Evolution of newborn neutron stars: role of quark matter nucleation

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Abstract. A phase of strong interacting matter with deconfined quarks is expected in the core of massive neutron stars. We study the quark deconfinement phase transition in cold (T = 0) and hot β-stable hadronic matter. Assuming a first order phase transition, we calculate and compare the nucleation rate and the nucleation time due to thermal and quantum nucleation mechanisms. We show that above a threshold value of the central pressure a pure hadronic star (HS) is metastable to the conversion to a quark star (QS) (i.e. hybrid star or strange star). We introduce the concept of critical mass $M_{cr}$ for cold HSs and proto-hadronic stars (PHSs), and the concept of limiting conversion temperature for PHSs. We show that PHSs with a mass $M < M_{cr}$ could survive the early stages of their evolution without decaying to QSs. Finally, we discuss the possible evolutionary paths of proto-hadronic stars.

1. Introduction
Neutron stars, the compact remnants of supernova explosions, are the densest macroscopic objects in the Universe. They represent the limit beyond which gravity overwhelm all the other forces of nature and lead to the formation of a black hole. In fact, neutron star structure calculations (see e.g. Refs. [1, 2, 3, 4, 5]) based on a large variety of modern equations of state (EOS) of hadronic matter, predict a maximum stellar central density (the one for the maximum mass star configuration) in the range of 4 – 8 times the saturation density ($\sim 2.8 \times 10^{14}$ g/cm$^3$) of nuclear matter. Thus the core of a neutron star is one of the best candidates in the universe where a phase of strong interacting matter with deconfined quarks could be found, and these compact stars can be viewed as natural laboratories to test the low temperature $T$ and high baryon chemical potential region of the QCD phase diagram.

At high baryon chemical potential and low $T$, many QCD inspired models suggest the deconfinement transition to be a first order phase transition [6, 7]. As it is well known, first order phase transitions are triggered by the nucleation of a critical size drop of the new (stable) phase in a metastable mother phase. In the last few years, we have investigated [8]–[17] (see also Refs. [18, 19, 20, 21]) the astrophysical consequences of the nucleation process of quark matter (QM) in the core of massive pure hadronic compact stars (hadronic stars, HSs) i.e. neutron
stars in which no fraction of QM is present. In this contribution, we report some of the main findings of these studies.

2. Equation of state of dense matter

All the results we report in the present paper are relative to the finite temperature version of the following models for the EOS of dense matter. For the hadronic phase we use the Glendenning–Moszkowski model [22, 23], and particularly the GM1 parametrization [22, 23, 14]. The nucleon coupling constants are fitted to the bulk properties of nuclear matter. The inclusion of hyperons involves new couplings, which can be written in terms of the nucleonic ones: $g_{\sigma Y} = x_\sigma g_\sigma$, $g_{\omega Y} = x_\omega g_\omega$, $g_{\rho Y} = x_\rho g_\rho$. In this model [22, 23] it is assumed that all the hyperons in the baryonic octet have the same coupling and, in addition, it is assumed that $x_\rho = x_\sigma$. The binding energy of the $\Lambda$ particle in symmetric nuclear matter $B_{\Lambda}/A = -28$ MeV = $x_\omega g_\omega \omega_0 - x_\sigma g_\sigma$ in used [22, 23] to determine $x_\omega$ in terms of $x_\sigma$. Measured neutron star masses can be used to put additional restrictions to the possible ranges of variability of the hyperon couplings. In this work we will consider the cases $x_\sigma = 0.6$ (hereafter GM1$_{0.6}$) and $x_\sigma = 0.7$ (GM1$_{0.7}$). Notice that the case with $x_\sigma = 0.6$ produces stars with a larger hyperon population (for a given stellar gravitational mass) with respect to the case $x_\sigma = 0.7$ [23, 14].

