Improved Distributed Event-Triggered Control for Networked Control System under Random Cyberattacks via Bessel–Legendre Inequalities

Hongqian Lu,1,2 Chaoqun Guo,1 Yue Hu,1 Wuneng Zhou,3 and Shihao Yan2

1School of Electrical Engineering and Automation, Qilu University of Technology (Shandong Academy of Sciences), Jinan 250353, China
2School of Engineering, Macquarie University, Sydney, NSW, Australia
3College of Information Science and Technology, Donghua University, Shanghai 201620, China

Correspondence should be addressed to Hongqian Lu; hqlu@163.com

Received 30 December 2019; Revised 21 February 2020; Accepted 2 March 2020; Published 25 March 2020

1. Introduction

In a practical NCS, it is inevitable that there exist some problems such as network-induced delay, perturbation, and packet dropouts. As for NCS with time delay, a large number of researchers have done many relevant investigations. Filtering for discrete NCS with random time delays has been studied in [1]. A fault detection filter has been designed in [2]. Control problem for delay-dependent NCS with actuator faults has been researched in [3, 4]. Recently, researchers began to pay a significant attention to cyberattacks, which can cause a serious network security issues [5–7]. Cyberattacks often lead to instability of system and deterioration of performance. In fact, cyberattacks include three forms: denial of service [8, 9], replay attacks [10], and deception attacks [11]. Kalman filtering for nonlinear systems with denial of service attack has been studied in [12]. For stochastic system with deception attacks, distributed filtering problem has been investigated in [13]. Performance analysis of NCS under replay attacks has been given in [14]. Although many research works on cyberattack problems have been conducted in the literature, cyberattack issue has not been fully addressed for various NCS, which serves as the main motivation of this work.

With the development of information technology, an ever-increasing amount of data needs to be sent through networks. Unfortunately, the bandwidth of the network channel is subject to limited resources. For the sake of reducing the burden of network transmission, it is preferred to utilize the event-based rules, which can release the data to controller only when the sampled state satisfies the event-based rule. Event-triggered scheme overcomes the shortcoming of traditional periodic-triggered scheme and derives extensive application [15, 16]. An event-triggered controller was designed via a delay system...
Event-triggered real-time scheduling method has been researched in [18]. For the stochastic Markovian jumping system, event-triggered state estimation has been considered in [19]. Event-triggered consensus control for multiagent systems has referred to in [20]. Recently, some modified event-triggered schemes have been proposed to adapt different system demands. Distributed event-triggered mechanism has been proposed in [21] for estimation of wireless sensor network system. Adaptive event-triggered mechanism has attracted comprehensive attention [22]. To stochastic state estimation problem, a deterministic event-triggered scheme has been proposed in [23]. In addition, in order to further reduce the release times, a dynamic event-triggered mechanism was put forward by introducing a dynamic variable [24]. Nowadays, people are not only committed to saving network transmission resources but also devoting to enhance the system dynamics behavior. For instance, a new static event-triggered scheme has been raised to accelerate the dynamic process by constructing a time-varying parameter in triggering rule [25]. Here, we will establish an improved distributed event-triggered scheme for cyber-attacked networked control system. This can not only reduce the load of the network communication but also enhance the property of system dynamics, which has never been tackled in the literature.

It is significant to guarantee that the system is stable within a certain range of delay. In order to reduce the conservatism of the upper bound of system delay, we usually expect the upper bound as large as possible. Lyapunov–Krasovskii functional method as a powerful method has two main matters to research to further increase the upper bound of time delay: constructing an appropriate Lyapunov–Krasovskii functional and estimating the derivative of the functional. In terms of more accurate estimation of derivative, over the past few years, many approaches have been proposed by researchers. For example, the model transformation method was employed in [26]. Free weighting matrix approach was researched in [27]. Later, researchers began to pay attention to Jensen inequality to derive the better upper bound of delay [28]. Wirtinger-based inequality has been utilized to estimate the derivative of Lyapunov–Krasovskii functional in [29]. In addition, the auxiliary function-based integral inequality has been introduced to various systems [30]. The methods mentioned above are all aiming to deal with the quadratic integral term such as \( - \int_{t-	au_M}^{\infty} \xi(t) W \dot{x}(t) ds \), which can be concluded in the derivative of Lyapunov–Krasovskii functional. Recently, a method called Bessel–Legendre inequality method was proposed in [31], which read as \( - \int_{t-	au_M}^{t} \dot{x}(t) W \dot{x}(t) ds \leq - \int_{t-	au_M}^{t} \dot{x}(t) W \dot{x}(t) ds \), where \( W_N = \text{diag}[W, 3W, \ldots, (2N+1)W] \), \( \Psi_N = (1/\tau_M) \int_{t-	au_M}^{t} L_N ((t-s) \tau_M) \xi(s) ds \), \( N \geq 0 \), and \( L_N \) is the “shifted” Legendre polynomial matrix. The criterion of stability is related to the order \( N \), and its conservatism will decrease as \( N \) grows. Now, a suitable Lyapunov–Krasovskii functional will be established, and a larger delay upper bound of the event-triggered cyber-attacked NCS will be got by means of Bessel–Legendre inequality.

Consequently, the following questions on the comprehensive NCS under cyberattacks will be addressed:

1. In order to save network transmission resources and improve dynamic property, we expect that suitable triggering scheme can realize that more triggers at the initial times and less triggers at the period tend to stable. How to devise an improved distributed event-triggered mechanism to the comprehensive delay-dependent NCS under cyberattacks for achieving above expectation?

2. Whether can we apply the Bessel–Legendre inequality approach to the investigation of stability for the system in this article? How to establish a powerful Lyapunov–Krasovskii functional applicable to the Bessel–Legendre inequality method?

3. Under the improved distributed event-triggered scheme, is it possible to design an effective controller to the NCS?

Motivated by the aforementioned challenges, the major contributions are listed as follows: (1) A more practical model of the networked control system subject to cyberattacks and time delay is constructed. A novel distributed event-triggered scheme is established for the comprehensive system researched in this paper, which can not only accelerate the system dynamics but also reduce communication burden. (2) For the analysis of cyber-attacked NCS, not alike previous researches, this paper constructs a Lyapunov–Krasovskii functional with respect to Legendre polynomials and applies Bessel–Legendre inequality approach to acquire a less conservative stability condition, which is related to the order \( N \). When \( N \) increases, the upper bound of delay increases. (3) An effective controller is devised.

The remainder of this article is organized as follows: definitions and problem formulation are described in Section 2. In addition, the proposed improved distributed event-triggered scheme is also given in Section 2. Section 3 gives the specific stability analysis process. A controller is designed in Section 4. In Section 5, numerical examples are shown to illustrate the effectiveness of the proposed method. At last, conclusions are described in Section 6.

Notations 1. \( \text{Prob}[X] \) means probability of event \( X \) occurring. \( \| \cdot \|_2 \) denotes the Euclidean vector norm. \( N \) means the natural number. For any matrix \( A, \text{He}(A) = A + A^T \). \( \mathbb{E}\{ \cdot \} \) means the mathematical expectation. \( \mathbb{S} \) expresses the set of positive definite matrices of \( \mathbb{R}^{n \times n} \), \( \mathbb{R}^{n \times n} \) denotes the set of all \( n \times n \) matrices.
2. Definitions and Problem Formulation

The considered state space model of NCS is described as follows:

$$\dot{x}(t) = Ax(t) + Bu(t),$$  \hspace{1cm} (1)

where $x(t) \in \mathbb{R}^n$ is the state of the system, $u(t) \in \mathbb{R}^p$ is the control input, $A$ and $B$ are the known constant matrices.

