Bulk Decay of \((4+n)\)-Dimensional Simply Rotating Black Holes: Tensor-Type Gravitons

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Abstract. We study the emission in the bulk of tensor-type gravitons by a simply rotating \((4+n)\)-dimensional black hole. The decoupling of the radial and angular part of the graviton field equation makes it possible to solve them analytically (in the limit of low-energy emitted particles and low-angular momentum of the black hole) and find the corresponding absorption probability. We also move to solve these equations numerically. The comparison between analytic and numerical results shows a very good agreement in low and intermediate energy regimes. Finally, the energy and angular momentum emission rates were calculated in order to explore their dependence on the number of additional spacelike dimensions of the spacetime background and the angular momentum of the black hole. Interesting conclusions about the significance of tensor-type gravitons as energy carriers in the context of Hawking radiation were reached.

1. Introduction
This work was motivated primarily by the exciting possibility that a mini-black hole could be created at the LHC should one of the theories, postulating the existence of additional spacelike dimensions in nature, is proven to be correct. All such scenarios, namely the Large Extra Dimensions \([1]\) and Warped Extra Dimensions \([2]\) ones, predict a new, lower than the four-dimensional, fundamental scale for gravity as low as one TeV. The most interesting phenomenological consequence of the new scale is the expectation that miniature black holes could be created during high-energy particle collisions at ground-based accelerators \([3]\). As these black holes would be higher-dimensional objects, their evaporation pattern by emission of Hawking radiation \([4]\), will strongly depend both on the number and nature of the extra dimensions. Therefore the comparison between the theoretically expected behavior and the measurements performed by our detectors, will reveal valuable information about spacetime.

Because of this possibility, there has been a considerable amount of interest in the study of the radiation emission spectra from a higher-dimensional black hole in the literature in recent years (for some reviews, see \([5,6]\)). Rotating and non-rotating black holes have been exhaustively studied both analytically and numerically \([7,8,9,11,12,13,14,15,16,17,18,19,20,21,22,23,24,25,26,27,28,29,30,31,32]\). These studies have shown that the brane channel is in most cases the dominant one, although at certain circumstances the bulk channel can be equally important at specific particle channels. Unfortunately, the question of the energy balance \([33]\) between the ‘bulk’ and ‘brane’ channel during the Hawking radiation has not been answered yet.
Nevertheless, this balance is our principal guide to correctly evaluate signals correlated to the black hole evaporation should ever our detectors record any, in order to find decisive evidence about the existence of extra dimensions and their nature. For the aforementioned balance to be calculated one has to include the contribution of gravitons to the overall evaporation process.

Here we study the emission of Hawking radiation in the bulk in the form of tensor-type gravitational modes by a higher-dimensional black hole with at least three additional spacelike dimensions and one angular-momentum component using the perturbation equations derived in Refs. [34, 35]. After presenting the geometrical set-up and theoretical framework, we solve analytically the radial part of the graviton field equation (in the of low-energy emitted particles and low-angular momentum of the black hole limit). We repeat these calculations using numerical methods. Next, we perform a comparison of the analytical and numerical results to check the validity of the analytic, approximate method. The exact form of the energy and angular-momentum emission rates in the bulk from the simply rotating black hole in the form of tensor-type gravitons are also computed and studied. Finally, a comparison with the corresponding results for the emission of scalars is presented. We finish with our conclusions.

2. Theoretical Framework

We will focus on the case of a spacetime described by a line-element of the form

\[ ds^2 = G_{MN} dz^M dz^N = g_{ab} dx^a dx^b + S^2(x) d\Omega_n^2, \]  

where \( \{a, b\} = (0, 1, 2, 3) \) and \( d\Omega_n^2 \) stands for the line-element of a \( n \)-dimensional unit sphere \( S^n \). The above line-element is a special \((4 + n)\)-dimensional case of a more general class of gravitational backgrounds where the spacetime can be written as the warped product of a \( m \)-dimensional spacetime \( \mathcal{N} \) and a \( n \)-dimensional space \( \mathcal{K} \) of constant curvature \( \mathcal{K} \). In these type of backgrounds it is possible to derive the perturbation equations for all types of gravitational modes in the case of a maximally-symmetric higher-dimensional black-hole background [38] where \( m = 2 \) and \( \mathcal{K} = S^n \).

