Quantum statistics and dynamics of nonlinear couplers with nonlinear exchange

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Abstract. In this paper we derive the quantum statistical and dynamical properties of nonlinear optical couplers composed of two nonlinear waveguides operating by second subharmonic generation, which are coupled linearly through evanescent waves and nonlinearly through non-degenerate optical parametric interaction. Main attention is paid to generation and transmission of non-classical light, based on a discussion of the squeezing phenomenon, the normalized second-order correlation function and quasiprobability distribution functions. Initially coherent, number and thermal states of optical beams are considered. In particular, results are discussed with dependence on the strength of the nonlinear coupling relatively to the linear coupling. We show that if the Fock state $|1\rangle$ enters the first waveguide and the vacuum state $|0\rangle$ enters the second waveguide, the coupler can serve as a generator of squeezed vacuum state governed by the coupler parameters. Further, if thermal fields enter initially the waveguides the coupler plays a similar role as a microwave Josephson-junction parametric amplifier to generate squeezed thermal light.

1. Introduction

In quantum optics many simple quantum systems have been examined from the point of view of a completely quantum statistical description, including not only amplitude and intensity (energy) development of such systems, but also higher-order moments and complete statistical behaviour. Such results have fundamental physical meaning for the interpretation of quantum theory [1] and they are useful for applications in optoelectronics and photonics as well. These results can be successfully transferred to more complicated and more practical systems, such as optical couplers composed of two or more waveguides connected linearly by means of evanescent waves. The waveguides used can be linear or nonlinear employing various nonlinear optical processes, such as optical para-

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metric processes, Kerr effect, Raman or Brillouin scattering, etc. Such devices play an important role in optics, optoelectronics and photonics as switching and memory elements for all-optical devices (optical processors and computers). When one linear and the other nonlinear waveguides are employed, we have a nonlinear optical coupler producing non-classical light in a nonlinear waveguide, which can be controlled from the linear waveguide, i.e. one can control light by light. The generation and transmission of non-classical light exhibiting squeezed vacuum fluctuations and/or sub-Poissonian photon statistics in nonlinear optical couplers can further be supported when all the waveguides are nonlinear. The possibility to generate and to transmit effectively non-classical light in this way is interesting especially in optical communication and high-precision measurements where the reduction of quantum noise increases the precision. In the present paper nonlinear couplers have been examined which are composed of linear and nonlinear waveguides [2] (and references therein), with particular attention paid to quantum statistical properties of such devices [3–7] related to quantum noise properties. These devices are useful for the generation and the transmission of non-classical light and new interesting effects can be obtained if phase mismatches are involved [8–10]. Also Schrödinger-cat states can be transmitted through nonlinear couplers [11] and stability analysis of such devices can be performed [12].

Nonlinear co-directional and contradirectional couplers composed of two nonlinear waveguides operating by second harmonic generation or by non-degenerate optical parametric processes can exhibit interesting switching properties [13, 14]. Quantum-consistent description of contrapropagating beams can be developed, which permits one to formulate the problem in the Hamilton formalism [15]. Phase mismatches inside the nonlinear waveguides and between them can be taken into account [10]. Interesting results can be obtained for the quantum statistical properties of nonlinear optical couplers operating by means of Raman and Brillouin scattering [7].

In this paper we continue the investigation of quantum statistical properties of nonlinear couplers composed of two waveguides operating by second subharmonic generation assuming strong coherent pumping and linear exchange of energy between waveguides by means of evanescent waves, however, we additionally take into account the influence of nonlinear coupling of the parametric type of both the waveguides. In section 2 we describe dynamics of the system under discussion together with the solution of the equations of motion, in section 3 we derive squeezing characteristics of generated light, section 4 is devoted to a discussion of sub-Poissonian statistics, section 5 includes results for quasidistribution function and finally we summarize our main conclusions in section 6.

2. Model description and exact solution

Let us consider a system described by the Hamiltonian $\hat{H}$ such that,

$$\frac{\hat{H}}{\hbar} = \sum_{j=1}^{2} \{\omega_j \hat{a}_j^\dagger \hat{a}_j + \lambda_j [\hat{a}_j^{12} \exp (i\mu_j t) + \text{h.c.}]\}$$

$$+ \lambda_3 \{\hat{a}_1 \hat{a}_2 \exp [i\phi_1(t)] + \text{h.c.}\} + \lambda_4 \{\hat{a}_1 \hat{a}_2 \exp [-i\phi_2(t)] + \text{h.c.}\},$$

(1)
where $\hat{a}_1(\hat{a}_1^\dagger)$, $\hat{a}_2(\hat{a}_2^\dagger)$ are annihilation (creation) operators of the fundamental modes in the first and second waveguides having frequency $\omega_1$ and $\omega_2$, respectively, $\mu_j$ are related with the frequency of the second-harmonic modes described classically as strong coherent fields, $\phi_j(t)$, $j = 1, 2$, are related to the difference- and sum-frequencies of modes 1 and 2, respectively, $\lambda_1$ and $\lambda_2$ are nonlinear coupling constants for the second subharmonic generation in the first and second waveguides, respectively, $\lambda_3$ is the coupling constant for linear exchange between waveguides through evanescent waves, $\lambda_4$ is the coupling constant for the nonlinear exchange through simultaneous annihilation or creation of a photon in both the subharmonic modes at the expense of pumping and h.c. denotes the Hermitian conjugate terms (for further details concerning the optical parametric processes, see [16] (Chap. 10)). When $\mu_j = 0$ and only the degenerate term is considered, we have the well-known Hamiltonian, in the interaction picture, for squeezed light generation [17], where $\lambda_1$ (or $\lambda_2$) represents the coupling constant proportional to the quadratic susceptibility, of the second-order nonlinear process (degenerate parametric down-conversion with classical coherent pumping), or the coupling constant proportional to the cubic susceptibility, of the third-order nonlinear process (degenerate four-wave mixing with classical coherent pumping) [18]. If additionally $\phi_1(t) = \phi_2(t) = 0$, Hamiltonian (1) represents a mixture of second subharmonic generation, frequency conversion and parametric amplification in the interaction picture [19–21].

It is important to mention that we treat the problem of propagation in the Hamiltonian formalism neglecting dispersion. Thus in the case that all waves are propagating with the same velocity, time $t$ and space $z$ are related by the velocity of propagation $v$, $z = vt$. Schematically, this Hamiltonian is represented in figure 1.

In fact Hamiltonian (1) can be regarded as a generalization of the models given in [19–24]. For example, if we take both $\lambda_1$ and $\lambda_2$ to be zeros, then we shall be left with the Hamiltonian which describes the back-action evading amplifiers, where

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{figure1.png}
\caption{Scheme of quantum nonlinear coupler with linear and nonlinear coupling formed from two nonlinear waveguides described by the quadratic susceptibility $\chi^{(2)}$. The beams are described by the photon annihilation operators as indicated; $z = vt$ is the interaction length. Both the waveguides are pumped by strong classical coherent waves. Outgoing fields are examined as single or compound modes by means of homodyne detection to observe squeezing of vacuum fluctuations, or by means of a set of photodetectors to measure photon anti-bunching and sub-Poissonian photon statistics in the standard ways.}
\end{figure}
the Hamiltonian in this case can be constructed by combining parametric amplifiers and parametric frequency converters with two different coupling parameters. On the other hand, if we take \( \mu = 0 \) and drop the time dependent phases, then Hamiltonian (1) will be consistent with the Hamiltonian given in [25], where the wave functions for both the number state and coherent state and the Green’s function have been obtained. It is also interesting to point out that Hamiltonian (1) contains ten generators based on the group \( sp(4, R) \), which represents the most general type of the two-mode quadratic Hamiltonian [26]. This will enable us to reconsider the problem from the Lie algebra point of view, where the most general solution for the wave functions may be obtained. For more details, see for example [27–29], where the wave function for some special cases of the above Hamiltonian has been obtained using the Lie algebra technique.

Annihilation and creation operators satisfy the boson commutation relations

\[
[a_i, a_j^\dagger] = \delta_{ij},
\]

where \( \delta_{ij} \) is the Kronecker delta.

