Analysis of Systematic Error Influences on Accuracy of Airborne Laser Scanning Altimetry

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ABSTRACT  The error sources related to the laser rangefinder, GPS and INS are analyzed in details. Several coordinates systems used in airborne laser scanning are set up, and then the basic formula of system is given. This paper emphasizes on discussing the kinematic offset correction between GPS antenna phase center and laser fired point. And kinematic time delay influence on laser footprint position, the ranging errors, positioning errors, attitude errors and integration errors of the system are also explored. Finally, the result shows that the kinematic time delay can be neglected as compared with other error sources. The accuracy of the coordinates is not only influenced by the amplitude of the error, but also controlled by the operation parameters such as flight height, scanning angle amplitude and attitude magnitude of the platform.

KEY WORDS  airborne laser scanning altimetry; kinematic offset correction; kinematic time delay; error analysis; imitation calculation

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Introduction

Airborne laser scanning altimetry has widely used for the derivation of topographic data with the development of related technologies and expansion of demands. There are some matured systems including HARC LIDAR terrain mapping system. The experiments of airborne laser scanning system are integrated with laser scanning rangefinder, GPS, INS and communication devices were conducted in 1998, and good results were achieved. Surveying and mapping sector in Netherlands has undertaken the research of DTM derivation with laser scanning altimetry since 1988. The ground mobile laser scanning system integration experiment has been made since 1999. LI Shukai et al. integrated the GPS, INS, laser scanning rangefinder and spectrum imager sensor into a new airborne 3D imager in 1996\(^1\). Matured laser scanning mapping systems products, such as TopEye, TopScan, Optech etc\(^{[3-4]}\), can be available in the market Ground-based laser scanner has been used for volume calculation\(^2\).

Since the airborne laser scanning altimetry system is integrated with laser rangefinder, attitude determination sensor and GPS receivers, the errors from different parts of the system will deteriorate the positioning accuracy. LI shukai et al. provided the geometric model in detail and the analysis of the underlying errors which degrade the positioning accuracy\(^3\). Auxiliary errors and the error from sea-water depths measurement in airborne laser depth sounding were analyzed by LIU Jiyu\(^4\). Other authors also analyzed the error sources which affect the positioning accuracy\(^2.7\). A series of position reduction formulas and their error equations in airborne laser depth sounding are derived by HUANG Motao\(^5\).
However, the kinematic offset correction between GPS antenna phase center and laser firing point and kinematic time delay error was not discussed in the previous literatures.

1 Geometric model

1.1 Local INS system

As regards local INS system we refer to a coordinate system that is centered at the INS system and orientated according to the INS reference frame, as installed on the platform. The origin point is coincided with the GPS antenna phase center. The X axis points toward north, the Y axis points toward east, the Z axis is vertical and downward (Fig. 1).

![Fig. 1 Reference frame of IMU platform](image)

1.2 Laser scanning coordinate system

This coordinate system is the reference frame for the laser scanning system. Its origin is at the laser’s firing point. The X axis points toward the flight direction and the Z axis is identical to the nadir position of the laser scanning system. It is also vertical and downwards. And X, Y, Z axes obey the right hand rule (shown in Fig. 2). The coordinate system is not geo-referenced and changes with the body movement.

The laser footprint position of \( p \) can be expressed in airborne laser scanning coordinate system as follows.

\[
\begin{align*}
x_p &= -S \sin \theta \sin \beta \\
y_p &= S \sin \theta \cos \beta \\
z_p &= S \cos \theta
\end{align*}
\]

where \( \theta \) is the scan angle; \( \beta \) is the azimuth of the scan line (the reference direction is identical to the Y axis direction for conic scan mode). It responds to the line scan when \( \beta \) is set to be zero.

1.3 Local geo-referenced coordinate system

Local geo-referenced coordinate system can be set up according to specific situation. Generally, the center of the concerned area can be as the origin, the X, Y, Z axes of the defined local system are parallel to the IMU platform frame axis respectively. In such a case, the laser footprint position can be transferred from local IMU system to local geo-referenced system with the following equation.

\[
\begin{bmatrix}
X_L \\
Y_L \\
Z_L
\end{bmatrix}
= \begin{bmatrix}
X_A \\
Y_A \\
Z_A
\end{bmatrix}
+ \begin{bmatrix}
X_G \\
Y_G \\
Z_G
\end{bmatrix}
\]

1.4 WGS-84 coordinate system

Finally, we need to establish a transformation between the local geo-referenced coordinate and the WGS-84 system.

