Research Article

Centralized Maintenance Time Prediction Algorithm for Freight Train Wheels Based on Remaining Useful Life Prediction

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Many freight trains for special lines have in common the characteristics of a fixed group. Centralized Condition-Based Maintenance (CCBM) of key components, on the same freight train, can reduce maintenance costs and enhance transportation efficiency. To this end, an optimization algorithm based on the nonlinear Wiener process is proposed, for the prediction of the train wheels Remaining Useful Life (RUL) and the centralized maintenance timing. First, Hodrick–Prescott (HP) filtering algorithm is employed to process the raw monitoring data of wheel tread wear, extracting its trend components. MK_hen, a nonlinear Wiener process model is constructed. Model parameters are calculated with a maximum likelihood estimation and the general deterioration parameters of wheel tread wear are obtained. Then, a nonlinear Wiener process model is constructed. Model parameters are calculated with a maximum likelihood estimation and the general deterioration parameters of wheel tread wear are obtained. Then, the updating algorithm for the drift coefficient is deduced using Bayesian formula. The online updating of the model is realized, based on individual wheel monitoring data, while a probability density function of individual wheel RUL is obtained. A prediction method of RUL for centralized maintenance is proposed, based on two set thresholds: “maintenance limit” and “the ratio of limit-arriving.” Meanwhile, a CCBM timing prediction algorithm is proposed, based on the expectation distribution of individual wheel RUL. Finally, the model is validated using a 500-day online monitoring data on a fixed group, consisting of 54 freight train cars. The validation result shows that the model can predict the wheels RUL of the train for CCBM. The proposed method can be used to predict the maintenance timing when there is a large number of components under the same working conditions and following the same path of degradation.

1. Introduction

There are three main maintenance policies for railway freight trains, including corrective maintenance, scheduled maintenance, and condition-based maintenance. The condition-based maintenance is actually getting increasing attention. One of the key points in condition-based maintenance of railway freight trains is Remaining Useful Life (RUL) prediction of vehicle parts, based on reliability theory and online monitoring data. Moreover, determining a reasonable maintenance time is essential for improving freight train operation efficiency and maintenance cost reduction. In a fixed group of a freight train consisting of 54 railway freight cars, there is a large number of parts of the same type and under the same working conditions, such as wheels and bogies. If the condition of each vehicle part is considered independently, then the train maintenance time will be significantly long, leading to high maintenance costs and low operation efficiency. Hence, Centralized Condition-Based Maintenance approach is proposed in this paper, which means to find a reasonable maintenance time point where all vehicle parts of the same type are repaired simultaneously, thus improving the maintenance efficiency.

Understandably, the wheel status will directly affect the train operation quality and safety, as an important part of any railway freight car. The wheel tread wear is one of the key parameters reflecting the wheel state in relation to the service time. The RUL of wheels can be predicted using a degradation model based on offline history tread wear data and online monitoring data, serving as an important basis for
vehicle CCBM. The dispersion of tread wear data for different wheels will gradually increase over operation time, producing different degradation curves for each individual wheel. A set reasonable maintenance time can avoid exceeding the operation limit of the wheel with a fast wear speed, thus avoiding compromising safety, while also avoiding over repair.

At present, there are many researches on the degradation process of railway vehicle wheels. Most of them are based on the mechanism to model the wheel wear, rolling contact fatigue and other degradation processes [1, 2]. Hossein-Nia et al. [3] developed a model to estimate the evolution of surface-initiated Rolling Contact Fatigue (RCF). In this model for RCF calculations, a shakedown-based theory is applied locally, and the FaStrip algorithm is used to estimate the tangential stresses. While considering the mechanism, the prediction of wheel wear needs to consider various factors such as lines, rails, vehicles, etc., and it will be difficult to update the model based on real-time degradation, which is easy to cause error accumulation. This study uses a data-driven approach that is flexible and easy to achieve real-time online prediction.

A lot of research has been carried out on condition-based maintenance, applying performance degradation data [4–6]. Wiener process is a commonly used method for remaining life modeling, as the first hitting time is inversely Gaussian-distributed and it can, furthermore, reflect nonmonotonic random changes [7–9]. The performance degradation model derived from standard Wiener process has been widely discussed in recent years and can be used to describe the performance degradation of many typical products. Si et al. [10] proposed a remaining life prediction model based on nonlinear Wiener process. The unknown parameters in the established model are estimated using the maximum likelihood estimation approach, leading to a probability density function for the RUL. The model is based on offline monitoring data, which means that no real-time updating of mode parameters is possible. Zhang et al. [11] incorporate the inspection influence into degradation modeling based on Wiener process. The impact of the inspection of the system in the process of obtaining degraded data on the degradation process is considered. The proposed approach was demonstrated by a case study using the mechanical gyroscopes. Peng et al. [12] proposed a switchable state-space degradation model to characterize degradation paths with non-deterministic switching manner dynamically. The proposed method was applied to a real bearing degradation process with phase transitions.

