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A new generalization of Gull Alpha Power Family of distributions with application to modeling COVID-19 mortality rates

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A B S T R A C T

This paper proposes a new generalization of the Gull Alpha Power Family of distribution, namely the exponentiated generalized gull alpha power family of distribution abbreviated as (EGGAPF) with two additional parameters. This proposed family of distributions has some well known sub-models. Some of the basic properties of the distribution like the hazard function, survival function, order statistics, quantile function, moment generating function are investigated. In order to estimate the parameters of the model the method of maximum likelihood estimation is used. To assess the performance of the MLE estimates a simulation study was performed. It is observed that with increase in sample size, the average bias, and the RMSE decrease. A distribution from this family is fitted to two real data sets and compared to its sub-models. It can be concluded that the proposed distribution outperforms its sub-models.

Introduction

Probability distributions have been of good use in various areas of science including reliability engineering, survival analysis, financial time series, econometrics, social sciences among others. Because of the applicability of probability distributions in real life, over the last decade researchers have dedicated efforts to modify and generate new distributions in order to address challenges of lifetime data. In order to guarantee flexibility of distributions, the number of shape parameters that characterize the cumulative distribution function matters. Many methods have been devised to add one or more parameters to the cumulative distribution function of a distribution which makes the resulting distribution to be flexible and rich for modeling data.

A list of the methods include the Beta generalized family developed by [1], Kumaraswamy-G family by [2], McDonald-G family by [3], Weibull X by [4], Gamma-X by [5], Beta Marshall–Olkin family of distributions by [6], a new Weibull G family by [7], the Alpha Power Transformation family by [8], the Marshall–Olkins Alpha Power family by [9], the Transmuted Alpha Power-G Family by [10], the Alpha-Power Marshall Olkins G distribution by [11], the Gull Alpha Power of the Ampadu Type by [12] and Families of distributions arising from the quantile of generalized lambda distribution by [13] are some of the notable generalizations and generators developed recently among others.

A new method called exponentiated method where the cumulative distribution function is raised to a parameter was introduced by [14]. Many traditional distributions have been extended using this method, the exponentiated exponential distribution by [15]. For other distributions extended using this criteria the reader is referred to the articles [16–18] and [19]. The addition of one shape parameter to the cumulative distribution function have a shortcoming in that it handles only the skewness, and fails to control kurtosis. [20] introduced a new method of adding two extra shape parameters called the exponentiated generalized class of distributions and studied its properties. Several authors have used this method to extend distributions for example [21] extended the Marshall Olkins family of distributions using this approach [22] developed the exponentiated generalized standardized half-logistic distribution.

Due to the availability of the above methods of generating new continuous distributions, the exponential distribution has been extended in many researches a few generalizations of the exponential distribution include the exponentiated generalized marshall olkins exponential distribution by [21], the Gompertz inverse exponential distribution...
studied by [23], odd generalized exponential distribution by [24], weibull exponential distribution by [25] among other generalizations. Recently, [26] developed the Gull Alpha Power Family (GAPF) with CDF and PDF:

\[
F_{\text{GAPF}}(y) = \begin{cases} 
W(y; \varphi) & \text{if } a > 0, a \neq 1 \\
W(y; \varphi) & \text{if } a = 1 
\end{cases}
\]

and Eq. (6) gives the PDF:

\[
f_{\text{GAPF}}(y) = \log(a)a^{1-W(y; \varphi)}(-u(y; \varphi)W(y; \varphi)) + \psi(y; \varphi)a^{1-W(y; \varphi)}; \ a > 0, a \neq 1
\]

New extension of the GAPF is proposed by considering its PDF and CDF as the baseline in the exponentiated generalized class studied by [20]. The new family of distributions is named the Exponentiated-Generalized Gull Alpha Power. The primary motivation to extend this family is:

1. To develop distributions that are characterized by different shapes of the hazard function
2. To generate distribution that offer a better fit compared to other models with the same baseline distribution
3. To construct distributions that are heavy tailed in order to model diverse real data sets.
4. To develop distributions that are characterized by different shapes.

Some special families from this new family of distributions are displayed in Table 1.

| a   | b   | Family                          |
|-----|-----|---------------------------------|
| 1   | 1   | \(W(y; \varphi)\)               |
| \(a\) | 1   | Gull Alpha Power                |
| \(a\) | \(b\) | Exponentiated class of distributions |
| \(a\) | \(b\) | Exponentiated Gull Alpha Power Family |

Table 1

Some special families.

