TRANSVERSAL ELECTRIC CONDUCTIVITY OF QUANTUM NON-DEGENERATE COLLISIONAL PLASMAS

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Аннотация

Formulas for calculation of transversal dielectric function and transversal electric conductivity in quantum non-degenerate collisional plasmas under arbitrary degree of degeneracy of the electron gas are received. The Wigner–Vlasov–Boltzmann kinetic equation with collision integral in BGK (Bhatnagar, Gross and Krook) form in coordinate space is used. Various special cases are investigated.

Key words: collisional non-degenerate plasma, Schrödinger equation, electric conductivity, dielectric function.

PACS numbers: 52.25.Dg Plasma kinetic equations, 52.25.-b Plasma properties, 05.30 Fk Fermion systems and electron gas

INTRODUCTION

In the present work formulas for calculation of electric conductivity and dielectric function in quantum non-degenerate collisional plasma under arbitrary temperature, i.e. under arbitrary degree of degeneration of the electron gas are deduced.

During the derivation of the kinetic equation we generalize the approach, developed by Klimontovich and Silin [1].

Dielectric function in the collisionless quantum gaseous plasma was studied by many authors (see, for example, [1] – [10]).
In the work [6], where the one-dimensional case of the quantum plasma is investigated, the importance of derivation of dielectric function with use of the quantum kinetic equation with collision integral in the form of BGK – model (Bhatnagar, Gross, Krook) [11], [12] was noted.

The present work is devoted to the performance of this task.

A dielectric function is one of the most significant characteristics of a plasma. This quantity is necessary for description of the skin effect [13], for analysis of surface plasmons [14], for description of the process of propagation and damping of the transverse plasma oscillations [10], the mechanism of electromagnetic waves penetration in plasma [9], and for analysis of other problems of plasma physics [15], [16], [17], [18] and [19].

Kliewer and Fuchs were the first who have noticed [4], that the dielectric function for quantum plasma deduced by Lindhard in collisional case does not pass into dielectric function for classical plasma in the limit when Planck’s constant $\hbar$ converges to zero. This means, that dielectric Lindhard’s function does not take into account electron collisions correctly. Kliewer and Fuchs have corrected Lindhard’s dielectric function so that it passed into classical one under condition $\hbar \to 0$.

In the works [14], [15] the dielectric function received by them was applied to consideration of various questions of metal optics.

In the work [5] the correct account of collisions in framework of the relaxation model in electron momentum space for the case of longitudinal dielectric function has been carried out. At the same time the correct account of influence of collisions for transversal dielectric function has not been implemented till now.

The aim of the present work is the elimination of this lacuna.

1. KINETIC EQUATION FOR THE WIGNER FUNCTION

Let’s consider the Schrödinger equation written for a particle in an elect-
romagnetic field in terms of density matrix \( \rho \)

\[
i\hbar \frac{\partial \rho}{\partial t} = H \rho - H^{*'} \rho. \tag{1.1}
\]

Here \( H \) is the Hamilton operator, \( H^* \) is the complex conjugate operator to \( H \), \( H^{*'} \) is the complex conjugate operator to the \( H \), which forces on primed spatial variables \( r' \).

Hamilton operator for the free particle which is in the field of the scalar potential \( U \) and in the field of vector potential \( A \), has the following form:

\[
H = \frac{(p - eA)^2}{2m} + eU = \frac{p^2}{2m} - \frac{e}{2mc}(pA + Ap) + \frac{e^2}{2mc^2}A^2 + eU. \tag{1.2}
\]

Here \( p \) is the momentum operator, \( p = -i\hbar \nabla \), \( e \) is the electron charge, \( m \) is the electron mass, \( c \) is the light velocity.

Let’s rewrite the Hamilton operator (1.2) in the explicit form:

\[
H = -\frac{\hbar^2}{2m} \Delta + \frac{ie\hbar}{2mc}(2A \nabla + \nabla A) + \frac{e^2}{2mc^2}A^2 + eU. \tag{1.3}
\]

Complex conjugate to the \( H \) operator \( H^* \) according to (1.3) has the form

\[
H^* = -\frac{\hbar^2}{2m} \Delta - \frac{ie\hbar}{2mc}(2A' \nabla + \nabla A') + \frac{e^2}{2mc^2}A'^2 + eU. \tag{1.4}
\]

Hence we can write down for \( H \rho \):

\[
H \rho = -\frac{\hbar^2}{2m} \Delta \rho + \frac{ie\hbar}{2mc}(2A \nabla \rho + \rho \nabla A) + \frac{e^2}{2mc^2}A^2 \rho + eU \rho \tag{1.4}
\]

and for \( H^{*'} \rho \):

\[
H^{*'} \rho = -\frac{\hbar^2}{2m} \Delta' \rho - \frac{ie\hbar}{2mc}(2A' \nabla' \rho + \rho \nabla' A) + \frac{e^2}{2mc^2}A'^2 \rho + eU' \rho. \tag{1.5}
\]

Operators \( \nabla \) and \( \Delta \) from Eqs (1.4) and (1.5) force on unprimed spatial variables of the density matrix, i.e. \( \nabla = \nabla_R \), \( \Delta = \Delta_R \). In the operator \( H^{*'} \) it is necessary to replace the operators \( \nabla = \nabla_R \) and \( \Delta = \Delta_R \) by operators \( \nabla' \equiv \nabla_{R'} \) and \( \Delta' \equiv \Delta_{R'} \), in addition we introduce the following designations

\[
A' \equiv A(R', t), \quad U' \equiv U(R', t).
\]
Let’s find the right-hand member of the equation (1.1), i.e. difference between relations (1.4) and (1.5): $H\rho - H^{*}\rho$. According to (1.4) and (1.5) we have:

$$H\rho - H^{*}\rho = -\frac{\hbar}{2m}(\Delta \rho - \Delta' \rho) +$$

$$+ \frac{i e \hbar}{2mc} \left[ 2 \left( A \nabla \rho + A' \nabla' \rho \right) + \rho \left( \nabla A + \nabla' A \right) \right] +$$

$$+ \frac{e^2}{2mc^2} \left[ A^2(\mathbf{R}, t) - A^2(\mathbf{R}', t) \right] + e[U(\mathbf{R}, t) - U(\mathbf{R}', t)] \rho.$$

The connection between density matrix $\rho(\mathbf{r}, \mathbf{r}', t)$ and Wigner function $f(\mathbf{r}, \mathbf{p}, t)$ is given by the inversion and direct Fourier conversions

$$f(\mathbf{r}, \mathbf{p}, t) = \int \rho(\mathbf{r} + \frac{\mathbf{a}}{2}, \mathbf{r} - \frac{\mathbf{a}}{2}, t) e^{-i \mathbf{p} \cdot \mathbf{a} / \hbar} d^3 a,$$

$$\rho(\mathbf{R}, \mathbf{R}', t) = \frac{1}{(2\pi \hbar)^3} \int f\left( \frac{\mathbf{R} + \mathbf{R}'}{2}, \mathbf{p}', t \right) e^{i \mathbf{p}' \cdot (\mathbf{R} - \mathbf{R}') / \hbar} d^3 \mathbf{p}'.$$

The Wigner function is analogue of distribution function for quantum systems. It is widely used in the diversified physics questions. Substituting the representation of the density matrix in terms of the Wigner function (1.2) into the equation for the density matrix (1.1), we obtain

$$i\hbar \frac{\partial \rho}{\partial t} = H \left\{ \frac{1}{(2\pi \hbar)^3} \int f\left( \frac{\mathbf{R} + \mathbf{R}'}{2}, \mathbf{p}', t \right) e^{i \mathbf{p}' \cdot (\mathbf{R} - \mathbf{R}') / \hbar} d^3 \mathbf{p}' \right\} -$$

$$- H^{*'} \left\{ \frac{1}{(2\pi \hbar)^3} \int f\left( \frac{\mathbf{R} + \mathbf{R}'}{2}, \mathbf{p}', t \right) e^{i \mathbf{p}' \cdot (\mathbf{R} - \mathbf{R}') / \hbar} d^3 \mathbf{p}' \right\}.$$

Let’s use the equalities written above. Thus the right–hand member of the previous equation we may present in explicit form. As a result we receive the following equation:

$$i\hbar \frac{\partial \rho}{\partial t} = \frac{1}{(2\pi \hbar)^3} \int \left\{ -\frac{i \hbar}{m} \mathbf{p}' \cdot \nabla f + \frac{i e \hbar}{2mc} \left[ \text{div} \mathbf{A}(\mathbf{R}, t) + \text{div} \mathbf{A}(\mathbf{R}', t) \right] f +$$

$$+ \frac{i e \hbar}{2mc} \left[ \mathbf{A}(\mathbf{R}, t) + \mathbf{A}(\mathbf{R}', t) \right] \nabla f - \frac{e}{mc} \left[ \mathbf{A}(\mathbf{R}, t) - \mathbf{A}(\mathbf{R}', t) \right] \mathbf{p}' f +$$

$$+ e[U(\mathbf{R}, t) - U(\mathbf{R}', t)] \rho.\right\}.$$
\[ + \frac{e^2}{2mc^2} [A^2(R, t) - A^2(R', t)] f + e [U(R, t) - U(R', t)] f \] \[ e^{ip'(R-R')/\hbar} d^3p'. \] (1.6)

