Fixed-time stability of stochastic nonlinear systems and its application into stochastic multi-agent systems

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Abstract
The paper devotes to study fixed-time stability of stochastic nonlinear systems and fixed-time consensus of stochastic multi-agent systems (SMASs). Firstly, a new fixed-time stability theorem is established and a high-precise estimation of settling time is provided. Secondly, as an application, the fixed-time consensus problem of SMASs is discussed in virtue of the established theorem. A new class of fixed-time nonlinear consensus protocols with stochastic perturbation is designed by employing the neighbour’s information. Based on graph theory, matrix theory and fixed-time stability theorem, we prove that fixed-time consensus of SMASs is achieved under the designed protocol. Moreover, some sufficient conditions are proposed to guarantee fixed-time consensus of SMAS. It is shown that the settling time is not only independent of the initial conditions, but also it has higher precise. In the end, three numerical examples are provided to illustrate correctness of our theoretical results.

1 | INTRODUCTION
Recent years have witnessed an enormous growth of research on finite-time stability and stabilisation of stochastic system. The reason is that finite-time stability system has better robustness and high convergence rate. Since Chen and Li [1] firstly proposed finite-time stability theorem of stochastic nonlinear system, many interesting results of finite-time stability of stochastic system were reported such as [2–8]. Moreover, lots of results about finite-time stabilisation of stochastic system were obtained as well, such as [9, 10] investigated finite time stabilisation of stochastic nonlinear systems, respectively; [11] dealt with finite-time stochastic input-to-state stability of impulsive switched stochastic nonlinear systems. By properties of homogeneous functions and homogeneous vector fields, [12] investigated finite-time stability of stochastic systems. It is worth noticing that the settling time functions obtained in above results are relative with initial conditions, which limited its practical applications because ideal characteristic may not be obtained in advance without initial conditions. To solve this problem, we proposed a new concept, which is said to be fixed-time stability of stochastic nonlinear systems; at the same time, a fixed-time stability theorem was proposed in [13]; in addition, a finite estimation of settling time was obtained from the fixed-time stability theorem, which can ensure estimation of settling time to be independent of initial conditions.

However, estimating settling time is one of much important tasks to stabilize system. To improve performance of system, obtaining more accurate estimation of settling time is one of the important aims. Up until now, for deterministic fixed-time stability system, there have been several interesting results on settling time estimation. For example, in [14], a high-precise estimation is obtained by means of the theory of optimum values. By technique of substitution integral, a least estimation of settling time is obtained in [15–17]. However, to the best of our knowledge, for fixed-time stability of stochastic system, how to improve precise of estimation of settling time is not involved in literature at present. The first objective in this paper is to explore this problem and establish a more accurate estimation of settling time for stochastic nonlinear systems.

With development of stochastic stability theory and its application in networks systems, consensus problem of networks systems has been paid to attention from many disciplines as well recently. A lot of results have been reported such as
Specially, as an application of finite/fixed-time stability of stochastic systems, based on results in [13], fixed-time stabilisation problem of stochastic nonlinear system was investigated in [22, 23]. Besides, fixed-time control of networked systems [24, 25] have been extensively researched in recent years. For example, in [26–28], fixed-time stability of stochastic systems is applied to deal with synchronisation of networks. Moreover, as a kind of important problem of networked system, finite-time and fixed-time consensus problem of SMASs have been paid attentions from many researchers recently. For instance, [29–32] investigated finite-time consensus problem of first-order SMASs and second-order SMASs, respectively. Compared with [29–32], [33–34] investigated fixed-time consensus of stochastic systems, and corresponding settling time satisfies conditions whose expectation is upper bounded by a constant regardless of initial conditions.

Note that the settling time mentioned in [33–34] depends on the parameters of controller and systems. From simulations in above references, it is easy to see that estimation of settling time is very conservative with respect to real stabilisation time, but it may be more desirable in application to obtain higher precise estimation of settling time. Too conservative estimation of settling time is meaningless to control system in practical application. Therefore, a meaningful and challenging issue is to design suitable control protocols to realize fixed-time consensus of SMASs and get higher precise estimation of settling time.

Motivated by above observations, the innovations of this paper are summed as follows: Firstly, a new fixed-time stability theorem of stochastic nonlinear system is established to improve estimation of settling time. Secondly, a new protocol is designed to realize fixed-time consensus of SMASs and get higher precise estimation of settling time. Moreover, some sufficient conditions are provided to guarantee fixed-time consensus of SMASs under detail balanced structure.

The rest of this paper is organised as follows: Some preliminaries and problem formulation are given in Section 2. The main results are presented in Section 3. Simulation examples are performed in Section 4. Some conclusions and future works are drawn in Section 5.

**Notation:** \( \mathbb{R} \), \( \mathbb{R}_{>0} \) and \( \mathbb{R}_{\geq 0} \) denote the set of real number, positive real number and nonnegative real number. \( \mathbb{R}^n \) stands for \( n \) dimensional real vector space and index set \( I_p \) = \{1, 2, ..., \( n \)\}. For a vector or matrix \( Y \), its transpose is denoted as \( Y^T \), and its trace is \( \text{Tr}(Y) \) if \( Y \) is a square matrix. \( \|Y\| \) is Euclidean norm of a vector \( Y \). \( I_p \) denotes a \( n \) dimensional vector with elements being 1. \( \text{diag}(a_1, a_2, ..., a_p) \) is diagonal matrix with diagonal entries being \( a_1, a_2, ..., a_p \). Matrix \( A = [a_{ij}] \) is called nonnegative matrix if \( a_{ij} \geq 0 \) for all \( i, j \in I_p \). \( \mathbb{P}(x) \) stands for the probability of random event \( x_i \). \( E(Y) \) is the expectation of stochastic variable \( Y \). Function \( \text{sign}(x)^2 = \text{sign}(x)\|x\|^2 \), where \( \text{sign}(\cdot) \) is sign function.

