A recent study of supersymmetric domain walls in $N = 1$ supergravity theories revealed a new class of domain walls interpolating between supersymmetric vacua with different non-positive cosmological constants. We classify three classes of domain wall configurations and study the geodesic structure of the induced space-time. Motion of massive test particles in such space-times shows that these walls are always repulsive from the anti-deSitter (AdS) side, while on the Minkowski side test particles feel no force. Freely falling particles far away from a wall in an AdS vacuum experience a constant proper acceleration, i.e. they are Rindler particles. A new coordinate system for discussing AdS space-time is presented which eliminates the use of a periodic time-like coordinate.
Domain walls in field theories have been understood for some time (see \cite{1} for a review). These objects are inherently relativistic since their surface tension is precisely equal their surface energy density. Such sources of nontrivial tension, or negative pressure, create a repulsive gravitational field in the sense that massive test particles are accelerated away from the wall. In addition, solving Einstein’s equations for an infinitesimally thin domain wall separating two Minkowski vacua reveals that the space-time metric is time dependent in a choice of coordinates which reflect the planar symmetry of the wall.

A recent study of domain walls arising in $N = 1, D = 4$ supergravity coupled to a chiral superfield discovered a class of vacuum domain walls with characteristically new features \cite{2,3}: (1) They interpolate between supersymmetric vacua with different non-positive cosmological constants; (2) They produce a space-time metric which is time-independent, \textit{i.e.} the domain walls are static; (3) They extend the notion of vacuum degeneracy to mean any supersymmetric vacuum in the theory, not just those with degenerate scalar potential energy.

The purpose of this paper is to classify and study the structure of the space-time induced by these domain walls by considering the motion of massive test particles. Such motion will fall into three classes depending on the three possible matter configurations compatible with the Bogomol’nyi equations. Comparisons with non-supersymmetric domain walls will be made as well. As a byproduct, we introduce a new choice of coordinates describing an AdS space-time which avoids the use of a periodic time-like coordinate.

First we review the relevant formulation \cite{3}. By exploiting the theory’s supersymmetry, a Bogomol’nyi bound for the ADM surface energy density of the planar (\textit{e.g.} in the $(x, y)$ plane) domain wall configuration has been derived. Using the technique of a generalized Nester’s form, we obtain the relation \cite{3}

\begin{equation}
\sigma - |C| = \int_{-\infty}^{\infty} \left[ -\delta\bar{\psi}_i \gamma^i g^{ij} \delta \psi_j + K_{TT} \delta \epsilon \chi \right] dz \geq 0 \tag{1}
\end{equation}
where $\sigma$ is the energy per unit area, $C$ is the topological charge, $g_{ij}$ is the metric of the space coordinates and $K_{T \bar{T}} > 0$ is the second derivative of the Kähler potential for the matter field $T$. $\delta_{\epsilon} \psi$ and $\delta_{\epsilon} \chi$ are the supersymmetry variations of the gravitino and supersymmetric partner of the matter chiral superfield, respectively. For supersymmetric bosonic backgrounds, $\delta_{\epsilon} \psi_{\mu} = \delta_{\epsilon} \chi = 0$. Thus the bound on the energy per area $\sigma$ is saturated by the absolute value of the topological charge $|C|$. Saturation of this bound yields two coupled first order differential equations for the matter and metric (the “square-root” of the Euler-Lagrange equations). We note that in order to ensure a time-like Killing vector, and thus a well defined notion of energy, we assume from the start that both the matter and metric are time independent.

