Strangeness Dependence in Radiative Hyperon Decay Amplitudes

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ABSTRACT

The radiative decays of the $\frac{3}{2}^+$ baryons are studied in the three flavor generalization of the Skyrme model. The kaon fields are treated in the slow rotator approach which properly accounts for the observed deviations from the $U$–spin relations for the hyperon magnetic moments. This makes possible a critical discussion of the $U$–spin selection rules for the radiative hyperon decays. The variation of the decay widths with strangeness is studied and a comparison with other treatments of the $SU(3)$ Skyrme model is performed in order to analyze the effects of flavor symmetry breaking.

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1. Introduction

At present, only few data are available on the electromagnetic decays of the $\frac{3}{2}^+$ baryons. Although recently the reaction $\Delta \rightarrow N\gamma$ has carefully been analyzed at MAMI \cite{1} the decay parameters are still unknown for those $J = \frac{3}{2}$ to $J = \frac{1}{2}$ transitions, which involve strange baryons. Upcoming experiments at CEBAF \cite{2} and Fermilab \cite{3} are expected to provide some data on these radiative decays soon and thus give more insight in the pattern of flavor symmetry breaking. Exhaustive studies of the radiative hyperon decays have been performed in a number of models which include the non–relativistic quark model \cite{4, 5}, the MIT bag model \cite{6}, heavy baryon chiral perturbation theory \cite{7}, a quenched lattice calculation \cite{5} as well as recent Skyrme model studies \cite{8, 9}.

A particular feature is that $U$–spin symmetry would imply that the transition matrix elements vanish for the processes which involve the negatively charged hyperons, \textit{i.e.} $\Sigma^*\rightarrow \gamma\Sigma^*$ and $\Xi^*\rightarrow \gamma\Xi^*$ \cite{10}. Although $U$–spin symmetry is not exactly realized in nature this result has recently been verified approximately \cite{8, 4} in both the bound state (BSA \cite{11}) and rigid rotor (RRA \cite{12}) approaches to the $SU(3)$ Skyrme model \cite{13, 14, 15, 16}. In the Skyrme model baryons emerge as solitons configurations of the pseudoscalar mesons. These two approaches conceptually differ in the way the kaon fields are treated. The RRA starts from a flavor symmetric formulation wherein non–vanishing kaon fields arise from a rigid rotation of the classical pion field. The associated collective coordinates, which parametrize these large amplitude fluctuations off the soliton, are canonically quantized to generate states which possess the quantum numbers of physical hyperons. It turns out that the resulting collective Hamiltonian can be diagonalized exactly even in the presence of flavor symmetry breaking \cite{17}. As a result the baryon wave–functions significantly deviate from those obtained in the flavor symmetric version of the model. On the other hand the BSA treats the kaons as small amplitude fluctuations off the soliton, \textit{i.e.} a flavor symmetric formulation is completely waived. As a consequence of flavor symmetry breaking a bound state with unit strangeness charge develops out of the zero mode \cite{11}. Hyperons are then constructed by pertinent occupations of this bound state while the $SU(2)$ quantum numbers are generated analogously to the two flavor Skyrme model \cite{14} from the large amplitude fluctuations in coordinate– and iso–space. Despite that in both the RRA and BSA the baryon wave–functions significantly deviate from the $SU(3)$ symmetric ones it is nevertheless not surprising that the $U$–spin selection rules for the radiative hyperon decays are almost preserved. The reason is that from studying the magnetic moments of the $\frac{1}{2}^+$ baryons it is known that the experimentally observed $U$–spin violation requires a strangeness dependent classical meson configuration \cite{18, 19}. Such a dependence is not incorporated in these two approaches. A similar statement can be made for the hyperon radii, for which also a sophisticated treatment of symmetry breaking is required to reproduce the empirical pattern \cite{20}. The main purpose
of the present study is to address the question of strangeness dependence of the decay widths for radiative hyperon decays. Of course, this also includes the role of the $U$–spin selection rule for the processes $\Sigma^* \to \gamma \Sigma^-$ and $\Xi^* \to \gamma \Xi^-$. As argued, a treatment should be employed, which is capable of reproducing the pattern of flavor symmetry breaking of other electromagnetic observables. For definiteness we will employ the slow rotator approach (SRA) to the Skyrme model which has been shown to reproduce the experimental pattern of both the hyperon magnetic moments and radii [18]. The starting point for the SRA essentially is the RRA, however, the stationary equation is solved for each orientation in flavor space. This procedure yields a strangeness dependent soliton. The underlying picture is that the collective rotation of the soliton proceeds slowly enough that the profile function can react according to the forces exerted by flavor symmetry breaking; whence the notion slow rotator.

The paper is organized as follows: In section 2 we will briefly review the appearance of electromagnetic fields in the Skyrme model as well as the SRA to the three flavor version of the model. Subsequently we will discuss the pertinent matrix elements in section 3 and then present the numerical results for the radiative hyperon decay widths in section 4. We also compare our predictions with other treatments of the Skyrme model as well as available predictions of other models. Concluding remarks may be found in section 5. Some technicalities are relegated to a short appendix.

