A multi-attribute decision making method based on the third generation prospect theory and grey correlation degree

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Abstract: Considering the uncertainty of the natural state and the convenience of calculation, based on the third generation prospect theory (3-PT) and grey correlation analysis (GRA), we propose a method to solve the multi-attribute decision-making (MADM) problems where the attributes are described by the linguistic intuitionistic fuzzy numbers (LIFNs). Firstly, we transform the LIFNs into the belief structure that includes identity value and belief degree. Then, the evaluation information represented by belief structure is calculated by using the 3-PT, and the prospect matrix is gotten. The alternatives are ranked by the GRA. Finally, we use the proposed method to calculate an example and compare it with other methods to prove its effectiveness and superiority.

Keywords: belief structure; third generation prospect theory; grey correlation analysis; multi-attribute decision making.

1. Introduction

Multi-attribute decision making (MADM) [1-5] is the problem of ranking finite alternatives or selecting the best one from multiple alternatives with multiple attributes. Nowadays, MADM problems are very common in everyday life and have received the attention of many researchers. For instance, Kannan [6] studied MADM methods for green supplier selection and Mardani [7] studied the application of MADM techniques in the field of sustainable and renewable energy. However, in real decision making, many problems cannot be described by accurate numbers, and can be expressed by fuzzy numbers. Zadeh [8] firstly proposed fuzzy sets (FSs) which are a good tool to describe the fuzzy information. However, the drawback was that they could only describe membership degree (MD) and could not describe non-membership degree (NMD). After that, the intuitionistic fuzzy set (IFS) proposed by Atanassov [9] had overcome this shortcoming. Both the MD and the NMD in IFS were real numbers defined on [0, 1], which could well describe the quantitative attributes but could not express the qualitative attributes well. Therefore, Chen [10] combined IFS with linguistic variables (LVs) and proposed the linguistic IFS (LIFS). After the LIFS was presented, it was studied and extended by many scholars. Zhang [11] proposed the distance formula of LIFNs and gave an extended outranking approach for MADM problems. Liu [12] extended the partitioned Heronian means (HM) to LIFNs. Ou et al. [13] proposed the TOPSIS method for LIFS. Peng et al. [14] proposed a linguistic intuitionistic MADM approach based on the Heronian operator with Frank operations and applied it to evaluate coal mine safety. Based on some new operational laws and entropy, Li [15] proposed an extended VIKOR method to solve decision making problems of attribute values as LIFNs.

On the basis of empirical analysis, Kahneman and Tversky [16] gave the prospect theory (PT) which was a combination of psychology, behavior and game theory. PT took into account the decision makers (DMs)’ bounded rationality and was more in line with the actual decision-making behavior of DMs. Further, Tversky and Kahneman [17] proposed cumulative PT (CPT). Value function, weight function and parameters of PT were important research contents. Tversky [18] and Fox [19] proposed a two-stage method to determine the decision weight. In the first stage, the DMs judged the probability
of the event based on the randomness of the event. In the second stage, the probability weight function was used to convert the probability into the decision weight. Zeng [20] designed an experiment to get the parameters of the value function and the weight function. Compared with Kahneman's results [16], it showed that different types of DMs had different parameters of the value function. Ma and Sun [21] improved the value function and extended the parameter range on the basis of Zeng [20], further explained that the different risk attitudes of DMs could determine the parameters of the value function. Wakker et al. [22] studied the simple preference foundation of CPT. In addition, different DMs might choose different reference points from different perspectives, so the choice of reference points also had an important impact on the PT. However, in the PT and CPT, the reference points were fixed and cannot be changed with the state. Therefore, on the basis of the PT and CPT, Schmidt [23] put forward the third-generation prospect theory (3-PT), which introduced the dynamic reference point considering the uncertainty of natural state. In recent years, the application of PT has received more and more attention. Birnbaum [24] conducted an empirical evaluation of 3-PT. Xiang [25] proposed a MADM method under risk based on 3-PT. Wu et al. [26] evaluated renewable power sources based on CPT. Jin et al. [27] proposed a method for MADM under uncertainty using evidential reasoning and PT. Zhang [28] proposed different situational emergency decision-making methods based on game theory and PT. Phochanikorn [29] proposed an integrated model based on PT for green supplier selection under uncertain environment.

Grey correlation analysis (GRA) was a very important method of MADM in grey system theory [30]. Because the grey correlation degree did not have much requirement on the sample size and the calculation amount was small, and it was easy to be combined with other decision methods, the GRA has been widely used in MADM problems. Liu [31] described the steps of MADM using grey correlation method in detail. Liu et al. [32] conducted GRA and grey cluster analysis on key indicators of the remanufacturing industry China. Based on GRA, Zhan et al. [33] studied factors that influence consumers’ loyalty towards geographical indication products.

Considering that GRA is easy to combine with other methods, the 3-PT fully considers the subjective preference of DMs and introduces dynamic reference points, this paper combines 3-PT and GRA to propose a new MADM method which can solve the MADM problem expressed by LIFNs. In the decision-making process, we firstly transform the LIFNs into belief structure, and then bring into the formula of the 3-PT to get the prospect matrix. At last, we use the grey correlation method to rank the alternatives and find the optimal one. The proposed method has three main advantages. (1) it is easier and more accurate for DMs to evaluate the decision making problems in the form of LIFNs; (2) it can consider the uncertainty of the natural state and subjective preferences of the human by 3-PT; (3) it is simple and more reasonable by the GRA based on the positive and negative ideal alternatives. In this article, we will accomplish the following goals.

1. Propose a transformation method to convert LIFNs to belief structure.
2. Proposed a new MADM method based on 3-PT and GRA, and an example is calculated by this method.
3. The effectiveness and superiority of the proposed method are illustrated by comparison with other methods.

The rest of this article is organized as follows. In the second part, we review some preliminaries, including LIFNs, 3-PT and GRA. In the third part, we present a method for transforming the LIFNs into belief structure. In the fourth part, based on PT and GRA, a method of MADM is given. In the fifth part, an example is calculated using the proposed method, and the effectiveness and superiority of
the proposed method are proved by comparison with the other two methods. In the sixth part, we summarize the article.

2. Preliminaries

2.1 LIFNs

Definition 1. [10] Let \( s_p, s_q \in S \) and \( g = (s_p, s_q) \), if \( p + q \leq t \), then the \( g \) is called a LIFN.

