HAVE WE AT LAST FOUND THE A-TOMS?

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Abstract

It is argued that the three families of quarks and leptons are the building blocks of all matter and all forces (leaving out gravity) among them are mediated by photons ($\gamma$), the weak bosons ($W^\pm$ and $Z$) and the gluons.
INTRODUCTION

From time immemorial, it has been the human endeavour to know what everything may be made up of and what is the form of all forces of nature. Can we understand a complex system – say the universe – in terms of its parts, which in turn is made up of further parts? Is there a basic set of constituents in terms of which, in principle, all matter and all phenomena of the physical world are understandable? Is it possible to reduce them to just a few principles that can be splashed on a T-Shirt?

Hindu texts refer to ‘Pancha Bhutas’: everything is a manifestation of a harmonious combination of Prithvi (earth), Jala (water), Vayu (air), Agni (fire) and Akasha (space?). Greek philosophers also mention four of these elements, leaving out Akasha to begin with and later realise that they need a ‘void’ to place the other elements in. While today we may recognise these not as basic constituents, but merely as different physical attributes of all matter, it would be nice if there are just a few ingredients from which everything can be built. It was Democritus of Mellitus, in Greece, who reasoned that as we keep dividing anything into smaller and yet smaller pieces, the process may terminate, yielding at the end ultimate constituents, that may not be cuttable any further. He named them a-tom (tom in Greek is to cut; a-tom means that which is uncuttable). Unfortunately the then dominant school of philosophy led by Aristotle largely ignored, in fact ridiculed, Democritus and his notion of ultimate constituents.

Several centuries later, we have the English chemist Dalton, who resurrected the idea of basic elements of nature and propounded that the large variety of chemical compounds can be understood in terms of just about 90 elements – each element being made up of its basic unit, which he called the atom, making use of the terminology of Democritus. All chemical compounds are molecules – made up of the atoms of the elements it is composed of. These atoms were indeed uncuttable in the length scales of chemistry (a few nanometres \((10^{-9}m)\)), but now we are aware that the atoms in Bohr model consist of a much tinier \((10^{-15}m)\) massive nucleus and very light electrons moving around the nucleus, somewhat like a miniature solar system. The nucleus has protons and neutrons as its constituents and they too reveal substructure. Indeed the name atom for these species was rather premature. They are certainly not the a-toms of Democritus.

So let us carry on the quest for the real a-toms of Democritus. Dalton’s atoms are made up of protons and neutrons in the nuclei and electrons which are bound to them by Coulomb forces. All chemical reactions are complex manifestation of the electromagnetic forces experienced by the electrons, so indeed is the short range van der Waals force between atoms and molecules – a consequence of the polarisability of the neutral atoms and molecules. The forces of elasticity of bulk material can be traced to the same electromagnetic origin. Indeed most of the chemical and physical phenomena are understandable in terms of the forces of electrodynamics, the classical content of which is summarised by the set of four
equations of Maxwell. Let us probe further to see what the protons are made up of. What is the force that holds the protons and neutrons together in the nucleus, overcoming the Coulomb repulsion among the protons? What are the analogues of Maxwell’s equation for the strong nuclear forces? What is the spectrum of states of which protons and neutrons are members.

In the sixties with the advent of proton and electron accelerators, there was a discovery of a plethora of states, which were either excited nucleons and similar states on the one hand or many mesons that could be playing the role of the strong nuclear force mediators on the other. Pion or $\pi$-meson was predicted by Yukawa as the particle that could be principally responsible for the attractive short range force between nucleons. Pions were seen during 50’s in cosmic ray experiments and were later produced copiously in the particle accelerators at Rochester, Columbia and Chicago. A prominent resonance in the pion-nucleon scattering was observed at a total energy of 1238 MeV and understood as a shortlived state $\Delta$, the lifetime being so small that it shows up as a resonance with a width of 120 MeV. More resonances were discovered and many new mesons were produced in the accelerator laboratories. Soon there was a flurry of activity in Particle Physics, classifying and jotting down the properties of the various species of particles and the many short lived and long lived states were systemetised. The particle spectroscopy was studied, naturally with a view to finding the clues for the underlying dynamics. In this way we discovered several new quantum numbers such as strangeness $S$ and charm $C$, that were preserved in the strongly interacting production processes, but violated in its weaker decays. The weak forces were the ones responsible for radioactivity in some (rather, many) nuclei. Particle Physics is the discipline that systematises the subconstituents of the subatomic world and strives to answer whether the a-toms of Democritus can indeed be real and arrived at.

