Matrix Black Holes

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Four and five dimensional extremal black holes with nonzero entropy have simple presentations in M-theory as gravitational waves bound to configurations of intersecting M-branes. We discuss realizations of these objects in matrix models of M-theory, investigate the properties of zero-brane probes, and propose a measure of their internal density. A scenario for black hole dynamics is presented.
1. Introduction

D-brane physics, and particular its embodiment in the matrix model of M-theory [1], leads to a radical change in our picture of gravitational physics. The static gravitational field is replaced by a gas of virtual open strings in a globally flat background Minkowski spacetime. Gravitational interactions turn off below a Planckian distance scale and are replaced by gauge field dynamics. In the light of this new perspective, it is important to revisit the major issues in gravitational physics.

In this article, we undertake a preliminary investigation of black hole dynamics in the context of matrix theory. We shall examine two well-known classes of black holes, the 5D black holes discussed in [2]-[5], and the 4D black holes discussed in [6]-[9]; section 2 contains a summary of the pertinent details of their geometry. U-duality transformations enable a uniform presentation of these objects as collections of membranes and fivebranes of M-theory intersecting along a common string, with the intersection string carrying a gravitational wave profile. These configurations have natural matrix model realizations, which we discuss in section 5. The natural probes of the geometry are the D0-brane/supergraviton ‘partons’ of the matrix formulation. The trajectories of these probes are by definition the light cones of the geometry as seen by low-energy observers (modulo the effects of the spin connection). We examine some of the properties of these probes in the black hole background in section 3. In many respects, the background geometry can be thought of as a kind of optical medium with spatially dependent refractive index, which is generated by integrating out heavy degrees of freedom. This optical analogy is a central theme of our work.

Section 4 introduces an intriguing quantity that may be a measure of the density of matter making up the black hole. It is the ‘volume’ of the black hole (in a nonrigorous sense to be explained below) in the directions transverse to the intersection strings. This quantity miraculously turns out to be independent of the moduli of the toroidal compactification, and is just the number of intersection strings times a factor of order one in Planck units. Thus it may be nearly as universal as the entropy (although it is not U-duality invariant).

In section 5, we present a scenario for black hole dynamics in the matrix formulation of M-theory. Because gravitational dynamics is embedded in a richer structure of noncommutative variables, there is a natural means to resolve the black hole evaporation problem

1 Or more precisely, their residual effects at strong coupling, which are the off-diagonal elements of the matrix.
by a version of ‘black hole complementarity’ [10]-[12]. Moreover, the global coordinates provided by the infinite momentum frame (IMF) of the matrix model should enable one to track the zero brane probes as they become stuck on a ‘stretched horizon’ [10] and rera-diate as Hawking particles. The fact that gravity turns off at short distances, becoming noncommutative Yang-Mills dynamics, also provides an elegant means of sustaining the matter making up the black hole against collapse to infinite density.

2. Review of 5d and 4d black holes

The popular version of five dimensional black holes discussed in [2]-[5] is framed in the IIB theory. Compactify the IIB theory on $T^4 \times S^1$, with the $S^1$ having coordinate $x_5$ and the $T^4$ having coordinates $x_6, \ldots, x_9$. There are three quantum numbers, $N_1$, $N_5$ and $n_R$, corresponding to the number of D1-branes wrapped around $S^1$, the number of D5-branes wrapped around $T^4 \times S^1$ and the right-moving momentum along $S^1$, respectively. We will often employ a notation

$$
\begin{bmatrix}
5 & 6 & 7 & 8 & 9 \\
5 & . & . & . & . \\
p5 & . & . & . & . \\
\end{bmatrix}
$$

to denote a set of brane orientations ($p$ denotes momentum along the corresponding direction). This is an extremal black hole with nonvanishing horizon area. The general nonextremal black hole is obtained by adding $N_1$ anti-D1-branes, $N_5$ anti-D5-branes and $n_L$ left-moving momentum. We refer to [3] for more details (see also [13]); in particular, the string metric is

$$
ds_{10B}^2 = H_5^{1/2} H_1^{1/2} [H_1^{-1} H_5^{-1} (dudv + Vdu^2) + H_5^{-1} (dx_6^2 + \ldots + dx_9^2) + (dx_1^2 + \ldots + dx_5^2)].
$$

(2.1)

T-dualizing along the 5 direction, one passes to the IIA theory. All D1-branes are mapped to D0-branes, D5-branes are mapped to D-4 branes, and the right-moving momentum modes become right-winding modes along $\tilde{S}^1$ and the left-moving momentum modes become left-winding modes. The configuration is now

$$
\begin{bmatrix}
. & 6 & 7 & 8 & 9 \\
. & . & . & . & . \\
w5 & . & . & . & . \\
\end{bmatrix}
$$
with the D0-branes strung along the winding strings \[5,14\]. We shall use \(N_0(N_0), N_4(N_4), w_R(w_L)\) to denote numbers of corresponding branes. The strong coupling limit yields M-theory on a circle of radius \(R\) (coordinate \(x_{11}\)), with configuration

\[
\begin{bmatrix}
. & 6 & 7 & 8 & 9 & 11 \\
. & . & . & . & p_{11} & . \\
5 & . & . & . & 11 \\
\end{bmatrix}
\]

An M-theory metric for this situation is known \[15\]:

\[
ds_{11}^2 = F^{2/3}T^{1/3}[F^{-1}T^{-1}(du^2 + K du^2) + T^{-1}dx_5^2 + F^{-1}(dx_6^2 + ... + dx_9^2) + (dx_1^2 + ... + dx_4^2)] .
\]  

(2.2)

Here \(F = 1 + Q_5 r^2\), \(T = 1 + Q_2 r^2\), \(K = \frac{P}{r^2}\) are harmonic functions, with \(Q_5, Q_2,\) and \(P\) the fivebrane, membrane, and ‘longitudinal wave’ momentum.

Horowitz and Marolf \[16\] have considered the geometry of the six-dimensional black string that results from decompactification (or large radius) of the longitudinal direction \(x_{11} = u + v\), in the particular case \(F = T\). They note a generalization \[17,18\] of (2.2) to include travelling waves with both ‘longitudinal’ and transverse polarizations:

\[
ds^2 = \left(1 + \frac{r_*^2}{r^2}\right)^{-1} \left[ du^2 + \frac{p(u) + r_*^2 (\dot{\phi}_i(u))^2}{r^2} \right] + \left(1 + \frac{r_*^2}{r^2}\right) \left( dr^2 + r^2 d\Omega_3^2 \right)
\]  

(2.3)

It is consistent to interpret the ‘longitudinal waves’ with a coarse-graining of the transverse waves (which are all one sees in D-brane physics) below some wavelength cutoff. The general features of the geometry are captured by keeping only the ‘longitudinal’ waves, and for the remainder of this section we shall do so, setting \(\dot{\phi}_i(u) = 0\); furthermore let \(p(u) = \text{const.} = r_*^4 \sigma^2\).

