Development of a radiation pressure noise interferometer

Akira Okutomi, K Yamamoto, S Miyoki, M Ohashi, K Kuroda
Institute for Cosmic Ray Research, University of Tokyo, 5-1-5 Kashiwanoha, Kashiwa, Chiba 277-8582, Japan
E-mail: okutomi@icrr.u-tokyo.ac.jp

Abstract. We are developing a small-scale interferometer in order to observe radiation pressure noise. This radiation pressure noise interferometer is composed of two cavities whose mirror mass is less than 1g. In this system, radiation pressure noise is dominant around at a few hundred Hz. The design and the current status of our prototype system are described.

1. Introduction

In advanced interferometric gravitational wave detectors, the quantum fluctuation of light will be serious noise sources, which are called the shot noise (SN) and the radiation pressure noise (RPN). SN corresponds to fluctuations of the number of photons detected by a photo-detector, while RPN is due to mirror motion caused by fluctuation from the back action of the reflected photons. The spectral density of the SN is inversely proportional to the square root of light power, $\sqrt{I_0}$, on the other hand, that of the RPN is in proportion to $\sqrt{I_0}$. Therefore, there is an optimum light power that makes the noise of the interferometer minimum considering only these quantum noises. Under this optimal light power, the minimum point of the noise spectrum, where SN and RPN balance, is believed to correspond to the standard quantum limit (SQL) that was deduced from the uncertainty principle of the quantum-mechanical behavior of the test masses [1]. This SQL was considered to be a principal limit, however Yuen showed that the deduction of the SQL is not rigorous and it is in principle possible to be overcome [2]. Moreover, recently Braginsky has shown that the quantum fluctuation of the test-mass does not practically affect the noise characteristic of the practical interferometer [1].

On the other hand, research on quantum noise reduction in actual interferometric gravitational wave detectors has been conducted for long time. Caves [3] presented that the SN of the interferometer is reduced by squeezed state of light. Unruh has reported that not only SN reduction, but also beating the SQL is possible by inputting squeezed states of light [4, 5]. Method for beating the SQL is to use the squeezed state of light generated from a moveable mirror. When the moveable mirror reflects the light, there are correlations between amplitude fluctuations (RPN) and phase fluctuations (SN) of light [6, 7, 8]. Squeezed light generation of this type is called ponderomotive squeezing effect [9]. Various ideas such as variational readout technique [10, 11], or a detuning signal recycling technique [12], which are associated with the RPN and ponderomotive squeezing effect, allow the interferometric gravitational wave detector to beat SQL. To test these theories [6-11, 13] measuring the RPN itself, has become a subject of growing interest.
Table 1. SN and RPN of Fabry-Perot Michelson interferometers with over-coupling (OC) and critical-coupling (CC) cavities, are given as displacement noise $[m/\sqrt{Hz}]$. Here $\Omega$ is angular frequency $\Omega = 2\pi f$, $L$ is the length of the cavity, $\omega_0$ is the angular frequency of light, $I_0$ is the intensity of the input light and $\gamma$ is the full width half maximum (FWHM) of the cavity. If OC and CC have the same $\gamma$, RPN of the CC case is smaller only by $1/\sqrt{2}$ than that of the OC. In this estimation a lossless system is assumed.

However, whilst there have been experiments demonstrating SN reduction [14, 15, 16, 17, 18] and a classical demonstration of noise cancellation using an optical cavity [19] to date there has been no experimental demonstration of quantum radiation pressure dominated optical system. This is because these effects are too small to be detected, and in order to detect the RPN, extremely high power laser with quite sophisticated suspension system is needed. Even if this condition was realized, the RPN would be overlapped by the seismic noise. We present the design and current status of the development of a radiation pressure noise interferometer (RNI) is able to measure the RPN.