2. The Model

Our starting point is a gauged effective chiral action with appropriate symmetry breaking terms. In the case of three flavors it is a functional of the pseudoscalar octet $\phi$ and the photon field $A_\mu$, the former is non–linearly represented by the chiral field $U = \exp(i\phi)$. For a convenient presentation we split the action into four pieces

\[ \Gamma = \Gamma_{SK} + \Gamma_{an} + \Gamma_{ab} + \Gamma_{non-min}. \]  

(2.1)

The first term represents the gauged Skyrme action

\[ \Gamma_{SK} = \int d^4x \left\{ \frac{f_\pi^2}{4} \text{Tr} \left[ D_\mu U (D^\mu U)^\dagger \right] + \frac{1}{32\epsilon^2} \text{Tr} \left[ \left[ U^\dagger D_\mu U, U^\dagger D_\nu U \right]^2 \right] \right\}. \]  

(2.2)

Here $f_\pi = 93\text{MeV}$ is the pion decay constant and $\epsilon$ is the dimensionless Skyrme parameter. The covariant derivative is defined via the electric charge matrix $Q$

\[ D_\mu U = \partial_\mu U + ie A_\mu \left[ Q, U \right], \quad Q = \frac{1}{2} \left[ \lambda_3 + \frac{1}{\sqrt{3}} \lambda_8 \right], \]  

(2.3)

Henceforth we adopt Gaussian units, i.e. $\epsilon^2 = 1/137$. $\Gamma_{an}$ is the Wess-Zumino action gauged to contain the photon field [21]:

\[ \Gamma_{an} = -\frac{iN_C}{240\pi^2} \int d^5x \epsilon^{\mu\nu\rho\sigma\tau} \text{Tr} \left[ L_\mu L_\nu L_\rho L_\sigma L_\tau \right] \]

\[ -\frac{N_C}{48\pi^2} \int d^4x \epsilon^{\mu\nu\rho\sigma} e A_\mu \text{Tr} \left[ Q (L_\nu L_\rho L_\sigma - R_\nu R_\rho R_\sigma) \right] + \mathcal{O}(\epsilon^2 A^2). \]  

(2.4)
Here we have used \( L_\mu = U^\dagger \partial_\mu U \) and \( R_\mu = U \partial_\mu U^\dagger \). Furthermore \( N_C = 3 \) is the number of colors. The flavor symmetry breaking terms are contained in \( \Gamma_{sb} \):

\[
\Gamma_{sb} = \int d^4x \left\{ \frac{f_\pi^2 m_\pi^2 + 2 f_K^2 m_K^2}{12} \text{Tr} \left[ U + U^\dagger - 2 \right] + \frac{f_\pi^2 m_\pi^2 - f_K^2 m_K^2}{2\sqrt{3}} \text{Tr} \left[ \lambda_8 \left( U + U^\dagger \right) \right] \right. \\
+ \frac{f_K^2 - f_\pi^2}{4} \text{Tr} \left[ \hat{S} \left( U(D_\mu U)^\dagger D^\mu U + U^\dagger D_\mu U(D^\mu U^\dagger) \right) \right] \right\}, \tag{2.5}
\]

where \( \hat{S} = \text{diag}(0,0,1) \) is the strangeness projector. In eq (2.3) \( f_K \) is the kaon decay constant while \( m_\pi \) and \( m_K \) are the pion and kaon masses, respectively. At non–vanishing momentum transfer non–minimal couplings of the photons to the pseudoscalar fields may be relevant

\[
\Gamma_{non–min} = i \int d^4x L_9 \left( \partial_\mu A_\nu - \partial_\nu A_\mu \right) \text{Tr} \left[ Q \left( L^\mu L^\nu + R^\mu R^\nu \right) \right]. \tag{2.6}
\]

In fourth order chiral perturbation this term is needed to correctly reproduce the electromagnetic pion radius thereby determining the dimensionless coefficient \( L_9 = (6.9 \pm 0.7) \times 10^{-3} \). The full action (2.1) defines the electromagnetic current \( J_\mu \) via the expansion in the photon field

\[
\Gamma[U, A_\mu] = \Gamma_{strong}[U] + e \int d^4x \ J_\mu A^\mu + \mathcal{O}(e^2 A^2). \tag{2.7}
\]

The resulting covariant expression for \( J_\mu \) in terms of the chiral field \( U \) may readily be taken from the literature [23, 24]:

\[
J_\mu = -\frac{f_\pi^2}{2} \text{Tr} \left\{ Q \left( L_\mu + R_\mu \right) \right\} + \frac{1}{8e^2} \text{Tr} \left\{ Q \left( \left[ L_\nu, [L_\mu, L^\nu] \right] + [R_\nu, [R_\mu, R^\nu]] \right) \right\} \\
- \frac{N_C}{48\pi^2} \epsilon_{\mu\rho\sigma\lambda} \text{Tr} \left\{ Q \left( L^\nu L^\rho L^\sigma - R^\nu R^\rho R^\sigma \right) \right\} - iL_9 \text{Tr} \left\{ Q \partial^\nu \left( [L_\nu, L_\mu] + [R_\nu, R_\mu] \right) \right\} \\
- \frac{f_K^2 - f_\pi^2}{4} \text{Tr} \left\{ Q \left\{ \hat{S} U + \hat{S} U^\dagger, L_\mu \right\} + \left\{ \hat{S} U + U^\dagger \hat{S}, R_\mu \right\} \right\}. \tag{2.8}
\]

