Neutral kaon mixing from new physics: matrix elements in $N_f = 2 + 1$ QCD

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We present results for matrix elements of $\Delta S = 2$ four-fermion operators arising generically in models of new physics. These are needed to constrain such models using the measured values of $\varepsilon_K$ and $\Delta M_K$. We use lattice QCD with $2 + 1$ flavors of improved staggered fermions on lattices generated by the MILC collaboration. We extrapolate to the continuum from three lattice spacings ranging down to $a \approx 0.045$ fm. Total errors are $\sim 5 - 6\%$, arising primarily from our use of one-loop matching between lattice and continuum operators. For two of the matrix elements, our results disagree significantly from those obtained using different fermion discretizations.

Processes that are highly suppressed in the standard model (SM) provide a window into beyond-the-standard model (BSM) physics complementary to that from direct searches. Indeed, in a given model, such processes can set lower limits on the scale of BSM physics which are beyond the reach of present accelerators (see, e.g., Refs. [1–3]). Here we focus on kaon mixing, which provides among the most powerful constraints. For both Refs. [1–3], processes that are highly suppressed in the standard model (BSM) provide a window into beyond-the-standard model (SM) physics complementary to that from direct searches.

We consider the BSM matrix elements $\langle K_0|Q_i(\mu)|K_0\rangle$ for $Q_i$ a four-fermion operator of the type $\bar{\psi} \gamma_{\mu \nu} \psi Q_i$, where $\mu$ and $\nu$ are color indices, $\gamma_{\mu \nu} = [\gamma_\mu, \gamma_\nu]/2$, and repeated indices are summed. The local $\Delta S = 2$ operator $Q_1$, is in the chiral basis of Ref. [4], $Q_1$ is the “left-left” operator arising in the SM contribution to $\varepsilon_K$, while $Q_{2-5}$ are the BSM operators. In previous lattice calculations, a different basis (the “SUSY basis”) for the operators has been used. We prefer the chiral basis as it has been used to calculate the two-loop anomalous dimensions which we use to run the results between different renormalization scales [4]. Results can be easily converted to the SUSY basis at the end—see below.

In our lattice set-up (described below) we obtain directly ratios of matrix elements. Specifically, we calculate the following $B$-parameters for the BSM operators:

$$B_i(\mu) = \frac{\langle K_0|Q_i(\mu)|K_0\rangle}{N_i(\bar{K}_0|\gamma_5\bar{d}(\mu)|0)(0|\bar{s}\gamma_5d(\mu)|K_0\rangle}, \quad (N_2, N_3, N_4, N_5) = (5/3, 4, -2, 4/3).$$

The operators in both numerator and denominator are renormalized in the MS scheme using naive dimensional regularization for $\gamma_5$ and other conventions described in Ref. [4]. The matrix elements of $Q_i$ can be obtained from the $B$-parameters since the denominators can be expressed in terms of known quantities ($M_K, f_K, m_s, m_d$). We also calculate $B_K$.

There have been two previous calculations of the BSM matrix elements using dynamical quarks, one using domain-wall fermions [3] and the other twisted-mass

1 Our normalization differs from Ref. [4], but this has no impact on the associated $B$-parameters.
2 In other calculations of the BSM matrix elements [3,4], different ratios have been used so as to avoid the need to use quark masses. However, quark masses are now quite well determined [4], with the $\sim 6\%$ error in $(m_s + m_d)^2$ being comparable to those we obtain for the $B_i$. 

$\varepsilon_K$.
fermions \(^3\). The former uses the physical complement of \(2+1\) light sea quarks, but has results only at a single lattice spacing. The latter uses 2 dynamical flavors, with the strange quark being quenched. Our calculation uses \(2+1\) flavors of dynamical staggered quarks, with multiple lattice spacings, quark masses and volumes. Their main disadvantage is that each flavor comes in 4 copies (“tastes”), with the associated \(SU(4)\) symmetry broken at non-zero lattice spacing \(a\). Removing the extra tastes requires rooting the fermion determinant, but we assume that the artifacts this introduces vanish in the continuum limit \(a\to 0\).

We use ensembles generated with the improved “asqtad” staggered action by the MILC collaboration \(^2\). Those used here are listed in Table I with \(m_f\) the average up/down sea-quark mass and \(m_s\) the strange sea-quark mass (which lies close to the physical value on all ensembles). For valence quarks, we use HYP-smeared staggered fermions \(^1\). These are known to substantially reduce both artifacts due to taste symmetry breaking and perturbative corrections to matching factors \(^3\).

