Resisting collapse: How matter inside a black hole can withstand gravity

Ram Brustein\(^{(1)}\), A.J.M. Medved\(^{(2,3)}\)

\(^{(1)}\) Department of Physics, Ben-Gurion University, Beer-Sheva 84105, Israel
\(^{(2)}\) Department of Physics & Electronics, Rhodes University, Grahamstown 6140, South Africa
\(^{(3)}\) National Institute for Theoretical Physics (NITheP), Western Cape 7602, South Africa

ramyb@bgu.ac.il, j.medved@ru.ac.za

Abstract

How can a Schwarzschild-sized matter system avoid a fate of gravitational collapse? To address this question, we critically reexamine the arguments that led to the “Buchdahl bound”, which implies that the minimal size of a stable, compact object must be larger than nine eighths of its own Schwarzschild radius. Following Mazur and Mottola, and in line with other counterexamples to the singularity theorems, we identify negative radial pressure as the essential ingredient for evading the Buchdahl bound. It is shown, in particular, that an external observer would attribute a negative pressure to a Schwarzschild-sized bound state of highly excited, long, closed, interacting strings and thus the ability to resist gravitational collapse.
1 Introduction

The tension between black hole (BH) evaporation and the rules of quantum mechanics [1, 2, 3, 4, 5, 6, 7], as well as the recent discovery of gravitational waves being emitted from colliding BHs and neutron stars, has reignited interest in the following question: What is the final state of matter after it collapses to form a BH?

The singularity theorems of Hawking and Penrose [8, 9] decree that the final state of collapsing matter, when considered within the purview of classical general relativity (GR), must be singular. An elegant discussion which preceded these theorems was provided in a simplified context by Buchdahl [10] (and later by Chandrasekhar [11, 12] and Bondi [13]). Buchdahl was able to show that a “conventional” matter system cannot be stable within its own Schwarzschild radius. In fact, the “Buchdahl bound” on the outermost radius of a stable fluid sphere $R$ is somewhat larger than the Schwarzschild limit, $R \geq \frac{9}{8} R_S$ [10]. This result puts a damper on the idea that an ultra-compact object could play the role of a BH while being fundamentally different from the BHs of GR.

Buchdahl, in his analysis, invoked the usual assumptions that lead to a Schwarzschild geometry. He further assumed both causality and a classical energy condition (the strong energy condition), as similarly required by the singularity theorems. These assumption were implemented by requiring that both the energy density and the pressure are positive. Regarding the matter distribution, Buchdahl made three additional assumptions. As more recently discussed in [14], these are (I) the isotropy and positivity of the pressure, (II) the monotonic dilution of the matter distribution when moving
outward from the center and (III) the continuity of the time-time component of the Schwarzschild metric and its first derivative across the boundary of the matter sphere. The rest of the proof relies only upon the Einstein field equations of GR.

The conclusion is that any reasonable BH substitute — meaning an ultracompact object which resembles a BH — has to evade the Buchdahl bound by invalidating at least one of its assumptions.

Following to some extent [14], our description of matter that can avoid gravitational collapse begins with a discussion about its pressure. We will, in particular, argue that (at least) the radial component of the pressure needs to be negative and large in magnitude. Such matter has to be perceived to have a solid composition and be under severe tension, as noticed long ago by Bondi [13]. Matter with negative pressure is important on three fronts: (a) It can explain why a system under collapse does not have to comply with the singularity theorems. This follows in analogy to the negative pressure of the cosmological dark energy. (b) Any negative component of pressure would lower an object’s gravitational or Komar mass density [15] and thus can impede collapse by weakening the gravitational field. (c) A negative radial pressure invalidates not only assumption (I) from the prior list but also Buchdahl’s assumption in (III) regarding the continuity of the first derivative of the metric.

Negative normal pressure, or tension, is quite common in real-world materials and should not be viewed as a particularly exotic property of solid matter. It is rather the response of gravity to negative pressure that is unusual. Many polymers in particular can have negative normal or radial
pressure, but what does this mean? To address this question, let us first remark that the term pressure is often reserved for fluids and gases, whereas the counterpart for solids is usually called stress, even though these amount to the same effect. The sign of the normal stress is taken to be positive when the object is compressed and negative when it is extended.

