Common Origin for CP Violation in Cosmology and in Neutrino Oscillations

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Abstract

We suggest predictive scenarios for neutrino masses which provide a common origin for CP violation in early universe cosmology and in neutrino oscillations. Our setup is the seesaw mechanism in the context of MSSM with two quasi–degenerate right–handed neutrinos, with baryon asymmetry generated via resonant leptogenesis. Three different models are found with specific textures in the Yukawa coupling matrices, each with a single phase which controls leptogenesis and neutrino CP violation. One model leads to normal hierarchy of light neutrino masses and the prediction $\tan \theta_{13} \approx \sin \theta_{12} \sqrt{\frac{m_2}{m_3}}$, resulting in a value of the reactor mixing angle $\theta_{13}$ very close to the current experimental lower limit. The other two models predict inverted hierarchical neutrino mass spectrum with the sum rules $\sin^2 \theta_{12} \approx \frac{1}{2} - \tan \theta_{23} \sin \theta_{13} \cos \delta$ and $\sin^2 \theta_{12} \approx \frac{1}{2} + \cot \theta_{23} \sin \theta_{13} \cos \delta$ respectively. We obtain a lower bound for the phase $|\delta|$ in the normal hierarchical model, and a narrow range for $|\delta|$ for the inverted hierarchical model from cosmology. In our scenario, the mass–splitting between the quasi–degenerate right–handed neutrinos arise via renormalization group flow, which provides a lower limit on the MSSM parameter $\tan \beta > 12$. The right–handed neutrino masses can be as low as TeV, which would avoid the gravitino problem generic to supersymmetric models.
1 Introduction

While the standard model (SM) of strong and electroweak interactions has been extremely successful in confronting experimental data, it leaves several questions unanswered. On the observational side, it does not provide a viable dark matter candidate, nor a dynamical mechanism to explain the observed baryon excess in the universe. Furthermore, the model needs to be extended, albeit in a minor way, to accommodate small neutrino masses as needed for atmospheric [1] and solar neutrino oscillation data [2]. On the theoretical side, the model suffers from the quadratic divergence problem, which destabilizes the Higgs boson mass.

An elegant synthesis of these issues is provided by low energy supersymmetry (SUSY) and the seesaw mechanism [3]. Low energy SUSY can cure the quadratic divergence problem for the Higgs boson mass. In its simplest form it also provides a natural dark matter candidate, the lightest SUSY particle (LSP). The seesaw mechanism assumes the existence of right–handed neutrinos (RHN) $N$ which facilitates small neutrino masses. It also provides a dynamical mechanism for baryon asymmetry generation via the lepton number violating decays of the $N$ [4]. The induced lepton asymmetry is converted into baryon asymmetry via the electroweak sphalerons [5] (for reviews of leptogenesis see Ref. [6, 7]).

Attractive as it is, the SUSY seesaw framework is not without its problems. First, the generic leptogenesis mechanism is impossible to test experimentally. This is primarily because the dynamics occurs at a very high energy scale, beyond reach of foreseeable experiments. The parameters that are relevant for leptogenesis are not the same that appear in low energy neutrino oscillation experiments. (The number of low energy observables in neutrino sector is nine, while leptogenesis in the general setting involves a total of eighteen parameters.) Second, in supergravity models, successful leptogenesis is in conflict with the gravitino abundance. This is because of the lower bound on the lightest RHN mass $M_{N_1} \gtrsim 10^9$ GeV, (assuming hierarchical masses for $N$) [8] which would suggest rather high reheat temperature, of order $10^9$ GeV. This conflicts with reheat temperature suggested by gravitino abundance $T_{\text{reheat}} < 10^8$ GeV [9, 10].

In this paper we suggest a scenario where the aforementioned problems of the SUSY seesaw framework are alleviated. The gravitino overproduction problem is avoided by resorting to resonant leptogenesis scenario [11–13] which assumes quasi–degenerate $N$ fields. In this case the mass of the $N$ fields can be as low as a TeV, consistent with successful leptogenesis, thus avoiding the gravitino problem. We supplement the resonant leptogenesis scenario with flavor symmetries which restrict the form of the neutrino Yukawa coupling matrices. Such flavor symmetries are anyway needed to guarantee the near degeneracy of the $N$ states. We identify three possible textures for the Dirac Yukawa couplings of the neutrinos that yield two quasi–degenerate $N$ fields and a sum rule for the neutrino oscillation angle $\theta_{13}$. Interestingly, in all three models, there is a single phase that controls cosmological CP asymmetry and CP violation in neutrino oscillations. We are able to constrain the range of the CP violation parameter $|\delta|$ from cosmology. Somewhat similar classification of textures has been recently pursued in Ref. [14] and earlier in Ref. [15], [16]. Our emphasis is on the connection between cosmological CP asymmetry and CP violation in neutrino oscillations. It turns out that, in our framework, there is a lower limit on the SUSY parameter $\tan\beta > 12$. This arises since the mass splitting between the quasi–degenerate $N$ fields is generated from renormalization group flow, which depends on $\tan\beta$. 


In our analysis we use the results of a global fit to the neutrino oscillation data [17]:

\[|\Delta m_{\text{atm}}^2| = (2.18 - 2.64) \times 10^{-3} \text{eV}^2 \ (2\sigma) , \]
\[\Delta m_{\text{sol}}^2 = (7.25 - 8.11) \times 10^{-5} \text{eV}^2 \ (2\sigma) , \]
\[
\sin^2 \theta_{23} = 0.39 - 0.63 \ (2\sigma) , \quad \sin^2 \theta_{12} = 0.27 - 0.35 \ (2\sigma) .
\]

Currently we do not know the sign of \(\Delta m_{\text{atm}}^2\), i.e. whether neutrinos have normal mass hierarchy or inverted mass hierarchy. Also, the value of the third mixing angle \(\theta_{13}\) is unknown. Only an upper bound [17]

\[\sin^2 \theta_{13} \leq 0.04 \ (2\sigma)\]

is available currently. Nothing is known about the CP violating phase \(\delta\) (and also about two ‘Majorana’ phases) of the leptonic mixing matrix.

We will identify explicit models wherein these unknown mixing parameters are significantly constrained. It will be highly desirable to relate the CP violation parameters in the leptonic mixing matrix with the cosmological CP asymmetry. Such a strategy was pursued successfully in Ref. [18]. While in Ref. [18] a close connection between cosmological CP violation and neutrino CP violation was realized, since the setup used hierarchical RHN masses, straightforward SUSY extension of that scenario would lead to gravitino overproduction. Our texture models are tailor-made for resonant leptogenesis, which would avoid this problem.\(^1\)

### 2 Texture Zeros for Predictive Models

Let us consider the lepton sector of MSSM augmented with two right–handed neutrinos (RHN) \(N_1\) and \(N_2\). The relevant Yukawa superpotential couplings are given by

\[W_{\text{lept}} = W_e + W_\nu ,\]

with

\[W_e = t^T Y_e e^c h_d , \quad W_\nu = t^T Y_\nu N h_u - \frac{1}{2} N^T M_N N ,\]

where \(h_d\) and \(h_u\) are up and down type MSSM Higgs doublet superfields respectively. We will work in a basis in which the charged lepton Yukawa matrix is diagonal:

\[Y_e = \text{Diag} (\lambda_e , \lambda_\mu , \lambda_\tau ) .\]

As far as the RHN mass matrix \(M_N\) is concerned, we will assume that at high scale (identified with the GUT scale later on) it has the form

\[M_N = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} M .\]

This form of \(M_N\) is crucial for our studies. It has interesting implications for resonant leptogenesis and also, as we will see below, for building predictive neutrino scenarios. Specific neutrino models consistent with resonant leptogenesis with a texture similar to (5) was investigated in [19]. Here we attempt to classify all possible scenarios with degenerate RHNs which lead to predictions consistent

\(^1\)For a concrete demonstration within predictive model see [19].
with experiments. Thus, with a basis (4) and the texture (5) we can discuss possible texture zeros
in the matrix \( Y_\nu \), which is of dimension \( 3 \times 2 \). One can easily verify that two (and more) texture
zeros in \( Y_\nu \) do not lead to results compatible with the neutrino data. However, with only one texture
zero, there are scenarios compatible with experiments and leading to interesting predictions.

