Meson wave function from holographic approaches

Alfredo Vega*, Ivan Schmidt*, Tanja Branz†, Thomas Gutsche† and Valery E. Lyubovitskij†

*Departamento de Física y Centro de Estudios Subatómicos,
Universidad Técnica Federico Santa María,
Casilla 110-V, Valparaíso, Chile
†Institut für Theoretische Physik, Universität Tübingen,
Kepler Center for Astro and Particle Physics,
Auf der Morgenstelle 14, D–72076 Tübingen, Germany

Abstract. We discuss the light-front wave function for the valence quark state of mesons using the AdS/CFT correspondence. We consider two kinds of wave functions obtained in different holographic Soft-Wall approaches.

Keywords: holographic model, light-front wave function, mesons

PACS: 11.25.Tq, 12.39.Ki, 14.40.Be

INTRODUCTION

The hadronic wave function in terms of quark and gluon degrees of freedom plays an important role in predictions for QCD process, but in a direct extraction several problems are encountered [1]. For this reason there are several non-perturbative approaches to obtain properties of distribution amplitudes and/or hadronic wave functions from QCD. Recently new techniques based on the Anti-de Sitter / Conformal Field Theory (AdS/CFT) correspondence have been developed, which allow to obtain mesonic Light-Front Wave Functions (LFWF) [2, 3, 4]. These holographic LFWFs are another application of Gauge/ Gravity ideas to QCD. In fact, bottom-up models have already been quite successful in the description of several QCD phenomena such as hadronic scattering processes [5], hadron spectra [6], hadronic couplings and chiral symmetry breaking [7] among others.

This work is a summary of the results presented in [4] and is structured as follows. In Sec. II we explicitly show the LFWFs according to two holographic models. In Sec. III we concentrate on the pion wave function, discussing the adjustment of the model parameters and distribution amplitudes, and in Sec. IV we present some conclusions.

MESON WAVE FUNCTION IN HOLOGRAPHIC MODELS

The comparison of form factors calculated both in the light-front formalism and in AdS offers the possibility to relate AdS modes to LFWFs. The main idea is that, with proper interpretation of certain quantities, the form factors in both approaches can look similar
FIGURE 1. The pion wave function $\psi(x,k_\perp)$ for $m = 4$ MeV. The left graph corresponds to Eq. (1) and the right one to Eq. (2).

and a mapping between the LFWF and AdS modes is possible. For details see [2, 3, 4]. Here we only consider two kinds of LFWFs obtained in two different holographic models (denoted by indices 1 and 2):

$$
\psi_{q_1q_2}^{(1)}(x,k_\perp) = \frac{4\pi A_1}{\kappa_1 \sqrt{x(1-x)}} e^{-\frac{k_\perp^2}{2\kappa_1^2 x(1-x)}} - \frac{1}{2\kappa_1^2} \left( \frac{m_1^2}{x} + \frac{m_2^2}{1-x} \right),
$$

(1)

$$
\psi_{q_1q_2}^{(2)}(x,k_\perp) = \frac{4\pi A_2}{\kappa_2 \sqrt{2}} \sqrt{\frac{2}{1-x}} e^{-\frac{k_\perp^2}{2\kappa_2^2 x(1-x)}} - \frac{1}{2\kappa_2^2} \left( \frac{m_1^2}{x} + \frac{m_2^2}{1-x} \right).
$$

(2)

In both cases massive quarks were included following a prescription suggested by Brodsky and de Téramond [8].

**EXAMPLE: THE PION**

The wave functions we consider depend on the parameters $(A_i, m_1, m_2, \kappa_i)$ which should be fixed. We work in the isospin limit assuming that the masses of the $u$ and $d$ quarks are equal: $m_d = m_u = m$. In this case we have a set of three free parameters $(A_i, m, \kappa_i)$ which is the same number of parameters considered in other models [9].

Using the quarks mass as an input, only two additional conditions are necessary. These are related to the decay amplitudes for $\pi \to \mu \nu$ and $\pi^0 \to \gamma\gamma$ [10]

$$
\int_0^1 dx \int \frac{d^2b}{16\pi^3} \psi_{q\pi}(x,k_\perp) = \frac{F_{\pi}}{2\sqrt{3}} \quad \text{and} \quad \int_0^1 dx \psi_{q\pi}(x,k_\perp = 0) = \frac{\sqrt{3}}{F_{\pi}},
$$

(3)

where $F_{\pi} = f_{\pi}/\sqrt{2} \approx 92.4$ MeV is the pion leptonic decay constant.

Sometimes the average transverse momentum squared of a quark in the pion, denoted as $\langle k^2_\perp \rangle_{\pi}$, with a value of about 300 MeV$^2$ [11] is used as an additional condition. Here we use this further constraint to check the validity of the model. The parameters $\kappa_1,2$, which are related to the Regge slopes, enter in the holographic model considered in Refs. [3, 4]. Both quantities could in principle be fixed by spectral data. Unfortunately,
The pion wave function $\psi_\pi(x,k_\perp)$ for $m = 330$ MeV. The left graph corresponds to Eq. (1) and the right one to Eq. (2).

