Uplink Channel Estimation for Double-IRS Assisted Multi-User MIMO

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Abstract—To achieve the more promising passive beamforming gains in the double-intelligent reflecting surface (IRS) assisted system over the conventional single-IRS system, channel estimation is practically indispensable but also a more challenging problem to tackle, due to the presence of not only the single- but also double-reflection links that are intricately coupled. In this paper, we propose a new and efficient channel estimation scheme for the double-IRS assisted uplink multiple-input multiple-output (MIMO) communication system to resolve the cascaded channel state information (CSI) of both its single- and double-reflection links. First, for the single-user case, the higher-dimensional double-reflection channel is efficiently estimated at the multi-antenna base station (BS) with low training overhead by exploiting the fact that its cascaded channel coefficients are scaled versions of those of a lower-dimensional single-reflection channel. Then, the proposed channel estimation scheme is extended to the multi-user case, where given an arbitrary user’s cascaded channel estimated as in the single-user case, the other users’ cascaded channels are scaled versions of it and thus can be estimated with reduced training overhead. Simulation results verify the effectiveness of the proposed channel estimation scheme as compared to the benchmark scheme.

I. INTRODUCTION

Intelligent reflecting surface (IRS) is an innovative solution to the realization of smart and reconfigurable environment for wireless communications [1]–[3]. Specifically, IRS consists of a large number of passive reflecting elements with ultra-low power consumption, each of which is capable of controlling the phase shift and/or amplitude of the incident signal in a programmable manner so as to collaboratively reshape the wireless propagation channel in favor of signal transmission. Moreover, as being light weight and free of radio frequency (RF) chains, large-scale IRS can be densely deployed in various wireless communication systems [4]–[6] with a low and scalable energy consumption and implementation cost.

Prior works on IRS mainly considered the wireless communication systems assisted by one or more distributed IRSs, each independently serving its nearby users without taking into account the inter-IRS signal reflection, which, however, fails to capture the cooperative beamforming gains between IRSs to further improve the system performance. Only recently, the cooperative beamforming gains over the inter-IRS channel has been explored in the double-IRS assisted system [7]–[9], which was shown to achieve a much higher-order passive beamforming gain than the conventional single-IRS system (i.e., $O(M^4)$ versus $O(M^2)$ with $M$ denoting the total number of reflecting elements in both systems). However, achieving such a more appealing passive beamforming gain requires more channel training overhead in practice, due to more channel coefficients to be estimated over the inter-IRS double-reflection link, in addition to the single-reflection links in the conventional single-IRS system. Existing works on IRS channel estimation mainly focused on the channel state information (CSI) acquisition for single-reflection links only [10]–[15], which, however, is inapplicable to the double-IRS assisted system with the co-existence of single- and double-reflection links as illustrated in Fig. 1. In [7], the authors assumed that the two IRSs are equipped with receive RF chains to enable the sensing capability for estimating their channels with the base station (BS)/user, separately. Nonetheless, even with receive RF chains integrated to IRSs, the channel estimation for the inter-IRS (i.e., IRS 1→IRS 2) link is still practically difficult. In contrast, the double-IRS channel estimation with fully passive IRSs was investigated in [9], but without the single-reflection links considered and for the single-user case only.

To overcome the above issues, we propose in this paper a new and efficient channel estimation scheme for the double-IRS assisted multi-user multiple-input multiple-output (MIMO) system shown in Fig. 1, where the communications between a multi-antenna BS and a cluster of nearby users are assisted by two fully passive IRSs, which are deployed near the BS and the cluster of users, respectively. First, for the single-user case, the cascaded channels of the two single-reflection links, each corresponding to one of the two IRSs respectively, are successively estimated at the multi-antenna BS with the other IRS turned OFF. Then, after canceling the signals over the two single-reflection channels estimated, the higher-dimensional double-reflection (i.e., user→IRS 1→IRS 2→BS) channel is efficiently estimated at the BS by exploiting the fact that its cascaded channel coefficients (through each subsurface of IRS 1) are the scaled versions of those of the single-reflection (i.e., user→IRS 2→BS) channel due to their commonly shared IRS 2→BS link; as a result, only the lower-dimensional scaling factors need to be estimated for the double-reflection channel, which substantially reduces the training overhead. Next, the proposed channel estimation scheme is extended to the general multi-user case, where given an arbitrary user’s cascaded channel estimated as in the single-user case, the other users’ cascaded channels are scaled versions of it and thus can be estimated with reduced training overhead. It is shown by simulation that the proposed channel
estimation scheme achieves lower training overhead and also improves channel estimation performance as compared to the existing scheme based on [9].

