DARK ENERGY AND NEUTRINO MASS CONSTRAINTS FROM WEAK LENSING, SUPERNOVA, AND RELATIVE GALAXY AGES

Yan Gong$^{1,2}$, Tong-Jie Zhang$^{3,4}$, Tian Lan$^{3,1}$ and Xue-Lei Chen$^{1,4}$

$^1$National Astronomical Observatories, Chinese Academy of Sciences, Beijing 100012, China
$^2$Graduate School of Chinese Academy of Sciences, Beijing 100049, China
$^3$Department of Astronomy, Beijing Normal University, Beijing, 100875, China, xiatj@bnu.edu.cn
$^4$Kavli Institute for Theoretical Physics China, CAS, Beijing 100190, China

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ABSTRACT

We use the current weak lensing data to constrain the equation of state of dark energy $w$ and the total mass of massive neutrinos $\sum m_{\nu}$. The constraint on $w$ would be weak if only the current weak lensing data are used. With the addition of other observational data such as the Type Ia supernovae, baryon acoustic oscillation, and the high redshift Hubble parameter data $H(z)$ derived from relative galaxy ages to break the degeneracy, the result is significantly improved. For the pure $\Lambda$CDM model without massive neutrinos, we find $w = -1.00_{-0.12}^{+0.10}$. For the $w$CDM model with the massive neutrino component, we show that the constraint on $w$ is almost unchanged, there is very little degeneracy between $w$ and $\sum m_{\nu}$. After marginalizing over other parameters, we obtain the probability distribution function of $\sum m_{\nu}$, and find that the upper limit is $\sum m_{\nu} \leq 0.8 \text{ eV}$ at 95.5% confidence level for the combined data sets. Our constraints of $w$ and $\sum m_{\nu}$ are both compatible and comparable with the constraints obtained from the WMAP 5-year data.

Subject headings: cosmology: theory — cosmological parameters — gravitational lensing — neutrinos

INTRODUCTION

Great progress is achieved in the study of modern cosmology in recent years. Thanks to the precision observations of the “distance indicator” type Ia supernovae (SN Ia) and the anisotropy of the cosmic microwave background (CMB), an accelerating Universe with a non-baryonic dark matter component is established (Riess et al. 1998; Perlmutter et al. 1999; de Bernardis et al. 2000; Spergel et al. 2003). Other cosmological probes such as the large scale structures (Tegmark et al. 2004), galaxy clusters (Allen et al. 2008), Hubble rate derived from relative galaxy ages (RGA) (Simon et al. 2005), provide further supporting evidence for this scenario, and help determine the model parameters with even better precision. However, the nature of the so-called dark energy which drives the acceleration of the Universe is still far beyond our understanding. The simple cosmological constant cold dark matter model (CDM) remains to be the most popular, with an equation of state (EOS) $w = p/\rho = -1$ for the dark energy. However, such a simple cosmological constant suffers from the fine tuning and coincidence problems (Weinberg 1989; Zlatev et al. 1999). Dynamical dark energy models typically have evolving EOS $w = w(z)$, a wide variety have been proposed and tested in literature (Caldwell 2002; Zhao et al. 2003; Feng et al. 2005; Xin et al. 2006; Zhao et al. 2007). To eventually solve the dark energy problem, it is very important to develop additional techniques and use more observational data to test the dark energy models and measure other cosmological parameters.

Another fundamental problem in modern physics which we would like to draw particular attention to in this paper is the mass of neutrinos. The neutrino oscillation experiments suggest that the mass differences between various types of neutrinos are $\Delta m_{12}^2 \approx 8 \times 10^{-5}$ eV$^2$ (from solar neutrino experiment) and $\Delta m_{12}^2 \approx 2.2 \times 10^{-3}$ eV$^2$ (from atmospheric neutrino experiment) (Maltoni et al. 2004). These results suggest that the masses of the neutrinos form a hierarchy of $m_1 \sim 0$, $m_2 \sim \Delta m_{\text{solar}}$ and $m_3 \sim \Delta m_{\text{atmospheric}}$, or an alternative an inverted order. However, it is also possible for the three types of neutrinos have almost degenerate masses $m_1 \sim m_2 \sim m_3 \gg \Delta m_{\text{atmospheric}}$. There are not absolute bounds on the neutrinos masses from oscillation experiments, while the cosmological observations can provide us effective ways to infer the abundance and total mass of neutrinos, since the kinematics of neutrinos affects the growth of cosmological structures (Hannestad 2006). For the $\Lambda$CDM cosmology, upper bounds on the sum of the neutrino masses have reached the level of $\sum m_{\nu} \leq 0.3 - 1$ eV at 2$\sigma$, depending on the data set used (Hannestad et al. 2006, and references therein). If the dark energy EOS is taken into account however, the upper bound is shown to be relaxed to about 1.5 eV at 2$\sigma$ level (Hannestad 2005). This may indicate a degeneracy between the neutrino mass and dark energy EOS $w$ (Hannestad 2005; Lesgourgues &Pastor 2006). The inclusions of baryon acoustic oscillation (BAO) data (Goobar et al. 2006) and the weak gravitational lensing (Hannestad et al. 2006) may help break such degeneracy, as we shall proceed to show in the remaining part of this paper.

