Ferromagnetic Semiconductor - Singlet (or Triplet) Superconductor - Ferromagnetic Semiconductor Systems as Possible Logic Circuits and Switches

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We consider thin superconducting (S) films of thickness $d \ll \xi_0$, sandwiched between two ferromagnetic semiconducting insulators (FI) with differently orientated magnetizations - the $FI - S - FI$ system. We calculate the dependence of the superconducting critical temperature on the orientation of the magnetization in the insulators and on the thickness of the superconducting film. The calculations are done for singlet as well as triplet superconductors. In the singlet case $T_c$ depends on the relative orientation of the left and right magnetization only, while in the triplet case $T_c$ depends on the absolute orientation of magnetization. The latter property can serve as a kind of spin-spectroscopy of triplet and unconventional superconductors, for instance in resolving the structure of the triplet order parameter in the recently discovered layered superconductor $Sr_2RuO_4$. The possibility of logic circuits and switches, which are based on the $FI - S - FI$ systems with arbitrary orientation of magnetizations in $FI$ films, is analyzed too.
I. INTRODUCTION

The physics of magnetic superconductors is especially interesting because of the competition between magnetic order and superconductivity. Important progress in the field started with the discovery of the magnetic superconductors \(RERh_4B_4\), \(REMo_6S_8\), \(REMo_6Se_8\), etc. where the rare earth ions, \(RE\), are regularly distributed in the crystal lattice. After intensive experimental [1] and theoretical [2] investigation it turned out that in many of these systems superconductivity (with the critical temperature \(T_c\)) coexists with antiferromagnetic order (with the critical temperature \(T_N\)). One usually has \(T_N < T_c\). However, due to their antagonistic characters singlet superconductivity and simple ferromagnetic order cannot coexist in bulk samples with realistic physical parameters as it was shown theoretically [2], [3], [4]. In fact, under certain physical conditions ferromagnetic order is transformed into a spiral or domain-like structure in the presence of superconductivity, as it was observed in \(ErRh_4B_4\), \(HoMo_6S_8\), \(HoMo_6Se_8\) [1], [2]. At the same time the superconducting order parameter is suppressed even in the presence of the modified ferromagnetism in such a way that they may coexist in some limited temperature interval (in \(ErRh_4B_4\), \(HoMo_6S_8\)) or even down to \(T = 0K\) (in \(HoMo_6Se_8\)) depending on system parameters. For more details see Refs. [2], [3], [4]. Recently, the coexistence of superconductivity and nuclear magnetism was found experimentally in \(AuIn_2\) [5] and explained theoretically [6] in terms of a spiral or a domain-like structure. There is also evidence for the coexistence of ferromagnetism, which appears at \(T_M = 137 K\), and superconductivity, which sets in at \(T_c = 45 K\), in the layered perovskite superconductor \(RuSr_2GdCu_2O_8\) [7], [8]. The specificity of this material is that the ferromagnetic (\(F\)) order is present in Ru-O planes while superconducting (\(S\)) pairing dominates in Cu-O planes.

Recently, various artificial multilayer \(SF\) systems were prepared in the form of bilayer or trilayer systems or superlattices consisting of ferromagnetic and superconducting films deposited on each other. In all of these artificial structures, for instance in \(Nb/Gd\) [9], [10] and \(Nb/CuMn\) multilayers [11], in \(Fe/Nb/Fe\) trilayers [12], [13], in \(Nb/Gd/Nb\) trilayers [14], superconductivity and ferromagnetism are spatially separated. In spite of this fact \(T_c\) is suppressed in these systems due to the combined paramagnetic and proximity effect in the normal metal. The theoretically predicted oscillation of \(T_c\) as a function of the thickness of the ferromagnetic film in \(SFS\) multilayers was observed in \(V/Fe\) [15] and in \(Nb/Gd\) [14], [20] superlattices, while in some artificial structures of \(V/Fe\) [21] and \(Nb/Fe\) [22] this effect was absent. The oscillation of \(T_c\) is due to the \(\pi\)-Josephson junction in the \(SFS\) system [15], [16]. Recently, several theoretical papers were published which study transport properties of \(FM - S\) systems, where \(FM\) is a ferromagnetic metal [17]. The critical temperature of \(FM - S - FM\) systems was studied recently in Ref. [18], where the (singlet) superconducting film and ferromagnetic metals are assumed to be dirty, but with (anti)parallel orientation of magnetization only.