II. SYSTEM MODEL AND PROBLEM DESCRIPTION

Consider a double-IRS assisted multi-user MIMO communication system shown in Fig. 1, in which the communications between a cluster of $K$ single-antenna users and an $N$-antenna BS are assisted by two distributed IRSs (referred to as IRS 1 and IRS 2). As in [8], the direct links between the users and the BS are assumed to be blocked due to obstacles (e.g., walls/corners in the indoor environment). To overcome the blockage as well as minimize the path loss, IRSs 1 and 2 are placed near the cluster of users and the BS, respectively, such that the $K$ users can be effectively served by the BS through the single- and double-reflection links created by them. Let $M$ denote the total number of passive subspaces for the two distributed IRSs, where IRSs 1 and 2 comprise $M_1$ and $M_2$ subspaces, respectively, with $M_1 + M_2 = M$. By adopting the element-grouping strategy in [6], [10], each of these IRS subspaces is composed of an arbitrary number of adjacent reflecting elements that share a common phase shift for reducing the channel estimation and reflection design complexity. In this paper, we assume the quasi-static flat-fading channel model for all the channels during each channel coherence interval.

Let $\mathbf{u}_k \triangleq [u_{k,1}, \ldots, u_{k,M_1}]^T \in \mathbb{C}^{M_1 \times 1}$, $\mathbf{d}_k \triangleq [d_{k,1}, \ldots, d_{k,M_2}]^T \in \mathbb{C}^{M_2 \times 1}$, $\mathbf{D} \triangleq [d_1, \ldots, d_{M_1}] \in \mathbb{C}^{M \times M_1}$, $\mathbf{G}_1 \in \mathbb{C}^{N \times M_1}$, and $\mathbf{G}_2 \in \mathbb{C}^{N \times M_2}$ denote the baseband equivalent channels in the uplink for the user $k$→IRS 1, user $k$→IRS 2, IRS 1→IRS 2, IRS 1→BS, and IRS 2→BS links, respectively, with $k = 1, \ldots, K$. Let $\mathbf{R} \triangleq [\mathbf{R}_{\beta_{\mu,1}, \theta_{\mu,1}}, \ldots, \mathbf{R}_{\beta_{\mu,M_1}, \theta_{\mu,M_1}}]^T$ denote the equivalent reflection coefficients of IRS $\mu$ with $\mu \in \{1, 2\}$, where $\beta_{\mu,m} \in [0,1]$ and $\theta_{\mu,m} \in [0,2\pi]$ are the reflection amplitude and phase shift of subspace $m$ at IRS $\mu$, respectively. Thus, the effective channel from user $k$ to the BS is the superposition of the double-reflection link and the two single-reflection links (see Fig. 1), which is given by

$$h_k = \mathbf{G}_2 \mathbf{F}_2 \mathbf{D} \mathbf{F}_1 \mathbf{u}_k + \mathbf{G}_2 \mathbf{F}_2 \mathbf{d}_k + \mathbf{G}_1 \mathbf{F}_1 \mathbf{u}_k$$

(1)

where $\mathbf{F}_\mu = \text{diag}(\mathbf{\theta}_\mu)$ denotes the diagonal reflection matrix of IRS $\mu$ with $\mu \in \{1, 2\}$. Since we consider the fully passive IRSs without any receiving/transmitting capability, it is infeasible to acquire the CSI between the two IRSs as well as that with the BS/users separately. Nonetheless, it was shown in [8] that the cascaded CSI (to be specified below) is sufficient for the cooperative reflection/passive beamforming design of the two IRSs to maximize the data transmission rate without loss of optimality. As such, let $\mathbf{R}_k = \mathbf{G}_1 \text{diag}(\mathbf{u}_k) \in \mathbb{C}^{N \times M_1}$ ($\mathbf{R}_k = \mathbf{G}_2 \text{diag}(\mathbf{d}_k) \in \mathbb{C}^{N \times M_2}$) denote the cascaded user $k$→IRS 1 (IRS 2)→BS channel (without taking the effect of IRS reflection yet), and $\mathbf{D}_k \triangleq [d_{k,1}, \ldots, d_{k,M_1}] = \mathbf{D} \text{diag}(\mathbf{u}_k) \in \mathbb{C}^{M \times M_1}$ denote the cascaded user $k$→IRS 1→IRS 2 channel (without taking the effect of IRS reflection yet)

$$\hat{d}_{k,m} = d_{m,u_{k,m}}, \forall m = 1, \ldots, M_1.$$ Then, the channel model in (1) can be equivalently expressed as

$$h_k = \mathbf{G}_2 \mathbf{F}_2 \mathbf{D}_k \mathbf{F}_1 \mathbf{u}_k + \mathbf{R}_k \mathbf{F}_1 \mathbf{d}_k + \mathbf{R}_k \mathbf{F}_1 \mathbf{u}_k$$

$$= \mathbf{G}_2 \mathbf{F}_2 \mathbf{D}_k \mathbf{F}_1 \mathbf{u}_k + \mathbf{G}_2 \mathbf{F}_2 \mathbf{d}_k + \mathbf{R}_k \mathbf{F}_1 \mathbf{d}_k + \mathbf{R}_k \mathbf{F}_1 \mathbf{u}_k$$

$$= \sum_{m=1}^{M_1} \mathbf{G}_2 \mathbf{F}_2 \mathbf{d}_k, m + \mathbf{R}_k \mathbf{F}_1 \mathbf{d}_k + \mathbf{R}_k \mathbf{F}_1 \mathbf{u}_k$$

