Contrast Measures based on the Complex Correlation Coefficient for PolSAR Imagery

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Abstract—We derive contrast measures which involve the number of looks and the complex correlation coefficient between polarization channels in PolSAR imagery. Using asymptotic results which characterize the behavior of these measures, we derive statistical regions of confidence which lead to test of hypothesis. An application to real data is performed, confirming the importance of the proposals.

I. INTRODUCTION

The aim of remote sensing is to capture and to analyze information scenes concerning the Earth surface. In this context, polarimetric synthetic aperture radar (PolSAR) has achieved a prominent position among the remote sensing technologies [1]. Such systems employ coherent illumination in its processing and, as a consequence, their resulting images are contaminated with fluctuations on its detected intensity called “speckle”. Speckle significantly degrades the perceived image quality, as much as the ability of extracting information from the data.

Speckle is well described by statistical models. Thus, two pre-processing steps are often sought: (i) the identification of a probability distribution for PolSAR image regions [2], and (ii) the derivation of statistical methods for quantifying contrast between such regions [3].

A successful statistical model for homogeneous regions in PolSAR images is the scaled complex Wishart law [2]. This distribution is equipped with two parameters: the number of looks and the complex covariance matrix. A rich discussion about estimation and interpretation of the number of looks was given by Anfinsen et al. [4]. In terms of the covariance matrix, it contains all necessary information to characterize the backscattered data. Conradsen et al. [5] discuss hypothesis tests based on the covariance matrix.

Lee et al. [6] proposed a reparametrization for the covariance matrix in terms of the complex correlation coefficient. Statistical models which describe the multilook phase difference, the magnitude of the complex product, and both intensity and amplitude ratios between two components of the scattering matrix are provided in that work. The resulting density functions have closed forms which depend on the complex correlation coefficient and on the number of looks.

Many authors have utilized the complex correlation coefficient as an important quantity for analyzing PolSAR images. For instance, Ainsworth et al. [7] presented evidence that the complex correlation coefficient between channels can be utilized to identify man-made targets.

In recent years, information-theoretic based measures have used to derive new PolSAR image processing methods. Literature in this field of research can be divided in two groups: (i) works that involve deterministic tools [8], and (ii) contributions that consider statistical properties of these measures [3], [9]. In this work, we advance the statistical inference based on the complex correlation coefficient in for PolSAR data — a contribution within the second group of works.

In summary, our contributions are two-fold:

1) Based on the parametrization proposed by Lee et al. [6], we derive four contrast measures which depend on the complex correlation coefficient. These measures were obtained considering four distances from the $h$-$\phi$ class of distances proposed by Salicrú et al. [10]: the Kullback-Leibler, Rényi (of order $\beta$), Bhattacharyya, and Hellinger distances between reparametrized scaled complex Wishart distributions.

2) We study the asymptotic properties of these measures, and we propose new confidence regions for these distances which allow comparing two PolSAR regions.

II. THE MODEL

If the complex return with $p$ polarization channels follows a complex Gaussian law [2], the multilook covariance matrix
return $Z$ follows a scaled complex Wishart distribution characterized by the following probability density function:

$$f_Z(Z; \Sigma, L) = \frac{L^p |Z|^{L-p} \exp(-L \text{tr}(\Sigma^{-1}Z))}{\Gamma_p(L)} \exp(-L \text{tr}((\Sigma^{-1}Z))),$$

where $\Gamma_p(L) = \pi^{p(p-1)/2} \prod_{l=1}^p \Gamma(1 - L - iL)$. $L$ is the number of looks, $L$, and the covariance matrix $\Sigma$. This situation is denoted $Z \sim W(\Sigma, L)$, and this distribution satisfies $E[Z] = \Sigma$, which is a Hermitian positive definite matrix [4].

Lee et al. [5] presented a reparametrization of a particular case of the complex Wishart distribution based on two-element scattering vector $y^{(k)} = [y_1^{(k)}, y_2^{(k)}]^\top$ at the $k$th look, for which the covariance matrix is written as

$$Z = \left[ \begin{array}{cc} z_{11} & \alpha e^{i\Delta} \\
\alpha e^{-i\Delta} & z_{22} \end{array} \right],$$

where $(\cdot)^\top$ is the transposition operator, $\alpha$ and $\Delta$ are the sample multilook magnitude and phase, respectively, and $z_{ii} = L^{-1} \sum_{k=1}^L y_i^{(k)} y_i^{(k)*}$, for $i = 1, 2$. The resulting covariance matrix is

$$\Sigma = \left[ \begin{array}{cc} \sigma_{11} & \sqrt{\sigma_{11}\sigma_{22}}|\rho_c|e^{i\delta} \\
\sqrt{\sigma_{11}\sigma_{22}}|\rho_c|e^{-i\delta} & \sigma_{22} \end{array} \right],$$

where $\sigma_{ii} = E(z_{ii})$, $\delta$ is the population multilook phase, and $\rho_c$ is the complex correlation coefficient between $z_{11}$ and $z_{22}$.

