Effects of Non-Standard Interactions for the Energy Spectrum of Secondary Leptons $e^+e^-\rightarrow t\bar{t}$

LONGIN BRZEZIŃSKI, BOHDAN GRZĄDKOWSKI

Institute of Theoretical Physics, Warsaw University
Hoża 69, PL-00-681 Warsaw, POLAND

ZENRŌ HIOKI

Institute of Theoretical Physics, University of Tokushima
Tokushima 770-8502, JAPAN

ABSTRACT

The process of top-quark pair production followed by semileptonic decays at future high-energy $e^+e^-$ linear colliders is investigated as a possible test of physics beyond the Standard Model. Assuming the most general non-standard forms for $\gamma t\bar{t}$, $Zt\bar{t}$ and $Wtb$ couplings, the energy spectrum of the single lepton $\ell^\pm$ and the energy correlation of $\ell^+$ and $\ell^-$ emerging from the process $e^+e^-\rightarrow t\bar{t}\rightarrow \ell^\pm\cdots/\ell^+\ell^-\cdots$ are calculated. Expected precision of the non-standard-parameter determination is estimated adopting the recently-proposed optimal method.
1. Introduction

High-energy $e^+e^-$ linear collider (NLC) can provide a very useful laboratory to study physics of the top quark. In spite of spectacular successes of experimental high-energy physics (e.g. precision tests of the Standard Model (SM) of electroweak interactions), the top-quark couplings have not been tested yet. We should not take it for granted from the beginning that the top-quark interactions obey the scheme provided by the SM. The aim of this paper is to consider possible non-standard effects in the energy spectrum of secondary lepton(s) emerging in the process $e^+e^- \rightarrow t\bar{t} \rightarrow \ell^\pm \cdots /\ell^+\ell^- \cdots$ at NLC. Since the top quark is heavy, $m_t^{exp} = 175.6\pm5.5$ GeV [1], it decays as a single quark before forming bound states. Thanks to this property it is possible to avoid complicated non-perturbative effects brought through fragmentation processes in contrast to a case of lighter quarks.

The leptonic energy spectrum has been studied in the existing literature [2]–[5]. However, none of those articles assumed the most general form for the interactions of $\gamma t\bar{t}$, $Zt\bar{t}$ and $Wtb$. Although our recent papers [4, 5] treated consistently non-standard effects in the production and decay of top quarks at NLC, we focused our discussion on CP-violating couplings only. In this paper we will present a comprehensive analysis taking into account the most general non-standard couplings with both CP-violating and CP-conserving terms.

The paper is organized as follows. In sec. 2 we will briefly describe a formalism for the energy-spectrum calculation. Section 3 will provide the differential cross section for polarized $t\bar{t}$ production assuming the most general $\gamma t\bar{t}$ and $Zt\bar{t}$ interactions. In sec. 4 we will consider the top-quark decay, where again we use the most general form for non-standard $Wtb$ interactions. Section 5 will contain the derivation of the single and the double lepton-energy spectrum. Then, in sec. 6, we will discuss how to measure all the non-standard couplings using the method

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$^{#1}$CP violation in top-quark production at NLC and in its decay has been discussed by very many authors, an incomplete list of references which are not cited in the text could be found in ref. [1].

$^{#2}$We treat all the other couplings as in the SM since it is well known that they are successfully described within the SM.
of optimal observables \[7\]. We summarize our results in sec. 7. There we also compare our results to those in other numerical analysis \[8\]–\[13\]. In the appendix, in order to provide readers some more concrete image, we show contributions of the dimension 6 operators to the form factors in the $\gamma t\bar{t}/Zt\bar{t}$ and $Wtb$ couplings in the framework of effective lagrangian approach \[14\].

2. The lepton-energy spectrum: standard-model results.

In this section we briefly present the formalism which will be used in this paper. For completeness we also show some standard-model results.

We will treat all the fermions except the top quark as massless and adopt the technique developed by Kawasaki, Shirafuji and Tsai \[15\]. This is a useful method to calculate distributions of final particles appearing in a production process of on-shell particles and their subsequent decays. This technique is applicable when the narrow-width approximation

$$\left| \frac{1}{p^2 - m^2 + im\Gamma} \right|^2 \simeq \frac{\pi}{m\Gamma} \delta(p^2 - m^2)$$

can be adopted for the decaying intermediate particles. In fact, this is very well satisfied for both $t$ and $W$ since $\Gamma_t \simeq 175(m_t/M_W)^3$ MeV $\ll m_t$ and $\Gamma_W = 2.07 \pm 0.06$ GeV \[16\] $\ll M_W$.

Adopting this method, one can derive the following formula for the inclusive distribution of the single-lepton $\ell^+$ in the reaction $e^+e^- \to t\bar{t}$ \[3\]:

$$\frac{d^3\sigma}{d^3p_\ell/(2p_\ell^0)}(e^+e^- \to \ell^+ + \cdots) = 4 \int d\Omega_t \frac{d\sigma}{d\Omega_t} (n, 0) \frac{1}{\Gamma_t} \frac{d^3\Gamma_t}{d^3p_\ell/(2p_\ell^0)} (t \to b\ell^+\nu),$$

(1)

where $\Gamma_\ell$ and $\Gamma_t$ are the leptonic and total widths of unpolarized top respectively, and $d\sigma(n, 0)/d\Omega_t$ is obtained from the angular distribution of $t\bar{t}$ with spins $s_+$ and $s_-$ in $e^+e^- \to t\bar{t}$, $d\sigma(s_+, s_-)/d\Omega_t$, by the following replacement:

$$s_+^\mu \to n^\mu = \left( g^{\mu\nu} - \frac{p_\ell^\mu p_\ell^\nu}{m_t^2} \right) \frac{m_t}{p_\ell} p_\ell \nu \quad \text{and} \quad s_- \to 0.$$
(Exchanging the roles of \(s_+\) and \(s_-\) and reversing the sign of \(n^\mu\), we get the distribution of \(\ell^-\).)

Following ref. [3], let us introduce the rescaled lepton-energy, \(x\), by
\[
x \equiv \frac{2E_\ell}{m_t} \left(\frac{1 - \beta}{1 + \beta}\right)^{1/2},
\]
where \(E_\ell\) is the energy of \(\ell\) in \(e^+e^-\) c.m. frame and \(\beta = \sqrt{1 - 4m_t^2/s}\) (\(s \equiv (p_{e+} + p_{e-})^2\)). We also define parameters \(D_V^{(0)}, D_A^{(0)}\) and \(D_{VA}^{(0)}\) as
\[
D_V^{(0)} = (v_e v_t d - \frac{2}{3})^2 + (v_t d)^2, \\
D_A^{(0)} = (v_e d)^2 + d^2, \\
D_{VA}^{(0)} = v_e d (v_e v_t d - \frac{2}{3}) + v_t d^2,
\]
by using the standard-model neutral-current couplings for \(e\) and \(t\): \(v_e = -1 + 4\sin^2 \theta_W\) and \(v_t = 1 - (8/3)\sin^2 \theta_W\), and a \(Z\)-propagator factor
\[
d \equiv \frac{s}{s - M_Z^2} \frac{1}{16 \sin^2 \theta_W \cos^2 \theta_W}.
\]

