BARYON SPECTROSCOPY AND THE CONSTITUENT QUARK MODEL

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We explore further the idea that the lattice QCD data for hadron properties in the region \( m_\pi > 0.2 \text{ GeV}^2 \) can be described by the constituent quark model. This leads to a natural explanation of the fact that nucleon excited states are generally stable for pion masses greater than their physical excitation energies. Finally we apply these same ideas to the problem of how pentaquarks might behave in lattice QCD, with interesting conclusions.

1. Introduction

Studies of lattice QCD have revealed a transition in the behavior of hadron properties in the region \( m_\pi \sim 0.4 - 0.5 \text{ GeV} \). Beyond this point hadron properties exhibit smooth, slowly varying behavior as a function of quark mass. The constituent quark mass is expected to depend linearly on the current quark mass, with \( M = M_0 + cm_\pi \) and \( c \sim 1 \). In addition, in a constituent quark model (CQM) hadron masses are roughly linear in the number of constituent quarks (\( \approx n_H M \)), magnetic moments are proportional to \( 1/M \) and so on. Since this is consistent with what is observed it is clear that the lattice simulations are qualitatively consistent with the constituent quark picture in the region \( m_\pi > 0.5 \text{ GeV} \).

On the other hand, in the region where the pion mass approaches zero, we know on model-independent grounds that all hadron properties exhibit rapidly varying, non-analytic behavior as a function of \( m_\pi \) – as a consequence of the pion loops resulting from spontaneous chiral symmetry breaking. These loops can change physical properties by up to 50% from the predictions based on naive chiral extrapolation. Since precise lattice computations at sufficiently low quark mass to see this curvature unambiguously are not yet possible, the accurate determination of physical hadron properties is not possible unless we know the corresponding chiral coefficients. Thus, while nucleon properties such as mass, magnetic moments, charge radii and moments of parton distribution functions can be extracted by chiral extrapolation, hadron spectroscopy presents a serious challenge. In the case of

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*One possible exception, which relies on the non-unitary nature of quenched QCD, is the comparison between \( N \) and \( \Delta \) properties in quenched QCD.*
pentaquarks, since if they exist their structure is completely unknown, nothing is known about their chiral behavior. Hence, even if one finds a signal at large $m_q$, the ambiguity in its physical mass is hundreds of MeV.

In this report we shall concentrate on the mass region in which constituent quark behavior dominates. We aim to understand the lattice data which has been obtained so far in terms of a constituent quark picture – with remarkable success.

2. Access Quark Model – AccessQM

In spite of the obvious, qualitative success of the CQM in describing the quark mass dependence of hadron properties calculated in lattice QCD in the currently accessible mass region, there has so far been only one quantitative application of the CQM to it! This approach, labelled AccessQM by the authors, involved the magnetic moments of the baryon octet. The procedure was to use the simplest CQM to give the octet moments in the mass region where $m_u \sim m_d \sim m_s$ and then to extrapolate to the region of physical, light quark masses using the correct chiral coefficients. The result was in excellent agreement with the experimental data. Further improvements in the CQM should be investigated in order to see just how good the agreement can be made.

On a more general level, this success suggests a novel approach to the CQM, where it would be developed to match lattice QCD data at relatively large quark mass, rather than trying to match it to experimental data where one knows that pion corrections can be very large. In addition, this work suggests that a comparison with lattice data at larger light quark mass would serve as a useful test of any model of hadron structure. So far only the chiral quark soliton model and the cloudy bag model have taken up this challenge.

2.1. $N$ and $\Delta$ Masses

As the next check on the physical consistency of the CQM at large quark mass we compare the data for $N$ and $\Delta$ masses from full QCD with expectations in AccessQM. In Ref. the constituent quark mass was written as

$$M = 0.421 + 0.301 m_\pi^2$$

with $m_\pi$ in GeV. In a careful comparison of the $N$ and $\Delta$ masses in full and quenched QCD, Young et al. found that the masses in full QCD (from the MILC Collaboration) were well fit by:

$$m_\Delta = 1.43(3) + 0.75(8) m_\pi^2 + \Sigma_\Delta (\text{chiral loops})$$
$$m_N = 1.24(2) + 0.92(5) m_\pi^2 + \Sigma_N (\text{chiral loops}),$$

where the chiral loops were calculated using a dipole form factor at the baryon-pion vertices with mass parameter 0.8 GeV. In the region where the pion mass is large these chiral loops are small and vary slowly. Thus it makes sense to compare the
slope of the average of the N and Δ masses in that region, namely 0.83±0.09 GeV⁻¹, with three times the coefficient of \( m_π^2 \) in Eq. (1), that is 0.90 GeV⁻¹. In terms of the absolute values, when the light quark mass is comparable with the strange quark mass, that is when \( m_π^2 \approx 0.48 \text{ GeV}^2 \), 3\( M \) is roughly 1.68 GeV, while the average of the N and Δ masses (from the MILC data) is of order 1.60 GeV. Clearly the CQM does a very reasonable job of describing the data quantitatively in the large mass region. A more sophisticated treatment would also model the hyperfine interaction associated with (say) gluon exchange with a term proportional to \( 1/M^2 \).

