Decay Rates, Structure Functions and New Physics Effects in Hadronic Tau Decays

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Abstract

Hadronic decays rates of the τ lepton into multi meson final states are presented. The structure of the hadronic matrix elements for various decay modes is discussed. The formalism of structure functions allows for a detailed test of these matrix elements. Various correlations are discussed which are sensitive to possible CP violation and new physics effects in the decay modes.

I. INTRODUCTION

The τ lepton is heavy enough to decay into a variety of hadronic final states. In particular, final states with kaons provide a powerful probe of the strange sector of the weak charged current. The Tau-Charm Factory operating at an $e^+e^-$ cms energy of around 4 GeV and a luminosity of $L = 10^{33} \text{cm}^{-2}\text{s}^{-1}$ with good π/K separation [4] would allow for high precision measurements of the hadronic matrix elements in all decay modes. Rare decay modes could be searched for at the level of about $10^{-7}$ in branching fraction. Of particular interest would also be the search for possible CP violation in the hadronic matrix elements.

In the present paper, we specify the general structure of the matrix elements for τ decays into various multi meson final states. We study angular correlations in the exclusive decay modes and show that the formalism of structure functions allows for a detailed model independent test of the hadronic matrix elements. Furthermore, the structure functions allow for a systematic analysis of possible CP violation effects in the matrix elements, which would have to come from new non-Standard Model contributions.

*Invited talk presented by E. Mirkes at the Workshop on the Tau/Charm Factory, Argonne National Laboratory, June 21-23, 1995
It is shown that CP violation effects are in principle observable in a Tau-Charm Factory (without polarized beams) for three meson decay modes with a nonvanishing vector and an axial vector current. CP violation effects originating from a charged Higgs could be detected only for decay modes with a nonvanishing vector current.

An observation of CP violation in two meson decays requires either polarized beams [2] or kinematical information from the second tau decay [3].

II. MATRIX ELEMENTS AND DECAY RATES

The matrix element $M$ for the hadronic $\tau$ decay into $n$ mesons $h_1, \ldots h_n$

$$\tau(l, s) \to \nu(l', s') + h_1(q_1, m_1) + \ldots h_n(q_n, m_n),$$

(1)

can be expressed in terms of a leptonic ($M_\mu$) and a hadronic current ($J^\mu$) as

$$M = \frac{G}{\sqrt{2}} (\cos \theta_c) M_\mu J^\mu.$$

(2)

In Eq. (2), $G$ denotes the Fermi-coupling constant and $\theta_c$ is the Cabibbo angle. The leptonic current is given by

$$M_\mu = \bar{u}(l', s') \gamma_\mu (g_V - g_A \gamma_5) u(l, s),$$

(3)

with $g_V = g_A = 1$ in the Standard Model. The hadronic current $J^\mu$ can in general be expressed in terms of a vector and an axial vector current

$$J^\mu(q_1, \ldots, q_n) = \langle h_1(q_1) \ldots h_n(q_n) | V^\mu(0) - A^\mu(0) | 0 \rangle.$$  

(4)

The simplest decay mode into a pion or a kaon proceeds only through the axial vector current whereas all decays into an even number of pions are expected to proceed through the vector current. In fact, the decay rates for $\tau \to 2n\pi, KK$ can be related through the conserved vector current (CVC) hypothesis to $e^+ e^- \to$ hadrons in the isovector state [4]. On the other hand, three body decay modes involving kaons allow for axial and vector current contributions at the same time. In the following, we specify the hadronic matrix elements for hadronic decays into multi meson final states as expected from the Standard Model.

A. One Meson Decays

The decay rate for the simplest decay mode with one pion or kaon is well predicted by the the pion or kaon kaon decay constants $f_\pi$ and $f_K$ defined by the matrix element of the axial vector currents

$$\langle \pi(q) | A^\mu(0) | 0 \rangle = i\sqrt{2} f_\pi q^\mu,$$

$$\langle K(q) | A^\mu(0) | 0 \rangle = i\sqrt{2} f_K q^\mu.$$

(5)

(6)

Both decay constants can be determined using the precisely measured pion (kaon) decay widths $\Gamma(\pi(K) \to \mu\nu_\mu)$. Radiative corrections $\delta R_{\tau/\pi} = (0.16 \pm 0.14)\%$ and $\delta R_{\tau/K} = (0.90 \pm$
0.22)% to the ratios \( \Gamma(\tau \to \pi\nu)/\Gamma(\pi \to \mu\nu) \) and \( \Gamma(\tau \to K\nu)/\Gamma(K \to \mu\nu) \) have been calculated recently [3]. Using the recent world average \( \tau = (291.6 \pm 1.6) \text{ fs} \) for the tau lifetime [4] one obtains the following theoretical predictions for the branching ratios

\[
\mathcal{B}(\pi\nu\tau) = (10.95 \pm 0.06)\%
\]

\[
\mathcal{B}(K\nu\tau) = (0.723 \pm 0.006)\%.
\]

These predictions agree within one standard deviation with the world averages as quoted in [4].

**B. Two Meson Decays**

The hadronic matrix element for the decay \( \tau \to h_1 h_2 \nu \) can be written as \( (Q^\mu = (q_1 + q_2)^\mu) \)

\[
\langle h_1(q_1)h_2(q_2)|V^\mu(0)|0 \rangle = [(q_1 - q_2)_\nu T^{\mu\nu} F^{h_1 h_2} + Q^\mu F_4^{h_1 h_2}]
\]

\( T^{\mu\nu} \) is the transverse projector, defined by

\[
T_{\mu\nu} = g_{\mu\nu} - \frac{Q_\mu Q_\nu}{Q^2}.
\]

The form factor \( F_4 \) describes the two mesons \( h_1 \) and \( h_2 \) in an \( s \) wave. As mentioned before, the form factor \( F^{\pi\pi} \) in \( \tau^{-} \to \rho^{+}\nu \to \pi^{-}\pi^{0}\nu \) can be obtained (using the CVC theorem) from the iso-vector part of the electromagnetic current for \( e^+e^- \to \pi^+\pi^- \) and the scalar form factor \( F_4 \) is expected to vanish. One has

\[
F^{\pi^{-}\pi^{0}} = \sqrt{2} T^{(1)}_\rho,
\]

where \( T^{(1)}_\rho \) is a normalized vector resonance form factor (two particle Breit-Wigner propagator) for the \( \rho \) resonance including the contribution from the radial excitations \( \rho' \) and \( \rho'' \). In general, the normalization of the form factors \( F^{h_1 h_2} \) is fixed by chiral symmetry constraints, which determines the matrix elements in the limit of soft meson momenta. The strong interaction effects beyond the low energy limit are taken into account by vector resonance factors with the requirement \( T^{(1)}_X(Q^2 = 0) = 1 \) (\( X = \rho, K^* \)). The hadronic matrix elements for the Cabibbo suppressed decay modes \( K^-\pi^0\nu\tau, \overline{K^0}\pi^-\nu\tau \) are dominated by the \( K^* \) resonance \( T^{(1)}_{K^*}(Q^2) \) [5], whereas the one for the Cabibbo allowed mode \( K^0K^- \) is dominated by the high energy tail of the \( \rho \). One has [10].

