Role of the Landau-Migdal Parameters with the Pseudovector and the Tensor
Coupling in Relativistic Nuclear Models
– The Quenching of the Gamow-Teller Strength –

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Role of the Landau-Migdal parameters with the pseudovector \( g_{\pi} \) and the tensor coupling \( g_{\rho} \) is examined for the giant Gamow-Teller (GT) states in the relativistic random phase approximation (RPA). The excitation energy is dominated by both \( g_{\pi} \) and \( g_{\rho} \), in a similar way, while the GT strength is independent of \( g_{\pi} \) and \( g_{\rho} \) in the RPA of the nucleon space, and is quenched, compared with that in non-relativistic one. The coupling of the particle-hole states with nucleon-antinucleon states is expected to quench the GT strength further through \( g_{\pi} \).

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I. INTRODUCTION

The long- and medium-range parts of the nuclear spin-isospin interaction are well described by the one-meson exchange approximation. In the non-relativistic limit, they are written in the momentum space as

\[
V_{\pi+\rho} = - \left[ \left( \frac{f_{\pi}}{m_{\pi}} \right)^2 \frac{\mathbf{q} \cdot \mathbf{q}}{m_{\pi}^2} \frac{1}{(\mathbf{q}^2 + m_{\pi}^2)} \right) \mathbf{\tau}_1 \cdot \mathbf{\tau}_2, \] (1)

where the first term stands for the one-pion exchange potential, while the second term the one-rho exchange potential. They are obtained from the pseudovector and tensor meson-nucleon couplings, respectively. In the short-range part of the interaction, on the other hand, many-body correlations should be taken into account. Their effects are approximately expressed by the Landau-Migdal (LM) parameter \( g' \),

\[
V_{LM} = \left( \frac{f_{\pi}}{m_{\pi}} \right)^2 g' \mathbf{\sigma}_1 \cdot \mathbf{\sigma}_2 \mathbf{\tau}_1 \cdot \mathbf{\tau}_2. \] (2)

Experimentally the value of \( g' \) is estimated to be about 0.6. This fact is understood in such a way that about a half of the value cancels the zero-range part of the one-pion exchange potential, while the rest the short-range part of the rho-meson exchange potential in Eq.(1). The contribution from the exchange terms of the one-pion and one-rho potential is also considered to be included in \( g' \). Indeed, if we calculate the LM parameter so as to cancel the zero-range part of the one-pion potential and to include the contribution from the exchange term of the finite-range part, we obtain from Eq.(1)

\[
g' = \frac{1}{3} \left( 1 + \frac{m_{\pi}^2}{16 k_F^2} \ln \frac{m_{\pi}^2}{m_{\pi}^2 + 4 k_F^2} \right). \] (3)

This value is about 0.318 for the pion mass \( m_{\pi} = 140 \) MeV and the Fermi momentum \( k_F = 1.36 \text{fm}^{-1} \). The corresponding expression from the rho-meson exchange potential is given by

\[
g'_{\rho} = \frac{2}{3} \left( \frac{f_{\rho}}{m_{\rho}} \right)^2 \left( \frac{m_{\pi}}{f_{\pi}} \right)^2 \left( 1 + \frac{m_{\rho}^2}{16 k_F^2} \ln \frac{m_{\rho}^2}{m_{\rho}^2 + 4 k_F^2} \right). \] (4)

If we simply calculate this value by using the strong coupling \( f_{\rho}^2/4\pi = 4.86 \) together with \( m_{\rho} = 770 \text{MeV} \), we obtain \( g'_{\rho} = 1.07 \). Refs. 3 and 4 have shown, however, that in the \( \rho \)-meson exchange, the short-range part of the finite range interaction should be also taken out, since the \( \rho \)-mass is much larger than the \( \pi \)-mass. This contribution makes the above value of \( g'_{\rho} \) smaller as 0.281, since the finite range part has an opposite sign to that of the zero-range part. Thus the experimental value about 0.6 is well understood by the sum of \( g'_{\pi} + g'_{\rho} \) in non-relativistic models.

Now, for the last 30 years it has been shown that nuclear structure is well explained phenomenologically by relativistic models. Not only the ground state, but also the excited states are well reproduced. So far, however, spin-dependent excitations of nuclei have not been studied in details. One of the reasons may be because there is no estimation of the LM parameter yet in relativistic models. Moreover, there is no unique way to extend the above non-relativistic \( g' \) to the relativistic one.

