Investigating early warning signs of oscillatory instability in simulated phasor measurements

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Abstract—This paper shows that the variance of load bus voltage magnitude in a small power system test case increases monotonically as the system approaches a Hopf bifurcation. This property can potentially be used as a method for monitoring oscillatory stability in power grid using high-resolution phasor measurements. Increasing variance in data from a dynamical system is a common sign of a phenomenon known as critical slowing down (CSD). CSD is slower recovery of dynamical systems from perturbations as they approach critical transitions. Earlier work has focused on studying CSD in systems approaching voltage collapse; in this paper, we investigate its occurrence as a power system approaches a Hopf bifurcation.

Index Terms—Critical slowing down, phasor measurement units, oscillatory stability, stochastic differential equations, Hopf bifurcation, Inter-area oscillations.

I. INTRODUCTION

After several recent blackouts in North America [1], [2] and Europe [3], there is increasing motivation to use phasor measurement unit (PMU) data for improving reliability in the electricity industry. For example, the September 8, 2011 Southwestern US blackout report [1] recommended the use of PMU data to increase situational awareness. It emphasized that PMUs may prove increasingly important in identifying and monitoring for signs of grid stress, such as dangerous oscillations.

One of the most important grid conditions that operators need to monitor for is oscillatory stability. Oscillatory stability problems are typically associated with a pair of complex eigenvalues crossing the imaginary axis of the complex plane after a system undergoes a contingency [4]. Oscillatory stability problems in a power system can be either local or global. Local plant mode involves rotor angle oscillations of a generator against the rest of the system. Global problems known as inter-area mode oscillations involve generator in one area swinging against generators in another area [5]. Inter-area oscillations can have widespread effects. Incidents of undamped inter-area oscillations have occurred in many different power grids such as August 10, 1996 blackout in Western North America [6] and May 1, 2005 in southeastern Europe [7]. There has been substantial research on fundamentals of this phenomenon in literature such as [8], [9]. Reference [8] analyzes the effects of excitation systems, loads and DC links on inter-area oscillations. Reference [9] reveals the existence of stable and unstable periodic solutions in power system models using the bifurcation theory.

Numerous methods are proposed for monitoring inter-area oscillations such as [10]–[12]. Reference [10] compares the Prony analysis and Hilbert transform methods for modal identification. In [11], a linear index is developed for identifying Hopf bifurcations, based on eigenvalues and singular values of state matrix. Reference [12] estimates frequency and damping of the inter-area mode from the Fourier spectrum of phasor measurements. These, while valuable, have limitations with regard to measurement noise, calculation speed, accuracy and discriminating between similar modes [10]. In this paper, we show that changes of statistics of some of system variables can be a potentially helpful complement to existing methods.

As dynamical systems approach a critical transition, they increasingly recover more slowly from perturbations. This phenomenon, known as critical slowing down (CSD), is primarily caused by reduced damping as a system’s eigenvalues approach the right-half plane. Increasing variance and autocorrelation are two common signs of CSD phenomenon in dynamical systems [13]. The increase in these two statistics have been observed in many different dynamical systems [13], and are frequently suggested as early warning signs of critical transitions [14]. However, recent research has shown that CSD signs are not universal [15], [16]; they may not be observable in all system variables and under all conditions. Therefore, it is important to carefully choose which variable(s) to monitor, in order to effectively use autocorrelation or variance as early warning signs of bifurcation.

Prior research [17]–[21] has shown that the signs of CSD occur in power systems, in the vicinity of saddle-node bifurcations. In [21], the present authors showed that CSD signs do not appear in all variables in the vicinity of a saddle-node bifurcation, in several power system models.

In this paper, we investigate changes in autocorrelation and variance of system variables as a power system model approaches a Hopf bifurcation. The results show that increasing variance of bus voltage magnitudes is a good early warning sign of Hopf bifurcation. Thus, monitoring for this value could potentially be used as an indicator of oscillatory stability problems in power system. Sec. II presents the simulation and results of study of changes in autocorrelation and variance of a Three-bus test case in the vicinity of Hopf bifurcation. Sec. III is a discussion on the results presented in Sec. II. Sec. IV highlights the results and contributions made in this paper.
II. SIMULATION AND RESULTS

This section presents the simulation of a small power system test case, with which we study the occurrence of CSD in the vicinity of a Hopf bifurcation. First, we present the test case and the simulation method used. Then, the changes of variances and autocorrelations of system variables in the vicinity of Hopf bifurcation are shown.