The Ansatz for the space-time metric due to a flat domain wall in the $(x, y)$ plane is

$$ds^2 = A(z)(dt^2 - dz^2) + B(z)(-dx^2 - dy^2).$$  \hspace{1cm} (2)

Saturating the Bogomol’nyi bound (1) yields the following constraints on the metric $A(z)$, $B(z)$ and the scalar component of the scalar superfield $T(z)$:

$$Im(\partial_z T \frac{D_T W}{W}) = 0,$$  \hspace{1cm} (3)

$$\partial_z T(z) = -\zeta \sqrt{A|W|} e^{\frac{2\kappa}{2K_T W}K_{T \bar{T}}D_T W},$$  \hspace{1cm} (4)

and

$$\partial_z A^{-1/2} = \partial_z B^{-1/2} = \kappa \zeta |W| e^{\frac{2\kappa}{2K}}$$  \hspace{1cm} (5)

where $W(T)$ is the superpotential, $K(T, \bar{T})$ is the Kähler potential, $D_T W = W_T + \kappa K_T W$, $K_{T \bar{T}} = (\partial_T \partial_{\bar{T}} K)^{-1}$, and $\kappa = 8\pi G_N$ with $G_N$ being Newton’s constant. $\zeta$ is either $+1$ or $-1$ and can change sign when and only when $W$ vanishes.

* We use the space-time signature $(+ - - -)$. 

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The explicit expression for the ADM mass density (energy per area or surface tension) of the supersymmetric domain wall configuration satisfying constraints (3), (4) and (5) is given by

$$\sigma = |C| \equiv 2|\zeta|W e^{\frac{sK}{2}}|_{z=+\infty} - |\zeta|W e^{\frac{sK}{2}}|_{z=-\infty}| \equiv \frac{2}{\sqrt{3}}\kappa^{-1}\Delta(\zeta|\Lambda|^{1/2})$$  \hspace{1cm} (6)

where $\Lambda \equiv -3|\kappa We^{\frac{sK}{2}}|^2$ is the cosmological constant for the supersymmetric vacuum. We now comment on eq. (6) and the eqs. (3), (4), and (5).

It follows from (6) that there are no static domain walls saturating the Bogomol’nyi bound that interpolate between two supersymmetric vacua with zero cosmological constant. In this case $W(\infty) = W(-\infty) = 0$ and thus there is no energy associated with such a domain wall since $|C| \equiv 0$. This result is in agreement with the results of Ref.\cite{4}, where for infinitesimally thin domain walls with asymptotically Minkowski space-times only time-dependent metric solutions were obtained. The result from (6) implies that static supersymmetric domain wall solutions exist only if at least one of the vacua is AdS.

Eq. (3) is a consistency constraint which specifies the geodesic path between two supersymmetric vacua in the supergravity potential space $e^{\frac{sK}{2}}W \in \mathbb{C}$ when mapped from the $z$-axis ($-\infty, +\infty$). This geodesic equation has qualitatively new features in comparison with the geodesic equation in the global supersymmetric case\cite{2,3,5}. While in the global case geodesics are arbitrary straight lines in the $W-$plane, the local geodesic equation in the limit $\kappa \to 0$ (global limit of the local supersymmetric theory) leads to the geodesic equation $Im(\frac{\partial W}{W}) \equiv \partial z \vartheta = 0$ where $W$ has been written as $W(z) = |W|e^{i\vartheta}$. This in turn implies that as $\kappa \to 0$ the local geodesic equation reduces to the constraint that $W$ has to be a straight line passing through the origin; i.e. the phase of $W$ has to be constant mod $\pi$.

\* We define the cosmological constant as follows. The energy momentum tensor when $T$ is at its vacuum value $(D_T W = 0)$ is $T_{\mu\nu} = -3\kappa|We^{\frac{sK}{2}}|^2g_{\mu\nu}$. Therefore, Einstein’s equation $R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = \kappa T_{\mu\nu}$ can be written $R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = \Lambda g_{\mu\nu}$ with $\Lambda = -3|\kappa We^{\frac{sK}{2}}|^2$.\hspace{1cm} (4)
This observation in turn implies that the introduction of gravity imposes a strong constraint on the type of domain wall solutions. In particular, domain wall solutions in the global case interpolating between vacua in the \( e^{\kappa W} \) plane that do not lie along a straight line passing through the origin do not have an analogous solution in the local case. This result is a manifestation of the singular nature of a perturbation in Newton’s constant; qualitatively different physics results when \( G_N \equiv 0 \) relative to \( G_N > 0 \).

