Hadron-quark phase transition: the QCD phase diagram and stellar conversion

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Abstract. Different extensions of the Nambu-Jona-Lasinio model, known to satisfy expected QCD chiral symmetry aspects, are used to investigate a possible hadron-quark phase transition at zero temperature and to build the corresponding binodal sections. We have shown that the transition point is very sensitive to the model parameters and that both pressure and chemical potential increase drastically with the increase of the vector interaction strength in the quark sector. Within the same framework, the possibility of quark and hybrid star formation is analyzed. The same conclusions drawn before with respect to the coexistence pressure and chemical potentials are reinforced. We conclude that even if a transition from a metastable hadronic star to a quark star is thermodynamically possible, it is either energetically forbidden or gives rise to a black hole. Nevertheless, conversions from metastable to hybrid stars are possible, but the mass difference between both compact objects is very small, never larger than 0.2 $M_\odot$.

Keywords: baryon asymmetry, massive stars, neutron stars

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1 Introduction

The complete understanding of the quantum chromodynamics (QCD) phase diagram represents a challenge in both theoretical and experimental physics. While many of the features it aims to describe can be tested in heavy-ion collision experiments, other aspects related to matter under extreme conditions can only be inferred from results of lattice QCD (LQCD) [1] or from observational results of astrophysical objects.

Effective models remain a good source of information about regions of the QCD phase diagram inaccessible by terrestrial experiments or by LQCD methods, providing qualitative results and theoretical insights. The present work intends to help in advancing our knowledge towards some of the regions of the QCD phase diagram through this strategy. When this approach is applied to the study of the transition of hadronic matter to the deconfined quark matter, it is suggested that the QCD phase diagram shows a first order phase transition [2].

From the LQCD perspective, the hadron-quark transition at zero chemical potential is believed to be a crossover. However the whole diagram is not expected to be covered within this approach in a near future, due to some numerical difficulties as the sign problem and to the huge computational cost, so that only the region close to zero chemical potential has been assessed so far. Effective models, such as the NJL [3] and PNJL [4], have, therefore, been used to study the phase diagram at high chemical potential. Within these models, it is expected that the crossover at zero chemical potential and high temperature goes into a first order phase transition at high chemical potential and low temperature [5, 6].

This idea is reinforced by experimental results, such as the Beam Energy Scan (BES-I, BES-II) programs at RHIC that have provided data signaling to a first order phase transition and pointed out the possible existence and location of the critical end point [7]. Future experiments to take place at the FAIR facility at GSI [8], NICA at the JINR [9] and the NA61/SHINE program at SPS (CERN) [10] will also contribute to shed some light into these still unknown aspects. Also, the improvement of the observational results of compact objects are expected from the new paradigm of multimessenger astronomy established by the gravitational waves detection of compact star mergers [11], and from x-ray telescopes, such as NICER [12] and, in the future, Athena [13], which will provide better constraints to the effective model parameterizations and thus contribute to the understanding of the low...
temperature and high density regime of the QCD phase diagram. A comprehensive historical perspective on the nuclear astrophysics aspects in the new era of multimessenger astronomy can be found in [14].

It is argued that the phase transition in QCD can take place in two different steps at low temperatures, first by the (partial) restoration of the chiral symmetry where chromodynamic matter is still confined, giving rise to the so-called quarkyonic phase [15], and only then by the deconfinement phase transition.

Considerations on the phase transition at zero temperature have already been done in many works [16–21], but we do believe the formalism we present next is more adequate, because the effective models employed here exhibit chiral symmetry in both hadronic and quark phases, which is demanded to take seriously the appearance of the quarkyonic phase. The models to be used here are all included in the Nambu-Jona-Lasinio (NJL) model framework [3] in order to naturally describe the chiral characteristics of QCD matter.

The two EoS model has been applied to the study of the hadron-quark phase transition in several studies, [18, 21–25]. In particular, in [24] the effect of the vector contribution in the quark-matter description was considered. In this studies the authors have discussed the hadron-quark phase transition taking for nuclear matter a RMF model and for quark matter the PNJL model including vector terms. A review of the effect of the symmetry energy on the hadron-quark phase transition was presented in [25] where the possible signatures of the hadron-quark phase transition in the range of the NICA program are also discussed. It is the aim of the present work to further study this phase transition taking the two EoS model but using, for the first time, the hadron and the quark phase chiral symmetric models for both EoS. This will be an exploratory study and, therefore, we will restrict ourselves to zero temperature.

In [18] and [21], the hadron-quark phase transition was investigated with the help of two different models, namely, the non-linear Walecka model (NLWM) for the hadronic phase and the MIT bag model for the quark phase. A formalism we understand as a more adequate one was used in [19, 26, 27] at zero temperature and in [20] for finite temperatures, all considering NJL-type models for the two phases. To describe the hadron phase, the standard NJL model with vector interaction is extended to include a scalar-vector channel in order to render the model capable of saturation at low densities [28]. In the present work, we revisit the approach of [19] and [20], but applying an extended NJL model for the hadron phase that includes additional channels to achieve a better description of important nuclear bulk properties [29]. Another extension of the NJL model for hadronic matter has been developed [30, 31] with a different choice of interaction channels. Recently, this version was also applied to investigate the hadron-quark phase transition [32], but the quark phase was still described by the MIT bag model. In [29], a special attention was paid to the correct description of the symmetry energy of nuclear matter. This was achieved by including a mixed vector-isovector–vector-isoscalar term or a mixed scalar-isoscalar–vector-isovector term in the Lagrangian density. The couplings associated with these terms were fixed so that neutron matter properties obtained from microscopic calculations based on chiral effective theories [33] and quantum Monte Carlo results [34] were reproduced. The symmetry energy within a similar model, including mixed vector-isovector–vector-isoscalar term or mixed scalar-isoscalar–vector-isovector terms, was also investigated in [35, 36]. When models proposed in [35, 36] are applied to the description of neutron stars, maximum mass configurations with a mass above 2M⊙ are obtained, however, for neutron Fermi momenta above the cut-off of the model. Albeit the authors of the afore mentioned papers also refer
to the hadronic extension as eNJL, here we name PPM NJL the extension developed in [29], to avoid confusion with the simpler versions proposed in [28] and [30, 31], or the alternative parameterizations obtained in [35, 36].

