Supersymmetry versus Gauge Symmetry on the Heterotic Landscape

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Abstract

One of the goals of the landscape program in string theory is to extract information about the space of string vacua in the form of statistical correlations between phenomenological features that are otherwise uncorrelated in field theory. Such correlations would thus represent predictions of string theory that hold independently of a vacuum-selection principle. In this paper, we study statistical correlations between two features which are likely to be central to any potential description of nature at high energy scales: gauge symmetries and spacetime supersymmetry. We analyze correlations between these two kinds of symmetry within the context of perturbative heterotic string vacua, and find a number of striking features. We find, for example, that the degree of spacetime supersymmetry is strongly correlated with the probabilities of realizing certain gauge groups, with unbroken supersymmetry at the string scale tending to favor gauge-group factors with larger rank. We also find that nearly half of the heterotic landscape is non-supersymmetric and yet tachyon-free at tree level; indeed, less than a quarter of the tree-level heterotic landscape exhibits any supersymmetry at all at the string scale.

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1 Introduction

Recent developments in string theory suggest that there exists a huge “landscape” of self-consistent string vacua [1]. The existence of this landscape is of critical importance for string phenomenology since the specific low-energy phenomenology that can be expected to emerge from string theory depends critically on the particular choice of vacuum state. Detailed quantities such as particle masses and mixings, and even more general quantities and structures such as the choice of gauge group, number of chiral particle generations, and the magnitude of the supersymmetry-breaking scale, can be expected to vary significantly from one vacuum solution to the next. Thus, in the absence of some sort of vacuum-selection principle, it is natural to determine whether there might exist generic string-derived statistical correlations between different phenomenological features that would otherwise be uncorrelated in field theory [2]. In this way, one can still hope to extract phenomenological predictions from string theory.

To date, there has been considerable work in this direction [2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12]; for recent reviews, see Ref. [13]. Collectively, this work addresses questions ranging from the formal (such as the finiteness of the number of string vacua and the methods by which they may be efficiently scanned and classified) to the phenomenological (such as the value of the cosmological constant, the scale of supersymmetry breaking, and the statistical prevalence of the Standard Model gauge group and three chiral generations).

In this paper, we shall undertake a statistical study of the correlations between two phenomenological features which are likely to be central to any description of nature at high energy scales: spacetime supersymmetry and gauge symmetry. Indeed, over the past twenty years, a large amount of theoretical effort has been devoted to studying string models with $N=1$ spacetime supersymmetry. However, it is important to understand the implications of choosing $N=1$ supersymmetry over other classes of string models (such as models with $N=2$ or $N=4$ supersymmetry, or even non-supersymmetric string models) within the context of the landscape. Moreover, since $N=1$ supersymmetry plays a huge role in current theoretical efforts to extend the Standard Model, we shall also be interested in understanding the statistical prevalence of spacetime supersymmetry across the landscape and the degree to which the presence or absence of supersymmetry affects other phenomenological features such as the choice of gauge group and the resulting particle spectrum.

In this paper, we shall investigate such questions within the context of the heterotic string landscape. There are several reasons why we shall focus on the heterotic landscape. First, heterotic strings are of tremendous phenomenological interest in their own right; indeed, these strings the framework in which most of the original work in string phenomenology was performed in the late 1980’s and early 1990’s. Second, heterotic strings have internal constructions and self-consistency constraints which are, in many ways, more constrained than those of their Type I (open) coun-
terparts. Thus, they are likely to exhibit phenomenological correlations which differ from those that might be observed on the landscape of, say, intersecting D-brane models or Type I flux vacua. Finally, in many cases these perturbative supersymmetric heterotic strings are dual to other strings (e.g., Type I orientifold models) whose statistical properties are also being analyzed in the literature. Thus, analysis of the perturbative heterotic landscape, both supersymmetric and non-supersymmetric, might eventually enable statistical tests of duality symmetries across the entire string landscape.

The first statistical study of the heterotic landscape appeared in Ref. [8]. This study, which focused exclusively on the statistical properties of non-supersymmetric (\(\mathcal{N}=0\)) tachyon-free heterotic string vacua, was based on a relatively small data set of four-dimensional heterotic string models [14] which were randomly generated using software originally developed in Ref. [15]. Since then, there have been several additional statistical examinations of certain classes of \(\mathcal{N}=1\) supersymmetric heterotic strings [10, 11]. Together, such studies can therefore be viewed as providing heterotic analogues of the Type I statistical studies reported in Refs. [5, 6].

Although the study we shall undertake here is similar in spirit to that of Ref. [8], there are several important differences which must be highlighted. First, as discussed above, we shall be focusing here on the effects of spacetime supersymmetry. Thus, we shall be examining models with all levels of spacetime supersymmetry (\(\mathcal{N}=0, 1, 2, 4\)), not just non-supersymmetric models, and examining how the level of spacetime supersymmetry correlates with gauge symmetry. Second, the current study is based on a much larger data set consisting of approximately \(10^7\) heterotic string models which was newly generated for this purpose using an update of the software originally developed in Ref. [15]. This data set is thus approximately two orders of magnitude larger than that used for Ref. [8], and represents literally the largest set of distinct heterotic string models ever constructed. Indeed, for reasons we shall discuss in Sect. 3, we believe that data sets of this approximate size are probably among the largest that can be generated using current computer technology.

But perhaps most importantly, because our heterotic-string data set was newly generated for the purpose of this study, we are able to quote results that take into account certain subtleties concerning so-called “floating correlations”. As discussed in Ref. [9], the problem of floating correlations is endemic to investigations of this type, and reflects the fact that not all physically distinct string models are equally likely to be sampled in any random search through the landscape. This thereby causes statistical correlations to “float” as a function of sample size. In Ref. [9], several methods were developed that can be used to overcome this problem, and it was shown through explicit examples that these methods allow one to extract correlations and statistical distributions which are not only stable as a function of sample size, but which also differ significantly from those which would have been naively apparent from a direct counting of generated models. We shall therefore employ these techniques in the current paper, extracting each of our statistical results in such a way that they...
represent stable correlations across the entire heterotic landscape we are examining.

As with most large-scale statistical studies of this type, there are several limitations which must be borne in mind. First, our sample size is relatively small, consisting of only \(\sim 10^7\) distinct models. However, although this number is minuscule compared with the numbers of string models that are currently quoted in most landscape discussions, we believe that the statistical results we shall obtain are stable as a function of sample size and would not change significantly as more models are added to the data sample. We shall discuss this feature in more detail in Sect. 3. Indeed, as mentioned above, data samples of the current size are likely to be the largest possible given current computer technology.

Second, the analysis in this paper shall be limited to correlations between only two phenomenological properties of these models: their low-energy gauge groups, and their levels of supersymmetry. More detailed examinations of the particle spectra of these models will be presented in Ref. [16].

Finally, the models we shall be discussing are stable only at tree level. For example, the models with spacetime supersymmetry continue to have flat directions which have not been lifted. Even worse, the non-supersymmetric models (even though tachyon-free) will generally have non-zero dilaton tadpoles and thus are not stable beyond tree level. Despite these facts, each of the string models we shall be studying represents a valid string solution at tree level, satisfying all of the necessary string self-consistency constraints. These include the requirements of worldsheet conformal/superconformal invariance, modular-invariant one-loop and multi-loop amplitudes, proper spacetime spin-statistics relations, and physically self-consistent layers of sequential GSO projections and orbifold twists. Thus, although such models may not represent the sorts of truly stable vacua that we would ideally like to be studying, it is reasonable to hope that any statistical correlations we uncover are likely to hold even after vacuum stabilization. Indeed, since no stable perturbative non-supersymmetric heterotic strings have yet been constructed, this sort of analysis is currently the state of the art for large-scale statistical studies of this type, and mirrors the situation on the Type I side, where state-of-the-art statistical analyses [5, 6, 7] have also focused on models which are only stable at tree level. Eventually, once the heterotic model-building technology develops further and truly stable vacua can be analyzed, it will be interesting to compare those results with these in order to ascertain the degree to which vacuum stabilization might affect these other phenomenological properties.

