A tale of two condensates: 
the odd ”Bose - Einstein” condensation 
of atomic Hydrogen.

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Abstract
The recent report of the observation of Bose-Einstein condensation in atomic Hydrogen, characterized by an ”anomalous” density spectrum, is shown to be in agreement with the prediction of the existence of two condensates for temperatures lower than a well defined temperature (which for Hydrogen is 105 $\mu K$ ), based on the QED coherent interaction in a gas of ultracold atoms at a density $n > n_0$ ($n_0 = \left( \frac{1}{\lambda} \right)^3$, $\lambda$ being the wave-length of the e.m. modes resonantly coupled to the Hydrogen atoms).
The last few years have seen a remarkable surge of activity in the physics of ultracold atoms that has led to the discovery of a phase transition from the gaseous state to a new state, characterized by a very narrow momentum distribution and (relatively) high density. By combining definite improvements in the techniques of magnetic trapping and r.f. evaporative cooling in 1995, within a few months, three independent groups were able to show such transition in the dilute alkali gases of \( \text{Rb} \), \( \text{Na} \) and \( \text{Li} \). Only a few months ago another group, through a remarkable experimental feat, was able to detect such transition in atomic Hydrogen, thus confirming the (apparent) universality of its nature.

An exciting aspect of such discoveries is that they appear to finally give, after 70 years of waiting, the confirmation of a fundamental prediction that, in the framework of the revolutionary quantum physics, S. N. Bose and A. Einstein were able to derive from the general ideas of the statistical thermodynamics of Maxwell and Boltzmann. According to Bose and Einstein, in order for the perfect gas to obey the "heat theorem" of W. Nernst (better known as the third principle of thermodynamics) above a certain (number) density, the Bose-Einstein density (we use throughout this paper the natural units system, where \( \hbar = c = k_B = 1 \))

\[
n_{BE} = 2.612 \left( \frac{mT}{2\pi} \right)^{\frac{3}{2}},
\]

where \( m \) is the mass of the atoms, a number of atoms (\( V \) is the volume of the gas)

\[
N_c = (n - n_{BE})V
\]

leaves the chaotic world of the gaseous state to populate the ground state with momentum \( \vec{p} = 0 \). In this way Bose and Einstein finally showed how nature manages to achieve in a continuous manner zero entropy at \( T = 0 \), as demonstrated by Nernst in a countless number of physical systems, in disagreement with the highly singular behavior predicted by classical physics.

The fact that the transition (apart from the notable exception of Ref.\[1\]) occurs at the predicted value (1), and its most remarkable signal is the dramatic narrowing of the momentum and space distributions of a number of atoms, in agreement with the predictions based on the structure of the magnetic trap, left no room for doubting that Bose-Einstein condensation finally belongs to the realm of natural phenomena. The equally dramatic discovery, a year later, of stunning interference patterns, with the typical de-Broglian modulation, has begun, however, to raise doubts in some of us that the observed condensation could in fact be exactly what was predicted by Bose and Einstein. The problem is an old one and in the case of superfluidity and superconductivity has been intensely argued by leading theoretical physicists: it boils down to the simple question: can a Bose-Einstein condensate (BEC) exhibit a macroscopic phase? A straightforward analysis of the Quantum Field Theory (QFT) of the statistical thermodynamics of a perfect gas (which the observed dilute atomic systems approximate very accurately) unambiguously shows that the BEC cannot posses a well defined phase for lack of "sufficient reason". Indeed the non-interacting nature of the condensed atoms prevents them from developing the peculiar phase relations that characterize a coherent state. The situation might change if some interaction is
introduced, but, as we have argued in Ref. 10, it appears impossible to generate a phase (and a robust one, as experimentally shown in a subsequent experiment [11]) with the short range interactions that are available in the generally accepted approach to condensed matter. Thus the surprising results of Ref. [8] and Ref. [11], in the light of the essentially negative answer to the question whether a BEC may acquire a macroscopic phase, appear to exclude that the condensates observed in Refs. [1], [2] and [3] are BEC's. If not BEC's what else can they be? In a recent paper [10] we have examined the problem in the framework of QED, and of its unexpected and neglected coherent long-range interactions [12]. We have shown that when the density $n_c$ of the atomic systems in the condensate is larger than

$$n_0 = \left(\frac{1}{\lambda}\right)^3,$$

(3)

