Distribution Model of Time Between Failures in CNC System

Guiyou Lu1,*
1Army Academy of Armored Forces, Changchun, China
*Corresponding author e-mail: jlyjlulu@163.com

Abstract. In this paper, the fault data of 50 CNC milling machine CNC systems are used. By observing the scatter plot of the probability density function and distribution function of the time between faults, the type of the distribution model is initially selected. After the inspection, the distribution model of CNC system failure interval time was established, and the three main reliability indexes were evaluated.

1. The introduction
The computer numerical control (CNC) system is an important part of the numerical control machine tool, and its general definition [1] is: the part other than the machine tool body. In the research of reliability of domestic CNC machine tools, the problems of CNC systems are particularly prominent. When researching the reliability growth technology of CNC system, we must understand its existing reliability level. Therefore, it is necessary to establish a reasonable CNC system failure interval time distribution model to evaluate its main reliability indicators.

2. The initial judgment of the time distribution model of fault interval
2.1. Scatter plot of probability density function
It is known from probability theory that the probability density function curves of the normal distribution and the lognormal distribution have a single peak shape, the probability density function curve of the exponential distribution has a monotonous downward shape, and the probability density function curve of the Weibull distribution depends on its shape parameters. Different or single peak shape or monotonous drop shape. From this we can know that according to the shape of the curve fitted from the observations, we can preliminarily determine what distribution a certain random variable follows. The data in this article was obtained from the on-site follow-up survey of 50 CNC milling machines of this type from January 1, 2017 to December 31, 2018. This type of CNC milling machine adopts an imported CNC system. After removing the non-associated faults, the faults belonging to the CNC system will be built into the database of the collected faults, and the maintenance time and fault interval time will be calculated. The following is the fault of the cnc system Observe the interval time to fit its probability density function. The observation value $t \in [18.71,4212.69]$ of the time between failures (cumulative time) is divided into 15 groups, as shown in Table 1:
Table 1 Observed values of time between failures

| NO | Upper   | Lower   | Median  | Time | frequency | total  |
|----|---------|---------|---------|------|-----------|--------|
| 1  | 18.71   | 318.2   | 168.5   | 13   | 0.2364    | 0.2364 |
| 2  | 318.2   | 617.8   | 468.0   | 9    | 0.1636    | 0.4    |
| 3  | 617.8   | 917.4   | 767.6   | 5    | 0.0909    | 0.4909 |
| 4  | 917.4   | 1216.9  | 1067.2  | 4    | 0.0727    | 0.5636 |
| 5  | 1216.9  | 1516.5  | 1366.7  | 4    | 0.0727    | 0.6364 |
| 6  | 1516.5  | 1816.1  | 1666.3  | 3    | 0.0545    | 0.6909 |
| 7  | 1816.1  | 2115.7  | 1965.9  | 2    | 0.0364    | 0.7273 |
| 8  | 2115.7  | 2415.2  | 2265.4  | 1    | 0.0182    | 0.7455 |
| 9  | 2415.2  | 2714.8  | 2565.0  | 1    | 0.0182    | 0.7636 |
| 10 | 2714.8  | 3014.4  | 2864.6  | 2    | 0.0364    | 0.8    |
| 11 | 3014.4  | 3313.9  | 3164.2  | 2    | 0.0364    | 0.8364 |
| 12 | 3313.9  | 3613.5  | 3463.7  | 3    | 0.0545    | 0.8909 |
| 13 | 3613.5  | 3913.1  | 3763.3  | 2    | 0.0364    | 0.9273 |
| 14 | 3913.1  | 4212.6  | 4062.9  | 3    | 0.0545    | 0.9818 |
| 15 | 4212.6  | 4512.2  | 4362.4  | 1    | 0.0182    | 1      |

2.2. Scatterplot of empirical distribution function

Taking the median of each group of time as the abscissa, the observed value of probability density \( \hat{f}(t) \) of each group as the ordinate, and the calculation of \( \hat{f}(t) \) is as follows:

\[
\hat{f}(t) = \frac{n_t}{n\Delta t_i}
\]

Where, \( n_t \)—The frequency of failures in each group of failure intervals;
\( n \)—The total frequency of failures is 55 times;
\( \Delta t_i \)—Group distance, 299.59 (h)

The scatterplot of the probability density function is shown in Figure 1. It can be seen from Figure 1 that the probability density curve of the time between failures shows a monotonous downward trend. It can be seen that the distribution obeyed by the failure time of the CNC system will not be a normal distribution or a logarithmic distribution, but may be an exponential distribution or a Wilbur distribution.

