Direct and inverse measurement of thin films magnetostriction.

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(Dated: November 10, 2009)

Two techniques of measurements of thin film magnetostriction are compared: direct, when changes of the substrate curvature caused by the film magnetization are controlled, and inverse ("indirect"), when the modification of the magnetic anisotropy induced by the substrate deformation (usually bending) is measured. We demonstrate how both the elastic strength of the substrate and the effective magneto-mechanical coupling between the substrate deformation and magnetic anisotropy of the film depend on different conditions of bending. Equations to be used for magnetostriction value determination in typical cases are given and critical parameters for the corresponding approximations are identified.

PACS numbers: 46.70.De; 62.20.dj; 75.70.Ak; 75.80.+q; 85.85.+j

I. INTRODUCTION

There are two alternatives for magnetostriction measurements of bulk ferromagnetic materials: it can be measured either directly, as it was initially observed from deformation under the magnetic field, or by an inverse way measuring the magnetic anisotropy induced by mechanical stress applied to the sample. In thin films the magnetostriction deformations are hindered by the substrate and only a secondary much smaller substrate deformation can be detected. By analogy to the bulk case, the direct technique uses the substrate deformation caused by the modification of the film magnetization. The inverse technique (often called "indirect", the term we find not very adequate) implies studies of the thin film anisotropy changes caused by the deformation of the substrate (see and references therein).

Description of these techniques involves elastic properties of both the thin film and the substrate and, in particular for the direct technique, it took a long time to find the right equation to extract the magnetostriction parameter. The entire mechanical problem is so complex that even after the right solution was well established other contradicting theories are published. The theory for the inverse method appears to be simpler because the shape of the substrate is controlled directly and one can consider only the thin film properties. Nevertheless, different expressions of the stress induced anisotropy can be found in different papers without a clear explanation of their origin. The difficulties come from an interplay between longitudinal and transverse deformations in response to uniaxial stresses related by Poisson’s ratio. The problem is crucial for applications of cantilevers as sensors and actuators where the role of the Poisson’s ratio was specially investigated.

The rigorous solutions for the bimorphous plate bending includes so heavy and not transparent equations that for scientists of the magnetic community it is difficult to understand the reason why the Poisson’s ratios of the substrate and of the film appear in so different forms. At the same time the direct experimental verification of the theory is not convincing due to the fact that the parameters of thin films can vary significantly compared to the bulk ones.

In this paper without repeating the complete derivation we present simple criteria for choosing the correct equation in every practical case. We pay particular attention to the limits of the validity of known solutions and we carry out comparative measurements of magnetostriction of the same thin film sample using both direct and inverse techniques.

II. REVISION OF THE THEORY

Already the existence of two "alternative" theories of the magnetostrictive effect creates some confusions. The anisotropic magnetostriction can be introduced in two ways. First, as the proper material deformations $\varepsilon^s_{ij}$ seen experimentally when the sample reaches the magnetic saturation (traditional technical definition). In the simplest case of an isotropic material $\varepsilon^s_{ij}$ are related only to the magnetization direction ( $x$ axis in Fig. 1h) and are given by a single magnetostrictive coefficient $\lambda_s$:

$$
\varepsilon^s_{xx} = +\lambda_s \\
\varepsilon^s_{yy} = \varepsilon^s_{zz} = -\frac{1}{2}\lambda_s
$$

In the second approach the magnetostriction is defined as the proper stresses originating from internal magnetic interactions (first principle approach, see Fig. 1b). For an isotropic material:

$$
\sigma^\lambda_{xx} = \sigma^\lambda \\
\sigma^\lambda_{yy} = \sigma^\lambda_{zz} = -\frac{1}{2}\sigma^\lambda
$$
These definitions are equivalent providing that the internal magnetostrictive stresses $\sigma^\lambda_{ij}$ produce elastic deformations equal to the magnetostrictive ones $e^\lambda_{ij}$, i.e.

$$\sigma^\lambda = \lambda_s \frac{E}{1 + \nu}$$

where $\nu$ is the Poisson’s ratio and $E$ is the Young modulus.

In general, the initial magnetic state of the sample is not known and the only way to obtain the magnetostriction value is to saturate magnetization successively in two orthogonal directions (say $x$ and $y$) and to measure the difference of deformation or stress between these two states (i.e. $e^\lambda_{xx} - e^\lambda_{yy} = \frac{3}{2} \lambda_s$ or $\sigma^\lambda_{xx} - \sigma^\lambda_{yy} = \frac{3}{2} \sigma^\lambda$).

The corresponding energy density of interaction between magnetostrictive deformation and elastic deformation (magneto-elastic energy) can be obtained from two equivalent general expressions: $w_{ms} = -\sum_{ij} e^\lambda_{ij} \sigma_{ij} = -\sum_{ij} e_{ij} \sigma^\lambda_{ij}$.

Very often the last expression is presented in a form where instead of $\sigma^\lambda_{ij}$ terms of type $B^{\gamma,2} \cos^2 \phi_i$ are used, where $\phi_i$ is the angle between the magnetization and the $i$-axis.

Combining all together for our isotropic case we obtain the angular variation of the magnetoelastic energy density for magnetization rotation in the basal plane ($x, y$):

$$w_{ms}(\phi) = -\frac{3}{2} \lambda_s (\sigma^\lambda_{xx} - \sigma^\lambda_{yy}) \cos^2(\phi)$$

$$= -\frac{3}{2} \lambda_s (e^\lambda_{xx} - e^\lambda_{yy}) \cos^2(\phi)$$

$$= B^{\gamma,2} (e^\lambda_{xx} - e^\lambda_{yy}) \cos^2(\phi)$$

where $\phi$ is the angle between the magnetization and $x$ axis.

From this equation one can see that the physical meaning of $B^{\gamma,2}$ referred usually as the magnetoelastic coupling constant represents the characteristic magnetostrictive stress $\sigma^\lambda$ illustrated in figure 1. $B^{\gamma,2} = -\frac{3}{2} \sigma^\lambda = -\frac{3}{2} \frac{E}{1 + \nu} \lambda_s$.

