Rain model for the transmission spectra of one dimensional disorder system

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Abstract

We imitate the spectrum character of one-dimensional disorder system with our new rain model. It has been shown that the transmission spectrum can be approximately characterized by the model, which include some coupled lorentzian transmission peaks on the spectrum and embody the coupling of the local modes in the disorder system.

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The transport of optical waves through disordered systems can exhibit remarkable interference effects, in analogy with the transport of electrons in solids. This leads to interesting optical phenomena\cite{1} of which the most surprising is that of Anderson localization of light\cite{2}. The observation of this extraordinary effect in three dimensional disordered optical systems requires very strong scattering that can be achieved only in selected materials\cite{3}. For lower dimensional systems, however, the situation is different. In one and two dimensional disordered systems, localization can always be reached for a sufficiently large sample size\cite{4}. However, not all modes are exponentially localized in 1D random systems, even though Anderson localization occurs. Pendry\cite{5, 6} and Tartakovskii\cite{7} predicted that even in one dimensional (1D) localized systems nonlocalized modes exist that extend over the sample via multiple resonances. These nonlocalized modes, called necklace states (NS), have a transmission coefficient close to 1 and become extremely rare upon increasing the sample thickness. Nevertheless, they dominate the average transmission coefficient, even at large thickness. Evidence for the existence of necklace states was recently found in time-resolved transmission experiments\cite{8}, and subsequently also observed in experiments with microwaves\cite{9}. It is noted that these works about NS are mainly concentrated on the one dimensional systems for its rather easily calculation and manufacture. The basic method and ways is commonly to analyze the transmission spectra of the system whether experimentally\cite{10, 11} or theoretically\cite{12, 13}, since the transmission spectra is essential for 1D disorder system and reveal important formation of the system. For example, when the phase of the transmission coefficient is the integral multiple of $2\pi$, it corresponds to the order of NS\cite{12} and the platforms in the spectrum usually can be used to identify the occurrence of NS. Since the transmission spectra directly implicate the intrinsic properties of the system, It is reasonable that one can artificially construct transmission spectra according to some aforehand rules inferred form the physical evaluation. Some statical properties of physical variables of the system can be explored based on the constructed spectra. It is equivalent that one is inquiring interactively the relations between the statical properties and the possible physical mechanism. Based on the above idea, In this letter we present a model called after rain model to imitate the transmission spectrum of the one dimensional disorder system. Generally, the transmission spectra of localized 1D systems exhibit many randomly distributed high-transmission peaks. These high-transmission peaks originate from resonances of the system with localized modes\cite{14} and result in big fluctuations in the transmission coefficient.
The localized modes decay exponentially and the ensemble average of over many realizations of the disorder decays linearly with the sample thickness $L$.

Our model is as follows. Consider a one-dimensional disorder system with length $L$. The localized states inhabiting in the system corresponding to the eigenfrequency $\omega_n$ usually have the form $E_n \propto e^{-|x-x_n|/\xi}$ except a oscillating factor, where $\xi$ is the localization length and $x_n$. For simplicity, omitting the fine structure of the field for the moment we assume the field localized at $x_n$ has simply the form of the envelope. Since the necklace states are usually constructed by a series of localized states close to each other spatially so that they seem like the necklace and the resultant transmission spectrum manifest a high lorentzian peaks bundle. Therefore, first of all we should investigate what the transmission spectrum would be like if the fields in system are superposed by multiple localized states. Here we have to use experiential conclusion that the transmission coefficient $t$ is expressed as $t = \frac{\min\{E_n(0), E_n(L)\}}{\max\{E_n(0), E_n(L)\}}$.

