A Highly Resolved Large-Eddy Simulation of a Wind Turbine using an Actuator Line Model with Optimal Body Force Projection

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Abstract. When representing the blade aerodynamics with rotating actuator lines, the computed forces have to be projected back to the CFD flow field as a volumetric body force. That has been done in the past with a geometrically simple uniform three-dimensional Gaussian at each point along the blade. We argue that the body force can be shaped in a way that better predicts the blade local flow field, the blade load distribution, and the formation of the tip/root vortices. In previous work, we have determined the optimal scales of circular and elliptical Gaussian kernels that best reproduce the local flow field in two-dimensions. In this work we extend the analysis and applications by considering the full three-dimensional blade to test our hypothesis in a highly resolved Large Eddy Simulation.

1. Introduction
The actuator line model (ALM) is a widely used tool for performing numerical simulations of wind turbine wakes [1, 2]. The length scale, \( \epsilon \), over which the forces of each actuator point are projected into volumetric body forces is an input parameter for the model [3, 4, 5]. Previous studies have looked at the effects of the smearing length scales in ALM and provided guidelines for its use [4, 5, 6]. In recent work, an optimal length scale, \( \epsilon \), has been determined for two-dimensional flow over an airfoil [7]. There, an optimal value of \( \epsilon/c \), where \( c \) is chord length, is established which minimizes the difference between a potential flow solution of flow over a Joukowsky airfoil and a new analytical solution for linearized inviscid flow with a Gaussian distributed lifting force. The optimal values used in this work are for an objective function which minimizes the difference between two flow fields, but other objective functions could be used. Also, a more general Gaussian function with stretching in both thickness and camber (\( \epsilon_c \) and \( \epsilon_t \)) directions is studied, which further minimizes the error in the solution. Here, we extend this study by simulating an NREL 5MW rotor [5] using the actuator line model with the optimal generalized Gaussian function for each section of the blade. The idea behind using a refined ALM, instead of a fully resolved rotor, is to avoid all the complexities associated with a body fitted grid and a rotating mesh. The ALM allows the use of a uniform Cartesian mesh, which simplifies the simulation process without compromising the accuracy of the solution. The proposed simulation will allow us to study how well the ALM with an improved body force
projection function and projection length scale replicates the flow fields of the blade without having to perform a fully resolved simulation.

The previous analytical study has shown that an ALM can reproduce a 2D flow field very accurately compared to the potential flow solution [7]. The flow field obtained using the optimal length scales is compared to the potential flow solution in Figure 1. The left figure shows streamlines and velocity magnitude contours of the potential flow solution over a Joukowsky airfoil with camber. The right figure shows the inviscid flow solution [7] over an elliptically shaped Gaussian lift force with the optimal length scales. In order to most closely replicate the flow field, the optimal length scale in the chord direction is found to be $\epsilon_c/c = 0.4$ for a cambered airfoil [7]. The value required for the thickness direction ($\epsilon_t$) is as small as the grid allows. In order to resolve reasonably well such small values of $\epsilon_t$ requires a very fine grid with several points across the chord of the blade. ALM simulations with such fine resolutions and shaped force distributions have not yet been attempted. In this work, we perform initial highly resolved Large Eddy Simulation (LES) with such force distributions to examine solution stability and to determine whether detailed features such as tip vortices can be produced realistically.

2. Simulations

The filtered Navier-Stokes equations are solved using the second-order finite-volume framework Simulator fOr Wind Farm Applications (SOWFA) from the National Renewable Energy Laboratory (NREL) [9]. It is 2nd-order accurate (time and space), collocated finite-volume, and can handle arbitrary unstructured meshes. It is built from the OpenFOAM CFD toolbox [10]. An actuator line model (ALM) is implemented with a generalized Gaussian shaping function.

2.1. An Optimal Body Force Projection

The NREL 5MW Rotor is used for the simulation with the lift and drag coefficient tables provided. ALM points along each blade determine a force (lift and drag) that is distributed onto the grid with a 2D Gaussian kernel in the blade cross plane

$$\eta_e = e^{-\left(x^2/\epsilon_c^2 + y^2/\epsilon_t^2\right)} \frac{1}{\epsilon_c \epsilon_t \pi},$$

with optimal kernel widths $\epsilon_c/c$ and $\epsilon_t/c$. 

Figure 1: Streamwise velocity ($U_x/U_\infty$) contours and streamlines for a potential flow solution (left) of a cambered Joukowsky airfoil at an angle of attack of 12° compared to a Gaussian body force solution (right) using the optimal scales $\epsilon_c$ and $\epsilon_t$. 

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**Figure 1**

![Streamwise velocity contours and streamlines](image)

Streamwise velocity ($U_x/U_\infty$) contours and streamlines for a potential flow solution (left) of a cambered Joukowsky airfoil at an angle of attack of 12° compared to a Gaussian body force solution (right) using the optimal scales $\epsilon_c$ and $\epsilon_t$. 

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**References**

[7] Reference 1

[9] Reference 2

[10] Reference 3
The optimal values of $\epsilon_c$ and $\epsilon_l$ can be determined according to the grid resolution $\Delta/c$. A lower bound limit has been identified in previous studies as $\epsilon/\Delta x \geq 2$ \[4, 3, 5\]. A set of initial simulations confirmed this limit, so the minimum value is set to $\epsilon_y/\Delta x \geq 2$. Figure 2 shows the optimal values of $\epsilon_*$ and flow field error compared to the potential flow solution as a function of grid size using the constraint that $\epsilon/\Delta x \geq 2$. The dashed vertical line indicates the grid resolution used in the study ($\Delta/c = 0.1$). This resolution requires about 10 points per chord for the smallest section of the blade (at the tip). With this resolution, a locally refined mesh is used with $\sim 150$ Million grid points.

