Measurement of Lagrangian velocity in fully developed turbulence

N. Mordant(1), P. Metz(1), O. Michel(2), J.-F. Pinton(1)

(1) CNRS & Laboratoire de Physique, École Normale Supérieure, 46 allée d’Italie, F-69007 Lyon, France
(2) Laboratoire d’Astrophysique, Université de Nice Parc Valrose, F-06108, Nice, France

We have developed a new experimental method to measure the Lagrangian velocity of tracer particles in a turbulent flow, based on ultrasonic Doppler tracking. This method yields a direct access to the velocity of a single particle at a turbulent Reynolds number $R_\lambda = 740$. Its dynamics is analyzed with two decades of time resolution, below the Lagrangian correlation time. We observe that the Lagrangian velocity spectrum has a Lorentzian form $E_L(\omega) = u_{rms}^2 T_L/(1 + (T_L \omega)^2)$, in agreement with a Kolmogorov-like scaling in the inertial range. The probability density function (PDF) of the velocity time increments displays a change of shape from quasi-Gaussian to a stretched exponential tail at the smallest time increments. This intermittency, when measured from relative scaling exponents of structure functions, is more pronounced than in the Eulerian framework.

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Lagrangian characteristics of fluid motion are of fundamental importance in the understanding of transport and mixing. It is a natural approach for reacting flows or pollutant contamination problems to analyze the motion of individual fluid particles. Another characteristic of mixing flows is their high degree of turbulence. For practical reasons, most of the experimental work concerning high Reynolds number flows has been obtained in the Eulerian framework. Lagrangian measurements are challenging because they involve the tracking of particle trajectories: enough time resolution, both at small and large scales, is required to describe the turbulent fluctuations.

Early Lagrangian information have been extracted from the dispersion of particles, following Taylor’s approach. Recently numerical and experimental studies have focused on resolving the motion of individual fluid or tracer particles. The emerging picture is as follows. The one-component velocity auto-correlation function is quasi-exponential with a characteristic time of the order of the energy injection scale. The velocity power spectrum is expected to have a scaling $E_L(\omega) \propto \omega^{-2}$, as recently reported and expected from a Kolmogorov similarity arguments. In the same spirit, the second order structure function should scale as $D_L^2(\tau) = C_6 \epsilon \tau$, where $\epsilon$ is the the power dissipation. Measurements of atmospheric balloons have given $C_6 = 4 \pm 2$ and a limit $C_6 \to 7$ has been suggested in stochastic models. Recent experiments using high speed optical techniques have shown that the statistics of the Lagrangian acceleration are strongly non-Gaussian.

We have developed a new experimental method, based on sonar techniques, in order to study in a laboratory experiment the Lagrangian velocity across the inertial range of time scales. We obtain the first measurement of single particle velocity for times up to the flow large scale turnover time, at high Reynolds number. In this Letter, we report the results of this measurements and compare with previous observations and numerical predictions.

Our technique is based on the principle of a continuous Doppler sonar. A small (2mm×2mm) emitter continuously insonifies the flow with a pure sine wave, at frequency $f_0 = 2.5$ MHz (in water). The moving particle backscatters the ultrasound towards an array of receiving transducers, with a Doppler frequency shift related to the velocity of the particle: $2\pi f_\Delta = \mathbf{q} \cdot \mathbf{v}$. The scattering wavevector $\mathbf{q}$ is equal to the difference between the incident and scattered directions. A numerical demodulation of the time evolution of the Doppler shift gives the component of the particle velocity along the scattering wavevector $\mathbf{q}$. It is performed using a high resolution parametric method which relies on an Approximated Maximum Likelihood scheme coupled with a generalized Kalman filter. The study reported here is made with a single array of transducers so that only one Lagrangian velocity component is measured.

The turbulent flow is produced in the gap between two counter-rotating discs. This setup has the advantage to generate a strong turbulence in a compact region of space, with no mean advection. In this way, particles can be tracked during times comparable to the large eddy turnover time. Discs of radius $R = 9.5$ cm are used to set water into motion inside a cylindrical vessel of height $H = 18$ cm. To ensure inertial entrainment, the discs are fitted with 8 blades with height $h_b = 5$ mm. In the measurement reported here, the power input is $\epsilon = 25$ W/kg. It is measured on the experiment cooling system, from the injection-dissipation balance. The integral Reynolds number is $Re = R^2 \Omega / \nu = 6.5 \times 10^4$, where $\Omega$ is the rotation frequency of the discs (7.2 Hz), and $\nu = 10^{-6}$ m$^2$/s is the kinematic viscosity of water. A conventional turbulent Reynolds number can be computed from the measured $rms$ amplitude of veloc-
ity fluctuations ($u_{rms} = 0.98 \text{ m/s}$) and an estimate of the Taylor microscale ($\lambda = \sqrt{15 u_{rms}^2/\epsilon} = 0.88 \text{ mm}$); we obtain $R_\lambda = 740$. This value is consistent with earlier studies in the same geometry; it corresponds to the range of turbulent Reynolds numbers where measurements of particle acceleration have been reported [11].