The purpose of modeling the degradation process is to guide the formulation and optimization of maintenance strategies. The Wiener process approach is often combined with the optimization problem of maintenance decision [13, 14]. Sun et al. [15] studied multicomponent systems, where the degradation of each component was assumed to comply with the Wiener process. In their study, the optimal maintenance decision-making problem of multunit systems was studied, under the premise of periodic inspection. Wei et al. [16] studied a binary degradation system affected by shock and used Wiener process to simulate degradation.

And according to the system state, the optimal action includes no action, imperfect repair, preventive replacement, or corrective replacement. Zhang et al. [17] published an overview of the current research status of the Wiener process. The paper reviews recent developments in the Wiener process-based modeling methods for degradation data analysis and RUL estimation, as well as their applications in the field of prognostics and health management (PHM). Specifically, the modifications in the Wiener process are introduced considering nonlinearity, multsource variability, covariates, and other multivariable factors involved in the degradation processes.

In this paper, a prediction algorithm of individual RUL, based on nonlinear Wiener process, is proposed for the prediction of centralized maintenance time of railway freight train wheels. The structure of the article is as follows: Section 2 introduces the HP filtering algorithm, which is used to extract the degradation trend of the monitoring data, in order to model the degradation process. In Section 3, the modeling process is described for the overall degradation model of the wheel tread circumferential wear, based on the nonlinear Wiener process. The model parameters are estimated by the maximum likelihood method, while the parameters updating algorithm is derived based on the Bayesian formula. Then, the real-time prediction of RUL for the individual wheel is provided, using the updated parameters. In Section 4, a method for determining the maintenance time of the whole train is proposed, by setting a maintenance limit and a ratio called limit-arriving. Weibull distribution is employed to fit the RUL distribution of different wheels on the same train, followed by a prediction for the centralized maintenance time. Finally, the algorithm was verified using 500-day data from a train.

2. Degradation Trend Extraction from Monitoring Data

In this paper, the monitoring data of wheel tread wear are obtained from an online train wheel monitoring system, called Train Wheel Detection System (TWDS), and mounted on the railway line. The system is used to detect wheel parameters such as tread wear, rim width, wheel diameter, etc. Particularly, as the operation mileage increases, the diameter of a wheel at different positions will vary. In addition, due to wheel vibration and sensor measurement errors, there is fluctuation in the monitoring data itself. The measurements show that even though the actual value of degradation is low, the relative fluctuation is quite large. Therefore, it is necessary to apply filtering on the degradation data, in order to extract the trend component, achieving better results for the RUL prediction.

HP filtering, proposed by Hodrick and Prescott in 1981, is widely used in economic analysis, but can be generally applied on data containing fluctuations, to extract trend components [18, 19]. Ouahilal et al. [20] used HP filter in the stock price forecasting field and proposed an approach combining Support Vector Regression with HP filter. The experimental results confirm that the proposed model is more powerful in terms of predicting stock prices. Poloni
and Sbrana [21] extend HP filtering to multidimensional conditions, offering an interesting option for industrial production analysis in several European countries. Dai et al. [22] first used HP filtering for degradation data processing in 2018 to extract the trend characteristics of solder joint failure data. Still, there are only a few studies on HP filtering for degradation data trend extraction.

In this paper, HP filter is used to process the monitoring data of wheel tread wear, while the trend component of wheel wear degradation is extracted for model parameters estimation and individual RUL prediction. The process of HP filtering algorithm is described as follows:

Suppose there is a time series \( Y = \{Y(1), Y(2), Y(3), \ldots, Y(m)\} \), which consists of a trend component \( X(t) \) and a fluctuation component \( C(t) \), as shown in

\[
Y(t) = X(t) + C(t). \tag{1}
\]

Trend component \( X(t) \) can be obtained by calculating the following equation, as described in [23]:

\[
\min \left\{ \sum_{t=1}^{m} (Y(t) - X(t))^2 + \beta \sum_{t=2}^{m-1} [(X(t+1) - X(t)) - (X(t) - X(t-1))]^2 \right\}. \tag{2}
\]

Equation (2) can be divided into two parts, the first part being

\[
\sum_{t=1}^{m} (Y(t) - X(t))^2. \tag{3}
\]

This part reflects the reductive degree of the original sequence. The lower the value of this part, the better tracking performance of \( X(t) \) to the original sequence is.

