PROBABILISTIC VERSUS FIELD-THEORETICAL DESCRIPTION OF
HEAVY FLAVOUR PRODUCTION OFF NUCLEI$^a$

M. A. Braun$^b$, C. Pajares and C. A. Salgado

Departamento de Física de Partículas, Universidade de Santiago de Compostela,
15706-Santiago de Compostela, Spain

N. Armesto$^c$ and A. Capella

Laboratoire de Physique Théorique et Hautes Energies$^d$, Université de Paris XI,
Bâtiment 211, F-91405 Orsay Cedex, France

Abstract

The absorptive corrections resulting from the rescattering of a heavy flavour
(the so-called nuclear absorption) are usually calculated with a probabilistic for-
mula valid only in the low energy limit. We extend this formula to all energies
using a quantum field theoretical approach. For charmonium and bottomium we
find that the absorptive corrections in the rigorous treatment are very similar to
the ones in the probabilistic approach. On the contrary, at sufficiently high energy,
open charm and bottom are absorbed as much as charmonium and bottomium - in
spite of the fact that their absorptive cross-sections are zero, and therefore they are
not absorbed in the probabilistic model. At high enough energies there are also ab-
sorptive corrections due to the shadowing of the nucleus structure function, which
are present for all systems including Drell-Yan pair production. These shadowing
corrections cancel in the low energy limit.

$^a$ Work supported by INTAS 93-79.

$^b$ Permanent address: Department of High-Energy Physics, University of St. Petersburg, St.
Petersburg, 198904 Russia.

$^c$ Now at II. Institut für Theoretische Physik, Universität Hamburg, Luruper Chaussee 149,
D-22761, Germany.

$^d$ Laboratoire associé au Centre National de la Recherche Scientifique - URA D0063.
1 Introduction

The study of rescattering effects in heavy flavour production in hadron-nucleus and nucleus-nucleus collisions is of particular interest for the interpretation of $J/\psi$ suppression found experimentally at CERN [1]. It is well known that an important part of this suppression is due to the rescattering of the pre-resonant $c\bar{c}$ system with the nucleons of the colliding nuclei. This phenomenon is known as nuclear absorption. To describe it the following probabilistic formula [2, 3] is currently applied

$$\sigma_{pA}^\psi = \sigma_{pN}^\psi A \int d^2b \int_{-\infty}^{+\infty} dz \rho_A(b, z) \exp \left(-\sigma_{abs}^\psi A \int_{z}^{+\infty} dz' \rho_A(b, z')\right)$$

$$= \sigma_{pN}^\psi (\sigma_{abs}^\psi)^{-1} \int d^2b \left[1 - \exp \left(-\sigma_{abs}^\psi T_A(b)\right)\right]. \quad (1)$$

In this formula $\rho_A$ and $T_A$ are the total and transverse nuclear densities respectively, $\sigma_{pA(N)}^\psi$ are the inclusive $J/\psi$ cross-section on $A(N)$ and $\sigma_{abs}^\psi$ is the absorptive $(c\bar{c})$-$N$ cross-section (i.e. corresponding to final states without $J/\psi$). For open heavy flavour production $\sigma_{abs} = 0$ and one obtains an $A^1$ behaviour in agreement with experiment [4]. For charmonium and bottomonium $\sigma_{abs} \neq 0$ and one obtains a behaviour $A^\alpha$ with $\alpha < 1$ - also in agreement with experiment [4, 5]. Eq. (1) has a probabilistic interpretation with a clear longitudinal ordering in $z$: in the first interaction at $z$ the heavy system is produced and in successive ones at $z' > z$ it rescatters with nucleons along its path. However, there is a fundamental change in the physical picture as one goes from low energies, for which the probabilistic formula (1) is derived, to high energies. Instead of successive interactions of the projectile with nucleons of the target, one has simultaneous interactions of particles into which the projectile has split. Thus the nuclear interaction responsible for the heavy particle production may occur after the interactions responsible for its absorption in nuclear matter. To illustrate this somewhat paradoxical point, compare Figs. 1a and 1b, in which the heavy particle production and its subsequent
interactions is presented as seen in the lab. system. The time goes from left to right. It is clear that in both cases the heavy particle is in fact created long before the actual interactions with the nucleus. In Fig. 1a the times of these interactions correspond to the common idea of production first, absorption after. However in Fig. 1b the order of interactions is the inverse one: the interaction of the (already formed) heavy particle precedes the one which is responsible for its formation.

In what follows we are going to study the changes in Eq. (1) resulting from this change in the physical picture at high energy. We shall derive an expression valid at all energies which exactly coincides with (1) in the low energy limit.

Our expression contains not only the rescattering of the heavy system but also the rescattering of light particles (gluons and light quarks). The latter can be interpreted as shadowing corrections to the nucleus structure function and cancel out in the low energy limit. Also our formula splits into pieces with no interactions of light particles, which we call the internal components and which are shown to be small in the central rapidity region, and a piece with at least one light particle interaction, which we call external and which corresponds to the usual production mechanisms such as gluon-gluon fusion.

At asymptotic energies the rescattering of the heavy flavour in the external component has a form different from (1). The change consists in the substitution

\[
(\sigma_{\text{abs}}^{\psi})^{-1} \left[1 - \exp \left(-\sigma_{\text{abs}}^{\psi} A T_A(b)\right)\right] \rightarrow A T_A(b) \exp \left(-\frac{1}{2} \bar{\sigma} A T_A(b)\right),
\]

where \(\bar{\sigma}\) is the \(c\bar{c} - N\) total cross-section. It is most interesting that for \(\bar{\sigma} \simeq \sigma_{\text{abs}}\), which is the case for charmonium or bottomonium production, the first rescattering correction (i.e. the term proportional to \(\bar{\sigma}\)) is the same in the two cases. Since \(\bar{\sigma}\) is small the change with increasing energy in the absorptive corrections due to the rescattering of the heavy system will be comparatively small. As a consequence, the probabilistic expression remains approximately valid even at LHC energies (if the shadowing of the nuclear structure
functions is neglected).

The situation is completely different for open charm or bottom production. In this case $\sigma_{\text{abs}} = 0$ and, as we said before, (1) leads to $A^1$. However with increasing energy there are shadowing corrections resulting from the rescattering of the heavy system (despite $\sigma_{\text{abs}} = 0$). At high energies ($\sqrt{s} \geq 200\text{GeV}$) they are practically identical to those for the $J/\psi$. This is a main prediction of our approach.

On top of these effects due to the rescattering of the heavy system, there are, of course, corrections due to the shadowing in the nucleus structure functions. The latter are present at high enough energy for all systems including Drell-Yan pair production, but vanish in the low energy limit.

The paper is organized as follows. In Section 2 we derive the expressions valid at asymptotic energies which show the change (2) in the absorptive series for the external component. These formulae are in fact applicable only at extremely high energies. Indeed, due to the presence of the heavy system there are finite energy corrections which are important up to energies of the order $M^2R_A/x_+$, where $M$ is the mass of the heavy system, $x_+$ its light cone momentum fraction and $R_A$ the nuclear radius [6, 7]. These finite energy corrections are explicitly computed in Section 3. In this Section we also show that, in the low energy limit, our formulae coincide exactly with the probabilistic formula (1). Section 4 contains our numerical results. Our conclusions are summarized in Section 5.

