Novel T-violation observable open to any decay channel at meson factories

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Abstract. Two genuine Quantum phenomena: Entanglement and Filtering Measurement are at the origin of the first direct observation of Time-Reversal-Violation in the time evolution of the B neutral meson system by BaBar. The used meson transitions are directly connected to semileptonic and CP-eigenstate decay channels. We analyse the possibility of extending the observable asymmetries to more decay channels. We propose an alternative T-Violation Asymmetry in the meson factories which allows its opening to any pair of decay channels. The new asymmetry needs also the measurement of the time dependent total survival probability of the initial tagged states. By combining several asymmetries these total survival probabilities can be avoided. We analyse carefully the channels that can be used and the sufficient conditions in order the proposed asymmetry be a genuine T violation measurement.

1. Introduction
The BABAR Collaboration reported, in the $B^0 - \bar{B}^0$ system, the first direct observation of T-violation [1], in the time evolution of any system with high statistical significance. The measurement is based in a method described in [2] following the concepts originally proposed in [3, 4]

In the $\{P^0, \bar{P}^0\}$ system with evolution operator $U(t, 0)$, where $P^0$ stands for a neutral meson $K^0, D^0, B^0$ or $B_s^0$ (and $\bar{P}^0$ for the corresponding antimeson), two arbitrary states are:

$$|P_1\rangle = p_1^0|P^0\rangle + \bar{p}_1^0|\bar{P}^0\rangle$$

$$|P_2\rangle = p_2^0|P^0\rangle + \bar{p}_2^0|\bar{P}^0\rangle$$

The probability that an initially prepared state $P_1$, evolving after time $t$ to $P_1(t)$, behave like state $P_2$ is

$$\Pr[P_1 \rightarrow P_2(t)] = |\langle P_2|P_1(t)\rangle|^2 = |\langle P_2|U(t, 0)|P_1\rangle|^2$$

The T-violation observable is related to compare the reversed probabilities

$$\Pr[P_1 \rightarrow P_2(t)] - \Pr[P_2 \rightarrow P_1(t)]$$
The BABAR asymmetry was built from $P_1 = B^0_d \rightarrow \bar{B}^0_d$ and $P_2 = B^0_\pm$ where $B^0_\pm$ are the B-states tagged or filtered by the decays to CP-eigenstates with definite flavour content. These asymmetries are experimentally independent of CP violation. The Kabir asymmetry [5], with $P_1, P_2 = K^0, \bar{K}^0$ was measured by the CPLEAR Collaboration [6] with a non-vanishing value near 4 standard deviations.

2. Reference transition at $C = -1$ initial Entangled State

We want to understand which is the Reference Transition $P_1 \rightarrow P_2(t)$, between meson states, associated to a given pair of decays $f_1$ at $t_1$ and $f_2$ at $t_2 > t_1$.

Two Quantum effects are relevant to answer this question:
- Entanglement between the two neutral mesons produced at a meson factory is essential to prepare the initial state.
- The filtering measurement induced by the meson decay is used to connect to the final state.

The entangled state of the two mesons is in an antisymmetric combination of individual orthogonal states

$$|\Phi(\text{c.e.-})\rangle = \frac{1}{\sqrt{2}} \{ |P^0(\bar{k})\rangle |P^0(-\bar{k})\rangle - |P^0(\bar{k})\rangle |P^0(-\bar{k})\rangle \}$$

If at time $t_1$ we observe at one side the decay product $f$, the (still living) meson at time $t_1$ is tagged as the state that does not decay into $f$ [7, 8]

$$|P_{\alpha f}\rangle = \frac{1}{\sqrt{(|A_f|^2 + |\bar{A}_f|^2)}} \{ \bar{A}_f |P^0\rangle - A_f |\bar{P}^0\rangle \}$$

Where $A_f$ ($\bar{A}_f$) is the decay amplitude from $P^0(\bar{P}^0)$ to $f$:

$$A_f = \langle f | W | P^0 \rangle; \bar{A}_f = \langle f | W | \bar{P}^0 \rangle$$

(to first order in the weak Hamiltonian $H_w$ and to all orders in strong interactions $W = U_S(\infty, 0)H_w$. $U_S(\infty, 0)$ is the strong evolution operator and is equal to the identity if we can neglect final state interactions. (For hadronic decays we will assume transitions with one helicity amplitude: $0 \rightarrow 0 + j$).

