Gauge/gravity dynamics for composite Higgs models and the top mass

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We provide gauge/gravity dual descriptions of the strong coupling sector of composite Higgs models using insights from non-conformal examples of the AdS/CFT correspondence. We calculate particle masses and decay constants for proposed Sp(4) and SU(4) gauge theories. Our results compare favourably to lattice studies and go beyond those due to a greater flexibility in choosing the fermion content. That content changes the running dynamics and its choice can lead to sizable changes in the bound state masses. We describe top partners by a dual fermionic field in the bulk. Including suitable higher dimension operators can ensure a top mass consistent with the Standard Model.

Extensions of the AdS/CFT correspondence [1–3] to less symmetric gauge/gravity duals have proved powerful in describing strongly coupled gauge theories. Adding probe branes allows quarks in the fundamental representation of the gauge group to be introduced [4] and to study the related meson operators [5, 6]. These methods were successfully used to obtain gravity duals of chiral symmetry breaking (χSB) and pseudo-Goldstone bosons in confining non-Abelian gauge theories [7, 8]. A natural extension is to apply this approach to other strongly coupled theories in the context of particle physics. Our attention here is on composite Higgs models (CHM) of Beyond the Standard Model physics, as reviewed in [9, 10].

The key element of CHM is a strongly coupled gauge theory, like QCD, causing χSB in the fermion sector and generating four or more Nambu-Goldstone bosons, or “pions” [11]. By weakly gauging the global chiral symmetries, four then pseudo-Nambu Goldstone bosons (pNGBs) can be placed in a doublet of SU(2)L to become the complex Higgs field. The composite nature of the Higgs removes the huge level of fine tuning in the Standard Model (SM) hierarchy problem. This strong dynamics would occur at a scale of 1-5 TeV, the expected scale for bound states. The LHC has started and, in future runs, will continue to search for such states.

There is lattice gauge theory work on CHMs [12–16]. It is limited though by the cost of numerics and the inability to unquench fields (i.e. to include the effect of flavour modes on the gauge fields) and to match the models’ precise fermion content. Here we use non-conformal gauge/gravity models that explicitly include the gauge theories’ dynamics through the running of the anomalous dimension γ of the fermion or ‘quark’ mass in the CHM. Our models are inspired by top-down models involving probe D-branes embedded into ten-dimensional supergravity. However, we combine this with a phenomenological approach and insert sensible guesses for the running of γ that are based on perturbation theory. Our models then predict some of the mesonic and baryonic spectrum of the theory. These top-down inspired holographic models are very different from previous analyses of CHM, e.g. the Randall-Sundrum [17] approach of [18].

We consider two models in particular: an Sp(4) theory with 4 fundamental and 6 sextet Weyl quarks [19]; and an SU(4) theory with five sextet Weyl and 3 fundamental Dirac quarks [20, 21]. Both models can incorporate a SM Higgs amongst their pNGBs. We present results for the mass spectrum of bound states analogous to mesons in these theories and compare to relevant lattice results: for the Sp(4) theory, lattice results were obtained for the quenched theory; for the SU(4) theory, they have a slightly different fermion content with an even number of multiplets. The pattern of masses is well reproduced by the holographic model, although precise values can differ by up to 20%. This gives us confidence to trust our predictions for how the masses change as we unquench, including the effect of flavours on the running coupling, and move to the true fermion content of the models. In particular, unquenching tends to separate the scales of the sextet and fundamental matter mesons. Slowing the running also significantly reduces the scalar meson masses [22].

Finally, the extra ingredient for which CHM were designed is the generation of the top quark’s large mass, while also keeping flavour changing neutral currents under control. This can be achieved by the mechanism of partial compositeness [23] to generate the top-Higgs Yukawa coupling with higher dimension operators (HDOs) from a flavour scale above the strong dynamics scale. These CHM use a mechanism where the left tL and right handed tR top quarks mix with baryon-like ‘top partner’ spin 1/2 states Tl, Tr in the gauge theory with the same quantum numbers [23]. The top partners are involved in the strong dynamics and so have an order one Yukawa coupling to the Higgs. HDOs then mix the top and top-partner fields to generate the top Yukawa coupling. To achieve the large top mass though requires top partner masses of the order of 800 GeV, which is below the natural baryon mass scale and not consistent with current LHC data [24 25, 26].

