Relativity of Causal Structure in Quantum Theory

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Quantum theory is a mathematical formalism to compute probabilities for outcomes happening in physical experiments. These outcomes constitute events happening in space-time. One of these events represents the fact that a system located in the region of space where is situated a physical device has a certain value of a physical observable at the time when the device fires the outcome corresponding to that value of the observable. The causal structure of these events is customarily assumed fixed in an absolute way. In this paper we show that this assumption cannot be substantiated on operational grounds proving that two observers looking at the same quantum experiment can calculate the probabilities of the experiment assuming a different causal structure for the space-time events constituted by the outcomes. We will thus say that in quantum theory we have relativity of causal structure.

In the standard, classical theory of probability, joint probability distributions on the values of two random variables are defined independently of the existence of a causal relationship between the values of one of the variables and those of the other. This is the case since a pair of random variables on which is defineable a joint probability distribution can represent something out of the domain of physics and thus not necessarily embedded in some given space-time (we will refer to space-time generically to a causal network of events, without specifying any other property such as discreteness, continuity, dimension etc.). In quantum theory, on the contrary, two random variables always represent two observables related to some physical system. The values of these variables are indeed associated to events embedded in some given space-time. In this case a given value of a random variable is in fact always perceived by a click of a detector that has revealed a property of a physical system in some position of space at some given instant of time. Hence the events on which are defined joint probability distributions for two physical observables always have a definite causal structure since any space-time ultimately constitutes a causal network of events. Quite recently, several authors have explored the statistics of different quantum experiments in which are investigated the same observables in physical experiments. These outcomes constitute events happening in space-time. One of these arrangements of the devices involved. In these papers it is found that there are a lot of analogies from a formal point view in describing quantum experiments differing only in the causal relations of the devices involved. Exploiting this fact, in [2], is formulated quantum theory as a theory of Bayesian inference in which the different causal relations between correlated regions are treated in a unified way. In this paper we show that the mathematical structure of quantum theory is such that two observers looking at the same quantum experiment can calculate the probabilities of the experiment assuming a different causal structure for the events on which is defined the probability distribution. This means that the causal structure of the events happening in a quantum experiment may not be regarded as absolutely fixed. Two observers can indeed obtain the information contained in a given experiment assuming a different causal structure for the events constituted by the outcomes involved. This result is likely to have implications in the search of a theory of quantum gravity. It suggests that to formulate a properly quantum theory of cosmological processes, we should look for a mathematical formalism to calculate probabilities of these processes such that the causal structure of the events on which is defined the probability distribution of a process can be regarded as a mathematical symmetry. This paper is organized as follows. We first introduce a general framework for quantum experiments whose information is contained in the joint probabilities of the values of a pair of physical observables. In this scenario, we formulate the property of relativity of causal structure. We show that in any quantum experiment performable in the above framework we have relativity of causal structure. We then discuss relativity of causal structure in relation with the no-signalling principle. We finally link our result to those obtained in [2].

A. Operational framework

In a generic quantum experiment one is interested in the joint probabilities of the outcomes happening on two devices $A, B$, that are able to analyze a set of physical observables generically pertaining to two quantum systems $\mathcal{S}_1$ and $\mathcal{S}_2$ respectively ($\mathcal{S}_1, \mathcal{S}_2$ can of course be the same system). The possible values of a given observable $A$ analyzed by device $A$ constitute a set of outcomes of this device $\{a_i\}_{i \in A}$ while the possible values of another observable $B$, analyzed by device $B$, constitute another set of outcomes $\{b_j\}_{j \in B}$. The information contained in the experiment is expressed by the probability distribution $\{p(a_i, b_j)\}_{i,j}$ for all $(a_i, b_j) \in A \times B$ where it holds the normalization condition: $\sum_{i \in A, j \in B} p(a_i, b_j) = 1$ This clearly holds for all the possible experiments performable with devices $A, B$, namely, for all the possible observables.
that $\mathcal{A}, \mathcal{B}$ can analyze. A simple example of this is an experiment involving two Stern-Gerlach apparatus $\mathcal{S}_1$, $\mathcal{S}_2$ analyzing the spin of an electron. In this case observables $A$ and $B$ represent two given orientations that the spin of an electron can have, for example $Z_1$ and $Z_2$.

