Investigation of the students’ number sense in seventh grade

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Abstract. 2013 curriculum is the latest center of attention, especially in emphasizing the students’ understanding in learning mathematics not just only counting. The students understanding especially about these numbers can be seen from the students’ ideas to solve a problem. The students’ skills in solving the problem can be seen from their sensitivity to the numbers. This number sense ability can help the students’ performance by using some strategies that are useful for solving problems. Therefore, this study aimed to know the extent of number sense component possessed by VII H students at SMPN 3 Ponorogo. The subjects in this research were 30 students. The data collection used in this research was written number sense test result. This research was a qualitative descriptive study. The data that was obtained in this research such as the description of students’ number sense. The results indicated the ability of students based on component number sense, the lowest component number sense in recognizing the relative size of number and the students’ ability to decide wisely the strategies used from the calculation results. Overall, the average of students’ numbers sense was 50.78. Thus, it could be concluded that the ability of number sense that possessed by VII H students could be categorized as medium. However, the results of students’ work were still dominated by procedural understanding. The often problem faced by students were the lack knowledge about numbers concepts, the students used procedural ways, and the students did some errors in calculations.

1. Introduction

2013 curriculum is the latest focus of attention in education sector, especially on the students’ understanding in learning mathematics now a days [1]. Therefore, the teachers are required to have a broad and a deep insight, creative, innovative, and a good way of communicating or asking the questions to their students, so that the teacher can help the students to participate actively and think critically throughout the learning process. Based on these teachers’ supports, the students’ achievement in some expected competencies can be achieved in the future [2, 3, 4]. The students’ competence achievement in mathematics is highly depend on the effectiveness of teaching instructions that used by their teachers and the use of available opportunities to continue the teaching and learning process [5, 6]. But in the fact, the obstacle that faced by students in the field is the lack of opportunity that given by the students to develop their thinking. This problem occurs because the teachers rush to equip their students with the use of written algorithms [7, 8]. As the result, the students will have a superficial mathematics understanding because of practical procedures that are often given by their teachers is in appropriate [9].

At the beginning of seventh grade numbers material became the first topic that is discussed by the teacher and the students in mathematics lesson. This numbers material understanding is an important foundation for advanced mathematics [10, 11, 12, 13]. This ability in processing numbers is very important for the basic mathematical calculations success in the beginning section which will be needed to solve even more for the complex problems [14]. The students’ numbers understanding is not...
only on mechanistic instructions such as algorithm procedure, but it is also influenced by the students’ sensitivity to the numbers itself [15].

So far, there had been many researches on the students’ number sense that was conducted by other researchers. One of them showed a result that the students’ numbers sense was low or did not have a good sensitivity to the numbers because they were always dominated by written algorithms [16,17,18]. Not yet clear on what were the indicators that the students did a lot of mistakes and what could cause these errors. This could be an early intervention for the teacher to look at the detail, to discover what were the students’ difficulties, so with that result, it could be corrected right on the obstacle with appropriate solution [19].

A good sensitivity or intuition for the numbers and the operations used to solve some problems of numbers in other ways and does not refers to the traditional algorithms, it can be called as number sense. This skill is used to develop problem solving strategies in a flexible and efficient way to deal with numbers problems [16, 20, 21, 22]. For example, to determine the fraction that has the greatest value from \( \frac{6}{7}, \frac{6}{8}, \frac{6}{9} \), without using the traditional algorithm, the students are able to answer which fraction that has the highest value without looking for the least common multiple (LCM) of denominator. This skill should be emphasized in learning process so that the use of traditional procedures can be reduced. Each student must have a different way to solve the problem in accordance with their sensitivity of numbers. The habit of using written calculation procedures not only inhibits the students’ thinking, but also prevents their sensitivity to the numbers [23].

In line with the statement above, the students are expected to have consciousness and sensitiveness from the variety of experiences and basic understandings that obtained by the students in learning process, which is in the future, it will become a series of insights or discoverledge that develops from the level of primary education to secondary education [24]. This numbers sense acts as a foundation to understand the concepts and abilities in mathematics formally. With this acts, the students can succeed in solving mathematical problems, especially in numbers material [5]. In addition, numbers sense can help the students in solving problems due to the adaptive skill of different strategies to produce the correct solutions [25].

The importance of identifying the students’ numbers sense at the beginning of learning that conducted by the teacher is as a tool for discovering to what extent the students’ understanding and the students’ mastery, especially on numbers material that had been discovered and studied since the basic education level. So that when the learning process takes place, the teacher can create the learning process that can improve the students’ lack competence. Later, it can be used to construct further knowledge simultaneously [26].