With the above approximation and assuming $x_n < L/2$, the transmission coefficient for a local state $x_n$ is $t = \frac{E_n(L)}{E_n(0)} = e^{-L/\xi}e^{2x_n/\xi}$. For the field of $N$ coupled states $E_N(x) = \sum_{n=1}^{N} e^{-|x-x_n|/\xi}$, the corresponding transmission coefficient $t_N = \frac{E_N(L)}{E_N(0)} = \sum_{n}^{N} t_n^{1/2} / \sum_{n}^{N} t_n^{-1/2}$. Since the transmittance peaks in the bundle are lorentzian and has the form $T(\omega) \propto \frac{(\Delta \omega/2)^2}{(\omega-\omega_n)^2 + (\Delta \omega/2)^2}$, where the center frequency $\omega_n$ and $x_n$ are all the random variables and the halfwidth $\Delta \omega$ can be determined by quality factor $Q$ with relation $Q = \frac{\omega_n}{\Delta \omega}$. According to the definition of $Q = \frac{\text{stored energy}}{\text{power loss}} = \omega_0 \frac{W_{\text{store}}}{P_{\text{loss}}} = \frac{\omega_n}{\Delta \omega}$, the halfwidth can be written as $\Delta \omega = \frac{P_{\text{loss}}}{W_{\text{store}}}$. In one dimensional system how is the quality factor correlated with the transmittance? Since quality factor $Q$ is the measure of the sharpness of response of the cavity to external excitation, it’s reasonable that the system is regarded a cavity enclosed by the vacuum and the transmitted energy corresponds the power loss.
Here consider a imaginary pipe with unit area though the system. So the power loss can be expressed transmitted average energy current $\langle S \rangle_{x=L} = \frac{E(L)}{2Z_0}$ while the stored energy in the pipe $W_{store} = \int \langle U \rangle dv = \int_0^L \Re(e|E|^2) dx$. It is found from above results that $\Delta \omega = \frac{eE(L)^2}{2Z_0 \int E^2 dx} = \frac{cE(L)^2}{2\epsilon_r \int E^2 dx}$, where $c$ is the light speed, $Z_0 = \sqrt{\mu_0 / \epsilon_0}$ is the vacuum impedance, $\epsilon_r$ is the average relative permittivity of the system. For the above field $E_n(x) = e^{-|x-x_n|/\xi}$, we obtain

$$\Delta \omega = \frac{e^{4x_n/\xi}}{\left( e^{2L/\xi} + e^{4x_n/\xi} - 2e^{2(L+x_n)/\xi} \right) 2\pi \epsilon_r \xi} \quad (1)$$

To imitate the spectrum of the necklace states we assume many lorentzian peaks randomly fall on the frequency axis with random center frequency $\omega_n$, which is related with another spatial random variable $x_m$ that is the center of local states. For each $(\omega_n, x_m)$, we first construct the transmittance coefficient

$$t_{nm}(\omega, \omega_n, x_m) = \frac{e^{(2x_m-L)/\xi} (\Delta \omega/2)}{\sqrt{(\omega - \omega_n)^2 + (\Delta \omega/2)^2}} \quad (2)$$

and the total transmittance spectrum can be obtained

$$t_{total}(\omega) = \frac{\sum_{n,m} t_{nm}^{1/2}}{\sum_{n,m} t_{nm}^{-1/2}} \quad (3)$$

To explore the possible mechanism we introduce the repulsive interaction between these ‘frequency raindrops’ which are characterized by the corresponding overlap integral between the two localized states. Specifically speaking, for the states localized at $x_i, x_j$, the overlap integral is $I_{i,j} = \int_{-\infty}^{\infty} e^{-|x-x_i| - |x-x_j|}/\xi dx = \left( \xi + \Delta x \right) e^{-\Delta x/\xi}$ and the modulus square $\int_{-\infty}^{\infty} |E(x)|^2 dx = \xi$. In the rain process the frequency points that satisfy the condition $I_{i,j} < \alpha \xi$ are permitted to be coupled into the $t_{total}$, where parameter $\alpha$ control the coupling strength. We call the process as two order process since it only involves two coupled frequencies and two localized states. To check the validity of the operation, we compared the correlation curves to frequency interval with those obtained from the real one dimensional disorder system by transfer matrix method. The correlation is defined as:

$$Cov(\Delta \omega) = \frac{\langle (\ln T(\omega + \Delta \omega) - \langle \ln T(\omega + \Delta \omega) \rangle)(\ln T(\omega) - \langle \ln T(\omega) \rangle) \rangle}{\sqrt{\langle (\ln T(\omega + \Delta \omega)^2 - \langle \ln T(\omega + \Delta \omega) \rangle^2)(\ln T(\omega)^2 - \langle \ln T(\omega) \rangle^2) \rangle}} \quad (4)$$

