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A superior extension for the Lomax distribution with application to Covid-19 infections real data

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1. Introduction

The world has suffered various pandemics and diseases throughout the history of humankind. One such was the recent
coronavirus (COVID-19), which has arisen over the last year and has been classified as a global public health emergency. This pandemic is thought to be among the most horrific disease outbreaks throughout all human history. COVID-19 has been slowed in its global spread by the implementation of “tight” health and safety precautions by most countries. They have also implemented a number of full restrictions on commerce and reduced business and evening school hours, for example. A complex mathematical and statistical model has also been used to predict the course of future infections related to the COVID-19 outbreak, along with numerous other variables. This idea was advanced by the Remuzzi and Remuzzi [1], who calculated how many nursing homes and employees would be required under an exponentially increasing trend in the numbers of the elderly. Maleki [2] used vector autoregression time series models to extrapolate and identify confirmed cases but they used the two-piece scale mixture normal distributions.

Caccavo [3] introduced the concept of the compartment susceptible-infected-COVID-19 (diseased model) that splits out cases between two compartments based on the state of illness, as used for data from Italy and China in terms of COVID-19 infections that are in terms of recovered and infected. Ayyoubazdeh et al. [4] used LSTM, a long-term memory model in order to reach their conclusions, for the prediction of COVID-19 cases in Iran. Alwendually et al. [5] implemented a new discrete distribution with a key property that are known as the Marshall–Olkin generalized exponential for the study of the newly added COVID-19 infected case. Al-Babtain et al. [24] analysis the Covid-19 spread in KSA. For the study of the newly added COVID-19 infected case. Alwendually et al. [5] implemented a new discrete distribution with a key property that are known as the Marshall–Olkin generalized exponential for the study of the newly added COVID-19 infected case. Al-Babtain et al. [24] analysis the Covid-19 spread in KSA. For more studied about COVID-19 see [34,35,22,14,18].

The Lomax model was studied by Lomax [19], as it is very efficient for modelling a lifetime data and business failure data. Sometimes we can name the Lomax distribution as Pareto Type II distribution. In addition, it is an important distribution for modelling different types of data in various contexts. The Lomax distribution is a heavy-tail probability distribution function (PDF) which is frequently used for business, economic and actuarial modelling. several generalizations for the Lomax distribution have been developed to add greater flexibility and possession to the new distribution in order to be able to model a larger number of phenomenal data. For examples, see Tahir et al. [7], who introduced Weibull-Lomax distribution, Fatima et al. [9] who introduced Rayleigh Lomax distribution, Baharith et al. [8] introduced the odds exponential-Pareto IV distribution, the Marshall–Olkin generalized Pareto as introduced by Haj Ahmad and Almetwally [17], etc.

Extended Odd Weibull Lomax (EOWL) distribution is to be considered in this study which has several attractive properties that will be obtained throughout this paper, and which can be summarized as follows:

- The EOWL distribution may be used to mimic a variety of real-world data types, including steady, declining, rising, and upside-down bathtub data. Its density function may be symmetrical or right-skewed. Additionally, the hazard rate function may be declining, inverted, or constant.
- The cumulative distribution function (CDF) of EOWL distribution presented in a closed form and also its hazard rate function (HR).

The primary purpose and motivation for this study is to develop a statistical model that more accurately characterises the daily number of fatalities and new infections caused by COVID-19. As a result, we utilised the model that we can state reflects optimum distributions for modelling the death rates associated with COVID-19 infections in a variety of nations and circumstances. To optimize the effectiveness of the lomax distribution, a novel modification dubbed the Odd Weibull-Lomax distribution will be created. To investigate the efficiency of any distribution, we must first estimate its parameter(s) using either a classical or a non-classical technique. In this study, we estimated a point using two traditional approaches, the MLE and the MPS. Additionally, we used non-traditional Bayesian approaches to estimate the unknown parameters. Through simulation, a statistical comparison of these techniques has been made to ascertain the performance of the estimators.

We can drive the EOWL distribution from the Extended Odd Weibull-G (EOW-G) family presented by Alizadeh et al. [11]. By assuming that \( G(x; \theta) = 1 - G(x; \theta) \) and \( g(x; \theta) = \frac{dG}{dx} \) which denotes the survival function (S) and probability density function (PDF) of a parameter vector-based model baseline \( \theta \), respectively. The CDF for the EOW-G family is as follows:

\[
F(x; \alpha, \beta, \theta) = 1 - \left\{ 1 + \beta \left[ \frac{G(x; \theta)}{G(x; \theta)} \right]^\frac{1}{\beta} \right\} , \quad x \in \mathbb{R}. \tag{1}
\]

Eq. (2) is corresponding PDF for Eq. (2)

\[
f(x; \alpha, \beta, \theta) = \frac{\alpha g(x; \theta) G(x; \theta)^{\alpha-1}}{G(x; \theta)^{\beta+1}} \left\{ 1 + \beta \left[ \frac{G(x; \theta)}{G(x; \theta)} \right]^\frac{1}{\beta} \right\} , \quad x \in \mathbb{R}, \tag{2}
\]

noting that \( \alpha \) and \( \beta \) are both greater than zero, and are considered to be shape parameters. We can now say that any random variable \( X \) possesses the PDF in Eq. (2), will follow the EOW-G(a distribution with three parameters: \( \alpha, \beta, \) and \( \theta \)).