The purpose of the present paper is to introduce phenomenologically the LM parameter in relativistic models, and to explore model-dependence of the excitation energy and strength of the giant Gamow-Teller (GT) resonance state. We will construct relativistic LM parameters so as to reproduce \( g' \) of Eq.(2) in the non-relativistic limit. Such candidates will be given by contact terms with the pseudovector \( g_a \) and the tensor \( g_t \) coupling in the nuclear Lagrangian.

In the previous paper, we investigated already the LM parameter of the pseudovector type for
the description of the giant GT resonance states. It was shown that the excitation energy in non-relativistic models is reproduced and that the GT excitation strength in the nucleon space is quenched, compared with the sum rule value by Ikeda-Fujii-Fujita (IFF)\textsuperscript{[11]}. In particular, the quenching, which is consistent with recent experiment\textsuperscript{[12]}, was an important conclusion, since in conventional non-relativistic models the value of the GT strength is fixed by the IFF sum rule. In this paper we will pay our attention mainly to whether or not those conclusions in the relativistic model are changed by the LM parameter\textsuperscript{[13]}.

In the following section, we will calculate the correlation function in the random phase approximation (RPA), where the correlations are induced through the LM parameters $g_a$ and $g_t$. In Sec\textsuperscript{III} and IV the excitation energy and strength of the GT state will be estimated, respectively. Since all calculations are performed for nuclear matter, discussions will be given according to analytic formulae of the excitation energy and strength. They make clear structure of the giant GT states in relativistic models. The final section will be devoted to a brief conclusion of the present paper.

II. RELATIVISTIC RPA CORRELATION FUNCTION

We assume that nucleons and antinucleons in nuclear matter are bounded in the mean field of the Lorentz scalar and vector potentials. The Lorentz scalar potential will be included into the nucleon effective mass $M^*$ below. On the other hand, we will not describe explicitly the Lorentz vector potential throughout the present paper, since it does not play any role in description of the GT states.

The particle-hole and nucleon-antinucleon correlations are assumed to be caused by the Lagrangian,

$$\mathcal{L} = \frac{g_a}{2} \overline{\psi} \Gamma_i^a \psi \overline{\psi} \Gamma_i \psi + \frac{g_t}{4} \overline{\psi} T_{\mu\nu}^i \psi \overline{\psi} T_{\mu\nu}^i \psi,$$

(5)

where the first term describes the pseudovector coupling and the second one the tensor coupling,

$$\Gamma_i^a = \gamma_5 \gamma^\mu \tau_i, \quad T_{\mu\nu}^i = \sigma^{\mu\nu} \tau_i.$$

We have also defined the isospin operators for convenience as

$$\tau_\pm = \frac{\tau_x \pm i \tau_y}{\sqrt{2}}, \quad \tau_0 = \tau_z.$$

The coupling constants $g_a$ and $g_t$ will be related later to the LM parameter $g$ in non-relativistic models.

The RPA correlation function for the external field, $\Gamma_A$ and $\Gamma_B$, is given by the mean field correlation function $\Pi$ as\textsuperscript{[8, 9, 10]}

$$\Pi_{\text{RPA}}(\Gamma_A, \Gamma_B) = \Pi(\Gamma_A, \Gamma_B) + \chi_a \Pi(\Gamma_A, \Gamma_i^a) \Pi_{\text{RPA}}(\Gamma_{ai}, \Gamma_B),$$

(6)

where we have used the notations,

$$\Gamma_i^a = \left\{ \begin{array}{ll} \gamma_5 \gamma^\mu \tau_i, & (a = a) \\ \sigma^{\mu\nu} \tau_i & (\mu > \nu), \end{array} \right.$$

$$\chi_a = \left\{ \begin{array}{ll} \frac{g_a}{(2\pi)^3}, & (a = a) \\ \frac{g_t}{(2\pi)^3}, & (a = t). \end{array} \right.$$

