A bias free true random number generator

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We propose a new approach to nondeterministic random number generation. The randomness originated from the uncorrelated nature of consecutive laser pulses with Poissonian photon statistics and that of photon number detections is used to generate random bit, and the von Neumann correction method is used to extract the final random bit. This method is proved to be bias free in randomness generation provided that the single photon detections are mutually independent. Further, it has the advantage in fast random bit generation since no postprocessing is needed. A true random number generator based on this method is realized and its randomness is tested and guaranteed using three statistical test batteries. © 2009 Optical Society of America

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Random numbers are essential in an extremely wide application range, such as statistical sampling [1], computer simulations [2], randomized algorithm [3] and cryptography [4]. Pseudo random numbers generated with certain algorithms are widely used in modern digital electronic information systems. Though the pseudo random number generators (PRNG), thanks to the development of computer science and technology, have been highly refined in terms of generation rate and robustness against random test, the algorithm based PRNG can not generate true randomness and so has the essential drawback in the applications which require physical randomness, such as quantum cryptography, which requires unpredictable bit strings to ensure the inability of estimation [5]. True random numbers should be unpredictable and unrepeatable. For this reason, physical random phenomena, such as the decay of radioactive nucleus [6], thermal noise in resistors [7], frequency jitter of electronic oscillators [8] and photon emission noise [9, 10], etc., are used as physical sources for nondeterministic random number generations. Random number generator based on these physical random processes are termed as true random number generators (TRNG), and those based on photon emission noise extract random bit either from the random time intervals between photon emissions of semiconductors [9], or from the collapse of the photon wave function on random gating cycle [10]. In these schemes, the timing precision is a limitation to the random bit generation rate.

In this Letter, we propose a new approach to realize a TRNG based on photon number detection of weak laser pulse. The main advantage of this new type of TRNG is that it has equal probabilities for the bits of ones and zeros and is bias free from the variability of the device and the environment. Moreover, this new type of TRNG can be realized with a simple and compact setup. It can generate random numbers at a high speed and is only limited by the repetition rate of the single photon detector (SPD). These advantages make it applicable for quantum cryptography and other applications which require fast generation of true random bits.

According to quantum theory of laser, the photon statistics for laser operating above threshold approaches to Poissonian [11], indicating that laser light has the unity second order correlation function [11] and the photon numbers of different laser pulses are mutually independent. A TRNG can be implemented based on this fact. Since the photon number distribution of partially absorbed light follows a Bernoulli transform of the initial field [12], provided that the detections are mutually independent, it can be proved that the photon numbers detected by a photodetector from weak laser pulses are also Poissonian distributed, i.e., \( P_{\eta}(n) = (\eta \lambda)^n \exp(-\eta \lambda)/n! \), where \( \lambda \) is the mean photon number of the weak laser pulses, and \( \eta \) is the detection efficiency of the photodetector. The probabilities of \( n_1 > n_2 \) and \( n_1 < n_2 \) are equal, where \( n_1 \) and \( n_2 \) are the photon numbers of two consecutive pulses. In experiment, we use an avalanche photodiode (APD) operating in gated Geiger mode for the photon detection, which does not distinguish the photon numbers above zero. Thus, we code “\( n_1 > 0 \) and \( n_2 = 0 \)” as bit 1 and “\( n_1 = 0 \) and \( n_2 > 0 \)” as bit 0.

![Fig. 1. (color online) Schematic experiment set up of our TRNG. FA: flexible attenuator. 1: bit 1. 0: bit 0. N: not used.](image-url)
Then, for two consecutive detections, the probabilities $P(1)$ and $P(0)$ of generating a bit 1 and a bit 0, respectively, follow

$$ P(1) = P(0) = P_n(n > 0) \times P_\lambda(n = 0) = e^{-\eta \lambda} (1 - e^{-\eta \lambda}). $$

(1)

It is then evident that the probabilities of generating the bits of ones and zeros are **intrinsically equal** and hence, a TRNG based on this method is physically unbiased.

The experiment setup of our TRNG is shown in Fig. 1. Laser pulses (300ps@1550nm) are generated by a pulsed distributed feedback (DFB) diode laser (id300, id Quantique), and the mean photon number of each pulse can be continuously adjusted by a flexible attenuator (FA). Photon number is detected by an InGaAs APD working in gated Geiger mode. The gate width of 2.5ns is chosen in experiment to minimize the probability of dark count. We develop an FPGA-based circuit for system synchronization and random bit extraction. The true random bits are derived from the sequence of detection signals using von Neumann correction method [13]. Using this method, we scan the random sequence from left to right reading successive pairs of photon detections. The outputs of “$n_1 > 0$ and $n_2 = 0$” and “$n_1 = 0$ and $n_2 > 0$” are adopted as the bits 1 and 0, respectively, while those of “$n_1 > 0$ and $n_2 > 0$” and “$n_1 = 0$ and $n_2 = 0$” are abandoned, though they also contain randomness that can be extracted by other correction procedures (see, e.g., [14]). However, for doing this, some postprocessing is needed, which may greatly reduce the random bit generation rate. We, instead, adopt von Neumann correction method for our TRNG because it is simple and easy to implement with digital logic and no postprocessing is needed. The repetition frequency of the generation and detection of laser pulses is chosen as 1 MHz, and the dark count probability for our SPD is measured to be $3 \times 10^{-5}$ with a gate width of 2.5ns. The detection efficiency of SPD is $\sim 10\%$ (specification of id Quantique).

