Static response of FGM porous rhombic conoidal shell

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Abstract. The porosity effect on static analysis of functionally graded porous doubly curved rhombic conoids was investigated. Porosities inside functionally graded materials can occur due to a technical issue during the fabrication process and lead to the occurrence of micro-voids in the materials. The porosity effect was accounted for using a modified power law. The presented mathematical formulation includes normal curvatures in displacement field, as well as cross curvature in strain-displacement field. The improvement in the 2D mathematical model enables to solve problems of moderately thick FGM porous conoids. The feature that distinguishes the presented shell from other is that the maximum transverse deflection does not occur at its centre. The improved mathematical model was executed in finite element code. The obtained numerical results were compared with the results available in the literature. Once validated, the current model was employed to study the effect of porosity, skew angle, boundary condition, volume fraction index, loading pattern and other geometric parameters.

1. Introduction

In the last two decades, shell structures made of functionally graded materials (FGM) have been broadly used by civil, mechanical, aeronautical and marine engineering. The FGM is an inhomogeneous material, composed of two (or more) materials, organised with a smooth gradation in the desired direction. However, at the time of fabrication, porosities are infused in the FGM which creates a major issue. Apart from this, the FGM manufactured using sintering process retain porosities because of different solidification rate of material constituents. Koizumi used FGM in advanced engineering structures experiencing elevated temperatures [1]. The bending analysis of FGM cylindrical shells using the element-free kp-Ritz method was analysed by Zhao [2]. An elastic solution for a sandwich panel with isotropic skins was presented by Kashtalyan and Menshykova [3]. The stresses in functionally graded doubly curved shells were calculated by incorporating the differential quadrature method in the first order shear deformation theory by Tornabene and Viola [4]. Viola used unconstrained third-order shear deformation theory for the static analysis of moderately thick functionally graded conical shells [5]. Dai and Dai developed an analytical solution based upon the classical shell theory for FGM cylindrical shell under thermomechanical loading [6]. Xiang and Liu used a meshless global collocation method with nth-order shear deformation theory for the static analysis of FGM sandwich plate [7].
Gutierrez Rivera and Reddy studied the large deformation behaviour of FGM shells by using seven-parameter continuum shell formulation [8]. Since casting and fabrication of conoids is easy due to its singly ruled surface, it is favoured in the construction industry. Conoids are structurally stiff, aesthetically appealing, and are used to cover the column-free large area in industrial structures, aircraft hangars and exhibition halls. The first study on simply supported and clamped conoids was carried out by Hadid [9]. A combined variational approach was used for the bending analysis of elastic conoids for both types of boundary condition. Finite difference method was carried out on the conoidal shell by Das and Bandyopadhyay for both experimental, as well as theoretical investigation [10]. Ghosh and Bandyopadhyay engaged their own formulation to examine the influence of cut-outs on the static analysis of conoidal shells [11]. The bending analysis of stiffened conoids was studied by Das and Chakravorty using a three-noded beam element [12]. Kumari and Chakravorty used FSDT for the study on the bending response of delaminated conoids [13]. Malekzadeh Fard and Baghestani explored the free vibration behaviour of moderately thick doubly curved shell based on FSDT with classic boundary conditions [14]. The bending and free vibration analysis of functionally graded porous plate was presented by Akbaş [15]. He used FSDT model and Navier solution for solving the problem. An analytical model was developed by Al Rjoub and Hamad to study the effect of porosity on free vibration analysis of porous beam [16]. Kiran et al. presented the effect of porosity on the static response of a functionally graded skew magneto-electro-elastic plate [17]. Gupta and Talha explored the influence of porosities on the flexure and free vibration response of graded plate using higher deformation theory [18,19].

The literature review reveals that no results for the static analysis of doubly curved FGM porous rhombic conoids are available. Therefore, an attempt was made in the present paper to study the bending behaviour of FGM porous conoidal shell with the help of an improved mathematical model. The FE coding was done by using a C0 nine-noded FE with seven nodal unknowns at each node for the presented improved mathematical model developed by the authors. C1 continuity requirement associated with the presented model is suitably circumvented. The present study facilitates the bending analysis by finite element (FE) modelling, keeping in mind the processing time using a computer and the simplicity of the approach. This study is the first step towards enhancing our understanding of the bending problem of FGM porous rhombic conoids.