When the external fields are written as $\Gamma_A = \gamma_a \tau_-, \gamma_B = \gamma_b \tau_+$, and $\Gamma_B = \gamma_b \tau_+$, $\gamma_a$ and $\gamma_b$ being some $4 \times 4$ matrices, Eq.(6) becomes to be

$$\Pi_{\text{RPA}}(\gamma_a \tau_-, \gamma_b \tau_+) = \Pi(\gamma_a \tau_-, \gamma_b \tau_+) + \chi_a \Pi(\gamma_a \tau_-, \Gamma_i^a) \Pi_{\text{RPA}}(\Gamma_{ai}, \gamma_b \tau_+).$$

(7)

In the same way, the last term of the above equation is described as,

$$\Pi_{\text{RPA}}(\Gamma_{ai}, \gamma_b \tau_+) = (U^{-1})_{ab} \Pi(\Gamma_{bi}, \gamma_b \tau_+),$$

(9)

with use of the dimesic function,

$$U_{ab} = \delta_{ab} - \chi_b \Pi(\Gamma_{ai}, \Gamma_{i+}).$$

(10)

From Eq.(9), Eq.(7) is expressed in the form,

$$\Pi_{\text{RPA}}(\gamma_a \tau_-, \gamma_b \tau_+) = \Pi(\gamma_a \tau_-, \gamma_b \tau_+) + \chi_a \Pi(\gamma_a \tau_-, \Gamma_i^a) (U^{-1})_{ab} \Pi(\Gamma_{bi}, \gamma_b \tau_+).$$

(11)

Thus, $\Pi_{\text{RPA}}$ can be described in terms of the mean field correlation function $\Pi$.

When we calculate the mean field correlation function, we neglect the divergent terms and keep all the nuclear density dependent terms including the Pauli blocking ones, as most of the previous authors did\textsuperscript{[13, 14]}. We call this approximation no free term approximation (NFA)\textsuperscript{[13]}. As discussed in details in Ref.\textsuperscript{[10]}, the Pauli blocking terms are necessary for keeping at least the IFF sum rule and the current conservation\textsuperscript{[12, 13]. The no-sea approximation (NSA)\textsuperscript{[16]}} is equivalent to NFA in the description of the low lying states where nucleon-degrees of freedom play a main role\textsuperscript{[10]. In the following calculations, we will use NFA, but will come back to the problem of this approximation at the end of Sec\textsuperscript{IV}.

In NFA, the mean field correlation function is given by

$$\Pi(\gamma_a \tau_-, \gamma_b \tau_+) = \int \frac{d^3p}{E_p} \left( \frac{i_{ab}(p, q)}{(p + q)^2 - M^2 + i\varepsilon} \theta_p^{(n)} + \frac{i_{ba}(p, -q)}{(p - q)^2 - M^2 + i\varepsilon} \theta_p^{(n)} \right)$$

$$+ i\pi \int \frac{d^3p}{E_p} \left( \frac{\delta(q_0 + E_p - E_{p+q})}{E_{p+q}} i_{ab}(p, q) \theta_p^{(n)} \theta_p^{(p)\dagger} \right)$$

(12)
where \( q \) stands for the four-momentum transfer from the external field to the nucleus. The notations are defined as \( p_0 = E_p = \sqrt{p^2 + M^*}, M^* \) being the nucleon effective mass, and \( \theta(i) = \theta(k_i - |p|), k_n \) and \( k_p \) being the Fermi momentum of the neutrons and protons, respectively. The step functions express the density-dependence. We have also used the abbreviation in the above equation,

\[
t_{ab}(p, q) = -\text{Tr}_\pi \left( \gamma_a (\not{q} + \not{g} + M^*) \gamma_b (\not{q} + \not{g} + M^*) \right). \tag{13}
\]

In the present paper, we study the GT transition at \( q = 0 \). In this case, Eq. (13) can be written, as shown later explicitly, in the following form,

\[
t_{ab}(p, q) = f(p) + g(p) q_0. \tag{14}
\]

Then, Eq. (14), for \( N \geq Z \) nuclei, can be expressed in terms of \( f(p) \) and \( g(p) \) as

\[
\Pi(\gamma_a \tau_-, \gamma_b \tau_+) = \int \frac{d^3p}{2p_0^2} \left[ \frac{f}{q_0 + i\varepsilon} \left( \theta^{(n)}_p - \theta^{(p)}_p \right) + \frac{2p_0 g - f}{2p_0} \left( \theta^{(n)}_p + \theta^{(p)}_p \right) \right]. \tag{15}
\]

