Introduction to the Effective Field Theory
Description of Gravity

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Abstract

This is a pedagogical introduction to the treatment of general relativity as a quantum effective field theory. Gravity fits nicely into the effective field theory description and forms a good quantum theory at ordinary energies.

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1 Introduction

The conventional wisdom is that general relativity and quantum mechanics are presently incompatible. Of the “four fundamental forces” gravity is said to be different because a quantum version of the theory does not exist. We feel less satisfied with the theory of gravity and exclude it from being recognized as a full member of the Standard Model. Part of the trouble is that we have tried to unnaturally force gravity into the mold of renormalizable field theories. In the old way of thinking, only the class of renormalizable field theories were considered workable quantum theories. For this reason, general relativity was considered a failure as a quantum field theory. However we now think differently about renormalizability. So-called non-renormalizable theories can be renormalized if treated in a general enough framework, and they are not inconsistent with quantum mechanics[1]. In the framework of effective field theories[2], the effects of quantum physics can be analyzed and reliable predictions can be made. We will see that in this regard the conventional wisdom about gravity is not correct; quantum predictions can be made.

The key point of effective field theory is the separation of known physics at the scale that we are working from unknown physics at much higher energies. Experience has shown that as we go to higher energies, new degrees of freedom and new interactions come into play. We have no reason to suspect that the effects of our present theory are the whole story at the highest energies. Effective field theory allows us to make predictions at present energies without making unwarranted assumptions about what is going on at high energies. In addition, whatever the physics of high energy really is, it will leave residual effects at low energy in the form of highly suppressed non-renormalizable interactions. These can be treated without disrupting the low energy theory. The use of effective field theory is not limited to non-renormalizable theories. Even renormalizable theories benefit from this paradigm. For example, there are the well-known divergences in all field theories. If these divergences were really and truly infinite, the manipulations that we do with them would be nonsense. However we do not believe that our calculations of these divergences are really correct, and if our theory is only a low energy effective field theory of the ultimate finite theory of everything, the manipulations are perfectly reasonable with the end predictions being independent of the physics at very high scales. Our feeling of the reality of
the radiative corrections has also convinced many that it is most natural if the present Standard Model is an effective theory which breaks down at the TeV scale, where Higgs self-interactions would otherwise become unnaturally large. We have even found it useful in Heavy Quark Effective Theory to convert a renormalizable theory into a non-renormalizable one in order to more efficiently display the relevant degrees of freedom and interactions.

In the case of gravity, we feel that the low energy degrees of freedom and interactions are those of general relativity. It would be a surprise if these could not be treated quantum mechanically. To be sure, radiative corrections appear to involve all energies, but this is a problem that the effective field theory formalism handles automatically. We will see that gravity very naturally fits into the framework of effective field theory[3]. In fact it is potentially even a better effective theory than the Standard Model as the quantum corrections are very small and the theory shows no hint of a breakdown before the Planck scale. If we insist on treating general relativity as the isolated fundamental theory even at very high energies, there will be the usual problems at high energy. However, the main point is that we can use the degrees of freedom that we have at ordinary energies to make quantum calculations relevant for those scales.

In these lecture notes, I will briefly review the structure of general relativity and its status as a classical effective field theory. Then I discuss the quantization of the theory, following the work of ’t Hooft and Veltman[4]. The methodology of effective field theory is explained in regards to the renormalization of the theory and the extraction of low energy quantum predictions. I describe the example of the gravitational interaction of two heavy masses to illustrate the method[3,5]. Some unique features of the gravitational effective theory are briefly mentioned.

2 Basic Structure of General Relativity

Since this is a school primarily for particle physicists, I need to briefly review general relativity[6,7]. This also allows us to specify our notation and make a few preliminary comments which are relevant for the effective field theory treatment. When a field theorist describes the ingredients of general relativity, it is interesting to see how much the description differs from that of conventional relativists. Below is a minimalist field theoretic presentation
of the structure of the theory.

Lorentz invariance is a global coordinate change which leaves the Minkowski metric tensor invariant.

\[ x'^{\mu} = \Lambda_{\nu}^{\mu} x^{\nu} \]
\[ \eta_{\mu\nu} = \Lambda^{\alpha}_{\mu} \Lambda^{\beta}_{\nu} \eta_{\alpha\beta} \]
\[ \eta_{\mu\nu} = \text{diag}(1, -1, -1, -1) \]
\[ d\tau^2 = \eta_{\mu\nu} dx'^{\mu} dx'^{\nu} = \eta_{\alpha\beta} dx^{\alpha} dx^{\beta} \]  

(1)

Fields transform as scalars, vectors, etc., under this change

\[ \phi'(x') = \phi(x) \]
\[ A'^{\mu}(x') = \Lambda^{\mu}_{\nu} A^{\nu}(x) \]  

(2)

This can be made into a local coordinate change

\[ x'^{\mu} = x^{\mu}(x) \]
\[ dx'^{\mu} = \Lambda^{\mu}_{\nu}(x) dx^{\nu} \]
\[ \bar{\Lambda}^{\nu}_{\mu}(x) \equiv [\Lambda^{\mu}_{\nu}(x)]^{-1} \]
\[ \Lambda^{\mu}_{\nu} \bar{\Lambda}^{\nu}_{\rho} = \delta^{\mu}_{\rho} \]  

(3)

but only if the metric is allowed to be a coordinate dependent field transforming as

\[ g'_{\mu\nu}(x') = \bar{\Lambda}^{\alpha}_{\mu} \bar{\Lambda}^{\beta}_{\nu} g_{\alpha\beta}(x) \]
\[ d\tau^2 = g'_{\mu\nu}(x') dx'^{\mu} dx'^{\nu} = g_{\alpha\beta}(x) dx^{\alpha} dx^{\beta} \]  

(4)

with inverse \( g^{\mu\nu} \)

\[ g^{\mu\nu}(x) g_{\nu\rho}(x) = \delta^{\mu}_{\rho} \]  

(5)

Scalar and vector fields are now defined with the properties
\[ \phi'(x') = \phi(x) \]
\[ A^{\mu'}(x') = A^{\mu}(x) A^{\nu}(x) \]  \hspace{1em} (6)

A covariant derivative can be defined with the right transformation property (i.e., \( D^{\prime}_{\mu} A^\lambda = \bar{\Lambda}^{\nu}_{\mu} \Lambda^\lambda_{\sigma} D_{\nu} A^\sigma \)) by

\[ D_{\mu} A^\lambda = \partial_{\mu} A^\lambda + \Gamma^{\lambda}_{\mu\nu} A^\nu \]  \hspace{1em} (7)

where the connection \( \Gamma^{\lambda}_{\mu\nu} \) is defined as

\[ \Gamma^{\lambda}_{\mu\nu} = \frac{1}{2} g^{\lambda\sigma} [\partial_{\mu} g_{\sigma\nu} + \partial_{\nu} g_{\sigma\mu} - \partial_{\sigma} g_{\mu\nu}] \]  \hspace{1em} (8)

It is important for the effective theory that the connection involves one derivative of the metric (\( \Gamma \sim \partial g \)).

