Profile of mathematical representation ability of junior high school students in Indonesia

B Y G Putra¹, N T Rosita² and W Hidayat³
¹Universitas Pasundan, Jl. Tamansari No. 6-8, Bandung 40117, Indonesia
²Sekolah Tinggi Keguruan dan Ilmu Pendidikan Sebelas April, Jl. Angkrek Situ No. 19, Sumedang 45323, Indonesia
³Institut Keguruan dan Ilmu Pendidikan Siliwangi, Jl. Terusan Jenderal Sudirman, Cimahi 40526, Indonesia

Email: pyusepa.fkip.pmat@unpas.ac.id

Abstract. This study aims to determine students' mathematical representation abilities. The research uses descriptive qualitative method. This ability is addressed after the students have answered five validated mathematical representation problems. The research subjects are 35 students of class VIII in one of State Junior High Schools in Bandung, West Java, Indonesia. The state junior high school is included in the medium level category according to the achievement of the National Final Examination score. The results of this study indicate that students still have difficulty in drawing the graph to solve Two-Variable Linear Equations, students have difficulty in making equations or mathematical expression and students have difficulty in solving and drawing conclusion from the problems of Two-Variable Linear Equation System in text.

1. Introduction
The process of learning mathematics that optimizes all of the students' abilities in the learning process becomes the attention of the world of mathematics education nowadays. Mathematical representation ability is one of the abilities a junior high school student must have [1]. The importance of mathematical representation ability is defined by NCTM which stated that students could make connections, develop, and deepen their understanding of mathematical concepts by using various representations. Representations such as physical objects, images, diagrams, graphics, and symbols also help students communicate their thoughts [2].

The facts in field show that students' mathematical representation ability tends to be lacking [3,4]. The lack of mathematical representation ability can be caused by various factors, from teaching materials, ways of learning, the students, teaching technique, and other supporting factors in the learning process. There is a problem in the delivery of mathematics learning material, such as the lack of development of students' representation ability, especially in junior high school students [5]. The teacher as a facilitator must be able to develop students’ abilities in the learning process so that students can interpret every material being studied including the development of students' mathematical representational ability.

Based on the elaboration that has been stated, it is necessary to have an in-depth analysis of students’ mathematical representation abilities. The analysis is in an effort to reveal the weakness or inaccuracies of students in answering the problems of mathematical representation ability. In addition, it will also reveal the other abilities that students have mastered in working on mathematical representation problems.
2. Method

The research uses descriptive qualitative method. This study aims to determine students’ mathematical representation ability. This ability can be known by describing the results of students’ works in answering problems about the mathematical representation ability which have been given in the Two-Variable Linear Equation System. This research consists of three main stages, namely: preparation (making the problems of mathematical representation ability according to indicators and have been validated), implementation (selecting research subjects and providing tests), and analysis (analysis of students’ answers and drawing conclusions). The instruments in this study are as follows (Table 1).

| Problem Number | Problem                                                                 |
|----------------|-------------------------------------------------------------------------|
| 1              | Find a solution to the system of the equations below using graphical method.  |
|                | \( \begin{align*}
|                | & 2x - y = 6 \\
|                | & 3x + y = 4
|                | \end{align*} \) |
| 2              | Here is a system of two-variable linear equations: \( \begin{align*}
|                | & 3x + 2y = 9000 \\
|                | & 2x + 4y = 10000
|                | \end{align*} \)  
|                | a. Write down a daily life problem using the system of equations above and propose a question related to the problem you created and make sure the question can be answered along while finding the solution to the system of the equation.  
|                | b. Find the solution to the system of the equations from the question you created in part a. |
| 3              | In the cartesian coordinates, there are 2 lines that pass certain points. the first line passes through the coordinates of points \( (5, 0) \) and \( (0, 5) \), while the second line passes through the coordinates of points \( (1, 0) \) and \( (0, -1) \). Determine the two-variable linear equation system from the graph? |
| 4              | On the way home from school, Farel and Aria stopped at a stationery shop with the intention of buying pencils and notebooks. Farel bought 2 pencils and 1 notebook with Rp20.000 bills and received a change of Rp11.000.00. Aria bought 2 pencils and 3 notebooks with Rp50.000 bills and received a change of Rp31.000.00. The next day, Azka intends to buy 1 pencil and 5 notebooks in the same brand as Farel and Aria bought at the store. Determine how much money Azka has to spend? Explain your answer? |

This research was conducted in one of state junior high schools in Bandung, West Java, Indonesia. The state junior high school is included in the medium level category according to the achievement of the National Final Examination score. The research subjects are 35 students of class VIII.