When the black hole rotates only in a single two-plane along the 4-dimensional spacetime, its line-element takes the well-known form [39]

\[ ds^2 = -\left(1 - \frac{\mu}{\Sigma r^{n-1}}\right) dt^2 - \frac{2a\mu \sin^2 \theta}{\Sigma r^{n-1}} dt d\varphi + \frac{\Sigma}{\Delta} dr^2 + \Sigma d\theta^2 
+ \left( r^2 + a_2^2 \frac{\mu \sin^2 \theta}{\Sigma r^{n-1}} \right) \sin^2 \theta d\varphi^2 + r^2 \cos^2 \theta d\Omega_n^2, \]

where \( \Delta = r^2 + a_2^2 - \frac{\mu}{\Sigma r^{n-1}} \), \( \Sigma = r^2 + a_2^2 \cos^2 \theta \),

\[ a_1 = a \quad \text{and} \quad a_2 = a_3 = \ldots = a_N = 0. \]

Since colliding particles have a non-zero impact parameter only along the usual 3-space which results into a single angular momentum component once the black hole is created, this line-element is absolutely suitable for our study. Then, the black hole’s mass \( M_{BH} \) and angular momentum \( J \) are related to the parameters \( a \) and \( \mu \) as follows [39]

\[ M_{BH} = \frac{(n + 2)A_{n+2}}{16\pi G_D} \mu, \quad J = \frac{2}{n + 2} M_{BH} a, \]

with \( G_D \) being the \((4 + n)\)-dimensional Newton’s constant, and \( A_{n+2} \) the area of a \((n + 2)\)-dimensional unit sphere given by \( A_{n+2} = 2\pi^{(n+3)/2}/\Gamma[(n + 3)/2] \).

The line-element (2) is described by Eq. (11) with \( S(x) = r \cos \theta \). Tensor-type gravitational perturbations for the line-element (2) – which exist only for \( n \geq 3 \) – can be expanded in terms
of a basis of transverse and traceless harmonic tensors \( T_{ij}^{(\ell, \alpha)} \) on the unit sphere \( S^n \) as follows

\[
\delta G_{ij} = 2S^2(x) \sum_{\ell, \alpha} H_{ij}^{(\ell, \alpha)}(x) T_{ij}^{(\ell, \alpha)}(y),
\]

where \( \{i, j\} \) refer to the \( y \)-coordinates along the sphere \( S^n \), and \( T_{ij}^{(\ell, \alpha)} \) satisfy the eigenvalue equation

\[
[\hat{\Delta} + \ell(\ell + n - 1) - 2] T_{ij}^{(\ell, \alpha)} = 0.
\]

\( \hat{\Delta} \) is the Laplace-Beltrami operator on \( S^n \), \( \ell = 2, 3, 4, \ldots \) labels the corresponding eigenvalues and \( \alpha \) is used to distinguish harmonic tensors with the same eigenvalue.

With the use of the expansion (5), the \((i, j)\)-component of the Einstein’s equation in vacuum leads to the following second-order hyperbolic equation for the amplitude \( H_{ij}^{(\ell, \alpha)}(x) \)

\[
-\Box H_T - \frac{n}{r \cos \theta} g^{ab} \partial_a (r \cos \theta) \partial_b H_T + \frac{\ell(\ell + n - 1)}{r^2 \cos^2 \theta} H_T = 0,
\]

where \( \Box \) is the d’Alembertian operator for the metric \( g_{ab}(x) \) (for simplicity, we have omitted the labels \( \{\ell, \alpha\} \)). Using the ansatz