For the deconfined quark phase we have used: (i) the MIT bag model EOS [24] with $m_s = 150$ MeV, $m_u = m_d = 0$, $\alpha_s = 0$ and different values for the bag constant $B$; (ii) the Nambu–Jona-Lasinio (NJL) model [25], with the lagrangian density given in Ref. [26]; (iii) the Chromo Dielectric model [27, 28].

3. Phase equilibrium

For a first-order phase transition the conditions for phase equilibrium are given by the Gibbs’ phase rule

$$T_H = T_Q \equiv T,$$

$$P_H = P_Q \equiv P_0,$$

$$\mu_H(T, P_0) = \mu_Q(T, P_0) \quad (1)$$

where

$$\mu_H = \frac{\varepsilon_H + P_H - s_H T}{n_H},$$

$$\mu_Q = \frac{\varepsilon_Q + P_Q - s_Q T}{n_Q} \quad (2)$$

are the Gibbs’ energies per baryon (average chemical potentials) for the hadron and quark phase respectively, $\varepsilon_H$ ($\varepsilon_Q$), $P_H$ ($P_Q$), $s_H$ ($s_Q$) and $n_H$ ($n_Q$) denote respectively the total (i.e., including leptonic contributions) energy density, total pressure, total entropy density, and baryon number density for the hadron (quark) phase.

Above the ”transition point” $P_0$ the hadronic phase is metastable, and the stable quark phase will appear as a results of a nucleation process. Virtual drops of the stable quark phase will arise from localized fluctuations in the state variables of the metastable hadronic phase. These fluctuations are characterized by a time scale $\nu_0^{-1} \sim 10^{-23}$ s. This time scale is set by the strong interactions (which are responsible for the deconfinement phase transition), and it is many orders of magnitude shorter than the typical time scale for the weak interactions. Therefore quark flavor must be conserved forming a virtual drop of QM. We will refer to this form of deconfined matter, in which the flavor content is equal to that of the $\beta$-stable hadronic system at the same pressure and temperature, as the Q*-phase. Soon afterward a critical size drop of Q*-matter is formed, the weak interactions will have enough time to act, changing the quark flavor fraction of the deconfined droplet to lower its energy, and a droplet of $\beta$-stable QM is formed (hereafter the Q-phase). This first seed of $\beta$-stable QM will trigger the conversion [29] of the pure hadronic star to a quark star (QS) (i.e. to a hybrid neutron star or to a strange star depending on the details of the EOS for quark matter used to model the phase transition).

The direct formation by fluctuations of a drop of $\beta$-stable QM is also possible in principle. However, it is strongly suppressed with respect to the formation of the Q*-phase drop by a factor
being $N$ the number of particles in the critical size quark drop. This is so because the formation of a $\beta$-stable drop will imply the almost simultaneous conversion of $\sim N/3$ up and down quarks into strange quarks. For a critical size $\beta$-stable drop at the center of a neutron star it is found $N \sim 100 - 1000$, and therefore the suppression factor is actually very tiny.

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{fig1.png}
\caption{Gibbs energy per baryon of $\beta$-stable hadronic phase and $Q^*$-phase, in neutrino-free matter, as a function of pressure, at different temperatures. Lines with the steeper slope refer to the hadronic phase. Full dots indicate the transition pressure $P_0$ for each temperature. GM1 EOS with $\sigma = 0.6$ for the hadronic phase and MIT bag model EOS with $B = 85$ MeV/fm$^3$ for the $Q^*$-phase (GM1$_{0.6}$–B85 EOS).}
\end{figure}

In Fig. 1 we plot the Gibbs’ energies per baryon for the $\beta$-stable hadronic phase and for the $Q^*$-phase in neutrino-free matter, at different temperatures (T = 0, 10, 20, 30 MeV). Results in Fig. 1 are obtained using the GM1 model with $\sigma = 0.6$ for the hadronic phase and the MIT Bag model with $B = 85$ MeV/fm$^3$ for the quark phase (hereafter the GM1$_{0.6}$–B85 EOS). Lines with the steeper slope refer to the hadron phase. As we see, the transition pressure $P_0$ (indicated by a full dot) decreases when the hadronic matter temperature is increased.