Hence, in the present work, we describe the hadronic matter with the PPM NJL model and the quark matter with the NJL in its SU(2) version in order to check for which parameters of these two models the phase transition is possible, considering both symmetric and asymmetric systems. Whenever possible, the binodal sections are obtained. We include in the quark model a vector contribution that has proved to make the quark EoS stiffer and may have important consequences on the structure of hybrid or pure quark compact stars [37–40]. In particular, the inclusion of this term gives rise to larger star masses although with smaller quark cores in the case of hybrid stars. The quark models that we use to describe the deconfined phase have been proposed in [41, 42] and in [40]. In particular, we are interested in testing quark models with a low vacuum quark constituent mass, which, according to [40] favors the appearance of a quark core in the center of neutron stars.

In the sequel, as an application of our two phase model and to compare our calculations with already existing results in the literature we impose $\beta$-equilibrium and charge neutrality conditions to obtain equations of state (EoS) for both phases in order to investigate the possibility of a hadron-quark phase transition to occur in the interior of compact stars. For this study, we consider the SU(3) version of the NJL model for the quark phase so that the strangeness demanded by the Bodmer-Witten conjecture for the stability of quark matter [43, 44] is considered, although we are aware that this model does not produce absolutely stable matter at zero temperature. We discuss this in more detail during the presentation of the results. These EoS are then applied to describe respectively hadronic, hybrid and quark stars to check when the former is metastable to decay into the two latter and the possibility of a hadron-quark conversion inside these objects is checked.

This paper is organized as follows: in section 2 we describe the basic formalism necessary to the understanding of the NJL model in all versions we need. In section 3 we discuss how to obtain the binodal points at zero temperature, display and comment our results. In section 4, we investigate if a hadron-quark phase transition can take place inside a compact star, the existence of metastable hadronic stars and the possible conversion to stable quark stars. Finally, in the last section, we make some final remarks and discuss the continuation of the present work.

2 Formalism

In the following we present the basic equations underlying the NJL models in three different versions: the usual SU(2) and SU(3) versions that describe quark matter and the PPM version of the NJL model that describes hadronic matter.

2.1 Quark matter — NJL SU(2)

The quark phase is described by a SU(2) NJL model Lagrangian including a vector term, given by [42]

$$\mathcal{L} = \bar{\psi}(i\gamma^\mu \partial_\mu - \hat{m}_0)\psi + G_s[(\bar{\psi}\psi)^2 + (\bar{\psi}\gamma_5 \overrightarrow{\tau}\psi)^2] - G_v(\bar{\psi}\gamma^\mu \psi)^2.$$  (2.1)

Here $\psi$ represents the quark field, $\hat{m}_0$ the quark bare mass, and $G_s$ and $G_v$ are coupling constants that are fitted by the pion mass $m_\pi = 135.0$ MeV and its decay constant $f_\pi = \ldots$
92.4 MeV. As a non-renormalizable theory, a momentum cutoff \( \Lambda \) must be employed in the momenta, which acts as a new free parameter of the model. In table 1, four possible parameter sets usually considered in the literature are given. Notice that the \( G_v \) parameter can be arbitrarily chosen, allowing to write \( G_v = x G_s \), where \( x \) is a free parameter varying in the range \( 0 \leq x \leq 1 \) [45]. Furthermore in text the parameterization choice is written as, e.g., PCP-0.1, which reads as the PCP parameter set taken with \( x=0.1 \).

From the Lagrangian \( \mathcal{L} \), one can obtain the thermodynamic potential per volume \( V \) at temperature \( T \) through the Hamiltonian density \( \mathcal{H} \), which leads to

\[
\Omega(T, \mu) = -\frac{T}{V} \ln \text{Tr} \exp \left[ -\frac{1}{T} \int d^3x (\mathcal{H} - \mu \psi \bar{\psi}) \right],
\]

where \( \text{Tr} \) stands for the trace over all states of the system, resulting in [41, 42]

\[
\Omega(M, \tilde{\mu}) = \sum_{i=u,d} \Omega_{M_i}(T, \mu_f) + G_s (\phi_u + \phi_d)^2 - G_v (\rho_u + \rho_d)^2,
\]

with, at the zero temperature limit in the mean-field approximation,

\[
\Omega_{M_i} = -2 N_c \int \frac{d^3p}{(2\pi)^3} \left[ E_p + (\tilde{\mu}_i - E_p) \theta(p_{F_i} - p) \right],
\]

\( \mu \) stands for the chemical potential and \( M_i \) are the quarks constituent masses. Here \( N_c \) stands for the number of colors and \( E_p = \sqrt{p^2_f + M_i^2} \). The Fermi momentum of the quarks is represented by \( p_{F_i} \) and \( \theta(p_{F_i} - p) \) stands for the step function.

The renormalized chemical potential \( \tilde{\mu}_i \) and the constituent mass \( M_i \) are respectively obtained by requiring \( \partial \Omega / \partial \tilde{\mu}_i = 0 \) and \( \partial \Omega / \partial M_i = 0 \), resulting in

\[
\tilde{\mu}_i = \mu_i - 2G_v \rho, \quad i = u, d,
\]

\[
M_i = m - 2G_s \phi, \quad i = u, d,
\]

with \( \rho = \rho_u + \rho_d, \phi = \phi_u + \phi_d, \) and

\[
\rho_i = \frac{N_c}{3\pi^2} p^3_{F_i},
\]

\[
\phi_i = \langle \bar{\psi} \psi \rangle = -2 N_c \int_0^\Lambda \frac{d^3p}{(2\pi)^3} \frac{M_i}{E_p} \left[ 1 - \theta(p_{F_i} - p) \right],
\]

\[
\tilde{\mu}_i = \sqrt{p^2_{F_i} + M_i^2}.
\]

A constant term in the potential has no physical meaning, consequently such term may be chosen so that the thermodynamic potential is zero at the value \( M = M_{\text{vac}} \) which minimizes \( \Omega \) at \( T = \mu = 0 \). This process may be represented by

\[
\bar{\Omega}(\mu; M, \tilde{\mu}) = \Omega(\mu; M, \tilde{\mu}) - \Omega(0, 0; M_{\text{vac}}, 0),
\]

so that the pressure \( P \) and the energy density \( \varepsilon \), the quantities we are interested in, are obtained through

\[
P = -\bar{\Omega}(\mu; M, \tilde{\mu}),
\]
\begin{table}[h]
\centering
\begin{tabular}{|c|c|c|c|c|c|c|c|c|c|}
\hline
Model & \(\Lambda\) & \(G_s (\text{fm}^2)\) & \(G_v (\text{fm}^2)\) & \(m_0\) (MeV) & \(M\) (MeV) \\
\hline
Buballa-1 & 650 & 0.19721 & — & 0 & 313 \\
Buballa-2 & 600 & 0.26498 & — & 0 & 400 \\
BuballaR-2 & 587.9 & 0.27449 & \(\propto G_s\) & 5.6 & 400 \\
PCP & 648 & 0.19565 & \(\propto G_s\) & 5.1 & 312.6 \\
\hline
\end{tabular}
\caption{Parameter sets for the SU(2) NJL Lagrangian density (2.1) [40–42].}
\end{table}