This paper is organized as follows. In Sect. 2 we describe the class of models that we shall be examining in this paper. In Sect. 3 we summarize our method of analysis which enables us to overcome the problem of floating correlations in order to extract statistically meaningful correlations. In Sect. 4 we present our results concerning the prevalence of spacetime supersymmetry across the heterotic landscape, and in Sect. 5 we present our results concerning correlations between spacetime supersymmetry and gauge groups. Finally, our conclusions are presented in Sect. 6.
2 The models

The models we shall be examining in this paper are similar to those studied in Ref. [8]. Specifically, each of the vacua we shall be examining in this paper represents a weakly coupled critical heterotic string compactified to four large (flat) spacetime dimensions. In general, such a string may be described in terms of its left- and right-moving worldsheet conformal field theories (CFT’s). For a string in four dimensions, these must have central charges \( (c_R, c_L) = (9, 22) \) in order to enforce worldsheet conformal anomaly cancellation, and must exhibit conformal invariance for the left-movers and superconformal invariance for the right-movers. While any such CFT’s may be considered, in this paper we shall focus on those string models for which these internal worldsheet CFT’s may be taken to consist of tensor products of free, non-interacting, complex (chiral) bosonic or fermionic fields.

As discussed in Ref. [8], this is a huge class of models which has been discussed and analyzed in many different ways in the string literature. On the one hand, taking these worldsheet fields as fermionic leads to the so-called “free-fermionic” construction \[17\] which will be our primary tool throughout this paper. In the language of this construction, different models are achieved by varying (or “twisting”) the boundary conditions of these fermions around the two non-contractible loops of the worldsheet torus while simultaneously varying the phases according to which the contributions of each such spin-structure sector are summed in producing the one-loop partition function. However, alternative but equivalent languages for constructing such models exist. For example, we may bosonize these worldsheet fermions and construct “Narain” models \[18, 19\] in which the resulting complex worldsheet bosons are compactified on internal lattices of appropriate dimensionality with appropriate self-duality properties. Furthermore, many of these models have additional geometric realizations as orbifold compactifications with appropriately chosen Wilson lines; in general, the process of orbifolding is quite complicated in these models, involving many sequential layers of projections and twists. All of these constructions generally overlap to a large degree, and all are capable of producing models in which the corresponding gauge groups and particle contents are quite intricate. Nevertheless, in all cases, we must ensure that all required self-consistency constraints are satisfied. These include modular invariance, physically sensible GSO projections, proper spin-statistics identifications, and so forth. Thus, each of these vacua represents a fully self-consistent string solution at tree level.

In order to efficiently survey the space of such four-dimensional string-theoretic vacua, we implemented a computer search based on the free-fermionic spin-structure construction \[17\]. Details of this study are similar to those of the earlier study described in Ref. [8], and utilize an updated version of the model-generating software that was originally written for Ref. [15]. In our analysis, we restricted our attention to those models for which our real worldsheet fermions can always be uniformly paired to form complex fermions, and therefore it was possible to specify the boundary con-
ditions (or spin-structures) of these real fermions in terms of the complex fermions directly. We also restricted our attention to cases in which the worldsheet fermions exhibited either antiperiodic (Neveu-Schwarz) or periodic (Ramond) boundary conditions around the non-contractible loops of the torus. Of course, in order to build a self-consistent string model in this framework, these boundary conditions must satisfy tight constraints. These constraints are necessary in order to ensure that the one-loop partition function is modular invariant and that the resulting Fock space of states can be interpreted as arising through a physically sensible projection from the space of all worldsheet states onto the subspace of physical states with proper spacetime spin-statistics. Thus, within a given string model, it is necessary to sum over appropriate sets of untwisted and twisted sectors with different boundary conditions and projection phases.

Our statistical analysis consisted of an examination of over $10^7$ distinct vacua in this class. Essentially, each set of fermion boundary conditions and GSO projection phases was chosen randomly in each sector, subject only to the required self-consistency constraints. However, in our statistical sampling, we placed essentially no limits on the complexity of the orbifold twisting (i.e., in the free-fermionic language, we allowed as many as sixteen linearly independent basis vectors). Thus, our statistical analysis included models of arbitrary intricacy and sophistication. We also made use of techniques developed specifically for analyzing string models generated in random searches, allowing for the mitigation of many of the effects of bias which are endemic to studies of this sort.

As part of our study, we generated string models with all degrees of spacetime supersymmetry ($N=0, 1, 2, 4$) that can arise in four dimensions. For $N=0$ models, we further demanded that supersymmetry be broken without introducing tachyons. Thus, the $N=0$ vacua are all non-supersymmetric but tachyon-free, and can be considered as four-dimensional analogues of the ten-dimensional $SO(16) \times SO(16)$ heterotic string [20] which is also non-supersymmetric but tachyon-free. However, other than this, we placed no requirements on other possible phenomenological properties of these vacua such as their possible gauge groups, numbers of chiral generations, or other aspects of the particle content. We did, however, require that our string construction begin with a supersymmetric theory in which the supersymmetry may or may not be broken by subsequent orbifold twists. (In the language of the free-fermionic construction, this is tantamount to demanding that our fermionic boundary conditions include a superpartner sector, typically denoted $W_1$ or $V_1$.) This is to be distinguished from a potentially more general class of models in which supersymmetry does not appear at any stage of the construction. This is merely a technical detail in our construction, and we do not believe that this ultimately affects our results.

As with any string-construction method, the free-fermionic formalism contains numerous redundancies in which different choices of worldsheet fermion boundary conditions and/or GSO phases lead to identical string models in spacetime. Indeed, a given unique string model can have many different representations in terms of
worldsheet constructions. For this reason, we judged string vacua to be distinct based on their spacetime characteristics — *i.e.*, their low-energy gauge groups and massless particle content.

| SUSY class | # distinct models |
|------------|------------------|
| $\mathcal{N}=0$ (tachyon-free) | 4,946,388 |
| $\mathcal{N}=1$ | 3,772,679 |
| $\mathcal{N}=2$ | 492,790 |
| $\mathcal{N}=4$ | 1,106 |
| Total: | 9,212,963 |

Table 1: The data set of perturbative heterotic strings analyzed in this paper. For each level of supersymmetry allowed in four dimensions, we list the number of corresponding distinct models generated. As discussed in the text, models are judged to be distinct based on their spacetime properties (*e.g.*, gauge groups and particle content). All non-supersymmetric models listed here are tachyon-free and thus are four-dimensional analogs of the $SO(16) \times SO(16)$ string model in ten dimensions.

Given this, our ultimate data set of heterotic strings is as described in Table 1. Note that all non-supersymmetric models listed in Table 1 are tachyon-free, and thus are stable at tree level. We should mention that while generating these models, we also generated over a million distinct non-supersymmetric tachyonic vacua which are not even stable at tree level. We therefore did not include their properties in our analysis, and recorded their existence only as a way of gauging the overall degree to which the tree-level heterotic string landscape is tachyon-free. Also note that as the level of supersymmetry increases, the number of distinct models in our sample set decreases. This reflects the fact that relatively fewer of these models exist, so they become more and more difficult to generate. This will be discussed further in Sects. 3 and 4.

Of course, the free-fermionic construction realizes only certain points in the full model space of self-consistent heterotic string models. For example, since each worldsheet fermion is nothing but a worldsheet boson compactified at a specific radius, a larger (infinite) class of models can immediately be realized through a bosonic formulation by varying these radii away from their free-fermionic values. However, this larger class of models has predominantly only abelian gauge groups and rather limited particle representations. Indeed, the free-fermionic points typically represent precisely those points at which additional (non-Cartan) gauge-boson states become massless, thereby enhancing the gauge symmetries to become non-abelian. Thus, the free-fermionic construction naturally leads to precisely the set of models which are likely to be of direct phenomenological relevance.

We should note that it is also possible to go beyond the class of free-field string models altogether, and consider models built from more complicated worldsheet
CFT’s (e.g., Gepner models). One could even go beyond the model space of critical string theories, and consider non-critical strings and/or strings with non-trivial background fields. Likewise, we may consider heterotic strings beyond the usual perturbative limit. However, although such models may well give rise to phenomenologies very different from those that emerge in free-field constructions, their spectra are typically very difficult to analyze and are thus not amenable to an automated statistical investigation.

3 Method of analysis

Each string model-construction technique provides a mapping between a space of internal parameters and a corresponding physical string model in spacetime. In the case of closed strings, for example, such internal parameters might include compactification moduli, boundary-condition phases, Wilson-line coefficients, or topological quantities specifying Calabi-Yau manifolds; in the case of open strings, by contrast, they might include D-brane dimensionalities and charges, wrapping numbers or intersection angles, fluxes, and the vevs of moduli fields. Regardless of the construction technique at hand, however, there is a well-defined procedure through which one can derive the spectrum and couplings of the corresponding model in spacetime.

