Testing Gaugino Mass Unification Directly at the LHC

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We report on the first step of a systematic study of how gaugino mass unification can be probed at the LHC in a quasi-model independent manner. Here we focus our attention on the theoretically well-motivated mirage pattern of gaugino masses, a one-parameter family of models of which universal (high scale) gaugino masses are a limiting case. Using a statistical method to optimize our signature selection we arrive at three ensembles of observables targeted at the physics of the gaugino sector, allowing for a determination of this non-universality parameter without reconstructing individual mass eigenvalues or the soft supersymmetry-breaking gaugino masses themselves. In this controlled environment we find that approximately 80% of the supersymmetric parameter space would give rise to a model for which our method will detect non-universality in the gaugino mass sector at the 10% level with \(O(10 \text{ fb}^{-1})\) of integrated luminosity.

1. MOTIVATION

Given that the LHC era is nearly here, it is hardly surprising to find members of the high energy theory community are increasingly turning their attention away from esoteric issues of model-building and towards issues of directly confronting theory with data. This activity has produced a number of new measurement techniques. Previously unconsidered signatures have been developed by studying certain “what-if” scenarios, and a new emphasis has been placed on making concrete predictions from famously nebulous string constructions. All of this represents important progress, but here I wish to consider another aspect of the challenge that lies ahead. Specifically, I wish to imagine the state of our field three to five years from now. Assuming something like the minimal supersymmetric standard model (MSSM) is discovered, as so many expect, then we can imagine being presented with a number of measurements that are indirectly related to the underlying soft supersymmetry-breaking mechanisms. Here I will discuss what is likely just the first step in a research program into what comes next: how to connect the multiple LHC observations to organizing principles in some (high-energy) effective Lagrangian of underlying physics.

In addressing this issue we must be careful to avoid the pitfalls of the “LHC Inverse” problem: the empirical fact that even in very restrictive model frameworks it is quite likely that more than one set of model parameters will give predictions for LHC observations that are in good agreement with the experimental data [2]. Much recent work has focused on how to address this issue [3,4,5,6,7,8], and we will borrow much of the philosophy and many of the useful techniques from this recent literature. The LHC inversion problem generally results from trying to use a large ensemble of (correlated) observations to constrain a multi-dimensional parameter space. But our ultimate goal as theorists is to understand broad properties of the underlying physics itself. Ever more precise measurement of a single parameter – say the gluino mass – does not necessarily further this goal. The most important such broad property is undoubtedly the issue of gaugino mass unification: few properties of the superpartner spectrum have more far-reaching implications for low-energy phenomenology, the nature of supersymmetry breaking, and the structure of the

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underlying physics Lagrangian [9,10]. But these soft parameters are not themselves directly measurable at the LHC [11]. One might consider performing a fit to some particular theory, such as minimal supergravity (mSUGRA), in which universal gaugino masses are assumed [12], but we are not so much interested in whether mSUGRA – or any other particular theory for which gaugino mass universality is a feature – is a good fit to the data. Rather, we wish to know whether gaugino mass universality is a property of the underlying physics independent of all other properties of the model.

We will therefore begin our attack by considering a concrete parametrization of non-universalities in soft gaugino masses. In recent work by Choi and Nilles [11] soft supersymmetry-breaking gaugino mass patterns were explored in a variety of string-motivated contexts. In particular, the so-called “mirage pattern” of gaugino masses [13,14,15] provides an interesting case study in gaugino mass non-universality. This paradigm gets its name from the fact that should the mirage pattern of gaugino masses be used as the low-energy boundary condition of the (one-loop) renormalization group equations then there will exist some high energy scale at which all three gaugino masses are identical [16]. The set of all such low-energy boundary conditions that satisfy the mirage condition defines a one-parameter family of models. In the parametrization we adopt from [10], the gaugino mass ratios at the electroweak scale take the form

\[ M_1 : M_2 : M_3 \simeq (1+0.66\alpha) : (2+0.2\alpha) : (6-1.8\alpha) \]

where the case \( \alpha = 0 \) is precisely the unified mSUGRA limit. In the limit of very large values for the parameter \( \alpha \) the ratios among the gaugino masses approach those of the anomaly-mediated supersymmetry breaking (AMSB) paradigm [17,18]. In fact, the mirage pattern is most naturally realized in scenarios in which a common contribution to all gaugino masses is balanced against an equally sizable contribution proportional to the beta-function coefficients of the three Standard Model gauge groups. Such an outcome arises in string-motivated contexts, such as KKLT-type moduli stabilization in D-brane models [19,20] and Kähler stabilization in heterotic string models [21,22,23,24,25,26,27]. Importantly, however, it can arise in non-stringy models, such as deflected anomaly mediation [29,30]. We note that in none of these cases is the pure-AMSB limit likely to be obtained, so our focus here will be on small to moderate values of the parameter \( \alpha \) in [1].

