$E_{6,7,8}$ Magnetized Extra Dimensional Models

Kang-Sin Choi$^1$, Tatsuo Kobayashi$^1$, Ryosuke Maruyama$^1$, Masaki Murata$^2$, Yuichiro Nakai$^2$, Hiroshi Ohki$^{1,2}$, Manabu Sakai$^2$

$^1$Department of Physics, Kyoto University, Kyoto 606-8502, Japan
$^2$Yukawa Institute for Theoretical Physics, Kyoto University, Kyoto 606-8502, Japan

Abstract

We study 10D super Yang-Mills theory with the gauge groups $E_6$, $E_7$ and $E_8$. We consider the torus/orbifold compactification with magnetic fluxes and Wilson lines. They lead to 4D interesting models with three families of quarks and leptons, whose profiles in extra dimensions are quasi-localized because of magnetic fluxes.
1 Introduction

Gauge theories with the gauge groups $E_6$, $E_7$ and $E_8$, are quite interesting as grand unified theory in particle physics, which would lead to the standard model at low energy. All of quarks and leptons are involved in the 16 representations of $SO(10)$ and such a 16 representation appears from the adjoint representation and 27 representation of $E_6$. Furthermore, these representations are included in adjoint representations of $E_7$ and $E_8$. These exceptional gauge theories can be derived in heterotic string theory, type IIB string theory with non-perturbative effects and F-theory. Indeed, interesting models have been studied e.g. in heterotic orbifold models [1-5] and F-theory [6-12].

Recently, extra dimensional field theories play an important role in particle phenomenology as well as cosmology. It is one of the most important issues how to realize a chiral spectrum in 4D effective field theory when we start with higher dimensional theory. One way to realize 4D chiral theory is to introduce non-trivial gauge backgrounds like magnetic fluxes. Indeed, several models with magnetic flux backgrounds have been studied in extra dimensional field theory and superstring theory [13-21]. Furthermore, T-dual of magnetized D-brane models are intersecting D-brane models and within the latter framework a number of interesting models have been constructed [16-18, 22-24].

The number of zero-modes, that is, the generation number, is determined by the magnitudes of magnetic fluxes. Indeed, one of important aspects in magnetized extra dimensional fields is that one can solve analytically zero-mode equations on the torus with magnetic fluxes. Their zero-mode profiles are non-trivially quasi-localized. Such a behavior of zero-mode wavefunctions would be phenomenologically important. For example, when zero-mode profiles are quasi-localized far away each other, their couplings would be suppressed. That could explain small Yukawa couplings for light quarks and leptons, and other couplings, which must be suppressed from the phenomenological viewpoint. At any rate, since we know zero-mode profiles explicitly, we can compute 4D effective theory concretely at least at the tree-level. (See for calculations of 4D effective couplings, e.g. [19, 26-28].) Orbifolding is another way to realize 4D chiral spectra. One can also solve zero-mode equations on the orbifold with magnetic fluxes and compute 4D effective theory [20, 30].

At the perturbative level, type II string theory can realize $U(N)$, $SO(N)$ and $Sp(N)$ gauge groups, but not exceptional groups, although exceptional gauge theories could be realized non-perturbatively. Thus, the former classes of gauge theories like $U(N)$ have been studied mainly with the magnetic flux backgrounds. At any rate, exceptional groups are quite interesting from the bottom-up phenomenological viewpoint.

Our purpose of this paper is to propose phenomenological model building from extra dimensional $E_{6,7,8}$ gauge theories with magnetic fluxes on the torus and orbifold backgrounds. Our starting point is 10D super Yang-Mills theory with gauge groups $E_{6,7,8}$. We compactify extra 6 dimensions on $(T^2)^3$ or the orbifold. Then, we introduce magnetic fluxes in $(T^2)^3$ as well as Wilson lines. These non-trivial gauge backgrounds, i.e. magnetic fluxes and Wilson lines, with/without orbifolding would lead to interesting particle phe-

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1 See for a review [25] and references therein.
nomenology. Through this type of model building, we show some semi-realistic massless spectra and phenomenological interesting aspects.

This paper is organized as follows. In section 2, we consider 4D effective theory derived from the torus/orbifold compactification with magnetic flux and Wilson line background. Most of them are already known results. (See e.g. [19,29–31].) However, we reconsider such backgrounds by emphasizing phenomenological implications of Wilson lines on magnetized torus models. In section 3, we study the $E_6$ models, and in section 4 we study the $E_7$ and $E_8$ models. Section 5 is devoted to conclusion and discussion.

2 Magnetized extra dimensions

Here we study magnetized torus models and orbifold models. We start with 10D super Yang-Mills theory with the gauge group $G$. Its Lagrangian is written as

$$\mathcal{L} = -\frac{1}{4g^2} \text{Tr} (F^{MN}F_{MN}) + \frac{i}{2g^2} \text{Tr} (\bar{\lambda}\Gamma^M D_M \lambda),$$

where $M, N = 0, \cdots, 9$. Here, $\lambda$ denotes gaugino fields, $\Gamma^M$ is the gamma matrix for ten-dimensions and the covariant derivative $D_M$ is given as

$$D_M \lambda = \partial_M \lambda - i[A_M, \lambda],$$

where $A_M$ is the vector field. Furthermore, the field strength $F_{MN}$ is given by

$$F_{MN} = \partial_M A_N - \partial_N A_M - i[A_M, A_N].$$

Although our starting point here is the 10D super Yang-Mills theory, the following discussions can be extended to other dimensions, e.g. 6D super Yang-Mills theory.

2.1 $T^6$ model

We consider the background $R^{3,1} \times (T^2)^3$, whose coordinates are denoted by $x_\mu$ ($\mu = 0, \cdots, 3$) for the uncompact space $R^{3,1}$ and $y_m$ ($m = 4, \cdots, 9$) for the compact space $(T^2)^3$. We often use complex coordinations $z_d$ ($d = 1, 2, 3$) for the $d$-th torus $T^2_d$, e.g. $z_1 = y_4 + \tau_1 y_5$. Here, $\tau_d$ denote complex structure moduli of the $d$-th $T^2_d$, while the area of $T^2_d$ is denoted by $A_d$. The periodicity on $T^2_d$ is written as $z_d \sim z_d + 1_d$ and $z_d \sim z_d + \tau_d$.

