Criteria for Evaluating Local Stability of Steel Beam Walls in Case of Uneven Stress Distribution

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Abstract. In the article, numerical studies are performed to assess the stress-strain state of the wall of a steel beam with an I-beam symmetrical cross-section. The determination of internal forces and stresses is performed in the SCAD Office computing complex. A new method is proposed for estimating the average stress values in the wall, taking into account the uneven distribution of stress over the area of the wall section where local stability is evaluated. Based on the results of calculations, stress plots and mosaics are constructed, showing their spatial distribution of stress on the controlled sections of the beam wall compartments. Analysis of the research results showed that the most appropriate application of the proposed approach is in areas with uneven distribution of beam stresses, especially in the support sections of the wall.

1. Introduction
At the present stage, steel composite beams of I-beam cross-section are widely used for the construction of buildings and structures. Ease of manufacture and adaptability made it possible to ensure high cost-effectiveness of these structures. The most important problem of beam components is an adequate assessment of local wall stability [2,9,10].

In engineering practice, to test the local stability of the wall as an element with a complex stress state, a criterion is used, which consists in analyzing the ratio of the average normal and tangent stresses on the tested section of the wall to the corresponding critical stresses [2]:

\[
\frac{\sigma}{\sigma_{cr}} + \frac{\sigma_{loc}}{\sigma_{loc,cr}} + \frac{\tau}{\tau_{cr}} \leq 1,
\]

where: \(\sigma, \sigma_{loc}, \tau, \sigma_{cr}, \sigma_{loc,cr}, \tau_{cr}\) - where-normal, normal local, tangent stresses and corresponding critical stresses.

In recent years, numerical methods for calculating construction structures, and especially the finite element method (FEM), have been used in computational practice. This computational approach allows us to accurately estimate the stress-strain state (VAT) of the structure in all its sections.
Previous studies have shown that criterion (1) is well suited for sections of composite beams with a uniform distribution of the stress state, but when this distribution becomes uneven, it has an ambiguous assessment.

Based on the above, it is important and relevant to develop computational approaches to assess the local stability of the walls of composite steel beams taking into account the uneven stress-strain state.

2. Object and method of research

As an object of comparative research, we consider a steel beam of I-beam cross-section, a composite of a symmetrical type. The wall of the beam is reinforced with transverse ribs of the same thickness, which are evenly spaced along the length of the beam. Local voltages are not taken into account. Under these initial conditions, criterion (1) will be formulated as follows:

$$\gamma_0 = \sqrt{\left(\frac{\sigma}{\sigma_{cr}}\right)^2 + \left(\frac{\tau}{\tau_{cr}}\right)^2} \leq 1.$$  (2)

In numerical studies, a steel composite beam is used, which is part of a simple beam overlap system. The basic layout parameters of the beam: the beams span L=12 m; step beams 6 m; material of steel I-beams – steel S255, contiguity to the cap beams of the columns hinge, continuous load case of 4.55 kN/m2; temporary estimated load of 2.4 kN/m2. According to the preliminary calculations, the cross-section of the beam was determined: the wall sheet 650x8 mm, the belt sheet 320x16 mm. Transverse ribs were installed in increments of 1700-1740 mm and divided the wall into six compartments.

The design scheme of the steel beam (Fig. 1) was formed in the environment of the SCAD Office computing complex, using the following finite element (FE): type 44-a universal quadrilateral four-node FE shell. In General, the design model of the beam consisted of 3452 specified plate elements. Connections were also modeled in the form of restrictions on linear movements: the left hinge-fixed support and the right hinge-movable support. In addition, the upper girder belt provides linear anchors "from the plane", which modeled the horizontal connections of the beam floor.

![Figure 1. Finite element design model of a composite beam.](image)

According to the regulated approach [2], normal and tangential stresses on the section of the I-th section of the beam being checked are determined by the following expressions:

$$\sigma = \frac{M}{J_x} y; \quad \tau = \frac{Q \cdot S}{J_x \cdot t_w},$$  (3)
where: \( Qz, My \) – average values of shear force and bending moment at a monitored site; \( Sx \) - static moment of cut off part of the cross section about the \( x \)-axis; \( Ix \) - is the moment of inertia of the full cross section about the \( x \)-axis; \( tw \) is the wall thickness of the; \( y \) – beam; the \( y \) coordinate counted from the neutral line to the fiber cross-section.

Note that with this approach, the averaging is performed by bending moments and transverse force. In finite element calculations, normal and tangent stresses are determined directly from the calculations, so they are averaged by voltage.

Note that both normal and tangent stresses have an uneven distribution of values over the height of the cross-section. For subsequent calculations, the maximum values of stresses are used: normal stresses located along the upper fibers of the wall, tangent stresses located in the middle of the cross section.

Based on the above, the average values of normal and tangent stresses are obtained using horizontal rows of finite elements, as follows:

\[
\sigma_{\text{mid}} = \frac{\sum_{i=1}^{n} (\sigma_i \cdot A_i)}{A_{\text{rank}}}; \quad \tau_{\text{mid}} = \frac{\sum_{i=1}^{n} (\tau_i \cdot A_i)}{A_{\text{rank}}},
\]

where \( \sigma_{\text{mid}}, \tau_{\text{mid}} \) – average values of normal and shear stresses in a certain set of FE; \( \tau_s, \sigma_i \) – shear and normal stresses in \( i \) FE; \( A_i \) – area of \( i \) FE; \( A_{\text{rank}} \) – total area of \( i \) FE located in a row of controlled section of a compartment or section of the beam; \( n \) – number of FE in the controlled population.

