Signal Space Separation Beamformer

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Abstract We have combined Signal Space Separation and beamformers (SSS beamformer). The SSS beamformer was tested by simulation in the presence of simulated brain noise. The SSS beamformer performs at least as well as the conventional beamformer, provided that the expansion order is sufficiently high. For beamformer outputs which depend on power or power difference normalized by the projected noise, the spatial resolution of the SSS beamformer is significantly better than that of the conventional beamformers if the sources are deeper, and about the same as that of the conventional beamformer when the sources are superficial. For beamformer outputs which depend on the ratio of powers, the spatial resolutions of the SSS and conventional beamformers are the same. The sensor noise covariance matrix in the SSS basis is non-diagonal. The SSS beamformers with diagonalized noise covariance matrix exhibit better spatial resolution than that with non-diagonal noise covariance matrix. The SSS beamformers are computationally more efficient than the conventional beamformers.

Keywords Beamformers · Signal space separation · Magnetoencephalography · MEG · Signal processing

Introduction

Beamformers have an important role in non-parametric analysis of the source activity underlying magnetoencephalography (MEG) data (Van Veen et al. 1997; Robinson and Vrba 1998; Gross et al. 2001; Sekihara et al. 2004). Beamformers are spatial filters designed to extract electrical activity from target brain location while suppressing contributions originating outside the target. Beamformer weights are based on data and they do not require a priori assumptions about the number of active sources, solutions are analytical (there is no need for global minimum searches of a cost function), estimates for each voxel can be made independent of other voxels, there is no tendency of source drift to surface (as in minimum norm based approaches), and the beamformers can be used to image spectral power which is not necessarily phase locked to stimulus (Barnes and Hillebrand 2003). A disadvantage of beamformers is that they tend to suppress spatially separate yet covariant sources, and in addition, the spatial resolution at a particular source space region depends on the data.

Biological magnetic fields are measured by multi-channel sensors. High spatial frequencies of the magnetic fields decay rapidly with increasing source depth and only relatively low spatial frequencies exceed the sensor noise levels, resulting in less than 200 degrees of freedom of the measurable MEG signals (Ahonen et al. 1993). The modern MEG devices employ over 250 channels and spatially oversample the signal. It was proposed (Taulu and Kajola 2005) to express the magnetic field by a truncated basis function expansion, which allows description of the magnetic field by fewer basis functions than the number of physical channels. The method is called signal space separation (SSS). The magnetic field can be represented as a
combination of two separate expansions: one corresponding to fields originating from the volume of interest (internal terms) and the second to fields arising outside the volume of interest (external terms). Elimination of the external terms from the expansion allows for suppression of the external interference without distorting the measured MEG signals.

After the introduction of the SSS method it was proposed to combine the beamforming and SSS approaches into an SSS beamformer (Nenonen and Taulu 2005). This approach utilizes harmonic function amplitudes and vector spherical harmonic leadfields obtained from the signal space separation method instead of the measured sensor values. Initially, it was thought that the method would provide effective data compression and improved computational efficiency—instead of 306 measured values at a given time the new method would typically need only about 100 harmonic amplitudes. However, it was found that in addition to reducing computational load, the method also improves spatial resolution of certain types of beamformers.

In this work we compare spatial resolution of the SSS and conventional beamformers by simulations. Other parameters, e.g., localization accuracy, are comparable for the two beamformer types and are not discussed in the paper.

Materials and Methods

This section describes the SSS formulation, conventional and SSS beamformers, the beamformer constructs used in the work, and simulation parameters.

Theoretical Background

Measurement at time \( t \) is a column vector \( \mathbf{m}(t) \) with dimensions \( M \times 1 \), where \( M \) is the number of MEG channels.