The possible outcomes happening on $\mathcal{S}_1$ are \{spin up along $Z_1 (Z_1 \uparrow)$, spin down along $Z_1 (Z_1 \downarrow)\} and those that can happen on $\mathcal{S}_2$ are \{spin up along $Z_2 (Z_2 \uparrow)$, spin down along $Z_2 (Z_2 \downarrow)\}$. The information contained in the experiment is in the joint probability distribution \{(p(Z_1 \uparrow, Z_2 \uparrow), p(Z_1 \uparrow, Z_2 \downarrow), p(Z_1 \downarrow, Z_2 \uparrow), p(Z_1 \downarrow, Z_2 \downarrow))\}. The events on which are defined probability distributions in quantum experiments always possess a definite causal structure.

**Definition 1** Given a pair of events, $\chi_a, \chi_b$ it is defined a causal structure for these events if one of the following holds:

- $\chi_a$ causes $\chi_b$
- $\chi_b$ causes $\chi_a$
- $\chi_a$ does not cause $\chi_b$ and $\chi_b$ does not cause $\chi_a$

This is the case since the random variables on which the probability distributions are defined, refer to observables pertaining to physical systems. Consider an arbitrary pair of outcomes $(a_i, b_j) \in \mathcal{A} \times \mathcal{B}$ such that $p(a_i, b_j) \neq 0$. These outcomes constitute two events that can happen in space-time. Outcome $a_i$ has associated an event $\chi_{a_i}$ representing that a system $\mathcal{S}_1$ has the value $a_i$ of observable $A$ in the region occupied by device $A$ with space coordinates $x_{a_i}$ at time $t_{a_i}$. In the same way outcome $b_j$ has associated an event $\chi_{b_j}$ saying that $\mathcal{S}_2$ has value $b_j$ of observable $B$ in a region occupied by $B$ with space coordinates $x_{b_j}$ at time $t_{b_j}$. To any quantum experiment is associated a specific dynamics of one or more systems $\mathcal{F}$. Whether for the events $\chi_{a_i}, \chi_{b_j}$ associated to the outcomes $(a_i, b_j)$ holds anyone of the alternatives in definition 1 clearly depends on the dynamics of the systems involved in the experiment. From an operational point of view, the assignment of a dynamics to the systems in a quantum experiment consists of a specification of the inputs and outputs for the devices involved in it. This point of view is first illustrated in 3. If we are interested in the joint probabilities of outcomes happening on devices $\mathcal{A}, \mathcal{B}$, the possible input/output combinations assigned to these devices are responsible for the different alternatives in definition 1 that can be associated to the pair of events $\chi_{a_i}, \chi_{b_j}$ corresponding to the pair of outcomes $(a_i, b_j)$. If the dynamics of the experiment is such that system $\mathcal{F}_1$ is the output of device $\mathcal{A}$ and $\mathcal{F}_2$ is the input of device $B$, we have that the pair of events $\chi_{a_i}, \chi_{b_j}$ associated to $(a_i, b_j)$ are such that $\chi_{a_i}$ causes $\chi_{b_j}$. If, conversely, the dynamics is the time reversal of the previous one, then system $\mathcal{F}_1$ is the input for $\mathcal{A}$, system $\mathcal{F}_2$ is the output for device $\mathcal{B}$ and the pair of events $\chi_{a_i}, \chi_{b_j}$ associated to $(a_i, b_j)$ are such that $\chi_{b_j}$ causes $\chi_{a_i}$. If the experiment is such that two causally independent systems are inputs (or outputs) for devices $\mathcal{A}$ and $\mathcal{B}$, then the causal structure of $\chi_{a_i}, \chi_{b_j}$ associated to outcomes $(a_i, b_j)$ is such that $\chi_{a_i}$, does not cause $\chi_{b_j}$ and $\chi_{b_j}$ does not cause $\chi_{a_i}$, namely, the two events are space-like. The assumption that these input/output associations can be done in an absolute way can hardly be motivated on operational grounds. There is instead no experiment that can probe that a quantum system is “escaped out from a device” in a given state and is “entered into another device” causing an outcome happening on it. This is the case since if it existed one such experiment, this should also make the system interact with another probe system; the interaction would perturb the dynamics of the original system and could in principle prevent it to enter the aperture of a physical device or even to escape out from it and would make the state and the measurement outcome change. A similar reasoning on the impossibility to “probe causal structure” in quantum theory can be found in 3. In what follows we will infant show that two different observers can compute the joint probabilities $p(a_i, b_j), \forall (a_i, b_j) \in \mathcal{A} \times \mathcal{B}$ in an experiment involving devices $\mathcal{A}$ and $\mathcal{B}$, assuming different input/output configurations for these devices. Since, from an operational point of view, the specific causal structure of events $\chi_{a_i}, \chi_{b_j}$ associated to the outcomes $(a_i, b_j)$ derives from the specification of the inputs and outputs of the devices involved, we will say that in quantum theory we have relativity of causal structure.