2. Method

This research aimed to determine the extent of numbers sense skill development that possessed by students at the beginning of VII grade in solving the numbers problems. From the result of research, teachers can use it to determine the improvement of learning steps that must be done by teacher. This research was a qualitative descriptive study. The data obtained were in the form of students’ number sense description. The selected subjects in this research were 30 students of VII H class at SMPN 3 Ponorogo. Selection was taken by using a purposive sampling technique. The research data were collected through written test results. These instruments were useful for identifying the students’ numbers sense. Before this research was conducted by the researchers, the test instruments that were used to collect the data had been validated by some experts. Written test given by the researchers was a description test with the total numbers was 12 questions. The test questions presented several indicators which included [16]:

| Table 1. The indicators framework of students’ numbers sense |
|-------------------------------------------------------------|
| **Test Items** | **The indicators framework of students’ numbers sense** |
| Understanding the “numbers” meaning, “numbers” operations and the relationship between “numbers” and operations | 2,4,9 |
The indicators framework of students’ numbers sense

| The indicators framework of students’ numbers sense | Test Items |
|-----------------------------------------------------|------------|
| Using various “numbers” representations and “numbers” operations | 5, 6       |
| Recognizing the relative size of “numbers”        | 3.7        |
| Elaborating and arranging the “numbers” flexibly   | 1, 11, 12  |
| Deciding the strategies that were used from the calculations results wisely | 8, 10      |

Assessment techniques for each item that was tested were: not answering got 0 score, answering but incorrectly got 1 score, answering correctly but giving less appropriate reason got 3 score, and answering correctly by using procedural calculations and the reasons got 4 score, and answering correctly using numbers sense got 5 score. The numbers of each question that was presented were different for each indicator based on the percentage of students’ correct answers.

3. Result and Discussion

To investigate the number sense ability possessed by students, the results that obtained from written test were analyzed and were grouped according to the number sense indicator. The first analysis was looking at each of the correct score (in percent) achieved by students on each item for each number sense indicator in Table 2.

Table 2. Student achievement for each item on each indicator

| Indicators of Number sense | The item numbers | Percentage | Mean Percentage |
|----------------------------|------------------|------------|----------------|
| Understanding the numbers meaning, numbers operations and the relationship between numbers and operations | 2, 4, 9         | 68 %, 63.3%, 52% | 61.1%, 41.3% |
| Using various “numbers” representations and numbers operations | 5, 6            | 62.6%, 41.3% | 52%, 40.3% |
| Recognizing the relative size of numbers | 3, 7             | 48%, 32.6% | 40.3%, 58.2% |
| Elaborating and arranging the numbers flexibly | 1, 11, 12       | 74.6%, 70.6%, 29.3% | 58.2%, 42.3% |
| Deciding the strategies that were used from the calculations results wisely | 8, 10           | 63.3%, 21.3% | 42.3%, 21.3% |

Based on the results in Table 2, it could be seen that in general, the results indicated that the students’ numbers sense performance of VII H class were low on the indicator recognizing the relative size of numbers that was obtained by the average of 40.3%. The highest average was in the indicator of understanding the numbers meaning, numbers operations and the relationship between numbers and operations that was obtained by the average of 61.1%. Overall, indicators the students’ numbers sense average of VII H class was 50.78. This assessment was conducted by using the assessment reference guidelines [27]. From this result, it could be classified that the students’ numbers sense of VII H class was in medium categories. This categorization could be seen in the Table 3 as follows:
Table 3. Number sense categories

| Number sense categories | IP          |
|-------------------------|-------------|
| High                    | $\bar{x} \geq 54.9$ |
| Medium                  | $46.6 \leq \bar{x} < 54.9$ |
| Low                     | $\bar{x} < 46.6$ |

In addition, Table 2 was used to know in detail the achievement of indicators for each item with the aim of focusing information gathering on the completion of low item. Following table were the students’ achievements for each item on each indicator.

The first indicators was understanding the numbers meaning, numbers operations and the relationship between numbers and operations that was obtained by 61.1%, the subjects were able to answer correctly. This first indicator was the highest indicator achievement that was compared to the other indicators. There were three items of questions on the first indicator, there were two questions in number 2 and 4 showed that more than 60% of students were able to answer correctly. In the number 9, the students faced the difficulty with the percentage obtained by 52%. In problem number 9, this question represented the relationship between numbers and the operations, which was determining the result of fractions multiplication. Almost the students were able to answer correctly but the students were unable to express their reasons correctly.

The second indicator, there were two question items for using various numbers representations and numbers operations indicator. The result presented in the mean percentage of 52%. In the question number 6, this item showed that the average of students’ score was 41.3%. This item indicated that there were difficulties in this item of test. The problem in number 6 showed that the representation of numbers that was packed out through the story problems.