Because the MEG sensors are located in a source-free volume, the magnetic field, \( \mathbf{B} \), can be expressed as a gradient of a scalar potential, \( \Psi \), i.e., \( \nabla^2 \Psi = 0 \). Such a solution can be represented as a linear combination of basis functions, e.g., spherical harmonics, as

\[
\Psi(r) = \sum_{n \geq 0} \sum_{m = -n}^{n} \tilde{a}_{nm} Y_{nm}(\theta, \phi) + \sum_{n \geq 0} \sum_{m = -n}^{n} \tilde{b}_{nm} r^n Y_{nm}(\theta, \phi)
\]

where \( Y_{nm} \) are spherical harmonic functions, \( \theta \) and \( \phi \) denote spherical angles, \( r = |r| \) is the distance from the expansion center, and \( \tilde{a}_{nm} \) and \( \tilde{b}_{nm} \) are expansion coefficients. The first term on the right-hand side of Eq. 1 diverges at the origin and it represents sources within the sensor shell; the second term diverges at infinity and corresponds to sources outside the sensor shell. Contributions of the internal and external sources can be separated and the external terms can be discarded to reduce the environmental noise.

The MEG is measured by SQUID sensors which typically consist of several sensing coils. Magnetic fields for a given sensor at all coil positions, as expressed by gradient of Eq. 1, are combined. Then, the sensor array measurement can be expressed in terms of spherical harmonics as \( \mathbf{m}(t) \approx \mathbf{S} \mathbf{X}(t) \), where the matrix \( \mathbf{S} = [\mathbf{S}_{in}, \mathbf{S}_{ext}] \) contains the basis vectors and has dimension \( M \times D \), \( D \) is the number of the basis vectors \( (D < M) \), and \( \mathbf{X}(t) \) is time dependent column vector of the basis vector amplitudes. The \( \mathbf{S}_{in} \) contains internal and \( \mathbf{S}_{ext} \) external expansion terms (see Eq. 1). The “\( \approx \)” sign is used because the expansion is truncated at \( n_{int} \) internal terms and \( n_{ext} \) external terms. The time dependent coefficients \( \mathbf{X}(t) \) can be estimated as \( \hat{\mathbf{X}}(t) \approx \mathbf{S}^+ \mathbf{m}(t) \), where \( \mathbf{S}^+ \) is pseudoinverse of \( \mathbf{S} \). The MEG measurement with external interference filtered out can be obtained as \( \hat{\mathbf{m}}(t) = \mathbf{V} \mathbf{X}(t) \), where the matrix \( \mathbf{V} = \mathbf{S}_m \mathbf{P} \mathbf{S}^+ \) and \( \mathbf{P} = [1 \ 0] \) (Taulu and Kajola 2005).

Only the scalar beamformers will be discussed. Equations for beamformer are well known (e.g., Sekihara et al. 2004) and general forms of the power and power normalized by noise (pseudo-\( Z^2 \)) are:

\[
P(r, \eta) = \frac{1}{\Phi(r, \eta) C_M^{-1} \Phi(r, \eta)}
\]

\[
Z^2(r, \eta) = \frac{\Phi(r, \eta) C_M^{-1} \Phi(r, \eta)}{\Phi(r, \eta) C_M^{-1} C_N C_M^{-1} \Phi(r, \eta)}
\]

where \( r \) is the vector of source position, and \( \eta \) is \( 3 \times 1 \) (or \( 2 \times 1 \)) source orientation vector. For conventional beamformers, \( \Phi(r, \eta) = \mathbf{L}(r, \eta) \) is \( M \times 1 \) lead field matrix, \( C_M = C_m \) is covariance matrix of the measurement with dimension \( M \times M \), and \( C_N = C_n \) is the noise covariance matrix computed from instrumental noise time courses, \( \nu(t) \). The lead field matrix can be separated into the position and orientation parts \( \mathbf{L}(r, \eta) = \mathbf{L}(r) \eta(r) \) and the source orientation which maximizes either the \( P \) or \( Z^2 \) can be found by procedure described in (Sekihara et al. 2004).