The remaining part of (2) is defined as Part Two; i.e.,

\[
\sum_{t=2}^{m-1} [(X(t+1) - X(t)) - (X(t) - X(t-1))]^2. \tag{4}
\]

Part Two measures the smoothness of the new sequence; that is, the lower it is, the higher the smoothness of \( X(t) \) is. \( \beta \) is a penalty factor to control the smoothness degree. Its value is a result of compromise between the fidelity to the raw data tracking and the smoothness degree of the raw data sequence. According to [24] and the monitoring frequency of tread wear monitoring data, \( \beta \) value is empirically set to 1000, as more suitable to the purpose of this study.

Partial derivatives of \( X(1), X(2), \ldots, X(1) \) are derived from (2) and set to 0, in order to solve for \( X(t) \), as shown in the following equation:

According to (5), the coefficient matrix \( F \) of dimensions \( N \times N \) can be obtained as follows:

\[
F = \begin{bmatrix}
1 & -2 & 1 & 0 & \cdots & 0 \\
-2 & 5 & -4 & 1 & 0 & \cdots & 0 \\
1 & -4 & 6 & -4 & 1 & 0 & \cdots & 0 \\
0 & 1 & -4 & 6 & -4 & 1 & 0 & \cdots & 0 \\
\vdots & & & & & & & & \\
0 & \cdots & 0 & 1 & -4 & 6 & 4 & 1 & 0 \\
0 & \cdots & 0 & 1 & -4 & 6 & 4 & 1 & 0 \\
0 & \cdots & 0 & 1 & -5 & 5 & -2 & 0 & \cdots \\
0 & \cdots & 0 & 1 & -2 & 1 & & & \\
\end{bmatrix}. \tag{6}
\]

Equation (5) can be expressed as a matrix, as shown in

\[
C = \beta FX, \tag{7}
\]

where,

\[
X = (X(1), X(2), \ldots, X(m))^\prime, \tag{8}
\]

\[
C = (C(1), C(2), \ldots, C(m))^\prime, \tag{9}
\]

where \( (\cdot)^\prime \) denotes the vector transposition. Combining (1) and (7), letting \( Y = (Y(1), Y(2), \ldots, Y(m))^\prime \), the following relation is derived:

\[
X = (\beta F + I)^{-1} Y, \tag{10}
\]

where \( I \) is the identity matrix.

The trend series \( X \) of time series \( Y \) can be obtained by (10).

The degradation trend data of wheel tread wear, after HP filtering, is used to establish the Wiener process model, as described in the next section.

3. Degradation Modeling and RUL Prediction Based on Wiener Process

The RUL prediction of wheels includes three steps: degradation model establishment, model parameters estimation, and calculation of the RUL. Because the wear rate of wheels increases as the service time progresses, a nonlinear Wiener
process model is selected as basis for the wheel tread wear model. The estimation of model parameters is carried out in two steps: (1) estimating the overall model parameters, using the degradation data of all wheels on the same train, to reflect the overall characteristics of the degradation process; (2) updating the model parameters, using Bayesian formula, based on the monitoring data of each individual wheel to better fit the degradation process of each individual unit. As new monitoring data are acquired, the individual degradation parameters are updated in the course of online RUL prediction. Then, the probability density distribution function (PDF) of the RUL is obtained, according to the concept of the first hitting time.

3.1. Nonlinear Wiener Process Modeling. A reasonable performance degradation model is the key to accurately predict the RUL of components. Due to the nonlinearity of wheel tread wear data, a nonlinear Wiener process model is selected as basis for the wheel tread wear model. 

\[ X(s) = X(0) + \lambda s^b + \sigma_B B(s), \]  

where \( X(s) \) is the degradation when the mileage is \( s \). \( X(0) \) is the initial degradation at the beginning of monitoring. \( \lambda \) is the drift coefficient, reflecting the degradation rate. It is often set as a random variable, subject to \( \lambda \sim N(\mu_\lambda, \sigma_{\lambda}^2) \), and \( b \) is a nonlinear coefficient, \( \sigma_B \) is a diffusion coefficient, and \( B(s) \) is a standard Brownian motion, subject to \( B(s) \sim N(0, s) \).

Most current studies often assume \( X(0) = 0 \) in order to facilitate the calculations. However, different wheels may have different initial degradation due to different service mileage; thus, \( X(0) \) cannot be assumed to be the same on all wheels, in this situation. Equation (10) is rewritten as (11), in order to simplify the model and take \( X(0) \) into account.

\[ \tilde{X}(s) = \lambda s^b + \sigma_B B(s), \]  

where

\[ \tilde{X}(s) = X(s) - X(0). \]  

In this way, the various values of initial degradation of different individual wheels are all converted to 0.