**STANDARD MODEL**

When the Russian chemist Mendeleev gave the periodic table of elements, he paved the way for the systematic understanding of the atomic structure of elements. Further during the first half of this century, the development of quantum mechanics yielded a fairly complete qualitative and quantitative confirmation of the underlying dynamics of all chemical phenomena.

In the world of Particle Physics we are now in a position of having got the analogue of the periodic table that classifies the ingredients of all matter and this clearly points towards the nature of the underlying forces. This is referred to as the Standard Model. The remarkable feature of the Model, which we claim explains all forces of interaction (except gravity) among all species of matter, is that the laws are simply suitable generalisation of the familiar Maxwell’s equations that govern all electromagnetic phenomena.

The main principle in the Standard Model is to elevate the notion of symmetry to the status of the essence of dynamics. Recall that the Maxwell’s Equations are:
\[ \nabla \cdot \mathbf{E} = \frac{\rho}{\varepsilon_0} \; ; \quad \nabla \cdot \mathbf{B} = 0 \]
\[ \nabla \times \mathbf{E} + \frac{\partial \mathbf{B}}{\partial t} = 0 \; ; \quad \nabla \times \mathbf{B} - \mu_0 \varepsilon_0 \frac{\partial \mathbf{E}}{\partial t} = \mu_0 \mathbf{j} \]

Electromagnetic fields $\mathbf{E}$ and $\mathbf{B}$ are given by these equations, once we give the distribution of the sources $\rho$ and $\mathbf{j}$, the electric charge and current densities carried by the matter. Since $\mathbf{E}$ and $\mathbf{B}$ are defined through the force experienced by the charges and currents of the matter, we can say that we have the complete description of the dynamics of the electromagnetic process. The underlying principle of electromagnetism is gauge invariance – more precisely the local gauge invariance. $\mathbf{E}$ and $\mathbf{B}$ can be expressed in terms of scalar ($\phi$) and vector ($\mathbf{A}$) potentials, specified together in the 4-vector notation as $A_\nu$. The field $F_{\mu\nu}(\equiv \partial_\mu A_\nu - \partial_\nu A_\mu)$ is invariant under the gauge transformation that takes $A_\mu \rightarrow A_\mu - ie \partial_\mu \Lambda(x)$. Indeed the Maxwell’s equations and all of physics is left invariant by the gauge transformation.

Symmetries in Physics, we recall are closely related to conservation laws. The symmetry of gauge invariance in electrodynamics is really the result of the conservation of electric charge. While on the mundane level this implies that the sum of the charges remains the same during an interaction, on a more subtle plane, this calls for an abstract operator $Q$, whose eigenvalue is indeed the charge of the state, and the gauge transformation is effected by the change of phase of the complex field that represents a charged particle: $\psi(x) \rightarrow e^{iQ\Lambda} \psi$ is a gauge transformation on $\psi$, that is labelled by the parameter $\Lambda$. It is easily seen that successive transformation with parameters $\Lambda_1$ and $\Lambda_2$ will also be another gauge transformation with parameter $\Lambda_1 + \Lambda_2$ and the set of all gauge transformations will constitute an abelian group. Since $\{e^{i\Lambda}\}$ is a set of all unitary unimodular matrices of dimension unity, the relevant group is known as $U(1)$.[1] The charge operator $Q$ is said to be the generator of the group.

When the symmetry is realised not only globally, but also at each space-time point, – that is when the parameter is a space-time dependent $\Lambda(x)$ – we have what used to be referred to as the gauge invariance of the second kind or more appropriately a local gauge symmetry. Roughly this means that the charge is conserved at each space-time point. Naturally there is a need to propagate the information of this gauge transformation over space and time and this necessitates the introduction of the gauge field $A_\mu(x)$; the electromagnetic interaction is indeed the coupling of $A_\mu$ to the electric current density $j^\mu(x)$ of the matter. When quantised, $A_\mu$ represents the field for photon, the quantum content of the electromagnetic waves as well as the messenger of the electromagnetic interaction.

