Topological solitons in optical oscillators

V V Yaparov and V B Taranenko

International Center ‘Institute of Applied Optics’ of the National Academy of Sciences of Ukraine

E-mail: yaparovwork@mail.ru

Received 29 February 2016, revised 12 April 2016
Accepted for publication 12 April 2016
Published 20 June 2016

Abstract
We present an overview of theoretical and experimental works on self-sustaining localized structures—spatial solitons—which can be formed in optical bistable oscillators with laser and/or parametric gain. The main attention is paid to the existence and dynamical properties of spatial solitons containing phase and polarization topological defects including vortices, points of circular polarizations and lines of linear polarization, domain walls and composed domain walls with Néel point topological defects.

Keywords: dissipative solitons, vortices, domain walls, vector solitons

(Some figures may appear in colour only in the online journal)

1. Introduction

Optical solitons are self-localized structures, which have been observed in various nonlinear optical systems including nonlinear optical cavities [1, 2]. The nonlinearity type defines the basic properties of optical solitons. Cavity spatial solitons, belonging to the dissipative nonlinear structures [3], are characterized by discrete parameters such as soliton width and its maximum intensity. Cavity solitons can be stored and manipulated in an optical cavity of a large Fresnel number, which makes them attractive for potential applications in all-optical storage and parallel processing as a switchable and movable carrier of information [2, 4].

In this paper we review studies of the main features of the spatial solitons existing in bistable optical oscillators with laser and/or parametric gain focusing on the existence and dynamical properties of the topological localized structures. In contrast to the passive cavities, for which the solitons are phase locked to the external driving beam, the optical oscillators can support formation of topological localized structures with free and bistable phases like vortex solitons and domain walls, respectively. The paper is organized as follows: in section 2 we provide a simple scalar model of a laser and describe laser solitons with phase point topological defects. In section 3 vector laser solitons with polarization topological defects are analyzed. In section 4 we focus on the domain wall structures formed in degenerate optical parametric oscillators. Finally in section 5 composite topological structures in optical oscillator with mixed laser and parametric gain are discussed.

2. Scalar laser solitons with phase topological defects

First, we consider a simple model of a transversely extended bistable laser containing purely dissipative nonlinearities, saturable gain and saturable absorber, placed inside a short length cavity arranged of plane parallel mirrors. The relaxation times for the gain and absorption are assumed to be small as compared with the light field lifetime in the empty cavity (the class-A laser). Since the field changes weakly during propagation between the cavity mirrors we use the mean field approximation whereby the optical field is averaged over the longitudinal coordinate $z$. The dynamics of the slow envelope (in space and time) of the optical field $E(x, y, t)$ inside such an active cavity can be well described by the generalized Ginzburg–Landau equation [2]:

$$\frac{\partial E}{\partial t} = \left( \frac{g_0}{1 + |E|^2} - \frac{a_0}{1 + b |E|^2} - 1 \right) E + (i + d) \Delta E.$$  

(1)

We use here the following notations: $t$ is the time measured in units of field decay time in the empty cavity, $g_0$ and $a_0$ are the unsaturated gain and absorption coefficients, respectively; $b$ is the ratio of gain and absorption saturation intensities; and...
nonresonant losses equal to $-1$, $d$ is the diffusion coefficient normalized to the diffraction coefficient, and $\Delta_c = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$ is the transverse Laplace operator. The transverse coordinates $x$ and $y$ are measured in units of Fresnel zone width: $w_F = \left[ \frac{L_c}{2k(1-R)} \right]^{1/2}$, where $L_c$ is the cavity length, $k$ is the wavenumber in a linear medium, and $R$ is the product of the cavity mirror reflectivity coefficients. Note that equation (1) describes a scalar case, assuming that the laser field polarization is fixed.

The homogeneous and stationary solution of equation (1) with properly selected parameters shows that the laser with a saturable absorber is intensity bistable and phase invariant. These properties are schematically illustrated in figure 1 where dependence of the complex intracavity field $E$ on the pump power $P$ is shown. One can see that there is a range of $P$ at which the laser can be in switched on or off states and the complex vectors $E$ lean on the circle manifesting the invariance to the phase shift.