(2)

where $Q_{k,m} \in \mathbb{C}^{N \times M_2}$ denotes the cascaded user $k$→IRS 1→IRS 2→BS channel associated with subsurface $m$ at IRS 1, $\forall m = 1, \ldots, M_1$ for the double-reflection link. According to (2), it is sufficient to acquire the cascaded CSI of $\{Q_{k,m}\}_{m=1}^{M_1}$, $\hat{R}_k$, and $R_k$ for jointly designing the passive beamforming $\{\theta_1, \theta_2\}$ for the uplink data transmission in the double-IRS assisted system [8].

However, in practice, the total number of channel coefficients in $\{Q_{k,m}\}_{m=1}^{M_1}$, $\hat{R}_k$, and $R_k$ is prohibitively large, which consists of two parts:

- The number of channel coefficients (equal to $K \times N(M_1 + M_2)$ for the high-dimensional double-reflection link, i.e., $\{Q_{k,m}\}_{m=1}^{M_1}$), which are newly introduced due to the double IRSs.
- The number of channel coefficients (equal to $K \times N(M_1 + M_2)$ for the two single-reflection links, i.e., $\hat{R}_k$ and $R_k$), which exist in the conventional single-IRS assisted system (with either IRS 1 or IRS 2 present).

As can be seen, the number of channel coefficients for the double-reflection link is of higher-order than that for the two single-reflection links due to the fact that $M_1 M_2 \gg M_1 + M_2$ in practice, which makes the channel estimation problem more challenging for the double-IRS assisted system, as compared to the single-IRS counterpart. Note that given a channel coherence interval, such a considerably larger number of channel coefficients may require significantly more training overhead that renders much less or even no time for data transmission, thus resulting in reduced achievable rate of the double-IRS assisted system (despite the higher passive beamforming gain over the double-reflection link assuming perfect CSI as shown in [8]).

To tackle the above challenge, we propose a new and efficient channel estimation scheme for the double-IRS assisted system to achieve minimum training overhead. In particular, we exploit the channel relationship between the double-reflection link $\{Q_{k,m}\}_{m=1}^{M_1}$ and the single-reflection link $\hat{R}_k$ due to the same (common) IRS 2→BS channel (i.e., $G_2$) shared by them to reduce training overhead. We first consider the single-user setup, i.e., $K = 1$, to illustrate the main idea of the proposed channel estimation scheme for the double-reflection link in Section III, and then extend the results to the general multi-user case in Section IV. To reduce the hardware cost, we consider the binary ON/OFF control for the training reflection amplitudes of the two IRSs, i.e., $\beta_{\mu,m} \in \{0,1\}, \forall m = 1, \ldots, M_\mu, \mu \in \{1, 2\}$ in our proposed channel estimation scheme.
III. CHANNEL ESTIMATION FOR SINGLE-USER CASE

In this section, we study the cascaded channel estimation for the single-user case with $K = 1$. For notational convenience, the user index $k$ is omitted in this section.

According to (2), the cascaded user→IRS 1→IRS 2→BS channel through each subsurface $m$ at IRS 1 is given by

$$Q_m = G_2 \text{diag}(\hat{d}_m), \quad m = 1, \ldots, M_1. \quad (3)$$

It is observed that all $Q_m$’s in (3) share the same (common) IRS 2→BS channel (i.e., $G_2$) as $\hat{R}$. As such, if given the single-reflection channel $\hat{R} = G_2 \text{diag}(\hat{u})$ as the reference CSI, we can re-express (3) as

$$Q_m = \frac{G_2 \text{diag}(\hat{u}) \cdot \text{diag}(\hat{u})^{-1} \text{diag}(\hat{d}_m)}{R \text{diag}(a_m)} \quad (4)$$

where $\text{diag}(a_m)$ is the diagonal matrix normalized by $\hat{u}$, with $a_m = \text{diag}(\hat{u})^{-1} \hat{d}_m \in \mathbb{C}^{M_2 \times 1}, \forall m = 1, \ldots, M_1$ being the scaling vector. By substituting $Q_m$ of (4) into (2), the channel model in (2) can be rewritten as

$$h = \sum_{m=1}^{M_1} \hat{R} \text{diag}(a_m) \theta_{2, 1, m} + \hat{R} \theta_2 + R \theta_1. \quad (5)$$

According to (5), it is sufficient to acquire the CSI of $R$, $\hat{R}$, and the scaling vectors $\{a_m\}_{m=1}^{M_1}$ for designing the passive beamforming for data transmission in the double-IRS assisted single-user system. Based on the channel relationship disclosed in (4), we propose to decouple the channel estimation for the single- and double-reflection links into three phases, for which the main procedures are described as follows and will be elaborated in the subsequent subsections.

