Scaling Tests of Some Lattice Fermion Actions

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I describe studies of quenched spectroscopy, using crude approximations to fixed point actions for fermions interacting with SU(3) gauge fields. These actions have a hypercubic kinetic term and a complicated lattice anomalous magnetic moment term. They show improved scaling compared to the conventional Wilson action.

1. INTRODUCTION

Fixed point actions of asymptotically free spin and gauge models show improved scaling behavior compared to standard discretizations. A natural extension of this work is to models with fermions interacting with gauge fields. I have been doing nonperturbative studies of fermion actions “inspired” by a perturbative fixed point (FP) vertex[1]. I have been testing scaling using quenched hadron spectroscopy. The calculation is not complete, but the results are encouraging.

The project is divided into two parts. First, one must construct approximate nonperturbative FP actions valid for rough gauge configurations, and second, one must perform scaling tests. I will describe the work in reverse order, however, since one does not have to know much about FP actions to look at the scaling tests.

In order not to entangle scaling tests with extrapolations to infinite volume or to zero quark mass, all tests are done in volumes of fixed physical size (defined through the critical temperature for deconfinement–all my tests are on lattice size \(L = 2/T_c\)) and at fixed physical quark mass (defined either by interpolating lattice data to a fixed value of \(m_\pi/m_\rho\) or to a fixed value of \(m_\pi/T_c\)). The tests are done at very coarse lattice spacing, where scaling violations from conventional actions are very large. Thus only modest statistics are required to identify improvement.

Another scaling test is the meson or baryon dispersion relation. In lattices of fixed physical volume set by \(T_c\), the physical momenta corresponding to the different allowed lattice modes are multiples of \(T_c\), \(a\vec{p} = 2\pi\vec{n}/L\) or \(\vec{p} = \pi T_c\vec{n}\), if \(L = 2/T_c\), and one can compare data at the same physical momentum with different lattice spacings. Wilson fermions at \(\beta < 6.0\) (using the Wilson gauge action) exhibit bad scaling or rotational invariance violations, which is often parameterized as “\(c_{eff}^2 < 1\),” with \(c_{eff}^2 = (E(p)^2 - m^2)/p^2\).

2. GENERIC ACTIONS

All the actions I have tested are Wilson-like: they have four-component spinors and no protection from additive mass renormalization. The “footprint” of the actions covers a hypercube. The free field limit of the action is parameterized as

\[
\Delta_0(x) = \lambda(x) + \sum_\mu \gamma_\mu \rho_\mu(x)
\]

with five nonzero \(\lambda\)'s and four nonzero \(\rho\)'s, corresponding to each of the generic types of offsets. These parameters are set by finding a particular FP action and truncating it to a hypercube. They vary smoothly with the bare mass \(m_0\) and in practice are parameterized as linear functions of \(m_0\).

The actions are made gauge invariant by connecting the fermion fields on different sites by sums of products of link variables. The actions differ in the specific choice of composition of the gauge connections.

The actions also include an anomalous magnetic moment term, or “Pauli term,” which is a very complicated extension of the standard “clover” term. Its effect is to correct lattice artifacts in magnetic interactions. This
term is part of the FP vertex. It consists of a large number of contributions of the form \( \sum c(r) \bar{\psi}(x)\sigma_{\mu\nu}F_{\mu\nu}\psi(x + r) \). The lattice analog of \( F_{\mu\nu} \) is made of oriented paths connecting sites \( x \) and \( x + r \). We can parameterize the magnetic interactions by comparing the inverse of the magnetic moment, labeled by the “magnetic mass” \( m_B \), to the pole mass \( m_0 \). My truncation of the FP vertex keeps only terms in which the fermion and antifermion lie in a cube, connected by minimum length gauge paths, and normalized so that \( m_0 = m_B \).

The cost of any of the actions I am testing is about 56 times as expensive as the usual Wilson action. About half of that (28) is due to the kinetic term and the rest is due to the complicated Pauli term. All the gauge connections are pre-computed.

### 3. TESTS OF ACTIONS

I have tested several related actions. All but one action use the “instanton-friendly” SU(3) gauge action of our recent work[2]. We have made rough measurements of its \( \beta_c(N_t) \) for deconfinement to set the scale. All the actions use gauge connections which are basically averages over the shortest paths connecting terms in the action.

