Negative tension branes as stable thin shell wormholes

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Abstract
We investigate negative tension branes as stable thin shell wormholes (TSWs) in Reissner–Nordström-(anti) de Sitter spacetimes in $d$ dimensional Einstein gravity. Imposing $Z_2$ symmetry, we construct and classify traversable static TSWs in spherical, planar (or cylindrical) and hyperbolic symmetries. In spherical geometry, we find the higher dimensional counterpart of Barceló and Visser’s wormholes, which are stable against spherically symmetric perturbations. We also find the classes of TSWs in planar and hyperbolic symmetries with a negative cosmological constant, which are stable against perturbations preserving symmetries. In most cases, stable wormholes are found with the combination of an electric charge and a negative cosmological constant. However, as special cases, we find stable wormholes even with vanishing cosmological constant in spherical symmetry and with vanishing electric charge in hyperbolic symmetry.

Keywords: wormholes, thin shell, stability

(Some figures may appear in colour only in the online journal)

1. Introduction

Wormholes are spacetime structures which connect two different universes or two points of our Universe. M Morris and K Thorne have pioneered a qualitative study for static spherically symmetric wormholes which are two-way traversable and discussed the traversable conditions in [1]. The most difficult requirement for satisfying the traversable conditions is an exotic matter which violates the null or weak energy condition. In general relativity, Morris and Thorne’s static spherically symmetric traversable wormholes need the stress-energy tensor that violates the energy condition at their throat. See details in [1].
An another class of traversable wormholes has been found by M Visser. This class of wormholes can be obtained by a ‘cut-and-paste’ procedure [2] and such structures are called thin shell wormholes (TSWs). E Poisson and M Visser first presented stability analysis for spherical perturbations around such TSWs, and found that there are stable configurations according to the equation of state of an exotic matter residing on the throats [3]. The work of Poisson and Visser has been extended in different directions; charged TSWs [4], TSWs constructed by a couple of Schwarzschild spacetimes of different masses [5] and TSWs with a cosmological constant [6]. TSWs in cylindrically symmetric spacetimes have also been studied [7, 8]. Garcia et al published a paper about stability for generic static and spherically symmetric TSWs [9]. Dias and Lemos studied stability in higher dimensional Einstein gravity [10].

TSW models are easy to construct, which certainly have a wormhole structure. TSWs have no differentiability for its metric at their throat, i.e. they are only $C^0$, contrary to smooth wormhole models such as the Ellis wormhole [11], which are smooth ($C^\infty$) at their throat. Though TSWs are not smooth, they indeed have qualitatively common features with smooth wormholes. Hence, investigating of properties of TSWs may prompt deeper understanding for properties of generic wormholes.

In this paper, we concentrate on stability against radial perturbations. The radial perturbation is important in the context of stability analysis in the following reasons: (i) since the stability analysis against radial perturbations is much simpler than nonradial perturbations which entail gravitational radiation, it is a natural first step to investigate radial stability of wormhole models. For thin shell models, the stability analysis against radial perturbations is particularly simple. (ii) The previous study suggests that the radial perturbation of wormhole spacetimes is most dangerous: the paper by Armendariz-Picon [12] showed that the Ellis wormhole is stable against metric perturbations including nonradial perturbations which do not change the throat radius. Subsequently, the Ellis wormhole turned out to be unstable against radial perturbation which changes the radius of the throat. The throat must shrink or inflate [13]. From the above, we can say that for the Ellis wormhole, the radial perturbation which changes the radius of the throat is the most ‘dangerous’ perturbation, as mentioned in Bronnikov’s book [14]. One can expect that this property applies to not only the Ellis wormhole but also other wormhole solutions.

We deal with higher dimensional wormholes with and without a cosmological constant in this paper. The notion of higher dimensional spacetimes was first introduced by Kaluza and Klein [15]. They found that the gravitational field and the electromagnetic field can be unified in five dimensional spacetimes. In their work, the length of the fifth dimension is confined in a very small scale. The gauge/gravity correspondence conjectured by Maldacena [16] generally shed light on structures in the anti de-Sitter (AdS) spacetime in higher dimensions. In the end of the twentieth century, Randall and Sundrum proposed an idea that we perhaps live in a (3+1) dimensional brane in the (4+1) dimensional spacetime whose extra dimension spreads widely with a negative cosmological constant, which is called a brane-world model [17]. In this model, the bulk five dimensional spacetime is the AdS spacetime. In a broader context, candidate theories for quantum gravity, such as superstring theory and M-theory, entail higher dimensional spacetimes.

The effects caused by the electromagnetic field on the stability of a TSW are not well known. One may wonder whether the existence of the electric charge of wormholes can stabilize the wormholes. Therefore, it is worth studying electrically charged TSWs. In spherically symmetric and hyperbolically symmetric spacetimes, the Reissner–Nordstöm AdS spacetime is the unique solution to the higher dimensional Einstein–Maxwell equation. In
cylindrically symmetric and planar symmetric spacetimes, the Reissner–Nordstöm AdS spacetime is not unique but one of the possible static solutions.

Pure tension branes, whether the tension is positive or negative, are particularly interesting because they have Lorentz invariance and have no intrinsic dynamical degrees of freedom. In the context of stability, pure negative tension branes have no intrinsic instability on their own, although they violate the weak energy condition. This is in contrast with the Ellis wormholes, for which a phantom scalar field is assumed as a matter content and it suffers the so-called ghost instability because of the kinetic term of a wrong sign [11, 13]. The construction of traversable wormholes by using negative tension branes have first been proposed by Barceló and Visser [18]. They analyzed dynamics of spherically symmetric traversable wormholes obtained by operating the cut-and-paste procedure for negative tension 2-branes (three dimensional timelike singular hypersurface) in four dimensional spacetimes. They found stable brane wormholes constructed by pasting a couple of Reissner–Nordström-AdS spacetimes. In their work [18], the charge is essential to sustain such wormholes. And in most cases, a negative cosmological constant tends to make the black hole (BH) horizons smaller. However, in exceptional cases, one can obtain wormholes with a vanishing cosmological constant, if the absolute value of the charge satisfies a certain condition.

