On the Spectrum Sensing, Beam Selection and Power Allocation in Cognitive Radio Networks Using Reconfigurable Antennas

Hassan Yazdani, Azadeh Vosoughi, Senior Member, IEEE
University of Central Florida
E-mail: h.yazdani@knights.ucf.edu, azadeh@ucf.edu

Abstract—In this paper, we consider a cognitive radio (CR) system consisting of a primary user (PU) and a pair of secondary user transmitter (SU$_{tx}$) and secondary user receiver (SU$_{rx}$). The SU$_{tx}$ is equipped with a reconfigurable antenna (RA) which divides the angular space into $M$ sectors. The RA chooses one sector among $M$ sectors for its data transmission to SU$_{rx}$. The SU$_{tx}$ first senses the channel and monitors the activity of PU for a duration of $T_{sen}$ seconds. We refer to this period as channel sensing phase. Depending on the outcome of this phase, SU$_{tx}$ stays in this phase or enters the next phase, which we refer to as transmission phase. The transmission phase itself consists of two phases: channel training phase followed by data transmission phase. During the former phase, SU$_{tx}$ sends pilot symbols to enable channel training and estimation at SU$_{rx}$. The SU$_{tx}$ selects the best beam (sector) for data transmission and feeds back the index of the selected beam as well as its corresponding channel gain. We also derive the probability of determining the true beam and take into account this probability in our system design. During the latter phase, SU$_{tx}$ sends data symbols to SU$_{rx}$ over the selected beam with constant power $\Phi$ if the gain corresponding to the selected beam is bigger than the threshold $\zeta$. We find the optimal channel sensing duration $T_{sen}$, the optimal power level $\Phi$ and a optimal threshold $\zeta$, such that the ergodic capacity of CR system is maximized, subject to average interference and power constraints. In addition, we derive closed form expressions for outage and symbol error probabilities of our CR system.

I. INTRODUCTION

The explosive rise in demand for high data rate wireless applications has turned the spectrum into a scarce resource. Cognitive radio (CR) is a promising solution which alleviates spectrum scarcity problem by allowing an unlicensed or secondary user (SU) to access licensed bands in a such way that its imposed interference on license holder primary users (PUs) is limited [1]–[5].

With the help of directional antennas, a system designer can go beyond just filling the spectrum holes in time and/or frequency domains and can further improve the spectral efficiency (well beyond what is attainable with omni-directional antennas), via utilizing the spectrum white spaces in spatial (angular) domain and allowing a PU and a SU use simultaneously the same frequency band, by steering their directional antenna beams to non-interfering directions. Consider an opportunistic CR network where SUs are equipped with directional antennas. SUs can identify the spectrum holes in spatial (angular) domain and use the identified unused directions for data communication. This significantly enhances the spectrum utilization, compared to the scenario where SUs in the same network are equipped with omni-directional antennas [6]–[8].

Reconfigurable antennas (RAs), with the capabilities of dynamically modifying their characteristics (e.g., operating frequency, radiation pattern, polarization) are emerging as promising solutions to efficiently utilize the spatial (angular) domain and beam steering in wireless communication systems, including CR networks [9]. RA has been used for identifying the directional spectrum sensing opportunities in CR networks as well as for directional wireless and millimeter wave communication systems and surveillance [10], [11]. An electrically steerable parasitic array radiator (ESPAR) antenna is a special kind of RAs, that has been used for identifying the spectral holes in spatial domain in CR networks. ESPAR divides the angular domain into several sectors (beams) and switches between beampatterns of sectors in a time-division fashion (only one of $M$ beams is active at a time). For CR networks, the RAs can provide an improved spectrum sensing and transmit/receive capability, due to a signal-to-noise ratio (SNR) increase for transmission and reception of directional signals, and can limit out-of-band interference to and from PUs [12]. It has been demonstrated that RAs have the ability to transmit multiple data streams by projecting on beamspace basis [13]. Also, they can be used for blind interference alignment through beampattern switching [14]. RAs have been used for performance enhancement of multiple-input multiple-output (MIMO) systems, via exploiting the additional degree of freedom provided by beam selection and enabling joint beam and antenna selection optimization [15]–[17].