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{fig2.png}
\caption{Phase equilibrium curve between the $\beta$-stable hadronic phase and the $Q^*$ phase. The continuous curve is relative to neutrino-free matter, the dashed curve to matter with trapped neutrinos. EOS: GM1 with $\sigma = 0.6$ plus MIT Bag model with $B = 85$ MeV/fm$^3$.}
\end{figure}

The phase equilibrium curve $P_0(T)$ between the $\beta$-stable hadronic phase and the $Q^*$-phase is shown in Fig. 2 for neutrino-free matter and matter with trapped neutrinos, making use of the GM1$_{0.6}$–B85 EOS. The region of the $P_0$–T plane above each curve represents the deconfined $Q^*$-phase. As expected [30, 31, 11, 32] neutrino trapping in $\beta$-stable hadronic matter inhibits the quark deconfinement phase transition, thus the global effect of neutrino-trapping is to produce a shift of the phase equilibrium curve toward higher values of the pressure in the $P_0$–T plane.

In Fig. 3, we show the phase equilibrium curve, for neutrino-free matter, in the case of the NJL model (left panel) or the Chromo Dielectric model (right panel) to describe the deconfined phase. For the hadron phase we take the GM1 model with $\sigma = 0.7$ (GM1$_{0.7}$) in both cases. Notice that in the case of the NJL model the transition pressure $P_0(T)$ is substantially higher than the one in the case of the MIT Bag or Chromo Dielectric models to describe the deconfined phase. As discussed in detail in Ref. [17], this behaviour can be traced back the large value of the strange quark effective mass in the NJL model.

As is well known, for a first-order phase transition the derivative $dP_0/dT$ is related to the
Figure 3. Phase equilibrium curve between the β-stable hadronic phase and the Q*-phase. The EOS for the hadronic phase is obtained using the GM1 model with $x_\sigma = 0.7$, the EOS for the Q*-phase using the NJL model (left panel) or the Chromo Dielectric model (right panel).

specific latent heat $Q$ of the phase transition by the Clapeyron-Clausius equation

$$\frac{dP_0}{dT} = -\frac{n_H n_{Q^*} Q}{n_{Q^*} - n_H T}$$

$$Q = \tilde{W}_{Q^*} - \tilde{W}_H = T(\tilde{S}_{Q^*} - \tilde{S}_H)$$

where $\tilde{W}_H$ ($\tilde{W}_{Q^*}$) and $\tilde{S}_H$ ($\tilde{S}_{Q^*}$) denote the enthalpy per baryon and entropy per baryon for the hadron (quark) phase, respectively. The specific latent heat $Q$ and the phase numbers densities $n_H$ and $n_{Q^*}$ at phase equilibrium are reported in Tables 1 and 2 in the case of the GM1$_{0.6}$-B85 equation of state. Results in Table 1 refer to neutrino-free matter, whereas those in Table 2 refer to matter with trapped neutrinos. As expected for a first-order phase transition, one has a discontinuity jump in the phase number densities: in our particular case $n_{Q^*}(T, P_0) > n_H(T, P_0)$. This result, together with the positive value of $Q$ (i.e. the deconfinement phase transition absorbs heat), tells us (see Eq. (3)) that the phase transition temperature decreases with pressure (as in the melting of ice).

The effect of neutrino trapping on the phase equilibrium properties of the system can be seen comparing the results reported in Tables 1 and 2. As we see, the phase number densities $n_H$ and $n_{Q^*}$ at phase equilibrium are shifted to higher values, and the specific latent heat $Q$ is increased with respect to the neutrino-free matter case.