Standard model enlarges the gauge symmetry to include both weak interactions and strong interactions. First let us take into account weak interactions and form Electro-weak gauge theory with the gauge group as $SU(2)_L \times U(1)_Y$. This has an $U(1)$ component, as

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[1] $U(n)$ denotes a group consisting of unitary unimodular matrices of dimension $n$ and $SU(n)$ implies further that the matrices are of unit determinant.
was encountered in the electrodynamics, but now related to another kind of charge; call it weak hypercharge $Y_W$. In addition, we have the $SU(2)$ component of the symmetry group that characterises yet another local symmetry, which we may refer as the weak isospin $I_W$.

There are four generators of the electroweak symmetry, three for $SU(2)_L$ and one for $Y_W$. The elements of the symmetry group are given as $\exp i \Lambda a I^a_W; a = 1, 2, 3$ and $\exp i \Lambda Y_W; R^a_W$ are like the angular momentum operator $J^i, i = 1, 2, 3$ and $Y_W$ is like the charge operator $Q_{em} ; \Lambda_a$ and $\Lambda$ are real space-time dependent parameters. The symmetry $SU(2)_L \times U(1)_Y$ is thus given by four generators $I^a_W$ and $Y_W$; further the states are labelled by the irreducible representations of $|I_W|^2, I^3_W$ and $Y_W$, very much like the symmetry under rotations imply states with definite $|J|^2$ and $J^3$ eigenvalues.

There is an important difference between this electroweak symmetry and the electromagnetic $U(1)_{em}$ symmetry. The electroweak symmetry is said to be a hidden symmetry, meaning that, while the dynamical Lagrangian is symmetric the solution to the equation of motion does not respect it entirely. We say that the symmetry is spontaneously broken down to $U(1)_{em}$ which is the residual symmetry that represents the electric charge conservation and gives rise to electrodynamics. To take an analogy, while roads admit traffic in either direction, we choose a convention, say in India, that the traffic moves along the left side of the road. Indeed it is equally possible to choose the opposite convention and have the traffic move along the right side, as in fact it does in several other countries. While the roads are symmetric, the ‘solution’ we pick makes us lose it spontaneously. In the electroweak case, the symmetry $SU(2)_L \times U(1)_Y$ is hidden in the sense that the gauge fields exist corresponding to each of the four generators, $I^1_W, I^2_W$ and $Y_W$, but only one of them (which corresponds to the residual conserved electric charge) survives as an unbroken symmetry.

The generator of this symmetry $Q_{em}$ is a particular combination of the generators of the electroweak symmetry:

$$Q_{em} = I^3_W + \frac{Y_W}{2}$$

Thus photon field ($A_\mu$) is a gauge boson, with a component from $SU(2)_L$ gauge field ($W^3_\mu$) and the $U(1)_Y$ field $B_\mu$.

$$A_\mu = \sin \theta_W W^3_\mu + \cos \theta_W B_\mu$$

where $\theta_W$ is called Weinberg angle, a parameter to be determined by experiment.

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2The weak isospin is different from the usual *isospin* which is believed to be the underlying symmetry responsible for the masses of proton and neutron to be nearly the same and also recognise them as two different *flavours* of Nucleon. Both weak isospin and (flavour) isospin have symmetry properties similar to the rotational symmetry that leads to the conservation of angular momentum.
The orthogonal combination $Z_\mu (= \cos \theta W^3 - \sin \theta B_\mu)$ as well as $W^\pm_\mu (= W^1_\mu \pm i W^2_\mu)$ will be different; for instance they acquire mass as a consequence of the symmetry breaking. They give rise to neutral ($Z$) and charged ($W^\pm$) vector bosons, responsible for mediating weak interactions. The relative weak strength of the weak processes is due to heavy mass of these bosons that occurs as the energy denominator in the relevant amplitudes. Standard Model predicts that such mesons will have masses in the region of 80 GeV and this was confirmed when CERN experiments were able to actually see them in a spectacular experiment in 1980. In the $Sp\bar{p}S$ collider, when protons and antiprotons were made to collide with a total energy of 540 GeV, it was enough to produce $W^+W^-$ pairs. Later at LEP (Large Electron Positron collider) it was possible to have $e^+$ and $e^-$ collide to produce $Z$ meson, when the energy reached 91 GeV. Thus we have the complete set of messengers of electroweak forces: Photon ($m_\gamma = 0$), Charged Vector Bosons ($m_W^\pm = 80$GeV) and Neutral Vector boson ($m_Z = 91$GeV). At energy scales large compared to 100 GeV, in the region when we may regard $W$ and $Z$ mass to be small compared to their momenta, the strength of electromagnetic and the ‘weak’ interactions will be of comparable order, thus resulting in the manifestation of the full electroweak symmetry.