The rest of this article is manuscript is as follows: We define the EOWL distribution and the graphical plot of its PDF and HR functions in Section 2. Section 3 examines the statistical properties of the EOWL distribution, where we introduced some important mathematical properties of the proposed distribution and explain their derivations mathematically, while Section 4 examines three methods of point estimation. We perform a numerical simulation in Section 5 to verify the effectiveness of the estimation techniques used in the simulation. In Section 6, COVID-19 data from various countries are also used to demonstrate the EOWL distribution’s efficiency in comparison to other distributions. Finally, Section 7 describes the conclusions that can be drawn from the paper and the major findings illustrated by this work. For more reading about this subject, please see [28,29,31,32,26,28]

2. EOWL distribution

This four-parameter expansion is a specific form of the EOW family of models with the Lomax distribution serving as a baseline function. PDF and CDF of the EOWL distribution are, respectively, obtained as mentioned below:
The PDF function (Q) of the EOWL distribution are given by:

\[
F(x; \alpha, \beta, \theta, \lambda) = 1 - \left[ 1 + \beta \left( \left( 1 + \frac{x}{\theta} \right)^{\theta} - 1 \right) \right]^{\frac{1}{\beta}}, \quad x > 0, \quad \alpha, \beta, \theta, \lambda > 0.
\]

\[
f(x; \alpha, \beta, \theta, \lambda) = \frac{x^{\theta - 1} \left( 1 + \frac{x}{\theta} \right)^{\theta - 1} \left( 1 + \beta \left( 1 + \frac{x}{\theta} \right)^{\theta} - 1 \right)^{\frac{1}{\beta}}}{1 + \beta \left( 1 + \frac{x}{\theta} \right)^{\theta} - 1}, \quad x > 0, \quad \alpha, \beta, \theta, \lambda > 0.
\]

As a result, a random variable that has a PDF as in Eq. (4) is denoted by \( X \sim \text{EOWL} (\alpha, \beta, \theta, \lambda) \). We can determine some special cases:

1. The EOWL model takes the form of the three-parameter Weibull Lomax model when \( \lambda \rightarrow 1 \).
2. The EOWL model tends to the three-parameter extended Odd Weibull Pareto model when \( \lambda \rightarrow 1 \).
3. The EOWL model takes the form of the three-parameter exponentiated Lomax model when \( \beta \rightarrow 1 \), as introduced by El-Bassiouny et al. [6].
4. We have two-parameter Rayleigh-Lomax model by setting \( \alpha = 2 \) and \( \beta \rightarrow 0 \).
5. We have two-parameter Weibull-Pareto model when \( \lambda \rightarrow 1 \) and \( \beta \rightarrow 0 \); for more information, see Alzaatreh et al. [16].

The HR function, the survival function (S), and the quantile function (Q) of the EOWL distribution are given by:

\[
h(x; \alpha, \beta, \theta, \lambda) = \frac{x^{\theta - 1} \left( 1 + \frac{x}{\theta} \right)^{\theta - 1} \left( 1 + \beta \left( 1 + \frac{x}{\theta} \right)^{\theta} - 1 \right)^{\frac{1}{\beta}}}{1 + \beta \left( 1 + \frac{x}{\theta} \right)^{\theta} - 1},
\]

\[
S(x; \alpha, \beta, \theta, \lambda) = \left[ 1 + \beta \left( \left( 1 + \frac{x}{\theta} \right)^{\theta} - 1 \right) \right]^{-\frac{1}{\beta}},
\]

and

\[
Q(u) = \lambda \left( 1 + \frac{1}{\beta} \left( 1 - u \right)^{-\beta} - 1 \right) - \frac{1}{\beta}, \quad 0 < u < 1,
\]

respectively.

Figs. 1 and 2 below describe the various shapes of the EOWL distribution’s PDF and HR. The PDF of the EOWL distribution can be right-skewed, symmetric, or declining curves, as illustrated in these figures. The HR of the EOWL distribution exhibits a variety of interesting shapes, such as steady, declining, and inverted curves, all of which are desirable characteristics for any life experience model. As demonstrated in the application section, the EOWL distribution is a highly versatile distribution that can be used to model skewed data. Consequently, it is widely applicable in various fields, including biomedical research, and other many fields in the sciences, and in various statistical models and regression models.

3. Properties of the proposed distribution

The following sections describe some of the EOWL distribution’s more significant statistical properties. These properties are the simple linear representation, moments, the incomplete moment, the moment generating function, the stress strength reliability function and the order statistic PDF and CDF of the EOWL distribution.

3.1. The linear representation of the EOWL density function

In this part, we generate a linear representation for the EOWL distribution, allowing us to easily estimate the number of statistical features of the proposed model using series expansions. Alzadeh et al. [11] demonstrated that the EOW-G family’s CDF and density have the following mixture representation.

\[
F(x) = 1 - \sum_{k,j=0}^{\infty} b_{k,j}[G(x)]^{(k+j)},
\]

where \( b_{k,j} = \frac{\theta^k \Gamma(\frac{k+1}{\theta})}{\Gamma(\frac{k+1}{\theta}+1)} \). Therefore, it is possible to rewrite the CDF of EOWL distribution as follows:

\[
F(x) = 1 - \sum_{k,j=0}^{\infty} b_{k,j} \left[ 1 - \left( \frac{x}{\theta} \right)^{\theta^{k+j}} \right],
\]

Using the binomial expansion to solve the problem as we used it to expand this bracket \( \left[ 1 - \left( 1 + \frac{x}{\theta} \right)^{-\beta} \right]^{k+j} \).
Now, we can express the random variable $X$'s $n$th central moments concerning the origin of the EOWL distribution.

The EOWL distribution's moment generating function has the following form:

$$M(t) = \sum_{k,j=0}^{\infty} W_{k,j} \int_0^\infty x^j g_k(x)dx$$

Its characteristic function may be found by substituting $it$ for $t$ in the previous equation.