In the past several years, the possibility of stable wormholes in various theories of modified gravity has gathered attention and has been extensively investigated by many authors [8, 19, 20]. However, the possibility of wormholes whose exotic matter is a pure tension brane has not yet fully been checked in those theories. To study such possibility, it is important and necessary to fully understand the existence and stability of all kinds of $Z_2$ symmetric RN-(A)dS TSWs in higher dimensional pure Einstein–Maxwell theory.

In this paper, we investigate negative tension branes as TSWs in spherical, planar (or cylindrical) and hyperbolic symmetries in $d$ dimensional Einstein gravity with an electromagnetic field and a cosmological constant in bulk spacetimes. In spherical geometry, we find the higher dimensional counterpart of Barceló and Visser’s wormholes which are stable against spherically symmetric perturbations. As the number of dimensions increases, larger charge is allowed to construct such stable wormholes without a cosmological constant. Not only in spherical geometry, but also in planar and hyperbolic geometries, we find static wormholes which are stable against perturbations preserving symmetries.

This paper consists of the following sections. In section 2, we present a formalism for wormholes, which is more general than previous formalisms and also obtain a stability condition against perturbations preserving symmetries. In section 3, we introduce wormholes with a negative tension brane and we analyze the existence of static solutions, stability and horizon avoidance in spherical, planar and hyperbolic symmetries. Section 4 is devoted to summary and discussion.

2. Wormhole formalism

2.1. Construction

The formalism for spherically symmetric $d$ dimensional TSWs has been developed first by Dias and Lemos [10]. We extend their formalism to more general situations. We obtain wormholes by operating three steps invoking junction conditions [21]. This approach to construct TSWs has been pioneered by Poisson and Visser [3].

Firstly, consider a couple of $d$ dimensional manifolds, $V_{\pm}$. We assume $d \geq 3$. The $d$ dimensional Einstein equations are given by
\[ G_{\mu\nu} + \frac{(d-1)(d-2)}{6} \Lambda_{\pm} g_{\mu\nu} = 8\pi T_{\mu\nu}, \]  
(2.1)

where \( G_{\mu\nu} \), \( T_{\mu\nu} \) and \( \Lambda_{\pm} \) are Einstein tensors, stress-energy tensors and cosmological constants in the manifolds \( \mathcal{V}_{\pm} \), respectively. The metrics on \( \mathcal{V}_{\pm} \) are given by \( g_{\mu\nu}^\pm (x^\pm) \). The metrics for static and spherically, planar and hyperbolically symmetric spacetimes on \( \mathcal{V}_{\pm} \) are written as

\[ ds^2 = -f_{\pm}(r_{\pm}) dt_{\pm}^2 + f_{\pm}(r_{\pm})^{-1} dr_{\pm}^2 + r_{\pm}^2 \left( d\Omega_{d-2}^\pm \right)^2, \]  
(2.2)

\[ f_{\pm}(r_{\pm}) = k - \frac{\Lambda_{\pm} r_{\pm}^2}{3} - \frac{M_{\pm}}{r_{\pm}^{d-3}} + \frac{Q_{\pm}^2}{r_{\pm}^{2(d-3)}}, \]  
(2.3)

respectively. \( M_{\pm} \) and \( Q_{\pm} \) correspond to the masses and charges in \( \mathcal{V}_{\pm} \), respectively. \( k \) is a constant that determines the geometry of the \( (d-2) \)-dimensional space and takes \( \pm 1 \) or \( 0 \). \( k = +1, 0 \) and \( -1 \) correspond to a sphere, plane (or cylinder) and a hyperboloid, respectively. \( \left( d\Omega_{d-2}^\pm \right)^2 \) is given by

\[
\begin{align*}
\left( d\Omega_{d-2}^{1_{\pm}} \right)^2 &= d\theta_1^2 + \sin^2 \theta_1 d\theta_2^2 + \ldots + \prod_{i=2}^{d-3} \sin^2 \theta_i d\theta_{d-2}^2, \\
\left( d\Omega_{d-2}^{0_{\pm}} \right)^2 &= d\theta_1^2 + d\theta_2^2 + \ldots + d\theta_{d-2}^2, \\
\left( d\Omega_{d-2}^{-1_{\pm}} \right)^2 &= d\theta_1^2 + \sinh^2 \theta_1 d\theta_2^2 + \ldots + \sinh^2 \theta_1 \prod_{i=2}^{d-3} \sin \theta_i d\theta_{d-2}^2.
\end{align*}
\]  
(2.4)

We should note that by generalized Birkhoff’s theorem [22], the metric \( 2.2 \) is the unique solution of Einstein equations of electrovacuum for \( k = \pm 1 \). However, this is not unique for \( k = 0 \). Therefore, we should regard equation \( 2.2 \) as a special electrovacuum spacetime for \( k = 0 \).

Secondly, we construct a manifold \( \mathcal{V} \) by gluing \( \mathcal{V}_{\pm} \) at their boundaries. We choose the boundary hypersurfaces \( \partial \mathcal{V}_{\pm} \) as follows:

\[ \partial \mathcal{V}_{\pm} \equiv \left\{ r_{\pm} = a | f_{\pm}(a) > 0 \right\}, \]  
(2.5)

where \( a \) is called the thin shell radius. Then, by gluing the two regions \( \tilde{\mathcal{V}}_{\pm} \) which are defined as

\[ \tilde{\mathcal{V}}_{\pm} \equiv \left\{ r_{\pm} > a | f_{\pm}(a) > 0 \right\} \]  
(2.6)

with matching their boundaries, \( \partial \mathcal{V}_{\pm} = \partial \mathcal{V}_{\pm} \equiv \partial \mathcal{V} \), we can construct a new manifold \( \mathcal{V} \) which has a wormhole throat at \( \partial \mathcal{V} \). \( \partial \mathcal{V} \) should be a timelike hypersurface, on which the line element is given by

\[ ds^2_{\partial V} = -dr^2 + a(\tau)^2 \left( d\Omega_{d-2}^\pm \right)^2. \]  
(2.7)

