Viable low-scale model with Universal and Inverse Seesaw Mechanisms

A. E. Cárcamo Hernández, 1,∗ Juan Marchant González, 1,† and U. J. Saldaña-Salazar 2,‡

1Universidad Técnica Federico Santa María and Centro Científico-Tecnológico de Valparaíso Casilla 110-V, Valparaíso, Chile.
2Max-Planck-Institut für Kernphysik, Postfach 103980, D-69029 Heidelberg, Germany.

(Dated: April 24, 2019)

We formulate a viable low scale seesaw model, where the masses for the Standard Model (SM) charged fermions lighter than the top quark emerge from a Universal Seesaw mechanism mediated by charged vector-like fermions. The small light active neutrino masses are produced from an Inverse Seesaw mechanism mediated by right-handed Majorana neutrinos. Our model is based on the $A_4$ family symmetry, supplemented by cyclic symmetries, whose spontaneous breaking produces the observed pattern of SM fermion masses and mixings. The model can accommodate the anomalous magnetic dipole moment of the muon and predicts strongly suppressed $\mu \to e\gamma$ and $\tau \to \mu\gamma$ decay rates, but allows a $\tau \to e\gamma$ decay within the reach of the forthcoming experiments.

I. INTRODUCTION

The Standard Model (SM) has offered us a theoretical framework with great experimental success. In spite of this, the observed values in quark mixing angles together with the pattern in the charged fermion masses find no explanation. Moreover, the observation of neutrino oscillations have augmented this puzzle as the theory must also be extended to incorporate neutrino masses along with the observed leptonic mixing parameters. The pattern in all the fermion masses may be described by three main aspects:

(i) Only one mass at the electro-weak (EW) scale while all others well below it,

$$m_t \sim \frac{v_{\text{EW}}}{\sqrt{2}} \gg \{m_b, m_\tau, m_e, m_\mu, m_s, m_d, m_u, m_e\}, \tag{1}$$

(ii) Neutrino masses are much smaller than the electron mass,

$$m_\nu \lesssim \left(\frac{m_e}{m_t}\right) m_e, \tag{2}$$

(iii) The charged fermion masses satisfy a hierarchical structure,

$$m_{f,3} \gg m_{f,2} \gg m_{f,1}, \quad (f = u, d, e). \tag{3}$$

Several attempts have been made to theoretically describe each of these aspects either individually [1–4] or various simultaneously, see for example [5–13]. In the following, to produce Eq. (1) and (3) we opt to work within a low-scale realization of a universal seesaw model whereas for Eq. (2) we consider an inverse seesaw mechanism.

In universal seesaw models [6, 14], the smallness of fermion masses except for the top quark, Eq. (1), can be easily explained by promoting parity symmetry ($L \leftrightarrow R$) to a fundamental symmetry at high energies, larger than the Fermi scale. These models are based on the $SU(2)_L \times SU(2)_R \times SU(3)_c \times U(1)_{B-L}$ gauge symmetry, where $B$ and $L$ stand for the baryon and lepton number, respectively. On the other hand, the matter content is enlarged by introducing

∗Electronic address: antonio.carca@gmail.com
†Electronic address: juan.marchantgonzalez@gmail.com
‡Electronic address: ulises.saldana@mpi-hd.mpg.de
vector-like fermions (singlets under the left and right isospin symmetries) whereas the scalar sector gets minimally
enlarged by mirroring the SM Higgs, $H \sim (2,1,1,1)$, to the right sector, $H_R \sim (1,2,1,1)$, transforming as a right
doublet. The conventional bidoublet in L-R symmetric theories is here missing. As a consequence neutrinos have no
masses and Yukawa interactions are now made with both scalars whereas the singlet fermions acquire their own mass
terms. Typically, after both scalars have acquired their vacuum expectation values (VEV), small fermion masses arise
as an admixture of both VEVs and the heavy mass of the singlet fermions, $m_f \sim v_{EW}(v_R/M_e)$, where $v_R \ll M_e$,
while the top quark mass has no vector-like fermion companion, and thus its mass is simply given by the standard
formula, $m_t \sim v_{EW}$. For last, the hierarchy given in Eq. (3) may be understood by considering exotic fermion masses
with an inverse hierarchy, $M_{e1} \gg M_{e2} \gg M_{e3}$. In the following, we mimic the main shared features among this class
of models and discuss a low-scale scenario.

The smallness of neutrino masses may have a different origin than that of the charged fermions. Already their
superlightness seems to point out to this possibility. Hence, here we consider that two different mechanisms are
responsible for the observed patterns in the fermion masses. We choose to study the mass nature of neutrinos via an
inverse seesaw [4, 15–17]. This mechanism leads to an effective mass parameter given by $m_\nu \sim (m_D/\Lambda)^2 \mu$ where $m_D$
is the typical scale of a Dirac mass, $M_E$ the heavy scale of the isosinglet leptons which conserve lepton number, and $\mu$
the mass scale of the gauge singlet neutrinos responsible for breaking lepton number. It follows that for small $\mu$ then
$m_\nu$ becomes small; which is opposite to the standard seesaw, where the smallness of neutrino masses is due to the
largeness of the right handed neutrino masses. The advantage of using an inverse seesaw is that lepton flavor violation
(LFV) rates do not depend on the small magnitude of the lepton number violating scale, $\mu$, while they vanish in
standard seesaw scenarios.

In this work we propose a low scale seesaw model with extended scalar and fermion sectors, consistent with the
current pattern of SM fermion masses and mixings. In our model, the masses of the SM charged fermions lighter than
the top quark are generated from a universal seesaw mechanism mediated by charged exotic vector-like fermions. The
small light active neutrinos masses arise from an inverse seesaw mechanism mediated by three sterile neutrinos. In our
model we use the $A_4$ family symmetry, which is supplemented by other auxiliary symmetries, thus allowing to have a
viable description of the current SM fermion mass spectrum and mixing parameters. We have chosen the $A_4$ family
symmetry since it is the smallest order discrete group with one three-dimensional and three distinct one-dimensional
irreducible representations, where the three families of fermions can be accommodated rather naturally. This group
was used for the first time in Ref. [18] and subsequently used in [19–34] to provide a viable and predictive description
of the SM fermion mass spectrum and mixing parameters.

The outline for the rest of this paper is as follows. In Section II we introduce the model, followed by discussions
on the quark and lepton masses and mixing in Sections III and IV, respectively. We devote Section V to study some
phenomenological aspects of our model. Finally, in Section VI we conclude.

II. THE MODEL

Our model is an explicit realization of a Tri-Permuting (TP) scenario wherein the neutrino sector is transformed into
the mass basis via a mixing matrix of the form [35],

$$|U^{TP}_\nu| = \frac{1}{3} \begin{pmatrix} 2 & 2 & 1 \\ 2 & 1 & 2 \\ 1 & 2 & 2 \end{pmatrix}. \quad (4)$$

In this type of scenarios, the charged lepton sector must be built in such a way that contributions arising from their
mixing matrix, $U_\ell$, may help us to reproduce the experimentally observed values as the full mixing matrix would
then be given by $U^{TP} = U_\nu U^{TP}_\nu$.

