EMPIRICAL CONSTRAINTS ON CONVECTIVE CORE OVERSHOOT

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Received 2001 May 1; accepted 2001 June 4

ABSTRACT

In stellar evolution calculations, the local pressure scale height is often used to empirically constrain the amount of convective core overshoot. However, this method brings unsatisfactory results for low-mass stars (≤1.1–1.2 M⊙ for Z = Z⊙), which have very small cores or no convective core at all. Following Roxburgh’s integral constraint, we implemented an upper limit of overshoot within the conventional method of z parameterization to remove an overly large overshoot effect on low-mass stars. The erroneously large effect of core overshoot due to the failure of z parameterization can be effectively corrected by limiting the amount of overshoot to ≤15% of the core radius; 15% of the core radius would be a proper limit of overshoot, which can be implemented in a stellar evolution code for intermediate- to low-mass stars. The temperature structure of the overshoot region does not play a crucial role in stellar evolution since this transition region is very thin.

Key words: color-magnitude diagrams — stars: evolution

1. INTRODUCTION

Understanding the physics of convective core overshoot is important in interpreting the color-magnitude diagrams (CMDs) and the luminosity functions of open star clusters and generally of intermediate-age stellar populations. This is due to the fact that increasing the mixed-core mass modifies the evolutionary tracks of stars (affecting the isochrone shape) and lengthens the evolutionary lifetimes of stars (affecting age determinations and luminosity functions near the main-sequence turnoff). Thus, in addition to the classic problem of establishing the chronology of star clusters in the Galaxy and nearby systems, core overshoot affects the spectral dating of young and intermediate-age galaxies observed at large redshifts.

Convective core overshoot is loosely understood here as the presence of material motions and/or mixing beyond the formal boundary for convection set by the classic Schwarzschild criterion (1906). An early study by Saslaw & Schwarzschild (1965), based on thermodynamic grounds (i.e., the edge of the convective core in massive stars is sharply defined in an entropy diagram), suggested that little overshoot takes place at the edge of convective cores. For this reason, it was believed that the presence of a gap in the CMD of open star clusters and its magnitude could be used as indicators of the age and chemical composition of the cluster (Aizenman, Demarque, & Miller 1969). However, Shaviv & Salpeter (1973) pointed out that if one takes into account the presence of hydrodynamic motions and turbulence, one might expect some nonnegligible amount of overshoot.

In fact, observational studies of the size of gaps near the turnoff in open star cluster CMDs suggest better agreement with theoretical isochrones that admit some amount of core overshoot (Maeder & Mermilliod 1981; Stothers & Chin 1991; Carraro et al. 1993; Daniel et al. 1994; Demarque, Sarajedini, & Guo 1994; Kozhurina-Platais et al. 1997; Nordström, Andersen, & Andersen 1997). Several computational schemes of various degrees of sophistication in treating the physics of overshoot have been developed for stellar evolution codes (Prather & Demarque 1974; Cogan 1975; Maeder 1975; Maeder & Meynet 1988; Bertelli et al. 1990).

As well, studies of detached eclipsing binaries in which the components have convective cores suggest some core overshoot of the order of 0.2 pressure scale height (Ribas, Jordi, & Gimenez 2000).

Stothers & Chin (1991) pointed out the high sensitivity of the amount of convective core overshoot to the adopted opacities. For example, increases in radiative opacities from the OPAL group (Iglesias & Rogers 1996) over the previous generation Los Alamos Opacity Library (Huebner et al. 1977) decrease the need for overshoot in comparison with observational data. In fact, the nature of the overshoot depends on the details of the local physics at the convective core edge. As pointed out by Zahn (1991), the local Péclet number, which characterizes the relative importance of radiative and turbulent diffusivity, determines whether penetration takes place in the mixed overshoot region (i.e., the temperature gradient is the adiabatic gradient) or whether the local radiative transfer dominates the energy transport (overmixing). In the latter case, the stable temperature gradient is unaffected by the mixing.

Roxburgh (1989, 1992) also considered the physics of convective core overshoot from a different point of view. His integral constraint argument places an upper limit on the amount of core overshoot that can take place in the stellar interior. Zahn (1991) argued that overshoot is practically adiabatic at the edge of the convective core, except in a thin transition layer, and therefore Roxburgh’s integral constraint defines the convective core size.

More information can be gained from numerical simulations of convection. Because of the long relaxation times involved, it is not possible to perform three-dimensional simulations of convective cores in full physical detail. Useful scaling information can, however, be derived from idealized three-dimensional simulations (Singh, Roxburgh, & Chan 1995, 1998).

