Revamped Bi-Large neutrino mixing with Gatto-Sartori-Tonin like relation

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Abstract

The Gatto Sartori Tonin (GST) relation which establishes the Cabibbo angle in terms of the quark mass ratio: $\theta_C = \sqrt{m_d/m_s}$, is instituted as $\theta_{13} = \sqrt{m_1/m_3}$ to a Bi-large motivated lepton mixing framework that relies on the unification of mixing parameters: $\theta_{13} = \theta_C$ and $\theta_{12} = \theta_{23}$. This modification in addition to ruling out the possibility of vanishing $\theta_{13}$, advocates for a nonzero lowest neutrino mass and underlines the normal ordering of the neutrino masses. The framework is further enhanced by the inclusion of a charged lepton diagonalizing matrix $U_{1L}$ with ($\theta_{12} \sim \theta_C$, $\delta = 0$). The model being architected at the Grand unification theory (GUT) scale is further run down upto the $Z$ boson scale to understand the universality of the GST relation and the Cabibbo angle.

Keywords: Neutrino mixing, Quark mixing, Cabibbo angle, Renormalization Group Equations, Bilarge neutrino mixing.

1. Introduction

The neutrinos are the most elusive fundamental particles available in Nature. The Standard model (SM) of particle physics fails to give a vivid picture of the same. The quest to understand the underlying first principle working behind the neutrino masses and mixing mechanism takes us beyond the SM. In this article, we emphasize on the significance of the simple unification schemes in terms of the common parameters and phenomenological relation that both the lepton and quark sectors may share.

The SM witnesses only the left-handed flavor neutrinos and the corresponding flavor eigenstates, $\nu_{eL}$, $\nu_{\mu L}$ and $\nu_{\tau L}$ are not identical to their mass eigenstates ($\nu_{1L}$, $\nu_{2L}$ and $\nu_{3L}$). If the charged lepton Yukawa mass matrix, $Y_l$ is diagonal, the neutrino flavor eigenstates are

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expressed as a linear superposition of the neutrino mass eigenstates in the following way,

$$\nu_{\alpha L} = \sum_{i=1}^{3} (U_\nu)_{\alpha i} \nu_{i L}, \quad (\alpha = e, \mu, \tau), \quad (1)$$

where, the matrix, $U_\nu$ is known as the Pontecorvo-Maki-Nakagawa-Sakata (PMNS) matrix\[1\] and it preserves the information of the Lepton mixing. The matrix $U_\nu$ is testable in the oscillation experiments and to parametrize $U_\nu$, we require three angles, and six phases. Out of the six phases, three are absorbed by the redefinition of the left handed charged lepton fields ($e_L$, $\mu_L$, and $\tau_L$). If the original framework beholds a non-diagonal charged lepton Yukawa matrix, $Y_l$, then the $U_\nu$ suffers a substantial amount of correction and the PMNS matrix is redefined as,

$$U = U_{iL}^\dagger U_\nu, \quad (2)$$

where, the $U_{iL}$ is the left handed unitary matrix that diagonalizes, $Y_l^\dagger Y_l$. The $U$ carries six observable parameters: three neutrino mixing angles: $\theta_{12}, \theta_{23}$ and $\theta_{13}$, often said as solar, atmospheric and reactor angles respectively, the Dirac-type CP violating phase ($\delta$) and two Majorana phases ($\psi_1$ and $\psi_2$). Following the particle data group PDG parametrization, the $U$ appears as shown below \[2\],

$$U = R_{23}(\theta_{23}).W_{13}(\theta_{13}; \delta).R_{12}(\theta_{12}).P, \quad (3)$$

where, $P = diag(e^{-i \frac{\psi_1}{2}}, e^{-i \frac{\psi_2}{2}}, 1)$. This is to be emphasized that the oscillation experiments can not witness the Majorana phases, $\psi_1$ and $\psi_2$ and the above parametrization ensures this fact. Moreover, the proper ordering and exact information of the neutrino mass eigenvalues are unavailable as the oscillation experiments can witness only two parameters: $\Delta m_{21}^2 = m_2^2 - m_1^2$ and $|\Delta m_{31}^2| = |m_3^2 - m_1^2|$. In short, the experimental results suggest: $\theta_{12} \approx 34^0$, $\theta_{23} \approx 47^0$, $\theta_{13} \approx 8^0$, $\Delta m_{21}^2 = 7.5 \times 10^{-5} \text{eV}^2$, $|\Delta m_{31}^2| = 2.5 \times 10^{-3} \text{eV}^2$ and $\delta_{CP} \sim 281^0$ \[3\].

