Modified Eigenspace Based Iterative Robust Capon Beamformer

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Abstract. To improve the performance of adaptive beamformer with large steering vector mismatch, a modified eigenspace based iterative robust capon beamformer (ME-IRCB) is proposed. The approach iteratively implements the robust Capon beamformer (RCB) with an adaptively adjusted uncertainty set. Based on the modified eigenspace method, the desired signal subspace is given, whereafter the size of the uncertainty set is adaptively adjusted according to the mismatch between the estimated and presumed steering vector of the desired signal at each iteration. The proposed approach gives two estimation methods about the uncertainty size, and both of them contribute to the robustness improving of the beamformer. Moreover, unlike other robust beamformers, the proposed approach does not take the integration of the steering vector’s outer product over the adjacent direction of the desired signal, and it works well with the adjacent directions of the desired signal and interference. This merit is due to that, the proposed approach only uses the fundamental sample covariance matrix during the subspace construction, therefore the influence of imprecise steering vector on the subspace is overcome. Simulation results show that the proposed approach performs better than the existing robust beamformers under different evaluation criteria.

1. Introduction

It is well known that as a data-dependent beamformer, the Capon beamformer has high resolution and good interference rejection ability [1]. However, it suffers performance degradation when the steering vector and covariance matrix are imprecise, hence a lot of parameter-based and parameter-free methods have been proposed to improve the robustness of adaptive beamformer [2],[3]. The early famous robust beamformers include [4]-[6]. The method in [4] provide some robustness, but this robustness is limited by the direction mismatch and the hard constraints consume a lot of degree-of-freedom, hence the modified version is proposed recently [7]. One shortfall with method [5] is that it is not clear how to choose the diagonal loading factor. The main drawback of eigenspace based beamformer [6] is that it can not work well when the signal-to-noise ratio (SNR) is low, where the modified method [8], [9] improved this in a way.

About decade ago, a varies of robust beamformers were proposed based on uncertainty set theory [10]-[14], in which the novel ideas injected the research on robust beamformer some fresh energy. But
the recent research [15]-[17] show that, the uncertainty set based methods can not effectively overcome the large steering vector mismatch. To overcome this problem, the iterative RCB (IRCB) method is proposed by the iteration implementation of RCB with small uncertainty set [15]. Other iterative uncertainty set methods are also proposed [16], [17]. However, the uncertainty set in these iterative methods is fixed. Using a signal subspace estimation algorithm, an adaptive uncertainty based IRCB (AU-IRCB) approach is proposed in [18], in which the size of the uncertainty set is adaptively estimated at each iteration.

However, in the aforementioned existing robust beamformers, one or more interferences are assumed directionally enough far from the desired signal [19]. In other words, these methods suffer when some interferences are close to the desired signal [19]. In some recent approach, the integration of the steering vector's outer product over the adjacent direction of the desired signal is used to construct a signal subspace [18]-[20]. This subspace may have a large mismatch with the adjacent directions of the desired signal and interference.

In this letter, we introduce a new approach to the design of robust beamformer. With the help of a modified eigenspace method, the desired steering vector is estimated by the IRCB beamformer, and the uncertainty set is adjusted at each iteration.

2. Background

The well known standard Capon beamformer is given by

\[
\min_w w^H R w \quad \text{s.t.} \quad w^H a = 1
\]

where \( w \) is the array weight vector, \( R \) is the covariance matrix of the array output, \( a \) is the steering vector of the desired signal and \( (\cdot)^H \) stands for the Hermitian transpose. It essentially minimizes the array output power subject to the constraint that the desired signal is passed undistorted. If the conditions are ideal, in other words, \( R \) and \( a \) are accurately known, (1) is equivalent to maximizing the output signal-to-interference-plus-noise ratio (SINR). However, \( R \) and \( a \) are often replaced by the sample covariance matrix \( \hat{R} = \frac{1}{N} \sum_{n=1}^{N} x(n)x^H(n) \) and presumed steering vector \( \hat{a} \) in practical applications, respectively. Here, \( x(n) \) and \( N \) are the array snapshot from \( M \) sensors and snapshots number, respectively.

The covariance matrix mismatch and steering vector mismatch result in great performance degradation of Capon beamformer, due to which the research on robust adaptive beamforming is pay attention to. In the RCB approach, the desired steering vector is constrained in an uncertainty set, and its actual value is estimated by maximize the power of the desired signal under this constraint [11]-[13]. The basic RCB approach can be given by

\[
\min_{\alpha} \alpha^H \hat{R} \alpha \quad \text{subjectto} \quad \| \alpha - \hat{a} \|^2 \leq \varepsilon
\]