The first action uses “tadpole-improved fat links” (action T). I begin with a vertex made of scalar and vector terms using the shortest gauge paths connecting the fermions, and then replace the individual link variable by a “fat link,”

\[
V_\mu(x) = (1 - \alpha)U_\mu(x) + \frac{\alpha}{6} \sum_\nu (U_\nu(x)U_\mu(x + \hat{\nu})U_\nu^\dagger(x + \hat{\mu}) + \ldots),
\]

taking \( \alpha = 1/2 \). The links are then tadpole-improved.

The second action (action A) replaces all the links by multi-level APE-blocked links: iterate Eqn. 2, with \( V_0^0(x) = U_j(x) \) and \( V_j^{n+1}(x) \) projected back onto \( SU(3) \). I used \( \alpha = 0.3 \) and seven blocking steps.

A third action (action F) uses different levels of APE-blocking for the links participating in the scalar and vector (“kinetic”) part of the action compared to the links used in the Pauli terms. The kinetic links have one level of APE-blocking with \( \alpha = 0.5 \). The links making up the Pauli terms are replaced by ten-level APE blocked links.

Figure 1 shows a comparison of Wilson action data and our test actions for the \( N/\rho \) mass ratio at one fixed \( \pi/\rho \) mass ratio. In this figure the diamonds are Wilson action data in lattices of fixed physical size \( (4^3 \text{ at } \beta = 5.1, 6^3 \text{ at } \beta = 5.54, 8^3 \text{ at } \beta = 5.7, 16^3 \text{ at } \beta = 6.0 \text{ and } 24^3 \text{ at } \beta = 6.3) \) and the crosses are data in various larger lattices. The bursts are from the nonperturbatively improved Wilson action of Ref. [3]. The other plotting symbols show our test actions.

The “fat links” considerably reduce the renormalization of bare parameters. For example, the bare mass at which \( m_\pi = 0 \) is \( m_0 = -0.13 \) at \( \beta_c(N_t = 2) \) and \( \beta_c(N_t = 4) \) for action A, in contrast to \(-1.58 \) and \(-1.04 \) for the standard Wilson action \( (m_0 = 1/2\kappa_c - 4) \).

I show one figure comparing dispersion rela-
tions. The result for test action $T$ is compared to the Wilson action dispersion relation (for heavy pseudoscalars) at $aT_c = 1/2$ in Fig. 2. All of the test actions I have studied have good dispersion relations even at $aT_c = 1/2$. I believe that is a generic feature of the hypercubic kinetic term. Leaving out the Pauli term gives noticeable scaling violations with a too-large $N/\rho$ ratio; probably one needs to keep some kind of explicit Pauli/clover term in the lattice action to boost the hyperfine splittings.

Figure 2. Dispersion relation for heavy hadrons at $aT_c = 1/2$ from a test action. The curves are the continuum dispersion relation for the appropriate (measured) hadron mass. A Wilson pseudoscalar is shown by the squares.

It appears that these actions are members of a family of actions which show improved scaling, even at $\beta_c(N_t = 2)$, about 0.4 fm lattice spacing. My data by themselves do not suggest a unique way to extrapolate to $a = 0$. However, if I linearly extrapolate the $\pi/\rho = 0.8$ Wilson results to $a = 0$, I get $N/\rho = 1.44(2)$, to be contrasted with quadratic extrapolations of 1.48(3) for the improved Wilson, 1.42(6) for action A, and 1.44(3) for action F.

4. TOWARDS A FP ACTION

These actions come from a program to find a nonperturbative FP action[1]. The present parameterizations began with a particular choice for an RGT. We then constructed (semi-analytically) a FP vertex, or FP action valid for smooth gauge configurations. By solving the RG equations, the vertex can then be used to find approximate actions valid for rough gauge configurations. The FP vertices we have found are far too complicated to simulate, and so I must truncate them before doing simulations. The fit to a FP action gradually worsens as I move towards the stronger couplings of interest to simulations. I believe a more sophisticated construction of the gauge connections is needed. This may involve looking at the coupling of instantons and fermions. This problem is under study.

5. TENTATIVE CONCLUSIONS

My goal is an action which scales at $aT_c = 1/2$. I clearly don’t have that (yet). However, it appears that I have gained a factor of perhaps 3 in lattice spacing for an equivalent amount of scale violation, compared to the Wilson action.

The generic feature of these actions which might be interesting to other “improvers” is the use of some sort of fat link in the gauge connections. The nonlocal Pauli terms are needed to satisfy the FP equations, but it is not clear to me that replacing the complicated Pauli term by an appropriately normalized simple clover term will seriously harm scaling.

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