In the equation (1.6) we will put
\[ R = r + \frac{a}{2}, \quad R' = r - \frac{a}{2}. \]

Then in this equation we obtain
\[ f \left( \frac{R + R'}{2}, p', t \right) e^{ip'(R-R')/\hbar} = f(r, p', t) e^{ip'a/\hbar}. \]

Let’s multiply the equation (1.6) by \( e^{-ip'a/\hbar} \) and let’s integrate it by \( a \). Then we will divide both parts of the equation by \( i\hbar \). As a result we receive
\[ \frac{\partial f}{\partial t} = \iint \left\{ -\frac{p'}{m} \nabla f + \frac{e}{2mc} [A(r + \frac{a}{2}, t) + A(r - \frac{a}{2}, t)] \nabla f + \right. \]
\[ + \frac{ie}{mch} [A(r + \frac{a}{2}, t) - A(r - \frac{a}{2}, t)] p' f + \]
\[ + \frac{e}{2mc} \left[ \text{div} A(r + \frac{a}{2}, t) + \text{div} A(r - \frac{a}{2}, t) \right] f - \]
\[ - \frac{ie^2}{2mc^2\hbar} [A^2(r + \frac{a}{2}, t) - A^2(r - \frac{a}{2}, t)] f - \]
\[ - \frac{ie}{\hbar} \left[ U(r + \frac{a}{2}, t) - U(r - \frac{a}{2}, t) \right] f \left\} e^{ip'(p-p)a/\hbar} \frac{d^3a d^3p'}{(2\pi\hbar)^3} \right. \] (1.7)

On the left-hand side of the equation (1.7) we have \( f = f(r, p, t) \), in the integral we have \( f = f(r, p', t) \).

We consider the integral
\[ \iint p'(\nabla f) e^{ip'(p-p)a/\hbar} \frac{d^3a d^3p'}{(2\pi\hbar)^3} = \nabla \iint p' f e^{ip'(p-p)a/\hbar} \frac{d^3a d^3p'}{(2\pi\hbar)^3} = \]
\[ = \nabla \iint p' f \delta(p' - p) d p' = p \nabla f(r, p). \]
Two following equalities can be verified similarly:

\[
\int \int \frac{e}{mc} A(\mathbf{r}, t)(\nabla f(\mathbf{r}, \mathbf{p}', t))e^{i(\mathbf{p}' - \mathbf{p})a/\hbar \frac{d^3a}{d^3p'}} = \frac{e}{mc} A(\mathbf{r}, t)\nabla f(\mathbf{r}, \mathbf{p}, t),
\]

and

\[
\int \int \frac{e}{mc}(\nabla A(\mathbf{r}, t))f(\mathbf{r}, \mathbf{p}', t)e^{i(\mathbf{p}' - \mathbf{p})a/\hbar \frac{d^3a}{d^3p'}} = \frac{e}{mc}(\nabla A(\mathbf{r}, t))f(\mathbf{r}, \mathbf{p}, t).
\]

Then the equation (1.6) can be rewritten as following

\[
\frac{\partial f}{\partial t} + \frac{1}{m}(\mathbf{p} - \frac{e}{c} \mathbf{A}) \nabla f - \frac{e}{mc}(\nabla A(\mathbf{r}, t))f(\mathbf{r}, \mathbf{p}, t) = W[f]. \quad (1.8)
\]

In the equation (1.8) the symbol \( W[f] \) is the Wigner — Vlasov integral, defined by the equality

\[
W[f] = \int \int \left\{ \frac{e}{2mc} \left[ A(\mathbf{r} + \frac{a}{2}, t) + A(\mathbf{r} - \frac{a}{2}, t) - 2A(\mathbf{r}, t) \right] \nabla f + \right.
\]

\[
\left. + \frac{ie}{mch} \left[ A(\mathbf{r} + \frac{a}{2}, t) - A(\mathbf{r} - \frac{a}{2}, t) \right] \mathbf{p}' f + \right.
\]

\[
\left. + \frac{e}{2mc} \left[ \text{div} A(\mathbf{r} + \frac{a}{2}, t) + \text{div} A(\mathbf{r} - \frac{a}{2}, t) - 2 \text{div} A(\mathbf{r}, t) \right] f - \right.
\]

\[
\left. - \frac{ie^2}{2mc^2\hbar} \left[ A^2(\mathbf{r} + \frac{a}{2}, t) - A^2(\mathbf{r} - \frac{a}{2}, t) \right] f - \right.
\]

\[
\left. - \frac{ie}{\hbar} \left[ U(\mathbf{r} + \frac{a}{2}, t) - U(\mathbf{r} - \frac{a}{2}, t) \right] f \right\} e^{i(\mathbf{p}' - \mathbf{p})a/\hbar \frac{d^3a}{d^3p'}}. \quad (1.9)
\]

The energy of the particle is equal to

\[
\mathcal{E} = \frac{1}{2m} \left( \mathbf{p} - \frac{e}{c} \mathbf{A} \right)^2 + eU.
\]

Then the velocity of the particle \( \mathbf{v} \) is equal to

\[
\mathbf{v} = \frac{\partial \mathcal{E}}{\partial \mathbf{p}} = \frac{1}{m} \left( \mathbf{p} - \frac{e}{c} \mathbf{A} \right),
\]
besides,

$$\nabla v = -\frac{e}{mc} \text{div} A.$$  

Hence, the left–hand part of the equation (1.9) equals to:

$$\frac{\partial f}{\partial t} + \frac{1}{m} (p - \frac{e}{c} A) \nabla f - f \frac{e}{mc} \text{div} A = \frac{\partial f}{\partial t} + v \nabla f + f \nabla A = \frac{\partial f}{\partial t} + \nabla (vf).$$

Therefore the equation (1.9) can be rewritten in standard for transport theory form

$$\frac{\partial f}{\partial t} + \nabla (vf) = W[f]. \quad (1.10)$$

In the case of collisional plasma we may write the kinetic equation (1.10) as following

$$\frac{\partial f}{\partial t} + \nabla (vf) = B[f, f] + W[f]. \quad (1.11)$$

In the equation (1.11) the symbol $B[f, f]$ represents the collision integral.

### 2. RELAXATION MODEL OF KINRTIC EQUATION

Under the electron scattering on impurity we will consider the equation (1.11) with collision integral in the form of relaxation $\tau$–model [11], [12]:

$$\frac{\partial f}{\partial t} + \nabla (vf) = \frac{f^{(0)} - f}{\tau} + W[f]. \quad (2.1)$$

In the equation (2.1) $\tau$ is the mean time between two consecutive collisions, $\tau = 1/\nu$, $\nu$ is the collision frequency, $f^{(0)}$ is the local equilibrium Fermi — Dirac distribution function,

$$f^{(0)} = \left[ 1 + \exp \left( \frac{E - \mu}{k_B T} \right) \right]^{-1}.$$  

Here $k_B$ is the Boltzmann constant, $T$ is the plasma temperature, $E$ is the electron energy, $\mu$ is the chemical potential of electron gas.

In an explicit form the local equilibrium distribution function has the following form

$$f^{(0)}(r, t) = \left[ 1 + \exp \left( \frac{p - (e/c)A(r, t)}{2mk_B T} + \frac{eU(r, t) - \mu}{k_B T} \right) \right]^{-1}.$$  

We introduce the dimensionless electron velocity $\mathbf{C}(\mathbf{r}, t)$, scalar potential $\phi(\mathbf{r}, t)$ and chemical potential $\alpha$:

$$\mathbf{C}(\mathbf{r}, t) = \frac{\mathbf{v}(\mathbf{r}, t)}{v_T}, \quad \phi(\mathbf{r}, t) = \frac{eU(\mathbf{r}, t)}{k_B T}, \quad \alpha = \frac{\mu}{k_B T},$$

where $v_T = \frac{1}{\sqrt{\beta}}$ is the thermal electron velocity, $\beta = \frac{m}{2k_B T}$.

