$B_s^0 - \overline{B}_s^0$ mixing and $B \to \pi K$ decays in stringy leptophobic $Z'$

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Abstract

We consider a leptophobic $Z'$ scenario in a flipped SU(5) grand unified theory obtained from heterotic string theory. We show that the allowed $Z'$ mass, flavor conserving and flavor changing couplings of the $Z'$ to the down-type quarks are strongly constrained by the mass difference in $B_s - \overline{B}_s$ system and the four branching ratios of $B \rightarrow \pi K$ decays. It is shown that even under these constraints large deviations in direct and/or indirect CP asymmetries of $B \rightarrow \pi K$ decays from the SM expectations are allowed. Especially it is possible to accommodate the apparent puzzling data in $B \rightarrow \pi K$ CP asymmetries.

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I. INTRODUCTION

The flavor changing neutral current (FCNC), in particular the $\bar{b} \to \bar{s}$ transition, is a sensitive probe of new physics (NP) beyond the standard model (SM) of particle physics. The processes, such as $B \to X_s \gamma$ [1], $B \to \pi K$ [2], $B \to \rho(\phi) K^*$ [3], $B \to \phi K_S$ [4], and $B_s \to \mu^+ \mu^-$ [5] which are dominated by the $\bar{b} \to \bar{s}$ transition have attracted much interest because they still allow much room for large NP contributions. The experimental data for some of them show apparent deviations from the SM predictions [2–4].

A viable NP scenario which may give large contribution to $\bar{b} \to \bar{s}$ FCNC is leptophobic $Z'$ [6]. Extra U(1) gauge groups appear naturally in many extensions of the SM. If some of them remain unbroken down to the electroweak scale, the $Z'$ can be light and affect low energy phenomenology. In addition, there is a possibility that the new neutral gauge boson does not couple to leptons. This kind of $Z'$ gauge boson is called leptophobic. In case the $Z'$ does not mix with the SM $Z$ boson, the strong constraints from the electroweak precision tests can be avoided. Explicit leptophobic $Z'$ model with these properties has been constructed by Lopez, Nanopoulos and Yuan [7] in heterotic string theory.

The model has gauge group, $G = G_{\text{obs}} \times G_{\text{hidden}} \times G_{U(1)}$, where $G_{\text{obs}} = SU(5) \times U(1)$, $G_{\text{hidden}} = SU(4) \times SO(10)$, and $G_{U(1)} = U(1)^5$. It also has 63 massless matter fields. It can be shown that the $Z'$ gauge boson can be light and leptophobic without mixing with $Z$. The additional feature of this model is that the $Z'$ coupling is generation dependent. Therefore tree-level FCNC is generated in general. Since $u^c$ and $L$ belong to the same multiplet which do not couple to the leptophobic $Z'$, the $Z'$ coupling to $u$-quarks maximally violates parity. And the disparity of the couplings to the $d^c$ and $u^c$ provides additional source of isospin breaking.

We study the contribution of leptophobic $Z'$ to $B^0_s - \bar{B}^0_s$ mixing and non-leptonic decays $B \to \pi K$‘s with this model in mind. However, our analysis can be easily extended to other (leptophobic) $Z'$ models allowing tree-level FCNC’s.

The measurement of the mass difference, $\Delta m_s$, in the $B^0_s - \bar{B}^0_s$ system CDF [8] collaborations

$$\Delta m_s^{\text{exp}} = 17.77 \pm 0.10(\text{stat}) \pm 0.07(\text{syst}) \text{ ps}^{-1}$$ (1)

is consistent with the SM calculations [9]

$$\Delta m_s^{\text{SM}} \bigg|_{(\text{HP+JL})\text{QCD}} = 22.57^{+5.88}_{-5.22} \text{ ps}^{-1}. \quad (2)$$

This constrains many NP models [10–13] including $Z'$ models, MSSM models, etc. In [6], we showed that the $\Delta m_s$ constraint on the leptophobic $Z'$ is much stronger than the previously considered one [14] from the semi-leptonic $B$-decays. In this paper we extend the analysis in [6]
to include the case where $Z'$ couples to both left-handed and right-handed quarks simultaneously. When both couplings exist at the same time, we will see that the $\Delta m_s$ constraint is not enough to set the upper bound on the sizes of the FCNC couplings. We go beyond the [6] and demonstrate that the additional constraints are available, i.e. the four branching ratios (BR) of $B \to \pi K$'s, which can give the limits even in the simultaneous existence of both left- and right-handed couplings.