Our mixed action set-up is identical to that we used previously to calculate \(B_K\), and the detailed lattice methodology is also very similar \(^1\). On each lattice, we place two wall sources separated by a fixed interval \(\Delta t\). These create kaons having taste \(\xi_5\) and zero spatial momenta. Lattice versions of the BSM operators are placed between the two wall sources, as are the pseudoscalar operators needed for the denominators of the \(B_i\) [see Eq. (7)]. We choose \(\Delta t\) such that the contamination from excited states and from kaons “propagating around the world” can be ignored. We use the same values as in our \(B_K\) calculation, the justification for which has been discussed in Ref. \(^1\). Multiple measurements with random time translations are carried out on each lattice (see Table I). After averaging over configurations, we form the ratios needed for the \(B_i\). The overlap of the wall sources with the kaon states cancels in these ratios, as well as some of the statistical error. Away from the sources, these ratios should be independent of \(t\). We find this to be the case within errors, and we fit them to a constant over a central “plateau” region.

Our lattice operators are matched to those in the continuum scheme of Ref. \(^4\) using 1-loop, mean-field improved perturbation theory. Previous one-loop matching calculations matched the lattice operators to those in a different continuum scheme \(^5\), but we have now determined the matching between the two continuum schemes \(^6\). At each lattice spacing, we match to the continuum scheme at scale \(\mu = 1/a\). The one-loop corrections are typically 10-20\%, with the largest being 30\%. This is in line with the expectation that the coefficient of \(\alpha\) should be of \(O(1)\) or smaller.

We use a partially quenched set-up with ten different valence quark masses: \(am_{x,y} = am_s \times (n/10)\) and \(n = 1,2, \ldots, 10\). Here \(x\) and \(y\) refer to valence \(d\) and \(s\) quarks, respectively. Our lightest valence pions have \(M_{\pi^\pm} \approx 200\) MeV. For our valence kaons, we use the lightest 4 values of \(am_K\) and the heaviest 3 of \(am_s\). These combinations satisfy \(m_x \ll m_y \sim m_{\text{phys}}\), so that our result lie in the regime in which heavy-kaon \(SU(2)\) chiral perturbation theory (ChPT) is applicable \(^8\).

For our chiral extrapolations, we use \(B_K\) (whose behavior is well understood from prior work) as well as the four “golden” combinations

\[
G_{23} = \frac{B_2}{B_3}, \quad G_{45} = \frac{B_4}{B_5}, \quad G_{24} = B_2B_4, \quad G_{21} = \frac{B_2}{B_K}.
\]

These have no chiral logarithms at next-to-leading order (NLO) \(^9\), and thus have simpler chiral extrapolations than the \(B_i\) and reduced sensitivity to taste-breaking lattice artifacts. At the end, we invert these relations to determine the \(B_i\). We have checked that extrapolating the \(B_i\) directly leads to compatible results \(^1\).

We extrapolate to physical quark masses and \(a = 0\) in three steps. The first two are done on each ensemble separately: we extrapolate \(m_x\) to \(m_{\text{phys}}\) (“X-fit”) and then \(m_y\) to \(m_{\text{phys}}\) (“Y-fit”). For \(B_K\) the fitting is as described in Refs. \(^1,13\). For the \(G_i\), we fit to

\[
c_1 + c_2 X + c_3 X^2 + c_4 X^2 \ln^2 X + c_5 X^2 \ln X + c_6 X^3, \tag{10}
\]

TABLE I. MILC ensembles used in this work, with “ens” the number of gauge configurations and “meas” the number of measurements per configuration. ID identifies the ensemble, with F=final, S=superfine and U=ultrafine.

| \(a (\text{fm})\) | \(am_f/am_s\) | size | \(\text{ens} \times \text{meas}\) | ID |
|-----------------|-----------------|------|-----------------|----|
| 0.09 | 0.0062/0.031 | 28 \(^3\times 96\) | 995 \(\times 9\) | F1 |
| 0.09 | 0.0093/0.031 | 28 \(^3\times 96\) | 949 \(\times 9\) | F2 |
| 0.09 | 0.0031/0.031 | 40 \(^3\times 96\) | 959 \(\times 9\) | F3 |
| 0.09 | 0.0124/0.031 | 28 \(^3\times 96\) | 1995 \(\times 9\) | F4 |
| 0.09 | 0.00465/0.031 | 32 \(^3\times 96\) | 651 \(\times 9\) | F5 |
| 0.06 | 0.0036/0.018 | 48 \(^3\times 144\) | 749 \(\times 9\) | S1 |
| 0.06 | 0.0072/0.018 | 48 \(^3\times 144\) | 593 \(\times 9\) | S2 |
| 0.06 | 0.0025/0.018 | 56 \(^3\times 144\) | 799 \(\times 9\) | S3 |
| 0.06 | 0.0054/0.018 | 48 \(^3\times 144\) | 582 \(\times 9\) | S4 |
| 0.045 | 0.0028/0.014 | 64 \(^3\times 192\) | 747 \(\times 1\) | U1 |

