exocartographer: A Bayesian Framework for Mapping Exoplanets in Reflected Light

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Abstract

Future space telescopes will directly image extrasolar planets at visible wavelengths. Time-resolved reflected light from an exoplanet encodes information about atmospheric and surface inhomogeneities. Previous research has shown that the light curve of an exoplanet can be inverted to obtain a low-resolution map of the planet, as well as constraints on its spin orientation. Estimating the uncertainty on 2D albedo maps has so far remained elusive. Here, we present exocartographer, a flexible open-source Bayesian framework for solving the exocartography inverse problem. The map is parameterized with equal-area Hierarchical, Equal Area, and isoLatitude Pixelation (HEALPix) pixels. For a fiducial map resolution of 192 pixels, a four-parameter Gaussian process describing the spatial scale of albedo variations, and two unknown planetary spin parameters, exocartographer explores a 198-dimensional parameter space. To test the code, we produce a light curve for a cloudless Earth in a face-on orbit with a 90° obliquity. We produce synthetic white-light observations of the planet: five epochs of observations throughout the planet’s orbit, each consisting of 24 hourly observations with a photometric uncertainty of 1% (120 data points). We retrieve an albedo map and—for the first time—its uncertainties, along with spin constraints. The albedo map is recognizable of Earth, with a typical 90% uncertainty of 0.14. The retrieved characteristic length scale is ~9800 km. The obliquity is recovered to be >87°9 at the 90% credible level. Despite the uncertainty in the retrieved albedo map, we robustly identify a high-albedo region (the Sahara desert) and a large low-albedo region (the Pacific Ocean).

Key words: methods: data analysis – planetary systems

1. Introduction

Next-generation space telescopes like HabEx and LUVOIR promise to directly image nearby Earth twins in reflected light, allowing astronomers to measure the reflectance spectra of these planets (Des Marais et al. 2002), as well as to monitor their time-varying brightness (Ford et al. 2001). We focus on this second goal, but note that it may be impossible to properly interpret spectra without context from time-resolved observations, and vice versa.

Using only time-resolved photometry, we would like to infer a planet’s spin and its albedo map (for a recent review of exoplanet mapping, see Cowan & Fujii 2017). Rotation and obliquity place strong constraints on the late stages of a planet’s formation (e.g., Schlichting & Sari 2007). Rotation also determines the frequency of diurnal radiative forcing and the amplitude of Coriolis forces, while obliquity determines the latitudinal distribution of insolation and the amplitude of seasons. Surface geography, on the other hand, bears witness to the geophysical/geochemical processes operating on a planet (e.g., Abbot et al. 2012; Cowan & Abbot 2014; Fujii et al. 2014), and—in the case of liquid water—can directly establish the habitability of a planet (Robinson 2017). Finally, the spatial and temporal distribution of clouds is determined by global climate, orography, and large-scale circulation (e.g., Showman et al. 2013).

Much progress has been made since Ford et al. (2001) showed that time-resolved photometry of Earth encodes information complementary to time-averaged spectroscopy. First of all, researchers demonstrated how to use the brightness variations of a planet to estimate its rotational period (Pallé et al. 2008; Oakley & Cash 2009). It was then shown that the rotational color variations of a planet can be used to infer the number, reflectance spectra, and longitudinal locations of major surface types (Cowan et al. 2009, 2011; Fujii et al. 2010, 2011, 2017; Cowan & Strait 2013). Meanwhile, the rotational and orbital color variations of an unresolved planet can be analyzed to create a two-dimensional multicolor map—equivalently a 2D map of known surfaces—and measure rotational obliquity (Kawahara & Fujii 2010, 2011; Fujii & Kawahara 2012; Kawahara 2016; Schwartz et al. 2016).