In this paper, we consider a CR system consisting of a PU and a pair of SU transmitter (SU$_{tx}$) and SU receiver (SU$_{rx}$). The SU$_{tx}$ is equipped with a reconfigurable antenna with the capability of choosing one sector among $M$ sectors for its data transmission to SU$_{rx}$. The SU$_{tx}$ first senses the channel and monitors the activity of PU for a duration of $T_{sen}$ seconds. We refer to this period as channel sensing phase. Depending on the outcome of this phase, SU$_{tx}$ stays in this phase or enters the next phase, which we refer to as transmission phase. The transmission phase itself consists of two phases: channel training phase followed by data transmission phase. During the former phase, SU$_{tx}$ sends pilot symbols to enable channel training and estimation at SU$_{rx}$. The SU$_{tx}$ selects the best beam (sector) for data transmission and feeds back the index of the
selected beam as well as its corresponding channel gain. We also derive the probability of determining the true beam and take into account this probability in our system design. During the latter phase, SU\textsubscript{tx} sends data symbols to SU\textsubscript{rx} over the selected beam with constant power $\Phi$ if the gain corresponding to the selected beam is bigger than a threshold $\zeta$. Our objective is to find the optimal channel sensing duration $T_{\text{sen}}$, the optimal power level $\Phi$ and the optimal threshold $\zeta$, such that the ergodic capacity of CR system is maximized, subject to average interference and power constraints. In addition, we derive closed form expressions for outage and symbol error probabilities of our CR system.

II. SYSTEM MODEL

Our CR system model is shown in Fig. 1, consisting of a PU and a pair of SU\textsubscript{tx} and SU\textsubscript{rx}. We note that PU in our system model can be a primary transmitter or receiver. Similar to [18], we assume when PU is active it is engaged in a bidirectional communication with another PU. The other PU is located far from SU\textsubscript{tx} and its activity is not considered in this paper. The SU\textsubscript{tx} is equipped with a RA (for both channel sensing and communication) with the capability of choosing one sector among $M$ sectors for its data transmission to SU\textsubscript{rx}, while SU\textsubscript{tx} and PU use omni-directional antennas\(^1\). Similar to [19], we model the radiation pattern of every sector of SU\textsubscript{tx}'s antenna in $x-y$ (azimuth) plane, with the Gaussian pattern as

$$ p(\phi) = A_1 + A_0 e^{-B \left( \frac{\pi(\phi - \Phi)}{2\pi} \right)^2}, $$

where

$$ M(\phi) = \text{mod}_{2\pi}(\phi + \pi) - 2\pi, $$

$\text{mod}_{2\pi}(\phi)$ denotes the remainder of $\frac{\phi}{2\pi}$. $B = \ln(2), \Phi_{\text{dB}}$ is the half-power beam-width, $A_1$ and $A_0$ are two constant antenna parameters. We set $A_1 = L/A_0$ where $L \ll 1$ is the antenna loss in side lobe. We denote the radiation pattern of $m$-th sector in angle $\phi$ by

$$ p_m(\phi) = p(\phi - \kappa_m) $$

where $\kappa_m = \frac{2\pi(m-1)}{M}$. In Fig. 2, the beampatterns of a RA with 8 sectors are shown. The orientation of PU and SU\textsubscript{tx} with respect to SU\textsubscript{rx} are denoted by $\phi_{\text{PU}}$ and $\phi_{\text{SR}}$, respectively, and we assume that SU\textsubscript{tx} knows $\phi_{\text{SR}}$. The fading coefficients from PU to SU\textsubscript{tx}, SU\textsubscript{tx} to SU\textsubscript{rx} and PU to SU\textsubscript{rx} are denoted by $h, h_{\text{rs}}$ and $h_{\text{sp}}$, respectively, if SU\textsubscript{tx} uses omni-directional antenna. We assume $\phi_{\text{PU}}$ and $\phi_{\text{SR}}$ are circularly symmetric complex Gaussian random variables, and $g = |h|^2, g_{\text{rs}} = |h_{\text{rs}}|^2$ and $g_{\text{sp}} = |h_{\text{sp}}|^2$ are mutually independent exponentially distributed random variables with mean $\gamma, \gamma_{\text{rs}}$ and $\gamma_{\text{sp}}$, respectively. In our problem, we assume that SUs and PU cannot cooperate and SUs cannot estimate $g$ and $g_{\text{sp}}$. We know that SU\textsubscript{tx} knows mean values $\gamma$ and $\gamma_{sp}$. Let $\psi_m$ and $\chi_m$ denote the fading coefficients of channel between $m$-th sector of SU\textsubscript{tx} and PU, and between $m$-th sector of SU\textsubscript{tx} and SU\textsubscript{rx}, respectively, where $\psi_m = h\sqrt{p_m(\phi_{\text{PU}})}$, $\chi_m = h_{\text{rs}}\sqrt{p_m(\phi_{\text{SR}})}$ and $p_m(\phi_{\text{PU}})$ and $p_m(\phi_{\text{SR}})$ indicate the radiation pattern of $m$-th sector at angles $\phi_{\text{PU}}$ and $\phi_{\text{SR}}$, respectively. Also, we assume that the channel gain $\nu_m = |\chi_m|^2$ is an exponential random variable with mean $\delta_{\text{rs}}$, and SU\textsubscript{tx} knows $\delta_{\text{rs}}$, for all $m$ [20].

