‘Nonlocal’ dispersion cancelation with classical light

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Abstract. We show a classical analog of the original nonlocal dispersion cancelation effect in intensity interferometry with stationary light obeying Gaussian statistics. The dispersion compensation is due to the uncorrelation of the spectral components of the radiation. Although this classical counterpart phenomenon is not nonlocal in a strict quantum mechanical sense, it suggests that some second-order interference devices relying on temporal entanglement do not require a quantum light source.

Nonlocal dispersion cancelation [1] is a quantum phenomenon that relies on the use of a quantum light source, e.g. spontaneous parametric down conversion (SPDC) in a nonlinear crystal [2], providing temporally entangled photon pairs. Each photon in the pair propagates through a dispersive medium endowed with suitable dispersion properties. Then, the dispersion experienced by one photon can cancel out the dispersion of the other one, so that their intensity coincidence probability remains unaffected. A similar dispersion cancelation effect [3, 4] can be found in a configuration based on a Hong–Ou–Mandel (HOM) interferometer [5]. In this case, only one dispersive medium is needed and the even-order dispersion terms are naturally canceled out, as they are not manifested in the HOM dip [3, 4]. Both nonlocal dispersion cancelation

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cancelation effects have triggered important applications in quantum information science, including quantum optical coherence tomography [6, 7] or distant clock synchronization [8, 9], among others. Considerable interest has lately appeared on whether classical sources can mimic the effects of quantum light in intensity interferometry. Ghost diffraction and photon-correlation imaging provide a good example in the spatial domain [10]–[12]. In a recent paper, Kaltenbaek et al [13] showed that dispersion cancelation is achieved in a time-reversed HOM interferometer using chirped classical coherent pulses. This result constitutes a classical analog of the results presented by Steinberg et al [3, 4] with two-photon light. It is the aim of this work to present a classical analog of the original dispersion cancelation effect of [1]. In a closely related work, we have demonstrated that the equations governing the temporal spreading of two-photon light in homogeneous, linearly dispersive media are, apart from a difference in a sign, the same as those for partially coherent classical pulses [14]. Hence, temporal second-order entanglement phenomena can be emulated with conventional partially coherent pulses under the assumption of Gaussian statistics. While it was shown in [1] that nonlocal dispersion cancelation cannot be achieved with classical coherent pulses, here we highlight the possibility of emulating this quantum effect with classical spectrally incoherent light obeying Gaussian statistics. Specifically, we demonstrate that such radiation in intensity correlation interferometry leads to similar dispersion compensation rules as the temporally entangled two-photon states obtained in continuous-wave (CW) pumped SPDC.

Let us now show the similarities and differences of the nonlocal dispersion cancelation schemes carried out with quantum and classical light. We start with the quantum case corresponding to figure 1(a), the configuration of the initial nonlocal dispersion cancelation idea [1]. A temporally entangled photon pair is obtained, for instance, from an SPDC source. Due to energy and momentum conservation in the photon-pair generation, the signal and
idler photons are broadband and frequency anti-correlated [2]. Each photon is then launched into different dispersive elements placed in the arms of an intensity interferometer device. After propagation, two square-law detectors measure the radiation and the signals are brought to a coincidence circuit [1]. If the detectors were ideal, the coincidence circuit would give a quantity proportional to the second-order correlation function [15, 16] $G^{(2)}(t_1, t_2) = |\langle 0|\hat{A}_{\alpha}(t_1)\hat{A}_{\alpha}(t_2)|\Psi\rangle|^2$, where $|0\rangle$ and $|\Psi\rangle$ are the vacuum and two-photon states, respectively, and $\hat{A}_{\alpha}(t)$ is the slowly varying (positive-frequency) amplitude operator [17] at the output. Basically, the second-order coherence function gives the probability of detecting one photon at time $t_1$ and the other at $t_2$.

