Review of the theory of hybrid modeling

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Abstract. This paper reviews the research theory of domestic and foreign hybrid modeling dynamic substructure and frequency response, and then introduces the basic principle of Jetmundsen, Ren substructure synthesis method, mechanical impedance admittance method, mechanical admittance method of interchange-ability and FBS substructure method. In the end, the scope and limitations of the method are compared and analyzed, and a new direction of hybrid modeling of dynamic substructure theory is put forward.

Keywords: FBS method, hybrid modelling, dynamic subsystem

1. Introduction

Hybrid modeling technology is a comprehensive technology that combines test method, CAE method and dynamic parameter correction method. In the process of hybrid modeling, how to deal with the connection stiffness and damping of dynamic substructures is an important factor. The dynamic structure modification theory can be applied to the dynamic analysis of complex structures and millions of degrees of freedom vehicle models, which can be accurately calculated and shorten the calculation time by reducing the degree of freedom. Based on the dynamic substructure modification theory and FRF test of hybrid modeling, the structural dynamic model characteristics are obtained, which have a good application prospect in the field of production and engineering practice. Fixed interface modal synthesis method, coupled interface modal synthesis method and mixed mode synthesis method have high order modal truncation error in modal synthesis, and it is difficult to deal with the substructure of the test. In recent years, the development of dynamic substructure FER has solved the above problems, especially for the calculation of large models, which has outstanding advantages. The dynamic substructure contains the frequency response function matrix of the substructure and the modal information needed for the solution, so the more accurate structure can be calculated without the influence of higher order submode truncation, and has a good application value. The significance of the dynamic substructure method is that for the complex structure and large-scale matrix calculation, the former modal truncation method needs to consume a lot of computer memory and calculation time, which is not good for the reverse modification and optimization of modern design, but the dynamic substructure method solves the above problems. Dynamic correction substructure method to solve the problem of the large scale matrix provides a new technical route, the dynamic structure is divided into multiple interconnected substructure, through the connection between the system and the system rigidity and damping, respectively calculation or test until the dynamic characteristics and transfer function of each
substructure, according to the connection relation between subsystems for the dynamic performance of the whole system.

2. Dynamic frequency domain substructure
Dynamic frequency domain substructure method is a comprehensive method that divides the whole structure into each interrelated substructure in the frequency domain and then connects each substructure according to the displacement coordination equation [1]. The methods used include system impedance, system admittance, system transfer function and FRF frequency response function. Duncan [2] integrated the basic theory of circuit system and first introduced the FRF method of transfer function into the analysis of multi-degree of freedom vibration system. Bishop and Johnson [3] improved the methods of mechanical impedance and mechanical admittance and applied them to the substructure analysis of simple structural beam elements. O’Hara, Rubin, Sykes [4] [5] [6] [7], based on Bishop and Johnson [8], innovated and improved the mechanical impedance and mechanical admittance amplification, and introduced the method into the multi-degree of freedom complex mechanical structures. Although mechanical admittance and mechanical impedance methods have been popularized and applied in certain engineering fields, they have some limitations. Mechanical admittance and mechanical impedance methods are not suitable for very large complex structures and structures with complex boundary conditions and uncertain boundary conditions. With the progress of science and technology and the development of testing technology, people rely more and more on testing technology to replace uncertain factors. On the other hand, with the increasing measurement accuracy of frequency response function, the application of direct measurement results of frequency response function FRF in frequency domain dynamic substructure synthesis becomes possible. Ewins [9] summarized and elaborated the measurement and synthesis methods of dynamic substructure frequency response function, which played a certain role in the direct application of frequency response function.

3. Frequency response function substructure
In the dynamic mechanical impedance of the system and mechanical admittance to solve in the process, need a frequency response function matrix of the whole subsystem matrix inversion, and each substructure test is in the region of the resonance frequency response matrix morbid matrix, the morbid matrix inversion in theory there is a certain error, which affects the whole system more fitting effect and operation precision. Therefore, Imregun, Gleeson, Ewins [10] [11] et al. proposed to reverse fit the modal of the substructure based on the test frequency response function of simple results, and to calculate the frequency response function based on the reverse operation of the modal of the simple substructure, which can improve the fitting accuracy between substructures. However, this method is only applicable to solving simple substructures with less complex degrees of freedom. Jutmunderson has carried on the induction to the frequency response function substructure method, the substructure between displacement and force equilibrium condition is deduced, the method of mechanical admittance and impedance method is greatly improved and don't need to the whole structure of the frequency response matrix inversion, only need to joint inversion of the frequency response function matrix is greatly reduced the difficulty of solving calculation, reduce the matrix to solve the order of time. However, this method is not suitable for the synthesis of coupling substructures, but only for the synthesis of simple independent substructures. Ren greatly improved the dynamic substructure method on the basis of Jutmunderson, and proposed a calculation method suitable for the synthesis of multiple substructures, and some preliminary calculations and applications could be carried out in the coupled independent substructures [12].