3. Investigation of local wall stability

Studies of local stability of the composite beam wall were performed for the following two compartments: the middle one – located near the middle of the beam span; the extreme one – located at the left support node of the beam. As a result of finite element calculation of a composite beam, the isofields of normal and tangent stresses are determined (Fig. 2, 3).

![Figure 2. Isofield distribution of tangent stresses in the beam.](image)

![Figure 3. Isofields of normal stress distribution in the beam.](image)

The analysis of the stress-strain state in the middle section of the beam revealed a practically uniform distribution over the horizontal rows of finite elements, both for normal and tangential stresses. For the left extreme compartment, a very uneven stress-strain state of the beam wall was observed, characterized by variability both along the wall height and along the length of the compartment (Fig. 4).
Figure 4. Diagrams of normal and tangent stresses (MPa) for the extreme compartment
a, b, c – average values of normal and tangent stresses in the wall for the series of FE;
d, e – normal and tangent stresses in the wall according to the regulated approach.

To assess the unevenness of the values in the stress samples under consideration, we use the following commonly used indicators: the span and average span:

\[ R = |X_{\text{max}} - X_{\text{min}}|; \quad \bar{R} = \frac{R}{X}; \]  

where: \( X_{\text{max}}, X_{\text{min}} \) - maximum and minimum values in the selection; \( \bar{X} \) - standard deviation in the sample.

In table 1 the calculated indicators of uneven stress distribution in the compartments of a composite beam are shown.

| Compartment | Normal stress range | Tangential stress range |
|-------------|---------------------|------------------------|
| Extreme     | 2.69                | 0.92                   |
| Average     | 0.047               | 0.66                   |

Note that in the middle compartment, the distribution of normal stresses along the height of the cross-section corresponded to the generally accepted character (the maximum values are located on the extreme fibers) and the values practically did not change along the length of the compartment. The distribution of tangential stresses along the height of the cross-section also did not deviate from the theoretical distribution (the maximum values are on the midline), but there was some variation in the values along the length of the compartment.

In the extreme compartment, due to the influence of anchorages, there was a significant change in the nature of stress distribution. Thus, the highest normal stresses were located near the Central line, and tangent stresses were about 0.10 – 0.15 \( \bar{h} \) below the Central line. This unevenness led to a rather large uncertainty in the estimation of the stress-strain state of the wall in this compartment.

Based on the results obtained on the stress-strain state of the beam, we proposed to perform averaging of normal and tangential stresses not along horizontal rows, but along a conditional line corresponding to the maximum compressive values of stresses in the wall.

For a wall, as for an element with a complex stress state, the maximum stresses are the main ones, which are calculated as follows:

\[ \sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \frac{1}{2} \sqrt{(\sigma_x + \sigma_y)^2 + 4 \cdot \tau_{xy}^2}. \]
In this case, the average values of normal and tangent stresses, as well as the criterion for the wall's stability, will be written as follows:

\[
\sigma_{mid,l} = \frac{\sum_{i=1}^{n}(\sigma_i \cdot A_l)}{A_l} ; \quad \tau_{mid,l} = \frac{\sum_{i=1}^{n}(\tau_i \cdot A_l)}{A_l} ;
\]

\[
\gamma_l = \sqrt{\left(\frac{\sigma}{\sigma_{cr}}\right)^2 + \left(\frac{\tau}{\tau_{cr}}\right)^2} / \gamma_c \leq 1 ,
\]

where: \( A_l \) - the total area of the finite elements located along the line of nice compressive stresses

Fig. 5 and 6 show mosaics of the highest compressive stresses in the middle and extreme sections of the composite beam, respectively. Table 2 shows the values of the criterion for evaluating local wall stability when averaging stresses along horizontal rows and the main compressive stress line.

From the analysis of the results of table 2 it can be seen that the largest deviations in the local stability criteria are observed in the extreme compartment. This fact is explained by the fact that when averaging along the line of reducing stresses in comparison with averaging along horizontal rows, the average normal stresses increased by about 1.5 times, while the average tangent stresses practically did not change. Also note that the comparison of criteria shows that the probability of loss of local stability in the extreme compartment of the composite beam is higher than in the average compartment.

![Table 2](image-url)

**Figure 5.** Mosaic distribution of the main compressive stresses for the middle compartment.

**Figure 6.** Mosaic distribution of the main compressive stresses for the leftmost compartment.
Table 2. The criterion of local buckling of the wall.

| Compartment | $\gamma_0$ | $\gamma_l$ | $\frac{\gamma_0 - \gamma_l}{\gamma_0} \times 100\%$ |
|-------------|------------|------------|------------------------------------------|
| Extreme     | 0.15       | 0.121      | 19.3%                                     |
| Average     | 0.22       | 0.204      | 7.27%                                     |

4. Conclusions

1. It is proposed to take into account the uneven distribution of stresses over the area of the compartment when determining the average values of normal and tangential stresses in the compartments of composite beams when evaluating local stability.

2. The proposed approach allows us to refine the criterion of local wall stability by 7-19% compared to the regulated approach.

3. Unevenness of stress distribution is most appropriate to take into account in the extreme sections of composite beams.

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