Eq. (4) for the \( T \) field (the “square root” of the equation of motion for \( T \)) and eq. (5) for the metric (the “square root” of the Einstein’s equation) are invariant under \( z \) translation as well as under rescalings of \((A,B) \rightarrow \lambda^2(A,B)\) and \( z \rightarrow \lambda^{-1} z \). Additionally, eq. (4) implies that \( \partial_z T(z) \rightarrow 0 \) as one approaches the supersymmetric minima which are points where \( D_T W = 0 \), thus indicating a solution smoothly interpolating between supersymmetric vacua. In general, the field \( T \) reaches the supersymmetric minimum exponentially fast as a function of \( z \).

We now concentrate the equation (5) for the metric. Our aim is to classify all the qualitatively different metric configurations.

First, we wish to emphasize in (5) the singular limit when gravity is turned off \((\kappa \rightarrow 0)\). As noted earlier, the same singular limit \((\kappa \rightarrow 0)\) is also responsible for the restrictive geodesics in the \( W \)-plane compared to a global theory which contains no gravitational information \((\kappa = 0)\). For \( \kappa = 0 \), the factors \( A \) and \( B \) are constant in the whole space; i.e. we have flat space-time everywhere. However, the moment \( \kappa > 0 \), \( A \) and \( B \) vary with \( z \).

We can set \( A(z) = B(z) \) without loss of generality which implies that the metric is conformally flat. Thus, our aim is to study the nature of the conformal factor \( A(z) \). We classify the following three types of static domain wall configurations which depend on the nature of the potential of the matter field.

\((I)\) A wall interpolating between a supersymmetric AdS vacuum \(|W_{+\infty}| \neq 0\) and a Minkowski supersymmetric vacuum \(|W_{-\infty}| = 0\). From (2) one sees that on the Minkowski side the conformal factor approaches a constant which can be
normalized to unity; \textit{i.e.}

\[ A(z) \to 1, \quad z \to -\infty. \quad (7) \]

On the AdS side \( A(z) \) falls off as \( z^{-2} \) with the strength of the fall-off determined by the strength of the cosmological constant; \textit{i.e.}

\[ A(z) \to \frac{3}{|\Lambda_{+\infty}|z^2}, \quad z \to +\infty. \quad (8) \]

The surface energy of this configuration as determined from (6) is:

\[ \sigma_I = \frac{2}{\sqrt{3}} \kappa^{-1} |\Lambda_{+\infty}|^{1/2} \quad (9). \]

Here, the cosmological constant of the supersymmetric AdS vacuum is \( \Lambda_{+\infty} = -3|\kappa W e^{\frac{z\kappa}{2}}|_{+\infty}^{1/2} \). Note also that the proper distance on the AdS side, \( d(z) = \int^z A^2(z')dz' \), grows as \( \ln z \) since \( A \propto z^{-2} \). This coordinate system therefore completely covers the space-time for points on the AdS side of the wall.

\textbf{(II)} A wall interpolating between two supersymmetric AdS vacua and where the superpotential passes through zero in between. The cosmological constant \textit{need not be the same} in both vacua. The point where \( W = 0 \) can be chosen at \( z = 0 \) without loss of generality. At this point \( \zeta \) changes sign and thus \( \zeta_{+\infty} = -\zeta_{-\infty} = 1 \). The conformal factor has the same asymptotic behaviour on both sides of the domain wall:

\[ A(z) \to \frac{3}{|\Lambda_{\pm\infty}|z^2}, \quad z \to \pm \infty \quad (10) \]

while at \( z = 0 \), \textit{i.e.} when \( W = 0 \), the conformal factor levels out, \textit{i.e.} \( \partial_z A(z)_{z=0} = 0 \). In other words \( A(z) \) has a characteristic (in general asymmetric) bell-like shape. Again asymptotically it corresponds to the AdS metric with infinite proper distance as \( z \to \pm \infty \).
The surface energy of this configuration is
\[ \sigma_{II} = \frac{2}{\sqrt{3}} \kappa^{-1} |\Lambda|_{-\infty}^{1/2} + |\Lambda|_{+\infty}^{1/2} \] (11).