2 Heavy particle production at asymptotic energies

In this Section we study the inclusive cross-sections for the production of a heavy flavour system in a high-energy collision, including the rescattering of the heavy system. We start with $pA$ collisions. The total cross-section for production of the heavy system
can then be represented by the diagram shown in Fig. 2. It has a clear meaning in the laboratory reference system: at some point the heavy system is born from light partons (gluons), whereafter both heavy and light particles scatter on the nucleus. In the probabilistic treatment leading to Eq. (1) there is a time ordering: first an interaction of a light particle occurs, in which the heavy system is created, and only afterwards the latter interacts with the nucleus. As explained in the Introduction this time ordering does not exist at high energies where all interactions of light and heavy particles are simultaneous. We shall treat interactions with the nucleus of both the light particle and heavy system in the eikonal approximation. We also take all participating particles as scalar, for simplicity.

2.1 The external contribution

Using an eikonal model for the multiple scattering (both of light and heavy systems) in Fig. 2b, we obtain for the external contribution to the inclusive cross-section of a heavy system, integrated over the transverse momenta:

$$I_A^{(ext)}(x) = \frac{\pi g^2}{M^2} F_p(x_1, M^2) F_N(x_2, M^2) \Phi_A,$$

where

$$\Phi_A = \frac{1}{\sigma} [\sigma_A(a + \tilde{a}) - \sigma_A(\tilde{a})].$$

Here $gM$ corresponds to the hard scattering vertex, $M$ is the heavy particle mass, $x_1$ and $x_2$ are the longitudinal momentum fractions of the projectile and target carried by the colliding partons, $x = x_1 - x_2$, $M^2 = x_1 x_2 s$ where $s$ is the overall energetic variable: $s = (p_1 + p_2)^2$, and $p_2$ is the momentum of a nucleon in the target nucleus. $F_{p(N)}(x, M^2)$ is the structure function of the projectile proton (nucleon) at $Q^2 = M^2$. $a$ is the light particle-nucleon amplitude, with $\sigma = 2 \text{Im } a$. The notation $\sigma_A(a)$ means the
total cross-section calculated with the amplitude $a$:

$$\sigma_A(a) = 2 \text{Re} \int d^2b \left( 1 - [1 + iaT_A(b)]^A \right) ,$$  \hspace{1cm} (5)

with $T_A(b)$ the nuclear profile function normalized to 1. Finally, we have introduced the amplitude $\tilde{a}$ for the scattering of the on-mass-shell heavy system on a nucleon from the target nucleus\footnote{In the case when the heavy particle is either a charmonium or bottomonium bound state there are two different time scales involved, the production of the heavy system containing the $c\bar{c}$ or $b\bar{b}$ pair and that of the bound state. Although we make no distinction in what follows between heavy system and heavy particle it has to be understood that what propagates through the nucleus and interacts with the nucleons is not the bound state but the pre-resonance heavy system which, for convenience, we denote $\Psi$ in the following.} with the cross-section $\tilde{\sigma} = 2 \text{Im} \tilde{a}$.

An explicit derivation of Eqs. (3) and (4) is given in Sect. 3. Expressions (3)-(4) have a simple physical meaning. Note first that for the production on a single nucleon they reduce to

$$I^{\text{ext}}_N(x) = \frac{\pi g^2}{M^2} F_p(x_1, M^2) \ F_N(x_2, M^2) .$$  \hspace{1cm} (6)

This expression corresponds to the standard QCD approach in which the heavy flavoured system is produced in a hard collision of two gluons. The nuclear effects are just given by the total cross-section $\sigma_A(a + \tilde{a})$ with an amplitude equal to the sum $a + \tilde{a}$. It corresponds to the scattering on the nucleus of a “projectile” containing light and heavy particles. The negative term in Eq. (4) eliminates the contribution with no interaction of the light particle, which, according to the definition given in the Introduction, is part of the internal contribution. As we shall see below the internal contributions have to be treated differently and turn out to be numerically small at mid-rapidities.

In order to see more clearly the physical meaning of Eqs. (3) and (4), we write them at fixed impact parameter, assuming that the amplitudes $a$ and $\tilde{a}$ are purely imaginary. Eqs. (3) and (4) reduce in this case to

$$I^{\text{ext}}_N(x) = \frac{\pi g^2}{M^2} F_p(x_1, M^2) \ F_N(x_2, M^2) .$$  \hspace{1cm} (6)
\[ I_A^{ext}(x, b) = \frac{2\pi g^2}{M^2} F_p(x_1, M^2) F_A(x_2, M^2, b) e^{-\frac{i}{2} \tilde{\sigma} A_L(b)}, \quad (7) \]

where

\[ F_A(x_2, M^2, b) = F_N(x_2, M^2) \frac{1}{\sigma} \left[ 1 - e^{-\frac{i}{2} \tilde{\sigma} A_L(b)} \right] \quad (8) \]

has the meaning of the structure function of the target nucleus at impact parameter \( b \) (see below). The last factor in Eq. (7) corresponds to the rescattering of the heavy system. By comparing with the corresponding rescattering in the probabilistic approach, Eq. (1), we obtain the substitution (2) mentioned in the Introduction if we neglect the shadowing corrections to the nuclear structure functions coming from (8). For \( J/\psi \) production, \( \tilde{\sigma} \approx \sigma_{abs} \) and the first absorptive correction (i.e. the term proportional to \( \tilde{\sigma} \)) is the same in the probabilistic approach and in the asymptotic formula. Since \( \sigma_{abs} \) is small, the absorptive corrections to \( J/\psi \) production due to the rescattering of the heavy system will be similar in the two cases (see Sect. 4 for a more detailed discussion). This result, which was first anticipated in Ref. [6], is by no means trivial. Indeed if one would naively remove the time ordering in the probabilistic expression (performing the integral in \( z \) from \(-\infty\) to \(+\infty\)) one would obtain an absorptive correction of the form (7) but with an exponent two times larger.

Another important point is that (7) contains the total cross-section \( \tilde{\sigma} \) instead of the absorptive part \( \sigma_{abs} \). As a result, the situation is very different for charmonium and for open charm production. In the latter case \( \sigma_{abs} = 0 \) and thus the probabilistic expression gives an \( A^1 \) behaviour. This result will also be obtained from our field theoretical treatment in the low energy limit. However, at high energies, as seen from (7), there will be absorptive corrections of the same magnitude as for charmonium. This is an interesting prediction of our approach.
In concluding this Subsection we would like to emphasize that the shadowing corrections to the nucleus structure function Eq. (8), have been described by an eikonal formula or, more precisely, by a factorized multi-Pomeron vertex. We could have used instead the sum of tree diagrams with the triple Pomeron coupling (eikonalized Schwimmer formula) or a pure eikonal formula. The smallness of the shadowing corrections in these approaches is due to the weakness of the triple Pomeron coupling or to off-mass-shell effects (in the latter case). Our treatment can certainly be generalized to include the pure eikonal mechanism and, possibly, other mechanisms as well. However our aim in this work is not to describe in detail these shadowing corrections but rather the nuclear effects due to the rescattering of the heavy system. So we do not elaborate on possible generalizations as far as the EMC effect at low $x$ is concerned. This effect is small at low energies and vanishes in the low-energy limit. This result, which will be derived explicitly in Sect. 3, is due to the fact that at low energies the momentum fraction $x_2$ associated to the target is large - outside the shadowing region. Technically, in the formalism of Sect. 3, this vanishing is due to $t_{\text{min}}$ effects.