To ensure that our TRNG generates true random bits, we must confirm that every single photon detection of the laser pulses is mutually independent. In detection, the after-pulse effect of APD may cause some correlation between two consecutive detection results. For an APD operating in gated Geiger mode, carriers are trapped after every avalanche. If some of them emit during the next gate-on time, it can trigger a new avalanche (after-pulse) even though no photon is absorbed. For the detection with InGaAs APDs, after-pulse is frequently found when the repetition frequency exceeds 1 MHz. Since only traps with an emission lifetime comparable to or longer than the time interval between two consecutive gates generate after-pulses, we can introduce a dead time following an avalanche, during which, no gate is applied to the APD. This is an effective way to eliminate the after-pulse effect [15]. For the SPD of our experiment, a dead time of 8 µs is chosen, and our experiment results show that the probability of after-pulse is negligible and the non-signal counts come only from the dark counts.

A technically important issue for TRNG is its efficiency to extract true random bit from random events, $\eta_{\text{TRNG}}$, which is defined as the number of random bits per random event. For our TRNG, a random event is a photon number detection of a weak laser pulse. Thus, $\eta_{\text{TRNG}}$ is equal to half of the probability that a random bit is generated by a pair of detections, i.e.,

$$ \eta_{\text{TRNG}} = \frac{P(1) + P(0)}{2} = e^{-\eta \lambda} (1 - e^{-\eta \lambda}). $$

(2)

When $\eta \lambda = \ln 2 \approx 0.693$, we get the optimal $\eta_{\text{TRNG}}$ as 0.25. This is also the point that the probability of detecting zero or above zero photons are equal. The value of $\eta_{\text{TRNG}}$ varying with $\eta \lambda$ is illustrated in Fig. 2. It shows that when $\eta_{\text{TRNG}}$ is at its maximum, it is not sensitive to slight fluctuations of $\eta \lambda$ around 0.693. Experimentally, we adjust the flexible attenuator to the state where the probabilities of detecting zero and above zero photons are approximately equal. In this case, our TRNG generates intrinsically unbiased random bits at the optimal rate permitted by the current setup.

We use three batteries of statistical tests to evaluate the performance of our TRNG. They are ENT [16], DIEHARD [17] and STS [18]. We record a bit file of $9.45 \times 10^8$ bits for the tests. The ENT results of our TRNG are: Entropy = 1.000000 bits per bit and the optimum compression would reduce the bit file by 0 percent, $\chi^2$ distribution is 1.49, and randomly would exceed this value 22.17% of the times, arithmetic mean value of data bits is 0.5000 (0.5 = random), Monte Carlo value for $\pi$ is 3.141747714 (error 0.00 percent), serial correlation coefficient is 0.000041 (totally uncorrelated = 0.0). A DIEHARD test, with a data sample of 10 to 11 Mbytes, is considered succeed if 0.01 $\leq p$ $\leq$ 0.99 is satisfied [17]. For multiple $p$-values cases, we use a KS test to obtain a final $p$-value and the test result is based on the final $p$-
value. The results of DIEHARD test for our TRNG are illustrated in Fig. 3 and show that all final $p$-values are within 0.01~0.99. In STS test, the large experimental file is divided into 920 separate smaller streams of $10^6$ bits. An individual bit stream is usually considered to pass a particular test when $p \geq 0.01$ and consequently, $98\% \sim 100\%$ of all the bit streams are expected to pass a particular test due to statistical fluctuations [18]. Fig. 4 shows that all tests are passed with the final $p$-values (after KS test) above the significance level, and all testing results are within the confidence interval for the proportion of pass. The above three random tests show that our TRNG has good quality in randomness.

![Fig. 3. (color online) Results of DIEHARD. All testes are passed at lower (upper) significance level of 0.01 (0.99).](image)

![Fig. 4. (color online) Results of STS. All testes are passed at the significance level of 0.01.](image)

We propose a new approach to realize a TRNG that is intrinsically bias free. Its randomness is based on the photon number detection of weak laser pulses. Compared with other bias free TRNGs, e.g., those based on photon emission noise, the complicated timing circuits with high precision is not needed in our TRNG and thus is convenient to be implemented. Currently, the random bit generation rate of our TRNG is restricted only by the repetition rate of our InGaAs APD based SPD. Hence, a faster TRNG can be experimentally implemented with a faster, say, silicon APD based SPD and the random bit generation rate can be up to Gbit/s, which is suitable for high speed applications.

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