The $\phi$ meson in nuclear matter with zero and non-zero momentum - recent results

Philipp Gubler
Advanced Science Research Center, Japan Atomic Energy Agency, Tokai, Ibaraki 319-1195, Japan
E-mail: gubler@post.j-parc.jp

Abstract. We review recent studies investigating the $\phi$ meson in nuclear matter from a theoretical point of view. We particularly focus on the most recent results dealing with the momentum dependence of the $\phi$ meson properties in medium, which is one of the targets of the future E16 experiment at J-PARC.

1. Introduction

The $\phi$ meson, whose valence quark structure is composed of a strange and an anti-strange quark, might be modified when inserted into nuclear matter. In the quark language, such a modification happens through interactions between the strange quarks with the surrounding nucleons of the nuclear medium [1, 2, 3, 4]. In terms of hadronic degrees of freedom, the $\phi$ meson consists of a bare state, modified by kaon loop self-energy corrections. In this picture, the in-medium modifications of the $\phi$ are dominated by interactions between kaons and the nuclei of the medium [3, 5, 6, 7, 8, 9].

The most commonly employed experimental approach to study this issue is the use of a collision between some particle (for instance a proton or a pion) and a nucleus of a sufficiently large size. In this collision, a $\phi$ meson is produced in the nuclear medium and subsequently decays into di-leptons which are then experimentally measured. The advantage of this approach is related to the fact that the di-leptons do not feel the strong interaction, therefore are not much disturbed by the presence of the nuclear medium and hence provide a relatively clear signal of the $\phi$ meson in the nucleus. One important issue, however, arises because the $\phi$ meson in such a reaction is almost never produced at rest with respect to the nuclear medium, but with some non-zero velocity. The effect of this velocity has to be well understood for obtaining a thorough understanding of the experimentally observed spectra.

The goal of these proceedings is to review the most recent results concerned with the behavior of the $\phi$ meson in nuclear matter, both with zero and especially with non-zero velocity. We will focus on the approach of QCD sum rules [10, 11], in which this problem can be studied in one consistent framework. The paper is organized as follows. After the introduction given in this Section, we will review the recent QCD sum rule calculations dealing with the $\phi$ meson in nuclear matter in Section 2. The paper is concluded with a brief summary in Section 3.

2. QCD sum rules in nuclear matter and with finite velocity

We start with a review of the QCD sum rule method at finite density. First, we prepare the correlator

$$\Pi_{\mu\nu}(q_0, q) = i \int d^4x e^{i q x} \langle T \left[ j_{\mu}(x) j_{\nu}(0) \right] \rangle_{\rho},$$

(1)
where in the present case \( j_\mu(x) \) is defined as \( j_\mu(x) = \overline{\psi}(x) \gamma_\mu \psi(x) \) as we study the \( \phi \) meson channel. \( \langle \cdots \rangle_\rho \) stands for the expectation value with respect to the nuclear matter ground state, the baryon density being fixed to \( \rho \). \( \Pi_{\mu\nu}(q_0, \mathbf{q}) \) for non-zero momentum \( \mathbf{q} \) can be decomposed into longitudinal and transverse components (see Refs.[12, 13]). We will denote the two independent components as \( \Pi_{L/T}(q_0^2 - \mathbf{q}^2, \mathbf{q}^2) \equiv \Pi_{L/T}(\mathbf{q}^2, \mathbf{q}^2) \). In this notation, the trivial momentum dependence, which in the vacuum is determined by Lorentz symmetry is included in the first variable \( q^2 = q_0^2 - \mathbf{q}^2 \), while the non-trivial momentum dependence, caused by the breaking of Lorentz symmetry due to the existence of a nuclear matter rest frame is parametrized in the second variable \( \mathbf{q}^2 \). For \( q = 0 \), both components are identical. With the help of the analyticity of \( \Pi_{L/T}(\mathbf{q}^2, \mathbf{q}^2) \) in the imaginary plane of \( \mathbf{q}^2 \), one can derive

\[
\Pi_{L/T}(q^2, q^2) = \frac{1}{\pi} \int_0^\infty ds \frac{\text{Im} \Pi_{L/T}(s, q^2)}{s - q^2 - i\varepsilon},
\]

which is the basis of the QCD sum rule approach. QCD sum rules make use of the asymptotic freedom of QCD, meaning that QCD can be treated perturbatively at large \(|q^2|\). Thus, \( \Pi_{L/T}(q^2, q^2) \) in this region can be computed with the help of perturbation theory and the operator product expansion (OPE), which results in a power series in \( 1/q^2 \), with coefficients involving QCD condensates and respective Wilson coefficients, computed as expansions in \( \alpha_s = g^2/(4\pi) \), \( g \) being the coupling constant of QCD. \( \text{Im} \Pi_{L/T}(s, q^2) \) represents the spectral function of the operator \( j_\mu(x) \), involving all physical states that couple to this operator. Obtaining this spectral function is the goal of QCD sum rules.