, where $\langle \cdot \rangle$ means the average over all realizations of disorder. For the two order process we introduce a coupled mode theory to phenomenologically characterize the effect on the
transmission coefficient by the fact that mode coupling transform the two random frequencies $\omega_m, \omega_n$ into the renormalized frequencies $\omega'_m, \omega'_n$:

$$\frac{d^2 a_1}{dt^2} + \gamma_1 \frac{da_1}{dt} + \omega_1 a_1 = q_{12} a_2 + e^{-i\omega't}$$

$$\frac{d^2 a_2}{dt^2} + \gamma_2 \frac{da_2}{dt} + \omega_2 a_2 = q_{21} a_1 \tag{5}$$

Where $a_1(t) = a_{10} e^{-i\omega_1 t}$, $a_2(t) = a_{20} e^{-i\omega_2 t}$ and $a_{10}, a_{20}, \omega'$ is to be determined. According to the condition of nonzero solution of $a_{10}, a_{20},$

$$(\omega_1^2 - (i\gamma_1 + \omega')\omega')(\omega_2^2 - (i\gamma_2 + \omega')\omega') - q_{12} q_{21} = 0 \tag{6}$$

we can get two renormalized peaks frequency $\omega'_1, \omega'_2$ with positive real parts, where the factor $e^{-i\omega't}$ describes the excitation of incident wave and for simplicity the coupling coefficients $q_{12} = q_{21} = q \propto I_{i,j}$, the damping factor $\gamma_n \propto \Delta \omega$

In Fig.2 we compare the $\ln T(\omega)$ and the correlation obtained by transfer matrix method for a 3000 layers disorder system with $d_1 = 200\text{nm}$, $\epsilon_A = 1$, $d_2 = 100\text{nm}$, $\epsilon_B = 2$ with the corresponding results obtained by the rain model. In Fig.2(b) for one order approximation where lorenztian peaks are summed directly the correlation by the rain model is only in good agreement with the system with low random strength where $w = 0.04$. However for higher random strength the one order approximation apparently fail in approximating the real disorder system. For the real system our calculation show that when random strength $w$ grow gradually the correlation for a fixed frequency interval rise rapidly and descends slowly after passing by a maximum, which is shown in Fig.3. Here for our the calculated 3000 layers’ system the critical random strength $w_c \approx 0.12$. It is found that when more and
more lorentzian peaks are coupled into the transmission coefficient spectrum, which means the higher order processes, the whole correlation curve also ascend rapidly as shown by the green dashed dotted line in Fig.2(b). For a given random strength \( w \) the correlation of the real system can be always approximated by the rain model with specific number of order process because it can be raised consecutively with the increment of number of order. For example, for \( w = 0.054 \) rain model can give a good approximation with \( \sim 20000 \) peaks and \( \alpha = 0.5, \ L/\xi = 8 \) which is marked with blue dashed line in Fig.2(b). The corresponding spectrum is shown in Fig.3(a). It is noted form Fig.3(a) that in the range \( 15.97 \sim 15.99 \) the spectrums keep the close similarity and some double peaks structure begin to occur, which imply the thrown coupling double peaks play a important role in the similarity of correlation curve.

In conclusion, we present a model to simulate the transmittance spectrum of the one dimensional disorder system. The model describe the construction of the spectrum based on a thought that many lorentzian peaks can be superposed in a way embodying some interaction between these single peaks. Furthermore, we introduce overlap integral between localized states and the coupling of modes to characterize the interaction phenomenologically. In the above way lorentzian peaks randomly drops on the frequency axis like raindrops and for each peak the characteristic parameter is determined by a localized states with a random position. Here it involves two aspects of randomness: frequency and spatial domain. During the sampling of the random variables uniform distribution is used. In the weak disorder the model can give rather better approximation. Because the model associate the form of transmittance spectrum with the interaction between single peaks and the position of the localized states it provide a intuitionistic tool for the subject that depends on the analysis.
of the transmittance spectrum such as the study of the necklace states.

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