The self-localized structures of the optical field were shown in [5, 6] to exist in such a bistable laser by solving equation (1) numerically using the split-step fast Fourier transform method with periodic boundary conditions. They can have both axially symmetric and asymmetric intensity profiles [7]. The phase invariance of equation (1) allows creation of phase topological laser solitons with different topological charges $(0, \pm 1)$ [8] by injection for a short time of narrow external beam into the laser cavity. The topological structure of this injecting beam defines the topology of the created soliton. If the injecting beam has a Gaussian shape, then the laser soliton has zero topological charge (figure 2(a), fundamental soliton). When the injecting beam has the phase point topological defect with positive/negative vorticity, then we obtain the laser soliton with topological charge $\pm 1$ (figure 2(b), vortex soliton). The sign of topological charge influences only the direction of the phase screw.

The fundamental laser solitons have been observed experimentally in a dye laser with a saturable absorber [6, 9, 10] in a bistable semiconductor laser [11] and in a laser-like two-wave-mixing oscillator with a saturable absorber [12]. It was shown that they can be switched on and off by external addressing pulses in different transverse locations and they can move in the phase gradient (figure 3).

The vortex solitons, carrying orbital angular momentum, have been observed experimentally in the semiconductor laser [13] and in the laser-like bistable two-wave mixing photorefractive oscillator [14] (figure 4).

We note that optical vortices can be converted into the optical domain wall structures (see section 4) via parametric rocking [15]. This has been implemented experimentally by

Figure 1. Dependence of complex intracavity electric field $E$ on unsaturated gain $g_0$ for the laser with a saturable absorber.

Figure 2. Transverse distributions of intensity (left) and phase (right) for the fundamental (a) and the vortex solitons (b).

Figure 3. Transverse motion of the fundamental soliton for the tilted resonator mirror of the multimode dye laser (top). Static soliton 1 and moving solitons 2, 3 in the near field NF corresponding to the first and second resonant rings in the far field FF in a single frequency two-wave-mixing oscillator (bottom).
injection of an amplitude modulated laser beam into the cavity of the two-wave-mixing photorefractive oscillator [16].

3. Vector laser solitons with polarization topological defects

So far, a scalar case was considered implying that the laser cavity contains polarization selective elements’ fixing polarization of the laser radiation. However, an isotropic laser, which is not only phase invariant but also polarization invariant, can support formation of transversely extended vector field with polarization topological defects [17]. Now we show that robust vector localized structures with polarization topological defects can be created in an isotropic laser when it operates in the bistable regime.

To calculate an evolution of the transverse structure for the slow-varying complex-valued vector field with x- and y-polarization components that occurs inside a laser cavity with a polarization isotropic active and passive saturable nonlinear media we modify equation (1) to the following system of equations:

\[
\frac{\partial E_x}{\partial t} = \left[ \frac{g_0}{1 + |E_x|^2 + \sigma_1 |E_y|^2} - \frac{a_0}{1 + b(|E_x|^2 + \sigma_2 |E_y|^2)} \right] E_x + (i + d) \Delta E_x,
\]

\[
\frac{\partial E_y}{\partial t} = \left[ \frac{g_0}{1 + |E_y|^2 + \sigma_1 |E_x|^2} - \frac{a_0}{1 + b(|E_x|^2 + \sigma_2 |E_y|^2)} \right] E_y + (i + d) \Delta E_y,
\]

where \(E_x\) and \(E_y\) are x- and y-polarization components of electric field complex envelope \(E\), \(\sigma_1\) and \(\sigma_2\) cross-saturation coefficients, other parameters are the same as that for equation (1). Laser dynamics described by equation (2) allows formation of topological defects both phase-type and polarization-type. Vector laser solitons can form when a proper balance between diffraction and self-/cross-saturation for the gain and absorption is achieved.

