Noisy channel effect on quantum correlations of two relativistic particles

M. Mahdian, T. Makaremi, and Sh. Salimi

1Faculty of Physics, Theoretical and astrophysics department, University of Tabriz, 51665-163 Tabriz, Iran
2Department of Physics, Faculty of Science, University of Kurdistan, Sanandaj, Iran

We study the quantum correlation dynamics of two relativistic particles which is transmitted through one of the Pauli channels $\sigma_x, \sigma_y, \text{and} \sigma_z$. We compare sudden death and robustness of entanglement and geometric discord and quantum discord of two relativistic particles under noisy Pauli channels. we find out geometric discord and quantum discord may be more robust than entanglement against decoherence.

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I. INTRODUCTION

In quantum information theory, quantifying of quantum correlation plays a important role. However, despite of researcher’s attention that have more focused on entanglement, the measures of non-classical correlation in quantum information theory is not restricted on quantum entanglement. Recently, quantum discord is introduced as a quantum correlation that isn’t vanished for mixed separable state, against quantum entanglement. Quantum discord define as difference of two natural quantum expansion of classical mutual information $I_{cl}$. Because of hard work to obtain quantum discord and most work has been done on specific models such as x-state $\rho^{x}$. Dakic, Vedral and Brukner [8] introduced another measure so-called geometric discord for a bipartite state. Next, Girolami and Adesso introduced function of geometric discord as an upper bound for quantum discord under Pauli noisy channels. We find out that under Pauli channels $\sigma_x, \sigma_y, \sigma_z$ quantum entanglement and geometric discord and quantum discord tend to death asymptotically related on the velocity of observer. This behavior occur with more gradient for the lower speed observers in $0 < \theta < \pi$ and in $\frac{\pi}{2} < \theta < \pi$ these are vice versa. The quantum entanglement sudden death (ESD) occur $\sigma_x, \sigma_z$ and we survey the time of ESD for them.

This paper is organized as follows: In Sec. 2, we review some of basic definitions to introduction to concurrence as a measure of entanglement and geometric discord and quantum discord. In Sec. 3 we discuss the computation of entanglement (by using the concurrence) and quantum discord of two maximally spin entangled states, respectively. In Sec. 4, we investigate how two relativistic particles as initial state evolve under noisy channels. In Sec. 5, 6, 7 we investigate noisy channel effect on entanglement and geometric discord and quantum discord of two relativistic particles state. Conclusion are then presented in Sec. 6.

II. BASIC DEFINITION

A. Concurrence

In this subsection, we present the concurrence which is a good measure of entanglement for two qubit subsystems. The spin-flipped state for two qubits is

$$\rho = (\sigma_y \otimes \sigma_y) \rho (\sigma_y \otimes \sigma_y).$$

Note that the complex conjugate is taken in the computational basis $|00\rangle, |01\rangle, |10\rangle, |11\rangle$. The concurrence is defined as follows

$$C = \max \left[0, (\chi_1 - \chi_2 - \chi_3 - \chi_4)\right].$$
where the $\chi_i$ are the square roots of the eigenvalues of the matrix $R = \rho \rho^T$ [29, 30].

B. Geometric Discord

Geometric discord is defined as the closest distance between an arbitrary state and a classical-quantum state (zero discord) [9]. Therefore, the density matrix in terms of Bloch states are expanded as follows:

$$\rho = \frac{1}{4}(I \otimes I + \sum_{i=1}^{3} x_i \sigma_i \otimes I + \sum_{i=1}^{3} y_i I \otimes \sigma_i + \sum_{i,j=1}^{3} t_{ij} \sigma_i \otimes \sigma_j)$$

(3)

where $x_i$ and $y_i$ are the three-dimensional Bloch vectors of subsystems and $t_{ij}$ are the elements of correlation matrix $T$. Therefore we can obtain geometric discord as:

$$DG(\rho) = \frac{1}{4}(\|\vec{y}\|\vec{y}^T\|_2 + \|T\|_2^2 - k)$$

(4)

where $k$ being the largest eigenvalue of matrix $\vec{y}\vec{y}^T + T^T T$ and $\|\cdot\|_2$ is the norm of Hilbert-Schmidt.