The equations of the motion in the Heisenberg picture for Hamiltonian (1) are

\[
\frac{d\hat{a}_1}{dt} = -i\omega_1 \hat{a}_1 - 2i\lambda_1 \hat{a}_1^\dagger \exp(it\mu_1) - i\lambda_3 \hat{a}_2 \exp[-i\phi_1(t)] - i\lambda_4 \hat{a}_2^\dagger \exp[i\phi_2(t)],
\]

\[
\frac{d\hat{a}_2}{dt} = -i\omega_2 \hat{a}_2 - 2i\lambda_2 \hat{a}_2^\dagger \exp(it\mu_2) - i\lambda_3 \hat{a}_1 \exp[i\phi_1(t)] - i\lambda_4 \hat{a}_1^\dagger \exp[i\phi_2(t)].
\]

Substituting \( \hat{a}_1 = \hat{A} \exp[(it/2)\mu_1] \) and \( \hat{a}_2 = \hat{B} \exp[(it/2)\mu_2] \), slowly varying forms of the operators, having the operators \( \hat{a}_j \) as ell as \( \hat{A} \) and \( \hat{B} \) time dependent, equations (3) take the form

\[
\frac{d\hat{A}}{dt} = -i(\omega_1 + \frac{\mu_1}{2})\hat{A} - 2i\lambda_1 \hat{A}^\dagger - i\lambda_3 \hat{B} \exp\left[-i(\frac{\mu_1 + \mu_2}{2})t + i\phi_2(t)\right],
\]

\[
\frac{d\hat{B}}{dt} = -i(\omega_2 + \frac{\mu_2}{2})\hat{B} - 2i\lambda_2 \hat{B}^\dagger - i\lambda_3 \hat{A} \exp\left[-i(\frac{\mu_1 + \mu_2}{2})t + i\phi_1(t)\right].
\]

Equations (4) with their Hermitian conjugates give a closed system of four differential equations with time-dependent coefficients which cannot be solved directly and hence some restrictions should be considered, so that we shall consider \( \phi_1(t) = \frac{1}{2}(\mu_2 - \mu_1)t \) and \( \phi_2(t) = \frac{1}{2}(\mu_2 + \mu_1)t \). Then the solutions of system (4), which yield the relations between input and output modes, can be obtained, after some tedious calculations, as

\[
\hat{a}_1(t) \exp\left(-it\frac{\mu_1}{2}\right) = \hat{a}_1(0)K_1(t) + \hat{a}_1^\dagger(0)L_1(t) + \hat{a}_2(0)M_1(t) + \hat{a}_2^\dagger(0)N_1(t),
\]

\[
\hat{a}_2(t) \exp\left(-it\frac{\mu_2}{2}\right) = \hat{a}_2(0)K_2(t) + \hat{a}_2^\dagger(0)L_2(t) + \hat{a}_1(0)M_2(t) + \hat{a}_1^\dagger(0)N_2(t),
\]
where time-dependent coefficients, which contain all the features of the structure, are given by

\[ K_1(t) = F_1(t) - \frac{i}{2} \left[ |k_+ + k_-|G_1(t) + \left( \lambda_+ \frac{g_2}{g_1} + \lambda_- \right) S(t) \right], \quad (6a) \]

\[ L_1(t) = -\frac{i}{2} \left[ |k_+ - k_-|G_1(t) + \left( \lambda_+ \frac{g_2}{g_1} - \lambda_- \right) S(t) \right], \quad (6b) \]

\[ M_1(t) = \frac{1}{2} \left\{ \left( 1 + \frac{g_2}{g_1} \right) C(t) - i \left[ |\lambda_+ + \lambda_-|G_1(t) + \left( J_+ \frac{g_2}{g_1} + J_- \right) S(t) \right] \right\}, \quad (6c) \]

\[ N_1(t) = \frac{1}{2} \left\{ \left( 1 - \frac{g_2}{g_1} \right) C(t) - i \left[ |\lambda_+ - \lambda_-|G_1(t) + \left( J_+ \frac{g_2}{g_1} - J_- \right) S(t) \right] \right\}, \quad (6d) \]

whereas

\[ K_2(t) = F_2(t) - \frac{i}{2} \left[ |J_+ + J_-|G_2(t) + \left( \lambda_+ \frac{g_2}{g_1} + \lambda_- \right) S(t) \right], \quad (7a) \]

\[ L_2(t) = -\frac{i}{2} \left[ |J_+ - J_-|G_2(t) + \left( \lambda_+ \frac{g_2}{g_1} - \lambda_- \right) S(t) \right], \quad (7b) \]

\[ M_2(t) = \frac{1}{2} \left\{ \left( 1 + \frac{g_2}{g_1} \right) C(t) - i \left[ |\lambda_+ + \lambda_-|G_2(t) + \left( k_+ + k_- \frac{g_2}{g_1} \right) S(t) \right] \right\}, \quad (7c) \]

\[ N_2(t) = \frac{1}{2} \left\{ \left( \frac{g_2}{g_1} - 1 \right) C(t) - i \left[ |\lambda_+ - \lambda_-|G_2(t) + \left( k_+ - k_- \frac{g_2}{g_1} \right) S(t) \right] \right\}. \quad (7d) \]

In the above equations we have defined

\[ \lambda_{\pm} = \lambda_3 \pm \lambda_4, \]
\[ k_{\pm} = \omega_1 + \frac{1}{2} \mu_1 \pm 2\lambda_1, \]
\[ J_{\pm} = \omega_2 + \frac{1}{2} \mu_2 \pm 2\lambda_2, \]
\[ g_1 = k_-\lambda_+ + \lambda_- J_+, \]
\[ g_2 = k_+ \lambda_- + \lambda_+ J_-, \]

and

\[ F_1(t) = \cos (t\bar{\Omega}_1) \cos^2 \theta + \cos (t\bar{\Omega}_2) \sin^2 \theta, \quad (9a) \]
\[ F_2(t) = \cos (t\bar{\Omega}_2) \cos^2 \theta + \cos (t\bar{\Omega}_1) \sin^2 \theta, \quad (9b) \]
\[ G_1(t) = \frac{\sin (t\bar{\Omega}_1)}{\bar{\Omega}_1} \cos^2 \theta + \frac{\sin (t\bar{\Omega}_2)}{\bar{\Omega}_2} \sin^2 \theta, \quad (9c) \]
\[ G_2(t) = \frac{\sin (t\bar{\Omega}_2)}{\bar{\Omega}_2} \cos^2 \theta + \frac{\sin (t\bar{\Omega}_1)}{\bar{\Omega}_1} \sin^2 \theta, \quad (9d) \]
\[ C(t) = \frac{1}{2} \left( \frac{g_1}{g_2} \right)^{1/2} \left[ \cos (t\bar{\Omega}_2) - \cos (t\bar{\Omega}_1) \right] \sin (2\theta), \quad (9e) \]
\[ S(t) = \frac{1}{2} \left( \frac{g_1}{g_2} \right)^{1/2} \left[ \frac{\sin (t\bar{\Omega}_2)}{\bar{\Omega}_2} - \frac{\sin (t\bar{\Omega}_1)}{\bar{\Omega}_1} \right] \sin (2\theta), \quad (9f) \]
where we have introduced the abbreviations
\[ \theta = \frac{1}{2} \tan^{-1} \left( \frac{2(g_{1} g_{2})^{1/2}}{J_{-} J_{+} - k_{-} k_{+}} \right), \]
\[ \bar{\Omega}_1 = \left[ \Omega_1^2 \cos^2 \theta + \Omega_2^2 \sin^2 \theta - (g_{1} g_{2})^{1/2} \sin (2\theta) \right]^{1/2}, \]
\[ \Omega_2 = \left[ \Omega_1^2 \cos^2 \theta + \Omega_2^2 \sin^2 \theta + (g_{1} g_{2})^{1/2} \sin (2\theta) \right]^{1/2}, \]
with \( \Omega_1^2 = \lambda_{-} \lambda_{+} + k_{-} k_{+} \) and \( \Omega_2^2 = \lambda_{-} \lambda_{+} + J_{-} J_{+} \).

One can see from this solution that when \( \bar{\Omega}_1 \) and \( \bar{\Omega}_2 \) are real, the coupler switches the energy between the modes which propagate inside since the solution will include trigonometric functions [19]. Nevertheless, if \( \bar{\Omega}_1 \) and \( \bar{\Omega}_2 \) are pure imaginary, the Heisenberg solutions attribute hyperbolic functions, which grow rapidly with time, and the coupler operates as an amplifier for the input modes [30]. Thus the behaviour of the coupler will be indicated essentially by the relation between coupling constants.

For the time-dependent coefficients, we can easily obtain the following relations
\[ |K_j(t)|^2 + |M_j(t)|^2 = 1 + |L_j(t)|^2 + |N_j(t)|^2; \quad j = 1, 2, \] (11a)
\[ K_1(t)N_2(t) + M_1(t)L_2(t) = N_1(t)K_2(t) + L_1(t)M_2(t), \] (11b)
\[ K_1(t)M^*_2(t) + M_1(t)K^*_2(t) = L_1(t)N^*_2(t) + N_1(t)L^*_2(t), \] (11c)
in correspondence to boson commutation rules (2).

In the following, we shall employ the results obtained in the present section to treat the squeezing phenomenon, normalized second-order correlation function, as well as quasiprobability distribution functions for the model under consideration.