4. Quark matter nucleation in cold hadronic stars

In a cold ($T = 0$) and neutrino-free pure hadronic star the formation of the first drop of QM could take place solely via a quantum nucleation process. The basic quantity needed to calculate the nucleation time is the energy barrier separating the Q*-phase from the metastable hadronic phase. This energy barrier, which represents the difference in the free energy of the system with and without a Q*-matter droplet, can be written as [33, 34]

$$U(R) = \frac{4}{3}\pi n_{Q^*}(\mu_{Q^*} - \mu_H)R^3 + 4\pi\sigma R^2$$

where $R$ is the radius of the droplet (supposed to be spherical), and $\sigma$ is the surface tension for the surface separating the hadron from the Q*-phase. The energy barrier has a maximum at the critical radius $R_c = 2\sigma/[n_{Q^*}(\mu_H - \mu_{Q^*})]$. We neglected the term associated with the
Table 1. The specific (i.e. per baryon) latent heat $Q$ and the phase number densities $n_H$ and $n_{Q^*}$ at phase equilibrium. GM1 EOS with $x_\sigma = 0.6$ for the hadronic phase, MIT bag model with $B = 85$ MeV/fm$^3$ for the quark phase. Results for neutrino-free matter.

| $T$ (MeV) | $Q$ (MeV) | $n_{Q^*}$ (fm$^{-3}$) | $n_H$ (fm$^{-3}$) | $P_0$ (MeV/fm$^3$) |
|----------|----------|------------------------|-------------------|---------------------|
| 0        | 0.00     | 0.453                  | 0.366             | 39.95               |
| 5        | 0.56     | 0.451                  | 0.364             | 39.74               |
| 10       | 2.40     | 0.447                  | 0.358             | 38.58               |
| 15       | 5.71     | 0.439                  | 0.348             | 36.55               |
| 20       | 10.60    | 0.428                  | 0.334             | 33.77               |
| 25       | 17.17    | 0.414                  | 0.316             | 30.36               |
| 30       | 25.44    | 0.398                  | 0.294             | 26.53               |

Table 2. Same as Table 1, but with trapped neutrinos.

| $T$ (MeV) | $Q$ (MeV) | $n_{Q^*}$ (fm$^{-3}$) | $n_H$ (fm$^{-3}$) | $P_0$ (MeV/fm$^3$) |
|----------|----------|------------------------|-------------------|---------------------|
| 0        | 0.00     | 0.603                  | 0.516             | 113.77              |
| 5        | 0.65     | 0.601                  | 0.514             | 113.11              |
| 10       | 2.87     | 0.594                  | 0.509             | 110.69              |
| 15       | 6.78     | 0.580                  | 0.499             | 106.17              |
| 20       | 12.65    | 0.560                  | 0.483             | 99.18               |
| 25       | 20.21    | 0.534                  | 0.462             | 90.12               |
| 30       | 29.88    | 0.502                  | 0.434             | 78.65               |

curvature energy and also the terms connected with the electrostatic energy, since they are known to only introduce small corrections [34, 10]. The value of the surface tension $\sigma$ for the interface separating the quark and hadron phase is poorly known, and typically values used in the literature range within $10^{-50}$ MeV fm$^{-2}$ [35]). We assume $\sigma$ to be temperature independent and we take $\sigma = 30$ MeV fm$^{-2}$.

The quantum nucleation time $\tau_q$ can be straightforwardly evaluated within a semi-classical approach [33, 34, 9] and it can be expressed as

$$\tau_q = (\nu_0 p_0 N_c)^{-1},$$

where $p_0$ is the probability of tunneling the energy barrier $U(R)$ in its ground state, $\nu_0$ is the oscillation frequency of a virtual drop of the Q*-phase in the potential well, and $N_c \sim 10^{48}$ is the number of nucleation centers expected in the innermost part ($r \leq R_{\text{nuc}} \sim 100$ m) of the HS, where the pressure and temperature (when we will consider the finite T case) can be considered constant and equal to their central values.