C. Quantum Discord

Quantum discord is defined in terms of difference between two expressions for mutual information that are same in classical form but are different in quantum form. These two classical expressions are:

$$I(A,B) = H(A) + H(B) - H(A,B),$$

(5)

$$J(A,B) = H(A) - H(A|B).$$

(6)

Where $H(\cdot)$ is Shannon entropy. We can see these two classical equation in above are equivalent. In quantum case, we are used an quantum entropy are so called Von Neumann entropy $S(\rho)$ . Therefore we can define two expressions for quantum mutual information:

$$I(\rho) = S(\rho^A) + S(\rho^B) - S(\rho),$$

(7)

$$J_A(\rho) = S(\rho^B) - S(\rho^B|\rho^A)$$

(8)

Despite of classical case, two equations in above are inequivalent. The difference between the two expressions $I(\rho) - J_A(\rho)$ defines the basis-dependent quantum discord which is asymmetrical in the sense that $D_A(\rho)$ can differ from $D_B(\rho)$ [3]. Classical correlation is defined as:

$$CC(\rho) = \max_{\{\sigma_i^A\}} J_{\{\sigma_i^A\}}(\rho).$$

(9)

Where maximum is over the set of all possible projective measurements on to the eigen basis [28]. According to Refs. [3, 4, 7], we can define quantum discord as:

$$D_A = I(\rho) - \max_{\{\sigma_i^A\}} J_{\{\sigma_i^A\}}(\rho) = S(\rho_A) - S(\rho) + \min_{\{\sigma_i^A\}} S(\rho_B|\{\sigma_i^A\}).$$

(10)

III. TWO RELATIVISTIC PARTICLES

Let us consider a maximally spin-entangled state for two $s = 1/2$ fermions or two photons A and B. Consider two particles are far apart

$$|\psi^-(p_a, p_b)\rangle = \frac{1}{\sqrt{2}}(\psi^+(p_a)\psi^-(p_b) - \psi^+(p_b)\psi^-(p_a)),$$

(11)

where $p_a$, $p_b$ are the corresponding momentum vectors of particles A and B and $\varphi^+(p)$, $\varphi^-(p)$ are spins of its particles which are in direction of z-axis. We choose equal interaction angles for the two particles, $\alpha_p^a = \alpha_p^b$, as a natural simplification. Based on computation in Ref. [27], we obtain the density matrix for this x-state as

$$\rho(0) = \begin{pmatrix} \eta & 0 & 0 & \eta \\ 0 & \frac{1}{2} - \eta / 2 & \frac{1}{2} - \eta & 0 \\ 0 & \frac{1}{2} - \eta & \frac{1}{2} - \eta & 0 \\ \eta & 0 & 0 & \eta \end{pmatrix},$$

(12)

where $\theta = (\alpha_1 - \alpha_2)$, and $\eta = \frac{1}{2}\sin^2 \theta$. It is easy to obtain the concurrence and geometric discord and quantum discord of this system as follow:

$$C = 2(|\xi| - |\eta|),$$

$$DG = \frac{1}{2}(1 - 4|\eta|^2),$$

$$D = -1 + \frac{8(\xi \ln 8 \xi + \eta \ln 8 \eta)}{\ln 16}$$

(13)

respectively, where $\xi = \frac{1}{8}(3 + \cos 2 \theta)$.

Dependence of all of these three measures of quantum correlation are sketched in Figs. [1][2][3]. According to these figures we can find out geometric discord and quantum discord of this case depend on angles of between momentum and spin of each particles as well as quantum entanglement. We can see in these figures that quantum entanglement and geometric discord and quantum discord in interval $0 < \theta < \frac{\pi}{2}$ are decreased by increase $\theta$ and for interval $\frac{\pi}{2} < \theta < \pi$ are vice versa.