In order to study the preservation of polarimetric properties along the process of filtering PolSAR imagery, Lee et al. [11] studied the correlation coefficient between polarization channels. It is given by

$$\rho_c = \frac{E(S_{HH}S_{VV})}{\sqrt{E(S_{HH}^2)E(S_{VV}^2)}}.$$  (3)

The normalized quantities

$$B_1 = \frac{z_{11}}{\sigma_{11}}, \quad B_2 = \frac{z_{22}}{\sigma_{22}}, \quad \eta = \frac{\alpha}{\sqrt{\sigma_{11}\sigma_{22}}},$$

and $\Delta$ obey the distribution characterized by the following joint probability density function:

$$f(B_1, B_2, \eta; \Delta; \rho_c, L) = \frac{\eta(B_1 B_2 - \eta^2)^{L-2}}{\pi(1 - |\rho_c|^2)\Gamma(L\Gamma(L - 1))} \times \exp\left\{-\frac{B_1 + B_2 - 2\eta|\rho_c|\cos(\Delta - \delta)}{1 - |\rho_c|^2} \right\}.$$  (4)

The complex correlation coefficient has been utilized as an important quantity for identifying contrast in PolSAR images. For instance, Lee et al. [11] presented results which provide evidences that changes of correlation coefficient between polarization channels can be captured when one considers pixels of different regions.

### III. Stochastic Distances Based on the Complex Correlation Coefficient

We adhere to the convention that a (stochastic) “divergence” is any non-negative function between two probability measures which obeys the identity of definiteness property [3]. If the function is also symmetric, it is called “distance” [12, ch. 1 and 14].

An image can be understood as a set of regions, in which the enclosed pixels are observations of random variables following a certain distribution. Therefore, stochastic dissimilarity measures can be used to assess the difference between the distributions that describe different image areas [13].

Dissimilarity measures were submitted to a systematic and comprehensive treatment in [10], leading to the proposal of the class of $(h, \phi)$-divergences. Stochastic distances applied to intensity SAR data were presented in [13], [14] and to PolSAR models in [3].

We consider here four stochastic distances between the models characterized by the densities $f_1$ and $f_2$ with the same support $A$:

i) Kullback-Leibler: $d_{KL} = \frac{1}{2} \int_A f_1 - f_2 \log \frac{f_1}{f_2}$.

ii) Rényi of order $\beta \in (0, 1)$: $d^\beta_R = (\beta - 1)^{-1}\log(\int_A f_1^{\beta} f_2^{1-\beta} + f_2^{\beta} f_1^{1-\beta})/2$.

iii) Bhattacharyya: $d_B = -\log \int_A \sqrt{f_1 f_2}$.

iv) Hellinger: $d_H = 1 - \int_A \sqrt{f_1 f_2}$.

In practical applications, the densities $f_1$ and $f_2$ are not known. Their parameters are usually estimated with samples of sizes $N_1$ and $N_2$, yielding $f_1 = f_1(\nu_1(N_1))$ and $f_2 = f_2(\nu_2(N_2))$. Whenever there is no risk of ambiguity, i.e., when the distribution is the same, only the estimators can be used to denote the distances.

These distances become more useful and comparable scaling them into test statistics, as discussed in [13]:

$$S_D(\hat{\theta}_1(N_1), \hat{\theta}_2(N_2)) = \frac{2N_1 N_2 v_D}{N_1 + N_2} d_D(\hat{\theta}_1(N_1), \hat{\theta}_2(N_2)),$$

where $v_D = 1, \beta^{-1}, 4$, and 4 for $D = KL$, R, BA, and H, respectively, and $\theta_1(N_1) = \{\Sigma_1(N_1), \tilde{L}(N_1)\}$ and $\theta_2(N_2) = \{\Sigma_2(N_2), \tilde{L}(N_2)\}$ are the maximum likelihood estimators for $\theta_1$ and $\theta_2$ using different random samples of sizes $N_1$ and $N_2$, respectively. Under mild conditions, $S_D(\theta_1(N_1), \theta_2(N_2))$ is asymptotically distributed as a $\chi^2_M$ random variable, where $M$ is the dimension of the parameter $\theta$.

### IV. Results

This section presents contrast measures based on the number of looks and the complex correlation coefficient, and provides probabilistic criteria for discriminating two PolSAR regions in terms of their sample correlation coefficients between polarization channels.

Since this seems untractable for random vectors equipped with densities given by Eq. (4), we derive the four distances discussed in the previous section between the random matrices.
and

\[ R_{KL}(\rho_1, \rho_2 \mid L) = \left\{ (\rho_1, \rho_2) \in \mathbb{C} \times \mathbb{C} : \frac{\sqrt{(1 - |\rho_1|^2)(1 - |\rho_2|^2)}}{4 - (|\rho_1| + |\rho_2|)^2} \leq \frac{(N_1 + N_2)}{2L N_1 N_2} \chi_1^2(\eta) \right\} \]

(8)

where \( \eta \) is the specified nominal level.