The flow is seeded with a small number of neutrally buoyant (density 1.06) polystyrene spheres with diameter $d = 250 \mu \text{m}$. It is expected that the particles follow the fluid motion up to characteristic times of the order of the turbulence eddy turnover time, at a scale corresponding to their diameter, i.e. $\tau_{\text{min}} \sim d/u_d \sim \epsilon^{-1/3} d^{2/3}$, using standard Kolmogorov phenomenology. For beads of diameter 250 $\mu \text{m}$, one estimates $\tau_{\text{min}} \sim 1.3 \text{ ms}$. This value is within the resolution of the demodulation algorithm whose cut-off frequency is at 3 kHz. Note that the Kolmogorov dissipative time ($\tau_\eta = \sqrt{\nu/\epsilon} = 0.2 \text{ ms}$) is smaller, so that we do not expect to resolve the dissipative region. The statistical quantities are calculated from $3 \times 10^6$ velocity data points, taken at a sampling frequency equal to 6500 Hz. The acoustic measurement zone is in central region of the flow, 10 cm thick in the axial direction and almost spanning the cylinder cross-section. In this region the flow is a good approximation to isotropic and homogeneous conditions: at all points, the mean velocity is non zero, but equal to about one tenth of its $rms$ value.

We first consider the Lagrangian velocity auto-correlation function:

$$R^L(\tau) = \frac{\langle v(t)v(t+\tau) \rangle_t}{\langle v^2 \rangle}. \quad (1)$$

We observe – Fig.1a – that it has a slow decrease which can be modeled by an exponential function $\rho^L(\tau) \sim e^{-\tau/TL}$. This expression defines an integral Lagrangian time scale $T_L = 22 \text{ ms}$. For comparison, the period of rotation of the discs is 140 ms and the sweeping period of the blades is 17 ms. The measured Lagrangian time scale thus appears as a time characteristic of the energy injection. The exponential reproduces extremely well the variation of the auto-correlation function, from about $5\tau_\eta$ at small scales to $4T_L$. These limits coincide with the upper and lower resolution of the technique, so that we observe an exponential decay over the entire range of our measurement. However, as the variance of the acceleration must be finite [12], there has to be some lower cut-off to this behavior, at times of order $\tau_\eta$. These observations extend and confirm previous numerical and experimental studies at moderate Reynolds numbers [2, 3, 6]. Note that the exponential decay of the Lagrangian velocity auto-correlation is a key feature of stochastic models of dispersion since it appears as a linear drift term in a Langevin model of particle dynamics [1, 8].

We show in Fig.1b the velocity power spectrum, computed both from the data and as the Fourier transform of the exponential decay of the auto-correlation function:

$$E^L(\omega) = \frac{u_{rms}^2 T_L}{1 + (T_L \omega)^2}. \quad (2)$$

We observe a clear range of power law scaling $E^L(\omega) \propto \omega^{-2}$. This is in agreement with a Kolmogorov K41 picture in which the spectral density at a frequency $\omega$ is a dimensional function of $\omega$ and $\epsilon$: $E^L(\omega) \propto \epsilon \omega^{-2}$. To our knowledge, this is the first time that it is directly observed at high Reynolds number and in a laboratory experiment, although it has been reported in oceanic studies [3, 4] and in lower Reynolds number direct numerical simulations [6]. Departure from the Kolmogorov behavior is observed at low frequencies in agreement with the exponential decay of the auto-correlation. At high frequencies, the spectrum deviates from the Lorentzian form due to the particle response. Note in Fig.1b that the measurement is made over a dynamical range of about 60 dB.

We now consider the second order structure function
of the velocity increment

$$D_2^v(\tau) = \langle (v(t+\tau) - v(t))^2 \rangle_t = \langle (\Delta_v v)^2 \rangle .$$

(3)

We emphasize that these are time increments, and not space increments as in the Eulerian studies. The profile $D_2^v(\tau)$ is shown in the inset of Fig.2. It is linked to the auto-correlation by $D_2^v(\tau) = 2u_{\text{rms}}^2 (1 - R^D(\tau))$: at small times one observes the trivial scaling $D_2^v(\tau) \propto \tau^2$ and at large times $D_2^v(\tau)$ saturates at $2u_{\text{rms}}^2$ (as $v(t)$ and $v(t+\tau)$ are uncorrelated).