By differentiating (13) we obtain:

\[
\psi(\gamma; \varphi) = \sum_{l=0}^{\infty}(-1)^l d \left( \frac{a}{a l} \right)^l W_{\text{EGAP}}(y; \varphi)^{l+1}
\]

The EGGAP family of distribution is obtained by inserting Eqs. (1) and (2) into Eqs. (3) and (4) respectively. The new family has CDF given as:

\[
F_{\text{EGGAP}}(y; \alpha, a, b, \varphi) = \left[1 - (W(y; \varphi))^{a b} \right] a > 0, \neq 1, \ a, b > 0
\]

and Eq. (6) gives the PDF:

\[
f_{\text{EGGAP}}(y; \alpha, a, b, \varphi) = \left( a b \log(a)a^{1-W(y; \varphi)} \right) \psi(y; \varphi)a^{1-W(y; \varphi)} \times \left( W(y; \varphi) \right)^{a b - 1} \ a > 0, \neq 1, \ a, b > 0
\]

Eq. (7) gives the survival function for the new family:

\[
\psi_{\text{EGGAP}}(y; \Theta) = \left[1 - (W(y; \varphi))^{a b} \right]^b a, b > 0
\]

The hazard function for the new family of distributions is

\[
\psi_{\text{EGGAP}}(y; \Theta) = \frac{f_{\text{EGGAP}}}{1 - (W(y; \varphi))^{a b}}
\]

EGGAP-exponential distribution

The EGGAPE\((a,b,a,\lambda)\) a special case of the EGGAP family is explained. The pdfs and the hrfs for some choices of parameter values are explored to determine the shapes that the family can assume. The exponential distribution has PDF \(\psi(y) = \lambda e^{-\lambda y}\) and CDF \(W(y) = 1 - e^{-\lambda y}, y > 0\). The extension of the exponential distribution EGGAPE\((a,b,a,\lambda)\) has the PDF, CDF and HRF as:

\[
W(y) = \left[1 - \left( 1 - \left( 1 - e^{-\lambda y} \right)^{a b} \right)^{a b} \right] \ a, b > 0, a > 0, \neq 1
\]

\[
\psi(y) = \frac{a b \lambda e^{-\lambda y}}{a b \lambda e^{-\lambda y} - a b \lambda e^{-\lambda y} \log(a)} \left( 1 - \left( 1 - e^{-\lambda y} \right)^{a b} \right)^{a b - 1}
\]

As observed in Fig. 2 the hazard shape of the new distributions can take various forms from the decreasing, increasing, inverted bathtub shape to bathtub. Results from Fig. 2 illustrates versatility of the new distribution in handling data characterized with several shapes of the hazard (see Fig. 1).

Mathematical properties of EGGAP family of distributions

Useful expansion

Consider:

\[
(1 - x)^d = \sum_{l=0}^{\infty}(-1)^l \left( \frac{d}{l} \right) x^l
\]

for any integer \(d\) and \(|x| < 1\). Using the expansion in Eq. (12) in Eq. (5) we can express the cdf of EGGAP\((a,b,a,\varphi)\) as:

\[
F_{\text{EGGAP}}(a,b,a,\varphi) = \left[1 - \left( W(y; \varphi) \right)^{a b} \right]^{b} = \sum_{l=0}^{\infty}(-1)^l \left( \frac{b}{l} \right) W(y; \varphi)^{b l}
\]

By differentiating (13) we obtain:

\[
f_{\text{EGGAP}}(a,b,a,\varphi) = f_{\text{EGGAP}}(y; \varphi) \sum_{l=0}^{\infty} a l \ W_{\text{EGAP}}(y; \varphi)^{l+1}
\]

\[
= \sum_{l=0}^{\infty} \frac{d}{l} \ G_{\text{EGAP}}(y; \varphi)^{l+1}
\]
where \( \omega_s = \sum_{l=0}^{\infty}(-1)^{l+1} \binom{b}{l} \left( \frac{a}{l} \right) \)

Eqs. (13) and (14) confirms that the CDF and PDF of \( EGGAP(a, b, \lambda, \varphi) \) are a linear combination of Exponentiated-Gull Alpha Power family of distributions. This result is not new, it has been shown by [20].