The output function $G^{(2)}$ can be calculated as [2]

$$G^{(2)}(t_1, t_2) \propto \left| \int d\omega_1 d\omega_2 \phi_\omega(\omega_1, \omega_2) \exp[i(\Phi_1(\omega_1) + \Phi_2(\omega_2))] \exp[-i(\omega_1 t_1 + \omega_2 t_2)] \right|^2.$$  

(1)

Here, $\phi_\omega(\omega_1, \omega_2)$ is the two-dimensional Fourier transform of the two-photon probability amplitude at the input state, $\Gamma_\omega(t_1, t_2) = \langle 0|\hat{A}_{\alpha}(t_1)\hat{A}_{\alpha}(t_2)|\Psi\rangle$. We refer the frequencies $\omega_1$ and $\omega_2$ with respect to the central frequency $\omega_0$. For the case of an SPDC process operating with CW pumping, $\phi_\omega(\omega_1, \omega_2) = F(\omega_1) \delta(\omega_1 + \omega_2)$. The function $F(\omega)$ represents the spectral amplitude of SPDC and it is determined by the wave-vector phase matching in the nonlinear crystal [2].

By expanding each phase factor, $\Phi_\omega$, of the dispersive medium in arm $a$ around $\omega_0$, we get $\Phi_\omega(\omega) = \sum_{n=1}^{N} n \cdot \Phi_{an}(\omega^a)$, where $\Phi_{an} = \frac{\partial^n \Phi_\omega(\omega)}{\partial \omega^n}|_{\omega = \omega_0}$. In the case of a single-mode fiber, $\Phi_{an} = \beta_{an} z_a$ [18], with $\beta_{an}$ and $z_a$ being the dispersion coefficient and length of fiber in arm $a$, respectively. Thus, equation (1) leads to

$$G^{(2)}(t'_2 - t'_1) \propto \left| \int d\omega F(\omega) \exp[-i\omega(t'_2 - t'_1)] \prod_{n=2}^{N} \exp[i\omega^n(\Phi_{1n} + (-1)^n \phi_{2n})/n!] \right|^2,$$

where $t'_j = t_j - \Phi_j t_1$ with $j = (1, 2)$. Unless one has two dispersive media adjusted to satisfy $\Phi_{1n} = (-1)^{n+1} \Phi_{2n} \forall n$, the effect of dispersion is to broaden the second-order correlation function [1, 16]. In the case of first-order dispersion ($N = 2$) and when the fibers are not matched, equation (2) becomes a Fresnel integral [16]. Interestingly, one can achieve a far-field condition whenever $\Phi_{12} + \Phi_{22} \gg \pi \sigma^2$, where $\sigma$ is the coherence time of the SPDC process, roughly given by the inverse of the bandwidth of the function $F(\omega)$. Then, a frequency-to-time mapping takes place, i.e. the output $G^{(2)}$ resembles the shape of $F(\omega)$ [2, 16]. This can be interpreted as time-domain van Cittert–Zernike theorem for the biphoton light.

Let us now consider a modified Hanbury Brown–Twiss interferometer [15] like the one shown in figure 1(b). The light source is assumed to be stationary and to obey Gaussian statistics, like the fields radiated by thermal sources. Then the hierarchy of all coherence properties can be expressed in terms of the first-order correlation function [15] $\Gamma_\omega(t_2 - t_1) = \langle e^{*}(t_1) e(t_2) \rangle$, where $e(t)$ is a random realization of the complex analytical signal and the angle brackets denote ensemble average. Light is split into two different arms, where similar dispersive elements as in the quantum case are located. Again, at the output of each arm the light intensity is photodetected and carried to a coincidence circuit. But now we assume that the averaged intensity can be independently measured at each arm and subtracted from the intensity correlation. This leads to the intensity interference term $\Delta G^{(2)}(t_1, t_2) = \langle I_1(t_1)I_2(t_2) \rangle - \langle I_1(t_1) \rangle \langle I_2(t_2) \rangle$. By recalling the assumption of Gaussian statistics, this quantity (the correlation
of intensity fluctuations) plays a role analogous to the second-order correlation function in the quantum case [19].