In addition to the usual SM particle content, in order to implement the universal seesaw mechanism producing the
masses for the SM charged fermions lighter than the top quark, we consider vector-like quarks and charged leptons,

$$T_{i,L(R)} \sim (3,1)_{2/3}, \quad B_{j,L(R)} \sim (3,1)_{-1/3}, \quad E_{k,L(R)} \sim (1,1)_{-1}, \quad (5)$$

where $i = 1,2$, $j, k = 1,2,3$ and their transformation assignment is given under the SM gauge group, $G_{SM} = SU(3)_c \times SU(2)_L \times U(1)_Y$. Also, the scalar sector is appropriately enlarged, apart from the Higgs doublet, $H$, with
Table I: Fermion content in the quark sector and its charge assignment under the discrete flavor symmetry group.

| \( \mathcal{G}_{\text{SM}} \) | (3, 2)_{1/6} | (3, 2)_{2/3} | (3, 2)_{1-3/2} | (3, 1)_{2/3} | (3, 1)_{1-3} |
|---|---|---|---|---|---|
| \( A_4 \) | 1 | 1" | 1′ | 1 | 1′ | 1" |
| \( Z_2 \) | 0 | 0 | 1 | 0 | 0 | 1 |
| \( Z_5 \) | 2 | 2 | 2 | 0 | 0 | 0 |
| \( Z_4 \) | 0 | 0 | 0 | 0 | 0 | 0 |
| \( Z_8 \) | 0 | 0 | 0 | 0 | 0 | 0 |

Table II: Fermion content in the lepton sector and its charge assignment under the discrete flavor symmetry group.

| \( \mathcal{G}_{\text{SM}} \) | (1, 2)_{1/2} | (1, 1)_{-1/2} | (1, 1)_{0} | (1, 1)_{-1} |
|---|---|---|---|---|
| \( A_4 \) | 3 | 1" | 1′ | 1 |
| \( Z_2 \) | 0 | 0 | 0 | 1 |
| \( Z_5 \) | 0 | 0 | 2 | 0 |
| \( Z_8 \) | -1 | -1 | -2 | -3 |

real scalar singlets (flavons),
\[
\text{Quark sector:} \quad \{ \chi_1, \chi_2, \Phi_{1k}, \Phi_{2k}, \Phi_{3k} \} \quad (6)
\]

\[
\text{Lepton sector:} \quad \{ \sigma, \eta_1 \eta_2, \rho_1, \rho_2, \xi_k, \zeta_k, S_k \} \quad (7)
\]

where \( k = 1, 2, 3 \) and we have classified the flavons into two different groups depending in which sector they are relevant.

To control the arbitrariness in the Yukawa interactions we introduce \( A_4 \) as our flavor symmetry. \( A_4 \) has been found to be phenomenologically interesting and successful in the neutrino sector [21, 36, 37]. Appendix A has a brief description of the group and multiplication rules. On the other hand, we have found to be insufficient this symmetry to fully develop a predictive theory. For this purpose, we employ the Abelian symmetry, \( Z_2 \times Z_5 \) with an additional \( Z_2 \) and \( Z_4 \times Z_8 \), in the quark and lepton sector, respectively. The charge assignments are given in Tables I, II, and III.

The full Yukawa terms can be written as \( \mathcal{L}_Y = \mathcal{L}_Y^u + \mathcal{L}_Y^d + \mathcal{L}_Y^\nu + \mathcal{L}_Y^\ell \) where for the up-type quarks,
\[
-\mathcal{L}_Y^u = y_t \bar{Q}_{3L} H u_{3R} + y_t \bar{Q}_{3L} H T_1 R + y_t \bar{Q}_{2L} H T_2 R + y_t \bar{Q}_{1L} \chi_1 u_{1R} + y_t \bar{Q}_{1L} \chi_2 u_{2R} + M_T T_1 L T_1 R + M_T T_2 T_2 T_2 R + \text{H.c.},
\]

down-type quarks,
\[
-\mathcal{L}_Y^d = y_t \bar{Q}_{3L} H B_1 R + y_t \bar{Q}_{2L} H B_2 R + y_t \bar{Q}_{1L} \chi_1 d_{1R} + y_t \bar{Q}_{1L} \chi_2 d_{2R} + y_t \bar{Q}_{1L} \chi_3 d_{3R} + M_B B_1 B_1 R + M_B B_2 B_2 R + M_B B_3 B_3 R + \text{H.c.},
\]

charged leptons,
\[
-\mathcal{L}_Y^\nu = y^{(l)}_t \left( \tilde{\nu}_L H \xi_1 \right) E_{1R} \frac{\sigma^2 \eta_1}{\Lambda^2} + y^{(l)}_t \left( \tilde{\nu}_L H \xi_1 \right) E_{2R} \frac{\sigma}{\Lambda^2} + y^{(l)}_t \left( \tilde{\nu}_L H \xi_1 \right) E_{3R} \frac{\eta_1^2}{\Lambda^2} + y^{(l)}_t \left( \tilde{\nu}_L H \xi_1 \right) E_{1R} \frac{\sigma^2 (\eta_2)^2}{\Lambda^5}
\]

\[
+ y^{(l)}_t \left( \tilde{\nu}_L H \xi_1 \right) E_{3R} \frac{\eta_2^2}{\Lambda^3} + x_1 \left( \tilde{\nu}_L \rho_1^2 e_{1R} \right) \frac{(\sigma^2)^4}{\Lambda^4} + x_2 \left( \tilde{\nu}_L \rho_2^2 e_{2R} \right) \frac{(\sigma^2)^2}{\Lambda^2} + x_3 \left( \tilde{\nu}_L \rho_3^2 e_{3R} \right) + \sum_{i=1}^{3} y^{(l)}_t \left( \tilde{\nu}_L \rho_i^2 e_{iR} \right) + \text{H.c.},
\]

and neutrinos,
\[
-\mathcal{L}_Y^\ell = y^{(\nu)} \left( \tilde{\nu}_L H N R \right)_{3s} \frac{\zeta^*}{\Lambda} + y^{(\nu)} \left( \tilde{\nu}_L H N R \right)_{3s} \frac{\zeta^*}{\Lambda} + y^{(\nu)} \left( \tilde{\nu}_L H N R \right)_{3s} \frac{S^*}{\Lambda} + y^{(\nu)} \left( \tilde{\nu}_L H N R \right)_{3s} \frac{S^*}{\Lambda}
\]

\[
+ y^{(N)} \left( \tilde{N}_R \varnothing \right) \frac{\rho_1 + y^{(l)} \left( \varnothing R \varnothing \right) \rho_2 \left( \rho_1^2 \right) \frac{(\sigma^4 \eta_1)}{\Lambda^7}}{\Lambda} + \text{H.c.},
\]

(10)
being the dimensionless couplings in Eqs. (8), (9), (10) and (11) $O(1)$ parameters.

