PID Tuning for Time-Varying Delay Systems
Based on Modified Smith Predictor

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Abstract: This paper presents a new linear matrix inequality (LMI) based control strategy for second-order systems with time-varying delay based on a modified Smith predictor with a proportional-integral-derivative (PID) controller. The main idea consists in representing the predictor model as a closed-loop observer, which takes into account only the estimated average value of the time-delay, such that no real time measurement of the delay is required. A numerical example is introduced to illustrate the effectiveness of the proposed method.

Keywords: Process time-delay, Smith predictor, PID controller, linear matrix inequalities.

1. INTRODUCTION

Time-delays appear naturally in different kinds of real world applications, in which the transportation or propagation of material, energy or information is present, as well as they can also be induced by sensors and actuators in the controller loop. As a matter of fact, time-delays can introduce instability and performance degradation and the control design should take it into account.

Among well-known methods in the literature to deal with time-delays, a class of them is based on the compensation of the time-delay in which the free delayed output of the system is predicted and feedbacked to the controller. The Smith predictor is one example of this kind of method and it has been largely used in the industry (Normey-Rico and Camacho, 2008). However, the main issue in the standard Smith predictor is that it is not robust to parameter uncertainties in the model as well as uncertain delays. Notice also that the standard Smith predictor is just applied to stable SISO (Single Input - Single Output) systems. For further details about advantages and disadvantages see Palmor (1996), Normey-Rico and Camacho (2008), and references therein.

As the Smith predictor has a great appeal for the industrial context, a lot of effort has been done to improve its robustness as, for example, including explicitly uncertain parameters (Santacesaria and Scattolini, 1993; Palmor and Blau, 1994; Lee et al., 1999), or changing its standard structure as in Normey-Rico et al. (1997) and Normey-Rico and Camacho (1999, 2009). More recently, different alternatives have also been introduced in the literature to deal explicitly with time-varying delay processes using, for example, linear matrix inequalities (LMIs) and \( H_\infty \) performance (Oliveira and Karimi, 2013; Normey-Rico et al., 2012; Bolea et al., 2014).

In this paper, we introduce a control strategy for second-order systems with time-varying delay based on a modified Smith predictor with a PID controller. The main contribution of the proposed approach is predicting the system output using a closed-loop observer, which takes into account only the constant estimated average value of the time-delay. Thus, real time measurement of the delay is not required. The main design results are derived as LMIs conditions. A numerical example is introduced to illustrate the effectiveness of the proposed method.

Notation: “*” denotes symmetric terms in matrices. The superscript “T” stands for transpose. For a real symmetric matrix \( M \), \( M > 0 \) stands for the positive definite matrix, \( M + M^T \), with \( I \) the identity matrix, and \( \text{diag} \{ \cdot \} \) stands for a diagonal matrix with the entries on the main diagonal.

2. PROBLEM FORMULATION

Consider second-order plants with time-delay in the form:

\[
G(s) = \frac{b_0 e^{-d_s s}}{s^2 + a_1 s + a_0}
\]

where the time-delay, \( d_s \), is assumed uncertain but belonging to a given interval, namely \( d_s \in [\tau - \mu, \tau + \mu] \), with \( \tau \) representing the estimated average delay and \( 0 \leq \mu \leq \tau \) is a scalar parameter used to define the time-delay range.

The aim is to design a PID controller to the system (1) modifying appropriately the standard Smith predictor, illustrated in Fig. 1, in order to design a control system more robust to uncertain delay.

In this paper, to compensate the time-delay uncertainties the predictor in the traditional Smith structure, which is

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* This work has been supported by the Brazilian agencies CAPES, CNPq, and FAPEMIG.
are given in (2). Further, for the observer only the estimator, and in the sequel, the observer design is performed considering the pre-tuned PID controller.

Moreover assume the PID controller

\[ C(s) = k_p + \frac{k_i}{s} + k_d \frac{\alpha s}{s + \alpha} \]

where \( \alpha \) is the derivative filter parameter.

One possible state-space realization for the PID controller (4) is given as:

\[
\begin{align*}
\dot{x}_e(t) &= Ax_e(t) + Bu(t) + L(y(t) - \hat{y}(t)) \\
y(t) &= C^\top\hat{x}_e(t - \tau)
\end{align*}
\]

with

\[ A_e = \begin{bmatrix} 0 & 1 \\ 0 & -\alpha \end{bmatrix}, B_e = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, C_e^\top = \begin{bmatrix} k_i \alpha \\ k_d \alpha \end{bmatrix}, D_e = k_p + k_d \alpha. \]

where the matrices \( A_e, B_e, C_e \), and \( D_e \) should be designed for the system free of time-delay previously than the observer in (3).

According Fig. 2, \( e(t) \) is defined as:

\[ e(t) = r(t) - \hat{y}(t + \tau) - y(t) + \hat{y}(t). \]

Using the fact that:

\[ x(t - d_r(t)) = x(t - \tau) - \int_{d_r(t)}^t \dot{x}(t - \xi)d\xi, \]

and defining \( \dot{\hat{x}}(t) = x(t) - \hat{x}(t) \), \( \dot{x}^T = [x^T(t) \hat{x}^T(t) x_e^T(t)]^T \), and \( z^T(t) = [y(t) \ r(t)] \), the closed-loop system, described from Fig. 2, is given by:

\[
\dot{\hat{x}}(t) = A\hat{x}(t) + A_d\hat{x}(t - \tau) + A_t\int_{d_r(t)}^t \hat{x}(t - \xi)d\xi + Dz(t),
\]

with

\[
A = \begin{bmatrix} A - BD_eC & BD_eC & BC_e \\ 0 & A & 0 \\ -B_eC & B_eC & A_e \end{bmatrix}, \quad A_d = \begin{bmatrix} 0 & -BD_eC & 0 \\ 0 & -LC & 0 \\ 0 & -B_eC & 0 \end{bmatrix}, \quad A_t = \begin{bmatrix} BD_eC & 0 & 0 \\ LC & 0 & 0 \\ B_eC & 0 & 0 \end{bmatrix}, \quad \text{and} \quad D = \begin{bmatrix} B & BD_e \\ B & 0 \\ -B_e & 0 \end{bmatrix}.
\]

The main results of this paper are stated in the next section.

3. MAIN RESULTS

Firstly an LMI delay dependent stability condition for the closed-loop system in (6) is presented. Afterwards such stability condition is used as the starting point to design the observer (3) used as the predictor model.

**Theorem 1.** Let \( \tau > 0 \), the estimated average time-delay, \( 0 \leq \mu \leq \tau \), the upper bound to its uncertainty range, be given. Then the closed-loop system in (6) is asymptotically stable if there exist matrices \( P = P^T \geq 0, Q, R_1 = R_1^T, R_2, R_3 = R_3^T, S = S^T, Z = Z^T, U = U^T, F, \) and \( G \in \mathbb{R}^{6 \times 6} \), such that the following LMIs hold:

\[
\begin{bmatrix} P & Q \\ * & \frac{1}{\tau} S \end{bmatrix} > 0, \quad \bar{R} = \begin{bmatrix} R_1 & R_2 \\ * & R_3 \end{bmatrix} > 0, \quad \Sigma = \begin{bmatrix} \Pi & \Phi^T \\ * & -\mu \delta \end{bmatrix} < 0,
\]

with

\[
\Phi = \begin{bmatrix} (\mu FA_t)^T & (\mu GA_t)^T & 0 & 0 \end{bmatrix},
\]

\[
\Pi = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \quad \delta = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}.
\]
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