We address this issue by first including a spinor of appropriate AdS mass into the holographic model, dual to
the top partner operator in the field theory. This allows us to calculate the top partners’ mass. A novel technical ingredient in this respect is the inclusion of spinor fields into a non-supersymmetric bulk theory, for which we adapt previous results for supersymmetric probe branes [27]. We also add appropriate strongly coupled HDOs to the holographic model using Witten’s double trace prescription [28]. The particular HDO we pick reduces to a shift in the top partners’ mass at low energies. We show that the top Yukawa coupling can be made of order one by lowering the top partners’ mass to roughly half the CHM’s vector meson mass. This is plausibly reconcilable with experimental constraints.

**DYNAMIC ADS/YM**

Our holographic model [29] is based on the Dirac-Born-Infeld (DBI) action of a top-down model with a D7-brane embedded in a (deformed) AdS5 geometry. The deformation is expanded to quadratic order in the embedding function $X$ (see e.g. [30, 31]). We add an axial gauge field similarly to AdS/QCD models [32, 33] and a spinor. The model describes either a single quark in the background of the gauge fields and other quarks; or, by placing the fields in the adjoint of flavour and tracing over the action, multiple mass-degenerate quarks.

The model has a dimension one field for each gauge invariant operator of dimension three in the dual field theory. $X = L e^{2\pi}$ is dual to the complex quark bilinear or any other suitable bilinear operator, depending on the group and representation considered. This field’s fluctuations are dual to the analog of the scalar and pseudoscalar $\sigma$ and $\pi$ mesons of the theory. $A^\mu_L$ and $A^\mu_R$ are dual to the analogs of the vector and axial vector mesons $V$ and $A$.

The gravity action of Dynamic AdS/YM is, including also a spinor field [34],

$$S_{\text{hologon}} = \int d^5x \rho^3 \left( \frac{1}{r^2} |DX|^2 + \frac{\Delta m^2}{\rho^2} |X|^2 ight.$$
$$+ \frac{1}{2g_5^2} (F_{LMN} F^{LMN}_L + (L \leftrightarrow R )) 
$$
$$+ \bar{\Psi} (\gamma_{\text{AdS}} - m) \Psi \big).$$

The five-dimensional coupling is obtained by matching to the UV vector-vector correlator [32]

$$g_5^2 = \frac{24\pi^2}{d(R) N_f(R)}. \quad (2)$$

d(R) is the dimension of the quark’s representation and $N_f(R)$ is the number of Weyl flavours in that representation. The model lives in a five-dimensional asymptotically AdS (AAdS) spacetime

$$ds^2 = r^2 dx_{(1,3)}^2 + \frac{d\rho^2}{r^2}, \quad (3)$$

with $r^2 = \rho^2 + L^2$ the holographic radial direction, corresponding to the energy scale, and $d\Sigma^2_{(1,3)}$ a four-dimensional Minkowski spacetime. The $\rho$ and $L$ factors in the action and metric are implemented directly from the top-down analysis of the D3/probe-D7 system. They ensure an appropriate UV behaviour. In the IR, the fluctuations know about any $\chi_{SB}$ through $L \neq 0$.

The dynamics of a particular gauge theory, including quark contributions to any running coupling, is included through $\Delta m^2$ in Eq. (1). To find the theory’s vacuum, with a non-zero chiral condensate, we set all fields to zero except for $L(\rho)$. For $\Delta m^2$ a constant, the equation of motion obtained from Eq. (1) is

$$\partial_\rho (\rho^3 \partial_\rho L) - \rho \Delta m^2 L = 0 . \quad (4)$$

The solution takes the form $L(\rho) = m_\rho^{-\gamma} + c_\rho^{-\gamma^2}$, with $\Delta m^2 = \gamma(\gamma - 2)$ in units of the inverse AdS radius squared. Here $\gamma$ is the anomalous dimension of the quark mass. The Breitenlohner-Freedman [35] bound, below which an instability to $\chi_{SB}$ occurs, is given by $\Delta m^2 = -1$.

We impose a particular gauge theory’s dynamics by using its running $\gamma$ to determine $\Delta m^2$ - as usual in holography, $M^2 = \Delta (\Delta - 4)$, with $\Delta$ the operator scaling dimension. For $\gamma < 1$, we find $\Delta m^2 = -2\gamma$, thus a theory triggers $\chi_{SB}$ if $\gamma$ passes through 1/2. Since the true running of $\gamma$ is not known non-perturbatively, we extend the perturbative results as a function of renormalization group scale to the non-perturbative regime. To find the running of $\gamma$ we use the one-loop anomalous dimension $\gamma = \frac{3}{2} \frac{C_{2f}(R)}{\pi} \alpha$, with a running $\alpha$ [36].

We numerically solve Eq. (4) with our ansatz for $\Delta m^2$, using the IR boundary conditions $L(\rho)|_{\rho = p_{\text{IR}}} = p_{\text{IR}}$, $\partial_\rho L(\rho)|_{\rho = p_{\text{IR}}} = 0$. They are imported from the D3/D7 probe brane system but imposed at the scale where the quarks go on mass shell. The spectrum is determined by considering fluctuations in all fields of (1) about the vacuum [29].