The matrix \( Y_\nu \) contains two columns. Since due to the form of \( M_N \) (5) there is exchange
invariance \( N_1 \rightarrow N_2, \ N_2 \rightarrow N_1 \), it does not matter in which column of \( Y_\nu \) we set one element to
zero. We choose here the second column of \( Y_\nu \) having one texture zero. This leads to the three
following possible forms for \( Y_\nu \):

\[
\text{Texture A : } \quad Y_\nu = \begin{pmatrix} a_1 & 0 \\ a_2 & b_2 \\ a_3 & b_3 \end{pmatrix},
\]

\[
\text{Texture B}_1 : \quad Y_\nu = \begin{pmatrix} a_1 & b_1 \\ a_2 & 0 \\ a_3 & b_3 \end{pmatrix}, \quad \text{Texture B}_2 : \quad Y_\nu = \begin{pmatrix} a_1 & b_1 \\ a_2 & b_2 \\ a_3 & 0 \end{pmatrix}.
\]

A few words about the parametrization, used in (6) and (7), are in order. With the basis (4) and
the form of \( M_N \) given in (5), the one texture zero \( 3 \times 2 \) matrix \( Y_\nu \) has only one physical phase.
Other phases can be rotated away by proper phase redefinitions of the fields. Moreover, in \( Y_\nu \) there
are five real parameters \(|a_{1,2,3}|\) and two absolute values of the \( b \)-entries. The mass parameter \( M \)
in (5) is in general complex, but its phase is not relevant for the physics of neutrino oscillations.
These systems lead to predictive scenarios with texture \( A \) corresponding to normal mass hierarchy
and textures \( B_1 \) and \( B_2 \) corresponding to inverted mass hierarchy. We will study these cases in
turn.

### 2.1 Texture A: Normal Hierarchical Case

We will discuss this case in details. With (5), (6) and using the seesaw formula for the light neutrino
mass matrix \( M_\nu = \langle h_u^0 \rangle^2 Y_\nu M_N^{-1} Y_\nu^T \), we arrive at

\[
M_\nu = \begin{pmatrix} 0 & a_1 b_2 \\ a_1 b_3 & a_2 b_2 + a_3 b_2 \end{pmatrix} \begin{pmatrix} a_1 b_3 \\ 2a_2 b_2 + 2a_3 b_2 \end{pmatrix} \frac{(v \sin \beta)^2}{M},
\]

where \( v \approx 174 \text{ GeV} \). The matrix in (8) is rank two and leads to the one massless neutrino and two
massive neutrinos labeled \( m_2 \) and \( m_3 \). This structure corresponds to the normal hierarchical case, i.e.

\[
M_\nu^\text{diag} = \text{Diag} \left( 0, m_2, m_3 \right),
\]

with \( m_3 \gg m_2 \). From (8) we can see that the mixings \( \theta_{12} \) and \( \theta_{23} \) are generated. The absolute
value of the overall factor \( a_3 b_3 (v \sin \beta)^2 / M \) determines one mass scale, say the value of \( m_3 \). Besides
this overall factor the matrix has four parameters: one phase and three real parameters. Three of
these parameters can be fixed from three observables \( \theta_{12} \), \( \theta_{23} \) and \( \frac{\Delta m^2_{\text{sol}}}{\Delta m^2_{\text{atm}}} \) (where \( \Delta m^2_{\text{sol}} = m^2_2 \) and
\( \Delta m^2_{\text{atm}} = m^2_3 - m^2_2 \)). Due to the condition \( m_1 = 0 \) we will still have one prediction (independent
from the value of the phase), which determines the angle \( \theta_{13} \).
One physical phase remains undetermined. Indeed this *single* phase will be directly related to the CP violation in neutrino oscillations and in leptogenesis. We will discuss this connection in more details in Sect. 3.

Now, let us derive the prediction of this model. To achieve this and also get other useful relations we will use the equality

$$M_\nu = P U^* P' M^\text{diag}_\nu U^\dagger P,$$

(10)

where $U$ is the lepton mixing matrix, given in a standard parameterization by:

$$U = \begin{pmatrix}
  c_{13} c_{12} & c_{13} s_{12} & s_{13} e^{-i\delta} \\
  -c_{23} s_{12} - s_{23} s_{13} c_{12} e^{i\delta} & c_{23} c_{12} - s_{23} s_{13} s_{12} e^{i\delta} & s_{23} c_{13} \\
  s_{23} s_{12} - c_{23} s_{13} c_{12} e^{i\delta} & -c_{23} c_{12} - c_{23} s_{13} s_{12} e^{i\delta} & c_{23} c_{13}
\end{pmatrix},$$

(11)

with $s_{ij} = \sin \theta_{ij}$ and $c_{ij} = \cos \theta_{ij}$. The $P$ and $P'$ are diagonal phase matrices $P = \text{Diag} (e^{i\omega_1}, e^{i\omega_2}, e^{i\omega_3})$, $P' = \text{Diag} (1, e^{i\rho_1}, e^{i\rho_2})$. Phases in $P$ can be removed by field redefinition, while $P'$ is physical, and contains the two Majorana phases. The matrix equation (10) gives six relations. One of them, namely the relation for the (1,1) elements of $M_\nu$ and the right hand side of (10) with the form of $U$ given in Eq. (11), gives

$$\tan \theta_{13} \simeq \sin \theta_{12} \sqrt{\frac{m_2}{m_3}}.$$ 

(12)

Since this case corresponds to the normal hierarchical neutrino mass spectrum (with $m_1 = 0$), with the help of (1) we have at 2σ level $m_2 = \sqrt{\Delta m^2_{\text{sol}}} \simeq (8.51 - 9.01) \cdot 10^{-3} \text{ eV}$ and $m_3 = \sqrt{|\Delta m^2_{\text{atm}}|^2 + \Delta m^2_{\text{sol}}} \simeq (4.7 - 5.2) \cdot 10^{-2} \text{ eV}$. Using these values in (12), together with 2σ accuracy value of $\theta_{12}$, we obtain range $\sin^2 \theta_{13} \simeq 0.042 - 0.062$. This fits well with an upper bound, within 3σ, given in Ref. [17], while the low limit (0.042) is pretty close to the 2σ upper bound of $\theta_{13}$. Future measurements of the $\theta_{13}$ will test the validity of this scenario. One more word about the neutrino sector: since the (1,1) element of the light neutrino mass matrix vanishes, the neutrino–less double $\beta$-decay ($0\nu2\beta$) does not take place in this scenario. That is, $m_{\beta\beta} = |U^2_{e2}m_2 e^{i\beta} + U^2_{e3}m_3| = 0$. There is only one Majorana phase, since $m_1 = 0$, which is $\bar{\rho} = \rho_2 - \rho_1$. This is determined from the phase $\delta$ as follows

$$\bar{\rho} = \pi - 2\delta.$$ 

(13)

### 2.2 Textures $B_1$ and $B_2$: Inverted Hierarchical Cases

The textures $B_1$ and $B_2$ both lead to the inverted hierarchical neutrino mass pattern. Using these textures (7), the form of $M_N$ given in (5) and the seesaw formula for the light neutrino mass matrices we obtain:

For Texture $B_1$ : 

$$M_\nu = \begin{pmatrix}
  2a_1 b_1 & a_2 b_1 & a_1 b_3 + a_3 b_1 \\
  a_2 b_1 & 0 & a_3 b_3 \\
  a_1 b_3 + a_3 b_1 & a_2 b_3 & 2a_3 b_3
\end{pmatrix} \frac{(v \sin \beta)^2}{M},$$

(14)

For Texture $B_2$ : 

$$M_\nu = \begin{pmatrix}
  2a_1 b_1 & a_1 b_2 + a_2 b_1 & a_3 b_1 \\
  a_1 b_2 + a_2 b_1 & 2a_2 b_2 & a_3 b_2 \\
  a_3 b_1 & a_3 b_2 & 0
\end{pmatrix} \frac{(v \sin \beta)^2}{M}.$$ 

(15)
In order to derive predictions for both cases, we can still use the relation (10), which is general, but for $M_{\nu}$ use the forms corresponding to the cases $B_{1,2}$, and for $M_{\nu}^{\text{diag}}$ an inverted hierarchical form:

$$M_{\nu}^{\text{diag}} = \text{Diag} (m_1, m_2, 0) .$$

We use the same form as before for the phase matrix $P$, while for the $P'$ we use $P' = \text{Diag} (e^{i\rho_1}, e^{i\rho_2}, 1)$. For cases $B_1$ and $B_2$ the predictive relations emerge by equating the (2, 2) and (3, 3) elements respectively (which are zero) of the expressions at the both sides of Eq. (10). Doing so we arrive at:

For texture $B_1$:

$$\sin^2 \theta_{12} \approx \frac{1}{2} - \frac{\sin \theta_{13} \tan \theta_{23} \cos \delta}{\tan^2 \theta_{23} \sin^2 \theta_{13} + e^{2i\delta}} + \frac{1}{8} \frac{\Delta m_{\text{sol}}^2}{\Delta m_{\text{atm}}^2},$$

For texture $B_2$:

$$\sin^2 \theta_{12} \approx \frac{1}{2} + \frac{\sin \theta_{13} \tan \theta_{23} \cos \delta}{\tan^2 \theta_{23} + \sin^2 \theta_{13} e^{2i\delta}} + \frac{1}{8} \frac{\Delta m_{\text{sol}}^2}{\Delta m_{\text{atm}}^2}.$$ 

As we see, for both cases, the deviation of $\sin^2 \theta_{12}$ from 1/2 (i.e. deviation of $\theta_{12}$ from $\pi/4$) is due to the non–zero value of $\theta_{13}$ and it also depends on $\cos \delta$. In fact, the product $\sin \theta_{13} \cos \delta$ should not be too small, otherwise the angle $\theta_{12}$ will be close to $\pi/4$ which is excluded. Using the current experimental data (1) (within 2$\sigma$-deviations) we obtain the following constraints for $\theta_{13}$ and $\cos \delta$:

**For texture B$_1$**: $\theta_{13} \gtrsim 0.12$, $\cos \delta \gtrsim 0.573$ ($|\delta| \lesssim 0.96$).

**For texture B$_2$**: $\theta_{13} \gtrsim 0.129$, $\cos \delta \lesssim -0.614$ ($|\pi - \delta| \lesssim 0.91$). (19)

The last terms in Eqs. (17), (18) are practically unimportant for the neutrino sector, but as we will see in section 3.1 they become crucial for the leptogenesis CP violation. The leptonic asymmetry will be determined by a phase $\propto \frac{\Delta m_{\text{sol}}^2}{i|\Delta m_{\text{atm}}^2|} \sin \delta$ which would vanish in the limit $\frac{\Delta m_{\text{sol}}^2}{|\Delta m_{\text{atm}}^2|} \to 0$.

By the fixed model parameters (see sect. 3.1 for relation between Yukawa couplings and the angles $\theta_{ij}, \delta$) we can compute one more observable. In contrast to the normal hierarchical neutrinos (corresponding to the texture A), cases $B_1$ and $B_2$ have non–zero $\beta_{\beta\nu}$ amplitudes, for both cases given by

$$m_{\beta\beta} = |U_{e1}^2 m_1 e^{i\rho} + U_{e2}^2 m_2| , \text{ with } \bar{\rho} = \rho_2 - \rho_1 .$$

For $m_{\beta\beta}$ and $\bar{\rho}$ for scenarios $B_1$ and $B_2$ respectively we derive:

For texture $B_1$:

$$m_{\beta\beta} = \sqrt{|\Delta m_{\text{atm}}^2| c_{13}^2 \frac{|t_{g12}^2 - 1 + 2t_{g12}t_{g23}s_{13}e^{i\delta}|}{|t_{g12} + t_{g23}s_{13}e^{i\delta}|^2} ,$$

$$\cot \frac{\bar{\rho}}{2} = -\frac{t_{g23}(1 + t_{g12}^2)s_{13} \sin \delta}{t_{g12}(1 - t_{g23}s_{13}^2) + t_{g23}(1 - t_{g12}^2)s_{13} \cos \delta} ,$$

For texture $B_2$:

$$m_{\beta\beta} = \sqrt{|\Delta m_{\text{atm}}^2| c_{13}^2 \frac{|t_{g12}^2 - 1 - 2t_{g12}ct_{g23}s_{13}e^{i\delta}|}{|t_{g12} - ct_{g23}s_{13}e^{i\delta}|^2} ,$$

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2Similar relation has been obtained in Ref. [19] within a specific model with $\theta_{23} \simeq \pi/4$. Here, since $\theta_{23}$ is not fixed from the model, we will have somewhat wider allowed ranges for $\theta_{13}$ and especially for $\delta$. Cases of texture zeros giving these relations have been identified recently in Ref. [14]. Correlations similar to Eqs. (17) and (18) have been obtained within scenarios with ‘quark-lepton complementarity’ [20].
\[
\cot \frac{\tilde{\rho}}{2} = \frac{\tan \theta_{12}}{\tan \theta_{13} - \tan \theta_{12}} \frac{s_{13} \sin \delta}{t_{12}} ,
\]

where \(t_{ij} \equiv \tan \theta_{ij}\) and \(ct_{ij} \equiv \cot \theta_{ij}\). Applying allowed ranges for \(\delta\) and \(\theta_{13}\) given in Eq. (19) and the measured neutrino oscillation parameters (1) (within 2\(\sigma\)) for \(m_{\beta\beta}\) we obtain:

For textures \(B_1 \& B_2\): 0.013 eV \(< m_{\beta\beta} \leq 0.023\) eV. (23)

Upper bounds for \(m_{\beta\beta}\) are obtained for \(|\delta| = 0.96\) and \(|\pi - \delta| = 0.91\) for cases \(B_1\) and \(B_2\) respectively, while lower limits correspond to \(\delta = 0\) and \(\delta = \pm \pi\). Planned experiments will certainly be able to test viability of these predictions. Note that the textures \(B_1\) and \(B_2\) in the neutrino sector give results which are practically indistinguishable (besides the allowed ranges for \(\delta\)). However, as we will see in the next section the scenario \(B_2\) fails to generate sufficient leptogenesis, while the texture \(B_1\) (and also the texture A) will work very well for this purpose.

### 3 Resonant Leptogenesis

Within the scenarios considered in the previous section, we have assumed an off-diagonal form for the RHN mass matrix \(M_N\). This gives the desired degeneracy between the two RHN states. The degeneracy will be lifted with small corrections to the \((1,1)\) and/or \((2,2)\) elements of \(M_N\). Even in the unbroken SUSY limit, 1-loop corrections (corresponding to the wave function renormalization) will split the degeneracy. The SUSY breaking effects has dramatic impact on the degeneracy of the scalar components of \(N_{1,2}\) superfields. This is discussed separately in the Appendix. As far as the fermionic RHN sector is concerned, the degeneracy there holds with pretty high accuracy. Therefore, this is an appealing framework for resonant leptogenesis, in which enhancement of the CP asymmetry happens because of quasi-degenerate RHN neutrinos [11–13]. One nice property of the resonant leptogenesis is that, it avoids the lower bound \((M_{N_1} \geq 10^9\) GeV) for the lightest RHN mass. This bound, called as Davidson-Ibarra bound, emerges within most of the scenarios with hierarchical right–handed neutrinos [8]. Once this bound is avoided, the reheat temperature can be sufficiently low to avoid the gravitino problem, which is common for low scale SUSY models [9, 10] with the gravity mediated SUSY breaking.

