Theoretical Field Limits for Multi-Layer Superconductors

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Multilayer Films

- SIS structure proposed for use in SRF cavities by A. Gurevich [1]
- Suggested advantage: Avoid risks of low $B_{c1}$ in alternative superconductors
- Above $B_{c1}$ superconductor is metastable state—only an energy barrier prevents vortex penetration
- Also suggestions that SIS structure could reach extremely high fields at RF frequencies

S. Posen - Theoretical Field Limits for Multi-Layer Superconductors

[1] A. Gurevich, App. Phys. Lett. 88, 012511 (2006)
[2] A. Gurevich, SRF Materials Workshop (2007).
Results to Be Shown in This Talk

- I will show that SIS films in fact have $B_{c1} = 0$
- Both SIS multilayers and bulk films rely on energy barrier – same vulnerability for small-$\xi$ alternative materials
- Looking at $B_{sh}$, no clear advantage for SIS films
- Adding more layers does not help: actually makes things worse

S. Posen - Theoretical Field Limits for Multi-Layer Superconductors
Recall: Flux vs Vortex

- **Flux** penetrates with $e^{-x/\lambda}$ into superconductor without strong dissipation
- A **vortex** is a normal conducting core with 1 quantum of flux
- Vortex penetration causes enormous dissipation in RF fields due to drag
$B_{c1}$ (or $H_{c1}$) in the SIS Structure
No Enhancement of $B_{c1}$

- “By definition, when $H = H_{c1}$ the Gibbs free energy must have the same value whether the first vortex is in or out of the sample” [1]

- Parallel $B_{c1}$ of a thin film is enhanced:

  $$B_{c1} \approx \frac{2\phi_0}{\pi d^2} \ln \frac{d}{\tilde{\xi}}, \tilde{\xi} = 1.07\xi, d < \lambda$$

- Does this $B_{c1}$ enhancement apply to SIS films as well?

- To find $B_{c1}$, calculate* $G(x)$ for a vortex (this is how above equation was derived)

[1] M. Tinkham, Introduction to Superconductivity (New York: Dover, 1996).
[2] C. Bean and J. Livingston, Phys. Rev. Lett. 12, 14-16 (1964).
[3] G. Stejic, et al, Phys. Rev. B 49,1274 (1994).

*Details of $G(x)$ calculation given in paper for this talk
Free Energy Calculations

**BULK SUPERCONDUCTOR**

- Exterior at $B_{\text{ext}}$

$B_{\text{ext}} = B_{\text{sh}} = 346$ mT

$B_{\text{c1}} < B_{\text{ext}} < B_{\text{sh}}$

$B_{\text{ext}} = B_{\text{c1}} \approx \frac{\phi_0}{4\pi\lambda^2} \ln\kappa = 43$ mT

Barrier to vortex penetration

Stable vortex position for $B_{\text{ext}} > B_{\text{c1}}$

$B_{\text{ext}} = 0$

$B_{\text{ext}} = B_{\text{sh}} = 346$ mT

**Nb$_3$Sn bulk**
Free Energy Calculations

Gibbs Free Energy of a Vortex Relative to Energy Outside Film [J]

\[ B_{ext} = B_{c1} \approx \frac{2\phi_0}{\pi d^2} \ln \frac{d}{\xi} = 1.2 \, T \]

\[ B_{ext} = B_{sh} = 1.3 \, T \]

**Thin Film**

- Exterior at \( B_{ext} \)
- Exterior at \( B_{ext} \)

**Expression**

\[ x \times 10^{-11} \]

**Barrier to vortex penetration**

**Stable vortex position**

\[ d = 50 \, nm, \lambda = 111 \, nm \]

**Exterior at \( B_{ext} \)**

**Exterior at \( B_{ext} \)**
Free Energy Calculations

**Thin Film/Insul./Bulk**

- $B_{\text{ext}} > B_{\text{sh}} = 331 \text{ mT}$
- $B_{\text{ext}} = B_{\text{sh}}$
- $B_{\text{ext}} > B_{c1}$

**Gibbs Free Energy of a Vortex [J]**

- $x \times 10^{-10}$
- $0 \rightarrow 6$
- $0 \rightarrow 600$

**Distance into Structure [nm]**

- $d = 50 \text{ nm}$
- $\lambda = 111 \text{ nm}$

**Stable “vortex” position**

- $B_{\text{ext}} = B_{c1} = 0$

**Nb$_3$Sn film**

**Ins. film**

**Exterior**

**Nb$_3$Sn bulk**
Conclusions

• Conclusion #1: SIS structure has $B_{c1} = 0$
  
  – $B_{c1}$ enhancement argument from thin films does not apply to SIS structures

  \[ B_{c1} \approx \frac{2\phi_0}{\pi d^2} \ln \frac{d}{\xi}, \tilde{\xi} = 1.07\xi, d < \lambda \]