III. OUR PROBLEM STATEMENT

We suppose the SUs employ a frame with a fixed duration of $T_1$ seconds, depicted in Fig. 3. We assume SU\textsubscript{tx} first senses the channel and monitors the activity of PU. We refer to this period as channel sensing phase (with a variable duration of $T_{\text{sen}}$ seconds). $T_{\text{sen}}$ is the sensing time duration for all $M$ sectors, i.e., every sector senses the channel for $T_{\text{sen}}/M$ seconds. Depending on the outcome of this phase, SU\textsubscript{tx} stays in this phase or enters the next phase, which we refer to as transmission phase. The transmission phase itself consists of two phases: channel training phase (with a fixed duration of $T_{\text{train}}$ seconds) followed by data transmission phase (with a variable duration of $T_1 - T_{\text{sen}} - T_{\text{train}}$ seconds). During the former phase, SU\textsubscript{tx} sends pilot symbols to enable channel training and estimation at SU\textsubscript{tx}. During the latter phase, SU\textsubscript{tx} sends data symbols to SU\textsubscript{rx}. Given $T_1$ and $T_{\text{train}}$ we have $0 < T_{\text{sen}} < T_1 - T_{\text{train}}$. In the following, we describe how SU\textsubscript{tx} operates during these three distinct phases. Based on these descriptions, we provide our problem statement.

\(^1\) Throughout this paper, “sector” and “beam” are used interchangeably.
A. Channel Sensing Using RA

During this phase, SU\textsubscript{tx} senses the channel and monitors the activity of PU. Suppose \( \mathcal{H}_1 \) and \( \mathcal{H}_0 \) represent the binary hypotheses of PU being active and inactive, respectively, with prior probabilities \( \Pr\{\mathcal{H}_1\} = \tau_1 \) and \( \Pr\{\mathcal{H}_0\} = \pi_0 \). SU\textsubscript{tx} applies a binary detection rule to decide whether or not PU is active. Let \( \mathcal{H}_1 \) and \( \mathcal{H}_0 \) denote the detector outcome. When active, PU transmits signal \( s(t) \) with power \( P_p \). We assume SU\textsubscript{tx} collects \( N = \lfloor T_T / (M T_s) \rfloor \) samples at each sector, where \( T_T \) is the sampling period. We denote the discrete-time symbol received in \( m \)-th sector of SU\textsubscript{tx} at time instant \( t = n T_s \) as

\[
y_m(n) = \psi_m(n) s(n) + w_m(n),
\]

We model the transmitted signal \( s(n) \) by PU as \( s(n) \sim \mathcal{CN}(0, P_p) \). The term \( w_m(n) \) is the additive noise at \( m \)-th sector of SU\textsubscript{tx}'s antenna and is modeled as \( w_m(n) \sim \mathcal{CN}(0, \sigma^2_w) \). It is assumed that \( \psi_m(n), w_m(n) \) are mutually independent. We assume \( w_m(n) \) are independent and thus uncorrelated in both time and space domains, i.e.,

\[
\mathbb{E}\{w_m(n) w^*_n(n')\} = \sigma^2_w \delta[m-m'] \delta[i-i']
\]

where \( \delta[.] \) is Kronecker’s delta. Since only one beam is active at a time, the channel gains \( \psi_m(n) \) are uncorrelated in both time and space domains, i.e.,

\[
\mathbb{E}\{\psi(n)\psi^*(n')\} = E_A \delta[m-m'] \delta[n-n']
\]

where \( E_A = \frac{1}{2\pi} \int_0^{2\pi} p(\theta) d\theta \). The hypothesis testing problem at \( n \)-th time instant for \( m \)-th sector is then given by

\[
\begin{align*}
\mathcal{H}_0 &: y_m(n) = w_m(n), \\
\mathcal{H}_1 &: y_m(n) = \psi_m(n) s(n) + w_m(n).
\end{align*}
\]

Suppose SU\textsubscript{tx} uses an energy detector to detect the activity of PU and let \( \varepsilon_m \) be the energy of received signal at sector \( m \).

