Probing the nuclear deformation with three-particle asymmetric cumulant in RHIC isobar runs

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Abstract

\(96^{\text{Ru}} + 96^{\text{Ru}}\) and \(96^{\text{Zr}} + 96^{\text{Zr}}\) collisions at \(\sqrt{s_{\text{NN}}} = 200\) GeV provide unique opportunities to study the geometry and fluctuations raised from the deformation of the colliding nuclei. Using iEBE-VISHNU hybrid model, we predict \(a_2(3)\) ratios between these two collision systems and demonstrate that the ratios of \(a_2(3)\), as well as the ratios of the involving flow harmonics and event-plane correlations, are sensitive to quadrupole and octupole deformations, which could provide strong constrains on the shape differences between \(96^{\text{Ru}}\) and \(96^{\text{Zr}}\). We also study the nonlinear response coefficients \(\lambda_{4,22}\), which show insensitivity to the deformation effect.

1. Introduction

Anisotropic flow observed in heavy-ion collisions at Relativistic Heavy-Ion Collider (RHIC) and Large Hadron Collider (LHC) indicate that the created quark-gluon-plasma (QGP) is a strongly coupled system with small specific shear viscosity [1-9]. Hydrodynamic simulations have successfully described the collective expansion of the QGP fireball and studied various flow observables at RHIC and the LHC [10-19]. After the hydrodynamic evolution, the initial stage geometry and fluctuations are translated into final stage correlations described by various flow observables such as different order flow harmonics, correlations between flow harmonics, event-plane correlations, etc. [19,29]. On the other hand, these flow observables raised from the collective expansion also depend on the properties of the QGP and the details of the initial profiles. The RHIC isobar runs with \(96^{\text{Zr}} + 96^{\text{Zr}}\) and \(96^{\text{Ru}} + 96^{\text{Ru}}\) collisions at \(\sqrt{s_{\text{NN}}} = 200\) GeV provide unique opportunities to probe the nuclear structure of the colliding nuclei from the initial stage, since the uncertainties from the bulk properties of the QGP can be largely reduced through the observable ratios between the two collision systems [30,31].

The \(96^{\text{Zr}} + 96^{\text{Zr}}\) and \(96^{\text{Ru}} + 96^{\text{Ru}}\) collisions at \(\sqrt{s_{\text{NN}}} = 200\) GeV originally aimed to search the chiral magnetic effect (CME). The observed differences in multiplicity distribution \(N_{(a)}\) and anisotropic flow observables between these two systems ruin the premise that isobar collisions can help identify the CME with enough precision [32,33], but provide a novel way to constrain the nuclear deformation from heavy ion collisions [34]. For the typical initial profile construction for A+A collisions, the

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calculated by the multi-particle azimuthal correlations \(20\,21\):

\[
\langle m_{n_1,n_2,...,n_n} \rangle \equiv \langle e^{i(n_1\phi_1+n_2\phi_2+...+n_n\phi_n)} \rangle,
\]

Here \(\langle \rangle\) denotes the sum of all particles of interest (POI) in a given event. The three-particle asymmetric cumulant can be calculated with \(23\,25\,27\,28\):

\[
a_{23} \equiv \langle (3)_{2,2,-4} \rangle = \langle (e^{i(2\phi_2+2\phi_4-4\phi_1)}) \rangle
\]

Here \(\langle (..)\rangle\) indicates the average of \(\langle \rangle\) over an ensemble of events. \(a_{23}\) is sensitive to the flow magnitudes and event-plane correlations. It is also directly related to the nonlinear response between the second and fourth order flow vector, which can also be used to extract the corresponding nonlinear response coefficients \(24\).

We will show in this work that the \(a_{23}\) ratios in isobar collisions are very sensitive to the deformation of the colliding nuclei. In the absence of non-flow effects, the \(a_{23}\) can be written as \(24\):

\[
a_{23} = (v_2^2 v_4 \cos 4(\Phi_2 - \Phi_4)),
\]

where \(\Phi\) is the event-plane of the related flow harmonic. Partly inherited from \(v_2\), \(a_{23}\) is sensitive to the deformation of the colliding nuclei \(20\,39\,41\). We will show that the normalized asymmetric cumulant

\[
n_{a_{23}} \equiv \frac{a_{23}}{\sqrt{(v_2(2)^2 - v_2(4)^2)v_4(2)^2}}
\]

is also sensitive to the nuclear deformation, even if the contributions from single flow harmonics have been scaled out. Here,

\[
v_2(2)^2 = \langle (2)_{2,-2} \rangle,
\]

\[
v_4(2)^2 = \langle (2)_{4,-4} \rangle,
\]

\[
v_2(4)^2 = 2v_2(2)^2 - \langle (4)_{2,2,-2} \rangle.
\]