## 2 PROBLEM FORMULATION

To state our main results, some basics on SMASs will be recalled in this section. Multi-agent systems is usually expressed by a graph. Thus, graph theory is a necessary tool to study consensus problem of multi-agent systems. Some basic definitions about graph is presented firstly.

### 2.1 Graph theory

A directed graph \( G(A) = (\mathcal{V}, \mathcal{E}, A) \) is composed of a vertex set \( \mathcal{V} = \{v_1, ..., v_N\} \), an edge set \( \mathcal{E} = \{(v_i, v_j) \in \mathcal{V} \times \mathcal{V}\} \) and a weighted adjacency matrix \( A \). Neighbors set of agent \( i \) is denoted by set \( N_i = \{v_j \in \mathcal{V} : (v_i, v_j) \in \mathcal{E}\} \). A path, which connects agent \( v_i \) and \( v_j \), is a sequence of distinct vertices \( v_i = v_i, v_{i_1}, ..., v_{i_k} = v_j \), where \( v_{i_1} = v_i, v_{i_k} = v_j \) satisfying \( (v_{i_l}, v_{i_{l+1}}) \in \mathcal{E}, 0 \leq l \leq k - 1 \). If \( (v_i, v_j) \in \mathcal{E} \) and only if \( (v_j, v_i) \in \mathcal{E} \) the graph is called as undirected graph. A directed (undirected) graph is called strongly connected (connected) if there exits a path between every pair vertices. The weighted adjacency matrix \( A = [a_{ij}] \in \mathbb{R}^{N \times N} \) is defined as \( a_{ij} > 0 \) if and only if \( v_i \in \mathcal{V} \), otherwise \( a_{ij} = 0 \), and assume \( a_{ii} = 0 \) in this paper. The degree matrix \( D = [d_{ij}] \in \mathbb{R}^{N \times N} \) is a diagonal matrix, where \( d_{ii} = \sum_{j=1}^n a_{ij} \). The Laplacian matrix of graph \( G(A) \) is denoted by \( L(A) \), where \( L(A) = D - A \). It is obvious that \( L(A) \) is symmetric if and only if graph \( G(A) \) is an undirected and connected graph. It is easy to verify that \( L(A) \) is semi-positive definite for an undirected and connected graph. Algebraic connectivity of graph \( G(A) \) is denoted by \( \Lambda_2(L(A)) \). If there exist a positive vector \( 0 < p = [p_1, ..., p_N]^T \in \mathbb{R}^N \) such that \( p_i a_{ij} = p_j a_{ji} \) for any \( i, j \in I_p \), the directed graph \( G(A) \) is called as detail-balanced graph.

### 2.2 Consensus protocol

Consider a multi-agent system, which includes \( n \) agents, labeled 1 through \( n \). The dynamics of \( i \)-th agent is described as follows:

\[
\dot{x}_i(t) = u_i(t), \quad i \in I_p, \quad (1)
\]

where \( u_i(t) \) is control input, which is called consensus protocol. The initial state is \( x_i(0) = x_0, i \in I_p \) and \( x_0 = [x_{01}, ..., x_{0n}]^T \).

To study the fixed-time consensus problem of system (1), we propose the following protocol with random perturbation:

\[
u_i(t) = \sum_{j=1}^n \left[ b_{ij} g(x_j - x_i) + b_{ij} g(x_j - x_i) + \bar{b}_{ij} g(x_j - x_i) \right]
\]

\[
+ \sum_{j=1}^n \left[ b_{ij} g(x_j - x_i) w_i(t), i \in I_p, \quad (2)
\]

where \( A = [a_{ij}] \in \mathbb{R}^{N \times N} \) is weighted adjacency matrix of graph \( G(A) \), \( B = [b_{ij}] \in \mathbb{R}^{N \times N} \) is the noise intensity matrix with \( b_{ij} \geq 0 \) and \( b_{ji} > 0 \) if and only if agent \( j \) is the neighbour of agent \( i \). \( w_i(t) \) is independent standard white noises satisfying \( w_i(t) = w_j(t) \) for \( \forall i, j \in I_p \).

**Remark 1.** In the same communication channel, the noise can be regarded as the same for agent \( i \) and agent \( j \). This claim is reasonable. Interested reader can refer to [30] for detail.
It is well known that networked systems usually work in environment with uncertainty. Stochastic perturbation exists in this uncertain environment. As statement in [29–34], the dynamic of the group is inevitably subject to stochastic perturbation. Thus, it is necessary to take the influence of uncertain disturbances into account. The uncertain disturbances are often modeled to be additive and multiplicative noises. Additive noise is often regarded as external interferences and independent of agents’ states. Therefore, we design controller (2) with noises. More detailed explanation can be found in [29–34].

