Alternative Estimation Method for a Three-Stage Cluster Sampling in Finite Population

L. A. Nafiu¹*, I. O. Oshungade², A. A. Adewara
¹Department of Mathematics and Statistics, Federal University of Technology, Minna, +234, Nigeria
²Department of Statistics, University of Ilorin, Ilorin, +234, Nigeria

Abstract This research investigates the use of a three-stage cluster sampling design in estimating population total. We focus on a special design where certain number of visits is being considered for estimating the population size and a weighted factor of / is introduced. In particular, attempt was made at deriving new method for a three-stage sampling design. In this study, we compared the newly proposed estimator with some of the existing estimators in a three-stage sampling design. Eight (8) data sets were used to justify this paper. The first four (4) data sets were obtained from [1],[2],[3] and [4] respectively while the second four (4) data sets represent the number of diabetic patients in Niger state, Nigeria for the years 2005, 2006, 2007 and 2008 respectively. The computation was done with software developed in Microsoft Visual C++ programming language. All the estimates obtained show that our newly proposed three-stage cluster sampling design estimator performs better.

Keywords Sampling, Three-stage, Cluster Design, Estimator, Bias and Variance

1. Introduction

In a census, each unit (such as person, household or local government area) is enumerated, whereas in a sample survey, only a sample of units is enumerated and information provided by the sample is used to make estimates relating to all units[5] and[6]. In designing a study, it can be advantageous to sample units in more than one-stage. The criteria for selecting a unit at a given stage typically depend on attributes observed in the previous stages[7]. Multistage sampling is where the researcher divides the population into clusters, samples the clusters, and then resample, repeating the process until the ultimate sampling units are selected at the last of the hierarchical levels[8]. If, after selecting a sample of primary units, a sample of secondary units is selected from each of the selected primary units, the design is referred to as two-stage sampling. If in turn a sample of tertiary units is selected from each selected secondary unit, the design is three-stage sampling[9].

The aim of this paper is to model a new estimator for three-stage cluster sampling scheme which is to be compared with the other existing seven conventional estimators.

2. Methodology

Sub sampling has a great variety of applications[3] and the reason for multistage sampling is administrative convenience[10]. The process of sub sampling can be carried to a third stage by sampling the subunits instead of enumerating them completely[11]. Comparing multistage cluster sampling with simple random sampling, it was observed that multistage cluster sampling is better in terms of efficiency[12]. Multistage sampling makes fieldwork and supervision relatively easy[4]. Multistage sampling is more efficient than single stage cluster sampling[13] and references had been made to the use of three or more stages sampling[9].

Let N denote the number of primary units in the population and n the number of primary units in the sample. Let Mi be the number of secondary units in the primary unit. The total number of secondary units in the population is

\[ M = \sum_{i=1}^{N} M_i \] (1)

Let \( y_{ij} \) denote the value of the variable of interest of the \( j \)th secondary unit in the \( i \)th primary unit. The total of the \( y \)-values in the \( i \)th primary unit is

\[ Y_i = \sum_{j=1}^{M_i} y_{ij} \] (2)

Accordingly, the population total for over-all sample in a two-stage is given as

\[ Y = \sum_{i=1}^{N} \sum_{j=1}^{M_i} y_{ij} \] (3)

For a three-stage sampling, the population contains \( N \)
first-stage units, each with M second-stage units, each of which has K third-stage units. The corresponding numbers for the sample are n, m and k respectively. Let \( Y_{ij} \) be the value obtained for the \( i \)th third-stage units in the \( j \)th secondary-stage units drawn from the \( i \)th primary units. The relevant population total for over-all sample in a three-stage is given as follows:

\[
Y = \sum^{n}_{i=1} \sum^{m}_{j=1} \sum^{K}_{k=1} Y_{ij}
\]  

(4)

For any estimation \( \hat{\Theta}_h \) in the \( h \)th cell based on \( Y_{ij} \) for the unbiased estimate of \( V \). We may then write

\[
V(\hat{\Theta}_h) = V(E(\hat{\Theta}_h)) + E(V(\hat{\Theta}_h))
\]  

(5)

The expression (5) may be written into three components as:

\[
V(\hat{\Theta}_h) = V(E(\hat{\Theta}_h)) + E(V(\hat{\Theta}_h)) + E(V(\hat{\Theta}_h))
\]  

(6)

where “\( \Rightarrow 1 \)” is the symbol to represent all stages of sampling after the first[3].