The gauge bosons that are responsible for the electroweak forces couple to the electromagnetic, charge-changing and neutral weak currents, the coupling parameter (which is actually mildly distance dependent) of $W^a_\mu$ and $B_\mu$ may be denoted as $g$ and $g'$. The electromagnetic coupling $\alpha \equiv (e^2/4\pi) = 1/137$ is related to them through

$$e = g \sin \theta_W = g' \cos \theta_W$$

(3)

From a variety of experimental inputs one finds that $\sin^2 \theta_W \simeq 0.23$.

Now let us turn to the matter content of the universe. Basic building blocks of matter come in two types of spin $\frac{1}{2}$ fermions – leptons and quarks. The most familiar leptons (which in Greek, means light particle) are electrons ($e^-$) and the associated neutrino $\nu_e$ that is emitted with an electron in a $\beta$-decay (say, when $n \rightarrow p + e^- + \bar{\nu}_e$). For a spin half state there are quantum states with spin up and down (with respect to some definite axis, say z-axis) or equivalently left and right handed helicity states. In discussing electroweak theory, it is convenient to group left helicity states and right helicity states seperately, since experiments show that the right helicity fermionic states do not take part in the weak interactions. The weak isospin ($I_W$) symmetry involves only left helicity fermions and hence is the subscript L in referring to the weak isospin $SU(2)_L$ symmetry. While the right helicity fermions such as $e^R_\mu$ is a $SU(2)_L$ singlet, left helicity fermions occur as $SU(2)_L$ doublets. The neutrino $\nu_e$, which has left helicity and the left handed $e^-_L$, form a electroweak $SU(2)_L$ doublet, very much like the spin up and down states of angular momentum representaion. This means that $I^3_W$ quantum numbers for $\nu_e, e^L$ and $e^R$ are
respectively $+\frac{1}{2}$, $-\frac{1}{2}$ and 0. Since $Q_{em} = I_{W}^{3} + \frac{Y_{W}}{2}$, we find that the doublet $\begin{pmatrix} \nu_{e} \\ e_{L}^{-} \end{pmatrix}$ has weak hypercharge $Y_{W} = -1$ and $e_{R}^{-}$ has $Y_{W} = -2$. That the left and right handed fermions have different electroweak group properties is at the root of the parity non-conservation of the weak interaction. In particular there is no right handed neutrino at all. Nevertheless, notice that both $e_{L}^{-}$ and $e_{R}^{-}$ have the same value for $Q_{em}$, which fact ensures that the electromagnetic part has parity invariant coupling.

Now turning to the strongly interacting states (collectively referred as hadrons) such as protons and neutrons, the sub constituents are $u$ and $d$ quarks. In a naive quark model proton is made up of two $u$-quarks and one $d$-quark and neutron has two $d$-quarks and one $u$-quark. Since nucleons have an attribute of baryon number, quarks must have nucleon or baryon number of $\frac{1}{3}$ unit; further $u$ has $+\frac{2}{3}$ units of electric charge and $d$ has $-\frac{1}{3}$ units. As far as the electroweak group properties are concerned, like leptons, left handed quarks are $SU(2)_{L}$ doublets and the right handed ones are neutral under $SU(2)_{L}$ group, again reflecting the feature that only left handed fermions have a role in weak interaction. We may read off their $Y_{W}$ quantum number from Eqn (2) and fill up the table 1.

| State               | $I_{W}^{3}$ of $SU(2)_{L}$ | $Y_{W}$ |
|---------------------|----------------------------|---------|
| $(u_{L}, d_{L})$    | $(\frac{1}{2}, -\frac{1}{2})$ | $\frac{1}{3}$ |
| $u_{R}$             | 0                          | $-\frac{2}{3}$ |
| $d_{R}$             | 0                          | $-\frac{1}{3}$ |
| $(\nu_{e}, e_{L}^{-})$ | $(\frac{1}{2}, -\frac{1}{2})$ | $-1$   |
| $e_{R}^{-}$         | 0                          | $-2$    |