3.2. Model Parameters Estimation. There are four unknown parameters \( \theta = [\mu_\lambda, \sigma_\lambda, b, \sigma_B] \) in the wheel degradation model. First, the degradation data of all wheels on the same train is taken as a sample set, where the overall model parameter \( \theta \) is estimated, according to the maximum likelihood estimation method. Following, the updating algorithm for the mean and variance of the drift coefficient in the model is deduced, according to the Bayesian formula, and then the parameters update of the individual model is realized.

3.2.1. Overall Model Parameter Estimation. The analytical solution of the parameters in the Wiener process model can be obtained using the maximum likelihood estimation method. It is assumed that \( N \) wheels are monitored \( m \) times up to the current mileage \( s_m \). The degradation data of the \( N \) wheels is denoted as \( X \), and the degradation of the \( n \)th wheel at the monitoring mileage \( s_1, s_2, \ldots, s_m \) is recorded as

\[ X_n = (X_n(s_1), X_n(s_2), \ldots, X_n(s_m))^T. \]

Let

\[ s = (s_1, s_2, \ldots, s_m)^T. \]

According to (12), \( \tilde{X}_n \) can be described as follows:

\[ \tilde{X}_n = (X_n(s_1) - X_n(0), X_n(s_2) - X_n(0), \ldots, X_n(s_m) - X_n(0))^T. \]

According to (11) and the independent incremental characteristics of the Wiener process, \( \tilde{X}_n \) follows the multivariate normal distribution, \( \tilde{X}_n \sim N(\tilde{\mu}, \tilde{\sigma}) \). The mean and variance are, respectively,

\[ \tilde{\mu} = \mu_\lambda s, \]

\[ \tilde{\sigma} = M + \sigma^2_B s's', \]

where

\[ M = \sigma^2_B T, \]

\[ T = \begin{bmatrix} 1 & s_1 & s_1 \cdots & s_1 \\ s_1 & 1 & s_2 & \cdots & s_2 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ s_1 & s_2 & \cdots & 1 \end{bmatrix}, \]

Assuming that the degradation of different wheels is independent of each other, the logarithmic likelihood function of \( \theta = [\mu_\lambda, \sigma_\lambda, b, \sigma_B] \) can be obtained [25] as

\[ L(\theta | \tilde{X}) = -\frac{Nm \ln (2\pi)}{2} - \frac{1}{2} N \ln(|M|(1 + \sigma^2_B s'M^{-1}s)), \]

where

\[ |M| = |M|(1 + \sigma^2_B s'M^{-1}s), \]

\[ \sigma^{-1} = M^{-1} - \frac{\sigma^2_B s'M^{-1}s}{1 + \sigma^2_B s'M^{-1}s}. \]

By calculating partial derivatives of logarithmic likelihood functions for \( \mu_\lambda \) and \( \sigma_\lambda \), we can derive the following:

\[ \frac{\partial l(\theta | \tilde{X})}{\partial \mu_\lambda} = \sum_{n=1}^{N} s^n M^{-1}\tilde{X}_n - N\mu_\lambda s'M^{-1}s, \]

\[ \frac{\partial l(\theta | \tilde{X})}{\partial \sigma_\lambda} = \frac{N\sigma_\lambda s'M^{-1}s}{1 + \sigma^2_B s'M^{-1}s} + \frac{\sigma^2_B \sum_{n=1}^{N} (\tilde{X}_n - \mu_\lambda s')M^{-1}s'M^{-1}(\tilde{X}_n - \mu_\lambda s)}{(1 + \sigma^2_B s'M^{-1}s)^2}. \]

For a specific set of \( b \) and \( \sigma_B \), given that the two partial derivatives of equations (20) and (21) equal zero, the
maximum likelihood estimates for \( \mu_\lambda \) and \( \sigma_\lambda \) can be calculated as

\[
\mu_\lambda = \frac{\sum_{n=1}^{N} s'M^{-1}\bar{X}_n}{Ns'M^{-1}X_n}, \tag{22}
\]

\[
\sigma_\lambda = \left\{ \frac{1}{N}\left[ \frac{1}{N s'M^{-1}X_n} \sum_{n=1}^{N} (\bar{X}_n - \bar{\mu}_s)^t M^{-1} s'M^{-1} (\bar{X}_n - \bar{\mu}_s) - \frac{1}{s'M^{-1}s} \right] \right\}^{1/2}. \tag{23}
\]

Equation (18) can be reformulated by introducing equations (22) and (23) into the maximum likelihood function:

\[
L(\theta | X_n, \bar{X}_n, \bar{\mu}_s) = -\frac{Nm \ln(2\pi)}{2} - \frac{N}{2} \ln |M| - \frac{1}{2} \ln \frac{\sum_{n=1}^{N} (s'M^{-1}\bar{X}_n)^2}{\sum_{n=1}^{N} (s'M^{-1}X_n)^2}.
\]