Where \( S \) is the set of the continuous linguistic terms (LTs) based on the discrete LTs \( S = \{ s_0, s_1, \ldots, s_t \} \). \( s_t \) is the upper limit of LTs. In general, we use \( \Gamma_{[0,1]} \) to express the set of all LIFNs.

Remark 1. [34] The uncertain LVs (ULVs) are equivalent to the LIFNs. If \( \tilde{s} = [s_p, s_q] \) is an ULV, where \( p, q \in [0, t] \) and \( p \leq q \), then LIFN \((s_p, s_{-q})\) is to equivalent to the \( \tilde{s} = [s_p, s_q] \).

Remark 2. [35] Suppose there is a linguistic set \( S = \{ s_i | i = 0, 1, 2 \cdots t \} \). When \( \theta_i \in [0, 1] \) is a numerical value, the linguistic scale function (LSF) is mapped from \( s_i \) to \( \theta_i (i = 0, 1, \cdots t) \). Based on subscript functions \( sub(s_i) = i \), the LSF is \( f(s_i) = \theta_i = \frac{i}{t} (i = 1, 2, \cdots t) \).

Using LSF, we can convert LVs in LIFNs and ULVs into real numbers.

2.2 Third-generation prospect theory

Kahneman and Tversky [16] firstly proposed the concept of PT in 1979. PT is chosen by prospect value (\( V \)) which is calculated by value function (\( v \)) and decision weight (\( \omega \)). After that, Tversky and Kahneman [17] proposed cumulative PT (CPT) in 1992. In PT or CPT, how to select the reference point was an important research problem. Schmidt et al. [23,36] introduced the concept of dynamic reference point based on PT and CPT, and proposed the third-generation prospect theory (3-PT).

Definition 2. [23,36] Suppose \( ST = \{ st_b | b = 1, 2, \cdots B \} \) are collections of natural states, and their probability of occurrence is \( P = \{ p_b | b = 1, 2, \cdots B \} \), \( \sum_{b=1}^{B} p_b = 1 \). The result of state \( ST \) at probability \( P \) is \( X = \{ x_b | b = 1, 2, \cdots B \} \), then \( \{ x_1, p_1; x_2, p_2; \cdots ; x_b, p_b; \cdots ; x_B, p_B \} \) means obtaining the result \( x_b \) with probability \( p_b \). Suppose that \( \forall b \in \{ 1, 2, \cdots B \} \), \( f(st_b) \in x, h(st_b) \in x \), \( h \) is the reference point, then value function is as follows.

\[
v(\Delta(f,h)) = \begin{cases} 
\epsilon^{+}(\Delta(f,h))^+, & \Delta(f,h) \geq 0 \\
-\epsilon^{-}(\Delta(f,h))^-, & \Delta(f,h) < 0 
\end{cases}
\]

\( \Delta(f,h) = f(st_b) - h(st_b) \). \( \epsilon^{+} \), \( \epsilon^{-} \) represent the sensitivity of DMs to gains or losses. If the DMs is more sensitive to the gain than the loss, then \( \epsilon^{+} > 1 \) and \( \epsilon^{-} = 1 \). If the loss is more sensitive than the gain, then \( \epsilon^{-} = 1 \) and \( \epsilon^{+} > 1 \). The DMs can be divided into three types of conservative, neutral and risky, and the values of \( \mu \) and \( v \) are different depending on the DMs’ attitude towards risk. For
conservative DMs, $\mu > 1, \nu > 1$; for neutral DMs, $\mu = \nu = 1$; for adventurous DMs, $\mu < 1, \nu < 1$.

The events are sorted according to the value function, satisfying if and only if $v(\Delta(f_m, h_n)) > v(\Delta(f_m, h_n)), m > n$. What’s more, $\Delta(f_m, h_n) < 0$ indicates that there is a strict loss in state $s_b$, $s^-$ is the number of states of strict loss; $\Delta(f_m, h_n) \geq 0$ indicates that there is a weak gain in state $s_b$, $s^+$ is the number of states of weak gain; and $s^+ + s^- = B$. Then the decision weight is defined as follows.

$$\omega(s_b; f, h) = \begin{cases} \omega^+(p_b) & b = B \\ \omega^-(\sum_{i \in b} p_i) - \omega^+(\sum_{i \in b} p_i) & s^- + 1 \leq b \leq B \\ \omega^-(\sum_{i \in b} p_i) - \omega^+(\sum_{i \in b} p_i) & 1 \leq b \leq s^- \\ \omega^+(p_b) & b = 1 \end{cases}$$

(2)

$\omega(p)$ is the probability weight function, and

$$\omega^+(p) = \begin{cases} \exp\left(-\xi^+ [-\ln(p)]^+\right) & p \neq 0 \\ 0 & p = 0 \end{cases}$$

(3)

$$\omega^-(p) = \begin{cases} \exp\left(-\xi^- [-\ln(p)]^+\right) & p \neq 0 \\ 0 & p = 0 \end{cases}$$

(4)

In $\omega(p)$, $p$ represents the probability of event $x$ occurring. The parameters $\xi^+, \xi^- > 0$. For conservative DMs, $0 < \tau^- < \tau^+ < 1$; for neutral DMs, $0 < \tau^- = \tau^+ < 1$; for adventurous DMs, $0 < \tau^+ < \tau^- < 1$.

The prospect value is calculated as follows.

$$V = \sum_{b=1}^{B} v(\Delta(f_m, h_n)) \omega(s_b; f_m, h_n)$$

(5)

### 2.3 Grey correlation analysis

Deng [30] firstly proposed grey system theory in 1983. Liu [31] described the steps of MADM using GRA method in detail.

Assume that the $j$th attribute value of the $i$th alternative is $g_{ij} (i = 1, 2, \ldots, m; j = 1, 2, \ldots, n)$, the weight vector of the attributes is $\omega = (\omega_1, \omega_2, \ldots, \omega_n)^T$ with $\omega_j \in [0, 1], (j = 1, 2, \ldots, n), \sum_{j=1}^{n} \omega_j = 1$. The decision steps are shown as follows.

1. Step 1. Get a normalized decision matrix $G = (g_{ij})_{m \times n}$.

2. Step 2. Determining the ideal solution and negative ideal solution as follows.

$$G^+_j = \max_i (g_{ij}), G^-_j = \min_i (g_{ij}) \quad j = 1, 2, \ldots, n$$

(6)
Step 3. Calculate the grey correlation degree between the $i_{th}$ and ideal solution on the $j_{th}$ index.