From a thermodynamic perspective, a negative pressure signifies a reversal in direction along some path in thermodynamic phase space when compared to, for instance, a box of gas molecules. For example, a negative component of pressure indicates that the entropy of an object decreases as its volume increases due to a deformation. (Recall the standard relation $pdV = TdS$ when all other variables are held constant.) A rubber band provides a textbook example of a typical polymer: If one tightens a rubber band by stretching it, the length of the band increases while its entropy decreases.

Let us now redirect the discussion back to BHs and call upon an analysis in [16]. There it was established that the radial pressure $p_r$ of any matter just outside of a Schwarzschild horizon has to satisfy the condition $p_r = -\rho$, with $\rho$ being the energy density. This condition is necessary to ensure the near-horizon regularity of curvature invariants like the Kretschmann scalar and, by continuity, has to hold all the way up to and including the horizon. The transverse components of pressure $p_\perp$, on the other hand, cannot be similarly constrained by appealing to regularity, as was also clarified in [16].

One way to enforce the condition $p_r = -\rho$ at the horizon is to insist that it be true throughout the interior; for instance, by replacing the GR description of the BH interior with a ball composed of some matter under tension. But what type of matter can satisfy $p_r = -\rho$ and, moreover, find
its way into the interiors of BH-like, ultra-compact objects? The answers to these puzzles will be presented later on in the paper.

Although we primarily argue against Buchdahl’s assumption of positive-pressure matter, we would like to point out an additional “hidden” assumption in his analysis. Namely, that the object’s interior can be accurately described by a curved spacetime geometry with a classical metric. We have previously argued that the geometry of the BH interior has no such semi-classical description \[17, 18, 19\]. But, since this breach is an implicit one and necessitates bringing in quantum mechanics, it will not be discussed any further.

Throughout the paper, we assume a Schwarzschild geometry in the exterior of the ultra-compact object and focus primarily on the case of \( D = 3+1 \) spacetime dimensions. This choice of \( D \) is made for the sake of clarity as none of our conclusions change for \( D > 4 \). Fundamental constants and numerical factors are often neglected unless needed to establish an exact equality and not just a scaling relation. A prime on a function denotes a radial derivative.

The rest of the content is organized as follows: First, the Buchdahl bound is directly addressed in Section 2, both in general and for a specific choice of model. We will, in particular, contend that the radial pressure of matter inside the surrogate BH is negative, contrary to two of Buchdahl’s assumptions. The issue of stability is considered in Section 3 and how this all works out for a Schwarzschild-sized bound state of highly excited, long, closed, interacting strings is answered, as promised, in Section 4. The paper concludes with a brief overview in Section 5.
2 Evading the Buchdahl bound with negative pressure

Let us start by specifying the geometry and matter for the inside of a spherical, ultra-compact object of radius $R$. We assume a “Schwarzschild-like” geometry for the interior (whereas that of the exterior is precisely Schwarzschild),

$$ds^2 = -f(r)dt^2 + \frac{1}{f(r)}dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2).$$ (1)

It is further assumed that $p_r = -\rho$, as motivated in the previous section, which translates into an energy–momentum–stress (EMS) tensor of the form

$$T^\mu_\nu = \begin{pmatrix} -\rho & 0 & 0 & 0 \\ 0 & -\rho & 0 & 0 \\ 0 & 0 & p_\perp & 0 \\ 0 & 0 & 0 & p_\perp \end{pmatrix}.$$ (2)

For this setup and in units with $8\pi G = 1$, the Einstein equations

$$\left(r\ddot{f}\right)' = 1 - \rho r^2,$$ (3)

$$\left(r f''\right) = 2rp_\perp,$$ (4)

can be combined into

$$\left(\rho r^2\right)' = -2rp_\perp,$$ (5)

which is guaranteed by the covariant conservation of the EMS tensor.

