Non-relativistic QCD (NRQCD) provides a rigorous framework to consistently separate high and low energy effects in quarkonium production and decay \[1\]. The factorization of the different energy scales in the effective non-relativistic theory is achieved through an expansion in \(v\), the relative velocity of the heavy quarks in the quarkonium state. If \(v\) is a sufficiently small parameter, the non-perturbative phenomena can be encoded into a few process-independent long-distance matrix elements (LDMEs) which can be extracted from experimental data.

Whether such an expansion is well-behaved for charmonium, and in particular for the \(J/\psi\), is still subject to debate. So far no complete experimental evidence for the universality of the LDMEs has been reached. In particular, the role of the color-octet transitions is not well assessed. Although these contributions seem to be required to account for the observed \(P_T\) spectrum in hadro-production \[2, 4\], they fail to explain the polarization measurement at large transverse momentum \[4\] and they do not seem to be needed to describe fixed-target experiments \[5\] and photo-production \[6, 7\] data.

Estimating the impact of the transitions at work for \(J/\psi\) production relies on the accuracy with which the corresponding short-distance coefficients can be computed. Cross sections at leading order in \(\alpha_S\) are normally affected by very large uncertainties and cannot give a reliable estimate of the yield. In these cases, gathering information on the underlying production mechanism(s) from data can be problematic. The recent computation of next-to-leading order (NLO) corrections in \(J/\psi\) electro-production \[6, 8, 10, 11\] has reduced the previous discrepancies between the NRQCD predictions and the data. In hadro-production, higher-order \(\alpha_S\) corrections to the color-singlet production have been shown to significantly increase the yield at large \(P_T\) \[12, 13, 14\], giving a better description of both the shape and the normalization of the \(P_T\) spectrum. Remarkably, the color-singlet prediction for the polarization as a function of the \(P_T\) also dramatically changes once corrections in \(\alpha_S\) are taken into account, leading to a better agreement with the data.

In view of the recent theoretical progress related to \(J/\psi\) hadro-production, it is worth reconsidering the photo-production data obtained at HERA. Differential cross sections and polarization observables for \(J/\psi\) photo-production have been measured both by the ZEUS and H1 collaborations. Previous comparisons of these measurements with the NRQCD predictions \[6, 7\] suggest that the color-singlet yield alone reproduces the experimental differential cross sections. However, theoretical uncertainties are too large to draw any definite conclusion.

For the purpose of identifying the transitions that dominate \(J/\psi\) production, it is often useful to analyze the polarization. Contrary to the differential cross sections, one can expect the polarization predictions to be less affected by the uncertainties associated with the theoretical inputs. It has been argued \[15\] that the \(J/\psi\) polarization could be a good discriminator between color-singlet and color-octet transitions in \(\gamma p\) collisions. However, only leading order predictions have been considered so far due to the lack of NLO calculations for the polarization for either the singlet \[16\] or the octets \[17\]. Since QCD corrections can be very large in some regions of phase space, especially for the color-singlet at large transverse momentum, comparisons with leading order predictions are of limited interest.

As new measurements of the \(J/\psi\) polarization at HERA are still being performed by the ZEUS and
H1 collaborations, we limit ourselves to a comparison with the preliminary data \cite{13}. For the color-singlet transition, polarization observables in photo-production are given here at NLO accuracy in $\alpha_S$ for the first time.

The relevant kinematic variables for $J/\psi$ photo-production are the energy of the photon-proton system ($W$) and the fraction of the photon energy carried by the $J/\psi$ in the proton rest frame ($z$),

$$W = \sqrt{(p_\gamma + p_p)^2}, \quad z = \frac{p_{\psi}p_p}{p_\gamma p_p},$$

as well as the transverse momentum of the $J/\psi$ ($p_T$). Near the end-point region $z \approx 1$, $p_T \approx 0$ GeV, $J/\psi$ production is enhanced by diffractive contributions characterized by the exchange of colorless states. These contributions cannot be correctly accounted for by the NRQCD factorization approach, and hence they must be excluded from experimental measurements in order to make a meaningful comparison. As diffractive production leads to a steeper slope for the differential cross sections into photo-production cross sections by dividing by the photon flux integrated in this range.