- **Estimation of the two single-reflection channels $\{R, \hat{R}\}$:**
  With all the subsurfaces at IRS 2 (IRS 1) turned OFF, the BS estimates $R$, $\hat{R}$ based on the time-varying training reflection of IRS 1 (IRS 2) and the pilot symbols sent by the user;

- **Estimation of the double-reflection channel $Q_m$ $\{m=1\}$:**
  After canceling the signals over the two single-reflection links and taking the estimated $\hat{R}$ as the reference CSI, the BS estimates $\{a_m\}_{m=1}^{M_1}$ for the double-reflection link.

A. Phase I: Estimation of $R$

With all the subsurfaces at IRS 2 turned OFF (i.e., $\theta_{2, 1}^{(i)} = \mathbf{0}_{M_2 \times 1}, \forall i$) in Phase I, the channel model in (5) reduces to the single-reflection channel related to IRS 1 only. In this case, the received signal of the BS at time slot $i$ of Phase I can be expressed as

$$y_i^{(i)} = R \theta_{1, i}^{(i)} x_i^{(i)} + v_i^{(i)}, \quad i = 1, \ldots, I_1. \quad (6)$$

where $I_1$ denotes the number of pilot symbols in Phase I, $x_i^{(i)}$ represents the pilot symbol transmitted by the user which is simply set as $x_i^{(i)} = 1$ for ease of exposition, and $v_i^{(i)} \sim \mathcal{N}(0, \sigma^2 I_N)$ is the additive white Gaussian noise (AWGN) vector at the BS with $\sigma^2$ being the normalized noise power.

By stacking the received signal vectors $\{y_i^{(i)}\}_{i=1}^{I_1}$ into $Y_1 = [y_1^{(i)}, \ldots, y_{I_1}^{(i)}]$, we obtain

$$Y_1 = R \Theta_{1,1} + V_1 \quad (7)$$

where $\Theta_{1,1} = \begin{bmatrix} \theta_{1, 1}^{(1)}, \ldots, \theta_{1, 1}^{(I_1)} \end{bmatrix}$ denotes the training reflection matrix at IRS 1 in Phase I and $V_1 = [v_1^{(1)}, \ldots, v_{I_1}^{(1)}] \in \mathbb{C}^{I_1 \times I_1}$ denotes the corresponding AWGN matrix. By properly constructing the training reflection matrix of IRS 1 such that rank($\Theta_{1,1}$) = $M_1$, the least-square (LS) estimate of $R$ based on (7) is given by

$$\hat{R} = Y_1 \Theta_{1,1}^H (\Theta_{1,1} \Theta_{1,1}^H)^{-1}. \quad (8)$$

Moreover, $I_1 \geq M_1$ is required to satisfy rank($\Theta_{1,1}$) = $M_1$, which can be achieved via different IRS training reflection designs (e.g., the ON/OFF based design [6]) or the more efficient orthogonal matrix-based designs [10]–[13].

B. Phase II: Estimation of $\hat{R}$

With all the subsurfaces at IRS 1 turned OFF (i.e., $\theta_{1, 1}^{(i)} = \mathbf{0}_{M_1 \times 1, \forall i}$) in Phase II, the channel model in (5) reduces to the single-reflection channel related to IRS 2 only. In this case, with $x_i^{(i)} = 1, \forall i$ being the pilot symbol transmitted by the user, the received signal matrix of the BS over $I_2$ pilot symbols of Phase II is similarly obtained as

$$Y_{II} = \hat{R} \Theta_{2,II} + V_{II} \quad (9)$$

where $\Theta_{2,II} = \begin{bmatrix} \theta_{2, 1}^{(i)}, \ldots, \theta_{2, 1}^{(I_2)} \end{bmatrix}$ is the training reflection matrix at IRS 2 in Phase II and $V_{II} \in \mathbb{C}^{I_2 \times I_2}$ is the corresponding AWGN matrix. Accordingly, the LS estimate of $\hat{R}$ based on (9) is given by

$$\hat{R} = Y_{II} \Theta_{2,II}^H (\Theta_{2,II} \Theta_{2,II}^H)^{-1}. \quad (10)$$

Similarly, $I_2 \geq M_2$ is required to achieve rank($\Theta_{2,II}$) = $M_2$ via different IRS training reflection designs [6], [10]–[13].

C. Phase III: Estimation of $\{a_m\}_{m=1}^{M_1}$

With the estimated $\{R, \hat{R}\}$ in Phases I and II, we further estimate each scaling vector $a_m$ for the double-reflection link. At time slot $i$ of Phase III with $\theta_{2, 1}^{(i)} = 1, \forall i$ being the pilot symbol transmitted by the user, the received signal at the BS based on the channel model in (5) can be expressed as

$$y_{III}^{(i)} = \sum_{m=1}^{M_1} \hat{R} \text{diag}(a_m) \theta_{2,III}^{(i)} \theta_{1,III}^{(i)} + \hat{R} \theta_{2,III}^{(i)} + \Theta_{2,III} + \psi_{III}^{(i)}$$

