ACCRETION OF HOLOGRAPHIC DARK ENERGY: DEPENDENCY ONLY UPON HORIZON RADIUS OF EXPANDING UNIVERSE

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Abstract

In this paper we deal with accretion of dark energy in the holographic dark energy model for a general non-rotating static spherically symmetric black hole. The mass of the black hole increases or decreases depending on the nature of the holographic dark energy (quintessence or phantom) as well as on some integration parameters. It is to be illustrated that the enhancement or reduction of mass of a black hole is independent of the mass or size of the black hole itself. Rather it depends only upon the radius of the event horizon of the universe. Finally, the generalized second law of thermodynamics has been studied on the event horizon to be assured that the law holds even if when the black hole mass is decreasing though it is engrossing some mass.

Key words: Black hole accretion, phantom energy, thermodynamics.

1 Introduction

It is popularly believed among the recent time astrophysicists and the cosmologists that our universe is experiencing an accelerated expansion. The strong supports of this idea of cosmic acceleration come from recent observational data of type Ia supernovae (Riess, A. G. et al. 1998; Perlmutter, S. et al. 1999; Astier, P. et al. 2006) in associated with Large Scale Structure (Tegmark, M. et al. 2004; Abazajian, K. et al. 2004, 2005) and Cosmic Microwave Background anisotropies (Spergel, D. N. et al. 2003, 2006) have provided main evidences. So there was a plenty of requirements for theories to be constructed which would be efficient enough to explain this interesting behavior. It was seemed that this way of explanation has taken two different paths namely introducing some modified gravity theory, such as \( f(R) \) gravity, where the extra terms in the modified Friedmann equations are responsible for the acceleration. The second way was to introduce some non-baryonic matter (known as dark energy (DE)) having negative pressure (violating the strong energy condition \( \rho + 3p > 0 \)) in the framework of general relativity. Though still the nature and cosmological origin of the DE have remained enigmatic at present, one recent proposal is the dynamical DE scenario (Copeland, E. J. 2006). The cosmological constant puzzles may be better interpreted by assuming that the vacuum energy is canceled to exactly zero by some unknown mechanism and introducing a DE component with a dynamically variable equation of state. From some scalar field mechanism which suggests that the energy form with a negative pressure is provided by a scalar field evolving under a suitable potential the dynamical DE paradigm is often realized. Careful analysis of
cosmological observations, in particular of the WMAP (Wilkenson Microwave Anisotropy probe) experiment (Spergel, D. N. et al. 2003; Bennett, C. et al. 2003; Peiris, H. V. et al. 2003) indicates that the two-thirds of the total energy of our universe is been occupied by the DE where as dark matter occupies almost the rest. This DE is thought to be responsible for the accelerated universe. Many models have been constructed for interpreting this component. Cosmological constant or vacuum energy (Weinberg, S. 1989; Carroll, S. M. 2004; Peebles, P. J. E. et al. 2003; Padmanabhan, T. 2003) and quintessence models (Wetterich, C. 1988; Peebles, P. J. E. et al. 1988; Ratra, B. et al. 1988; Frieman, J. A. et al. 1995; Turner, M. S. et al. 1997; Caldwell, R. R. et al. 1998; Liddle, A. R. et al. 1999; Zlatev, L. et al. 1999; Steinhardt, P. J. et al. 1999; Torres, D. F. 2002) are two very much popular models among these. However, as is well known, there are two difficulties which arise from all of these scenarios, the fine-tuning problem and the cosmic coincidence problem. The fine-tuning problem asks why the DE density today is so small compared to typical particle scales. The DE density of order $10^{-47} GeV^4$, which appears to require the introduction of a new mass scale. The second difficulty, the cosmic coincidence problem, states: Since the energy densities of DE and dark matter scale so differently during the expansion of the universe, why are they nearly equal today? To get this coincidence, it appears that their ratio must be set to a specific, infinitesimal value in the very early universe. Recently, considerable interest has been stimulated in explaining the observed DE by the Holographic DE (HDE) model. For an effective field theory in a box of size $L$, with UV cut-off $\Lambda$, the entropy $S$ scales extensively, $S \sim L^3 \Lambda^3$. However, the peculiar thermodynamics of black hole (BH) (Bekenstein, J. D. 1973, 1974, 1981, 1994; Hawking, S. W. 1975, 1976) has led Bekenstein to postulate that the maximum entropy in a box of volume $L^3$ behaves nonextensively growing only as the area of the box, i.e., there is a so called Bekenstein entropy bound, $S \leq S_{BH} \equiv \pi M_p^2 L^2$. This nonextensive scaling suggests that quantum field theory breaks down in large volume. To reconcile this breakdown with the success of local quantum field theory in describing observed particle phenomenology, Cohen et. al. (1999) proposed a more restrictive energy bound. They pointed out that in quantum field theory a short distance (UV) cut off is related to a long distance (IR) cut off due to the limit set by forming a black hole. In the other words, if the quantum zero point energy density $\rho$ is relevant to a UV cut-off, the total energy of the whole system with size $L$ should not exceed the mass of a BH of the same size, thus $L^3 \rho L \leq L M_p^2$, this means that the maximum entropy is in order of $S_{BH}^4$. When we take the whole universe into account, the vacuum energy related to this holographic principle (Hooft, G. T. 1993; Susskind, L. 1994) is viewed as DE, usually dubbed HDE. The largest IR cut-off $L$ is chosen by saturating the inequality so that we get the HDE density