**Quantile function**

Inverting the cdf of EGGAP family of distribution using mathematica software we get

\[
Q(p) = G^{-1}\left( W(y) \left( -1 + \left( 1 - \frac{1}{\varphi} \right)^{\frac{1}{\varphi}} \right) \log(\alpha) \times \frac{1}{\alpha} \right), \quad \alpha > 0, \alpha \neq 1
\]

\( W(y) \) is the product log function defined as:

\[
W(y) = \sum_{n=1}^{\infty}(-1)^{n-1}a^{n-2} \frac{1}{(n-1)!} z^n
\]

Using this formula, random numbers \( y \) can be generated from \( EGGAP(a, b, \lambda, \varphi) \) as:

\[
y = G^{-1}\left( \frac{W(y) \left( -1 + \left( 1 - \frac{1}{\varphi} \right)^{\frac{1}{\varphi}} \right) \log(\alpha) \times \frac{1}{\alpha} }{\log(\alpha)} \right)
\]

For example the quantile function for the EGGAPE(\( a, b, \alpha, \lambda \)) is given as:

\[
Q_y(u) = \frac{1}{\lambda} \log \left( \frac{\log(\alpha)}{\log(\alpha) + W_{-1}\left( -1 + \left( 1 - u \right)^{\frac{1}{\varphi}} \right) \log(\alpha)} \right), \quad \alpha > 0, \alpha \neq 1
\]

\[ (16) \]

Table 2 shows the quantile values for some parameter values with relation to Eq. (16).

To provide a numerical example on how to simulate random observations from the EGGAPE distribution Eq. (16) is used and the R language to generate 1000 EGGAPE(1.4,1.3,0.8,0.3) random variables.

As shown in Fig. 3, the plot show the adequacy of the model for practical application.
The kurtosis of the EGGAPE distribution showing the impact of the parameters. The extra shape parameters have an effect on skewness which exists even if the distribution does not have moments and they are less sensitive to outliers.

Some quantile values.

| \( y \) | \( Q(y) \) of EGGAPE(1.5,0.8,0.5,0.2) |
|---|---|
| 0.1 | 0.373692 |
| 0.2 | 0.889970 |
| 0.3 | 1.490652 |
| 0.4 | 2.175624 |
| 0.5 | 2.964027 |
| 0.6 | 3.896447 |
| 0.7 | 5.053373 |
| 0.8 | 6.67515 |
| 0.9 | 9.17003 |

Plots of the EGGAPE(1,4,1,3,0,8,0,3) for simulated data with \( n = 1000 \).

The Bowley’s skewness defined by \([27]\) is given as:

\[
Q(0.75) + Q(0.25) - 2Q(0.5)
\]

Moors Kurtosis coefficient, \([28]\) is defined as:

\[
Q(0.375) - Q(0.125) + Q(0.875) - Q(0.25)
\]

As shown in Fig. 4 shows the Moors Kurtosis with different baseline parameter values. The extra shape parameters have an effect on the kurtosis of the EGGAPE distribution showing the impact of the parameters.

The Bowley’s skewness is given in Fig. 5. It is clear that the extra shape parameters have an effect on the skewness of the EGGAPE distribution showing the impact of the parameters.

Order statistics

Assume \( y_1, y_2, \ldots, y_n \) is a sample from the EGGAP(\(a, b, a, \phi\)) family. Eq. (17) shows the \( r \)th order statistic PDF:

\[
f_{r,n}(y, \Theta) = \frac{n!}{(r-1)!(n-r)!} f_{EGGAPE}(y, \Theta) \left[ F_{EGGAPE}(y, \Theta) \right]^{r-1} \times \left[ 1 - F_{EGGAPE}(y, \Theta) \right]^{n-r} \times F_{EGGAPE}(y, \Theta)^{b_{r+1}}
\]

Next, making use of the expansions described in Section “Useful Expansion”, \( EGGAP(a, b, a, \phi) \) is derived and given as:

\[
f_{r,n}(y, \Theta) = \frac{n!}{(r-1)!(n-r)!} \sum_{i=0}^{\infty} (-1)^i \left( \frac{n-r}{h} \right)^i f_{EGGAPE}(y, \Theta) \times \sum_{k=0}^{\infty} a_{2k} W_{EGGAPE}(y; \phi)^{i+k}
\]