Kawahara & Fujii (2010) demonstrated the retrieval of a planet’s surface albedo map and rotational obliquity from simulated one-year light curves of an atmosphere-less Earth. Mapping the surface from light curves is an ill-conditioned inverse problem that is unstable to noise. To overcome it, they adopted the physical condition that the surface albedo is between 0 and 1 and recovered the rough surface features of Earth. The obliquity was measured by minimizing $\chi^2$ or the extended information criterion, along with its uncertainty by bootstrap resampling.
Kawahara & Fujii (2011) and Fujii & Kawahara (2012) applied the concept of 2D mapping to simulated light curves of a realistic cloudy Earth, with an updated inversion technique. In these studies, they employed Tikhonov regularization for the albedo map instead of the bounding condition. This method discards the components associated with small singular values of the design matrix and is equivalent to adopting a Gaussian prior in a Bayesian framework. Their recovered 2D surface maps at different photometric bands exhibit features consistent with the actual maps of cloud/snow cover, continents, and vegetation. However, evaluating the uncertainty of the recovered map and obliquity was left aside.

In this paper, we improve on the work of Kawahara & Fujii (2011) in three important ways: (1) we base our map on only five epochs of 24 hr each, rather than a year’s worth of exposures; (2) we fit for the characteristic length scale of albedo markings on the planet, rather than adopting an a priori spatial scale; and (3) we quantify the uncertainty on the albedo map.

2. exocartographer

exocartographer is a fully Bayesian framework for retrieving the albedo map and spin geometry of a planet based on time-resolved photometry. The Python code is open source and available at https://github.com/bfarr/exocartographer.

2.1. The Forward and Inverse Problems

We cannot hope to extract the same detailed information from the light curves as went into them. We will instead use a relatively simple surface integral that captures the essential physics governing the light curves. Following Cowan et al. (2013), we describe the time-resolved reflectance of the planet as

\[ R(t) = \int_0^1 A(\theta, \phi)K(\theta, \phi, S, t)\sin\theta d\theta d\phi, \]

where \( A(\theta, \phi) \) is the 2D albedo map of the planet, which we assume to be fixed; \( K(\theta, \phi, S, t) \) is the convolution kernel; \( \theta \) is co-latitude; \( \phi \) is longitude; and \( S \) represents planetary spin parameters that are not known a priori, namely, obliquity and its orientation with respect to the observer. Computing \( R(t) \) given everything on the right-hand side of Equation (1) is the forward problem.

The inverse problem is to determine \( A(\theta, \phi) \) and \( S \), given \( R(t) \), the photometric uncertainty \( \sigma \), and a parameterization of \( K \). In practice, this entails repeatedly solving the forward problem with varying parameters to see which ones best match the data in hand. In order for this to be computationally feasible, one must make simplifying assumptions: the model used for retrieving the albedo map and planetary spin is essentially a toy model, albeit one that captures the first-order physics. We adopt the kernel for diffuse reflection from Cowan et al. (2013): \( K = \frac{1}{2} V(\theta, \phi, t)I(\theta, \phi, t) \), where \( V \) and \( I \) denote the visibility and illumination functions. All of the time dependence—and the dependence on planetary spin—enter the forward problem through the visibility and illumination. They can be expressed compactly in terms of the angles between the local normal and the vector pointing toward the observer and the star: \( V = \max(\cos \gamma_o, 0) \) and \( I = \max(\cos \gamma_s, 0) \), where \( \gamma_o \) and \( \gamma_s \) are the observer and stellar zenith angles, both of which are a function of time and location on the planet.

The visibility and illumination functions are more usefully expressed in terms of latitude and longitude of the subobserver and substellar positions:

\[ V = \max \begin{cases} \sin\theta\sin\phi \cos\gamma_o + \cos\theta \cos\phi \sin\gamma_o + c\theta c\theta, & 0 \end{cases}, \]

\[ I = \max \begin{cases} \sin\theta\sin\phi \cos\gamma_s + \cos\theta \cos\phi \sin\gamma_s + c\theta c\theta, & 0 \end{cases}, \]

where we have used \( s \) and \( c \) to denote sine and cosine, and the \( o \) and \( s \) subscripts denote the subobserver and substellar location, which are functions of time. The computational crux of the forward problem is thus to quickly compute the sines and cosines of these time-varying angles as a function of the orbital and spin parameters (e.g., Appendix A of Schwartz et al. 2016).