We have

\[
\varepsilon_m = \frac{1}{N} \sum_{n=1}^{N} |y_m(n)|^2.
\]

We consider the summation of energies of received signals over all sectors as the decision statistics as:

\[
T = \frac{1}{M} \sum_{m=1}^{M} \varepsilon_m \geq \frac{\mathcal{H}_1}{\mathcal{H}_0} \eta.
\]

Under hypothesis \( \mathcal{H}_0 \), for large \( N \) we invoke the central limit theorem (CLT), to approximate \( T \) as Gaussian with distribution \( T \sim \mathcal{N}(\sigma^2_T|\mathcal{H}_0) \), where

\[
\sigma^2_T|\mathcal{H}_0 = \frac{\sigma^2_w}{M N}.
\]

Similarly, under hypothesis \( \mathcal{H}_1 \) for large \( N \), \( T \) can be approximated with another Gaussian with the distribution \( T \sim \mathcal{N}(\mu, \sigma^2_T|\mathcal{H}_1) \) where \( \mu = P_p E_A + \sigma^2_w \) and

\[
\sigma^2_T|\mathcal{H}_1 = \frac{1}{M N} \left[ \sigma^2_w + 2 \rho P_p E_A \sigma^2_w + \gamma^2 P_p^2 (3 E_B - M N E_A^2) \right] + \frac{\gamma^2 P_p^2}{M^2} \sum_{m=1}^{M} \sum_{m'=1}^{M} E_{mm'},
\]

and \( E_{mm'} = \frac{1}{2\pi} \int_0^{2\pi} |p_m(\theta) p_{m'}(\theta)| d\theta \) and \( E_B = E_{mm} \). Then, the false alarm and detection probabilities of this energy detector are given as follows

\[
P_a = Q\left( \frac{\eta - \mu}{\sigma_T|\mathcal{H}_1} \right), \quad P_d = Q\left( \frac{\eta - \mu}{\sigma_T|\mathcal{H}_0} \right).
\]

For a given value of \( P_d = P_{\text{false alarm}} \), the probability of false alarm can be written as

\[
P_a = Q\left( \frac{\sigma_T|\mathcal{H}_1 Q^{-1}(P_{\text{false alarm}}) + \mu - \sigma^2_w}{\sigma_T|\mathcal{H}_0} \right).
\]

where \( Q(\cdot) \) is the Q-function. Therefore, the probabilities of events \( \mathcal{H}_0 \) and \( \mathcal{H}_1 \) become \( \pi_0 = \Pr\{\mathcal{H}_0\} = \pi_1 (1 - P_d) + \pi_0 (1 - P_{\text{false alarm}}) \) and \( \pi_1 = \Pr\{\mathcal{H}_1\} = \pi_1 P_d + \pi_0 P_{\text{false alarm}} \), respectively. The accuracy of channel sensing impacts the maximum information rate that SU\textsubscript{tx} can transmit reliably to SU\textsubscript{rx}. Our problem formulation incorporates the effect of imperfect channel sensing on the constrained ergodic capacity maximization.

B. Estimating the orientation of PU (\( \phi_{\text{PU}} \))

As long as the channel is sensed busy, SU\textsubscript{tx} stays in channel sensing phase. While being in this phase, SU\textsubscript{tx} estimates the location (orientation) of PU using one of the methods explained in [19], [21]–[24]. SU\textsubscript{tx} uses \( \phi_{\text{PU}} \) for adapting its transmit power during data transmission phase. We note that, there is a non-zero error when SU\textsubscript{tx} estimates \( \phi_{\text{PU}} \) and we can incorporate this error in our system design and performance analysis. However, in this paper we assume that SU\textsubscript{tx} can estimate \( \phi_{\text{PU}} \) perfectly (with no error).