$$\rho = \frac{3c^2 M_p^2 L^2}{\sqrt{8 \pi G}}$$

where $M_p$ is the reduced Planck mass, $c$ is a numerical constant. If we take $L$ as the size of the current universe for instance the Hubble scale $H^{-1}$, then the DE density will be close to the observed data.

In HDE paradigm (Cohen et. al. 1999; Horava, P. et al. 2000; Thomas, S. D. 2002; Hsu, S. D. H. 2004; Li, M. 2004; Pavon, D. et. al. 2005) one determines an appropriate quantity to serve as an IR cut off for the theory and imposes the constraint that the total vacuum energy in the corresponding maximum value must not be greater than the mass of a BH of the same size. By saturating the inequality one identifies the acquired vacuum energy as HDE. Although the choice of the IR cut off has raised on discussion in the literature (Li, M. 2004; Pavon, D. et. al. 2005; Gong, Y. 2004; Guberina, B. et. al. 2005; Setare, M. R. 2007A, 2007B), it has been shown, and it is generally accepted, that
the radius of the event horizon of the universe \( R_h \) the most suitable choice for the IR cut off where \( R_h \) is defined as (Hsu, S. D. H. 2004)

\[
R_h = a \int_t^\infty \frac{dt}{a} = a \int_t^\infty \frac{da}{Ha^2}
\]

which leads to results compatible with observations. Here ‘a’ is the scale factor of the background metric of the universe and \( H \) is the corresponding Hubble parameter.

The holographic energy density \( \rho \) is then given by (taking \( 8\pi G = 1 \))

\[
\rho = \frac{3c^2}{R_h^2}, \tag{3}
\]

Furthermore, we can define the dimensionless dark energy as:

\[
\Omega \equiv \frac{\rho}{3H^2} = \frac{c^2}{R_h^2H^2} \tag{4}
\]

In the case of a dark-energy dominated universe, dark energy evolves according to the conservation law

\[
\dot{\rho} + 3H(p + \rho) = 0 \tag{5}
\]

or equivalently:

\[
\dot{\Omega} = H\Omega(1 - \Omega) \left(1 + \frac{2\sqrt{\Omega}}{c}\right) \tag{6}
\]

where the equation of state is

\[
p = \omega_D \rho \tag{7}
\]

and consequently the index of the equation of state is of the form:

\[
\omega_D = -\frac{1}{3} \left(1 + \frac{2\sqrt{\Omega}}{c}\right) \tag{8}
\]

As we can clearly see, \( \omega \) depends on the parameter \( c \). In recent observational studies, different groups have ascribed different values to \( c \). A direct fit of the present available SNe Ia data indicates that the best fit result is the best-fit value \( c = 0.21 \) within \( 1 - \sigma \) error range (Huang, Q. G.et. al. 2004A). In addition, observational data from the X-ray gas mass fraction of galaxy clusters lead to \( c = 0.61 \) within \( 1 - \sigma \) (Chang, Z. et. al. 2006). Similarly, combining data from type Ia supernovae, cosmic microwave background radiation and large scale structure give the best-fit value \( c = 0.91 \) within \( 1 - \sigma \) (Kao, H. C. et. al. 2005; Zhang, X. et. al. 2005, 2007), while combining data from type Ia supernovae, X-ray gas and baryon acoustic oscillation lead to \( c = 0.73 \) as a best-fit value within \( 1 - \sigma \) (Wu, Q. et. al. 2007; Ma, Y-Z. et. al. 2007). Finally, the study of the constraints on the dark energy arising from the holographic connection to the small 1 CMB suppression, reveals that \( c = 2.1 \) within \( 1 - \sigma \) error (Shen, J. et. al. 2004). In conclusion, \( 0.21 \leq c \leq 2.1 \), and HDE provides the mechanism for the \( w = 0.1 \) crossing and the transition to the accelerating expansion of the Universe.