The theoretical works above study singlet superconductors in contact with ferromagnetic metals where the superconducting proximity effect dominates. Here, we consider \(FI - S - FI\) sandwiches where \(FI\) stands for a ferromagnetic insulator (semiconductor). In such systems conduction electrons penetrate the magnetic layers on much smaller distances than in the case of metals and are totally reflected at the \(FI - S\) boundary. Additionally, the boundaries in \(FI - S - FI\) systems are magnetically active and rotate spins
of reflected electrons. The $FI-S-FI$ systems, with $FI$ a ferromagnetic insulator, might be of practical interest. In contrast to ferromagnetic metals, where the proximity effect is pronounced, this effect is drastically reduced in ferromagnetic insulators (semiconductors) and the physics depends on fewer parameters. Note, the ferromagnetic semiconductors are already realized in systems like EuO, EuS with the Curie temperature $T_F = 66.8 \, K$ and 16.3 $K$, respectively. In the magnetic semiconductor EuSe, which shows metamagnetic behavior, the antiferromagnetic and ferromagnetic order are realized at $T_{AF} = 4.6 \, K$ and $T_F = 2.8 \, K$.

In this paper the critical temperature of the $FI-S-FI$ sandwich with singlet and triplet superconductors is calculated for various orientations of the magnetization in the $FI$ films. It turns out that the effect of magnetically active boundaries on the superconducting critical temperature $T_c$ strongly depends on the orientation of magnetization. For the parallel orientation, for instance, $T_c$ is reduced much more than for the perpendicular one. This property opens the possibility of switches and logic operations based on $FI-S-FI$ systems, which is also briefly discussed here.

II. THEORY OF $FI-S-FI$ SYSTEMS

In the following we study systems consisting of a singlet or a triplet superconductor and a ferromagnetic insulator as shown in Fig.1.

**FIGURE 1.** Thin film ($d \ll \xi_0$) geometry of a $FI-S-FI$ system. A superconducting film (S) is sandwiched between two ferromagnetic insulators ($F_l,F_r$). The magnetization, $\mu_{l,r}$, is parallel to the $xy$-plane. The subscripts label the left (l) and the right (r) side of the sandwich. $\hat{p}_{in}$ and $\hat{p}_{out}$ refer to incoming and specular reflected (outgoing) trajectories, respectively.
At the boundaries superconducting electrons penetrate the ferromagnetic insulator only on short distances and are totally reflected. During this short penetration time the electron spin is rotated by the exchange field, $h_{ex}$, of the ferromagnetic insulator. The $FI-S$ boundary can be described by appropriate boundary conditions (see Eq. (3) below) for the quasiparticle Green’s function $\hat{g}(\hat{p}_F, \mathbf{R}, \omega_n)$, which were introduced in Ref. [23] and applied to a $FI-S-I$ ($I$ is a nonmagnetic insulator) sandwich in Ref. [24].

In this paper it is assumed that the exchange fields, $h_{ex,l}$ and $h_{ex,r}$, in the ferromagnetic insulator as well as the corresponding magnetization, characterized by unit vectors $\mu_l$ and $\mu_r$ ($h_{ex,l,r} \parallel \mu_{l,r}$), are parallel to the boundaries, but otherwise have arbitrary orientation. We describe the superconducting film by the quasiclassical equations of Eilenberger [25] and Larkin-Ovchinnikov [26], generalized to problems where spins of Cooper pairs are affected by magnetic perturbations [3]. In the presence of magnetic perturbations acting on quasiparticle spins, the quasiclassical Green’s function is a $4 \times 4$ matrix, $\hat{g}(\hat{p}, \mathbf{R}, \omega_n)$, in the spin⊗particle-hole product space

$$\hat{g}(\hat{p}, \mathbf{R}, \omega_n) = \left( \begin{array}{cc} \tilde{g}(\hat{p}, \mathbf{R}, \omega_n) & \tilde{f}(\hat{p}, \mathbf{R}, \omega_n) \\ \tilde{f}(-\hat{p}, \mathbf{R}, \omega_n)^* & \tilde{g}^{\text{tr}}(-\hat{p}, \mathbf{R}, -\omega_n) \end{array} \right),$$

where the normal and anomalous Green’s functions, $\tilde{g}(\hat{p}, \mathbf{R}, \omega_n)$ and $\tilde{f}(\hat{p}, \mathbf{R}, \omega_n)$, are $2 \times 2$ matrices in spin space (see Ref. [27])

$$\tilde{g} = \hat{g}_s + \hat{g}_t \sigma_i$$

$$\tilde{f} = (f_s + f_t \sigma_i) i \sigma_2.$$  (2)

