Large phase shift of spatial soliton in lead glass by cross-phase modulation in pump-signal geometry

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We investigate the large phase shifts of the bi-color spatial soliton pair in a cylindrical lead glass rod. The theoretical study suggests a synchronous propagation of a strong pump beam and a weak signal beam under the required initial condition. We experimentally obtain a π-phase shift of the signal beam by changing the power of the pump beam by about 14 mW around the soliton critical power, which agrees qualitatively with our theoretical result. The ratio of the phase shift rate of the signal soliton to that of the pump soliton shows a close match to the theoretical estimation.

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I. INTRODUCTION

In possession of infinite nonlocality in nature, lead glass is one of the most promising candidate to serve as the propagation media for the strongly nonlocal spatial optical soliton (SNSOS) predicted by Snyder and Mitchell [1]. High-order solitons [2], vortex solitons CR-pRL-05, surface solitons [4] have been realized in lead glass since nonlocal nonlinear response can suppress soliton transverse instabilities; long-range interaction between solitons [3] as well as soliton and boundaries [6, 7] have also been carried out in lead glass. Recently, the large phase shift of nonlocal solitons in lead glass along with the propagation distance and the soliton power was investigated by Shou et al [8]. This is the first theoretically and experimentally study on the nonlocal soliton phase shift or phase modulation itself which is often regarded as a parameter or causation in the study of the soliton interaction.

The theoretical prototype of Shou’s work was addressed by Guo et al [9, 10]. Based on the phenomenological Gaussian response function, Guo et al. predicted a large phase shift in nonlocal media which is \((w_m/w_0)^2\) times larger than that of the local counterpart, where \(w_m\) and \(w_0\) are the characteristic length of the response function and the beam width, respectively. In actual strongly nonlocal media, lead glass, Shou predicted the times is much smaller, but the phase shift rate is still more than one order larger than the result for local solitons. They conducted the experiment in lead glass to obtain a π-phase shift by changing the soliton power around its critical power [3].

However, Shou’s work is not practical in real optical fiber communication system. Firstly, it is not usual to obtain phase shift of the carrier wave by mean of self-phase-modulation. Secondly, the soliton critical power in lead glass is much stronger than the power of the carrier wave. In this paper a π-phase shift of the signal SNSOS is obtained by adjusting the pump SNSOS power with the aid of the cross modulation between the SNSOSs. Although the power of the signal soliton is much smaller than its critical power, trapping in the pump-soliton-induced waveguide, its phase shift can be sensitively and linearly modulated by the power of the pump soliton. The ratio of the phase shift of the signal beam to that of the pump beam is determined by the ratio of their wavelengths.

II. THE SOLUTION OF THE COUPLED EQUATIONS

The system we study is described by the nonlocal non-linear Schrodinger equations for the two slowly varying light fields amplitude \(A_p\) and \(A_s\) coupled to the steady-state heat transfer equation. In cylindrical coordinate for z-axis symmetry, the three coupled equations can be expressed [11]:

\[
2i k_p \frac{\partial A_p}{\partial Z} + \frac{1}{R} \frac{\partial}{\partial R} \left( R \frac{\partial A_p}{\partial R} \right) + 2 k_p^2 \frac{\Delta N_p}{n_{op}} A_p = 0, \quad (1a)
\]

\[
2i k_s \frac{\partial A_s}{\partial Z} + \frac{1}{R} \frac{\partial}{\partial R} \left( R \frac{\partial A_s}{\partial R} \right) + 2 k_s^2 \frac{\Delta N_s}{n_{os}} A_s = 0, \quad (1b)
\]

\[
\frac{1}{R} \frac{\partial}{\partial R} \left( R \frac{\partial T}{\partial R} \right) = - \frac{1}{\kappa} (\alpha_p I_p + \alpha_s I_s), \quad (1c)
\]

where \(I_{p,s} = |A_{p,s}|^2\) is the light intensity for pump and signal beams respectively. A slight portion of the light energy is absorbed by the glass with the absorption coefficient \(\alpha_{p,s}\) and is conducted transversely to the boundary of the glass rod with the thermal conductivity \(\kappa\). \(\Delta N_{p,s}\) is the light induced refractive index which is proportional to the change of temperature

\[
\Delta N_{p,s} = \beta_{p,s} (T - T_0), \quad (2)
\]
where \( T_0 \) is the fixed temperature at the boundary and \( \beta_{p,s} \) is the thermal-optical coefficient.

