INTERACTION OF THE $\Theta^+$ WITH THE NUCLEAR MEDIUM

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We study the selfenergy of the $\Theta^+$ pentaquark in nuclei associated with two types of interaction: the $KN$ decay of the $\Theta^+$ and two meson baryon decay channels of the $\Theta^+$. Whereas the potential related to the first source is quite weak, the second kind of interaction produces a large and attractive potential that could lead to the existence of $\Theta^+$ nuclear bound states.

1. Introduction

The discovery of the $\Theta^+$ resonance\(^1\) opens the possibility of forming exotic $\Theta^+$ hypernuclei, which, like in the case of negative strangeness, could provide complementary information on the structure and properties of the pentaquark to that obtained from elementary reactions.

In Ref. 2, a schematic model for quark-pair interaction with nucleons was used to describe the $\Theta^+$, which suggested that $\Theta^+$ hypernuclei, stable against strong decay, might exist. Later, in Ref. 3, the $\Theta^+$ selfenergy in the nuclei is calculated, based on the $\Theta^+KN$ interaction. The results show a selfenergy too weak to bind the $\Theta^+$ in nuclei.

In this talk, we present some selected results from Ref. 4. There, we redo the calculations of Ref. 3 modifying some of the assumptions. The results are similar to those of Ref. 3 and a quite small potential is obtained from this source. Additionally, we consider other new selfenergy pieces related to the coupling of the $\Theta^+$ resonance to a baryon and two mesons under certain assumptions. We find that the in-medium renormalization of the two meson cloud of the $\Theta^+$ could lead to a sizable attraction, enough to produce bound and narrow $\Theta^+$ states in nuclei.

The coupling of the $\Theta^+$ to two mesons and a nucleon is studied using
a $SU(3)$ symmetric Lagrangian\(^5\), constructed to account for the coupling of the antidecuplet to a baryon and two pseudoscalar mesons. With this Lagrangian an attractive selfenergy is obtained for all the members of the antidecuplet coming from the two meson cloud.

2. The $\Theta^+$ selfenergy in nuclear matter

2.1. Selfenergy from the $KN$ decay channel

The $\Theta^+$ selfenergy diagram in vacuum is shown in Fig. 1. For the $L = 0$ case, the free $\Theta^+$ selfenergy from this diagram is given by

$$-i\Sigma_{KN} = 2 \int \frac{d^4q}{(2\pi)^4} \frac{M}{p^0 - q^0 - E_N + i\epsilon} \frac{1}{q^2 - m_K^2 + i\epsilon}, \quad (1)$$

where $M$ is the nucleon mass, $E_N(k) = \sqrt{M^2 + \vec{k}^2}$. The results for $L = 1$ are obtained by the substitution $g_{K+n}^2 \rightarrow \bar{g}_{K+n}^2 \bar{q}^2$.

Next, we evaluate the $\Theta^+$ selfenergy in infinite nuclear matter with density $\rho$. The nucleon propagator is changed in the following way,

$$\frac{1}{p^0 - q^0 - E_N + i\epsilon} \rightarrow \frac{1 - n(\vec{p} - \vec{q})}{p^0 - q^0 - E_N + i\epsilon} + \frac{n(\vec{p} - \vec{q})}{p^0 - q^0 - E_N - i\epsilon}, \quad (2)$$

where $n(\vec{k})$ is the occupation number. The vacuum kaon propagator is replaced by the in-medium one,

$$\frac{1}{q^2 - m_K^2 + i\epsilon} \rightarrow \frac{1}{q^2 - m_K^2 - \Pi_K(q, \rho)}, \quad (3)$$