A specific model predicts a testable $U$. One of the many popular mixing schemes, the Tri-Bimaximal (TBM) \[4\] mixing scheme is still relevant as a first approximation because what the TBM model predicts: $\theta_{12} = 35.26^0$ and $\theta_{23} = 45^0$, fit well within the $3\sigma$ range \[3\]. But the prediction that $\theta_{13} = 0$, is strictly ruled out by the recent experiments \[5, 6\]. One can see that,

$$\theta_{13} \sim O(\theta_C), \quad (4)$$

where, the parameter, $\theta_C$ is the Cabibbo angle \[7\] and this is considered as the most important parameter of the quark sector. On the other hand, instead of introducing a correction of the order of $\theta_C$, another promising mixing scheme termed as Bi-large (BL) neutrino mixing \[8\] \[14\] is proposed which shelters $\theta_C$ as an inherent parameter of the neutrino sector. The angle, $\theta_{13}$ is visualized as: $\sin \theta_{13} \sim \lambda$, where $\lambda = \sin \theta_C$, is called the Wolfenstein parameter \[15\]. Therefore, it hints for new unification possibilities. The BL framework is further strengthened by the fact that in the $SO(10)$ or $SU(5)$ inspired Grand Unified Theories (GUT), a single operator generates the Yukawa matrices for the down type quarks $Y_d$. 

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and charged leptons, $Y_l$[16][22]. In that case, a matrix element of $Y_l$ are proportional to that of the $Y_d$ which in turn suggests that,

$$U_{IL} \sim V_{CKM}$$

(5)

where, the $V_{CKM}$ is called the Cabibbo-Kobayashi-Maskawa matrix [15][23]. A non-diagonal textured $Y_l$ is craved in those models where the reactor angle in the neutrino sector is vanishing. But in the present work, the appearance of a nondiagonal $Y_l$ is a natural consequence of the GUT motivation.

Interestingly, the role of the Cabibbo angle is not limited in defining the quark mixing only, but it describes the masses also. The Gatto-Sartori-Tonin (GST) relation establishes $\theta_C$ in terms of the mass ratio of up and down quarks [24]:

$$\sin \theta_C \simeq \sqrt{m_d/m_s},$$

(6)

The question appears whether in case of lepton sector, the masses and the mixing angles are somehow related or not. In deed, the quark and the neutrino sector differs a lot than being similar. The $V_{CKM}$ is too close to an Identity matrix, whereas the PMNS matrix $U$, is far from being an Identity matrix. Although the mixing schemes differ a lot, but believing on the unification framework like GUT, there lies enough reasons to explore similar signatures in both quark and lepton sectors. Following the footprints of GST relation in eq.(6), the viability of a similar GST like relation:

$$\sin \theta_{ij} = \sqrt{m_i/m_j},$$

(7)

is explored in the neutrino sector in Ref. [25], where the analysis is done in a basis where the $Y_l$ is diagonal and the CP violation is absent. One sees that based on the phenomenology only two GST like relations such as,

$$\sin \theta_{13} = \sqrt{m_1/m_3} \quad \text{or} \quad \sin \theta_{23} = \sqrt{m_3/m_2},$$

(8)

are possible in the neutrino sector. Needless to mention that the two relations can not be experienced simultaneously. The first relation can be enhanced in the light that $\theta_{13}$ and $\theta_C$ are of same order and at certain energy scale these two parameters may unify. If the first relation were true, the model will lean towards the normal ordering of the neutrino masses and this possibility is indicated recently by the experimental results [3]. The vindication of nonzero $\theta_{13}$, its proximity towards the Cabibbo angle and the hint for normal ordering of neutrino masses make the foundation of unification schemes stronger. In the next section we shall try to explore how the GST relation can be invoked in the framework of Bi-large neutrino mixing.
2. Modified bilarge ansatz