**B. Relativity of Causal Structure**

Consider an arbitrary pair of outcomes $(a_i, b_j)$ having non zero probability of jointly happening $p(a_i, b_j)$. An observer $O_\alpha$ assumes that a quantum system $\mathcal{F}_1$ is the output of device $A$ on which $a_i$ happens, is subject to an evolution $\mathcal{F}$ (eventually transforming $\mathcal{F}_1$ in system $\mathcal{F}_2$) and then constitutes the input of a measurement device $B$ on which $b_j$ happens. This implies that the space-time events $\chi_{a_i}, \chi_{b_j}$ associated to outcomes $a_i, b_j$ are assumed such that $\chi_{a_i}$ causes $\chi_{b_j}$. A second observer $O_\beta$ looking at the same quantum experiment of $O_\alpha$ assumes that system $\mathcal{S}_2$ is the output of device $B$ where $b_j$ happens, is subject to an evolution $\mathcal{F}'$ (eventually transforming $\mathcal{F}_2$ in $\mathcal{F}_1$) and then constitutes the input of a measurement device $A$ on which it happens $a_i$. Since this constitutes the time reversal of the dynamics assumed by observer $O_\alpha$, space-time events $\chi_{a_i}, \chi_{b_j}$ are assumed by $O_\beta$ in such a way that $\chi_{b_j}$ causes $\chi_{a_i}$. A third observer, $O_\gamma$, looking at the same experiment, is indeed assuming that systems $\mathcal{F}_1$ and $\mathcal{F}_2$ are both causally independent inputs respectively of two measurement devices $\mathcal{A}$ and $\mathcal{B}$ on which happen $a_i$ and $b_j$. The two systems are both outputs of a preparation device for the composite system $\mathcal{F}_1\mathcal{F}_2$ that prepares a state $\tau_2$. Observer $O_\gamma$ thus assumes that $\chi_{a_i}, \chi_{b_j}$ are two space-like events, namely, $\chi_{a_i}$ does not cause $\chi_{b_j}$ and $\chi_{b_j}$ does not cause $\chi_{a_i}$.

**Definition 2** We have relativity of causal structure if, given a choice of mathematical objects performed by any-
one of $O_a, O_β, O_γ$ to calculate probability $p(a_i, b_j)$, there exist unique choices of mathematical objects for the remaining two observers that permit them to calculate $p(a_i, b_j)$. This must hold for all $(a_i, b_j) \in A \times B$ and for all $(A, B)$.

