Chiral-spin symmetry of the meson spectral function above $T_c$

C. Rohrhofer,\textsuperscript{1,2} Y. Aoki,\textsuperscript{3} L.Ya. Glozman,\textsuperscript{2} and S. Hashimoto\textsuperscript{4,5}

\textsuperscript{1}Department of Physics, Osaka University, Toyonaka 560-0043, Japan
\textsuperscript{2}Institute of Physics, University of Graz, 8010 Graz, Austria
\textsuperscript{3}RIKEN Center for Computational Science, Kobe 650-0047, Japan
\textsuperscript{4}KEK Theory Center, High Energy Accelerator Research Organization (KEK), Tsukuba 305-0801, Japan
\textsuperscript{5}School of High Energy Accelerator Science, The Graduate University for Advanced Studies (Sokendai), Tsukuba 305-0801, Japan

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Abstract

Recently, via calculation of spatial correlators of $J = 0, 1$ isovector operators using a chirally symmetric Dirac operator within $N_F = 2$ QCD, it has been found that QCD at temperatures $T_c - 3T_c$ is approximately $SU(2)_{CS}$ and $SU(4)$ symmetric. The latter symmetry suggests that the physical degrees of freedom are chirally symmetric quarks bound by the chromoelectric field into color singlet objects without chromomagnetic effects. This regime of QCD has been referred to as a Stringy Fluid. Here we calculate correlators for propagation in time direction at a temperature slightly above $T_c$ and find the same approximate symmetries. This means that the meson spectral function is chiral-spin and $SU(4)$ symmetric.
I. INTRODUCTION

Artificial truncation of the near-zero modes of the Dirac operator at zero temperature results in the emergence of a large degeneracy in the hadron spectrum, larger than implied by the chiral symmetry of the QCD Lagrangian [1–4]. A symmetry group of this degeneracy, the chiral-spin $SU(2)_{CS}$ group and its flavor extension $SU(2N_F)$, contains chiral symmetries as subgroups [5, 6]. These symmetries are not symmetries of the Dirac Lagrangian. However they are symmetries of the electric interaction in a given reference frame, while the magnetic interaction as well as the quark kinetic term break them. Consequently these symmetries allow us to separate the electric and magnetic interactions in a given frame. The emergence of the $SU(2)_{CS}$ and $SU(2N_F)$ symmetries in the hadron spectrum upon the low mode truncation means that while the confining chromoelectric interaction is distributed among all modes of the Dirac operator, the chromomagnetic interaction contributes only (or at least predominantly) to the near-zero modes. Some unknown microscopic dynamics should be responsible for this phenomenon.

At high temperatures, above the pseudocritical temperature $T_c$, chiral symmetry is restored due to the near-zero modes of the Dirac operator being naturally suppressed by temperature effects [7–10]. Then one could expect a natural emergence of the $SU(2)_{CS}$ and $SU(2N_F)$ symmetries in QCD above $T_c$ [11].

In [12, 13] we have studied a complete set of $J = 0$ and $J = 1$ isovector correlation functions in $z$-direction for a system with $N_F = 2$ dynamical quarks in simulations with the chirally symmetric domain wall Dirac operator at temperatures up to $5.5T_c$. Similar ensembles have been used previously for the study of the $U(1)_A$ restoration in $t$-correlators and via the Dirac eigenvalue decomposition of correlators [9, 14]. We have observed emergence of approximate $SU(2)_{CS}$ and $SU(4)$ symmetries in the spatial correlators in the temperature range $T_c – 3T_c$. While the spatial correlators do not have a special physical meaning, their symmetries at $T_c – 3T_c$ do reflect symmetries of the QCD action since the correlation functions are driven only by the action of the theory. Observation of approximate $SU(2)_{CS}$ and $SU(4)$ symmetries at $T_c – 3T_c$ suggests that the physical degrees of freedom in this temperature range are chirally symmetric quarks bound by the chromoelectric field into color-singlet compounds without the chromomagnetic effects. Such a system is reminiscent of a “string”, that is why the corresponding regime of QCD at $T_c – 3T_c$ is referred to as a Stringy Fluid.
The chemical potential term in the QCD action has precisely the same symmetries [15], so one can expect that the symmetries observed in the lattice calculations at zero chemical potential will persist at \( \mu > 0 \) as well.