This is to be emphasized that even though there are reasons to demarcate the quark and the lepton sectors, yet we can see that both sectors may confront similar relations or parameters motivated in GUT. Several BL schemes are proposed in the Refs. [8–14], out of which we adopt the original one [8, 12] which stresses on the unification of the atmospheric angle and solar angle in addition to that between Cabibbo and reactor. We extend the BL framework with an additional GST like relation at the GUT scale ($\sim 10^{16}$ GeV) and the unification ansatz is presented as in the following,

$$\theta_{13}^\nu = \theta_C = \sqrt{\frac{m_1}{m_3}} = \sqrt{\frac{m_4}{m_5}},$$

$$\theta_{12}^\nu = \theta_{23}^\nu = \sin^{-1}(\psi\lambda), \quad \text{(where, } \psi \sim 3),$$

$$\theta_{12}^l \approx \theta_C, \quad \theta_{23}^l = A\lambda^2, \quad \text{(where, } A \approx 0.813),$$

where, the $\theta_{ij}^\nu$ and $\theta_{ij}^l$ stands for the mixing angles for neutrino and charged lepton sectors. It is worth mentioning that this proposition at the outset favors the normal ordering of the neutrino masses and rules out any possibility concerning $m_1 = 0$.

We identify, $U_{\nu}$, the diagonalizing matrix of the neutrino mass matrix, $m_{\nu}$ in the basis where, $Y_l$ is diagonal as shown in the following,

$$U_{\nu} = \left( \begin{array}{ccc} c - \frac{c\lambda^2}{2} & \frac{s - \frac{s\lambda^2}{2}}{s^2 - c^2 e^{i\delta_0} \lambda} & e^{-i\delta_0} \lambda \\ -cs \left(e^{i\delta_0} \lambda + 1\right) & c^2 - e^{i\delta_0} s^2 \lambda & s - \frac{s\lambda^2}{2} \\ s^2 - c^2 e^{i\delta_0} \lambda & -cs \left(e^{i\delta_0} \lambda + 1\right) & c - \frac{c\lambda^2}{2} \end{array} \right) P,$$

where, $s = \psi\lambda$ and $s = \cos(\sin^{-1}(\psi\lambda))$ and following the same, we define the neutrino mass matrix, $m_{\nu}$ as,

$$m_{\nu}(m_2, m_3, \psi, \psi_1, \psi_2, \delta_0, \lambda) = U_{\nu}^* \text{diag}\{\lambda^2, m_2', 1\} U_{\nu}^{\dagger} m_3,$$

where, $m_2' = m_2 / m_3$. The $m_{\nu}$ contains four free parameters: $m_3, m_2, \psi, \delta_0$ two Majorana phases $\psi_1$ and $\psi_2$.

The choice of the Dirac neutrino Yukawa matrix, $Y_{\nu}$ is arbitrary and we fix it as per Ref. [26] as shown

$$Y_{\nu} = \frac{1}{2} \begin{pmatrix} \nu_{11}\lambda^3 & 0 & 0 \\ \nu_{12}\lambda^6 & \nu_{22}\lambda & 0 \\ 0 & 0 & 1 \end{pmatrix},$$

where, the coefficients $\nu_{ij}$’s are illustrated in Table. (1).
On the other hand, we draw the motivation from the $SU(5)$ GUT to describe the Yukawa matrices of down quarks ($Y_d$), up quarks ($Y_u$) and $Y_l$. One finds that the unification possibilities emphasize that respective matrix elements of ($Y_l$)$_{ij}$ are linearly dependent on ($Y_d$)$_{ij}$ such that the proportionality factors are chosen fractions which arise in the $SU(5)$ phenomenology [27–31]. We propose,

$$Y_d = \begin{pmatrix} d_{11}\lambda^8 & d_{12}\lambda^5 & 0 \\ d_{21}\lambda^5 & d_{22}\lambda^4 & -d_{23}\lambda^3 \\ d_{31}\lambda^7 & d_{32}\lambda^6 & d_{33}\lambda \end{pmatrix},$$