2.3. Scatterplot of empirical distribution function

The theoretical distribution function of the time between failures can be defined as:
\[ F(t) = P\{T < t\} \]

Where, \(T\)—Total time between failures;
\(t\)—Any time between failures.

Let \(t_1, t_2, t_3, \ldots, t_n\) be the observation value of the time between failures, and the order statistics of the time between failures obtained from the observations of this group are \(t_{(1)}, t_{(2)}, t_{(3)}, \ldots, t_{(n)}\), then the empirical distribution function of the CNC system fault interval time is:

\[
F_{(n)}(t) = \begin{cases} 
0 & t < t_{(1)} \\
\frac{i}{n} & t_{(i)} \leq t < t_{(i+1)}, i = 1, 2, \ldots, n \\
1 & t_{(n)} \leq t 
\end{cases}
\]

According to Grivenco's theorem, when the sample size \(n\) is large enough, the difference between the empirical distribution function \(F_{(n)}(t)\) and the theoretical distribution function \(F(t)\) obtained from the sample observations is small enough \(F(t)\) can be estimated from \(F_{(n)}(t)\).

It can be seen from the above formula that the graph of \(F_{(n)}(t)\) is a stepped line graph, which is a continuous curve of \(F_{(n)}(t)\) fitted, and the formula is simplified to

\[ F_{(n)}(t) = \frac{i}{n}, i = 1, 2, \ldots, \]

The \(F_{(n)}(t)\) is fitted according to the above formula, with the median of each group of time as the abscissa, and the cumulative frequency of each group as the ordinate, resulting from \(F_{(n)}(t)\) The scatter plot is shown in Figure 2.

It can be seen from Figure 2 that the empirical distribution function \(F_{(n)}(t)\) of the time between failures is convex and has no inflection point. It can be seen that the distribution obeyed by the failure time of the CNC system will not be a normal distribution or a lognormal distribution, but may be an exponential distribution or a Wilbur distribution.

![Figure 2 Scatter plot of empirical function](image)

It can be seen from Figures 1 and 2 that the probability density function and the empirical distribution function of the time between failures are strictly monotonous and have no inflection points. It can be seen that the distribution followed by the CNC system's failure interval may be the Weibull distribution in the typical distribution. In practical applications, the Weibull distribution can be simplified to:

\[
f(t) = \frac{\beta}{\alpha} \left(\frac{t}{\alpha}\right)^{\beta-1} \exp\left[-\left(\frac{t}{\alpha}\right)^{\beta}\right], t \geq 0
\]

\[
F(t) = 1 - \exp\left[-\left(\frac{t}{\alpha}\right)^{\beta}\right], t \geq 0
\]
3. Fit test of time interval model of fault interval

From the above discussion, it can be seen that the time between failures may follow an exponential distribution or a Wilbur distribution. Assuming that the time between failures follows the Weibull distribution, the parameters are estimated by the least square method and tested using the correlation coefficient method to determine the distribution law of the time between failures.

3.1. Parameter estimation

3.1.1. Parameter estimation of Weibull distribution

1) Formula simplification

The probability density function of the Weibull distribution is:

$$ f(t) = \begin{cases} \frac{\beta}{\alpha} \left(\frac{t-\gamma}{\alpha}\right)^{\beta-1} \exp \left[-\left(\frac{t-\gamma}{\alpha}\right)^\beta\right], & t \geq \gamma \\ 0, & t < \gamma \end{cases} $$

The distribution function is:

$$ F(t) = \begin{cases} \int_0^t f(t) \, dt = 1 - \exp \left[-\left(\frac{t-\gamma}{\alpha}\right)^\beta\right], & t \geq \gamma \\ 0, & t < \gamma \end{cases} $$