Since our models of neutrino masses and mixings are predictive and involve very limited number of parameters, we expect that we will not have much freedom in the calculation of leptogenesis. As we have already mentioned, an important ingredient for the resonant leptogenesis is the form of \(M_N\) given in Eq. (5). Note that the mass matrix of the fermionic RHNs coincides with \(M_N\) of the superpotential mass term. First we will discuss radiative corrections to the superpotential mass matrix \(M_N\), which directly can be applied to the fermionic RHNs. This structure can be justified by some symmetry at high scale. However, at low energies, due to the radiative corrections the \((1,1)\) and \((2,2)\) entries in \(M_N\) will receive non-zero corrections. These corrections are calculable thanks to the well defined neutrino models we have presented above. To be brief, eventually two RHNs are become quasi-degenerate, and the CP asymmetries \(\epsilon_1\) and \(\epsilon_2\) generated by out-of-equilibrium decays of the fermionic components of \(N_1\) and \(N_2\) states respectively are given by [12, 13]

\[
\epsilon_1 = \frac{\text{Im}(\hat{Y}_\nu^\dagger \hat{Y}_\nu)_{21}^2}{(\hat{Y}_\nu^\dagger \hat{Y}_\nu)_{11}(\hat{Y}_\nu^\dagger \hat{Y}_\nu)_{22}} \frac{(M_2^2 - M_1^2)}{M_1^2} \frac{M_1 M_2}{\Gamma_3 \Gamma_2} , \quad \epsilon_2 = \epsilon_1(1 \leftrightarrow 2) .
\]
Here $M_1$ and $M_2$ (we will use the convention $M_2 > M_1$) are the mass eigenvalues of the RH neutrino mass eigenstates. $\hat{Y}_\nu = Y_\nu U_N$ is the Dirac Yukawa coupling matrix in the basis where RH neutrino mass matrix is diagonal and real: $U_N^T M_N U_N = \text{Diag}(M_1, M_2)$. $\Gamma_i$ is the tree level decay width of $\tilde{N}_i$ (mass eigenstates of RHN) and is given by $\Gamma_i = \frac{M_i}{\pi} (Y^*_\nu Y_\nu)_{ii}$. From (24) we see that in order to have non-zero CP asymmetry two conditions need to be satisfied. First, the RHN masses should be split, and secondly the element $(\hat{Y}_\nu^\dagger \hat{Y}_\nu)_{12}$ must be complex. To realize both of these conditions, we need to include radiative corrections into our study. As we will see shortly, the desired result can be obtained only at two-loop level. In our treatment we assume that the textures we have considered are realized at the GUT scale $M_G \simeq 2 \cdot 10^{16}$ GeV. At low scales, due to the renormalization group effects the zero entries in the flavor matrices will receive some corrections. To compute these corrections we set up the RG equation for the matrix $M_N$ (only its renormalization has relevance for us), which at two-loop order is given by [21]:

$$16\pi^2 \frac{d}{dt} M_N = 2 M_N Y_\nu^\dagger Y_\nu + 2 Y_\nu^T Y_\nu^* M_N$$

$$- \frac{1}{8\pi^2} M_N \left( Y_\nu^\dagger Y_e^\dagger Y_e + Y_\nu^\dagger Y_\nu^T Y_\nu + Y_\nu^\dagger Y_\nu^* (3\lambda_i^2 + \text{tr}(Y_\nu^\dagger Y_\nu)) \right)$$

$$- \frac{1}{8\pi^2} \left( Y_\nu^T Y_e^* Y_e^\dagger Y_\nu^* + Y_\nu^T Y_\nu^* Y_\nu^T Y_\nu^* \right) Y_\nu^* (3\lambda_i^2 + \text{tr}(Y_\nu^T Y_\nu)) M_N + \frac{3}{20\pi^2} M_N \left( g_1^2 + 5g_2^2 \right),$$

(25)

where $t = \ln \mu$. The first line in (25) corresponds to the 1-loop correction and will be responsible for the mass splitting between RHNs. However, the two-loop correction, presented in a second line of Eq. (25), will be crucial for the CP phase of $(\hat{Y}_\nu^\dagger \hat{Y}_\nu)_{12}$. Since we intend to have $M_{1,2} \lesssim 10^7$ GeV, in order to get reasonable scale for the light neutrino mass, the matrix elements of $Y_\nu$ should be much less than unity. Thus, we can solve the RG equation analytically to a good approximation. One-loop correction to the $M_N$ can be found from (25) to be

$$\delta M_N^{1\text{-loop}} \simeq - \frac{1}{8\pi^2} \left( M_N Y_\nu^\dagger Y_\nu + Y_\nu^T Y_\nu^* M_N \right) \mu = M_G \ln \frac{M_G}{M}.$$ 

(26)

From this we see that at scale $\mu = M$ the form of $M_N$ will become

$$M_N = M \begin{pmatrix} -\delta_N & 1 \\ 1 & -\delta_N^* \end{pmatrix}. \quad \quad \quad (27)$$

Interestingly enough, this structure, of correlated phases of $(1,1)$ and $(2,2)$ entries of $M_N$, persists also at two-loop order. What is more important, one can see that at the one-loop level the phase of $\delta_N$ is determined by the phase of $(Y_\nu^\dagger Y_\nu)_{12}$ and therefore $(\hat{Y}_\nu^\dagger \hat{Y}_\nu)_{12}$ will be real at this level. This property can be easily seen also from different angle. Regardless of the form of $Y_\nu$ (including all possible radiative corrections to it), it can be written in the form

$$Y_\nu = U \begin{pmatrix} 0 & 0 \\ \hat{a}_2 & 0 \\ \hat{a}_3 & \hat{b}_3 \end{pmatrix} \tilde{P}, \quad \quad \quad (28)$$

with $U$ some unitary matrix, $\hat{a}_{1,2,3}$ all real parameters and $\tilde{P} = \text{Diag}(1,e^{i\xi})$. Using now the form (28) in the first line of (25), one can show that $U$ drops out and we remain with the non
physical phase $\xi$ which can be absorbed in $N_2$. With this, any complexity in $\delta M_{N}^{1\text{-}\text{loop}}$ and $(Y_{\nu}^\dagger Y_{\nu})_{12}$ disappears and we have no CP violation at the one–loop level. That is why it is important to include two–loop radiative corrections for the renormalization of $M_{N}$. Indeed, the term $M_{N}Y_{\nu}^\dagger Y_{e}^\dagger Y_{e}$ in the second line of Eq. (25) is important. The appearance of the combination $Y_{e}Y_{\nu}^\dagger Y_{e}Y_{\nu}$ plays an important role. With the basis (28) we see that the matrix $U$ does not disappear, and thus we expect to have CP violation (induced through two–loop correction). From (25), this correction can be approximated as follows:

$$\delta M_{N}^{2\text{-}\text{loop}} \simeq \frac{2}{(16\pi^2)^2} \left( M_{N}Y_{\nu}^\dagger Y_{e}^\dagger Y_{e} + Y_{\nu}^T Y_{e}^T Y_{e}^* Y_{\nu} M_{N} \right) \mu = M_{G} \frac{R_{\ell} \ln \frac{M_{G}}{M}}{M}. \quad (29)$$

where we have suitably absorbed CP conserving and flavor universal corrections (coming from the entries $\text{Tr}(Y_{\nu}^\dagger Y_{\nu}), g_{i}, \lambda_{i}^2$ etc.) in the overall scale $M$. The RG factor $R_{\ell}$ ($\ell_i = (e, \mu, \tau)$) is for the running of $Y_e$ Yukawa couplings and can strongly deviate from one. It is defined as:

$$R_{e,\mu,\tau} = \int_{M}^{M_{G}} \frac{\lambda_{e,\mu,\tau}^2(t)dt}{\lambda_{e,\mu,\tau}^2(M_{G}) \ln \frac{M_{G}}{M}}. \quad (30)$$

In the approximation (29), the fourth powers of $Y_{\nu}$ have been neglected. Actually, for calculating the mass splitting $M_2^2 - M_1^2$ - the combination appearing in (24) - it is enough to keep only correction $\delta M_{N}^{1\text{-}\text{loop}}$ of (26). However, to deal with the CP violating effect, we need to include also two–loop effects. Thus, at the scale $\mu = M$ for $M_{N}$ we use

$$M_{N}(M) = M_{N}(M_{G}) + \delta M_{N}^{1\text{-}\text{loop}} + \delta M_{N}^{2\text{-}\text{loop}}, \quad (31)$$

with $M_{N}(M_{G}), \delta M_{N}^{1\text{-}\text{loop}}$ and $\delta M_{N}^{2\text{-}\text{loop}}$ given by Eqs. (5), (26) and (29) respectively. This completes the calculation of supersymmetric part, which will be useful for calculation of leptogenesis via fermionic RHN decays. However, inclusion of soft SUSY breaking terms, in general, may affect the leptogenesis induced through the right–handed sneutrino decays. In an Appendix we studied this case separately and shown that under plausible assumptions the right–handed sneutrino decays practically do not contribute to the net baryon asymmetry. Thus, we should rely on the fermionic RHN decays which, as we show below, generate sufficient baryon asymmetry.

