Enhancement of the retrapping current of superconducting microbridges of finite length

D.Y. Vodolazov and F.M. Peeters

1 Institute for Physics of Microstructures, Russian Academy of Sciences, 603950, Nizhny Novgorod, GSP-105, Russia
2 Departement Fysica, Universiteit Antwerpen (CGB), Groenenborgerlaan 171, B-2020 Antwerpen, Belgium

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We theoretically find that the resistance of a superconducting microbridge/nanowire decreases while the retrapping current $I_r$ for the transition to the superconducting state increases when one suppresses the magnitude of the order parameter $|\Delta|$ in the attached superconducting leads. This effect is a consequence of the increased energy interval for diffusion of the ’hot’ nonequilibrium quasiparticles (induced by the oscillations of $|\Delta|$ in the center of the microbridge) to the leads. The effect is absent in short microbridges (with length less than the coherence length) and it is relatively weak in long microbridges (with length larger than the inelastic relaxation length of the nonequilibrium distribution function). A nonmonotonous dependence of $I_r$ on the length of the microbridge is predicted. Our results are important for the explanation of the enhancement of the critical current and the appearance of negative magnetoresistance observed in many recent experiments on superconducting microbridges/nanowires.

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I. INTRODUCTION

Recently, several experimental groups observed a negative magnetoresistance (NMR) of superconducting nanowires/microbridges at temperatures lower than the critical temperature $T_c$. In Refs. [3–6] authors demonstrated that in their case the effect is connected with the suppression of superconductivity in the bulk superconductor caused by the applied magnetic field. Moreover in Refs. [2, 6] it was shown that this NMR appears together with an enhancement of the critical current of the nanowires in weak magnetic fields.

Presently there exist several theories [8–12] which propose different mechanisms for the NMR. Ref. [8] claims that it is connected with the suppression of a new channel of dissipation in superconducting wires by an applied magnetic field, while Refs. [3–6] argue that the suppression of the intrinsic pair-breaking resulting from total ’spin-flip’+’non-spin-flip’ rate is responsible for the effect. In Ref. [11] the stabilization of the superconducting phase due to magnetic field induced normal metal/superconductor boundaries at the ends of the microbridge was offered as the main mechanism for the negative magnetoresistance while in Refs. [1, 12, 13] the effect was explained as due to a decrease of the charge imbalance decay length $\lambda_Q$ in weak magnetic fields.

Taking into account the strong dependence of both the NMR and the enhancement of $I_r$ on the length of the superconducting microbridge/nanowire (i.e. the effect does not exist in relatively long and short samples - see Refs. [3–6]) we argue that the proposed mechanisms as put forward in Refs. [3–11] are not relevant for these experiments. Indeed their applicability is not limited by the length of the superconductors. Due to the same reason a decrease of $\lambda_Q$ [1, 12, 13] cannot explain the length dependence of the effect because it should lead to NMR in samples of arbitrary length. Besides at currents close to the depairing current, $\lambda_Q$ starts to depend on the current [14] and it smears its dependence on $H$. As a result this mechanism cannot explain the increase of $I_r$ in weak magnetic fields.

In Ref. [12] another mechanism for the enhancement of $I_r$ was proposed connected with the complete suppression of the order parameter in the superconducting leads. But this mechanism is not able to explain the monotonous enhancement of $I_r$ in weak magnetic fields (see [2, 3, 6]) when the superconducting leads are still in the superconducting state with weakly suppressed order parameter.

Here, we offer a new mechanism that leads to the negative magnetoresistance of microbridges/nanowires and the enhancement of the critical (retrapping) current. Our explanation is based on the following idea: even a weak suppression of the order parameter in the leads opens new energy channels for the diffusion of the ’hot’ quasiparticles from the microbridge where they are induced by oscillations of the order parameter $\lambda_Q$. As a result the effective ’temperature’ of the quasiparticles in the microbridge decreases and the retrapping current for the transition to the superconducting state increases. Simultaneously it leads to decrease of the resistance of the microbridge at fixed current. Our proposed mechanism leads to no effect in very short and very long microbridges/nanowires (in agreement with the experiments [3–6]). Besides we find that the retrapping current is a nonmonotonous function of the length of the microbridge.

The paper is organized as follows. In section II we present the theoretical model. The current-voltage (IV) characteristics of microbridges of different length are pre-
presented in Section III for different values of the order parameter in the leads. In section IV we discuss our result and in Sec. V we present our conclusions.