\[
F^{\overline{K^0}\pi^-} = \frac{1}{\sqrt{2}} T^{(1)}_{K^*}(Q^2),
\]

\[
F^{K^-\pi^0} = T^{(1)}_{K^*}(Q^2),
\]

\[
F^{K^0K^-} = T^{(1)}_{\rho}(Q^2).
\]

In the \( \tau \to K\pi \) decay mode, \( F_4 \) gets a contribution from the off-shellness \( (m_{K^*}^2 - Q^2) \) of the \( K^* \). However, this scalar contribution is strongly suppressed compared to the contribution of \( F \). As we will see in the last section, the form factor \( F_4 \) allows also for a possible contribution from a charged Higgs exchange and is therefore of special interest.
We use the following form for the two particle Breit-Wigner propagators with an energy dependent width $\Gamma_X(s)$ throughout this paper:

$$\text{BW}_X(s) \equiv \frac{M_X^2}{[M_X^2 - s - i\sqrt{s}\Gamma_X(s)]},$$

(15)

where $X$ stands for the various resonances of the two meson channels. The following parametrization is used for the $\rho$ resonance:

$$T_\rho^{(1)}(s) = \frac{1}{1 + \beta_\rho} \left[ \text{BW}_\rho(s) + \beta_\rho \text{BW}_\rho(s) \right],$$

(16)

where $\beta_\rho = -0.145, m_\rho = 0.773 \text{ GeV}, \Gamma_\rho = 0.145 m_\rho = 1.370 \text{ GeV}, \Gamma_\rho' = 0.510 \text{ GeV}$. These are the values which have been determined from $e^+e^- \rightarrow \pi^+\pi^-$ in $[9]$. The parameterization for $T_{K^*}(Q^2)$ allows for a contribution of the first excitation $K^*(1410)$ in analogy to Eq. (16):

$$T_{K^*}^{(1)}(s) = \frac{1}{1 + \beta_{K^*}} [\text{BW}_{K^*}(s) + \beta_{K^*} \text{BW}_{K^*}(s)],$$

(17)

where $\beta_{K^*} = -0.135, m_{K^*} = 0.892 \text{ GeV}, \Gamma_{K^*} = 0.050 \text{ GeV}, m_{K^*'} = 1.412 \text{ GeV}, \Gamma_{K^*'} = 0.227 \text{ GeV}$. The parameter $\beta_{K^*}$ was fixed in $[10]$ by comparing the theoretical results to the recent experimental branching ratio for $B(K^*\nu_\tau) = 1.36 \pm 0.08 [7]$. The value $\beta_{K^*} = -0.135$ is remarkably close to the strength of the $\rho'$ contribution to the $\rho$ Breit-Wigner, supporting the use of approximate $SU(3)$ flavour symmetry.

The branching ratios based on these parametrizations are $B(\pi^-\pi^-\nu_\tau) = 23.5\%, B(K^0\pi^-\nu_\tau) = 0.45\%, B(K^-\pi^0\nu_\tau) = 0.9\%$. For the decay into two kaons we obtain $B(K^0K^-\nu_\tau) = 0.11\%$, in good agreement with the recent world average $B(K^0K^-\nu_\tau) = 0.13 \pm 0.04\%$ $[7]$.

C. Three Meson Decays

The hadronic matrix elements for three meson final states have a much richer structure. The decay modes involving kaons allow for axial and vector current contributions at the same time $[11,12]$

$$J^\mu(q_1, q_2, q_3) = \langle h_1(q_1)h_2(q_2)h_3(q_3)|V^\mu(0) - A^\mu(0)|0 \rangle.$$  

(18)

The most general ansatz for the matrix element of the quark current $J^\mu$ in Eq. (18) is characterized by four form factors $F_i$ $[13]$, which are in general functions of $Q^2$, $s_1 = (q_2 + q_3)^2, s_2 = (q_1 + q_3)^2$ and $s_3 = (q_1 + q_2)^2$

$$J^\mu(q_1, q_2, q_3) = V_1^\mu F_1 + V_2^\mu F_2 + i V_3^\mu F_3 + V_4^\mu F_4,$$

(19)

with

$$V_1^\mu = (q_1 - q_3)_\nu T^{\mu\nu},$$

$$V_2^\mu = (q_2 - q_3)_\nu T^{\mu\nu},$$

$$V_3^\mu = \epsilon^{\mu\alpha\beta\gamma} q_1_\alpha q_2_\beta q_3_\gamma,$$

$$V_4^\mu = q_1^\mu + q_2^\mu + q_3^\mu,$$

(20)
\( T^{\mu\nu} \) denotes again the transverse projector as defined in Eq. (10). The form factors \( F_1 \) and \( F_2(F_3) \) originate from the axial vector hadronic current (vector current) and correspond to a hadronic system in a spin one state, whereas \( F_4 \) is due to the spin zero part of the axial current matrix element. In the limit of vanishing quark masses, the weak axial-vector current is conserved and this implies that the scalar form factor \( F_4 \) vanishes. The massive pseudoscalars give a contribution to \( F_4 \), however, the effect is very small [14] and we will neglect this contribution in the subsequent discussion of this section. Note however that the form factor \( F_4 \) in the \( \tau \rightarrow (3\pi)\nu_\tau \) decay mode could receive a sizable contribution due to the \( J^P = 0^- \) resonance of the \( \pi' \) [15,13]. Furthermore, the form factor \( F_4 \) allows also for a possible contribution from a charged Higgs exchange. We will consider this in more detail in the last section.

The form factors \( F_1 \) and \( F_2 \) can be predicted by chiral lagrangians, supplemented by informations about resonance parameters. Parametrizations for the \( 3\pi \) final states based on this model can be found in [13,11,16]. In this case the vector form factor is absent due to the \( G \) parity of the pions. On the other hand, the decay mode \( \tau^- \rightarrow \eta\pi^-\pi^0\nu_\tau \) has a vanishing contribution from the axial vector current [14,13,12]. The vector form factor is related to the Wess-Zumino anomaly [19,17] whereas the axial-vector form factors are again predicted by chiral Lagrangians as mentioned before. 