For a thin magnetostrictive film deposited onto a substrate the planar components ($e^\lambda_{xx}, e^\lambda_{yy}$ and $e^\lambda_{xy}$) of the elastic deformation are fixed by the corresponding components of elastic deformations on the substrate surface. For a cantilever substrate elastically bent along its length $x$ these elastic deformations are proportional to the curvature $1/R$. A general expression of the energy of the unit area of such a sample in terms of $1/R$ can be given as follows:

$$W = W_{el} + W_{ms} + W_{an}$$

$$= \frac{1}{2} A d \left( \frac{1}{R} \right)^2 - B t \left( \frac{1}{R} \right) \cos^2(\phi) - K t \cos^2(\phi)$$

where $W_{el}$ is the integral of the elastic energy density over the substrate thickness $d$; $W_{ms} = t w_{ms}$ ($t$ - thin film thickness) and the magnetic anisotropy energy density of the film is $W_{an}/t$.

For simplicity we consider only the uniaxial magnetic anisotropy with easy axis along the sample length. Here $K$ is the usual anisotropy constant.

By analogy with the bulk measurements the direct way to get the films magnetostriction is to turn the magnetization direction from $\phi = 0$ to $90^\circ$ and to observe the corresponding change of the equilibrium substrate curvature that can be calculated by minimization of eq. 5:

$$\Delta \left( \frac{1}{R} \right) = \frac{B t}{A d}$$

Practically this effect can be measured by deviation of a laser beam reflected from a point near to the free sample end: $\Delta \theta = 2L \Delta \left( \frac{1}{R} \right)$. Here $L$ is the sample length (more precisely the distance between the support and the laser spot on the sample).

Alternative way to measure the magnetostriction is the more frequently used inverse technique: $\left( \frac{1}{R} \right)$ is modified.
and according to eq. (5) the corresponding changes of the effective uniaxial anisotropy constant is measured:

\[ K_{\text{eff}} = K + B \frac{1}{R} \]  

(7)

As follows from eq. (6) and (7) both the direct and the inverse techniques should give the same result: the value of coefficient \( B \) where the magnetostrictive parameters of the film are "hidden". The equivalence of these techniques is rarely discussed because they are practically never used together.

A closer look to the problem shows that the parameters \( A \) and \( B \) may be different for different experiments and thus comparison of data obtained from different techniques is not so obvious. Moreover not only direct and inverse experiments should be compared with care but also data obtained from the same technique should be treated differently depending on the geometry of the substrate bending in the given experiment.

In general, the Young moduli and Poisson’s ratios of the substrate and of the film material are different and should be distinguished when used together. Below we demonstrate how different algebraic forms of the Poisson’s ratio of the cantilever substrate \( \nu_s \) enters to \( A \) and \( B \) in two characteristic cases: anticlastic bending and cylindrical bending. And we show in a simple way why the Poisson’s ratio of film \( \nu_f \) and Young moduli of both the substrate \( E_s \) and the film \( E_f \) appear always in the same form for all kinds of deformations. One can easily understand the variety of these forms of \( \nu \) (\( \nu_s \) or \( \nu_f \)) by reviewing characteristic cases of elementary deformations presented in Appendix A.

Small bending of a cantilever in one direction inevitably induces an opposite bending in perpendicular direction in order to satisfy the absence of transversal stresses at its free side edges. In this case the principal surface deformation is followed by the transversal deformation as for an elastic element under uniaxial stress (configuration 1 in Appendix A). These two opposite bending produces an anticlastic shape of the cantilever that can not develop simultaneously with increasing deformation. Due to the corresponding geometrical constrain the process becomes non linear and a much more complicated deformation appears for large cantilever curvature. This effect is usually not considered in the papers on thin film magnetostriction while it can be non negligible as shown below.

The conditions when bending can be considered small and the corresponding equations are linear can be found only in specialized papers on the mechanics of thin plates. The relevant critical parameter for thin cantilever with large length \( L \) to width ratio \( W \) is \( \beta \sim W/\sqrt{Rd} \).

The linear transversal anticlastic deformation across the sample width \( z = y^2 \nu_s/R \) is valid when this value at the sample edge \( y = W/2 \) is less than about 0.1d for \( \nu \approx 1/3 \) i.e. \( \beta < 1 \) (see Appendix B).

At larger \( \beta \) the anticlastic profile cannot progress anymore in the same way because an additional edge axial tension created by it can no longer be neglected. The shape of the plate flattens and for \( \beta \gtrsim 3 \) one can consider the bending as purely cylindrical. Another case of the cylindrical bending is when transverse deformation is blocked near to the sample fixed end by the support (clamping effect). An immediate consequence of the fixed transversal deformations is the appearance of the transversal stresses \( \sigma_{yy} \) (configuration 2 in Appendix A).

It is obvious that both the effective elasticity coefficient of the plate bending \( A \) and the coupling coefficient \( B \) between the bending and the magnetostrictive stress in the film are different in these two limit cases.

Pure bending in the principal cantilever direction \( x \) produces the longitudinal elastic deformations \( e_{xx}(z) = z/R \) (see Fig. 4).

In the anticlastic case the elastic energy of the unit area of the curved plate is

\[
W_{el} = \int_{-d/2}^{d/2} \frac{1}{2} \sigma_{xx} e_{xx} dz = \frac{1}{24} E_s d^3 \left( \frac{1}{R} \right)^2
\]

(8)

taking into account the relation between \( e_{xx} \) and \( \sigma_{xx} \) (configuration 1 in Appendix A). \( \sigma_{xx} = E e_{xx} \). This gives \( A = 1/12 \ E_s d^2 \).

The magnetoelastic coupling energy of the thin film with planar magnetization is fully determined by the surface deformation of the much thicker substrate:

\[
W_{ms} = tB^{\gamma,2} (\varepsilon_{xx} - \varepsilon_{yy}) = tB^{\gamma,2} \frac{d}{2R} (1 + \nu_s) \cos^2 \phi
\]

(9)

that gives \( B = dB^{\gamma,2} (1 + \nu_s) \). Thus expression (7) of the effective magnetic anisotropy for inverse magnetostriction measurements in the anticlastic case includes the Poisson’s ratio of the substrate in the form \((1 + \nu_s)\).

From eq. (6) we obtain the variation of the curvature \( 1/R \) produced by the magnetization rotation from \( \phi = 0 \) to \( 90^\circ \) to be used in the direct technique:

\[
\Delta \left( \frac{1}{R} \right)_{\text{anticlastic}} = \frac{Bt}{Ad} = \frac{6}{E_s} \frac{1 + \nu_s t}{d^2} B^{\gamma,2}
\]

(10)

This result exactly corresponds to the solution given by Trémolet de Lacheisserie.