It is well known that the event-triggered scheme can compensate the shortcoming of traditional periodic-triggered scheme. For example, under the traditional periodic-triggered scheme, some unnecessary signals can be sent to the channel, which places a burden on the limited bandwidth. Consider that the system has multiple sensors. In order to not only reduce the network transmission burden but also improve the system dynamics, a novel distributed event-triggered scheme will be introduced. The schematic diagram of distributed event-triggered NCS with cyberattacks is shown in Figure 1.

Define that $t_{lh}, t_{jh}, t_{h}t_{h}, \ldots$ as release times, which means that the sampled states at $t_{lh}, t_{jh}, t_{h}t_{h}, \ldots$ satisfy event-triggering condition and can be sent to the transmission channel. System (1) should be described as

$$\dot{x}(t) = Ax(t) + Bu(t_k h).$$  \hspace{1cm} (2)

Due to $u(t) = \text{K}x(t)$, we have

$$\dot{x}(t) = Ax(t) + B\text{K}x(t_k h),$$ \hspace{1cm} (3)

where $x(t_k h) = \begin{bmatrix} x_1^l(t_k h)^T & x_2^l(t_k h)^T & \cdots & x^n(t_k h)^T \end{bmatrix}^T$ and $K = \text{diag}(K_1, K_2, \ldots, K_n)$.

In fact, there exists a time delay during the process of signal transmission. Assume that $t^l_k \in (0, \tau)$ is the transmission delay, where $\tau$ is a positive scalar, $l = 1, 2, \ldots, n$. The released state $x^l(t^l_k h)$ arrives at the actuator at the time $t^l_k h + r_k$. Next, we will establish the system model with network transmission delay. Define that

$$\phi_k = \min \{\alpha | t^l_k h + r^l_k + ah \geq t^l_{k+1} h + r^l_{k+1}, \alpha = 0, 1, 2, \ldots \}.$$ \hspace{1cm} (4)

Let

$$l^l_u = \left[ t^l_k h + r^l_k + (\alpha - 1)h, t^l_k h + r^l_k + ah \right], \hspace{1cm} \alpha = 1, 2, \ldots, \phi_k - 1,$$
$$l^l_{pk} = \left[ t^l_k h + r^l_k + (\phi_k - 1)h, t^l_{k+1} h + r^l_{k+1} \right].$$  \hspace{1cm} (5)

Then,

$$\left[ t^l_k h + r^l_k, t^l_{k+1} h + r^l_{k+1} \right] = \bigcup_{\alpha = 1}^{\phi_k} l^l_u.$$ \hspace{1cm} (6)

For $t \in \left[ t^l_k h + r^l_k, t^l_{k+1} h + r^l_{k+1} \right)$, define that

$$t' = \begin{cases} t - t^l_k h, & t \in l^l_1, \\ t - t^l_k h - h, & t \in l^l_2, \\ \vdots & \\ t - t^l_k h - (\phi_k - 1)(h), & t \in l^l_{\phi_k}. \end{cases}$$ \hspace{1cm} (7)

$$e^l_k(t) = \begin{cases} 0, & t \in l^l_1, \\ x(t^l_k h) - x(t^l_k h + h), & t \in l^l_2, \\ \vdots & \\ x(t^l_k h) - x(t^l_k h + h + (\phi_k - 1)h), & t \in l^l_{\phi_k}. \end{cases}$$

Apparently, $0 < t^l_k \leq t' \leq \tau + h$. Let $\tau_M = \tau + h$, then $0 < t^l_k \leq t' \leq \tau_M$. In order to shorten the system dynamic process, we expect that there are more packets transmitted at the initial times and the triggering frequency lowers when the system gets close to the steady state. Thus, we introduce the time-varying parameter $\sigma_k(t)$, $\sigma_k(t) = \text{diag}[\sigma_1^k(t), \sigma_2^k(t), \ldots, \sigma_n^k(t)]$. Set $l = 1, 2, \ldots, \phi_k$, and

$$\sigma^l_k(t) = \sigma^l_{ka}, \hspace{1cm} t \in l^l_u, \alpha = 1, 2, \ldots, \phi_k,$$ \hspace{1cm} (8)

where

$$\sigma^l_{k+1} = \sigma + \frac{(\sigma^l_{ka})^T e^l_{ka} e^l_{ka}}{\varepsilon + (e^l_{ka})^T e^l_{ka} (\sigma_{ka} - \sigma)},$$ \hspace{1cm} (9)

$$\sigma^l_{ka} = x^l(t^l_k h) - x^l \left( t^l_k h + (\alpha - 1)h \right), \alpha = 1, 2, \ldots, \phi_k,$$ \hspace{1cm} (10)

known constant $\varepsilon > 0$, $\sigma_{ka} = \text{diag}[\sigma_{1_{ka}}, \sigma_{2_{ka}}, \ldots, \sigma_{n_{ka}}]$, $\sigma$ is the upper bound of $\sigma_{ka}$, and $\sigma \in [0, 1]$, $0 < \sigma_{1_{ka}} \leq \sigma$. Next, we put forward the following improved static distributed event-triggered scheme which contains the time-varying parameter $\sigma_k(t)$ for $t \in [t_k h + r_k, t_{k+1} h + r_{k+1})$:

$$e^l_k(t) \Delta \dot{e}^l_k(t) \leq \sigma_k(t) \Lambda x(t - \tau(t)) \Lambda x(t - \tau(t)),$$ \hspace{1cm} (11)
where \( x(t - \tau(t)) = [x^1(t - \tau^1(t))^T \ldots x^n(t - \tau^n(t))^T]^T \) and \( \Lambda \) is the symmetric positive definite matrix satisfying \( \Lambda = \text{diag}[\Lambda_1, \Lambda_2, \ldots, \Lambda_n] \).

Using \( c_i(t) \) and \( \tau(t) \), for \( t \in [t_0 h + \tau_k, t_{k+1} h + \tau_{k+1}] \), we rewrite \( u(t) \) as

\[
    u(t) = BKx(t - \tau(t)) + BKc_k(t). \tag{12}
\]

Consider cyberattacks launched by adversaries whose aim is to attack the controller. Thus, \( u(t) \) can be described as follows:

\[
    u(t) = (1 - \beta(t))BK[x(t - \tau(t)) + c_k(t)] + \beta(t)f(x(t)), \tag{13}
\]

where the variable \( \beta(t) \) satisfies the Bernoulli distribution. \( \text{Prob}[\beta(t) = 1] = \beta \), \( \text{Prob}[\beta(t) = 0] = 1 - \beta \), \( 0 \leq \beta \leq 1 \). When \( \beta = 1 \), cyberattacks occur. When \( \beta = 0 \), the released signals will be sent through network without cyberattacks.

Nonlinear function \( f(x(t)) \) denotes the cyberattack characteristics. Next, with the novel distributed event-triggered mechanism, the complete model of cyber-attacked NCS with time delay is

\[
    \dot{x}(t) = Ax(t) + (1 - \beta(t))[BKx(t - \tau(t)) + BKc_k(t)] + \beta(t)f(x(t)),
\]

\[
    x(t) = \phi(t), t \in [-\tau_M, 0], \tag{14}
\]

where the function \( \phi(t) \) is continuous on \([-\tau_M, 0]\). Note that

\[
    0 < \tau_k \leq \tau(t) \leq \tau_M, \quad d_1 \leq \tau(t) \leq d_2, \tag{15}
\]

where scalars \( d_1 < 0 \) and \( d_2 > 0 \).