A general parameterization of the form factors for various three meson decays modes with pions and kaons was proposed in [12]. The parameterization has been extensively reanalyized in [10] which lead to sizable differences in the predictions of the decay rates compared to [12]. Furthermore, a parameterization for the final states with two neutral kaons \( \tau^- \rightarrow K_S\pi^-K_S\nu_\tau \), \( \tau^- \rightarrow K_L\pi^-K_L\nu_\tau \), and \( \tau^- \rightarrow K_S\pi^-K_L\nu_\tau \) was derived in [11]. The results for the form factors \( F_i \) in Eq. (13) for the decay modes \( \tau \rightarrow abc\nu_\tau \) are summarized by

\[
F_1^{(abc)}(Q^2, s_2, s_3) = \frac{2\sqrt{2}A^{(abc)}}{3f_\pi} G_1^{(abc)}(Q^2, s_2, s_3), \tag{21}
\]

\[
F_2^{(abc)}(Q^2, s_1, s_3) = \frac{2\sqrt{2}A^{(abc)}}{3f_\pi} G_2^{(abc)}(Q^2, s_1, s_3), \tag{22}
\]

\[
F_3^{(abc)}(Q^2, s_1, s_2, s_3) = \frac{A^{(abc)}}{2\sqrt{2}\pi f_\pi^3} G_3^{(abc)}(Q^2, s_1, s_2, s_3). \tag{23}
\]

The Breit-Wigner functions \( G_{1,2}(G_3) \) and the normalizations \( A^{(abc)} \) are listed in Tab. I [11] for the various decay modes. Note that by convenient ordering of the mesons, the two body resonances in \( F_1 \) (\( F_2 \)) occur only in the variables \( s_2, s_3 \) \( (s_1, s_3) \).

Let us briefly discuss the three particle resonances in Tab. I and II (for details see [10]). We use the \( A_1 \) resonance in the non-strange case with energy dependent width \( BW_{A_1}(s) = \frac{m_{A_1}^2}{m_{A_1}^2 - s - im_{A_1} \Gamma_{A_1} g(s)/g(m_{A_1})} \), with \( m_{A_1} = 1.251 \text{ GeV} \), \( \Gamma_{A_1} = 0.475 \text{ GeV} \). The function \( g(s) \) has been calculated in [11]. The three particle resonances with strangeness are

\[
T_{K_1}^{(a)}(s) = \frac{1}{1 + \xi} \left[ BW_{K_1(1400)}(s) + \xi BW_{K_1(1270)}(s) \right],
\]

\[
T_{K_1}^{(b)}(s) = BW_{K_1(1270)}(s). \tag{24}
\]
excitations $\rho$ and $\xi$ with $\lambda$ and $\epsilon$ in Eqs. (21, 22) for the vector form factor $m$ and for the branching ratios in Tab. III based on this model for the form factors. The pre-
TABLE I. Parameterization of the form factors $F_1$ and $F_2$ in Eqs. (27) for the matrix elements of the weak axial-vector current for the various channels.

| channel   | $A^{(abc)}$ | $G_1^{(abc)}(Q^2, s_2, s_3)$ | $G_2^{(abc)}(Q^2, s_1, s_3)$ |
|-----------|-------------|-----------------------------|-----------------------------|
| $\pi^-\pi^-\pi^+$ | $\cos \theta_c$ | $\mathrm{BW}_{A_1}(Q^2)T_\rho(1)(s_2)$ | $\mathrm{BW}_{A_1}(Q^2)T_\rho(1)(s_1)$ |
| $\pi^0\pi^0\pi^0$ | $\cos \theta_c$ | $\mathrm{BW}_{A_1}(Q^2)T_\rho(1)(s_2)$ | $\mathrm{BW}_{A_1}(Q^2)T_\rho(1)(s_1)$ |
| $K^-\pi^-K^+$ | $-\cos \theta_c$ | $\mathrm{BW}_{A_1}(Q^2)T_\rho(1)(s_2)$ | $\mathrm{BW}_{A_1}(Q^2)T_K^*(s_1)$ |
| $K^0\pi^-K^0$ | $-\cos \theta_c$ | $\frac{1}{2}$ $\mathrm{BW}_{A_1}(Q^2)T_\rho(1)(s_2)$ | $\mathrm{BW}_{A_1}(Q^2)T_K^*(s_1)$ |
| $K_S\pi^-K_S$ | $-\cos \theta_c$ | $\frac{1}{4}$ $\mathrm{BW}_{A_1}(Q^2)T_K^*(s_3)$ | $\frac{1}{4}$$\mathrm{BW}_{A_1}(Q^2)\times$ $[T_K^*(s_1) + T_K^*(s_3)]$ |
| $K_S\pi^-K_L$ | $-\cos \theta_c$ | $\frac{1}{4}$ $\mathrm{BW}_{A_1}(Q^2)\times$ $[2T_\rho(1)(s_2) + T_K^*(s_3)]$ | $\frac{1}{4}$$\mathrm{BW}_{A_1}(Q^2)\times$ $[T_K^*(s_1) - T_K^*(s_3)]$ |
| $K^-\pi^0K^0$ | $\frac{3}{2} \cos \theta_c$ | $\frac{1}{2}$$\mathrm{BW}_{A_1}(Q^2)\times$ $T_\rho(1)(s_2)$ | $\frac{1}{2}$$\mathrm{BW}_{A_1}(Q^2)\times$ $T_K^*(s_1)$ |
| $\pi^-\pi^0K^-$ | $\frac{3}{4} \sin \theta_c$ | $\frac{1}{3}$$\mathrm{BW}_{A_1}(Q^2)\times$ $T_K^*(s_3)$ | $\frac{1}{3}$$\mathrm{BW}_{A_1}(Q^2)\times$ $T_K^*(s_1)$ |
| $K^-\pi^-\pi^+$ | $-\sin \theta_c$ | $\frac{1}{2}$$\mathrm{BW}_{A_1}(Q^2)\times$ $T_K^*(s_3)$ | $\frac{1}{2}$$\mathrm{BW}_{A_1}(Q^2)\times$ $T_K^*(s_1)$ |
| $\pi^-K^0\pi^0$ | $\frac{3}{2} \sin \theta_c$ | $\frac{2}{3}$$\mathrm{BW}_{A_1}(Q^2)\times$ $T_K^*(s_3)$ | $\frac{2}{3}$$\mathrm{BW}_{A_1}(Q^2)\times$ $T_K^*(s_1)$ |