2. Formulation

2.1. Porosity inclusion

The FGM porous conoidal shell of sides a, b and thickness h are depicted in Figure 1. As presented in Figure 1(b), the even type of porosity distributions along the thickness coordinate is considered in the present work. Using the modified power law for the inclusion of porosity effect, the effective material properties of the FGM porous conoidal shell at any point in thickness coordinate (z) can be stated as:

\[ P(z) = P_c V_c(z) + P_m V_m(z) - (P_c + P_m) \frac{\beta}{2} \]

where, \( V_c(z) = \left( \frac{1}{2} + \frac{z}{h} \right)^n, \quad (0 \leq n \leq \infty), V_c(z) + V_m(z) = 1 \) (1)

where \( \beta (\beta \ll 1) \) is the volume fraction of porosities. \( P(z) \) implies the material properties like the Young’s modulus of elasticity \( (E) \), material density \( (\rho) \) and Poisson’s ratio \( (\nu) \) of the FGM porous conoidal shell. Subscripts \( c \) and \( m \) stand for ceramic and metallic constituents, respectively.
2.2. Displacement field and strains

In order to develop the mathematical model, the displacement fields \((u, v, w)\) along the \((x, y, z)\) coordinates for FGM conoidal shell are considered on the basis of the third order shell theory and are as follows:

\[
\begin{align*}
\mathbf{u}(x,y,z) &= \left(1 + \frac{z}{R_x}\right) u_0 + z \theta_x + z^2 \xi_x + z^3 \zeta_x \\
\mathbf{v}(x,y,z) &= \left(1 + \frac{z}{R_y}\right) v_0 + z \theta_y + z^2 \xi_y + z^3 \zeta_y \\
\mathbf{w}(x,y,z) &= w_0
\end{align*}
\]  

where \((u_0, v_0, w_0)\) are the corresponding displacements of any point on the mid-plane along the \((x, y, z)\) coordinates. \((\theta_x, \theta_y)\) are the rotations of normal to the mid-plane about the \(y\) and \(x\)-axes, respectively. The functions \((\xi_x, \xi_y, \zeta_x, \zeta_y)\) are higher order terms of Taylor’s series expansion at the mid-plane of the conoidal shell. By imposing the boundary condition (zero transverse shear strain at top and bottom) in equation (2), the function \(\xi_x, \xi_y, \zeta_x\) and \(\zeta_y\) are calculated as

\[
\begin{align*}
\mathbf{u}(x,y,z) &= \left(1 + \frac{z}{R_x}\right) u_0 + \theta_x \left(z - \frac{4z^3}{3h^2}\right) - \frac{\partial w_0}{\partial x} \left(\frac{4z^3}{3h^2}\right) + v_0 \left(\frac{4z^3}{3h^2 R_y}\right) \\
\mathbf{v}(x,y,z) &= \left(1 + \frac{z}{R_y}\right) v_0 + \theta_y \left(z - \frac{4z^3}{3h^2}\right) - \frac{\partial w_0}{\partial y} \left(\frac{4z^3}{3h^2}\right) + u_0 \left(\frac{4z^3}{3h^2 R_y}\right) \\
\mathbf{w}(x,y,z) &= w_0
\end{align*}
\]  

For omitting \(C^1\) continuity problem associated with TSDT, the out of plane derivatives are replaced by the following relations

\[
\psi_x = \frac{\partial w_0}{\partial x}, \quad \psi_y = \frac{\partial w_0}{\partial y}
\]
Hence, the field variables per node used in the present mathematical model are \(u_0, v_0, w_0, \theta_x, \theta_y, \psi_x\) and \(\psi_y\). Mathematically, it may be expressed as

\[
\{\delta\} = \{u_0, v_0, w_0, \theta_x, \theta_y, \psi_x, \psi_y\}^T
\]

(5)

where \(\{\delta\}\) is termed as a displacement vector.

The strain vector may be expressed as

\[
\{\varepsilon\} = \{\varepsilon_x, \varepsilon_y, \gamma_{xy}, \gamma_{xz}, \gamma_{yz}\}^T
\]

(6)

Further, the strain vector \(\{\varepsilon\}\) can be associated with global displacement vector \(\{X\}\) by the subsequent relationship

\[
\{\varepsilon\} = [B]\{X\}
\]

(7)

Here, matrix \([B]\) is known as strain-displacement matrix that contains the derivatives of shape functions.