Similarly we can define tensor and scalar fields, the curvatures, which depend only on two derivatives of the metric

\[ [D_{\mu}, D_{\nu}] A_\alpha \equiv R^\beta_{\alpha\mu\nu} A_\beta \]

\[ R^\beta_{\alpha\mu\nu} = \partial_{\mu} \Gamma^\beta_{\alpha\nu} - \partial_{\nu} \Gamma^\beta_{\alpha\mu} + \Gamma^\lambda_{\alpha\nu} \Gamma^\beta_{\lambda\mu} - \Gamma^\lambda_{\alpha\mu} \Gamma^\beta_{\lambda\nu} \]

\[ R_{\alpha\mu} \equiv R^\lambda_{\alpha\mu\lambda} \]

\[ R \equiv g^{\alpha\mu} R_{\alpha\mu} \]  \hspace{1em} (9)

The curvature is nonlinear in the field \( g_{\mu\nu} \). Despite the similarity to the construction of the field strength tensor of Yang Mills field theory, there is the important difference that the curvatures involve two derivatives of the basic field (\( R \sim \partial g \)).

It is easy to construct an action for the matter fields which is invariant under the general coordinate transformation, simply by modifying the usual Lagrangian to use covariant derivatives and to raise and lower Lorentz indices with \( g_{\mu\nu}(x) \). In addition, if we want the metric to be a dynamical field we need an action involving derivatives on the metric. The simplest invariant function is the scalar curvature so that one would postulate

\[ S_g = \int d^4x \sqrt{g} \frac{2}{\kappa^2} R \]  \hspace{1em} (10)
with $\kappa^2$ presently an unknown constant. We will return to this step in the next section. Variation of the full action leads to Einstein’s Equation

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = -8\pi G T_{\mu\nu}$$

$$\sqrt{g} T^{\mu\nu} \equiv -2 \frac{\partial}{\partial g_{\mu\nu}} (\sqrt{g} L_m)$$

where $L_m$ is the Lagrange density for matter, and $T^{\mu\nu}$ is the corresponding energy momentum tensor, and $\kappa^2 \equiv 32\pi G$. By investigating solutions to this equation, it can be seen to describe Newtonian gravity in the appropriate limit if $G$ is identified as the Cavendish constant.

In this summary, invariance requirements take precedence over geometrical ideas and indeed the fact that this is a good theory for gravity appears only at the end of this construction.

We will use a few other facts of general relativity which deserves to be mentioned in this section. In the weak field limit we can expand the metric around Minkowski space introducing the dynamical part of the metric as $h_{\mu\nu}$

$$g_{\mu\nu} \equiv \eta_{\mu\nu} + \kappa h_{\mu\nu} \quad (\text{exactly})$$
$$g^{\mu\nu} = \eta^{\mu\nu} - \kappa h^{\mu\nu} + \kappa^2 h^{\mu\lambda} h_{\lambda\nu} + \ldots$$

The weak field gauge invariance is given by

$$x'^\mu = x^\mu + \epsilon^\mu(x) \quad \epsilon << 1$$
$$h_{\mu\nu}'(x') = h_{\mu\nu}(x) - \partial_\mu \epsilon_\nu - \partial_\nu \epsilon_\mu$$

and the curvatures are

$$R_{\mu\nu} = \frac{\kappa}{2} \left[ \partial_\mu \partial_\nu h^\lambda + \partial_\lambda \partial^\lambda h_{\mu\nu} - \partial_\mu \partial_{\lambda\nu} h^\lambda - \partial_\lambda \partial_{\mu\nu} h^\lambda \right] + O(h^2)$$

$$R = \kappa \left[ \square h^\lambda - \partial_\mu \partial_\nu h^{\mu\nu} \right] + O(h^2)$$

where indices are raised and lowered with $\eta_{\mu\nu}$. This can equally well be done around any fixed smooth background space time metric.
The Greens function does not exist without a gauge choice and it is most convenient to use harmonic gauge

\[ \partial^{\lambda} h_{\mu \lambda} = \frac{1}{2} \partial_{\mu} h^{\lambda} \]  

which reduces Einstein’s Equation in the weak field limit to

\[ \Box h_{\mu \nu} = -16\pi G \left( T_{\mu \nu} - \frac{1}{2} \eta_{\mu \nu} T^{\lambda}_{\lambda} \right) \]  

This has the solution for a static point mass of

\[ h_{\mu \nu} = \text{diag}(1, 1, 1, 1) \left( \frac{2GM}{r} \right) \]  

There are also plane wave solutions. These satisfy

\[ R_{\mu \nu} = 0 = \Box h_{\mu \nu} \]  

resulting in

\[ h_{\mu \nu} = N \epsilon_{\mu \nu} e^{-ip \cdot x} + \text{h.c.} \]  

with \( p^2 = 0 \). The harmonic gauge choice plus residual gauge freedom can reduce the polarization vector to two transverse traceless degrees of freedom appropriate for a massless spin two degrees of freedom.

The gravity waves also carry energy and momentum; hence gravity is nonlinear. The energy momentum tensor of the gravity waves simplifies a bit in harmonic gauge and can be put in the form

\[
T_{\mu \nu} = -\frac{1}{4} h_{\alpha \beta} \partial_\mu \partial_\nu h^{\alpha \beta} + \frac{1}{8} h \partial_\mu \partial_\nu h - \frac{1}{8} \eta_{\mu \nu} \left( h_{\alpha \beta} \Box h_{\alpha \beta} - \frac{1}{2} h \Box h \right) - \frac{1}{4} \left( h_{\mu \rho} \Box h^\rho_\nu + h_{\nu \rho} \Box h^\rho_\mu - h_{\mu \nu} \Box h \right) - \frac{1}{8} \partial_\mu \partial_\nu \left( h_{\alpha \beta} h^{\alpha \beta} - \frac{1}{2} hh \right) - \frac{1}{16} \eta_{\mu \nu} \Box \left( h_{\alpha \beta} h^{\alpha \beta} - \frac{1}{2} hh \right) - \frac{1}{4} \partial_\alpha \left[ \partial_\nu \left( h_{\mu \beta} h^{\alpha \beta} \right) + \partial_\mu \left( h_{\nu \beta} h^{\alpha \beta} \right) \right]
\]
\[ + \frac{1}{2} \partial_\alpha \left[ h^{\alpha\beta} \left( \partial_\nu h_{\mu\beta} + \partial_\mu h_{\nu\beta} \right) \right] \]  

(20)

with \( h \equiv h^\lambda_\lambda \). In this form only the first term contributes to the forward matrix element of a physical transverse traceless mode.