The gravitational weak lensing (WL) effect generates small distortions (of the order 1%) on the images of distant galaxies as the light passed through inhomogeneous matter distribution along the line of sight. This effect can be measured statistically to yield information on the density field and the geometry of the Universe. A great advantage of WL is that it relies on the total matter content of the Universe directly, thus the prob-
lem of galaxy-to-matter bias in the large structure survey is avoided. The WL surveys provide a powerful and precise probe of the dynamical property of the Universe at redshift $z \lesssim 3$, and it is an independent technique to measure the properties of dark energy (Munshi et al. 2008). “Cosmic shear” is recently reported in galaxy WL surveys, which provides important cosmological implications (Wittman et al. 2000, Bacon et al. 2000, Van Waerbeke et al. 2000, Kaiser et al. 2000).

A direct measurement of the Hubble expansion rate at different redshifts can be obtained from the relative galaxy ages (RGA), i.e., the differential age of galaxies which are passively evolving (Jimenez & Loel 2002). This method was verified at low redshift with SDSS data (Jimenez et al. 2003), and has been used in the reconstruction of scalar field potential of dark energy (Simon et al. 2005) and the constraint on the cosmological model firstly (Yi & Zhang 2007). It is potentially competitive with other methods which probe cosmic expansion history, such as SN Ia and BAO. Indeed, it measures $H(z)$ directly while SN Ia measures only the distance and is related to $H(z)$ by integration, hence the RGA method could be an even more sensitive probe in some cases (Lin et al. 2008).

In this work, we focus on using WL data to constrain the equation of state of dark energy and the masses of neutrinos. With the current WL survey data, the SN Ia data, the baryon acoustic oscillation (BAO) data, and the Hubble parameter $H(z)$ derived with the RGA method, we constrain the EOS $w$ of the dark energy in a constant $w$ model, the sum of the neutrino mass $\sum m_{\nu}$, and the other cosmological parameters. We also discuss the degeneracy of the parameters, especially for $\sum m_{\nu}$ and $w$ with the different data sets. The CMB data are not included in this work, as we wish to compare the results from the weak lensing with that of the CMB data (WMAP 5-year measurement). We shall assume the geometry of the Universe is flat in this work.

The outline of this paper is as follows. In Sec. 2 we present our methods and the observational data sets. The results are given in Sec. 3. Finally we draw conclusions in Sec. 4.

2. METHODS AND OBSERVATIONAL DATA

In this section, we briefly introduce the theoretical predictions of the observables. We employ the Markov Chain Monte Carlo (MCMC) technique to constrain the parameters. Eight MCMC chains are generated for each combination of our data sets, and after the convergence each chain contains about 100000 points to sample the probability distribution in the parameter space. Then, these chains are thinned and joined together, and there are about 12000 points left to be utilized to perform the constraints (Gong & Chen 2008).

2.1. Weak lensing

As we know, the dark energy could affect the expansion rate and the large scale structure (LSS) of the Universe, from observation of these two aspects of cosmological evolution we can measure determine the property of dark energy very well. The WL provides us a powerful probe for both aspects (Huterer 2002). The massive neutrinos could suppress the matter power spectrum on small scales, due to their free streaming, thus reducing the convergence power spectrum of the weak lensing, which is sensitive to the small scale matter distribution. Weak lensing is therefore a powerful measurement for both the dark energy and the massive neutrinos.