with $\xi = 0.33$ [10]. The three body vector resonances $T_\rho^{(2)}$ and $T_K^{(2)*}$ include the higher radial excitations $\rho'$ and $\rho''$ and $K^{*'}$ and $K^{*''}$

\[
T_\rho^{(2)} = \frac{1}{1 + \lambda + \mu} \left[ \mathrm{BW}_{\rho}(s) + \lambda \mathrm{BW}_{\rho'}(s) + \mu \mathrm{BW}_{\rho''}(s) \right],
\]

\[
T_K^{(2)*} = \frac{1}{1 + \lambda + \mu} \left[ \mathrm{BW}_{K^{*}}(s) + \lambda \mathrm{BW}_{K^{*'}}(s) + \mu \mathrm{BW}_{K^{*''}}(s) \right],
\]

with $\lambda = -0.25, \mu = -0.038$. The $\omega$ resonance $T_\omega(s) = \frac{1}{1 + \epsilon} \left[ \mathrm{BW}_{\omega}(s) + \epsilon \mathrm{BW}_{\phi}(s) \right]$ in the vector form factor $F_3$ in Tab. II allows for a contribution of the $\phi$ with a relative strength \( \epsilon = 0.05 \) [14].

Numerical results for the hadronic decay widths $\Gamma^{(abc)}$ normalized to the leptonic width $\Gamma_e$ and for the branching ratios in Tab. [11] based on this model for the form factors. The predictions for the branching ratios use $\Gamma_e/\Gamma_{tot} = 17.8\%$, as calculated from the experimental values for the tau mass $m_\tau = 1.7771$ GeV and lifetime $\tau_\tau = 291.6$ fs [1].
TABLE II. Parameterization of the form factor $F_3$ in Eq. (23) for the matrix elements of the
weak vector current for the various channels.

| channel (abc) | $A^{(abc)}$ | $G_3^{(abc)}(Q^2, s_1, s_2, s_3)$ |
|---------------|-------------|---------------------------------|
| $K^−π^−K^+$  | $−\cos \theta_c$ | $T_ρ^{(2)}(Q^2)(\sqrt{2} - 1) \left[ 2\sqrt{2}T_ω(s_2) - T_K^{(1)}(s_1) \right]$ |
| $K^0π^−K^0$  | $\cos \theta_c$  | $T_ρ^{(2)}(Q^2)(\sqrt{2} - 1) \left[ 2\sqrt{2}T_ω(s_2) + T_K^{(1)}(s_1) \right]$ |
| $K_Sπ^−K_S$  | $−\cos \theta_c$ | $T_ρ^{(2)}(Q^2)(\sqrt{2} - 1) \left[ T_K^{(1)}(s_1) - T_K^{(1)}(s_3) \right]$ |
| $K_Sπ^−K_L$  | $\cos \theta_c$  | $T_ρ^{(2)}(Q^2)(\sqrt{2} - 1) \left[ 2\sqrt{2}T_ω(s_2) + T_K^{(1)}(s_1) + T_K^{(1)}(s_3) \right]$ |
| $K^−π^0K^0$  | $−\frac{\cos \theta_c}{\sqrt{2}}$ | $T_ρ^{(2)}(Q^2)(\sqrt{2} - 1) \left[ T_K^{(1)}(s_3) - T_K^{(1)}(s_1) \right]$ |
| $ηπ^−π^0$    | $\sqrt{2}\sin \theta_c$ | $T_ρ^{(2)}(Q^2)T_ρ^{(1)}(s_1)$ |
| $π^0π^0K^−$  | $\sin \theta_c$ | $\frac{1}{4}T_K^{(2)}(Q^2) \left[ T_K^{(1)}(s_1) - T_K^{(1)}(s_2) \right]$ |
| $K^−π^−π^+$  | $\sin \theta_c$ | $\frac{1}{4}T_K^{(2)}(Q^2) \left[ T_ρ^{(1)}(s_1) + T_K^{(1)}(s_2) \right]$ |
| $π^−K^0π^0$  | $\sqrt{2}\sin \theta_c$ | $\frac{1}{4}T_K^{(2)}(Q^2) \left[ 2T_ρ^{(1)}(s_2) + T_K^{(1)}(s_1) + T_K^{(1)}(s_3) \right]$ |

TABLE III. Predictions for the normalized decay widths $Γ(abc)/Γ_e$ and the branching ratios
$B(abc)$ for the various channels. The contribution from the vector current is listed in column 3 and
available experimental data are listed in column 5. The later are taken from [20, 21].

| channel (abc) | $Γ(abc)/Γ_e$ | $B(abc)_{(pred.)}$ | $B(abc)_{(expt.)}$ |
|---------------|-------------|----------------|----------------|
| $π^−π^−π^+$   | 0.48        | 0.86%          | (8.64 ± 0.24)% |
| $π^0π^0π^−$   | 0.48        | 0.86%          | (9.09 ± 0.14)% |
| $K^−π^−K^+$   | 0.011       | 0.20%          | (0.20 ± 0.07)% |
| $K^0π^−K^0$   | 0.011       | 0.20%          | (0.021 ± 0.006)% |
| $K_Sπ^−K_S$   | 0.0027      | 0.048%         | (0.12 ± 0.04)% |
| $K_Sπ^−K_L$   | 0.0058      | 0.10%          | (0.170 ± 0.028)% |
| $K^−π^0K^0$   | 0.0090      | 0.16%          | (0.09 ± 0.03)% |
| $ηπ^−π^0$     | 0.0108      | 0.19%          | (0.41 ± 0.07)% |
| $π^0π^0K^−$   | 0.0080      | 0.14%          | (0.40 ± 0.09)% |
| $K^−π^−π^+$   | 0.043       | 0.77%          | (0.41 ± 0.07)% |
| $π^−K^0π^0$   | 0.054       | 0.96%          | (0.41 ± 0.07)% |
D. Four Pion Decays

In order to predict the two tau decays into four pions, $\tau^- \rightarrow \nu_\tau \pi^- \pi^- \pi^+ \pi^0$ and $\tau^- \rightarrow \nu_\tau \pi^0 \pi^0 \pi^0 \pi^-$, there are two possible approaches.

The first approach is based on the fact that these tau decays are again related through CVC to corresponding $e^+e^-$ annihilation channels, namely to $e^+e^- \rightarrow 2\pi^+2\pi^-$ and $e^+e^- \rightarrow \pi^+\pi^-2\pi^0$. And so by using the measured $e^+e^-$ cross sections as input, the tau decays can be predicted [4,8,28], and the results are in good agreement with the $\tau$ data [7,23–25]. This approach, however, allows only to predict the integrated decay rates and the four pion invariant mass distributions. In order to predict the various two and three pion differential distributions, or in order to understand angular distributions, a dynamical model is need.

