The reliability model of the fault-tolerant border routing with two Internet services providers in the enterprise computer network

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Abstract. This paper deals with the reliability model of the fault-tolerant border routing with two Internet Services Providers in the enterprise computer network. Developed by authors, the generalized mathematical model of the fault-tolerant routing with two Internet providers and derivation of the calculation formula for the availability factor are presented. The calculation examples of the availability factor for the different cases with the specific routing schemes, which are used in modern border routing, are also provided. Calculation results show more realistic values of the availability factor for fault-tolerant border routing with two Internet Services Providers, rather than values, which can be obtained by well-known simplified formula for the duplex system in reliability theory.

1. Introduction
In the last three decades, world saw a rapid development of the information technologies and their application in various spheres of human activities. Computer networks [1, 2] became an essential part of the modern telecommunication systems, without which the data exchange is impossible in most cases. A special place in modern world is assigned to the worldwide area network - Internet, which became a key part of the human life and enterprise business processes. In this situation, availability of the Internet resources takes a special place, and therefore modern IT professionals implement different fault-tolerant schemes to border routing [3, 4] of the enterprise computer network with two or more Internet Services Providers (ISP). Therefore, development of the reliability model for the fault-tolerant routing with two or more Internet Services Providers is an urgent scientific task.

However, we must mention the fact that most academic books [5, 6, 7, 8], dedicated to reliability theory, are deeply concerned with the theoretical and mathematical aspects, but they do not consider the specific issues of modern computer network topologies, routing protocols and fault-tolerant border routing schemes. In this situation, a wide gap between the simplified theoretical reliability models and real picture of the fault-tolerant border routing is observed. Therefore, in the last several years, a set of scientific research works in the field of reliability of the modern data storing, processing and exchanging systems was provided by the authors [9, 10]. Finally, on basis of the practical experience in the field of implementation of the fault-tolerant routing for enterprise computer networks, a
scientific task of development of the realistic mathematical model for the fault-tolerant routing with two Internet Services Providers was raised and solved by authors.

2. The reliability model of the fault-tolerant border routing with two Internet providers

In first, let assume that we have an enterprise network with two Internet services providers (fig. 1). An enterprise network may use different routing schemes to provide fault-tolerance using at each time moment only one of two ISP in accordance with the primary / backup scheme with priorities (to make one of the ISP more preferable than the other one) or without prioritization, or using both ISPs simultaneously with traffic-balancing between them.

![Diagram of the enterprise network with two Internet services providers](image)

**Figure 1.** The enterprise network with two Internet services providers.

Next, we should take in consideration the finite time of switching between the providers in fault-tolerant schemes. So, let assume that each ISP can stay in one of three states from the viewpoint of the reliability model, based on well-known Markov chain representation (fig. 2):

- Passive state – ISP is available, but it is passive. ISP does not provide the Internet traffic exchange because of the initialization or reconfiguration of the routing protocols after startup, failure or restore of one or both ISPs, or, in accordance with the specific routing scheme, configured for enterprise, especially in the schemes with primary and backup ISPs. From this state, ISP can pass either to the unavailable state with failure rate \( \lambda \), or to the active one with activation rate \( \gamma \), if the ISP is assigned to become active by the routing protocol.

- Active state – ISP is available and active. ISP provides the Internet traffic exchange for an enterprise. From this state, ISP can pass to the unavailable state with failure rate \( \lambda \).

- Unavailable state – ISP is unavailable for an enterprise because of some kind of failure (we will not go deep to the details of various types of failures and just assume some «abstract» failure with some given rate, the value for which is obtained by IT professionals from the practical monitoring of the specific ISP in the specific enterprise network). From this state, ISP can pass to the passive-state with recovery rate \( \mu \).

![Diagram of the reliability model for the border routing with the stand-alone Internet services provider](image)

**Figure 2.** The reliability model for the border routing with the stand-alone Internet services provider.

The system with the two ISP produces a more complex model, as each of the ISPs can be in one of three states with different kinds of dependencies between them in different fault-tolerant routing scenarios. Moreover, the base three reliability parameters, which are observed above, can be different for each ISP: \( \lambda_1, \mu_1, \gamma_1 \) for the first ISP and \( \lambda_2, \mu_2, \gamma_2 \) for the second ISP.

