PROBABILISTIC ROBUST ANTI-DISTURBANCE CONTROL OF UNCERTAIN SYSTEMS

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Abstract. We propose a novel method for constructing probabilistic robust disturbance rejection control for uncertain systems in which a scenario optimization method is used to deal with the nonlinear and unbounded uncertainties. For anti-disturbance, a reduced order disturbance observer is considered and a state-feedback controller is designed. Sufficient conditions are presented to ensure that the resulting closed-loop system is stable and a prescribed $H_\infty$ performance index is satisfied. A numerical example is presented to illustrate the effectiveness of the techniques proposed and analyzed.

1. Introduction. In the past decade, uncertain systems with unconstructed or constructed uncertain parameters have attracted much attention from worldwide researchers, and robust control [1] has been used for controller design for such a system [19, 7, 13, 10], controller design for constrained systems is still addressed in some work [11]. In the classical robust control methods, the controllers designed are robust against all the uncertainties so that the system has a good performance under the control, see [20, 21, 22].

However, robust controllers are obtained under the assumption that the numerical ranges of uncertain parameters in the system structure are known or available.

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Under this assumption, a worst case controller is designed so that all the performance requirements for the closed-loop system are satisfied. Clearly, it is not easy to determine the bounds of complex systems, such as high dimensional systems. Even a rough bound is given, it will still lead to conservativeness of robustness.

Semi-definite programming (SDP) has become a crucial tool for control and optimization problems of complex systems [3, 4, 8, 15, 17, 12, 18]. Along with the development, probabilistic analysis and design techniques have also been used successfully to cope with control problems of systems with uncertain parameters, especially in robust controller design of control systems with uncertainty.

In the probabilistic control method, either the sequential randomized algorithm or scenario approach may be applied to solve convex optimization problems [14, 2, 5], the main advantage of these methods is appearance- very general uncertainty structures of the feedback system can be represented by random sampling in the uncertainty space and no bounds or estimation of domains are necessary. It is a relaxation of the solution of the worst case robust optimization problem.

On the other hand, as we know, disturbances may affect the performance of closed-loop system, and sometimes make a stable system unstable one. Normally, such noises are unbounded and not easy to tolerate. Anti-rejection is an effective method to estimate the disturbance accurately. Some attempted work has been done in [9]. Following this work, many results are obtained in control community [16, 23]. In this approach, a reduced order observer is used to estimate the disturbance, and then a feed-forward compensator is used to compensate the disturbance based on the observer output.

In this paper, the scenario optimization approach is used for the design of a disturbance rejection probabilistic stabilization controller for uncertain systems, containing nonlinear uncertain parameters. Random sampling and reduced order observer are addressed, and sufficient conditions under which anti-rejection controller are proposed to guarantee that the corresponding closed-loop system is probabilistically $H_\infty$ stable.

The remainder of the paper is organized as follows. Problem statement, basic definitions and preliminaries are given in Section 2. In Section 3, probabilistic stability analysis of the closed-loop system is carried out and sufficient conditions are established. In Section 4, a scenario method is addressed to design a disturbance rejection state feedback controller. Section 5 presents simulation results to show the effectiveness. Section 6 concludes the paper.

**Notation.** Throughout the paper, $\mathbb{R}^n$ stands for the $n$-dimensional Euclidean space, the transpose of a matrix $A$ is denoted by $A^T$, a positive (negative) definite matrix $P$ is denoted by $P > 0$ ($P < 0$), $I$ is an identity matrix of appropriate dimension, and $*$ indicates a symmetric element in a symmetric matrix.

2. Problem statement and preliminaries. We consider an underlying discrete-time uncertain linear systems:

$$x(k+1) = (A+\Delta A)x(k) + B(u(k)+n_1(k)) + Cn_2(k), \quad (1)$$

where $x(k) \in \mathbb{R}^n$ is the state, $u(k) \in \mathbb{R}^m$ is the control input, $A$, $B$ and $C$ are constant matrices of appropriate dimensions, $\Delta A$ is uncertain system matrix of appropriate dimensions such that $\Delta A = f(\delta)$, where $\delta \in \mathbb{R}^p$ is an uncertainty vector and $f(\cdot)$ is a given nonlinear function. $n_1(k)$ is an external noise which comes from system (2) and $n_2(k) \in L^2_{\mathbb{R}}[0, \infty)$ is noise to the system.

