Hadronic transition $\chi_{c1} \to \eta_c \pi \pi$ at the Beijing Spectrometer BES and the Cornell CLEO-c

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Hadronic transitions of the $\chi_{cJ}$ states have not been studied yet. We calculate the rate of the hadronic transition $\chi_{c1} \to \eta_c \pi \pi$ in the framework of QCD multipole expansion. We show that this process can be studied experimentally at the upgraded Beijing Spectrometer BES III and the Cornell CLEO-c.

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It has been shown that the theory of hadronic transitions based on QCD multipole expansion \cite{1, 2, 3, 4} can make quite successful predictions for many hadronic transition rates in the $c\bar{c}$ and $b\bar{b}$ systems \cite{5, 6}. So far, hadronic transitions of the $\chi_{cJ}$ states have not been studied yet. The observed $\chi_{cJ}$ decays are mainly hadronic decays (decaying into light hadrons), and the hadronic widths of the three $\chi_{cJ}$ states are rather different. From the point of view of perturbative QCD, the hadronic decay rates of $\chi_{c0}$ and $\chi_{c2}$ are proportional to $\alpha_s^2$, while that of $\chi_{c1}$ is proportional to $\alpha_s^3$. Hence $\chi_{c1}$ has the smallest hadronic decay rate, and is thus the most interesting one among the three $\chi_{cJ}$ states for studying hadronic transitions. The main hadronic transition process of $\chi_{c1}$ is $\chi_{c1} \to \eta_c \pi \pi$. In the framework of QCD multipole expansion, this is dominated by the E1-M1 transition, and the rate is significantly smaller than those of E1-E1 transitions such as $\psi' \to J/\psi \pi \pi$. In addition, $\chi_{c1}$ cannot be directly produced in $e^+e^-$ collision. It can only be produced via the radiative transition $\psi' \to \gamma \chi_{c1}$. This is why $\chi_{c1} \to \eta_c \pi \pi$ is not easy to detect at the original Beijing Spectrometer BES II. The designed luminosity of the upgraded Beijing Electron-Positron Collider BEPC II is about 1–2 orders of magnitude higher than that of the original BEPC I, and the ability of the upgraded Beijing Spectrometer BES III will be significantly improved compared with the original BES II, especially its photon detector. So that BES III will be able to detect processes like $\chi_{c1} \to \eta_c \pi \pi$ which BES II can hardly do. In this paper, we calculate the transition rate of $\chi_{c1} \to \eta_c \pi \pi$ in the framework of QCD multipole expansion, and our result shows that both the upgraded BES III and the Cornell CLEO-c have a good chance to study $\chi_{c1} \to \eta_c \pi \pi$.

The hadronization factor $\langle \pi \pi | E_i^a B_j^a | 0 \rangle$ is a second rank pseudo tensor at the light hadron scale. In the rest frame of $\chi_{c1}$, it is a function of the two pion momenta $q_1$ and $q_2$. Since the mass difference $M_{\chi_{c1}} - M_{\eta_c} = 530.3$ MeV is small, the pions are soft. Taking the PCAC and soft pion approach, the hadronization factor is of the form

$$\langle \pi \pi | E_i^a B_j^a | 0 \rangle = C_{ij} (q_1 \cdot q_2 + q_2 \cdot q_1),$$

where $C$ is approximately a constant characterizing the size of the matrix element. Unfortunately, there is no experimental result serving as the input data to determine $C$ at present. Therefore we have to take an alternative approach to make predictions. Now we take the two gluon approximation used in Refs. \cite{2, 3}, which is
shown to be reasonable so far as only the transition rates are concerned [2, 3]. In this approach, we approximately express \( \langle \pi \pi | E_i^a B_j^a | 0 \rangle \) as

\[
\langle \pi \pi | E_i^a B_j^a | 0 \rangle \approx \langle \pi \pi | gg \rangle \langle gg | E_i^a B_j^a | 0 \rangle.
\]

