NONZERO $\Omega_\Lambda$ AND A NEW TYPE OF THE DISSIPATIVE STRUCTURE

Yasunori Fujii

Nihon Fukushi University, Handa, 475-0012 Japan
and
ICRR, University of Tokyo, Tanashi, Tokyo, 188-8502 Japan

Abstract
We revisit the proposed theoretical model for a small but nonzero cosmological constant which seems supported increasingly better by recent observations. The model features two scalar fields which interact with each other through a specifically chosen nonlinear potential. We find a very sensitive dependence of the solutions of the scalar field equations on the initial values. We discuss how the behavior is similar to and different from those in well-known chaotic systems, coming to suggest an interesting new type of the dissipative structure.

The cosmological constant problem has two faces; an upper bound and a lower bound. The former is given by the critical density $\rho_{cr}$, or $\Omega_\Lambda \equiv \Lambda/\rho_{cr} \lesssim 1$. In the Planckian unit system with $8\pi G = c = \hbar = 1$, we know $\rho_{cr} \equiv 3H_0^2 \sim t_0^{-2}$, and the present age of the universe $t_0 \sim 10^{10}$Gy is of the order of $10^{60}$; hence the upper bound $\Lambda_{ob} \lesssim 10^{-120}$. On the other hand, almost any theoretical models of unification gives $\Lambda_{th} \sim 1$. The discrepancy is the well-known disaster. Recent observations seem to indicate a lower bound as well; $\Omega_\Lambda \sim 0.7$. Let us try to understand this “small but nonzero cosmological constant.”

A solution for the upper bound may come from a scalar field $\phi$ of the Brans-Dicke type:

$$\mathcal{L} = \sqrt{-g}\left(\frac{1}{2}\xi \phi^2 R - \frac{1}{2}\epsilon g^{\mu\nu} \partial_{\mu}\phi \partial_{\nu}\phi + \Lambda + L_{\text{matter}}\right),$$

where $\xi$ is a dimensionless constant related to BD’s parameter $\omega$ by $\omega \xi = 1/4$, and $\epsilon = \pm 1$. The scalar field contribution is shown to cancel the effect of $\Lambda$ asymptotically. To see this, it is most convenient to apply a conformal transformation moving to the Einstein frame (as denoted by $\ast$) with nonminimal coupling removed. The same Lagrangian as Eq. (1) is now put into

$$\mathcal{L} = \sqrt{-g_\ast}\left(\frac{1}{2}R_\ast - \frac{1}{2}\partial_\mu\sigma \partial_\nu\sigma - V(\sigma) + L_{\ast \text{matter}}\right),$$

*Based on the talk delivered at XXXIIIrd Rencontres de Moriond, Fundamental Parameters in Cosmology (Les Arcs, France, January 17-24, 1998)
†E-mail address: fujii@handy.n-fukushi.ac.jp
where the new canonical scalar field $\sigma$ is related to the original $\phi$: $\phi = \xi^{-1/2} e^{\xi \sigma}$, with $\xi = (6 + \epsilon \xi^{-1})^{-1/2}$. We also find that $\Lambda$ now acts as a potential $V(\sigma)$:

$$V(\sigma) = \Lambda e^{-4\zeta \sigma},$$

(3)

which pushes $\sigma$ toward infinity; the effect of $\Lambda$ continues to decrease.

In spatially flat RW cosmology, the cosmological equations are

$$3H^2 = \rho_\sigma + \rho_m \equiv \rho, \quad \text{and} \quad \ddot{\sigma} + 3H\dot{\sigma} + V'(\sigma) = 0,$$

(4)

where we omit the symbol $*$ to simplify the notation. We should also supply the equation for the matter density $\rho_m$ as usual. Notice that $\rho_\sigma = \frac{1}{2}\dot{\sigma}^2 + V(\sigma)$ is the energy density of the scalar field to be interpreted as an “effective cosmological constant” $\Lambda_{\text{eff}}$.