Upon further simplification, Eq. (18) reduces to:

\[
f_{r,n}(y, \Theta) = \frac{n!}{(r-1)!(n-r)!} \sum_{i=0}^{\infty} (-1)^i \left( \frac{n-r}{h} \right)^i f_{EGGAPE}(y, \Theta) \times \sum_{k=0}^{\infty} d_{h+r-1,k} W_{EGGAPE}(y; \phi)^{i+k}
\]

Moment generating function

MGF of EGGAP family of distributions is derived. By definition the MGF is given by:

\[
M_{Y}(t) = E[e^{yt}]
\]
The EGGAPE distribution’s moments.

\[
\begin{array}{cccccc}
\text{r} & I & II & III & IV & V \\
\hline
\mu' & 0.4296 & 0.5825 & 0.2580 & 0.1505 & 0.6271 \\
\mu'' & 1.1481 & 0.3722 & 0.1021 & 0.0313 & 0.4505 \\
\mu''' & 4.8405 & 0.2594 & 0.0571 & 0.0085 & 0.3651 \\
\mu'''' & 27.3025 & 0.1964 & 0.0429 & 0.0029 & 0.3301 \\
\mu''''' & 191.849 & 0.1609 & 0.0419 & 0.0023 & 0.3308 \\
\end{array}
\]

The effect of skewness on the EGGAPE distribution’s moments.

The Renyi Entropy for different parameter values is displayed in Table 4.

| s | I   | II  | III |
|---|-----|-----|-----|
| 0.1 | 2.4980 | 2.9219 | 1.6761 |
| 0.5 | 0.7697 | 1.9139 | 0.4364 |
| 0.9 | 0.2433 | 1.6650 | 0.1182 |
| 1.5 | -0.1974 | 1.4967 | -0.0880 |
| 2.5 | -0.7559 | 1.3667 | -0.2405 |

Estimation

The maximum likelihood estimation approach is used to estimate the parameters of the new family. Assume \( y_1, y_2, y_3, \ldots, y_n \) is a sample of size \( n \) from EGGAPE family of distribution where \( a, a, b, \varphi \) and \( \Psi \) are parameters and \( \varphi \) is a vector of parameters which is associated with the baseline distribution. Let \( \Theta = (a, a, b, \varphi) \) be the \( t \times 1 \) vector of parameters, then the corresponding log-likelihood function is:

\[
L = n \log(ab) + \log(\log(a)) + \sum_{i=1}^{n} \left[ 1 - W(y_i; \varphi) \right] \log(a) + \sum_{i=1}^{n} \log W(y_i; \varphi) + \frac{n}{(a - 1)} \sum_{i=1}^{n} \log[1 - W(y_i; \varphi)]^s
\]

where

\[
\Theta = \left( \delta, \alpha, a, b, \varphi \right)
\]

and for \( w = 1, 2, \ldots, p \)

\[
\frac{\partial l}{\partial \varphi_k} = -2 \log(ab) \sum_{i=1}^{n} W_i^w(y_i; \varphi) + \frac{W_i^w(y_i; \varphi)}{W_i^{(w)}(y_i; \varphi)} + \frac{(a - 1)}{W_i^{(w)}(y_i; \varphi)} W_i^{(w)}(y_i; \varphi)
\]

where

\[
W_i^w(y_i; \varphi) = \frac{\partial W_i^{(w)}(y_i; \varphi)}{\partial \varphi_k}, \quad W_i^{(w)}(y_i; \varphi) = \frac{\partial W_i^{(w)}(y_i; \varphi)}{\partial \varphi_k}
\]

The MLEs \( \hat{\Theta} = (\hat{\delta}, \hat{\alpha}, \hat{a}, \hat{b}, \hat{\varphi}) \) can be obtained by equating Eqs. (25) to (28) to zero and obtaining solutions through the numerical method using an appropriate software.

Monte Carlo Simulation study

The effectiveness of the MLE method to estimate the parameters of the EGGAPE distribution by the use of Monte Carlo Simulation is discussed.
In order to conduct the simulation study, Eq. (16) was utilized to generate different samples of different lengths using different parameter values. The simulation study made 2000 iterations for the following choices of sample sizes $n = 100, ..., 2000$ and the initial parameter values $a = 0.9, b = 0.3, \alpha = 0.6$ and $\lambda = 0.5$ in set I and $a = 0.7, b = 0.8, \alpha = 0.4$ and $\lambda = 0.3$ in set II.