The input disturbance $n_1(k)$ in system (1) is the output of the following system.
\[
\begin{align*}
\{ & w(k + 1) = Dw(k) + En_3(k), \\
& n_1(k) = Fw(k),
\end{align*}
\]

(2)

where \( D, E \) and \( F \) are constant matrices of appropriate dimensions and \( n_3(k) \in L^2_2(0, \infty) \) is an external disturbance to the system due to uncertainties or system noises.

For a given positive constant \( \gamma \), system (2) satisfies an \( H_\infty \) performance index \( \gamma \), if it holds:

\[
\sum_{k=0}^{\infty} n_1^T(k)n_1(k) \leq \gamma^2 \sum_{k=0}^{\infty} n_3^T(k)n_3(k).
\]

(3)

**Assumption 1.** For systems (1) and (2), it holds that 1) \((A,B)\) is controllable; and 2) \((D,BF)\) is observable.

Under the assumption that all the system states are available, we only need to estimate \( n_1(k) \). To achieve this, we consider the following reduced-order observer.

\[
\begin{align*}
\dot{n}_1(k) &= F\dot{w}(k), \\
\dot{w}(k) &= v(k) - Lx(k), \\
\dot{v}(k) &= (D + LBF)(v(k) - Lx(k)) + L(\dot{A}x(k) + Bu(k)),
\end{align*}
\]

(4)

where \( \dot{n}_1(k) \) and \( \dot{w}(k) \) are estimations of \( n_1(k) \) and \( w(k) \), respectively, and \( \dot{A} = (A + \Delta A) \).

The controller in this system is chosen to be:

\[
u(k) = -\dot{n}_1(k) + Kx(k).
\]

Let

\[
e(k) = w(k) - \dot{w}(k).
\]

Then, we have

\[
e(k + 1) = (D + LBF)e(k) + En_3(k) + LCn_2(k).
\]

Introducing \( \eta^T(k) = [x^T(k) \quad e^T(k)] \), combining systems (1), (2) and (3), we obtain an error estimation system:

\[
\eta(k + 1) = \bar{A}\eta(k) + \bar{S}n(k),
\]

(5)

where \( n^T(k) = [n_2^T(k) \quad n_3^T(k)] \),

\[
\bar{A} = \begin{bmatrix}
\dot{A} + BK & BF \\
0 & D + LBF
\end{bmatrix},
\]

\[
\bar{S} = \begin{bmatrix}
C & 0 \\
LC & E
\end{bmatrix}.
\]

The reference output of system (5) is set as:

\[
z(k) = C\eta(k),
\]

where \( C = [C_1 \quad C_2] \).

As there are some uncertainties in the system parameters, we will devise an algorithm for solving the probabilistic control problem of system (5) based on scenario optimization approach.
Let Prob denote a given probability measure, and let $K$ be a gain matrix of the controller $u(k)$. We consider a binary function $g(\cdot)$ for system (5) defined by,

$$g(\cdot) = \begin{cases} 
0 & \text{if system (5) is stable under } K, \\
1 & \text{otherwise}.
\end{cases} \quad (6)$$

For a given $K$,

$$P(K) = \text{Prob}\{g(\cdot) = 1\}$$

is the probability for system (5) failing to be stable under $K$ for some uncertainties.

Our objective in this paper is to find a $K$ for system (5) such that

$$P(K) \leq \epsilon, \quad (7)$$

where $\epsilon \in (0, 1)$ is a priority confident probability level.

However, condition (7) may be intractable. Thus, we shall use the scenario optimization method to deal with it. Given a confidence parameter $\beta \in (0, 1)$ and $\epsilon \in (0, 1)$, we find a suitable $K$ for system (5) such that the probability for the satisfactory of $\{g(\cdot) = 1\}$ is not smaller than $1 - \beta$.

**Remark 1.** As in [6], let the level parameter $\epsilon \in (0, 1)$ and a confidence parameter $\beta \in (0, 1)$ be given. Choose an $N$ such that

$$N \geq N_{\text{gen}}(\epsilon, \beta) = \left\lceil \inf_{\varsigma \in (0, 1)} \frac{1}{1 - \varsigma} \left( \frac{1}{\epsilon} \ln \frac{1}{\beta} \right) + n_\theta \frac{1}{\varsigma \epsilon} + \frac{1}{\epsilon} \ln \left( \frac{n_\theta}{\epsilon} \right)^{n_\theta} n_\theta! \right\rceil,$$

where $\lceil \cdot \rceil$ denotes the ceiling function and $n_\theta$ is the number of free variables for the constraint

$$g(\cdot) = 1. \quad (8)$$

Then, select $N$ independent identically distributed samples $\delta^1, \delta^2, \ldots, \delta^N$ such that a $K$ can be obtained to ensure $P(K) \leq \epsilon$ is satisfied with probability not smaller than $1 - \beta$.

