Estimation of Average Paddy Production of Pira Nagar Village at Barabanki District in India

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Abstract
The idea of the present paper is the use of the known information on study variable for enhanced estimation of average paddy production of Pira Nagar village at Barabanki District in India under the Simple Random Sampling Scheme. This known information is utilized in the form of median of primary variable as it is readily available and does not require every unit of the population to be inquired. The Bias and MSE of the suggested estimator are derived up to approximation of degree one. The minimum value of the MSE of suggested estimator is also obtained by optimizing the characterizing scalar. The MSE has also been compared with the considered competing estimators both theoretically and empirically. The theoretical efficiency conditions of the suggested estimator to be better than the considered estimators are verified using natural population on primary data collected from Pira Nagar Village at Barabanki District of Uttar Pradesh state in India.

Keywords: Study variable, median of study variable, simple random sampling, bias, MSE, PRE.

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1 Introduction

Cochran (1940) was the first who used the auxiliary information for enhancing the efficiency of the estimator of population mean. Since then a number of researchers have modified the usual ratio estimator, utilizing the auxiliary information in the form of its various parameters. Taking data on auxiliary variable requires extra cost. Sometimes it is not feasible to bear the extra cost and sometimes it becomes impossible to get information on auxiliary variable. In such situations we need to find an alternative to the auxiliary information. One of the solutions for such situations is to obtain some easily accessible characteristics on study variable itself as supplementary information. For example, let the study variable is monthly salary of the workers working at a place. Most of the workers are unwilling to reveal their exact salary or in case of free lancers they don’t have a fixed salary. But they can tell whether their salary lies between 10000–15000, 15000–20000 and so on. Hence it is easier for them to tell the range within which their earning lies. In such kind of situations median can be obtained for the whole data and can be used for elevated estimation of average salary of the workers. In the present investigation, we make use of the known median of the main variable to obtain the estimate of average production of paddy crop at Pira Nagar Village of Barabanki District of Uttar Pradesh state in India.

Let \( U = U_1, U_2, \ldots, U_N \) be population containing \( N \) units which are distinct and may be identified. The problem is to estimate the population mean \( \bar{Y} = \frac{1}{N} \sum_{i=1}^{N} Y_i \) of the main variable \( Y \) with higher efficiency. The most suitable estimator for \( \bar{Y} \) is the sample mean \( \bar{y} \). When it costs high to get auxiliary information, we may consider additional information on \( Y \) and may suggest a modified ratio type estimator for enhanced estimation of \( \bar{Y} \). If we go for biased estimator, we can obtain much lesser MSE than the variance of \( \bar{y} \) and MSE/Variance of other existing biased and unbiased estimators of \( \bar{Y} \).

1.1 Notation

- \( N \): Population Size
- \( n \): Sample Size
- \( f = \frac{n}{N} \): Sampling Fraction
- \( \mathcal{N}_n \): All possible samples of size \( n \)
- \( Y \): Study variable
- \( M \): Median of the \( Y \)
- \( X \): Auxiliary variable
\( \overline{Y}, \overline{X} \): Population means
\( \bar{y}, \bar{x} \): Sample means
\( \rho \): Correlation coefficient between X and Y
\( \beta \): Regression coefficient of Y on X
\( \beta_1 \): Coefficient of Skewness of X
\( \beta_2 \): Coefficient of Kurtosis of X
\( \overline{M} \): Average of sample medians of Y
\( m \): Sample median of Y
\( Q_r \): Interquartile range
\( B(\cdot) \): Bias of the estimator
\( V(\cdot) \): Variance of the estimator
\( \text{MSE}(\cdot) \): Mean squared error of the estimator
\( Q_1, Q_2, Q_3 \): Quartiles of X
\( C_y, C_x, C_m \): Coefficient of variation of x, y and m respectively
\( C_{yx}, C_{ym} \): Relative Covariances
\( \lambda : \frac{1-f}{n} \)
\( \text{PRE}(e,p) : \) Percentage relative efficiency of the proposed estimator(p) with respect to the existing estimator(e)

1.2 Formulae
Variance of Study Variable:

\[
V(\bar{y}) = \frac{1}{\binom{N}{n}} \sum_{i=1}^{\binom{N}{n}} (y_i - \overline{Y})^2 = \frac{1-f}{n} S_y^2
\]

Variance of Auxiliary variable:

\[
V(\bar{x}) = \frac{1}{\binom{N}{n}} \sum_{i=1}^{\binom{N}{n}} (x_i - \overline{X})^2 = \frac{1-f}{n} S_x^2
\]

where

\[
S_y^2 = \frac{1}{N-1} \sum_{i=1}^{N} (y_i - \overline{Y})^2 \quad \text{and} \quad S_x^2 = \frac{1}{N-1} \sum_{i=1}^{N} (x_i - \overline{X})^2
\]

