The effect of snow covers properties and air temperature on the ice freezing thickness near Yakutsk

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Abstract. This study is devoted to the possible Yakutsk Lena–river automobile and railway bridge construction and estimation of icing and ice blockage formation possibility variation. So, according to developed algorithm and calculating scheme the calculations of ice freezing thickness variations for observation site of Yakutsk Lena-river hydrological station were performed on basis of meteorological data on air temperature and snow cover thickness for winter periods of 1980/81-2018/19. The comparison of calculated and available in open access observation data on ice freezing thickness for these winter periods was also conducted and the coefficient of correlation of equal 0.7-0.8 for them was stated. The calculating scheme for ice freezing is constructed on basis of two-layer media heat conductivity problem (snow cover, ice) with phase transition on the boundary of ice and unfrozen water. Heat balance equation includes phase transition energy on the interface and outflow of heat through ice and snow cover into the atmosphere. The heat flux is calculated on basis of Fourier law as a product of heat conductivity and temperature gradient. It is supposed, that temperature changes in each media linearly.

1. Introduction

The work was devoted to the possible Yakutsk Lena–river automobile and railway bridge construction and estimation of icing and ice blockage formation possibility variation. Ice blockage happens on the rivers because of their latitudinal extent, nonsimultaneous debacle and filling of the river’s cross-section with ice. Determining factor of ice blockage formation is the ice thickness on the watercourse in the end of winter season. This ice thickness can be determined by the thermal regime of winter season and peculiarities of snow cover accumulation regime. Ice blockage events are typical for the rivers where the debacle happens as destruction of relatively firm ice cover by the stream force and where it takes place when the flood starts from the upper part of the basin. This is in particular possible for such large rivers, flowing from south to north, as Lena, Yenisei, Irtysh, Pechora, and North Dvina. In our case calculating scheme for ice thickness was constructed on basis of two-layer media heat conductivity problem (snow cover, ice) with phase transition on the boundary of ice and unfrozen water. Heat balance equation included phase transition energy, inflow of heat from unfrozen water and outflow through the ice and snow cover to the atmosphere. The heat flux was calculated on basis of Fourier law as a product of heat conductivity and temperature gradient. It was supposed, that temperature changes in each media linearly. For snow cover and ice the formula of heat conductivity of two-layer media was used. It was supposed, that temperature varies in each media linearly.
2. Materials and methods

So in this work according to the data on the snow cover thickness and air temperature at a weather station in Yakutsk (http://meteo.ru/) and described by J. Stefan [1, 2] equation for ice cover thickness dynamics the calculations of ice thickness for the river Lena near Yakutsk for the winter seasons 1980/81-2018/19 were conducted. Also, the calculated maximum values of the ice thickness at this observation point for these winter seasons are compared with the actual data available on the AIS GWMO site. The calculation scheme used for the dynamics of ice thickness formation was built on the basis of the heat balance equation, taking into account the phase transition at the interface between water and ice and the outflow of heat from the boundary of the phase transition through ice and snow cover into the atmosphere. This outflow of heat to the atmosphere was calculated according to the Fourier law taking into account the heat conduction formula for a combination of two media: snow cover and ice. It was assumed that the temperature in each of the media varied linearly. The obtained ordinary differential equation was approximated and solved by the explicit numerical Euler method.

Figure 1. Linear temperature distribution in the media, consisting of snow thickness (1), layer of ice (2) and unfrozen water (3).

The calculations of ice freezing for covered with snow cover ice surface in winter period on basis of daily data on air temperature and snow thickness allows the estimation of the movement rate of ice freezing interface during the winter period. The rate of movement of ice freezing interface under assumption of zero heat flow into unfrozen water can be expressed with the formulas:

The equation of heat balance can be written as:

\[ F_1 = \rho V = \frac{L \rho dh_{ice}}{d\tau} \text{or } dh_{ice} = \frac{V}{\rho L} \]

- \( F_1 \) is heat outflow through snow cover and ice from ice freezing interface through ice and snow cover into atmosphere (W/m²);
- \( \rho \) - density of ice (916 kg/m³);
- \( L \) - energy of H₂O phase transition (335 kJ/kg);
- \( V = \frac{dh_{ice}}{d\tau} \) - rate of movement of ice freezing interface (cm/s);

Heat flux is expressed according to Fourier law by means of temperature gradient and heat conductivity as \( F = \lambda (\text{grad } T) \).

Heat conductivity and heat flux through combination of two media (snow and ice) according to [3] can be expressed as:

\[ F_1 = \lambda \frac{\Delta T}{\Delta x} = \frac{-\Delta T}{\left(\frac{\lambda_s}{\lambda_s} + \frac{\lambda_{ice}}{\lambda_{ice}}\right)} = \frac{-T_{air}}{\left(\frac{h_s}{\lambda_s} + \frac{h_{ice}}{\lambda_{ice}}\right)} \]

Here \( T_{air} \) – air temperature, \( h_s \) and \( h_{ice} \) – snow cover and ice thickness, and \( \lambda_s \) and \( \lambda_{ice} \) – heat conductivity of snow and ice. Averaged heat conductivity of snow and ice \( \lambda_s \) and \( \lambda_{ice} \) are assumed to be equal 0.3 and 2.3 W/m °C correspondingly according to [4].

The differential scheme \( h_{ice}(t_{n+1}) = h_{ice}(t_n) + \Delta t \ V(t_n) \) was constructed by Euler approximation for the equation for the rate of ice freezing.

By elaborated differential scheme the calculations of ice freezing thickness were carried out for the winter seasons of 1980/81-2018/19. The calculations were performed with the time step of one day. For initial conditions, it was supposed that ice thickness \( h_{ice} \) was equal 0.5 cm. For each time step (each day) the rate of movement of freezing interface \( V \) and the value ice thickness \( h_{ice} \) for the next day (time-step) were calculated.
3. Results and discussion
Calculations of the ice freezing thickness dynamic are done according to the constructed calculating scheme on basis of heat conductivity Stephan problem with multilayer heat conductivity. For this reason on basis of knowledge about winter period temperature and snow cover thickness are used.

Calculations scheme was constructed by means of approximation of derived ordinary differential equation for ice thickness variation with numerical scheme. On basis of obtained numerical scheme the calculations were performed and the comparison of obtained results with observed values were done.

The results of ice freezing thickness calculations according to the constructed calculating scheme on basis of heat conductivity Stephan problem with multilayer heat conductivity for Yakutsk Lena-river hydrological station for winter seasons 1980/81-2018/19 is shown on Fig. 2 (maximal thickness).

![Figure 2. Variations of calculated maximal ice thickness for hydrological station Yakutsk Lena-river for winter periods 1980/81-2018/19.](image)

At the same time, the correlation between observed and calculated maximal ice freezing thickness value for hydrological station Yakutsk Lena-river is shown on Fig. 3. The correlation coefficient of the calculated and actual data for the maximum ice thickness for these winter seasons had the value of 0.7.

![Figure 3. Correlation between observed and calculated maximal ice freezing thickness value for hydrological station Yakutsk Lena-river.](image)
4. Conclusion
The applied method is well physically consistent. The obtained by method solution indicates good correspondence with the process of ice freezing variation during the winter period. The correlation coefficient of the calculated and actual data for the maximum ice thickness had the value of 0.7. The important point for successful work of method is good setting up of initial conditions. Solving of this first order ODE equation is simple and could be done within Excel program.

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