In order to generate baryon states of good spin and flavor quantum numbers we consider the solitonic meson configuration that corresponds to an arbitrary orientation of the hedgehog \( U_0 \) in flavor space, i.e. \( U = AU_0 A^\dagger \). For the time being we confine ourselves to static rotations in order to establish the slow rotator approach. Due to spin and isospin invariance this corresponds to the ansatz [18]

\[
U(r, \nu) = \exp \left( -i\nu \lambda_4 \right) \exp \left( i\mathbf{r} \cdot \mathbf{F}(r, \nu) \right) \exp \left( i\nu \lambda_4 \right) \tag{2.9}
\]

for the chiral field. As already indicated this ansatz has the remarkable feature that the chiral angle may depend on the flavor orientation, which is characterized by the strangeness changing angle \( \nu \in [0, \pi/2] \). Substituting this ansatz into the strong interaction part of the action, \( \Gamma_{strong} \), yields the classical energy \( E(\nu, F) \) as a functional of the
chiral angle $F$ and a function of the strangeness changing angle $\nu$. It is important to note that the explicit dependence on $\nu$ originates from the symmetry breaking terms in $\Gamma_{sb}$. Upon extremizing $E(\nu, F)$ for a given value $\nu \in [0, \pi/2]$ the chiral angle depends on $\nu$ in a parametrical way. This treatment is to be compared with that of the rigid rotator approach where the chiral angle is fixed to $F(r, \nu = 0)$. These two approaches mainly differ by the large distance behavior of the chiral angle: In the RRA the chiral angle decays with the pion mass for every flavor orientation. On the other hand the SRA exhibits the desired feature that the chiral angle decays with the pion mass only for $\nu = 0$ while the configuration which is maximally rotated into strange direction ($\nu = \pi/2$) indeed has the kaon mass entering the Yukawa tail.

In the next step the time independence of the flavor rotations is waived by substituting the time dependent meson configuration

$$U(r, t) = A(t) \exp\left(i \tau \cdot \hat{F}(r, \nu)\right) A(t)$$

into the action (2.1). This allows us to extract a Lagrangian, which apparently is a function of the time derivative of the collective rotation $A$. This derivative is most conveniently presented by introducing the angular velocities $A_\dagger \dot{A} = (i/2) \sum_{a=1}^{8} \lambda_a \Omega_a$. The canonical quantization introduces the right generators of flavor $SU(3)$ via $R_a = -\partial L/\partial \Omega_a$ and leads to the collective Hamiltonian

$$H = E(\nu) + \left(\frac{1}{2\alpha^2(\nu)} - \frac{1}{2\beta^2(\nu)}\right) J^2 + \frac{1}{2} \left\{ \frac{1}{2\beta^2(\nu)}; C_2 [SU(3)] \right\} - \frac{3}{8\beta^2(\nu)} \tag{2.11}$$

together with the constraint $R_8 = \sqrt{3}/2$. This constraint stems from the Wess–Zumino term (2.4) and guarantees that the eigenstates of $H$ possess half–integer spin [21]. In eq (2.11) $J_i = -R_i$ denotes the spin operator for $i = 1, 2, 3$ while $C_2 [SU(3)] = \sum_{a=1}^{8} R^2_a$ refers to the quadratic Casimir operator of $SU(3)$. It should be stressed that the coefficients in eq (2.11), which are functionals of the chiral angle, have both an explicit as well as an implicit dependence on the strangeness changing angle $\nu$. While the former is due to the symmetry breaking part of the action (2.5) the latter stems from the (parametrical) $\nu$ dependence of the chiral angle. The RRA corresponds to omitting this implicit dependence and computing the coefficients as radial integrals over $F(r, \nu = 0)$; the explicit dependence in $E(\nu)$ is kept, though. Note that we have adopted a symmetric operator ordering to render the Hamiltonian Hermitian. As has been shown previously [18] this Hamiltonian can be diagonalized exactly. A suitable technique is to express the generators $R_a$ as differential operators with respect to the eight “Euler angles” parametrizing the rotation matrix $A$. The eigenfunctions $\Psi_B(A) = \langle A|B \rangle$ of the collective Hamiltonian (2.11) are identified as the wave–functions corresponding to baryon $B$. These are distorted $SU(3)$ D–functions reflecting that in the presence of flavor symmetry breaking the resulting baryon eigenstates are no longer pure octet (for $J = 1/2$) or decouplet (for $J = 3/2$).
states but rather contain sizable admixtures of baryon states with appropriate spin and flavor quantum numbers in higher dimensional representations of $SU(3)$ like for example $10$ or $27$.

Adjusting the Skyrme parameter to $\epsilon = 3.46$ reproduces the observed mass differences for the $\frac{1}{2}^+$ and $\frac{3}{2}^+$ baryons when the pion and kaon masses are taken at their physical values $m_\pi = 138$MeV and $m_K = 495$MeV while the kaon decay constant is chosen to be $f_K = 118$MeV which is only slightly larger than the experimental number 113MeV. Unless otherwise noted we will always adopt this set of parameters.

3. Current Matrix Elements

Having obtained both the covariant form of the electromagnetic current (2.8) and the baryon wave–functions as eigenfunctions of the collective Hamiltonian (2.11) it is straightforward to compute electromagnetic properties of the baryons by evaluating the appropriate matrix elements.

