THREE FLAVOR NEUTRINO OSCILLATION ANALYSIS OF THE KAMIOKANDE ATOMSPHERIC NEUTRINO DATA

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Abstract

Using the published Kamiokande binned data for sub-GeV and multi-GeV atmospheric neutrinos, we have searched for the optimum set of three flavor neutrino oscillation parameters within the constraints of reactor experiments. It is found that $\chi^2$ is minimized for $(\Delta m^2_{21}, \Delta m^2_{32}, \theta_{12}, \theta_{13}, \theta_{23}) = (7.4 \times 10^{-1} \text{eV}^2, 2.6 \times 10^{-2} \text{eV}^2, 2^\circ, 3^\circ, 45^\circ)$ and $(2.6 \times 10^{-2} \text{eV}^2, 7.4 \times 10^{-1} \text{eV}^2, 0^\circ, 87^\circ, 46^\circ)$ with $\chi^2_{\text{min}} = 87.8$ (76%CL). The sets of parameters which are suggested by the two flavor analysis turn out to be close to the minimum (0.2$\sigma$).

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There has been much interest in atmospheric neutrinos \cite{1, 2, 3, 5, 4, 6}, which might give evidence for neutrino oscillations. While NUSEX \cite{4} and Frejus \cite{5} have reported consistency between the data and the predictions of the atmospheric neutrino flux \cite{7, 8, 9}, Kamiokande, IMB and Soudan-2 have reported a discrepancy. In particular, the Kamiokande group claimed that their data suggests that the mass squared difference is of order $10^{-2}\text{eV}^2$ \cite{2}. The reason that they have obtained a narrow region for the mass squared difference is because they have used both the binned data in the sub-GeV and the multi-GeV energy regions. Namely, while the multi-GeV data show remarkable zenith angle dependence, the sub-GeV data have little zenith angle dependence. This indicates that the mass squared difference relevant to the neutrino oscillations in the atmospheric neutrinos cannot be much smaller than or much larger than $10^{-2}\text{eV}^2$. The momentum spectra of the sub-GeV atmospheric neutrinos (Fig.1 in \cite{1}) also supports this argument.

People have studied neutrino oscillations among three flavors \cite{10}, and it has been shown recently \cite{11} that one can easily get strong constraints for the mass squared differences and the mixing angles if one assumes a mass hierarchy. The original analysis of the atmospheric neutrino data by the Kamiokande group \cite{1, 2} was based on the framework of neutrino oscillation between two flavors and it is important to see what happens if we analyze the data in the three flavor framework. Much work has been done on the analysis of atmospheric neutrinos \cite{12, 13, 14, 15} and among those that discuss quantitatively the binned data of the multi-GeV neutrinos are \cite{13, 14, 15}. However, these works did not take the sub-GeV binned data of Kamiokande \cite{1} into account, and the allowed regions for the relevant mass squared difference obtained in there are wider than those in \cite{2}. In this paper\cite{2} we will perform a three flavor analysis of the published data \cite{1, 2} of the

\textsuperscript{2}This paper supersedes \cite{15}. 

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Kamiokande atmospheric neutrino experiment, along with the data of the reactor neutrino experiments [16, 17]. We will make full use of both the sub-GeV and the multi-GeV binned data as well as the data of reactor neutrino experiments [16, 17], and as we will see below, we have strong constraints on the mass squared differences and some of the mixing angles. We will take the smearing effects into consideration, and evaluate the number of events by summing over the energy and the zenith angle of neutrinos, to reproduce the original analysis by the Kamiokande group as much as possible. Throughout this paper we will restrict our discussions only to the data by the Kamiokande group [1, 2], not only because the Monte Carlo result for the neutrino energy spectrum is available only in Ref. [1, 2], but also because this is the only data which gives both the upper and the lower bound on the mass squared difference of neutrinos.