3. Squeezing phenomenon

Squeezing is a pure non-classical phenomenon and squeezed states have less noise in one field quadrature than a coherent state. On the other hand, this means that there is an excess of noise in conjugate quadrature, since the product of canonically conjugate variances must satisfy the uncertainty relation. This light has a lot of applications, e.g. in optical communication networks [31], in interferometric techniques [32] and in optical waveguide tap [33]. Generation of squeezed light has been observed in many optical processes [34, 35]. Investigation of the squeezing properties of the radiation field is a central topic in quantum optics which can be measured by homodyne detection where the signal is superimposed on a strong coherent beam of the local oscillator.

For this purpose we define the position and momentum operators, which are related to the conjugate electric and magnetic field operators \( \hat{E} \) and \( \hat{H} \) of an electromagnetic field, for each mode in terms of \( \hat{a}_j(t) \) and \( \hat{a}^\dagger_j(t) \) as
\[ \hat{X}_j(t) = \frac{1}{2} \left[ \hat{a}_j(t) \exp \left( \frac{i t \mu_j}{2} \right) + \hat{a}^\dagger_j(t) \exp \left( -\frac{i t \mu_j}{2} \right) \right], \] (12)
\[ \hat{Y}_j(t) = \frac{1}{2i} \left[ \hat{a}_j(t) \exp \left( \frac{i t \mu_j}{2} \right) - \hat{a}^\dagger_j(t) \exp \left( -\frac{-i t \mu_j}{2} \right) \right], \] (13)
where we have considered $j(t)$ to be the phase of the local oscillator, without loss of generality, to cancel the high frequency terms, and $j = 1, 2$ stands for mode 1 and mode 2, respectively. These operators satisfy the commutation relations

$$[\hat{X}_j(t), \hat{Y}_j(t)] = \frac{i}{2},$$

so that the uncertainty relations are

$$\Delta \hat{X}_j(t) \Delta \hat{Y}_j(t) \geq \frac{1}{4},$$

with $\Delta \hat{X}_j(t) = [\langle \hat{X}_j^2(t) \rangle - \langle \hat{X}_j(t) \rangle^2]^{1/2}$.

One of the following squeezing conditions for each mode can occur,

$$S_j(t) = 4\Delta \hat{X}_j(t)^2 - 1 < 0,$$

$$Q_j(t) = 4\Delta \hat{Y}_j(t)^2 - 1 < 0,$$

i.e. negative values of these quantities express squeezing of vacuum fluctuations. Here we study the squeezing phenomenon when the modes are initially prepared in thermal states (or in number states since both of these two cases, number states and thermal states, have identical quadrature variances) with the average thermal photon numbers $\bar{n}_j$, $j = 1, 2$ as well as in the coherent states. More details on the evolution of thermal light in the model under discussion will be adopted in section 5. Now for the quantities $S_j(t)$ and $Q_j(t)$, provided that both the modes are initially in the thermal states, we have for the first mode the following expressions

$$S_1(t) = 2\bar{n}_1 [L_1(t)^2 + |K_1(t)|^2] + 2\bar{n}_2 [N_1(t)^2 + |M_1(t)|^2] + 2[L_1(t)^2 + N_1(t)^2] + (2\bar{n}_1 + 1)[L_1(t)K_1(t) + c.c.] + (2\bar{n}_2 + 1)[M_1(t)N_1(t) + c.c.],$$

$$Q_1(t) = 2\bar{n}_1 [L_1(t)^2 + |K_1(t)|^2] + 2\bar{n}_2 [N_1(t)^2 + |M_1(t)|^2] + 2[L_1(t)^2 + N_1(t)^2] - (2\bar{n}_1 + 1)[L_1(t)K_1(t) + c.c.] - (2\bar{n}_2 + 1)[M_1(t)N_1(t) + c.c.],$$

where c.c. denotes the complex conjugate terms. The corresponding expressions for the second mode can be obtained from (17) and (18) by using the interchange $1 \leftrightarrow 2$. However, the other expressions related to the injection coherent light initially in the coupler are the same as (17) and (18) but we have to put $\bar{n}_j = 0$.

It is known that the nonlinear coupler is a source of optical fields, the statistical properties of which are changed as a result of the linear and nonlinear interaction inside and between waveguides. Consequently, one can generate non-classical light from one input and, in addition, it can be switched.

We have plotted $S_1(t)$, $Q_1(t)$ in figures 2 (a) and (b) and $S_2(t)$, $Q_2(t)$ in figures 3 (a) and (b), when the initial light is coherent, for different values of $\lambda_k$. Further we have chosen $\lambda_3 = 1$ for all curves and for curve A: $\lambda_1 = \lambda_2 = \lambda_4 = 0.25$; for the curve B: $\lambda_1 = \lambda_2 = \lambda_4 = 0.20$, and for the curve C: $\lambda_1 = 0.17$, $\lambda_2 = \lambda_4 = 0.2$. On the other hand, figure 2 (c) gives $S_1(t)$ (first mode) when the initial light is thermal light with coupling constants as those for curve C, where $\bar{n}_1 = 0.5$ and $\bar{n}_2 = 0.5$ (solid curve), 1.5 (dashed curve); and the straight line shows the bound of squeezing of the curves. Firstly, we start our discussion by studying the case of input coherent light. From these figures we can see how the coherent states, which
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2.2.1.0.0.-0.40

Figure 2. Squeezing phenomenon for mode 1 when the modes are initially in coherent light and in thermal light. For initial coherent light: (a) for the first component $S_1(t)$; (b) for the second component $Q_1(t)$; $\lambda_3 = 1$ for all curves; curves A, B and C correspond to $\lambda_1 = \lambda_2 = \lambda_4 = 0.25$, $\lambda_1 = \lambda_2 = \lambda_4 = 0.2$ and $\lambda_1 = 0.17$, $\lambda_2 = \lambda_4 = 0.2$, respectively. For initial thermal light: (c) the first component $S_1(t)$ with $\bar{n}_1 = 0.5$, $\bar{n}_2 = 0.5$ (solid curve), 1.5 (dashed curve) and the coupling constants $\lambda_j$ are the same as those for curve C when the light is initially coherent; a straight line has been put to show the bound of squeezing.

are minimum-uncertainty states, evolve in the coupler to produce squeezed light. We can observe the oscillatory behaviour in these curves, showing that squeezing can be switched from one waveguide to the other over the course of time during power transfer. Moreover, squeezing can be interchanged between the two quadratures of the same waveguide. More precisely, for model 1, squeezing can occur for all selected values of $\lambda_k$ in $S_1(t)$, but in $Q_1(t)$ only curves A and B can exhibit squeezing, as shown in figures 2(a) and (b), which reflects the dependence of non-classical behaviour on the strength of subharmonic generation. For mode 2 we can see squeezing in all curves in both the quadratures, as shown in figures 3(a) and (b). It can be easily seen that the amount of squeezing is sensitive to the strength of coupling $\lambda_k$ and that in general its values in the second component are more pronounced than those in the first one. Now if we turn our attention to the case of injected thermal light, i.e. figure 2(c), we can observe that squeezing is available for the large interaction time. Further, $S_1(t)$ exhibits oscillatory behaviour and it evolved from unsqueezed values during the short range of interaction time, owing to the fact that thermal states are not minimum-uncertainty states,
Figure 3. Squeezing phenomenon for mode 2: (a) for the first component $S_2(t)$; (b) for the second component $Q_2(t)$; the vaules of the parameters $\lambda_k$ are as in figure 2.
into squeezed values and eventually unsqueezed values can be recovered. Indeed, we noted numerically that this behaviour is periodically recovered with time. Moreover, by comparing the dashed curve with the solid one, we can see that an increase of the photon number in the second waveguide causes a decrease of the amount of squeezing in the first one. This is related to the effect of evanescent waves between waveguides and shows how one can control light by light in the coupler. Finally, we can conclude that by controlling the input average thermal photon number and the interaction time (or on the length of the coupler), the interaction under consideration can generate squeezed thermal light. It is worthwhile to refer to [36], where more discussions related to squeezed thermal states are given. Furthermore, squeezing of the thermal radiation field already has been produced in a microwave Josephson-junction parametric amplifier [37], where a thermal input field was introduced to the squeezing device and the generated field exhibited noise reduction.

