Charged-particle multiplicity and transverse-energy distribution using the Weibull-Glauber approach in heavy-ion collisions

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The charged-particle multiplicity distribution and the transverse-energy distribution measured in heavy-ion collisions at top RHIC and LHC energies are described using the two-component model approach based on a convolution of the Monte Carlo Glauber model with the Weibull model for particle production. The model successfully describes the multiplicity and transverse-energy distribution of minimum-bias collision data for a wide range of energies. The Weibull-Glauber model can be used to determine the centrality classes in heavy-ion collisions as an alternative to the conventional negative binomial distribution (NBD)-Glauber approach.

I. INTRODUCTION

The multiplicity distribution of charged particles emitted in ultrarelativistic nucleus-nucleus collisions constitutes an important global observable, which has been widely measured and studied to understand the properties of the hot and dense matter created in such collisions. Similarly, the transverse-energy ($E_T$) distribution is also considered an additional global observable well suited for probing the QCD medium. Both observables are connected to the collision geometry, the entropy, and the initial energy density of the system created in heavy-ion collisions [1–3]. Various experimental measurements suggest that charged-particle pseudorapidity density can be successfully used to study the expansion dynamics by Landau hydrodynamics [2, 4–7]. Alternatively, the transverse-energy measurements can also be used to probe the longitudinal expansion of the produced system at midrapidity [8, 9]. These observations establish a strong correlation between the two and their measurements provide significant constraints on the collision dynamics. The experimental observation of a linear correlation between the mean charged-particle multiplicity and the mean transverse energy further suggests the apparent equivalence. Hence, both of these global observables are usually the first measurements carried out in heavy-ion collision experiments to understand the underlying mechanism of particle production in heavy-ion collisions. In view of this, it is crucial to understand the charged-particle multiplicity and the transverse-energy distribution measured in such collisions.

It is customary to understand the particle production mechanism in nucleon-nucleon collisions before exploring complex systems like nucleus-nucleus collisions. Recently, the Weibull model of particle production based on the fragmentation and sequential branching mechanism has been quite successful in describing the overall features of multiparticle production in hadronic and leptonic collisions [8–10]. Previously, it was emphasized that the main features of multiplicity distribution in nucleus-nucleus collisions were just a consequence of the initial geometry of the collisions. The interaction volume depends on the impact parameter (distance between the centers of two colliding nuclei at the closest approach) of the colliding nuclei and therefore on the number of participating nucleons in that volume. However, it is impossible to measure the impact parameter of the collision directly in experiments. Experimentally, these quantities are best estimated by measuring the number of charged particles produced or the transverse energy of the produced particles in the collision. To establish a correspondence between these geometrical quantities, like impact parameter, initial overlap volume, etc., with measured charged-particle multiplicity, a Monte Carlo (MC)-based Glauber model is widely used. In the MC Glauber model [11, 12], a nuclear collision (in pA and AA systems) is modeled as a superposition of individual nucleon-nucleon interactions [13]. The initial overlap volume of two colliding nuclei can be expressed in terms of the number of wounded nucleons (nucleons which have undergone one or more binary collisions). The number of such nucleons is known as the number of participant nucle-
ons, \( N_{\text{part}} \). The number of binary collisions (\( N_{\text{coll}} \)) among the nucleons depends on inelastic nucleon-nucleon cross section (\( \sigma_{NN}^{\text{inel}} \)). The MC Glauber model can be used to calculate both these quantities for a Woods-Saxon type of initial nuclear density distribution at a given value of the impact parameter. This geometrical approach helps to provide a consistent description of nuclear collisions in different systems (\( p-A, d-A, \) and \( A-A \) ) when comparing data from different experiments to theoretical calculations \[14, 18\].

In this work, the two-component approach based on the Glauber model, combined with the Weibull model of multiparticle production in hadronic interactions, is implemented to calculate the multiplicity distribution in heavy-ion collisions. The obtained distribution is compared with the experimental data measured at RHIC and LHC energies. The Weibull-Glauber approach is similar to the procedure adopted by the ALICE experiment for the centrality determination in Pb-Pb collisions at 2.76 and 5.02 TeV using the negative binomial distribution (NBD) \[14, 13\]. The transverse-energy distribution in heavy-ion collisions was also described by using the same formalism. An alternative method has been proposed to determine the centrality in heavy-ion collisions using the model and HIJING event generator \[19\].