Thus a pure hadronic star, having a central pressure $P_c$ larger than the static transition pressure $P_0$ for the formation of the Q*-phase, is metastable to the “decay” (conversion) to a quark star (QS) i.e. to a more compact stellar configuration in which deconfined quark matter is present. These metastable HSs have a mean-life time which is related to the nucleation time to
Figure 4. Mass-radius relation for a pure hadronic stars (HS) and hybrid star configurations (QS). The configuration marked with an asterisk represents the HS for which $\tau_q = \infty$ (i.e. $P_c = P_0$). The conversion process of the HS, with a gravitational mass equal to $M_{cr}$, into the final QS is denoted by the full circles connected by a dashed line. Results are relative to the GM1 model with $x_\sigma = 0.6$ for the hadronic phase and the MIT Bag model EOS with $B = 85$ MeV/fm$^3$ for the quark phase. The surface tension is $\sigma = 30$ MeV/fm$^2$. Stellar masses are in unit of the mass of the sun, $M_{\text{sun}} = 1.989 \times 10^{33}$ g.

form the first critical-size drop of deconfined matter in their interior (the actual mean-life time of the HS will depend on the mass accretion or on the spin-down rate which modifies the nucleation time via an explicit time dependence of the stellar central pressure). Following Refs. [8, 9, 10] we define as critical mass $M_{cr}$ of the metastable HSs, the value of the gravitational mass for which the nucleation time is equal to one year: $M_{cr} \equiv M_{HS}(\tau_q = 1\text{yr})$. Pure hadronic stars with $M_{HS} > M_{cr}$ are very unlikely to be observed. Thus $M_{cr}$ plays the role of an effective maximum mass for the hadronic branch of compact stars. Notice that the Oppenheimer–Volkov maximum mass $M_{max}^{HS}$ is determined by the overall stiffness of the EOS for hadronic matter, whereas the value of $M_{cr}$ will depend in addition on the bulk properties of the EOS for quark matter and on the properties at the interface between the confined and deconfined phases of matter (e.g., the surface tension $\sigma$).

These findings are exemplified in Fig. 4, where we show the mass-radius (MR) curve for hadronic stars (HS) and that for quark stars (QS). The configuration marked with an asterisk on the hadronic MR curve represents the HS for which the central pressure is equal to $P_0$ and thus $\tau_q = \infty$. The full circle on the HS sequence represents the critical mass configuration $M_{cr}$, in the case $\sigma = 30$ MeV/fm$^2$. The full circle on the QS mass-radius curve represents the hybrid star which is formed from the conversion of the hadronic star with $M_{HS} = M_{cr}$. We assume [29] that during the stellar conversion process the total number of baryons in the star is conserved. The stellar conversion process, described so far, will start to populate the new branch of quark stars (the part of the QS sequence above the full circle). Long term accretion on the QS can next produce stars with masses up to the limiting mass $M_{max}^{QS}$ for the quark star configurations.

5. Quark matter nucleation in proto-hadronic stars

A neutron star at birth (proto-neutron star) is very hot ($T = 10 – 30$ MeV) with neutrinos being still trapped in the stellar interior [36, 37, 30, 38, 39, 40]. Subsequent neutrino diffusion causes deleptonization and heats the stellar matter to an approximately uniform entropy per baryon $S = 1 – 2$ (in units of the Boltzmann’s constant $k_B$). Depending on the stellar composition,
Figure 5. Energy barrier for a virtual drop of the $Q^*$-phase in $\beta$-stable neutrino-free hadronic matter as a function of the droplet radius and for different temperatures for a fixed pressure $P = 57$ MeV/fm$^3$. Results are relative to the GM1.6–B85 equation of state and surface tension $\sigma = 30$ MeV/fm$^2$.

during this stage neutrino escape can lead the more “massive” stellar configurations to the formation of a black hole [41, 30]. However, if the mass of the star is sufficiently small, the star will remain stable and it will cool to temperatures well below 1 MeV within a cooling time $t_{cool} \sim$ a few $10^2$ s, as the neutrinos continue to carry energy away from the stellar material [36, 30, 38]. Thus in a proto-neutron star, the quark deconfinement phase transition will be likely triggered by a thermal nucleation process [42, 43, 44, 45]. In fact, for sufficiently high temperatures, thermal nucleation is a much more efficient process with respect to the quantum nucleation mechanism. In the following we will establish the physical conditions under which a newborn hadronic star (proto-hadronic star, PHS) could survive the early stages of its evolution without “decaying” to a quark star.