4. Second-order correlation function

Starting with the experiment of Hanbury-Brown and Twiss, strong interest in the photon-counting statistics of optical fields began. Traditional diffraction and interference experiments and spectral measurements may be considered as being performed in the domain of one photon or linear optics. The theory of higher-order optical phenomena, described by higher-order correlation functions of the electromagnetic field, was founded by Glauber [38], who introduced the measure of super-Poissonian statistics (classical phenomenon) and sub-Poissonian statistics (non-classical phenomenon) of photons in any state, which is given by the normalized normal second-order correlation function defined as

\[ g_j^{(2)}(t) = \frac{\langle \hat{a}_j\dagger(t)\hat{a}_j^2(t) \rangle}{\langle \hat{a}_j\dagger(t)\hat{a}_j(t) \rangle^2} = 1 + \frac{\langle (\Delta \hat{n}_j(t))^2 \rangle - \langle \hat{a}_j\dagger(t)\hat{a}_j(t) \rangle}{\langle \hat{a}_j\dagger(t)\hat{a}_j(t) \rangle^2}, \]  

(19)

where the subscript \( j \) relates to the \( j \)th mode and \( \langle (\Delta \hat{n}_j(t))^2 \rangle \) are the photon number variances, which can be obtained from the relation

\[ \langle (\Delta \hat{n}_j(t))^2 \rangle = \langle (\hat{a}_j\dagger(t)\hat{a}_j(t))^2 \rangle - \langle \hat{a}_j\dagger(t)\hat{a}_j(t) \rangle^2. \]  

(20)

Then it holds that \( g_j^{(2)}(t) < 1 \) for sub-Poissonian distribution of photons, \( g_j^{(2)}(t) > 1 \) for super-Poissonian distribution of photons and when \( g_j^{(2)}(t) = 1 \) Poissonian distribution occurs. The degree of coherence \( g_j^{(2)}(t) \) can be measured by a set of two detectors. An application of radiation exhibiting the sub-Poissonian statistics to optical communications has been considered in [39].

The most familiar quantum states from the earlier days of quantum mechanics are coherent and number states. Following the development of the quantum theory of radiation and with the advent of the laser, the coherent states of the field, that mostly describe a classical electromagnetic field, were widely studied. These states are minimum-uncertainty states and have a Poissonian distribution of photons and they may be evolved in the nonlinear optical coupler to generate non-classical
light. On the other hand, number states are purely non-classical states (they always exhibit sub-Poissonian statistics) and there is great interest for their preparation and quantum non-demolition detection \[40-42\], because they exhibit the maximum channel capacity, i.e. they provide the maximum of information that can be transmitted by a single photon, and the minimum time-energy product in optical communications \[43\].

Here we shall study the intensities of the field as well as the normalized normal second-order correlation function for mode 1 when both the modes are initially in the coherent states \(|\alpha\rangle_1, |\beta\rangle_2\) or in the number states \(|n\rangle_1, |m\rangle_2\). Then the photon number variance in the coherent state is given by

\[
\langle(\Delta \hat{n}_j(t))^2\rangle_{\text{coh}} = \left[V_1^{(j)}(t) + 4|V_4^{(j)}(t)|^2 \right]|\alpha|^2 + \left[V_2^{(j)}(t) + 4|V_5^{(j)}(t)|^2 \right]|\beta|^2 
+ \left[|V_7^{(j)}(t)|^2 + |V_6^{(j)}(t)|^2 \right](|\alpha|^2 + |\beta|^2) + \left[|V_4^{(j)}(t)|^2 + 2|V_5^{(j)}(t)|^2 \right] 
+ 2|V_6^{(j)}(t)|^2 + \{\alpha^2[2V_1^{(j)}(t)V_6^{(j)}(t) + 2V_7^{(j)}(t)V_6^{(j)}(t)] 
+ \beta^2[2V_2^{(j)}(t)V_5^{(j)}(t) + V_6^{(j)}(t)V_6^{(j)}(t)] + \alpha \beta[V_1^{(j)}(t)V_6^{(j)}(t)] 
+ 2V_4^{(j)}(t)V_7^{(j)}(t) + 2V_5^{(j)}(t)V_6^{(j)}(t) + V_2^{(j)}(t)V_6^{(j)}(t)] 
+ \alpha^* \beta[V_1^{(j)}(t)V_7^{(j)}(t) + V_4^{(j)}(t)V_6^{(j)}(t) + 2V_5^{(j)}(t)V_6^{(j)}(t)] 
+ V_2^{(j)}(t)V_7^{(j)}(t)\} + \text{c.c.},
\] (21)

while the expectation value of the photon number is

\[
\langle \hat{a}_j^\dagger(t)\hat{a}_j(t) \rangle_{\text{coh}} = |\alpha|^2V_1^{(j)}(t) + |\beta|^2V_2^{(j)}(t) + V_3^{(j)}(t) 
+ \alpha^2V_4^{(j)}(t) + \beta^2V_5^{(j)}(t) + \alpha \beta V_6^{(j)}(t) + \alpha^* \beta V_7^{(j)}(t) + \text{c.c.}
\] (22)

For an initial number state we find the photon number variance in the form

\[
\langle(\Delta \hat{n}_j(t))^2 \rangle_n = 2|V_4^{(j)}(t)|^2(n^2 + n + 1) + 2|V_5^{(j)}(t)|^2(m^2 + m + 1) 
+ (|V_6^{(j)}(t)|^2 + |V_7^{(j)}(t)|^2)(n + m + 2mn),
\] (23)

while the expectation value of the photon number is

\[
\langle \hat{a}_j^\dagger(t)\hat{a}_j(t) \rangle_n = nV_1^{(j)}(t) + mV_2^{(j)}(t) + V_3^{(j)}(t),
\] (24)

where

\[
V_1^{(j)}(t) = |K_j(t)|^2 + |L_j(t)|^2, \quad (25a)
V_2^{(j)}(t) = |M_j(t)|^2 + |N_j(t)|^2, \quad (25b)
V_3^{(j)}(t) = |N_j(t)|^2 + |L_j(t)|^2, \quad (25c)
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\[ V_4^{(j)}(t) = K_j(t)L_j(t), \]  
\[ V_5^{(j)}(t) = M_j(t)N_j(t), \]  
\[ V_6^{(j)}(t) = K_j(t)N_j(t) + L_j(t)M_j(t), \]  
\[ V_7^{(j)}(t) = M_j(t)K_j(t) + N_j(t)L_j(t), \]

and \( j = 1, 2 \) corresponding to first and second mode, respectively.

It is important to study the evolution of the mean photon numbers (intensities) \( \langle a_j^\dagger(t)a_j(t) \rangle \) inside the waveguides of the coupler to visualize how the energy is exchanged between the waveguides. For this purpose we show figure 4 in which the mean photon number (22) of the beams is plotted against time \( t \) for values of the parameters shown. The solid and dashed curves are related to the first and second beams, respectively. We note that essential to the behaviour of the coupler under consideration is the relation of powers of the linear \( \lambda_3 \) and nonlinear \( \lambda_4 \) coupling constants. To be more specific, for \( \lambda_3 > \lambda_4 \) (figure 4(a)), the intensities evolve oscillatory with time \( t \), which means that the periodic power transfer occurs between waveguides and the coupler operates as an optical switcher. Further, at certain values of time, corresponding to intersections of the two curves, all energy in the coupler becomes equally shared between the propagating modes. However, for \( \lambda_3 < \lambda_4 \) (figure 4(b)), the initial intensities are amplified over the course of time and the coupler operates as an amplifier for input modes.

![Figure 4(a). Mean photon number against time t for mode 1 (solid curve) and mode 2 (dashed curve) when both modes are initially in the coherent states with \( \alpha = 20, \beta = 5, \lambda_1 = 0.17, \lambda_2 = 0.2, \lambda_3 = 1: \lambda_4 = 0.2 \).](image-url)
A similar behaviour is expected for the normalized normal second-order correlation function for mode 1 if initially both the modes are in coherent states (figure 5 for values of the parameters shown). In other words, for $\lambda_3 > \lambda_4$, we observe that $g_1^{(2)}(t)$ has oscillatory behaviour between Poissonian and super-Poissonian statistics, i.e. coherent light can be approximately recovered at certain values of time. This behaviour is independent of the initial amplitudes of the input light (compare solid and dashed curves). On the other hand, for $\lambda_3 < \lambda_4$, the oscillatory behaviour disappears and the fields begin to be localized in the waveguides into which they were initially launched. The interesting point, which could be realized here, is that there is a possibility to generate sub-Poissonian light from the initial Poissonian light input into the coupler provided that $\alpha > \beta$ (figure 5(b)).

The situation will be quite different if we inject initially number states in the coupler, as is illustrated in figure 6, where we see that the initial sub-Poissonian statistics for the Fock state are not recovered with the progress of time $t$ and super-Poissonian statistics dominate. However, $g_1^{(2)}(t)$ exhibits oscillatory behaviour under the condition provided that the linear coupling is stronger than the non-linear coupling (figure 6(a)).