Numerical analysis of equation (2) shows that the vector solitons can exist in such an isotropic laser in roughly the same parameter range, in which the fundamental and vortex laser solitons are stable. The vector solitons are composite mutually trapped orthogonally polarized localized structures with axially symmetric intensity profiles, and non-uniform distribution of polarization states. We found four types of the vector laser solitons (figures 5 and 6), for which at least one of the polarization components carries a vorticity. They have distinct dynamical polarization structures with rotating polarization defects: points of right-handed circular polarization (C+ points), points of left-handed circular polarization (C− points) and lines of linear polarization (L-lines).

The type I structure (figures 5(a), (b)) is composed of the vortex soliton with the phase point defect and the defect free fundamental soliton, each of them can exist separately and independently in the scalar case (see section 2). It contains a C+ point defect surrounded by the pattern of ellipses having the star morphology with three radial directions of the ellipses’ major axes and the C− point defect having the lemon morphology with one radial direction of the ellipses’ major axes [18]. In the type II structure (figures 5(c), (d)) one polarization component has also vortex soliton structure (as in the type I vector soliton), but another polarization component represents the second order axisymmetric localized structure that has no scalar counterpart and can exist only in bound state. C point defects have the same morphology as that of the type I structure. The vector soliton components have different spatial amplitude and phase structures and also different frequencies. Therefore, their beating produces complex dynamic of the polarization structure, which contains rotating polarization defects (C− points and L-lines). We note that the polarization defects of the soliton type I and soliton type II perform rotational movements in opposite directions, even though their vortex components have the same sign.

The type III and type IV vector solitons are composed of asymmetric polarization components each carrying vortices of

---

Figure 4. Transverse distributions of intensity (a) and phase reconstructed from interferogram by Fourier-filtering method (b) for the vortex soliton in bistable two-wave mixing photorefractive oscillator.
the first order and the same sign. The type III composed structure (figures 6(a), (b)) contains a pair of C− points with the lemon morphology and a pair of C+ points with the star morphology [18]. The type IV structure (figures 6(c), (d)) contains a C-point with the lemon morphology. Polarization defects of the vector structures type III/IV and the asymmetric polarization components rotate in the same direction. We note that such localized structures of the vector soliton type III/IV components cannot exist separately.

Change the sign of the topological charges of the vector laser solitons leads to a change in morphology of C point polarization defects (from the lemon to the star and vice versa) and to a change in the direction of their rotation.

4. Phase domain walls in optical parametric oscillators

In contrast to lasers, which are phase invariant, optical parametric oscillators, such as a degenerate four-wave-mixing oscillator, exhibit phase bistability and their dynamics can be described by a real Swift–Hohenberg equation [19, 20]. A bifurcation diagram for this case is shown in figure 7, which shows that the phase-sensitive oscillator can amplify only two real-valued fields with opposite signs. Thus, the typical radiation field structures of such oscillators, when it has transversely extended resonator, are domains of equal light intensity and opposite (shifted by \( \pi \)) phases separated by domain walls (figure 8). Depending on the cavity, detuning the domain walls can be in two states: in static domain wall state of Ising-type for which the phase jumps abruptly by \( \pi \) and in the moving domain wall state of Bloch-type with a smooth change in the phase [21, 22].

Note that the phase domain walls are topological objects and their formation is based on a phase bistability. Therefore the domain walls can be self-contained or closed on the borders of the oscillator aperture (figure 8) and they cannot be removed in a continuous way but through the borders. Depending on resonator detuning the domain walls expand or shrink as it was experimentally observed in a degenerate four-wave-mixing photorefractive oscillator operating in a single

Figure 5. Vector solitons of type I (a), (b) and type II (c), (d). Distributions of the total intensity and major axes of polarization ellipses (polarization azimuths) in the cross-section of the laser (a), (c). Distribution of polarization states (b), (d), C+ and C− points are labeled with blue and red circles, respectively. Domains of left-handed (red) and right-handed (blue) polarization states are separated by L-lines (green).
When domain wall shrinking is balanced by diffraction expanding, the phase ring soliton can form spontaneously from noise (figure 8(a)) or in a controlled way by addressing coherent laser pulse injection at a particular desired location (figure 9(a)).

