Theoretical and Experimental Study of Multipactor Discharge on a Dielectric

V A Ivanov\textsuperscript{1,2}, A S Sakharov\textsuperscript{1,3} and M E Konyzhev\textsuperscript{1}

\textsuperscript{1} Prokhorov General Physics Institute of the Russian Academy of Sciences, Moscow, 119991 Russia
\textsuperscript{2} National Research Nuclear University “MEPhI,” Moscow, 115409 Russia
\textsuperscript{3} E-mail: sakharov_as@mail.ru

Abstract. A review of theoretical and experimental studies of secondary electron emission microwave discharge (multipactor) on a dielectric is presented. The coefficient of microwave power absorption by a single-surface multipactor on a dielectric is found as a function of the incident microwave power and secondary electron emission properties of the dielectric. Results of experimental studies of a single-surface multipactor on a lithium fluoride (LiF) single crystal in a rectangular waveguide are presented and compared with theoretical results. The dependence of the microwave power absorbed by a single-surface multipactor on the incidence angle of microwave radiation is studied numerically and analytically. The influence of the metal waveguide walls on a single-surface multipactor in a parallel-plate waveguide is analyzed. It is shown that, at a sufficiently high microwave intensity, a combined single + double-surface multipactor develops in the region adjacent to the dielectric window.

1. Introduction

Suppression of microwave discharges in the vacuum transmission lines of high-power microwave devices is a very challenging problem in various fields of science and technology, such as space and ground-based microwave communication \cite{1, 2} and systems for microwave plasma heating in magnetic confinement devices \cite{3, 4}. Microwave discharges deteriorate the transmission properties of vacuum waveguides, lead to intermodulation and generation of microwave harmonics, and may cause the damage of the elements of transmission lines, including destruction of the input and output dielectric windows \cite{5–12}. On the other hand, microwave discharges are widely used in modern technologies for material processing \cite{13, 14} and the knowledge of their properties and conditions for their excitation is of great practical significance.

An important stage of a microwave discharge on a dielectric or metal surface is the so-called electron multipactor—an electron avalanche caused by secondary electron emission (SEE) from the surface under the bombardment by electrons accelerated in the microwave field. Two main types of multipactor discharge are traditionally considered in the scientific literature: single-surface multipactor on a dielectric and double-surface multipactor between two metal walls \cite{15–19}.

In the classical single-surface multipactor on a dielectric \cite{16} (Figure 1a), the external microwave electric field $E_0 \sin \omega t$ is directed along the dielectric surface and the emitted electrons return back to the surface under the action of the restoring force, caused by the positive charge accumulated on the dielectric due to SEE. For this type of discharge to develop, it is necessary that the electron oscillation energy in the microwave field $\varepsilon_{osc} = (eE_0/\omega)^2/2m_e$ be higher than the first crossover energy $\varepsilon_1$ (the
energy of an incident electron above which the secondary emission yield (SEY) $\delta$ is larger than unity) [15, 16]. Here, $e$ and $m_e$ are the electron charge and mass, respectively; $E_0$ is the microwave electric field amplitude; and $\omega$ is the microwave field circular frequency.

The bombardment of the dielectric by the electrons generated in a single-surface multipactor leads to the heating of a thin (~1 $\mu$m) surface layer of the dielectric. High temperature gradients ($\geq 10^4$ K/cm) arising in this layer result in the appearance of microcracks on the dielectric surface. The multipactor can also reduce the threshold for the development of a surface microwave breakdown due to either ionization of the residual gas near the dielectric surface [20] or a short-term increase in the conductivity of the dielectric surface caused by the accumulation of point defects of the crystal lattice (color centers) under bombardment by electrons accelerated in the microwave field [7, 9, 21].

In the simplest model of a double-surface multipactor [17, 18] (Figure 1b), the external microwave field is directed perpendicular to the waveguide walls and the discharge develops if the electron transit time between the walls satisfies the resonance condition $t_{\text{transit}} \approx (2n + 1)\pi/\omega$, where $n$ is an integer.

![Figure 1. Two types of multipactor discharge: (a) single-surface multipactor on a dielectric and (b) two-surface multipactor between two metal walls.](image)

In this paper, we present a brief review of theoretical and experimental studies of multipactor discharges carried out in recent years at the Plasma Physics Department of the Prokhorov General Physics Institute, Russian Academy of Sciences. In Section 2, the single-surface multipactor on a dielectric is analyzed theoretically and simulated numerically [22, 23]. Main attention is paid to the saturated multipactor at microwave intensities substantially exceeding the threshold intensity. The power absorbed by the saturated multipactor is calculated. In Section 3, the dependence of the microwave power absorbed by a single-surface multipactor on the incidence angle of microwave radiation is analyzed [24]. It is found that the power deposited per unit area of the dielectric surface has a minimum at a certain incidence angle, which also depends of the SEE properties. Section 4 is devoted to numerical simulations of a single-surface multipactor on a dielectric window in a parallel-plate waveguide [25]. It is shown that, in this case, a combined single + double-surface multipactor can develop in the region adjacent to the dielectric window. In Section 5, the influence of elastic and inelastic electron reflections from the dielectric surface on the multipactor parameters is examined [22, 23, 25, 26]. It is shown the coefficient of microwave power absorption increases significantly in the presence of electron reflections. In Section 6, results of experimental studies of a single-surface multipactor on a lithium fluoride (LiF) single crystal in a rectangular waveguide are presented and compared with theoretical results [22, 23].

2. Microwave power absorption by a single-surface multipactor on a dielectric

Let us consider the classical single-surface multipactor on a dielectric (Figure 1a). When the electron oscillation energy $\varepsilon_{\text{osc}}$ in the microwave field is higher than the first crossover energy $\varepsilon_1$, the emitted
electrons return back to the surface with energies sufficient to knock out a larger number of secondary electrons, i.e., an electron avalanche develops, during which an electron sheath forms near the dielectric surface, whereas the surface itself is charged positively. As the positive charge on the dielectric increases, the time during which the emitted electrons return back to the surface decreases, which leads to a decrease in the energy acquired by the electrons in the microwave field. Hence, as time elapses, the electron avalanche gradually decelerates and then reaches saturation.

In the saturated (quasi-steady) multipactor, the $\delta$ value averaged over the microwave oscillation period is equal to 1. If the oscillation energy is much higher than first crossover energy ($\varepsilon_{\text{osc}} \gg \varepsilon_1$), then the electron flight time $\Delta t$ in the saturated multipactor must be much shorter than $\omega^{-1}$, otherwise the emitted electrons would acquire an energy larger than $\varepsilon_1$ and the number of electrons would continue to grow. This means that the external microwave field varies only slightly during the flight time of secondary electrons and the multipactor characteristics have time to adjust to the instantaneous value of the microwave field. Hence, we can use the so-called “constant-field” approximation, in which the multipactor parameters are determined by the instantaneous value of the microwave electric field $E_y(t)$, i.e., depend on time parametrically. In this approximation, the power deposited per unit area of the dielectric surface can be estimated analytically [23].