2.2. Basis Maps

In order to make the inverse problem tractable, we need to adopt a parameterization for the albedo map, \( A(\theta, \phi) \). As discussed in Cowan & Fujii (2017), there are two complementary classes of basis maps one can adopt: pixels and spherical harmonics. For the current application it is necessary to switch back and forth between these two representations to take advantage of both of their strengths.

Spherical harmonics are an orthonormal basis and are complete for any continuous map on a sphere. This means that the coefficients we derive for an expansion up to, say, \( l = 3 \) should remain unchanged if we extend the expansion to higher \( l \). (In practice, this is only strictly true if one is decomposing a map rather than a light curve, but it is roughly true for light curves, too: adding higher-order spherical harmonics should only produce small changes in the lower-order coefficients.) Moreover, the spherical harmonic basis set exhibits a null space: certain maps have no light curve signature. By using spherical harmonic basis functions, we can trivially quantify the extent to which our Gaussian process prior constrains otherwise unconstrained coefficients. The spherical harmonic representation is convenient for mapping because it enables coherent jumps in large regions of the planet and enables a straightforward application of regularization, as described below.

Pixels are also an orthonormal basis. They have a more intuitive null space (e.g., latitudes more than 90° from the subobserver latitude are simply unobservable and hence in the null space). The pixel representation is convenient because the albedo of a pixel must be between 0 and 1, which makes it easy to propose parameter jumps.

We use Hierarchical, Equal Area, and isoLatitude Pixelation (HEALPix)\(^{10} \) pixels (Górski et al. 2005), so our map parameters are the albedo of each pixel. HEALPix is superior to a regular lat-lon grid because all pixels are the same area, there are no singularities at the poles, and there exist relatively simple transformations between HEALPix and spherical harmonics, as discussed below.

The base HEALPix resolution is 12 pixels. Higher resolutions are defined by \( N_{\text{side}} \), the number of divisions along the side of a base pixel. The number of HEALPix pixels in a map is therefore \( N_{\text{pix}} = 12N_{\text{side}}^2 \). We treat Equation (1) as a simple

\(^{10} \text{http://healpix.sourceforge.net/} \)
inner product of the albedo map and the kernel, rather than integrating over each finite pixel. We test the necessary resolution of the kernel by producing light curves with the same map, but with kernels of different resolution. We find that \( N_{\text{side}} = 8 \) and 4 introduce pixelation errors near quadrature of \( 10^{-3} \) and a few \( \times 10^{-3} \), respectively. Photometry of directly imaged planets is unlikely to be better than 1% in the foreseeable future (Cowan et al. 2009; Fujii & Kawahara 2012), so we deem that \( N_{\text{side}} = 4 \) is sufficient for our purposes (Fujii & Kawahara 2012 use \( N_{\text{side}} = 8 \)).

A heuristic can be devised for the \( N_{\text{side}} \) that one should adopt as a function of orbital phase. The kernel is a lune and therefore has a height of \( \pi \) and a width of \( \pi - \alpha \), where \( \alpha \) is the star–planet–observer “phase angle.” Each pixel has an angular area of \( 4\pi/N_{\text{pix}} \) sr and an angular size of \( \sqrt{4\pi/N_{\text{pix}}} = \sqrt{\pi/N_{\text{side}}} \approx N_{\text{side}}^{-1} \) radians. In order to properly resolve the kernel, one would like at least three pixels across the narrow width of the kernel\(^{11} \), \( N_{\text{side}}^{-1} \lesssim (\pi - \alpha)/3 \), suggesting that the minimum resolution at a given orbital phase is \( N_{\text{side}} \gtrsim 3/(\pi - \alpha) \). At quadrature (\( \alpha = \pi/2 \)), for example, we find that the minimum \( N_{\text{side}} \) is \( 6/\pi \approx 2 \), justifying our use of \( N_{\text{side}} = 4 \).