The gaugino fields $\lambda$ and the vector fields $A_\mu$ and $A_m$ are decomposed as

$$\lambda(x, z) = \sum_n \chi_n(x) \otimes \psi_n(z),$$

$$A_\mu(x, z) = \sum_n A_{n, \mu}(x) \otimes \phi_{n, \mu}(z),$$

$$A_m(x, z) = \sum_n \varphi_{n, m}(x) \otimes \phi_{n, m}(z).$$
Here, we concentrate on zero-modes, $\psi_0(z)$ and we denote them as $\psi(z)$ by omitting the subscript “0”. Furthermore, the internal part $\psi(z)$ is decomposed as a product of the $T^2_d$ parts, i.e. $\psi_{(d)}(z_d)$. Each of $\psi_{(d)}(z_d)$ is two-component spinor,

$$\psi_{(d)} = \begin{pmatrix} \psi_{+}\,(d) \\ \psi_{-}\,(d) \end{pmatrix},$$

and their chirality for the $d$-th part is denoted by $s_d$. We choose the gamma matrix for $T^2_d$ as

$$\tilde{\Gamma}^1_{(d)} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \tilde{\Gamma}^2_{(d)} = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}. \quad (6)$$

We introduce the magnetic flux along the $U(1)_a$ (Cartan) direction of $G$ on $T^2_d$,

$$F = \frac{\pi i}{\text{Im}\tau_d} m^a_{(d)} \ (dz_d \wedge d\bar{z}_d), \quad (7)$$

where $m^a_{(d)}$ is an integer $[32]$. Here, we normalize $U(1)_a$ charges $q^a$ such that all $U(1)_a$ charges are integers and the minimum satisfies $|q^a| = 1$. We assume that 4D N=1 supersymmetry (SUSY) is preserved. The above magnetic flux can be obtained from the vector potential,

$$A(z_d) = \frac{\pi m^a_{(d)}}{\text{Im}\tau_d} \text{Im}(\bar{z}_d \ dz_d). \quad (8)$$

This form of the vector potential satisfies the following relations,

$$A(z_d + 1_d) = A(z_d) + \frac{\pi m^a_{(d)}}{\text{Im}\tau_d} \text{Im}(dz_d),$$

$$A(z_d + \tau_d) = A(z_d) + \frac{\pi m^a_{(d)}}{\text{Im}\tau_d} \text{Im}(\bar{\tau}_d \ dz_d). \quad (9)$$

Furthermore, these can be represented as the following gauge transformations,

$$A(z_d + 1_d) = A(z_d) + d\chi_{1}\,(d), \quad A(z_d + \tau_d) = A(z_d) + d\chi_{2}\,(d), \quad (10)$$

where

$$\chi_{1}\,(d) = \frac{\pi m^a_{(d)}}{\text{Im}\tau_d} \text{Im}(z_d), \quad \chi_{2}\,(d) = \frac{\pi m^a_{(d)}}{\text{Im}\tau_d} \text{Im}(\bar{\tau}_d \ z_d). \quad (11)$$

Then, the fermion field $\psi_{(d)}(z_d)$ with the $U(1)_a$ charge $q^a$ must satisfy

$$\psi_{(d)}(z_d + 1_d) = e^{iq^a\chi_{1}\,(d)}\psi_{(d)}(z_d), \quad \psi_{(d)}(z_d + \tau_d) = e^{iq^a\chi_{2}\,(d)}\psi_{(d)}(z_d). \quad (12)$$

\footnote{4D N=1 SUSY is preserved by choosing proper values of area $A_d$ as well as $\tau_d$ [19, 21, 33].}
By the magnetic flux (7) along the $U(1)_a$ direction, all of 4D gauge vector fields $A_\mu$, which have $U(1)_a$ charges, become massive, that is, the gauge group is broken from $G$ to $G' \times U(1)_a$ without reducing its rank\(^3\) where 4D gauge fields $A_\mu$ in $G' \times U(1)_a$ have vanishing $U(1)_a$ charges and their zero-modes $\phi_\mu(z)$ have a flat profile. Since the magnetic flux has no effect on the unbroken gauge sector, 4D N=4 supersymmetry remains in the $G' \times U(1)_a$ sector, that is, there are massless four adjoint gaugino fields and six adjoint scalar fields\(^4\).

In addition, matter fields appear from gaugino fields corresponding to the broken gauge part, that is, they have non-trivial representations under $G'$ and non-vanishing $U(1)_a$ charges $q^a$. The Dirac equations for their zero-modes become

\[
\begin{alignat}{2}
\left( \partial_{z_d} + \frac{\pi q^a m^a_{(d)}}{2 \text{Im}(\tau_d)} z_d \right) \psi_{+(d)}(z_d, \bar{z}_d) &= 0, \\
\left( \partial_{z_d} - \frac{\pi q^a m^a_{(d)}}{2 \text{Im}(\tau_d)} z_d \right) \psi_{-(d)}(z_d, \bar{z}_d) &= 0,
\end{alignat}
\]

for $T^2_a$. When $q^a m^a_{(d)} > 0$, the component $\psi_{+(d)}$ has $M = q^a m^a_{(d)}$ independent zero-modes and their wavefunctions are written as \(^5\)

\[
\Theta^{j,M}(z) = N_M e^{i \pi M z / \text{Im}(\tau)} \vartheta \left[ \begin{array}{c} j/M \\ 0 \end{array} \right] (M z, M \tau),
\]

where $N_M$ is a normalization factor, $j$ denotes the flavor index, i.e. $j = 1, \cdots, M$ and

\[
\vartheta \left[ \begin{array}{c} a \\ b \end{array} \right] (\nu, \mu) = \sum_n \exp \left[ \pi i (n + a)^2 \mu + 2 \pi i (n + a)(\nu + b) \right],
\]

that is, the Jacobi theta-function. Note that $\Theta^{0,M}(z) = \Theta^{M,M}(z)$. Furthermore, for $q^a m^a_{(d)} > 0$, the other component $\psi_{-(d)}$ has no zero-modes. On the other hand, when $q^a m^a_{(d)} < 0$, the component $\psi_{-(d)}$ has $|q^a m^a_{(d)}|$ independent zero-modes, but the other component $\psi_{+(d)}$ has no zero-modes.

