Can we observe galaxies that recede faster than light? — A more clear-cut answer

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Abstract A more clear-cut answer to the title question is: Yes, if the expanding universe started with a big bang; No, if it started infinitely slowly.

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1. Formulations of the Question The present Letter is a follow-up of a previous, much longer paper (Kiang 1997) on the same question stated in the title. The verbs in the question were left deliberately vague as to tense, because the previous paper dealt with both of the two formulations, (i) “Can we now observe galaxies that were receding faster than light at the time of emission?” and (ii) “Can we eventually observe galaxies that are now receding faster than light?”

In the present Letter I shall concentrate on the first formulation, which seems to be of more practical interest, for it can be re-stated in a sharper form: “Among the objects we now observe can we identify those that were receding faster than light at the time of emission?”

It appears that, when so formulated, the question is capable of quite a clear-cut answer. That is the main thrust of this Letter. But before I present it, I shall (1) summarize the relevant results so far obtained and (2) consider the question for the currently favored ($\Omega_M = 0.3$, $\Omega_\Lambda = 0.7$, $\Omega_K = 0$) model (the “30/70 model” hereinafter). It was the result obtained with the 30/70 model that prompted me to seek, and eventually to find, the more general result.

2. Previous Results The main relevant results of my (1997) paper are as follows.

1. This question of observability has nothing to do with horizon.
2. It depends solely on the form of the scale factor, $R(t)$, of the cosmological model. More precisely, it simply depends on whether or not two well-defined curves in the $r - t$ plane ($r$ the coordinate distance, $t$ the cosmic time) intersect at some point in the past. The two curves are, 1) the curve that separates the regions of subluminal and superluminal recession velocities,

$$r_{v=c}(t) = 1/\dot{R}(t),$$

(1)
and 2) the equation for our past light cone:

\[ r_{PLC}(t) = \int_t^{t_0} \frac{dt}{R(t)}, \]

\( t_0 \) being the present epoch. Here and throughout this Letter, we always use \( c = 1 \) units.

3. It then followed that for the steady-state model, the answer is “no”; on the other hand, for all the three varieties (\( k = 0, \pm 1 \)) of the big bang (\( \Lambda = 0 \)) model, the answer is “yes”; in particular, for the \( k = 0 \) model (the “standard” big bang), we have the result that all quasars with redshifts greater than 1.25 (and we now know thousands of such objects) had superluminal recession velocities at the time of emission.

3. The 30/70 Model

The equations appear simplest when we use the Hubble units (Kiang 1987), i.e., when all times are in units of the current Hubble time \( H_0^{-1} \) and all distances, the current Hubble radius \( cH_0^{-1} \). Then, \( c \) and \( H_0 \) will not appear in the equations (being equal to 1), and all the physical quantities are non-dimensional. In these units, the Friedmann equation for the 30/70 model is, (cf. e.g., Davis et al., 2003, Equation (11); Bondi 1960, p.80),

\[ \dot{R}^2 = 0.3 R^{-1} + 0.7 R^2, \]

and we have the explicit solution (Bondi 1960, p.82, putting \( C = 0.3, \lambda = 3 \times 0.7 = 2.1 \) in the Bondi formula for his case 2 (i)),

\[ R^3(t') = 0.2143 [\cosh(2.510 t') - 1], \]

where \( t' \) is time reckoned from the big bang. Since for \( t' = 0.964 \) we have \( R = 1 \), the age of the universe in this model is \( t_0 = 0.964 H_0^{-1} \). In the figures that follow, I shall be using time reckoned from the present epoch, \( t \), rather than \( t' \); we have \( t' = t + 0.964 \).

From these expressions it is easy to calculate the two curves (1) and (2). These are shown in Fig. 1, along with the scale factor \( R \). The scale factor for the standard big bang model is included for interest;—it is labelled \( R_1 \):

The \( v = c \) locus (1) and our past light cone (2) intersect at point A (\( t = -0.681, r = 1.063 \)). At \( t = -0.681 \), we have \( R = 0.383 \) (Point B). This last value corresponds to \( z = 1.61 \). Hence we may assert that according to the 30/70 model, all objects with redshifts greater than 1.61 have the property that, at the time of emission, they were receding from us faster than light.

This result is qualitatively the same as for the standard big bang model; the only difference is that, in the latter, the dividing redshift is even lower, at 1.25.

4. Towards a More Clear-Cut Answer. It thus appears that the answer to the stated question may not depend on whether or not there is a non-zero
Fig. 1 The scale factor $R(t)$, and the $r_{v=c}(t)$ and $r_{PLC}(t)$ curves for the 30/70 model.

