Understanding quantum polarized-light interference experiments through electromagnetic energy flow lines

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Abstract

General expressions to obtain the electromagnetic energy flow lines behind interference gratings are derived in the case where the incident light consists of a polarized monochromatic plane wave. These flow lines show how the electromagnetic energy redistributes in space (behind the grating) until the Fraunhofer regime is reached, thus providing an interpretation based on photon paths for the physics underlying interference phenomena with light. Within this interpretation, one finds that the outcome from a Young’s experiment is related in a simple manner to how the electromagnetic energy flux is influenced by the experimental setup, specifically, how the presence of polarizers on each slit and the boundaries imposed by having one or both slits open affect at each time the electromagnetic energy flow, which is directly linked to the Arago-Fresnel laws.

Key words: Electromagnetic energy flow line, Photon path, Maxwell equations, Bohmian mechanics, Quantum trajectory

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1. Introduction

Nowadays it is possible to carry out interference experiments with low intensity beams where the appearance of the final pattern, built up by accumulating single particle counts, can be monitored in time. In the pattern

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formation sequence, initially, one can appreciate a seemingly random distribution of points on a screen (when the number of particle registered is still low), which evolves toward the well-known light and dark interference fringe-like structure (when the number of particles detected is already very large) as time proceeds. Moving from the darker regions of the pattern to the lighter ones thus means that the density of particles increases. Reconstructions of interference patterns in this way have been carried out, for instance, using electrons [1] and sodium atoms [2]. This kind of experiments constitute a nice manifestation of the statistical nature of Quantum Mechanics [3], which, in the large count-number limit, establishes that quantum massive particles distribute according to the quantum probability density.

In order to build a bridge between the single counts observed experimentally and the statistical quantum-mechanical description in terms of smooth, continuous probability densities, different trajectory-based approaches have been provided in the literature [4, 5, 6, 7, 8, 9]. One of the better known is Bohmian mechanics [10, 11], a reformulation of standard Quantum Mechanics in terms of trajectories, which, to some extent, at the same level as Newtonian mechanics with respect to classical Liouvillian mechanics. Bohmian trajectories evolve following the streamlines associated with the quantum probability current density (quantum flow) and, therefore, reproduce exactly the quantum-mechanical probability distribution in both the near and the far field when a large number of them is considered, in particular, in interference experiments [5, 7]. Alternatively, the emergence of interference patterns in the far field has also been simulated by using trajectories determined from the momentum distribution (MD trajectories) associated with the particle wave function [8].

The extensive use of such trajectory-based descriptions in problems involving massive particles strikingly contrasts with its lack of analogous experiments with light or, for a better analogy, with photons. In Optics it is well known that, like massive particles, photons also behave statistically [12]. Indeed, some experiments carried out recently show how the interference pattern described by standard (“classical”) Electromagnetism are reconstructed photon by photon [13]. Among the reasons for such a lack, one might argue that there is a difficulty involved in defining a wave function for a photon [14, 15, 16, 17, 18, 19, 20, 21]. Nevertheless, some attempts to explain interference and diffraction experiments with weak intensity light beams, where the final patterns are reconstructed by detecting photon counts one by one, can be found in the literature. One approach consists of associating photon
paths with electromagnetic energy (EME) flow lines, which can be traced backwards to Braunbek and Laukien \cite{22} and Prosser \cite{23}, and that we have considered recently \cite{24}. Within this approach, the EME flow lines are determined after solving the path equation arising from the Poynting energy flow vector. This approach is thus based on a “classical” treatment of Electromagnetism, which is the limit of a large photon number (the photon number is a non well-defined magnitude), contrary to what happens in Quantum Optics \cite{12}. In a similar direction (i.e., in the large photon number limit), but within the relativistic limit, Ghose et al. \cite{25} have determined photon trajectories.

In \cite{24}, we considered EME flow lines associated with a linearly polarized electromagnetic (EM) field that was diffracted by interference gratings. The components of both the electric and the magnetic vector fields satisfied Maxwell’s equations as well as the boundary conditions imposed by the grating. These EME flow lines showed how the EME redistributes in space from the grating to the detection screen, located far away from the former in the Fraunhofer region \cite{5}. By properly sampling the field at the slits, one could also observe how, as time proceeds, the accumulation of trajectory arrivals, which eventually led to the appearance of the well-known fringe-like pattern, just as in the analogous real experiment \cite{13} or as in quantum mechanics, where the formation of similar patterns arises after a large count of particle trajectories. Here, we generalize this work, extending it to the case where the EM field is polarized. Experiments with high-intensity polarized beams are well-known since the beginning of the XIXth century, when Arago and Fresnel enunciated their laws on Young’s interference experiment with this kind of light \cite{26}. According to these laws, no interference pattern will be observed if, for instance, we have two interfering beams linearly polarized in orthogonal directions \cite{27} or both are elliptically polarized but one is left-handed and the other right-handed \cite{28}. In the case of low intensity beams (i.e., within the domain of the Quantum Optics), polarized light is used in experiments such as the so-called interference quantum eraser \cite{29,30}, which are very important nowadays because of their implications at a fundamental level \cite{31} and also from the viewpoint of quantum information \cite{32}.

\footnote{Throughout this work, the concept of path will be used to denote the time-independent track eventually pursued by a quantum of electromagnetic energy or photon, while the concept of trajectory will refer to a time-dependent track either pursued by either photon or by a massive particle.}
our opinion, we think that it is important to have a path-based picture of this kind of experiments, where patterns are reconstructed from one-per-one photon count processes, in order to better understand the physics contained in them. Note that, according to the conventional or more standard viewpoint, this kind of experiments are interpreted following “which-way”-like arguments, which appeal directly to the observer’s will to interact with the experiment. However, when the reality is brought back to EME flows, one finds that the outcome from a Young’s experiment can be easily related to how such a flow is influenced by the whole experimental setup [33, 34]. In other words, how the presence of polarizers on the slits and the boundaries imposed by having one or both slits open affect at each time the EME flow.

The organization of this paper is as follows. In order to be self-contained, in Section 2 we introduce some fundamental theoretical grounds related to the EME flow-line formalism, specifically focused to our analysis of experiments with quantum polarized light. In Section 3 we describe the incident EM field and its polarization, as well as the derivation of the corresponding EME flow lines before reaching the interference grating. In Section 4 the derivation of the EM field at and behind the grating, which will be used to calculate the associated EME flow lines, is explained. In Section 5 we present and discuss the EME flow lines behind a two-slit grating in the case of an incident circularly polarized EM field. In Section 6 an analogous analysis is carried out, but when the slits are covered with polarizers with orthogonal polarization axes. Calculations of EME flow lines for the cases of incident linearly and circularly polarized EM fields are shown. Finally, in Section 7 the conclusions arisen from this work are summarized.

2. Maxwell’s equations and EME flow lines

In vacuum, the electric and magnetic fields can be expressed as harmonic waves,

$$\tilde{\mathbf{E}}(\mathbf{r}, t) = \mathbf{E}(\mathbf{r}) e^{-i\omega t}, \quad \tilde{\mathbf{H}}(\mathbf{r}, t) = \mathbf{H}(\mathbf{r}) e^{-i\omega t}, \quad (1)$$

where the space dependent parts of these fields satisfy the time-independent Maxwell’s equations,

$$\nabla \cdot \mathbf{E}(\mathbf{r}) = 0, \quad (2)$$
$$\nabla \cdot \mathbf{H}(\mathbf{r}) = 0, \quad (3)$$
$$\nabla \times \mathbf{E}(\mathbf{r}) = i\omega\mu_0 \mathbf{H}(\mathbf{r}), \quad (4)$$
$$\nabla \times \mathbf{H}(\mathbf{r}) = -i\omega\varepsilon_0 \mathbf{E}(\mathbf{r}), \quad (5)$$

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as well as the boundary conditions associated with the particular problem under study. Equivalently, from (2)-(5), it is readily shown that both $E(r)$ and $H(r)$ satisfy the Helmholtz equation,

\begin{align}
\nabla^2 E(r) + k^2 E(r) &= 0, \\
\nabla^2 H(r) + k^2 H(r) &= 0,
\end{align}

where $k = \omega/c$.