### 3 Classical Effective Field Theory

Let us revisit a crucial step in the derivation of general relativity. What is the rationale for choosing the gravitational action proportional to \( R \) and only \( R \)? It is not due to any symmetry and, unlike other theories, cannot be argued on the basis of renormalizability. However physically the curvature is small so that in most applications \( R^2 \) terms would be yet smaller. This leads to a rationale based on classical effective field theory.

There are in fact infinitely many terms allowed by general coordinate invariance, i.e.,

\[ S = \int d^4 x \sqrt{g} \left\{ \Lambda + \frac{2}{\kappa^2} R + c_1 R^2 + c_2 R_{\mu\nu} R^{\mu\nu} + \ldots + \mathcal{L}_{\text{matter}} \right\} \]  

(21)

Here the gravitational Lagrangians have been ordered in a derivative expansion with \( \Lambda \) being of order \( \partial^0 \), \( R \) of order \( \partial^2 \), \( R^2 \) and \( R_{\mu\nu} R^{\mu\nu} \) of order \( \partial^4 \) etc. Note that in four dimensions we do not need to include a term \( R_{\mu\nu\alpha\beta} R^{\mu\nu\alpha\beta} \) as the Gauss Bonnet theorem allows this contribution to the action to be written in terms of \( R^2 \) and \( R_{\mu\nu} R^{\mu\nu} \).

The first term in Eq.21, i.e., \( \Lambda \), is related to the cosmological constant, \( \lambda = -8\pi G \Lambda \), with Einstein’s equation becoming

\[ R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = -8\pi G T_{\mu\nu} - \lambda g_{\mu\nu} \]  

(22)

This is a term which in principle should be included, but cosmology bounds \( | \lambda | < 10^{-56} \text{cm}^{-2}, | \Lambda | < 10^{-46} \text{GeV}^{-4} \) so that this constant is unimportant at ordinary energies[8]. We then set \( \Lambda = 0 \) from now on.

In contrast, the \( R^2 \) terms are able to be shown to be unimportant in a natural way. Let us drop Lorentz indices in order to focus on the important elements, which are the numbers of derivatives. A \( R + R^2 \) Lagrangian
\[ \mathcal{L} = \frac{2}{\kappa^2} R + cR^2 \]  

has an equation of motion which is of the form

\[ \Box h + \kappa^2 c^2 \Box h = 8 \pi GT \]  

The Greens function for this wave equation has the form

\[ G(x) = \int \frac{d^4 q}{(2\pi)^4} \frac{e^{iq \cdot x}}{q^2 + \kappa^2 c q^4} \]
\[ = \int \frac{d^4 q}{(2\pi)^4} \left[ \frac{1}{q^2} - \frac{1}{q^2 + 1/\kappa^2 c} \right] e^{-iq \cdot x} \]  

The second term appears like a massive scalar, but with the wrong overall sign, and leads to a short-ranged Yukawa potential

\[ V(r) = -Gm_1 m_2 \left[ \frac{1}{r} - \frac{e^{-r/\sqrt{\kappa^2 c}}}{r} \right]. \]  

The exact form has been worked out by Stelle\[9\], who gives the experimental bounds \( c_1, c_2 < 10^{74} \). Hence, if \( c_i \) were a reasonable number there would be no effect on any observable physics. [Note that if \( c \sim 1, \sqrt{\kappa^2 c} \sim 10^{-35} m \). Basically the curvature is so small that \( R^2 \) terms are irrelevant at ordinary scales.

As a slightly technical aside, in an effective field theory we should not treat the \( R^2 \) terms to all orders, as is done above in the exponential of the Yukawa solution, but only include the first corrections in \( \kappa^2 c \). This is because at higher orders in \( \kappa^2 c \) we would also be sensitive to yet higher terms in the effective Lagrangian (\( R^3, R^4 \) etc.) so that we really do not know the full \( r \to 0 \) behavior. Rather, for \( \sqrt{\kappa^2 c} \) small we can note the Yukawa potential becomes a representation of a delta function

\[ \frac{e^{-r/\sqrt{\kappa^2 c}}}{r} \to 4\pi \kappa^2 c \delta^3(\vec{r}) \]  

Alternatively in the Greens function we could note that
\[ \frac{1}{q^2 + \kappa^2 c q^4} = \frac{1}{q^2} - \kappa^2 c + \cdots \] (28)

and that the Fourier transform of a constant is a delta function. Either way, one is lead to a form of the potential

\[ V(r) = -G m_1 M_2 \left[ \frac{1}{r} + 128 \pi^2 G (c_1 - c_2) \delta^3(\vec{x}) \right] \] (29)

\( R^2 \) terms in the Lagrangian lead to a very weak and short range modification to the gravitational interaction.

Thus when treated as a classical effective field theory, we can start with the more general Lagrangian, and find that only the effect of the Einstein action, \( R \), is visible in any test of general relativity. We need not make any unnatural restrictions on the Lagrangian to exclude \( R^2 \) and \( R_{\mu\nu} R^{\mu\nu} \) terms. J. Simon[10] has shown that the standard problems with classical \( R + R^2 \) gravity are not problems when one restricts oneself to the low energy domain appropriate for an effective field theory.

4 Quantization

There is a beautiful and simple formalism for the quantization of gravity. The most attractive variant combines the covariant quantization pioneered by Feynman and De Witt[11] with the background field method[12] introduced in this context by 't Hooft and Veltman[4]. The quantization of a gauge theory always involves fixing a gauge. This can in principle cause trouble if this procedure then induces divergences which can not be absorbed in the coefficients of the most general Lagrangian which displays the gauge symmetry. The background field method solves this problem because the calculation retains the symmetry under transformations of the background field and therfor the loop expansion will be gauge invariant, retaining the symmetries of general relativity.