In the Table 3, it could be seen that in the third indicator recognizing the relative size of numbers, the students faced many difficulties that proved by the lowest percentage achievement from all existing indicators, which was only 40.3%. Focus on the lowest item question number 7, it showed that the students had difficulty in estimating the calculation results from mixed numbers.

Without calculating, what is the approximate calculation result from $10.2 \div 0.5 - \frac{3}{6} \times 6.2 + 23$?

Figure 1. Question number 7

Figure 2. The answer of S1 on question number 7

It could be seen from the results of students’ answers in Figure 2, the method used by S1 to complete the question number 7 using written algorithms, the first mistake was made by S1 in calculating the results of decimal numbers division was $10.2 \div 0.5$ and this could made error in the next calculation. Because of this error, the students would get the false result. The students would not get score. Following were the result of interview between researcher and S1 with the answers were shown above.

Researcher : “In your opinion, what is the estimated yield of $10.2 \div 0.5$, if it is done without counting?”

S1 : “I don’t know miss, maybe I must use divided operation.”

Researcher : “how do you complete the calculation?”
S1 : “The first step I used divided operation to complete the division of 10.2 ÷ 0.5 and \( \frac{3}{6} \times 6.2 \)

Researcher : ”And then?”

S1 : “I got the results for the division, after that I reduced the multiplication of \( \frac{3}{6} \times 6.2 \)

Researcher : ”Why you had to use the method of divided operation to complete this decimal division?”

S1 : “Because when decimal number were divided mutually, it was workable only by using divided operation.”

Based on the results obtained from the interview, S1 did not recognize the relative size of numbers, it could cause the students used the procedural or rote memorization procedures when it came to decimal operation problems. The second mistake made by the students was the calculation for the reduction of decimal numbers. It had an impact on the results that obtained by the students. From these mistakes, it could be seen in the calculation especially for the decimal number, the students still faced some difficulties. These difficulties could obstruct the students’ skills in processing numbers. One of the students’ inability caused the results of mixed fractions calculation without counting the calculation. The students’ insensitivity to the relative numbers could be used to estimate the results of the calculation, so to solve these problems, the students always used written algorithms.

The fourth indicator in elaborating and arranging the numbers flexibly, the results in numbers 1 and 11 reached more than 70% of students answered it correctly, but in question number 12 there were less than 30% of students answered it correctly. This showed a very significant difference. In questions 1 and 11, the students were asked to make sort mixed fraction numbers that had been presented. For question number 12, the problems were presented with existing contextual problems.

To install a water pipe, Budi get an assignment from his father to buy three types of pipes with the specified size type, pipe 1 has a length of \( \frac{11}{22} \) meters from pipe 3, Pipe 2 has a length of 1.53 cm more than pipe 1, where as pipe 3 has a length of 1 meters. Without calculating, which pipe has the smallest length and largest length? Please give your reason!

**Figure 3. Question 12**

Pipe 3 is 1 meter
Pipe 2 is 1.53 + \( \frac{11}{22} = \frac{153}{100} + \frac{11}{22} = \frac{3366}{2200} + \frac{11}{2200} = \frac{3377}{2200} = 1.535 \)
Pipe 1 is \( \frac{11}{22} = 0.5 \)
So, the smallest length of pipe is pipe I and largest length of pipe is pipe 2

**Figure 4. The answer of S2 on question number 12**

It could be seen from the results of the student’s answers in Figure 4, S2 still used traditional algorithms by using calculations to determine the smallest and largest pipe lengths. When knowing the form \( \frac{11}{22} \), S2 was not had sensitivity to these numbers, so it made the subject thought to use calculations to find the length of the pipe. In pipe 1, it was actually known that the form of \( \frac{11}{22} \) had the same value as \( \frac{1}{2} \) or 0.5. Based on the result, the length of pipe 3 could be known as 1 meter which had an amount equal to 100 cm. It was immediately known that the length of pipe 1 was half of the length of pipe 3, if
S2 had sensitivity toward these numbers, the subject would definitely reduced the use of calculations. Then for the second statement with pipe 2 which was 1.53 cm longer than pipe 1. From this statement, it could be concluded that the length of pipe 2 would definitely be longer than pipe 1. So, without did any calculations, S2 should be able to know the length of pipe from the largest until the smallest one.