In the case of \( I_s \ll I_p \), we only consider the light energy contributing from the pump beam. Simplifying Eq. (1) in a dimensionless form and using Eq. (2), we can obtain,

\[
2i \frac{\partial a_p}{\partial z} + \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial a_p}{\partial r} \right) + 4\Delta n a_p = 0, \quad (3a)
\]

\[
2i \frac{\partial a_s}{\partial z} + \frac{1}{\mu r} \frac{\partial}{\partial r} \left( r \frac{\partial a_s}{\partial r} \right) + 4\mu \Delta n a_s = 0, \quad (3b)
\]

\[
\frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial \Delta n}{\partial r} \right) = -|a_p|^2, \quad (3c)
\]

where \( r = R/w_0, z = Z/(2k_p w_0^2) \), \( \Delta n = k_p^2 w_0^2 \Delta N_p/\left(2n_{op}\right) \), \( a_{p,s} = A_{p,s}/A_0 \), \( A_0^2 = 2n_{op}/(\alpha_p \beta_p k_p^2 w_0^2) \), \( \mu = k_p^2/k_p = \lambda_p/\lambda_s \), and \( \rho = n_{op}/\beta_p/\rho_{ns}/\beta_{p,s} \). It is indicated by Eq. (3a) and Eq. (3b) that the signal beam propagates in a pump-beam-induced index distribution, say a waveguide. Therefore the propagation behavior of the signal beam is totally determined by that of the pump beam.

According to the Snyder’s method the nonlinear index can be expanded in a Taylor series and only the first two terms are kept. This indicates the light-induced index acts as a parabolic waveguide in the region of the pump beam center.

\[
\Delta n = \Delta n^{(0)} - r^2 \Delta n^{(2)}. \quad (4)
\]

Assuming the solutions of Eq. (1) are in the forms of Gaussian functions,

\[
a_p = \frac{\sqrt{p_{op}}}{\sqrt{\pi} w_p(z)} \exp \left[ -\frac{r^2}{2w_p^2(z)} + ic_p(z)r^2 \right], \quad (5a)
\]

\[
a_s = \frac{\sqrt{p_{os}}}{\sqrt{\pi} w_s(z)} \exp \left[ -\frac{r^2}{2w_s^2(z)} + ic_s(z)r^2 \right], \quad (5b)
\]

where \( p_{op,os} = \int |a_p(s(x',x, y'))|^2 dx' dy' \) is the normalized light power. By integrating Eq. (5a) twice \( s \), we obtain \( \Delta n^{(0)} = p_{op}/(4\pi) \{ \Gamma[0, r_0^2/w^2(z)] + \ln[r_0^2/w^2(z)] \} \}, \) where \( \Gamma \) is the Gamma function, \( \gamma = 0.58 \) is Euler’s constant, \( r_0 \) is the normalized radius of the cross section of the glass rod.

\( \theta_{p,s}(z) \) in Eq. (5a) and (5b) is phase shift of the pump beam and the signal beam, respectively. We can directly obtain the phase shift associated with \( \Delta n^{(0)} \), called the zero-order phase shift to be

\[
\theta^{(0)}_{p,s}(z) = n^{(0)}_p z, \quad \mu \theta^{(0)}_{p,s}.
\]

It is obvious that the zero-order phase shifts of the pump and signal beams are proportional to the pump power. The total phase shift can be rewritten as, according to the two terms of the nonlinear index designating in Eq. (4),

\[
\theta_{p,s}(z) = \theta^{(0)}_{p,s}(z) + \theta^{(2)}_{p,s}(z).
\]

