Modelling and extraction technique for micro-doppler signature of aircraft rotor blades

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Abstract. The process of detecting and distinguishing between different aircrafts has been a major point of interest in Defence applications. Micro-Doppler effect is one such phenomenon unique for aircrafts with different rotor dynamics and design. In this paper, we focus on deducing a mathematical model for micro-Doppler signature, of aircraft rotor blades assumed to be rotating in a plane perpendicular to the flying direction, induced on the incident radar signal. Also, we use the Wigner-Ville Distribution (WVD) to extract this signature from the radar return. This mathematical model is compared with the simulation results obtained from MATLAB, to validate the results and show the accurateness of the developed model.

1. Introduction
The use of Radars for target detection dates back to early 1900’s, with the focus being on defense applications. With the advancements in technology and powerful mathematical tools, discrimination of different targets has been made possible. Micro-Doppler is one such phenomenon unique to different targets with different micro-motion, apart from the overall movement, such rotor blade rotation, vibration etc. In the recent years, Micro-Doppler based Radar signal processing has been proved effective in target detection and distinguishing.

The first mathematical model for Micro-Doppler frequency induced by a point scatter undergoing rotation and/or vibration was given by V.C.Chen et al [1]. This can be further extended to coin the model for different rotating/vibrating parts of different aircrafts, as proposed by Baoshuai Wang et al, in their paper [2]. Also, micro-Doppler signature of a rotating object being time varying in nature, its extraction calls for powerful time-frequency analysis tools such as Wigner-Ville Distribution (WVD), Wavelet transforms etc. Roger Tan et al, in their paper [3], highlight the accurateness of one such transform – Psuedo WVD in extracting micro-Doppler features.

This paper throws light on a mathematical model for micro-Doppler signature of an aircraft rotor blade, from the point scatter model. The plane of rotation of the blades is assumed to be perpendicular to the direction of motion of the aircraft. The model is then used to synthesize a Radar echo signal on MATLAB, which is then processed further using different time-frequency analysis tools. The extracted micro-Doppler spectrum further aids in extraction of different features of the rotor such as the blade length, rotational speed etc.

The mathematical expression for micro-Doppler frequency induced due to pure rotation, of a point scatter and a rotor with N blades, has been derived in the following section. Section III highlights about WVD and pseudo-WVD. The results and extraction using different time frequency methods are discussed in section IV, followed by the conclusion in section V.
2. Mathematical model of micro-Doppler return for Rotor Blades

In fig.1, the range of point P, considered on a blade in air, is given by:

\[ R_p(t) = \sqrt{(R_t \cos \beta' \cos \alpha' + l \cos \theta_t)} + (R_t \cos \beta' \sin \alpha' + l \sin \theta_t) \]  

(1)

![Figure 1. Rotor in Radar coordinate system](image)

We know that,

\[ f_d = \frac{2v}{\lambda} \]  

(2)

Thus, a rotating point scatter in space, would introduce a micro-Doppler and corresponding Radar return given by:

\[ s_p(t) = \exp[-j\left(\frac{4\pi}{\lambda}\right)R_p(t)] \times \exp[j2\pi f_0 t] \]  

(3)

where, \( f_0 \) is the transmitted pulse carrier frequency and \( \lambda \) is the corresponding wavelength.

Integrating the above expression over the length of the blade, \( L=L_2-L_1 \), where \( L_2 \) is the distance from the blade bottom to center of rotation and \( L_1 \) is the distance from blade tip to the center of rotation, we get:

\[ s_\theta(t) = (L_2 - L_1) \text{sinc} \left[ \left(\frac{2\pi}{\lambda}\right)(L_2 - L_1) \cos \beta' \cos(\theta_0 + \omega_r t - \alpha') \right] \times \exp[-j\frac{4\pi}{\lambda}\left(\frac{L_1 + L_2}{2}\cos \beta' \cos(\theta_0 + \omega_r t - \alpha')\right)] \times \exp[j2\pi(f_0 - f_d)t] \]  