Given this, one generally conducts a random search through the space of models by randomly choosing self-consistent values of these internal parameters, and then deriving the physical properties of the corresponding string models. Questions about statistical correlations are then addressed in terms of the relative abundances of models that emerge with different spacetime characteristics. Indeed, if \{\alpha, \beta, \gamma, ...\} denote these different spacetime characteristics (or different combinations of these characteristics), then we are generally interested in extracting ratios of population abundances of the form \(N_\alpha / N_\beta\), where \(N_\alpha\) and \(N_\beta\) are the numbers of models which exhibit physical characteristics \(\alpha\) and \(\beta\) across the landscape as a whole.

Clearly, we cannot survey the entire landscape, and thus we are forced to attempt to extract such ratios with relatively limited information. In particular, let us assume that our search has consisted of analyzing \(D\) different randomly generated sets of internal parameters, ultimately yielding a set of different models in spacetime exhibiting varying physical characteristics. Let \(M_\alpha(D)\) denote the number of distinct models which are found which exhibit characteristic \(\alpha\). Our natural tendency is then to attempt to associate

\[
\frac{N_\alpha}{N_\beta} \approx \frac{M_\alpha(D)}{M_\beta(D)} \quad (3.1)
\]

for some sufficiently large value of \(D\). While this relation might not hold exactly for relatively small values of \(D\), the expectation is that we might be able to reach sufficiently large values of \(D\) for which we might hope to extract reasonably accurate predictions for \(N_\alpha / N_\beta\).
Unfortunately, as has recently been discussed in Ref. [9], Eq. (3.1) does not generally hold for any reasonable value of $D$ (short of exploring the full landscape). Indeed, the violations of this relation are striking, even in situations in which sizable fractions of the landscape are explored, and will ultimately doom any attempt at extracting population fractions in this manner. In the remainder of this section, we shall first explain why Eq. (3.1) fails. We shall then summarize the methods which were developed in Ref. [9] for circumventing these difficulties, and which we will be employing in the remainder of this paper.

As stated above, each string model-construction technique provides a mapping between a space of internal parameters and a physical string model in spacetime. However, this mapping is not one-to-one, and there generally exists a huge redundancy wherein a single physical string model in spacetime can have multiple realizations or representations in terms of internal parameters. For this reason, the space of internal parameters is usually significantly larger than the space of obtainable distinct models.

The failure of this mapping to be one-to-one is critical because any random statistical study of the string landscape must ultimately take the form of a random exploration of the space of internal parameters that lead to these models. First, one must randomly choose a self-consistent configuration of internal parameters; only then can one derive and tabulate the spacetime properties of the corresponding model. But then we are faced with the question of determining whether spacetime models with multiple internal realizations should be weighted more strongly in our statistical analysis than models with relatively few realizations. In other words, we must decide whether our landscape measure should be based on internal parameters (wherein each model is weighted according to its number of internal realizations) or based on spacetime properties (wherein each physically distinct model is weighted equally regardless of the number of its internal realizations).

If we were to base our landscape measure on internal parameters, then these redundancies would not represent problems; they would instead become vital ingredients in our numerical analysis. However, if we are to perform statistics in the space of models in a physically significant way, it is easy to see that we are forced to count distinct models rather than distinct combinations of internal parameters. The reason for this is as follows. In many cases, these redundancies arise as the result of worldsheet symmetries (e.g., mirror symmetries), and even though such symmetries may be difficult to analyze and eliminate analytically for reasonably complicated models, their associated redundancies are similar to the redundancies of gauge transformations and do not represent new physics. In other cases, such redundancies are simply reflections of the failures or limitations of a particular model-construction technique; once again, however, they do not represent new physics, but rather reflect a poor choice of degrees of freedom for our internal parameters, or a mathematical difficulty or inability to properly define their independent domains. Finally, such redundancies can also emerge because entirely different model-construction techniques can often
lead to identical models in spacetime. Thus, two landscape researchers using different construction formalisms might independently generate random sets of models which partially overlap, but once again this does not mean that the models which are common to both sets should be double-counted when their statistical results are merged. Indeed, in all of these cases, redundancies in the mapping between internal parameters and spacetime properties do not represent differences of physics, but rather differences in the description of that physics. We thus must use spacetime characteristics (rather than the parameters internal to a given string construction) as our means of counting and distinguishing string models.

Many of these ideas can be illustrated by considering the $E_8 \times E_8$ heterotic string in ten dimensions. As is well known, this string model can be represented in many ways: as a $\mathbb{Z}_2$ orbifold of the $SO(32)$ supersymmetric string, as a $\mathbb{Z}_2 \times \mathbb{Z}_2$ orbifold of the non-supersymmetric $SO(32)$ heterotic string, and so forth. Likewise, this model can be realized through an orbifold construction, through a free-fermionic construction, through a bosonic lattice construction, and through other constructions as well. Yet, there is only a single $E_8 \times E_8$ string model in ten dimensions. It is therefore necessary to tally distinct string models, and not distinct internal formulations, when performing landscape calculations and interpreting their results.

Unfortunately, this redundancy inherent in the mapping between internal parameters and their corresponding string models implies that in any random exploration of the space of models, certain string models are likely to be sampled much more frequently than other models. Thus, one must filter out this effect by keeping a record of each distinct model that has already been sampled so that each time an additional model is generated (i.e., each time there is a new “attempt”), it can be compared against all previous models and discarded if it is not new. Although this is a memory-intensive and time-consuming process which ultimately limits the sizes of the resulting data sets that can be generated using current automated technology, this filtering can successfully be employed to eliminate model redundancies.

However, there remains the converse problem: because some models strongly dominate the random search, others effectively recede and are therefore extremely difficult to reach. They therefore do not tend to show up during the early stages of a random search, and tend to emerge only later in the search process after the dominant models have been more fully tallied. Indeed, as the search proceeds into its later stages, it is only the models with “rare” characteristics which increasingly tend to be generated, precisely because those models with “common” characteristics will have already been generated and tabulated. Thus, the proportion of models with “rare” characteristics tends to evolve rather dramatically as a function of time through the model-generation process.

This type of bias is essentially unavoidable, and has the potential to seriously distort the values of any numerical correlations that might be extracted from a random search through the landscape. In particular, as discussed in Ref. [9], this type of bias generally causes statistical correlations to “float” or evolve as a function of the
sample size of models examined. Moreover, since one can ultimately explore only a limited portion of the landscape, there is no opportunity to gather statistics at the endpoint of the search process at which these correlations would have floated to their true values. This, then, is the problem of floating correlations.

Fortunately, as discussed in Ref. [9], there are several statistical methods which can be used in order to overcome this difficulty. These methods enable one to extract statistical correlations and distributions which are stable as a function of sample size and which, with some reasonable assumptions, represent the statistical results that would be obtained if the full space of models could be explored. We shall now describe the most important of these methods, since we shall be using this technique throughout the rest of this paper.

In general, a model search proceeds as follows. One randomly generates a self-consistent set of internal parameters, and calculates the properties of the corresponding string model. One then compares this model against all models which have previously been generated: if the model is distinct, it is recorded and saved; if it is redundant, it is discarded. One then repeats this process. Early in the process, most attempts result in new distinct models because very few models have already been found. However, as the search proceeds, an increasing fraction of attempts fail to produce new models. This rise in the ratio of attempts per new model indicates that the space of models is becoming more and more explored. Thus, attempts per model can be used as a measure of how far into the full space of corresponding models our search has penetrated.