For details of the WL technique the readers can refer to Bartelmann & Schneider (2001). We start with the power spectrum of the convergence $\kappa$, which describes the strength of the lensing effect. Under Limber’s approximation, the convergence power spectrum $P_{\kappa}$ can be related to the matter power spectrum $P(k, z)$ as

$$P_{\kappa}(l) = \frac{c^2}{H_0} \int_0^z \frac{W^2(z)}{r^2(z) E(z)} P(l/r(z), z) dz$$

$$= \frac{2\pi^2}{l^3} \int_0^z \frac{W^2(z) r(z)}{H_0 E(z)} \Delta^2(k, z) dz,$$

where $\Delta^2(k, z) = \frac{\delta^2}{\Delta^2}$, $P(k, z)$ is the dimensionless matter power spectrum, $k = l/r(z)$ with $l$ the multipole and $r(z)$ the comoving distance defined as $\int_0^z \frac{dz}{H(z)}$, $H_0$ is the Hubble constant, and $E(z)$ is the expansion rate of the Universe.

$$E(z) = \frac{H(z)}{H_0} = \sqrt{\Omega_m (1+z)^3 + (1 - \Omega_m)(1+z)^3(1+w)},$$

where $\Omega_m$ is the matter density parameter and $w$ is the EOS of dark energy. We give the detailed formula for the calculation of $P_{\kappa}$ in the Appendix. The shear correlation functions can be defined as (Munshi et al. 2008, Daniel et al. 2008)

$$\xi_{+}(\theta) = \frac{1}{2\pi} \int_0^\infty dl J_0(l) P_{+}(l, \theta),$$

$$\xi_{-}(\theta) = \frac{1}{2\pi} \int_0^\infty dl J_0(l) P_{-}(l, \theta),$$

$$\xi_{\ell}(\theta) = \xi_{-}(\theta) + 4 \int_0^\infty d\theta^2 \frac{\xi_{+}(\theta^2)}{\theta^2} - 12(\theta^2)^2 \int_0^{\infty} d\theta^2 \frac{\xi_{+}(\theta^2)}{(\theta^2)^3},$$

$$\xi_{E}(\theta) = \frac{\xi_{+}(\theta) + \xi_{\ell}(\theta)}{2} = \frac{\xi_{+}(\theta) - \xi_{\ell}(\theta)}{2},$$

where $J_0$ and $J_4$ are the zeroth and forth order Bessel functions of the first kind respectively. The shear correlation functions can then be compared with the measurements directly. In this work we use the $\xi_{E}(\theta)$ data from Benjamin et al. (2007), which contains two wide sky survey: the Canada-France-Hawaii Telescope Legacy Survey of Wide fields (CFHTLS-Wide, sky coverage 22 deg$^2$, Fu et al. 2008) and the Red-Sequence Cluster Survey (RCS, sky coverage 53 deg$^2$, Hoekstra et al. 2002).

The WL observations play the key role in our constraint on the neutrino mass. We modify the code of calculating matter power spectrum to account for the suppression due to neutrino mass, the details of our calculation can be found in the Appendix.

2.2. Type Ia Supernovae

The SN Ia are widely used as standard candles to measure the luminosity distance (Riess et al. 1998, Perlmutter et al. 1999). The redshift-dependent luminosity distances $d_L(z)$ of the SN Ia are determined by the expansion history and geometry of the Universe. In a spatially flat Friedmann-Robertson-Walker (FRW) Universe, the luminosity distance with redshift $z$ is given by

$$d_L(z) = \frac{c(1+z)}{H_0} \int_0^z \frac{dz'}{E(z')}.$$
Then the distance modulus of the SN Ia can be written as

\[ \mu_{\text{th}}(z) = 5 \log d_L(z) + 25. \]  

(8)

We use here the SN Ia data recently published by the Supernova Cosmology Project (SCP) team (Kowalski et al. 2008). This data set contains 307 SN Ia, selected from several current widely used SN Ia data sets, including the Hubble Space Telescope (HST, Riess et al. 2004), Supernova Legacy Survey (SNLS) (Astier et al. 2006) and the Equation of State: SupErNovae trace Cosmic Expansion (ESSENCE, Wood-Vasey et al. 2007). They were re-analysed by the SCP team with the same procedure to get a consistent and high-quality “Union” SN Ia data set.