Assuming that the external tangential field $E_y$ is constant and the secondary electrons have a nearly Maxwellian distribution with an effective temperature $T$ (typically, about 1–2 eV [27]), we find from Poisson’s equation and equations of motion for electrons that the profiles of the electric potential and electron density along the normal coordinate $z$ are

$$e\Phi = -2T\ln\left(1 + z/\Delta z\right), \quad n_z = n_0\left(1 + z/\Delta z\right)^{-2}. \quad (1)$$

Here, $n_0$ is the electron density at $z = 0$ and $\Delta z = \sqrt{2}\nu_e / \omega_p^0$ is the characteristic sheath thickness (here, $\nu_e = \sqrt{T / m_e}$ and $\omega_p^0 = (4\pi n_e\mu/m_e)^{1/2}$). Solving the equation of motion for an emitted electron moving in the field of electric potential (1), we find that the flight time of an emitted electron up to the maximum height above dielectric surface and back is

$$\Delta t(\varepsilon_{z0}) = \sqrt{2\pi\Phi} \left(\sqrt{\varepsilon_{z0} / 2T}\right) \exp\left(\varepsilon_{z0} / 2T\right) / \omega_p^0, \quad (2)$$

where $\varepsilon_{z0}$ is the $z$ component of the initial electron energy and $\Phi(x) = (2/\sqrt{\pi})\int_0^x \exp(-x^2)dx$ is the error function.

The electron density $n_0$ just near dielectric surface can be found from the flux balance of incident and emitted electrons. Taking into account that, in the saturated multipactor, the characteristic energy of incident electrons is on the order of $\varepsilon_1$, which is usually much higher the temperature of emitted electrons ($\varepsilon_1 \gg T$), we can neglect the initial velocities $v_{z0}$ and $v_{y0}$ of a secondary electron compared to the velocity $-eE_y\Delta t/m_e$, acquired by the electron over time interval (2). Then, the flux balance equation can be written as

$$\int \delta(\varepsilon_{z0}) \exp\left(-\varepsilon_{z0} / T\right) d\varepsilon_{z0} = \int \exp\left(-\varepsilon_{z0} / T\right) d\varepsilon_{z0}, \quad (3)$$

where

$$\varepsilon_{\text{inc}}(\varepsilon_{z0}) = \left(eE_y\Delta t(\varepsilon_{z0})\right)^2 / 2m_e = E_y^2\Phi^2 \left(\sqrt{\varepsilon_{z0} / 2T}\right) \exp\left(\varepsilon_{z0} / T\right) / 4n_0 \gg T. \quad (4)$$

is the energy with which the electron returns to the dielectric surface.

To solve integral equation (3) with respect to $n_0$, it is necessary to explicitly specify the energy dependence $\delta(\varepsilon_{\text{inc}})$. In theoretical studies on multipactor discharges, the SEY is customarily described by Vaughan’s empirical formula [28] (Figure 2, curve 1),
\[ \delta = \delta_{\text{max}} (V e^{1-V})^k, \]  
(5)\

where \( V = (\epsilon_{\text{inc}} - \epsilon_0) / (\epsilon_{\text{max}} - \epsilon_0) \). Here, \( \epsilon_{\text{inc}} \) is the energy of an incident (primary) electron; \( \epsilon_0 \) is the cutoff energy, below which \( \delta \) is zero; \( \epsilon_{\text{max}} \) is the energy corresponding to the maximum SEY at a given incident angle \( \theta \); \( k = 0.62 \) for \( V < 1 \); \( k = 0.25 \) for \( V > 1 \); \( \delta_{\text{max}} = \delta_{\text{max}0}(1 + \theta^2/2\pi) \); and \( \epsilon_{\text{max}} = \epsilon_{\text{max}0}(1 + \theta^2/\pi) \), with \( \delta_{\text{max}0} \) and \( \epsilon_{\text{max}0} \) being the peak value of the SEY at \( \theta = 0 \) and the incident energy corresponding to this peak. In theoretical works and numerical simulations, a simplified formula with \( \epsilon_0 = 0 \) (Figure 2, curve 2) is widely used (see, e.g., [29–33]).

**Figure 2.** SEY as a function of the incident electron energy: (1) Vaughan’s formula and (2) simplified Vaughan’s formula with \( \epsilon_0 = 0 \). The unit step at \( \epsilon_{\text{inc}} < \epsilon_0 \) corresponds to the reflection coefficient \( R = 1 \), introduced by Vicente et al. [34] (see Section 5).

Using the simplified Vaughan formula and taking into account that the first crossover energy \( \epsilon_1 \) is usually much less than \( \epsilon_{\text{m0}} \), we can approximately write

\[ \delta = \left( \frac{\epsilon_{\text{inc}}}{\epsilon_{10}} \right)^{0.62}, \]  
(6)\

where \( \epsilon_{10} = \epsilon_{\text{m0}} / (\delta_{\text{m0}})^{0.62} \) is the first crossover energy corresponding to Vaughan’s formula with \( \epsilon_0 = 0 \). Then, from (3) we obtain

\[ n_0 = 0.7 E_r^2 / \epsilon_{10}. \]  
(7)\

The power \( W_{\text{abs}} \) deposited per unit area of the dielectric surface is found by integrating the electron energy flux onto the surface over \( \epsilon_{z0} \),

\[ W_{\text{abs}} = \int_{\epsilon_{10}}^{\epsilon_{\text{osc}}} \frac{n_0}{\sqrt{2\pi v_T}} \epsilon_{\text{inc}} (\epsilon_{z0}) \exp(-\epsilon_{z0} / T) d\epsilon_{z0} = v_T \frac{E_r^2 \sqrt{2\pi}}{8\pi} \int_0^{\epsilon_{\text{osc}}} \Phi^2 \left( \frac{\epsilon_{z0}}{2T} \right) d\epsilon_{z0}. \]  
(8)\

The integral in (8) formally diverges, because the decrease in the distribution function at \( \epsilon_{z0} \to \infty \) is balanced by the exponential increase in the incident energy \( \epsilon_{\text{inc}} \) (see (4)). However, it is clear that \( \epsilon_{\text{inc}} \) cannot be higher than \( \epsilon_{\text{osc}} \). Therefore, the integral should be truncated at the energy \( \epsilon_{z0\text{max}} \) at which incident energy (4) becomes comparable with \( \epsilon_{\text{osc}} \) and, accordingly, the electron flight time becomes comparable with \( \omega^{-1} \), i.e., the constant-field approximation is violated. Substituting (7) into (4) and setting \( \epsilon_{\text{inc}} = \epsilon_{\text{osc}} \), we find

\[ \epsilon_{z0\text{max}} = T \ln(\epsilon_{\text{osc}} / \epsilon_{10}). \]  
(9)\

Then, for \( \epsilon_{\text{osc}} >> \epsilon_{10} \), we have

\[ W_{\text{abs}} = v_T \sqrt{2\pi} \left( E_r^2 / 8\pi \right) \ln \left( \epsilon_{\text{osc}} / \epsilon_{10} \right). \]  
(10)
Averaging (10) over time and defining the microwave absorption coefficient $\kappa$ as

$$\kappa = \langle W_{ab} \rangle \left( cE_0^2 / 8\pi \right)^{\frac{1}{4}},$$

we obtain the following analytical expression for $\kappa$:

$$\kappa = \frac{\sqrt{c}}{\sqrt{2}} \sqrt{\frac{\pi}{2}} \ln(\varepsilon_{osc} / \varepsilon_{10}),$$

according to which, the absorption coefficient at high electron oscillation energies increases in proportion to the logarithm of $\varepsilon_{osc}$, which, in turn, is proportional to the intensity of the incident microwave radiation, $\varepsilon_{osc} \sim cE_0^2 / 8\pi$.