In this letter, we will implement iEBE-VISHNU hybrid model to calculate the flow observables and demonstrate that the nuclear deformation not only influences the magnitude of anisotropic flow but also their correlations, which can be reflected by \(a_{23}\) and \(n_{a_{23}}\) correlations. Specifically, the effect on flow harmonics is amplified in the ratio of \(a_{23}\), and the residual effect from even-plane correlation is reflected by the ratio of \(n_{a_{23}}\), where the ratio is defined as:

\[
R(X) = \frac{X_{Ru}}{X_{Zr}}.
\]
can give a better description of elliptic flow ratio \( R(v_2) \) and triangle flow ratio \( R(v_3) \), although the motivation of this study is not focused on quantitative prediction of flow ratios. The Woods-Saxon parameter sets for different deformation factors are listed in Tab. 2.

In this study, the azimuthal correlations are calculated using the standard Q-cumulant method \([20]\) with \( Q_n \equiv \langle e^{i n \phi} \rangle \). All charged particles with \( 0.2 < p_T < 2 \text{ GeV}/c \) are used. To reduce statistical uncertainties, an \(|\eta| < 2\) pseudorapidity cut is used. iEBE-VISHNU simulations contain part of the non-flow effect from resonance decays. While the standard Q-cumulant method can not fully reduce the non-flow effect, especially for the ac_2(3) which has a large non-flow subtraction method dependence \([27, 28]\). The large pseudorapidity cut used in this study can reduce the non-flow contributions to some extent, and we will discuss this in the next section.

### 3. Results and discussions

Figure 1 shows the ratio of ac_2(3), \( R(\text{ac}_2(3)) \), as a function of centrality in isobar collisions at \( \sqrt{sNN} = 200 \text{ GeV} \), calculated from iEBE-VISHNU model. The comparison of \( R(\text{ac}_2(3)) \) at most central collisions with different combination of the deformation parameters \( \beta_2 \) and \( \beta_3 \) demonstrates that \( R(\text{ac}_2(3)) \) is sensitive to the nuclear deformation. The trend is similar to the one of \( R(v_2(2)) \) as shown in Fig. 2(a), which decreases from most central to semi-central collision and then increases from semi-central to peripheral collisions. The large \( R(\text{ac}_2(3)) \) and \( R(v_2(2)) \) in the most central collisions are mostly due to the large quadrupole deformation in \(^{96}\text{Ru}\) \([39]\), while the enhancement trend from semi-central to peripheral collisions is due to the thick halo-type neutron skin in \(^{96}\text{Zr}\) \([41]\). The octupole deformation in \(^{96}\text{Zr}\) give some contributions to \( R(\text{ac}_2(3)) \) and \( R(v_2(2)) \) in the most central collision, and lead to the valley structure in semi-central collisions \([32]\). Compared with \( R(v_2(2)) \), \( R(\text{ac}_2(3)) \) is more sensitive to nuclear deformation and contains more information on flow fluctuations and correlations. Description of their sensitivities on the nuclear structure at a quantitative level can help us to precisely constrain the nuclear deformation factors in isobar collisions. Note that the data of \( R(v_2(2)) \) in Fig. 2(a) prefer \( \beta_{2,Ru} = 0.12 \) and \( \beta_{3,Zr} = 0.16 \). While, we should also emphasis that this paper is not aimed to precisely describe the flow data in isobar collisions. More sophisticated extractions of the deformation parameters will be given in the following study \([54]\).

Besides \( v_2(2), \, v_3(2) \) and \( v_3(2) \) also contribute to ac_2(3). Due to flow fluctuations, \( v_2(2) \) from the two-particle correlation is larger than \( v_2(4) \) from the four-particle correlation for different collision systems. However, as shown in Fig. 2(a) and (b), the ratios \( R(v_2(2)) \) and \( R(v_2(4)) \) in isobar collisions present opposite behavior as observed in experiment \([30]\), which indicates the importance of initial state deformation and fluctuations \([55]\). Note that the \( R(v_2(4)) \) and \( R(v_2(2)) \) in the most central collisions also depend on the deformation. We observe \( R(v_2(4)) \) deviates from unity in the most central isobar collisions, while firm conclusion needs high statistical runs. We note that, comparing to high order flow observable \( R(v_2(2)) \), the \( R(\text{ac}_2(3)) \) shows stronger dependence on nuclear deformation with smaller statistical uncertainties, indicate that \( R(\text{ac}_2(3)) \) is statistical friendly observable which are very important for model study and data analysis.