The charmless non-leptonic decays $B \to \pi K$ have been measured precisely enough to probe the electroweak amplitudes [2]. The experimental data indicate that while the BRs are consistent with the SM expectations, some of direct and indirect CP asymmetries show apparent (but still debatable) deviations from the SM [15]. Accepting this discrepancy seriously, we can see the electroweak penguin sector is the best place to search for NP [15]. We will see that in our model the leptophobic $Z'$ can give large contributions to the electroweak penguin amplitudes while satisfying the $\Delta m_s$ and the $BR(B \to \pi K)$'s. In addition we will see that the predicted direct and indirect CP asymmetries can accommodate the discrepancies between the SM predictions and measurements simultaneously.

We note that the merit of our model in explaining the data comes from (i) it automatically evades the stringent constraints involving leptons, such as LEP I data, $B_s \to \mu^+ \mu^-$, (ii) there are new CP violating phases, and (iii) the characteristic isospin breaking interaction in this model can generate large electroweak penguins.

The paper is organized as follows: In section II we briefly describe our model, the leptophobic $Z'$ model in the stringy flipped SU(5) theory. In section III we calculate the $Z'$ contribution to the $\Delta m_s$ when $Z'$ couplings to the quarks have both handedness. In section IV we consider the constraints imposed by the $BR(B \to \pi K)$ and predict the deviations in the direct and indirect CP asymmetries from the SM expectation and compare with the experimental results. We conclude in section V.

II. THE MODEL

The leptophobic $Z'$ model can occur naturally in some grand unified theories (GUTs) and string theories. It naturally avoids stringent low energy constraints thanks to the absence of couplings to charged leptons and light neutrinos. There are at least two known mechanisms that can generate the leptophobic $Z'$. The first one is obtained via dynamical mixing between the U(1) and U(1)' in the $E_6$ GUT [16]. The other scenario of leptophobia is obtained in the stringy flipped SU(5) GUT in the heterotic string theory [7]. In this paper we consider only
the latter scenario only because it is more relevant to the $B \rightarrow \pi K$ decays.

In the flipped SU(5), the SM particles appear in three copies of the representations

$$F = (10, \frac{1}{2}) = \{Q, d^c, \nu^c\}, \quad \bar{f} = (\overline{5}, -\frac{3}{2}) = \{L, u^c\}, \quad \ell^c = (1, \frac{5}{2}) = \{e^c\}. \quad (3)$$

The new neutral gauge boson $Z'$ can be leptophobic if it does not couple to $\overline{5}$ and $1$, while the quarks in $10$ still couple to it [7].

In addition to its own beauty this scenario has the following phenomenologically interesting features:

- The new $Z'$ coupling is generation dependent and can generate FCNC processes.
- The FCNC couplings allow large CP violation.
- It violates the isospin symmetry in the right-handed up- and down-quarks.
- The new gauge boson interaction maximally violates the parity in the up-quark sector.

In the mass eigenstates the interactions of $Z'$ gauge boson with the quarks can be written as

$$\mathcal{L} = -\frac{g_2}{\cos \theta_W} \delta Z'_\mu \left( \overline{u} \gamma^\mu P_L \left[V_L^u \overline{c} V_L^{u\dagger}\right] u + \overline{d} \gamma^\mu P_L \left[V_L^d \overline{c} V_L^{d\dagger}\right] d + \overline{d} \gamma^\mu P_R \left[V_R^d \overline{c} V_R^{d\dagger}\right] d \right) + h.c., \quad (4)$$

where $\delta$ parameterizes the size of the new gauge coupling relative to the SM coupling and is expected to be of $\mathcal{O}(1)$. The $\hat{c} = \text{diag}(c_1, c_2, c_3)$ represent the generation-dependent U(1)' quantum numbers [7], and $V_{L,R}^q (q = u, d)$ are unitary matrices diagonalizing the quark mass matrices. The explicit sets of values for $c_i (i = 1, 2, 3)$ derived from heterotic string theory are given in [7]. Since $V_L^u, V_L^d, V_R^d$ are unknown, we do not take specific values of $c_i$'s in [7] and take them as free parameters.

