Modelling and Application of Output Current of the Solar Array for Satellite in Sun Synchronous Orbit

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Abstract. The output current of the solar array for satellite in sun synchronous orbit is a critical parameter for performance in-orbit analyzing. In order to analysis the rules of the solar array output current accurately, some influential factors, such as the incidence angle of the sun, the temperature of the solar array, the earth-sun distance factor and the fixed offset angle to the sun of solar wing, are comprehensively researched. Then a model of output current is established. Results in-orbit show that the model has high accuracy, which can provide a reference for optimization design of satellite solar array.

1. Introduction
The sun synchronous orbit satellite needs sunlight to provide energy, and it can use the solar array for transforming light energy into electrical energy. So the output power of solar array is a very important basis for energy budget and balance 0, which decides the work condition of the spacecraft. The output power of the solar array is reflected by the output current, which is mainly affected by the incidence angle of the sun, the earth-sun distance factor, the solar array temperature and the attenuation factor of the solar array.

Johnson et al. obtained the decay rules of the solar cell with the time of 15 months’ in-orbit operation by the direct measurement and calculation of the in orbit equipment. However, the study focused on the attenuation characteristics. Zhang et al. considered the influence of the solar incident light intensity and the working temperature of the solar array on the output power of the solar array on solar aircraft, whose research had certain reference value to the satellite. Fabio and Fabrizion analyzed the UNISAT-3 solar array in-orbit performance. Their study focused on the decaying rate of different solar array materials. Jing et al. used the normalization method to obtain the influence rules of earth albedo, stars occlusion, attenuation factors on the output power of the solar array. But the lack of precise modelling of the output current makes it can only reflect the general trend. In addition, the existing studies did not consider the influence of the fixed offset angle to the sun of solar wing on the output current in the actual projects.

In this paper, the relationship between the solar array output current, the incidence angle of the sun, the temperature of the solar array, the earth-sun distance factor, the attenuation factor of the solar array and the fixed offset angle to the sun of solar wing is comprehensively analyzed. Then the Poisson series is used and improved to fit their relationship. Finally the model of the solar array output current is established, and it is verified in practical application.
2. Output power of the solar array
On the basis of in-orbit data statistics, the range of the bus voltage of the sun synchronous orbit satellite is very small. The output voltage of the solar array can be regarded as a constant. So, the variation rule of the output power of the solar array is reflected by the variation of the output current of the solar array $I$. The output current of the power supply array of the solar array is directly affected by the illumination change of the sun, which is used for characteristic analysis in this paper $0$. The output current of the solar array $I$ is related to the incidence angle of the sun, the temperature of the solar array, the earth-sun distance factor and the attenuation factor, which is given by $0$

$$I = \left[ I_{bol} + \eta (T - 25) \right] \cos \beta \cdot F \cdot F$$  

(1)

where $I_{bol}$ (A) is the output current of the solar array with the incidence angle of $0^\circ$ and the working temperature of 25°C at the beginning of life. $\eta$ is the current-temperature coefficient. $T$ is the temperature of the solar array. $\beta$ is the incidence angle of the sun. $F_0$ is the earth-sun distance factor. $F$ is the attenuation factor of the solar array, which includes the attenuation factor of space particle radiation, the attenuation factor of ultraviolet radiation, the attenuation factor affected by the collision of meteoroid and the alternation of heat and cold, and et al $0$. Besides, earth albedo and stellar occultation can also influence the output power. $I_{bol}$ and $\eta$ in equation (1) can be obtained by multi-group data at the beginning of satellite life using the least square method.

The angle of incidence of the sun's rays relative to the orbital plane, i.e., the angle between the sun's rays and the normal plane of the orbit, is given by $0$

$$u(t) = \arccos \left[ \cos i \sin \left( \arctan \left( \frac{\sin \left( \frac{2\pi}{365} \right)}{\cot e} \right) \right) + \sin i \cos \left( \arctan \left( \frac{\sin \left( \frac{2\pi}{365} \right)}{\cot e} \right) \right) \sin \left( \frac{t_0}{12} - \Delta \alpha(t) \right) \right]$$  

(2)

where $t \in [0, 365]$ and $t=0$ means vernal equinox. The sun moves eastward along the equator from the vernal equinox. $t_0$ is the local time of descending node of satellite in sun synchronous orbit, and $t_0 \in [0, 24]$. $e$ is the obliquity and $e = 23^\circ 26'$. $\Delta \alpha(t)$ is the accessional difference of the real sun and the mean sun. If the angle between the solar array and the orbit plane is $\kappa$, the angle between the solar ray and the normal vector of the solar array, i.e., $\beta = u(t) - \kappa$.