and hence,

$$Y_l = \begin{pmatrix} -\frac{3}{2} d_{11}\lambda^8 & 6 d_{12}\lambda^5 & 0 \\ -\frac{1}{2} d_{21}\lambda^5 & 6 d_{22}\lambda^4 & \frac{3}{2} d_{23}\lambda^3 \\ -\frac{1}{2} d_{31}\lambda^7 & 6 d_{32}\lambda^6 & -\frac{3}{2} d_{33}\lambda \end{pmatrix}^T,$$

where, $d_{ij}$s are $O(1)$ coefficients (See Table. [1]). It is worth mentioning that the RL convention is adopted in the present article which says, $U_{(x)R}^\dagger Y_{(x)} U_{(x)L} = Y_{(x)diag}$, where, $x = d, u$ and $l$ and the above proposition of $Y_d$ and $Y_l$ gives the ratio $(y_{\mu}y_{d})/(y_{\tau}y_{e}) \approx 11.57$ [30] and $|V_{us}| = 0.2254$ [2]. We choose the up-quark yukawa matrix in the following manner,

$$Y_u = \begin{pmatrix} u_{11}\lambda^8 & 0 & 0 \\ 0 & u_{22}\lambda^4 & -u_{23}\lambda^6 \\ 0 & u_{32}\lambda^2 & u_{33} \end{pmatrix},$$

where, $u_{ij}$s appear as $O(1)$ coefficients. The above preparation proclaims,

$$U_{uL} \approx \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & -A\lambda^2 \\ 0 & A\lambda^2 & 1 \end{pmatrix},$$

$$U_{dL} \approx \begin{pmatrix} 1 - \lambda^2 & \lambda & 0 \\ -\lambda & 1 - \lambda^2 & 0 \\ 0 & 0 & 1 \end{pmatrix},$$

and therefore,

$$V_{CKM} = U_{uL}^\dagger U_{dL} \approx \begin{pmatrix} 1 - \frac{\lambda^2}{2} & \frac{\lambda}{2} & 0 \\ -\lambda & 1 - \frac{\lambda^2}{2} & A\lambda^2 \\ 0 & -A\lambda^2 & 1 \end{pmatrix},$$

the left-handed diagonalizing matrix of $Y_l$, appears as in the following,

$$U_{lL} \approx \begin{pmatrix} 1 - \frac{a^2\lambda^2}{2} & a\lambda & 0 \\ -a\lambda & 1 - \frac{a^2\lambda^2}{2} & -A\lambda^2 \\ 0 & A\lambda^2 & 1 \end{pmatrix},$$
where, $a = 1.03$. That the parameter is not exactly equal to unity, shifts $U_{IL}$ a little from $V_{CKM}$. The parameter $a = 1$, is true if the correlation between $Y_l$ and $Y_d$ were, $Y_l = Y_d^T$. This is to be noted that the $U_{IL}$ in the present work does not contain any complex CP phase and hence will not contribute towards the CP violation in the lepton sector. Here we wish to mention that in our earlier work\cite{12} and in the Refs. \cite{13,14}, the $U_{IL}$ shelters an arbitrary CP violating phase which is associated with the 1-2 rotation of $U_{IL}$. But in $V_{CKM}$ matrix, the CP phase is related with the 1-3 rotation. In this work, we insist on the similarity of the mixing angles along with proper placement of the CP phase in the $U_{IL}$ as per $V_{CKM}$. The CP phase does not appear in $U_{IL}$ as in the latter the $\mathcal{O}(\lambda^3)$ contribution is neglected.

As it is mentioned that the above framework is considered at the GUT scale $M_{GUT}$, we run the neutrino mass matrix $m_\nu$ following a top-down approach\cite{12,11} upto the level of $M_z$ scale. The analysis involves the heavy right-handed neutrino singlets and we see that the Renormalization Group Equations (RGE) for running the $m_\nu$ in the interval of different thresholds are different\cite{12,48}. At each threshold, the heavy right handed neutrinos has to be integrated out(These analysis involve rigorous mathematics and for necessary details see Ref.\cite{26}). In order to deal with the RGE evolution of the neutrino mass matrix and other observational parameters related to the neutrinos, we extensively use the a mathematica package \textbf{REAP} (Renormalisation group Evoluion of Angles and Phases)\cite{26} which takes care of running the neutrino mass matrix, the Yukawa matrices and the gauge couplings. This package is capable of integrating out categorically the heavy neutrinos at the appropriate thresholds. But the \textbf{REAP} rather takes the heavy right handed neutrino mass matrix $M_R$ as an input than $m_\nu$. For this, we invert the Seesaw formula\cite{49,50} and get $M_R$,