II. MODEL

![Diagram](image)

FIG. 1: (Color online) (a) Schematic illustration of the finite width Dayem microbridge connected with wide superconducting leads. (b) Dependence of the magnitude of the order parameter along the center line of the microbridge for different magnetic fields. (c) Contour plot of $|\Delta|$ in the leads and microbridge at different H (blue(red) color corresponds to low(high value for $|\Delta|$)). The length of the full system (leads+microbridge) is $27\xi$; width $15\xi$; length of microbridge is $7\xi$ and width of microbridge is $\xi$.

To study the resistive state of the microbridge we use the set of equations (time-dependent Ginzburg-Landau equation for the superconducting order parameter coupled with the kinetic equations for the quasiparticle distribution function and the Usadel equations for the Green’s functions) derived for ‘dirty’ superconductors near the critical temperature of the superconductor

\[ N_1 \frac{\partial \delta f_L}{\partial t} - D \nabla ((N_1^2 - R_2^2) \nabla \delta f_L) = D \nabla (j_L f_T) - \frac{N_1}{\tau_{in}} \delta f_L - R_2 \frac{\partial f_L^0}{\partial \epsilon} \frac{\partial |\Delta|}{\partial t}, \quad (1a) \]

\[ \frac{\partial}{\partial t} N_1 (f_T + e \varphi \frac{\partial f_T^0}{\partial \epsilon}) - D \nabla ((N_1^2 + N_2^2) \nabla f_T) = D \nabla (j_L \delta f_L) - \frac{N_1}{\tau_{in}} \left( f_T + e \varphi \frac{\partial f_T^0}{\partial \epsilon} \right) - N_2 |\Delta| \left( 2 f_T + h \frac{\partial f_T^0}{\partial \epsilon} \right), \quad (1b) \]

\[ hD \left( \frac{d^2 \Theta}{dx^2} + \frac{d^2 \Theta}{dy^2} \right) + \left( \left( 2i\epsilon - h \right) \tau_{in} \right) - hDQ \cos \Theta \sin \Theta + 2|\Delta| \cos \Theta = 0, (2) \]

\[ \xi_{GL}^2 \left( \nabla - i \frac{2eA}{hc} \right)^2 \Delta + \left( 1 - \frac{T}{T_c} - \frac{|\Delta|^2}{\xi_{GL}^2} \right) \Delta. \quad (3) \]

Here $\Delta = |\Delta| e^{i\phi}$ is a complex order parameter, $\xi_{GL}^2 = \pi h D/8 k_B T_c$ and $\Delta_{GL}^2 = 8 \pi^2 (k_B T_c)^2 / 7 \zeta(3)$ are the zero temperature Ginzburg-Landau coherence length and the order parameter correspondingly. $Q = (\partial \phi / \partial x - 2eA/hc)$ is a quantity which is proportional to the superfluid velocity ($v_s = D Q$) with $A$ the vector potential taken in the Landau gauge, $\varphi$ is the electrostatic potential, $f_L(\epsilon) = \delta f_L(\epsilon) + \xi f_L(\epsilon)$ is the longitudinal and $f_T(\epsilon)$ is the transverse parts of the quasiparticle distribution function $2f(\epsilon) = 1 - f_L(\epsilon) - f_T(\epsilon)$ (in equilibrium $f_L = f_L^0(\epsilon) = \tanh(\epsilon/2k_B T_c)$, $f_T^0(\epsilon) = 0$). $N_1, N_2, R_2$ are the spectral functions which are determined by the Usadel equation for the normal $\alpha(\epsilon) = \cos \Theta = N_1(\epsilon) + i R_1(\epsilon)$ and anomalous $\beta_1 = e^{i\phi}, \beta_2 = e^{-i\phi} (\beta(\epsilon) = \sin \Theta = N_2(\epsilon) + i R_2(\epsilon))$ Green functions.

Nonequilibrium corrections to the quasiparticle distribution function enters Eqs. (3) via the potentials $\Phi_1 = \int_0^\infty R_2 \delta f_L \text{d} \epsilon / |\Delta|$ and $\Phi_2 = \int_0^\infty N_2 f_T \text{d} \epsilon / |\Delta|$. Eqs. (1a,b) are coupled due to the finite spectral supercurrent $j_c = Re(\beta \nabla \beta_2 - \beta_2 \nabla \beta_1)/2 = 2N_2 R_2 Q$.