IV. TIME EVOLUTION OF TWO RELATIVISTIC PARTICLE STATE TRANSMITTED THROUGH NOISY CHANNEL

For open systems that are coupled to their environment, decoherence occurs. For this purpose, researchers has investigated a lot of noise models such as Pauli channels, in recent decades. In this Section, we shall give a survey of the various types of channels which can be affected on the quantum dynamics of open system as an initially entangled state $\rho(0)$ for two relativistic particles
that was supposed to be transmitted through these channels for the time $t$, respectively and it’s time evolution $\rho(t)$ will obtain as a solution of a master equation:

$$\frac{\partial \rho}{\partial t} = -i[H, \rho] + \sum_k \frac{1}{2} \gamma_k (2L_k \rho L_k^\dagger - \{L_k L_k^\dagger, \rho\}).$$  \hspace{1cm} (14)

In this equation $L_k \equiv 1 \otimes \sigma_k$ describes the decoherence of first qubit under $\sigma_k$ and $\gamma_k$ is the coupling constant and this master equation should be solved at $t > 0$ [20]. 

In this paper we assume $H_k = 0$ and one of particles is subject to the local noises[21].

V. TIME EVOLUTION OF QUANTUM ENTANGLEMENT OF TWO RELATIVISTIC PARTICLES STATE TRANSMITTED UNDER PAULI CHANNELS

A. Pauli channel $\sigma_x$

If one of two relativistic particles with initial state as Eq.(12) is subject to a local bit-flip noise, the time evolution is obtain by the solution of the master equation Eq.(14) with Lindblad operator $L_k \equiv 1 \otimes \sigma_x$. So after transmission of two relativistic particles through the Pauli channel $\sigma_x$ the density matrix is given as follows:

$$\rho_x = \frac{1}{4} \begin{pmatrix}
1 - \lambda & 0 & 0 & 1 + \mu_1 - 4\xi \\
0 & 1 + \lambda & 1 - \mu_1 - 4\xi & 0 \\
0 & 1 - \mu_1 - 4\xi & 1 + \lambda & 0 \\
1 + \mu_1 - 4\xi & 0 & 0 & 1 - \lambda
\end{pmatrix}$$  \hspace{1cm} (15)

where $\rho_x$ is the density matrix after transmission through Pauli channel $\sigma_x$ and $\mu_1 = \exp(-2\gamma_1 t)$ and $\lambda_1 = \mu_1(1 - 4\eta)$. We consider concurrence as a measure of quantum entanglement in this section. Therefore, for this case concurrence is given as:

$$C = \text{Max} \left[ 0, \frac{1}{2}(\mu_1 + \lambda_1 + 4(8\xi^2 - 3\xi + 1)) \right].$$  \hspace{1cm} (16)

As we see in Figs. 4 crossing the Pauli channel $\sigma_x$ concurrence for the observer in the interval $0 < \theta < \frac{\pi}{2}$ is decreased by increase of velocity of observer. It means that in this interval ESD is happened for higher speed observers more sudden. On the other hand, in the interval $\frac{\pi}{2} < \theta < \pi$ concurrence is increased by increase of velocity of observer. In the other words in this interval the concurrence reduction with observer speed is slower. It means that ESD occur for lower speed observers faster.

We can obtain the ESD time for this case as:

$$t_{ESD} = \frac{1}{\gamma_1} \ln \left( \sqrt{\frac{\xi}{\eta}} \right)$$  \hspace{1cm} (17)