For the interferometer in figure 1(b), we obtain

\[ \Delta G^{(2)}(t_1, t_2) = \left| \int d\omega_1 d\omega_2 W(\omega_1, \omega_2) \exp\left[i(\Phi_2(\omega_2) - \Phi_1(\omega_1))\right] \exp\left[i(\omega_1 t_1 - \omega_2 t_2)\right] \right|^2. \]  

(3)

Here, \( W(\omega_1, \omega_2) = \langle \tilde{e}^*(\omega_1) \tilde{e}(\omega_2) \rangle \), where \( \tilde{e}(\omega) \) denotes the Fourier transform of the field realization \( e(t) \), is the cross-spectral density function of the input source, which is related to the mutual coherence function through the generalized Wiener–Khintchine theorem [20]. This dual relationship holds even for the case of nonstationary pulsed light beams [21]. The strong similarity between equations (1) and (3) originates from the integration of the fundamental equations that govern the evolution of quantum entanglement [17] and classical partially coherent light pulses [14]. For the stationary case the frequencies are uncorrelated, i.e. \( W(\omega_1, \omega_2) = S(\omega_1) \delta(\omega_2 - \omega_1) \) [15], and we find

\[ \Delta G^{(2)}(t_2 - t_1) = \left| \int d\omega S(\omega) \exp\left[-i\omega(t'_2 - t'_1)\right] \prod_{n=2}^N \exp[i\omega^n(\Phi_{2n} - \Phi_{1n})/n!] \right|^2. \]  

(4)

where \( S(\omega) \) is the spectral density function of the source centered at the baseband, \( S(\omega) = s(\omega + \omega_0) \). In view of the Wiener–Khintchine theorem, \( S(\omega) \) forms a Fourier transform pair with \( \Gamma_c(t_2 - t_1) \) [15]. It is important to note that having a broad spectrum \( S(\omega) \) with uncorrelated frequency components does not imply frequency entanglement. In equation (4), time is also expressed in the retarded frame, \( t'_j = t_j - \Phi_j \) with \( j = (1, 2) \). One readily observes that it is possible to achieve all-order dispersion compensation with classical light, simply by selecting the different fibers to satisfy \( \Phi_{2n} = \Phi_{1n} \forall n \).

By comparing equation (4) with (2) we see that \( S(\omega) \) plays the role of the spectrum \( F(\omega) \) of the SPDC process. The coherence time of the field is roughly given by the spectral width of \( F(\omega) \) in the two-photon case [2] and of \( S(\omega) \) for the classical light [15]. The first-order coherence properties of each beam (quantum or classical) traveling individually in each arm are not affected by dispersion; the dispersive effects are only observed in second-order interference [16]. It is of interest to note also that a nonlinear interferometric scheme for dispersion cancelation with classical light has been proposed, in which optical phase conjugation with broadband noise is applied between two passages of a field in a dispersive medium [22]. In another recent method frequency correlations of the field are employed to obtain a second-order signal by multiplying pairs of intensity measurements from an Mach–Zehnder interferometer, and dispersion cancelation is then achieved by digital filtering [23]. Furthermore, new schemes for compensating for the effects of dispersion have been recently suggested based on a numerical method [24] and through manipulating the spectral phase of coherent classical pulses [13]. In contrast to all these approaches, our method gets the dispersion cancelation in the scheme of [1] thanks to the nature of the spectral uncorrelation of statistically stationary Gaussian light.

The temporal coherence properties of thermal light are known not to change on propagation in a dispersive medium [25]. For this reason, in the appendix of the original work on nonlocal cancelation [1], Franson pointed out that with stationary light, the intensity coincidence detection could be affected by dispersion, but any manipulation of peak coincidences by altering
the dispersive coefficients should not be considered as nonlocal cancelation, since dispersion takes place and the correlation time of each beam is unaffected. However, that is indeed the case in the quantum regime too: the first-order correlation function of the signal (idler) photons from the SPDC process remains unaffected by dispersion, and only $G^{(2)}$ broadens [2, 16]. Nevertheless, there is a sign difference in the dispersion compensation laws between the classical and quantum cases. The origin of this difference is twofold. Firstly, because of the definitions of the functions $\Gamma_q$ and $\Gamma_c$, there is a difference in the Green propagators in equations (1) and (3). Secondly, the signal and idler photons in SPDC are initially frequency anti-correlated, whereas the stationary case corresponds to fully uncorrelated frequencies. One might wonder if the same dispersion rules between the classical and quantum schemes could be achieved by synthesizing the correlation between signal and idler beams, so that they appear to be correlated [26], i.e. $\phi_e(\omega_1, \omega_2) = F(\omega_1)\delta(\omega_1 - \omega_2)$. Even in this case, the dispersion requirements to get a $G^{(2)}$ signal unaffected are different to those for the classical scheme. Again, this is due to the difference in the propagation equations of $\Gamma_q$ and $\Gamma_c$.