In nature, the compact objects, particularly the black holes (BHs), are not visible but can be detected by the presence of the accretion disc around them. By analyzing light rays off an accretion disc, one can speculate the properties of the central compact object.

Although the accretion phenomena around compact objects (particularly BHs) have been extensively discussed over the last three decades (e.g. Mukhopadhyay, B. 2003), it was started long ago in 1952 by Bondi (1952). He studied stationary spherical accretion
problem by introducing formal fluid dynamical equations in the Newtonian framework.
In the framework of general relativity, the study of accretion was initiated by Michel (1972). By choosing the Newtonian gravitational potential, Shakura and Sunyaev (1973) formulated very simplistic but effective model of the accretion disc. Some aspects of the accretion disc in fully relativistic framework had been studied by Novikov and Thorne (1973) and Page and Thorne (1974). Subsequently, various aspects related to the critical behavior of general relativistic flows in spherical symmetry have been studied (Begelman, M. C. 1978; Brinkmann, W. 1980; Malec, E. 1999; Das, T. K. 2001). Although there are a few steps forward, still it is extremely difficult to simulate the full scale realistic accretion discs including outflows in a full general relativistic framework.

One may note that in BH accretion, an important issue is that the flow of accreting matter must be transonic in nature, i.e., there should be sonic point(s) (Chakrabarti, S. K. 1990, 1996A, 1996B) in the flow. On the other hand, accretion flow around a neutron star is not necessarily transonic (i.e., sonic point may or may not exist).

Due to present accelerated expansion of the universe, the matter in the universe is dominated by DE (almost $\frac{2}{3}$ of the matter is in the form of DE). Therefore it is reasonable to assume the accreting matter is in the form of DE. Babichev et.al. (Babichev, E. et al. 2004,2006) were the pioneers to think about the DE accretion upon a BH, in the framework of Bondi accretion (Bondi, H. 1952).

In this paper we will study the accretion of HDE upon BH where in section 2 we will calculate the expression for the mass change of the BH. Section 3 contains the thermodynamical analysis. Finally in the chapter 4 we will discuss the whole thing derived in the previous sections.

2 Equation governing the accretion of HDE on general static non rotating BH

In this section, we shall calculate the rate of mass accretion using the conservation equations in fluid dynamics. For that let us consider the spherical accretion of HDE onto BH. For simplicity, we assume a non-rotating spherically symmetric BH having metric ansatz

$$ds^2 = -h(r)dt^2 + \frac{dr^2}{h(r)} + r^2d\Omega^2$$

(9)

The lapse function $g_{00} = h(r)$ identifies the event horizon ($r_e$) by setting $h(r) = 0$ (i.e., $h(r_e) = 0$).

Suppose the HDE is represented by a perfect fluid for which the energy momentum tensor be:

$$T_{\mu\nu} = (\rho + p) u_\mu u_\nu + pg_{\mu\nu}$$

(10)

where, $\rho$ and $p$ are respectively the energy density and pressure of the HDE and $u^\mu = \frac{dx^\mu}{dt}$ is the four velocity of the flow. Assuming the fluid flow in the radial direction to be $u$ (note that $u < 0$ as the fluid flow towards the BH). The explicit form of the components of $u^\mu$ is :

$$(u^0, u^1, 0, 0)$$

It is to be noted that the third and the fourth component of $u^\mu$ are zero due to spherical symmetry. Using the normalization rule $u^\mu u_\mu = -1$, we have,

$$u_0 = -\sqrt{h(r) + u^2}, \quad u_1 = \frac{u}{h(r)}$$

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\[ u^0 = \frac{\sqrt{h(r) + u^2}}{h(r)} , \quad u^1 = u \]  

(11)