Defining

$$m(r) = 4\pi \int_0^r dx x^2 \rho(x)$$ (6)
and taking into account that \( m(R) = M \) is the total mass of the ultra-compact object, one finds that solving Eqs. (3) and (4) leads to

\[
f(r) = \tilde{f}(r) = 1 - \frac{2m(r)}{r}.
\]

(7)

The value of \( p_\perp \) is determined by the function \( m(r) \), which is determined in turn by \( \rho(r) \). Additionally, \( \rho \) (and likewise for \( p_r \) and \( p_\perp \)) has to vanish for \( r \geq R \). According to Buchdahl, the function \( f(r) \) and its first derivative are supposed to be continuous across \( r = R \).

It is instructive to consider some explicit examples that satisfy \( p_r = -\rho \). To this end, let us then parametrize \( \rho = Br^\alpha \), where \( B \) is a dimensional constant. A well-known example is the gravastar solution [20], for which \( \alpha = 0 \) or \( \rho = \text{const.} \). In this case, \( p_\perp = -\rho \), so that the interior of the ultra-compact object, \( r \leq R \), is part of a de Sitter spacetime. It follows that \( f(r) = 1 - r^2/R^2 \) on the inside, whereas \( f(r) = 1 - 2M/r \) with \( M = \frac{4}{3}\pi BR^3 \) is the outer solution. It is clear that the function \( f(r) \) is continuous across the outermost surface but its derivative is not. In [14], this issue was resolved by adding a surface layer of matter with a surface tension that is determined by the continuity condition on \( f'(r) \).

Another solution of this sort is described by our “collapsed polymer” model [18, 21, 22] (and see Section 4), for which \( \alpha = -2 \). In this case and for \( r \leq R \), then \( f = 0 \) and

\[
\begin{align*}
r^2\rho &= \frac{1}{8\pi G}, \quad (8) \\
r^2p_r &= -\frac{1}{8\pi G}, \quad (9) \\
p_\perp &= 0, \quad (10)
\end{align*}
\]
with the units chosen to yield \( m(r) = \frac{1}{2} r \). Continuity across the outer surface then requires \( \rho \) and \( p_r \) to vanish as \( r \to R \). This ensures that the function \( m(r) \) approaches a constant at the surface, which then allows \( f'(r)|_{r \to R} \sim 1/R^2 \) on the inside to be matched to its outer Schwarzschild value of \( 2M/R^2 \). Consistency further requires the energy density of the interior matter to transition from a power law to zero in a smooth way as \( r \to R \). This is accompanied by some positive transverse pressure which also vanishes smoothly in the same limit (cf. Eq. (5)) but, unlike the gravastar, no additional layer of matter is needed.

The vanishing of the metric function \( f \) everywhere inside is not a problem because the Einstein equations, curvature tensors, curvature invariants and metric determinant are all as regular as they would be at the horizon of a conventional Schwarzschild BH. However, as far as solutions of GR go, this one does seem rather peculiar, which might be why Bondi chose not to pursue it any further \([13]\). A heuristic way of understanding this solution goes as follows.

Let us recall that the radial pressure of any form of matter just outside a Schwarzschild horizon has to satisfy the condition \( p_r = -\rho \) \([16]\). A subtle consequence of this constraint is that each spherical slice of the solution has to behave just like a horizon if its outer “skin” is peeled away (as one would peel an onion). And yet, by Gauss’ Law along with spherical symmetry, the presence of the outer skin is irrelevant to the inside. It can then be concluded that \( p_r = -\rho \) has to be the physical radial pressure throughout the interior matter. The vanishing of the transverse pressure and the metric function then follow from the Einstein equations.
Staying with the polymer model, one can also calculate the gravitational, or Komar, mass density $\rho_{\text{grav}}$ in the standard way \cite{15},

$$
\rho_{\text{grav}} = \rho + \sum p_i = 0.
$$

(11)

This density determines, locally, the strength of the gravitational field that is produced by an object (see, e.g., Section 6 of \cite{10}) and is thus relevant to an object’s self-gravitation. The vanishing of the Komar mass density provides yet another hint as to how the matter can avoid collapse. There is, however, still the issue of stability, which we address next.