As a result, we can realize a chiral spectrum when we introduce magnetic fluxes on all of three $T^2_a$. That is, since the ten-dimensional chirality of gaugino fields is fixed, zero-modes for either $q^a > 0$ and $q^a < 0$ appear with a fixed four-dimensional chirality. For example, when $q^a > 0$ and $m^a_{(d)} > 0$ for all of $d = 1, 2, 3$, only the combination $\psi_{+(1)} \psi_{+(2)} \psi_{+(3)}$ has zero-modes and the number of their zero-modes is equal to $(q^a m^a_{(1)} m^a_{(2)} m^a_{(3)})^3$.

Now, let us introduce Wilson lines along the $U(1)_b$ direction of $G'$. That breaks further the gauge group $G'$ to $G'' \times U(1)_b$ without reducing its rank\(^5\) All of the $U(1)_b$-charged

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\(^3\)For example, when $G = SU(N)$, $G'$ would correspond to $SU(N-1)$.

\(^4\) In string terminology, these adjoint scalar fields correspond to open string moduli, that is, D-brane position moduli. How to stabilize these moduli is one of important issues.

\(^5\)For example, when $G = SU(N)$ and $G' \times U(1)_a = SU(N-1) \times U(1)_a$, these matter fields have $(N-1)$ fundamental representation under $SU(N-1)$ and $U(1)_a$ charge $q^a = 1$ and their conjugates.

\(^6\) For example, when $G' = SU(N-1)$, the Wilson line breaks it to $SU(N-2) \times U(1)_b$. 

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fields including 4D vector, spinor and scalar fields become massive because of the Wilson line, when they are not charged under $U(1)_a$ and their zero-mode profiles are flat. On the other hand, the matter fields with non-trivial profiles due to magnetic flux have different behavior. For matter fields with $U(1)_a$ charge $q^a$ and $U(1)_b$ charge $q^b$, the Dirac equations of the zero-modes are modified by the Wilson line background, $C^b_d = C^b_{d,1} + \tau_d C^b_{d,2}$ as

$$\begin{align*}
(\partial_{z_d} + \frac{\pi}{2\text{Im}(\bar{\tau}_d)}(q^a m^a_{(d)} z_d + q^b C^b_d)) \psi_{+ (d)}(z_d, \bar{z}_d) &= 0, \\
(\partial_{\bar{z}_d} - \frac{\pi}{2\text{Im}(\bar{\tau}_d)}(q^a m^a_{(d)} \bar{z}_d + q^b \bar{C}^b_d)) \psi_{- (d)}(z_d, \bar{z}_d) &= 0,
\end{align*}$$

where $C^b_{d,1}$ and $C^b_{d,2}$ are real parameters. That is, we can introduce Wilson lines along the $U(1)_b$ direction by replacing $\chi^{(d)}_i$ in (11) as

$$\chi^{(d)}_1 = \frac{\pi}{\text{Im}\tau_d} \text{Im}(m^a_{(d)} z_d + q^b C^b_d/q^a), \quad \chi^{(d)}_2 = \frac{\pi}{\text{Im}\tau_d} \text{Im}(\bar{\tau}_d (m^a_{(d)} \bar{z}_d + q^b \bar{C}^b_d/q^a)).$$

Because of this Wilson line, the number of zero-modes does not change, but their wave functions are shifted as

$$\Theta^{i,M}(z_d) \rightarrow \Theta^{i,M}(z_d + q^b C^b_d/(q^a m^a_{(d)})).$$

Note that the shift of zero-mode profiles depend on $U(1)_b$ charges of matter fields. Similarly, we can introduce the Wilson line $C^a_d$ along the $U(1)_a$ direction. Then, the zero-mode wavefunctions shift as

$$\Theta^{i,M}(z_d + q^b C^b_d/(q^a m^a_{(d)})) \rightarrow \Theta^{i,M}(z_d + C^a_d/m^a_{(d)} + q^b C^b_d/(q^a m^a_{(d)})).$$

However, the shift due to $C^a_d$ is rather universal shift, but the shift by $C^b_d$ depends on the charges $q^b$ of matter fields. Thus, the shift by $C^b_d$ would be much more important than one by $C^a_d$, in particular from the phenomenological viewpoint.

Let us explain more about its phenomenological implications. Suppose that we introduce magnetic fluxes in a model with a larger gauge group $G$ such that they break $G$ to a GUT group like $SO(10)$ and this model includes three families of matter fields like the $16$ representation, corresponding to all of quarks and leptons. Their 3-point couplings and higher order couplings in 4D effective field theory can be computed by overlap integral of wavefunctions. Then, we assume that the $SO(10)$ gauge symmetry is broken to $SU(3) \times SU(2) \times U(1)_Y$ by some mechanism. If zero-mode profiles of quarks and leptons are degenerate even after such $SO(10)$ breaking, couplings in 4D effective field theory are constrained (at the lowest level) by the $SO(10)$ symmetry. For example, Yukawa matrices have the $SO(10)$ relation, that is, Yukawa matrices would be the same between the up-sector, the down-sector and the lepton sector. However, when we break $SO(10)$ to $SU(3) \times SU(2) \times U(1)_Y \times U(1)$ by introducing Wilson lines along the $U(1)_Y \times U(1)$ direction, these Wilson lines resolve the degeneracy of zero-mode profiles among quarks and leptons. That is, zero-mode profiles of quarks and leptons split depending on their
2.2 $T^6/Z_2$ model and $T^6/(Z_2 \times Z'_2)$ model

Here, we consider orbifold models. For simplicity, we start with the $T^2/Z_2$ orbifold. The $Z_2$ twist acts on the coordinate $z_d$ as

$$z_d \rightarrow -z_d,$$  \hspace{1cm} (21)

and the $T^2/Z_2$ orbifold is constructed through identifying $z_d \sim -z_d$ on $T^2_d$. We impose the $Z_2$ boundary condition as

$$\psi_{\pm(d)}(-z_d) = \pm P \psi_{\pm(d)}(z_d),$$  \hspace{1cm} (22)

where $P$ is the embedding of the $Z_2$ twist into the gauge space. For non-trivial embedding, the gauge group is broken further.