C. Determining the Sector Corresponding to SU\textsubscript{rx} and Finding the Strongest Channel between SU\textsubscript{tx}-SU\textsubscript{rx}

When the channel is sensed idle, SU\textsubscript{tx} leaves channel sensing phase and enters channel training phase. Suppose SU\textsubscript{tx} knows SU\textsubscript{rx} is located between two adjacent sectors, however, it does not know which one is better, in terms of enabling a higher transmission rate. During channel training phase, SU\textsubscript{tx} sends pilot symbols over these two sectors, to enable channel training and estimation at SU\textsubscript{tx}. Without loss of generality, suppose SU\textsubscript{tx} is located between the first and second sectors as shown in Fig. 4a. Using the received training signal, SU\textsubscript{tx} estimates the channel gains \( \nu_m = |\chi_m|^2, m = 1, 2 \) and determines the strongest channel \( \nu^* = \max\{\nu_1, \nu_2\} \) and the corresponding beam index \( m_{\text{SR}}^* = \arg\max\{\nu_1, \nu_2\} \). For example in Fig. 4b, we have \( m_{\text{SR}}^* = 2 \), i.e., the second beam has the strongest channel gain. Then, SU\textsubscript{tx} feeds back \( m_{\text{SR}}^* \) as well as the value of \( \nu^* \) to SU\textsubscript{rx}. We take into account the probability of determining the true sector corresponding to SU\textsubscript{tx} on the constrained capacity maximization.

We denote the correlation coefficient between channel gains of sectors \( m \) and \( m' \) by \( \rho_{mm'} \). The correlation coefficient depends on the structure and design of RA [16] and we assume SU\textsubscript{tx} has complete knowledge of the correlation coefficient \( \rho_{mm'} \). Let \( \rho \) represent the correlation coefficient between two adjacent sectors which SU\textsubscript{tx} is located. The joint probability distribution function (PDF) and cumulative density function (CDF) of channel gains \( \nu_1 \) and \( \nu_2 \) can be written as following

\[
f_{\nu_1, \nu_2}(y_1, y_2) = \frac{\alpha_1}{\delta_2} e^{-\alpha_1 y_1} e^{-\alpha_2 y_2} f_0 \left( 2 \sqrt{\rho^2 \alpha_1 \alpha_2 y_1 y_2} \right)
\]
where $D_t = (T_i - T_{sen} - T_{train})/T_i$ is the fraction of time in which SU$_{rx}$ sends data to SU$_{tx}$, $c_{i,0}$ is the instantaneous capacity of this link corresponding to the event $\mathcal{H}_i$ and $\bar{\mathcal{H}}_0$, given as

$$c_{0,0} = \log_2 \left( 1 + \frac{\nu^* P(\nu^*)}{\sigma_w^2} \right),$$  \hspace{1cm} (13)$$

$$c_{1,0} = \log_2 \left( 1 + \frac{\nu^* P(\nu^*)}{\sigma_w^2 + P_{\phi_PU}} \right),$$  \hspace{1cm} (14)

and $\alpha_0 = \pi_0 (1 - P_{\phi_PU})$, $\beta_0 = \pi_1 (1 - P_{\phi_PU})$. Let $\bar{T}_{av}$ indicate the maximum allowed interference power imposed on PU and $\overline{T}_{av}$ denote the maximum allowed average transmit power of SU$_{tx}$. To satisfy the average interference constraint (AIC), we have

$$D_t \delta_0 \mathbb{E}\{g_{sp} p(\kappa_{SR}^* - \phi_{PU}) P(\nu^*)\} \leq \overline{T}_{av},$$  \hspace{1cm} (15)

and to satisfy the average power constraint (APC), we have

$$D_t \bar{\pi}_0 \mathbb{E}\{P(\nu^*)\} \leq \overline{T}_{av}.$$  \hspace{1cm} (16)

Notice that for a given $T_i$, as we increase the sensing time $T_{sen}$, the spectrum sensing will be more accurate and data communication between SU$_{tx}$ and SU$_{rx}$ would cause less interference on PU. On the other hand, the available time for data transmission to SU$_{tx}$ decreases. Therefore, a trade-off exists between sensing and transmission capacity in terms of $T_{sen}$. Our main objective is to find the optimal channel sensing duration $T_{sen}$, the optimal power level $\Phi$ and the optimal threshold $\zeta$, such that the ergodic capacity $C$ in (12) is maximized, subject to AIC and APC given in (15) and (16), respectively. In other words, we are interested in solving the following constrained optimization problem

$$\text{Maximize } C = D_t \mathbb{E}\{\alpha_0 c_{0,0} + \beta_0 c_{1,0}\}$$  \hspace{1cm} (P1)

subject to:

$$0 < T_{sen} < T_i - T_{train},$$

$$\Phi \geq 0, \zeta \geq 0,$$  \hspace{1cm} (15) and (16) are satisfied.