In the formula, $\beta$ is the shape parameter, $\beta > 0$; $\alpha$ is the size parameter, $\alpha > 0$; $\gamma > 0$ is the position parameter; $\gamma > 0$; in the failure analysis of the product, $\beta$ is related to the failure mechanism of the product, and different $\beta$ values with different failure mechanisms. When $\beta < 1$, it shows the life distribution of the early failure period; when $\beta = 1$, it shows the life distribution of the accidental failure period; when $\beta > 1$, it shows the life distribution of the wear-out failure period. $\alpha$ is related to the load of the working condition. If the load is large, the corresponding $\alpha$ is small; vice versa. The change of $\gamma$ affects the translation position of the probability density curve. The product does not fail before $t = \gamma$, and fails after $t > \gamma$. In practical applications, it is often assumed that the product fails when $t = 0$, so that the formula is simplified to:

$$ f(t) = \frac{\beta}{\alpha} \left(\frac{t}{\alpha}\right)^{\beta-1} \exp \left[-\left(\frac{t}{\alpha}\right)^\beta\right], \quad t \geq 0 $$

$$ F(t) = 1 - \exp \left[-\left(\frac{t}{\alpha}\right)^\beta\right], \quad t \geq 0 $$

2) Linear correlation test

Here, the two-parameter Weibull distribution is used to study the distribution law of the time between failures.

$$ \hat{\beta} = \frac{\sum_{i=1}^n x_i y_i - n\bar{x}\bar{y}}{\left(\sum_{i=1}^n x_i^2 - n\bar{x}^2\right)\left(\sum_{i=1}^n y_i^2 - n\bar{y}^2\right)} = 0.991683 $$

When $|\rho| > \rho_\alpha$, then $x$ and $y$ are considered to be linearly related. When the significance level $\alpha = 0.1$, the minimum value of the correlation coefficient

$$ \rho_\alpha = \frac{1.645}{\sqrt{n-1}} = 0.0305 $$

3) Linear regression equation

$$ \hat{B} = \frac{\sum_{i=1}^n x_i y_i - n\bar{x}\bar{y}}{\sum_{i=1}^n x_i^2 - n\bar{x}^2} = 1.129699 $$

$$ \hat{A} = \bar{y} - \hat{B}\bar{x} = -6.84200 $$

$$ \hat{y} = \hat{A} + \hat{B}x = -6.84200 + 1.129699x $$

4) Parameter estimation

$$ \hat{\beta} = 1.129699 $$
The calculation shows that $\hat{\rho} = 0.991683 > \rho_a = 0.0305$, so $x$ is linearly related to $y$, and the linear correlation equation is: $\hat{y} = \hat{A} + \hat{B}x = -6.84200 + 1.129699x$

The estimated parameters are: $\hat{k} = \hat{B} = 1.129699$

### 3.1.2. Parameter estimation of exponential distribution

1) Formula simplification

The probability density function of the exponential distribution is:

$$f(t) = \lambda e^{-\lambda t}$$

The distribution function is:

$$F(t) = 1 - e^{-\lambda t}$$

2) Linear correlation test

Since $\hat{\rho} = 0.991683 > \rho_a = 0.0305$, $x$ and $y$ are linearly related.

3) Linear regression equation:

$$\hat{B} = \frac{\sum_{i=1}^{n} x_i y_i - n \bar{x} \bar{y}}{\sum_{i=1}^{n} x_i^2 - n \bar{x}^2} = 0.000667$$

$$\hat{A} = \bar{y} - \hat{B} \bar{x} = 0.0113065$$

$$\hat{y} = \hat{A} + \hat{B}x = 0.0113065 + 0.000667x$$

4) Parameter estimation

$$\lambda = \hat{B} = 0.000667$$

### 3.2. Hypothesis testing

Commonly used hypothesis test methods are $d$ test method and $\chi^2$ test method, but the $d$ test method is finer than the $\chi^2$ test method, and it is also applicable to the case of small samples.

#### 3.2.1. Performing a $d$-test on the Weibull distribution

The rejection domain is:

$$D_n = \sup_{-\infty < x < \infty} |F_n(x) - F_0(x)| = \max\{d_i\} \geq D_{n,a}$$

Where, $F_0(x)$—Null hypothesis distribution function;

$F_n(x)$—Empirical distribution function with sample size $n$;

$D_{n,a}$—Critical value.

When $n=55$, $\alpha = 0.1$,

$$D_{55,0.1} = 0.1645$$

After comparison:

$$d_{max} = 0.07063194 < D_{55,0.1} = 0.1645$$. Therefore, the distribution function of the time between failures conforms to the Weibull distribution assumed above.

#### 3.2.2. Performing $d$-test on the exponential distribution

$d_{max} = 0.08712027 < D_{55,0.1} = 0.1645$, so the distribution of time between failures also conforms to the exponential distribution assumed above.