3. Result and Discussion

The measurement of mathematical representation ability includes students’ ability to make mathematical models, to draw graphs or tables, to solve a problem Two-Variable Linear Equation System, students’ ability to compose stories or written texts based on the representations presented, students’ ability to make equations or mathematical expressions from a given graph, and students’ ability to solve and conclude a problem students’ with written texts. The materials which were tested are the Straight-Line Equation and the Two-Variable Linear Equation. To find out the accuracy and inaccuracy or mistakes
of students in the process of solving mathematical representation ability, an analysis of students’ answers was conducted.

The results of the analysis of students’ answers show a description of the number of students in each criterion (value weight) in solving problems of mathematical representation ability. Further details will explain the results of the analysis of students’ works based on the indicators of mathematical representation ability.

3.1. Students’ ability to draw a graph or a table to solve a two-variable linear equation system

Problem number 1 (Table 1) measures the aspect of visual representation (graphs, tables or figures) with the indicator of Students’ ability to draw a graph or table to solve a two-variable linear equation system.

In problem number 1, two-variable linear equations were given as a system of equations. Students were asked to solve the equation system by the graphical method. Students’ answers to problem number 1, there are 36.36% of students who did not give answers, 48.48% of students gave answers by drawing incomplete graphs, and only 15.15% of students’ gave almost completed answers.

Based on students’ answers that are almost complete, students have understood how to do the problems as requested. Students began with the first equation by replacing y with 0 in the first equation (2x – y = 6). The students’ goal is to get the value of x if y = 0. This means that students already know the first step to draw a graph by determining the intersection point of the axis. After substituting y = 0 into the first equation, from the students’ calculation, x = 3 is obtained so that the point (3, 0) is obtained. With a similar step, students replaced the variable x in the first equation with 0. The students’ goal is to get the value of y if x = 0. From the results of the calculation there were errors in changing the equation 0 – y = 6 so that they obtained y = 6. This can happen because students were tricked by the negative sign (−) after zero, or students still do not understand how to simplify the equation to get the value of y if what he previously obtained was − y. Error in getting the value of y will lead to wrong set of settlement.

The next step of solving the problem was the students had to draw the Cartesian axis. Students know that the Cartesian axis consists of the X-axis (horizontal) and the Y-axis (vertical) with positive numbers and the greater the value when moving up and right, while the value will get smaller when it is going down or left. From the points obtained in the first step in determining points on the line, students plotted the points on the Cartesian axis. From the results of plotting the points, it appears that students still did not understand to determine the position of these points in Cartesian coordinates, and were still incorrect in drawing lines from known points. This means students still did not understand how to draw lines or determine the intersection of the two lines, so students can draw the right conclusion.

3.2. Students’ ability to compose stories or written texts based on the representations presented and students’ ability to solve and conclude the problems of two-variable linear equation system with written texts

Problem number 2 (Table 1) measures aspects of verbal representation (words or written text) with the indicator of students’ ability to compose word problems based on the representations presented in problem 2a and the indicator of students’ ability to solve and conclude the problems of two-variable linear equation system with written text in problem 2b. In problem number 2, it is given a two-variable linear equation system. Problem 2a, students were asked to make a daily word problem in accordance with the two-variable linear equation system given, then make problems in accordance with the problems that have been made. Next to problem 2b, students were asked to determine the set of solutions for problem 2a. In this third indicator, 51.52% of students could answer completely and correctly, while 27.27% of students have not answered completely and 21.21% of students did not answer.

Based on the sample results of students’ answers, students were able to answer problem number 2a by making word problems based on the given representation. Students answered to problem number 2a completely as requested. Students were able to construct word problem that are connected with the real world problem in everyday life. The problems were related to shopping activities. Carrots and tomatoes become part of shopping activities and is a routine activity in the family. This means students already have the ability to represent mathematical indicators on compiling stories or written texts based on the given representation.
However, in problem number 2b there was a mistake for the problem of determining each price each kilogram. Students did not interpret the questions that were made by themselves, as they try assuming the variables. The student considers carrots and tomatoes as variables, not carrot prices and tomato prices. In determining the set of solutions to the equation, students began by making an example of each variable, \( x \) for carrots and \( y \) for tomatoes, which is not right. The purpose of students making examples is to bridge between real-life word problems with mathematical symbols called mathematical models, so that the word problems can be solved algebraically. Most students often do mistakes when making a mathematical model of a real word problem.