2.2. Unstable Hadrons – Δ(1232)

Now we compare the behavior of the mass of the Δ(1232) resonance with the mass of the corresponding decay channel, namely a pion and a nucleon. Provided that the self-energies of the N and Δ associated with chiral loops are either nearly equal or small, we can write the difference between the Δ mass and that of the threshold for the open Nπ channel as:

\[
m_Δ - m_N - m_π = 0.19 - 0.17m_π^2 - m_π.
\]

This would vanish and hence the Δ would become bound at \( m_π \sim 0.18 \text{ GeV} \). However, at such a low mass the self-energies associated with chiral loops cannot be neglected and it is not hard to see that to a good approximation we would expect the lhs of Eq. (3) to vanish when \( m_π \) is approximately equal to the physical Δ-N mass difference. This is indeed consistent with the lattice results.

2.3. Other Excited States

In terms of the application of lattice QCD to hadron spectroscopy, the result of the previous subsection, namely that the Δ resonance actually becomes a stable state when the pion mass exceeds the physical Δ-N mass difference, is an important result. It is far easier to use lattice techniques to find a stable particle, which is a true eigenstate of the QCD Hamiltonian, than to extract information about a resonance which is not the lowest energy state with a particular set of quantum numbers. Of course, in the case of the Δ the situation is even better because the π-N channel must have angular momentum one (\( L = 1 \)) for which the minimum, non-zero momentum on a lattice of size \( N_a \) is \( 2\pi/N_a \). Hence the Δ is stable on the lattice unless \( m_Δ < \sqrt{m_N^2 + (2\pi/N_a)^2 + m_π^2 + (2\pi/N_a)^2} \), which is typically several hundred MeV higher than the naive threshold, \( m_N + m_π \).

Another case of considerable interest involving \( L = 1 \) is the \( ρ \) meson which has a p-wave decay into two pions. Of course, for the physical \( ρ \) this decay involves momenta that are too high for the comfortable application of effective field theory. Nevertheless, it must be accounted for if one hopes to obtain a meaningful value of the physical mass. It helps that the width of the \( ρ \) is well known and this provides a powerful constraint on the phenomenological treatment of this channel. In
any case, for the present discussion what matters is that the $\rho$ is actually stable on the lattice even when $m_\rho$ is below $2m_\pi$ and a data point at such a low mass provides a powerful constraint on the chiral extrapolation needed to obtain the physical mass of the $\rho$.

However, the challenge of greatest interest to us here is baryon spectroscopy and it is interesting to see whether it is more generally true that nucleon excited states become stable as the quark mass increases. Figure 1 from Lasscock et al. shows the difference between the mass of the $1/2^- N^*(1535)$ and its s-wave decay threshold, $m_N + m_\pi$, as a function of pion mass. It is clearly stable for $m_\pi^2 > 0.3$–0.4 GeV$^2$. As in the case of the $\Delta$ this is very close to the point where the pion mass is equal to the difference between the physical resonance and nucleon masses ($m_\pi \sim 0.6$ GeV).

A similar result seems to hold for all nucleon excited states which look as though they might match the corresponding experimental state after a reasonable chiral extrapolation. As we have seen earlier, this is what we expect as a first approximation in the case of any nucleon excited state, given that lattice QCD shows that the CQM seems to describe the behavior of non-perturbative QCD in the region where the light quark masses are large ($m_\pi > 0.5$ GeV). The key to this behavior is the rapid variation of the pion mass and hence the corresponding threshold energy, while the energy difference between the mass of the nucleon and the excited state varies relatively slowly.
3. Possible Pentaquarks

It is difficult to imagine a more important discovery in modern strong interaction physics should the pentaquark story have a positive conclusion. At the present time the experimental situation is certainly confusing, with almost as many negative findings as positive and some earlier announcements now contradicted with much higher statistics. Theoretically the situation is just as inconclusive. It is difficult to take predictions based on various quark models seriously for a state where we have so little experience. As a result there has been a particular interest in studies based on lattice QCD, with a considerable amount of effort already applied to the problem.

Apart from the challenge of finding an appropriate source which has a large amplitude for producing a pentaquark, one has the bigger challenge of deciding exactly what signal one is expecting to find. We saw in the previous section that it has been possible to study nucleon excited states on the lattice because they become stable as the light quark mass increases. Without this property it would have been a much harder job. The question then is what one might expect with respect to the stability of the pentaquark.