As an asymptotic solution, we obtain (for assumed radiation dominance)

$$a(t) = t^{1/2}, \quad \text{and} \quad \sigma(t) = \bar{\sigma} + \frac{1}{2}\zeta^{-1}\ln t,$$

(5)

$$\rho(t) = \frac{3}{4}\left(1 - \frac{1}{4}\zeta^{-2}\right)t^{-2}, \quad \text{with} \quad \rho_\sigma(t) = \frac{3}{16}\zeta^{-2}t^{-2}.$$

(6)

The last equation shows that $\Lambda_{\text{eff}} = \rho_\sigma$ does decay like $t^{-2}$, hence implementing the scenario of a “decaying cosmological constant.” Today’s $\Lambda$ is small only because our universe is old [1].

We notice, however, that $\Lambda_{\text{eff}}(t)$ falls off in the same way as the ordinary matter density $\rho_{\text{matter}}(t)$ does. Not qualified to be called a “constant,” it is simply another form of dark matter. It does not help understanding the lower bound. We must include something that behaves more or less like a constant or at least falls off more slowly than $t^{-2}$. On the other hand, the behavior $\sim t^{-2}$ was the key to have a “small” $\Lambda$. As a compromise we expect that $\Lambda_{\text{eff}}(t)$ would decay like $\sim t^{-2}$ as an overall behavior, but with some “landings,” in one of which we today happen to be, and which mimics a cosmological “constant” for some duration of time.

Trying to implement the idea, we came across a model on a try-and-error-basis, by introducing another scalar field $\Phi$ which couples to $\sigma$ [2,3]. The Lagrangian is the same as Eq. 2 with the added kinetic term of $\Phi$ and $V(\sigma)$ replaced by (see Fig. 2 of Ref. 2)

$$V(\sigma, \Phi) = e^{-4\zeta \sigma}\left(\Lambda + \frac{1}{2}m^2\Phi^2 [1 + \beta \sin(\kappa \sigma)]\right),$$

(7)

where $m, \beta, \kappa$ are the constants naturally of the order unity. It has a “central valley” given by Eq. [3], but shows an oscillatory behavior $\sim \sin(\kappa \sigma)$ if we climb the wall in the direction of $\Phi$. 
A typical solution is shown in Fig. 1. We started the integration at some time in the post-inflation era. We also mildly fine-tuned initial conditions such that $\Omega_\Lambda$ somewhat smaller than 1, say, at $t_0 \approx 10^{60}$. Notice a sequence of alternate occurrence of a rapid change and a nearly standstill of the scalar fields. Also toward the end of each “landing” of $\rho_s(= \Lambda_{\text{eff}}(t))$, which is now the energy density of the coupled $\sigma$-$\Phi$ system, the scale factor $a(t)$ shows a mini-inflation, an expected behavior. Magnifying a portion of Fig. 1(b) around the present time ($\log t_0 \sim 60$) extending to a tiny interval ($\Delta \log t \sim 1$) yields a “practical” plot going back to $z \sim 5$, showing $\Lambda_{\text{eff}}$ which looks nearly constant.

These landings, as indicated also by other examples, tend to occur nearly periodically with respect to $\ln t$. This seems to suggest that some cosmological anomalies might have taken place nearly periodically with respect to $\ln t$.

As we find, the results depend on the choice of the parameters quite sensitively, particularly on the initial conditions. In this connection, we first point out that the most important ingredients are present in the dynamics of the system of the two scalar fields. We find the alternate occurrence of the “wake-up phase” and the “dormant phase” of the scalar fields even without the cosmological environments, provided the frictional coefficients $3H$ vary as $\sim t^{-1}$.

Fig. 2 is one of the examples of the solutions of the isolated 2-scalar system with the same potential with the frictional constants $\Gamma = \gamma/t$. Suppose we start from an initial position off the central valley on the potential Eq. 7. $\sigma$ will be pushed in the positive direction. The potential will decrease quickly because it falls off like $e^{-4\zeta \sigma}$. Soon both fields will be virtually free. Due to the frictional forces they will be decelerated almost to a complete stop. However, as the time goes on, the frictional forces will also decrease according to $\sim t^{-1}$. They will become even weaker than the forces derived from the potential which had much dwindled. Then the fields are suddenly pushed again. But the potential will be weakened soon again leading to the next dormant period.