Consider the expansion of the metric about a smooth background field \( \bar{g}_{\mu\nu}(x) \),

\[ g_{\mu\nu}(x) = \bar{g}_{\mu\nu}(x) + \kappa h_{\mu\nu} \] (30)
Indices are now raised and lowered with $\bar{g}$. The Lagrangian may be expanded in the quantum field $h_{\mu\nu}[4,13]$.

\[
\frac{2}{\kappa^2} \sqrt{\bar{g}} R = \sqrt{\bar{g}} \left\{ \frac{2}{\kappa^2} \bar{R} + \mathcal{L}_g^{(1)} + \mathcal{L}_g^{(2)} + \cdots \right\}
\]

\[
\mathcal{L}_g^{(1)} = \frac{h_{\mu\nu}}{\kappa} [\bar{g}^{\mu\nu} \bar{R} - 2 \bar{R}^{\mu\nu}]
\]

\[
\mathcal{L}_g^{(2)} = \frac{1}{2} D_\alpha h_{\mu\nu} D^\alpha h^{\mu\nu} - \frac{1}{2} D_\alpha h D^\alpha h + D_\alpha h D_\beta h^{\alpha\beta}
\]

\[
- D_\alpha h_{\mu\nu} D^\beta h^{\mu\nu} + \bar{R} \left( \frac{1}{2} h^2 - \frac{1}{2} h_{\mu\nu} h^{\mu\nu} \right)
\]

\[
+ \bar{R}^{\mu\nu} \left( 2h^\lambda_{\mu} h_{\nu\lambda} - hh_{\mu\nu} \right)
\]

(31)

Here $D_\alpha$ is a covariant derivative with respect to the background field. The total set of terms linear in $h_{\mu\nu}$ (including those from the matter Lagrangian) will vanish if $\bar{g}_{\mu\nu}$ satisfies Einstein’s equation. We are then left with a quadratic Lagrangian plus interaction terms of higher order.

However, the quadratic Lagrangian cannot be quantized without gauge fixing and the associated Feynman-DeWitt-Fadeev-Popov ghost fields[11,14]. Let us briefly recall the logic, which is the same as for the quantization of Yang Mills theories[14,15]. The path integral overcounts fields which are equivalent under a gauge transformation and we need to divide out extra gauge copies. Carrying out the procedure leads to a path integral

\[
Z = \int dh_{\mu\nu} \delta(G_\alpha(h)) \det | \partial G_\alpha / \partial \epsilon_\beta | e^{iS}
\]

(32)

where $G_\alpha(h)$ is the gauge constraint to be imposed and $\partial G_\alpha / \partial \epsilon_\beta$ refers to the variation of the constraint under infinitesimal gauge transformation as in Eq.13. Exponentiation of $\delta(G_\mu(h))$ leads to the addition of a gauge fixing term, $\mathcal{L}_{gf}$, to the quadratic Lagrangian. Exponentiation of $\det | \partial G / \partial \epsilon |$ by introducing new fermion fields and using

\[
\det M = \int d\eta d\bar{\eta} e^{iS} \int d^4x d^4\eta M_\eta
\]

(33)

brings in the ghost Lagrangian and completes the procedure of producing a quadratic Lagrangian with gauge fixing.
In this case, we would like to impose the harmonic gauge constraint in the background field, and can choose the constraint\[4\]

\[ G^\alpha = \sqrt{g} \left( D^\nu h_{\mu \nu} - \frac{1}{2} D_\mu h^\lambda_\lambda \right) t^{\nu \alpha} \]  

(34)

where

\[ \eta_{\alpha \beta} t^{\mu \alpha} t^{\nu \beta} = \bar{g}^{\mu \nu} \]  

(35)

This leads to the gauge fixing Lagrangian\[4\]

\[ \mathcal{L}_{gf} = \sqrt{\bar{g}} \left\{ \left( D^\nu h_{\mu \nu} - \frac{1}{2} D_\mu h^\lambda_\lambda \right) \left( D_{\sigma} h^{\mu \sigma} - \frac{1}{2} D_\mu h^\sigma_\sigma \right) \right\} \]  

(36)

Because the gauge constraint contains a free Lorentz index, as does the gauge transformation variable \( \epsilon_\beta \), the ghost field will carry a Lorentz label, i.e., they will be fermionic vector fields. After a bit of work the ghost Lagrangian is found to be

\[ \mathcal{L}_{gh} = \sqrt{\bar{g}} \eta^{* \mu} \left[ D_\lambda D^{\lambda} \bar{g}_{\mu \nu} - \bar{R}_{\mu \nu} \right] \eta^\nu \]  

(37)

The full quantum action is then of the form

\[ S = \int d^4 x \sqrt{\bar{g}} \left\{ \frac{2}{k^2} \bar{R} - \frac{1}{2} h_{\alpha \beta} D^{\alpha \beta \gamma \delta} h_{\gamma \delta} \right. \\
+ \left. \eta^{* \mu} \left\{ D_\lambda D^{\lambda} \bar{g}_{\mu \nu} - \bar{R}_{\mu \nu} \right\} \eta^\nu + \mathcal{O}(\hbar^3) \right\} \]  

(38)

with \( D^{\alpha \beta \gamma \delta} \) an invertible differential operator formed using Eq.31 and Eq.36. This can then be used to define the propagator and the Feynman rules in a straightforward fashion.

Despite the conceptual simplicity, explicit formulas in gravity have a notational complexity due to the proliferations of Lorentz indices. Around flat space, the momentum space propagator is relatively simple in this gauge

\[ i D_{\mu \nu \alpha \beta} = \frac{i}{q^2 + i \epsilon} P_{\mu \nu, \alpha \beta} \]

\[ P_{\mu \nu, \alpha \beta} = \frac{1}{2} \left[ \eta_{\mu \alpha} \eta_{\nu \beta} + \eta_{\mu \beta} \eta_{\nu \alpha} - \eta_{\mu \nu} \eta_{\alpha \beta} \right] \]  

(39)
The coupling to matter of the one graviton and two graviton vertices are respectively

\[ \tau_{\mu\nu} = -\frac{i\kappa}{2} \left( p \mu p' \nu + p' \mu p \nu - g_{\mu\nu} [p \cdot p' - m^2] \right) \]

\[ \tau_{\eta\lambda,\rho\sigma} = \frac{i\kappa^2}{2} \left\{ I_{\eta\lambda,\alpha\beta} \delta_{\beta,\rho\sigma} \left( p^\alpha p'^\beta + p'^\alpha p^\beta \right) - \frac{1}{2} \left( \eta_{\eta\lambda} I_{\rho\sigma,\alpha\beta} + \eta_{\rho\sigma} I_{\eta\lambda,\alpha\beta} \right) p^\alpha p'^\beta \right. \]

\[ \left. - \frac{1}{2} \left( I_{\eta\lambda,\rho\sigma} - \frac{1}{2} \eta_{\eta\lambda} \eta_{\rho\sigma} \right) [p \cdot p' - m^2] \right\} \]

with

\[ I_{\mu\nu,\alpha\beta} \equiv \frac{1}{2} \left[ \eta_{\mu\alpha} \eta_{\nu\beta} + \eta_{\mu\beta} \eta_{\nu\alpha} \right] \]