3-1. Firstly, calculate the grey correlation coefficient.

$$q^+_i = \frac{m + \eta M}{\Delta^+_j + \eta M}, \eta \in (0,1)$$  \hspace{1cm} (7)

$$\Delta^+_j = |G_j - g_j|, m = \min \Delta^+_j, M = \max \Delta^+_j, \eta \text{ is the coefficient, we usually take } \eta = 0.5.$$  

3-2. Then the coefficient matrix $Q^+$ of each alternative and the ideal solution is given as follows.

$$Q^+ = \begin{pmatrix} q^+_{11} & q^+_{12} & \cdots & q^+_{1m} \\ q^+_{21} & q^+_{22} & \cdots & q^+_{2m} \\ \vdots & \vdots & \ddots & \vdots \\ q^+_{n1} & q^+_{n2} & \cdots & q^+_{nm} \end{pmatrix}$$  \hspace{1cm} (8)

3-3. The grey correlation degree between each alternative and the ideal solution is $Q^+_i \ (i = 1,2,\cdots m)$.

$$Q^+_i = \sum_{j=1}^{n} \omega_j q^+_{ij} \ (i = 1,2,\cdots m)$$  \hspace{1cm} (9)

Step 4. Calculate the grey correlation degree of the $i_{th}$ and negative ideal solution on the $j_{th}$ index.

4-1. Firstly calculate the grey correlation coefficient.

$$q^-_i = \frac{m + \eta M}{\Delta^-_j + \eta M}, \eta \in (0,1)$$  \hspace{1cm} (10)

$$\Delta^-_j = |G_j - g_j|, m = \min \Delta^-_j, M = \max \Delta^-_j, \eta \text{ is the coefficient, we usually take } \eta = 0.5.$$  

4-2. Then the coefficient matrix of each alternative and the negative ideal solution is shown in $Q^-$.

$$Q^- = \begin{pmatrix} q^-_{11} & q^-_{12} & \cdots & q^-_{1m} \\ q^-_{21} & q^-_{22} & \cdots & q^-_{2m} \\ \vdots & \vdots & \ddots & \vdots \\ q^-_{n1} & q^-_{n2} & \cdots & q^-_{nm} \end{pmatrix}$$  \hspace{1cm} (11)

4-3. Finally, the grey correlation degree between each alternative and the negative ideal solution is $Q^-_i \ (i = 1,2,\cdots m)$.

$$Q^-_i = \sum_{j=1}^{n} \omega_j q^-_{ij} \ (i = 1,2,\cdots m)$$  \hspace{1cm} (12)

Step 5. Calculate the relative closeness of each alternative.
\[ C_i = \frac{Q_i'}{Q_i' + Q_i} (i = 1, 2, \cdots, m) \]

Step 6. Rank all alternatives.

Rank all alternatives according to the relative closeness. The better alternative with the higher relative closeness.

3. Transformation method of belief structure

Shortliffe and Buchanan [37] proposed certainty factor for MYCIN expert system. Based on the MYCIN certainty factor, a new form called belief structure is proposed by Jin [27]. Belief structures are used to describe the uncertainty of events and the uncertainty of human perceptions.

**Definition 3.** [27] Suppose there is an event \( \lambda \), which is given a value of \([0,1]\) according to subjective cognition or objective analysis, indicating that the event \( \lambda \) is true. This value is called the belief degree of \( \lambda \), recorded as \( cd(\lambda) \).

**Definition 4.** [38] Assuming that there is an event \( \lambda \), the belief degree of \( \lambda \) is \( cd(\lambda) \in [0,1] \), then \( (\lambda, cd(\lambda)) \) is called the belief structure. \( \lambda \) is also called the identity value.

Identity value can be numerical, fuzzy numbers, linguistic variables, etc., and Jin et al. [27] has proposed belief structure transformation methods of real numbers, interval numbers, IFNs, and linguistic variables. But how the LIFNs translate into belief structure has not yet been proposed. For any LIFN \( \gamma = (s_p, s_q) \in \Gamma_{[0,1]} \), it consists of two parts: MD and N-MD. But identity value in the belief structure is determined by a number. The degree of certainty can be measured by the similarity (already proved in [38]), and the method of transforming the interval value is also proposed. According to the above two points, the method of transforming the LIFNs is proposed.

1. Identity value

First, according to **remark 1**, we change the LIFNs into ULVs. \( \gamma = (s_p, s_q) \) becomes \( \gamma' = [s_p, s_{-q}] \).

Then use the linguistic scale function to convert the ULVs into interval numbers. Based on LSF \( f(s) = \theta_i = \frac{i}{t} (i = 1, 2, \cdots, t) \), \( \gamma' = [s_p, s_{-q}] \) converts into \( \gamma'' = [\frac{p}{t}, \frac{q}{t}] \).

Finally, we find the midpoint value of the interval number, which is the belief degree.

\[ \lambda = \frac{p - q + t}{2t} \]

2. Belief degree

The degree of certainty can be measured by the similarity (already proved in [38]), and the similarity can be obtained by the distance formula, so we first calculate the distance of the LIFNs. Here the distance is defined as the difference between the attribute values \( \gamma = (s_p, s_q) \) and the optimal
attribute value \( \gamma' = (s_m, s_n) \). For benefit attributes, the optimal value is \( \gamma' = (s_i, s_j) \), for cost attributes, we can convert them to a benefit type. Liu et al. [39] proposed a distance formula for LIFNs.

Let \( \gamma_1 = (s_{p1}, s_{q1}), \gamma_2 = (s_{p2}, s_{q2}) \in \Gamma_{[0,1]} \), then the distance between \( \gamma_1 \) and \( \gamma_2 \) is shown as follows:

\[
d(\gamma_1, \gamma_2) = \frac{|p_1 - p_2| + |q_1 - q_2|}{2t}
\]

Liu's method has some minor drawbacks, for example, \( \gamma_1 = (s_i, s_j), \gamma_2 = (s_i, s_k), \gamma_3 = (s_i, s_l) \), \( t = 8 \), then \( d(\gamma_1, \gamma_2) = d(\gamma_1, \gamma_3) = 0.375 \). Obviously, this is not reasonable because it does not consider the hesitation of LIFNs. Referring to the Hamming distance of the IFNs proposed by Szmidt [40], we propose a distance formula for the LIFNs.