Here, the subscripts $s$ and $t$ correspond to singlet and triplet superconductivity, respectively. $\sigma_i$ are Pauli matrices in spin space, $\hat{p} = \mathbf{p}_F/p_F$ and $\mathbf{p}_F$ is the Fermi momentum. In the following, we assume the weak coupling limit for a clean superconductor. The equation for $\hat{g}$ then reads

$$[i\omega_n \tau_3 - \hat{\Delta}(\hat{p}, \mathbf{R}, \omega_n), \hat{g}(\hat{p}, \mathbf{R}, \omega_n)]$$

$$+ i V_F \nabla \hat{g}(\hat{p}, \mathbf{R}, \omega_n) = 0,$$

where $\tau_i = 1 \otimes \tau_i$ and $\tau_i$, $i = 1, 2, 3$, are Pauli matrices in particle-hole space. For later purposes we also define $\hat{\sigma}_i = 1 \otimes \sigma_i$ for $i = 1, 2, 3$. The normalization condition for $\hat{g}$ is given by

$$\hat{g}^2(\hat{p}, \mathbf{R}, \omega_n) = -\hat{1},$$  (4)

while the order parameter $\hat{\Delta}(\hat{p}, \mathbf{R})$ is the solution of the self-consistency equation

$$\hat{\Delta}(\hat{p}, \mathbf{R}) = N(0)T \sum_n \int \frac{dQ'}{4\pi} V(\hat{p}, \hat{p}') \tilde{f}(\hat{p}', \mathbf{R}, \omega_n).$$  (5)

Here, $V(\hat{p}, \hat{p}')$ is the pairing potential, $\tilde{f}(\hat{p}, \mathbf{R}, \omega_n)$ is the off-diagonal part of $\hat{g}(\hat{p}, \mathbf{R}, \omega_n)$ and $\Omega$ characterizes integration over the Fermi surface. The structure of the $4 \times 4$ matrix $\hat{\Delta}$, which is different for singlet and triplet pairing, will be given below.
To the equations above one needs to add boundary conditions for \( \hat{g}(\hat{p}, \mathbf{R}, \omega_n) \) on the left and on the right \( FI - S \) surface \( \mathbf{R}^{l,r}_s \), respectively. These relate incoming, \( \hat{p}_{in} \), and outgoing, \( \hat{p}_{out} \), quasiparticles [24]

\[
\hat{g}(\hat{p}_{in}, \mathbf{R}^{l,r}_s, \omega_n) = \hat{S}_{l,r}\hat{g}(\hat{p}_{out}, \mathbf{R}^{l,r}_s, \omega_n)\hat{S}_{l,r}^{-1}.
\]

(6)

In the following we assume totally (specular) reflecting boundaries. \( \hat{p}_{in} \) and \( \hat{p}_{out} \) are then related by

\[
\hat{p}_{out} = \hat{p}_{in} - 2\hat{z}(\hat{p}_{in}\hat{z}z).
\]

(7)

The unit vector \( \hat{z} \) is normal to the \( FI - S \) surface - see Fig.1. The boundary scattering (rotation) matrix \( \hat{S} \equiv \hat{S}(\hat{p}, \mathbf{\mu}) \), which characterizes the magnetically active \( FI - S \) surface, has the form [23], [24]

\[
\hat{S} = \begin{pmatrix}
\tilde{S}(\hat{p}, \mathbf{\mu}) & 0 \\
0 & \tilde{S}^*(-\hat{p}, \mathbf{-\mu})
\end{pmatrix}
= \begin{pmatrix}
e^{-i\hat{\Delta}}(\mathbf{\mu}\sigma_{l,r}) & 0 \\
0 & e^{-i\hat{\Delta}}(\mathbf{\mu}\sigma_{r,l})
\end{pmatrix}.
\]

(8)