The earth-sun distance factor is given by $0$:

$$F_0 = 1.00011 + \eta(0.034221\cos \varphi + 0.00128\sin \varphi + 0.000719\cos 2\varphi + 0.000077\sin 2\varphi)$$  

(3)

where $\varphi = 2\pi t/365$, and $t \in [0, 365]$.

The attenuation factor of the solar array is fitted by $0$:

$$F = ax^3 + bx^2 + cx + d$$  

(4)

where $a$, $b$, $c$ and $d$ are polynomial variables. $x$ is the working days of the in-orbit satellite.

When the fixed angle of the solar array to the sun is $\theta$ (0 $\leq \theta \leq 90^\circ$), the output current of the power supply array can be calculated by:

$$I(\theta) = \left[ I_{bol} + \eta (T - 25) \right] \cos \beta \cdot F_0 \cdot F \cdot \cos \theta$$  

(5)

3. Poisson series fitting of output current
In this study, the temperature of the solar array $T$, the incidence angle of the sun $\beta$, the earth-sun distance $F_0$, the attenuation factor $F$ and the fixed angle of the solar array to the sun $\theta$ are taken as variables, so equation (2) can be changed to:

$$\frac{I(\theta)}{I_{bol}} = F \cdot F_0 \cdot \cos \theta \cdot \cos \beta + \frac{\eta (T - 25)}{I_{bol}} \cdot F \cdot F_0 \cdot \cos \theta \cdot \cos \beta$$  

(6)
From equation (6) we can see that \( I(\theta) \) is the combination function of polynomials \((F, F \text{ and } T)\) and trigonometric functions \((\theta \text{ and } \beta)\). A special series is introduced to fit the output current \( I(\theta) \), i.e., Poisson series. The combination of polynomials and trigonometric functions are used in the theory of perturbations of celestial mechanics, orbital mechanic, nonlinear mechanics and nonlinear differential equation, which is called Poisson series 00 and also called polynomial-trigonometric function. Its general form is given by:

\[
P = \sum_{i_1,\ldots,i_n} \sum_{k=0}^{m} C_{i_1,\ldots,i_n}^{k} x_1^{i_1} x_2^{i_2} \cdots x_n^{i_n} \left( \sin(j_1 \phi_1 + \cdots + j_n \phi_n) \right)
\]

where \( x_1, x_2, \ldots, x_n \) are polynomial variables, \( \phi_1, \phi_2, \ldots, \phi_n \) are trigonometric variables. \( C_i \) are Poisson coefficients. \( i_1, i_2, \ldots, i_n \) and \( j_1, j_2, \ldots, j_n \) are all integers. There are three polynomial variables and two trigonometric variables in equation (6), so equation (7) can be simplified into a \((3, 2)\) type of Poisson series:

\[
P(x_1, x_2, x_3, \phi_1, \phi_2) = \sum_{a,b=0}^{n} \sum_{k=0}^{m} x_1^{a} x_2^{b} x_3^{k} \left[ A_{a,b,k}^{i,j,k} \cos(j_1 \phi_1 + j_2 \phi_2) + B_{a,b,k}^{i,j,k} \sin(j_1 \phi_1 + j_2 \phi_2) \right]
\]

where \( x_1 = F, x_2 = F, x_3 = \eta(T-25) \) is used to minimize the following objective function:

\[
F = \sum_{i=1}^{N} \left( \frac{I(k)}{I_{\text{bol}}} - P(k) \right)^2
\]

Each set of data can be written into a linear equation about the unknown coefficient:

\[
I(k)/I_{\text{bol}} = A_{0,0,0}^{0} + A_{0,0,0}^{1} \cos(\phi_1(k)) + \cdots + A_{m,n,0}^{0} \cos(m \phi_1(k)) + B_{0,0,0}^{1} \sin(\phi_1(k)) + \cdots + B_{m,n,0}^{0} \sin(m \phi_1(k))
\]