\[
H_T(x) = e^{-i\omega t} e^{im\phi} R(r) Q(\theta),
\]

the above partial differential equation reduces to a set of radial and angular equation, namely

\[
\frac{1}{r^n} \partial_r (r^n \Delta \partial_r R) + \left( \frac{K^2}{\Delta} - \frac{\ell(\ell + n - 1)a^2}{r^2} - \Lambda_{j\ell m} \right) R = 0,
\]

\[
\frac{1}{\sin \theta \cos^n \theta} \partial_\theta (\sin \theta \cos^n \theta \partial_\theta Q) + \left( \omega^2 a^2 \cos^2 \theta - \frac{m^2}{\sin^2 \theta} - \frac{\ell(\ell + n - 1)}{\cos^2 \theta} + E_{j\ell m} \right) Q = 0
\]

where

\[
K = (r^2 + a^2) \omega - am, \quad \Lambda_{j\ell m} = E_{j\ell m} + a^2 \omega^2 - 2am\omega,
\]

with \( E_{j\ell m} \) being the separation constant of the two equations, and \( j \) a new quantum number that labels the eigenvalues of the angular function \( Q(\theta) \).

One needs to solve the above set of second-order ordinary differential equations (9)-(10) for \( R \) and \( Q \). The solution of the radial equation will yield the expression for the corresponding absorption probability and the angular equation will provide us with the value of the separation constant \( E_{j\ell m} \).

3. Analytic Solution

In spacetimes of the form of Eq. (1), the tensor-type gravitational perturbations are found to satisfy the same field equations that a massless scalar field obeys in the same background. Equations (9)–(10) are identical to the ones that follow from the scalar field equation

\[
\frac{1}{\sqrt{-G}} \partial_M \left( \sqrt{-G} G^{MN} \partial_N \Phi \right) = 0,
\]

using the expansion of \( \Phi \) in terms of the hyperspherical harmonics \( \Upsilon^{(\ell, \alpha)}(y) \) on \( S^n \)

\[
\Phi(x, y) = e^{-i\omega t} e^{im\phi} R(r) Q(\theta) \Upsilon^{(\ell, \alpha)}(y).
\]
Because of the different nature of the scalar and gravitational degrees of freedom, \( \ell \) is constrained to values \( \geq 2 \) in the case of gravitons whereas \( \ell \geq 0 \) in the scalar case [41].

The aim of the analytic approach is to find the asymptotic solutions near the horizon of the black hole \( (r \simeq r_h) \), and far away from it \( (r \gg r_h) \) and then match them in an intermediate zone, to create an analytical solution for \( R(r) \) over the whole radial regime. The black hole’s horizon radius \( r_h \) follows from the equation \( \Delta(r_h) = 0 \), and is given by the relation \( r_h^{n+1} = \mu/(1 + a_r^2) \), where \( a_r \equiv a/r_h \).

In the near-horizon regime \( (r \simeq r_h) \), Eq. (19) takes the form

\[
f (1-f) \frac{d^2 R}{df^2} + (1-D_s f) \frac{dR}{df} + \left[ \frac{K^2_+}{A^2_+ f(1-f)} - \frac{[\ell(\ell+n-1)a_r^2 + \Lambda_{j\ell m}]}{A^2_r (1+f)} \right] R = 0, \tag{14}
\]

with the use of the variable \( f(r) = \Delta(r)/(r^2 + a_r^2) \), where

\[
A_s = (n+1) + (n-1)a_r^2, \quad K_s = (1 + a_r^2)\omega_s - a_s m \tag{15}
\]

\[
\omega_s \equiv \omega r_h \quad \text{and} \quad D_s \equiv 1 - 4a_r^2/A^2_r. \tag{16}
\]