According to the Langer theory [46, 47] of homogeneous nucleation the thermal nucleation rate can be written [47] as

$$I = \frac{\kappa}{2\pi} \Omega_0 \exp\left(-U(R_c, T)/T\right)$$

where $\kappa$ is the so-called dynamical prefactor, which is related to the growth rate of the drop radius $R$ near the critical radius ($R_c$), and $\Omega_0$ is the so-called statistical prefactor, which measures the phase-space volume of the saddle-point region around $R_c$. We have used [15, 16] for $\kappa$ and $\Omega_0$ the expressions derived in Refs. [48, 49], where the Langer nucleation theory has been extended to the case of first order phase transitions occurring in relativistic systems, as in the case of the quark deconfinement transition. The dominant factor in the nucleation rate (7) is the exponential, in which $U(R_c, T)$ is the activation energy, $i.e.$ the change in the free energy of the system required to activate the formation of a critical size droplet. This quantity has been calculated [15, 16] using the generalization of Eq. (5) to the case of $T \neq 0$ (thus using finite temperature EOS for the hadronic and the quark phases), and assuming a temperature independent surface tension.

The thermal nucleation time $\tau_{th}$, relative to the innermost stellar region ($V_{nuc} = (4\pi/3)R_{nuc}^3$, with $R_{nuc} \sim 100$ m) where almost constant pressure and temperature occur, can thus be written as

$$\tau_{th} = (V_{nuc} I)^{-1}$$

In Fig. 5, we represent the energy barrier for a virtual drop of the $Q^*$-phase in the neutrino-free hadronic phase as a function of the droplet radius and for different temperatures at a fixed
Figure 6. Thermal ($\tau_{th}$) and quantum ($\tau_q$) nucleation time of quark matter (Q*-phase) in $\beta$-stable neutrino-free hadronic matter as a function of temperature at fixed pressure $P = 57$ MeV/fm$^3$. The crossover temperature is $T_{co} = 7.05$ MeV. The limiting conversion temperature for the proto-hadronic star is, in this case, $\Theta = 10.3$ MeV, obtained from the intersection of the thermal nucleation time curve (continuous line) and the dot-dashed line representing $\log_{10}(\tau/s) = 3$. The surface tension is $\sigma = 30$ MeV/fm$^2$. Results are relative to GM1.06–B85 EOS.

Figure 7. The limiting conversion temperature $\Theta$ for a newborn hadronic star as a function of the central stellar pressure. Newborn hadronic stars with a central temperature and pressure located on the right side of the curve $\Theta(P)$ will nucleate a Q*-matter drop during the early stages of their evolution, and will finally evolve to cold and deleptonized quark stars, or will collapse to black holes. The lines labeled $T_S$ represent the stellar matter temperature as a function of pressure at fixed entropies per baryon $\bar{S}/k_B = 1$ (dashed line) and 2 (solid line). Results are relative to GM1.06–B85 EOS.
pressure \( P = 57 \text{ MeV/fm}^3 \). Results in Fig. 5 are obtained using the GM1\(_{0.6}\)-B85 equation of state. As expected, from the results plotted in Fig. 1, the energy barrier \( U(R, T) \) and the droplet critical radius \( R_c \) decrease as the matter temperature is increased. This effect favors the Q\(^*\)-phase formation and, in particular, increases (decreases) the quantum nucleation rate (nucleation time \( \tau_q \)) with respect to the corresponding quantities calculated at \( T = 0 \).