The EME flow lines are obtained from the real part of the time-averaged complex Poynting vector,

\begin{equation}
S(r) = \frac{1}{2} \text{Re}[E(r) \times H^*(r)],
\end{equation}

as follows. We know that this vector describes the flow of the time-averaged EME density through space,

\begin{equation}
U(r) = \frac{1}{4} \left[ \epsilon_0 E(r) \cdot E^*(r) + \mu_0 H(r) \cdot H^*(r) \right].
\end{equation}

Therefore, one can then assume that the EME density is carried across space along paths described by the solutions or characteristics of the equation

\begin{equation}
\frac{d r}{d s} = \frac{1}{c} \frac{S(r)}{U(r)},
\end{equation}

where $ds$ denotes the infinitesimal element of an arc-length or metric distance. These solutions are the EME flow lines, which can also be understood as the paths pursued by EME quanta or photons.

In the particular case of interference experiments, a functional form for (10) can be found by means of the following considerations. The screen containing the grating is on the $XZ$ plane, at $y = 0$, and the slits are parallel to the $z$-axis, their width ($\delta$) being much larger along this direction than along the $x$-direction (i.e., $\delta_z \gg \delta_x$). Accordingly, we can assume that the EME density is independent of the $z$-coordinate and, therefore, the electric and magnetic fields will not depend either on this coordinate. This allows us to consider a simplification in the analytical treatment, for, as mentioned in [35], a problem independent of one Cartesian coordinate is essentially scalar and, therefore, can be formulated in terms of one dependent variable. Thus, if we introduce here

\begin{equation}
\frac{\partial H}{\partial z} = 0 = \frac{\partial E}{\partial z}
\end{equation}

into Eqs. (4) and (5), we obtain two independent sets of equations,

\[
\begin{align*}
\frac{\partial E_z}{\partial y} &= \frac{i\omega}{\epsilon_0 c^2} H_x, \\
\frac{\partial E_x}{\partial x} &= -\frac{i\omega}{\epsilon_0 c^2} H_y,
\end{align*}
\]

and

\[
\begin{align*}
\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y} &= -i\omega \epsilon_0 E_z, \\
\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y} &= \frac{i\omega}{\epsilon_0 c^2} H_z.
\end{align*}
\]

The set (12) only involves \(H_x, H_y\) and \(E_z\), and therefore is commonly referred as a case of E-polarization, while the set (13), which only involves \(E_x, E_y\) and \(H_z\), is referred to as H-polarization.

More specifically, as infers from the set of equations (12), in the case of E-polarization the electric field is polarized along the \(z\)-direction, while the magnetic field is confined to the plane \(XY\). That is, \(E_{e,x} = E_{e,y} = H_{e,z} = 0\), with the components of the magnetic field satisfying

\[
H_{e,x} = -\frac{i\epsilon_0 c^2}{\omega} \frac{\partial E_{e,z}}{\partial y}, \quad H_{e,y} = \frac{i\epsilon_0 c^2}{\omega} \frac{\partial E_{e,z}}{\partial x}.
\]

Substituting these expressions for \(H_x\) and \(H_y\) into the second line of (12) yields

\[
\frac{\partial^2 E_{e,z}}{\partial x^2} + \frac{\partial^2 E_{e,z}}{\partial y^2} + k^2 E_{e,z} = 0.
\]

We thus have

\[
\begin{align*}
\mathbf{E}_e &= E_{e,z} \hat{z}, \\
\mathbf{H}_e &= H_{e,x} \hat{x} + H_{e,y} \hat{y},
\end{align*}
\]

with \(E_{e,z}\) satisfying Helmholtz’s equation, according to (15). Analogously, in the case of H-polarization the magnetic field is polarized along the \(z\)-direction and the electric one confined to the plane \(XY\) (i.e., \(H_{h,x} = H_{h,y} = E_{h,z} = 0\), with the components of the latter being

\[
E_{h,x} = \frac{i}{\omega \epsilon_0} \frac{\partial H_{h,z}}{\partial y}, \quad E_{h,y} = -\frac{i}{\omega \epsilon_0} \frac{\partial H_{h,z}}{\partial x}.
\]

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Substituting now these relations into the second line of (13) yields
\[
\frac{\partial^2 H_{h,z}}{\partial x^2} + \frac{\partial^2 H_{h,z}}{\partial y^2} + k^2 H_{h,z} = 0,
\]
which allows us to characterize \(H\)-polarization as
\[
E_h = E_{h,z} \hat{x} + E_{h,y} \hat{y},
\]
\[
H_h = H_{h,z} \hat{z},
\]
with \(H_{h,z}\) satisfying the Helmholtz equation (19). Therefore, any general (time-independent) solution will be expressible as
\[
E = E_e + E_h = E_e + \frac{i}{\omega \epsilon_0} \left[ \nabla \times H_h \right],
\]
\[
H = H_e + H_h = -\frac{i}{\omega \mu_0} \left[ \nabla \times E_e \right] + H_h.
\]

Since \(E_z\) and \(H_z\) satisfy Helmholtz’s equation, consider now that both are proportional to a scalar field, \(\Psi(r)\), which also satisfies this equation, i.e.,
\[
E_e = \alpha \Psi \hat{z},
\]
\[
H_h = \beta \sqrt{\frac{\epsilon_0}{\mu_0}} e^{i\phi} \Psi \hat{z}.
\]
Here, \(\alpha\) and \(\beta\) are real quantities and the phase shift between both components is given by \(\phi\); (5) has been used to obtain the correct dimensionality in the r.h.s. of (25). If (24) and (25) are substituted into Eqs. (22) and (23), respectively, the latter become
\[
E = \frac{i \beta e^{i\phi}}{k} \frac{\partial \Psi}{\partial y} \hat{x} - \frac{i \beta e^{i\phi}}{k} \frac{\partial \Psi}{\partial x} \hat{y} + \alpha \Psi \hat{z},
\]
\[
H = -\frac{i \alpha}{\omega \mu_0} \frac{\partial \Psi}{\partial y} \hat{x} + \frac{i \alpha}{\omega \mu_0} \frac{\partial \Psi}{\partial x} \hat{y} + \frac{k \beta e^{i\phi}}{\omega \mu_0} \Psi \hat{z},
\]
with their time-dependent counterparts being
\[
\tilde{E}(r,t) = \left[ \frac{i \beta e^{i\phi}}{k} \frac{\partial \Psi}{\partial y} \hat{x} - \frac{i \beta e^{i\phi}}{k} \frac{\partial \Psi}{\partial x} \hat{y} + \alpha \Psi \hat{z} \right] e^{-i\omega t},
\]
\[
\tilde{H}(r,t) = \left[ -\frac{i \alpha}{\omega \mu_0} \frac{\partial \Psi}{\partial y} \hat{x} + \frac{i \alpha}{\omega \mu_0} \frac{\partial \Psi}{\partial x} \hat{y} + \frac{k \beta e^{i\phi}}{\omega \mu_0} \Psi \hat{z} \right] e^{-i\omega t},
\]
Equations (28) and (29) are general time-dependent solutions for a problem which can be described in terms of superpositions, as also happens with (22) and (23). Once this set of equations is set up, the whole problem reduces to finding $\Psi$ and its propagation along $x$ and $y$ (by the above hypothesis, the set of equations does not depend on $z$), which is a boundary condition problem.

3. Incident EM plane wave, its polarization and EME flow lines

Before reaching the two slits, we assume that both the electric and magnetic fields propagate along the $y$-direction. Moreover, we also assume that the incident scalar field is a monochromatic plane wave,

$$\Psi_0(r) = e^{iky}. \tag{30}$$

Introducing (30) into (28) and (29) yields

$$\tilde{E}_0(r,t) = \left[ E_{0,h,x}\hat{x} + E_{0,e,z}\hat{z} \right] e^{i\omega t} = \left[ -\beta e^{i(ky+\phi)}\hat{x} + \alpha e^{iky}\hat{z} \right] e^{-i\omega t}, \tag{31}$$

$$\tilde{H}_0(r,t) = \left[ H_{0,h,x}\hat{x} + H_{0,e,z}\hat{z} \right] e^{i\omega t} = \sqrt{\frac{\varepsilon_0}{\mu_0}} \left[ \alpha e^{iky}\hat{x} + \beta e^{i(ky+\phi)}\hat{z} \right] e^{-i\omega t}. \tag{32}$$

From these solutions, it follows that the polarization is going to play an important role in the interference patterns observed, and also in the topology displayed by the EME flow lines.