The in-plane and transverse shear strains are

\[
\varepsilon_x = \frac{\partial u}{\partial x} + \frac{w}{R_x}
\]

\[
\gamma_{xy} = \frac{\partial v}{\partial x} + \frac{\partial u}{\partial y} + \frac{2w}{R_y}
\]

\[
\gamma_{xz} = \frac{\partial u}{\partial z} + \frac{\partial w}{\partial x} - \frac{u_0}{R_x} - \frac{v_0}{R_y}
\]

\[
\gamma_{yz} = \frac{\partial v}{\partial z} + \frac{\partial w}{\partial y} - \frac{u_0}{R_x} - \frac{v_0}{R_y}
\]

(8)

The strain relationship can be written as:

\[
\begin{bmatrix}
\varepsilon_x \\
\varepsilon_y \\
\gamma_{xy}
\end{bmatrix} = \begin{bmatrix}
\varepsilon_x^0 \\
\varepsilon_y^0 \\
\gamma_{xy}^0
\end{bmatrix} + Z \begin{bmatrix}
k_1 \\
k_2 \\
k_3
\end{bmatrix} - \frac{4Z^2}{3h^2} \begin{bmatrix}
k_1^3 \\
k_2^3 \\
k_3^3
\end{bmatrix}
\]

\[
\begin{bmatrix}
\gamma_{xz} \\
\gamma_{yz}
\end{bmatrix} = \begin{bmatrix}
\gamma_{xz}^0 \\
\gamma_{yz}^0
\end{bmatrix} + \frac{4Z^2}{h^2} \begin{bmatrix}
k_4 \\
k_5
\end{bmatrix}
\]

(9)

where

\[
\gamma_{yz}^0 = \left(\frac{\partial w_0}{\partial y} + \theta_y\right) - \frac{u_0}{R_{sy}}, \quad \gamma_{xz}^0 = \left(\frac{\partial w_0}{\partial x} + \theta_x\right) - \frac{v_0}{R_{sx}}
\]

\[
k_1 = \frac{\partial \theta_x}{\partial x} + \frac{\partial u_0}{\partial x} \frac{1}{R_x}, \quad k_2 = \frac{\partial \theta_y}{\partial y} + \frac{\partial v_0}{\partial y} \frac{1}{R_y}
\]
2.3. Constitutive relationship

The linear stress-strain constitutive correlation for the FGM conoids are

\[
\{\sigma\} = [Q]\{\varepsilon\}
\]

where the constitutive matrix

\[
[Q] = \begin{bmatrix}
Q_{11} & Q_{12} & 0 & 0 & 0 \\
Q_{21} & Q_{22} & 0 & 0 & 0 \\
0 & 0 & Q_{66} & 0 & 0 \\
0 & 0 & 0 & Q_{44} & 0 \\
0 & 0 & 0 & 0 & Q_{55}
\end{bmatrix}
\]

and

\[
Q_{66} = Q_{44} = Q_{55} = \frac{E(z)}{2(1+\nu)}
\]

\[
Q_{21} = Q_{22} = \frac{E(z)}{1-\nu^2}, \quad Q_{12} = Q_{21} = \frac{\nu E(z)}{1-\nu^2}
\]

3. Finite element modelling

3.1. Introduction

For the present \(C^0\) finite element (FE) model, nine-noded isoparametric Lagrangian element with seven degrees of freedom at each node was utilised in the present investigation.

3.2. Transformation of skew boundary of conoids

The plan of rhombic conoid is shown in Figure 2. In order to transform the global axes to local axes, transformation matrix \([T]\) was used.
where \( s = \sin \alpha \), \( c = \cos \alpha \) and \( \alpha \) is the skew angle of the conoidal shell.

3.3. Governing equation
The strain energy may be expressed as

\[
U = \frac{1}{2} \int \int \int \{ \varepsilon \}^T \{ \sigma \} \, dx \, dy \, dz
\]  

(15)

By using the equation (11), the expression above can be represented as

\[
U = \frac{1}{2} \int \int \{ \varepsilon \}^T \{ D \} \{ \varepsilon \} \, dx \, dy
\]  

(16)

where, \( [D] = \int [H]^T [Q][H] \, dx \) in which \( [H] \) is the matrix that contains terms involving \( z \) and \( h \).

By utilising equation (7) the stiffness matrix \( [K] \) is written as

\[
[K] = \int \int \{ B \}^T \{ D \} [B] \, dx \, dy
\]  

(17)

4. Numerical results and discussion
The bending analysis of FGM porous rhombic conoidal shells was performed under various types of mechanical loading. Parameters such as thickness ratio, aspect ratio, \( hl/hh \) ratio, porosity volume fraction and volume fraction index are also accomplished for numerical results. Unless stated otherwise, the following non-dimensional factors are utilised in the upcoming examples.