### 3.3. Incomplete moments

The EOWL distribution’s $s^{th}$ incomplete moment is provided by:

$$\Psi_s(t) = \int_0^s x^s f(x)dx = \sum_{k=0}^{\infty} W_{k,j} \int_0^s x^j g_k(x)dx$$

By taking into account that $B_s(a,b) = \int_0^s t^{a-1}(1-t)^{b-1}dt$, is the first incomplete moment that has a significant use in relation to the Bonferroni and Lorenz curves.

### 3.4. Stress-strength reliability

Consider the independent random variables $X$ and $Y$, representing intensity and stress, respectively, as obtained from the EOWL distribution. The following formula calculates it

$$R = P(Y < X) = \int_0^\infty f(x; \alpha, \beta, \theta_1, \lambda) \cdot F(x; \alpha_2, \beta_2, \theta_2, \lambda)dx.$$  

By using a linear representation for the EOWL equations (10, 9), the stress-strength reliability of the EOWL distribution can be given as:

$$R = P(Y < X) = 1 - \eta \int_0^\infty \frac{\partial h_1}{\partial \lambda} \left(1 + \frac{x}{\lambda}\right)^{-\theta_1 h_1 - \theta_2 h_2 - 1} dx,$$

where $\eta = \sum_{i=1}^{\infty} W_{k,i}h_i \sum_{k=1}^{\infty} W_{k,j}h_j$. Then, the stress-strength reliability of the EOWL distribution can be written as:

$$R = P(Y < X) = 1 - \eta \frac{\partial h_1}{\partial \lambda} + \frac{\partial \beta_2}{\partial \lambda},$$

$$R(t) = \int_0^\infty f(x; \alpha, \beta, \theta_1, \lambda) \cdot F(x; \alpha_2, \beta_2, \theta_2, \lambda)dx.$$
3.5. Order statistics

The PDF and CDF forms of the EOWL distribution’s \(i\)th order statistic are as follows:

\[
f_{x,i}(x) = \frac{\exp\left[-F(X_i)^{\alpha}a\right]}{s\left((\frac{o}{o}\left(F(X_i)^{\alpha}a\right))\right)}
\]

\[
F_{x,i}(x) = \sum_{i=1}^n \left(1 - \left(\frac{o}{o}\left(F(X_i)^{\alpha}a\right)\right)^{-1}\right)
\]

where \(2F_1\) is a hypergeometric function.

With \(i = 1\), we get the PDF and CDF representations of the minimal order statistics \((Wn)\). The distribution of \(Wn\)’s limit values decreases to

\[
\lim_{n \to \infty} P(W_n < d_n) = 1 - \exp(-x^a), \quad x > 0, d_n = F^{-1}\left(\frac{1}{n}\right).
\]

With \(i = n\), we have the maximum order PDF and CDF \((Z_n)\). The limiting distribution for \(Z_n\) has the following representation:

\[
\lim_{n \to \infty} P(Z_n < b_n) = \exp(-x^a), \quad b_n = F^{-1}\left(\frac{1}{n}\right).
\]

4. Parameter estimation

This section will explore various techniques that can be used to estimate unknown parameters. As it is known, not all of these estimators are equal in their efficiencies at estimating the unknown parameters. We employ the MLE as a first method. Then, we used another alternative classical method, namely the MPS. The last method used for statistical inference is the Bayesian method with the aid of the squared error loss function. In recent years, many authors, such as ([20,17,12,10], and [25]), have focused their attention on parameter estimation using various estimation techniques that utilize either classical or non-classical methods.

4.1. The MLE method

Suppose we randomly pick a sample from a continuous population. This randomly selected sample is made up of \(n\) elements which follows our proposed model, and are independent and identically distributed of size \(n\) and ordered as \(x_1, \ldots, x_n\). The following equation denotes the likelihood function:

\[
L(\Omega) = s^r \left(\frac{r}{n}\right) \prod_{i=1}^n \left(1 + \frac{x_i^a}{2}\right)^{a-1}/\left(1 + \frac{x_i^a}{2}\right) - 1\right)^{-1} \left(1 + \beta \left(\frac{x_i^a}{2}\right) - 1\right)^{-1}
\]

hence for \(\Omega = (x, \beta, \theta, \lambda)\). The EOWL distribution’s log-likelihood function is as follows:

\[
\ell(\Omega) = n[\log(z) + \log(\theta) - \log(\lambda)]
\]

\[
+ (\theta - 1) \sum_{i=1}^n \log \left(1 + \frac{X_i}{z}\right) + (z - 1) \sum_{i=1}^n \log(W_i^z) - \frac{1 + \beta}{\beta} \sum_{i=1}^n \log(1 + \beta(W_i)^z)
\]

(14)

where \(W_i = \left(1 + \frac{X_i^a}{2}\right)^{a - 1}\). The derivative of last equation with respect to \(\Omega\) is obtained as follows:

\[
\frac{\partial(\ell(\Omega))}{\partial \theta} = \frac{n}{\theta} + \sum_{i=1}^n \log(W_i) - (1 + \beta) \sum_{i=1}^n \frac{W_i^z \log(W_i^z)}{1 + \beta W_i^z}
\]

(15)

\[
\frac{\partial(\ell(\Omega))}{\partial \theta} = \frac{n}{\beta} + \sum_{i=1}^n \log(1 + \beta W_i^z) - \frac{1 + \beta}{\beta} \sum_{i=1}^n \frac{W_i^z \log(W_i^z)}{1 + \beta W_i^z},
\]

\[
\frac{\partial(\ell(\Omega))}{\partial x} = \frac{n}{x} + \sum_{i=1}^n \frac{\log(W_i)}{W_i^z} + \frac{1}{x} \sum_{i=1}^n \frac{(1 + \frac{X_i^a}{2})^a - 1}{1 + \beta(W_i)^z} + \theta x (1 + \beta) \sum_{i=1}^n \frac{(1 + \frac{X_i^a}{2})^{a - 1} \frac{X_i^a}{2}}{1 + \beta(W_i)^z}.
\]

These equations are equated to zero, and unfortunately cannot be solved explicitly. Hence we will use the R language, and will use the implemented functions incorporated into this language which use the Newton–Raphson approach for solving this kind of equation.