In what follows we will prove that in quantum theory we have relativity of causal structure. Before doing this we will state a rule of transformation from mathematical objects describing physical objects (i.e. evolutions, preparations and measurement outcomes) used by an observer $O$ to the corresponding mathematical objects used by another observer $O'$.

**Transformation Rule -** Whenever a system $\mathcal{S}$ for which is defined a physical object (i.e. preparation, evolution or measurement outcome) is seen as an input (output) by observer $O$ and as an output (input) by observer $O'$, the operator used to describe that object by $O$ is the transpose on the Hilbert space of system $\mathcal{S}$ of the operator used to describe the corresponding object seen by $O'$.

Observer $O_α$ assumes that $a_i$ is an element of a preparations ensemble represented by a density matrix $ρ$ and a POVM $\{a_i\}_{i ∈ A}$ such that:

$$ρ = \sum_{i ∈ A} \text{Tr}[a_i ρ] \sqrt{ρ} a_i \sqrt{ρ} / \text{Tr}[a_i ρ]$$  \hspace{1cm} (1)

It is easy to see that $ρ$ is a convex combination of density operator used to describe the corresponding object seen by observer $O_α$. We will first explic- itly write the evolution by means of transformation $\mathcal{S}$ of ensemble $ρ$ seen by $O_α$. The density matrix obtained after the evolution by $O_α$ is:

$$\mathcal{S}(ρ) = \sum_{m, ab, cd} K^{m}_{ab} K^{m*}_{cd} |a_{21} b_{c2} c_{12} d_{22} \rangle$$  \hspace{1cm} (2)

Using the fact that $\sum_m K^m \otimes K^m$ can be written as:

$$\sum_{m, ab, cd} K^{m}_{ab} K^{m*}_{cd} |c_{11} b_{c2} a_{22} d_{22} \rangle$$  \hspace{1cm} (3)

and the polar decomposition of $ρ$ we have:

$$\mathcal{S}(ρ) = \text{Tr}_1[ ∑_{m, ab, cd} K^{m}_{ab} K^{m*}_{cd} |c_{11} b_{c2} c_{12} d_{22} \rangle]$$  \hspace{1cm} (4)

Note that, for the polar decomposition of $ρ$ to be uniquely defined, one must assume $ρ$ to be full rank in the Hilbert space corresponding to $A$. This assumption is consistent with the fact that we are considering that observer $O_α$ describes the possible values of an observable $A$ as an ensemble of preparations representing the system having all the different values of $A$. The density matrix obtained by $O_α$ after the evolution can thus be written as $\mathcal{S}(ρ) = \text{Tr}_1[\mathcal{S}_ρ]$ where we define:

$$\mathcal{S}_ρ := √ρ ⊗ I_2[∑_{m} (K^m \otimes K^m)] √ρ ⊗ I_2$$  \hspace{1cm} (5)

where $I_2$ is the identity matrix on system $\mathcal{S}_2$. From (5) we see that the evolution of ensemble $ρ$ can be represented as an operator acting on Hilbert spaces of systems $\mathcal{S}_1$ and $\mathcal{S}_2$. The evolution represented by $\mathcal{S}_ρ$ is seen as a bipartite state $ρ_{12}$ by $O_α$ since he assumes that the output of device $A$ seen by $O_α$, $\mathcal{S}_1$, is indeed an input for $A$. According to the transformation rule stated above, $O_α$ uses the following mathematical object to represent the bipartite state $ρ_{12}$:

$$ρ_{12} = \mathcal{S}_ρ^{11} = ρ^{1T} ⊗ I_2[∑_{m} (K^m \otimes K^m)^{1T}] √ρ^{1T} ⊗ I_2$$  \hspace{1cm} (6)