2.3. Baryonic Acoustic Oscillation

The acoustic oscillations in the plasma of the early Universe are imprinted on the matter power spectrum. This signatures in the large-scale clustering of galaxies yield additional cosmological tests. Using a large spectroscopic sample of 46748 luminous red galaxies covering 3816 square degrees out to z = 0.47 from the Sloan Digital Sky Survey (SDSS), Eisenstein et al. (2005) successfully found the acoustic peak in the matter power spectrum. The position of the feature can be described by the model-independent A-parameter

\[ A = \sqrt{\Omega_m} \left[ \frac{1}{z_1 E(z_1)} \int_0^{z_1} \frac{dz'}{E(z')} \right]^{2/3}, \]  

(9)

with \( z_1 = 0.35 \) the redshift at which the measurement is taken. The value of the A parameter is measured as \( A = 0.469 \pm 0.017 \) (Eisenstein et al. 2005).

If the neutrinos are taken into account, the SDSS constraint on BAO can be approximated as (Goobar et al. 2006)

\[ A = 0.469 \left( \frac{n_s}{0.98} \right)^{-0.35} \left( 1 + 0.94 f_\nu \right) \pm 0.017, \]  

(10)

where \( n_s \) is the primordial power spectrum of fluctuations (see the Appendix), \( f_\nu = \Omega_\nu / \Omega_m \) is the fraction of neutrinos relative to matter density.

2.4. \( H(z) \) from relative galaxy ages

The Hubble parameter \( H(z) \) is related with the differential age of the Universe by

\[ H(z) = -\frac{1}{1+z} \frac{dz}{dt}, \]  

(11)

so through the determination of \( dz/dt \) it can be measured directly. The later can be determined by using the differential ages of passively evolving galaxies observed in the Gemini Deep Deep Survey (GDDS, Simon et al. 2005) obtained a set of 9 \( H(z) \) measurement in the redshift range \( 0 \sim 1.8 \) using this method. Various cosmological models were tested using this data set in the last few years (Yu & Zhang 2007, Samushia & Ratra 2006, Wei & Zhang 2007, Qiang et al. 2007, Lin et al. 2008, Carvalho et al. 2008, Figueroa et al. 2008). The constraint derived using this method is compatible and comparable to that of SN Ia (Lin et al. 2008, Carvalho et al. 2008, Figueroa et al. 2008).

3. Constraints on cosmological parameters

In this section, we first study the case of pure \( w \)CDM model, which contains a dark energy component with constant EOS \( w \) and the cold dark matter. The separate constraints for each data set are produced to illustrate their power of constraints on the parameters and the degeneracy directions. Next we include the massive neutrinos component, first in the \( \Lambda \)CDM model (henceforth \( \Lambda \)CDM model), and then in the \( w \)CDM cosmology (\( w \)CDM model). We give upper limits on \( \sum m_\nu \) from the marginalized probability distribution function (PDF), and discuss the degeneracy between the \( \sum m_\nu \) and \( w \).

3.1. EOS of dark energy in \( w \)CDM model

The \( 1 \sigma \) and \( 2 \sigma \) contours of the \( \Omega_m-w \) plane are shown in Fig.\( \text{1} \)B. We find that the WL data is sensitive to the matter density parameter \( \Omega_m \), but less effective in constraining the dark energy EOS \( w \). Also, the best-fit \( \Omega_m \) is nearly independent of the \( w \) for the WL data, this is consistent with what were found in other works (Hoekstra et al. 2006, Benjamin et al. 2007), so the degeneracy direction of the WL is different from other cosmological observations. It is also shown that the degeneracy directions of the SN Ia data and the RGA data are almost the same, which again agrees with the conclusions reached in the recent works of Lin et al. (2008); Carvalho et al. (2008); Figueroa et al. (2008). The constraint from the SN Ia data is much tighter than that of the RGA data due to the much larger sample of the former. If we combine all of the data sets, the results are significantly improved. We measure from the global fit of WL+SN Ia+RGA+BAO that, \( \Omega_m = 0.28^{+0.03}_{-0.02} \) and \( w = -1.06^{+0.10}_{-0.12} \), which are consistent with the recent reported results from WMAP 5-year data (Komatsu et al. 2008).
This result shows that the cosmological constant is an excellent candidate of dark energy.