Magnetic fluxes are invariant under the $Z_2$ twist, and we can solve the Dirac equation for zero-modes in a way similar to the previous subsection. Note that

$$\Theta^{j,M}(-z) = \Theta^{M-j,M}(z),$$  \hspace{1cm} (23)

where $\Theta^{M,M}(z) = \Theta^{0,M}(z)$. Thus, we can write the $Z_2$ eigenstates as

$$\Theta^{j,M}_\pm(z) = \frac{1}{\sqrt{2}} \left( \Theta^{j,M}(z) \pm \Theta^{M-j,M}(z) \right),$$  \hspace{1cm} (24)

for $j \neq M/2, M$. The wavefunctions $\Theta^{j,M}(z)$ for $j = M/2, M$ are the $Z_2$ eigenstates with the $Z_2$ even parity. Either even or odd modes are chosen by the boundary condition (22). Suppose that $q^a m^a_{(d)} > 0$. Then, only the $+$ component $\psi_{+(d)}$ has zero-modes. If $P = 1$ in Eq. (22), even zero-modes $\Theta^{j,M}_+(z)$ are chosen. On the other hand, if $P = -1$, odd zero-modes $\Theta^{j,M}_-(z)$ are chosen. Thus, the number of zero-modes are reduced by orbifolding.

$U(1)_Y \times U(1)$ like Figure 1. Then, Yukawa matrices would become different from each other among the up-sector, the down-sector and the lepton sector.

Similarly we can analyze 4D massless scalar modes [19]. We are assuming that 4D N=1 supersymmetry is preserved [19,21]. Thus, the number of zero-modes and the profiles for 4D scalar fields are the same as those for their superpartners, i.e. the above spinor fields. For example, for Higgs fields, we study zero-modes and their profiles of Higgsino fields.
|                | $M = \text{even}$ | $M = \text{odd}$ |
|----------------|------------------|------------------|
| even zero-modes| $M/2 + 1$        | $(M + 1)/2$      |
| odd zero-modes | $M/2 - 1$        | $(M - 1)/2$      |

Table 1: The numbers of zero-modes for even and odd wavefunctions.

When $M = q^a m_{(d)}^a$ is even, the number of even and odd zero-modes are equal to $M/2 + 1$ and $M/2 - 1$, respectively. When $M$ is odd, the number of even and odd zero-modes are equal to $(M + 1)/2$ and $(M - 1)/2$, respectively. These numbers are shown in Table 1. For example, three families can be obtained from even (odd) zero-modes with $M = 4$ and $M = 5$ (7 and 8).

Now let us consider the $T^6/Z_2$ orbifold, where the $Z_2$ twist acts as

$$z_1 \rightarrow -z_1, \quad z_2 \rightarrow -z_2, \quad z_3 \rightarrow z_3.$$ (25)

Before orbifolding, the gauge sector has 4D N=4 SUSY, but this orbifolding reduces it to 4D N=2 SUSY. For spinor fields, we impose the following $Z_2$ boundary condition,

$$\psi_{s_1(1)}(-z_1)\psi_{s_2(2)}(-z_2)\psi_{s_3(3)}(z_3) = s_1 s_2 P \psi_{s_1(1)}(z_1)\psi_{s_2(2)}(z_2)\psi_{s_3(3)}(z_3),$$ (26)

where $s_d$ denotes the chirality for the $d$-th $T^2_d$, i.e. $s_d = \pm 1$.

Furthermore, with the above $Z_2$ twist we can consider another $Z_2'$ twist on the $T^6/(Z_2 \times Z'_2)$ orbifold. The $Z'_2$ twist acts as

$$z_1 \rightarrow -z_1, \quad z_2 \rightarrow z_2, \quad z_3 \rightarrow -z_3.$$ (27)

Through $Z_2 \times Z'_2$ orbifolding, only 4D N=1 SUSY remains even in the gauge sector. For spinor fields, we impose the following $Z'_2$ boundary condition,

$$\psi_{s_1(1)}(-z_1)\psi_{s_2(2)}(z_2)\psi_{s_3(3)}(-z_3) = s_1 s_3 P' \psi_{s_1(1)}(z_1)\psi_{s_2(2)}(z_2)\psi_{s_3(3)}(z_3),$$ (28)

where $P'$ can be independent of $P$. Then, depending on the projections $P$ and $P'$, even or odd modes for the $d$-th torus remain such as $\Theta_+^M(z_d)$ or $\Theta_-^M(z_d)$ and their products provide with zero-modes on the $T^6/(Z_2 \times Z'_2)$ orbifold.

### 3 E₆ model

Here, we consider 10D super Yang-Mills theory with the $E_6$ gauge group.

#### 3.1 $T^6$ model

We compactify the extra six-dimensions on $T^6$. We introduce magnetic fluxes (7) along the $U(1)_a$ direction, which breaks the gauge group, $E_6 \rightarrow SO(10) \times U(1)_a$. The $E_6$ adjoint representation is decomposed as

$$78 = 45_0 + 1_0 + 16_1 + \overline{16}_{-1}.$$ (29)
for $SO(10) \times U(1)_{a}$. Here, $16_1$ and $\overline{16}_{-1}$ correspond to the broken part and the corresponding gaugino fields appear as matter fields.

For example, we assume magnetic fluxes,

$$m_{(1)}^a = 3, \quad m_{(2)}^a = 1, \quad m_{(3)}^a = 1.$$  \hfill (30)

Then, the chiral matter fields corresponding to $16_1$ and $s_d = (+, +, +)$ have zero-modes, but there are no massless modes for $\overline{16}_{-1}$. Furthermore, the number of $16_1$ is equal to $m_{(1)}^a m_{(2)}^a m_{(3)}^a = 3$, that is, the model with three families of $16_1$. Their wavefunctions are written as

$$\Theta^{j,3}(z_1)\Theta^{1,1}(z_2)\Theta^{1,1}(z_3).$$  \hfill (31)

The flavor structure is determined by the first torus $T_1^2$. Thus, the massless matter spectrum is realistic, although there is no Higgs fields and the gauge sector has 4D N=4 SUSY.

The $U(1)_a$ symmetry is anomalous. We assume that its gauge boson become massive by the Green-Schwarz mechanism. Hereafter, we also assume that if other $U(1)$ symmetries become anomalous they become massive by the Green-Schwarz mechanism.

Here, we break the $SO(10)$ gauge group further to the standard model gauge group up to $U(1)$ factors, i.e. $SU(3) \times SU(2) \times U(1)_Y \times U(1)_b$, by introducing Wilson lines along $U(1)_Y$ and $U(1)_b$ directions. The $16$ representation of $SO(10)$ is decomposed under $SU(3) \times SU(2) \times U(1)_Y \times U(1)_b$ as

$$16 = (3, 2)_{1,-1} + (3, 1)_{-4,-1} + (1, 1)_{6,-1} + (3, 1)_{2,3} + (1, 2)_{-3,3} + (1, 1)_{0,-5},$$  \hfill (32)

where we normalize $U(1)_Y$ and $U(1)_b$ charges, such that minimum charges satisfy $|q^Y| = 1$ and $|q^b| = 1$.