Equation (14) can be brought to the form of a hypergeometric differential equation with general solution [42]

\[
R_{NH}(f) = A_- f^\alpha (1-f)^\beta F(a, b, c; f) + A_+ f^{-\alpha} (1-f)^\beta F(a-c+1, b-c+1, 2-c; f), \tag{17}
\]

where the indices \( (a, b, c) \) are defined as: \( a \equiv \alpha + \beta + D_s - 1, b \equiv \alpha - \beta, \) and \( c \equiv 1 + 2\alpha \) and \( A_\pm \) are integration constants. Parameters \( \alpha \) and \( \beta \) are given by \( \alpha_\pm \equiv \pm iK_s/A_s \) and

\[
\beta = \frac{1}{2} \left[ (2-D_s) - \sqrt{(D_s - 2)^2 - 4 \frac{K^2_+ - [\ell(\ell+n-1)a_r^2 + \Lambda_{j\ell m}]}{A^2_r (1+f)}} \right]. \tag{18}
\]

Close to the horizon, we employ the convenient transformation of the radial variable \( y = r_h(1 + a_r^2)\ln(f/A_s) \) and impose the boundary condition that no outgoing modes exist near the black hole’s horizon. Then the near-horizon solution takes the form

\[
R_{NH}(f) = A_- f^{\alpha} (1-f)^{\beta} F(a, b, c; f). \tag{19}
\]

In the far-field regime \( (r \gg r_h) \), Eq. (9) can easily be brought into the form of a Bessel differential equation by making the substitution \( R(r) = r^{-(\frac{\alpha+1}{2})} \tilde{R}(r) \) and employing a new radial variable \( z = \omega r \), with general solution

\[
R_{FF}(r) = \frac{B_1}{r^{\frac{\alpha+1}{2}}} J_\nu(\omega r) + \frac{B_2}{r^{\frac{\alpha+1}{2}}} Y_\nu(\omega r), \tag{20}
\]

where \( J_\nu \) and \( Y_\nu \) are the Bessel functions of the first and second kind, respectively, and

\[
\nu = \sqrt{E_{j\ell m} + a_r^2 \omega^2 + (\frac{n+1}{2})^2}.
\]

In order to match the two asymptotic solutions [10] and [20], they both need to be expanded for intermediate values of the radial variable. First we shift the hypergeometric function [15] so that its argument changes from \( f \) to \( 1-f \) by using a well-known relation [12] [30]. Then in the limit \( r \gg r_h \) (or, equivalently, \( f \to 1 \)) the near-horizon solution takes the ‘stretched’ form

\[
R_{NH}(r) \simeq A_1 r^{-(n+1)\beta} + A_2 r^{(n+1)(\beta + D_s - 2)}, \tag{21}
\]
with \( A_1 \) and \( A_2 \) defined as
\[
A_1 = A_- \left[ (1 + a_s^2) r_h^{n+1} \right]^\beta \frac{\Gamma(c)\Gamma(c-a-b)}{\Gamma(c-a)\Gamma(c-b)},
\]
\[
A_2 = A_- \left[ (1 + a_s^2) r_h^{n+1} \right]^{-(\beta+D_s-2)} \frac{\Gamma(c)\Gamma(a+b-c)}{\Gamma(a)\Gamma(b)}.
\]

The far-field solution \([20]\) in the limit of \( r \to 0 \) takes the polynomial form
\[
R_{FF}(r) \simeq \frac{B_1 \left( \frac{\omega r}{a_s^2} \right)^\nu}{r^{\frac{n+1}{2}} \Gamma(\nu+1)} - \frac{B_2 \Gamma(\nu)}{\pi r^{\frac{n+1}{2}} \left( \frac{\omega r}{a_s^2} \right)^\nu}.
\]