**Definition 5.** Let \( \gamma_1 = (s_{p1}, s_{q1}), \gamma_2 = (s_{p2}, s_{q2}) \in \Gamma_{[0,1]} \) be any two LIFNs, \( s_i \) and \( s_j \) represent the degree of hesitation of \( \gamma_1 \) and \( \gamma_2 \) respectively, \( \pi_1 = t - p_1 - q_1, \pi_2 = t - p_2 - q_2 \), then the distance between \( \gamma_1 \) and \( \gamma_2 \) is

\[
d(\gamma_1, \gamma_2) = \frac{|p_1 - p_2| + |q_1 - q_2| + |\pi_1 - \pi_2|}{2t}
\]

According to (16), we can calculate the similarity between the required attribute value \( \gamma = (s_r, s_s) \) and the optimal attribute value \( \gamma' = (s_m, s_n) \) as

\[
S_{(\gamma', \gamma)} = 1 - d(\gamma, \gamma') = 1 - \frac{|p - m| + |q - n| + |m - p + n - q|}{2t}
\]

and this is the belief degree. 

**Example 1.** In a MADM problem, there is a benefit-type attribute \( c_i \), and the evaluation value of \( c_i \) is \( \gamma_i = (s_i, s_j) \), \( t = 8 \). Next we convert it into the belief structure \( (\lambda, \text{cd}(\lambda)) \).

For identity value, \( \lambda = \frac{p - q + t}{2t} = \frac{4 - 3 + 8}{2 \times 8} = 0.5625 \); for belief degree, the optimal attribute value \( \gamma' = (s_i, s_j) \), so \( S_{(\gamma', \gamma)} = 1 - d(\gamma, \gamma') = 1 - \frac{|4 - 8| + |3 - 0| + |8 - 4 + 0 - 3|}{2 \times 8} = 0.5 \). Then the final result is \( (\lambda, \text{cd}(\lambda)) = (0.5625, 0.5) \).

4 A MADM method based on prospect theory and grey correlation analysis

In this section, we will present a method to solve the MADM problem described by the form of LIFNs based on prospect theory, grey correlation degree and belief structure. The specific method is described as follows.

Suppose \( A = \{a_1, a_2, \ldots, a_m\} \) is a set of alternatives, and \( C = \{c_1, c_2, \ldots, c_n\} \) is a collection of attributes. The weight vector of the attributes is \( \omega = (\omega_1, \omega_2, \ldots, \omega_n)^T \) with \( \omega_j \in [0,1](j = 1, 2, \ldots, n) \),
\[ \sum_{j=1}^{n} \omega_j = 1. \] Assume that there are three natural states in the process of selecting a plan: good, medium and poor, and different states will have an impact on the final benefit. The three states are represented by \( ST = \{st_1, st_2, st_3\} \), and the probability of occurrence is \( W = \{w_1, w_2, w_3\} \), and satisfying \( w_1, w_2, w_3 \in [0, 1] \) and \( w_1 + w_2 + w_3 = 1 \). In state \( st_b \), each attribute value \( c_j \) of each alternative \( a_i \) is represented by the form of LIFN \( \gamma_{ij}^b = (s_{ij}^b, s_{ij}^b) \) (\( i = 1, 2, \cdots, m; j = 1, 2, \cdots, n; b = 1, 2, 3 \)), then the decision matrix \( R'_b = [\gamma_{ij}^b]_{m \times n} \) \( (b = 1, 2, 3) \) is constructed. The decision goal is to give a ranking of all alternatives.

In the following, we will give the decision-making steps.

**Step 1:** Normalize the decision matrix.

Because there are two types of attributes, cost or benefit types, we must first convert the cost type into benefit type. The standardized matrix can be given as \( \tilde{R}_b = [\tilde{\gamma}_{ij}^b]_{m \times n} \) \( (b = 1, 2, 3) \). The specific conversion method is shown as follows:

\[
\tilde{\gamma}_{ij}^b = \begin{cases} 
(s_{ij}^b, s_{ij}^b) & \text{for benefit-type attribute } c_j \\
(s_{ij}^b, s_{ij}^b) & \text{for cost-type attribute } c_j
\end{cases}
\]  \( (18) \)

**Step 2:** Convert standardized LIFNs into belief structure.

Convert LIFNs to belief structure according to the formulas (14) and (17) described in Section 3, and the decision matrix is changed from \( \tilde{R}_b = [\tilde{\gamma}_{ij}^b]_{m \times n} = \left[ \begin{array}{cccc} s_{11}^b & \cdots & s_{1n}^b \\ \vdots & \ddots & \vdots \\ s_{m1}^b & \cdots & s_{mn}^b \end{array} \right] \) to \( R_b = \left[ (\lambda_{ij}^b, cd(\lambda_{ij}^b)) \right]_{m \times n} \). \( \lambda_{ij}^b \) represents the identity value of the attribute \( c_j \) of the alternative \( a_i \) in the state \( st_b \). \( cd(\lambda_{ij}^b) \) denotes the degree to which the attribute \( c_j \) of the alternative \( a_i \) in the state \( st_b \) takes the value \( \lambda_{ij}^b \).

**Step 3:** Calculate the prospect matrix.

3-1. Get belief structure attribute values that take into account future state uncertainties.

Because the belief degree \( cd(\lambda_{ij}^b) \) reflects the uncertainty of the attribute value, and the probability \( w_b \) represents the uncertainty of the future state, and they are independent of each other, we use their product to combine the uncertainty of the attribute value and the uncertainty of the future state [2] The calculation formula is shown as follows:

\[
\text{cd}(\lambda_{ij}^b)^\ast = \text{cd}(\lambda_{ij}^b) \times w_b \ (b = 1, 2, 3)
\]  \( (19) \)

The belief structure is changed from \( (\lambda_{ij}^b, \text{cd}(\lambda_{ij}^b)) \) to \( (\lambda_{ij}^b, \text{cd}(\lambda_{ij}^b)^\ast) \).

3-2. Calculate the value function according to formula (1).

3-3. Calculate the decision weights according to formulas (2) (3) (4).

3-4. Calculate the prospect value according to formula (5), and obtain the prospect matrix.

**Step 4:** The final ordering of the alternatives is obtained by using the grey correlation method.
4-1. Calculate the ideal solution and negative ideal solution of the prospect matrix.

4-2. Calculate the grey correlation degree of the $i$th and ideal solution on the $j$th index.

4-3. Calculate the grey correlation degree of the $i$th and negative ideal solutions on the $j$th index.

4-4. Calculate the relative closeness of each alternative and get the final order. The higher the relative closeness, the better the alternative is.

