Quantum chromodynamic quark benzene *

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A six-quark state with the benzene-like color structure based on a color string model is proposed and studied. Calculation with quadratic confinement with multi-string junctions shows that such a state has a ground state energy similar to that of other hidden color six-quark states proposed so far. Its possible effect on $NN$ scattering is discussed.

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In the QED world, there are almost countless structures of matter. Among organic molecules, there are very interesting varieties such as methane, benzene, fullerene, etc. In the QCD world, up to now, only very limited structures of matter have been discovered: quark-antiquark mesons, three-quark baryons, nuclei and neutron stars. However, the QCD interaction is richer and more varied than QED so one would expect there to be even more varieties of QCD matter. In pentaquark studies, various color structures have been proposed: color singlet hadron molecules [K($q\bar{q}$)N($q^3$)], color antitriplet diquarks [(qq)(qq)q], quark methane [q\bar{q}q], etc. Although the high statistics experimental data of JLab and COSY \textsuperscript{1} did not reveal a pentaquark signal, the theoretical view point of multi-quark structures has been broadened in the course of pentaquark studies.

Lattice QCD calculation does not rule out the existence of glueballs, quark-gluon hybrids, multi-quark states, etc. An interesting question is whether there is a peculiar property of low energy non-perturbative QCD that is responsible for hiding or even excluding these otherwise possible structures of matter. Lattice QCD calculations for mesons, baryons, tetraquarks and pentaquarks reveal a flux-tube or string like structure \textsuperscript{2,3}. The naive flux-tube or string model \textsuperscript{4,5}, after adjusting the model parameters, gives qualitatively similar estimates of ground state energies of hadrons compared to other quark models. We therefore use this model to study the possible structures of multi-quark systems. Other models, such as the bag model \textsuperscript{6}, quark compound model \textsuperscript{7}, potential model \textsuperscript{8}, Skyrmion model \textsuperscript{9}, chiral soliton model \textsuperscript{10} and others \textsuperscript{11} have all been used in the study of multi-quark systems. In general they give similar results and mutually support the existence of multi-quark systems.

In this letter, we propose and study a new color string structure, a benzene-like structure, for the six-quark system. We estimate the ground state energy of this system, which we call QCD benzene, using a non-relativistic color string model \textsuperscript{2} with harmonic confinement.

Strings are actually dynamical variables to which kinetic and potential energies can be attributed \textsuperscript{4}. The potential energy is proportional to the total length of strings. String tensions can be estimated from the bag model or deduced from lattice QCD calculations. They can also be determined empirically by fitting hadron masses. We take the last, \textit{i.e.} a purely phenomenological approach. Since we are only interested in qualitative properties of QCD benzene, we assume a quadratic confinement potential for the color string rather than linear as this simplifies the calculation significantly.

A comparative study showed the inaccuracy of this choice is quite small \textsuperscript{6}. There are two reasons to expect this: One is that the spatial variations in separation of the quarks (lengths of the string) do not differ significantly, so the difference between the two functional forms is small and can be absorbed in the adjustable parameter, the stiffness. The second is that we are using a non-relativistic description of the dynamics and, as was shown long ago \textsuperscript{12}, an interaction energy that varies linearly with separation between fermions in a relativistic, first order differential dynamics has a wide region in which a harmonic approximation is valid for the second order (Feynman-Gell-Mann) reduction of the equations of motion. A numerical check (see Table 1 below) confirmed this expectation.

For the six-quark system, there are four possible string structures as shown in Fig.1. The first three were discussed in many papers in the past but we have not found any previous discussion for the last one. In these figures, $r_i$ represents the position coordinate of the quark $q_i$ which is denoted by a black dot, $y_i$ represents a junction where three flux tubes meet. A thin line connecting a quark and a junction represents a fundamental, \textit{i.e.} color triplet, representation and a thick line connecting two junctions is for a color sextet, octet or others, namely a compound string. The different types of string may have differing stiffness \textsuperscript{13,14}. An inverted line represents a conjugate $SU(3)$ color representation. Both the overall color singlet nature of a quark system and the $SU(3)$ color coupling rule at each junction must be satisfied. Then various types of color coupling are allowed in QCD, including that shown in figure 1d of a benzene-like form.