\(^3\) There have also been earlier quenched calculations, which established the basic methodology \(^8\).

\(^4\) Staggered fermions also preserve part of the continuum chiral symmetry, although this is less important for the BSM matrix elements than for \(B_K\), as the former are not constrained by chiral symmetry to vanish in the \(SU(3)\) chiral limit.
where $X \equiv X_P/\Lambda^2$, with $X_P = M_{xx}^2$ and $\Lambda = 1$ GeV. This incorporates the absence of the NLO logarithm. The NNLO chiral logarithms are not known, so we use the generic form of such terms. We also include a single analytic NNLO term, which is required for good fits. Our X-fits include the full correlation matrix, and constrain the coefficients $c_{3-6}$ with Bayesian priors: $c_i = 0 \pm 1$. Examples of X-fits for $G_{23}$ are shown in Fig. 1. We see that the chiral extrapolations are short and the dependence mild. This holds also for the other $G_i$. Systematic errors are estimated by doubling the widths of the Bayesian priors, and also by comparing with fits using the eigenmode shift method [22]. These two estimates are then combined in quadrature.

The $m_y$ dependence is close to linear, so we use a linear extrapolation for our central value and a quadratic fit to estimate a systematic error. After these Y-fits, we evolve results from all lattices to a common renormalization scale, either $\mu = 2$ or 3 GeV, using the two-loop anomalous dimensions given in Ref. 4. It turns out that the standard form of solution is singular, but this can be removed by the analytic continuation method proposed in Ref. 23. Details will be given in Ref. 17.

In our final extrapolation we simultaneously extrapolate sea-quark masses to their physical values and $a \to 0$. As for $B_K$, we use only the three finest lattice spacings. For our central values we fit to

$$ f_1 = d_1 + d_2 (a\Lambda_Q)^2 + d_3 L_P/\Lambda^2 + d_4 S_P/\Lambda^2, \quad (11) $$

with $L_P$ ($S_P$) the squared masses of the taste $\xi_5$ pseudoscalars composed of light (strange) sea quarks. We set $\Lambda_Q = 0.3$ GeV and expect $d_2 \sim O(1)$. In fact, for the $G_i$ we find $|d_2| \sim 2 - 7$, indicating enhanced discretization errors. We also find that $d_3 \neq d_4$, indicating substantial SU(3) breaking, although both coefficients are of the expected size $|d_{3,4}| \ll 1$. Examples of fits are shown in Figs. 2 and 3, for $B_K$ and $G_{23}$ respectively. These fits have $\chi^2/\text{dof} = 1.9$ and 2.7 respectively. Those for the other $G_i$ are slightly better, with $\chi^2/\text{dof} = 1.6 - 1.7$.

We also consider more elaborate fits, using

$$ f_2 = f_1 + d_5 (a\Lambda_Q)^2 \alpha_s + d_6 \alpha_s^2 + d_7 (a\Lambda_Q)^4, \quad (12) $$

where $\alpha_s$ is the strong coupling constant. Figs. 2 and 3 show the fits with $d_2$, $d_3$, and $d_4$. Examples of fits are shown in Figs. 2 and 3, for $B_K$ and $G_{23}$ respectively. These fits have $\chi^2/\text{dof} = 1.9$ and 2.7 respectively. Those for the other $G_i$ are slightly better, with $\chi^2/\text{dof} = 1.6 - 1.7$.