Mean of medians of possible samples

\[
\overline{M} = \frac{1}{\binom{N}{n}} \sum_{i=1}^{\binom{N}{n}} m_i
\]
Variance of Sample Median of $Y$

\[ V(m) = \frac{1}{(\frac{N}{n})} \sum_{i=1}^{\frac{N}{n}} (m_i - M)^2 \]

Covariance of $\bar{y}$ and $\bar{x}$

\[ Cov(\bar{y}, \bar{x}) = \frac{1}{(\frac{N}{n})} \sum_{i=1}^{\frac{N}{n}} (y_i - \bar{Y})(x_i - \bar{X}) = \frac{1 - f}{n} S_{yx} \]

where
\[ S_{yx} = \frac{1}{N - 1} \sum_{i=1}^{N} (y_i - \bar{Y})(x_i - \bar{X}) \]

Covariance of $\bar{y}$ and $m$

\[ Cov(\bar{y}, m) = \frac{1}{(\frac{N}{n})} \sum_{i=1}^{\frac{N}{n}} (y_i - \bar{Y}) (m_i - M) \]

Coefficient of Variations

\[ C_{xx} = \frac{V(\bar{x})}{\bar{X}^2} = C_x^2 \]
\[ C_{yy} = \frac{V(\bar{y})}{\bar{Y}^2} = C_y^2 \]
\[ C_{mm} = \frac{V(m)}{M^2} = C_m^2 \]
\[ C_{ym} = \frac{Cov(\bar{y}, m)}{M\bar{Y}} \]
\[ C_{yx} = \frac{Cov(\bar{y}, \bar{x})}{\bar{Y}\bar{X}} \]

2 Literature Review of Existing Estimators

Under this section, various estimators of $\bar{Y}$ along with their biases and MSEs are presented in Table 1. It is well known that in simple random sampling technique ($\bar{y}, \bar{x}$) are unbiased estimators for ($\bar{Y}, \bar{X}$) respectively.
### Table 1 Various existing estimators of $\bar{Y}$ along with their biases & MSEs

| S.No. | Estimators                                                                 | Bias                                                                 | MSE/Variance                                      |
|-------|-----------------------------------------------------------------------------|----------------------------------------------------------------------|---------------------------------------------------|
| 1     | $t_0 = \bar{y} = \frac{1}{n} \sum_{i=1}^{n} y_i$                          | -                                                                   | $\frac{1}{n} \bar{Y}^2 C_y^2$                     |
|       | Sample Mean                                                                |                                                                      |                                                   |
| 2     | $\bar{Y}_{1p} = \bar{y} + \beta (\bar{X} - \bar{x})$                     | -                                                                   | $\frac{1}{n} \bar{Y}^2 C_y^2 (1 - \rho^2)$       |
|       | Watson(1937)                                                                |                                                                      |                                                   |
| 3     | $t_1 = \bar{y} \left( \frac{X}{x} \right)$                               | $\frac{1}{n} \bar{Y} (C_x^2 - \rho C_y C_x)$                       | $\frac{1}{n} \bar{Y}^2 (C_y^2 + C_x^2 - 2 \rho C_y C_x)$ |
|       | Cochran(1940)                                                               |                                                                      |                                                   |
| 4     | $t_2 = \bar{X}\bar{r}$                                                    | -                                                                   | $\frac{1}{n} \bar{Y}^2 (C_y^2 + C_x^2 - 2 \rho C_y C_x)$ |
|       | $t_2' = \bar{X}\bar{r} + \frac{n (N - 1)}{N (n - 1)} (\bar{y} - \bar{x})$ |                                                                      |                                                   |
|       | Goodman(1958)                                                               |                                                                      |                                                   |
| 5     | $t_3 = (1 - \alpha)\bar{y} + \alpha \bar{X} \bar{x}$                     | $\frac{1}{n} \bar{Y} \left( \frac{\alpha}{2} C_x^2 - \alpha \rho C_y C_x \right)$ | $\frac{1}{n} \bar{Y}^2 \left( C_y^2 + \alpha^2 C_x^2 - 2 \alpha \rho C_y C_x \right)$ |
|       | Chakrabarty(1979)                                                           |                                                                      |                                                   |
| 6     | $t_4 = \bar{y} \left\{ 2 - \left( \frac{\bar{X}}{\bar{x}} \right)^{\frac{N}{2}} \right\}$ | $\frac{1}{n} \bar{Y} (-w(1-w) C_x^2 - w \rho C_y C_x)$ | $\frac{1}{n} \bar{Y}^2 \left( C_y^2 + w^2 C_x^2 - 2 w \rho C_y C_x \right)$ |
|       | Sahai (1980)                                                                |                                                                      |                                                   |
| 7     | $t_5 = \bar{y} \left( \frac{X + C_x}{\bar{X} + C_x} \right)$             | $\frac{1}{n} \bar{Y} (R_5^2 C_x^2 - R_5 \rho C_y C_x)$             | $\frac{1}{n} \bar{Y}^2 (C_y^2 + R_5^2 C_x^2 - 2 R_5 \rho C_y C_x)$ |
|       | Sisodia (1981)                                                              |                                                                      |                                                   |
| 8     | $t_6 = \bar{y} \exp \left( \frac{X}{\bar{X} + \bar{x}} \right)$         | $\frac{1}{8n} \bar{Y} (3 C_x^2 - 4 \rho C_y C_x)$                  | $\frac{1}{n} \bar{Y}^2 \left( C_y^2 + \frac{C_x^2}{4} - \rho C_y C_x \right)$ |
|       | Tuteja (1991)                                                               |                                                                      |                                                   |
| 9     | $t_7 = \bar{y} \exp \left( \frac{X \beta_2 - C_x}{\bar{X} \beta_2 + C_x} \right)$ | $\frac{1}{n} \bar{Y} (R_7^2 C_x^2 - R_7 \rho C_y C_x)$             | $\frac{1}{n} \bar{Y}^2 (C_y^2 + R_7^2 C_x^2 - 2 R_7 \rho C_y C_x)$ |
|       | Upadhyaya(1999)                                                             |                                                                      |                                                   |