### 3 Hydrodynamic equilibrium and stability

In the previous section, it was shown that the solution in Eqs. (8)-(10) can describe a Schwarzschild-sized, spherical mass distribution. We will now show, following Chandrasekhar \cite{11, 12}, that such a distribution can be both in hydrodynamic equilibrium and stable against radial oscillations. In fact, it will be further shown that these oscillations are completely absent in the equilibrium state.

The relativistic hydrodynamic equations can be cast in the form of conservation equations,

$$
\partial_{\mu} (\sqrt{-g} \rho u^\mu) = 0,
$$

(12)

$$
\partial_{\mu} (\sqrt{-g} T^\mu_{\nu}) = 0.
$$

(13)

where $u^\mu = (\gamma, \gamma \vec{v})$ is the 4-velocity of the fluid, $\vec{v}$ is the 3-velocity and $\gamma = 1/\sqrt{1 - \vec{v}^2}$.
In our case, the standard expression for the EMS tensor of a perfect fluid, 
\( T^{\mu \nu} = (\rho + p) u^\mu u^\nu + pg^{\mu \nu} \), reduces to (cf, Eq. (2)),

\[
T^0_0 = -\rho , \tag{14}
\]
\[
T^r_r = -\rho , \tag{15}
\]
\[
T^r_0 = 0 . \tag{16}
\]

Equations (12), (13) lead to three more equations,

\[
\partial_t (r^2 \rho \gamma) + \partial_r (r^2 \rho \gamma v_r) = 0 , \tag{17}
\]
\[
\partial_t (r^2 \rho) = 0 , \tag{18}
\]
\[
\partial_r (r^2 \rho) = 0 . \tag{19}
\]

Hydrodynamic equilibrium requires that the 4-velocity remains constant, from which \( \gamma, v_r = \text{const.} \) follows as well. Amazingly, each of the terms in Eqs. (17)-(19) then vanishes for hydrodynamic equilibrium simply because \( r^2 \rho \) in Eq. (8) is a temporal and radial constant.

The stability analysis proceeds by perturbing the metric, the velocity and the EMS tensor by small perturbations that depend on \( r \) and \( t \). The full process is described explicitly by Chandrasekhar in Sect. IV of [12]. This analysis is straightforward but quite long and technical and will not be repeated here. Rather, we will briefly explain our results while referring to some expressions in [11], which is a shorter companion paper to [12].

For our case with \( p_r + \rho = 0 \) and \( g_{tt} = g^{rr} = 0 \) for the background solution, the perturbations to the off-diagonal elements of the EMS tensor are trivially vanishing at linear order, whereas the linear perturbations \( \delta \rho \), \( \delta g_{rr} \) and \( \delta p_r \) vanish according to Eqs. (10), (6) and (7) in [11], respectively.
Let us recall these expressions,

\[ \delta \rho = \left[ r^2 (p_{r0} + \rho_0) \xi \right]' / r^2, \]  
\[ (r g_{0r}^r \delta \lambda)' = -r^2 \delta \rho, \]  
\[ r^2 \delta p_r = g_{0r}^r / \left(1 + r \nu'_0\right) \delta \lambda - r g_{0r}^r \delta \nu', \]  

where \( g_{rr} = e^\lambda, \) \( g_{tt} = -e^{\nu}, \) a subscript 0 indicates a background quantity and \( \xi \) is defined implicitly through \( v_r = \frac{\partial \xi}{\partial t}. \)

The vanishing of \( \delta g_{tt} \) is more subtle as \( \delta \nu \) is always accompanied by a factor of either \( p_{r0} + \rho_0 \) or \( g_{0r}^r. \) The easiest way to see that \( \delta g_{tt} \) similarly vanishes is to impose that the determinant of the perturbed metric is regular, which necessitates that \( g_{tt} = -g_{rr} \) to all orders. Alternatively, one could regularize the leading-order components by regarding \( g_{0r}^r \) and \( g_{tt}^0 \) as small constants and take them to zero at the end. One then finds that \( \delta g_{tt} \) vanishes given that all the other perturbations do.

Now, since the perturbations are all vanishing and the velocity must be constant, there is no opportunity for radial oscillations to occur. One can further verify this result by inspecting some additional equations in [11]. We conclude that the equilibrium configuration of a spherical matter distribution with \( p_r = -\rho \) is indeed stable.