Such a dynamical model has be constructed in [26] which uses the other possible approach. One follows along the lines which have been used above to obtain the hadronic current in the three meson modes. Again one starts from the structure of the hadronic current in the chiral limit and then implements low lying resonances in the various channels ($\rho$, $\rho'$, $\rho''$, $A_1$ and $\omega$ mesons). There are a few free parameters, which are fixed using the experimental $e^+e^- \rightarrow 2\pi^+2\pi^-$ cross sections and the measured decay rate of the $\tau \rightarrow \omega\pi\nu_\tau$ sub-mode. After parameter fixing, predictions for $e^+e^- \rightarrow \pi^+\pi^-2\pi^0$ and for the four pion decay modes of the $\tau$ are obtained, including detailed two, three and four pion differential mass distributions. The various predictions agree well with the available experimental data.

The $\omega\pi$ contribution to the $4\pi$ final state is expected to proceed via a vector current. However, a violation of $G$-parity would allow the $\omega\pi$ system to be in an axial vector state, which could be revealed by an analysis of the angular distribution in the $\omega\pi$ mode as introduced in [27].

III. ANGULAR DISTRIBUTIONS AND STRUCTURE FUNCTIONS IN TWO AND THREE MESON DECAY MODES

In this section, we study angular distribution of the hadronic system of two and three meson final states which are accessible in a future $\tau$-charm factory. We will assume that the direction of the $\tau$ in the hadronic rest frame is known and that no spin informations of the decaying $\tau$ can be used in the analysis. Of particular interest in the three meson case are the distributions of the normal to the Dalitz plane and the distributions around this normal. It is shown that the most general distribution in the three meson case can be characterized by 16 structure functions most of which can be determined under the conditions mentioned above. The study of angular correlations of the hadronic system allows for much more detailed studies of the hadronic charged current than it is possible by rate measurements alone. Special emphasis is put on $T$-odd triple momentum correlations, which allow for the observation of $CP$-violating contributions beyond the Standard Model.

A. Two Body Decays

Of particular interest in the two body decays is the distribution of the direction of $h_1$ ($\vec{q}_1 = \vec{q}_1 / ||\vec{q}_1||$) and the direction of the $\tau$ (denoted by $\vec{n}_\tau$) viewed from the hadronic rest frame
cos β = \vec{n}_\tau \cdot \vec{q}_1. After integration over the unobserved neutrino direction, the differential decay rate for a two meson final state is given by

\[ d\Gamma(\tau \to 2h) = \left\{ \bar{L}_B W_B + \bar{L}_{SA} W_{SA} + \bar{L}_{SF} W_{SF} + \bar{L}_{SG} W_{SG} \right\} \times \]

\[ \frac{G^2}{4m_\tau} (g_V^2 + g_A^2) (\cos^2 \theta_c) \frac{1}{(4\pi)^3} \frac{1}{m^2_\tau - Q^2} \frac{1}{m^2_\tau} |\vec{q}_1| \frac{dQ^2}{\sqrt{Q^2}} \frac{d\cos \beta}{2} \]

with \( \vec{q}_1 = \frac{1}{2\sqrt{Q^2}} \left( [Q^2 - m_1^2 - m_2^2]^2 - 4m_1^2m_2^2 \right)^{1/2} \). The hadronic structure functions \( W_X \) can be expressed in terms of the form factors \( F \) and \( F_4 \) as defined in Eqs. (9) as follows:

\[ W_B = 4(\vec{q}_1)^2 |F|^2 \]  
\[ W_{SA} = Q^2 |F_4|^2 \]  
\[ W_{SF} = 4\sqrt{Q^2} \vec{q}_1 | \text{Re}[F F_4^*] \]  
\[ W_{SG} = -4\sqrt{Q^2} \vec{q}_1 | \text{Im}[F F_4^*] \]

The leptonic coefficients are

\[ \bar{L}_B = K_1 \sin^2 \beta + K_2 \] \[ \bar{L}_{SA} = K_2 \] \[ \bar{L}_{SF} = -K_2 \cos \beta \] \[ \bar{L}_{SG} = 0 \]

with

\[ K_1 = 1 - (m_\tau^2/Q^2); \quad K_2 = (m_\tau^2/Q^2); \]

Note that the coefficient \( \bar{L}_{SG} \) vanishes, if only the \( \beta \) dependence of the decay is analyzed. In the case of a polarized \( \tau \) (as it is the situation at LEP) one can use the direction of the \( \tau \) spin-vector \( \vec{s} \) in the lab to define a further angle \( \alpha \) by \( \cos \alpha = \frac{(\vec{n}_\tau \times \vec{s}) \cdot (\vec{n}_\tau \times \vec{q}_1)}{|\vec{n}_\tau \times \vec{s}| |\vec{n}_\tau \times \vec{q}_1|} \) (see also Fig. 5 in [13]). Taking into account the distribution with respect to this angle would allow to measure also the structure function \( W_{SG} \). Note that the structure function \( W_{SG} \) is proportional to the imaginary part of the form factors \( (F F_4^*) \) and requires nontrivial phases of the amplitudes resulting from final state interactions. These strong interaction phases are essential for the observation of possible CP violation effects in the hadronic decay amplitudes. However, in our case the angle \( \alpha \) is not observable and has to be averaged out. Hence, the \( T \)-odd correlation \( \bar{L}_{SG} W_{SG} \) vanishes and no test of CP violation is possible. However, a nonvanishing contribution to the distributions \( \bar{L}_{SA} W_{SA} \) or \( \bar{L}_{SF} W_{SF} \) would be a clear signal of a scalar contribution (parametrized by \( F_4 \)) to the two meson decay modes.