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In the string model with quadratic confinement, the potential energy of the benzene-like string configuration for a six-quark state has the following form,

$$U = \frac{1}{2}k \left( \sum_{i=1}^{6} (r_i - y_i)^2 + \kappa \sum'_{i < j} (y_i - y_j)^2 \right),$$  \hspace{1cm} (1)$$

where the $\sum'$ means the summation is over the adjacent pairs. The string stiffness constant of an elementary or color triplet string is $k$, while $\kappa k$ is the compound string stiffness. In fig.1d, each quark is connected by a different compound string and each string with a different dimension of the color SU(3) group representation will have a different stiffness \[13, 14\]. However, due to quarks being fermions, the Hamiltonian for a multi-quark system must be properly antisymmetrized under quark exchange. In QED benzene, each carbon is connected to the neighboring carbons with a single and a double bond, but it has been proven that QED benzene has $D_{6h}$ symmetry. Therefore, it is natural to use a common stiffness, $\kappa k$, for the compound string for each color configuration, after taking into account the symmetry requirement.

For given quark positions $r_i$, $i = 1, \cdots, 6$, in the center-of-mass (CM) system, we can fix the position coordinates of those junctions $y_i$, $i = 1, \cdots, 6$ by minimizing the energy of the system. The result is,

$$y_1 = \frac{(\kappa + 4\kappa^2)(r_2 + r_6) + \kappa^2(r_4 + r_5) + (1 + 3\kappa)^2r_1}{(1 + \kappa)(1 + 3\kappa)(1 + 4\kappa)},$$  \hspace{1cm} (2)$$

and other configurations have a similar form. Thus we can define a set of canonical coordinates,

$$R_1 = \frac{1}{2}(r_6 + r_3 - r_1 - r_4),$$

$$R_2 = \frac{1}{2}(r_6 - r_3 + r_1 - r_4),$$

$$R_3 = \sqrt{\frac{1}{12}}(2r_2 - r_6 - r_1 + 2r_5 - r_3 - r_4),$$  \hspace{1cm} (3)$$

$$R_4 = \sqrt{\frac{1}{12}}(2r_2 - r_6 + r_1 - 2r_5 + r_3 - r_4),$$

$$R_5 = \sqrt{\frac{1}{6}}(r_2 + r_4 + r_6 - r_1 - r_3 - r_5),$$

$$R_6 = \sqrt{\frac{1}{6}}(r_1 + r_2 + r_3 + r_4 + r_5 + r_6).$$

Finally, the potential energy $U$ can be written as,

$$U = \frac{k}{2} \left( \frac{3\kappa}{1 + 3\kappa} R_1^2 + \frac{\kappa}{1 + \kappa} R_2^2 + \frac{3\kappa}{1 + 3\kappa} R_3^2 + \frac{\kappa}{1 + \kappa} R_4^2 + \frac{4\kappa}{1 + 4\kappa} R_5^2 \right)$$  \hspace{1cm} (4)$$

Using this string potential in a non-relativistic Hamiltonian with effective quark mass $m$, \[3\], we find the ground-state energy of QCD benzene to be

$$M_6 = 6m + \frac{3}{2} \omega \left( \sqrt{\frac{\kappa}{1 + \kappa}} + 2 \sqrt{\frac{3\kappa}{1 + 3\kappa}} + \sqrt{\frac{4\kappa}{1 + 4\kappa}} \right),$$  \hspace{1cm} (5)$$
where $\omega$ is equal to $\sqrt{k/m}$.