5. An illustrate example

5.1 decision-making problem

In this part, we will solve a practical problem using the method presented in the previous section. Suppose an investment company wants to select an investment target from four candidate companies $A = \{ a_1, a_2, a_3, a_4 \}$ which are evaluated from five aspects $C = \{ c_1, c_2, c_3, c_4, c_5 \}$: economic benefits, risk controllable analysis, social impact analysis, company policy, and development sustainability. The attribute weights are equal, that is $\omega_1 = \omega_2 = \omega_3 = \omega_4 = \omega_5 = 0.2$. In the investment process, there are three possible natural states $ST = \{ st_1, st_2, st_3 \}$, which are good, medium, and poor, and the probability of occurrence is $W = \{ w_1, w_2, w_3 \}$, and specifically $w_1 = 0.3, w_2 = 0.5, w_3 = 0.2$. Based on the linguistic set $S = \{ s_0 = \text{extremely poor}, s_1 = \text{very poor}, s_2 = \text{poor}, s_3 = \text{slightly poor}, s_4 = \text{fair}, s_5 = \text{slightly good}, s_6 = \text{good}, s_7 = \text{very good}, s_8 = \text{extremely good} \}$, in different natural states, the evaluation values of each attribute of each company are different, and are expressed by the form of LIFNs. The decision matrix $R^* = \left[ \gamma^b_{ij} \right]_{mn}$ ($b = 1, 2, 3$) is shown in tables 1-3.

Table 1 Decision matrix $R^*_1$

Table 2 Decision matrix $R^*_2$

Table 3 Decision matrix $R^*_3$

Next we will give a specific calculation process.

Step 1: standardize the evaluation matrix.

Because all attributes are benefit type, we don’t need to standardize the matrix $R^*_1 \sim R^*_3$.

Step 2: Convert standardized LIFNs into belief structure.

The identity value and the belief degree are transformed separately using Equations 14 and 17. The matrix $R^*_1 \sim R^*_3$ are transformed from the LIFNs to the form of belief structure. The details are shown in Tables 4-6.
Step 3: Calculate the prospect matrix.

3-1. Belief structure is calculated by formula (19).

The probability of occurrence of the three states is $w_1 = 0.3, w_2 = 0.5, w_3 = 0.2$. Considering the probability of occurrence of the natural state, we get the new belief structures shown in tables 7-9.

3-2. Use formula (1) to get the value function.

We believe that the reference points of each attribute in the same state are the same, and give the reference points of each state after the transformation. $h(st_1) = 0.8125, h(st_2) = 0.75, h(st_3) = 0.6875$. Suppose decision makers are neutral and are more sensitive to losses than earnings, then we use parameters $\xi^+ = 1, \xi^- = 2.25, u = 1, v = 1$ [41] to obtain the value function shown in table 10.

3-3. Calculate the decision weights according to formulas (2)-(4).

We need to sort the alternatives in descending order according to the size of the value function, and then calculate the decision weights shown in table 11. Where we use the parameters $\xi^+ = 1, \xi^- = 1, r^+ = 0.604, r^- = 0.604$ given by Prelec [42].

3-4. Calculate the prospect value according to formula (5), and the prospect matrix is shown in table 12.

Step 4: The final ordering of the alternatives is obtained by using the grey correlation method.

4-1. Determining the ideal solution and negative ideal solution.

$$G^+ = (0.0453, -0.0575, 0.0759, 0.0570, -0.0861)$$

$$G^- = (-0.0865, -0.2319, -0.1633, -0.1976, -0.2783)$$

4-2. Calculate the grey correlation degree of the $i_a$ and ideal solutions on the $f_a$ index.
\[Q^* = (0.6302, 0.8602, 0.6276, 0.6801)\]

4.3. Calculate the grey correlation degree of the \(i_{th}\) and negative ideal solutions on the \(j_{th}\) index.

\[Q^* = (0.6635, 0.4375, 0.6454, 0.6528)\]

4.4. Calculate the relative closeness of each alternative and get the final order.

After calculation, the relative closeness of each alternative is \(C_a = 0.4871\), \(C_a = 0.6629\), \(C_a = 0.4930\), and \(C_a = 0.5103\), respectively, and the final ordering is \(a_2 > a_4 > a_3 > a_1\), the optimal solution is \(a_2\).

In order to better explain the influence of the decision makers’ type on final decision results, we change the parameters of the value function and weight function and recalculate this example. The six representative combinations of parameters are obtained as follows and shown in table 13.

The value function parameters \(\varepsilon^- = 1, \varepsilon^+ = 2.25, \mu = 0.89, \nu = 0.92\) were given by Tversky and Kahneman [17]. Xu [43] got \(\varepsilon^- = 1, \varepsilon^+ = 1.51, \mu = 0.37, \nu = 0.59\). Zeng [20] obtained the parameters \(\varepsilon^- = 1, \varepsilon^+ = 2.25, \mu = 1.21, \nu = 1.02\) through experiments. The four different weight function parameters \(\xi^+ = 0.938, \xi^- = 0.9381, \tau^+ = 0.603, \tau^- = 0.605\); \(\xi^+ = 1.083, \xi^- = 1.083, \tau^+ = 0.533, \tau^- = 0.535\); \(\xi^+ = 0.938, \xi^- = 0.938, \tau^+ = 0.605, \tau^- = 0.603\); \(\xi^+ = 1.083, \xi^- = 1.083, \tau^+ = 0.535, \tau^- = 0.533\) are the research results of Prelec [42] and Bleichrodt et al. [44].

Table 13 Representative parameter combination

According to the decision method given above, six kinds of parameter combinations are respectively used, and the final orders are obtained as shown in Table 14.

Table 14 Calculation results and ranking

According to Tables 13 and 14, we find that the ordering obtained by the MADM method proposed in this paper is basically the same for different parameters. Specifically, the results obtained by the five combinations are the same as the previous calculation, which are \(a_2 > a_4 > a_3 > a_1\). When the parameters are \(\varepsilon^- = 1, \varepsilon^+ = 1.51, \mu = 0.37, \nu = 0.59\), the ordering becomes \(a_2 > a_3 > a_4 > a_1\), that is, the two alternatives in the middle of the ranking exchange positions. But the optimal solution is unchanged. We believe that the ordering is stable and has certain reference value.

5.2 Comparison with other methods

In this section, we will use the other two methods to calculate the examples given above and analyze the orderings. The first comparison method is a stochastic intuitionistic fuzzy decision-making method based on PT proposed by Li et al. [45], and the second method is to use the dynamic multiple-attribute grey correlation decision model proposed by Dang et al. [46].

