Multiple-quasiparticle agglomerates at $\nu = 2/5$

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Abstract

We investigate the dynamics of quasiparticle agglomerates in edge states of the Jain sequence for $\nu = 2/5$. Comparison of the Fradkin-Lopez model with the Wen one is presented within a field theoretical construction, focusing on similarities and differences. We demonstrate that both models predict the same universal role for the multiple-quasiparticle agglomerates that dominate on single quasiparticles at low energy. This result is induced by the presence of neutral modes with finite velocity and is essential to explain the anomalous behavior of tunneling conductance and noise through a point contact.

Key words: Fractional quantum Hall, edge theories, quasiparticles.

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1. Introduction

Noise experiments in point contacts have been crucial to demonstrate the existence of fractionally charged quasiparticles in fractional quantum Hall systems [1]. In particular, it was proved that for filling factor $\nu = \frac{p}{2np + 1}$, with $n, p \in \mathbb{N}$, (Jain series [2]), the quasiparticle (qp) charge is given by $e^* = \frac{e}{2np + 1}$ [3,4,5]. A suitable framework for the description of these phenomena is provided by the theory of edge states [6,7]. For the Laughlin series ($p = 1$) a chiral Luttinger Liquid theory ($\chi$LL) with a single mode was proposed and shot-noise signatures of fractional charge were observed [3,8]. For the Jain series ($p > 1$) extensions were introduced either by considering $p - 1$ additional hierarchical fields, propagating with finite velocity (Wen model [9]), or by considering two fields, one charged and one neutral and topological (Fradkin-Lopez model [10,11]). Recently, transport experiments performed on a point contact at extremely low temperatures have shown unexpected change in the temperature power-law of the conductance and tunneling particles with a charge that can reach $p$ times the single quasiparticle charge [12,13]. In [14] we proposed a possible explanation of these effects by introducing a generalized Fradkin-Lopez model (GFL) with neutral modes propagating at finite velocity. This assumption was crucial in order to lead the agglomerates of qps to dominate the tunneling processes[14].

In this paper we will address the question about the robustness of this important result. For this reason we will compare the above mentioned GFL model with the one proposed by Wen [9]. We will focus on filling factor $\nu = 2/5$, which is one of the available case considered in experiments. We will prove that both models predict the same universal role for the qp agglomerates, and that are able to explain, with similar level of accuracy, the experimental observations. These facts further sup-
port our interpretation of experiments, confirming the importance of agglomerates in tunneling processes for the Jain series [14].

2. Quasiparticle agglomerate field construction

Let us start this section with some general remarks. The bulk excitation wave functions in the Hall fluid have to satisfy the no-monodromy requirement [15,16] and have to be of single-valuedness with respect to the electrons. This means that the phase acquired by any excitation in a loop around an electron must be a multiple of $2\pi$. Considering edge excitations, the "holo-

graphic principle" shows that any bulk excitation can be mapped into operators defined at the boundary [17]. So the above no-monodromy condition, for an excitation at the edge, can be expressed in terms of constraint on the mutual statistical angle between the excitation itself and the electron. In general, the mutual statistical angle $\Theta$ between two edge operators $\Psi(x)$ and $\Psi'(x)$ is defined as

$$
\Psi(x)\Psi'(x') = \Psi'(x')\Psi(x)e^{-i\Theta \text{ sgn}(x-x')}.
$$

The no-monodromy condition requires that the mutual statistical angle between any quasiparticles operator and the electronic operator must be an integer multiple of $\pi$. Note that $\Theta$ corresponds to the usual definition of the statistical angle $\theta$ if the two operators in (1) coincide [18].

In the following, the field representation of edge excitations will be obtained along two main steps. First, one has to identify the possible electron operators, which in the end will form the electron field. They must have unit charge $e$, fermionic statistics and they have to mutually satisfy the no-monodromy condition. Second, one has to identify the operators of the qps and of the agglomerates with the appropriate charge, statistics and no-monodromy requirement.

We will classify all the excitations in terms of their charge that has to be an integer multiple of the quasi-

particle (qp) charge $e = e/5$. We will refer to an $m$-agglomerate as an excitation with a charge $me^*$. Note that the electron is the agglomerate with $m = 5$.

2.1. General theoretical framework at $\nu = 2/5$

We start with the description of a single infinite edge at $\nu = 2/5$. Both GFL and Wen models are described in terms of two bosonic fields. Here, we do not consider the details of the derivation, but we refer to the available literature [6,7,10,11,14].