In our photo-production analysis, we fix $\langle O_{J/\psi}[^3S_1^1] \rangle = 1.16$ GeV$^3$ and we use the CTEQ6M pdf set \cite{22}. Given the small range in $p_T$ available and for the sake of simplicity we prefer to use fixed scales. As the central value we choose $\mu_0 = 4m_c$, i.e., where the sensitivity of the cross section $\sigma(p_T > 1 \text{ GeV})$ to scale variations is minimal (see Table \ref{table:1}). Since a large variation is observed when we vary the scales in opposite directions, we impose the additional condition $0.5 < \frac{\mu}{\mu_T} < 2$ in our prediction for the differential cross sections. We also vary the charm quark mass in the range $1.4 - 1.6$ GeV. Scale and mass uncertainties are combined in quadrature. The resulting distributions are displayed in Fig. \ref{fig:1}.

\begin{table}[h]
\centering
\begin{tabular}{ccc}
\hline
$\mu$ & $\mu_T$ & 0.5 $\mu_0$ & $\mu_0$ & 2 $\mu_0$
\hline
2 $\mu_0$ & 10.6 & 8.28 & 5.96 & \\
$\mu_0$ & 10.6 & 9.45 & 10.2 & 10.3 \\
0.5 $\mu_0$ & 10.6 & 8.28 & 5.96 & \\
\hline
\end{tabular}
\caption{Scale dependence of the total cross section \(\sigma(p_T > 1 \text{ GeV})\) (expressed in nb). $W$ is set to 100 GeV. Other input parameters are explained in the text.}
\end{table}

\section{Results}

The cross section $\sigma(\gamma p \rightarrow J/\psi + X)$ in the color-singlet model \cite{20} has been computed in Ref. \cite{16} at NLO accuracy in $\alpha_S$. We reproduced this computation by using the method and the results presented in Ref. \cite{12} for the hadro-production case, which also allows tracking of the $J/\psi$ polarization through the angular correlations of the $J/\psi$ decay products, $J/\psi \rightarrow \ell^+\ell^-$. We have compared the results for several inclusive observables, finding always very good agreement. As a further check and application of our results we also calculated the NLO corrections to the decay width of a $[^3S_1^1]$ into $\gamma +$ hadrons. Our result is in excellent agreement with that of Ref. \cite{21}.

In our photo-production analysis, we fix $\langle O_{J/\psi}[^3S_1^1] \rangle = 1.16$ GeV$^3$ and we use the CTEQ6L1 pdf set \cite{22}. Given the small range in $p_T$ available and for the sake of simplicity we prefer to use fixed scales. As the central value we choose $\mu_0 = 4m_c$, i.e., where the sensitivity of the cross section $\sigma(p_T > 1 \text{ GeV})$ to scale variations is minimal (see Table \ref{table:1}). Since a large variation is observed when we vary the scales in opposite directions, we impose the additional condition $0.5 < \frac{\mu}{\mu_T} < 2$ in our prediction for the differential cross sections. We also vary the charm quark mass in the range $1.4 - 1.6$ GeV. Scale and mass uncertainties are combined in quadrature. The resulting distributions are displayed in Fig. \ref{fig:1}.

For comparison, we also plot the color-singlet prediction at leading order in $\alpha_S$, for which we use the CTEQ6L1 pdf set. As can be seen from Fig. \ref{fig:1}, the $\alpha_S$ corrections increase the differential cross section in the high $p_T$ region, where the yield is dominated by the new channels that open up at order $\alpha_S^3$. Nevertheless, the color-singlet yield at NLO clearly undershoots the ZEUS data. The plots in Fig. \ref{fig:1} differ from the comparison presented in Ref. \cite{1}, where rather extreme values for the renormalization scale.
were used that have the effect of artificially increasing the normalization. We do not display here the comparison with the $P_T^2$ and $z$ distributions measured by the H1 collaboration [6] since it shares the same features.

We now turn to the polarization. Experimentally, the polarization of the $J/\psi$’s can be traced back by analyzing the angular distribution of the leptons originating from the decay of the $J/\psi$. It is convenient to decompose this angular distribution in terms of the polar and azimuthal angles $\theta$ and $\phi$ in the $J/\psi$ rest frame:

$$
\frac{d\sigma}{d\Omega dy} \propto 1 + \lambda(y) \cos^2 \theta + \mu(y) \sin 2\theta \cos \phi + \frac{\nu(y)}{2} \sin^2 \theta \cos 2\phi
$$

(2)

where $y$ stands for a certain (set of) variable(s) (either $P_T$ or $z$ in the following). If the polar axis coincides with the spin quantization axis, the parameters $\lambda$, $\mu$, $\nu$ can be related to the spin density matrix elements:

$$
\lambda = \frac{\rho_{1,1} - \rho_{0,0}}{\rho_{1,1} + \rho_{0,0}}, \quad \mu = \frac{\sqrt{2} Re(\rho_{1,0})}{\rho_{1,1} + \rho_{0,0}}, \quad \nu = \frac{2 \rho_{1,-1}}{\rho_{1,1} + \rho_{0,0}}.
$$

(3)

The spin information that we extract in this way depends on the choice of the quantization axis. Here, we decide to work in the target frame ($\hat{z} = \frac{P_T}{|P_T|}$) as in the recent analysis performed by the ZEUS collaboration.