4. Typical methods of FRF theory

4.1. Singular value decomposition algorithm
Jetmundsen is the originator of the dynamic substructure decomposition method. He was first engaged in the research work in this field and applied this method into engineering practice. This method is also
called the substructure mechanical admittance method. The Jetmundsen method is applicable to independent dynamic substructures, and the substructures are decoupled from each other. This method is not applicable to independent dynamic substructures. In the mechanical admittance method, the frequency response matrix of each substructure is obtained by testing the dynamic substructure, and the independent subsystems are synthesized by using the related theory of connecting degrees of freedom and the equation of displacement and force coordination. The frequency response matrix of the total structure is obtained, and the substructure matrix is shown in Figure 1. If the degree of freedom B and D of the connection interface are connected purely rigorously (this method does not consider the flexible processing of the connection interface):

\[
\begin{bmatrix}
H^{A} & \Pi_A \\
\Pi_A^T & H^{B}
\end{bmatrix}
= \begin{bmatrix}
H^{A} & \Pi_A \\
\Pi_A^T & H^{B}
\end{bmatrix}
\]

\[
H^{C} = \begin{bmatrix}
H^{A} & \Pi_A \\
\Pi_A^T & H^{B}
\end{bmatrix}
\]

\[(1)\]

\[X^{A}_{\Pi} = X^{B}_{\Pi} \overset{\text{def}}{=} X^{C}_{\Pi}\]

\[(2)\]

**Figure 1.** Jetmundsen substructure synthesis

The displacement and force coordination equation of Jetmundsen dynamic substructure decomposition method is shown in Equation 2, which indicates that the internal degrees of freedom of independent substructures are equal to the degrees of freedom and displacements after connection.

In the formula, the letters marked A, B and C on the dynamic substructure represent the independent substructure A, B and the combined structure C synthesized by independent substructure respectively; The letter subscript j of dynamic substructure represents the connected freedom of two substructures A and B after connection, and the letter subscript A and c of dynamic substructure represents the independent internal freedom of independent substructures A and B without connection. The frequency response matrix of structure C obtained by Jetmundsen dynamic substructure synthesis is shown in Equation 3.
The difficulty of Jetmundsen dynamic substructure synthesis method is to solve the inverse operation of substructure frequency response function. In this method, the inverse of dynamic substructure frequency response matrix is reduced to the inverse of substructure interface coordinate frequency response matrix, which reduces the difficulty of matrix inverse and the ill-posed matrix problem of matrix inverse. Jetmundsen dynamic substructure synthesis can be calculated by singular value decomposition [13].

4.2. Ren synthesis method
Ren dynamic substructure synthesis method is a seed structure synthesis theory of dynamic internal connection method proposed on the basis of Jetmundsen substructure method. This method introduces the internal correlation terms of connection freedom between substructures [14], and a sub-method of non-independent substructure connection is obtained in Jetmundsen theory. The premise of REN dynamic substructure synthesis method is: it is assumed that a part of the degrees of freedom between dynamic subsystems are connected to each other. After connecting a part of the degrees of freedom, the complete system needs to further connect the degrees of freedom, as shown in Fig. 2. The dynamic substructure C is composed of the original substructure A and B by connecting point B and point C. The frequency response function matrix of substructure O is:

$$H^O = \begin{bmatrix} H^O_{\pi \pi} & H^O_{\pi \tau} & H^O_{\pi \tau} \\ H^O_{\sigma \tau} & H^O_{\sigma \tau} & H^O_{\sigma \tau} \\ H^O_{\tau \tau} & H^O_{\tau \tau} & H^O_{\tau \tau} \end{bmatrix}$$  (4)

If the connection interface B and C are rigidly connected, there is:

$$x^O_{\tau} = x^C_{\tau} = x^B_{\tau}$$  (5)

In the formula, the superscript O and C of dynamic substructure denotes the combined structure O and C connected by substructure; Substructure subscript j represents the degree of freedom b and degree of freedom C are connected to form the degree of freedom j. Substructure subscript a and subscript b represent internal degrees of freedom respectively. The frequency response function matrix is obtained by the independent substructure of Ren method.
$$H^C = \begin{bmatrix} H^C_{\pi \pi} & H^C_{\pi \zeta} \\ H^C_{\zeta \pi} & H^C_{\zeta \zeta} \end{bmatrix} = \begin{bmatrix} H^0_{\pi \pi} & H^0_{\pi \zeta} \\ H^0_{\zeta \pi} & H^0_{\zeta \zeta} \end{bmatrix} - \begin{bmatrix} H^0_{\pi \pi} - H^0_{\pi \zeta} \\ H^0_{\zeta \pi} - H^0_{\zeta \zeta} \end{bmatrix} \left( H^0_{\pi \pi} + H^0_{\pi \zeta} - H^0_{\pi \zeta} - H^0_{\zeta \pi} \right)^{-1} \begin{bmatrix} H^0_{\pi \pi} - H^0_{\pi \zeta} \\ H^0_{\zeta \pi} - H^0_{\zeta \zeta} \end{bmatrix} \right)^T$$ (6)

Ren method can also be applied to the synthesis between independent substructures. This method assumes that there are still rigid connections between the degree of freedom of connection, and does not consider the flexible connections of the interface.

5. Conclusions
In this paper, the research status of dynamic substructure and frequency domain substructure is reviewed, and the typical hybrid modeling substructure theory is introduced. By comparing several seed structure theories, the following conclusions can be drawn: 1) The limitation of Jetmundsen's method is that only pure rigid connections are considered between substructures, and the method is only applicable to independent substructures. 2) Ren has carried out the connection method between non-independent subsystems on the basis of his predecessors. Although this method extends Jetmundsen's method and introduces correlation terms between substructures, that is, associational degrees of freedom, the connection method only considers pure rigid connection as the first method.

Acknowledgements
The research was financially supported by the National Natural Science Foundation of China (51275541).

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