Non-dimensional quantities used are

\[
\bar{w} = 100 \frac{wh^3E_a}{q_o a^4}, \quad \bar{\sigma}_x = \sigma_x \left( \frac{a \ b \ h}{2 \cdot 2 \cdot 2} \right) 100 \frac{h^2}{q_o a^3}
\]
Loading patterns used are

\[ q = q_0, \quad q = q_0 \sin \left( \frac{\pi x}{a} \right) \sin \left( \frac{\pi y}{b} \right), \quad q = q_0 \cos \left( \frac{\pi x}{a} \right) \sin \left( \frac{\pi y}{b} \right), \quad q = q_0 \cos \left( \frac{\pi x}{a} \right) \cos \left( \frac{\pi y}{b} \right) \]

The details of some of the boundary condition (BC) used are as follows:

1. Simply supported (SSSS):
   - At \( x = 0 \), \( a = w = \theta_y = \psi_y = 0 \)
   - At \( y = 0 \), \( b = u = \theta_x = \psi_x = 0 \)

2. Clamped and simply supported (CCSS):
   - At \( x = 0 \), \( a = u = v = w = \theta_x = \psi_x = 0 \)
   - At \( y = 0 \), \( b = u = \theta_x = \psi_x = 0 \)

### 4.1. Comparison and convergence study

Three appropriate examples have been solved to check consistency and stability of the presented FE results. No results for FGM porous rhombic conoidal shell were found in the literature; hence, effectiveness of the present formulation has been tested by relating dimensionless deflection for FGM porous plate and isotropic conoids. The properties of the FGM constituents at room temperature used in the present study are as follows:

- Al/ZrO\(_2\): \( E_c = 151 \text{ GPa}, \quad E_m = 70 \text{ GPa}, \quad v_c = v_m = 0.3, \quad \rho_c = 3000 \text{ kg/m}^3, \quad \rho_m = 2707 \text{ kg/m}^3 \)

Example 1. The convergence study of dimensionless maximum deflection of FGM (Al/ZrO\(_2\)) plate was conducted for four power law indices \( (n = 0, 0.5, 1, \infty) \) and presented in Table 1. For this particular example, we used \( hl = 0 \) and \( hh = 0 \) in the present FE code of the FGM-1 conoidal shell for converting it into FGM plate. It was found that at \( 16 \times 16 \) mesh, the results converge for the present nine-noded isoparametric elements. From Table 1, it was noted that our numerical results are consistent with the ones obtained by Ferreira et al. [20].

Example 2. Figure 3 represents the comparison of dimensionless maximum transverse displacement of simply supported FGM porous plate. The bottom layer of FGM porous plate is metallic and the top layer is ceramic. For various values of volume fraction indices, the results were compared with Akbaş and again it showed a decent agreement [15].

Example 3. No results for FGM porous conoidal shell exists in the literature; therefore, the presented FE result of FGM porous conoidal shell were compared with numerical result of isotropic conoidal shell. The Poisson’s ratio \( \nu = 0.15, \quad hl/hh = 0.50 \) and side-to-thickness ratio \( a/h = 19 \) was used to validate the presented result. Table 2 shows the validation of the presented FE formulation with Hadid and Bakshi and Chakravorty. The validation study confirms the improvement of the present obtained FE result over Bakshi and Chakravorty as the current numerical results are closer to Hadid [9,21].

| Mesh  | \( n = 0 \) | \( 0.5 \) | \( 1 \) | \( \infty \) |
|-------|------------|----------|--------|-----------|
| 6x6   | 0.02476    | 0.03326  | 0.03680| 0.05348   |
| 8x8   | 0.02480    | 0.03331  | 0.03685| 0.05352   |
| 12x12 | 0.02482    | 0.03333  | 0.03688| 0.05353   |
| 16x16 | 0.02482    | 0.03333  | 0.03688| 0.05353   |
| Ferreira et al. [20] | 0.0248    | 0.0330   | 0.0368   | 0.0536    |
Figure 3. Comparison of dimensionless deflection of FGM porous plate subjected to uniform loading.

