Shear Wave Velocity for Evaluation of State of Cohesionless Soils with Fines

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Abstract. The paper concerns evaluation of cohesionless soils containing fines. In clean sands, state of soil is usually quantified by relative density $D_R$ with use of field techniques like static or dynamic probes. However, in cohesionless soils containing considerable amount of fines, relative density alone, which is based solely on void ratio values, is not representative. This results from the fact that in case of cohesionless soil there is no unique intrinsic compressibility line, like it is in case of cohesive soils. Thus state of soil depends not only on void ratio but also state of stress. For this reason it is necessary to look for an alternative means to quantify state of soils with fines. The paper concerns possibility of evaluation of state of soil containing various amount of fines on the basis of shear wave velocity measurement. The idea rests on the fact that void ratio and state of stress are the major factors which contribute to a state of soil and shear wave velocity as well. When measured shear wave velocities are normalised with respect to stresses the resulting values might be strictly correlated to void ratio. To validate this approach, an experimental test programme (based on series of sophisticated triaxial tests) was carried out on four kinds of sandy material containing various amount of fines up to 60%. The experimental data made possible to establish basic correlation between soil states and shear wave velocity for each kind of soil. Normalized shear wave velocity was compared with void ratio and state parameter as well. The obtained results revealed that determination of void ratio on the basis of shear wave velocity in a certain range of fines can be much more adequate than for clean sands. However, if the fines content exceeds certain value, the obtained correlation is no longer as good.

1. Introduction
In cohesionless soils, evaluation of soil state is much more challenging task than for cohesive soils. The main problem in such soils consists in difficulty of getting undisturbed samples. The only reliable technique, which makes possible to obtain undisturbed sample of sand, is a freezing of soil in situ, however it is very expensive it can’t be used for cohesionless soils containing considerable amount of fines, since freeze-thaw cycle in such soils is not reversible with respect to soil structure.

In such a situation there is a need to investigate alternative methods which might ensure reliably evaluation of state of cohesionless soils containing fines. This is especially important in engineering projects where liquefaction of soil is a major concern, because state of soil controls susceptibility to liquefaction.
Some premises for successful application of shear wave velocity measurement in evaluation of a soil state are discussed in the paper. Up to date, the existing experience in the subject concerns mainly clean sands. The paper presents results of shear wave velocity measurement in cohesionless tailings sand containing 10, 17, 36 and 60% of fines. Fines content in this paper is understood as a percent of soil mass passing sieve 0.075 mm (No. 200 according to ASTM). When measured $V_s$ values are normalized with respect to state of stress, it is possible to obtain quite good correlation between normalized shear wave velocity and void ratio. Some remarks concerning accuracy of the method are also given.

2. Description of state of soil
In evaluation of state of soil, it is very important to identify major factors contributing to its value. Void ratio or relative density alone are not sufficient parameters to describe a state of soil. Test results documented by [1] shown that behaviour of dense sand under high pressure can be very similar to the behaviour of loose sand. This fact can be illustrated by characteristics presented in Figure 1 where results of shearing stage of two specimens carried out on clean sand are shown [2]. The specimens had very similar void ratio but various mean effective stress i.e.: 

- $(e = 0.863, p' = 196 \text{ kPa})$
- $(e = 0.849, p' = 98 \text{ kPa})$

![Figure 1. Influence of stress level on undrained response of sand.](image)

The difference in void ratio at the end of consolidation between two specimens is very small (0.014), which justifies the assumption of very similar relative density. The results are shown in the form of deviatoric stress, effective stress paths and pore pressure change. The shown characteristics clearly indicate that increase in stress level results in increase of peak deviatory stress and pore pressure change. Effective stress paths are similar in shape in initial state of shearing but having achieved peak value, they become different. Direct comparison of stress strain characteristics as well as pore pressure change indicate that a specimen consolidated to higher stress exhibits more contractive behaviour than specimen consolidated to smaller stress. This example clearly proves that proper description of soil state should account not only state of density but also for state of stresses.

In the light of the above, it is advisable to adopt a steady state concept and based on it a measure $\psi$ capable of scaling a state of soil accounting for void ratio and state of stress proposed by [3]. The state parameter $\psi$ is a difference between void ratio at an initial state prior to shearing and void ratio at the same mean effective stress but projected on a steady state $e_{ss}$ (Figure 2).

![Figure 2. Definition of state parameter $\psi$.](image)
ψ = e₀ - e₁

3. Premises for use of shear wave velocity measurement to detect soil state

In the last 50 years a great interest can be observed in correlation of body wave velocity to numerous geotechnical parameters. Among various application (e.g. resulting from theory of elasticity) shear wave velocity has a potential to evaluate state of soil. Pioneering work by [4] who first measured shear wave velocity in laboratory showed that $V_s$ is primarily a function of both the void ratio $e$ and mean normal effective stress $p'$. They suggested the following relationship:

$$V_s = (m_1 - m_2 \cdot e) (p')^{0.25}$$  \hspace{1cm} (2)

where:

- $e$ - void ratio
- $p'$ - effective mean normal stress
- $m_1, m_2$ - material constants (represent soil fabric)

Due to the fact that shear wave velocity is propagated through a soil skeleton and depends on void ratio and current stresses, it is conceivable that the actual state of material prior to destructive loading commencement, can be evaluated.