We denote by $\langle \chi_i \rangle = v_{\chi_i}$ ($i = 1, 2$), and assume the following VEV patterns for the $A_4$ triplet SM singlet scalars $\Phi_{1,2,3}$, $\xi$, $\zeta$ and $S$,

$$\langle \Phi_1 \rangle = \frac{v_1}{\sqrt{2}}(0,1,1), \quad \langle \Phi_2 \rangle = \frac{v_2}{\sqrt{3}}(1,1,1), \quad \langle \Phi_3 \rangle = v_3(0,0,1),$$

$$\langle \xi \rangle = \frac{v_5}{\sqrt{3}}(1,1,1), \quad \langle \zeta \rangle = v_c(0,0,1), \quad \langle S \rangle = v_S(1,0,0),$$

which are natural solutions of the scalar potential minimization equations for a large region of the parameter space as shown in Refs. [22, 38–42]. As the hierarchy among charged fermion masses and quark mixing angles mass emerges from the spontaneous breaking of the $A_4 \times Z_2 \times Z_3 \times Z_4 \times Z_3$ discrete group, we set the VEVs of the SM singlet scalar fields $\sigma$, $\xi_i$, $\zeta_i$ ($i = 1, 2, 3$) with respect to the Wolfenstein parameter $\lambda = 0.225$ and the model cutoff $\Lambda$, as follows

$$\{v_{\rho_1}, v_{\rho_2}\} \sim O(1) \text{ TeV} \ll \{v_{\zeta}, v_S, v_\xi, v_\sigma, v_\eta_1, v_\eta_2\} \sim \lambda \Lambda. \quad (14)$$

### III. Quark Masses and Mixings

Due to the symmetry assignments, the top quark does not mix with the exotic vector-like quarks and thus its mass is simply given as in the SM via its Yukawa interaction with the Higgs doublet, $m_t \sim y_t v_{EW}$. On the other hand, the vector-like quarks do mix with all the others SM quarks. Then, from Eqs. (8) and (9), the quark mass matrices take the form,

$$M_{up}^{3 \times 3} = \begin{pmatrix} 0_{3 \times 3} & M_{\nu}^u & \end{pmatrix}, \quad \text{and} \quad M_{down}^{3 \times 3} = \begin{pmatrix} 0_{3 \times 3} & M_{\nu}^d \end{pmatrix},$$

where

$$M_{\nu}^u = \frac{v_{EW}}{\sqrt{2}} \begin{pmatrix} y_{11} & 0 & 0 \\ 0 & y_{22} & 0 \\ 0 & 0 & y_{33} \end{pmatrix}, \quad M_{\nu}^d = \begin{pmatrix} 0 & y_{14} y_{12} & y_{14} y_{13} \\ y_{24} y_{22} & 0 & y_{24} y_{23} \\ y_{34} y_{32} & y_{34} y_{33} & 0 \end{pmatrix},$$

$$M_T = \begin{pmatrix} 0 & M_{T1} & 0 \\ M_{T2} & 0 & 0 \end{pmatrix}, \quad M_B = \begin{pmatrix} M_{B1} & 0 & 0 \\ 0 & M_{B2} & 0 \\ 0 & 0 & M_{B3} \end{pmatrix}.$$ 

As the masses of the vector-like quarks are much larger than the employed VEVs, $M_T, M_B \gg \{v_{EW}, v_\nu, v_t\}$, the implementation of the Universal Seesaw yields the following $3 \times 3$ low-scale quark mass matrices,

$$M_u \simeq \begin{pmatrix} m_u & 0 & 0 \\ 0 & m_c & 0 \\ 0 & 0 & m_t \end{pmatrix}, \quad \text{and} \quad M_d \simeq \begin{pmatrix} 0 & a_d & \omega a_d \\ \omega b_d & b_d & \omega^2 (b_d + c_d) \\ d_d & d_d & d_d + e_d \end{pmatrix},$$

#### Table III: Scalar content and its charge assignment under the discrete flavor symmetry group.

| $G_{SM}$ | $H$ | $\chi_1$ | $\chi_2$ | $\Phi_1$ | $\Phi_2$ | $\Phi_3$ | $\sigma$ | $\eta_1$ | $\eta_2$ | $\rho_1$ | $\rho_2$ | $\xi$ | $\zeta$ | $S$ |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| $(1,2)_{1/2}$ | | $1$ | $1'$ | $3$ | $3$ | $3$ | $1$ | $1$ | $1'$ | $1$ | $3$ | $3$ | $3$ | 
| $A_4$ | $1$ | $0$ | $0$ | $0$ | $0$ | $0$ | $0$ | $0$ | $0$ | $-1$ | $0$ | $0$ | $-1$ | $-1$ |
| $Z_2$ | $0$ | $0$ | $0$ | $0$ | $0$ | $0$ | $0$ | $0$ | $0$ | $-1$ | $3$ | $0$ | $-2$ | $-2$ |
| $Z_3$ | $2$ | $0$ | $0$ | $0$ | $0$ | $0$ | $0$ | $0$ | $0$ | $0$ | $0$ | $0$ | $0$ | $0$ |
| $Z_4$ | $0$ | $0$ | $0$ | $0$ | $0$ | $0$ | $0$ | $0$ | $0$ | $0$ | $-2$ | $-1$ | $0$ | $0$ |
| $Z_8$ | $0$ | $0$ | $0$ | $0$ | $0$ | $0$ | $0$ | $-1$ | $0$ | $0$ | $0$ | $0$ | $-1$ | $-1$ |
where the up-type quark masses are given by,

\[ m_u = \frac{v_{EW} y^u_1 v_{x_1}}{\sqrt{2} M_{T1}}, \quad m_c = \frac{v_{EW} y^c_2 v_{x_2}}{\sqrt{2} M_{T2}}, \quad m_t = \frac{v_{EW} y^t}{\sqrt{2}}, \]

and we have defined the parameters,

\[ a_d = \frac{v_{EW} y^d_1 v_1}{\sqrt{2} \sqrt{2} M_{B1}}, \quad b_d = \frac{v_{EW} y^d_2 v_2}{\sqrt{2} \sqrt{3} M_{B2}}, \quad c_d = \frac{v_{EW} y^d_3 v_3}{\sqrt{2} M_{B2}}, \quad d_d = \frac{v_{EW} y^d_4 v_2}{\sqrt{2} \sqrt{3} M_{B3}}, \quad e_d = \frac{v_{EW} y^d_5 v_3}{\sqrt{2} M_{B3}}, \]

with \( y^u_1 \) denoting the product of two different Yukawa couplings which can be merged into a single one as both are \( O(1) \) parameters. From Eq. (19) and the known hierarchy in the up-quark masses we can estimate the ratio among the heavy masses and VEVs to be,

\[ \frac{v_{x_1}}{M_{T1}} \sim 10^{-5} \quad \text{and} \quad \frac{v_{x_2}}{M_{T2}} \sim 10^{-2}, \]

which if we assume a single heavy scale, \( M_T \equiv \{M_{T1}, M_{T2}\}, \) then \( v_{x_2} \sim \lambda^3 M_T \) and \( v_{x_1} \sim \lambda^7 M_T. \) For example, for the range \( M_T \sim [1, 100] \) TeV one gets \( v_{x_2} \sim [0.01, 1] \) TeV and \( v_{x_1} \sim [0.1, 10] \) MeV.