We introduce complex parameters, $L$ and $R$,

$$\left[ V_L^d \overline{c} V_L^{d\dagger} \right]_{23} \equiv \frac{1}{2} L_{sb}^Z, \quad \left[ V_R^d \overline{c} V_R^{d\dagger} \right]_{23} \equiv \frac{1}{2} R_{sb}^Z, \quad (5)$$

to represent the $b \rightarrow s$ FCNC couplings. For comparison with [6], we kept the factor 2 in (5).

Then

$$\mathcal{L}_{\text{FCNC}}^{Z'} = -\frac{g_2}{2 \cos \theta_W} \left[ L_{sb}^Z \overline{s} \gamma_\mu b_L Z'^\mu + R_{sb}^Z \overline{s} \gamma_\mu b_R Z'^\mu \right] + h.c., \quad (6)$$

where $\delta$ in (4) is absorbed into $L$ and $R$. To calculate the $B \rightarrow \pi K$ decay amplitudes we also need the $Z'$ couplings to the first generation quarks. Although $Q$ and $d^c$ have the same $U'(1)$ charges, the mixing effect in (4) can give different couplings to the $Z'$ in general

$$\mathcal{L}(Z' \overline{q} q) = -\frac{g_2}{\cos \theta_W} \delta Z' \left[ \overline{u} \gamma_\mu c_L^u P_L u + \overline{d} \gamma_\mu (c_L^d P_L + c_R^d P_R) d \right], \quad (7)$$
where we defined
\[ c_u^L \equiv \left[ V_L^\dagger c V_L^d \right]_{11}, \quad c_d^L \equiv \left[ V_L^\dagger c V_L^d \right]_{11}, \quad c_R^d \equiv \left[ V_R^\dagger c V_R^d \right]_{11}. \] (8)

From the structure of CKM matrix, we assume the couplings to the left-handed quarks are approximately equal, i.e. \( c_u^L = c_d^L \equiv c_q^L \). However, in general \( c_d^R \) can be different from \( c_q^L \).

As mentioned above, the absence of \( c_u^R \) is the characteristic feature of the leptophobic flipped SU(5) scenario. Note that since \( \delta \) and \( c_q^L \) are unknown, \( c_q^L \) can always be absorbed to \( \delta \). In addition, \( \delta \) does not appear in the expression for \( \Delta m_s \) and it can absorbed into \( L_{Z'}^{sb} \) or \( R_{Z'}^{sb} \) in the \( B \to \pi K \) amplitudes. So we fix \( \delta = c_d^l = 1 \) from now on.

### III. \( B_s^0 - \bar{B}_s^0 \) MIXING

In general the \( Z' \) can couple to both left- and right-handed quarks simultaneously as can be seen in (4). Then we need to extend the operator basis beyond the SM one in the effective Hamiltonian describing \( B_s^0 - \bar{B}_s^0 \) mixing.

The most general \( \Delta B = \Delta S = 2 \) process is described by the effective Hamiltonian [17]:
\[ H_{\text{eff}} = \sum_{i=1}^{5} C_i Q_i + \sum_{i=1}^{3} \tilde{C}_i \tilde{Q}_i + h.c., \] (9)

where
\[
\begin{align*}
Q_1 & = \bar{s}_L^\alpha \gamma_\mu b_L^\alpha \bar{s}_L^\beta \gamma_\mu b_L^\beta \\
Q_2 & = \bar{s}_R^\alpha b_L^\alpha \bar{s}_R^\beta b_L^\beta \\
Q_3 & = \bar{s}_R^\alpha b_L^\beta \bar{s}_R^\beta b_L^\alpha \\
Q_4 & = \bar{s}_R^\alpha b_L^\alpha \bar{s}_R^\beta b_L^\beta \\
Q_5 & = \bar{s}_R^\alpha b_L^\beta \bar{s}_R^\beta b_L^\alpha
\end{align*}
\] (10)

and the operators \( \tilde{Q}_{1,2,3} \) are obtained from the \( Q_{1,2,3} \) by the exchange \( L \leftrightarrow R \). Here \( q_{R,L} = P_{R,L} q \), with \( P_{R,L} = (1 \pm \gamma_5)/2 \), and \( \alpha \) and \( \beta \) are color indices.

