Massive selfdual perturbed gauge theory

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**Abstract**

Spontaneously broken gauge theories are described as a perturbation of selfdual gauge theory. Instead of the incorporation of scalar degrees of freedom, the massive component of the gauge field is obtained from an anti-selfdual field strength consisting of three components before gauge fixing. The interactions describe a massive gauge theory that is non-polynomial with an expansion containing an infinite number of terms. The Lagrangian generalizes the form of the axial anomaly in two dimensions. Unitary propagation of the tensor field occurs upon gauge fixing an additional symmetry.
Selfdual quantum field and string theories describe a sector of both gauge and gravity theories [1]. Perturbations of these selfdual models generate reformulations of quantum gauge theories that have simplified expansions about the selfdual limit [2, 4, 3, 5]. In this work, we examine a selfdual inspired reformulation of quantum gauge theory possessing a covariant mass term that is renormalizable and unitary. Selfdual systems are quasi two-dimensional in the sense that the analytic properties of the scattering are contained in one half of the Lorentz group. The mass term that we examine in this work is a generalization of the integrated axial anomaly in two dimensions; it has a local description via the implementation of auxiliary fields, describing a perturbation of selfduality.

A Lorentz covariant form of selfdual quantum field theory is described by the lagrangians

$$L = \text{Tr} \ G^{\alpha\beta} F_{\alpha\beta}$$

for spin one and

$$L = \rho^{\alpha\beta} R_{\alpha\beta}$$

for spin two theories (proposed in [3] and quantized in [1]). We adopt the conventions of those in [7]. Non-lorentz covariant versions [8] involving one degree of freedom give identical one-loop results (modulo a factor of half) but are inconsistent at higher loop orders. The selfdual field strengths are the selfdual projections with $R_{\alpha\beta}$ the selfdual component of the Weyl tensor. The quantum theories may be solved exactly in perturbation theory with the former described by the S-matrix [11, 12],

$$A_{[J]}^{n;1} = -\frac{i}{48\pi^2} \sum_{1 \leq i < j < k < l \leq n} \frac{\langle ij \rangle \langle jk \rangle \langle kl \rangle \langle ln \rangle}{\langle 12 \rangle \langle 23 \rangle \cdots \langle n1 \rangle},$$

with $[J]$ denoting the spin of the internal virtual particle. The tree-level amplitudes of the selfdual gauge theory are equal to zero except for a nonvanishing three-point vertex. The expression is written in spinor helicity form [13, 14, 15], in which $\langle ij \rangle = k_i^\alpha k_j^\beta$ and $[ij] = k_i^{\alpha} k_j^{\beta}$, are inner products with respect to the covering of the Lorentz group; they are complex conjugates in $d = 3+1$ dimensions, but real and independent in $d = 2+2$ dimensions. The subscript on $A_{[J]}^{n;1}$ denotes the leading in color component of the quantum amplitude in color-ordered form [16, 10], and is expressed in terms of the basis,

$$N^2 \text{Tr} \ T^{a_1} T^{a_2} \cdots T^{a_n},$$

for an internal state of spin $[J]$ in the adjoint representation. Sub-leading in color partial amplitudes may be deduced via permutations of the indices of the former [17].

1Reviews of the techniques in calculating gauge theory amplitudes are found in [8] for tree-level processes and in [10] at the one-loop level.
A similar exact expression follows for the gravitational theory \[18\]. Furthermore, these S-matrices have a description in terms of the $N = 2$ quantum string theory defined by self-consistency between different orders in the loop expansion \[19\] rather than the usual path integral quantization \[20, 21, 22, 23\]. Several exact sequences of amplitudes have compact forms, including the tree-level next-to-maximal Parke-Taylor helicity violating amplitudes \[24\] and those in supersymmetric theories \[25, 26\].

