I. INTRODUCTION

The thermal and field emission of electrons from metal surfaces is a limiting factor on the performance and operation of high-gradient room temperature accelerator structures. In practice, metal surfaces are not perfectly flat and clean. The microscopic imperfections cause a large variation in the local surface electric field. Since the field emission is an exponentially increasing function of surface field, strongly emitting sites form where the field is enhanced due to local imperfections. If the imperfections are due to contaminants, oxides and/or adsorbates, then there is variation in the work function as well. For purposes of this paper, work function variation due to adsorbate or crystal face variation are not considered. Metallic protrusions are a type of surface imperfection that can cause enhanced field emission and heating. Here, we particularly focus on heating and the possibility of thermal runaway and melting of protrusion tips, as these are put forward as precursors to breakdown in the community. From these considerations we want to model the processes in a micro-protrusion to answer the following questions: i) How does the geometry of the protrusion affect the heating and emission? ii) How does the temperature of the protrusion evolve and is there a possibility of thermal runaway and melting of protrusion tips, as these are put forward as precursors to breakdown in the community? iii) How does the RF frequency affect the temperature rise?

Field emitting structures are widely analyzed both in the context of dark current, device breakdown and field emitter technology. Previous studies that inspired our work can be listed as follows. Wang and Loew gave a tutorial account of field emission and RF breakdown and showed elementary calculations related to field enhancement and subsequent emission in RF fields. Chatterton, in his theoretical study published in 1965, established limits for the applied electric field, by considering various geometries for the protrusions. Ancona numerically analyzed failure in molybdenum field emitters taking Nottingham heating into account which lays down the methodology that we followed in this study, and finally, Jensen et. al. built an analytically tractable model to describe the geometry and heating of micro protrusions and their contribution to dark current.

Our approach to determine the temperature rise in micro-protrusions is to break the problem into several distinct, but interrelated sub-problems. These include: i) the determination of the electric field outside the protrusion, ii) the determination of the rate of electron emission from the surface of the protrusion including an estimate of the effect of the electron space charge on the electric field near the surface, iii) the determination of the conduction current in the protrusion and iv) the determination of the temperature rise due to Joule heating.
implied by the conduction current and the Nottingham effect implied by the electron emission.

The physical processes governing these problems occur on different time scales and this will motivate a number of simplifying approximations that will allow these problems to be solved sequentially. First, because the protrusion is small in comparison to the vacuum wavelength of the RF field in the structure, fields outside the protrusion will be essentially electrostatic and will oscillate following the RF field. We will assume that the electron space charge field is a small perturbation to the applied field (except near the surface of the protrusion) so that the electric field outside the protrusion is essentially a vacuum field. Finally, we will assume that the electrical conductivity of the material composing the protrusion is high such that the potential on the surface of the protrusion can be taken to be a constant (zero) in the calculation of the field outside the protrusion.

The rate of electron emission from the surface of the protrusion will be calculated assuming that the surface is locally flat. Here it is assumed that the variation of the electric field along the surface occurs on a scale length that is greater than the atomic dimensions that define the emitting regions. Further, we assume that the electrons transit through the emitting region rapidly compared with the time scale for variation of the electric field. The result will be an emission law that gives the current density emitted from the surface of the protrusion as a function of the local normal component of the vacuum electric field, calculated previously, and the local temperature of the surface. This emission law will be modified to account for shielding of the emitting surface by the electron space charge layer. Associated with the emitted current density, there is a heat flux leaving the surface. The heat flux may be either positive or negative due to the so-called Nottingham effect [8].

The electric field and conduction current inside the protrusion will be calculated under the quasi-static assumption that the relaxation time of the conduction current is much shorter than the period of the RF fields. Further, we assume that the electrons transit through the emitting region rapidly compared with the time scale for variation of the electric field. The result will be an emission law that gives the current density emitted from the surface of the protrusion as a function of the local normal component of the vacuum electric field, calculated previously, and the local temperature of the surface. This emission law will be modified to account for shielding of the emitting surface by the electron space charge layer. Associated with the emitted current density, there is a heat flux leaving the surface. The heat flux may be either positive or negative due to the so-called Nottingham effect [8].