$$= \sum_{m=1}^{M_1} \hat{R} \text{diag}(a_m) \theta_{2,III}^{(i)} \theta_{1,III}^{(i)} + \hat{R} \theta_{2,III}^{(i)} + \Theta_{2,III} + \psi_{III}^{(i)} \quad (11)$$

where $\psi_{III}^{(i)} \sim \mathcal{N}(0, \sigma^2 I_N)$ is the AWGN vector. Given the estimated $\{R, \hat{R}\}$, the pilot signals over the two single-reflection links can be removed from (11), and thus the effective received signal over the double-reflection link at the BS is given by

$$\tilde{y}_{III}^{(i)} = y_{III}^{(i)} - \Theta_{2,III} - \Theta_{1,III}$$

$$= \sum_{m=1}^{M_1} \hat{R} \text{diag}(a_m) \theta_{2,III}^{(i)} \theta_{1,III}^{(i)} + \Theta_{2,III} + \psi_{III}^{(i)} \quad (12)$$

For ease of exposition, we assume perfect cancellation of the pilot signals over the two single-reflection links; while the residual interference due to imperfect cancellation with estimated $\{R, \hat{R}\}$ will be taken into account for evaluating the channel estimation performance via simulations in Section V.
For the estimation of \( \{a_m\}_{m=1}^{M_1} \), we consider the following two cases.

**Case 1:** \( N \geq M_2 \). In this case, we consider the fixed full-ON reflection of IRS 2 (say, \( \theta_{2,111}^{(i)} = 1_{M_2 \times 1}, \forall i \)) and turn ON one out of the \( M_1 \) subsurfaces at IRS 1 (say, \( \theta_{1,111}^{(i)} = 1 \) and \( \theta_{1,m,111}^{(i)} = 0, \forall m \neq i \)) sequentially to estimate each \( a_i \), for which the received signal of the BS at time slot \( i \) of Phase III is given by

\[
y^{(i)}_{III} = \hat{R}_a + v^{(i)}_{III}, \quad i = 1, \ldots, M_1.
\]

Based on (13), the LS estimate of each \( a_i \) is given by

\[
a_i = (\hat{R}^H \hat{R})^{-1} \hat{R}^H y^{(i)}_{III}, \quad i = 1, \ldots, M_1.
\]

**Case 2:** \( N < M_2 \). In this case, we cannot estimate each \( a_m \) separately according to (14), since \( \hat{R} \) is (column) rank-deficient, i.e., \( \text{rank}(\hat{R}) = N < M_2 \). Alternatively, we consider the joint estimation of \( \{a_m\}_{m=1}^{M_1} \) by stacking the received signal vectors \( \{y^{(i)}_{III}\} \) in (12) over \( I_3 \) time slots of Phase III, which is given by

\[
\hat{\eta} = \begin{bmatrix}
\hat{y}^{(1)}_{III} \\
\vdots \\
\hat{y}^{(I_3)}_{III}
\end{bmatrix} = \begin{bmatrix}
\begin{bmatrix}
\theta_{1,111}^{(1)} \\
\vdots \\
\theta_{1,111}^{(I_3)}
\end{bmatrix}^T \hat{R} \begin{bmatrix}
\theta_{2,111}^{(1)} \\
\vdots \\
\theta_{2,111}^{(I_3)}
\end{bmatrix} + \begin{bmatrix}
v^{(1)}_{III} \\
\vdots \\
v^{(I_3)}_{III}
\end{bmatrix}
\end{bmatrix}
\end{bmatrix}_{C_{I_3} \times M_1 M_2}.
\]

As such, by properly designing the training reflection coefficients \( \{\theta_{1,111}^{(i)}\}_{i=1}^{I_3} \) and \( \{\theta_{2,111}^{(i)}\}_{i=1}^{I_3} \) of IRSs 1 and 2 such that \( \text{rank}(\hat{C}) = M_1 M_2 \), the LS estimate of \( \eta \) can be obtained as

\[
\hat{\eta} = (\hat{C}^H \hat{C})^{-1} \hat{C}^H \hat{y}_{III}.
\]

Note that \( I_3 N \geq M_1 M_2 \) is the necessary condition to achieve \( \text{rank}(\hat{C}) = M_1 M_2 \) with \( I_3 \) being an integer, and thus we have \( I_3 \geq \lceil \frac{M_1 M_2}{N} \rceil \). Moreover, we can design the training reflections of IRSs 1 and 2 based on the orthogonal matrices as in [10]–[13]. It is worth pointing out that the channel estimation based on (15) and (16) with the orthogonal matrix-based training design can also be applied to the case of \( N < M_2 \) to achieve better channel estimation performance at the expense of higher complexity due to the higher-dimensional matrix inversion operation and the joint estimation of \( \{a_m\}_{m=1}^{M_1} \).

Finally, with the estimated CSI of \( \hat{R} \) and \( \{a_m\}_{m=1}^{M_1} \), we can obtain the estimated CSI of \( \{Q_{m}\}_{m=1}^{M_1} \) according to (4).