We start with the Dirac equation for three flavors of neutrinos with mass in matter [18]

\[ i \frac{d}{dx} \Psi(x) = \left[ U \text{diag} \left( 0, \Delta m^2_{21}/2E, \Delta m^2_{31}/2E \right) U^{-1} + \text{diag} \left( A(x), 0, 0 \right) \right] \Psi(x), \]  

(1)

where we have taken the ultra-relativistic limit and have subtracted a term proportional to the unit matrix, \( \Delta m^2_{ij} \equiv m_i^2 - m_j^2 \) is the mass squared difference of the neutrinos with energy \( E \), \( \Psi(x) \equiv (\nu_e(x), \nu_\mu(x), \nu_\tau(x))^T \) is the wave function of the neutrinos in the flavor basis, and \( A(x) \equiv \sqrt{2} G_F N_e(x) \) stands for the effect due to the charged current interactions between \( \nu_e \) and electrons in matter [18]. Here,

\[
U \equiv (U_{\alpha j}) \equiv \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13} & c_{12}c_{23} - s_{12}s_{23}s_{13} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13} & -c_{12}s_{23} - s_{12}c_{23}s_{13} & c_{23}c_{13} \end{pmatrix} \]  

(2)

with \( c_{ij} \equiv \cos \theta_{ij}, \ s_{ij} \equiv \sin \theta_{ij} \ (\alpha = e, \mu, \tau; \ i, j = 1, 2, 3) \) is the orthogonal mixing matrix of neutrinos. For simplicity, we will not discuss the CP vio-
lating phase of the mixing matrix here. Without loss of generality we can assume \(m_1^2 < m_2^2 < m_3^2\), so that we have \(\Delta m_{31}^2 > \max(\Delta m_{21}^2, \Delta m_{32}^2)\), and we will use \(\Delta m_{21}^2, \Delta m_{32}^2 > 0\) as two independent parameters.

The number of the expected charged leptons \(\ell_\alpha\) (\(\ell_\alpha = e\) or \(\mu\)) with energy \(q\) from a scattering \(\nu_\alpha N \rightarrow \ell_\alpha N'\) (\(\alpha = e, \mu\)) is given by

\[
N(\ell_\alpha) = n_T \sum_{\beta=e,\mu} \int_0^{q_{\text{max}}} dE \int_0^1 dq \int_{-1}^1 d\cos \Theta \int_{-1}^1 d\cos \theta \int_0^{2\pi} d\varphi \, \epsilon_\alpha(q) \times \frac{d^3 F_\beta(E, \theta)}{dE \, d\cos \theta \, d\varphi} \cdot \frac{d^2 \sigma_\alpha(E, q)}{dq \, d\cos \psi} \cdot \frac{d \cos \psi}{d \cos \theta} \, P(\nu_\beta \rightarrow \nu_\alpha; E, \theta) \tag{3}
\]

Here \(d^3 F_\beta/dE d\cos \theta d\varphi\) is the flux of atmospheric neutrinos \(\nu_\beta\) with energy \(E\) from the zenith angle \(\theta\), \(n_T\) is the effective number of target nucleons, \(\epsilon_\alpha(q)\) is the detection efficiency function for charged leptons \(\ell_\alpha\), \(d\sigma_\alpha/dqd\cos \psi\) is the differential cross section of the interaction \(\nu_\alpha N \rightarrow \ell_\alpha N'\) (\(\alpha = e, \mu\)), and \(\Theta\) is the zenith angle of the direction from which the charged lepton \(\ell_\alpha\) comes (See Fig. 1).

\[\text{(Insert Fig.1 here.)}\]

\(P(\nu_\beta \rightarrow \nu_\alpha; E, \theta)\) is the probability of \(\nu_\beta \rightarrow \nu_\alpha\) transitions with energy \(E\) after traveling a distance \(L = \sqrt{(R + h)^2 - R^2 \sin^2 \theta - R \cos \theta}\), where \(R\) is the radius of the Earth, \(h \sim 15\text{Km}\) is the altitude at which atmospheric neutrinos are produced.

As for the analysis of the sub-GeV data, almost all the information which is necessary to get the right hand side of (3) is available in the published references. We have used the differential cross section \(d\sigma_\alpha/dqd\cos \psi\) in \([19]\), and the detection efficiency function \(\epsilon_\alpha(q)\) is given in \([20]\). The flux of atmospheric neutrinos \(d^3 F_\beta/dE d\cos \theta d\varphi\) without the geomagnetic effects is given

\[\text{\footnote{\text{Even if we include the CP violating phase \(\delta\) of the mixing matrix, the effect of \(\delta\) always appears in the combination of \(c_{13} \sin 2\theta_{13} \sin \delta\). \(\sin 2\theta_{13}\) has to be small because of the constraints from the reactor experiments.}}}

\]
in [8] but we have used the flux which has been obtained with the geomagnetic effects [9]. There are two important types of binned data in [1] from which we derive the dependence of neutrino oscillations on the energy and the path length of neutrinos: Fig.1 (a) and (b) in [1] give the binned data with respect to energy of the outgoing charged leptons, while Fig.3 in [1] gives the binned data (the ratio \( \frac{\mu/e_{\text{data}}}{\mu/e_{\text{MC}}} \)) with respect to the zenith angle.

