Upgrading geometry conceptual understanding and strategic competence through implementing rigorous mathematical thinking (RMT)

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Abstract. To reach higher order thinking skill, needed to be mastered the conceptual understanding and strategic competence as they are two basic parts of high order thinking skill (HOTS). RMT is a unique realization of the cognitive conceptual construction approach based on Feurstein with his theory of Mediated Learning Experience (MLE) and Vygotsky’s sociocultural theory. This was quasi-experimental research which compared the experimental class that was given Rigorous Mathematical Thinking (RMT) as learning method and the control class that was given Direct Learning (DL) as the conventional learning activity. This study examined whether there was different effect of two learning model toward conceptual understanding and strategic competence of Junior High School Students. The data was analyzed by using Multivariate Analysis of Variance (MANOVA) and obtained a significant difference between experimental and control class when considered jointly on the mathematics conceptual understanding and strategic competence (shown by Wilk’s $\Lambda = 0.84$). Further, by independent t-test is known that there was significant difference between two classes both on mathematical conceptual understanding and strategic competence. By this result is known that Rigorous Mathematical Thinking (RMT) had positive impact toward Mathematics conceptual understanding and strategic competence.

1. Introduction
Geometry is a branch of mathematic that comes from the needs of human beings. Basic ideas of Geometry, i.e lines, surfaces, angels, polygons, cubes, spheres, etc was taken from reality [1,2]. Geometry is an important part of mathematics curriculums [3]. Therefore, it seems necessary to raise geometry skill in learning mathematics. On learning of geometry needed the ability of visualization, verbalisation, picturing, logical and application. This was notion which caused most of students get difficulty in learning geometry.

Conceptual understanding was the comprehension which concerned with mathematical concept, operations, and relations [4,5]. The indicators of conceptual understanding based on previous discussion were (1) performing mathematical operations related to appropriate concepts; (2) classifying the objects based on whether the requirements to formulate the concept was fulfilled; (3) providing examples or non-examples of the concept learned; (4) representing mathematical concepts into different ways; (5) linking interrelated concepts; (6) developing the necessary terms or adequately terms of a concept.

Strategic competence was the comprehension which included problem solving and problem formulation which required performing mathematical, numerical, symbolic, oral, or graphical
representation to solve kind of problems. Strategic competence as strategic thinking which is selecting or devising, and implementing a mathematical strategy to solve problems arising from the task or context [5]. Thus, in this study, strategic competence is defined as a skill about the use of strategies in problem-solving activities, which includes understanding, representing, and solving a problem. The indicator of strategic competence consists of: (a) understand the problem, (b) represent the problem mathematically, symbolically, verbally, or graphically, (c) determining the right strategy to solve problems, (d) apply formulas or approaches to solve problems, (e) re-examine the solution obtained.

Based on National Examination in Indonesia, known that students experience difficulties at performing geometry task. It might because students had low conceptual understanding and strategic competence, which were started by students underwent difficulties in learning geometry. To help students overcome the difficulties in learning geometry an innovative learning activity need to be developed. This learning activity has to base on combining teaching strategies, through mediation, such as using mathematical and geometric language properly, including visual marking and identifying memory supports to prior knowledge and using it in the process of finding a solution [6].

Rigorous Mathematical Thinking (RMT) as the learning method was invented by James Kinard [7]. RMT is a learning theory which was developed by Kinard based on sociocultural theory by Vygotsky [8-10] about psychology tools [11] and Zone of Proximal Development (ZPD) [8] and mediated learning theory by Feurstein [12,13]. The practice of RMT focuses on mediating the learner in constructing robust cognitive process while building mathematical concept using the three phrases (i.e. cognitive development, content as process development, cognitive conceptual construction practice). Teacher-mediation take important part on RMT learning process as it contain language which is based on Vygotsky, language was both a cultural tool and psychological tool [14]. The goal of RMT as learning method is to equip the learner with the capacity and motivation to construct and apply deep mathematical conceptual understanding. Therefore, considering that mathematics achievement of Indonesian students’ was in the lower grade than the other countries, it was necessary to conduct the research in purpose to upgrade students’ mathematics skill particularly in conceptual understanding and strategic competence by implementing Rigorous Mathematical Thinking (RMT) as learning method. Thereby, this research focused on upgrading geometry conceptual understanding and strategic competence by implementing Rigorous Mathematics Thinking as learning method.

2. Method
This research used experimental method, which was comparing the effect of learning method between experimental class which used RMT as experimental learning method and control class which used DL as conventional learning method. The population was 7th-grade students of all Junior High School in Ngawi Regency. The selection of the population was based on the latest data which showed that Ngawi was regency with lowest rank of National Examination on Academic Year 2015/2016 in East Java, Indonesia.

The sample of this research was 79 students in experimental class and 86 students in control class. The sample was taken by using probability sampling and the technique used was stratified cluster random sampling. Stratified cluster random sampling was modification of cluster random sampling and stratified sampling. This technique was used because of the population was a part of classes and schools. Steps of sampling in this research consist of:

1. Categorizing all Junior High School in Ngawi in to 3 level i.e. low, medium, and high.
2. Choosing randomly 1 school in each level to obtain 3 schools as the object of research.
3. Choosing randomly 2 classes in each school obtained as the experimental class and the control class.

