LETTER

Polymer-loaded three dimensional microwave cavities for hybrid quantum systems

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Abstract
Microwave cavity resonators are crucial components of many quantum technologies and are a promising platform for hybrid quantum systems, as their open architecture enables the integration of multiple subsystems inside the cavity volume. To suspend these subsystems within the centre of a cavity where field strengths are strong and uniform, auxiliary support structures are often required, but the effects of these structures on the microwave cavity mode are difficult to predict due to a lack of a priori knowledge of the materials’ response in the microwave regime. Understanding these effects becomes even more important when frequency matching is critical and tuning is limited, for example, when matching microwave modes to atomic resonances for atomic vapour cells inside enclosed microwave cavities. Here, we study the microwave cavity mode in the presence of three commonly-used machinable polymers, paying particular attention to the change in resonance and the dissipation of energy. We demonstrate how to use the derived dielectric coefficient for cavity design in a test case, wherein we match a polymer-filled 3D microwave cavity to a hyperfine transition in rubidium.

1. Introduction
The high-quality factors and open architecture of three dimensional (3D) microwave cavities [1–8] has enabled hybridization with a wide range of classical and quantum systems, including 3D transmon qubits [9–11], magnonic resonators [12–18], neutral atoms and molecules [4, 19–23], quantum dots [24], and piezoelectric optomechanical resonators [25]. In many of these systems, it is important that—due to a resonant interaction—the microwave cavity frequency be identical to (or well-controlled near) the desired transitions in the system of interest: for example, in a vapour cell of neutral atoms inside a microwave cavity, the microwaves can drive electronic transitions [23]; or in piezo-optomechanics, the microwave can drive the mechanical modes [25]. While in some scenarios, the hybrid system is easily tunable over a wide range, such as the magnon frequencies in cavity magnon polaritons [13], these are the exception and generally, the tunability is limited [23] or zero [2]. Hence, to achieve resonant interaction and efficient coupling, all components of the 3D microwave cavity must be precisely understood at the design stage to accurately produce a microwave cavity with the desired frequency. This includes any materials inserted into the cavity, often machinable polymers, used to physically support the resonating devices.

Values for the dielectric properties of common machinable polymers are available [26], but are generally measured over broad (several GHz) frequency ranges, and it can be difficult to pinpoint values at the exact frequencies of interest. Here, we report frequency-dependent values of the dielectric constant for three common machinable polymers, as well as the relative microwave cavity loss due to these materials at particular modes within a narrow microwave range of interest, from approximately 3-10 GHz. These measurements will assist those seeking to construct resonant microwave cavities, especially in hybrid systems with limited tunability.

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2. Methods

In this work, we focus on measurements of three commercially available polymer materials, polytetrafluoroethylene (PTFE), polypropylene (PP), and polyethylene (PE), with densities \( \rho_{\text{PTFE}} = 2.14(5) \text{ g/cm}^3 \), \( \rho_{\text{PP}} = 0.902(2) \text{ g/cm}^3 \), and \( \rho_{\text{PE}} = 0.924(2) \text{ g/cm}^3 \). All measurements were performed at room temperature. [Note that we also tested an additional six materials (nylon, anti-static polyethylene, moisture-resistance polyethylene, polyester, polycarbonate, and Garolite LE), but we found that these suffered from considerable dielectric loss at GHz-frequencies and therefore made microwave modes generally unobservable.]

We determine the effective energy loss of several cavity modes for the three materials by comparing cavity measurements to finite-element simulations. Using a cylindrical copper cavity [2, 23], we measure the frequencies and internal quality factors of multiple modes, both for air-filled cavities, and for cavities with tight-fitting polymer plugs, as shown in figure 1. Any air gaps between the polymer and the cavity are negligible, and we neglect any air-gap effect in the simulations and analysis [27]. Microwave reflection (S\(_{11}\)) measurements are performed on a vector network analyser (VNA). In order to couple microwaves into and out of the cavity, an SMA pin-coupler acted as a waveguide mounted at the midpoint of the cylindrical axis and offset 15 mm tangentially; this was modified to be flush with the cavity wall so the entire void of the cavity is filled with material.