The edge consists of a charged mode $\phi^+$ and a neutral mode $\phi^-$, mutually commuting. The real-time action $S$ is ($\hbar = 1$)

$$
S = \frac{1}{4\pi\nu_+} \int dt dx \partial_x \phi^+ (-\eta^+ \partial_t - \nu_+ \partial_x)\phi^+ + \frac{1}{4\pi\nu_-} \int dt dx \partial_x \phi^- (-\eta^- \partial_t - \nu_- \partial_x)\phi^- ,
$$

with $\nu_+$ the charge and neutral mode velocities. The charge parameters are common in both models with $\eta^+ = 1$ and $\nu_+ = 2/5$, while the coefficients associated to the neutral mode are model dependent [10,14,19]

$$
\eta^- = -1 \quad \nu_- = 1 \quad \text{GFL model}
$$

$$
\eta^- = 1 \quad \nu_- = 2 \quad \text{Wen model}.
$$

One can see that the neutral mode is counter-

propagating in the GFL model and co-propagating in the Wen one. The electron number density is

$$
\rho(x) = \frac{1}{2\pi} \partial_x \phi^+ (x) - 2\pi \eta^+ \nu_+ \text{sgn}(x-x').
$$

Using the bosonization technique the edge operators are expressed as exponential combinations of the bosonic edge fields. We start with the operator associated to the $m$-agglomerate [19]

$$
\Psi^{(m)}(x) = \frac{1}{\sqrt{2\pi a}} e^{i(\alpha_m \phi^+(x) + \beta_m \phi^-(x))},
$$

here, $a$ is the short cut-off length and $\alpha_m, \beta_m$ represent the coefficients of the charged and neutral modes respectively. The pair of their values $(\alpha_m, \beta_m)$ determines uniquely an edge excitation and it will be used as an alternative notation for the operator $\Psi^{(m)}(x)$. The physical properties of the operator (5) can be now obtained as follows. The charge coefficient is derived using the commutation with the electron density

$$
[\rho(x), \Psi^{(m)}(x')] = -Q_m \delta(x-x')\Psi^{(m)}(x') ,
$$

The original Fradkin-Lopez theory postulates two neutral modes but, for infinite edges, it is possible to consider an “effective” theory with a single neutral mode [11].)
with $Q_m = me^*/e$ the $m$-agglomerate charge in unit of $e$. Applying the commutations among the bosonic fields one has

$$\alpha_m = \frac{Q_m}{\nu_+} = \frac{m}{2}. \quad (7)$$

Statistical properties between different operators are characterized by the mutual statistical angle in Eq.(1). This angle depends on the coefficients ($\alpha$, $\beta$) and ($\alpha'$, $\beta'$) that define two different operators. By using the commutation rules one obtains

$$\Theta \left[ (\alpha, \beta), (\alpha', \beta') \right] = \pi \left( \eta_+ \nu_+ \alpha \alpha' + \eta_- \nu_- \beta \beta' \right). \quad (8)$$

Statistical properties among equal operators are defined in terms of the statistical angle $\theta_m$ which is fixed in the Chern-Simons effective theory of hierarchical fractional quantum Hall states. For $\nu = 2/5$ it is [10,14]

$$\theta_m = -\pi m^2 \left( \frac{7}{5} \right) - 2\pi k, \quad (9)$$

with $k \in \mathbb{Z}$, encoding the $2\pi$ periodicity. For the $m$-agglomerate in (5) the explicit expression in terms of the pair $(\alpha_m, \beta_m)$ is

$$\theta_m = \pi \left[ \nu_+ (\alpha_m)^2 + \eta_- \nu_- (\beta_m)^2 \right]. \quad (10)$$

Note that once the charge of the agglomerate is fixed, one can still choose operators differing in the statistical angle by an integer of $2\pi$. If they satisfy the no-monodromy condition with electrons they are indeed admissible operators.

Hereafter we will apply these general results to the Fradkin-Lopez model in order to derive the set of admissible operators for the agglomerates.

2.2. Fradkin-Lopez model

Let us start to identify the possible electron operators that correspond to an agglomerate with $m = 5$. The parameters for the Fradkin-Lopez model are in Eq.(3). The electron charge fixes the charge coefficient (7) to be $\alpha_e = 5/2$. The possible values of $\beta_e$ will be determined, first of all, by comparing the expressions (9) and (10). This constrain restricts the possible solutions $^3$ to $\beta_e(k) = \pm \sqrt{3/2 + 2k}$ with $k \in \mathbb{N}$.