Furthermore, by calculating the distance between two adjacent quarks and the distances between quarks and the CM, the spatial structure of QCD benzene can be obtained. The distances between any two adjacent quarks can be shown to be equal and so are the distances between any quark and the CM. These are

$$
\langle r_i - r_j \rangle^2 = \frac{1}{2m\omega\sqrt{\kappa(1 + 4\kappa)}} \left( \sqrt{(1 + \kappa)(1 + 4\kappa)} + 1 + 4\kappa + \sqrt{3(1 + 3\kappa)(1 + 4\kappa)} \right),
$$

where $j = i + 1 \pmod{6}$, and

$$
\langle r_i^2 \rangle = \frac{1}{24m\omega\sqrt{\kappa(1 + 4\kappa)}} \left( 12\sqrt{(1 + \kappa)(1 + 4\kappa)} + 3 + 12\kappa + 4\sqrt{3(1 + 3\kappa)(1 + 4\kappa)} \right).
$$

Clearly these two distances generally are not equal, so the QCD benzene cannot be planar. However, we can show that the two equilateral triangles, namely $\triangle_{135}$ and $\triangle_{246}$, lie on two parallel planes. The length of the side of the equilateral triangle is

$$
\langle L^2 \rangle = \frac{3}{2m\omega} \left( \sqrt{\frac{1 + 3\kappa}{3\kappa}} + \sqrt{\frac{1 + \kappa}{\kappa}} \right)
$$

and the distance between the two parallel planes is

$$
\langle (R_{135} - R_{246})^2 \rangle = \frac{1}{2m\omega} \sqrt{\frac{1 + 4\kappa}{\kappa}}
$$

$R_{135}$ and $R_{246}$ represent the centers-of-mass of $q_1q_3q_5$ and $q_2q_4q_6$, respectively. The spatial structure of QCD benzene is shown in Fig.2. Projecting the $\Delta_{135}$ plane onto the $\Delta_{246}$ plane, we obtain a planar benzene.

Fig. 2. The spatial structure of the benzene-like state.

To check this result, the same model has been applied to the tetraquark and pentaquark systems. Three-dimensional configurations are also favored as shown in Figs.3 and 4, where a solid-dot and a hollow-dot represent a quark and antiquark, respectively, while $R_{12} = \frac{r_1 + r_2}{2}$ and $R_{34} = \frac{r_3 + r_4}{2}$ represent the midpoints of $r_1$, $r_2$ and $r_3$, $r_4$, respectively. Please note Figs.3 and 4 only express the coordinates of the quarks; the string configurations are not shown.

In Figs.3 and 4, $r_{12} = r_1 - r_2$ is perpendicular to $r_{34} = r_3 - r_4$, and $r_{12,34} = R_{12} - R_{34}$ is perpendicular to $r_{12}$ and $r_{34}$. The lengths of $r_{12}$ and $r_{34}$ can be shown to be equal,

$$
\langle (r_{12})^2 \rangle = \langle (r_{34})^2 \rangle = \frac{1}{m\omega}
$$

For the tetraquark and pentaquark states, the distances between $R_{12}$ and $R_{34}$ are respectively:

$$
\langle (R_{12} - R_{34})^2 \rangle = \frac{1}{m\omega} \sqrt{\frac{1 + \kappa}{\kappa}}
$$

$$
\langle (R_{12} - R_{34})^2 \rangle = \frac{1}{m\omega} \sqrt{\frac{2 + \kappa}{\kappa}}
$$

In the pentaquark state, the distance between the antiquark and the CM of the 4-quark subset is

$$
\langle (r_{5,1234})^2 \rangle = \frac{1}{m\omega} \sqrt{\frac{2 + 5\kappa}{5\kappa}}
$$

Consequently, multiquark states in our quark model must form three-dimensional configurations which is due to the dynamics of the systems: The string shrinks the distance between any two connected quarks to as short a distance as possible to minimize the potential energy, while the kinetic motion expands the distance between any two quarks to as long a distance as possible to minimize the kinetic energy: The stereo structures meet this requirement better than a planar one does.