2. METHOD

The ultimate goal is to ask whether or not soft supersymmetry breaking gaugino masses obey some sort of universality condition independent of all other facts about the supersymmetric model. We begin by asking a simpler question: assuming the world is defined by the MSSM with gaugino masses obeying the relation (1), how well can we determine the value of the parameter \( \alpha \). At the very least we would like to be able to establish that \( \alpha \neq 0 \) with a relatively small amount of integrated luminosity. The first step in such an incremental approach is to demonstrate that some set of “targeted observables” [31] (we will call them “signatures” in what follows) is sensitive to small changes in the value of the parameter \( \alpha \) in a world where all other parameters which define the SUSY model are kept fixed.

2.1. Setting Up the Problem

We will construct what we will call a “model line” by specifying the supersymmetric model in all aspects other than the gaugino sector. We choose a simplified set of 17 input parameters given by

\[ \begin{cases} \tan \beta, m_H^2, m_D^2, \\ M_3, A_t, A_b, A_\tau, \\ m_{Q_{i,2}}, m_{U_{i,2}}, m_{D_{i,2}}, m_{L_{i,2}}, m_{E_{1,2}}, \\ m_{Q_3}, m_{U_3}, m_{D_3}, m_{L_3}, m_{E_3} \end{cases} \]

which are understood to be taken at the electroweak scale (specifically \( \Lambda_{EW} = 1000 \text{ GeV} \)) so no renormalization group evolution is required. The gluino soft mass \( M_3 \) will set the overall scale for the gaugino mass sector. The other two gaugino masses \( M_1 \) and \( M_2 \) are then determined relative to \( M_3 \) via (1). A model line will take the inputs of (2) and then construct a family of theories by varying the parameter \( \alpha \) from \( \alpha = 0 \) (the
mSUGRA limit) to some non-zero value in even increments.

For each point along the model line we passed the model parameters to PYTHIA 6.4 for spectrum calculation and event generation. Events are then sent to the PGS4 package to simulate the detector response. Additional details of the analysis will be presented in later sections. The end result of our procedure is a set of observable quantities that have been designed and (at least crudely) optimized so as to be effective at separating α = 0 from other points along the model line in the least amount of integrated luminosity possible.

2.2. Distinguishability

The technique we employ to distinguish between candidate theories using LHC observables involves the construction of a variable similar to a traditional chi-square statistic

\[(\Delta S_{AB})^2 = \frac{1}{n} \sum_i \left[ \frac{S_i^A - S_i^B}{\delta S_{iAB}} \right]^2,\]

where \(S\) is some observable quantity (or signature). The index \(i = 1, \ldots, n\) labels these signatures, with \(n\) being the total number of signatures considered. The labels \(A\) and \(B\) indicate two distinct theories which give rise to the signature sets \(S_i^A\) and \(S_i^B\), respectively. Finally, the error term \(\delta S_{iAB}\) is an appropriately-constructed measure of the uncertainty of the term in the numerator, i.e. the difference between the signatures. In this work we will always define a signature \(S\) as an observation interpreted as a count (or number) and denote it with capital \(N\). One example is the number of same-sign, same-flavor lepton pairs in a certain amount of integrated luminosity. Another example is taking the invariant mass of all such pairs and forming a histogram of the results, then integrating from some minimum value to some maximum value to obtain a number. In principle there can be an infinite number of signatures defined in this manner. In practice experimentalists will consider a finite number and many such signatures are redundant.