In fact, ac_2(3) is largely influenced by individual flow harmonics, while the correspondent normalized asymmetric cumulant nac_2(3) could reduce such flow contributions. nac_2(3) directly reflect the correlation between second and fourth order event-plane \( \cos 4(\Phi_2 - \Phi_4) \), after neglecting the correlations between different flow harmonics. In Fig. 3(a) and (b), we plot the ratios of normalized asymmetric cumulant \( R(\text{nac}_2(3)) \) and \( R(\cos 4(\Phi_2 - \Phi_4)) \) with \( \Phi_2 = (1/2) \arctan (\text{Im} Q_2/\text{Re} Q_2) \) and \( \Phi_4 = (1/4) \arctan (\text{Im} Q_4/\text{Re} Q_4) \). The \( R(\text{nac}_2(3)) \) depends on \( \beta_{2,Ru} \) and \( \beta_{3,Zr} \) with large statistical uncertainties which is inherited from \( v_2(2) \) and \( v_3(4) \) shown in Fig. 2. We found the event-plane correlations ratio \( R(\cos 4(\Phi_2 - \Phi_4)) \) also depend on \( \beta_{2,Ru} \) and \( \beta_{3,Zr} \), which show similar trend as \( R(\text{ac}_2(3)) \) and \( R(v_2(2)) \). We note that the event plane...
correlations can also be calculated by [22]

\[ c[2, 2, -4] = \frac{\langle O_{4A}^2 O_{4B}^2 \rangle}{\sqrt{\langle O_{4A}^2 O_{4B}^2 \rangle \langle O_{2A}^2 O_{2B}^2 \rangle}} \]  \tag{9}

with the two sub-event method, and we have checked that the \( R(c[2, 2, -4]) \) consist with \( R(nac_2[3]) \) and \( R(\cos 4(\Phi_2 - \Phi_4)) \) within large statistical errors.

The similar trends for \( R(ac_2[3]), R(\cos 4(\Phi_2 - \Phi_4)) \), and \( R(v_2[2]) \) indicate that both the nuclear deformation and the resulting fluctuations are important to understand the observed flow differences in isobar collisions. In Fig. 4 as a summary, we compare those ratios with two sets of deformations, i.e., (Ru-para-I, Zr-para-I) v.s. (Ru-para-II, Zr-para-II). We find that the \( R(ac_2[3]), R(\cos 4(\Phi_2 - \Phi_4)) \), and \( R(v_2[2]) \) show different response to the deformation. We expected that our proposed observables, together with other observables like \( R(v_3) \), can be used to constrain the initial deformation and fluctuations for relativistic isobar collisions.

Note that the non-flow contributions have not been fully included in our study, since non-flow contribution from iEBE-VISHNU simulations are mainly from resonance decay. If the non-flow contributions are the same for the two colliding system, the observed differences should be diluted to some extend. We have checked that the \( |\eta|<2 \) used in this study make the \( R(v_2[2]) \) a little bit larger deviate from unity than the one using a smaller pseudorapidity cut \( |\eta|<1 \). We also find that the sub-event method (e.g. \( \Delta\eta>0.4 \)) can also suppress the non-flow effect, which make the \( R(v_2[2]) \) further deviate from unity. These effects are considerably small on \( R(v_2[2]) \), and even not visible on other observables, partly due to large statistical uncertainties. For the three particle cumulant \( ac_2[3] \), the three sub-event method can largely suppress the non-flow contributions but restricted by statistics. Besides resonance decay, further studies with more non-flow effects included are needed. The dataset collected in the experiment (about 2 billion events for each collision system) is 14 times larger than the model study used in this work (about 3 million (hydro) \times 50(URQMD oversamplings) \approx 150 million events for each collision system), with which observables are expected to be measured more precisely with various non-flow subtraction methods. The comparison between model study and the experiment data can provide more insights on the non-flow contributions.

