An experimental study of flow over V- shape crump weir crest

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Abstract. Weirs are considered important structures controlling flow conditions and measuring flow rates in open channels. Crump weir represent good solution tool to control flow depth in streams, especially where controlling fish zones is required. However, installing crump weirs in rivers and relatively wide streams may be accompanied with difficulty of controlling same water level over the entire weir crest length. This study aimed to modify the crest of crump weir to be V-shaped rather than normal crest. Four wooden crump weir models were adopted. Each of the four models had longitudinal equilateral triangle section with the same horizontal base length, L (78.5cm), width, W (29.5cm) and height, P (20cm), but they differ by the middle crest height, P' as (20, 17.5, 15 and 12 cm) according to models 1, 2, 3 and 4, respectively.. The three v-shaped crested crumps and the normal crump were subjected to eight flow rates and five flow depths at steady state modular flow conditions. Data gathered were used to conclude a nonlinear multiple regression formula of discharge coefficient for the modified models tested. The formula concluded was calibrated and verified with very good determination factor (0.958). Effect of crest modification on water level was clear at relatively low flow rates and vice versa.

1-Introduction

Weirs are considered as hydraulic structures usually used for indirect flow measurement and/or controlling water elevation in basins and channels [1],[2]. When installing a weir in an open channel, a relationship between flow rate and water flow height above weir may be derived according to weir geometry on the basis of the Bernoulli equation [3]. Water viscosity and surface tension as well as weir surface roughness and geometry are all parameters included in that relationship. However, surface tension effect is very small compared to the other parameters and can be neglected [4]. The relationship “flow rate equation” is, principally, consist of dimensions related to weir geometry and upstream water depth, whereas the other parameters are included in a term called “discharge coefficient, Cd” in the equation. The Cd is assumed to cover effects of vertical curvature of the upstream flow streamlines in the vicinity of weir which cause head differences at both upstream and over weir. It also covers the assumption of considering pressure distribution upstream weir is hydrostatic [5]. According to downstream flow conditions, flow over weir may be modular or non-modular. In modular flow, the weir operates undrowned, and the upstream head is not affected by the downstream head, whereas in non-modular, the weir operates drowned, and the upstream head is affected by changes in the downstream head. Therefore, it is possible to determine the flow rate depending on only upstream head measurement in modular flow in contrast to non-modular, which has to include downstream head by means of “submergence factor” [6],[7], [8].

Weirs installed in open channels can take a verity of applications, such as in rivers, irrigation channels, water treatment plants, industries…etc. However, the most popular type of weirs adopted in rivers and relatively wide streams are long base weirs for their structural stability and simplicity, and also comparative good accuracy in flow measurement. Crump weir is considered one of the Broad crested weirs (long base weirs)[9],[10]. Its hydraulic behaviour approaches to the stepped weirs that considerably dissipate hydraulic jump energy in downstream side that may endanger stability of the weir.
Standard crump weir has a triangular section along flow direction with an upstream slope of 1 (vertical) to 2 (horizontal) and a downstream slope of 1 to 5, and the intersection of upstream and downstream surfaces forms a straight line crest, horizontal and at right angles to the direction of flow in the approach channel [10]. However, some modifications for the geometry of the weir have been investigated for certain purposes. Servais carried out a study through which several physical models of low-cost modifications to the Crump weir investigated in order to improve fish passage in England and Wales. Her study included the hydraulic investigation for installing baffles with different numbers, geometry and arrangement [13]. AL-Naely et. al. studied the effect of longitudinal flow openings (holes) penetrating upstream and downstream faces of crump weir, horizontally, to do as energy dissipaters, and as an improver for the discharge coefficient [14].

Triangular profile flat-V weir is a modification for the crump weir to measure a wider range of discharges by adopting a transverse symmetrical V-shaped crest, having small side-slopes (typically 1:10, 1:20 or 1:40) in the planes normal to the flow direction (upstream and downstream faces)[15],[16],[17],[18],[19]. Its longitudinal section is triangular same as that of crump weir but with possibility of equal (1:2) longitudinal slopes rather than the standard slopes (1:2) and (1:5) of upstream and downstream faces of standard crump weir [20]. Keller studied a standing inclined crest Crump weir. He concluded that at relatively large heads, the weir behaves as one half of a flat-V Crump weir for the same transversal crest slope. At lower heads, the flow cross section becomes strongly asymmetrical with a significant decrease in discharge coefficient value, [21].