### 2.2 The internal component

We turn next to the internal component. As defined in the Introduction the internal component of the projectile (target) corresponds to the case when the heavy system is produced from partons belonging to the projectile (target) and no interaction takes place between light partons of projectile and target. It turns out that in the central region this component is very small at high energies. At low energies it is also small for charmonium. For open charm it is not small but is absorbed exactly in the same way as the external component. Therefore, in the central region one would obtain the same results for the $A$-dependence by neglecting this internal component. In the projectile
fragmentation region, the internal component from the target becomes dominant, since other components evidently vanish in the limit \( x_F \to 1 \). To calculate it correctly, however, one has to properly take into account shadowings corrections to the nuclear structure function at extremely low \( x \), which are presumably very important. This will be done in our numerical calculations as discussed in Sect. 4.

The internal contribution evidently involves the probability to find the heavy system with a given scaling variable \( x'_1 \) in the projectile - described by the part of the projectile structure function contributed by the heavy flavoured particle \( F_{\Psi/p}(x'_1, Q^2) \). Since the heavy system formed in the projectile may have an arbitrary scaling variable \( x'_1 \geq x_1 \), integration over \( x'_1 \) is expected. Upon colliding with the nucleus the heavy system with the scaling variable \( x'_1 \) has to transform into the observed heavy system with the scaling variable \( x_1 \). This process is described by the inclusive cross-section \( I_{\Psi A \to \Psi}(x'_1 \to x_1) \).

From the diagrams in Fig. 2 one finds

\[
I_A^{(\text{int.p})}(x_1) = \int_{x_1}^{1} \frac{dx'_1}{x'_1} F_{\Psi/p}(x'_1, M^2) I_{\Psi A \to \Psi}(x'_1 \to x_1) .
\] (9)

The heavy particle component of the structure function can be calculated via the light particle one assuming that the heavy particle is produced inside the projectile by a hard scattering mechanism (i.e. that there is no intrinsic heavy particle component). In our simplified scalar case we find

\[
F_{\Psi/p}(x, Q^2) = \frac{g^2 x^2}{16\pi^2} \int_{x}^{1} \frac{dx_1}{x_1^3} F_{p}(x_1, Q^2) .
\] (10)

From (10) one concludes that the heavy flavour structure function is smaller than the ordinary one by a factor \( g^2 \) (actually taken at the scale \( Q^2 \sim M^2 \) and therefore small). It is also damped in the region \( x \sim 1 \) by an extra power of \((1 - x)\).

As to the inclusive cross-section \( I_{\Psi A \to \Psi}(x_1 \to x) \), we can estimate it knowing that in the rescattering the heavy system tends to conserve its longitudinal momentum. Then
we approximately have

\[ I_{\Psi A \rightarrow \Psi}(x_1 \rightarrow x) = x_1 \sigma_{\Psi A} \delta(x - x_1) \].

This simple result is however valid only for open heavy flavour (\(D \bar{D}\), etc.). For hidden heavy flavour (e.g. \(J/\psi\)) one expects an extra absorption in each inelastic collision due to transition into open heavy flavour channels. To describe this absorption we introduce a factor \(\varepsilon\) which represents the part of the initial heavy flavoured cross-section which contains the observed heavy system. For open flavour \(\varepsilon = 1\) and for charmonium or bottomonium \(\varepsilon \ll 1\). After introducing \(\varepsilon\), (11) changes to

\[ I_{\Psi A \rightarrow \Psi}(x_1 \rightarrow x) = x_1 \delta(x - x_1) \sum_{n=0}^{A} \epsilon^n \sigma_{\Psi A}^{(n)}, \]

where \(\sigma_{\Psi A}^{(n)}\) denotes the \(\Psi-A\) cross-section with \(n\) inelastic interactions.

Putting (12) into (9) we obtain the internal contribution from the projectile in a simple form (an explicit derivation is given in Sect. 3):

\[ I_A^{(int,p)}(x) = \sigma_{\Psi A}^{(e)} F_{\Psi/p}(x_1, M^2), \]

where

\[ \sigma_{\Psi A}^{(e)} = \sigma_{\Psi A}^{(el)}(\tilde{a}) + \sigma_{\Psi A}^{(in)}(\tilde{a}) - \sigma_{\Psi A}^{(in)}(\tilde{a} \rightarrow \tilde{a}(1 - \varepsilon)) \]

and \(\sigma_{\Psi A}^{(in)}(\tilde{a})\) is the \(\Psi A\) inelastic cross-section calculated with the \(\Psi-N\) amplitude \(\tilde{a}\). For open flavour production (\(\varepsilon = 1\)) we evidently have \(\sigma_{\Psi A}^{(e)} = \sigma_{\Psi A}^{(tot)}\), whereas for hidden flavour with large absorption (\(\varepsilon \simeq 0\)) \(\sigma_{\Psi A}^{(e)} \simeq \sigma_{\Psi A}^{(el)}\). In the last case the internal contribution from the projectile is therefore substantially reduced.

Apart from the internal flavour in the projectile, we have also to take into account the internal flavour in the target nucleus, which upon scattering off the projectile will also contribute to the cross-section. Its expression can be easily obtained. We get

\[ I_A^{(int,A)}(x) = \varepsilon \tilde{\sigma} F_{\Psi/N}(x_2, M^2) \Phi_A, \]
where $\tilde{\sigma}$ is the $\Psi N$ cross-section and the factor $\Phi_A$ is given by (4).

For the production on a single nucleon we find

$$I_N^{(\text{int},p)}(x) = \varepsilon \tilde{\sigma} F_{\Psi/p}(x_1, M^2)$$  \hspace{1cm} (16)$$

and

$$I_N^{(\text{int},N)}(x) = \varepsilon \tilde{\sigma} F_{\Psi/N}(x_2, M^2) .$$  \hspace{1cm} (17)$$

To have a more explicit form we rewrite (16) and (17) for fixed impact parameter $b$. Assuming that $a$ and $\tilde{a}$ are purely imaginary and restricting ourselves to the case of open flavour ($\varepsilon = 1$) we have

$$I_A^{(\text{int},p)}(x, b) = 2 F_{\Psi/p}(x_1, M^2)(1 - e^{-(1/2)\tilde{\sigma} AT_A(b)})$$  \hspace{1cm} (18)$$

and

$$I_A^{(\text{int},A)}(x, b) = \tilde{\sigma} F_{\Psi/A}(x_1, M^2, b)e^{-(1/2)\tilde{\sigma} AT_A(b)},$$  \hspace{1cm} (19)$$

where $F_{\Psi/p}(x_1, M^2, b)$ is the heavy flavoured part of the nucleon structure function, Eq. (10), at fixed $b$, and

$$F_{\Psi/A}(x_2, M^2, b) = F_{\Psi/N}(x_2, M^2) \frac{1}{\tilde{\sigma}}[1 - e^{-(1/2)\tilde{\sigma} AT_A(b)}] .$$  \hspace{1cm} (20)$$

As it was the case for the external component, (18) and (19) actually describe two different physical effects: a change of the gluon distribution in the colliding nucleus as compared to the free nucleon (shadowing part of the EMC effect) and the rescattering of the heavy system in the nucleus target. The former effect corresponds to terms of the second and higher powers in the light-particle interaction with the target, i.e. in the amplitude $a$ or cross-section $\sigma$. Rescattering of the heavy system is described by the last factor in (18) and (19).