B. Pauli channel $\sigma_y$

If one of two relativistic particles with initial state as Eq.(12) is subject to a local Pauli channel $\sigma_y$, the time evolution is obtain by the solution of the master equation Eq.(14) with Lindblad operator $L_k \equiv 1 \otimes \sigma_y$. So after transmission of two relativistic particles through the Pauli channel $\sigma_y$ the density matrix is given as follows:

$$\rho_y = \frac{1}{4} \begin{pmatrix}
1 - \lambda_2 & 0 & 0 & 1 - \lambda_2 \\
0 & 1 + \lambda_2 & -(1 + \lambda_2) & 0 \\
0 & -(1 - \lambda_2) & 1 - \lambda_2 & 0 \\
1 - \lambda_2 & 0 & 0 & 1 - \lambda_2
\end{pmatrix}$$  \hspace{1cm} (18)

where $\rho_y$ is the density matrix after transmission through Pauli channel $\sigma_y$ and $\mu_2 = \exp(-2\gamma_2 t)$ and $\lambda_2 = \mu_2(1 - 4\eta)$. So, for this case we can obtain concurrence as:

$$C = \frac{1}{2} (|\lambda_2 + 1| - |\lambda_2 - 1|).$$  \hspace{1cm} (19)

As we see in Figs. 5 under the Pauli channel $\sigma_y$ the gradient reduction of concurrence for the observer in the interval $0 < \theta < \frac{\pi}{2}$ is more for lower speed observers. It means that in this interval system tend to ESD for lower speed observers more sudden. On the other hand, in the interval $\frac{\pi}{2} < \theta < \pi$ the concurrence gradient reduction is more for higher speed observers. In the other words if the observer tend faster to ESD occur for him faster in this interval.

C. Pauli channel $\sigma_z$

If one of two relativistic particles with initial state as Eq.(12) is subject to a local dephasing noisy channel, the time evolution is obtain by the solution of the master equation Eq.(14) with Lindblad operator $L_k \equiv 1 \otimes \sigma_z$. So after transmission of two relativistic particles through the Pauli channel $\sigma_z$ the density matrix is given as follows:

$$\rho_z = \begin{pmatrix}
\eta & 0 & 0 & \eta \mu_3 \\
0 & \xi & -\xi \mu_3 & 0 \\
0 & -\xi \mu_3 & \xi & 0 \\
\eta \mu_3 & 0 & 0 & \eta
\end{pmatrix}$$  \hspace{1cm} (20)

where $\rho_z$ is the density matrix after transmission through dephasing channel and $\mu_3 = \exp(-2\gamma_3 t)$ and $\lambda_3 = \mu_3(1 - 4\eta)$. According to density matrix in above concurrence could be obtain as:

$$C = \text{Max} \left[ 0, 2 (|\mu_3 \xi| - \eta) \right].$$  \hspace{1cm} (21)

According to Fig.6 we can find out in the interval $0 < \theta < \frac{\pi}{2}$ the concurrence leads to ESD faster by increase speed of observer but for the observers in the interval $\frac{\pi}{2} < \theta < \pi$ ESD is happen later for high speed observer. The time of ESD could be obtain as:

$$t_{ESD} = \frac{1}{4\gamma_3} \ln \left( 1 - 2 \csc^2 \theta \right)^2.$$  \hspace{1cm} (22)
VI. TIME EVOLUTION OF GEOMETRIC DISCORD OF TWO RELATIVISTIC PARTICLE STATE TRANSMITTED UNDER PAULI CHANNELS

A. Pauli channel $\sigma_x, \sigma_z$

If the time evolution of density matrix when one of two relativistic particles is subject to a local bit-flip noise and dephasing channel are in the form of \([15, 20]\) respectively then the geometric discord is obtained as:

$$DG_{x(z)} = \frac{1}{4} \left[ (1 - 4\eta)^2 + \mu_{1(3)}^2 (1 + (1 - 4\eta)^2) \right]$$

$$-\frac{1}{4} \text{Max} \left[ \mu_{1(3)}^2, (1 - 4\eta)^2, \mu_{1(3)}^2 (1 - 4\eta)^2 \right]$$  \(23\)

where $\mu_{1(3)}$ is related on when Pauli channels $\sigma_{x(z)}$ act on system. According to Fig.\[\text{IV}\] we could see in the interval $0 < \theta < \frac{\pi}{2}$ geometric discord for the observers with a lower rate is more. So geometric discord death in this interval occur more sudden for the lower speed observers but in the interval $\frac{\pi}{2} < \theta < \pi$ geometric discord death occur more sudden for higher speed observers.

