Deformation of electron holes in phase space as prerequisite for narrow band maser emission: A qualitative discussion

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A qualitative discussion is given of the role electron holes play in generating fine structure on the electron cyclotron maser radiation. It is argued that electron holes become deformed in phase space when interacting with an incomplete ring or horseshoe distribution which occurs in the presence of strong field aligned electric fields in the upward current region and in the presence of a loss cone. This interaction is based on momentum balance considerations. Deformed narrow electron holes cause steep velocity space gradients on the ring distribution that lead to intense but narrow band emissions from their high speed sides and absorption at slightly higher frequency from their low speed sides. The twins of banded emission and absorption move in frequency space due to the average real space displacement of the deformed electron hole.

It has been firmly established [Ergun et al., 2001] by now that the auroral kilometric radiation – and probably also a number of radio emissions from other celestial objects [Ergun et al., 2000; Begelman et al., 2005; Treumann, 2006] – is due to the (relativistic) electron cyclotron maser in the presence of a magnetic-field-aligned electrostatic potential drop. In its embryonic version [Wu and Lee, 1979; Melrose et al., 1982] the mechanism referred to loss cone distributions. However, the potential drop locally evacuates the plasma making it manifestly underdense with plasma-to-cyclotron frequency ratio \( \omega_c/\omega_{pe} < 1 \) and, at the same time, transforms the electron distribution into a ring in velocity space. Realizing this important modification, Pritchett [1984] put the mechanism back on its feet. In this realistic version, fundamental emission is generated in the \( x \)-mode below but (for not too small frequency ratios) close to the (non-relativistic) cyclotron frequency \( \omega_c = eB/m_e \) and propagates strictly perpendicular to the magnetic field.

Controversy concerns the persistent observation of very narrow-band, very intense emissions of sometimes less than 100 Hz bandwidth, requiring extraordinarily steep positive perpendicular phase-space density gradients \( \partial f_e/\partial v_{\perp} > 0 \) covering a large phase-space volume. The steep gradient is responsible for the narrow bandwidth, while the large phase-space volume assures high emissivity. Both conditions are difficult to maintain simultaneously and globally. Some models of phase space distributions, meeting these conditions and providing marginally high emissivities, have indeed been proposed [cf., e.g., Yoon and Weatherwax, 1998; Pritchett et al., 2002]. These models refer to the global horseshoe distribution imposing severe conditions on its shape. Global phase space gradients as sharp as assumed have, to our knowledge, never been observed. To our opinion the observation of the fast spectral displacement of the emission bands suggests that the radiation sources occupy only a very small volume in real space. We therefore suggested [Pottelette et al., 2001; Pottelette and Treumann, 2005] that the narrow-band spectral fine structure detected on the auroral kilometric radiation is not the result of a global steep gradient in the distribution; rather it is due to emission from ‘elementary radiation sources’. We tentatively identified these with phase-space (ion and electron) holes of the kind of Bernstein-Green-Kruskal (or BGK) modes investigated by Muschietti et al. [1999a, b], Goldman et al. [2003], and others. Justification has been derived from the real-space velocity of the sources deduced from their spectral displacements of the emissions. Even the entire auroral kilometric emission spectrum may be due to the superposition of the contributions from such tiny ‘elementary radiators’ [Pottelette et al., 2001], a proposal that so far is lacking attention.

Existence of phase-space holes in the downward current region is by now a well established observational fact backed by numerous numerical simulations [e.g., Ergun et al., 2002; Goldman et al., 2003]. Observational evidence has also been presented [Pottelette and Treumann, 2005] for the existence of BGK modes in the low-density upward current region.

‘Elementary radiators’ must be electron holes since ion holes do not directly contribute to radiation even though being important in the dynamics of dilute plasmas [Goldman et al., 2003]. Being Debye-scale structures, they are beyond resolution of particle detectors and thus invisible on the distribution function. Their phase space extension is determined by their capability of trapping particles. This is restricted to the range \( \Delta v_H = \pm \sqrt{2\phi_0/m_e} \), where \( \phi_0 \) is the amplitude of the hole potential. Since \( \phi_0 \) is of the order of, say, \( \sim 10 \) mV/m, the hole is a narrow (and most probably also shallow) distortion on the electron distribution of width \( \sim 2\Delta v_H \) (Figure 1b). Any gradients it generates on the distribution function will necessarily be sharp. However, in all models such gradients exist only in the parallel direction. In the following we argue (qualitatively) that electron holes, when suffering deformation in phase space, are capable of providing the required steep perpendicular phase-space density gradients while occupying modestly large phase-space volumes and in this way directly produce radiation.