We performed 1D3V (one-dimensional in coordinate space and three-dimensional in velocity space) particle-in-cell (PIC) simulations of a single-surface multipactor excited on the surfaces of a LiF single crystal, NaCl single crystal, and amorphous quartz (SiO$_2$), because, for these materials, there is a large database on the SEE properties in the literature (see, e.g., [27, 35]).

Figure 3 presents simulation results obtained for a LiF crystal ($\varepsilon_{mb} = 1000$ eV, $\delta_{mb} = 7.5$ [35]). The single-surface multipactor was excited by a microwave field with an intensity substantially exceeding the threshold value ($\varepsilon_{osc} = 450$ eV, $\varepsilon_{10} = 14.5$ eV). The SEY was described by the simplified Vaughan’s formula with $e_0 = 0$, the frequency of the microwave field was $f = \omega / 2\pi = 1.95$ GHz, and the temperature of secondary electrons was $T = 1$ eV. The multipactor was initiated by injecting seed electrons with the areal density $S_{inj} = 0.05n_{cr}v_T / \omega = 0.8 \times 10^7$ cm$^2$ from the dielectric (where $n_{cr} = m_e \omega^2 / 4\pi e^2$). The figure shows typical time dependences of the (a) effective thickness of the electron sheath, $\delta z = S/n_0$ (where $S(t) = \int_0^n n_e(z,t)dz$ is the areal density of emitted electrons), and (b) the electron density $n_0$ near the dielectric. Here, $n_0$ and $\delta z$ are normalized to $n_{cr} = m_e \omega^2 / 4\pi e^2 = 4.7 \times 10^{10}$ cm$^{-3}$ and $z_0 = v_T / \omega = 3.4 \times 10^{-3}$ cm, respectively.

Figure 3. Time dependences of the parameters of a single-surface multipactor excited on the surface of a LiF single crystal for the electron oscillation energy substantially exceeding the first crossover critical energy ($\varepsilon_{osc} = 450$ eV, $\varepsilon_{10} = 14.5$ eV): (a) effective thickness of the electron sheath, $\delta z = S/n_0$, and (b) electron density $n_0$ just near the dielectric.

It is seen that, in the stage of electron avalanche, the number of electrons in the sheath grows monotonically, while the sheath thickness decreases to $\sim 10^{-3}$ cm, after which the discharge saturates. In the saturated (quasi-steady) regime, the electron areal density, the electron density just near the dielectric, and the effective thickness of the electron sheath oscillate with the doubled frequency of the external field, $2\omega$, following the oscillations of $E^2$. It should be noted that, in the saturated multipactor, the electron density in the sheath substantially exceeds the critical density. Nevertheless, the electron
sheath is almost fully transparent for microwave radiation, because, in this case, the skin depth $z_{\text{skin}} = c/\omega p_0 \approx 0.25$ cm is much larger than the sheath thickness.

Figure 4 shows the absorption coefficient $\kappa$ calculated using 1D3V simulations as a function of the oscillation energy for different dielectric materials [35]: amorphous SiO$_2$ (quartz glass), LiF single crystal, and NaCl single crystal, assuming that $T = 2$ eV and the SEY is described by Vaughan’s formula with $\varepsilon_0 = 0$. It can be seen that the simulation results agree satisfactorily with analytical formula: all the curves group around dependence (12). It is worth noting that analytical dependence (12) satisfactorily agrees with the simulation results not only at $\varepsilon_{\text{osc}} > \varepsilon_{10}$ (i.e., when the constant-field approximation is valid), but also near the threshold for the onset of a single-surface multipactor ($\varepsilon_{\text{osc}} \sim \varepsilon_{10}$). The larger deviation of the calculated curve for the NaCl single crystal from the analytical dependence compared to that for the LiF single crystal is probably related to the larger ratio $T_e/\varepsilon_{10}$ for the former (remember that, in deriving dependence (12), this ratio was assumed to be zero). The deviation of the calculated curve from the analytical dependence for amorphous SiO$_2$ may be attributed to the deflection of the SEY from reduced formula (6), because the first crossover energy $\varepsilon_{10}$ for SiO$_2$ is much closer to $\varepsilon_{m0}$ than for LiF and NaCl crystals [35].

3. Single-surface multipactor on an inclined dielectric surface

Single-surface multipactor on a dielectric can also develop when the microwave electric field is inclined with respect to the dielectric surface [36, 37]. Such conditions can occur on the surfaces of dielectric inserts on the waveguide walls and also when the microwave window is inclined relative to the axis of the microwave beam. Such a situation can also take place in the course of multipactor processing of dielectric crystals with the purpose of modifying their surface properties [21] (coloring of jewelry crystals, creation of a surface layer of laser-active medium for the use in integrated circuits, etc.). In this case, the optimum regime of crystal processing can be determined not only by the power deposited in the discharge, but also the direction of the microwave electric field. In this context, it is of interest to examine how the coefficient of microwave power absorption in the multipactor depends on the inclination angle $\alpha$ of the microwave electric field relative to the dielectric surface. In what follows, by the coefficient of microwave power absorption in the case of an inclined microwave field we will mean the quantity defined by formula (11).

In the two limiting cases, the multipactor is excited by the microwave field directed parallel or perpendicular to the dielectric surface. Further, we will refer to these types of multipactor as “parallel” and “perpendicular” multipactors, respectively. Parallel multipactor was considered in the previous section. Let us now turn to perpendicular multipactor. As an example, let us consider perpendicular multipactor on the surface of a LiF single crystal ($\delta_{m0} = 7.5$, $\varepsilon_{m0} = 1000$ eV, $\varepsilon_{10} = 14.5$ eV), the
numerical simulations [24] show that the coefficient of microwave power absorption in the perpendicular multipactor increases somewhat faster with increasing electron oscillation energy, but only insignificantly exceeds the absorption coefficient in the parallel multipactor. However, in its structure, the perpendicular multipactor differs fundamentally from the parallel multipactor and, in essence, is quite a different type of discharge.

Figure 5 shows the electron phase portraits in the \((z, v_z)\) plane in the perpendicular multipactor on the surface of a LiF crystal for \(f_0 = 1.95 \text{ GHz\ and } \varepsilon_{\text{osc}} = 200 \text{ eV} \gg \varepsilon_{10}\) for different instants of time during one microwave period \((- \pi \leq \omega t < \pi)\). It is seen from the figure that the electrons are periodically injected from the dielectric surface and further propagate in the form of jets in the phase plane. Most of injected electrons return back to the surface during one oscillation period, whereas the remaining electrons have time to execute several oscillations and move away from the dielectric surface over a distance of about several millimeters, which is several hundred times larger than the characteristic width of the electron sheath in the parallel multipactor.

![Figure 5](image.png)

**Figure 5.** Electron phase portraits in the \((z, v_z)\) plane in the perpendicular multipactor on the surface of a LiF crystal at \(f = 1.95 \text{ GHz\ and } \varepsilon_{\text{osc}} = 200 \text{ eV} \gg \varepsilon_{10}\) for different instants during one microwave period \((- \pi \leq \omega t < \pi)\). Here, \(v_T = (T/m_e)^{1/2}\), where \(T = 1 \text{ eV}\) is the effective temperature of secondary electrons. For illustrativeness, particles with positive and negative values of \(v_z\) are plotted in blue and red, respectively.