**B. Three Body Decays**

Like in the two body case, the three meson decay modes are most easily analyzed in the hadronic rest frame \( \vec{q}_1 + \vec{q}_2 + \vec{q}_3 = 0 \). The orientation of the hadronic system is in general
characterized by three Euler angles \((\alpha, \beta, \gamma)\) as introduced in [13,16]. Performing the analysis of \(\tau \to \nu_\tau + 3\) mesons in the hadronic rest frame has the advantage that the product of the hadronic and the leptonic tensors reduce to a sum \[13\] 
\[L^{\mu\nu} H_{\mu\nu} = \sum_X \tilde{L}_X W_X.\]
In this system the hadronic tensor \(H_{\mu\nu}\) is decomposed into 16 hadronic structure functions \(W_X\) corresponding to 16 density matrix elements for a hadronic system in a spin one \{contributions proportional to \(V_1^\mu F_1, V_2^\mu F_2, V_3^\mu F_3\) in Eq.\}(20) and spin zero state \([V_4^\mu F_4]\) (nine of them originate from a pure spin one and the remaining are pure spin zero or interference terms). The 16 structure functions contain the dynamics of the three meson decay and depend only on the hadronic invariants \(Q^2\) and the Dalitz plot variables \(s_i\). The leptonic factors \(\tilde{L}_X\) factorize the dependence on the Euler angles and also depend on the chirality parameter \(\gamma_{VA} = \frac{2g_Y g_A}{g_Y^2 + g_A^2}\). In our case, one can measure two Euler angles \(\beta\) and \(\gamma\) defined by \(\cos \beta = \vec{n}_\tau \cdot \vec{n}_\perp, \cos \gamma = -\frac{\vec{n}_\tau \cdot \hat{\vec{q}}_3}{|\vec{n}_\tau \times \vec{n}_\perp|}, \sin \gamma = |\vec{n}_\tau \times \vec{n}_\perp| \). The vector \(\vec{n}_\tau\) denotes the \(\tau\) direction in the hadronic rest frame. The \((x, y)\) plane is aligned with the hadron momenta, \(i.e. \vec{n}_\perp = (\vec{q}_1 \times \vec{q}_2)/|\vec{q}_1 \times \vec{q}_2|\) \(\) (the normal to the hadronic plane) pointing along \(Oz\). The \(Ox\) axis is defined by the direction of \(\vec{q}_3 = \vec{q}_3/|\vec{q}_3|\). In the three pion case \(\pi^-\pi^-\pi^+\) we choose \(\vec{q}_3 = \vec{q}_{\pi^+}\) and \(|\vec{q}_2| > |\vec{q}_1|\).

The differential decay rate with respect to these two angles is then given by

\[d\Gamma(\tau \to 3h) = \frac{G^2}{2m_\tau} \cos^2 \theta_e \left\{ \sum_X \tilde{L}_X W_X \right\} \times \]
\[\frac{1}{(2\pi)^5} \frac{1}{64} \frac{1}{m_\tau} \frac{(m_\tau^2 - Q^2)^2}{Q^2} \frac{dQ^2}{dQ^2} \frac{ds_1}{ds_1} \frac{ds_2}{ds_2} \frac{d\gamma}{2\pi} \frac{d\cos \beta}{2}.\]

The leptonic coefficients \(\tilde{L}_X\) will be discussed below. The dependence of the structure functions on the form factors \(F_i\) reads \[13\]:

\[W_A = (x_1^2 + x_2^2)[F_1]^2 + (x_1^2 + x_3^2)[F_2]^2 + 2(x_1 x_2 - x_3^2) \Re(F_1 F_2^*)\]

\[W_B = x_1^2 |F_3|^2\]

\[W_C = (x_1^2 - x_2^2)[F_1]^2 + (x_2^2 - x_3^2)[F_2]^2 + 2(x_1 x_2 + x_3^2) \Re(F_1 F_2^*)\]

\[W_D = 2[x_1 x_3 |F_1|^2 - x_2 x_3 |F_2|^2 + x_3 (x_2 - x_1) \Re(F_1 F_2^*)]\]

\[W_E = -2x_3 (x_1 + x_2) \Im(F_1 F_2^*)\]

\[W_F = 2x_4 [x_1 \Im(F_1 F_3^*) + x_2 \Im(F_2 F_3^*)]\]

\[W_G = -2x_4 [x_1 \Re(F_1 F_3^*) + x_2 \Re(F_2 F_3^*)]\]

\[W_H = 2x_3 x_4 [\Im(F_1 F_3^*) - \Im(F_2 F_3^*)]\]

\[W_I = -2x_3 x_4 [\Re(F_1 F_3^*) - \Re(F_2 F_3^*)]\]

\[W_{SA} = Q^2 |F_4|^2\]

\[W_{SB} = 2\sqrt{Q^2} [x_1 \Re(F_1 F_4^*) + x_2 \Re(F_2 F_4^*)]\]

\[W_{SC} = -2\sqrt{Q^2} [x_1 \Im(F_1 F_4^*) + x_2 \Im(F_2 F_4^*)]\]
\[ W_{SD} = 2\sqrt{Q^2x_3} \left[ \text{Re} \left( F_1 F_4^* \right) - \text{Re} \left( F_2 F_4^* \right) \right] \]
\[ W_{SE} = -2\sqrt{Q^2x_3} \left[ \text{Im} \left( F_1 F_4^* \right) - \text{Im} \left( F_2 F_4^* \right) \right] \]
\[ W_{SF} = -2\sqrt{Q^2x_4} \text{Im} \left( F_3 F_4^* \right) \]
\[ W_{SG} = -2\sqrt{Q^2x_4} \text{Re} \left( F_3 F_4^* \right) \]

The variables \( x_i \) are defined by \( x_1 = V_1^x = q_1^x - q_3^x, x_2 = V_2^x = q_2^x - q_3^x, x_3 = V_1^y = q_1^y = -q_2^y, x_4 = V_3^z = \sqrt{Q^2x_3q_3^x} \), where \( q_i^x (q_i^y) \) denotes the \( x (y) \) component of the momentum of meson \( i \) in the hadronic rest frame. They can easily be expressed in terms of \( s_1, s_2 \) and \( s_3 \).

Note that the first 9 structure functions originate from the hadronic system in a spin one state (\( W_A, W_C, W_D, W_E \) from the axial vector current, \( W_B \) from the vector current and \( W_F, W_G, W_H, W_I \) from the interference of the axial vector and vector current). \( W_{SA} \) originates only from a hadronic system in a spin zero state and the remaining six structure functions are interference terms between the spin one and spin zero states.

An inspection of Eq. (34) shows also that the structure functions \( W_E, W_F, W_H, W_{SC}, W_{SE}, W_{SF} \) require nontrivial phases of the amplitudes resulting from final state interactions. Only the \( T \)-odd correlations \( \bar{L}_X W_X, X \in \{ E, F, H, SC, SE, SF \} \) allow in principle for a measurement of CP violating effects in the hadronic matrix elements (see next section).