For the two stretched solutions to perfectly match, the power coefficients of \( r \) need to be the same. This is indeed the case in the limit of \( a_s < 1 \) and \( \omega_s < 1 \). Then, by ignoring terms of order \( \omega_n^2, a_s^2, \omega_s \) or higher, we find that \( (n+1) \beta \simeq -j, \) \( (n+1) (\beta + D_s - 2) \simeq -(j + n + 1) \) and \( \nu \simeq j + \frac{n+1}{2} \). Identifying the coefficients of the same powers of \( r \), gives us the ratio
\[
\frac{B_1}{B_2} = \frac{(2/\omega r_h)^{2j+n+1} \nu \Gamma^2(\nu) \Gamma(\alpha + \beta + D_s - 1) \Gamma(\alpha + \beta) \Gamma(2 - 2\beta - D_s) \Gamma(2 + \alpha - \beta - D_s) \Gamma(1 + \alpha - \beta)}{\pi (1 + a_s^2)^{2j+n+1} \Gamma(2\beta + D_s - 2) \Gamma(2 + \alpha - \beta - D_s) \Gamma(1 + \alpha - \beta)}.
\]

that guarantees the existence of a smooth, analytic solution for the radial part of the tensor-type graviton wavefunction for all \( r \) (given that \( a_s \) and \( \omega_s \) remain small).

Finally, we need to calculate the absorption probability \( |A_{j\ell m}|^2 \) or greybody factor, since it is the reason for the deviation of the black-hole spectrum from a pure black-body one. We derive it, by expanding the far-field solution \([20]\) for \( r \to \infty \), in which case we find
\[
R_{FF}(r) \simeq \frac{1}{r^{\frac{n+1}{2}} \sqrt{2\pi \omega}} \left[ (B_1 + iB_2) e^{-\left(\frac{\omega r}{a_s^2} - \frac{\nu}{2} \nu - \frac{\pi}{2} \right)} + (B_1 - iB_2) e^{i \left(\omega r - \frac{\nu}{2} \nu - \frac{\pi}{2} \right)} \right].
\]

The greybody factor is then determined via the amplitudes of the outgoing and incoming spherical waves, as
\[
|A_{j\ell m}|^2 = 1 - |R_{j\ell m}|^2 = 1 - \left| \frac{B_1 - iB_2}{B_1 + iB_2} \right|^2 = \frac{2i (B^* - B)}{BB^* + i (B^* - B) + 1},
\]

where \( B \equiv B_1 / B_2 \) is given by Eq. \([23]\). The above result can be used to evaluate the absorption probability for the emission of tensor-type gravitons in the bulk, from a simply rotating black hole, in the low-energy and low-angular momentum limit.

4. Numerical Analysis
With the use of numerical analysis we manage to solve both the angular and radial equations for any value of the energy of the emitted particles and angular-momentum of the black hole. Consequently we were able to calculate the greybody factor and both energy and angular momentum emission spectrum for tensor-type gravitational modes in the bulk. Details concerning our method are extensively presented in our published article \([43]\).

4.1. Absorption Probability
The results obtained analytically and numerically should agree in the low-energy and low-angular-momentum limit but are expected to deviate once we move outside these regimes. Fig. \([4][a] \) depicts the results for the indicative modes \((j = 2, \ell = 2, m = 0)\) and \((j = 5, \ell = 2, m = 1)\):
Figure 1. (a) Comparison between our analytical (solid lines) and numerical (data points) results for the greybody factor for the modes \((j = \ell = 2, m = 0)\) and \((j = 5, \ell = 2, m = 1)\), for \(a_s = 0.5\) and \(n = 3\). (b) Absorption probabilities for the sets of modes with \(\ell = 2\) and \((j = 2, m = 0)\) (red), \((j = 3, m = -1, 1)\) (green), \((j = 4, m = -2, 0, 2)\) (blue), and \((j = 5, m = -3, -1, 1, 3)\) (magenta), for \(a_s = 0.5\) and \(n = 3\).