### 3.2. Constraints on neutrino mass

Now we investigate the constraints on neutrino mass for the employed data sets. We use the WL, the WL+RGA+BAO, the WL+SN Ia+BAO and the WL+SN Ia+RGA+BAO data set combinations to perform the constraints, so that we could compare the power of parameter constraint for the different data set combinations.

Due to parameter degeneracy, with WL only there is very little constraining power. With the addition of BAO and RGA data, a 2σ limit of 0.6 eV can be obtained. With SN Ia instead of RGA the constraint is better (0.4 eV). If all (WL, BAO, SN Ia and RGA) data are combined to make the constraint, it is further slightly improved. It is also notable that the peak of the PDF distribution is not at 0 eV but at about 0.1 eV for the last three data set combinations.

In Fig. 3, we show the contour map of the Ω_m-w plane in the wNCDM model. As can be seen, the results do not change much compared with that of the wCDM model. The constraint of w for WL+RGA+BAO is looser than that for WL+SN Ia+BAO, but these two combined data sets yield similar constraint on Ω_m. This comparison actually reflects the constraining ability of the SN Ia data and the RGA data. As we shall also see in other results presented below, except for w, the WL+RGA+BAO almost have about the same constraining power as the WL+SN Ia+BAO data [Lin et al. 2008], even though the number data points in the RGA data set is much less than that of the SN Ia data set.

The contour maps for w-∑m_ν are shown in Fig. 4. The constraint is not good if only the WL data are employed. After including the other data sets, the constraints of ∑m_ν are improved much, because the SN Ia, RGA and BAO data could remarkably improve the constraints on the other cosmological parameters such as Ω_m and w. We can also see that ∑m_ν tends to be greater when the w becomes more negative; however, there is no strong degeneracy between w and ∑m_ν. The reason of this may be, on one hand, the current WL data are not yet accurate enough to indicate such relations; on the other hand, the P_κ, which is used to constrain the parameters for the WL data, is the integral of the matter power spectrum P(k, z), so it is not as powerful as P(k, z) to reflect the influence of w on the formation of the LSS [Hannestad 2005, Spergel et al. 2007].

Finally, we marginalize over the other parameters and get the PDF of ∑m_ν shown in Fig. 5, the 2σ C.L. is also shown as a vertical line. We find ∑m_ν ≤ 0.8 eV at
In the ΛCDM model with massive neutrinos, the cosmological constant is weak if only the WL data are used, but after inclusion of the other observational data, the constraints on $w$ and $\sum m_\nu$ are fairly large if we just use the current WL data, but the PDF for the WL+RGA+BAO data set is almost identical to that for the WL+SN Ia+BAO, as we have discussed earlier.

4. CONCLUSION AND DISCUSSION

In this work we use the WL survey data, combined with SN Ia, RGA and BAO data to constrain the EOS of dark energy $w$ and the sum of the neutrinos mass $\sum m_\nu$. The MCMC method is employed to give the PDF of the parameters.

We first study the $w$CDM model without the neutrinos, and obtain the constraint on $\Omega_m-w$ for different cosmological observations. We find that the constraint for $w$ is weak if only the WL data are used, but after inclusion of the other observational data, the constraints are improved remarkably, and the cosmological constant ($w=-1$) is favored.

Then we considered constraints on the neutrino masses. In the LCDM model with massive neutrinos, WL along does not provide a constraint, but if one combine WL with SN, BAO or RGA data, the neutrino mass is well constraint. The $2\sigma$ limit with all of these observations together is about 0.4 eV, and the peak of the fit is around 0.1 eV. We also considered the $w$CDM model with massive neutrinos. However, we find that $w = 1$ (cosmological constant) is still favored in this model, and a weak degeneracy is found between $w$ and $\sum m_\nu$. It shows that WL can indeed break the degeneracy between $w$ and $\sum m_\nu$ effectively (Hannestad et al. 2006). After marginalizing over the other parameters we obtain the PDF for $\sum m_\nu$. The upper limit of the neutrino mass is obtained as $\sum m_\nu \leq 0.8$ eV at $2\sigma$, which is comparable with the recent combined analysis of CMB+SN Ia+BAO data (Komatsu et al. 2008). Moreover, except for $w$, the constraint ability for the WL+SN Ia+BAO and the WL+RGA+BAO data set is nearly the same, which means that RGA data can play a similar role in cosmological study as SN Ia (Lin et al. 2008; Carvalho et al. 2008; Figueras et al. 2008), the point of which is clearly demonstrated by Lin et al. (2008).

