Generating sequences of phonon wave packets by optical excitation of a quantum dot

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Abstract. Phonons can play an active role in controlling solid state systems or in the transport of energy and information. Thus, it is vital to understand generation processes and properties of phonons. By optical excitation of a semiconductor quantum dot, a sequence of phonon wave packets can be created. In this paper we focus on the fluctuation properties of lattice displacement and momentum of these phonon wave packets. For detuned excitation the fluctuations may fall below their respective vacuum values and squeezing occurs.

1. Introduction

While phonons, which are always present in any solid state system, have mostly been regarded as unwanted sources of dissipation or decoherence of the electronic system [1, 2], now the focus shifts towards the active use of phonons. Examples are the phonon assisted state preparation of excitons in quantum dots [3–6] or ultrafast manipulation of lasing processes by strain pulses [7]. Phonons also offer the possibility to study fundamental quantum effects like squeezing in a mechanical system [8–14]. To actively use phonons, it is mandatory to understand how phonons are generated and what the properties of the created phonons are. In this paper we will investigate the properties of acoustic phonons generated by the optical excitation of a semiconductor quantum dot (QD). Longitudinal acoustic (LA) phonons couple strongly to the electronic states of the QD, which makes a QD also a good candidate for a phonon generator. Sequences of phonon wave packets can be generated by switching on a continuous wave (CW) excitation [14, 15]. The properties of these wave packets depend strongly on the excitation conditions like excitation strength and detuning. In previous studies we have analyzed the lattice displacement and its properties [14, 15]. To complement these studies, in this paper we will discuss the momentum and its fluctuations. This is particularly important for squeezing as displacement and momentum are a pair of conjugate operators that enter the Heisenberg uncertainty principle. From the large variety of excitation conditions we will pick a strong excitation which results in the generation of a long sequence of phonon wave packets. We will consider excitations with three different detunings and discuss the resulting phonon dynamics including the behavior of the fluctuations.
2. Theory
A detailed description of the electron and phonon system can be found in previous publications, e.g. Refs. [14, 15], such that we limit ourself to a brief summary here. We treat the QD as an electronic two-level system consisting of the ground and a single exciton state. The excitation is continuous and switched on smoothly. The switch on is simulated by an error function. The strength of the CW field is characterized by its Rabi frequency $\Omega_R$. A resonant excitation refers to the case when the frequency of the light matches the transition energy (including the polaron shift) of the electronic system. We also consider detuned excitations where the detuning $\Delta$ between laser frequency and transition energy has a finite value. The LA phonons are treated in harmonic approximation. From the phonon creation and annihilation operators, we calculate the lattice displacement $u(r)$, the lattice momentum $\pi(r)$ and furthermore the squared fluctuations $(\Delta u)^2$ and $(\Delta \pi)^2$, respectively. For simplicity, the QD is taken to be spherical such that all quantities are radially symmetric and only depend on the distance $r$ from the dot center. Squeezing is achieved when the fluctuations fall below their respective vacuum values. To easily identify if the phonons are squeezed we introduce the normalized difference of the fluctuation to the vacuum fluctuations as $D_u = ((\Delta u)^2 - (\Delta u)^2_{\text{vac}})/(\Delta u)^2_{\text{vac}}$ and $D_\pi = ((\Delta \pi)^2 - (\Delta \pi)^2_{\text{vac}})/(\Delta \pi)^2_{\text{vac}}$ for displacement and momentum, respectively. If one of these quantities is smaller than zero, i.e., $D_u < 0$ or $D_\pi < 0$, the phonons are squeezed.

3. Results
For our calculations we consider a 5 nm large GaAs QD which is excited by a CW laser pulse with $\Omega_R = 6 \text{ ps}^{-1}$ at temperature $T = 0 \text{ K}$. The switch on takes place on the time scale of 1 ps. Our results are summarized in Fig. 1. In the three columns we consider three different detunings, an excitation above the transition energy with $\Delta = +0.5 \text{ meV}$ (left column), resonant excitation with $\Delta = 0$ (middle column) and excitation below the transition energy with $\Delta = -0.5 \text{ meV}$ (right column).

The first row shows the dynamics of the population $f$ of the exciton state (solid red line) as well as the dynamics of the laser amplitude (blue dashed line). As soon as the laser sets in, the population exhibits Rabi oscillations. For $\Delta = 0$ the period of the Rabi oscillation is $T_R = 2\pi/\Omega_R \approx 1 \text{ ps}$, which is determined by the strength of the laser excitation. For QDs, the coupling to the phonons is non-monotonic in energy [15] and the maximal coupling for our parameters is found at $\Omega_R = 3 \text{ ps}^{-1}$. An efficient coupling is reflected in the electronic system by strongly damped Rabi oscillations. Accordingly, for $\Omega_R = 6 \text{ ps}^{-1}$ the Rabi oscillations are almost undamped within the time window considered here, because the phonon system cannot react quickly enough to the fast change in the electronic system. Since in all cases studied here $\Delta^2/\hbar^2 \ll \Omega_R^2$, the detuning plays a minor role for the oscillation period and only slightly affects the long time values of the occupation. Nevertheless it turns out that it strongly influences the phonon properties, as will be discussed below.