The next step, students consider \( 3x + 2y = 9000 \) as the first equation and \( 2x + 4y = 10000 \) as the second equation. Next students equate the coefficient of variable \( x \) by multiplying the first equation by 2 and multiplying the second equation by 3. The goal is to obtain the value of \( y \) by removing the variable \( x \). This means students have understood one way to determine the value of a variable from an equation by eliminating another variable called the elimination method. Up to this stage, students have understood the procedure or the steps to solve the word problems.

From the sample of students’ answers, students are able to perform algebraic operations in two variables, \((6x + 4y - 18000) - (6x + 12y - 30000) = -8y + 12000\). But there was a mistake when students tried simplifying the algebraic expression \(-8y = -12000\). The expression \(-8y = -12000\) was simplified to \( y = -12000 + 8 \) which should be \( y = -12000 / -8 \). This can happen because students forgot or did not understand correctly in simplifying the algebraic expression. The result obtained was \( y = 11992 \) and it was incorrect. The next step, students substituted the value of \( y \) that had been obtained to the equation \( 3x + 2y = 9000 \). The goal was to get the value of \( x \) because the value of \( y \) was already known. Up to this step, students have understood the concept of the substitution method. From the calculation results, it was obtained that \( 3x = 14984 \) which was then simplified to \( x = -14984/3 \) so that the value of \( x = -4994.6 \) was obtained. Another error occurred again when students tried multiplying by -1, because only the right side was multiplied by -1 while the left was not. This shows that students were still weak in algebraic arithmetic operations. With the right side multiplied by -1, it was obtained \( x = 4994.6 \).

Students’ mistakes at the initial stage of determining the value of \( y \) was not repeated when determining the value of \( x \), but the error of the value of \( y \) at the beginning causes them to obtain a wrong value of \( x \). The positive thing about a students’ mistake while working on the problem was that the student understood that the price of an item cannot be negative. This means that the student had understood that it was not possible that \( x \) which was a variable assumed as a carrot is negative, because the price of an object is always positive. After obtaining the values of the two variables, students conclude for the price of 1 kilogram of carrots was Rp 4,994.6. Up to this stage, students already have procedural knowledge in solving word problems. Because there was a misconception, the conclusions obtained were not exactly in accordance with the problems made.

3.3. Students’ ability to make equations or mathematical expressions from a given graph

Problem number 3 (Table 1) measures aspects of symbolic representation (equations or mathematical expressions) with the indicator of students’ ability to make equations or mathematical expressions from a given graph. In problem number 3, there was a Cartesian coordinate which was passed by two straight lines, the first line went through the coordinates of points (5 , 0) and (0 , 5), while the second line through the coordinates of points (1 , 0) and (0 , -1). Students were asked to determine the two-variable linear equation system from the graph. The results of the analysis of students’ answers, 57.58% of students did not give answers, 15.15% of students gave incomplete answers, and 21.21% of students gave complete and correct answers.

Based on the sample answers, students have written correctly what is known, namely by specifying the coordinate point \( A \) and coordinate point \( B \) located on line \( k \). Students assumed correctly the coordinate point \( C \) and coordinate point \( D \) which are located on line \( l \). A \((0 , 5)\) and B \((5 , 0)\) as two points on line \( k \) and C \((1 , 0)\) and D \((0 , -1)\) as two points on line \( l \). Furthermore, students could connect the coordinates of known points with the concept of making line equations. By using the formula \((y - y_1)/(y_2 - y_1) = (x - x_1)/(x_2 - x_1)\) we get the equation of line \( k, y = -1x + 5 \) and line \( l, y = 1x - 1 \). This means students understand the steps of drawing a line if the points are known, so students can determine the equation of the line intended in the problem. Students already have conceptual knowledge about
Cartesian coordinate, coordinate points, simplifying algebraic shapes and determining straight line equations with formulas. In addition, students also have possessed procedural knowledge so that students were able to solve this problem with the right steps.

Examples of other student’s answer begin to work by making graph sketches in accordance with the problems given. Then the student wrote correctly what was known and substitute the equation for the line equation \((y_2-y_1)/(x_2-x_1) = (x-x_1)/(y_2-y_1)\). Students get the \(5x + 5y = 25\). But there was a mistake in the calculation to determine the second equation. So, the answer was not complete.