As a consequence the explicit components of \( T_{\mu\nu} \) (from (10)) are given by,

\[ T^0_0 = -\rho - \frac{u^2}{h(r)} (\rho + p) \]

\[ T^1_1 = p + \frac{u^2}{h(r)} (\rho + p) \]

So, we have got the stress energy tensor components and the velocity components. Now to determine different dynamical parameters we need to construct and solve different differential equations. These will be provided from the conservation relations.

\[ T^2_2 = p \]  

(12)

\[ T^3_3 = p \]

\[ T^0_0 = u (\rho + p) \sqrt{h(r) + u^2} \]

In the present problem we have two conservation relations namely,

(I) Conservation of mass flux :

\[ J^\mu; a = 0, \]  

(13)

where \( J^\mu \) is the current density. Explicitly it gives (after integrating with respect to 'r')

\[ pur^2 = \lambda \]  

(14)

with \( \lambda \), an arbitrary constant of integration. We have to note that as \( u < 0 \) so \( \lambda \) is also < 0.

(II) Energy-momentum conservation relation :

\[ T_{\mu\nu} = 0. \]  

(15)

In particular \( T^{\nu}_{0\nu} = 0 \) characterizes the energy flux across the horizon. A first integral (w.r.t. 'r') of \( T^{\nu}_{0\nu} = 0 \) gives

\[ (\rho + p) \left( h(r) + u^2 \right) \frac{d}{dr} ur^2 = a_1 \]  

(16)

here \( a_1 \) is the arbitrary constant of integration.

We have three variables, namely \( u, \rho \) and \( p \) (the EoS parameter \( \omega_D \) rather). Now as we got two integration of motion only still we are underdetermined.

We can have another integral of motion by projecting the energy-momentum conservation relation (7) along the four velocity \( u_\mu \), i.e.,

\[ u^\mu T^{\nu}_{\mu\nu} = 0 \] or in explicit form

\[ u^\rho \rho_{,\rho} + (\rho + p) u_{,\rho}^\rho = 0 \]

Integrating once gives (Babichev, E. et. al. 2004)

\[ ur^2 \exp \left\{ \int_{\rho_{\infty}}^{\rho} \frac{d\rho}{\rho^2 + p(\rho)} \right\} = -b_1 \]  

(17)

This is also known as energy-flux equation. Here for convenience the negative sign is chosen in front of the constant 'b_1'.

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Now removing $ur^2$ from (15) and (17) we obtain

$$(\rho + p) \left( h(r) + u^2 \right)^{\frac{1}{2}} \exp \left\{ - \int_{\rho_\infty}^{\rho} \frac{d\rho'}{\rho' + p(\rho')} \right\} = c_1$$

with $c_1 = -\frac{2a_1}{c_1} = \rho_\infty + p(\rho_\infty)$. Now from the relations (17) and (18), the fluid velocity $u_H$ and the density $\rho_H$ at the event horizon are related by the relation (Babichev, E. et. al. 2004) (with $r_e = 1$)

$$b_1 \frac{\rho_H + p(\rho_H)}{\rho_\infty + p(\rho_\infty)} = -\frac{b_1^2}{u_H^2} \exp \left\{ 2 \int_{\rho_\infty}^{\rho_H} \frac{d\rho'}{\rho' + p(\rho')} \right\}$$

Now eliminating $ur^2$ from equations (14) and (16) we have

$$(\rho + p\frac{\rho}{\rho_\infty}) \sqrt{u^2 + h(r)} = a_1$$

Taking $r$ as the independent variable if we take differentials of eq. (14) and (20) and eliminate $d\rho$ from them, then after simplification we obtain

$$\frac{du}{u} \left[ -c_2^2 + \frac{u^2}{h(r) + u^2} \right] + \frac{dr}{r} \left[ -2c_2^2 + \frac{1}{2} \frac{h'(r)}{h(r)} + u^2 \right] = 0$$

where, $1 + c_2^2 = \frac{4\ln(\rho + p)}{d\rho/d\rho'}$. Here the solution will be feasible if it passes through a critical point and it characterizes the fluid falling into the BH with monotonically increasing velocity. However the critical point corresponds to vanishing of both the square brackets in equation (21) and the parameters at the critical point ($r_*$) as (Babichev, E. et. al. 2004).

$$u_*^2 = \frac{1}{4} r_* h'(r_*) \quad , \quad c_{_{**}}^2 = \frac{u_*^2}{h(r_*) + u_*^2}$$