For pentaquark systems, lattice QCD obtains almost degenerate energies for both planar and 3-dimensional configurations. However, the entropy of a $5q$ system
is found to be larger in a three-dimensional configuration [3]. For tetraquark systems, a three-dimensional tetrahedral structure is rather stable against the transition into two mesons [12].

The non-zero value of $(r_{5,1234})^2$ is due to the fact that the anti-quark oscillates around the CM of the four quarks and the value of $(r_{5,1234})^2$ is calculated with a harmonic oscillator wave function.

For comparison, we also present results for two other color structures of the six quark system [3],

\[
M_{33} = 6m + \frac{3}{2} \omega \left( 2 + \frac{3\kappa}{2 + 3\kappa} + \sqrt{\frac{\kappa(2\kappa + 0.6277)}{2\kappa^2 + 7\kappa + 2}} \right) + \frac{2\kappa(\kappa + 3.186)}{2\kappa^2 + 7\kappa + 2} \]

(14)

\[
M_{222} = 6m + \frac{3}{2} \omega \left( 3 + 2 \sqrt{\frac{\kappa}{2 + \kappa}} \right) \]

(15)

Here, $M_{33}$ and $M_{222}$ are ground-state energies of the two color-octet, hidden color state (Fig.1b) and the three di-quark state (Fig.1c) respectively.

In our calculation, there are three free parameters: The non-strange quark mass $m$ and the harmonic oscillator stiffnesses, $k$ and $\kappa k$. The first two can be fixed from the masses of the non-strange hadrons. We take $M_{3}$ as the average mass of $N + \Delta$ and take $M_{2}$ as the average mass of light, non-strange mesons

\[
\bar{M}_3 = \frac{1}{2}(N + \Delta) = 3m + 3\omega, \]

(16)

\[
\bar{M}_2 = \frac{1}{4} \left( \frac{3}{4} \eta^3 + \frac{1}{4} \eta \right) + \frac{3}{4} \left( \frac{3}{4} \rho^3 + \frac{1}{4} \rho \right) = 2m + \frac{3}{2} \omega. \]

(17)

From this, we obtain the effective quark mass

\[
m = 2\bar{M}_2 - \bar{M}_3 = 0.19GeV. \]

(18)

The constant $\kappa$ of an elementary string can be eliminated in favor of the experimental baryon mass $M_{3}$. The inter-junction string parameter $\kappa$ depends on the color dimension, $d$, of the string. In the MIT bag model [13], for example, one finds effectively

\[
\kappa_d = \sqrt{\frac{C_d}{C_3}}, \]

(19)

where $C_d$ is the eigenvalue of the Casimir operator associated with the $SU(3)$ color representation $d$ on either end of the string. However lattice QCD calculations find that the stiffness, $\kappa_d$, of higher dimensional color charge is scaled by $C_d$ instead of $\sqrt{C_d}$ [14]. The compound strings of six-quark systems have five possible color structures: $3, 3^*, 6, 6^*$ and $8$, so each geometric configuration may have different states when the color dimension dependence of the string stiffness is included. This means that

the minimum string energy also varies in general from one state to another. We shall not include all these (small) variations, but use instead an average compound string constant $\kappa k$.

Numerical results for these three hidden color structures are shown in the Table 1. To check the approximation introduced by quadratic confinement the linear confinement results with the same multi-quark wave functions obtained from quadratic confinement model are listed in Table 1 as a comparison.

| confinement | Three-di-quark | Color-octet | Benzene-like |
|-------------|---------------|-------------|--------------|
| quadratic   | 1.0           | 2.22        | 2.21         | 2.19         |
|             | 1.5           | 2.26        | 2.26         | 2.25         |
| linear      | 1.0           | 2.31        | 2.31         | 2.38         |
|             | 1.5           | 2.31        | 2.31         | 2.37         |

From the Table, it can be seen that while the different color structures have similar masses (around 2.2 to 2.4 GeV), the benzene-like structure has the lowest mass whereas the three di-quark structure has the highest mass for quadratic confinement. On the other hand for linear confinement, the benzene-like structure has the highest mass, whereas the other two have smaller masses. After taking into account the model uncertainties, we can only conclude that the three hidden color states have similar masses. With an increase of $\kappa$, the differences between the masses decreases. In the limit $\kappa \to \infty$, they have the same mass due to having the same flux-tube structure in which the compound flux-tube shrinks to zero, leaving a hub and spokes configuration with one junction.