\begin{table}[h]
\centering
\begin{tabular}{|c|c|c|c|c|c|c|c|c|c|}
\hline
Model & \(G_s\) & \(G_v\) & \(m_{u,d}\) (MeV) & \(m_s\) (MeV) & \(M_{u,d}\) (MeV) & \(M_s\) (MeV) \\
\hline
HK & 631.4 & 1.93 & 2.09 & 135.7 & 335.5 & 528 \\
PCP & 630.0 & 1.781 & 2.09 & 135.7 & 312.2 & 508 \\
\hline
\end{tabular}
\caption{Parameter sets for the SU(3) NJL Lagrangian density (2.15) [40, 46].}
\end{table}

\begin{table}[h]
\centering
\begin{tabular}{|c|c|c|c|c|c|c|c|c|c|}
\hline
Model & \(G_s\) & \(G_v\) & \(G_{sv}\) & \(G_{ho}\) & \(G_{v\rho}\) & \(G_{s\rho}\) & \(\Lambda\) (MeV) & \(m\) (MeV) \\
\hline
eNJL3 & 1.93 & 3.0 & -1.8 & 0.0269 & 0 & 0.5 & 534.815 & 0 \\
eNJL2 & 1.078 & 1.955 & -2.74 & -0.1114 & 0 & 1 & 502.466 & 500 \\
\hline
\end{tabular}
\caption{Parameter sets for the PPM NJL Lagrangian density (2.22) [29]. (\(G_s\), \(G_v\), and \(G_{sv}\) values are in \(\text{fm}^2\); \(G_{ho}\), \(G_{v\rho}\), and \(G_{s\rho}\) in \(\text{fm}^8\).)}
\end{table}

\[\varepsilon = -P + \sum_{i=u,d} \mu_i \rho_i, \quad (2.12)\]

respectively resulting in

\[P = 2N_c \sum_{i=u,d} \left[ \int_{p_{F_i}}^{\Lambda} \frac{d^3p}{(2\pi)^3} E_p \left( \frac{\tilde{\mu}_i p_{F_i}}{6\pi^2} \right) \right.\]
\[\left. - G_s (\phi_u + \phi_d)^2 + G_v (\rho_u + \rho_d)^2 + \Omega(0, 0; M_{\text{vac}}, 0) \right] (2.13)\]

and

\[\varepsilon = 2N_c \sum_{i=u,d} \left[ - \int_{p_{F_i}}^{\Lambda} \frac{d^3p}{(2\pi)^3} E_p \left( \mu_i - \tilde{\mu}_i \right) \frac{p_{F_i}}{6\pi^2} \right] \left. + G_s (\phi_u + \phi_d)^2 - G_v (\rho_u + \rho_d)^2 - \Omega(0, 0; M_{\text{vac}}, 0). \right] (2.14)\]

The equations are then solved self-consistently for each value of \(\rho_i\) or \(\mu_i\), noticing that one has \(p_{F_i} = \sqrt{\tilde{\mu}_i^2 - M_i^2}\) for \(\tilde{\mu}_i^2 \geq M_i^2\) in this case. All the parameters are presented in table 1.

2.2 Quark matter — NJL SU(3)

Dense matter in a quark phase can also be described with the SU(3) version of NJL model, which incorporates the \(s\)-quark, with the repulsive vector interaction. In this case, the Lagrangian density is given by [40, 47]

\[\mathcal{L} = \bar{\psi} (i\gamma_\mu \partial^\mu - \hat{m}_f) \psi + \mathcal{L}_{\text{sym}} + \mathcal{L}_{\text{det}} + \mathcal{L}_{\text{vec}}, \quad (2.15)\]
with $\mathcal{L}_{\text{sym}}$, $\mathcal{L}_{\text{det}}$ and $\mathcal{L}_{\text{vec}}$ given by

$$
\mathcal{L}_{\text{sym}} = G_s \sum_{a=0}^{8} \left[ (\bar{\psi} \lambda_a \psi)^2 + (\bar{\psi} i \gamma_5 \lambda_a \psi)^2 \right],
$$

$$
\mathcal{L}_{\text{det}} = -K \{ \det[\bar{\psi}(1 + \gamma_5)\psi] + \det[\bar{\psi}(1 - \gamma_5)\psi] \},
$$

$$
\mathcal{L}_{\text{vec}} = -G_v (\bar{\psi} \gamma^\mu \psi)^2,
$$

where $\psi(u, d, s)$ represents the three flavor quark field, $\hat{m}_f = \text{diag}(m_u, m_d, m_s)$ is the quark current mass matrix, $\lambda_0 = \sqrt{2/3}I$ where $I$ is the U(3) unit matrix, and $\lambda_a$, with $a = 1, \ldots, 8$, are the Gell-Mann flavor matrices.

To obtain effective quark masses $M_i$ we must minimize the thermodynamic potential given by

$$
\Omega(\mu; M, \tilde{\mu}) = \sum_{i=u,d,s} \Omega_{M_i}(T, \mu_f) + 2G_s(\phi_u^2 + \phi_d^2 + \phi_s^2) - 4K\phi_u \phi_d \phi_s - 2G_v(\rho_u + \rho_d + \rho_s)^2,
$$

with, at the zero temperature limit in the mean-field approximation,

$$
\Omega_{M_i} = -2N_c \int_{p_F^i}^{\Lambda} \frac{d^3p}{(2\pi)^3} \frac{p^2 + m_i M_i}{E_p^i}
$$

where $\phi_i$ and $\rho_i$ are the same as in the SU(2) case. The renormalized chemical potential $\tilde{\mu}_i$ is given by

$$
\tilde{\mu}_i = \mu_i - 2G_v \rho, \quad \rho = \rho_u + \rho_d + \rho_s,
$$

where $i$ refers to the flavor and $\rho_i$ refers to the respective quark number density. Thus, minimizing $\Omega$, we obtain in mean-field approach the following gap equations

$$
M_i = m_i - 4G_s \phi_i + 2K \phi_j \phi_k,
$$

with $(i, j, k)$ being any permutation of $(u, d, s)$.