Consider the electric field (31), whose real components are

$$\tilde{E}_{0,h,x}^r = -\beta \cos(ky - \omega t + \phi), \tag{33}$$

$$\tilde{E}_{0,e,z}^r = \alpha \cos(ky - \omega t). \tag{34}$$

Since the magnetic field displays the same polarization properties as the electric field due to their relationship through the Maxwell equations (4) and (5), we will only argue in terms of the electric field without loss of generality. Thus, expressing (33) and (34) as

$$\frac{\tilde{E}_{0,h,x}^r}{\beta} + \frac{\tilde{E}_{0,e,z}^r}{\alpha} \cos \phi = \sin(ky - \omega t) \sin \phi, \tag{35}$$

$$\frac{\tilde{E}_{0,e,z}^r}{\alpha} \sin \phi = \cos(ky - \omega t) \sin \phi. \tag{36}$$
and then squaring and rearranging terms, we reach

\[
\left( \frac{E_{0,h,x}}{\beta} \right)^2 + \left( \frac{E_{0,e,z}}{\alpha} \right)^2 + 2 \left( \frac{E_{0,h,x}}{\beta} \right) \left( \frac{E_{0,e,z}}{\alpha} \right) \cos \phi = \sin^2 \phi.
\] (37)

According to this relation, several cases are possible depending on the value of the phase-shift \( \phi \):

(a) When \( \phi = 0 \) or \( \pi \),

\[
\left( \frac{E_{0,h,x}}{\beta} \pm \frac{E_{0,e,z}}{\alpha} \right)^2 = 0 \quad \Rightarrow \quad \frac{E_{0,h,x}}{\beta} = \pm \frac{E_{0,e,z}}{\alpha}.
\] (38)

This case describes linear polarization, for any arbitrary \( \alpha \) and \( \beta \).

(b) For any other value of \( \phi \), there is elliptic polarization. In this case, (37) is the equation of an ellipse inscribed in a rectangle parallel to the \( XZ \) plane, with sides \( 2|\alpha| \) and \( 2|\beta| \) (see Fig. 1). The electric field (and also the magnetic one, which is perpendicular) can move clockwise or anticlockwise, as seen by an observer toward whom the EM wave is moving. These rotations define, respectively, right-handed or
left-handed polarization states. The polarization handedness can be determined by computing the time derivative of the angle formed by the two components of the electric field, \( \theta_E = (\tan^{-1}(\frac{\vec{E}_{0,e,z}}{\vec{E}_{0,h,x}})) \),

\[
\frac{d\theta_E}{dt} = \frac{1}{1 + (\frac{\vec{E}_{0,e,z}}{\vec{E}_{0,h,x}})^2} \frac{d}{dt} \left( \frac{\vec{E}_{0,e,z}}{\vec{E}_{0,h,x}} \right) \\
= -\frac{\beta \sin \phi}{\alpha} \frac{\omega [1 + \tan^2(ky - \omega t)]}{1 + (\frac{\vec{E}_{0,e,z}}{\vec{E}_{0,h,x}})^2}.
\] (39)

As can be noticed, the information about the handedness is contained in the prefactor of this expression, since the second factor (the time-dependent one) is always positive. Thus, if \( \alpha \) and \( \beta \) are chosen positive, \( \frac{d\theta_E}{dt} \) will be positive for \(-\pi < \phi < 0\) and negative for \(0 < \phi < \pi\). In the first case, the field is right-handed polarized (\( \theta_E \) increases with time) and, in the second one, it is left-handed (\( \theta_E \) decreases with time).

(c) In the particular case \( \phi = \pm \pi/2 \), we have

\[
\left( \frac{\vec{E}_{0,h,x}}{\beta} \right)^2 + \left( \frac{\vec{E}_{0,e,z}}{\alpha} \right)^2 = 1.
\] (40)

If \( \alpha = \beta \), the ellipse described by (40) reduces to the equation of circle. We then have circular polarization.

The particular values of \( \phi \), \( \alpha \) and \( \beta \), which define the polarization state of the incident plane wave, are very important regarding the observation of interference patterns behind the slits according to the Arago-Fresnel laws [26]. But they are also going to be very important with respect to the topology of the corresponding EME flow lines, as shown below.

In Sec. 4 we will tackle the calculation of the EM field and EME flow lines behind gratings with \( N \) and two slits. In the second case, we will not only consider free passage through the slits, but also when they are covered by linear polarizers parallel to the \( XZ \) plane, whose polarization axes are oriented along the \( z \)-axis in one of the slits and along the \( x \)-axis in the other. Moreover, and without loss of generality, we are going to assume that the polarizer oriented along the \( z \)-axis produces \( E \)-polarization, while the polarizer along the \( x \)-axis gives rise to \( H \)-polarization. As can be shown, the action of these polarizers on an incident plane wave which propagates
along the $y$-axis, such as (30), justifies this specific labelling. Note that, after passing through such polarizers, the EM field described by Eqs. (31) and (32) gives rise to transmitted fields, $E_{tr}$ and $H_{tr}$, with the following polarizations:

a) If the polarization axis is oriented along the $x$-axis:

\[ E_{tr}(r) = -\beta e^{i(ky+\phi)}\hat{x} = E_{0,h,x}\hat{x}, \quad (41) \]
\[ H_{tr}(r) = \sqrt{\frac{\varepsilon_0}{\mu_0}} \beta e^{i(ky+\phi)}\hat{z} = H_{0,h,z}\hat{z}. \quad (42) \]

b) If the polarization axis is oriented along the $z$-axis:

\[ E_{tr}(r) = \alpha e^{iky}\hat{z} = E_{0,e,z}\hat{z}, \quad (43) \]
\[ H_{tr}(r) = \sqrt{\frac{\varepsilon_0}{\mu_0}} \alpha e^{iky}\hat{x} = H_{0,e,x}\hat{x}. \quad (44) \]

Regarding the EME flow lines, before the EM field reaches the grating, they are obtained from

\[ S_0(r) = \frac{1}{2} \text{Re}[E_0(r) \times H_0^*(r)] \]
\[ = \frac{1}{2} \sqrt{\frac{\varepsilon_0}{\mu_0}} (\alpha^2 + \beta^2)\hat{y}, \quad (45) \]
\[ U_0(r) = \frac{1}{4} [\varepsilon_0 E(r) \cdot E^*(r) + \mu_0 H(r) \cdot H^*(r)] \]
\[ = \frac{\varepsilon_0}{2} (\alpha^2 + \beta^2), \quad (46) \]

where the time-independent parts of (31) and (32) have been considered. Substituting (45) and (46) into (10), we obtain

\[ \frac{dr}{ds} = \hat{y}, \quad (47) \]

which, after integration, yield

\[ x(s) = x_0, \quad z(s) = z_0, \quad (48) \]
\[ y(s) = y_0 + s. \quad (49) \]
That is, the EME flow evolves along the $y$-direction and, therefore, photons pursue straight lines parallel to the $y$-axis. Since we are in vacuum, we can assume that the distance $s$ travelled by a photon during a time $t$ is given by $s = ct$ and, therefore, (49) can also be expressed as

$$y(t) = y_0 + ct. \quad (50)$$

4. EM field behind a grating

According to Sec. 2, the EM field behind a grating can be described by Eqs. (26) and (27) (or Eqs. (28) and (29), respectively, if we consider time-dependence), with the scalar function $\Psi(\mathbf{r}) = \Psi(x, y)$ satisfying both Helmholtz’s equation and the boundary conditions at the grating. If the incident EM field is monochromatic, we can also assume that the incident scalar function is given by (30).