The description of quantum gauge theory may be simplified by perturbing the selfdual gauge theory by dualizing the theory with $G^{\alpha\beta}$ transforming as the antiselfdual $(0, 1)$ tensor \[3, 27\],

$$L = \text{Tr} \left( \frac{1}{2} G^{\alpha\beta} G_{\alpha\beta} + G^{\alpha\beta} F_{\alpha\beta} \right) ,$$  \quad (1.3)

which generates $\text{Tr} F^2$. The perturbations of this theory and its massive version describing spontaneously broken gauge theories \[3\] generate simplified Feynman diagrammatics when expanded around the selfdual helicity configuration; This expansion is one in helicity flips around the maximally helicity violating amplitude in (1.1). Interestingly, these S-matrices also have a description in terms of the $\mathcal{N} = 2$ quantum string theory (and non-maximal helicity violating amplitudes may be obtained by deformations) and maps a direct perturbative QCD/string relation \[19\]. In this work we examine more general deformations of selfdual quantum field theories that describe massive gauge theories. As the Higgs particle has not been found it is of interest to generate alternative means to describe massive vector theories.

This work is organized as follows. In section 2 we dualize the massive theory beginning with a local representation of the theory and generate the interactions, comparing with previous work. In section 3 we list the diagrammatic rules and analyze briefly the renormalizability and unitarity properties of the theory.

## 2 Dual Lagrangian

Self-dual field theories, quantum field theories with selfdual equations of motion, have been extensively studied and multiple actions have been proposed. A Lorentz covariant version is described by the Lagrangian,

$$\mathcal{L} = \text{Tr} \ G^{\alpha\beta} F_{\alpha\beta} ,$$  \quad (2.1)

and admits an exact solution which consists of a one-loop amplitude together with a non-vanishing three-point tree vertex. Gauge theory in general may be dualized by
expressing the theory via

$$\mathcal{L} = \frac{1}{2} \text{Tr} \ F^{\alpha \beta} F_{\alpha \beta} \rightarrow \mathcal{L} = \text{Tr} \left( -\frac{1}{2} G^{\alpha \beta} G_{\alpha \beta} + G^{\alpha \beta} F_{\alpha \beta} \right), \quad (2.2)$$

that is, as a deformation around the self-dual sector. More general deformations of self-duality are also obtained and may be used to formulate a massive version of gauge theory.

The modified self-dual gauge theory we examine is given by,

$$\mathcal{L} = \text{Tr} \left( \frac{1}{2} G^{\alpha \beta} \square G_{\alpha \beta} + M G^{\alpha \beta} F_{\alpha \beta} + \frac{1}{2} \tilde{G}^{\alpha \beta} \square \tilde{G}_{\alpha \beta} + M \tilde{G}^{\alpha \beta} F_{\alpha \beta} \right) + \frac{1}{2} \text{Tr} \left( \tilde{F}^{\alpha \beta} F_{\alpha \beta} + \tilde{F}^{\alpha \beta} \tilde{F}_{\alpha \beta} \right), \quad (2.3)$$

with the field strengths defined as

$$F_{\alpha \beta} = i \frac{1}{2} \partial^{\alpha} A_{\beta} - A_{(\alpha} \tilde{A}_{\beta)} - A_{(\alpha} \tilde{A}_{\beta)} \quad \text{which differs from (2.2)}$$

via a covariant 'de Lambertian on the dual fields. As the theory in (2.3) has terms of mass dimension less than or equal to four the theory is renormalizable. To implement the dualization we need to add in a counterterm $$Z_m \frac{m^2}{2} \left( G^{\alpha \beta} G_{\alpha \beta} + G^{\alpha \beta} \tilde{G}_{\alpha \beta} \right)$$ so that the renormalized mass is equal to zero. The theory in (2.3) is distinguished from a propagating Kalb-Ramond field with kinetic term $$\mathcal{L} = -\frac{1}{4} H^{\mu \nu \rho} H_{\mu \nu \rho}$$ and field strength $$H_{\mu \nu \rho} = \partial_{\mu} G_{\nu \rho} + \partial_{\nu} G_{\mu \rho} + \partial_{\rho} G_{\mu \nu}$$.