To facilitate the analysis, some assumptions and lemmas are given as follows.

**Assumption 1.** The nonlinear function \( f(x(t)) \) which determines stochastic cyberattacks satisfies the following condition:

\[
    \|f(x(t))\|_2 \leq \|Gx(t)\|_2, \tag{16}
\]

where \( G \) is a known constant matrix.

**Lemma 1** (see [32]). Suppose that symmetric positive matrices \( W_1, W_2 \in \mathbb{R}^n \). For \( \epsilon = 0, 1 \), if there exist the symmetric matrices \( X_1, X_2 \in \mathbb{R}^n \) and \( Y_1, Y_2 \in \mathbb{R}^n \) such that the following inequality holds:

\[
    \begin{bmatrix}
        W_1 & 0 \\
        0 & W_2
    \end{bmatrix}
    - \epsilon
    \begin{bmatrix}
        X_1 & Y_1 \\
        Y_1^T & 0
    \end{bmatrix}
    - (1 - \epsilon)
    \begin{bmatrix}
        0 & Y_2 \\
        Y_2^T & 0
    \end{bmatrix}
    \geq 0, \tag{17}
\]

Then, for all \( \epsilon \in (0, 1) \),

\[
    \begin{bmatrix}
        1 & W_1 \\
        0 & 1 - \epsilon
    \end{bmatrix}
    \geq
    \begin{bmatrix}
        W_1 & 0 \\
        0 & W_2
    \end{bmatrix}
    + \epsilon
    \begin{bmatrix}
        0 & Y_1 \\
        Y_1^T & X_2
    \end{bmatrix}
    + (1 - \epsilon)
    \begin{bmatrix}
        0 & X_1 \\
        X_1^T & 0
    \end{bmatrix}
    \tag{18}
\]

is true.

The definitions about Legendre polynomials and the properties of polynomials matrix will be presented as below.

**Definition 1.** For any \( u \in [0, 1] \), \( f, q \in \mathbb{N} \), the “shifted” Legendre polynomial is

\[
    L_f(u) = (-1)^f \sum_{q=0}^{f} \rho_f^q u^q, \tag{19}
\]

where \( \rho_f^q = (-1)^q \binom{f}{q} \binom{f + q}{q} \) and the binomial coefficient \( \binom{f}{q} = \binom{f}{f-q} q^q \).

Correspondingly, the polynomial matrix \( L_N \) is described as

\[
    L_N(u) := \left[ L_0(u) I_n, L_1(u) I_n, \ldots, L_N(u) I_n \right]^T, \tag{20}
\]

where \( n \in \mathbb{N}, N \in \mathbb{N} \). Due to that, the Legendre polynomials have the orthogonality property. Thus, for any symmetric positive definite matrix \( W \), the equation

\[
    \int_{0}^{1} L_N(u) W^{-1} L_N^T(u) du = W_N^{-1}, \tag{21}
\]

is true, where \( W_N = \text{diag}[W, 3W, \ldots, (2N+1)W] \).

The evaluation values of the polynomial matrix boundaries \( L_N(0) \) and \( L_N(1) \) are shown as follows:

\[
    L_N(0) = \begin{bmatrix}
        I_n \\
        -I_n \\
        \vdots \\
        (-1)^N I_n
    \end{bmatrix} = \overline{X}_N, \tag{22}
\]

\[
    L_N(1) = \begin{bmatrix}
        I_n \\
        I_n \\
        \vdots \\
        I_n
    \end{bmatrix} = X_N. \tag{23}
\]

Next, we give the derivative about the Legendre polynomials matrix which will be employed in the proof process of system stability.
defined as 
\[ (s-a) x(s) ds, \]
where 
\[ \Psi_N = \frac{1}{b-a} \int_a^b L_N \left( \frac{s-a}{b-a} \right) x(s) ds, \]
holds, where 
\[ W_N = \text{diag}(W, 3W, \ldots, (2N+1)W). \]

Remark 1. For \(-\int_a^b x^T(s)Wx(s) ds\), Bessel–Legendre inequality can estimate a tighter upper bound than other methods. In addition, the obtained upper bound can be as tight as possible along with \(N\) approaching to infinity. Consequently, the stability criterion to be obtained next will be less conservative, and the effectiveness will be verified in final example.

3. Stability Analysis

With the improved distributed event-triggered mechanism, the less conservative stability criterion of cyber-attacked NCS (14) is obtained via Bessel–Legendre inequalities. The main results are shown in Theorem 1.

**Theorem 1.** Given \(N \in \mathbb{N}\), scalar \(\varepsilon > 0\), \(0 \leq \beta \leq 1\), if there exist matrix \(P_N \in \mathbb{S}^{(2N+3)n}\), matrices \(Q_1, Q_2, W \in \mathbb{S}_n\), and \(Y_1, Y_2 \in \mathbb{R}^{(N+1)n} \) for all \((\tau(t), \dot{\tau}(t)) \in \mathcal{H}\) making the following inequality:

\[
\begin{bmatrix}
\Psi_N & \Pi_N^T Y_1 \\
\Pi_N Y_1^T & \tau(t) Y_2^T \end{bmatrix} \geq \begin{bmatrix}
\tau_M - \tau(t) & 0 \\
0 & \tau_M \end{bmatrix} \begin{bmatrix}
Y_2 & Y_1^T \\
Y_1 & \tau_M \end{bmatrix} \geq 0,
\]