We can conclude that this structure can be used to generate non-classical light from classical light, e.g. coherent light, by controlling the device design and the initial input field. Of course, this is based on the fact that when electromagnetic fields are guided inside the structure, exchange of energy between the two waveguides is possible because of the evanescent field between the waveguides [44].
Figure 5. Normalized normal second-order correlation function $g_1^{(2)}(t)$ for mode 1 when both the modes are initially in the coherent states with $\alpha = 5$, $\beta = 20$ (solid curve) and $\alpha = 20$, $\beta = 5$ (dashed curve): (a) for both curves $\lambda_1 = 0.17$, $\lambda_2 = 0.2$, $\lambda_3 = 1$ and $\lambda_4 = 0.2$; (b) for both curves $\lambda_1 = 0.17$, $\lambda = 0.2$, $\lambda_3 = 1$ and $\lambda_4 = 2$. 
Figure 6. Normalized normal second-order correlation function $g_1^{(2)}(t)$ for mode 1 when both the modes are initially in the number states with $n = 5$, $m = 50$ (solid curve) and $n = 50$, $m = 5$ (dashed curve): (a) $\lambda_k$ have the same values as in figure 5(a); (b) $\lambda_k$ have the same values as in figure 5(b).
5. **Quasiprobability functions**

Here we shall continue in our investigation for the statistical properties of the system under discussion on the basis of quasiprobability distribution functions for compound modes when both the modes are initially in number, coherent and thermal states.

There are three types of these functions: Wigner W-, Husimi Q- and Glauber P-functions. These functions give a complete description for the statistical properties of a microscopic system and provide insight into the non-classical features of the radiation fields. For example, the density operator for the quantum mechanical system can be expressed in terms of them and the various moments of the system operators may be obtained by appropriate integration in phase space using these functions [16]. Furthermore, these quasidistributions can be determined in homodyne tomography [45].

On the other hand, as we have mentioned before, propagation of waves inside the nonlinear directional coupler causes energy exchange between the waveguides owing to the evanescent waves and hence if the measurement of an observable in the first waveguide is performed, this projects the state of the other waveguide into a new state; so it would be convenient to consider in our investigation not only the joint quasiprobability functions but also these functions for single modes.

The starting point for our analysis is the s-parametrized characteristic function which is complex in its nature and may be used also to generate the different moments of the quantum system by means of differentiation. The two-mode s-parametrized characteristic function is given by

\[
C^{(2)}(\zeta_1, \zeta_2, s, t) = \text{Tr} \left\{ \hat{\rho}(0) \exp \left[ \frac{s}{2} |\zeta_i|^2 + \zeta_i \hat{a}_i^\dagger(t) - \zeta_i^* \hat{a}_i(t) \right] \right\},
\]

where \( s \) takes on values 1, 0 and -1 corresponding to normally, symmetrically and anti-normally ordered characteristic functions, respectively, \( \hat{\rho}(0) \) is the initial density operator for the model and \( \text{Tr} \) denotes trace of the operator.

The s-parametrized quasiprobability distribution functions are defined as the Fourier transform of the s-parametrized characteristic function by

\[
W^{(2)}(\alpha_1, \alpha_2, s, t) = \frac{1}{\pi^4} \int d^2 \zeta_1 d^2 \zeta_2 C^{(2)}(\zeta_1, \zeta_2, s, t) \exp \left[ \sum_{i=1}^2 (\alpha_i \zeta_i^* - \alpha_i^* \zeta_i) \right],
\]

where \( C^{(2)}(\zeta_1, \zeta_2, s, t) \) is given by (26). When \( s = 1, 0, -1 \), equation (27) gives formally \( P- \), \( W- \) and \( Q- \) functions, respectively.

The corresponding single-mode s-parametrized characteristic and quasiprobability functions are

\[
C^{(1)}(\zeta_j, s, t) = \text{Tr} \left\{ \hat{\rho}(0) \exp \left[ \frac{s}{2} |\zeta_i|^2 + \zeta_i \hat{a}_i^\dagger(t) - \zeta_i^* \hat{a}_i(t) \right] \right\},
\]

\[
W^{(1)}(\alpha_j, s, t) = \frac{1}{\pi^2} \int d^2 \zeta_j C^{(1)}(\zeta_j, s, t) \exp (\alpha_j \zeta_j^* - \zeta_j^* \alpha_j), \quad j = 1, 2.
\]

The superscripts (1) and (2) in the above equations stand for single-mode case and two-mode case, respectively.

The various moments of the bosonic operators for the system, using the characteristic functions and quasiprobability functions, in the normal form (N),
anti-normal form (A) and symmetrical form (S), corresponding to \( s = 1, -1, 0 \), respectively, can be obtained by

\[
\langle \prod_{j=1}^{2} \hat{a}_j^{m_j}(t) \hat{a}_j^{n_j}(t) \rangle \bigg|_{N,A,S} = \prod_{j=1}^{2} \frac{\partial^{m_j+n_j}}{\partial \xi_j^{m_j} \partial (-\zeta_j)^{n_j}} C^{(2)}(\zeta, s, t)_{s=1,-1,0|\xi=\zeta=0}
\]

\[
= \int W^{(2)}(\alpha, s, t)_{(s=1,-1,0)} \prod_{j=1}^{2} \alpha_j^{m_j} \alpha_j^{n_j} d^2 \alpha_j,
\]

where \( n_j, m_j \) are positive integers, \( \zeta = (\zeta_1, \zeta_2), \alpha = (\alpha_1, \alpha_2) \), and the integral is taken over \( \alpha_1, \alpha_2 \) in phase space. For example, when \( n_1 = m_1 = 1 \) and \( n_2 = m_2 = 0 \), then \( (\hat{a}_1^n(t) \hat{a}_1^n(t))_N = (\hat{a}_1^n(t) \hat{a}_1^n(t))_A = (\hat{a}_1^n(t) \hat{a}_1^n(t))_S = \frac{1}{2} (\hat{a}_1^n(t) \hat{a}_1^n(t) + \hat{a}_1^n(t) \hat{a}_1^n(t)) \). The formula (30) is valid for the single and compound modes owing to the normalization of quasiprobability functions and taking into account that the single mode characteristic function can be obtained from that for two modes by simply setting one of the parameters \( (\zeta_1 \text{ or } \zeta_2) \) equal to zero.

5.1. Input Fock states

It is known that the nonlinear directional coupler is an important optical device for generating non-classical light in the context of control of light in the nonlinear medium. So the initial input light has a direct relation to the output light. In fact, investigation of output light from the coupler, when the number states are initially injected [43, 46–48], was paid little attention compared with the injected coherent states. However, some interesting results have been extracted by considering such a situation [46, 48]. For example, we can mention, in the linear directional coupler, displaced number states can be generated if a number state enters waveguide 1 and a strong coherent field enters waveguide 2 [46]; also a coherent state has been obtained in the non-degenerate optical parametric symmetric coupler when one of the modes enters the coupler in the Fock state [1] and the other modes are in vacuum states [48]. Here we shall turn our attention to deduce the quasiprobability functions for Hamiltonian (1) when the two modes are initially uncorrelated and enter the coupler in number states. Of course, this will give general formulas having wide applicability for special cases [6, 19–21] by appropriate choice of the parameters. It is important to mention that some of these special cases have not been considered before [6, 19–21].

The density operator for two-mode number states is

\[
\hat{\rho}_n(0) = |n\rangle_1 \langle n| \otimes |m\rangle_2 \langle m|.
\]

Inserting (31) into (26), the two-mode \( s \)-parametrized characteristic function takes the form

\[
C^{(2)}_{n,m}(\zeta_1, \zeta_2, s, t) = \exp \left[ \frac{s}{2} \left( |\zeta_1|^2 + |\zeta_2|^2 \right) - \frac{1}{2} (|\eta_1(t)|^2 + |\eta_2(t)|^2) \right]

\times L_n(|\eta_1(t)|^2) L_m(|\eta_2(t)|^2),
\]

where
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\[ \eta_1(t) = \zeta_1 K_1(t) - \zeta_1^* L_1(t) + \zeta_2 M_2(t) - \zeta_2^* N_2(t), \quad (33a) \]
\[ \eta_2(t) = \zeta_1 M_1(t) - \zeta_1^* N_1(t) + \zeta_2 K_2(t) - \zeta_2^* L_2(t), \quad (33b) \]

and \( L_n \) represents the Laguerre polynomial.

Equations (32) and (27) yield the Wigner function for the two-mode number states; after some manipulations, we have the following expression

\[
W^{(2)}_{n,m}(\alpha_1, \alpha_2, s = 0, t) = \frac{4}{\pi^2} (-1)^{(n+m)} L_n(4|A_1(t)|^2) L_m(4|A_2(t)|^2) 
\times \exp \left[ -2(|A_1(t)|^2 + |A_2(t)|^2) \right],
\]

where

\[
\begin{align*}
A_1(t) &= \alpha_1 K_1(t) - \alpha_1^* L_1(t) + \alpha_2 M_2(t) - \alpha_2^* N_2(t), \\
A_2(t) &= \alpha_1 M_1(t) - \alpha_1^* N_1(t) + \alpha_2 K_2(t) - \alpha_2^* L_2(t).
\end{align*}
\]

Equation (34) cannot be factorized owing to the intermodal correlation between the propagating modes inside the coupler and this is clear since (34) includes terms like \( \alpha_1 \alpha_2, \alpha_1^* \alpha_2, \) etc.