## II. THE MODEL

The simple model aims to describe the qualitative features of the charged-particle multiplicity and transverse-energy distribution in heavy-ion collisions. In a nucleus-nucleus collision, each of the wounded constituents gives rise to an “ancestor,” which then fragments into the final state hadrons. The MC Glauber model \[11, 12\] is used to simulate the collision process of two nuclei on an event-by-event basis. The position of the nucleons inside the nucleus is determined by the nuclear density function, modeled by the available functional forms (Fermi function, Hulthén form, Woods-Saxon, uniform, etc). The impact parameter is chosen randomly and the maximum value of the same is fixed to twice the radius of the nucleus. The collision of two nucleis is treated as a sequence of individual and independent collisions of the nucleons, where the nucleons travel undeflected in a straight path. The inelastic cross section, \( \sigma_{NN}^{\text{inel}} \), is treated as independent of the number of collisions a nucleon underwent previously. The value of \( \sigma_{NN}^{\text{inel}} \) is given as an input for the MC Glauber model, which depends on the collision energy as used by various experiments \[11, 14, 17\]. The model provides the number of participants, \( N_{\text{part}} \), and the number of binary collisions, \( N_{\text{coll}} \), for an event with a given impact parameter and collision energy. To determine the particle multiplicity for a single event, one defines the number of independent particle-emitting sources, also known as ancestors \[14\]. The number of ancestors can be parametrized by assuming a suitable dependence on \( N_{\text{part}} \) and \( N_{\text{coll}} \), as the final multiplicity of an event depends on the impact parameter of the collision. The event multiplicity (or transverse energy) is expected to scale with \( N_{\text{part}} \) where the particle production is dominated by soft processes, while the \( N_{\text{coll}} \) scaling is observed where hard processes dominate over soft particle production \[20, 21\]. A two-component approach has been assumed \[14\] where the number of ancestors have been parametrized in terms of both \( N_{\text{part}} \) and \( N_{\text{coll}} \) as the following:

\[
N_{\text{ancestors}} = xN_{\text{part}} + (1-x)N_{\text{coll}}. \tag{1}
\]

The expressed dependence takes care of the relative contribution \( x \) of both hard and soft processes in the final multiplicity. The approach as described by Eq. \(1\) in convolution with the NBD has been very successful in describing the charged-particle multiplicity densities at RHIC and LHC energies \[14\]. But in the present model, the charged-particle multiplicity per nucleon-nucleon collision is parametrized by the Weibull distribution. This latter assumption is motivated by the fact that in minimum-bias \( pp (\bar{p}p) \) collisions, the charged-particle multiplicity distribution is nicely described by the Weibull function for a wide range of energies \[8, 9\]. Therefore, one can use the Weibull distribution as the statistical model of particle production in nucleon-nucleon collision, with the Glauber model of nucleus-nucleus collision to simulate the multiplicity distribution in heavy-ion collisions.

The probability of producing \( n \) particles per ancestor is given by the two-parameter Weibull distribution

\[
P(n; \lambda, k) = \frac{k}{\lambda} \left( \frac{n}{\lambda} \right)^{k-1} e^{-\left( \frac{n}{\lambda} \right)^{k}}, \tag{2}
\]

where \( \lambda \) is related to the mean multiplicity per ancestor and \( k \) is related to the dynamics of particle production. For each event generated by the Glauber model, the Weibull distribution is randomly sampled \( N_{\text{ancestors}} \) times to obtain the am-
plitude of the number of particles produced in that event. The process of obtaining the multiplicity (or transverse-energy) distribution is repeated for a large pool of events and for different values of $\lambda$, $k$ and the two-component parameter $x$. This is done to simulate an experimental multiplicity (or transverse-energy) distribution, which can be compared with measured experimental data. A $\chi^2$-square minimization method is employed to obtain the best values of free parameters ($\lambda$, $k$ and $x$) for which the simulated distribution has good agreement with the measured one. Alternatively, the goodness of the agreement was also cross-checked by obtaining the ratio of measured amplitude to that of the generated one. The obtained values of $\lambda$, $k$ and $x$ for different collision systems and energies from the Weibull-Glauber model are tabulated in Table II.