\[ W_N = \text{diag}(W, 3W, \ldots, (2N+1)W). \]
true; then, system (14) is stable, where
\[
\mathcal{K} = \text{Col}\{0,0,(d_2), (\tau_M, 0), (\tau_M, d_1)\},
\]
\[
\mathcal{N} = \text{He}\left( (E_{1N} + \tau(t))E_{2N} \right) \Phi_N + E_{3N} - \Pi_N^T \Lambda_N \Pi_N,
\]
\[
\Pi_N = \begin{bmatrix}
\chi_N & -\bar{\chi}_N & 0 & 0 & 0 \\
0 & 0 & -\bar{\chi}_N & \chi_N & 0 \\
0 & 0 & \chi_N & -\bar{\chi}_N & 0 \\
0 & 0 & -\bar{\chi}_N & \chi_N & 0 \\
0 & 0 & \chi_N & -\bar{\chi}_N & 0 \\
\end{bmatrix},
\]
\[
\mathcal{W}_N = \text{diag}(W, 3W, \ldots, (2N + 1)W),
\]
\[
E_{1N} = \begin{bmatrix}
A & 0 & 0 & (1 - \bar{\beta})BK & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
\end{bmatrix},
\]
\[
E_{2N} = \begin{bmatrix}
0 & \Xi_N & 0 & 0 & 0 \\
0 & 0 & \Xi_N & 0 & 0 \\
0 & 0 & \Xi_N & 0 & 0 \\
0 & 0 & \Xi_N & 0 & 0 \\
0 & 0 & \Xi_N & 0 & 0 \\
\end{bmatrix},
\]
\[
\Phi_N = \begin{bmatrix}
I_n & 0 & 0 & 0 & 0 & 0 \\
0 & \tau(t)I_{nN} & 0 & 0 & 0 & 0 \\
0 & 0 & (\tau_M - \tau(t))I_{nN} & 0 & 0 & 0 \\
\end{bmatrix},
\]
\[
\Lambda_N = \begin{bmatrix}
\mathcal{W}_N & 0 \\
0 & \mathcal{W}_N \\
0 & \mathcal{W}_N \\
0 & \mathcal{W}_N \\
\end{bmatrix} + \tau(t) \begin{bmatrix}
0 & Y_1 \\
Y_1^T & \mathcal{W}_N \\
0 & \mathcal{W}_N \\
0 & \mathcal{W}_N \\
\end{bmatrix} + \left[ \begin{array}{ccc}
\tau_M - \tau(t) \\
\tau_M - \tau(t) \\
\tau_M - \tau(t) \\
\tau_M - \tau(t) \\
\end{array} \right] \begin{bmatrix}
\mathcal{W}_N & Y_2 \\
0 & Y_2 \\
0 & Y_2 \\
0 & Y_2 \\
\end{bmatrix},
\]
\[
E_{3N} = \text{diag}\{Q_1 + G^T \Gamma, 0, 0, (1 - \bar{\beta})BK - \Lambda, -\bar{\Lambda}, -I_n\},
\]
\[
+ \sigma_h(t)\Lambda, -\bar{\Lambda}, -I_n, \gamma_1 = \begin{bmatrix}
A & 0 & 0 & (1 - \bar{\beta})BK & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
\end{bmatrix},
\]
\[
\gamma_2 = \begin{bmatrix}
0 & 0 & 0 & \bar{\beta}BK & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
\end{bmatrix},
\]
\[
\eta_N^T(t) = \begin{bmatrix}
x^T(t) & \Psi_{1N}^T(t) & \Psi_{1N}^T(t) & x^T(t - \tau(t)) & x^T(t - \tau_M) & \sigma_h^T(t) & f^T(x(t)) \end{bmatrix},
\]
\[
\xi_N^T(t) = \begin{bmatrix}
x^T(t) & \frac{d}{dt}\left[ \tau(t)\Psi_{1N}(t) \right] & \frac{d}{dt}\left[ (\tau_M - \tau(t))\Psi_{2N}(t) \right] \end{bmatrix},
\]
\[
\mathcal{N} \in \mathbb{R}^{(N+1)nx(N+1)n} \text{ and } I_n \in \mathbb{R}^{n}.
\]

Proof. Establish a Lyapunov–Krasovskii functional with respect to Legendre polynomials matrix as
\[
V_N(x(t), \dot{x}(t), t) = V_{1N}(x(t), t) + V_{2N}(x(t), t)
\]
\[
+ V_{3N}(x(t), \dot{x}(t), t),
\]
where
\[
V_{1N}(x(t), t) = \xi_N^T(t)P_N \xi_N(t),
\]
\[
V_{2N}(x(t), t) = \int_{t-\tau(t)}^t x^T(s)Q_1x(s)ds + \int_{t-\tau_M}^{t-\tau(t)} x^T(s)Q_2x(s)ds,
\]
\[
V_{3N}(x(t), \dot{x}(t), t) = \tau_M \int_{t-\tau_M}^t \dot{x}^T(s) W \dot{x}(s) ds dt,
\]
where
\[
\Psi_{1N}(t) = \begin{bmatrix}
x^T(t) & \tau(t) \Psi_{1N}(t) & (\tau_M - \tau(t)) \Psi_{2N}(t) \end{bmatrix},
\]
\[
\Psi_{1N}(t) = \begin{bmatrix}
\frac{1}{\tau(t)} \int_{t-\tau(t)}^t L_N(s - t + \tau(t))x(s)ds, \xi_N(t)
\end{bmatrix},
\]
\[
\Psi_{2N}(t) = \begin{bmatrix}
\frac{1}{\tau_M - \tau(t)} \int_{t-\tau_M}^{t-\tau(t)} L_N(s - t + \tau_M)\xi_N(s)ds
\end{bmatrix},
\]

Before obtaining the derivative of $V_{1N}$, define that

(30)
\[ \dot{x}(t) = Ax(t) + (1 - \beta(t))[BKx(t - \tau(t)) + BK\varepsilon_k(t)] \\
+ \beta(t)f(x(t)), \]
\[ \Psi_{1,N}(t) = \frac{1}{\tau(t)} \int_{t-\tau(t)}^{t} L_N(\frac{s-t+\tau(t)}{\tau(t)}) x(s) ds, \]
\[ \Psi_{2,N}(t) = \frac{1}{\tau_M - \tau(t)} \int_{t-\tau_M}^{t-\tau(t)} L_N(\frac{s-t+\tau_M}{\tau_M - \tau(t)}) x(s) ds. \]

Using the subsection integration method and employing (23), (25), we can get
\[ \tau(t) \int_{0}^{1} \lambda L_N(\lambda) \dot{x}(s(\lambda)) d\lambda = \int_{0}^{1} \lambda L_N(\lambda) dx(s(\lambda)) \]
\[ = L_N(1)x(t) - \int_{0}^{1} x(s(\lambda)) \frac{d}{d\lambda} (\lambda L_N(\lambda)) d\lambda \] (40)
\[ = L_N(1)x(t) - \Psi_{1,N}(t) - \Xi_N \Psi_{1,N}(t) \]
\[ = \chi_N x(t) - \Psi_{1,N}(t) - \Xi_N \Psi_{1,N}(t). \]

Correspondingly, for \( \tau(t) \int_{0}^{1} \lambda L_N(\lambda) \dot{x}(s(\lambda)) d\lambda \), using the subsection integration and employing (20), (24), we obtain
\[ \tau(t) \int_{0}^{1} L_N(\lambda) \dot{x}(s(\lambda)) d\lambda = \int_{0}^{1} L_N(\lambda) dx(s(\lambda)) \]
\[ = L_N(1)x(t) - L_N(0)x(t-\tau(t)) - \int_{0}^{1} x(s(\lambda)) \frac{d}{d\lambda} (L_N(\lambda)) d\lambda \]
\[ = \chi_N x(t) - \Xi_N x(t-\tau(t)) - \Psi_{1,N}(t). \]

Thus,
\[ \frac{d}{dt}[\tau(t)\Psi_{1,N}(t)] = \dot{\tau}(t)\Psi_{1,N}(t) + \dot{\tau}(t)[\chi_N x(t) - \Psi_{1,N}(t)] \\
- \Xi_N \Psi_{1,N}(t)] + (1 - \dot{\tau}(t))[\chi_N x(t) - \Xi_N x(t-\tau(t)) - \Psi_{1,N}(t)]. \]

Set \( \lambda = s - t + \tau(t)/\tau(t) \), then \( s = \lambda \tau(t) + t - \tau(t) \), and rewriting \( \Psi_{1,N}(t) \) as
\[ \Psi_{1,N}(t) = \int_{0}^{1} L_N(\lambda) x(s(\lambda)) d\lambda, \] (38)
then,
\[ \Psi_{2,N}(t) = \int_{0}^{1} L_N(u) x(u) d\lambda. \] (39)

\[ u = (s - t + \tau(t))/\tau_M - \tau(t) \] and \( s = ut_{\tau_M} - ut - t - \tau_M \), then we rewrite \( \Psi_{2,N}(t) \) as
\[ \Psi_{2,N}(t) = \int_{0}^{1} L_N(u) x(s(u)) du. \] (43)