Further due to fluid accretion the rate of change of BH mass (Huang, Q. G. 2004B)

$$\dot{M} = 4\pi b_1 (\rho_\infty + p(\rho_\infty))$$

It is to be noted that the above equation is independent of the mass of the BH (contrary to the Schwarzschild and Reissner-Nordström BHs (Jamil, M. et. al. 2008; Gonzalez-Diaz, P.F. 2004; Cai R-G.et. al. 2006; Jimenez Madrid, J. A.et. al. 2008; Zhang, X. 2009; Guariento, D.C.et al 2008)). When the mass of the BH concerned is not present or the expression is true for any arbitrary BH mass we will not confine our system at a infinite distance only(as it is not dependent upon the potential itself which is a function of BH mass). So equation (23) holds at any finite point. So the above equation can be written for any general $\rho$ and $p$ as done in (Jamil, M. et. al. 2008, 2011A, 2011B)(satisfying the holographic equation of state and violating weak energy condition), i.e.,

$$\dot{M} = 4\pi b_1 (\rho + p(\rho))$$

When BH accretes fluid simultaneously it also radiates energy known as Hawking Radiation. This radiation (Saskind, L. 1992) causes the evaporation of the BH which is balanced by the accretion of matter into the BH and as a result the total system is supposed to be under equilibrium. But when we analyze the parameters, for example temperature of the accreting fluid at very far from the BH with very near to the BH there will be a huge
difference. But in local cells the parameters show equilibrium nature. Such an equilibrium is called quasi equilibrium. In this paper we have considered large BHs in general. Now if we think about small BHs then as the Hawking temperature of the BH, $T = (8\pi M)^{-1}$ is inversely proportional to its mass, temperature will increase for the small BHs. This will cause more radiation according to the standard fourth order rule of black body radiation. under such huge amount of Hawking radiation the accretion-radiation equilibrium may not be the equilibrium one. In such a case we will not be able to speculate whether the accretion procedure is at all independent of the mass or not. For very small BHs the procedure of accretion is still a fact be research with.

From equation (20) we have (with $r_e = 1$) the index of equation of state as

$$
\omega_D = -1 + \frac{a_1}{\lambda \{u^2 + h (r)\}^2}
$$

Note that $\omega_D > 0$ or $\omega_D < -1$ depends on the sign of the constant $a_1$, i.e., the HDE is of phantom nature or not will depend crucially on $a_1 < 0$ or $a_1 > 0$.

3 Thermodynamical analysis of accreting matter

We shall now discuss the thermodynamics of the DE accretion that crosses the event horizon of the BH given by equation $p = \omega_D \rho$. Actually, we have two way motives for doing this thermodynamical analysis. The first, is to determine the value of $b_1$ such that we can determine the sign of $\dot{M}$ and second one is to examine the validity of the generalized second law of thermodynamics which is an invariant law and to find any restriction on the equation of state $\omega_D$ from thermodynamical point of view. Let us rewrite the BH metric in the form

$$
\text{ds}^2 = h_{ab} dx^a dx^b + r^2 d\Omega^2
$$

where $(a, b) = (0, 1)$ and $h_{ab} = \text{diag} \left(-h(r), \frac{1}{h(r)}\right)$.

For thermodynamical analysis we start with the work density ($W$) and energy supply vector $\psi_{a}$ which are defined as (Cai, R. G. et. al. 2007, Chakraborty, S.et. al. 2010)

$$
W = -\frac{1}{2} \text{Trace} \; T^b_{a} = \frac{1}{2} (\rho - p)
$$

and

$$
\psi_{a} = T^b_{a} \partial_b r + W \partial_a r
$$

i.e.,

$$
\psi_0 = T^0_{a} \partial_a r = -u (\rho + p) \sqrt{u^2 + h (r)}
$$

and

$$
\psi_1 = T^1_{a} + W = (\rho + p) \left(\frac{1}{2} + \frac{u^2}{h(r)}\right)
$$

Where $T_{ab}$ is the projected energy-momentum tensor, normal to the 2-sphere. Then the change of energy across the event horizon is given by (Chakraborty, S.et. al. 2010)

$$
dE \equiv -A \Psi = -A [\psi_0 dt + \psi_1 dr]
$$

Hence the energy crossing the event horizon is (Chakraborty, S.et. al. 2010; Mazumder, N. et. al. 2009) (choosing $r_e = 1$)

$$
dE = 4\pi u^2 (\rho + p) dt
$$

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so comparing equations (24) and (28) (as \( E = mc^2 \) and \( c = 1 \)) the arbitrary constant \( b_1 \) is given by

\[
b_1 = u^2
\]

i.e.,

\[
\dot{M} = 4\pi u^2 (\rho + p)
\]

