three-fold: diverse prompts, voting verifier, and step-level correctness. Experimental results clearly show that DiVerSe can bring significant and consistent improvements. For example, with code-davinci-002, DiVerSe achieves new state-of-the-art results in most of the reasoning tasks, outperforming the 540B PaLM model combined with previous prompting approaches.

There are many directions for our future works. (1) As discussed in Section 5.3, we will continue to investigate how to reduce or recognize false positive pseudo exemplars. (2) We plan to investigate mechanisms to produce better diverse prompts than simple sampling. (3) We will extend DiVerSe to other tasks and continue to design better prompting techniques to elicit the power of gigantic language models.

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