Traditionally, the exact solution for the slit array that we are going to consider here arises from Helmholtz’s equation (see Eqs. (6) and (7)) and is expressed as a Fresnel-Kirchhoff integral [35],

$$\Psi(x, y) = \sqrt{\frac{k}{2\pi y}} e^{-i\pi/4} e^{iky} \int_{-\infty}^{\infty} \Psi(x', 0^+) e^{ik(x-x')^2/2y} dx'. \quad (51)$$

Then, after assuming some appropriate approximations, expressions valid to describe Fresnel and Fraunhofer diffraction can be obtained from the corresponding general solutions. Alternatively, Arsenović et al. [41] have shown that the solution behind a grating can also be expressed as a superposition of transverse modes of the fields multiplied by an exponential function of the longitudinal coordinate, i.e.,

$$\Psi(x, y) = \frac{1}{\sqrt{2\pi}} e^{iky} \int c(k_x) e^{ik_x x-i k_x^2 y/2} dk_x, \quad (52)$$

where

$$c(k_x) = \frac{1}{\sqrt{2\pi}} \int \Psi(x, 0^+) e^{-ik_x x} dx. \quad (53)$$

As also shown by Arsenović et al. [42, 43], the solution (52) is equivalent to the Fresnel-Kirchhoff integral (51) whenever the wave number associated with the transverse mode $k_x$ of a general slit satisfies the condition $k \gg k_x$, which occurs in most cases of physical interest.
For a grating which is totally transparent inside the slits and completely absorbing outside them, the boundary conditions are: \( \Psi(x,0^+) = 0 \) for any \( x \) belonging to the slit support and \( \Psi(x,0^+) = \Psi(x,0^-) \) for any \( x \) within the apertures, with \( \Psi(x,0^-) \) being the wave function incident on the grating, here given by (30). Thus, in the case of a grating with \( N \) openings of width \( \delta \) and mutual distance \( d \), a simple analytical calculation [44] renders

\[
c_N(k_x) = \sqrt{\frac{\delta}{2\pi N}} \left[ \frac{\sin(k_x\delta/2)}{k_x\delta/2} \right] \left[ \frac{\sin(Nk_xd/2)}{\sin(k_xd/2)} \right].
\] (54)

In the particular case \( N = 2 \), i.e., the well-known double-slit experiment, it is useful to express Eqs. (51) and (53) as

\[
\Psi(x,y) = \sqrt{\frac{k}{2\pi y}} e^{-i\pi/4} e^{iky} \int_{A_1} \Psi(x',0^-) e^{ik(x-x')^2/2y} dx' \\
+ \sqrt{\frac{k}{2\pi y}} e^{-i\pi/4} e^{iky} \int_{A_2} \Psi(x',0^-) e^{ik(x-x')^2/2y} dx' \\
\equiv \psi_1(x,y) + \psi_2(x,y),
\] (55)

and

\[
c_2(k_x) = \frac{1}{\sqrt{2\pi}} \int_{A_1} \Psi(x',0^-) e^{-ik_x x} dx \\
+ \frac{1}{\sqrt{2\pi}} \int_{A_2} \Psi(x',0^-) e^{-ik_x x} dx \\
\equiv \frac{1}{\sqrt{2}} (c_{1,d/2} + c_{1,-d/2}),
\] (56)

respectively, where \( \psi_1 \) refers to the scalar field coming from slit 1, centered at \( x = d/2 \), and \( \psi_2 \) is the scalar field coming from slit 2, at \( x = -d/2 \). After carrying out each integral in (56), we obtain

\[
c_{1,d/2}(k_x) = \sqrt{\frac{2}{\pi \delta}} \left[ \frac{\sin(k_x\delta/2)}{k_x} \right] e^{-ik_x d/2},
\]
\[
c_{1,-d/2}(k_x) = \sqrt{\frac{2}{\pi \delta}} \left[ \frac{\sin(k_x\delta/2)}{k_x} \right] e^{ik_x d/2},
\] (57)
which, when they are added, yield
\[ c_2(k_x) = \sqrt{\frac{\delta}{\pi}} \left[ \frac{\sin(k_x \delta/2)}{k_x \delta/2} \right] \cos(k_x d/2). \] (58)

In the space behind the grating, the EM field is given by Eqs. (26) and (27), where \( \Psi \) is described by (55). That is, the resulting EM field behind the grating consist of a superposition of two fields propagating from each slit, which reads as

\[
E = \frac{i \beta e^{i\phi}}{k} \frac{\partial \psi_1}{\partial y} \hat{x} - \frac{i \beta e^{i\phi}}{k} \frac{\partial \psi_1}{\partial x} \hat{y} + \alpha \psi_1 \hat{z} \\
\quad + \frac{i \beta e^{i\phi}}{k} \frac{\partial \psi_2}{\partial y} \hat{x} - \frac{i \beta e^{i\phi}}{k} \frac{\partial \psi_2}{\partial x} \hat{y} + \alpha \psi_2 \hat{z} \\
\equiv E_1 + E_2, \tag{59}
\]

\[
H = -\frac{i \alpha}{\omega \mu_0} \frac{\partial \psi_1}{\partial y} \hat{x} + \frac{i \alpha}{\omega \mu_0} \frac{\partial \psi_1}{\partial x} \hat{y} + \frac{k \beta e^{i\phi}}{\omega \mu_0} \psi_1 \hat{z} \\
\quad - \frac{i \alpha}{\omega \mu_0} \frac{\partial \psi_2}{\partial y} \hat{x} + \frac{i \alpha}{\omega \mu_0} \frac{\partial \psi_2}{\partial x} \hat{y} + \frac{k \beta e^{i\phi}}{\omega \mu_0} \psi_2 \hat{z} \\
\equiv H_1 + H_2. \tag{60}
\]

5. EME flow lines behind the two slits

In order to obtain the EME flow lines behind the grating, first we substitute (26) and (27) into (8), which yields the components of the Poynting vector along the different directions,

\[
S_x = \frac{i(\alpha^2 + \beta^2)}{4\omega \mu_0} \left( \Psi \frac{\partial \Psi^*}{\partial x} - \Psi^* \frac{\partial \Psi}{\partial x} \right), \tag{61}
\]

\[
S_y = \frac{i(\alpha^2 + \beta^2)}{4\omega \mu_0} \left( \Psi \frac{\partial \Psi^*}{\partial y} - \Psi^* \frac{\partial \Psi}{\partial y} \right), \tag{62}
\]

\[
S_z = -\frac{i \alpha \beta \sin \phi}{2k \omega \mu_0} \left( \frac{\partial \Psi}{\partial x} \frac{\partial \Psi^*}{\partial y} - \frac{\partial \Psi}{\partial y} \frac{\partial \Psi^*}{\partial x} \right). \tag{63}
\]

Proceeding similarly with (9) leads us to the EME density,

\[
U = \frac{(\alpha^2 + \beta^2)}{4\omega^2 \mu_0} \left( \frac{\partial \Psi}{\partial x} \frac{\partial \Psi^*}{\partial x} + \frac{\partial \Psi}{\partial y} \frac{\partial \Psi^*}{\partial y} + k^2 \Psi \Psi^* \right), \tag{64}
\]
which describes the interference pattern at the observation screen.

Before computing the EME flow lines, it is interesting to note the following feature about the interference pattern. Consider (64) expressed in terms of the two scalar fields, $\psi_1$ and $\psi_2$, i.e.,

$$U = \left(\frac{\alpha^2 + \beta^2}{4 \omega^2 \mu_0}\left(\frac{\partial \psi_1}{\partial x} \frac{\partial \psi_1^*}{\partial x} + \frac{\partial \psi_1}{\partial y} \frac{\partial \psi_1^*}{\partial y} + k^2 \psi_1 \psi_1^*\right) + \frac{\alpha^2 + \beta^2}{4 \omega^2 \mu_0}\left(\frac{\partial \psi_2}{\partial x} \frac{\partial \psi_2^*}{\partial x} + \frac{\partial \psi_2}{\partial y} \frac{\partial \psi_2^*}{\partial y} + k^2 \psi_2 \psi_2^*\right) + \frac{\alpha^2 + \beta^2}{4 \omega^2 \mu_0}\left(\frac{\partial \psi_1}{\partial x} \frac{\partial \psi_1^*}{\partial x} + \frac{\partial \psi_1}{\partial y} \frac{\partial \psi_1^*}{\partial y} + k^2 \psi_1 \psi_1^*\right) + \frac{\alpha^2 + \beta^2}{4 \omega^2 \mu_0}\left(\frac{\partial \psi_2}{\partial x} \frac{\partial \psi_2^*}{\partial x} + \frac{\partial \psi_2}{\partial y} \frac{\partial \psi_2^*}{\partial y} + k^2 \psi_2 \psi_2^*\right)\right). \quad (65)$$

In short-hand notation, (65) can also be expressed as

$$U = U_1 + U_2 + U_{12}, \quad (66)$$

where $U_1$ and $U_2$ are the EME densities associated with $\psi_1$ and $\psi_2$ (the first and second terms in (65)), respectively. On the other hand, $U_{12}$ (the last two terms in (65)) is the EME density arising from the interference of these waves. Since the polarization part (prefactor in terms of $\alpha$ and $\beta$) and the space part (depending on $\Psi$) appear factorized in (64), the interference pattern observed will not depend on the polarization state of the incident field, in agreement with the Arago-Fresnel laws.