Our map parameters are the pixel albedos. The kernel of the convolution is computed on the HEALPix map, so the transformation from map to light curve is simply matrix multiplication. But the GP prior is applied on the spherical harmonic coefficients, \( a_l^m \), so we must convert from pixels to spherical harmonics at each step in the Markov Chain Monte Carlo (MCMC). At first blush, it seems that one could decompose the kernel into spherical harmonics as well and perform the convolution in \( Y_l^m \) space, but this is no faster because there are roughly as many spherical harmonics as there are pixels. Unfortunately, one cannot have exactly the same number of pixels as spherical harmonics. In particular, for \( N_{\text{side}} = \{1, 2, 4, 8\} \) the number of pixels is \( N_{\text{pix}} = \{12, 48, 192, 768\} \). The number of spherical harmonics is \( N_{\text{SH}} = (l_{\text{max}} + 1)^2 \), which corresponds to \( N_{\text{SH}} = \{9, 16, 36, 49, 169, 196, 729, 784\} \) for \( l_{\text{max}} = \{2, 3, 5, 6, 12, 13, 26, 27\} \). Our chosen HEALPix resolution of \( N_{\text{side}} = 4 \) roughly translates to \( l_{\text{max}} = 13 \). Since \( N_{\text{SH}} \) is slightly greater than \( N_{\text{pix}} \), we are slightly overconstraining the map, as described below.

### 2.3. Gaussian Process

Our adopted resolution of \( N_{\text{side}} = 4 \) means that we have 192 map parameters, namely, the albedo of each pixel. This is more than the 120 data points we will fit below, so we must apply additional constraints on the pixel values to regularize the inferred map. We choose to do this by imposing a GP prior on the map that imparts a preferred length/angular scale to the albedo structures. This scale could correspond to the typical size of clouds or continents on a planet. Unlike other regularization procedures, we do not have to choose the angular scale a priori, rather it is a fitted parameter along with the pixel values themselves and the viewing geometry parameters.

At each step in the fit, all of the pixels, viewing parameters, and GP parameters are varied independently. We then use the current GP model to evaluate the prior on the current map. The GP parameters themselves have priors, so there are two layers of priors, making this a “hierarchical” model. The GP parameters and their priors are the mean albedo (flat prior), which is subtracted from the pixel values before transformation to the spherical harmonic basis (see below); the standard deviation of the albedo (prior is flat in log); the preferred angular scale (flat prior between 1/3 of a pixel to 3\( \pi \)); and the relative amplitude of spatially uncorrelated, white-noise, albedo variations (flat prior between 0 and 1). Since the priors on the GP parameters only indirectly affect the posterior, these choices are not critical.

The posterior, the probability of parameters, \( \theta = (\theta_{\text{GP}}, \theta_{\text{map}}, \theta_{\text{orb}}) \), given data, \( d \), is given by

\[
p(\theta|d) \propto p(\theta_{\text{GP}})p(\theta_{\text{orb}}) \times p(\theta_{\text{map}}|\theta_{\text{GP}})p(d|\theta_{\text{map}}, \theta_{\text{orb}}),
\]

where the last term is the likelihood of the data given the map, orbital, and GP parameters. Evaluating the GP prior—a multivariate Gaussian—in pixel space is expensive, so we take advantage of the property that any Gaussian map whose correlation matrix depends only on angular separation between points—i.e., any statistically isotropic map—will have a diagonal covariance matrix in \( Y_l^m \) space (e.g., Wandelt 2013).\(^{12} \)

Instead of computing a covariance matrix and using a multivariate Gaussian distribution in pixel space (an operation whose computational cost would scale as \( N_{\text{pix}}^2 \)), we compute \( a_l^m \) spherical harmonic coefficients from the pixel values using the fast HEALpix algorithm and apply separate univariate Gaussian distributions in spherical harmonic space. Each \( a_l^m \) for \( l \geq 1 \) is given a Gaussian prior with a scale given by (the square root of) an angular power spectrum, \( C_l \): \( a_l^m \sim N(0, \sqrt{C_l}) \). The choice of function for \( C_l \) implements the desired angular correlation scale in pixel space. We use