3. Proposed Three-Stage Cluster Sampling Design

To estimate the population size at different hospitals using three-stage sampling, the unbiased estimator of population total can be derived as follows. In a three-stage sampling without replacement design supported by[3],[4] and[14]: a sample of primary units is selected, then a sample of secondary units is chosen from each of the selected primary units and finally, a sample of tertiary units is chosen from each selected secondary unit. For instance, the state consists of \( N \) number of local government areas out of which a simple random sampling of \( n \) number of local government areas is selected. Each local government area consists of \( M_i \) number of cities out of which a simple random sampling without replacement of \( m_i \) number of cities is selected. Finally, from the selected sample of city containing \( K_{ij} \) number of hospitals, \( k_{ij} \) number of hospitals is selected at random without replacement and the number of diabetic patients in this hospital is collected.

Then;

\[
y = \sum^{n}_{i=1} \sum^{m_i}_{j=1} Y_{ij}
\]  

(7)

Again, let \( n \) be the number of primary units (local government areas) sampled without replacement, \( m_i \) be the number of secondary units (cities) selected without replacement from the \( i \)th sampled primary unit (local government area) and \( k_{ij} \) be the number of tertiary units (hospitals) selected from the \( j \)th secondary unit (city) in the \( i \)th primary unit (local government area). An unbiased estimator of the population total at \( j \)th secondary unit in the \( i \)th primary unit in the sample is:

\[
\hat{y}_{ij} = \frac{k_{ij}}{k_{ij}} \sum^{n}_{i=1} \frac{y_{ij}}{n}
\]  

(8)

where \( y_{ij} = \frac{k_{ij}}{k_{ij}} \) is the known sampling fraction for tertiary units in the \( j \)th secondary unit of the \( i \)th primary unit.

Also, let \( y_{ij} \) denote the number of individuals (tertiary units) in the sample from the \( j \)th secondary unit of the \( i \)th primary unit who engage in the treatment of diabetes. An unbiased estimator of the population total in the \( i \)th primary unit in the sample is:

\[
\hat{y}_i = \frac{M_i}{m_i} \sum^{m_i}_{j=1} \hat{y}_{ij}
\]  

(9)

Finally, an unbiased estimator of the population total of the diabetic patients undergoing treatment in all the hospitals at the \( j \)th secondary unit (city) in the \( i \)th primary unit (local government area) is:

\[
\hat{y}_{3NPE} = \frac{N}{n} \sum^{n}_{i=1} \hat{y}_i
\]  

(10)

4. Theorems

4.1. Theorem 1: \( \hat{y}_{3NPE} \) is Unbiased for the Population total \( Y \)

Proof:

We know that expectation of \( \hat{y}_{ij} \) given by equation (8) conditional on samples \( s_1 \) and \( s_2 \) of primary units and secondary units respectively equals \( y_{ij} \) of engaging in the variable of interest in each primary unit and each secondary unit [15]. That is;

\[
E(\hat{y}_{ij} | s_1, s_2) = y_{ij}
\]  

(11)

Also, the expectation of \( \hat{y}_i \) given by equation (9) conditional on sample \( s_1 \) of primary units equals \( y_i \) of engaging in the variable of interest in each primary unit. That is;

\[
E(\hat{y}_i | s_2) = y_i
\]  

(12)

To obtain the expected value of \( \hat{y}_{3NPE} \) given by equation (10) over all possible samples of primary units.