Following the method in Ref. [9], by substituting Eq. (4)-(7) into Eq. (3), we can obtain the evolution equations of the beam widths and the second-order phase shifts of pump and signal beams,

\[
\frac{d^2 w_p}{dz^2} = \frac{4}{w_p^3} - \frac{4p_{op}}{\pi w_p^2}, \quad (8a)
\]

\[
\frac{d^2 w_s}{dz^2} = \frac{4}{\mu w_s^3} - \frac{4p_{op}w_s}{\pi w_p^2}, \quad (8b)
\]

\[
\frac{d\theta^{(2)}_p}{dz} + \frac{2}{\mu w_p^2} = 0, \quad (8c)
\]

\[
\frac{d\theta^{(2)}_s}{dz} + \frac{2}{\mu w_s^2} = 0. \quad (8d)
\]

When there is not much difference between the wavelengths of the two beams, \( \rho \approx 1 \). Thereby Eq. (8a) become

\[
\frac{d^2 w_s}{dz^2} = \frac{4}{\mu w_s^3} - \frac{4p_{op}w_s}{\pi w_p^2}. \quad (9)
\]

Compare Eq. (8a) with Eq. (9), replace \( w_s \) with \( 1/\sqrt{\mu}w_p \), these two equations turn to be equivalent mathematically. In such case \( \theta^{(2)}_s(z) \) and \( \theta^{(2)}_p(z) \) have the same evolution equations revealed in Eq. (8c) and Eq. (8d).

Satisfying the initial condition \( w_{op} = (1/\sqrt{\mu})w_{op} \), we can achieve the synchronous propagations of the pump beam and the signal beam.

The critical power \( p_c = \pi \) for the pump beam to propagate in the form of soliton can be easily obtained based on Eq. (8a) supposing \( w_p(z) = 1 \). The signal beam also forms a soliton in the "soliton-waveguide", though its power is far less than its own critical power. The beam widths and the second-order phase shifts of pump and signal beams are of the forms

\[
w_s(z) = \frac{1}{\sqrt{\mu}}w_p(z) = \frac{1}{\sqrt{\mu}}[\sigma + (1 - \sigma) \cos(bz)], \quad (10)
\]

\[
\theta^{(2)}_s(z) = -\frac{2}{2\sigma - 1} \left\{ \frac{(1 - \sigma) \sin(bz)}{\sigma + (1 - \sigma) \cos(bz)} - \frac{2\sigma}{\sqrt{2\sigma - 1}} \left[ \arctan\left( \sqrt{2\sigma - 1} \tan \frac{bz}{\sqrt{2\sigma - 1}} \right) + \pi \right] \right\}, \quad (11)
\]

where \( \sigma = \sqrt{p_c/p_{op}}, b = 2\sqrt{2}/\sigma^2 \). Fig. 4 show comparison between the analytical results and numerical results, while the numerical results is obtained by simulating Eq. (5a) and (5c). The disagreement between the
analytical result and the numerical result is attributed to the limited expanding of the nonlinear index. Ma et al. also numerically obtained a bigger change rate of the propagating constant with power than the analytical result when they investigated the surface soliton in lead glass [12].

![Graph](image1.png)

**FIG. 1:** Phase shifts of pump beam and signal beam versus $p_0/p_c$. Solid thick line and thin line are respectively analytical results of pump and signal beam in case of $r_0 = 200$ and $z = 2$, which accords to the following experiment situation. Triangle and squares are respectively numerical results of pump and signal beam in the same case.

### III. EXPERIMENT IN LEAD GLASS

We carry out the experiment in a lead glass rod using the Mach-Zehnder interferometer technique to measure the phase shift of the signal beam modulated by the power of the pump beam. The glass rod has a radius of 7.5 mm and length of 60 mm. Its absorption coefficient changes strongly with wavelength which is of the value of 0.07 cm$^{-1}$ at the pump wavelength 532 nm and 0.03 cm$^{-1}$ at signal wavelength 790 nm. The sketch map of the experimental setup is detailed in Fig. 2.

