DeepGalaxy: Testing Neural Network Verifiers via Two-Dimensional Input Space Exploration

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Abstract—Deep neural networks (DNNs) are widely developed and applied in many areas, and the quality assurance of DNNs is critical. Neural network verification (NNV) aims to provide formal guarantees to DNN models. Similar to traditional software, neural network verifiers could also contain bugs, which would have a critical and serious impact, especially in safety-critical areas. However, little work exists on validating neural network verifiers. In this work, we propose DeepGalaxy, an automated approach based on differential testing to tackle this problem. Specifically, we (1) propose a line of mutation rules, including model level mutation and specification level mutation, to effectively explore the two-dimensional input space of neural network verifiers; and (2) propose heuristic strategies to select test cases. We leveraged our implementation of DeepGalaxy to test three state-of-the-art neural network verifies, Marabou, Eran, and Neurify. The experimental results support the efficiency and effectiveness of DeepGalaxy. Moreover, five unique unknown bugs were discovered.

Index Terms—Deep Learning, Neural Network Verification, Differential Testing

I. INTRODUCTION

Deep neural networks are becoming more and more popular due to their remarkable performance in dealing with challenging problems, such as machine translation [16], speech recognition [17], and autonomous driving [18], [19]. Despite their tremendous success, it is difficult to formally verify that neural networks satisfy desired specifications. This hinders the deployment of neural networks in safety-critical domains.

In order to mitigate this problem, neural network verification (NNV) [1], [2] is proposed to verify properties of neural networks, e.g., adversarial robustness [20], [21], and aims at providing formal guarantees to DNNs. Figure 1 shows the overview of NNV. Given a network \( N \) under verification and a specification \( \varphi \), neural network verification seeks an answer for the question: whether the network \( N \) satisfies \( \varphi \)? Given the aforementioned input, neural network verifiers can either provide a proof of correctness of \( \varphi \), or find a counterexample that violates the specification \( \varphi \). Recently, there has been a plethora of research on neural network verification. For instance, adversarial robustness certification [1], [2] tries to prove the absence of adversarial examples, and fairness analysis [22] verifies whether DNN models satisfy certain fairness properties.

Nevertheless, like all traditional software, NNV implementations (i.e., neural network verifiers) also contain bugs and may return wrong verification results. It is possible that fatal errors of DNNs still remain after verification, which could cause severe consequences for DNNs in safety-critical domains, e.g., autonomous driving. However, no previous work exists on automatically and systematically giving quality assurance of neural network verifiers.

In this work, we aim to validate the correctness of neural network verifiers. Nevertheless, there are several challenges, and we see the main challenges for testing neural network verifiers as being two-fold. (1) Neural network verifier testing remains largely untouched, and challenges come while generating test suite. Different from classical software testing, which takes files or concrete values as input and is typically one dimensional, the inputs to neural network verifiers are DNN models and desired specifications, as shown in Figure 1. Thus, the input space to the verifiers has two dimensions, which makes the input space exploration more difficult. There is a lack of systematic methods to generate effective test cases to explore verifiers’ behavior. (2) There is no oracle to distinguish whether indeed neural network verifiers give wrong verification results. Although in traditional software testing, there are many test oracles to identify if a real bug is detected, it is hard to expose the bug in neural network verifier testing because of the absence of test oracles.

Towards solving the aforementioned challenges, we propose DeepGalaxy, a novel approach for detecting bugs in neural network verifiers. DeepGalaxy is designed to overcome the above mentioned challenges. To address the first challenge, we...
design two levels of mutation operators to produce various test cases to explore the two-dimensional input space. In particular, we propose (1) DNN model level mutation, which mutates the weights and activation functions in each layer of the model, (2) specification level mutation, which mutates the specification of interest. With the help of two-dimensional mutations, DeepGalaxy is able to cover more behavior of the verifiers under test, and expose the underlying bugs. To address the second challenge, we leverage the classical idea of differential testing to identify potential bugs. In other words, we input the same DNN models and specifications to multiple neural network verifiers, and observe whether there are inconsistencies in the output of the verifiers. If one verifier gives an answer different from all the other verifiers, we consider that as a bug.