We will use this model to explore two gauge theories that have been proposed to underlie CHM. We denote them by their gauge group and the number of Weyl flavours in the fundamental (F), and two-index antisymmetric ($A_2$) representations.

**Sp(4) gauge theory - Sp(4) 4F, 6A2:** This CHM, proposed in [19], has top partners. Lattice studies related to this model were performed in [12, 13]. The fundamental of Sp(2N) is pseudo-real, hence the $SU(2)_L \times SU(2)_R$ symmetry of a model with four Weyl quarks in the fundamental representation is enhanced to an $SU(4)$ flavour symmetry [37]. The condensation pattern is the same as in QCD, and breaks the $SU(4)$ flavour symmetry to Sp(4) with 5 pNGBs.

The top partners are introduced by including three additional Dirac fermions in the $A_2$ representation of the gauge group for $N > 1$. It is natural to concentrate on
the minimal, $Sp(4)$, gauge group case. The top partners are $FA_2F$ bound states. From the point of view of the $Sp(4)$ dynamics, there is an $SU(6)$ symmetry on the 6 $A_2$ Weyl fermions that are in a real representation. The $A_2$ degeneracy between the $F$ and $A_2$, from holography and the lattice. Also the pion decay constant and two estimates of the top partner baryon mass.

| $f_{\gamma A_2}$ | $M_{VA_2}$ | $M_{VA}$ | $M_{MS}$ | $M_{MTA_2}$ | $M_{TF}$ |
|------------------|------------|---------|---------|-------------|---------|
| 0.120            | 1.025      | 0.117   | 0.238   | 1.15        | 1.93    |
| $f_{\gamma F}$  | 0.0701     | 0.962   | 0.132   | 1.28        | 1.75    |
| $M_{V A_2}$      | 1*         | 1*      | 1.000   | 1.28        | 1.75    |
| $M_{VF}$         | 0.814      | 0.962   | 0.83(19) | 1.28(13)   | 1.75    |
| $MA_{A_2}$       | 1.35       | 1.36    | 1.52(11) | 1.33(18)   | 1.75    |
| $MA$             | 1.91       | 1.44    | 1.65(15) | 1.33(18)   | 1.75    |
| $MS_{A_2}$       | 0.375      | 1.14    | 1.25     | 1.52(11)   | 1.75    |
| $MSF$            | 0.902      | 1.25    | 1.52     | 1.33(18)   | 1.75    |
| $MTA_2$          | 1.85       | 1.86    | 1.52     | 1.33(18)   | 1.75    |
| $MTF$            | 1.53       | 1.79    | 1.52     | 1.33(18)   | 1.75    |

TABLE I. $AdS/Sp(4)$ $4F, 6A_2$. Ground state masses for the vector, axial-vector, and scalar mesons for the two representations, $F$ and $A_2$, from holography and the lattice. Also the pion decay constant and two estimates of the top partner baryon mass.

| $f_{\gamma A_2}$ | $M_{VA_2}$ | $M_{VA}$ | $M_{MS}$ | $M_{MTA_2}$ | $M_{TF}$ |
|------------------|------------|---------|---------|-------------|---------|
| 0.15(4)          | 1.004(4)   | 1*      | 1*      | 1.28        | 1.75    |
| $f_{\gamma F}$  | 0.11(2)    | 0.939   | 0.93    | 1.28        | 1.75    |
| $M_{VA_2}$       | 1.37       | 1.37    | 1.37    | 1.28        | 1.75    |
| $M_{VF}$         | 0.873      | 1.02    | 1.02    | 1.28        | 1.75    |
| $MA_{A_2}$       | 1.40       | 1.40    | 1.40    | 1.28        | 1.75    |
| $MS_{A_2}$       | 1.02       | 1.02    | 1.02    | 1.28        | 1.75    |
| $MSF$            | 1.85       | 1.85    | 1.85    | 1.28        | 1.75    |
| $MTA_2$          | 1.53       | 1.53    | 1.53    | 1.28        | 1.75    |
| $MTF$            | 1.75       | 1.75    | 1.75    | 1.28        | 1.75    |

TABLE II. $SU(4)$ theories - the spectrum in two scenarios and lattice data for comparison.