Inserting (2) into (1), we can obtain the following Itô type SMASs:
\[
dx_i(t) = \sum_{j=1}^{n} \left[ p_{ij} a_j \left( f(x_j - x_i) + b(x_j - x_i) \right) + b^2_i g(x_j - x_i) \right] dt + \sum_{j=1}^{n} b_{ij} g(x_j - x_i) dw_j(t), \quad i \in I_a.
\]

To give out our main results, the following assumptions are necessary for (2):

**Assumption (i):** \( f(\cdot) : \mathbb{R} \to \mathbb{R}, b(\cdot) : \mathbb{R} \to \mathbb{R} \) and \( g(\cdot) : \mathbb{R} \to \mathbb{R} \) are continuous, differential and odd functions, which satisfy \( f(\theta) = b(\theta) = g(\theta) = 0 \iff \theta = 0 \), and for \( \theta \neq 0, f(\theta) > 0, b(\theta) > 0; \)

**Assumption (ii):** There exist positive real numbers \( \beta_j > 0, i = 1, 2 \) and \( 0 < \alpha < 1 \), such that \( \| f(\theta) \| \leq \beta_1 \| \theta \|^\alpha, \| b(\theta) \| \leq \beta_2 \| \theta \|^{1-\alpha}; \)

**Assumption (iii):** \( g(\theta)(g(\theta) - \theta) \leq 0. \)

Above Assumptions are reasonable, for example, \( f(\theta) = \text{sign}(\theta)^\alpha, b(\theta) = \text{sign}(\theta)^{2-\alpha} \) and \( g(\theta) = \theta \), where \( \alpha \in (0, 1). \)

To state our main results, some necessary definitions are to be listed subsequently. Similar to finite-time consensus in probability [29–31, –34], the definition of fixed-time consensus in probability is described by means of consensus with probability one and the stochastic settling time function.

**Definition 1.** (Consensus with probability one [29–34]: For system (3), the system is said to be consensus with probability one if for any initial states, there exists a \( \bar{x} \) such that
\[
\lim_{t \to \infty} P(\| x_i(t) - \bar{x} \| = 0) = 1 \quad \forall i \in I_a.
\]

**Definition 2.** (Fixed-time consensus in probability [34]): Given the protocol \( u_i(t), i \in I_a \), the system is said to solve fixed-time consensus in probability problem of (3) if for any initial state \( x_0 \), there exist some stochastic settling time function \( T(x_0, w) \) such that \( E(T(x_0, w)) < \infty \), where \( \varepsilon > 0 \) is a positive constant, which is independent of initial state \( x_0 \), and \( \bar{x} \) is same to Definition 1. Moreover,
\[
\lim_{t \to T(x_0, w)} P(\| x_i(t) - \bar{x} \| = 0) = 1,
\]
\( x_i(t) = \bar{x}, \) for \( t \geq T(x_0, w) \) \( \forall i \in I_a. \)

**Remark 2.** The fixed-time consensus in probability is different from the finite-time consensus in probability because the former is upper bounded by a constant independent of initial state \( x_0 \). However, for finite-time consensus, \( E(T(x_0, w)) \) often depends on initial state \( x_0 \). It may become large when \( x_0 \) grows.

In fact, the main difference between fixed-time consensus and finite-time consensus is the estimation of settling time. For the fixed-time consensus, corresponding estimation is independent of initial state of systems. Moreover, the estimation is up bounded by a positive constant, which just depends on parameters of protocol and information of underlying topology. However, usually the estimation of finite-time consensus depends on the initial states of systems; it is often an increasing function of initial states. Estimation may become larger as increasing of norm of initial state. This character limits many practical applications, specially, for high precise control systems, such as navigation control.

Before proceeding, some necessary lemmas, which are necessary to prove our main results as well, are presented as follows:

**Lemma 1.** [35]: Let \( L(A) = [l_{ij}]_{n \times n} \) be Laplacian of graph \( \mathcal{G}(A) \), where \( A \in \mathbb{R}^{n \times n} \). We have the following conclusions:

(i) 0 is an eigenvalue of \( L(A) \) and \( 1_n \) is associated eigenvector.

(ii) If \( \mathcal{G}(A) \) has a spanning tree, then eigenvalue 0 is algebraically simple and all the other eigenvalues have positive real parts.

(iii) If \( \mathcal{G}(A) \) is strongly connected, then there exists a positive column vector \( \omega \in \mathbb{R}^n \) satisfying \( \omega^T L(A) = 0 \).

(iv) If \( \mathcal{G}(A) \) is undirected and connected, then \( L(A) \) satisfies the following equality:
\[
\xi^T L(A) \xi = \frac{1}{2} \sum_{i,j=1}^{n} a_{ij} (\xi_j - \xi_i)^2
\]
where vector \( \xi = [\xi_1, \xi_2, \ldots, \xi_n]^T \in \mathbb{R}^n \), that is to say, matrix \( L(A) \) is positive semi-definite.

(v) The second smallest eigenvalue \( \lambda_2(L(A)) \) of \( L(A) \), which is said to be algebraic connectivity of graph \( \mathcal{G}(A) \), is larger than zero.