Now local equilibrium function can be presented in terms of the electron velocity as follows

$$f^{(0)}(\mathbf{r}, t) = \left[1 + \exp \left(\frac{mv^2(\mathbf{r}, t)}{2k_B T} + \frac{eU(\mathbf{r}, t) - \mu}{k_B T}\right)\right]^{-1},$$

or, in dimensionless parameters,

$$f^{(0)}(\mathbf{r}, t) = \frac{1}{1 + \exp \left[C^2(\mathbf{r}, t) + \phi(\mathbf{r}, t) - \alpha\right]. \quad (2.2)$$

We designate $\chi = \alpha - \phi$. Then we have

$$f^{(0)} = \frac{1}{1 + e^{C^2 - \chi}}.$$

The quantity $\chi$ is defined from the conservation law of number of particles

$$\int f d\Omega_F = \int f^{(0)} d\Omega_F.$$

Here $d\Omega_F$ is the quantum measure for electrons,

$$d\Omega_F = \frac{2d^3p}{(2\pi\hbar)^3}.$$

Let’s note, that in the case of constant potentials $U = \text{const}$, $A = \text{const}$ the equilibrium distribution function (2.2) is the solution of the equation (2.1).

Let’s find the electron concentration (numerical density) $N$ and mean electron velocity $\mathbf{u}$ in an equilibrium state. These macroparameters are defined as follows:

$$N(\mathbf{r}, t) = \int f(\mathbf{r}, p, t) d\Omega_F,$$

$$\mathbf{u}(\mathbf{r}, t) = \frac{1}{N(\mathbf{r}, t)} \int \mathbf{v}(\mathbf{r}, t) f(\mathbf{r}, p, t) d\Omega_F.$$
For calculation of these macroparameters in equilibrium condition it is necessary to put \( f = f^{(0)} \), where \( f^{(0)} \) is defined by equality (1.13). We designate these macroparameters in equilibrium condition through \( N^{(0)}(\mathbf{r}, t) \) and \( \mathbf{u}^{(0)}(\mathbf{r}, t) \).

Let’s carry out the replacement of the integration variable \( \mathbf{p} - (e/c)A(\mathbf{r}, t) = \mathbf{p}' \) in the previous equalities. Then, passing to integration in spherical coordinates, for numerical density in an equilibrium state we get:

\[
N^{(0)} = \frac{m^3 v_F^3}{\pi^2 \hbar^3} f_2(\alpha - \phi), \tag{2.3}
\]

where

\[
f_2(\alpha - \phi) = \int_0^\infty \frac{x^2 \, dx}{1 + \exp(x^2 + \phi - \alpha)} = \int_0^\infty x^2 f_F(\alpha - \phi) \, dx.
\]

In the same way, as for numerical density, for mean velocity in equilibrium state we derive

\[
\mathbf{u}^{(0)}(\mathbf{r}, t) = \frac{1}{N^{(0)}} \int \mathbf{v}(\mathbf{r}, t) f^{(0)}(\mathbf{r}, \mathbf{p}, t) \, d\Omega_F,
\]

or, in explicit form,

\[
\mathbf{u}^{(0)}(\mathbf{r}, t) = \frac{2}{N^{(0)}(2\pi \hbar)^3} \int \frac{[\mathbf{p} - (e/c)A] \, d^3p}{1 + \exp \left[ \frac{(\mathbf{p} - (e/c)A)^2}{2k_B T m} + \frac{eU - \mu}{k_T m} \right]}.
\]

After the same change of variables \( \mathbf{p} - (e/c)A(\mathbf{r}, t) = \mathbf{p}' \) we receive:

\[
\mathbf{u}^{(0)}(\mathbf{r}, t) = \frac{2}{N^{(0)}(2\pi \hbar)^3} \int \frac{\mathbf{p}' \, d^3p'}{1 + \exp \left[ \frac{p'^2}{2k_B T m} + \frac{eU - \mu}{k_T m} \right]} = 0. \tag{2.4}
\]

So, the electron velocity in an equilibrium state according to (2.4) is equal to zero.

Let’s note, that numerical electron density and their mean velocity satisfy the usual continuity equation:

\[
\frac{\partial N}{\partial t} + \text{div}(N \mathbf{u}) = 0. \tag{2.5}
\]
For the derivation of the continuity equation (2.5) it is necessary to integrate
the kinetic equation (2.1) by quantum measure for electrons $d\Omega_F$ and to use
the definition of numerical density and mean velocity. Then it is necessary
to use the conservation law of number of particles and to check up, if the
integral by quantum measure $d\Omega_F$ of Wigner — Vlasov integral is equal to
zero. Indeed, we have

$$
\int W[f] \frac{2 \, d^3 p}{(2\pi \hbar)^3} = 2 \iint \left\{ \cdots \right\} e^{i \mathbf{p'} \mathbf{a}/\hbar} \delta(\mathbf{a}) \, d^3 \mathbf{a} \, d^3 \mathbf{p'} =
$$

$$
= 2 \int \left\{ \cdots \right\} \bigg|_{\mathbf{a}=0} \, d^3 \mathbf{p} \equiv 0,
$$
as after some algebra,

$$
\left\{ \cdots \right\} \bigg|_{\mathbf{a}=0} \equiv 0.
$$

Here the symbol $\left\{ \cdots \right\}$ means the same expression, as in the right-hand
side of the equation (1.9).

Let’s note, that the left-hand side of the kinetic equation (1.11) or (2.1)
takes standard form for transport theory under the following gauge condition:

$$
\text{div} \mathbf{A}(\mathbf{r}, t) = 0. \quad (2.6)
$$

Thus, i.e. in case of gauge (2.6), the kinetic equation has the following
form:

$$
\frac{\partial f}{\partial t} + \mathbf{v} \nabla f = B[f, f] + W[f]. \quad (2.7)
$$

Here the Wigner — Vlasov integral equals to:

$$
W[f] = \iint \left\{ \frac{e}{2mc} \left[ \mathbf{A}(\mathbf{r} + \frac{\mathbf{a}}{2}, t) + \mathbf{A}(\mathbf{r} - \frac{\mathbf{a}}{2}, t) - 2\mathbf{A}(\mathbf{r}, t) \right] \nabla f +
\right. \left. + \frac{ie}{mch} \left[ \mathbf{A}(\mathbf{r} + \frac{\mathbf{a}}{2}, t) - \mathbf{A}(\mathbf{r} - \frac{\mathbf{a}}{2}, t) \right] \mathbf{p'} f - \frac{i e^2}{2mc^2 \hbar} \left[ \mathbf{A}^2(\mathbf{r} + \frac{\mathbf{a}}{2}, t) - \mathbf{A}^2(\mathbf{r} - \frac{\mathbf{a}}{2}, t) \right] f -
\right. \left. - \frac{i e}{\hbar} \left[ U(\mathbf{r} + \frac{\mathbf{a}}{2}, t) - U(\mathbf{r} - \frac{\mathbf{a}}{2}, t) \right] \right\} e^{i (\mathbf{p'} - \mathbf{p}) \mathbf{a}/\hbar} \, d^3 \mathbf{a} \, d^3 \mathbf{p'} \right. \left. \left( \frac{2\pi \hbar}{2\pi \hbar} \right)^3. \quad (2.8)
$$
3. LINEARIZATION OF THE KINETIC EQUATION AND ITS SOLUTION

Let’s consider the kinetic equation with collision integral in the form of \(\tau\)-model and suppose, that the scalar potential is equal to zero: \(U(\mathbf{r}, t) \equiv 0\).

We take vector potential which is orthogonal to the direction of the wave vector \(\mathbf{k}: \mathbf{kA} = 0\) in the form of a running harmonic wave:

\[
\mathbf{A}(\mathbf{r}, t) = A_0 e^{i(\mathbf{k}\cdot \mathbf{r} - \omega t)}. 
\]

We suppose that the vector potential is small enough. This assumption allows us to linearize the equation and to neglect terms quadratic in electric field.

Then the equation (2.7) can be reduced to:

\[
\frac{\partial f}{\partial t} + \mathbf{v} \nabla f = \frac{f^{(0)} - f}{\tau} + W[f]. \tag{3.1}
\]

In this case chemical potential is equal to a constant.