**IV. CHANNEL ESTIMATION FOR MULTI-USER CASE**

For the multi-user channel estimation, a straightforward method is by adopting the single-user channel estimation design in Section III to estimate the channels of \( K \) users separately over consecutive time, which, however, increases the total training overhead by \( K \) times as compared to the single-user case and thus is practically prohibitive if \( K \) is large. To reduce the overall training overhead, we extend the channel estimation scheme for the single-user case in Section III to the general multi-user case in this section. By exploiting the fact that the other users’ cascaded channels are scaled versions of the cascaded channel of an arbitrary user (referred to as the reference user) [11], [14], the channel training overhead can be substantially reduced, which is elaborated in the following.

After estimating an arbitrary user’s cascaded channel as in the single-user case of Section III, the cascaded channels of the other users can be efficiently obtained by exploiting the fact that all the users share the common IRS 2→BS (i.e, \( G_2 \)), IRS 1→BS (i.e, \( G_1 \)), and IRS 1→IRS 2 (i.e, \( D \)) links in (1) in their respective single- and double-reflection channels. In particular, if given the cascaded CSI of any user (say, \( R_1, R_1 \), and \( \{Q_{m}\}_{m=1}^{M_1} \) of user 1) as the reference CSI, we can rewrite the two single-reflection channels \( \{R_k, \tilde{R}_k\} \) in (2) as

\[
R_k = G_1 \text{diag}(u_1) \cdot \text{diag}(u_1)^{-1} u_k = R_1 \text{diag}(b_k)
\]

\[
\tilde{R}_k = G_2 \text{diag}(\tilde{u}_1) \cdot \text{diag}(\tilde{u}_1)^{-1} \tilde{u}_k = \tilde{R}_1 \text{diag}(\tilde{b}_k)
\]

and the double-reflection channel \( \{Q_{k,m}\}_{m=1}^{M_1} \) in (3) as

\[
Q_{k,m} = G_2 \text{diag}(d_{k,m}) = G_2 \text{diag}(d_{m,u_1,m})^{-1} u_k = Q_{1,m,b_k,m}
\]

where \( b_k = [b_{k,1}, \ldots, b_{k,M_1}]^T = \text{diag}(u_1)^{-1} u_k \) and \( \tilde{b}_k = \text{diag}(\tilde{u}_1)^{-1} \tilde{u}_k \) are the user \( k \)-IRS 1 and user \( k \)-IRS 2 channel vectors normalized by \( u_1 \) and \( \tilde{u}_1 \), respectively. By substituting (17)-(19) into (2), we can re-express the channel model in (2) as

\[
h_k = \sum_{m=1}^{M_1} Q_{1,m} \theta_{2,1} b_{k,m} + \tilde{R}_1 \text{diag}(\tilde{b}_k) \theta_{2} R_1 \text{diag}(b_k) \theta_{1}
\]

\[
= \sum_{m=1}^{M_1} Q_{1,m} \theta_{2,1} b_{k,m} + \tilde{R}_1 \text{diag}(\tilde{b}_k) \theta_{2} R_1 \text{diag}(\tilde{b}_k) b_k
\]

As such, after acquiring the cascaded CSI of user 1 as in Section III, we only need to further estimate \( \{b_k\}_{k=2}^{K} \) and \( \{\tilde{b}_k\}_{k=2}^{K} \) for the remaining \( K - 1 \) users according to (17)-(20). In the following, we propose the decoupled channel estimation for \( \{b_k\}_{k=2}^{K} \) and \( \{\tilde{b}_k\}_{k=2}^{K} \) in Phases IV and V following Phases I-III for estimating the cascaded CSI of the single user (i.e., user 1) in Section III.

**A. Phase IV: Estimation of \( \{b_k\}_{k=2}^{K} \)**

In this phase, we turn OFF all the subsurfaces at IRS 2. Based on the channel model in (20) and denoting \( \{x_{k}^{(i)}\}_{k=2}^{K} \) as pilot symbols transmitted by the remaining \( K - 1 \) users, the received signal of the BS at time slot \( i \) of Phase IV can be expressed as

\[
y_{IV}^{(i)} = \sum_{k=2}^{K} x_{k}^{(i)} R_1 \text{diag}(\theta_{1,i}^{(i)}) b_k + v_{IV}^{(i)}
\]

with \( v_{IV}^{(i)} \sim N_c(0, \sigma^2 I_N) \) being the AWGN vector. For the estimation of \( \{b_k\}_{k=2}^{K} \), we consider the following two cases.