The corresponding orthogonal state $\langle P^0_{\alpha f} | P_{\alpha f} \rangle = 0$ is given by

$$|P^0_{\alpha f}\rangle = \frac{1}{\sqrt{(|A_f|^2 + |\bar{A}_f|^2)}} \{ A_f^* |P^0\rangle + \bar{A}_f^* |\bar{P}^0\rangle \}$$

and it is the one filtered by the decay. What we call the "filtering identity" defines the precise meaning of this statement:

$$|\langle P^0_{\alpha f_2} | P_1(t) \rangle|^2 = \frac{|\langle f_2 | W | P_1(t) \rangle|^2}{(|A_{f_2}|^2 + |\bar{A}_{f_2}|^2)}$$
Experimentally, the Reference Transition $P_1 \rightarrow P_2(t)$ is therefore directly connected to $P_1 = P_{\alpha f_1}$ and $P_2 = P_{\alpha f_2}$, i.e., $P_{\alpha f_1}(t_1) \rightarrow P_{\alpha f_2}(t_2)$. And the T transformed transition (reversed) $P_{\alpha f_2}(t_1) \rightarrow P_{\alpha f_1}(t_2)$ does not correspond to the pair of decays $f_2$ at $t_1$, $f_1$ at $t_2 > t_1$, neither in the initial nor in the final decays. This is the orthogonality problem – previously defined in reference [9] - that prevent taking an arbitrary pair of decay channels. To connect the T transformed transition with experiment, we need to find a pair of decay channels such that, for each of them:

$$\text{Given } f \rightarrow \exists f' | P_{\alpha f_1} = | P_{\alpha f} \rangle$$

(9)

This orthogonality condition is satisfied by either CP conjugate decay channels ($P^0, \bar{P}^0$) or CP eigenstates of opposite sign with the same flavour content ($P_+, P_-$) and no direct CP violation. The precise condition to be fulfilled by $f$ and $f'$ with the definitions provided in the next section is $(\lambda_f, \lambda_{f'}^* = -|q/p|^2 \sim -1)$. Hence the exceptionality of the transitions between semileptonic and CP eigenstate decays. As a consequence, the orthogonality condition limits the pair of decay channels suitable for T-symmetry tests if we start, as a Reference, from the "experimental" transition $P_{\alpha f_1}(t_1) \rightarrow P_{\alpha f_2}(t_2)$.

Having identified the orthogonality problem, we give a bypass consisting in having an alternative reference transition once we fix the two channels $f_1, f_2$. We make the following replacements

| REFERENCE TRANSITION | BABAR | NEW |
|----------------------|-------|-----|
| $P_{\alpha f_1}(t_1) \rightarrow P_{\alpha f_2}(t_2)$ | $\Rightarrow$ | $P_{\alpha f_1}(t_1) \rightarrow P_{\alpha f_2}(t_2)$ |

Whereas now the two initial $P$-states are directly connected to experiment, the price to be paid is that the two final states are not. One has to work out what is the connection of the novel genuine theoretical T-asymmetry observable to experimental measurements.