Then, the \(x\) spectrum is given in terms of these quantities by
\[
\frac{1}{B_\ell \sigma_{e^-e^+\rightarrow t\bar{t}}} \frac{d\sigma^\pm}{dx} = \frac{1}{B_\ell \sigma_{e^-e^+\rightarrow t\bar{t}}} \frac{d\sigma}{dx}(e^+e^- \rightarrow \ell^\pm + \cdots) = f(x) + \eta g(x).
\]
Here \(\sigma_{e^-e^+\rightarrow t\bar{t}} \equiv \sigma_{\text{tot}}(e^+e^- \rightarrow t\bar{t})\), \(B_\ell\) is the branching ratio for \(t \rightarrow \ell + \cdots\) (\(\simeq 2/9\) for \(\ell = e, \mu\)). \(f(x)\) and \(g(x)\) are functions introduced in ref. [3]:
\[
f(x) = \frac{3}{W} \frac{1 + \beta}{\beta} \int d\omega \omega, \quad g(x) = \frac{3}{W} \frac{1 + \beta}{\beta} \int d\omega \omega \left[1 - \frac{x(1 + \beta)}{1 - \omega}\right],
\]
where
\[
W \equiv (1 - r)^2(1 + 2r), \quad r \equiv (M_W/m_t)^2, \quad \omega \equiv (p_t - p_e)^2/m_t^2.
\]
\(f(x)\) and \(g(x)\) satisfy the following normalization conditions:
\[
\int f(x)dx = 1 \quad \text{and} \quad \int g(x)dx = 0.
\]
The explicit form of \(f(x)\) and \(g(x)\) could be found in refs. [3] and [4]. \(\eta\) is defined as
\[
\eta \equiv 4 a_{VA} D_{VA}^{(0)},
\]
where $a_{VA} \equiv 1/[(3 - \beta^2)D_V^{(0)} + 2\beta^2D_A^{(0)}]$. Applying the same technique, we get the following energy correlation of $\ell^+$ and $\ell^-$:

\[
\frac{1}{B^2_{\ell^+}\sigma_{e\bar{e}\to t\bar{t}}^\ell} \frac{d^2\sigma}{dx\,d\bar{x}} = S_0(x, \bar{x}),
\]

where $x$ and $\bar{x}$ are the rescaled energies of $\ell^+$ and $\ell^-$ respectively, and

\[
S_0(x, \bar{x}) = f(x)f(\bar{x}) + \eta [f(x)g(\bar{x}) + g(x)f(\bar{x})] + \eta'g(x)g(\bar{x})
\]

with $\eta'$ being defined as

\[
\eta' \equiv \beta^{-2}a_{VA}[ (1 + \beta^2)D_V^{(0)} + 2\beta^2D_A^{(0)} ].
\]

Clearly, the $(x, \bar{x})$ distribution is symmetric in $x$ and $\bar{x}$, which is a sign of the standard-model $CP$ symmetry.

In the following, we use $M_W = 80.43$ GeV, $M_Z = 91.1863$ GeV, $m_t = 175.6$ GeV, $\sin^2\theta_W = 0.2315$ and $\sqrt{s} = 500$ GeV. For these inputs, we have

\[
\eta = 0.2074, \quad \eta' = 1.2720 \quad \text{and} \quad a_{VA} = 0.7545.
\]

### 3. Non-standard effects in the production of polarized $t\bar{t}$

We will assume that all non-standard effects in the production of $t\bar{t}$ can be represented by the following corrections to the photon and $Z$-boson vertices contributing to the s-channel diagrams:

\[
\Gamma_{\gamma t\bar{t}}^\mu = \frac{g}{2} \bar{u}(p_t) \left[ \gamma^\mu \{ A_\gamma + \delta A_v - (B_v + \delta B_v)\gamma_5 \} + \frac{(p_t - p_{\bar{t}})^\mu}{2m_t} (\delta C_v - \delta D_v\gamma_5) \right] v(p_{\bar{t}}),
\]

where $g$ denotes the $SU(2)$ gauge coupling constant, $v = \gamma, Z$, and

\[
A_\gamma = \frac{4}{3} \sin\theta_W, \quad B_\gamma = 0, \quad A_Z = \frac{v_t}{2\cos\theta_W}, \quad B_Z = \frac{1}{2\cos\theta_W}.
\]

In addition, contributions to the vertex proportional to $(p_t + p_{\bar{t}})^\mu$ are also allowed, but their effects vanish in the limit of zero electron mass.\(^{24}\) Among the above new

\(^{23}\)The SM requires at least two-loops to generate $CP$-violating energy distributions.

\(^{24}\)These contributions are essential for the $U(1)_{EM}$ gauge invariance. We discuss this point briefly in the appendix.
form factors, $\delta A_{\gamma,Z}$, $\delta B_{\gamma,Z}$, $\delta C_{\gamma,Z}$ and $\delta D_{\gamma,Z}$ are parameterizing $CP$-conserving and
$CP$-violating non-standard interactions, respectively. In the appendix, we show
how they receive contributions from effective operators of dimension 6.

On the other hand, interactions of initial $e^+e^-$ have been assumed untouched
by non-standard interactions:

- $\gamma e^+e^-$ vertex

$$\Gamma_{\gamma e^+e^-}^\mu = -e \bar{\nu}(p_{e^+}) \gamma^\mu u(p_{e^-}),$$

- $Ze^+e^-$ vertex

$$\Gamma_{Ze^+e^-}^\mu = \frac{g}{4\cos\theta_W} \bar{\nu}(p_{e^+}) \gamma^\mu (v_e + \gamma_5) u(p_{e^-}).$$

A tedious but straightforward calculation leads to the following formula for
the angular distribution of polarized top-quark pair in presence of the above non-
standard interactions:

$$\frac{d\sigma}{d\Omega}(e^+e^- \rightarrow t(s_+\bar{t}(s_-))$$

$$= \frac{3\beta\alpha^2}{16s^3} \left[ D_V \left\{ 4m_t^2 s + (lq)^2 \right\} (1 - s_+ s_-) + s^2 (1 + s_+ s_-) \right.$$  
$$+ 2s(ls_+ ls_- - Ps_+ Ps_-) + 2lq(ls_+ Ps_- - ls_- Ps_+) \right]$$  
$$+ D_A \left[ (lq)^2 (1 + s_+ s_-) - (4m_t^2 s - s^2) (1 - s_+ s_-) \right.$$  
$$- 2(s - 4m_t^2)(ls_+ ls_- - Ps_+ Ps_-) - 2lq(ls_+ Ps_- - ls_- Ps_+) \right]$$  
$$- 4 \text{Re}(D_{VA}) \left[ m_t \left( s(Ps_+ - Ps_-) + lq(ls_+ + ls_-) \right) \right.$$  
$$+ 2 \text{Im}(D_{VA}) \left[ lq \epsilon(s_+, s_-, q, l) + ls_- \epsilon(s_+, P, q, l) + ls_+ \epsilon(s_-, P, q, l) \right] \right.$$  
$$+ 4 E_V m_t ls_+ ls_- + 4 E_A m_t lq(Ps_+ - Ps_-) \right.$$  
$$+ 4 \text{Re}(E_{VA}) \left[ 2m_t^2 (ls_+ Ps_- - ls_- Ps_+) - lq s \right] \right.$$  
$$+ 4 \text{Im}(E_{VA}) \left[ m_t \epsilon(s_+, P, q, l) + \epsilon(s_-, P, q, l) \right]$$  
$$- \text{Re}(F_1) \frac{1}{m_t} \left[ lq s(ls_+ - ls_-) - \{(lq)^2 + 4m_t^2 s\}(Ps_+ + Ps_-) \right]$$  
$$+ 2 \text{Im}(F_1) \left[ s \epsilon(s_+, s_-, P, q) + lq \epsilon(s_+, s_-, P, l) \right]$$  
$$+ 2 \text{Re}(F_2) s(Ps_+ ls_- + Ps_- ls_+).$$
\[- \text{Im}(F_2) \frac{s}{mt} [\epsilon(s_+, P, q, l) - \epsilon(s_-, P, q, l)] \]
\[-2 \text{Re}(F_3) \frac{lq}{mt} (Ps_+ ls_- + Ps_- ls_+) \]
\[+ \text{Im}(F_3) \frac{lq}{mt} [\epsilon(s_+, P, q, l) - \epsilon(s_-, P, q, l)] \]
\[- \text{Re}(F_4) \frac{s}{mt} [lq (Ps_+ + P_{s_-}) - (s - 4m_t^2)(ls_+ - ls_-)] \]
\[-2 \text{Im}(F_4) [Ps_+ \epsilon(s_-, P, q, l) + Ps_- \epsilon(s_+, P, q, l)] \]
\[+ 2 \text{Re}(G_1) [\{4m_t^2 s + (lq)^2 - s^2\}(1 - s_+s_-) - 2sPs_+ Ps_- \]
\[+ lq(ls_+ Ps_- - ls_- Ps_+) \]
\[- \text{Im}(G_1) \frac{lq}{mt} [\epsilon(s_+, P, q, l) + \epsilon(s_-, P, q, l)] \]
\[- \text{Re}(G_2) \frac{s}{mt} [(s - 4m_t^2)(ls_+ + ls_-) - lq (Ps_+ - Ps_-)] \]
\[-2 \text{Im}(G_2) [Ps_+ \epsilon(s_-, P, q, l) - Ps_- \epsilon(s_+, P, q, l)] \]
\[- \text{Re}(G_3) \frac{lq}{mt} [lq (Ps_+ - Ps_-) - (s - 4m_t^2)(ls_+ + ls_-)] \]
\[-2 \text{Im}(G_3) lq \epsilon(s_+, s_-, q, l) \]
\[+ 2 \text{Re}(G_4) [(s - 4m_t^2)(Ps_+ ls_- - P_{s_-} ls_+) + 2lq P_{s_+} P_{s_-} \]
\[+ \text{Im}(G_4) \frac{1}{mt} (s - 4m_t^2)[\epsilon(s_+, P, q, l) + \epsilon(s_-, P, q, l)] \]
\]