The electric field outside the protrusion is determined by Laplace’s equation with the following boundary conditions. The potential on the surface of protrusion and on the surface of the supporting conductor is taken to be zero. Far above the protrusion, the potential matches that of a uniform electric field, where \( \mathbf{F}_0 = F_0 \hat{z} \) represents the time dependent, normal component of the RF electric field that would be present if the surface was flat (i.e. no protrusion).

We will solve this problem using a point charge approach based on the method of images [5]. In this approach the potential is represented as the sum of the potential generating the applied field (\( \phi = -zF_0 \)) and the potential due to a set of point charges along the \( z \)-axis located in the protrusion, and their images located along the \( z \)-axis in the conductor below the protrusion. The charge and its image constitute a dipole. The number of dipoles in the model is given by the parameter \( N \).

We place the first charge at \( z_1 = a_o \). The rest of the charges are placed at steps with sizes decreasing geometrically with a constant factor \( r \), so that the position of \( n^{th} \) charge becomes: \( z_n = \sum_{j=1}^{n} r^{j-1} a_o \) where \( 1 < n < N \). The image charges are placed at \( -z_n \). As a result we obtain the following formula for the potential:

\[
V_N(\rho, z) = F_o a_o \left\{ -\frac{z}{a_o} + a_o \sum_{j=1}^{N} \frac{\lambda_j}{\sqrt{\rho^2 + (z-z_j)^2}} \right\}
\]

To determine the relative strength of charges, \( \lambda_n \), we demand that the potential satisfies the following recursion relation: for each \( n, n = 1 \) to \( N, \lambda_n \) is determined by \( V_n(0, z_{n+1}) = 0 \). For example, if we have the model with a single dipole, \( N = 1 \), to find \( \lambda_1 \), we demand \( V_1(0, z_2) = F_o a_o \left\{ -2z/a_o + a_o[\lambda_1/(z_2 - z_1) - \lambda_1/(z_2 + z_1)] \right\} = 0 \). In the case \( N = 2 \), to find \( \lambda_1 \) and \( \lambda_2 \), we solve \( V_1(0, z_2) = 0 \) and \( V_2(0, z_3) = 0 \), sequentially.

An example is shown in Fig. I for a particular choice of these parameters as described in the caption. The thick
line shows the surface of the protrusion (zero potential) and the thin lines show level curves for other values of the potential. Only the potential outside the protrusion is of interest to us. We note that the potential curves outside the protrusion, near its tip, are crowded together indicating a strong enhancement of the electric field near the tip. Most of the electron emission will occur there. Fig. 1 also shows the boundary of protrusions for three different choices of point charge parameters. For each protrusion, the field enhancement factor $\beta$, that is defined as the ratio of geometrically enhanced electric field at the tip to $F_o$, is calculated. It can be seen that varying these parameters, potentials can be constructed for protrusions of differing degrees of pointedness. Finally, we plot in Fig. 2 the field enhancement factor $\beta$ for different combinations of the point charge model parameters.

### III. EMISSION LAW

The second step in our procedure is to determine the emitted current density on the surface of the protrusion. The emission current is a combination of thermal and field emission, which we take to be described by a general thermal field (GTF) emission formula [9].

In this treatment the free electrons in the metal face a square potential, which may be modified by external fields and the Schottky effect. One then computes the wave function in the presence of this potential and calculates the transmission probability. Given the Fermi-Dirac distribution of electrons, the current can be expressed as an integral over energy of the product of supply function and the transmission probability.

$$J(F,T) = \frac{q}{2\pi \hbar} \int_0^\infty D(E) f(E) dE$$

(2)

where $D(E)$ denotes the tunneling or transmission probability as a function of energy and $f(E)$ denotes the electron supply function as a function of energy. Expressions for $D(E)$ and $f(E)$ can be found in Ref. [9]. The resulting integral in (2) is a function of the local electric field, $F$, and the temperature, $T$, on the surface.

