STRONGLY INTERACTING VECTOR BOSONS
AT TeV $e^\pm e^-$ LINEAR COLLIDERS

— ADDENDUM —

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ABSTRACT

Extending earlier investigations, we analyze the quasi-elastic scattering of strongly interacting electroweak bosons at high-energy $e^\pm e^-$ colliders. The three processes $e^+e^- \rightarrow \bar{\nu}\nu W^+W^-$, $\bar{\nu}\nu ZZ$ and $e^-e^- \rightarrow \bar{\nu}\nu W^-W^-$ are examined at a c.m. energy of 1 TeV for high-luminosity runs. The expected experimental error on the scattering amplitude, parameter-free to leading order in the chiral expansion of the $WW$ interactions, is estimated for 1 TeV colliders at the level of ten percent, providing a stringent test of strong interaction mechanisms for breaking the electroweak symmetries.
1. Unitarity leads to the alternative scenarios that either a light Higgs boson is realized
in the electroweak sector of the Standard Model (SM), or that the electroweak $W^\pm, Z$ gauge
bosons become strongly interacting at high energies $[1]$. Within the canonical formulation of
the Standard Model, analyses of the high-precision electroweak data are in striking agreement
with the existence of a light Higgs boson $[2]$. However, if the SM interactions are supplemented
by low-energy remnants of new interactions at high energy scales, alternatives to the light Higgs
scenario are still viable (see, e.g., Ref.$[3]$).

In a preceding investigation $[4]$ we have analyzed the quasi-elastic scattering of $W^\pm, Z$
bosons,

$$WW \rightarrow WW$$

at TeV $e^\pm e^-$ linear colliders in the high-energy range where the strong interactions between
the electroweak gauge bosons become effective in the absence of a light Higgs boson. The
strong interactions of the $W$ bosons can, in a natural way, be traced back to the interactions
of Goldstone bosons which are associated with the spontaneous breaking of a chirally invariant
theory, characterized by an energy scale $\Lambda \sim \mathcal{O}(1 \text{ TeV})$. As formulated by the equivalence
theorem $[5]$, the Goldstone bosons are absorbed by the gauge bosons to build up the longitudinal
degrees of freedom $[6]$.

Such a theory can be described by an effective Lagrangian, expanded in the dimensions
of the field operators, or equivalently the energy in momentum space $[7]$. This systematic
expansion gives rise to a parameter-free prediction of the $WW$ scattering amplitudes to leading
order; the leading-order predictions therefore reflect the basic dynamical mechanism which
breaks the electroweak symmetries. Higher orders in the expansion are determined by the
detailed structure of the underlying new strong-interaction theory. The effective Lagrangian
can, in unitary gauge, be written as

$$\mathcal{L} = \mathcal{L}_g + \mathcal{L}_0 + \mathcal{L}_4 + \mathcal{L}_5 + \ldots$$

$\mathcal{L}_g$ describes, in standard notation $[4]$, the kinetic terms of the gauge fields:

$$\mathcal{L}_g = -\frac{1}{8} \text{tr} [W_{\mu\nu}^2] - \frac{1}{4} B_{\mu\nu}^2$$

$^1$For a recent theoretical summary see Ref.$[8]$ which includes also triple $W$ production in the $e^+e^-$ annihilation
channels $[9]$, supplementing the present analysis.
\( \mathcal{L}_0 \), the lowest-order term in the chiral expansion, corresponds to the mass terms:

\[
\mathcal{L}_0 = M_W^2 W^+_{\mu} W^-_{\mu} + \frac{1}{2} M_Z^2 Z^-_{\mu} Z^+_{\mu} 
\]  

(4)