  – Both SIS multilayers and bulk films rely on energy barrier in RF fields to prevent vortex penetration: same vulnerability for small-$\xi$ alternative materials

• Conclusion #2: No clear $B_{sh}$ advantage for SIS films
  
  – SIS layers need correct thicknesses for high $B_{sh}$
  
  – Optimal SIS film about as good as bulk film

  – Multiple layers are worse: smaller maximum field
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Vortex Dissipation

• Can SIS or bulk superconductors survive vortex penetration at RF freq?
  • No: heating is enormous if vortices pass through the film every half cycle

• Calculation from Gurevich:
  \[ \frac{P}{A} = \frac{2\omega d}{\pi \mu_0 \lambda_f} \left(\lambda_b + \delta + \frac{d}{2}\right) B_v (B_0 - B_v) \]

• 1 mT above \( B_{sh} \) for 50 nm film \( \text{Nb}_3\text{Sn}/\text{I}/ \) bulk \( \text{Nb}_3\text{Sn} \) at 1.3 GHz = \(~9\) W/cm\(^2\) of heating

• Above \( B_{c1} \), in RF fields, we have to rely on metastability (both SIS and bulk)
$B_{sh}$ in the SIS Structure
Energy Barrier

• Both SIS and bulk films rely on energy barrier to prevent flux penetration up to $B_{sh}$

• Can ideal SIS reach higher maximum fields than ideal bulk film?
Superheating Field

Note: Similar $B_{sh}$ calculations done previously by Kubo, Iwashita, and Saeki
Superheating Field

Gain is significant only for relatively small range in d

Maximum gain for Nb₃Sn/I/Nb compared to Nb₃Sn bulk is ~12%

Single-material SIS has smaller $B_{sh}$ than bulk for any d

To left of peak, $B_{max}$ limited by bulk $B_{sh}$
To right of peak, $B_{max}$ limited by film $B_{sh}$

Note: Similar $B_{sh}$ calculations done previously by Kubo, Iwashita, and Saeki
Superheating Field

Gain is significant only for relatively small range in d

Maximum gain for Nb\textsubscript{3}Sn/I/Nb compared to Nb\textsubscript{3}Sn bulk is \(~12\%\)

Multilayer with 5 S-I films (d = total thickness of S layers) over bulk Nb has smaller $B_{sh}$ than single layer

Single-material SIS has smaller $B_{sh}$ than bulk for any d

To left of peak, $B_{max}$ limited by bulk $B_{sh}$

To right of peak, $B_{max}$ limited by film $B_{sh}$

Note: Similar $B_{sh}$ calculations done previously by Kubo, Iwashita, and Saeki
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Choose bulk films over SIS films for SRF
DC – Enhanced Screening

• From these arguments, SIS multilayers are not superior for SRF applications

• In DC and low frequency AC, vortex penetration can be tolerated without excessive heating
  – SIS multilayers can be useful in DC and low frequency AC applications
Hope for the Future
Hope for the Future

• Both SIS and bulk films rely on operation above $B_{c1}$
• Can superconductors survive in the metastable state when the coherence length is small?
• Will small surface defects cause vortex penetration in alternative materials?
• Is there hope for $\text{Nb}_3\text{Sn}$, $\text{Nb}(\text{Ti})\text{N}$, $\text{MgB}_2$?
I designed, assembled, and commissioned a Nb$_3$Sn coating chamber for cavities.

I coated and tested a single cell Nb$_3$Sn cavity, which showed exceptional RF performance.
Bulk Film Experiment

- Small $\xi$ (3.2 ± 0.2 nm) bulk film, but far exceeds $B_{c1}$ with no indication of vortex penetration
- Q-slope in previous Nb$_3$Sn cavities not fundamental—proof that even for small $\xi$, $B_{c1}$ is NOT a limit

Clearly above $B_{c1} = 27 \pm 5$ mT for Nb$_3$Sn cavity without strong Q-slope!

See also TUP027 by D. Gonnella for small-$\xi$ Nb well above $B_{c1}$ with no indication of vortex penetration
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There is hope for alternative materials!