The likely presence of differential rotation deep within stars brings further complexity to the overshoot problem, as one might expect a shear layer to develop at the edge of a convective core (Pinsonneault, Deliyannis, & Demarque 1991). Deupree (1998, 2000) considered the combined effects of rotation and convective overshoot in massive stars,
which have large convective cores, with the help of two-dimensional hydrodynamic calculations. Rotation likely plays a role in the case of less massive stars as well (in the range 1.5–2.0 \( M_\odot \)), with shear-driven turbulence near the edge of the convective core inducing mixing from the helium-enriched core into the envelope.

Since a complete physical theory predicting the amount of overshoot for a star of a given mass and chemical composition, free of arbitrary parameters, is not available at this point, in this paper we approach the problem of core overshoot in the spirit of the work of Rosvick & VandenBerg (1998). We consider and test parameterizations of convective overshoot, which are compatible with sound physical principles and with the growing data available on open cluster CMDs. These simple parameterizations have the merit of being readily implemented in a stellar evolution code.

2. TEST OF OVERSHOOT TREATMENTS

The local pressure scale height is often used to constrain empirically the amount of convective core overshoot in stellar evolution calculations. This conventional method assumes that the radial size of the overshoot region is proportional to the pressure scale height at the edge of a convective core, i.e., \( d_{\text{core}} = \alpha H_p \).

Observational studies of open clusters show that a moderate amount of core overshoot is essential to explain the observed gap near the turnoff in CMDs. In a study of NGC 2420, Demarque et al. (1994) found that \( \alpha = 0.23 \) is required for the best-fit CMD. Also, Kozhurina-Platais et al. (1997) estimated \( \alpha = 0.20 \) and 0.25 for NGC 3680 and NGC 752, respectively, by isochrone fitting (cf. Nordström et al. 1997 for NGC 3680). Overshoot effects appear to decrease because of the decreasing mass of stars near the turnoff in older clusters (Carraro et al. 1994; Sarajedini et al. 1999). Although open clusters do not have many member stars, these studies of nearly solar metallicity clusters show that the conventional method constrains the amount of core overshoot relatively well for intermediate-mass stars.

However, this pressure scale height method brings unsatisfactory results for low-mass stars (\( \lesssim 1.1 \)–1.2 \( M_\odot \)) for \( Z = Z_\odot \). These stars have very small cores or no convective core at all; thus the local pressure scale height at the convective core edge would be very large, implying a large amount of overshoot. Figure 1 illustrates the effects of the convective core and core overshoot on stellar evolution. With no overshoot, models of mass \( \geq 1.2 \) \( M_\odot \) show a main-sequence (MS) hook, which is caused by abrupt contraction of a convective core due to the flat hydrogen profile. When overshoot is included, evolutionary tracks show a redder MS hook due to the hydrogen supply from mixing in the overshoot region, which lengthens the hydrogen core-burning lifetime. However, the effect of overshoot on low-mass stars seems too large compared with observations. If the amount of overshoot is set to 0.2 \( H_p \), low-mass stars with 1.0 and 1.1 \( M_\odot \) show a prominent overshoot effect in their evolutionary shape. However, these low-mass stars are not expected to have large overshoot regions since relatively old open clusters do not show an overshoot gap near the turnoff. Therefore, simple use of the \( \alpha \) parameter cannot give consistent results for a wide range of masses and ages.

Roxburgh's integral constraint (Roxburgh 1989; Zahn 1991) is another way to quantify the amount of core overshoot since it provides an upper limit to the extent of convective penetration in the following formula:

\[
\int_0^{r_c} (L_{\text{rad}} - L_{\text{tot}}) \frac{1}{T^2} \frac{dT}{dr} dr = 0 ,
\]

where \( L_{\text{tot}} \) is the total luminosity produced by nuclear reactions, \( L_{\text{rad}} \) is the radiative luminosity, and \( r_c \) is the radius of the effective core. Since the energy is transported by both radiation and convection inside a convective core, the integral is positive until it reaches the Schwarzschild boundary, where \( L_{\text{tot}} = L_{\text{rad}} \). The region of convective overshoot is located beyond this boundary, where \( L_{\text{rad}} > L_{\text{tot}} \) up to the point \( r = r_c \), which satisfies the constraint.

However, Canuto (1997) pointed out in his theoretical investigation that Roxburgh's integral gives only an upper limit to the overshoot extent. Rosvick & VandenBerg (1998) used this constraint on NGC 6819 and found that a factor of 0.5 in Roxburgh's integral is required for the best-fit CMD. Therefore, simply adopting Roxburgh's integral to quantify the amount of overshoot is not valid, and the fuzzy factor (analogous to the \( \alpha \) parameter in the pressure scale height method) should be tested for clusters of various ages and metallicities.