(III) A wall interpolating between two AdS vacua, while the superpotential does not pass through zero. Again, the cosmological constant need not be the same in both vacua. In this case, since $|W|$ is never zero, $\zeta$ has the same sign in the whole region, say, $+1$. Eq.(2) in turn implies that the conformal factor necessarily blows up at some coordinate $z^*$. In general, the matter field $T$ has long since interpolated between the two vacua by the time the metric reaches $z^*$. Thus, the domain wall, defined as the region over which $T$ moves from one vacuum to another, lies entirely within the coordinate region $z^* < z < +\infty$. The conformal factor has the asymptotic behaviour:
\[
A(z) \rightarrow \begin{cases} 
\frac{3}{|\Lambda_{+\infty}|z^2}, & z \rightarrow +\infty \\
\frac{3}{|\Lambda_{z^*}|(z - z^*)^2}, & z \rightarrow z^*
\end{cases}
\] (12)

The surface energy of this configuration is
\[ \sigma_{III} = \frac{2}{\sqrt{3}} \kappa^{-1} ||\Lambda|_{z^*}^{1/2} - |\Lambda|_{+\infty}^{1/2}||. \] (13)

Note that the point $z^*$ is an infinite proper distance away from any other point $z > z^*$ since $\int dz A^{1/2} \rightarrow |\ln(z - z^*)|$.

In order to understand this singularity as well as the distinctive $z^{-2}$ behaviour of the conformal factor on the AdS side of a wall, it is appropriate at this point to study AdS space-time in a coordinate system which singles out the $z$ direction. For this purpose, we consider the metric
\[ ds^2 = (\alpha z)^{-2}(dt^2 - dx^2 - dy^2 - dz^2) \] (14)
with $z > 0$. As noted above, this is the form of the metric on the AdS side of the domain wall when the $T$ field has reached its supersymmetric vacuum. In this context, $\alpha$ is related to the cosmological constant by $\Lambda = -3\alpha^2$.  

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Eq. (14) is the form for the metric describing AdS space-time where the translational invariance is broken in the $z$ direction. The curvature tensor satisfies the maximally symmetric condition $R_{\mu\nu\sigma\rho} = \alpha^2 (g_{\mu\sigma} g_{\nu\rho} - g_{\mu\rho} g_{\nu\sigma})$, thus ensuring the existence of 10 Killing vectors. It follows that one can represent four dimensional AdS space-time as the hyperboloid $\eta_{AB} Y^A Y^B = \alpha^{-2}$ embedded in the five dimensional space with flat metric $\eta^{AB} = \text{diag}(+---+)$. We found that the following choice of coordinates

\begin{align*}
Y^0 &= te^{\alpha \tilde{z}} \\
Y^1 &= xe^{\alpha \tilde{z}} \\
Y^2 &= ye^{\alpha \tilde{z}} \\
Y^3 &= (\alpha)^{-1} \sinh(\alpha \tilde{z}) - \frac{1}{2} \alpha e^{\alpha \tilde{z}} (x^2 + y^2 - t^2) \\
Y^4 &= (\alpha)^{-1} \cosh(\alpha \tilde{z}) + \frac{1}{2} \alpha e^{\alpha \tilde{z}} (x^2 + y^2 - t^2)
\end{align*}

