STOCHASTIC INCLUSIONS WITH CURRENT VELOCITIES HAVING DECOMPOSABLE RIGHT-HAND SIDES

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An existence of solution theorem is obtained for stochastic differential inclusions given in terms of the so-called current velocities (symmetric mean derivatives, a direct analogs of ordinary velocity of deterministic systems) and quadratic mean derivatives (giving information on the diffusion coefficient) on the flat \(n\)-dimensional torus. Right-hand sides in both the current velocity part and the quadratic part are set-valued, lower semi-continuous but not necessarily have convex images. Instead we suppose that they are decomposable.

Keywords: mean derivatives; current velocities; decomposable set-valued mappings; differential inclusions.

Introduction

The notion of mean derivatives was introduced by Edward Nelson [1–3] for the needs of stochastic mechanics (a version of quantum mechanics). The equation of motion in this theory (called the Newton–Nelson equation) was the first example of equations in mean derivatives. Later it turned out that the equations in mean derivatives arose also in many other branches of science (mechanics, hydrodynamics, Navier–Stokes vortices, gauge fields, economics, etc.).

Nelson introduced forward and backward mean derivatives while only their half-sum, symmetric mean derivative called current velocity, is a direct analog of ordinary velocity for deterministic processes. In [4] another mean derivative called quadratic, is introduced. It gives information on the diffusion coefficient of the process and using Nelson’s and quadratic mean derivatives together, one can in principle recover the process from its mean derivatives.

Since the current velocities are natural analogs of ordinary velocities of deterministic processes, investigation of equations and especially inclusions with current velocities is very much important for applications since there are a lot of models of various physical, economical etc. processes based on such equations and inclusions.

Here we investigate inclusions with current velocities who’s right hand sides are lower semi-continuous. Unlike previous publications we do not suppose that the images of points are convex sets. Instead we suppose that they are decomposable. This property yields serious modification of all proofs and constructions. We obtain an existence of solution theorem for such inclusions.

Some words about notation. By \(S(n)\) we denote the space of symmetric \(n \times n\) matrices, by \(S_+(n)\) the subset of positive defined symmetric matrices, and by \(\bar{S}_+(n)\) its closure, the set of positive semi-definite symmetric matrices.
1. Preliminaries on the Mean Derivatives

Let $\xi(t)$ be a stochastic process in $T^\prime$, $t \in [0, T]$, that is given on a certain probability space $(\Omega, \mathcal{F}, P)$ so that $\xi(t)$ is $L_1$-random variable for all $t$. The minimal complete $\sigma$-algebra that contains the preimages of all Borel sets in $T^\prime$ with respect to $\xi(t) : \Omega \to T^\prime$ is called "the present" $N_t^\xi$. We denote by $E_t^\xi$ the conditional expectation with respect to $N_t^\xi$.

Strictly speaking, almost surely (a.s.) the sample paths of $\xi(t)$ are not differentiable for almost all $t$. Thus, its "classical" derivatives exist only in the sense of generalized functions. To avoid using the generalized functions, following Nelson (see, e.g., [1–3]) we give

**Definition 1.** [1,4] (i) Forward mean derivative $D_1(t)$ of $\xi$ at time $t$ is an $L_1$-random variable of the form

$$D_1(t) = \lim_{\Delta t \to +0} E_t^\xi \left( \frac{\xi(t + \Delta t) - \xi(t)}{\Delta t} \right),$$

where the limit is supposed to exists in $L_1(\Omega, \mathcal{F}, P)$ and $\Delta t \to +0$ means that $\Delta t$ tends to $0$ and $\Delta t > 0$.

(ii) Backward mean derivative $D_\ast(t)$ of $\xi$ at time $t$ is an $L_1$-random variable

$$D_\ast(t) = \lim_{\Delta t \to +0} E_t^\xi \left( \frac{\xi(t) - \xi(t - \Delta t)}{\Delta t} \right),$$

where the conditions and the notation are the same as in (i).