where stands for the size of the uncertainty set. The research in [15] indicates that when the steering vector mismatch is large caused by the signal source movement or antenna array motion, the RCB with large uncertainty set will suffer performance degradation and the iterative RCB with a fixed small uncertainty set may have a better capability to deal with this large mismatch. To accelerate the convergence rate, the AU-IRCB approach designs a method to adaptively adjusting the size of uncertainty set based on the estimated signal subspace \( \hat{U}_i(\Delta) = [\mathcal{P}(Q(\Delta))] \left[ u_1 \cdots u_K \right] \) [18]. In \( \hat{U}_i(\Delta) \), \( \left[ u_1 \cdots u_K \right] \) are the \( K \) eigenvectors of \( \hat{R} \) with the corresponding values sorted in decreasing order, and \( \hat{K} \) is assumed to be estimated from the MDL or AIC approaches [21]. In addition, \( \mathcal{P}(Q(\Delta)) \) is the principal eigenvector of a positive definite matrix

\[
Q(\Delta) = \int_{\Delta} \hat{\sigma}^H(\theta + \phi) \hat{\sigma}(\theta + \phi) d\phi
\]

(3)
Where $\theta$ denotes the look-direction, and $\Delta$ defines the spatial sector assumed to represent the range of the angular location of the desired signal. To ensure that the reconstructed signal space $\hat{U}_s(\Delta)$ include both the desired signal and interferences components and the length of the steering vector mismatch is minimum, $\Delta$ can be found from

$$\hat{\Delta} = \arg\min_{\Delta} \| \hat{e}(\hat{U}_s(\Delta)) \|$$  \hspace{1cm} (4)

where $\hat{e}$ is the steering vector mismatch estimated by subspace projection based on $\hat{U}_s(\Delta)$ (one can see details in [18]).

3. Proposed modified eigenspace based iterative robust capon beamformer

In this section, we introduce a modified eigenspace based iterative robust capon beamformer. The basic idea behind our approach is to estimate the desired signal subspace based on a modified eigenspace and to correct the presumed steering vector using IRCB with an adaptively adjusted size of uncertainty set which can be calculated by this signal subspace.

3.1. Signal subspace estimation

The AU-IRCB approach in [18] proposed an excellent method to estimate signal-plus-interference subspace $\hat{U}_s(\Delta)$. We observe that the principal eigenvector $P(\Omega(\Delta))$ of $\Omega(\Delta)$ in (3) can improve the poor performance of original eigenspace based method at low SNRs. Based on the reconstructed signal space $\hat{U}_s(\Delta)$, the AU-IRCB approach really performs well at low SNRs. In this subsection, we use another method to estimate the signal subspace. To overcome the demerit of original eigenspace based beamformer, a modified eigenspace (ME) approach [8] which can work well even at low SNRs is used in our approach.

The eigen-decomposition of the received data covariance matrix $R$ can be given by $R - \sum_{i = 1}^{K} \lambda_i \varepsilon_i \varepsilon_i^H$, where $\varepsilon_i$ for $i = 1,2,...,M$ are the eigenvectors, and $\lambda_1 \geq \lambda_2 \geq ... \geq \lambda_{K+1} = \ldots = \lambda_M = \sigma^2$ are the corresponding eigenvalues of $R$ sorted in decreasing order. Here, $K$ is the dimension of signal-plus-interference, and $\sigma^2$ stands for the power of noise. To overcome the influence of small snapshot, $R$ is pretreated, and it is replaced by an enhanced covariance matrix $\tilde{R} = \beta \hat{R} + \alpha I$ which is estimated by a linear shrinkage approach, where $\alpha \geq 0$ and $\beta \geq 0$ are the shrinkage parameters and can be automatically estimated from the received data by minimizing the mean square error (MSE) of the $\tilde{R}$ [22].

With the small mismatch assumption between $a$ and $\tilde{a}$, the new estimated desired signal subspace $\hat{U}_s$ is reconstructed by $\hat{U}_s = [\varepsilon_{[m]},...,\varepsilon_{[M-1]},\varepsilon_{[M]}]$ [8]. How to choosing the eigenvectors according to the the projections $\rho(i) = \varepsilon_{[i]}^H \tilde{a}(i = 1,2,...,M)$ which are arranged in descending order as $P_{[M]} \geq P_{[M-1]} \geq ... \geq P_{[1]}$, and the ones reach the energy percentage criterion $(P_{[M]} + P_{[M-1]} + ... + P_{[1]})/\sum_{i=1}^{M} \rho(i) > \rho$ can be selected. Thus the desired signal subspace projection matrix can be constructed by $P_s = \hat{U}_s \hat{U}_s^H$.

3.2. Uncertainty set adjusting

In this subsection, we will give two methods for uncertainty set adjusting. The basic constraint of uncertainty set based methods is that the actual steering vector of desired signal should be constrained in the uncertainty set. In other words, it should be in the feasible set of the optimization problem. If the projection of $\tilde{a}$ onto the desired signal subspace $\hat{U}_s$ is taken as the actual steering vector of desired signal, the size of the uncertainty set at each iteration can be adjusted according to the steering vector mismatch between the actual value and the presumed value, i.e.
The formulation (5) is the first method to adjust the uncertainty set. Meanwhile, the norm constraint of steering vector \( M \) which is generally considered in [11]-[13], [15]-[18] and so on. If we consider this constraint, another more precise adjusting method for uncertainty set can be derived. After projection of \( \tilde{a} \) onto the desired signal subspace and norm normalization, the actual desired steering vector can be estimated by \( \| P_s^n a \| \tilde{P}_s^n \tilde{a} \). Subsequently, the steering vector mismatch can be used to estimate the uncertainty size \( \varepsilon = \| P_s^n a - \tilde{a} \| \). The formulation (6) is the second method to adjust the uncertainty set.