In addition, one has still to impose the no-monodromy requirement with any other electron operators

$$\Theta \left[ (\alpha_e, \beta_e(k)), (\alpha_e, \beta_e(k')) \right] = \pi l \quad (11)$$

with $l \in \mathbb{Z}$. Here, $k$ and $k'$ identify two different electron fields. By using the expressions (8) and (11) and substituting the above values for $\alpha_e, \beta_e(k), \beta_e(k')$ one obtains the condition for $k$ and $k'$

$$(3 + 4k)(3 + 4k') = (2h + 1)^2 \quad (12)$$

with $h \in \mathbb{N}$. The more general solution in (12) can be represented in terms of two integers $r$ and $q$ as $k_r[q] = r + (3 + 4r)(q^2 + q)$. Any $r$-family defines an admissible set of electrons operators. In the FL model one selects the $r = 0$ family $^{10}$ with $k_0(q) = 3q(q + 1)$. In this case, the neutral field coefficient is $\beta_e(q) = \sqrt{6(q + 1/2)}$ with $q \in \mathbb{Z}$.

It is important to note that the operators associated to $\beta_e(q)$ can be further classified into two sub-classes that differ only in the parity of $q$: $\beta_{e, +}(q)$ for odd $q$ and $\beta_{e, -}(q)$ for $q$ even. Within the same class the operators anticommute, while they commute if they belong to two different classes, as can be easily verified using Eq.(8). This last property defines a parafermionic statistics. In order to write the more general expression for the electron operator one has to convert the parafermionic set of operators in a fermionic one via a Klein transformation [20]. This is achieved by defining two bosonic Klein factors for the even and odd sub-classes respectively $\mathcal{F}^{(+)}$ and $\mathcal{F}^{(-)}$ mutually anticommuting $\{\mathcal{F}^{(+)}, \mathcal{F}^{(-)}\} = 0$. It is easy to verify that this restore the anticommutation properties between operators of the two different classes. The more general electron operator can be then written as a linear combination of all the possible electron representatives

$$\Psi_e(x) = \sum_{q \in \mathbb{Z}} c_q \mathcal{F}^{(-)} e^{i \phi^+(x)} e^{i \phi^-(x)} \quad (13)$$

where $c_q$ are the coefficients of the linear superposition. Let us consider now the $m$-agglomerate with charge $me^*$ and $\alpha_m = m/2$ (cf. Eq.(7)). The neutral mode

Note that if $\beta(q)$ corresponds to an admissible value also $-\beta(q)$ will be admissible. We encode this property by admitting $q \in \mathbb{Z}$.

Note that the condition of no-monodromy requires only that the mutual statistical angle must be integer and not an odd integer as the fermionic statistic would imply.
coefficient $\beta_m(k)$ is again determined by the statistical angle $\theta_m$ given in Eq.(9). One has
\[ \beta_m(k) = \pm \sqrt{2m^2 + 2k}, \quad (14) \]
with $k \geq k_{\text{min}}$ and $k_{\text{min}} = -\text{Int}[3m^2/4]$. As already pointed out the agglomerate operators have to be also compatible with the no-monodromy condition with the electron operators of Eq.(13). Imposing this we find that, for a given $m$-agglomerate, $k$ is not an arbitrary integer but it assumes specific values only. In particular, for odd agglomerates with $m = 2s + 1$, the admissible $k$ are given by $k = 3q^2 + 3q - 3s^2 - 3s$ being $q \in \mathbb{N}$, while for even agglomerates, $m = 2s$, the possible values are $k = 3q^2 - 3s^2$. This implies the following values for $\beta_m(q)$
\[ \beta_{2s+1}(q) = \sqrt{6} \left[q + \frac{1}{2}\right], \quad \beta_{2s}(q) = \sqrt{6}q \quad (15) \]
with $q \in \mathbb{Z}$. Note that the single qp and the electron excitations belong to the odd family with $(s = 0)$ and $(s = 2)$.

Substituting the results for $\alpha_m$ and $\beta_m(q)$ in the operator expression (5) we obtain the general form
\[ \Psi^{(2s+1)}(x) = \frac{1}{\sqrt{2\pi a}} e^{i(s+\frac{1}{2})\phi^+(x) + (q + \frac{1}{2})\phi^-(x)}, \quad (16) \]
for odd agglomerates, and
\[ \Psi^{(2s)}(x) = \frac{1}{\sqrt{2\pi a}} e^{i(s+\frac{1}{2})\phi^+(x) + (q + \frac{1}{2})\phi^-(x)}, \quad (17) \]
for even ones.