The covariant derivatives are defined as,

$$\square = \partial^{\alpha} \partial_{\alpha} + \left\{ A^{\alpha}, \partial_{\alpha} \right\} + A^{\alpha} A_{\alpha} \quad (2.4)$$

which requires expansions in the forms of the interactions that follow in the denominator. The Lagrangian has equations of motion,

$$\square G^{\alpha \beta} + M F^{\alpha \beta} = 0 \quad G^{\alpha \beta} = -\frac{M}{\square} F_{\alpha \beta}, \quad (2.5)$$

which generates the classical theory,

$$\mathcal{L} = \text{Tr} \left( \frac{1}{2} F^{\alpha \beta} F_{\alpha \beta} + \frac{1}{2} \tilde{F}^{\alpha \beta} \tilde{F}_{\alpha \beta} - \frac{M^2}{6} F_{\alpha \beta} - \frac{M^2}{6} \tilde{F}_{\alpha \beta} \right), \quad (2.6)$$

with a covariant mass term,

$$F_{\alpha \beta} \frac{1}{\square} F_{\alpha \beta} = \frac{1}{4} \partial^{(\alpha} A^{\beta)\gamma} \frac{1}{\square_0} \partial_{(\alpha} A^{\beta) \gamma} + \ldots = A^{\alpha \beta} A_{\alpha \beta} + \ldots \quad (2.7)$$

The form in four-component vector notation follows from replacing $$G_{\alpha \beta} \rightarrow G_{\mu \nu} + i \tilde{G}_{\mu \nu}$$ and $$G_{\alpha \beta} \rightarrow G_{\mu \nu} - i \tilde{G}_{\mu \nu}$$ with $$\tilde{G}_{\mu \nu} = \frac{1}{2} \epsilon_{\mu \nu \rho \sigma} G^{\rho \sigma}$$. 

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after integrating by parts. The theory is a four-dimensional version of the chiral anomaly in two dimensions. The quantum extension of (2.6) has a different form that we next analyze. Because the theory has a local form that is manifestly real, the theory is renormalizable and unitary (after projecting out the three negative norm states of the tensor fields $G^{\alpha\beta}$ and $G^{\alpha\beta}$). The three-component anti-selfdual tensor $G_{\alpha\beta}$ gives the degrees of freedom describing the massive vector. One may consider processes with external $G_{\alpha\beta}$ fields as required by unitarity; we integrate them out in the loop to generate the massive component of the gauge fields.

The theory in four-vector notation has the form,

$$
\mathcal{L} = \text{Tr} \left( -\frac{1}{2} G^{\mu\nu} \Box G_{\mu\nu} - \frac{1}{4} F^{\mu\nu} F_{\mu\nu} + MG^{\mu\nu} F_{\mu\nu} \right). \tag{2.8}
$$

(We could write the gauge field kinetic term as $-\frac{1}{8} F^2 - \frac{1}{8} \tilde{F}^2$ as in (2.3).) We conclude this section with a discussion of the unitarity and the unphysical modes of the tensor field. The action in (2.8) has the linearized gauge invariance

$$
G^{\mu\nu} \rightarrow G^{\mu\nu} + \partial^{\mu} \theta^{\nu} - \partial^{\nu} \theta^{\mu} \tag{2.9}
$$

and

$$
A^{\mu} \rightarrow \frac{1}{M} \Box \theta^{\mu}, \tag{2.10}
$$

upon restricting to

$$
M = -1 \quad M^2 \theta^{\mu} + \frac{1}{2} \Box \theta^{\mu} = 0. \tag{2.11}
$$

This invariance possesses three components as the latter equation restricts the vector to be massive. The action is invariant under $G \rightarrow \lambda G$, $x^{\mu} \rightarrow \lambda^{-1} x^{\mu}$, $A^{\mu} \rightarrow \lambda A$, $M \rightarrow \lambda M$ and $g \rightarrow g$; as a result the invariance holds for all values of the mass parameter as this scaling results in $\lambda M = -1$. The three components are sufficient to cancel the three modes of the anti-symmetric $G^{\mu\nu}$ tensor possessing negative norm, rendering the propagator well-defined. After the dualization, one mode is absorbed by the gauge the field and two remain. The gauge covariantization of this symmetry lifts to the full theory.