1.5 Offset correction between the laser firing point and IMU reference point

The offset \((\Delta x; \Delta y; \Delta z)\) between GPS antenna phase center and laser firing point are constant in the laser scanning coordinate system, which can be determined beforehand with calibration procedure, but their values denoted as
\[ (\Delta X_i; \Delta Y_i; \Delta Z_i) \] in local INS coordinate system change with the attitude change of the plane. Provided that the aircraft is in a status with the attitude angle of \( H \) (heading), \( P \) (pitch), and \( R \) (rolling). The transformation laser scanning coordinate system to the local INS coordinate system is done with the attitude angles \( H, P, R \). Then the offset can be expressed with the following equation in local INS system:

\[
\begin{bmatrix}
\Delta X_o \\
\Delta Y_o \\
\Delta Z_o
\end{bmatrix} = \mathbf{R}(H,P,R)
\begin{bmatrix}
\Delta x \\
\Delta y \\
\Delta z
\end{bmatrix}
\]

(3)

The laser footprint \( P \) coordinate in local INS coordinate system becomes:

\[
\begin{bmatrix}
\cos H \cos P \\
\sin H \cos P + \sin P \sin R \\
\sin H \sin P \cos R + \cos H \cos P \sin R
\end{bmatrix}
\]

Define:

\[
\begin{bmatrix}
a_1 \\
a_2 \\
a_3
\end{bmatrix} = \begin{bmatrix} b_1 \\
b_2 \\
b_3 \\
c_1 \\
c_2 \\
c_3
\end{bmatrix}
\]

then we can get

\[
\begin{bmatrix}
\Delta X_f \\
\Delta Y_f \\
\Delta Z_f
\end{bmatrix} = \begin{bmatrix} a_1 \Delta x + a_2 \Delta y + a_3 \Delta z \\
b_1 \Delta x + b_2 \Delta y + b_3 \Delta z \\
c_1 \Delta x + c_2 \Delta y + c_3 \Delta z
\end{bmatrix}
\]

(6)

Substituting Eq. (1) into Eq. (6), we get:

\[
\begin{align*}
X_f &= a_1 (\Delta x - \sin \theta \sin \phi) + a_2 (\Delta y + \sin \theta \cos \phi) + a_3 (\Delta z + \cos \phi) \\
Y_f &= b_1 (\Delta x - \sin \theta \sin \phi) + b_2 (\Delta y + \sin \theta \cos \phi) + b_3 (\Delta z + \cos \phi) \\
Z_f &= c_1 (\Delta x - \sin \theta \sin \phi) + c_2 (\Delta y + \sin \theta \cos \phi) + c_3 (\Delta z + \cos \phi)
\end{align*}
\]

(7)

Setting \( \beta \) to be zero, we can get the equation in line scanning mode:

\[
\begin{align*}
X_f &= a_1 \Delta x + a_2 \Delta y + a_3 \Delta z + a_4 \sin \theta + a_5 \cos \theta \\
Y_f &= b_1 \Delta x + b_2 \Delta y + b_3 \Delta z + b_4 \sin \theta + b_5 \cos \theta \\
Z_f &= c_1 \Delta x + c_2 \Delta y + c_3 \Delta z + c_4 \sin \theta + c_5 \cos \theta
\end{align*}
\]

(8)

2 Error sources in airborne laser altimetry

2.1 Laser ranging errors

(1) Instrument error

Instrument errors mainly include the following aspects: scanner mirror vibration, laser firing point ambiguity, laser transmitted time and receipt time determination etc.

(2) Atmospheric refraction error

The laser signal is refracted in the atmosphere as the GPS signal, the degree of refraction depends on the wavelength of the laser pulse. The refraction error is mainly related to the tempera-
ture, pressure, humidity factors. The wavelength of the laser pulse is about 1 \( \mu \)m, while the GPS signal wavelength is about 2 dm. Therefore, the atmospheric error of laser is far smaller than that of GPS. The zenith tropospheric delay magnitude is only a few millimeters, which is far less than the achievable accuracy presently. The remainder errors can be ignored after model correction.

(3) Error sources related to the surface
The amount of light that is returned from a target’s surface is characterized by the reflection coefficient. For a diffusely reflecting target, the maximum value of coefficient is 100\% and the returned signal is noisy. For mirror-like or retroreflecting targets, the (theoretical) value of reflectivity can exceed 100 \% by far, the reflected signal may lose. In addition, some pulses return after multiple reflection.