Before the end of this study, it is also interested to study the nonlinear response coefficient \( \chi_{4,22} \) ratios in isobar collisions, which is defined as [24]:

\[ \chi_{4,22} \equiv \frac{ac_2[3]}{\langle v_2^4 \rangle} = \frac{nac_2[3]}{\langle v_2^4 \rangle} \sqrt{\frac{v_4[2]}{2v_2[2]^2 - v_2[4]^2}}. \]  \tag{10}

The results are shown in Fig. 5, which are consistent with unity within errors. It indicates that the nonlinear coefficient \( \chi_{4,22} \) is not sensitive to the nuclear deformation, although the top 5% results show some weak sensitivities with large uncertainties. Note that earlier study also found that the nonlinear coefficients are not sensitive to impact parameter and initial model, but mostly determined by the freezeout temperature in hydrodynamic simulation [24, 55].

In the most central collisions, the \( v_2[4] \) is significantly smaller than \( v_2[2] \), then \( \chi_{4,22} \) can be approximately expressed as:

\[ \chi_{4,22,\text{Approx}} = \frac{ac_2[3]}{v_2[2]^2}. \]  \tag{11}

Fig. 5 also shows \( R(\chi_{4,22,\text{Approx}}) \), calculated with \( \beta_{Zr} = 0.16 \) and \( \beta_{Ru} = 0.12 \), with open black circles. We find that this approximation works well for top 10% centrality, but failed at semi-central and peripheral collisions. Such deviation at non-central collisions indicates that fluctuations are essential to understand the flow differences in relativistic isobar collisions.

### 4. Summary

The observed differences between flow harmonics for \(^{96}\text{Ru} + ^{96}\text{Ru}\) and \(^{96}\text{Zr} + ^{96}\text{Zr}\) collisions at \( \sqrt{s_{NN}} = 200 \text{ GeV} \) pro-

\[^{1}\text{We thank G. Giacalone for valuable discussion on this point.}\]
Figure 4: The comparison among $R(\Delta c_3)$, $R(\cos 4(\Phi_2 - \Phi_3))$ and $R(\kappa_2[2])$ at relativistic isobar collisions with different set of nuclear deformations: (a) (Ru-para-I, Zr-para-I) and (b) (Ru-para-II, Zr-para-II). The observables are calculated by the standard Q-cumulant method with $0.2 < p_T < 2$ GeV/c and $|y| < 2$.

Figure 5: The centrality dependent $R(\chi_{4,22})$ calculated by iEBE-VISHNU model with different sets of nuclear deformations. The approximated $R(\chi_{4,22})$ calculated by Eq. (11) is shown as open black circles. The observables are calculated by the standard Q-cumulant method with $0.2 < p_T < 2$ GeV/c and $|y| < 2$.

Figure 6: The centrality dependent $R(\chi_{4,22})$ calculated by iEBE-VISHNU model with different sets of nuclear deformations. The approximated $R(\chi_{4,22})$ calculated by Eq. (11) is shown as open black circles. The observables are calculated by the standard Q-cumulant method with $0.2 < p_T < 2$ GeV/c and $|y| < 2$.

provide unique opportunities to probe the nuclear structure of the colliding nuclei. In this letter, we proposed that the asymmetric cumulant ratio $R(\Delta c_3)$, together with the corresponding individual flow harmonic ratios $R(\kappa_2)$ and event-plane correlation ratio $R(\Delta nac_3)$ ($R(\cos 4(\Phi_2 - \Phi_3))$), can simultaneously constrain the nuclear deformation and the resulting fluctuations. Our iEBE-VISHNU hybrid model simulations indicate that the statistical friendly observable $R(\Delta c_3)$ is very sensitive to the quadrupole and octupole deformation of $\beta_2$ and $\beta_3$. To further investigate this sensitivity, we divided the $\Delta c_3$ into three parts, i.e., $v_2$ ($v_2[2]$ and $v_2[4]$), $v_4$ ($v_4[2]$), and the normalized asymmetric cumulants $nac_3$, but ignore their correlations and non-flow effect. We found that both the flow harmonics differences and event-plane correlation differences in the isobar collisions depend on $\beta_2$ and $\beta_3$. The event-plane correlation differences on the nuclear structure could be larger than the elliptic flow difference, indicating the importance of initial fluctuations. The $R(\Delta c_3) = \chi_{4,22}$ works well in the most central collisions, but show obvious deviation in non-central collisions. We found the nonlinear coefficients extracted from the $\Delta c_3$ are identical in the two systems, indicating the insensitivity of $\chi_{4,22}$ to the initial state and the details of nuclear deformation.