4.2. Maximum product spacing

We will use one of the most well-known methods for solving equations in this subsection. The maximum product spacing approach is effective concerning other point estimating methods. For more information, please refer to Cheng and Amin [13]. Thus, we approximate the estimates of the unknown parameters using MPS. This can be found by taking the logarithm of the spacing function’s product \(G(\Theta)\) written, for instance:

\[
G(\Theta) = \left(1 - \left[1 + \beta(W_i)^z\right]\right) \prod_{i=1}^n \left(1 + \beta(W_i)^z\right) + \left(1 + \beta(W_i)^z\right)
\]

and compute the function \(G(\Omega)\) logarithmically, as follows:

\[
\log G(\Omega) \propto \log \left[1 - \left(1 + \beta(W_i)^z\right)\right] + \log \left[1 + \beta(W_i)^z\right]
\]

\[
+ \sum_{i=1}^n \log \left[1 + \beta(W_i)^z\right] + \log \left[1 + \beta(W_i)^z\right]
\]

(15)

Also, by differentiating the log-product equation in Eq. (15) with respect to each parameter in the distribution, the MPS estimators for \(\Omega\) can be achieved, and the resulting nonlinear system of equations can be resolved via iterative procedures.
4.3. Bayesian estimation

Here, we studied the concept of Bayesian inference, which is considered a type of statistical inference with a prior distribution for the model parameters and the loss function used in the estimation process, whether the latter is symmetric or asymmetric. In the Bayesian methods, we treat the parameters as random variables rather than as constants in the manner of all classical methods. The researcher must choose a specific distribution based effectively on prior information derived from their own informed, if subjective, opinions. If there is no prior information or it is absent, one would have to rely on noninformative prior. Because of this, our choice of prior information is significant, and our parameters do not depend on the sample distribution. However, the loss function is important in Bayesian models. For the Bayesian inference in this paper, we used only symmetric and loss functions, the latter being the squared error loss function, which is considered as one of the most well known functions in mathematics.

In this paper we used independent gamma prior as the prior in action. Hence the joint prior density function \( \Omega \) can be given in the following manner:

\[
\pi(\Omega) = \frac{b_1^3}{\Gamma(a_1)} \frac{b_2^8}{\Gamma(a_2)} \frac{b_3^1}{\Gamma(a_3)} \times \frac{b_4^4}{\Gamma(a_4)} e^{a_1-1} \beta e^{-1} \beta^{a-1-1} e^{-(b_1+b_2+b_3+b_4)}.
\]

From the posterior density function of \( \Omega \), the joint posterior density function can be concluded using (13) and (16)

\[
\pi(\Omega|x) = \int_{\Omega} \ell(x|\Omega) \pi(\Omega) d\Omega.
\]

**Table 1** Numerical results for the simulation study can be found in this table for classical and non-classical methods of estimation with true values for \( x = 1.5, \theta = 2 \).

| \( \beta \) | \( \lambda \) | \( n \) | MLE | MPS | Bayesian |
|---|---|---|---|---|---|
| | | | | | |
| 1.5 | 2 | 50 | a | -0.3916 0.3180 | -0.5213 0.3267 | -0.3141 0.1035 |
| | | | \( \beta \) | 0.4048 3.8574 | -0.2275 0.5090 | 0.3509 0.2355 |
| | | | \( \theta \) | 1.0204 3.1824 | 0.1798 0.1800 | 0.4550 0.2563 |
| | | | \( \lambda \) | -0.0026 0.6307 | -0.4155 0.4886 | -0.4319 0.2326 |
| 100 | | | a | -0.4536 0.2742 | -0.5238 0.2982 | -0.2430 0.0656 |
| | | | \( \beta \) | 0.1391 1.3692 | -0.2304 0.2900 | 0.3478 0.2275 |
| | | | \( \theta \) | 0.9141 1.8485 | 0.2518 0.1582 | 0.4183 0.2113 |
| | | | \( \lambda \) | -0.1181 0.3470 | -0.4066 0.3368 | -0.4311 0.2227 |
| | 5 | 50 | a | -0.4886 0.2562 | -0.5215 0.2823 | -0.1760 0.0365 |
| | | | \( \beta \) | -0.0918 0.5237 | -0.2500 0.1788 | 0.2374 0.1427 |
| | | | \( \theta \) | 0.5714 0.7470 | 0.3098 0.1515 | 0.3609 0.1638 |
| | | | \( \lambda \) | -0.1721 0.2650 | -0.3678 0.2252 | -0.3740 0.1977 |
| 100 | | | a | -0.3867 0.3065 | -0.5262 0.3214 | -0.4960 0.2514 |
| | | | \( \beta \) | 0.4436 4.8846 | -0.0924 0.6026 | -0.3132 0.1321 |
| | | | \( \theta \) | 1.1387 3.2335 | 0.5368 0.5762 | 0.3572 0.1399 |
| | | | \( \lambda \) | -0.2992 1.7058 | -0.6299 0.9813 | -0.2012 0.0496 |
| | 200 | | a | -0.4505 0.2629 | -0.5225 0.2976 | -0.4546 0.2168 |
| | | | \( \beta \) | 0.1669 1.7990 | -0.1004 0.3549 | -0.2578 0.1175 |
| | | | \( \theta \) | 0.9063 1.8060 | 0.6048 0.5340 | 0.2899 0.0965 |
| | | | \( \lambda \) | -0.3025 1.0407 | -0.5328 0.5910 | -0.1479 0.0292 |
| | | | a | -0.4803 0.2509 | -0.5209 0.2814 | -0.3819 0.1611 |
| | | | \( \beta \) | 0.0416 0.6033 | -0.1155 0.1896 | -0.1519 0.0872 |
| | | | \( \theta \) | 0.7236 0.9502 | 0.6621 0.5295 | 0.2269 0.0625 |
| | | | \( \lambda \) | -0.2493 0.5513 | -0.4495 0.3376 | -0.1050 0.0171 |
| 4 | 2 | 50 | a | -0.8964 0.9262 | -0.8703 0.9104 | -0.5504 0.3085 |
| | | | \( \beta \) | 0.2674 4.1715 | -0.6779 1.0711 | 0.7325 0.6067 |
| | | | \( \theta \) | 1.2992 3.9079 | 1.0770 1.7278 | 0.6821 0.5128 |
| | | | \( \lambda \) | 0.2533 1.0249 | -0.2461 0.7071 | -0.7732 0.6328 |
| 100 | | | a | -0.7978 0.8003 | -0.7805 0.7915 | -0.4243 0.1863 |
| | | | \( \beta \) | -0.0398 0.9516 | -0.6605 0.7469 | 0.6545 0.4987 |
| | | | \( \theta \) | 1.0191 2.4787 | 0.9207 1.2741 | 0.4993 0.2882 |
| | | | \( \lambda \) | 0.1852 0.9108 | -0.1670 0.4372 | -0.7524 0.6196 |
| 200 | | | a | -0.6004 0.7029 | -0.5935 0.6081 | -0.2915 0.0912 |
| | | | \( \beta \) | -0.6211 0.7505 | -0.6211 0.5656 | -0.5196 0.3280 |
| | | | \( \theta \) | 0.8661 0.9522 | 0.4257 0.7505 | 0.4158 0.2182 |
| | | | \( \lambda \) | 0.0469 0.4156 | -0.1701 0.2284 | -0.6494 0.4833 |
The Bayes estimators of $\Omega$, say $\hat{\beta}_B, \hat{\theta}_B, \hat{\lambda}_B$, as based on
the squared error loss function, are given by:

$$
\hat{\beta}_B, \hat{\theta}_B, \hat{\lambda}_B = \min_{\theta, \psi, \lambda} \mathbb{E}[L(x, \beta, \theta, \lambda)]
= \int_0^\infty \int_0^\infty \int_0^\infty \mathbb{E}[L(x, \beta, \theta, \lambda)] d\lambda d\theta dx.
$$

For more details about Bayesian estimation, see, for example, Almetwally et al. [15] and Afify et al. [33]. It is very clear
and noticeable that the integrals given by (18) is difficult to
achieve explicitly. As a consequence, we use the Monte Carlo
Markov Chain approach (MCMC) to obtain a reasonable
approximation of a given integral’s value as per (18), we will
use the MH algorithm to evaluate the integration in Eq. (18).

5. Simulation analysis

The discussion in this section presents the classical estimators
MLE and MPS, and the non classical Bayesian estimators with
MCMC under a squared loss function of the EOWL distribution
for the EOWL distribution using the R language. Monte Carlo experiments are
conducted using 10,000 data that we generated randomly from the
EOWL distribution, such that $x$ represents the EOWL lifetime
for various model parameters and various allocated sample
sizes, for instance, $n = 50, n = 100$, and $n = 200$. The optimal
estimator methods are those with the lowest bias and mean
squared error (MSE).

Tables 1 and 2 summarise the simulation of the proposed
point estimation methods in this article, which produces the
following results.

1. With increasing sample size, bias and MSE diminish.
2. In Table 1, we show that the MFS technique outperforms
MLE in terms of MSE and bias for predicting the EOWL
distribution parameters.
3. To assess the performance between classical estimation
methods and the Bayesian method under the squared error
loss function, we look for the values of bias and MSE