The power absorbed in the perpendicular multipactor can be estimates as follows. Let the electrons be periodically injected in the discharge in the phase corresponding to the accelerating external field \((E_z = E_0 \sin \omega t < 0)\) and further propagate in the form of a gradually expanding layer containing \(N\) electrons per unit area. Between the layer and the dielectric surface, the charge separation field \(E_\sigma = 4\pi eN\) arises, which decelerates the electrons and returns them back to the dielectric surface. In order for most electrons to return back to the surface over one microwave period, it is necessary that \(E_\sigma \sim E_0\). From here, we find that \(N \sim E_0/4\pi e\). Then, the average (over the microwave period) density of
the electron flux onto the surface is \( n_{v_{\parallel}} \sim N/\pi \sim E_0/4\pi \tau \sim n_{cr}v_{osc} \) (where \( \tau = 2\pi/\omega \) is the microwave period and \( v_{osc} = eE_0/m_e\omega \) is the electron oscillation velocity in the microwave field). Since the electrons return back to the surface with an energy of about \( \epsilon_{osc} \) (note that only some of them fall onto the surface in the phase favorable for the departure of secondary electrons), the mean density of the energy flux onto the surface is on the order of \( \langle W_{abs} \rangle = n_{cr}v_{osc} \epsilon_{osc} \sim (v_{osc}/c)(cE_0^2/8\pi) \) and, accordingly, the absorption coefficient is (see (11))

\[
\kappa = \beta \left( v_{osc}/c \right),
\]

where \( \beta \) is a numerical factor on the order of unity. The 1D2V PIC simulations performed for a LiF crystal [24] confirm that \( \kappa \) in the perpendicular multipactor is closely proportional to the oscillation velocity, the proportionality factor \( \beta \) being about 0.3.

The fundamental difference between the perpendicular and parallel multipactors is clearly illustrated in Figure 6. Figure 6a shows the mean flux density of secondary electrons \( \langle n_{v_{\parallel}} \rangle \) as a function of \( \epsilon_{osc} \) for the parallel (curve 1) and perpendicular (curve 2) multipactors on the surface of a LiF single crystal obtained by means of 1D3V PIC simulations [24]. For the parallel multipactor, taking into account formula (7), we have \( \langle n_{v_{\parallel}} \rangle \sim n_{cr}v_T \sim (E_0^2/\epsilon_{10})v_T \sim (\epsilon_{osc}/\epsilon_{10})n_{cr}v_T \), i.e., the mean flux density of secondary electrons is proportional to \( \epsilon_{osc} \). In the perpendicular multipactor, the current density of secondary electrons is much lower and is proportional to \( v_{osc} \). In contrast, it is seen from Figure 6b that the mean energy of incident electrons in the perpendicular multipactor, defined as \( \langle \epsilon_{inc} \rangle = \langle W_{abs} \rangle/\langle n_{v_{\parallel}} \rangle \), is much higher than that in the parallel multipactor and increases linearly with increasing \( \epsilon_{osc} \), whereas in the parallel multipactor, \( \langle \epsilon_{inc} \rangle \) grows very slowly (logarithmically) with increasing \( \epsilon_{osc} \). It is noteworthy that the absolute values of the mean power \( \langle W_{abs} \rangle \sim \langle n_{v_{\parallel}} \epsilon_{inc} \rangle \) absorbed per unit area of the dielectric surface and, accordingly, absorption coefficients (11) are close to one another for both types of multipactor.

![Figure 6](image_url)

**Figure 6.** (a) Mean flux density \( \langle n_{v_{\parallel}} \rangle \) of secondary electrons and (b) mean energy \( \langle \epsilon_{inc} \rangle \) of electrons incident on the dielectric surface as functions of \( \epsilon_{osc} \) for the (1) parallel and (2) perpendicular multipactors on the surface of a LiF crystal. Here, \( v_T = (T/m_e)^{1/2} \), where \( T = 1 \) eV is the effective temperature of secondary electrons.

Thus, the energy of incident electrons is larger for the perpendicular multipactor, whereas the flux density of incident electrons is larger for the parallel multipactor. These features of the perpendicular and parallel multipactors can be used when choosing modes of microwave processing of crystal dielectrics with the purpose of modifying their surface properties.
Let us now consider the “inclined” multipactor, which develops at intermediate inclination angles of the microwave electric field with respect to the dielectric surface, $0^\circ < \alpha < 90^\circ$. Such a situation can take place when a $p$-polarized microwave beam (the microwave electric field vector lies in the incidence plane) passes through an inclined output window of a high-power microwave source. In this case, the amplitude of the normal component of the microwave electric field is $E_0 = E_0 \sin \alpha$.

At sufficiently small inclination angles, such that $\sin \alpha < (T/\varepsilon_{10})^{1/2}$, the discharge structure remains close to that of the parallel multipactor, because, in this case, the normal component of the microwave electric field $E_0$ is smaller than the characteristic value of the electrostatic field in the parallel multipactor, $|\partial \varphi/\partial z| \sim 4\pi e_0 \Delta z \sim (T/\varepsilon_{10})^{1/2} E_0$. Accordingly, at small inclination angles, the absorption coefficient $\kappa$ in the inclined multipactor is close to absorption coefficient (12) in the parallel multipactor.

At $\sin \alpha > (T/\varepsilon_{10})^{1/2}$, the space charge sheath near the dielectric surface is destroyed and the discharge structure becomes similar to that in the perpendicular multipactor. In this case, however, the electron motion along the $z$ axis is determined by the field $E_0 = E_0 \sin \alpha$ (instead of $E_0 = E_0$ in the perpendicular multipactor). Estimating, as was done above, the number of secondary electrons emitted over one microwave period per unit area of the dielectric surface, we obtain $N \sim E_0 \sin \alpha / 4\pi e$ and, accordingly,

$$\kappa = \beta \sin \alpha \left( v_{osc} / c \right), \quad (14)$$

where $\beta$ is the proportionality factor on the order of unity ($\beta = 0.3$ for $\alpha = 90^\circ$).

Figure 7a shows the absorption coefficient $\kappa$ as a function of $\alpha$ for a multipactor on the surface of a LiF crystal for different values of $\varepsilon_{osc}$. It is seen that the absorption coefficient decreases with decreasing $\alpha$ in accordance with formula (14) and reaches its minimum value at $\sin \alpha = (T/\varepsilon_{10})^{1/2} = 0.35$, after which it increases to its value in the parallel multipactor (see (12)).

Figure 7b shows similar dependences for a multipactor on the SiO$_2$ surface. It is seen that the angular dependences of $\kappa$ for SiO$_2$ also have minima at angles approximately corresponding to $\sin \alpha = (T/\varepsilon_{10})^{1/2}$. Note that, at $\varepsilon_{osc} = 800$ eV, the absorption coefficient for the perpendicular multipactor on SiO$_2$ is reduced substantially, because, at such high oscillation energies, a considerable fraction of electrons return back to the dielectric surface with energies significantly exceeding the energy $\varepsilon_{m0} = 400$ eV, above which the SEY of SiO$_2$ decreases exponentially.