The advantage of Eqs. (1-3) in comparison with the ordinary or the extended time-dependent Ginzburg-Landau equations is that they allow to take into account nonlocal effects. Here, under nonlocality we mean that if in one place of the superconductor and in some moment in time the quasiparticle distribution function $f(\epsilon)$ becomes nonequilibrium (due to some kind of perturbation) then nonequilibrium correction to $f(\epsilon)$ will be nonzero over a distance $\sim L_{in} = (D \tau_{in})^{1/2}$ around that point and during a time $\sim \tau_{in}$ after turning off the perturbation.
Before solving Eqs. (1-3) numerically we calculate the order parameter in a model 2D system (see Fig. 1) in the stationary state in the presence of an applied magnetic field. This result demonstrates the suppression of $|\Delta|$ in the leads by increasing H and the weak influence on $|\Delta|$ in the microbridge. It also shows that instead of the 2D model we may use a 1D model where the suppression of the order parameter in the superconducting leads could be simulated by introducing locally a lower critical temperature. Thus we may use the model of Ref. \[15\] where we introduce a different critical temperature in the leads (see Fig. 2). Correlation between $T'_c$ and H is clear: smaller $T'_c$ corresponds to larger H.

The self-consistent set of Eqs. (1-3) was solved numerically using the method and boundary conditions presented in Ref. \[15\]. Further, we use the following dimensionless units. The order parameter is scaled by $\Delta_0$ ($\Delta_0 = 1.76k_BT_c$), distance is in units of the zero temperature coherence length $\xi_0 = \sqrt{\hbar D/\Delta_0}$, time in units of $\tau_0 = \hbar/\Delta_0$ and temperature in units of the critical temperature $T_c$. The current is scaled in units of $j_0 = \Delta_0\sigma_n/(\xi_0\epsilon)$ and the electrostatic potential is in units of $\varphi_0 = \Delta_0/\epsilon$. It is useful to introduce the dimensionless inelastic relaxation time $\tilde{\tau}_n = \tau_n/\tau_0$ which is the main control parameter in the model described by Eqs. (1-3). For example in MgB$_2$ $\tilde{\tau}_n \approx 4$ \[20\], in Nb and Pb $\tilde{\tau}_n \approx 40$, in Sn $\tilde{\tau}_n \approx 70$, in Al $\tilde{\tau}_n \approx 3500$ and in Zn $\tilde{\tau}_n \approx 2 \cdot 10^3$ \[14\].

### III. RESULTS

In Fig. 3 we present the IV characteristics of the superconducting microbridge with length $L = 21\xi_0$ calculated for different values of the order parameter in the leads \[24\] (in the inset we show distribution of the time averaged order parameter in the microbridge and leads at $I = 0.75I_c$). Notice that the retrapping current increases and the resistance of the microbridge decreases when the order parameter is slightly suppressed in the leads. We should stress here that the critical current $I_c$ of the microbridge (at which the superconducting state becomes unstable) monotonically decreases with decreasing $T'_c$.

Our result can be explained as due to the enhanced diffusion of the 'hot' quasiparticles induced by oscillations of the order parameter in the center of the microbridge \[15\] when $\Delta_{lead}$ becomes smaller. Indeed, in this case the energy barrier connected with the finite energy gap at $\epsilon < |\Delta|_{lead}$ decreases and nonequilibrium quasiparticles in the wider energy interval can diffuse to the leads. Here we should remind the reader that the local energy gap in the microbridge is smaller than the local value of the order parameter due to the finite supercurrent and the spatial variation of $|\Delta|$. \[14, 21\]. Therefore, 'hot' quasiparticles may diffuse in the energy interval $\epsilon > |\Delta|_{lead}$ even when the local $|\Delta|$ in the microbridge is larger than $|\Delta|_{lead}$ (see inset in Fig. 3).

In order to illustrate the above effect we show in Fig. 4 the energy dependence of the nonequilibrium correction to $f_L$ in the center of the microbridge averaged over one period of Josephson oscillations. In the inset to Fig. 4 we present the spatial dependence of the time-averaged potential $\Phi_1$ which corresponds to the effective 'temperature' $T_{eff} = T + T_\Phi \Phi_1$ of quasiparticles in the microbridge \[15\]. One can easily see that with decreasing $\Delta_{lead}$ the energy interval where the quasiparticles are 'heated' (corresponds to a negative sign of $\delta f_L$) decreases and it results in drastic changes of $\Phi_1$.