Substituting now (61)-(64) into (10) renders the corresponding path equations along each direction,

$$\frac{dx}{ds} = ik \left\{ \frac{\Psi \frac{\partial \Psi^*}{\partial x} - \Psi^* \frac{\partial \Psi}{\partial x}}{\frac{\partial \Psi}{\partial x} \frac{\partial \Psi^*}{\partial x} + \frac{\partial \Psi}{\partial y} \frac{\partial \Psi^*}{\partial y} + k^2 \Psi \Psi^*} \right\}, \quad (67)$$

$$\frac{dy}{ds} = ik \left\{ \frac{\Psi \frac{\partial \Psi^*}{\partial y} - \Psi^* \frac{\partial \Psi}{\partial y}}{\frac{\partial \Psi}{\partial x} \frac{\partial \Psi^*}{\partial x} + \frac{\partial \Psi}{\partial y} \frac{\partial \Psi^*}{\partial y} + k^2 \Psi \Psi^*} \right\}, \quad (68)$$

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\[
\frac{dz}{ds} = -\frac{2i\alpha \beta \sin \phi}{(\alpha^2 + \beta^2)} \left\{ \frac{\partial \Psi}{\partial x} \frac{\partial \Psi^*}{\partial y} - \frac{\partial \Psi}{\partial y} \frac{\partial \Psi^*}{\partial x} \right\}.
\]

(69)

As can be noticed from these expressions, all the information about the polarization state of the diffracted EM wave is contained in the prefactor of (69). Thus, regardless of the polarization of the initial EM wave, since the diffracted waves arising from each slit have the same polarization state, one will always observe interference fringes, which is in agreement with the Arago-Fresnel laws [26].

In the case of linear polarization, (69) vanishes [24] and we can solve the EME flow-line equations simply by parametrizing, for instance, \( y \) as a function of \( x \), i.e.,

\[
\frac{dy}{dx} = \frac{\left( \Psi \frac{\partial \Psi^*}{\partial y} - \Psi^* \frac{\partial \Psi}{\partial y} \right)}{\left( \Psi \frac{\partial \Psi^*}{\partial x} - \Psi^* \frac{\partial \Psi}{\partial x} \right)},
\]

(70)

while the solution of (63) is simply \( z = z_0 \). On the contrary, in the case of elliptic polarization, the \( z \)-component does play an important role, as can be noticed when the EME flow-line equations are computed,

\[
\frac{dy}{dx} = \frac{\left( \Psi \frac{\partial \Psi^*}{\partial y} - \Psi^* \frac{\partial \Psi}{\partial y} \right)}{\left( \Psi \frac{\partial \Psi^*}{\partial x} - \Psi^* \frac{\partial \Psi}{\partial x} \right)},
\]

(71)

\[
\frac{dz}{dx} = -\frac{2\alpha \beta \sin \phi}{(\alpha^2 + \beta^2)k} \left\{ \frac{\partial \Psi}{\partial x} \frac{\partial \Psi^*}{\partial y} - \frac{\partial \Psi}{\partial y} \frac{\partial \Psi^*}{\partial x} \right\}.
\]

(72)

In Fig. 2, the EME flow lines associated with the diffraction of an incident EM field circularly polarized are plotted. The projections of these flow lines on the \( XY \) plane, shown in Fig. 2(b), are identical to those for incident linearly polarized light [24] (note that (71) is exactly the same as (70)). As shown elsewhere [40] within the context of Bohmian mechanics, for equations like (71), which describes the EME flow-line projections on the \( XY \), trajectories exiting through one slit never cross the trajectories coming up from the other.
Moreover, if both slits are identical (here, this means the same with and transmission function), the EME fluxes coming out from each slit are symmetric with respect to the axis $y = 0$. Now, although neither the electric nor the magnetic field depend on the $z$-coordinate, the EME flow lines display some remarkable features along this direction, as seen in Figs. 2(c) and 2(d). This is an effect of having circular (or elliptical, in general) polarization, which vanishes in the case of linear polarization, when $\phi = 0$ or $\pi$.

In order to understand the somewhat unexpected motion along the $z$-direction, let us go back to (63). Rearranging terms and using (61) and (62), this equation can be rewritten as

$$S_z = -\frac{\alpha \beta \sin \phi}{(\alpha^2 + \beta^2)k} \left( \frac{\partial S_y}{\partial x} - \frac{\partial S_x}{\partial y} \right) = \left[ -\frac{\alpha \beta \sin \phi}{(\alpha^2 + \beta^2)k} \right] \vec{\zeta} \cdot \hat{z}, \quad (73)$$

where

$$\vec{\zeta} \equiv \begin{pmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & 0 \\ S_x & S_y & 0 \end{pmatrix}. \quad (74)$$

According to (73), the presence of a polarization state gives rise to a flow along the $z$-direction in terms of the vorticity manifested by the fields $S_x$ and $S_y$, which may lead the EME flow lines to display loops out of the $XY$ plane. Nodal structures and other singularities and topological structures may then appear, as shown by Nye [36] within the context of wave dislocations [37] and by other authors within the context of the Riemann-Silberstein complex formulation of Maxwell’s equations [19, 38, 39] (see Appendix A). Experimentally, what one would observe on the $XZ$ plane is simply the typical fringe-like interference pattern constituted by dark and light parallel strips, which results from the accumulation of photons arriving at this plane. Note that (64) describes the interference pattern and is the result of transporting the EME density from the slits to some detection screen in accordance to the guidance or continuity equation [35]

$$S(\mathbf{r}) = U(\mathbf{r}) \mathbf{v}, \quad (75)$$

which is an alternative way to express (10). In this expression, $\mathbf{v}$ is an effective vector velocity field that transports the EME density through space in the form of the Poynting vector (i.e., the EME current density). This means that, if we make a histogram with the arrivals of a statistical distribution of
Figure 2: EME flow lines (15 for each slit) behind a two-slit grating associated with an incident EM plane wave is circularly polarized ($\alpha = \beta, \phi = \pi/2$): (a) 3D view, (b) $XY$ projection, (c) $XZ$ projection and (d) $YZ$ projection. The parameters considered in the simulation are: $\lambda = 500$ nm, $d = 20\lambda = 10$ µm and $\delta = d/2 = 5$ µm.
EME flow lines along the \( x \)-direction, we will observe the well-known interference pattern, as can be seen in Fig. 3(a). However, from (74), all the arrivals at a certain height \( z_f \) will not arise from positions at the slits at the same height \( z_0 = z_f \), but there is a flux upwards and downwards which breaks the longitudinal (along the \( z \)-direction) symmetry of the experiment when it is studied from the viewpoint of EME flow lines. This gives rise to a certain distribution of arrivals along the \( z \)-direction, as shown in Fig. 3(b). Since the EME flow lines distribute evenly around \( z_0 \) (here, along positive and negative \( z \), since we have chosen \( z_0 = 0 \)), as can be appreciated in Fig. 2(d), their distribution is also going to be symmetric with respect to \( z_0 \) in the histogram of Fig. 3(b).