IV. FORMALIZING AND SOLVING (P1)

Since SUs and PU cannot cooperate, SU$_{tx}$ cannot estimate the channel gain $g_{sp}$ and thus $c_{1,0}$ cannot be directly maximized at SU$_{tx}$. Instead, we consider a lower bound on its average over $g_{sp}$, denoted as $\mathbb{E}_{g_{sp}}\{c_{1,0}\}$. Using the Jensen’s inequality [26], the lower bound on $\mathbb{E}_{g_{sp}}\{c_{1,0}\}$ becomes

$$\mathbb{E}_{g_{sp}}\{c_{1,0}\} \geq \log_2 \left( 1 + \frac{\nu^* P(\nu^*)}{\sigma_p^2 + \sigma_w^2} \right) = c_{LB}^{1,0},$$  \hspace{1cm} (17)

where $\sigma_p^2 = P_{\phi_PU}$. Let $C_{LB} = D_t \mathbb{E}_{g_{sp}}\{\alpha_0 c_{0,0} + \beta_0 c_{1,0}\}$ where $C_{LB}$ is the lower bound on $C$ in (12). From now on, we focus on $C_{LB}$. Next, we focus on the AIC in (15) and find the term $\mathbb{E}\{p(\kappa_{SR}^* - \phi_{PU})\}$. By using the average probabilities derived in (10) we have

$$\mathbb{E}\{p(\kappa_{SR}^* - \phi_{PU})\} = \Delta_1 p(\kappa_1 - \phi_{PU}) + \Delta_2 p(\kappa_2 - \phi_{PU}).$$

D. Data Transmission

When the channel is sensed idle, SU$_{tx}$ sends data to SU$_{rx}$ over the selected sector ($m_{SR}^*$) using the following power control policy

$$P(\nu^*) = \begin{cases} \Phi, & \text{if } \nu^* \geq \zeta \\ 0, & \text{if } \nu^* < \zeta \end{cases}$$  \hspace{1cm} (11)

According to (11), when the selected channel (sector) is too weak ($\nu^*$ is less than the cut-off threshold $\zeta$), SU$_{tx}$ does not transmit data. However, when channel is strong enough, SU$_{tx}$ sends data with constant power $\Phi$. We find $\Phi$ and $\zeta$ such that the ergodic capacity of SU$_{tx}$-SU$_{rx}$ link is maximized. The ergodic capacity of SU$_{tx}$-SU$_{rx}$ link is [8]

$$C = D_t \mathbb{E}\{\alpha_0 c_{0,0} + \beta_0 c_{1,0}\},$$  \hspace{1cm} (12)
By defining $b_0 = \beta_0 \gamma_{sp} [\Delta_1 p (\kappa_1 - \rho_{PR}) + \Delta_2 p (\kappa_2 - \rho_{PR})]$, the optimal power $\Phi$, given $T_{sen}$ and $\zeta$, can be easily obtained as

$$\Phi = \frac{1}{1 - F_{\nu, \zeta}} \min \left\{ \frac{P_{av}}{D_{\nu}}, \frac{T_{av}}{D_{\nu} b_0} \right\}. \quad (18)$$

To obtain a closed form for ergodic capacity, we expand the PDF in (9) using the following equation

$$Q_1(\sqrt{\lambda}, \sqrt{\gamma}) = \sum_{j=0}^{\infty} \left( \frac{\Phi}{\gamma} \right)^j e^{-\frac{j}{\gamma}} \sum_{i=0}^{j} \left( \frac{\gamma}{i!} \right)$$

and rewrite $f_{\nu, \zeta}(x)$ as the following form

$$f_{\nu, \zeta}(x) = \sum_{m=1}^{2} \frac{1}{\delta_m} e^{-\frac{x}{\delta_m}} - \sum_{j=0}^{\infty} \sum_{i=0}^{j} D_{ij} x^{i+j} e^{-x}. \quad (19)$$

where $\omega = \alpha_1 + \alpha_2$ and $D_{ij} = \frac{\alpha_1 \alpha_2}{\delta_1 \delta_2} [e^{-\frac{\alpha_1}{\delta_1}} + e^{-\frac{\alpha_2}{\delta_2}}]$.