### 3.1 Asymmetry Via Fermionic RHN Decays

**Leptogenesis for Normal Hierarchical Case**

For this case we will take the form of $Y_{\nu}$ given by Eq. (6). For leptogenesis study, it is convenient to parameterize this Yukawa matrix as follows:

$$\text{Texture A : } \quad Y_{\nu} = \begin{pmatrix} x\alpha_1 & 0 \\ x\alpha_2 & b \\ xe^{i\phi} & 1 \end{pmatrix} \cdot \bar{\beta}, \quad (32)$$

where the couplings $\alpha_{1,2}, b, \bar{\beta}$ and $x$ are real parameters. Only single phase $\phi$ appears. This has been achieved by suitable redefinition of phases of $l_{1,2,3}$ and $N_{1,2}$ superfields. First we will relate the parameters appearing in $Y_{\nu}$ to some observables. The relation (10) enables us to express $\alpha_1, \alpha_2, b$...
and $\bar{\beta}$ in terms of $x$, neutrino mass, the scale $|M|$ and lepton mixing matrix elements. Also $\phi$ can be determined by the phase $\delta$ and the leptonic mixing angles. Doing so, we arrive to the following relations

$$
\alpha_1 = 2 \left| \frac{A_2}{A_1} \right|, \quad \alpha_2 = \left| \frac{A_2 A_1}{A_1 A_3} \right|, \quad b = \frac{A_3}{A_2}, \quad \bar{\beta} = \frac{1}{v \sin \beta} \left( \frac{m_3}{2} \left| \frac{A_1 M}{x} \right| \right)^{1/2},
$$

$$
\phi = \text{Arg} \left( \frac{A_2 A_4}{A_1 A_3} \right),
$$

where

$$
\mathcal{A}_1 = U_{\tau 3}^2 - U_{\tau 2}^2 \frac{U_{e 3}^2}{U_{e 2}^2}, \quad \mathcal{A}_2 = U_{e 3} U_{\tau 3} - U_{e 2} \frac{U_{e 3}}{U_{e 2}}, \quad \mathcal{A}_3 = U_{e 3} U_{\mu 3} - U_{\mu 2} \frac{U_{e 3}}{U_{e 2}}, \quad \mathcal{A}_4 = U_{\mu 3}^2 - U_{\mu 2}^2 \frac{U_{e 3}^2}{U_{e 2}^2}.
$$

These will be useful upon studying the leptogenesis. As we have already mentioned, remarkable thing is the fact that there is a single CP violating phase $\phi$ which is related to the phase $\delta$ controlling the CP violation in the neutrino oscillations. The same phase will appear in the CP asymmetry of the resonant leptogenesis. In Fig. 1 we show correlation between $\phi$ and $\delta$.

Furthermore, applying expressions (26), (29), for the splitting parameter $\delta_N$ of (27) we obtain

$$
\delta_N \simeq \left( b \alpha_2 + e^{i \phi} - \frac{\lambda_3^2 R_\tau}{16 \pi^2} e^{i \phi} \right) \frac{x \beta^2}{4 \pi^2} \ln \frac{M_G}{M},
$$

where we have ignored the couplings $\lambda_e$ and $\lambda_\mu$ because the main effect is obtained by the tau Yukawa coupling. In Eq. (36), the coupling $\lambda_\tau$ is defined at $M_Z$ scale, and therefore the quantity $R_\tau$ accounts for the renormalization effects mostly due to $\lambda_\tau$, running, and is given in Eq. (30). Now we can give the unitary matrix $U_N$ diagonalizing $M_N$ (by the transformation $U_N^T M_N U_N = M_N^\text{diag}$):

$$
U_N \simeq \frac{1}{\sqrt{2}} \begin{pmatrix} e^{-i \eta/2} & -ie^{-i \eta/2} \\ e^{i \eta/2} & ie^{i \eta/2} \end{pmatrix},
$$

Figure 1: Correlation between $\phi$ and $\delta$. Left side: normal hierarchical case (texture A). Right side: inverted hierarchical case (texture $B_1$). The vertical lines, for right panel, correspond to the maximal allowed value of $|\delta| = 0.96$. 
where
\[ \eta = \text{Arg} \left( b\alpha_2 + e^{i\phi} - \frac{\lambda^2 R_\tau}{16\pi^2} e^{i\phi} \right). \] (38)

At the same time we have
\[ (Y^\dagger_\nu Y_\nu)_{21} = \beta^2 x \left| b\alpha_2 + e^{i\phi} \right| e^{i\eta'}, \quad \text{with} \quad \eta' = \text{Arg} \left( b \alpha_2 + e^{i\phi} \right). \] (39)

Therefore, the complex phase appearing in \((Y^\dagger_\nu Y_\nu)_{21}\) will be proportional to the mismatch \(\eta - \eta'\), which using (38) and (39) takes the form
\[ \eta - \eta' \simeq -\lambda^2 R_\tau \frac{b\alpha_2}{16\pi^2} \left| b\alpha_2 + e^{i\phi} \right|^2 \sin \phi. \] (40)

Note once again that the phase \(\eta - \eta'\), determining the lepton asymmetry, is proportional to \(\sin \phi\), which itself is related to the phase \(\delta\) of the lepton mixing matrix. The model gives the relation between them by Eq. (34). Also, it is rather impressive that other parameters, \(b\) and \(\alpha_2\), appearing in (40) can be calculated by the lepton mixing matrix elements through the relations (33), (35).

The masses of two right handed neutrinos are
\[ M_1 = |M|(1 - \kappa), \quad M_2 = |M|(1 + \kappa), \quad \text{with} \quad \kappa \simeq \left| x(b\alpha_2 + e^{i\phi}) \right| \frac{\beta^2}{4\pi^2} \ln \frac{M_G}{M}. \] (41)

Here, the unknown parameter \(x\) appears which is free and can be varied upon numerical calculations. Finally, we give expressions built from the elements of the matrix \((Y^\dagger_\nu Y_\nu)\) appearing in the expressions of the CP asymmetries of Eq. (24). These are:
\[ (Y^\dagger_\nu Y_\nu)_{11} \simeq \frac{\beta^2}{2} \left( x^2(1 + 2\alpha_1^2 + 2\alpha_2^2) + 1 + b^2 + 2x|b\alpha_2 + e^{i\phi}| \right). \]
In order to compute generated baryon asymmetry of the Universe, recall that the lepton asymmetry is converted to the baryon asymmetry via sphaleron processes \[5\] and is given by \(\frac{n_B}{s} \simeq -1.48 \times 10^{-3}(\kappa_f^{(1)} \epsilon_1 + \kappa_f^{(2)} \epsilon_2)\), where the efficiency factors \(\kappa_f^{(1,2)}\) are given by the extrapolating formula \[6\]:

\[
\kappa_f^{(1,2)} = \left( \frac{3.3 \times 10^{-3} \text{eV}}{\tilde{m}_{1,2}} + \left( \frac{\tilde{m}_{1,2}}{0.55 \times 10^{-3} \text{eV}} \right)^{1.16} \right)^{-1},
\]

with \(\tilde{m}_1 = \frac{(v \sin \beta)^2}{M_1} (\hat{Y}_\nu^\dagger \hat{Y}_\nu)_{11}\) and \(\tilde{m}_2 = \frac{(v \sin \beta)^2}{M_2} (\hat{Y}_\nu^\dagger \hat{Y}_\nu)_{22}\).