For the SSS beamformer, \( C_M = C_s \) is the covariance matrix of the time dependent expansion coefficients \( \mathbf{X}(t) \) with dimension \( D \times D \) and is related to the covariance matrix of measurement by \( C_s = \mathbf{S}^+ C_m \mathbf{S}^+ \mathbf{T} \), \( C_N = C_{sv} \) is the noise covariance matrix computed from instrumental noise transformed into the SSS basis and is related to the conventional noise covariance matrix by \( C_{sv} = \mathbf{S}^+ \mathbf{C_n} \mathbf{S}^+ \mathbf{T} \), and \( \Phi(r, \eta) = \Gamma(r, \eta) \) is \( D \times 1 \) SSS lead field matrix. The \( \Gamma \) is related to \( \mathbf{L} \) by \( \Gamma(r, \eta) = \mathbf{S}^+ \mathbf{L}(r, \eta) \). The \( \Gamma \) can also be decomposed into the position and orientation parts, and...
source orientation which maximizes $P$ or $Z^2$ can be found
the same way as for the conventional beamformers. To
calculate the SSS beamformer, the data and noise are first
transformed into the SSS basis and then the covariance
matrices $C_x$ and $C_{xx}$ are computed. Instead of directly
computing the lead field matrix in the SSS basis, we have
used the relationship $\Gamma = S^+ L$. The SSS noise covariance
matrix $C_{xx}$ is non-diagonal. External terms were omitted in
$S^+$, and simulations were done either with the non-diagonal
form of $C_{xx}$, or the $C_{xx}$ was diagonalized by setting the off-
diagonal terms to zero.

Computation of $S^+$ requires inversion of $S^T S$, which
may be ill-conditioned. The inversion can be successfully
completed by regularization. But, it was found that the
standard regularization procedures (SVD truncation and
Tikhonov regularizations) cannot simultaneously maintain
low sensor noise and large SSS interference attenuation. To
avoid this problem, we have removed from $S$ all basis
vectors, one at a time, and each time re-computed the
condition number of the $S^T S$ based on the remaining vec-
tors. We then removed the basis vector which reduced the
condition number most. The procedure was repeated until
the condition number was less than a specified value. Such
procedure maintains specified span of singular values and
yet reduces the SSS sensor noise and maintains large SSS
interference attenuation. The vectors which were removed
from the matrix $S$ either have low amplitudes or are only
slightly different from a linear combination of other vec-
tors. Removal of these vectors will not result in a loss of
important spatial topographies, at least within the accuracy
of the specified span of singular values.

The SSS is known to attenuate the sensor noise, es-
specially if the condition number of matrix $S^T S$ is small. We
have adjusted the condition number limit to $10^3$, because at
this value the sensor noise attenuation by the SSS for a
reasonable range of spherical harmonic expansion orders
is $\approx 1$.

We report results only on pseudo-t and f dual state
beamformer constructs (Vrba and Robinson 2001), but
statistically normalized (Barnes and Hillebrand 2003) and
event related (Robinson 2004; Cheyne et al. 2007) beam-
formers were also simulated and exhibit similar behaviour.

Description of Simulations

A realistic, helmet shaped sensor array with 306 triple
sensors (102 radial magnetometers and 204 planar gradi-
ometers with 1.7 cm baseline) was simulated with random
sensor gain error of 0.1%. The spontaneous brain activity,
the “brain noise”, was modeled by 5000 dipole sources,
present in all the following simulations, randomly distrib-
uted in a shell bounded by concentric spherical surfaces
with 5 and 8 cm radii. Dipoles had random orientations and
random amplitudes. After the brain noise simulation was
completed, the rms brain noise density over all samples and
all channels was normalized to 14 fT/sqrt/Hz for planar
gradiometers, which resulted in 33.8 fT/sqrt/Hz noise density
for magnetometers.