To solve (P1), we consider two initial values for $\zeta$ and $T_{sen}$ and obtain $\Phi$ using (18). Then, $\zeta_{opt}$ and $T_{sen}$ can be obtained by maximizing the ergodic capacity given in (20) using searching methods such as bisection, where

$$\text{SNR}^{(0)} = \frac{\Phi}{\sigma^2}, \quad \text{SNR}^{(1)} = \frac{\Phi}{\sigma_u^2 + \sigma_p^2}.$$ 

$$G(\delta, S, \zeta) = e^{-\frac{x}{\delta}} \ln (1 + S) - e^{-\frac{x}{\delta}} \text{Ei} \left( \frac{-1}{\delta} (1 + S \zeta) \right),$$

and Ei$(\cdot)$ is the exponential integral [25] and $V(\cdot)$ can be calculated using the recursive equation in (21). In (21b), $\Gamma(\cdot, \cdot)$ is the incomplete Gamma function.

V. OUTAGE AND SYMBOL ERROR PROBABILITIES

Two other relevant metrics to evaluate the performance of our CR system with the RA at SU$_{rx}$ are outage probability and symbol error probability (SEP), denoted as $P_{out}$ and $P_e$, respectively. We define $P_{out}$ as the probability of SU$_{tx}$ not transmitting data due to the weak SU$_{tx}$-SU$_{rx}$ channel. In the following, we derive closed-form expressions for $P_{out}$ and $P_e$. The outage probability $P_{out}$ can be directly obtained using the CDF of $\nu^*$ as

$$P_{out} = \Pr \{ P(\nu^*) = 0 \} = F_{\nu^*}(\zeta). \quad (22)$$

For many digital modulation schemes SEP can be written as a function with the following form [17]

$$P_e = \mathbb{E} \left\{ Q(\sqrt{\Psi} \text{SNR}_{rx}) \right\}, \quad (23)$$

where $\Psi$ is a constant parameter related to the type of modulation and SNR$_{rx}$ is the received signal-to-noise-ratio (SNR) at SU$_{rx}$ given as

$$\text{SNR}_{rx} = \begin{cases} \frac{\Phi \nu^*}{\sigma_u^2 + \sigma_p^2}, & \text{if } \nu^* \geq \zeta, (H_0, \tilde{H}_0) \\ \frac{\Phi \nu^*}{\sigma_u^2 + \sigma_p^2}, & \text{if } \nu^* \geq \zeta, (H_1, \tilde{H}_0) \\ 0, & \text{else} \end{cases} \quad (24)$$

After some manipulation, SEP can be written as (25), where

$$J(k, S, \Psi, \omega, \zeta) = \frac{1}{S} \left( \frac{\omega}{2D} \right)^{-k-\frac{1}{2}} \Gamma \left( k + \frac{1}{2}, \frac{S \Psi}{2} + \omega \right)$$

and $E_{ij} = \frac{\alpha_i^j \alpha_j^i}{\delta j}$.

VI. SIMULATION RESULTS

In this section, we illustrate the effect of reconfigurable antennas on the ergodic capacity and outage and symbol error probabilities of our secondary network by Matlab simulations. Assume $\gamma_1 = 1, \gamma_{sp} = \gamma = 1, \pi_1 = 0.4, L = 0.01, T_i = 10$ ms, $f_s = 100$ KHz, $P_d = 0.85, P_p = 0.2$ watts, $\Psi = 4$.

When beam-width of sectors increases, RA spreads electromagnetic power in a wider area. To fairly compare the performance of our system with different $\phi_{3MB}$, we choose $A_0$ such that $E_A = 1$. The beampatterns of a sector of RA for $\phi_{3MB} = 25^\circ, 35^\circ$ and the beampattern of a traditional omnidirectional antenna are shown in Fig. 5a. We can see that by decreasing $\phi_{3MB}$ and setting $E_A = 1$, the gain of antenna in $0^\circ$ increases. Fig. 5b shows the probability of detection versus the probability of false alarm for $N = 16, M = 8$ and $\phi_{3MB} = 20^\circ, 25^\circ, 30^\circ$. We observe that as $\phi_{3MB}$ increases, $P_{out}$ increases.