(4)

The above return is then summed up over the number of blades, \( N \), to get the returns from the entire rotor.
\[ s_N(t) = \sum_{k=0}^{N-1} \left( L_2 - L_1 \right) \text{sinc} \left[ \left( \frac{2\pi}{\lambda} \right) \left( L_2 - L_1 \right) \cos \beta' \cos(\theta_k + \omega_r t - \alpha') \right] \]
\[ \times \exp \left[ -j \frac{4\pi}{\lambda} \left( R_0 + \frac{L_1 + L_2}{2} \cos \beta' \cos(\theta_k + \omega_r t - \alpha') \right) \right] \times \exp[j2\pi(f_0 - f_d)t] \]

(5)

3. WVD and pseudo-WVD

Wigner Ville distribution is one of the powerful time frequency analysis tools and is a Cohen’s class member. It is given by [4]:

\[ W(t, \omega) = \frac{T}{\pi} \sum_{k=-\infty}^{\infty} s^*(t - kT)e^{-2j\omega k} s(t + kT) \]

(6)

where, s is the analytical signal and 1/T the sampling frequency

In discrete form, it is given by:

\[ W(n, \theta) = \frac{1}{\pi} \sum_{k=-\infty}^{\infty} s^*(n - k)e^{-2j\theta k} s(n + k) \]

(7)

Comparing WVD with the expression for Short Time Fourier Transform:

\[ \text{STFT} \{x[n]\} = X(n,k) = \sum_{n=-\infty}^{\infty} x(n)\omega(n - k)e^{-j2\pi nk} \]

(8)

We then go for another enhanced transform of WVD class, the pseudo-WVD, given by:

\[ \text{SPWVD} \{x[n]\} = X(n,k) = \frac{1}{2} \sum_{n=-N+1}^{N+1} |h(n)|^2 \sum_{p=-M+1}^{M-1} g(p)x(n + p + k)x^*(n + p - k)e^{-j4\pi nk} \]

(9)

A comparative analysis using the above three transforms is done in the following sections.

4. Results

The simulated radar return signal with respect to the equation developed in sec II, is subject to different time frequency analysis such as STFT, WVD and pseudo-WVD. The corresponding results are shown in the following figures.

![Figure 2. FFT spectrum of Radar return](image-url)
Figure 3. STFT Spectrogram

Figure 4. WVD spectrum of received signal (image to the right is the magnified version of spectrum’s positive peak)

Figure 5. pseudo-WVD spectrum of the received signal (image to the right is the magnified version of spectrum’s positive peak)
From the above results, it is evident that pseudo-WVD gives us a better spectrum in the time-frequency analysis. Also, from the obtained spectrum, we can derive the frequency of rotation of the blade and the Length (L) of each blade, given by:

\[ L = \frac{B}{2\pi f} \]  

(10)

where, \( B \) is the bandwidth of the p-WVD spectrum and \( 1/f \) is the time at which the spectrum peak repeats itself, indicating the frequency at which the rotor blade flashes.

Table 1. Feature Extraction

| Feature               | Actual Value | Extracted Value |
|-----------------------|--------------|-----------------|
| No. of Blades         | 10           | 10              |
| Blade Speed (Rpm)     | 1000         | 878             |
| Blade Length (m)      | 5            | 5.729           |

5. Conclusion and future scope

The mathematical model for the micro-Doppler signature of an aircraft rotor blade was analyzed and a return signal using the analyzed model was synthesized. The synthesized signal was then subject to different time-frequency tools, to study the time varying micro-Doppler. It is observed that p-WVD proves to be a powerful tool in extracting the m-D spectrum. Also, from the obtained m-D spectrum, the features of the rotor were derived and verified.

This work is to be further extended using a real-time simulator to synthesize the radar returns/echo signal. The obtained signal is to be subject to various time frequency transformations and the extracted features are to be compared with the one from the mathematically developed signal’s data.

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