Then, the expectation of \( \hat{y}_{3NPE} \) is :

\[
E(\hat{y}_{3NPE}) = E_1[E_2[E_3(\hat{y}_{3NPE} | s_1, s_2) | s_2]]
\]  

(13)

where \( s_1 \) and \( s_2 \) denote the samples of primary units and secondary units respectively.

\[
E(\hat{y}_{3NPE}) = E[E(E(\frac{n}{n} \sum^{n}_{i=1} \hat{y}_i | s_1, s_2) | s_2)]
\]  

(14)

\[
= E[E(\frac{n}{n} \sum^{n}_{i=1} \hat{y}_i | s_2)]
\]  

(15)

\[
= E(\frac{n}{n} \sum^{n}_{i=1} y_i)
\]  

(16)
known for \( i = 1,2,\cdots, n \) and \( j = 1,2,\cdots, m_j \). The third term contains variance due to estimating the \( y_{ij} \)'s from a subsample of tertiary units within the selected secondary units. An unbiased estimator of the variance of \( \hat{y}_{3Ne} \) given in equation (18) is obtained by replacing the population variances with the sample variances as follows:

\[
\hat{V}(\hat{y}_{3Ne}) = N(N-n)\frac{s_i^2}{n} + \frac{2}{n} \sum_{i=1}^{n} \frac{M_i(M_i - m_i)\sigma_i^2}{m_i} + \frac{N(N-n)}{n} \sigma^2_{ij}
\]

Next, we note that:

\[
E\left( \sum_{i=1}^{n} y_i^2 \right) = E\left( \sum_{i=1}^{n} (V(y_i | s_i) + (E(y_i | s_i))^2) \right)
\]

\[
= E\left( \sum_{i=1}^{n} \frac{\sigma_i^2 + \sum_{j=1}^{N} Y_j^2}{m_i} \right)
\]

\[
= E\left( \sum_{i=1}^{N} Y_i^2 \right) + \frac{n}{N} \sum_{i=1}^{N} \sigma_i^2 + \frac{N}{n} \sum_{i=1}^{N} Y_i^2
\]

In addition:

\[
E(\hat{y}_{3Ne}) = V(\hat{y}_{3Ne}) + \{E(\hat{y}_{3Ne})\}^2
\]

\[
= \frac{n}{N} \sum_{i=1}^{N} \sigma_i^2 + \frac{N}{n} \sum_{i=1}^{N} Y_i^2 + Y^2
\]

Combining equations (27) and (28), we have:

\[
E(s_i^2) = \frac{n}{N(n-1)}\left( \sum_{i=1}^{n} \sigma_i^2 + \sum_{i=1}^{n} Y_i^2 \right) - \frac{n}{N^2(n-1)} \left( \frac{N(N-n)}{n} \sigma^2 - \frac{N}{n} \right)
\]

\[
= \frac{1}{N} \sum_{i=1}^{N} \sigma_i^2 + \frac{(N-1)n}{(N-1)n} \left( \frac{N}{n} \right) - \frac{(N-n)}{N} \sigma^2
\]

Using the fact that:

\[
\sigma_i^2 = \frac{1}{N-1} \left( \sum_{i=1}^{N} Y_i^2 - \frac{Y^2}{N} \right)
\]

Equation (29) becomes:

\[
E(s_i^2) = \frac{1}{N} \sum_{i=1}^{N} \sigma_i^2 + \frac{(N-1)n}{(N-1)n} \sigma_i^2 - \frac{(N-n)}{N} \sigma^2
\]

\[
= \frac{1}{N} \sum_{i=1}^{N} \sigma_i^2 + \frac{N(N-n)}{n} \sigma^2
\]

Next, we note that:
\[
E(s_i^2) = E(E(\sum_{i=1}^{n} s_i^2)) = E(\sum_{i=1}^{n} s_i^2) = E(\sum_{i=1}^{n} \frac{M_i^2}{m_i} \sigma_i^2) = E(\sum_{i=1}^{n} \frac{M_i^2}{m_i} \sigma_i^2)
\]

\[
= \frac{n}{N} \sum_{i=1}^{N} M_i (M_i - m_i) \sigma_i^2 - \frac{1}{n} \sum_{i=1}^{N} M_i (M_i - m_i) \sigma_i^2
\]

Therefore,

\[
E(s_i^2) = \frac{k}{n} \sum_{i=1}^{k} M_i (M_i - m_i) \sigma_i^2 - \frac{1}{N} \sum_{i=1}^{N} M_i (M_i - m_i) \sigma_i^2
\]