yield the metric intrinsic to the surface

$$ds^2 = e^{2\alpha \tilde{z}} (dt^2 - dx^2 - dy^2) - d\tilde{z}^2.$$  

This choice of intrinsic coordinates is motivated from the cosmological form for the metric in deSitter space (see, for example\cite{6}). By choosing $z = \alpha^{-1} e^{-\alpha \tilde{z}}$ we recover the form of the metric in (14). The 10 Killing vectors can be written as $L_{AB} = z_A \partial_B - z_B \partial_A$ and satisfy the $SO(3,2)$ Lie algebra \cite{7} $[L_{AB}, L_{CD}] = \eta_{BC} L_{AD} - \eta_{AC} L_{BD} - \eta_{BD} L_{AC} + \eta_{AD} L_{BC}$.

One should note that this choice of intrinsic coordinates covers only one-half of the full AdS space-time since $Y^3 + Y^4 > 0$. By choosing $(Y^3, Y^4, z) \rightarrow (-Y^3, -Y^4, -z)$, we cover the $Y^3 + Y^4 < 0$ region and have the metric (14) for $z < 0$. This choice should be contrasted with the standard set of coordinates

* We would like to thank F. Wilczek for pointing out this analogy to us.
respecting spherical symmetry about an origin which also completely covers AdS space-time\(^7\). In this case the metric has the form

\[ ds^2 = (\alpha \cos \rho)^{-2}(d\eta^2 - d\rho^2 - \sin^2 \rho (d\theta^2 + \sin^2 \theta d\phi^2)) \] (17)

with \(0 \leq \rho \leq \pi/2\), \(0 \leq \theta \leq \pi\), \(0 \leq \phi \leq 2\pi\).

The time-like coordinate \(\eta\) in (17) is periodic. However, the coordinates (15), in which time ranges over \(-\infty < t < \infty\), exhibit no periodic structure. What we have effectively done in choosing the planar coordinates (15) is to sacrifice a complete covering of AdS for a non-periodic time-like variable. Observers restricted to one half of AdS and who use the line element (14) to describe the space-time will see a flat Minkowski causality structure. And, as we will see, the time-like geodesics as seen by these observers will be hyperbolas in the \(t, z\) plane; i.e., freely falling test particles moving in the \(z\) direction are Rindler particles. Further study of AdS space-time in this coordinate system is being pursued\(^8\).

We return now to the study of the domain wall configurations. First we note that the previous discussion of the metric (14) allows for a straightforward interpretation of singular wall (type \(III\)) configuration. What we have is a domain wall separating two distinct regions of a generalized AdS space-time possessing a \(z\) dependent cosmological parameter which never passes through zero. The singular point \(z^*\) corresponds to the origin \(z = 0\) in the metric (14). On the “other side” of \(z^*\) lives an AdS space-time symmetric to the \(z > z^*\) side. Together these two sides completely cover the whole of the generalized AdS space-time just as the regions \(z > 0\) and \(z < 0\) in the planar coordinates leading to (14) cover all of AdS.

The above discussion of the three types of domain walls is illustrated by a simple polynomial form for the superpotential, a flat Kähler manifold: \(K = T\bar{T}\), and a real \(T\). We choose the superpotential

\[ W = \gamma T \left[ \frac{1}{5} T^4 - \frac{1}{3} T^2 (a^2 + b^2) + a^2 b^2 \right]. \] (18)

where \(\gamma\) is a mass dimension \(-2\) parameter which we set to unity and \(a^2\) and \(b^2\) are
positive dimension 2 parameters. The Kähler potential is chosen to be \( K = T \bar{T} \). Depending on the value of the parameters \( a \) and \( b \), the superpotential (18) provides us with a set of theories which accommodate the above three classes of the domain walls.