3 Quantum Extension

The quantum extension of (2.6) is found by summing the one-particle reducible bubble diagrams and covariantizing. It has the form,

$$
\mathcal{L} = \text{Tr} \left( \frac{1}{2} F^{\alpha\beta} F_{\alpha\beta} - \frac{M^2}{2} \sum_{k=0}^{\infty} F^{\alpha\beta} \frac{1}{\Box} \left( \frac{M^2}{2} \ln \Lambda^2 / \Box \right)^k F_{\alpha\beta} \right) + \text{h.c.}
$$
There are two regimes in which we may expand the denominator in (3.1). In the momentum range \( k^2 >> M^2 \) we obtain the interaction from the series

\[
L^{(a)}_{\text{int}} = -\frac{M^2}{2} \text{Tr} \, F^{\alpha\beta} \sum_{n=0}^{\infty} \frac{M^2}{\Lambda^2} \ln \frac{\Lambda^2}{\Lambda^2} \, \frac{1}{n!} \frac{1}{M^2} \ln \Lambda - M^2 \ln \Lambda + h.c. \quad (3.2)
\]

and in the range \( M^2 >> k^2 \),

\[
L^{(b)}_{\text{int}} = \text{Tr} \, \frac{1}{2 \ln (\Lambda^2/\Lambda^2)} \, F^{\alpha\beta} \sum_{k=0}^{\infty} \left( \frac{\Lambda^2}{M^2} \ln \frac{\Lambda^2}{\Lambda^2} \right)^k \, F^{\alpha\beta} + h.c. \quad (3.3)
\]

Both expanded forms permit a representation in diagrammatic rules. The logarithmic terms in \( L^{(a)}_{\text{int}} \) and \( L^{(b)}_{\text{int}} \) are non-analytic and appear not to generate local interactions; however, they can be mapped into local interactions via analytic continuation or through the use of auxiliary fields. At high energies, greater than \( M^2 \) the theory consists of massive vector bosons. There is a Landau pole located to this order in the quantum expansion at approximately \( k^2 \sim -M^2 \ln (-M^2/k^2) \).

The expansion of the inverse of the covariant box is,

\[
\frac{1}{1} = \frac{1}{\Lambda_0} + \left( A^{\alpha\dot{\alpha}}, \partial_{\alpha\dot{\alpha}} \right) + A^{\alpha\dot{\alpha}} A_{\alpha\dot{\alpha}}.
\]

\[
= \frac{1}{\Lambda_0} \sum_{k=0}^{\infty} \left[ -\frac{1}{\Lambda_0} \left( \left( A^{\alpha\dot{\alpha}}, \partial_{\alpha\dot{\alpha}} \right) + A^{\alpha\dot{\alpha}} A_{\alpha\dot{\alpha}} \right) \right]^k
\]

and generates the interactions in the second regime in which \( k^2 >> M^2 \) from (3.1)

\[
L_1 = -\frac{M^2}{2} \text{Tr} \, F^{\alpha\beta} \frac{1}{\Lambda_0} \sum_{k=1}^{\infty} \left[ \frac{\Lambda^{\alpha\dot{\alpha}}, \partial_{\alpha\dot{\alpha}}}{\Lambda_0^2} + A^{\alpha\dot{\alpha}} \right]^k
\]

\[
\times \sum_{p=0}^{\infty} \left[ \frac{1}{1 - \frac{M^2}{\Lambda_0^2} \ln \frac{\Lambda^2}{\Lambda^2}} \sum_{k=0}^{\infty} M_k \right]^p F^{\alpha\beta}
\]

(3.5)

with terms in \( M_k \) containing products of \( k \) gauge fields. The complete theory includes also the interaction \( \text{Tr} \, F^{\alpha\beta} F^{\alpha\dot{\beta}} + \text{Tr} \, F^{\alpha\dot{\beta}} F^{\alpha\beta} \).