\[
C_l \propto \exp \left[ -\frac{1}{2} \left( \frac{l \lambda}{\pi} \right)^2 \right] + \epsilon,
\]

where \( \lambda \) is the desired angular scale and \( \epsilon \) is chosen to give the desired level of uncorrelated variation (i.e., white noise) in the pixels. The overall multiplicative scale of the \( C_l \) is set to ensure the desired variance over the pixels. This effectively imposes a squared-exponential isotropic GP covariance kernel in pixel space with a standard deviation (in angle) \( \approx \lambda \) plus a white-noise term for the uncorrelated pixel variance that is proportional to \( \epsilon \).

There is only one final wrinkle: the number of degrees of freedom in the \( a_l^m \) coefficients cannot be made equal to the number of pixels, as described above. We can either (1) underconstrain the pixels by using an \( l_{\text{max}} \) that corresponds to fewer \( a_l^m \) coefficients than pixels, which will result in some linear combinations of pixels (corresponding to the \( Y_l^m \) with \( l > l_{\text{max}} \)) that are unconstrained by the prior, or (2) overconstrain the pixels by choosing an \( l_{\text{max}} \) that corresponds to more \( a_l^m \) coefficients than pixels, meaning the effective prior imposed in pixel space is not exactly the squared-exponential kernel. We choose to do the latter, so the pixels are overconstrained. This makes sampling easier, since no combinations of pixels can run away to infinity.

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11. One can roughly think of the longitudinal cross section of the kernel as an unnormalized Gaussian, described by a mean, width, and height.

12. Plate tectonics are unaffected by Coriolis forces, so we expect the surface map of Earth to be statistically isotropic; this assumption might break down for maps that include clouds.
2.4. Viewing Geometry

We assume that the orbital parameters of the planet are constrained via other means. In particular, the semimajor axis and orbital inclination can be measured via radial velocity (Lovis & Fischer 2010), stellar astrometry (tracking the host star’s reflex motion in response to the planet’s gravitational tug; Quirrenbach 2010), or planetary astrometry (tracking the planet’s motion via direct imaging; Blunt et al. 2017).

The spin of a planet is described by three scalars: rotational period (or angular frequency), obliquity (angle between the planet’s spin vector and its orbital angular momentum vector), and equinox (orientation of spin vector within the orbital plane). Previous authors have shown that a planet’s rotational period can be extracted from time-resolved photometry via autocorrelation functions, fast Fourier transforms, or periodograms (Pallé et al. 2008; Oakley & Cash 2009; Jiang et al. 2018). We therefore presume that the rotational period is known a priori and focus on the two angles—obliquity and equinox—describing the orientation of the planet’s spin.

The two remaining geometrical parameters are arbitrary: we do not worry about offending extraterrestrials by redefining their Greenwich (zero longitude), nor do we care about the exact date at which the planet appears at a given point in its orbit.

In principle, the time-resolved photometry we consider below could be used to determine the orbit of the planet and its spin rate, but in practice we expect that actual exoplanet observations will be iterative: a few images at different epochs to establish a comoving companion, a few more to establish the planet’s orbit, an intensive multiday campaign to establish the planet’s rotational period, leading to the dedicated exocartographer campaign we have considered here.

2.5. Parameter Sampling

For a given light curve we use Powell’s method (Powell 1964) to maximize the posterior probability density to determine best-fit map and viewing geometry parameters, which we use to seed a thorough exploration of parameter space to determine parameter uncertainties.

In order to explore parameter space, exocartographer uses the MCMC sampler emcee (Foreman-Mackey et al. 2013). In particular, we use parallel tempering (Vosseun et al. 2016) to improve sampling efficiency of the high-dimensional, nonlinearly correlated posterior. For the analysis in this paper we initialize the ensemble using Gaussian draws about the best-fit parameters found using Powell’s method.