These binned data are expressed as

\[
Y^\alpha_j = (1 + \alpha) n_T \sum_{\beta=e,\mu} C^\beta \int_{\cos \Theta_j}^{\cos \Theta_{j+1}} d \cos \Theta \int_0^\infty dE \int_0^{q_{\text{max}}} dq \int_{-1}^1 d \cos \theta \int_0^{2\pi} d\varphi \\
\times \epsilon_\alpha(q) \cdot \frac{d^3 F_\beta(E, \Theta)}{dE d\cos \theta d\varphi} \cdot \frac{d^2 \sigma_\alpha(E, q)}{dq d\cos \psi} \cdot \frac{d \cos \psi}{d \cos \Theta} P(\nu_\beta \rightarrow \nu_\alpha; E, \theta)
\]

\[
Y^\alpha_a = (1 + \alpha) n_T \sum_{\beta=e,\mu} C^\beta \int_{q_{a+1}}^{q_a} dq \int_0^\infty dE \int_{-1}^1 d \cos \theta \int_{-1}^1 d \cos \theta \int_0^{2\pi} d\varphi \\
\times \epsilon_\alpha(q) \cdot \frac{d^3 F_\beta(E, \Theta)}{dE d\cos \theta d\varphi} \cdot \frac{d^2 \sigma_\alpha(E, q)}{dq d\cos \psi} \cdot \frac{d \cos \psi}{d \cos \Theta} P(\nu_\beta \rightarrow \nu_\alpha; E, \theta),
\]

respectively, where \((1 + \alpha)\) is a factor for the uncertainty in the absolute normalization, \(C_\mu \equiv 1 + \beta/2, C_e \equiv 1 - \beta/2\) for the uncertainty in the relative normalization of the \(\nu_\mu\) and \(\nu_e\) flux, and \(\cos \Theta_j \equiv (2j - 7)/5\) \((1 \leq j \leq 5)\), \(q_a \equiv (a + 1)/10\) GeV \((1 \leq a \leq 10)\).

To reproduce the analysis of the multi-GeV data by the Kamiokande group, one needs the detection efficiency function \(\epsilon_\alpha(q)\), which is not given in [2]. However, the quantity

\[
g_\alpha(E) = n_T \int_0^{q_{\text{max}}} dq \epsilon(q) \int_{-1}^1 d \cos \Theta \int_{-1}^1 d \cos \theta \int_0^{2\pi} d\varphi \\
\times \frac{d^3 F_\alpha(E, \Theta)}{dE d\cos \theta d\varphi} \cdot \frac{d^2 \sigma_\alpha(E, q)}{dq d\cos \psi} \cdot \frac{d \cos \psi}{d \cos \Theta}
\]

is given in Fig.2 (d)–(f) in Ref. [2]. The zenith angle dependence \(n_\beta(E, \Theta)\) of the atmospheric neutrino flux for various neutrino energy \(E\) has been given in Ref. [8] in detail. Here we multiply the quantity \(g_\alpha(E)\) by the
zenith angle dependence $n_\alpha(E, \Theta)$ in Ref. \[8, 9\] with suitable normalization $N' \equiv 1/ \int_0^\pi d \cos \Theta n_\alpha(E, \Theta)$, and insert the Gaussian factor with $1 \sigma \equiv 17^\circ$ to simulate a smearing effect of the detecting neutrinos. Thus we adopt the following quantity for the multi-GeV analysis.

$$X_j^\alpha \equiv NN'(1 + \alpha) \sum_{\beta = \mu, \tau} C_\beta \int_0^{\Theta_j+1} d \cos \Theta \int_0^\infty dE \int_0^\infty d \cos \theta d \varphi \times \frac{g_\beta(E)n_\beta(E, \Theta) \exp \left( -\tan^2 \psi/2/\tan^2 \psi_0/2 \right)}{(1 + \cos \psi)\sqrt{\sin^2 \Theta - \sin^2 \theta \sin^2 \varphi}} P(\nu_\beta \to \nu_\alpha; E, \theta),$$  