In total we have 5 complex parameters, that is, 10 free parameters to fit 7 observables. We restrict the number of free parameters to 7. For this purpose, we only keep free 2 of the 5 phases, and limit ourselves to the particular case where,

\[ \arg(a_d) = -\arg(b_d) = \arg(c_d) \quad \text{and} \quad \arg(d_d) = 0. \]

Hence, the set of independent parameters becomes,

\[ \{ |a_d|, |b_d|, |c_d|, |d_d|, |e_d|, \arg(a_d), \arg(e_d) \}. \]

We then perform a numerical fit to the set of parameters. The experimental input parameters are the three down-type quark masses, the magnitude of the three independent mixing matrix elements, and the Jarlskog invariant. The masses are taken at the \( M_z \) scale with a symmetrised 1\( \sigma \) error taken to be the larger one. The employed input parameters are summarized in Table IV. To measure the quality of the fit we use the function,

\[ \chi^2 = \frac{(m_d^{1\text{th}} - m_d^{\exp})^2}{\sigma_d^2} + \frac{(m_s^{1\text{th}} - m_s^{\exp})^2}{\sigma_s^2} + \frac{(m_b^{1\text{th}} - m_b^{\exp})^2}{\sigma_b^2} + \frac{(|V_{12}^{1\text{th}}| - |V_{12}^{\text{ckm}}|)^2}{\sigma_{12}^2} + \frac{(|V_{23}^{1\text{th}}| - |V_{23}^{\text{ckm}}|)^2}{\sigma_{23}^2} + \frac{(|V_{13}^{1\text{th}}| - |V_{13}^{\text{ckm}}|)^2}{\sigma_{13}^2} + \frac{(J_q^{1\text{th}} - J_q^{\exp})^2}{\sigma_J^2}. \]

Its minimization leads to the best-fit values,

\[ |a_d| = 0.0114225 \text{ GeV}, \quad |b_d| = 0.0215709 \text{ GeV}, \quad |c_d| = 0.130513 \text{ GeV}, \quad |d_d| = 0.765595 \text{ GeV}, \]
\[ |e_d| = 1.97927 \text{ GeV}, \quad \arg(a_d) = 5.39151 \text{ rad}, \quad \arg(e_d) = 0.605986 \text{ rad}, \]

implying the observed down-type quark masses and mixing shown in Table IV. Moreover, we find that the two smallest mixing angles are correlated among them and also with the Jarlskog invariant, see Fig. 1. For last, notice that our model prefers small values of the Jarlskog invariant compared to the latest fit from the PDG [43].
Figure 1: Correlation plots between the two smallest quark mixing angles (top panel) and between the Jarlskog invariant with each of them (bottom panels). All the (red) points in the background are in agreement with the experimental values within 3σ deviations. The (blue) star points represent the best fit point of our minimization whereas the black points show the most recent values as taken from the PDG-2018 with their 1σ deviations in (yellow) lines.

From the best fit values, Eq. (24), we can estimate the required ratio among the heavy masses and VEVs in order to reproduce the observed mild hierarchy among the fitted parameters,

\[
\frac{v_1}{M_{B_1}} \sim 10^{-4}, \quad \frac{v_2}{M_{B_2}} \sim 10^{-4}, \quad \frac{v_3}{M_{B_3}} \sim 10^{-2}, \quad \frac{v_2}{M_{B_3}} \sim 10^{-2}, \quad \frac{v_3}{M_{B_2}} \sim 10^{-3.5},
\]

in such a way that all Yukawa couplings may still remain as O(1) parameters. All these ratios can be rewritten in terms of the heaviest mass,

\[
M_B \equiv \{M_{B_1}, M_{B_2}\}, \quad M_{B_3} \sim \lambda^3 M_B, \quad \{v_1, v_2, v_3\} \sim \lambda^6 M_B.
\]

These relations imply, for example, for the range \( M_B \sim [100, 1000] \) TeV, then \( M_{B_3} \sim [1, 10] \) TeV and \( \{v_1, v_2, v_3\} \sim [10, 100] \) GeV.

Models with vector-like fermions are being tested at the LHC. The ATLAS collaboration has reported several analyses, in particular [45–47]. At the moment, mass exclusion limits for exotic isosinglet quarks give \( M_B \gtrsim 1.22 \) TeV and \( M_T \gtrsim 1.31 \) TeV as found in Ref. [47]. These lower bounds were set by only assuming that the exotic quarks can decay on SM particles. That is, the vector-like quarks would first be produced at collider experiments via pair-production, a process dominated by the strong interactions, \( gg \to B \overline{B} (T \overline{T}) \). Then, each exotic quark would decay to: \( T \to W_b, Z_l, H_t \) or \( B \to W_t, Z_b, H_b \), where it has only been considered the third fermion family. Now, in our case, in the interaction basis, our model has no initial mixing between the top-quark and the vector- and top-like fermions, but only between the up and charm quarks with the exotic partners. On the other hand, in the down-quark
IV. LEPTON MASSES AND MIXINGS

Using Eq. (11) we get the following mass matrix for charged leptons:

$$\mathcal{M}_{6 \times 6}^{(E)} = \begin{pmatrix} M_1^{(l)} & M_1^{(l)} & M_1^{(l)} \\ M_2^{(l)} & M_2^{(l)} & M_2^{(l)} \\ M_3^{(l)} & M_3^{(l)} & M_3^{(l)} \end{pmatrix},$$

(27)

where the different submatrices are given by:

$$M_1^{(l)} = \frac{v_{EW} \eta_1}{\sqrt{2}} \left( \begin{array}{ccc} 1 & 1 & 1 \\ 0 & \omega^2 & \omega \\ 0 & \omega & \omega^2 \end{array} \right) \left( \begin{array}{ccc} \cos \alpha & 0 & -e^{-i\gamma} \sin \alpha \\ 0 & 1 & 0 \\ e^{i\gamma} \sin \alpha & 0 & \cos \alpha \end{array} \right),$$

$$M_2^{(l)} = \left( \begin{array}{ccc} y_{11} \lambda_1 \alpha & 0 & 0 \\ 0 & y_{22} \lambda_2 \alpha & 0 \\ 0 & 0 & y_{33} \lambda_3 \alpha \end{array} \right),$$

$$M_3^{(l)} = \left( \begin{array}{ccc} y_{11}^{(E)} \alpha & 0 & 0 \\ 0 & y_{22}^{(E)} \alpha & 0 \\ 0 & 0 & y_{33}^{(E)} \alpha \end{array} \right) v_{\rho_2},$$

(28)

Here we have adopted a simplifying benchmark scenario with the following particular assumptions about the charged lepton sector model parameters and VEVs of some of the gauge singlet scalars:

$$y_{41}^{(l)} = e^{i\gamma} y_{11}^{(l)}, \quad y_{51}^{(l)} = -e^{-i\gamma} y_{33}^{(l)}, \quad v_{\eta_1} = \lambda \cos \alpha, \quad v_{\eta_2} = \sqrt{\lambda} \sin \alpha \Lambda, \quad v_{\sigma} = \lambda \Lambda.$$

(29)

Thus, the universal seesaw mechanism gives rise to the following SM charged lepton mass matrix:

$$M_L = M_1^{(l)} M_3^{(l)} M_2^{(l)} = R_{dL} \text{diag} (m_e, m_\mu, m_\tau),$$

$$R_{dL} = \frac{1}{\sqrt{3}} \left( \begin{array}{ccc} 1 & 1 & 1 \\ 1 & \omega^2 & \omega \\ 1 & \omega & \omega^2 \end{array} \right) \left( \begin{array}{ccc} \cos \alpha & 0 & -e^{-i\gamma} \sin \alpha \\ 0 & 1 & 0 \\ e^{i\gamma} \sin \alpha & 0 & \cos \alpha \end{array} \right), \quad \omega = e^{2\pi i / 3},$$

(30)

where the charged lepton masses are:

$$m_e = x_1^{(l)} y_{11} \lambda_1 \alpha \eta_1^{(E)} \Lambda = a_1 \lambda \eta^{(E)} \Lambda \sqrt{2}, \quad m_\mu = x_2^{(l)} y_{22} \lambda_2 \eta_1^{(E)} \Lambda = a_2 \lambda \eta^{(E)} \Lambda \sqrt{2}, \quad m_\tau = x_3^{(l)} y_{33} \lambda_3 \eta_1^{(E)} \Lambda = a_3 \lambda \eta^{(E)} \Lambda \sqrt{2}.$$  

(31)

Here we have considered $x_i^{(l)} \sim y_i^{(l)} \lesssim O(1) \ (i = 1, 2, 3)$ and $y_i^{(E)} \lesssim O(\sqrt{\pi})$. Let us note that the charged lepton masses are linked with the scale of electroweak symmetry breaking through their power dependence on the Wolfenstein parameter $\lambda = 0.225$, with $O(1)$ coefficients.