3. Testing exocartographer

3.1. Synthetic Light Curve

We produce an idealized light curve with an hourly cadence (Figure 1) using a cloud-free toy model of Earth (a composite image of Earth,13 HEALPix-pixelized with $N_{\text{side}} = 64$; see the top panel of Figure 2), adopting Lambertian reflection, and adopting a face-on orbit ($i = 0$) and $90^\circ$ obliquity, but otherwise Earth-like values (1 day rotation and 365 day orbit). This is the most favorable viewing geometry for exoplanet mapping: (1) the planet–star projected separation is constant and hence the planet is visible throughout its orbit; (2) since it is always at quadrature, scattering phase effects can be neglected; (3) the planet is viewed equator-on and therefore the entire planet can be mapped; and (4) the large obliquity ensures that all latitudes are well illuminated at at least one point in the orbit, making it possible to recover a faithful map of the entire planet. Indeed, this idealized scenario is precisely what Kawahara & Fujii (2010) adopted for their seminal paper.

Leveraging the work of Schwartz et al. (2016), we only consider five epochs, and in deference to the fact that these missions will be horribly oversubscribed, we only observe the planet for slightly more than a planetary rotation at each epoch. Observing the planet for a full rotation at each epoch is the minimum one can do and still expect to properly map the planets. Real observations would likely include multiple rotations per epoch for three reasons: to detect the periodic variations and hence establish the planetary rotation period in the first place, to bolster the signal-to-noise ratio of the variations, and to enable the study of time-varying clouds.

Considering five days’—rather than a year’s—worth of data has the ancillary benefit of reducing the run time of forward model calls by almost two orders of magnitude. Generating a light curve across five days with a 1 hr cadence from a 192 pixel map takes 1.6 ms with a 3 GHz Intel Xeon W processor, and a full posterior probability density evaluation (including light curve generation) takes 2.3 ms.

3.2. Retrieval Results

The retrieval exercise was a surprising success. We can successfully model the simulated photometry (Figure 1), but this is not particularly noteworthy: as stated above, the problem would be underconstrained if it were not for the GP prior. More

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13 Blue Marble Next Generation w/ Topography and Bathymetry, June: https://visibleearth.nasa.gov/view.php?id=73726.
importantly, we accurately recover the planet’s spin and albedo map, shown in Figures 2, 3, and 4, and described in more detail below.

3.2.1. Planetary Spin

Using only five days’ worth of photometry, we recover the planet’s obliquity and its orientation (i.e., equinox) to high precision. Figure 4 shows the 2D marginal posterior distribution for these quantities. We constrain the equinox to be $9.8^{\pm 1.3}$ degrees (median and bounds of the central 90% credible interval) and the obliquity to be $\theta > 87.9^\circ$ at the 90% credible level (we quote only the lower bound here since the posterior has support up to the maximum allowed value of $90^\circ$). The simulated values for obliquity and equinox were $10^\circ$ and $90^\circ$, respectively.

The obliquity of a planet dictates the insolation pattern on the planet (Pierrehumbert 2010), which, in tandem with the
rotational period and hence Coriolis forces, dictates the large-scale atmospheric flow (Showman et al. 2013). Obliquity is also the source of seasons for planets with small orbital eccentricity and hence controls (de)glaciations (Spiegel et al. 2009). Planetary obliquity is also a tribute to the last major impact in the planet’s formation, moderated by subsequent tidal evolution. Hence, measuring the obliquities for a sample of terrestrial planets would provide insight into planetary formation and evolution.

3.2.2. Albedo Map

We recover the albedo map of the planet with enough precision to robustly identify a high-albedo region (the Sahara Desert) and a low-albedo region (the Pacific Ocean). The 5th percentile, mean, and 95th percentile maps are shown in Figure 2, and the posterior distributions for the albedo of several selected pixels are shown in Figure 3. Over the 192 pixels, the average width of the central 90% credible interval for pixel albedo is 0.14. As seen in the recovered distribution for the albedo of pixel 10, the simulated albedo can fall in regions of low posterior probability for individual pixels. If the simulated map were a fair draw from our prior (i.e., a GP), then we would expect faithful marginal posterior distributions where the simulated albedo of an individual pixel would lie within a 90% credible interval for 90% of simulations. The map used for this simulation is a very unlikely draw from a GP; thus, we do not expect this property to hold for our simulation. Instead, we see the result of contention between the GP prior and the data; where the data are not informative enough to infer features at the resolution of a single pixel, the GP prior tends to smooth out the features over surrounding pixels.