The horizon at \(r = 0\) is a nonsingular null surface, as may be seen by passing to coordinates

\[
U = \frac{1}{2\sigma} e^{2\sigma u}
\]

\[
V = v - \tilde{R}^2 \sigma
\]

(2.4)

\[
W = e^{-\sigma u} \tilde{R}^{-1}
\]

where \(\tilde{R} = r_* \left(\frac{r_*^2 + r^2}{r^2}\right)^{1/2}\). In these coordinates, the horizon at \(U = 0\) looks like \(adS_3 \times S^3 \times \mathbb{R}^5\):

\[
ds^2 \sim r_*^2 (-W^2 dU dV + (d\log W)^2 + d\Omega_3^2) + (dx_5^2 + ... + dx_9^2) .
\]  

(2.5)
The radius of curvature at the horizon, \( r_\ast \), is quite large for large black holes, and the horizon geometry is smooth \( [10] \). The interior geometry is of the same form as (2.2), with the replacements \( 1 + \frac{Q}{r^2} \rightarrow -1 + \frac{Q}{r^2} \), etc. Timelike geodesics reach the horizon in finite proper time, and null geodesics reach the horizon at finite values of their affine parameter. The continuation of probe motion beyond the horizon (ignoring back-reaction, etc.) sees the probe reach a minimum radius, bounce, and hit a Cauchy horizon \( [16] \).

The four dimensional extremal black hole has a similar M-theory interpretation as a configuration \[ \begin{pmatrix} 4 & 5 & 6 & 7 & \ldots & 11 \\ \ldots & 6 & 7 & 8 & 9 & 11 \\ 4 & 5 & \ldots & 8 & 9 & 11 \\ \ldots & \ldots & \ldots & \ldots & \ldots & p_{11} \end{pmatrix} \]

of intersecting fivebranes. A metric involving longitudinal waves is \( [13, 19] \)
\[
\begin{aligned}
\text{ds}_{11}^2 &= (F_1 F_2 F_3)^{2/3} \left[ (F_1 F_2 F_3)^{-1} (dudv + Kdu^2) + (dx_1^2 + \ldots + dx_3^2) \right. \\
&\quad + (F_3 F_1)^{-1} (dx_4^2 + dx_5^2) + (F_1 F_2)^{-1} (dx_6^2 + dx_7^2) + (F_2 F_3)^{-1} (dx_8^2 + dx_9^2) \left. \right] .
\end{aligned}
\] (2.6)

3. Geometry and probes

The geometry of the black holes (2.2), (2.4) is measured in different ways by various probes. We will mostly be interested in 0-brane probes, which can be statically supported by a BPS cancellation of gravitational and gauge forces. The cancellation is spoiled when the black hole and probe have relative velocity, causing the probe to be attracted to the hole. The Born-Infeld action for D0-branes is a simple consequence of massless particle dynamics in eleven dimensions. Begin with the massless particle action in 11d:
\[
S = \int p_M \dot{x}^M - \frac{1}{2} eG^{MN} p_M p_N ,
\] (3.1)
where \( M = 0, 1, \ldots, 9, 11 \) are 11d coordinates; 10d labels will be \( \mu = 0, 1, \ldots, 9 \). Take the metric to have the Kaluza-Klein form
\[
ds^2 = e^{-2\phi/3} g_{\mu\nu} dx^\mu dx^\nu + e^{4\phi/3} (dx_{11} - A_\mu dx^\mu)^2 ;
\] (3.2)
eliminating \( p_\mu \) (note that it is the inverse metric which appears in (3.1)), one finds
\[
S = \int \frac{1}{2e} e^{-2\phi/3} g_{\mu\nu} \dot{x}^\mu \dot{x}^\nu - p_{11} A_\mu \dot{x}^\mu - \frac{e^{-4\phi/3}}{2} p_{11}^2 + p_{11} \dot{x}_{11} .
\] (3.3)
Finally, solving for the einbein \( e \) yields the D0-brane action
\[
S = \int p_{11} \left[ e^{-\phi} \sqrt{g_{\mu\nu} \dot{x}^\mu \dot{x}^\nu} - A_\mu \dot{x}^\mu + \dot{x}_{11} \right] .
\] (3.4)
The last term is a total derivative when \( p_{11} \) is constant, but contributes to the eikonal phase of the particle. The inclusion of the fermionic terms in the D0-brane action simply generates the coupling of the background fields to the intrinsic spin of the 11d supergraviton multiplet.
3.1. 5D black holes

The action for a nonrelativistic D1-brane probe derived in [20,21] can be recast as the D0-brane action

\[ S = -\frac{1}{R} \int d\tau + \frac{1}{2R} \int d\tau \left( FT v^2 + Tw^2 \right), \tag{3.5} \]

where \( R \) is the radius of the eleventh dimension and

\[ T = 1 + \frac{v^2}{r^2}, \quad r_w = \frac{l^6 w R}{(2\pi)^4 V}, \]
\[ F = 1 + \frac{v_4^2}{r^2}, \quad r_4 = \frac{l^6 N_4}{(2\pi)^2 R_5}. \]

Here \( v \) is the velocity in the macroscopic dimensions, while \( w \) is that in the internal torus; we shall set \( w = 0 \). It was shown by [20] that a nonrelativistic zero-brane probe is captured if its impact parameter is less than \( r_w + r_4 \). Note that this is independent of the zero-brane charge \( Q \) carried by the black hole.

The properties of this probe can be recast in eleven-dimensional form as follows: Let us assume that the action (3.5) results from an eikonal equation for the 11d supergraviton of the form

\[ n^2 (E^2 - p_{11}^2) = p_i^2, \tag{3.6} \]

where \( p_i = \partial_i \psi, p_{11} = \partial_{11} \psi \) and \( E = \partial_t \psi \); \( \psi \) is the eikonal of the wave function, \( \Phi \sim e^{i\psi} \), and \( n(r) \) is a spatially dependent ‘refractive index’. Let \( p_+ = E + p_{11} \) and \( p_- = E - p_{11} \); both of these quantities are conserved in the black hole background (2.2). The light-cone energy is

\[ p_+ = \frac{p_i^2}{n^2 p_-} = \frac{R p_i^2}{2n^2}, \]

where we used the approximation \( p_+ = 2/R \). This implies the action

\[ S = \frac{1}{2R} \int d\tau n^2 v^2, \tag{3.7} \]

compared with (1.5) we find \( n^2 = FT \). Thus, for the super-graviton propagating in the background of the 11D black hole, we have the eikonal equation

\[ n^2 ( (\partial_t \psi)^2 - (\partial_{11} \psi)^2 ) = (\partial_i \psi)^2, \tag{3.8} \]
with \( n^2 = FT \).

Indeed, the massless particle action (3.1) in the background (2.2) yields the scalar Laplacian

\[
FT(\partial_u \partial_v - K \partial^2_v) + (\partial_1^2 + \ldots + \partial_4^2) + T \partial_5^2 + F(\partial_6^2 + \ldots + \partial_9^2)
\]  

(3.9)

Inclusion of fermionic terms in (3.1) will generate the spin connection terms of the higher spin wave equations obeyed by supergravitons. It is possible to choose polarizations such that these terms vanish (for example the \( A_{567} \) component of the antisymmetric tensor field). This Laplacian reduces to (3.8) for waves with \( \partial_u \psi \gg \partial_v \psi \), and no dependence on internal coordinates, unless one is close to the horizon. Thus the corresponding wave equation for a D0-brane at low energies is

\[
n^2(\partial_t^2 - \partial_{11}^2)\Phi = \partial_t^2 \Phi .
\]  

(3.10)

Consider S-wave scattering of the D0-brane off the black hole. Since both energy and \( p_{11} \) are conserved, we can replace the l.h.s. of (3.10) by \(-n^2(\omega^2 - p_{11}^2)\Phi = -n^2\omega'^2\Phi \). The relevant wave equation is then

\[
r^{-3} \partial_r (r^3 \partial_r \Phi) + \omega'^2 n^2 \Phi = 0.
\]  

(3.11)

The low energy limit requires \( \omega' r_w, \omega' r_4 \ll 1 \). As in [22] and [23], we shall solve this equation approximately in two regions. Region I: \( r \gg \omega' r_w r_4 \). Region II: \( [Q(\omega - p_{11})/\omega + p_{11})]^{1/2} \ll r \ll r_w, r_4 \). The low energy limit ensures that there is an overlap between the two regions. The region inside of region II does not substantially affect the results, as one may see by comparing our answer below with a similar calculation of [23], which finds the same result for low frequency waves by a more involved computation using the exact Laplacian (3.9).