2. Experimental design

Our experimental setup is shown in Fig. 1. We use a small-scale laser interferometer which has two suspended Fabry-Perot cavities, and measure the RPN in the differential lengths of the two cavities. To simplify the control system, reflected light from the cavities is not recombined at the beam-splitter. We readout the fluctuation of each cavity length independently by the Pound-Drever-Hall (PDH) technique [20], and subtract these two lengths electrically. To prevent large amounts of DC light from the cavity going into the photo-detector, the mirrors of the cavities were designed to have the same reflectivity.

This kind of a cavity is called critical coupling (CC) in which the reflectivity is zero on resonance. For the GW interferometer, over coupled (OC) cavity whose reflectivity of the end mirror is almost unity is commonly used. We compare the difference of the quantum noises of Fabry-Perot interferometer of CC cavities and OC cavities. Table 1 shows the quantum noises of the two type cavities. Shot noise in the CC cavities is two times larger than that of the OC. On the other hand the RPN in the CC is smaller only $1/\sqrt{2}$ than that of OC case. Thus, there is no serious difference between CC and OC for the RPN measurement.

The specification of our experimental setup is summarized in Table 2. We use 0.4g fused silica mirrors with high reflectivity of over 99.99% with 7.75 mm in diameter. Near and end mirrors use the same reflectivity, $R_n = R_e = R$. The sensitivity to RPN is inversely proportional to the reduced mass $\mu$. The suspension system of our cavities is shown in Figure 2. The four main mirrors are individually suspended. For common-mode rejection of seismic noise, the top platform plate is shared by the two pendulums. These mirrors are suspended as triple pendulum and the cavity length is 300 mm. The beam waist size at the near mirror is about 300 $\mu$m in radius. To suspend the test-mass, we use two tungsten wires of $\phi = 5\mu$m which are glued to the mirror. We use the coil-magnet actuator to control the length of cavity; tiny magnets (0.5mm in diameter), are attached to the back sides of the mirror.

We estimate the sensitivity of this RNI as shown in Figure 3. The displacement from RPN
Figure 1. Conceptual design of the experimental setup; Using the PDH technique, two test cavities are locked to the laser. Comparing the signals from each cavities, the radiation pressure noise is measured.

Figure 2. Our suspension system; The four main mirrors are individually suspended. For common-mode rejection of seismic noise, the top platform plate is shared by the two pendulums.

| Parameter                                | symbol | practical design |
|------------------------------------------|--------|-----------------|
| near mirror mass                         | \(m_n\) | 0.4g            |
| end mirror mass                          | \(m_e\) | 0.4g            |
| reflectivity                             | \(R_n = R_e\) | 0.9999         |
| beam spot size                           | \(w_0\) | 300 µm (radius) |
| reduced mass                             | \(\mu \equiv \frac{m_e m_n}{2(m_e + m_n)}\) | 0.1 g          |
| half magnitude bandwidth of the cavity   | \(\gamma_c\) | 50000          |
| light angular frequency                  | \(\omega_0\) | 1.77 × 10^{15} rad s\(^{-1}\) |
| Input LASER power                        | \(I_0\) | 100 mW          |
| cavity length                            | \(L\)   | 0.3 m           |
| common mode rejection ratio              | \(\epsilon\) | 0.01           |
| Q factor of the pendulum                 | \(Q_{sus}\) | 4 × 10^{5}     |
| Q factor of the mirror                   | \(Q_{mirror}\) | 10^{6}        |

Table 2. The specification of our experimental parts.

is around \(10^{-16} \text{m}/\sqrt{\text{Hz}}\) at 100Hz, which is about 10-times larger than the other noises.