We can identify any signature \(i\) with an effective cross section \(\bar{\sigma}_i\) via the relation

\[\bar{\sigma}_i = N_i/L,\]

where \(L\) is the integrated luminosity. We refer to this as an effective cross-section as it is defined by the counting signature \(N_i\) which contains in its definition such things as the geometric cuts that are performed on the data, the detector efficiencies, and so forth. Furthermore these effective cross sections, whether inferred from actual data or simulated data, are subject to statistical fluctuations. The transformation in (4) allows for a comparison of two signatures with differing amounts of integrated luminosity. This will prove useful in cases where the experimental data is presented after a limited amount of integrated luminosity \(L_A\), but the simulation being compared to the data involves a much higher integrated luminosity \(L_B\). We will assume that the errors associated with the signatures \(N_i\) are purely statistical in nature and that the integrated luminosities \(L_A\) and \(L_B\) are precisely known, so that \((\Delta S_{AB})^2\) is given by

\[(\Delta S_{AB})^2 = \frac{1}{n} \sum_i \left[ \frac{\bar{\sigma}_i^A - \bar{\sigma}_i^B}{\sqrt{\bar{\sigma}_i^A/L_A + \bar{\sigma}_i^B/L_B}} \right]^2,\]

where each cross section includes the (common) Standard Model background, i.e. \(\bar{\sigma}_i = \bar{\sigma}_i^{susy} + \bar{\sigma}_i^{SM}\).

The variable \((\Delta S_{AB})^2\) forms a measure of the distance between any two theories in the space of signatures defined by the \(S_i\). We can use this metric on signature space to answer the following question: how far apart should two sets of signatures \(S_i^A\) and \(S_i^B\) be before we conclude that theories \(A\) and \(B\) are truly distinct? To answer this question we note that in the limit in which the luminosities \(L_A\) and \(L_B\) are large the probability distribution for the quantity \((\Delta S_{AB})^2\) given by

\[P(\Delta S^2) = n \chi^2_{n,\lambda}(n\Delta S^2),\]

where \(\chi^2_{n,\lambda}\) is the non-central chi-squared distribution for \(n\) degrees of freedom. The non-
centrality parameter $\lambda$ is given by

$$\lambda = \sum_i \frac{(\sigma_i^A - \sigma_i^B)^2}{\sigma_i^A/\lambda_i + \sigma_i^B/L_B},$$

and now the $\sigma_i$ represent exact cross sections. From (6) and (7) it is apparent that all the physics behind the distribution of possible $(\Delta S_{AB})^2$ values is contained in the values of $\lambda$ and $n$.

The problem of defining “distinguishability” has now been reduced to a well-understood problem in statistics which can be solved analytically. To say that two potential models have been distinguished – or that a set of $n$ observations are inconsistent with those derived from a simulation – we first demand that the quantity $(\Delta S_{AB})^2$ constructed from the $n$ observations satisfy $(\Delta S_{AB})^2 > (\Delta S_{AA})^2|_{5\text{th}}$. In other words, we first require that the distance in signature space be larger than that expected simply by quantum fluctuations with 95% confidence. The values of $(\Delta S_{AA})^2|_{5\text{th}}$ for $\lambda = 0$ are easily computed and tabulated for any value of $n$.

### 2.3. Optimization

Now return to the basic problem: using experimental data to distinguish two models that truly are distinct. Though we have $\lambda \neq 0$, there is always a finite chance that an experimental measurement will not reveal this fact due to quantum fluctuations. For any given value of $\lambda \neq 0$, the probability that a measurement of $(\Delta S_{AB})^2$ will fluctuate to a value so small that it is not possible to separate two distinct models (to confidence level $p$) is simply the fraction of the probability distribution in (7) that lies to the left of the value $(\Delta S_{AA})^2|_{5\text{th}}$. There is always some minimum value of the non-centrality parameter that can be chosen so that this fraction is below some pre-determined value $\lambda_{\text{min}}$. Let us call that value $\lambda_{\text{min}}(n, p)$. Again, these values are simply determined by integrating the probability distributions (6) and are independent of the physics of the problem. So, for example, given two distinct models $A$ and $B$, any combination of $n$ experimental signatures such that $\lambda > \lambda_{\text{min}}(n, p = 0.95)$ will be effective in demonstrating that the two models are indeed different 95% of the time, with a confidence level of 95%.

