Buczyński, Jarosław; Januszkiewicz, Tadeusz; Jelisiejew, Joachim; Michałek, Mateusz
Constructions of $k$-regular maps using finite local schemes. (English)
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Summary: A continuous map $\mathbb{R}^m \to \mathbb{R}^N$ or $\mathbb{C}^m \to \mathbb{C}^N$ is called $k$-regular if the images of any $k$ points are linearly independent. Given integers $m$ and $k$ a problem going back to Chebyshev and Borsuk is to determine the minimal value of $N$ for which such maps exist. The methods of algebraic topology provide lower bounds for $N$, but there are very few results on the existence of such maps for particular values $m$ and $k$. Using methods of algebraic geometry we construct $k$-regular maps. We relate the upper bounds on $N$ with the dimension of the locus of certain Gorenstein schemes in the punctual Hilbert scheme. The computations of the dimension of this family is explicit for $k \leq 9$, and we provide explicit examples for $k \leq 5$. We also provide upper bounds for arbitrary $m$ and $k$.

MSC:
53A07 Higher-dimensional and -codimensional surfaces in Euclidean and related n-spaces
57R42 Immersions in differential topology
14C05 Parametrization (Chow and Hilbert schemes)
13H10 Special types (Cohen-Macaulay, Gorenstein, Buchsbaum, etc.)

Keywords:
$k$-regular embeddings; secants; punctual Hilbert scheme; finite Gorenstein schemes

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