**Definition 2.1.** Let $\epsilon \in (0, 1)$ and $\beta \in (0, 1)$ be given. Set $\delta^1, \delta^2, \ldots, \delta^N$ to be the $N$ possible extractions amongst uncertainty region according to the procedure detailed in Remark 1. For a given initial state $\eta(0)$, suppose that there exists a feasible controller $u(k)$ and a positive number $N(\xi_0)$, such that system (5) with $n(k) = 0$ satisfies (10) and (11), then, system (5) is said to be asymptotically probabilistically stable with an $H_\infty$ performance index $\gamma$, if the following two inequalities are satisfied:

The aim of our work is to design an $H_\infty$ anti-disturbance controller for system (5) to ensure that the error system (5) is probabilistically stable and satisfies a prescribed $H_\infty$ performance index. To proceed further, some definitions are needed.

**Definition 2.2.** Let $\epsilon \in (0, 1)$ and $\beta \in (0, 1)$ be given. Set $\delta^1, \delta^2, \ldots, \delta^N$ to be the $N$ possible extractions in uncertainty region according to the procedure detailed in Remark 1. For a given initial state $\eta(0)$, suppose that

$$\lim_{m \to \infty} \left\{ \sum_{k=0}^{m} \eta(k)^T \eta(k) | \eta(0), \delta^l \right\} < \infty, \quad l \in I[1, N]. \quad (9)$$

Then $K$ is called an $\epsilon$-level robust solution, and system (5) is said to be robustly $\epsilon$-level probabilistically stable.
\[
\lim_{m \to \infty} \left\{ \sum_{k=0}^{m} \eta(k)^T \eta(k) \eta(0), \delta^l \right\} < \infty, \quad l \in I[1, N],
\]
(10)
\[
\sum_{k=0}^{\infty} z^T(k) z(k) \leq \gamma^2 \sum_{k=0}^{\infty} n^T(k) n(k).
\]
(11)

3. Stability analysis. In this section, sufficient conditions are given under which system (5) is probabilistically asymptotically stable.

**Theorem 3.1.** Let \( n(k) = 0, \epsilon \in (0, 1) \) and \( \beta \in (0, 1) \) be given. Let \( \delta^1, \delta^2, \ldots, \delta^N \) be the \( N \) possible extractions amongst uncertainty region chosen according to the procedure detailed in Remark 1. System (5) is probabilistically asymptotically stable if there exists a positive definite symmetric matrix \( P \) such that
\[
\bar{A}^T P \bar{A} - P < 0.
\]
(12)

**Proof of Theorem 3.1.** Construct a Lyapunov function as follows
\[
V(k) = \eta^T(k) P \eta(k),
\]
then we have
\[
\Delta V(k) = \eta^T(k+1) P \eta(k+1) - \eta^T(k) P \eta(k)
\]
\[
= \eta^T(k) (\bar{A}^T P \bar{A} - P) \eta(k)
\]
\[
= \eta^T(k) \Omega \eta(k).
\]
Let
\[
\rho = \min_k \{ \lambda_{\text{min}}(-\Omega) \},
\]
where \( \lambda_{\text{min}}(-\Omega) \) denotes the minimal eigenvalue of \( -\Omega \).
Thus,
\[
\Delta V(k) \leq -\rho \eta^T(k) \eta(k).
\]
Then, we have
\[
V(T+1) - V(0) \leq -\rho \left\{ \sum_{k=0}^{T} \| \eta(k) \|^2 \right\}.
\]
It follows
\[
\left\{ \sum_{k=0}^{\infty} \| \eta(k) \|^2 \right\} \leq \frac{1}{\rho} V(0) - \frac{1}{\rho} \{ V(k) \} \leq \frac{1}{\rho} V(0) < \infty.
\]
Therefore, system (5) is asymptotically stable. The proof is completed. \( \Box \)