4 Matter with negative pressure: the inside story

We have presented arguments supporting the idea that matter with negative radial pressure can resist gravitational collapse, even when it is confined to a
Schwarzschild-sized region. In light of these arguments, it is worth recalling the earlier-posed questions: What type of matter can satisfy \( p_r = -\rho \) and how can such matter find its way inside of BH-like, ultra-compact objects?

Our proposed answer to either query is to consider a bound state of long, closed, interacting strings at temperatures just above the Hagedorn temperature; what we have been calling the collapsed polymer model \[21\]. This proposal was inspired in part by \[23, 24\] and motivated by the observation that the BH interior has to be in a strongly non-classical state \[17, 18, 19\], even at times well before the Page time \[25\]. Fundamental strings could be produced out of whatever form of matter that collapses to form a BH, as long as the resulting string state is entropically favored. This answers the latter of the above questions, but addressing the former will require some additional work.

The “elephant in the room” is that such stringy matter actually possesses positive pressure \( p_r = +\rho \) \[21\]. How then can this be reconciled with the requirement that \( p_r = -\rho \)? As discussed in the Introduction, negative normal pressure, or tension, is a common phenomenon in solid matter and especially in polymers. It is then a natural tendency to expect the collapse-resistant matter to be composed of tensile material. The purpose of this section is to explain that this tendency is unwarranted.

The outside observer, who knows only about GR and Einstein’s equations, will need to create a narrative explaining why the matter does not collapse. She reaches the conclusion that the radial pressure is negative. But an observer who knows about the microphysical properties of the material and, in particular, its entropy density and temperature, would naturally use
the free energy density to decide on the local value of the pressure. Because of this disparity in perspectives, the relationship between \( p_r \) and \( \rho \) and, indeed, even the sign of \( p_r \) could be different for different observers.

The intrinsic reason that a state of highly excited, interacting, long, closed strings does not collapse can be traced to its entropy. For such a state, the entropy \( S \) is equal to the total length of the strings \( L \) in units of the string scale \( l_s \), \( S = L/l_s \), and the spatial configuration of the strings can be viewed as an \( N \)-step random walk with \( N \sim S \). Free strings occupy a region in space whose linear size \( R \) is the random-walk scale, \( R \sim \sqrt{N} \), but attractive interactions result in a smaller value of \( R \) \[26, 27\]. And so, for (attractively) interacting strings, one can expect that \( R \sim N^\nu \) with \( \nu \leq 1/2 \) and, for an area law (as in the case of BHs), the condition becomes \( \nu = 1/(d-1) \) in \( D = d+1 \) dimensions. This may be \( \nu = 1/2 \) when \( D = 4 \), but the size of the random walk is still parametrically smaller than that of the free-string case, as now one finds \( R \sim g\sqrt{N} \) with \( g < 1 \) being the strength of the string coupling.

The only way that the strings can collapse further and occupy a smaller region in space is by splitting up into a collection of parametrically smaller strings. However, a configuration of many smaller strings is strongly disfavored in comparison to a few long strings because the entropy of latter is substantially larger.

Let us briefly summarize the calculation of the radial pressure \( p_r \) from the internal perspective. (We assume \( p_\perp = 0 \) on the basis of spherical symmetry.) At equilibrium, the energy density \( \rho \) and entropy density \( s \) in
the collapsed polymer are, for a certain choice of units, expressible as [18, 21]

\[ \rho = \frac{1}{2} g^2 r^2 , \quad (23) \]

\[ s = \frac{1}{g^2 r} , \quad (24) \]

with neither of the relations depending on \( D \). Note that, here, \( r \) is the radial coordinate for a fiducial coordinate system.