Then,
\[ \frac{d}{dt}[(\tau_M - \tau(t))\Psi_{2,N}(t)] = -\dot{\tau}(t)\Psi_{2,N}(t) + (\tau_M - \tau(t)) \\
\cdot \int_{0}^{1} L_N(u) x(s(u))[-u\dot{t}(t) + 1] du \]
\[ = -\dot{\tau}(t)\Psi_{2,N}(t) + (\tau_M - \tau(t)) \int_{0}^{1} u L_N(u) x(s(u)) du + (\tau_M - \tau(t)) \int_{0}^{1} L_N(u) x(s(u)) du, \]
\[ \text{where} \]
\[ (\tau_M - \tau(t)) \int_{0}^{1} u L_N(u) x(s(u)) du = \int_{0}^{1} u L_N(u) x(s(u)) du \]
\[ = L_N(1)x(t - \tau(t)) - \Psi_{2,N}(t) - \Xi_N \Psi_{2,N}(t) \]
\[ = \chi_N x(t - \tau(t)) - \Psi_{2,N}(t) - \Xi_N \Psi_{2,N}(t), \]
\[ (\tau_M - \tau(t)) \int_{0}^{1} L_N(u) x(s(u)) du = \int_{0}^{1} L_N(u) x(s(u)) du \]
\[ = L_N(1)x(t - \tau(t)) - L_N(0)x(t - \tau_M) - \Psi_{2,N}(t) \]
\[ = \chi_N x(t - \tau(t)) - \Xi_N x(t - \tau_M) - \Psi_{2,N}(t). \] (44)

Thus, \( (\tau_M - \tau(t))\Psi_{2,N}(t) \) becomes
\[ \frac{d}{dt}[(\tau_M - \tau(t))\Psi_{2,N}(t)] \]

Obviously, by integral calculation, \( (d/dt)\tau(t)\Psi_{1,N}(t) \) in \( \xi_N(t) \) is derived formed by the elements in \( \eta_N(t) \), which can facilitate the realization of inequality in Theorem 1. Next, we deal with \( (d/dt)(\tau_M - \tau(t))\Psi_{2,N}(t) \). Set
where the matrix $E_{3,N}$ is defined in Theorem 1:
\[
\dot{V}_{3,N} = r_{2M}^2 \dot{x}(t) W \dot{x}(t) - \tau_{M} \int_{t-M}^{t} \chi^T(s) W \dot{x}(s) ds.
\]

Due to that, $\dot{x}(t)$ can be written as
\[
\dot{x}(t) = \Omega_1 + (\bar{\beta} - \beta(t)) \Omega_2,
\]
where
\[
\Omega_1 = A x(t) + (1 - \bar{\beta}) [B K x(t - \tau(t)) + B K e_k(t)] + \bar{\beta} f(x(t)),
\]
\[
\Omega_2 = (\bar{\beta} - \beta(t))[B K x(t - \tau(t)) + B K e_k(t) - f(x(t))].
\]

Employing $\mathcal{E} \{ \dot{\beta} - \beta(t) \} = 0$, $\mathcal{E} \{ (\bar{\beta} - \beta(t)) \dot{x} \} = \bar{\beta}(1 - \bar{\beta})$, then we get
\[
\mathcal{E} \{ r_{2M}^2 \dot{x}(t) W \dot{x}(t) \} = \tau_{M} r_{1M}^2 W \Omega_1 + \tau_{M}^2 (1 - \bar{\beta}) \Omega_2^T W \Omega_2.
\]

Using Lemma 2, we have
\[
\tau_{M} \int_{t-M}^{t} \chi^T(s) W \dot{x}(s) ds \geq \left[ \begin{array}{c} \Psi_{1,N}^T(t) \\ \Psi_{2,N}^T(t) \end{array} \right]^T \left[ \begin{array}{cc} \Psi_{1,N}^T(t) & 0 \\ 0 & \tau_{M} (\tau_{M} - \tau(t)) W N^{-1} \Psi_{2,N}^T(t) \end{array} \right] \\
\left[ \begin{array}{c} \Psi_{1,N}^T(t) \\ \Psi_{2,N}^T(t) \end{array} \right],
\]
where
\[
\Psi_{1,N} = \frac{1}{\tau(t)} \int_{t-\tau(t)}^{t} N^{-1} \left( s - t + \tau(t) \right) \dot{x}(s) ds,
\]
\[
\Psi_{2,N} = \frac{1}{\tau_{M} - \tau(t)} \int_{t-M}^{t} N^{-1} \left( s - t + \tau_{M} \right) \dot{x}(s) ds.
\]

Employing the subsection integration method, we obtain
\[
\mathcal{E} \{ \dot{V}_{1,N} \} = 2 \xi_{N}^{T}(t) P_{N}^{-1} \tilde{\xi}_{N}(t) = \eta(t) H \left( E_{1,N} + \dot{\tau}(t) E_{2,N} \right) P_{N}^{-1} \Phi_{N} \eta(t).
\]

Next, we deal with $\mathcal{E} \{ \dot{V}_{2,N} \}$. According to Assumption 1, the following inequality is true:
\[
\xi^T(t) G x(t) - f^T x(t) f(x(t)) \geq 0.
\]

Add (49) and term $e_k^T(t) A e_k(t) - e_k^T(t) A e_k(t)$ on $\dot{V}_{2,N}$, then
\[
\mathcal{E} \{ \dot{V}_{2,N} \} \leq x^T(t) Q_N x(t) + (1 - \bar{\beta}) x^T(t - \tau(t)) (Q_2 - Q_1) x(t - \tau(t))
\]
\[
- x^T(t - \tau_M) Q_N x(t - \tau(t)) + e_k^T(t) A e_k(t) - e_k^T(t) A e_k(t) + x^T(t) G^T G x(t) - f^T x(t) f(x(t))
\]
\[
\leq \eta(t) H \left( E_{1,N} + \dot{\tau}(t) E_{2,N} \right) P_{N}^{-1} \Phi_{N} \eta(t).
\]
\[
\Lambda_N = \begin{bmatrix} W_N & 0 \\ W_N & \tau(t) Y_1 \\ 0 & W_N \end{bmatrix} + \frac{\tau(t) - \tau_1}{\tau_M} \begin{bmatrix} 0 & Y_1 \\ Y_1 & W_N \end{bmatrix} + \frac{\tau_M - \tau(t) - \tau_1}{\tau_M} \begin{bmatrix} W_N & Y_2 \\ Y_2 & 0 \end{bmatrix}.
\]

\[
\Lambda'_N = \begin{bmatrix} \frac{\tau(t) - \tau_1}{\tau_M} Y_1 W_N & 0 & \tau(t) Y_1 W_N \\ \frac{\tau(t) - \tau_1}{\tau_M} Y_2 & 0 & \tau(t) Y_2 W_N \end{bmatrix}.
\]

Then, using equations (54) and (60), we obtain \(E\{V_{3N}\}\) as follows:

\[
E\{V_{3N}\} = \eta^T(t) \left[ \Pi^T N_A N N - \Pi^T N_A N N \right] \eta(t) + \tau^2 M \Omega^T_1 W \Omega_1 + \tau^2 M \beta \Omega^T_2 W \Omega_2.
\]

Combining (48), (50) and (62), \(E\{V_N\}\) satisfies that

\[
E\{V_N\} \leq \eta^T(t) \left[ H\left( E_{1, N} + \hat{\tau}(t) E_{2, N} \right)^T P_N \Phi N \right] + E_{3, N} + \Pi^T N_A N N - \Pi^T N_A N N
\]

\[
\eta(t) + \tau^2 M \beta \Omega^T_2 W \Omega_2
\]

\[
= \eta^T(t) \Theta N \eta(t),
\]

where

\[
\Theta N = \Theta N + \Pi^T N_A N N + \tau^2 M \Gamma^T_1 W \Gamma_1 + \tau^2 M \beta \Omega^T_2 W \Omega_2.
\]

\[
\Theta_N = H\left( E_{1, N} + \hat{\tau}(t) E_{2, N} \right)^T P_N \Phi N \right] + E_{3, N} + \Pi^T N_A N N
\]

\[
\Gamma_1 = \begin{bmatrix} A & 0 & 0 & (1 - \beta) BK & 0 & (1 - \beta) BK \end{bmatrix},
\]

\[
\Gamma_2 = \begin{bmatrix} 0 & 0 & BK & 0 & BK & -I_n \end{bmatrix}.
\]