In region I, the wave equation is approximately

\[
r^{-3} \partial_r (r^3 \partial_r \Phi) + \omega'^2 n^2 \Phi = 0 ,
\]  

(3.12)

the general solution is

\[
\Phi_I = \sqrt{\omega'} r (\alpha J_1(\omega' r) + \beta N_1(\omega' r)).
\]

In region II, the equation reduces to

\[
r \partial_r (r^3 \partial_r \Phi) + \omega'^2 r_w^2 r_4^2 \Phi = 0 .
\]  

(3.13)

\footnote{One can arrange a hierarchy of scales so that the region where the \( \partial_t^2 \) term becomes important is much inside \( r_w, r_4 \).}
Letting $\rho = 1/r$ and $\Phi = \rho \Psi$, we have an equation
\[
\partial_\rho^2 \Psi + \frac{1}{\rho} \partial_\rho \Psi + (\omega'^2 r_w^2 r_4^2 - \frac{1}{\rho^2}) \Psi = 0 ,
\] (3.14)
whose solution is again given by Bessel functions. If we demand that there is only incoming wave near the horizon ($\rho = \infty$), we have to choose $\Psi = J_1(\omega' r_w r_4 \rho) - i N_1(\omega' r_w r_4 \rho)$. So in region II the approximate solution is
\[
\Phi_{II} = A \rho (J_1(\omega' r_w r_4 \rho) - i N_1(\omega' r_w r_4 \rho)) .
\]

Matching the two solutions in the overlapping region, one finds that $|\alpha| \gg |\beta|$ as usual, and
\[
\alpha = \frac{4 i A}{\pi \omega'^5/2 r_w r_4} .
\] (3.15)

The incoming flux at $r = \infty$ is given by
\[
f_{in} = \frac{1}{2i} (\Phi_{in}^* r^3 \partial_r \Phi_{in} - \text{c.c.}) = \frac{\omega'}{2\pi} |\alpha|^2 ,
\]
and the absorption flux at the horizon is
\[
f_{abs} = \frac{1}{2i} (\Phi^* \rho^{-1} \partial_\rho \Phi - \text{c.c.}) = \frac{2}{\pi} |A|^2 .
\]

Thus the absorption ratio is
\[
\sigma_{abs} = \frac{f_{abs}}{f_{in}} = \frac{1}{4} \pi^2 \omega'^2 r_w^2 r_4^2 ,
\] (3.16)
and the absorption cross section
\[
\sigma_{abs} = \frac{4\pi}{\omega'^5} \sigma_{abs} = \pi^3 \omega'^2 r_w^2 r_4^2 .
\] (3.17)

This is to be contrasted to the results for a minimally coupled scalar [23], and for a fixed scalar [24]. For a minimally coupled scalar, in the low energy limit the absorption cross section is independent of $\omega$ but proportional to the horizon area. For a fixed scalar, the absorption cross section goes as $\omega^2$ in the low energy limit. We will discuss the interpretation of the result (3.17) in section 4.
3.2. 4D black holes from intersecting 5-branes

Consider three sets of 5-branes intersecting along a string as in (2.6), and take the string as the longitudinal direction. Let the compact space transverse to the string be $T^6$, each of whose circles has size $L$. The 11D Einstein metric is (2.6) with

$$F_i = 1 + \frac{r_i}{r}, \quad r_i = \frac{l_i^2 N_i}{2L^2},$$

$$K = Q/r, \quad Q \sim N_0,$$

where $N_i$ are numbers of 5-branes, and $N_0$ the number of D0-branes. It is easy to read off the relevant quantities from (2.6):

$$e^{-\phi} = (F_1 F_2 F_3)^{1/4} (1 + K)^{3/4}, \quad G_{00} = ((1 + K) F_1 F_2 F_3)^{-1/2},$$

$$G_{ij} = \delta_{ij} ((1 + K) F_1 F_2 F_3)^{1/2}, \quad A_0 = (1 + K)^{-1} - 1,$$

where $G_{\mu\nu}$ is the string metric.

The action of a probing nonrelativistic D0-brane can be derived from the Dirac-Born-Infeld action (3.4). Again, just like (3.5), there is no static potential

$$S = -\frac{1}{R} \int d\tau + \frac{1}{2R} \int d\tau F_1 F_2 F_3 v^2.$$  \hfill (3.20)

Consequently, the wave equation is the same as (3.8) with $n^2 = F_1 F_2 F_3$.

The approximate wave equation

$$n^2(\omega^2 - p_{11}^2) \Phi + r^{-2} \partial_r (r^2 \partial_r \Phi) = 0$$

for the S-wave is identical to the one treated in [25] in the limit $r_0 = 0$. Again this is an approximation (for wavefunctions with $\partial_u \Phi \gg \partial_v \Phi$) to the exact wave equation, which is

$$F_1 F_2 F_3 (\partial_u \partial_v - K \partial_u^2) + (\partial_1^2 + \ldots + \partial_5^2) + F_3 F_1 (\partial_4^2 + \partial_5^2) + F_1 F_2 (\partial_6^2 + \partial_7^2) + F_2 F_3 (\partial_8^2 + \partial_9^2).$$

The s-wave absorption cross section in the low energy limit of the 4d black hole is

$$\sigma_{abs} = 4\pi^2 r_1 r_2 r_3 \omega',$$

where again $\omega' = \omega^2 - p_{11}^2$. The above scalar wave equation as well as (3.9) are straightforward consequences of the ‘harmonic function rule’ [15]; for modes independent of internal coordinates, they both take the form

$$\Delta = n^2(\partial_u \partial_v - K \partial_u^2) + \partial_i^2.$$
Douglas, Polchinski, and Strominger [20] have reproduced the probe action (3.5) up to terms of order $1/r^2$ in a D-brane calculation. The gravitational field of the black hole is generated by the exchange of closed strings between the D-branes; in the dual open string channel, these are virtual loops of open strings whose mass is $r$ in string units. Thus in a very direct sense the background geometry is a spatially dependent permeability contributed by the vacuum polarization. A profound feature of this picture is that a curved space geometry is generated by virtual effects of objects in flat space. Note also that the mass of the strings being integrated out is determined by their length in flat space, not the spacelike distance to the horizon in the black hole metric (which is infinite, of course). There appear to be some subtleties [20] in reproducing the $1/r^4$ terms in (3.3).