Table 2. Comparison of deflection (x10-2) of isotropic conoid along y/b = 0.50 subjected to uniformly distributed load.

| x/a | Bakshi and Chakravorty [21] | Hadid [9] | Present |
|-----|-----------------------------|-----------|---------|
| 0.10| 2.3680                      | 2.5231    | 2.5621  |
| 0.40| 5.1557                      | 4.6333    | 4.7609  |
| 0.60| 4.0510                      | 3.5448    | 3.8575  |
| 0.70| 3.4129                      | 3.3149    | 3.2970  |
| 0.80| 2.4809                      | 2.5544    | 2.5949  |

4.2. Results and discussion

In order to analyse FGM porous conoids under various type of transverse loading, different composition of material constituents, different combination of boundary condition, numerous values of volume fraction index, side-to-thickness ratio, aspect ratio and hl/hh ratio were considered. Table 3 shows dimensionless maximum deflection and their location subjected to uniform loading for the FGM porous rhombic conoids, respectively. The numerical results were computed for a/h = 10, a/b = 1, hl/hh = 0.25 (hl = 0.05, hh = 0.2), and α = 15°, 30°, 45° and 60°. It can be seen that as we move from n = 0 to n = ∞, dimensionless deflection of the porous conoidal shell is increased and it may be attributed to the higher volume fraction index which leads to a lesser ceramic content, thus reducing its stiffness. Interestingly, around 1.1 times increase in maximum dimensionless deflection was noticed for all skew angle as the volume fraction index changed from 0 to ∞ for perfect conoidal shell, 1.4 times for porous conoidal shell with porosity volume fraction β = 0.1 and 1.7 times for porous shell with porosity volume β = 0.2.

Table 4 depicts dimensionless maximum deflection of FGM porous rhombic conoids with skew angle for CCCC, CCSS, CSCS, CCFF and CFCF boundary condition. It was noted that the deflection of the porous conoidal shell increased along with the porosity volume and it may be attributed to the decrease in the equivalent modulus of elasticity. The maximum dimension-less deflection decreases along with skew angle. Due to the fact that an increase in skew results in the reduction of length of shorter diagonal and shortening of diagonal leads to improvement in stiffness of the rhombic conoids; thus, deflection reduces. Interestingly, highest deflection was found for CFCF type end condition while CCCC retains lowest deflection values among all types of end conditions. The effect of loading pattern on dimensionless maximum deflection of simply supported FGM porous rhombic conoids is presented in Table 5. For all considered types of loading pattern, the non-dimensional value of deflection decreases with an increase in skew angle. It can be noted that the effect of the skew angle is greater for cos-sin loading. The numerical results of dimensionless deflection of FGM porous rhombic conoids with a/h ratio are plotted in Figure 4. These numerical results highlighted that non-dimensional deflection value decreases with
an increase in the a/h ratio. The dimensionless deflexion decreases up to a/h = 50; furthermore, no significant changes in dimensionless deflexion were noticed.

**Table 3.** Variation of dimensionless deflexion of FGM porous rhombic conoids subjected to uniform loading.

| n  | β  | Skew angle | 15°  | 30°  | 45°  | 60°  |
|----|----|------------|------|------|------|------|
| 0  | 0  |            | 0.9342 | 0.5291 | 0.2214 | 0.0567 |
|    | 0.1|            | 1.0238 | 0.5808 | 0.2435 | 0.0623 |
|    | 0.2|            | 1.1269 | 0.6405 | 0.2691 | 0.0689 |
| 0.2| 0  |            | 1.0225 | 0.5785 | 0.2410 | 0.0613 |
|    | 0.1|            | 1.1307 | 0.6406 | 0.2672 | 0.0679 |
|    | 0.2|            | 1.2577 | 0.7136 | 0.2980 | 0.0756 |
| 0.5| 0  |            | 1.1042 | 0.6249 | 0.2597 | 0.0659 |
|    | 0.1|            | 1.2313 | 0.6977 | 0.2901 | 0.0735 |
|    | 0.2|            | 1.3837 | 0.7851 | 0.3267 | 0.0826 |
| 1  | 0  |            | 1.2062 | 0.6832 | 0.2834 | 0.0719 |
|    | 0.1|            | 1.3592 | 0.7707 | 0.3199 | 0.0810 |
|    | 0.2|            | 1.5479 | 0.8789 | 0.3650 | 0.0923 |
| ∞  | 0  |            | 2.0151 | 1.1413 | 0.4776 | 0.1222 |
|    | 0.1|            | 2.4366 | 1.3818 | 0.5798 | 0.1485 |
|    | 0.2|            | 3.0326 | 1.7235 | 0.7242 | 0.1854 |

