Quantum non-locality vs. quasi-local measurements in the conditions of the Aharonov-Bohm effect

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Abstract. Theoretical explanation of the Meissner effect involves proportionality between current density and vector potential, which has many deep consequences. As noticed by de Gennes, superconductors in a magnetic field “find an equilibrium state where the sum of kinetic and magnetic energies is minimum” and this state “corresponds to the expulsion of the magnetic field”. This statement still leaves an open question: from which source is the superconducting current acquiring its kinetic energy? A naïve answer, perhaps, is from the energy of the magnetic field. However, one can consider situations (Aharonov-Bohm effect), where the classical magnetic field is locally absent in the area occupied by the current. Experiments demonstrate that despite the local absence of the magnetic field, current is, nevertheless, building up. From what source is it acquiring its energy then? Locally, only a vector potential is present. How does the vector potential facilitate the formation of the current? Is the current formation a result of a truly non-local quantum action, or does the local action of the vector potential have experimental consequences? We discuss possible experiments with a hybrid normal-metal superconductor circuitry, which can clarify this puzzling situation. Experimental answers will be important for further developments.

1. Introduction
Superconductors occupy a peculiar position among solid-state objects: they demonstrate quantum properties not only at the microscopic and mesoscopic levels, but also – and most importantly – at the macroscopic level. One encounters this feature when considering superconductors main property: charge transport without resistance. It turns out [1] (F. London and H. London, 1935) that one should adopt a relation \( j \propto -A \) (we drop unimportant factors in intermediate expressions) to avoid infinitely large values of conductivity and to describe the Meissner effect [2]. This immediately separates superconductors from classical objects: in classical physics observables cannot depend on \( A \), since \( A \) is not gauge invariant. As soon as superconductors belong to the quantum world, they can “supply” a quantity, which will make the expression for the current gauge-invariant. Such a quantity is the phase \( \theta \) of the quantum wave function \( \psi = |\psi| \exp(i\theta) \). At the gauge transformation \( A \rightarrow A + \nabla f \), with an arbitrary function \( f(x, y, z, t) \), the phase transforms as \( \theta \rightarrow \theta + f \), so that \( j \propto -A + \nabla \theta \) is then gauge invariant. Does this indicate “sensitivity” of the superconductor to the vector potential? If so, then how could one measure the response of superconductors to the value of \( A \)? These questions are beyond pure theoretical analysis. In typical situations, when one uses superconductor-based
electronics, such as SQUIDs, the responses are being expressed in terms of magnetic fluxes, \( \phi \). That is easily doable using Stokes’ theorem, according to which
\[
\int_S \mathbf{A} \cdot d\mathbf{s} = \int_C \mathbf{curl}\mathbf{A} \cdot d\mathbf{l} = \int_S \mathbf{H} \cdot d\mathbf{s} = \phi ,
\]
where \( S \) is a surface bounded by the contour \( C \). However, the measurement of magnetic fields \( H \) and corresponding fluxes is always possible using closed superconducting circuits. Such measurements in our case would be categorized as “non-local” responses. Non-locality is at the heart of quantum mechanics. Our task here is not in debating that principle. The intention is to find out whether or not superconductors react locally to the presence of the vector potential.

Classical electromagnetic field theory operates with the local fields \( E \) and \( H \). It is customary to express these fields via the 4-vector potential \( A_i = \{ \phi, A \} \), \( i=0,1,2,3 \): \( H = \mathbf{\nabla} \times \mathbf{A} \), \( E = \mathbf{\nabla} \phi - \partial \mathbf{A} / \partial t \). In classical electrodynamics, the value of \( A \) is defined up to the gradient of an arbitrary (gauge) function \( f(\mathbf{x},y,z,t) \). Indeed, one can always perform a transformation \( A = A^\prime + \mathbf{\nabla} f \), which will not change the magnetic, or the electric field provided the scalar potential \( \phi \) is also transformed as \( \phi = \phi^\prime - \partial f / \partial t \).