Also:

\[
E(s_{ij}^2) = E[E(s_{ij}^2 | s_i, s_j)] = E[E(\frac{M_i^2}{m_i} \sum_{j=1}^{N} Y_{ij}} | s_i, s_j])
\]

\[
= \frac{M_i^2}{m_i} E[E(\sum_{j=1}^{N} Y_{ij} | s_i, s_j])
\]

\[
= \frac{M_i^2}{m_i} E[\sum_{j=1}^{N} Y_{ij}^2]
\]

\[
= \frac{M_i^2}{m_i} E[\sum_{j=1}^{N} \sigma_i^2]
\]

\[
= \frac{M_i^2}{m_i} \sum_{j=1}^{N} \sigma_i^2
\]

Therefore;

\[
E(s_{ij}^2) = \frac{n}{N} \sum_{i=1}^{N} M_i \sum_{j=1}^{N} K_{ij} (K_{ij} - k_{ij}) \frac{\sigma_i^2}{k_{ij}}
\]

Combining equations (30), (31) and (32), we have:

\[
E(\mathcal{V}(\mathcal{Y}_{3NPE})) = N(N - n) \frac{\sigma_i^2}{n} + \frac{N}{n} \sum_{i=1}^{N} M_i (M_i - m_i) \frac{\sigma_i^2}{m_i}
\]

\[
+ \frac{N}{n} \sum_{i=1}^{N} M_i \frac{M_i}{m_i} \sum_{j=1}^{N} K_{ij} (K_{ij} - k_{ij}) \frac{\sigma_i^2}{k_{ij}}
\]

That is;

\[
E(\mathcal{V}(\mathcal{Y}_{3NPE})) = \mathcal{V}(\mathcal{Y}_{3NPE})
\]

Hence, \(\mathcal{V}(\mathcal{Y}_{3NPE})\) is an unbiased sample estimator of the proposed estimator \(\mathcal{Y}_{3NPE}\) in three-stage cluster sampling design.

This estimator, \(\mathcal{Y}_{3NPE}\), is then compared with these seven conventional three-stage cluster sampling design estimators:

i. \(\hat{\mathcal{Y}}_{HH} = \frac{1}{n} \sum_{i=1}^{n} \frac{\bar{Y}_{ij}}{p_{ij}}\)

ii. \(\hat{\mathcal{Y}}_{HHG} = M_0 \sum_{g=1}^{G} \sum_{i=1}^{N_g} \bar{Y}_{ij} \)

iii. \(\hat{\mathcal{Y}}_{HT} = \frac{1}{n} \sum_{i=1}^{n} \frac{\bar{Y}_{ij}}{z_{ij}}\)

iv. \(\hat{\mathcal{Y}}_{BHC} = \frac{1}{n} \sum_{g=1}^{G} \frac{\bar{Y}_{ij}}{z_{ij}}\)

v. \(\hat{\mathcal{Y}}_{C} = \sum_{g=1}^{G} \sum_{j=1}^{K_g} \frac{\bar{Y}_{ij}}{z_{ij}}\)

vi. \(\hat{\mathcal{Y}}_{T} = \sum_{i=1}^{n} \sum_{j=1}^{K_{ij}} \frac{\bar{Y}_{ij}}{z_{ij}}\)

and

vii. \(\hat{\mathcal{Y}}_{O} = \frac{N}{n} \sum_{i=1}^{N} \frac{M_i}{m_i} \sum_{j=1}^{K_{ij}} \frac{\bar{Y}_{ij}}{z_{ij}}\)

5. Data Used for this Study

There are eight (8) categories of data sets used in this paper. The first four (4) data sets were obtained from [1],[2],[3] and [4] respectively. The second four (4) data sets used are of secondary type and were collected from [16] and [17] where we constructed a sampling frame from all diabetic patients with chronic eye disease (Glaucoma and Retinopathy) in the twenty-five (25) Local Government Areas of the state between 2005 and 2008.