We next expand the theory in a series in the gauge couplings in the first regime. The theory expanded to order \( O(A) \) contains the zeroth order term,

\[
\mathcal{L} = \text{Tr} \, F^{\alpha\beta} F^{\alpha\dot{\beta}} - \frac{M^2}{2} F^{\alpha\beta} \frac{1}{\Lambda_0} \left[ 1 - \frac{1}{\Lambda^2 - \ln \Lambda^2} \right] F^{\alpha\beta} + h.c.
\]

(3.6)
and the first order correction,

\[ \mathcal{L}_a = -\frac{M^2}{2} \text{Tr} \, F^{\alpha\beta} \frac{1}{\ln \Lambda^2 / \Box} \left( -\frac{1}{\Box} \{A, \partial \} \right) F_{\alpha\beta} + \text{h.c.} \]  

(3.7)

The expanded form to order \( \mathcal{O}(A^2) \) has the form,

\[ \mathcal{L}_b = \frac{M^2}{2} \text{Tr} \, F^{\alpha\beta} \frac{1}{\ln \Lambda^2 / \Box} \left[ 1 \right] \left( \frac{1}{\ln \Lambda^2 / \Box} \left( -\frac{1}{\Box} \{A, \partial \} \right) F_{\alpha\beta} + \right. \]  

\[ \times \left[ \frac{M^2}{\Box} A^2 \ln \Lambda^2 - \left( \ln \Lambda^2 \right) \{A, \partial\} \frac{1}{\Box} \{A, \partial\} \right] F_{\alpha\beta} , \]  

(3.8)

\[ \]  

together with the hermitian conjugate. There are an infinite number of higher order terms in the expansion but these vertices are sufficient to generate the four-point function.

### 4 Diagrammatic Rules

We first specify the line factors associated with the massive vector bosons \[14, 28\].

There are three independent polarizations that satisfy the completeness relation,

\[ \sum_{\lambda=\pm,0} \epsilon_{\lambda \alpha}^* (k; M) \epsilon_{\lambda \beta} (k; M) = C_{\alpha\beta} C_{\alpha\beta} + \frac{k_{\alpha \dot{\alpha}} k_{\beta \dot{\beta}}}{2M^2} . \]  

(4.1)

The polarizations written in terms of spinor inner products of legs with an arbitrary massive momentum \( k_{\alpha \dot{\alpha}} = k_{-\alpha \dot{\alpha}} + k_{+\alpha \dot{\alpha}} \), and \( k_2^2 = k_1^2 = 0 \) (so that \( k_{\pm \pm} = k_{\pm \pm} \)), have the explicit form,

\[ \epsilon_{+ \dot{\alpha}} (k; M) = \frac{k_{+ \dot{\alpha}}}{M} \]  
\[ \epsilon_{- \dot{\alpha}} (k; M) = \frac{k_{- \dot{\alpha}}}{M} , \]  

(4.2)

and

\[ \epsilon_{0 \dot{\alpha}} (k; M) = \frac{1}{\sqrt{2}M} \left( k_{+ \dot{\alpha}} - k_{- \dot{\alpha}} \right) , \]  

(4.3)

and momentum in spinor form,

\[ M^2 = -k^2 = \langle k_+ k_- \rangle \]  

(4.4)