Although the capability of the current WL data on cosmological parameter constraints is still not very good, and our constraints on $w$ and $\sum m_\nu$ are not as strong as the other works (Seljak et al. 2005a,b, 2006; Kristiansen et al. 2007; Komatsu et al. 2008; Fogli et al. 2008), it shows that WL is a good and powerful complementary for the other observations, and plays a more and more important role for the cosmological study. When the next generation WL surveys, such as the Super-Nova/Acceleration Probe1 (SNAP) and the Large Synoptic Survey Telescope2 (LSST), are put to work, the WL observation would be more accurate and become an indispensable measurement for the cosmology study.

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APPENDIX

CALCULATION OF LENSING POWER SPECTRUM

The weight function $W(z)$ in Eq. (1) is written as (Huterer 2002)

$$W(z) = \frac{3}{2} \left( \frac{H_0}{c} \right)^2 \Omega_m f(z)(1+z),$$  \hspace{1cm} (A1)

with function

$$f(z) = r(z) \int_z^\infty \frac{r(z') - r(z)}{r(z')} n(z') dz'.$$  \hspace{1cm} (A2)

1 http://snap.lbl.gov
2 http://www.lsst.org
in which \( n(z) \) is the normalized number density distribution (i.e., \( \int n(z) \, dz = 1 \)) of source galaxies. The \( n(z) \) we use here is proposed in \((\text{Benjamin et al.} \, 2007)\), which takes the form as

\[
n(z) = N \frac{z^a}{z^b + c}, \tag{A3}
\]

where \( a, b \) and \( c \) are free parameters, and \( N \) is a normalizing factor,

\[
N = (\int_0^\infty dz' \frac{z'^{2n}}{z'^b + c})^{-1}. \tag{A4}
\]

The linear matter power spectrum \( \Delta_L^2(k, z) \) is parameterized as

\[
\Delta_L^2(k, z) = A k^{n_s + 3} T^2(k) D^2(z), \tag{A5}
\]

where \( D(z) = g(z)/(1 + z) g(0) \) is the linear growth factor, \( T(k) \) is the transfer function, \( A \) is the normalization factor and \( n_s \) is the primordial fluctuation spectrum. Hereafter we use a Harrison-Zeldovich spectrum \( n_s = 1 \). For the \( \Lambda \text{CDM} \) model \((w = -1)\), the relative growth factor \( g(z) \) is found to be well approximately by \((\text{Carroll et al.} \, 1992)\)

\[
g_\Lambda(z) = \frac{(5/2) \Omega_m(z)}{\Omega_m^{1/4}(z) - \Omega_\Lambda(z) + (1 + \Omega_m(z)/2)(1 + \Omega_\Lambda(z)/70)}, \tag{A6}
\]

with

\[
\Omega_m(z) = \frac{\Omega_m(1 + z)^3}{\Omega_\Lambda(1 + z)^3 + \Omega_\Lambda}, \quad \Omega_\Lambda(z) = \frac{\Omega_\Lambda}{\Omega_m(1 + z)^3 + \Omega_\Lambda}. \tag{A7}
\]

For the transfer function, we adopt the fitting result of \((\text{Bardeen et al.} \, 1986)\) for an adiabatic \( \Lambda \text{CDM} \) model

\[
T_\Lambda(q) = \frac{\ln(1 + 2.34q)}{2.34q} \left[ 1 + 3.89q + (16.1q)^2 + (54.6q)^3 + (67.4q)^4 \right]^{-1/4}, \tag{A8}
\]

where \( q = k/h \Gamma \), and \( h = H_0/(100 \, \text{km} \, \text{s}^{-1} \, \text{Mpc}^{-1}) \). \( \Gamma = \Omega_m h \exp[-\Omega_b(1 + \sqrt{2}h/\Omega_m)] \) is the shape parameter with baryon density \( \Omega_b \). \( \Omega_b = 0.0462 \) is adopted according to the recent analysis of CMB, SN Ia and BAO data \((\text{Komatsu et al.} \, 2008)\). For the extension of the growth factor and transfer function from \( \Lambda \text{CDM} \) model to any dark energy case with constant EOS \( w \), we use the fitting form of \((\text{Ma et al.} \, 1999)\)

\[
g_Q = g_\Lambda(-w)^t,
\]

\[
t = -(0.255 + 0.305w + 0.0027/w)[1 - \Omega_m^w(z)] - (0.366 + 0.266w - 0.07/w) \ln \Omega_m^w(z),
\]

where \( \Omega_m^w(z) = \Omega_m/[\Omega_m + (1 - \Omega_m)(1 + z)3w] \). This fitting formula is accurate to 2\% for \( 0.2 \lesssim \Omega_m \lesssim 1 \) and \(-1 \leq w \lesssim -0.2 \).