Table 1: Quantum numbers of basic fermionic states

While protons and neutrons are three quark states, the mesons, such as pions ($\pi^{\pm}, \pi^{0}$ of mass 139 and 135 MeV respectively) are quark-antiquark bound states. Indeed the plethora of resonant states that were observed in the high energy collisions in the 50’s and 60’s can all be classified as either $(qqq)$ baryons or $(q\bar{q})$ mesons. Before proceeding further, we need to address an issue that arises since quarks are fermions and hence should satisfy Pauli’s exclusion principle. Most prominent excited state of nucleon is a spin $(\frac{3}{2})$ and isospin $(\frac{3}{2})$ quartet $(\Delta^{++}, \Delta^{+}, \Delta^{0}, \Delta^{-})$ with mass $M_{\Delta} = 1238$ MeV and width $\Gamma_{\Delta} = 120$ MeV. This is, like nucleon, a three quark s-wave bound state. In particular $\Delta^{++}$ state with $m_{s} = \frac{3}{2}$ will be represented by a symmetric wave function of three $u$-quarks, with all the spins aligned along the same direction. Such a state cannot be there, forbidden by Pauli’s exclusion
principle, except if the quarks possess yet another quantum number, in the internal space of which the wave function of the $\Delta$- state should be antisymmetric. This space is referred to as the color space and let the color index of quark $q_i$ take values 1, 2, and 3. In order that $\Delta^{++}$ wave function is antisymmetric under the interchange of all quantum numbers of constituent quarks, it is necessary that it is of the form $\epsilon_{ijk}u_iu_ju_k$ in the color space.

This color symmetry can be promoted as a local gauge symmetry, with the gauge group as $SU(3)$ and it is remarkable that the forces related to this symmetry are the basic dynamics underlying all strong interactions – a further example of symmetry being the essence of dynamics. Quarks are color triplets and strangely they are never observed in its free form, and all known strongly interacting states like nucleons and mesons are color singlet bound states of quarks and antiquarks. That is, the physical states that are asymptotically realised are either states that do not have any color substructure, such as electrons and neutrinos or hadrons (eventhough the strong interaction properties of them are due to color forces of the components) that are color neutral states of the type $\sum_{ijk}^{}\epsilon_{ijk}q_iq_jq_k$ and $\sum_{i}^{}q_iq_i^\dagger$. The strong interactions between nucleons and mesons are the color analogues of van der Waals forces that were seen responsible for the residual electromagnetic effects between charge neutral atoms and molecules. The role of photon is played in the color interaction by an octet of colored massless vector bosons that are known picturesquely as gluons. Being colored states gluons are also not observable as asymptotic physical states.

| Vector Meson | Associated generators |
|--------------|-----------------------|
| photon($\gamma$) | $Q_{em}$ |
| $W^\pm$ | $I_W^{1\pm 2}$ of $SU(2)_L$ |
| $Z$ | $I_3^W, Y_W$ |
| gluons | $T^a a = 1, \ldots 8$ of $SU(3)_C$ |

Table 2: Gauge Bosons

In summary, the Standard Model is a gauge field theory with the local symmetry of $U(1)_Y \times SU(2)_L \times SU(3)_C$ spontaneously broken down to $U(1)_{em} \times SU(3)_C$. The twelve (1+3+8) generators of the symmetry group are related to the 12 vector bosons (photon ($\gamma$), $W^\pm$, $Z$, octet of gluons) that mediate various interactions. Of these, photon and gluons are massless and represent the unbroken symmetries of Quantum Electro Dynamics (QED) and Quantum Chromo Dynamics (QCD). The matter consists of a set of leptons ($e^-, \nu_e$), color triplet quarks ($u_i, d_i$) and the corresponding antiparticles. Color confinement

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3We deliberately use the American spelling to denote that this has nothing to do with the usual colour that stands for the variety of hues.
is believed to be an important, still to be properly understood property of the \( SU(3) \) gauge theory.