The neutrino Yukawa terms of Eq. (11) give origin to the following neutrino mass terms:

$$-\mathcal{L}^{\nu}_{\text{mass}} = \frac{1}{2} \left( \nu^c_L \ N_R \ \overline{\nu_R} \right) \mathcal{M}_\nu \left( \begin{array}{c} \nu^c_L \\ N^c_R \end{array} \right) + \text{H.c},$$

(32)

where the neutrino mass matrix reads:

$$\mathcal{M}_\nu = \begin{pmatrix} M_1^{(l)} & M_2^{(l)} & M_3^{(l)} \\ M_1^{(l)^T} & M_2^{(l)^T} & M_3^{(l)^T} \\ M_1^{(l)^T} & M_2^{(l)^T} & M_3^{(l)^T} \end{pmatrix},$$

(33)
and the submatrices read:

\[
\begin{align*}
M_1 & = \frac{v_C v_{\text{EW}}}{\sqrt{2} \Lambda} \begin{pmatrix}
0 & y_1^{(\nu)} + y_2^{(\nu)} & 0 \\
y_1^{(\nu)} - y_2^{(\nu)} & 0 & r(y_1^{(\nu)} + y_2^{(\nu)}) \\
0 & r(y_1^{(\nu)} - y_2^{(\nu)}) & 0
\end{pmatrix} \\
& = f \begin{pmatrix}
x & 0 & 0 \\
y & 0 & rx \\
0 & ry & 0
\end{pmatrix}, \quad f = \frac{v_C v_{\text{EW}}}{\sqrt{2} \Lambda}, \quad r = \frac{v_S}{v_C}, \quad x = y_1^{(\nu)} + y_2^{(\nu)}, \quad y = y_1^{(\nu)} - y_2^{(\nu)}, \\
M_2 & = m_N \begin{pmatrix}
1 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & 1
\end{pmatrix}, \quad \mu = \frac{y^{(\nu)} v_{\rho_2} v_{\rho_1} v_{\sigma} v_{\eta_1}}{\Lambda^2} \begin{pmatrix}
0 & 1 & -1 \\
1 & 0 & 1 \\
-1 & 1 & 0
\end{pmatrix}, \quad m_N = y^{(N)} v_{\rho_1}.
\end{align*}
\]

The light active masses arise from an inverse seesaw mechanism and the resulting physical neutrino mass matrices take the form:

\[
\tilde{M}_\nu = M_1 (M_1^T)^{-1} \mu M_2^{-1} M_1^T, \quad \tilde{M}_\nu^{(1)} = -\frac{1}{2} (M_2 + M_2^T) + \frac{1}{2} \mu, \quad \tilde{M}_\nu^{(2)} = \frac{1}{2} (M_2 + M_2^T) + \frac{1}{2} \mu,
\]

where \( \tilde{M}_\nu \) corresponds to the active neutrino mass matrix whereas \( \tilde{M}_\nu^{(1)} \) and \( \tilde{M}_\nu^{(2)} \) are the sterile mass matrices.

Thus, the light active neutrino mass matrix is given by:

\[
\tilde{M}_\nu = \frac{y^{(\nu)} v_{\rho_2} v_{\rho_1} v_{\sigma} v_{\eta_1} f^2}{m_N^2 \Lambda^2} \begin{pmatrix}
0 & x(y + rx) & 0 \\
x(y + rx) & -2xy & ry(y + rx) \\
0 & ry(y + rx) & 0
\end{pmatrix}, \quad m_\nu = \frac{y^{(\nu)} v_{\rho_2} v_{\rho_1} v_{\sigma} v_{\eta_1} f^2}{m_N^2 \Lambda^2}.
\]

The full neutrino mass matrix given by Eq. (33) can be diagonalized by the following rotation matrix [48]:

\[
R = \begin{pmatrix}
R_\nu & R_1 R_M^{(1)} & R_2 R_M^{(2)} \\
-R(R_1^2 + R_1^2) \frac{1}{\sqrt{2}} & R_1 R_M^{(1)} (1 - S) \frac{1}{\sqrt{2}} & R_2 R_M^{(2)} (1 + S) \frac{1}{\sqrt{2}} \\
-R(R_1^2 - R_1^2) \frac{1}{\sqrt{2}} & -R_1 R_M^{(1)} (1 - S) \frac{1}{\sqrt{2}} & R_2 R_M^{(2)} (1 - S) \frac{1}{\sqrt{2}}
\end{pmatrix},
\]

where

\[
S = -\frac{1}{4} M_2^{-1} \mu, \quad R_1 \simeq R_2 \simeq \frac{1}{\sqrt{2}} M_1^* M_1^{-1}. \]

Notice that the physical neutrino spectrum is composed of three light active neutrinos and six exotic neutrinos. The exotic neutrinos are pseudo-Dirac, with masses \( \sim \pm \frac{1}{2} (M_2 + M_2^T) \) and a small splitting \( \mu \). Furthermore, \( R_\nu, R_M^{(1)}, \) and \( R_M^{(2)} \) are the rotation matrices which diagonalize \( \tilde{M}_\nu, \tilde{M}_\nu^{(1)} \) and \( \tilde{M}_\nu^{(2)} \), respectively.

On the other hand, using Eq. (40) we find that the neutrino fields \( \nu_L = (\nu_{1L}, \nu_{2L}, \nu_{3L})^T, \nu_R^C = (\nu_{1R}^C, \nu_{2R}^C) \) and \( N_R^C = (N_{1R}^C, N_{2R}^C) \) are related with the physical neutrino fields by the following relations:

\[
\begin{pmatrix}
\nu_L \\
\nu_R^C \\
N_R^C
\end{pmatrix} = \mathbb{R} \Omega L \simeq \begin{pmatrix}
R_\nu^C & R_1 R_M^{(1)} & R_2 R_M^{(2)} \\
-R(R_1^2 + R_1^2) \frac{1}{\sqrt{2}} & R_1 R_M^{(1)} (1 - S) \frac{1}{\sqrt{2}} & R_2 R_M^{(2)} (1 + S) \frac{1}{\sqrt{2}} \\
-R(R_1^2 - R_1^2) \frac{1}{\sqrt{2}} & -R_1 R_M^{(1)} (1 - S) \frac{1}{\sqrt{2}} & R_2 R_M^{(2)} (1 - S) \frac{1}{\sqrt{2}}
\end{pmatrix} \begin{pmatrix}
\psi_L^{(1)} \\
\psi_L^{(2)} \\
\psi_L^{(3)}
\end{pmatrix},
\]

where \( \psi_L^{(j)} \) and \( \psi_R^{(j)} \) are the three active neutrinos and six exotic neutrinos, respectively.
Majorana neutrino mass parameter takes the form:

\[ m_{\nu\beta\beta} \]

Another relevant observable that can be determined in this model, is the effective Majorana neutrino mass parameter in the range 0\( \lesssim m \lesssim 27\) meV, which provides information on the Majorana nature of neutrinos. The effective Majorana neutrino mass parameter takes the form:

\[ m_{ee} = \left| \sum_j U_{e j}^2 m_{\nu j} \right|, \]

By varying the lepton sector model parameters, we find values for the neutrino mass squared splittings, i.e., \( \Delta m_{21}^2 \) and \( \Delta m_{31}^2 \), lepton mixing angles \( \theta_{12}^{(l)} \), \( \theta_{23}^{(l)} \), \( \theta_{13}^{(l)} \) and the Dirac leptonic CP violating phase consistent with the neutrino oscillation experimental data, as indicated in Table V.