Here, \( \hat{S} \) is the 4 \times 4 scattering matrix (\( \tilde{S} \) is the 2 \times 2 spin scattering matrix) which describes rotation in spin space, i.e. it rotates spins around the vector \( \mathbf{\mu} \) by the spin-mixing angle \( \Theta \). The mixing angle depends on physical quantities in the superconducting and the magnetic layer, like for instance \( \hat{p} \), the exchange field, \( h_{ex} \), and the semiconducting gap, \( E_g \). A model calculation for \( \Theta(\hat{p}, h_{ex}, E_g, p_F) \) in a ferromagnetic semiconductor is given in Ref. [24]. For a ferromagnetic material with a large semiconducting band gap one has \( \Theta \sim (h_{ex}/E_g) \ll 1 \). For our purposes the mixing angle is considered a phenomenological parameter.

In the following, we study a \( FI - S - FI \) system with a thin superconducting film, \( d \ll \xi_0 \), where \( \xi_0 \) is the superconducting correlation length. In that case the solution of Eq.(3) is searched for in the form

\[
\hat{g}(\hat{p}, \mathbf{R}, \omega_n) \approx \hat{g}_0(\hat{p}, \omega_n) + (z - \frac{d}{2})\hat{g}_1(\hat{p}, \omega_n),
\]

(9)

with \( |\hat{g}_0| \gg |d\hat{g}_1| \). By using the boundary conditions given in Eq.(3) one can eliminate \( \hat{g}_1 \) in terms of \( \hat{g}_0 \), \( \hat{S}_l \) and \( \hat{S}_r \), which leads to an equation for \( \hat{g}_0 \)

\[
\{[i\omega_n\hat{r}_3 - \hat{\Delta}, \hat{g}_0], \hat{S}_l\hat{S}_r\} + 2i\frac{\nu_F}{d}[\hat{p}_z, [\hat{g}_0, \hat{S}_l\hat{S}_r]] + \\
+ \hat{S}_l[i\omega_n\hat{r}_3 - \hat{\Delta}, \hat{S}_l^\dagger\hat{g}_0\hat{S}_l + \hat{S}_l^\dagger\hat{g}_0\hat{S}_r]\hat{S}_r = 0
\]

(10)

The brackets \( \{\ldots\} \) and \( [\ldots] \) mean anticommutator and commutator, respectively. In the following, we solve Eq.(10) near the critical temperature, \( T_c \), for singlet as well as triplet superconductors and for various orientations of \( \mathbf{\mu}_l \) and \( \mathbf{\mu}_r \).
A. $T_c$ of $FI - S - FI$ systems with singlet superconductivity

In the case of singlet superconductivity the $4 \times 4$ matrix $\hat{\Delta}$ is given by

$$\hat{\Delta} = i\Delta_s \hat{\sigma}_2 \hat{\tau}_1. \tag{11}$$

Here, $\Delta_s$ is chosen real and in the following we omit the subscript $s$, i.e. $\Delta_s \equiv \Delta$. Near $T_c$ the solution for $\hat{g}_0$ in Eq.(10) is searched for in the form

$$\hat{g}_0 = \hat{g}_0^{(0)} + \hat{f}_0^{(1)}, \tag{12}$$

where $\hat{g}_0^{(0)}$ is independent of the order parameter, $\Delta$, while $\hat{f}_0^{(1)}$ is linear in $\Delta$. From Eq.(4) it follows

$$[\hat{g}_0^{(0)}]^2 \approx -\hat{1}, \tag{13}$$

$$\{\hat{g}_0^{(0)}, \hat{f}_0^{(1)}\} = 0. \tag{14}$$

For a singlet superconductor the relative orientation of magnetization is relevant and therefore the transition temperature only depends on the relative orientation of the exchange fields. Below, $T_c$ is calculated for fields which are parallel, antiparallel or perpendicular to each other. Particular orientations with respect to the $x$- and $y$-axis are chosen to perform the calculations. These choices are for convenience only and do not affect the final results. Furthermore, it is assumed that the mixing angles are identical, i.e. $\Theta_l = \Theta_r = \Theta$.