Then, using (2.11) and (2.12), the pressure and energy density may be written as

$$
P = 2N_c \sum_{i=u,d,s} \left[ \int_{p_F^i}^{\Lambda} \frac{d^3p}{(2\pi)^3} \frac{p^2 + m_i M_i}{E_p^i} \right] - 2G_s(\phi_u^2 + \phi_d^2 + \phi_s^2) + 2K \phi_u \phi_d \phi_s$$

$$
+ 2G_v(\rho_u + \rho_d + \rho_s)^2 + \Omega(0, 0; M_{\text{vac}}, 0)
$$

$$
\varepsilon = 2N_c \sum_{i=u,d,s} \left[ \mu_i \rho_i - \int_{p_F^i}^{\Lambda} \frac{d^3p}{(2\pi)^3} \frac{p^2 + m_i M_i}{E_p^i} \right] + 2G_s(\phi_u^2 + \phi_d^2 + \phi_s^2) - 2K \phi_u \phi_d \phi_s$$

$$
- 2G_v(\rho_u + \rho_d + \rho_s)^2 - \Omega(0, 0; M_{\text{vac}}, 0),
$$

The parameter sets used for the SU(3) NJL are shown in table 2, where again $G_v = xG_s$. 


where the kinetic energy contribution is given by
\begin{equation}
\dot{\rho} = \frac{\partial}{\partial p} \left[ \frac{1}{2} \sum_{i=p,n} \mu_i \right] \frac{p_i^2}{E_p} \left[ 1 - \theta(p_{F_i} - p) \right],
\end{equation}
and 
\begin{equation}
M = m - 2G_s \phi + 2G_{sv} \phi \rho_B^2 + 2G_{sp} \phi \rho_3^2,
\end{equation}
where \( \phi = \phi_p - \phi_n \) and \( \rho_B = \rho_p + \rho_n \).

The thermodynamic potential is obtained from (2.22) in the same way as for the quark case, and is given by
\begin{equation}
\Omega(\mu) = \varepsilon_{\text{kin}} + M \phi - \mu_p \rho_p - \mu_n \rho_n - G_s \phi^2 + G_v \rho_B^2 + G_{sv} \phi \rho_B^2 + G_{sp} \phi \rho_3^2 + G_{s \omega} \rho_B^2 + G_{s \rho} \rho_3^2 + G_{s \phi} \rho^2_3 \rho_3^2.
\end{equation}

The effective mass \( M \) and the chemical potentials appearing in the thermodynamic potential \( \Omega \) are also here determined by requiring that \( \partial \Omega / \partial M = 0 \) and \( \partial \Omega / \partial p_{F_i} = 0 \), resulting in
\begin{equation}
\mu_i = \frac{E_i^2}{p} + 2G_v \rho_B + 2G_{sv} \rho_B \phi^2 + 2G_{sp} \rho_3 \phi_3^2 + 2G_{s \omega} \rho^2_B + 2G_{s \rho} \rho_3 \phi_3^2 + 2G_{s \phi} \rho^2_3 \phi_3^2,
\end{equation}
where \( \mu_i \) stands for \( i = p \) and \( \mu_i \) stands for \( i = n \), and \( E_i^2 = M^2 + (p_{F_i})^2 \). The equations of state can then be obtained using (2.11) and (2.12), taking \( M_{\text{vac}} = m_N \) representing the nucleon mass. More details on the PPM NJL model and its parameterizations can be obtained in [29].
3 Binodals

Before we start our discussion on the phase transition itself, we display in figure 1, the EoS of both phases for two specific choices of parameters. The discontinuities are related to the points where chiral symmetry is restored and the points where the pressure becomes negative are omitted. In table 4 we show, for each of the quark and the hadronic parameterizations used in the present work, the density and chemical potential for which chiral symmetry is restored.

The QCD phase-diagram is characterized by potentially multiple phases, whose phase separation boundaries are referred as binodals [48]. Over those boundaries, the phases from the regions of either side of the boundary can coexist. The binodals may be determined using the Gibbs conditions [18]:

\[
\begin{align*}
\mu_Q^B &= \mu_H^B, \\
T_Q &= T_H, \\
P_Q &= P_H,
\end{align*}
\]

where the indexes \( H \) and \( Q \) refer to the hadronic and quark phases. The chemical potentials are given by

\[
\begin{align*}
\mu_H^B &= \frac{\mu_p + \mu_n}{2}, \\
\mu_Q^B &= \frac{3}{2}(\mu_u + \mu_d) = 3\mu_q.
\end{align*}
\]

At a certain fixed temperature (\( T = 0 \) in the present context), the phase coexistence condition may be obtained by plotting \( P^i \times \mu_B^i \), \( i = Q, H \), and looking for the intersection of both curves. See figure 2 for an example, where the hadron pressure given by the eNJL3\( \sigma_p1 \) parameterization is plotted together with the quark pressure given by the PCP parameterization of the NJL model for several choices of the vector interaction strength \( x \) such the...
Table 4. Values of $\rho_B$ and $\mu_B^i$ at the onset of chiral restoration for different parameterizations. $i=H,Q$.

| Set       | $\rho_B$ (fm$^{-3}$) | $\mu_B$ (MeV) |
|-----------|------------------------|----------------|
| Buballa-1 | 0.27                   | 941            |
| Buballa-2 | 0.36                   | 1035           |
| PCP-0.0   | 0.29                   | 1005           |
| PCP-0.1   | 0.24                   | 1011           |
| PCP-0.2   | 0.20                   | 1020           |
| PCP-0.3   | 0.17                   | 1032           |
| PCP-0.4   | 0.17                   | 1047           |
| PCP-0.5   | 0.17                   | 1059           |
| eNJL3$\sigma\rho_1$ | 1.0               | 1674           |
| eNJL2$m\sigma\rho_1$ | 1.0               | 1568           |

Figure 2. Examples of combinations of parameter sets for which hadron-quark phase transitions are allowed to happen in symmetric matter.

Figure 3. Example of a combination of parameter sets for which hadron-quark phase transitions are not allowed to happen in symmetric matter.

The coexistence of the hadron and the quark phases occurs, allowing the phase transition to happen. Otherwise, the absence of intersections imply that there are no phase transitions allowed between the phases considered within a specific pair of models, as shown in figure 3, where the hadronic matter is always more stable. The existence of a hadron-quark phase transition depends on both the quark matter and hadronic matter EoS: the same quark matter EoS, PCP-0.2, that predicts a phase coexistence with one hadronic EoS (eNJL3$\sigma\rho_1$) ceases to predict with a different one.