So we can say that \( \dot{M} > 0 \) in quintessence era, i.e., the BH mass is increasing there though the rate of increasing is slowing down as we move towards the phantom barrier line. While in phantom era \( \dot{M} < 0 \), i.e., the mass of the BH is decreasing. Where the value of \( \dot{M} \) starts to decrease is a point of interest. To calculate that we will use (3) and (8) in (31) and with the help of (4) we have

\[
\dot{M} = 8\pi u^2 \frac{c^2}{R_h^4} \left( 1 - \frac{1}{R_h H} \right)
\]

which on differentiation gives

\[
\frac{d\dot{M}}{dR_h} = 8\pi u^2 \frac{c^2}{R_h^4} \left( \frac{3}{H} - 2R_h \right)
\]

So, \( \dot{M} \) increases when \( R_h < \frac{3}{2} R_H \) where \( R_H \) is the Hubble radius, \( R_h \) is the radius of the event horizon (defined in (2)) and \( \dot{M} \) decreases when \( R_h > \frac{3}{2} R_H \).

In fig.1 we have plotted \( \dot{M} \) with the variation of \( \omega_D \) and \( \rho \). This is not a graph to be scaled actually. The graph shows that \( \dot{M} \) is negative when \( \omega_D < -1 \).
Now it is quite awkward to hear that the BH is absorbing fluid but the mass of the BH is decreasing. Obviously the negative pressure of the dark energy model is responsible for this incidence. We shall now examine the validity of the generalised second law of thermodynamics in the present case. Using Clausius relation, the time variation of horizon entropy is given by

\[ \dot{S}_h = \frac{4\pi u^2}{T_e} (\rho + p) \]  

where \( T_e \) is the temperature of the event horizon. We now study the entropy variation of the matter in the form of HDE bounded by the event horizon. We assume that the thermodynamical system bounded by the event horizon is an equilibrium one and hence the temperature of the matter inside the event horizon is same as \( T_e \). From the Gibb’s equation

\[ T_e dS_I = dE + pdV \]

we obtain

\[ T_e \dot{S}_I = 4\pi u (\rho + p) \left[ \sqrt{h(r)} + u^2 + \frac{up}{\rho} \right] \]

Thus the total entropy change is given by

\[ \left( \dot{S}_h + \dot{S}_I \right) = \frac{16\pi^2 u^2 \rho (1 + \omega_D)}{|h'(r)|} \left[ 1 + \omega_D + \sqrt{1 + \frac{h(r)}{u^2}} \right] \]

From the above expression of the total entropy change (i.e., eq. (35)) we see that the term outside the square bracket in the right hand side is positive in phantom era. So to make the time variation of the total entropy to be positive the square bracket must be a positive quantity also and it is true again in the phantom era. So GSLT is valid in the phantom era. But in quintessence era the term outside the square bracket is negative. So the square bracket must be negative to ensure the validity of the GSLT, which lead us to a lower boundary for the equation of state parameter as

\[ \omega_D > -1 + \sqrt{1 + \frac{h(r)}{u^2}}. \]

4 Conclusion :

The work deals with the possibility of DE accretion onto a general static spherically symmetric BH. The analysis shows that the evolution of mass of a static BH does not depend on its mass rather it depends on the equation of state of the HDE. Due to static non rotating BH the accreting DE falls radially \((u < 0)\) on the BH. Although, the accreting HDE satisfies GSLT but the BH evaporates in phantom era while in quintessence era BH only accretes but the equation of state must have an lower bound for the validity of GSLT. The figure shows the variation of \( M \) against energy density \( \rho \) and equation of state parameter \( \omega_D \). As expected we see that in phantom era whatever be the value of \( \rho \), \( M \) is negative while in quintessence era \( M \) is positive. Further, the variation of the BH mass does not depend on any BH parameters, rather it depends on the horizons of the space time. Therefore, we may conclude that the geometry of the background space-time has a signified effect on the BH accretion of DE. For further work it will be interesting to study the accretion of HDE for rotating BH and examine the influence of rotation on the
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