III. RESULTS

The method was used to describe the charged-particle multiplicity and transverse-energy distribution measured by various experiments at RHIC and LHC energies. The model is compared to the uncorrected charged-particle multiplicity distribution measured in $|\eta| < 0.5$ in Au-Au collisions at $\sqrt{s_{NN}} = 200$ GeV by STAR Time Projection Chamber (TPC) detector, as shown in Figure 1 [22]. The lower panel of Figure 1 shows the ratio between the experimentally measured multiplicity distribution and the one obtained from Weibull-Glauber model. In ALICE, the amplitude of the VZERO detector is proportional to the charged-particle multiplicity. The ALICE VZERO detector consists of two scintillator arrays placed asymmetrically to the interaction point: VZERO-A ($2.8 < \eta < 5.1$) and VZERO-C ($-3.7 < \eta < 1.7$). The model was fitted to the VZERO detector amplitude measured in Pb-Pb collisions at $\sqrt{s_{NN}} = 2.76$ TeV by ALICE [14], which is shown in Figure 2. The ratio between VZERO multiplicity and model results are illustrated in the lower panel. It can be seen from Figure 1 and Figure 2 that the distributions obtained from the Weibull-Glauber model successfully describe the charged-particle multiplicity distribution in Au-Au and Pb-Pb collisions at $\sqrt{s_{NN}} = 200$ GeV and 2.76 TeV, respectively. The model shows some deviation from the charged-particle multiplicity distributions for most peripheral and central events. This is because the most peripheral events are contaminated by electromagnetic interactions and by trigger inefficiency. The most central events are affected by fluctuations of $N_{\text{part}}$ as well as by detector acceptance and resolution effects [14], which have not been taken into account in the model.

The model was also used to fit the minimum-bias transverse-energy distribution measured within the acceptance of $|\eta| < 1.0$ by PHENIX experiment in Au-Au collisions at $\sqrt{s_{NN}} = 200$ GeV as shown in Figure 3 [22]. Recently, the transverse energy distribution was also measured by ALICE experiment for Pb-Pb collisions at $\sqrt{s_{NN}} = 2.76$ TeV [24] at mid-rapidity ($|\eta| < 0.6$). Figure 3 compares the $E_T$ distribution generated by the Weibull-Glauber model with the measured distribution in Pb-Pb collisions at 2.76 TeV. The ratio of transverse-energy distribution obtained from experiments to the model calculations is shown in the lower panels of Figure 3 and 4. Furthermore, all the distributions are also compared with the estimations of the NBD-Glauber approach. It should be noted that the value of the two-component $x$ parameter is the same for both the approaches for a given distribution type and energy. Thus, the two-component Weibull-Glauber approach provides a nice description of the measured minimum-bias data for charged-particle multiplicity and transverse-energy distribution, in addition to the NBD-Glauber model.
TABLE I. The values of $\lambda$, $k$ and $x$ obtained from the Weibull-Glauber model to describe the minimum-bias charged-particle multiplicity and transverse-energy distributions measured in different collision systems at RHIC and LHC energies.

| System      | Energy | Distribution       | $\lambda$ | $k$   | $x$  |
|-------------|--------|--------------------|-----------|-------|------|
| Au–Au       | 200 GeV| Charged particle   | 0.685     | 1.17  | 0.805|
| Pb–Pb       | 2.76 TeV| Charged particle   | 33.20     | 1.17  | 0.8015|
| Au–Au       | 200 GeV| Transverse energy  | 4.58      | 1.17  | 0.802 |
| Pb–Pb       | 2.76 TeV| Transverse energy  | 1.81      | 1.17  | 0.799 |

FIG. 2. (Color online) The distribution of the sum of amplitudes in the VZERO scintillators in Pb-Pb collisions at $\sqrt{s_{NN}} = 2.76$ TeV [14] shown by open circle. The distribution is compared with the Weibull-Glauber and NBD-Glauber model as shown by dashed line and open triangles, respectively. The ratio of the ALICE VZERO amplitude to that of the Weibull-Glauber (NBD-Glauber) model is shown in the lower panel, which is represented by open squares (open circles).

FIG. 3. (Color online) The open circle represents the transverse energy recorded in mid-rapidity by PHENIX lead-scintillator detector in minimum-bias Au-Au collision events at $\sqrt{s_{NN}} = 200$ GeV [23]. The data are compared with Weibull-Glauber and NBD-Glauber models, shown by a dashed line and open triangles, respectively. The open squares (open circles) in the lower panel represent the value of the ratio of experimentally measured transverse energy to the Weibull-Glauber (NBD-Glauber) model.