![Figure 2](image_url)

Figure 2: Left panel: Optimal values of $\epsilon_{xz}$ and $\epsilon_{yz}$ for a cambered airfoil at $\alpha = 6^\circ$ according to the LES resolution and the limit $\epsilon/\Delta x \geq 2$. Right panel: Square error between potential flow and inviscid flow over Gaussian lift force distribution \[7\], as function of resolution when using the associated optimal body force projection (from left panel).

The optimal values used are $\epsilon_c/c = 0.4$ and $\epsilon_l/c = 0.2$. The error could be further reduced by decreasing the value of $\epsilon_y$, but the resolution required would exceed the computational resources available. Time-stepping is limited by the tip of the blade not being able to go through more than one grid cell every time-step \[4\]. For the cylindrical sections near the shaft, the momentum thickness, i.e. $\epsilon = \frac{1}{2} d c_d$, where $d$ is the cylinder diameter and $c_d$ the drag coefficient, will be used as the optimal value, given that in this case, the flow will be dominated by drag \[7\].

### 3. Results

Two main cases are compared, one with the optimal body force, and another one with a typically used value of $\epsilon = 1.5m$, which gives accurate power estimates for the NREL 5MW \[8, 9\]. We first compare the quantities along the blade to results from Blade Element Momentum Theory (BEM). A tip loss correction has been used in the BEM algorithm \[11\]. Figure 3 shows angle of attack $\alpha$, axial velocity $U_x/U_\infty$, and lift ($F_L$) and drag ($F_D$) forces as a function of blade radius. The results from the simulation with the optimal body force projection agree very well with BEM Theory. The tip effects are difficult to capture by the LES, especially when these are diffused by using a large smearing parameter $\epsilon$. With the optimal values of $\epsilon$, the tip-vortices are resolved more accurately by the LES. Using the optimal body force projection, the forces applied do not extend past the blade, and there is a sharp drop in lift and angle of attack as we would expect in the real blade. We emphasize the fact that the ALM with the optimal body force projection does not need a tip-loss correction, as the tip-losses will be resolved by the LES.

The flow field reproduced by using the optimal smoothing function closely approximates the shape of the blade. Figure 4 shows the axial velocity in the rotor plane. The shape of the wind turbine blade can be seen as the flow moves along what would be the blade in the simulation.
Figure 3: Angle of attack, Axial velocity, Drag and Lift along the blade compared with results from Blade Element Momentum.

The optimal body force projection allows the streamlines to curve following (approximately) the path of the airfoil sections. Figure 5 shows the vorticity in the wake, where we can clearly identify the tip vortices. The case for the optimal body force projection shows tighter tip vortices, which take longer to diffuse. These tip vortices create the right amount of losses near the tip, such that a tip loss correction is not needed.

Figure 4: Axial velocity contour ($U_x$) at the rotor plane for a standard ALM (left) and an ALM with optimal body force projection (right).
Taking a cross-sectional cut along the blade can help visualize the 2D flow along the actuator point. Figure 6 shows a cross-sectional plane at 75% radius. The velocity has been projected such that the inflow is coming from left to right. The flow around the actuator point agrees very well with the analytical solution used to obtain the optimal body force projection. This flow, corresponds to the closest approximation of flow over a 2D airfoil with an optimal body force projection.

Figure 6: Velocity contours ($U_x'/U_{\infty}'$) and streamlines projected onto a cross sectional plane for the blade at 75% radius. Left panel: Results from LES using the standard ALM. Middle panel: LES results using the optimal ALM with optimal $\epsilon/c = 0.4$ and $\epsilon/c = 0.2$. Right panel: Analytical solution of inviscid flow with Gaussian lift force, also with optimal scales.

4. Conclusion
A simulation of an actuator line model (ALM) with an optimal body force projection has been performed. The optimal body force projection is based on an analytical approach which aims at mimicking the flow around Joukowsky airfoils. The optimal force projection is a 2D Gaussian kernel with widths of $\epsilon_c/c = 0.4$ and $\epsilon_c/c = 0.2$. These values require very fine resolutions in
order to satisfy the numerical limits established in the literature. A set of initial tests confirmed
the numerical limit for the optimal body force projection is given by $\epsilon_t/\Delta x \geq 2$, where the width
in thickness direction is smallest.

This study is a first attempt to run an optimal formulation based on 2D aerodynamics on a
3D rotor. The results look promising but there is still more work to be done in order to validate
it and improve on it. The study focused on the quantities at the rotor plane and in the near
wake very close to the rotor. Further studies are required to study the the effects of the optimal
body force projection on the far wake.

The simulation with the optimal value is able to resolve very small structures, such as the
flow around the tip. For this reason, the effects on the tip are captured by the LES and no tip
loss correction is needed in the ALM with optimal body force projection. The velocities and
loads along the blades for the case of an optimal body force projection agree very well with Blade
Element Momentum Theory (BEM). The standard case with a 1D coarser Gaussian kernel gives
good agreement with BEM except near the tip. The 1D kernel extends past the tip, and smears
the force applied in the radial direction. This diffuses the tip vortex and is not resolved by the
LES. The resolutions required for this type of simulations are on the order of $\sim$10 grid points
per chord ($\sim$ 100 Million grid points per turbine). With these fine resolutions, the flow field
along the blades is reproduced as accurately as possible with the ALM.

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