Figure 7. Absorption coefficient $\kappa$ as a function of the angle $\alpha$ for multipactors on the (a) LiF ($\varepsilon_{m0} = 1000$ eV, $\delta = 7.5$, $\varepsilon_{10} = 14.5$ eV) and (b) SiO$_2$ ($\varepsilon_{m0} = 400$ eV, $\delta = 2.4$, $\varepsilon_{10} = 40$ eV) surfaces.

The presence of a minimum in the angular dependence of the absorption coefficient at $\alpha \sim 20^\circ - 30^\circ$ indicates that the power released in the multipactor on the input/output microwave window can be reduced appreciably by tilting the window with respect to the axis of the microwave beam.
4. Single-surface multipactor on a dielectric window in a parallel-plate waveguide

The development of a multipactor discharge on a dielectric surface results in the formation of an electron sheath near this surface. If the dielectric is adjacent to the waveguide walls, the properties of the sheath can be affected by these walls. Moreover, at sufficiently high microwave intensities, both types of multipactor discharge can develop simultaneously. Hence, it is of interest to consider such a combined multipactor discharge and examine how the above two types of multipactor discharge affect one another. To take these effects into account in simulations, the numerical code should be at least two-dimensional in coordinate space. Therefore, we have developed an original 2D3V (two-dimensional in coordinate space and three-dimensional in velocity space) PIC code for modeling the effect of the waveguide walls on a single-surface multipactor on a dielectric [25, 26].

The simulation geometry (see Figure 8) was chosen to be similar to the geometry of the experiments carried out on the BRUS device [8, 9, 12, 21] (see Section 6). The problem was solved in a rectangular box $0 < y < h$, $0 < z < z_{\text{max}}$. The $z$ coordinate was directed along the waveguide. The metal walls were at $y = 0$ and $y = h$. Along the $x$ axis, the problem was homogeneous. In fact, such geometry corresponds to a parallel-plate waveguide unbounded in the $x$ direction. An infinitely thin dielectric plate was placed at $z = 0$. The external microwave electric field $E_0 \sin \omega t$ was directed along the $y$ axis and was independent of $z$. The microwave magnetic field was disregarded. This implies that we consider a region located near the antinode of the standing wave ($|z| \leq c/\omega$). The system was assumed to be symmetric with respect to $z = 0$. A more detailed description of the numerical model can be found in [25].

![Figure 8. Geometry of the model used for 2D3V PIC simulations.](image)

Figure 9 presents results of 2D3V PIC simulations of a multipactor discharge on a LiF crystal ($\delta_{\text{abs}} = 7.5$, $\epsilon_{\text{ns}} = 1000$ eV) in the presence of bounding walls with different SEYs. The interwall distance is $h = 1.5$ cm. At a low electron oscillation energy ($\epsilon_{\text{osc}} = 18$ eV, Figure 9a), drift of secondary electrons onto the absorbing ($\delta_{\text{abs}} = 0$) or low-emitting ($\delta_{\text{abs}} = 0.5$) walls results in the suppression of the single-surface multipactor on the dielectric, whereas in the presence of copper walls with a moderate SEY ($\delta_{\text{abs}} = 2$), the multipactor develops even at microwave intensities only slightly exceeding the threshold intensity in the 1D case ($\epsilon_{\text{thr}}^{\text{D}} = 16$ eV).

At a moderate oscillation energy ($\epsilon_{\text{osc}} = 50$ eV, Figure 9b), the SEE multiplication of electrons on the dielectric surface begins to prevail over their escape onto the waveguide wall and the single-surface multipactor develops for any wall material, the areal electron density in the saturated regime being practically independent of the wall material, although saturation in the case of fully absorbing walls ($\delta_{\text{abs}} = 0$) is reached somewhat later.

Finally, at a very high oscillation energy ($\epsilon_{\text{osc}} = 450$ eV, Figure 9c), secondary electrons drift onto the walls so fast that absorbing walls again suppress the development of a single-surface multipactor on a LiF crystal.
Figure 9. Influence of the bounding waveguide walls on the development of a single-surface multipactor. Time evolution of the $y$-averaged areal density of electrons $\overline{S}$ near a LiF surface at different electron oscillation energies for walls with different SEYs: (1) fully absorbing walls ($\delta = 0$), (2) soot ($\delta_m = 0.5$, $\varepsilon_m = 500 \text{ eV}$), and (3) copper ($\delta_m = 2$, $\varepsilon_m = 500 \text{ eV}$). The interwall distance is 1.5 cm; the areal density of seed electrons is $S_{inj} = 0.05 n_{cr} z_0$; and the temperatures of secondary electrons emitted from the dielectric and metal surfaces are $T_d = 1 \text{ eV}$ and $T_m = 3 \text{ eV}$, respectively.

Figure 10 shows the threshold oscillation energy for the development of a single-surface multipactor on a LiF crystal in the presence of bounding walls as a function of the interwall distance $h$. The SEE properties of the walls are the same as in Figure 9. The domains corresponding to the onset of a multipactor lie to the right of the corresponding curves. It is seen from the figure that the threshold oscillation energy $\varepsilon_{thr}$ depends substantially on the wall material at interwall distances of $h \leq 2 \text{ cm}$ (i.e., in a low-profile waveguide). At $h \geq 3 \text{ cm}$, $\varepsilon_{thr}$ is practically independent of the wall material and is close to $\varepsilon_{thr}^{1D}$. It should be noted, however, that the threshold oscillation energy also depends on the areal density of seed electrons, approaching $\varepsilon_{thr}^{1D}$ with increasing the areal density of seed electrons $S_{inj}$. Moreover, after the discharge has reached saturation, the influence of the walls decreases and the interwall distance can be reduced substantially without quenching the single-surface multipactor on the dielectric even in the case of fully absorbing walls.

In a 1D single-surface multipactor, the total negative surface charge in the electron sheath is exactly equal to the positive surface charge accumulated on the dielectric surface; therefore, the total surface charge density is zero. In the 2D case, a fraction of emitted electrons escape onto the metal walls, so it is natural to expect that the multipactor will acquire an excess positive charge and the electric potential of the dielectric surface will grow until it reaches the maximum electron drift energy equal to $4\varepsilon_{osc}$. Figure 11 shows the time evolution of the $y$-averaged relative (with respect to the metal walls) electric potential $\overline{\phi}_0$ of a LiF surface bounded by copper walls for $h = 1.5 \text{ cm}$ and two values of the electron oscillation energy. It is seen that, for $\varepsilon_{osc} = 24.5 \text{ eV}$, the electric potential of the dielectric surface, as was expected, grows monotonically in time, whereas for $\varepsilon_{osc} = 128 \text{ eV}$, it drops to about

\[ \varepsilon_{thr} \approx 16 \text{ eV} \]
−170 V, which, as will be shown below, is related to the accumulation of a negative space charge in the interwall space due to the development of a double-surface multipactor between the waveguide walls.

Figure 11. Time evolution of the y-averaged relative (with respect to the metal walls) electric potential \( \bar{\phi}_0 \) of the LiF surface bounded by copper walls for \( h = 1.5 \text{ cm} \) and two values of the electron oscillation energy: \( \varepsilon_{\text{osc}} = \) (a) 24.5 and (b) 128 eV.