The energy momentum tensor for gravitons leads to the interaction of gravitons with an external field which, with the harmonic gauge fixing, is of the form

\[ \tau_{\mu\nu,\alpha\beta,\gamma\delta} = \frac{i\kappa}{2} \left\{ \eta_{\alpha\beta} \eta_{\gamma\delta} k^\mu k^\nu + (k - q)^\mu (k - q)^\nu + q^\mu q^\nu - \frac{3}{2} \eta_{\mu\nu} q^2 \right\} \]

\[ + 2q_\lambda q_\sigma \left[ I_{\lambda\sigma,\alpha\beta} I_{\mu\nu,\gamma\delta} + I_{\lambda\sigma,\gamma\delta} I_{\mu\nu,\alpha\beta} - I_{\lambda\sigma,\alpha\beta} I_{\mu\nu,\gamma\delta} + I_{\lambda\sigma,\gamma\delta} I_{\mu\nu,\alpha\beta} \right] \]

\[ + \eta_{\mu\nu} q^2 \left( \eta_{\alpha\beta} I_{\mu\nu,\gamma\delta} + \eta_{\gamma\delta} I_{\mu\nu,\alpha\beta} \right) + q_\lambda q_\sigma \left( \eta_{\alpha\beta} I_{\lambda\sigma,\mu\nu} + \eta_{\gamma\delta} I_{\lambda\sigma,\mu\nu} \right) - \frac{1}{2} \left( \eta_{\gamma\delta} I_{\mu\nu,\eta\sigma} + \eta_{\mu\nu} I_{\alpha\beta,\eta\sigma} \right) \]

\[ + \left\{ \left( k^2 + (k - q)^2 \right) \left( I_{\mu\nu,\alpha\beta} I_{\lambda\sigma,\gamma\delta} + I_{\mu\nu,\alpha\beta} I_{\gamma\delta,\lambda\sigma} \right) - \frac{1}{2} \eta_{\mu\nu} P_{\alpha\beta,\gamma\delta} \right\} \]

As a simple example of the use of these Feynman rules, consider the interaction of two heavy masses. This is written as
\[ M = \frac{1}{2} \tau_{\mu\nu}(q) D^{\mu\nu,\alpha\beta}(q) \tau_{\alpha\beta}(q) \]  \hspace{1cm} (43)

Taking the nonrelativistic limit \( p_\nu \sim (m, \vec{0}) \), and accounting for the normalization of the states leads us to

\[ \frac{1}{2m_1} \frac{1}{2m_2} M = 4\pi G \frac{m_1 m_2}{q^2} \]  \hspace{1cm} (44)

which leads to the usual potential energy function.

\[ V(r) = -G \frac{m_1 m_2}{r} \]  \hspace{1cm} (45)

Of course, this is a classical result which did not require us to go through the quantization procedure. True quantum effects will be discussed later.

5 Quantum Effective Field Theory-Overview

While the quantization of gravity proceeds in an almost identical fashion to that of Yang Mills theory, it is what happens next which has been troublesome for gravity. Because of the dimensionful coupling \( \kappa \) and the nonlinear interactions to all orders in \( h_{\mu\nu} \), gravity does not belong to the class of renormalizable field theories. As we will see, loop diagrams generate divergences which cannot be absorbed in only a renormalization of \( G \), but require increasing numbers of renormalized parameters with increasing numbers of loops. However this pattern is typical of effective field theories and is not an obstacle to making quantum predictions.

The procedure for carrying out this program involves the following steps. The action is organized in full generality in an energy expansion. The vertices and propagators of the theory start with terms from the lowest order Lagrangian, and higher order Lagrangians are treated as perturbations. The quantum corrections are calculated and since the ultraviolet divergences respect the symmetry of the theory and are local, they can be absorbed into the parameters of the action. These renormalized parameters must be determined from experiment. The remaining relations between amplitudes are the predictions of the theory. In the following sections I describe in more detail the steps sketched above.
6 The Energy Expansion

We have already partially discussed this in previous sections. In addition to the gravitational Lagrangian (with $\Lambda = 0$)

$$\mathcal{L}_g = \sqrt{g} \left\{ \frac{2}{\kappa^2} R + c_1 R^2 + c_2 R_{\mu\nu} R^{\mu\nu} + \mathcal{O}(R^3) \right\}$$  \hspace{1cm} (46)

the matter Lagrangian must also be written with general couplings in increasing powers of the curvature

$$\mathcal{L}_m = \sqrt{g} \left\{ \frac{1}{2} (g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - m^2 \phi^2) \\
+ d_1 R^{\mu\nu} \partial_\mu \phi \partial_\nu \phi + R (d_2 g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi + d_3 m^2 \phi^2) + \ldots \right\}$$  \hspace{1cm} (47)

The couplings $c_i$ are dimensionless and in matrix elements the expansion will be of the form $1 + Gq^2 c_i$. The coefficients $d_i$ are a bit more subtle. They have dimension $d_i \sim 1/(\text{mass})^2$. In a theory where gravity is the only low energy interaction, a point particle would be expected to have $d_i$ of order $G$. However for interacting theories or composite particles the coefficients can be much larger. In matrix elements of the energy momentum tensor, $d_i$ play the role analogous to the charge radius. In QED, the photonic radiative corrections to the energy-momentum charge radius of a charged particle will generate $d_i$ of order $\alpha/m^2$. In the case of bound states, a composite particle will have $d_i$ of order the physical spatial extent of the particle $d_i \sim \langle r^2 \rangle$.

If we just had gravity plus a single type of matter field, we could use the equations of motion to eliminate some terms in these effective Lagrangian, as the equations of motion relate the curvatures to the matter field. When treated as an effective field theory, it is fair to use the lowest order equations of motion to simplify the next order Lagrangian. However, in practice we have several types of possible matter fields, as well as interactions among these fields, so that the equations of motion would vary according to which fields were included. I have therefore not eliminated any terms by the equations of motion.
7 Renormalization

The one loop divergences of gravity have been studied in two slightly different methods. One involves direct calculation of the Feynman diagrams with a particular choice of gauge and definition of the quantum gravitational field [16]. The background field method, with a slightly different gauge constraint, allows one to calculate in a single step the divergences in graphs with arbitrary numbers of external lines and also produces a result which is explicitly generally covariant[4]. In the latter technique one expands about a background spacetime $\bar{g}_{\mu\nu}$, fixes the gauge as we described above and collects all the terms quadratic in the quantum field $h_{\mu\nu}$ and the ghost fields. For the graviton field we have

$$Z[\bar{g}] = \int [dh_{\mu\nu}] \exp \left\{ i \int d^4 x \sqrt{g} \left\{ \frac{2}{\kappa^2} \bar{R} + h_{\mu\nu} D^{\mu\nu\alpha\beta} h_{\alpha\beta} \right\} \right\}
= \det D^{\mu\nu\alpha\beta}
= \exp \text{Tr} \ln(D^{\mu\nu\alpha\beta})$$