The maximum likelihood estimated value of \( \sigma_B^b \) and \( b \) can be derived by calculating the maximum value of equation (24), applying two-dimensional search method. The estimated values of \( \mu_\lambda \) and \( \sigma_\lambda \) can be obtained, using equations (22) and (23) with the estimated values of \( \sigma_B^b \) and \( b \), as previously calculated. The overall model can only describe the general trend of the degradation process. The case of the degradation process of each individual wheel is quite different, so the model parameters need to be updated accordingly, in order to accurately characterize the degradation process of any individual wheel. In this paper, the Bayesian formula is used to solve the posterior distribution of drift coefficients \( \lambda \).

**3.2. Online Updating of Model Parameters.** Assuming that at the mileage point \( s_m \), the model parameters of the \( n \)th sample are updated with the \( m \) elements of the monitoring data time series. Degradation increments are represented as follows:

\[
x_i = X_n(s_i) - X_n(s_{i-1}) = \mu(s_i - s_{i-1}) + \sigma_B(B(s_i) - B(s_{i-1})). \tag{25}
\]

The updated model parameter \( \lambda \) can be calculated based on given degradation data and using the conditional distribution \( P(\lambda | x_1, \ldots, x_m) \), as this is expressed according to the Bayesian theory:

\[
P(\lambda | x_1, \ldots, x_m) \propto f(x_1, \ldots, x_m | \lambda) \pi(\lambda). \tag{26}
\]

The following equation can be acquired based on the independent incremental properties of the Wiener process:

\[
f(x_1, x_2, \ldots, x_m | \lambda) = \frac{1}{\prod_{i=1}^{m} 2\pi \sigma_B^2 (s_i - s_{i-1})} \exp \left( -\frac{1}{2\sigma_B^2} \left( s_{i-1}^2 - 2\lambda (s_i^2 - s_{i-1}^2) \right) \right). \tag{27}
\]

According to equation (27), we can get

\[
P(\lambda | x_1, x_2, \ldots, x_m) \propto f(x_1, x_2, \ldots, x_m | \lambda) \pi(\lambda)
\]

\[
\propto \exp \left( -\frac{1}{2\sigma_B^2} \left( s_{i-1}^2 - 2\lambda (s_i^2 - s_{i-1}^2) \right) \right) \exp \left( -\frac{1}{2\sigma_B^2} \left( s_{i-1}^2 - 2\lambda (s_i^2 - s_{i-1}^2) \right) \right).
\]

Since the parameter \( \lambda \) is assumed to be normally distributed, the posterior distribution is also normal. Considering the monitoring data, the following relation is derived:

\[
P(\lambda | x_1, \ldots, x_m) = \frac{1}{\sqrt{2\pi\sigma^2_{\lambda, sm}}} \exp \left( -\frac{1}{2\sigma^2_{\lambda, sm}} \left( \lambda - \mu_{\lambda, sm} \right)^2 \right).
\]

The posterior expression of the parameter is obtained by comparing equations (28) and (29).

\[
\mu_{\lambda, sm} = \mu_B \sigma_B^2 + X(s_m)\sigma^2_{\lambda}, \tag{30}
\]

\[
\sigma^2_{\lambda, sm} = \frac{\sigma_B^4}{\sigma^2_{m} \sigma^2_{\lambda} + \sigma_B^2}. \tag{31}
\]

**3.3. Individual RUL Prediction.** Based on the concept of the first hitting time, the remaining life \( L_m \) of the component, at the mileage \( s_m \), is defined as

\[
L_m = \inf \left\{ t_m : x(s_m + t_m) \geq w \right\}, \tag{32}
\]

where \( w \) is the preset degradation threshold, which refers to the “maintenance limit,” as it is defined in this paper. As the current degradation \( X(s_m) \) is known, the PDF of the remaining life \( L_m \) can be obtained [10].
\[ f_{L_m}(l_m) = \frac{1}{\sqrt{2\pi l_m^2} \sigma^2 l_m} \left( \frac{(w - X(s_m) - \eta(l_m)) - b l_m (l_m + s_m)^{b-1}}{\sigma^2 l_m^2 + \sigma^2 l_m} \right) \times \exp \left[ \frac{((w - X(s_m) - \eta(l_m)) - b l_m)^2}{2(\sigma^2 l_m^2 + \sigma^2 l_m)} \right], \]  
\tag{33}

where
\[ \eta(l_m) = (l_m + s_m)^b - s_m^b \]  
\tag{34}

The updated model parameters \( \mu_{l_m}, \sigma_{l_m} \) are brought into equation (33) in order to obtain the individual PDF of the RUL, after the parameters update.