The accuracy of the laser footprint position is also related to the surface roughness, terrain slope as well as other disturbance. In addition, water surface will absorb the ultrared light, and there is few to be reflected. The calm water surface reflects the signal away from the receiver device. The terrain discontinuity and the mobile objects such as pedestrian, cars, animals etc also affect the accuracy of laser footprint positioning.

2.2 **Errors induced by GPS**

Errors induced by GPS mainly include orbital error, satellite clock error, receiver clock error, atmospheric delay, multipath, phase center instability, geometric configuration of the constellation, observation noise, and ambiguity solution. The details will be found in the following section.

2.3 **Errors induced by INS**

Initialization errors, misalignment, and gyro drifts, measurement noise contribute to the systematic INS errors. The details will be found in the following section.

2.4 **Kinematic time delay errors**

Kinematic time delay results from the disagree-
3 Error propagation and accuracy analysis

Assuming that all errors are independent. According to the error propagation law we can get the errors’ equations from Eqs. (7) and (2).

\[
\Delta X = \delta\Delta x \quad \Delta Y = \delta\Delta y \quad \Delta Z = \delta\Delta z
\]

And the ranging error \( \delta S \) influence on laser footprint position is

\[
\Delta X = 0; \quad \Delta Y = \delta S \sin \theta; \quad \Delta Z = \delta S \cos \theta
\]

In this case, set \( \delta S = 0.08 \) m, and the maximum scan angle is equal to 30 degree, we can get the position error plots with respect to different scan angles. The relationship is illustrated in Fig. 5. It indicates that the ranging errors have no impact on X component. The ranging error influence on Y component increases as the scan angle increases. However, the ranging error influence on Z component decreases as the scan angle increases.

4 Errors simulation analysis

The accuracy of laser footprint positioning is determined by the following factors: \( m_{x_1}, m_{x_2}, m_{x_3} \) are the variances of the laser footprint with respect to X, Y, Z components, respectively, in local coordinate system; \( m_{y_1}, m_{y_2}, m_{y_3} \) are the offset determination variances; \( m_{z_1}, m_{z_2}, m_{z_3} \) are the scan angle variances, scan azimuth variance and ranging variance, respectively; \( m_{\alpha}, m_{\beta}, m_{\gamma} \) are the attitudes angle variances; \( a_{ij} (i=1,2,3; \ j=1,2,3,...,12) \) is the first order partial derivative with respect to every variant.

where \( m_{x_1}, m_{y_1}, m_{z_1} \) are the variances of the laser footprint with respect to X, Y, Z components, respectively, in local coordinate system; \( m_{x_2}, m_{y_2}, m_{z_2} \) are the offset determination variances in the three components; \( m_{x_3}, m_{y_3}, m_{z_3} \) are the scan angle variances, scan azimuth variance and ranging variance, respectively; \( m_{\alpha}, m_{\beta}, m_{\gamma} \) are the attitudes angle variances; \( a_{ij} (i=1,2,3; \ j=1,2,3,...,12) \) is the first order partial derivative with respect to every variant.

![Fig. 5 Ranging error influence on laser footprint coordinates](image-url)
attitude errors are $\delta H, \delta R, \delta P$, with no consideration of ranging error and scan angle error, then we have:

\[
\begin{align*}
    a_1 & \approx 1, \\
    a_2 & \approx -\delta H + \delta P \delta R, \\
    a_3 & \approx \delta H \delta R + \delta P, \\
    b_1 & \approx \delta H, \\
    b_2 & \approx 1 + \delta H \delta P \delta R, \\
    b_3 & \approx \delta H \delta R - \delta R, \\
    c_1 & \approx -\delta P, \\
    c_2 & \approx \delta R, \\
    c_3 & \approx 1.
\end{align*}
\]

Then we have:

\[
\begin{align*}
    X_p &= \Delta x + (\delta P \delta R - \delta H)(\Delta y + \sin\theta) + (\delta H \delta R + \delta P)(\Delta z + \cos\theta), \\
    Y_p &= \delta H \Delta x + (1 + \delta P \delta R \delta H)(\Delta y + \sin\theta) + (\delta H \delta R - \delta R)(\Delta z + \cos\theta), \\
    Z_p &= -\delta P \Delta x + \delta R(\Delta y + \sin\theta) + (\Delta z + \cos\theta). \\
\end{align*}
\]