In the equation (3.1) local equilibrium Fermi — Dirac distribution is simplified as following:

\[
f^{(0)} = \left[ 1 + \exp \left( C^2(\mathbf{r}, t) - \alpha \right) \right]^{-1}. \tag{3.2}
\]

The Wigner — Vlasov integral (2.8) also can be simplified essentially and has the following form:

\[
W[f] = \frac{ie}{mch} \int \int \left[ \mathbf{A}(\mathbf{r} + \frac{\mathbf{a}}{2}, t) - \mathbf{A}(\mathbf{r} - \frac{\mathbf{a}}{2}, t) \right] \mathbf{p'} f e^{i(\mathbf{p'} - \mathbf{p})a/\hbar} d^3\mathbf{a} d^3\mathbf{p'}/(2\pi\hbar)^3. \tag{3.3}
\]

We notice, that

\[
\mathbf{A}(\mathbf{r} + \frac{\mathbf{a}}{2}, t) - \mathbf{A}(\mathbf{r} - \frac{\mathbf{a}}{2}, t) = \mathbf{A}(\mathbf{r}, t) \left[ e^{i\mathbf{k}\mathbf{a}/2} - e^{-i\mathbf{k}\mathbf{a}/2} \right].
\]

Calculating the integral in (3.3), we find, that

\[
W[\mathbf{A}, f] = \frac{ie}{mch} \mathbf{A}(\mathbf{r}, t) \int \int \left[ e^{i\mathbf{k}\mathbf{a}/2} - e^{-i\mathbf{k}\mathbf{a}/2} \right] e^{i(\mathbf{p'} - \mathbf{p})a/\hbar} d^3\mathbf{a} d^3\mathbf{p'}/(2\pi\hbar)^3.
\]
The internal integral is equal to:

\[
\frac{1}{(2\pi \hbar)^3} \int \left\{ \exp \left( i \left[ p' - p + \frac{\hbar k}{2} \right] \frac{a}{\hbar} \right) - \exp \left( i \left[ p' - p + \frac{\hbar k}{2} \right] \frac{a}{\hbar} \right) \right\} d^3a = \]

\[= \delta \left( p' - p + \frac{\hbar k}{2} \right) - \delta \left( p' - p - \frac{\hbar k}{2} \right). \]

We calculate the Wigner – Vlasov integral

\[
W[f, A] = \]

\[= A(r, t) \frac{ie}{mc} \int \left[ \delta \left( p' - p + \frac{\hbar k}{2} \right) - \delta \left( p' - p - \frac{\hbar k}{2} \right) \right] p' f(r, p', t) d^3p' = \]

\[= A(r, t) \frac{ie}{mc} \left[ \left( p - \frac{\hbar k}{2} \right) f(r, p - \frac{\hbar k}{2}, t) - \left( p + \frac{\hbar k}{2} \right) f(r, p + \frac{\hbar k}{2}, t) \right] = \]

\[= A(r, t) \frac{ie}{mc} \left[ p f(r, p - \frac{\hbar k}{2}, t) - f(r, p + \frac{\hbar k}{2}, t) \right] - \]

\[\frac{\hbar k}{2} \left[ f(r, p - \frac{\hbar k}{2}, t) + f(r, p + \frac{\hbar k}{2}, t) \right] \right\} = \]

\[= A(r, t) \frac{ie}{mc} p \left( f_+ - f_- \right), \]

where

\[f_\pm \equiv f(r, p \mp \frac{\hbar k}{2}, t). \]

Consequently, the Wigner – Vlasov integral is equal to

\[
W[f] = \frac{ie}{mc} pA(r, t) \left[ f_+ - f_- \right] = \frac{iev_T}{mc} PA(r, t) \left[ f_+ - f_- \right] = \]

\[= \frac{iev_T}{ch} PA(r, t) \left[ f_+ - f_- \right]. \tag{3.4} \]

Here and below the expression \( PA \) means scalar production.

Further we will use dimensionless velocity \( C \) in the form

\[ C = \frac{\mathbf{v}}{v_T} = \frac{\mathbf{P}}{p_T} - \frac{e}{c} \mathbf{A}(r, t) \equiv \mathbf{P} - \frac{e}{c} \mathbf{A}(r, t), \]
where $\mathbf{P} = \frac{\mathbf{P}}{p_T}$ is the dimensionless momentum.

In linear approximation it is possible to replace the function $f$ in Wigner–Vlasov integral by the absolute Fermi–Dirac distribution, i.e. we put $f = f_F(P)$, where

$$f_F(P) = \frac{1}{1 + \exp(P^2 - \alpha)}, \quad \alpha = \text{const}.$$

Here Wigner–Vlasov integral (3.4) has the following form:

$$W[f_F] = \frac{i e v r}{\hbar} \mathbf{PA}(\mathbf{r}, t) \left[ f_F^+ - f_F^- \right],$$

where

$$f_F^\pm = f_F^\pm(P) = \frac{1}{1 + \exp \left[ \left( P \mp \frac{\hbar \mathbf{k}}{2p_T} \right)^2 - \alpha \right]},$$

and $p_T = mv_T$ is the thermal electron momentum, or,

$$f_F^\pm = \frac{1}{1 + e^{p^2_\pm - \alpha}}.$$

Here

$$P^2_\pm = \left( P \mp \frac{\hbar \mathbf{k}}{2p_T} \right)^2 = (P_x \mp \frac{\hbar k_x}{2p_T})^2 + (P_y \mp \frac{\hbar k_y}{2p_T})^2 + (P_z \mp \frac{\hbar k_z}{2p_T})^2,$$

or

$$p^2_\pm = \left( \frac{p_x \mp \frac{\hbar k_x}{2}}{p_T} \right)^2 + \left( \frac{p_y \mp \frac{\hbar k_y}{2}}{p_T} \right)^2 + \left( \frac{p_z \mp \frac{\hbar k_z}{2}}{p_T} \right)^2.$$

The linearization of the Wigner equilibrium function (3.2) we will carry out in terms of vector potential $\mathbf{A}(\mathbf{r}, t)$:

$$f^{(0)} = f^{(0)} \bigg|_{\mathbf{A}=0} + \frac{\partial f^{(0)}}{\partial \mathbf{A}} \bigg|_{\mathbf{A}=0} \mathbf{A}(\mathbf{r}, t),$$

or, in explicit form:

$$f^{(0)} = f_F(P) + g(P) \frac{2e}{c p_T} \mathbf{PA}(\mathbf{r}, t), \quad (3.5)$$

$$g(P) = \frac{e^{p^2 - \alpha}}{(1 + e^{p^2 - \alpha})^2}.$$
Considering decomposition (3.5), we will search for Wigner’s function in the form:

\[ f = f_F(P) + g(P) \frac{2e}{cP_T} PA(r, t) + g(P)(PA(r, t))h(P). \] \tag{3.6}

We receive the following equation

\[ PA(r, t) g(P)(\nu - i\omega + ikv)h(P) = \]

\[ = PA \left\{ \frac{2ie}{cP_T} g(P)(\omega - v_T kP) + \frac{iev_T}{c\hbar} (r, t)(f^+_F - f^-_F) \right\}. \]

From this equation we find

\[ PA(r, t)g(P)h(P) = PA(r, t) \frac{2ie}{cP_T} \frac{\omega - v_T kP}{\nu - i\omega + iv_T kP}g(P) + \]

\[ + \frac{iev_T}{c\hbar} PA(r, t) \frac{f^+_F - f^-_F}{\nu - i\omega + iv_T kP}. \] \tag{3.7}

With the help of (3.6) and (3.7) we construct the full distribution function

\[ f = f^{(0)} + g(P)PAh(P) = \]

\[ = f^{(0)} + \frac{2ie}{cP_T} \frac{\omega - v_T kP}{\nu - i\omega + iv_T kP}g(P)PA + \frac{iev_T}{c\hbar} \frac{PA(f^+_F - f^-_F)}{\nu - i\omega + iv_T kP}, \]

or

\[ f = f^{(0)} + PA \left[ \frac{2ie}{cP_T} \frac{\omega - k_1 P}{1 - i\omega T + ik_1 \hbar} g(P) + \frac{iel}{c\hbar} \frac{f^+_F - f^-_F}{1 - i\omega T + ik_1 \hbar} \right]. \] \tag{3.8}

Here \( k_1 = kl, \ l \) is the electron mean free path, \( l = v_T T, \ k_1 \) is the dimensionless wave vector.

\section*{4. DENSITY OF ELECTRIC CURRENT}

We consider the connection between electric field and potentials

\[ E(r, t) = -\frac{1}{c} \frac{\partial A(r, t)}{\partial t} - \frac{\partial U(r, t)}{\partial r}, \]
or

\[ \mathbf{E}(\mathbf{r}, t) = \frac{i\omega}{c} \mathbf{A}(\mathbf{r}, t). \]

Hence, the current density is connected with vector potential as:

\[ \mathbf{j}(\mathbf{r}, t) = \sigma_{tr} \frac{i\omega}{c} \mathbf{A}(\mathbf{r}, t). \]

By definition, the current density is equal to

\[ \mathbf{j}(\mathbf{r}, t) = e \int \mathbf{v} f \frac{2 d^3 p}{(2\pi\hbar)^3}. \]

Let’s note, that the current density in the equilibrium state is equal to zero:

\[ \mathbf{j}^{(0)}(\mathbf{r}, t) = e \int \mathbf{v}(\mathbf{r}, \mathbf{v}, t) f^{(0)}(\mathbf{P}) \frac{2 p^3_T}{(2\pi\hbar)^3} = 0. \]

Indeed, considering that mean electron velocity in the equilibrium state is equal to zero, according to (2.4) we have:

\[ \mathbf{j}^{(0)}(\mathbf{r}, t) = e N^{(0)} \mathbf{u}^{(0)}(\mathbf{r}, t) \equiv 0. \]