1. $FI - S - FI$ sandwich with $\mu_l = \mu_r = \hat{y}$

For parallel magnetization the rotation matrices at the left and the right boundary are the same, $\hat{S}_l = \hat{S}_r$, and Eq.(14) then reduces to

$$[i\omega_n \hat{\tau}_3 - \hat{\Delta} - \alpha \hat{\sigma}_2 \hat{\tau}_3, \hat{g}_0] = 0, \tag{15}$$

with

$$\alpha = \frac{v_F |\hat{p}_z|}{2d} \tan \Theta.$$

$T_c$ is determined by

$$\ln t_c = -\sum_{n=0}^{\infty} \frac{1}{n + \frac{1}{2}} \left[1 - (n + \frac{1}{2}) \frac{t_c}{\rho} \arctan \frac{\rho}{t_c(n + \frac{1}{2})}\right]. \tag{16}$$

Here, $t_c = T_{c\|}/T_{c0}$ and $\rho = \rho_0 \tan \Theta / \tan(\Theta/2)$, where $\rho_0$ is defined by

$$\rho_0 = \frac{\xi_0}{2d} \tan \frac{\Theta}{2}. \tag{17}$$

$T_{c0}$ is the critical temperature of the bulk and $\rho_0$ a parameter which describes pair-breaking for a sandwich with a single magnetically active boundary, i.e. for the FI-S-I system$(|0\rangle)$. If $\rho \ll 1$, one has $t_c = 1 - (7\zeta(3)/3)\rho^2 = 1 - 4(7\zeta(3)/3)\rho_0^2$, i.e.
\[
\frac{\delta T_{c,\parallel}}{T_{c0}} \equiv \frac{T_{c,\parallel} - T_{c0}}{T_{c0}} = -\frac{28\zeta(3)}{3} \rho_0^2 = 4 \frac{T_{c,\parallel0} - T_{c0}}{T_{c0}} \equiv 4 \frac{\delta T_{c,0}}{T_{c0}} \quad (18)
\]

The functions \(T_{c,\parallel}(\rho_0)\) and \(T_{c,\parallel0}(\rho_0)\) are shown in Fig.2.

2. \(FI - S - FI\) with \(\mu_l = -\mu_r = \hat{y}\)

If the exchange fields are antiparallel the spin scattering matrices are related by \(\hat{S}_i \hat{S}_r = \hat{1}\). As a result Eq.(10) has the form

\[
[i\omega_n \hat{\tau}_3 - a\hat{\Delta} - b\hat{\sigma}_2\hat{\Delta}, \hat{g}_0] = 0,
\]

where \(a = \cos^2(\Theta/2)\) and \(b = i\sin(\Theta/2)\). The linearization of Eq.(12), as in Eq.(13), leads to

\[
T_{c,a} = T_{c0} e^{-\frac{\tan^2 \theta/2}{2\lambda}}.
\]

Here, \(\lambda\) is the superconducting coupling constant, i.e. \(T_{c0} = 1.13\omega_c e^{-1/\lambda}\). In the antiparallel case the effect of ferromagnetic boundaries is pair-weakening, which means that \(T_{c,a}\) goes to zero when the pair-weakening parameter \(\tan^2(\theta/2)/\lambda \to \infty\). It is interesting to note that the pair-weakening does not depend on the thickness of the superconducting film in the limit \(d \ll \xi_0\). However, for the more realistic case when \(\Theta \ll 1\) the inequality \(|\delta T_{c,a}| \ll |\delta T_{c,0}|\) holds if \(\lambda \gg (d/\xi_0)^2/3\). The condition on \(\lambda\) is fulfilled for thin films of most low-temperature superconductors. So, in the antiparallel case, \(T_c\) is practically unchanged, i.e. \(T_{c,a} \approx T_{c0}\). This property of the \(FI - S - FI\) system is in contrast to the antiparallel case of the \(FM - S - FM\) systems where \(T_{c,a}\) depends strongly on the thickness \(d\) \[18\]. Note, the same results also hold for the case when \(\mu_l\) and \(\mu_r\) are in opposite direction but along the \(x\)-axis.

