Neutron skin $r_{\text{skin}}^{48\text{Ca}}$ determined from reaction cross section of proton+$^{48}\text{Ca}$ scattering

Shingo Tagami
Department of Physics, Kyushu University, Fukuoka 819-0395, Japan

Maya Takechi
Niigata University, Niigata 950-2181, Japan

Jun Matsui
Department of Physics, Kyushu University, Fukuoka 819-0395, Japan

Tomotsugu Wakasa
Department of Physics, Kyushu University, Fukuoka 819-0395, Japan

Masanobu Yahiro
Department of Physics, Kyushu University, Fukuoka 819-0395, Japan
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Background: Using the chiral (Kyushu) g-matrix folding model with the densities calculated with Gongny-HFB (GHFB) with the angular momentum projection (AMP), we determined the neutron skin $r_{\text{skin}}^{208\text{Pb}} = 0.25$ fm from the central values of reaction cross sections $\sigma_R(\text{EXP})$ of $p^+208\text{Pb}$ scattering in $E_{\text{in}} = 40 - 81$ MeV. As for $^{48}\text{Ca}$, the high-resolution $E1$ polarizability experiment ($E1p$E) yields $r_{\text{skin}}^{48\text{Ca}}(E1pE) = 0.14 - 0.20$ fm. Meanwhile, $\sigma_R(\text{EXP})$ are available as a function of incident energy $E_{\text{in}}$ for $p^+208\text{Pb}$ scattering.

Aim: Our aim is to determine $r_{\text{skin}}^{48\text{Ca}}$ from the central values of $\sigma_R(\text{EXP})$ for $p^+48\text{Ca}$ scattering by using the Kyushu g-matrix folding model.

Results: As for $^{48}\text{Ca}$, we first determine neutron radius $r_n(E1pE) = 3.56$ fm from the central value 0.17 fm of $r_{\text{skin}}^{48\text{Ca}}(E1pE)$ and the proton radius $r_p(\text{EXP}) = 3.385$ fm of electron scattering, and evaluate matter radius $r_m(E1pE) = 3.385$ fm from the $r_n(E1pE)$ and the $r_p(\text{EXP})$ of electron scattering. The Kyushu g-matrix folding model with the GHFB+AMP densities reproduces $\sigma_R(\text{EXP})$ in $23 \leq E_{\text{in}} \leq 25.3$ MeV in one-$\sigma$ level, but slightly overestimates the central values of $\sigma_R(\text{EXP})$ there. In $23 \leq E_{\text{in}} \leq 25.3$ MeV, the small deviation allows us to scale the GHFB+AMP densities to the central values of $r_p(\text{EXP})$ and $r_n(E1pE)$. The reaction cross sections $\sigma_R(\text{EXP})$ obtained with the scaled densities almost reproduce the central values of $\sigma_R(\text{EXP})$ when $E_{\text{in}} = 23 - 25.3$ MeV, so that the $\sigma_R(\text{AMP})$ obtained with the GHFB+AMP densities and the $\sigma_R(E1pE)$ are in one $\sigma$ of $\sigma_R(\text{EXP})$ there. In $E_{\text{in}} = 23 - 25.3$ MeV, we determine the $r_m(\text{EXP})$ from the central values of $\sigma_R(\text{EXP})$ and take the average for the $r_m(\text{EXP})$. The averaged value is $r_m(\text{EXP}) = 3.471$ fm. Eventually, we obtain $r_{\text{skin}}^{48\text{Ca}}(\text{EXP}) = 0.146$ fm from $r_m(\text{EXP}) = 3.471$ fm and $r_p(\text{EXP}) = 3.385$ fm.

I. INTRODUCTION AND CONCLUSION

Background: As for neutron skin thickness $r_{\text{skin}} = r_n - r_p$, Horowitz, Pollock and Souder proposed a direct measurement [1]. The measurement composes of parity-violating and elastic electron scattering. The neutron radius $r_n$ is determined from the former, whereas the proton radius $r_p$ is from the latter. In fact, the $^{208}\text{Pb}$ Radius EXperiment (PREX) [2][3] yields

$$r_p(\text{PREX}) = 5.45 \text{ fm},$$

$$r_n(\text{PREX}) = 5.78^{+0.16}_{-0.18} \text{ fm},$$

$$r_{\text{skin}}^{208\text{Pb}}(\text{PREX}) = 0.33^{+0.18}_{-0.18} = 0.15 - 0.49 \text{ fm}. \quad (3)$$

For $^{208}\text{Pb}$, the $r_p(\text{PREX})$ and $r_n(\text{PREX})$ are most reliable at the present stage. For $^{48}\text{Ca}$, the $^{48}\text{Ca}$ Radius EXperiment (CREX) is ongoing at Jefferson Lab [2].