We should remember here that the Young’s modulus \( E_s \) and Poisson’s ratio \( \nu_s \) of the substrate enters into \( W_{el} \) and \( W_{ms} \) while \( B^{\gamma,2} \) includes the Young’s modulus \( E_f \) and the Poisson’s ratio \( \nu_f \) of the film.

For the cylindrical curvature, one have to use the relation between \( \sigma_{xx} \) and \( e_{xx} \) corresponding to the fixed transversal deformation (\( \sigma_{yy} = 0 \), configuration 2 in Appendix A). \( \sigma_{xx} = E/(1 - \nu^2) e_{xx} \). In this case, the integration (8) gives harder plate bending coefficient \( A = 1/12 \ E d^2 / (1 - \nu^2) \). At the same time the elastic coupling
becomes smaller $B = \frac{1}{2} dB^{2}$. Consequently, $\nu_s$ does not enter into the equation (7) of $K_{eff}$ used for inverse measurements unlike the previous anticlastic case. 

As to the direct measurements, eq. (9) provides:

$$\Delta \left( \frac{1}{R} \right)_{cylindrical} = Bt \frac{E_s}{Ad} = 0 \frac{1 - \nu^2}{d^2} B^{2/3} \text{(11)}$$

This equation giving $(1 - \nu_s)$ smaller substrate curvature than in the anticlastic case can be found in the literature, often without any justification. In reality, it has no practical application because in direct measurements the cantilever curvature is extremely small ($\beta \ll 1$) and clamping effect can be neglected for cantilevers with large $L/W$ ratio.

One should note that the elastic deformation $e_{xx}$ due to the plate bending is not the only deformation produced by the magnetostrictive stress during direct measurements of magnetostriction. This stress also changes the sample length creating homogeneous deformation $e_{xx}^L$. Consequently the position of the neutral line – where total longitudinal deformation $e_{xx}(z) + e_{xx}^L = 0$ – is shifted as discussed by many authors (see for example ref. 21). The elastic energy corresponding to this additional deformation does not interfere with the energy of the pure substrate bending and, thus eq. (9) is unaffected. The shift of the neutral line can be illustrated by considering separately symmetrical and antisymmetrical deformations produced by films deposited on both surfaces of substrates (see Appendix [C]). This small $e_{xx}^L \approx \lambda_s t/d$ usually does not produce any measurable effect.

III. EXPERIMENTAL DETAILS

We apply both direct and inverse techniques to the same sample. For illustration we have chosen a $(\text{[Tb34%Co66%]}60 \text{ Å/}[\text{Co12%Fe58%}]50\text{ Å}) \times 10 \text{ multilayer}$ deposited onto rectangular Corning glass substrate $(22 \times 5 \times 0.145 \text{ mm}^3)$ from CoFe and TbCo mosaic 4 inch targets using a Z550 Leybold RF sputtering equipment with a rotary table technique. An uniaxial anisotropy along sample length was induced by the field of a permanent magnet applied during deposition process. The direct measurements were realized using cantilever deflectometry technique. Details of the sample preparation and of the cantilever measurements can be found in [22].

For the inverse measurement we have studied local magnetization curves of bent sample using the magneto-optical Kerr effect (MOKE) as shown in figure 2. We have chosen the longitudinal Kerr effect geometry for which the planar magnetization component parallel to the plane of incidence is measured. The incident laser light is polarized vertically and the light reflected from the film surface is split by a Wollaston prism to 2 photodetectors so that their signal difference is proportional to the Kerr rotation. The incidence plane and the sample surface are parallel to the direction of the applied magnetic field. The sample can be rotated in its plane in order to measure the film local magnetization either along its length (as shown in the figure) or along its width.

The sample is mounted on a special bending jig shown Fig.2. One end of the sample is glued on a support so that the film is on the top free surface from which the laser is reflected. The other sample end can be moved vertically ($z_L$) by a screw in order to bend it up or down. To respect the sign given in the theoretical equations we define positive curvature when the sample is bent downwards, and negative when the sample is bent upwards. We have found that the most precise way to get the sample curvature in our experimental setup is to analyze the modification of the reflected laser beam. The light spot observed on a screen after reflection from curved surface is spread in two directions proportionally to the principal curvatures. This method is similar to the industrial multi-beam technique of measuring curved surfaces. With a single beam we can measure the curvature at the same small area where the MOKE hysteresis is studied. In order to justify and calibrate our single beam technique we have analyzed the reflection from a number of cylindrical objects of known radius. Measuring our sample we could see that the light reflected near to the support forms on the screen a narrow line thus showing that the sample is curved here only along its length and remains flat in the transverse direction (cylindrical curvature). Further from the support we could see that the laser spot becomes elliptical thus demonstrating the anticlastic shape of the sample deformation (see Appendix [D]).

FIG. 2: (Color on line) Schematic diagram of the indirect measurements of the film magnetostriction using MOKE magnetometer. The geometry of sample bending is shown at the top of the figure. The dashed line shows the neutral line where the longitudinal deformation is zero.
IV. RESULTS AND DISCUSSION

We have studied the variation of the MOKE hysteresis with curvature at $x = 2$ mm from the support where the maximum curvature is achieved. Characteristic results from a sequence of measurements done for different bending in positive and negative directions are shown in Fig. 3. For every curvature the sample has been rotated so as to obtain magnetization cycles both for the magnetic field applied along the sample large axis (full red square symbol) and perpendicularly to it (open blue circle symbol).

As observed with VSM similarly to our previous study\textsuperscript{23}, the unstressed sample has a uniaxial anisotropy: rectangular magnetization along the sample length shows the easy axis and slanted magnetization along the sample width (hard axis magnetization) provides the initial anisotropy value (see Fig. 3(a)).

For positive curvatures (left column), the large axis hysteresis loop shape does not vary, keeping the rectangular shape characteristic for the easy axis magnetization, whereas the short axis hysteresis loop is getting more and more slanted when curvature increases. Thus we clearly see that a positive curvature (stress) reinforce the anisotropy. Negative curvature produces the opposite effect: the uniaxial anisotropy, first, is reduced by the applied stress (the slope of the short axis hysteresis loop is reduced by stress counterbalancing the initial anisotropy) and, then, the easy anisotropy axis switches from longitudinal to transversal direction and the anisotropy increases again. In our sample we observe that this switching is not abrupt: the long axis hysteresis starts to slant before the short axis hysteresis becomes completely rectangular (see Fig. 3 (b')). As we show below, this effect appears if the sample is not perfectly homogeneous.