For the non-linear power spectrum, we adopt the formula given by \((\text{Peacock & Dodds} \, 1996)\),

\[
\Delta_{NL}^2(k_{NL}) = f_{NL} \Delta_L^2(k_L),
\]

\[
k_L = (1 + \Delta_{NL}(k_{NL}))^{-1/3} k_{NL},
\]

\[
f_{NL}(x) = x \left[ 1 + B x^\alpha \right] / \left[ 1 + (A x)^\beta \right]^{1/\beta}. \tag{A11}
\]

The parameters in the non-linear function \( f_{NL} \) are

\[
A = 0.428(1 + n_s/3)^{-0.947},
\]

\[
B = 0.226(1 + n_s/3)^{-1.778},
\]

\[
\alpha = 3.310(1 + n_s/3)^{-0.244},
\]

\[
\beta = 0.862(1 + n_s/3)^{-0.287},
\]

\[
V = 11.55(1 + n_s/3)^{-0.423},
\]

which are fitted to numerical simulation results.

Actually, the non-linear matter power spectrum we employ can be seen as the Peacock-Dodds (PD96) fitting formula with a modified growth factor proposed by \((\text{Ma et al.} \, 1999)\). There is not yet a reliable analytical fit of the non-linear power spectrum for the \( w \text{CDM} \) model \((\text{Hoekstra et al.} \, 2006)\), so now it is usually obtained from fitting to the N-body simulations, the applicable range in the parameter space is always small. For instance, an accurate fitting non-linear matter power spectrum with varying \( w \) was proposed by \((\text{McDonald et al.} \, 2006)\), but unfortunately, its applicable range was \( \Omega_m \in [0.211, 0.351] \). However, We have tested our power spectrum and find it matches well with the simulation results of \((\text{McDonald et al.} \, 2006)\), especially for \( w < -0.5 \) (see Fig.6).
to constrain the sum of the neutrino mass be modified as $P_\nu$ (Brandbyge et al. 2008). Hence, When we consider the neutrino component the linear matter power spectrum should be accurate for the linear theory regime with relative large scales is calculated by our non-linear power spectrum code. The dash-dotted, dashed and dotted lines represent respectively, and we set $\Omega_m = 0.281$ which is to match the parameter value for the black solid line in Fig. 1 of McDonald et al. (2006). We find our results match well with that of McDonald et al. (2006) when $w = -0.5$, even for $w = -0.5$, our $P_\nu/P_\Lambda$ is about 1.6 at $k = 10$ Mpc$^{-1}$ while it is about 1.35 in McDonald et al. (2006), that still does not deviate much.

If there is a fraction of massive neutrinos in the matter components of the Universe, the growth of the structure is suppressed by the free streaming of neutrinos. The transition scale is the horizon scale when the neutrinos become non-relativistic: $k_{nr} \approx 0.026 (E_{\nu}/2eV)^{1/2} \Omega_m^{1/2} h$ Mpc$^{-1}$, below which (Hu & Eisenstein 1998)

$$\frac{\Delta P_L}{P_L} \approx -8 \frac{\Omega_\nu}{\Omega_m}. \quad (A12)$$

where $\Omega_\nu = \sum m_\nu/(93.2$ eV h$^2$) is the neutrino matter density (Goobar et al. 2006). This approximation is shown to be accurate for the linear theory regime with relative large scales $k \leq 0.2$ Mpc$^{-1}$ and small neutrino fraction $f_\nu$ (Brandbyge et al. 2008). Hence, When we consider the neutrino component the linear matter power spectrum should be modified as $P'_L = P_L + \Delta P_L (k > k_{nr})$. We add this modification into our non-linear matter power spectrum code to constrain the sum of the neutrino mass $\sum m_\nu$.

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