$$M_R = -\frac{2}{v^2}(Y^T_\nu)^{-1}m_\nu.Y_\nu. \quad (22)$$

where, $v$ is the Higgs' vev. We shall work in the light of the minimal supersymmetric extension of the standard model(MSSM)\cite{51,53}. The analysis involves a parameter known as supersymmetry breaking scale ($m_s$) which is still unknown. In fact it can take values from a few Tev to hundred Tev.

In the next section we shall discuss about the numerical results of the neutrino physical parameters.

3. Numerical Analysis

To exemplify, let us take the input set of parameters at the the GUT scale, $M_{GUT} = 4.577 \times 10^{16} GeV$, as shown in the following,

$$\psi = 3, \ m_2 = 0.0131 \ eV, \ m_3 = 0.074 \ eV,$$
$$\delta_0 = 318^\circ, \ \psi_1 = 0^\circ, \ \psi_2 = 180^\circ$$
$$g_1 = 0.7063, \ g_2 = 0.7065, \ g_3 = 0.7069,$$
along with the vacuum expectation value, \( v = 246 \, GeV \), \( \tan \beta = 60 \), and SUSY breaking scale \( m_s \) set at 3 \( TeV \).

The observable neutrino mass and mixing parameters are run down upto the scale of \( M_Z = 91.19 \, GeV \) and we extract the information of the observable parameters as shown below,

\[
\begin{align*}
\theta_{12} &= 33.34^\circ, \quad \theta_{13} = 8.67^\circ, \quad \theta_{23} = 46.95^\circ, \\
\delta &= 268.27^\circ, \quad \Delta m^2_{\text{sol}} = 7.40 \times 10^{-5} \, eV^2, \\
\Delta m^2_{\text{atm}} &= 2.44 \times 10^{-3} \, eV^2, \quad \sum m_{\nu_i} = 0.062 \, eV, \\
\psi_2 &= 0.277^\circ, \quad \psi_1 = 182.76^\circ
\end{align*}
\]

We see that the two angles \( \theta_{13} \) and \( \theta_{23} \) are well fitted within the 1\( \sigma \) bound and \( \theta_{23} > 45^\circ \) \cite{3}. Also, \( \theta_{12} \) lies little below the 1\( \sigma \) bound but within the 2\( \sigma \) \cite{3}. The solar and the atmospheric mass squared difference are consistent within the 1\( \sigma \) and 2\( \sigma \) bounds respectively\cite{3}. According to the recent analysis in Refs. \cite{53, 54}, the observational parameter, \( \sum m_{\nu_i} \) has got an upper bound of 0.154 \( eV \) to 0.270 \( eV \) and the most stringent upper bound is 0.078 \( eV \) as per Ref. \cite{55}. The lower bound is predicted as \( \sum m_{\nu_i} < 0.058 \, eV \) in Refs. \cite{54, 55} or \( \sum m_{\nu_i} < 0.060 \, eV \) according to the Ref. \cite{2}. We see that prediction of \( \sum m_{\nu_i} \) in our analysis lies slightly above the prescribed lower bound.

This is to be noted that in the present analysis, we see that the prediction of \( \theta_{13} \) at \( M_Z \), unlike the other mixing angles, changes appreciably with the variation of the unphysical phase parameter \( \delta_0 \) at the GUT scale. To illustrate, in the above example, keeping all the input parameters fixed, if we change \( \delta_0 \) a little from 318\( ^\circ \) to 323\( ^\circ \), we see that the \( \theta_{13} \) at the \( M_Z \) scale changes from 8.67\( ^\circ \) to 7.80\( ^\circ \) (which lies outside the 3\( \sigma \) range).