**Theorem 3.2.** Let \( n(k) = 0, \epsilon \in (0, 1) \) and \( \beta \in (0, 1) \) be given. Choose \( N \) possible extractions \( \delta^1, \delta^2, \ldots, \delta^N \) from uncertainty region according to the procedure detailed in Remark 1. System (5) is probabilistically asymptotically stable if there exists a positive symmetric matrix \( Q \) and a matrix \( G \) such that it holds
\[
\begin{bmatrix}
Q - G^T & G^T \bar{A}^T \\
* & -Q
\end{bmatrix} < 0.
\]
(13)
Proof of Theorem 3.2. Condition (12) is written as
\[
\begin{bmatrix}
-P & \bar{A}^T \\
* & -Q
\end{bmatrix} < 0,
\]
where \( Q = P^{-1} \). Multiplying the above inequality by \( \text{diag} \{ G^T, I \} \) and \( \text{diag} \{ G, I \} \) on the right side and left side, respectively, we have
\[
\begin{bmatrix}
-G^T P G & \bar{A}^T \\
* & -Q
\end{bmatrix} < 0.
\]
Recalling \( G^T P G \geq G^T - P^{-1} + G \), we have condition (13). The proof is completed.

4. Controller design. Sufficient conditions for the existence of an admissible mode-dependent \( H_\infty \) state-feedback controller for system (5) are presented in the following theorem.

**Theorem 4.1.** Let \( \epsilon \in (0, 1) \) and \( \beta \in (0, 1) \) be given. Set \( \delta_1, \delta_2, \cdots, \delta_N \) being the \( N \) possible extractions amongst uncertainty region according to the procedure detailed in Remark 1. Assuming \( \gamma \) be a given positive constant. If there exist positive definite symmetric matrices \( P \) and \( Q \) such that
\[
\begin{bmatrix}
-P & 0 & \bar{A}^T & \bar{C}^T \\
* & -\gamma^2 I & \bar{A}^T & \bar{S}^T \\
* & * & -Q & 0 \\
* & * & * & -I
\end{bmatrix} < 0,
\]
then, system (5) is probabilistically asymptotically stable and also satisfies a prescribed \( H_\infty \) performance index.

**Proof of Theorem 4.1.** Introduce the Lyapunov function
\[
V(k) = \eta^T(k) P \eta(k).
\]
We consider the following cost function
\[
J = \sum_{k=0}^{\infty} (z^T(k) z(k) - \gamma^2 n^T(k) n(k)).
\]
Under the homogeneous initial condition, \( J \) can be written as
\[
J \leq E \left\{ \sum_{k=0}^{\infty} [z^T(k) z(k) - \gamma^2 n^T(k) n(k) + \Delta V(k)] \right\}
= \eta^T(k) \left[ \bar{C}^T \bar{C} + \bar{A}^T P \bar{A} - P \right] \eta(k)
+ 2\eta^T(k) \left[ \bar{A}^T P \bar{S}(k) \right] n(k)
+ n^T(k) \left[ \bar{S}^T \bar{P} \bar{S} - \gamma^2 I \right] n(k).
\]
The above inequality can also be written as
\[
J \leq \left[ \begin{array}{cc} \eta^T(k) & n^T(k) \end{array} \right] \Lambda \left[ \begin{array}{c} \eta(k) \\ n(k) \end{array} \right],
\]
where
\[
\Lambda = \begin{bmatrix}
C^T \bar{C} + A^T P \bar{A} - P & A^T P \bar{S} \\
* & \bar{S}^T \bar{P} \bar{S} - \gamma^2 I
\end{bmatrix}.
\]
Using Schur complement, we have condition (15).
Theorem 4.2. Let $\epsilon \in (0,1)$ and $\beta \in (0,1)$ be given, and $\delta_1, \delta_2, \cdots, \delta^N$ be $N$ possible extractions chosen from uncertainty region according to the procedure detailed in Remark 1. Assuming $\gamma$ is a given positive constant. If there exist positive definite symmetric matrices $P$ and $Q$ such that

$$
\begin{bmatrix}
Q - g^T - g & 0 & 0 & 0 & (A_{g} + B_{g}K)^T & 0 & (C_{g}^T)^T \\
* & Q - 2I & 0 & 0 & (B_{g}F)^T & (D + LB_{g}F)^T & (C_{g}^T)^T \\
* & * & -\gamma^2 I & 0 & C^T & (L_{g}C)^T & 0 \\
* & * & * & -\gamma^2 I & 0 & E^T & 0 \\
* & * & * & * & -Q & 0 & 0 \\
* & * & * & * & * & -Q & 0 \\
* & * & * & * & * & -Q & 0 \\
* & * & * & * & * & -Q & 0 \\
* & * & * & * & * & -Q & 0 \\
* & * & * & * & * & -Q & 0
\end{bmatrix} < 0,
$$

(16)

then, system (5) is probabilistically asymptotically stable and satisfies an $H_\infty$ performance index, and the controller gain is $K = \hat{K}G^{-1}$.