In our model the nonvanishing Wilson coefficients at \( M_{Z'} \) scale are simply given by
\[
\begin{align*}
C_1(M_{Z'}) & = \frac{g_2^2}{8M_{Z'}^2} \left( L_{Z'}^{sb} \right)^2, \\
\tilde{C}_1(M_{Z'}) & = \frac{g_2^2}{8M_{Z'}^2} \left( R_{Z'}^{sb} \right)^2, \\
C_5(M_{Z'}) & = \frac{g_2^2}{8M_{Z'}^2} \left( -2L_{Z'}^{sb} R_{Z'}^{sb} \right). \quad (11)
\end{align*}
\]
Here $Q_5$ is the additionally generated operator compared with [6]. The renormalization group running down to $m_b$ scale mixes $Q_5$ with $Q_4$ and we get [17]

$$C_1(\mu_b) \simeq 0.801 C_1(M_{Z'}) ,$$
$$C_4(\mu_b) \simeq 0.697 C_5(M_{Z'}) ,$$
$$C_5(\mu_b) \simeq 0.886 C_5(M_{Z'}) .$$

(12)

The other operators are not generated at all and we get $C_2(\mu_b) = C_3(\mu_b) = 0$.

Now we can calculate the $B_s^0 - \bar{B}_s^0$ mixing matrix element

$$M_{12}^s = M_{12}^{s,SM} + M_{12}^{s,Z'} \equiv M_{12}^{s,SM}(1 + R) ,$$

(13)

where $R \equiv M_{12}^{s,Z'}/M_{12}^{s,SM}$. The SM contribution $M_{12}^{s,SM}$ is given by [18],

$$M_{12}^{s,SM} = \frac{G_F^2 M_B^2}{12\pi^2} M_{B_s} \left( f_{B_s} \hat{B}_{B_s}^{1/2} \right)^2 \eta_B S_0(x_t) (V_{tb}V_{ts}^*)^2 ,$$

(14)

involves additional hadronic parameters, $B_4(\mu_b)$ and $B_5(\mu_b)$.

The mass difference in the $B_s^0 - \bar{B}_s^0$ system, $\Delta m_s$ is obtained by

$$\Delta m_s = 2 |M_{12}^s| .$$

(16)

In the SM, we get

$$\Delta m_s^{SM} = (22.5 \pm 5.5) \text{ ps}^{-1} ,$$

(17)

where the nonperturbative hadronic parameters $f_{B_s}$ and $\hat{B}_{B_s}$ are the main sources of the uncertainty. We used the value

$$f_{B_s} \hat{B}_{B_s}^{1/2} \bigg|_{(HP+JL)QCD} = (0.295 \pm 0.036) \text{ GeV} ,$$

(18)

which is the combined lattice result [9] from JLQCD and HPQCD. For other parameters, we used $\alpha_s(\mu_b) = 0.22$, $\eta_B = 0.551$, $\overline{m}_t^{\overline{MS}}(m_t) = 162.3$ GeV and $V_{ts} = 0.04113$ [19].

In Figure 1 the allowed region by $\Delta m_s$ alone is shown. We fixed $m_{Z'} = 700$ GeV which is above the experimental lower bound [20] and scanned the weak phases $\phi_{L(R)} \equiv \arg(L_{ss}^{Z'}(R_{ss}^{Z'}))$.
from 0 to $2\pi$ independently. We can see that the sizes of FCNC couplings, $|L_{sb}'|$ and $|R_{sb}'|$, are restricted typically to be less than $\sim 0.1$ for most values of the scanned parameters, which is consistent with [6]. However, the two additional bands appear in this case due to the cancelation between $L_{sb}'$ and $R_{sb}'$. These regions extend indefinitely and show that the $\Delta m_s$ alone is not enough to constrain both $L_{sb}'$ and $R_{sb}'$ simultaneously. We will show that the $BR(B \to \pi K)$ can give upper bounds on both $|L_{sb}'|'$s and $|R_{sb}'|$ in the next section.