As mentioned above, in our NLO computation the $J/\psi$ decays into leptons. In order to obtain $\lambda$ (resp. $\nu$) we have integrated the cross sections over $\phi$ (resp. $\theta$) and extracted the polarization parameters by fitting the resulting distributions. In so doing, we have used the same kinematic conditions as those considered by the ZEUS collaboration: $E_p = 920$ GeV, $E_e = 27.5$ GeV, $P_T > 1$ GeV, $z > 0.4$ and $50 \text{ GeV} < W < 180$ GeV. Notice that the cut $z < 0.9$ previously considered for the differential cross sections has been relaxed here. The measurement of the polarization versus $P_T$ is therefore subject to a larger contamination from diffractive contributions.

The NLO predictions of the polarization parameters associated with color-singlet production are displayed in Fig. 2, together with the LO predictions and the preliminary ZEUS measurements. The band comes from the uncertainties associated with the choice of the scales – varied in the range defined by $0.5 \mu_f < \mu_f, \mu_r < 2 \mu_0$ and $0.5 < \frac{\mu_f}{\mu_f} < 2$ – which is much larger than the mass uncertainty. For some specific values of the scales (namely $\mu_r = 0.5$), the $\lambda$ and $\nu$ parameters appear to be unphysical in some bins. This is due to the fact that in these cases our calculation leads to a negative value for the diagonal components of the spin density matrix at $P_T \lesssim 1$ GeV, and hence cannot be trusted in this region. Therefore, in the error bands in Fig. 2 we have disregarded scale choices leading to unphysical predictions.

QCD corrections to the color-singlet production have a strong impact on the polarization prediction. The most spectacular effect comes from the behavior of the $\lambda$ parameter at large transverse momentum, for which the prediction is rather stable under variation of the scales. At leading order in $\alpha_S$, the color-singlet transition gives a transverse $J/\psi$ at large $P_T$. Once QCD corrections are included, the $\lambda$ parameter decreases rapidly and has a large negative value above $P_T = 4 - 5$ GeV. This situation is similar to that in hadro-production where the $J/\psi$ produced via a color-singlet transition is longitudinal at large transverse momentum $[13, 14]$. Such a correction for the $\lambda$ parameter at moderate and high $P_T$ as well as the decrease at $z \approx 0.8$, is not supported by the preliminary data from the ZEUS collaboration and therefore suggests the presence of other mechanisms for $J/\psi$ production. In the low $z$ region, the scale uncertainty is too large to draw any conclusion. QCD corrections to color-singlet production

FIG. 1: Differential cross sections in $P_T^2$ and $z$, compared with the ZEUS data [7].
FIG. 2: Polarization parameters for color-singlet production at leading order and next-to-leading order in $\alpha_S$, compared with the ZEUS preliminary measurement.

also affect the value of the $\nu$ parameter, which is closer to the experimental data in comparison with the prediction at leading order.

In this letter, we have studied $J/\psi$ photoproduction via a color-singlet transition at HERA and presented for the first time a comparison with the predicted polarization at NLO accuracy. Taking into account the $\alpha_S$ corrections, color-singlet production alone does not describe all features of the data collected at HERA. With a natural choice for the renormalization scale, the predicted rate is smaller than data, even though the shapes of the differential distributions are well described. Moreover, the preliminary measurement of the $J/\psi$ polarization by the ZEUS collaboration as a function of the $P_T$ shows a very different trend with respect to the theoretical predictions.

New channels appearing at NNLO in $\alpha_S$ (including fragmentation processes) may increase the differential cross section, as has been pointed out in the hadroproduction case \cite{14}. Although the kinematics differ in photoproduction, such contributions could give an enhancement at large transverse momentum ($P_T > 5$ GeV). However, no reliable estimate of NNLO contributions in the $P_T$ region accessible at HERA is presently available.

The current discrepancies could possibly be solved by invoking color-octet transitions, i.e. contributions from the intermediate states $1S_0^{[8]}$ and $3P_J^{[8]}$. Unfortunately, any phenomenological analysis of the impact of these contributions on differential cross sections and polarization observables is limited by the omission of higher-order corrections that are currently unknown. A complete $\alpha_S^3$ computation, particularly for the prediction of the polarization of the $J/\psi$ produced via a $P-$wave color-octet state, would be welcome in order to shed further light on the mechanisms at work in photo-production.

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