4. The ‘transverse volume’ of a black hole

In this section we will view the above black holes as black strings wrapped around $x^{11}$ in M-theory. The area of the horizon is then nine-dimensional including the longitudinal direction. Let $A^E_9$ denote the area of the 9D horizon, measured against the 11D Einstein metric; it is not difficult to show that the proper definition of the entropy is

$$S = \frac{A^E_9}{\pi l_p^9}. \quad (4.1)$$

Here $l_p$ is the 11D Planck length, in terms of which the membrane tension is given by $T_2 = l_p^{-3}$. If more dimensions are compactified, the formula (4.1) can be re-written as $A_{D-2}/(4G_D)$, where $A_{D-2}$ is the horizon area of the D dimensional black hole, and $G_D$ is the D dimensional Newton constant.

While the entropy is a pure number independent of both $R$, the radius of the longitudinal direction, and $l_p$, one does not expect it be invariant when the whole system is boosted along the longitudinal direction. Indeed, the longitudinal momentum is proportional to $N_0 - \bar{N}_0$, the difference of the number of 0-branes and the number of anti-0-branes, and $S$ in all cases is proportional to $\sqrt{N_0} + \sqrt{\bar{N}_0}$. On the other hand, there is an intriguing ‘geometric’ quantity that is invariant under longitudinal boosts:

$$\Sigma = \frac{V^E_9}{\pi l_p^9}, \quad (4.2)$$

where $V^E_9$ is the transverse ‘volume’ enclosed by the horizon, in some sense measured against the 11D Einstein metric. We define it as the $r \to r_0$ limit ($r_0$ is the horizon
radius) of the volume of a Euclidean (D-2)-sphere of radius \( r \), times the part of the eleven-dimensional volume element \( \sqrt{G_\perp} \) transverse to the eleventh dimension. Roughly speaking, \( V^{9}_{E} \) is proportional to the optical transverse cross section (see below). According to the standard infinite momentum frame physics, this quantity is boost invariant, and therefore will have a simple description in the matrix model. We shall see that, rather surprisingly, the ratio (4.2), not only is independent of \( N_0 \) and \( \bar{N}_0 \) but also independent of \( R \) and \( l_p \) just like the entropy. Moreover, it has a simple dependence on numbers of other types of branes. In the extremal limit, it is linear in the number of branes of any type other than D0-branes. The definition of \( \Sigma \) in (4.2) is such that in cases examined here it is always a rational number for extremal black holes. Given these properties, clearly it is an important quantity to study in addition to the entropy. On the downside, \( V^{9}_{E} \) is not U-duality invariant, selecting as it does 0-branes for special treatment.

The relation between the 11D Einstein metric and the IIA string metric is

\[
G^{E}_{\mu\nu} = e^{-2\phi/3}G^{s}_{\mu\nu}, \quad \mu = 0,1,...,9.
\]

Our convention for the dilaton is such that its asymptotic value is always zero, so the effective string coupling constant is \( g = g_se^{\phi} \), \( g_s \) is the asymptotic value of the string coupling constant. The transverse volume viewed in terms of the IIA theory is just the spatial volume enclosed by the horizon. Thus

\[
V^{9}_{E} = e^{-3\phi}V^{s}_{9},
\]

where \( V^{s}_{9} \) is the volume measured in the string metric.

We assume the geometry of the horizon is always the tensor product of a \( D - 2 \) dimensional sphere and some compactified space of dimension \( 10 - D \), so the black hole is really a hole in \( D \) dimensional spacetime. We ignore the curved space geometry and take as a measure of the volume enclosed by the horizon that of the standard Euclidean ball enclosed by the \( D - 2 \) sphere times the volume of the compact space. This is not completely unreasonable, given the way that D-branes (and matrix theory) reproduce curved space geometry from Euclidean matrix dynamics (see [26,20] and below). Let \( A^{s}_{8} \) be the horizon area viewed in 10D, then \( V^{s}_{9} = \frac{1}{D-1} A^{s}_{8} r_{hor} \); \( r_{hor} \) is the radius of the horizon. We thus have

\[
\Sigma = \frac{1}{(D-1)\pi l_p^{9}} e^{-3\phi} 4G(r_0)S r_{hor}, \quad (4.3)
\]
where we used the formula \( S = A_s^8/(4G(r_0)) \), \( G(r_0) \) is the 10D Newton constant at the horizon. Its relation with \( \phi \) is \( G(r_0) = 8\pi^6(\alpha')^4g_s^2e^{2\phi} \). Substituting this relation into (4.3),

\[
\Sigma = \frac{32\pi^6}{(D-1)\pi l_p^9}(\alpha')^4g_s^2Se^{-\phi}r_{hor}.
\]

This is a rather obscure relation. Now we make a simple observation about the condition for \( \Sigma \) to be a number independent of \( l_p \) and \( R \). As is to be seen, for the black holes we are considering, \( e^{-\phi} \) at the horizon is always a number independent of \( l_p \) and \( R \). If the dependence of \( r_{hor} \) on \( g_s \) and \( \alpha' \) is always linear in the combination \( g_s\sqrt{\alpha'} = R \), then combined with (4.4) this gives rise to a number

\[
(\alpha')^4g_s^2\sqrt{\alpha'} = (\alpha')^3R^3 = \frac{l_p^9}{(2\pi)^6},
\]

where we have used the relation \( \alpha' = \frac{l_p^3}{(4\pi^2)R} \). This together with (4.4) implies that \( \Sigma \) is a number independent of \( l_p \) and \( R \). For the time being, we have no general argument in terms of the usual string theory for why \( r_{hor} \) is always proportional to \( g_s\sqrt{\alpha'} \), let alone the fact that \( \Sigma \) is always independent of the 0-brane charge. In the following we shall examine the 5D black holes and the 4D black holes separately.

### 4.1. 5D Black Holes

Consider again an extremal black hole (2.1) of the IIB theory on \( T^4 \times S^1 \). This is an extremal black hole with nonvanishing horizon area. The general nonextremal black hole is obtained by adding \( N_1 \) anti-D1-branes, \( N_5 \) anti-D5-branes and \( n_L \) left-moving momentum. We refer to [5] for formulas for the string metric and more details. For our purposes, we need to know the dilaton, the space dependent radius \( R_5 \) of \( S^1 \) and the horizon radius \( r_{hor} \):

\[
e^{2\phi} = f_0 f_4^{-1} \]

\[
R_5(r_0) = f_0^{-1/4} f_4^{-1/4} f_w^{1/2}R_5(\infty)
\]

\[
r_{hor} = f_0^{1/4} f_4^{1/4} r_0,
\]

where \( f_i = \cosh \alpha_i \). All scales are measured in the string metric.