In Fig. 6, we plot the quantum and thermal nucleation times of the Q\(^*\)-phase in \( \beta \)-stable neutrino-free hadronic matter as a function of temperature and at a fixed pressure \( P = 57 \text{ MeV/fm}^3 \). As expected, we find a crossover temperature \( T_{co} \) above which thermal nucleation is dominant with respect to the quantum nucleation mechanism. For the case reported in Fig. 6, we have \( T_{co} = 7.05 \text{ MeV} \) and the corresponding nucleation time is \( \log_{10}(\tau/s) = 54.4 \).

Having in mind the physical conditions in the interior of a PHS [36, 30], to establish if this star will survive the early stages of its evolution without decaying to a quark star, one has to compare the quark matter nucleation time \( \tau = \min(\tau_q, \tau_{th}) \) with the cooling time \( t_{cool} \sim \text{a few } 10^2 \text{ s} \). If \( \tau >> t_{cool} \) then quark matter nucleation will not likely occur in the newly formed star, and this star will evolve to a cold deleptonized configuration. We thus introduce the concept of limiting conversion temperature \( \Theta \) for the proto-hadronic star and define it as the value of the stellar central temperature \( T_c \) for which the Q\(^*\)-matter nucleation time is equal to \( 10^3 \text{ s} \). The limiting conversion temperature \( \Theta \) will clearly depend on the value of the stellar central pressure (and thus on the value of the stellar mass).

The limiting conversion temperature \( \Theta \) is plotted in Fig. 7 as a function of the stellar central pressure. A proto-hadronic star with a central temperature \( T_c > \Theta \) will likely nucleate a Q\(^*\)-matter drop during the early stages of its evolution, and will finally evolve to a cold and deleptonized quark star, or will collapse to a black hole (depending on the value of the stellar baryonic mass \( M_B \) and on the EOS).

In Fig. 8 we plot the limiting conversion temperature \( \Theta \) for a newborn HS for different values of the bag constant. The increase in \( B \) produces a growth of the region of the P-T plane where the proto-hadronic star could survive Q\(^*\) nucleation and thus evolve to a cold hadronic star.

For an isentropic stellar core [36, 30], the central temperature of the proto-hadronic star is given, for the GM1\(_{0.6}\) EOS model, by the lines labeled by \( T_S \) in Fig. 7, relative to the case \( \tilde{S} = 1 \ k_B \) (dashed curve) and \( \tilde{S} = 2 \ k_B \) (continuous curve). The intersection point \( (P_S, \Theta_S) \) between the two curves \( \Theta(P) \) and \( T_S(P) \) thus gives the central pressure and temperature of the configuration that we denote as the critical mass configuration of the proto-hadronic stellar sequence. Taking \( \sigma = 30 \text{ MeV/fm}^2 \) and \( \tilde{S} = 2 \ k_B \) we get \( M_{cr} = 1.390 \ M_{sun} \) for the gravitational critical mass and \( M_{B,cr} = 1.492 \ M_{sun} \) for the baryonic critical mass (being \( M_{sun} = 1.989 \times 10^{33} \text{ g} \)
The evolution of a PHS within our scenario is delineated in Fig. 9, where we plot the appropriate stellar equilibrium sequences in the gravitational–baryonic mass plane for the GM1_{0.6}–B85 EOS and $\sigma = 30$ MeV/fm$^2$. The upper (red) line represents the PHS sequence, i.e. isoentropic HSs ($\tilde{S} = 2 \ k_B$) and neutrino-free matter. The middle (blue) line represents the cold HS sequence. The asterisk and the full circle on these lines represent respectively the stellar configuration with $\tau = \infty$ and the critical mass configuration $M_{cr}$. The lower (green) line represent the cold QS sequence. We assume $M_B = \text{const}$ during these stages of the stellar evolution. Thus according to the results in Fig. 9, proto-hadronic stars with a baryonic mass $M_B < 1.492 \ M_{\odot}$ will survive Q*-matter early nucleation (i.e. nucleation within the cooling time $t_{\text{cool}} \sim \text{a few } 10^2 \text{ s}$) and in the end they will form stable ($\tau = \infty$) cold hadronic stars. Proto-hadronic stars with $1.492 \ M_{\odot} \leq M_B < 1.813 \ M_{\odot}$ (the maximum baryonic mass $M_{B,QS}^{\text{max}}$ of the cold QS sequence for the present EOS) will experience early nucleation of a Q*-matter drop and will ultimately form a cold deleptonized quark star. The last possibility is for PHSs having $M_B > 1.813 \ M_{\odot}$. In this case the early nucleation of a Q*-matter drop will trigger a stellar conversion process to a cold QS configuration with $M_B > M_{B,QS}^{\text{max}}$, thus these PHSs will finally forms black holes.