We determine the value of chemical potential $\mu_B^i$ for which the phase transition takes place for all combinations of parameter sets given in tables 1 and 3. At this point, we still restrict our treatment of the hadron phase to symmetric matter due to the fact that in our treatment of the quark phase the proportion of $u$ and $d$ quarks is always 50% of each particle. This reflects the fact that both particles are assumed to have the same bare masses and the
same chemical potentials. The results so obtained are displayed in Table 5. In particular we may note that no combinations involving the BuballaR-2 set with $G_v \neq 0$ give rise to a phase transition. It should be pointed out the large differences among the chemical potential and density at the hadron-quark transition predicted by the models considered. Compatibility constraints between the hadronic and quark model should be imposed when describing the hadron-quark phase transition within a two-model description, which may reduce the phase transition uncertainties. In the present study, chiral symmetry is present in both the hadron and quark model. Several compatibility constraints could be considered: i) the quark phase should not be in a chiral broken phase at deconfinement if the hadronic phase is already in a chiral symmetric phase. This condition is fulfilled for all cases discussed above. ii) a more restrictive constraint would be that at deconfinement the hadron and the quark phase have the same chiral symmetry. From tables 4 and 5, we may conclude that for symmetric matter only quark models that predict a deconfinement chemical potential above 1674 (1568) MeV are compatible with eNJL3$\sigma\rho$1 (eNJL2$m\sigma\rho$1), i.e. Buballa-2, Buballa-R2, and PCP-0.2; iii) however, we may also interpret that the deconfinement coincides with chiral symmetry restoration. Moreover, in fact the eNJL2$m\sigma\rho$1 model has no chiral symmetric phase because this is a model with a term breaking explicitly the chiral symmetry, and the chemical potential indicated corresponds to half the vacuum mass. In this scenario the mixed phase between a pure hadronic and a pure quark matter phase would be constituted by clusters of non-chiral symmetric hadronic matter in a background of chiral symmetric quark matter, or the other way around; iv) for asymmetric matter the possible scenarios are much more complex because two or more conserved charges may be considered, and the restoration of chiral symmetry will occur at different baryonic densities or chemical potentials for different species. In the following our discussion is based on interpretation iii) and we do not discuss different scenarios corresponding to point iv).

Also constraints coming from experiments are needed to reduce the uncertainty between models. One possibility is to use freeze-out information.

Up to this point, as stated previously, only symmetric matter was considered, since the masses and chemical potentials of the $u$ and $d$ quarks in the SU(2) NJL are identical. However, the conditions of phase coexistence are also important in asymmetric matter and to obtain the binodal sections as a function of the system asymmetry, we use the prescription

| NJL SU(2) | Hadronic | $\mu_B$ (MeV) | $P$ (MeV/fm$^3$) | $\rho_B$ (fm$^{-3}$) |
|-----------|----------|---------------|------------------|-------------------|
| Buballa-2 | eNJL2$m\sigma\rho$1 | 1674 | 504 | 1.420 |
|           | eNJL3$\sigma\rho$1 | 1567 | 356 | 0.812 |
| BuballaR-2 | eNJL2$m\sigma\rho$1 | 1729 | 586 | 1.497 |
|           | eNJL3$\sigma\rho$1 | 1585 | 373 | 0.839 |
| PCP-0.0  | eNJL2$m\sigma\rho$1 | 1312 | 158 | 0.506 |
|           | eNJL3$\sigma\rho$1 | 1348 | 185 | 0.553 |
| PCP-0.1  | eNJL3$\sigma\rho$1 | 1544 | 336 | 0.780 |
| PCP-0.2  | eNJL3$\sigma\rho$1 | 1787 | 576 | 1.088 |

Table 5. Chemical potential, pressure, and barionic density at the coexistence point for different parameterization combinations in symmetric matter. $\rho_B$ refers to the hadronic phase. For BuballaR-2, in this table, $G_v = 0$ and eNJL2$m\sigma\rho$1 presents no chiral symmetric phase (see the text for details).
Figure 4. Baryonic chemical potentials at the coexistence point as a function of $\mu_3$ for: BuballaR-2 and eNJL2m$\rho_1$ (black line), BuballaR-2 and eNJL3$\sigma\rho_1$ (blue long-dashed line), PCP-0.0 and eNJL2m$\rho_1$ (red short-dashed line), PCP-0.0 and eNJL3$\sigma\rho_1$ (black double-dot dashed line), PCP-0.1 and eNJL3$\sigma\rho_1$ (magenta long-dash dotted line), and PCP-0.2 and eNJL3$\sigma\rho_1$ (orange short-dash dotted).

Figure 5. Pressure at the coexistence points for different asymmetry parameters using PCP-$x$ for the quark phase and eNJL3$\sigma\rho_1$ for the hadron phase. The thick (internal) curve corresponds to the hadron phase, while the external one corresponds to the quark phase. The lines corresponds to: PCP-0.0 (black double-dot dashed), PCP-0.1 (magenta long-dash dotted line), and PCP-0.2 (orange short-dash dotted).

given in [16]. The isospin chemical potentials are defined as

$$\mu^H_3 = \mu_p - \mu_n,$$

$$\mu^Q_3 = \mu_u - \mu_d,$$

and enforced to be identical according to the Gibbs conditions. The asymmetry parameters of the hadron and quark phases are respectively

$$\alpha^H = \frac{\rho_n - \rho_p}{\rho_n + \rho_p}, \quad \alpha^Q = \frac{3\rho_d - \rho_u}{\rho_d + \rho_u},$$

in such a way that $0 \leq \alpha^H \leq 1$ (just nucleons) and $0 \leq \alpha^Q \leq 3$ (just quarks).

To obtain the binodals we choose values of $\mu_B$ and $\mu_3$, which determine the proton and neutron chemical potentials and, through equation (3.1), the chemical potentials of the quarks. The $\mu_3$ parameter directly controls the proton fraction of both phases. For each pair of values ($\mu_B, \mu_3$) we test the difference in pressure of both phases. If this difference is smaller than a tolerance of 0.1 MeV, we assume that there is a phase transition. This procedure leads to the binodals shown in figures 4 and 5. The pressures shown for $\alpha = 0$ in figure 5 correspond to the intersections marked in figure 2.
From table 5 and figure 5, we can clearly see that the increase in the value of $G_v$ causes a substantial modification on the transition point, which reflects in the values of the pressure in the binodal sections.

As clearly stated in the Introduction, our aim is to obtain the QCD phase diagram with both hadronic and quark models based on the same underlying formalism, i.e., within different versions of the NJL model. The binodal section at zero temperature is the first step, but the inclusion of temperature and eventually, magnetic field will be performed. We next make a simple application of the phase transition to stellar matter to compare the results obtained with our formalism with the ones already existing in the literature.