### 4. Determination of probability density function and distribution function

The probability density function $f(t)$ and distribution function $F(t)$ of the CNC system failure interval time in this experiment are:

$$f(t) = \frac{1.1297}{1537.87} \left(\frac{t}{1537.87}\right)^{0.1297} \exp\left[-\left(\frac{t}{1537.87}\right)^{1.1297}\right]$$

$$F(t) = 1 - \exp\left[-\left(\frac{t}{1537.87}\right)^{1.1297}\right]$$

Therefore, the reliability function $R(t)$ and the failure rate function $\lambda(t)$ are respectively obtained as follows:
$$R(t) = 1 - F(t) = \exp \left[ -\left(\frac{t}{1537.87}\right)^{1.1297}\right]$$

$$\lambda(t) = \frac{f(t)}{R(t)} = \frac{1.1297}{1537.87} \left(\frac{t}{1537.87}\right)^{0.1297}$$

5. Evaluation of reliability indicators

5.1. Estimation of the average failure-free working time $F$ degree

MTBF is estimated by the formula: $MTBF = \int_0^\infty t \cdot f(t) dt = E(t) = b\Gamma(1 + \frac{1}{k})$, and the estimated point value obtained by the CNC machine tool reliability information management system is:

$$MTBF = 1537.87 \times 0.956701 = 1471.28 (h)$$

5.2. Interval estimation of mean trouble-free working time $F$

The interval estimation of MTBF is a confidence interval for obtaining the reliability feature quantity MTBF according to the data. This interval includes the true value of the unknown parameter MTBF with a certain probability (ie, confidence level). Due to the randomness of the life test data, the total test time $T^*$ can be regarded as a random variable. $2T^*/\theta$ obtained by multiplying $T^*$ by $2/\theta$ can also be regarded as a random variable.

According to the requirements of GB5080.4-85 Point Estimation and Interval Estimation of Reliability Test of Equipment Reliability Testing", the time-censored test index interval estimation is used, and the confidence level is $1-\alpha=90\%$(ie, $\alpha=10\%$), from which we can get

1) MTBF's unilateral confidence interval

$$m > \frac{2T^*}{\chi^2_{0.99}(2r + 2)} = 1184.1482 (h)$$

2) Bilateral confidence interval of MTBF

$$\frac{2T^*}{\chi^2_{0.95}(2r + 2)} < m < \frac{2T^*}{\chi^2_{0.05}(2r)}$$

1132.3625(h) < m < 1806.4450(h)

Where, $r$—Number of failures

5.3. Mean time to repair $MTTR$

For repairable products, in addition to reliability, maintainability is also considered. Although some products are not faulty, once a fault occurs, it will take a long time to be repaired. It may often be in a repaired state, and the utilization rate is obviously not high. Therefore, we should not only pay attention to whether the product is easily damaged, but also whether the product is easy to repair. The maintenance time $t_{Mi}$ is the actual maintenance time after the exponentially controlled lathe fails and can be directly obtained from the database.

The corresponding maintainability index has the average maintenance time, which is the mathematical expectation of the maintenance time of CNC machine tools. The average maintenance time can also be obtained from the observation value as follows:

$$MTTR = \frac{1}{N_0} \sum_{i=1}^{n} t_{Mi}$$

Where,$N_0$—The total number of failures;
$t_{Mi}$—The actual repair time (h) of the i-th CNC lathe in the assessment period.

Available from the reliability information management system:

$$MTTR = 3.25 (h)$$
5.4. Inherent availability $A_i$

The inherent availability combines the generalized reliability feature quantities of reliability and maintenance, and its expression is:

$$A_i = \frac{MTBF}{MTBF + MTTR}$$

It can be seen that the larger the $A_i$, the higher the effective working degree of the whole machine. The method to increase the inherent availability is to increase MTBF and shorten MTTR.

The inherent availability of the CNC system is:

$$A_i = \frac{MTBF}{MTBF + MTTR} = 0.9978$$

$$MTTR = \frac{1}{N_0} \sum_{i=1}^{n} t_{Mi} = 3.25\text{ (h)}$$

6. Conclusion

The data used in this evaluation is in accordance with the broad definition of the CNC system. Among the collected machine faults, after removing the non-associated faults, the faults belonging to the CNC system are established. The calculation result of the self-edited software "Reliability Information of CNC Machine Tools" is obtained after processing by the management system. But according to its other definition [2] (that is, the part that specifies the execution instruction-the servo system, the drive system, and the detection device does not belong to the CNC system) to calculate, the index of the average trouble-free working time will increase. Therefore, when evaluating the reliability index, it is necessary to clarify the composition of the CNC system.

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