Hence, with the use of equality (3.8) we have the following equality:

\[ \mathbf{j}(\mathbf{r}, t) = \frac{2 e p^3_T}{(2\pi\hbar)^3} \int (\mathbf{PA}) \mathbf{v}(\mathbf{r}, \mathbf{v}, t) \times \]

\[ \times \left[ \frac{2ie}{c \mathbf{p}_T} \frac{\omega \tau - i \mathbf{k}_1 \mathbf{P}}{1 - i \omega \tau + i \mathbf{k}_1 \mathbf{P}} g(\mathbf{P}) + \frac{i e l}{c \hbar} \frac{f^+_F - f^-_F}{1 - i \omega \tau + i \mathbf{k}_1 \mathbf{P}} \right] d^3 \mathbf{P}. \]

Substituting obvious expression for the velocity into this equality

\[ \mathbf{v}(\mathbf{r}, t) = \frac{\mathbf{p}}{m} - \frac{e \mathbf{A}(\mathbf{r}, t)}{mc} = \frac{p_T \mathbf{P}}{m} - \frac{e \mathbf{A}(\mathbf{r}, t)}{mc}, \]

and, after linearization of it by vector field, we receive

\[ \mathbf{j}(\mathbf{r}, t) = \frac{2 e p^4_T}{(2\pi\hbar)^3 m} \int (\mathbf{A}(\mathbf{r}, t) \mathbf{P}) \mathbf{P} \times \]

\[ \times \left[ \frac{2ie}{c \mathbf{p}_T} \frac{\omega \tau - i \mathbf{k}_1 \mathbf{P}}{1 - i \omega \tau + i \mathbf{k}_1 \mathbf{P}} g(\mathbf{P}) + \frac{i e l}{c \hbar} \frac{f^+_F(\mathbf{P}) - f^-_F(\mathbf{P})}{1 - i \omega \tau + i \mathbf{k}_1 \mathbf{P}} \right] d^3 \mathbf{P}. \] (4.1)
We notice that
\[
\left( P \mp \frac{\hbar k}{2p_T} \right) = P^2 \mp \frac{\hbar}{p_T} Pk + \left( \frac{\hbar}{2p_T} \right)^2 k^2 =
\]
\[
= P^2 \mp \frac{\hbar \nu}{2\mathcal{E}_T} Pk_1 + \left( \frac{\hbar \nu}{4\mathcal{E}_T} \right)^2 k_1^2.
\]

Let’s copy the previous equality in the form
\[
j(r, t) = \frac{2ie^2 p_T^4}{(2\pi \hbar)^3 m} \int [PA(r, t)]PS(P, Pk_1)d^3P,
\]
where
\[
S(P, Pk_1) = \frac{2}{cp_T} \frac{\omega \tau - k_1 P}{1 - i\omega \tau + ik_1 P} g(P) + \frac{l}{ch} \frac{f^+_F(P) - f^-_F(P)}{1 - i\omega \tau + ik_1 P}.
\]

We take the unit vector \( e_1 = \frac{A}{A} \), direct lengthwise the vector \( A \). Then the equality (4.2) we may write in the form:
\[
j(r, t) = \frac{2ie^2 p_T^4}{(2\pi \hbar)^3 m} \int (Pe_1)PS(P, Pk_1)d^3P,
\]
(4.3)

In view of the symmetry the value of integral will not change, if the vector \( e_1 \) is replaced by any other unit vector \( e_2 \), perpendicular to the vector \( k_1 \), i.e.
\[
e_2 = \frac{A \times k_1}{|A \times k_1|} = \frac{A \times k_1}{Ak_1},
\]
and \( A \times k_1 \) is the vector product.

Let’s spread out a vector \( P \) in three orthogonal directions \( e_1, e_2 \) and \( n = k_1/k_1 \):
\[
P = (Pn)n + (Pe_1)e_1 + (Pe_2)e_2.
\]

By means of this decomposition we receive that
\[
(PA)P = A(PE_1)P =
\]
\[
= A(PE_1)(Pn)n + A(PE_1)^2e_1 + A(PE_1)(PE_2)e_2.
\]
Substituting this decomposition in (4.3), and, considering, that integrals from odd functions on a symmetric interval are equal to zero, we receive that

\[ j(r, t) = \frac{2ie^2p_T^4A(r, t)}{(2\pi\hbar)^3m} \int (P e_1)^2 S(P, Pn) d^3P, \]  

(4.4)

or, in the explicit form,

\[ j(r, t) = \frac{2ep_T^4A(r, t)}{(2\pi\hbar)^3m} \int \left( e_1 P \right)^2 \left[ \frac{2ie}{c p_T} \frac{\omega - k_1 P}{1 - i\omega + ik_1 P} g(P) + \frac{i e l f_F^+(P) - f_F^-(P)}{c h} \right] d^3P. \]

Here \( e_1 = A/A \) is the unit vector directed lengthwise \( A \). Therefore in view of symmetry

\[ \int \left( e_1 P \right)^2 [S] d^3P = \int \left( e_2 P \right)^2 [S] d^3P = \]

\[ = \frac{1}{2} \int \left[ \left( e_1 P \right)^2 + \left( e_2 P \right)^2 \right] [S] d^3P. \]

We will notice that the square of length of a vector \( P \) is equal

\[ P^2 = (P e_1)^2 + (P e_2)^2 + (P n)^2, \]

therefore

\[ (P e_1)^2 + (P e_2)^2 = P^2 - \frac{(P k_1)^2}{k_1^2} = P^2 - (P n)^2 = P'_\perp, \]

where \( P'_\perp \) is the projection of the vector \( P \) to direct, perpendicular planes \( (e_1, e_2) \), and the vector \( n \) is the unit vector directed along the vector \( k_1 \), \( n = \frac{k}{k_1} \).

Hence for the current density we receive the following expression

\[ j(r, t) = \frac{ep_T^4A(r, t)}{(2\pi\hbar)^3m} \int \left[ P^2 - (P n)^2 \right] \left[ \frac{2ie}{c p_T} \frac{\omega - k_1 P}{1 - i\omega + ik_1 P} g(P) + \right. \]

\[ + \frac{i e l f_F^+(P) - f_F^-(P)}{c h} \left. \right] d^3P. \]
Replacing the current density in the left-hand side of this equality by the expression in terms of field, we receive:

\[ \sigma_{tr} \frac{i\omega}{c} A(\mathbf{r}, t) = \frac{e p_T^4 A(\mathbf{r}, t)}{(2\pi \hbar)^3 m} \int \left[ P^2 - (\mathbf{Pn})^2 \right] \times \]

\[ \times \left[ \frac{2ie}{c p_T} \frac{\omega \tau - k_1 \mathbf{P}}{1 - i\omega \tau + ik_1 \mathbf{P}} g(\mathbf{P}) + \frac{i e l f^+_E(\mathbf{P}) - f^-_E(\mathbf{P})}{ch 1 - i\omega \tau + ik_1 \mathbf{P}} \right] d^3 P. \]

5. ELECTRIC CONDUCTIVITY AND DIELECTRIC FUNCTION

From the last formula we receive the following expression for the transversal electric conductivity in quantum non-degenerate plasma:

\[ \sigma_{tr} = \frac{e^2 p_T^4}{(2\pi \hbar)^3 m_\omega} \int \left[ P^2 - \left( \frac{\mathbf{P} k_1}{k_1^2} \right)^2 \right] \left[ \frac{2}{p_T} \frac{\omega \tau - k_1 \mathbf{P}}{1 - i\omega \tau + ik_1 \mathbf{P}} g(\mathbf{P}) + \right. \]

\[ + \frac{l f^+_E(\mathbf{P}) - f^-_E(\mathbf{P})}{\hbar 1 - i\omega \tau + ik_1 \mathbf{P}} \right] \int d^3 P. \quad (5.1) \]

We will transform expression for transversal conductivity and we will bring it to the form:

\[ \sigma_{tr} = \frac{2e^2 p_T^3}{(2\pi \hbar)^3 m_\omega} \int \left[ (\omega \tau - k_1 \mathbf{P}) g(\mathbf{P}) + \frac{E_T}{\hbar \nu} (f^+_E - f^-_E) \right] \frac{P^2 d^3 P}{1 - i\omega \tau + ik_1 \mathbf{P}}, \]

where \( E_T \) is the thermal electron energy,

\[ E_T = \frac{m v_T^2}{2}. \]

With the use of the equality (2.3) we will present the previous formula in the form:

\[ \sigma_{tr} = \frac{\sigma_0}{4\pi f_2(\alpha)} \int \left[ \left( 1 - (\omega \tau)^{-1} k_1 \mathbf{P} \right) g(\mathbf{P}) + \right. \]

\[ \left. \right] \int d^3 P. \]
Here the function $f_2(\alpha)$ has been entered above and in the absence of the scalar potential it is defined by equality:

$$f_2(\alpha) = \int_0^\infty x^2 f_F(x) \, dx = \int_0^\infty \frac{x^2 \, dx}{1 + e^{x^2 - \alpha}} = \frac{1}{2} \int_0^\infty \ln(1 + e^{\alpha - x^2}) \, dx.$$ 