See also TUP027 by D. Gonnella for small-$\xi$ Nb well above $B_{c1}$ with no indication of vortex penetration
Summary and Outlook

- SIS multilayer films have $B_{c1} = 0$
- They rely on energy barrier as the bulk does
  - SIS $B_{sh}$ is very close to bulk $B_{sh}$
  - Adding more layers does not help
  - Small potential gain but very difficult to fabricate
- Not superior for SRF applications—they are useful in DC applications
- $B_{c1}$ is not a limit for cavities made from small-\(\xi\) superconductors! No need for $B_{c1}$ enhancement!
- SIS multilayers may not protect alternative SRF materials, but new developments give reason for strong optimism for bulk films
http://www.icmmo.u-psud.fr/Labos/LCI/Service_SQUID/squid.php

- External field can “sneak” between layers
- Each layer acts independently – SQUID sees vortex penetration into most vulnerable layer, likely Nb bulk

A. Gurevich, TFSRF Workshop, 2005
SQUID Measurements

W. Roach et al., IEEE Trans. App. S. 8600203 (2013)

- External field can “sneak” between layers
- Each layer acts independently – SQUID sees vortex penetration into most vulnerable layer, likely Nb bulk
External field can “sneak” between layers
With no bulk, simply observe well understood $B_{c1}$ enhancement, just for several layers at once
Clear that NbN helps and that more NbN is better (due to more total thickness? Importance of perpendicular fields?)

C. Antoine, APL 102603 (2013) and TFSRF Workshop 2010
DC and Low Frequencies

- In general, one must be careful when conducting measurements of multilayers at low frequencies.
- Vortices can pass through the superconducting films into the insulating region with minimal dissipation.
- At RF frequencies the vortex dissipation would be intolerable (linear with $f$).
Material Properties

| Material | $\lambda$ [nm] | $\xi$ [nm] | $B_{c1}$ [T] | $B_{sh}$ [T] |
|----------|----------------|------------|--------------|-------------|
| Nb       | 40             | 27         | 0.13         | 0.24        |
| Nb$_3$Sn | 111            | 4.2        | 0.042        | 0.36        |
| NbN      | 375            | 2.9        | 0.006        | 0.15        |
| MgB$_2$  | 185            | 4.9        | 0.017        | 0.19        |

$\lambda$ is calculated using Eqn 3.131 in [1]. $\xi$ is calculated using the equations in [2]. For Nb a RRR of 100 was assumed. For MgB$_2$, $\lambda$ and $\xi$ are not calculated, as the experimental values are given in the reference. For calculations, $B_c = \phi_0/(2\sqrt{2}\pi \xi \lambda)$ is used [1]. $B_{c1}$ for Nb found from power law fit to numerically computed data from [3] and for strongly type II materials is found from Eqn 5.18 in [1]. $B_{sh}$ for Nb is found from [4] and for others calculated from $B_c \sqrt{20}/6$ (valid only for strongly type II materials near $T_c$) [5]. Nb data from [6], Nb$_3$Sn data from [3], NbN data from [7], and MgB$_2$ data from [8]. Note that the two gap nature of MgB$_2$ may require more careful analysis than is performed here.

[1] M. Tinkham, Introduction to Superconductivity (New York: Dover, 1996).
[2] T. Orlando, et al., Phys. Rev. B 19, 4545 (1979).
[3] M. Hein, High-Temperature Superconductor Thin Films at Microwave Frequencies (Berlin: Springer, 1999).
[4] A. Dolgert, S. Bartolo, and A. Dorsey, Erratum [Phys. Rev. B 53, 5650 (1996)], Phys. Rev. B 56, 2883 (1997).
[5] M. Transtrum, G. Catelani, and J. Sethna, Phys. Rev. B 83, 094505 (2011).
[6] B. Maxfield and W. McLean, Phys. Rev. 139, A1515 (1965).
[7] D. Oates, et al., Phys. Rev. B 43, 7655 (1991).
[8] Y. Wang, T. Plackowski, and A. Junod, Physica C 355, 179 (2001).
No $B_{c1}$ Enhancement

- Can also show $B_{c1} = 0$ from simple argument
- Free energy for flux quantum in vacuum or insulator is $B\phi_0/\mu_0$ (invalid for superconductor)
- Field and therefore free energy is higher in external region than in insulator
- Structure is clearly above $B_{c1}$, metastable

Lowest energy position for vortex is in film
GL $B_{sh}$ numerical calculations: max 8% gain for $\text{Nb}_3\text{Sn}/\text{I}/\text{Nb}$ vs $\text{Nb}_3\text{Sn}$ bulk---only a few % off from London limit calculations shown earlier.