A. Test Case and Simulation

Fig. 1 shows the single-line diagram of a Three-bus test system model under study. The two generators in this system are modeled with a standard sixth order generator model [4], and are equipped with excitors. A governor is connected to the first generator. The system data are given in Appendix A. We simulated this system using the power system analysis toolbox (PSAT) [23]. Here, we assume that the load power varies stochastically, with normally distributed fluctuations. However, since the variance of white noise is infinite, we assumed that the load perturbations have finite correlation time. We also assumed that the correlation time of the load power varies stochastically, with normally distributed fluctuations. However, since the variance of white noise is infinite, we assumed that the load perturbations have finite correlation time. Numerically, this correlation time is assumed to be equal to the integration time step of (1) below. Adding noise to the system load adds randomness to the system. Therefore, a set of stochastic differential-algebraic equations (SDAEs) describe this system:

\[
\dot{x} = f(x, y) \tag{1}
\]

where \(x, y\) represent the vector of differential and algebraic variables respectively, \(\eta\) is the gaussian random variable added to the load \(\eta \sim \mathcal{N}(0, 0.01)\). \(f, g\) represent the set of differential and algebraic equations of the system, respectively. A subset of algebraic equations are power flow equations, into which the noise is added:

\[
P_k - P_{k0}\eta = V_k \cdot \sum_{m=1}^{n} (G_{km}V_m \cos \theta_{km} + B_{km}V_m \sin \theta_{km}) \tag{3}
\]

\[
Q_k - Q_{k0}\eta = V_k \cdot \sum_{m=1}^{n} (G_{km}V_m \sin \theta_{km} - B_{km}V_m \cos \theta_{km}) \tag{4}
\]

where \(n = 3; P_k\) and \(Q_k\) are injected active and reactive power at each bus; \(P_{k0}, Q_{k0}\) are constant values; \(G_{km}\) and \(B_{km}\) are the conductance and the susceptance of the line between bus \(k\) and bus \(m\); \(V_m\) is the voltage magnitude of bus \(m\); \(\theta_{km} = \theta_k - \theta_m\), where \(\theta_k, \theta_m\) are voltage angles of buses \(k, m\). The differential and algebraic equations that describe the generator, exciter and turbine governor are available in [24].

We solved the resulting SDAEs using a fixed-step trapezoidal differential-algebraic equations solver for different load levels. For each load level, we simulated the system around the equilibrium. Each load’s active and reactive powers fluctuate around their mean values. The time for each simulation was 120s and the integration step size was 0.01s. We assumed that the noise level is constant (i.e. \(P_{k0}, Q_{k0}\) are constant in (3), (4)) when the load is varied, so as to make sure that the increase of variables’ variances is not due to the increase of the noise level. At the end of the simulation, we subtracted means from the time-series of the algebraic and differential variables, before calculating their variances and autocorrelations. For each load level, we ran simulations 100 times, and calculated the average of variances and autocorrelations of variables. In this work, we vary \(P\) and \(Q\) proportionally, so that the power factor remains constant.

As the load increases, the system passes through a Hopf bifurcation. Fig. 2 shows the PV curve for this system. Hopf bifurcation occurs before the maximum power transfer limit (the nose point of the PV curve). Fig. 3 shows the trajectory of the eigenvalues of the system as the load increases. Only the three pairs of eigenvalues closest to the right-half plane are shown. In this and subsequent figures, the dotted line shows a point close to the bifurcation at which we did eigenvalue analysis (see Sec. III) to find out why variances and autocorrelations of the variables show different patterns in the vicinity of the bifurcation.

B. Autocorrelations and variances of the system variables

Figs. 4-7 show variance and autocorrelation of several variables that can be measured in real time. Before calculating the variances and autocorrelations, the variables’ means were subtracted from their values. Horizontal axis is the ratio of \(P_d\) to the nominal load \(P_{dn}\). These figures demonstrate that the bus voltage magnitudes are the only variables whose variances show a monotonic and, importantly, gradual increase over the entire range of load values. Among the angle variables,
only the variance of \( \theta_3 \) shows a monotonic and gradual increase starting with \( P_{d}/P_{dn} \approx 0.6 \), and its increase is less pronounced than that of \( \sigma_{2}^{2} \). In contrast to the variances, the autocorrelations of the voltage magnitudes increase conspicuously only very near the bifurcation, while variances and autocorrelations of other variables (Figs. 5–7) are not even spicuously only very near the bifurcation, while variances and autocorrelations of the voltage magnitudes increase conspicuously only very near the bifurcation.