| Table 2 | Numerical results for the simulation study can be found in this table for classical and non-classical methods of estimation with true values for $\alpha = 2, \beta = 4$. |
|---------|-----------------|-----------------|-----------------|-----------------|
| $\theta$ | $\lambda$ | $n$ | MLE | Bias | MSE | Bias | MSE | Bayesian | Bias | MSE |
| 2 | 4 | 50 | $\alpha$ | $-0.9021$ | 0.9688 | $-0.9504$ | 0.9124 | $-0.7053$ | 0.5529 |
| | | | $\beta$ | 0.1722 | 3.8543 | $-0.6735$ | 0.9905 | 0.5987 | 0.4314 |
| | | | $\theta$ | 1.7988 | 4.2856 | 1.0634 | 1.5955 | 0.6021 | 0.4236 |
| 100 | | | $\lambda$ | 0.1864 | 4.5185 | $-0.5545$ | 1.3080 | $-0.8366$ | 0.7594 |
| | | | $\alpha$ | $-0.8545$ | 0.8523 | $-0.8603$ | 0.8079 | $-0.6029$ | 0.3677 |
| | | | $\beta$ | 0.0231 | 2.3587 | $-0.6110$ | 0.6208 | 0.5382 | 0.3551 |
| | | | $\theta$ | 1.1826 | 1.3493 | 0.9620 | 1.3660 | 0.8031 | 0.6931 |
| | | | $\lambda$ | 0.0324 | 2.4703 | $-0.4051$ | 0.9173 | $-0.5235$ | 0.3350 |
| 200 | | | $\alpha$ | $-0.6500$ | 0.6224 | $-0.4039$ | 0.6809 | $-0.2325$ | 0.1127 |
| | | | $\beta$ | $-0.2707$ | 0.9548 | $-0.5834$ | 0.4787 | 0.4919 | 0.2305 |
| | | | $\theta$ | 0.8660 | 0.7430 | 0.6289 | 0.3764 | 0.5999 | 0.3423 |
| | | | $\lambda$ | $-0.0348$ | 1.2820 | $-0.1429$ | 0.5416 | $-0.4584$ | 0.2165 |
| 5 | 2 | 50 | $\alpha$ | $-0.9747$ | 1.0858 | $-1.0574$ | 1.1597 | $-0.5728$ | 0.3322 |
| | | | $\beta$ | $-0.6899$ | 2.4906 | $-0.9745$ | 2.0290 | 0.6590 | 0.5145 |
| | | | $\theta$ | 1.0693 | 1.7901 | 0.6562 | 0.8909 | 0.3702 | 0.1527 |
| | | | $\lambda$ | 0.6445 | 1.0445 | $-0.5294$ | 0.7378 | $-0.4930$ | 0.8846 |
| 100 | | | $\alpha$ | $-0.8904$ | 0.9114 | $-0.8037$ | 0.9135 | $-0.4437$ | 0.2027 |
| | | | $\beta$ | $-0.5941$ | 1.7809 | $-0.9859$ | 1.0510 | 0.6447 | 0.4910 |
| | | | $\theta$ | 0.9180 | 1.1560 | 0.5841 | 0.7891 | 0.2687 | 0.0837 |
| | | | $\lambda$ | $-0.1460$ | 0.6100 | $-0.4583$ | 0.5402 | $-0.8556$ | 0.7621 |
| 200 | | | $\alpha$ | $-0.7342$ | 0.8169 | $-0.0606$ | 0.5132 | $-0.3113$ | 0.1034 |
| | | | $\beta$ | $-1.2531$ | 1.1633 | $-0.7085$ | 0.9482 | 0.5059 | 0.3237 |
| | | | $\theta$ | 0.6639 | 0.5316 | 0.4938 | 0.6968 | 0.1980 | 0.0504 |
| | | | $\lambda$ | $-0.1267$ | 0.3602 | $-0.4253$ | 0.3983 | $-0.6833$ | 0.5381 |
| 4 | 50 | $\alpha$ | $-0.9733$ | 1.2906 | $-0.9509$ | 1.0848 | $-0.6263$ | 0.3967 |
| | | | $\beta$ | $-0.6495$ | 2.7832 | $-0.6165$ | 2.0762 | 0.2241 | 0.1155 |
| | | | $\theta$ | 0.8773 | 1.0485 | 0.8652 | 0.9892 | 0.6852 | 0.4883 |
| | | | $\lambda$ | $-0.2861$ | 2.0120 | $-1.0059$ | 1.9021 | $-1.2005$ | 1.4880 |
| 100 | | | $\alpha$ | $-1.0430$ | 1.1034 | $-0.9090$ | 1.0204 | $-0.4843$ | 0.2411 |
| | | | $\beta$ | $-0.9986$ | 1.4031 | $-0.8239$ | 1.2792 | 0.2132 | 0.0977 |
| | | | $\theta$ | 0.6946 | 0.6050 | 0.6016 | 0.5151 | 0.5319 | 0.3014 |
| | | | $\lambda$ | $-0.2473$ | 1.9022 | $-0.9458$ | 1.3544 | $-0.9747$ | 1.0130 |
| 200 | | | $\alpha$ | $-0.8607$ | 0.7156 | $-0.8082$ | 0.5179 | $-0.3337$ | 0.1191 |
| | | | $\beta$ | $-1.1932$ | 1.0257 | $-0.8220$ | 0.6780 | 0.1538 | 0.0822 |
| | | | $\theta$ | 0.6826 | 0.5870 | 0.5087 | 0.4237 | 0.3723 | 0.1573 |
| | | | $\lambda$ | $-0.4316$ | 1.1515 | $-0.4496$ | 1.1378 | $-0.6947$ | 0.5561 |
through which we realize that Bayesian estimation performs better than all other classical approaches for estimating the EOWL distribution parameters.

6. Data analysis of COVID-19 mortality rates

This section examines three real-world data sets for the COVID-19 data on death rates in various nations. We studied the model's efficiency by evaluating the goodness measure of the EOWL distribution.

The EOWL model is validated to many other competing models, including the Weibull-Lomax model (Tahir et al. [7]), Exponential Lomax (El-Bassiouny et al. [6]) and the Odds Exponential-Pareto IV distributions (Baharith et al. [8]).

Tables 3–5 gives the MLE estimates and standard errors (SE) for all model parameters and Kolmogorov–Smirnov (KS) statistics along with their P-values, values for the log-likelihood ($\ell$), the Akaike information criterion (AIC), the corrected Akaike information criterion (CAIC), Bayesian information criterion (BIC), Hannan-Quinn information criterion (HQIC), Cramér-von Mises (W*), and Anderson–Darling (A*) for all competitive models using the data used in the application section.