(vi) \( \lambda_2(L(A)) \) is equal to \( \min_{\{\xi \neq 0, \sum_{i=1}^{n} \xi_i = 0\}} \frac{\xi^T L(A) \xi}{\xi^T \xi} \), and thus, if \( 1^T \xi = 0 \), one has
\[
\xi^T L(A) \xi \geq \lambda_2(L(A)) \xi^T \xi.
\]

**Lemma 2.** [36]: Let real numbers \( \xi_1, \ldots, \xi_n \geq 0 \), then following inequalities always hold
\[
\sum_{i=1}^{n} \xi_i^p \leq \left( \sum_{i=1}^{n} \xi_i \right)^p, \quad 0 < p \leq 1,
\]
\[
\sum_{i=1}^{n} \xi_i^p \geq n^{-p} \left( \sum_{i=1}^{n} \xi_i \right)^p, \quad p > 1.
\]

**Lemma 3.** [1, 13 29–34]: For the following stochastic differential equation:
\[
dx = g(x)dx + b(x)d\omega(t), \quad x(0) = x_0,
\]

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\]
Lemma 5. : If inequality (10) changes to

\[ \mathcal{L}V(x) \leq -k_1 V(x)^\alpha - k_2 V(x)^\beta, \]

where second-order differential operator \( \mathcal{L} \) of \( V \) is finite-time stable in probability and corresponding stochastic settling time satisfies

\[ E[T(x_0, w)] \leq T_1 = \frac{(V(x_0))^{1-\alpha}}{k_1(1-\alpha)}. \]  

Lemma 4. [13] : If inequality (10) changes to

\[ \mathcal{L}V(x) \leq -k_1 V(x)^\alpha - k_2 V(x)^\beta, \]

where real numbers \( k_i > 0 \), \( i = 1, 2 \), \( 0 < \alpha < 1 < \beta \) and other conditions are same to Lemma 3, then the origin of (8) is fixed-time stable in probability, and the stochastic settling time satisfies

\[ E[T(x_0, w)] \leq T_2 = \frac{1}{k_1(1-\alpha)} + \frac{1}{k_1(\beta - 1)}. \]  

Lemma 5. [30] : If inequality (10) changes to

\[ \mathcal{L}V(x) \leq -k_1 V(x)^\alpha - k_2 V(x), \]

where real numbers \( k_i > 0 \), \( i = 1, 2 \), \( 0 < \alpha < 1 \) and other conditions are same to Lemma 3, then the origin of (8) is finite-time stable in probability, and the stochastic settling time satisfies

\[ E[T(x_0, w)] \leq T_3 = \frac{1}{k_2(1-\alpha)} \ln \left( \frac{k_2 V(x_0)^{1-\alpha} + k_1}{k_1} \right). \]  

3 MAIN RESULTS

In this section, in order to investigate fixed-time consensus problem of SMASs (1) with (2), a new fixed-time stability theorem is established as follows:

**Theorem 1.** Consider stochastic nonlinear system (8) with unique global solution. If there exists a positive definite, twice continuous differentiable and radially unbounded Lyapunov function \( V : \mathbb{R}^n \to \mathbb{R}_{\geq 0} \) and real numbers \( k_i > 0 \), \( i = 1, 2 \), \( 0 < \alpha < 1 \) such that

\[ \mathcal{L}V(x) \leq -k_1 V(x)^\alpha - k_2 V(x)^{2-\alpha} \]

and other conditions are same to Lemma 3, then the origin of system (8) is globally fixed-time stable in probability and associated stochastic settling time \( T(x_0, \omega) \) satisfies:

\[ E[T(x_0, w)] \leq \frac{1}{k_2(1-\alpha)} \pi \arctan \left( \frac{\sqrt{k_2}}{\sqrt{k_1}} (V(x_0))^{1-\alpha} \right) \]

\[ \leq \frac{1}{\sqrt{k_1 k_2}} \frac{\pi}{2}. \]

Proof. Let function \( r(v) = k_1 v^\alpha + k_2 v^{2-\alpha} \). It is easy to verify that the conditions of Lemma 5 and Lemma 4 are satisfied. So one can conclude that the origin of (8) is globally finite-time stable in probability, and the settling time \( T(x_0, \omega) \) satisfies

\[ E[T(x_0, w)] \leq \int_0^{V(x_0)} \frac{1}{k_1 v^\alpha + k_2 v^{2-\alpha}} dV'. \]  

Next, we will prove that \( E(T(x_0, \omega)) < +\infty \). According to the proof theorem 3.1 [1], one has:

\[ E[T(x_0, w)] \leq \int_0^{V(x_0)} \frac{1}{k_1 v^\alpha + k_2 v^{2-\alpha}} dV' \]

\[ = \frac{1}{k_1(1-\alpha)} \frac{1}{1 + \frac{k_1}{k_2} v^{2-2\alpha}} \int_0^{V(x_0)} d(V^{1-\alpha}) \]

\[ = \frac{1}{\sqrt{k_1 k_2}} \pi \arctan \left( \frac{\sqrt{k_2}}{\sqrt{k_1}} (V(x_0))^{1-\alpha} \right) \]

\[ \leq \frac{1}{\sqrt{k_1 k_2}} \frac{\pi}{2}. \]

Obviously, (19) means that \( E[T(x_0, \omega)] \) is finite and up bounded by constant \( \frac{1}{\sqrt{k_1 k_2}} \frac{\pi}{2} \), which is independent of initial state. Thus, the origin of (8) is fixed-time stable. This completes the proof.

**Remark 3.** Compared to Lemma 4, \( T_4 \) only depends on parameters \( \alpha, k_1, k_2 \); this character is significant for controller design of stochastic nonlinear systems.