Where $τ_1$ denotes partial transpose on space 1 corresponding to $\mathcal{S}_1$. To see that (6) is a normalized bipartite state we define the normalized bipartite state on two copies of $A_1, |Φ⟩_{11'}$:

$$|Φ⟩_{11'} = √ρ^{1T} ⊗ I_1' ∑_{j} |j⟩_{1} ⊗ |j⟩_{1'}$$  \hspace{1cm} (7)

where $∑_{j} |j⟩_{j=1}^{d_1}$ is an orthonormal basis for Hilbert space of system $A_1$. Exploiting (6) we can write:

$$|Φ⟩_1 ⊗ |Φ⟩_1' = \mathcal{S}_ρ^{11}$$  \hspace{1cm} (8)

where $|Φ⟩$ is the identity map on system $A_1$ and $\mathcal{S}_ρ$ represent the evolution defined above. From (8) we can see that $τ_{12}$ is a normalized bipartite state since $\mathcal{S}_ρ$ is a TPCP map acting on system $A_1$ and $|Φ⟩_1 ⊗ |Φ⟩_1$ is a normalized bipartite state. The element of ensemble of preparations corresponding to $a_i$ for $O_α$ is seen by $O_γ$ as a measurement outcome. In consequence of this it is represented as $a_i^T$ by $O_γ$ since he assumes system $A_1$ as an input contrary to $O_α$. The probability $p_α(a_i, b_j)$ calculated by observer $O_α$ is:

$$p_α(a_i, b_j) = \text{Tr}_2[b_j \text{Tr}_1[\mathcal{S}_ρ a_i]]$$  \hspace{1cm} (9)

The probability calculated by $O_β$ is:

$$p_β(a_i, b_j) = \text{Tr}_1[a_i^T \text{Tr}_2[\mathcal{S}_ρ^T b_j^T]]$$  \hspace{1cm} (10)
The probability calculated by $O_a$ is:

$$p_\gamma(a_i, b_j) = \text{Tr}_{12}[a_i^T \otimes b_j \mathcal{T}_\rho^{T_1}]$$  \hspace{1cm} (11) \hspace{1cm}

It can be easily verified that $p_\alpha(a_i, b_j) = p_\beta(a_i, b_j) = p_{\gamma_i}(a_i, b_j)$. Since given an operator corresponding to a preparation, transformation or measurement outcome seen by a given observer, its transpose and its partial transpose on any of its subspaces are uniquely defined and we assumed $(a_i, b_j)$ arbitrary, we have relativity of causal structure by definition 3.

C. Discussion and related work

First we discuss the assumption done after (1) that $p_\gamma$ is not a pure state. This is necessary for $\tau_{12}$ in (3) to be normalized. If we have $\rho = |a\rangle\langle a|$ then observer $O_a$ is interested only in joint probabilities of the type $p(a_i, b_j)$ with $b_j \in \{b_j\}_{j \in B}$ and $a$ fixed value of observable $A$. This is equivalent to state that the uncertainty in observable $A$ is 0 and $p(a_i, b_j) = p(b_j|a)$. Observers $O_a$ and $O_\beta$ clearly cannot assume that the uncertainty in $A$ is 0, since from their point of view this represents a measurement whose outcomes are random. Nevertheless they can compute the above probability as

$$p(b_j|a) = p(a, b_j) / \sum_{b_j} p(a, b_j).$$  \hspace{1cm} (12) \hspace{1cm}

This is the fraction of times $b_j$ happens given that $a_i = a$ has happened and is the probability obtained by $O_a$ assuming $\rho = |a\rangle\langle a|$. The probability $p(a_i, b_j)$ written in (2) for $O_\beta$ and $O_\gamma$ can be calculated for an arbitrary probability distribution on the values $\{a_i\}_{i \in A}$ of observable $A$. We should now discuss the relationship between relativity of causal structure and the "no-signalling principle".