Contrary to what was observed on the VSM cycles, back and forth branches in the hard axis loop are not fully superimposed and the cycle is somewhat asymmetrical. This is not an unusual feature of MOKE measurements caused by a non linear interference with the transverse magnetization signal\textsuperscript{22}. We have found that for magnetostrictive films this MOKE loop feature may have an additional reason. We observed a superposition of the magnetization and magnetostrictive response when the free sample end did not touch the screw at all and, thus, could move freely under the magnetostrictive stress in the film. Probably a small part of the latter effect can persist even when the sample end is touching the screw.

This small distortion of the hard axis magnetization loop can be excluded by the following procedure of the determination of the uniaxial anisotropy field $H_K$.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{fig3}
\caption{(Color on line) MOKE magnetization loops measured close to the sample support ($x = 2$ mm) for different positive (left column) and negative (right column) cantilever curvatures. Red full square symbols - magnetic field applied along the sample. Blue open circle symbols - magnetic field applied across the sample as shown in c) and c') near to the corresponding curves.}
\end{figure}

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{fig4}
\caption{(Color on line) Dependence of anisotropy field measured as a function of curvature for fixed position ($x = 2$ mm). Positive values of $H_K$ (open blue circles are obtained for $H \parallel x$) and negative values of $H_K$ (full red squares are obtained for $H \parallel y$).}
\end{figure}
The loop is firstly "symmetrized" by addition of the back and forth branches. Then the central part of it \([-20 \text{ Oe}; +20 \text{ Oe}]\) is linearly fitted and \(H_K\) is determined as the intersection of the fitted straight line with the saturation magnetization level. The anisotropy field \(H_K\) and the corresponding anisotropy constant \(K\) \((H_K = 2K/(\mu_0M_s)\) where \(M_s\) is the saturation magnetization) are defined positive when the hard axis loop is seen for applied field parallel to the small sample axis (see Fig. 3a-c), and negative if the hard axis loop is seen for applied field parallel to the large sample axis (see Fig. 3c').

For a perfectly homogeneous sample with a well defined uniaxial intrinsic anisotropy direction (i.e. long sample axis), we would observe an abrupt reversal between slanted and rectangular loops at the moment of 90° rotation of the easy axis when the initial anisotropy is compensated by the stress induced anisotropy. In reality, as we indicated above (Fig. 3') the transition is gradual. There is an interval around the compensation point where \(H_K\) can be formally measured for both directions. This is due to small variations of both magnitude and direction of the intrinsic anisotropy in the sample area probed by laser. This kind of anisotropy inhomogeneity was already observed and quantified in our previous work. There we have shown that large domains of opposite magnetization exist where maximum easy axis distribution is about ±6° from the average direction over the whole sample. In the intermediate region every measured \(H_K\) corresponds not only to the average anisotropy of given sign but also to its relative occurrence.

Figure 4 shows the evolution of the obtained positive and negative \(H_K\) with the substrate curvature \((R^{-1})\) or the equivalent surface strain \(\varepsilon_{xx} = d/2R\).

In order to get the expected linear variation of \(H_s\) with curvature one has to consider only points outside the intermediate compensation region shaded gray in the figure. Inside this region the slope is about twice smaller because the repartition between the areas with positive and negative \(H_K\) also varies with curvature. So if this region is not identified the resulting magnetostriction value can be considerably reduced. The slope of the line fitting the points outside the compensation region \(\mu_0M_s = 8.2\) Oe.m = 650 A allows us to determine the magnetoelastic coupling coefficient of the film \(B^{γ,2}\).

As mentioned above for this measurement the deformation is cylindrical close to the support. So, for eq. 14 we have to take the corresponding form of the effective magnetoelastic coefficient \(B\):

\[
B^{γ,2} = \frac{\mu_0M_s}{d} \frac{\partial H_K}{\partial (1/R)}
\]  

(12)

With \(\mu_0M_s = 1.4 \text{ ~T}\) obtained from the magnetic measurements and \(d = 145 \text{ ~μm}\) we found \(B^{γ,2} = 6.3 \text{ ~MPa}\).

Far from the support the deformation of the sample gets an anticlastic shape if the bending is not too strong (see Appendix B). As was mentioned above, we observe this anticlastic deformation as the dispersion of the reflected laser beam: the laser spot is spread to an ellipse after reflection from the sample. For the deformation being equivalent to the deformation of narrow cantilevers the magnetoelastic coefficient \(B\) is by \((1 + ν_s)\) larger. Correspondingly one have to use for this case another expression relating \(B^{γ,2}\) and the induced anisotropy:

\[
B^{γ,2} = \frac{1}{1 + ν_s} \frac{\mu_0M_s}{d} \frac{\partial H_K}{\partial (1/R)}
\]  

(13)

In order to observe the effect of this shape modification we have carried out measurements at different points from the support.

MOKE hysteresis loops obtained for a given deformation \(z_L = +1.6 \text{ mm}\), left column, and \(z_L = -1.6 \text{ mm}\) right column, when laser spot is swept along the sample length from a position close to the clamped tip \((x = 2 \text{ mm})\) to \(x = 14 \text{ mm}\) through \(x = 8 \text{ mm}\) are presented Fig. 5.

For positive deviation \((z_L = +1.6 \text{ mm})\), the closer to the clamped sample tip \((x = 2 \text{ mm})\) the more anisotropic the sample is: The small axis hysteresis loop is more slanted when recorded closer to the clamped tip. The hard axis (small side of the sample) is thus getting harder when \(x\) decreases. \(H_K\) as defined above remains positive decreasing with \(x\) from its maximal value to its initial value close to the free end. The reason for this is that the created curvature and the corresponding induced anisotropy reduce progressively with the distance \(x\) from the support completely disappearing at the free end \(x = L\) (see Appendix C).