where $\lambda$ is the wave-length of the e.m. transition between the ground state and the first excited state, a completely new type of condensation may occur, driven by the energy gain that the system can achieve by letting its atoms oscillate in phase with resonant modes (of wave-length $\lambda$) of the e.m. field. Let's now summarize the main results of the analysis of Ref. [10]:

i) in the new condensed phase, which we shall call Coherent Electrodynamic Condensate (CEC), an average energy gap $\bar{\delta}$ develops, whose value is given by

$$\bar{\delta} = \frac{3}{16\pi^2 m_e} \omega^2 \ell x_c = \delta_0 x_c,$$

(4)

where $\omega = \frac{2\pi}{\lambda}$ is the energy of the atomic transition, $f$ its oscillator strength and $x_c (0 \leq x_c \leq 1)$ the fraction of atoms that belong to the CEC;

ii) the fraction $x_c$ is given by the solution of the following equation

$$1 - x_c = \frac{1}{n_c} \left(\frac{mT}{2\pi}\right)^\frac{3}{2} f \left(\frac{\delta_0 x_c}{T}\right),$$

(5)

where

$$f(x) = \frac{2}{\sqrt{\pi}} \int_0^\infty \frac{dt t^\frac{3}{2}}{(e^{t+x} - 1)}.$$  

(6)

By use of Eq. [10] we can cast (5) in the form

$$\frac{1 - x_c}{x_c} = \frac{n_{BE}}{n_0} \frac{f \left(\frac{\delta_0 x_c}{T}\right)}{2.612};$$

(7)

iii) when $n_{BE} < n_0$, and this happens when $T < T_{BEC}$, where

$$T_{BEC} = \frac{2\pi}{m} \left(\frac{n_0}{2.612}\right)^\frac{2}{3},$$

(8)
there is a density interval

\[ n_{BE} < n < n_0 \]  

where our theory predicts an incoherent BEC, in full agreement with the theory of Bose and Einstein.

In table I we report the relevant parameters of our theory for H, Li, Na and Rb respectively, while in Figs. 1, we show the phase plane \( T - n \) for the different atomic species. Noting that in the actual experiments \( T_{Na} = 2 \mu K \) \cite{2}, and \( T_{Rb} = 140 nK \) \cite{3}, are all above their respective \( T_{BEC} \), our theory predicts that what has been observed in those experiments is indeed not a BEC but a CEC, and the observed macroscopic phases are in complete agreement with our predictions.

Let’s now focus our attention upon the very recent report \cite{4} of condensation in atomic Hydrogen, where the temperature at which the transition occurs is \( T = 44 \mu K \), definitely lower than \( T_{BEC} = 105 \mu K \). Thus our theory predicts the existence of two condensates: the BEC for

\[ 1.48 \times 10^{14} \text{ cm}^{-3} < n < 5.57 \times 10^{14} \text{ cm}^{-3}, \]  

and the CEC for

\[ n > 5.57 \times 10^{14} \text{ cm}^{-3}. \]  

Is there any evidence for this prediction in the reported data? Before we answer this question, let us try to follow the steps of the transition to the condensed phase(s). When lowering the temperature from \( T = 53 \mu K \) to \( T = 44 \mu K \) the MIT group finds an extremely suggestive change in their spectra of the two-photon \( 1S - 2S \) transitions, signalling that a number \( N_c \) of the initial atoms have made a transition to the BEC. In a time determined by the typical trap parameters (milliseconds) the system reaches an equilibrium distribution \cite{4} (\( V \) is the profile of the magnetic trap)

\[ n = n_p - \frac{V(\rho, z)}{\tilde{U}} \]  

where the peak density \( n_p \) is related to the (unknown) number \( N_c \) by the formula \cite{4}

\[ N_c = \frac{16\pi \sqrt{z}}{15} \frac{\tilde{U}^3 n_p^2}{\omega_\rho^2 \omega_z m^2}, \]  

where \( \omega_\rho \) and \( \omega_z \) are the radial and longitudinal trap "pulsations" respectively, and \( \tilde{U} = \frac{4\pi a}{m} \) describes the short-range repulsion between the cold atoms. From the experimental determination \cite{4}

\[ n_p = (4.8 \pm 1.1) \times 10^{15} \text{ cm}^{-3}, \]  