Let us assume for the moment that “model $A$” is the experimental data, which corresponds to an integrated luminosity of $L_{\text{exp}}$. Our “model $B$” can then be a simulation with integrated luminosity $L_{\text{sim}} = qL_{\text{exp}}$. Define the quantity

$$R_{AB} = \sum_i (R_{AB})_i = \sum_i \frac{(\sigma_i^A - \sigma_i^B)^2}{\sigma_i^A + \frac{1}{q} \sigma_i^B}$$

where $R_{AB}$ has the units of a cross section. Our condition for 95% certainty that we will be able to separate two truly distinct models at the 95% confidence level becomes

$$L_{\text{exp}} \geq \frac{\lambda_{\text{min}}(n, 0.95)}{R_{AB}}.$$ 

Given two models $A$ and $B$ and a selection of $n$ signatures both $\lambda_{\text{min}}(n, 0.95)$ and $R_{AB}$ are completely determined. Therefore the minimum amount of integrated luminosity needed to separate the models experimentally will be given by

$$L_{\text{min}}(p) = \frac{\lambda_{\text{min}}(n, p)}{R_{AB}}.$$ 

Equation (10) provides us with a quantitative measure of the efficacy of any particular set of $n$ (uncorrelated) signatures. We therefore wish to select a set of $n$ signatures $S_i$ such that the quantity $L_{\text{min}}(p)$ as defined in (10), for a given value of $p$, is as small as it can possibly be over the widest possible array of model pairs $A$ and $B$. We must also do our best to ensure that the $n$ signatures we choose to consider are reasonably uncorrelated with one another so that the statistical treatment of the preceding section is applicable.

The optimal strategy is generally to choose a rather small subset of the total signatures one could imagine. In part this is because in any set of observations the probability that any two are highly correlated with one another will only grow with the size of the set. But in addition not all possible signatures are equally effective at separating models. An absolute measure of the “power” of any given signature to separate two models $A$ and $B$ can be provided by considering the condition in (10). For any signature $S_i$ we can define an individual $L_{\text{min}}$ by

$$(L_{\text{min}})_i = \lambda_{\text{min}}(1, p) \frac{\sigma_i^A + \frac{1}{q} \sigma_i^B}{(\sigma_i^A - \sigma_i^B)^2}.$$ 

(11)
where, for example, \( \lambda_{\min}(1,0.95) = 12.99 \). This quantity is exactly the integrated luminosity required to separate models \( A \) and \( B \), to confidence level \( p \), by using the single observable \( S_i \). As more signatures are added, the threshold for adding the next signature in the list gets steadily stronger. Mathematically, the ratio \( \lambda_{\min}(n,p)/\lambda_{\min}(1,p) \) grows with increasing \( n \). This indicates that as we add signatures with ever diminishing \( (L_{\min})_i \) values we will eventually encounter a point of negative returns, where the resulting overall \( L_{\min} \) starts to grow again.

For a particular pair of models, \( A \) and \( B \), it is always possible to find the optimal list of signatures from among a given grand set by ordering the resulting \( (L_{\min})_i \) values and adding them sequentially until a minimum of \( L_{\min} \) is observed. To do so, we note that kinematic distributions must be converted into counts (and all counts are then converted into effective cross sections). This conversion requires specifying an integration range for each histogram. The choice of this range can itself be optimized, by considering each integration range as a separate signature and choosing the values such that \( (L_{\min})_i \) is minimized. By repeating this optimization procedure over many background model lines it is possible to construct a list that will be at least close to optimal over a wide range of supersymmetric model space.

3. SIMULATION AND SIGNATURE SELECTION

To perform the optimization procedure we constructed a large number of model families in the manner described in Section 2.1, each involving the range \(-0.5 \leq \alpha \leq 1.0 \) for the parameter \( \alpha \) in steps of \( \Delta \alpha = 0.05 \). For each point along these model lines we generated 100,000 events using PYTHIA 6.4 and PS4 using the default level-one triggers. To this we added an appropriately-weighted Standard Model background sample consisting of 5 fb\(^{-1} \) each of t\( \bar{t} \) and b\( \bar{b} \) pair production, high-\( p_T \) QCD dijet production, single \( W^\pm \) and \( Z \)-boson production, pair production of electroweak gauge bosons (\( W^+W^- \), \( W^\pm Z \) and \( ZZ \)), and Drell-Yan processes. Further object-level cuts were then performed, followed by an event-level cut on the surviving detector objects similar to those used in [2]. Specifically we required all events to have missing transverse energy \( E_T > 150 \) GeV, transverse sphericity \( S_T > 0.1 \), and \( H_T > 600 \) GeV (400 GeV for events with 2 or more leptons) where \( H_T = E_T + \sum_{\text{jets}} P_T^{\text{jet}} \).