where
\[P \equiv p_e + p_{\bar{e}} (= p_t + p_{\bar{t}}), \quad l \equiv p_e - p_{\bar{e}}, \quad q \equiv p_t - p_{\bar{t}}, \]
the symbol \(\epsilon(a, b, c, d)\) means \(\epsilon_{\mu
u\rho\sigma}a^\mu b^\nu c^\rho d^\sigma\) for \(\epsilon_{0123} = +1,\)

\[D_V \equiv C [A_\gamma^2 - 2A_\gamma A_Z v_d d' + A_Z^2 (1 + v_e^2)d^2 + 2(A_\gamma - A_Z v_e d') \text{Re}(\delta A_\gamma)] \]
\[-2\{A_\gamma v_e d' - A_Z (1 + v_e^2)d^2\} \text{Re}(\delta A_Z)], \]
\[D_A \equiv C [B_Z^2 (1 + v_e^2)d^2 - 2B_Z v_e d' \text{Re}(\delta B_\gamma) + 2B_Z (1 + v_e^2)d^2 \text{Re}(\delta B_Z)], \]
\[D_{VA} \equiv C [-A_\gamma B_Z v_e d' + A_Z B_Z (1 + v_e^2)d^2 - B_Z v_e d'(\delta A_\gamma)^* \]
\[+(A_\gamma - v_e d' A_Z) \delta B_\gamma + B_Z (1 + v_e^2)d^2(\delta A_Z)^* \]
\[-\{A_\gamma v_e d' - A_Z (1 + v_e^2)d^2\} \delta B_Z], \]
\[E_V \equiv 2C [A_\gamma A_Z d' - A_Z^2 v_e d'^2 + A_Z d' \text{Re}(\delta A_\gamma) + (A_\gamma d' - 2A_Z v_e d'^2) \text{Re}(\delta A_Z)], \]
\[E_A \equiv 2C [-B_Z^2 v_e d'^2 + B_Z d' \text{Re}(\delta B_\gamma) - 2B_Z v_e d'^2 \text{Re}(\delta B_Z)], \]
\[E_{VA} \equiv C [A_\gamma B_Z d' - 2A_Z B_Z v_e d'^2 + B_Z d'(\delta A_\gamma)^* + A_Z d' \delta B_\gamma] \]
-2B_Z v_e d^2 (\delta A_Z)^* + (A_\gamma d' - 2A_Z v_e d^2) \delta B_Z ,

F_1 \equiv C \left[ -(A_\gamma - A_Z v_e d') \delta D_\gamma + \{A_\gamma v_e d' - A_Z (1 + v_e^2) d^2 \} \delta D_Z \right],

F_2 \equiv C \left[ -A_Z d' \delta D_\gamma - (A_\gamma d' - 2A_Z v_e d^2) \delta D_Z \right],

F_3 \equiv C \left[ B_Z v_e d' \delta D_\gamma - B_Z (1 + v_e^2) d^2 \delta D_Z \right],

F_4 \equiv C \left[ -B_Z d' \delta D_\gamma + 2B_Z v_e d^2 \delta D_Z \right],

G_1 \equiv C \left[ (A_\gamma - A_Z v_e d') \delta C_\gamma - \{A_\gamma v_e d' - A_Z (1 + v_e^2) d^2 \} \delta C_Z \right],

G_2 \equiv C \left[ A_Z d' \delta C_\gamma + (A_\gamma d' - 2A_Z v_e d^2) \delta C_Z \right],

G_3 \equiv C \left[ -B_Z v_e d' \delta C_\gamma + B_Z (1 + v_e^2) d^2 \delta C_Z \right],

G_4 \equiv C \left[ B_Z d' \delta C_\gamma - 2B_Z v_e d^2 \delta C_Z \right],

\text{and}

C \equiv 1/(4 \sin^2 \theta_W), \quad d' \equiv d \cdot 4 \sin \theta_W \cos \theta_W .

D^{(0)}_{V,A,V A} \text{ used in §2 (eq.3) are the standard-model parts of the above } D_{V,A,V A} . \ D_{V,A,V A} \text{ and } E_{V,A,V A} \text{ are both defined the same way as in ref.[3], however } F_i \ (i = 1 \sim 4) \text{ differ by a factor } -2i m_t \ (\text{i.e., our } F_i = -2i m_t \times \text{their } F_i) . \text{ Finally it should be emphasized that only linear terms in the non-standard couplings have been kept.}

4. Non-standard effects in } \text{ and } \bar{t} \text{ decays}

We shall next consider non-standard effects in } \text{ and } \bar{t} \text{ decays. We will adopt the following parameterization of the } Wtb \text{ vertex suitable for the } \to W^+ b \text{ and }
\bar{t} \to W^- \bar{b} \text{ decays for the on-shell } W :

\Gamma_{Wtb}^\mu = -\frac{g}{\sqrt{2}} V_{tb} \bar{u}(p_b) \left[ \gamma^\mu (f_1^L P_L + f_1^R P_R) - i\sigma^{\mu\nu} k_\nu (f_2^L P_L + f_2^R P_R) \right] u(p_t),

(12)

\bar{\Gamma}_{Wtb}^\mu = -\frac{g}{\sqrt{2}} V_{tb}^* \bar{v}(p_t) \left[ \gamma^\mu (\bar{f}_1^L P_L + \bar{f}_1^R P_R) - i\sigma^{\mu\nu} k_\nu (\bar{f}_2^L P_L + \bar{f}_2^R P_R) \right] v(p_b),

(13)

\text{where } P_{L/R} = (1 \mp \gamma_5)/2, V_{tb} \text{ is the } (tb) \text{ element of the Kobayashi-Maskawa matrix and } k \text{ is the momentum of } W . \text{ Because } W \text{ is on shell}^3 \text{ the two additional form factors do not contribute. We show in the appendix how dimension 6 operators affect these form factors.}