(Continued)
Table 1 Continued

| S.No. | Estimators | Bias | MSE/Variance |
|-------|------------|------|--------------|
| 11    | \( t_9 = \tilde{y} \left( \frac{X^2}{\bar{X}^2} \right) \) | \( \frac{1-f}{n} \bar{Y} (3C_x^2 - 2\rho C_y C_x) \) | \( \frac{1-f}{n} \bar{Y}^2 (C_y^2 + 4C_x^2 - 4\rho C_y C_x) \) |
| 12    | \( t_{10} = \tilde{y} \left( \frac{\bar{X}\beta_1 + S_x}{\bar{x}\beta_1 + \bar{x}} \right) \) | \( \frac{1-f}{n} \bar{Y} \left( R_{1(10)}^2 C_x^2 - R_{1(10)} \rho C_y C_x \right) \) | \( \frac{1-f}{n} \bar{Y}^2 (C_y^2 + R_{1(10)}^2 C_x^2 - 2R_{1(10)} \rho C_y C_x) \) |
| 13    | \( t_{11} = \tilde{y} \left( \frac{\bar{X} + \rho}{\bar{x} + \rho} \right) \) | \( \frac{1-f}{n} \bar{Y} \left( R_{1(11)}^2 C_x^2 - R_{1(11)} \rho C_y C_x \right) \) | \( \frac{1-f}{n} \bar{Y}^2 (C_y^2 + R_{1(10)}^2 C_x^2 - 2R_{1(10)} \rho C_y C_x) \) |
| 14    | \( t_{12} = \tilde{y} \left( \frac{\bar{X} + \beta_2}{\bar{x} + \beta_2} \right) \) | \( \frac{1-f}{n} \bar{Y} \left( R_{1(11)}^2 C_x^2 - R_{1(11)} \rho C_y C_x \right) \) | \( \frac{1-f}{n} \bar{Y}^2 (C_y^2 + R_{1(12)}^2 C_x^2 - 2R_{1(12)} \rho C_y C_x) \) |
| 15    | \( t_{13} = \tilde{y} \left( \frac{\bar{X} + \beta_1}{\bar{x} + \beta_1} \right) \) | \( \frac{1-f}{n} \bar{Y} \left( R_{1(13)}^2 C_x^2 - R_{1(13)} \rho C_y C_x \right) \) | \( \frac{1-f}{n} \bar{Y}^2 (C_y^2 + R_{1(13)}^2 C_x^2 - 2R_{1(13)} \rho C_y C_x) \) |
| 16    | \( t_{14} = \tilde{y} \left( \frac{\bar{X}\beta_1 + \beta_2}{\bar{x}\beta_1 + \bar{x}} \right) \) | \( \frac{1-f}{n} \bar{Y} \left( R_{1(14)}^2 C_x^2 - R_{1(14)} \rho C_y C_x \right) \) | \( \frac{1-f}{n} \bar{Y}^2 (C_y^2 + R_{1(14)}^2 C_x^2 - 2R_{1(14)} \rho C_y C_x) \) |
| 17    | \( t_{15} = \tilde{y} \left( \frac{\bar{X}C_x + \beta_1}{\bar{x}C_x + \beta_1} \right) \) | \( \frac{1-f}{n} \bar{Y} \left( R_{1(15)}^2 C_x^2 - R_{1(15)} \rho C_y C_x \right) \) | \( \frac{1-f}{n} \bar{Y}^2 (C_y^2 + R_{1(15)}^2 C_x^2 - 2R_{1(15)} \rho C_y C_x) \) |
| 18    | \( t_{16} = \tilde{y} \left( \frac{\bar{X}\beta_2 + \beta_1}{\bar{x}\beta_2 + \bar{x}} \right) \) | \( \frac{1-f}{n} \bar{Y} \left( R_{1(16)}^2 C_x^2 - R_{1(16)} \rho C_y C_x \right) \) | \( \frac{1-f}{n} \bar{Y}^2 (C_y^2 + R_{1(16)}^2 C_x^2 - 2R_{1(16)} \rho C_y C_x) \) |
Estimation of Average Paddy Production of Pira Nagar Village