The second row depicts the lattice displacement $\tilde{u}$ (top) and the lattice momentum $\tilde{\pi}$ (bottom), which have been scaled to compensate for the radial decay. Each change in the electronic system is accompanied by the emission of a phonon wave packet, because the change in the electronic system leads to a rapid shift of the equilibrium position for the phonons. The creation of the exciton leads to the emission of a wave packet with negative amplitude in the displacement and one with alternating amplitude in the momentum. The switch on of the laser pulse leads to a rather intense wave packet, which is emitted from the QD. The amplitude of the following wave packet is lower and remains almost the same for all following wave packets, which is in agreement with the weak dephasing of the Rabi oscillations. The repetition rate of the wave packet emission matches the Rabi frequency. The shape of the wave packets is mostly independent of the detuning. In addition to the emitted wave packets, a polaron is created in the QD region which causes a localized lattice distortion which is persistent as long as the exciton occupation.
is non-vanishing. The third row displays the fluctuations of the lattice displacement \( \tilde{D}_u \) (top) and the lattice momentum \( \tilde{D}_\pi \) (bottom), which have again been scaled to compensate for the radial decay. The fourth and last row shows the fluctuations of the lattice displacement \( \tilde{D}_u \) (solid orange) and the lattice momentum \( \tilde{D}_\pi \) (dotted green) at \( r = 20 \text{ nm} \). Let us first consider the zero detuning case. Apart from the fluctuations of the initial wave packet, both fluctuations exhibit regular oscillations with twice the Rabi frequency. The fluctuations of the displacement \( \tilde{D}_u \) clearly oscillate out of phase with the fluctuations of the momentum \( \tilde{D}_\pi \). Note that oscillations with double frequency are not a clear proof for squeezing [10], i.e., it is not directly related to the fact that \( \tilde{D}_{u/\pi} \) become negative. For zero detuning, the both fluctuations \( \tilde{D}_u \) and \( \tilde{D}_\pi \) are always positive, thus, no squeezing occurs.

Figure 1. First row: occupation of the exciton state (solid red) and laser amplitude (dashed blue); second row: lattice displacement \( \tilde{u} = u/r \) and momentum \( \tilde{\pi} = \pi/r \); third row: scaled relative fluctuations of the displacement \( \tilde{D}_u = D_u/r^2 \) and of the momentum \( \tilde{D}_\pi = D_\pi/r^2 \); fourth row: cut of the fluctuations at \( r = 20 \text{ nm} \). All displayed for three different detunings: positive detuning \( \Delta = +0.5 \text{ meV} \), resonant excitation \( \Delta = 0 \text{ meV} \) and negative detuning \( \Delta = -0.5 \text{ meV} \). The scaling of the plotted quantities by \( r^{-1} \) and \( r^{-2} \), respectively, compensates for the radial decay.
For detuned excitation, the oscillation is less regular, because in addition to the double Rabi frequency single Rabi frequency components appear [14]. The amplitude of the fluctuations for displacement and momentum are still out of phase, i.e., when $\tilde{D}_u$ has a maximum $\tilde{D}_\pi$ has a minimum and vice versa. For negative detuning, we find that both fluctuations $\tilde{D}_u$ and $\tilde{D}_\pi$ fall below zero, i.e., the phonons exhibit displacement and momentum squeezing. This is in agreement with findings from other systems, where squeezed states can be generated by negative detuning [8, 16, 17]. Previous studies have shown that for negative detuning squeezing is found for all excitation strengths [14]. It is interesting to note that for positive detunings, the appearance of squeezing depends on the excitation strength, i.e., on the Rabi frequency. If the Rabi frequency is too low, phonon emission processes dominate and $\tilde{D}_u$ becomes entirely positive [14]. For the high Rabi frequency taken here, we find that the fluctuations of the displacement $\tilde{D}_u$ fall below zero also for $\Delta > 0$. In contrast, the fluctuations of the momentum $\tilde{D}_\pi$ stay positive for all excitation strengths in this regime. Because for squeezing it is sufficient, when the fluctuations of one variable fall below their vacuum value, also here squeezed phonon wave packets emerge.

4. Conclusions
In summary, we have presented a detailed analysis of the properties of the phonons generated by the optical excitation of a QD. In particular we have compared the lattice momentum and its fluctuations with their respective displacement counterparts. Similar to the fluctuations of the displacement, the fluctuations of the momentum fall below their vacuum value for negative detuned excitation, confirming that in this case squeezed phonons are created. For positive detuning, squeezing has been found, because the lattice fluctuations fall below their vacuum value, however, the fluctuations of the momentum stay above their vacuum value and thus there is no momentum squeezing.

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References
[1] Ramsay A J 2010 Semicond. Sci. Technol. 25 103001
[2] Reiter D E, Kuhn T, Glässl M and Axt V M 2014 J. Phys. Cond. Matter 26 423203
[3] Reiter D E et al. 2012 Acta Phys. Pol. A 122 1065
[4] Glässl M, Barth A M and Axt V M 2013 Phys. Rev. Lett. 110 147401
[5] Quilter J H et al. 2015 Phys. Rev. Lett. 114 137401
[6] Boumonar S et al. 2015 Phys. Rev. B 91 161302
[7] Brüggemann C et al. 2011 Nature Photon. 6 30
[8] Garrett G A et al. 1997 Science 275 1638
[9] Hu X and Nori F 1997 Phys. Rev. Lett. 79 4605
[10] Sauer S et al. 2010 Phys. Rev. Lett. 105 157401
[11] Hussain A and Andrews S R 2010 Phys. Rev. B 81 224304
[12] Misochko O V, Hu J and Nakamura K G 2011 Phys. Lett. A 375 4141
[13] Wigger D, Reiter D E, Axt V M and Kuhn T 2013 Phys. Rev. B 87 085301
[14] Wigger D et al. 2015 Photonics 2 214
[15] Wigger D et al. 2014 J. Phys. Cond. Matter 26 355802
[16] Papenkort T, Axt V M and Kuhn T 2012 Phys. Rev. B 85 235317
[17] Gerry C C and Knight P L 2005 Introductory Quantum Optics (Cambridge: Cambridge University Press)