**Remark 4.** Let \( T_\alpha = \frac{1}{\sqrt{k_1 k_2}} \pi \frac{\pi}{2} \). It is easy to calculate that \( \frac{dT_\alpha}{d\alpha} = \frac{\pi}{2(1-\alpha)^2 \sqrt{k_1 k_2}} > 0 \), for any \( \alpha \in (0, 1) \). Therefore, \( T_\alpha \) is an increasing function with respect to parameter \( \alpha \). To obtain the smaller settling time, \( \alpha \) should be chosen as small as possible.

**Remark 5.** It is worth noticing that the difference between Theorem 1 and Lemma 4 in [13] is the precision of settling time.
Corollary 1. \( T_4 \leq T_1 \) when \( k_2 > 0 \).

Proof: According to inequality \( z > \arctan(z) \), \( \forall z > 0 \), one has
\[
T_4 = \frac{1}{\sqrt{k_1 k_2 (1 - \alpha)}} \arctan \left( \sqrt{\frac{k_2}{k_1}} \left( V(x) \right)^{1 - \alpha} \right) < \frac{1}{\sqrt{k_1 k_2 (1 - \alpha)}} \left( \sqrt{\frac{k_2}{k_1}} \left( V(x) \right)^{1 - \alpha} \right) = T_1.
\]
This is complete proof.

Corollary 2. \( T_4 < T_3 \) when \( \nabla < 4 \) and \( T_4 > T_3 \) when \( \nabla \geq 4 \), where parameter
\[
\nabla = \frac{\arctan \left( \sqrt{\frac{k_2}{k_1}} \left( V(x) \right)^{1 - \alpha} \right)}{\ln \left( \frac{k_2}{k_1} \left( V(x_0) \right)^{1 - \alpha} + k_1 \right)}.
\]
Proof: Obviously, \( T_4 \) and \( T_3 \) are both positive numbers. Therefore, if the ratio of \( T_4 \) and \( T_3 \) is larger than 1, one can obtain that \( T_4 > T_3 \), at the same time, the ratio is smaller than 1 leads to \( T_4 < T_3 \). It is easy to calculate that \( \nabla = \frac{T_4}{T_3} \). Thus, one has \( T_4 > T_3 \) when \( \nabla > 4 \) and \( T_4 < T_3 \) when \( \nabla < 4 \). This is complete proof of Corollary 2.

Corollary 3. \( T_4 < T_2 \) when \( \beta = 2 - \alpha \).

Proof. Due to \( \beta = 2 - \alpha \), one has
\[
T_2 = \frac{1}{k_1 (1 - \alpha)} + \frac{1}{k_1 (1 - \alpha)} = \left( \frac{k_1 + k_2}{k_1 k_2} \right) \frac{2}{1 - \alpha} \frac{1}{\sqrt{k_1 k_2 (1 - \alpha)}} \arctan \left( \sqrt{\frac{k_2}{k_1}} \left( V(x) \right)^{1 - \alpha} \right) = T_4.
\]
This is complete proof.

Above corollaries show that the precision of settling time estimates are different because that estimation methods are different. To obtain higher precise settling time estimation, a suitable estimation method should be chosen according to conditions.

Next, based on Theorem 1 and (2), we are to deal with the fixed-time consensus problem of (3). In the first place, in order to clear our idea, we will discuss consensus of system (3) under a special detail-balanced graph, that is, undirected and connected graph. For this case, vector \( p = [1, \ldots, 1]^T \in \mathbb{R}^n \), and the protocol (2) can be rewritten as follows:
\[
u_i(t) = \sum_{j=1}^{n} \left[ a_{ij} \left( f(x_j - x_i) + b(x_j - x_i) \right) + b_{ij}^2 g(x_j - x_i) \right] + \sum_{j=1}^{n} \left[ b_{ij} g(x_j - x_i) u_{ij}(t) \right], i \in I_e.
\]

Remark 6. Protocol (21) is superior to the ones in [29–31]. The reasons is that associated settling time function is to be upper bounded by a constant, which is independent of initial conditions. However, the ones in [29–31] are unbounded functions with respect to initial states. That is to say, if norm of initial states grows up, estimation of settling time may become infinite.

Theorem 2. Consider multi-agent systems (3) under undirected and connected topology \( \mathcal{G}(A) \). If \( B = B^T \) and the Assumptions (i)–(iii) hold, then the multi-agent systems (3) can achieve fixed-time consensus in probability, where \( A = [a_{ij}]_{\infty \times \infty} \) is weighted adjacency matrix, \( B = [b_{ij}]_{\infty \times \infty} \) is the noise intensity matrix.

Proof. Let \( \tilde{x} = \frac{1}{\nabla} \sum_{i=1}^{n} x_i(t) \). Then one has
\[
d\tilde{x} = \frac{1}{\nabla} \sum_{i=1}^{n} dx_i(t) = \frac{1}{\nabla} \sum_{i=1}^{n} \sum_{j=1}^{n} \left[ a_{ij} \left( f(x_j - x_i) + b(x_j - x_i) \right) \right] dt + \frac{1}{\nabla} \sum_{i=1}^{n} \sum_{j=1}^{n} \left[ b_{ij}^2 g(x_j - x_i) \right] dt + \frac{1}{\nabla} \sum_{i=1}^{n} \sum_{j=1}^{n} \left[ b_{ij} g(x_j - x_i) u_{ij}(t) \right] dt = 0.
\]