Note that the internal part coming from the projectile, Eq. (18), is absorbed in a different manner. It is proportional to the total cross-section for $\Psi-A$ scattering and
behaves like $A^{2/3}$ whereas the two other parts behave like $A^{1/3}$ (at very large $A$). However, as we shall see numerically, both internal contributions are much smaller than the external one at $x_F \sim 0$.

In concluding this Subsection we would like to emphasize that, while the internal contribution does contain $\varepsilon$, the external one does not. This is due to the following. In order to compute the inclusive cross-section for the production of the heavy system we have to fix an intermediate on-mass-shell heavy system in Fig. 2 (i.e. "cut" the diagram through the heavy system line). This cut may pass either through the rescattering blob $A$ or through one of the two heavy particle lines with which the blob is attached to the rest of the diagram. If the cut passes through the blob $A$ then the lower blob $B_t$ may be both cut or uncut. The corresponding contributions cancel in this sum due to the well-known AGK cancellations. Therefore we have to consider only the two cases when the cut passes either through the left or through the right heavy particle line in Fig. 2b. Due to this important result the blob $A$, containing the inclusive cross-section of the heavy system, is never cut - and thus $\varepsilon$ never appears. However, this AGK cancellation is only true for the external component. Clearly, it does not take place in the case when the light particle does not interact with the nucleus at all. Therefore the internal contribution does involve the cut blob $A$ and has to be treated separately. In particular it contains the factor $\varepsilon$.

2.3 Generalization to nucleus-nucleus collisions

In spite of a considerable complication in the dynamics, it can be shown that the AGK cancellation which governs the contributions to the inclusive cross-sections discussed in the last paragraph of Sect. 2.2 remains valid for $AB$ collisions. Namely, emission of the heavy system from the rescattering blob is cancelled out, unless there is no interaction of
the light particle with one of the nuclei. As a result, in the \(AB\) case we have to consider diagrams of the same type as for \(hA\) scattering and, correspondingly, the contributions to the heavy system inclusive cross-section are divided into an external part, with interactions of at least one light particle of each nucleus with nucleons of the other nucleus, and two internal parts: that of the nucleus \(B\), with no light particle interaction with nucleus \(A\), and that of the nucleus \(A\), with no light particle interaction with nucleus \(B\). All three parts are calculated quite similarly to the \(hA\) case. We present here only the final results for the inclusive cross-sections.

For the external part we find an expression which is a generalization of Eqs. (3) and (4):

\[
I_A^{(ext)}(x) = \frac{\pi g^2}{M^2} F_N(x_1, M^2) F_N(x_2, M^2) \Phi_A \Phi_B .
\]

(21)

As one observes, the inclusive cross-section (21) is factorized in the two nuclei, \(A\) and \(B\). Recalling (3) and (6) it can be rewritten as a simple relation

\[
\frac{I_{AB}^{(ext)}}{I_N^{(ext)}} = \frac{I_A^{(ext)}}{I_N^{(ext)}} \frac{I_B^{(ext)}}{I_N^{(ext)}},
\]

(22)

where in the \(hA\) and \(hB\) collisions the nuclei are taken in the same kinematics as in the \(AB\) collision. Evidently (22) means that the total absorption is just the product of absorptive factors coming from both nuclei. As is well-known this relation also holds in the probabilistic approach [2, 3].

For the internal parts we also obtain expressions factorized in the two colliding nuclei. The internal heavy flavour present in the nucleus \(B\) gives a contribution which generalizes Eqs.(13) and (14):

\[
I_{AB}^{(int,B)}(x) = \sigma_{\Psi N}^{(c)}(x_1, M^2) \Phi_B .
\]

(23)

To this internal contribution a similar one \(I_{AB}^{(int,A)}\) has to be added, which takes into account the internal heavy flavour of the target. It is given by (23) with the exchanges
\[ A \leftrightarrow B \text{ and } x_1 \leftrightarrow x_2. \]

3 Finite energy corrections

3.1 The formalism

As discussed in the Introduction, finite energy effects are very important in heavy flavour production. These effects are present up to rather high energies especially at small \( x_F \) (see below). In this Section we are going to consider these corrections. By introducing them we obtain equations valid at all energies. At asymptotically high energies they coincide with the ones derived in the previous Section. In the low energy limit, they coincide with the result of the probabilistic approach, Eq. (1). As discussed in Refs. [3] and [4] the finite energy corrections have a clear origin. Due to the presence of the heavy system, some of the contributions to the inclusive cross-section have a non-vanishing minimal transverse momentum \( (t_{\text{min}} \neq 0) \) and are suppressed by the nuclear form factor in a well-defined way. These modified cutting rules have been computed in Ref. [7] in the framework of a specific parton model. However, their physical content is so transparent that they presumably have a more general validity. They can be summarized in the following way. Let us consider a particular ordering of the longitudinal coordinates \( z_j \) of \( n \) interactions with the nucleus of the light particle and the heavy system, \( n = l + h \), where \( l(h) \) is the number of the interactions of the light particle (heavy system) and

\[
z_1 \leq z_2 \leq \cdots \leq z_n. \tag{24}
\]

In this case, at finite energies, the \( n \)-th power of the nucleus profile function \( T_A^n \) which appears in the expansion in the number of interactions of the cross-section \( \sigma_A \), Eq. (4), has to be replaced [7] by one of the following integrals:

\[
T_A^{(j)}(b) = n! \int_{-\infty}^{+\infty} dz_1 \int_{z_1}^{+\infty} dz_2 \cdots \int_{z_{n-1}}^{+\infty} dz_n \exp(i\Delta(z_1 - z_j)) \prod_{i=1}^{n} \rho_A(b, z_i), \tag{25}
\]
where $j = 1, 2, ..., n$ and
\[
\Delta = m_N M^2 / s x_+ .
\]

Note that for $\Delta = 0$, corresponding to asymptotic energies, all integrals $T^{(j)}_n$ are equal to $T^n_A$ and are independent of $j$.

For $\Delta$ non zero the value of $j$ to be taken in Eq. (25) depends on the particular discontinuity of the scattering amplitude we are considering. Namely, all the discontinuities containing $T^{(j)}_n$ are of the following type. Interactions with the nucleus from 1 to $j - 1$ have to be located to the left of the considered cutting line (“cut to the right”). Interaction $j$ may either be cut or be located to the right of the cutting line (“cut to the left”). All the other interactions, from $j + 1$ to $n$ may be cut in all possible ways. To this contribution one has to add its complex conjugate.

The physical content of these rules is quite clear. When the first interaction, in the ordering (24), is cut the exponential damping factor in (25) is not present, i.e. $T^{(1)}_n = T^n_A$. Clearly this is the only case where $t_{\min} = 0$. In all other cases the exponential damping factor is present and depends on the longitudinal distance $z_j - z_1$ between the first interaction 1 and the first one $j$ which is either cut or cut to the left.