3. \(FI - S - FI\) with \(\mu_l = \hat{x}\) and \(\mu_r = \hat{y}\)

In the case of perpendicular exchange fields Eq.(10) reads

\[
[i\omega_n \hat{\tau}_3 - \hat{\Delta} - (\alpha_1 \hat{\sigma}_1 + \alpha_2 \hat{\sigma}_2 \hat{\tau}_3 + \alpha_3 \hat{\tau}_3 \hat{\tau}_3), \hat{g}_0] = 0,
\]

where \(\alpha_i = \alpha_0 t_i, i = 1, 2, 3, t_1 = t_2 = \tan(\Theta/2), t_3 = \tan^2(\Theta/2), \alpha_0 = v_F |\hat{p}_z|/2d\). Near \(T_c\) the anomalous function \(\hat{f}_0^{(1)}\) is searched for in the form

\[
\hat{f}_0^{(1)}(\hat{\tau}_3) = A\hat{\Delta} + B\hat{\tau}_3\hat{\Delta} + C\hat{\sigma}_1\hat{\Delta} + D\hat{\sigma}_2\hat{\Delta} + F\hat{\sigma}_1\hat{\tau}_3\hat{\Delta} + G\hat{\tau}_3\hat{\tau}_3\hat{\Delta}.
\]

Note, \(A \equiv A(\omega_n, |\hat{p}_z|)\) determines \(T_c\). Straightforward but cumbersome calculations give for \(T_c\)

\[
\ln t_c = -c \sum_{n=0}^{\infty} \frac{1}{n + \frac{1}{2}} \left[1 - (n + \frac{1}{2}) \frac{t_c}{\rho} \arctan \frac{\rho}{t_c(n + \frac{1}{2})}\right],
\]

where \(c = 1/[1 + (\tan^2(\Theta/2))/2]\), \(\rho = \sqrt{2}\rho_0/\sqrt{c}\). For \(\rho \ll 1\) one obtains

\[
\delta T_{c,\perp} = 2\delta T_{c,0} = \frac{\delta T_{c,\parallel}}{2}.
\]
In Fig. 2 $T_c(\rho_0)$ is shown for the cases when $\mu_l$ and $\mu_r$ are parallel or perpendicular to each other and for the situation that only one boundary is magnetically active.

**FIGURE 2.** The critical temperature $T_c(\rho_0)$ of a $FI-S-FI$ sandwich with a thin ($d \ll \xi_0$) singlet superconductor and various orientations of the exchange fields $h_{\text{ex},l,r}$; $S_\parallel$ - parallel orientation; $S_\perp$ - perpendicular orientation; $S_0$ - the FSI system. Note, $\rho_0 \equiv \rho_0$ given by Eq. 17.

**B. $T_c$ of $FI-S-FI$ systems with triplet superconductivity**

Let us consider $FI-S-FI$ systems with triplet superconductivity in which case the $\hat{\Delta}$ matrix reads

$$\hat{\Delta} = \begin{pmatrix} 0 & (\Delta_t \cdot \sigma)i\sigma_2 \\ i\sigma_2(\Delta^*_t \cdot \sigma) & 0 \end{pmatrix}. \tag{25}$$

Between various possible triplet states we choose the pairing potential in Eq.(3) which gives real $\Delta_t = \Delta^*_t$, with $\Delta_t = \Delta(\hat{p})\hat{x}$ (Note, $\Delta(-\hat{p}) = -\Delta(\hat{p})$). This means that $\Delta_t$ is parallel to the boundaries. The order parameter $\hat{\Delta}$ in this case is given by

$$\hat{\Delta} = -i\Delta(\hat{p})\hat{\sigma}_3\hat{r}_2. \tag{26}$$

Note, if $\Delta_t = \Delta(\hat{p})\hat{y}$ is realized the physics is similar but with appropriately chosen orientation of magnetization at the boundaries. Since the physics of the problem does not depend on $\Delta(\hat{p})$, we take $\Delta(\hat{p}) = \hat{p}_x\Delta$, where $\Delta$ is constant, to simplify the calculation.

For triplet pairing the absolute orientation of $h_{\text{ex},l}(||\mu_l)$ and $h_{\text{ex},r}(||\mu_r)$ plays a very important role and different results are obtained when $h_{\text{ex},l,r}$ are orientated along the $x$-
or \(y\)-axis. If, for instance, both fields are perpendicular to the order parameter \(\Delta_t\) the transition temperature is the same as in a bulk superconductor, while in other cases it is not - see below. This property opens a new possibility for testing the structure of the order parameter in triplet superconductors - see discussions below. As above, the mixing angles are assumed equal(\(\Theta_l = \Theta_r = \Theta\)).