Note that above equality holding is based on the symmetry of matrices \( A, B \) and Assumptions (i)–(iii). The derivative of equality (22) is same to that of theorem 3.1 in [29], where \( p_j = 1 \) and \( \omega_{ij} = \omega_{ji} \) for any \( i, j \in I_e \) applied in mathematical operation. Above mathematical operating leads to \( d\tilde{x} = 0 \). More details can be found in [29]. To simplify our proof, let disagreement variable \( \delta_i = x_i - \tilde{x}, i \in I_e, \delta = [\delta_1, \ldots, \delta_n]^T \). Then, it is easy to verify that \( V_0 = \delta = 0 \), and \( \delta - \delta_i = x_i(t) - x_i(t) \), for \( \forall i, j \in I_e \). According to controller (21) and (22), one has the following equality:
\[
d\delta_i(t) = d\delta_i(t) = 0.\]

Therefore
\[
d\delta_i(t) = \sum_{j=1}^{n} \left[ a_{ij} \left( f(\delta_j - \delta_i) + b(\delta_j - \delta_i) \right) \right] dt + \sum_{j=1}^{n} \left[ b_{ij} g(\delta_j - \delta_i) \right] dt + \sum_{j=1}^{n} \left[ b_{ij} g(\delta_j - \delta_i) u_{ij}(t) \right].
\]
Obviously, fixed-time consensus of SMAS (23) is equivalent to fixed-time consensus of SMAS (3). Subsequently, we will prove that (23) can reach fixed-time consensus in probability. To do this, the candidate Lyapunov function is taken as

\[ V(\delta) = \sum_{i=1}^{n} \delta_i^2. \]  

(24)

From Ho formula, one has

\[
\begin{align*}
dV(\delta) &= \sum_{i=1}^{n} 2\delta_i \left[ \sum_{j=1}^{n} \left[ a_{ij}(f(\delta_j - \delta_i) + b(\delta_j - \delta_i)) \right] \right] dt \\
&\quad + \sum_{i=1}^{n} 2\delta_i \left[ \sum_{j=1}^{n} b_{ij}g(\delta_j - \delta_i) \right] dt \\
&\quad + \sum_{i=1}^{n} \left[ \sum_{j=1}^{n} (\delta_j - \delta_i) b_{ij}g(\delta_j - \delta_i) \right] dw_i(t) \\
&= \sum_{i=1}^{n} \left[ \sum_{j=1}^{n} (\delta_j - \delta_i) \left[ a_{ij}(f(\delta_j - \delta_i) + b(\delta_j - \delta_i)) \right] \right] dt \\
&\quad + \sum_{i=1}^{n} \left[ \sum_{j=1}^{n} (\delta_j - \delta_i) b_{ij}g(\delta_j - \delta_i) \right] dt \\
&\quad + \sum_{i=1}^{n} \left[ \sum_{j=1}^{n} (\delta_j - \delta_i) b_{ij}g(\delta_j - \delta_i) \right] dw_i(t). \\
&\quad (25)
\end{align*}
\]

For \( J_1 \), to simplify expression, some notations are defined as following:

Matrix \( C_f = [a_{ij}^{\frac{1}{\alpha}}]_{\alpha > 0} \) denotes the Laplacian matrix of graph \( G_f(C_f) \). Due to symmetry of matrix \( A \), one can obtain \( C_f \) is symmetric matrix as well. From case (v) in Lemma 1, one has

\[
\sum_{i=1}^{n} \sum_{j=1}^{n} a_{ij}^{\frac{1}{\alpha}} (\delta_j - \delta_i)^2 = 2 \delta^T(t) L(C_f) \delta(t). 
\]

(27)

In addition, based on \( V(\delta(t)) = 0 \) and case (vii) in Lemma 1, one has

\[
\delta^T(t) L(C_f) \delta(t) \geq \lambda_2(L(C_f)) V(\delta(t)).
\]

(28)

Invoking Lemma 2, we can obtain

\[
\begin{align*}
J_1 &= -\beta_1 \sum_{i=1}^{n} \sum_{j=1}^{n} \left[ (a_{ij})^{\frac{1}{\alpha}} (\delta_j - \delta_i)^2 \right]^\frac{1+\alpha}{\alpha} \\
&\leq -\beta_1 \left( \sum_{i=1}^{n} \sum_{j=1}^{n} \left[ a_{ij}^{\frac{1}{\alpha}} (\delta_j - \delta_i)^2 \right] \right)^\frac{1+\alpha}{\alpha}. \\
&\quad (29)
\end{align*}
\]

Combining (27), (28) and (29), one can obtain

\[
\begin{align*}
J_1 &\leq -\beta_1 (2\lambda_2(L(C_f)))^{\frac{1+\alpha}{\alpha}}, \\
&= -\beta_1 (2\lambda_2(L(C_f)))^{\frac{1+\alpha}{\alpha}} \left( V(\delta) \right)^\frac{1+\alpha}{\alpha}, \\
&= -\eta_f \left( V(\delta) \right)^\frac{1+\alpha}{\alpha}, \\
&\quad (30)
\end{align*}
\]

where parameters \( \eta_f = \beta_1 (2\lambda_2(L(C_f)))^{\frac{1+\alpha}{\alpha}} \), \( \theta = \frac{1+\alpha}{\alpha} \).