In a parametric oscillator with a one-dimensional cavity and a certain fixed cavity detuning [22, 24] both isolated (figures 9(b), (d)) and coupled (figure 9(c)) straight static domain walls of Ising-type can be created. When cavity detuning is varied, Ising–Bloch transition occurs and the domain wall becomes movable. Bloch walls are chiral topological objects and therefore Bloch walls with opposite chirality move in opposite directions [22].

5. Domain walls with Néel defects

It has been shown numerically and experimentally [25] that the optical oscillator with combined laser and parametric gains supports formation of composite topological structures appearing in the form of optical domain walls with the point defects similar to Néel topological defects available in ferromagnetics and liquid crystals [26].

Now we consider a bistable optical oscillator with combined laser and parametric gains and with saturable absorber is well described by modified equation (1) by addition of a
term that is responsible for parametric gain $\gamma$:

$$\frac{\partial E}{\partial t} = \frac{g_0 E + \gamma E^*}{1 + |E|^2} - \left( \frac{a_0}{1 + b |E|^2} + 1 \right) E + (i + d) \Delta_z E.$$  

Such a generator can simultaneously have both intensity bistability and phase anisotropy, which is manifested by the fact that the complex electric field vectors are based on the ellipse. The numerical calculations of equation (2) show that under certain phase anisotropy (eccentricity of the ellipse) one can excite localized structure with composite topological structure consisting of domain wall with a Néel-type point defect in the center [27] (figure 10).

This Néel topological soliton possesses properties of both optical vortex and optical phase domain wall. To demonstrate this fact more clearly we trace the change of intensity and phase in the circle line around the central point of the Néel soliton (figure 11, black lines). For comparison the red lines (figure 11) represent the same dependences for the laser

![Figure 8](image8.png)  
**Figure 8.** Calculated (a), (b) and experimental (c), (d) intensity (a), (c), phase (b) and interferogram (d) for phase domain walls.

![Figure 9](image9.png)  
**Figure 9.** Ring phase soliton (a), isolated one-dimensional wall (b), coupled (c) and uncoupled (d) one-dimensional walls.

![Figure 10](image10.png)  
**Figure 10.** Transverse distribution of intensity (a) and phase (b) for the Néel topological soliton. The soliton rotates around its centre as shown by the arrow.
vortex soliton. One can see that there is a certain similarity between these structures. However, the difference is that the Néel soliton has two dips in intensity over which phase changes smoothly by \(\pi\) as it takes place for Bloch-type domain wall. The phase in a three-dimensional representation has the form of propeller blades. The phase histogram (figure 12) confirms that we deal with the Bloch-type domain wall. Note that the Néel soliton rotates around its centre (arrow in figure 10(a)).

6. Conclusion

In this review, we have discussed questions of existence, bistability and dynamical properties of various topological solitons (vortex solitons, vector solitons, domain walls and Néel solitons) by analyzing simple models of experimentally realized optical oscillators with laser and/or parametric gains. These solitons are of interest from both a fundamental viewpoint for physics of non-equilibrium systems and from an applied viewpoint as a nontrivial (switchable and mobile) carrier of information.