Correlators along the time direction have a direct physical meaning since they are connected to the spectral density in Minkowski space via an integral transformation. Observation of the \( SU(2)_{CS} \) and \( SU(4) \) symmetries in \( t \)-correlators would imply that the spectra of the corresponding color-singlet states in Minkowski space have the same symmetry. The symmetries of the \( z \)-correlators do suggest the same symmetries in the spectra, albeit indirectly. A direct observation of these symmetries in \( t \)-correlators in practice is a priori not obvious since on the lattice one has only a few lattice sites along the time direction at high \( T \) and large discretization errors as well as a small evolution time can easily spoil the real picture. Here we use \( N_t = 12 \) ensembles at \( T = 1.2T_c \) and observe very clear \( SU(2)_{CS} \) and \( SU(4) \) symmetries in \( t \)-correlators. This implies that the corresponding spectral functions in Minkowski space are also \( SU(2)_{CS} \) and \( SU(4) \) symmetric.

II. CHIRAL-SPIN SYMMETRY

The \( SU(2)_{CS} \) chiral-spin transformations for quarks are defined by [5, 6]

\[
\psi(x) \rightarrow \exp \left( \frac{i}{2} \Sigma \epsilon \right) \psi(x), \quad \bar{\psi}(x) \rightarrow \bar{\psi}(x) \gamma_4 \exp \left( -\frac{i}{2} \Sigma \epsilon \right) \gamma_4 ,
\]

where \( \epsilon \in \mathbb{R}^3 \) are the rotation parameters. For the generators \( \Sigma \),

\[
\Sigma = \{ \gamma_k, -i\gamma_5 \gamma_k, \gamma_5 \},
\]

one has four different choices \( \Sigma = \Sigma_k \) with \( k = 1, 2, 3, 4 \). Here \( \gamma_k, k = 1, 2, 3, 4 \), are hermitian Euclidean gamma-matrices, obeying the anticommutation relations

\[
\gamma_i \gamma_j + \gamma_j \gamma_i = 2\delta_{ij}; \quad \gamma_5 = \gamma_1 \gamma_2 \gamma_3 \gamma_4 .
\]

The \( su(2) \) algebra

\[
[\Sigma^a, \Sigma^b] = 2i\epsilon^{abc} \Sigma^c
\]

is satisfied for any \( k \).

The choice of \( k \) is fixed by the requirement that the \( SU(2)_{CS} \) transformation does not mix operators with different spin, i.e., respects the rotational \( O(3) \) symmetry in Minkowski space. For the propagators in time direction, defined below, \( k = 4 \).
\( U(1)_A \) is a subgroup of \( SU(2)_{CS} \). The \( SU(2)_{CS} \) transformations mix the left- and right-handed fermions and different representations of the Lorentz group.

The direct product \( SU(2)_{CS} \times SU(N_F) \) can be embedded into a \( SU(2N_F) \) group. The chiral symmetry group of QCD, \( SU(N_F)_L \times SU(N_F)_R \times U(1)_A \), is a subgroup of \( SU(2N_F) \).

The \( SU(2)_{CS} \) and \( SU(2N_F) \) groups are not symmetries of the Dirac equation as well of the QCD Lagrangian as a whole. In a given reference frame the quark-gluon interaction Lagrangian in Minkowski space can be split into temporal and spatial parts:

\[
\bar{\psi} \gamma^\mu D_\mu \psi = \bar{\psi} \gamma^0 D_0 \psi + \bar{\psi} \gamma^i D_i \psi. \tag{5}
\]

Here \( D_\mu \) is a covariant derivative that includes interaction of the quark field \( \psi \) with the gluon field \( A_\mu \),

\[
D_\mu \psi = (\partial_\mu - ig \frac{t \cdot A_\mu}{2}) \psi. \tag{6}
\]

The temporal term includes an interaction of the color-octet charge density

\[
\bar{\psi}(x) \gamma^0 \frac{t}{2} \psi(x) = \psi(x)^\dagger \frac{t}{2} \psi(x) \tag{7}
\]

with the electric part of the gluonic gauge field. It is invariant under any unitary transformation acting in the Dirac and/or flavor spaces. In particular it is a singlet under \( SU(2)_{CS} \) and \( SU(2N_F) \) groups. The spatial part consists of a quark kinetic term and interaction with the magnetic part of the gauge field. It breaks \( SU(2)_{CS} \) and \( SU(2N_F) \). We conclude that interaction of electric and magnetic components of the gauge field with fermions can be distinguished by symmetry.

In order to discuss the notions “electric” and “magnetic” one needs to fix a reference frame. The invariant mass of the hadron is the rest frame energy. Consequently, to discuss physics of hadron mass generation it is natural to use the hadron rest frame.