**MORE A-TOMS?**

In the preceeding section, we have described the vector bosons as responsible for all interactions and the main matter ingredients to be one set of quarks and leptons. As we probe shorter distances, through high energy collisions, we find that there appears to be two more copies of the set of fermions and they are referred to as the second and third generations of matter. First member of the second set to be discovered (during 50’s in cosmic ray studies) was the muon \( (\mu) \), which is just a fat electron, about 200 times heavier and this was seen accompanied by a nearly massless neutrino, call it \( \nu_\mu \). Like \( u \) and \( d \) quarks of the electron family, in the second family, we have a pair of quarks, which are denoted as \( c \) and \( s \), and they carry Charm and Strangeness quantum numbers respectively. Since the quarks are not realisable as asymptotic states one can not talk about their mass; nevertheless we may associate with \( s \)-quark an effective mass of about 160 MeV and \( c \)-quark about 1500 MeV, gleaned from the analysis of deep inelastic scattering probes or mass content of the states carrying strangeness and charm. Apart from the mass parameters and the flavour quantum numbers \( S, C \) etc, all fermions of the 2nd generation have identical structure and properties with respect to the symmetries of the Standard Model. In particular the left handed fermions are \( SU(2)_L \) doublets and the right handed fermions are singlets. Muon, that has a life time of about \( 10^{-6} \)s, undergoes decay to electron and neutrinos; \( \mu^- \to \nu_\mu + e^- + \bar{\nu}_e \), similar to the interactions \( d \to u + e^- + \bar{\nu}_e \), which happens during the \( \beta^- \) decay of free neutron \( (n \to p + e^- + \bar{\nu}_e) \). Strangeness carrying hadrons arise when we have a strange quark in the state. For example, some of the strangeness carrying hadrons are \( \Lambda(1115\text{MeV}) \), which is a \((uds)\) bound state and \( K^{+,0} \) (495 MeV), which are \((u\bar{s})\) and \((d\bar{s})\) mesons. From the fact that a non-strange vector meson \( \phi(1020\text{MeV}) \) decays almost entirely into the channels \( K^+K^- \) and \( K^0\bar{K}^0 \), we conclude that it is mostly a \((s\bar{s})\) state. In 1974, the new flavour charm was discovered, when the charmonium state \((cc)\) was observed as a very narrow resonance at about 3097 MeV in the \((e^+e^-)\) collision. Subsequently charmed baryon \( \Lambda_c(2281)\) MeV as a \((udc)\) bound state, \( D^{\pm,0} \) (1869 MeV) mesons of \((c\bar{d})\) and \((c\bar{u})\) states etc. were discovered.

The Third set of fermions starts with the \( \tau^- \) lepton at 1760 MeV and the related neutrino \( \nu_\tau \) in the lepton sector. In the quark part, in the late seventies, a new flavour B was found as a consequence of yet another quark, call it \( b \)-quark. We may call it ‘beauty’ or ‘bottom’ (to denote that it is a quark with charge \(-\frac{1}{3}\) and occurs as the bottom part of the \( SU(2)_L \) doublet). Since Beautonium is a \((bb)\) state with 9650 MeV, it is natural that ‘beautiful’ states will have masses in the range of 5 GeV. \( B^{+,0} \) mesons (5275 GeV) are \((u\bar{b})\) and \((d\bar{b})\)

\(^4\text{The flavour of the state is given by the type of the constituent quarks. The } u \text{ and } d \text{ quark indicate the flavour isospin doublet. Strangeness } S \text{ and Charm } C \text{ flavours are carried by } s \text{ and } d \text{ quarks.}\)
states, while $\Lambda_b (4750 \text{ MeV})$ is a $(udb)$ bound state.

The third set needs another quark to complete the picture. Such a quark will occupy the top part of the $SU(2)_L$ doublet, of which the $b$-quark is the bottom. The quest for the top quark $t$ (which is also referred to as truth, the related flavour) had been elusive, mainly because the top quark turns out to be much heavier than was expected. Whereas $u, d, s$ are relatively light quarks $c$ and $b$ were found to be 1.5 and 5 times as heavy as nucleon (whose mass is about 1GeV). It was not possible to sight $t$-quark until we increased the collision energies considerably; the Fermilab results last year suggest that the $t$-quark is about 175 GeV. With the discovery of $t$-quark the third set of fundamental fermionic matter appears complete. Notwithstanding the much heavier masses, the third set of leptons $\nu_\tau$ and $\tau$ and the third set of quarks $(t, b)$ have identical structure and couplings with respect to the Standard Model interactions as the first and second generations of fermions.