To sum up, this paper makes the following key contributions:

- We propose a novel blackbox mutational testing technique, DeepGalaxy, for detecting bugs in neural network verifiers. To the best of our knowledge, our work is the first to automatically validate the correctness of neural network verifiers.
- We design a set of mutation rules to effectively and efficiently generate DNN model mutants and specification mutants, which facilitates the test case generating process for triggering bugs in neural network verifiers.
- We implemented DeepGalaxy, tested three state-of-the-art neural network verifiers (Marabou, ERAN, Neurify), and found five unique bugs.

II. BACKGROUND AND PROBLEM DEFINITION

In this section, we introduce the background and define the problems that we aim to solve.

Deep Neural Networks. A deep neural network (DNN) consists of multiple layers, including an input layer, one or more hidden layers, and an output layer. Each of the layers is composed of multiple neurons, and each neuron gets input values from the previous layer and gives the computed output value to the next layer. Typically, the computation of a neuron includes an affine transformation followed by a non-linear transformation, which is called activation function. Some popular activation functions are ReLU and Sigmoid. There are many kinds of DNN models, e.g., Convolutional Neural Networks (CNNs), which is good at image classification, and Recurrent Neural Networks (RNNs), which performs well in audio recognition.

Neural Network Verification (NNV). NNV [1, 2] aims at making sure DNNs run correctly and giving formal guarantees to neural network models. For a DNN model $N$ and a specification $\phi$, a neural network verifier checks whether $\phi$ is satisfied on $N$ for all inputs $x$. The specification $\phi$ holds if there are no counterexamples. Otherwise, the specification is violated and a counterexample is returned by the verifier.

A specification can be expressed in the form:

$$\forall x, \phi_{\text{pre}}(x) \land y \leftarrow N(x) \implies \phi_{\text{post}}(x, y).$$

Here, we use $\phi_{\text{pre}}(x)$ to specify the preconditions on the input of $N$. We use $\phi_{\text{post}}(x, y)$ to express the postconditions on the input and the output of $N$. We use $y \leftarrow N(x)$ to encode the process of calculating the output of $N$ for input $x$ and assigning the output value to $y$.

Recent researches have already developed many verification techniques. For example, Planet [26] encodes the linear approximation of the overall network behavior into SMT or integer linear programming problems, and Zhang et al. [25] proposed CROWN, which computes the certified lower bound of minimum perturbation in DNNs for any given data point.

Problem Definition. Similar to the traditional software, neural network verifiers could also contain bugs, which is fatal in safety-critical domains. Notice that there are two inputs to neural network verifiers, the formal specification, i.e., the properties to be verified, and DNN models. We define the test cases in neural network verifier testing as follows.

Definition 1 (Test Case). A test case for neural network verifier testing is a tuple $(N, \varphi)$, where $N$ is a DNN model and $\varphi$ stands for the desired specification.

Having defined the test cases in neural network verifier testing, we now describe the main problem we target at.

Problem 1 (Neural Network Verifier Testing). Given a neural network verifier $V$, generate a set of valid test cases $T$ to detect bugs in $V$.

In general, this problem is very challenging because of the intractability of the input space exploration and the absence of the test oracle. In the following section, we propose a novel technique, called DeepGalaxy, to tackle this problem.

III. APPROACH

In this section, we describe our testing framework, DeepGalaxy, in more details.

Figure 2 presents the overall workflow. Given a set of seed test cases $T_{\text{seed}}$, DeepGalaxy produces potential bugs in the neural network verifiers. First, we put $T_{\text{seed}}$ into the test case pool $P$. Then, the test cases in $P$ are selected based on designed heuristics. The test case $(N, \varphi)$ is mutated by using different levels of mutation operators, and neural network verifiers verify whether $\varphi$ holds for $N$. In the end, the verification results
from different verifiers under test are compared with each other. If one of the results from verifier \(V\) is inconsistent than the others, a bug is found. In the following three subsections, we describe the predominant parts of DeepGalaxy: test case selection, mutation, and bug report.

A. Test Case Selection

Since the two-dimensional input space, including DNN model space and specification space, is prohibitively large, even infinite. It is infeasible to explore the space completely. Therefore, we design three model selection approaches to generate the DNN models and the specification towards the direction of exposing the potential bugs in various neural network verifiers.