In this document, we assume that loss angles of all components relating to thermal noise are independent of the frequency. As for the thermal noise from the mirror, our system satisfies the relation \(\phi_{\text{Sub}}/\phi_{\text{coat}} > d_{\text{coat}}/w_0\) [21], which shows that the thermal noise from the coating loss is larger than that from the substrate. Here, we have assumed that the thickness of coating \(d_{\text{coat}}\) is about 5 µm and \(w_0\) is 300µm, the coating loss angle (\(\phi_{\text{coat}}\)) is about \(5 \times 10^{-4}\), and the loss angle of the fused silica substrate (\(\phi_{\text{Sub}}\)) is about \(10^{-6}\), respectively. Figure 3 shows the thermal noise from the coating becomes larger than RPN over 1 kHz. The power spectrum of suspension thermal noise, \(S_{\text{sus}}\), is

\[
S_{\text{sus}}(\omega) \sim \frac{4kBT_{\text{tmp}}\Omega_{\text{sus}}}{\mu Q_{\text{sus}}\omega^5}, \quad \omega_{\text{sus}} \ll \omega,
\]
where $k_B$ is the Boltzmann constant and $T_{\text{tmp}}$ is the temperature, and $\Omega_{\text{pen}}$ is the resonance angular frequency of the suspension, and $Q_{\text{sus}}$ is the quality factor of the suspension which is estimated as follows,

$$Q_{\text{sus}} = \frac{16l_{\text{sus}}}{\phi_{\text{wire}}d_{\text{wire}}^2} \sqrt{\frac{mg}{\pi nE_{\text{wire}}}} + \frac{1}{\phi_{\text{wire}}}$$

where $n$ is the number of wire; $l_{\text{sus}}, E_{\text{wire}},$ and $d_{\text{wire}}$ are the length, the Young modulus, and the diameter of the wires of the pendulum. From (2), $Q_{\text{sus}}$ depends on not only $\phi_{\text{wire}}$ but also the diameter of the suspension wires. In Figure 3, the suspension thermal noise of $Q_{\text{sus}} = 4 \times 10^5$ is shown. This corresponds to the pendulum using two ($n = 2$) tungsten wires of diameter of 20 $\mu$m and $\phi_{\text{wire}}^{-1} = 2000$.

Using thin wire is also advantageous for seismic isolations. For the small test-mass, the mass of the wire is not negligible, and the transfer function of mechanical vibration of the pendulum does not fall like $1/\omega^2$ for the frequencies which are higher than $\omega_{\text{1st}}$ that is the angular frequency of the first violin mode of the wire. This means that the mirror does not act as a free mass for the frequencies of $\omega > \omega_{\text{1st}}$. Therefore, we are suspending the mirror by two thin wires ($n = 2$, $d_{\text{wire}} = 5\mu$m), and we have $\omega_{\text{1st}}/2\pi \sim 300Hz$.

In Figure 3, the frequency noise is not considered because our cavity is short ($L = 0.3m$); frequency stability of $0.1Hz/\sqrt{Hz}$ around at 100 Hz is required if the common mode rejection ratio (CMRR) of two cavities can be attained to be $\epsilon = 0.01$. This can be easily achieved by a rigid reference cavity.

3. Current status of RNI

Currently, we have already checked the vacuum system, and the suspension system has been assembled at the laboratory of ICRR in the suburbs of Tokyo. If the seismic motion turns out to be larger than the expected RPN, our interferometer will be moved to Kamioka mine [22].

Our test-mass is currently suspended with a mirror-holder of 0.2g mass as shown in Figure 4. The holder has four magnets on their backside, and it is designed to hold the mirror firmly by very small claws. Using this holder, we succeeded in suspending the test-mass by wires of $5\mu$m in diameter. There are frictions at the points where the holder touched the mirror, and this friction cause a reduction of the $Q_{\text{mirror}}$. Our estimation shows that $Q_{\text{mirror}}$ is reduced to $1 \times 10^5$ and the thermal noise from the substrate and the coating balance. If $Q_{\text{mirror}}$ is reduced to $5 \times 10^3$, RPN and thermal noise balance at 300Hz. Levin [23] indicated that thermal noise from mechanical inhomogeneous losses depends on the distance between the beam spot and the points where losses are localized. Using the numerical estimation of Yamamoto et.al. [24], it is possible to measure the RPN if we can make $Q_{\text{mirror}}$ over $10^3$. Besides using such a mirror-holder, we are also developing more sophisticated suspension system to suspend and control the mirrors.

