Estimation of the output parameters of a numerical model of a cantilever-type piezoelectric generator with attached mass and active termination upon pulsed excitation

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Abstract. A simulation of a piezoelectric generator (PEG) of energy is given. A feature of this generator is the presence of two types of piezo elements: plate-type elements experiencing bending deformations and cylindrical-type elements experiencing compression deformations. The simulation in the finite element complex is presented. The results of the analysis of unsteady oscillations during vibrational pulsed excitation of the PEG base are presented. Variations in the time range of PEG loading are considered. The analysis of output power and operation of individual elements of the generator is given.

1. Introduction
A huge number of devices use piezoelectric materials acting as actuators, sensors and energy generators. Such devices are used as in everyday life, such as sensors for monitoring the state of surrounding structures, sensors for detecting the movements of specialized devices, use in industries such as engineering, space engineering, vibration monitoring of the surrounding state of structures, indication of movements of robotic machines and other complex products. Such widespread use is achieved due to its good electromechanical properties, flexibility in the design process, ease of production, and high energy conversion efficiency. One of the options for using piezoelectric materials is the introduction of complex devices as actuators. Deformations of elements in such devices can be controlled by changing the magnitude of the applied electric potential at the electrodes of such elements. The negative side is that the piezoelectric elements have small deformations. The inverse mechanism is used in displacement sensors in such systems. These measurements are indirectly a multiple of the acceleration and are recorded by measuring the potential at the electrodes of such devices. Another area of use of piezoelectric materials is the creation of energy generation devices. This direction is quite relevant. Using specialized devices that use certain schemes for switching on piezoelectric materials, free mechanical energy that is present in the environment into electrical potential is converted. A review of the work is given in [1-5]

Simulation of the performance of PEG energy harvesting devices can be carried out on the base of finite element modeling in computer software such as ACelan, ANSYS, etc. Experimental and numerical studies of PEG of stack and bimorph types are discussed in [6-13].
Piezoelectric materials and devices can be preliminarily investigated in laboratory conditions. In the works, studies of the characteristics of PEG with the use of measuring devices and the corresponding experimental techniques are presented [14-17]. Moreover, in these and other well-known works, descriptions of experimental installations for measuring vibration parameters and recording PEG characteristics of a mainly cantilever type are given [15].

Among the various problems that arise during the development of PEG, the most significant of them can be distinguished, including the choice of energy-efficient piezoceramic materials (PCM), the development of electrical circuits for electric energy storage with minimal charge leakage, the search for geometric configurations and assembly technologies for the PEG sensitive element, which cause maximum power output. The solution to these problems we use various approaches that vary depending on the type and application of PEG. The reviews [18–24] contain materials on PEG systems.

2. Formulation of the problem
The goal of the work is numerical modeling of cantilever-fixed PEG energy having active structures in the form of a bimorph and compressible piezocylinders under pulsed excitation at the base.

3. Description of the physical simulation of the model
The numerical model was an analog of the full-scale prototype of the generator model. In Fig. 1. shows the scheme of PEG. The scheme consists of two parts: in Fig. 1a is a circuit diagram of a device. Fig. 1b shows the PEG model.

PEG consists of the following parts: (1) generator mounting leg, (2) fixing frame, (3) substrate, (4) attached mass, (5, 6) piezoelectric elements (PE) in the form of plates, (7, 8) cylindrical PE forms, $R_1-R_4$ active electric load of the corresponding PE. PE cylindrical volumetric configuration $PE_1-PE_4$ have respectively pairwise parallel electrical connections. The lower $PE_1$ have a codirectional polarization, the upper $PE_2$ have a polarization directed in opposite directions. All cylindrical PEs are polarized in thickness. Thin symmetrical $PE_3, PE_4$, have a unidirectional polarization scheme relative to the OY axis. PE plate configuration $PE_3, PE_4$, are fixed on the base console in a specific area. A detailed diagram of the model is presented in [4]. Table 1 presents some parameters of the corresponding materials of PE plates. The electrical diagram of the connection of PEG with an active load is shown in Fig. 1. The value of the connected mass in the calculations was assumed constant and amounted to $m = 17.6$ grams. The sizes of the piezoelectric plates: $l_p \times b_p \times h_p$ (mm$^3$) = 50x10x0.45 piezoelectric cylinders $RxH$ (mm$^2$) = 10 x 10, substrate $l \times b \times h$ (mm$^3$) = 135x13.2x1.5, cargo fixation $l_m = 146.5$ mm.