Turning to the inclusive cross-sections, we have to stress that for $\Delta > 0$ the AGK cancellation discussed at the end of Sect. 2.2 is no more valid. Therefore we have to consider all possible cuttings on the same footing.

In treating the emission from the rescattering blob we apply the same approximation as was used in Sect. 2 for the internal contribution from the projectile. Namely we assume that the observed heavy system conserves the original longitudinal momentum. As in Sect. 2 we introduce the factor $\varepsilon \leq 1$ for each inelastic interaction of the heavy system to take into account a possible leakage of the hidden flavour into the open one.

Using these simple rules it is now easy to write the expression for the inclusive cross-
section at finite energies (i.e. $\Delta \neq 0$). In order to do so one has to remember that when cutting an interaction to the right (left) the amplitude $ia$ is replaced by $ia$ ($-ia^*$), irrespective of whether the particle is light or heavy. Cutting an interaction of a light particle one has to replace $i\bar{a}$ by $\varepsilon\bar{\sigma}$. Cutting that of a heavy particle one has to replace $i\bar{a}$ by $\varepsilon\bar{\sigma}$.

The expression for the external part of the inclusive cross-section can then be written in the same form (3),(4), where now

$$\sigma_A(a + \bar{a}) - \sigma_A(\bar{a}) \rightarrow \tilde{\sigma}_A(a, \bar{a}) - \tilde{\sigma}_A(a = 0, \bar{a}), \quad (27)$$

with

$$\tilde{\sigma}_A(a, \bar{a}) = \text{Re} \sum_{n=1}^{A} C_A^n \sum_{j=1}^{n} \int d^2 b T_n^{(j)}(b) \sigma_n^{(j)}(b). \quad (28)$$

Here for $j > 1$

$$\sigma_n^{(j)}(b) = 2 (ia + i\bar{a})^{j-1}(-ia - i\bar{a} - (1 - \varepsilon)\bar{\sigma})[-(1 - \varepsilon)\bar{\sigma}]^{n-j} \quad (29)$$

and for $j = 1$

$$\sigma_n^{(1)}(b) = (\sigma + \varepsilon\bar{\sigma})[-(1 - \varepsilon)\bar{\sigma}]^{n-1}. \quad (30)$$

Eq. (29) is quite obvious: the factor $(ia + i\bar{a})^{j-1}$ results from the cutting to the right of the first $j - 1$ interactions; the second factor corresponds to the cutting of the $j$-th interaction and the last one to the cutting of interactions from $j + 1$ to $n$. The first factor is trivial since a right cutting does not change the amplitudes $ia$ and $i\bar{a}$. The cutting of the interaction plus its cutting to the left that appears in the second factor leads to $-ia$ for a light particle and to $-i\bar{a} - (1 - \varepsilon)\bar{\sigma}$ for a heavy one. Indeed, the cutting of a light particle interaction in all possible ways (right + left + interaction itself) gives
\( i a - i a^* + \sigma = 0 \). The cutting of a heavy system interaction in all possible ways gives 
\((\varepsilon - 1)\tilde{\sigma}\). This also explains the last factor of (29).

Eq. (30) is obtained after putting \( j = 1 \). In this case the \( j \)-th interaction is the first one, and thus it has to be cut either to the left or through its interaction. It can be seen that the former (cutting to the left) is already included in the complete conjugate term responsible for the factor 2 in (29). Only the cut interaction remains in this factor. No complex conjugate term appears in this case.

For the internal part from the projectile, applying the same rules, we obtain Eq. (13) where the cross-section \( \sigma^{(e)}_\Psi \) is now replaced by \( \tilde{\sigma}^{(e)}_\Psi \), with

\[
\tilde{\sigma}^{(e)}_\Psi = \text{Re} \sum_{n=1}^A C_A^n \sum_{j=1}^n \int d^2b T_n^{(j)}(b) \sigma_n^{(j)},
\]

(31)

where for \( j > 1 \)

\[
\sigma_n^{(j)} = 2(i\tilde{a})^{j-1}(-i\tilde{a} - (1 - \varepsilon)\tilde{\sigma})[(-(1 - \varepsilon)\tilde{\sigma})]^{n-j}
\]

(32)

and for \( j = 1 \)

\[
\sigma_n^{(1)} = \varepsilon\tilde{\sigma}[-(1 - \varepsilon)\tilde{\sigma}]^{n-1}.
\]

(33)

The internal part coming from the target contains the same rescattering diagrams as the external part. This resulted in the same rescattering factor at asymptotic energies, (cfr. Eqs. (3) and (15)). Therefore at finite energies it is given by the same Eq. (15) with the substitution (27) in \( \Phi_A \).

Since, by construction, each possible discontinuity has been included in one and only one term of (28)-(33), it is clear that the above formulae provide an explicit derivation of the asymptotic energy results given in Sect. 2. Let us consider the asymptotic case when \( \Delta = 0 \) and \( T_n^{(j)} = T_A^n \). It can be easily checked that one recovers the results of that Section. Take Eq. (28). One can notice that, with \( T_n^{(j)} = T_A^n \), there is a cancellation between the term of \( \sigma^{(j)} \) proportional to \(-ia - i\tilde{a}\) and the term of \( \sigma^{(j+1)} \) proportional
to \(-(1 - \varepsilon)\bar{\sigma}\). As a result, in the sum over \(j\), one is left with the term proportional to \(-ia - i\bar{a}\) from \(\sigma^{(n)}\), the one proportional to \(-(1 - \varepsilon)\bar{\sigma}\) from \(\sigma^{(2)}\) and the term with \(j = 1\).

The first term gives \(\sigma_A(a + \bar{a})\) and the sum of the other two terms gives \(-\sigma_A^{(in)}((1 - \varepsilon)\bar{\sigma})\).

Subtracting \(\bar{\sigma}_A(a = 0, \bar{a})\) (see Eq. (27)) one obtains Eqs. (3) and (4). Likewise one can show that for \(\Delta = 0\) one recovers Eqs. (13) and (15) for the internal components.

Finally we shall study the low energy limit by taking \(\Delta \to \infty\). In this case only terms with \(j = 1\) survive in Eqs. (28) and (31). We obtain from (30), for the external contribution,

\[
I_{(A,\Delta \to \infty)}^{(ext)}(x) = \frac{\pi g^2}{M^2} F_p(x_1, M^2) F_p(x_2, M^2) \Phi_A^{(0)} .
\] (34)

For the internal contribution of the projectile we obtain from (33)

\[
I_{(A,\Delta \to \infty)}^{(int,p)}(x) = \varepsilon \bar{\sigma} F_{\Psi/p}(x_1, M^2) \Phi_A^{(0)}
\] (35)

and for the contribution of the internal heavy flavour of the target nucleus we get from (30)

\[
I_{(A,\Delta \to \infty)}^{(int,A)}(x) = \varepsilon \bar{\sigma} F_{\Psi/p}(x_2, M^2) \Phi_A^{(0)} .
\] (36)

In all cases

\[
\Phi_A^{(0)} = \frac{\sigma_A^{(in)}((1 - \varepsilon)\bar{\sigma})}{(1 - \varepsilon)\bar{\sigma}} .
\] (37)

Two important observations can be made upon inspection of Eqs. (34)-(37). First, in the cross-sections (34) and (36) only terms linear in \(a\) (or \(\sigma\)) have survived. This means that in the low energy limit the screening corrections to the nuclear structure function disappear, as expected. Second, comparison of Eqs. (34)-(36) shows that the absorptive corrections for all parts of the inclusive cross-section, external and internal, turn out to be the same in the low energy limit. This allows to write for the total inclusive cross-section on the nucleus in this limit

\[
I_{(A,\Delta \to \infty)}^{(tot)}(x) = I_N^{(tot)}(x) \Phi_A^{(0)} = I_N^{(tot)}(x) \frac{\sigma_A^{(in)}((1 - \varepsilon)\bar{\sigma})}{(1 - \varepsilon)\bar{\sigma}} .
\] (38)
Since $\sigma_{\text{abs}} = (1 - \varepsilon)\bar{\sigma}$ we recover exactly the probabilistic expression (1). As far as we know this is the first time that this expression has been derived in a field theoretical approach.