It is convenient to define the function \( F(x) \equiv r - a(\tau) \) so that \( \partial \mathcal{V} \) is given by

\[ F = r - a(\tau) = 0. \]  
(2.8)

\( \tau \) stand for proper time on the junction surface \( \partial \mathcal{V} \) whose position is described by the coordinates \( x^\mu(\xi^\nu) = x^\mu(\tau, \theta_1, \theta_2, \ldots, \theta_{d-2}) = (t(\tau), a(\tau), \theta_1, \theta_2, \ldots, \theta_{d-2}) \), where Greek indices run over \( 1, 2, \ldots, d \) and Latin indices run over \( 1, 2, \ldots, d-1 \). \( \{ \xi^\mu \} \) are the intrinsic coordinates on \( \partial \mathcal{V} \).
Thirdly, by using the junction conditions, we derive the equations for the manifold $\partial \mathcal{V}$. To achieve this, we define unit normals to hypersurfaces $\partial \mathcal{V}_\pm$. The unit normals are defined by

$$n_{a\pm} \equiv \pm \frac{F_{a\sigma}}{|g^{a\mu}F_{\mu\sigma}|^{1/2}}. \quad (2.9)$$

To construct TSWs, we make the unit normals on $\partial \mathcal{V}_\pm$ take different signs, while to construct normal thin shell models, the unit normals are chosen to be of same signs. We define vectors tangent to $\partial \mathcal{V}$ as

$$e_{(i)\pm}^a \equiv \frac{\partial x^a}{\partial y^i}. \quad (2.10)$$

We also define the four-velocity $u_{(i)\pm}^a$ of the boundary as

$$u_{(i)\pm}^a = e_{(i)\pm}^a = (i_{\pm}, \dot{a}, 0, \ldots, 0)$$

$$= \begin{pmatrix} \frac{1}{f_\pm(a)} \sqrt{f_\pm(a) + \dot{a}^2}, & \dot{a}, & 0, & \ldots, & 0 \end{pmatrix}. \quad (2.11)$$

where $\cdot \equiv \partial/\partial \tau$ and $u^a u_a = -1$ is satisfied. From equations (2.8) and (2.9) we have

$$n_{a\pm} = \pm \begin{pmatrix} -\dot{a}, & \sqrt{f_\pm + \dot{a}^2}, & 0, & \ldots, & 0 \end{pmatrix} \quad (2.12)$$

and the unit normal satisfies $n_a n^a = 1$ and $u^a n_a = 0$.

### 2.2. Equations for the shell

In general, there is matter distribution on $\partial \mathcal{V}$. The equations for the shell are given by the junction conditions of the Einstein equations [21] as follows:

$$S_{ij}^0 = -\frac{1}{8\pi} \left( \kappa_i^j - \delta_i^j \kappa_0^j \right), \quad (2.13)$$

$$\kappa_i^j = \left( K_i^j - K_j^i \right)_{\partial \mathcal{V}}, \quad (2.14)$$

where $S_{ij}^0$ is the surface stress-energy tensor residing on $\partial \mathcal{V}$. $K^\pm_{ij}$ are the extrinsic curvatures defined by

$$K^\pm_{ij} \equiv \left( V_{(i)\pm}^\mu n_\mu^j \right) e_{(i)\pm}^\mu e_{(j)\pm}^\nu. \quad (2.15)$$

The nonzero components of $K^\pm_{ij}$ are the following:

$$K^\pm_{ij} = \pm \left( f_\pm + \dot{\alpha}^2 \right)^{-1/2} \left( \dot{\alpha} + \frac{1}{2} f'_\pm \right). \quad (2.16)$$

$$K^0_\theta^A = K^\theta_0^A = \ldots = K^0_{\theta_2^A} = \ldots = K^0_{\theta_{-1}^A} = \pm \frac{1}{a} \sqrt{f_\pm + \dot{\alpha}^2}, \quad (2.17)$$

where $\cdot \equiv d/\alpha$. Consequently, the nonzero components of $\kappa_i^j$ are

$$\kappa_i^j = \frac{B_\pm(a)}{A_\pm(a)} + \frac{B_\pm(a)}{A_\pm(a)}, \quad (2.18)$$


\[ \kappa^{b_1}_{b_1} = \cdots = \kappa^{b_{d-1}}_{b_{d-1}} = \frac{A_+(a) + A_-(a)}{a}, \quad (2.19) \]

where
\[ A_\pm (a) \equiv \sqrt{f_\pm + \dot{a}^2}, \quad B_\pm (a) \equiv \pm \frac{1}{2} \frac{f'_\pm}{a}. \quad (2.20) \]

Since our metrics (2.2) are diagonal, \( S^i_j \) is also diagonalized and written as
\[ S^i_j = \text{diag}(\ldots - \sigma, p, \ldots, p), \quad (2.21) \]

where \( p \) is the surface pressure (of opposite sign to surface tension) and \( \sigma \) is the surface energy density living on the thin shell. From equations (2.13) and (2.21), we obtain
\[ \sigma = -\frac{d-2}{8\pi a}(A_+ + A_-), \quad (2.22) \]
\[ p = \frac{1}{8\pi} \left[ \frac{B_+}{A_+} + \frac{B_-}{A_-} + \frac{d-3}{a}(A_+ + A_-) \right]. \quad (2.23) \]

Thus, we deduce a critical property of wormholes that \( \sigma \) must be negative.