Let us introduce the next state-space for the reliability model with two ISPs:

- State 0 – both of the ISPs are available, but both are passive from the viewpoint of the enterprise network, as the initialization or reconfiguration of the routing protocol is not complete yet.
• State 1 – the first ISP is unavailable, the second ISP is available, but passive, and the routing protocol is preparing to activate it as a primary ISP for the traffic exchange.
• State 2 – the second ISP is unavailable, the first ISP is available, but passive, and the routing protocol is preparing to activate it as a primary ISP for the traffic exchange.
• State 3 – the first ISP is unavailable, the second ISP is available and active, and the enterprise network uses it as a primary ISP for the traffic exchange.
• State 4 – the second ISP is unavailable, the first ISP is available and active, and the enterprise network uses it as a primary ISP for the traffic exchange.
• State 5 – both of the ISPs are available, but the enterprise network uses only the second ISP as active, because the second ISP is configured as a priority primary or the first ISP is configured as a priority primary. But the routing protocol is not switched yet to it after last recovery of the first ISP, or traffic-balancing between the two ISPs is configured, but the first ISP is not activated yet.
• State 6 – both of the ISPs are available, but the enterprise network uses only the first ISP as active, because the first ISP is configured as a priority primary or the second ISP is configured as a priority primary. But the routing protocol is not switched yet to it after last recovery of the second ISP, or traffic-balancing between the two ISPs is configured, but the second ISP is not activated yet.
• State 7 – both of the ISPs are unavailable and cannot be used for traffic exchange in an enterprise.
• State 8 – both of the ISPs are available and actively used in accordance with the routing scheme based on traffic-balancing. However, they do not exchange the same Internet traffic and each of them transfers different Internet traffic, so this fault-tolerance scheme is significantly different from the well-known duplex system, in which both elements process same data.

Also let us introduce three boolean parameters, which will be used to mathematically describe the routing schemes within the reliability model:

• Parameter $\beta$. If $\beta = 1$, then the fault-tolerant routing scheme with the traffic-balancing between ISPs is used. Otherwise, if $\beta = 0$, then one of the ISPs is assumed as primary and another ISP is assumed as secondary.
• Parameter $\alpha_1$. If $\alpha_1 = 1$, then the first ISP is the prioritized primary, and is always used as a primary ISP when both providers are available. The parameter is used only when $\beta = 0$.
• Parameter $\alpha_2$. If $\alpha_2 = 1$, then the second ISP is the prioritized primary, and is always used as a primary ISP when both providers are available. The parameter is used only when $\beta = 0$.

Finally, taking into consideration all reliability aspects, mentioned above, we can introduce the next generalized reliability model, represented as a Markov chain, using the six reliability parameters of two ISPs and three boolean parameters of the fault-tolerant routing scheme (fig. 3).
Figure 3. A generalized reliability model for the border routing with two Internet services providers. Accordingly, the differential equations system of Kolmogorov-Chapman for this Markov chain is as follows:

\begin{align*}
P_0(0) &= 1; \quad P_1(0) = 0; \quad \ldots \quad P_8(0) = 0; \\
P_0(t) + \ldots + P_8(t) &= 1; \\
\frac{dP_0(t)}{dt} &= -(\lambda_1 + \lambda_2 + \gamma_1 + \gamma_2)P_0(t) + \mu_1P_1(t) + \mu_2P_2(t); \\
\frac{dP_1(t)}{dt} &= \lambda_1P_0(t) - (\mu_1 + \lambda_2 + \gamma_2)P_1(t) + \lambda_1P_0(t) + \lambda_1P_6(t) + \mu_2P_7(t); \\
\frac{dP_2(t)}{dt} &= \lambda_2P_0(t) - (\mu_2 + \lambda_1 + \gamma_1)P_2(t) + \lambda_2P_5(t) + \lambda_2P_8(t) + \mu_1P_7(t); \\
\frac{dP_3(t)}{dt} &= \gamma_2P_1(t) - (\mu_1 + \lambda_2)P_3(t) + \lambda_1P_5(t); \\
\frac{dP_4(t)}{dt} &= \gamma_1P_2(t) - (\mu_2 + \lambda_1)P_4(t) + \lambda_2P_6(t); \\
\frac{dP_5(t)}{dt} &= \gamma_2P_0(t) + \mu_1P_3(t) - (\lambda_1 + \lambda_2 + \beta\gamma_1 + \alpha\gamma_1)P_5(t) + \alpha_2\gamma_2P_6(t); \\
\frac{dP_6(t)}{dt} &= \gamma_1P_0(t) + \mu_2P_4(t) - (\lambda_1 + \lambda_2 + \beta\gamma_2 + \alpha_2\gamma_2)P_6(t) + \alpha_1\gamma_1P_5(t); \\
\frac{dP_7(t)}{dt} &= \lambda_2P_1(t) + \lambda_1P_2(t) + \lambda_2P_3(t) + \lambda_1P_4(t) - (\mu_1 + \mu_2)P_7(t); \\
\frac{dP_8(t)}{dt} &= \beta\gamma_1P_5(t) + \beta\gamma_2P_6(t) - (\lambda_1 + \lambda_2)P_8(t).
\end{align*}