It is important to observe that the 5-agglomerate family corresponds exactly to the series of electron operators we found before in Eq.(13). As a final remark we would like to comment on the parafermionic statistics. All the $m$-agglomerate with $m$ odd, similarly to the electron case, can be divided in two classes. Agglomerates within one class have the appropriate statistical angle $\theta_m$ while agglomerates in different classes have the mutual statistical angle equal to $\theta_m \pm \pi$.

The proper fractional statistics of the $m$-agglomerate can be then recovered by using the bosonic Klein operators $\mathcal{F}^{(+)}$ and $\mathcal{F}^{(-)}$ previously defined. The more general operator of $m$-agglomerate will be then written as a linear combination of exponential operators similar to Eq.(13).

To conclude we note the symmetries of the coefficients $(\alpha, \beta)$ in Eqs.(16) and (17). When the charge coefficient $\alpha$ is half-integer (integer) the neutral coefficient $\beta$ is always given by $\sqrt{6}$ multiplied with an half-integer (integer). These numbers play a role analogous to the integers $m_1$ and $m_2$ that appear in the original Fradkin-Lopez model [10].

2.3. Wen model

Here, we would like to compare the results obtained for the GFL model with the one deriving from the Wen model. Field theoretical description of the Wen theory was extensively treated in the literature [6,7,9,21] and also presented in textbooks [19]. We assume that the reader is familiar to the basic results of the theory.

Wen showed that is possible to write all the physical operators as $\Psi_l(x) \propto e^{[\varphi_1(x) + \varphi_2(x)]}$ where $l = (l_1, l_2)$ is a vector with $l_1, l_2 \in \mathbb{Z}$ and $\varphi_r(x)$ are the edge bosonic fields with $r = 1, 2$ defined on the symmetric base. It is easy to show that these states correspond to the $m$-agglomerate fields obtained in the GFL model if we use the base of charged and neutral modes $\phi^\pm = \varphi_1 \pm \varphi_2$. In this case the physical states are $\Psi_l(x) = e^{[2\varphi_1(x) + \frac{1}{2}\phi^+(x) \pm \frac{1}{2}\phi^-(x)]}$, where we introduced the two variables: $m = l_1 + l_2$ as the number of qp in the excitation, and the $j = l_1 - l_2$ that plays the role of neutral isospin. These two variables are integers and have the same parity. For odd agglomerates with $m = 2s + 1$ and $j = 2q + 1$ the corresponding operators $(q \in \mathbb{Z})$ are
\[ \Psi^{(2s+1)}(x) = \frac{1}{\sqrt{2\pi a}} e^{i(s+\frac{1}{2})\phi^+(x) + (q + \frac{1}{2})\phi^-(x)}. \quad (18) \]

For even agglomerates $(m = 2s$ and $j = 2q)$ it is
\[ \Psi^{(2s)}(x) = \frac{1}{\sqrt{2\pi a}} e^{i(s+\frac{1}{2})\phi^+(x) + i(q + \frac{1}{2})\phi^-(x)}, \quad (19) \]
where we used the notation of Eq.(16) and Eq.(17).

Note that the above field operators could have been obtained by following similar steps applied in the previous section for GFL model.

From the comparison of Eq.(16) and Eq.(17) with previous formulas we note that the GFL has exactly the same operatorial structure of the Wen model. This indicates that the electrons and all the $m$-agglomerates with odd $m$ have the peculiar parafermational statistics found in the GFL model. Introducing again the two Klein operators $\mathcal{F}^{(\pm)}$ we can restore the correct statistical properties for all the representatives and write the most general $m$-agglomerate as a linear combina-
tion of all the operators with a final result very similar to Eq.(13).

In conclusion, we observe that the difference between the GFL and the Wen model at the level of \( m \)-agglomerate operators is present in the factor \( \sqrt{6} \) in the neutral coefficient. As we will see in the next section, this will play an important role in the evaluation of the scaling properties.