3. The New T-violating Asymmetry

The probability associated to the new reference transition is

$$P_{12}(t) \equiv \left| \langle P_{\alpha f_1} | P_{\alpha f_2}(t) \rangle \right|^2$$

(10)

It is not directly connected to the double decay rate as measured at meson factories

$$I_{12}(t) \equiv \left| \langle f_2 | W | P_{\alpha f_1}(t) \rangle \right|^2 \left( |A_{f_2}|^2 + |\bar{A}_{f_2}|^2 \right) = \left| \langle P_{\alpha f_2} | P_{\alpha f_1}(t) \rangle \right|^2$$

(11)

but using closure it easy to relate both quantities:

$$P_{12}(t) = \langle P_{\alpha f_1}(t) | P_{\alpha f_2} \rangle \langle P_{\alpha f_2} | P_{\alpha f_1}(t) \rangle$$

$$= \langle P_{\alpha f_1}(t) | [I - | P_{\alpha f_2} \rangle \langle P_{\alpha f_2} |] P_{\alpha f_1}(t) \rangle$$

(12)

The key result is
\[ P_{12}(t) = N_1(t) - I_{12}(t) \]  

(13)

Where \( N_1(t) \) is the total survival probability of the \( P_{\alpha f_1} \) state after a time \( t \):

\[ N_1(t) = \langle P_{\alpha f_1} | P_{\alpha f_1} \rangle(t) \]  

(14)

a well-defined and measurable quantity: the ratio between the number of mesons that have not decayed after a time \( t \) and the initial number tagged at \( t = 0 \) by the observation of the first decay \( f_1 \). We may then call this term the total survival probability of the state \( | P_{\alpha f_1} \rangle \).

With the probability of the reference transition and its time reversed theoretically and experimentally well-defined we present the Motion Reversal Asymmetry [9]:

\[ A_R(f_1, f_2; t) = P_{12}(t) - P_{21}(t) \]  

(15)

This new observable becomes entirely measurable in a \( \Lambda^- \) meson factory for any pair of decay channels \( f_1, f_2 \). We have not imposed any particular condition to the pair of decay channels \( f_1, f_2 \), so one is not forced to use flavour specific or CP eigenstate decay channels. We will discuss later whether the measurable \( A_R(f_1, f_2; t) \) becomes a genuine Time-Reversal-Violating Asymmetry for any pair of decay channels. This asymmetry can be written explicitly as

\[ A_R(f_1, f_2; t) = P_{12}(t) - P_{21}(t) = [N_1(t) - N_2(t)] + [I_{21}(t) - I_{12}(t)] \]  

(16)

The second piece in the r.h.s. corresponds to the typical \( t \leftrightarrow -t \) asymmetry of the double decay rate in the meson factory: comparing the rate of a process decaying first one side to the final state \( f_1 \) and after a time \( t \) the other side decays to \( f_2 \) with the one with \( f_1 \) reversed. The first piece is the difference of the total survival probability of the two states \( | P_{\alpha f_1} \rangle \); this is the new piece to be measured that allows working with the here proposed new reference transition.

4. The theoretical expressions

Using the time evolution imposed by Quantum Mechanics we know the time dependent structure of the needed measurable quantities in terms of \( \Delta m, \Gamma \) and \( \Delta \Gamma \), determined by the eigenvalues of the entire Hamiltonian for the \( (\mathcal{P}^0, \overline{\mathcal{P}}^0) \). The motion reversal asymmetry is then given by:

\[ A_R(f_1, f_2; t) = e^{-\Gamma t} \left\{ C^N[f_1, f_2] \left( \cosh \left( \frac{\Delta \Gamma t}{2} \right) - \cos(\Delta m t) \right) + S^N[f_1, f_2] \sinh \left( \frac{\Delta \Gamma t}{2} \right) + S^N[f_1, f_2] \sin(\Delta m t) \right\} \]  

(17)

and depends on three "asymmetry parameters": the non-vanishing value of any of these asymmetry parameters would be a signal of Time-Reversal-Violation.