Collecting all this, we can now calculate \(\frac{n_B}{s}\). One can try the different values of \(M\) in a mass range which would not cause the gravitino problem. We can also try different values of the phase \(\delta\), relevant also for the CP violation into the neutrino oscillations. As we have already mentioned, there is one more free parameter \(x\), which we will vary. It is quite interesting that this system, by requiring to have baryon asymmetry in the range of the observed amount \((\frac{n_B}{s})_{\exp} = (8.75 \pm 0.23) \times 10^{-11}\), dictates the preferred range for the MSSM parameter \(\tan \beta\). The reason for this is simple. The strength of the Yukawa coupling \(\lambda_\tau\), determining the amount of the CP violation \[see Eq. (40)\], depends on the value of \(\tan \beta\): \(\lambda_\tau = \frac{m_\tau}{v} \sqrt{1 + \frac{t^2}{\beta}}\). By simple but quite complete numerical simulation we obtain, in this model, the low bound on the \(\tan \beta\). Upon calculations we take into account the renormalization effects. Namely, the running of \(\lambda_\tau\). Obtained low bound for \(\tan \beta\) is: \(\tan \beta \gtrsim 12\) (corresponds to \(|\delta| \simeq 1.3\) and \(M = 10^7 \text{ GeV}, R_\tau = 0.617\)). Smaller values of \(\tan \beta\) do not give sufficient baryon asymmetry. This also indicates that the non SUSY version (i.e. SM augmented by two RHNs) of this scenario will fail to generate baryon asymmetry through the leptogenesis. The presented scenario also allows to derive the low bound for the absolute value of the phase \(\delta\). This comes out from the maximal allowed value of \(\tan \beta \lesssim 58\) (from the requirement that \(\lambda_{b,\tau} \lesssim 1\) all the way up to the GUT scale). With \(\tan \beta = 58\), \(M = 10^7 \text{ GeV}\) \(\left( R_\tau = 2.17\right)\) in order to have needed baryon asymmetry we should have \(|\delta| \gtrsim 0.012\). It is interesting to note that for \(\tan \beta \lesssim 35\), for generating the baryon asymmetry we need \(|\delta| \gtrsim 0.1\). This limit for the CP violating phase is within the reach of future experiments. In Figs. 2 and 3 we plot \(\frac{n_B}{s}\) for different choices of the model parameters.

**Leptogenesis for Inverted Hierarchical Case**

Now we study the leptogenesis for the inverted hierarchical case. We note right away that the scenario with texture \(B_2\) of (7) does not work for the leptogenesis. The reason is following. Due to the zero in (3, 2) entry of this texture, it is easy to see from Eq. (29) that the \(\lambda_\tau\) coupling do not contribute to the CP asymmetry induced at 2-loop level. The couplings \(\lambda_e\) and \(\lambda_\mu\) do contribute, however they are small and can not induce needed asymmetry.

Thus, we focus here only on case with texture \(B_1\). For this case, it is convenient to write \(Y_\nu\) with the parameterization

\[
\text{Texture } B_1: \quad Y_\nu = \begin{pmatrix} x_1 & b \\ x_2 & 0 \\ x e^{i\phi} & 1 \end{pmatrix},
\]
where, as in case of texture A, by suitable phase redefinition of \( l_{1,2,3}, N_{1,2} \) superfields we left with only single phase \( \phi \). Remaining parameters are real. First we will express the model parameters \( \alpha_1, b, \bar{\beta} \) in terms of matrix elements of \( U \), neutrino mass \( m_2 \), the \( M \), and \( x \). By solving the equations derived from the relation (10) we obtain

\[
\alpha_1 = \left| \frac{B_2}{B_1} \right|, \quad \alpha_2 = 2 \left| \frac{B_3}{B_2} \right|, \quad b = \left| \frac{B_3}{B_2} \right|, \quad \bar{\beta} = \frac{1}{\sin \beta} \left( \frac{m_2}{2} \frac{B_1 M}{x} \right)^{1/2},
\]

\[
\phi = \arg \left( \frac{B_2^2 B_4}{B_1 B_3} \right),
\] (45)

where

\[
B_1 = U_{\tau 2} - U_{\tau 1} \frac{U_{\mu 2}^2}{U_{\mu 1}^2}, \quad B_2 = U_{\mu 2} U_{\tau 2} - U_{\tau 1} \frac{U_{\mu 2}^2}{U_{\mu 1}^2}, \quad B_3 = U e_2 U_{\mu 2} - U e_1 \frac{U_{\mu 2}^2}{U_{\mu 1}^2}, \quad B_4 = U e_2 - U e_1 \frac{U_{\mu 2}^2}{U_{\mu 1}^2}. \quad (46)
\]

As we see, also in this case the phase \( \phi \) is related to the \( \delta \)-phase of the leptonic mixing matrix \( U \) (see Eq. (11)). In particular using the relation (17) in (45) for \( \phi \) and performing simple algebra we derive

\[
\phi \simeq \arg \left( \frac{\Delta m^2_{\text{sol}}}{\Delta m^2_{\text{atm}}} e^{i\delta} - 1 \right) \quad \Rightarrow \quad \phi \simeq \pi - \frac{\Delta m^2_{\text{sol}}}{\Delta m^2_{\text{atm}}} \frac{\cot \theta_{23}}{4 \sin \theta_{13}} \sin \delta. \quad (47)
\]

Since the phase \( \phi \) will appear in the leptonic CP asymmetry, with relation (47) we will be able to make calculations in terms of measured neutrino oscillation parameters and the CP phase \( \delta \). In Fig. 1 the correlation between \( \phi \) and \( \delta \) is shown.

Now we are ready to investigate the leptogenesis for the inverted hierarchical scenario \((B_1)\). The way of calculation is same as was presented in the previous subsection, so we will keep discussion...
short and give only several expressions and final results. Using the expressions of Eqs. (26)-(31) and the form of the texture $B_1$ in (44), for the phase mismatch (in analogy of Eq. (40) we obtain

$$ (\eta - \eta')^{(B_1)} \simeq -\frac{\lambda_2^2 R_\tau}{16 \pi^2} \frac{b \alpha_1}{|b \alpha_1 + e^{i \phi}|^2} \sin \phi ,$$

(48)

where here and below we will use superscript ‘$B_1$’ in order to distinguish expressions corresponding to the scenario $B_1$ from those of the texture $A$. Moreover, for the splitting parameter (in analog to Eq. (41)) we have

$$ \kappa^{(B_1)} \simeq |x(b \alpha_1 + e^{i \phi})| \frac{\beta^2}{4 \pi^2} \ln \frac{M_G}{M} .$$

(49)

We will also give the expression for Im($\hat{Y}^\dagger_\nu \hat{Y}_\nu$)$_{21}^2$ which will help to understand some physics. We have

$$ \left( \text{Im} (\hat{Y}^\dagger_\nu \hat{Y}_\nu)_{21}^2 \right)^{(B_1)} \simeq \frac{\lambda_2^2 R_\tau}{16 \pi^2} \beta^4 x b \alpha_1 \left( x^2 (1 + \alpha_1^2 + \alpha_2^2) - 1 - b^2 \right) \frac{\sin \phi}{|b \alpha_1 + e^{i \phi}|} .$$

(50)

Note, that according to (47) the phase $\phi$ is close to $\pi$ and one may suspect that also final result for the CP violation should be suppressed by the factor $\sim \Delta m_{\text{sol}}^2/((\Delta m_{\text{atm}}^2)|4 \sin \theta_{13}) \approx 1/20$. However, such suppression do not takes place because the combination $|b \alpha_1 + e^{i \phi}|$, appearing in the denominator of the last multiplier of (50), is suppressed by precisely same factor! Indeed, using the relations of Eqs. (17), (45)-(47) we derive

$$ |b \alpha_1 + e^{i \phi}| \simeq \frac{\Delta m_{\text{sol}}^2}{\Delta m_{\text{atm}}^2} \cot \theta_{23} .$$

(51)

With these for the combination appearing in (50) we get

$$ \frac{\sin \phi}{|b \alpha_1 + e^{i \phi}|} \simeq \sin \delta ,$$

(52)
showing that suppression factors mentioned above drop out and it is maximized with $|\delta| \simeq 1.115$ (maximal allowed value which is acceptable for viable neutrino sector). Moreover, because of the suppression of the combination $|b_{\alpha 1} + e^{i\phi}|$, also the RHN mass splitting parameter in (49) gets additional suppression, which makes two RHNs more degenerate. This also gives some enhancement of the (resonant) CP asymmetry factors $\epsilon_{1,2}$.

Without bothering to give other expressions, we will move to the presentation of the main results. In this scenario, from the requirement of needed baryon asymmetry, the $\tan\beta$ is bounded from below. Interestingly thing is that the leptogenesis dictates $\tan\beta \gtrsim 21$ (lower values do not give sufficient baryon asymmetry) For obtaining this low bound we have taken $M = 10^4$ GeV ($R_\tau \simeq 0.71$) and maximal allowed value for the $\delta \simeq 0.96$. Note that within this scenario low values of $M$ give larger lepton asymmetries. It is also possible to derive low bound for $|\delta|$. This is obtained by largest allowed (from requirement $\lambda_{\beta\tau} \lesssim 1$ up to the GUT scale) value of $\tan\beta$. Namely, with $\tan\beta \simeq 58, M = 10^4$ GeV (we have for these choices $R_\tau \simeq 1.95$), needed baryon asymmetry can be generated with $|\delta| \gtrsim 0.021$ (note that for $|\delta| = 0.021$, for acceptable solar mixing angle we should choose $\sin^2\theta_{23} \simeq 0.6$ and $\sin^2\theta_{13} \simeq 0.04$). Worthwhile for noting that for $\tan\beta \lesssim 45$ for generating sufficient baryon asymmetry we need $|\delta| \gtrsim 0.1$. The latter value is within the reach of planned experiments. We have performed numerical calculations without approximations and made sure that our analytical expressions, presented above, are good approximations. In Figs. 4 and 5 we show baryon asymmetries for several different choices of the model parameters.