A small portion of energy launched from a coherent Verdi V12 laser contributes to act as the pump beam of orthogonal polarization. The most portion of energy of Verdi V12 serves as the pump source for a tunable, single-frequency ring Ti:Sapphire laser MBR 110 which produces the signal beam of parallel polarization. The pump and signal beams are focused by $L_1$ and $L_2$ with different beam widths of 75µm and 90µm, respectively, but the same positions of the focal points. $B_3$ is a polarizing cube beamsplitter which allows signal beam transmits and pump beam reflects into the Mach-Zehnder interferometer, respectively. The pump beam is absorbed totally by a filter on one arm after coaxially propagating through the lead glass with the signal beam. On the other arm the pump beam is absorbed by another filter just after being reflected by the non-polarizing beamsplitter B2. The beam spot at the output side of the glass rod is imaging by lens $F_3$ onto the CCD where it interferes with a large-diameter beam spot acting as a phase reference. The interference fringes move along with the change of the pump power indicates the phase shift of the signal beam modulated by the pump power. The phase shift of the pump power modulated by the power of its own can be obtained in the same way but after removing $F_1$ and $F_2$ and blocking the signal beam. The rest detail of the experimental setup can be found in Ref. [8].

![Diagram](image2.png)

**FIG. 2:** Experiment setup. TA is tunable attenuator; $B_1$ is a polarizing cube beamsplitter; $B_2$ and $B_3$ are non-polarizing cube beamsplitters; $F$ is beam filter; $L_1$, $L_2$, $L_3$, $L_4$ are lenses.

![Graph](image3.png)

**FIG. 3:** Experimental phase shift of pump beam and signal beam versus $P_0/P_c$. Pump beam wavelength is 532nm, and signal beam wavelength 790nm; critical power $P_c$ is measured to be 260 mW. Triangle and squares are, respectively, the experimental data of pump and signal beams. The solid curves are the linear fits.

By tuning the input power of pump beam from 190mW to 340mW, we obtain the phase shifts of the the pump beam or the signal beam as function of the input power normalized by the critical power in lead glasses. Ref. [8] gives a detailed demonstration how to obtain the phase shift from the interference fringes captured by CCD. As shown in Fig 4 linear fits of the data is good, while the
slopes of the two lines are $90.7 \,(rad/P_c)$ and $61.5 \,(rad/P_c)$.
Due to some reasons mentioned in Ref. [8], phase shift of the two beams in our experimental is bigger than the numerical and analytical results, in other words, the modulation sensitivity suggested by the experimental results is about twice that predicted by the theoretical curves in Fig.[1]. However, The ratio of the two slopes in the experiment is 1.47, which is in very good agreement with the numerical result. This means that the ratio of the two slopes is only determined by the ratio of their wavelength.

Fig. 4 shows the phase shifts of the soliton pair when the beam with wavelength of 790nm serves as the pump soliton, while beam with wavelength of 532nm serves as signal soliton. We do not know the critical power of the pump soliton because the absorption coefficient $\alpha$ at 780nm is so small that the critical power for soliton propagation is too higher to be obtained. Therefore the changing power in Fig. 4 is not normalized by critical power. The ratio between the slopes of the two fit lines is 1.45, which also approaches to the numerical result.

**IV. CONCLUSION**

In conclusion, we investigate the large phase shifts of the bi-color spatial soliton pair in lead glass. A $1.5\pi$-phase shift of the pump soliton and a $\pi$-phase shift of the signal soliton are simultaneously obtained by tuning the power of the pump beam around its critical power for about 14 mW via self-phase-modulation and cross-phase-modulation, respectively. The linear modulation result is much sensitive than the theoretical prediction, while the ratio of the phase shift rate of the signal beam to that of the pump beam agrees quantitatively with the result for the thermal theoretical model. This suggests that we can obtain bigger phase shift rate by choosing pump beam with longer wavelength. Effectively realizing the $\pi$-phase shift is important for processing and controlling an optical signal based on the principle of interference.

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