Design of LQR Excitation Controller of Synchronous Condenser in HVDC System

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Abstract. As a kind of reactive power compensation equipment, a synchronous condenser is widely used in High Voltage Direct Current (HVDC) transmission systems due to its small influence on the regulation ability and high strong excitation ability. In this paper, the practical state space model of the synchronous condenser is established and the parameters of the model are calculated. The linear quadratic regulator (LQR) controller for excitation system is designed and the controllability and stability are analyzed. In order to verify the voltage support ability and dynamic response performance of LQR excitation controller, the simulation is carried out in MATLAB/SIMULINK. The simulation results show that the grid voltage fluctuation can be quickly quelled and the strong excitation function of synchronous condenser is quickly exerted.

1. Introduction

The energy and load are distributed in reverse to a certain extent in China. The optimal allocation of energy in a large range can be realized by the long-distance HVDC transmission which has become an important component of the power grid on account of its low cost, good interconnection and simple control[1]. With the continuous improvement of transmission power, the characteristics of "strong DC and weak AC" are further highlighted, and the impact of DC power grid on weak AC power grid of the receiving converter station is becoming increasingly more and more serious. The part of the reactive power is needed to be consumed during the operation of the converter station, and the voltage instability will be caused by the insufficient reactive power reserves of the receiving converter station. What’s more, in case of DC commutation failure, the problem of voltage stability will become more prominent. In principle, large-scale dynamic reactive power compensation devices must be matched in HVDC power transmission. There are many reactive power compensation methods based on power electronics technology, such as static var compensator (SVC) and static synchronous compensator (STATCOM) etc[2]. Compared with these methods, synchronous condenser is widely used in HVDC transmission system because of its small influence on the regulation ability of the system and high strong excitation ability. When the grid system fault occurs, the short-circuit capacity for the system as dynamic reactive power compensation is provided. As a dynamic reactive power compensation device, it is an effective scheme to solve the reactive power compensation.

The excitation system of synchronous condenser is the key of its strong support. In[3], the existing excitation control strategies of condenser are divided into two categories: generator terminal voltage
regulation control and grid voltage regulation control. In[4], a coordinated reactive power and voltage control strategy is proposed aiming at solving the lack of reactive power support ability of terminal voltage regulation control strategy. In[5], the modeling process of the synchronous generator can be referred to for the synchronous condenser since it can be used as a synchronous generator without load. In[6], the linear quadratic optimal control method to design the synchronous generator excitation control system is adopted. However, the linear quadratic optimal control method is rarely used in the excitation control system of synchronous condenser.

The practical state space model of the synchronous condenser, and the parameters of the model are calculated in this paper. Then the Linear Quadratic Regulator(LQR) controller for the excitation system of the condenser is designed based on the multi-variable excitation control mode, and the controllability and stability are analyzed. Combined with VSC-HVDC(Voltage Sourced converters) system, the simulation experiment of synchronous condenser is carried out. The simulation results show that the grid voltage fluctuation can be quickly quelled, the strong excitation function of synchronous condenser is quickly exerted and the strong support and fast stability of the weak AC system voltage are realized.

2. Practical State Space Model of Synchronous Condenser

2.1. Basic Equation

For a single-machine infinity system, the motion equation of the condenser rotor is as follows[7]:

\[ \dot{\delta} = \omega - \omega_0 \text{ or } \Delta \dot{\delta} = \Delta \omega \]  (1)

\[ \dot{\omega} = -\frac{D}{H} (\omega - \omega_0) - \frac{\alpha_0}{H} (P_e - P_m) \]  (2)

The electromagnetic dynamic equation is:

\[ \dot{E}_q' = \frac{1}{T_{\delta_0}} (V_i - E_q) \]  (3)

The non-salient pole structure is adopted by the large-scale condenser, so the equation is[8]:

\[ P_e = \frac{E_q V_s}{x_{ds}} \sin \delta + \frac{V_s^2}{2 x_{ds} x_{qs}} \sin 2 \delta \]  (4)

Where, \( P_e = f_1(E_q, \delta) = f_1(E_q', \delta) \). From the total differential equation[9]:

\[ \Delta P_e = \frac{\partial f}{\partial E_q} \Delta E_q + \frac{\partial f}{\partial \delta} \Delta \delta \]  (5)

The higher-order terms are ignored to get:

\[ \Delta P_e = S_q \Delta \delta + R_q \Delta E_q \text{ or } \Delta P_e = S_q' \Delta \delta + R_q' \Delta E_q' \]  (6)