In quantum theory, this function \( f \) couples with the phase \( \theta \) of the wave function \( \psi \). As was mentioned above, this opens opportunities to measure the value of the vector potential quasi-locally using specific devices with quantum elements. We will describe the details below.

2. The Aharonov-Bohm potential
In the Coulomb gauge \( \mathbf{\nabla} \cdot \mathbf{A} = 0 \), \( A_{\text{int}}(x,y,z) = (\alpha / R^2) \{ -y,0,0 \} \) inside the solenoid \( x^2 + y^2 < R^2 \), which corresponds to a uniform magnetic field \( H_0 = \mathbf{\nabla} \times \mathbf{A} \) (see, e.g., [3,4]). Of immediate interest is the external field described by \( A_{\text{ext}}(x,y,z) = \alpha \{ -y/(x^2 + y^2), y/(x^2 + y^2), 0 \} \), \( x^2 + y^2 > R^2 \). Direct calculation yields \( H = \mathbf{\nabla} \times \mathbf{A}_{\text{ext}} = 0 \) at any point outside the solenoid – no magnetic field associated locally with \( A_{\text{ext}} \neq 0 \). In cylindrical coordinates, \( A_{\text{ext}}(\rho,\theta,\phi) = (0,0,\alpha / \rho) \), which will be used in subsequent analysis. Here \( \alpha = H_0 R^2 / 2 \), where \( R \) is the radius of solenoid.

3. Basic idea of the measuring device
Let us first consider a loop in the plane orthogonal to the axis of the solenoid (figure 1).

Using this figure, one can describe a simple device for measuring the \( A \)-potential. Suppose that the loop is made of a superconducting wire. We consider a very thin wire so that the motion of the electrons is quasi-one dimensional and homogeneous throughout its cross section (i.e., the diameter of the wire is much smaller than the London penetration depth \( \lambda_L \)). Then, if the loop is cooled down below the superconducting transition temperature \( T_c \) in a pre-existent solenoid field, a stationary supercurrent will settle in the wire. This effect has been demonstrated experimentally [5].

It is possible to detect the current in the looped wire by classical instruments of the magnetic field induced in the vicinity of the wire. The whole system represents then a primitive detector of the \( A \)-potential. At the same time, the topology of the detector allows us to integrate along the wire (the
closed contour C in Fig. 1), and express the answer via the magnetic flux inside the solenoid. Then it may be regarded as a measuring device of a non-local influence of the magnetic field \( H_0 \) inside of the solenoid onto the superconducting loop.

To exclude such options, one should not work with a closed loop. Accordingly, let us consider a finite superconducting bar, such as the one shown between points 1 and 2 in figure 2.

The current density in superconducting wire is described by the usual (Ginzburg-Landau-type) equation

\[
 j = j_s = (\hbar e^*/m^*)(\nabla \theta - A)
\]

(see, e.g., [6]), which is a standard quantum-mechanical expression for the current density of charged particles (Cooper pairs) in the magnetic field \( \psi \), normalized by the density \( n_s \) of superconducting electrons: \( |\psi|^2 = n_s \). The velocity of superfluid motion is \( v_s = (\hbar /m^*)(\nabla \theta - (e^*/hc)A) \) and the effective charge and mass are \( e^* = 2e \) and \( m^* = 2m \). In the 1D-motion of electrons, assuming a constant cross section for the conductor, one can deal with the current density instead of the current (i.e., \( |\psi| = \text{const}, |A| \) and \( |j| \) are constant on the contour \( C \)). Since \( j \) is obviously zero, \( A = (hc / e^*)\nabla \theta \), or in other words, the bar possesses a phase difference at its ends \( \oint C A \cdot dl = \int^2_1 A \cdot dl \), which compensates the influence of the \( A \)-field. Is there any physical consequence of this phase difference \( \delta \theta \), and if yes, how to measure it?