4 Metastable stars

In order to describe compact star matter, leptons are included and electric charge neutrality and chemical equilibrium must be taken enforced. Leptons are introduced in the system by adding them in the model Lagrangian density as a free fermionic Lagrangian, i.e.,

$$\mathcal{L} = \bar{\psi}_l (i\gamma^\mu \partial_\mu - m_l) \psi_l,$$

(4.1)

where $l$ refers to the leptons, and unless stated otherwise, electrons and muons are considered, whose masses are, respectively, 0.511 MeV and 105.66 MeV. Thus, the following constraints on chemical potential and baryonic number density have to be imposed for hadronic star matter

$$\mu_n = \mu_p + \mu_e,$$

(4.2)

$$\rho_p = \rho_e + \rho_\mu,$$

(4.3)

and similarly, for quark star matter,

$$\mu_s = \mu_d = \mu_u + \mu_e,$$

(4.4)

$$\rho_e + \rho_\mu = \frac{1}{3}(2\rho_u - \rho_d - \rho_s).$$

(4.5)

In both cases, $\mu_e = \mu_\mu$.

We next study the possibility of a hadron-quark phase transition to take place in the interior of compact stars. Thus, we consider the EoS obtained from the model presented in section 2.3, with $\beta$-equilibrium and electric charge neutrality enforced, in the description of hadronic stars. As for the quark matter, we consider the EoS derived in section 2.2 to describe deconfined quark stars, also imposing $\beta$-equilibrium and electric charge neutrality. During the hadron-quark phase transition process, the composition of quark matter is not expected to be $\beta$-stable [49]. However, as we are mainly interested in the energetical content of the final quark or hybrid star, this intermediate stage is disregarded in what follows. It is worth noting that, generally, the use of the quark NJL model does not predict a pure quark core inside hybrid stars. This feature can be circumvented if an extra bag pressure is introduced in the model and adjusted in order to comply with an extra constraint such as the chiral restoration density [50] or the hadron-quark phase transition [39, 51]. We do not include an extra parameter in the quark model but instead, use parameterizations fitted to vacuum mesonic properties that simultaneously predict a low quark constituent mass.

Figure 6 shows the quark matter EoS for some parameter choices, from where one can see the hardening effect of the vector interaction in both situations, the same well known effect
encountered in the SU(2) model for hadronic matter without equilibrium conditions [47]. The small bumps present in figure 6 are a characteristic of the chiral symmetry restoration associated with the $s$ quarks. Moreover, at large densities, after the total restoration of the chiral symmetry, the densities of the three quarks are the same (1/3 of the total baryonic number density each).

In the same way as previously shown in section III, we obtain the transition pressure and chemical potential which satisfy the Gibbs conditions (3.1)–(3.3), now enforcing $\beta$-equilibrium and charge neutrality within both phases. In stellar matter, the baryonic and quark chemical potentials are usually defined in terms of the EoS variables as [49]

$$\mu_H^B = \frac{\varepsilon_H + P_H}{\rho}, \quad \mu_Q^B = \frac{\varepsilon_Q + P_Q}{\rho},$$

(4.6)

taking $T = 0$. The results obtained for the coexistence points of hadron and quark stellar matter are displayed in table 6. From it, we can see that the effect of the vector interaction on the phase transition is a displacement of the phase transition point towards higher pressures and higher chemical potentials.

Three different internal structures are next considered for the compact star families: (i) hadronic stars modeled by the PPM NJL SU(2) equations of state; (ii) bare quark stars modeled by the NJL SU(3) EoS; and (iii) hybrid stars, constituted by hadronic matter in its outer region and deconfined quark matter in the center. The equation of state for hybrid stars is built from the hadronic and quark EoS by performing a Maxwell construction. This method might seem naive since charge neutrality is imposed only locally and results in the fact that the leptonic chemical potential suffers a discontinuity. But, as we aim to study the macroscopic properties and the energetical content of the compact stars, this construction suffices as shown in [52]. The BPS EoS [53] is also included to the hadronic matter results to account for the description of the low-density matter in the hadronic and hybrid stars outer crusts.

The family of possible compact stars are straightforwardly obtained by using the equations of state as input to the Tolman-Oppenheimer-Volkoff (TOV) equations for the relativistic hydrostatic equilibrium [54, 55]. To solve the TOV equations we need to impose
Table 6. Chemical potential and pressure at the coexistence point for different parameterization combinations for hadronic and three flavor quark stellar matter with equilibrium conditions enforced.

| NJL SU(3)  | Hadronic | $\mu_0$ (MeV) | $P_0$ (MeV/fm$^3$) |
|------------|----------|---------------|-------------------|
| HK-0.0     | eNJL2$\sigma_1$ | 1399          | 196               |
| HK-0.1     | eNJL2$\sigma_1$ | 1529          | 297               |
| HK-0.2     | eNJL2$\sigma_1$ | 1710          | 482               |
| HK-0.3     | eNJL2$\sigma_1$ | 2122          | 1144              |
| HK-0.0     | eNJL3$\sigma_1$ | 1349          | 154               |
| HK-0.1     | eNJL3$\sigma_1$ | 1462          | 227               |
| HK-0.2     | eNJL3$\sigma_1$ | 1579          | 313               |
| HK-0.3     | eNJL3$\sigma_1$ | 1709          | 422               |
| HK-0.4     | eNJL3$\sigma_1$ | 1863          | 571               |
| PCP-0.0    | eNJL2$\sigma_1$ | 1209          | 83                |
| PCP-0.1    | eNJL2$\sigma_1$ | 1420          | 211               |
| PCP-0.2    | eNJL2$\sigma_1$ | 1594          | 356               |
| PCP-0.0    | eNJL3$\sigma_1$ | 1170          | 64                |
| PCP-0.1    | eNJL3$\sigma_1$ | 1328          | 143               |
| PCP-0.2    | eNJL3$\sigma_1$ | 1481          | 239               |
| PCP-0.3    | eNJL3$\sigma_1$ | 1617          | 344               |
| PCP-0.4    | eNJL3$\sigma_1$ | 1768          | 477               |
| PCP-0.5    | eNJL3$\sigma_1$ | 1949          | 663               |

boundary conditions given by $P(R) = 0$ and $P(0) = P_c$, where $R$ is the star radius and $P_c$ is the central pressure. In the following, $M(R)$ and $M_B(R)$ are the respectively the total gravitational mass and the total baryonic.