Through the present study, a modified crump weir is adopted. The modified weir have triangular section along flow direction with equal longitudinal slopes as (1 vertical: 2 horizontal) for the upstream and downstream plane faces, with a transverse symmetrical V-shaped crest of three slopes (1:8, 1:4, 1:2.5). So, the invert of the V-shaped crest of the adopted modified weir represent horizontal line parallel to flow direction rather than the common triangular profile flat-V weir which has a point invert not as line, Figure 1. According to available literature, such modified weir may be investigated for the first time through present study. Justification for such study is that it investigate a weir that provide solution for wider flow range in open channels. This modified weir if replicated to form a structure consisting of successive attached weir segments, transversally, can provide a solution for the problem of uneven distribution of crest water depth along wide section channels. Test experiments of four weir models for different water flow rates and depths (upstream and downstream) have been evaluated. The results used to conclude a multiple regression equation for the discharge coefficient of the modified weir for variables driven from dimensional analysis.

2. Methodology
2.1. Hypothesis and constrains
This study assumes constant values as constrains for some variables related to the models tested and conditions of experiments. The variables assumed to have constant values are: weir length, width, crest height of sides and longitudinal slopes (upstream and downstream slopes), whereas middle crest height is variable. Hydraulic variables considered are flow rate and flow depth in upstream and downstream sides. Water temperature was nearly constant throughout experiments (around 25 ±1 °C). Channel bottom slope is zero (horizontal).

2-2- Dimensional Analysis
Variables effecting flow over weirs may be categorized into three groups; weir geometry, flow conditions and flowing fluid (water) properties. Table (1) lists the variables related to the testing conditions:-
Table 1. Principal variables related to the testing conditions.

| Variable     | Meaning                          | Dimensions |
|--------------|----------------------------------|------------|
| L            | Weir length                      | L          |
| P            | Weir height at crest terminals   | L          |
| P'           | Weir height at weir centerline   | L          |
| W            | Weir width                       | L          |
| C            | Submergence ratio                | unit less  |
| g            | Gravitational acceleration       | LT^{-2}    |
| H            | Total upstream head              | L          |
| h_d          | Total downstream head            | L          |
| Q            | Flow rate                        | L^1 T^{-1} |
| S            | Channel bed slope                | unit less  |
| ρ            | Water mass density               | ML^{-3}    |
| µ            | Water dynamic viscosity          | ML^{-1} T^{-1} |
| σ            | Water surface tension            | MT^{-1}    |

Discharge over the weir according to modular flow may be expressed as:

\[ Q = f(L, P, P', W, g, H, S, \rho, \mu, \sigma) \]  

or

\[ f(Q, L, P, P', W, g, H, S, \rho, \mu, \sigma) = 0 \]  

According to the π-theorem:

m = 11 (number of variables)  
n = 3 (number of the primary units involved in the equations above, MLT)  
π-terms = 11 − 3 = 8 (number of dimensionless terms)

Considering g, H and ρ as repeating variables, eq. (2) can be expressed as:

\[ f_1(\pi_1, \pi_2, \pi_3, \pi_4, \pi_5, \pi_6, \pi_7, \pi_8) = 0 \]  

where:

\[ \pi_1 = g^{a_1} H^{b_1} \rho^{c_1} Q \quad \pi_5 = g^{a_1} H^{b_1} \rho^{c_1} W \]
\[ \pi_2 = g^{a_1} H^{b_1} \rho^{c_1} \mu \quad \pi_6 = g^{a_1} H^{b_1} \rho^{c_1} S \]
\[ \pi_3 = g^{a_1} H^{b_1} \rho^{c_1} \sigma \quad \pi_7 = g^{a_1} H^{b_1} \rho^{c_1} L \]
\[ \pi_4 = g^{a_1} H^{b_1} \rho^{c_1} P \quad \pi_8 = g^{a_1} H^{b_1} \rho^{c_1} P' \]