\[ t_{17} = \frac{\bar{X}}{\bar{Y}} \left( 1 - \frac{k \sigma^2}{\mu^2} \right)^{-1} \]

Pandey (2011)

\[ t_{18} = \frac{\bar{X}}{\bar{Y}} \left( \frac{\bar{X} + Q_3}{\bar{Y} + Q_3} \right) \]

Al-Omari (2012)

\[ t_{19} = \frac{\bar{X}}{\bar{Y}} \left( \frac{\bar{X} + Q_3}{\bar{Y} + Q_3} \right) \]

Al-Omari (2012)

\[ t_{20} = \frac{\bar{X}}{\bar{Y}} \left( \frac{\bar{X} + M_d}{\bar{Y} + M_d} \right) \]

Subramani and Kumarpandiyan (2012)

\[ t_{21} = \frac{\bar{X}}{\bar{Y}} \left( \frac{\bar{X} + M_d}{\bar{Y} + M_d} \right) \]

Subramani and Kumarpandiyan (2012)

\[ t_{22} = \left( \frac{\bar{X}}{\bar{Y}} \right)^{1/2} \]

Swain (2014)

\[ t_{23} = \alpha' + \frac{t_{Re} + (1 - \alpha')t_{Pe}}{Y} \]

Yadav and Mishra (2015)

Estimation of Average Paddy Production of Pira Nagar Village
| S.No. | Estimators | Bias | MSE/Variance |
|-------|------------|------|--------------|
| 26    | \( t_{24} = \bar{y} \left( \frac{X + n}{\overline{x} + n} \right) \) | \( 1 - \frac{f}{n} \bar{y} \left( R_{24}^2 C_y^2 C_x - R_{24} \rho C_y C_x \right) \) | \( 1 - \frac{f}{n} \bar{y}^2 \left( C_y^2 + R_{24}^2 C_y^2 - 2 R_{24} \rho C_y C_x \right) \) |
|       | Jerjuddin and Kishun (2016) | | |
| 27    | \( t_{25} = \bar{y} \left( \frac{X + C_x}{\overline{x} + C_x} \right)^{b_1} \) | \( 1 - \frac{f}{n} \bar{y} \left[ \frac{b_1 (b_1 + 1)}{2} R_{11}^2 C_y^2 - b_1 R_{11} \rho C_y C_x \right] \) | \( 1 - \frac{f}{n} \bar{y}^2 C_y^2 (1 - \rho^2) \) |
|       | Soponviwatkul and Lawson (2017) | | |
| 28    | \( t_{26} = \bar{y} \left( \frac{X + \rho}{\overline{x} + \rho} \right)^{b_2} \) | \( 1 - \frac{f}{n} \bar{y} \left[ \frac{b_2 (b_2 + 1)}{2} R_{11}^2 C_y^2 - b_2 R_{11} \rho C_y C_x \right] \) | \( 1 - \frac{f}{n} \bar{y}^2 C_y^2 (1 - \rho^2) \) |
|       | Soponviwatkul and Lawson (2017) | | |
| 29    | \( t_{27} = \omega_1 \bar{y} + (1 - \omega_1) \left( \frac{X}{\overline{x}} \right) \) | \( 1 - \frac{f}{n} \bar{y} \rho C_y (C_x - \rho) \) | \( 1 - \frac{f}{n} \bar{y}^2 C_y^2 (1 - \rho^2) \) |
|       | Ijaz and Ali (2018) | | |
| 30    | \( t_{28} = \omega_2 \bar{y} + (1 - \omega_2) \left( \frac{\exp X}{\overline{x}} \right) \) | \( 1 - \frac{f}{n} \bar{y} \rho C_y \left( \frac{1}{4} C_x - \rho C_y \right) \) | \( 1 - \frac{f}{n} \bar{y}^2 C_y^2 (1 - \rho^2) \) |
|       | Ijaz and Ali (2018) | | |
| 31    | \( t_{29} = \bar{y} \left( \frac{abX + cd}{abx + cd} \right) \) | - | \( 1 - \frac{f}{n} \bar{y}^2 \left( C_y^2 - C_y^2 \right) \) |
|       | Yadav et al. (2019) | | |
Note: We will denote our measures in terms of Coefficient of Variation because it is a relative measure and most suitable to compare two series.
where,
\[ R_5 = \frac{X}{X + C_x}, R_7 = \frac{XC_x}{XC_x + \beta_2}, R_8 = \frac{X\beta_2}{X\beta_2 + C_x}, R_{10} = \frac{X\beta_1}{X\beta_1 + S_x}, \]
\[ R_{11} = \frac{X}{X + \rho}, R_{12} = \frac{X}{X + \beta_2}, R_{13} = \frac{X}{X + \beta_1}, R_{14} = \frac{X\beta_1}{X\beta_1 + \beta_2}, \]
\[ R_{15} = \frac{XC_x}{XC_x + \beta_1}, R_{16} = \frac{X\beta_2}{X\beta_2 + \beta_1}, k = \frac{C_y}{C_x}, R_{18} = \frac{X}{X + Q_3}, \]
\[ R_{19} = \frac{X}{X + Q_r}, R_{20} = \frac{X}{X + M_d}, R_{21} = \frac{XC_x}{XC_x + M_d}, \alpha' = \frac{\rho C_y}{4 C_x}, \]
\[ \omega_1 = w/R_5, \omega_2 = w/R_{11}, w = \frac{C_y}{C_x}. \]