^n3\text{Note that we use the narrow-width approximation also for the } W \text{ propagator as mentioned in §2.
It is worth to know that the form factors for top and anti-top satisfy the following relations [17]:

\[ f_{1}^{L,R} = \pm \bar{f}_{1}^{L,R}, \quad f_{2}^{L,R} = \pm \bar{f}_{2}^{R,L}, \]  

(14)

where upper (lower) signs are those for CP-conserving (-violating) contributions.

\[ W \ell \nu \] couplings are treated within the SM as mentioned before:

\[ \Gamma_{W \ell \nu}^{\mu} = -\frac{g}{\sqrt{2}} \bar{u}(p_{\nu}) \gamma^{\mu}(1 - \gamma_{5}) v(p_{\ell}^{+}), \]  

(15)

\[ \bar{\Gamma}_{W \ell \nu}^{\mu} = -\frac{g}{\sqrt{2}} \bar{u}(p_{\ell}^{-}) \gamma^{\mu}(1 - \gamma_{5}) v(p_{\nu}). \]  

(16)

Assuming that \((-1) f_{1}^{L,-1}, (-1) f_{1}^{R}, (-1) f_{2}^{L}\) and \((-1) f_{2}^{R}\) are small and keeping only their linear terms, we obtain for the double differential spectrum in \(x\) and \(\omega\) the following result:

\[ \overline{1} \frac{d^{2} \Gamma_{t}}{\Gamma_{t} dx d\omega} (t \rightarrow b\ell^{+}\nu) = \frac{1 + \beta}{\beta} 3 B_{t} \frac{1}{W_{\omega}} \left[ 1 + 2 \text{Re}(f_{2}^{R}) \sqrt{r} \left( \frac{1}{1 - \omega} - \frac{3}{1 + 2r} \right) \right]. \]  

(17)

An analogous formula for \(\bar{t} \rightarrow \bar{b}\ell^{-}\bar{\nu}\) holds with \(f_{2}^{R}\) replaced by \(\bar{f}_{2}^{L}\):

\[ \frac{1}{\overline{\Gamma}_{\bar{t}} dx d\omega} (\bar{t} \rightarrow \bar{b}\ell^{-}\bar{\nu}) = \frac{1 + \beta}{\beta} 3 B_{\bar{t}} \frac{1}{W_{\omega}} \left[ 1 + 2 \text{Re}(\bar{f}_{2}^{L}) \sqrt{r} \left( \frac{1}{1 - \omega} - \frac{3}{1 + 2r} \right) \right]. \]  

(18)

Here let us give some comments on the CP-violating parameters. Combining the above results with eq.(14), we find that CP-violating quantities in the decay processes are proportional to \(\text{Re}(f_{2}^{R} - \bar{f}_{2}^{L})\) within our approximation. As will be found in the appendix (eqs.(49) and (52)), \(\text{Re}(f_{2}^{R} - \bar{f}_{2}^{L})\) becomes zero in the effective lagrangian approach where only the SM particles are taken into account. Indeed CPT symmetry also requires this. That is, CPT tells us that order by order in perturbation expansion \(\Gamma(t \rightarrow \text{all}) = \Gamma(\bar{t} \rightarrow \text{all})\). Then, if we assume only the standard top-quark decay channel \(t \rightarrow bW^{+}\) at the lowest order, one is led to \(\text{Re}(f_{2}^{R} - \bar{f}_{2}^{L}) = 0\) through the above (17) and (18). Non-zero \(\text{Re}(f_{2}^{R} - \bar{f}_{2}^{L})\) may emerge at the one-loop level, e.g., in SUSY with light neutralinos and stops such that appropriate absorptive parts of vertex corrections appear. This means observing non-zero \(\text{Re}(f_{2}^{R} - \bar{f}_{2}^{L})\) would be an evidence of not only some new interactions but also some new relatively light particles.

\(^{26}\)Assuming CP-conserving Kobayashi-Maskawa matrix.
5. The lepton-energy spectrum: non-standard results.

Combining the results of the previous sections, we obtain the single lepton-energy spectrum for $e^+e^- \rightarrow \ell^\pm + \cdots$

$$\frac{1}{B_{\ell^\pm}} \frac{d\sigma_{e^+e^- \rightarrow \ell^\pm}}{dx} = \sum_{i=1}^{3} c^\pm_i f_i(x),$$

where $\pm$ corresponds to $\ell^\pm$,

$$c^\pm_1 = 1,$$

$$c^\pm_2 = \lambda_1 \text{Re}(G_1) + \lambda_2 \text{Re}(\delta A_\gamma) + \lambda_3 \text{Re}(\delta B_\gamma) + \lambda_4 \text{Re}(\delta A_Z) + \lambda_5 \text{Re}(\delta B_Z) \mp \xi,$$

$$c^+_3 = \text{Re}(f_2^R), \quad c^-_3 = \text{Re}(f_2^L),$$

and

$$f_1(x) = f(x) + \eta g(x), \quad f_2(x) = g(x), \quad f_3(x) = \delta f(x) + \eta \delta g(x).$$

$\xi$ is the CP-violating parameter\(^\#\) in the production process used in refs. [3, 4]

$$\xi \equiv 2 \text{Re}(F_1) a_{VA},$$

while $\lambda_i$ are defined as

$$\lambda_1 \equiv 2 \beta^2 \eta a_{VA},$$

$$\lambda_2 \equiv -2a_{VA} \left[ \eta(3 - \beta^2)(A_\gamma - A_Zv_e d') + 2B_Zv_e d' \right],$$

$$\lambda_3 \equiv 4a_{VA} \left[ A_\gamma - v_e d'(A_Z - \eta \beta^2 B_Z) \right],$$

$$\lambda_4 \equiv 2a_{VA} \left[ \eta(3 - \beta^2) \{ A_\gamma v_e d' - A_Z(1 + v_e^2)d'^2 \} + 2B_Z(1 + v_e^2)d'^2 \right],$$

$$\lambda_5 \equiv -4a_{VA} \left[ A_\gamma v_e d' - (1 + v_e^2)d'(A_Z - \eta \beta^2 B_Z) \right].$$

(\(a_{VA}\) was defined just before eq.(6) in §2.) For the present input parameters, they are

$$\lambda_1 = 0.1586, \quad \lambda_2 = -0.4303, \quad \lambda_3 = 1.9578, \quad \lambda_4 = 0.5635, \quad \lambda_5 = 0.2684.$$

\(^\#\)Similarly to $\text{Re}(f_2^R - \bar{f}_2^L)$, it will be seen in the appendix that within the effective lagrangian scenario $\xi = 0$ (eqs. (39) and (40)). Non-zero $\xi$ would also be generated through absorptive parts of loop diagrams involving relatively light non-standard particles.
Figure 1: The functions $f_i(x)$ defined in eq. (20).

$F_1$ and $G_1$ in $c_2^\pm$ are numerically related to the non-SM parameters as

\[
\begin{pmatrix} F_1 \\ G_1 \end{pmatrix} = 0.7035 \begin{pmatrix} -\delta D_\gamma \\ \delta C_\gamma \end{pmatrix} + 0.1205 \begin{pmatrix} -\delta D_Z \\ \delta C_Z \end{pmatrix}.
\]

$\delta f(x)$ and $\delta g(x)$ are the functions derived in our previous work, and their explicit form could be found in the appendix of ref. [4]. The functions $f_i(x)$ are shown in fig. [4].

