Study on Evolvement Complexity in an Artificial Stock Market

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An artificial stock market is established based on multi-agent. Each agent has a limit memory of the history of stock price, and will choose an action according to his memory and trading strategy. The trading strategy of each agent evolves ceaselessly as a result of self-teaching mechanism. Simulation results exhibit that large events are frequent in the fluctuation of the stock price generated by the present model when compared with a normal process, and the price returns distribution is Lévy distribution in the central part followed by an approximately exponential truncation. In addition, by defining a variable to gauge the “evolvement complexity” of this system, we have found a phase cross-over from simple-phase to complex-phase along with the increase of the number of individuals, which may be a ubiquitous phenomenon in multifarious real-life systems.

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In social systems, such as insect societies, increased colony size is associated with profound and wide-ranging changes in “internal” organization and operation. For instance, larger colony size is correlated with increased homeostasis, cooperative activity, spatial organization of work, and caste polymorphism to name but a few “social correlates” [1, 2]. Gautrais et al catch up with a model demonstrating a proximate mechanisms to emerge polymorphism in insect societies, which indicates that specialization only occurs above a critical colony size such that smaller colonies contain a set of undifferentiated equally inactive individuals while larger colonies contain both active specialists and inactive generalists, as has been found in empirical studies [3]. The specialization of workers upon certain tasks can increase colony productivity. The experimentation Weidenmüller made indicated that the dynamics of the colony response changed as colony size increased: colonies responded faster to perturbations of their environment when they were large (60 or more individuals) than when they were small [4]. These findings provide intriguing new examples of the ways in which individuals, each using only local information, acting simply and independently and not subject to any central or hierarchical control, can coordinate group-level behavior which differs from that of each individual, as does economical system. Every economical agent behaves simply contrast to the system which is composed of them. Economical system complex behaviors also result from repeated nonlinear interaction between each others. But, does the social economical system have similarity to insect societies that macro-properties have to do with participants size? In this letter, we have found a phase cross-over from simple-phase to complex-phase along with the increase of the number of individuals based on our model, which may be a ubiquitous phenomenon in multifarious real-life systems.

There are many modelling methods to explain origins of the observed behavior of market price as emerging from simple behavioral rules of a large number of heterogeneous market participants, such as behavior-mind model [5, 6], dynamic-games model [7], multi-agent model [8, 9, 10, 11, 12, 13, 14] and so on. The mainstream method is agent-based modelling because of its simplicity, agility and verisimilitude and which based on a stylized description for the behavior of agents. Here, we proposed a stock market model based on multi-agent that incorporates the feedback between the price trend and agent’s trading strategy. Therefore, our model will demonstrate that each agent has a limit memory of the history of stock price and will choose an action according to his memory, and that the trading strategy of each agent evolving ceaselessly as a result of self-teaching mechanism will influence the price trend inversely, which resemble the minority game [15].

In our model, before a trade, each agent should choose an action: to buy, to sell or to ride the fence, the former two should determine the price and amount of the trading-application. The buyer with higher price and the seller with lower price will trade preferentially, and the trading-price is the average of selling-price and buying-price. The stock price is the weighted average of trading-price according to the corresponding trading-amount [16].

Each agent holds a so-called decision-matrix, which can tell him how to do according to the history of stock price. Let \( p(t) \) be the stock price at time step \( t \), then the range of fluctuation is

\[
f(t) = (p(t) - p(t-1))/p(t-1) \in (-1, +\infty) \quad (1)
\]

For the sake of simplification, the range of fluctuation is categorized into 5 types: drastic fall \( f \in (-1, -0.05) \), fall \( f \in [-0.05, -0.01] \), near immovability \( f \in (-0.01, 0.01) \), rise \( f \in [0.01, 0.05] \) and drastic

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rise \( (f \in (0.05, +\infty)) \), which are denoted by -2, -1, 0, 1 and 2 respectively. The agent’s memory is limited to the current 5 fluctuations, thus there are \( 5^5 = 3125 \) different fluctuation-patterns. The decision-matrix contains the probabilities of trading strategies according to the different fluctuation-patterns. For instance, table 1 shows a decision-matrix of an agent named John. Based on this matrix, John will choose to sell half of his shares in hand at probability 0.40 when the present fluctuation-pattern is “1, 0, -1, 0, 2”. If an agent decides to buy or to sell, the buying-price or selling-price will be chosen completely randomly in the interval \([p(t), 1.1p(t)]\) or \([0.9p(t), p(t)]\) respectively.

After a trade, each decision-matrix will change as a result of self-teaching mechanism. For each agent, if his action made his money increase, the corresponding probability in his decision-matrix will be doubled, contrarily, it will be halved. After that, the probabilities under the very pattern will be normalized. For instance, if John’s action were an unsuccessful one, the probabilities under the fluctuation-pattern “1, 0, -1, 0, 2” would become “0.125, 0.25, 0.25, 0.125, 0.25” after normalization. Apparently, there will be no changes if the agent did nothing or his action kept his money unaltered. In order to mimic the “bounded rationality” and “inductive thinking” of investors\([\ref{14}, \ref{15}]\), we set a very small probability \( \gamma \), which is called the reversal parameter. Agents may change their decision-matrix in completely contrary direction at the probability \( \gamma \).