For negative deviation \((z_L = -1.6 \text{ mm})\) we observe the inversion of easy and hard direction at small \(x\) where the induced anisotropy overcomes the initial one. Correspondingly, \(H_K\) varies from a negative to a positive value along the sample length. This transition is not abrupt similarly to the modification of \(H_K\) as function of the bending curvature discussed above and there is an intermediate compensation area approximatively in the middle of the sample. We represent the evolution of the induced anisotropy along the sample length by the difference between \(H_K\) obtained for positive \((H_K^+)\) and negative \((H_K^-)\) bending (see Fig. 6). Such presentation allows us to exclude the inhomogeneity of the initial anisotropy of the undeformed sample. We can distinguish two kinds of points: when \(H_K^-\) and \(H_K^+\) have different signs in the part of the sample close to the support (open circles in the figure) and when \(H_K^-\) and \(H_K^+\) remain both positive in the second part of the sample (closed square in the figure). Similarly to the evolution with \(1/R\) discussed before these two kinds of points coexist in the intermediate compensation area and expected theoretical value should lie somewhere between them. Except of the inconvenience caused by the sample inhomogeneity, there are two effects to be considered for quantitative analysis of these data: the effect of the support clamping and the
edge blocking for large curvature.

The cylindrical shape imposed by the clamping at the support extends to some distance thus producing in this region the strengthening of its bending and the reduction of the induced anisotropy as discussed above (see discussion before eq. (11)). The total shape of the clamped sample does not depend on the bending amplitude until the curvature is small enough and the linear equations can be used. The variation of the induced anisotropy due to clamping effect can be illustrated by eq. (C9) of Appendix C.

\[ R_d \lesssim 1 \] is valid, for our sample is \((1/R_{\text{max}}) = 6 \text{ m}^{-1}\). In our experiment the largest sample curvature considerably overcomes this value near to the support (see Fig. 3) and a further gradual modification of the anisotropy with curvature should be added according to the calculated correction \(\kappa(1/R)\) (see Appendix B, eq. (B7) and Fig. 9b)). Since \(1/R\) reduces almost linearly along the sample length, the low curvature approximation can be used till maximal allowed bending only close to the sample free end \((x \gtrsim 14 \text{ mm})\) where, unfortunately, the induced anisotropy is too small for precise measurements of the magnetostriction.

Thus, in the main part of the sample we have a complicated superposition of at least three effects: the compensation effect with the revealed inhomogeneous initial anisotropy, the clamping effect and the non linear shape effect.

Interestingly, our experimental measurements near to the cantilever support discussed above (Fig. 3, 4) have no such complication. The clamping of the cantilever end imposes the cylindrical curvature at any width to length ratio and at any bending amplitude, and excludes all other effects considered above. Consequently the determination of the magnetoelastic coefficient \(B^{\gamma,2}\) has less uncertainty.

![FIG. 5: (Color on line) MOKE magnetization loops measured for opposite sample bending \(z_L = +1.6 \text{ mm}\) (left column) and \(z_L = -1.6 \text{ mm}\) (right column) at different position on the sample: \(x = 2 \text{ mm}\), \(x = 8 \text{ mm}\) and \(x = 14 \text{ mm}\). Red full square symbols - magnetic field applied along the sample length and blue open circle symbols - magnetic field applied across the sample.](image)

An example of such an estimation is shown by the dashed curve in Fig. 6 calculated with the following parameters of our sample: sample length \(L = 18.5 \text{ mm}\), \(\nu_s = 0.2\), characteristic length of the clamping effect \(a \approx W/2 = 2.5 \text{ mm}\) and amplitude coefficient \(6FB^{\gamma,2}/E_dW = 1.2 \times 10^6 \text{ A/m}^2\). The theoretical predicted deviation from the linear dependence is clearly visible close to the sample support.

The maximum curvature, for which the condition of this linear approximation \(\beta \sim W/\sqrt{Rd} \lesssim 1\) is valid, for our sample is \((1/R)_{\text{max}} = 6 \text{ m}^{-1}\). In our experiment the largest sample curvature considerably overcomes this value near to the support (see Fig. 3) and a further gradual modification of the anisotropy with curvature should be added according to the calculated correction \(\kappa(1/R)\) (see Appendix B, eq. (B7) and Fig. 9b)). Since \(1/R\) reduces almost linearly along the sample length, the low curvature approximation can be used till maximal allowed bending only close to the sample free end \((x \gtrsim 14 \text{ mm})\) where, unfortunately, the induced anisotropy is too small for precise measurements of the magnetostriction.

For our direct measurements of magnetostriction by deflectometry the anticlastic shape of the sample can be always considered: The aspect ratio \(L/W \sim 4\) of the cantilever is sufficiently large to neglect the effect of clamping and the curvature is extremely small. Thus corresponding anticlastic parameter \(\beta \lesssim 0.01\) and only eq. (10) have to be used.

![FIG. 6: (Color on line) Modification of the induced anisotropy along the sample length. Open circles shows results involving anisotropy field \(H^+\) created by positive bending and \(H^-\) created by negative bending. Results shown by full squares involve positive anisotropy fields \(H^+\) and \(H^-\) for both opposite bending. A theoretical estimation taking into account the clamping effect is shown by the dashed line.](image)
parallel to the sample length (easy axis) and perpendicularly to it. The easy axis magnetization, as should be, does not produce any magnetostrictive effect. The maximum laser deflection in the saturating perpendicular field is \( \Delta \theta = 0.15 \) mrad giving the sample curvature \( \Delta (1/R) = \Delta \theta / 2L = 4.3 \times 10^{-3} \text{m}^{-1} \) (distance between the support and the laser spot on the sample is \( L = 17.5 \) mm). From these measurements we obtain \( B_s^{\gamma, 2} = 6.8 \) MPa using eq. (10) with following parameters: \( E_s = 60 \) GPa, \( \nu_s = 0.2, \ d = 145 \) \( \mu \)m, \( t = 110 \) nm.

![Graph](image.png)

**FIG. 7**: (Color on line) Deflectometry measurements of the magnetostrictive bending under the field applied along the sample length and perpendicularly to it as schematically shown near each corresponding curve. The geometry of the laser beam deflection is sketched in the inset.