### Table 3

| EOWL | WL | OEPIV | EL | APL |
|------|----|-------|----|-----|
| $\alpha$ | 2.1044 | 0.3123 | 10.2170 | 12.5942 | 55.2170 |
| $\beta$ | 0.5390 | 0.2588 | 2.4775 | 0.5234 | 0.4451 |
| $\theta$ | 1.5328 | 1.2842 | 0.2246 | 0.0856 | 0.0379 |
| $\lambda$ | 0.2071 | 0.2104 | 0.0406 | 0.0341 | 0.1668 |
| KS test, P-value | 0.0732 | 0.6155 | 0.0884 | 0.3733 | 0.0741 |

### Table 4

| EOWL | WL | OEPIV | EL | APL |
|------|----|-------|----|-----|
| $\alpha$ | 1.7803 | 0.2817 | 3.7535 | 9.2768 |
| $\beta$ | 0.4663 | 0.3727 | 2.1601 | 0.4423 |
| $\theta$ | 2.3305 | 3.7987 | 0.3813 | 0.2959 |
| $\lambda$ | 0.1356 | 0.2481 | 0.0250 | 0.0181 |
| KS test, P-value | 0.0707 | 0.5903 | 0.0775 | 0.5718 | 0.0832 |

This is really a COVID-19 dataset for the United Kingdom that covers 107 days, from 12 March to 28 June 2020. This dataset is comprised of the number of deaths as reported on a daily basis divided by new daily cases. For instant access to the data, see the following: 0.0149, 0.0096, 0.0322, 0.0467, 0.0541, 0.0717, 0.0558, 0.0610, 0.0282, 0.0540, 0.0627, 0.1021, 0.0859, 0.0687, 0.1266, 0.1718, 0.0917, 0.1537, 0.1896, 0.1052, 0.1155, 0.2163, 0.1708, 0.2028, 0.1886, 0.1052, 0.1394, 0.1651, 0.1481, 0.1653, 0.1865, 0.1513, 0.1607, 0.1019, 0.0960, 0.1925, 0.1714, 0.1392, 0.1426, 0.1655, 0.0925, 0.0835, 0.1466, 1.0842, 0.1117, 0.1192, 0.1292, 0.0726, 0.0723, 0.1573, 0.1045, 0.0978, 0.1347, 0.0888, 0.0683, 0.0542, 0.1842, 0.1524, 0.1242, 0.1079, 0.1357, 0.0481, 0.0745, 0.0331, 0.2047, 0.1998, 0.1547, 0.1340, 0.0584, 0.3541, 0.1960, 0.1919, 0.0975, 0.2164, 0.1310, 0.0581, 0.0456, 0.1643, 0.2443, 0.1193, 0.1311,
Table 5  In this table, we tabulate the estimated parameters with their standard errors as generated using a real dataset along with information criterion and goodness of fit measures for COVID-19 data for Italy.

|     | EOWL | WL  | OEPIV | EL   | APL  |
|-----|------|-----|-------|------|------|
| $\alpha$ | 2.2483 | 0.2791 | 4.9203 | 16.1873 | 6.2835 | 4.9405 | 0.7700 | 1498.9223 | 157.3563 |
| $\beta$  | 0.5318 | 0.2949 | 2.8565 | 0.6976 | 0.3744 | 0.0428 | 60.3105 | 57.3884 | 1934.1413 | 91.9834 |
| $\theta$ | 13.9638 | 7.0319 | 0.4480 | 0.3237 | 0.3230 | 0.1935 | 4.2246 | 4.1302 | 122.3563 | 58.8497 |
| $\lambda$ | 3.1813 | 6.7366 | 0.1025 | 0.1224 | 0.2172 | 0.0647 | 155.7425 | 153.9973 | 154.9973 | 154.9973 |

KS test, P-value 0.0408 | 0.9840 | 0.0590 | 0.7680 | 0.0491 | 0.9196 | 0.0852 | 0.3157 | 0.0541 | 0.8509 |

|     | AIC  | CAIC | BIC  | HQIC | W*  | A*  |
|-----|------|------|------|------|-----|-----|
| $\ell$ | 155.7425 | 153.9973 | 154.9973 | 149.7287 | 149.7287 | 154.9973 |
| AIC | $-303.4851$ | $-301.6464$ | $-293.2622$ | $-298.9907$ | $-302.0421$ |
| CAIC | $-293.1083$ | $-290.5975$ | $-284.9248$ | $-289.9907$ | $-292.5996$ |
| BIC | $-298.8628$ | $-295.3724$ | $-297.3520$ | $-289.9907$ | $-298.5755$ |
| HQIC | $-293.1572$ | $-290.6668$ | $-293.6464$ | $-289.9907$ | $-291.8470$ |

|     | 0.0457 | 0.0871 | 0.0696 | 0.2300 | 0.0994 |
|-----|------|------|------|------|-----|

Fig. 3  The fitted PDF, CDF, SF, and P–P plots for COVID-19 data of the United Kingdom.

Fig. 4  The fitted PDF, CDF, SF, and P–P plots for COVID-19 data for the United States of America.
6.2. Data analysis for the United States of America’s mortality rate due to COVID-19 infections

This is a 102-day COVID-19 dataset for the United States of America, spanning the period 28 March to 7 July 2020. This information was compiled from the daily number of deaths divided by daily new cases. For instant access to the data, see the following:

0.1270, 0.0238, 0.0393, 0.1822, 0.1650, 0.1108, 0.1285, 0.0988, 0.0352, 0.0157, 0.3040, 0.2362, 0.1333, 0.1333 and 0.1124.