For the term \( f_2 \), according to the same process of dealing with \( f_1 \), let matrix \( D_h = [a_{ij}^{\frac{1}{\alpha \alpha}}]_{\alpha > 0} \) denote the Laplacian matrix of graph \( G(D_h) \). From Lemma 2 and Assumption (i)–(iii), one has

\[
\begin{align*}
f_2 &\leq -n^{1-\alpha} \beta_2 \left( \sum_{i=1}^{n} \sum_{j=1}^{n} \left[ a_{ij}^{\frac{1}{\alpha \alpha}} (\delta_j - \delta_i)^2 \right] \right)^\frac{3-\alpha}{2} \\
&= -\eta_h \left( V(\delta) \right)^\frac{3-\alpha}{2}, \\
&\quad (31)
\end{align*}
\]

where \( \eta_h = \beta_2 n^{1-\alpha} (2\lambda_2(L(D_h)))^{\frac{3-\alpha}{2}}, \theta = \frac{1+\alpha}{2} \). Combining (26), (30) and (31) yields to

\[
\begin{align*}
L^V(\delta(t)) &\leq -\eta_f \left( V(\delta) \right)^\theta - \eta_h \left( V(\delta) \right)^\frac{3-\alpha}{\theta}. \\
&\quad (32)
\end{align*}
\]

From Theorem 1 and (32), one can conclude that the SMASs (23) achieves fixed-time consensus in probability, and the associated E\{T(x_0, w)\]
satisfies:

$$E[T(x_0, w)] \leq \frac{1}{\eta_f} \frac{1}{\alpha - \eta} \arctan \left( \sqrt{\frac{\eta_f}{\eta}} (V(x_0))^{\alpha - 1} \right)$$

$$\leq \frac{1}{\sqrt{\eta_f/\eta}} \frac{1}{\alpha - \eta} \pi = \frac{1}{\sqrt{\eta_f/\eta}} \frac{1}{\alpha - \eta} \pi. \quad (33)$$

This is complete proof of Theorem 2.

Remark 7. (33) means that the upper bound of $E[T(x_0, w)]$ is independent of initial state $x_0$, and it is an increasing function with respect to parameter $\alpha$ in the open interval $(0, 1)$ according to Remark 5.

The communication topology discussed in Theorem 1 is undirected and connected graph, but digraphs are more common for real network systems. Therefore, next, we will consider consensus problem of SMASs (3) under a special digraph, which is called as detail-balanced graph.

**Theorem 3.** Consider SMASs (3) under a detail-balanced graph $G(A)$.

If Assumptions (i)–(iii) hold, then SMASs (3) with the protocol (2) can achieve fixed-time consensus in probability, and associated settling time $T(x, \omega)$ satisfies $E(T(x, \omega)) \leq \epsilon$, where $\epsilon$ is a positive constant independent of initial conditions.

**Proof.** Let $D = \{p_i, a_j\} \in \mathbb{R}^{\infty \times 2}$, $y = \frac{1}{n} \sum_{i=1}^{n} \lambda_i(t)$. Due to $p_i a_j = p_j a_j, \forall i, j \in \mathcal{I}_n$, one has $D = D^T$, the protocol (2) can be rewritten as follows:

$$u_i(t) = \sum_{j=1}^{n} \left[ d_{ij}(f(x_j - x_i) + b(x_j - x_i)) + b_{ij}g(x_j - x_i) \right]$$

$$+ \sum_{j=1}^{n} \left[ b_{ij}g(x_j - x_i)w_{ij}(t) \right], i \in \mathcal{I}_n \quad (34)$$

Based on the fact that $D = D^T$, $B = B^T$, and $w_{ij}(t) = w_{ji}(t), \forall i, j \in \mathcal{I}_n$, it is easy to obtain

$$dy = \frac{1}{n} \sum_{i=1}^{n} u_i(t)dt$$

$$= \frac{1}{n} \sum_{i=1}^{n} \sum_{j=1}^{n} \left[ d_{ij}(f(x_j - x_i) + b(x_j - x_i)) + b_{ij}^2g(x_j - x_i) \right] dt$$

$$+ \frac{1}{n} \sum_{i=1}^{n} \sum_{j=1}^{n} \left[ b_{ij}g(x_j - x_i)dw_{ij}(t) \right] = 0. \quad (35)$$

Therefore, $y(t)$ is a constant, and $y(t) = y(0)$ for $t \geq 0$. It is easy to verify that graph $G(D)$ is an undirected and connected topology. According to Theorem 2, the protocol (34) can solve fixed-time consensus problem of SMASs (3). That is to say, agents states converge to final state $y(0)$ in fixed-time in probability, and associated $T(x_0, w)$ satisfies the following inequality:

$$E[T(x_0, w)] \leq \frac{1}{\eta_f} \frac{1}{\alpha - \eta} \arctan \left( \sqrt{\frac{\eta_f}{\eta}} (V(x))^{\alpha - 1} \right)$$

$$= \frac{1}{\sqrt{\eta_f/\eta}} \frac{1}{\alpha - \eta} \pi = \epsilon. \quad (36)$$

where $\eta_f' = \beta_2 n^{1-\alpha}(2\lambda_2 L(D_b)^{\alpha-2})^{-\frac{1}{2}}, \text{matrix } C_f' = \left[ \frac{d_{ij}^2}{\alpha \delta} \right]_{\mathcal{I}_n}, \eta_f' = \beta_1 (2\lambda_2 (L(C_f')^2))^{\frac{1}{2}}, D_b' = \left[ \frac{d_{ij}^2}{\alpha \delta} \right]_{\mathcal{I}_n}$. It is obvious that $\epsilon$ is a constant regardless of initial conditions. This completes the proof of Theorem 3.