The found effect depends strongly on the length of the microbridge. When $L \lesssim 2\xi(T)$ the value of $\Delta_{lead}$ has the strongest effect on the value of $I_c$ (which itself is
close to the critical current of the microbridge \( I_c \) - see Fig. 5) while the effect of 'heating' is relatively weak (see Ref. [22]). With decreasing \( \Delta_{\text{lead}} \) both \( I_c \) and \( I_r \) monotonically decreases and the resistance increases - see Fig. 5. The critical length \( L_c \) for which \( I_r \) starts to increase depends on the inelastic relaxation time - the larger \( \tilde{\tau}_{\text{in}} \) the shorter \( L_c \). For example \( 11\xi_0 < L_c < 15\xi_0 \) for \( \tilde{\tau}_{\text{in}} = 250 \) while \( 17\xi_0 < L_c < 21\xi_0 \) for \( \tilde{\tau}_{\text{in}} = 60 \) at \( T = 0.92T_c \).

There is one interesting effect when \( L \lesssim L_c \). We find a decrease of \( I_r \) and an increase of the resistance at \( I \sim I_r \) but starting from some current \( I > I_r \), the resistance of the microbridge decreases when \( \Delta_{\text{lead}} \) is suppressed - see Fig. 7. At this length there is a competition between the influence of the order parameter in the leads and the 'heating' of the quasiparticles on the IV curves. At low currents \( I \sim I_r \) the value of \( I_r \) is determined mainly by \( \Delta_{\text{lead}} \) and \( I_r \) decreases with decreasing \( \Delta_{\text{leads}} \). At larger currents the 'heating' of the quasiparticles becomes stronger because it increases with decreasing Josephson period. As a result decreasing \( \Delta_{\text{lead}} \) weakens the 'heating' effects and the voltage at fixed current decreases.

FIG. 4: (color online). Energy dependence of the time averaged \( \delta \Phi \) in the center of the microbridge with length \( L = 21\xi_0 \approx 4.9\xi(T) \), \( \tilde{\tau}_{\text{in}} = 250 \) and \( T = 0.92T_c \) calculated for \( I = 0.75I_c \).

FIG. 5: (color online). Current voltage characteristics of the superconducting microbridge with length \( L = 21\xi_0 \approx 4.9\xi(T) \), \( \tilde{\tau}_{\text{in}} = 250 \) and \( T = 0.92T_c \) calculated for \( I = 0.75I_c \).

FIG. 6: (color online). Current voltage characteristics of the superconducting microbridge with length \( L = 31\xi_0 \approx 7.3\xi(T) \) calculated at \( T = 0.92T_c \) and \( \tilde{\tau}_{\text{in}} = 250 \). The inset shows the distribution of the time-averaged order parameter in the microbridge at different \( T_c' \). Current is normalized in units of the critical current of the microbridge with \( T_c' = T_c \).

IV. DISCUSSION

A typical feature of many experiments about the enhancement of the critical current and the negative magnetoresistance is presence of finite resistance of the nanowires/microbridges even at low temperatures [3,7] which implies a strong influence of fluctuations. It can explain the absence of the hysteresis of IV curves observed...
FIG. 7: (color online). Current voltage characteristics of the superconducting microbridge with length $L = 11\xi_0 \approx 2.6\xi(T)$ calculated at $T = 0.92T_c$ and $\tau_{in} = 250$. The inset shows the distribution of the time-averaged order parameter in the microbridge at different $T_c$. Current is normalized in units of the critical current of the microbridge with $T_c = T_c$.

in Refs. 3 7. Indeed, it is known for example from the theory of Josephson junctions that fluctuations may completely destroy the hysteretic behavior 25. Therefore the current measured in that experiments is probably not $I_c$ but $I_r$. And indeed, the estimations made in Ref. 6 showed that the measured critical current was much smaller (more than 10 times) than the depairing current. Therefore we believe that our results can be directly applied for the explanation of the enhancement of the critical current found in Refs. 3 7. Taking into account that in zinc $L_{in} \approx 25\mu m$ 14 (when $\xi_0 \approx 250$ nm 6) it becomes clear why the effect was weak in relatively long microbridge with $L = 10\mu m \approx 0.4L_{in}$ and in short one with $L = 1\mu m \approx 4\xi_0$ 6. In Sn the effect was absent for nanowires with $L = 6 - 35\mu m$ 6 because in tin $\xi_0 \approx 55$ nm and $L_{in} \approx 470$ nm 14. The applicability of our result to the experiment of Rogachev et. al 2 is questionable because those authors claimed that their critical current is about the depairing current and furthermore experimental IV curves were strongly hysteretic (see inset in Fig. 2(c) in Ref. 2). Unfortunately no length dependence of the effect was studied in that work.