Therefore, if we are interested in extracting the ratio \( N_\alpha / N_\beta \) for two physical characteristics \( \alpha \) and \( \beta \), as discussed above Eq. (3.1), the solution is not to extract this ratio through Eq. (3.1) because such a relation assumes that the spaces of \( \alpha \)-models and \( \beta \)-models are being penetrated at exactly the same rates during the random search process. Rather, the solution [9] is to keep a record not only of the models generated as the search proceeds, but also of the cumulative average attempts per model that are needed in order to generate these models. We then extract the desired ratio \( N_\alpha / N_\beta \) through a relation of the form

\[
\frac{N_\alpha}{N_\beta} = \left| \frac{M_\alpha(d_\alpha)}{M_\beta(d_\beta)} \right| \frac{d_\alpha}{M_\alpha} = \frac{d_\beta}{M_\beta}
\]

(3.2)

where \( d_\alpha \) and \( d_\beta \) respectively represent the numbers of attempts that resulted in \( \alpha \)-models and \( \beta \)-models, regardless of whether the models in each class were distinct. Thus, we must essentially perform two independent search processes, one for \( \alpha \)-models and one for \( \beta \)-models, and we terminate these searches only when they have each reached the same degree of penetration as measured through their respective numbers of attempts per model \( d_\alpha / M_\alpha \). The value of \( N_\alpha / N_\beta \) obtained in this way should then be independent of the chosen reference value of \( d_\alpha / M_\alpha \) for sufficiently large \( d_\alpha / M_\alpha \). This method of extracting \( N_\alpha / N_\beta \) is discussed more fully in Ref. [9], where the derivation and limitations of this method are outlined in detail.
Of course, in the process of randomly generating string models, we cannot normally control whether a random new model is of the $\alpha$- or $\beta$-type. Both will tend to be generated together, as part of the same random search. Thus, our procedure requires that we completely disregard the additional models of one type that might be generated in the process of continuing to generate the required, additional models of the other type. This is the critical implication of Eq. (3.2). Rather than let our model-generating procedure continue for a certain duration, with statistics gathered at the finish line as in Eq. (3.1), we must instead establish two separate finish lines for our search process, one for $\alpha$-models and one for $\beta$-models. Of course, these finish lines are not completely arbitrary, and must be chosen such they correspond to the same relative degree of penetration of the $\alpha$- and $\beta$-model spaces. Indeed, these finish lines must be balanced so that they correspond to points at which the same ratio of attempts per model has been reached. However, these finish lines will not generally coincide with each other, which requires that some data actually be disregarded in order to extract meaningful statistical correlations.

As discussed in Ref. [9], Eq. (3.2) will enable us to extract a value for the ratio $N_\alpha/N_\beta$ which is stable as a function of sample size only when the biases within the $\alpha$-model space are the same as those within the $\beta$-model space. In such cases, we can refer to the physical characteristics $\alpha$ and $\beta$ as being in the same universality class. However, for a given model-generation method (such as the free-fermionic construction which we shall be employing in this paper), it turns out that many physical characteristics of interest $\{\alpha, \beta, \ldots\}$ have the property that they are in the same universality class. In the rest of this paper, correlations for physical quantities will be quoted only when the physical characteristics being compared are in the same universality class. The above method is then used in order to extract these correlations.

4 Supersymmetry on the heterotic landscape

In this section, we begin our analysis of the structure of the heterotic string landscape. In so doing, we shall also provide an explicit example of the method described in Sect. 3. Our focus in this section is to determine the extent to which string models with different levels of unbroken supersymmetry ($N=0, 1, 2, 4$) populate the tree-level four-dimensional heterotic landscape. For $N=0$ models, we shall further distinguish between models which are tachyon-free at tree level, and those which are tachyonic. Note that these characteristics are all mutually exclusive and together span the entire landscape of heterotic string models in four dimensions. Thus, our goal is to achieve nothing less than a partitioning of the full set of tree-level heterotic string models according to their degrees of supersymmetry. (We stress that this analysis will be the only case in which unstable tachyonic $N=0$ string models will be considered in this paper.) We will then proceed in Sect. 5 to examine questions related to correlations between the numbers of unbroken supersymmetry generators.
and the corresponding gauge groups.

The landscape of four-dimensional heterotic strings is a relatively large and complex structure. It may therefore be useful, as an initial step, to quickly recall the much smaller “landscape” of ten-dimensional heterotic strings. In ten dimensions, the maximal allowed supersymmetry is $\mathcal{N}=1$, and thus our tree-level ten-dimensional landscape may be partitioned into only three categories: $\mathcal{N}=1$ models, $\mathcal{N}=0$ tachyon-free models, and $\mathcal{N}=0$ tachyonic models. Note that since the $\mathcal{N}=0$ tachyonic models are not even stable at tree level, the tree-level “landscape” actually consists only of models in the first two categories. However, for convenience, in this section we shall use the word “landscape” to describe the full set of heterotic vacuum solutions regardless of stability.

| SUSY class | % of 10D landscape | % of reduced 10D landscape |
|------------|---------------------|---------------------------|
| $\mathcal{N}=0$ (tachyonic) | 66.7 | 62.5 |
| $\mathcal{N}=0$ (tachyon-free) | 11.1 | 12.5 |
| $\mathcal{N}=1$ | 22.2 | 25.0 |

Table 2: Classification of the ten-dimensional tree-level heterotic “landscape” as a function of the number of spacetime supersymmetries and the presence/absence of tachyons at tree level. As always, models are judged to be distinct based on their gauge groups and particle contents. The full ten-dimensional heterotic landscape consists of nine distinct string models, while the landscape of models accessible through our random search methods is reduced by one model. In either case, we see that two thirds of the tachyon-free portion of the ten-dimensional landscape is supersymmetric. Thus unbroken supersymmetry tends to dominate the “landscape” consisting of ten-dimensional models which are stable at tree level.

As is well known [21], the full set of $D = 10$ heterotic strings consists of nine distinct string models: two are supersymmetric [these are the $SO(32)$ and $E_8 \times E_8$ models], one is non-supersymmetric but tachyon-free [this is the $SO(16) \times SO(16)$ string model [20]], and six additional models are non-supersymmetric and tachyonic. Expressed as proportions of a full ten-dimensional heterotic landscape, we therefore find the results shown in the middle column of Table 2. It is important to note, however, that not all of these models would be realizable through the methods we shall be employing in this paper (involving a construction in which all degrees of freedom are represented in terms of complex worldsheet fermions). Indeed, one of the tachyonic non-supersymmetric models exhibits rank-reduction and thus would not be realizable in a random search of the sort we shall be conducting. Statistics for the corresponding “reduced” landscape of accessible models are therefore listed along the third column of Table 2; these are the statistics which will form the basis for future comparisons. Note that in either case, the tachyon-free portion of the ten-dimensional landscape is dominated by supersymmetric models. This suggests
that breaking supersymmetry without introducing tachyons is relatively difficult in ten dimensions.

Our goal is to understand how this picture changes after compactification to four dimensions. Towards this end, one procedure might be to randomly generate a large set of string models, and see how many models one obtains of each type after a certain fixed time as elapsed. However, as discussed in Sect. 3, these percentages will generally float or evolve as a function of the total number of models examined. This behavior is shown in Fig. 1 and we see that while the non-supersymmetric percentages seem to be floating towards greater values, the supersymmetric percentages seem to be floating towards lesser values.

Figure 1: The numbers of distinct string models exhibiting different amounts of spacetime supersymmetry, plotted as functions of the total number of distinct string models examined. Models exhibiting $N=4$ supersymmetry are too few to appear on this figure.

As discussed in Sect. 3, it is easy to understand the reason for this phenomenon. Clearly, as we continue to generate models randomly, an ever-increasing fraction of these models consists of models without supersymmetry. This in turn suggests that at any given time, we have already discovered a greater fraction of the space of supersymmetric models than non-supersymmetric models. This would explain why it becomes increasingly more difficult to randomly generate new, distinct supersymmetric models as compared with non-supersymmetric models, and why their relative
percentages show the floating behavior illustrated in Fig. 1.

How then can we extract meaningful information? As discussed in Sect. 3, the remedy involves keeping track of not only the total numbers of distinct models found in each supersymmetric class, but also the total number of attempts which yielded a model in each class, even though such models were not necessarily new. This information is shown in Table 3 for our total sample of $\gtrsim 10^7$ models.

| SUSY class | # distinct models | # attempts | avg. attempts/model |
|------------|-------------------|------------|---------------------|
| $N=0$ (tachyonic) | 1 279 484 | 3 810 838 | 2.98 |
| $N=0$ (tachyon-free) | 4 946 388 | 18 000 000 | 3.64 |
| $N=1$ | 3 772 679 | 24 200 097 | 6.41 |
| $N=2$ | 492 790 | 13 998 843 | 28.41 |
| $N=4$ | 1106 | 6 523 277 | 5 898.08 |
| Total: | 10 492 447 | 66 533 055 | 6.34 |

Table 3: This table expands on Table 1 by including the numbers of attempts to generate models in each class as well as the corresponding average numbers of attempts per distinct model. We also include information about the attempts which resulted in non-supersymmetric models whose spectra are tachyonic at tree level. It is apparent that the number of attempts per model increases rather dramatically as the level of supersymmetry increases, indicating that our heterotic string sample has penetrated further into the spaces of models with greater numbers of supersymmetries than into those with fewer.