The part of the matrix element associated with the out–going photon corresponds to taking the Fourier–transform of the electromagnetic current $J_\mu$ with respect to the photon momentum $q$ measured in the rest frame of the decaying $\frac{3}{2}^+$ baryon. Upon substituting the time dependent meson configuration (2.10) into the covariant expression for the electromagnetic current (2.8) one extracts two operators $\hat{E}$ and $\hat{M}$ which have non–vanishing transition matrix elements between $\frac{1}{2}^+$ and $\frac{3}{2}^+$ baryons. The operators $\hat{E}$ and $\hat{M}$ respectively stem from the quadrupole part of the time component $J_0$ and the spatial component $J_i$: 

$$\hat{E} = \int d^3r j_2(qr) \left( \frac{z^2}{r^2} - \frac{1}{3} \right) J_0$$

$$\hat{M} = \frac{1}{2} \int d^3r j_1(qr) \epsilon_{3ij} \hat{r}_i J_j .$$

Here $j_l(qr)$ denote the spherical Bessel functions corresponding to orbital angular momentum $l$. Replacing the angular velocities $\Omega_a$ in favor of the $SU(3)$ generators $R_a$, $\hat{E}$ and $\hat{M}$ read

$$\hat{E}(q, A, R_a) = -\frac{8\pi}{15\alpha^2(\nu)} D_{em,3}(A) R_3 \int_0^\infty dr \, r^2 j_2(qr)V_0(r, \nu)$$

and

$$\hat{M}(q, A, R_a) = -\frac{4\pi}{3} \int_0^\infty dr \, r^3 j_1(qr) \left[ V_1(r, \nu) D_{em,3}(A) + V_3(r, \nu) d_{33,\beta} D_{em,\alpha}(A) D_{8\beta}(A) \right.$$

$$\left. - \frac{1}{2} \left\{ \frac{1}{\beta^2(\nu)} V_2(r, \nu, d_{33,\beta} D_{em,\alpha}(A) R_\beta \right\} \right] ,$$

where $\alpha, \beta = 4, \ldots, 7$. Explicit expressions for $V_i(r, \nu)$ are listed in the appendix. We have also introduced the adjoint representations of the collective rotations $D_{ab} = (1/2)$.
Tr \left( \lambda_a A \lambda_b A^\dagger \right). In particular the subscript “em” refers to the electromagnetic direction: \( D_{em,a} = D_{3a} + D_{8a}/\sqrt{3} \). Again we have chosen a symmetric ordering for those terms which develop ambiguities when elevating the (classical) angular velocities to operators in the space of the collective coordinates.

While \( \hat{M} \) represents the operator whose matrix elements directly yield the amplitudes for the \( M1 \) channel, the operator \( \hat{E} \) can be associated with the \( E2 \) channel only after some approximations according to Siegert’s Theorem \cite{25}. When employing the continuity equation \( \partial_\mu J^\mu \) one has to assume that \( j_{i-1} \gg j_{i+1} \) and to expand \( j_2(qr) \approx (qr)^2/15 \). For typical photon momenta for the radiative hyperon decays \( q \sim 200\text{MeV} \) this introduces errors as little as 10% since the spatial extension is of the order \( \langle r^2 \rangle \approx 1\text{fm}^2 \) or less. In section 4 we will give an estimate of these kinematical corrections. Recently the deviation from Siegert’s Theorem has been studied in the two flavor reduction of the model for the \( \Delta - \)nucleon transition \cite{26}. In addition to the before-mentioned kinematical corrections these authors introduce pion fluctuations off the rotating soliton \( (2.10) \) to consistently satisfy the continuity equation \( \partial_\mu J^\mu = 0 \) at subleading order in \( 1/N_C \). These induced fields account for shortcomings\(^1\) in the collective quantization and brings into the game contributions to \( E2 \) which are not only two orders down in \( 1/N_C \) compared to those parts which are associated with the pure soliton configuration \( (2.10) \) but also have an additional factor \( q^2 \). Hence one is inclined to assume that these corrections are negligibly small. However, the numerical studies indicate that this is not quite the case. For for the \( \Delta - \)nucleon transition it has been observed \cite{26} that altogether corrections of the order of 30% may arise. They may be split up into those associated with the induced field components \( \sim 25\% \) and smaller kinematical ones \( \sim 5\% \), see below.

Unfortunately such an inclusion of induced fields seems to be unfeasible in the three flavor model with symmetry breaking included and in particular in the framework of the SRA. In any event, the main purpose of the present investigation is to study the interrelations between various radiative hyperon decays. In this regard the structure of the baryon wave–functions \( \Psi(A) \) appears to be significantly more important than the precise form of the radial functions \( V_i \). We therefore approximate the \( E2 \) and \( M1 \) decay widths by

\[
\Gamma_{E2} = \frac{675}{8} e^2 q \left| \langle \Psi_{J=\frac{1}{2}} | \hat{E}(q) | \Psi_{J=\frac{3}{2}} \rangle \right|^2 \tag{3.5}
\]

\[
\Gamma_{M1} = 18 e^2 q \left| \langle \Psi_{J=\frac{1}{2}} | \hat{M}(q) | \Psi_{J=\frac{1}{2}} \rangle \right|^2 . \tag{3.6}
\]

These matrix elements are computed as integrals over the collective coordinates \( A \) using the exact eigenfunctions \( \Psi_j(A) \) of the collective Hamiltonian \( (2.11) \). For details we refer\(^2\) the matrix elements of \( \hat{E} \) are related to the \( C_2 \) channel.