The MLEs are estimated for each item that is $(\hat{a}, \hat{b}, \hat{\alpha}, \hat{\lambda})$ for $n = 100, ..., 2000$ and the AB and RMSE calculated by:

$$AB = \frac{1}{M} \sum_{i=1}^{M} (\hat{\varphi} - \varphi)$$

(29)

and

$$RMSE = \sqrt{\frac{1}{M} \sum_{i=1}^{M} (\hat{\varphi} - \varphi)^2}$$

(30)

where $\varphi = a, b, \alpha, \lambda$

**Simulation algorithm**

The following simulation algorithm was used

i. Specification of the initial values of the model parameters
ii. Using Eq. (19) generate a random sample of size $n$
iii. Evaluate the values of the estimates using the maximum likelihood method
iv. Calculate the AB and the RMSE for all the parameters

**Conclusions from simulation study**

Figs. 6 to 11 summarize the findings of the simulation study. The mean of the estimated parameters, the average bias and the RMSE are determined. From the simulation graphs we can confidently say that the MLE estimates the unknown parameters effectively and the resulting parameters are close to the actual values when the sample size increases. Moreover, average bias and RMSE decrease with increase in sample size.

**Practical illustration using real life data**

This section examines real world data set for COVID-19 mortality rates to show the value and the flexibility of the EGGAPE distribution. The study compared the proposed distribution’s goodness-of-fit test results and criterion measures to those of its sub-models.

The motivation of the EGGAPE distribution is its usefulness in data analysis problems in various fields especially survival data analysis. Recently a number of distributions have been developed to model mortality rates of COVID-19. The CDFs of the competing models include:

1. The exponential(E) distribution

$$W(y, \lambda) = 1 - e^{-\lambda y}$$

(31)

2. Gull Alpha Power Exponential(GAPE)

$$W(y, \alpha, \lambda) = \frac{\alpha(1 - e^{-\lambda y})}{\alpha y e^{\lambda y}}$$

(32)
The shape of the TTT curve enables researchers to decide on the failure rate of the data, [30]. As seen in Fig. 12 the data has a decreasing failure rate that is recommended for using a modification of the exponential distribution.

For UK-COVID data it is platykurtic and right-skewed, as can be seen in Table 6.

The MLEs and standard errors are portrayed in Table 7. Most parameters of the fitted distributions were significant at 5% level since by the standard error test.

From Table 8, in comparison to the other models, the AIC, K–S, and W* values for the EGGAPE distribution are the lowest, while the log-likelihood is the highest. The EGGAPE distribution, according to formal goodness-of-fit tests, gives a better fit than its sub-models since its K–S, Anderson–Darling, and W* values are the least.

The results of the LRT test are displayed in Table 9. The results revealed that the EGGAPE is a better fit than its sub-models at 5% and 10% level of significance.

Fig. 13 shows the densities of the fitted distributions.

Data set 2: Kenya COVID-19 daily cases

Table 10 represents the daily number of cases of COVID-19 for Kenya for the time span of 56 days starting at 28 March 2020 and ending at 22 May 2020. Source of the data was https://covid19.who.int.

Table 11 displays the descriptive statistics for the Kenya daily COVID-19 data. From the descriptive statistic, the data is right skewed and platykurtic.

The TTT is a pictorial representation of the failure time. The shape of the TTT curve enables researchers to decide on the failure rate of the data. As seen in Fig. 14 the data has a modified bathtub failure rate that is recommended for using a modification of the exponential distribution.

The MLEs for the parameters and their standard errors are shown in Table 12. Parameters for most of the fitted distributions were significant at 5% level since by the standard error test, the values of the standard errors of the parameters are less than half the value of the parameter. As depicted in Table 13, in comparison to the other models, the AIC, K–S, and W* values for the EGGAPE distribution are the lowest, while the log-likelihood is the highest. The EGGAPE distribution, according to formal goodness-of-fit tests, gives a better fit than its sub-models since its K–S, Anderson–Darling, and W* values are the least.

The visual display for the estimated PDF of the competing models is presented in Fig. 15.