IV. $B \to \pi K$ DECAYS

The $B \to \pi K$ decays are dominated by the $\bar{b} \to \bar{s}$ QCD penguin diagrams. The subdominant electroweak penguin contribution is also sizable and may play important role in probing the NP as mentioned in the Introduction. The current experimental data in Table I show the branching ratios are quite precisely measured and the so-called $R_c/R_n$ puzzle [21] has disappeared. Therefore we take the four BRs as the additional constraints to the $\Delta m_s$ constraint considered in section III. As we will see in a moment, they are orthogonal to and as strong as $\Delta m_s$ constraint.

| Mode          | $BR[10^{-6}]$ | $A_{CP}$     | $S_{CP}$     |
|---------------|---------------|--------------|--------------|
| $B^+ \to \pi^+ K^0$ | 23.1 ± 1.0    | 0.009 ± 0.025 |              |
| $B^+ \to \pi^0 K^+$ | 12.9 ± 0.6    | 0.050 ± 0.025 |              |
| $B^0 \to \pi^- K^+$ | 19.4 ± 0.6    | −0.097 ± 0.012 |            |
| $B^0 \to \pi^0 K^0$ | 9.9 ± 0.6     | −0.14 ± 0.11  | 0.38 ± 0.19  |

TABLE I: Branching ratios, direct CP asymmetries $A_{CP}$, and mixing-induced CP asymmetry $S_{CP}$ (if applicable) for the four $B \to \pi K$ decay modes. The data are taken from Refs. [22] and [23].
In the SM the $B \to \pi K$ decay amplitudes can be written in terms of topological amplitudes:

\[
A(B^+ \to \pi^+ K^0) = -P'_{tc} - \frac{1}{3} P'_{EW} + P'_{uc} e^{i\gamma},
\]

\[
\sqrt{2}A(B^+ \to \pi^0 K^+) = P'_{tc} - P'_{EW} - \frac{2}{3} P'_{EW} - \left(T' + C' + P'_{uc}\right) e^{i\gamma},
\]

\[
A(B^0 \to \pi^- K^+) = P'_{tc} - \frac{2}{3} P'_{EW} - \left(T' + P'_{uc}\right) e^{i\gamma},
\]

\[
\sqrt{2}A(B^0 \to \pi^0 K^0) = -P'_{tc} - P'_{EW} - \frac{1}{3} P'_{EW} - \left(C' - P'_{uc}\right) e^{i\gamma},
\]  \hspace{1cm} (19)

where other small annihilation and exchange amplitudes are neglected. Here the weak phase, $\gamma$, dependence has been explicitly written. The primes denote the $\bar{b} \to \bar{s}$ transition. The $P'_{tc}$ ($P'_{uc}$) is the QCD penguin amplitude with $t, c$ ($u, c$) quarks running inside the loop. The tree (color-suppressed tree) diagrams are represented by $T'$ ($C'$). The $P'_{EW}$ is the electroweak (color-suppressed electroweak) penguins and related to the $T'$($C'$) by flavor SU(3) symmetry [24]:

\[
P'_{EW} = \frac{3 C_9 + C_{10}}{4 C_1 + C_2} R(T' + C') + \frac{3 C_9 - C_{10}}{4 C_1 - C_2} R(T' - C'),
\]

\[
P'_{EW} = \frac{3 C_9 + C_{10}}{4 C_1 + C_2} R(T' + C') - \frac{3 C_9 - C_{10}}{4 C_1 - C_2} R(T' - C'),
\]  \hspace{1cm} (20)

where $C_i$ (i=1,2,9,10) are the Wilson coefficients and $R = |V_{ts}V_{tb}^*/V_{us}V_{ub}|$.

In the SM, from the loop-, color-factor and the hierarchy of CKM matrix elements, we expect the following hierarchies:

\[
O(1) \quad |P'_{tc}|,
\]

\[
O(\lambda) \quad |T'|, |P'_{EW}|,
\]

\[
O(\lambda^2) \quad |C'|, |P'_{uc}|, |P'_{EW}|,
\]

\[
O(\lambda^3) \quad |A'|.
\]  \hspace{1cm} (21)

However the experimental data in Table I are not fully consistent with these hierarchies. Specifically $A_{CP}(B^+ \to \pi^0 K^+) \neq A_{CP}(B^0 \to \pi^- K^+)$ and $S_{CP}(B^0 \to \pi^0 K^0) \neq \sin 2\beta$ require $|C'/T'| = 1.6 \pm 0.3$ [15]. This large ratio is inconsistent with the SM expectation (21) which is supported by theoretical calculations [25–27]. And the implications of this apparent discrepancy have been considered in many NP models [2]. This puzzle can be solved most naturally if NP is introduced in the electroweak penguin amplitude [15].