Similarly, for the all the input parameters fixed, if the SUSY breaking scale \( m_s \) is varied a little, the predictions of the mass parameters at \( M_Z \) are affected. To illustrate, we study in details the variation of all the mixing angles and the mass parameters at the \( M_Z \) scale, with respect to the variation of \( \delta_0 \) and for different values of \( m_s \). We take different values of \( m_s \) ranging from 1 \( TeV \) to 14 \( TeV \). The analysis requires the knowledge of numerical values of the three gauge coupling constants and three Yukawa couplings at the GUT scale \cite{10, 11}. For this, the respective RGE equations are run in the bottom-up approach for different values of \( m_s \) (See Table. (2)). For further discussion, we fix the input parameters at, \( \psi = 3, \, m_2 = 0.0131 \, eV, \, m_3 = 0.074 \, eV, \, \psi_1 = 0^\circ, \, \psi_2 = 180^\circ \).

As the observable \( \theta_{13} \) at \( M_Z \) varies a lot with respect to the unphysical phase \( \delta_0 \), we restrict the numerical input of the latter (See Figure (1a)) with respect to the 3\( \sigma \) bound of the former\cite{3}. We find two bounds of \( \delta_0 \) which are: 37\( ^\circ \) \( \leq \) \( \delta_0 \) \( \leq \) 45\( ^\circ \) and 315\( ^\circ \) \( \leq \) \( \delta_0 \) \( \leq \) 324\( ^\circ \) out of which the first bound is rejected in the light of 3\( \sigma \) range of \( \delta \) at \( M_Z \) scale (See Fig. (1b)). The Dirac CP violation Phase \( \delta \) is predicted to lie within a range, 267\( ^\circ \) \( \leq \) \( \delta \) \( \leq \) 276\( ^\circ \) which is true upto the 2\( \sigma \) range \cite{3}. With respect to the allowed range of \( \delta_0 \), one sees that in Figs. (2a) and (2b), the mixing angles \( \theta_{12} \) and \( \theta_{23} \) are predicted to lie within 2\( \sigma \) and 1\( \sigma \) bounds respectively. It is found that the mixing angles are less sensitive towards the variation of \( m_s \). On the contrary, the mass parameters hence the related observational parameters drifts a lot if \( m_s \) is varied (See Figs. (3a), (3b), (3c), (2e) and (2d)). We see that the numerical values \( \Delta m^2_{\text{sol}} \)
with respect to the allowed bound of $\delta_0$ agrees well within the $3\sigma$ range for variation of the $m_s$ from $1\,\text{TeV}$ to $14\,\text{TeV}$. In contrast, the same for $\Delta m^2_{\text{atm}}$ goes outside the $3\sigma$ range if $m_s \geq 5\,\text{TeV}$. The $\sum m_{\nu_i}$ (at $M_Z$), though varies with respect to $m_s$ but stays within the bound (see Fig. 3d).

4. Summary

Through this article, we have tried establish a pathway to realize the theory of neutrinos based on the ansatze inspired by unification. Its found that a simple extension of the bi-large model in terms of two unification strategies: $\theta_{13}^v = \theta_C$ and $\theta_{13}^d = \sqrt{m_1/m_3}$ can lead to a successful prediction of the observables through running the RGEs following a top-down approach. The inclusion of the GST like relation ensures the normal ordering of the neutrino masses. The model shows the variation in the prediction of the Dirac CP violation phase, $\delta$ against that of $\theta_{13}$. This in turn results in constraining the predictions of the other observable parameters within a smaller bound. The model predicts the atmospheric mixing angle, $\theta_{23}$ to strictly lie within the second octant. While running the neutrino observable parameters, we have taken care of the variation of the SUSY breaking scale. We see that the effect of this variation is more on the mass parameters and least on the mixing angles.

The present article emphasizes on the simplicity of the Bi-large mixing proposition which unifies the solar and the atmospheric angles and underlines its relevance. Based on the GUT motivation, we have formulated an $U_{\ell L}$ which in addition to being CKM like disallows the presence of any arbitrary complex phase. This distinguishes our present work from the earlier works on Bi-large model[11–14]. The present work once again justifies the universality of the Cabibbo angle in terms of featuring the mixing angles and the masses.