(48)

where $D^{\mu\nu\alpha\beta}$ is a differential operator made up of derivatives as well as factors of the background curvature. The short distance divergences of this object can be calculated by standard techniques once a regularization scheme is chosen. Dimensional regularization is the preferred scheme because it does not interfere with the invariances of general relativity. First calculated in this scheme by ’t Hooft and Veltman[4], the divergent term at one-loop due to graviton and ghost loops is described by a Lagrangian

$$\mathcal{L}_{1\text{loop}}^{(\text{div})} = \frac{1}{8\pi^2\epsilon} \left\{ \frac{1}{120} \bar{R}^2 + \frac{7}{20} \bar{R}_{\mu\nu} \bar{R}^{\mu\nu} \right\}$$

(49)

with $\epsilon = 4 - d$. Matter fields of different spins will also provide additional contributions with different linear combinations of $R^2$ and $R^\mu_\nu R^\nu_\mu$ at one loop.

The fact that the divergences is not proportional to the original Einstein action is an indication that the theory is of the non-renormalizable type. Despite the name, however, it is easy to renormalize the theory at any given order. At one loop we identify renormalized parameters.
\[ c_1^{(r)} = c_1 + \frac{1}{960\pi^2\epsilon} \]
\[ c_2^{(r)} = c_2 + \frac{7}{160\pi^2\epsilon} \]

which will absorb the divergence due to graviton loops. Alternate but equivalent expressions would be used in the presence of matter loops.

A few comments on this result are useful. One often hears that pure gravity is one loop finite. This is because the lowest order equation of motion for pure gravity is \( R_{\mu\nu} = 0 \) so that the \( O(R^2) \) terms in the Lagrangian vanish for all solutions to the Einstein equation. However in the presence of matter (even classical matter) this is no longer true and the graviton loops yield divergent effects which must be renormalized as described above. At two loops, there is a divergence in pure gravity which remains even after the equations of motion have been used \[17\].

\[ \mathcal{L}^{(\text{div})}_{2\text{loop}} = \frac{209\kappa^2}{2880(16\pi^2)} \frac{1}{\epsilon} R^{\alpha\beta\gamma\delta} R^{\epsilon\sigma\eta\tau} R^{\eta\sigma}_{\alpha\beta} \]

For our purposes, this latter result also serves to illustrate the nature of the loop expansion. Higher order loops invariably involve more powers of \( \kappa \) which by dimensional analysis implies more powers of the curvature or of derivatives in the corresponding Lagrangian (i.e., one loop implies \( R^2 \) terms, 2 loops imply \( R^3 \) etc.). The two loop divergence would be renormalized by absorbing the effect into a renormalized value of a coupling constant in the \( O(R^3) \) Lagrangian.

8 Quantum Predictions in An Effective Theory

At this stage it is important to be clear about the nature of the quantum predictions in an effective theory. The divergences described in the last sections come out of loop diagrams, but they are not predictions of the effective theory. They are due to the high energy portions of the loop integration, and we do not even pretend that this portion is reliable. We expect the real divergences (if any) to be different. However the divergences do not in any
case enter into any physical consequences, as they absorbed into the renormalized parameters. The couplings which appear in the effective Lagrangian are also not predictions of the effective theory. They parameterize our ignorance and must emerge from an ultimate high energy theory or be measured experimentally. However there are quantum effects which are due to low energy portion of the theory, and which the effective theory can predict. These come because the effective theory is using the correct degrees of freedom and the right vertices at low energy. It is these low energy effects which are the quantum predictions of the effective field theory.

It may at first seem difficult to identify which components of a calculation correspond to low energy, but in practice it is straightforward. The effective field theory calculational technique automatically separates the low energy observables. The local effective Lagrangian will generate contributions to some set of processes, which will be parameterized by a set of coefficients. If, in the calculation of the loop corrections, one encounters contributions which have the same form as those from the local Lagrangian, these cannot be distinguished from high energy effects. In the comparison of different reactions, such effects play no role, since we do not know ahead of time the value of the coefficients in $L$. We must measure these constants or form linear combinations of observables which are independent of them. Only loop contributions which have a different structure from the local Lagrangian can make a difference in the predictions of reactions. Since the effective Lagrangian accounts for the most general high energy effects, anything with a different structure must come from low energy.

A particular class of low energy corrections stand out as the most important. These are the nonlocal effects. In momentum space the nonlocality is manifest by a nonanalytic behavior. Nonanalytic terms are clearly distinct from the effects of the local Lagrangian, which always give results which involves powers of the momentum.

Let us illustrate the nature of the quantum corrections by considering the interactions of two loop heavy masses. In coordinate space we can consider possible power modifications at order $G$ of the interactions of the form

$$V(r) = -\frac{G m_1 m_2}{r} \left(1 + a \frac{G m}{r c^2} + b \frac{G h}{r^2 c^3} + \ldots \right)$$

The form of these corrections is fixed strictly by dimensional analysis. The
first, $\frac{Gm}{r^2}$, is the classical expansion parameter for the nonlinear effects in classical general relativity. In contrast, the second is the unique form linear in $G\hbar$ and is the quantum expansion parameter. In momentum space, obtained by the Fourier transform of the potential, one has the corresponding expansion (up to constants of order 1)

$$V(q) \sim \frac{Gm_1m_2}{q^2}\left(1 + aGq^2\sqrt{\frac{m^2}{-q^2}} + bGq^2\ln(-q^2) + \ldots\right)$$  \hspace{1cm} (53)

The nonanalytic term of the form $Gq^2\sqrt{\frac{m^2}{-q^2}}$ corresponds to the classical expansion parameter, while the nonanalytic form $Gq^2\ln(-q^2)$ corresponds to the quantum expansion parameter. If we now turn to a calculation in an effective field theory (to be described in the next section) we directly find the desired terms, with an analytic contribution also

$$V(q) \sim \frac{Gm_1m_2}{q^2}\left(1 + aGq^2\sqrt{\frac{m^2}{-q^2}} + bGq^2\ln(q^2) - q^2 + cGq^2 + \ldots\right)$$  \hspace{1cm} (54)

The analytic term $Gq^2$ receives contributions from the local effective Lagrangian, and also from the quantum loops. It is therefore not a quantum prediction of the low energy effective theory. On the other hand the one loop calculation shows that the nonanalytic terms are finite and independent of the coefficients in the effective Lagrangian (aside from $G$). These are then predictions of the low energy theory. Note that the analytic term Fourier transforms to a delta function $\delta V \sim G^2m_1m_2\delta^3(r)$, which at finite $r$ is smaller than any power correction. Thus the leading power corrections to the gravitational potential are reliably predicted by the effective theory, including the quantum correction!