IV. DETERMINING CENTRALITY CLASSES

The experimental charged-particle multiplicity distribution can be divided into different classes of geometrical collision by defining sharp cuts in multiplicity or any other suitable detector variable to define the same. These centrality classes also define the intervals of hadronic cross section. It is very important to determine the correct centrality class to study various physics observables as a function of centrality or impact parameter and to compare the same at different collision energies in heavy-ion experiments.

To determine the centrality classes, the charged-particle multiplicity distribution was obtained for Pb-Pb collisions at $\sqrt{s_{NN}} = 2.76$ TeV, using HIJING event generator [19]. The same was simulated using the Weibull-Glauber model. Figure 4 shows the comparison of the multiplicity distribution obtained from HIJING with the one simulated with the Weibull-Glauber approach. This creates a relationship between an experimental observable and the phenomenological model of particle multiplicity in nucleus-nucleus collisions using the Weibull-Glauber approach. Thus, a given centrality class, defined by sharp cuts in the geometrical properties [like $N_{\text{part}}$, $N_{\text{coll}}$, impact parameter($b$), etc.] in the HIJING multiplicity distribution corresponds to the same centrality class obtained using the Weibull-Glauber model. The HIJING distribution was divided into well-defined centrality in-
Using the obtained impact parameter values from the HIJING model, the multiplicity distribution is generated by the Weibull-Glauber approach in that range of impact parameter. The values of \( \langle N_{\text{part}} \rangle \) and \( \langle N_{\text{coll}} \rangle \) obtained from the model are then compared with the previously retained values from HIJING. This is shown in Table II. One can observe that the values obtained from the model and HIJING are in close agreement with each other. Figure 6 shows the good agreement between the multiplicity distribution obtained from HIJING and the one simulated with the Weibull-Glauber approach for two different centrality classes. Hence, this method serves as an alternative way to determine the approximate value of \( \langle N_{\text{part}} \rangle \) and \( \langle N_{\text{coll}} \rangle \) for a given centrality class, for a given collision system and energy.

| Centrality (in %) | \( b_{\text{min}} \) (fm) | \( b_{\text{max}} \) (fm) | \( N_{\text{part}} \) (HIJING) | \( N_{\text{part}} \) (Model) |
|-------------------|----------------|----------------|----------------|----------------|
| 0 - 5%            | 0             | 5.0           | 364.7          | 354.9          |
| 20 - 30%          | 6.6           | 10.8          | 144.6          | 144.0          |
| 50 - 60%          | 11.0          | 13.8          | 34.01          | 34.24          |
| 70 - 80%          | 13.6          | 18.6          | 6.052          | 6.713          |

FIG. 4. (Color online) Mid-rapidity transverse-energy distribution of minimum-bias collision events in Pb-Pb collisions at \( \sqrt{s_{\text{NN}}} = 2.76 \) TeV shown by open circles [24]. The dashed line (open triangles) represents the Weibull-Glauber (NBD-Glauber) model. The ratio of ALICE transverse-energy data and Weibull-Glauber (NBD-Glauber) model results are shown by open squares (open circles) in the lower panel.

FIG. 5. (Color online) The charged particle distribution obtained from HIJING for Pb-Pb collisions at \( \sqrt{s_{\text{NN}}} = 2.76 \) TeV is shown by open stars. The distribution is compared with the Weibull-Glauber model represented by the dashed line. The ratio between HIJING charged-particle multiplicity and the Weibull-Glauber model results is represented by open squares in the lower panel.

TABLE II. Values of \( N_{\text{part}} \) from HIJING and Weibull-Glauber model for Pb–Pb collisions at \( \sqrt{s_{\text{NN}}} = 2.76 \) TeV.
V. SUMMARY

The charged-particle multiplicity and transverse-energy distributions in heavy-ion collisions at RHIC and LHC energies are well described by the two-component Glauber approach using the Weibull model of particle production. The Weibull-Glauber approach can be used to determine the centrality classes of heavy-ion collisions at a given energy. This is particularly significant regarding the applicability of the Weibull distribution to characterize the system created in heavy-ion collisions.

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