**Case 1:** \( N \geq M_1 \). In this case, the remaining \( K - 1 \) users send pilot symbols sequentially for the BS to estimate each \( b_k \) with \( k = 2, \ldots, K \). Specifically, with \( x_{k}^{(i)} = 1 \) and \( x_{k}^{(i)} = 0, \forall k \neq i + 1 \) and the fixed full-ON reflection of IRS 1 (say, \( \theta_{1,1}^{(i)} = 1_{M_1 \times 1}, \forall i \)) in Phase IV, the received signal in (21) can be rewritten as

\[
y_{IV}^{(i)} = R_1 b_{i+1} + v_{IV}^{(i)}, \quad i = 1, \ldots, K - 1
\]
and the LS estimate of $b_{i+1}$ is thus given by

$$
\hat{b}_{i+1} = \left( R_i^H R_i \right)^{-1} R_i^H y_{i+1}^T, \quad i = 1, \ldots, K - 1. \quad (23)
$$

**Case 2:** $N < M_1$. In this case, since $R_1$ is not of full-column rank, i.e., $\text{rank}(R_1) = N < M_1$, we cannot estimate each $b_k$ separately according to (22). As such, we consider the joint estimation of $\{b_k\}_{k=2}^K$ with concurrent pilot symbols sent by the remaining $K - 1$ users, for which the received signal vector at the BS over $I_4$ pilot symbols is given by

$$
\begin{align*}
\begin{bmatrix}
    y_{1V}^T \\
    \vdots \\
    y_{IV}^T
\end{bmatrix}
=& \begin{bmatrix}
    \left( x_1^{(1)} \right)^T \otimes R \text{ diag} \left( \theta_{1,1V}^{(1)} \right) \\
    \vdots \\
    \left( x_4^{(4)} \right)^T \otimes R \text{ diag} \left( \theta_{4,1V}^{(4)} \right)
\end{bmatrix}
\begin{bmatrix}
    b_2 \\
    \vdots \\
    b_K
\end{bmatrix} + \\
& \begin{bmatrix}
    v_{1V}^{(1)} \\
    \vdots \\
    v_{IV}^{(4)}
\end{bmatrix},
\end{align*}
$$

where $x_k^{(i)} = \begin{bmatrix} x_2^{(i)} \ldots x_K^{(i)} \end{bmatrix}^T$ denotes the pilot symbol vector. As such, by properly designing the training reflection coefficients $\{\theta_{i,1V}^{(i)}\}_{i=1}^{I_4}$ of IRS 1 and the pilot symbol vectors $\{x_k^{(i)}\}_{i=1}^{I_4}$ such that rank $(F) = (K-2)M_1$, the LS estimate of $\lambda$ is given by

$$
\hat{\lambda} = \left( F^H F \right)^{-1} F^H y_{IV}. \quad (25)
$$

Since $I_4 N \geq (K-2)M_1$ is required to ensure the condition of rank $(F) = (K-2)M_1$ with $I_4$ being an integer, we have $I_4 \geq \left\lceil \frac{(K-2)M_1}{N} \right\rceil$. Furthermore, we can construct the training reflection coefficients $\{\theta_{i,1V}^{(i)}\}_{i=1}^{I_4}$ of IRS 1 and the pilot symbol vectors $\{x_k^{(i)}\}_{i=1}^{I_4}$ from some orthogonal matrices as in [10]–[13] to achieve rank $(F) = (K-2)M_1$.

**B. Phase V: Estimation of $\{\hat{b}_k\}_{k=2}^K$**

Similarly, based on the channel model in (20) with all the subsurfaces at IRS 1 turned OFF, the received signal of the BS at time slot $i$ of Phase V can be expressed as

$$
y_{IV}^{(i)} = \sum_{k=2}^{K} x_k^{(i)} \tilde{R}_i \text{ diag} \left( \theta_{2,V}^{(i)} \right) \hat{b}_k + v_{IV}^{(i)}, \quad (26)
$$

with $\{x_k^{(i)}\}_{k=2}^{K}$ being pilot symbols transmitted by the remaining $K-1$ users and $v_{IV}^{(i)} \sim \mathcal{N}_c(0, \sigma^2 I_N)$ being the AWGN vector. As such, following the similar procedures in Section IV-A, we can estimate $\{\hat{b}_k\}_{k=2}^K$ for the two cases of $N \geq M_2$ and $N < M_2$ with minimum training overhead of $K - 1$ and $\left\lceil \frac{(K-1)M_1}{N} \right\rceil$, respectively, whose details are thus omitted for brevity.

**V. SIMULATION RESULTS**

In this section, we present simulation results to numerically validate the effectiveness of the proposed channel estimation scheme for the double-IRS assisted multi-user MIMO system. Under a three-dimensional (3D) Cartesian coordinate system, we assume that the central (reference) points of the BS, IRS 2, IRS 1, and user cluster are located at $(1, 0, 2), (0, 0.5, 1), (0, 49.5, 1), (0, 49.5, 1)$ in meter (m), respectively. Moreover, we assume that the BS is equipped with a uniform linear array (ULA); while the two distributed IRSs are equipped with uniform planar arrays (UPAs). As the element-grouping strategy in [6], [10], each IRS subsurface is a small-size UPA composed of $5 \times 5$ adjacent IRS elements that share a common phase shift for reducing design complexity. The distance-dependent channel path loss is modeled as $\gamma = \gamma_0/(d^\alpha)$, where $\gamma_0$ denotes the path loss at the reference distance of 1 m which is set as $\gamma_0 = -30$ dB for all individual links, $d$ denotes the individual link distance, and $\alpha$ denotes the path loss exponent which is set as 2.2 for the link between the user cluster/BS and its nearby serving IRS (due to the short distance) and set as 3 for the other links (due to the relatively large distance).