6. Results

The estimates obtained with the aid of software developed using Visual Basic C++ Programming Language [18] are given in tables 1 - 12 for the illustrated and the real-life data respectively.
Table 1. Estimated Population Totals using Illustrated Data

| Estimator | POP1 | POP2 | POP3 | POP4 |
|-----------|------|------|------|------|
| \( \hat{y}_{HH} \) | 26,022 | 25,019 | 24,355 | 25,162 |
| \( \hat{y}_{HT} \) | 27,917 | 27,313 | 27,048 | 27,451 |
| \( \hat{y}_{RH} \) | 26,428 | 26,851 | 25,321 | 26,022 |
| \( \hat{y}_{C} \) | 27,197 | 26,831 | 25,214 | 25,804 |
| \( \hat{y}_{O} \) | 27,048 | 27,451 | 25,321 | 26,022 |

Table 2. Estimated Population Totals using Real Life Data

| Estimator | POP1 | POP2 | POP3 | POP4 |
|-----------|------|------|------|------|
| \( \hat{y}_{HH} \) | 26,022 | 25,019 | 24,355 | 25,162 |
| \( \hat{y}_{HT} \) | 27,917 | 27,313 | 27,048 | 27,451 |
| \( \hat{y}_{RH} \) | 26,428 | 26,851 | 25,321 | 26,022 |
| \( \hat{y}_{C} \) | 27,197 | 26,831 | 25,214 | 25,804 |
| \( \hat{y}_{O} \) | 27,048 | 27,451 | 25,321 | 26,022 |

Table 3. Biases of Estimated Population Totals using Illustrated Data

| Estimator | Case I | Case II | Case III | Case IV |
|-----------|-------|--------|----------|---------|
| \( \hat{y}_{HH} \) | 29 | 516 | 8 | 446 |
| \( \hat{y}_{HT} \) | 15 | 364 | 2 | 461 |
| \( \hat{y}_{C} \) | 33 | 463 | 9 | 369 |
| \( \hat{y}_{O} \) | 13 | 261 | 3 | 476 |

Table 4. Biases of Estimated Population Totals using Real Life Data

| Estimator | POP1 | POP2 | POP3 | POP4 |
|-----------|------|------|------|------|
| \( \hat{y}_{HH} \) | 26,022 | 25,019 | 24,355 | 25,162 |
| \( \hat{y}_{HT} \) | 27,917 | 27,313 | 27,048 | 27,451 |
| \( \hat{y}_{RH} \) | 26,428 | 26,851 | 25,321 | 26,022 |
| \( \hat{y}_{C} \) | 27,197 | 26,831 | 25,214 | 25,804 |
| \( \hat{y}_{O} \) | 27,048 | 27,451 | 25,321 | 26,022 |

Table 5. Variances of the Estimated Population Totals using Illustrated Data

| Estimator | Case I | Case II | Case III | Case IV |
|-----------|-------|--------|----------|---------|
| \( \hat{y}_{HH} \) | 584.3718 | 163.955.9954 | 3.075 | 2.012.5838 |
| \( \hat{y}_{HT} \) | 498.8637 | 1467.797.7757 | 2.9402 | 1.9582.3921 |
| \( \hat{y}_{RH} \) | 444.7838 | 131.942.6350 | 1.6598 | 1.8614746 |
| \( \hat{y}_{C} \) | 425.8846 | 109.764.4333 | 1.6226 | 1.84458793 |
| \( \hat{y}_{O} \) | 418.4156 | 107.030.5504 | 1.6201 | 1.84240448 |
| \( \hat{y}_{SNE} \) | 315.7266 | 96.800.8501 | 1.5117 | 1.8344763 |
| \( \hat{y}_{SNE} \) | 296.8283 | 94.624.7532 | 1.1674 | 1.81055856 |
| \( \hat{y}_{SNE} \) | 274.6806 | 91.963.5764 | 1.1105 | 1.6195559 |

Table 6. Variances of the Estimated Population Totals using Real Life Data

| Estimator | POP1 | POP2 | POP3 | POP4 |
|-----------|------|------|------|------|
| \( \hat{y}_{HH} \) | 17.46 | 16.636 | 15.36 | 15.146 |
| \( \hat{y}_{HT} \) | 13.419 | 14.626 | 11.493 | 13.315 |
| \( \hat{y}_{RH} \) | 11.684 | 11.788 | 11.398 | 12.396 |
| \( \hat{y}_{C} \) | 9.985 | 11.532 | 10.938 | 12.239 |
| \( \hat{y}_{O} \) | 9.749 | 11.441 | 10.532 | 12.069 |