B. Pauli channel $\sigma_y$

If the time evolution of density matrix when one of two relativistic particles is subject to a Pauli channel $\sigma_y$, is in the form of \([18]\) then the geometric discord is obtained as:

$$DG_y = \frac{1}{2} \mu_{2}^2 (1 - 4\eta)^2.$$  \(24\)

As we seen in Fig.\[\text{V}\] in $0 < \theta < \frac{\pi}{2}$ by increase of speed of observer geometric discord death is happen slower. In the other words geometric discord for slower observers is more faster. On the other hand in $\frac{\pi}{2} < \theta < \pi$ by increase of speed of observers geometric discord leads to death by more gradient. It means that geometric discord death in this interval is occur later for higher speed observers. According to this figure for every velocity of observers unless $\theta = \frac{\pi}{2}$ geometric discord tend to death asymptotically.

VII. TIME EVOLUTION OF QUANTUM DISCORD OF TWO RELATIVISTIC PARTICLE STATE TRANSMITTED UNDER PAULI CHANNELS

A. Pauli channel $\sigma_x, \sigma_z$

If two relativistic particles transmitted through Pauli channel $\sigma_x$ and $\sigma_z$ then the time evolution of it is as Eq.\([15, 20]\), respectively. There are x-states. We choose the algorithm of Ref.\[\text{6}\] to compute the quantum discord. Therefore the eigenvalues of Eq.\([15, 20]\) are:

$$\varepsilon_1 = \xi (1 - \mu_{1(3)})$$

$$\varepsilon_2 = \xi (1 + \mu_{1(3)})$$

$$\varepsilon_3 = \eta (1 - \mu_{1(3)})$$

$$\varepsilon_4 = \eta (1 + \mu_{1(3)}).$$  \(25\)

We can obtain the optimal conditional Von Neumann entropy as:

$$SC = - \frac{1 + \phi}{2} \log_2 \left( \frac{1 + \phi}{2} \right) - \frac{1 - \phi}{2} \log_2 \left( \frac{1 - \phi}{2} \right)$$  \(26\)

where $\phi = \text{max} \left[ 1 - 4\eta, \mu_{1(3)}^2, \mu_{1(3)}^2 (1 - 4\eta) \right]$. Since the Von Neumann entropy for the two subsystems of this state are as: $S(\rho_{x}^2) = S(\rho_{z}^2) = 1$. Therefore the quantum mutual information as a measure of total correlation is as:

$$I(\rho_x) = 2 + \sum_{i=1}^{4} \varepsilon_i \log_2 \varepsilon_i$$  \(27\)

where $\varepsilon_i$’s are the eigenvalues in Eq.\([20]\). So the quantum discord of two relativistic particles after transmit trough Pauli channels $\sigma_x, \sigma_z$ is given as:

$$D(\rho_x) = -1 + \frac{\nu}{\ln 16} \left( \ln \left[ (8\eta)^4 \xi^3 \eta \right] + (8\xi - 3) \ln \frac{\xi}{\eta} \right)$$

$$+ \frac{\vartheta}{\ln 16} \left( \ln \left[ (8\theta)^4 \xi^3 \eta \right] + (8\xi - 3) \ln \frac{\xi}{\eta} \right) + SC$$  \(28\)

where $\nu = \sqrt{\mu_{1(3)} \cosh (\gamma_{1(3)} t)}$ and $\vartheta = \sqrt{\mu_{1(3)} \sinh (\gamma_{1(3)} t)}$. As we see in Fig.\[\text{VI}\] quantum discord is invariant for different velocity of observers at first then it is decreased until it tend to DSD. It is interesting that this reduction is more in $0 < \theta < \frac{\pi}{2}$ for lower speed observers but in $\frac{\pi}{2} < \theta < \pi$ the quantum discord reduction is more for higher speed observers.