Therefore, among all the measurable quantities in the Three-bus system, only the load bus voltage variance is a reliable early sign of the bifurcation.

Figure 4. Variance and autocorrelation of the voltage magnitude of the load bus versus load level. Note that the autocorrelation \( \langle \Delta V(t) \Delta V(t + \Delta t) \rangle \) is shown only for \( \Delta V_3 \); it is similar for the other two voltages.

Figure 5. Variance and autocorrelation of the voltage angle of the load bus versus load level. Note that the autocorrelation \( \langle \Delta \theta(t) \Delta \theta(t + \Delta t) \rangle \) is shown only for \( \Delta \theta_3 \); it is similar for the other two angles.

III. DISCUSSION

From the description in Sec. [1][2] we conclude that a good early warning sign of a Hopf bifurcation in power systems is the same as for saddle-node bifurcation [21]. We emphasize the word “early”. Indeed, it is well-known that variance and autocorrelation of most variables increases according to certain universal laws near a bifurcation [13]. However, our results demonstrate that only a small subset of such variables - namely, the bus voltage magnitudes’ variance - exhibits a consistent increase sufficiently far from the bifurcation. Therefore, only these variables can serve as a useful warning sign, which can potentially be detected early enough to avert a system collapse.

Let us point out that this conclusion is supported by the eigenvalue analysis. It is well-known that right eigenvectors of the state matrix give the relative activity of state (i.e. differential) variables when the corresponding mode is excited [4]. Eigenvectors are obtained from linearization of (1), (2), whereby these equations reduce to:

\[ \Delta \dot{x} = A \Delta x \]  \hspace{1cm} (5)

where \( A \) is:

\[ A = f_x - f_y g_y^{-1} g_x \]  \hspace{1cm} (6)

...
where \( f_x, f_y, g_x, g_y \) are matrices of partial derivatives of \( u \) and \( v \) with respect to the differential and algebraic variables. The solution of \( \mathbf{5} \) is represented as:

\[
\Delta \mathbf{z} = \mathbf{\phi z}
\]

where \( \mathbf{\phi} \) is a matrix whose columns are right eigenvectors of the state matrix, \( \mathbf{z} \) is the time-dependent vector of transformed state variables such that each variable is associated with only one mode \( \mathbf{4} \). In order to determine the relative activity of algebraic variables, we linearized \( \mathbf{5} \) (with no noise in the load):

\[
\Delta \mathbf{y} = -g_y^{-1}g_x\Delta \mathbf{z}
\]

Then \( \mathbf{7} \) and \( \mathbf{8} \) yield:

\[
\Delta \mathbf{y} = \mathbf{C z}
\]

where \( \mathbf{C} = -g_y^{-1}g_x\mathbf{\phi} \). The columns of \( \mathbf{C} \) give the relative activity of algebraic variables in corresponding modes.

To identify the most “active” variables, one looks for a group of the entries of the eigenvectors (columns of \( \mathbf{\phi} \)) for the state variables, or the columns of matrix \( \mathbf{C} \) for the algebraic variables. Strictly speaking, one should do so for all the dominant modes, whose eigenvalues have the smallest real part, because their response to an external disturbance (e.g. noise in the load) would decay most slowly. However, out of the three dominant modes shown in Fig. \( \mathbf{3} \) we have focused only on the one with the smallest real part for the specific value of load \( P_d = 10.4 \)pu (see the vertical dotted line in Fig. \( \mathbf{3} \)), and demonstrate that even such restricted analysis agrees with the results provided by Figs. \( \mathbf{4} \) and \( \mathbf{7} \) in Table \( \mathbf{1} \) we show the magnitudes of only those entries of the corresponding column of \( \mathbf{\phi} \) and of \( \mathbf{C} \) which can be directly measured. There are other entries as well, which explains why the displayed entries do not satisfy the conventional normalization:

\[
|u_1|^2 + |u_2|^2 + \ldots = 1
\]