- **Random Selection**: to randomly select a test case from the test case pool, which utilizes the advantage of exploration of the space.
- **Recency-aware**: to select the test case that is last added to the pool.
- **Mixed**: to combine the recency-aware selection and the random selection, which balance exploration and exploitation. We randomly choose recency-aware selection and random selection.

With the help of the model selection approach, the test case that can exploit the unused code in the verifier under test can be selected more frequently. Next, we introduce the two levels of mutation, DNN model level mutation and specification level mutation, which is the key to expose the underlying bug.

B. Mutation

The input space to be explored is extremely large, since that includes the space of DNN models and the specifications. In order to explore the space thoroughly, we propose two levels of mutation operators.

**DNN Model Level Mutation.** The goal of the model level mutation is to test neural network verifiers by exploring the underlying un-triggered operation of the verifier or diverse combination of behaviors of the verifier. In particular, we adopt five mutation rules from existing work [14] to fulfill our mutation task.

- **Gaussian Fuzzing (GF)**: to fuzz the value of the weights following the Gaussian distribution \(N(\mu, \delta^2)\), where \(\mu\) is the mean value and \(\delta\) is the standard deviation. We set \(\mu\) as zero and \(\delta\) as 0.5.
- **Weight Shuffling (WS)**: to randomly choose a neuron and shuffles the weights of its connections to the previous layer.
- **Neuron Effect Blocking (NEB)**: to block neuron effects to all of the connected neurons in the next layers by resetting its connection weights of the next layer to be zero.
- **Neuron Activation Inverse (NAI)**: to change the sign of the output value before giving it to the corresponding activation function. The goal is to invert the activation status of a neuron.
- **Neuron Switch (NS)**: to randomly switch two neurons within a layer to change their effects to the next layers.

**Specification Level Mutation.** We leverage four newly designed mutation rules for specification level mutation.

- **Constant Addition (CA)**: to select a constant in the specification and add a random value to it.
- **Constant Removal (CR)**: to remove a constant in the specification.
- **Constant Subtraction (CSb)**: to select a constant in the specification and subtract a random value on it.
- **Constant Switch (CSw)**: to switch two constants in the specification.

We take adversarial robustness as an example to illustrate how to conduct specification level mutation. Suppose that we have a neural network \(N\) trained on MNIST dataset and we want to verify adversarial robustness on \(N\). Given the DNN \(N\), a fixed input \(x'\), a region \(R \subset \mathbb{R}^n\), distance function \(dist\), and \(\varepsilon = 0.1\), the seed adversarial robustness specification is

\[
\forall x \in R : dist(x, x') \leq 0.1 \implies N(x) = N(x').
\]  

To mutate the above specification using the mutation rule of CA, we randomly change the value of \(\varepsilon\) to 0.15. The mutated specification becomes

\[
\forall x \in R : dist(x, x') \leq 0.15 \implies N(x) = N(x').
\]  

This new mutated specification can then be used as the input to the verifiers under test.

C. Bug Identification

One of the main challenges of testing neural network verifiers is that it is difficult to expose and identify the potential bugs. Different from classical software testing, where typically there are oracles to indicate the ground truth, there exist no oracles to distinguish whether the detected bug is a real bug. Therefore, we leverage differential testing to identify whether a real bug is detected. In particular, DeepGalaxy compares the results returned from different verifiers. If one of them is inconsistent with the others, we consider that a bug is detected.

Besides the inconsistency bugs, we also consider crash, e.g., the neural network verifier exits abnormally, as a kind of bug.

D. Algorithm

Algorithm 1 shows our neural network verifier testing approach to capturing underlying bugs. The inputs are a seed DNN model \(N\), a seed specification \(\varphi\), the set of verifiers under test \(V\), maximum iteration \(maxIter\), and test case selection method \(selection\), as described in subsection III-A. The output is the bug report \(b\).

First, we initialize the number of iteration \(iter\), the result set \(R\), and the test case pool \(P\) (Line 1-3). Then, DeepGalaxy iterates for \(maxIter\) times (Line 4). In each iteration, it selects the test case from \(P\) (Line 6), and performs the mutation by randomly selecting the mutation operator (Line 7). For each test case, DeepGalaxy input it to each verifier and collect the result (Line 8-11). If any of the results is inconsistent with the
For the seed specification, we choose adversarial robustness for DNN models, e.g., Alexnet [13], as initial seed models. We trained a fully connected network verifiers as the bug-hunting object. In particular, we use equation 1 as the seed specification. All experiments are conducted on a machine with Intel (R) Core (TM) i5 CPU @2.4 GHz and 16 GB RAM, equipped with a GNU/Linux system.