Due to the very limited work on channel estimation for the double-IRS assisted system, we extend the channel estimation method proposed in [9] as the benchmark scheme for comparison, where the double-reflection channel is estimated at each BS antenna in parallel without exploiting the (common) channel relationship with the single-reflection channels, and the cascaded channels of $K$ users are separately estimated over consecutive time. Moreover, as the single-reflection channels were ignored in [9], the same channel estimation procedures for the single-reflection channels in Sections III-A and III-B are adopted for each user in the benchmark scheme. The channel training overhead comparison between the proposed and benchmark schemes is shown in Table I, where $M_1 = M_2 = M/2$ is assumed for ease of exposition. As can be seen, by exploiting the peculiar channel relationship over double-reflection channels and multiple users, the proposed channel estimation scheme incurs much lower training overhead than the benchmark scheme.

| Proposed scheme | Minimum number of pilot symbols |
|-----------------|--------------------------------|
| $N = M_2$       | $M + 2$                       |
| Benchmark scheme based on [9] | $K M^2$                       |

In the following simulations, we calculate the normalized mean squared error (MSE) for the single- and double-reflection channels over 1,000 independent fading channel realizations. For example, the normalized MSE of the cascaded user $k$-IRS 1-BS channel $R_k$ is given by

$$
\varepsilon = \frac{1}{K N M_1} \sum_{k=1}^{K} \mathbb{E} \left\{ \| \tilde{R}_k - R_k \|_F^2 / \| R_k \|_F^2 \right\}. \quad (27)
$$

The normalized MSE of other channels can be similarly calculated as in the above. Given the total number of subsurfaces $M = 40$, we set $M_1 = M_2 = M/2 = 20$ for the two distributed IRSs. Without loss of generality, all the users are assumed to have equal transmit power, i.e., $P_k = P, \forall k$ and the noise power at the BS is set as $\sigma_N^2 = -65$ dBm. Accordingly, the normalized noise power at the BS is given by $\sigma^2 = \sigma_N^2 / P$.

In Fig. 2(a), we show the required training overhead versus the number of antennas, $N$, at the BS. It is observed that for the proposed channel estimation scheme, the total training overhead decreases with the number of BS antennas $N$, which is in sharp contrast to the benchmark scheme where its training overhead is independent of $N$. This is expected since the proposed channel estimation scheme exploits the multiple
antennas at the BS with joint IRS channel estimation to reduce training overhead substantially, whereas in the benchmark scheme the BS estimates its channels associated with different antennas independently in parallel without exploiting the channel relationship among them. When the number of BS antennas is sufficiently large (i.e., $N \geq \max\{M_1, M_2\} = 20$), the minimum training overhead in the proposed scheme reaches its lower bound of $2M_1 + M_2 + 2(K - 1)$ pilot symbols.

In Fig. 2(b), we show the required training overhead versus the number of users $K$. On can observe that the training overhead increment is marginal in the proposed channel estimation scheme as the number of users $K$ increases. In contrast, the training overhead required by the benchmark scheme increases dramatically with $K$ since it does not exploit the (common) channel relationship among different users. As such, by fully exploiting the channel relationship between the single- and double-reflection channels as well as among different users, the proposed scheme achieves much lower training overhead than the benchmark counterpart.

In Figs. 3(a) and 3(b), we compare the normalized MSE performance of different schemes versus user transmit power $P$ for the single- and multi-user cases, respectively. For fair comparison, we proportionally increase the training overhead of each phase in the proposed channel estimation scheme until reaching the same total training overhead as the benchmark scheme. It is observed that the proposed scheme achieves much lower MSE than the benchmark scheme, especially for the multi-user case. Moreover, the MSE performance gap between the single- and double-reflection channels in the benchmark scheme is much larger than that in the proposed scheme. This can be understood since the training overhead for the double-reflection channel is of higher order than that for the single-reflection channels in the benchmark scheme ($4KM^2$ versus $KM$). In contrast, the proposed scheme achieves balanced MSE performance for the single- and double-reflection channels with the proper proportional training time allocation.

VI. CONCLUSIONS

In this paper, we proposed an efficient uplink channel estimation scheme for the double-IRS assisted multi-user MIMO system. For the single-user case, the higher-dimensional double-reflection channel was efficiently estimated with substantially reduced training overhead by exploiting the property that its cascaded channel coefficients are the scaled versions of those of a lower-dimensional single-reflection channel. The proposed channel estimation scheme was then extended to the multi-user case by exploiting the fact that the other users’ cascaded channels are scaled versions of that of an arbitrary (reference) user’s cascaded channel for training overhead reduction. Simulation results demonstrated the effectiveness of the proposed channel estimation scheme as compared to the existing scheme.

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