Faced with the current situation of the core overshoot treatments, we developed a consistent approach, which can be used regardless of stellar mass. Our simple approach of core overshoot is discussed in the next section. All stellar evolutionary tracks are constructed with the Yale Rotating Evolution Code (YREC) with OPAL opacities (Iglesias & Rogers 1996). The general description of stellar evolution models can be found in Yi et al. (2001).

2.1. Maximum Amount of Core Overshoot (\( \beta \) Limit)

Roxburgh (1992) showed that his integral constraint gives an upper limit on the extent of convective penetration (\( \delta_{\text{max}} = \beta R_{\text{core}} \)). For very small convective cores, the maximum amount of the extent of penetration is \( \sim 18\% \) of the core radius, independent of the details of energy gener-
ation and opacity. The maximum amount, the $\beta$ limit, varies from 0.18 to 0.4, depending on the size of the convective core.

Following Roxburgh’s integral constraint, we implemented an upper limit of overshoot within the conventional pressure scale height method to remove the erroneously large overshoot effect on low-mass stars. In other words, we limited the core overshoot so that the amount of overshoot, $d_{\text{ov}} (= \alpha H_p)$, cannot be larger than a portion of the core radius, $d_{\text{max}} (= \beta R_{\text{core}})$. The merit of this modification is that the ad hoc size of overshoot regions for low-mass stars can be effectively corrected without affecting the evolution of higher mass stars.

Since the integral constraint is obtained for convective penetration, which maintains a nearly adiabatic temperature structure, we used the adiabatic temperature gradient in the overshoot region. The effect of the temperature structure will be discussed in the following section. Accepting the result of open cluster studies, we used $\alpha = 0.2$ and performed numerical experiments with various $\beta$ limits for solar-metallicity stars.

The effect of the $\beta$ limit on stellar evolution is presented in Figure 2. For low-mass stars, the presence of the overshoot effect is very sensitive to the choice of the upper limit of overshoot. It can be easily noted that the overshoot effect on low-mass stars (mass $\lesssim 1 M_\odot$) becomes negligible for $\beta \leq 0.15$. However, if the upper limit is greater than 20% of the core radius, the overshoot effect is still too large for low-mass stars. This result is consistent with Roxburgh’s value of 0.18 for a nearly zero convective core radius. Thus, the erroneously large effect of core overshoot due to the failure of $\alpha$ parameterization for low-mass stars can be effectively corrected by limiting the amount of overshoot to $\lesssim 15\%$ of the core radius.

The choice of $\beta$ does not significantly affect the evolution of intermediate-mass stars ($1.5-2.0 M_\odot$) when $\beta \geq 0.15$ is used. These models are consistent with those of the same $\alpha$ without limiting the overshoot extent. However, if the amount of overshoot is restricted to much less than 15% of the core radius, the overall overshoot effect would be significantly reduced. Therefore $\beta \leq 0.15$ for intermediate-mass stars gives effectively the same result with a smaller $\alpha$ of the conventional method, which is undesirable in our effort to modify the $\alpha$ parameterization with the $\beta$ limit for a given $\alpha$ value. Combining the criteria for low-mass and intermediate-mass stars, we conclude that 15% of the core radius ($\beta = 0.15$) is a proper limit of overshoot for intermediate- to low-mass stars.

Because of the effects of overshoot on stellar evolution, such as the shape of evolutionary locus near turnoff and timescale in the hydrogen-burning stage, the ensuing isochrones are expected to show a different shape. We used 1.0–2.0 $M_\odot$ evolutionary tracks with mass bin size 0.1 $M_\odot$ to generate isochrones and test the effect of the $\beta$ limit. We compared $\alpha = 0.2$ isochrones with and without limiting the amount of overshoot (Fig. 3). As expected from the comparison of evolutionary tracks, $\beta = 0.2$ isochrones are not

![Fig. 2.—Effect of the $\beta$ limit on stellar evolution for $Z = 0.018$. The amount of core overshoot is limited to $\beta$ factor times $R_{\text{core}}$. Note that models with $\beta \geq 0.15$ show nearly identical tracks for intermediate-mass stars (top). However, the overshoot effect is not present for low-mass stars when $\beta \leq 0.15$ (bottom), which suggests that the erroneously large effect of overshoot on low-mass stars can be corrected by limiting the extent of core overshoot.](image1)

![Fig. 3.—Effect of the $\beta$ limit on the isochrones of $Z = 0.018$. $\beta = 0.2$ models are not much different from the models without limiting core overshoot and repeat the failure of $\alpha$ parameterization (top). When overshoot is limited to 10% of the core radius, overall isochrones are dimmed, which is similar to the result of smaller $\alpha$ (bottom).](image2)
much different from the isochrones without the $\beta$ limit. When overshoot is limited to 10% of the core radius, overall isochrones are dimmed because of the reduced size of the overshoot region, especially for younger ages. This result is similar to that when a smaller $\alpha$ is used. Thus, $\beta = 0.1$ is too small to keep the amount of overshoot of intermediate-mass stars given by $\alpha = 0.2$.