The general form of the OPE in the \( \phi \) channel considered here (or any other vector channel), can be given as

\[
\Pi_{L/T}(q^2, q^2) = \epsilon^{0,0}_{L/T} \log \left( -\frac{q^2}{\mu^2} \right) + \epsilon^{2,0}_{L/T} \frac{q^2}{q^2} + \epsilon^{4,0}_{L/T} \frac{4q^4}{q^2} + \epsilon^{6,0}_{L/T} \frac{6q^6}{q^2} + \epsilon^{4,2}_{L/T} \frac{q^2}{q^2} + \epsilon^{6,2}_{L/T} \frac{q^4}{q^2} + \epsilon^{8,2}_{L/T} \frac{q^6}{q^2} + \ldots \]

For the coefficients \( \epsilon^{n,m}_{L/T} \), \( n \) represents the dimension and \( m \) the spin of the involved operator. Because of limitations in space and for simplicity, we do not provide their concrete form here. They can however be extracted from Ref.[13]. Note that operators with \( m > 0 \) break Lorentz symmetry and thus generally vanish in vacuum.

To improve the properties of the sum rules, the dispersion relation of Eq. (2) is usually further modified with the help of the Borel transform. We will not go into the details here, but just mention that the application of the Borel transform to Eq. (2) leads to

\[
\Pi_{L/T}^B(M^2, q^2) = \frac{1}{M^2} \int_0^\infty ds e^{-s/M^2} \rho_{L/T}(s, q^2),
\]

where \( \rho_{L/T}(s, q^2) = \frac{1}{\pi} \text{Im} \Pi_{L/T}(s, q^2) \). In Ref.[4], the maximum entropy method (MEM) was used to analyze the \( q = 0 \) case of Eq. (4) (for more details about MEM, see Ref.[14]).

The OPE input at finite density is modified due to changing operator expectation values/condensates. Unfortunately, this density dependence is not well known, especially for operators with higher dimensions. At present, one usually makes use of the linear density approximation, which for the strange quark dimension 3 chiral condensate reads

\[
\langle \bar{s}s \rangle_\rho \simeq \langle \bar{s}s \rangle_0 + \langle N|\bar{s}s|N \rangle \rho = \langle \bar{s}s \rangle_0 + \frac{\sigma_{SN}}{m_s} \rho.
\]
Figure 1. $\phi$ meson mass as a function of density for two typical values of the strange sigma term $\sigma_{sN}$ obtained in the lattice QCD calculations of Refs. [17, 18]. The $\phi$ meson mass is normalized to its vacuum value, while the density is given in units of the nuclear matter density $\rho_0$. Adapted from Ref. [4].

$\sigma_{sN}$ here is the strange nucleon sigma term, while $\langle N \cdots | N \rangle$ represents the expectation value with respect to a one-nucleon state. The linear density approximation ignores correlations between nuclei and only takes into account independent and separate contributions from each nucleon. Besides the condensates already present in the vacuum, new non-scalar condensates can appear at finite density. The following spin-2 (or twist-2) condensates are for instance generated at dimension 4:

$$\langle N \mathcal{F} \mathcal{F} \gamma^\mu iD^\nu s | N \rangle = A_s^2 \left( p^\mu p^\nu - \frac{1}{4} M_N^2 g^{\mu\nu} \right),$$

(6)

$$\langle N \mathcal{F} \mathcal{F} g^{\alpha\beta} G^{\alpha\beta} | N \rangle = A_g^2 \left( p^\mu p^\nu - \frac{1}{4} M_N^2 g^{\mu\nu} \right).$$

(7)

$\mathcal{F} \mathcal{F}$ stands for the following operation on the Lorentz indices: $\mathcal{F} \mathcal{F} A^{\mu\nu} \equiv \frac{1}{2} (A^{\mu\nu} + A^{\nu\mu}) - \frac{1}{4} g^{\mu\nu} A_\alpha^\alpha$. $M_N$ represents the (isospin averaged) nucleon mass, while $p^\mu$ is the nucleon four-momentum ($p^2 = M_N^2$). $A_s^2$ and $A_g^2$ are defined as the first moments of the strange quark and gluon parton distributions of the nucleon. They can be given as follows:

$$A_s^2 = 2 \int_0^1 dx [s(x) + \bar{s}(x)],$$

(8)

$$A_g^2 = \int_0^1 dx x g(x).$$

(9)

A complete list of independent scalar and non-scalar condensates (up to dimension 6) appearing in the OPE of the vector correlator and their respective Wilson coefficients are provided at leading order in $\alpha_s$ in Ref. [13]. Some of them are not well known even at the level of the linear density approximation. Others have been constrained recently with the help of newly reported experimental data [15]. For a recent survey of our current knowledge of these condensates, see Ref. [16].