This value is in good agreement with the value obtained above (6.3 MPa) using the inverse technique. It is difficult to get better correspondence of two techniques taking into account that there are parameters which enter to the calculation for only one of the two techniques: for the direct technique - elastic moduli of the substrate \( E_s \) and \( \nu_s \), sample length \( L \), other dimensions of the installation and the film thickness \( t \); for the inverse technique - saturation magnetization of the film \( M_s \), curvature calibration \( 1/R \) and determination of \( H_K \). Another important difference is that the substrate thickness enters as \( d^2 \) to the direct technique and as \( 1/d \) to the inverse one. Eventual influence of the inhomogeneity on \( B_s^{\gamma, 2} \) also can not be excluded: the direct technique provides the average value over the whole sample whereas the inverse technique measures a local value.

V. CONCLUSION

The comparison of direct and inverse ("indirect") techniques of the film magnetostriction measurements shows that both techniques give exactly the same magnetostrictive stress \( \sigma^\lambda \) (or \( B_s^{\gamma, 2} = -2/3\sigma^\lambda \)) provided all different experimental conditions are correctly taken into account.

It is natural to characterize the thin films magnetostriction by \( \sigma^\lambda \) since the lateral deformations of the film are blocked by the substrate and one can measure only the stress (see Fig. 1b). This is in contrast to measurements of the magnetostrictive deformation \( \lambda_s \) of bulk materials (see Fig. 11).

The key factor to determine the exact expression for \( \sigma^\lambda \) to be used is the shape of the curved cantilever realized in the particular conditions of the experiment. Both the elastic and the magnetoelastic energies depend on the peculiarities of the curved cantilever shape. Normally a small cantilever bending involves anticlastic motion of its side edges.

This motion can be hindered by two effects: clamping by the support and non linear geometrical edge blocking for large cantilever bending. The critical curvature above which this cylindrical bending with completely flat transversal profile appears is \( W^2/Rd \geq 10 \) (more precisely anticlastic parameter \( \beta \geq 3 \)). For the direct measurements the critical curvature can be achieved only for extremely thin cantilevers with \( d \leq \sqrt{\lambda_s W^3 t} \). This critical value is obtained from eq. (10) neglecting the difference of the elastic moduli of the film and the substrate. For \( \lambda_s = 10^{-4}, \ W = 5 \) mm and \( t = 110 \) nm, as an example, this gives \( d \leq 6 \) \( \mu \)m. The indirect measurements are limited by the maximum strain that can be obtained on the cantilever surface without risk to break it \((\epsilon_{\text{max}} \sim 2 \times 10^{-3})\). So, in this case the critical limit to the cantilever thickness is less severe: \( d \leq \sqrt{0.2\epsilon_{\text{max}}W} \) i.e. \( d \leq 100 \) \( \mu \)m for the same numerical values.

In the extreme case of strongly curved thin cantilever clamped at one end the result of the local measurements of the induced anisotropy will vary not only with the distance from the support but also across the sample width: \( K = K(x, y) \) (Appendix C).

Comparing the advantages of the two techniques one can think that the indirect technique is more sensitive and easier to realize and practically unlimited by the film thickness. This is true only for the materials with weak magnetic crystalline anisotropy. When this anisotropy is much larger than the maximum induced anisotropy \( \lambda_s E_f \epsilon_{\text{max}} \) this technique loses its advantages. The magnetostriction measurement of highly anisotropic materials by the direct technique is limited mainly by the availability of strong enough magnetic field \((H > H_K)\). In order to measure materials with low \( \lambda_s \) or very thin films by the direct technique one can reduce the substrate thickness.

**Acknowledgments**

D. Dekadjevi is acknowledged for the development of the MOKE experiment.
APPENDIX A: EFFECTS OF THE POISSON’S RATIO

| Configuration | Description | Equations |
|---------------|-------------|-----------|
| 1             | Free edge load: definition of the Young’s modulus $E$ and the Poisson ratio $\nu$ | $\sigma_{xx} = \frac{E}{1+\nu} \epsilon_{xx}$, $\epsilon_{yy} = -\nu \epsilon_{xx}$ |
| 2             | Fixed edge load | $\sigma_{xx} = \epsilon_{yy} = 0$, $\epsilon_{yy} = -\nu \epsilon_{xx}$ |
| 3             | Antisymmetric load/deformation | $\sigma_{xx} = -\epsilon_{yy} = \frac{1+\nu}{\nu} \epsilon_{xx} = \frac{\sigma_{xx}}{\nu}$ (G–shear modulus) |
| 4             | Symmetric load/deformation | $\sigma_{xx} = \epsilon_{yy} = \frac{1-\nu}{\nu} \epsilon_{xx}$ (Case of isotropic lateral stresses) |

TABLE I: Summary of characteristic appearances of Poisson’s ratio $\nu$

Even in the simple case of isotropic materials the appearance of the Poisson’s ratio $\nu$ in different formula related to the magneto-elastic interactions presents sometimes a "mystery". In order to simplify this issue we give here a simple summary of relations between stresses $\sigma_{ij}$ and deformations $\epsilon_{ij}$ ($i, j = x, y, z$) of an isotropic material for typical conditions of mechanical loads useful for the thin film geometry with their proper characteristic appearance of $\nu$. Let’s consider a small square element with the principal load/deformation along the horizontal $x$ axis and $z$ axis perpendicular to the film surface where the normal stress $\sigma_{zz} = 0$.

When the side edges of the element are free ($\sigma_{yy} = 0$) the lateral linear deformations $\epsilon_{xx}$ and $\epsilon_{yy}$ under uniaxial stress $\sigma_{xx}$ define the Young’s modulus $E$ and the Poisson’s ratio $\nu$ (see configuration 1 in table A). The ratio $\epsilon_{yy}/\epsilon_{xx} = -\nu$ is directly related to the antilastic bending of narrow cantilevers with free side edges.

The elastic properties of the element with fixed side edges $\epsilon_{yy} = 0$ can be obtained through a linear superposition of two crossed loads ($\sigma_{xx}$ and $\sigma_{yy}$) with free edge conditions presented above, where $\sigma_{yy}$ compensates the transverse deformation produced by principal load $\sigma_{xx}$: $\sigma_{yy} = \nu \sigma_{xx}$ (configuration 2 in table A). The resulting strengthening $\sigma_{xx}/\epsilon_{xx}$ of the axial deformation $(1 - \nu^2)^{-1}$ corresponds to the strengthening of strongly bent wide cantilevers where the condition $\epsilon_{yy} = 0$ is due to the transverse flattening caused by the geometrical edge blocking (see Appendix B).