As we see from Table 3, the number of required attempts per model increases dramatically with the level of supersymmetry. This in turn implies, for example, that although we may have generated many fewer distinct $N=4$ models than $N=1$ models, the full space of $N=4$ models has already been penetrated much more fully than the space of $N=1$ models. Thus, as we continue to generate more models, it should become relatively easier to generate non-supersymmetric models than supersymmetric models. If true, this would imply that the relative proportion of non-supersymmetric models should increase as we continue to generate more models, while the relative proportion of supersymmetric models should decrease. This is, of course, exactly what we have already seen in Fig. 1.

In order to extract final information concerning the relative sizes of these spaces, the procedure outlined in Sect. 3 instead requires that we do something different, and compare the numbers of distinct models generated in each class at those points in our model-generating process when their corresponding numbers of attempts per model are equal. It is only in this way that we can overcome the effects of floating correlations and extract stable relative percentages which do not continue to evolve as functions of the total sample size.

For example, let us consider the relative numbers of $N=1$ and $N=2$ models. Although we see from Table 3 that our full sample of $\gtrsim 10^7$ models contains approx-
approximately 7.66 times as many $\mathcal{N}=1$ models as $\mathcal{N}=2$ models, this is not the relative size of their corresponding model spaces because the $\mathcal{N}=2$ space of models has already been explored more fully than the $\mathcal{N}=1$ model space, with 6.41 attempts per $\mathcal{N}=1$ model compared with 28.41 attempts per $\mathcal{N}=2$ model. However, at an earlier point in our search, we found that it took an average of approximately 6.41 attempts to generate a new, distinct $\mathcal{N}=2$ model: this occurred when we had generated only approximately 90,255 models with $\mathcal{N}=2$ supersymmetry. This suggests that the space of $\mathcal{N}=1$ models is actually \( \frac{3772679}{90255} \approx 41.8 \) times as large as the space of $\mathcal{N}=2$ models.

Moreover, we can verify that this ratio is actually stable as a function of sample size. For example, at an even earlier point in our search when we had generated only \( \approx 2.22 \times 10^6 \) $\mathcal{N}=1$ models, we found that an average of 3.64 attempts were required to generate a new, distinct $\mathcal{N}=1$ model. However, this same average number of attempts per model occurred in our $\mathcal{N}=2$ sample when we had generated only \( \approx 53,000 \) $\mathcal{N}=2$ models. Thus, once again, the $\mathcal{N}=1$ and $\mathcal{N}=2$ model spaces appear to have a size ratio of \( \approx 41.8 : 1 \).

In this way, by comparing total numbers of models examined at equal values of attempts per model, we can extract the relative sizes of the spaces of models with differing degrees of supersymmetry and verify that these results are stable as functions of sample size (i.e., stable as functions of the chosen value of attempts per model). Our results are shown in Table 4. As far as we can determine, the percentages quoted in Table 4 represent the values to which the percentages in Fig. 1 would float if we could analyze what is essentially the full landscape. However, short of examining the full landscape, we see that there is no single point at which these percentages would simultaneously appear in any finite extrapolation of Fig 1. Instead, it is only by comparing the numbers of models obtained at different points in our analysis that the true ratios quoted in Table 4 can be extracted.

| SUSY class          | % of heterotic landscape |
|---------------------|--------------------------|
| $\mathcal{N}=0$ (tachyonic) | 32.1                     |
| $\mathcal{N}=0$ (tachyon-free) | 46.5                     |
| $\mathcal{N}=1$     | 20.9                     |
| $\mathcal{N}=2$     | 0.5                      |
| $\mathcal{N}=4$     | 0.003                    |

Table 4: Classification of the four-dimensional tree-level heterotic landscape as a function of the number of unbroken spacetime supersymmetries and the presence/absence of tachyons at tree level. This table is thus the four-dimensional counterpart of Table 2, which quoted analogous results for ten dimensions. Relative to the situation in ten dimensions, we see that compactification to four dimensions tends to favor breaking all spacetime supersymmetries without introducing tachyons at tree level.
Table 4 thus represents our final partitioning of the tree-level four-dimensional landscape according to the amount of supersymmetry exhibited. There are several rather striking facts which are evident from these results:

- First, we see that nearly half of the heterotic landscape is non-supersymmetric and yet tachyon-free.
- Second, we see that the supersymmetric portion of the heterotic landscape appears to account for less than one-quarter of the full four-dimensional heterotic landscape.
- Finally, models exhibiting extended (\(\mathcal{N} \geq 2\)) supersymmetries are exceedingly rare, representing less than one percent of the full landscape.

Of course, we stress once again that these results hold only for the tree-level landscape, i.e., models which are stable at tree level only. It is not clear whether these results would persist after full moduli stabilization. However, assuming that they do, these results lead to a number of interesting conclusions.

The first conclusion is that the properties of the tachyon-free heterotic landscape as a whole are statistically dominated by the properties of string models which do not have spacetime supersymmetry. Indeed, the \(\mathcal{N}=0\) string models account for over three-quarters of this portion of the heterotic string landscape. The fact that the \(\mathcal{N}=0\) string models dominate the tachyon-free portion of the landscape suggests that breaking supersymmetry without introducing tachyons is actually favored over preserving supersymmetry for this portion of the landscape. Indeed, we expect this result to hold even after full moduli stabilization, unless an unbroken supersymmetry is somehow restored by stabilization.

The second conclusion which can be drawn from these results is that the supersymmetric portion of the landscape is almost completely comprised of \(\mathcal{N}=1\) string models. Indeed, only 2% of the supersymmetric portion of the heterotic landscape has more than \(\mathcal{N}=1\) supersymmetry. This suggests that the correlations present for the supersymmetric portion of the landscape can be interpreted as the statistical correlations within the \(\mathcal{N}=1\) string models, with the \(\mathcal{N}=2\) correlations representing a correction at the level of 2% and the \(\mathcal{N}=4\) correlations representing a nearly negligible correction.

It is natural to ask what effects are responsible for this hierarchy. As was discussed in Sect. 3, two string models are considered distinct if any of their spacetime properties are found to be different. Two models which have the same number of unbroken spacetime supersymmetries must therefore differ in other features, such as their gauge groups and particle representations. Thus, if there exist more models with one level of supersymmetry than another, this must mean that there are more string-allowed configurations of gauge groups and particle representations with one level of supersymmetry than the other. Indeed, given the results of Table 4, our
expectation is that increasing the level of supersymmetry will have the effect of decreasing the number of distinct models with a given gauge group, and possibly even the range of allowed gauge groups. We shall test both of these expectations explicitly in Sect. 5.

5 Supersymmetry versus gauge groups

Within the heterotic string, worldsheet self-consistency conditions arising from the requirements of conformal anomaly cancellation, one-loop and multi-loop modular invariance, physically sensible GSO projections, etc., impose many tight constraints on the allowed particle spectrum. These constraints simultaneously affect not only the spacetime Lorentz structure of the theory (such as is involved in spacetime supersymmetry), but also the internal gauge structure of the theory. Thus, it is precisely within the context of string theory that we expect to find correlations between supersymmetries and gauge symmetries — features which would otherwise be uncorrelated in theories based on point particles.

In general, these correlations can lead to certain tensions in a given string construction. Models exhibiting large numbers of unbroken supersymmetries may be expected to have relatively rigid gauge structures, and vice versa. There are two specific types of correlations which we shall study. First, we shall analyze how the degree of supersymmetry affects the range of possible allowed gauge groups. For example, in extreme cases it may occur that certain gauge symmetries may not even be allowed for certain levels of spacetime supersymmetry. Second, even within the context of a fixed gauge group, we can expect the degree of spacetime supersymmetry to affect the range of allowed particle representations which can appear at the massless level. In other words, the number of distinct string models with a given fixed gauge group may be highly sensitive to the degree of spacetime supersymmetry.

Some of these features are already on display in the ten-dimensional heterotic “landscape”. For example, no gauge group is shared between those ten-dimensional models with supersymmetry and those without. Moreover, in each case, there is only a single model with each allowed gauge group. Thus, in ten dimensions, the specification of the level of supersymmetry (and/or the gauge group) is sufficient to completely fix the corresponding particle spectrum.