In this paper, as mentioned above, we use the four $BR(B \to \pi K)$’s to constrain the sizes of $Z'$ couplings. Using the remaining parameters after imposing $\Delta m_s$ and the $BR(B \to \pi K)$’s, we predict the direct and indirect CP asymmetries which show apparent deviations from the SM. We show that the predictions for $A_{CP}(B^+ \to \pi^0 K^+), A_{CP}(B^0 \to \pi^- K^+)$ and $S_{CP}(B^0 \to \pi^0 K^0)$ in our model are in the right directions to the experimental results.
To calculate the SM predictions for the BR’s and CP asymmetries, we use the NLO calculations in the perturbative QCD (PQCD) results [26]. In our model the \( Z' \) contributions can change the amplitudes, \( P'_{tc(uc)} \), \( P'^{(C)}_{EW} \). The NP contributions are calculated using the naive factorization method and their strong phases are assumed to be equal to the corresponding SM diagrams. We will also discuss the effect of the NP strong phase later. The \( Z' \) contribution to the topological amplitudes are written in terms of the Wilson coefficients in the standard operator basis [28] as

\[
P'(Z') = -\lambda_t \left[ (a_4 + r^K_s a_6) - (\bar{a}_4 + \bar{r}^{K}_s \bar{a}_6) \right] A_{\pi K} e^{i\delta_{tc}}
\]

\[
P'_{EW}(Z') = \frac{3}{2} \lambda_t \left[ (-a_7 + a_9) - (-\bar{a}_7 + \bar{a}_9) \right] A_{K\pi} e^{i\delta_{EW}}
\]

\[
P'^{(C)}_{EW}(Z') = \frac{3}{2} \lambda_t \left[ (a_{10} + r^K_s a_8) - (\bar{a}_{10} + \bar{r}^{K}_s \bar{a}_8) \right] A_{\pi K} e^{i\delta_{EW}^{(C)}},
\]

where \( \lambda_t = V_{ts}^* V_{tb} \), \( a_i = C_i + C_i \pm 3 \) \((\pm)\) for odd (even) \( i \), \( r^K_s = 2m^2_s/m_b(m_s + m_q) \) (with \( m_q = (m_u + m_d)/2 \)), \( A_{\pi K(K\pi)} = G_F(m^2_B - m^2_\pi)F_0^{\pi(K)}f_{K(\pi)}/\sqrt{2} \), and the \( \delta \)'s are the corresponding strong phases obtained in [26]. The \( \bar{a}_i \)'s are Wilson coefficients for the chirality flipped operators.

In our model the Wilson coefficients at \( M_{Z'} \) scale are

\[
-\lambda_t C_3(M_{Z'}) = \delta \frac{m^2_Z L_{sb} Z'_{sb} c^d_L + 2 c^d_L}{3}, \quad C_4(M_{Z'}) = 0,
\]

\[
-\lambda_t C_5(M_{Z'}) = \delta \frac{m^2_Z L_{sb} Z'_{sb} c^d_R + 2 c^d_R}{3}, \quad C_6(M_{Z'}) = 0,
\]

\[
-\frac{3}{2} \lambda_t C_7(M_{Z'}) = \delta \frac{m^2_Z L_{sb} Z'_{sb} (c^d_L - c^d_R)}{3}, \quad C_8(M_{Z'}) = 0,
\]

\[
-\frac{3}{2} \lambda_t C_9(M_{Z'}) = \delta \frac{m^2_Z L_{sb} Z'_{sb} (c^d_L - c^d_R)}{3}, \quad C_{10}(M_{Z'}) = 0.
\]

There are also the chirality flipped operators to the SM operators. Their Wilson coefficients, \( \tilde{C}_i \)'s are obtained by exchanging \( L_{sb} Z'_{sb} \leftrightarrow R_{sb} Z'_{sb} \), \( c^d_L \leftrightarrow c^d_R \).