Future work

It should be noted that the proposed generator of distributions only looks at cases which were complete (uncensored). However, in real life scenarios, incomplete cases arise especially in real medical research. Therefore, the study recommends that for future works, censored data may be used to demonstrate the applicability of the developed models. In this study, the maximum likelihood estimation has been used to develop the estimators of the parameters, for further studies a comparison of the parameter estimation procedures; maximum likelihood estimation, method of moments and the maximum probability space can be considered. In many instances, covariates exists and they may have an effect on the outcome variable. Therefore, in this context parametric regression models for studying the relationship between the output and input variables may be constructed using the proposed distributions. Further, Bayesian estimation and prediction for the EGGAPE distribution using informative priors can be considered as an open area for future research.
Table 5
UK COVID-19 mortality rates data.

|             | 0.0587 | 0.0863 | 0.1165 | 0.1247 | 0.1277 | 0.1303 | 0.1652 | 0.2079 | 0.2395 | 0.2751 |
|-------------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|
|             | 0.2845 | 0.2992 | 0.3188 | 0.3217 | 0.3446 | 0.3553 | 0.3622 | 0.3926 | 0.3926 | 0.4110 |
|             | 0.4633 | 0.4690 | 0.4954 | 0.5139 | 0.5686 | 0.5837 | 0.6197 | 0.6365 | 0.7096 | 0.7193 |
|             | 0.7444 | 0.8590 | 1.0438 | 1.0602 | 1.1305 | 1.1468 | 1.1533 | 1.2260 | 1.2707 | 1.3423 |
|             | 1.4149 | 1.5709 | 1.6017 | 1.6083 | 1.6324 | 1.6998 | 1.8164 | 1.8392 | 1.8721 | 1.9844 |
|             | 2.1360 | 2.3987 | 2.4153 | 2.5225 | 2.7087 | 2.7946 | 3.3609 | 3.3715 | 3.7840 | 3.9042 |
|             | 4.1969 | 4.3451 | 4.4627 | 4.6477 | 5.3664 | 5.4500 | 5.7522 | 6.4241 | 7.0657 | 7.4456 |

Fig. 12. TTT plot, boxplot and histogram of the UK-COVID 19 data.

Fig. 13. Fitted densities plot for UK COVID-19 data set.
Fig. 14. TTT plot, boxplot and histogram of the Kenya-Covid 19 daily cases data.

Fig. 15. Fitted Densities plot Kenya daily cases for COVID-19.

Table 6
COVID-19 mortality rates in the United Kingdom summary statistics.

| Statistic | Minimum | Maximum | Mean | Std.dev | Median | CK | CS |
|-----------|---------|---------|------|---------|--------|----|----|
| Value     | 0.0587  | 11.458  | 2.437| 2.936   | 1.957  | 1.669|

Conclusions

The EGGAP family of distributions has been developed and studies. The expressions for the basic statistical properties including the probability distribution function, hazard rate function, survival function, order statistics and the quantile have been derived successfully. A special case of the family called EGGAPE distribution has been explored and the estimates of the parameters of the sub-family have been obtained through the maximum likelihood method. The AB and the RMSE were used to evaluate the performance of the estimators of the new

Table 7
The estimates and SEs for first data set.

| Model  | $\hat{\alpha}$ | $\hat{\beta}$ | $\hat{\alpha}$ | $\hat{\lambda}$ |
|--------|-----------------|----------------|-----------------|-----------------|
| EGGAPE | 0.141(0.020)    | 1.220(0.191)   | 2.660(0.025)    | 2.300(0.031)    |
| EGAPE  | $-$             | 0.939(0.156)   | 1.739(0.502)    | 0.282(0.079)    |
| GAPE   | $-$             | $-$            | 0.291(0.073)    | 1.781(0.442)    |
| GEG    | 0.378(0.732)    | 0.934(1.807)   | $-$             | 0.805(0.118)    |
| EEE    | $-$             | 0.353(0.056)   | $-$             | 0.800(0.116)    |
| E      | $-$             | $-$            | $-$             | 0.410(0.047)    |
distribution. We can conclude that the MLE estimates are consistent and both the AB and the RMSE decrease with increase in sample size. It has been shown that the new distribution provides a better fit than that of its sub models in data that is characterized by monotone and non-monotone hazard rates through application to real data.

**CRediT authorship contribution statement**

**Mutua Kilai:** Supervision, Validation, Visualization, Writing – original draft, Writing – review editing. **Gichuhi A. Waititu:** Supervision, Validation, Visualization, Writing – original draft, Writing – review editing. **Wanjoyia A. Kibira:** Supervision, Validation, Visualization, Writing – original draft, Writing – review editing. **Huda M. Alshambari:** Supervision, Validation, Visualization, Writing – original draft, Writing – review editing.

**Declaration of competing interest**

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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