The quantity $\sigma_0$ is defined by classical expression for the static electric conductivity

$$\sigma_0 = \frac{e^2 N(0)}{m \nu}.$$ 

Dielectric function we will find according to the formula:

$$\varepsilon_{tr} = 1 + \frac{4\pi i}{\omega} \sigma_{tr}.$$ 

Substituting electric conductivity (3.1) into this equality, we receive the expression for dielectric permittivity in quantum non-degenerate collision plasma:

$$\varepsilon_{tr} = 1 + \frac{\omega^2}{\omega^2 + \frac{4\pi i}{\omega}} \int \left[ \frac{\omega \tau - k_1 P}{f_2(\alpha)} \right] g(P) +$$

$$+ \frac{\mathcal{E}_T}{\hbar \nu} \left[ f_F^+(P) - f_F^-(P) \right] \frac{P^2 \, d^3 P}{1 - i\omega \tau + i k_1 P}.$$ 

We investigate some special cases of electroconductivity. In the long-wave limit (when $k \to 0$) from (3.1) we receive the well known classical expression:

$$\sigma_{tr}(k = 0) = \sigma_0 \frac{\nu}{\nu - i\omega} = \frac{\sigma_0}{1 - i\omega \tau}.$$ 

Let’s consider the quantum mechanical limit of the conductivity in the case of arbitrary values of wave number, i.e. conductivity limit in the case, when Planck’s constant $\hbar \to 0$, and the quantity $k$ is arbitrary.

Now we consider the case, when values of the wave number are arbitrary, but Planck’s constant converges to zero: $\hbar \to 0$. 
When the values of $\hbar$ are small we have:

$$f_0^\pm(P) = f_F(P) \pm g(P)2P\frac{\hbar k}{2m\nu_T},$$

hence

$$f_F^+(P) - f_F^-(P) = 2g(P)2P\frac{\hbar k}{2m\nu_T}.$$ 

Therefore

$$(\omega - v_T kP)g(P) + \frac{p_T^2}{2m\hbar}[f_F^+(P) - f_F^-(P)] = \omega g(P).$$

Thus, in linear approximation at small $\hbar$ (independently of the quantity $k$) for transverse conductivity we receive:

$$\sigma_{tr} = \sigma_{tr}^{\text{classic}},$$

where

$$\sigma_{tr}^{\text{classic}} = \frac{\sigma_0}{4\pi f_2(\alpha)} \int \frac{g(P)P_+ d^3P}{1 - i\omega\tau + i\mathbf{k}_1\mathbf{P}}. \quad (5.3)$$

The expression (5.3) accurately coincides with the expression of the transversal conductivity for classical plasma with arbitrary temperature.

Let’s return to the expression (5.1’). We present it in the form of the sum of two components

$$\sigma_{tr} = \sigma_{tr}^{\text{classic}} + \sigma_{tr}^{\text{quant}}, \quad (5.4)$$

where $\sigma_{tr}^{\text{classic}}$ is defined by the equality (5.3), and second component $\sigma_{tr}^{\text{quant}}$ corresponds to quantum properties of the plasma under consideration

$$\sigma_{tr}^{\text{quant}} = \frac{\sigma_0}{4\pi f_2(\alpha)} \int \left[ - \frac{\mathbf{k}_1\mathbf{P}}{\omega\tau} g(P) + \frac{\mathcal{E}_T}{\hbar\omega}[f_F^+(P) - f_F^-(P)] \right] \frac{P_+ d^3P}{1 - i\omega\tau + i\mathbf{k}_1\mathbf{P}}. \quad (5.5)$$

The quantum summand $\sigma_{tr}^{\text{quant}}$ we will present in the form, proportional to a square of the Planck’s constant $\hbar$. 
For this aim we use cubic expansion of $\sigma_{tr}^{\text{quant}}$ by powers of $\hbar$. We will remind, that in linear approximation by $\hbar$, as it was already specified, the quantity $\sigma_{tr}^{\text{quant}}$ disappears. We will direct an axis $x$ along the wave vector $\mathbf{k}$.

Let’s expand the Fermi — Dirac distribution by degrees of dimensionless wave number $q = \frac{k}{k_T} = \frac{k_1 \hbar \nu}{m v_T^2}$, where $k_T = \frac{p_T}{\hbar}$ is the thermal wave number.

We receive that

$$f_F^\pm(P) = f_F(P) \pm g(P) P_x q - \left[ g'_{p2}(P) P_x^2 + \frac{1}{2} g(P) \right] \frac{q^2}{2} \pm$$

$$\pm \left[ g''_{p2p2}(P) P_x^2 + \frac{3}{2} g'_{p2}(P) \right] P_x \frac{q^3}{6} + \cdots .$$

Here $g'_{p2}(P) = g'(P^2)$, $g''_{p2p2}(P) = g''(P^2)$,

$$g'(P^2) = g(P) \frac{1 - e^{P^2 - \alpha}}{1 + e^{P^2 - \alpha}}, \quad g''(P^2) = g(P) \left[ \left( \frac{1 - e^{P^2 - \alpha}}{1 + e^{P^2 - \alpha}} \right)^2 - 2 g(P) \right].$$

Now it is easy to find the difference

$$f_F^+(P) - f_F^-(P) = 2g(P) P_x q + \left[ g''(P^2) P_x^2 + \frac{3}{2} g'(P^2) \right] P_x \frac{q^3}{3} + \cdots .$$

By means of this expression we find, that

$$- \frac{k_1 P_x^3}{\omega \tau} g(P) + \frac{\mathcal{E}_T}{\hbar \omega} [f_F^+(P) - f_F^-(P)] = G(P) \frac{k_1^3 \hbar^2 v^3}{6 \omega m^2 v_T^4} + \cdots ,$$

where

$$G(P) = P_x \left[ g''(P^2) P_x^2 + \frac{3}{2} g'(P^2) \right].$$

Substituting this expression into (5.5), we obtain, that the quantum summand is proportional to the square of Planck’s constant and it is defined by expression

$$\sigma_{tr}^{\text{quant}} = \hbar^2 \sigma_0 \frac{k_1^3 v^3}{24 \pi \omega m^2 v_T^4 f_2(\alpha)} \int \frac{G(P) (P^2 - P_x^2) d^3 P}{1 - i \omega \tau + i k_1 P_x} .$$

Let’s similarly (5.4) we present the formula for calculation of dielectric permeability in the form

$$\varepsilon_{tr} = 1 + \frac{4 \pi i}{\omega} \left( \sigma_{tr}^{\text{classic}} + \sigma_{tr}^{\text{quant}} \right) = \varepsilon_{tr}^{\text{classic}} + \varepsilon_{tr}^{\text{quant}} ,$$

(5.6)
where

\[ \varepsilon_{tr}^{\text{classic}} = 1 + \frac{4\pi i}{\omega} \sigma_{tr}^{\text{classic}}, \quad \varepsilon_{tr}^{\text{quant}} = \frac{4\pi i}{\omega} \sigma_{tr}^{\text{quant}}. \]

Thus, in an explicit form dielectric permeability of classical non-degenerate collisional plasmas is equal

\[ \varepsilon_{tr}^{\text{classic}} = 1 + \frac{i}{\omega} \frac{\omega^2}{2\pi} \int \frac{g(P) P^2 d^3 P}{1 - i\omega \tau + i\mathbf{k}_1 \mathbf{P}}. \tag{5.7} \]

And the component of dielectric permeability answering to quantum properties of non-degenerate collisional plasmas is defined by equality

\[ \varepsilon_{tr}^{\text{quant}} = \frac{i}{\omega} \frac{\omega^2}{2\pi} \int \frac{-\mathbf{k}_1 \mathbf{P} g(P) + (\mathcal{E}_T/\hbar \nu)(f_1^+ - f_1^-)}{1 - i\omega \tau + i\mathbf{k}_1 \mathbf{P}}. \tag{5.8} \]

6. CALCULATION OF ELECTRIC CONDUCTIVITY AND DIELECTRIC PERMEABILITY

In the expressions for classical and quantum components of the conductivity we can simplify several integrals.

We break the triple integral to external one-dimensional integration by the variable \( P_x \) from \(-\infty\) to \(+\infty\) and internal double integration by plane orthogonal to the axis \( P_x \) in the expression (5.3). The internal integration we carry out in polar coordinates. Here we obtain that

\[ P^2 = P_x + P_{\perp}^2, \quad d^3 P = dP_x \, d\mathbf{P}_\perp, \quad d\mathbf{P}_\perp = P_\perp \, dP_\perp \, d\chi, \]

where \( P_\perp \) is the polar radius, and \( \chi \) is the polar angle.