The equation of motion for the surface stress-energy tensor \( S^i_j \) is given by
\[ S^i_j \mid_v + \left[ T^i_j_{\bar{\mu} \bar{\nu}} \right]_{\bar{\mu} \bar{\nu}} = 0 \quad (2.24) \]
as in [23], where \( ' \mid_v \) denotes the difference between the quantities in \( V_+ \) and in \( V_- \). Since the stress-energy tensor \( T^i_j \) in the bulk spacetime only contains the electromagnetic field,
\[ T^i_j = \frac{Q^2}{8\pi r^4} \text{diag}(-1, -1, 1, 1), \quad (2.25) \]
one can find \( T^i_j_{\bar{\mu} \bar{\nu}} = 0 \). Hence equation (2.24) yields
\[ \frac{d}{dt} \left[ \sigma a^{d-2} \right] + p \frac{d}{dt} \left( a^{d-2} \right) = 0. \quad (2.26) \]
Equation (2.26) corresponds to the conservation law. For later convenience for calculations, we recast equation (2.26) as
\[ \sigma' = -\frac{d-2}{a} (\sigma + p). \quad (2.27) \]

We can get the conservation law of mechanical energy for the exotic matter on the thin shell throat by recasting equation (2.22) as follows:
\[ \dot{a}^2 + V(a) = 0, \quad (2.28) \]
\[ (a) = \left( \frac{4\pi a^2}{d-2} \right)^2 - \left( \frac{f_+ - f_-}{2} \right)^2 \left( \frac{d-2}{8\pi a^2} \right)^2 + \frac{1}{2} \left( f_+ + f_- \right). \quad (2.29) \]
where \( V(a) \) or just the potential. From equation (2.28), the range of \( a \) which satisfies \( V(a) \leq 0 \) is the movable range for the shell. Since we obtained equation (2.28) by twice squaring of equation (2.22), there is possibility that we take wrong solutions which satisfy equation (2.28) but do not satisfy equation (2.22). See the appendix for the condition of right solutions. By differentiating equation (2.28) with respect to \( \tau \), we get the equation of motion for the shell as
Suppose a thin shell throat be static at $a = a_0$ and its throat radius satisfy the relation

$$f(a_0) > 0.$$  \hfill (2.31)

This condition is called the horizon-avoidance condition in [18]. We analyze stability against small perturbations preserving symmetries. To determine whether the shell is stable or not against the perturbation, we use Taylor expansion of the potential $V(a)$ around the static radius $a_0$ as

$$V(a) = V(a_0) + V'(a_0)(a - a_0) + \frac{1}{2}V''(a_0)(a - a_0)^2 + O((a - a_0)^3).$$  \hfill (2.32)

From equations (2.28) and (2.30), $a_0 = 0$, $\dot{a}_0 = 0 \Rightarrow V(a_0) = 0$, $V'(a_0) = 0$ at $a = a_0$ so the potential given by equation (2.32) reduces to

$$V(a) = \frac{1}{2}V''(a_0)(a - a_0)^2 + O((a - a_0)^3).$$  \hfill (2.33)

Therefore, the stability condition against radial perturbations for the thin shell is given by

$$V''(a_0) > 0.$$  \hfill (2.34)

### 3. Wormholes with a negative tension brane with Z2 symmetry

#### 3.1. Effective potential

From now on, for simplicity, we assume Z2 symmetry, that is, we assume $M_+ = M_-$, $Q_+ = Q_-$ and $A_+ = A_-$ and hence $f_+(r) = f_-(r)$. We denote $M_+ = M_- = M$, $Q_+ = Q_-$, $A_+ = A_- = A$ and $f(r) := f_+(r) = f_-(r)$. Then equation (2.32) reduces to

$$V(a) = f(a) - \left(\frac{4\pi\sigma}{d-2}\right)^2.$$  \hfill (3.1)

We investigate wormholes which consist of a negative tension brane. From equations (2.22) and (2.23), the surface energy density and surface pressure for the negative tension brane are represented as

$$\sigma = -\frac{d-2}{4\pi a}A = \alpha,$$  \hfill (3.2)

$$p = \frac{1}{4\pi} \left(\frac{B}{A} + \frac{d-3}{4\pi a}A\right) = -\alpha,$$  \hfill (3.3)

where $\alpha < 0$. By substituting equations (3.2) and (3.3) into equation (2.27), we obtain $\sigma' = 0$ irrespective of $\dot{a}$. Therefore, $\alpha$ must be constant and then both $p$ and $\sigma$ must be constant, too. So the effective potential reduces to

$$V(a) = f(a) - \left(\frac{4\pi\alpha}{d-2}\right)^2 a^2.$$  \hfill (3.4)

#### 3.2. Static solutions, stability criterion and horizon-avoidance condition

The present system may have static solutions $a = a_0$. We define $p_0 := p(a_0)$ and $\sigma_0 := \sigma(a_0)$, where $a_0$ satisfies equation (2.31):
\[
\sigma_0 = -\frac{d - 2}{4\pi a_0} A_0 = \alpha, \quad (3.5)
\]
\[
p_0 = \frac{1}{4\pi} \left( \frac{B_0}{A_0} + \frac{d - 3}{a_0} A_0 \right) = -\alpha, \quad (3.6)
\]
where
\[
A_0 \equiv \sqrt{f(a_0)}, \quad B_0 \equiv \frac{1}{2} f''(a_0). \quad (3.7)
\]
Eliminating \(\alpha\) from equations (3.5) and (3.6), we can obtain the equation for the static solutions,
\[
\frac{a_0}{2} f''(a_0) - f(a_0) = 0. \quad (3.8)
\]
The explicit form of equation (3.8) is
\[
2ka_0^{2(d-3)} - (d - 1)Ma_0^{d-3} + 2(d - 2)Q^2 = 0. \quad (3.9)
\]
The stability conditions for wormholes are shown in the previous section as \(V^\prime(a_0) > 0\). The relation between \(\alpha\) and \(a_0\) is given by equation (3.5). By substituting equation (3.5) into equation (3.4), we obtain stability conditions as
\[
\begin{align*}
V^\prime(a_0) &= f_0^\prime - 2\left(\frac{4\pi a_0}{d - 2}\right)^2 = f_0^\prime - 2\frac{f_0}{a_0^2} > 0 \\
\Leftrightarrow (d - 3) \left[ 4k - (d - 1)\frac{M}{a_0^{d-3}} \right] &< 0. \quad (3.10)
\end{align*}
\]
We used equation (3.9) to derive equation (3.10). As one can see, since static solutions of equation (3.9) and stability conditions of equation (3.10) do not contain \(\Lambda\), the cosmological constant only affects the horizon-avoidance condition of equation (2.31). By studying both the existence of static solutions and stability conditions, we can search static and stable wormholes.