Eq. (13) leads to the evaluation

\[ N_c = (1.1 \pm 0.6) \times 10^9. \]
Now, only if \( n_p > n_0 = 5.57 \times 10^{14} \text{ cm}^{-3} \) the CEC will after a while develop. Due to the average gain of \( \bar{\delta}_0 \simeq 100 \mu K \) per atom, in fact, that portion of the original BEC that lies between \( n_p \) and \( n_0 \) will fall in the CEC, leaving the rest, for which \( n < n_0 \) in the BEC phase. The experimental determination (14) shows that we are indeed in the situation where two distinct condensates coexist around the bottom of the magnetic trap. Recalling that in the frequency shift analysis of the densities of ref. [1] \( n_p \) corresponds to \( \Delta \nu_p = -(9. \pm .15) \text{ MHz} \), \( n_0 \) to \( \Delta \nu_0 = -(106.\pm20) \text{ kHz} \), while \( n_{BE} \) corresponds to \( \Delta \nu_{BE} = -(28.\pm6) \text{ kHz} \), the observed Doppler-free spectrum (Figs 2 and 3 of Ref. [4]) can be easily understood. Indeed the CEC matches perfectly the distribution of Fig. 2, including its stopping at about 150 kHz (\( \Delta \nu_0 = -(106. \pm 20) \text{ kHz} \)), the (almost) empty region in Fig. 3 between \(-100\) and \(-80\) kHz can be understood by the fact that the gas fluctuations of typical density \( n_{BE} \) (\( \Delta \nu_{BE} = -30. \text{ kHz} \) ) will cause the BEC condensate at densities within \( n_{BE} \) from \( n_0 \) to fall sooner or later in the "deeper trap" (by \(~100\mu K\)) of the CEC condensate. Finally the gas fluctuations around \( n = n_{BE} \) will tend to replenish this region of the BEC condensate. The net consequence of this (semiquantitative) discussion is that we can now explain "...spectral weight at frequency shifts much larger than expected for the maximum density in the normal gas. The origin of this effect is (not yet) understood " [4]. And we hope that our parentheses will be reckoned by the reader as an adequate, necessary modification of the discussion of the Authors of Ref. [4].

We hope that the natural, quantitative explanation of the odd-looking results on the condensation of cold atomic Hydrogen [4] achieved by our theory, rigorously based on QED, will contribute to open the eyes of the condensed matter physics community to an approach to the subject which, without violating any fundamental laws of physics, enriches the field with an extremely wide range of new interaction mechanisms, whose real potential for new, exciting developments remains still largely undisclosed.

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1 Even though at present we have not carried out this analysis, we expect that it will take a relatively long time before the CEC phase emerges from an incoherent system of ordered atoms (BEC).
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| Atom | $\lambda(A)$ | $\omega(eV)$ | $f$ | $n_0\text{ cm}^{-3}$ | $\delta_0(\mu K)$ | $\nu_{BE}\text{ cm}^{-3}(\mu K)^{\frac{3}{2}}$ | $T_{BEC}(\mu K)$ |
|------|------------|-------------|----|---------------------|-----------------|---------------------------------|-----------------|
| H    | 1215.67    | 10.34       | .44| 5.57 $10^{14}$      | 200             | 5.08 $10^{11}$                  | 105             |
| Na   | 5889.9     | 2.13        | .64| 4.89 $10^{12}$      | 18.7            | 5.62 $10^{13}$                  | 0.20            |
|      | 5895.5     | 2.13        | .32| 4.87 $10^{12}$      | 9.35            |                                 |                 |
| Li   | 6707       | 1.87        | .50| 3.31 $10^{12}$      | 11.25           | 9.40 $10^{12}$                  | 0.49            |
|      | 6707       | 1.87        | .25| 3.31 $10^{12}$      | 5.623           |                                 |                 |
| Rb   | 7800       | 1.61        | .67| 2.11 $10^{12}$      | 11.17           | 4.02 $10^{14}$                  | 0.03            |
|      | 7947       | 1.58        | .33| 2.12 $10^{12}$      | 5.58            |                                 |                 |

The CEC parameters for different atomic species.
FIGURE CAPTION

Figure 1: The phase diagram for the condensation of H Atoms. The lines are the boundaries of the regions where the Coherent Electrodynamics Condensation (CEC) occurs.