According to dimensional form, \( \pi_1 \) may be written as:

\[ \pi_1 = \rho^{a_1} g^{b_1} H^{c_1} Q \Rightarrow M^0 L^0 T^0 = (ML^{-3})^{a_1} (LT^{-2})^{b_1} (L)^{c_1} L^1 T^1 \]

hence, for \( M: a_1 = 0 \)

for \( T: -2b_1 - 1 = 0 \) thus \( b_1 = -0.5 \)

for \( L: -3a_1 + b_1 + c_1 + 3 = 0 \), thus \( c_1 = -2.5 \)
therefore, \( \pi_1 = \rho^0 g^{0.5} H^{2.5} Q \Rightarrow \pi_1 = \frac{Q}{\sqrt{gH^5}} \)

following the same procedure, the other \( \pi \)-terms can be written as:-

\[
\pi_2 = \frac{\mu}{\rho \sqrt{g H^5}}, \quad \pi_3 = \frac{\sigma}{\rho g H^2}, \quad \pi_4 = \frac{P}{H}, \quad \pi_5 = \frac{W}{H}, \quad \pi_6 = \frac{S}{H}, \quad \pi_7 = \frac{L}{H}, \quad \pi_8 = \frac{P'}{H}
\]

But, \( \pi_2 = \frac{\mu}{\rho \sqrt{g H^5}} = \frac{1}{Re} \), and \( \pi_3 = \frac{\sigma}{\rho g H^2} = \frac{1}{We} \)

Where, Re and We are Reynolds number and Weber number, respectively.

Both of Re and We can be ignored in present study. This due to that flow over weirs with a depth exceeding 3 cm involves ignorable We [22] and Re [23]. For horizontal channel bed of present study, \( \pi_6 \) of bed slope can be ignored. Simplifying and combining some of \( \pi \)-terms, such as \( \pi_4 \) and \( \pi_8 \), eq. (3) can be written as:

\[
f_1 \left( \frac{Q}{\sqrt{g H^5}}, \frac{P}{P'}, \frac{W}{H}, \frac{L}{H} \right) = 0 \quad (4)
\]

or

\[
\frac{Q}{\sqrt{g H^5}} = f \left( \frac{P}{P'}, \frac{W}{H}, \frac{L}{H} \right) \quad (5)
\]

Rewriting eq. (5) in the form of coefficient of discharge \( C_d \) as:

\[
C_d = f \left( \frac{P}{P'}, \frac{W}{H}, \frac{L}{H} \right) \quad (6)
\]

2.3. Experimental Work

2.3.1. Modified Crump Weir Models

Four wooden models have been adopted in present study. Wood sheets of 5mm thick cut into desired dimensions and assembled through fixing with nails and pasted together by silicon to produce the models shown in Fig. 1. Each of the four models had longitudinal equilateral triangle section with the same base length, L (78.5cm), width, W (29.5cm) and height, P (20cm), but they differ by the middle height, \( P' \) (20, 17.5, 15 and 12 cm) according to models 1, 2, 3 and 4, respectively. Wooden stiffeners fixed inside the models to strengthen them against flow stresses. Through experiment runs, each model fixed in the experimental flume by means of two terminal screws and adequate silicon rubber to prevent movement and provide water tightness, Fig. 2. The silicon rubber margins fixing the model in the flume have thickness of 2.5mm to get total weir width of 30cm. All models painted with red color to be clearly distinguished in the flume. According to ratio of \( P/P' \) the models 1, 2, 3 and 4 denoted as \( P/P'=1, \ P'/P=0.875, \ P/P=0.75, \ P'/P=0.6 \), respectively.

2.3.2. Flume and accessories

All experiments of the present study carried out using a rectangular flume of 15m length, 0.3m width and 0.45m depth. Walls of the flume are made of acrylic glass to enable visual observation. Flume bed is made of stainless steel. Flume operation is controlled by an electrical board. Sluice gate was situated at the entrance and downstream gate at the end to control water depth in the flume. The flume provided with point gauges to measure water depth. Water is circulating from main downstream tank into upstream tank by means of variable speed pump forcing water through steel pipe at which float-type
flow meter is installed. Wave suppressor made of vertical steel piers is installed close to the entrance to distribute the flow and break waves, Fig.3.