From the properties of conditional expectation (see [5]) it follows that $D_1(t)$ and $D_\ast(t)$ can be represented as compositions of $\xi(t)$ and Borel measurable vector fields (regressions)

$$a(t, x) = \lim_{\Delta t \to +0} E \left( \frac{\xi(t + \Delta t) - \xi(t)}{\Delta t} \bigg| \xi(t) = x \right),$$

$$a_\ast(t, x) = \lim_{\Delta t \to +0} E \left( \frac{\xi(t) - \xi(t - \Delta t)}{\Delta t} \bigg| \xi(t) = x \right)$$

on $\mathbb{R}^n$. This means that $D_1(t) = a(t, \xi(t))$ and $D_\ast(t) = a_\ast(t, \xi(t))$.

**Definition 2.** [1,4] The derivative $D_2 = \frac{1}{2}(D + D_\ast)$ is called symmetric mean derivative. The derivative $D_1 = \frac{1}{2}(D - D_\ast)$ is called anti-symmetric mean derivative.

Consider the vector fields

$$v^\xi(t, x) = \frac{1}{2}(Y^0(t, x) + Y^0_\ast(t, x))$$

and

$$u^\xi(t, x) = \frac{1}{2}(Y^0(t, x) - Y^0_\ast(t, x)).$$

**Definition 3.** $v^\xi(t) = v^\xi(t, \xi(t)) = D_2\xi(t)$ is called current velocity of $\xi(t)$;

$u^\xi(t) = u^\xi(t, \xi(t)) = D_1\xi(t)$ is called osmotic velocity of $\xi(t)$.

Following [4,6] we introduce the differential operator $D_2$ that differentiates an $L_1$ random process $\xi(t)$, $t \in [0, T]$ according to the rule

$$D_2\xi(t) = \lim_{\Delta t \to +0} E_t^\xi \left( \frac{(\xi(t + \Delta t) - \xi(t))(\xi(t + \Delta t) - \xi(t))^\ast}{\Delta t} \right),$$

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where \((\xi(t + \Delta t) - \xi(t))\) is considered as a column vector (vector in \(\mathbb{R}^n\)), \((\xi(t + \Delta t) - \xi(t))^*\) is a row vector (transposed, or conjugate vector) and the limit is supposed to exists in \(L_1(\Omega, \mathcal{F}, P)\). We emphasize that the matrix product of a column on the left and a row on the right is a matrix so that \(D_2 \xi(t)\) is a symmetric positive semi-definite matrix function on \([0, T] \times \mathbb{R}^n\). We call \(D_2\) the quadratic mean derivative.

Let \(v(t, m)\) be a vector field and \(\alpha(t, m)\) be a symmetric positive semi-definite \((2, 0)\)-tensor field on \(\mathcal{T}^n\). The system

\[
\begin{align*}
D_2 \xi(t) &= v(t, \xi(t)), \\
D_1 \xi(t) &= \alpha(t, \xi(t))
\end{align*}
\]

is called the first order differential equation with current velocities.

**Definition 4.** We say that (5) on \(\mathcal{T}^n\) has a solution on \([0, T]\) with initial condition \(\xi(0) = \xi_0\) if there exists a probability space \((\Omega, \mathcal{F}, P)\) and a process \(\xi(t)\) given on \((\Omega, \mathcal{F}, P)\) and taking values in \(\mathcal{T}^n\) such that \(\xi(0) = \xi_0\) and for almost all \(t \in [0, T]\) equation (5) is satisfied \(P\)-a.s. by \(\xi(t)\).