### 3.3. \(d = 3\)

We first analyze static solutions and stability for \(d = 3\). In this case, the stability analysis is simple. The metric (2.3) becomes
\[
f(a) = k - M + Q^2 - \frac{\Lambda}{3} a^2, \quad (3.11)
\]
so the potential is
\[
V(a) = k - M + Q^2 - \Lambda_{\text{eff}} a^2, \quad (3.12)
\]
where
\[
\Lambda_{\text{eff}} \equiv \frac{\Lambda}{3} + \left( \frac{4\pi a_0}{d - 2} \right)^2. \quad (3.13)
\]
Since the shell is static, \(V^\prime(a_0) = 0\), we obtain
\[
\Lambda_{\text{eff}} = 0. \quad (3.14)
\]
Therefore the potential is \( V(a) = k - M + Q^2 \). Besides, \( V(a_0) = 0 \) yields \( k - M + Q^2 = 0 \) so we obtain
\[
f(a) = -\frac{\Lambda}{3} a^3, \quad V(a) = 0. \tag{3.15}\]
Therefore any radius \( a_0 \) is static. We find \( V'(a_0) = 0 \), which means the wormholes is marginally stable. The horizon-avoidance condition \( f(a_0) > 0 \Leftrightarrow \Lambda < 0 \) is satisfied because of (3.13) and (3.14). This wormhole is constructed by pasting a couple of AdS spacetimes.

3.4. \( d \geq 4 \)

From now on, we assume \( d \geq 4 \). For \( k \neq 0 \) and \( M \neq 0 \), equation (3.9) is a quadratic equation. The static solutions are then given by
\[
a_{d-3}^{\pm} = \frac{d - 1}{4k} M (1 \pm b), \tag{3.16}\]
where
\[
b := \sqrt{1 - k \frac{q^2}{q_c^2}}, \quad q := \frac{|Q|}{|M|}, \quad q_c := \frac{(d - 1)}{4\sqrt{d - 2}}. \tag{3.17}\]
Combining equations (3.16) and (3.10), we can see that for \( b = 0 \), the positive and negative sign solutions coincide and their stability depends on higher order terms. For \( b > 0 \) and \( k = +1 \), the negative sign solution is stable, while the positive sign solution is unstable. For \( k = -1 \), we can conclude \( b \geq 1 \) and stability depends on the sign of mass \( M \).

The horizon-avoidance condition (2.31) reduces to
\[
\frac{1}{3} \Lambda a_{d-3}^2 < -\frac{(d - 3)k}{(d - 1)(d - 2)(1 \pm b)} [2 - (d - 1)(1 \pm b)]. \tag{3.18}\]
We investigate \( k = +1 \) and \( k = -1 \) cases, separately.

3.4.1. \( k = 1 \) and \( M \neq 0 \). Though the original range for \( b \) is \( 0 \leq b \leq 1 \), \( b = 0 \Leftrightarrow q = q_c \) does not satisfy the stability condition: For \( q = q_c \), there is the only one static solution,
\[
a_0^{d-3} = \frac{d - 1}{4} M. \tag{3.19}\]
This double root solution is linearly marginally stable but nonlinearly unstable because \( V''(a_0) \neq 0 \). From equation (3.19), we must have positive mass \( M > 0 \) to make sure \( a_0 \) to be positive. The horizon-avoidance condition reduces to
\[
\lambda < R(d), \tag{3.20}\]
where \( \lambda \) is a dimensionless quantity corresponding to \( \Lambda \) and \( R(d) \) is defined by
\[
\lambda := \frac{\Lambda}{3} |M|^\frac{1}{2d^2}, \quad R(d) \equiv \left( \frac{4}{d - 1} \right)^\frac{1}{2d} \frac{(d - 3)^2}{(d - 1)(d - 2)}. \tag{3.21}\]
Since \( R(d) \) is positive, we can have the wormhole even without \( \Lambda \).

If \( b = 1 \Leftrightarrow Q = 0 \), the negative sign solution vanishes. The positive sign solution which does not satisfy the stability condition is
\[ a_{d^3} = \frac{d - 1}{2} M. \]  

(3.22)

From equation (3.22), \( M \) must be positive. One can verify that the horizon-avoidance condition is satisfied even without \( \Lambda \).

When \( 0 < b < 1 \iff 0 < q < q_c \), there are two static solutions

\[ a_{d^3} = \frac{d - 1}{4} M (1 \pm b). \]  

(3.23)

The stability condition is satisfied if we take the negative sign solution of equation (3.23). The positive sign solution is unstable. Since the static solution must be positive, we must have \( M > 0 \). The following transformation helps us to understand the potentials:

\[ \hat{V}(a) \equiv k \frac{d}{a^2} = \frac{M}{a^{d-1}} + \frac{Q^2}{a^{2(d-2)}} \]

so that

\[ \hat{\dot{a}}^2 + \hat{V}(a) = 0 \iff \left( \frac{d \ln a}{dr} \right)^2 + \hat{V}(a) = \Lambda_{\text{eff}}. \]

The potentials \( \hat{V}(a) \) are plotted in figures 1 and 2 for \( d = 4 \) and \( d = 5 \), respectively. The horizon-avoidance condition (3.18) reduces to

\[ \lambda < H_{q^2}(d, q), \]  