Focusing on the modification of the $\alpha$ parameter method, we increased $\beta$ with increasing stellar mass to minimize the effect of limiting the extent of overshoot for stars of mass $\gtrsim 1.5 \, M_\odot$. Since the maximum extent of core overshoot depends on the size of a convective core, higher mass stars would have larger limits, according to Roxburgh’s integral constraint. Thus, we assumed a simple linear relation between stellar mass and the upper limit of overshoot ($\beta = aM + b$). The coefficients are determined for $\beta = 0.1$ and 0.4 for 1 and 2 $M_\odot$ stars, respectively. Figure 4 demonstrates how the mass-$\beta$ relation changes stellar evolution. Compared with $\beta = 0.15$ models, models with increasing $\beta$ show slightly bluer and brighter isochrones for younger ages. Although increased $\beta$ models are closer to the models without the $\beta$ limit, the difference is not significant.

2.2. Penetration and Overmixing

The overshoot region generally represents a well-mixed zone between convective and radiative zones. The temperature structure of the overshoot region depends on the physical conditions such as the ratio of radiative to kinetic timescales at the convective core boundary. According to Zahn (1991; also see Demarque et al. 1994), if the adiabatic cells penetrate into the radiative zone sufficiently far, then the temperature gradient of the overshooting zone would be adiabatic also (convective penetration). Or, if the adiabatic cells dissolve rapidly, then only mixing would occur with the remaining radiative gradient (overmixing).

However, it is not well known whether the temperature structure of the overshoot region is adiabatic or radiative. To test the effects of the temperature structure on stellar evolution, we generated evolutionary tracks and isochrones with and without forcing the temperature gradient of the overshoot region to be adiabatic.

Stellar evolutionary tracks of solar metallicity show almost no difference between overmixing and convective penetration when the conventional method of $\alpha = 0.1$–0.3 is used. This is also true when the amount of overshoot is limited by any factor of the core radius (Fig. 5). We conclude that temperature structure of the overshoot region does not play a crucial role in stellar evolution, at least when the local pressure scale height method is used. This is due to the fact that this transition region is very thin. Note that the effect is slightly larger in low-mass stars.

3. CONCLUSION

This paper presents an approach to the treatment of convective core overshoot in stellar evolutionary models for stars with convective cores. By limiting the amount of convective core overshoot, the failure of the pressure scale height method can be effectively corrected. A proper limit of core overshoot for intermediate- to low-mass stars would be 15% of the core radius. We intend to use evolutionary tracks based on this approach to explore population synthesis and the spectral dating of high-redshift galaxies.

It is crucial to test stellar evolution by observation. Several observational approaches to test stellar models with convective overshoot are under way. The systematic study of open star clusters that exhibit hydrogen exhaustion phases provides important constraints that stellar models must satisfy. The WIYN Open Cluster Study (WOCS)
project provides a comprehensive study of open clusters in the Galaxy selected to provide a range of chemical compositions and ages. In addition to photometric and spectroscopic observations of individual stars, WOCS includes an astrometric proper-motion study to establish cluster membership, a necessary condition. Because the number of stars in open clusters is small, establishing membership is essential for carrying out detailed tests of stellar evolutionary models using a cluster CMD and luminosity function. This advantage is demonstrated in recent deep proper-motion studies of NGC 188 (Dinescu et al. 1996) and NGC 3680 (Kozhurina-Platais et al. 1997). An excellent overview of WOCS objectives and results obtained so far has been written by Mathieu (2000).

Observations of the CMDs of some of the more populous star clusters in the Magellanic Clouds will provide more information on core overshoot in intermediate-age stellar populations. Although more distant than open clusters in the Galaxy, the Magellanic Cloud clusters have the advantage of testing the evolution of stars with a lower metallicity.

Finally, stellar seismology, by analyzing oscillations that propagate deep in stellar interiors, promises to probe sensitively the convective cores of stars evolving off the main sequence in the intermediate-mass range. Similarly, the mass of the exhausted helium core in subgiants and giants can be derived on the basis of physical models. Already observations of oscillations of nearby stars near the main sequence and in the subgiant and giant evolutionary phases have been done from the ground (Kjeldsen et al. 1995; Martic et al. 1999; Bedding et al. 2001) and in space (Buzasi et al. 2000). Preliminary analysis of some of these observations with the help of stellar models show the unmistakable signature (mixed $p$- and $g$-modes) of exhausted core size in the case of the more evolved stars (Christensen-Dalsgaard, Bedding, & Kjeldsen 1995; Guenther & Demarque 1996).

This research has been supported in part by NASA grant NAG 5-8406.

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