By introducing Wilson lines along $U(1)_Y$ and $U(1)_b$ directions, the generation number does not change, but the zero-mode profiles of three families of $16$ split differently each other among quarks and leptons. Furthermore, their splitting behaviors depend on which torus $T_2^2$ we introduce Wilson lines. Recall that in this model the flavor structure is determined by the first torus $T_1^2$. For example, when we introduce Wilson lines along $U(1)_Y$ and $U(1)_b$ directions on the second torus $T_2^2$, the zero-mode profiles of quarks $(Q, u, d)$ and leptons $(L, e, \nu)$ split as

$$Q : \Theta^{j,3}(z^1)\Theta^{1,1}(z^2 + C^Y - C^b)\Theta^{1,1}(z^3),$$

$$u^c : \Theta^{j,3}(z^1)\Theta^{1,1}(z^2 - 4C^Y - C^b)\Theta^{1,1}(z^3),$$

$$d^c : \Theta^{j,3}(z^1)\Theta^{1,1}(z^2 + 2C^Y + 3C^b)\Theta^{1,1}(z^3),$$

$$L : \Theta^{j,3}(z^1)\Theta^{1,1}(z^2 - 3C^Y + 3C^b)\Theta^{1,1}(z^3),$$

$$e^c : \Theta^{j,3}(z^1)\Theta^{1,1}(z^2 + 6C^Y - C^b)\Theta^{1,1}(z^3),$$

$$\nu^c : \Theta^{j,3}(z^1)\Theta^{1,1}(z^2 - 5C^b)\Theta^{1,1}(z^3),$$  \hfill (33)

where $C^Y$ and $C^b$ are the Wilson lines along $U(1)_Y$ and $U(1)_b$ directions. On the other hand, when we introduce Wilson lines on the first torus $T_1^2$, the zero-mode profiles of
quarks \((Q, u, d)\) and leptons \((L, e, \nu)\) split as

\[
\begin{align*}
Q & : \Theta^{j,3}(z^1 + C^Y/3 - C^b/3)\Theta^{1,1}(z^2)\Theta^{1,1}(z^3), \\
\bar{u} & : \Theta^{j,3}(z^1 - 4C^Y/3 - C^b/3)\Theta^{1,1}(z^2)\Theta^{1,1}(z^3), \\
\bar{d} & : \Theta^{j,3}(z^1 + 2C^Y/3 + C^b/3)\Theta^{1,1}(z^2)\Theta^{1,1}(z^3), \\
L & : \Theta^{j,3}(z^1 - C^Y + C^b)\Theta^{1,1}(z^2)\Theta^{1,1}(z^3), \\
\bar{e} & : \Theta^{j,3}(z^1 + 2C^Y - C^b/3)\Theta^{1,1}(z^2)\Theta^{1,1}(z^3), \\
\nu & : \Theta^{j,3}(z^1 - 5C^b/3)\Theta^{1,1}(z^2)\Theta^{1,1}(z^3).
\end{align*}
\]

Since the flavor structure is determined by the first torus \(T^2_1\), the first case \((33)\) preserves the \(SO(10)\) flavor structure. However, such flavor structure is deformed in the second case \((34)\) by Wilson lines.

In order to make this point clear, for the moment we assume that our model has electro-weak Higgs fields with certain zero-mode profiles, although the present model does not include 4D massless Higgs fields. In general, Yukawa couplings are computed by the overlap integral of three zero-mode profiles, \(\psi_i(z), \psi_j(z)\) and \(\psi_k(z)\),

\[
y_{ijk} = g \int d^6z \psi_i(z)\psi_j(z)\psi_k(z),
\]

where \(g\) denotes the corresponding coupling in the higher dimensional theory. Such overlap integral for extra 6 dimensions are factorized as products of overlap integrals for \(T^2\) in our case. Obviously, when we do not introduce Wilson lines along \(U(1)_Y\) and \(U(1)_b\) directions, the zero-mode profiles of quarks \((Q, u, d)\) and leptons \((L, e, \nu)\) do not split and the above overlap integral leads to the \(SO(10)\) GUT relation among quark and lepton Yukawa matrices. When we introduce the Wilson lines like \((33)\), the overall factors between up and down quark Yukawa matrices as well as lepton Yukawa matrices change, but the ratios of Yukawa matrix elements do not change. On the other hand, in the case with \((34)\), ratios of Yukawa matrix elements are deformed.

Obviously, other configurations of Wilson lines are possible, e.g. \(C^Y\) on \(T^2_1\) and \(C^b\) on \(T^2_2\) and so on. In any case, the flavor structure is determined by which Wilson lines we introduce on the first \(T^2_1\). For example, if we introduce only \(C^b\) on \(T^2_1\), the resultant Yukawa matrices would have the \(SU(5)\) GUT relation.

Thus, the above model is interesting. Its chiral matter spectrum is realistic and the model has the interesting flavor structure, although electro-weak Higgs fields do not appear and the gauge sector has 4D \(N=4\) SUSY.

### 3.2 Orbifold model

Here, let us study the orbifold model. First, we introduce the following magnetic fluxes,

\[
m_{(1)} = 4, \quad m_{(2)} = 1, \quad m_{(3)} = 0,
\]

in order to break \(E_6\) to \(SO(10) \times U(1)_a\). We consider \(T^6/(Z_2 \times Z_2)\) orbifold with the trivial twists, \(P = P' = 1\). Note that all of zero-mode profiles on the third torus are
flat. Then, even zero-modes for both first and second tori survive through the $Z_2 \times Z_2'$ projection and we can realize three families of 16. Their wavefunctions for $T_1^2$ and $T_2^2$ are written as

$$\Theta_i^{4,1}(z_1)\Theta^{1,1}(z_2),$$

and they have flat profiles for $T_3^2$. Note that $\Theta^{1,1}(z_2) = \Theta^{1,1}(z_2)$. The flavor structure is determined by the first torus $T_1^2$. Furthermore, the $Z_2 \times Z_2'$ twists reduce 4D N=4 SUSY to 4D N=1 SUSY in the gauge sector. Hence, the 4D massless spectrum of this orbifold model is quite realistic, although electro-weak Higgs fields do not appear.