3. Dielectric constant

We describe the dielectric properties of the polymers using the Drude-Lorentz oscillator model [28]. In this model, the electrons within the material are treated as damped harmonic oscillators. Therefore, due to the polarizability of the material, the dielectric constant can be written as a function of the material’s complex susceptibility, with \( \varepsilon_r = 1 + \varepsilon'' \) and [29]

\[
\varepsilon_r = \varepsilon' + i\varepsilon'',
\]

where the real part \( \varepsilon' \) is associated with the index of refraction of the material, and the imaginary part \( \varepsilon'' \) describes the energy loss due to absorption.

To study the microwave-frequency dielectric response of various materials, we measure the properties of resonant modes of a microwave cavity with and without a dielectric filling the interior volume. The resonance frequencies of the transverse electric (TE) modes of a cylindrical cavity filled with a dielectric are

\[
f_{\text{mode}} = \frac{c}{2\pi} \sqrt{\frac{\mu_r}{\mu_0}} \sqrt{\left( \frac{n \pi}{a} \right)^2 + \left( \frac{m \pi}{d} \right)^2},
\]

where \( f_{\text{mode}} \) is the frequency of the mode with indices \((n, m, \ell)\); \( c \) is the speed of light in vacuum; \( a \) is the internal radius of the cavity; \( d \) is the internal height; and \( \mu_r \) is relative permeability (for non-magnetic materials, \( \mu_r = 1 \)).
The cavity mode is defined through the Bessel function parameter $p'_{nm}$, where $p'_{nm}$ is the $m$th root of $J'_n$ \[29\]. We emphasize that to study the effect of changing materials, and thus $\varepsilon'$, we must consider the same mode (with indices $(n, m, \ell)$) under both air- and polymer-filled conditions.

When transitioning between an empty cavity and one filled with dielectric, tracking the resonances can be difficult due to mode-dependent changes in the external coupling, in addition to changes in their resonance frequencies. In order to ensure proper mode identification, a combination of preliminary finite element simulations using approximate values of the dielectric constant, as well as tracking mode frequencies relative to the highest-$Q$ mode (the TE$_{011}$ mode), was performed.

Using this tracking, we measure and identify the mode frequencies and quality factors for 7-10 cavity resonances in each material (table 2), then extract values of $\varepsilon'$ calculated from (2), as shown in figure 2. A linear fit well represents the frequency dependence of the dielectric constant in this range, and allows for the extrapolation to frequencies needed for novel designs. From the fit results, the frequency dependence of the dielectric constants are tabulated in table 1. In the case of PE, the frequency-dependence is null, within the uncertainty of the fit, which may have advantages in some experimental designs. The choice of dielectric material will influence cavity designs, with higher-dielectric materials allowing for smaller cavities and greater mode confinement, while lower-dielectric materials allow for the design of larger cavities to accommodate particular experiments.

4. Dissipation

The quality factor of a resonator depends on the stored energy compared to the rate at which energy is dissipated. An $S_{11}$ measurement can be fit to extract the internal quality factor of the cavity ($Q_i$) and the external quality factor ($Q_{ex}$) \[31\]:

$$Q_{\text{total}} = \left(\frac{1}{Q_i} + \frac{1}{\text{Re}(Q_{ex})}\right)^{-1}.$$  (3)
Q_{ext} is a measure of the external coupling into and out of the cavity and in general is complex valued due to impedance mismatch, whereas Q_{i} is the relevant value to measure the cavity performance. The parameter Q_{i} accounts for loss due to the resistance of the conducting walls of the cavity (associated with Q_{conductor}), as well as the dissipation of energy in the dielectric inside the cavity (Q_{dielectric}), and the additional resistance across the seam between the cavity end-caps and walls (Q_{seam}), such that

\[ Q_i = \left( \frac{1}{Q_{\text{conductor}}} + \frac{1}{Q_{\text{dielectric}}} + \frac{1}{Q_{\text{seam}}} \right)^{-1}. \] (4)

To determine the dielectric properties of materials (1) using data from different resonant modes, care must be taken to account for the varying mode volumes and external couplings that affect loss channels. For example, a higher-dielectric material may result in the localization of a resonant mode to the center of a cavity, where the fields interact less with the walls and have lower conductor-induced losses. Using measurements that extract the internal and external quality factors separately, we use finite-element simulations (via COMSOL, in our case) to identify the mode-dependent contributions from each loss channel. By measuring the empty cavity, the internal quality factor Q_i has contributions from only Q_{conductor} and Q_{seam}, which allows us to extract the cavity-only material properties. Next, by comparing the loaded-cavity Q_i to the empty-cavity value, and taking into account dielectric-dependent changes to the mode through simulations, we extract the polymer’s dielectric properties \( \varepsilon' \) and \( \varepsilon'' \) for each mode frequency.