Where, \( \delta \) is used the concept of generator as power angle, \( \omega \) is the rotor angular velocity, \( \omega_0 \) is the synchronous speed, \( D \) is the damping coefficient, \( H \) is inertial time constant, \( P_e \) is the electromagnetic power, \( P_m \) is the mechanical power. The condenser operates without load, \( P_m = 0 \). \( E_q \) is the no-load electromotive force, \( E_q' \) is the transient electromotive force, \( T_{\delta_0} \) is the time constant of the field winding.
when the stator winding is open, $V_f$ is the generator excitation voltage, $V_s$ is the bus voltage of HVDC transmission system.

\[ R_e = \frac{V}{x_{de}} \sin \delta \]  
\[ R'_e = \frac{V}{x_{de}} \sin \delta \]  
\[ \frac{S'_e}{T_{do}} = \frac{x_{de}}{x_{de}} \]  

For non-salient pole machine:

\[ S_e = \frac{E_s V_e}{x_{de}} \cos \delta \]  
\[ S'_e = \frac{E'_s V_e}{x_{de}} \cos \delta + V'_s \frac{x'_{de} - x_{de}}{x'_{de} x_{de}} \cos 2\delta \]  

In the above equation (10) and (11):

\[ x_{de} = x_d + x_s \]
\[ x'_{de} = x'_d + x_s \]
\[ x_{qe} = x_q + x_s \]
\[ x_s = x_s + x_l \]  

Where, $x_d$ is the d-axis reactance, $x_q$ is the q-axis reactance, $x'_d$ is the d-axis transient reactance, $x_s$ is reactance of main transformer, $x_l$ is the line reactance.

2.2. Practical State Space Model

In order to obtain a more practical state-space model and to facilitate the design of LQR, the equation (2) is transformed into a variable form as shown in equation (13):

\[ \Delta \dot{\omega} = \frac{\omega_0}{H} \Delta \dot{\omega} + \frac{D}{H} \Delta \omega \]  

The equation (5) is substituted into (13):

\[ \Delta \dot{\omega} = -\frac{\omega_0}{H} S'_e \Delta \delta - \frac{\omega_0}{H} R'_e \Delta E'_q - \frac{D}{H} \Delta \omega \]  

The differential equation of $E'_q$ variation is shown in equation (15):

\[ \Delta \dot{E}'_q = \frac{S_s - S'_e}{T_{do} R_e} \Delta \delta - \frac{1}{T'_e} \Delta E'_q + \frac{1}{T_{do}} \Delta E'_f \]  

Under the condition that the armature reaction is ignored, the electromagnetic inertia increment equation can be described by equation (16):

\[ \Delta E'_f = \frac{1}{T_{es} + 1} \Delta V_R \]  

The equation (16) is converted into time domain:
From equations (1), (14), (15) and (17), the state space equation of the system can be obtained as follows:

\[
\Delta \dot{E}_t = -\frac{1}{T_e} \Delta E_t + \frac{1}{T_e} \Delta V_g
\]  

(17)

The following problems exist in the actual use of the state variables in equation (18):
1. \(\delta\) is not very difficult to measure, but it is often replaced by \(\Delta P_e\) in practice;
2. \(\Delta E'\) is the variation of transient electromotive force of excitation winding, and its value is not easy to measure. In order to collect the value of state variable in practice, the terminal voltage deviation \(\Delta V_s\) of synchronous condenser is introduced.

The practical state space model of the condenser can be obtained by the linear transformation of the state variables:

\[
X = [\Delta \delta \; \Delta \omega \; \Delta E'_q \; \Delta E_t]^T, \quad A = \begin{bmatrix}
0 & 1 & 0 & 0 \\
\frac{\alpha_0}{H} S'_E & -\frac{D}{H} & -\frac{\alpha_0}{H} R'_E & 0 \\
\frac{S_E - S'_E}{T_{d0} R_E} & 0 & -\frac{1}{T_d'} & \frac{1}{T_{d0}} \\
0 & 0 & 0 & -\frac{1}{T_e}
\end{bmatrix}, \quad B = \begin{bmatrix}
0 \\
0 \\
0 \\
\frac{1}{T_e}
\end{bmatrix}, \quad U = \Delta V_g
\]

The general form of state space is written:

\[
\dot{X} = AX + BU
\]

(19)