Equation (10) can be expressed in matrix equation with \( n=1 \) and \( m=1 \):

\[
S_{N\times1} = H_{N\times60} \phi_{49\times1} + \epsilon_{N\times1}
\]

where

\[
S = \left( I(1), I(2), \ldots, I(N) \right)^T
\]

\[
\epsilon = \left( \epsilon_1, \epsilon_2, \ldots, \epsilon_N \right)^T
\]

\[
\phi = \left( A_{0,0,0}^{0}, A_{0,0,0}^{1}, \ldots, A_{0,0,0}^{I}, B_{0,0,0}^{0}, B_{0,0,0}^{1}, \ldots, B_{0,0,0}^{I} \right)^T
\]

So the estimated value of \( A_{0,0,0}^{i,j,k} \) and \( B_{0,0,0}^{i,j,k} \) can be calculated by:

\[
H = \begin{bmatrix}
1 & \cos(\phi_1(1)) & x_1(1) \cos(\phi_1(1)) + \phi_1(1)) \\
1 & \cos(\phi_1(2)) & x_1(2) \cos(\phi_1(2)) + \phi_1(2)) \\
\vdots & \vdots & \vdots \\
1 & \cos(\phi_1(N)) & x_1(N) \cos(\phi_1(N)) + \phi_1(N))
\end{bmatrix}
\]
\[
\hat{\theta} = (\mathbf{H}^T\mathbf{H})^{-1}\mathbf{H}^T\mathbf{S}
\]  

(16)

There are 49 Poisson coefficients in equation (9), which makes the matrix in equation (16) close to singular. Each variable in equation (8) is multiplied by a scaled factor, i.e., \( x_1 = k_1 F \), \( x_2 = k_2 F_e \), \( x_3 = k_1 \eta(T - 25)/I_{Inc} \), \( \phi_1 = k_3 \theta \), and \( \phi_2 = k_4 \beta \).

4. Experimental results and analysis

Some in-orbit satellite, which has been working normally for eighteen months, is selected for our experiment. 256 sets of in-orbit data in two months about the solar array output current, the incidence angle of the sun, the temperature of the solar array, the earth-sun distance factor, the attenuation factor of the solar array and the fixed offset angle to the sun of solar wing, are used to verify our model. The earth-sun distance factor, the incidence angle of the sun, and the attenuation factor of the solar array are showed in figure 1, figure 2 and figure 3, respectively. The fixed angle of the solar array to the sun is set to be 45°, i.e., \( \theta = 45^\circ \).

Table 1. Data statistics of different parameters.

| Parameter | Numerical value | Amplitude of variation |
|-----------|----------------|-----------------------|
| \( F_s \) | 1.035050 1.035020 | 2.65% |
| \( F \)  | 0.986039 0.985532 | 0.05% |
| \( \cos \beta \) | -18.05 -19.39 | 5.67% |
| \( T(\circ C) \) | 54.86 11.37 | 79.27% |
| \( I(A) \) | 47.69 22.34 | 53.16% |

Figure 1. The earth-sun distance factor for two months.

Figure 2. The incidence angle of the sun for two months.

Figure 3. The attenuation factor of the solar array for two months.

The earth-sun distance factor increases progressively while the attenuation factor of the solar array decreasing progressively. However, their changes are very small, which are 2.65% and 0.05% respectively. As is showed in table 1, the temperature of the solar array has the largest influence on the
output current compared with the other parameters. So, more attention should be paid on the change of the temperature of the solar array for in-orbit satellites.

![Figure 4](image1.png)

**Figure 4.** Model results compared to output current with different solar wing temperatures.

![Figure 5](image2.png)

**Figure 5.** Error distribution of model calculation results.

The calculation results are showed in figure 4. Our model fits the trend of the output current well. The mean error is $\mu=-0.3711$ A, and the standard deviation is $\sigma=2.6208$ A. The error distribution of model calculation results is showed in figure 5. The confidence interval of experimental results is 98.83% ($3\sigma$). The calculation results of our model show that the trend to be slightly fluctuated. However, there is a sharp increase point of the temperature (55 °C) in figure 4, which makes the calculation error of our model very big. The average value of the temperature is about 37 °C, which makes the sharp point to be a noise. Except that our model fits the output current well.

5. Conclusions

The relationship between the solar array output current, the incidence angle of the sun, the temperature of the solar array, the fixed offset angle to the sun of solar wing and so on is comprehensively analyzed. Then Poisson series is improved to fit their relational expression according to the in-orbit working condition. And then a model with multiple variables is established. Experimental results show that our model fits the output current well, which can be used in the related studies. Besides, the temperature of the solar wing has a great influence on the output current (i.e., the output power). Therefore, more attention should be paid on the change of the temperature in satellite management.
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