Figure 12 shows the 2D distributions of the electric potential in the simulation region for \( \varepsilon_{\text{osc}} = 24.5 \) and 128 eV at different times. It is seen that, for \( \varepsilon_{\text{osc}} = 24.5 \text{ eV} \), the positive potential formed on the dielectric surface penetrates into the waveguide up to \( z = h \). For \( \varepsilon_{\text{osc}} = 128 \text{ eV} \), the electric potential in the region adjacent to the dielectric surface rapidly drops and, then, the region with the negative potential expands along the \( z \) axis until it occupies the entire simulation region.

Figure 12. 2D distributions of the electric potential in the simulation region for \( \varepsilon_{\text{osc}} = 24.5 \) and 128 eV at different times (the LiF surface bounded by copper walls).

Comparison of the time-averaged \( z \) profiles of the electric field component \( E_z \) in a 1D single-surface multipactor and the \( y \)-averaged field \( \bar{E}_z \) near the surface of the LiF crystal in the parallel-plate waveguide shows that, although the potential of the dielectric surface becomes negative with respect to the metal walls, both profiles almost coincide, i.e., in the 2D case, the dielectric surface, as before, has a positive charge and the structure of the electron sheath near the dielectric remains practically unchanged.
Figure 13. Distributions of model particles in the simulation region for $\varepsilon_{osc} = 128$ eV at two times (LiF surface bounded by copper walls, $h = 1.5$ cm).

Figure 14. Dynamics of electron bunches in the two-surface multipactor in the time intervals $\omega t/2\pi = (a) 20-24$ and (b) $100-104$ (LiF surface bounded by copper walls, $h = 1.5$ cm, $\varepsilon_{osc} = 128$ eV).

Thus, analysis of the distribution of the electron density in the simulation region shows that the drop in the electric potential in the interwall space is caused by the development of a two-surface multipactor, the negative space charge of which also leads to the drop in the electric potential on the dielectric surface. Figure 13 shows the distributions of model particles in the simulation region for $\varepsilon_{osc} = 128$ eV at two times. At $\omega t/2\pi = 20$, one can see two electron bunches, the densities of which decrease with distance from the dielectric surface, which means that the two-surface multipactor begins to develop near the dielectric surface and then gradually expands along the $z$ axis. At $\omega t/2\pi = 100$, the bunches have already become almost uniform along the $z$ axis. Here, the single-surface multipactor is seen as a very narrow red strip adjacent to the left boundary.

The dynamics of electron bunches in the two-surface multipactor is illustrated in Figure 14, which shows the relief of the function $S_1(y, t) = \int_{z_1}^{z_{\text{max}}} \left( n_e(y, z, t) / z_e n_{e0} \right) dz$ in the $(y, t)$ plane in the time intervals $\omega t/2\pi = (a) 20-24$ and (b) $100-104$. Here, $z_{\text{max}} = 3$ cm and integration over $z$ is performed from $z_1 = 0.2$ cm in order to exclude the contribution from the electron sheath of the single-surface multipactor on the dielectric. It is seen that, in the early stage of the two-surface multipactor (Figure 14a), the bunches are periodically injected from both walls and then propagate into the interwall space, gradually spreading due to the finite electron temperature and electrostatic repulsion between the bunch electrons. In Figure 14b, the two-surface multipactor has already reached saturation and transformed into two “perpendicular” single-surface multipactors (see Section 3) operating almost independently on each waveguide wall.

It should be noted that, although the saturated multipactors on the dielectric and metal surfaces operate almost independently of one another, the single-surface multipactor on the dielectric accelerates the development of the two-surface multipactor, serving as a source of seed electrons for the latter, i.e., in this case, we can speak of a combined single + double-surface multipactor. For the
same number of initial electrons in the interwall space, but without a dielectric, the two-surface multipactor develops at an appreciably slower rate.

5. Influence of electron reflections on multipactor

The analytical expression for the SEY proposed by Vaughan [28] refers to the so-called “true” secondary electrons, which are emitted from a narrow surface layer of a dielectric or metal under bombardment by primary electrons. The energy spectrum of true secondaries is usually close to Maxwellian with a temperature of several electronvolts [27]. The total energy spectrum of secondary electrons can also be significantly contributed by elastically reflected (scattered) and inelastically reflected (rediffused) primaries [27, 38]. A typical energy spectrum of secondary electrons for \( \varepsilon_{\text{inc}} = 180 \text{ eV} \) is schematically shown in Figure 15 [27].

The significant effect of electron reflections on the single-surface multipactor is illustrated in Figure 16, which presents results of 1D3V simulations of a “parallel” \( (\alpha = 0) \) multipactor on a SiO\(_2\) surface [22, 23]. Upon elastic reflection, the energy of the reflected electron \( \varepsilon_{\text{ref}} \) was set equal to the energy of the incident electron \( \varepsilon_{\text{inc}} \), whereas upon inelastic scattering, it could take any value between zero and \( \varepsilon_{\text{inc}} \) with equal probabilities. In both cases, the reflected electrons were distributed over velocity directions according to the law \( dN/d\Omega \sim \cos\theta \) [27, 35].

The heavy line in Figure 16 shows analytical dependence (12), the dotted line show the result of simulations by the simplified Vaughan’s formula with \( \varepsilon_0 = 0 \), and curve 1 shows the result obtained using Vaughan’s formula with a finite cutoff energy (\( \varepsilon_0 = 30 \text{ eV} \)) without allowance for electron reflections. It is seen that the absorption coefficient somewhat decreases when the finite cutoff energy is taken into account.

Curve 2 in Figure 16 shows the absorption coefficient calculated under the assumption that the coefficient of elastic electron reflections at \( \varepsilon < \varepsilon_0 \) is \( R = 1 \) (the unit step in \( \delta \) in Figure 2), as was proposed by Vicente et al. [34] in order to describe the experimentally observed high reflection coefficient of primary electrons at very low energies (see also [39]). According to experimental data [35], the coefficient of electron reflection from various dielectrics (including alkali halide crystals, such as NaCl or LiF) increases substantially at energies lower than the energy corresponding to the long-wavelength edge of crystal fundamental absorption. Moreover, this edge itself was in [40] associated with the boundary energy above which true SEE takes place (i.e., in fact, with \( \varepsilon_0 \)). It is seen that reflections of low-energy electrons substantially increase the absorption coefficient. It is seen that reflections of low-energy electrons substantially increase the absorption coefficient.

Finally, curve 3 in Figure 16 shows the absorption coefficient calculated with additionally introduced small elastic and inelastic electron reflections at \( \varepsilon > \varepsilon_0 \) with the coefficients \( R = 0.1 \) and \( \eta = 0.1 \), respectively. It is seen that even small reflections in this energy range additionally increase the absorption coefficient, especially at lower oscillation energies.
We also considered the influence of electron reflections from the dielectric and metal surfaces on the electron energy distribution function (EEDF) in the combined single + double-surface multipactor in a parallel-plate waveguide. The coefficients of elastic and inelastic reflections from the dielectric were assumed to be $R = \eta = 0.05$, and those on the metal walls, $R = \eta = 0.1$. Figure 17 shows the distribution functions of electrons incident on the dielectric surface and bounding metal walls for two values of $\varepsilon_{osc}$ without and with allowance for electron reflections. It is seen that the introduction of even a small electron reflection (which insignificantly affects the general properties of the multipactor) results in the appearance of high-energy tails in the EEDF, which extend up to $\varepsilon \sim 3\varepsilon_{osc}$. The effective temperature of the tails is about $3\varepsilon_{osc}$. It is also seen that the development of the two-surface multipactor at $\varepsilon_{osc} = 128 \text{ eV}$ results in a relative increase in the number of electrons incident onto the metal walls in comparison with those incident onto the dielectric.