When proper initial condition and parameters have been chosen, the artificial stock market can generate its stock price. In figure 1, we report a typical simulation result about price time series generated by our model, which is similar to the reality (inset). In this simulation we set the market size as 1000 (i.e. 1000 stockholders), the initial stock price as 50. The initial quantity of fund and shares owned follows uniform distribution in the interval \([0, 1000000]\) and \([0, 10000]\) respectively, and the original fluctuation-pattern are randomly selected from the 3125 candidates. Notice that an agent’s action may be restricted by his wealth. In other words, he may be prevented from buying or selling because of, respectively, a shortage of fond or shares in hand.

Large numbers of simulations have been performed to check if the model can generate price time series of key characteristics according with the reality. Since chaotic characteristic is one of complex dynamical properties of economical system, which has been demonstrated by previous studies\([\ref{14}, \ref{20}]\). We have calculated the Lyapunov exponent and correlative dimension of the stock price time series generated by the model, carried out principal component analysis, and drawn the conclusion that our model can not only create stock price trends rather similar to the real, but also show the chaotic behavior in deep consistency with the real stock market. The details are omitted, and can be referred to the corresponding reference\([\ref{21}]\).

In addition, we have calculated the distribution of price return \( r(t) \), where \( r(t) \) is defined as the difference between two successive logarithms of the price:

\[
r(t) = \beta (\log p(t + \Delta t) - \log p(t))
\]

Here, \( \beta \) is a positive constant. The corresponding price returns with \( \Delta t = 2 \) are shown in figure 2, from which one can see that large events are frequent in the fluctuation of the stock price generated by the present model when compared with a normal process, which agrees with the previous empirical studies\([\ref{22}, \ref{23}, \ref{24}, \ref{25}, \ref{26}]\). In figure 3, we plot the probability distribution of price returns, and the fitted Gaussian curve for the case \( \Delta t = 1 \). Comparing with normal distribution, the present returns distribution is of more peaked center and fatter tail, according with the empirical studies that suggest the distribution of returns in real-life financial market is a Lévy distribution in the central part followed by an approximately exponential truncation\([\ref{22}, \ref{23}, \ref{24}, \ref{25}, \ref{26}]\). Since our main goal in this letter is not to show the comparison between price time series generated by our model and the reality, more details are omitted here.

In succession, let’s discuss whether macro-properties have to do with participants size. As far as whether the

| Patterns | Action A | Action B | Action C | Action D | Action E |
|----------|----------|----------|----------|----------|----------|
| 1, 0, -1, 0, 2 | 0.10 | 0.40 | 0.20 | 0.10 | 0.20 |
| ... | ... | ... | ... | ... | ... |

FIG. 1: Time series of the typical evolution of the stock price in the interval \( t \in [0, 40000] \), where \( \gamma = 0.01 \) and the elements of initial decision-matrices are chosen completely randomly in the range \([0, 1]\) before normalization. The inset is the Dow Jones Industrial Average (DJIA) from 01-02-1931 to 12-31-1987.
is the fitted Guassian curve for the case $\Delta = 1$.

FIG. 3: The normalized probability density of price returns over different time scale $\Delta t = 1, 2, 4, 8, 16, 32$. The solid curve is the fitted Guassian curve for the case $\Delta t = 1$.

FIG. 4: The evolvement complexity of size $N$, where $L = 10000$, $n = 100$ and $\gamma$ is randomly chosen in the interval $[0, 0.1]$.

reaches a approximate fixed point. Such a price time series $p(t)$ generated by our model with length $L$ is defined to be simple. Here, “tail” means the last $0.1L$ points, and “it reaches a approximate fixed point” means the ratio of its range to its average is smaller than 0.05. Let the market size be fixed and other parameters be variable, if $m$ simple time series are generated by $n$ independent experiments, then the evolvement complexity of size $N$ is loosely defined as $C(N) = \frac{n - m}{n}$. Note that the length of tail ($0.1L$) and the criterion of what is an approximate fixed point ($< 0.05$) are not especially selected to generate the simulation results shown in the present letter. One can write a program and easily check that the simulation results are robustious for a wide parameter space, thus the following phenomena are.

As is shown in figure 4, the system evolutive behavior is evidently divided into 3 areas. When $N \leq 50$, the behavior is simple, and its evolvement complexity increases rather slowly with the increasing of the number of individuals. When $N \geq 100$, the system has a great evolvement complexity, but its “complex degree” does not increase with the increasing of the number of individuals. Therefore the areas $N \leq 50$ and $N \geq 100$ can be considered as the simple-phase and complex-phase respectively. Between these, the complexity increases fiercely with the increasing of the number of individuals, thus it is called the critical interval with the inf-critical-point 50 and sup-critical-point 100. Here, we have found a phenomenon that the behavior of our artificial economical system is correlated with increased participants size, which is similar to that of insect societies.

The economical system constitutes one among many other systems exhibiting a complex organization and dynamics with similar behavior, which, with large number of mutually interacting parts, self-organize their dynamics with novel and sometimes surprising microscopic
properties. The phenomenon that the evolvement complexity can be divided into 3 parts owing to the increasing of the number of individuals maybe one of the common characteristics among various complex systems that do not seem alike at all in appearance. The conception “evolvement complexity” is novel and interesting, but it is hard for us to give out a strict and appropriate definition. Therefore, it is an innovative point as well as a shortcoming in this letter. Although the definition is rough, we believe it will enlighten physicists on how to measure the complexity of complex systems.

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