Table 7. Standard Error for Estimated Population Total using Illustrated Data

| Estimator | Case I | Case II | Case III | Case IV |
|-----------|-------|--------|----------|---------|
| \( \hat{y}_{HH} \) | 24.1738 | 404.9148 | 1.7537 | 44.8618 |
| \( \hat{y}_{HT} \) | 22.3173 | 383.1420 | 1.7147 | 44.2520 |
| \( \hat{y}_{RH} \) | 21.0899 | 363.2391 | 1.2841 | 43.1480 |
| \( \hat{y}_{C} \) | 20.6370 | 331.3072 | 1.2738 | 49.5213 |
| \( \hat{y}_{O} \) | 20.4552 | 327.1522 | 1.2728 | 42.9232 |
| \( \hat{y}_{SNE} \) | 17.6787 | 311.1283 | 1.2295 | 42.8308 |

Table 8. Standard Error for Estimated Population Total using Real Life Data

| Estimator | POP1 | POP2 | POP3 | POP4 |
|-----------|------|------|------|------|
| \( \hat{y}_{HH} \) | 132.1502 | 128.9710 | 130.6399 | 123.0702 |
| \( \hat{y}_{HT} \) | 125.3573 | 127.6163 | 125.3413 | 122.8823 |
| \( \hat{y}_{RH} \) | 115.8417 | 120.9390 | 113.0051 | 115.3810 |
| \( \hat{y}_{C} \) | 108.0937 | 108.5724 | 112.5340 | 111.3351 |
| \( \hat{y}_{O} \) | 99.9287 | 107.3827 | 110.2437 | 110.6305 |
| \( \hat{y}_{SNE} \) | 98.7379 | 106.9641 | 108.1752 | 108.8593 |
| \( \hat{y}_{SNE} \) | 97.8150 | 102.5056 | 107.1344 | 105.2964 |
| \( \hat{y}_{SNE} \) | 95.4893 | 98.6229 | 104.7944 | 103.0137 |
Table 9. 95% Confident Intervals for Estimated Population using Illustrated Data

| Estimator | Case I | Case II | Case III | Case IV |
|-----------|--------|---------|----------|---------|
| $\hat{Y}_{HH}$ | (353,447) | (98189,9977) | (33,39) | (13810,13986) |
| $\hat{Y}_{HHH}$ | (396,484) | (110559,112061) | (19,25) | (13913,14087) |
| $\hat{Y}_{HT}$ | (438,520) | (138815,140239) | (24,30) | (14706,14876) |
| $\hat{Y}_{RHC}$ | (397,477) | (98181,99479) | (29,33) | (15130,15298) |
| $\hat{Y}_{C}$ | (345,425) | (131034,132316) | (37,41) | (13879,14047) |
| $\hat{Y}_{T}$ | (421,491) | (102025,103245) | (32,36) | (14492,14660) |
| $\hat{Y}_{O}$ | (363,431) | (140591,141797) | (22,26) | (13867,14033) |
| $\hat{Y}_{3NPE}$ | (460,524) | (98542,99730) | (40,44) | (14939,15095) |

Table 10. 95% Confident Intervals for Estimated Population Totals using Real Life Data

| Estimator | Population 1 | Population 2 | Population 3 | Population 4 |
|-----------|--------------|--------------|--------------|--------------|
| $\hat{Y}_{HH}$ | (25760,26290) | (26290,26790) | (28170,28680) | (29110,29600) |
| $\hat{Y}_{HHG}$ | (24110,24600) | (24770,25270) | (25800,25410) | (28370,28850) |
| $\hat{Y}_{HT}$ | (25290,25740) | (26960,27430) | (27800,27520) | (26870,27320) |
| $\hat{Y}_{RHC}$ | (25830,26250) | (26220,26640) | (27400,27820) | (27230,27670) |
| $\hat{Y}_{C}$ | (24220,24620) | (25110,25530) | (26300,27070) | (28370,28850) |
| $\hat{Y}_{T}$ | (25610,26000) | (26990,27410) | (27400,27820) | (29260,29690) |
| $\hat{Y}_{O}$ | (27010,27400) | (27920,28320) | (28400,29460) | (29040,29460) |
| $\hat{Y}_{3NPE}$ | (25960,26340) | (26410,26800) | (27310,27720) | (27890,28290) |