3.2 Computational methods

As we see from the above formulae, in order to compute nuclear effects at finite energies we have to deal with the new profile functions $T_n^{(j)}$ which depend on the longitudinal order of the collisions and involve the parameter $\Delta$ (Eq. (26)). At first sight it seems an impossible task due to multiple longitudinal integrations in (25). However the situation improves if we take a Fourier transform with respect to $\Delta$. Then instead of $T_n^{(j)}$ we find integrals

$$F_n^{(j)}(b, \xi) = \int \frac{d\Delta}{2\pi} T_n^{(j)}(b, \Delta) \exp(i\Delta\xi) =$$

$$n! \int_{-\infty}^{+\infty} dz_1 \int_{z_1}^{+\infty} dz_2 \cdots \int_{z_{n-1}}^{+\infty} dz_n \delta(z_1 - z_j + \xi) \prod_{i=1}^{n} \rho_A(b, z_i).$$

(39)

Evidently $F_n^{(j)}(\xi) = 0$ for $\xi < 0$ and

$$F_n^{(1)}(\xi) = \delta(\xi) T^n$$

(40)

(here and in the following we suppress the argument $b$ and the subindex $A$). For $j > 1$ and $\xi > 0$ simple calculations lead to

$$F_n^{(j)}(b) = \frac{n!}{(n - j)!(j - 2)!} \int_{-\infty}^{+\infty} dz \rho(z) \rho(z + \xi) T^{j-2}(z, z + \xi) T^{n-j}(z + \xi),$$

(41)

where

$$T(z) = \int_{z}^{+\infty} dz' \rho(z')$$

(42)

and

$$T(z_1, z_2) = \int_{z_1}^{z_2} dz' \rho(z') = T(z_1) - T(z_2).$$

(43)

Thus the calculation of all nontrivial $F_n^{(j)}$ reduces to the one-dimensional integral (41).
Moreover, using the representation (41), one can perform the sums over \( n \) and \( i \) in the formulae for the inclusive cross-sections (28) and (31) and thus obtain for them closed expressions as a function of \( \xi \). We restrict ourselves to the pure rescattering contributions, that is to terms linear in \( a \).

For the external contribution we then obtain
\[
I_A^{(ext)} = I_N^{(ext)}(\Phi_A^{(0)} + X),
\]
where the first term in the brackets, with \( \Phi_A^{(0)} \) given by (37), represents the contribution from the term with \( j = 1 \) (this is the contribution (34), which survives in the low-energy limit) and the second term represents all the terms with \( j > 1 \). It is given by an integral
\[
X = A(A-1)\text{Re} \int d^2b \int d\xi e^{-i\Delta \xi} \int_{-\infty}^{+\infty} dz \rho(b,z)\rho(b,z+\xi)
(2i\tilde{a} + \eta\tilde{\sigma} + i\tilde{a}T(b,z,z+\xi)(A\tilde{a} + (A-1)\eta\tilde{\sigma}) - \eta\tilde{\sigma}(2i\tilde{a} + \eta\tilde{\sigma})T(b,z+\xi)) (1 + w)^{A-3},
\]
where we have put
\[
w = i\tilde{a}T(b,z,z+\xi) - \eta\tilde{\sigma}T(b,z+\xi)
\]
(46)
and \( \eta = 1 - \varepsilon \).

A similar result holds for the internal contribution from the target nucleus:
\[
I_A^{(int,A)}(x) = I_N^{(int,N)}(x)(\Phi_A^{(0)} + X).
\]
(47)

For the internal contribution from the projectile we obtain
\[
I_A^{(int)} = I_N^{(int,p)}(\Phi_A^{(0)} + Y),
\]
(48)
where again the first term comes from \( j = 1 \) and is the one that survives at low energies and the rest comes from \( j > 1 \) and is given by an integral
\[
Y = -2A(A-1)\text{Re} \frac{i\tilde{a} (i\tilde{a} + \eta\tilde{\sigma})}{\varepsilon\tilde{\sigma}}
\]
(46)
\[
\int d^2 b \int d\xi e^{-i\Delta \xi} \int_{-\infty}^{+\infty} dz \rho(b, z) \rho(b, z + \xi) (1 + w)^{A-2} .
\] (49)

The calculations although rather cumbersome are now feasible. We have taken a standard Saxon-Woods form for the nuclear density \(\rho(b, z)\) and replaced all \((1 + i\tilde{a}T)^A\) by \(\exp(Ai\tilde{a}T)\).

4 Numerical results and discussion

In this Section we present our results for \(pPb\) collisions at several energies. For onium production we take \(\varepsilon = 0.001\) and for open charm and bottom \(\varepsilon = 0.999\). In the calculations we take the amplitudes \(a\) and \(\tilde{a}\) purely imaginary. First, we restrict ourselves to the nuclear effects due to the rescattering of the heavy system, i.e. we take only linear terms in \(a\) or \(\sigma\).

We start with the asymptotic formulae of Sect. 2. Neglecting the EMC effect at low \(x\)

\[\Phi_A \equiv AR_A \simeq A \int d^2 b T_A(b) \exp \left( -\frac{1}{2} \frac{\tilde{\sigma}}{\bar{\sigma}} AT_A(b) \right).\]

(50)

We present our results in terms of \(A_{\text{eff}} = I_{pPb}/I_{pN}\). The three parts of \(A_{\text{eff}}\), the external, internal from the projectile and internal from the target, are given by

\[A_{\text{eff}}^{(\text{ext})} = \frac{AR_A}{1 + r_1 + r_2}, A_{\text{eff}}^{(\text{int},p)} = \frac{\sigma_{\Psi/N}/\tilde{\sigma}}{1 + 1/r_1 + r_2/r_1}, A_{\text{eff}}^{(\text{int},A)} = \frac{AR_A}{1 + 1/r_2 + r_1/r_2},\]

(51)

where \(r_1\) and \(r_2\) are ratios to the external part of the internal ones contributed by the projectile and target, respectively, for a single nucleon as a target. These ratios are shown in Fig. 3 for open charm \((\varepsilon = 0.999)\) taking \(\tilde{\sigma} = 7\) mb at \(s = 60\) GeV\(^2\) in accordance with the data [1, 10]. For the structure functions \(F_{p,N}\) we have taken the GRV LO parametrization [11] and the structure function \(F_{\Psi/p}\) has been calculated using Eq. (10). One observes that \(r_1\) is very small for all \(x_F > 0\), whereas \(r_2\) grows with \(x_F\).  