Fig. 2 shows the correlation between the solar mixing parameter \( \sin^2 \theta_{12} \) and the leptonic CP violating phase. To obtain this figure the lepton sector model parameters were randomly generated in a range of values where the neutrino oscillation experimental data, as indicated in Table V.

Table V: Model predictions for the scenario of normal (NH) neutrino mass hierarchy. The experimental values are taken from Refs. [49, 51].

| Observable | range | \( \Delta m_{21}^2 \) [10^{-2} eV^2] | \( \Delta m_{31}^2 \) [10^{-2} eV^2] | \( \theta_{12}^{(l)}(\circ) \) | \( \theta_{23}^{(l)}(\circ) \) | \( \theta_{13}^{(l)}(\circ) \) | \( \delta_{CP}^{(l)}(\circ) \) |
|------------|-------|---------------------------------|---------------------------------|-----------------|-----------------|-----------------|-----------------|
| Experimental | 1\( \sigma \) | 7.39\( ^{+0.21}_{-0.20} \) | 2.52\( ^{+0.033}_{-0.032} \) | 33.8\( ^{+0.78}_{-0.76} \) | 8.61\( ^{+0.13}_{-0.13} \) | 49.6\( ^{+1.0}_{-1.2} \) | 215\( ^{+40}_{-29} \) |
| Value from Ref. [49] | 3\( \sigma \) | 6.79\( ^{-0.01}_{+0.02} \) | 2.427\( ^{-0.043}_{+0.025} \) | 31.6\( ^{-0.27}_{+0.26} \) | 8.2\( ^{-0.08}_{+0.09} \) | 40.3\( ^{-0.12}_{+0.15} \) | 125\( ^{-0.5}_{+0.4} \) |
| Experimental | 1\( \sigma \) | 7.55\( ^{-0.16}_{+0.16} \) | 2.50\( ^{-0.03}_{+0.03} \) | 34.5\( ^{-0.12}_{+0.12} \) | 8.45\( ^{-0.16}_{+0.16} \) | 47.7\( ^{-1.7}_{+1.7} \) | 218\( ^{-3.8}_{+3.8} \) |
| Value from Ref. [50] | 3\( \sigma \) | 7.05\( ^{-0.14}_{+0.14} \) | 2.41\( ^{-0.20}_{+0.20} \) | 31.5\( ^{-0.30}_{+0.30} \) | 8.0\( ^{-0.9}_{+0.9} \) | 41.8\( ^{-0.30}_{+0.30} \) | 157\( ^{-3.9}_{+3.9} \) |
| Fit | 1\( \sigma \) | 7.55 | 2.50 | 34.45 | 8.45 | 43.1 | 218.2 |

Figure 2: Correlation between the solar mixing parameter \( \sin^2 \theta_{12} \) and the leptonic CP violating phase.

By varying the lepton sector model parameters, we find values for the neutrino mass squared splittings, i.e., \( \Delta m_{21}^2 \) and \( \Delta m_{31}^2 \), lepton mixing angles \( \theta_{12}^{(l)} \), \( \theta_{23}^{(l)} \), \( \theta_{13}^{(l)} \) and the Dirac leptonic CP violating phase consistent with the neutrino oscillation experimental data, as indicated in Table V.

As seen from Fig. 2, our model predicts a solar mixing parameter \( \sin^2 \theta_{12} \) and leptonic Dirac CP violating phase in the range 0.27 \( \lesssim \sin^2 \theta_{12} \lesssim 0.38 \) and 140° \( \lesssim \delta \lesssim 260^\circ \), respectively.

Another relevant observable that can be determined in this model, is the effective Majorana neutrino mass parameter of neutrinoless double beta decay, which provides information on the Majorana nature of neutrinos. The effective Majorana neutrino mass parameter takes the form:

\[ m_{ee} = \left| \sum_j U_{e j}^2 m_{\nu j} \right|, \]

where \( U_{ej} \) and \( m_{\nu j} \) are the PMNS leptonic mixing matrix elements and the neutrino Majorana masses, respectively. The neutrinoless double beta \( (0\nu\beta\beta) \) decay amplitude is proportional to \( m_{ee} \). In Figure 3 we display the correlation between the effective Majorana neutrino mass parameter \( m_{ee} \) and the leptonic Dirac CP violating phase \( \delta_{CP} \). As indicated by Figure 3, our model predicts an effective Majorana neutrino mass parameter in the range 0.020 eV \( \lesssim m_{ee} \lesssim 0.040 \) eV, thus implying that the values for the effective Majorana neutrino mass parameter predicted by our model are within the reach of the next-generation bolometric CUORE experiment [52], as well as the next-to-next-generation ton-scale \( 0\nu\beta\beta \)-decay experiments [53–56]. The current most stringent experimental upper bound on the effective Majorana neutrino mass parameter, i.e., \( m_{ee} \leq 160 \) meV arises from the KamLAND-Zen limit on the \( ^{136}Xe \) \( 0\nu\beta\beta \) decay half-life \( T_{1/2}^{0\nu\beta\beta} \) \( (^{136}Xe) \geq 1.07 \times 10^{26} \) yr [53], which corresponds to the upper bound of \( |m_{\beta\beta}| \leq (61 – 165) \) meV at 90% C.L. For information about those other experiments see Refs. [54, 57–59].
Figure 3: Correlation of the effective Majorana neutrino mass parameter $m_{ee}$ with the leptonic Dirac CP violating phase $\delta_{CP}$.

V. PHENOMENOLOGY

As previously stated, the physical sterile neutrino spectrum contains six almost degenerate TeV scale neutrinos, which mix the active ones, with mixing angles of the order of $\frac{(M_1)_{ij}}{\sqrt{2}y^v(N)_{1\nu_1}}$ (i, j = 1, 2, 3). In return, these new couplings will induce one-loop level phenomena through which we may impose constraints to our model.

In this section we will discuss the implications of our model in the lepton flavor violating decays and in the anomalous magnetic dipole moment of the muon.