a,b,c,d = Constants or Parametric Value

3 Proposed Estimator Based on Median
If the median M of Y is known so by utilizing it, we may suggest an elevated estimator of Y as,
\[ t = \overline{y} + \alpha \log \frac{m}{M} \]  
(1)
Where \( \alpha \) is chosen such that MSE(t) is minimum. The bias and MSE of t up to approximation of order one is given by,
\[ B(t) = \frac{\alpha B(m)}{M} - \frac{\alpha}{2} \lambda C_m^2 \]  
(2)
\[ MSE(t) = \lambda \overline{Y^2} C_y^2 + \lambda \alpha^2 C_m^2 + 2\alpha \overline{Y} \lambda C_{ym} \]  
(3)
where \( \lambda = \frac{1-f}{n} \)
Hence
\[ \alpha_{min} = \frac{-\overline{Y} C_{ym}}{C_m^2} \]  
(4)
and \( MSE_{min}(t) = \lambda \overline{Y^2} \left( C_y^2 - \frac{C_{ym}^2}{C_m^2} \right) \)  
(5)
3.1 Proposed Estimator Based on Median: Detailed Study

Let

\[ e_0 = \frac{\bar{y} - \bar{Y}}{\bar{Y}} \quad \text{and} \quad e_1 = \frac{m - M}{M} \]

then \( \bar{y} = \bar{Y}(1 + e_0) \) and \( m = M(1 + e_1) \)

So,

\[ E(e_0) = E\left( \frac{\bar{y} - \bar{Y}}{\bar{Y}} \right) = E\left( \frac{\bar{y}}{\bar{Y}} \right) - 1 = 0 \quad \text{or} \quad E(e_0) = 0 \quad (6) \]

\[ E(e_1) = E\left( \frac{m - M}{M} \right) = E\left( \frac{m}{M} \right) - 1 = B(m) \quad \text{or} \quad E(e_1) = B(m) \quad (7) \]

\[ E(e_0^2) = E\left( \frac{\bar{y} - \bar{Y}}{\bar{Y}} \right)^2 = E\left( \frac{\bar{y}}{\bar{Y}} \right)^2 = V(\bar{y})/\bar{Y}^2 = \frac{1 - f}{n} C_y^2 \quad (8) \]

\[ E(e_1^2) = E\left( \frac{m - M}{M} \right)^2 = E\left( \frac{m}{M} \right)^2 = V(m)/M^2 = \frac{1 - f}{n} C_m^2 \quad (9) \]

\[ E(e_0 e_1) = E\left( \frac{(\bar{y} - \bar{Y})(m - M)}{\bar{Y} M} \right) = Cov(\bar{y}, m) = \lambda C_{ym} \]

\[ E(e_0 e_1) = C_{ym} \quad (10) \]

Hence the Estimator can be rewritten as,

\[ t = \bar{Y} + \bar{Y} e_0 + \alpha = \left( e_1 - \frac{e_1^2}{2} \right) \quad (11) \]