3.4. Students’ ability to solve and conclude the two-variable linear equation system problems with written texts

Problem number 4 (Table 1) measures the verbal aspect (words or written text) with the indicator of students’ ability to solve and conclude the two-variable linear equation system problems with written text. In problem number 4, it is given a story about three children who bought pencils and notebooks. Both children have bought the same pencil and book, but the numbers are different, so it is apparent how much money each child must spend. The next day, the third child intends to buy the same pencil and book with different volumes from the previous two children. Students were asked to determine how much money the third child must spend. The results of the analysis of student answers, 34.85% of students did not give answers, 43.95% of students gave incomplete answers, and 21.21% of students answered completely and precisely.

Based on the sample of student’s answer, student was able to write correctly what was known, namely Farel bought 2 pencils and 1 notebook with Rp 20,000.00 and received a change of Rp 11,000.00. Aria bought 2 pencils and 3 notebooks with Rp 50,000.00 and got a change of Rp 31,000.00. Azka intends to buy 1 pencil and 5 notebooks. The students wrote correctly what was asked and how much money Azka had to spend. In addition, students already know the next step to solve the given problem, namely by determining the mathematical model. This means students already have procedural knowledge. Before you can determine the mathematical model, students must first write the variables exactly according to the given problem. At this stage students made the mistake of making incorrect examples, namely\( pencil = x \) and \( notebook = y \). Furthermore, students made mistakes in making mathematical models. This means that students have not been able to interpret the problems perfectly. This can be seen from the answers given. Students wrote the equation \(2x + y = Rp 20,000.00\) and \(2x + 3y = Rp 50,000.00\). It should be \(2x + y = Rp 9,000.00\) because the money spent was \(Rp 20,000.00\) and got a change of \(Rp 11,000.00\) and \(2x + 3y = Rp 19,000.00\) because the money spent was \(Rp 50,000.00\) and got a change of \(Rp 31,000.00\).

The author tries to follow the mindset of students in answering the problems given so that the author can understand the root of students’ errors. Procedurally students have understood what must be done to solve the given problem. Students started by making an example for each object with a variable, a pencil with \(x\) and a notebook with \(y\). From this step the students obtained two equations namely \(2x + y = Rp 20,000.00\) and \(2x + 3y = Rp 50,000.00\).

The next step taken by students was to eliminate one of the variables to get the value of another variable. Students immediately subtracted the first equation with the second equation. This means that students had already known that the coefficient value of variable \(x\) was the same so it did not have to be multiplied by a number for each equation. From the calculation results, it was obtained that \(y = Rp 15,000.00\). These results were substituted into the equation \(2x + y = Rp 9,000.00\). The students’ goal was to find out the value of \(x\) if the value of \(y\) was already known. So, the students got \(x = Rp 2,500.00\). After obtaining the value of both variables, students determined the amount of money that must be spent on new conditions according to the problem requested. Up to this stage students had been able to reconnect the results of variable initiation at the beginning with the results obtained. Thus, students could determine the amount of money that must be spent was \(Rp 77,500.00\). From the results of the work, students have understood the concept of operating algebraic expressions in two variables, understood the concept of solving a two-variable linear equation system with the substitution method and elimination method and have been able to make conclusions and to understand procedural knowledge in solving the problem. Error in determining the mathematical model of the problem causes students to obtain incorrect answers.
Based on the analysis of all students’ answers, the mistakes made by some students was when working on the first step in answering problems, which is students did not make an example first. This has to do with procedural knowledge. Conceptual mistakes made by students are when translating everyday language into mathematical language by making incorrect examples and errors in making calculations in algebra. For example, the mistake of students when they were assuming that \( strawberry\ juice = x \) and \( avocado\ juice = y \), the variables should be about the price of one glass of strawberry juice and the price of one glass of avocado juice. This means that there are still students who have difficulty working on non-routine problems. The students’ mistakes in working on the given problems show that the students’ ability to solve non-routine problem is still weak [6,7]. Representations constructed by students when solving problems and investigating mathematical ideas are important habits in helping students to understand and to solve problems, as well as providing meaningful ways to write solutions and to describe methods for others [8]. Most of the difficulties in solving problems occur at the stage of representation. As a result, the process of translating problems into internal representations is a key for students to successfully solve problems [9]. This also means that students must have mathematical abstraction ability. Abstract-thinking ability in mathematics learning is very important for students to develop. Mathematics abstraction is a process of building a continuous mathematical knowledge from concrete to abstract [10].