B. Pauli channel $\sigma_y$

If two relativistic particles transmitted through Pauli channel $\sigma_y$ then the time evolution of it is as Eq.\([18]\) which is a x-state. As in the previous section we can obtain the eigenvalues for Eq.\([18]\) as:

$$\epsilon_1 = \epsilon_2 = 0$$
\[ \epsilon_3 = \frac{1}{2} (-\mu_2 (4\xi - 1) + 1) \]
\[ \epsilon_4 = \frac{1}{2} (\mu_2 (4\xi - 1) + 1). \]  
(29)

On one hand the Von Neumann entropy for marginal subsystems are given as:

\[ S(\rho^A_y) = S(\rho^B_y) = 1 \]

Therefore the quantum mutual information for this system is as:

\[ I_y = \frac{2}{\ln 16} \left[ \lambda_2 \ln \left( \frac{1 + \lambda_2}{1 - \lambda_2} \right) + \ln 4 (1 + \lambda_2) (1 - \lambda_2) \right] . \]  
(30)

On the other hand the classical correlation is 1. Therefore we can obtain the quantum discord as:

\[ QD(\rho_y) = \frac{2}{\ln 16} \left[ \lambda_2 \ln \left( \frac{1 + \lambda_2}{1 - \lambda_2} \right) + \ln (1 + \lambda_2) (1 - \lambda_2) \right] . \]  
(31)

As seen in Fig.10 in the interval 0 < \theta < \frac{\pi}{2} the quantum discord gradient reduction is more for lower speed observers. In the other words lower speed observers tend to quantum discord death with more gradient. On the other hand in the interval \frac{\pi}{2} < \theta < \pi by increase of speed of observer the quantum discord gradient reduction become more and more. In the other words the higher speed observers tend to quantum discord death with more gradient in this interval.

**VIII. CONCLUSIONS**

In this paper we have investigated the evolutions of quantum entanglement and geometric discord and quantum discord dynamics of two relativistic particles under noisy Pauli channels. We have found out these quantum correlations dependence on speed of observer in Pauli channels. We have shown that under each of Pauli channels \sigma_y, \sigma_x, \sigma_z quantum entanglement and geometric discord and quantum discord tend to death. This behavior occur for the lower speed observers in 0 < \theta < \frac{\pi}{2} and in \frac{\pi}{2} < \theta < \pi these are vice versa. The quantum entanglement sudden death (ESD) occur under Pauli channels \sigma_x, \sigma_z.

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FIG. 1: Concurrence for $0 < \theta < \frac{\pi}{2}$ (left panel) and concurrence for $\frac{\pi}{4} < \theta < \pi$ (right panel) measures versus $\theta$ for different values of $\theta$ before noise is sketched in this figure. As we seen by increase of $\theta$ in $0 < \theta < \frac{\pi}{2}$ concurrence is decreased but for $\frac{\pi}{2} < \theta < \pi$ is vice versa.

FIG. 2: Geometric discord (DG) for $0 < \theta < \frac{\pi}{2}$ (left panel) and geometric discord for $\frac{\pi}{4} < \theta < \pi$ (right panel) measures versus $\theta$ for different values of $\theta$ before noise is sketched in this figure. As we seen by increase of $\theta$ in $0 < \theta < \frac{\pi}{2}$ geometric discord is decreased but for $\frac{\pi}{2} < \theta < \pi$ is vice versa.
FIG. 3: Quantum discord (QD) for $0 < \theta < \frac{\pi}{2}$ (left panel) and quantum discord for $\frac{\pi}{2} < \theta < \pi$ (right panel) measures versus $\theta$ for different values of $\theta$ before noise is sketched in this figure. As we seen by increase of $\theta$ in $0 < \theta < \frac{\pi}{2}$ quantum discord is decreased but for $\frac{\pi}{2} < \theta < \pi$ is vice versa.