For giant GT states with \( |q_0| \ll M^* \), the above equation may be written as

\[
\Pi(\gamma_a \tau_-, \gamma_b \tau_+) = \int \frac{d^3p}{2p_0^2} \left[ \frac{f}{q_0 + i\varepsilon} \left( \theta^{(n)}_p - \theta^{(p)}_p \right) + \frac{2p_0 g - f}{2p_0} \left( \theta^{(n)}_p + \theta^{(p)}_p \right) \right]. \tag{16}
\]

As shown in the previous paper [2], the GT states can be described by taking the component, \( \gamma_a = \gamma_b = \gamma_5 \gamma_2 \), as the external field. This component has non-zero values of \( t_{ab} \) in the following correlation functions,

\[
\Pi_{11} = \Pi(\gamma_5 \gamma_2 \tau_-, \gamma_5 \gamma_2 \tau_+) = -\Pi(\gamma_5 \gamma_2 \tau_-, \gamma_5 \gamma_2 \tau_+),
\]

\[
\Pi_{12} = \Pi(\gamma_5 \gamma_2 \tau_-, \sigma_3 \tau_+) = \Pi(\gamma_5 \gamma_2 \tau_-, \sigma_3 \tau_+) = -\Pi(\sigma_3 \tau_-, \gamma_5 \gamma_2 \tau_+). \tag{17}
\]

Therefore, in writing

\[
\Pi_{22} = \Pi(\sigma_3 \tau_-, \sigma_3 \tau_+) = \Pi(\sigma_3 \tau_-, \sigma_3 \tau_+), \tag{18}
\]

the dimeson function is provided as

\[
U = \delta_{\alpha\beta} - \chi_{\alpha\beta} \Pi(\Gamma_{\alpha-}, \Gamma_{\beta+}) = \left( 1 + \chi_a \Pi_{11} - \chi_t \Pi_{12} - 1 - \chi_t \Pi_{22} \right). \tag{19}
\]

Calculations of \( t_{ab} \) for \( \Pi_{ij} \) are straightforward. For \( \Pi_{11} \), we obtain

\[
t_{ab} = -4 \left( 2M^*2 + 2p_y^2 + E_p q_0 \right). \tag{20}
\]

This, together with Eq. (15), gives

\[
\Pi_{11} = -\frac{4}{q_0 + i\varepsilon} \int d^3p \frac{M^*2 + p_y^2}{E_p^2} \left( \theta^{(n)}_p - \theta^{(p)}_p \right) - 2 \int d^3p \frac{p_y^2}{E_p^2} \left( \theta^{(n)}_p + \theta^{(p)}_p \right). \tag{21}
\]

In the case of \( \Pi_{12} \), \( t_{ab} \) is calculated to be

\[
t_{ab} = -4M^* \left( 2E_p + q_0 \right),
\]

which yields

\[
\Pi_{12} = -\frac{4M^*}{q_0 + i\varepsilon} \int d^3p \frac{\theta^{(n)}_p - \theta^{(p)}_p}{E_p}, \tag{22}
\]

In the same way, \( \Pi_{22} \) is described as

\[
\Pi_{22} = -\frac{4}{q_0 + i\varepsilon} \int d^3p \frac{M^*2 + 2p_y^2}{E_p^2} \left( \theta^{(n)}_p - \theta^{(p)}_p \right) - 2 \int d^3p \frac{p_y^2}{E_p^2} \left( \theta^{(n)}_p + \theta^{(p)}_p \right),
\]

by using

\[
t_{ab} = -4 \left( 2M^*2 + 4p_y^2 + E_p q_0 \right). \tag{23}
\]

In order to express the above equations in a simpler form, let us employ the following notations,

\[
Q_a(k_p) = \frac{4}{3} \int_0^{k_p} d^3p \frac{M^*2 + p_y^2}{E_p^2}, \tag{24}
\]

\[
Q_m(k_p) = \frac{4}{3} \int_0^{k_p} d^3p \frac{M^*}{E_p}, \tag{25}
\]

\[
\kappa = \frac{2}{3} \int d^3p \frac{p_y^2}{E_p^2} \left( \theta^{(n)}_p + \theta^{(p)}_p \right), \tag{26}
\]

and, moreover,

\[
q_a = Q_a(k_n) - Q_a(k_p), \tag{27}
\]

\[
q_m = Q_m(k_n) - Q_m(k_p), \tag{28}
\]

\[
q_t = Q_t(k_n) - Q_t(k_p). \tag{29}
\]