### 4 Summary

In this paper we have considered an extension of MSSM with two quasi-degenerate right–handed neutrinos. Our motivation was to realize resonant leptogenesis which avoids the gravitino problem.
generic for low scale SUSY scenarios. With this setup we have classified all viable texture zeros of the neutrino Dirac Yukawa matrices which lead to consistent predictions. We find three predictive scenarios, each with one texture zero. One model has normal hierarchical neutrino mass spectrum, while the remaining two have inverted hierarchical mass pattern. The predictive power of these models show up also in the resonant leptogenesis. The model with the normal mass hierarchy (texture $A$) and one of the inverted hierarchical scenario (with texture $B_1$) lead to the successful leptogenesis. In Appendix we have discussed the impact of the soft SUSY breaking terms on the CP asymmetry generated by RH sneutrino decays and concluded that with natural choice of the soft SUSY breaking terms, scalar RH neutrinos do not contribute sizably to the total baryon asymmetry. Thus, the baryon asymmetry is due to fermionic RHN decays and the leptonic CP phase is directly related to to CP violation in neutrino oscillation. Putting together the predictions from the neutrino sector and the results from leptogenesis calculations, we have obtained the following predictions:

For normal hierarchical case (texture $A$)

$$\sin^2 \theta_{13} \gtrsim 0.05 , \quad |\delta| \gtrsim 0.012 , \quad m_{\beta\beta} = 0 , \quad \tan \beta \gtrsim 12 ;$$

with \( \tan \beta \lesssim 35 , \quad |\delta| \gtrsim 0.1 \).

For the inverted hierarchical case corresponding to texture $B_1$:

$$\theta_{13} \gtrsim 0.12 , \quad 0.021 \lesssim |\delta| \lesssim 0.96 , \quad 0.013 \text{ eV} \lesssim m_{\beta\beta} \lesssim 0.023 \text{ eV} , \quad \tan \beta \gtrsim 21 ;$$

with \( \tan \beta \lesssim 45 , \quad |\delta| \gtrsim 0.1 \).

The texture $B_2$ do not generate the baryon asymmetry within this scenario and other mechanism need to be invoked [23]. However, from the viewpoint of the neutrino sector the texture $B_2$ is viable and gives:

$$\theta_{13} \gtrsim 0.129 , \quad |\pi - \delta| \lesssim 0.91 , \quad 0.013 \text{ eV} \lesssim m_{\beta\beta} \lesssim 0.023 \text{ eV} .$$

Future experiments will examine the viability of these scenarios.

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Appendix: Asymmetry Via $\tilde{N}$ Decays

In this appendix we will discuss the contribution to the net baryon asymmetry from the out of equilibrium resonant decays of the right handed sneutrinos (RHS). With inclusion of the soft SUSY breaking terms, the RHS mass spectrum and couplings will be altered and one should expect result different from that corresponding to the fermionic RHN decays. Besides soft SUSY breaking couplings, there are other particularities, highlighted below, which distinguish cases of RHN and RHS decays. We are considering the system with two RHN superfields $N_{1,2}$ which have two complex scalar components $\tilde{N}_{1,2}$. With SUSY breaking term, the masses of RHS’s will differ from their fermionic partners’ masses. Thus we will have four real mass-eigenstate RHS’s $\tilde{n}_{i=1,2,3,4}$
with masses $\tilde{M}_{i=1,2,3,4}$ respectively. Assuming that the SUSY scale is smaller (at least by factor of 10) than the scale $M$ (the overall tree level mass for the RHN superfields) we expect that the states $\tilde{n}_i$ remain quasi-degenerate. To study the resonant $\tilde{n}$-decays we will apply ressumed effective amplitude technic [12]. An effective amplitudes for the real $\tilde{n}_i$ decay, say into the lepton $l_\alpha$ ($\alpha = 1, 2, 3$ is a generation index) and antilepton $\bar{l}_\alpha$ respectively are given by [12]

$$
\hat{S}_{\alpha i} = S_{\alpha i} - \sum_j S_{\alpha j} \frac{\Pi_{ji}(\tilde{M}_i)(1 - \delta_{ij})}{\tilde{M}_i^2 - M_j^2 + \Pi_{ji}(\tilde{M}_i)} , \quad \hat{S}_{\alpha i}^* = S_{\alpha i}^* - \sum_j S_{\alpha j}^* \frac{\Pi_{ji}(\tilde{M}_i)(1 - \delta_{ij})}{\tilde{M}_i^2 - M_j^2 + \Pi_{ji}(\tilde{M}_i)} ,
$$

(53)

where $S_{\alpha i}$ is a tree level amplitude and $\Pi_{ij}$ is a two point Green function’s (polarization operator of $\tilde{n}_i - \tilde{n}_j$) absorptive part. The CP asymmetry is then given by

$$
\epsilon_i^{sc} = \frac{\sum_\alpha (|\hat{S}_{\alpha i}|^2 - |\hat{S}_{\alpha i}^*|^2)}{\sum_\alpha (|\hat{S}_{\alpha i}|^2 + |\hat{S}_{\alpha i}^*|^2)} .
$$

(54)

We will apply (53) and (54) for our scenario, however, also derive general expressions applicable for different models.

Together with superpotential couplings (3) we include the following soft SUSY breaking terms

$$
V_{SB}^\nu = \tilde{I}A_\nu \tilde{N}h_u - \frac{1}{2} \tilde{N}^T B_N \tilde{N} + \text{h.c.}
$$

(55)

We do not display here soft mass$^2$ terms, such as $\tilde{n}_{1,2}^2, [\tilde{N}_{1,2}]^2$, because $B_N$ plays much more significant role in the splitting of RHS masses. For simplicity we will assume at GUT scale ($M_G$) the ‘proportionality’ $A_\nu \propto Y_\nu$ and degeneracy in $B_N \propto M_N$. Thus,

$$
\text{at } \mu = M_G : \quad A_\nu = m_A Y_\nu , \quad B_N = m_B M_N .
$$

(56)

Similarly, for the charged lepton sector we can assume $A_\epsilon = m_E Y_\epsilon$. Performing RG studies, similar way as we have done in section 3, we will have

$$
\text{at } \mu = M : \quad B_N \simeq m_B M \left( -\frac{1}{\tilde{M}_N} \frac{1}{m_B} \delta_N^4 \right) .
$$

(57)

Note that with $A_\nu = m_A Y_\nu$ and $A_\epsilon = m_E Y_\epsilon$ at high scale, the $A_\nu$ will remain well aligned with $Y_\nu$ also at low scales. With diagonalization of total mass matrix of the RHS’s, for mass-eigenstate ($\tilde{n}_i$) masses we get

$$
\tilde{M}_1^2 = |M|^2 (1 - |\delta_N|^2) - |M||m_B - (m_B + 2m_A)|\delta_N| ,
\tilde{M}_2^2 = |M|^2 (1 - |\delta_N|^2) + |M||m_B - (m_B + 2m_A)|\delta_N| ,
\tilde{M}_3^2 = |M|^2 (1 + |\delta_N|^2) - |M||m_B + (m_B + 2m_A)|\delta_N| ,
\tilde{M}_4^2 = |M|^2 (1 + |\delta_N|^2) + |M||m_B + (m_B + 2m_A)|\delta_N| .
$$

(58)

Interaction of $\tilde{n}$ states with leptons and sleptons has the form

$$
\tilde{h}_u Y_\nu \tilde{n} + \tilde{h}_u \tilde{Y}_B \tilde{n} + \text{h.c.}
$$

(59)
where
\[ Y_F = Y_\nu \bar{V} \, , \quad Y_B = Y_\nu M_N^* \bar{V} + A_\nu \bar{V} \]where \( \bar{V} = U_N \left( \rho_u e^{i \bar{\theta}} , \rho_d \right) \),
with \( \rho_u = \frac{1}{\sqrt{2}} \left( \begin{array}{c} 1 \\ 0 \\ 0 \end{array} \right) \), \( \rho_d = \frac{1}{\sqrt{2}} \left( \begin{array}{c} 0 \\ 0 \\ 1 \end{array} \right) \), \( \bar{\theta} \approx 2 \text{Im} \left( \frac{m_A}{m_B} \right) |\delta_N| \) .