To examine the effectiveness of our candidate signature sets at measuring the value of the parameter \( \alpha \) we fixed “model \( A \)” to be the point on each of the model lines with \( \alpha = 0 \) and then treated each point along the line with \( \alpha \neq 0 \) as a candidate “model \( B \)” Clearly each model line we investigated – and each \( \alpha \) value along that line – gave slightly different sets of maximally effective signatures. The lists we will present represent an ensemble average over these model lines.

As a straw man we may consider the single most effective signature at separating models with different values of the parameter \( \alpha \). It is the effective mass formed from all objects in the event

\[
M_{\text{eff}} = E_T + \sum_{\text{all}} p_T^{\text{all}},
\]

where we form the distribution from all events which pass our initial cuts. To turn this distribution into a count we integrate from \( M_{\text{eff}} = 1250 \) GeV to the end of the distribution. We will refer to the variable in (12) as signature “list” A. That this one signature would be the most powerful is not a surprise given the way we have set up the problem. It is the most inclusive possible signature one can imagine (apart from the overall event rate itself) and therefore has the largest overall cross-section. Furthermore, the variable in (12) is sensitive to the mass differences between the gluino and the lighter electroweak gauginos – precisely the quantity that is governed by the parameter \( \alpha \). Yet as we will see in Section 4 this one signature can often fail to be effective at all in certain circumstances, resulting in a rather large required \( L_{\min} \) to be able to separate \( \alpha = 0 \) from non-vanishing cases. In addition it is built from precisely the detector objects that suffer the most from experimental uncertainty. This suggests a larger and more varied set of signatures would be preferable.

We next consider the five signatures in Table [1]. These signatures were chosen by taking our most
Table 1

Signature List B. Distributions are integrated from “Min Value” to “Max Value”.

| Description                  | Min Value | Max Value |
|------------------------------|-----------|-----------|
| 1 $M_{\text{jets}}^{\text{eff}}$ [0 leptons, $\geq 5$ jets] | 1100 GeV  | End       |
| 2 $M_{\text{any}}^{\text{eff}}$ [0 leptons, $\leq 4$ jets] | 1450 GeV  | End       |
| 3 $M_{\text{any}}^{\text{eff}}$ [$\geq 1$ leptons, $\leq 4$ jets] | 1550 GeV  | End       |
| 4 $p_T$(Hardest Lepton) [$\geq 1$ lepton, $\geq 5$ jets] | 150 GeV   | End       |
| 5 $M_{\text{inv}}^{\text{jets}}$ [0 leptons, $\leq 4$ jets] | 0 GeV     | 850 GeV   |

effective observables and restricting ourselves to that set for which $\epsilon = 10\%$. We again see the totally inclusive effective mass variable of (12) as well as the more traditional effective mass variable, $M_{\text{jets}}^{\text{eff}}$, defined via (12) but with the scalar sum of $p_T$ values now running over the jets only. We now include the $p_T$ of the hardest lepton in events with at least one lepton and five or more jets, as well as the invariant mass $M_{\text{inv}}^{\text{jets}}$ of the jets in events with zero leptons and 4 or less jets. The various jet-based effective mass variables would normally be highly correlated with one another if we were not forming them from disjoint partitions of the overall data set. The favoring of jet-based observables to those based on leptons is again largely due to the fact that jet-based signatures will have larger effective cross-sections for reasonable values of the SUSY parameters in (2) than leptonic signatures. The best signatures are those which track the narrowing gap between the gluino mass and the electroweak gauginos and the narrowing gap between the lightest chargino/second-lightest neutralino mass and the LSP mass. In this case the first leptonic signature to appear -- the transverse momentum of the leading lepton in events with at least one lepton -- is an example of just such a signature.

Finally, let us consider the larger ensemble of signatures in Table 2. In this final set we have relaxed our concern over the issue of correlated signatures, allowing as much as 30% correlation between any two signatures in the list. This allows for a larger number as well as a wider variety of observables to be included. This can be very important in some cases in which the supersymmetric model has unusual properties, or in cases where the two $\alpha$ values being considered give rise to different mass orderings (or hierarchies) in the superpartner spectrum. In displaying the signatures in Table 2 we find it convenient to group them according to the partition of the data being considered. Note that the counting signatures are taken over the entire data set.