The learning activity conduct for 8 times meeting, and at the end of the meeting, students were given Geometry scholastic task which was represented the indicator of conceptual understanding and strategic competence. The data obtained were analyzed by using one-way Multivariate Analysis of Variance (one-way MANOVA). It was used to know whether there was difference mean value when considering jointly on the mathematics conceptual understanding and strategic competence. To know whether there was difference mean value in each dependent variable used independent t-test.
3. Result and Discussion
This research aims to know the implementation result of RMT as learning method toward conceptual understanding and strategic competence in geometry material. By the preface test known that the mean value of jointly conceptual understanding and strategic competence in each class was equal. It shows that both class had the same ability on conceptual understanding and strategic competence.

On learning process, students in experimental class were treated by using RMT as learning method. In the first phase (i.e cognitive development), teacher mediated the students to define the concept of area by using picture and asked the students to colour which part was the area of each shape (see Figure 1). Then, teacher asked students to identify which colour showed the shape’s area concepts (see Figure 2). Through these activities, students could imagine what the shape’s area is. At the end of this activity, teacher mediated students to define the concept of area. Teacher stimulated students to recall the idea based on the activities they did. Thus, they could define the area concept was the shape’s part which is bordered by its sides.

![Figure 1. RMT activity sheet of colouring the shape’s area](image)

The second phase (i.e content as process development), could be said as the basic stage because this concept would be used to construct the area concepts of others planes. In this phase, teacher had to mediate the students to construct their conceptual understanding of rectangular area. In this step, teacher had to mediate students to analyze the relation between rectangular area and the blue square (see Figure 3).

![Figure 2. RMT activity sheet of identifying the shape’s area](image)

![Figure 3. Basic concept of rectangle area](image)
Teacher mediated students to connect the amount of blue square needed and the area of rectangular by asking “How many blue square needed to cover the rectangular?” and “Is there any relationship between rectangular area and the amount of blue square to cover the rectangular?”. To develop the formula of rectangular area, teacher mediated students to connect the area of rectangular and the multiple of its sides, by asking “Is there any relation between the area of rectangular and the multiple of length and width side? thus, what is the relationship?”. Therefore, by this step students could appropriate mathematically specific psychological tools (i.e the formula of rectangular area).

In the next phase (i.e., cognitive conceptual construction practice), teacher mediated students to construct the formula area of other planes which was approached by rectangular area concept as psychological tools. Teacher mediated students by asking the relation between paralelogram area (see Figure 4.a) and the rectangular area (see Figure 4.b). Teacher mediated students by asking “Is there any relation between paralelogram area and rectangular area? What is the relation?”. In this condition, to help students found the relationships teacher mediated by making various devices e.g. by using colour paper in form of paralelogram (as in the Figure 4.a) and cut it into 2 trapezium parts then reconstruct them in to rectangle (as in the Figure 4.b). Thus, students understand that the paralelogram’s area (see Figure 4.a) was equal to rectangle’s area (see Figure 4.b). By this mediation, students could construct the planes area concepts (i.e paralelogram’s area) by using concept of rectangular area.

Figure 4. The Illustration of paralelogram area formula approached by rectangular area

By these three RMT phase, students were expected to be able to construct their conceptual understanding of planes area. Then, to gain higher quality of though, students were given various daily problems which represented the planes area. This step aimed at upgrading students’ strategic competence which related to problem solving ability. Thus, by this step teacher expected students not to have only better conceptual understanding but also better strategic competence.

After finished the learning process (8 times meeting), conducted post-test in each class and the data obtained would be analyzed by using MANOVA and independent t-test. By prerequisite test (i.e. normality multivariate and homogeneity variance-covariance or Box’s M test) known that the data distribution was normal and the variance-covariance was homogenous. Wilks’ Lambda (i.e. 0.841) showed that there was significant difference mean value between experimental and control class when considered jointly on the geometry conceptual understanding and strategic competence. Then, independent t-test was used to consider whether there was significant difference mean value between experimental and control class in each dependent variable (see Table 1). Based on table 1, known that there was significant difference mean value between experimental and control class when consider partly both on geometry conceptual understanding and strategic competence. Based on the marginal mean known that in each dependent variable (i.e. geometry conceptual understanding and strategic competence) RMT as learning method gave better effect than conventional learning method (i.e. Direct Learning). By this stage, known that RMT as learning method effective to upgrade students’
conceptual understanding and strategic competence. It was in line with the previous research (see [15,16]).

| Class      | N  | Conceptual Understanding | Strategic Competence |
|------------|----|--------------------------|----------------------|
|            |    | Mean | Sig. 2 tails | Mean | Sig. 2 tails |
| Experimental | 79  | 71.288 | 0.001       | 58.724 | 0.000       |
| Control     | 86  | 61.549 | 0.001       | 41.362 | 0.000       |

4. Conclusion
Based on the result and discussion described in the previous section, RMT as learning method accommodates and stimulates students to construct the conceptual understanding and develop strategic competence of geometry. By statistical analysis known that RMT as learning method gave better effect than conventional method toward geometry conceptual understanding and strategic competence. It might because teacher had an important role as mediator who mediated students in every phase of RMT. However, this research limited on implementing RMT as learning method toward conceptual understanding and strategic competence. Therefore, we recommended other researchers to explore the impact of RMT on other mathematical cognitive skills. We also recommended other researchers to explore the implementation of RMT as learning method in other mathematics materials.

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