Either one or two tangential target sources were placed
into the model sphere. Single source was positioned at
$(0, 0, a)$ and the two sources were positioned at $(0, \pm d/2,
(a^2 - d^2/4)^{0.5})$, where $a$ is the source distance from the
model sphere center and $d$ is the source separation.
Parameters used were $a = 3, 5, 7, 9$ cm and $d = 0.5, 1,
2$ cm, and source orientations were $(1, 0, 0)$. The duration
of the time series was $T = 100$ s, sample rate $f_s = 150$ Hz,
there were 156 triggers associated with the simulated
source activity, and 0.2-s pre- and 0.2-s post-trigger
intervals.

Source magnitudes were adjusted for signal-to-noise ratio (SNR) measured relative to the brain noise of
SNR = 0.1, 1, and 10, with SNR defined as weighted
averages $(M_m \text{SNR}_m + M_p \text{SNR}_p)/(M_m + M_p)$, where $M_m$
and $M_p$ are the numbers of magnetometer and planar gra-
diometer channels, and $\text{SNR}_m$ and $\text{SNR}_p$ are the magne-
tometer and planar gradiometer SNRs. The SNRs were
defined as $\text{SNR} = q^2 |L|^2 / (v_{rms} M)$, where $q$ is source
magnitude, $L$ is lead field vector (sensor response to a unit
source), $v_{rms}$ is nominal brain noise, and $M$ is the number of
magnetometer or planar gradiometer channels.

For single source, the signal was 25 ms wide general-
ized Lorentzian peak with 50 ms latency relative to trig-
gers, and 30% random amplitude variation. When two
sources were used, the signal of the first was the same as
that of the single source, and the signal of the second was
40 ms wide peak with 90 ms latency and 40% random
amplitude variation.

To assess the spatial resolution of the beamformer, its
output peak dimensions were measured for single source
simulations. The beamformer scan covered a volume
which contained the source. Number of voxels, in which
the beamformer output was larger than $1/2$ the associated
peak amplitude, was counted. The peak volume was
obtained by multiplying this count by voxel volume, and
the peak dimension was approximated as a cube root of
the peak volume. Voxels were kept sufficiently small such
that at least 50 voxels were counted within the peak
volume.

Results

Comparison of conventional and SSS beamformer peak
dimensions for single source as a function of expansion
order for different values of $a$ are shown in Fig. 1 for di-
agonalized instrumental noise covariance matrix and in
Fig. 2 for non-diagonal noise covariance matrix. The conventional beamformers do not depend on the expansion order and their values are shown as horizontal lines spanning all expansion orders (gray solid and dashed lines).

Direct comparison of peak dimensions of conventional and SSS beamformers with diagonalized and non-diagonal noise covariance matrices is shown in Fig. 3a for dual state t beamformer and \( a = 5 \) cm. Similar comparison for the dual state f beamformer would show that for non-diagonal noise covariance matrix, the SSS and conventional peak dimensions are practically identical for all \( n_{\text{int}} \), for the diagonalized noise covariance matrix the peak dimensions are the same for \( n_{\text{int}} = 7 \) and 14, and the SSS peak dimension is slightly smaller than the conventional one for intermediate values \( 7 < n_{\text{int}} < 14 \).

Actual shapes of the dual state beamformer t responses to single source are shown in Figs. 3b–f as contour maps corresponding to intersection of the beamformer scan with model center. Solid gray—Conventional, t; dashed gray—conventional, f; solid black—SSS, t; dashed black—SSS, f.

Dashed black and gray lines for \( a = 3 \) and 5 cm are nearly identical
Behavior is quite different for the dual state $f$-beamformers and deep sources. There the difference between the SSS and conventional beamformer resolutions is small for diagonalized noise covariance matrix (dashed lines in Fig. 1a, b, and 2a, b). For deep sources, the SSS beamformer peak dimension increases with increasing expansion order. For very superficial sources, the spatial resolution of the SSS beamformer is roughly the same as that of the conventional beamformer if the expansion order is sufficiently high (Figs. 1d, 2d) and slightly worse if the expansion order is low (but still only about 0.5 mm); the peak dimension monotonically decreases with increasing expansion order and attains the spatial resolution of the conventional beamformer for expansion order $n_{int} = 14$. For intermediate source depths in Fig. 1c the SSS beamformer resolution is better than that of the conventional beamformer for $n_{int} \geq 10$, and in Fig. 2c for all expansion orders. Improvement of the SSS beamformer resolution for deep sources is larger for diagonalized than for non-diagonal noise covariance matrix.