The first counting signature is simply the total size of the partition in which the events have at least one lepton and 4 or less jets. The next two signatures are related to “spoiler” modes for the trilepton signal. Note that the trilepton signal itself did not make the list: this is a wonderful discovery mode for supersymmetry, but the event rates between a model with $\alpha = 0$ and one with non-vanishing $\alpha$ were always very similar (and low). This made the trilepton counting signature ineffective at distinguishing between models. By contrast, counting the number of b-jet pairs (a proxy for counting on-shell Higgs bosons) or the number of opposite-sign electron or muon pairs whose invariant mass was within 5 GeV of the Z-mass (a proxy for counting on-shell Z-bosons) were excellent signatures for separating models from time to time. This was especially true when the two models in question had very different values of $\alpha$ such that the mass differences between $\tilde{N}_2$ and $\tilde{N}_1$ were quite different in the two cases. Signature #8 is defined as the following ratio

$$r_{\text{jet}} \equiv \frac{p_T^{\text{jet3}} + p_T^{\text{jet4}}}{p_T^{\text{jet1}} + p_T^{\text{jet2}}}$$

(13)

where $p_T^{\text{jet}i}$ is the transverse momentum of the $i$-th hardest jet in the event. For this signature we require that there be at least three jets with $p_T > 200$ GeV. This signature, like the $p_T$ of the
Table 2: Signature List C. Distributions are integrated from “Min Value” to “Max Value”.

| Description | Min Value | Max Value |
|-------------|-----------|-----------|
| Counting Signatures | | |
| 1 | \(N_t\) | \([\geq 1\text{ leptons}, \leq 4\text{ jets}]\) | |
| 2 | \(N_{t\pm t^-}\) \((M_{\text{inv}}^{t^\pm t^-} = M_Z \pm 5\text{ GeV})\) | |
| 3 | \(N_B\) | \([\geq 2\text{ B-jets}]\) | |
| & \([0\text{ leptons, } \leq 4\text{ jets}]\) & | |
| 4 | \(M_{\text{any eff}}\) | 1000 GeV | End |
| 5 | \(M_{\text{jets inv}}\) | 750 GeV | End |
| 6 | \(E_T\) | 500 GeV | End |
| & \([0\text{ leptons, } \geq 5\text{ jets}]\) & | |
| 7 | \(M_{\text{any eff}}\) | 1250 GeV | 3500 GeV |
| 8 | \(r_{\text{jet}}\) \([\geq 3\text{ jets } > 200\text{ GeV}]\) | 0.25 | 1.0 |
| 9 | \(p_T\) \((4\text{th Hardest Jet})\) | 125 GeV | End |
| 10 | \(E_T/M_{\text{any eff}}\) | 0.0 | 0.25 |
| & \([\geq 1\text{ leptons, } \geq 5\text{ jets}]\) & | |
| 11 | \(E_T/M_{\text{any eff}}\) | 0.0 | 0.25 |
| 12 | \(p_T\) \((\text{Hardest Lepton})\) | 150 GeV | End |
| 13 | \(p_T\) \((4\text{th Hardest Jet})\) | 125 GeV | End |
| 14 | \(E_T + M_{\text{jets eff}}\) | 1250 GeV | End |

hardest lepton or the \(p_T\) of the 4th hardest jet, was effective at capturing the increasing softness of the products of cascade decays as the value of \(\alpha\) was increased away from \(\alpha = 0\).

4. RESULTS

In this section we examine how well our signature lists in perform in measuring the value of the parameter \(\alpha\) which appears in (1). These lists were designed through the analysis of several hundred “model lines” as described previously. To test the approach we generated a random collection of 500 new models with six points on the \(\alpha\)-lines ranging from 0 to 0.5. In this case a 16-dimensional parameter space defined by the quantities in (2) was considered. Specifically, slepton and squark masses were allowed to vary in the range 300 GeV to 1200 GeV with the masses of the first and second generation scalars kept equal. The gaugino mass scale given by \(M_3\) and the \(\mu\)-parameter were also allowed to vary in this range. The pseudoscalar Higgs mass \(m_A\) was fixed to be 850 GeV and the value of \(\tan\beta\) was allowed to vary from 2 to 50. If all points along the \(\alpha\)-line satisfied all experimental constraints on the superpartner mass spectrum, then 100,000 events were generated for each of the six points along the \(\alpha\)-line in the manner described in Section 3. Using this data the value of \(L_{\text{min}}\) was computing using (10) for each of our three signature sets.