A. Charged LFV decays

The heavy sterile neutrinos together with the W gauge boson induce the one loop level decay $l_i \rightarrow l_j \gamma$, whose corresponding branching ratio reads [15, 60, 61]:

$$Br (l_i \rightarrow l_j \gamma) = \frac{\alpha_3^2 s_W^2 W_m^5 |G_{ij}|^2}{256 \pi^2 M_W^4 \Gamma_{l_i}}, \quad (44)$$

where $s_W = \sin(\theta_W)$,

$$G_{ij} = \sum_k (R^*)_{ik} (R)_{jk} G_{\gamma} \left( \frac{m_{N_k}^2}{M_W^2} \right) \simeq 2 (R_1 R_1^T)_{ij} G_{\gamma} \left( \frac{m_{N_k}^2}{M_W^2} \right) = \frac{(M_1^* M_1^T)_{ij}}{m_{N_k}^2} G_{\gamma} \left( \frac{m_{N_k}^2}{M_W^2} \right),$$

$$G_{\gamma}(z) = -\frac{2z^2 + 5z^2 - z}{4(1-z)^2} \ln z, \quad m_N = y^{(N)} v_{\rho_1}, \quad (45)$$

and

$$M_1^* M_1^T = f^2 \begin{pmatrix} 0 & x & 0 \\ y & 0 & rx \\ 0 & ry & 0 \end{pmatrix} \begin{pmatrix} 0 & y & 0 \\ x & 0 & ry \\ 0 & rx & 0 \end{pmatrix} = f^2 \begin{pmatrix} x^2 & 0 & rxy \\ 0 & r^2 x^2 + y^2 & 0 \\ rxy & 0 & r^2 y^2 \end{pmatrix}. \quad (46)$$

Thus, the charged lepton flavor violating processes $\mu \rightarrow e\gamma$, $\tau \rightarrow \mu\gamma$ and $\tau \rightarrow e\gamma$ have the following branching ratios:

$$Br (\mu \rightarrow e\gamma) \simeq 0, \quad Br (\tau \rightarrow \mu\gamma) \simeq 0, \quad Br (\tau \rightarrow e\gamma) = \frac{\alpha_3^2 s_W^2 m_{N}^5 r^2 y^2 f^4}{256 \pi^2 M_W^4 T_r m_{N}^4} \left| G_{\gamma} \left( \frac{m_{N}^2}{M_W^2} \right) \right|^2, \quad (47)$$
where $\Gamma_{\tau} = 2.27 \times 10^{-12}$ GeV is the tau decay width. On the other hand, the upper experimental bound of the charged lepton flavor violating process $\tau \rightarrow e\gamma$ is given by:

$$\text{Br} (\tau \rightarrow e\gamma)_{\exp}^{\text{max}} = 3.3 \times 10^{-9}. \quad (48)$$

In Figure 4 we display the allowed parameter space in the $m_N - f$ plane consistent with the constraints arising from charged lepton flavor violating decays. As seen from Figure 4, the obtained values for the branching ratio of $\tau \rightarrow e\gamma$ decay are located in the range $5 \times 10^{-10} \lesssim \text{Br} (\tau \rightarrow e\gamma) \lesssim 3 \times 10^{-9}$, for sterile neutrino masses $m_N$ lower than about 1 TeV. Consequently, our model is compatible with the charged lepton flavor violating decay constraints provided that the sterile neutrinos are lighter than about 1 TeV.

**B. Contributions to $(g - 2)_{\mu}$**

The current discrepancy between the experimental and predicted value is still inconclusive and amounts to 3.5 standard deviations [43],

$$\Delta a_{\mu} \equiv a_{\mu}^{\exp} - a_{\mu}^{\text{SM}} = 268(63)(43) \times 10^{-11}, \quad (49)$$

where the errors at 1$\sigma$ are from experiment and theory, respectively. In the following, we consider the average error between the theoretical and experimental one.

Contributions to $\Delta a_{\mu}$ arising from scenarios like this one where the active neutrinos mix with heavy-right handed neutrinos have been already computed. The relevant expression is given by [61, 62],

$$\Delta a_{\mu} = \frac{-1}{8\pi^2 \kappa_{\mu}^2} \int_0^1 dz \sum_f \frac{|R_{f\mu}^v|^2 P_3^+(z) + |R_{f\mu}^a|^2 P_3^+(z)}{\epsilon_f^2 \kappa_{\mu}^2 (1 - z)(1 - \epsilon_f^2 z) + z}, \quad (50)$$

with

$$P_3^\pm(z) = -2z^2(1 + z \mp 2\epsilon_f) + \kappa_{\mu}^2 z(1 - z)(1 \mp \epsilon_f)^2(z \pm \epsilon_f), \quad (51)$$
and $\epsilon_f = \frac{m_N f}{m^2}$ and $\kappa_\mu = \frac{m_\mu}{m^2}$. In our particular case, the vector and axial-vector couplings to the $W$ bosons are identical and thus the expression is reduced to,

$$\Delta a_\mu = \frac{-1}{4\pi^2} \kappa_\mu^2 \sum_f |R^\nu_{f\mu}|^2 \int_0^1 dz \frac{-z^2(1+z) + \kappa_\mu^2 z(1-z)[z + \epsilon_f(z - 2)]}{\epsilon_f^2 \kappa_\mu^2 (1-z)(1-\epsilon_f^2 z) + z} .$$  (52)

Notice that only two couplings will contribute, $R^\nu_{1\mu}$ and $R^\nu_{3\mu}$. Both of them can be approximated to $R^\nu_{(3)\mu} \simeq \frac{f}{\sqrt{2} m_N}$ times order one parameters, $\{x, y, r\} \sim O(1)$. Figure 5 exemplifies the available parameter space in the $m_N - f$ plane which accommodates $\Delta a_\mu$ at $3\sigma$. Variations of the set of parameters in the range $0.3 \lesssim \{x, y\} \lesssim 1$ and $-4.5 \lesssim r \lesssim 2.9$ are shown in the gray background whereas for the particular case in which all order one parameters are taken equal to one are depicted by the colored bands.

![Figure 5: Available parameter space in the $m_N - f$ plane accommodating $\Delta a_\mu$ at $3\sigma$. The gray background considers variations of the set of parameters in the range $0.3 \lesssim \{x, y\} \lesssim 1$ and $-4.5 \lesssim r \lesssim 2.9$ while the colored bands depict a particular scenario with $\{x, y, r\} = 1$ where one may more easily appreciate the dependence of $f$ and $m_N$ in $\Delta a_\mu$.]

VI. CONCLUSIONS

We have proposed a viable low scale seesaw model based on the $A_4$ family symmetry and other auxiliary cyclic symmetries, where the SM particle spectrum is enlarged by the inclusion of several charged vector-like fermions, right-handed Majorana neutrinos and scalar singlets, consistent with the low energy SM fermion flavor data. The masses for the SM charged fermions lighter than the top quark emerge from a Universal Seesaw mechanism mediated by charged vector-like fermions, whereas the small light active neutrino masses are generated from an Inverse Seesaw mechanism. The smallness of the $\mu$ parameter of the inverse seesaw is attributed to a right-handed neutrino non-renormalizable mass term, generated after the spontaneous breaking of the discrete symmetries of the model. The spontaneous breaking of these discrete symmetries takes place at large energies and gives rise to the observed SM fermion mass spectrum and mixing parameters. We have studied the implications of our model in the lepton flavor violating decays and in the anomalous magnetic dipole moment of the muon. We have found that the $\mu \to e\gamma$ and $\tau \to \mu\gamma$ are strongly suppressed in our model, whereas the $\tau \to e\gamma$ decay can attain values within the reach of the current sensitivity of the forthcoming charged lepton flavor violation experiments. Furthermore, the obtained values
for the branching ratio for the $\tau \rightarrow e\gamma$ are lower than its current experimental bound for sterile neutrino masses lower than about 1 TeV. Finally, we have found that our model successfully accommodates the experimental value of the anomalous magnetic dipole moment of the muon.