Then, \( \Pi_{ij} \) is described as

\[
\Pi_{11} = \frac{q_a}{q_0 + i\varepsilon} - 2\kappa, \tag{30}
\]

\[
\Pi_{12} = \frac{q_m}{q_0 + i\varepsilon}, \tag{31}
\]

\[
\Pi_{22} = -\frac{q_t}{q_0 + i\varepsilon} - \kappa. \tag{32}
\]
The functions in Eq. (27) can be expanded in terms of $(k_n - k_p)$, for example, as
\[ q_a \approx \frac{dQ(k_F)}{dk_F} (k_n - k_p), \quad \frac{dQ(k_F)}{dk_F} = 16\pi k_F^2 \left( 1 - \frac{2}{3} v_F^2 \right), \]
where $v_F$ stands for the Fermi velocity, $k_F/E_F$, with $E_F = \sqrt{k_F^2 + M^2}$. When we use the relationship, as usual,
\[ k_n - k_p \approx \frac{2}{3} k_F (N - Z)/A, \]
the function $q_a$ is written in terms of $(N - Z)$ as
\[ q_a \approx \alpha \left( 1 - \frac{2}{3} v_F^2 \right), \quad \alpha = 32\pi k_F^3 N - Z/3A. \] (33)

In the same way, the functions, $q_m$ and $q_t$, are expressed approximately as
\[ q_m \approx \alpha \left( 1 - v_F^2 \right), \quad q_t \approx \alpha \left( 1 - 3 v_F^2 \right). \] (34)

We note again that the above equations are obtained by their expansion in terms of $(N - Z)$, but not in terms of $v_F$. On the other hand, if we expand $\kappa$ in Eq. (20) in terms of $v_F$, we obtain
\[ \kappa \approx \frac{8\pi}{15} k_F^2 v_F^3 \left( 1 + \frac{3}{7} v_F^2 + \cdots \right). \] (35)

This comes from Pauli blocking terms due to nucleon-antinucleon excitations. Since $\kappa$ is of order $v_F^3$, the Pauli blocking effect is negligible in the case of the GT excitations.

### III. EXCITATION ENERGY OF THE GT STATES

The excitation energy of the GT state is determined by the determinate of the dimesic function,
\[ \det U = 0, \] (36)
which gives $q_0 = \omega_{\pm}$,
\[ \omega_{\pm} = \frac{1}{2} \left( \omega_a + \omega_t \pm \sqrt{(\omega_a - \omega_t)^2 - 4\chi_a \chi_t q_m^2} \right). \] (37)

with
\[ \omega_a = \tilde{\chi}_a q_a, \quad \omega_t = -\tilde{\chi}_t q_t, \]
\[ \tilde{\chi}_a = \frac{\chi_a}{1 - 2\kappa\chi_a}, \quad \tilde{\chi}_t = \frac{\chi_t}{1 + \kappa\chi_t}. \] (38)

It is obvious that $\omega_a$ and $\omega_t$ are the solutions of Eq. (30), when there is no mixing between the pseudovector and tensor couplings. According to Eqs. (33) and (34), they are given approximately as
\[ \omega_a \approx \alpha \tilde{\chi}_a \left( 1 - \frac{2}{3} v_F^2 \right), \quad \omega_t \approx -\alpha \tilde{\chi}_t \left( 1 - \frac{1}{3} v_F^2 \right). \] (39)

In non-relativistic models, it has been shown that the LM parameter $g'$ provides us with the excitation energy of the GT state,
\[ \omega_{GT} = g' \left( \frac{f_\pi}{m_\pi} \right)^2 \frac{4k_F^4 (N - Z)}{3\pi^2 A}. \] (41)