## 9 Quantum corrections to the Gravitational Potential

The only complete example of this program is the calculation of the effect of quantum physics on the gravitational interaction of two heavy masses. The power-law corrections to the usual $\frac{1}{r}$ potential are calculable. While
there is more than one way to define what one means by the potential when one is working beyond leading order\cite{5,18}, the calculation of the quantum corrections to that object are well defined. I will describe the specific one-particle-reducible potential\cite{5} defined by including the vertex and propagator modifications of one graviton exchange.

The vertex corrections for a scalar particle have the most general form

\[
V_{\mu\nu} \equiv \langle p' | T_{\mu\nu} | p \rangle = F_1(q^2) \left[ p_\mu p'_\nu + p'_\mu p_\nu + q^2 \eta_{\mu\nu} \right] 
+ F_2(q^2) \left[ q_\mu q_\nu - g_{\mu\nu} q^2 \right]
\]

with \( F_1(0) = 1 \). Including the contributions of higher order effective Lagrangians as well as graviton loops one will obtain corrections of the form

\[
F_1(q^2) = 1 + d_1 q^2 + \kappa^2 q^2 \left( \ell_1 + \ell_2 \ln \left( -\frac{q^2}{\mu^2} \right) + \ell_3 \sqrt{\frac{m^2}{-q^2}} \right) + \ldots
\]

\[
F_2(q^2) = -4(d_2 + d_3) m^2 + \kappa^2 m^2 \left( \ell_4 + \ell_5 \ln \left( -\frac{q^2}{\mu^2} \right) + \ell_6 \sqrt{\frac{m^2}{-q^2}} \right) + \ldots
\]

where the \( d_i \) are defined in Eq. 47 and the \( \ell_i (i = 1, 2 \ldots 6) \) are dimensionless numbers from the loop diagrams. This general structure can be gotten from dimensional analysis. Note that the \( d_i \) can be interpreted as the charge radii for the energy-momentum tensor. For the propagator correction, allow me to drop Lorentz indices here in order to see the physics rather than the indices. In the vacuum polarization there is no mass, hence no correction of the form \( \sqrt{\frac{m^2}{-q^2}} \). The propagator plus vacuum polarization is of the form

\[
\frac{1}{q^2} + \frac{1}{q^2} \pi(q^2) \frac{1}{q^2} + \ldots = \left\{ \frac{1}{q^2} + \kappa^2 \left[ c_1 + c_2 + \ell_7 + \ell_8 \ln(-q^2) \right] \right\}
\]

With the definition of the one-particle-reducible interaction, we form

\[
- \frac{\kappa^2}{4} \frac{1}{2m_1} V^{(1)}_{\mu\nu}(q) \left[ i D^{\mu\nu,\alpha\beta}(q) + i D^{\mu\nu,\rho\sigma} i \Pi_{\rho\sigma,\eta\lambda} i D^{\eta\lambda,\alpha\beta} \right] V_{\alpha\beta}(q) \frac{1}{2m_2}
\approx 4\pi Gm_1 m_2 \left[ \frac{i}{q^2} - i\kappa^2 \left[ a \ln(q^2) + \frac{b(m_1 + m_2)}{\sqrt{-q^2}} \right] + \text{const} \right]
\]

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where \( a \) is a linear combination of \( \ell_2, \ell_5, \ell_8 \) and \( b \) is a combination of \( \ell_3, \ell_6 \). The factors of \( \frac{1}{2m} \) account for the normalization used in our states. When this is turned into a coordinate space potential the constant terms yield zero at any finite radius, since

\[
\int \frac{d^3 q}{(2\pi)^3} e^{-i\vec{q} \cdot \vec{r}} = \delta^3(x)
\]

while the non analytic terms however lead to power law behavior since

\[
\int \frac{d^3 q}{(2\pi)^3} e^{-i\vec{q} \cdot \vec{r}} \frac{1}{q} = \frac{1}{2\pi^2 r^2}
\]

\[
\int \frac{d^3 q}{(2\pi)^3} e^{-i\vec{q} \cdot \vec{r}} \ln q^2 = -\frac{1}{2\pi^2 r^3}
\]

(60)

From our discussion in the previous section, we know that constant terms in \( V(r) \) correspond to a delta function potential and can be dropped at any finite \( r \). The factors of \( \sqrt{\frac{m^2 - q^2}{-q^2}} \) lead to the classical power-law corrections and the \( \ln(-q^2) \) terms lead to the quantum power-law corrections, as in Eq. 52. Therefore we need to calculate the constants \( \ell_2, \ell_5 \) and \( \ell_8 \) and \( \ell_3, \ell_6 \).

The calculation is a bit tedious because of all the Lorentz indices, but one finds for the non-analytic terms in the vertex[3,5]

\[
F_1(q^2) = 1 + \frac{\kappa^2}{32\pi^2} q^2 \left[ -\frac{3}{4} \ln(-q^2) + \frac{1}{16} \frac{\pi^2 m}{\sqrt{-q^2}} \right]
\]

\[
F_2(q^2) = \frac{\kappa^2 m^2}{32\pi^2} \left[ -\frac{4}{3} \ln(-q^2) + \frac{7}{8} \frac{\pi^2 m}{\sqrt{-q^2}} \right]
\]

(61)

The non-analytic terms in the vacuum polarization can be obtained from the work of 't Hooft and Veltman[4] by noting that, in a massless theory, the \( \ln(-q^2) \) terms are always related to the coefficient of \( \frac{1}{d-4} \) in dimensional regularization. The appropriate combination is

\[
P_{\mu\nu,\alpha\beta} \Pi_{\alpha\beta,\gamma\delta} P_{\gamma\delta,\rho\sigma} = \frac{\kappa^2 q^4}{32\pi^2} \left[ \frac{21}{120} \left( \eta_{\mu\rho} \eta_{\nu\sigma} + \eta_{\nu\rho} \eta_{\mu\sigma} \right) + \frac{1}{120} \eta_{\mu\nu} \eta_{\rho\sigma} \right] \left[ -\ln(-q^2) \right] + \ldots
\]