Figure 2. Absorption probabilities for the mode \((j = 2, \ell = 2, m = 0)\) as (a) a function of \(n = 3, 4, 5, 6, 7\), for \(a_s = 0.5\), and (b) a function of \(a_s = 0, 0.5, 1, 1.5, 2, 2.5\), for \(n = 3\).

the analytical results are given by the solid lines and the numerical ones are presented as data points - both sets of results correspond to the case with \(n = 3\) and \(a = 0.5\) (in units of \(r_h\)). The agreement between the two curves is indeed very good in the low-energy and even intermediate-energy regime and, as expected, breaks down as we move towards the high-energy one. Furthermore, the agreement is better for the lowest modes and it worsens for higher modes for which the greybody factor raises to a significant value and approaches unity at an increasingly higher value of the energy parameter \(\omega r_h\).

In Fig. II(b), we examine the behavior of the greybody factors for different tensor modes. The constraints \(j \geq \ell + |m|\) and \(\frac{j-(\ell+|m|)}{2} \in \{0, Z^+\}\) dictate that for each value of \(j, \ell\) can take values in the range \([2, j]\) while, for given \(j\) and \(\ell\), \(m\) can take \(j - \ell + 1\) values in total \((j: \text{total angular momentum}, \ell: \text{angular-momentum along the compact space } S^m, m: \text{angular-momentum in the plane of rotation of the black hole})\). As either \(j\) or \(m\) increases, the corresponding greybody curve shifts to the right and to higher-energies. Note that we get a similar behavior if we vary also \(\ell\).
Finally, we investigate the dependence of the greybody factors on the spacetime parameters of the theory, namely the number of additional spacelike dimensions $n$ and the angular-momentum parameter of the black hole $a$ by studying the indicative mode ($j = 2, \ell = 2, m = 0$). Fig. 2(a) clearly shows that the greybody factors for the tensor-gravitational modes in the bulk decrease as the number of transverse-to-the-brane spacelike dimensions increases. In Fig. 2(b) the greybody factors are enhanced as the angular-momentum of the black hole increases. This behaviour is in total agreement with the one observed for bulk scalar fields [30, 32] propagating in the same background.

### 4.2. Energy and Angular Momentum Emission Rates

We now proceed to compute the differential emission rates of energy and angular momentum in the bulk in the form of tensor-type gravitons. These are given by the expressions

$$\frac{d^2 E}{d\omega dt} = \frac{1}{2\pi} \sum_{j, \ell, m} \omega \frac{N_{ST}^\ell(S^n) |A_{j\ell m}|^2}{c^2/\tau_H - 1},$$  

$$\frac{d^2 J}{d\omega dt} = \frac{1}{2\pi} \sum_{j, \ell, m} m \frac{N_{ST}^\ell(S^n) |A_{j\ell m}|^2}{c^2/\tau_H - 1},$$

where $\tilde{\omega} \equiv \omega - \frac{am}{a^2 + r_h^2}$ and $T_H$ the temperature of the black hole given by

$$T_H = \left(\frac{n + 1}{n - 1}\right) \left(\frac{a^2}{4\pi(1 + a^2)r_h}\right).$$

$N_{ST}^\ell$ is the multiplicity of the 2nd-rank symmetric, traceless and divergence-free tensor harmonics $T_{AB}$ that satisfy Eq. (6). This quantity first calculated by Rubin and Ordonez in [45], was reproduced in [43] via an alternative method and found to be

$$N_{ST}^\ell(S^n) = \frac{(n + 1)(n - 2)(n + \ell)(\ell - 1)(n + 2\ell - 1)(n + \ell - 3)!}{2(\ell + 1)!(n - 1)!},$$

for the $\ell$-th eigenvalue.

Particular care was given to sum up to an appropriate maximum value of $j$ which ensures that the derived emission rates are as close as possible to the real ones. In practise, we stopped the summation when the contribution to the total curve of the modes corresponding to the next higher value of $j$ was negligibly small. Fig. 4 shows explicitly that as we sum up to $j_{\text{max}} = 5, 8, 10, 12$ and, finally, 15 the energy emission rate curve becomes wider and the slope of the tail decreases, whereas the low-energy behavior and the peak remains unchanged. In general, as either $n$ or $a^*$ increases, the number of modes that needed to be summed increases too. In the cases considered, we had to sum up to $j_{\text{max}} = 22$, i.e. $m = 20$ in order to obtain the optimum results.