The matrix element \( \langle \pi \pi | gg \rangle \) is at the scale of \( M_{\chi_1} - M_{\eta_c} \approx 530 \) MeV, and is independent of the multipole gluon emission. If we take the same approximation to the process \( \psi' \rightarrow J/\psi \pi \pi \), i.e., expressing the corresponding hadronization factor as \( \langle \pi \pi | gg \rangle \langle gg | E_i^a B_j^a | 0 \rangle \), the matrix element \( \langle \pi \pi | gg \rangle \) in this process is at the scale of \( M_{\psi'} - M_{J/\psi} \approx 590 \) MeV which is not much different from 530 MeV in the case of \( \chi_{c1} \rightarrow \eta_c \pi \pi \). Therefore we can neglect the running of \( \langle \pi \pi | gg \rangle \) from 590 MeV to 530 MeV, and regard it as the same matrix element in both \( \psi' \rightarrow J/\psi \pi \pi \) and \( \chi_{c1} \rightarrow \eta_c \pi \pi \). Then \( \langle \pi \pi | gg \rangle \) can be absorbed into the definition of the phenomenological coupling constant \( g_E \), and we can determine \( g_E \) by taking the experimental datum of the transition rate \( \Gamma(\psi' \rightarrow J/\psi \pi \pi) \) as input. The remaining factor \( \langle gg | E_i^a B_j^a | 0 \rangle \) in (3) is then easy to evaluate. It has been shown in Refs. [2, 3] that such an approach gives predictions for the transition rates quite close to those in the soft pion approach [9]. So we take this approximation and obtain the following transition rate [2]

\[
\Gamma(\chi_{c1} \rightarrow \eta_c \pi \pi) = \frac{4\alpha_E \alpha_M}{8505 \pi m_c^2} |f_{1110}^{10}|^2 |f_{1110}^{101}|^2 \times (M_{\chi_{c1}} - M_{\eta_c})^7,
\]

where \( \alpha_E = \frac{g_E^2}{4\pi}, \alpha_M = \frac{g_M^2}{4\pi} \), and

\[
\begin{align*}
f_{1110}^{10} & = \eta_c \pi \pi, \\
f_{1110}^{101} & = \chi_{c1} \pi \pi,
\end{align*}
\]

in which \( \chi_{c1} = \eta_c \pi \pi \). The potential is

\[
V(r) = kr - \frac{16\pi}{25} \frac{1}{rf(r)} \left[ 1 + \frac{2\gamma_E + \frac{53}{75}}{f(r)} - \frac{462}{625} \ln f(r) \right],
\]

where \( k = 0.1491 \) GeV\(^2\) is the string tension related to the Regge slope [11], \( \gamma_E \) is the Euler constant, and \( f(r) \) is

\[
f(r) = \ln \left[ \frac{1}{\Lambda_{MS}^4} + 4.62 - \left( 1 - \frac{1}{4 \Lambda_{MS}^4} \right) \right] + \exp \left\{ - \left[ 15 \left( \frac{3\Lambda_{MS}^4}{\Lambda_{MS}^4} - \frac{1}{15} \lambda_{MS}^4 \right) \right] r^2 \right\}.
\]

in which \( \lambda_{MS}^4 = 180 \) MeV. In this model, the c quark mass is \( m_c = 1.478 \) GeV, and we take \( \Lambda_{MS} = 200 \) MeV in the following calculation.

With this potential model, we can calculate the transition amplitudes \( f_{n1l1n2l2}^{LP1P_F} (K) \) defined in Eq. (9). To see the convergence of the summation \( \sum_K f_{n1l1n2l2}^{LP1P_F} (K) \) in (9), we list the values of the first eight amplitudes in TABLE I. We see that taking the first eight terms in the summation is enough, so we obtain

\[
f_{1110}^{10} + f_{1110}^{101} = -7.21402 \text{ GeV}^2.
\]
Next we consider the determination of $\alpha_E$ and $\alpha_M$. Taking the same approximation to $\psi' \rightarrow J/\psi \pi\pi$, we can obtain $\Gamma(\psi' \rightarrow J/\psi \pi\pi)$ which is \[ \alpha_E \] 

\[ \text{(9)} \]

The updated Particle Data Group (PDG) best fit experimental values \[ \Gamma(\psi') = 337 \pm 13 \text{ keV}, \]

\[ B(\psi' \rightarrow J/\psi \pi\pi) = (48.26 \pm 0.95)\%, \] 

leads to \[ \Gamma(\psi' \rightarrow J/\psi \pi\pi) = 162.6 \pm 9.4 \text{ keV}. \] 

\[ \text{(11)} \]

