Small signal stability analysis of paralleled inverters for multiple photovoltaic generation units connected to weak grid

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Abstract: In this study, a small signal model of paralleled inverters for multiple photovoltaic (PV) generation units (MPGUs) connected to weak grid is developed. Based on the proposed small signal model, eigenvalue analysis is employed to study the stability of MPGUs within different grid strength, different control parameters in each phase-locked loop (PLL) controller and varying operating points in each PV generation unit. Furthermore, based on the transfer function of the power control loops in dq rotation frame, the coupling mechanism of operating point in each PV unit is revealed. The results show that: with the increasing of output power of PV unit among the PV generation and the decreasing of the grid strength, it may occur oscillation phenomena, the oscillation frequency is about 5 Hz; tuning gains of PLL has noticeable effect on the damping characteristic of the system, and larger PLL gains can improve system damping; each PV unit power control loop is coupled by the point of common coupling voltage, increasing the output power of PV unit, it may reduce the phase margin of the system transfer function and deteriorate the system stability. Theoretical analysis is verified with simulation of MPGUs connected to weak grid.

1 Introduction

PV power generation is an important way to promote transformation of energy and deal with environmental challenges. By the end of 2015, the national cumulative installed PV capacity was 43.18 GW in China and 200 GW worldwide [1]. The global installed PV capacity is expected to exceed 450 GW by 2017; the trend of PV development is rapid.

Due to the restriction of resource endowment, large scale PV generation is mostly located in deserts or semi-deserts where the grid structure is relatively weak, and integrated to power systems through voltage source converter (VSCs). Ideally, the point of common coupling (PCC) voltage is stable, the weak coupling between VSCs and the PCC voltage, they have less interactions with each other. With the increasing capacity of grid-connected PV generation, the grid impedance cannot be ignored, it may lead to VSCs connected to weak grid [2, 3], and the ideal condition is destroyed. The PCC voltage is unstable, it will be affected by the output power disturbance of PV array and the grid disturbance. With the strong coupling between VSCs and the PCC voltage, they have more interactions with each other. Therefore, this situation poses challenges on the control and safe operation of PV generation.

Stability of PV generation connected to weak grid has been investigated in some papers [3–7]. In [4], an impedance-based stability criterion for grid-connected VSCs is proposed; it reveals that the system will be unstable if the ratio between the grid impedance and the VSC output impedance cannot satisfy the Nyquist stability criterion. Yan et al. [5] proposed a small signal model to analyse the stability of DC-link voltage in VSC and discussed the effect of P–V characteristic of PV array on the VSC stability. Agorreta et al. [6] proposed an equivalent VSC that models the N-paralleled grid-connected VSCs in PV plants and the coupling effect due to grid impedance is described.

The above studies have explained the operation stability of PV generation qualitatively under the condition of weak grid; however, the control system plays an important role which has not been widely considered. In [8], it was found that the instability phenomenon is quite related with converter control loops such as AC terminal control loop and phase-locked loop (PLL). In addition, most of these studies adopt the single-machine-infinite-bus model, few literatures have adopted the paralleled model for multiple photovoltaic generation units (MPGUs), it cannot represent the PV generation stability within interactions among machines, the research results are limited.

This paper establishes a small signal model of paralleled inverters for MPGUs connected to weak grid, eigenvalue analysis is employed to study the stability of MPGUs within different grid strength, different control parameters in each PLL controller and varying operating points in each PV generation unit. Based on the transfer function of the power control loops in dq rotation frame, the coupling mechanism of operating point in each PV unit is revealed.

2 Configuration of MPGUs connected to weak grid

Fig. 1 shows the configuration of MPGUs connected to AC grid. In Fig. 1, \( C_i \) is the input filter capacitor; \( U_{dcj} \) is the DC voltage; \( L_n \) is the output filter inductor; \( C_{d} \) is the output filter capacitor; \( R_{s}\) and \( L_{s} \) are the resistor and inductor of the collector system, respectively; \( R_{i} \) and \( L_{i} \) are the resistor and inductor of the grid, respectively; \( U_{fi} \) is the terminal voltage; \( U_{fci} \) is the PCC voltage; \( U_{gi} \) is the grid voltage.