\(^1\)The rotating hedgehog \( (2.10) \) does not represent a solution to the time dependent equations of motion, even in the simpler two flavor case.
to appendix A of ref [16]. These analyses also allow us to compute the ratio [27]

\[ \frac{E^2}{M1} = \frac{5}{4} \frac{\langle \Psi_{J=1/2} | \hat{E}(q) | \Psi_{J=1/2} \rangle}{\langle \Psi_{J=1/2} | M(q) | \Psi_{J=1/2} \rangle} . \]  

(3.7)

At this point it is interesting to note that the magnetic moments of the baryons can be obtained from the diagonal matrix elements of \( \hat{M} \). The computation of the magnetic moments indeed provides the strongest support for the SRA because it has the distinctive feature of reproducing the experimentally observed deviation from the \( U \)-spin symmetric relations between the baryon magnetic moments [18]. For example, for the parameters listed above the ratio \( \mu(\Sigma^+)/\mu(p) = 0.85 \) compares well with the experimental number of 0.87 while assuming \( U \)-spin symmetry gives unity. Similarly, the SRA gives sizable \( U \)-spin violations for the charge radii which in the SRA are predicted to decrease with strangeness, similar to the pattern discussed in ref [20]³. The comparison with the RRA, wherein \( U \)-spin relations are approximately preserved despite of sizable distortions of the collective wave–functions, indicates that only minor contributions of the \( U \)-spin violations stem from configuration mixing but rather are due to the influence of symmetry breaking on the chiral angle. Similar results are found when the influence of symmetry breaking on the extension of the soliton is treated quantum mechanically [19]. These inter–relations strongly motivate the study of radiative hyperon decays in the SRA. Unfortunately, the absolute magnitude of the magnetic moments is underestimated by about 30%. In the two flavor reduction it has been shown that this shortcoming is cured when quantum corrections \( \mathcal{O}(1/N_C) \) are taken into account [28]. Of course, this problem is expected to contaminate the predictions for at least the \( M1 \) transition amplitudes such that decay widths are underestimated by up to 50%. However, from quite general arguments it has been found that the \( 1/N_C \) corrections enter the coefficient \( V_1 \) in eq (3.4) multiplicatively [29]. Hence our conclusions concerning the \( U \)-spin violation and strangeness dependencies of the decay widths, which rely on the comparison of various matrix elements, will not be effected crucially.

4. Numerical Results

Here we will discuss the numerical results for the electromagnetic transitions as defined in the preceding section.

In order to estimate the kinematical corrections to Siegert’s Theorem we also consider a modification of the operator \( \hat{E} \). According to eq (3.14) of ref [26] we include a Bessel

³Although these authors study the strong interaction radii, which do not need to be the same as the electromagnetic ones, they also confirm that “wherever the comparison is possible, the strong interaction radii are very similar to the charge radii.”
Table 4.1: SRA predictions for the electromagnetic decay widths of the $\frac{3}{2}^+$ baryons. Here we consider the case $L_9 = 0$. The data in parentheses correspond to the operator $\hat{E}'(q, A, R_a)$ cf. eq \eqref{4.3}.

| Transition       | $\Gamma_{E2}(eV)$ | $\Gamma_{M1}(keV)$ | $E2/M1(\%)$ |
|------------------|-------------------|-------------------|-------------|
| $\Delta \to \gamma N$ | 416.3 (354.6)     | 326.0             | -2.06 (-1.90) |
| $\Sigma^{*0} \to \gamma \Lambda$ | 157.8 (142.2)     | 163.9             | -1.78 (-1.70) |
| $\Sigma^{*-} \to \gamma \Sigma^-$ | 1.82 (1.70)       | 1.72              | -1.88 (-1.81) |
| $\Sigma^{*0} \to \gamma \Sigma^0$ | 2.35 (2.25)       | 7.79              | -1.00 (-0.98) |
| $\Sigma^{*+} \to \gamma \Sigma^+$ | 19.5 (18.5)       | 47.5              | -1.17 (-1.14) |
| $\Xi^{*-} \to \gamma \Xi^-$ | 1.65 (1.54)       | 1.35              | -2.02 (-1.95) |
| $\Xi^{*0} \to \gamma \Xi^0$ | 29.8 (28.3)       | 64.5              | -1.24 (-1.21) |

function associated with orbital angular momentum $l = 3$ into the integrand of $\hat{E}$ \eqref{3.3}:

$$
\hat{E}'(q, A, R_a) = -\frac{8\pi}{5\alpha^2(\nu)}D_{em,3}(A)R_3 \int_0^\infty dr r_2^2 \left( 3j_2(qr) - qrj_3(qr) \right)V_0(r, \nu).
$$

(4.1)

From the numerical results displayed in tables 4.1 and 4.2 we conclude that the kinematical corrections lower the $E2$ amplitude by no more than about 5%. The total corrections found in ref \cite{26} by the complicated treatment of the continuity equation were of the order 30%. As mentioned above the remaining 25% are due to the induced fields\footnote{The effect of these fields associated with the modification of the quantization rule $R_i = -\partial L/\partial \Omega_i = -\alpha^2 \Omega_i + \ldots$ was omitted in ref \cite{26}.} at subleading order in $1/N_C$.