FIG. 4: Concurrence for $0 < \theta < \frac{\pi}{2}$ (left panel) and concurrence for $\frac{\pi}{2} < \theta < \pi$ (right panel) measures versus $\gamma_1 t$ for different values of $\theta$ under Pauli noisy channel $\sigma_x$ is sketched in this figure. As we seen in $0 < \theta < \frac{\pi}{2}$ ESD is more sudden for the lower speed observer but for $\frac{\pi}{2} < \theta < \pi$ is vice versa.
FIG. 5: Concurrence for $0 < \theta < \frac{\pi}{2}$ (left panel) and concurrence for $\frac{\pi}{2} < \theta < \pi$ (right panel) measures versus $\gamma_2 t$ for different values of $\theta$ under Pauli noisy channel $\sigma_x$ is sketched in this figure. As we seen ESD don’t occur but all of them tend to quantum entanglement death asymptotically for lower speed observers more sudden in $0 < \theta < \frac{\pi}{2}$ but for $\frac{\pi}{2} < \theta < \pi$ is vice versa.

FIG. 6: Concurrence for $0 < \theta < \frac{\pi}{2}$ (left panel) and concurrence for $\frac{\pi}{2} < \theta < \pi$ (right panel) measures versus $\gamma_3 t$ for different values of $\theta$ under Pauli noisy channel $\sigma_z$ is sketched in this figure. As we seen ESD in $0 < \theta < \frac{\pi}{2}$ is happen for lower speed observers more sudden but for $\frac{\pi}{2} < \theta < \pi$ is vice versa.
FIG. 7: Geometric discord (DG) for $0 < \theta < \frac{\pi}{2}$ (left panel) and geometric discord for $\frac{\pi}{2} < \theta < \pi$ (right panel) measures versus $\gamma_1 t$ for different values of $\theta$ under Pauli noisy channel $\sigma_x$ is sketched in this figure. As we seen geometric discord sudden death don’t occur but geometric discord tend to death in for lower speed observers more sudden $0 < \theta < \frac{\pi}{2}$ but for $\frac{\pi}{2} < \theta < \pi$ is vice versa.

FIG. 8: Geometric discord (DG) for $0 < \theta < \frac{\pi}{2}$ (left panel) and geometric discord for $\frac{\pi}{2} < \theta < \pi$ (right panel) measures versus $\gamma_2 t$ for different values of $\theta$ under Pauli noisy channel $\sigma_y$ is sketched in this figure. As we seen geometric discord tend to death for lower speed observers asymptotically more sudden in $0 < \theta < \frac{\pi}{2}$ but for $\frac{\pi}{2} < \theta < \pi$ is vice versa.
FIG. 9: Quantum discord (QD) for $0 < \theta < \frac{\pi}{2}$ (left panel) and quantum discord for $\frac{\pi}{2} < \theta < \pi$ (right panel) measures versus $\gamma_1 t$ for different values of $\theta$ under Pauli noisy channel $\sigma_x$ and $\sigma_z$ which is same is sketched in this figure. As we seen at first quantum discord is invariant then quantum discord tend to death accordance together for all observers. This reduction of quantum discord is more for lower speed observers in $0 < \theta < \frac{\pi}{2}$ but for $\frac{\pi}{2} < \theta < \pi$ is vice versa.

FIG. 10: Quantum discord (QD) for $0 < \theta < \frac{\pi}{2}$ (left panel) and quantum discord for $\frac{\pi}{2} < \theta < \pi$ (right panel) measures versus $\gamma_2 t$ for different values of $\theta$ under Pauli noisy channel $\sigma_y$ is sketched in this figure. As we seen in the interval $0 < \theta < \frac{\pi}{2}$ quantum discord tend to death asymptotically with more gradient for lower speed observers but in the interval $\frac{\pi}{2} < \theta < \pi$ is vice versa.