We are mostly interested in exploring the dependence of the energy and angular momentum emission rates of the black hole on the spacetime parameters, namely $n$ and $a_*$. In Figs. 4(a,b), we illustrate their dependence on the number of additional spacelike dimensions and the angular momentum of the black hole, respectively. As in the case of scalars [30, 32], the energy emission rate has a very strong dependence on $n$ with an enhancement of almost two orders of magnitude as $n$ changes from $n = 3$ to $n = 7$ and a more complex correlation with $a_*$. More specifically, the energy emission curve becomes taller and wider with the increase of $n$ in the whole energy spectrum. On the other hand, Fig. 4(b) ($n = 3$) shows that, as $a_*$ goes from 0 to 2.5, the emission rate of high-energy modes increases, whereas the one for low- and intermediate-energy modes decreases.
Figure 3. Energy emission of tensor-type gravitons for \( n = 3, a_* = 1 \). Upper curves, from left to right, correspond to the highest value of \( j \) considered in the sum: \( j = 6 \) (red), \( j = 8 \) (green), \( j = 10 \) (blue), \( j = 12 \) (magenta) and \( j = 15 \) (black) [43].

Table 1. Comparison between total energy emissivities in the bulk through scalars and tensor-gravitons

| \( n \)  | Scalars  | Tensor-Gravitons | ratio  |
|---------|----------|------------------|--------|
| \( n = 3 \) | 0.1646   | 0.0013           | 0.0018 |
| \( n = 4 \) | 0.3808   | 0.0222           | 0.058  |
| \( n = 5 \) | 0.7709   | 0.1853           | 0.24   |

Figs. 5(a,b) depict the dependence of the angular momentum emission rate on \( n \) and \( a_* \). As either parameter gets larger, the angular momentum emission rate becomes more and more enhanced (even though the enhancement due to the increase of \( a_* \) is milder) and in all cases leads to the emission of larger number of modes in all energy regimes.

Finally, we move on to compare the significance of the emission of scalars and tensor-gravitons as independent channels, through which energy goes away from our brane into the bulk. We use the results found in [32] for the contribution of scalars to this procedure, in order to find the ratio of the total energy carried away by tensor-gravitons over the one carried away by scalars. Table 1 shows that scalar modes are the principal energy carriers in all cases. However, tensor-gravitons play an increasingly important role as the number of extra dimensions gets larger. Consequently, one has to carefully include their contribution as well to the overall process of energy being radiated away into the bulk by the black hole.

5. Conclusions

The potential creation of miniature black holes at the LHC and future high-energy experiments, means that we will also witness their evaporation through the emission of Hawking radiation. Of course, our detectors will record only the on-brane emitted degrees of freedom. Therefore, it is of absolute importance to have a clear knowledge of the balance between the amount of energy expected to go into the bulk and the one expected to rest on our brane, in order to evaluate any
relevant data and find decisive evidence of the existence of extra dimensions and their nature as well. Previous studies \cite{30,32} have shown that the bulk emission of scalars, allowed to propagate in the whole spacetime, is subdominant compared to the on-brane restricted particles (scalars and all fermions and bosons of SM). However, the contribution of gravitons (scalar- vector- and tensor-type ones) has to be considered as well in order to calculate the actual brane-bulk channel balance. In this work we have shown that tensor-type gravitons do make the bulk channel less subdominant compared to the brane one (especially when the number of extra spacelike dimensions is large), even though the overall picture remains essentially unchanged. Hopefully, the contribution of scalar- and vector-type gravitons will also be calculated in the near future, to obtain full knowledge of the energy-loss-in-the-bulk procedure via of hawking radiation.

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