In an earlier paper \[ \text{[4]} \], we calculated the related branching ratios by using QCD multipole expansion and the Gross-Treiman-Wilczek formula \[ \text{[14]} \] for obtaining the transition rate $\Gamma(\psi' \rightarrow h_c\pi^0)$, and by using perturbative QCD for obtaining the hadronic decay width $\Gamma(h_c \rightarrow \text{hadrons})$. Our obtained result is \[ B(\psi' \rightarrow h_c\pi^0) \times B(h_c \rightarrow \eta_c\gamma) = (3.5 \pm 1.0 \pm 0.7) \times 10^{-4}. \] 

\[ \text{(13)} \]

Comparing these two results, we obtain \[ \frac{\alpha_M}{\alpha_E} = 1.8 \pm 0.9. \] 

\[ \text{(15)} \]

Note that there is a 50% error in \[ \text{[13]} \] coming from the errors in \[ \text{[13]} \].
present calculation.

With all these, we obtain the transition rate

$$\Gamma(\chi_{c1} \to \eta_c \pi \pi) = 11.0 \left(\frac{\alpha_M}{\alpha_E}\right) \text{keV} = 19.8 \pm 9.9 \text{keV}. \quad (16)$$

In (16), the 50% uncertainty comes from the error in (15). For checking the model dependence of this prediction, we also calculated the transition rate using the well-known Cornell model with Coulomb plus linear potential [17]. and the result is $$\Gamma(\chi_{c1} \to \eta_c \pi \pi) = 9.4 \left(\frac{\alpha_M}{\alpha_E}\right) \text{keV} = 17.0 \pm 8.5 \text{keV}$$. So that the model dependence is about 14% which is not significant considering the large uncertainty in (16). The total width of $$\chi_{c1}$$ is $$\Gamma(\chi_{c1}) = 0.89 \pm 0.05 \text{MeV}$$ [12]. So that the branching ratio is

$$B(\chi_{c1} \to \eta_c \pi \pi) = (2.22 \pm 1.24)\% \quad (17)$$

At $$e^+e^-$$ colliders, the state $$\chi_{c1}$$ can be produced via $$\psi' \to \gamma \chi_{c1}$$ at the $$\psi'$$ peak. In the rest frame of $$\psi'$$, the momentum of $$\chi_{c1}$$ is 171 MeV which is only 5% of its mass $$M_{\chi_{c1}} = 3510.66 \text{MeV}$$. Therefore we can neglect the motion of $$\chi_{c1}$$, and simply take the branching ratio (17) to estimate the event numbers in the experiments.

The detection of the process

$$\psi' \to \gamma \chi_{c1} \to \gamma \eta_c \pi \pi \quad (18)$$

can be performed in two ways, namely the inclusive and the exclusive detections [18]. In the inclusive detection, only the photon and the two pions are detected, while $$\eta_c$$ is regarded as a missing energy. In the exclusive detection, all the photon, the two pions, and the decay products of $$\eta_c$$ are detected. Reconstruction of $$\eta_c$$ and $$\chi_{c1}$$ from the measured final state tagging particles can suppress the backgrounds.

The inclusive detection requires measuring the momenta of the photon and the pions to certain precision. It is difficult to do this kind of analysis with the BES II data because the BES II photon detector is not efficient enough. At BES III and CLEO-c, this kind of detection is possible.

For BES III, it is not difficult to accumulate $$10^8$$ of $$\psi'$$ events. The branching ratio of $$\psi' \to \gamma \chi_{c1}$$ is $$B(\psi' \to \gamma \chi_{c1}) = (8.7 \pm 0.4)\%$$ [12]. Taking account of a 15% detection efficiency, we obtain the number of events of (19) at BES III

$$N_{incl}(\psi' \to \gamma \chi_{c1} \to \gamma \eta_c \pi \pi) = (2.90 \pm 1.74) \times 10^4 \quad (19)$$

This is quite a large number.

CLEO-c is now running at the $$\psi'$$ peak again, and will soon accumulate $$3 \times 10^7$$ $$\psi'$$ events. The CLEO-c detection efficiency is around 15% [13]. For such a sample, the corresponding number of events of (18) is

$$N_{incl}(\psi' \to \gamma \chi_{c1} \to \gamma \eta_c \pi \pi) = (8.7 \pm 5.2) \times 10^5 \quad (20)$$

So the transition (18) can also be clearly studied at CLEO-c.