As an indirect measurement, the high-resolution $E1$ polarizability experiment ($E1pE$) yields

$$r_{\text{skin}}^{208\text{Pb}}(E1pE) = 0.156^{+0.025}_{-0.021} = 0.135 - 0.181 \text{ fm} \quad (4)$$

for $^{208}\text{Pb}$ [4]

$$r_{\text{skin}}^{48\text{Ca}}(E1pE) = 0.17 \pm 0.03^{+0.025}_{-0.021} = 0.14 - 0.20 \text{ fm} \quad (5)$$

for $^{48}\text{Ca}$. As for $r_{\text{skin}}$, the value for $^{48}\text{Ca}$ is slightly larger than that for $^{208}\text{Pb}$.

The central value of reaction cross section $\sigma_R$ is a standard observable to determine matter radius $r_m$. One can evaluate $r_{\text{skin}}$ and $r_n$ from the $r_m$ and the $r_p(\text{EXP})$ of the electron scattering. Eventually, one can determine $r_{\text{skin}}$ from the central value of $\sigma_R(\text{EXP})$. The $\sigma_R(\text{EXP})$ are available for $p^+48\text{Ca}$ scattering [7].

In the previous work [3], we determined the neutron skin $r_{\text{skin}}^{208\text{Pb}} = 0.25$ fm from the central values of $\sigma_R(\text{EXP})$ of $p^+208\text{Pb}$ scattering in $E_{\text{in}} = 40 - 81$ MeV, using the chiral (Kyushu) g-matrix folding model with the densities calculated with Gongny-HFB (GHFB) with the angular momentum projection (AMP).

The g-matrix folding model is a standard way of obtaining microscopic optical potential for proton scattering and
nucleus-nucleus scattering \[^9]_\[^13\]. Applying the folding model with the Melbourne \(g\)-matrix \[^12\] for interaction cross sections \(\sigma_I\) for Ne isotopes and \(\sigma_R\) for Mg isotopes, we found that \(^{31}\)Ne is a deformed halo nucleus \[^19\], and deduced the matter radii \(r_m\) for Ne isotopes \[^20\] and for Mg isotopes \[^21\].

Kohno calculated the \(g\) matrix for the symmetric nuclear matter, using the Brueckner-Hartree-Fock method with chiral N\(^3\)LO 2NFs and NNLO 3NFs \[^22\]. He set \(c_D = -2.5\) and \(c_E = 0.25\) so that the energy per nucleon can become minimum at \(\rho = \rho_0\). Toyokawa \textit{et al.} localized the non-local chiral \(g\) matrix into three-range Gaussian forms \[^17\], using the localization method proposed by the Melbourne group \[^12\] \[^23\] \[^24\]. The resulting local \(g\) matrix is called “Kyushu \(g\)-matrix”.

The Kyushu \(g\)-matrix folding model is successful in reproducing \(\sigma_R\) and differential cross sections \(d\sigma / d\Omega\) for \(^4\)He scattering in \(E_{lab} = 30 \sim 200\) MeV per nucleon \[^12\]. The success is true for proton scattering at \(E_{lab} = 65\) MeV \[^15\].

**Proton and neutron densities used in the folding model:** In Ref. \[^18\], GHFB and GHFB+AMP reproduce the one-neutron separation energy \(S_1\) and the two-neutron separation energy \(S_2\) in \(^{41-58}\)Ca \[^25\] \[^27\]. We found, with \(S_1\) and \(S_2\), that \(^{46}\)Ca is an even-dripline nucleus and \(^{50}\)Ca is an odd-dripline nucleus. Our results are consistent with the data \[^23\] in \(^{40-58}\)Ca for the binding energy \(E_B\). This means that the proton and neutron densities are good.

**Aim:** Our aim is to determine a central value of \(r_{48}^{48}\) skin from the central values of measured \(\sigma_R\) for \(p^{48}\)Pb scattering, using the Kyushu \(g\)-matrix folding model with the GHFB+AMP densities.