To obtain the formula for the stationary availability factor of the border routing, we do not have to solve the differential equations system, and we can take into consideration stationary case \( t \to \infty \), when the derivatives tend to zero, and we obtain the system of linear algebraic equations.
It is also important to mention, that only states 3, 4, 5, 6 and 8 are considered as the up-states from the viewpoint of enterprise networks, in which at least one of the ISPs is active and provides the Internet traffic exchange for the enterprise network. So, the stationary availability factor of border routing with two ISPs is equal to the sum of stationary probabilities of the up-states:  
\[ K_{SYS} = P_a + P_b + P_5 + P_6 + P_8. \]

However, to find the stationary probabilities, we also need to solve the system of linear algebraic equations. We will omit the huge intermediate algebraic transforms and just introduce the obtained calculation formula for the stationary availability factor:

\[
K_{SYS} = \frac{\mu_1 \mu_2 (\gamma_1 + \gamma_2) + \lambda_1 \mu_2 (\mu_1 + \lambda_1 + \lambda_2 + \gamma_1 + \gamma_2)}{(\mu_1 + \lambda_1)(\mu_2 + \lambda_2)(\lambda_1 + \lambda_2 + \gamma_1 + \gamma_2)} \frac{M_1}{D} + \frac{\lambda_2 \mu_1 (\mu_2 + \lambda_1 + \lambda_2 + \gamma_1 + \gamma_2)}{(\mu_1 + \lambda_1)(\mu_2 + \lambda_2)(\lambda_1 + \lambda_2 + \gamma_1 + \gamma_2)} \frac{M_2}{D}. \tag{1}
\]

Here, the coefficients \( M_1, M_2 \) and \( D \) can be calculated by the author’s next formulas:

\[
M_1 = (\lambda_2 (\lambda_1 + \gamma_1) + (\mu_2 + \lambda_1 + \gamma_1)(\lambda_1 + \beta \gamma_1))(\mu_1 \lambda_1 \gamma_2 + (\lambda_1 + \lambda_2 + \gamma_1 + \gamma_2) \gamma_2 (\lambda_1 + \lambda_2 + \alpha_1 \gamma_1 + \beta \gamma_2)) + (\mu_2 + \lambda_1 + \gamma_1)(\mu_1 \lambda_1 \gamma_2 + (\lambda_1 + \lambda_2 + \gamma_1 + \gamma_2) \gamma_2 (\lambda_1 + \lambda_2 + \alpha_1 \gamma_1 + \beta \gamma_2)).
\]

\[
M_2 = (\lambda_1 (\lambda_2 + \gamma_2) + (\mu_1 + \lambda_2 + \gamma_2)(\lambda_2 + \beta \gamma_1))(\mu_2 \lambda_2 \gamma_2 (\lambda_1 + \lambda_2 + \gamma_1 + \gamma_2) \gamma_1 (\lambda_1 + \lambda_2 + \alpha_2 \gamma_2 + \beta \gamma_2)) + (\mu_1 + \lambda_2 + \gamma_2)(\mu_2 \lambda_2 \gamma_2 (\lambda_1 + \lambda_2 + \gamma_1 + \gamma_2) \gamma_1 (\lambda_1 + \lambda_2 + \alpha_2 \gamma_2 + \beta \gamma_2)).
\]

\[
D = (\lambda_1 + \lambda_2 + \gamma_1 + \gamma_2)(\lambda_1 (\lambda_2 + \gamma_2) + (\mu_1 + \lambda_2 + \gamma_2)(\lambda_2 + \beta \gamma_1))(\lambda_2 \lambda_1 + \gamma_1) + (\mu_2 + \lambda_1 + \gamma_1)(\lambda_1 + \beta \gamma_2) + (\mu_1 \lambda_1 \gamma_2 + \alpha_1 \gamma_1)(\mu_1 + \lambda_2 + \gamma_2)(\lambda_1 + \lambda_2 + \gamma_1 + \gamma_2) \gamma_2 (\lambda_1 + \lambda_2 + \alpha_1 \gamma_1 + \beta \gamma_2)) + (\mu_1 + \lambda_2 + \gamma_2)(\lambda_2 + \beta \gamma_1) + (\lambda_1 + \gamma_1)(\lambda_1 + \beta \gamma_2) + (\mu_1 \lambda_1 \gamma_2 + \alpha_2 \gamma_2)(\mu_2 + \lambda_1 + \gamma_1)(\lambda_1 + \lambda_2 + \gamma_1 + \gamma_2) \gamma_2 (\lambda_1 + \lambda_2 + \alpha_2 \gamma_2 + \beta \gamma_2)).
\]

Note 1. If the ISP activation rates are \( \gamma_1 = 0 \) and \( \gamma_2 = 0 \), then \( M_3/D = 0 \) and \( M_4/D = 0 \), then availability factor \( K_{SYS} = 0 \).