For the exclusive detection, suitable decay modes of $$\eta_c$$ should be taken for identifying the $$\eta_c$$ in (18). Some feasible decay modes with reasonable branching ratios are [12, 13]

$$\eta_c \to \rho \rho : \quad B(\eta_c \to \rho \rho) = (2.0 \pm 0.7)\% \quad (21)$$
$$\eta_c \to K^+K^- : \quad B(\eta_c \to K^+K^-) = (1.03 \pm 0.26)\% \quad (22)$$
$$\eta_c \to \phi \phi : \quad B(\eta_c \to \phi \phi) = (0.27 \pm 0.09)\% \quad (23)$$
$$\eta_c \to K^+(892)^0K^-\pi^+ : \quad B(\eta_c \to K^+(892)^0K^-\pi^+) = (2.0 \pm 0.7)\% \quad (24)$$
$$\eta_c \to K^+K^-\pi^+\pi^- : \quad B(\eta_c \to K^+K^-\pi^+\pi^-) = (1.5 \pm 0.6)\% \quad (25)$$
$$\eta_c \to 2(\pi^+\pi^-) : \quad B(\eta_c \to 2(\pi^+\pi^-)) = (1.20 \pm 0.30)\% \quad (26)$$
$$\eta_c \to \eta\pi\pi \to \gamma\gamma\pi\pi : \quad B(\eta_c \to \eta\pi\pi \to \gamma\gamma\pi\pi) = (1.9 \pm 0.7)\% \quad (27)$$
$$\eta_c \to K_SK^+\pi^- : \quad B(\eta_c \to K_SK^+\pi^-) = (1.9 \pm 0.5)\% \quad (28)$$
$$\eta_c \to K_LK^+\pi^- : \quad B(\eta_c \to K_LK^+\pi^-) = (1.9 \pm 0.5)\% \quad (29)$$

The modes (28) and (29) have been used by CLEO-c in the search for the $$h_c$$ state [13].

In the exclusive detection, the requirement of the precision of the photon momentum measurement in (18) is not so strict. So it is possible to do this kind of analysis with the BES II data except for those modes with photons in $$\eta_c$$ decays. Therefore we also calculate the event numbers at BES II. The BES II data contains $$1.4 \times 10^7$$ $$\psi'$$ events. Considering the ability of the BES II detector, we take a detection efficiency of 10% for BES II. The Predicted event numbers for BES II, BES III, and CLEO-c are listed in TABLE II. We see that the exclusive detection of the process (18) can be well studied at BES III and CLEO-c. Considering the theoretical uncertainty, the events at BES II are marginal.
TABLE II: Predictions for the exclusive detection event numbers for the process (18) in the CK potential model using the $\eta_c$ decay modes shown in $^{21-29}$ at BES II, BES III, and CLEO-c. The accumulated numbers of the $\psi'$ events are taken to be $1.4 \times 10^7$ for BES II, $10^6$ for BES III, and $3 \times 10^7$ for CLEO-c. The detection efficiency is taken to be 10% for BES II, and 15% for BES III and CLEO-c.

| modes          | BES II    | BES III   | CLEO-c    |
|----------------|-----------|-----------|-----------|
| $\rho\rho$     | $54 \pm 51$ | $579 \pm 552$ | $174 \pm 165$ |
| $K^+K^*$       | $28 \pm 24$ | $298 \pm 255$ | $90 \pm 76$ |
| $\phi\phi$     | $7 \pm 7$  | $78 \pm 73$  | $23 \pm 22$ |
| $K^+(892)0^0K^-\pi^+$ | $54 \pm 51$ | $579 \pm 552$ | $174 \pm 165$ |
| $K^+K^-\pi^+\pi^-$ | $41 \pm 41$ | $435 \pm 435$ | $130 \pm 130$ |
| $2(\pi^+\pi^-)$ | $32 \pm 28$ | $348 \pm 296$ | $104 \pm 89$ |
| $\eta\pi\pi \rightarrow \gamma\gamma\pi\pi$ | $51 \pm 50$ | $550 \pm 534$ | $165 \pm 160$ |
| $K_SK^+\pi^+\pi^+$ | $51 \pm 44$ | $550 \pm 476$ | $165 \pm 143$ |
| $K_LK^+\pi^+\pi^+$ | $51 \pm 44$ | $550 \pm 476$ | $165 \pm 143$ |

We have seen that the theoretical uncertainty mainly comes from the experimental errors in $^{13}$. If the product of branching ratios $B(\psi' \rightarrow h_c\pi^0) \times B(h_c \rightarrow \eta_c\gamma)$ can be measured to higher precision at CLEO-c or BES III, the predictions for the event numbers of the process $^{18}$ will be more definite.