From the above discussions, we can summarize the proposed ME-IRCB algorithm as follows:

- **Initial values:** \( \hat{a}, \rho \)
- **Algorithm at the \( i \) th iteration:**
  1. Calculate \( \hat{R} = \beta \hat{R} + \alpha I \) base on [22].
  2. With \( \rho \) and \( \hat{R} \), estimate the desired signal subspace \( \hat{U} \), and calculate \( \hat{P}_s = \hat{U}, \hat{U}_s^H \) according the method in subsection 1 of section 3.
  3. Calculate the uncertainty size according to (5) or (6).
  4. Calculate \( \hat{a} \) by solving (2) according to the well known RCB method in [12].
  5. Update the presumed desired steering vector with \( \tilde{a} = \hat{a} \).
  6. Calculate the new uncertainty size from the updated \( \tilde{a} \) using (5) or (6). Check if the uncertainty size is not decreasing or after 20 iterations, go to step 7. Otherwise, repeat step 4 to 6.
  7. Calculate the proposed ME-IRCB weight vector \( \hat{w}_{\text{ME-IRCB}} = \hat{R}^{-1} \hat{a} \).

**4. Simulation results**

In this section, Monte Carlo simulation is carried out to evaluate the performance of the proposed ME-IRCB. The proposed ME-IRCB is compared with the RCB [12], ME [8], IRCB [13], and AU-IRCB [15]. Through the simulations, a uniform linear array (ULA) of 10 omni-directional antenna elements with the inter-element spacing of half wavelength is considered. Additive noise in antenna elements is modelled as spatially and temporally independent complex Gaussian noise with zero mean and unit variance. Two interfering sources are assumed to impinge on the antenna from the directions \( 15^\circ \) and \( 30^\circ \) with 20 dB interference-to-noise ratio (INR). The desired signal is assumed to be a plane-wave from the presumed direction \( \theta_0 = 0^\circ \). For obtaining each point in the simulation results, 200 independent runs are used and the sample data covariance matrix is estimated using \( N = 100 \) snapshots. In IRCB, the size of small uncertainty set, tolerance parameter for iteration stopping, and the DOA uncertainty region are chosen as \( \varepsilon = 0.1 \), \( \delta = 10^{-5} \) and \( \Delta \theta = 4^\circ \), respectively. The parameter \( \rho = 0.9 \) is used for ME and proposed ME-IRCB, and the second method (6) is used to calculate uncertainty size for ME-IRCB.

In the first example, we compare the output SINR performance of the beamformers versus SNR with a fixed DOA mismatch and random array steering vector mismatch. The true DOA of the desired signal is fixed to be \( \theta_0 = 4^\circ \). The random steering vector mismatch is assumed as a zero-mean complex random vector with unit variance for desired and interferences. The results shown in Fig. 1 shows the good performance of ME-IRCB. Meanwhile, we observe that the ME method performs awfully at high SNRs.
In the second example, we investigate the performance of beamformers with different DOA mismatch. The true DOA of the desired signal is changed from \(-8^\circ\) to \(8^\circ\), and the SNR is 10 dB. The interferences are set closer to the desired signal as example 3, and the results are shown in Fig. 2.

From the results shown in Fig. 2, we can see that the larger of the direction mismatch, the worse performance of all the beamformers. However, the proposed ME-IRCB performs better than other methods and has a lower performance degradation than other beamformers with the large mismatch. Especially, when the desired signal is close to the interference, AU-IRCB suffers great degradation while the proposed ME-IRCB does not. In Fig. 2, we can clearly see that RCB with large uncertainty set \(\varepsilon = 8.5\) performs quite well with large mismatch, but its performance at small mismatch is poor, and the result is that the large uncertainty set consumes the degree-of-freedoms of RCB and weakens its interference-plus-noise suppression ability. Meanwhile, when the direction mismatch is large, the ME beamformer performs badly, and this result confirms the precondition of ME in [8] that the steering vector mismatch should not be too large. But with the help of IRCB with small adaptively adjusted uncertainty set, the proposed ME-IRCB can effectively overcome this problem of ME.

5. Conclusion
We have introduced a new ME-IRCB approach to the design of robust adaptive beamforming. A modified eigenspace method in tandem with IRCB is proposed in the proposed method. The ME-IRCB can adaptively adjust the uncertainty set according to arbitrary one of our methods given in this letter, and it performs better than other existing method with a large range of arbitrary mismatches. Moreover, the proposed ME-IRCB can also work well with the adjacent directions of the desired signal and interference. The effectiveness of the proposed algorithm has been shown using simulations results.
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