Thus we receive, that

\[ \sigma_{tr}^{\text{classic}} = \sigma_0 \frac{\omega}{4\pi f_2(\alpha)} \int_{-\infty}^{\infty} dP_x \int_{0}^{2\pi} \int_{0}^{\infty} \frac{g(P) P_\perp^3 dP_\perp d\chi}{1 - i\omega \tau + i\mathbf{k}_1 P_x}, \]

where

\[ g(P) = \frac{e^{P_x^2 + P_{\perp}^2 - \alpha}}{(1 + e^{P_x^2 + P_{\perp}^2 - \alpha})^2}. \]
Internal double integral we calculate in polar coordinates

\[
\int_{0}^{2\pi} \int_{0}^{\infty} g(P) P_{\perp}^3 dP_{\perp} d\chi = \pi \ln(1 + e^{\alpha - P_{x}^2}).
\] (6.1)

Hence, the expression for the classical component is simplified to one-dimensional integral

\[
\sigma_{tr}^{\text{classic}} = \frac{\sigma_0}{4f_2(\alpha)} \int_{-\infty}^{\infty} \frac{\ln(1 + e^{\alpha - P_{x}^2})}{1 - i\omega \tau + ik_1 P_x} dP_x.
\] (6.2)

The quantum item (5.5) we present in the form of the sum of two items

\[
\sigma_{tr}^{\text{quant}} = \sigma_1 + \sigma_2.
\] (6.3)

Here

\[
\sigma_1 = -\frac{\sigma_0 k_1}{4\pi f_2(\alpha) \omega \tau} \int \frac{P_x (P^2 - P_{x}^2) g(P) d^3 P}{1 - i\omega \tau + ik_1 P_x},
\]

and

\[
\sigma_2 = \frac{\sigma_0 E_T}{4\pi f_2(\alpha) \hbar \omega} \int \frac{f_F^+(P) - f_F^-(P)}{1 - i\omega \tau + ik_1 P_x} (P^2 - P_{x}^2) d^3 P.
\] (6.4)

With the help of the equality (6.1) the expression for \(\sigma_1\) can be rewritten in the following form

\[
\sigma_1 = -\frac{\sigma_0 k_1}{4f_2(\alpha) \omega \tau} \int_{-\infty}^{\infty} \frac{P_x \ln(1 + e^{\alpha - P_{x}^2}) dP_x}{1 - i\omega \tau + ik_1 P_x}.
\] (6.5)

After change of variable

\[
P_x \mp \frac{\hbar k}{2p_T} \equiv P_x \mp \frac{k_1 \hbar \nu}{2mv_T^2} \equiv P_x \mp \frac{k_1 \hbar \nu}{4E_T} \rightarrow P_x
\]

the difference of integrals from (6.4) will be transformed to one integral and we receive

\[
\sigma_2 = -\frac{i \sigma_0 k_1^2}{8\pi f_2(\alpha) \omega \tau} \int \frac{f_F(P)(P^2 - P_{x}^2) d^3 P}{(1 - i\omega \tau + ik_1 P_x)^2 + (k_1^2 \hbar \nu/4E_T)^2}.
\] (6.6)
In the same way, as well as during the derivation of the formula (6.1), double internal integral in (6.6) we reduce to the one-dimensional integral

\[
\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f_F(P)[P^2 - P_x^2] dP_\perp = \int_{0}^{2\pi} \int_{0}^{\infty} f_F(P) P_\perp^3 dP_\perp d\chi =
\]

\[
= 2\pi \int_{0}^{\infty} P_\perp \ln(1 + e^{\alpha-P_x^2-P_\perp^2}) dP_\perp.
\]

Now the expression (6.6) can be written in the following form (replacing a variable of integration \(P_\perp = \rho\))

\[
\sigma_2 = -\frac{i\sigma_0 k_1^2}{4f_2(\alpha)\omega\tau} \int_{-\infty}^{\infty} \int_{0}^{\infty} \frac{\rho \ln(1 + e^{\alpha-\rho^2-P_x^2})d\rho dP_x}{(1 - i\omega\tau + ik_1 P_x)^2 + (k_1^2 \hbar\nu/4E_T)^2}. \tag{6.7}
\]

Thus, it is possible to present the expression for transverse conductivity in the form of the sum of one-dimensional (6.2), (6.5) and two-dimensional (6.7) integrals

\[
\sigma_{tr} = \frac{\sigma_0}{4f_2(\alpha)} \int_{-\infty}^{\infty} \frac{[1 - (k_1/\omega\tau)P_x] \ln(1 + e^{\alpha-P_x^2})dP_x}{1 - i\omega\tau + ik_1 P_x} -
\]

\[
-\frac{i\sigma_0 k_1^2}{4f_2(\alpha)\omega\tau} \int_{-\infty}^{\infty} \int_{0}^{\infty} \frac{\rho \ln(1 + e^{\alpha-\rho^2-P_x^2})d\rho dP_x}{(1 - i\omega\tau + ik_1 P_x)^2 + (k_1^2 \hbar\nu/4E_T)^2}.
\]

In the expression (6.6) for \(\sigma_2\) the thruple integral can be reduced to one-dimensional integral. For this purpose in (6.5) we pass to integration in spherical coordinates and present this expression in the form

\[
\sigma_2 = -\frac{i\sigma_0 k_1^2}{4f_2(\alpha)\omega\tau} \int_{0}^{\infty} f_F(P) P^4 J(P) dP,
\]

where

\[
J(P) = \int_{-1}^{1} \frac{(1 - \mu^2) d\mu}{(1 - i\omega\tau + ik_1 P\mu)^2 + (k_1^2 \hbar\nu/4E_T)^2}.
\]
Let’s designate temporarily

\[ a = 1 - i\omega\tau, \quad b = \text{i}k_1P, \quad d = \frac{\hbar\nu k_1^2}{4\mathcal{E}_T}, \]

and rewrite the integral \( J(P) \) in the form:

\[
J = \int_{-1}^{1} \frac{(1 - \mu^2)d\mu}{(a + b\mu)^2 + d^2}.
\]

After change of variable \( a + b\mu = t \) this integral will be rewritten in the form

\[
J = \frac{1}{b^3} \int_{a-b}^{a+b} \frac{b^2 - (t-a)^2}{t^2 + d^2}dt.
\]

This integral equals to

\[
J = -\frac{2}{b^2} + \frac{d^2 + b^2 - a^2}{b^3} \frac{1}{2id} \ln \frac{(a + b - id)(a - b + id)}{(a + b + id)(a - b - id)} + \\
+ \frac{a}{b^3} \ln \frac{(a + b - id)(a + b + id)}{(a - b - id)(a - b + id)},
\]

or

\[
J = -\frac{2}{b^2} + \frac{d^2 + b^2 - a^2}{2idb^3} \ln \frac{a^2 - (b - id)^2}{a^2 - (d + id)^2} + \frac{a}{b^3} \ln \frac{(a + b)^2 + d^2}{(a - d)^2 + d^2}.
\]

Considering designations for \( a, b, d \), we receive

\[
J(P) \equiv J(P; \omega\tau, k_1) = \frac{2}{(k_1P)^2} - \\
- \frac{(1 - i\omega\tau)^2 + (k_1P)^2 - (\hbar\nu k_1^2/4\mathcal{E}_T)^2}{k_1^5P^3(\hbar\nu/2\mathcal{E}_T)} \ln \frac{(1 - i\omega\tau)^2 + (k_1P - \hbar\nu k_1^2/4\mathcal{E}_T)^2}{(1 - i\omega\tau)^2 + (k_1P + \hbar\nu k_1^2/4\mathcal{E}_T)^2} + \\
+i \frac{1 - i\omega\tau}{(k_1P)^3} \ln \frac{(1 - i\omega\tau + ik_1P)^2 + (\hbar\nu k_1^2/4\mathcal{E}_T)^2}{(1 - i\omega\tau - ik_1P)^2 + (\hbar\nu k_1^2/4\mathcal{E}_T)^2}.
\]

(6.8)

Thus, the expression of quantum transverse conductivity is defined by one-dimensional integral

\[
\sigma_{tr} = \frac{\sigma_0}{4f_2(\alpha)} \int_{-\infty}^{\infty} \frac{1 - (k_1/\omega\tau)P_x}{1 - i\omega\tau + ik_1P_x} \ln(1 + e^{a-P_x^2})dP_x -
\]
\[-\frac{i\sigma_0 k_1^2}{4f_2(\alpha)\omega_T} \int_0^\infty f_F(P) P^4 J(P) dP,\]

where the function \(J(P)\) is defined by expression (6.8).