1. \(T_c\) of a \(FI - S - FI\) sandwich with \(\mu_l = \mu_r = \hat{x}\)

If the exchange fields are both parallel to the \(x\)-axis \(\hat{g}_0\) fulfills Eq.\((21)\). After linearization one obtains an equation for \(T_c\)

\[
\ln t_c = -\frac{3}{4} \sum_{n=0}^{\infty} \frac{1}{n + \frac{1}{2}} [I(a_n) - \frac{2}{3}]
\]

with

\[
I(a_n) = 2(1 + a_n^2)(1 - a_n \arctan \frac{1}{a_n}).
\]

\(a_n = (n + 1/2)t_c/\rho\) and \(\rho = (\tan \Theta/\tan \Theta/2)\rho_0\) with \(\rho_0\) given by Eq.\((17)\). The function \(T_{c,\parallel}(\rho_0)\) is shown in Fig.\(3\). However, in the case when \(\rho \ll 1\) one has \(\rho = 2\rho_0\) and Eq.\((27)\) can be solved analytically. The result for \(\delta T_{c,\parallel}(\equiv T_{c,\parallel} - T_{c0})\) reads

\[
\frac{\delta T_{c,\parallel}}{T_{c0}} = -\frac{7\zeta(3)}{5} \rho^2 = -\frac{28\zeta(3)}{5} \rho_{0}^2
\]

Assuming that \(T_{c0}\) for singlet and triplet pairing is the same, this result means that ferromagnetic boundaries are less detrimental for triplet than for singlet pairing - compare Eq.\((18)\) and Eq.\((29)\). Note, in a \(FI - S - I\) system with triplet superconductivity and with a magnetization which is parallel to the order parameter Eq.\((27)\) holds if \(\rho\) is replaced by \(\rho_0\).

2. \(T_c\) of a \(FI - S - FI\) sandwich with \(\mu_l = -\mu_r = \hat{x}\)

In the antiparallel case the ferromagnetic boundaries are pair-weakening and \(T_c\) is given by the same expression, Eq.\((21)\), as for singlet pairing. The pair-weakening parameter does again not depend on the thickness of the superconducting film in the limit \(d \ll \xi_0\).

3. \(T_c\) of a \(FI - S - FI\) sandwich with \(\mu_l = \hat{x}\) and \(\mu_r = \hat{y}\)

In the perpendicular (\(\perp\)) geometry Eq.\((21)\) holds with \(\hat{\Delta}\) from Eq.\((23)\). \(\hat{f}_0^{(1)}\) is searched for in the form of Eq.\((22)\). \(T_c\) is given by

\[
\ln t_c = -\frac{3}{2} \sum_{n=0}^{\infty} \frac{1}{n + \frac{1}{2}} \int_0^{1} (1 - x^2) \frac{b_n(x)}{1 + b_n(x)} dx,
\]

\[
b_n(x) = x^2 \frac{(n + \frac{1}{2})^2 y^2 + 2x^2}{(n + \frac{1}{2})^4 y^4 + 3(n + \frac{1}{2})^2 y^2 x^2 + 2x^4},
\]

with \(y = t_c/\rho_0\). For \(\rho_0 \to 0\) one obtains

\[
\frac{\delta T_{c,\perp}}{T_{c0}} = -\frac{7\zeta(3)}{5} \rho_{0}^2,
\]
i.e. \( \delta T_{c,\parallel} = 4\delta T_{c,\perp} \). The calculations above show that the ferromagnetic boundaries are much less detrimental for the perpendicular geometry than for the parallel one. This can also be seen in Fig. 3 where it is apparent that \( T_{c,\parallel} \) vanishes at smaller \( \rho_0 \) than \( T_{c,\perp} \). It is interesting to note that in the perpendicular case there is a reentrant behavior of triplet superconductivity, i.e. superconductivity disappears in some interval \( T_{c,\perp}^{(2)} < T < T_{c,\perp}^{(1)} \) while it reappears at \( T < T_{c,\perp}^{(3)} \), where \( T_{c,\perp}^{(3)} < T_{c,\perp}^{(2)} < T_{c,\perp}^{(1)} \). This result might mean that instead of the assumed second-order phase transition a first-order phase transition takes place in some region of the \( (T_{c,\perp}, \rho_0) \) phase diagram.