The single-mode \( s \)-parametrized characteristic function for the first mode can be obtained, in a similar way as for the two-mode case, from (28) as

\[
C^{(1)}_{n,m}(\zeta_1, s, t) = \exp \left[ \frac{s}{2} \left| \zeta_1 \right|^2 - \frac{1}{2} \left( |\nu_1(t)|^2 + |\nu_2(t)|^2 \right) \right] L_n(|\nu_1(t)|^2) L_m(|\nu_2(t)|^2),
\]

where

\[
\begin{align*}
\nu_1(t) &= \zeta_1 K_1^*(t) - \zeta_1^* L_1(t), \\
\nu_2(t) &= \zeta_1 M_1^*(t) - \zeta_1^* N_1(t).
\end{align*}
\]

Inserting (36) into (29), carrying out the integration and taking \( s = 0 \) and \( s = -1 \), the W-function and Q-function for the single-mode case can be obtained:

\[
W^{(1)}_{n,m}(\alpha_1, s, t) = \frac{2(n!m!)}{\pi[(\tau(t) - s)^2 - 4|\psi(t)|^2]^{1/2}} 
\times \exp \left[ \frac{(\alpha_1 \exp (-i\epsilon/2) - \alpha_1^* \exp (i\epsilon/2))^2}{2(\tau(t) - s - 2|\psi(t)|)} 
\right. 
- \left. \frac{(\alpha_2 \exp (-i\epsilon/2) + \alpha_2^* \exp (i\epsilon/2))^2}{2(\tau(t) - s + 2|\psi(t)|)} \right] 
\times \sum_{l_1=0}^{n} \sum_{l_2=0}^{n} \sum_{m=0}^{n} \sum_{m+1=0}^{n} \sum_{k_1=0}^{n} \sum_{k_2=0}^{n} (-2)2n - 2r + m_1 + n_1 - l_1 - l_2 
\times \frac{r!(m-n+m_1+l_2-k_1-r)!}{k_1!k_2!(l_1-n_1)!(l_2-m_1)!(n-r)!(n-l_1)!} 
\times \frac{[\eta_1^2(t) - \eta_2^2(t)]^{l_1-n_1/2} [\eta_1^2(t) - \eta_2^2(t)]^{l_2-m_1/2}} 
\times [\zeta_1^2(t) - \zeta_2^2(t)]^{k_1/2}[\zeta_1^2(t) - \zeta_2^2(t)]^{k_2/2} 
\times [\eta_1^2(t) - \zeta_2^2(t)]^{k_1/2}[\zeta_1^2(t) + \zeta_2^2(t)]^{k_2/2} 
\times \frac{[\eta_1^2(t) - \eta_2^2(t)]^{l_1-n_1/2} [\eta_1^2(t) - \eta_2^2(t)]^{l_2-m_1/2}} 
\times [\zeta_1^2(t) - \zeta_2^2(t)]^{k_1/2}[\zeta_1^2(t) - \zeta_2^2(t)]^{k_2/2}.
\[ z(t) = \left\{ 1 - 2(1 + |\eta_+(t)|^2 + |\eta_-(t)|^2) \right\}^{1/2}, \]
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\( r = \frac{1}{2} |n_1 + m_1 - |n_1 - m_1||, H_m \) is the Hermite polynomial of order \( m \) and \( P_{r(a,b)}(x) \) is the Jacobi polynomial which is defined as

\[
P_{r(c,d)}(x) = \sum_{k=0}^{r} (-1)^{(r-k)} \binom{r + d}{r - k} \binom{r + k + c + d}{k} \left( \frac{x + 1}{2} \right)^k.
\]

Equation (38) is real in spite of its complex form, which can be seen explicitly in the summations where we can find each term with its complex conjugate.

We can easily check the limits of equations (36) and (38) as \( t \to 0 \), which give the corresponding well-known quantities for the Fock state \( |n\rangle \) appropriate for the description before the interaction starts. In fact, this is clear also from the solutions of the Heisenberg equations of motion, where at \( t = 0 \), all factors reduce to zero except \( K_1(0) \) which equals 1. So we get

\[
C^{(1)}(\zeta, s) = \exp \left[ \frac{1}{2} (s - |\zeta|^2) \right] L_n(|\zeta|^2),
\]

\[
W(\alpha) = -\frac{2}{\pi} (-1)^n \exp (-2|\alpha|^2) L_n(4|\alpha|^2),
\]

\[
Q(\alpha) = \frac{1}{\pi} \frac{|\alpha|^{2n}}{n!} \exp (-|\alpha|^2),
\]

which are the \( s \)-parametrized characteristic function, \( W \)-function and \( Q \)-function for the Fock state \( |n\rangle \).

As we have mentioned before, the nonlinear directional coupler can be used as a source of quantum states [48]. This may be illustrated by displaying one of the quasiprobability functions [49]. The best quasiprobability functions for this task are \( W \)- and \( Q \)-functions since they are not singular and may contain oscillatory fringes (particularly the \( W \)-function) that are indicative of non-classical behaviours. So we have plotted the \( W \)-function and the \( Q \)-function using (38) in figures 7 and 8, respectively, against \( x = \text{Re} \alpha_1 \) and \( y = \text{Im} \alpha_1 \), when the first state is the Fock state \( |1\rangle \) and the second one is the vacuum state \( |0\rangle \), i.e. \( n = 1, m = 0; \lambda_1 = 1, \lambda_2 = \lambda_3 = \lambda_4 = 0.25 \) and for values of time shown. We have considered quasiprobability functions at \( t \in [0, \pi] \). In figure 7 (a) we have the \( W \)-function for \( t = \pi/100 \), i.e. after short time interaction between the two modes we observe similar behaviour as for the \( W \)-function of state \( |1\rangle \) (see figure 2 of [50]), which means that pronounced negative values are exhibited. This behaviour of the \( W \)-function is completely different with increasing time \( (t = \pi/2) \); we see the disappearance of negative values of the quasidistribution and a stretched positive peak occurs (figure 7 (b)). This form of \( W \)-function is close to that of squeezed vacuum states [17], i.e. squeezed vacuum states can be generated, in principle, in our model. It should be borne in mind that the specific direction of stretching for the quasiprobability function of squeezed states may be achieved by choosing a suitable value for the phase of the squeezing parameter. Of course, in figure 7 (b), there is a negligible spike at the top of the peak which can be smoothed out by governing the coupler parameters. After a larger time interaction \( t = \pi \), the negative values are reached again but they are less pronounced and asymmetry can be observed due to stretching (figure 7 (c)). So we meet a time development of the \( W \)-function as a result of the power transfer between the two modes inside the coupler.
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Figure 7. $W$-function for the single mode (mode 1) for different values of time $t$ when both the modes are initially in the number states; the first mode is in the state $|1\rangle$ and the second mode is in the state $|0\rangle$ and $\lambda_1 = 1$, $\lambda_2 = \lambda_4 = 0.25$: (a) for $t = \pi/100$; (b) for $t = \pi/2$; (c) for $t = \pi$.

The $Q$-function is the quasiprobability function which is always positive definite, however, it can be used as an indicator for the squeezing in the model by including stretching in the phase space. In figure 8 we can see a kind of relation for the behaviour of the $W$-function and the $Q$-function and we can observe the top hole peak for short and long time interaction, which does not appear for intermediate interaction times. For all cases the stretching is remarkable.

5.2. Input coherent light

In a similar way to that followed in section 5.1, we can study the same quantities when both the modes are initially in coherent states. In this case the density operator is given by

$$\rho_{\text{coh}}(0) = |\alpha\rangle_1 \langle \alpha|_2 (|\alpha\rangle \langle \alpha|). \quad (44)$$

Then the two-mode $s$-parametrized characteristic function is derived in the form

$$C_{\text{coh}}^{(2)}(\zeta_1, \zeta_2, s, t) = \exp \left\{ \sum_{j=1}^{2} \left[ \frac{s}{2} (1 - s + 2|L_j(t)|^2 + 2|N_j(t)|^2) \right] |\zeta_j|^2 \right\}$$

$$\times \exp \left\{ \sum_{j=1}^{2} \frac{1}{2} [\zeta_j^2 (N_j^*(t)M_j^*(t) + L_j^*(t)K_j^*(t)) + \text{c.c.}] \right\}$$

$$\times \exp \{ \zeta_1 \zeta_2 [K_1^*(t)N_2^*(t) + M_1^*(t)L_2^*(t)] + \text{c.c.} \}$$
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Figure 8. $Q$-function for the single mode (mode 1) for different values of time $t$ when both the modes are initially in the number states; the values of the parameters are as in figure 7.

\[ \times \exp \left\{ -\zeta_1^* \zeta_2 [L_2^*(t)N_1(t) + N_2^*(t)L_1(t)] - \text{c.c.} \right\} \]

\[ \times \exp \left\{ \sum_{j=1}^{2} (\zeta_j \hat{\alpha}_j^* (t) - \zeta_j^* \hat{\alpha} (t)) \right\} \]

(45)

where $\hat{\alpha}_j(t)$ are the mean values of the operators $\hat{\alpha}_j(t)$ given by equation (5) with respect to the coherent states.