\(^2\)This is the center of mass energy of the collision \(\Psi-N\) for experiments at \(\sqrt{s}=20\) GeV and \(x_F=0\).
and becomes greater than unity for $x_F > 0.4 \div 0.5$. From (51) we then conclude that the internal contribution from the projectile proton is very small for all $x_F$ and can safely be neglected. The internal contribution from the nuclear target, though also very small in the central region, becomes dominant at high $x_F$, since the rest vanishes as $x_F \to 1$.

The total $A_{eff}$ is given by

$$A_{eff} \simeq A_{eff}^{(ext)} + A_{eff}^{(int,A)} = AR_A,$$

(52)

with the absorptive factor $R_A$ irrespective of the relative weight of the two remaining contributions.

As commented in the Introduction, this absorptive factor is different from the probabilistic one. However for $J/\psi$ production this difference is quite small numerically. One can see this from Table 1, where we present $A_{eff}$ and its three components compared to the probabilistic value $A^{prob}$ at $x_F = 0$ and 0.5 and various energies. The dependence of $\tilde{\sigma}$ on the energy was taken in accordance with the soft Pomeron picture: $\tilde{\sigma} \sim s^{0.08}$.

A completely different result follows for open charm production, for which the probabilistic approach gives no absorption altogether. Our asymptotic formulas, on the contrary, lead to considerable absorption, as presented in Table 2. However these results can only be trusted at very high energies, as we shall presently see.

To see the finite energy effects we calculated the same quantities using our formalism of Section 3. The results are presented in Tables 3 and 4 for $J/\psi$ and open charm, respectively. Comparing with Tables 1 and 2, we observe that finite energy effects are much stronger for the open charm than for the hidden one. Indeed, for open charm at low energies the finite energy corrections make the absorption quite small ($\alpha \sim 0.98$ in the $A$-dependence $A^\alpha$), bringing the results in accordance with the experimental data [4]. This effect only dies out at $\sqrt{s} \geq 200$ GeV, when our model predicts an absorption for the open charm of the same order as for the hidden one. For $J/\psi$ the finite energy effects
turn out to be rather small. They also reduce absorption, but the effect is insignificant ($\sim 3\%$).

As to the $x_F$-dependence, in all cases absorption grows with $x_F$ due to the growth with energy of the $\Psi-N$ cross-section, although this growth is rather mild. However it is strengthened if one takes into account the shadowing corrections to the nuclear structure function, which are quite large at very small $x_2$ relevant for heavy flavour production at large $x_F$ and $s$. This has been done in a simplified manner multiplying our results by the absorption factor due to nuclear corrections to structure functions taken from Refs. [12] and [13]. The results are shown in the last but one columns of Tables 1 and 2 and in the last columns of Tables 3-6. We see that the $x_F$ dependence of the EMC effect is quite important. Once it is taken into account, the resulting $x_F$-dependence of $J/\psi$ suppression turns out to be consistent with the experimental one [5, 13]. Note, however, that at the two lower energies of the Tables, the experimentally observed $A$-dependence of Drell-Yan pair production is very close to $A^1$ at all values of $x_F$. Thus, a detailed check of this $A$-dependence, with our formalism for the EMC effect, is needed before we can claim to have a consistent explanation of the $x_F$-dependence of $J/\psi$ suppression. This interesting point is, however, beyond the scope of the present work.

In Tables 5 and 6 we present the same results for hidden and open bottom production respectively taking $\bar{\sigma} = 2$ mb at $\sqrt{s} = 20$ GeV and assuming the same energy dependence as in the case of charm production. The $A$-dependence of $\Upsilon$ at LHC energies and $x_F=0$ corresponds to $\alpha = 0.96$, which value is lowered to $\alpha = 0.91$ when shadowing in the nuclear structure functions is introduced.

Finally results for $J/\psi$ production at $x_F = 0$ in $Pb-Pb$ collisions are presented in Table 7. The difference between the asymptotic formula and the probabilistic one is larger than for $pA$ but still small.
5 Conclusions

The probabilistic formula used up to now to describe heavy flavour production off nuclei has been generalized to all energies using a quantum field theoretical approach. For $J/\psi$ and $\Upsilon$ production it gives practically the same results as the probabilistic formula up to $\sqrt{s} \simeq 6$ TeV. For open heavy flavour production we predict nuclear absorption for $\sqrt{s} \gtrsim 40$ GeV. For $\sqrt{s} \geq 200$ GeV this absorption turns out to be almost the same as the suppression of the $J/\psi$. Our formalism also predicts an increase of the $J/\psi$ suppression with increasing $x_F$.

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Figure captions

**Fig. 1a.** Low-energy heavy flavour production amplitude with rescattering.

**Fig. 1b.** High-energy heavy flavour production amplitude with rescattering.

**Fig. 2a.** Factorized diagram for heavy flavour production without rescattering.

**Fig. 2b.** Same as Fig. 2a but with rescattering of the heavy system with partons from the nucleus $A$.

**Fig. 3.** Ratios of the internal parts over the external one for projectile (dashed line) and target (solid line) contributions for open charm.
Table 1: Effective atomic numbers for $J/\Psi$ production in $pPb$ collisions with asymptotic formulae.

| $\sqrt{s}, \text{GeV}$ | $A_{\text{ext eff}}$ | $A_{\text{int p eff}}$ | $A_{\text{int A eff}}$ | $A_{\text{tot eff}}$ | $A_{\text{tot + EMC}}$ | $A_{\text{prob}}$ |
|------------------------|-----------------------|-----------------------|-----------------------|-----------------------|-----------------------|-------------------|
| $x_F=0$                |                       |                       |                       |                       |                       |                   |
| 20                     | 128.9                 | 3.8                   | 0.016                 | 132.7                 | 144.6                 | 134.4             |
| 39                     | 125.7                 | 2.5                   | 0.010                 | 128.2                 | 134.5                 | 131.7             |
| 200                    | 117.6                 | 1.0                   | 0.004                 | 118.6                 | 101.8                 | 124.7             |
| 6000                   | 99.4                  | 0.3                   | 0.001                 | 99.8                  | 71.0                  | 109.5             |
| $x_F=0.5$              |                       |                       |                       |                       |                       |                   |
| 20                     | 122.3                 | 0.50                  | 0.4                   | 123.2                 | 117.2                 | 129.1             |
| 39                     | 115.9                 | 0.27                  | 0.3                   | 116.5                 | 96.9                  | 123.6             |
| 200                    | 98.6                  | 0.08                  | 0.2                   | 98.9                  | 70.3                  | 109.1             |
| 6000                   | 61.4                  | 0.01                  | 0.2                   | 61.6                  | 43.1                  | 78.6              |
Table 2: Effective atomic numbers for open charm production in $pPb$ collisions with asymptotic formulae.