The presence of surface liquid water is the canonical definition of planetary habitability (Kasting et al. 1993), and the presence of surface water reservoirs and dry land is likely necessary to maintain a stable climate on geological timescales (Abbot et al. 2012). Although previous mapping efforts had proven that the best-fit map bears a resemblance to Earth (Kawahara & Fujii 2010, 2011; Fujii & Kawahara 2012), we have now shown that—even with very limited orbital coverage—the map uncertainties are small enough to make robust inferences about the surface character of the planet.

3.2.3. Characteristic Length Scale

The characteristic length scale of albedo features, one of the parameters of the GP, is found to be 9775 km (see Figure 5). The characteristic length scale of continents on Earth today range from 3000 to 6000 km (Antarctica and Asia, respectively), where we report the square root of each continent’s surface area in deference to their irregular shapes. Oceans are somewhat bigger, ranging from 4000 to 13,000 km (Arctic and Pacific Oceans, respectively).

These characteristic sizes—and indeed the total area of continental crust—change on geological timescales due to continental drift, so it may be a coincidence that continents and oceans are comparable to the planetary radius (e.g., Hawkesworth et al. 2017). Nonetheless, one might expect that constraining a planet’s characteristic surface length scale to be on the order of the planetary radius suggests Earth-like plate tectonics.

4. Discussion and Future Work

We have built an open-source, modular code that builds on Fujii & Kawahara (2012). Crucially, we use an MCMC to extract albedo maps and planet spin and their uncertainties, whereas they used optimization to determine best-fit parameters. Also, we use GPs to enforce smooth maps rather than Tikhonov regularization. The latter has a tunable regularization parameter, λ, which is chosen based on the “L-curve criterion.” By contrast, GPs uses the data to fit for the characteristic length scale of the map.

Although this was a useful numerical experiment, we made many simplifying assumptions, all of which should eventually be relaxed in future efforts. exocartographer is a solid platform on which to build more sophisticated mapping capabilities. We now discuss possible improvements, starting with relatively straightforward considerations and ending with more challenging improvements.

4.1. Low-hanging Fruit

Our assumed photometric uncertainty of 1% for one-hour integrations is at the optimistic end of what LUVOIR might achieve for nearby targets (∼10 pc). Nonetheless, smaller telescope diameters might be able to achieve comparable photometric precision for planets larger than Earth that rotate more slowly. In any case, we expect that somewhat larger photometric uncertainties would produce proportionally larger parameter and map uncertainties. Significantly larger photometric uncertainties would wash out much of the variations in the third and fourth epochs, potentially scuttling the retrieval of spin orientation and the 2D mapping. However, we estimate that the lowest-order aspect of the map might still be recovered: the dark hemisphere (Pacific Ocean) and brighter hemisphere with continents. This question is of primary importance to the design of proposed missions like LUVOIR and HabEx, so we will tackle it quantitatively in a future study.

We assumed a known rotational period for the planet, whereas in practice this would have to be extracted from the data themselves. Previous efforts have shown that this can be done independently of exocartography (e.g., by computing the autocorrelation function of the time-resolved photometry; Pallé et al. 2008; Oakley & Cash 2009). It is unlikely that an MCMC initialized near the correct rotational period will jump to one of
its harmonics, unless the time sampling is very poor, in which case planet mapping is hopeless. By forcing the albedo to be between 0 and 1, we have implicitly assumed that the planetary radius is known. In reality, the radii of directly imaged planets will be poorly constrained (Guimond & Cowan 2018). Allowing for unknown planetary radius effectively removes the $A \leq 1$ constraint: is the planet very reflective or simply very big? Even in that case there is a physical limit to the size of planets, roughly that of Jupiter. The $A > 0$ constraint would be unaffected by an unknown radius: flux cannot be negative, regardless of planet size. For the particular retrieval exercise we have performed, the albedo of cloudless Earth is quite low, so we expect that our results would be robust to within a scale factor; we would not know the absolute albedo of the Sahara, merely that it is some factor greater than the planetary mean.