Let us T-dualize along the \( S^1 \) to go to the IIA theory. We still use \( R_5(r_0) \) to denote the new radius in the T-dual theory, and \( \phi \) the dilaton in the IIA theory. Using the standard T-duality transformation we find

\[
e^{2\phi} = f_0^{3/2} f_4^{-1/2} f_w^{-1},
\]

\[
R_5(r_0) = f_0^{1/4} f_4^{1/4} f_w^{-1/2}R_5(\infty),
\]

\[
r_{hor} = f_0^{1/4} f_4^{1/4} r_0.
\]
The scale factors $f_i$ can be read off from relations in (5):

\[ f_0^{1/2} = \frac{1}{2} (N_0 N_\bar{0})^{-1/4} (\sqrt{N_0} + \sqrt{N_\bar{0}}), \]
\[ f_4^{1/2} = \frac{1}{2} (N_4 N_\bar{4})^{-1/4} (\sqrt{N_4} + \sqrt{N_\bar{4}}), \]
\[ f_w^{1/2} = \frac{1}{2} (w_R w_L)^{-1/4} (\sqrt{w_R} + \sqrt{w_L}). \]

We also need a formula for $r_0$ which is completely determined in terms of the other parameters:

\[ r_0 = 2g_s \sqrt{\alpha'} \left( \frac{N_4 N_\bar{4} w_R w_L}{N_0 N_\bar{0}} \right)^{1/4}, \]  

(4.8)

where $g_s$ is the coupling constant for the IIA theory. The above formula together with the last relation in (4.5) is precisely what we need to get a pure number $\Sigma$. To compute $\Sigma$ using (4.4), we first compute the combination $e^{-\phi} r_{\text{hor}} = f_0^{-1/2} f_4^{1/2} f_w^{1/2} r_0$. It is quite remarkable that all the factors such as $N_0 N_\bar{0}$ cancel out, thanks to eq.(4.8). Moreover, due to the factor $f_0^{-1/2}$ we obtain a factor $1/\left(\sqrt{N_0} + \sqrt{N_\bar{0}}\right)$ which is to cancel the factor $\left(\sqrt{w_R} + \sqrt{w_L}\right)$ in $S$. The final result is

\[ \Sigma = \frac{1}{4} (\sqrt{N_4} + \sqrt{N_\bar{4}})^2 (\sqrt{w_R} + \sqrt{w_L})^2, \]

(4.9)

where we used the formula $S = 2\pi (\sqrt{N_0} + \sqrt{N_\bar{0}})(\sqrt{N_4} + \sqrt{N_\bar{4}})(\sqrt{w_R} + \sqrt{w_L})$. Eq.(4.9) is a remarkably simple result compared to any geometric datum involved in the definition of $\Sigma$. As we shall see, there is a similar formula for 4D black holes.

We have emphasized that the relation $r_{\text{hor}} \sim g_s \sqrt{\alpha'} = R$ is crucial for $\Sigma$ to be independent of $l_p$ and $R$. In no way is this relation a consequence of string theory, since in the IIB picture, the relation becomes $r_{\text{hor}} \sim \sqrt{g_s \alpha'}$, where now $g_s$ is the IIB string coupling constant.

To show that $\Sigma$ is the only quantity independent of $N_0$, we list all three scales first in string metric

\[ r_{\text{hor}}^s = f_0^{1/4} f_4^{1/4} r_0 = 2R \left( \frac{f_0 f_4 N_4 N_\bar{4}}{N_0 N_\bar{0}} \right)^{1/4}, \]
\[ V^s = \alpha^2 f_0 f_4^{-1} \left( \frac{N_0 N_\bar{0}}{N_4 N_\bar{4}} \right)^{1/2}, \]
\[ R_5^s = \frac{\sqrt{\alpha'}}{g_s} f_0^{1/4} f_4^{1/4} f_w^{-1/2} \left( \frac{N_0 N_\bar{0}}{w_R w_L} \right)^{1/2}, \]

(4.10)
where \((2\pi)^4V^s\) is the volume of \(T^4\). We see that all three scales in string metric depend on \(N_0\). In the 11D Einstein metric, these become

\[
\begin{align*}
r_{\text{hor}}^E &= 2Rf_4^{1/3}f_w^{1/6} \left( \frac{N_4N_4 w_R w_L}{N_0 N_{\bar{0}}} \right)^{1/4}, \\
V_E &= \sqrt{\alpha'} f_4^{-1/3}f_w^{2/3} \left( \frac{N_0 N_{\bar{0}}}{N_4 N_{\bar{4}}} \right)^{1/2}, \\
R_5^E &= \sqrt{\alpha'} f_4^{1/3}f_w^{-1/3} \left( \frac{N_0 N_{\bar{0}}}{w_R w_L} \right)^{1/2}.
\end{align*}
\tag{4.11}
\]

Now all three scales still depend on \(N_0\). Interestingly there is no dependence on \(f_0\). These three scales assume a more obscure form in the 10D Einstein metric.

From (4.10) and (4.11) we see that both internal scales expand with \(N_0 \to \infty\), if one holds other numbers fixed. The horizon size \(r_{\text{hor}}\) contracts in the large \(N_0\) limit. The extremal limit corresponds to vanishing \(N_{\bar{0}}, N_{\bar{4}}\) and \(w_L\). In this limit scales in (4.10) and in (4.11) become free parameters, except that their combination \(\Sigma\) is fixed as in (4.9).

In computing \(A_{9}^E\), the quantity associated to entropy (4.1), we trade the radial size \(r_{\text{hor}}^E\) in \(V_9^E\) with the longitudinal size \(R\). Since \(V_9^E\) is independent of \(N_0\), one might conclude that \(A_{9}^E\) is also independent of \(N_0\), in contradiction with the entropy formula. The resolution to this puzzle is obvious. One should not use \(2\pi R\) in computing \(A_{9}^E\), but the effective longitudinal size at the horizon: \(2\pi R \exp(2\phi/3) = 2\pi R f_4 f_w^{1/6} f_w^{-1/3}\). This formula shows that in the large \(N_0\) limit, the longitudinal size grows as \(N_0^{1/2}\). In terms of the longitudinal momentum \(p_{11}\), we have \(R^E \sim (p_{11})^{1/2}\), an interesting result to be explained in matrix theory.

The transverse volume defined here has an interesting connection to the capture cross-section computed in section 3. Substituting the formulas for \(r_w\) and \(r_4\) into (3.17) and defining a new quantity \((2\pi)^4V2\pi R_5 \lambda \sigma_{abs}\), one finds

\[
(2\pi)^4 V 2\pi R_5 \lambda \sigma_{abs} = \pi^3 l_p^3 w_R N_4 ,
\tag{4.12}
\]

where \(\lambda = 2\pi/\omega'\). This is just the transverse volume. The formula (4.12) also has an optics interpretation. In the long wavelength limit, the absorption cross-section of a dielectric body of volume \(V\) is (see for instance [27])

\[
\sigma_{abs} = \frac{8\pi^2}{\lambda} \cdot V \cdot \text{Im}(4\pi \chi) ,
\tag{4.13}
\]

\[\text{[Page 13]}\]
where $4\pi\chi$ is the dielectric susceptibility. Thus $\Sigma$ does indeed play the role of the transverse volume of the black hole!

Given this interpretation, one expects a similar formula for the absorption cross section of a nonextremal black hole, in which instead of the product $w_R N_4$ there is a factor $(\sqrt{w_R} + \sqrt{w_L})^2(\sqrt{N_4} + \sqrt{N_4})^2$. Thus the formula contains no thermal factors as for other scalars (such as $\exp(\omega/T_H) - 1$). This must be the case since temperatures usually depend on the D0-brane charge carried by the black hole, while the probing D0-brane decouples from this charge up to order $v^2$.