**Theorem 1.** [7] Let \(v : [0, T] \times \mathbb{R}^n \rightarrow \mathbb{R}^n\) be smooth and \(\alpha : [0, T] \times \mathbb{R}^n \rightarrow S_+(n)\) be smooth. Let them also satisfy the estimates

\[
\|v(t, x)\| < K(1 + \|x\|),
\]

\[
\text{tr } \alpha(t, x) < K(1 + \|x\|^2)
\]

and for all indices \(ij\) let the elements of matrix \(\alpha(x)\) satisfy the inequality

\[
\left| \frac{\partial \alpha_{ij}}{\partial x^j}(x) \right| < K(1 + \|x\|)
\]

for some \(K > 0\). Let \(\xi_0\) be a random element with values in \(\mathbb{R}^n\) whose probability density \(\rho_0\) is smooth and nowhere equal to zero. Then for the initial condition \(\xi(0) = \xi_0\) equation (5) has a solution that is well posed on the entire interval \(t \in [0, T]\) and unique as a diffusion process.

**Corollary 1.** Let \(v : [0, T] \times \mathcal{T}^n \rightarrow \mathbb{R}^n\) be smooth and \(\alpha : [0, T] \times \mathcal{T}^n \rightarrow S_+(n)\) be smooth. Let \(\xi_0\) be a random element with values in \(\mathcal{T}^n\) whose probability density \(\rho_0\) on \(\mathcal{T}^n\) is smooth and nowhere equal to zero. Then for the initial condition \(\xi(0) = \xi_0\) equation (5) has a solution that is well posed on the entire interval \(t \in [0, T]\) and unique as a diffusion process.

Indeed, the boundedness estimates (6), (7) and (8) follow from the fact that \(v, \alpha\) and \(\frac{\partial \alpha_{ij}}{\partial x^j}(x)\) are smooth ant \(\mathcal{T}^n\) is compact.

Consider a sequence of equations

\[
\begin{align*}
D_2 \xi(t) &= v_k(t, \xi(t)), \\
D_1 \xi(t) &= \alpha_k(t, \xi(t))
\end{align*}
\]

on \(\mathcal{T}^n\) of (5) type such that they all have solutions with the same initial condition. Denote by \(\mu_k\) the measure on \((C^0([0, T], \mathcal{T}^n), C)\) generated by the solution of the \(k\)-th equation from (9).
Below we will use the following result:

**Lemma 1.** [8] Let all \( v_k \) and \( \alpha_k \) be uniformly bounded on \( T^n \) by the same upper bound. Then the set of measures \( \{\mu_k\} \) on \( (C^0([0,T], T^n), \mathcal{C}) \) is weakly compact.

2. The Main Result

If the values of a lower semi-continuous set-valued mapping (generally speaking) are not convex, it may not have continuous selectors. Then the following construction is often very much useful.

**Definition 5.** Let \( E \) be a separable Banach space. A non-empty set \( M \in L^1([0,l]; E) \) is called decomposable if \( f \cdot \chi_{2\mathbb{R}} + g \cdot \chi_{[0,l] \setminus 2\mathbb{R}} \in M \) for all \( f, g \in M \) and for every measurable subset \( \mathfrak{M} \) in \( [0,l] \) where \( \chi \) is the characteristic function of the corresponding set.

The reader can find more details about decomposable sets in [9] and [10].

**Theorem 2.** (Bressan-Colombo Theorem) Let \( (\Omega, d) \) be a separable metric space, \( X \) be a Banach space and \( (J, \mathcal{A}, \mu) \) be a measurable space with a \( \sigma \)-algebra \( \mathcal{A} \) and a non-atomic measure \( \mu \) such that \( \mu(J) = 1 \). Consider the space \( Y = L^1_X(J, \mathcal{A}, \mu) \) of integrable mappings from \( (J, \mathcal{A}, \mu) \) into \( X \). If a set-valued mapping \( F : \Omega \rightarrow Y \) is lower semicontinuous and has close decomposable values, \( F \) has a continuous selector.

The assertion of Theorem 2 is proved, e.g., as Lemma 9.2 in [9].

Recall some facts and notions involved in further considerations. Specify \( l > 0 \). In what follows we denote by \( \lambda \) the normalized Lebesgue measure on \( [0,l] \), i.e., such that \( \lambda([0,l]) = 1 \).