Equation (2) describes electron field emission and reduces to the Fowler-Nordheim formula in the low temperature limit [13]. In the high temperature low field limit it describes thermionic emission and reduces to the Richardson-Laue-Dushman formula [10]. However, there is a transition region in which both formulas fail to apply. By using series expansion techniques Murphy and Good proposed a formula valid for the transition region [4]. (See Coulombe and Meunier for a comparison of Murphy-Good formula with its predecessors [11].) Finally in 2007 Jensen proposed a general thermal-field emission (GTF) formula for a wide range of temperatures and fields [12]. Here, instead of employing the analytical expressions for the GTF current, we resort to numerical evaluation of Eq. (2). In Fig. 3 we plot the GTF current density obtained from Eq. (2) at different temperatures for copper, versus peak electric field at the tip. It should be noted that there is a bound to the maximum electric field that can be treated in this model. It should be noted that there is a bound to the maximum electric field that can be treated in this model. First, the maximum of the Schottky modified potential barrier should not drop below the Fermi level. For example, copper has a work function of 4.5 eV, and the barrier maximum is $-\sqrt{e^3F/(4\pi\epsilon_o)}$ measured from the vacuum energy [4]. The formula (2) is a good approximation provided the barrier maximum is above the Fermi level. This gives a maximum field of around 14 $GV/m$. For a background field of 300 $MV/m$ this corresponds to a field enhancement factor of 47. As an example,
probability distribution function to calculate the average we look at Eq. (2). The integrand of (2), is used as a through the surface. To see the direction of heat flux

The flow of electrons brings about a convective heat flux surface. Each electron emitted from the conduction band emitting body due to the flow of electrons through its

The Nottingham effect is the heating or cooling of an sion and the Nottingham effect that acts at the surface. Therefore, we use the general thermal-field (GTF) emission in this study.

We use the thermionic and studies[9, 11, 15] already established, the thermionic and field emissions formulas miscalculate the correct emission from a heated field enhancing structure. Therefore, we use the general thermal-field (GTF) emission in this study.

The emission current leads to two mechanisms of heating, namely the Joule heating in the bulk of the protrusion and the Nottingham effect that acts at the surface. The Nottingham effect is the heating or cooling of an emitting body due to the flow of electrons through its surface. Each electron emitted from the conduction band of the metal surface is replaced by a Fermi level electron. The flow of electrons brings about a convective heat flux through the surface. To see the direction of heat flux we look at Eq. (2). The integrand of (2), is used as a probability distribution function to calculate the average energy per electron leaving the surface. Eq. (3) describes the Nottingham flux \( Q \), the heat flux escaping through the surface due to the emitted electrons. If \( \mu \) is the Fermi energy:

\[
Q(F,T) = \frac{1}{2\pi h} \int_0^\infty (E - \mu) D(E) f(E) dE \quad [\text{W/m}^2].
\]

The temperature at which the average energy of emitted electron matches the Fermi energy is called the inversion temperature, \( T^* \). The Nottingham effect causes heating of the surface for temperatures below the inversion temperature and cooling otherwise[13, 16]. We use the Nottingham heat flux as a boundary condition in our solution of the heat diffusion equation to specify the gradient of temperature along the surface normal[3].

Figure 5 shows the behavior of the Nottingham flux with respect to external electric field and temperature. Some immediate observations are the following. i) The transition temperature is above the melting temperature, therefore the Nottingham effect is always a heating effect as far as the solid state is concerned. ii) The Nottingham flux is almost constant over the range of temperature below melting, which means heating is constant over time and no run away is possible due to the Nottingham effect. iii) The Nottingham flux falls sharply as the electric field decreases, which means, Nottingham effect contributes only at the tip.

**IV. CURRENT DENSITY IN THE PROTRUSION**

Inside the protrusion we assume the current density and electric field are related by Ohm’s Law \( \mathbf{J} = \sigma \mathbf{E} \) where \( \sigma \) is a spatially uniform electrical conductivity conductivity. The condition that charge density does not accumulate in time, \( \nabla \cdot \mathbf{J} = -\partial \rho / \partial t = 0 \) then yields a Laplace

![FIG. 3. Comparison of Fowler-Nordheim (FN) and General Thermal Field Current Densities for copper work function taken as \( \Phi = 4.5 \text{ eV} \).](image1)

![FIG. 4. Electric Field Magnitude in the presence of space charge effect \( F_S \) compared to its vacuum value \( F_V \) for various values of tip radius \( r_T \). The dotted line shows the case with no space charge.](image2)
equation for the electrostatic potential. Here we assume
the dimensions of the protrusion are less than the skin
depth. We apply the following boundary conditions to
the assumed azimuthally symmetric protrusion. The ra-
dial current density at the axis is zero due to symme-
try. The electric potential is assumed to be zero deep
in the base supporting the protrusion. Finally, the nor-
mal component of the electric field at the surface inside
the protrusion is determined such that the normal cur-
dent density matches the emission current calculated in
Eq. (2).