### 4.2. 4D black holes

Compactifying the IIA theory on $T^4 \times S^1 \times S'^1$, the 4D black holes considered in [7] and [9] in general carry 4 different charges. These are charges associated to D6-branes wrapped around $T^4 \times S^1 \times S'^1$, NS5-branes wrapped around $T^4 \times S^1$, D2-branes wrapped around $S^1 \times S'^1$, and string momentum modes flowing around $S^1$. The configuration is

\[
\begin{bmatrix}
4 & 5 & 6 & 7 & 8 & 9 \\
5 & 6 & 7 & 8 & 9 \\
4 & . & . & . & 9 \\
. & . & . & . & p_9 \\
\end{bmatrix};
\]

fat characters denote NS fivebranes. There are 8 independent integers, 4 of them are numbers of branes, and 4 are numbers of anti-branes. To put this class of black-holes in the context of matrix theory, we need to T-dualize along both $S^1$ and $S'^1$. D6-branes become D-4 branes wrapped around $T^4$, D2-branes become D0-branes, NS5-branes become the so-called ‘non-marginal solitonic branes’ [21], and finally momentum modes flowing along $S^1$ become winding modes along $S^1$. The strong coupling limit yields the configuration

\[
\begin{bmatrix}
. & 5 & 6 & 7 & 8 & 11 \\
5 & 6 & 7 & 8 & 9 & . \\
. & . & . & . & . & p_{11} \\
. & . & . & . & 9 & 11 \\
\end{bmatrix}.
\]

We can say less about this system in matrix theory (although (2.6) provides a similar equivalent system; see below). The main roadblock is that we do not quite understand the transverse 5-brane which is T-dual of one of the constituents in the black hole. However, as we shall see, there is a formula for $\Sigma$ similar to that in (4.9), thus crying for some simple explanation.
Before the T-duality transformation, the dilaton and string metric are [28]

\[ e^{2\phi} = f_0^{-1/2} f_5^{-1/3} f_4^{3/2}, \]
\[ ds^2_{str} = f_0^{-1/2} f_4^{-1/2} (-dt^2 + dx_9^2 + k(dt - dx_9)^2) + f_0^{-1/2} f_5^{-1/2} f_5 dx_4^2 \]
\[ + f_0^{1/2} f_4^{-1/2} (dx_5^2 + \ldots + dx_8^2) + f_0^{1/2} f_4^{1/2} f_5 (dx_1^2 + \ldots + dx_3^2), \]

(4.14)

where the indices of \( f_i \) are named after branes in the T-dual theory. \( T^4 \) is parametrized by \((x_5, \ldots, x_8)\), \( S^1 \) is parametrized by \( x_9 \), \( S'^1 \) parametrized by \( x_4 \). Factors \( f_i \) are harmonic functions, and at horizon assume forms similar to those in (4.7). It follows from the metric (4.14)

\[ R_4(r_0) = f_0^{-1/4} f_4^{-1/4} f_5^{1/2} R_4(\infty), \]
\[ R_9(r_0) = f_0^{-1/4} f_4^{-1/4} f_5^{1/2} R_9(\infty). \]

(4.15)

Again we still use \( R_4 \) and \( R_9 \) to denote radii of \( S'^1 \) and \( S^1 \) in the T-dual theory. These and the new dilaton are

\[ e^{2\phi} = f_0^{3/2} f_4^{-1/2} f_w^{-1}, \]
\[ R_4(r_0) = f_0^{1/4} f_4^{1/4} f_5^{-1/2} R_4(\infty), \]
\[ R_9(r_0) = f_0^{1/4} f_4^{1/4} f_w^{-1/2} R_9(\infty). \]

(4.16)

Note that the dilaton has the same form as in the 5D case (4.6). After certain amount of calculation, we find \( R_i(\infty) \) and the horizon size

\[ R_4(\infty) = R \left( \frac{N_4 N_4 w_R w_L}{N_0 N_0 N_5 N_5} \right)^{1/4}, \]
\[ R_9(\infty) = \frac{\alpha'}{R} \left( \frac{N_0 N_0}{w_R w_L} \right)^{1/2}, \]
\[ r_{hor} = 2 R f_0^{1/4} f_4^{1/4} f_5^{1/2} \left( \frac{N_4 N_4 N_5 N_5 w_R w_L}{N_0 N_0} \right)^{1/4}. \]

(4.17)

So indeed \( r_{hor} \) scales as \( R \) to ensure that \( \Sigma \) be independent of \( R \). We see that three scales \( R_4, R_9 \) and \( r_{hor} \) in the string metric all depend on \( f_0 \) therefore on \( N_0 \). Let \((2\pi)^4 V(r_0)\) denote the volume of \( T^4 \) in string metric, then

\[ V(r_0) = f_0 f_4^{-1} (\alpha')^2 \left( \frac{N_0 N_0}{N_4 N_4} \right)^{1/2}, \]

(4.18)

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also depends on $f_0$. In the 11D Einstein metric, these four scales are
\[
\begin{align*}
  r_{\text{hor}}^E &= 2R f_4^{1/3} f_5^{1/2} f_w^{1/6} \left( \frac{N_4 N_4 N_5 N_5 w_{R} w_{L}}{N_0 N_0} \right)^{1/4}, \\
  R_4^E &= f_4^{1/3} f_5^{-1/2} f_w^{1/6} R_4(\infty), \\
  R_9^E &= f_4^{1/3} f_w^{-1/3} R_9(\infty), \\
  V^E &= f_4^{-2/3} f_w^{2/3} (\alpha')^2 \left( \frac{N_0 N_0}{N_4 N_4} \right)^{1/2}.
\end{align*}
\] (4.19)

They no longer depend on $f_0$, but still depend on $N_0$ through factors such as $R_4(\infty)$. Given the above data, it is straightforward to compute $\Sigma$; the answer is
\[
\Sigma = \frac{1}{6} (\sqrt{N_4} + \sqrt{N_4})^2 (\sqrt{N_5} + \sqrt{N_5})^2 (\sqrt{w_R} + \sqrt{w_L})^2,
\] (4.20)
a result similar to (4.9). We have used formulas for $f_i$ at the horizon which are given by (4.7). The simple result was not expected before all other factors magically cancel. In the extremal limit, $\Sigma$ again is the product of numbers of different branes. This certainly hints at some simple origin.

Another 4D example comes from intersecting 5-branes, equation (2.6). There are three sets of 5-branes intersecting along $N_1 N_2 N_3$ intersection strings, with gravitational waves travelling along the the intersections. Taking the direction of these strings as the longitudinal one, then one obtains $\Sigma = (1/6) N_1 N_2 N_3$, exactly the same formula. In this case, the reduction to IIA along $x_{11}$ gives only fourbranes, i.e. longitudinal fivebranes, which have a simple description in matrix theory [29]. Thus the result (4.20) does not seem so dependent on the particular configuration of branes, so long as it contains D0-branes.

As in the 5d black hole case, we may define a quantity $L^6 \lambda \sigma_{\text{abs}}$ in terms of the absorption cross-section of supergravitons of wavelength $\lambda$; we have
\[
L^6 \lambda \sigma_{\text{abs}} = \pi^3 l_p^3 N_1 N_2 N_3,
\] (4.21)
the same as in (4.12) and agreeing with the transverse volume up to a trivial numerical factor.

The D0-brane as a probe of the 4d black hole is different than the scalar considered in [25]. Note that even in the extremal limit, the absorption cross section computed in [25] depends on the longitudinal momentum carried by the black hole, namely the D0-brane charge. Here again the probing D0-brane decouples from $N_0$ at the order $v^2$. 