Finally, we would like to mention that the numbers listed in TABLE II are obtained by taking the PDG best fit experimental value $\Gamma(\psi') = 337 \pm 13$ keV in Eq. $^{10}$ as input. Actually, the Particle Data Book also provides a world averaged value $\Gamma(\psi') = 277 \pm 22$ keV in Meson Particle Listing $^{12}$. If we take this world averaged value as input instead of the PDG best fit value in $^{10}$, all the numbers in TABLE II will be smaller by a factor of 0.82. This is another uncertainty caused by the input datum of $\Gamma(\psi')$. We expect further improved measurement of $\Gamma(\psi')$ to reduce this kind of uncertainty.

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[1] T.M. Yan, Phys. Rev. D 22, 1652 (1980).
[2] Y.P. Kuang and T.M. Yan, Phys. Rev. D 24, 2874 (1981).
[3] Y.P. Kuang, S.F. Tuan and T.M. Yan, Phys. Rev. D 37, 1210 (1988).
[4] Y.-P. Kuang, Phys. Rev. D 65, 094024 (2002).
[5] Y.-P. Kuang, Front. Phys. China, 1, 19 (2006) [hep-ph/0601044].
[6] N. Brambilla et al., CERN Yellow Report, CERN-2005-005 [hep-ph/0412158].
[7] See for example T. Appelquist, R.M. Barnett, and K. Lane, Ann. Rev. Nucl. Part. Sci. 28, 387 (1978); T. Appelquist and H.D. Politzer, Phys. Rev. D 12, 1404 (1975); R. Barbieri, R. Gatto, and R. Kögerler, Phys. Lett. B 60, 183 (1976); R. Barbieri, R. Gatto, and E. Remiddi, Phys. Lett. B 61, 465 (1976).
[8] S.-H. H. Tye, Phys. Rev. D 13, 3416 (1976); R.C. Giles and S.-H. H. Tye, Phys. Rev. Lett. 37, 1175 (1976); Phys. Rev. D 16, 1079 (1977); W. Buchmüller and S.-H. H. Tye, Phys. Rev. Lett. 44, 850 (1980).
[9] Although there has been a comment on this approximation $^{10}$, it has been discussed in Ref. $^2$ why this approximation can lead to reasonable order of magnitude estimates of the transition rates (except the case of very small phase space), and the predicted transition rates in this approximation are shown to be close to those obtained in the soft pion approach $^2$. Thus the two gluon approximation can be taken as a possible approximation for making order of magnitude estimate of transition rates when there is no reliable way of calculating the highly nonperturbative hadronization factor from QCD, and no suitable experimental input to determine the unknown coefficient in the soft pion formula $^2$. Considering the large uncertainty in the input datum (cf. Eq. $^{15}$ below), taking this approximation is reasonable in the present calculation.
[10] M.B. Voloshin, Sov. J. Nucl. Phys. 43, 1011 (1986); Yad.
Fiz. 43, 1571 (1986).

[11] Y.-Q. Chen and Y.-P. Kuang, Phys. Rev. D 46, 1165 (1992); 47, 350 (1993)(E).

[12] W.-M. Yao et al. (Particle Data Group), J. Phys. G 33, 1 (2006).

[13] P. Robin et al. (CLEO Collaboration), Phys. Rev. D 72, 092004 (2005); J.L. Rosner et al. (CLEO Collaboration), Phys. Rev. Lett. 95, 102003 (2005).

[14] D.J. Gross, S.B. Treiman, and F. Wilzcek, Phys. Rev. D 19, 2188 (1979).

[15] V.A. Novikov and M.A. Shifman, Z. Phys. C 8, 43 (1981).

[16] M.B. Voloshin, Mod. Phys. Lett. A 17, 1533 (2002).

[17] E. Eichten et al., Phys. Rev. D 21, 203 (1980).

[18] See for example, J. Green et al., Phys. Rev. Lett. 49, 617 (1982); T. Bowcock et al., ibid. 58, 307 (1987).