The double lepton-energy spectrum is given by the following formula:

\[
\frac{1}{B^2 \epsilon_{ee\rightarrow tt}} \frac{d^2 \sigma}{dx dx} = \sum_{i=1}^{6} c_i f_i(x, \bar{x}),
\]

where

\[
c_1 = 1, \quad c_2 = \xi, \quad c_3 = \frac{1}{2} \text{Re}(f_2^R - f_2^L),
\]

\[
c_4 = \lambda_1 \text{Re}(G_1) + \lambda_2 \text{Re}(\delta A_\gamma) + \lambda_3 \text{Re}(\delta B_\gamma) + \lambda_4 \text{Re}(\delta A_Z) + \lambda_5 \text{Re}(\delta B_Z),
\]
\[ c_5 = \lambda_1 \text{Re}(G_1) + \lambda_2 \text{Re}(\delta A_\gamma) + \lambda_3 \text{Re}(\delta B_\gamma) + \lambda_4 \text{Re}(\delta A_Z) + \lambda_5 \text{Re}(\delta B_Z), \]

\[ c_6 = \frac{1}{2} \text{Re}(f_2^R + f_2^L), \]

and

\[ f_1(x, \bar{x}) = f(x)f(\bar{x}) + \eta \left[ f(x)g(\bar{x}) + g(x)f(\bar{x}) \right] + \eta'g(x)g(\bar{x}), \]
\[ f_2(x, \bar{x}) = f(x)g(\bar{x}) - g(x)f(\bar{x}), \]
\[ f_3(x, \bar{x}) = \delta f(x)f(\bar{x}) - f(x)\delta f(\bar{x}) \]
\[ + \eta \left[ \delta f(x)g(\bar{x}) - f(x)\delta g(\bar{x}) + \delta g(x)f(\bar{x}) - g(x)\delta f(\bar{x}) \right] \]
\[ + \eta'\left[ \delta g(x)g(\bar{x}) - g(x)\delta g(\bar{x}) \right], \]
\[ f_4(x, \bar{x}) = g(x)g(\bar{x}), \]
\[ f_5(x, \bar{x}) = f(x)g(\bar{x}) + g(x)f(\bar{x}), \]
\[ f_6(x, \bar{x}) = \delta f(x)f(\bar{x}) + f(x)\delta f(\bar{x}) \]
\[ + \eta \left[ \delta f(x)g(\bar{x}) + f(x)\delta g(\bar{x}) + \delta g(x)f(\bar{x}) + g(x)\delta f(\bar{x}) \right] \]
\[ + \eta'\left[ \delta g(x)g(\bar{x}) + g(x)\delta g(\bar{x}) \right]. \]  

(22)

\( \lambda'_i \) are defined similarly to \( \lambda_i \):

\[ \lambda'_1 \equiv 2(1 + \beta^2 \eta') a_{VA}, \]
\[ \lambda'_2 \equiv 2\beta^{-2}[1 + \beta^2 - \eta'\beta^2(3 - \beta^2)] a_{VA} (A_\gamma - A_Z v_e d''), \]
\[ \lambda'_3 \equiv -4(1 - \eta'\beta^2) a_{VA} B_Z v_e d'', \]
\[ \lambda'_4 \equiv 2\beta^{-2}[1 + \beta^2 - \eta'\beta^2(3 - \beta^2)] a_{VA} [-A_\gamma v_e d'' + A_Z(1 + v_e^2)d'^2], \]
\[ \lambda'_5 \equiv 4(1 - \eta'\beta^2) a_{VA} B_Z(1 + v_e^2)d'^2, \]

and their values are

\[ \lambda'_1 = 2.4814, \quad \lambda'_2 = -0.1943, \quad \lambda'_3 = 0.0278, \quad \lambda'_4 = -0.0333, \quad \lambda'_5 = 0.2313. \]

The functions \( f_i(x, \bar{x}) \) are plotted in fig. 2.
6. The optimal observables

Let us briefly summarize the main points of the optimal-observables technique [7]. Suppose we have a cross section

\[ \frac{d\sigma}{d\phi} (\equiv \Sigma(\phi)) = \sum_i c_i f_i(\phi) \]

where \( f_i(\phi) \) are known functions of the location in final-state phase space \( \phi \) and \( c_i \)'s are model-dependent coefficients. The goal would be to determine \( c_i \)'s. It can be
done by using appropriate weighting functions \( w_i(\phi) \) such that \( \int w_i(\phi) \Sigma(\phi) d\phi = c_i \).

Generally, different choices for \( w_i(\phi) \) are possible, but there is a unique choice such that the resultant statistical error is minimized. Such functions are given by

\[
  w_i(\phi) = \sum_j X_{ij} f_j(\phi) / \Sigma(\phi),
\]

(23)

where \( X_{ij} \) is the inverse matrix of \( M_{ij} \) which is defined as

\[
  M_{ij} \equiv \int \frac{f_i(\phi)f_j(\phi)}{\Sigma(\phi)} d\phi.
\]

(24)

When we take these weighting functions, the statistical uncertainty of \( c_i \) becomes

\[
  \Delta c_i = \sqrt{X_{ii} \sigma_T / N},
\]

(25)

where \( \sigma_T \equiv \int (d\sigma/d\phi) d\phi \) and \( N \) is the total number of events.

**Single distribution**

Let us consider first the single-lepton inclusive process:

\[
  e^+e^- \rightarrow t\bar{t} \rightarrow \ell^+ \cdots,
\]

(26)

where \( l = e \) or \( \mu \) and dots stand either for jets or leptons. The \( M \) and \( X \) matrices obtained from eq.\((19)\) are the same as presented in ref.\([5]\) (the numerical values are a bit different since the input data are not the same):

\[
  (M_{ij}) = \begin{bmatrix}
  1 & 0 & 0 \\
  0 & 0.099 & 0.15 \\
  0 & 0.15 & 0.24
  \end{bmatrix},
\]

\[
  (X_{ij}) = \begin{bmatrix}
  1 & 0 & 0 \\
  0 & 128.79 & -79.30 \\
  0 & -79.30 & 52.99
  \end{bmatrix}.
\]

Since \( M \) and \( X \) are the same for \( \ell^+ \) and \( \ell^- \) we obtain the following statistical errors:

\[
  \Delta c_2^\pm = 11.35/\sqrt{N_\ell}, \quad \Delta c_3^\pm = 7.28/\sqrt{N_\ell},
\]

(27)

where \( N_\ell \) is the expected number of detected single-lepton events. This quantity is obtained from the integrated luminosity \( L \) and lepton-tagging efficiency \( \epsilon_\ell \) as
$N_{\ell} = B_\ell \sigma_{ee \to \ell\ell} L_{\text{eff}}^{\ell} = 127.9 L_{\text{eff}}^{\ell}$, where $L_{\text{eff}}^{\ell} \equiv \epsilon_\ell L$ (in fb$^{-1}$ units) and we estimated $\sigma_{ee \to \ell\ell}$ to be 581.5 fb using $\alpha(s) = 1/126$. Since $\ell^+$ and $\ell^-$ events are statistically independent, we can combine them when necessary. For example, we have $\Delta \xi = 8.03/\sqrt{N_{\ell}}$ since $\xi = (c_2^- - c_2^+)/2$. Moreover, if we were not interested in differences ($CP$-violation) between the $\ell^+$ and $\ell^-$ distributions but just concentrated on determination of form factors (the same for leptons and antileptons) we could allow for both signs of leptons at the same time, which would increase the branching-ratio-suppression factor from $18/81$ to $36/81$ and increase expected statistics.

3$\sigma$ effects will be observable at $\sqrt{s} = 500$ GeV if the following relations are satisfied:

$$|c_2^\pm| \geq 3.01 \text{ fb}^{-1/2}/\sqrt{L_{\text{eff}}^{\ell}}, \quad |c_3^\pm| \geq 1.93 \text{ fb}^{-1/2}/\sqrt{L_{\text{eff}}^{\ell}}.$$  \hfill (28)

In table 1 we show the square root of the effective luminosity $\sqrt{L_{\text{eff}}^{\ell}}$ obtained for some typical representative values of $L$ and $\epsilon_\ell$ expected at planned $\sqrt{s} = 500$ GeV $e^+e^-$ linear colliders [18, 19].

| $\epsilon_\ell$ | $L[\text{ fb}^{-1}]$ |
|---|---|---|---|---|
| 20 | 40 | 100 | 200 |
| 0.6 | 3.46 | 4.90 | 7.75 | 11.0 |
| 0.8 | 4.00 | 5.66 | 8.94 | 12.6 |
| 1.0 | 4.47 | 6.32 | 10.0 | 14.1 |

Table 1: Square root of the single-lepton effective luminosity: $\sqrt{L_{\text{eff}}^{\ell}}$.