Acknowledgments

AECH has received funding from Fondecyt (Chile), Grants No. 1170803, CONICYT PIA/Basal FB0821. UJSS acknowledges financial support from the early stage of the work from a DAAD One-Year Research Grant and during the late stages from CONACYT-México. UJSS is grateful to FCFM (BUAP) for hospitality during the completion of this work.

Appendix A: The product rules for $A_4$

Alternating symmetry groups, $A_n$, describe the even permutations of a given number, $n$, of indistinguishable objects. The smallest one is $A_4$. It has one triplet $\mathbf{3}$ and three distinct one-dimensional $\mathbf{1}$, $\mathbf{1}'$ and $\mathbf{1}''$ irreducible representations, satisfying the following product rules,

$$\mathbf{3} \otimes \mathbf{3} = \mathbf{3}_s \oplus \mathbf{3}_a \oplus \mathbf{1} \oplus \mathbf{1}' \oplus \mathbf{1}'', \quad \mathbf{1} \otimes \mathbf{1} = \mathbf{1}, \quad \mathbf{1}' \otimes \mathbf{1}'' = \mathbf{1}'', \quad \mathbf{1}' \otimes \mathbf{1}' = \mathbf{1}'', \quad \mathbf{1}'' \otimes \mathbf{1}' = \mathbf{1}'. \quad (A1)$$

Considering $(a_1, a_2, a_3)$ and $(b_1, b_2, b_3)$ as basis vectors for two $A_4$-triplets $\mathbf{3}$, the following relations are fulfilled,

$$\begin{align*}
(\mathbf{3} \otimes \mathbf{3})_{\mathbf{1}_1} &= a_1 b_1 + a_2 b_2 + a_3 b_3, \\
(\mathbf{3} \otimes \mathbf{3})_{\mathbf{3}_1} &= (a_2 b_3 + a_3 b_2, a_3 b_1 + a_1 b_3, a_1 b_2 + a_2 b_1), \\
(\mathbf{3} \otimes \mathbf{3})_{\mathbf{3}_a} &= (a_2 b_3 - a_3 b_2, a_3 b_1 - a_1 b_3, a_1 b_2 - a_2 b_1), \\
(\mathbf{3} \otimes \mathbf{3})_{\mathbf{1}'} &= a_1 b_1 + \omega a_2 b_2 + \omega^2 a_3 b_3, \\
(\mathbf{3} \otimes \mathbf{3})_{\mathbf{1}''} &= a_1 b_1 + \omega^2 a_2 b_2 + \omega a_3 b_3,
\end{align*} \quad (A2)$$

where $\omega = e^{\frac{2\pi i}{3}}$. The representation $\mathbf{1}$ is trivial, while the non-trivial $\mathbf{1}'$ and $\mathbf{1}''$ are complex conjugate to each other.

[1] P. Minkowski, Phys. Lett. 67B, 421 (1977). doi:10.1016/0370-2693(77)90435-X
[2] R. N. Mohapatra and G. Senjanovic, Phys. Rev. Lett. 44, 912 (1980). doi:10.1103/PhysRevLett.44.912
[3] J. Schechter and J. W. F. Valle, Phys. Rev. D 22, 2227 (1980). doi:10.1103/PhysRevD.22.2227
[4] R. N. Mohapatra and J. W. F. Valle, Phys. Rev. D 34, 1642 (1986). doi:10.1103/PhysRevD.34.1642
[5] C. D. Froggatt and H. B. Nielsen, Nucl. Phys. B 147, 277 (1979). doi:10.1016/0550-3213(79)90316-X
[6] A. Davidson and K. C. Wali, Phys. Rev. Lett. 59, 393 (1987). doi:10.1103/PhysRevLett.59.393
[7] L. E. Ibáñez and G. G. Ross, Phys. Lett. B 332, 100 (1994) doi:10.1016/0370-2693(94)90865-6 [hep-ph/9403338].
[8] N. Arkani-Hamed and M. Schmaltz, Phys. Rev. D 61, 033005 (2000) doi:10.1103/PhysRevD.61.033005 [hep-ph/9903417].
[9] J. Kubo, A. Mondragon, M. Mondragon and E. Rodríguez-Jauregui, Prog. Theor. Phys. 109, 795 (2003) Erratum: [Prog. Theor. Phys. 114, 287 (2005)] doi:10.1143/PTP.109.795 [hep-ph/0302196].
[10] I. de Medeiros Varzielas, S. F. King and G. G. Ross, Phys. Lett. B 648, 201 (2007) doi:10.1016/j.physletb.2007.03.009 [hep-ph/0607045].
[11] A. E. Cárcamo Hernández, Eur. Phys. J. C 76, no. 9, 503 (2016) doi:10.1140/epjc/s10052-016-4351-y [arXiv:1512.09092 [hep-ph]].
[12] A. E. Cárcamo Hernández, S. Kovalenko and I. Schmidt, JHEP 1702, 125 (2017) doi:10.1007/JHEP02(2017)125 [arXiv:1611.09797 [hep-ph]].
[13] W. Rodejohann and U. Saldàña-Salazar, arXiv:1903.00983 [hep-ph].
[14] F. F. Deppisch, C. Hati, S. Patra, P. Pritimita and U. Sarkar, Phys. Rev. D 97, no. 3, 035005 (2018) doi:10.1103/PhysRevD.97.035005 [arXiv:1701.02107 [hep-ph]].
[15] F. Deppisch and J. W. F. Valle, Phys. Rev. D 72, 036001 (2005) doi:10.1103/PhysRevD.72.036001 [hep-ph/0406040].
[16] F. Deppisch, T. S. Kosmas and J. W. F. Valle, Nucl. Phys. B 752, 80 (2006) doi:10.1016/j.nuclphysb.2006.06.032 [hep-ph/0512360].
[17] A. Abada and M. Lucente, Nucl. Phys. B 885, 651 (2014) doi:10.1016/j.nuclphysb.2014.06.003 [arXiv:1401.1507 [hep-ph]].
doi:10.1103/PhysRevLett.120.132502 [arXiv:1710.11608 [nucl-ex]].

[58] C. Alduino et al. [CUORE Collaboration], Phys. Rev. Lett. 120, no. 13, 132501 (2018) doi:10.1103/PhysRevLett.120.132501 [arXiv:1710.07988 [nucl-ex]].

[59] R. Arnold et al. [NEMO-3 Collaboration], Phys. Rev. D 95, no. 1, 012007 (2017) doi:10.1103/PhysRevD.95.012007 [arXiv:1610.03226 [hep-ex]].

[60] A. Ilakovac and A. Pilaftsis, Nucl. Phys. B 437, 491 (1995) doi:10.1016/0550-3213(94)00567-X [hep-ph/9403398].

[61] M. Lindner, M. Platscher and F. S. Queiroz, Phys. Rept. 731, 1 (2018) doi:10.1016/j.physrep.2017.12.001 [arXiv:1610.06587 [hep-ph]].

[62] J. P. Leveille, Nucl. Phys. B 137, 63 (1978). doi:10.1016/0550-3213(78)90051-2