Therefore the two-mode $s$-parametrized quasiprobability function is

\[ W_{\text{coh}}^{(2)}(\alpha_1, \alpha_2, s, t) = \frac{1}{\pi^2} \left| \frac{|L_1(t)|^2 + |N_1(t)|^2 - |B_1(t)|^2}{S_+(t)S_-(t) - T_2^2(t)} \right|^{-1/2} \]

\[ \times \exp \left[ \frac{S_-(t)X_+^2(t) + S_+(t)X_-^2(t) + 2X_-(t)X_+(t)T(t)}{S_+(t)S_-(t) - T_2^2(t)} \right] \]

\[ \times \exp \left[ \frac{|B_1(t)| \left[ E_1^2(t) + E_-^2(t) \right] - \left[ |L_1(t)|^2 + |N_1(t)|^2 \right] |E_1(t)|^2}{|L_1(t)|^2 + |N_1(t)|^2 - |B_1(t)|^2} \right], \]

(46)

where we have used the following abbreviations...
\[ A_j(t) = \frac{1}{2}(1 - s + 2|L_j(t)|^2 + 2|N_j(t)|^2), \]
\[ B_j(t) = N_j^*(t)M_j^*(t) + L_j^*(t)K_j^*(t) = |B_j(t)| \exp [2i\delta_j(t)], \]
\[ D(t) = K_j^*(t)N_j^*(t) + M_j^*(t)L_j^*(t) = |D(t)| \exp [i\chi(t)], \]
\[ \tilde{C}(t) = L_j^*(t)N_j(t) + N_j^*(t)L_j(t) = |\tilde{C}(t)| \exp [i\gamma(t)], \]
\[ E_j(t) = (\alpha_j(t) - \alpha_j) \exp [i\delta_j(t)], \]
\[ F_\pm(t) = D(t) \sin [\delta_1(t) + \delta_2(t) - \chi(t)] \pm \tilde{C}(t) \sin [\delta_1(t) - \delta_2(t) + \gamma(t)], \]
\[ R_\pm(t) = D(t) \cos [\delta_1(t) + \delta_2(t) - \chi(t)] \pm \tilde{C}(t) \cos [\delta_1(t) - \delta_2(t) + \gamma(t)], \]
\[ S_+(t) = A_2(t) + |B_2(t)| - \frac{F_+^2(t)}{A_1(t) - |B_1(t)|} - \frac{R_+^2(t)}{A_1(t) + |B_1(t)|}, \]
\[ S_-(t) = A_2(t) - |B_2(t)| - \frac{R_-^2(t)}{A_1(t) - |B_1(t)|} - \frac{F_-^2(t)}{A_1(t) + |B_1(t)|}, \]
\[ T(t) = \frac{R_-(t)F_+(t)}{A_1(t) - |B_1(t)|} - \frac{R_+(t)F_-(t)}{A_1(t) + |B_1(t)|}, \]
\[ X_+(t) = i[E_2(t) + E^*_2(t)] + \frac{F_+(t)[E_1^*(t) - E_1(t)]}{A_1(t) - |B_1(t)|} - \frac{R_+(t)[E_1^*(t) + E_1(t)]}{A_1(t) + |B_1(t)|}, \]
\[ X_-(t) = [E_2^*(t) - E_2(t)] + \frac{R_-(t)[E_1^*(t) - E_1(t)]}{A_1(t) - |B_1(t)|} + \frac{F_-(t)[E_1^*(t) + E_1(t)]}{A_1(t) + |B_1(t)|}, \] (47)

with the following condition \( |A_j(t)| > |B_j(t)| \) for the Glauber \( P \)-function and no additional constraints.

From equation (46) we can see that \( W^{(2)}(\alpha_1, \alpha_2, t, s) \) includes the non-classical correlation nature due to the presence of the terms \( \alpha_1\alpha_2, \alpha_1^*\alpha_2, \) etc. These mode correlations have been used in a number of studies on non-classical aspects of light including questions such as violations of Bell inequalities [51]. The amount of correlation between the waveguides inside the coupler is governed by the coupler parameters, i.e. \( \alpha_j, \lambda_j, t \). Further, the \( P \)-function does not exist for \( |A_j(t)| < |B_j(t)| \), and this should be reflected as a non-classical effect in the behaviour of the compound modes inside the coupler. The physical reason for this is that the modes may no longer fluctuate independently in even the small amount allowed in a pure state.

For the single-mode case the \( s \)-parametrized characteristic function and the \( s \)-parametrized quasiprobability function are given, respectively, as

\[ C^{(1)}_{\text{coh}}(\zeta, s, t) = \exp \left[-\frac{1}{2}(1 - s + 2|L_1(t)|^2 + 2|N_1(t)|^2)[\zeta^2 + \zeta^*\alpha_1(t) - \zeta^*\alpha_1(t)] \times \exp \left\{ \frac{1}{2}[N_1^*(t)M_1^*(t) + L_1^*(t)K_1^*(t)] \right\} \times \exp \left\{ \frac{1}{2}[N_1(t)M_1(t) + L_1(t)K_1(t)] \right\}, \] (48)
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\[ W_{\text{coh}}^{(1)}(\alpha, s, t) = \frac{1}{\pi} \left\{ \left[ \frac{1-s}{2} + |L_1(t)|^2 + |N_1(t)|^2 \right] - |B_1(t)|^2 \right\}^{-1/2} \]

\times \exp \left\{ - \frac{\left[ \frac{1-s}{2} + |L_1(t)|^2 + |N_1(t)|^2 \right]^2 |\bar{\alpha}_1 - \alpha_1|^2}{\left[ \frac{1-s}{2} + |L_1(t)|^2 + |N_1(t)|^2 \right]^2 - |B_1(t)|^2} \right\} \times \exp \left\{ - \frac{1}{2} |B_1(t)| \left[ E_1^2(t) + E_1^*2(t) \right] \right\} \right\} \quad (49) \]

and \( |L_1(t)|^2 + |N_1(t)|^2 > |B_1(t)| \) must hold for the Glauber P-function. It is known that the correspondence between quantum and classical theories can be established via the Glauber-Sudarshan P-representation. But the P-representation does not possess all the properties of a classical distribution function for quantum fields. More precisely, light fields for which the P-representation is not a well-behaved distribution (in most processes in interaction at least for some values of interaction time, including the process under consideration) exhibit non-classical features. From (49), the P-function is not well defined as an ordinary function for \( |L_1(t)|^2 + |N_1(t)|^2 < |B_1(t)| \) and hence the non-classical effects, e.g. squeezing of vacuum fluctuations and sub-Poissonian statistics can occur, as we have seen before.

Furthermore, the non-classical effect, especially squeezing of vacuum fluctuations in the case of our system, can be recognized in the behaviour of \( W \)-function (and/or \( Q \)-function) in phase space as shown in figure 9 for the values of the parameters shown. For \( t = 0 \), i.e. when there is no interaction between the two modes, the \( W \)-function is identical with that shown for a single mode representing a symmetric Gaussian bell in phase space. As soon as the interaction switches on (\( t > 0 \)), we observe that the Gaussian centre is shifted and the rotationally symmetric function of the initial state at \( t = 0 \) gets to be squeezed in various phase space directions with dependence on time, as demonstrated in figures 9 (a) and (b). In other words, the initial symmetric contour of the \( W \)-function has been stretched as the interaction switches on, i.e. noise ellipse characterizing squeezed light appears, which rotates in the phase space as the interaction time progresses.