Writing the entropy density as a function of the energy density,

\[ g^2 s = \sqrt{2g^2 \rho} , \quad (25) \]

one can obtain the (inverted) temperature

\[ \frac{1}{T} = \frac{\partial s}{\partial \rho} = \frac{s}{2\rho} , \quad (26) \]

which leads to

\[ sT = 2\rho . \quad (27) \]

The pressure can now be directly evaluated,

\[ p_r = sT - \rho = +\rho . \quad (28) \]

And so \( p_r = +\rho \), as could have been anticipated [28]. This “internal” equation of state reveals that signals propagate at the speed of light along a closed string. It is not, however, a statement about the interior geometry, as a semiclassical description of the metric is invalidated by strong quantum fluctuations [18, 19] (and see below). In short, what one perceives from the outside as being matter under tension turns out to be a completely different form of matter.

14
Let us remark, in passing, that Eq. (25) leads to the scaling relation

$$S \sim \sqrt{\frac{EV}{G}}$$

where $S = sV$, $E = \rho V$, $V \sim R^d$ and $G \sim g^{2d-1}$. In other words, the internal matter saturates the causal entropy bound [29] throughout its volume, signaling the breakdown of classical GR.

5 Conclusion

The moral of this paper is as follows: Negative pressure is, in all likelihood, a necessary condition for avoiding gravitational collapse. The same conclusion was reached (and then abandoned) long ago by Bondi [13] and recently revisited in [14]. Whereas the focus of [14] was on the gravastar solution, we illustrated the idea with our model for the BH as a bound state of highly excited, interacting, long, closed strings.

We have seen, however, that the pressure is positive from the perspective of an observer who could (somehow) probe the interior. Given this discrepancy, how does one know that our external and internal descriptions apply to the same matter system? The answer is simple: These are both describing a compact object whose every spherically concentric layer behaves just like a BH horizon. This follows externally from the perceived form of the inside metric, $g_{tt} = g_{rr} = 0$, and internally by virtue of the area law being saturated throughout, $s(r)r^d \sim r^{d-1}$.

The observation that negative pressure need not be associated with a tensile material could have far-reaching implications, as the meaning of $p < 0$ in a solution to Einstein’s equations is no longer certain. For instance, the
accelerated expansion of the Universe may not be due to a negative-pressure fluid — the so-called dark energy — after all. The identity $p = -\rho$ may instead be a relation that internal de Sitter observers introduce to explain the accelerated expansion. On the other hand, if these observers are missing out on some contribution to the entropy, there might be others who attribute the expansion of spacetime to matter with a positive pressure. Indeed, signals propagate through the “cosmological fluid” at the speed of light, same as for the interior of our polymer model. We hope to further pursue this line of inquiry at a later time.

**Acknowledgments**

We would like to thank Emil Motolla for explaining his work to us and critically reading ours, and also numerous colleagues who encouraged us to pursue and answer the question in the title. The research of AJMM received support from an NRF Incentive Funding Grant 85353, an NRF Competitive Programme Grant 93595 and Rhodes Research Discretionary Grants. The research of RB was supported by the Israel Science Foundation grant no. 1294/16. AJMM thanks Ben Gurion University for their hospitality during his visit.

**References**

[1] S. W. Hawking, “Breakdown of Predictability in Gravitational Collapse,” Phys. Rev. D 14, 2460 (1976).
[2] N. Itzhaki, “Is the black hole complementarity principle really necessary?,” arXiv:hep-th/9607028.

[3] S. D. Mathur, “What Exactly is the Information Paradox?,” Lect. Notes Phys. 769, 3 (2009) [arXiv:0803.2030 [hep-th]].

[4] S. L. Braunstein, S. Pirandola and K. Zyczkowski, “Entangled black holes as ciphers of hidden information,” Physical Review Letters 110, 101301 (2013) [arXiv:0907.1190 [quant-ph]].

[5] A. Almheiri, D. Marolf, J. Polchinski and J. Sully, “Black Holes: Complementarity or Firewalls?,” JHEP 1302, 062 (2013) [arXiv:1207.3123 [hep-th]].

[6] D. Marolf and J. Polchinski, “Gauge/Gravity Duality and the Black Hole Interior,” Phys. Rev. Lett. 111, 171301 (2013) [arXiv:1307.4706 [hep-th]].

[7] S. D. Mathur, “What does strong subadditivity tell us about black holes?,” Nucl. Phys. Proc. Suppl. 251-252, 16 (2014) [arXiv:1309.6583 [hep-th]].