Note that the geodesic constraint \( \text{Im}(\partial_z T \frac{D_T W}{W}) = 0 \) is always satisfied for \( T = \bar{T} \). The supersymmetric vacuum satisfies \( D_TW \equiv W_T + \kappa K_TW = 0 \), where \( W_T = (T^2 - a^2)(T^2 - b^2) \). Thus, for \( a,b < < 1/\sqrt{\kappa} \), the supersymmetric vacua take place for real values of \( T \) near \( \pm a, \pm b \). By continuously changing the value of parameters \( a \) and \( b \), (18) provides us with a set of theories which can accommodate all the three classes of the domain walls discussed above. Figures 1, 2 and 3 display the conformal factor \( A \) for the these three classes of the domain walls. Each example corresponds to a different choice of the parameters \( a \) and \( b \), which we took for simplicity to be in the range \( << 1/\sqrt{\kappa} \).

We now turn to the study of the geodesic structure for the induced space-time. To do so, we analyze the motion of test particles in the background of a supersymmetric domain wall*. We will find that a test particle living in an AdS side of a type I or type II wall is always repelled from the wall. However, on a Minkowski side there will be no force on the test particle. We will show that the three classes of domain walls discussed above yield three distinct time-like geodesic motions of test particles.

The motion of massless particles is trivial since the metric is conformally flat; they simply define the usual 45° null rays in a space-time diagram. Particles moving in constant \( z \) planes will feel no force since the conformal factor is only a function of the transverse coordinate \( z \). Therefore, the only interesting geodesics will come from the \( 1 + 1 \) metric \( ds^2 = A(z)(dt^2 - dz^2) \). For massive particles, which live on

* Note, in general there is a coupling between test particles and the \( T \) field. E. g., in string theory where the \( T \) field is the modulus field of the internal compactification, the charged matter field couples in a very specific way to the \( T \) field. Another interesting example of the coupling is that of the supersymmetric partner \( \chi \) to the \( T \) field. The study of such test particles is under way [8]. Here we simply use test particles to map out the geodesics of the space-time.
time-like geodesics, we can parametrize the motion with the proper-time element
\[ ds^2 = d\tau^2 > 0. \]
Rearranging the metric and introducing the conserved energy per mass \( \epsilon \equiv A dt/d\tau \) of the particle yields the equation
\[
\left( \frac{dz}{dt} \right)^2 + \frac{A}{\epsilon^2} = 1.
\]  (19)

On a time-like geodesic, \( 0 \leq \left( \frac{dz/dt}{dt} \right)^2 < 1 \), and so the turning point, \( i.e. \ v \equiv dz/dt = 0 \), of the motion is where \( A/\epsilon^2 = 1 \).

A convenient way to understand massive particle motion is to consider a particle with a given initial coordinate velocity \( v_o \) at some coordinate \( z_o \); from (19) \( \epsilon \) for such a particle is \( \epsilon^2 = A(z_o)(1 - v_o^2)^{-1} \). Equation (19) can be thought of as the conservation of energy with an effective potential \( V(z) \equiv (1 - v^2) = \frac{A(z_o)}{A(z)}(1 - v_o^2) \). Again, points where \( V(z) = 1 \) are turning points.

The three classes of domain walls discussed above yield three time-like geodesics.

(I) Consider a particle with initial coordinate velocity \( -v_o \leq 0 \) moving towards a Minkowski-AdS wall (class I; see Figure 1) centered at the origin and let the approach be from the AdS side (\( z > 0 \)). There will be a turning point if the initial velocity of the test particle satisfies \( v_o^2 \leq 1 - A(z_o) \equiv v_c \). Particles with \( v_o^2 \) above the critical value \( 1 - A(z_o) \) will penetrate the wall after a period of slowing down and move at constant velocity in the Minkowski side. Figure 4 shows the phase plane \( (v = \frac{dz}{dt} \text{ vs. } z) \) for particles incident on a Minkowski-AdS wall from the \( z > 0 \) AdS side. Note the particles asymptote to the null ray \( v = \frac{dz}{dt} = 1 \) as \( z \rightarrow +\infty \) since \( A(z) \rightarrow 0 \) as \( z \rightarrow +\infty \).