| BOSONS          | FERMIONS          |
|-----------------|-------------------|
| **Charge**      | **Charge**        |
| ±1 $W^\pm$      | 0 $\nu_e$        |
| $(80.2 \text{GeV})$ | $(0)$ $\nu_\mu$ |
| 0 $Z$           | −1 $\nu_\tau$   |
| $(91.2 \text{GeV})$ | $(0)$ $e$       |
| 0 Photon        | 2/3 $u$          |
| $(0)$           | $(5 \text{MeV})$ |
| 0 Gluons        | −1/3 $c$         |
| $(0)$           | $(1.5 \text{MeV})$ |
|                  | 2/3 $d$          |
|                  | $(8 \text{MeV})$ |
|                  | −1/3 $s$         |
|                  | $(160 \text{MeV})$ |
|                  | 2/3 $b$          |
|                  | $(4.250 \text{GeV})$ |

Table 3: The a-toms of the Standard Model

In summary, we can now assert that, the a-toms of Democritus are the three sets of fermions; each set consists of a left helicity lepton doublet \( \left( \frac{\nu_E}{E_L} \right) \), right helicity lepton \( E_R^- \) and a color triplet of left handed quark doublet \( \left( \frac{U_L}{D_L} \right) \) and right handed singlets \( U_R \) and \( D_R \) with $E, U$ and $D$ as the generic labels. All physical states are made up of these fundamental sets of fermions and their dynamics is given by the Lagrangian that displays the gauge dogma. The forces of interactions are due to the messenger vectors bosons ($\gamma, W^\pm, Z$ and gluons) which are also a-toms.
EPILOGUE

Can there be more than three families of matter? Why are there three generations? Can we understand the reason why top quark is much heavier than the rest. How many parameters do we need to specify in describing the Standard Model? Why should the gauge group be $U(1) \times SU(2) \times SU(3)$? Is there at least a partial answer to all these queries?

Strictly speaking in our description of the Standard Model, we are yet to introduce mass parameters of the constituents. The gauge fields, like photons, to begin with are massless. The symmetry does not admit a mass term for the chiral fermions in the Lagrangian. Indeed, if the gauge symmetry is exact, there is no scope for any of the masses. Then how are the masses generated. Recall that we mentioned that the $U(1) \times SU(2) \times SU(3)$ is spontaneously broken down to $U(1)_{em} \times SU(3)_{c}$. The mechanism that achieves this should be the key to understanding the various mass values. While the full Lagrangian is manifestly $U(1) \times SU(2) \times SU(3)$ symmetric, the solutions (that includes the ground state vacuum) has less symmetry. For this purpose, we need one more species of particles, which will be responsible for the symmetry breaking and has an added role to make many particles massive. The agent for this is a $SU(2)_L$ doublet scalar field. The Higgs doublet $\Phi = \frac{1}{\sqrt{2}} \left( \begin{array}{c} \phi_1 + i\phi_2 \\ \phi_3 + i\phi_4 \end{array} \right)$ has $Y_W = 1$. The Higgs self interaction is such that it has a nonvanishing vacuum expectation value $<\Phi> = \left( \begin{array}{c} 0 \\ v/\sqrt{2} \end{array} \right)$, where $v$ is an important real parameter, the non zero value of which indicates the spontaneous breaking of the symmetry. To understand this, let us consider as example, a single complex field $\phi$ and let the interaction be given by $V(\phi)$. Let $V(\phi) = \frac{1}{2}m^2\phi^{*}\phi + \frac{\lambda}{4}(\phi^{*}\phi)^2$. If, as usual, $m^2 > 0$ and $\lambda > 0$, this represents scalar meson of mass $m$ and quartic interaction of strength $\lambda$. The ground state will have $\phi = 0$ and is unique. If, on the other hand $m^2 < 0$, then $\phi = 0$ is no longer a minimum of $V(\phi)$. The minima, on the contrary are given by $|\phi| = (-m^2/\lambda)^{1/2} = v$. The ground state solutions for $\phi$ could assume values $ve^{i\theta}$; $\theta$ arbitrary. Hence the ground state is degenerate. While the full symmetry is reflected in the complete set of the solutions, as soon as we pick a particular solution, say $\phi = v$, then the symmetry, in this case $U(1)$, is spontaneously broken. In the Standard Model, similar process occurs and we have the solution for $\Phi$ reflect the symmetry breaking from $SU(2)_L \times U(1)_Y$ to $U(1)_{em}$. From $\mu$-decay rates, we determine that $v \simeq 250$ GeV. It is possible to recast so that, three of the four real scalar fields of $\Phi$ play the role of the longitudinal polarisation modes of $W^+, W^-$ and $Z$ bosons and in this way make these gauge bosons massive. (Recall photon has only two transverse polarisation modes while the massive vector mesons should have longitudinal polarisation as well.) The last remaining field of $\Phi$ should show up as a massive scalar meson, called Higgs, that remains to be