The results of this analysis are given in Figures 1 and 2. In Figure 1 the integrated luminosity needed to detect \(\alpha \neq 0\) for 95% of our random models is given as a function of the five non-vanishing \(\alpha\) values simulated. Since the random model sample includes examples with very different superpartner mass scales, the overall supersymmetric production cross-section varies much more across this sample than in the controlled model sample described above. We therefore take this into account by plotting the required number
of supersymmetric events in Figure 2. While our single best signature (“List A”) has some resolving power, it is the larger sets that will prove most effective in establish the non-universality of gaugino masses, as to be expected. Yet going from the 5 signatures of List B to the 14 signatures of List C produces only a marginal increase in resolving power. This is evidence of the increased correlations amongst the signatures of List C. It is also demonstrative of the general principle of diminishing returns to adding new observables to any particular set. Even using our best set of signatures (List C) it will require nearly 100 fb\(^{-1}\) to be able to detect non-universality at the level of \(\alpha \approx 0.1\) for an arbitrary supersymmetric model. Yet for the vast majority of models the departure from universality should become apparent after just 10-20 fb\(^{-1}\). Departures from universality at the level of \(\alpha \approx 0.3\) should be apparent using this method for most supersymmetric models after just a few fb\(^{-1}\).

To understand why this approach works, it is useful to examine the signature results themselves. As an example, consider a pair of signatures drawn from List C in Table 2. Figure 3 shows a two-dimensional slice of the signature space “footprint” for our large set of model variations \([32,33,34]\). In these figures the results have been normalized to 5 fb\(^{-1}\) of data. The count rate for signature #9 is shown versus that of signature #11 of List C for the case \(\alpha = 0\) and \(\alpha = 0.33\) (top panel), \(\alpha = 0.66\) (middle panel) and \(\alpha = 1\) (bottom panel). We have chosen this pair for the dramatic separation that can be achieved, though similar results can be obtained with other pairs of signatures.

The power of our inclusive signature list approach lies in the choice of signatures and their ability to remain highly sensitive to changes in the physical behavior of each model. This feature is reflected qualitatively in the visual clustering of the data points, which become progressively more distinct as the parameter \(\alpha\) is increased. As the regions separate it becomes increasingly less likely that a model from one class can be confused with a model from the other class, even when considering statistical fluctuations. In our approach this manifests itself when one computes \(R_{AB}\), which reflects the “distance” in signature space between the two models under comparison, and which becomes large when the models are sufficiently different from one another.

5. CONCLUSIONS

If supersymmetry is discovered at the LHC the high energy community will be blessed with a
large number of new superpartners whose masses and interactions will need to be measured. Undoubtedly performing global fits of the many observables to the parameter space of certain privileged and well-defined benchmark models will be of great help in making sense of this embarrassment of richness. It is an interesting question to ask whether it is possible to fit to certain model characteristics rather than to any particular model itself. Perhaps the most important such characteristic is the pattern of soft supersymmetry-breaking gaugino masses. No other property of the low-energy soft Lagrangian is more easily linked to underlying high-scale physics, particularly if that high-scale physics is of a string-theoretic origin. We are thus interested in asking whether we can identify the presence on non-universalities in the gaugino sector independent of all other properties of the superpartner spectrum. In the present work we have decided to begin this process with a simple parametrization in terms of model “lines”, in the spirit of previous benchmark studies such as the Snowmass Points & Slopes in which only the single non-universality parameter is varied. By understanding how the observable physics at the LHC is affected by this parameter – and then repeating the analysis many times with the other supersymmetric parameters varied – we were able to obtain two sets of observables that fared well in detecting the presence of non-universalities with relatively small amounts of integrated luminosity. The larger of the two sets generally performed slightly better, but at the expense of allowing signatures that are correlated with one another at the 30% level. Requiring a correlation at only the 10% level (and thus using a shorter list of observables) had only a small effect on the ability to distinguish the size of the non-universality parameter $\alpha$. Broadly speaking, we find that a non-universality at the 10% level can be measured with 10-20 fb$^{-1}$ of integrated luminosity over approximately 80% of the supersymmetric parameter space relevant for LHC observables. If we are interested in measurements at only the 30% level these numbers change to 5-10 fb$^{-1}$ over approximately 95% of the relevant parameter space.