0.0149, 0.0235, 0.0230, 0.0159, 0.0200, 0.0413, 0.0360, 0.0378, 0.0363, 0.0399, 0.0453, 0.0436, 0.0598, 0.0624, 0.0546, 0.0607, 0.0609, 0.0521, 0.0615, 0.0928, 0.2232, 0.0620, 0.0812, 0.0629, 0.0651, 0.0840, 0.1072, 0.0821, 0.0567, 0.0559, 0.0606, 0.0380, 0.0586, 0.0980, 0.0925, 0.0631, 0.1869, 0.0049, 0.0176, 0.0495, 0.1112, 0.0890, 0.0940, 0.0600, 0.0652, 0.0413, 0.0588, 0.0665, 0.0816, 0.0753, 0.0579, 0.0436, 0.0527, 0.0382, 0.0568, 0.0613, 0.0531, 0.0767, 0.0400, 0.0406, 0.0237, 0.0471, 0.0722, 0.0595, 0.0597, 0.0389, 0.0265, 0.0518, 0.0419, 0.0566, 0.0516, 0.0390, 0.0245, 0.0266, 0.0314, 0.0701, 0.0410, 0.0436, 0.0320, 0.0255, 0.0171, 0.0268, 0.0259, 0.0333, 0.0318, 0.0188, 0.0172, 0.0112, 0.0155, 0.0229, 0.0184, 0.0621, 0.0146, 0.0114, 0.0216, 0.0103, 0.0129, 0.0134, 0.0117, 0.0143, 0.0032 and 0.0054.

Fig. 5  The fitted PDF, CDF, SF, and P–P plots for COVID-19 data for Italy.

Fig. 6  Plots of the profile-likelihood functions for the four parameters from the three datasets.
6.3. Data analysis for Italy’s Mortality Rate due to COVID-19 infections

This is a COVID-19 dataset for Italy that encompasses 127 days, from 1 March to 6 July 2020. This statistic is calculated by dividing daily new deaths by daily new cases. For instant access to data, see the following: 0.0107, 0.0490, 0.0601, 0.0460, 0.0533, 0.0630, 0.0297, 0.0885, 0.0540, 0.1720, 0.0847, 0.0713, 0.0989, 0.0495, 0.1025, 0.1079, 0.0984, 0.1124, 0.0806, 0.1044, 0.1212, 0.1167, 0.1255, 0.1416, 0.1315, 0.1073, 0.1629, 0.1485, 0.1453, 0.2000, 0.2070, 0.1520, 0.1628, 0.1666, 0.1417, 0.1221, 0.1767, 0.1987, 0.1408, 0.1456, 0.1443, 0.1319, 0.1053, 0.1789, 0.2032, 0.2167, 0.1387, 0.1646, 0.1375, 0.1421, 0.2012, 0.1957, 0.1297, 0.1754, 0.1390, 0.1761, 0.1119, 0.1915, 0.1827, 0.1548, 0.1522, 0.1369, 0.2495, 0.1253, 0.1597, 0.2195, 0.2555, 0.1956, 0.1831, 0.1791, 0.2057, 0.2406, 0.1227, 0.2196, 0.2641, 0.3067, 0.1749, 0.2148, 0.2195, 0.1993, 0.2421, 0.2430, 0.1994, 0.1779, 0.0942, 0.3067, 0.1965, 0.2003, 0.1180, 0.1686, 0.2668, 0.2113, 0.3371, 0.1730, 0.2212, 0.4972, 0.1641, 0.2667, 0.2690, 0.2321, 0.2792, 0.3515, 0.1398, 0.3436, 0.2254, 0.1302, 0.0864, 0.1619, 0.1311, 0.1994, 0.3176, 0.1856, 0.1071, 0.1041, 0.1593, 0.0357, 0.1149, 0.1176, 0.0457, 0.1264, 0.0476, 0.1620, 0.1154, 0.1493, 0.0673, 0.0894 and 0.0365, for more information see [21].

6.4. Concluding observations from using the three datasets

1. From Tables 3–5 It is abundantly evident that the EOWL distribution includes the fewest data when compared to the other distributions across all information criteria.

2. Furthermore, when the lifespan is EOWL, the P-value for the KS statistic is the largest. As a result, we find that the EOWL more closely matches the three real-world data sets from the United Kingdom, the United States of America, and Italy.

3. The estimated PDF and CDF values for the model plots are shown in Figs. 3–5 suggest that the proposed distribution is a good match and model for the COVID-19 data presented above.

4. The existence of MLEs is confirmed by Fig. 6, the fact that the log-likelihood function crosses the x-axis at only one point Furthermore:

5. As seen in Fig. 7, the MLEs are unique because the log-likelihood function has a global maximum number of roots. As shown in the graph, the log-likelihood function is a decreasing function that intersects the x-axis only once. As a result, as illustrated in the graph, the log-likelihood function has unique roots that are also the global maximum.

6. Fig. 8 includes the estimated hazard rate function plots of the EOWL model and TTT plots for three datasets.

7. Conclusion

We proposed a novel Lomax-Weibull distribution generalization, we called it the EOWL distribution, in this paper. We derived its statistical properties and a linear representation for its CDF that effectively determined the linear representations of the PDF, moments, moment generating function, and stress-strength reliability. Several approaches such as Bayesian estimation, and classical, were examined in order to acquire point estimates for the unknown EOWL parameters, \( \alpha, \beta, \gamma, \) and \( \lambda. \) The performance of several estimation methodologies was compared using a simulated study. This was accomplished using the Markov chain Monte Carlo technique, and we determined that the Bayesian approach surpasses all other traditional methodologies studied. Three different COVID-19 data sets from the United Kingdom, the United States of America, and Italy were assessed. EOWL was shown...
Fig. 8  Estimated hazard rate function plots of the EOWL model and TTT plots for the three datasets.
to fit the data more successfully than the majority of other rival distributions. Also, by referring to Fig. 8 we can conduct that the estimated hazard rate function plots of the EOWL model and TTT plots for three data sets provides a very good fitting for the three data sets.

8. Future work

Our point of is very interesting, in future we will make extension fro this work, as will apply censored sample on covid 19 infections to avoid highly cast of the waiting time to get mortality. Also we will apply different regression models to predict the future coming infections and mortality numbers in many countries also we will extend our work to make a time series modelling for the infections through the proposed distribution.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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