These are the most precise results which we can draw from the single distribution [19] alone. In order to get a higher precision, therefore, it is important to combine it with other independent information. For example, if we knew $c_3^\pm$ from

\footnote{Here, the double-leptonic-decay modes were also included. In order to remove them one should replace the branching-ratio-suppression factors 18/81 and 36/81 by 12/81 and 24/81 respectively.}
some other data and we only had to determine \( c_2^\pm \) here, then the corresponding matrices would become

\[
(M_{ij}) = \begin{bmatrix} 1 & 0 \\ 0 & 0.099 \end{bmatrix}, \quad (X_{ij}) = \begin{bmatrix} 1 & 0 \\ 0 & 10.10 \end{bmatrix},
\]

which leads to

\[
\Delta c_2^\pm = 3.18/\sqrt{N_\ell}.
\]

(29)

Similarly, if we already had \( c_3^\pm \), then

\[
\Delta c_3^\pm = 2.04/\sqrt{N_\ell}.
\]

(30)

Discussion of the above results is left to the next section.

**Double distribution**

The double-lepton spectrum eq. (21) leads to the following \( M \) and \( X \):

\[
(M_{ij}) = \begin{bmatrix}
1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0.23 & -0.34 & 0 & 0 & 0 \\
0 & -0.34 & 0.54 & 0 & 0 & 0 \\
0 & 0 & 0 & 0.011 & -0.0038 & -0.0029 \\
0 & 0 & 0 & -0.0038 & 0.18 & 0.26 \\
0 & 0 & 0 & -0.0029 & 0.26 & 0.44
\end{bmatrix},
\]

\[
(X_{ij}) = \begin{bmatrix}
1 & 0 & 0 & 0 & 0 & 0 \\
0 & 42.27 & 26.09 & 0 & 0 & 0 \\
0 & 26.09 & 17.94 & 0 & 0 & 0 \\
0 & 0 & 0 & 96.88 & 9.16 & -4.88 \\
0 & 0 & 0 & 9.16 & 47.42 & -28.55 \\
0 & 0 & 0 & -4.88 & -28.55 & 19.49
\end{bmatrix}.
\]

The statistical errors for the determination of the coefficients \( c_i \) calculated according to the formula (25) for the double-lepton spectrum are shown in table 2. Table 3 provides the square root of the effective luminosity \( \sqrt{L_{\ell\ell}^{\text{ef}}} \) similarly to table 1, where \( L_{\ell\ell}^{\text{ef}} \equiv \epsilon_\ell^2 L \).
Table 2: Standard deviations $\Delta c_i$ expected for measurements of $c_i$ defined for the double energy spectrum by eq.(21) could be read from the second row, where $N_{\ell\ell} = B^2_\ell \sigma_{e\bar{e} \rightarrow \ell\bar{\ell}}^{} L_{\ell\ell}^{\text{eff}} = 28.14 L_{\ell\ell}^{\text{eff}}$ denotes the expected number of double-lepton events. $L_{\ell\ell}^{\text{eff}} \equiv \epsilon_\ell^2 L$ (in fb$^{-1}$ units) stands for the effective integrated luminosity. The last row shows the minimal value for $|c_i|\sqrt{L_{\ell\ell}^{\text{eff}}}$ (in fb$^{-1/2}$ units) necessary for an observation of 3$\sigma$ effects at $\sqrt{s} = 500$ GeV.

| $i$ | 2   | 3   | 4   | 5   | 6   |
|-----|-----|-----|-----|-----|-----|
| $\Delta c_i \sqrt{N_{\ell\ell}}$ | 6.50 | 4.24 | 9.84 | 6.89 | 4.41 |
| $|c_i|\sqrt{L_{\ell\ell}^{\text{eff}}}$ | 3.68 | 2.40 | 5.57 | 3.89 | 2.50 |

Table 3: Square root of the double-lepton effective luminosity: $\sqrt{L_{\ell\ell}^{\text{eff}}}$.  

| $\epsilon_\ell$ | $L$ [fb$^{-1}$] |
|-----------------|-----------------|
|                 | 20              | 40              | 100             | 200             |
| 0.6             | 2.68            | 3.79            | 6.00            | 8.49            |
| 0.8             | 3.58            | 5.06            | 8.00            | 11.3            |
| 1.0             | 4.47            | 6.32            | 10.0            | 14.1            |

Again we can find from the diagonal elements of $M_{ij}$ what precision we can get when other information is available and we have here only one undetermined parameter left.

7. Summary, discussion and comments

Next-generation linear colliders of $e^+e^-$, NLC, will provide a cleanest environment for studying top-quark interactions. There, we shall be able to perform detailed tests of the top-quark couplings to the vector bosons and either confirm the SM simple generation-repetition pattern or discover some non-standard interactions. In this paper, assuming the most general ($CP$-violating and $CP$-conserving) couplings for $\gamma t\bar{t}$, $Zt\bar{t}$ and $Wtb$, we have calculated in a model-independent way the single- and the double-leptonic spectra. Then, the recently proposed optimal-observables technique [7] have been adopted to determine non-standard couplings both through the single- and double-leptonic-spectra measurements. The effective luminosity
necessary for an observation of $3\sigma$ effects at $\sqrt{s} = 500$ GeV for given values of non-standard couplings have been found. It would be very interesting if we observed non-standard couplings to be non-zero. In particular, finding non-zero $CP$-violating parameters must be exciting since in that case not only new interactions but also new relatively-light particles are required as discussed in §4 and 5.

The results we have presented are the most precise ones which could be obtained from the single or double distribution alone. As pointed out in the previous section, combining various independent data is important to achieve a higher precision, for which we gave a simple example there. Indeed there are several articles in which such comprehensive analysis has been performed. In ref.[8] full reconstruction of events was assumed, and helicity amplitudes for the production and decay processes were adopted to construct a likelihood. Then the likelihood has been investigated varying a single form factor at a time. Therefore one can compare expected precision for a measurement of form factors with our example eqs.(29, 30) for $\Delta c_{2/3}^{\pm}$ while $c_{3/2}^{\pm}$ is known. In spite of the fact that the method adopted here requires just lepton-energy measurement, it is seen that the obtained precision has not been substantially reduced (notice that in ref.[8] integrated luminosity of $10 \text{ fb}^{-1}$ has been applied).

Ladinsky and Yuan discussed prospects for measurements of top-quark non-standard form factors at future linear colliders [9]. Their analysis based on the top-quark angular distribution assumes however that its decays do not involve any non-standard couplings. They provide algorithms to determine top-quark momentum through its decay products, where full reconstruction of events is needed as in ref.[8]. Taking into account that we measure here only lepton energy, the results are consistent even though our precision is slightly lower. Similar analysis including some non-standard effects in top-quark decays has been performed in ref.[10]. The precision for measurements of non-standard couplings estimated there is again at the level of a few per cents.