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5. Matrix black holes

D-brane technology has enabled a remarkable window into the physics of the extremal and near-extremal black holes under consideration. Entropy, Hawking temperature, absorption cross-sections, and greybody factors all agree with those of macroscopic black holes. However, the D-brane calculations are valid in the regime $g_s Q \ll 1$, while semiclassical black hole physics holds when $g_s Q \gg 1$, where $Q$ is a typical charge. Use of D-brane probes and the large-$N$ limit may enable a partial bridge of this gap \cite{20}, since although the hole is much smaller than the string scale, so is the size of the probe. Nevertheless, to address the issue of Hawking evaporation and quantum coherence, one would like to directly formulate black hole dynamics in a nonperturbative framework. Black holes smaller than the string scale are not useful in this regard, as one knows that light cones – and therefore horizons – are fuzzy on the string scale \cite{30}. Indeed, in the picture of \cite{3}, the information carried by the black hole resides in the ‘stringy halo’ surrounding the D-branes.

The matrix model of \cite{1} appears to be such a nonperturbative formulation in the infinite momentum frame (IMF). It consists of matrix quantum mechanics of $N$ D0-branes; the D0-branes are the partons of the IMF description of eleven-dimensional supergravity. Lorentz invariance and the properties of toroidal compactification involve subtleties of the large $N$ limit, which we will assume can be brought under control. However, to minimize the effects of any resulting modifications of our current understanding of the model, it is perhaps best to choose judiciously the orientation of the branes in the black hole. The usual continuum limit is $N \to \infty$, $R \to \infty$, $p_{11} = N/R$ fixed; although there are claims \cite{31} that some properties may continue to hold even at finite $N$ and $R$. Finally, we will use the apparent correspondence between compactification on a torus, and Yang-Mills theory on the dual torus \cite{1,32}.

There are several ways one might imagine embedding the black holes (2.2) and (2.6) into this construction:

a) $x_{11}$ is the coordinate along which the various branes intersect. This is the orientation used in (2.2) and (2.6). In this case, the usual large $N$ limit decompactifies the solution to a 5d or 6d black string.

Note that in the large $R$ limit, the internal wave profile $\varphi_i(u)$ that distinguishes various black holes of given charges becomes visible to the asymptotic observer, who can now resolve the profile using low-energy experiments in the asymptotic region. However, if one is merely interested in sending probes into the black object to learn about Hawking
evaporation, what matters is that there is a finite capture cross-section for a probe which reaches the classical horizon in finite proper or affine time coordinate. The zero-brane partons of the matrix model describe the eleven-dimensional supergravity multiplet, and we have seen that they have these properties. A second possibility is to choose (for the 5d black hole)

b) \( x_{11} \) as a longitudinal coordinate on the 5-brane, not the direction of the intersection string.

The 2-brane is transverse, and the waves travelling down the intersection string are not the matrix model partons, but rather ‘\( \tilde{D}0 \) branes’ – carriers of electric flux in the Yang-Mills theory on the dual torus. The 2-branes are torons of this Yang-Mills theory \([24]\). The \( R \rightarrow \infty \) limit again gives a black string. In this case, the internal waves remain ‘invisible’ to the macroscopic observer, since the internal wave profile remains microscopic. Probes that preserve some supersymmetry are for example the \( \tilde{D}0 \) branes – the matrix D0-brane partons break the supersymmetry of this configuration (it is not difficult to show that their low-energy Lagrangian has a static potential). Both cases (a) and (b) have the advantage that the 5-branes involved in the black hole configuration are longitudinal. A third orientation is

c) \( x_{11} \) is one of the noncompact coordinates in which the black hole is a localized object; the large \( N \) limit does not give a black string. In this case all the 5-branes and 2-branes composing the black hole are transverse.

Here there is no problem of principle; the main difficulty is the lack of a concrete description of the transverse fivebrane, since it is a magnetic object in the dual Yang-Mills theory. A similar problem arises in case (b) for the 4d black hole, where two of the three sets of fivebranes would be transverse. One can imagine understanding enough of the properties of these objects to make the same qualitative statements about black hole dynamics as one can for cases (a) and (b); we leave this issue for future research.

In the remainder of this article, we will concentrate on the first case – black strings whose longitudinal coordinate is \( x_{11} \) – as these have perhaps the most straightforward interpretation in matrix theory. An additional advantage is the simple behavior of the black hole \((2.2), (2.6)\) under longitudinal boosts. Finally, one might be able to make contact with the ‘transverse volume’ measure introduced in section 4. Since this quantity is independent of zero-brane charge, it is an \( N \)-independent, boost-invariant quantity and therefore is maximally insensitive to any modifications of the matrix theory which might
be required to implement Lorentz invariance; it might even survive the truncation to finite $N$.

A remarkable picture of probe interaction with a black hole is developed in [20] (see also [29]). The static gravitational field seen by the probe arises from integrating out the massive open strings that stretch between the probe and the hole. The resulting moduli space metric is that which should be seen at long distances and low velocities, in the static coordinates of an asymptotic observer. The horizon is a singularity in the description that appears because the open strings become massless when the probe reaches the D-brane configuration making up the black hole. In D-brane language the horizon is the confluence of the Coulomb branch of the probe dynamics, describing separate motion of probe and hole, with the Higgs branch describing probe-hole bound states. The vicinity of this juncture is the so-called ‘stadium’ region [20], where light probe-hole strings contribute important dynamical effects; in black hole language, one would call this the ‘stretched horizon’ [11].

The transcription of the configuration to matrix theory is straightforward. In case (a), all membranes and fivebranes are longitudinal. Longitudinal fivebranes wrapped around a torus are described in matrix theory as instantons in the dual Yang-Mills theory. Longitudinal membranes are states carrying a momentum flux $T_{0i}$ along the $i^{th}$ internal coordinate [33]. The waves along the string are the zero-brane partons themselves. The 5d black hole turns into a 6d black string; the dual Yang-Mills description is a bound state of instantons carrying momentum on the dual $\tilde{T}^5$. The 4d black hole becomes a 5d black string; the corresponding instantons in the Yang-Mills theory on the dual $\tilde{T}^6$ are three sets of two-dimensional objects occupying mutually orthogonal tori. Shrinking any transverse circle to a size much smaller than the 11d Planck scale, the dual Yang-Mills theory lives on a large circle, times a remaining torus of moderate size. In this limit, one recovers the light-cone description of the IIA string [33]-[37], and hence in principle the results of [20] (see also [31]). The matrix description is not limited to weak string coupling, however.

The translation of the dynamics to matrix theory is as follows: The D-brane bound state we are describing has a good semiclassical limit in the dual Yang-Mills theory as a bound state carrying various perturbatively visible charges. The probe is a D0-brane parton (or bound state thereof). The open strings stretching between the bound state and probe become off-diagonal elements of the matrices. As the probe approaches the horizon, it slows down due to its interaction with these coordinates. This is because the D-brane description is intrinsically in static coordinates. When the probe reaches the black hole
‘horizon’, the off-diagonal entries are easily excited, and the moduli space metric obtained by integrating them out becomes singular. The full matrix dynamics should be perfectly regular, however. The hole-probe system becomes an excited bound state; it can relax to a BPS state through the emission of low-momentum partons, which the asymptotic observer interprets as Hawking radiation. One expects the spectrum to be thermal, since the bound state has a long time to explore its phase space and equilibrate before reradiating.