Because Li’s method [45] uses the form of IFNs, and our example gives the evaluation matrix in the form of LIFNs, we must use the LSF (introduced in section 2.1) to convert the LIFNs into IFNs before the calculation. What’s more, we take the parameter values of the PT formula appearing in Li’s paper [45] as \(\gamma = 0.604, \delta = 1.21, \beta = 1.02, \sigma = 2.25\). Dang’s method [46] only use the evaluation value as real number. The identity value in the belief structure mentioned in this paper is the real number obtained through subjective cognition or objective evaluation, so we use the belief structure transformation method to convert the LIFNs into real numbers. The final calculation results and ordering are shown in the table 15.
Next we will conduct a detailed analysis for the results in Table 15.

The ordering of Li’s method [45] is roughly the same as that obtained by the method proposed in this paper. Specifically, the ordering of Li is $a_2 > a_1 > a_4 > a_3$, and our ordering is $a_1 > a_2 > a_3 > a_4$. The optimal solution obtained by the two methods is the same, both are $a_2$, which can explain the effectiveness of the proposed method, and the advantages of the proposed method are shown as follows. (1) Li’s method [45] is to evaluate the alternatives by IFNs, and we use the LIFNs. In real life, it is difficult to evaluate some alternatives with accurate numbers, and it is much more convenient to use LIFNs. When evaluating qualitative attributes, Li’s method is difficult to do, and our method is much easier. (2) Although both methods use PT, the method in this paper uses the 3-PT. The reference point of the 3-PT is dynamic, and the reference points in different states can be different. In Li’s method, the reference points in different cases are the same, and the value is zero. So we think our approach is more reasonable. (3) Li’s method uses only PT, and our method combines PT with GRA method, which is more reasonable in the final ranking result. In summary, the proposed method is reasonable and superior.

The order obtained by the method in [46] is the same as that obtained in this paper, which can explain the rationality of the proposed method. The superiority of the method in this paper is mainly manifested in two aspects. (1) Dang et al.’s method [46] solves the problem of MADM in which the information form is real numbers, but in the real problems, many qualitative attributes cannot be evaluated with accurate figures. The method proposed in this paper solves the MADM problem in which the information form is LIFNs, and the scope of application is wider. Using the LSF, the LIFNs can be converted into the IFNs. When the NMD is zero, the IFNs is converted into real numbers. Therefore, compared with the method of Dang et al. [46], our method can deal with a variety of information forms of MADM problems, and the scope of adaptation is wider. (2) The second point is that Dang et al.’s method [46] only uses the GRA method to evaluate the dynamic multi-indicator problem, and does not use the 3-PT. Considering that the DM is a limited rational person, it is not always to pursue the effect maximization, but to choose a satisfactory plan according to the actual situation. Our method uses the 3-PT to consider the actual situation of the difference in reference points in different natural states. At the same time, different parameters can be selected according to the sensitivity of decision makers to risks and benefits. At this point our approach is also more reasonable.

In summary, by comparing with two existing methods, the method proposed in this paper is reasonable and superior.

6. Conclusion

In this paper, we propose a MADM method based on 3-PT and GRA to solve the problem with dynamic reference points. First, we use the proposed transformation method to convert the LIFNs into a form of belief structure. Second, the decision matrix in different states is calculated by using the formula of 3-PT to obtain a prospect matrix. Finally, the GRA method is used to calculate the prospect matrix, and the alternatives are ranked. The effectiveness and superiority of the proposed method are proved by comparison with the other two methods. On the one hand, the form of information we use is the LIFNs, which is convenient for DMs to evaluate qualitative attributes from both MD and NMD. And LIFNs is a generalized form of information that can be transformed into other forms. On the other hand, the 3-PT is adopted in our method, which takes into account the limited rationality of DMs and dynamic reference point, which is more in line with the actual situation. In the future, we hope that our
method can be applied to practical problems such as medical diagnosis, supplier selection, or extending methods from MADM to multi-attribute group decision making [47,48].

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Table 1 Decision matrix $R^i_j$

|      | $c_1$ | $c_2$ | $c_3$ | $c_4$ | $c_5$ |
|------|-------|-------|-------|-------|-------|
| $a_1$ | $(s_7, s_8)$ | $(s_6, s_2)$ | $(s_5, s_1)$ | $(s_6, s_1)$ | $(s_5, s_2)$ |
| $a_2$ | $(s_6, s_2)$ | $(s_5, s_2)$ | $(s_6, s_1)$ | $(s_6, s_2)$ | $(s_7, s_1)$ |
| $a_3$ | $(s_5, s_1)$ | $(s_6, s_1)$ | $(s_7, s_1)$ | $(s_5, s_2)$ | $(s_6, s_2)$ |
\begin{table}[h]
\centering
\begin{tabular}{|c|c|c|c|c|c|}
\hline
\(a_i\) & \(s_1, s_2\) & \(s_2, s_3\) & \(s_3, s_4\) & \(s_4, s_1\) & \(s_5, s_5\) \\
\hline
\(a_1\) & \(s_1, s_1\) & \(s_2, s_2\) & \(s_3, s_3\) & \(s_4, s_4\) & \(s_5, s_5\) \\
\hline
\(a_2\) & \(s_1, s_2\) & \(s_2, s_3\) & \(s_3, s_4\) & \(s_4, s_5\) & \(s_5, s_5\) \\
\hline
\(a_3\) & \(s_1, s_3\) & \(s_2, s_4\) & \(s_3, s_5\) & \(s_4, s_1\) & \(s_5, s_5\) \\
\hline
\(a_4\) & \(s_1, s_4\) & \(s_2, s_5\) & \(s_3, s_1\) & \(s_4, s_2\) & \(s_5, s_3\) \\
\hline
\end{tabular}
\caption{Decision matrix \(R_1\)}
\end{table}

\begin{table}[h]
\centering
\begin{tabular}{|c|c|c|c|c|c|}
\hline
\(a_i\) & \(s_1, s_1\) & \(s_2, s_2\) & \(s_3, s_3\) & \(s_4, s_4\) & \(s_5, s_5\) \\
\hline
\(a_1\) & \(s_1, s_2\) & \(s_2, s_3\) & \(s_3, s_4\) & \(s_4, s_5\) & \(s_5, s_5\) \\
\hline
\(a_2\) & \(s_1, s_3\) & \(s_2, s_4\) & \(s_3, s_5\) & \(s_4, s_1\) & \(s_5, s_5\) \\
\hline
\(a_3\) & \(s_1, s_4\) & \(s_2, s_5\) & \(s_3, s_1\) & \(s_4, s_2\) & \(s_5, s_3\) \\
\hline
\end{tabular}
\caption{Decision matrix \(R_2\)}
\end{table}