**4. Two experimental opportunities: oscillating and pulsed fields**

Let a superconducting bar (1-2), located at a distance \( r \) from the axis of solenoid on a circumference shown in Fig. 2, be a part of a normal circuit containing classical measurement devices. These external devices put current into the circuit, and are able to measure the energy required for the motion of charged carriers. If the superconducting bar has a thickness \( \lambda_L \) and a height \( w \), then kinetic energy of the current is \( \delta E = \varepsilon A \lambda_L A \cdot dl / |A| \), where \( \varepsilon = n_s m v_s^2 / 2 \) is related to the \( A \)-potential via \( v_s = -(e / mc)A \) in the case of unrestricted motion of the supercurrent (\( \nabla \theta = 0 \)). Our suggestion is that the external current source mentioned above mimics the unrestricted motion by removing charges from the ends of the bar. If there is no flux in the solenoid, the current source provides the energy \( \delta E \) when switched on, and sets up a superfluid motion in the bar. Suppose now that there is a flux \( \phi \) in the solenoid (and a corresponding \( A \)-field, figure 1). Because of the action of the \( A \)-field, at a fixed value of current \( j \) in our measuring device, the contribution of the external source should be less by an amount of \( \delta E \), if the direction of the current it is initiating matches the direction of flow being initiated by the \( A \)-field. Reciprocally, the source should provide \( \delta E \) more energy if the directions are opposite. By the same logic, in the case of symmetric AC excitation of the device, the presence of the \( A \)-field will cause an asymmetric response in the circuit (figure 3a).

Our second approach is a pulse-type process. A strip of a superconductor with a cross-section \( S \) and length \( L \) is placed in the \( AB \)-field (figure 2), so that the \( A \)-vector is either parallel or anti-parallel to the length \( L \) (figure 3b). For a given orientation and value of the \( A \)-potential due to the solenoid,
Figure 3. (a)-AC and (b)-pulse detection schemes of the $A$-potential.

one charges the capacitor $C$ to a level $Q$ (with a chosen voltage $V$ and polarity), then opens switch 1 to disconnect the voltage source, and closes switch 2, thus putting current trough the superconducting strip. If the current is less than the critical value, then voltmeter will show no voltage. One can repeat the operations, and increase the voltage, thus increasing the energy $E_c=Q^2/2C=CU^2/2$ stored in the capacitor. When the energy reaches the amount required to destroy the superconducting state in the strip, one stops, and repeats the same procedure with the reversed polarity of the battery. Asymmetry between the threshold values will reveal the role of the vector potential, thus detecting it quasi-locally.

5. Discussion and conclusions

Let us now make some estimates. In the case (a), performing integration, one obtains

$$\delta E = \left(\frac{e^2}{h} R^4 n \lambda L w \vartheta \right) / (8r m e^2),$$

where $\vartheta = f_2 - f_1$. Let us estimate the value of this energy. We choose $\vartheta \sim 1$, $R \sim 0.1 cm$, $r \sim 3R$, $\lambda \sim 10^{-5} cm$, $w \sim 10^{-5} cm$, $n \sim 10^{23} cm^{-3}$, and $H_{0} \sim 10 Oe$. Then $\delta E \sim 1 nJ$, which is a detectable value. For comparison, the energy of the so-called RSFQ pulses used in superconducting electronics is $\sim 0.2 aJ$, i.e., $\delta E$ is equivalent to five billion RSFQ pulse energies.

In the case (b), suppose the bar has $L=1000 \mu m$ and $S=0.01 \mu m^2$, as in the previous case. Then the number of Cooper pairs is about $10^{12}$. Each Cooper pair has $\sim 1 meV$ energy, which means that the upper limit to the energy stored in the capacitor should be about $10^9 eV$, or about $100 pJ$. In principle, the energy can be delivered using a more sophisticated device, such as a pulsing electronics circuit.

These conceptually simple experiments can be used to detect the local influence of the Aharonov-Bohm vector potential on superconductors. The outcome of these experiments can demonstrate whether or not quantum systems can sense the vector potential locally, thus serving the perplexed [9].

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