These leg factors have the ambiguity in the representation of breaking the off-shell momentum into two null vectors, which may be utilized to simplify calculations.
The local form of the massive theory \([2.2]\) has the Feynman rules, with the gauge-fixing term for the \(A^{\alpha\beta}\) field specified with \(\mathcal{L}_{gf} = \lambda^2 / 2(\partial \cdot A)^2\) and \(\lambda = 1\), consisting of the propagators,

\[
\langle G^{\alpha\beta}(k)G^{\mu\nu}(-k) \rangle = \frac{1}{k^2 + m_0^2} \left( C^{\alpha\mu}C^{\beta\nu} + C^{\alpha\nu}C^{\beta\mu} \right),
\]

\[
\langle A^{\alpha\beta}(k)A^{\beta\gamma}(-k) \rangle = \frac{1}{k^2} C^{\alpha\beta}C^{\beta\gamma},
\]

and the two-point interaction,

\[
\langle G^{\alpha\beta}(k)A^{\mu\nu}(-k) \rangle = M k^{(\alpha\beta)} \mu .
\]

Summing the diagrams with the \(\langle AG \rangle\) vertex generates the mass term for \(A\). The three- and four-point vertices are,

\[
\langle A^{\mu\nu}(k_1)A^{\alpha\beta}(k_2)G^{\gamma\delta}(k_3) \rangle = C^{\alpha\beta} \left( C^{\mu\nu}C^{\gamma\delta} + C^{\mu\delta}C^{\alpha\gamma} \right)
\]

\[
\langle A^{\alpha_1\beta_1}(k_1)A^{\alpha_2\beta_2}(k_2)A^{\alpha_3\beta_3}(k_3) \rangle = k^{\beta_3(\alpha_1}C^{\alpha_2\alpha_3}C^{\beta_1\beta_2)} + \text{perms} ,
\]

and

\[
\langle \prod_{j=1}^{4} A^{\alpha_j\beta_j}(k_j) \rangle = C^{\beta_1\beta_2C^{\beta_3\beta_4} (C^{\alpha_1\alpha_3}C^{\alpha_2\alpha_4} + C^{\alpha_2\alpha_3}C^{\alpha_1\alpha_4}) + \text{perms} .
\]

These interactions generate the local form of the dualized massive theory.

The quantum one-loop effective action following from integrating out the auxiliary \(G^{\alpha\beta}\) fields in the two-point function and covariantizing is listed in \((3.1)\). These vertices follow from a resummation of one loop diagrams. The propagator is,

\[
V_2 = \left(-\frac{1}{2}\right) \left(1 + 11 \frac{M^2}{k^2 + M^2 \ln(-k^2/\Lambda^2)}\right) \left( k^{\alpha\beta}k^{\beta\alpha} - \frac{1}{4} k^2 C^{\alpha\beta}C^{\alpha\beta} \right)
\]

\[
+ \frac{1}{2} \left(1 + 9 \frac{M^2}{k^2 + M^2 \ln(-k^2/\Lambda^2)}\right) \left( k^{\alpha\beta}k^{\beta\alpha} + \frac{k^2}{4} C^{\alpha\beta}C^{\alpha\beta} \right)^{-1}
\]

The third variation of the Lagrangian generates the color ordered three-point vertices,

\[
V_3^{(a)}(\alpha) = \lambda \left[ k_1^{\alpha_1\alpha_2}C^{\alpha_2\alpha_3}C^{\alpha_1\alpha_3} + k_2^{\alpha_2\alpha_3}C^{\alpha_3\alpha_1}C^{\alpha_2\alpha_1} + k_3^{\alpha_3\alpha_1}C^{\alpha_1\alpha_2}C^{\alpha_3\alpha_2} \right]
\]

\[
V_3^{(b)} = \frac{M^2}{k_f^2 + M^2 \ln(-\Lambda^2/k_f^2)} \frac{1}{k_f^2} \left[ k_1^{\alpha_1(\alpha}C^{\beta)\alpha_1}k_3^{\alpha_3}C^{\beta_4}C^{\alpha_4} \frac{1}{k_f^2} (k_3 + 2k_2)^{\alpha_2\alpha_2} \right]
\]
and the fourth, the color ordered four-point vertices,