## 4 | SIMULATIONS

In this section, we present three examples to demonstrate the theoretical results in Section 3. Our aim is to compare with results in [13] and [29]; therefore, our simulation method applied here is same as in [13] and [29]. Detailed statement about the simulation method is same as in [13] and [29].

**Example 4.1.** To compare with [13], consider the following stochastic nonlinear system:

$$dx = \left( -x^3 - x^5 \right) dt + x^2 d\omega(t), \quad (37)$$

where state $x \in \mathbb{R}, \omega(t) \in \mathbb{R}$ denotes one dimensional independent standard Wiener process (Brown motion). The candidate Lyapunov function is chosen as $V(x) = \frac{1}{2} x^4$. Associated with system (37), the second differential operator $L V(x)$ can be calculated as follows:

$$L V(x) = x \left( -x^3 - x^5 \right) + \frac{1}{2} x^4$$

$$= -\frac{1}{2} x^4 (V(x))^{\frac{3}{2}} - 2 x^2 (V(x))^{\frac{1}{2}}.$$

According to Theorem 1, the corresponding settling time satisfies $E(T(x_0, \omega)) \leq T_1 = 3.332$ s. However, according to corollary 3.4 in [13], corresponding settling time satisfies $E(T(x_0, \omega)) \leq T_2 = 4.27$ s. Obviously, the estimation $T_1$ is smaller than $T_2$ obtained from [13]. Fig. 1 is corresponding simulation results, where the initial states are same to [13]. It is easy to see that state of system converges to zero in fixed time $T_1 = 3.332$ s. Moreover, real stability time is about 2 s. The simulation result supports our theoretical analysis.

To compare with [29], consider SMASs in [29] as following:

**Example 4.2.** An SMAS, which is formed by five autonomous agents with dynamics (1) under communication
FIGURE 1 The states response of system (37)

![Figure 1](image1)

FIGURE 2 The comparison of system states with initial state $x_1$

![Figure 2](image2)

FIGURE 3 The comparison of system states with initial state $x_2$

![Figure 3](image3)

FIGURE 4 The comparison of system states with initial state $x_3$

![Figure 4](image4)

The topology graph $G(A_1)$, is considered, where $A_1$ is defined as $a_{ij} = b_{ij} = 1$, if $(v_j, v_i) \in E$, otherwise $a_{ij} = b_{ij} = 0$, where $i, j \in I_5$,

$$A_1 = \begin{bmatrix} 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 & 0 \end{bmatrix}, f(\Theta) = \text{sig}(\Theta)\top, h(\Theta) = \text{sig}(\Theta)\top,$$

$g(\Theta) = \Theta$, $w_{ij}$ are equal for $\forall i, j \in I_5$. It is easy to verify that Assumptions (i)–(iii) hold. It can be seen from structure of matrix $A_1$ that corresponding communication topology is an undirected and connected graph. Three sets of initial states are chosen randomly as follows: $x_1 = [1, -5, 5, 3, 2]\top$, $x_2 = [2, -50, 50, 30, -20]\top$ and $x_3 = [2, -60, 40, 10, -15]\top$. To compare with [29], state trajectories of agents with initial state $x_i$, $i = 1, 2, 3$ are provided in Figures 2–4, respectively. Note that the upper subfigure corresponds to protocol (21), and the below one corresponds to protocol in [29]. Figures 2–4 show that protocol (21) has higher convergence rate than the one in [29].

To verify Theorem 3, we consider the following example:

Example 4.3. Under a detail-balanced graph $G(A_1)$, consider SMASs (3) with protocol (34). An SMAS is formed by three agents with the dynamics (1), associated adjacency matrix $A$ and positive vector $p$ are given as following:

$$A = \begin{bmatrix} 0 & 0.4 & 2.4 \\ 0.6 & 0 & 0.9 \\ 1.2 & 0.3 & 0 \end{bmatrix}, \quad p = [1.5, 1, 3]\top.$$

We choose randomly three sets of initial states as following $x_1 = [-15, 10, -4]\top$, $x_3 = [-5, 1, 4]\top$ and $x_3 = [-5, 15, 2]\top$. Figures 5–7 demonstrate the state trajectories of the three agents. According to Theorem 3, one can obtain the settling time satisfies $E(T(x_i, \omega)) \leq 0.8727$ s. The simulation results shown in Figures 5–7 are consistent with Theorem 3.
5 | CONCLUSION

In this paper, fixed-time stability of stochastic nonlinear system and fixed-time consensus of SMASs are investigated. Firstly, a new fixed-time stability theorem of stochastic nonlinear system is established, and a less conservative estimation of stochastic settling time is obtained by theoretical derivation. Moreover, comparisons between the obtained estimation and existing results are carried out. Secondly, a new class of fixed-time consensus protocols is proposed. This class protocol can not only solve fixed-time consensus problem, but also corresponding settling time has higher precise than existing finite and fixed-time consensus protocol, specially when initial states are larger. At the same time, the corresponding estimation \( E[T(x_0, \omega)](\forall x_0) \) is upper bounded by a constant regardless of initial conditions. However, there are much works to be dealt with in future. For example, fixed-time consensus problem of SMASs under the general topology graph or time-varying topology, which has not been involved in this paper. This is our next work as well.

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COMPLIANCE WITH ETHICAL STANDARDS

CONFLICT OF INTEREST

The authors declare that they have no conflict of interest.

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