Comparing the above equation to Eq. (40), it is seen that both pseudovector and tensor coupling can reproduce individually the non-relativistic result by the relationship,
\[ g_a = g' \left( \frac{f_\pi}{m_\pi} \right)^2, \quad g_t = -g' \left( \frac{f_\pi}{m_\pi} \right)^2, \] (42)
but they produce a different relativistic correction of order $v_F^2$ from each other. The above relationship is also verified from the comparison of Eq. (28) with the one from the non-relativistic reduction of the space part of the Lagrangian Eq. (3),
\[ \mathcal{L} \approx -\frac{g_a - g_t}{2} \psi^\dagger \sigma_{\tau i} \psi \cdot \psi^\dagger \sigma_{\tau i} \psi. \] (43)

If we take both the pseudovector and tensor couplings, $\omega_{\pm}$ in Eq. (37) up to second order of $v_F$ is written as
\[ \omega_{\pm} \approx \alpha \left( \chi_a - \tilde{\chi}_a - \frac{2\chi_a - \chi_t}{3} v_F^2 \right), \] (44)
while $\omega_-$ is of fourth order,
\[ \omega_- \approx -\alpha \frac{2\chi_a \chi_t}{9 (\chi_a - \chi_t)} v_F^2. \] (45)

As mentioned in Sec. 1, the non-relativistic LM parameter works in a way that a half of $g'$ is for the $\pi$-meson exchange potential, and the rest is for the $\rho$-meson exchange one. In this sense, it may be reasonable to use the pseudovector and the tensor coupling with equal weight in relativistic models. In assuming that
\[ g_a = -g_t = \frac{1}{2} g' \left( \frac{f_\pi}{m_\pi} \right)^2 \] (46)
the above $\omega_-$ becomes
\[ \omega_- \approx \omega_{GT} \left( 1 - \frac{1}{2} v_F^2 \right). \] (47)

### IV. THE EXCITATION STRENGTH OF THE GT STATES

The GT strength is given by the imaginary part of the RPA correlation function. The RPA correlation function is written, using Eq. (7), as
\[ \Pi_{\text{RPA}}(\gamma_5 \gamma_2 \tau^-, \gamma_5 \gamma_2 \tau^+) = (U^{-1})_{1a} \Pi (\Gamma_{\alpha^-}, \gamma_5 \gamma_2 \tau^+). \]

The explicit expression of the dimesic function in Eq. (19) yields
\[ \Pi_{\text{RPA}}(\gamma_5 \gamma_2 \tau^-, \gamma_5 \gamma_2 \tau^+) = \frac{1}{\chi_a} \left( 1 - \frac{1 - \chi_t \Pi_{22}}{\det U} \right). \] (48)
The last term in the parentheses can be expressed in terms of the eigenvalues in the preceding section,

\[
\frac{1 - \chi_t \Pi_{22}}{\det U} = \frac{1}{1 - 2\kappa \chi_a} \left( 1 + \frac{\omega_a - \omega_-}{\omega_+ - \omega_-} \frac{\omega_a - \omega_-}{\omega_+ - \omega_-} q_0 - \omega_+ + i\varepsilon \right.
\]

\[
+ \frac{\omega_+ - \omega_a}{\omega_+ - \omega_-} q_0 - \omega_- + i\varepsilon \right) \right). 
\] (49)

The response function for the external field, \( \gamma_5 \gamma_2 \tau_+ \), therefore, is described as

\[
R_a(q_0) = \frac{3}{16\pi^2 k_F^2} A \Im \Pi_{RPA} \left( \frac{\omega_a - \omega_-}{\omega_+ - \omega_-} \delta(q_0 - \omega_+) + \frac{\omega_+ - \omega_a}{\omega_+ - \omega_-} \delta(q_0 - \omega_-) \right). 
\] (50)

The above equation provides us with the GT strengths of the two states with the excitation energy, \( \omega_+ \) and \( \omega_- \), respectively,

\[
S_+ = \frac{3}{16\pi^2 k_F^2} A \frac{1}{1 - 2\kappa \chi_a} \frac{\omega_a - \omega_-}{\omega_+ - \omega_-} \frac{\omega_a - \omega_-}{\omega_+ - \omega_-} \left( 1 - \frac{2}{3} v_F^2 \right) 2 (N - Z), 
\] (51)

\[
S_- = \frac{3}{16\pi^2 k_F^2} A \frac{1}{1 - 2\kappa \chi_a} \frac{\omega_+ - \omega_a}{\omega_+ - \omega_-} \frac{\omega_+ - \omega_a}{\omega_+ - \omega_-} \left( 1 - \frac{2}{3} v_F^2 \right) 2 (N - Z). 
\] (52)