At this point, a word of caution related to the bare quark stars is important. It is well known that the NJL SU(3) does not satisfy the Bodmer-Witten conjecture [56]. However, the effects of a magnetic field not necessarily too strong and a small increase of temperature [57] seem to be enough to guarantee that the quark matter acquires stability.

In the following, we investigate the conversion mechanism of hadronic to hybrid stars. Similar analysis already exist in the literature [49, 51, 52], but models based on the same underlying field theory class in both hadron and quark phases were never considered.

If a compact star consisting only of hadrons and leptons in $\beta$-equilibrium, electrically neutral and with no fraction of deconfined quark matter, sustains a central pressure $P_C$ larger than the coexistence pressure of the hadron and quark phases, i.e. $P_0$, the hadronic star is said to be metastable to conversion to a quark or hybrid star [21, 49, 58, 59]. The possibility of the conversion depends on the values of the hadronic star central pressure, $P_C$, and the pressure that satisfies the condition of phase coexistence, $P_0$, for a given pair of EoS obtained from the respective models.

In table 7 we show some basic properties of hadronic stars modeled with the parameterizations of the PPM NJL models discussed, for stars with the maximum mass and for canonical stars with $M = 1.4 \, M_\odot$. Two of these results are of special relevance following recent observational and theoretical advances, namely the radius of the canonical neutron.
Table 7. Stellar macroscopic properties obtained with the two PPM eNJL parameterizations. The first set of values refers to the maximum mass star and the later to the canonical star.

|                  | eNJL2mσρ1 | eNJL3σρ1 |
|------------------|-----------|----------|
| $M_{\text{max}}$ ($M_\odot$) | 2.02      | 2.19     |
| $M_B$ ($M_\odot$)       | 2.33      | 2.56     |
| $R$ (km)             | 11.19     | 11.37    |
| $C_{M_{\text{max}}}$ ($M_\odot$/km) | 0.180    | 0.192    |
| $\rho_C$ (fm$^{-3}$)  | 0.981     | 0.966    |
| $\mu_C$ (MeV)        | 1623      | 1781     |
| $P_C$ (MeV/fm$^3$)    | 363       | 489      |
| $R_{1.4M_\odot}$ (km) | 12.20     | 12.94    |
| $C_{1.4M_\odot}$ (km/$M_\odot$) | 0.114    | 0.108    |


star ($R_{1.4M_\odot}$) and the compactness of the maximum mass and the canonical star ($C_{M_{\text{max}}}$ and $C_{1.4M_\odot}$), defined as the ratio between masses and radii of the respective compact stars. Both properties have been extensively discussed in the recent literature [60, 61]. Different hypotheses lead to predictions of the radii of the canonical neutron star varying from 9.7–13.9 km [33] to 10.4–12.9 km [62] and from 10.1 to 11.1 km [63]. The results we show for the radii are not compatible with the predictions of very small radii of [63] but lie within the other two constraints, as also obtained in [61] for a very large number of models. Similarly, properties of maximum mass configuration of quark and hybrid stars for some parameter choices are shown in table 8. It is worth noticing that larger vector interaction parameters in the quark matter model result in more massive hybrid stars with smaller quark cores, reflecting the stiffening of the EoS discussed in [47]. Indeed, following the effect of the vector interaction in the displacement of the phase transition point to higher pressures, as $P_0$ approaches the maximum $P_C$ of the metastable star family, the deconfined quark matter core is possible only inside the most massive stars. As a result, the TOV stable solutions for hadronic and hybrid EoS differ only for a narrow set of stars where the condition $P_C \geq P_0$ is fulfilled. The compactness of both pure hadronic and hybrid canonical star are close to the one recently measured for an isolated neutron star [64] as being equal to $0.105 \pm 0.002$.

Moreover, we can see that the central pressures $P_C$ of the hadronic stars are larger than some of the coexistence pressure values $P_0$, as shown in table 6, notably for smaller values of the vector interaction parameter $\lambda$ in the quark matter modeling. This is the first condition that enables the conversion of a metastable neutron star into a quark or hybrid star. The other condition is that the gravitational mass of the initial metastable hadronic star must be bigger than the gravitational mass of the final star, either quark or hybrid star, for a given baryonic mass, so that the conversion can be exothermal in rest while respecting the baryonic number conservation [21]. In figure 7 we illustrate the results by plotting the ratio between the gravitational and baryonic masses with respect to the baryonic mass, in a way that highlights the small differences between the curves while preserving the interpretation that the conversion is energetically allowed only if the final configuration is below the initial one for $M_B$ fixed. The very small gravitational masses attained in the conversion from the metastable to the stable stars are related to the differences between the hadronic and the quark EOS. The more similar the EOSs, the smaller the mass.

The gravitational masses of quark stars are bigger than the gravitational mass of the hadronic star with the same baryonic mass, which is already expected from previous results.
Table 8. Stellar macroscopic properties of quark and hybrid stars, obtained with some different EoS parameterizations for the phases. The first set of values refers to the maximum mass star and the second to the canonical star. For the hybrid stars, $\rho_H$ and $\rho_Q$ denote the densities of the metastable and quark matter at the phase coexistence point, and $M_{H-Q}$ denotes the gravitational mass of the less massive star that sustains a deconfined quark core. The units are the same as the table 7.

| Properties | Quark Star | Hybrid Star |
|------------|------------|-------------|
| PCP-x:     | 0.0 0.2 0.5 | 0.0 0.2 0.0 0.1 0.2 0.4 |
| $M_{\text{max}}$ | 1.63 1.79 1.97 | 1.80 2.02 1.63 1.97 2.18 2.19 |
| $M_B$ | 1.81 1.97 2.15 | 2.03 2.33 1.81 2.25 2.55 2.57 |
| $R$ | 9.90 10.19 10.79 | 11.60 11.23 12.02 12.25 12.13 11.39 |
| $C_{M_{\text{max}}}$ | 0.164 0.175 0.182 | 0.155 0.179 0.135 0.160 0.179 0.192 |
| $\rho_C$ | 1.035 0.995 0.915 | 0.910 1.084 1.021 0.834 0.820 1.118 |
| $\mu_C$ | 1408 1527 1667 | 1380 1594 1408 1408 1481 1768 |
| $P_C$ | 230 283 332 | 202 356 227 205 239 477 |
| $R_{1.4M_\odot}$ | 10.00 10.43 11.05 | 12.20 12.20 12.94 12.94 12.94 12.94 |
| $C_{1.4M_\odot}$ | 0.140 0.134 0.126 | 0.114 0.114 0.108 0.108 0.108 0.108 |
| $\rho_H$ | 0.487 0.979 | 0.421 0.564 0.700 0.955 |
| $\rho_Q$ | 0.527 1.185 | 0.477 0.648 0.873 1.234 |
| $M_{H-Q}$ | 1.62 — | 1.57 1.93 — — |

Figure 7. Ratio between the gravitational and baryonic masses versus baryonic mass of hadronic, hybrid and quark stars, for different EoS parameterizations.