(3.24)

where

\[ H_{q^2}(d, q) \equiv -\left\{ \frac{4}{(d - 1)(1 \pm b)} \right\}^{\frac{2d}{d - 3}} \frac{d - 3}{(d - 1)(d - 2)(1 \pm b)} [2 - (d - 1)(1 \pm b)]. \]  

(3.25)

The positive and the negative sign corresponds with the sign of equation (3.23) in same order. When one take the positive sign, \( H_{q^2}(d, q) \), one can find the inside of the square brackets of equation (3.25) is negative, then \( H_{q^2}(d, q) \) is positive. Therefore equation (3.24) is satisfied even with \( \Lambda = 0 \) in the case of \( H_{q^2}(d, q) \). Similarly, in the case of the negative sign, \( H_{q^2}(d, q) \), if the inside of the square brackets of equation (3.25) can be negative, equation (3.24) is satisfied even with \( \Lambda = 0 \). In this stable case, one can achieve this situation if and only if

\[ \frac{1}{2} < q < q_c, \]  

(3.26)

is satisfied. \( H_{q^2}(d, q) \) are plotted in figure 3. Therefore, we can construct a stable TSW without \( \Lambda \) when the condition equation (3.26) is satisfied. So the extremal or subextremal charge \( q \leq 1/2 \) of the Reissner–Nordström spacetime cannot satisfy equation (3.26). We can reconfirm the previous result by taking \( d = 4 \) and \( M = 2m \) for equation (3.26) as

\[ 1 < \left( \frac{|Q|}{m} \right)^2 < \frac{9}{8}. \]  

(3.27)

This coincides with the previous result by Barceló and Visser [18]. From equation (3.26), as \( d \) increases, larger charge is allowed to construct a stable wormhole without \( \Lambda \). This class of wormholes are constructed by pasting a couple of over-charged higher dimensional Reissner–Nordström spacetimes.
and $M \neq 0$. The static solution is given by

$$a_{d \pm}^{d-3} = -\frac{d-1}{4} M (1 \pm b),$$

(3.28)

where

$$b = \sqrt{1 + \frac{q^2}{q_c^2}}.$$  \hspace{1cm} (3.29)

$b$ is more than than or equal to unity i.e. $b \geq 1$. If $M > 0$ and $Q \neq 0$, the stability condition equation (3.10) is satisfied, if we take the negative sign solution of equation (3.28). If $M > 0$ and $Q = 0$, since the negative sign solution vanishes and the positive sign solution is negative, there is no static solution. Therefore, if $M > 0$ and $Q \neq 0$, we can have stable wormholes. In this case the horizon-avoidance condition equation (3.18) reduces to

$$\lambda < 1(d, q),$$  \hspace{1cm} (3.30)

Figure 1. The potential $\tilde{V}(a)$ for $d = 4$, $k = +1$ and $M = 2$. The dashed line is the potential for the critical value defined in equation (3.17).

Figure 2. The potential for $d = 5$, $k = +1$ and $M = 2$. 3.4.2. $k = -1$ and $M \neq 0$. The static solution is given by
where

\[
I(d, q) = -\frac{d-3}{(d-1)(d-2)} \left\{ \frac{4}{(d-1)(b-1)} \right\}^{-\frac{1}{2}} \left[ \frac{2}{b-1} + d - 1 \right].
\]

(3.31)

Equation (3.30) can be satisfied only if \( \Lambda \) is negative and \(|\Lambda|\) is sufficiently large. One can find that the inside of the square brackets on the right hand side of equation (3.31) is positive, so the vanishing cosmological constant cannot satisfy equation (3.31) unlike for \( k = +1 \). The value of \( I(d, q) \) determines the maximum value for \( \lambda \) which is needed to achieve the horizon-
avoidance condition equation (3.30). $I(d, q)$ is plotted in figure 4. Since $I(d, q) < 0$, we need a negative cosmological constant for the horizon-avoidance condition to be satisfied.

If $M < 0$, the stability condition is satisfied if we take the positive sign solution equation (3.28) whether it is with or without charge. The negative sign solution contradicts $a_0 > 0$. The positive sign solution is

$$a_0^{d-3} = \frac{d - 1}{4} |M|(1 + b).$$

(3.32)

In this case, the horizon-avoidance condition equation (3.18) reduces to

$$\lambda < N(d, q),$$

(3.33)

where

$$N(d, q) \equiv \left\{ \frac{4}{(d - 1)(1 + b)} \right\}^{\frac{1}{d - 3}} \frac{d - 3}{(d - 1)(d - 2)(1 + b)} [2 - (d - 1)(1 + b)].$$

(3.34)

$N(d, q)$ is plotted in figure 5. Since $N(d, q) < 0$, we find that a negative cosmological constant is needed to achieve the horizon avoidance.

3.4.3. $k \neq 0$ and $M = 0$. For $k = +1$, from equation (3.9), we find there is no static solution. For $k = -1$, equation (3.9) has a double root solution:

$$a_0^{d-3} = \sqrt{d - 2} |Q|,$$

(3.35)

where $Q \neq 0$ must hold for the positivity of $a_0$. One can easily verify the stability condition is satisfied in this case. The horizon avoidance equation (2.31) reduces to

$$\frac{\Lambda}{3} < S(d, q),$$

(3.36)

where

$$S(d, q) \equiv -|Q|^{\frac{2}{d - 3}} (d - 3)(d - 2)^{d-4}.$$  

(3.37)
Since the right hand side of equation (3.36) is negative, we need a negative cosmological constant for the stable wormhole in this case. However, even an arbitrarily small $\Lambda$ can satisfy equation (3.36), if $|Q|$ is sufficiently large.