(7)

where $D$ is the region of $(\theta, \varphi)$ in which the argument of $\sqrt{\sin^2 \Theta - \sin^2 \theta \sin^2 \varphi}$ becomes positive, $N \equiv 2/\sqrt{\pi} \tan(\psi_0/2)$ is the normalization factor such that

$$N \int_{-1}^1 d \cos \psi \exp\left( -\tan^2 \psi/2/\tan^2 \psi_0/2 \right) (1 + \cos \psi)^{3/2}(1 - \cos \psi)^{1/2} = 1,$$

(8)

and we put $\psi_0 = 17^\circ$. The relation between $\psi$ and $\Theta$

$$\cos \psi = \frac{\cos \theta \cos \Theta - \sin \theta \cos \varphi \sqrt{\sin^2 \Theta - \sin^2 \theta \sin^2 \varphi}}{1 - \sin^2 \theta \sin^2 \varphi}$$

(9)

can be obtained by spherical trigonometry, and the measure $d \cos \psi/d \cos \Theta$ gives factors in the numerator in (7). (7) is not exactly the same quantity as the one in the original analysis [2], but this is almost the best which can be done with the published data in [2].

Several groups \[7, 8\] have given predictions on the flux of atmospheric neutrinos but they differ from one another in the magnitudes, and the Kamiokande group assumed that the errors of the overall normalization $1 + \alpha$ and the relative normalization $1 + \beta/2$ are $\sigma_\alpha = 30\%$ and $\sigma_\beta = 12\%$, respectively. Thus we define the total $\chi^2$ as [21]

$$\chi^2 = \frac{\alpha^2}{\sigma_\alpha^2} + \frac{\beta^2}{\sigma_\beta^2} + \chi^2_{\text{sub-GeV}} + \chi^2_{\text{multi-GeV}} + \chi^2_{\text{Bugey}} + \chi^2_{\text{Krasnoyarsk}},$$

(10)
where

\[
\chi^2_{\text{sub-GeV}} = 2 \sum_{\alpha=e,\mu} \sum_{a=1}^{10} \left( Y^\alpha_a - y^\alpha_a - y^\alpha_a \ln \frac{Y^\alpha_a}{y^\alpha_a} \right) + \sum_{j=1}^{5} \frac{1}{\sigma^2_j} \left( \frac{Y^\mu_j / y^\mu_j - y^\mu_j}{Y^\mu_j / y^\mu_j - y^\mu_j} \right)^2,
\]

(11)

\[
\chi^2_{\text{multi-GeV}} = 2 \sum_{\alpha=e,\mu} \sum_{j=1}^{5} \left( X^\alpha_j - x^\alpha_j - x^\alpha_j \ln \frac{X^\alpha_j}{x^\alpha_j} \right),
\]

(12)

\(\chi^2_{\text{Bugey}}\) is given by (9) in [16], and \(\chi^2_{\text{Krasnoyarsk}}\) is defined in terms of the eight data and the three parameters \(N_1, N_3, N_b\) defined in [17]. Here \(x^\alpha_j, y^\alpha_j\) (\(\alpha = e, \mu; 1 \leq j \leq 5\)) are the multi-GeV and the sub-GeV data for each zenith angle \(\cos \Theta_j < \cos \Theta < \cos \Theta_{j+1}\), respectively, and \(y^\alpha_a\) (\(\alpha = e, \mu; 1 \leq a \leq 10\)) are the sub-GeV data of the charged leptons with the energy \(q_a < q < q_{a+1}\). It is understood that \(\alpha^2/\sigma^2_\alpha + \beta^2/\sigma^2_\beta + \chi^2_{\text{sub-GeV}} + \chi^2_{\text{multi-GeV}}\) is the optimal value with respect to \(\alpha, \beta\), \(\chi^2_{\text{Bugey}}\) with respect to the parameters \(A, b, a_j (j = 1, 2, 3)\) defined in [16], and \(\chi^2_{\text{Krasnoyarsk}}\) with respect to the parameters \(N_1, N_3, N_b\).