In literature, e.g., [56], follows that the conversion of a hadronic star to a bare quark star is always energetically forbidden for the parameterizations considered in this work, even in cases where it would be allowed by the Gibbs thermodynamic condition. This feature can be better understood looking for some notable cases. For the PCP-0.5 case, which results in a quark star with $P_C = 332$ MeV/fm$^3$, we see in table 6 that the conversion is allowed by the Gibbs criteria only if the eNJL3$\sigma\rho_1$ hadronic matter is used in the modeling of the metastable star. However, the coexistence pressure is much higher than the ones sustained by the compact
stars described by each phase. This feature prevents the conversion to take place, since it would occur at constant $P$, i.e., both initial and final should sustain $P_C \geq P_0$. Taking the PCP-0.2 case, instead, the quark star sustains $P_C = 283$ MeV/fm$^3$, which allows a hadron-quark coexistence point with both hadronic matter parameterizations, as seen in table 6. If the eNJL2m$\rho_1$ hadronic matter is considered, we have $P^H_C = 363 > P_0 = 356 > P^Q_C = 283$ MeV/fm$^3$. It means that, despite the metastable hadronic star bulk is overpressured enough to allow the phase transition to the PCP-0.2 quark matter, there are not such final compact object constituted by the latter phase. The metastable star decays into a black hole. In other words, this set fulfills the thermodynamic criteria but the astrophysical conditions do not allow the formation of a stable quark star. The last set to be analyzed is when the PCP-0.2 quark matter is compared with the eNJL3$\sigma \rho_1$ hadronic matter. In this case $P^H_C = 489 > P_0 = 239$ MeV/fm$^3$ and $P^Q_C = 283 > P_0 = 239$ MeV/fm$^3$. The main imposition to the hadron-quark phase transition to take place inside metastable compact stars is to have $P_C \geq P_0$ for both stars, which is fulfilled by this choice of models. Nevertheless, a conversion process that preserves baryonic mass, requires that the final state has the same baryonic mass and a smaller gravitational mass. Since the quark star has a larger gravitational mass, the conversion is forbidden due to energy arguments.

A different situation occurs when hybrid stars are considered. In figure 7, we can see the hybrid star family curve differs from the respective pure hadronic star family for stars with a central density above $P_0$, i.e., for hadronic metastable stars massive enough to sustain the conversion of their core from the hadronic matter to a deconfined quark matter bulk. It follows from previous results that the branches where the conversion is allowed are bigger for smaller values of the vector interaction parameter $x$ in the quark matter modeling. In fact, we get a quark core only for a low enough $x$ value, which is 0.12 for nuclear matter model eNJL3$\sigma \rho_1$ [29] and 0.1 for eNJL2m$\rho_1$, as can be seen from table 8 by comparing the values of $\rho_C$ with the values of $\rho_Q$. Stable stars are only possible if $\rho_C$ is larger than $\rho_Q$ and $P_0$ larger than $P_C$. Again, analysing the results shown in tables 6 and 8, we see that in most cases these pressures are identical and a stable star with a quark core is not sustainable. Another feature worth noticing is that, even when the conversion from a hadronic to a hybrid star is allowed, the mass-energy difference of the initial and final objects are always small (a narrow gap of the order of $10^{-3}$–$10^{-2} M_\odot$).

5 Conclusions

In this work we have revisited the study of hadron-quark phase transition at zero temperature with different extensions of the NJL model, which are more appropriate to describe systems where chiral symmetry is an important ingredient.

We have first analyzed possible phase transitions from a hadron phase described by the PPM NJL with nucleons to a quark phase described by the SU(2) NJL with the inclusion of a vector interaction whose strength is arbitrary. It is the first time that the PPM NJL model is used in the context that may be of interest to heavy ion collisions when it is extrapolated to finite temperature, the next step in our work. We have considered symmetric matter and verified that the existence of a hadron-quark phase transition depends on both the quark matter and hadronic matter EoS. For a given hadronic EoS there is a limiting $G_V$ value above which the transition ceases to exist. For a given quark EoS the deconfinement transition does not occur if the hadronic EoS is too soft. We have considered two models that reproduce well accepted properties of nuclear matter at saturation, as well as experimental results obtained
from collective flow data in heavy-ion collisions [65] and from the KaoS experiment [66].

The existing high density constraints are, however, too much model dependent and it is not clear how reliable they are. Another manifestation of the dependence of the results on the choice of parameters is the range of chemical potentials for which the transition takes place: it spans from 1312 MeV to 1787 MeV, a 30% difference in relation to the lowest value. In terms of the transition densities, this translates into a density that can be as low as 0.5 fm$^{-3}$ or as high as 1.5 fm$^{-3}$, with one of the quark models predicting much smaller transition densities. This indicates a strong parameter dependence and experimental constraints are needed. Chiral symmetry may impose some constraints. In our study deconfinement and chiral symmetry restoration are coincident transitions, but different scenarios could occur, as the chiral symmetry restoration before the deconfinement transition, corresponding to a quarkyonic phase, and this scenario would certainly impose strong constraints on the hadronic EoS. We have next analyzed asymmetric systems and, whenever possible, binodal sections were obtained. Both pressure and chemical potential increase drastically with the increase of the vector interaction strength in the quark sector. For asymmetric matter we may expect the appearance of quark matter at smaller densities according to [18]. As already mentioned, as the next step on this analysis, we plan to expand our results to include finite temperature in the system and obtain the complete binodal sections.

Afterwards, for a more complete treatment, we have investigated phase transitions from hadronic stellar matter (same PPM NJL model, but subject to $\beta$-equilibrium and charge neutrality) to quark stellar matter. In this case, flavor conservation is not possible because the hadronic phase only includes nucleons and the quark phase should contain strangeness to satisfy (although barely) the Bodmer-Witten hypothesis. We have seen that, in general, the phase transition pressure and chemical potentials increase with the increase of the vector interaction strength in the quark sector, as before. Then, we have seen that the conversion of metastable stars to quark stars within the studied models is virtually impossible if the condition that the gravitational mass of the hadronic star has to be larger than the gravitational mass of the quark star at the same baryonic mass is imposed. Nevertheless, the conversion from hadronic to hybrid stars is possible, but the mass-energy difference between both objects is very small.

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