4. Conclusion

To confirm theoretical prediction for the quantum fluctuation of interferometric GW detectors [6-11,13], we are developing a small prototype interferometer whose sensitivity is limited by RPN in a frequency band form 100Hz to 1kHz. Using light mirrors for all mirrors, RPN is estimated to be observed between 100 Hz and 1 kHz without using additional technique such as optical rigidity [25] which was firstly observed by Dorsel et.al. [26].

5. Acknowledgments

We are pleased to thank T. Tomaru (KEK) and T. Suzuki (KEK) for helpful support preparing our vacuum system and suspensions. This study was supported by a Grant-in-Aid for Scientific
Figure 3. Sensitivity of the radiation pressure noise interferometer. The displacement noise from RPN is around $10^{-16}$ m$/\sqrt{\text{Hz}}$ at 100Hz, which is about 10 times larger than other noises. The suspension thermal noise is estimated for tungsten wires of 20 $\mu$m in diameter. Thermal noise from the coating is calculated by using a frequency independent loss angle ($\phi_{\text{coat}}$) of $5 \times 10^{-4}$. $Q$-factor of the mirror and the pendulum are assumed to be $Q_{\text{mirror}} = 10^6$, $Q_{\text{sus}} = 4 \times 10^5$. The seismic noise, which is the dominant noise below 100 Hz, is not shown.

Figure 4. A mirror-holder which has magnets on there backside.

Figure 5. Our suspesion system.

Research on Priority Areas by the Ministry of Education, Culture, Sports, Science and Technology.
6. References

[1] V.B. Braginsky, et al 2003 Phys. Rev. D 67 82001
[2] H.P. Yuen, 1983 Phys. Lett. 51 719
[3] C. Caves 1981 Phys. Rev. D 23 1693
[4] W.G Unruh 1983 Quantum Optics, Experimental Gravitation and Measurement Theory ed. P. Meystre, M.O. Scully (Plenum, New York) p 647
[5] M. T. Jaekel et al 1990 Europhys. Lett. 13 301
[6] S. Mancini et al 1994 Phys. Rev. A 49 4055
[7] C. Fabre et al 1994 Phys. Rev. A 49 1337
[8] A.B. Heidmann et al 1994 Phys. Rev. A 50 4237
[9] V. B. Braginsky et al 1988 Sov. Phys. JETP 67 84
[10] S.P. Vyatchanin et al 1995 Phys. Lett. A 201 269
[11] H. J. Kimble et al 2002 Phys. Rev. D 65 22002
[12] A. Buonanno et al 2001 Phys. Rev. D 64 42006
[13] J.M. Courty et al 2003 Phys. Rev. Lett. 90 83601
[14] M. Xiao et al 1984 Phys. Rev. Lett. 59 2153
[15] P. Grangier et al 1984 Phys. Rev. Lett. 59 2566
[16] K. McKenzie et al 2002 Phys. Rev. Lett. 88 231102
[17] S. Chelkowski, et al 2005 Phys. Rev. A 71 13806
[18] H. Vahlbruch, et al 2005 Phys. Rev. Lett. 95 211102
[19] C.M. Mow-Lowry et al 2004 Phys. Rev. Lett. 92 161102
[20] R. W. Drever et al 1983 Appl. Phys.B 31 j97
[21] N. Nakagawa et al 2002 Phys. Rev. D 65 102001
[22] K. Sato et al 2002 Phys. Rev. D 69 102005
[23] Y. Levin 1998 Phys. Rev. D 57 659
[24] K. Yamamoto et al 2002 Phys. Lett. A 305 18
[25] T. Corbitt et al 2003 gr-qc/0511001
[26] A. Dorsel et al 1983 Phys. Rev. lett. 24 1550