**Table 4.** Variation of dimensionless deflexion of FGM porous rhombic conoids subjected to uniform loading.

| BC  | β  | Skew angle | 15°  | 30°  | 45°  | 60°  |
|-----|----|------------|------|------|------|------|
| CCC  | 0  |            | 0.9544 | 0.5615 | 0.2367 | 0.0604 |
|     | 0.1|            | 1.0814 | 0.6361 | 0.2681 | 0.0684 |
|     | 0.2|            | 1.2404 | 0.7291 | 0.3072 | 0.0783 |
| CCSS | 0  |            | 0.9970 | 0.5773 | 0.2433 | 0.0622 |
|     | 0.1|            | 1.1236 | 0.6508 | 0.2743 | 0.0701 |
|     | 0.2|            | 1.2807 | 0.7420 | 0.3129 | 0.0798 |
| $\alpha$ | $B$ | Loading pattern |
|---------|-----|-----------------|
|         |     | uniform | sin-sin | cos-sin | cos-cos |
| 15°     | 0   | 0.9544  | 0.7191  | 0.1070  | 0.1830  |
|         |     | (0.58,0.48) | (0.55,0.48) | (0.29,0.34) | (0.84,0.72) |
|         | 0.1 | 1.0814  | 0.8146  | 0.1200  | 0.2064  |
|         |     | (0.58,0.48) | (0.55,0.48) | (0.29,0.34) | (0.84,0.72) |
|         | 0.2 | 1.2404  | 0.9347  | 0.1366  | 0.2360  |
|         |     | (0.58,0.48) | (0.55,0.48) | (0.29,0.34) | (0.84,0.72) |
| 30°     | 0   | 0.5615  | 0.4043  | 0.0432  | 0.1612  |
|         |     | (0.69,0.41) | (0.61,0.41) | (0.31,0.24) | (0.92,0.65) |
|         | 0.1 | 0.6361  | 0.4573  | 0.0480  | 0.1815  |
|         |     | (0.69,0.41) | (0.61,0.41) | (0.31,0.24) | (0.92,0.65) |
|         | 0.2 | 0.7291  | 0.5239  | 0.0542  | 0.2073  |
|         |     | (0.69,0.41) | (0.61,0.41) | (0.31,0.24) | (0.92,0.65) |
| 45°     | 0   | 0.2367  | 0.1655  | 0.0180  | 0.0938  |
|         |     | (0.74,0.32) | (0.65,0.30) | (0.35,0.18) | (0.95,0.53) |
|         | 0.1 | 0.2681  | 0.1867  | 0.0199  | 0.1054  |
|         |     | (0.74,0.32) | (0.65,0.30) | (0.35,0.18) | (0.95,0.53) |
|         | 0.2 | 0.3072  | 0.2133  | 0.0223  | 0.1200  |
|         |     | (0.74,0.32) | (0.65,0.30) | (0.35,0.18) | (0.95,0.53) |
| 60°     | 0   | 0.0064  | 0.0438  | 0.0059  | 0.0381  |
|         |     | (0.77,0.21) | (0.67,0.20) | (0.37,0.11) | (0.97,0.37) |
|         | 0.1 | 0.0684  | 0.0493  | 0.0065  | 0.0425  |
|         |     | (0.77,0.21) | (0.67,0.20) | (0.37,0.11) | (0.97,0.37) |
|         | 0.2 | 0.0783  | 0.0561  | 0.0072  | 0.0480  |
|         |     | (0.77,0.21) | (0.67,0.20) | (0.37,0.11) | (0.97,0.37) |

Table 5. Variation of dimensionless deflexion of FGM porous rhombic conoids subjected to uniform and trigonometrical loading.
Table 6. Effect of $hl/hh$ ratio on dimensionless maximum deflexion of FGM porous rhombic conoids subjected to uniform loading.