**Lemma 2.** Let \( (\Xi, d) \) be a separable metric space, \( X \) be a Banach space. Consider the space \( Y = L^1([0,l], \mathcal{B}, \lambda, X) \) of integrable maps from \( [0,l] \) into \( X \). If a set-valued map \( G : \Xi \rightarrow Y \) is lower semicontinuous and has closed decomposable images, it has a continuous selector.

This is a particular case of Bressan-Colombo Theorem 2.

Let \( v(t, m) \) be a set-valued vector field and \( \alpha(t, m) \) be a set-valued symmetric positive semi-definite \((2,0)\)-tensor field on \( T^n \). The system of the form

\[
\begin{align*}
D_S \xi(t) &\in v(t, \xi(t)), \\
D_2 \xi(t) &\in \alpha(t, \xi(t))
\end{align*}
\]

is called a first order differential inclusion with current velocities.

The definition of solution of inclusion (10) is quite analogous to Definition 4.

**Theorem 3.** Let the set-valued fields \( v \) and \( \alpha \) on \( T^n \) be lower semicontinuous, uniformly bounded and have closed decomposable images of points.

Consider a random element \( \xi_0 \) with values in \( T^n \) that has the density \( \rho_0 \) smooth and nowhere equal to zero. Then for the initial condition \( \xi(0) = \xi_0 \) inclusion (10) has a solution well-posed on the entire interval \( t \in [0,T] \).
Proof.

By Lemma 2 set-valued fields \( \mathbf{v} \) and \( \mathbf{\alpha} \) have continuous selectors \( v \) and \( \alpha \), respectively. Construct a sequence of their smooth approximations \( v_k \) and \( \alpha_k \) that converge to \( v \) and \( \alpha \) with respect to the supremum norm. In addition, it is possible to construct \( \alpha_k \) to be symmetric and positive definite since we deal with approximations in the space of symmetric matrices and any positive semi-definite matrix is a limit point of the space of positive definite matrices.

Consider the sequence of equations of (5) type:

\[
\begin{align*}
D_2 \xi_k(t) &= v_k(\xi_k(t)), \\
D_1 \xi_k(t) &= \alpha_k(\xi_k(t)).
\end{align*}
\]

For all those equations we consider the same initial condition \( \xi_0 \).

The approximations \( v_k \) and \( \alpha_k \) are uniformly bounded by the same constant by the construction. Thus, all the above equations satisfy the conditions of Theorem 1, i.e., for every equation there exists a solution \( \xi_k \).

Introduce on the Banach manifold \( C^0([0, T], T^n) \) of continuous curves in \( T^n \) the \( \sigma \)-algebra \( C \) generated by cylinder sets and denote by \( \mu_k \) the measure on \( (C^0([0, T], T^n), C) \), generated by the solution \( \xi_k(t) \). Introduce also the family of complete \( \sigma \)-sub-algebras \( \mathcal{P}_t \), generated by cylinder sets with bases over \([0, t], t \in [0, T] \), and the family of complete \( \sigma \)-sub-algebras \( \mathcal{N}_t \), generated by primages of Borel sets in \( T^n \) under the mapping \( x(\cdot) \mapsto x(t) \).

It is clear that \( \mathcal{N}_t \) is a \( \sigma \)-sub-algebra in \( \mathcal{P}_t \) and that \( \mathcal{P}_t \) is the "past" while \( \mathcal{N}_t \) is the "present" for the coordinate process on \( (C^0([0, T], T^n), C, \mu_k) \).

By Theorem 1 the set \( \{\mu_k\} \) of measures on \( (C^0([0, T], T^n), C) \) is weakly compact. Hence, we can select a subsequence that weakly converges to some measure \( \mu \). Without loss of generality we can suppose that the sequence \( \mu_k \) weakly converges to \( \mu \). Consider the coordinate process \( \xi(t) \) on the probability space \((C^0([0, T], T^n), C, \mu)\), i.e., for any elementary event \( x(\cdot) \in C^0([0, T], T^n) \) by definition \( \xi(t, x(\cdot)) = m(t) \). Recall that \( \mathcal{P}_t \) is the "past" for \( \xi(t) \), and \( \mathcal{N}_t \) is the "present" for this coordinate process.