To solve the Laplace equation we use a finite element
method (FEM) with triangular elements [17, 18]. The
triangular grid is employed so that variations of the po-
tential near the tip can be resolved accurately and effi-
ciently. The system is solved and the solution is plotted
using MATLAB [19]. Fig. 6 shows the false color im-
ages of the square of the current density inside the metal.
The figure indicates that the Joule heating term is con-
centrated in the tip area. As an example, for a protrusion
with 1 µm base radius, half maximum of the heating term
occurs at 12 nm below the tip of the protrusion.

V. TEMPERATURE RISE IN THE
PROTRUSION

In the previous section, the current density inside the
metal was obtained. Using this we calculate the volumet-
ric Joule heating. The temperature rise in the protrusion
as a function of time, \((\Delta T(r, z, t))\) is then obtained by
solving the heat diffusion equation with the volumetric
Joule heating term as a source and the following bound-
ary conditions. The radial temperature gradient on the
axis is zero and the temperature rise is zero on the bottom
of the base. The boundary condition on the protrusion
surface is determined by the Nottingham heat flux de-
fined in Eq. (3). Specifically, \(\kappa \partial \Delta T / \partial x_n = Q\), where \(\kappa\)
is the thermal conductivity. The thermal conductivity,
\(kappa\), depends on the relaxation time of scattering thus
the temperature. However, here, \(\kappa\) is assumed to be tem-
perature independent. This assumption is validated both
through simulations and another study by Jensen et. al.
[20]. The space dependent part of the heat equation is
calculated by using the finite element method. The re-
resulting ordinary differential equations are solved in time
using MATLAB® ode15s [19].

The temporal variation of the RF electric field comes
into play while solving these equations. Specifically, both
the Joule heating term and the Nottingham heat flux oscil-
culate periodically in time with a period equal to that
of the RF signal. The result is that the temporal varia-
tion of the temperature rise, \(\Delta T\), consists of a periodic
variation superimposed on a slow temperature rise due
to the average heating. Figure 7 shows the tempera-
ture at the tip of a protrusion during the last cycle of
a 100 ns pulse for two different frequencies, \(f = 1\) GHz
and \(f = 10\) GHz. Other parameters are indicated in
the caption, and time is normalized to the RF period.
The temporal pulse envelope of the RF in this case is a
square wave. As a consequence the temperature variation
during cycle is nearly the same for all cycles in a pulse.
Thus, the maximum tip temperature is essentially deter-
mined during the course of a single cycle. Also shown
in Fig. 7 are the cycle averaged temperature rise, which
is the same for both the 1 GHz and 10 GHz cases, and
the temperature rise during the first cycle if space charge
affects are not included in the model. In this latter case
the temperature quickly rises to the melting tempera-
ture \(T_m = 1357^\circ K\). Thus space charge is an essen-
tial component of the model. The maximum temperature
excursion depends weakly on frequency, with the tem-
perature rise in the 1 GHz case exceeding that of the
10 GHz case. The trend with frequency is less dramatic
than would be expected based on dimensional analysis of
the one dimensional heat equation. Specifically, if \(T_{RF}\)
is the period of the RF, one would expect the amount of
heat deposited during a cycle, \(U\) to scale as \(U \sim T_{RF}\)
The two cases. For these cases the ratio of temperature rise at the time of the peak excursion in the two cases, $f=1$ GHz on the left and $f=10$ GHz on the right.

FIG. 8. Spatial distribution of the temperature rise at the time of the peak excursion in the two cases, $f=1$ GHz on the left and $f=10$ GHz on the right.

FIG. 7. Comparison of $\Delta T$, change in tip temperature during the last RF cycle of a 100 ns pulse versus normalized time, between 1 GHz and 10 GHz cases. $\beta \approx 47$ and $E_0 = 300 \text{MV/m}$. RF mean of the change in temperature is shown with the dashed line. The dotted lines show the evolution of the tip temperature during the first cycle in the 1 GHz and 10 GHz cases, respectively, when space charge is neglected.