The theoretical connection of these measurable parameters to the matrix element of the meson evolution Hamiltonian \( H_{ij} \) involved in the Weisskopf-Wigner Approach (WWA) [10, 11, 12] for the \( (\mathcal{P}^0, \overline{\mathcal{P}}^0) \) system is
\[ C_N[f_1, f_2] = \delta(C_{f_1} - C_{f_2}) - \text{Im}(\theta)(S_{f_1} - S_{f_2}) \]
\[ S_N^R[f_1, f_2] = \delta(C_{f_1} R_{f_2} - C_{f_2} R_{f_1}) - \text{Im}(\theta)(S_{f_1} R_{f_2} - S_{f_2} R_{f_1}) \]
\[ S_N^A[f_1, f_2] = (C_{f_1} S_{f_2} - C_{f_2} S_{f_1})\{1 + \delta(C_{f_1} + C_{f_2})\} + + \delta(S_{f_1} - S_{f_2}) - \text{Re}(\theta)(S_{f_1} R_{f_2} - S_{f_2} R_{f_1}) \]

(18)

We have presented the results at leading order in terms of the usual mixing parameters -see the book by G. Branco et al. [8]- \( \delta = (1 - |q/p|^2) / (1 + |q/p|^2) \), that is, T and CP violating. As usual \( (q/p)^2 = \frac{H_{21}}{H_{12}} \), and the complex parameter \( \theta \), that is CPT and CP violating, \( \theta = \frac{(H_{22} - H_{11})}{(\Delta m - i\Delta^T/2)} \). We also use the parameter \( R_i, S_i \) and \( C_i \) defined in terms of the well-known \( \lambda_i = q\bar{A}_{f_i}/pA_{f_i} \)

\[ C_i = \frac{(1 - |\lambda_i|^2)}{(1 + |\lambda_i|^2)} \quad S_i = \frac{2\text{Im}(\lambda_i)}{(1 + |\lambda_i|^2)} \quad R_i = \frac{2\text{Re}(\lambda_i)}{(1 + |\lambda_i|^2)} \]

(19)

Note that these parameters are not all independent \( C_i^2 + S_i^2 + R_i^2 = 1 \).

It is worthwhile to point out that all these pieces in equation (18) should be small, except the term \( (C_{f_1} S_{f_2} - C_{f_2} S_{f_1}) \) that could be of order 1. In fact this term is the reminiscent of the BaBar T-violating measurement translated to the present context. If the asymmetry \( A_R(f_1, f_2; t) \) were always a time reversal asymmetry this observable would represent the generalization of the T violating program to "any pair of decay channels"

5. Conditions to have a true T violating asymmetry

The principle of microreversibility [13] strictly means that T-invariance implies the cancellation of the asymmetry

\[ A_T(f_1, f_2; t) = P_{12}(t) - P_{21-1T}(t) \]

(20)

Where

\[ P_{1T2T}(t) = |(U_TP_{\alpha f_1}|U_TP_{\alpha f_1}(t))|^2 \]

(21)

This asymmetry is not exactly \( A_R(f_1, f_2; t) \). \( U_T \) is the antiunitary time reversal operator \( U_T = \bar{U}_T K \), where \( \bar{U}_T \) is unitary and \( K \) is the complex conjugation operator. For pseudoscalars, the effect of \( \bar{U}_T \) is safe with the only change \( \bar{p} \rightarrow -\bar{p} \) and using rotational invariance. The possible difficulty in assigning a genuine character of T-violation to the asymmetry \( A_R(f_1, f_2; t) \) is concentrated on \( K \). It is therefore clear that a sufficient condition that must satisfy both channels in order to be \( A_R(f_1, f_2; t) \) a true T-violating asymmetry is that in the T invariant limit

\[ U_T |P_{\alpha f_1} \rangle = e^{i\phi} |P_{\alpha f_1} \rangle \]

(22)

In the \( (P^0, \bar{P}^0) \) meson space one can take

\[ \bar{U}_T = \begin{pmatrix} e^{i(\nu - \xi)} & 0 \\ 0 & e^{i(\nu + \xi)} \end{pmatrix} \]