Note 2. If the ISP activation rates are \( \gamma_1 \to \infty \) and \( \gamma_2 \to \infty \), then \( M_3/D \to 1 \) and \( M_4/D \to 1 \). Accordingly, the formula for the availability factor is simplified to the well-known formula for the availability factor of duplex systems with two independent nodes:

\[
K_{SYS} = \frac{\mu_1 \mu_2 + \lambda_1 \mu_2 + \lambda_2 \mu_1}{(\mu_1 + \lambda_1)(\mu_2 + \lambda_2)}. \tag{5}
\]

3. Availability factor calculation examples
An enterprise network is connected to the two Internet services providers. The failure rate of the first ISP is \( \lambda_1 = 1/1440 \) hour\(^{-1} \) (on average, one failure per 2 months), recovery rate \( \mu_1 = 1 \) hour\(^{-1} \) (on average, one recovery per 1 hour) and activation rate \( \gamma_1 = 20 \) hour\(^{-1} \) (on average, one activation per 3
minutes). The failure rate of the second ISP is $\lambda_2 = 1/1800$ hour$^{-1}$ (on average, one failure per 2.5 months), recovery rate $\mu_2 = 2$ hour$^{-1}$ (on average, one recovery per 0.5 hour) and activation rate $\gamma_2 = 20$ (on average, one activation per 3 minutes).

Let us calculate the availability factor of fault-tolerant border routing with two ISPs in different routing schemes. By formula 1, we obtain the following values for the different routing schemes:

- Traffic-balancing mode between two providers, $\beta = 1$:
  $$K_{\text{SYS}} \approx 0.999939431$$

- Primary / Backup mode without priorities, $\beta = 0$, $\alpha_1 = 0$, $\alpha_2 = 0$:
  $$K_{\text{SYS}} \approx 0.9999700545$$

- Primary / Backup mode with priority for the first ISP, $\beta = 0$, $\alpha_1 = 1$, $\alpha_2 = 0$:
  $$K_{\text{SYS}} \approx 0.9999659179$$

- Primary / Backup mode with priority for the second ISP, $\beta = 0$, $\alpha_1 = 0$, $\alpha_2 = 1$:
  $$K_{\text{SYS}} \approx 0.9999732905$$

- Primary / Backup mode with priority conflict between the ISPs, $\beta = 0$, $\alpha_1 = 1$, $\alpha_2 = 1$:
  $$K_{\text{SYS}} \approx 0.9999696042$$

Calculations show, that in this example, the highest value for the availability factor is reached at the Primary / Backup mode with priority for the second ISP: $K_{\text{SYS}} \approx 0.9999732905$.

It is necessary to mention that, if we try to ignore the reconfiguration time of the routing protocol and just assume the immediate activation rate for providers ($\gamma_1 \to \infty$ and $\gamma_2 \to \infty$), we degrade our model to the well-known simplified reliability model for the duplex system with two independent elements and obtain a highly overestimated value for the availability factor by formula 5: $K_{\text{SYS}} = 0.9999998073$ (six «nines» instead of more realistic four «nines» by formula 1).

Therefore, the generalized reliability model and the calculation formula, obtained by authors, give significantly more realistic values of the availability factor for fault-tolerant border routing with two Internet services providers, taking into consideration the finite time of ISP activation after startup, failure and recovery events, and specific issues of the different routing schemes.

4. Conclusion
Within the scope of this article, a reliability model of the generalized mathematical model of the fault-tolerant routing with two Internet Providers and derivation of the end calculation formula for the availability factor are observed. The calculation examples of the availability factor for different cases with the specific routing schemes and policies, which are used in modern border routing, are also provided. Calculation results show significantly more realistic values of the availability factor for fault-tolerant border routing with two Internet Services Providers, rather than values, which can be obtained by the well-known simplified formula for the duplex system.

Scientific results, obtained by the authors, were used for designing of enterprise networks with fault-tolerant border routing with two Internet Service Providers in Moscow Power Engineering Institute, Nuclear Power Plant “Balakovo” and several other medium-scale enterprises.

5. Acknowledgements
The authors are grateful to S. N. Khorkov, a chief network administrator, Moscow Power Engineering Institute, for technical support and to V. A. Tushavin, an associate professor, Saint-Petersburg State University of Aerospace Instrumentation, for scientific support.

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