6. The EM field and EME flow lines behind two slits, each followed by a linear polarizer

According to the Arago-Fresnel laws \([26]\), if two diffracted beams with different polarization states interfere, the visibility of the interference pattern will decrease. Indeed, if the polarization states are orthogonal the pattern will disappear totally. In order to describe this effect with EME flow lines, we consider the EM field behind a grating with two slits, each followed by a linear polarizer, such that behind the slit 1 there is a polarizer with its polarization axis oriented along the \( z \)-axis and behind the slit 2 the polarizer is oriented along the \( x \)-axis. As in Sec. 4, we shall write the total EM field as a sum of the electric and magnetic fields. However, due to the presence of the polarizers and their filtering effect on the incident EM field, instead of Eqs. (24) and (25), now we will express the electric and magnetic fields in terms of \( \psi_1 \) and \( \psi_2 \), described by (55), i.e.,

\[
E_e = \alpha \psi_1(\mathbf{r}) \mathbf{\hat{z}}, \tag{76}
\]

\[
H_h = \beta e^{i\phi} \sqrt{\frac{\epsilon_0}{\mu_0}} \psi_2(\mathbf{r}) \mathbf{\hat{z}}. \tag{77}
\]

By substituting (76) and (77) into (22) and (23), we will obtain the EM field behind the two slits covered by the polarizers,

\[
E = \frac{i \beta e^{i\phi}}{k} \frac{\partial \psi_2}{\partial y} \mathbf{\hat{x}} - \frac{i \beta e^{i\phi}}{k} \frac{\partial \psi_2}{\partial x} \mathbf{\hat{y}} + \alpha \psi_1 \mathbf{\hat{z}}, \tag{78}
\]

\[
H = -\frac{i \alpha}{\omega \mu_0} \frac{\partial \psi_1}{\partial y} \mathbf{\hat{x}} + \frac{i \alpha}{\omega \mu_0} \frac{\partial \psi_1}{\partial x} \mathbf{\hat{y}} + \frac{k \beta e^{i\phi}}{\omega \mu_0} \psi_2 \mathbf{\hat{z}}. \tag{79}
\]

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Figure 3: Histograms built up by counting the end points of individual EME flow lines associated with the incident EM field circularly polarized of Fig. 2. The detection screen is at $L = 1$ mm from a two-slit grating and the histograms represent counts: (a) along the $x$-direction and (b) along the $z$-direction. In part (a), the red solid line indicates the theoretical curve predicted by standard Electromagnetism, according to (64). The total number of paths considered is 5,000, with initial conditions homogenously distributed along each slit.
From these fields, we obtain the expressions for the EME density and the Poynting vector,

\[ U = \frac{\alpha^2}{4\omega^2\mu_0} \left( \frac{\partial \psi_1}{\partial x} \frac{\partial \psi_1^*}{\partial x} + \frac{\partial \psi_1}{\partial y} \frac{\partial \psi_1^*}{\partial y} + k^2 \psi_1 \psi_1^* \right) + \beta^2 \frac{\psi_2}{4\omega^2\mu_0} \left( \frac{\partial \psi_2}{\partial x} \frac{\partial \psi_2^*}{\partial x} + \frac{\partial \psi_2}{\partial y} \frac{\partial \psi_2^*}{\partial y} + k^2 \psi_2 \psi_2^* \right) \]

\[ = U_1 + U_2 \]  

(80)

and

\[ S_x = \frac{i\alpha^2}{4\omega\mu_0} \left( \psi_1 \frac{\partial \psi_1^*}{\partial y} - \psi_1^* \frac{\partial \psi_1}{\partial y} \right) + \frac{i\beta^2}{4\omega\mu_0} \left( \psi_2 \frac{\partial \psi_2^*}{\partial y} - \psi_2^* \frac{\partial \psi_2}{\partial y} \right), \]

(81)

\[ S_y = \frac{i\alpha^2}{4\omega\mu_0} \left( \psi_1 \frac{\partial \psi_1^*}{\partial x} - \psi_1^* \frac{\partial \psi_1}{\partial x} \right) + \frac{i\beta^2}{4\omega\mu_0} \left( \psi_2 \frac{\partial \psi_2^*}{\partial x} - \psi_2^* \frac{\partial \psi_2}{\partial x} \right), \]

(82)

\[ S_z = \frac{\alpha\beta e^{i\phi}}{4k\omega\mu_0} \left( \frac{\partial \psi_2}{\partial y} \frac{\partial \psi_1^*}{\partial x} - \frac{\partial \psi_2}{\partial x} \frac{\partial \psi_1^*}{\partial y} \right) \]

\[ - \frac{\alpha\beta e^{-i\phi}}{4k\omega\mu_0} \left( \frac{\partial \psi_1}{\partial y} \frac{\partial \psi_2^*}{\partial x} - \frac{\partial \psi_1}{\partial x} \frac{\partial \psi_2^*}{\partial y} \right), \]

(83)

respectively. As can be noticed in (80), \( U \) describes the bare addition of EME densities associated with the fields diffracted by each slit, with no interference term. This means that no interference pattern is going to be observed at the detection screen, in accordance to the Arago-Fresnel law for the interference of two beams with orthogonal polarization states (perpendicularly polarized in the case of linear polarization \([27]\) or with opposite handedness in the case of elliptic or circular polarization \([28]\)). Regarding the Poynting vector, we note that the EME flux along the \( x \) and \( y \)-direction is also given by the simple addition of fluxes coming from each slit. Thus, unlike the diffraction problem dealt with in the previous section, now it is not possible to factorize the flux (neither the EME density) in terms of its spatial and polarization parts. This has a consequence on the EME flux along the \( z \)-direction, which does not satisfy the rotational character described by (74). On the other hand, since the third and forth terms in (83) do not vanish, there will be an EME flux along the \( z \)-direction even in the case of linear polarization (remember from Sec. 5 that, for linear polarization, \( S_z = 0 \)). This can also be seen from the
EME flow-line equations,

\[
\frac{dz}{dx} = \frac{i\alpha\beta e^{i\phi}}{k} \left( \frac{\partial \psi_2 \partial \psi_1^*}{\partial x} - \frac{\partial \psi_2^* \partial \psi_1}{\partial y} \right) - \frac{\alpha^2}{k} \left( \psi_1 \frac{\partial \psi_1^*}{\partial y} - \psi_1^* \frac{\partial \psi_1}{\partial y} \right) + \frac{\beta^2}{k} \left( \psi_2 \frac{\partial \psi_2^*}{\partial y} - \psi_2^* \frac{\partial \psi_2}{\partial y} \right),
\]

(84)

\[
\frac{dy}{dx} = \frac{\alpha^2}{\psi_1^* \frac{\partial \psi_1^*}{\partial x} - \psi_1^* \frac{\partial \psi_1}{\partial x} + \beta^2 \left( \psi_2 \frac{\partial \psi_2^*}{\partial y} - \psi_2^* \frac{\partial \psi_2}{\partial y} \right),
\]

(85)

Note in (84) that \(dz/dx\), effectively, does not vanish, not only for \(\phi = 0\) or \(\pi\) (the conditions for linear polarization), but neither for any other \(\phi\) value.

In Fig. 4 we observe an ensemble of EME flow lines for an incident EM field linearly polarized. As can be noticed, when polarizers with orthogonal polarization directions act on the diffracted wave, the topology of the flow lines changes dramatically when compared with that observed in Fig. 2. When one looks at the \(XY\) projection (see Fig. 4(b)), the wiggling behavior that gives rise to the different interference fringes of the pattern in the Fraunhofer region are lacking. This reflects the fact that the EME density, described by (80), is just the sum of the EME densities associated with the components of the wave that arise from each slit, which can be appreciated in the histogram presented in Fig. 5(a). This histogram reproduces the distribution pattern of the EME flow lines in the Fraunhofer region, which consists of the sum of the two single-slit diffraction patterns associated with the EM field arising from each slit (the different maxima and minima are related to the fact that the single-slit diffraction patterns here are sinc-functions, rather than to a two-slit interference pattern). This damping of the interference fringes as well as the lack of wiggling features in the paths is similar to the behavior found in trajectories for massive particles in the case of decoherence [33, 34].