### 4. Centralized Maintenance Timing Prediction

The calculations, as described in the previous section, provide the PDF of the RUL of each individual wheel at the current moment. Since the differences between the various wheels are significant, they should be taken into account, in order to determine a reasonable time for centralized maintenance. This paper proposes that the time of centralized maintenance of train wheels is determined by the maintenance limit \( w \) and the ratio of limit-arriving \( p \). The ratio of limit-arriving \( p \) is the proportion of individuals who exceed the maintenance limit \( w \) in all wheels. It is suggested that the CCBM be carried out, when the proportion of individual wheels, with degradation greater than \( w \), reaches \( p \). Among others, the setting method for the \( w \) and \( p \) values should take into account the relevant technical specifications, equipment maintenance capability, equipment operation safety, maintenance economy, etc., matters that are not discussed in this paper. In order to ensure the safety, \( w \) should be selected less than the use limit, as specified in the respective technical specifications.

In the case where \( w \) and \( p \) are known, a maintenance timing prediction algorithm, based on the distribution of remaining life expectation, is proposed. Suppose, that at some point, \( N_1 \) out of \( N \) wheels, have reached \( w \), while \( N_2 \) wheels have not reached this limit. The remaining life of \( N_2 \) individual wheels was predicted, under the condition that the threshold is \( w \). The expectation of the remaining life probability density is taken as RUL prediction value.

\[ L_n = \int_{0}^{\infty} l_m f_{L_m}(l_m) dl_m, \]  
\tag{35}

where \( f_{L_m}(l_m) \) represents the PDF of the RUL of the \( n \)th individual wheel, after parameter updating.

The set of RUL prediction value of the \( N_2 \) individual wheels will be recorded as

\[ L = (L_1, L_2, L_3, \ldots, L_{N_2}). \]  
\tag{36}

The distribution function of \( L \) is required, in order to accurately find the remaining life, when the proportion of individual wheels whose deterioration reaches \( w \) is \( p \). Weibull distribution, having high applicability in product failure and life analysis, is used to fit the set \( L \). The distribution function and PDF of the set of individual remaining life prediction values are obtained as

\[ F(l) = 1 - e^{-(\mu_l^\alpha l)^\beta}, \]  
\[ f(l) = \frac{a}{m} (1 - m)^{a-1} e^{-(\mu_l^\alpha l)^\beta}. \]  
\tag{37}

The process of solving parameters \( a \) and \( m \) in Weibull distribution, by maximum likelihood method, is relatively simple and thus it is not discussed here.

The quantile \( I_R \) of the probability distribution function must be calculated, in order to find a remaining life value, when the percentage of degradation, exceeding the threshold \( w \) in all individuals at that mileage, reaches exactly \( p \).

\[ \frac{N_1 + a \cdot N_2}{N} = p, \]  
\tag{38}

\[ a = \frac{N \cdot p - N_1}{N_2}, \]

where \( I_R \) corresponds to the remaining life of the train, for centralized maintenance.

Let \( \alpha = F(I_R) \) provide the remaining life of centralized maintenance as follows:

\[ I_R = \frac{(-\ln (1 - a))^{1/\alpha}}{m}. \]  
\tag{39}

### 5. Model Verification

Tread wear is one of the most important characteristics of wheel degradation. As shown in Figure 1, the measuring point of tread wear is at the tread 70 mm from the end face of wheel flange. The distance between the current tread wear measurement point and the original profile measurement point is defined as tread wear.

The wheel wear monitoring data comes from the monitoring system called TWDS (Train Wheel Detection System), which is mounted on the railway line, as shown in Figure 2. TWDS system uses structural optical sensors to measure the profile size of wheels as the trains pass by. TWDS system is mainly composed of a laser source and a digital camera with filter plate in front of the lens. When the lowest point of the wheel set reaches the laser plane, the system issues a shooting command to shoot the image. Through the image processing algorithm, the contour size of the wheel is measured. The measurement error of TWDS system is ±0.3 mm.

Taking the 500-day monitoring data of tread wear, in a 54 fixed group of railway freight cars, as an example, based
on the prediction of individual RUL, we further forecast the
timing of wheel centralized maintenance. The 54 fixed group
railway freight cars are special trains for coal transportation
of the same type, with rated load of 80 tons. All the freight
cars are used under the same conditions and run on fixed
railway lines with an average daily mileage of 550 km.

According to the HP filtering algorithm, introduced in
Section 2, the online monitoring data are processed to ex-
tract the trend components of the degradation data. The filtering effect of three wheels wear data is shown in Figure 3.

It can be seen from the graph that the new sequence, after
HP filtering, provides a better sense of the trend in the
original sequence, while the fluctuation obviously decreases. The trend components obtained after filtering can better
model the degradation process.