TABLE II. Final results for the $B_i$ and $G_i$ at two renormalization scales. Errors are respectively statistical and systematic.

|     | $\mu = 2$ GeV | $\mu = 3$ GeV |
|-----|---------------|---------------|
| $B_K$ | 0.537 (7)(24) | 0.519 (7)(23) |
| $B_2$ | 0.620 (4)(31) | 0.549 (3)(28) |
| $B_3$ | 0.433 (3)(19) | 0.390 (2)(17) |
| $B_4$ | 1.081 (6)(48) | 1.033 (6)(46) |
| $B_5$ | 0.853 (6)(49) | 0.855 (6)(43) |
| $G_{23}$ | 1.430 (3)(63) | 1.405 (2)(62) |
| $G_{45}$ | 1.267 (2)(72) | 1.209 (1)(60) |
| $G_{24}$ | 0.670 (4)(30) | 0.567 (4)(27) |
| $G_{21}$ | 1.155 (12)(66) | 1.058 (11)(62) |
with $\alpha_s = \alpha_s(N_S, 1/a)$. This form includes all terms expected at NLO in staggered ChPT [24] plus one NNNLO term. We impose Bayesian constraints, $d_{2-7} = 0 \pm 2$, and find improved fits with $\chi^2/\text{dof} = 0.8 - 1.5$. We take the difference between $f_2$ and $f_1$ fits as the systematic error in the chiral-continuum extrapolation.

We present our final results and error budgets in Tables II and III respectively. Statistical errors are estimated by bootstrap and are at the percent-level or smaller. Of the errors not discussed above, the dominant one comes from our use of one-loop matching. We estimate this as $\delta B/B = \alpha_s^2$, with $\alpha_s$ evaluated on our finest lattice [12]. Support for this estimate comes from a recent comparison of perturbative and non-perturbative renormalization using staggered bilinears [24]. Since this error is also accounted for by the $d_8$ term when fitting to Eq. (12), to avoid double-counting we take the largest of "$f_2 - f_1$ error" and the "$\alpha^2$ error" for our combined chiral-continuum-matching error. Finite volume errors are estimated by comparing fits using ChPT with and without finite-volume corrections. Since the chiral-logarithms in all BSM $B$-parameters have the same relative magnitude as that in $B_K$, we take this estimate from our earlier work on $B_K$ [11]. Specifically, the largest error estimate is from the F1 ensemble (see Table I of Ref. [24]).

We close by comparing our results to those of Ref. [8]. As noted above, the results for $B_K$ agree. For the BSM $B$-parameters, Ref. [8] finds, at $\mu = 3$ GeV, $B_i^{\text{SUSY}} = 0.43(5), 0.75(9), 0.69(7), 0.47(6)$. Only $B_3$ differs between SUSY and chiral bases, with $B_i^{\text{SUSY}} = (5B_i^{\text{chiral}} - 3B_i^{\text{chiral}})/2$. Our results convert to $B_i^{\text{SUSY}} = 2(3)$ GeV = 0.79(3). Using this result and Table II we find that $B_2$ and $B_3$ are consistent with Ref. [8] (the former only at $2\sigma$), while $B_4$ and $B_5$ differ significantly (by 4$s$ and $5\sigma$, respectively). Our $B_4$ and $B_5$ are larger than those of Ref. [8] by 50% and 80%, respectively.

Given this difference, we have cross-checked the components of our calculation in several ways e.g. comparing our RG running matrices with those in Refs. [1-5]. Clearly, further investigation is needed to resolve the disagreement with Refs. [3, 4]. One possibility that we are investigating is that the true truncation errors in perturbative matching are larger than our estimate, which can be checked by renormalizing our operators non-perturbatively. Another useful test would be for the other calculations to calculate directly the golden combinations, so as to pinpoint the source of the disagreement.

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| TABLE III. Error budgets (in percent) for the $B_i$(2 GeV). |
|-----------------------------|
| source of error | $B_1$ | $B_2$ | $B_3$ | $B_4$ | $B_5$ |
| statistics | 1.37 | 0.64 | 0.63 | 0.60 | 0.66 |
| matching | 4.40 | 4.95 | 4.40 | 4.40 | 5.69 |
| cont-extrap. | 0.10 | 0.10 | 0.12 | 0.12 | 0.12 |
| X-fit (F1) | 0.62 | 0.12 | 0.19 | 0.22 | 0.16 |
| Y-fit (F1) | 0.50 | 0.50 | 0.50 | 0.50 | 0.50 |
| $r_1 = 0.3117(22)$ fm | 0.34 | 0.18 | 0.17 | 0.05 | 0.02 |
| $f_S = 132$ vs. 124 MeV (F1) | 0.46 | 0.46 | 0.46 | 0.46 | 0.46 |

5 We use this work rather than Ref. [8], since the latter quenches the strange quark. This choice is not quantitatively important, however, since the results from these two works are consistent.

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