We can construct similar models. For example, we introduce the magnetic fluxes,

$$m_a(1) = 5, \quad m_a(2) = 1, \quad m_a(3) = 0,$$

on the $T^6/(Z_2 \times Z_2')$ orbifold. That also leads to three families of 16 with the wavefunctions $\Theta^{i,4}(z_1)\Theta^{1,1}(z_2)$ and the flat profile along $T_3^2$. These wavefunctions, in particular the first $T_1^2$ part, are different from (37), and they lead to different aspects, e.g. in the flavor structure.

Similarly, we can consider the $T^6/Z_2$ orbifold (25) with the magnetic fluxes,

$$m_a(1) = 4, \quad m_a(2) = 1, \quad m_a(3) = 1,$$

and

$$m_a(1) = 5, \quad m_a(2) = 1, \quad m_a(3) = 1.$$

Both of them lead to three families of 16 with the wavefunctions $\Theta^{i,4}(z_1)\Theta^{1,1}(z_2)\Theta^{1,1}(z_3)$ and $\Theta^{i,5}(z_1)\Theta^{1,1}(z_2)\Theta^{1,1}(z_3)$. Note that the third plane is just $T^2$, but not orbifold. For the gauge sector, 4D N=2 SUSY remains and for $T_3^2$ we can introduce Wilson lines, which we discussed in section 2.1. Moreover, on the $T^6/Z_2$ orbifold, the following magnetic flux

$$m_a(1) = 1, \quad m_a(2) = 1, \quad m_a(3) = 3,$$

also leads to three families of 16 with the wavefunctions $\Theta^{1,1}(z_1)\Theta^{1,1}(z_2)\Theta^{1,3}(z_3)$. Recall that on the $T^6/Z_2$ orbifold, the third torus $T_3^2$ and the others have different behaviors from each other.

Now, let us study the possibility for introducing electro-weak Higgs fields. The orbifold has fixed points. It is possible to put certain modes on such fixed points. Here, we assume electro-weak Higgs fields as such localized modes, although the gauge multiplets and three families are originated from 10D bulk modes. If the gauge symmetry is broken to the $SU(3) \times SU(2) \times U(1)_Y \times U(1)_a \times U(1)_b$ on orbifold fixed points, we do not need to introduce full multiplets of $E_6$, $SO(10)$ or $SU(5)$. Thus, we could assume only one pair of Higgs doublets on orbifold fixed points on some of $T_d^2$, and they may be bulk modes on other of $T_d^2$. The Yukawa couplings among Higgs fields and matter fields could be allowed only on fixed points. Thus, Yukawa couplings are determined by magnitudes of zero-mode profiles of quarks and leptons on such a fixed point. Note that all of matter zero-modes are even functions and they have non-vanishing values on fixed points, although odd wavefunctions vanish on fixed points.

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7Some $U(1)$ factors may be broken.
4 $E_7$ and $E_8$ models

4.1 $T^6$ model

Similarly, we can study $E_7$ and $E_8$ models. Their ranks are larger than $E_6$ and their adjoint representations include several representations. The $E_8$ adjoint representation $248$ is decomposed under $E_7 \times U(1)_{E8}$ as

$$248 = 133_0 + 1_0 + 56_1 + 56_{-1} + 1_2 + 1_{-2}.$$  \hspace{1cm} (42)

Note that we are using $U(1)$ charge normalization such that the minimum charge except vanishing charge is equal to one, $|q| = 1$. Then, the $E_7$ adjoint representation $133$ is decomposed under $E_6 \times U(1)_{E7}$ as

$$133 = 78_0 + 1_0 + 27_{-2} + 27_{-2},$$  \hspace{1cm} (43)

and the $56$ representation of $E_7$ is decomposed under $E_6 \times U(1)_{E7}$ as

$$56 = 27_1 + 27_{-1} + 1_2 + 1_{-2}.$$  \hspace{1cm} (44)

Furthermore, the $27$ representation of $E_6$ is decomposed under $SO(10) \times U(1)_{E6}$ as

$$27 = 16_1 + 10_{-2} + 1_4.$$  \hspace{1cm} (45)

Thus, we can construct various models from $E_7$ and $E_8$ models. Quark and lepton matter fields can be originated from several sectors, although such matter fields are originated from $16$ of the $E_6$ adjoint sector in the models of the previous section. In addition, the $E_7$ and $E_8$ adjoint representations include exotic representations. Hence, exotic matter fields, in general, appear in 4D massless spectra. Instead of $U(1)$ charge normalization, we use the $U(1)_{c} \times U(1)_{d}$ basis, such that those charges are related as

$$q_c = \frac{1}{2} q_{E8} + \frac{1}{2} q_{E7}, \quad q_d = -\frac{1}{2} q_{E8} + \frac{1}{2} q_{E7},$$ \hspace{1cm} (46)

where $q_c$, $q_d$, $q_{E8}$ and $q_{E7}$ denote $U(1)_c$, $U(1)_d$, $U(1)_{E8}$ and $U(1)_{E7}$ charges, respectively. In addition, we denote $U(1)_{E6}$ by $U(1)_a$ as in section 3. Also, as in section 3, we use the notation $U(1)_b$, which appears through the $SO(10)$ breaking as $SO(10) \rightarrow SU(5) \times U(1)_b$.

Here, we show just simple illustrating models. First of all, we can construct almost the same model as the $E_6$ models. For example, we start with the 10D $E_7$ super Yang-Mills theory. We can introduce magnetic fluxes with the same form in $U(1)_{E6}$ as (30). Furthermore, we introduce Wilson lines such that the gauge group is broken down to $SU(3) \times SU(2) \times U(1)_Y$ up to $U(1)$ factors. Then, we realize three families of quarks and leptons under the standard model gauge group, that is, the same 4D massless spectrum as one in section 3.1, although the gauge sector has partly 4D N=4 SUSY and there is no Higgs fields. Similarly, the same model can be derived from the 10D $E_8$ super Yang-Mills theory. Also the same orbifold models as one in section 3.2 can be derived from 10D $E_7$ and $E_8$ super Yang-Mills theories.
Now, let us consider another illustrating model with different aspects. We start with the 10D $E_8$ super Yang-Mills theory. When $E_8$ is broken to the standard model gauge group, there are five $U(1)$’s including $U(1)_Y$, i.e., $U(1)_I$ ($I = a, b, c, d, Y$). We introduce magnetic fluxes $m^I_{(d)}$ along these five $U(1)_I$ directions. Then, the sum of magnetic fluxes $M = \sum_I q^I m^I_{(d)}$ appears in the zero-mode Dirac equation for the matter field with charges $q^I$. We require that \( \sum_I q^I m^I_{(d)} \) should be integer for all of matter fields, that is, the quantization condition of magnetic fluxes [32].