(23)

where \( \nu \) is the phase of the CPT operator \( U_{\text{CPT}}|P^0 \rangle = e^{i\nu}|P^0 \rangle \) and \( U_{\text{CP}}|P^0 \rangle = e^{-i\xi} |\bar{P}^0 \rangle(U_{\text{CP}}|\bar{P}^0 \rangle = e^{-i\xi} |P^0 \rangle) \) defines the phase of the CP operator [8]. Therefore we can write
And imposing equation (22) we can conclude that in order the motion reversal asymmetry $A_R(f_1, f_2; t)$ be a true T-violating asymmetry both channels $f_i$ should satisfy, in the T invariant limit, the relation:

$$A_f^* \tilde{A}_f = \tilde{A}_f^* A_f e^{-2i\xi}$$  \hspace{1cm} (25)$$

In the limit of T invariance it is well-known that $(g/p)^2 = e^{2i\xi}$ [8] therefore we have to choose decay channels $f_i$ in $A_R(f_1, f_2; t)$ such that they verify in the T invariant limit

$$\lambda_{f_i} = \lambda_{f_i}^*$$  \hspace{1cm} (26)

Or equivalently: channels that in the T invariant limit verifies

$$S_{f_i} = 0$$  \hspace{1cm} (27)

This condition seems quite reasonable but reveals the difficulties associated to T violating observables. Namely in the absence of final state interactions there only remains weak phases that should vanish in the T-invariant limit: therefore $S_{f_i} = 0$. But by the same token we discovered that also in these observables final state interactions can give fake T violation. If we look at the expression of $A_R(f_1, f_2; t)$ in equations (17) and (18) we discovered that $A_R(f_1, f_2; t) = 0$ provided the quantity $\delta$ - violating $T$ is equal to zero, together with the condition $S_{f_i} = S_{f_i} = 0$. Therefore if the chosen channels $f_i$ are such that $S_{f_i}$ are T violating quantities, then $A_R(f_1, f_2; t)$ is a T violating quantity.

6. Channels that make the Motion Reversal Asymmetry a True T-Violation Asymmetry

1. Flavour specific channels verify $S_{f_i} = 0$ always. Therefore flavour specific channels can be used in $A_R(f_1, f_2; t)$.
2. If $U_T |f\rangle = e^{i\nu f} |f\rangle$, and we take rotationally invariant states $f = f'$ (for example a two body state with one spin zero particle and total spin zero) and neglecting final state interactions (FSI) $W = H_w$ one can show that in the T invariant limit

$$\tilde{A}_f = \langle f | U_T^\dagger U_T H_w U_T^\dagger U_T | P^0 \rangle = e^{-i\nu_f A_f^*} e^{-i(\nu + \xi)}$$

$$A_f = \langle f | U_T^\dagger U_T H_w U_T^\dagger U_T | P^0 \rangle = e^{-i\nu_f A_f^*} e^{-i(\nu - \xi)}$$  \hspace{1cm} (28)

that reproduces equation (25). This confirms our previous comment in a precise way: any channel $f_i$ where we can neglect FSI can be used in $A_R(f_1, f_2; t)$.

3. As in the precedent case but including final state interactions one get under T invariance [13]...
\[ A_f^* = \langle U_T f | S^\dagger U(\infty, 0) H_w | U_T p^0 \rangle = \sum_\beta \langle U_T f | S^\dagger | \beta \rangle e^{i(y-\xi)} A_\beta \]
\[ \bar{A}_f^* = \langle U_T f | S^\dagger U(\infty, 0) H_w | U_T \bar{p}^0 \rangle = \sum_\beta \langle U_T f | S^\dagger | \beta \rangle e^{i(y+\xi)} \bar{A}_\beta \]  

(29)

where we have used the strong scattering matrix \( S = U(+\infty, -\infty) \). If \( f \) is an eigenstate of the S matrix \( \langle \beta | S | f \rangle = e^{-i2\phi f_\beta} \), then it is straightforward for rotationally invariant \( f \) states to reproduce equation (25). We conclude that one can also use in \( A_R(f_1, f_2; t) \) \( f_\ell \) eigenstates of the strong scattering \( S \) matrix. This can be relevant to study \( T \) violation at \( \Phi \) factories [14] where kaons can decay to a two pion state with well-defined isospin: \( (\pi\pi)_I \) that is an eigenstate of the strong S matrix.