Let us now illustrate the expected classical dispersion cancelation effect by a numerical simulation whose results are shown in figure 2. We assume a stationary source with Gaussian frequency spectrum of 1 nm width at half-maximum, centered at 1550 nm. The light is assumed to propagate a distance $z = 10$ km in the two arms of the interferometer of figure 1(b), and the signals corresponding to the intensity correlation function normalized to the constant background, i.e. $\langle I_1(t_1)I_2(t_2) \rangle / \langle I_1(t_1) \rangle \langle I_2(t_2) \rangle$, are shown for three cases: (a) input field before propagation, (b) dispersion only in one arm, and (c) dispersion compensated by selecting $\Phi_{2n} = \Phi_{1n}$ in the two arms. For simplicity, only the second-order dispersion coefficient, with a typical value of single-mode fibers $\beta_2 = -21$ ps$^2$ km$^{-1}$, is taken into account in the simulation.

For detecting the ideal signals illustrated by solid lines in figure 2, the temporal response of the photodiodes should be infinitely fast. While the finite response time of real detectors limits the measurement accuracy, the difference between the dispersed and dispersion-compensated signals is expected to remain clear in suitable experimental conditions. The simulated effect of the detectors’ finite response on the measured signal is illustrated in figure 2 by dashed lines.

Figure 2. The normalized intensity correlation signal versus the time difference (a) at the input of the interferometer, (b) propagated with dispersive media in one arm, and (c) propagated with dispersive media in both arms satisfying the compensation rule. The results are shown for the ideal case (solid line) and assuming detectors with finite response time (dashed lines). See the text for the simulation parameters. Note the different scaling of the axes in subplot (b).
assuming a Gaussian temporal response function of 20 ps full-width at half-maximum. Such
temporal response can be obtained by state-of-the-art photodiodes featuring hundreds of GHz
of electronic bandwidth. It can be seen that with the chosen parameters the narrow dispersionless
signals in (a) and (c) are significantly broadened due to the insufficient temporal resolution of
the detectors, whereas for the broadened signal in figure 2(b) the difference between the ideal
signal and the expected measurement result is not remarkable. However, when the dispersive
broadening is compensated, the simulated measurable signal is still over four times narrower
and higher than the dispersed peak. Moreover, the width of the signals in (a) and (c) is set only
by the spectral width of the source and the temporal response of the detectors. In contrast, the
signal in (b) depends also on the propagation distance and the dispersive properties of the media,
and thus with different parameters it could easily get more broadened, up to unresolvable.

Generally, the feasibility of detecting the classical intensity interference term defined by
equation (4) depends on the ratio between the source coherence time and the temporal resolution
of the detectors [15]. Thus, if the spectrum of the source is wide, the detection of the intensity
correlation signal may become impossible using currently available photodiodes. However, the
finite resolution problem can be circumvented with ultrafast all-optical correlation technique
using, for example, second-harmonic-generation in a nonlinear crystal, as discussed in [27].
In this way, the dashed and solid lines of the example in figure 2 would fit perfectly, and the
difference in the amount of dispersion added in both arms would be the limiting factor to resolve
the correlation of the intensity fluctuations over the background terms.

In summary, we have shown that nonlocal dispersion cancelation has a classical counterpart
situation when one considers statistically stationary classical light with Gaussian statistics. For
the initial configuration [1], the role of the two-photon spectral amplitude of the SPDC process
is assumed by the spectral density function of the source. In contrast to the quantum case, we
have found that all-order dispersion cancelation can be obtained if the fibers match each other.
The origin of this slight difference is in the definition of the coherence functions and in the
fact that the signal and idler photons are frequency anti-correlated, whereas the frequencies
of a stationary source are uncorrelated. As previously demonstrated, this difference is also
present in the differential equations that determine the evolution of the temporal entanglement
and partially coherent light pulses in homogeneous, linearly dispersive media [14]. Temporal
entanglement produced by an SPDC process pumped with CW light and stationary light is a
particular example of this analogy. As a direct consequence, some applications based on the
Hanbury Brown–Twiss-type intensity interferometers, such as the one presented in figure 1,
initially developed for entangled-photon sources with frequency anti-correlation, can be realized
also with spectrally incoherent classical light provided that the Gaussian statistics assumption
holds.

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