For example, five $(3,2)_1$ representations under $SU(3) \times SU(2) \times U(1)_Y$ as well as their conjugates appear from the 248 adjoint representation. Three of them appear from three 27 representations of 248, i.e., Eqs. (42), (43) and (44). In the zero-mode equations of such three $(3,2)_1$ matter fields, the following sum of magnetic fluxes $\sum_I q^I m^I_{(d)}$ appear

\[
\begin{align*}
m^{Q1}_{(d)} &= m^c_{(d)} + m^a_{(d)} - m^b_{(d)} + m^Y_{(d)}, \\
m^{Q2}_{(d)} &= m^d_{(d)} + m^a_{(d)} - m^b_{(d)} + m^Y_{(d)}, \\
m^{Q3}_{(d)} &= -m^c_{(d)} - m^d_{(d)} + m^a_{(d)} - m^b_{(d)} + m^Y_{(d)}.
\end{align*}
\]

(47)

In addition, one $(3,2)_1$ representation appears from 16 of the $E_6$ adjoint 78 representation [29] as section 3. In the zero-mode equation of such $(3,2)_1$ matter field, the following sum of magnetic fluxes $\sum_I q^I m^I_{(d)}$ appears

\[
m^{Q4}_{(d)} = -3m^a_{(d)} - m^b_{(d)} + m^Y_{(d)}.
\]

(48)

Moreover, the $SO(10)$ adjoint 45 representation also include a $(3,2)_1$ representation and the corresponding matter field has the sum of magnetic fluxes $\sum_I q^I m^I_{(d)}$: 8

\[
m^{Q5}_{(d)} = 4m^b_{(d)} + m^Y_{(d)},
\]

(49)

in the zero-mode equation. Here, we require that all of $m^{Q1}_{(d)}$, $m^{Q2}_{(d)}$, $m^{Q3}_{(d)}$, $m^{Q4}_{(d)}$ and $m^{Q5}_{(d)}$ should be integers. Similarly, we require that $\sum_I q^I m^I_{(d)}$ should be integers for all of matter fields with charges $q^I$, which appear from the $E_8$ adjoint 248 representation. By an explicit computation, it is found that the sum $\sum_I q^I m^I_{(d)}$ for any charge $q^I$ appearing from 248 can be written as a linear combination of $m^{Q1}_{(d)}$, $m^{Q2}_{(d)}$, $m^{Q3}_{(d)}$, $m^{Q4}_{(d)}$ and $m^{Q5}_{(d)}$ with integer coefficients. Thus, when all of $m^{Q1}_{(d)}$, $m^{Q2}_{(d)}$, $m^{Q3}_{(d)}$, $m^{Q4}_{(d)}$ and $m^{Q5}_{(d)}$ are integers, the sum $\sum_I q^I m^I_{(d)}$ for any charge $q^I$ of 248 is always integer.

8The $SO(10)$ adjoint 45 representation includes another $(3,2)$ representation but its $U(1)_Y$ charge is different.
Using the above notation, we introduce the magnetic fluxes such as,

\[ m_{Q_1}^{(1)} = 1, \quad m_{Q_1}^{(2)} = -1, \quad m_{Q_1}^{(3)} = -3, \]
\[ m_{Q_2}^{(1)} = -1, \quad m_{Q_2}^{(2)} = 0, \quad m_{Q_2}^{(3)} = 1, \]
\[ m_{Q_3}^{(1)} = -1, \quad m_{Q_3}^{(2)} = 0, \quad m_{Q_3}^{(3)} = 1, \]
\[ m_{Q_4}^{(1)} = -1, \quad m_{Q_4}^{(2)} = 0, \quad m_{Q_4}^{(3)} = 1, \]
\[ m_{Q_5}^{(1)} = -2, \quad m_{Q_5}^{(2)} = -1, \quad m_{Q_5}^{(3)} = 0. \] (50)

In addition, we also introduce all possible Wilson lines on each torus along five \( U(1) \) directions. Then, the gauge group is \( SU(3) \times SU(2) \times U(1)_Y \) with \( U(1) \) factors.

The 4D massless spectrum of this model includes the following matter fields under the standard gauge group, \( SU(3) \times SU(2) \times U(1)_Y \),

\[
3 \times \left[ (3, 2)_1 + (\overline{3}, 1)^{-4} + (\overline{3}, 1)_2 + (1, 2)_{-3} + (1, 1)_6 \right] \\
+ 8 \left[ (1, 2)_3 + (1, 2)_{-3} \right] \\
+ 15 \times \left[ (3, 1)_4 + (\overline{3}, 1)^{-4} \right] + 6 \times \left[ (3, 1)^{-2} + (\overline{3}, 1)_2 \right] + 27 \times \left[ (1, 1)_6 + (1, 1)_{-6} \right],
\] (51)

and \( SU(3) \times SU(2) \) singlets with vanishing \( U(1)_Y \) charges. That is, this massless spectrum includes three families of quarks and leptons as well as eight pairs of up- and down-sectors of electroweak Higgs fields. In addition, many vector-like matter fields appear, but matter fields with exotic representations do not appear even in vector-like form. Such exotic matter fields have (effectively) vanishing magnetic flux on one of \( T^2_d \). Then, such fields become massive when we switch on proper Wilson lines. Thus, this model has semi-realistic massless spectrum, although the gauge sector still has 4D \( N=4 \) SUSY. We can write the wavefunctions of these zero-modes. For example, the zero-mode wavefunctions of left-handed quarks are written as

\[
\Theta^{1,1}(z_1 + C_1)\Theta^{1,1}(z_2 + C_2)\Theta^{(3)}(z_3 + C_3/3),
\] (52)

for \( j = 1, 2, 3 \), where \( C_d \) denote Wilson lines along five \( U(1) \) directions. Thus, the flavor structure for the left-handed quarks is determined by the third torus. Similarly, we can write zero-mode wavefunctions of the other matter fields. The above massless spectrum includes several vector-like generations of right-handed quarks as well as right-handed leptons. These vector-like generations may gain mass terms. Thus, the flavor structure of chiral right-handed quarks depends on mass matrices of vector-like generations.