Given two polarimetric data samples, their correlation coefficients can be estimated according to Eq. (3). Quantities \( \hat{\xi}_1 \), \( \hat{\xi}_2 \), \( t_{KL} \), and \( t_{H} \) are evaluated by means of (7) and (8), replacing \( \rho_1 \) and \( \rho_2 \) by their estimates (or sample counterparts) \( \hat{\rho}_1 \) and \( \hat{\rho}_2 \). We then propose the following decision rules:

- **Kullback-Leibler criterion:** If \( \hat{\xi}_1 \leq t_{KL} \), then we have statistical evidence that the samples come from populations with similar correlation between HH and VV channels.
- **Hellinger criterion:** If \( \hat{\xi}_2 \geq t_{H} \), the samples present equivalent correlation between channels.

Thus, regions \( R_{KL}(\rho_1, \rho_2 \mid L) \) and \( R_{H}(\rho_1, \rho_2 \mid L) \) along with (3) are methods for comparing two regions based on their estimates for the correlation coefficient.

In order to illustrate this methodology, consider the samples highlighted in Fig. 1 (which is extracted from an E-SAR image from surroundings of Weßling, Germany). Table I lists the quantities observed, along with the decisions they led to. Notice that both rules discriminate well.

| Regions | \( \hat{\xi}_1 \) | \( t_{KL} \) | Decision | \( \hat{\xi}_2 \) | \( t_{H} \) | Decision |
|---------|-----------------|----------------|-----------|----------------|----------------|-----------|
| D1-D2   | 2.0220 2.0016    | Distinct     | 0.2486    | 0.2499         | Distinct     |
| D1-D3   | 2.0068 2.0012    | Distinct     | 0.2495    | 0.2499         | Distinct     |
| D1-D4   | 2.0734 2.0014    | Distinct     | 0.2455    | 0.2499         | Distinct     |
| D2-D3   | 2.0534 2.0018    | Distinct     | 0.2467    | 0.2499         | Distinct     |
| D2-D4   | 2.0149 2.0020    | Distinct     | 0.2490    | 0.2498         | Distinct     |
| D3-D4   | 2.1254 2.0016    | Distinct     | 0.2424    | 0.2499         | Distinct     |

Fig. 1. PolSAR image (HH channel) with four samples.
correlation coefficient in Z. Fig. 2 illustrates the resulting statistics for $|\rho_1|^2 = 0.5$ and $L_1 = L_2 = L \in \{2, 5\}$ (Figures 2(a) and 2(b) respectively), while $|\rho_2|$ varies on $[0, 1]$. As expected, the curves have their minimum value, zero, at $\rho_1 = \rho_2$. Notice, however, that the curves are steeper on the interval $(|\rho_1|, 1)$ than on $(|\rho_1|, 0)$. Thus for discrimination purposes, the statistics provide better sensitivity capabilities when $|\rho_2| > |\rho_1|$. Additionally, the increasing of the number of looks tends to correct this behaviour.

$$d_{KL}(\rho_1, \rho_2 \mid L) = L \left[ \frac{(1 - |\rho_1|^2)(2 - |\rho_1|^2 - |\rho_2|^2)^2}{(1 - |\rho_1|^2)(1 - |\rho_2|^2)} - 2 \right]$$

(9)

$$d_R^\beta(\rho_1, \rho_2 \mid L) = \frac{\log 2}{1 - \beta} + \frac{1}{\beta - 1} \log \left\{ \left( \frac{1 - |\rho_1|^2}{1 - \{1 - |\rho_1|^2\}(1 - |\rho_2|^2)} \right)^L \right\} + \left( \frac{1 - |\rho_2|^2}{1 - \{1 - |\rho_2|^2\}(1 - |\rho_1|^2)} \right)^L$$

(10)

$$d_H(\rho_1, \rho_2 \mid L) = L \left\{ \frac{\log(1 - |\rho_1|^2) + \log(1 - |\rho_2|^2)}{2} - 2 \log 2 + \log \left( \frac{4 - (|\rho_1| + |\rho_2|)^2}{(1 - |\rho_1|^2)(1 - |\rho_2|^2)} \right) \right\}$$

(11)

$$d_B(\rho_1, \rho_2 \mid L) = \left[ \frac{4(1 - |\rho_1|^2)(1 - |\rho_2|^2)}{4 - (|\rho_1| + |\rho_2|)^2} \right]^L$$

(12)

V. CONCLUSION

In this paper, we have derived four contrast measures in terms of the correlation coefficient and of the number of looks. Using asymptotic results for these measures, two methodologies based on the Kullback-Leibler and Hellinger distances were proposed as new discrimination techniques between PolSAR image regions. These methods were applied to actual data and the obtained results present evidence in favor of both the proposals.

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