Clearly, in four dimensions, things will be far more complex. In particular, we shall study three correlations in this section:

• First, we shall focus on the number of allowed gauge groups as a function of the degree of supersymmetry. We shall also study gauge-group multiplicities — i.e., the probabilities that there exist distinct string models with the same gauge group but different particle spectra. This will be the focus of Sect. 5.1.

• Second, as a function of the degree of supersymmetry, we shall investigate “shatter” — i.e., the degree to which our total (rank-22) gauge group is “shat-
tered” into distinct irreducible factors, or equivalently the average rank of each irreducible gauge-group factor. This will be the focus of Sect. 5.2.

- Finally, as a function of the degree of supersymmetry, we shall study the probabilities of realizing specific (combinations of) gauge-group factors in a given string model. This will be the focus of Sect. 5.3.

As we shall see, these studies will find deep correlations which ultimately reflect the string-theoretic tension between supersymmetry and the string consistency conditions.

5.1 Numbers and multiplicities of unique gauge groups

We begin by studying the total numbers of distinct gauge groups which can be realized as a function of the number of unbroken supersymmetries in a given string model.

To do this, one direct approach can might be to classify models according to their numbers of unbroken spacetime supersymmetries, and tabulate the numbers of distinct gauge groups which appear as functions of the total number of models in each class. As we continue to generate more and more models, we then obtain the results shown in Fig. 2.

It is evident from Fig. 2 that for a fixed sample size, models with more unbroken supersymmetries tend to exhibit larger numbers of distinct gauge groups, or equivalently smaller numbers of model multiplicities per gauge group. For example, we see from Fig. 2 that when each class of models has reached a sample size of 500,000 models, the tachyon-free $\mathcal{N}=0$ models have a greater multiplicity per gauge group than $\mathcal{N}=1$ models by an approximate factor $\approx 1.4$, while the $\mathcal{N}=2$ models have a smaller multiplicity per gauge group than the $\mathcal{N}=1$ models by an approximate factor $\approx 0.8$. However, it is easy to understand this behavior. As the level of supersymmetry increases, there are more constraints on the possible particle spectra that can emerge for a given gauge group. This in turn implies that there are likely to be fewer ways for two models with the same gauge group to be distinct, which in turn implies that there is a greater chance that distinct models will be forced to exhibit distinct gauge groups. Thus, models exhibiting greater amounts of supersymmetry are likely, on average, to exhibit greater numbers of gauge groups amongst a fixed number of models.

Of course, as also evident from Fig. 2 the multiplicity of distinct models per gauge group exhibits a strong, floating dependence on the sample size. Therefore, in order to extract a stable ratio of multiplicity ratios — one which presumably represents the values of these ratios when extrapolated to the full landscape — we must employ the methods described in Sect. 3. We then obtain the results shown in the middle column of Table 5. Using these results in conjunction with the corresponding ratios of landscape magnitudes in Table 4, we can also calculate the relative numbers of
Figure 2: Numbers of distinct gauge groups obtained as functions of the number of distinct string models generated. Each curve corresponds to models with a different number of unbroken spacetime supersymmetries, with $\mathcal{N}=0$ signifying models which are non-supersymmetric but tachyon-free. We see that for a fixed sample size, models with more unbroken supersymmetries tend to exhibit a larger number of distinct gauge groups. (Note that models with $\mathcal{N}=4$ supersymmetry are too few to be shown in this plot.)

distinct gauge groups realizable within each SUSY class of models. These results are shown in the final column of Table 5. Note that in each case, these quantities are quoted as ratios relative to their $\mathcal{N}=1$ values; this represents the most detailed information that can be extracted using the methods of Sect. 3.

We see from Table 5 that both the average multiplicities per gauge group and the total numbers of realizable gauge groups are monotonically decreasing functions of the number of unbroken supersymmetries. While this is to be expected on the basis of the arguments described above, we must realize that our class of $\mathcal{N}=0$ models does not consist of all non-supersymmetric models, but merely those which are tachyon-free. Thus, the requirement of avoiding tachyons could have turned out to be more stringent than the requirement of maintaining an unbroken supersymmetry, at least as far as generating a variety of gauge groups is concerned. This is indeed what happens in the ten-dimensional landscape, where there are fewer realizable gauge groups for non-supersymmetric tachyon-free models than for models with $\mathcal{N}=1$ supersymmetry.
Table 5: The average relative multiplicities (distinct models per gauge group) and total numbers of realizable gauge groups, evaluated for heterotic string models with $\mathcal{N} = 0, 1, 2$ unbroken spacetime supersymmetries. In each case, these quantities are normalized to their $\mathcal{N}=1$ values.

| SUSY class | avg. multiplicity per gauge group | # of realizable gauge groups |
|------------|----------------------------------|------------------------------|
| $\mathcal{N}=0$ (tachyon-free) | 1.65 | 1.35 |
| $\mathcal{N}=1$ | 1.00 | 1.00 |
| $\mathcal{N}=2$ | 0.89 | 0.03 |

However, the results in Table 5 indicate that the opposite is true in $D = 4$.

Note that in Table 5, we do not quote results for the $\mathcal{N}=4$ portion of the heterotic landscape because the absolute numbers of models in this class are so small that no stable numerical results can be extracted relative to the other levels of supersymmetry. However, it is worth noting that literally each $\mathcal{N}=4$ model in our sample has a unique gauge group, so the absolute (rather than relative) gauge-group multiplicity in the $\mathcal{N}=4$ case is exactly 1.000. This only reinforces our general observation that increased levels of supersymmetry reduce the gauge-group multiplicity; indeed, we now see that the case of maximal supersymmetry appears to result in the minimal allowed gauge-group multiplicity. It is likely that this result can be proven analytically for the $\mathcal{N}=4$ landscape as a whole.

5.2 Shatter/average rank

Having studied the numbers of different possible gauge groups, we now turn our attention to the gauge groups themselves. Once again, our goal is to study how these gauge groups depend on the presence or absence of spacetime supersymmetry.

To begin the discussion, our focus in this section will be on what we call “shatter” [8]. Recall that the heterotic string models we are considering all have gauge groups with total rank 22. This stretches from models with gauge group $SO(44)$ all the way down to models with gauge groups of the form $U(1)^n \times SU(2)^{22-n}$ with potentially all values of $n$ in the range $0 \leq n \leq 22$. Following Ref. [8], we shall define the “shatter” for a given string model as the number of distinct irreducible gauge-group factors into which its total rank-22 gauge group has been shattered. Note that for this purpose, factors of $SO(4) \sim SU(2) \times SU(2)$ contribute two units to shatter. Since the total rank of the gauge group is fixed at 22 for such models, this means that shatter is also a measure of the average rank of the individual group group factors, with $\langle \text{rank} \rangle = 22/\text{shatter}$. Roughly speaking, shatter can also be taken as a measure of the degree of complexity needed for the construction of a given string model, with increasingly smaller individual gauge-group factors tending to require increasingly
many non-overlapping sequences of orbifold twists and Wilson lines.

Given this definition of shatter, we may then calculate the distribution of shatter across the landscape of heterotic strings. We may calculate, for example, the relative probabilities that models with certain levels of shatter emerge across the landscape, and ask how these probability distributions vary with the amount of spacetime supersymmetry present in the model.

Our results are shown in Fig. 3. Once again, we stress that our raw data tends to evolve significantly as a function of the sample size of models considered. It is therefore necessary to employ the techniques described in Sect. 3 in order to extract stable results which should apply across the landscape as a whole. In practice, this requires a difficult and time-consuming process in which each of the data points shown in Figs. 3 for $\mathcal{N}=0, 1, 2$ has individually been extracted through the limiting procedure described in Sect. 3. Only then is an entire “curve” constructed for each level of supersymmetry, as shown.

For the $\mathcal{N}=4$ case, by contrast, our sample size is too small to permit stable results to be extracted. However, the fact that the attempts per model count in Table 3 is so large for the $\mathcal{N}=4$ models suggests that our $\mathcal{N}=4$ sample has already explored a significant fraction (and perhaps even most) of the corresponding landscape. The $\mathcal{N}=4$ curve in Fig. 3 thus represents a direct tally of our $\mathcal{N}=4$ sample set.

As evident from Fig. 3, certain features of these plots are independent of the level of spacetime supersymmetry. These therefore represent general trends which hold across the entire tachyon-free heterotic string landscape. For example, one general trend is a strong preference for models with relatively high degrees of shatter and correspondingly small average ranks for individual gauge-group factors — models exhibiting shatters near or in the teens clearly dominate. On the other hand, this preference for highly shattered gauge groups does not appear to extend to the limit of completely shattered models with shatter=22; indeed, the set of models with only rank-one gauge-group factors seems to represent a fairly negligible portion of the landscape regardless of the degree of supersymmetry. This indicates that most models in this class have gauge groups which contain at least one factor of rank greater than one.