Why hasn’t the information encoded in the infalling probe state been lost behind an event horizon? The basic reason is that the matrix dynamics is framed in a background Minkowski spacetime, which has no horizons. What is conventionally thought of as the causal structure of spacetime is an effective concept determined by the moduli space metric of D0-brane/supergravitons, which breaks down near the horizon. One can map out this causal structure via the trajectories of the massless zero-brane test particles, which the low-energy observer interprets as lightlike geodesics. Far from the hole, this provides an accurate picture of information propagation. Lightlike geodesics are bent by the ‘optical medium’ of off-diagonal matrix elements, mocking up curved space geometry. These trajectories are dramatically affected when the D-brane ‘stretched horizon’ is approached. The true path taken by such a massless particle involves a period of thermalization on the stretched horizon, perhaps followed by reradiation as a Hawking particle (the Hawking particles may come from other D0-branes present in the black hole bound state). Barring a singularity in the large-N physics, it would appear consistent to interpret the infalling data as getting stuck on the stretched horizon, thermalized, and reradiated to the asymptotic region.

5.1. Crossing the horizon

How then can one reconstruct the infalling observer’s experience? There ought not to be any such object as a collection of D-branes that one runs into as one crosses the horizon in finite proper time. Therefore, let us examine the change of variables (2.4) needed to pass from static to infalling coordinates. There are several important features.

First, the redefinition $u \to U = \frac{1}{2\sigma} e^{2\sigma u}$ brings the horizon $u \to \infty$ to a finite coordinate value, thereby undoing the exponential redshift of infalling proper time. There is a nonlocal relation between the matrix description and spacetime coordinates. The IMF description
of the matrix model utilizes \( t = x^+ = v \) and \( N/R \sim p_- \sim \frac{\partial}{\partial x} \) as coordinates. The other light-front coordinate \( x^- = u \) is thus dual to \( N \). To localize physics in \( x^- \) requires introduction of a ‘chemical potential’ for \( N \), followed by a Legendre transformation. Thus the horizon, at \( x^- = \infty \) as well as \( r = 0 \), is diffused across the probe part of the matrix.

The second important feature is that crossing the horizon at \( r = 0 \) \((r = r_0 \text{ for a nonextremal hole})\) is an analytic continuation of the static radial coordinate to complex values. The static interior geometry replaces \( f_i^{\text{ext}} = [1 + r_i^2] \) by \( f_i^{\text{ext}} = [1 - r_i^2] \) (5d black holes); or \( f_i^{\text{ext}} = [1 + \frac{r_i}{r}] \) by \( f_i^{\text{ext}} = [1 - \frac{r_i}{r}] \) (4d black holes). In both cases, the matrix eigenvalues describing probe physics must be continued into the complex domain.

The ‘internal clock’ and other structure of a macroscopic probe falling into a macroscopic black hole is thus an approximate construct. Its evolution inside the horizon is essentially an analytic continuation – or rather an extrapolation – of the probe moduli space approximation (Coulomb branch), beyond the horizon (juncture with the Higgs branch). In static coordinates, the probe wavefunction becomes strongly entangled with that of the black hole, due to their mutual interaction with the light off-diagonal matrix variables. The passage to infalling coordinates must represent an approximate rediagonalization of the matrices, separating probe and black hole degrees of freedom. The integrity of the probe wavefunction in these approximate time and space coordinates can be maintained in a limited domain. At the singularity (or Cauchy horizon, if that is the disease), the needed transformation becomes singular. Thus one might regard the infalling description

\footnote{Note that our choice of IMF time variable is just the opposite of that used in, \textit{e.g.} \cite{38}, which uses \( u \) as the light-cone time of an infalling test string. Instead, we wish to take \( v \) as time, since it is in this variable that stationary D0-brane probes are BPS saturated. A small radial D0-brane velocity represents a slight departure from BPS, or in other words a lightlike geodesic that adiabatically crosses the horizon. Note that this means that Hawking radiation will be a low-energy (small longitudinal momentum) process, rather than a short-time phenomenon at the horizon as in \cite{38}.}

\footnote{We are only interested in transforming the description of the probe to its proper time or affine coordinate evolution. Hence the appropriate procedure is to consider the family of probe experiments at different probe longitudinal momenta \( N_{\text{probe}} \), and then Legendre transform this variable to determine the probe wavefunction’s dependence on \( u \).}

\footnote{Much as a heavy object interacting with a bath of massless objects is not disturbed, so long as the energies of the light objects does not approach the gap in the excitation spectrum of the heavy one. One might then regard the singularity as the place where this separation fails, due to high energy processes.}
as a kind of saddle point or collective field approximation (in complexified matrix space), which breaks down at singularities of the effective geometry.

It is crucial that the infalling coordinate frame, and in particular the infalling observer’s proper time, is built out of matrix observables. Such quantities can be understood as conventional geometry only when commutators are small; reaching the singularity (or a Cauchy horizon), this observer’s proper time becomes ‘noncommutative’. One can no longer use it to describe a simple semiclassical evolution equation. Nevertheless, this is a singularity of the description, not the physics. The static (IMF) frame provides global coordinates for the full evolution, from infall to evaporation.

This picture of the dynamics may be regarded as a form of ‘black hole complementarity’ [10]-[12]. Passage between the static and infalling frame involves a transform of the matrix variables by left and right multiplication, and the two sets of observables will not commute.

5.2. Fat black holes

Nonextremal (and even ‘fat’) black holes can also be described in the matrix framework: They are essentially a ‘plasma’ of instantons, anti-instantons, gluons, etc., of the dual Yang-Mills theory. The relative amounts of each are controlled by the various charges $Q_i \propto N_i - \bar{N}_i$; and by the shape parameters of the internal torus, which act as chemical potentials by adjusting the masses of the various branes, *c.f.* equations (4.11), (4.19). This localized lump of plasma is very long-lived; for example, a graviton attempting to escape encounters an ‘optical medium’ whose ‘refractive index’ $n^2 = \prod f_i$ has a very strong radial gradient near the horizon. If the graviton has any angular momentum whatsoever, the optics will bend the trajectory and refocus the escaping wave back onto the hole. The plasma is supported from indefinite collapse because gravity is an effective interaction that turns off at short distances.\footnote{For example, even the instantaneous boost relating a static and infalling observer near the horizon involves an exponentiation of longitudinal boost operators which are built out of $U(N)$ generators.} This picture fits nicely with the transverse volume measure proposed in section 4. There it was seen that each ‘intersection string’ of the constituent branes occupies on the order of one Planck volume in the transverse space, for any black

\footnote{G. Polhemus has suggested to us that eigenvalue repulsion may play an important role (c.f. [14]) in determining the density at which the plasma stabilizes.}
hole in the class described by [8]. This suggests that the ‘plasma’ indeed stabilizes at Planckian densities.

Finally, it seems that there is a version of Mach’s principle at work here. There is a preferred frame imposed on the theory by the underlying Minkowski space of the Yang-Mills dynamics. It is the causal structure of this dynamics, and not the effective trajectories of D0-brane/supergravitons, that ensures unitary evolution. The causal boundaries of the effective gravitational physics are failures only of the description of the dynamics, and not of the dynamics itself.

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