**Results:** We first determine neutron radius \(r_n(E_{1pE}) = 3.56\) fm from the central value \(0.17\) fm of \(r_{48}^{48} (E_{1pE}) \[^5\]\) and the proton radius \(r_p(E_{1pE}) = 3.885\) fm of electron scattering, and evaluate matter radius \(r_m(E_{1pE}) = 3.485\) fm from the \(r_n(E_{1pE})\) and the \(r_p(E_{1pE})\).

The Kyushu \(g\)-matrix folding model with the GHFB+AMP densities produces \(\sigma_R(\text{EXP})\) in \(23 \leq E_{in} \leq 25.3\) MeV in one-\(\sigma\) level. In \(23 \leq E_{in} \leq 25.3\) MeV, the small deviation allows us to scale the GHFB+AMP densities to the central values \(r_p(\text{EXP})\) and \(r_n(E_{1pE})\). The Kyushu \(g\)-matrix folding model with the scaled densities almost reproduces the central values of \(\sigma_R(\text{EXP})\) when \(E_{in} = 23 - 25.3\) MeV, so that both the \(\sigma_R(\text{AMP})\) obtained with the GHFB+AMP densities and the \(\sigma_R(E_{1pE})\) calculated with the scaled densities are in one \(\sigma\) of \(\sigma_R(\text{EXP})\) there. In \(E_{in} = 23 - 25.3\) MeV, we determine the \(r_n(\text{EXP})\) from the central values of \(\sigma_R(\text{EXP})\) by using \(\sigma_R(\text{EXP}) = c r_n^2(\text{EXP})\) with \(c = \sigma_R(E_{1pE}) / r_m(E_{1pE})^2\), and take the average for the \(r_m(\text{EXP})\). In \(E_{in} = 23 - 25.3\) MeV, we determine the \(r_m(\text{EXP})\) from the central values of \(\sigma_R(\text{EXP})\) and take the average for the \(r_m(\text{EXP})\). The averaged value \(r_m(\text{EXP}) = 3.471\) fm leads to \(r_{48}^{48}\) skin \(= 0.146\) fm.

**Conclusion:** Our conclusion is that the central value of \(r_{48}^{48}\) skin \(= 0.146\) fm. This result is consistent with \(r_{48}^{48} E_{1pE}\) of Eq. \[^5\].

**II. MODEL**

Our model is the Kyushu \(g\)-matrix folding model \[^17\] with densities calculated with GHFB+AMP \[^18\]. The folding model itself is clearly shown in Ref. \[^14\]. The Kyushu \(g\)-matrix is constructed from chiral interaction with the cutoff 550 MeV.

**III. RESULTS**

Figure 1 shows reaction cross sections \(\sigma_R\) as a function of incident energy \(E_{in}\), for \(p^{48}\)Ca scattering. In one-\(\sigma\) level, the Kyushu \(g\)-matrix folding model with the GHFB+AMP densities (circles) reproduces measured reaction cross sections \(\sigma_R(\text{EXP})\) in \(23 \leq E_{in} \leq 25.3\) MeV. Our results show that our framework is reliable there. This allows us to scale the proton and neutron densities so that the proton and neutron radii, \(r_{48}^{48} p\) and \(r_{48}^{48} n\), calculated with GHFB+AMP may agree with the central values of \(r_{48}^{48} p(\text{EXP})\) and \(r_{48}^{48} n(\text{EXP})\). For \(E_{in} = 23, 25.3\) MeV, the Kyushu \(g\)-matrix folding model with the scaled densities (squares) slightly overestimates the central values of \(\sigma_R(\text{EXP})\). In \(E_{in} = 23, 25.3\) MeV, we determine the \(r_m(\text{EXP})\) from the central values of \(\sigma_R(\text{EXP})\) and take the average for the \(r_m(\text{EXP})\); see results of Sec. \[^1\] for how to determine \(r_m(\text{EXP})\) from the central values of \(\sigma_R(\text{EXP})\). The averaged value is \(r_m(\text{EXP}) = 3.471\) fm. Using the \(r_m(\text{EXP})\) and \(r_p(\text{EXP}) = 3.385\) fm, we can get \(r_{48}^{48}\) skin \(= 0.146\) fm.

![FIG. 1.](image-url)

**FIG. 1.** \(E_{in}\) dependence of reaction cross sections \(\sigma_R\) for \(p^{48}\)Ca scattering. Circles denote results of the GHFB+AMP densities, and squares correspond to the folding model with the scaled densities. The data (crosses) are taken from Refs. \[^7\].
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