4. If the CP operator acting on the state \( f \) is such that \( CP|f\rangle = e^{-i\xi_f}|f\rangle \) it is easy to prove that in the CP invariant limit in the decay amplitudes we get

\[ \bar{A}_f = e^{i(\xi_f - \xi)} A_f \quad ; \quad A_f = e^{i(\xi_f + \xi)} \bar{A}_f \]  

(30)

therefore in the case of CP eigenstates \( f = \bar{f} \), and \( e^{-i\xi_f} = \eta_f = \pm 1 \) and we reproduce again the sufficient condition equation (25). Therefore we can use in our \( T \) violating asymmetry all the CP eigenstates channels \( f \) where we can neglect CP violation in the corresponding decay amplitudes. This argument can be transformed in a more general one related to \( CPT \).

5. If we consider just decays to stays \( f = \bar{f} \) mediated by \( CPT \) invariant decay amplitudes, then in the \( T \) invariant limit we also have CP invariance and consequently we get again the desired condition equation (25). An important difference with the previous case is that assuming CP invariance in the decay we just can use the decay \( B^0 \to (\pi\pi)_I = 2 \) in the two pion mode. Under \( CPT \) invariance in the decay we can use both \( B^0 \to \pi^+\pi^- \) and \( B^0 \to \pi^0\pi^0 \) [15]. Assuming \( CPT \) invariance in the decay amplitude we can use any CP eigenstate in our \( T \) violating asymmetry \( A_R(f_1, f_2; t) \). Even if the asymmetries we are studying here are different from the one measured by BabBar, this result is conceptually in agreement with the conditions obtained in reference [16].

6. It is clear that if \( f \) is not a CP eigenstates, strong phases can make \( S_f \neq 0 \) even in the absence of weak phases (in the \( T \) invariant limit). This means that our asymmetry \( A_R(f_1, f_2; t) \) cannot be used alone with this non CP eigenstate \( f \). But let us assume CP invariance of the decay amplitudes, then it can be shown [8] that

\[ \lambda_f = \frac{q A_f}{p A_f} = \frac{q}{p} e^{i(\xi_f - \xi)} A_f = e^{-2i\xi} \frac{q}{p} A_f = e^{-2i\xi} \left( \frac{q}{p} \right)^2 \frac{1}{\lambda_f} \]  

(31)

and in the \( T \) invariant limit we get

\[ \lambda_f \bar{\lambda}_f = 1 \]  

(32)

From here it is straightforward to prove that assuming CP invariance in the decay amplitudes \( A_f \) and \( \bar{A}_f \) the violation of the following identities will be a clear signal of \( T \) violation.

\[ \lambda_f \bar{\lambda}_f = 1 \]
\[ C_f + C_{\bar{f}} = 0 \; ; \; S_f + S_{\bar{f}} = 0 \; ; \; R_f - R_{\bar{f}} = 0 \]  
(33)

7. As in the case of CP eigenstates we can exchange the role of CP by CPT and still use the violation of equation (33) as a departure of T invariance. The fact that for example \( S_f + S_{\bar{f}} \neq 0 \) is a signal of T violation for non CP eigenstates suggest us to combine two asymmetries:

\[ \langle A_R(f, g; t) \rangle_f = A_R(f, g; t) + A_R(\bar{f}, g; t) \]  
(34)

in such a way that if \( g \) fulfill the conditions to participate then the new averaged asymmetry is a T violating one provided the decay amplitudes \( A_f \) and \( \bar{A}_f \) are invariant under CPT.

7. Avoiding the measurement of total survival probabilities

The T violating program here presented relays on the measurement of the asymmetry

\[ A_R(f_1, f_2; t) = [N_1(t) - N_2(t)] + [I_{21}(t) - I_{12}(t)] \]  
(35)