The observed scaling with period is $U/\Delta \sim T_{RF}^{1/2}$. The expected temperature rise, in one dimension, will scale as $U/\Delta \sim T_{RF}^{1/2}$. The observed scaling with period is weaker. This is due to the pointed nature of the tip. As the width of the protrusion increases with distance from the tip, the volume over which the heat is spread, $\Delta V$, increases with penetration depth. We expect the temperature rise to scale as $\Delta T \sim U/\Delta V$, where for a conical tip $\Delta V \sim \Delta^3$. Figure 8 shows the spatial distribution of the temperature rise at the time of the peak excursion in the two cases. For these cases the ratio of temperature rise is $\Delta T_{1 \text{GHz}} / \Delta T_{10 \text{GHz}} = 1.70$. Based on a conical tip we could expect $\Delta T_{1 \text{GHz}} / \Delta T_{10 \text{GHz}} = 10 / (1.7)^3 \approx 2.0$ which is consistent with the simulation.

To have a better understanding of the role of the Nottingham flux we compare the volumetric equivalent of heating ($P_N$) caused by it, to the volumetric Joule heating $P_J = J^2/\sigma$. Assuming a semi-spherical tip of radius $r_T$, integrating the heat flux over the surface and dividing by the volume of the tip we find an equivalent volumetric heating rate $S_N = 3Q/r_T$. As seen in Fig. 9 $Q \approx 30 \text{W/\mu m}^2$, for an enhancement factor of $\beta \approx 44$ and background field of $E_0 = 300 \text{ MV/m}$. Assuming a tip radius of 100 nm, we find $3Q/r_T \approx 900 \text{W/\mu m}^3$. We now take the ratio of the two volumetric heat sources: $P_N/P_J \approx 3Q/r_T J^2$. For the same value of $\beta$ background field and tip radius, we find $P_J = J^2/\sigma \approx 13.5 \text{W/\mu m}^3$ giving $P_N/P_J \approx 60.6$. This ratio decreases as temperature or electric field on the tip increases. When the range of our data is concerned, its minimum value is approximately 52. This implies that Nottingham heating dominates Joule heating at the tip. The dominance of Nottingham heating over the Joule heating was shown before also by Ancona in the treatment of thermal failure of field emitter tips. The above order of magnitude calculation was validated by comparing the temperature at the tip with and without Joule heating; the difference is less than 0.5%.

Figure 9 summarizes the dependence of the maximum change in tip temperature on various parameters, such as protrusion size, field enhancement factor and RF frequency. Notice that, although the emission law is independent of RF frequency, high frequency causes less heat accumulation and hence lower swings of tip temperature. Similarly, the quantity $a_o$ that determines the protrusion size as seen in eq. (4), does not affect emission and the vacuum electric fields. However, it affects the tip temperature by modifying the space charge behavior.

VI. CONCLUSIONS

In this study we have performed extensive numerical simulations of electrical and thermal behavior of field emission on metal protrusions, that are believed to be a source of dark current and a cause of breakdown in high-gradient accelerating structures. After unifying the ther-
nal and field emission in the same numerical framework, we calculated the Nottingham and Joule heating terms and solved the heat equation to characterize the thermal evolution of emitters under RF electric field. Noteworthy conclusions can be listed as follows:

i) The contribution of Joule heating is negligible compared with the contribution of the Nottingham effect. The Nottingham heating effect is approximately independent of temperature in the region of interest. ii) The largest temperature rise occurs at the tip of the protrusion. The temperature excursion is nearly periodic in times, with period given by the RF. The peak temperature rise is several times the average rise, and the peak rise is relatively insensitive to RF period. Due to both (i) and (ii), scenarios involving thermal runaway are not plausible, unless some external phenomena that strongly couple heating to electric fields are present. iii) Field emission is high enough to induce a space charge effect. The depression of electric field due to space charge causes a 5 to 7 fold reduction in emitted current. This eliminates possibility of tip melting, including the case with the maximum emission allowed by the free electron model. iv) The RF frequency effects the peak temperature. High frequency causes less temperature swing, therefore less peak temperature. v) Both heating and emission are functions of field enhancement factor $β$. The geometric scaling factor $a_o$ slightly modifies heating and emission through the space charge mechanism. Bigger protrusions emit more current in total. The RF averaged current scales with $a_o$ as seen in Fig. 11. vi) Time evolution of temperature and emission current is critically dependent on phenomena that modifies the field enhancement factor. In this study, apart form the geometry, space charge played a decisive role. Gas discharge, plasma formation and ion bombardment mechanisms can be investigated by using our protrusion model, which is fairly simplified due to the afore mentioned considerations.