In general, students’ mistakes in working on representation abilities are conceptual and procedural errors. Though understanding of mathematical concepts is the basis for learning mathematics meaningfully [11-15]. There are three types of errors of junior high school students in translating algebraic statements: (a) relative to the completeness of the statement. This error refers to whether there are symbols or words that are lacking in algebraic expressions that must be corrected. (b) derived from arithmetic, this error comes from sign interpretation or is incorrect in operation, and (c) originates from the characteristics of algebraic symbolism, this error comes from the symbolic characteristics of algebra very specific to the use of symbolic representation systems [16].

4. Conclusion
Based on the results and discussion, it can be concluded that the students’ mistakes in working on mathematical representation ability are conceptual and procedural errors. Students still have difficulty drawing a graph to solve two-variable linear equations, difficulty in making equations or mathematical expressions given, and difficulties in solving then inferring the problem of two-variable linear equation system with written text. However, in general students can already make mathematical models from the given problems.

5. Acknowledgments
In this study the author received a lot of assistance from various parties. The author expressing her gratitude to Mr. Prof. Dr. Ir. H. Eddy Justuf, Sp., M.Sc., M.Kom as the Rector of Pasundan University, Mr. H. Uus Toharudin, M.Pd. as Dean of FKIP Pasundan University, Mr. H. Bana G. Kartasasmita, Ph.D. and Mrs. Prof. Dr. Hj. R. Poppy Yaniawati, M.Pd. who always provide motivation and guidance to the authors.

6. References
[1] Yusepa B G P, Kusumah Y S, and Kartasasmita B G 2018 The enhancement of students’ mathematical representation in junior high school using cognitive apprenticeship instruction (CAI). J. Phys. Conf. Ser. 983 012100
[2] National Council of Teachers of Mathematics 2000 Principles and Standards for School Mathematics. (Reston. VA: NCTM)
[3] Laila N, Hidayat W, and Hendriana H 2018 Kemampuan Representasi dan Keaktifan Belajar Siswa. JPMI-J. Pembelajaran Matematika Inovatif 1 395-400
[4] Murni A 2014 Peningkatan Kemampuan Representasi matematis Siswa SMP Melalui Pembelajaran Metakognitif dan Pembelajaran Metakognitif Berbasis Soft Skill. J. Pendidikan 4 96-107
[5] Hutagaol K 2013 Pembelajaran Kontekstual untuk Meningkatkan Kemampuan Representasi
Matematis Siswa Sekolah Menengah Pertama *Infinity* 2 85-99

[6] Hidayat W, Wahyudin W and Prabawanto S 2018 The mathematical argumentation ability and adversity quotient (AQ) of pre-service mathematics teacher *J. Math. Educ.* 9 239–248

[7] Sari V T A and Hidayat W 2019 The students’ mathematical critical and creative thinking ability in double-loop problem solving learning *J. Phys. Conf. Ser.* 1315 12024

[8] Dahlan J A and Juandi D 2011 Analisis Representasi Matematika Siswa Sekolah Dasar dalam Penyelesaian masalah matematika Konstekstual *J. Pengajaran MIPA* 16 128-138

[9] Chen M J, Lee C Y, and Hsu W C 2015 Influence of Mathematical Representation and Mathematics Self-Efficacy on the Learning Effectiveness of Fifth Graders in Pattern Reasoning. *Int. J. of Learning, Teaching and Educational Research* 13 1-16

[10] Yusepa B G P, Kusumah, Y S, and Kartasasmita, B G 2018 Promoting middle school students’ abstract thinking ability through cognitive apprenticeship instruction in mathematics learning *J. Phys. Conf. Ser.* 948 012051

[11] Hidayat W and Aripin U 2019 The improvement of students’ mathematical understanding ability influenced from argument-driven inquiry learning *J. Phys. Conf. Ser.* 1157 32085

[12] Hidayat W and Husnussalam H 2019 The adversity quotient and mathematical understanding ability of pre-service mathematics teacher *J. Phys. Conf. Ser.* 1315 12025

[13] Hidayat W, Noto M S and Sariningsih R 2019 The influence of adversity quotient on students’ mathematical understanding ability *J. Phys. Conf. Ser.* 1157 32077

[14] Lee C, Li H C, and Shahrill M 2018 Utilising the Think-Pair-Share Technique in the Learning of Probability *Int. J. Emerg. Math. Educ.* 2 49-64

[15] Lestari R M and Prahmana R C I 2018 Desain Pembelajaran Logaritma untuk Siswa SMA Kelas X *J. Gantang* 3 31-39

[16] Molina M, Rodríguez-Domingo S, Cañadas M C, and Castro, E 2017 Secondary school students’ errors in the translation of algebraic statements *Int. J. of Science and Math. Educ.* 15 1137-1156