[8] R. Penrose, “Gravitational Collapse and Space-Time Singularities,” Phys. Rev. Lett. 14, 57 (1965).

[9] S. W. Hawking and R. Penrose, “The singularities of gravitational collapse and cosmology,” Proc. R. Soc. Lond. A 314, 529 (1970).

[10] H. Buchdahl, “General Relativistic Fluid Spheres,” Phys. Rev. 116, 1027 (1959).
[11] S. Chandrasekhar “Dynamical Instability of Gaseous Masses Approaching the Schwarzschild Limit in General Relativity,” Phys. Rev. Lett. **12**, 114 (1964).

[12] S. Chandrasekhar, “The Dynamical Instability of Gaseous Masses Approaching the Schwarzschild Limit in General Relativity,” Astrophys. J. **140**, 417 (1964).

[13] H. Bondi, “Massive spheres in general relativity,” Proc. Roy. Soc. Lond. A **282**, 303 (1964).

[14] P. O. Mazur and E. Mottola, “Surface tension and negative pressure interior of a non-singular black hole,” Class. Quant. Grav. **32**, no. 21, 215024 (2015) [arXiv:1501.03806 [gr-qc]].

[15] A. Komar, “Positive-Definite Energy Density and Global Consequences for General Relativity”, Phys. Rev. **129**, 1873 (1963).

[16] A. J. M. Medved, D. Martin and M. Visser, “Dirty black holes: Space-time geometry and near horizon symmetries,” Class. Quant. Grav. **21**, 3111 (2004) [gr-qc/0402069].

[17] L. Alberete, R. Brustein, A. Khmelnitsky and A. J. M. Medved, “Density matrix of black hole radiation,” JHEP **1508**, 015 (2015) doi:10.1007/JHEP08(2015)015 [arXiv:1502.02687 [hep-th]].

[18] R. Brustein and A. J. M. Medved, “Quantum state of the black hole interior,” JHEP **1508**, 082 (2015) [arXiv:1505.07131 [hep-th]].
[19] R. Brustein, A. J. M. Medved and Y. Zigdon, “The state of Hawking radiation is non-classical,” JHEP 1801, 136 (2018) [arXiv:1707.08427 [hep-th]].

[20] P. O. Mazur and E. Mottola, “Gravitational condensate stars: An alternative to black holes,” arXiv:gr-qc/0109035.

[21] R. Brustein and A. J. M. Medved, “Black holes as collapsed polymers,” Fortsch. Phys. 65, no. 1, 1600114 (2017) [arXiv:1602.07706 [hep-th]].

[22] R. Brustein and A. J. M. Medved, “Emergent horizon, Hawking radiation and chaos in the collapsed polymer model of a black hole,” Fortsch. Phys. 65, no. 2, 1600116 (2017) [arXiv:1607.03721 [hep-th]].

[23] P. Salomonson and B.-S. Skagerstam, “On superdense superstring gases: A heretic string model approach,” Nucl. Phys. B 268, 349 (1986).

[24] D. A. Lowe and L. Thorlacius, “Hot string soup,” Phys. Rev. D 51, 665 (1995) [hep-th/9408134].

[25] D. N. Page, “Average entropy of a subsystem,” Phys. Rev. Lett. 71, 1291 (1993) [arXiv:gr-qc/9305007]; “Information in black hole radiation,” Phys. Rev. Lett. 71, 3743 (1993) [arXiv:hep-th/9306083].

[26] G. T. Horowitz and J. Polchinski, “Selfgravitating fundamental strings,” Phys. Rev. D 57, 2557 (1998) [hep-th/9707170].

[27] T. Damour and G. Veneziano, “Selfgravitating fundamental strings and black holes,” Nucl. Phys. B 568, 93 (2000) [hep-th/9907030].
[28] J. J. Atick and E. Witten, “The Hagedorn transition and the number of degrees of freedom in string theory,” Nucl. Phys. B 310, 291 (1988).

[29] R. Brustein and G. Veneziano, “A Causal entropy bound,” Phys. Rev. Lett. 84, 5695 (2000) [hep-th/9912055].