However, in order for the fields to be pushed strongly after the dormant phase, their positions and velocities must “match” the previous values to some extent. Otherwise, the fields will not be waken up, beginning simply to fall the potential slope slowly toward and along the central valley. This is the asymptotic behavior, as we find in this example, reached commonly for any initial conditions. Obviously this corresponds to a “fixed-point attractor,” like in standard damped oscillators.

The “matching condition” is rather subtle. This is the reason why a slight change of initial values may sometimes result in a considerable change in the behaviors afterward. This sensitivity to the initial values reminds us of a well-known chaotic behavior in nonlinear systems. Sudden change of the number of repetitions before reaching the
smooth asymptotic behavior, for example, might be compared to sudden occurrence of “bifurcation” in typical chaotic solutions.

However, there is an important difference from purely chaotic systems. Usually a chaotic behavior is characterized by “strange attractors,” while our solutions tend to a simple fixed-point attractor. Complicated hard-to-predict states occur only during a finite transient period. In accordance with this no fractal structure is produced in the orbits in phase space; fractal structure might be present only to a limited “depth.”

We thus find what might be called an “incomplete chaos” or an “incomplete fractals.” They can be still interesting because genuine chaos might be a mathematical idealization for natural phenomena. Also chaotic behaviors are seen almost every aspect of life. But obviously life lasts only during a limited lifetime; the ultimate destination is a non-chaotic death, corresponding to a fixed-point attractor. In this sense, our model might provide a primitive example of a living system. On the other hand, it seems to give a model of pattern formation (with variable period).

In our model, near periodicity in $\ln t$ is a consequence of $\Gamma \sim t^{-1}$. There are many examples of pattern formation with some periodicity. Patterns on sea shells, for example, can be reproduced by certain nonlinear equations of rather simple structure [4]. Periods of these patterns are not determined by the restoring force but, unlike in simple systems, primarily by friction or dissipation in addition to the coupling strength. This feature is shared in our model. In this sense also, our nonlinear equations might provide another example of “dissipative structure.” It is interesting to notice that a model of game dynamics with more than three species also generates periodicity with respect to $\ln t$ [4].

Did we introduce as many degrees of freedom and parameters as we need? The answer: the cosmological constant problem is so challenging that we may not have reached even close to any promising result no matter how many ingredients we introduced if we have come in a wrong direction. It seems we are on a right track.

We thank Kunihiko Kaneko and Takashi Ikegami for enlightening discussions on nonlinear systems.

References

[1] Y. Fujii and T. Nishioka, Phys. Rev. D42(1990), 361, and papers cited therein.

[2] Y. Fujii and T. Nishioka, Phys. Lett. 254(1991), 347.

[3] Y. Fujii, Astrop. Phys. 5(1996), 133.

[4] H. Meinhardt, The algorithmic beauty of sea shells, Springer-Verlag, 1995.
[5] T. Chawanya, Prog. Theor. Phys. 94(1995), 163.
Figure 1: (a) $\ln a$ (solid; $a$ for the scale factor), $\sigma$ (dotted), $2\Phi$ (broken) against $\log t$. (b) $\log \rho_s$ (solid), $\log \rho_m$ (dotted). The parameters are $\Lambda = 1, \zeta = 1.582, m = 4.75, \beta = 0.8, \kappa = 10$. The initial values at $t_1 = 10^{10}$ are $\sigma_1 = 6.75442, \dot{\sigma}_1 = 0, \Phi_1 = 0.212, \dot{\Phi}_1 = 0$. At $\log t_0 = 60.15$ corresponding to the present age chosen to be $1.21 \times 10^{10}$y, we obtain $H_0 = 81$km/s/Mpc and $\Omega_\Lambda = 0.67$. See also Captions to Figs. 1 and 2 in Ref. 3.
Figure 2: An example of the two-scalar system, in which the solution enters the asymptotic behavior after two repeated patterns. $\sigma$ (solid) and $2\Phi$ (dotted) and the broken line for the asymptotic behavior of $\sigma$. $\Phi$ will tend to zero slowly. The parameters and the initial values are the same as those in Fig. 1 except for the frictional coefficient chosen to be $(3/2)t^{-1}$. See also Caption to Fig. 3 in Ref. 3. A slight change of the initial values may result in a sudden change of the repetition number to three, for example, also changing the time of the end of the repetition period.