So, for electric conductivity and dielectric function we have received following expressions

\[
\sigma_{tr} = \frac{\sigma_0}{4i f_2(\alpha)} \left[ \int_{-\infty}^\infty \left( \frac{1}{q} - \frac{\tau}{x} \right) \frac{y \ln(1 + e^{\alpha - \tau^2}) d\tau}{\tau - z/q} - \frac{y}{x} \int_0^\infty \frac{u^4 J(u) du}{1 + e^{u^2 - \alpha}} \right] \tag{6.9}
\]

and

\[
\varepsilon_{tr} = 1 + \frac{x_p^2}{4f_2(\alpha) x^2} \left[ \int_{-\infty}^\infty \left( x - q\tau \right) \frac{\ln(1 + e^{\alpha - \tau^2})}{\tau - z/q} d\tau - q \int_0^\infty \frac{u^4 J(u) du}{1 + e^{u^2 - \alpha}} \right]. \tag{6.10}
\]

In formulas (6.9) and (6.10) following designations are accepted

\[
q = \frac{k}{k_T}, \quad z = x + iy = \frac{\omega + i\nu}{k_Tv_T}, \quad x = \frac{\omega}{k_Tv_T}, \quad y = \frac{\nu}{k_Tv_T}, \quad x_p = \frac{\omega_p}{k_Tv_T},
\]

\[
J(u) = -\frac{2}{u^2} - \frac{(z/q)^2 + (q/2)^2 - u^2}{w^3 q} \ln \left( \frac{(u - q/2)^2 - (z/q)^2}{(u + z/q)^2 - (q/2)^2} \right) - \frac{z}{w^3 q} \ln \left( \frac{(u - z/q)^2 - q/2)^2}{(u + z/q)^2 - (q/2)^2} \right) = \int_{-1}^1 \frac{(1 - \tau^2) d\tau}{(u\tau - z/q)^2 - (q/2)^2}.
\]

Besides these formulas (6.9) and (6.10) we can give and other formulas for electric conductivity and dielectric function

\[
\sigma_{tr} = \frac{\sigma_0}{4i f_2(\alpha)} \left[ \int_{-\infty}^\infty \left( \frac{y \ln(1 + e^{\alpha - \tau^2}) d\tau}{\tau - z/q} \right) - \frac{y}{x} \int_0^\infty \frac{d\tau}{(\tau - z/q)^2 - (q/2)^2} \right] \int_0^\infty \rho \ln(1 + e^{\alpha - \tau^2 - \rho^2}) d\rho
\]

and

\[
\varepsilon_{tr} = 1 + \frac{x_p^2}{4f_2(\alpha) x^2} \left[ \int_{-\infty}^\infty \left( \frac{x}{\tau - z/q} \right) \frac{\ln(1 + e^{\alpha - \tau^2}) d\tau}{\tau - z/q} \right].
\]
\[- \frac{x_p^2}{4 f_2(\alpha) x^2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \frac{d\tau}{(\tau - z/q)^2 - (q/2)^2} \int_{0}^{\infty} \rho \ln(1 + e^{\alpha - \rho^2 - \tau^2}) d\rho.\]

7. THE SUM RULES

Let’s check up performance of one of the parities, named the rule $f$-sums (see, for example, [23]) for transversal dielectric permeability (6.14). This rule is expressed by the formula (4.200) of the monography [23]

\[\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \varepsilon_{tr}(q, \omega, \nu) \omega d\omega = \pi \omega_p^2. \quad (7.1)\]

As shown in [23], for the proof of the parity (7.1) it is enough to prove performance of the limiting parity

\[\varepsilon_{tr}(q, \omega, \nu) = 1 - \frac{\omega_p^2}{\omega^2} + o\left(\frac{1}{\omega^2}\right), \quad \omega \to \infty. \quad (7.2)\]

From expression (6.10) it is visible that

\[\varepsilon_{tr}^{\text{quant}} = o\left(\frac{1}{\omega^2}\right), \quad \omega \to \infty.\]

By means of this limiting parity we will present the parity (6.10) at big $\omega$ ($\omega \gg 1$) in the form

\[\varepsilon_{tr} = 1 + \frac{i \omega_p^2 \omega \tau}{4\pi f_2(\alpha) \omega^2} \int \frac{g(P) P_{\perp}^2 d^3P}{1 - i \omega \tau + i k_1 P}. \quad (7.3)\]

From comparison (7.2) and (7.3) it is visible, that now it is required to prove equality

\[\frac{i \omega \tau}{4\pi f_2(\alpha)} \int \frac{g(P) P_{\perp}^2 d^3P}{1 - i \omega \tau + i k_1 P} = -1,\]

or, considering once again, that $\omega \gg 1$, is required to prove equality

\[\frac{1}{4\pi f_2(\alpha)} \int g(P) P_{\perp}^2 d^3P = 1. \quad (7.4)\]

Passing in equality (7.4) to spherical coordinates and integrating once in parts, we receive

\[\frac{2}{3 f_2(\alpha)} \int_0^{\infty} \frac{e^{P^2 - \alpha} P^4 dP}{(1 + e^{P^2 - \alpha})^2} = 1. \quad (7.5)\]
Of justice of equality (7.5) we are convinced, having integrated once by parts.

8. THE ANALYSIS OF RESULTS AND THE CONCLUSION

The analysis of graphics on fig. 1–7 shows, that at great values $q$ the quantum conductivity does not decrease to zero (as classical conductivity), and tends for collisional plasma to the finite limit. Really, from the formula (6.13) follows, that

$$ \lim_{q \to \infty} \sigma_{tr} = i \frac{y}{x} = i \frac{\nu}{\omega}. $$

(8.1)

On Figs 1–7 all curves 1 answer to classical plasma, and curves 2 to the quantum plasma.

On Figs. 1–3 are presented dependence $|\sigma_{tr}/\sigma_0|$ quantum and classical conductivity from quantity $q = k/k_T$ at the various values of the resulted chemical potential $\alpha$: $\alpha = 0$ (fig. 1), $\alpha = 6$ (fig. 2) and $\alpha = -5$ (fig. 3).

The analysis of graphics and numerical calculations show at negative values of chemical potential weak dependence $|\sigma_{tr}/\sigma_0|$ on quantity $q$, and strong dependence at positive values of chemical potential. From fig. 1–3 it is visible, that while the module of classical conductivity has only one maximum, the module of quantum conductivity has also a minimum. The range of values the module quantum conductivity decreases at decrease chemical potential.

On fig. 4 and 5 dependence $\text{Re}(\sigma_{tr}/\sigma_0)$ is presented for quantum and classical conductivity. From the parity (8.1) it is visible, that the real part of quantum conductivity tends to zero at $q \to \infty$ the same as also the real part of classical conductivity. However, decrease of quantum conductivity occurs much faster, than decrease of classical conductivity that is visible from fig. 4 and 5. At the real parts of quantum conductivity there is a maximum, the minimum is absent.

On fig. 6 and 7 dependence $\text{Im} (\sigma_{tr}/\sigma_0)$ is presented quantum and classical conductivity on quantity $q$. Qualitatively imaginary part behaves the same
as also the module. From fig. 1 - 7 it is visible, that at $q \rightarrow \infty$ quantum and classical conductivity coincide.

On fig. 8 dependence graphics $|\sigma_{tr}/\sigma_0|$ are presented quantum plasma from quantities $q$ at various values of quantity $x$ (dimensionless frequency oscillations of vector potential). The analysis of graphics shows, that with growth quantity $x$ the range of values of the module of quantum plasma decreases, thus the maximum and module minimum are levelled, i.e. values of the module in points maximum and minimum approach, reducing range of values of the module of conductivity.

In the present work the correct formula for calculation of transversal electric conductivity in the quantum non-degenerate collisional plasma is deduced. For this purpose the Wigner—Vlasov—Boltzmann kinetic equation with collisional integral in the form of BGK—model (Bhatnagar, Gross and Krook) in coordinate space is used.
Рис. 1: Case: $x = 1, y = 0.01, \alpha = 0$. Dependence $|\sigma_{tr}/\sigma_0|$ on quantity $q$.

Рис. 2: Case: $x = 1, y = 0.01, \alpha = 6$. Dependence $|\sigma_{tr}/\sigma_0|$ on quantity $q$. 
Рис. 3: Case: $x = 1, y = 0.01, \alpha = -5$. Dependence $|\sigma_{tr}/\sigma_0|$ on quantity $q$.

Рис. 4: Case: $x = 1, y = 0.01, \alpha = 0$. Dependence $\text{Re}(\sigma_{tr}/\sigma_0)$ on quantity $q$. 
Рис. 5: Case: $x = 1, y = 0.01, \alpha = 5$. Dependence $\text{Re}(\sigma_\text{tr}/\sigma_0)$ on quantity $q$.

Рис. 6: Case: $x = 1, y = 0.01, \alpha = 0$. Dependence $\text{Im}(\sigma_\text{tr}/\sigma_0)$ on quantity $q$. 
Рис. 7: Case: $x = 1, y = 0.01, \alpha = 5$. Dependence $\text{Re}(\sigma_{tr}/\sigma_0)$ on quantity $q$.

Рис. 8: Case: $x = 1, y = 0.01, \alpha = 0$. Dependence $|\sigma_{tr}/\sigma_0|$ on quantity $q$. 

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