$R_1(t)$ is the scale factor for the standard big bang model.

cosmological constant $\Lambda$, that the answer is always “yes”, as long as the universal expansion started with a big bang, that is, as long as there was a point in the finite past, $t_{BB}$, when $R = 0$ and $\dot{R} = +\infty$. This conjecture is easily proved to be true as follows. Consider the behavior of the two curves (1) and (2) as we move backward in time from $t = 0$ to $t = t_{BB}$. Consider curve (2) first. Since $R(t)$ is always positive, $r_{PLC}$ must start from 0 and increase monotonically to some finite value at $t_{BB}$ (The integral in (2) converges at $t_{BB}$, because generally, $R \rightarrow (t')^{2/3}$ as $t' \rightarrow 0$). On the other hand, the $r_{v=c}$ curve (1) always starts at unity (when the Hubble units are used) at $t = 0$, and ends at 0 at $t = t_{BB}$, whether the decrease is monotonic, as in $\Lambda = 0$ cases, or whether it first increases then decreases, as may happen in a $\Lambda \neq 0$ case. In all cases, the two curves must intersect at some time between the present epoch and $t_{BB}$, and we have proved

**Lemma 1** If the universal expansion started with a big bang, then of the objects we now observe, those with redshifts above a certain calculable value have the property that, at emission, they were receding from us at speeds greater than $c$.

Now, what happens if the universe did not start with a bang, but very, very gently from time minus infinity? The steady-state model is one such model and I found in my 1997 paper that here, the $r_{v=c}$ curve lies consistently one Hubble
radius above the $r_{\text{PLC}}$ curve; in fact, for all $t < 0$,

$$r_{v=c}(t) = 1 + e^{|t|}, \quad r_{\text{PLC}}(t) = e^{|t|}.$$  \hfill (5)

Hence, as already stated, the answer is “no” here.

Now, the steady-state model is a non-relativistic model. Of the relativistic models (more precisely, Friedmann models), only those with a non-negative mass parameter ($\Omega_M \geq 0$) and a non-negative curvature parameter ($\Omega_K \geq 0$) and the corresponding critical dark-energy parameter ($\Omega_{\Lambda} = \Omega_{\Lambda_c} = (4 \Omega_K^3)/(27 \Omega_M^2)$) start expansion infinitely slowly from a value of $R$ equal to $(3 \Omega_M)/(2 \Omega_K)$ (Bondi p.82). Felten and Isaacman (1986) have comprehensively reviewed all Friedmann models with non-negative mass density $\Omega_M$ for all values of the cosmological constant $\Lambda$.

Fig. 2 shows the $R(t)$ for the model ($\Omega_M = 0.1, \Omega_K = 0.45, \Omega_{\Lambda_c} = 1.35$), as a typical representative of $\Lambda = \Lambda_c$ models. Also shown is the scale factor, $R^*(t)$, for the limiting case ($\Omega_M = \Omega_K = 0, \Omega_{\Lambda_c} = 1$).

The limiting (0,0,1) model is just the de-Sitter model. Its scale factor is the same as that of the steady-state model: $R^*(t) = \exp(t)$ for all $t$ between $-\infty$ and $+\infty$. Hence, always using the asterisk to mark the limiting case, we have, from (5),

$$r_{v=c}^*(t) > r_{\text{PLC}}^*(t)$$  \hfill (6)

for all $t < 0$. Now, from Fig. 2 we see that, for all $t < 0$,

$$R(t) > R^*(t), \quad \dot{R}(t) < \dot{R}^*(t)$$  \hfill (7)
hence, from the definitions (1) and (2), we have, for all \( t < 0 \),
\[
r_{v=c}(t) > r^*_{v=c}(t), \quad r_{PLC}(t) < r^*_{PLC}(t).
\]
(8)

Combining the inequalities (8) and (6), we have, for all \( t < 0 \),
\[
r_{v=c}(t) > r_{PLC}(t).
\]
(9)

This result is valid for all \((\Lambda_c, \Omega_M > 0)\) models. Combining this with (6),
we have that for all \((\Lambda_c, \Omega_M \geq 0)\) models, the two curves (1) and (2) never intersected at any time in the past. Hence we have

**Lemma 2** If the universe started expansion infinitely slowly from
either zero or a finite size, then all the objects we observe now had
subluminal recession velocities at the time of emission

Combining the two Lemmas stated above, I now conclude with the following
statement: “to the question whether among the objects we now see, there are
some that were receding faster than light at the time of emission, the answer is
‘Yes’, if the universe started the expansion with a bang; ‘No’, if the expansion
started infinitely slowly.”

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