In Fig. 1, the strength of the AC system is generally described by a short-circuit ratio (SCR), as shown in (1). The AC system is regarded as a weak grid under the condition that the SCR is < 3. From (1), it can be obtained that SCR is decreased with increase of grid impedance.

\[
SCR = \frac{S_{nic}}{S_{N}} = \frac{U_{g}^{2}}{Z_{g}S_{N}} \tag{1}
\]
where $S_{ac}$ is the short-circuit capacity of the AC system; $S_N$ is the rated power of the PV generation; $Z_d$ is the grid impedance.

### 3 Modelling of MPGUs connected to weak grid

A small signal model is derived for stability analysis and some assumptions are made as follows: (1) $C_p$ and $R_p$ are not included in the stability analysis; (2) The system is lossless.

#### 3.1 PV array model

This part is intended to derive the mathematical model of PV array. Fig. 2 shows the configuration of PV array, $N_p$ and $N_s$ are the number of parallel and series connected cells, $U_{dc1}$ is the DC voltage, $I_{PV}$ is the DC current. The $U_{dc1}$-$I_{PV}$ characteristic of a PV array is shown in (2), which can be obtained from the manufacturer

$$I_{PV} = N_p f_{sc} [1 - C_1(e^{U_{dc1}/C_2U_{oc}}) - 1]$$

where $I_{sc}$ is the short-circuit current; $U_{oc}$ is the open-circuit voltage; $C_1$ and $C_2$ are constants.

#### 3.2 VSC model and control

Fig. 3 shows the topology of VSC, the mathematical model of VSC is shown in (3), which is transformed into $dq$ frame.

A typical grid voltage oriented-based vector control system of VSC is shown in Fig. 4. The outer loop in the $d$ frame is the DC voltage controller which produces the current reference $i_{dref}$ for the current inner loop. The outer loop in the $q$ frame is the AC-bus voltage controller which produces the current reference $i_{qref}$ for the current inner loop. The inner current controllers produce the modulation signals in $dq$ frame which are then translated back to the $abc$ frame

$$\begin{align*}
L_d \frac{di_d}{dt} &= U_{di} - U_{ldi} + \omega L_{di}i_q \\
L_q \frac{di_q}{dt} &= U_{dq} - U_{ldq} + \omega L_{dq}i_d \\
U_{dc1}C_1 \frac{di_{dref}}{dt} &= i_{PV}U_{dc1} - U_{ldi}i_d
\end{align*}$$

In Fig. 4, $U_{dref}$ and $U_{qref}$ are the DC voltage reference value and the actual value, respectively; $U_{ldi}$ and $U_{ldq}$ are the terminal voltage reference value and the actual value, respectively; $i_{dref}$ and $i_{qref}$ are the current reference values in $dq$ frame; $i_{di}$ and $i_{dq}$ are the actual values in $dq$ frame; $U_d$ and $U_q$ are the modulation voltages in $dq$ frame; $\theta_{PLL}$ is the output of PLL.

According to Fig. 4, the following equations can be obtained:

$$\begin{align*}
\frac{dx_{d}}{dt} &= U_{d} - U_{dref} \\
\frac{dx_{q}}{dt} &= k_p(U_{d} - U_{dref}) + k_t x_{11} - i_d \\
\frac{dx_{\theta}}{dt} &= U_{\theta} - U_{\theta ref} \\
\frac{dx_{\phi}}{dt} &= k_p(U_{\theta} - U_{\theta ref}) + k_t x_{31} - i_q \\
U_{d} &= k_p[1(U_{d} - U_{dref}) + k_t x_{11} - i_d] + k_3 x_{21} + U_{ldi} - \omega L_{di}i_q \\
U_{q} &= k_p[1(U_{d} - U_{dref}) + k_t x_{11} - i_d] + k_3 x_{31} + U_{ldq} - \omega L_{dq}i_d
\end{align*}$$

where $x_{11}, x_{21}, x_{31}$ and $x_{13}$ are the state vectors; $k_{pi}, k_{pqi}, k_{pi}, k_{pqi}$ and $k_{pi}$ are the proportional gains of the controllers; $k_{ti}, k_{ti}, k_{3i}$ and $k_{3i}$ are the integral gains of the controllers.