Another useful superposition of two uniaxial loads $\sigma_{xx}$ and $\sigma_{yy} = -\sigma_{xx}$ (antisymmetric case) is equivalent to the pure shear deformation along the diagonal of the element (configuration 3 in table A). It appears in the magnetoelastic coupling where only the difference between two states with stress axes rotating by $90^\circ$ is essential.

Consequently the elastic parameters of the material appear in the form $E/(1 + \nu)$ in these cases (see eq. (9) and (10)).

For completeness we present also the symmetric superposition of $\sigma_{xx}$ and $\sigma_{yy} = \sigma_{xx}$ which correspond to the thermal and epitaxial stresses of thin films (configuration 4 in table A). Factor $1 - \nu$ obtained here instead of the factor $1 + \nu$ in the previous antisymmetric configuration enter to the expression of the substrate curvature (eq. (10)) for this case well known as Stoney’s formula.

APPENDIX B: EVOLUTION OF ANTICLEASTIC DEFORMATION

The details of induced anisotropy of magnetostrictive films on curved substrates related to their antilastic deformation, clamping effect were largely discussed in the literature. All these numerical and analytical solutions were limited to very small deformations. It is interesting to consider the effect of the evolution of the form of a cantilever with increasing curvature. The antilastic deformation can not be maintained for strong curvatures because of appearance of tear stresses at the side edges of the cantilever. General equations describing thin plate 2D profile $Z(x, y)$ are non-linear. For our cantilever with principal curvature $1/R$ along its length $(Z(x, y) = -x^2/2R + z(y))$ there is an analytical solution: the transverse profile of the vertical deviation $z(y)$ can be obtained from the 4th order differential equation:

$$\frac{d^4 z}{dy^4} + 4 \left( \frac{\beta}{W} \right)^4 z = 0$$  \hspace{1cm} (B1)

where

$$\beta^2 = \frac{W^2}{Rd} \left[ 3(1 - \nu^2) \right]^{1/2}$$ \hspace{1cm} (B2)

is a parameter (antilastic parameter) which determines the degree of the non-linearity $\nu$. $W$ is the cantilever width and $d$ its thickness.

The solution of this equation is:
width and its amplitude saturates (Fig. 9a). This edge deformation region is clearly demonstrated on images of bent metallic sheets shown by Conway and Nikola. The transition from anticlastic to cylindrical bending produces corresponding modification of the ratio between induced anisotropy and the principal cantilever curvature $1/R$ [See changes of the forms of the effective coefficient of magnetostriective coupling $B$ in eq. (10) and (11)].

It is interesting to follow the whole evolution of this ratio from small (linear regime) to large (non linear regime) cantilever bending.

The induced anisotropy is determined by $(e_{xx} - e_{yy})$ at the cantilever surface where the film is deposited:

\[
e_{xx} = \frac{d}{2} \frac{\partial^2 Z}{\partial x^2} = -\frac{d}{2} \frac{1}{R} (B5)
\]

\[
e_{yy} = \frac{d}{2} \frac{\partial^2 Z}{\partial y^2} = \frac{d}{2} \frac{\partial^2 z}{\partial y^2} (B6)
\]

For a narrow cantilever $e_{yy} = -\nu e_{xx}$ is valid for any curvature ($\beta \ll 1$ see eq. (11)).

The induced anisotropy measured in the center of wide cantilever of the same curvature must be corrected by a factor:

\[
\kappa(1/R) = \frac{e_{xx} - e_{yy}}{(1+\nu)e_{xx}} = \frac{1}{1+\nu} \left(1 + R \frac{d^2 z}{dy^2} \right|_{y=0} \right) (B7)
\]

For $\nu = 1/3$ the evolution of $\kappa$ is shown in Fig. 9 b).

When bending is small $(1/R < W/d^2$ or $\beta < 1$ ) the anticlastic shape does not depend on the cantilever width $d^2 z/dy^2 = \nu/R$ and $\kappa = 1$. When bending is large $(1/R > 10W/d^2$ or $\beta > 3$) and the center of the cantilever becomes flat ($d^2 z/dy^2 \approx 0$) we have to correct the usual expression of the induced anisotropy by $\kappa = 1/(1+\nu)$.

### APPENDIX C: STRESS VARIATION BY THE CANTILEVER THICKNESS AND LENGTH

There is a large variety of the stress distribution for different cases of the cantilever deformation. We consider here two specific cases relevant to our experiment. In this appendix, for simplicity, we discuss only the free edge linear curvature of the cantilever corresponding to configuration 1 in Appendix A with $e_{xx} = E_s e_{xx}$. Here we have to distinguish Young modulus and Poisson’s ratio for the substrate ($E_s$ and $\nu_s$) and for the magnetostrictive film ($E_f$ and $\nu_f$). The latter appear only in $B_{xx} = -3/2 \lambda \nu E_f/(1+\nu_f)$.

The deformation of the cantilever under the magnetostrictive stress of the film with magnetostriction $+\lambda_s$ deposited onto only one surface is not symmetrical with respect to its midplane. The corresponding effect of the shift of the neutral line of the elastic plate deformation somewhat complicates the analysis provided in different papers[20]. This practical problem can be considered as

$FIG. 8$: (Color on line) Anticlastic deformation of a bent cantilever for different values of the reduced curvature $W^2/Rd$. a) small enough bending $W^2/Rd = 1$ (anticlastic parameter $\beta = 1.28$) b) strong bending $W^2/Rd = 10$ ($\beta = 4$) with a flattening in the center. Transverse profile calculated from eq. (B3) $(z/d)$ is differently magnified in a) and b) for clearness. We added to this figure a gradual flattening qualitatively reproducing the clamping effect of the support (at $x = 0$).

\[
\frac{z}{d} = \sqrt{\frac{\nu}{2(1-\nu)}} \left\{ C_- \cosh (\beta \frac{y}{W}) \cos (\beta \frac{y}{W}) + C_+ \sinh (\beta \frac{y}{W}) \sin (\beta \frac{y}{W}) \right\} (B3)
\]

with

\[
C_- = \frac{\sinh(\beta/2) \cos(\beta/2) \mp \cosh(\beta/2) \sin(\beta/2)}{\sinh(\beta) + \sin(\beta)}
\]

For $\beta \ll 1$, it gives the anticlastic transversal curvature $(\nu/R)$ opposite to the principal curvature of the cantilever $(-1/R)$:

\[
z(y) = \frac{\nu}{2R} (y^2 - W^2/8) (B4)
\]

as illustrated in Fig. 8. It is interesting to note that this parabolic transversal deformation proportional to $1/R$ persists till $\beta \sim 1$.