![Figure 1. PEG: a) electrical diagram; b) model](image-url)
Table 1. Properties of PEG’s elements

| №  | PEG Elements      | Material     | ρ, kg/m³ | E, Pa  | ν    |
|----|-------------------|--------------|----------|--------|------|
| 1  | Leg               | Duralumin    | 2800     | 0.33e11 | 0.33 |
| 2  | Fixing frame      | Duralumin    | 2800     | 0.33e11 | 0.33 |
| 3  | Attached Mass     | Metal        | 7700     | 2.1e11  | 0.33 |
| 4  | Основание         | Duralumin    | 2800     | 0.33e11 | 0.33 |
| 5, 6| PE plate configuration | CTS-19 (in Russian) | 7280 | [9] | 0.33 |
| 7, 8| PE cylindrical configuration | CTS-19 (in Russian) | 7280 | [9] | 0.33 |
| 9  | Active electrical load | Resistor | 1000000 Ohm |

In figure 1 shows a model sample of the PEG cantilever type of mechanical energy into electrical energy. The cantilever beam 4 (generator substrate) of the PEG consists of an elastic material on which 5.6 piezoelectric elements are fixed on both sides (bimorph), one end of the console 3 is fixed in the frame base 2, and an additional attached mass 3 is fixed at the free end, in the base In addition, four 7.8 piezoelectric elements were installed, two from above and two from below relative to the plane of the cantilever beam 4, having the directions of the polarization vector according to the shown diagram in Fig. 1. The voltage is removed from the nodes on the surface of the PE having the properties of electrical conductivity, simulating the electrodes of a real PEG device. The active load in this case for each block of piezoelectric elements was \( R_1..R_4 = 1 \) MΩ.

4. Numerical simulation

PEG modeling was carried out in the ANSYS FE complex. To build the model involved elements of a three-dimensional configuration SOLID5 and SOLID45. The first type of elements is designed to model piezoelectric and elastic media, the second type of element has a simplified stiffness matrix and is designed to simulate elastic media. In the process of building the model, 4100 nodes and more than 9200 elements were created. An unsteady statement of the solution of the problem was posed. The loading took place by simulating the movement of the PEG leg according to a linear law simulating impact loading.

At the first stage, a modal analysis of the structure was performed. In the FE model, the displacements of nodes on the surface of the legs of the FixP generator in three directions were recorded. 4 natural frequencies of PEG vibrations were obtained. The first two oscillation frequencies are \( \omega_1 = 36.928 \) Hz, \( \omega_2 = 133.63 \) Hz. The calculation results are presented in figure 2.
At the second stage, harmonic analysis was modeled for stationary PEG excitation by displacement relative to the $OY$ axis, applied to nodes located on the FixP plane at the generator base. The displacements of the nodes of this plane in the remaining directions were fixed. For analysis, we considered the frequency spectrum $\omega_i$ from 1 to 200 Hz. Graphs were plotted (figure 3) for the dependences of the amplitude of the oscillations of the points $P_1 - P_3$ located on the cantilever of the generator and the voltage across the electrodes of the piezoelectric elements of the generator $PE_1 - PE_4$ on the frequencies during harmonic analysis.

**Figure 2.** The first 4 natural frequencies and the corresponding waveforms of the PEG model

**Figure 3.** a) the dependence of the amplitude of oscillations of the points $P_1 - P_3$ in the vertical direction $OY$ located on the cantilever of the generator, and the voltage on the electrodes of the piezoelectric elements of the generator $PE_1 - PE_4$, depending on the frequencies during harmonic analysis
An analysis of the graphs shows that in the frequency range from 1 to 200 Hz in the excited direction $OY$ of the $PEG$ base vibrations, the maximum output voltage was reached with resonance of the cantilever bending vibrations at a frequency of 33 Hz. In this case, the voltage on the PE plates will be equal, respectively: $V_1 = 8.69E-02 \, V$, $V_2 = 0.138775 \, V$, $V_3 = -4.17596 \, V$, $V_4 = 4.17596 \, V$. One of the electrodes based on the cantilever will have a minimum output voltage, thereby working in antiphase antiresonance. The maximum vibrational parameters are achieved at the bimorph on the base at a frequency of 48 hertz. In this case, the output voltage parameters of the corresponding piezoelectric elements are equal to $V_1 = 3.61E-02 \, V$, $V_2 = 7.69E-02 \, V$, $V_3 = 4.65054 \, V$, $V_4 = -4.65054 \, V$.

Peak characteristics of the output voltage of a cylindrical type PE are achieved at a resonance frequency $w_1 = 36.928 \, Hz$. In this case, the parameters of the output voltage characteristics were: $V_1 = 0.1345734 \, V$, $V_2 = 0.214175 \, V$, $V_3 = 1.56457 \, V$, $V_4 = 1.564575 \, V$.

Numerical simulation of a piezoelectric generator with active termination during pulsed excitation. The problem of pulsed excitation of a piezoelectric generator with active termination is considered. As input parameters, the displacement field at the base of the structure as a result of short-term pulse exposure is considered. A pulse in the form of a displacement of a certain amplitude $U = 0.1 \, mm$ is applied to the $FixP$ surface of the base 1 of the $PEG$ (Fig. 3a). The application time of the $T_{imp}$ displacement was assumed to be 0.025 s, 0.01355 s, 0.01 s, 2 s. This corresponded to the work of $PEG$ at half-wave frequencies of pulsed action 0.25 Hz, 20 Hz, 37 Hz, 50 Hz. Unsteady oscillations of the generator design are analyzed at certain points $P_1-P_3$ and the voltage on the plates of the generator electrodes is fixed for a time $t = 5 \, s$.

| №    | Pulse excitation time $T_{imp}, s$ | Wave frequency $f_{imp}, Hz$ |
|------|---------------------------------|-------------------------------|
| 1    | 2                               | 0.25                          |
| 2    | 0.025                           | 20                            |
| 3    | 0.01355                         | 36.9                          |
| 4    | 0.01                            | 50                            |

In ANSYS FE packets, to describe the losses of mechanical energy, the attenuation coefficients $\alpha$ and $\beta$ are introduced, which, assuming the same $Q$ factor at the first two $f_{r1, r2}$ resonance frequencies, are expressed through the $Q$ factor as follows [25]. In this case, the quality factor of all $PEG$ materials was taken equal to $Q = 10$.