(II) A wall separating two AdS space-times with \( W \) passing through zero (class II; see Figure 2) will have turning points on both sides if an initial velocity of a test particle incident on the wall is below the critical velocity. A particle with enough energy can pass from one side to the other after having slowed down in the transition region.

(III) Particles incident on the singular wall configuration (class III; see Figure 3) from the \( z > z^* \) side will always reach a turning point no matter how much initial
energy it has. One should not think of this behaviour as being due to an infinitely repulsive domain wall. Indeed, as discussed above, the wall region (around \( z = 0 \)) is well away from \( z^* \) which is actually an infinite proper distance away. This behaviour is a reflection of the repulsive nature of AdS space-time. A similar behaviour is seen in test particles moving radially outwards in pure AdS space whose metric is written in the spherically symmetric form (17) as well as particles directed towards \( z = 0 \) in the planar form (14). Both of these points correspond to proper distance infinity. In this sense AdS space can be thought of as a contracting space-time since particles can never reach proper radial \( \infty (\rho = \pi/2) \) in the spherical case or proper \( -\infty (z = 0) \) in the planar case. We note that a particle in the planar coordinates (14) can, however, reach proper distance \( +\infty \).

Another way to understand the repulsive nature of these space-times to calculate the force on a test particle which has a fixed position \( z \) (also known as a fiducial observer). This force can be obtained through the geodesic equation \( p^\alpha p^\beta_{;\alpha} = m f^\beta \) with \( p^\alpha = m \frac{dx^\alpha}{d\tau} \). The gravitational force acting on the fiducial observer is

\[
f^\beta = (0, 0, 0, -\frac{m}{2} A^{-2} \partial_z A).
\] (20)

For a metric which falls off as on the AdS side of a wall, this force is directed towards the AdS vacuum (e.g. \( z = +\infty \) in the type I wall depicted in figure 1). The magnitude of the acceleration is given by

\[
|a|^2 \equiv |f_{\alpha} f^\alpha|/m^2 = \left( \frac{1}{2} \frac{\partial_z \ln A}{A^{1/2}} \right)^2 = (\kappa|W|e^{\frac{\kappa}{2}})^2
\] (21)

where eq. (5) was used in the last equality. For fiducial observers in the region where \( T \) is essentially at its vacuum value; \( i.e. \) far away from the wall, the proper acceleration has the constant magnitude \( |a|^2 = |\Lambda_{\pm\infty}/3| \). Additionally, integration of (19) yields the hyperbolic world line for freely falling test particles \( z^2 - t^2 = \)

\* Our convention for the gravitational covariant derivative is \( p^\alpha_{;\mu} = \frac{dp^\alpha}{dx^\mu} + \Gamma^\alpha_{\mu\nu} p^\nu \) with \( \Gamma^\alpha_{\mu\nu} = \frac{1}{2} g^{\alpha\beta} (g_{\beta\mu,\nu} + g_{\beta\nu,\mu} - g_{\mu\nu,\beta}) \).
Therefore, a fiducial observer situated far away from a type $I$ or $II$ wall in a $\Lambda_{\pm\infty} \neq 0$ region will feel a constant acceleration $|\Lambda_{\pm\infty}/3|^{1/2}$ directed away from the wall as well as see test particles of energy per mass $\epsilon$ moving away from the wall with acceleration $|\Lambda_{\pm\infty}/3|^{1/2}\epsilon$. Thus, asymptotically in the AdS space-times test particles moving in the $z$ direction are Rindler particles\cite{9} whose constant proper acceleration is proportional to the square root of the cosmological constant, while on the Minkowski side they experience no gravitational force.