The leptonic coefficients \( \bar{L}_X \) depend on the two angles \( \beta, \gamma \) and on \( \gamma_{VA} \):

\[
\begin{align*}
\bar{L}_A &= 1/2 \ K_1 (1 + \cos^2 \beta) + K_2; \\
\bar{L}_B &= K_1 \sin^2 \beta + K_2; \\
\bar{L}_C &= -1/2 \ K_1 \sin^2 \beta \cos 2\gamma; \\
\bar{L}_D &= 1/2 \ K_1 \sin^2 \beta \sin 2\gamma; \\
\bar{L}_E &= \gamma_{VA} \cos \beta; \\
\bar{L}_F &= 1/2 \ K_1 \sin 2\beta \cos \gamma; \\
\bar{L}_G &= -\gamma_{VA} \sin \beta \sin \gamma; \\
\bar{L}_H &= -1/2 \ K_1 \sin 2\beta \sin \gamma; \\
\bar{L}_I &= -\gamma_{VA} \sin \beta \cos \gamma;
\end{align*}
\]

The coefficients \( K_i \) are defined in Eq. (32). Note that the coefficients \( \bar{L}_{SC}, \bar{L}_{SE}, \bar{L}_{SG} \) vanish if only the two Euler angles \( \beta \) and \( \gamma \) are considered. It has been shown in [13] that in the case of a polarized \( \tau \) (as it is the situation at LEP) one can use the direction of the \( \tau \) spin-vector in the lab to define a further Euler angle \( \alpha \). If this additional angle is considered, all 16 coefficients \( \bar{L}_X \) in Eqs. (35) are nonvanishing enabling the measurement of all 16 structure functions \( W_X \).

The coefficients \( \bar{L}_{SC}, \bar{L}_{SE} \) are of particular importance for the detection of possible CP violation originating from a charged Higgs exchange (see below).

Numerical results for the nonvanishing structure functions in the \( 3\pi \) decay mode are discussed in [13, 14]. Furthermore, it has been shown in [13] that the technique of the structure functions allows for a model independent test of possible spin zero components.
(parametrized by $F_4$) in the hadronic current by analyzing the structure functions $W_{SB}$ and $W_{SD}$. Note that the $\cos \beta$ distribution allows already for a model independent separation of the axial-vector and the vector current contribution in the decay modes with different mesons, i.e. the structure functions $W_A$ and $W_B$ in Eq. (34) can be disentangled due to the different $\beta$ dependence of $\tilde{L}_A$ and $\tilde{L}_B$. Numerical results of the structure functions for several three meson decay modes with different mesons based on the model in [12] are discussed in [29]. A more detailed analysis (including the full $Q^2$ and $s_i$ dependence of the structure functions) based on the parameterization in [10] is in preparation [30].

### IV. CP VIOLATION EFFECTS

Currently CP violation has been experimentally observed only in the $K$ meson system. The effect can be explained by a nontrivial complex phase in the Kobayashi-Maskawa flavour mixing matrix. However, the fundamental origin of this CP violation is still unknown. CP-odd correlations of the $\tau^-$ and $\tau^+$ decay products, which originate from an electric dipole moment in the $\tau$ pair production, have been discussed in [31]. In this paper, we investigate the effects of possible non-Kobayashi-Maskawa-type of CP violation, i.e. CP violation effects beyond the Standard Model. Such effects could originate for example from multi Higgs boson models [32], scalar leptoquark model [33] or left-right symmetric models [34].

Any possible observation of these CP violation effects needs not only a CP-violating complex phase (parametrized as $\eta$ and $\chi$ below) in the hadronic matrix elements but also the interference with a CP conserving phase resulting from final state interactions. Therefore, only the correlations involving structure functions proportional to the imaginary part of the form factors $F_i$ allow in principle for an observation of CP violation effects by taking the difference of $d\Gamma[\tau^-] - d\Gamma[\tau^+]$ of the corresponding $T$-odd correlations (see below).

In the two meson decay modes, the only structure function which is sensitive to CP-violation effects is $W_{SG}$ in Eq. (30) [proportional to $\text{Im} [F_4^*]$]. Unfortunately, this structure function is not observable if only distributions of the angle $\beta$ are considered, i.e. the coefficient $\tilde{L}_{SG}$ vanishes. However, $W_{SG}$ could in principle be measured by taking into account additional distributions with respect to the $\tau$ spin vector (assuming polarized incident beams).

CP violation effects in the $\tau \rightarrow 2\pi\nu$ decay mode from the scalar sector (e.g. the multi Higgs boson models) have recently been discussed in terms of “stage-two spin correlation functions” in [3] and in the case of polarized electron-positron beams at $\tau$ charm factories in [2]. In [3], the decay products of the second tau decay are used to define a $T$-odd correlation whereas the $\tau$ polarization (assuming a polarized incident electron beam) is used in [2] to define a $T$-odd triple correlation. In fact, the correlations in [3] are equivalent to the product $\tilde{L}_{SG}W_{SG}$ as discussed before in the two meson case, if the angle $\alpha$ is defined with respect to the $\tau$ spin as described after Eq. (32).

In the three meson case, the structure functions $W_E, W_F, W_H, W_{SC}, W_{SE}, W_{SF}$ in Eq. (34) require nontrivial phases of the amplitudes resulting from final state interactions. Only the $T$-odd correlations $\tilde{L}_X W_X, X \in \{E, F, H, SC, SE, SF\}$ allow therefore in principle for a measurement of CP violating effects in the hadronic matrix elements. As can be seen from Eq. (35) the coefficients $\tilde{L}_{SC}, \tilde{L}_{SE}, \tilde{L}_{SG}$ vanish if only the two Euler angles $\beta$ and $\gamma$ are
considered. However, the structure functions $W_E, W_F, W_H, W_{SF}$ can be measured through the $\beta$ and $\gamma$ dependence encoded in the coefficients $L_E, L_F, L_H, L_{SF}$.

Let us therefore parametrize possible CP violation effects in the hadronic decay amplitudes by replacing Eqs. (19) by

$$J^\mu(q_1, q_2, q_3) = [(V_1^\mu F_1 + V_2^\mu F_2) (1 + \chi_A) + V_4^\mu F_4 (1 + \chi_V + \eta)$$

$$+ i V_3^\mu F_3 (1 + \chi_V)]$$

where $V_i^\mu$ are given in Eq. (20).

The term proportional to $\eta$ parametrizes the effect of a possible charged Higgs boson [32], whereas the complex numbers $\chi_A$ and $\chi_V$ parametrize any new physics that would arise from vector or scalar boson exchange motivated by left-right symmetric models [34].