**FIGURE 2.**

**TABLE 1.** Parameters defining the LFWFs given by Eqs. (1) and (2) and predictions for $\sqrt{\langle k_\perp^2 \rangle}$ and $P_{q\bar{q}}$.

| Model | $\psi(x,k_\perp)$ | $m$ (MeV) | $A$  | $\kappa$ (MeV) | $\sqrt{\langle k_\perp^2 \rangle}$ (MeV) | $P_{q\bar{q}}$ |
|-------|-------------------|-----------|------|-----------------|--------------------------------------|---------------|
| 1     | $\psi_{1c}$       | 4         | 0.452| 951.043         | 388.319                              | 0.204         |
|       | $\psi_{1c}$       | 330       | 0.924| 787.43          | 356.478                              | 0.279         |
| 2     | $\psi_{2c}$       | 4         | 0.486| 921.407         | 376.222                              | 0.236         |
|       | $\psi_{2c}$       | 330       | 0.965| 781.218         | 353.877                              | 0.299         |

The pion mass is an exception since it falls outside the Regge trajectories. Therefore, the values $\kappa_{1,2}$ have been fixed by using the previous conditions. The resulting pion wave functions $\psi_\pi(x,k_\perp)$ for different values of the quark mass ($m = 4$ and 330 MeV) are displayed in Figs.1 and 2.

As an example we consider the meson distribution amplitude which is calculated using [12]

$$\phi(x,q) = \int q^2 \frac{d^2k_\perp}{16\pi^3} \psi_{val}(x,k_\perp).$$

(4)

Fig. 3 shows the distribution amplitudes obtained using (1) and (2) in the previous expression. They are also compared to the prediction of PQCD with $\phi(x,Q \to \infty) = \sqrt{3}F_\pi x(1-x)$ both for current and constituent quark masses. As can be seen, an increase of the quark mass reduces the differences between the two variants of the LFWFs.

**CONCLUSIONS**

We have considered two kinds of wave functions for mesons in the light-front formalism obtained by the AdS/CFT correspondence within two soft wall holographic models. These wave functions have different $x$ dependence, which is less pronounced when the quark masses are increased. The parameters $\kappa_{1,2}$ used in the holographic models can be fixed by spectroscopic data. Taking quark masses as an initial input only one parameter remains (the normalization constant $A_{1,2}$), which can be further fixed by the
FIGURE 3. Pion distribution amplitudes using holographic LFWFs. Solid lines correspond to the PQCD prediction, dashed lines to model 1 and dotted ones to model 2, both for \( m = 4 \) MeV (left panel) and for \( m = 330 \) MeV (right panel).

normalization condition. Since pions do not really follow a Regge trajectory, \( \kappa_1,2 \) must be fixed in a different way. Due to the importance of the hadronic wave function in QCD the versions considered here represent a clear example of the usefulness of the AdS/CFT ideas in QCD.

ACKNOWLEDGMENTS

A. V. acknowledges the financial support from Fondecyt grants 3100028 (Chile). T. B., T. G. and V. E. L. were supported by the DFG under Contract No. FA67/31-2 and No. GRK683 and the European Community-Research Infrastructure Integrating Activity “Study of Strongly Interacting Matter” (HadronPhysics2, Grant No. 227431).

REFERENCES

1. S. J. Brodsky, C. R. Ji and M. Sawicki, Phys. Rev. D 32, 1530 (1985); O. C. Jacob and L. S. Kisslinger, Phys. Lett. B 243, 323 (1990).
2. S. J. Brodsky and G. F. de Teramond, Phys. Rev. Lett. 96, 201601 (2006).
3. S. J. Brodsky and G. F. de Teramond, Phys. Rev. D 77, 056007 (2008).
4. A. Vega, I. Schmidt, T. Branz, T. Gutsche and V. E. Lyubovitskij, Phys. Rev. D 80, 055014 (2009).
5. J. Polchinski and M. J. Strassler, Phys. Rev. Lett. 88, 031601 (2002); R. A. Janik and R. B. Peschanski, Nucl. Phys. B 565, 193 (2000); S. J. Brodsky and G. F. de Teramond, Phys. Lett. B 582, 211 (2004); E. Levin, J. Miller, B. Z. Kopeliovich and I. Schmidt, JHEP 0902, 048 (2009).
6. A. Vega and I. Schmidt, Phys. Rev. D 78, 017703 (2008); A. Vega and I. Schmidt, Phys. Rev. D 79, 055003 (2009); G. F. de Teramond and S. J. Brodsky, Phys. Rev. Lett. 94, 201601 (2005); A. Karch, E. Katz, D. T. Son and M. A. Stephanov, Phys. Rev. D 74, 015005 (2006); S. J. Brodsky and G. F. de Teramond, arXiv:0802.0514 [hep-ph].
7. L. Da Rold and A. Pomarol, Nucl. Phys. B 721, 79 (2005); J. Erlich, E. Katz, D. T. Son and M. A. Stephanov, Phys. Rev. Lett. 95, 261602 (2005); P. Colangelo, F. De Fazio, F. Giannuzzi, F. Jugeau and S. Nicotri, Phys. Rev. D 78, 055009 (2008).
8. See, e.g., S. J. Brodsky and G. F. de Teramond, arXiv:0802.0514 [hep-ph].
9. T. Huang, B. Q. Ma and Q. X. Shen, Phys. Rev. D 49, 1490 (1994).
10. S. J. Brodsky, T. Huang and G. P. Lepage, Proceedings of the Banff Summer Institute “Particles and Fields 2”, Banff, Alberta, 1981, edited by A. Z. Capri and A. N. Kamal (Plenum, New York, 1983), p. 143; P. Lepage, S. J. Brodsky, T. Huang and P. B. Mackenzie, ibid., p. 83; T. Huang, AIP Conf. Proc. 68, 1000 (1981).
11. See, e.g., W. J. Metcalf et al., Phys. Lett. B 91, 275 (1980).
12. G. P. Lepage and S. J. Brodsky, Phys. Rev. D 22, 2157 (1980).