| $hl/hh$ | $\beta$ | $\alpha$ | $\beta$ | $\alpha$ | $\beta$ | $\alpha$ | $\beta$ | $\alpha$ |
|--------|---------|----------|---------|----------|---------|----------|---------|----------|
| 0.20   | 0       | 1.2599   | 0.7067  | 0.2918   | 0.0741  |
|        |         | (0.52,0.46) | (0.60,0.39) | (0.63,0.28) | (0.63,0.17) |
|        | 0.1     | 1.4195   | 0.7970  | 0.3291   | 0.0834  |
|        |         | (0.52,0.46) | (0.60,0.39) | (0.63,0.28) | (0.63,0.17) |
|        | 0.2     | 1.6162   | 0.9087  | 0.3754   | 0.0950  |
|        |         | (0.52,0.46) | (0.60,0.39) | (0.63,0.28) | (0.63,0.17) |
| 0.15   | 0       | 1.3157   | 0.7309  | 0.3004   | 0.0764  |
|        |         | (0.52,0.46) | (0.60,0.39) | (0.63,0.28) | (0.63,0.17) |
|        | 0.1     | 1.4821   | 0.8242  | 0.3388   | 0.0860  |
|        |         | (0.52,0.46) | (0.60,0.39) | (0.63,0.28) | (0.63,0.17) |
|        | 0.2     | 1.6873   | 0.9395  | 0.3861   | 0.0978  |
|        |         | (0.52,0.46) | (0.60,0.39) | (0.63,0.28) | (0.63,0.17) |
| 0.10   | 0       | 1.3735   | 0.7559  | 0.3093   | 0.0787  |
|        |         | (0.52,0.46) | (0.60,0.39) | (0.63,0.28) | (0.63,0.17) |
|        | 0.1     | 1.5471   | 0.8522  | 0.3487   | 0.0886  |
|        |         | (0.52,0.46) | (0.60,0.39) | (0.63,0.28) | (0.63,0.17) |
|        | 0.2     | 1.7610   | 0.9712  | 0.3973   | 0.1007  |
|        |         | (0.52,0.46) | (0.60,0.39) | (0.63,0.28) | (0.63,0.17) |
| 0.05   | 0       | 1.4336   | 0.7816  | 0.3184   | 0.0812  |
|        |         | (0.52,0.46) | (0.60,0.39) | (0.63,0.28) | (0.63,0.17) |
|        | 0.1     | 1.6146   | 0.8810  | 0.3589   | 0.0913  |
|        |         | (0.52,0.46) | (0.60,0.39) | (0.63,0.28) | (0.63,0.17) |
|        | 0.2     | 1.8378   | 1.0039  | 0.4089   | 0.1038  |
|        |         | (0.52,0.46) | (0.60,0.39) | (0.63,0.28) | (0.63,0.17) |
| 0.0    | 0       | 1.4955   | 0.8080  | 0.3277   | 0.0837  |
|        |         | (0.52,0.46) | (0.60,0.39) | (0.63,0.28) | (0.63,0.17) |
|        | 0.1     | 1.6841   | 0.9107  | 0.3693   | 0.0941  |
|        |         | (0.52,0.46) | (0.60,0.39) | (0.63,0.28) | (0.63,0.17) |
|        | 0.2     | 1.9167   | 1.0375  | 0.4207   | 0.1069  |
|        |         | (0.52,0.46) | (0.60,0.39) | (0.63,0.28) | (0.63,0.17) |

Figure 4. Variation of dimensionless maximum deflexion of FGM porous rhombic conoids subjected to uniform loading.
The correlation between $hl/hh$ ratio and dimensionless deflection of FGM porous rhombic conoids subjected to uniform loading was tested in Table 6. Interestingly, for higher value of $hl/hh$ ratio, lower value of dimensionless deflection was noticed. The decrease in $hl/hh$ ratio reduces the curvature of the lower end ($hl$) of the conoidal shell. Thus, the stiffness of shell reduces, while deflection increased. The results were calculated for $a/b = 1$ and $a/h = 10$. Figure 5 shows the dimensionless value of axial stresses along with thickness coordinate for FGM porous rhombic conoids having $\alpha = 15^\circ$ and $\alpha = 30^\circ$.

**Figure 5.** Effect of porosity on dimensionless axial stress of FGM porous rhombic conoids subjected to uniform loading.

### 5. Conclusion
This paper focuses on the influence of porosity on the static analysis of FGM porous rhombic conoids based on improved TSDT subjected to the various types of loads using an efficient $C^0$ FE model. The subsequent outcomes of the present study were written below for the volume fraction indices, skew angles, thickness ratios, $hl/hh$ ratios and various types of end support.

- The dimensionless deflection of the FGM porous conoidal shell increases along with the porosity volume fraction and volume fraction index.
- With an increase in skew angle, dimensionless deflection and normal stress decreases.
- The inclusion of porosity brings up deflection in comparison with the perfect conoidal shell.
- The dimensionless deflection increases along with thickness of conoids.
- For all considered skew angles, a decrease in $hl/hh$ ratio results in an increase in dimensionless deflection.