Angular-energy distribution of charged leptons originating from the top-quark decay measured in the top-quark rest frame is known to be very sensitive to the top-
quark spin direction \[11\]. Therefore one can adopt it as analyzer of the top-quark polarization vector. Since the angular distribution of polarized quarks, eq.(10), is sensitive to non-standard interactions, it is understandable that the energy spectrum of leptons does in fact carry information on non-standard couplings in the production process. The angular-energy distribution measured in the top-quark rest frame is of course also sensitive to non-standard interactions in the decay. Therefore it has been advocated by Ježabek and Kühn in ref.[12] as a possible test of $V \pm A$ charged current structure. Numerical analysis performed by Schmitt [13] shows that the expected precision for right-handed charged current should be about 2% of the left-handed coupling.

Finally, let us give a brief comment on the effects of radiative corrections. All the non-standard couplings considered here may be generated at the multi-loop level within the SM. As it has been already mentioned, $CP$-violating couplings $\delta D_\gamma$, $\delta D_Z$ and $Re(f^R_2 - f^L_2)$ requires at least two loops of the SM, so they are negligible. On the other hand, $CP$-conserving couplings $\delta A_{\gamma,Z}$, $\delta B_{\gamma,Z}$, $\delta C_{\gamma,Z}$ and $Re(f^R_2 + f^L_2)$ could be generated already at the one-loop level approximation of QCD. Therefore, in order to disentangle non-standard top-quark interactions and such QCD effects it is important to calculate and subtract the QCD contributions from the lepton-energy spectrum, this is however beyond the scope of this paper. One should however remember that the QCD corrections include also an emission of real gluons and therefore not only form factors would be corrected but also the structure of matrix elements\(^{59}\) and the phase space would be different. Consequently, optimal observables would need to be modified. Ref.[11, 20] provides literature on QCD corrections to $t\bar{t}$ production and/or decay at linear colliders.

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\(^{59}\)One could also allow for modification of the standard $g\bar{t}\bar{t}$ coupling.
Appendix

The form factors discussed in the text could be derived within the framework of the effective lagrangian parameterizing non-standard corrections to the SM.

The effective lagrangian approach requires a choice of the low-energy particle content. In this paper we assume that the SM correctly describes all such excitations (including the Higgs particle).\(^{10}\) Thus we imagine that there is a scale \(\Lambda\), independent of the Fermi scale, at which the new physics becomes apparent.

Since the SM is renormalizable and \(\Lambda\) is assumed to be large, the decoupling theorem \(^{21}\) is applicable and requires that all new-physics effects be suppressed by inverse powers of \(\Lambda\). All such effects are expressed in terms of a series of local gauge invariant \((\text{SU}(2)_L \times \text{U}(1))\) operators of canonical dimension \(>4\); the catalogue of such operators up to dimension 6 is given in ref.\(^{14}\) (there are no dimension 5 operators respecting the global and local symmetries of the SM).

Adopting the notation of Buchmüller and Wyler we are listing all the dimension 6 operators contributing to \(\gamma t\bar{t}\) and \(Zt\bar{t}\) vertices:

\[
\begin{align*}
\mathcal{O}_{qW} &= i\bar{q}\gamma^\mu D\nu q W^{i\mu\nu}, & \mathcal{O}_{qB} &= i\bar{q}\gamma^\mu D\nu q B^{\mu\nu}, \\
\mathcal{O}_{uB} &= i\bar{u}\gamma^\mu D\nu u B^{\mu\nu}, & \mathcal{O}^{(1)}_{\phi q} &= i(\phi^\dagger D\mu \phi)(\bar{q}\gamma^\mu q), \\
\mathcal{O}^{(3)}_{\phi q} &= i(\phi^\dagger D\mu \tau^i \phi)(\bar{q}\gamma^\mu \tau^i q), & \mathcal{O}_{\phi u} &= i(\phi^\dagger D\mu \phi)(\bar{u}\gamma^\mu u), \\
\mathcal{O}_{Du} &= (\bar{q} D\mu u) D^\mu \phi, & \mathcal{O}_{D\bar{u}} &= (D\mu \bar{q} u) D^\mu \bar{\phi}, \\
\mathcal{O}_{uW\phi} &= (\bar{q} \sigma_{\mu\nu} \tau^i u) \bar{\phi} W^{i\mu\nu}, & \mathcal{O}_{uB\phi} &= (\bar{q} \sigma_{\mu\nu} u) \bar{\phi} B^{\mu\nu},
\end{align*}
\]

where \(\tau^i\) is the Pauli matrices. Some of them also contribute to \(Wtb\) vertex. In addition, \(Wtb\) vertex receives corrections from the following dimension 6 operators:

\[
\begin{align*}
\mathcal{O}_{\phi\phi} &= i(\phi^\dagger e D\mu \phi)(\bar{u}\gamma^\mu d), & \mathcal{O}_{dW\phi} &= (\bar{q} \sigma_{\mu\nu} \tau^i d) \phi W^{i\mu\nu}, \\
\mathcal{O}_{Dd} &= (\bar{q} D\mu d) D^\mu \phi, & \mathcal{O}_{D\bar{d}} &= (D\mu \bar{q} d) D^\mu \bar{\phi}.
\end{align*}
\]

Given the above list, the whole lagrangian is written as

\[
\mathcal{L} = \mathcal{L}^{SM} + \frac{1}{\Lambda^2} \sum_i (\alpha_i \mathcal{O}_i + \text{h.c.}).
\]
We have used the following parameterization of $\gamma$ and $Z$ vertices relevant for the production process:

$$
\Gamma^\mu_{\nu t} = \frac{g}{2} \bar{u}(p_t) \left[ \gamma^\mu \left\{ A_v + \delta A_v - (B_v + \delta B_v)\gamma_5 \right\} + \frac{(p_t - p_\ell)^\mu}{2m_t} (\delta C_v - \delta D_v\gamma_5) + \frac{(p_t + p_\ell)^\mu}{2m_t} (\delta E_v - \delta F_v\gamma_5) \right] v(p_\ell) \tag{32}
$$

($v = \gamma/Z$). Here we kept the terms of $\delta E_v$ and $\delta F_v$ for the later discussion though they do not contribute to our processes as mentioned in the main text. Direct calculation leads to the following results for the non-standard contributions to the form factors:

$$
\delta A_\gamma = \frac{1}{A^2} \left[ \frac{s}{g} \left\{ \cos \theta_W \text{Im}(\alpha_u + \alpha_{qB}) + \sin \theta_W \text{Im}(\alpha_{qW}) \right\} + \frac{8m_t v}{g} \left\{ \cos \theta_W \text{Re}(\alpha_{uB\phi}) + \sin \theta_W \text{Re}(\alpha_{uW\phi}) \right\} \right], \tag{33}
$$

$$
\delta A_Z = \frac{1}{A^2} \left[ -\frac{v^2}{\cos \theta_W} \text{Re}(\alpha_{\phi u} + \alpha^{(1)}_{\phi q} - \alpha^{(3)}_{\phi q}) - \frac{s}{g} \left\{ \sin \theta_W \text{Im}(\alpha_{uB} + \alpha_{qB}) - \cos \theta_W \text{Im}(\alpha_{qW}) \right\} - \frac{8m_t v}{g} \left\{ \sin \theta_W \text{Re}(\alpha_{uB\phi}) - \cos \theta_W \text{Re}(\alpha_{uW\phi}) \right\} \right], \tag{34}
$$

$$
\delta B_\gamma = \frac{1}{A^2} \left[ \frac{s}{g} \left\{ \sin \theta_W \text{Im}(\alpha_{qW}) + \cos \theta_W \text{Im}(\alpha_{qB} - \alpha_{uB}) \right\} \right], \tag{35}
$$

$$
\delta B_Z = \frac{1}{A^2} \left[ \frac{v^2}{\cos \theta_W} \text{Re}(\alpha_{\phi u} - \alpha^{(1)}_{\phi q} + \alpha^{(3)}_{\phi q}) + \frac{s}{g} \left\{ \sin \theta_W \text{Im}(\alpha_{uB} - \alpha_{qB}) + \cos \theta_W \text{Im}(\alpha_{qW}) \right\} \right], \tag{36}
$$