In Figure 2, we show the allowed topological amplitudes in \((P'_{tc}, P'_{EW})\) plane (a) and in \((P'_{EW}, P'^{(C)}_{EW})\) plane (b). We can see that \( P'_{tc} \) is strongly constrained to lie in the region \((49,52)\) eV by the \( BR(B \to \pi K) \)'s, whereas sizable deviation from the SM predictions are possible for \( P'_{EW} \) and \( P'^{(C)}_{EW} \).

As mentioned in the previous section, the Figure 3 shows that, given \( c^d_L \) and \( c^d_R \), the flavor changing couplings, \(|L'_{sb}|\) and \(|R'_{sb}|\), are constrained by the four \( BR(B \to \pi K) \)'s in addition to the \( \Delta m_s \). Since the experimental measurements are quite precise now and the theoretical calculations have still large errors, we allowed 3-\( \sigma \) range for the BRs. For the plot, we set \( M_{Z'} = 700 \) GeV and \( c^d_R = 1, 0.5, 0 \) from the left, respectively. Since the \( BR(B \to \pi K) \) decays are most sensitive to the \( P'_{EW} \) which is maximized at \( c^d_R = 1 \) and vanishes at \( c^d_R = 0 \), the constraint is strongest for \( c^d_R = 1 \).
FIG. 2: The correlations between $P_{tc}'$ and $P_{EW}'$ (a) and between $P_{EW}'$ and $P_{EW}^{C}$ (b) for $M_{Z'} = 700$ GeV and $c_R^d = 1$.

FIG. 3: The allowed region in $(|L\phi_{Z'}|, |R\phi_{Z'}|)$ plane by $\Delta m_s$ and the four $BR(B \to \pi K)$’s. We fixed $c_R^d = 1.0, 0.5, 0.0$ from the left.

Now we predict the direct and indirect CP asymmetries using the parameter set allowed by $\Delta m_s$ and the four $BR(B \to \pi K)$’s. We are especially interested in the correlation between the two direct CP asymmetries, $A_{CP}(B^{+} \to \pi^{0}K^{+})$ and $A_{CP}(B^{0} \to \pi^{-}K^{+})$, and the indirect CP asymmetry, $S_{CP}(B^{0} \to \pi^{0}K^{0})$ because they show the apparent deviations from the SM predictions.

The predictions for the $A_{CP}(B^{+} \to \pi^{0}K^{+})$ and $A_{CP}(B^{0} \to \pi^{-}K^{+})$ are shown in Figure 4 for $M_{Z'} = 700$ GeV. In these figures the errors for the SM predictions are also obtained from [26]. For the NP predictions we fixed the SM to the central values and we did not include the hadronic uncertainties. Although the SM results are consistent with the experimental data at 2-σ level, the $Z'$ contribution can accommodate the current data at 1-σ level for $c_R^d = 1.0, 0.5$. 
The predictions for $A_{\text{CP}}(B^+ \to \pi^0 K^+)$ and $A_{\text{CP}}(B^0 \to \pi^- K^+)$ for $M_{Z'} = 700$ GeV and (a) $c_R^d = 1.0$, (b) $c_R^d = 0.5$, (c) $c_R^d = 0.0$.

The predictions for the correlation between $A_{\text{CP}}(B^+ \to \pi^0 K^+)$ and $S_{\text{CP}}(B^0 \to \pi^0 K^0)$ for $M_{Z'} = 700$ GeV and (a) $c_R^d = 1.0$, (b) $c_R^d = 0.5$, (c) $c_R^d = 0.0$.

The value $c_R^d = 0.0$ cannot explain the data. These results are consistent with [15] which claims that the current data require large NP contributions at the electroweak penguin sector.

The predictions for the correlation between $A_{\text{CP}}(B^+ \to \pi^0 K^+)$ and $S_{\text{CP}}(B^0 \to \pi^0 K^0)$ are shown in Figure 5. While it is difficult to get $S_{\text{CP}}(B^0 \to \pi^0 K^0)$ as low as $\sim 0.38$ which is the central value for the current experiments, it is possible to accommodate both $S_{\text{CP}}(B^0 \to \pi^0 K^0)$ and $A_{\text{CP}}(B^+ \to \pi^0 K^+)$ simultaneously for $c_R^d = 1.0, 0.5$ (Figure 5(a),(b)). Again it is difficult to get large deviations from the SM prediction for the $S_{\text{CP}}(B^0 \to \pi^0 K^0)$ for $c_R^d = 0.0$. Therefore the value $c_R^d = 0.0$, corresponding to $P_{\text{EW}}'(Z') = 0$, is disfavored even if we include the hadronic uncertainties in the calculation.