Similarly, various models can be constructed within the framework of \( E_7 \) and \( E_8 \) models with magnetic flux and Wilson line backgrounds. This type of model building would lead to quite interesting models. We would study these types of model building systematically elsewhere.

\[ ^9 \text{In the limit of vanishing Wilson lines, such exotic fields appear in the vector-like form, but they become massive for finite values of Wilson lines.} \]
4.2 Orbifold model

The orbifold background with $E_7$ and $E_8$ gauge groups can also be studied. As the $E_6$ model, we can consider the orbifold models with the trivial twist $P = 1$. However, here we study the orbifold models with non-trivial twists and Wilson lines, which break the gauge groups with reducing their ranks, in order to show the variety of model building on the backgrounds with magnetic fluxes, Wilson lines and orbifolding.

As an illustrating model, we start with the 10D $E_7$ super Yang-Mills theory on the $T^6/Z_2$ orbifold of section 2.2. The $E_7$ adjoint representation $133$ is decomposed under $SO(10) \times SU(2) \times U(1)$ as

$$133 = (45, 1)_0 + (1, 3)_0 + (1, 1)_0 + (16, 2)_1 + (\overline{16}, 2)_{-1} + (10, 1)_2 + (10, 1)_{-2}. \quad (53)$$

For example, we assume the following magnetic fluxes along this $U(1)$ direction,

$$m_{(1)} = 3, \quad m_{(2)} = 1, \quad m_{(3)} = 1. \quad (54)$$

Then, three $(16, 2)_1$ zero-modes as well as 24 $(10, 1)_2$ zero-modes would appear without orbifolding. Now, let us study non-trivial orbifold twists and Wilson lines, e.g. in the $SU(2)$ part. We concentrate on $(16, 2)_1$ matter fields, because only these matter fields have non-trivial representations under the $SU(2)$ group, that is, the doublet,

$$\begin{pmatrix} \lambda_{1/2} \\ \lambda_{-1/2} \end{pmatrix}. \quad (55)$$

Before orbifolding, two components, $\lambda_{1/2}$ and $\lambda_{-1/2}$, in the $SU(2)$ doublet have the zero-mode wavefunctions,

$$\lambda_{1/2} : \Theta_{1/2}^{j,3}(z^1)\Theta_{1/2}^{1,1}(z^2)\Theta_{1/2}^{1,1}(z^3),$$
$$\lambda_{-1/2} : \Theta_{-1/2}^{j,3}(z^1)\Theta_{-1/2}^{1,1}(z^2)\Theta_{-1/2}^{1,1}(z^3), \quad (56)$$

with $j = 0, 1, 2$. Here, these forms of wavefunctions are the same, i.e. $\Theta_{1/2}^{j,3}(z^1) = \Theta_{-1/2}^{j,3}(z^1)$, $\Theta_{1/2}^{1,1}(z^2) = \Theta_{-1/2}^{1,1}(z^2)$ and $\Theta_{1/2}^{1,1}(z^3) = \Theta_{-1/2}^{1,1}(z^3)$, but we put the indices $\pm 1/2$ to show that they correspond to $\lambda_{1/2}$ and $\lambda_{-1/2}$ components, respectively. We embed the $Z_2$ twist $P$ in the $SU(2)$ gauge space as

$$P = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad (57)$$

for the $SU(2)$ doublet. In addition, we introduce a Wilson line along the Cartan direction of $SU(2)$, i.e.

$$\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \quad (58)$$
e.g. on the first torus. Since this Wilson line is not commutable with the orbifold twist $P$, the $SU(2)$ gauge group is completely broken. Through this breaking, only three $16$ matter fields among three $(16, 2)$ fields remain and their wavefunctions are obtained as

$$\Theta^{j, 3}_{1/2}(z^1 + C/2M_3)\Theta^{1, 1}_{1/2}(z^2)\Theta^{1, 1}_{1/2}(z^3) + \Theta^{3-j, 3}_{-1/2}(z^1 - C/2M_3)\Theta^{1, 1}_{-1/2}(z^2)\Theta^{1, 1}_{-1/2}(z^3),$$

(59)

up to a normalization factor, where $C$ denotes the continuous Wilson line on the first torus. Similarly, we can introduce Wilson lines on other tori. The orbifold twist $P$ acts trivially on the $(10, 1)_2$ fields, because they are singlets. Then, six $(10, 1)_2$ fields among 24 modes remain after orbifolding.

This model can realize the three families of $16$ matter fields as well as several (would-be Higgsino) fields $10$. Furthermore, if we break $SO(10)$ to $SU(3) \times SU(2) \times U(1)_Y$ by Wilson lines on the third torus, the wavefunction profiles of $16$ matter fields would split such that the profiles of quarks and leptons have peaks at different points.

In the above example, we have embedded the non-trivial combination between orbifold twists and Wilson lines only into the $SU(2)$ part. One can embed them into other parts and break e.g. $SO(10)$ by the orbifold twist. Of course we can start with the $E_8$. At any rate, non-trivial combinations of magnetic fluxes, Wilson lines and orbifold twists could lead to various interesting models. Thus, it would be interesting to study more on this type of model constructions.

5 Conclusion and discussion

We have studied 10D super Yang-Mills theory with the gauge groups, $E_6$, $E_7$ and $E_8$. On the torus and orbifold compactifications, we have considered the magnetic flux background as well as Wilson lines. This type of model building leads to interesting models. One of simple examples is the $E_6$ model on the torus and orbifold. The $E_7$ and $E_8$ models would lead to various interesting models. Thus, it would be interesting to investigate more on $E_7$ and $E_8$ models. We would study them systematically elsewhere.

Also it would be interesting to study more on 4D effective theory, e.g. their flavor structure and predictions on quark/lepton masses and mixing angles after we would construct 4D models with realistic spectra. Although we have started with 10D theory, we can start with other extra dimensions, e.g. 6D super Yang-Mills theory.

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[10] They have no couplings such as $161610$ (of the superpotential) in bulk, because all of remaining matter fields have the same 6D chirality $(s_1, s_2, s_3) = (+, +, +)$ and their couplings are not allowed in the 10D Lagrangian. Their couplings could be allowed on the orbifold fixed points, where the 10D Lorentz symmetry is violated.
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