Due to

\[
(I_n - W)^{-1} (I_n - W) > 0,
\]

\[
-W^{-1} \leq -(2I_n - W).
\]

By Schur complement, if \(\Theta N \leq 0\) for all \((\tau(t), \hat{\tau}(t)) \in \mathcal{R} = \mathcal{R}[0, 0], (0, d_2), (\tau_M, 0), (\tau_M, d_1)\), then \(\Theta N \leq 0\), which guarantees \(E\{V_N\} \leq 0\) for all \((\tau(t), \hat{\tau}(t)) \in \mathcal{R} \).

\(\square\)

**Remark 2.** \(\Theta N\), which is defined in Theorem 1 is multi-affine on \((\tau(t), \hat{\tau}(t))\), where \((\tau(t), \hat{\tau}(t)) \in \mathcal{R} = \mathcal{R}[0, 0], (0, d_2), (\tau_M, 0), (\tau_M, d_1)\). It is deserved to mention that the solution of LMI (29) with allowable delay set \(\mathcal{R}\) has lower conservatism than that with allowable delay set \([0, \tau_M] \times [d_1, d_2]\). Because of the impossible situations that \(\hat{\tau}(t)\) is negative when \(\tau(t) = 0\) and \(\hat{\tau}(t)\) is positive when \(\tau(t) = \tau_M\), the vertices \((0, d_1)\) and \((\tau_M, d_2)\) will never be reached at any time.

### 4. Stabilization Analysis

This part will design a controller for system (14) under the distributed event-triggered mechanism. Specific results are in Theorem 2.

**Theorem 2.** Given \(N \in \mathbb{N}\), scalar \(\epsilon > 0\), \(0 \leq \beta \leq 1\), system (14) is stable if there exist \(\hat{P}_{1, N} \in \mathbb{S}^n_+\), \(\hat{P}_{2, N} \in \mathbb{S}^{(N+1)n}_+\), \(\hat{P}_{3, N} \in \mathbb{S}_+^{(N+1)n}\), matrices \(Q_1, Q_2, W \in \mathbb{S}^n_+\), and \(Y_1, Y_2 \in \mathbb{R}^{(N+1)n_\infty (N+1)n}\) such that the following inequality:

\[
\begin{bmatrix}
\hat{\Sigma}_N & \Pi^T N_A N N & \Pi^T N_A N N & \Pi^T N_A N N
\\
\Pi^T N_A N N & \Pi^T N_A N N & \Pi^T N_A N N & \Pi^T N_A N N
\\
\Pi^T N_A N N & \Pi^T N_A N N & \Pi^T N_A N N & \Pi^T N_A N N
\\
\Pi^T N_A N N & \Pi^T N_A N N & \Pi^T N_A N N & \Pi^T N_A N N
\\
\end{bmatrix}
\leq 0,
\]

is true for all \((\tau(t), \hat{\tau}(t)) \in \mathcal{R}, \) where
and other terms are defined in Theorem 1.

Proof. From (29) in Theorem 1, we can find the existence of nonlinear terms such as $E_{12}^T P_N \Phi_N$. Divide the matrix $P_N$ into block matrix as

$$P_N = \begin{bmatrix} \tilde{P}_{1N} & 0 & 0 \\ 0 & \tilde{P}_{2N} & 0 \\ 0 & 0 & \tilde{P}_{3N} \end{bmatrix},$$  \hspace{1cm} (69)$$

where $P_{1N} \in S_+^n$, $P_{2N} \in S_+^{(N+1)n}$, and $P_{3N} \in S_+^{Nn}$. The concrete expressions of the nonlinear terms become $K^T B^T \tilde{P}_{1N}$ and $\tilde{P}_{1N} BK$ in $E_{12} P_N$. To eliminate the nonlinear terms, set $K = -B^{-1} \tilde{P}_{1N}^{-1}$. Then, $K^T B^T \tilde{P}_{1N}$ and $\tilde{P}_{1N} BK$ become $-I_n$.

Moreover, $BK$ will be replaced by $\tilde{P}_{1N}^{-1}$. Due to

$$\left(I_n - \tilde{P}_{1N}^{-1}\right) \tilde{P}_{1N}^{-1} (I_n - \tilde{P}_{1N}) \geq 0,$$  \hspace{1cm} (70)$$

$$-\tilde{P}_{1N}^{-1} \leq \tilde{P}_{1N}^{-1} - 2I_n.$$  \hspace{1cm} (71)$$

We replace $BK$ with $\tilde{P}_{1N} - 2I_n$, then we obtain

$$\tilde{\Gamma}_1 = \begin{bmatrix} A & 0 & 0 \\ 0 & (1 - \tilde{\beta})(\tilde{P}_{1N}^{-1} - 2I_n) & 0 \\ 0 & 0 & (1 - \tilde{\beta})(\tilde{P}_{1N}^{-1} - 2I_n) \end{bmatrix},$$

$$\tilde{\Gamma}_2 = \begin{bmatrix} 0 & 0 & \tilde{P}_{1N}^{-1} - 2I_n \end{bmatrix}.$$  \hspace{1cm} (72)$$

Thus, we get

$$\tilde{\Theta}_N = \tilde{\Theta}_N + \Pi_N^T \Lambda_N \Pi_N + \tau_M \tilde{\Gamma}_2^T W \Gamma + \tau_M \tilde{\beta} (1 - \tilde{\beta}) \tilde{\Gamma}_2^T W \Gamma,$$  \hspace{1cm} (73)$$

where

$$\tilde{\Theta}_N = \Phi_N^T \left( \Phi_N + \tilde{\Theta}_N \right)^{\top} P_N \Phi_N + E_{3N} - \Pi_N^T \Lambda_N \Pi_N.$$  \hspace{1cm} (74)$$

By Schur complement, if $\tilde{\Theta}_N \leq 0$, then $\tilde{\Theta}_N \leq 0$ in (67) is true. Due to $\tilde{\Theta}_N \geq \tilde{\Theta}_N$, according to Theorem 1, system (14) is stable for any delay in allowable delay set $\mathcal{H}$. Thus, the proof ends.

To check the inequalities in Theorem 1 and Theorem 2, Algorithm 1 is presented.

5. Numerical Examples

Example 1. Consider the following distributed event-triggered delay-dependent NCS with cyberattacks:

$$A = \begin{bmatrix} -0.21 & 1.6 & 0 & 0 \\ 1.55 & -1.9 & 0 & 0 \\ 0 & 0 & -0.72 & 0.40 \\ 0 & 0 & 0.25 & -0.56 \end{bmatrix},$$  \hspace{1cm} (75)$$

$$B = \begin{bmatrix} -0.5 & 0 & 0 & 0 \\ 0 & 1.8 & 0 & 0 \\ 0 & 0 & -1.2 & 0 \\ 0 & 0 & 0 & 0.5 \end{bmatrix}.$$  \hspace{1cm} (76)$$

Take $n = 2$, $x(t) = \begin{bmatrix} x^1(t) & x^2(t) \end{bmatrix}^T$, $\bar{\sigma} = 0.3$, $\sigma_{k1} = 0.01$, $\sigma_{k1} = 0.01$, $\epsilon = 0.01$, $h = 0.36$, $\tilde{\beta} = 0.5$, and $N = 0$. The designed controller gain $K$ and triggering parameter $\Lambda$ are shown in Table 1. The state trajectories are shown in Figure 2.