The two terms, \( \mathcal{L}_g + \mathcal{L}_0 \), generate the parameter-free \( WW \) scattering amplitudes to leading order in the energy region where the \( WW \) interactions become strong. The dimension-4 operators \( \mathcal{L}_4 \) and \( \mathcal{L}_5 \) are new quadrilinear contact interactions of the \( W^\pm \) and \( Z \) bosons:

\[
\mathcal{L}_4 = \alpha_4 \left[ \frac{g^4}{2} \left[ (W^+_{\mu} W^-_{\mu})^2 + (W^+_{\mu} W^+_{\nu})(W^-_{\nu} W^-_{\mu}) \right] + \frac{g^4}{c_w^2} (W^+_{\mu} Z_{\mu})(W^-_{\nu} Z_{\nu}) + \frac{g^4}{4c_w^4} (Z_{\mu} Z_{\mu})^2 \right] 
\]

\[
\mathcal{L}_5 = \alpha_5 \left[ g^4 (W^+_{\mu} W^-_{\mu})^2 + \frac{g^4}{c_w^2} (W^+_{\mu} W^-_{\mu})(Z_{\nu} Z_{\nu}) + \frac{g^4}{4c_w^4} (Z_{\mu} Z_{\mu})^2 \right] 
\]  

(5)

with \( c_w^2 = 1 - \sin^2 \theta_w \) and \( g^2 = e^2 / \sin^2 \theta_w \). \( \alpha_4 \) and \( \alpha_5 \) are the parameters of the next-to-leading order terms in the expansion. These contact terms introduce all possible quartic couplings compatible with the custodial \( SU(2)_c \) symmetry. The amplitudes for the \( WW \) scattering processes may be expressed in terms of a master amplitude \( A \) which is a function of the Mandelstam variables \( s, t \) and \( u \):

\[
A(W^+ W^- \rightarrow ZZ) = A(s, t, u) 
\]  

(6)

\[
A(W^+ W^- \rightarrow W^+ W^-) = A(s, t, u) + A(t, s, u) 
\]  

(7)

\[
A(W^- W^- \rightarrow W^- W^-) = A(t, s, u) + A(u, t, s) 
\]  

(8)

The dominating strong-interaction part of the master amplitude is given by the expansion

\[
A(s, t, u) = \frac{s}{v^2} + \alpha_4 \frac{t^2 + u^2}{v^4} + \alpha_5 \frac{8s^2}{v^4} 
\]  

(9)

with \( v^2 = 1 / (\sqrt{2}G_F) = (246 \text{ GeV})^2 \). The leading-order term \( s/v^2 \) of the expansion is parameter free.

It is generally expected that \( e^\pm e^- \) linear colliders will in a first step be realized for a total c.m. energy up to about 1 TeV, see Refs.\[10\]. Moreover, a high integrated luminosity of \( \int \mathcal{L} = 1 \text{ ab}^{-1} \) may be reached within two years of operation with TESLA. Since due to the complicated mixture of signal and background mechanisms, simple scaling laws are not trustworthy \textit{a priori}, we have updated the \( WW \) scattering analysis of Ref.\[4\] for a total c.m. \( e^\pm e^- \) energy of \( \sqrt{s} = 1 \text{ TeV} \) and integrated luminosities of \( \int \mathcal{L}_{e^+ e^-} = 1 \text{ ab}^{-1} \) for \( e^+ e^- \) collisions, and \( \int \mathcal{L}_{e^- e^-} = 100 \text{ fb}^{-1} \) for \( e^- e^- \) collisions. Electron and positron polarizations are assumed to be 100\% and 50\%, respectively.
Using the Lagrangian of Eqs. (2–5), the cross sections have been determined for the processes

\[ e^+e^- \rightarrow \bar{\nu}\nu W^+W^- \quad \text{and} \quad \bar{\nu}\nu ZZ \]  
(10)
\[ e^-e^- \rightarrow \nu\nu W^-W^- \]  
(11)

by calculating the amplitudes analytically and performing the phase space integrations numerically. The analysis includes the signal diagrams Fig.1 as well as all relevant background diagrams (a few important examples are depicted in Fig.2).