**RQ1: Efficiency of DeepGalaxy.** We investigate the efficiency of DeepGalaxy, and use the execution time as the metrics. On average, the execution time of each testing round of DeepGalaxy is 39.6 seconds. Notice that some verification procedure takes more time, because the speed of different verifiers for tackling verification is different. For example, ERAN, based on abstract interpretation, conducts fast but not complete verification, whereas Marabou, based on SMT solving, is relatively slow.

**Answer to RQ1:** DeepGalaxy is efficient in terms of generating mutation test cases, which indicates the usefulness when applying it in practice.

**RQ2: Bug Hunting of DeepGalaxy.** We use bug hunting numbers as the measuring metrics in this research question. In total, DeepGalaxy discovered five bugs in three state-of-the-art neural network verifiers, where two were confirmed. More specifically, we captured two bugs in Marabou, two bugs in ERAN, and one bug in Neurify. Table I is overview of bugs found by DeepGalaxy. The numbers in the table represent the bugs that DeepGalaxy detected, confirmed, unconfirmed, and fixed in the three verifiers, respectively. Detected means the bugs found by DeepGalaxy detected, confirmed, unconfirmed, and fixed in the three verifiers, respectively. Confirmed means the bugs found by DeepGalaxy. Confirmed means the bugs have been confirmed by the developers. Unconfirmed means we reported the bugs to the developers, but have not received a confirmation yet. Fixed means the developers confirmed the bugs and fixed them. After DeepGalaxy detects the bugs, we also use the manual inspection to facilitate bug reporting and enhance the quality of the bug report.

**Answer to RQ2:** Our approach can detect real bugs in the state-of-the-art neural network verifiers. Several bugs have been reported, and one bug has been fixed by the developer.

### Table I

| Bugs Status | Marabou | ERAN | Neurify | Total |
|-------------|---------|------|---------|-------|
| Detected    | 2       | 2    | 1       | 5     |
| Confirmed   | 0       | 2    | 0       | 2     |
| Unconfirmed | 2       | 0    | 1       | 3     |
| Fixed       | 0       | 1    | 0       | 1     |

**V. Related Work**

**Testing Classical Software Verifiers.** In classical software, software verifiers are broadly utilized to enhance the quality...
assurance of software systems. There is a considerable amount of study on the correctness and reliability of software verifiers. Zhang et al. propose MCFuzz [6], which is an automated fuzzing technique to test software model checkers, e.g., CPAchecker [11]. It leverages branch reachability to tackle scalability and the oracle problem. Yinang [7] is designed to validate the correctness of SMT solvers, e.g., Z3 [10] and CVC4 [27], via fusing pairs of equisatisfiable formulas. Please note that, different from them, our work takes the first step to validate the correctness of neural network verifiers with both DNN model level mutation and specification level mutation.

**Testing Deep Neural Networks.** There are also many works on giving quality assurance on DNN models. For instance, Pei et al. [28] propose to leverage neuron coverage to guide the generation of test cases. Ma et al. [8] propose multiple granularity coverage criteria to evaluate the adequacy of testing DNN models. Odena et al. [24] first design a coverage-guided fuzzing framework for testing deep neural networks. Zhang et al. [28] develop a fuzzing-based blackbox adversarial attack for DNNs. Moreover, Khmelnitsky et al. [9] design an technique to extract a surrogate automata model to analyze and verify regular properties for recurrent neural networks. Notice that our work is orthogonal to these methods, because we primarily focus on detecting bugs in neural network verifiers, whereas they focus on measuring the quality of DNN models.

**VI. CONCLUSION**

In this paper, we present a novel approach, DeepGalaxy, to automatically and systematically validating neural network verifiers. In particular, we design a set of mutation operators to generate effective DNN model mutants and specification level verifiers. In particular, we design a set of mutation operators to automatically and systematically validating neural network verifiers with both DNN model level mutation and specification level mutation.

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