Since 1963 considerable progress has been done in improvement of equation 3. An alternative equation, called individual stress method was proposed by [5], who carried out tests on cubic samples with use of load system enabling independent stress application. Roesler proposed the following formulae:

$$V_s = (m_1 - m_2 \cdot e) (\sigma_a)^{m_a} (\sigma_p)^{m_p}$$  \hspace{1cm} (3)

where:

- $\sigma_a$ - the effective stress in the direction of wave propagation
- $\sigma_p$ - the effective stress in the direction of particle motion
- $m_1, m_2, m_a, m_p$ - constants

It is interesting to note that the third principal effective stress perpendicular to direction of wave propagation and particle motion does not influence shear wave velocity and the sum of exponents of acting stresses is around 0.25 as in the case of equation 3.

Large number of data that have been accumulated during the next years e.g. [2], [6], [7], [8], [9], [10], [11], [12] This created a firm basis for use of shear wave velocity application in recognition of a soil state. However, since void ratio and state of stress contribute to a state of soil, it is necessary to eliminate influence of state of stress on measured value of shear wave velocity. This can be done by normalisation of shear wave velocity with respect to stress. One of the possible approach was proposed by [13].

$$V_{s1} = V_s \cdot \left( \frac{P_a}{\sigma_v} \right)^{0.25}$$  \hspace{1cm} (4)

where:

- $V_{s1}$ - normalised shear wave velocity (m/s)
- $V_s$ - shear wave velocity (m/s)
- $\sigma_v$ - vertical effective stress (kPa)
- $P_a$ - reference pressure (usually 100 kPa)
Another possibility of normalisation shear wave velocity is based on work [14] proposition. This approach accounts for two components of stress state.

\[ V_{S1} = V_S \left( \frac{100}{\sigma_v} \right)^{0.125} \left( \frac{100}{\sigma_h} \right)^{0.125} \]  \hspace{1cm} (5)

where

\( \sigma_v \) - horizontal effective stress (kPa)

The above formulae creates some premises for successful use of shear wave velocity in detecting a state of soil, however it is not certain if this approach can be used to detect state of soil containing fines. In order to verify applicability of this concept for various materials, more consistent data are required for sands with various amount of fines.

### 4. The material and test procedure

To recognize possibility of application shear wave velocity based concept to detect state of soil containing fines, laboratory test programme was undertaken based on triaxial tests where on various stages of consolidation shear wave velocity was measured. For granular material were tested. They had various amount of fines 10, 17, 36 and 60%. Grain size distribution of five tested soils are shown in Figure 3.

![Grain size distribution of tested materials.](image)

Figure 3. Grain size distribution of tested materials.

Triaxial tests were carried out on reconstituted specimens which were 50 mm in diameter and 100 mm in height. Specimen were prepared by moist tamping by under compaction method as described [15]. Moist tamping was selected in order to be able prepare samples of wide range in void ratio. When a triaxial cell was assembled a specimen was saturated by back pressure until Skempton's parameter \( B \) exceeded value 0.96. After saturation specimens were consolidated anisotropically with zero lateral strain, thus the relation between principal effective stresses at the end of consolidation stage depended on initial density of soil. When consolidation was terminated, shear wave velocity was measured. Piezoelectric transducer of bender type was used for sending and receiving the signals.
5. Results for soils containing fines

In order to check to what extent shear wave velocity is capable to reflect state of soil of various fines content the data obtained for all tested material were presented in figure 4.

For each material tested to charts were shown. In chart a, raw results of measured $V_s$ were shown against void ratio, while in chart b a measured shear wave velocity was normalized with respect to vertical effective stress. Eight charts were assembled in one figure 4 in order to compare results for all materials. At the first glance it that materials can be divided into two groups. The first group is represented by sandy materials which contain 10 and 17% of fines. The second group is represented by soils containing 36 and 60% of fines. Regarding charts were raw shear wave velocity $V_s$ is shown for sandy soils, large scatter of data is observed. The same data depicted in charts b where normalized shear wave velocity $V_{s1}$ is shown against void ratio entire different result is obtained. The fact that

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scatter of data is small proves that in sandy soils shear wave velocity can be successfully applied for reflecting soil state.

A different pictures is observed for soils containing 36 and 60% of fines. Here the raw data did not exhibit large as it was for sandy soil. In case of fine grained soils substantial difference can be observed for charts were normalized shear wave velocity is shown. Addition observation is that the results for soil containing 36% of fines can't be put to the same group with the finest tested material. For soil containing 60% of fines no correlation between normalized shear wave velocity and void ratio is observed. Therefore it can concluded that for such fine grained soil resolution of method resting on shear wave velocity is not satisfactory. Especially separation of contribution of state of stress and void ratio to the whole state of soil is doubtful. Other factors e.g. as stress history may play important role here and they are not accounted for.