To further describe the statistics of the Lagrangian velocity fluctuations, we have analyzed the statistics of the velocity increments $\Delta_v v$. Their PDF $\Pi_\tau$ for $\tau$ covering the accessible range of time scales is shown in Fig.3.

![FIG. 3: PDF $\sigma_\tau \Pi_\tau$ of the normalized increment $\Delta v / \sigma_v$. The curves are shifted for clarity. From top to bottom: $\tau = [0.15, 0.3, 0.6, 1.2, 2.5, 5, 10, 20, 40]$ ms.](image)

To emphasize the functional form, the velocity increments have been normalized by their standard deviation so that all PDFs have unit variance. A first observation is that the PDFs are symmetric, in agreement with the local symmetries this flow. Another is that the PDFs almost Gaussian at integral time scales and progressively develop stretched exponential tails for small time increments. At the smallest increment, the stretched exponential shape is in agreement with measurements of the PDF of Lagrangian acceleration at identical Reynolds numbers [10]. In our case, the limit form of the velocity increments PDF is not as wide as that of the acceleration because the Kolmogorov scale is not resolved. Note that in regards of the evolution of the PDF, the intermittency is at least as developed in the Lagrangian frame as it is in the Eulerian one [15].

![FIG. 4: Evolution of the excess kurtosis factor $K(\tau) = \langle \Delta_v v \rangle^4 / \langle \Delta_v v \rangle^2 - 3$ for the PDFs of the time velocity increments.](image)

The continuous evolution with scale can be quantified
using the flatness factor. We show in Fig.4 the
variation excess kurtosis \( K(\tau) = \left< (\Delta_t v)^4 \right>/\left< (\Delta_t v)^2 \right>^2 - 3. \)
It is null at integral scale as expected from the Gaus-
sian shape of the PDF and increases steeply at small
scales. Below about 5\( \tau_\eta \), the increase is limited by the
cut-off of the particle; an extrapolation of the trend to \( \tau_\eta \)
yields \( K(\tau_\eta) \sim 40 \) in agreement with acceleration mea-
surements in [10].

More generally, one can choose to describe the evolu-
tion of the PDFs by the behavior of their moments (or
‘structure functions’) \( D_q^E(\tau) = \left< |\delta \tau v|^p \right> \). Indeed, a con-
sequence of the change of shape of the PDFs with scale
is that their moments, as the flatness factor above, vary
with scale. Classically in the Eulerian picture, one ex-
pects scaling in the inertial range, \( D_q^E(\tau) \propto \tau^{p-3} \), at least in the limit of very large Reynolds numbers. At the fi-
nite Reynolds number where most experiments are made,
the lack of a true inertial range is usually compensated by
studying the relative scaling of the structure func-
tions – the ESS ansatz [10]. We use the second order
structure function as a reference. Indeed the dimension-
eal estimation of \( D_2^L(\tau) \) (as that of \( D_2^E \)) depends linearly on the
increment and on the dissipation. Fig.5 shows that,
as in the Eulerian frame, a relative scaling is observed
for the Lagrangian structure functions of orders 1 to 5,
\( D_q^L(\tau) \propto D_q^L(\tau)^{\xi_q} \). We observe that the relative expo-
nents follow a sequence close to, but more intermittent
than the corresponding Eulerian quantity. Indeed, we
obtain: \( \xi_1^L/\xi_4^L = 0.42, \xi_2^L/\xi_4^L = 0.75, \xi_4^L/\xi_4^L = 1.17, \)
\( \xi_5^L/\xi_4^L = 1.28 \) to be compared to the commonly ac-
cepted Eulerian values \( \xi_1^E/\xi_4^E = 0.36, \xi_2^E/\xi_4^E = 0.70, \)
\( \xi_4^E/\xi_4^E = 1.28, \xi_5^E/\xi_4^E = 1.53 \).

In conclusion, using a new experimental technique,
we have obtained a Lagrangian velocity measurement
that covers the inertial range of scales. Our results are
consistent with Kolmogorov-like dimensional predictions
for second order statistical quantities. At higher orders,
the observed intermittency is very strong. How the
Lagrangian intermittency is related to the statistical
properties of the energy transfers is an open question.
From a dynamical point of view, the Navier-Stokes
equation in Lagrangian coordinates could be modeled
using stochastic equations. Work is currently underway
to compare the dynamics of the Lagrangian velocity to
predictions of Langevin-like models.

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