The spectral density \( \rho(\omega) \) is an integral transform

\[
C_\Gamma(t) = \int d\omega \frac{\cosh(\omega(t - \frac{1}{T}))}{\sinh(\omega \frac{1}{2T})} \rho_\Gamma(\omega) \tag{8}
\]

of the rest frame \( t \)-direction Euclidean correlator

\[
C_\Gamma(t) = \sum_{x,y,z} \langle O_\Gamma(x, y, z, t) O_\Gamma(0, 0)^\dagger \rangle, \tag{9}
\]

where \( O_\Gamma(x, y, z, t) \) is an operator that creates a quark-antiquark pair for mesons with fixed quantum numbers.
FIG. 1. Transformations between interpolating vector operators, $i = 1, 2, 3$. The left columns indicate the chiral representation for each operator. Red and blue arrows connect operators that transform into each other under $SU(2)_L \times SU(2)_R$ and $U(1)_A$, respectively. Green arrows connect operators that form triplets of $SU(2)_{CS}, k = 4$. The $f_1$ and $a_1$ operators are the $SU(2)_{CS}, k = 4$ – singlets. Purple arrows show the 15-plet of $SU(4)$. The $f_1$ operator is a $SU(4)$-singlet.

Transformation properties of the local $J = 1$ quark-antiquark bilinears $\mathcal{O}_i(x, y, z, t)$ with respect to $SU(2)_L \times SU(2)_R$ and $U(1)_A$ are given on the left side of Fig. 1 and those with respect to $SU(2)_{CS}, k = 4$ and $SU(4)$ on the right side of Fig. 1 [6]. Emergence of the respective symmetries is signalled by the degeneracy of the correlators (9) calculated with operators that are connected by the corresponding transformations.

III. METHODOLOGY

The lattice data presented in the next section is calculated on JLQCD gauge configurations with $N_F = 2$ fully dynamical domain wall fermions ([9][16]). The length of the fifth dimension for the fermions is chosen as $L_5 = 16$, to ensure good chiral symmetry [14].

The quark propagators are computed on point sources after three steps of stout smearing. The fermion fields obey anti-periodic boundary conditions in time direction. For the gauge part we use the Symanzik-improved gauge action with an inverse gauge coupling $\beta_g = 4.3$ ($a = 0.075$ fm). The time extent of the lattices is $N_t = 12$, which corresponds to a temperature of $T \simeq 220$ MeV ($\sim 1.2T_c$). We calculate the data on three spatial volumes, $N_s = 24, 32, 48$, with a quark mass of $m_{ud} = 0.001$. Measurements are performed on $\mathcal{O}(50)$ independent configurations.
The main observables are correlation functions of local isovector bilinears

\[ O_\Gamma = \bar{\psi} (\tau/2 \otimes \Gamma) \psi, \]

where the \( \Gamma \) structures from Fig. 1 determine the resulting quantum numbers. To extract correlation functions of states with definite, i.e. zero, momentum, we perform a momentum projection according to Eq. (9).

Finally, the data shown in the next section is rescaled to a dimensionless variable

\[ tT = \frac{n_t a}{N_t a} = \frac{n_t}{N_t}, \]

(10)

where \( t \) is the measured lattice distance in time direction, \( T \) the temperature, \( a \) the lattice constant, and \( N_t \) the overall temporal lattice extent. For spatial correlators in \( z \)-direction the same rescaling is done with \( z = n_z a \) instead of \( t \).

IV. RESULTS

On the right side of Fig. 2 we show \( t \)-correlators (9) normalized at \( n_t = 1 \) calculated on \( 48^3 \times 12 \) lattices at \( T = 1.2 T_c \). The results obtained on \( N_s = 32, 24 \) lattices are similar and agree within statistical errors, they are omitted for clarity.

Specifically we calculate the correlators of \( J = 0 \) isovector scalar \( \bar{\psi} \tau/2 \psi (S) \) and pseudoscalar \( \bar{\psi} \gamma_5 \tau/2 \psi (PS) \) operators, where \( \tau \) are isospin Pauli matrices as well as correlators of isovector operators \( \{ b_1, (1/2, 1/2)_a \}, \{ \rho, (1/2, 1/2)_b \}, \{ \rho, (1, 0) \oplus (0, 1) \} \) and \( \{ a_1, (1, 0) \oplus (0, 1) \} \) from Fig. 1. A degeneracy of scalar and pseudoscalar correlators reflects restoration of \( U(1)_A \) symmetry, since the corresponding operators are connected by the \( U(1)_A \) transformation, observed already previously in Refs. [9, 14]. Since the \( \{ b_1, (1/2, 1/2)_a \} \) and \( \{ \rho, (1/2, 1/2)_b \} \) operators are also connected by the \( U(1)_A \) transformation, the degeneracy of the corresponding correlators also signals the \( U(1)_A \) symmetry. A degeneracy of \( \{ \rho, (1, 0) \oplus (0, 1) \} \) and \( \{ a_1, (1, 0) \oplus (0, 1) \} \) correlators evidences the restoration of chiral \( SU(2)_L \times SU(2)_R \) symmetry.