where \( u_i \) is the \( i \)-th entry of a column of \( \mathbf{\phi} \) or \( \mathbf{C} \). We see that the “activity” of the state variables \( \delta_{1,2}, \delta_{1,2} \) is too small compared to that of other state variables. This is reflected in Figs. \( \mathbf{4} \) and \( \mathbf{7} \) by the fact that these variables do not show any substantial increase in variance except perhaps very near the bifurcation point. The “activity” of voltage magnitudes \( V_{1,2,3} \) and angles \( \theta_{1,2,3} \) is also not very high (in light of the normalization \( \mathbf{10} \)). Yet, the “activity” of \( V_{1,2,3} \) relative to other variables is, apparently, high enough for the variance of \( V_{1,2,3} \) to exhibit conspicuous increase near the bifurcation. Note that of the three angles, \( \theta_3 \) has the largest “activity”, and its variance’s increase is comparable to that of \( V_{1,2,3} \). The other two angles have too small “activities”, and their variances do not show monotonic growth as \( P_d \) approaches the Hopf bifurcation value.

**IV. CONCLUSION**

In this paper, we showed that critical slowing down occurs in power system for Hopf bifurcation. As previously shown for the saddle-node bifurcation, the results show that CSD signs are better observable in some variables than others. We showed that this occurs because fluctuations of some variables

![Table 1](image)

are more aligned with the direction of dominant mode. Specifically, we found that variance of load bus voltage magnitude is a good early warning sign of Hopf bifurcation. This property along with the availability of fast PMU measurements can potentially help in developing a method for monitoring of oscillatory stability in power grid using phasor measurements.

**APPENDIX A**

**SYSTEM DATA**

System base power is 100 MVA.
Nominal load and generation:

\( P_d = 900 \text{ MW}, Q_d = 300 \text{ MVAR}, P_{g2} = 400 \text{ MW} \)

**Synchronous generator:**

| Bus no. | Base MVA | \( r \) (pu) | \( X_d \) (pu) | \( X'_d \) (pu) |
|---------|----------|-------------|--------------|--------------|
| 1       | 555.5    | 0           | 1.81         | 0.3          |
| 2       | 700      | 0           | 1.81         | 0.3          |

| \( X''_d \) (pu) | \( T'_{do} \) (s) | \( T''_{do} \) (s) | \( X'_q \) (pu) |
|------------------|------------------|------------------|----------------|
| 2               | 0.217            | 7.8              | 0.022          | 1.76          |
| 2               | 0.217            | 7.8              | 0.022          | 1.76          |

| \( X''_q \) (pu) | \( T'_{qo} \) (s) | \( T''_{qo} \) (s) |
|------------------|------------------|------------------|
| 1                | 0.61             | 0.9              | 0.074          |
| 2                | 0.61             | 0.9              | 0.074          |

| \( M \) (s) | \( D \) (pu) |
|-------------|-------------|
| 1           | 9.06        | 0               |
| 2           | 13.06       | 0               |

**Exciter:**

Exciter model is PSAT’s Type III model \[23\].

| Gen. no. | \( v_{\theta} \) max | \( v_{\phi} \) min | \( K_0 \) | \( T_2 \) | \( T_1 \) |
|----------|-----------------------|-------------------|--------|--------|--------|
| 1        | 40                    | -40               | 20     | 12     | 1      |
| 2        | 40                    | -40               | 20     | 12     | 1      |

| \( v_{\theta} \) | \( S_0 \) | \( T_n \) | \( T_r \) |
|------------------|--------|--------|--------|
| 1                | 0      | 0.04   | 0.05   |
| 2                | 0      | 0.04   | 0.05   |

**Turbine Governor:**

Turbine Governor model is PSAT’s Type II model \[24\].

| Gen. no. | \( R \) | \( P_{\text{max}} \) | \( P_{\text{min}} \) | \( T_2 \) | \( T_1 \) |
|----------|--------|----------------------|----------------------|--------|--------|
| 1        | 0.2    | 10                   | 0.3                   | 5      | 0      |

**REFERENCES**

[1] Staff, “Arizona–Southern California outages on September 8, 2011: causes and recommendations," FERC and NERC Staff, Tech. Rep., Apr. 2012.

[2] S. Abraham and J. Efford, “Final report on the August 14, 2003 blackout in the United States and Canada: causes and recommendations,” US–Canada Power Syst. Outage Task Force, Tech. Rep., 2004.

[3] R. Bacher, U. Naf, M. Renggli, W. Balthmann, and H. Glavitsch, “Report on the blackout in Italy on 28 september 2003,” Swiss Federal Office of Energy, Tech. Rep., Nov. 2003.
A. Hastings and D. B. Wysham, “Regime shifts in ecological systems.”