**Definition 3** No-signalling principle

*If two devices $A$ and $B$ are space-like separated originating outcomes corresponding to space-like events, then*

$$p(b_j) = \sum_{a_i} p(a_i, b_j) = \sum_{a_i} p(a_i', b_j) \quad \forall \{a_i\}_{i \in A}, \{a_i'\}_{i \in A'}$$  \hspace{1cm} (13) \hspace{1cm}

*for all $b_j \in \{b_j\}_{j \in B}$ where $B$ is a measurement on device $A$ and $A'$ and $A''$ are any two different measurements performed on device $A$.*

Since this principle holds in quantum theory, the quantum correlations between space-like devices cannot be used by an agent operating on $B$ to become aware of the actions of an agent operating on device $A$. Note that (13) is only a necessary condition that joint probabilities of outcomes happening on two space-like separated devices must satisfy. Hence there are no contradictions for an observer $O_a$ assuming that for every pair of outcomes $(a_i, b_j) \in A \times B$ the associated space-time events $\chi_{a_i}, \chi_{b_j}$ are such that, say, $\chi_{a_i}$ causes $\chi_{b_j}$. On the other hand, if $\sum a_i p(a_i, b_j) \neq \sum a_i' p(a_i', b_j)$ for some $b_j$ and a pair $(a_i)_{i \in A}, (a_i')_{i \in A'}$ then an observer $O_a$ establishes that an agent operating on $A$ must have changed the ensemble of preparations from $\rho = \sum_{i \in A} \text{Tr}[a_i^T \sqrt{\rho_a} a_i^T / \text{Tr}[a_i^T \sqrt{\rho_a}]]$ to $\rho' = \sum_{i' \in A'} \text{Tr}[a_i'^T \sqrt{\rho}} a_i'^T / \text{Tr}[a_i'^T \sqrt{\rho}]]$. An observer $O_a$ looking at the same experiments establishes that correlations on devices $A$ and $B$ are in one case due to a bipartite state $\tau = \mathcal{T}_\rho^{T_1}$ (with $\mathcal{T}$ evolution of ensemble $\rho$ seen by $O_a$ and $\mathcal{T}_\rho$ defined in (3)) and in the other case to a bipartite state $\tau' = \mathcal{T}_{\rho'}^{T_1}$; in both cases $T_1$ means transposition on Hilbert space on which $\rho, \rho'$ are defined.

In (3) it is introduced an isomorphism between bipartite states and evolutions of preparations ensembles via partial transposition of the corresponding operators. This isomorphism is also introduced in (3) where it is invented the formalism of *quantum conditional states*. Quantum conditional states are used to formulate a theory of Bayesian inference for random variables representing physical observables pertaining to two regions that have a definite causal relationship. The peculiarity of this theory is a tool called *star product*. Star product permits to perform statistical inference for two correlated regions $A$ and $B$ in strict analogy with the ordinary theory of probability in which there is no dependence on the causal relationship between the regions. Quantum conditional states are divided into causal conditional states and acasual conditional states depending on whether the two regions are causally related regions (an outcome in one region causes that in the other region or vice versa) or not. A CPTP map, $\mathcal{F}_{AB}$ from region $A$ to region $B$, is related to an *acasual* conditional state $\rho_{AB}$, by means of the Choi isomorphism (3):

$$\mathcal{F}_{AB} \leftrightarrow \mathcal{F}_{A'} \otimes \mathcal{F}_{A''}(|\Phi^+\rangle\langle\Phi^+|) = \rho_{AB}^*$$  \hspace{1cm} (14) \hspace{1cm}

where $|\Phi^+\rangle = \frac{1}{\sqrt{d_A d_B}} \sum_{i, i'} |i\rangle_{A'} \otimes |i\rangle_{A''}$ and $\{|i\rangle\}_{i = 1}^{d_A}$ is a basis for Hilbert space pertaining to the system in region $A$ and $A'$, $A''$ two copies of the system in region $A$. The rule of belief propagation is used to find the joint state $\rho_{AB}$ for two systems in space-like separated regions, $A$ and $B$, starting from the prior pertaining to one of the two regions, $\rho_A$; this is expressed via the star product:

$$\rho_{AB}^s = \rho_A \ast \rho_{AB}^s = d_A \sqrt{\rho_A} \otimes I_B \rho_{AB}^s \sqrt{\rho_A} \otimes I_B$$  \hspace{1cm} (15) \hspace{1cm}