Figure 1. $\Theta^+$ selfenergy diagram related to the $KN$ decay channel.
where $\Pi_K(q^0, |q|, \rho)$ is the kaon selfenergy. The $s-$wave part of this selfenergy is well approximated by\(^\text{\textsuperscript{6,7}}\) $\Pi_K^{(s)}(\rho) = 0.13 m_K^2 \rho / \rho_0$ , where $\rho_0$ is the normal nuclear density. The $p-$wave part is taken from the model of Ref. 8, which accounts for $\Lambda h$, $\Sigma h$ and $\Sigma^*(1385)h$ excitations. After some approximations, the $q^0$ integration leads to

$$\Sigma_{KN}(p^0, \vec{p}; \rho) = \frac{M_0 \Gamma}{M q_{\text{on}}} \int \frac{d^3 q}{(2\pi)^2} \frac{M}{E_N} F_L(q) \frac{1}{2\tilde{\omega}(q)} \left( p^0 - \tilde{\omega}(q) - E_N - V_N + i\epsilon \right)$$

with $q^0 = p^0 - E_N(\vec{p} - \vec{q}) - V_N, V_N = -k_F^2/2M$, $F_0 = 1$, $F_1 = q^2/4m$ and $q_{\text{on}}$ the on shell kaon momentum. In Eq. 4 we have taken into account the nucleon binding by using the Thomas-Fermi potential, $V_N = -k_F^2(r)/2M$. For the calculations, we have taken an average value of the momentum of the $\Theta^+$ in eventual bound states of $p = 200$ MeV, similar to that of bound nucleons in nuclei.

We show the results in Figs. 2, 3, where we assume that $\Gamma = 15$ MeV. As it is obvious from Eq. 4, the in medium selfenergy is proportional to the vacuum width, thus the results should be scaled when the width is better determined. In any case, even if $\Gamma = 15$ MeV in vacuum, inside the nucleus the width is small, basically because of the Pauli blocking. For 20 MeV of $\Theta^+$ binding, the width would go down from 15 MeV to less than 6 MeV at $\rho = \rho_0$. This width could be smaller than the separation between different bound levels. As for the real part of the $\Theta^+$ selfenergy, we find, in qualitative agreement with Ref. 3, that the $\Theta^+$ potential in the medium is small, of the order of 1 MeV or less and not enough to bind $\Theta^+$ in nuclei.

2.2. The $\Theta^+$ selfenergy tied to the two-meson cloud

In this section we consider contributions to the $\Theta^+$ selfenergy from diagrams in which the $\Theta^+$ couples to a nucleon and two mesons, like the one in Fig. 4. There is no direct information on these couplings since the $\Theta^+$ mass is below the two-meson decay threshold.

To proceed, we do several assumptions. First, the $\Theta^+$ is assumed to have $J^P = 1/2^+$ associated to an $SU(3)$ antidecuplet\(^\text{\textsuperscript{9}}\). Also the $N^*(1710)$ is supposed to couple strongly to the same antidecuplet.

From the PDG data on $N^*(1710)$ decays we determine the couplings to the two-meson channels, and using $SU(3)$ symmetry obtain the corre-
Figure 2. Imaginary part of the $\Theta^+$ selfenergy associated to the $KN$ decay channel for $L = 0$.

Figure 3. Imaginary part of the $\Theta^+$ selfenergy associated to the $KN$ decay channel for $L = 1$. 
Figure 4. Two-meson $\Theta^+$ selfenergy diagram.

Corresponding couplings for the $\Theta^+$ pentaquark.

In order to account for the $N^*(1710)$ decay into $N(\pi\pi, p -\text{wave}, I = 1)$ and $N(\pi\pi, s -\text{wave}, I = 0)$ we use the following lagrangians

$$L_1 = ig_{i0}^\epsilon\epsilon^{ilm}T_{ijk}\gamma^\mu B_j^i(V_\mu)_m^k,$$  

with

$$V_\mu = \frac{1}{4f^2}(\phi\partial_\mu\phi - \partial_\mu\phi\phi),$$

where $f = 93$ MeV is the pion decay constant and $T_{ijl}, B_j^i, \phi_m^k$ $SU(3)$ tensors which account for the antidecuplet states, the octet of $\frac{1}{2}^+$ baryons and the octet of $0^-$ mesons, respectively. The second term is given by

$$L_2 = \frac{1}{2\tilde{g}_{i0}}\epsilon^{ilm}T_{ijk}(\phi\cdot\phi)^j_iB_m^k,$$  

which couples two mesons in $L = 0$ to the antidecuplet and the baryon and they are in $I = 0$ for the case of two pions. From the Lagrangian terms of Eqs. (5, 7) we obtain the transition amplitudes $N^* \rightarrow \pi\pi N$. Taking the central values from the PDG$^{10}$ for the $N^*(1710) \rightarrow N(\pi\pi, p -\text{wave}, I = 1)$ and for the $N^*(1710) \rightarrow N(\pi\pi, s -\text{wave}, I = 0)$, the resulting coupling constants are $g_{i0} = 0.72$ and $\tilde{g}_{i0} = 1.9$.