For larger $\beta$ the cantilever starts to flatten in the center. For $\beta \geq 3$ (see Fig. 8) the anticlastic deformation remains only at the sample edges in a region of $W/\beta$.  

\[
\frac{z}{d} = \frac{\nu}{2R} (y^2 - W^2/8) (B4)
\]
the linear superposition of 2 simpler problems: symmetric - the deformation of the substrate by 2 equivalent films deposited on both surfaces with magnetostriction coefficient $+1/2 \lambda_s$, and antisymmetric - 2 films with opposite magnetostrictions coefficients $+1/2 \lambda_s$ and $-1/2 \lambda_s$ on opposite cantilever surfaces (see Fig. 10).

Let’s consider the influence of the longitudinal stress $\sigma_{xx}^l = \sigma^l$ of the film magnetized along the $x$ axis as indicated in Fig. 10.

In the symmetric case, evidently, the only deformation is lengthening $e_{xx}^L$ and there is no curvature. From the balance of the forces between the constant magnetostrictive stress on both surfaces and the Hooke’s force of the sample lengthening one gets:

$$t \sigma^l = d E_s e_{xx}^L$$  \hspace{1cm} (C1)

In the antisymmetric case the pure curvature $(1/R)$ with the neutral line at the middle of cantilever thickness is produced without any lengthening of the sample, i.e. $e_{xx}(z) = z/R$. Here one should balance the momentum of surface magnetostrictive forces and the momentum of the bulk Hooke’s forces relative to the neutral central line:

$$t \sigma^l d/2 = \int_{-d/2}^{d/2} z E_s e_{xx}(z) dz$$  \hspace{1cm} (C2)

that gives the resulting curvature:

$$\frac{1}{R(\sigma_{xx}^l)} = \frac{6t\sigma^l}{E_s d^2}$$  \hspace{1cm} (C3)

For the initial combined problem, the deformation of curvature and lengthening have the same origin and are related by the following relation:

$$e_{xx}(z) = 6 \left( \frac{z}{d^2} \right) e_{xx}^L$$  \hspace{1cm} (C4)

Thus the maximal value of $e_{xx}(z)$ is $3|e_{xx}^L|$ and the neutral line of the total deformation $e_{xx}(z) + e_{xx}^L$ is located at $z = -d/6$ (See Fig. 10).

For complete description of the effect of the magnetization rotation more components of the stress should be considered: the transversal stress for the magnetization along $x$ axis $\sigma_{yy}^l = -1/2 \sigma^l$ and the stresses for

FIG. 9: (Color on line) Effects of the reduced curvature $W^2/Rd$: a) Transverse deviation $z$ at the edge of the cantilever $(y = W/2)$ relatively to its thickness $d$. b) Correction for induced anisotropy in the center of a wide cantilever compared to an equally bended narrow cantilever. $\nu = 1/3$ is used for the calculations.

FIG. 10: The practical experimental configuration with a film deposited on one substrate surface can be considered as the superposition of a symmetric problem with 2 equivalent films deposited on both surfaces (sample lengthening only) and an antisymmetric problem with 2 films of opposite magnetostriction on both surfaces (sample curvature only).
the perpendicular magnetization $\sigma_{yx} = -\frac{1}{2} \sigma^\lambda$ and $\sigma_{yy} = \sigma^\lambda$. The position of the neutral plane for each component is the same and their total influence increases curvature $1/R(\sigma_{yy})$ by $3(1 + \nu_s)/2$ found in eq. (10).

The shift of the neutral plane is found not only for magnetostrictive film stresses $\sigma_{yy}$ but also for thermal or epitaxial stresses (15 and references therein).

Second question we have to address is the stress distribution and the corresponding curvature variation along the cantilever length. The simplest way to obtain this is to balance the momentum of forces around a given point of the cantilever curved by force $F$ applied to its free end ($x = L$), the other end ($x = 0$) being clamped.

Let’s consider the equilibrium of part $[x, L]$ of the cantilever around the middle point at its left edge. The momentum of bulk Hooke’s forces (see the second term of eq. (C2)) applied to $x$ should be equal to the applied force momentum:

$$\int_{-d/2}^{d/2} \int_{-d/2}^{d/2} z E_s e_{xx}(z) dz = \frac{F}{W}(L - x) \quad \text{(C5)}$$

that gives:

$$\frac{1}{R} = \frac{12F}{E_s W d^3}(L - x) \quad \text{(C6)}$$

This simple solution is valid when the sample side edges are free and the anticlastic deformation occurs everywhere along the sample length. In reality the anticlastic deformation is blocked at the sample support (clamping) with corresponding bending hardening by a factor $1 - \nu_s^2$ (see configuration 2 in Appendix A and modification of elastic coefficient $A$ in eq. (5) from anticlastic to cylindrical bending). This phenomena can be described qualitatively by introducing an effective Poisson’s ratio correction $\nu(x) = \nu_s \exp(-x/a)$ where $a \sim W/2$ is the characteristic relaxation length of the clamping effect. With this correction:

$$\frac{1}{R} = \frac{12F}{E_s W d^3}(1 - \nu^2(x))(L - x) \quad \text{(C7)}$$

The clamping produces even stronger influence onto the induced magnetic anisotropy that has to be multiplied by $(1 + \nu)$ according to modification of the effective magnetoelastic coefficient $B$ in eq. (5) from anticlastic to cylindrical bending. So we obtain:

$$B(x) = \frac{1}{2} dB_{\gamma,2} \frac{1 + \nu_s}{1 + \nu(x)} \quad \text{(C8)}$$

(see eq. (41)).

Combining $1/R(x)$ and $B(x)$ in eq. (7) we get a non-linear variation of the anisotropy with the distance from the cantilever support:

$$K_{\text{eff}}(x) = K + \frac{6FB_{\gamma,2}}{E_s W d^2}(1 + \nu_s)(1 - \nu(x))(L - x) \quad \text{(C9)}$$

instead of the traditionally considered linear variation without clamping effect ($\nu = 0$).

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