![Figure 17](image)

**Figure 17.** Distribution functions of electrons incident on the dielectric (LiF) surface (curves $l'$, $l$) and bounding metal (Cu) walls ($h = 1.5 \text{ cm}$) (curves $2'$, $2$), calculated without and with allowance for electron reflections (dashed and solid curves, respectively) for $\varepsilon_{osc} = (a) 24.5$ and (b) $128 \text{ eV}$.

### 6. Measurements of the microwave power absorbed by a multipactor

We performed experimental measurements of microwave power absorption by a multipactor discharge excited on a dielectric surface [23]. The experiments were carried out at the BRUS device [7–9, 12, 21]. The scheme of the experiment is shown in Figure 18. The dielectric target (LiF single crystal, NaCl single crystal, or SiO$_2$ plate) was placed in the antinode of the H$_{10}$ standing mode of a $6 \times 12\text{-cm}$
evacuated ($p \sim 10^{-6}$ Torr) rectangular waveguide. The input microwave power $P_{\text{inp}}$ was from several tens of kilowatts to 2 MW, the duration of the microwave pulse being of up to 25 $\mu$s.

At high microwave powers ($P_{\text{inp}} \geq 1$ MW), three stages of the discharge on the dielectric target were observed: (i) multipactor discharge (which lasted for several microseconds), (ii) filamentary microwave breakdown (in which up to 70% of the microwave power was absorbed), and (iii) plasma-flare microwave discharge (in which the absorption coefficient dropped to 20–30%) [12]. At moderate microwave powers ($P_{\text{inp}} \leq 100$ kW), only the first stage was observed. The onset of a multipactor discharge was detected by the appearance of a feeble glow on the dielectric surface and the current onto the electron collector installed under the dielectric plate (Figure 19) [21–23].

Figure 18. Arrangement of the experiment: (1) evacuated waveguide, (2) microwave discharge, (3) 24-mm-diameter below-cutoff circular waveguide, (4) dielectric plate, (5) diagnostic window, (6) photo camera, and (7) 10-mm-diameter below-cutoff circular waveguide. The free-space wavelength is $\lambda_0 = 15.4$ cm and the waveguide wavelength is $\lambda_g = 20$ cm.

Figure 19. Current density measured by the electron collector installed under the LiF plate as a function of the input microwave power $P_{\text{inp}}$. The vertical dashed line shows the threshold power $P_{\text{thr}} = 32$ kW ($\varepsilon_{\text{osc}} = 20$ eV) obtained from 1D3V PIC simulations for a LiF crystal ($\varepsilon_{\text{in}} = 1000$ eV, $\delta_{\text{in}} = 7.5$, $\varepsilon_{\text{i}} = 14.5$ eV) by using Vaughan’s formula with $\varepsilon_0 = 0$.

Taking into account the geometry of the waveguide and dielectric plate, the ratio $\Delta P/P_{\text{inp}}$ can be recalculated into the absorption coefficient $\kappa$ defined by expression $\kappa = \langle W_{\text{abs}} \rangle \left( c E_0^2 / 8\pi \right)^{-1}$. On one hand, the input microwave power $P_{\text{inp}}$ is expressed through the amplitude of the input wave $E_{\text{inp}}$ as $P_{\text{inp}} = 0.5c(E_{\text{inp}}^2 / 8\pi)ab\cos\chi$, where $a$ and $b$ are the width and height of the waveguide, $\cos\chi = (1 - (\lambda_0 / 2a)^2)^{1/2}$, and $\lambda_0$ is the microwave wavelength in free space. On the other hand, according to definition (11), the absorbed power is equal to $P_{\text{abs}} = \kappa c(E_0^2 / 8\pi)S_{\text{tot}}$, where $S_{\text{tot}}$ is the total area of the dielectric plate (the multipactor discharge develops on both sides of the plate). Then,
taking into account that, in the antinode of the standing mode, \( E_0 = 2E_{\text{inp}} \), we find that, for the parameters of our experiment, \( \kappa \approx 0.58 \Delta P/P_{\text{inp}} \).

![Figure 20. Typical signals of the (a) reflected microwave power and (b) current measured by the electron collector installed under the SiO\(_2\) plate for the input microwave power \( P_{\text{inp}} = 85\) kW, slightly exceeding the threshold power \( P_{\text{thr}} = 65\) kW for the onset of a multipactor discharge.](image)

Unfortunately, we could measure the ratio \( \Delta P/P_{\text{inp}} \) only for input microwave powers of \( P_{\text{inp}} \leq 85\) kW, slightly exceeding the threshold power for the onset of a multipactor discharge on a SiO\(_2\) plate (\( P_{\text{thr}} = 65\) kW), because, at higher powers, the multipactor discharge developed already at the front of the microwave pulse and no step was observed at the top of the reflected microwave signal. The range of \( \kappa \) values obtained from the experimental measurements of \( \Delta P/P_{\text{inp}} \) at \( P_{\text{inp}} = 85 \) kW is shown in Figure 16 by the vertical bar. It is seen that agreement between the theoretical results and the experimental data is achieved only if electron reflections from the dielectric surface are taken into account.

7. Conclusions

The results of these studies can be summarized as follows.

(i) An analytical expression for the coefficient of microwave power absorption by a single-sided multipactor discharge on a dielectric surface has been derived under the assumption that the SEY is described by the simplified Vaughan's formula with a zero cutoff energy, widely used in theoretical and numerical studies of multipactor discharges. The analytical expression satisfactorily agrees with the simulation results obtained under the same assumptions on the shape of the SEY. Although the obtained expression is of limited applicability, it can be used as a reference formula when analyzing the influence of various factors (such as electron reflections and the shape of the energy dependence \( \delta(\varepsilon_{\text{inc}}) \)) on the coefficient of microwave power absorption by a multipactor discharge on a dielectric.

(ii) Multipactors developing on a dielectric surface inclined with respect to the microwave electric field have been studied analytically and numerically. It is shown that the structure of the “parallel” multipactor differs fundamentally from that of the “perpendicular” multipactor. The parallel multipactor is concentrated in a narrow layer adjacent to the dielectric, the layer thickness being about \( v_T/\omega \approx 30 \) \( \mu \text{m} \) for \( f = 2\) GHz, whereas the perpendicular multipactor extends from the solid surface to a distance of about several electron oscillation amplitudes in the microwave field (\( \alpha_0 = v_{\text{osc}}/\omega \geq 1\) mm for \( f = 2\) GHz and \( \varepsilon_{\text{osc}} \geq 100\) eV). It is found that the relation between the average energy of electrons bombarding the dielectric surface and their average flux density in the perpendicular multipactor differs radically from that in the parallel multipactor, which should be taken into account when choosing modes for multipactor processing of crystal dielectrics.