\[ V_4^{(a)} = \frac{\lambda^2}{2} - \frac{1}{2} \left( \frac{M^2}{k_1^2 + M^2 \ln \left( -\Lambda^2 / k_1^2 \right)} \right) \left( \frac{1}{k_2^2} - \frac{M^2}{k_4^2} \right) k_1^{\alpha_1} C^{(\beta)\alpha_1} k_4^{\alpha_4} (\alpha C_{\beta_4}^{\alpha_2} \alpha_2 C_{\alpha_3}^{\alpha_2} C_{\alpha_3}^{\alpha_2} ) \]

\[ V_4^{(b)} = \left( \frac{M^2}{k_1^2 + M^2 \ln \left( -\Lambda^2 / k_1^2 \right)} \right) \left( \frac{1}{k_2^2} - \frac{M^2}{k_4^2} \right) k_1^{\alpha_1} C^{(\beta)\alpha_1} k_4^{\alpha_4} (\alpha C_{\beta_4}^{\alpha_2} \alpha_2 C_{\alpha_3}^{\alpha_2} C_{\alpha_3}^{\alpha_2} ) - (\ln \Lambda^2 / k_1^2) \frac{M^4}{k_1^4 (k_3 + k_4)^2} (k_2 + 2k_3 + 2k_4)^{\alpha_2} (k_4 + 2k_3)^{\alpha_3} \]

The ordering of the inverse boxes follows from the expansion of the one-loop effective action. There are an infinite number of higher-point interaction vertices in the fully expanded form; for simplicity we derive the vertices up to the four-point order.

## 5 Summary

In this work we examine a selfdual inspired reformulation of a theory of massive vector bosons found by perturbing a Lorentz covariant selfdual theory \[ L = \text{Tr} \left( \frac{1}{2} G^{\alpha\beta} \Box G_{\alpha\beta} + MG^{\alpha\beta} F_{\alpha\beta} + \frac{1}{2} G^{\alpha\beta} \Box G_{\alpha\beta} + MG^{\alpha\beta} F_{\alpha\beta} \right) \]

\[ + \frac{1}{2} \text{Tr} \left( F^{\alpha\beta} F_{\alpha\beta} + F^{\alpha\beta} F_{\alpha\beta} \right) \]

in which the selfdual theory is described by \[ L = \text{Tr} G^{\alpha\beta} F_{\alpha\beta} \]. At low-energies the dynamics is governed by an exactly solvable selfdual system and at higher energies, above the energy scale set by the dimensionful coupling \( M \), the theory consists of massive vector bosons in which the additional degrees of freedom are absorbed by the auxiliary field imposing the selfduality constraint. The Lagrangian (5.1) possesses a gauge symmetry. This symmetry is sufficient to gauge away half of the components of the tensor fields and allows for unitary propagation of the modes.

In previous works selfdual inspired reformulations of gauge theories has led to improved Feynman rules and expansions [1]. These include second order reformulations of fermionic couplings [4] and spontaneously broken gauge theories [5]. In
this work the additional degrees of freedom required to compose a massive vector
are replaced by a Lagrange multiplier field that imposes the selfduality constraint in
the undeformed case. The Lagrangian we consider in this work has a non-local form
with an expansion containing an infinite number of interactions; it is an analog of the
chiral anomaly in two dimensions. Furthermore the theory permits a local form \([5.1]\)
in which the renormalizibility and unitarity properties are manifest. We examine the
interactions up to four-point order.

**Acknowledgements**

The work of GC is supported in part by the US Department of Energy, Division of
High Energy Physics, contract W-31-109-ENG-38. GC thanks Cosmas Zachos and
the referee for helpful comments.

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