### 6. References
[1] Koizumi M 1997 FGM activities in Japan Compos Part B 28B 1-4 https://doi.org/10.1016/S1359-8368(96)00016-9
[2] Zhao X, Lee Y and Liew K 2009 Thermoelastic and vibration analysis of functionally graded cylindrical shells Int J Mech Sci 519-10 694-707 https://doi.org/10.1016/j.ijmecsci.2009.08.001
[3] Kashtalyan M and Menshykova M 2009 Three-dimensional elasticity solution for sandwich panels with a functionally graded core Compos Struct. 871 36-43 https://doi.org/10.1016/j.compstruct.2007.12.003
[4] Tornabene F and Viola E 2013 Static analysis of functionally graded doubly-curved shells and panels of revolution Meccanica 48 901-30. https://doi.org/10.1007/s11004-012-9643-1
[5] Viola E, Rossetti L, Fantuzzi N and Tornabene F 2014 Static analysis of functionally graded conical shells and panels using the generalized unconstrained third order theory coupled with the stress recovery Compos Struct. 112 44-65. https://doi.org/10.1016/j.composites.2014.01.039

[6] Dai H and Dai T 2014 Analysis for the thermoelastic bending of a functionally graded material cylindrical shell Meccanica 49 1069-81 https://doi.org/10.1007/s11012-013-9853-1

[7] Xiang S and Liu Y 2016 An nth-order shear deformation theory for static analysis of functionally graded sandwich plates J. Sandw. Struct. Mater. 185 579-96 https://doi.org/10.1177/1099636216647928

[8] Gutierrez Rivera M and Reddy J 2016 Stress analysis of functionally graded shells using a 7-parameter shell element Mech. Res. Commun. 78 60-70 https://doi.org/10.1016/j.mechrescom.2016.02.009

[9] Hadid H 1964 An analytical and experimental investigation into the bending theory of elastic conoidal shell. PhD Dissertation, University of Southampton

[10] Das A K and Bandyopadhyay J N 1993 Theoretical and experimental studies on conoidal shells Computers and Structures 493 531-36 https://dx.doi.org/10.1016/0045-7949(93)90054-H

[11] Ghosh B and Bandyopadhyay J 1994 Bending Analysis of Conoidal Shells With cut-outs Computers and Structures 531 9-18

[12] Das H S and Chakravorty D 2009 Composite full conoidal shell roofs under free vibration ADV VIB ENG 84 321-28

[13] Kumari S and Chakravorty D 2010 Finite element bending behaviour of discretely delaminated composite conoidal shell roofs under concentrated load Int. J. Eng. Sci. 24 54-70

[14] Malekzadeh Fard K and Baghestani A 2017 Free vibration analysis of deep doubly curved open shells using the Ritz method Aerosp Sci Technol 69 136-48 https://doi.org/10.1016/j.ast.2017.06.021

[15] Akbas S D 2017 Vibration and Static Analysis of Functionally Graded Porous Plates Journal of Applied and Computational Mechanics 33 199-207 https://dx.doi.org/10.22055/jacm.2017.21540.1107

[16] Al Rjoub Y S and Hamad A G 2017 Free vibration of functionally Euler-Bernoulli and Timoshenko graded porous beams using the transfer matrix method KSCE Int. J. Civ. Eng. 213 792-806. https://doi.org/10.1007/s12205-016-0149-6

[17] Kiran M, Kattimani S and Vinyas M 2018 Porosity influence on structural behaviour of skew functionally graded magneto-electro-elastic plate Compos Struct. 191 36-77 https://doi.org/10.1016/j.composites.2018.02.023

[18] Gupta A and Talha M 2017 Influence of porosity on the flexural and vibration response of gradient plate using nonpolynomial higher-order shear and normal deformation theory INT J MECH MATER DES 14 https://doi.org/10.1007/s10999-017-9369-2

[19] Gupta A and Talha M 2018 Influence of Porosity on the Flexural and Free Vibration Responses of Functionally Graded Plates in Thermal 18 https://doi.org/10.1142/S021945541850013X

[20] Ferreira A, Roque C, Jorge R, Fasshauer G and Batra R 2007 Analysis of Functionally Graded Plates by a Robust Meshless Method MECH ADV MATER STRUC 148 577-87 https://doi.org/10.1080/15376490701672732

[21] Bakshi K and Chakravorty D 2014 First ply failure study of thin composite conoidal shells subjected to uniformly distributed load Thin-Walled Structures 76 1-7 https://doi.org/10.1016/j.tws.2013.10.021