\begin{table}[h]
\centering
\begin{tabular}{|c|c|c|c|c|c|}
\hline
\(a_i\) & \(c_1\) & \(c_2\) & \(c_3\) & \(c_4\) & \(c_5\) \\
\hline
\(a_1\) & (0.875,0.875) & (0.75,0.75) & (0.8125,0.75) & (0.875,0.875) & (0.6875,0.625) \\
\hline
\(a_2\) & (0.75,0.75) & (0.6875,0.625) & (0.8125,0.75) & (0.75,0.75) & (0.875,0.875) \\
\hline
\(a_3\) & (0.8125,0.75) & (0.625,0.625) & (0.875,0.875) & (0.6875,0.625) & (0.75,0.75) \\
\hline
\(a_4\) & (0.6875,0.625) & (0.875,0.875) & (0.625,0.625) & (0.8125,0.75) & (0.75,0.75) \\
\hline
\end{tabular}
\caption{Decision matrix \(R_4\)}
\end{table}

\begin{table}[h]
\centering
\begin{tabular}{|c|c|c|c|c|c|}
\hline
\(a_i\) & \(c_1\) & \(c_2\) & \(c_3\) & \(c_4\) & \(c_5\) \\
\hline
\(a_1\) & (0.875,0.875) & (0.5,0.5) & (0.75,0.75) & (0.6875,0.625) & (0.375,0.375) \\
\hline
\(a_2\) & (0.875,0.875) & (0.75,0.625) & (0.8125,0.75) & (0.6875,0.625) & (0.5625,0.5) \\
\hline
\(a_3\) & (0.6875,0.625) & (0.8125,0.75) & (0.875,0.875) & (0.625,0.625) & (0.5,0.5) \\
\hline
\(a_4\) & (0.75,0.75) & (0.5625,0.5) & (0.6875,0.625) & (0.875,0.875) & (0.625,0.625) \\
\hline
\end{tabular}
\caption{Decision matrix \(R_5\)}
\end{table}

\begin{table}[h]
\centering
\begin{tabular}{|c|c|c|c|c|c|}
\hline
\(a_i\) & \(c_1\) & \(c_2\) & \(c_3\) & \(c_4\) & \(c_5\) \\
\hline
\(a_1\) & (0.625,0.625) & (0.5,0.5) & (0.875,0.875) & (0.75,0.625) & (0.625,0.5) \\
\hline
\(a_2\) & (0.8125,0.75) & (0.75,0.75) & (0.8125,0.75) & (0.6875,0.625) & (0.8125,0.75) \\
\hline
\(a_3\) & (0.6875,0.625) & (0.4375,0.375) & (0.75,0.75) & (0.5,0.375) & (0.6875,0.625) \\
\hline
\(a_4\) & (0.5625,0.5) & (0.75,0.625) & (0.625,0.5) & (0.75,0.75) & (0.6875,0.625) \\
\hline
\end{tabular}
\caption{Decision matrix \(R_6\)}
\end{table}

\begin{table}[h]
\centering
\begin{tabular}{|c|c|c|c|c|c|}
\hline
\(a_i\) & \(c_1\) & \(c_2\) & \(c_3\) & \(c_4\) & \(c_5\) \\
\hline
\(a_1\) & (0.875,0.2625) & (0.75,0.225) & (0.8125,0.225) & (0.875,0.2625) & (0.6875,0.1875) \\
\hline
\(a_2\) & (0.75,0.225) & (0.6875,0.1875) & (0.8125,0.225) & (0.75,0.225) & (0.875,0.2625) \\
\hline
\(a_3\) & (0.8125,0.225) & (0.625,0.1875) & (0.875,0.2625) & (0.6875,0.1875) & (0.75,0.225) \\
\hline
\(a_4\) & (0.6875,0.1875) & (0.875,0.2625) & (0.625,0.1875) & (0.8125,0.225) & (0.75,0.225) \\
\hline
\end{tabular}
\caption{Belief structure matrix under \(s_i\)}
\end{table}
Table 8 Belief structure matrix under $M_2$

| $\alpha_i$ | $c_1$   | $c_2$   | $c_3$   | $c_4$   | $c_5$   |
|------------|---------|---------|---------|---------|---------|
| $\alpha_1$ | (0.875,0.4375) | (0.5,0.25) | (0.75,0.375) | (0.6875,0.3125) | (0.375,0.1875) |
| $\alpha_2$ | (0.875,0.4375) | (0.75,0.3125) | (0.8125,0.375) | (0.6875,0.3125) | (0.5625,0.25) |
| $\alpha_3$ | (0.6875,0.3125) | (0.8125,0.375) | (0.875,0.4375) | (0.625,0.3125) | (0.5,0.25) |
| $\alpha_4$ | (0.75,0.375) | (0.5625,0.25) | (0.6875,0.3125) | (0.875,0.4375) | (0.625,0.3125) |

Table 9 Belief structure matrix under $M_3$

| $\alpha_i$ | $c_1$ | $c_2$ | $c_3$ | $c_4$ | $c_5$ |
|------------|-------|-------|-------|-------|-------|
| $\alpha_1$ | (0.625,0.125) | (0.5,0.1) | (0.875,0.175) | (0.75,0.125) | (0.625,0.1) |
| $\alpha_2$ | (0.8125,0.15) | (0.75,0.15) | (0.8125,0.15) | (0.6875,0.125) | (0.8125,0.15) |
| $\alpha_3$ | (0.6875,0.125) | (0.4375,0.075) | (0.75,0.15) | (0.5,0.075) | (0.6875,0.125) |
| $\alpha_4$ | (0.5625,0.1) | (0.75,0.125) | (0.625,0.1) | (0.75,0.15) | (0.6875,0.125) |

Table 10 Value function matrix

| $c_i$ | $c_1$ | $c_2$ | $c_3$ | $c_4$ | $c_5$ |
|-------|-------|-------|-------|-------|-------|
| $\alpha_1$ | 0.0625 | -0.1406 | 0 | 0.0625 | -0.2813 |
| $\alpha_2$ | 0.125 | -0.5625 | 0 | -0.1406 | -0.8438 |
| $\alpha_3$ | -0.1406 | -0.4219 | 0.1875 | 0.0625 | -0.1406 |
| $\alpha_4$ | -0.0625 | -0.2813 | 0 | -0.1406 | 0.0625 |