Figure 3. Footprint-style plot for a pair of signatures from List C. Total counts for signature #11 versus signature #13 of List C is given for the case $\alpha = 0$ (green triangles) $\alpha = 0$ (black squares). The cases shown are for $\alpha = 0$ versus $\alpha = 0.33$ (top panel), $\alpha = 0.66$ (middle panel) and $\alpha = 1$ (bottom panel). The axes measure the number of events for which the kinematic quantity was in the range given in Table 2. Larger values of the non-universality parameter $\alpha$ correspond to a greater degree of separation between the two model “footprints.”
REFERENCES

1. B. Altunkaynak, P. Grajek, M. Holmes, G. Kane and B. D. Nelson, JHEP 0904, 114 (2009).
2. N. Arkani-Hamed, G. L. Kane, J. Thaler and L. T. Wang, JHEP 0608, 070 (2006).
3. G. L. Kane, P. Kumar, D. E. Morrissey and M. Toharia, Phys. Rev. D 75, 115018 (2007).
4. C. F. Berger, J. S. Gainer, J. L. Hewett, T. G. Rizzo and B. Lillie, Phys. Lett. B 677, 48 (2009).
5. C. F. Berger, J. S. Gainer, J. L. Hewett, B. Lillie and T. G. Rizzo, arXiv:0712.2965 [hep-ph].
6. B. Altunkaynak, M. Holmes and B. D. Nelson, JHEP 0810, 013 (2008).
7. G. Kane and S. Watson, Mod. Phys. Lett. A 23, 2103 (2008).
8. C. Balazs and D. Kahawala, arXiv:0904.0128 [hep-ph].
9. P. Binetruy, G. L. Kane, J. D. Lykken and B. D. Nelson, J. Phys. G 32, 129 (2006).
10. K. Choi and H. P. Nilles, JHEP 0704, 006 (2007).
11. J. L. Kneur and G. Moultaka, Phys. Rev. D 59, 015005 (1999).
12. R. L. Arnowitt and P. Nath, Phys. Rev. Lett. 69, 725 (1992).
13. K. Choi, A. Falkowski, H. P. Nilles, M. Olechowski and S. Pokorski, JHEP 0411, 076 (2004).
14. K. Choi, A. Falkowski, H. P. Nilles and M. Olechowski, Nucl. Phys. B 718, 113 (2005).
15. A. Falkowski, O. Lebedev and Y. Mambrini, JHEP 0511, 034 (2005).
16. K. Choi, K. S. Jeong and K. Okumura, JHEP 0509, 039 (2005).
17. G. F. Giudice, M. A. Luty, H. Murayama and R. Rattazzi, JHEP 9812, 027 (1998).
18. L. Randall and R. Sundrum, Nucl. Phys. B 557, 79 (1999).
19. S. Kachru, R. Kallosh, A. Linde and S. P. Trivedi, Phys. Rev. D 68, 046005 (2003).
20. M. Grana, Phys. Rept. 423, 91 (2006).
21. M. K. Gaillard and B. D. Nelson, Int. J. Mod. Phys. A 22, 1451 (2007).
22. P. Binetruy, M. K. Gaillard and Y. Y. Wu, Nucl. Phys. B 481, 109 (1996).
23. P. Binetruy, M. K. Gaillard and Y. Y. Wu, Nucl. Phys. B 493, 27 (1997).
24. J. A. Casas, Phys. Lett. B 384, 103 (1996).
25. P. Binetruy, M. K. Gaillard and Y. Y. Wu, Phys. Lett. B 412, 288 (1997).
26. M. K. Gaillard and B. D. Nelson, Nucl. Phys. B 571, 3 (2000).
27. P. Binetruy, M. K. Gaillard and B. D. Nelson, Nucl. Phys. B 604, 32 (2001).
28. G. L. Kane, J. D. Lykken, S. Mrenna, B. D. Nelson, L. T. Wang and T. T. Wang, Phys. Rev. D 67, 045008 (2003).
29. E. Katz, Y. Shadmi and Y. Shirman, JHEP 9908, 015 (1999).
30. R. Rattazzi, A. Strumia and J. D. Wells, Nucl. Phys. B 576, 3 (2000).
31. P. Binetruy, G. L. Kane, B. D. Nelson, L. T. Wang and T. T. Wang, Phys. Rev. D 70, 095006 (2004).
32. J. L. Bourjaily, G. L. Kane, P. Kumar and T. T. Wang, arXiv:hep-ph/0504170.
33. G. L. Kane, P. Kumar and J. Shao, J. Phys. G 34, 1993 (2007).
34. G. L. Kane, P. Kumar and J. Shao, Phys. Rev. D 77, 116005 (2008).
35. B. C. Allanach et al., Eur. Phys. J. C 25, 113 (2002).