Electron holes are excited by a Buneman-like instability [cf., e.g., Goldman et al., 2003] between the (downward) energetic ring (or horseshoe) electrons and the (upward) cold ion beam. At a given perpendicular electron velocity \( v_{\perp} = v_{\perp 0} = \text{const} \), the bulk parallel electron velocity of the ring is \( v_\parallel (v_{\perp 0}) = v_R \cos \alpha \); \( v_R \) is the bulk ring velocity, and \( \alpha = \tan^{-1} (v_{\perp 0}/v_\parallel) \) is the angle between velocity and magnetic field \( \mathbf{B} \). For electrons and ions accelerated by the same parallel electric field the ion beam speed is a factor of \( \sqrt{m_i/m_e} \) smaller and can be neglected. The unstable frequency and growth rate of the Buneman instability are
tively, and the unstable wave number satisfies the condition
\[ k_x \sim \beta B / k_B \sim 0.03 \beta \omega_p, \] and \( \omega_p \sim \omega_B, \) respectively, and the unstable wave number satisfies the condition \( k_x (v_x, \alpha) \sim \omega_p / \alpha R \cos \alpha. \) The initial velocity of the hole at any given angle \( \alpha \) is \( v_H (\alpha) \approx \omega_p / k_B \sim 0.03 \beta \omega_p \cos \alpha, \) much less than \( v_F. \) For constant \( v_H \) it decreases with increasing angle \( \alpha. \) The Buneman instability ceases when the parallel electron drift velocity approaches the electron thermal velocity. For a ring (or horseshoe) distribution of given thermal spread \( v_t \), this occurs at some critical angle \( \alpha_c \) above that the instability switches off and the ring excites ion acoustic waves. Thus the shape of an electron hole in velocity space is cosinoidal for \( \alpha < \alpha_c \) as shown in Figure 1a. At larger angles the hole dissolves into trains of ion acoustic waves propagating parallel to the magnetic field at the corresponding ion-sound speeds. Because of this cosinoidal shape an electron hole in phase-space is in fact a two-dimensional bent entity itself exhibiting steep gradients in both directions, parallel and perpendicular to the magnetic field.

These perpendicular gradients already contribute to radiation. However, being located on the low-phase-space density low-velocity side of the distribution, the phase-space gradients are moderate and the phase-space volume is too small for them to provide high emissivities. For this to achieve one wishes the hole to be displaced deep into the bulk of the distribution. Steep gradients evolve for constant hole-phase-space density.

\[ \omega_B \sim \left( m_e / 16 m_i \right)^{1/2} \omega_{pe} \sim 0.03 \omega_{pe} \] and \( \gamma_B \sim \omega_B \), respectively, and the unstable wave number satisfies the condition \( k_x (v_x, \alpha) \sim \omega_p / \alpha R \cos \alpha. \) The initial velocity of the hole at any given angle \( \alpha \) is \( v_H (\alpha) \approx \omega_p / k_B \sim 0.03 \beta \omega_p \cos \alpha, \) much less than \( v_F. \) For constant \( v_H \) it decreases with increasing angle \( \alpha. \) The Buneman instability ceases when the parallel electron drift velocity approaches the electron thermal velocity. For a ring (or horseshoe) distribution of given thermal spread \( v_t \), this occurs at some critical angle \( \alpha_c \) above that the instability switches off and the ring excites ion acoustic waves. Thus the shape of an electron hole in velocity space is cosinoidal for \( \alpha < \alpha_c \) as shown in Figure 1a. At larger angles the hole dissolves into trains of ion acoustic waves propagating parallel to the magnetic field at the corresponding ion-sound speeds. Because of this cosinoidal shape an electron hole in phase-space is in fact a two-dimensional bent entity itself exhibiting steep gradients in both directions, parallel and perpendicular to the magnetic field.