**FIGURE 3.** The critical temperature \( t_c(\rho_0) \) of a \( FI-S-FI \) sandwich for a thin\( (d \ll \xi_0) \)triplet superconductor and various orientations of the exchange fields, \( h_{ex,l,r} \); \( T_{\parallel} \) - parallel orientation; \( T_{\perp} \) - perpendicular orientation; \( T_{0} \) - the FSI system with magnetization along the \( x \)-axis. Note, \( \rho_0 \equiv \rho_{|0} \) given by Eq.\((17)\).

### III. DISCUSSION

We have shown that the critical temperature of a ferromagnetic semiconductor - superconductor - ferromagnetic semiconductor sandwich depends on the orientation of the exchange fields, \( h_{ex,l,r} \). In case of singlet superconductivity \( T_s \) depends on their relative orientation only, while for triplet superconductivity it depends on their absolute orientation. Significant depression of \( T_s \) for singlet superconductors starts for a pair-breaking parameter \( 2\rho_{|0} \geq 0.3 \) if the exchange fields are parallel to each other. For such values of \( \rho_{|0} \) there is significant anisotropy of \( T_s \) in this system, i.e. \( T_{c,\parallel} < T_{c,\perp} < T_{c,|0} < T_{c,a} \approx T_{c,0} \) as shown in Fig.2. Hence, the superconductor can be switched from its normal to its superconducting state or vice versa by changing the relative orientation of the magne-
tization in the magnetic layers. If, for instance, the exchange fields of a $FI - S - FI$ sandwich (with a singlet superconductor) are perpendicular to each other at a given temperature $T$, which fulfills $T_{c,\parallel} < T < T_{c,\perp}$, the superconductor is in its superconducting state. Rotation of one of the fields by ninety degrees to the parallel configuration then switches the superconductor to its normal state. This means that in the perpendicular configuration the state ”1” is realized, while in the parallel one the state ”0” is realized. Depending on the magnetic properties of the ferromagnetic insulator (like for instance magnetic anisotropy) it may happen that energy losses of such a switch are minimized if it operates between the perpendicular and the parallel configuration. By combining many such switches various logic circuits can be realized what will be analyzed elsewhere.

The reorientation of magnetization in thin ferromagnetic films is already realized experimentally [28]. This result opens the possibility of $FI - S - FI$ switches.

There is another possibility of a realization of logic circuits. If one keeps, for instance, the temperature fixed, $T < T_{c,\parallel} < T_{c,\perp} < T_{c,a}$, in a cryotron-like device, which consists of several $FI - S - FI$ switches with fixed orientation of the exchange fields, $h_{ex,l,r}$, one can then reach that some switches pass to normal state while others stay in the superconducting state by changing the current in the control device. Various designs based on these switches are possible (imaginable), also by combining these two possibilities.

Similar applications are possible with $FI - S - FI$ switches based on triplet superconductivity as well as with combined singlet and triplet superconducting $FI - S - FI$ switches.

Note, triplet superconductivity is probably realized in the layered perovskite superconductor $Sr_2RuO_4$ [29] with $T_c \cong 1.5\thinspace K$, while the structure of the order parameter is still unknown. However, there is some evidence that the order parameter belongs to the $E_u$ irreducible representation of the $D_{4h}$ group, i.e. $\Delta (\hat{\mathbf{p}}) = \Delta (\hat{p}_a \pm i\hat{p}_b)\hat{c}$, where the unit vectors $\hat{a}, \hat{b}$ lie in the $RuO_2$ plane and $\hat{c}$ is perpendicular to it [30]. In the analysis of $FI - S - FI$ systems with triplet superconductivity it is shown that $T_c$ strongly depends on the absolute orientation of magnetization with respect to the order parameter $\Delta$. According to this result we expect that similar behavior will be realized in $Sr_2RuO_4$ with the order parameter proposed above [30]. The possibility of resolving its structure by using $FI - S - FI$ systems will be analyzed elsewhere.

In conclusion, the $FI - S - FI$ switches and logic circuits would be advantageous compared to $FM - S - FM$ devices, because in the former fewer physical parameters need to be controlled than in the latter. The $FI - S - FI$ systems can be used in spin-dependent superconducting spectroscopy especially in resolving the structure of the order parameter in triplet and unconventional superconductors like, for instance, the newly discovered $Sr_2RuO_4$.

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