For Biasedness,

\[ t = \bar{Y} + \bar{Y} e_0 + a(\bar{e}_1 - \frac{e_1^2}{2}) \]

\[ (t - \bar{Y}) = \bar{Y} e_0 + ae_1 - \frac{ae_1^2}{2} \]

\[ E(t - \bar{Y}) = \bar{Y} E(e_0) + a E(e_1) - \frac{a}{2} E(e_1^2) \]

\[ B(t) = 0 + a \frac{B(m)}{M} - \frac{a}{2} \lambda C_m^2 \]

(from Equation (6), (7) and (8))
Hence Biasedness is:

\[ B(t) = \alpha \frac{B(m)}{M} - \frac{\alpha}{2} \lambda C_m^2 \]  \hspace{1cm} (12)

For MSE,

\[
\text{MSE}(t) = E(t - \overline{Y})^2 = E(Y e_0 + \alpha e_1)^2 \\
\text{(second and higher order terms are ignored)} \\
= E \left( Y^2 e_0^2 + \alpha^2 e_1^2 + 2Y \alpha e_0 e_1 \right) \\
= Y^2 E(e_0^2) + \alpha E(e_1^2) + 2 \alpha E(e_0 e_1)
\]

Hence,

\[
\text{MSE}(t) = \lambda Y^2 C_y^2 + \lambda \alpha^2 C_m^2 + 2 \alpha Y \lambda C_{ym} 
\]  \hspace{1cm} (13)

(from equation (8), (9) and (10))

**Minimum Value of \( \alpha \)**

For Minimum value of \( \alpha \), we should have

\[
\frac{\partial \text{MSE}(t)}{\partial \alpha} = 0 \quad \text{and} \quad \frac{\partial^2 \text{MSE}(t)}{\partial \alpha^2} > 0
\]

So,

\[
\frac{\partial \text{MSE}(t)}{\partial \alpha} = 0 \lambda C_m^2 + 2 \lambda Y \lambda C_{ym} = 0 \\
\alpha_{\min} = -\frac{Y C_{ym}}{C_m^2} 
\]  \hspace{1cm} (14)

For Minima,

\[
\frac{\partial^2 \text{MSE}(t)}{\partial \alpha^2} = 2 \lambda C_m^2 > 0
\]

Hence \( \alpha \) has a minimum value.

**Minimum Value of MSE**

Hence Minimum value of MSE(t) is obtained by putting the Value of \( \alpha \) in Equation (3.3)

\[
\text{MSE}_{\min}(t) = \lambda Y^2 C_y^2 + \lambda Y^2 C_{ym}^2 C_m^2 C_{ym}^2 - 2 \frac{Y C_{ym}}{C_m^2} Y \lambda C_{ym} 
\]
\begin{align*}
= & \lambda \bar{Y}^2 C^2_y + \lambda \bar{Y}^2 \frac{C^2_{ym}}{C^2_m} - 2\lambda \frac{\bar{Y}^2 C^2_{ym}}{C^2_m} \\
MSE_{\text{min}}(t) = & \lambda \bar{Y}^2 \left( C^2_y - \frac{C^2_{ym}}{C^2_m} \right) \tag{15}
\end{align*}

4 Efficiency Comparison

In this section the suggested estimator is being compared with the competing estimators of \( \bar{Y} \) and the efficiency conditions are presented in Table 2.

| S.No. | \( \text{MSE}(t) \) Condition |
|-------|--------------------------------|
| 1     | \( \text{MSE}(t) \leq \text{var}(\bar{Y}) \) \( \frac{C^2_{ym}}{C^2_m} \geq 0 \) |
| 2     | \( \text{MSE}(t) \leq \text{MSE}(t_2) \) \( \frac{C^2_{ym}}{C^2_m} \geq C_x(2\rho C_y - C_x) \) |
| 3     | \( \text{MSE}(t) \leq \text{MSE}(t_{ij}), \text{MSE}(\bar{Y}_j) \) \( \frac{C^2_{ym}}{C^2_m} \geq \rho^2 C^2_y; j = 3, 4, 25, \ldots, 29 \) |
| 4     | \( \text{MSE}(t) \leq \text{MSE}(t_5) \) \( \frac{C^2_{ym}}{C^2_m} \geq 2R_x C_x(2\rho C_y - \frac{R_x}{2} C_x) \) |
| 5     | \( \text{MSE}(t) \leq \text{MSE}(t_6) \) \( \frac{C^2_{ym}}{C^2_m} \geq C_x(\rho C_y - C_x) \) |
| 6     | \( \text{MSE}(t) \leq \text{MSE}(t_j) \) \( \frac{C^2_{ym}}{C^2_m} \geq 2R_x C_x(2\rho C_y - \frac{R_x}{2} C_x); \) \( j = 7, 8, 10 \ldots, 16, 18 \ldots, 21 \) |
| 7     | \( \text{MSE}(t) \leq \text{MSE}(t_9) \) \( \frac{C^2_{ym}}{C^2_m} \geq 4C_x(\rho C_y - C_x) \) |
| 8     | \( \text{MSE}(t) \leq \text{MSE}(t_{17}) \) \( \frac{C^2_{ym}}{C^2_m} \geq \rho^4 C^2_y \) |
| 9     | \( \text{MSE}(t) \leq \text{MSE}(t_{22}) \) \( \frac{C^2_{ym}}{C^2_m} \geq C_x(\rho C_y - \frac{C_x}{4}) \) |
| 10    | \( \text{MSE}(t) \leq \text{MSE}(t_{23}) \) \( \frac{C^2_{ym}}{C^2_m} \geq \rho C_y C_x \) |
| 11    | \( \text{MSE}(t) \leq \text{MSE}(t_{24}) \) \( \frac{C^2_{ym}}{C^2_m} \geq 2R_{24} C_x(\rho C_y - \frac{R_{24}}{2} C_x) \) |
5 Numerical Study