(10)

Define: $\delta P = \delta R = \delta H = 0.03$ degree; definitely $\delta P \delta R \delta H$ is apt to zero. Eq. (10) condenses to

\[
\begin{align*}
    X_p & \approx \Delta x + (\delta P \delta R - \delta H) \sin\theta + (\delta H \delta R + \delta P) \cos\theta, \\
    Y_p & \approx \Delta y + (1 + \delta P \delta R \delta H) \sin\theta + (\delta H \delta R - \delta R) \cos\theta, \\
    Z_p & \approx \Delta z + \delta R \sin\theta + \cos\theta.
\end{align*}
\]

(11)

Subtract Eq. (9) from Eq. (11), then the attitude errors influence on laser footprint position can be expressed as

\[
\begin{align*}
    \delta X_p & \approx (\delta P \delta R - \delta H) \tan\theta + (\delta H \delta R + \delta P) \tan\theta, \\
    \delta Y_p & \approx (\delta P \delta R \delta H) \tan\theta + (\delta H \delta R - \delta R) \tan\theta, \\
    \delta Z_p & \approx \delta R \tan\theta.
\end{align*}
\]

(12)

Set different simulated value to the variants. The results are listed in Table 1.

### Table 1: Attitude error influences on laser footprint coordinates

| Height/m | Scan angle | Attitude errors/degree | $\delta X_p$/m | $\delta Y_p$/m | $\delta Z_p$/m |
|----------|------------|------------------------|----------------|----------------|----------------|
| 400      | 0          | $\delta P = \delta R = \delta H = 0.03$ | 0.21           | 0.21           | 0.00           |
|          | 15         | $\delta P = \delta R = \delta H = 0.03$ | 0.26           | 0.21           | 0.06           |
|          | 30         | $\delta P = \delta R = \delta H = 0.03$ | 0.33           | 0.21           | 0.12           |
| 1000     | 0          | $\delta P = \delta R = \delta H = 0.03$ | 0.52           | 0.52           | 0.00           |
|          | 15         | $\delta P = \delta R = \delta H = 0.03$ | 0.66           | 0.52           | 0.14           |
|          | 30         | $\delta P = \delta R = \delta H = 0.03$ | 0.83           | 0.52           | 0.30           |

The results show that the attitude errors’ influence on laser footprint position increases gradually as the altitude increases. And the laser footprint position error increases as the scan angle increases. Vertical accuracy is higher than horizontal accuracy.

4) If the attitude angle is not equal to zero, e.g., $\theta = 10^\circ, R = 10^\circ, H = 30^\circ$, and the attitude of aircraft $h = 400$ m, and the other parameters are the same as in 2). The simulation result is demonstrated in Fig. 6.

Generally, if the high precise sensors are used, the system provides high accuracy coordinates and the result is stable. On the contrary, poor integrated sensors attain low accuracy. According to Eq. (8), GPS error is linear to the laser point position. In addition, laser point position accuracy is related to flight height, attitude magnitude and scan angle. Ordinarily, the errors occur at the same time and relate to each other.

## 5 Conclusions

The accuracy of laser altimetry is related to various factors as described above. The achieved accuracy for the even, smooth and hard surface is pretty better than that vegetation coverage. The slope terrain will cause secondary error as the laser spot is not a point. Different transformation model from WGS-84 to local coordinate system gives slight different result. The geoid accuracy also affects the height accuracy. The accuracy of laser footprint position derived from
one reference station is better than that derived from multi-reference station. The accuracy of the laser point coordinates located on overlap area relies on the relative accuracy between reference stations. The reference station coordinate errors deteriorate the laser footprint coordinate. The absolute accuracy of laser altimetry is up 15 cm; relative accuracy achieved is up to 5 cm. Absolute accuracy of laser point in the horizontal relies on the flight height parameter (the horizontal accuracy is about one thousandth of height). Kinematic GPS position error is the biggest one among the error sources. Normally, 7-8 cm accuracy level in the vertical can be achieved if we optimize the flight operation and improve the kinematic GPS solution. The potential higher accuracy will be achieved if all errors source can be modeled and canceled out in the future.

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Notes to Contributors

Contributions are welcomed on one of the following subjects or in related areas:

- GIS
- Geodynamic
- Geophysical geo-surveying
- Engineering surveying
- GPS
- Geo-surveying
- Photogrammetry
- Mapping apparatus
- RS
- Cartology
- Graphics

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