References

[1] Kivshar Yu S and Agraval GP 2003 Optical Solitons: From Fibers to Photonic Crystals (Amsterdam: Academic)
[2] Rosanov NN 2002 Spatial Hysteresis and Optical Patterns (Berlin: Springer)
[3] Dissipative solitons 2005 ed N Akhmediev and A Ankiewicz Lecture Notes in Physics 661 (Berlin: Springer)
[4] McDonald GS and Firth WJ 1990 Spatial solitary wave optical memory J. Opt. Soc. Am. B 7 1328–35
[5] Fedorov SV, Khodova GV and Rosanov NN 1992 Soliton-like field transverse structures in passive and active optical bistable systems Proc. SPIE 1840 208–15
[6] Bazhenov YV, Taranenko VB and Vasnetsov MV 1992 Transverse optical effects in bistable active cavity with nonlinear absorber on bacteriorhodopsin Proc. SPIE 1840 183–9
[7] Fedorov SV, Rosanov NN, Shatsev AN, Veretenov NA and Vladimirov AG 2003 Topologically multicharged and multihumped rotating solitons in wide-aperture lasers with a saturable absorber IEEE J. Quantum Elect. 39 197–205
[8] Rosanov NN, Fedorov SV and Shatsev AN 2005 Curvilinear motion of multivortex laser-soliton complexes with strong and weak coupling Phys. Rev. Lett. 95 1–053903
[9] Taranenko VB, Staliunas K and Weiss CO 1997 Spatial soliton laser: localized structures in a laser with a saturable absorber in a self-imaging resonator Phys. Rev. A 56 1582–91
[10] Taranenko VB, Sleikys G and Weiss CO 2003 Spatial resonator solitons Chaos 13 777–90
[11] Genetev P, Barland S, Giudici M and Tredicce JR 2008 Cavity soliton laser based on mutually coupled semiconductor microresonators Phys. Rev. Lett. 101 123905
[12] Staliunas K, Taranenko VB, Sleikys G, Viselga R and Weiss CO 1998 Moving spatial solitons in active nonlinear-optical resonators Phys. Rev. A 57 599–604
[13] Genetev P, Barland S, Giudici M and Tredicce JR 2010 Bistable and addressable localized vortices in semiconductor lasers Phys. Rev. Lett. 104 223902
[14] Yaparov VV, Taranenko VB, Rosanov NN and Fedorov SV 2012 Experimental observation of a vortex dissipative soliton at amplification on the basis of two-wave mixing with saturable absorption Opt. Spectrosc. 112 601–3
[15] de Valcárcel G and Staliunas K 2003 Excitation of phase patterns and spatial solitons via two-frequency forcing of a 1:1 resonance Phys. Rev. E 67 026604
[16] Esteban-Martín A, Martínez-Quesada M, Taranenko V, Roldán E and Valcárcel G 2006 Bistable phase locking of a nonlinear optical cavity via rocking: transmuting vortices into phase patterns Phys. Rev. Lett. 97 093903
[17] Gil L 1993 Vector order parameter for an unpolarized laser and its vectorial topological defects Phys. Rev. Lett. 70 162–5
[18] Nye JF 1999 Natural Focusing and Fine Structure of Light (Bristol: IOP Publishing)
[19] Staliunas K and Sanchez-Morcillo VJ 1997 Localized structures in degenerate optical parametric oscillators Opt. Commun. 139 306–12
[20] Staliunas K and Sánchez-Morcillo VJ 1998 Spatial-localized structures in degenerate optical parametric oscillators Phys. Rev. A 57 1454–7
[21] Coullet P, Lega J, Houchmanzadeh B and Lajzerowicz J 1990 Breaking chirality in nonequilibrium systems Phys. Rev. Lett. 65 1352–5
[22] Esteban-Martín A, Taranenko VB, García J, de Valcárcel GJ and Roldán E 2005 Controlled observation of a nonequilibrium Ising-Bloch transition in a nonlinear optical cavity Phys. Rev. Lett. 94 223903–6
[23] Taranenko VB, Staliunas K and Weiss CO 1998 Pattern formation and localized structures in degenerated optical parametric mixing Phys. Rev. Lett. 81 2236–8
[24] Esteban-Martín A, Taranenko VB, Roldán E and de Valcárcel GJ 2005 Control and steering of phase domain walls Opt. Express 13 3631–6
[25] Yaparov VV and Taranenko VB 2015 Moving domain walls with Néel defects in optical oscillator Ukr. J. Phys. Opt. 16 159–64
[26] Kawagishi T, Mizugushi T and Sano M 1995 Points, walls, and loops in resonant oscillatory media Phys. Rev. Lett. 75 3768–71
[27] Yaparov VV and Taranenko VB 2011 Topological solitons in active optical cavities: fundamental properties and possible applications Bull. Lebedev Phys. Inst. 38 28–9