This result should be contrasted with the observation in Ref.\cite{4}, where infinitesimally thin reflection symmetric domain walls with asymptotically Minkowski space-times always repel the fiducial observer with a constant acceleration $\kappa\sigma/4$. Here, $\sigma$ is the energy per unit area of the domain wall.\footnote{Domain walls which separate two Minkowski vacua yet satisfy the nonstandard relation $\sigma = 2\tau$, where $\tau$ is the surface tension of the wall, produce no gravitational force on test particles. Walls of isotropically and uniformly distributed cosmic strings produce such an equation of state\cite{10}.} Note that these domain walls always produce a time dependent metric. In our case everything is static. In particular, for the type ($I$) domain walls interpolating between AdS and Minkowski space-times, the asymptotic acceleration on the AdS side can be written as $a = \kappa\sigma_I/2$, where $\sigma_I$ is the energy per unit area of the domain wall ($I$) defined in (9). However, on the Minkowski side $a \rightarrow 0$. For the type $II$ domain wall when the potential has $Z_2$ symmetry (see Fig. 2: both AdS vacua are degenerate), the energy per unit area (11) is $\sigma_{II} = 4\kappa^{-1}|\Lambda_{\pm\infty}/3|^{1/2}$ and the fiducial observer is repelled on both sides of the domain wall with the same acceleration $a_{\pm\infty} \rightarrow \kappa\sigma_{II}/4$ which resembles remarkably the form for the acceleration for the domain walls discussed in Ref.\cite{4}. In our case the domain wall also respects the $Z_2$ symmetry, however, it is completely static and its repulsive nature is due to the AdS nature of the asymptotic space-time.

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Figure Captions

Figure 1: Type (I) conformal factor $A(z)$ for a space-time with $\Lambda_{-\infty} = 0$ (Minkowski: $z < 0$) separated by a domain wall from a space-time with $\Lambda_{+\infty} < 0$ (AdS: $z > 0$). The wall, i.e. the region over which the matter field $T$ changes is centered at $z = 0$ and has thickness $\approx 200$ in $\sqrt{\kappa}$ units. The superpotential (18) has parameters $a^2 = 0, b^2 = 0.1$ and $T$ interpolates between $T_{-\infty} = 0 = a$ and $T_{\infty} = .318 \approx b$.

Figure 2: Type (II) Conformal factor $A(z)$ for a space-time with negative cosmological constant separated by a domain wall from its mirror image (i.e. a $Z_2$ configuration). The wall is centered at $z = 0$ and has thickness $\approx 200$ in $\sqrt{\kappa}$ units. The superpotential (18) has parameters $a^2 = .025, b^2 = 0.1$, and $T$ interpolates between $T_{\pm\infty} = \pm .1598 \approx \pm a$.

Figure 3: Type (III) conformal factor $A(z)$ for a space with negative cosmological constant separated by a domain wall from a space with a different negative cosmological constant. The superpotential $W$ never passes through a zero as $T$ interpolates from one vacuum to another. The domain wall is centered at $z = 0$ and has thickness $\approx 200$ where $zq$ is measured in $\sqrt{\kappa}$ units. The singularity is at $z^* \approx -5600$. The superpotential (18) has parameters $a^2 = .025, b^2 = 0.1$ and $T$ interpolates between $T_{-\infty} = .315 \approx b$ and $T_{\infty} = .160 \approx a$.

Figure 4: Phase plane ($v \equiv \frac{dz}{dt}$ vs. $z$) for massive test particles incident on a Minkowski-AdS (class (I)) wall from the AdS side. Particles which start at $z_o$ with an initial velocity $v_o$ will reach a turning point if $v_o^2 \leq v_c^2 \equiv 1 - A(z_o)$. Particles with a turning point will return to $z_o$ with the same velocity $v_o$ now directed away from the wall and will continue to accelerate toward the speed of light. Particles with $v_o^2 > v_c$ will penetrate the wall region after slowing down and continue on in the Minkowski side with a constant velocity.