Be the construction of \( \xi_k(t) \), its quadratic derivative equals \( \alpha_k(\xi_k(t)) \). This means that for any bounded continuous real function \( f \) on \( C^0([0, T], T^n) \), measurable with respect to \( \mathcal{N}_t \), for all \( k \) the equality

\[
\lim_{\Delta t \to 0} \int_{C^0([0, T], T^n)} \left[ \frac{(m(t + \Delta t) - m(t))(m(t + \Delta t) - m(t))^*}{\Delta t} - \alpha_k(m(t)) \right] f(m(\cdot)) d\mu_k = 0 \tag{11}
\]

holds. Since \( \alpha_k(t, m) \) converge uniformly to \( \alpha(t, m) \) as \( k \to \infty \), we derive that \( \alpha_k(t, m(t)) \) tends to \( \alpha(t, m(t)) \) uniformly for all \( \mu_k \) including \( \mu \).

The field \( \alpha(m(t)) \) is continuous on some set of complete measure in \( C^0([0, T], T^n) \) since \( \alpha(m) \) is continuous on \( T^n \).

Since we have uniform convergence (see above) for all \( k \), we derive from boundedness of \( f(m(\cdot)) \) that for \( k \) large enough we get

\[
\left\| \int_{C^0([0, T], T^n)} [\alpha_k(m(t)) - \alpha(m(t))] f(m(\cdot)) d\mu_k \right\| < \delta.
\]
Since $f(m(\cdot))$ is bounded, there exists a certain number $\Xi > 0$ such that $|f(m(\cdot))| < \Xi m(\cdot)$. Recall that all $\alpha_k(m)$ and $\alpha(m)$ are uniformly bounded, i.e., their norms are not greater than a certain $Q > 0$. Then

$$\left\| \int_{C^0([0,T], T^n)} [\alpha_k(m(t)) - \alpha(m(t))] f(m(\cdot)) d\mu_k \right\| < 2\delta Q \Xi$$

for all $k$ large enough. Since $\delta$ is an arbitrary positive number,

$$\lim_{k \to \infty} \int_{C^0([0,T], T^n)} [\alpha_k(m(t)) - \alpha(m(t))] f(m(\cdot)) d\mu_k = 0.$$

The function $\alpha(m(t))$ is $\mu$ a.s. continuous and bounded on $C^0([0,T], T^n)$ (as it is shown above). Since the measures $\mu_k$ weakly converge to $\mu$, by Lemma [11, Section VI.1]

$$\lim_{k \to \infty} \int_{C^0([0,T], T^n)} \alpha(m(t)) f(m(\cdot)) d\mu_k = \int_{C^0([0,T], T^n)} \alpha(m(t)) f(m(\cdot)) d\mu.$$

Evidently

$$\lim_{k \to \infty} \int_{C^0([0,T], T^n)} [(m(t + \Delta t) - m(t))(m(t + \Delta t) - m(t))^*] f(m(\cdot)) d\mu_k =$$

$$= \int_{C^0([0,T], T^n)} [(m(t + \Delta t) - m(t))(m(t + \Delta t) - m(t))^*] f(m(\cdot)) d\mu.$$

Thus,

$$\lim_{\Delta t \to 0} \int_{C^0([0,T], T^n)} \left[ \frac{(m(t + \Delta t) - m(t))(m(t + \Delta t) - m(t))^*}{\Delta t} - \alpha(m(t)) \right] f(m(\cdot)) d\mu = 0. \quad (12)$$

Since $f(m(\cdot))$ is an arbitrary bounded continuous function that is measurable with respect to $\mathcal{N}_t$, this means that $D_2\xi(t) = \alpha(\xi(t))$. But by construction $\alpha(\xi(t)) \in \alpha(\xi(t)) \mu$-a.s.