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absorption occurs at a few Hz up to a few 10 Hz above the frequency ωce by the small amount ω⊥ > ω∥ > ωce. The emission/absorption features just above ωce will preferentially be strictly perpendicular, while radiation at slightly higher frequency could originate from slightly beneath spacecraft, the 'elementary radiator' mechanism offers another explanation based on the deformed electron hole. This might be the case for the observed narrow-band emission/absorption features just above ωce in Figure 3 as well as in other places. In the more general case, on the other hand, it will not be possible to map strongly deformed holes to one single resonance ellipse nor resonance circle. In this case their edge regions might piecewise contribute to radiative emission and absorption at different frequencies. Nevertheless, because of the very steep velocity space gradients involved at the different edge pieces (see Figure 2), the emissivity at each of the frequencies might not be small even though only small phase space volumes become involved in the generation of the radiation. Oblique radiation of this kind can much easier escape from the large-scale density cavity into free space due to multiple reflection at the cavity boundaries than strictly perpendicular emissions.

All the narrow emission bands observed so far exhibit a well pronounced even though apparently irregular motion across the spectrum. These spectral displacements have traditionally been identified with the real-space motion of the radiation source [cf., e.g., Pottelette et al., 2001]. In the deformation model such a motion could also be caused by the phase-space displacement. In fact, momentum exchange between fHS and fH is expected to be strongest at small v⊥. Hence, the hole will move initially fastest at low v⊥ reaching its final v⊥-position near v⊥ ∼ 0. Only after having settled there the hole shape will deform at higher v∥. At fixed (final) v∥ the holes will preferentially be strictly perpendicular; while radiation at strictly perpendicular radiation at the frequency should decrease. However, since this effect is small in a weakly relativistic plasma, it can probably be neglected. Therefore, any observed spectral motion of the emission may still be attributed to the real-space displacement of the hole which is calculated from the second moment of the hole distribution function fH as

\[ n_{HH} = \int v_\perp^3 d^2 v_\perp f_H(v_\parallel, v_\perp) \]  

where the hole distribution is subject to the reduced kinetic equation

\[ \frac{df_H}{dt} = -\frac{e}{m_e} E_\parallel(z - z_H) \frac{\partial f_{HS}}{\partial v_\parallel} \bigg|_{v_\perp} \]
which follows from the Vlasov equation and the definition of electron distribution $f_e = f_{HS} - f_H$. $E_{\parallel}$ is the self-consistent parallel electric field centered at the position of the hole, $z_H$ (which itself is a function of time). This equation must be solved for the dynamics of the passing electrons, keeping the density $n_H$ of the hole constant, which follows from $m_e v = -eE_{\parallel}(z - z_H)$ and $v = \dot{z}$ with $\epsilon_0 \nabla \cdot E_{\parallel} = e(n - n_{HS} + \int \text{d}v f_H) \approx e \int \text{d}v f_H$. In order to account for the de-correlation of the hole response the above mentioned condition on the transition time must be worked out. Thus integration extends only over times $t < \omega_{pe}^{-1}$ for particles not becoming trapped. The integral in Eq. (3) is to be taken over the entire deformed shape of the hole in phase space. Hereby the average motion of the hole can be in any direction, upward or downward, depending on the shape and contribution of the hole distribution $f_H$ to the integral.

The present discussion has been entirely qualitative. It supports the view that deformation of electron holes in phase space is responsible for AKR fine structure. So far we have not specified the very process of momentum exchange in phase space. This can only be done by numerical simulation and will be the purpose of a follow-up communication on this subject [C.-H. Jaroschek et al., in preparation, 2007]. The discussion neglected a large number of effects which happen to take place in hole dynamics: particle acceleration and retardation when interacting with the hole, particle trapping, hole stability and internal dynamics, dissipation of the hole electric field, and the various processes of wave excitation in the hole interior and in hole interaction. We are very well aware of the importance of these effects in the required momentum exchange based on the proposed de-correlation. Various authors have investigated one or the other of them, but the important question of hole deformation in phase space has to our knowledge not yet been treated anywhere.

This research is part of a Visiting Scientist Programme at ISSI, Bern. RT thanks the ISSI staff and directors for their hospitality and support. It has also benefitted from a Gay-Lussac-Humboldt award of the French Government. The data access was through the French-Berkeley cooperative program on FAST supported by the French FNST program.

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