3.4.4. $k = 0$ and $M \neq 0$. There is the only one static solution that is

$$a_0^{d-3} = 2d - 2 |Q|^2 / M^{d-1}. \quad (3.38)$$

$M > 0$ and $Q \neq 0$ must hold since $a_0$ must be positive. The stability condition is satisfied in this case. The horizon-avoidance condition reduces to

$$\lambda < J(d, q), \quad (3.39)$$

where

$$J(d, q) \equiv -1 / q \left( d - 1 \right)^{d-1} \left( d - 2 \right)^{d-1} (d-1)(d-3) / (d-2)^2. \quad (3.40)$$

Since the right hand side of (3.39) is negative, $\Lambda$ should be negative. Taking a limit of $d \to \infty$ leads to

$$\lambda \left( |Q| / M \right)^2 < -1 / 4 \quad \text{as} \quad d \to \infty. \quad (3.41)$$

The function $J(d, q)$ is plotted in figure 6. Since $J(d, q)$ is negative, we need a negative cosmological constant for the horizon avoidance.

3.4.5. $k = 0$ and $M = 0$. In this case, we must have $Q = 0$ to satisfy equation (3.9). Then we get $V(a) = -\Lambda_{\text{eff}} a^2$, where $\Lambda_{\text{eff}}$ is defined in equation (3.13). Since the shell is static, $V'(a_0) = 0$, we find

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{fig6.png}
\caption{The function $J(d, q)$ defined in equation (3.40) is plotted. The regions below these curves represent the regions of $\lambda$ which satisfy equation (3.39). $J(d, q)$ is negative. As the number of dimensions increases, it approaches $-1/4q^2$.}
\end{figure}
Then, the potential vanishes, i.e., \( V(a) = 0 \), which means the wormhole can be static at an arbitrarily radius and is marginally stable. Since the cosmological constant turned out to be negative from equation (3.42), we have \( f(a_0) > 0 \) is automatically satisfied. This solution is what we can call an another side of Randall–Sundrum (RS) II brane world model [24]. In both cases, the ingredients are a couple of AdS spacetimes. The RS II model is a compactified spacetime by pasting the interiors of a couple of AdS spacetimes at the boundaries, while the wormhole solution is a noncompactified spacetime by pasting the exteriors of a couple of AdS spacetimes at the boundaries.

### 4. Summary and discussion

We developed the thin shell formalism for \( d \) dimensional spacetimes which is more general than Dias and Lemos formalism [10]. We investigated spherically, planar (cylindrically) and hyperbolically symmetric wormholes with a pure negative tension brane and found and

### Table 1. The existence and stability of \( Z_2 \) symmetric static wormholes in three dimensions. \( k = 1, 0 \) and \(-1\) correspond to spherical, planar (cylindrical) and hyperbolic symmetries, respectively.

| Static solution \( k - M + Q^2 = 0 \) | Horizon avoidance \( \forall a_0 > 0 \) | Stability | Marginal stability |
|----------------------------------------|---------------------------------|-----------|-------------------|
| \( k - M + Q^2 \neq 0 \)               | None                            | \( - \)    | \( - \)            |

### Table 2. The existence and stability of \( Z_2 \) symmetric static wormholes in spherical symmetry in four and higher dimensions. \( q \), \( \lambda \) and \( q_c \) are defined as \( q = |Q/M| \), \( \lambda = (\Lambda/3) |M|^{2/d-2} \) and \( q_c = (d - 1)/(4\sqrt{d - 2}) \), respectively. The expressions for the static solutions \( a_{0z} \) (0 < \( a_{0z} < a_{0+} \)) are given by equations (3.16) and (3.17) with \( k = 1 \). \( H_2(d, q) \) are given by equation (3.25) and plotted in figure 3. \( R(d) \) is positive and given by equation (3.21). Note that \( H_0 > 0 \) for 0 < \( q \leq q_c \), while \( H_- \) > 0 only for 1/2 < \( q \leq q_c \). Therefore, if \( \Lambda = 0 \), the horizon-avoidance condition holds for \( a_{0z} \) for 0 < \( q \leq q_c \), while it does for \( a_{0+} \) only for 1/2 < \( q \leq q_c \). For \( M > 0 \) and \( q = q_c \), the double root solution \( a = a_{0z} \) is linearly marginally stable but nonlinearly unstable.

\[
A_{\text{eff}} = 0 \Leftrightarrow \frac{\Lambda}{3} = -\left(\frac{4\pi\alpha}{d - 2}\right)^2. \tag{3.42}
\]

Then, the potential vanishes, i.e., \( V(a) = 0 \), which means the wormhole can be static at an arbitrarily radius and is marginally stable. Since the cosmological constant turned out to be negative from equation (3.42), we have \( f(a) = (4\pi\alpha/d^2 - 2)^2 \), then the horizon avoidance \( f(a_0) > 0 \) is automatically satisfied. This solution is what we can call an another side of Randall–Sundrum (RS) II brane world model [24]. In both cases, the ingredients are a couple of AdS spacetimes. The RS II model is a compactified spacetime by pasting the interiors of a couple of AdS spacetimes at the boundaries, while the wormhole solution is a noncompactified spacetime by pasting the exteriors of a couple of AdS spacetimes at the boundaries.
classified Z2 symmetric static solutions which are stable against radial perturbations. We found that in most cases charge is needed to keep the static throat radius positive and that a negative cosmological constant tends to decrease the radius of the BH horizon and then to achieve the horizon avoidance. So the combination of an electric charge and a negative cosmological constant makes it easier to construct stable wormholes. However, a negative cosmological constant is unnecessary in a certain situation of $k = +1$ and $M > 0$ and a charge is unnecessary in a certain situation of $k = -1$ and $M < 0$. We summarize the results in tables 1, 2, 3 and 4.

In three dimensions, it is only possible to have a marginally stable wormhole. The ingredients of this wormhole are a couple of AdS spacetimes.

Then, we restrict the spacetime dimensions to be higher than or equal to four. For $k = +1$, spherically symmetric TSWs which are made with a negative tension brane are
investigated. It turns out that the mass must be positive, i.e., $M > 0$. The obtained wormholes can be interpreted as the higher dimensional counterpart of Barceló–Visser wormholes [18]. As a special case, if $1/2 < q < q_c$, one can obtain a stable wormhole without a cosmological constant. This wormhole consists of a negative tension brane and a couple of over-charged Reissner–Nordström spacetimes.