The color magnetic interaction (CMI), which is important for hadron spectroscopy, has not yet been included in our model calculation. The main reason for this is that we have not found a method to calculate the matrix element of CMI for benzene-like structure. In our previous pentaquark calculation, we found that the tetrahedron has a lower mass than the Jaffe-Wilczek di-quark structure even after taking into account of CMI [16]. To give an estimate of the effect of CMI, we calculated CMI for hidden color states consisted of two color-octet baryons. The results are in the range of 20-120 MeV for two color-octet nucleons or $\Delta$’s systems. We expect the benzene-like structure should have energy around 2.21-2.50 GeV, at 0.33-0.62 GeV above the two-nucleon mass.

The true six quark state will, of course, involve a mixture of these differing string structures. In general, channel coupling will reduce the ground state energy of the system. Therefore, for channel coupling calculations in QCD models, these color channels should be included. Unfortunately, at present we do not have any reliable information about the transition interaction between different color structures, especially from a color singlet hadron state to genuine hidden color multi-quark states. The transition interaction is very much needed for the study of multi-quark states.

In nucleon-nucleon ($NN$) scattering, QCD benzene
should be one of the possible intermediate states as well as the other color structures shown in Fig.1. In order to simplify our argument, we neglect the 6 and 6* strings temporarily. Then, if we cut the two color 8 strings, the QCD benzene will shrink into two color singlet baryons (which we take to be two nucleons for convenience in this discussion). Except for exchanges in quark labels, these two nucleons are the initial ones, as shown in Fig.1a, in the $NN$ scattering process. It should be possible for two nucleons to transform into QCD benzene and other genuine hidden color six quark states due to flux tube fluctuations when they are close enough to be within the range of confinement (~ $1\text{fm}$). For example, the creation of two gluons can simultaneously excite the two color singlet nucleons into two color octet baryons which are coupled together to be overall color singlet – this is nothing else but a QCD benzene. If the scattering energy is near that of these hidden color states ($2.2$ to $2.5 \text{ GeV}$), then the intermediate six quark state should be dominantly in a hidden color state because the number of hidden color states is much larger than color singlet one so the entropy of hidden color state is much higher than the color singlet one and it cannot decay into two colorful nucleons directly due to color confinement. It must transform back into two color singlet nucleons before decaying. Such a process is similar to compound nucleus formation and therefore should induce a resonance around the energies of these hidden color states in $NN$ scattering. We call this state a “color confined, multi-quark resonance” and it is different from all of the microscopic resonances discussed by S. Weinberg [17]. Our present understanding of QCD does not obviate such a back and forth transition between color singlet hadron and genuine hidden color multi-quark states. An open question remains as to whether or not experiments have observed such a color confined, multi-quark resonance. In fact there is a structure in the $pp$ scattering cross section around $2.2$ to $2.4 \text{ GeV}$ which has been explained in terms of $\Delta$ production [18]. However in Our quark model calculations, we found that there are several $N-\Delta$, $\Delta-\Delta$ broad resonances in this energy range, in addition to the benzene-like structure. Can it also be described by broad overlapping resonances due to these structures? This possibility seems not to have been ruled out and it is under study in our group.

The possible existence of benzene-like structure is based on the description of quarks interaction via flux tubes. Although the lattice QCD calculation gave some supports to the idea, its validity is not beyond doubt. Other studies are needed to check if the QCD benzene structure obtained from color flux tube model is really a QCD result.

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