Now let us turn to the current velocity of solution.

By construction $D_2\xi_k(t) = v_k(t, \xi_k(t))$ for all $k$. This means that for any real bounded continuous function $f$ on $C^0([0,T], T^n)$, measurable with respect $\mathcal{N}_t$, for any $k$ the equality

$$\lim_{\Delta t \to 0} \int_{C^0([0,T], T^n)} \left[ \frac{m(t + \Delta t) - m(t - \Delta t)}{\Delta t} - v_k(m(t)) \right] f(m(\cdot)) d\mu_k = 0.$$

holds.
Specify an arbitrary \( \varepsilon > 0 \). Since \( \mu_k \) weakly converges to \( \mu \), there exists \( K(\varepsilon) \) such that for \( k > K(\varepsilon) \)

\[
\left\| \int_{C^0([0,T],T^n)} [m(t + \Delta t) - m(t - \Delta t)] f(m(\cdot))d\mu_k - \int_{C^0([0,T],T^n)} [m(t + \Delta t) - m(t - \Delta t)] f(m(\cdot))d\mu \right\| < \varepsilon
\]

and

\[
\left\| \int_{C^0([0,T],T^n)} f(m(\cdot))v(m(t))d\mu_k - \int_{C^0([0,T],T^n)} f(m(\cdot))v(m(t))d\mu \right\| < \varepsilon.
\]

By analogous arguments, as above, we can show that

\[
\lim_{k \to \infty} \int_{C^0([0,T],T^n)} [v_k(m(t)) - v(m(t))]f(m(\cdot))d\mu_k = 0.
\]

and that \( v \) is continuous. Recall that \( v \) is bounded since it is a selector of the bounded set-valued mapping.

Then by Lemma [11, Section VI.1] we obtain that

\[
\lim_{k \to \infty} \int_{C^0([0,T],T^n)} v(m(t))f(m(\cdot))d\mu_k = \int_{C^0([0,T],T^n)} v(m(t))f(m(\cdot))d\mu.
\]

It is obvious that

\[
\lim_{k \to \infty} \int_{C^0([0,T],T^n)} [(m(t + \Delta t) - m(t - \Delta t))]f(m(\cdot))d\mu_k =
\]

\[
= \int_{C^0([0,T],T^n)} [(m(t + \Delta t) - m(t - \Delta t))]f(m(\cdot))d\mu.
\]

Thus,

\[
\lim_{\Delta t \to 0} \int_{C^0([0,T],T^n)} \left[ \frac{(m(t + \Delta t) - m(t - \Delta t))}{\Delta t} - v(m(t)) \right] f(m(\cdot))d\mu = 0.
\]

Since \( f(m(\cdot)) \) is an arbitrary bounded function, measurable with respect to \( N_t \), this means that \( D_S \xi(t) = v(\xi(t)) \). But by construction \( v(\xi(t)) \in v(\xi(t)) \mu\text{-a.s.} \)

\[ \Box \]

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СТОХАСТИЧЕСКИЕ ВКЛЮЧЕНИЯ С ТЕКУЩИМИ СКОРОСТЯМИ, ИМЕЮЩИЕ РАЗЛОЖИМЫЕ ПРАВЫЕ ЧАСТИ

Ю. Е. Гликлих, А. В. Макарова

Получена теорема существования для стохастических дифференциальных включений, заданных в терминах так называемых текущих скоростей (симметрических производных в среднем, прямых аналогов обычных скоростей детерминированных систем) и квадратичных производных в среднем (дающих информацию о коэффициенте диффузии) на плоском n-мерном торе. Правые части и с текущей скоростью, и с квадратичной производной многозначны, полунепрерывны снизу, но не обязательно имеют выпуклые образы. Вместо этого мы предполагаем, что они разложимы.

Ключевые слова: производные в среднем; текущие скорости; разложимые многозначные отображения; дифференциальные включения.

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