**Fig. 1:** Modified Crump Weir synthesized for present study.

**Fig. 2:** Fixing the model in the experimental flume.

**Fig. 3:** Sketch of the experimental flume
2.3.3. Testing procedure
First, flow meter readings checked and calibrated with quantitative method (flow volume with time watch). The bed of the flume was set horizontal for all runs. Models installed at the middle of flume length. For each model, eight flow rates (8, 23, 40, 70, 95, 110, 117, 123 m3/hr) were passed. For each of the eight flow rates, downstream gate rise (hs) was maintained for five depths (20, 17.5, 15, 12 and 0.0 cm). Net of 168 experiment runs carried out throughout the present study. In each run, flow depths at four locations were recorded after 5 minutes passed of continuous operation. The locations agreed with WMO [10]. They were at the weir crest (from water surface to weir crest center hcr), at 0.48m upstream weir center (from water surface to channel bottom, H), at 0.8m downstream weir center (from water surface to channel bottom, hd1) and at 1.14m downstream weir center (from water surface to channel bottom, hd2). At the same location where (H) measured, water depth from water surface to weir crest upper level (y) was calculated by subtracting weir crest height (P=20cm) from (H) reading. Also, length of hydraulic jump (Lj) was measured when occurred downstream the weir, Fig.4 and Fig.5.

3. Results and discussion
3.1. Flow over weir evaluation
3.1.1. Approach water depth at weir upstream, y.
The transversal V- modification of weir crest facilitates passage of water over the crest lowering approach water surface level for a given flow rate. Fig.6 illustrates results of water depth at approach (y) verses flow rate (Q) range for the four models. It is obvious that increasing (Q) lead to increase (y), and this is logical as higher flow rate requires higher upstream water depth for a given weir. But, for a given flow (Q) when comparing (y) for different models (P'/P), it is clear that decreasing weir crest middle height (P') lead to enlarge crest triangular open flow area that facilitating flow over weir and resulted in lower water depths (y). The overall average reduction in (y) can be calculated as ratio of average decrease in (y) values to average of (y) of model 1 (P'/P=1). The overall average reduction is 7% for model 2 (P'/P=0.875), 17.3% for model 3 (P'/P=0.75) and 32% for model 4 (P'/P=0.6). This reduction may effect downstream flow condition including strength (turbulence) and length of hydraulic jump as well as energy dissipation as illustrated in following sections below.

3.1.2. Hydraulic Parameter H/P versus Froude Number Fr.
The parameter (H/P) effects the discharge coefficient of any weir, directly. As this parameter increases, discharge coefficient increases too. In other words, minimizing weir height (P) to ignorable value leading to ignoring effect of the weir as retarding to flow, and vice versa. On the other hand, Froude number (Fr) indicates local ratio between gravitational and inertial forces of flow. So, as (Fr) of approaching flow upstream the weir increases, that mean water flow depth (hcr) decreases and/or flow velocity increases. Logically, for a given weir height (p) in modular flow, increasing upstream water flow depth (H) mean increasing total upstream head, so, it lead to increase flow rate and hence velocity rather than increase flow depth at weir crest section which is mean increase (Fr). But, as continuing with
increasing (H) reaching certain values, the rate of increase in (Fr) decreases due to flow depth increase at weir crest section. That is why trend of the relationship between (H/P) verse (Fr) is approximately linear for certain limits then changes to curvilinear as illustrated in a through d of Fig. 7 for all models. However, comparing the four curves for similar (H/P) values indicates that the modified models resulted in decreasing (Fr) for a given (H/P) with maximum decrease in Model 3 of (P'/P=0.75). Values of (Fr) was calculated from the well-known formula (Fr= v/√(g*D)), in which, (v) is flow velocity at weir crest section, and (D) the hydraulic depth which was calculated by dividing flow cross sectional area at the weir crest centre over the top width of flow. The eight (Fr) values of each of the four curves belong to the same flow rates.

**Fig. 6:** Approach water depth (y) verses flow rate (Q)

**Fig. 7:** H/P vs (Fr) for (a) model 1, (b) model 2, (c) model 3 and (d) model 4.
3.1.3. Length of Hydraulic Jump (Lj).

Fig. 8 (a through d) illustrates the relationship between the length of hydraulic jump (Lj) and the flow rate for downstream water depth ratios hs/P as 1, 0.875, 0.75 and 0.6, respectively. For (hs/P= 1 and 0.875), model 4 (of P'/P=0.6) resulted in the lowest length, because downstream water depth (hs) equals and exceeds (P') that lead to reduce jump length. For (hs/P= 0.75 and 0.6) all modified models (2, 3 and 4) resulted in jump length longer than that of model 1 of (P'/P=1) with highest values for model 3 of (P'/P=0.75).