The implementation of the medium effects is done by including the medium selfenergy of the kaon and modifying the nucleon propagator, as before. On the other hand, for the pion we modify the propagator using the $p -\text{wave}$ selfenergy from $ph$ and $\Delta h$ excitations$^{11,12,13}$. Once the $\Theta^+$ selfenergy at a density $\rho$ is evaluated, the optical potential felt by the $\Theta^+$ in the medium is obtained by subtracting the free $\Theta^+$ selfenergy.

We should also note that while the $\Theta^+ \rightarrow K\pi N$ decay is forbidden, in the medium the $\pi$ can lead to a $ph$ excitation and this opens a new decay
channel $\Theta^+ N \rightarrow NNK$, which is open down to 1432 MeV, quite below the free $\Theta^+$ mass. We have shown\cite{footnote1} that the width from this channel is also very small, but should the $\Theta^+$ free width be of the order of 1 MeV as suggested in Refs. 14, 15, the new decay mode would make the width in the medium larger than the free width.

We present the results in Figs. 5 and 6. We can see there that the potential for $\rho = \rho_0$ is sizable and attractive and goes from $-70$ to $-120$ MeV depending on the cut-off used in the selfenergy evaluation.

![Real part of the two-meson contribution to the $\Theta^+$ selfenergy at $\rho = \rho_0$.](image)

Even with the quoted large uncertainties we conclude that there could be a sizable attraction of the order of magnitude of 50-100 MeV at normal nuclear density, which is more than enough to bind the $\Theta^+$ in any nucleus. In Fig. 6 we show the imaginary part of the $\Theta^+$ selfenergy related to the two-meson decay mechanism. We find that $\Gamma$ would be smaller than 5 MeV for bound states with a binding of ~20 MeV and negligible for binding energies of ~40 MeV or higher. This, together with the small widths associated to the $KN$ decay channel, would lead to $\Theta^+$ widths below 8 MeV, assuming a free width of 15 MeV, and much lower if the $\Theta^+$ free width is of the order of 1 MeV. In any case, for most nuclei, this width would be smaller than the separation of the deep levels\cite{footnote2}.
Figure 6. Imaginary part of the two-meson contribution to the $\Theta^+$ selfenergy at $\rho = \rho_0$.

3. Conclusions

The selfenergy of the $\Theta^+$ in the nuclear medium associated to the $KN$ decay channels is quite weak, even assuming a large free width of around 15 MeV for the $\Theta^+$. However, there is a large attractive $\Theta^+$ potential in the nucleus associated to the two meson cloud of the antidecuplet. New decay channels open for the $\Theta^+$ in the medium, $\Theta^+N \rightarrow NNK$, but the width from this new channel, together with the one from $KN$ decay, is still small compared to the separation of the bound levels of the $\Theta^+$ in light and intermediate nuclei.

This conclusions depend on several assumptions, namely: the $\Theta^+$ is assumed to be $1/2^+$ associated to an $SU(3)$ antidecuplet, the $N^*(1710)$ is supposed to couple largely to the same antidecuplet, two specific Lagrangians have been chosen, the average value of the $N^*(1710)$ width and the partial decay ratios, which experimentally have large uncertainties, have been taken to fix the couplings. It is clear that with all these assumptions one must accept a large uncertainty in the results. So we can not be precise on the binding energies of the $\Theta^+$. However, the order of magnitude obtained for the potential is such that even with a wide margin of uncertainty, the conclusion that there would be bound states is quite safe. In fact, with potentials with a strength of 20 MeV or less one would already get bound
Indeed, since the strength of the real part and the imaginary part from the $NKph$ decay are driven by the same coupling, a reduction on the strength of the potential would also lead to reduced widths such that the principle that the widths are reasonably smaller than the separation between levels still holds.

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