For $k = -1$, though it is hard to imagine how such symmetry is physically realized, they are interesting from the viewpoint of stability analyses. It turns out that $M$ can be positive, zero and negative for stable wormholes. In this geometry, there is no upper limit for $|Q|$ for stable wormholes. There is a possibility for a stable wormhole without a charge if $M < 0$ and $\lambda < N(d, 0)$ is satisfied.

For $k = 0$, the geometry is planar symmetric or cylindrically symmetric. In this case, since the generalized Birkhoff’s theorem does not apply [22], we should regard the Reissner–Nordström-AdS spacetime as a special solution to the electrovacuum Einstein equations. This means that the present analysis only covers a part of the possible static TSWs and the stability against only a part of possible radial perturbations. Under such a restriction, we find that we need $Q \neq 0$ and $\Lambda < 0$ to have stable wormholes. There is no upper limit for $|Q|$. In the zero mass case, the wormhole is marginally stable.

We would note that the existence and stability of negative tension branes as TSWs crucially depend on the curvature of the maximally symmetric $(d-2)$ dimensional manifolds. On the other hand, they do not qualitatively but only quantitatively depend on the number of space time dimensions.

In this paper, we focused on the radial stability of TSWs. In a realistic situation, a wormhole in the Universe would be nonspherically perturbed by particles and waves going through or around the wormhole. Therefore, the nonradial stability of wormholes is of great physical interest. So far, no stability analysis against nonspherical perturbations has been done for TSWs. A linear stability analysis of a Morris–Thorne type wormhole against axial-mode perturbations was made in [26], where the matter field is not distributional but continuous. Since the matter fields of TSWs are localized on their throat as a junction surface, the perturbation analysis for the TSWs should be very different from that for the wormholes with continuous matter fields. To deal with this problem, we can employ the perturbed junction condition developed by Gerlach and Sengupta [27]. Using this, Ishibashi and Kodama investigated whether a domain wall emits gravitational waves or not. In their work, a domain pastes a couple of Minkowski spacetimes [28]. Since their situation is similar to that of the TSW, we expect their study to extend to our TSWs.

Stability of BHs may give us suggestions about stability of TSWs. The linearized perturbations of BHs are decomposed into scalar, vector and tensor modes in terms of their tensorial behaviour against rotation on a unit sphere [29]. The Schwarzschild BH has been proven to be stable for all modes [29, 30]. The $d$-dimensional extension of the Schwarzschild BH, so called the Tangherlini BH [31], is also stable, including those foliated by any $(d-2)$-dimensional maximally symmetric spaces $(k = 1, 0, -1)$ [32, 33]. The Schwarzschild-AdS BHs and their topological extension $(k = 0, -1)$ are stable in four dimensions [33]. The higher dimensional counterpart of them are also stable for vector and tensor modes. Although the scalar-mode stability of them has not yet been proven, numerical calculations show that the Tangherlini-de Sitter BHs are stable even for scalar modes for $d = 5, 6, ..., 11$ [35].

The Reissner–Nordström BHs $(k = 1, 0, -1)$ have been proven to be stable for all modes even for any values of the cosmological constant [33, 36] in four dimensions but not in arbitrary dimensions. Numerically, spherically symmetric Reissner–Nordström BHs with a negative cosmological constant (RNAdS BHs) have been shown to be stable for
\( d = 5, 6, \ldots, 11 \) [37]. However, spherically symmetric Reissner–Nordström BHs with a positive cosmological constant (RNdS BH) have a peculiar behaviour in higher dimensions. It is also numerically shown that although the RNdS BHs are stable for \( d = 5 \) and 6, they are unstable for the large values of the electric charge and the cosmological constant for \( d \geq 7 \) [38].

From the above point of view, we should note that our radially stable TSWs are made of a couple of nonextremal RNAdS BHs in most cases. Since nonextremal RNAdS BHs are stable against nonspherical perturbations, we can expect that if our radially stable TSWs are unstable against nonspherical perturbations, that instability should be directly induced by the nonspherical instability of the negative tension brane itself. If the negative tension brane does not suffer from nonspherical instability, we expect that our radially stable TSWs are stable for all kinds of perturbations. This interesting issue is far beyond the scope of the current paper and will be addressed in the future.

Finally, we note that Einstein–Gauss–Bonnet (GB) gravity is one of the plausible extended gravity models in higher dimensions [25]. GB gravity is a higher dimensional modified theory of gravity which has Einstein–Hilbert action plus an additional term constructed from curvature tensors in a specific manner. It is interesting that in this theory the stability of spacetimes is highly nontrivial [20, 39, 40]. It is under investigation to study the existence and stability of TSWs in GB gravity.

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Appendix A. The conditions for the right solutions

We obtained the equation of motion (2.28) by twice squaring equation (2.22). Equation (2.22) is equivalent to

\[
\begin{align*}
C^2 \sigma^2 &= (A_+ + A_-)^2, \\
\sigma &< 0.
\end{align*}
\]

(A.1)

The above is also equivalent to

\[
\begin{align*}
\left( C^2 \sigma^2 - A_+^2 - A_-^2 \right)^2 &= (2A_+ A_-)^2, \\
C^2 \sigma^2 - A_+^2 - A_-^2 &> 0, \quad \sigma < 0.
\end{align*}
\]

(A.2)

The first equation of equations (A.2) is obtained by squaring the first equation of equations (A.1). By recasting the first equation of equations (A.2), we obtain the equation of motion equation (2.28). However, the solution of equation (2.28) is valid if and only if the second and third inequalities of equations (A.2) are satisfied. The second inequalities of equations (A.2) is represented explicitly as
\[ A_0^2 > 0, \quad \text{(A.3)} \]

where equation (3.5) is used. Since \( A_0^2 = f(a_0) > 0 \) is guaranteed by the horizon-avoidance condition, our analysis does not contain wrong solutions of equation (2.28).

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