Fig. 5 shows the working principle of PLL, which is employed to make sure that $d$ frame is always aligned with the terminal voltage.
The equivalent circuit of the AC side of MPGUs connected to AC grid is shown in Fig. 7, the KVL equation of the AC side in the $xy$ frame can be written as

\[
\begin{align*}
U_{ti} &= U_{gi} - \sum_{i=1}^{n} l_x x_i - i_x c_x \\
U_{ti} &= U_{gi} + \sum_{i=1}^{n} l_y x_i + i_y c_y
\end{align*}
\]  

(6)

where $x_y$ is the grid impedance; $x_i$ is the collector system impedance; $U_{ti}$ and $U_{gi}$ are the terminal voltage in $xy$ frame; $U'_{xi}$ and $U'_{yi}$ are the grid voltage in $xy$ frame.

The transformation for the terminal voltage and current from $dq$ frame to $xy$ frame is given as

\[
\begin{align*}
U'_{ti} &= U_{di} \cos \theta_{dli} - U_{qi} \sin \theta_{dli} \\
U'_{ti} &= U_{di} \sin \theta_{dli} + U_{qi} \cos \theta_{dli} \\
l_{di} &= i_d \cos \theta_{dli} - i_q \sin \theta_{dli} \\
l_{di} &= i_d \sin \theta_{dli} + i_q \cos \theta_{dli}
\end{align*}
\]  

(7)

Equations (2)–(7) are the differential algebraic equations, which can describe the MPGUs connected to weak grid accurately. Taking the two photovoltaic (PV) generation units paralleled as an example, from linearisation of the equations, a small signal model of the system is deduced.

The state vectors are defined as

\[
\Delta x = \begin{bmatrix}
\Delta x_{11}, \Delta x_{12}, \Delta x_{13}, \Delta x_{14}, \Delta x_{21}, \Delta x_{22}, \\
\Delta x_{23}, \Delta x_{24}, \Delta x_{25}, \Delta x_{26}, \Delta x_{27}, \Delta x_{28}, \\
\Delta x_{29}, \Delta x_{30}, \Delta x_{31}, \Delta x_{32}, \Delta x_{33}, \Delta x_{34}, \\
\Delta x_{35}, \Delta x_{36}, \Delta x_{37}, \Delta x_{38}, \Delta x_{39}, \Delta x_{40}, \\
\Delta x_{41}, \Delta x_{42}, \Delta x_{43}, \Delta x_{44}, \Delta x_{45}, \Delta x_{46}
\end{bmatrix}^T
\]

The small signal model of the system is expressed in state-space equations as

\[
\Delta x = A \Delta x + B \Delta u
\]  

(8)

4 Small signal stability analysis

4.1 Eigenvalues analysis

In order to obtain the eigenvalues, taking the two PV generation units paralleled as an example, the system parameters are shown in Table 1. Table 2 shows all the system modes with frequency and damping of the oscillation, under the condition that SCR is equal to 1.5. As can be seen From Table 2, the system has four oscillation modes and 10 damping modes. All the eigenvalues of the system are distributed on the left side of the complex plane, the system is stable.

4.2 Effect of grid strength

Influence factors on stability of the system involve grid strength, different control parameters in each PLL controller and varying operating points in each PV generation unit. Based on the small signal model of MPGUs connected to weak grid, we discuss the effects of above factors on the stability of the system.