From Figure 2, we can see that the system states achieve stability in the fourth second. Figure 3 shows the time-varying parameter $\sigma_k(t)$ and the release situation of states $x^1(t), x^2(t)$. Figure 4 shows the time-varying parameter $\sigma_k(t)$ and the release instants of $x^1(t)$. From Figures 3 and 4, $\sigma_k(t)$ and $\sigma_k(t)$ vary from 0.01 to 0.3. The release frequency of the $x(t)$ is higher at the beginning times than other times. When the states tend to be stable, the release times gradually decrease. For state $x^1(t)$, only 47% of the sampled data is sent. For state $x^2(t)$, there is 39% of the sampled data being sent. Thus, the proposed event-triggered scheme shortens the system dynamic process and reduces the transmission burden.

Next, set $h = 0.25$. Under the improved distributed event-triggered scheme with time-varying parameter $\sigma_k(t)$, the release instants and release interval of states $x^1(t)$ and $x^2(t)$ are given in Figures 5 and 6, respectively. If we replace $\sigma_k(t)$ with the constant $\sigma$, the corresponding release instants and
(1) Set the system model parameters $A$ and $B$, the probability parameter $\beta$, and $N=0$. Assume the upper bound of the improved event-triggered scheme $\bar{\sigma}=0.3$, sampling period $h=0.36$, $\epsilon=0.01$, $\sigma_{k1}^1=0.01$, and $\sigma_{k1}^2=0.01$.

(2) Set the $\sigma_k$ satisfying equations (8)–(10). Use LMI toolbox in MATLAB to construct the linear matrix inequality (29) or (67).

(3) Set the initial condition $\tau_M$ and the constrained condition $\tau(t)$, $\dot{\tau}(t)$.

(4) Adjust the value of $\tau_M$ to calculate whether the solution of LMI exists.

(5) If the feasible solution does not exist, stop the calculation and obtain the maximum value of $\tau_M$, the controller gain $K$, and the triggering parameter $\Lambda$, else go to step 4 and increase the value of $\tau_M$.

(6) Stop.

**Algorithm 1:** The algorithm for solving linear matrix inequality.

| $t$ | 0 | 1 | ... | 20 |
|-----|---|---|-----|----|
| $\Lambda$ | $\begin{bmatrix}10.1 & 0.04 & 0.13 & 0.62 \\ 0.04 & 12.3 & 0.64 & 0.78 \\ 0.13 & 0.64 & 21.9 & 0.28 \\ 0.62 & 0.78 & 0.28 & 6.50\end{bmatrix}$ | $\begin{bmatrix}1.47 & 0.01 & 0.17 & 0.33 \\ 0.01 & 6.22 & 0.12 & 0.04 \\ 0.17 & 0.12 & 2.90 & 0.40 \\ 0.33 & 0.04 & 0.40 & 17.3\end{bmatrix}$ | $\begin{bmatrix}0.29 & 0.03 & 0.11 & 0.17 \\ 0.03 & 9.58 & 0.78 & -2 \\ 0.11 & 0.78 & 11.2 & 0 \\ 0.17 & -2 & 0 & 1\end{bmatrix}$ |

| $K$ | $\begin{bmatrix}-1.87 & 0 & 0 & 0 \\ 0 & 0.34 & 0 & 0 \\ 0 & 0 & -0.91 & 0 \\ 0 & 0 & 0 & -0.44\end{bmatrix}$ | $\begin{bmatrix}-0.71 & 0 & 0 & 0 \\ 0 & -0.14 & 0 & 0 \\ 0 & 0 & -2.10 & 0 \\ 0 & 0 & 0 & 1.09\end{bmatrix}$ | $\begin{bmatrix}-2.34 & 0 & 0 & 0 \\ 0 & 0.47 & 0 & 0 \\ 0 & 0 & 0 & 1.00 \\ 0 & 0 & 0 & 0.97\end{bmatrix}$ |

*Figure 2: State trajectories $x(t)$ of the cyber-attacked networked control system.*

*Figure 3: The parameter $\sigma_1^k(t)$ and the release instants of $x^1(t)$.*
The release interval of states $x^{1}(t)$ and $x^{2}(t)$ are shown in Figures 7 and 8, respectively. Based on the improved distributed event-triggered scheme, transmission frequency at the initial times is higher than that under the general distributed event-triggered strategy.

**Example 2.** Take the same system parameters in Example 1. Set different $N$, get the corresponding upper bounds of network time delay with $(\tau(t), \dot{r}(t)) \in \mathcal{H}$. Table 2 shows the comparison of the upper bounds of delay with other papers. If $N = 0$, then $\tau_M = 2.2161$, and it is bigger than other papers.
If $N = 1$, then $\tau_M = 2.3521$. If $N = 2$, then $\tau_M = 2.9552$. Apparently, $\tau_M$ increases with $N$ increasing, which signifies conservativeness of stability criterion decreasing.

### 6. Conclusions

The distributed event-triggered control problem for NCS with time delay was investigated in this paper. We considered a scenario where NCS may suffer from deception cyberattacks. The distributed event-triggered scheme is improved by introducing a time-varying parameter in triggered strategy to achieve that there is a higher frequency of release at the beginning times than other times. To obtain a less conservative stability criterion, the Bessel–Legendre inequality method was applied. In addition, a Lyapunov–Krasovskii functional-related Legendre polynomial was constructed. Consequently, we get a larger upper bound on the time delay. At last, a distributed event-triggered controller was also designed.

Future research directions include, but not limited to, filter research of NCS with multiple cyberattacks and time delay based on novel distribute event-triggered strategy, distributed event-triggered stabilization of cyber-attacked NCS with serious uncertainties, and further improvement of an event-triggered scheme for various specific requirements.

### Data Availability

All data generated or analyzed during this study are included in this article.

### Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

### Acknowledgments

This work was supported by the National Natural Science Foundation of China under Grant No. 61573095 and the Natural Science Foundation of Shandong Province under Grant No. ZR2014FM019.

### References

[1] Q. Xu, Y. Zhang, and S. Xiao, “Networked $H_\infty$ filtering for discrete-time nonlinear singular system with mixed random delays and packet dropouts,” in Proceedings of the 36th Chinese Control Conference (CCC), Dalian, China, July 2017.

[2] Y. L. Wang, T. B. Wang, and Q. L. Han, “Fault detection filter design for data reconstruction-based continuous-time networked control systems,” Information Sciences, vol. 328, pp. 577–594, 2016.

[3] E. Tian, D. Yue, and C. Peng, “Reliable control for networked control systems with probabilistic actuator fault and random delays,” Journal of the Franklin Institute, vol. 347, no. 10, pp. 1907–1926, 2010.

[4] S. Q. Wang, J. Feng, and H. Zhang, “Robust fault tolerant control for a class of networked control systems with state delay and stochastic actuator failures,” International Journal of Adaptive Control and Signal Processing, vol. 28, no. 9, pp. 798–811, 2014.

[5] J. L. Liu, J. Xia, J. Cao, and E. Tian, “Quantized state estimation for neural networks with cyber attacks and hybrid triggered communication scheme,” Neurocomputing, vol. 291, pp. 35–49, 2018.