To begin this determination, we consider the two dominant loss channels of the empty cavity; the bulk-copper conductivity (for which we use the literature value [32] 5.89 \times 10^{7} \Omega/m) and the seam losses between the end caps and the cylindrical body [29, 33]. Seam losses must be manually added to the simulations, where the quality factor for the seam can be calculated using the line integral form [33],

\[ \frac{1}{Q_{\text{seam}}} = \frac{1}{G_{\text{seam}}} \left( \frac{L \int_{\text{seam}} |\vec{J}_s| \times |\vec{H}|^2 dV}{\omega \mu_0 \int_{\text{int}} |\vec{H}|^2 dV} \right), \] (5)

where, \( G_{\text{seam}} \) is the conductivity across the seam, the numerator \( L \int_{\text{seam}} |\vec{J}_s| \times |\vec{H}|^2 dV \) is the energy loss due to the seam in the conductor, \( L \) is the length of the seam along which the line integral is taken, \( J_s \) is the surface current across the seam, \( \omega = 2\pi f \) is the angular frequency of the resonance, and \( \mu_0 \int_{\text{int}} |\vec{H}|^2 dV \) is the volume integral of the magnetic field \( \vec{H} \) inside the cavity, and \( \mu_0 \) is the vacuum permeability.

By adjusting values of \( G_{\text{seam}} \) in the simulations to give a calculated \( Q_i \) that matches the measured empty-cavity value, we extract a value of \( G_{\text{seam}} \) for each mode, as shown in table 2. We then apply these results to determine the dielectric properties of each material at each mode by comparing measured quality factors \( Q_i \) with simulations (maintaining self-consistent results by allowing the simulator to adjust the mode geometry due to the presence of the dielectric), and extracting the dielectric constant \( \varepsilon'' \) for which the simulation reproduces the measured \( Q_i \) for each mode. Finally, we present our values in table 2 as

\[ \tan \delta = \frac{\varepsilon''}{\varepsilon'} = \frac{1}{Q_{\text{dielectric}}}. \] (6)

From these results we note that, for example, while PTFE is commonly used in microwave cavities, PP and PE generally have lower loss and may therefore be better candidates for some applications.

We also note that these loss values will depend on factors such as the humidity, temperature, and material densities, and that the particular material should be characterized whenever precise engineering is required.

5. Test case: cavity design

As a test of the accuracy and usefulness of these findings, we set out to design a ‘science’ cavity with a different size and resonant frequency than the one used for the characterizations. The target was to hold a sealed ^85Rb vapour cell for microwave coupling to the 3.035 732 GHz hyperfine transition. The vapour cell is a box made of borosilicate glass and filled with a low-pressure Rb vapour, and it is to fit inside a hollow machined into the centre of the polymer plug, allowing the atoms to sit at the region of strongest and most uniform oscillating magnetic field. The cavity itself was to be made from aluminum. All of these geometric and material details are inputs to the finite-element simulations; to match the smaller transition frequency (corresponding to this atomic isotope) to the high-\( Q \) TE_{011} mode, this cavity is considerably larger than an empty cavity; the internal height is 96.5 mm and the internal diameter is 90 mm. All of these differences from our test cavity provide an opportunity to use the results from above—the dielectric properties of PTFE—in a dissimilar environment.

An important consideration is that once a cylindrical cavity is built, the cavity can be machined shorter in length to increase its target frequency, if the resonance is too low. Tuning the opposite direction, that is lowering
a too-high frequency can be achieved by lengthening the cavity, but only if the end caps are designed to slide outwards smoothly. In our design, this end-cap extension provides roughly 40 MHz of frequency reduction. It is clear that $\varepsilon'$ must be accurate to roughly 1% to ensure that such hybrid cavities can be accurately produced with this level of tunability.