\footnote{The term chiral fermion implies that the left and right helicity states of the fermion have different properties.}
discovered. Higgs mechanism also lets the fermions acquire mass through the coupling of the Higgs doublet to fermions $\hat{\psi}_L \Phi \psi_R$. Closely related to the fermion masses is the fermion mixing angles, three in number and the possibility of intrinsic CP violating phase angle in the mass matrix.

Standard Model thus consists of a gauge symmetry with 12 gauge vector bosons, of which all but three ($W^\pm, Z$) are massless, three sets of quarks and leptons and one neutral Higgs Scalar meson, that is yet to be discovered. It is specified by three gauge coupling parameters ($\alpha_{em}, \alpha_s$ and $\sin^2 \theta_W$) and Higgs meson vacuum expectation value that characterises the masses of $W^\pm$ and $Z$, Higgs meson coupling to fermions that could be parametrised in terms of the masses of the 3 charged leptons and 6 quarks, 3 mixing angles, one CP violating phase and one yet to determined Higgs meson mass. Thus we count in all merely 18 parameters that are needed to define the Standard Model.

Perhaps there are no more than three generations. $Z$ boson width, which is a measure of the sum total of probabilities of $Z$ decays into various modes, is known so well that it can accommodate no more than three species of neutrinos. Also there will be problems with cosmology if there are more than four types of neutrinos. If there were only two families, there is no room for the CP violating phase - whose presence was indicated in the 1965 experiment that found that there is a tiny amount of CP violation in the neutral $K$- meson decays. Further the fact that the universe consists of only matter and no antimatter can be triggered only if we may have the possibility of an intrinsic CP violation. Are these then, the reasons why we have three generations of matter?

We may have to go beyond the Standard Model to get any clues about why this specific group is chosen or why top quark, is so heavy. Is there a hint in the fact that the top quark, has mass in the same region as weak interaction symmetry breaking scale? Then, could it be that Higgs scalar (which is the agent for breaking symmetry), is not an a-tom, but a ($t\bar{t}$) bound state? We have to wait for about a decade, when the Large Hadron Collider will be completed at CERN, and hopefully produce Higgs mesons copiously. Until then, we have just about three sets of fermions, and the twelve vector bosons of the gauge interaction as the a-toms that make up all of the universe and let the future decide whether the Higgs Scalar boson is elementary or composite.

For further reading:

1. Dreams of the final theory - Steven Weinberg (Pantheon Books, New York 1992). A popular account at the Standard Model and journey towards a final theory.

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\(^6\)C and P here denote two discrete symmetries - Charge conjugation that interchanges particles and anti-particles and Parity that reflects the spatial co-ordinates. They were believed to be good symmetries until they were both seen violated in the weak interaction, leaving however the combination CP intact, except in very few processes involving the neutral $K$ mesons. The phase angle is a measure of the CP violation.
2. The God Particle - Leon B. Lederman (Dell Paperback 1995)  
The Higgs Scalar meson is referred to by Lederman as the God Particle and the popular book is on the history of developments in Particle Physics and the expectation for the discovery of Higgs in the first decade of the 21st century.

3. Building up the Standard Gauge Model of the High Energy Physics - G. Rajasekaran in Gravity, Gauge theories and the Early Universe (UGC Instructional Conference) Ed. B.R.Iyer et al (Kluwer Academic Publishers 1989) p.185-236.  
Lectures on the Standard Model and beyond.

4. High Energy Physics - Special Section, Current Science 71 (1996) p.109-127.  
Proceedings of a symposium on High Energy Physics in 21st Century held as a part of the 61st Annual Meeting of the Indian Academy of Science during 10-12 November 1995 at Madras. Articles are by G. Rajasekaran, D. P. Roy, Romesh K. Kaul, Abhijit Sen and R. Ramachandran.