M. Scheffer, J. Bascompte, W. A. Brock, V. Brovkin, S. R. Carpenter, M. C. Boerlijst, T. Oudman, and A. M. de Roos, “Catastrophic collapse from second-order phase transitions in large complex systems,” Phys. Rev. Lett., vol. 89, no. 19, p. 198701, 2002.

D. Kosterev, S. Yirga, and V. Venkatasubramanian, “Validation report of the austgust 10, 1996 wsc disturbance,” IEEE Trans. Power Syst., vol. 20, no. 3, pp. 1296–1303, 2005.

J. C. Mantzaris, A. Metsiou, and C. D. Vournas, “Analysis of interarea oscillations including governor effects and stabilizer design in southeastern europe,” IEEE Trans. Power Syst., vol. 22, no. 2, pp. 771–780, May 2007.

M. Klein, G. Rogers, and P. Kundur, “A fundamental study of inter-area oscillations in power systems,” Int. J. Electr. Power Energy Syst., vol. 11, no. 5, pp. 299–307, 1989.

V. Ajjarapu and B. Lee, “Bifurcation theory and its application to norm auto-dynamical phenomena in an electrical power system,” IEEE Trans. Power Syst., vol. 7, no. 1, pp. 11–22, Feb. 1992.

T. J. Browne, V. Vittal, G. T. Heydt, and A. R. Messina, “A comparative assessment of two techniques for modal identification from power system measurements,” IEEE Trans. Power Syst., vol. 23, no. 3, pp. 1408–1415, 2008.

C. A. Canizares, N. Mithulananthan, F. Milano, and J. Reeve, “Linear performance indices to predict oscillatory stability problems in power systems,” IEEE Trans. Power Syst., vol. 19, no. 2, pp. 1104–1114, 2004.

N. Kakimoto, M. Sugumi, T. Makino, and K. Tomiyama, “Monitoring of interarea oscillation mode by synchronized phasor measurement,” Power Systems, IEEE Transactions on, vol. 21, no. 1, pp. 260–268, 2006.

M. Scheffer, J. Bascompte, W. A. Brock, V. Brovkin, S. R. Carpenter, V. Dakos, H. Held, E. H. Van Nes, M. Rietkerk, and G. Sugihara, “Early-warning signals for critical transitions,” Phys. Rev. Lett., vol. 105, no. 19, p. 198701, 2099.

C. Kuehn, “A mathematical framework for critical transitions: Bifurcations, fast-slow systems and stochastic dynamics,” J. Math. Biol., vol. 69, no. 1-2, pp. 1–39, 2014.

M. C. Boerlijst, T. Oudman, and A. M. de Roos, “Catastrophic collapse can occur without early warning: examples of silent catastrophes in structured ecological models,” PloS one, vol. 8, no. 4, p. e62033, 2013.

A. Hastings and D. B. Wysham, “Regime shifts in ecological systems can occur with no warning,” Ecology Lett., vol. 13, no. 4, pp. 464–472, 2010.

E. Cotilla-Sanchez, P. D. Hines, and C. M. Danforth, “Predicting critical transitions from time series synchrophasor data,” 2012.

D. Podolsky and K. Turitsyn, “Random load fluctuations and collapse probability of a power system operating near codimension 1 saddle-node bifurcation,” arXiv preprint arXiv:1212.1224, 2012.

G. Ghanavati, P. D. Hines, T. Lakoba, and E. Cotilla-Sanchez, “Calculation of the autocorrelation function of the stochastic single machine infinite bus system,” arXiv preprint arXiv:1308.1925, 2013.

D. Podolsky and K. Turitsyn, “Critical slowing-down as indicator of approach to the loss of stability,” arXiv preprint arXiv:1307.4318, 2013.

G. Ghanavati, P. D. Hines, T. I. Lakoba, and E. Cotilla-Sanchez, “Understanding early indicators of critical transitions in power systems from autocorrelation functions,” arXiv preprint arXiv:1309.7306, 2013.

H. Ghassemi, “On-line monitoring and oscillatory stability margin prediction in power systems based on system identification,” Ph.D. dissertation, University of Waterloo, 2006.

F. Milano, “An open source power system analysis toolbox,” IEEE Trans. Power Syst., vol. 20, no. 3, pp. 1199–1206, 2005.

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