5.3. Input thermal light

Signal beams are usually accompanied by thermal noise, so that examination of quantum fields with thermal noise is an important problem from both theoretical and practical points of view. Such a thermal field can be generated by a thermal source composed of many independent atomic radiators and consists of the superposition of waves of many different frequencies within some continuous range. These waves can be regarded as independent waves with random phases [16]. This field possesses uniform phase distribution (it is described by normal distribution), exhibits thermal statistics, i.e. \( g^{(2)}(0) = 2 \), and its photon distribution is the Bose–Einstein distribution.
Figure 9. $W$-function for the single mode (mode 1) for different values of time $t$ when both the modes are initially in the coherent states; $|\alpha_1|^2 = |\alpha_2|^2 = 2$ and $\lambda_k$ are the same as in figure 7: (a) for $t = \pi$; (b) for $t = 2\pi$. 
Here we study the quasiprobability functions for two modes as well as for a single mode as before, when both the modes are thermal. In this case the density operator takes the form

\[ \hat{\rho}_T(0) = \frac{1}{(\bar{n}_1 + 1)(\bar{n}_2 + 1)} \sum_{n,m=0}^\infty \left( \frac{\bar{n}_1}{\bar{n} + 1} \right)^n \left( \frac{\bar{n}_2}{\bar{n} + 1} \right)^m |n\rangle_1 |m\rangle_2 \langle m|_1 \langle n|, \]  

(50)

where \( \bar{n}_1 (\bar{n}_2) \) is the average thermal photon number for mode 1 (2). It is clear that the thermal distribution has a diagonal expansion in terms of the Fock states. This diagonality causes the electric field expectation value inside the coupler to vanish in thermal equilibrium at all times. This, of course, is related with the linearity of the relations (5) in terms of creation and annihilation operators.

The two-mode s-parametrized characteristic function is given as

\[ C_{\text{th}}^{(2)}(\zeta_1, \zeta_2, s, t) = \exp \left[ -(\bar{n}_1 + \frac{1}{2})|Z_1(t)|^2 - (\bar{n}_2 + \frac{1}{2})|Z_2(t)|^2 + \frac{s}{2}(|\zeta_1|^2 + |\zeta_2|^2) \right], \]

(51)

where

\[ Z_1(t) = \zeta_1 K_1^* (t) - \zeta_1^* L_1(t) + \zeta_2 M_1^* (t) - \zeta_2^* N_2(t), \]

(52a)

\[ Z_2(t) = \zeta_2 K_2^* (t) - \zeta_2^* L_2(t) + \zeta_1 M_1^* (t) - \zeta_1^* N_1(t). \]

(52b)

Therefore the two-mode s-parametrized quasiprobability function equals

\[ W_{\text{th}}^{(2)}(\alpha_1, \alpha_2, s, t) = \frac{1}{\pi^2 \{[\bar{A}_1^2(t) - |C_1(t)|^2][\bar{A}_2^2(t) - |C_2(t)|^2]\}^{1/2}} \times \exp \left\{ \frac{C_1(t)\alpha_1^2 + C_1^*(t)\alpha_2^2}{2[\bar{A}_1^2(t) - |C_1(t)|^2]} - \frac{|D_1(t) - \alpha_2|^2}{[\bar{A}_2^2(t) - |C_2(t)|^2]} \right\} \times \exp \left\{ \frac{C_2^*(t)|D_1(t) - \alpha_2|^2 + C_2(t)|D_2^*(t) - \alpha_2^2|^2}{2[\bar{A}_2^2(t) - |C_2(t)|^2]} \right\}, \]

(53)

where we have defined

\[ \bar{A}_1(t) = (\bar{n}_1 + \frac{1}{2})[|L_1(t)|^2 + |K_1(t)|^2] + (\bar{n}_2 + \frac{1}{2})[|M_1(t)|^2 + |N_1(t)|^2] - \frac{s}{2}, \]

\[ \bar{A}_2(t) = (\bar{n}_1 + \frac{1}{2})[|M_2(t)|^2 + |N_2(t)|^2] + (\bar{n}_2 + \frac{1}{2})[|K_2(t)|^2 + |L_2(t)|^2] - \frac{s}{2}, \]

\[ C_1(t) = 2[(\bar{n}_1 + \frac{1}{2})L_1^* (t)K_1^*(t) + (\bar{n}_2 + \frac{1}{2})M_1^*(t)N_1^*(t)], \]

\[ C_2(t) = \frac{1}{\bar{A}_1^2(t) - |C_1(t)|^2} \left[ C_1(t)l_1^2(t) + C_1^*(t)l_2^2(t) - 2\bar{A}_1(t)l_1^*(t)l_2(t) \right] + 2[(\bar{n}_2 + \frac{1}{2})L_2(t)K_2(t) + (\bar{n}_1 + \frac{1}{2})M_2(t)N_2(t)], \]

\[ D_1(t) = \frac{1}{\bar{A}_1^2(t) - |C_1(t)|^2} \{ \bar{A}_1(t)[\alpha_1 l_1^*(t) + \alpha_1^* l_2^2(t)] - \alpha_1 l_2^2(t)C_1(t) - \alpha_1^* l_1^2(t)C_1^*(t) \}, \]

(54)

such that \( |\bar{A}_j(t)| > |C_j(t)| \).
In equation (54) we have defined $l_1(t)$, and $l_2(t)$ as follows:

$$l_1(t) = (\bar{n} + \bar{n}_2 + 1)\left[L_1^*(t)M_2^*(t) + K_1^*(t)N_2^*(t)\right],$$

$$l_2(t) = (\bar{n}_1 + \bar{n}_2 + 1)\left[L_1^*(t)N_2^*(t) + K_1^*(t)M_2^*(t)\right] + (\bar{n}_2 + \frac{1}{2})[M_1^*(t)K_2(t) + N_1^*(t)L_2(t)].$$

We can see from (53) that the thermal light (classical light) propagating through the system under consideration can exhibit non-classical effects, since the $P$-function can be singular under some constraints. Further we can see also that the non-classical correlation between modes is available.

For the single-mode case the $\xi$-parametrized characteristic and quasiprobability functions are

$$C_{\text{th}}^{(1)}(\zeta, \alpha, s, t) = \exp \left[ -|\zeta|^2 \left( J(t) - \frac{s}{2} \right) + \frac{\alpha^2}{\beta} \frac{U(t) + \alpha^2 U^*(t)}{2} \right],$$

then the $\xi$-parametrized distribution functions can be written in the form

$$W_{\text{th}}^{(1)}(\alpha, \xi, s, t) = \frac{1}{\pi \left\{ \left[ J(t) - \frac{s}{2} \right]^2 - |U(t)|^2 \right\}^{1/2}} \exp \left\{ -|\alpha|^2 \left[ J(t) - \frac{s}{2} \right] - \frac{1}{2} \left[ U(t)\alpha^2 + U^*(t)\alpha^2 \right] \right\},$$

where we have denoted

$$U(t) = L_1^*(t)K_1^*(t)(2\bar{n}_1 + 1) + M_1^*(t)N_1^*(t)(2\bar{n}_2 + 1),$$

$$J(t) = ([L_1(t)]^2 + [K_1(t)]^2)\bar{n}_1 + ([M_1(t)]^2 + [N_1(t)]^2)\bar{n}_2 + \frac{1}{2} + |L_1(t)|^2 + |N_1(t)|^2,$$

with $[J(t) - (s/2)]^2 > |U(t)|^2$.

It is well known for the thermal optical cavity that photons have a tendency to bunch each other, when photon distribution is described by the Bose–Einstein distribution (super-Poissonian statistics). However, as we have shown in section 3 the single mode thermal light can display squeezing of thermal fluctuations under this interaction, e.g. one can derive that the coupler is the source for squeezed thermal light. This can also be recognized in the behaviour of $W$-function (see figure 10 where the cut through the $W$-function is displayed). In this figure one can see the noise ellipse for squeezed thermal light with the centre at the origin.

6. Conclusions

In this paper we have examined the quantum statistical properties of radiation generated and propagated in the nonlinear optical coupler composed of two nonlinear waveguides operating by the second subharmonic processes, coupled linearly by evanescent waves and nonlinearly by a non-degenerate optical parametric process. We have demonstrated regimes for generation and propagation of non-classical light exhibited by squeezing of vacuum fluctuations and/or antibunching of photons (sub-Poissonian photon statistics). We have also obtained quasidistribution functions for the initial light beams which are in coherent states, Fock states and thermal states. Compared to earlier results for nonlinear optical
couplers we have shown that the nonlinear coupling increases, in general, quantum noise in the device even if in some cases it can support the generation of non-classical light.

The motivation for examination of the system under consideration arises from the previous investigations of the nonlinear couplers as promising devices to produce non-classical light. When coherent light is injected initially in the system, squeezed as well as sub-Poissonian light can be generated. For injected number states, squeezed vacuum states are produced. When thermal light initially enters the coupler, the coupler can operate as a microwave Josephson-junction parametric amplifier [37]. These effects have been recognized as a result from the competition between linear and nonlinear properties of the system and are dependent on the initial amplitudes of the input fields. The mechanism of the energy exchange between waveguides plays here the crucial role.

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