**SU(4) gauge theory - SU(4) $3F, 3F, 5A_2$:** We report on a second model [20, 21], for which there has also been related lattice work. The gauge group is $SU(4)$. There are 5 Weyl fields in the $A_2$ representation. When these $A_2$ condense they break their $SU(5)$ symmetry to $SO(5)$ - the pNGBs include the Higgs. To include top partner baryons, fermions in the fundamental are added, allowing $FA_2F$ states. The number of flavours is fixed to be three Dirac spinors, since we need to be able to weakly gauge the flavour symmetry to become the SU(3) of QCD. When these fields condense, the chiral $SU(3)_L \times SU(3)_R$ symmetry is broken to the vector $SU(3)$ subgroup. Note that this model is hard to simulate on the lattice due to the fermion doubling problem, so instead lattice work [14, 15] has focused on the case with just 2 Dirac $A_2$S and 2 fundamentals, which allows consideration of the unquenched case.

We display our holographic results in Table II with lattice data from [14]. The holographic approach and the lattice agree in the $V$ sector and the pion decay constants lie close to or just below the lower lattice error bar. We obtain additional masses that have not been computed on the lattice. Crucially, we move to the $5A_2, 3F, 3F$ fermion content by adding in the additional fields needed for the CHM of [20, 21]. We see that the extra fermions slow the running between the $A_2$ and $F$ mass scales, reducing the $F$ sector masses by 10%. The scalar masses again fall (by 20%) in response to the slower running.

**TOP PARTNERS**

Top partners in both models we consider are $FA_2F$ states, spin-1/2 baryons of the strong dynamics. We describe them by a spinor fluctuating in AdS [38], dual to the baryon operator. From Eq. (1), we derive a second order wave equation for the spinor, given in [39], based on [27]. The main features are discussed below.

States with $\Delta = 9/2$, appropriate for three-quark states, have an AdS mass $m = 5/2$. The spinor can be written as eigenstates of the $\gamma_\rho$ projector, with $\gamma_\rho \psi_\pm =
We have computed $y_t$ from the full set of factors in Fig. 1 in both models. If we set a cut off for the HDOs roughly 6 times the vector meson mass, we find the top Yukawa coupling is only of order 0.01, far below the needed value of 1. This is the standard problem when trying to generate the top mass - it is suppressed both by the HDO scale $\Lambda$ and top partner mass squared.

Our new solution to this is to enhance $y_t$ by including a further HDO given by

$$L_{HDO} = \frac{g_T^2}{\Lambda^5} (FA_2F)^2.$$  \hspace{1cm} (8)

As the operator $FA_2F$ becomes the top partner, this is directly a shift in its mass. To include this holographically, we use Witten's double trace prescription [28]. If the operator $O$ were to acquire a vacuum expectation value then, via Eq. (8), there would be an effective source at the boundary $J = \frac{y_t^2}{\Lambda^5} (FA_2F)$. Until now we have considered sourceless theories, $J = 0$. Now, for the baryons, we allow the solutions with $J \neq 0$ and reinterpret them as part of the sourceless theory, but with the HDO present. Asymptotically we read off $J$, $O$ and then compute $g_T$. From these solutions we can find the masses of the top partner for a particular $g_T$.

The HDO can indeed be used to reduce the top partner’s mass as was shown in a more formal setting [27] - for small $g_T$ the effect is linear and small, but after a critical value the effect is much larger. The same conclusion was reached in a simple bottom-up model for the scalar mode as well [41]. In the two models above, using either of the baryon mass’ estimates, one can reduce the top mass to half the V meson mass (which is likely 0.5 TeV or above in CHM) for $7 < g_T < 10$.

Caution is needed in computing the top Yukawa. As the top partner’s mass changes, so do the $Z$ factors in Eqs. (6) and (7). In particular, the HDO in Eq. (8) plays a large role since it induces a sizable non-normalizable piece in the UV holographic wave function of the top partner. The integrals in the normalization factors for the state, which enter directly in the expressions for the $Z$ factors, are more dominated by the UV part of the integral. The overlap between different states can also change substantially. We therefore plot an example of the full expression for the Yukawa coupling from Fig. 1 against the top partner mass (which changes with $g_T$) in Fig. 2. We see that the top Yukawa grows as the top partner’s mass falls and can become of order 1 as the top partners mass falls to about half of the vector meson mass. This suggests one might be able to realize a phenomenological viable top partner mass of 1 TeV or so and the required top mass.

To conclude, we have demonstrated that gauge/gravity duality methods are a powerful tool for obtaining sensible estimates in strongly coupled gauge theories relevant to CHM, and are thus a resource for model builders. A model such as the AdS/YM theory presented is fast to
compute with. The fermion content can be changed in a simple way to obtain further models. Our results for CHM suggest how the spectrum will change as lattice simulations are unquenched to the correct flavour content and HDOs introduced.

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