Proof of Theorem 4.2. First, condition (15) is rewritten as

$$
\begin{bmatrix}
-P & 0 & 0 & 0 & (A_{g} + B_{g}K)^T & 0 & (C_{g}^T)^T \\
* & -P & 0 & 0 & (B_{g}F)^T & (D + LB_{g}F)^T & (C_{g}^T)^T \\
* & * & -\gamma^2 I & 0 & C^T & (L_{g}C)^T & 0 \\
* & * & * & -\gamma^2 I & 0 & E^T & 0 \\
* & * & * & * & -Q & 0 & 0 \\
* & * & * & * & -Q & 0 & 0 \\
* & * & * & * & -Q & 0 & 0 \\
* & * & * & * & -Q & 0 & 0 \\
* & * & * & * & -Q & 0 & 0 \\
* & * & * & * & -Q & 0 & 0
\end{bmatrix} < 0,
$$

(17)

Multiplying the above inequality by $\text{diag}\{G^T, I, \cdots, I\}$ and $\text{diag}\{G, I, \cdots, I\}$ on the right side and left side, respectively, and $G = \text{diag}\{g^T, I\}$. By Schur complement, recalling $G^TPG \geq G^T - P^{-1} + G$, we have Condition (16), thus it is omitted.

Remark 2. The input disturbance $n_1(k)$ in this paper is a non-bounded noise, which is more realistic in practical systems.

5. Numerical example. We consider the discrete-time uncertain system with the following given coefficient matrices:

$$
A = \begin{bmatrix}
1.2 & -0.17 \\
0.15 & -0.92
\end{bmatrix}, 
B = \begin{bmatrix}
0.21 \\
0.15
\end{bmatrix},
$$

$$
D = \begin{bmatrix}
0 & -1 \\
1 & 0
\end{bmatrix}, 
C = \begin{bmatrix}
0.1 \\
-0.1
\end{bmatrix},
$$

$$
E = \begin{bmatrix}
0.1 \\
0.1
\end{bmatrix}, 
F = \begin{bmatrix}
0.3 & 0.5
\end{bmatrix},
$$

$$
C_1 = \begin{bmatrix}
0.5 \\
0.1
\end{bmatrix}, 
C_2 = \begin{bmatrix}
0.1 & 0
\end{bmatrix}.
$$

The uncertainties in system matrices are given below:

$$
\Delta A = \delta_1 \begin{bmatrix}
6 * (1.8 + \delta_2) \Upsilon & 0 \\
0 & -2.2 * \delta_1
\end{bmatrix},
$$

where $\Upsilon = \cos(0.195 + \delta_3) - \sin(0.863 + \delta_3)$ and $\Psi = \{\delta_i : |\delta_i| \leq 0.2, i = I[1,3]\}$. 

Choose the probabilistic levels $\epsilon = 0.1$, and $\beta = 0.1\%$. To verify the closed-loop performance, we run simulations for the cases and the feedback gain obtained is, respectively,

$$K = \begin{bmatrix} -5.5483 & 1.1415 \\ 3.4297 & 2.5863 \\ 2.5863 & 1.5720 \end{bmatrix}$$

$$L = \begin{bmatrix} -5.5483 & 1.1415 \\ 3.4297 & 2.5863 \\ 2.5863 & 1.5720 \end{bmatrix}$$

We run an a posteriori Monte-Carlo analysis for a larger number of systems (200) under controller designed, it is arising that under the controller obtained, the probability of stabilization is bigger than desired. State trajectories of the closed-loop system and the controlled output of the system are given in Figure 1. The disturbance $n_1(k)$, the estimation disturbance $\hat{n}_1(k)$, and the error disturbance $n_1(k) - \hat{n}_1(k)$ are shown in Figure 2 and the controlled output of the system are given in Figure 3 which shows the effectiveness comparing with single controller.

Clearly, the system (5) is probabilistically asymptotically stable under such a controller, which means that it satisfies the probability levels.

6. **Conclusion.** In this paper, the problem of probabilistic disturbance rejection controller design for uncertain systems with input disturbances is studied. Conditions under which the closed-loop system is probabilistic stable are established by employing a scenario optimization approach. Based on these conditions, the problem for design of an reduced order based state feedback controller is formulated and solved as a constrained optimization problem. A numerical example illustrates the
effectiveness of the proposed design procedure and the performance of the resulting closed-loop system.

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