### Appendix A: The Space Charge Effect

In this study we use a modified version of the Child Langmuir Law to account for electron space charge. The Child Langmuir Law describes the steady current that flows from a planar cathode plate to a planar anode a distance $x_D$ from the cathode at fixed potential $V_o$ when emission is space charge limited. The model includes the Poisson equation for the electrostatic potential $U(x)$.

$$\frac{d^2}{dx^2} U(x) = \frac{|J|}{\epsilon_0 e(x)} = \alpha |J| U^{-1/2}, \quad (A1)$$

along with boundary condition $U(x = 0) = 0$ and $U(x_D) = U_D$, where, $J$ is the current density emitted from the surface of the cathode and $e(x)$ is the velocity of electrons at point $x$. Energy conservation dictates: $e(x) = (2eU(x)/m)^{1/2}$ where $e$ is the fundamental charge and $m$ the electron mass. Introducing the constant $\alpha = (m/(2e))^{1/2}/e_o$ leads to the second equality in (A1). In the space charge limited case the value of current density is determined by the condition $dU/dx|_{x=0} = 0$. To incorporate emission one can use a relation of the form of (2) where we set $J = J(F_s = dU/dx|_{x=0}, T)$.

Barbour et al. have done this by using the Fowler Nordheim emission and Child’s equation together and provided a comparison with experiment[12]. For a historical review and a general mathematical formulation, the reader is encouraged to read the recent article by Forbes[21]. Numerical approaches are also employed to solve equilibrium equations for space charge limited flow for sharp edges and conical points of emitters [22].

In our case, we have to address two issues to incorporate space charge in our model. First, we have a 3D emitter with nontrivial geometry, and second, there is no well defined anode that fixes the domain and boundary condition as in [A1]. We address these as follows: Noting that the protrusion emits most of the current from the tip, we will ignore the emission from the rest of the protrusion. Moreover, for a highly curved tip, the electron current will spread radially as it flows away from the tip. Consequently, the electron density will drop rapidly with distance along the vertical axis of the protrusion away from the tip. Hence, the tip radius will serve as a scale with which we measure the region where the space charge is assumed to affect the emission. We will, thus, assume that one tip radius away from the tip, the electric field reaches its vacuum value. This will serve as a boundary condition on the electric field, in the one dimensional equation. The separation distance $x_D$, is now equal to the tip radius $r_T$.

In summary we solve Eq. (A1) with the boundary conditions

$$U(0) = 0, \quad (A2a)$$

$$\partial_x U|_{x_D} \approx F_v, \quad (A2b)$$

![FIG. 10. The RF averaged current versus $β$ for various values of scaling factor $a_o$.](image)
where $F_V$ is the vacuum field, and we define

$$F_S = dU/dx|_0.$$  \hfill (A2c)

The emission law is used to determine $|J| = J(F_S, T)$. The solution of \((A1)\) is standard. Writing $d(dU/dx)^2 = 2\alpha JU^{-1/2}dU$ and integrating, one finds: $dU/dx = \sqrt{4\alpha JU^{1/2} + F_S^2}$. Integration from $x = 0$ (where $U(0) = 0$) to $x = r_T$ (where $dU/dx = F_V$) yields

$$12\alpha^2 J^2 r_T = (F_V^3 - F_S^3) - 3F_S^2(F_V - F_S).$$ \hfill (A3)

which can be viewed as a transcendental equation for the field on the emitter ($F_S$) once the emission law $J(F, T)$ is prescribed.

The right hand side of \((A3)\) is a monotonically decreasing function of $F_S$ and the left hand side is a monotonically increasing function of $F_S$. Thus we can expect a single solution satisfying $0 < F_S < F_V$. Solution of the transcendental equation for three different tip radii $r_T$, and the generalized emission law Eq. (2) at room temperature is shown in Fig. 4. We note in the limit in which the emission current is large such that $F_S \ll F_V$, we recover a form of the Child-Langmuir law, $J = F_V^{3/2}/(\alpha(12r_T)^{1/2})$.

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