The obtained trend component data are translated
according to (12) and substituted into the parameters esti-
imation algorithm of Section 3.2.1. The estimated values of
the parameters of the overall degradation process are ob-
tained as shown in Table 1.

From the overall model parameters, the overall degra-
dation process can be simulated, as demonstrated in Fig-
ure 4, along with some individual degradation paths.

It can be seen from Figure 4 that the degradation model,
as derived from the overall parameters, can reflect the
degradation trend, but there is still a certain gap between
the individual degradation paths and the overall degradation
path. Therefore, the model parameters should be updated
according to the data of the individual wheels. The individual
degradation model parameters and the PDF of the RUL,
after parameter updating, can be obtained by the algorithms
presented in Sections 3.2 and 3.3.

Each time a group of real monitoring data is obtained,
the model parameters are updated according to equations
(36) and (37). Tables 2 and 3 show the updates of model
parameters of four wheels at different mileage. For different
wheels, the mean values of drift parameters have their own
update paths, and the standard deviation of drift parameters
is related to monitoring mileage.

The degradation paths of these four wheels simulated
by real-time updating model parameters are shown in
Figure 5, where the red line represents the overall deg-
radation path, the blue line is the true degradation path of
various individual wheels, and the green lines represent
degradation paths simulated by real-time updating model
parameters of individual wheels. Each time a real mon-
itoring data is acquired, the parameters are updated. It can
be seen that the parameter updating method can better
modify the model parameters, to make them consistent
with the real state.

In order to verify the prediction effect of the proposed
method on the wheel tread wear degradation path, it was
compared with the modeling methods in other references.
The measurement data of the wheels with 160,000 kilometers
is used for model parameter estimation and parameters
update. The model parameters at 160,000 kilometers are
used to predict the wear at different mileages thereafter, and
the model prediction accuracy is measured by calculating the

| Table 1: Model parameters of overall degradation process. |
|-----------------|-----------------|-----------------|-----------------|-----------------|
| Parameters | $\mu_1$ | $\sigma_1$ | $b$ | $\sigma_b$ |
| Estimated values | $1.54 \times 10^{-6}$ | $1.26 \times 10^{-6}$ | 1.19 | $3.69 \times 10^{-3}$ |
Table 2: The mean value of drift parameters at different mileage after parameter update of four wheels.

| Monitoring mileage (km) | 5.81 × 10^4 | 1.10 × 10^5 | 1.51 × 10^5 | 1.91 × 10^5 | 2.27 × 10^5 | 2.70 × 10^5 |
|-------------------------|--------------|--------------|--------------|--------------|--------------|--------------|
| Wheel (a)               | 1.09 × 10^{-6} | 1.03 × 10^{-6} | 9.05 × 10^{-7} | 8.88 × 10^{-7} | 8.18 × 10^{-7} | 7.76 × 10^{-7} |
| Wheel (b)               | 1.55 × 10^{-6} | 1.22 × 10^{-6} | 9.97 × 10^{-7} | 8.76 × 10^{-7} | 8.14 × 10^{-7} | 7.97 × 10^{-7} |
| Wheel (c)               | 9.74 × 10^{-7} | 9.29 × 10^{-7} | 8.34 × 10^{-7} | 7.56 × 10^{-7} | 6.96 × 10^{-7} | 6.70 × 10^{-7} |
| Wheel (d)               | 6.62 × 10^{-7} | 8.43 × 10^{-7} | 8.38 × 10^{-7} | 8.32 × 10^{-7} | 8.06 × 10^{-7} | 7.96 × 10^{-7} |

Table 3: The standard deviation of drift parameters at different mileage.

| Monitoring mileage (km) | 106 | 200 | 276 | 348 | 415 | 492 |
|-------------------------|-----|-----|-----|-----|-----|-----|
| σ_{λ,w}                 | 2.20 × 10^{-7} | 1.07 × 10^{-7} | 7.00 × 10^{-8} | 5.32 × 10^{-8} | 4.20 × 10^{-8} | 3.47 × 10^{-8} |

Figure 4: Overall and individual degradation processes.

Figure 5: Continued.
RMSE (root-mean-square error) at different mileages. RMSE is defined as

$$\text{RMSE}_k = \sqrt{\frac{\sum_{i=1}^{n} (x_{i,k} - x_{i,k}^p)^2}{n}}. \quad (40)$$

RMSE$_k$ represents root-mean-square error of predicted value of wheel wear at mileage $s_k$. $x_{i,k}$ is the trend component value of the $i$th wheel tread wear monitoring data after HP filtering when the mileage is $s_k$, and $x_{i,k}^p$ is the predicted value at this time.