If we take into account order up to \( v_F^2 \), they become

\[
S_+ \approx \frac{3}{16\pi^2 k_F^2} A \frac{\omega_a}{1 - 2\kappa \chi_a} = \frac{1}{1 - 2\kappa \chi_a} \left( 1 - \frac{2}{3} v_F^2 \right) 2 (N - Z), 
\] (53)

\[
S_- \approx 0. 
\] (54)

Thus, the strength of the GT state does not depend on the tensor coupling, up to \( v_F^2 \), although the excitation energy does, as shown in the preceding section. The sum of the two strengths in Eqs. (51) and (52) is given, without expansion in terms of \( v_F \), as

\[
S_+ + S_- = \int dq_0 R_a(q_0) = \frac{3}{16\pi^2 k_F^2} A \frac{\omega_a}{1 - 2\kappa \chi_a} = \frac{1}{1 - 2\kappa \chi_a} \left( 1 - \frac{2}{3} v_F^2 \right) 2 (N - Z), 
\] (55)

which is independent of the tensor coupling. If we discuss the GT strength in the nucleon space only, then the sum is independent of \( g_a \) also, because of \( \kappa = 0 \), and is quenched by the factor \( (1 - 2v_F^2/3) \), compared with IFF sum rule value \( 2(N - Z) \) in the present definition \[8, 10\]. The last equation is what we obtained in our previous paper \[8, 9, 11\].

It may be worthwhile noting the response function for the external field, \( \sigma_3 \tau_+ \). The RPA correlation function in this case can be written as

\[
\Pi_{RPA}(\sigma_3 \tau_-, \sigma_3 \tau_+) = - \frac{\kappa}{1 + \kappa \chi \ell} \left( \frac{\omega_- - \omega_-}{\omega_+ - \omega_-} q_0 - \omega_+ + i\varepsilon \right.
\]

\[
+ \frac{\omega_+ - \omega_-}{\omega_+ - \omega_-} q_0 - \omega_- + i\varepsilon \right). 
\] (56)

This gives the response function,

\[
R_\ell(q_0) = \frac{3}{16\pi^2 k_F^2} A \Im \Pi_{RPA} \left( \frac{\omega_- - \omega_-}{\omega_+ - \omega_-} \delta(q_0 - \omega_+) + \frac{\omega_+ - \omega_-}{\omega_+ - \omega_-} \delta(q_0 - \omega_-) \right). 
\] (57)

The sum of the strengths expanded in terms of \( (N - Z) \), therefore, is obtained as

\[
\int dq_0 R_\ell(q_0) = - \frac{3}{16\pi^2 k_F^2} A \frac{\omega_-}{1 + \kappa \chi \ell} = \frac{1}{(1 + \kappa \chi \ell)^2} \left( 1 - \frac{1}{3} v_F^2 \right) 2 (N - Z), 
\] (58)

which is independent of the pseudovector coupling. As expected, the relativistic correction in Eq. (58) is different from that in Eq. (55). Finally effects of the Dirac sea in the relativistic model should be mentioned in more detail. In the above calculations, effects of the Dirac sea are included in \( \kappa \) in Eq. (26), which comes from the Pauli blocking terms in NFA. Since its value is positive, the GT strength in Eqs. (53) and (55) is a little increased owing to the coupling of the particle-hole states with the nucleon-antinucleon ones. This increasing is, however, due to a poor approximation of NFA where the divergent term is simply neglected. The no-sea approximation, which has been extensively used so far, is essentially the same as NFA, and provides us with Eqs. (53) and (55), as shown in Ref. [10]. If we keep the divergent term in the RPA correlation function, \( \kappa \) in Eq. (26) is replaced by \( \kappa' \),

\[
\kappa' = \frac{2}{3} \int dp \frac{p^2}{E_p} \left( \theta^{(n)}_p + \theta^{(p)}_p - 2 \right). 
\] (59)

Thus, the divergent term has an opposite sign to that of the density-dependent part. This fact implies that if
we treat the divergent terms properly, the GT strength may be quenched more than in Eqs. (53) and (55). In this sense, the factor \((1 - 2v_F^2/3)\) yields the minimum quenching of the present relativistic model. We note that in nuclear matter the quenching due to the antinucleon-degrees of freedom is caused only through the pseudovector coupling. In finite nuclei, where the momentum is not a good quantum number, there may be a small contribution from the tensor coupling. It is important future work to investigate how much the quenching is increased by the coupling of the particle-hole states with the nucleon-antinucleon states. In that case, it may be also required to take into account the energy-dependence of the LM parameter which has not been well studied yet.