Due to pulsed loading, a wave of displacements of the points of the structure occurs. In the process of solving the problem, displacements are analyzed at control points 1-3 of the structure located on the basis of the $PEG$ cantilever. In figure 3 shows the transverse absolute displacements $P_1-P_3$ at points of surface points.
Figure 4. The oscillation amplitudes of the points on the cantilever upon pulsed excitation at the points: during time $T$. The time of application of the load: $a) T_{imp}=2 \text{ s}; b),c) T_{imp}=0.025 \text{ s}; d),e) T_{imp}=0.01355 \text{ s}; f),g) T_{imp}=0.01 \text{ s}$
Figure 5. The output parameters of the excitation voltage of the piezoelectric elements in the cantilever design during pulsed excitation for a time $T = 1$ s (a, b, c, d). Accordingly, colors indicate $PE$. Load application time: a) $T_{imp} = 2$ s; b) $T_{imp} = 0.025$ s; c) $T_{imp} = 0.01355$ s; d) $T_{imp} = 0.01$ s.

The average values of the instantaneous power $\overline{P}$ over a period of time $T = 1$ s were calculated by formula (1), the work $A$ performed by various corresponding piezoelectric elements for these time periods was calculated by formula (2).

$$\overline{P} = \frac{1}{T} \int_0^T p(t) dt = \frac{1}{T} \int_0^T \frac{V(t)}{R} dt$$

$$A = \int_0^T p(t) dt$$

Table 3. The calculated values of the average instantaneous power $\overline{P}$ and the work $A$ performed by various $PE$s over a certain period of time $T=1$s

| № | Parameter | $PE_1$ | $PE_2$ | $PE_3$ | $PE_4$ |
|---|-----------|--------|--------|--------|--------|
| 1 | $T_{imp_1}$ | $\overline{P}$, $m$kW | 0.000006460 | 0.00000313 | 0.00000526 | 0.00000526 |
|  | $A$, $m$kVA | 0.000006460 | 0.00000313 | 0.00000526 | 0.00000526 |
| 2 | $T_{imp_2}$ | $\overline{P}$, $m$kW | 0.73800 | 0.00603 | 0.00430 | 0.00430 |
|  | $A$, $m$kVA | 0.73800 | 0.00603 | 0.00430 | 0.00430 |
| 3 | $T_{imp_3}$ | $\overline{P}$, $m$kW | 1.7100 | 0.0218 | 0.1900 | 0.1900 |
|  | $A$, $m$kVA | 1.7100 | 0.0218 | 0.1900 | 0.1900 |
| 4 | $T_{imp_4}$ | $\overline{P}$, $m$kW | 2.550 | 0.041 | 0.288 | 0.288 |
|  | $A$, $m$kVA | 2.550 | 0.041 | 0.288 | 0.288 |

Analysis of vibration damping data shows that vibration damping for 1 case at $T_{imp_1} = 2$ s. The process of attenuation occurs within 5 s. As a result of this, the comparative period was chosen for the average power and perfect work 5 s. An analysis of the average power $P$ and the perfect work for various $PE$s shows that its highest values are given by a pair of cylindrical $PE_1$, for all variations of the $PEG$
excitation time. Due to the neglect of the scale factor of the piezoelectric element of cylindrical shape and the attached mass, which is the inertial load for PE at the base of the structure, additional calculations are required to establish the influence of these factors. So, under the pulsed action $T_{imp} = 2c$, the minimum values of the average power were obtained, although in the case of the pulsed effect $T_{imp} = 0.01\,s$, the values of both the average power and the perfect work were maximum for all PEs.

**Summary**

Numerical simulation of a cantilever PEG with an attached mass and an active base in the Ansys FE complex is considered. The numerical model was an analog of the full-scale prototype of the generator model. The analysis of modal, harmonic and non-stationary calculations of PEG oscillations is given. The first 4 forms of oscillation of the generator are given. The output parameters of the generator are calculated for 4 piezoelectric electrodes with harmonic excitation of the base of the generator. The results of harmonic analysis show that the maximum, output oscillation parameters are achieved in the stationary calculation, it was shown that the maximum average power value for a cylindrical PE with unidirectional polarization of the declared configuration at a resistance of 1 Mom can be $2.550\,mkW$, and a multidirectional polarization of 0.041 $mkW$. Moreover, for a PE plate configuration located on the base, the power is 0.288 $mkW$. These power values were obtained during shock excitation at the generator base at $T_{imp1} = 0.01\,s$ and the base displacement amplitude $A = 0.1\,mm$.

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