In table 4.1 the resulting decay widths for the radiative transitions of the $\frac{3}{2}^+$ baryons are displayed as obtained in the SRA. Apparently these widths follow the pattern

$$
\Gamma_{\Delta \to \gamma N} > \Gamma_{\Sigma^{*0} \to \gamma \Lambda} > \Gamma_{\Xi^{*0} \to \gamma \Xi^0} > \Gamma_{\Sigma^{*-} \to \gamma \Sigma^-} \gg \Gamma_{\Sigma^{*+} \to \gamma \Sigma^+} \gg \Gamma_{\Xi^{*-} \to \gamma \Xi^-} \approx \Gamma_{\Xi^{*0} \to \gamma \Xi^0} \approx 0.
$$

(4.2)

for both the $M1$ and $E2$ channels. In particular this pattern implies that the transition amplitudes for the negatively charged $\frac{3}{2}^+$ baryons turn out to be negligibly small. Thus this $U$–spin relation is maintained in the SRA as well. The non–minimal term \eqref{2.6} has only moderate effects on the transition matrix elements as can be observed from the comparison of tables 4.1 and 4.2. Since the integrands entering the evaluation of the charge radii and the $E2$ amplitudes differ only by the additional Bessel function the rise in the electric amplitude is expected. However, the total decay widths get reduced because the $M1$ amplitudes become smaller when including the non–minimal term. Since this
Table 4.2: Same as table 4.1 for $L_9 = 0.0069$.

| Transition         | $\Gamma_{E2}$ (eV) | $\Gamma_{M1}$ (keV) | $E2/M1$ (%) |
|--------------------|--------------------|--------------------|-------------|
| $\Delta \rightarrow \gamma N$ | 455.0 (410.7) | 308.9 | -2.22 (-2.11) |
| $\Sigma^{*0} \rightarrow \gamma \Lambda$ | 168.7 (158.4) | 157.4 | -1.89 (-1.83) |
| $\Sigma^{*-} \rightarrow \gamma \Sigma^-$ | 1.90 (1.82) | 1.67 | -1.95 (-1.91) |
| $\Sigma^{*0} \rightarrow \gamma \Sigma^0$ | 2.43 (2.37) | 7.66 | -1.02 (-1.01) |
| $\Sigma^{*+} \rightarrow \gamma \Sigma^+$ | 20.2 (19.6) | 46.6 | -1.20 (-1.18) |
| $\Xi^{*-} \rightarrow \gamma \Xi^-$ | 1.72 (1.66) | 1.30 | -2.10 (-2.06) |
| $\Xi^{*0} \rightarrow \gamma \Xi^0$ | 30.9 (30.0) | 63.2 | -1.28 (-1.26) |

decrease effects all channels approximately equally we conclude that the $V_1$ contribution in eq (3.4) dominates the magnetic channel.

As expected from the above discussion on the absolute values of the magnetic moments, the total decay width for the $\Delta \rightarrow \gamma N$ is about 50% smaller than the empirical value $\Gamma_{\Delta \rightarrow \gamma N} = 610\text{keV} \ldots 730\text{keV}$ given by the PDG [30]. Note that the rescaling motivated by the $1/N_C$ analysis [29] of the coefficient $V_1$ in eq (3.4) to account for the absolute value of the proton magnetic moment solves this discrepancy as well. Our prediction for the ratio $E2/M1$ for the radiative decay of the $\Delta$ resonance is surprisingly close to the empirical value $-2.5\pm0.2$ which has recently been extracted from the pion–photoproduction experiment performed at MAMI [1]. However, this agreement should be taken with a grain of salt since according to the above discussions we expect modifications to the absolute values of both the electric and the magnetic transition amplitudes. On the other hand, we think that the statement that all $E2/M1$ ratios are small and negative is save against the indicated corrections.

Several models [7] as well as dispersion relation analysis [31] predict complex values for the ratio $E2/M1$. As the subtracted background may be different in these two channels their ratio is in general not taken at the resonance position. In all collective approaches to soliton models, however, the $J = 3/2$ baryons have zero widths as long as meson fluctuations off the soliton are omitted. Hence the resulting ratio $E2/M1$ is real.

To estimate the model dependence we also consider the effects of a sixth–order stabilizing term

$$L_6 = -\frac{\epsilon_6^2}{2} B_{\mu} B^\mu$$

where

$$B_{\mu} = \frac{1}{24\pi^2} \epsilon_{\mu\nu\rho\sigma} \text{Tr} \left[ (U^\dagger D^\nu U) (U^\dagger D^\rho U) (U^\dagger D^\sigma U) \right].$$

(4.3)

Note that in the absence of the photon field, $B_{\mu}$ represents the topological current. In the same way in which the Skyrme term mocks up the $\rho$ meson exchange the strong interaction part of $L_6$ can be interpreted as the exchange of an $\omega$ meson.