(62)
This leads to

\[
- \frac{\kappa^2}{4} \frac{1}{2 m_1} V_{\mu \nu}^{(1)}(q) \left[ i D_{\mu \nu, \alpha \beta}(q) + i D_{\mu \nu, \rho \sigma} i \Pi_{\rho \sigma, \eta \lambda} i D_{\eta \lambda, \alpha \beta} \right] V_{\alpha \beta}(q) \frac{1}{2 m_2}
\]

\approx 4 \pi G m_1 m_2 \left[ \frac{i}{q^2} \right. - \frac{i \kappa^2}{32 \pi^2} \left[ - \frac{127}{60} \ln(q^2) + \frac{\pi^2 (m_1 + m_2)}{2 \sqrt{q^2}} \right] + \text{const} \left. \right]

(63)

and the corresponding Fourier transform

\[
V(r) = - \frac{G m_1 m_2}{r} \left[ 1 - \frac{G (m_1 + m_2)}{r c^2} - \frac{127}{30 \pi^2} \frac{G h}{r^2 c^3} \right]
\]

(64)

Photons and massless neutrinos also modify the vacuum polarization at large distance, generating \( \ln(-q^2) \) corrections[16]. If we include these, with \( N_\nu \) massless neutrinos, the quantum piece becomes

\[
- \frac{135 + 2 N_\nu}{30 \pi^2} \frac{G h}{r^2 c^3}
\]

(65)

When we perform a perturbative expansion, we want the corrections to be small and well behaved. In this regard, gravity is the best perturbative theory in Nature! The quantum correction \( \frac{G h}{r^2 c^3} \) is about \( 10^{-38} \) at \( r = 1 \text{ fm} \), and is unmeasurably tiny on macroscopic scales. This of course is a success; we want quantum gravity to have a well behaved classical limit. However it indicates that this is unlikely to generate an active phenomenology. Overall though, the number is not important; the methodology is. Quantum predictions can be made in general relativity.

10 Issues in the Structure of Gravitational Effective Field Theory

In this section I would like to describe two ways in which the gravitational effective field theory superficially appears different from standard expectations in other effective field theories.

i) The extreme low energy limit.

The considerations above concern what could be called “ordinary” distance scales, i.e. fermis to kilometers, and regions where the curvature is
small. Certainly as one goes to high energies, the methodology is no longer applicable. However, gravity may also behave oddly in the limit of extreme low energy, i.e. as the wavelength probed becomes comparable to the size of the universe or the distance to the nearest black hole. For example, there are singularity theorems[19] which state that for most reasonable matter distributions, spacetime evolved by Einstein’s equations has a singularity in the future or past. The singularity itself is not necessarily a problem; most likely the singularity is smoothed out in the full high energy theory. However, it is unusual that the low energy theory evolves into a state where it is no longer valid. The G.E.F.T. would not work in the neighborhood of a singularity. Therefore we could have a reasonable calculation which works for some ordinary wavelengths but cannot be applied for \( \lambda \to \infty \) because of the presence of large curvature. The existence of a horizon around black holes may also be a problem as \( \lambda \to \infty \). The horizon itself is not a problem, in the sense that the curvature can be very small at the horizon so that a freely falling observer would be able to apply the effective theory locally. However, problems associated with the horizon could possible arise as \( \lambda \to \infty \) since regions at spatial infinity are inaccessible to processes inside the horizon. I do not know if in fact there are real problems with the infinite wavelength limit, but at least this is a limit where our past experience with effective theories is not applicable. In many ways, it would be more interesting to have a problem at the extreme low energy end rather than the better known problems at high energy, since we could not rely on new physics at high scales to resolve the issue.

ii) Naive power counting

The second point involves a technical issue, i.e. the correct counting of momenta in loops. In chiral theories, Weinberg[20] proved an elegant power counting theorem which shows that higher order loop diagrams always generate higher orders in the energy expansion. Explicit calculations in gravity have also followed this pattern. However, there does not yet exist the equivalent of Weinberg’s power counting theorem. In fact an attempt to do naive power counting runs into an obstacle in that it seems to give a dangerous behavior for some diagrams. For example if one calculates the box diagram where two heavy particles interact by the exchange of two gravitons, both
naive power counting and explicit calculation give a correction of the form

\[ \frac{\kappa^2 m_1^2 m_2^2}{q^2} (\kappa^2 m^2) \]  

(66)
i.e. an apparent expansion in the dimensionless variable \( \kappa^2 m^2 \). However this would be a disaster as \( \kappa^2 m^2 \) can be large for classical objects and because it does not allow a classical limit. When one puts in factors of \( \hbar \) and \( c \) one has \( \frac{\kappa^2 m^2 c}{\hbar} \), i.e. the \( \hbar \) is in the denominator. Direct calculation shows that this bad behavior is canceled by the crossed box diagram, and that the sum of the two fulfills standard expectation. This is similar to the cancellation if infrared divergences, which is known to occur in gravity in the same way as in QED\[21\]. With some work, some of the systematics of this cancellation in other diagrams can be worked out, and T. Torma and I will soon have a paper describing these issues\[22\]. We have not found any breakdown of the standard expectation of the nature of the quantum expansion; but the field still lacks a general proof comparable to Weinberg’s.

11 Future Directions

The gravitational effective field theory provides a new technology for quantum gravity. Most discussions of the topic of gravity and quantum mechanics do not keep track of the high vs low scales. This “effective” way of thinking can be very useful in deciding which aspects are trustworthy and which are speculative.

Within the effective theory, there are several possible directions for future work. It may be possible to compare the quantum predictions with the results of computer simulations of lattice gravity. It is too early to entirely give up on all hopes for real phenomenology. Perhaps quantum effects can build up and be visible as deviations from various null effects of the classical theory\[23\]. Perhaps these ideas may be useful in describing the very early universe. There are also potential theoretical applications, such as the effects of anomalies or the quantum influence on the development of singularities. Finally there exist in the literature various suggestions for unusual gravitational effects such as phase transitions, running \( G \) at low energy, solutions to the dark matter problem etc. Effective field theory should be able to support or refute these
suggestions. At the least, we will put our standard expectations on a stronger footing.

Nature has apparently given us the fields and interactions of the electroweak gauge theory, quantum chromodynamics and general relativity at present energies. All are treatable at those energies by the techniques of quantum mechanics. It remains a formidable challenge to construct the ultimate high energy structure of Nature. We expect it to be quite different from what we presently have. Because physics is an experimental science, it is possible that we will not be able to truly know the ultimate theory, but impressive attempts are underway. However we do have a right to expect that our theories are consistent at the energies that we use them. In this regard, the effective field theory framework is the appropriate description of quantum gravity at ordinary energies.

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