We believe that our result gives a clue in the understanding of the origin of the negative magnetoresistance at low currents $I \ll I_r$. It is believed that the finite resistance of the superconducting nanowires/microbridges at low currents originates from the finite rate of thermo-activated and/or quantum phase slips (see for example review 26). Each phase slip event is connected with one oscillation of the magnitude of the order parameter which provides the ‘heating’ of the quasiparticles. Therefore, during a finite time $\min\{\tau_{in}, \tau_{diff} \sim L^2/D\}$ after the phase slip event the effective ‘temperature’ of quasiparticles will be larger than the bath temperature and the probability of the next phase slip becomes higher. It creates the condition for phase slip avalanches. By decreasing the order parameter in the leads one increases the flux of ‘hot’ quasiparticles from the microbridge and therefore decreases the effective ‘temperature’ and the probability of phase slip avalanches. The effect should be strongest in microbridges with $L \lesssim L_{in}$ where such a diffusion is the most effective one. In short microbridges with $L \lesssim 2\xi(T)$ the suppression of the order parameter in the leads suppresses also $\Delta$ in the microbridge (due to the proximity effect - see Figs. 1(b,c)) and it gives the leading contribution to the probability for phase slips - i.e. it considerably increases. Therefore one should observe positive magnetoresistance in short microbridges.

From our theoretical calculations we found that the enhancement of $I_r$ is absent in the temperature interval $0.92 < T/T_c < 0.99$ for $\tau_{in} \lesssim 20$ for all considered lengths of the microbridges $L = 7 - 51\xi_0$. This is not surprising because in this temperature interval the corresponding $L_{in}$ is about the coherence length and the period of oscillations of the order parameter is about $\tau_{in}$. For these conditions the effective ‘heating’ is rather weak (see 22) and hence diffusion of ‘hot’ quasiparticles does not play any role. For $\tau_{in} = 60$ the effect is practically absent at $T/T_c > 0.96$ (in this case $L_{in} \approx 7.5\xi_0 \sim \xi(T)$) and it becomes noticeable at $T/T_c = 0.92$ for microbridges with length $17\xi_0 < L < 31\xi_0$.

One more interesting effect which comes from our calculations is the nonmonotonous dependence of $I_r$ on the length of the microbridge at zero magnetic field (when $T_c = T_c$). One can see from comparing the values of $I_r$ in Figs. 3-7 that $I_r$ is minimal for $L = 21\xi_0$. The reason for such a behavior is the following - in a very short microbridge the ‘heating’ of quasiparticles is weak and $I_r \sim I_c$ and $I_r$ decreases with decreasing $L$. In a very long microbridge $L \gg L_{in}$ the ‘hot’ quasiparticles relax on the length scale $\sim L_{in}$ near the phase slip core while in microbridge with length $L < L_{in}$ such a relaxation is less effective. This results in the following main tendency: the retrapping current $I_r$ first decreases with increasing length of the microbridge, reaches the minimal value at $L \gtrsim 4\xi$ (when the influence of the leads becomes sufficiently weak) and then increases and saturates at $L \gg L_{in}$.
V. CONCLUSION

The ‘heating’ of quasiparticles, which occurs in the phase slip core due to oscillations of the order parameter can be reduced by diffusion of the quasiparticles to the outside regions. It results in an enhancement of the retrapping current when one slightly suppresses the order parameter in the superconducting leads. The enhancement of \( I_r \) strongly depends on the length of the microbridge - the effect is absent in short microbridges with length \( L \lesssim 2\xi(T) \) and it is weak in relatively long samples with length \( L \gtrsim L_{in} \). Our results predict also a nonmonotonous dependence of the retrapping current \( I_r \) on the length of the microbridge - it is minimal when \( 4\xi(T) \lesssim L < L_{in} \). We should note that our results cannot be obtained in the framework of ordinary \cite{Tinkham} or extended time-dependent Ginzburg-Landau equations and one need to solve Eqs. (1-3) where nonlocal effects connected with a time delay of the response and the diffusion of nonequilibrium quasiparticles are taken into account.

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