The outcomes of this scenario are not altered by neutrino trapping effects in hot $\beta$-stable hadronic matter. In fact, for proto-hadronic stars with trapped neutrinos (νPHSs) (with a lepton fraction $Y_L = 0.4$ and $\tilde{S} = 2 \ k_B$) we find [16] (for the same EOS and surface tension $\sigma$ used in Fig. 9) a critical baryonic mass $M_{\text{cr}} = 1.96 \ M_{\odot}$. Thus νPHSs with $M_B \geq M_{B,\text{cr}}$ after neutrino escape and cooling will finally evolve to black holes. The fate of a νPHS with $M_B < M_{B,\text{cr}}$ is the same as the corresponding neutrino-free PHS with equal baryonic mass.

In Fig. 10 we plot the PHS, cold HS, and cold QS sequences in the gravitational–baryonic mass plane for the case of the NJL model for the quark phase and the GM1 model in the case of pure nucleonic matter (right panel) or hyperonic matter with $\sigma = 0.7$ (left panel). It is clearly...
seen that in the case of the NJL model it is almost impossible to populate the QS branch. Cold quark stars can be formed in the case of \( x_\sigma = 0.7 \) (left panel) for a very narrow range of baryonic stellar masses \( 2.20 < M_B/M_\odot < 2.23 M_\odot \).

6. Conclusions

In summary, in this work we have studied the quark deconfinement phase transition in hot \( \beta \)-stable hadronic matter and explored some of its consequences for the physics of neutron stars at birth. We calculated and compared the nucleation time due to thermal and quantum nucleation mechanisms, and computed the crossover temperature above which thermal nucleation dominates the finite temperature quantum nucleation mechanism. In addition, we introduced the new concept of limiting conversion temperature \( \Theta \) for proto-hadronic stars and extended the concept of critical mass (introduced in Refs. [8, 9, 10]) to the case of finite temperature hadronic stars.

Our main finding is that proto-hadronic stars with a gravitational mass lower than the critical mass \( M_{cr} \) could survive the early stages of their evolution without decaying to a quark star. This outcome contrasts with the predictions of earlier studies [42, 43, 44, 45] where it was inferred that all the pure hadronic compact stars, with a central temperature above 2 – 3 MeV, are converted to quark stars within the first seconds after their birth. However, the prompt formation of a critical size drop of quark matter could take place when \( M > M_{cr} \). These proto-hadronic stars evolve to cold and deleptonized quark stars or collapse to a black holes.

We found that for the NJL model the presence of hyperons disfavour the phase transition pushing the transition point to very high densities. In addition, in the case of the NJL model it is almost impossible to populate the quark star branch and quark matter nucleation will lead to the formation of a black hole. Thus within the NJL model for the quark phase, all compact stars are pure hadronic stars.

Accurate measurements of both the mass and radius of a few individual “neutron stars” [50, 51] or measurements of temperature profiles of accretion discs around rapidly spinning compact stars [52] could shed light on the validity of the scenario discussed in the present work.
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