On the fifth indicator, deciding the strategies used by the students from the calculations results wisely showed that the second lowest position indicator after the third indicator. It was proved by the result that was obtained from the students’ scores was 42.3%. There was a significant difference between 2 items that was presented, which were the difference scores that obtained from 2 items were very large. Item number 10 was the estimation results of calculations involving fractions. Following were the results of students’ work.

| Question in number 10 |
|------------------------|
| $1 - 2 + 3 - 4 + 5 - 6 + \cdots + 95 - 96 + 97 - 98 + 99 - 100 = \cdots$ |

**Figure 5.** Question in number 10

| The answer of S3 on question number 10 |
|---------------------------------------|
| $1 - 2 + 3 - 4 + 5 - 6 + \cdots + 95 - 96 + 97 - 98 + 99 - 100 = \cdots$  
$-1 + 3 = 2 - 4 = -2 + 5 = -3 + 7 = 4 - 8 = \ldots$ |

The numbers are a lot, Mom, so I can only get that answer.

**Figure 6.** The answer of S3 on question number 10

In Figures 6, it appeared that the students calculated one by one the number operation, so it made difficulty for students to solve the problems for the numbers that had a large number of series. The students were accustomed to use written algorithms so the students did not have other strategies to solve this problem. The following were the results of interview with S3

Researcher : “Why did not you solve this problem (Figure 6)’’?
S3 : “The addition was too long, miss, the number was too much when you calculate it’’

Researcher : “May be there are other ways or strategies that you can use to solve this problem easily?’’
S3 : “I don’t know miss’’

Researcher : “Have you ever encountered the problems like this?’’
S3 : “Never miss.’’

Researcher : “What question you often do?’’
S3 : “Not long like this miss, maybe usually there are only 4 or up to 5 numbers not up to one hundred like this’’

Researcher : “When you are facing with this problem, what do you do first?’’
S3 : “I count from 1 – 2 then the result –1 and add it by 3, then like that for the next step miss.’’

Researcher : “Do you feel some difficulties?’’
S3 : “Yes’’

Researcher : “What are your difficulties?’’
S3 : “To calculate it miss, it takes a long time, I am impatient.’’

Actually S3 was able to solve these problems by compiling and counting one by one, because S3 did not have a strategy that makes it easy. To solve the problem, S3 selected to give up and did not solve the problem. In addition, the lack of number sense possessed by students could cause the students only be able to work on problems of that type. Overall, this number sense had an important role to be able to solve more complex problems, it can be seen from the results that there was also a number sense could show an understanding of the concept of numbers [16,30].

Although number material had been taught since elementary school, the results of this study showed that most students used procedural understanding rather than conceptual understanding when
solving the number problems. This step could be seen from the results of student’s answers. The problem solving strategies that students did more directed to standard procedural understanding. The result showed that if there was no broad and deep conceptual understanding, the students could not find the new procedures for solving the problems [26]. The lowest performance was in the component of recognizing the relative size of the location where the difficulty is that students do not recognize the relative size of the numbers so students could not determine the level of accuracy that was appropriate for the situation. One of the most difficult concepts related to the estimation was the distinction between relative and absolute errors [26].

Another disadvantage was that students were slow in responding one to another’s material relationship because all of time in the learning process that takes place to present new material and sustainable material. It made the students stunned to use only methods taught by their teacher and tended to use it in various situations. Of course, this made it difficult for students when facing the different problem situations, especially in problem solving. Lack of sensitivity to numbers in contextually presented problems also made it difficult for students to solve problems correctly. This was consistent with research that the ability of number sense was useful for solving problems [28,29].

In addition, learning activities did not direct the students to make completion patterns that certainly inhibit the students’ number sense. The freedom of students to solve each problem with their own ideas turned out to affect students’ enthusiasm in learning. Based on the statement above, the teacher might be able to facilitate, develop, and complete the knowledge possessed by students such as when learning happened, the teacher could direct the students to be able to use numbers in more meaningful ways. In addition, the classroom learning must be more conceptual than procedural way [26, 31]. Therefore, this investigation was important as a method for teachers to strengthen those weaknesses so that the expected competencies were truly owned by the students.

4. Conclusion
Based on the result, it could be concluded that the ability of number sense possessed by students can be categorized as medium category. Indicated the ability of students based on number sense component shows the lowest performance was in the component of recognizing the relative size of the location where the difficulty is that students do not recognize the relative size of the numbers so students could not determine the level of accuracy that was appropriate for the situation. One of the most difficult concepts related to the estimation was the distinction between relative and absolute errors.

Therefore, the importance for knowing the students’ number sense ability can help the teacher’s performance in increasing the number sense that possessed by the students. The teacher must be able to strengt the basic skill of students about numbers with learning process. Not only prioritizes the skills of arithmetic, but also it is better to deepen the students’ understanding of numbers concept that is presented by the teacher through existing contextual problems. Through basic skill of numbers sense, the students can develop their reasoning and creative ideas. The teachers can develop their teaching strategies to solve this problem. In addition, learning activities did not direct the students to make completion patterns that certainly inhibit the students’ number sense. The freedom of students to solve each problem with their own ideas turned out to affect students’ enthusiasm in learning.

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