The practical model of the condenser can be obtained, the \(\vec{X}=[\Delta P_e \; \Delta \omega \; \Delta V_s \; \Delta E_t]^T\)

\[
X = \begin{bmatrix}
\frac{1}{S_N} & 0 & -\frac{R_v}{S_N} & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{bmatrix}, \quad \vec{X} = \begin{bmatrix}
\frac{1}{S_N} & 0 & -\frac{R_v}{S_N} \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{bmatrix}, \quad \vec{A} = P^{-1} A P, \quad \vec{B} = P^{-1} B, \quad P = \begin{bmatrix}
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{bmatrix}

For non-salient pole machine:
\[
S_v = S_e - R_v \frac{\partial V_i}{\partial \delta} \\
R_v = \frac{R_e}{\partial V_i / \partial E_q}
\]

\[
\frac{\partial V_i}{\partial \delta} = -\frac{1}{x_{d\Sigma}} x_d x_{d} E_q V_i \sin \delta (E_q^2 x_i^2 + V_i^2 x_i^2 + 2 x_d x_i E_q V_i \cos \delta)^{\frac{1}{2}}
\]

\[
\frac{\partial V_i}{\partial E_q} = \frac{1}{x_{d\Sigma}} (x_i^2 E_q + x_d x_i E_q V_i \cos \delta) (E_q^2 x_i^2 + V_i^2 x_i^2 + 2 x_d x_i E_q V_i \cos \delta)^{\frac{1}{2}}
\]

A self shunt excitation system is adopted in the excitation system in this paper, so the time constant of the exciter \( T_e \approx 0 \). The control quantity is transformed into the electric potential change of the excitation winding of the condenser \( \Delta E_i \), The equation of the system is transformed into a three order form, and the practical equation of state is obtained, as shown in equation (21).

\[
\begin{bmatrix}
\Delta P_e \\
\Delta \omega \\
\Delta V_i
\end{bmatrix} =
\begin{bmatrix}
S_e - S_v \\
-T_e S_v \\
T_d R_v S_v
\end{bmatrix}
\begin{bmatrix}
\Delta P_e \\
\Delta \omega \\
\Delta V_i
\end{bmatrix} +
\begin{bmatrix}
R_e \\
0 \\
0
\end{bmatrix}
\frac{T_{d\alpha}}{T_{d\alpha} R_v} \Delta E_i
\]

\[
(21)
\]

3. LQR Controller Design

The state space model of the condenser system is established according to equation (21):

\[
X = [\Delta P_e \ \Delta \omega \ \Delta V_i]^T
\]

\[
U = \Delta E_i
\]

\[
J = \frac{1}{2} \int_0^t [x^T Q x + u^T R u] dt
\]

The purpose of LQR design of the condenser is to design the optimal excitation control system, so that the electromagnetic power deviation \( \Delta P_e \), rotor speed deviation \( \Delta \omega \) and voltage deviation \( \Delta V_i \) of the condenser terminal can be adjusted to zero state, and the square sum integral to time can be minimized, and the control energy is appropriate.

The design steps of LQR are as follows:
1. Judging the controllability of the system by rank criterion
2. Selecting the appropriate weight matrix
3. The Raccati equation is solved by MATLAB to judge whether \( P \) is positive definite, and the system is stable if \( P \) is positive definite
4. Solving the state feedback matrix \( K = R^{-1} B^T P \)
5. Constructing the control law by the deviation

4. Simulation analysis

4.1. System Simulation Model and Parameters

In order to verify the LQR performance of the condenser and the voltage support function of the condenser when the system voltage drops or short circuit occurs, the condenser is added in the
The background of VSC-HVDC System in MATLAB/SIMULINK, as shown in Figure 1. The whole simulation system includes HVDC system, 300Mvar synchronous condenser, transformer and adjustable voltage source. The static self shunt excitation mode is adopted, and the main parameters of the simulation model are as follows:

The synchronous condenser parameters:

\[ P_n = 300 \text{Mvar}, V_n = 20 \text{kV}, f_n = 50 \text{Hz}, T_{do}^* = 9.41 \text{s}, T_{do}^- = 0.05 \text{s}, T_{dq}^* = 0.83 \text{s}, T_{dq}^- = 0.096 \text{s} \]

\[ X_d = 1.96 \text{pu}, X_d' = 0.229 \text{pu}, X_d'' = 0.1856 \text{pu}, X_q = 1.79 \text{pu}, X_q' = 0.378 \text{pu}, X_q'' = 0.234 \text{pu}, \]

\[ X_i = 0.0023 \text{pu}, H = 1.49 \text{s}, D = 0, \text{non-salient pole machine}. \]