The decay parameters resulting from the inclusion of $L_6$ are shown in table 4.3. Ob-
Table 4.3: Same as table 4.1 for $f_K = 108\text{MeV}$, $\epsilon = 5.61$, $\epsilon_6 = 0.0118\text{MeV}^{-1}$ and $L_9 = 0.0069$. With the sixth order term (4.3) included this set provides a fit to the baryon mass differences [18].

| Transition | $\Gamma_{E2}(\text{eV})$ | $\Gamma_{M1}(\text{keV})$ | $E2/M1(\%)$ |
|------------|----------------|----------------|-------------|
| $\Delta \to \gamma N$ | $531.1 \ (478.5)$ | $319.3$ | $-2.35 \ (-2.24)$ |
| $\Sigma^* \to \gamma \Lambda$ | $210.9 \ (198.2)$ | $185.8$ | $-1.94 \ (-1.89)$ |
| $\Sigma^- \to \gamma \Sigma^-$ | $1.91 \ (1.83)$ | $1.17$ | $-2.34 \ (-2.29)$ |
| $\Sigma^0 \to \gamma \Sigma^0$ | $3.83 \ (3.74)$ | $12.3$ | $-1.02 \ (-1.01)$ |
| $\Sigma^+ \to \gamma \Sigma^+$ | $28.1 \ (27.3)$ | $65.6$ | $-1.19 \ (-1.18)$ |
| $\Xi^- \to \gamma \Xi^-$ | $1.88 \ (1.80)$ | $1.02$ | $-2.48 \ (-2.43)$ |
| $\Xi^0 \to \gamma \Xi^0$ | $44.2 \ (43.0)$ | $94.4$ | $-1.25 \ (-1.23)$ |

Table 4.4: Same as table 4.1 for the rigid rotator approach: $f_K = 113\text{MeV}$, $\epsilon = 3.9$, $\epsilon_6 = 0$ and $L_9 = 0$.

| Transition | $\Gamma_{E2}(\text{eV})$ | $\Gamma_{M1}(\text{keV})$ | $E2/M1(\%)$ |
|------------|----------------|----------------|-------------|
| $\Delta \to \gamma N$ | $942.0 \ (648.4)$ | $319.4$ | $-3.14 \ (-2.60)$ |
| $\Sigma^* \to \gamma \Lambda$ | $567.9 \ (403.4)$ | $208.9$ | $-3.01 \ (-2.54)$ |
| $\Sigma^- \to \gamma \Sigma^-$ | $74.7 \ (6.04)$ | $2.27$ | $-3.31 \ (-2.98)$ |
| $\Sigma^0 \to \gamma \Sigma^0$ | $12.0 \ (9.71)$ | $11.3$ | $-1.89 \ (-1.70)$ |
| $\Sigma^+ \to \gamma \Sigma^+$ | $93.4 \ (75.5)$ | $67.5$ | $-2.15 \ (-1.93)$ |
| $\Xi^- \to \gamma \Xi^-$ | $12.8 \ (10.0)$ | $3.42$ | $-3.54 \ (-3.12)$ |
| $\Xi^0 \to \gamma \Xi^0$ | $168.3 \ (131.1)$ | $100.2$ | $-2.37 \ (-2.09)$ |

Previously there are no qualitative changes to the ordinary Skyrme model. We find a small increase for most of the decay widths. For the $M1$ channel this is expected since in this model the predicted proton magnetic moment is $\mu(p) = 1.90$ [18] and thus a little larger than in the model with only the fourth order stabilizer. However, the magnetic widths for the $U$–spin forbidden channels decrease when the sixth order term is included. This corroborates our conclusion that these decays remain (at least approximately) forbidden even when the effects of flavor symmetry breaking are incorporated into the soliton configuration.

The electromagnetic transitions of the $\frac{3}{2}^+$ baryons have been computed in several other approaches and models. In table 4.4 we display the results from the RRA, i.e. we have omitted the implicit dependence on the strangeness changing angle $\nu$. Note also that for simplicity we have not included the induced kaon fields. Hence these data differ slightly from those in ref. [4]. Apparently the RRA predicts larger $E2$ amplitudes than the SRA...
Table 4.5: Total decay widths normalized to that of the $\Delta \to \gamma N$ transition in various models. The results for SRA and RRA correspond to tables 4.2 and 4.4, respectively. The BSA results have been obtained with the empirical values for the meson masses and decay constants together with $\epsilon = 4.25$ and $\epsilon_6 = 0$. The entry $SU(3)$ denotes the predictions from a flavor symmetric formulation of the Skyrme model. The data for the quark model (QM) and lattice calculation (Lat.) are taken from refs [4, 5].

| Transition       | SRA  | RRA  | BSA  | $SU(3)$ | QM   | Lat. |
|------------------|------|------|------|---------|------|------|
| $\Sigma^* \to \gamma \Lambda$ | 0.509 | 0.653 | 0.765 | 3/4     | –    | 0.703 |
| $\Sigma^* \to \gamma \Sigma^-$ | 0.005 | 0.007 | 0.010 | 0       | 0.007| 0.006 |
| $\Sigma^0 \to \gamma \Sigma^0$ | 0.024 | 0.035 | 0.037 | 1/4     | 0.040| 0.055 |
| $\Sigma^+ \to \gamma \Sigma^+$ | 0.152 | 0.210 | 0.233 | 1       | 0.233| 0.303 |
| $\Xi^- \to \gamma \Xi^-$ | 0.004 | 0.011 | 0.039 | 0       | 0.009| 0.012 |
| $\Xi^0 \to \gamma \Xi^0$ | 0.205 | 0.313 | 0.412 | 1       | 0.300| 0.415 |