[6] D. Ding, Z. Wang, D. W. C. Ho, and G. Wei, “Observer-based event-triggering consensus control for multiagent systems with lossy sensors and cyber-attacks,” IEEE Transactions on Cybernetics, vol. 47, no. 8, pp. 1936–1947, 2017.

[7] D. R. Ding, Q.-L. Han, Z. Wang, and X. Ge, “A survey on model-based distributed control and filtering for industrial cyber-physical systems,” IEEE Transactions on Industrial Informatics, vol. 15, no. 5, pp. 2483–2499, 2019.
[8] S. Amin, G. A. Schwartz, and S. S. Sastry, “Security of interdependent and identical networked control systems,” *Automatica*, vol. 49, no. 1, pp. 186–192, 2013.

[9] M. Long, C.-H. Wu, and J. Y. Hung, “Denial of service attacks on network-based control systems: impact and mitigation,” *IEEE Transactions on Industrial Informatics*, vol. 1, no. 2, pp. 85–96, 2005.

[10] F. Miao, M. Pajic, and G. J. Pappas, “Stochastic game approach for replay attack detection,” in *Proceedings of the 52nd IEEE Conference on Decision and Control*, Florence, Italy, December 2013.

[11] J. L. Liu, E. Tian, X. Xie, and H. Lin, “Distributed event-triggered control for networked control systems with stochastic cyber-attacks,” *Journal of the Franklin Institute*, vol. 356, no. 17, pp. 10260–10276, 2019.

[12] S. Liu, G. Wei, Y. Song, and Y. Liu, “Extended Kalman filtering for stochastic nonlinear systems with randomly occurring cyber attacks,” *Neurocomputing*, vol. 207, pp. 708–716, 2016.

[13] D. Ding, Z. Wang, D. W. C. Ho, and G. Wei, “Distributed recursive filtering for stochastic systems under uniform quantizations and deception attacks through sensor networks,” *Automatica*, vol. 78, pp. 231–240, 2017.

[14] M. H. Zhu and S. Martinez, “On the performance analysis of resilient networked control systems under replay attacks,” *IEEE Transactions on Automatic Control*, vol. 59, no. 3, pp. 804–808, 2014.

[15] H. Yu and P. J. Antsaklis, “Event-triggered output feedback control for networked control systems using passivity: time-varying network induced delays,” in *Proceedings of the IEEE Conference on Decision and Control and European Control Conference*, Orlando, FL, USA, December 2011.

[16] X. P. Xie, Y. Dong, and P. Chen, “Relaxed real-time scheduling stabilization of discrete-time takagi-sugeno fuzzy systems via an alterable-weights-based ranking switching mechanism,” *IEEE Transactions on Fuzzy Systems*, vol. 26, no. 6, pp. 3808–3819, 2018.

[17] D. Yue, E. Tian, and Q.-L. Han, “A delay system method for designing event-triggered controllers of networked control systems,” *IEEE Transactions on Automatic Control*, vol. 58, no. 2, pp. 475–481, 2013.

[18] X. P. Xie, Q. Zhou, D. Yue, and H. Li, “Relaxed control design of discrete-time takagi-sugeno fuzzy systems: an event-triggered real-time scheduling approach,” *IEEE Transactions on Systems, Man, and Cybernetics: Systems*, vol. 48, no. 12, pp. 2251–2262, 2018.

[19] Q. Li, B. Shen, Y. Liu, and F. E. Alsaadi, “Event-triggered $H_{\infty}$ state estimation for discrete-time stochastic genetic regulatory networks with Markovian jumping parameters and time-varying delays,” *Neurocomputing*, vol. 174, pp. 912–920, 2016.

[20] D. Ding, Z. Wang, B. Shen, and G. Wei, “Event-triggered consensus control for discrete-time stochastic multi-agent systems: the input-to-state stability in probability,” *Automatica*, vol. 62, pp. 284–291, 2015.

[21] J. Weimer, J. Araújo, and K. H. Johansson, “Distributed event-triggered estimation in networked systems,” *IFAC Proceedings Volumes*, vol. 45, no. 9, pp. 178–185, 2012.

[22] J. Liu, Q. Liu, J. Cao, and Y. Zhang, “Adaptive event-triggered $H_{\infty}$ filtering for T-S fuzzy system with time delay,” *Neurocomputing*, vol. 189, pp. 86–94, 2016.

[23] J. F. Wu, Q.-S. Jia, K. H. Johansson, and L. Shi, “Event-based sensor data scheduling: trade-off between communication rate and estimation quality,” *IEEE Transactions on Automatic Control*, vol. 58, no. 4, pp. 1041–1046, 2012.

[24] A. Girard, “Dynamic triggering mechanisms for event-triggered control,” *IEEE Transactions on Automatic Control*, vol. 60, no. 7, pp. 1992–1997, 2015.

[25] E. Tian, Z. Wang, L. Zou, and D. Yue, “Probabilistic-constrained filtering for a class of nonlinear systems with improved static event-triggered communication,” *International Journal of Robust and Nonlinear Control*, vol. 29, no. 5, pp. 1484–1498, 2019.

[26] S.-I. Niculescu, “Optimizing model transformations in delay-dependent analysis of neutral systems: a control-based approach,” *Nonlinear Analysis*, vol. 47, no. 8, pp. 5379–5390, 2001.

[27] Y. He, Q.-G. Wang, L. Xie, and C. Lin, “Further improvement of free-weighting matrices technique for systems with time-varying delay,” *IEEE Transactions on Automatic Control*, vol. 52, no. 2, pp. 293–299, 2007.

[28] L. V. Hien and H. Trinh, “Refined Jensen-based inequality approach to stability analysis of time-delay systems,” *IJ Control Theory and Applications*, vol. 9, no. 14, pp. 2188–2194, 2015.

[29] A. Seuret and F. Gouaisbaut, “Wirtinger-based integral inequality: application to time-delay systems,” *Automatica*, vol. 49, no. 9, pp. 2860–2866, 2013.

[30] P. Park, W. I. Lee, and S. Y. Lee, “Auxiliary function-based integral inequalities for quadratic functions and their applications to time-delay systems,” *Journal of the Franklin Institute*, vol. 352, no. 4, pp. 1378–1396, 2015.

[31] A. Seuret and F. Gouaisbaut, “Hierarchy of LMI conditions for the stability analysis of time-delay systems,” *Systems & Control Letters*, vol. 81, pp. 1–7, 2015.

[32] P. Park, J. W. Ko, and C. Jeong, “Reciprocally convex approach to stability of systems with time-varying delays,” *Automatica*, vol. 47, no. 1, pp. 235–238, 2011.

[33] X. M. Zhang, Q. L. Han, A. Seuret, and F. Gouaisbaut, “An improved reciprocally convex inequality and an augmented lyapunov-krasovskii functional for stability of linear systems with time-varying delay,” *Automatica*, vol. 84, pp. 221–226, 2017.

[34] B. Shen, L. Sun, and Z. Zhao, “Wirtingers inequality is applied to event-driven output control of networked control systems with $H_{\infty}$ performance,” in *Proceedings of the 2018 Chinese Control and Decision Conference (CCDC)*, Shenyang, China, July 2018.

[35] X.-M. Zhang and Q.-L. Han, “Event-triggered $H_{\infty}$ control for a class of nonlinear networked control systems using novel integral inequalities,” *International Journal of Robust and Nonlinear Control*, vol. 27, no. 4, pp. 679–700, 2017.