V. CONCLUSIONS

In the previous papers \([8, 9]\), we investigated the excitation energy and strength of the giant Gamow-Teller (GT) states in the relativistic model by introducing the Landau-Migdal (LM) parameter \(g_\pi\) with the pseudovector coupling. The pseudovector coupling is chosen so as to reproduce the non-relativistic LM parameter \(g'\) in the non-relativistic limit. The main conclusions were that the relativistic correction to the excitation energy is 14% and that the GT strength in the nucleon sector is quenched by 12% in nuclear matter, compared with the Ikeda-Fujii-Fujita (IFF) sum rule value. The quenching factor is given by \((1 - 2v_F^2/3)\), \(v_F\) being the Fermi momentum. In finite nuclei, the quenched amount has been estimated to be about 6% in Ref. \([8, 9]\). The reduction of the quenching is due to the larger effective mass near the nuclear surface than in nuclear matter. Among the above results, the quenching of the strength is especially important. This prediction is a rare example to distinguish the relativistic model from conventional non-relativistic models, and is related to an important observation of the recent experiment \([12]\).

Recently, Tokyo group has observed 10% quenching of the IFF sum rule value in \(^{90}\text{Nb}\) through charge-exchange reaction \([12]\). Since nuclear models assuming the nucleus to be composed of nucleons satisfy the model-independent sum rule, the observed quenching implies that additional degrees of freedom play a role to reduce the GT strength. So far, one candidate has been proposed, which is the \(\Delta\)-degrees of freedom \([2, 4]\). Particle-hole states are assumed to couple with \(\Delta\)-hole states through the Landau-Migdal (LM) parameter \(g'_{N\Delta}\). Since the coupling force is expected to be repulsive, a part of the strength in the nucleon space is taken by the high excited states. Of course, it depends on the value of \(g'_{N\Delta}\) how much strength is taken out from the low excitation energy region.

The relativistic model predicted another source of the quenching. If the 6% quenching is due to the relativistic effect, as estimated in the previous papers \([8, 10]\), then it is about 4% what is left for the coupling with the \(\Delta\)-hole states. In this case, the value of \(g'_{N\Delta}\) becomes less than 0.2, which is much smaller than the one estimated before \([2, 4, 10]\). The small value changes our previous understanding of the spin-dependent structure of nuclei. For example, the critical density of the pion condensation becomes less than two times of the normal density \([20]\). Thus, it was required to investigate model-dependence of the previous result \([8, 9]\) in the relativistic model.

In the present paper, we have explored whether or not the LM parameter with the tensor coupling changes the previous conclusions. The tensor coupling is another candidate which can reproduce the non-relativistic result of the GT states. We have described the GT states in the two cases. One is to use the tensor coupling instead of the pseudovector coupling. The other is to take into account both pseudovector and tensor couplings.

If we use the only tensor coupling, the relativistic effect on the excitation energy is reduced by a half, as shown in Eq. (44), while the quenching of the GT strength remains in the same way as in other cases. This fact is shown by the factor \((1 - 2v_F^2/3)\) in Eq. (55).

In conclusion, the pseudovector and tensor coupling play a role to determine the excitation energy of the GT state in a similar way, but do not change the previous conclusion \([8, 9]\) that the GT strength in the relativistic model is quenched by 12% in nuclear matter and by 6% in finite nuclei, compared with the IFF sum rule value. The quenched amount may be increased by the coupling of the particle-hole states with nucleon-antinucleon states through the pseudovector coupling, as discussed at the end of the preceding section. The present result is consistent with the recent experiment \([12]\), but in future work, other spin-dependent structure of nuclei, like the pion condensation and response functions of the charge-exchange reaction \([21]\), should be understood consistently. It is also important to study the present relativistic effects on the \(\beta\) decay for its precise discussions in neutrino physics.

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