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References

[1] J. Adams, et al., Experimental and theoretical challenges in the search for the quark gluon plasma: The STAR Collaboration’s critical assessment of the evidence from RHIC collisions, Nucl.Phys. A757 (2005) 102–183. arXiv:nucl-ex/0501009 doi:10.1016/j.nuclphysa.2005.03.086
[2] K. Ackox, et al., Formation of dense partonic matter in relativistic nucleus-nucleus collisions at RHIC: Experimental evaluation by the PHENIX collaboration, Nucl.Phys. A757 (2005) 184–283. arXiv:nucl-ex/0410003 doi:10.1016/j.nuclphysa.2005.03.086
[3] K. Aamodt, et al., Elliptic flow of charged particles in Pb-Pb collisions at 2.76 TeV, Phys. Rev. Lett. 105 (2010) 252302. arXiv:1011.3914 doi:10.1103/PhysRevLett.105.252302
[4] P. Romatschke, U. Romatschke, Viscosity Information from Relativistic Nuclear Collisions: How Perfect is the Fluid Observed at RHIC?, Phys.Rev.Lett. 99 (2007) 172301. arXiv:0706.1522 doi:10.1103/PhysRevLett.99.172301
[5] D. A. Teaney, Viscous Hydrodynamics and the Quark Gluon Plasma, 2010, pp. 207–266. arXiv:0905.2433 doi:10.1142/9789814293297_0004
[46] C. Shen, Z. Qiu, H. Song, J. Bernhard, S. Bass, U. Heinz, The iEBE-VISHNU code package for relativistic heavy-ion collisions, Comput. Phys. Commun. 199 (2016) 61–85. arXiv:1409.8164, doi:10.1016/j.cpc.2015.08.039

[47] H. Song, S. A. Bass, U. Heinz, Viscous QCD matter in a hybrid hydrodynamic+Boltzmann approach, Phys. Rev. C 83 (2011) 024912. arXiv:1012.0555, doi:10.1103/PhysRevC.83.024912

[48] H. Song, U. W. Heinz, Causal viscous hydrodynamics in 2+1 dimensions for relativistic heavy-ion collisions, Phys. Rev. C 77 (2008) 064901. arXiv:0712.3715, doi:10.1103/PhysRevC.77.064901

[49] H. Song, U. W. Heinz, Suppression of elliptic flow in a minimally viscous quark-gluon plasma, Phys. Lett. B 658 (2008) 279–283. arXiv:0709.0742, doi:10.1016/j.physletb.2007.11.019

[50] S. A. Bass, et al., Microscopic models for ultrarelativistic heavy ion collisions, Prog. Part. Nucl. Phys. 41 (1998) 255–369. [Prog. Part. Nucl. Phys.41,225(1998)]. arXiv:nucl-th/9803035, doi:10.1016/S0146-6410(98)00058-1

[51] M. Bleicher, et al., Relativistic hadron hadron collisions in the ultrarelativistic quantum molecular dynamics model, J. Phys. G 25 (1999) 1859–1896. arXiv:hep-ph/9909407, doi:10.1088/0954-3899/25/9/308

[52] J. S. Moreland, J. E. Bernhard, S. A. Bass, Alternative ansatz to wounded nucleon and binary collision scaling in high-energy nuclear collisions, Phys. Rev. C 92 (1) (2015) 011901. arXiv:1412.4708, doi:10.1103/PhysRevC.92.011901

[53] G. Nijs, W. van der Schee, Inferring nuclear structure from heavy isobar collisions using Trajectum [arXiv:2112.13771]

[54] S. Zhao, et al., Extracting the nuclear structure parameters in relativistic isobar collisions.

[55] J. Wang, et al., Importance of initial fluctuations on anisotropic flow in relativistic isobar collisions.

[56] M. Luzum, C. Gombeaud, J.-Y. Ollitrault, $v_4$ in ideal and viscous hydrodynamics simulations of nuclear collisions at the BNL Relativistic Heavy I on Collider (RHIC) and the CERN Large Hadron Collider (LHC), Phys. Rev. C 81 (2010) 054910. arXiv:1004.2024, doi:10.1103/PhysRevC.81.054910