The theoretical predictions \(X^\alpha_j, Y^\alpha_j\) (\(1 \leq j \leq 5\), \(1 \leq a \leq 10\)), \((\alpha = e, \mu)\) depend on five free parameters \((\Delta m^2_{21}, \Delta m^2_{32}, \theta_{12}, \theta_{13}, \theta_{23})\), where \(\Delta m^2_{ij} \equiv m_i^2 - m_j^2\), so (10) is expected to obey a \(\chi^2\) distribution with 103−5=98 degrees of freedom (10\times2+5=25 for sub-GeV, 5\times2=10 for multi-GeV, 60 for Bugey, and 8 for Krasnoyarsk). The number of degrees of freedom in the present analysis for the sub-GeV and the multi-GeV data is smaller than in the original one by the Kamiokande group (10\times11=110 for sub-GeV, 5\times8+5=85 for multi-GeV).

The value of \(\chi^2\) is affected to some extent by the presence of matter, and it is necessary to take into consideration the contribution of the second term in (1). In the case of the multi-GeV analysis we have solved (1) numerically for each \(E\) (0.9 GeV \(\leq E \leq 100\) GeV) and evaluated the number of events for a given range of the zenith angle. As for the sub-GeV case, however, it turned out that the second term in (1) is negligible for the entire energy region of
the sub-GeV neutrinos with $\Delta m^2_{ij}$ under consideration ($A(x) \ll \Delta m^2_{ij}/E$), so we have ignored the matter contribution in the sub-GeV analysis. We have also taken into account the effects of the particle misidentification error and the backgrounds ($Y^e \rightarrow Y^e + 0.04Y^\mu$, $Y^\mu \rightarrow 0.96Y^\mu$ for sub-GeV [1], $X^e \rightarrow X^e + 0.08X^\mu$, $X^\mu \rightarrow 0.92X^\mu$ for multi-GeV [22]). We have almost reproduced the Monte Carlo results by the Kamiokande group, such as the energy spectrum of the sub-GeV (Fig. 1 in [1]) the zenith angle distributions of the sub-GeV (Fig. 2 in [1]) and the multi-GeV data (Fig. 3 in [2]).

We have meshed each $\Delta m^2_{ij}$ region into 18 points ($\Delta m^2_{ij} = 10^{(2\ell-24)/5}$, $0 \leq \ell \leq 17$) and each $\theta_{ij}$ region into 4 points $\theta_{ij} = \ell \pi/6$, $0 \leq \ell \leq 3$, and evaluated the value of $\chi^2$. Furthermore, using the grid-search and the gradient-search methods described in Ref. [24], we have found that $\chi^2$ has the minimum value for

$$\left(\Delta m^2_{21}, \Delta m^2_{32}\right) = (7.4 \times 10^{-1}\text{eV}^2, 2.6 \times 10^{-2}\text{eV}^2)$$
$$\left(\theta_{12}, \theta_{13}, \theta_{23}\right) = (2^\circ, 3^\circ, 45^\circ)$$
$$\left(\alpha, \beta\right) = (3.1 \times 10^{-1}, -6.5 \times 10^{-2})$$

with $\chi^2_{\text{min}} = 87.8$ and

$$\left(\Delta m^2_{21}, \Delta m^2_{32}\right) = (2.6 \times 10^{-2}\text{eV}^2, 7.1 \times 10^{-2}\text{eV}^2)$$
$$\left(\theta_{12}, \theta_{13}, \theta_{23}\right) = (0^\circ, 87^\circ, 46^\circ)$$
$$\left(\alpha, \beta\right) = (3.1 \times 10^{-1}, -6.9 \times 10^{-2})$$

with $\chi^2_{\text{min}} = 87.8$.

The number of degrees of freedom of our analysis is 98, so the value of the reduced chi square is 0.9, which corresponds to 76 % confidence level. This suggests that our fit in the present analysis is good. The 68 % CL, 90 % CL