The probability of first beam selection ($\Delta_1$) versus $\phi_{3MB}$ is plotted in Fig. 6 for $M = 8$ when SU$_{rx}$ is located at $\phi_{SR} = 0^\circ, 10^\circ, 15^\circ$. If we increase the beam-width $\phi_{3MB}, \Delta_1$ decreases, i.e., to increase $\Delta_1$, we need a narrower beam-width. Also, we note that when $\phi_{SR}$ approaches $0^\circ$, the beam selection is more accurate.
\[ P_e = \frac{1}{2\sqrt{2\pi}} \left[ \alpha_0 J(0, \text{SNR}^{(0)}, \Psi, 0, \zeta) + \beta_0 J(0, \text{SNR}^{(1)}, \Psi, 0, \zeta) \right] \left( 1 - F_{\nu'}(\zeta) \right) - \alpha_0 J(0, \text{SNR}^{(0)}, \Psi, \frac{1}{\delta_2}, \zeta) \]
\[ \quad - \beta_0 J(0, \text{SNR}^{(1)}, \Psi, \frac{1}{\delta_2}, \zeta) \sum_{i=0}^{\infty} \sum_{j=0}^{\infty} E_{ij} \left( \alpha_0 J(i+j, \text{SNR}^{(0)}, \Psi, \omega, \zeta) + \beta_0 J(i+j, \text{SNR}^{(1)}, \Psi, \omega, \zeta) \right) \] \tag{25}

Fig. 5: (a) Beampattern in polar coordination, (b) \( P_t \) versus \( P_{ta} \) for \( \phi_{\text{AB}} = 20^\circ, 25^\circ, 30^\circ \).

Fig. 6: \( \Delta_1 \) versus \( \phi_{\text{AB}} \).

Fig. 7: Optimal Capacity averaged over orientation of \( \text{SU}_{\text{tx}} \) and PU versus \( P_{av} \).

Fig. 8: (a) \( P_{\text{out}} \) versus \( P_{av} \), (b) \( P_e \) versus \( P_{av} \).

Assume \( C_{\text{LB}}^\text{Opt} \) is the maximized capacity averaged over all possible orientations of \( \text{SU}_{\text{tx}} \) (\( \phi_{\text{SR}} \)) and PU (\( \phi_{\text{PU}} \)). Fig. 7 illustrates \( C_{\text{LB}}^\text{Opt} \) versus \( P_{av} \) when \( M = 8, P_{av} = 0 \) dB, \( \phi_{\text{AB}} = 20^\circ, 30^\circ \). For comparison, the maximized capacity when \( \text{SU}_{\text{tx}} \) equipped with omni-directional antenna and \( \Phi, \zeta \) and \( T_{\text{sen}} \) are optimized, is plotted. We can see that, in average, RA yields a higher capacity compared to omni-directional antenna. Also, a RA with smaller \( \phi_{\text{AB}} \) yields a higher capacity, because it can cancel more interference imposed on PU from \( \text{SU}_{\text{tx}} \).

The averaged outage and symbol error probabilities over \( \phi_{\text{SR}} \) and \( \phi_{\text{PU}} \) (denoted as \( P_{\text{out}} \) and \( P_e \)) are shown in Figs. 8a and 8b, respectively. Regarding the maximized capacity shown in Fig. 7, we observe that increasing the beam-width from \( 20^\circ \) to \( 30^\circ \) yields lower outage and symbol error probabilities. However, the other performance metric, i.e. the ergodic capacity, decreases.

VII. CONCLUSION

We considered a CR system consisting of a PU and a pair of \( \text{SU}_{\text{tx}} \) and \( \text{SU}_{\text{rx}} \). The \( \text{SU}_{\text{tx}} \) is equipped with a RA which divides the angular space into \( M \) sectors. The \( \text{SU}_{\text{tx}} \) first senses the activity of PU and transmits data to \( \text{SU}_{\text{rx}} \) (if the channel is sensed idle) over the strongest channel (sector) of the RA. We obtained the optimal channel sensing duration, the optimal power level and the optimal threshold, such that the ergodic capacity of CR system is maximized, subject to average interference and power constraints. In addition, we derived closed form expressions for outage and symbol error probabilities of our CR system.

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