The strategy for isolating the signal from the background has been described in Ref.[4] in detail. For the present analysis we have used the following cuts on the final-state particles:

\[ C: \quad M(\nu\bar{\nu}) > 150 \text{ GeV} \]
\[ |\cos \theta(W/Z)| < 0.8 \quad \text{and} \quad p_{\perp}(W/Z) > 100 \text{ GeV} \]
\[ p_{\perp}(WW) > 40 \text{ GeV} \quad \text{resp.} \quad p_{\perp}(ZZ) > 30 \text{ GeV} \]
\[ 400 \text{ GeV} < M(WW/ZZ) < 800 \text{ GeV} \]
The efficiency for the detection of vector bosons and the probability of $W/Z$ misidentification are determined by the decay branching ratios and by the detector resolution for invariant jet pair masses. Taking into account both leptonic and hadronic decays, we adopt the numbers from Ref.\[4\] which amount to an overall detection efficiency of 33\% for both $WW$ and $ZZ$ pairs in the final state.

3. The results of this analysis are summarized in Fig.\[4\]. Exclusion contours at the 1$\sigma$ level are shown for the parameters $[\alpha_4, \alpha_5]$ as derived from the three processes introduced above. The highest sensitivity is predicted for the $W^+W^-$ and $ZZ$ channels; the additional $W^-W^-$ channel, however, is useful for resolving the two-fold ambiguity and singling out the unique solution. For an energy of 1 TeV and luminosities as specified above, the dynamical parameters $\alpha_4$ and $\alpha_5$ can be measured to an accuracy

\begin{align}
\alpha_4 &\lesssim 0.010 \\
\alpha_5 &\lesssim 0.007 
\end{align}

When compared with the results of Ref.\[4\] for higher energy but reduced luminosity, $\alpha_{4,5} \lesssim 0.002$, the bounds follow roughly the scaling law $\alpha_{4,5} \propto s^{-1} \times (\int \mathcal{L})^{-1/2}$ which may be used for qualitative inter- and extrapolations. As a threshold effect, the sensitivity improves dramatically with rising energy.

Assuming the same scaling law in luminosity also for LHC analyses \[12\] one finds bounds on $\alpha_4$ and $\alpha_5$ which are about a factor 2.5 and 3 less stringent after two years of high-luminosity running for a total equivalent of $\int \mathcal{L} = 200 \text{ fb}^{-1}$, and provided the systematic errors can be kept under control at this level. Nevertheless, the correlation between the parameters in individual channels is different so that independent information can be obtained from experiments at lepton and hadron colliders.

The sensitivity bounds on $\alpha_{4,5}$ can be rephrased in bounds on the errors with which the lowest-order part of the master amplitude

$$A(s, t, u)_{\text{LO}} = s/v^2$$

can be determined experimentally\[2\]. Taking proper account of the angular dependence of the

\footnote{These experimental analyses will only be carried out in the future for a physical scenario in which light Higgs bosons have experimentally been proven not to exist. The comparison of $WW$ scattering amplitudes between theories without and with light Higgs bosons is therefore a $\text{res vacua}$ in this specific context.}
coefficients coming with $\alpha_4$ and $\alpha_5$, the accuracy on the master amplitude is given by

$$\langle \delta A / A \rangle \lesssim 0.15$$  \hspace{1cm} (15)

for an average $WW$ invariant mass of $\sim 600$ GeV, corresponding to a total $e^+e^-$ energy of 1 TeV, and an integrated luminosity of $\int \mathcal{L} = 1 \text{ ab}^{-1}$.

Thus high-luminosity $e^+e^-$ colliders allow us to test the basic mechanism for electroweak symmetry breaking even in the absence of a light Higgs boson quite stringently at a collider energy of 1 TeV.

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Figure 3: Exclusion contours for the hypothesis $\alpha_{4,5} = 0$, assuming $\sqrt{s} = 1$ TeV and an integrated $e^+e^-$ luminosity of $\int L = 1$ ab$^{-1}$ (50%/100% polarization). The 90% exclusion line has been obtained by combining the $W^+W^-$ and $ZZ$ channels (dark gray). The contour for the $W^-W^-\bar{\nu}\nu$ channel (light gray) corresponds to an integrated $e^-e^-$ luminosity of $\int L = 100$ fb$^{-1}$ (100% polarization).