The Standard Model prediction is obtained from Eq. (36) by setting $\chi$ and $\eta$ to zero. Let us now assume that the complex numbers $\chi_A, \chi_V$ and $\eta$ transform like

$$\chi_A \xrightarrow{CP} \chi_A^*; \quad \chi_V \xrightarrow{CP} \chi_V^*; \quad \eta \xrightarrow{CP} \eta^*.$$  \hspace{1cm} (37)

The hadronic structure functions $\tilde{W}_X$, which include the new physics effects parametrized by the numbers $\eta$ and $\chi$ are easily obtained from Eq. (34) using the transformation

$$F_1 \rightarrow \tilde{F}_1 = F_1(1 + \chi_A),$$

$$F_2 \rightarrow \tilde{F}_2 = F_2(1 + \chi_A),$$

$$F_3 \rightarrow \tilde{F}_3 = F_3(1 + \chi_V),$$

$$F_4 \rightarrow \tilde{F}_4 = F_4(1 + \chi_A + \eta).$$

The hadronic structure functions are affected by the sign change in the weak phases under CP transformation as described in Eq. (37). Note that the strong (complex) phases due to final state interactions [given by Breit-Wigner propagators for the two body resonances] are not changed, because the strong interaction is invariant under charge conjugation. Besides of the sign change in the weak phases, the structure functions $\tilde{W}_F, \tilde{W}_G, \tilde{W}_H, \tilde{W}_I, \tilde{W}_{SF}, \tilde{W}_{SG}$, which originate from the interference of the axial vector and vector current, change sign. Furthermore, the amplitude for the CP conjugated process $\tau^+$ can be obtained from the results for $\tau^-$ by reversing all momenta and spins of the particles. Thus, $\cos \beta \rightarrow - \cos \beta$ and $\gamma_{VA} = - \gamma_{VA}$. CP invariance therefore relates the differential decay rates for $\tau^+$ and $\tau^-$ as:

$$d\Gamma[\tau^-](\cos \beta, \gamma_{VA}, \tilde{W}_X) \overset{CP}{\xrightarrow{\Delta \tilde{W}_X}} d\Gamma[\tau^+](\cos \beta, - \gamma_{VA}, a_X \tilde{W}_X)$$

with $a_X = -1$ for $X \in \{\tilde{W}_F, \tilde{W}_G, \tilde{W}_H, \tilde{W}_I, \tilde{W}_{SF}, \tilde{W}_{SG}\}$ and $a_X = 1$ else.

If CP is not violated, the difference $d\Gamma[\tau^-] - d\Gamma[\tau^+]$ should vanish. From the $T$-odd correlations $\bar{L}_X \tilde{W}_X, X \in \{E, F, H, SC, SE, SF\}$, one can construct CP-violating quantities by taking the difference of these correlations for $\tau^-$ and $\tau^+$.

$$\Delta_X = \frac{1}{2} \left(\bar{L}_X(\cos \beta, \gamma_{VA}) \tilde{W}_X[\tau^-] - \bar{L}_X(\cos \beta, - \gamma_{VA}) a_X \tilde{W}_X[\tau^+]\right)$$

$$= \bar{L}_X(\cos \beta, \gamma_{VA}) \left(\tilde{W}_X[\tau^-] - \tilde{W}_X[\tau^+]\right) \equiv \bar{L}_X \Delta \tilde{W}_X,$$

$$\quad (43)$$
where

$$\Delta \tilde{W}_X = \tilde{W}_X[\tau^-] - \tilde{W}_X[\tau^+] \quad (44)$$

The nonvanishing CP-violating differences can be calculated from Eqs. (34,38-41) and expressed in terms of the form factors $F_i$ and the complex numbers $\chi_A, \chi_V$ and $\eta$ as follows:

$$\Delta \tilde{W}_F = 2x_4 [x_1 \text{Re} (F_1 F_3^*) + x_2 \text{Re} (F_2 F_3^*)] \text{Im} (\chi_A - \chi_V + \chi_A \chi_V^*) \quad ,$$

$$\Delta \tilde{W}_H = 2x_3 x_4 [\text{Re} (F_1 F_3^*) - \text{Re} (F_2 F_3^*)] \text{Im} (\chi_A - \chi_V + \chi_A \chi_V^*) \quad ,$$

$$\Delta \tilde{W}_{SF} = -2\sqrt{Q^2} x_4 \text{Re} (F_3 F_4^*) \text{Im} (\chi_V - \chi_A - \eta + \chi_V (\chi_A^* + \eta^*)) .$$

An observed nonzero values for these differences would signal a true CP-violation. Note that all CP-violating differences are proportional to the imaginary part $\eta$ and $\chi$. Note also that $\Delta \tilde{W}_E$ vanishes, because the form factors $F_1$ and $F_2$ multiply the same complex weak phase. Eqs. (45,46) show that CP violation effects parametrized by $\chi_A$ and $\chi_V$ are in principle observable in a Tau-Charm Factory for three meson decay modes with a nonvanishing vector (proportional to $F_3$) and axial vector current (proportional to $F_1, F_2$). CP violation effects from a charged Higgs could be detected through $\Delta \tilde{W}_{SF}$ only for decay modes with a nonvanishing vector current. Therefore, CP-violation tests in the three pion decay mode are not possible, if only the decay distribution with respect to the angles $\beta$ and $\gamma$ are taken into account.

As mentioned before, it has been shown in [13] that in the case of a polarized $\tau$ one can use the direction of the $\tau$ spin-vector in the lab to define a further Euler angle $\alpha$. This additional angular dependence allows in principle for the measurement of the two additional CP-violating differences

$$\Delta \tilde{W}_{SC} = 2\sqrt{Q^2} x_4 \text{Re} (F_3 F_4^*) \text{Im} (-\eta + \chi_A \eta^*) \quad ,$$

$$\Delta \tilde{W}_{SE} = 2\sqrt{Q^2} x_3 [\text{Re} (F_1 F_4^*) - \text{Re} (F_2 F_4^*)] \text{Im} (-\eta + \chi_A \eta^*) \quad .$$

and hence for CP violation tests originating from a charged Higgs in the three pion decay mode.

The authors in [35] studied the effects of $T$-odd triple correlations (as derived in [13]) in the decay modes $\tau \rightarrow K\pi\pi\nu$ and $\tau \rightarrow KK\pi\nu$ using the model for the hadronic form factors as suggested in [12]. They found that CP violation effects in some extensions of the Standard Model could be as big as 0.1%. CP violating effects in the $\tau \rightarrow 3\pi\nu$ decay mode have also been discussed in [36].

**ACKNOWLEDGEMENTS**

We would like to thank J.H. Kuehn for collaboration on part of the work presented here. The work of E. M. was supported in part by the U. S. Department of Energy under Grant No. DE-FG02-95ER40896. Further support was provided by the University of Wisconsin Research Committee, with funds granted by the Wisconsin Alumni Research Foundation. The work of M.F. has been supported in part by the National Science Foundation Grant PHY-9218167.
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