| Parameters | Values |
|------------|--------|
| rating power of PV array | 500 kW |
| DC-link rated voltage of converter | 600 V |
| grid voltage | 380 V |
| input filter capacitor | 0.02 F |
| output filter inductor | 0.5 mH |
| grid inductor | 0.3 mH |
| DC-link voltage control loop ($k_{p1}, k_{i1}$) | (2200) |
| AC voltage control loop ($k_{p2}, k_{i2}$) | (2100) |
| current control loop ($k_{p3}, k_{i3}$) | (2100) |
| phase-locked loop ($k_{p4}, k_{i4}$) | (50,1500) |

| Table 2 Eigenvalues of PV system |
| Mode | Eigenvalues | Frequency, Hz | Damping |
|------|-------------|---------------|---------|
| $\lambda_1$ | $-5720.9$ | 0 | 1 |
| $\lambda_2$ | $-4197.6$ | 0 | 1 |
| $\lambda_3$ | $-3885.9$ | 0 | 1 |
| $\lambda_4$ | $-3885.9$ | 0 | 1 |
| $\lambda_{5,6}$ | $-30.8 \pm 74.3$ | 11.8 | 0.38 |
| $\lambda_{7,8}$ | $-42.8 \pm 69.8$ | 11.1 | 0.53 |
| $\lambda_{9,10}$ | $-3.1 \pm 27.8$ | 4.4 | 0.11 |
| $\lambda_{11,12}$ | $-23.6 \pm 29.4$ | 4.7 | 0.63 |
| $\lambda_{13}$ | $-20.3$ | 0 | 1 |
| $\lambda_{14}$ | $-3$ | 0 | 1 |
| $\lambda_{15}$ | $-50.2$ | 0 | 1 |
| $\lambda_{16}$ | $-50.3$ | 0 | 1 |
| $\lambda_{17}$ | $-50.6$ | 0 | 1 |
| $\lambda_{18}$ | $-50.6$ | 0 | 1 |
The system parameters are shown in Table 1; the eigenvalues locus for SCR varies from 4 to 1.2 is shown in Fig. 8. As the grid strength decreases, it can be seen that there are eight eigenvalues changing, $\lambda_1$, $\lambda_2$, $\lambda_3$, and $\lambda_{14}$ move to left-half plane; $\lambda_9$, $\lambda_{10}$, $\lambda_{11}$ and $\lambda_{12}$ move to right-half plane. When SCR turns to be $1.2$, $\lambda_9$ and $\lambda_{10}$ enter the unstable region, the system becomes unstable. Therefore, the stability of the MPGUs connected to weak grid becomes worse with reduction of grid strength.

4.3 Effect of PLL gains

When SCR turns to be $1.5$ and the proportional gain of PLL $k_{p42}$ turns to be $50$, the eigenvalues locus for another proportional gain of PLL $k_{p41}$ varies from $10$ to $100$ is shown in Fig. 9. As $k_{p41}$ increases, it can be seen that there are seven eigenvalues changing, $\lambda_7$, $\lambda_8$, $\lambda_9$, $\lambda_{10}$, $\lambda_{11}$ and $\lambda_{12}$ move to left-half plane; $\lambda_{14}$ moves to right-half plane. When $k_{p41}$ turns to be $10$ and $k_{p42}$ turns to be $50$, $\lambda_9$ and $\lambda_{10}$ enter the unstable region, the system becomes unstable. The eigenvalues locus for both proportional gains of PLL $k_{p41}$ and $k_{p42}$ vary from $10$ to $100$ is shown in Fig. 10. As $k_{p41}$ and $k_{p42}$ increase, it can be seen that there are also seven eigenvalues changing, $\lambda_7$, $\lambda_8$, $\lambda_9$, $\lambda_{10}$, $\lambda_{11}$ and $\lambda_{12}$ move to left-half plane; $\lambda_{14}$ moves to right-half plane. When both $k_{p41}$ and $k_{p42}$ turn to be $20$, $\lambda_9$ and $\lambda_{10}$ enter the unstable region, the system becomes unstable.

The oscillation frequency and damping ratio are shown in Figs. 11 and 12. As can be seen from Fig. 12, tuning gains of PLL has noticeable effect on the damping characteristic of the system, and larger PLL gains can improve system damping, enhance the system stability.

4.4 Effect of operating point

The MPGUs change their operating point through the active power output varies. The eigenvalues locus for active power output varies from $0.45$–$1.0$ pu is shown in Fig. 13. As the active power output increases, it can be seen that there are also seven eigenvalues changing, $\lambda_7$, $\lambda_8$ and $\lambda_{14}$ move to left-half plane; $\lambda_9$, $\lambda_{10}$, $\lambda_{11}$ and $\lambda_{12}$ move to right-half plane.