With these we can calculate the absorptive part of the polarization diagram with external legs \( \bar{n}_i \) and \( \bar{n}_j \). At 1-loop level it is given by

\[ \Pi_{ij}(p) = \frac{i}{8\pi} \left( p^2 Y^\dagger_F Y_F + p^2 Y^T_F Y_F^* + Y^\dagger_B Y_B + Y^T_B Y_B^* \right)_{ij} \]where \( p \) denotes external momentum in the diagram.

Now we are ready to calculate the lepton asymmetry. Note that in unbroken SUSY limit, neglecting finite temperature effects \((T \to 0)\), the \( \bar{N} \) decay does not produce lepton asymmetry. The reason for this is following. The decay of \( \bar{N} \) in two fermion is \( \bar{N} \to l \bar{h}_u \), while in two scalars is \( \bar{N} \to l^* h^*_u \). Since the rates of these processes are same due to SUSY \((T = 0)\), the lepton asymmetries created from these decays cancel each other. However, with \( T \neq 0 \) the cancelation is partial and one has

\[ \bar{\epsilon}_i = \epsilon_i (\bar{n}_i \to l \bar{h}_u) \Delta_{BF} \]
with temperature dependent factor \( \Delta_{BF} \) given in [22]. We note that Eq. (62) is valid when we have the alignment \( A_\nu = m_A Y_\nu \). Without this alignment other terms in r.h.s of (62) proportional to \( m_A/M \) will appear. Since we are assuming the alignment and \( m_A/M \prec 0.1 \), the SUSY breaking effects would not affect decay amplitudes significantly and we can apply (62) for our study. Thus, we just need to compute \( \epsilon_i (\bar{n}_i \to l \bar{h}_u) \) - the asymmetry created by \( \bar{n}_i \) decays in two fermions. Using in (53) \( S_{ai} = (Y_F)_{ai} \), with (54) after straightforward calculation we obtain

\[ \epsilon_i (\bar{n}_i \to l \bar{h}_u) \simeq \frac{1}{(Y^\dagger_F Y_F)_{ii}} \left\{ 2 \sum_j \frac{(M_{ij}^2 - \bar{M}_{ij}^2) \text{Im}(\Pi_{jj})}{(M_j^2 - \bar{M}_j^2)^2 + |\Pi_{jj}|^2} \text{Im}(Y^\dagger_F Y_F)_{ji} + \right. \]

\[ \left. \sum_{j,k} \text{Im}(Y^\dagger_F Y_F)_{kj} \left( \frac{M_j^2 - \bar{M}_j^2) \text{Im}(\Pi_{kk}) - (M_k^2 - \bar{M}_k^2) \text{Im}(\Pi_{jj})}{(M_j^2 - \bar{M}_j^2)^2 + |\Pi_{jj}|^2} \right) \right\} \Pi_{ji} \Pi_{ki} . \]

In (63) for the absorptive part \( \Pi \) we should use (61) with \( p = \bar{M}_i \). Now, the baryon asymmetry created from the lepton asymmetry due to \( \bar{n}_i \) decays is:

\[ \frac{\bar{n}_B}{s} \simeq -8.46 \cdot 10^{-4} \sum_{i=1}^{4} \bar{\epsilon}_i \Delta_{BF} \eta_i = -8.46 \cdot 10^{-4} \sum_{i=1}^{4} \epsilon_i (\bar{n}_i \to l \bar{h}_u) \eta_i , \]

where we have taken into account that an effective number of degrees of freedom, including two RHN superfields, is \( g_s = 228.75 \). \( \eta_i \) are an efficiency factors which depend on \( \bar{m}_i \simeq \frac{(v \sin \beta)^2}{M} 2(Y^\dagger_F Y_F)_{ii} \), and take into account temperature effects by integrating the Boltzmann equations [22]. Before discussing this in more details, it is more instructive to see what are the effects of the soft SUSY breaking terms in the CP asymmetry given by Eq. (63). The parameter \( \epsilon_i \) is controlled by the imaginary parts of the elements of the matrix \( Y^\dagger_F Y_F \). First note that the phase \( \bar{\theta} \) appearing in this
matrix (see Eq. (60)) for $M \lesssim 10^7$ GeV is $\bar{\theta} \lesssim 10^{-10}$ and can be safely ignored. With this, the matrix $Y^\dagger_F Y_F$ has the form

$$Y^\dagger_F Y_F = \begin{pmatrix}
\hat{\sigma}(\bar{Y}_\nu^\nu Y_\nu^\nu)_{11} & \hat{\sigma}(\bar{Y}_\nu^\nu Y_\nu^\nu)_{12} \\
\hat{\sigma}(\bar{Y}_\nu^\nu Y_\nu^\nu)_{21} & \hat{\sigma}(\bar{Y}_\nu^\nu Y_\nu^\nu)_{22}
\end{pmatrix}, \quad \text{with} \quad \hat{\sigma} = \frac{1}{2} \begin{pmatrix} 1 & i \\ -i & 1 \end{pmatrix},$$

where $\bar{Y}_\nu = Y_\nu U_N$ is the same matrix appearing in the CP asymmetries (24) induced by fermionic RHN decays. Note that the matrix $\hat{\sigma}$ has purely imaginary entries and they can be new sources for the CP violation. For instance, the element $(Y^\dagger_F Y_F)_{12}$ has the large phase. This means that there happens the ‘conversion’ between $\bar{n}_1$ and $\bar{n}_2$ states. On the other hand, from Eq. (58) one can see that the degeneracy of $\bar{M}_1$ and $\bar{M}_2$ is split by the $B$-term and unless $m_B \lesssim 10$ MeV the resonant enhancement does not happen (similar to the case of soft leptogenesis [22]). Since the natural value of $m_B$ is from few×100 GeV to few TeV, we conclude that this channel does not give important contribution to the CP asymmetry. For those states amongst which degeneracy is not ruined (the ‘pairs’ $\bar{n}_1 - \bar{n}_3$ and $\bar{n}_2 - \bar{n}_4$) by the $B$-terms, the CP asymmetry is controlled not by imaginary components of $\hat{\sigma}$ but by Im$(\bar{Y}_\nu^\nu Y_\nu^\nu)_{12}$ (like to those corresponding to the fermionic RHN decays, Eq. (24)). Thus, the CP asymmetry via $\bar{n}_i$ decays would not be larger than asymmetry generated due to their fermionic partners. Moreover, due to the efficiency factors $\eta_i$, the $\bar{n}_B/s$ turns out to get additional suppression in comparison to the $\bar{n}_B/s$ (the total baryon asymmetry due fermionic RHNs). We have checked this on two examples corresponding to the textures of $A$ and $B_1$. Namely, we have performed calculations for $(m_A, m_B) = (10^3 i, 10^3)$ GeV and for several choice of model parameters (tan $\beta, M, \delta$). For a given set of these parameters, for fixed $x$ we can calculate the values of the masses $\bar{m}_i = (\frac{\sin \beta}{M})^2 (2(Y^\dagger_F Y_F)_{ii})$. With given values of $\bar{m}_i$, according to Ref. [22] we picked up the corresponding values of $\eta_i$ and with help of Eqs. (63), (64) calculated $\bar{n}_B/s$. For the texture $A$ we obtained $\frac{\bar{n}_B}{\bar{n}_B} < 4 \cdot 10^{-3}$, while for the texture $B_1$: $\frac{\bar{n}_B}{\bar{n}_B} < 10^{-2}$. These confirm that the baryon asymmetry via $\bar{n}$ decays is a negligible effect. For completeness we also examined the case corresponding to texture $B_2$. The latter does not give relevant asymmetry also through $\bar{n}$ decays.

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