The second piece on the r.h.s. corresponds to the typical \( t \leftrightarrow -t \) asymmetry of the double decay rate in the meson factory. The first piece is the difference of the total survival probability of the two states \( |P_{\alpha f_1} \rangle \). In order to measured \( N_i(t) \) one has to have a good control of all decay channels \( f_i \) to be sure that our \( |P_{\alpha f_1} \rangle \) has not decay at time \( t \). In several cases a very good approximation is \( N_i(t) \sim e^{-\Gamma t} \) and therefore the contribution of the total survival probability almost cancels out. Nevertheless we have presented a method to eliminate these \( N_i(t) \) contributions. The idea is to sum three T violating asymmetries to cancel out theses unwanted pieces in the following way:

\[ A_3(f_1, f_2, f_3; t) = A_R(f_1, f_2; t) + A_R(f_2, f_3; t) + A_R(f_3, f_1; t) = I_{21}(t) - I_{12}(t) + I_{32}(t) - I_{23}(t) + I_{13}(t) - I_{31}(t) \]  
(36)

As long as the three asymmetries are T violating, we have constructed a T violating observable by comparing three time-inverted and normalized doubled decay rate processes at a meson factory. The result we obtain do not contain terms whose time dependence goes with \( \cosh(\Delta \Gamma t/2) \) or \( \cos(\Delta m t) \), these pieces coming from the total survival probabilities cancel out. We get therefore

\[ A_3(f_1, f_2, f_3; t) = e^{-\Gamma t} \left\{ S_R^N [f_1, f_2, f_3] \sinh \left( \frac{\Delta \Gamma t}{2} \right) + S_R^N [f_1, f_2, f_3] \sin(\Delta m t) \right\} \]  
(37)

To simplify we choose Flavour Specific Channels (FS); \( f_3 = FS \) and \( f_2 = \bar{FS} \). Then if we chose \( f \) a CP eigenstates and assume CPT invariance in the decay to this channel, for any channel \( f = f_{CP} \) it turns out that \( A_3(f_{CP}, \bar{FS}, FS; t) \) is a genuine T violating asymmetry. This procedure allows extending to any CP eigenstate the T violating program initiated by Babar- in any meson factory like Belle II, Daphne II etc... The result in this case is

\[ A_3(f, \bar{FS}, FS; t) = e^{-\Gamma t} \left\{ 2 \delta R_f \sinh \left( \frac{\Delta \Gamma t}{2} \right) + 2 S_f (1 + \delta C_f) \sin(\Delta m t) \right\} \]  
(38)
Controlled by $\delta$ and $S_f$ as it should. The program can be extended to non CP eigenstates with the combination

$$A_3(f,\bar{f};t) + A_3(\bar{f},f;\bar{t}) = e^{-\Delta t} \left\{ 2 \delta (R_f + R_{\bar{f}}) \sinh \left( \frac{\Delta \Gamma t}{2} \right) + 2 (S_f + S_{\bar{f}}) \left( 1 + \delta (C_f + C_{\bar{f}}) \right) \sin (\Delta mt) \right\}$$

where the $T$ violating pieces are proportional to $\delta$ or $(S_f + S_{\bar{f}})$ consistently with our previous analysis. This second formula represents the extension of the $T$ violating program to any non CP eigenstate.

We have to stress that equations (17) and (34) already represent the generalizations of the BaBar program to any channel, with the appropriate assumptions. The difference with equations (37) and (39) involves the measurement of the total survival probabilities defined in equation (14) instead of combining several motion reversal asymmetries $A_R(f,\bar{f};t)$.

8. Conclusions

To summarize, we have extended the $T$ -Violation tests to any decay channel by means of a novel strategy: instead of searching which is the pair of decay channels associated to the time-reversed meson transition, we build a new asymmetry by selecting the decay channels in such a way that automatically tags the initial states of both the Reference and the $T$ -reverse meson transitions. The connection to the experimental quantities requires the determination of the total survival probability of the tagged meson, as well as the filtering probability for the considered decay. We have analyzed the precise condition under which the proposed asymmetry is a genuinely $T$ violating one. By combining appropriate asymmetries we can avoid the measurement of the total survival probabilities. With the proposal discussed here, the way is open to a full experimental program of studies of $T$ -Violation observables at meson factories [17], using the quantum principles of entanglement and the decay as a filtering measurement.

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