Exploring the gluon plasma

Saumen Datta and Sourendu Gupta

Department of Theoretical Physics, Tata Institute of Fundamental Research, Homi Bhabha Road, Mumbai 400005, India

Abstract

We report lattice computations in SU($N_c$) pure gauge theory, where $N_c$ is increased beyond the physical value of 3. We demonstrate two-loop scaling of $T_c$, thus obtaining the variation of $T_c/\Lambda_{\overline{MS}}$ with $N_c$, and fixing the temperature scale. We study the equation of state of the gluon plasma, the conformal anomaly, and the approach to the weak coupling theory. We find that the weak-coupling prediction is always closer to the lattice data than the conformal field theory is.

1. The phase diagram at large $N_c$

![Figure 1: The flag diagram of QCD in the plane of $N_c$ and the quark mass $m$, with two flavours of degenerate quarks, i.e., with a flavour SU(2) group. For physical values of $N_c$ and $m$ there is only a crossover at finite temperature. For each fixed value of the quark mass, $m$, there is a critical end point of the first order deconfining line, $N^*_c$. As $m$ changes, this end point changes continuously, going away to infinity in the chiral limit $m \to 0$. This line of the critical end point of the deconfinement transition is in the Ising universality class. The line $m = 0$ is a critical line for the chiral phase transition and is expected to lie in the O(4) universality class.](image-url)

The phase diagram of large $N_c$ QCD can be patched together from two different universality arguments. The first is about the pure gauge theory [1]. The order parameter for the finite temperature ($T$) transition in this theory is the expectation value of the Wilson line, $\mathcal{L}$. This vanishes at small $T$, but could be non-vanishing at high $T$ [2]. The loop is unchanged under the center symmetry of the gauge group, $Z(N_c)$. Therefore, if there is a phase transition, then it lies in the universality class of $Z(N_c)$. Since there is no critical point for any $N_c > 2$, such transitions,
if they occur, must be of first order. The SU(2) theory is an exception, and would have a second order transition. In other words, we might expect a line of first order transitions ending in a critical point at $N_c = 2$ which is in the Z(2) universality class. There is evidence for such a line from many studies with $N_c = 3$ [3], and several with $N_c = 4$ [4] and higher [5], ending with a critical point in the correct universality class [6].

Since quark loops can be neglected to an accuracy of $1/N_c$ in QCD at large $N_c$, the quark determinant in the QCD partition function is suppressed by a power of $N_c$, and hence can be neglected. As long as the quark mass, $m$, is large enough to ensure this, continuity arguments (i.e., the Gibbs’ phase rule) ensures that one should find a line of first order phase transitions ending in a critical point in the Ising universality class. In other words, the $N_c = 2$ critical point in the quenched theory develops into an Ising critical line with changing quark mass mass. This critical line bounds a region of first order transitions.

In the chiral limit, on the other hand, the effect of the quark determinant on thermodynamics cannot be neglected. Here one expects chiral symmetry breaking for two flavours of massless quarks, and hence three massless pions at low temperature. This leads to a second universality argument [7]. For generic $N_c$ one might expect that chiral symmetry is restored at finite temperature, the order parameter being the chiral condensate, $\bar{\psi}\psi$. Since SU(2) flavour symmetry restoration is in the same universality class as the O(4) spin model, we expect a line of second order phase transitions in this universality class for generic $N_c$. For $N_c = 2$, since all representations are real, the symmetry is enhanced to the Pauli-Güersey symmetry, U(4), spontaneously broken to Sp(4). The finite temperature symmetry restoring transition is critical, and in a different universality class, possibly O(6) [8].

The quark determinant adds terms of the kind $(\log \lambda_i)/N_c$ to the QCD action, where $\{\lambda_i\}$ are eigenvalues of the Dirac operator. As $m$ decreases, the lowest eigenvalue, $\lambda_0$, moves to zero linearly with $m$, and a finite density of eigenvalues develops near $\lambda_0$. This mechanism is intimately related to chiral symmetry breaking [9]. Then, as $m$ decreases, one must move to larger and larger $N_c$ in order to be able to apply the large $N_c$ arguments. For $N_c = 3$ and roughly physical quark masses one has a thermal crossover, so this “physical” point lies outside the boundary where the large $N_c$ arguments about the phase diagram apply.