To investigate this question we begin with an expression for the mass of a pentaquark as a function of light quark mass motivated by a CQM which should be valid at sufficiently large $m_q$:  

$$m_5 = M_s + 4M + H_{5}^{\text{hyp}}.$$  

(4)

Here $M_s$ is the constituent mass of the strange quark, which is fixed (at 0.565 GeV in the AccessQM) in this study of light quark mass variation. Using Eq.(1) we find

$$m_5 = 2.25 + 1.20m_{\pi}^2 + H_{5}^{\text{hyp}}.$$  

(5)

In order to compare with the corresponding threshold, in this case $KN$, we need an expression for the kaon mass for fixed strange quark mass. Using the Gell-Mann–Oakes–Renner relation one can write $m_K^2 = (m_K^{(0)})^2 + m_s^2/2$, where $m_K^{(0)}$ is the kaon mass in the SU(2) chiral limit. To keep the discussion simple, we expand the kaon mass to leading order in $m_{\pi}^2$

$$m_K = m_K^{(0)} + m_s^2/(4m_K^{(0)}) = 0.485 + 0.515m_{\pi}^2 + \delta_K.$$  

(6)

All higher order terms are collected into the term $\delta_K$, where to indicate the scale at $m_{\pi}^2 = 0.4$ GeV$^2$, $\delta_K = 30$ MeV and at $m_{\pi}^2 = 0.8$ GeV$^2$, $\delta_K = 100$ MeV.

Finally, we can now compute the difference between the pentaquark mass and the $KN$ threshold (ignoring issues of finite lattice volume for the moment) at sufficiently large $m_q$ that we can ignore chiral loops:

$$m_5 - m_N - m_K = 0.53 - 0.24m_{\pi}^2 + H_{5}^{\text{hyp}} - \delta_K.$$  

(7)

The clear difference between the situation for nucleon excited states and pentaquark states is that in the region of large quark mass, where the chiral loops for
the nucleon and (presumably) the hyperfine interaction (including chiral loops) for the pentaquark are smooth and slowly varying, there is not expected to be any rapid variation with $m_\pi$. In particular, there is no term linear in $m_\pi$ and the coefficient of $m_\pi^2$ is relatively small.

A plot of $m_5 - m_N - m_K$ would be expected to show only a very smooth variation with $m_\pi$ in the region above $m_\pi^2 \sim 0.2\text{GeV}^2$, with typically rapid chiral variation only below this region if this region is accessible.

The only way that the pentaquark could be stable (i.e., the lhs of Eq. (7) negative) in the large mass region explored by lattice QCD is if the hyperfine interaction in the pentaquark were very large and attractive in that region — an order of magnitude larger than the estimate reported recently in Ref. 20. If this were the case, then it suggests that the only way that this state could become unbound at the physical point would be through the increase at low $m_\pi$ in the known, large chiral corrections to the nucleon mass. These would need to cancel a good part of the large pentaquark hyperfine interaction at the physical point. In this case the interaction responsible for producing the pentaquark is unlikely to be chiral in origin. Rather it is more likely to have as its origin something like gluon exchange, which is expected to exhibit a relatively smooth variation as in the CQM (typically like $1/M$).

The other alternative, should the pentaquark exist, is that the hyperfine interaction is relatively weak. In this case, Eq. (7) indicates that the pentaquark is likely to remain unstable at larger quark masses. This would force one into the situation of needing to provide a careful analysis to identify one- and two-particle states in a finite-volume lattice simulation. To make a connection with the physical limit, the chiral corrections must necessarily be important, and one must consider

$$m_5 - m_N - m_K = 0.53 - 0.24m_\pi^2 + \Sigma_\pi(\text{chiral loops}) - \Sigma_N(\text{chiral loops}), \quad (8)$$

where $H_5^{\text{hyp}}$ and $\delta_K$ are assumed negligible. At the physical point, $\Sigma_N \sim -300\text{ MeV}$ and thus for the pentaquark to exist at $\sim 100\text{ MeV}$ above threshold, the chiral corrections must be of order $\sim -700\text{ MeV}$. The dynamical origin of the pentaquark in this case would therefore be chiral in nature.

4. Conclusion

We have explored the degree to which the constituent quark model can quantitatively describe the behavior of baryon properties in the region of relatively large quark mass currently accessible to lattice QCD. In fact it works very well, providing further support to calls to develop modern constituent quark models in this mass regime and to make the connection with experimental data through chiral extrapolation. Of particular interest with respect to baryon spectroscopy is a very natural explanation of the reason why nucleon excited states are stable for $m_\pi$ greater than roughly the difference of the physical mass of the excited state and the mass of the nucleon.

When the same ideas are applied to the possible existence of a pentaquark, the
conclusions are somewhat different. For example, a plot of $m_5 - m_N - m_K$ would be expected to show only a very smooth variation with $m_\pi$ in the region above $m_\pi^2 \sim 0.2 \text{GeV}^2$, with typically rapid chiral variation only below this region – if it is accessible. It is interesting that this is precisely the type of behavior seen in the recent investigation of a possible spin-3/2, positive parity pentaquark state by the CSSM-JLab collaboration – see Fig. 2. Clearly this needs further investigation.

Furthermore, these ideas lead us to the conclusion that, if the signal observed in Fig. 2 holds under further study, the interaction responsible for producing pentaquarks is unlikely to be chiral in origin. Rather it is more likely to have as its origin something like gluon exchange, which is expected to exhibit a relatively smooth variation as in the CQM (typically like $1/M$). Finally, we note that while the slope of $\Delta M$ at large $m_\pi$ in Fig. 2 differs from that in Eq. (7), this may be consistent with a $1/M$ dependence of a large, attractive hyperfine interaction.
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