The face-on orbital geometry is favorable for mapping because the planet is never inside the coronagraph/starshade inner working angle, provided it is visible at all. The number of epochs—five—was motivated by previous theoretical work (Schwartz et al. 2016) and is thought to be close to the minimum number that enables retrieval of the planet’s spin orientation. With fewer epochs, we expect the obliquity constraints and 2D map constraints to be severely handicapped. More epochs should merely serve to tighten the constraints on the map, albeit at the expense of computational burden. The exact time of the epochs, however, were chosen randomly. So we expect that any five epochs that are well spaced throughout the orbit would yield similar constraints on the albedo map and spin orientation of the planet. The 90° obliquity of the planet facilitated our task in two ways: all latitudes receive significant sunlight at one or more orbital phases, and all latitudes are visible to the observer. The combination of these two factors mean that we can, in principle, map the entire planet. For different viewing geometries we would only be able to map certain latitudes (e.g., an observer at 45° north cannot map locations below 45° south) and some latitudes would be poorly constrained (for low-obliquity planets, the poles are poorly illuminated and hence hard to map, even for a polar observer; Cowan et al. 2011). In the exocartographer framework, however, those regions would still be constrained due to the GP, which enforces a smooth albedo map. Nonetheless, the uncertainty on those unseen regions would be large, especially those more than the characteristic angular scale away from observed latitudes.

Recent work (Haggard & Cowan 2018) has provided completely analytic solutions to the forward problem of exocartography introduced below Equation (1), that is, to the problem of computing the time-resolved reflectance given the albedo map. These results may be of help in speeding up exocartographer, but more likely will be useful in completely understanding the null space of the spherical harmonics. This will clarify what aspects of the albedo map we cannot hope to resolve from the light curve, will further clarify the uncertainties, and may be useful in optimizing the extraction of the viewing geometry and the spin parameters.

### 4.2. Harder Nuts to Crack

We have assumed that the surface of the planet is a nonuniform Lambertian reflector. In fact, real planets are non-Lambertian, notably due to forward scattering from molecules and aerosols, as well as due to specular reflection by surface liquids (Robinson et al. 2010, 2014). In principle, one could use a parameterized scattering phase function to fit for this behavior. Of course, the parameters would be different for different surface types. Although this would add a few model parameters, the main challenge would be that forward model calls might become more computationally expensive due to the nontrivial convolution kernel.

Finally, we have adopted a cloudless planet. To first order, clouds are just another bright surface, and one can differentiate between clouds and, say, continents with multiwavelength data—even two broad bands can do the trick (Cowan et al. 2009; Fujii & Kawahara 2012). But clouds further complicate the mapping exercise in two ways: they mask underlying surfaces, and they change with time. One can make a map of surfaces and average cloud cover (Fujii & Kawahara 2012), and cloud variability will show up as residual structured noise. In this scheme, regions that are essentially always shrouded in clouds, e.g., rain forests, are treated as if the clouds were indeed glued to the surface.

Is there a better way to deal with clouds? The very feature that makes clouds permicious—their time variability—may also be their undoing: since most regions are cloud free, at least occasionally, then it may be possible to construct a cloud-corrected map. This would require multiple rotations at each epoch; the duration of the observations must be greater than the characteristic weather timescale on the planet (e.g., a few days for Earth; Peixoto & Oort 1992).

A possible strategy for dealing with both non-Lambertian scattering and clouds is using multiwavelength data. In this paper, we limited ourselves to white-light photometry. Simultaneously analyzing multiband data has the advantage that the maps can look very different at different wavelengths, while the planetary spin parameters must be common (e.g., Fujii & Kawahara 2012). One can even use the color variations of the planet to identify the number and colors of surfaces, even if these are a priori unknown (Cowan & Strait 2013; Fujii et al. 2017).

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