What can one learn about QCD from a study of the large $N_c$ pure gauge theory? It would seem that there is little that one can learn quantitatively about the phase diagram of interest, i.e., for QCD with light quark masses, since the pure gauge theory has a first order deconfinement transition whereas that theory has a crossover. However, the high temperature phase over the full flag diagram is connected. As a result, one thing that one might learn about this more physically relevant theory is the nature of the high temperature phase. In particular we ask questions about the approach to the ideal gas, and the relevance of strongly coupled conformal theories and the weak coupling expansion in this phase. Such questions are likely to have qualitatively similar answers in every part of the flag diagram.

2. The temperature scale

The computations reported here were performed with $N_c = 4$ and 6 on $N_t \times N_s^3$ lattices with $N_t = 4, 6$ and 8, and $N_s \geq 2N_t$. These include the finest lattice spacings used in simulations in four dimensions for $N_c > 3$ until now. The Wilson action was used to study the deconfinement transition temperature, $T_c$. In the Euclidean thermal field theory $N_c a(g) = 1/T$, where $a(g)$ is the lattice spacing when the gauge coupling is $g$. By making an independent measurement of $g$ and then using the renormalization group equation (RGE) for finding $a(g)$, one can set the scale
of $T$. The RGE needs a boundary condition, which is supplied by the determination of $T_c$. This process gives the temperature scale $T/T_c$ \cite{10}.

The renormalized coupling was defined by a measurement of the $T = 0$ plaquette. The weak coupling expansion of the plaquette to second order was inverted to obtain the running coupling at the scale of the plaquette, which is proportional to $a$. This is called the V-scheme \cite{11}. Then the two-loop RGE was solved to give $T/T_c$. The adequacy of the two-loop RGE can be gauged by comparing the scales obtained by simulations with different $N_t$. Figure 2 shows that two-loop scaling is adequate for $N_t = 6$ or more when $N_c = 4$.

The scale setting of $T/T_c$ also allows us to invert the RGE and find $T_c/\Lambda_{\overline{MS}}$, in other words, to determine $\Lambda_{\overline{MS}}$. This is done in the V-scheme for each $N_t$. The result scales to a finite value in the $N_c \to \infty$ limit with $1/N_c$ corrections. The determination of $\Lambda_{\overline{MS}}$ could be more sensitive to the scheme choice than the temperature scale because of the logarithmic running of the coupling.

3. The equation of state

The equation of state was computed for SU(4) pure gauge theory. The interaction measure, $\Delta = E - 3P$, computed using the two-loop beta function, scales as $T^2$ \cite{12} in the high-temperature phase (see Figure 3). Computing the pressure using the integral method, we found that the continuum limit is reached at $N_t = 6$.

We show the equation of state of the SU(3) \cite{3} and SU(4) theories in Figure 4. We see that the lattice data are closer to the weak-coupling results \cite{13} than to conformal field theory ($\Delta = 0$) at all temperatures. At high temperatures, of course, the weak coupling theory tends to an ideal gas, which in turn is a conformal theory. There seems to be no window in temperature where a non-trivial conformal theory describes the equation of state better than weak-coupling theory.

References

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Figure 3: The equation of state for SU(4) pure gauge theory. On the left is the interaction measure $\Delta/(T^2T_c^2)$, showing that it scales as $T^2$. On the right is the pressure, $P$, evaluated at two different lattice spacings, showing that the continuum result is obtained.

Figure 4: The equation state of SU(3) (left) and SU(4) (right) pure gauge theory. The diagonal is the line of all possible conformal theories ($\Delta = 0$), and the curve is the weak coupling resummed result from [13].

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