$$
\delta C_\gamma = \frac{1}{A^2} \left[ -\frac{8m_t v}{g} \left\{ \cos \theta_W \text{Re}(\alpha_{uB\phi}) + \sin \theta_W \text{Re}(\alpha_{uW\phi}) \right\} \right], \tag{37}
$$

$$
\delta C_Z = \frac{1}{A^2} \left[ \frac{8m_t v}{g} \left\{ \sin \theta_W \text{Re}(\alpha_{uB\phi}) - \cos \theta_W \text{Re}(\alpha_{uW\phi}) \right\} - \frac{m_t v}{\cos \theta_W} \text{Re}(\alpha_{Du} - \alpha_{D\mu}) \right], \tag{38}
$$

$$
\delta D_\gamma = \frac{1}{A^2} \left[ \frac{i 4m_t^2}{g} \left\{ \cos \theta_W \text{Re}(\alpha_{uB} - \alpha_{qB}) - \sin \theta_W \text{Re}(\alpha_{qW}) \right\} + \frac{i 8m_t v}{g} \left\{ \cos \theta_W \text{Im}(\alpha_{uB\phi}) + \sin \theta_W \text{Im}(\alpha_{uW\phi}) \right\} \right]. \tag{39}
$$

\footnote{Renormalization of the gauge-boson fields have been omitted for simplicity. For details, see ref. \cite{14}.}
\[
\delta D_Z = \frac{1}{A^2} \left[ -\frac{4m_t^2}{g} \left\{ \sin \theta_W \text{Re}(\alpha_{uB} - \alpha_{qB}) + \cos \theta_W \text{Re}(\alpha_{qW}) \right\} \\
- i\frac{8m_t v}{g} \left\{ \sin \theta_W \text{Im}(\alpha_{uB}) - \cos \theta_W \text{Im}(\alpha_{aW}) \right\} \\
+ i\frac{m_t v}{\cos \theta_W} \text{Im}(\alpha_{Du} - \alpha_{\bar{D}u}) \right], \tag{40}
\]

\[
\delta E_\gamma = 0, \tag{41}
\]

\[
\delta E_Z = \frac{1}{A^2} \left[ i\frac{m_t v}{\cos \theta_W} \text{Im}(\alpha_{Du} + \alpha_{\bar{D}u}) \right], \tag{42}
\]

\[
\delta F_\gamma = \frac{1}{A^2} \left[ -\frac{4m_t^2}{g} \left\{ \sin \theta_W \text{Im}(\alpha_{qW}) + \cos \theta_W \text{Im}(\alpha_{qB} - \alpha_{uB}) \right\} \right], \tag{43}
\]

\[
\delta F_Z = \frac{1}{A^2} \left[ -\frac{4m_t^2}{g} \left\{ \cos \theta_W \text{Im}(\alpha_{qW}) - \sin \theta_W \text{Im}(\alpha_{qB} - \alpha_{uB}) \right\} \\
- \frac{m_t v}{\cos \theta_W} \text{Re}(\alpha_{Du} + \alpha_{\bar{D}u}) \right], \tag{44}
\]

where the Higgs vacuum expectation value is \( v \) (not \( v/\sqrt{2} \)) as in ref. [14].

Let us briefly discuss constraints on the general form of \( \Gamma_\gamma \) from \( U(1)_{\text{EM}} \) symmetry. This coupling, obtained as the matrix element of the EM current \( j^\mu(x) \) at \( x = 0 \), is given as

\[
\langle p_t, p_\bar{t}| j^\mu(0)|0 \rangle = \bar{u}(p_t) \left[ \gamma^\mu (A_\gamma + \delta A_\gamma - \delta B_\gamma \gamma_5) + \frac{(p_t - p_\bar{t})^\mu}{2m_t} (\delta C_\gamma - \delta D_\gamma \gamma_5) \right. \\
\left. + \frac{(p_t + p_\bar{t})^\mu}{2m_t} (\delta E_\gamma - \delta F_\gamma \gamma_5) \right] v(p_\bar{t}).
\]

From the current conservation (\( U(1)_{\text{EM}} \) symmetry), \( \partial_a j^\mu(x) = 0 \),

\[
0 = \langle p_t, p_\bar{t}| \partial_\mu j^\mu(0)|0 \rangle = i(p_t + p_\bar{t})_\mu \langle p_t, p_\bar{t}| j^\mu(0)|0 \rangle \\
= i\bar{u}(p_t) \left[ -2m_t \delta B_\gamma \gamma_5 + s \frac{2m_t}{4m_t^2} (\delta E_\gamma - \delta F_\gamma \gamma_5) \right] v(p_\bar{t}).
\]

As it is seen from above equation \( U(1)_{\text{EM}} \) gauge symmetry requires the following relations to hold:

\[
\delta E_\gamma = 0, \quad \delta B_\gamma = -\frac{s}{4m_t} \delta F_\gamma. \tag{45}
\]

That is, \( U(1)_{\text{EM}} \) could be maintained even with non-zero axial coupling \( B_\gamma \). Indeed, it could be directly checked that the above relations are satisfied among eqs. (35), (41) and (43).
Using the notation defined by eqs. (12) and (13), the non-standard parts of the form factors contributing to the top-quark decays read:

\begin{align}
\bar{f}_1^L &= \frac{1}{V_{tb} A^2} \left[ -\frac{2M_W^2}{g} \text{Im}(\alpha_{qW}) + \frac{m_t v}{2} (\alpha_{Du} - \alpha_{Dd}) - 2v^2 \alpha_{d\phi}^{(3)} \right], \\
\bar{f}_1^R &= \frac{1}{V_{tb} A^2} \left[ v^2 \alpha_{d\phi}^{*} + \frac{m_t v}{2} (\alpha_{Du}^* - \alpha_{Dd}^*) \right], \\
\bar{f}_2^L &= \frac{1}{V_{tb} A^2} \left[ -\frac{4M_W v}{g} \alpha_{dW\phi}^* - \frac{M_W v}{2} (\alpha_{Du}^* - \alpha_{Dd}^*) \right], \\
\bar{f}_2^R &= \frac{1}{V_{tb} A^2} \left[ -\frac{4M_W v}{g} \alpha_{uW\phi}^* - \frac{M_W v}{2} (\alpha_{Du}^* - \alpha_{Dd}^*) + i\frac{2m_t M_W}{g} \text{Re}(\alpha_{qW}) \right], \\
\bar{f}_1^L &= \frac{1}{V_{tb}^* A^2} \left[ -\frac{2M_W^2}{g} \text{Im}(\alpha_{qW}) + \frac{m_t v}{2} (\alpha_{Du}^* - \alpha_{Dd}^*) - 2v^2 \alpha_{u\phi}^{(3)*} \right], \\
\bar{f}_1^R &= \frac{1}{V_{tb}^* A^2} \left[ v^2 \alpha_{u\phi}^{*} + \frac{m_t v}{2} (\alpha_{Du}^* - \alpha_{Dd}^*) \right], \\
\bar{f}_2^L &= \frac{1}{V_{tb}^* A^2} \left[ -\frac{4M_W v}{g} \alpha_{dW\phi}^* - \frac{M_W v}{2} (\alpha_{Du}^* - \alpha_{Dd}^*) - i\frac{2m_t M_W}{g} \text{Re}(\alpha_{qW}) \right], \\
\bar{f}_2^R &= \frac{1}{V_{tb}^* A^2} \left[ -\frac{4M_W v}{g} \alpha_{dW\phi}^* - \frac{M_W v}{2} (\alpha_{Du}^* - \alpha_{Dd}^*) \right].
\end{align}

Similar studies for the form factors have been recently performed in ref. [22].

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