Table 11. Coefficient of Variation for Estimated Population Totals using Illustrated Data

| Estimator | Case I | Case II | Case III | Case IV |
|-----------|--------|---------|----------|---------|
| $\hat{Y}_{HH}$ | 6.04% | 0.41% | 4.87% | 0.32% |
| $\hat{Y}_{HHG}$ | 5.07% | 0.34% | 7.79% | 0.32% |
| $\hat{Y}_{HT}$ | 4.40% | 0.26% | 4.76% | 0.29% |
| $\hat{Y}_{RHC}$ | 4.72% | 0.34% | 4.11% | 0.28% |
| $\hat{Y}_{C}$ | 5.31% | 0.25% | 3.26% | 0.31% |
| $\hat{Y}_{T}$ | 3.90% | 0.30% | 3.62% | 0.29% |
| $\hat{Y}_{O}$ | 4.34% | 0.22% | 4.50% | 0.31% |
| $\hat{Y}_{3NPE}$ | 3.98% | 0.31% | 2.51% | 0.27% |

Table 12. Coefficient of Variation for Estimated Population Totals using Real Life Data

| Estimator | Populations |
|-----------|-------------|
| $\hat{Y}_{HH}$ | 0.44% | 0.43% | 0.42% | 0.42% |
| $\hat{Y}_{HHG}$ | 0.51% | 0.51% | 0.44% | 0.43% |
| $\hat{Y}_{HT}$ | 0.45% | 0.44% | 0.37% | 0.34% |
| $\hat{Y}_{RHC}$ | 0.42% | 0.41% | 0.37% | 0.41% |
| $\hat{Y}_{C}$ | 0.37% | 0.42% | 0.38% | 0.33% |
| $\hat{Y}_{T}$ | 0.38% | 0.33% | 0.36% | 0.37% |
| $\hat{Y}_{O}$ | 0.36% | 0.36% | 0.36% | 0.36% |
| $\hat{Y}_{3NPE}$ | 0.34% | 0.33% | 0.35% | 0.37% |

7. Discussion of Results

The estimation methods given in equation (10) was applied to four different illustrated data (Cases I – IV) and four real life data (Populations 1 - 4). The population totals obtained for illustrated data are given in table 1 while the population totals obtained for real life data are given in table 2. Table 3 gives the biases of the estimated population totals for illustrated data for our own estimator as 11, 219, 1, and 112 for cases I – IV respectively while table 4 gives that of the four life data sets as 112, 104, 103, and 107 respectively. This implies that our own estimator has the least biases using both data sets. Table 5 shows the variances obtained using illustrated data for our own estimator as 274.6806, 91963.5764, 1.1105 and 1619.5559 for cases I – IV respectively while table 6 shows that of life data sets as 9118.2037, 9726.4809, 9883.6215 and 10611.8216 respectively meaning that our own estimator has the least variances using both data sets. Table 7 shows the standard errors for the estimated population totals using illustrated data for our own estimator as 16.5785, 302.2597, 1.0538 and 40.2437 for cases I – IV respectively while table 8 shows that of life data sets as 95.4893, 98.6229, 104.7944 and 103.0137 respectively meaning that our own estimator have the least standard errors using both data sets. The confidence intervals of the estimated populations in table 1 are given in table 9 for α=, 5%. The confidence intervals of the estimated populations in table 2 are given in...
8. Conclusions

The alternative estimation method of population allows the use of certain number of visits to the venues (hospitals) within the clusters (cities) and a more precise (minimum mean square error) estimate was obtained and the estimates presented indicate that substantial reduction in the variances was obtained through the use of newly proposed estimator. We also observed that irrespective of the data considered, the variance of newly proposed estimator is always less than those of already existing estimators in three-stage cluster sampling designs. The newly proposed estimator ($P_{3NPE}$) is preferred to the already existing estimators considered in this study and is therefore recommended.

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