(iii) It is shown that the dependence of the absorption coefficient on the inclination angle \( \alpha \) of the dielectric surface relative to the microwave electric field has a minimum at \( \alpha \approx 20^\circ - 30^\circ \). This effect
can be used to minimize the microwave power released in multipactor discharges on the surfaces of input/output microwave windows, as well as to choose optimal regimes of processing of crystal dielectrics by electron fluxes generated in such discharges.

(iv) 2D3V PIC simulations of a multipactor discharge on a dielectric placed in a parallel-plate waveguide with allowance for SEE from the dielectric surface and waveguide walls have shown that the threshold for the development of a single-surface multipactor on a dielectric in a low-profile waveguide with absorbing walls increases compared to that on an unbounded dielectric due to escape of secondary electrons onto the bounding walls. It is found that, depending on the amplitude of the microwave field, the discharge can operate in two modes: (a) a single-surface multipactor on the dielectric, in the course of which the dielectric acquires a positive potential with respect to the waveguide walls, and (b) a combined (single + double-surface) multipactor, in which the dielectric and the interwall space acquire a negative potential.

(v) The results of PIC simulations have shown that the coefficient of microwave power absorption increases substantially when electron reflections from the dielectric surface are taken into consideration. Taking into account electron reflections also leads to the appearance of high-energy tails in the energy distribution function of electrons bombarding the surface.

(vi) The experiments on a multipactor discharge on a dielectric in a rectangular waveguide have shown that the coefficient of microwave power absorption by such a discharge can reach several percent, which should lead to various destructive processes on the dielectric and distortion of signals in microwave communication systems. Comparison of experimental data with results of theoretical studies and numerical simulations show that agreement between the calculated and measured values of the absorption coefficient is achieved only if the theoretical model takes into account the entire complex of SEE phenomena, such as emission of true secondaries and elastic and inelastic reflections of incident electrons in a wide energy range.

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References
[1] Puech J, Anderson D, Buyanova M, Dorozhkina D, Jordan U, Lapierre L, Lisak M, Semenov V, Sombrin J, Udiljak R 2005 Proc. MULCOPIM-2005 (Noordwijk) 235
[2] Piro F, Brand Y 2014 Proc. 8th Europ. Conf. on Antennas and Propagation (Hague) 1643
[3] Goede A P H, Bongers W A, Elzendoorn B S Q, Graswinckel M F, de Baa M R 2010 Fusion Eng. Des. 85 1117
[4] G. Giruzzi et al 2011 Nucl. Fusion 51 063033
[5] Neuber A, Dickens J, Hemmert D, Krompholz H, Hatfield L L, Kristiansen M 1998 IEEE Trans. Plasma Sci. 26 296
[6] Ivanov V A, Konyzhev M E 2002 Proc. XX Int. Symp. on Discharges and Electrical Insulation in Vacuum (Tours) 499
[7] Ivanov V A, Konyzhev M E 2003 Proc. V Int. Workshop Microwave Discharges: Fundamentals and Applications (Greifswald) 247
[8] Ivanov V A, Konyzhev M E, Gavrilenko V P, Oks E 2005 Proc. MULCOPIM-2005 (Noordwijk) 169
[9] Ivanov V A, Sakharov A S, Konyzhev M E 2009 Proc. VII Int. Workshop Microwave Discharges: Fundamentals and Applications (Hamamatsu) 34
[10] Hoff B W, Mardahl P J, Gilgenbach R M, Haworth M D, French D M, Lau Y Y, Franzi M 2009 Rev. Sci. Instrum. 80 094702
[11] Strong Microwaves: Sources and Applications 2009 ed A G Litvak (Nizhny Novgorod: Inst. of Applied Physics, Russ. Acad. Sci.)
[12] Ivanov V A, Sakharov A S, Konyzhev M E 2015 IEEE Trans. Plasma Sci. 43 1871
[13] Singh S, Gupta D, Jain V, Sharma A K 2015 Mater. Manuf. Process. 30 1
[14] Sun J, Wang W, Yue Q 2016 Materials 9 231
[15] Vaughan J R M 1988 IEEE Trans. Electron Dev. 35 1172
[16] Kishek R A, Lau Y Y 1988 Phys. Rev. Lett. 80 193
[17] Kishek R A, Lau Y Y, Ang L K, Valfells A, Gilgenbach R M 1998 Phys. Plasmas 5 2120
[18] Semenov V E, Rakova E I, Anderson D, Lisak M, Puch J 2007 Phys. Plasmas 14 033501
[19] Kossyi I A, Lukyanchikov G S, Semenov V E, Rakova E I, Anderson D, Lisak M, Puch J 2008 J. Phys. D 41 065203
[20] Kim H C, Verboncoeur J P 2006 Phys. Plasmas 13 123506
[21] Batanov G M, Ivanov V A, Konyzhev M E, Letunov A A 1997 JETP Lett. 66 170
[22] Ivanov V A, Sakharov A S, Konyzhev M E 2012 Proc. Proc. VIII Int. Workshop Microwave Discharges: Fundamentals and Applications (Zvenigorod) 69
[23] Sakharov A S, Ivanov V A, Tarbeeva Yu A, Konyzhev M E 2012 Plasma Phys. Rep. 38 1090
[24] Sakharov A S, Ivanov V A, Konyzhev M E 2013 Plasma Phys. Rep. 39 1122
[25] Sakharov A S, Ivanov V A, Konyzhev M E 2016 Plasma Phys. Rep. 42 610
[26] Ivanov V A, Sakharov A S, Konyzhev M E 2012 Proc. Proc. VIII Int. Workshop Microwave Discharges: Fundamentals and Applications (Zvenigorod) 75
[27] Hachenberg O, Brauer W 1959 Adv. Electron. El. Phys. 11 413
[28] Vaughan J R M 1989 IEEE Trans. Electron Dev. 36 1963
[29] Kishek R A, Lau Y Y 1995 Phys. Rev. Lett. 75 1218
[30] Ang L K, Lau Y Y, Kishek R A, Gilgenbach R M 1998 IEEE Trans. Plasma Sci. 26 290
[31] Valfells A, Verboncoeur J P, Lau Y Y 2000 IEEE Trans. Plasma Sci. 28 529
[32] Kim H C, Verboncoeur J P 2005 Phys. Plasmas 12 123504
[33] Sazonov A, Semenov V, Buyanova M, Vdovicheva N, Anderson D, Lisak M, Puch J, Lapierre L 2005 Phys. Plasmas 12 093501
[34] Vicente C, Mattes M, Wolk D, Hartnagel H L, Mosig J R, Raboso D 2005 Proc. MULCOPIM-2005 (Noordwijk) 11
[35] Bronshtein I M, Fraiman B S 1969 Secondary Electron Emission (Moscow: Nauka) [in Russian]
[36] Grishin L V, Dorofeyuk A A, Kossyi I A, Luk’yanchikov G. S, Savchenko M M 1977 Tr. FIAN 92 82
[37] Valfells A, Ang L K, Lau Y Y, Gilgenbach R M 2000 Phys. Plasmas 7 750
[38] Dekker A J 1958 Solid State Physics (London: Macmillan) Chap 17
[39] Cimino R 2006 Nucl. Instrum. Meth. A 561 272
[40] Fridrikhov S A 1960 Sov. Phys. Solid State 2 157