Mechanism of the beamformer spatial resolution in the presence of brain noise is complex. We speculate that the improvement of the spatial resolution by the SSS beamformer for the deep sources may have the following origins: First, it is observed that the angle between signal space vectors corresponding to two deep sources is larger when calculated in the SSS expansion basis than when calculated in sensor space. For superficial sources, this difference diminishes. Second, if the expansion order is lower than necessary for superficial brain noise sources, but adequate for deeper target source, then the deep target sources will be described well by the SSS expansion, but the more superficial brain noise sources will not. Such “spatial low-pass filtering” of the superficial brain noise sources will alter their signal space vectors by decreasing their amplitudes and smoothing the corresponding field distributions. All these effects will improve spatial resolution of the beamformer.
covariance matrix (Fig. 2a, b). For superficial sources, the character of the SSS $f$-beamformers is similar to that of the $t$-beamformers, the conventional and SSS resolutions are the same only if the SSS expansion order is high. Lack of significant spatial resolution improvement for deep sources by the SSS $f$-beamformers is possibly caused by cancelation when forming power ratios.

The diagonalization of the noise covariance matrix significantly improves the SSS $t$-beamformer resolution. This is illustrated in Fig. 3a. The SSS beamformer with both non-diagonal and diagonalized noise covariance matrices exhibit sharper peaks than the conventional beamformer, but the peaks with the diagonalized noise covariance matrix are significantly sharper than those with non-diagonal matrix. Reasons for this behavior are presently under investigation.

The beamformer peak contours in Fig. 3b–f explicitly illustrate the spatial resolution improvement by the SSS type $t$ beamformers. For non-diagonal covariance matrix, the improvement is modest (compare Fig. 3c, d with b) and the peak dimension increases with increasing expansion order, as predicted by the dashed line in Fig. 3a. For diagonalized noise covariance matrix in Fig. 3e, f the SSS peak size reduction over that of the conventional beamformer is dramatic.

Volume or dimension of a single peak was used as a measure of spatial resolution and was systematically investigated. Another possible measure of the spatial resolution is a distance between two sources at which the sources are resolved. This measure of the spatial resolution has not been investigated. However, it was shown that the better spatial resolution of the SSS beamformers is maintained even when two sources are present in the system, as shown in Fig. 4. Similar to the conventional beamformers, the spatial resolution of two sources by the SSS beamformers will depend on the source SNR, depth, and orientation.

Beamformers already are spatial filters which are sensitive to a target source and suppress contributions from other sources with forward solutions which do not match the target forward solution. In addition, the SSS $t$ beamformers can be adjusted to exhibit increased resolution for deeper sources. This increased resolution is achieved by lowering the expansion order to match the source depth. At the same time the lower expansion order will make the beamformer mismatched for superficial sources and will attenuate them. This mechanism effectively acts as an additional spatial filter which can be tuned to enhance deep sources and attenuate superficial sources.

Sensitivity of the SSS beamformers to the lead field inaccuracy has not been investigated. It could be speculated that the SSS beamformers would be less sensitive to the lead field inaccuracy because their dimensionality is lower. But this sensitivity will have to be established more rigorously by simulations.

In conclusion, beamformers were constructed which take advantage of the lesser number of the SSS expansion basis vectors than the physical MEG channels. The SSS beamformers perform as well as the conventional beamformers for superficial sources, provided that the expansion order is sufficiently high. But for deeper sources and the beamformer outputs which depend on power or power difference normalized by the projected noise, the SSS beamformers exhibit much better spatial resolution than the conventional beamformers. In addition, because the SSS beamformers operate on a lower dimensional system and do not require integration over the sensor area, the computational speed is increased.

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