On the other hand, if we look at the topology of the EME flow lines along the \(z\)-direction (see Figs. 4(c) and (d)), we note that there is a symmetry
Figure 4: EME flow lines (15 for each slit) behind a two-slit grating, where each slit is followed by polarizers with orthogonal axes and the incident EM field is linearly polarized ($\alpha = \beta$, $\phi = 0$): (a) 3D view, (b) XY projection, (c) XZ projection and (d) YZ projection. Because of incident EM field is linearly polarized, the ensembles leaving each slit behave exactly the same and, therefore, the paths exiting through slit 1 look the same as those exiting through slit 2. The parameters considered in the simulation are: $\lambda = 500$ nm, $d = 20\lambda = 10$ $\mu$m and $\delta = d/2 = 5$ $\mu$m.
Figure 5: Histograms built up by counting the end points of individual EME flow lines associated with the incident EM field linearly polarized of Fig. 4. The detection screen is at $L = 1$ mm from a two-slit grating and the histograms represent counts: (a) along the $x$-direction and (b) along the $z$-direction. In part (a), the red solid line indicates the theoretical curve predicted by standard Electromagnetism, according to (80). The total number of paths considered is 5,000, with initial conditions homogeneously distributed along each slit.
with respect to \( x = 0 \) (see Fig. 4(c)). This is the result of the breaking of the rotationality property mentioned in the previous section. The same “symmetry” breaking can also be observed by looking at the distribution of the trajectories along the \( z \)-direction, as shown in Fig. 5(b). If we changed the initial polarization state, from \( \phi = 0 \) to \( \phi = \pi \), we would obtain the same pattern, but inverted with respect to \( z = 0 \).

In the case of an incident EM field circularly polarized, displayed in Fig. 6, we find that the EME flow-line projections on the \( XY \) plane (see Fig. 6(b)) look exactly the same as those for the incident EM field linearly polarized (see Fig. 4(b)), in accordance to Eq. (85). However, as seen in Sec. 5 due to the rotationality associated with circular polarization, a breaking of the specular symmetry with respect to \( x = 0 \) will take place, and the EME flux leaving each slit is going to be different, as shown in Figs. 6(c) and 6(d). This will give rise to a histogram like the one shown in Fig. 5(a) when photon are counted along the \( x \)-direction and another similar to that of Fig. 3(b) when they are counted along the \( z \)-direction.

7. Final discussion and conclusions

Interference experiments with massive particles and photons can be well understood on the theoretical grounds provided by the Schrödinger and Maxwell equations, respectively. Thus, if one wishes to determine photon paths on the same footing as Bohmian trajectories, the most appropriate theoretical framework is the one based on Maxwell’s equations. Within this formulation, a close analogy can be established between the path equations based on the latter and Bohm’s approach when the particle aspect of both light and matter is taken into consideration. This allows to compare on the same grounds the (Bohmian) trajectories for massive particles with the paths derived for photons from classical electromagnetism. The photon paths (EME flow lines) are determined from the Poynting vector, with the components of the electric and magnetic vector fields expressed in terms of a function that explicitly takes into account the boundary conditions imposed by the grating. It is remarkable that, in the case of photons, any path-based interpretation will be complementary to the standard Huyghens’ one, based on the superposition of secondary wavelets. Furthermore, the topology displayed by the photon paths is strikingly similar to that displayed by massive particles. As happens in the case of massive particles [7], such a topology can also be inferred and explained from the corresponding path equation.
Figure 6: EME flow lines (15 for each slit) behind a two-slit grating, where each slit is followed by polarizers with orthogonal axes and the incident EM field is circularly polarized ($\alpha = \beta$, $\phi = \pi/2$): (a) 3D view, (b) $XY$ projection, (c) $XZ$ projection and (d) $YZ$ projection. The parameters considered in the simulation are: $\lambda = 500 \text{ nm}$, $d = 20\lambda = 10 \mu\text{m}$ and $\delta = d/2 = 5 \mu\text{m}$.
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A. Appendix A: EME flow lines in the Riemann-Silberstein formulation of Electromagnetism

Both Maxwell’s and Schrödinger’s equations describe the evolution of fields (EM and probability fields, respectively) in configuration space. From these fields, one can obtain paths which show how they evolve in space, namely EME flow lines or Bohmian trajectories, respectively. However, Maxwell’s equations look quite different formally from the Schrödinger equation as they are usually given. A formulation that allows one to put Maxwell’s equations on similar formal grounds as Schrödinger’s one (but without $\hbar$) is the complex form of Maxwell’s equations [45], which is based on the so-called Riemann-Silberstein complex EM vector [15, 16],

$$\tilde{F}(r, t) = \frac{1}{\sqrt{2}} \left[ \sqrt{\varepsilon_0} \tilde{E}(r, t) + i \sqrt{\mu_0} \tilde{H}(r, t) \right], \quad (86)$$

with the change of variable

$$\tilde{E}(r, t) = \frac{1}{\sqrt{2 \varepsilon_0}} \left( \tilde{F} + \tilde{F}^* \right), \quad (87)$$

$$\tilde{H}(r, t) = \frac{1}{i \sqrt{2 \mu_0}} \left( \tilde{F} - \tilde{F}^* \right), \quad (88)$$

where $\tilde{E}$ and $\tilde{H}$ are real fields. Introducing (87) and (88) into Maxwell’s equations in the absence of electrical charge densities, we obtain

$$i \frac{\partial \tilde{F}}{\partial t} = c \nabla \times \tilde{F} \quad (89)$$

$$\nabla \cdot \tilde{F} = 0. \quad (90)$$

As can be noticed, (89) is the analog for photons of the Schrödinger equation for massive particles, while (90) describes the conservation of the EME density through space. This analogy between Schrödinger’s equation and the EM
equations within the Riemann-Silberstein formulation becomes more apparent by gathering (89) and (90) in a single equation. This is done by applying the operator $-i\partial/\partial t$ to both sides of (89) and then rearranging terms taking into account (90) and the vectorial relation

\[ \nabla \times (\nabla \times \mathbf{A}) = \nabla(\nabla \cdot \mathbf{A}) - \nabla^2 \mathbf{A}, \quad (91) \]

where $\mathbf{A}$ is a general vector field. This renders

\[ \frac{\partial^2 \mathbf{F}}{\partial t^2} = c^2 \nabla^2 \mathbf{F}, \quad (92) \]

which has the well-known form of a wave equation. However, unlike Schrödinger’s equation, here we find that the time-derivative is of second order; this is a manifestation of the fact that energy is proportional to the momentum for radiation ($E \sim cp$), while the dependence is quadratic for matter ($E \sim p^2/2m$) [46]. In the particular case of diffraction problems, which can be reduced to boundary condition problems because of their time-independence, the space and time parts of (92) are separable. Thus, $\mathbf{F}_i$ can be decomposed as

\[ \mathbf{F}_i(r, t) = \mathbf{F}_i(r) \phi_i(t) = \mathbf{F}_i(r) e^{-i\omega t}, \quad (93) \]

where the space part, $\mathbf{F}_i$, satisfies Helmholtz’s equation,

\[ \nabla^2 \mathbf{F}_i(r) + k^2 \mathbf{F}_i(r) = 0, \quad (94) \]

and its time-dependent part the differential equation

\[ \frac{\partial^2 \phi_i(t)}{\partial t^2} = -\omega^2 \phi_i(t), \quad (95) \]

with $c = \omega/k$.

Within this formulation, the EME density is given by

\[ \mathcal{U} = \frac{1}{2} \left( \epsilon_0 \mathbf{E} \cdot \mathbf{E} + \mu_0 \mathbf{H} \cdot \mathbf{H} \right) = \mathbf{F} \cdot \mathbf{F}^* \quad (96) \]

and its flux, described by the Poynting vector, is straightforwardly obtained after developing the time-derivative of $\mathcal{U}$ to yield

\[ \mathbf{S} = \mathbf{E} \times \mathbf{H} = ic\mathbf{F} \times \mathbf{F}^*, \quad (97) \]
where the relation
\[ \nabla (A \times B) = B \cdot \nabla \times A - A \cdot \nabla \times B \] (98)
for any two vectors, \( A \) and \( B \), has been taken into account. From these magnitudes, we can now obtain the stationary EME flow lines within the Riemann-Silberstein formulation as,
\[ \frac{dr}{ds} = \frac{1}{c} \frac{S}{U} = \frac{i}{c} \frac{\langle \tilde{F} \times \tilde{F}^* \rangle}{\langle \tilde{F} \cdot \tilde{F}^* \rangle} \] (99)
(here, \( \langle \tilde{A} \rangle \) denotes the time-average of the magnitude \( \tilde{A} \)), which transport the time-averaged EME density, \( U \), as described by the (also time-averaged) Poynting vector, \( S \).