while the changes for the $M1$ amplitudes are not as drastic. Hence the $E2/M1$ ratios increase. It should, however, be noted that the $M1$ widths involving strange baryons are appreciably smaller in the SRA than in the RRA while for the $\Delta \to \gamma N$ transition the magnetic widths do not seem to significantly depend on the approach. The latter result is due to the fact that this channel is dominated by the two flavor portion. As we favor the SRA over the RRA from their predictions on the magnetic moments and the hyperon radii we are lead to the assessment that the hyperon radiative decay widths should actually be smaller than previously assumed when normalized to the width of the $\Delta \to \gamma N$ decay. This is shown in table 4.7. It should be stressed that these numbers will not suffer from eventual $1/N_C$ corrections to the $M1$ channel as discussed at the end of section 3. Except for the $U$–spin forbidden channels we observe strong deviations from the flavor symmetric formulation. Note that the symmetric formulation also requires to assume a common photon momentum for all channels. We see that the inclusion of symmetry breaking effects is crucial to maintain the previously established order of the decay widths (4.2). The deviation from the symmetry relations is strongest in the SRA where the normalized decay widths apparently decrease much more quickly with strangeness than in all other available computations. To analyze this behavior it is illuminating to recall the predictions of these models on the extensions of the baryons. Except of the SRA the radii show only a moderate variation with hypercharge [23, 32, 33, 34] although the analysis of ref [20] indicates that the radii should decrease with strangeness. In the BSA [32] and the lattice calculation [33] the electric radius of the $\Sigma^+$ is even slightly larger than that of the proton, although in the BSA the $\Xi^-$ radius is predicted surprisingly small. The radiative hyperon decays and the radii are strongly related as both probe moments of the electromagnetic
current. It is hence not surprising that in the BSA and the lattice calculation the decay width \( \Gamma_{\Sigma^{*+} \rightarrow \gamma\Lambda} \) is significantly larger than in the RRA and especially in the SRA. From the discussions on the magnetic moments and on the hyperon radii it seems fair to conclude that pattern of the hyperon decays widths should follow the SRA prediction, in particular one would expect a strong decrease with strangeness.

6. **Summary and Conclusions**

We have presented a Skyrme model calculation of the widths for radiatively decaying \( \frac{3}{2}^+ \) baryons which will be experimentally available soon. Here we have emphasized on discussing the flavor symmetry breaking pattern for these widths. From the comparison of the model predictions on other observables (magnetic moments, radii) we have argued that the slow rotator approach (SRA), wherein the stationary equation is solved for each flavor orientation of the hedgehog field, is most suitable to address this question. In particular this assessment has to be concluded from the observed deviations from the \( U^- \)-spin relations among the hyperon magnetic moments. By including symmetry effects not only in the baryon wave–functions but also in the soliton field, which enters the electromagnetic current, the SRA properly accounts for these deviations. This feature made very interesting to examine the \( U^- \)-spin predictions on the radiative hyperon decays within this approach. The \( U^- \)-spin predictions state that the processes \( \Sigma^{*-} \rightarrow \gamma\Sigma^- \) and \( \Xi^{*-} \rightarrow \gamma\Xi^- \) are forbidden. Somewhat surprisingly we have found that also in the SRA these decay widths are negligibly small. Our main conclusion, however, is that the widths should exhibit a stronger dependence on strangeness than previously deduced from other model calculations. This is meant in the sense that the SRA predicts the decay widths to decrease more quickly with strangeness than any other model. Since the matrix elements entering the computation of the decay widths are sensitive to the extension of the hyperons involved this result is linked to and supported by the empirical pattern of the hyperon radii which are also assumed to decrease with strangeness.

**Appendix**

Here we present the explicit expressions for the radial functions entering the collective operators \( \vec{E} \) and \( \vec{M} \) in equations (3.3) and (3.4), respectively. We also include the contributions stemming from the sixth order term (4.3). To simplify the presentation we introduce the abbreviations \( s = \sin F \) and \( c = \cos F \).

\[
V_0(r, \nu) = s^2 \left[ f_\pi^2 + \frac{1}{\epsilon^2} \left( F_{\nu}^2 + \frac{s^2}{r^2} \right) + \frac{\epsilon_6^2}{4\pi^2} F_{\nu}^2 \frac{s^2}{r^2} + \sin^2 \nu \left( f_K^2 - f_\pi^2 \right) c \right]
- 4L_9 \left( s^2 + \frac{s}{r} (rs)^{\nu} - 3 \frac{s^2}{r^2} \right),
\]
\[ V_1(r, \nu) = \frac{s^2}{r^2} \left[ f_\pi^2 + \frac{1}{\epsilon^2} \left( F^2 + \frac{s^2}{r^2} \right) + \frac{\epsilon^2}{4\pi^2} F^2 \frac{s^2}{r^2} + \sin^2 \nu \left( f_\pi^2 - f_\pi^2 \right) c \right] \]
\[ - \frac{L_9}{r^2} \left( (e^2 - s^2)^\prime + 4 \frac{s^2}{r^2} \right), \]
\[ V_2(r, \nu) = \frac{1}{4\pi^2 r^2} F^\prime s^2 \quad \text{and} \quad V_3(r, \nu) = -\frac{2}{\sqrt{3}} \left( f_\pi^2 - f_\pi^2 \right) \frac{s^2}{r^2}. \quad (A.1) \]

In these expressions a prime indicates the derivative with respect to the radial coordinate, 
\textit{i.e.} \( F' = \partial F(r, \nu)/\partial r \). Note, that the radial functions \( V_i \) not only have an explicit 
dependence on the strangeness changing angle \( \nu \) but also an implicit one via the chiral 
angle \( F(r, \nu) \).

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