As shown in Figure 6, the red line is the prediction error using the method in [10]. This method used the overall model parameters to predict the degradation path of different individuals, and the model parameters are not updated. The blue line is the prediction error obtained with the method in [8]. Reference [8] adopted a linear Wiener process model and updated the model parameters using monitoring data based on the overall model parameters. The green line is the prediction error obtained by our proposed method. It can be seen that all the prediction errors of three methods gradually increase with the increase of the predicted mileage, but the method proposed in this paper has smaller prediction error than the other two methods.

After the model parameters are updated, the PDF of the remaining life of the wheel individuals can be obtained according to (39). For example, the RUL probability density value of one wheel at different mileage is shown in Figure 7. If the expectation value of remaining life is taken as the predicted value, then the predicted remaining life under different using mileage can be obtained. The predictions of two individual wheels are shown in Figure 8.

Figure 8 shows that the variance of the probability density of remaining life decreases as the updated data increase. At the initial stage of just a small volume of monitoring data, there is a certain deviation between the predicted value and the real value of remaining life. As monitoring data are accumulated, the model parameters are constantly updated to improve the prediction accuracy.

In order to evaluate the effect of this algorithm on the prediction of centralized maintenance time, the whole train is predicted to be under maintenance, after monitoring for 191000 km. The parameters of each individual wheel are updated, using the monitoring data of the first 191000 km, and the probability density of the RUL of each wheel, under the limit $w$, is obtained, as shown in Figure 9.

Using Section 4 method, a set of expected values of remaining life and the distribution functions of remaining life of each wheel are calculated and demonstrated in Figure 10.

In order to verify the accuracy of the proposed algorithm, the prediction of centralized maintenance mileage under different maintenance thresholds was carried out after being monitored for 191000 km. When the predicted maintenance mileage was reached, the accuracy of the algorithm was judged by comparing the real arrival limit ratio to the set value. For example, $w$ is set to 4 mm; the predicted centralized maintenance mileage is set to 25700 km and 82700 km, while $p$ is set to 40% and 50%, respectively. When the recommended maintenance mileage is reached, the true arrival limit ratio is 38.3% and 48.9%, respectively. Similarly, $w$ can be set to 3.5 mm. The experimental prediction results are as shown in Table 4.

From the results, we can see that when the scheduled maintenance mileage is reached, there is a gap between the true arrival limit ratio and the expected value. And the average error of the four test results is 1.28%. The error
Figure 6: Prediction effect comparison.

Figure 7: The RUL probability density value of one wheel at different mileage.

Figure 8: RUL prediction results. (a) and (b) are the remaining life prediction effects of the two wheels, respectively.
comes from the randomness of wheel wear degradation path. The degradation model obtained from the monitoring data of the first 191000 km may be different from the real degradation path behind. Therefore, there are some differences between the predicted and actual values of remaining life. Such prediction accuracy can meet the requirements for the organization and arrangement of railway freight car maintenance. According to the experimental results, it is evident that by setting thresholds \( w \) and \( p \), the model can better predict the remaining life of CCBM of train wheels.

6. Conclusions

Predicting the time of centralized maintenance of wheels based on the degradation is of great significance for realizing condition-based maintenance of railway freight cars. This paper presents a prediction algorithm for CCBM timing of freight train wheels, based on the nonlinear Wiener process and, more specifically, on the basis of solving the problem of RUL prediction for individual wheels.

In order to solve the problem of fluctuations in the directly collected monitoring data, this paper uses the HP filtering algorithm to extract the trend components in the monitoring data. Using trend components for modeling provides a better fit to the real degradation path. According to the nonlinearity of wheel degradation, a nonlinear Wiener process model is constructed to describe the wheel tread wear, with model parameters that are solved by maximum likelihood estimation method. In order to make the model fit the degradation paths of different individuals better, based on the Bayesian formula, the updating algorithm of model parameters is deduced, and the real-time updating of individual model parameters is realized, providing the PDF of individual RUL. A prediction method of centralized maintenance timing is proposed based on two set thresholds: “maintenance limit” and “the ratio of limit-arriving.” The remaining life of each wheel can be obtained by setting “maintenance limit,” and the predicted mileage for CCBM can be obtained by setting “the ratio of limit-arriving.” The accuracy of the algorithm is verified by using 500-day monitoring data from a 54 fixed group of railway freight cars. According to the test results, it is considered the fact that the prediction accuracy of this algorithm can meet the requirements of application.

The method can be extended to other equipment with multiple parts of the same type and under the same working conditions, to determine the time for centralized maintenance of these parts, reducing thus downtime, increasing operational lifetime, and improving operation efficiency.

Data Availability

The data used to support the findings of this study are available from the corresponding author upon reasonable request.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

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