If the electric and magnetic fields are complex and the definition of the Riemann-Silberstein vector is kept as in (86), i.e., the real and imaginary parts are given by the electric and magnetic fields, respectively, then we need to include into the formulation two of these vectors in order to have a complete description of the problem, each one associated with the real or the imaginary parts of the electric and magnetic fields. That is, if
\[ \tilde{E} = \tilde{E}_1 + i\tilde{E}_2, \quad \tilde{H} = \tilde{H}_1 + i\tilde{H}_2, \] (100)
with \( \tilde{E}_i \) and \( \tilde{H}_i \) \((i = 1, 2)\) being real vector fields satisfying the corresponding Maxwell equations, we will have
\[ \tilde{F}_1 = \frac{1}{\sqrt{2}} \left( \sqrt{\epsilon_0} \tilde{E}_1 + i\sqrt{\mu_0} \tilde{H}_1 \right), \]
\[ \tilde{F}_2 = \frac{1}{\sqrt{2}} \left( \sqrt{\epsilon_0} \tilde{E}_2 + i\sqrt{\mu_0} \tilde{H}_2 \right). \] (101)
The EME density (96) and the Poynting vector (97) then read as
\[ U = \frac{1}{2} \left( \epsilon_0 \tilde{E} \cdot \tilde{E}^* + \mu_0 \tilde{H} \cdot \tilde{H}^* \right) = \sum_{i=1,2} \tilde{F}_i \cdot \tilde{F}_i^*, \] (102)
\[ S = \text{Re} \left( \tilde{E} \times \tilde{H}^* \right) = \text{Re} \left( ic \sum_{i=1,2} \tilde{F}_i \times \tilde{F}_i^* \right), \] (103)
respectively, and their time-averaged homologous, assuming the decomposition (93), as

\begin{equation}
U = \frac{1}{2} \sum_{i=1,2} F_i \cdot F^*_i, \quad (104)
\end{equation}

\begin{equation}
S = \frac{1}{2} \text{Re} \left( ic \sum_{i=1,2} F_i \times F^*_i \right). \quad (105)
\end{equation}

Apart from the interest of this formulation within the field of the Fundamental Physics, it has also been considered in a more applied way, for instance, in Solid State Physics and Condensed Matter [47].

References

[1] A. Tonomura, J. Endo, T. Matsuda, T. Kawasaki, H. Ezawa, Am. J. Phys. 57 (1989) 117.

[2] F. Shimizu, K. Shimizu, H. Takuma, Phys. Rev. A 46 (1992) R17.

[3] M. Born, Z. Phys. 37 (1926) 863 (1926); 38 (1926) 803; Nature 119 (1927) 354.

[4] C. Philippidis, C. Dewdney, B.J. Hiley, Nuovo Cimento B52 (1979) 15.

[5] A.S. Sanz, F. Borondo, S. Miret-Artés, Phys. Rev. B 61 (2000) 7743; A.S. Sanz, F. Borondo, S. Miret-Artés, J. Phys.: Condens. Matter 14 (2002) 6109.

[6] R. Guantes, A.S. Sanz, J. Margalef-Roig, S. Miret-Artés, Surf. Sci. Rep. 53 (2004) 199; A.S. Sanz, S. Miret-Artés, in: D. Micha, I. Burghardt (Eds.), Quantum Dynamics of Complex Molecular Systems, Vol. 83, Springer, New York, 2006, p. 343.

[7] A.S. Sanz, S. Miret-Artés, J. Chem. Phys. 126 (2007) 234106.

[8] M. Božič, D. Arsenović, Acta Phys. Hung. B 26 (2006) 219.

[9] M. Davidović, D. Arsenović, M. Božič, A.S. Sanz, S. Miret-Artés, Eur. Phys. J. Spec. Top. 160 (2008) 95.
[10] D. Bohm, Phys. Rev. 85 (1952) 166; 85 (1952) 184.

[11] P.R. Holland, The Quantum Theory of Motion, Cambridge University Press, Cambridge, 1993.

[12] R. Loudon, The Quantum Theory of Light, Oxford University Press, Oxford, 1983.

[13] T.L. Dimitrova, A. Weis, Am. J. Phys. 76 (2008) 137.

[14] P.A.M. Dirac, The Principles of Quantum Mechanics, Clarendon Press, Oxford, 1958.

[15] I. Bialynicki-Birula, Prog. Opt. 36 (1996) 245, and references therein.

[16] I. Bialynicki-Birula, in: J.H. Eberly, L. Mandel, E. Wolf (Eds.), Coherence and Quantum Optics, Vol. 8, Plenum, New York, 1996, p. 313, and references therein.

[17] M.O. Scully, M.S. Zubairy, Quantum Optics, Cambridge University Press, Cambridge, 1997.

[18] D.H. Kobe, Found. Phys. 29 (1999) 1203.

[19] M.V. Berry, J. Opt. A 6 (2004) S175.

[20] P.R. Holland, Proc. R. Soc. A 461 (2005) 3659.

[21] B.J. Smith, M.G. Raymer, New J. Phys. 9 (2007) 414.

[22] W. Braunbek, G. Laukien, Optik 9 (1952) 174.

[23] R.D. Prosser, Int. J. Theor. Phys. 15 (1976) 169; 15 (1976) 181.

[24] M. Davidović, A.S. Sanz, D. Arsenović, M. Božić, S. Miret-Artés, Phys. Scr. T135 (July, 2009); Preprint arXiv:0805.3330.

[25] P. Ghose, A.S. Majumdar, S. Guha, J. Sau, Phys. Lett. A 290 (2001) 205.

[26] R. Barakat, J. Opt. Soc. Am. A 10 (1993) 180.

[27] J.L. Hunt, G. Karl, Am. J. Phys. 38 (1970) 1249.
[28] D. Pescetti, Am. J. Phys. 40 (1972) 735.

[29] M.O. Scully, B.G. Englert, H. Walther, Nature (London) 351 (1991) 111.

[30] S.P. Walborn, M.O. Terra-Cunha, S. Pádua, C.H. Monken, Phys. Rev. A 65 (2002) 033818.

[31] V. Scarani, A. Suarez, Am. J. Phys. 66 (1998) 718.

[32] M.A. Nielsen, I.L. Chuang, Quantum Computation and Quantum Information, Cambridge University Press, Cambridge, 2000.

[33] A.S. Sanz, F. Borondo, Eur. Phys. J. D 44 (2007) 319.

[34] A.S. Sanz, F. Borondo, Chem. Phys. Lett. (accepted for publication, 2009); Preprint [arXiv:0803.2581].

[35] M. Born, E. Wolf, Principles of Optics, Pergamon Press, Oxford, 2002, 7th ed.

[36] J.F. Nye, Proc. R. Soc. Lond. A 387 (1983) 105; 389 (1983) 279.

[37] J.F. Nye, M.V. Berry, Proc. R. Soc. Lond. A 336 (1974) 165.

[38] I. Bialynicki-Birula, Z. Bialynicka-Birula, Phys. Rev. A 67 (2003) 062114.

[39] G. Kaiser, J. Opt. A: Pure Appl. Opt. 6 (2003) S243.

[40] A.S. Sanz, S. Miret-Artés, J. Phys. A 41 (2008) 435303.

[41] M. Božić, D. Arsenović, L. Vušković, Z. Naturf. A56 (2001) 173.

[42] D. Arsenović, M. Božić, O.V. Man’ko, V.I. Man’ko, J. Russ. Laser Res. 26 (2005) 94.

[43] M. Davidović, M. Božić, D. Arsenović, J. Russ. Laser Res. 27 (2006) 220.

[44] D. Arsenović, M. Božić, L. Vušković, J. Opt. B: Quantum Semiclass. Opt. 4 (2002) S358.

[45] L. Silberstein, Ann. Phys. 22 (1907) 579.
[46] L.I. Schiff, Quantum Mechanics, McGraw-Hill, Singapore, 1968, 3rd ed.

[47] A.G. Borisov, S.V. Shabanov, J. Comp. Phys. 209 (2005) 643; 216 (2006) 391; A.G. Borisov, F.J. García de Abajo, S.V. Shabanov, Phys. Rev. B 71 (2005) 075408.