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\[ O(1) \text{ coefficients appearing in } Y_{\nu,d,u} \]

\[
\begin{align*}
\nu_{11} &= 0.8733, \quad \nu_{21} = 0.7626, \quad \nu_{22} = 0.4437 \\
d_{11} &= 0.676, \quad d_{12} = 0.717, \quad d_{21} = 0.730, \quad d_{22} = 0.676, \\
d_{23} &= 1.037, \quad d_{31} = 0.594, \quad d_{32} = 0.550, \quad d_{33} = 1.274 \\
u_{11} &= 0.898, \quad u_{22} = 0.672, \quad u_{23} = 0.547, \quad u_{32} = 0.603, \quad u_{33} = 0.7411
\end{align*}
\]

Table 1: The coefficients of \( Y_{\nu}, Y_{d} \) and \( Y_{u} \) as shown in eqs. (14), (15) and (17) respectively are described in this table.

| \( m_s (TeV) \) | \( M_{GUT} (10^{16} GeV) \) | \( g_1 \) | \( g_2 \) | \( g_3 \) |
|----------------|-----------------|---------|---------|---------|
| 1              | 4.090           | 0.7151  | 0.7154  | 0.7158  |
| 3              | 4.577           | 0.7063  | 0.7065  | 0.7069  |
| 5              | 4.790           | 0.7028  | 0.7031  | 0.7034  |
| 7              | 4.848           | 0.7007  | 0.7009  | 0.7010  |
| 9              | 4.912           | 0.6987  | 0.6987  | 0.6987  |
| 11             | 5.112           | 0.6973  | 0.6975  | 0.6977  |
| 14             | 7.211           | 0.6954  | 0.6957  | 0.6916  |

Table 2: The list of the gauge coupling constants \( g_1, g_2, g_3 \) and the \( M_{GUT} \) for different values of the SUSY breaking scales ranging from 1\( TeV \) to 14\( TeV \) is given.

Figure 1: (1a) and (1b) show how the \( \theta_{13}(M_Z) \) changes with respect to variation of \( \delta_0 \) at the GUT scale respectively for different values of the SUSY breaking scale, \( m_s \) ranging from 1\( TeV \) to 14\( TeV \) (All the graphs are merged almost together). In both of the plots, black horizontal line, purple and orange bands signify the best fit value, 1\( \sigma \) and 3\( \sigma \) ranges of the concerned parameter. In Fig. (1a), with respect to the 3\( \sigma \) range\(^{[3]} \) of \( \theta_{13} \), two possible ranges of input parameter \( \delta_0: 37^\circ \leq \delta_0 \leq 45^\circ \) and \( 315^\circ \leq \delta_0 \leq 324^\circ \) (shown by two vertical grey bands) are obtained. In Fig. (1b) we see that only the second range is allowed in the light of the 3\( \sigma \) bound of \( \delta \). This range \( 315^\circ \leq \delta_0 \leq 324^\circ \) predicts the Dirac CP phase \( (\delta) \) within the 2\( \sigma \) bound\(^{[3]} \).
Figure 2: (2a), (2b), (2c) and (2d) show the variation of $\theta_{12}$, $\theta_{23}$, $\Delta m^2_{\text{sol}}$ and $\Delta m^2_{\text{atm}}$ at the $M_Z$ scale respectively with respect to the variation of $\delta_0$ at the GUT scale for different values of the SUSY breaking scale, $m_s$, ranging from 1 $TeV$ to 14 $TeV$. The plots in Figs (2a) and (2b) merge almost together. The Black line, purple and the orange band represent the best-fit, 1$\sigma$ and 3$\sigma$ bounds [3] respectively for the concerned observational parameters. The vertical grey band represents the allowed bound of $\delta_0$ at GUT scale which is $315^\circ \leq \delta_0 \leq 324^\circ$. The $\theta_{12}$ is predicted around 33.5$^\circ$ (2$\sigma$) and that for $\theta_{23}$ is around 47$^\circ$ [3].
Figure 3: (3a), (3b), (3c) and (3d) show the variation of $m_1$, $m_2$, $m_3$ and $\sum m_{\nu_i}$ at the $M_Z$ scale respectively with respect to the variation of $\delta_0$ at the GUT scale for different values of the SUSY breaking scale, $m_s$ ranging from 1 TeV to 14 TeV. The vertical grey band represents the allowed bound of $\delta_0$ at GUT scale which is $315^\circ \leq \delta_0 \leq 324^\circ$. In Fig. (3d), the bound on $\sum m_{\nu_i}$ is prescribed with respect to the ref. [55].