Until now we fixed the NP strong phases to be equal to the corresponding SM strong phases. In general, they may not be equal to each other. To see the effect of the NP strong phases, now we allow the strong phase of the dominant NP electroweak penguin $\delta_{\text{EW}}'$ to take arbitrary
FIG. 6: The $|P'_{EW}|$ (a), $A_{CP}(B^+ \rightarrow \pi^0 K^+)$ (b), and $S_{CP}(B^0 \rightarrow \pi^0 K^0)$ (c) as a function of strong phase, $\delta_{EW}'$, of the electroweak penguin. We fixed $c^d_R = 1$, $M_{Z'} = 700$ GeV.

FIG. 7: A scattered plot in $(M_{Z'}, |R_{sb}^{Z'}|)$ plane. For this plot we imposed $A_{CP}(B^+ \rightarrow \pi^0 K^+)$, $A_{CP}(B^0 \rightarrow \pi^- K^+)$, and $S_{CP}(B^0 \rightarrow \pi^0 K^0)$ constraints as well as the $\Delta m_s$ and $BR(B \rightarrow \pi K)$’s.

values. Figure 6 shows that it can give strong impact on the $A_{CP}(B^+ \rightarrow \pi^0 K^+)$. (For these plots we fixed $c^d_R = 1$, $M_{Z'} = 700$ GeV.) The reason is that if $\delta_{EW}'$ is equal to the strong phase of the dominant QCD penguin, the NP electroweak penguin contribution to $A_{CP}(B^+ \rightarrow \pi^0 K^+)$ vanishes at the leading order of $P'_{EW}(Z')/P_{tc'}$, independent of the weak phases $\phi_{L(R)}^{Z'}$. The effects of $\delta_{EW}'$ on $|P'_{EW}|$ and $S_{CP}(B^0 \rightarrow \pi^0 K^0)$ are rather minor, and any value of $\delta_{EW}'$ can successfully explain the $S_{CP}(B^0 \rightarrow \pi^0 K^0)$ anomaly.

Now we consider the mass dependence of the $Z'$ gauge boson. In Figure 7 we can see that the $M_{Z'}$ as large as 5 TeV which is beyond the LHC reach can accommodate the data with $|R_{sb}^{Z'}| \lesssim 0.3$. (We fixed $c^d_R = 1$ for the plot.) The parabolic shape can be understood from (23) because the parameter $|L_{sb}^{Z'}|$ can be fixed in terms of $|R_{sb}^{Z'}|$ by $\Delta m_s$. 
V. CONCLUSIONS

We considered the leptophobic $Z'$ model in the flipped SU(5) GUT obtained from heterotic string theory [7]. This is phenomenologically interesting because it contains ingredients which can possibly explain the apparent deviations from the SM predictions in the $B \to \pi K$ decays:

- The new $Z'$ coupling is generation dependent and can generate FCNC processes.
- The FCNC couplings allow large CP violation.
- The couplings also violate the isospin symmetry and can give large contributions to the electroweak penguins, $P_{EW}'$ and $P_{EW}^{IC}$.

We found that if we include the left- and right-handed FCNC couplings $L_{sb}^{Z'}$ and $R_{sb}^{Z'}$ simultaneously, we cannot obtain the absolute upper bounds for them contrary to [6] where it was assumed that only a single coupling exists at a time. If we impose the additional constraints, $BR(B \to \pi K)$’s, with some reasonable assumptions, we can constrain $|L_{sb}^{Z'}|$ and $|R_{sb}^{Z'}|$.

We predicted the CP asymmetries, $A_{CP}(B^+ \to \pi^0 K^+)$, $A_{CP}(B^0 \to \pi^- K^+)$, and $S_{CP}(B^0 \to \pi^0 K^0)$. Interestingly enough all of them are consistent with the current experimental results when the isospin breaking coupling, $c_{R}^{d}$, is non-vanishing. The case for $c_{R}^{d} = 0$ where there is no $Z'$ contribution to the electroweak penguin is disfavored from the current data.

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