Let us now turn to the results obtained by analyzing the sum rules. The $\phi$ meson mass is given as a function of density in Fig. 1 for two typical $\sigma_{sN}$ values, which were obtained in earlier lattice QCD calculations [17, 18]. The mass shift at finite density roughly depends linearly on $\sigma_{sN}$, is positive for $\sigma_{sN} < 35$ MeV, and turns negative for $\sigma_{sN} > 35$ MeV. For more details, see Ref. [4].

Next, we consider the case of finite momentum, which was studied already some time ago in Ref. [19]. We will here discuss parts of updated results, that were obtained only recently [20]. The longitudinal and transverse components can behave differently for $q \neq 0$. As shown in Fig. 2, the mass shifts induced...
Figure 2. $\phi$ meson mass as a function of momentum $|q|$ for the longitudinal (red curve) and transverse (blue curve) modes, computed at normal nuclear matter density $\rho_0$.

by finite momentum effects indeed have an opposite sign. It is interesting to note that the mass shifts of the longitudinal and transverse modes are caused by different condensates. The decreasing mass of the longitudinal mode is dominated by the gluonic spin 2 condensate $\langle N | T G_{\mu}^{\alpha} G^{\nu\alpha} | N \rangle$, while the increasing mass of the transverse mode is mainly related to the strange quark spin 2 condensate, $\langle N | T \gamma^\mu iD^\mu s | N \rangle$. For a more detailed discussion about the momentum dependence and the range of reliability of the results shown in Fig. 2, see Ref. [20].

One might wonder whether the splitting between the longitudinal and transverse components could be measured in a real experiment. The two modes should in principle show up in a measured di-lepton spectrum as two distinct peaks if their distance is large enough (e.g. larger than the peak width). Simply increasing the momentum however would not necessarily improve the visibility of the two peaks, as only $\phi$ mesons with a sufficiently small velocity $v$ are affected by finite density effect, because $\phi$s with large $v$ will likely travel outside of the target nucleon before they decay into di-leptons. For a $\phi$ peak width of 15.3 MeV at normal nuclear matter density (reported by the E325 experiment at KEK [21]), the two modes will hence presumably be seen as a single peak with a slightly increasing mass and increasing width with increasing momentum. This situation is illustrated in Fig.3, where we show the behavior of the sum of one longitudinal and two transverse peaks (corresponding to the number of modes in each channel). To be specific, we depict in these plots the peak mass and width, both obtained by fitting the combined peak by a single Breit-Wigner.

3. Summary and conclusions

In these proceedings we have summarized and reviewed recent results of studies investigating the $\phi$ meson in nuclear matter by a QCD sum rule approach. We have especially discussed the non-trivial momentum dependence of the $\phi$ meson spectrum, that is, its splitting into longitudinal and transverse modes and their respective mass shifts with increasing momentum. We found that while the mass shifts of the individual modes can be quite large, they largely cancel in their combined spectrum once broadening effects are considered. What is left is a small positive mass shift and increasing effective width. It remains to be seen whether the E16 experiment at J-PARC [22], which plans to study the momentum dependence of the $\phi$ meson spectrum, will have a good enough precision to observe such effects.

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\[ \Gamma = 15.3 \text{ MeV} \]

preliminary

\begin{align*}
|q| \text{ [GeV]} & \\
\text{longitudinal} & \\
\text{transverse} & \\
\text{one-peak fit} & \\
1.004 & 1.006 & 1.008 & 1.01 & 1.012 & 1.014 & 1.016 & 1.018 & 1.02
\end{align*}

\[ \text{Width of the same single peak fit as a function of momentum } |q|. \]

**Figure 3.** Left plot: Longitudinal (red curve) and transverse (blue curve) modes (same as in Fig.2) and the result of a single peak fit to the sum of one longitudinal and two transverse peaks with a constant width of 15.3 MeV (green curve). Right plot: Width of the same single peak fit as a function of momentum |q|.

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