The star product used here involves also a normalization factor $d_A$ that cancels with the factor $1/d_A$ arising from the definition of conditional state involving $|\Phi^+\rangle$. Note that $\rho_{AB}^s$ is equal to $\tau_{12}$ defined in (3) with $\mathcal{F}_1$ and $\mathcal{F}_2$ pertaining to devices $A$ and $B$ respectively in regions $A$ and $B$. The map $\mathcal{F}_{AB}$ is related to a *causal* conditional state $\rho_{AB}$ by means of the Jamiołkowski isomorphism (3):

$$\mathcal{F}_{AB} \leftrightarrow [\mathcal{F}_{A'} \otimes \mathcal{F}_{A''}(|\Phi^+\rangle\langle\Phi^+|)]^{T_{A'}} = \rho_{AB}^s$$  \hspace{1cm} (16) \hspace{1cm}

where $T_{A'}$ denotes partial transposition on Hilbert space of system $A'$ pertaining to region $A$. The rule of belief
propagation is used to find the joint state $\rho_{AB}^t$ for two systems in two causally related regions A and B (or equivalently for one system at two different times) starting from the prior pertaining to region A, $\rho_A$. This is expressed with the star product as above:

$$
\rho_{AB}^t = \rho_A^T \star \rho_A^t = d_A \sqrt{\rho_A^T} \otimes I_B \rho_A^t \sqrt{\rho_A^T} \otimes I_B
$$

(17)

where $T$ denotes transposition. Note that here $\rho_{AB}^t$ is equal to $\mathcal{J}_p$ defined in (5) with $\mathcal{J}_1$ and $\mathcal{J}_2$ pertaining to devices $\mathcal{A}$ and $\mathcal{B}$ respectively in regions A and B.

### D. Conclusion

In conclusion we have showed that the assumption of an absolute causal structure for the space-time events associated to the outcomes in a quantum experiment cannot be motivated on operational grounds. We in fact showed that two observers looking at the same quantum experiment can compute the relevant probabilities assuming a different causal structure for the events on which is defined the probability distribution. This can have implications in quantum gravity. In light of this result, a possible way to conceive a theory of quantum gravity is to look for a formalism to compute probabilities of cosmological processes such that the causal structure of the events on which is defined the probability distribution of a process can be regarded as a mathematical symmetry.

[1] B. de Finetti, Theory of Probability, vol. 1 (Wiley 1974)
[2] M.S. Leifer, R. W. Spekkens, arXiv:1107.5849v1
[3] M.S. Leifer, Phys. Rev. A 74, 042310 (2006), arXiv:quant-ph/0606022v2
[4] S. Taylor, S. Cheung, C. Brukner, V. Vedral, Proceedings of Quantum Communication, Measurement and Computing, (AIP, 2004), vol 734 of AIP Conference Proceedings, arXiv:quant-ph/0611233
[5] P. Evans, H. Price, K.B. Wharton, arXiv:1001.5057.
[6] S. Marchovitch, B. Reznik, arXiv:1103.2557.
[7] R. M. Wald, General Relativity, The University of Chicago Press, 1984.
[8] L. Hardy, arXiv:0912.4740
[9] L. Hardy, arXiv:gr-qc/0509120
[10] K. Kraus, States, Effects and Operations: Fundamental Notions of Quantum Theory, Springer Verlag 1983
[11] M. Choi, Completely Positive Linear Maps on Complex Matrices, Lin. Alg. and App., 285290, 1975
[12] A. Jamiołkowski, Rev. Math. Phys. 3, 275 (1972).