Under this section the efficiency conditions of the suggested estimator over competing estimators are verified using real data sets.

5.1 Data Collection

To verify the results we have obtained for the Paddy Production data from the Pira Nagar Village of Barabanki District. The details of the obtained data is as follows:

| Village       | Pira Nagar       |
|---------------|------------------|
| District      | Barabanki        |
| Time          | March 2018       |
| Production    | Paddy Production |
| Type of Data  | Primary Data     |
| Information Taken | Name of Resident |
| Their Area of Cultivation (Unit in Hectares) | |
| Yield obtained for each area (Unit in Quintals): | |
| one Quintal=100 Kilogram | |

For our Numerical Justification we have taken:

| Study Variable     | Yield | denoted as Y |
|--------------------|-------|--------------|
| Auxiliary Variable | Area of Cultivation | denoted as X |
| Population Size    | 52    |
| Sample Size        | 3     |

5.2 Population Parameters

| Parameter      | Value     | Units in |
|----------------|-----------|----------|
| N              | 52        | –        |
| n              | 3         | –        |
| \( \frac{N}{n} \) | 22100    | –        |
| \( \bar{Y} \)  | 14.721    | Quintal  |
| \( \bar{X} \)  | 0.46227   | Hectare  |
| M              | 10        | Quintal  |
| \( \rho \)     | 0.8046229 | –        |
| \( S_{y}^{2} \) | 190.8668  | Quintal  |
| \( S_{x}^{2} \) | 0.15675383 | Hectare  |
5.3 Measurement on Population Parameters

On the basis of the data, the numerical values related to proposed estimators are obtained as follows:

| Parameter | Value | Units in |
|-----------|-------|----------|
| $S_{yx}^2$ | 4.401155 | – |
| $C_{y}^2$ | 0.8807379 | – |
| $C_{x}^2$ | 0.7335474 | – |
| $C_{yx}$ | 4.401155 | – |
| $\beta_1$ | 8.1028894 | – |
| $\beta_2$ | 14.146291 | – |
| $\beta$ | 28.0769 | – |
| $Q_1$ | 0.4040 | Hectare |
| $Q_3$ | 0.5050 | Hectare |
| $Q_r$ | 0.2525 | Hectare |
| $\overline{M}$ | 12.0119 | Quintal |
| $V(m)$ | 37.7429 | Quintal |
| $C_{ym}$ | 0.2536 | – |
| $C_{m}$ | 0.3774 | – |

5.4 Numerical Comparison

We have constructed a table for numerical comparison of the suggested estimator with the estimators in competition. The following table gives:

| Parameter | Value |
|-----------|-------|
| $t$ | 14.022176 |
| $B(t)$ | 1.5962096 |
| $a_{min}$ | 9.8921161 |
| $MSE_{min}(t)$ | 14.211079 |

- The values of the suggested and competing estimators on the Basis of the data obtained and sample taken.
- The Biases of competing and suggested Estimators.
- Mean Square Error (MSE) of competing and Proposed Estimators.
- The percentage relative efficiencies (PRE) of the suggested over competing estimators of $\overline{Y}$. 
Table 4  Biases, MSE’s and respective PRE’s