The results for soil containing 36% are not similar to those for sandy soils and neither the finer material. Analysing chart with raw measurement of shear wave velocity it van be observe that for loose state very good correlation is obtained, while for dense material the scatter of data is significant. In turn for normalized data the accuracy is good with moderate resolution. In conclusion of the above observation it can be stated that soil containing 36% of fines reveals transition behaviour between sands and clays. For wide range of void ratio, what results from specimen preparation method used, the resolution of applying shear wave velocity is sufficient but in some application can be not satisfactory.

6. The role of normalization technique

As mentioned in chapter 3 there are two different approaches to normalization of shear wave velocity. They use one or two components of stress. The first one uses only vertical effective stress, while the second uses vertical and horizontal one. Since both technique are used in elaboration of shear wave velocity data, it is of interest to what extent they influence relation with void ratio. This is especially important in engineering structure where potential problem of static liquefaction might appear. Simplified way of thinking postulates that once the correlation between void ratio and normalized shear wave velocity is settled then single value of $V_s$ correlated to threshold void ratio might be considered as value below which problem of potential liquefaction exists.

In the light of the above consideration, it is worth to make a comparison between shear wave velocity normalized with respect to vertical effective stress $\sigma'_v$ with that where two components of stress ($\sigma'_v, \sigma'_h$) are used. For the sake of better resolution the data were presented in two figures for sandy soil (figure 5) and for fine grained soils (figure 6). The first observation common for all charts presented in both figures is that shear wave velocity data normalized with respect to both stress components plot above those normalized to vertical stress alone. This can be easy explained by fact that coefficient of earth pressure at rest $K_0$ for tested soils is less than 1, so with the fixed sum of exponents, when effective horizontal stress is smaller than vertical one, the calculated normalized shear wave velocity is higher. The second common feature is that data normalized with respect to both stress components have less scatter. It is even more pronounced for fine grained soils, however, it might result from the fact that data for sandy soils have higher value of $R^2$. The third observation concerns the difference between to values of normalization which is bigger for dense material and diminish for high values of void ratio. This fact can also be explained by $K_0$ consolidation in which final effective stress ratio depends of the state of density.

As regards differences between sandy soils and fine grained soils, it is certainly range of absolute values of void ratio change. In presented data these differences are no so pronounced can be not so evident since applied specimen preparation technique i.e. moist tamping by undercompaction artificially changes these ranges and does not replicate natural conditions. In spite of this fact the ranges and absolute values of void ratio changes are smaller for fine grained soils than for sandy soils. Another observation concern the data for fine grained soils presented in figure 6. In spite of the fact that data for shear wave normalized with respect of both components of stress are less scattered, it is quite obvious that regression obtained for soils containing 36% of fines can be considered as having
sufficient resolution while regressions (slope) and scatter data for soils containing 60% of fines is not acceptable.

Figure 5. Normalized shear wave velocity against void ratio for sandy soils.

Figure 6. Normalized shear wave velocity against void ratio for soils containing 36 and 60% of fines.

In order to present the above data in more versatile way, void ratio can be substituted by state parameter $\psi$ which determines if a soil might have contractive or dilative response under loading. Such data are presented in figure 7 and 8 were normalized shear wave velocity (in both ways) are shown against state parameter $\psi$. In principal only the first observation concerning figures 5 and 6 holds true i.e. normalization with respect to both components of stress plots above the one in which only vertical effective stress was considered. In data presented against state parameter, scatter for shear wave velocity normalized with respect to one stress component is smaller than for two components, what is different than for data presented against void ratio. It can be also noticed that regression lines for both ways of normalization are almost parallel. It suggests that susceptibility to liquefaction do not change with stress. This is not true since compressibility line and steady state line for a given material are not parallel. This proves that state parameter $\psi$ as defined by Been and Jefferies does not account for this fact. It implies that normalized shear wave velocity read for state parameter $\psi$ equal 0 can't be considered as threshold value for commencement of contractive response. However, it should also be noticed that these normalized shear wave velocity for $\psi = 0$ are much higher for fine grained soils than for sandy ones.

It should also be emphasised that slope of regression lines for soil containing 36% of fines are definitely bigger than for other materials what confirms transition features between sands and clays. This kind of material deserves more attention in further research.

7. Conclusions
The test programme carried out allowed to formulate the following conclusions:

- State of cohesionless soils, which controls stress strain characteristic of a material, is represented not only by state of density (void ratio) but also depends to large extent on state of stress.
- Shear wave velocity is capable to described state of soil, since it reflects change in density and state of stresses as well, however it is restricted by fines content.
If measured values of shear wave velocity are normalized with respect to stress components, the resulting number can be directly correlated to void ratio or state parameter. The approach to what stress components shear wave velocity is normalized plays an important role in the interpretation of the data. Normalization of shear wave velocity with respect to both stress components seems superior to the one which relies on effective vertical stress only.

Identification of state of soil on the basis of shear wave velocity measurement depends on fines content. For sandy soils containing up to 17% of fines, the resolution of the method is satisfactory. For soils containing 60% of fines, this method of state identification has not sufficient resolution.

Soils containing 36% of fines have transition features. Further research which should be carried on samples prepared by other specimen preparation methods is necessary to verify the capability of shear wave velocity to detect this kind of soil.

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