The main transformer parameters:

\[ P_n = 360 \text{Mw}, f_n = 50 \text{Hz}, \] transformation ratio: 20kv/230kv, Delta-Yg, grounded, \( R_1 = R_2 = 0.05 \text{pu}, L_1 = L_2 = 0.02 \text{pu}. \]

![Figure 1. The system simulation model.](image)

4.2. Establishment of LQR Controller for Condenser

According to the practical state space model and parameters of the condenser, the system matrix \( A \) and input matrix \( B \) are calculated:

\[
\begin{bmatrix}
-0.08 & 1.675 & -2.368 \\
-0.671 & 0 & 0 \\
-0.001 & 0.012 & -0.026
\end{bmatrix}
\begin{bmatrix}
0.096 \\
0 \\
0.001
\end{bmatrix}
\]

Controllability discriminant matrix \( Q_c \):

\[
Q_c \equiv \begin{bmatrix} A^2B & AB & B \end{bmatrix} =
\begin{bmatrix}
0.096 & -0.01 & -0.1068 \\
0 & -0.0644 & 0.0067 \\
0.001 & 0.0001 & 0.0008
\end{bmatrix}
\]

The rank of \( Q_c \) is 3, and the system is fully controllable. Then for the selected weighting matrices \( Q \) and \( R \). There is a unique solution to the Raccati equation. The lqr function is used in MATLAB (selecting \( R = 1, Q = \text{diag}[1 \ 100 \ 1000] \)) . The feedback gain matrix \( K = [5.149 \ -2.7497 \ 13.3242] \).

The matrix \( P \) is:
The eigenvalues of symmetric matrix $P$ are all greater than 0, it can be determined that $P$ is a positive definite matrix and the closed-loop system is asymptotically stable.

The LQR control law is:

$$U = \Delta E_f$$

$$= -Kx$$

$$= -5.149\Delta P_e + 2.7497\Delta \omega - 13.3242\Delta V_f$$  \hspace{1cm} (22)$$

4.3. Simulation Results and Analysis

According to the LQR controller, as shown in Figure 1, the stator d-axis voltage $V_d$, q-axis voltage $V_q$, rotor angular velocity $\omega$ and electromagnetic power $P_e$ are led out from the condenser terminal. The dynamic deviation is used as the feedback quantity for each control quantity, and the variation is realized by engineering differential. The calculated feedback gain $K$ is brought into the simulation system to complete the design of LQR excitation controller and packaged into a subsystem. The LQR excitation control is shown in Figure 2.

![Figure 2. LQR excitation control system.](image)

For verifying the voltage support ability and dynamic response performance of LQR excitation controller of condenser, three-phase short circuit fault is set when VSC-HVDC System runs to 1.35s and lasts for 0.25s. As is shown in Figure 3, no_sc means that the voltage change of AC system at receiving converter station without the condenser, sc means it with condenser and LQR_sc means that adjusting the LQR excitation controller that is shown in Figure 2, and the reactive power output of LQR excitation controller of condenser is shown in Figure 4.
As shown in Figure 3, it takes 0.25 s for LQR based excitation controller to recover the expected value from the maximum overshoot, and it takes 3s to install a condenser without the condenser. The LQR excitation controller is superior to the original system in suppressing system disturbance and the voltage support can be quickly provided. The LQR system provides ideal artificial damping, greatly improves the ability of suppressing oscillation of HVDC system, and has better system regulation ability.

As shown in Figure 4, the reactive power in case of system short circuit fault can be quickly provided by the LQR excitation controller, so that the over excitation also can be quickly realized, and the over excitation can reach 4.5 times. That the condenser can support the grid voltage obviously under the action of LQR excitation controller is shown in the system.

5. Conclusion
The state space model of the large-scale condenser is established, and the practical model of the condenser is also established through the linear transformation of the state variables in this paper. The design steps of the LQR excitation controller for the synchronous condenser are described. Based on this, the LQR excitation controller for the synchronous condenser is designed and verified by the simulation experiments under the background of HVDC transmission. The simulation results show that the reactive power is quickly provided, and the over excitation is also fast realized based on the LQR excitation controller. The voltage fluctuation of weak AC system is quelled and the strong
support can be provided for the grid voltage and even the power system. A reference for the excitation optimal control of large-scale synchronous condenser is carried out in this paper.

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