An approximate degeneracy of \( \{ b_1, (1/2, 1/2)_a \}, \{ \rho, (1/2, 1/2)_b \} \) and \( \{ \rho, (1, 0) \oplus (0, 1) \} \) correlators signals emergence of the \( SU(2)_{CS} \) symmetry, since all three operators belong to the same irreducible representation (triplet) of \( SU(2)_{CS} \). Finally a degeneracy of all four correlators \( \{ b_1, (1/2, 1/2)_a \}, \{ \rho, (1/2, 1/2)_b \}, \{ \rho, (1, 0) \oplus (0, 1) \} \) and \( \{ a_1, (1, 0) \oplus (0, 1) \} \) is due to the emergence of \( SU(4) \) symmetry.
FIG. 2. Temporal correlation functions for $48^3 \times 12$ lattices. The l.h.s. shows correlators calculated with free noninteracting quarks on the same lattice, and features a symmetry pattern expected from chiral symmetry. The r.h.s. presents full QCD data at a temperature of $T = 220\text{MeV} = (1.2T_c)$, which shows multiplets of all $U(1)_A$, $SU(2)_L \times SU(2)_R$, $SU(2)_{CS}$ and $SU(4)$ groups.

On the left side of Fig. 2 we show the correlators calculated with free, noninteracting quarks on the same lattice with the same Dirac action (the gauge operator $U$ is set to 1). Dynamics of free quarks are governed by the Dirac equation and only chiral symmetries exist. Indeed, a multiplet structure in this case is very different as compared to right side of Fig. 2 and only degeneracies due to $U(1)_A$ and $SU(2)_L \times SU(2)_R$ symmetries are seen in meson correlators calculated for free quarks. The pattern seen on the left of Fig. 2 reflects correlators at a very high temperature, since due to the asymptotic freedom at a very high $T$ the quark-gluon interactions can be neglected.

While we observe practically exact chiral symmetries, the $SU(2)_{CS}$ and $SU(4)$ symmetries are only approximate. A degree of the symmetry breaking can be evaluated via the parameter $\kappa$,

$$\kappa = \frac{C^{(1.0)\oplus(0,1)} \rho - C^{(1/2,1/2)} \rho}{C^{(1.0)\oplus(0,1)} \rho - C_S},$$

that measures the splitting within the $SU(2)_{CS}$ multiplet relative to the distance between different multiplets. With this definition, good symmetry implies $|\kappa| \ll 1$.

The degree of the symmetry breaking obviously depends on the dimensionless variable $tT$. At $tT \sim 0.5$ the breaking is tiny, as can be seen from Fig. 3. For the noninteracting quarks there is no $SU(2)_{CS}$ symmetry and in infinite volume $|\kappa| \sim 1$. [13]
It is instructive to compare the scale dependence of the symmetry breaking parameter $\kappa$ extracted from the $t$-correlators and from the $z$-correlators \cite{13} since the $t$- and $z$-correlators probe QCD at different dimensionless “distance” $tT$, $zT$ (the time extent of the lattice is smaller than its spatial extent). Apriori one would expect that at the same $tT = zT$ a degree of the symmetry breaking would be similar as measured by the $t$- and $z$-correlators. As it can be seen from Fig. 3, both measurements show the same level of symmetry breaking.

V. CONCLUSIONS

We have calculated meson rest-frame correlators of $J = 0$ and $J = 1$ isovector operators along the time-direction with $N_F = 2$ QCD with physical masses with the chirally symmetric domain wall Dirac operator at $T = 1.2T_c$. We have observed a very clear emergence of approximate chiral-spin $SU(2)_{CS}$ and $SU(4)$ symmetries in these correlators. The $t$-correlators are connected via an integral transform with the measurable spectral density in Minkowski
space. Approximate $SU(2)_{CS}$ and $SU(4)$ symmetries of the $t$-correlators imply the same symmetries of spectral densities. This result reinforces our findings in Refs. [12] [13].

These symmetries are incompatible with free deconfined quarks and suggest that the physical degrees of freedom are chirally symmetric quarks bound into color-singlet compounds by the chromoelectric field without the chromomagnetic effects. This result is model-independent and relies solely on lattice results and symmetry classification of the QCD Lagrangian. Such relativistic objects are reminiscent of “strings” since they are purely electric and we refer to the corresponding regime of QCD as a Stringy Fluid.

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