Another universal trend implied by (though not explicitly shown in) Fig. 3 is that string models with shatters of less than four accrue relatively little measurable amount of probability. Even in the $\mathcal{N}=4$ case, these models are thus actually quite rare across the landscape as a whole. In some sense, this too is to be expected, since

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* Of course, we stress that this conclusion applies only for models in the free-fermionic class. In general, it is always possible to deform away from the free-fermionic limit by adjusting the internal radii of the worldsheet fields away from their free-fermionic values; in such cases, we expect all gauge symmetries to be broken down to $U(1)^2$. However, as noted earlier, the free-fermionic points typically represent precisely those points at which additional (non-Cartan) gauge-boson states become massless, thereby enhancing the gauge symmetries to become non-abelian. Thus, as discussed more fully in Sect. 2, the free-fermionic construction naturally leads to precisely the set of models which are likely to be of direct phenomenological relevance.
Figure 3: The absolute probabilities of obtaining distinct four-dimensional heterotic string models with different numbers of unbroken supersymmetries, plotted as functions of the degree to which their gauge groups are “shattered” into separate irreducible factors. The total value of the points (the “area under the curve”) in each case is 1. Here $N=0$ refers to models which are non-supersymmetric but tachyon-free.
there are many more ways of breaking a large gauge symmetry through orbifolds and non-trivial Wilson lines than of preserving it.

Despite these universal features, we see that spacetime supersymmetry nevertheless does have a significant effect on the shapes of these curves. In this regard, there are two features to note.

First, we observe that as the degree of unbroken supersymmetry increases, the range of probable shatter values also tends to increase, with probability shifting from models with high shatters to models with lower shatters. This is especially noticeable when comparing the distribution of the \( \mathcal{N}=2 \) and \( \mathcal{N}=4 \) models with those of the \( \mathcal{N}=0 \) and \( \mathcal{N}=1 \) models. These results indicate that models exhibiting smaller amounts of shatter (i.e., models whose gauge-group factors have larger individual average ranks) become somewhat more probable as the level of supersymmetry increases. Ultimately, this correlation between unbroken supersymmetry and unbroken gauge symmetry emerges since both have their underlying origins in how our orbifold twists and Wilson lines are chosen.

Second, and perhaps more unexpectedly, we see that the degree of supersymmetry also affects the overall profiles of these curves. While the \( \mathcal{N}=0 \) curve is relatively smooth, exhibiting a single peak at shatter=20, these curves begin to experience even/odd oscillations as the degree of supersymmetry increases, with odd values of shatter significantly favored over even values when supersymmetry is present. The origins of this phenomenon are less apparent, and perhaps lie in the modular invariance and anomaly cancellation constraints which correlate the orders of the allowed twists leading to self-consistent string models. Interestingly, this even/odd behavior continues into the \( \mathcal{N}=4 \) case, although these oscillations are significantly less pronounced and flip sign, with evens now dominating over odds.

One notable feature of the \( \mathcal{N}=4 \) curve is its approximate reflection symmetry around shatter=10. It is unclear whether this is an exact symmetry which holds in situations with maximal supersymmetry, or whether this is merely an accident.

5.3 Specific gauge-group factors

Finally, we turn to an analysis of the probabilities of realizing individual gauge-group factors. Just how likely is it, say, that a randomly chosen heterotic string model will exhibit an \( SU(3) \) factor in its gauge group, and how does this probability correlate with the spacetime supersymmetry of the model?

Just as with previous questions, addressing this issue requires a detailed analysis along the lines discussed in Sect. 3. This is because the probabilities of realizing different gauge-group factors also float quite strongly as a function of sample size. As dramatic illustration of this fact, let us restrict our attention to models with \( \mathcal{N}=1 \) spacetime supersymmetry and calculate the probability that a given model will exhibit an \( SU(3) \) gauge-group factor as a function of the number of models we have examined. We then obtain the result shown in Fig. 4, and it is clear that the
Figure 4: The percentage of distinct four-dimensional $\mathcal{N}=1$ supersymmetric heterotic string models exhibiting at least one $SU(3)$ gauge-group factor, plotted as a function of the number of models examined for the first 1.25 million models. We see that as we generate further models, $SU(3)$ gauge-group factors become somewhat more ubiquitous — i.e., the fraction of models with this property floats. One must therefore account for this floating behavior using the methods described in Sect. 3 in order to extract meaningful information concerning the relative probabilities of specific gauge-group factors.

percentage of models with $SU(3)$ gauge-group factors floats rather significantly as a function of the sample size. Indeed, on the basis of this information alone, it would be quite impossible to determine the final value to which this curve might float. Just as with previous examples, this floating behavior ultimately occurs because models with $SU(3)$ gauge-group factors are relatively difficult to generate using the construction methods we are employing; thus, they tend to emerge in increasing numbers only after other models are exhausted. As discussed more fully in Ref. [9], this does not imply that there are fewer of these models or that our construction method cannot ultimately reach them — all we can conclude is that they are less likely to be generated in a random search than other models, and thus they tend to emerge only later in the search process. Indeed, as we shall shortly see, models with $SU(3)$ gauge-group factors actually tend to dominate the landscape.

Therefore, in order to extract meaningful results, we again employ the methods
discussed in Sect. 3. We then obtain the final percentages quoted in Table 6. We observe, in particular, that the probability of models with $\mathcal{N}=1$ supersymmetry exhibiting at least one $SU(3)$ gauge-group factor has actually risen all the way to 98%. The fact that this probability has floated from nearly 55% to 98% only reinforces the importance of the analysis method presented in Sect. 3, and illustrates the need to properly account for floating correlations when quoting statistical results for such studies.

| gauge group | $\mathcal{N}=0$ | $\mathcal{N}=1$ | $\mathcal{N}=2$ | $\mathcal{N}=4$ |
|-------------|-----------------|-----------------|-----------------|-----------------|
| $U_1$       | 99.9            | 94.5            | 68.4            | 89.6            |
| $SU_2$      | 62.46           | 97.4            | 64.3            | 60.9            |
| $SU_3$      | 99.3            | 98.0            | 93.0            | 45.1            |
| $SU_4$      | 14.46           | 30.0            | 39.0            | 53.5            |
| $SU_5$      | 16.78           | 43.5            | 66.3            | 33.8            |
| $SU_>5$     | 0.185           | 1.7             | 10.6            | 73.0            |
| $SO_8$      | 0.482           | 1.6             | 6.2             | 21.1            |
| $SO_{10}$   | 0.084           | 0.2             | 1.6             | 18.7            |
| $SO_{>10}$  | 0.005           | 0.038           | 0.77            | 7.5             |
| $E_{6,7,8}$ | 0.0003          | 0.03            | 0.16            | 11.5            |

Table 6: Percentages of heterotic string models exhibiting specific gauge-group factors as functions of their spacetime supersymmetry. Here $SU_>5$ and $SO_{>10}$ collectively indicate gauge groups $SU(n)$ and $SO(2n)$ for any $n > 5$, while $\mathcal{N}$ refers to the number of unbroken supersymmetries at the string scale. Note that the $\mathcal{N}=0$ models are all tachyon-free.

As we see from Table 6, supersymmetry can have quite sizable effects upon the probability of realizing specific groups. However, there are some general trends that hold for the full heterotic landscape. These trends include:

- A preference for $SU(n+1)$ over $SO(2n)$ groups for each rank $n$. Even though these two groups have the same rank, it seems that $SU$ groups are more common than the $SO$ groups for all levels of supersymmetry.

- Groups with smaller rank are much more common than groups with larger rank. Once again, this also appears to hold for all levels of supersymmetry.

- Finally, the gauge-group factors comprising Standard-Model gauge group $G_{SM} \equiv SU_3 \times SU_2 \times U_1$ are particularly common, much more so than those of any of its grand-unified extensions.