Table 11 Decision weight matrix

| $c_i$ | $c_1$ | $c_2$ | $c_3$ | $c_4$ | $c_5$ |
|-------|-------|-------|-------|-------|-------|
| $\alpha_1$ | 0.1688 | 0.2799 | 0.1413 | 0.3036 | 0.1280 |
| $\alpha_2$ | 0.4101 | 0.2958 | 0.3722 | 0.3343 | 0.2554 |
| $\alpha_3$ | 0.2110 | 0.0620 | 0.2469 | 0.0757 | 0.1911 |
| $\alpha_4$ | 0.2799 | 0.2554 | 0.2799 | 0.1461 | 0.1608 |
| $\alpha_5$ | 0.4101 | 0.1933 | 0.2336 | 0.3343 | 0.2958 |
| $\alpha_6$ | 0.0922 | 0.2294 | 0.2294 | 0.2110 | 0.2294 |
| $\alpha_7$ | 0.2799 | 0.1164 | 0.1655 | 0.2554 | 0.1461 |
| $\alpha_8$ | 0.3343 | 0.3722 | 0.4101 | 0.1933 | 0.2958 |
| $\alpha_9$ | 0.0757 | 0.1692 | 0.2294 | 0.1692 | 0.2110 |
| $\alpha_{10}$ | 0.1164 | 0.3036 | 0.2554 | 0.2799 | 0.1461 |
| $\alpha_{11}$ | 0.3722 | 0.2958 | 0.3343 | 0.4101 | 0.3343 |
| $\alpha_{12}$ | 0.1911 | 0.0764 | 0.0605 | 0.0922 | 0.2110 |
### Table 12 Prospect matrix

| $\alpha_1$ | $c_1$ | $c_2$ | $c_3$ | $c_4$ | $c_5$ |
|-----------|------|------|------|------|------|
| $\alpha_1$ | 0.0321 | -0.2319 | 0.0463 | -0.0233 | -0.2783 |
| $\alpha_2$ | 0.0453 | -0.0575 | 0.0433 | -0.0676 | -0.0861 |
| $\alpha_3$ | -0.0470 | -0.1210 | 0.0759 | -0.1976 | -0.1869 |
| $\alpha_4$ | -0.0865 | -0.1010 | -0.1633 | 0.0570 | -0.1146 |

### Table 13 Representative parameter combination

| Parameter of value function | Parameter of weight function |
|----------------------------|------------------------------|
| $\varepsilon^+ = 1, \varepsilon^- = 2.25$ | $\xi^+ = 0.938, \xi^- = 0.938$ |
| $u = 0.89, v = 0.92$ | $\tau^+ = 0.603, \tau^- = 0.605$ |

| Parameter of value function | Parameter of weight function |
|----------------------------|------------------------------|
| $\varepsilon^+ = 1, \varepsilon^- = 2.25$ | $\xi^+ = 1.083, \xi^- = 1.083$ |
| $u = 0.89, v = 0.92$ | $\tau^+ = 0.535, \tau^- = 0.535$ |

| Parameter of value function | Parameter of weight function |
|----------------------------|------------------------------|
| $\varepsilon^+ = 1, \varepsilon^- = 2.25$ | $\xi^+ = 0.938, \xi^- = 0.938$ |
| $u = 1.21, v = 1.02$ | $\tau^+ = 0.605, \tau^- = 0.603$ |

| Parameter of value function | Parameter of weight function |
|----------------------------|------------------------------|
| $\varepsilon^+ = 1, \varepsilon^- = 2.25$ | $\xi^+ = 1.083, \xi^- = 1.083$ |
| $u = 1.21, v = 1.02$ | $\tau^+ = 0.535, \tau^- = 0.533$ |

### Table 14 Calculation results and ranking

| Parameter of value function | Parameter of weight function |
|----------------------------|------------------------------|
| $C_n = 0.4939$ | $C_n = 0.6503$ |
| $C_n = 0.4961$ | $C_n = 0.5093$ |
| $a_1 > a_4 > a_1 > a_1$ |

| Parameter of value function | Parameter of weight function |
|----------------------------|------------------------------|
| $C_n = 0.4944$ | $C_n = 0.6507$ |
| $C_n = 0.4951$ | $C_n = 0.5091$ |
| $a_2 > a_1 > a_1 > a_1$ |

| Parameter of value function | Parameter of weight function |
|----------------------------|------------------------------|
| $C_n = 0.4989$ | $C_n = 0.6604$ |
| $C_n = 0.5018$ | $C_n = 0.5217$ |
| $a_2 > a_1 > a_1 > a_1$ |

| Parameter of value function | Parameter of weight function |
|----------------------------|------------------------------|
| $C_n = 0.4995$ | $C_n = 0.6604$ |
| $C_n = 0.5009$ | $C_n = 0.5215$ |
| $a_2 > a_1 > a_1 > a_1$ |

| Parameter of value function | Parameter of weight function |
|----------------------------|------------------------------|
| $C_n = 0.4868$ | $C_n = 0.6120$ |
| $C_n = 0.4914$ | $C_n = 0.4913$ |
| $a_2 > a_1 > a_1 > a_1$ |

| Parameter of value function | Parameter of weight function |
|----------------------------|------------------------------|
| $C_n = 0.4870$ | $C_n = 0.6130$ |
| $C_n = 0.4906$ | $C_n = 0.4911$ |
| $a_2 > a_1 > a_1 > a_1$ |

### Table 15 Ranking results by different methods

| Methods | Values | Ranking |
|---------|--------|---------|
| Li et al.’s method [45] based on PT | $W_1 = 0.3292, W_2 = 0.4778$ | $a_2 > a_4 > a_1 > a_1$ |
| | $W_3 = 0.3137, W_4 = 0.3554$ | |
Dang et al.’s method [46] based on grey correlation method

|  |  |  |  |
|---|---|---|---|
|  | $u_1 = 0.4226, u_4 = 0.6797$ | $a_2 > a_4 > a_3 > a_1$ |
|  | $u_1 = 0.4623, u_4 = 0.4819$ |  |

The method proposed in this paper

|  |  |  |  |
|---|---|---|---|
|  | $C_n = 0.4871, C_n = 0.6629$ | $a_2 > a_4 > a_3 > a_1$ |
|  | $C_n = 0.4930, C_n = 0.5103$ |  |

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