\footnote{The 4% uncertainty (out of 8%) of the multi-GeV data is due to the $\mu$-like neutral current events [23].}
allowed regions of the parameters $\Delta m^2_{21}, \Delta m^2_{32}$ with unconstrained $\theta_{12}, \theta_{13}, \theta_{23}$ are given by $\chi^2 \leq \chi^2_{\text{min}} + 5.9$, $\chi^2 \leq \chi^2_{\text{min}} + 9.2$ respectively, and are given in Fig. 2, where the asterisks stand for the sets of the parameters for $\chi^2_{\text{min}}$. The reason that we have obtained a rather narrow region for $(\Delta m^2_{21}, \Delta m^2_{32})$ is because we have taken into consideration the multi-GeV binned data, the sub-GeV binned data and the reactor data together: The significant zenith angle dependence of the multi-GeV data gives certain lower and upper bounds on $\Delta m^2_{ij}$ and the little zenith angle dependence of the sub-GeV data gives more constraint on the lower bound. In general $\chi^2_{\text{multi-GeV}}$ prefers a large value of $\sin^2 2\theta_{13}$ (if $\Delta m^2_{21} < \Delta m^2_{32}$) or a large value of $4|U_{e1}|^2(1 - |U_{e1}|^2)$ (if $\Delta m^2_{21} > \Delta m^2_{32}$), but the reactor experiments exclude a region with large value of either factor if the larger $\Delta m^2_{ij}$ is of order $10^{-2}\text{eV}^2$.

(Insert Fig.2 here.)

In Fig. 3 we have plotted $\chi^2$ as a function of $\theta_{13}$, putting $\Delta m^2_{21} = 0$ with $\theta_{23}$ unconstrained. This is a situation which is realized by the two flavor analysis of the solar neutrino problem [18, 25]: Irrespective of whether we consider the vacuum solution ($\Delta m^2_{21} \sim \mathcal{O}(10^{-11}\text{eV}^2)$) or the MSW solution ($\Delta m^2_{21} \sim \mathcal{O}(10^{-5}\text{eV}^2)$) for the solar neutrino, the mass squared difference $\Delta m^2_{21}$ is negligible compared to the contribution of the matter effect $A(x)$ in (1) and the other mass squared difference $\Delta m^2_{32}$, which should be at least of order $10^{-2}\text{eV}^2$ to account for the zenith angle dependence of the multi-GeV data. In this case $\theta_{12}$ does not appear in $\chi^2$. In particular, for $\theta_{13} = 0$ we have $\chi^2 - \chi^2_{\text{min}} = 2.2$, so we conclude that this set of parameters falls within
0.2σ for all three types of the solar neutrino solutions.\cite{18, 25}

(Insert Fig.3 here.)

In this paper we have analyzed the sub-GeV and the multi-GeV atmospheric neutrino data by the Kamiokande group based on the framework of three flavor neutrino oscillations along with constraints from the reactor experiments, and have shown that at least one of $\Delta m^2_{ij}$ should be of order $10^{-2}$eV$^2$ at 90% confidence level. We have also shown that the popular set of parameters $((\Delta m^2_{21}, \sin^2 2\theta_{12}) = (\Delta m^2, \sin^2 2\theta)_{\odot}, (\Delta m^2_{32}, \sin^2 2\theta_{23}) = (\Delta m^2, \sin^2 2\theta)_{\text{atm}}, \theta_{13} = 0)$ fall within 0.2σ, which is very close to the best fit. The minimum value of $\chi^2$ is 87.8 for 98 degrees of freedom, and the fit based on the hypothesis of neutrino oscillations is good. If we combine the present results with the solar neutrino experiments, then we get even stronger constraints, which will be reported somewhere.\cite{26}

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\footnote{In fact any set of the parameters which satisfies $\Delta m^2_{21} \ll \Delta m^2_{32} < \Delta m^2_{31}$ and $\theta_{13} = 0$ falls within 0.2σ.}
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Figures

Fig.1 The parametrization of angles in the interaction $\nu_\alpha + N \rightarrow \ell_\alpha + N'$.

Fig.2 The allowed region for $\Delta m^2_{21}, \Delta m^2_{32}$ with $\theta_{ij}$ unconstrained. The solid and dashed lines stand for the regions at 90 % and 68 % confidence level, respectively. The asterisks stand for the sets of the parameters for the best fit.

Fig.3 $\chi^2$ as a function of $\theta_{13}$ with $\Delta m^2_{21} = 0$ and $\theta_{23}$ unconstrained. $\chi^2$ is independent of $\theta_{12}$ in this limit.
Fig. 1
Fig. 2

\[ \Delta m_{32}^2 [\text{eV}^2] \]

\[ \Delta m_{21}^2 [\text{eV}^2] \]

\[ \theta_{12}, \theta_{13}, \theta_{23} \text{ unconstrained} \]

90\% CL

68\% CL

best fit *

\[ \times \times \times \times \times \times \times \times \times \times \]
\( \Delta m_{21}^2 = 0; \Delta m_{32}^2, \theta_{23} \) unconstrained