As we found in Sect. 4, the $\mathcal{N}=0$ string models dominate the tachyon-free portion of the heterotic landscape. Similarly, the $\mathcal{N}=1$ string models are the dominant part of the supersymmetric portion of the landscape. Nevertheless, it is interesting to
Table 7: Percentage of heterotic string models exhibiting specific gauge-group factors, quoted across the entire landscape of tachyon-free models (both supersymmetric and non-supersymmetric) as well as across only that subset of models with at least $N \geq 1$ spacetime supersymmetry. These results are derived from those of Table 6 using the landscape weightings in Table 4.

| gauge group | entire landscape | SUSY subset |
|-------------|-----------------|-------------|
| $U_1$       | 98.00           | 93.89       |
| $SU_2$      | 73.22           | 96.62       |
| $SU_3$      | 98.85           | 97.88       |
| $SU_4$      | 19.42           | 30.21       |
| $SU_5$      | 25.37           | 44.03       |
| $SU_{>5}$   | 0.73            | 1.92        |
| $SO_8$      | 0.87            | 1.71        |
| $SO_{10}$   | 0.13            | 0.23        |
| $SO_{>10}$  | 0.02            | 0.06        |
| $E_{6,7,8}$ | 0.01            | 0.03        |

examine the gauge-group probabilities across both of these portions of the landscape. These probabilities are easy to calculate by combining the results in Tables 4 and 6, leading to the results shown in Table 7.

Several features are immediately apparent from Table 7. First, gauge groups with larger ranks appear to be favored more strongly across the supersymmetric subset of the landscape than across the tachyon-free landscape as a whole. Since each of our heterotic string models in this class has a gauge group of fixed total rank, this preference for higher-rank gauge groups necessarily comes at the price of sacrificing smaller-rank gauge groups. Indeed, it often happens that this preference for larger-rank gauge groups actually precludes the appearance of any small-rank gauge groups whatsoever. Interestingly, the supersymmetric portion of the landscape seems to sacrifice $U(1)$ primarily and $SU(3)$ to a lesser extent. This is in contrast to $SU(2)$, which is actually more strongly favored in the supersymmetric portion of the landscape than in the general tachyon-free landscape as a whole.

Second, the level of supersymmetry also appears to affect the probability distributions across the different possible gauge-group factors. The supersymmetric portion of the landscape has a much greater representation of the large rank groups. This suggests that the constraints placed on the string spectrum in order to preserve spacetime supersymmetry also have the effect of favoring larger gauge symmetries, a fact already noted in Sect. 5.2. In other words, there tends to be a decrease in the gauge-group multiplicity for highly shattered gauge groups which consist of only very small gauge-group factors, and thus the larger-rank gauge groups make up a
larger proportion of the whole landscape. Indeed, this effect is particularly acute for that subset of the landscape exhibiting maximal $\mathcal{N}=4$ supersymmetry, where the larger-rank $SU$ gauge groups are particularly well represented.

6 Discussion

In this paper, we have examined both the prevalence of spacetime supersymmetry across the heterotic string landscape and the statistical correlations between the appearance of spacetime supersymmetry and the gauge structure of the corresponding string models. Somewhat surprisingly, we found that nearly half of the heterotic landscape is non-supersymmetric and yet tachyon-free at tree level; indeed, less than a quarter of the tree-level heterotic landscape exhibits any supersymmetry at all at the string scale. Moreover, we found that the degree of spacetime supersymmetry is strongly correlated with the probabilities of realizing certain gauge groups, with unbroken supersymmetry at the string scale tending to favor gauge-group factors with larger rank.

There are several extensions to these results which are currently under investigation. For example, we would like to understand how the presence of supersymmetry affects the statistical appearance of the entire composite Standard-Model gauge group $G_{\text{SM}} \equiv SU_3 \times SU_2 \times U_1$, and not merely the appearance of its individual factors. We would also like to understand how the presence or absence of supersymmetry affects other features which are equally important for the overall architecture of the Standard Model: these include the appearance of three chiral generations of quarks and leptons, along with a potentially correct set of gauge couplings and Yukawa couplings. This work has already been completed, and will be reported shortly [16].

Despite this progress, such studies have a number of intrinsic limitations which must continually be borne in mind. A number of these have been emphasized by us in recent articles (see, e.g., the concluding sections of Refs. [8, 9]) and will not be repeated here. However, other limitations are particularly relevant for the results we have quoted here and thus deserve emphasis.

First, we must continually bear in mind that our study has been limited to models in which rank-cutting is absent. Thus, all of the four-dimensional heterotic string models we have examined exhibit a fixed maximal rank=22. This has the potential to skew the statistics of the different gauge-group factors. For example, it is possible that gauge-group factors with smaller ranks might be over-represented in this sample simply because the appearance of such groups is often the only way in which a given model can precisely saturate the total rank bound. By contrast, for models which can exhibit rank-cutting, this saturation would not be needed and it is therefore possible that lower-rank groups are consequently less abundant.

A second limitation of this study stems from the nature of performing random search studies in general. In Sect. 3 we summarized several methods by which the problematic issue of floating correlations can be transcended, and this paper has
provided several examples of not only the need for such methods but also of the means by which they are implemented. As more fully discussed in Ref. [9], such problems are going to arise — and such methods are going to be necessary — whenever we attempt to extract statistical correlations from a large data set to which our computational access is necessarily limited. However, despite the apparent success of such methods, it is always a logical possibility that there exists a huge pool of string models with non-standard physical characteristics remaining just beyond our computational power to observe. The existence of such a pool of models would completely change the nature of our statistical results, to an extent which is essentially unbounded, yet we may miss this completely because of limited computational power. Although this becomes increasingly unlikely as our search through the landscape becomes larger and increasingly sophisticated, this nevertheless always remains a logical possibility which cannot be discounted.

But finally, perhaps the most serious limitation of our study is the fact that we are analyzing the statistical properties of string models which are not necessarily stable beyond tree level. Indeed, since none of our non-supersymmetric string models has a vanishing one-loop cosmological constant, these models in particular necessarily have non-zero dilaton tadpoles at one-loop order and thus become unstable. Even our supersymmetric models have flat directions which have not been lifted. Thus, as we have stressed throughout this paper, the “landscape” we have examined in this paper is at best a tree-level one. Despite this fact, however, it is important to realize that these models do represent self-consistent string solutions at tree level. Specifically, these models satisfy all of the constraints needed for worldsheet conformal/superconformal invariance, modular-invariant one-loop and multi-loop amplitudes, proper spacetime spin-statistics relations, and physically self-consistent layers of sequential GSO projections and orbifold twists. Indeed, since no completely stable perturbative heterotic strings have yet been constructed, this sort of analysis is currently the state of the art for large-scale statistical studies of this type. This mirrors the situation on the Type I side, where state-of-the-art statistical analyses have also focused on models which are only stable at tree level.

Nevertheless, we are then left with the single over-arching question: to what extent can we believe that the results we have found for this “tree-level” landscape actually apply to the true landscape that would emerge after all moduli are stabilized? The answer to this question clearly depends on the extent to which the statistical correlations we have uncovered here are likely to hold even after vacuum stabilization.

A priori, this is completely unknown. However, one surprising result of this paper is the observation that the string self-consistency requirements themselves — even merely at tree-level — do not preferentially give rise to supersymmetric solutions at the string scale. Indeed, as we discussed in Sect. 4, less than a quarter of the tree-level heterotic landscape appears to exhibit any supersymmetry at all at the string scale. Thus, breaking supersymmetry without introducing tachyons is actually statistically favored over preserving supersymmetry, even at the string scale and even
when the requirements of avoiding tachyons are implemented. Observations such as these tend to shift the burden of proof onto the SUSY enthusiasts, and perhaps reframe the question to one in which we might ask whether an unbroken supersymmetry is somehow restored by modulus stabilization. This seems unlikely, especially since most modern methods of modulus stabilization rely on breaking rather than introducing supersymmetry. In either case, however, this shows how the results of such studies — even though limited to only the tree-level landscape — can have the power to dramatically reframe the relevant questions. Indeed, once the technology for building heterotic string models develops further and truly stable vacua can be statistically analyzed in large quantities, it will be interesting to compare the statistical properties of those vacua with these in order to ascertain the degree to which vacuum stabilization might affect these other phenomenological properties.

Thus, it is our belief that such statistical landscape studies of this sort have their place, particularly when the results of such studies are interpreted correctly and in the proper context. As such, we hope that this study of the perturbative heterotic landscape may represent one small step in this direction.

Acknowledgments

The work of KRD, ML, and VW is supported in part by the U.S. National Science Foundation under Grant PHY/0301998, by the U.S. Department of Energy under Grant DE-FG02-04ER-41298, and by a Research Innovation Award from Research Corporation.
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