![Graphs showing Lj vs Q for different hs/P ratios](image)

Fig. 8: (Lj) vs (Q) for a: hs/P=1, b: hs/P=0.875, c: hs/P=0.75 and d: hs/P=0.6

3.1.4. Energy Dissipation (ΔE)

Energy dissipation (ΔE) of flow within weir zone is important to reduce downstream turbulence, hence, reduce scour and erosion. Usually, inertial forces (kinetic energy) is proposed to be reduced more than gravitational (potential energy). The dissipation is calculated as a ratio of energy difference between upstream and downstream sides to the upstream one as [ΔE = (Eu-Ed)/Eu]. The upstream energy is (Eu=H +3yc /2), where (yc) is the critical flow depth as [yc=(Q^2/gW^2)(1/3)]. The downstream energy is [Ed=y1+ 1.1 Q^2/(A^2 g)], where (y1) is downstream flow depth before hydraulic jump, and (A) is the flow cross sectional area at (y1) location [24]. In present study, focusing was on energy dissipation when the downstream gate was lowered down, completely, (hs=0). Values of (hd1) were considered as the needed (y1) values. Fig.9 (a through d) shows the results of energy dissipation calculation for the four models. The eight points of each of the four curves belong to the same flow rates. However, values of upstream approach (Fr) decreased with decreasing (p') according to models 2, 3 and 4, respectively when compared with that of model 1 of (P'/P=1). This is because of decreasing flow velocity and increasing flow depth with decreasing (p'). At relatively low (Fr), effect of the weir in dissipating flow energy...
decreases. This is clear in models 3 and 4, and consequently, less energy dissipation. Model 2 with 
\(P'/P=0.875\) showed similar energy dissipation as Model 1 with no opening \((P'/P=1)\).

\[\begin{array}{c}
\text{(a) Model 1} \\
\text{(b) Model 2} \\
\text{(c) Model 3} \\
\text{(d) Model 4}
\end{array}\]

**Fig. 9:** \((\Delta E)\) vs \((Fr)\) for (a) model 1, (b) model 2, (c) model 3 and (d) model 4.

### 3.2. Regression formula for discharge coefficient

On the basis of dimensional analysis results explained in section (2-2) above, and in particular eq. (5), multiple regression analysis was carried out for about 80% of the experimental results obtained, whereas the rest 20% were used for verification. The selection of the 80% and 20% was random process. All results adopted were belong to modular flow conditions. Statistical computer software SPSS was used to perform the non-linear multiple regression. The resulted basic equation is:

\[Cd = \left(\frac{\rho}{\rho}\right)^a \times \left(\frac{w}{H}\right)^s \times \left(\frac{L}{H}\right)^z + b\]  

(7)

The factors \(a\), \(s\), \(z\) and \(b\) produced by the software to have the developed formula as:

\[Cd = \left(\frac{\rho}{\rho}\right)^{6.09} \times \left(\frac{w}{H}\right)^{0.606} \times \left(\frac{L}{H}\right)^{-0.003} + 0.712\]  

(8)

The coefficient of determination \(R^2\) of this formula is (0.958). To test (verify) this formula, the rest 20% of the observed experimental results were compared with the corresponding results of the formula. Fig. 10 show a comparison between results predicted by the formula (theoretical) and those observed (actual). Good agreement between the predicted and observed results that justify using this formula to estimate the discharge coefficient \(Cd\) of the weirs modified in this study.
4. Conclusions
The following conclusions may derived from the present study:-

a- The overall average reduction in approach water depth upstream the modified crump weir models is 7% for model 2 (P'/P=0.875), 17.3% for model 3 (P'/P=0.75) and 32% for model 4 (P'/P=0.6).

b- The modified models resulted in increasing (Fr) for a given (H/P) with maximum increase in Model 3 of (P'/P=0.75).

c- For relatively high downstream water depth (hs/P= 1 and 0.875), model 4 (of P'/P=0.6) resulted in lowering the length of hydraulic jump, while for (hs/P= 0.75 and 0.6) all modified models (2,3 and 4) resulted in jump length longer than that of model 1 of (P'/P=1) with highest values for model3 of (P'/P=0.75).

d- Model 2 with (P'/P=0.875) showed similar energy dissipation as Model 1 with no opening (P'/P=1).

e- The non-linear multiple regression formula developed in this study to estimate coefficient of discharge Cd of the modified models 2, 3 and 4 has good coefficient of determination $R^2$ as (0.958) and good agreement with observed data in verification process.

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