In order to reveal the coupling mechanism of operating point in each PV unit, according to the method in [9], neglecting the effect of current control loop dynamic and PLL dynamic. The power control loops of paralleled inverters for MPGUs connected to weak grid is simplified as shown in Fig. 14. In Fig. 14, $P_{pv1}$ and $k_{p41}$...
$P_{\text{PV2}}$ are the active power of PV units; $P_{e1}$ and $P_{e2}$ are the output power of VSCs; $k$ is the output power ratio.

Simplified the small signal model further, which is described in Fig. 15. The open loop transfer function $G_o(s)$ can be obtained in (10)

$$G_o(s) = \frac{\Delta P_{e1}}{\Delta P_{\text{PV1}}} = \frac{1}{1 + k} \cdot \frac{k}{C \cdot U_{dc,20} s} \cdot \frac{U_{gd0}}{1 - k_1 \cdot U_{gd0} s} \cdot \text{PI}_1$$

According to (10), the bode plot of $G_o(s)$ is given in Fig. 16. It can be seen from Fig. 16, with the output power ratio $k$ increases, the phase margin of $G_o(s)$ decreases and the stability of the system becomes weaken.

5 Simulation results

In order to verify the effectiveness of the small signal analysis, a simulation model of MPGUs connected to weak AC grid is built in power systems computer aided design (PSCAD)/electromagnetic transients (EMTDC). The parameters are shown in Table 1.

When the proportional gains of PLLs $k_{p41}$ and $k_{p42}$ turn to be 50, the integral gains of PLLs $k_{i41}$ and $k_{i42}$ turn to be 1500. Figs. 17 and 18 show DC voltages ($U_{dc1}$ and $U_{dc2}$) response and terminal voltages ($U_{t1}$ and $U_{t2}$) response of a corresponding detailed model for a step change of illumination at $t = 8$ s with different grid strengths. Fig. 17 illustrates that the system becomes stable after a step change of illumination in SCR = 1.5. Fig. 18 illustrates that the system becomes unstable after a step change of illumination in SCR = 1.2 and the oscillation frequency is about 5 Hz. The
simulation results are coherent with the eigenvalue locus analysis in Fig. 8.

When SCR turns to be 1.5, Figs. 19 and 20 show DC voltages ($U_{dc1}$ and $U_{dc2}$) response and terminal voltages ($U_{t1}$ and $U_{t2}$) response of a corresponding detailed model for a step change of illumination at $t=8$ s with different PLL gains. In Fig. 19, $k_{p41} = 15$, $k_{i41} = 1500$, $k_{p42} = 50$ and $k_{i42} = 1500$; in Fig. 20, $k_{p41} = 30$, $k_{i41} = 1500$, $k_{p42} = 50$, $k_{i42} = 1500$.

Fig. 19 illustrates that the system becomes unstable after a step change of illumination and the oscillation frequency is about 5 Hz. Fig. 20 illustrates that the system becomes stable after a step change of illumination. The simulation results are coherent with the eigenvalue locus analysis in Fig. 9.

6 Conclusions

A small signal model of paralleled inverters for MPGUs connected to weak grid is presented in this paper. Eigenvalue analysis is employed to study the stability of MPGUs within different grid strength, different control parameters in each PLL controller and varying operating points in each PV generation unit. The following conclusions can be drawn:

(i) With the increasing of output power of PV unit among the PV generation and the decreasing of the grid strength, it may occur oscillation phenomena (SCR < 1.2), the oscillation frequency is about 5 Hz.
(ii) Tuning gains of PLL has noticeable effect on the damping characteristic of the system, and larger PLL gain can improve system damping (30–100), enhance the system stability.
(iii) Each PV unit power control loop is coupled by the PCC impedance due to parallel grid-connected inverters with LCL filter coupled due to grid impedance in PV plants, ‘IEEE Trans. Power Electron., 2011, 26, pp. 770–785

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