In light of this, we initially targeted a resonance of roughly 3.0 GHz with a height of 104 mm and 90 mm diameter, allowing for subsequent trimming modifications to match the target rubidium transition. From the data in figure 2 for $\varepsilon'$, we find that $\varepsilon'(3 \text{ GHz}) = 2.03(4)$ and simulation of the COMSOL design resulted in a cavity frequency of 3.014 GHz. After construction, our measured TE011 mode was at 3.0141 GHz, allowing for careful shortening of the cavity to get in range of the tunability of the end-caps. Simulation of a shortened cavity to 96.5 mm gave a resonance of 3.0439 GHz (measured via the method shown in figure 1(c)), where the final measurement of the constructed cavity yielded a frequency of 3.0449 GHz (within 0.03% of our final target). The small tunability of the cavity end-caps then allows us to precisely match the 3.035 73 GHz transition.

6. Conclusion

In this paper we report the dielectric constants and loaded-cavity loss tangents for three relevant machinable polymers in 3D microwave cavities, tracked over a large number of microwave resonances in the GHz range. This allowed us to extract the frequency dependence of the dielectric constants, useful for designing of microwave cavities including these dielectric materials, as shown in a test case targeting a hyperfine resonance in rubidium. Additionally, by comparison between finite element simulations that accounted for a variety of loss channels, we were able to extract loaded cavity loss tangents for these materials, which could influence material

| Mode  | Material | $Q_i$ ($\times 10^3$) | $f$ (GHz) | $G_{\text{seam}}$ | $\varepsilon'$ | $\tan \delta$ ($\times 10^{-3}$) |
|-------|----------|----------------------|----------|----------------|--------------|-------------------------------|
| TE211 | Air      | 9.81(4)              | 5.675 42 | 1.44           |              |                               |
|       | PTFE     | 0.926(3)             | 3.976 92 | 2.0322         | 0.985        |                               |
|       | PP       | 2.16(1)              | 4.631 22 | 1.4985         | 0.280        |                               |
|       | PE       | 1.04(1)              | 4.889 20 | 1.3446         | 0.272        |                               |
| TE011 | Air      | 27.6(5)              | 6.822 11 | 5.13           |              |                               |
|       | PTFE     | 2.38(3)              | 4.834 47 | 1.9949         | 0.386        |                               |
|       | PP       | 4.86(1)              | 5.648 05 | 1.4616         | 0.133        |                               |
|       | PE       | 2.97(3)              | 5.866 69 | 1.3547         | 0.123        |                               |
| TE212 | Air      | 4.48(9)              | 7.294 24 | 2.87           |              |                               |
|       | PTFE     | 1.96(1)              | 5.182 15 | 1.96 94        | 0.441        |                               |
|       | PP       | 3.97(1)              | 5.939 77 | 1.49 90        | 0.098        |                               |
|       | PE       | 1.93(1)              | 6.202 85 | 1.3746         | 0.071        |                               |
| TE012 | Air      | 33.6(3)              | 8.202 74 | 4.47           |              |                               |
|       | PTFE     | 3.53(5)              | 5.909 71 | 1.9290         | 0.258        |                               |
|       | PP       | 3.40(2)              | 6.812 13 | 1.4518         | 0.197        |                               |
|       | PE       | 3.15(2)              | 6.946 66 | 1.3961         | 0.192        |                               |
| TE312 | Air      | 7.28(10)             | 8.708 11 | 2.56           |              |                               |
|       | PTFE     | 3.33(4)              | 6.212 68 | 1.9533         | 0.228        |                               |
|       | PP       | 1.61(1)              | 7.249 01 | 1.4347         | 0.252        |                               |
|       | PE       | 2.95(2)              | 7.474 94 | 1.3493         | 0.308        |                               |
| TE013 | Air      | 22(1)                | 10.089 70 | 3.39          |              |                               |
|       | PTFE     | 3.40(1)              | 7.118 81 | 2.0117         | 0.213        |                               |
|       | PP       | 3.79(2)              | 8.413 41 | 1.4402         | 0.171        |                               |
|       | PE       | 1.48(1)              | 8.749 71 | 1.3317         | 0.123        |                               |
selections in future designs. These techniques and measurements should aid researchers designing 3D microwave cavities for hybrid systems, especially those with limited tunability, or where the lowest loss must be attained. Future work could explore the temperature dependence of these material properties, both in ambient and cryogenic conditions, to aid in the design of future quantum technologies.

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Data availability statement

The data that support the findings of this study are available upon reasonable request from the authors.

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