3. Scaling dimension of the agglomerates

In this section we select, among the different excitations, the ones that are dominant in tunneling processes. We introduce the local scaling dimension \( \Delta_m \) of the \( m \)-agglomerate, defined as half of the power-law exponent at long times (\( |\tau| \to \infty \)) in the two-point imaginary time Green function [22] \( \tilde{G}_m(\tau) = \langle T_\tau \tilde{\psi}^{(m)}(0, \tau) \tilde{\psi}^{(m)}(0, 0) \rangle \propto \tau^{-2\Delta_m} \). For the operator in Eq.(5) it is at \( T = 0 \)

\[
\tilde{G}_m(\tau) \propto \left( \frac{1}{1 + \omega_+|\tau|} \right)^{g_+ \nu_+ \alpha_m^2} \left( \frac{1}{1 + \omega_-|\tau|} \right)^{g_- \nu_- \beta_m^2}.
\]

Here, \( \omega_\pm = v_\pm / a \) represent the mode bandwidths setting the high energy cutoff. The scaling dimension is then

\[
\Delta_m = \left[ g_+ \nu_+ (\alpha_m)^2 + g_- \nu_- (\beta_m)^2 \right] / 2.
\]

Note that the presence of the neutral mode contribution \( \beta_m \) in (21) is induced by a finite bandwidth \( \omega_- \) and a finite velocity \( v_- \). This is the main generalization and difference with respect to the Fradkin-Lopez model which assumes topological neutral mode with \( \omega_- = v_- = 0 \).

In order to take into account possible additional interaction effects we considered in Eq. (20) renormalized parameters with \( g_\pm \geq 1 \). They correspond to the renormalization of the dynamical exponents induced by a coupling of the fields with independent dissipative baths [23]. The microscopic models underlying these renormalizations were extensively treated in literature [23,24,25,26] and will not be specifically discussed here. Note that the renormalizations do not affect the statistical properties of the fields, which depend only on the equal-time commutation relations, i.e. the field algebra.

Calculating the scaling dimension in Eq.(21) for the operators in GFL model, described by Eqs.(16)-(17) and in the Wen model with Eqs.(18)-(19), it is easy to map the two models for example choosing the substitution

\[
g_+^W = g_+^{GFL} \quad g_-^W = 3g_-^{GFL},
\]

where \( g_\pm^{GFL} \) and \( g_\pm^W \) are respectively the generalized Fradkin-Lopez and Wen \( g_\pm \) parameters. Having in mind this relation we will now analyze the scaling behavior of the two models.

The most relevant operator with the minimal scaling dimension will dominate the tunneling processes between two edges in the weak-backscattering limit [14]. For a given \( m \)-family, the minimal value \( \Delta_m^{\text{min}} \) corresponds to the minimal value of \( (\beta_m(q))^2 \) in Eq.(15), this is for \( q = 0 \).

In Fradkin-Lopez one has \( \beta_{2+1}(0) = \sqrt{3/2} \) and \( \beta_{2}(0) = 0 \). Agglomerates with \( m > 2 \) are never dominant because the charge coefficient grows with \( m \) and consequently from Eq.(21) we have \( \Delta_{m>2}^{\text{min}} > \Delta_{m=2}^{\text{min}} \). So we need to compare only the minimal scaling dimension of the single qp \( \Delta_{1}^{\text{min}} \) with the two-agglomerate \( \Delta_2^{\text{min}} \). Simple calculations show that the two qp agglomerates are always dominant in the parameter region \( g_+^{GFL} / g_-^{GFL} < 5 \), otherwise the single qp tunneling prevails [14]. Using the mapping in Eq.(22) this means for the Wen model \( g_+^W / g_-^W < 5/3 \). Note that the above conditions are fulfilled in the case of unrenormalized parameters \( (g_+^{GFL} = g_-^{GFL} = 1) \) for both models. As a result the scenario, previously used to explain the experimental anomalies [12,14], can be qualitatively applied to both models. From the above analysis we conclude that the agglomerates are important excitations both for the Fradkin-Lopez and the Wen models at \( \nu = 2/5 \) in the weak back-scattering limit.

To summarize, we demonstrated that the family of operators are essentially the same in the two models. The mapping of the scaling dimension, demonstrating in both models the relevance of agglomerates is in agreement with the recent scenario introduced to explain experimental observations [14]. These result suggest that there is an urgent need of a renewed interest from the experimental community in the quantum Hall effect.

\[\text{In the unrenormalized Wen model the two qp agglomerate dominance at low energy was reported long time ago. [21]}\]
system in the Jain series to better clarify the agglomerate physics.

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