| $\sqrt{s}, GeV$ | $A_{ext}^{eff}$ | $A_{int,p}^{eff}$ | $A_{int,A}^{eff}$ | $A_{tot}^{eff}$ | $A_{tot+EMC}^{eff}$ | $A^{prob}$ |
|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|------------|
| $x_F=0$         |                 |                 |                 |                 |                 |            |
| 20              | 103.1           | 16.5            | 12.9            | 132.5           | 144.4           | 207.9      |
| 39              | 108.3           | 11.3            | 8.7             | 128.3           | 131.9           | 207.9      |
| 200             | 110.6           | 4.7             | 3.5             | 118.8           | 101.9           | 207.9      |
| 6000            | 97.9            | 1.2             | 0.8             | 99.9            | 71.0            | 207.8      |
| $x_F=0.5$       |                 |                 |                 |                 |                 |            |
| 20              | 28.3            | 0.58            | 93.9            | 122.8           | 116.8           | 207.9      |
| 39              | 32.5            | 0.36            | 83.4            | 116.3           | 96.8            | 207.9      |
| 200             | 28.5            | 0.09            | 70.4            | 98.9            | 70.3            | 207.8      |
| 6000            | 17.1            | 0.01            | 44.4            | 61.6            | 43.1            | 207.7      |

Table 3: Finite Energy effective atomic numbers for $J/\Psi$ production in $pPb$ collisions.

| $\sqrt{s}, GeV$ | $A_{ext}^{eff}$ | $A_{int,p}^{eff}$ | $A_{int,A}^{eff}$ | $A_{tot}^{eff}$ | $A_{tot+EMC}^{eff}$ |
|-----------------|-----------------|-----------------|-----------------|-----------------|---------------------|
| $x_F=0$         |                 |                 |                 |                 |                     |
| 20              | 134.9           | 0.1             | 0.017           | 135.0           | 147.2               |
| 39              | 133.2           | 0.6             | 0.011           | 133.8           | 137.5               |
| 200             | 118.4           | 1.0             | 0.004           | 119.4           | 102.4               |
| 6000            | 99.6            | 0.3             | 0.001           | 99.9            | 71.0                |
| $x_F=0.5$       |                 |                 |                 |                 |                     |
| 20              | 126.4           | 0.33            | 0.42            | 127.2           | 121.0               |
| 39              | 116.5           | 0.26            | 0.30            | 117.0           | 97.3                |
| 200             | 98.7            | 0.08            | 0.24            | 99.1            | 70.5                |
| 6000            | 61.4            | 0.01            | 0.16            | 61.6            | 43.1                |
### Table 4: Finite Energy effective atomic numbers for open charm production in $pPb$ collisions.

| $\sqrt{s}, \text{GeV}$ | $A_{\text{eff}}$ | $A_{\text{eff}}^{P}$ | $A_{\text{eff}}^{A}$ | $A_{\text{eff}}^{T}$ | $A_{\text{eff}}^{T+EMC}$ |
|------------------------|------------------|----------------------|----------------------|----------------------|--------------------------|
| $x_F=0$                |                  |                      |                      |                      |                          |
| 20                     | 162.7            | 20.6                 | 20.4                 | 203.7                | 222.0                    |
| 39                     | 155.8            | 13.5                 | 12.5                 | 181.8                | 186.9                    |
| 200                    | 114.1            | 4.7                  | 3.6                  | 122.5                | 105.1                    |
| 6000                   | 97.8             | 1.2                  | 0.8                  | 99.8                 | 71.0                     |
| $x_F=0.5$              |                  |                      |                      |                      |                          |
| 20                     | 33.8             | 0.63                 | 112.2                | 146.7                | 139.5                    |
| 39                     | 33.2             | 0.36                 | 85.1                 | 118.6                | 98.7                     |
| 200                    | 28.5             | 0.09                 | 70.3                 | 98.9                 | 70.3                     |
| 6000                   | 17.0             | 0.01                 | 44.1                 | 61.1                 | 42.7                     |

### Table 5: Effective atomic numbers for $\Upsilon$ production in $pPb$ collisions.

| $\sqrt{s}, \text{GeV}$ | $A_{\text{eff}}$ | $A_{\text{eff}}^{P}$ | $A_{\text{eff}}^{A}$ | $A_{\text{eff}}^{T}$ | $A_{\text{eff}}^{T+EMC}$ |
|------------------------|------------------|----------------------|----------------------|----------------------|--------------------------|
| $x_F=0$                |                  |                      |                      |                      |                          |
| 20                     | 181.5            | 0.02                 | 0.006                | 181.5                | 197.1                    |
| 39                     | 180.3            | 0.06                 | 0.004                | 180.4                | 186.5                    |
| 200                    | 175.9            | 0.12                 | 0.002                | 176.0                | 157.7                    |
| 6000                   | 166.9            | 0.04                 | 0.000                | 166.9                | 127.5                    |
| $x_F=0.5$              |                  |                      |                      |                      |                          |
| 20                     | 178.4            | 0.040                | 0.2                  | 178.7                | 173.8                    |
| 39                     | 175.1            | 0.034                | 0.1                  | 175.3                | 153.6                    |
| 200                    | 166.5            | 0.012                | 0.1                  | 166.6                | 126.7                    |
| 6000                   | 142.5            | 0.003                | 0.1                  | 142.6                | 82.8                     |

Table 4: Finite Energy effective atomic numbers for open charm production in $pPb$ collisions.

Table 5: Effective atomic numbers for $\Upsilon$ production in $pPb$ collisions.
| $\sqrt{s}, GeV$ | $A_{\text{ext}}^{\text{eff}}$ | $A_{\text{eff}}^{\text{int},p}$ | $A_{\text{eff}}^{\text{int},A}$ | $A_{\text{eff}}^{\text{tot}}$ | $A_{\text{eff}}^{\text{tot}+\text{EMC}}$ |
|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| $x_F=0$         |                 |                 |                 |                 |                 |
| 20              | 193.3           | 7.0             | 6.9             | 207.2           | 224.4           |
| 39              | 192.5           | 4.5             | 4.4             | 201.4           | 208.2           |
| 200             | 174.4           | 1.7             | 1.5             | 177.6           | 159.4           |
| 6000            | 166.2           | 0.4             | 0.3             | 167.1           | 127.6           |
| $x_F=0.5$       |                 |                 |                 |                 |                 |
| 20              | 96.5            | 0.45            | 91.4            | 188.3           | 186.2           |
| 39              | 101.6           | 0.26            | 74.4            | 176.3           | 156.6           |
| 200             | 97.7            | 0.07            | 69.0            | 166.7           | 127.6           |
| 6000            | 81.9            | 0.01            | 60.7            | 142.7           | 83.4            |

Table 6: Effective atomic numbers for open bottom production in $pPb$ collisions.

| $\sqrt{s}, GeV$ | $A_{\text{ext}}^{\text{eff}}$ | $A_{\text{eff}}^{\text{int},p}$ | $A_{\text{eff}}^{\text{int},A}$ | $A_{\text{eff}}^{\text{tot}}$ | $A_{\text{prob}}^{\text{eff}}$ |
|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|
|                 |                 |                 |                 |                 |                 |
| Asymptotic Energies: |
| 20              | 16625.5         | 492.8           | 2.1             | 17120.4         | 18078.1         |
| 39              | 15809.0         | 316.6           | 1.3             | 16126.9         | 17341.2         |
| 200             | 13825.1         | 123.0           | 0.5             | 13948.6         | 15552.8         |
| 6000            | 9896.7          | 29.8            | 0.1             | 9926.6          | 11997.9         |

Table 7: Effective atomic numbers for $J/\Psi$ production in $PbPb$ collisions at $x_F=0$. 

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Fig. 1a.

Fig. 1b.
Fig. 2a.

Fig. 2b.