| Estimator | Bias | MSE         | PRE         |
|-----------|------|-------------|-------------|
| $t_0$     | 0    | $V(t_0) = 59.951751$ | 421.866     |
| $y_{ir}$  | 0    | $V(y_{ir}) = 21.137907$ | 148.74245   |
| $t_1$     | 0.40139256 | 21.837168 | 153.66298  |
| $t_2$     | 0    | 21.837168 | 153.66298  |
| $t_3$     | -1.1203056 | 21.137907 | 148.74245  |
| $t_4$     | -2.3629292 | 21.137907 | 148.74245  |
| $t_5$     | -0.63149751 | 35.223464 | 247.8592  |
| $t_6$     | -0.22328973 | 28.411333 | 199.92382  |
| $t_7$     | -0.078904053 | 57.591622 | 405.25862  |
| $t_8$     | 0.0076015701 | 21.138228 | 148.74471  |
| $t_9$     | 4.1946732 | 83.587598 | 588.18614  |
| $t_{10}$  | 0.06976912 | 21.163736 | 148.9242  |
| $t_{11}$  | -0.63958709 | 34.472784 | 242.5768  |
| $t_{12}$  | -0.091234028 | 57.215612 | 402.6127  |
| $t_{13}$  | -0.15151962 | 55.345217 | 389.4512  |
| $t_{14}$  | -0.47740469 | 43.707413 | 307.5587  |
| $t_{15}$  | -0.086059435 | 57.373669 | 403.7249  |
| $t_{16}$  | -0.65903753 | 30.58859 | 215.2447  |
| $t_{17}$  | 0.27660967 | 21.493713 | 151.2462  |
| $t_{18}$  | -0.65448695 | 29.277583 | 206.0194  |
| $t_{19}$  | -0.51534035 | 23.893597 | 168.13359  |
| $t_{20}$  | -0.62993932 | 27.185932 | 191.30097  |
| $t_{21}$  | -0.64921611 | 28.605068 | 201.2871  |
| $t_{22}$  | -0.22328973 | 28.411333 | 199.92382  |
| $t_{23}$  | 1.7460142 | 15.928206 | 112.0830  |
| $t_{24}$  | -0.33881397 | 49.086159 | 345.4077  |
| $t_{25}$  | -0.7940365 | 21.137907 | 148.74245  |
| $t_{26}$  | -0.8804347 | 21.137907 | 148.74245  |
| $t_{27}$  | 0.1809299 | 21.137907 | 148.74245  |
| $t_{28}$  | -1.888758 | 21.137907 | 148.74245  |
| $t_{29}$  | - | 21.137907 | 148.74245  |
| (t)       | 1.5962096 | 14.211079 |           |

6 Results and Discussion

From the Table 4 it is observed from all the estimators that,

6.1 Biasednesses are ranging from $-2.3629$ to $4.1947$.
6.2 Mean Square Errors are ranging from $14.2111$ to $83.5875$.
6.3 The PREs of the suggested estimator over estimators in competition are ranging from $112.0830$ to $588.1861$. 
6.4 The sample mean \((t_0)\), Usual Regression Estimator (Watson, 1937) \(\bar{y}_{lr}\), Goodman and Hartely’s revised estimator (1958), \(t_2\); are unbiased for \(Y\).

6.5 Among the existing estimators, the MSE of the estimator of Yadav and Mishra (2015), \(t_{23}\) is minimum i.e. 15.9282 and MSE of the estimator of Kadilar and Kingi (2003), \(t_9\) is maximum i.e. 83.5875.

6.6 The suggested estimator is 1.12\% more efficient than \(t_{23}\) and 5.88\% more efficient than \(t_9\).

6.7 Since Efficiency is stronger property than the unbiasedness. Hence here we prefer the biased estimator with minimum MSE instead of unbiased estimator with higher MSE.

6.8 It is observed from data that the given below inequalities hold good;

\[
MSE(t) \leq MSE(t_{23}) \leq V(\bar{y}_{lr}) \leq MSE(t_1) \leq V(\bar{y})
\]

6.10 The proposed estimator has minimum MSE and comes out to be more efficient than the other estimators which was our aim of study.

7 Conclusion

7.1 We have applied our proposed estimator successfully for the estimation of Average Paddy production. We can also use it for other agricultural areas and productions with larger sample size and population size.

7.2 In case the exact values are not known but we may know the ranges within which our values may be supposed to lie, this Median based estimator will give a precise result.

7.3 The use of this estimator can be extended to other fields also and can be used as an alternative of SRSWOR sample mean when exact values of characteristic under study are not known.

7.4 Whenever taking the auxiliary information involves very high cost we can use the proposed estimator as an alternative of Ratio and Regression type estimators.

7.5 It has been shown theoretically as well as numerically that the suggested estimator is better than the competing estimators for the estimating \(Y\).

7.6 Mostly the minimum MSE of any ratio type estimator is equal to the variance of the regression estimator but the suggested ratio type estimator has its MSE less than the variance of the usual regression estimator.

7.7 Therefore, the suggested estimator is highly recommended for the practical applications.
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