Basic Constituents of Matter and their Interactions – A Progress Report

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Our concept of the basic constituents of matter has undergone two revolutionary changes – from atoms to proton & neutron and then onto quarks & leptons. Indeed all these quarks and leptons have been seen by now along with the carriers of their interactions, the gauge bosons. But the story is not complete yet. A consistent theory of mass requires the presence of Higgs bosons along with SUSY particles, which are yet to be seen. This is a turn of the century account of what has been achieved so far and what lies ahead.
Introduction

Our concept of the basic constituents of matter has undergone two revolutionary changes during this century. The first was the Rutherford scattering experiment of 1911, bombarding $\alpha$ particles on the Gold atom. While most of them passed through straight, occasionally a few were deflected at very large angles. This was like shooting bullets at a hay stack and finding that occasionally one would be deflected at a large angle and hit a bystander or in Rutherford’s own words “deflected back and hit you on the head”! This would mean that there is a hard compact object inside the hay stack. Likewise the Rutherford scattering experiment showed the atom to consist of a hard compact nucleus, surrounded by a cloud of electrons. The nucleus was found later to be made up of protons and neutrons.

The second was the electron-proton scattering experiment of 1968 at the Stanford Linear Accelerator Centre, which was awarded the Nobel Prize in 1990. This was essentially a repeat of the Rutherford scattering type experiment, but at a much higher energy. The result was also similar as illustrated below. It was again clear from the pattern of large angle scattering that the proton is itself made up of three compact objects called quarks.

![Diagram](image)

Fig. 1. The SLAC electron-proton scattering experiment, revealing the quark structure of the proton.

We now know from many such experiments that the nuclear particles (proton, neutron and mesons, which are collectively called hadrons) are all made up of quarks – i.e. they are all quark atoms.

The main difference between the two experiments comes from the fact that, while the dimension of the atom is typically $1A^o = 10^{-10}m$ that of the proton is about $1fm$ (fermi or femtometer) $= 10^{-15}m$. It follows from the Uncertainty Principle,

$$\Delta E \cdot \Delta x > \hbar c \sim 0.2 \text{ GeV.fm},$$  \hspace{1cm} (1)

that the smaller the distance you want to probe the higher must be the beam energy. Thus probing inside the proton ($x \ll 1fm$) requires a beam energy $E \gg 1 \text{ GeV}(10^9 \text{ eV})$, which is the energy acquired by the electron on passing through a billion (Gega) volts. It is this multi-GeV acceleration technique that accounts for the half a century gap between the two experiments.
Following the standard practice in this field we shall be using the so-called natural units,

\[ h = c = 1, \]  

so that the mass of particle is same as its rest mass energy \((mc^2)\). The proton and the electron masses are

\[ m_p \sim 1 \text{ GeV}, \ m_e \sim 1/2 \text{ MeV}. \]  

The GeV is commonly used as the basic unit of mass, energy and momentum.

**The Standard Model:**

As per our present understanding the basic constituents of matter are a dozen of spin-1/2 particles (fermions) along with their antiparticles. These are the three pairs of leptons (electron, muon, tau and their associated neutrinos) and three pairs of quarks (up, down, strange, charm, bottom and top) as shown below. The masses of the heaviest members are shown parenthetically in GeV units.

| Basic Constituents of Matter |
|-----------------------------|
| **Leptons**                 |
| \( \nu_e \) \quad \nu_\mu \quad \nu_\tau \quad 0 | \( e \) \quad \mu \quad \tau(2) \quad -1 |
| **Quark**                   |
| \( u \) \quad c \quad \bar{t}(175) \quad 2/3 |
| \( d \) \quad s \quad b(5) \quad -1/3 |

The members of each pair differ by 1 unit of electric charge as shown in the last column – i.e. charge 0 and -1 for the neutrinos and charged leptons and 2/3 and -1/3 for the upper and lower quarks. This is relevant for their weak interaction. Apart from this electric charge the quarks also possess a new kind of charge called colour charge. This is relevant for their strong interaction, which binds them together inside the nuclear particles (hadrons).

There are four basic interactions among these particles – strong, electromagnetic, weak and gravitational. Apart from gravitation, which is too weak to have any practical effect on their interaction, the other three are all gauge interactions. They are all mediated by spin 1 (vector) particles called gauge bosons, whose interactions are completely specified by the corresponding gauge groups.
Each of the three interactions is represented below by the corresponding Feynman diagram, which is simply a space-time picture of scattering with time running vertically upwards. The 4-momentum squared transferred between the particles is denoted by $Q^2$, which is a Lorentz invariant quantity. The corresponding scattering amplitudes, representing the square-roots of the scattering cross-sections (rates), are denoted below each diagram.

![Feynman Diagrams](image)

Fig. 2. The scattering diagrams and amplitudes for (a) strong, (b) electromagnetic and (c) weak interactions.

The strong interaction between quarks is mediated by the exchange of a massless vector boson called gluon. This is analogous to the photon, which mediates the electromagnetic interaction between charged particles (quarks or charged leptons). The gluon coupling is proportional to the colour charge just like the photon coupling is proportional to the electric charge. The constant of proportionality for the strong interaction is denoted by $\alpha_s$ in analogy with the fine structure constant $\alpha$ in the EM case. And the theory of strong interaction is called quantum chromodynamics (QCD) in analogy with the quantum electrodynamics (QED). The major difference of QCD with respect to the QED arises from the nonabelian nature of its gauge group, $SU(3)$. This essentially means that unlike the electric charge the colour charge can take three possible directions in an abstract space. These are rather whimsically labelled red, blue and yellow as illustrated below. Of course the cancellation of
the colour charges of quarks ensure that the nuclear particles (hadrons) composed of them are colour neutral just like the atoms are electrically neutral.

![Diagram of quark structure of proton, neutron, π and K mesons along with their colour charges](image)

Fig. 3. The quark structure of proton, neutron, π and K mesons along with their colour charges (the bar denotes antiparticles).

A dramatic consequence of the nonabelian nature of the QCD is that the gluons themselves carry colour charge and hence have self-interaction unlike the photons, which have no electric charge and hence no self-interaction. Because of the gluon self-interaction the colour lines of forces between the quarks are squeezed into a tube as illustrated below.

![Diagram of squeezed (1-dim.) lines of force between colour charges contrasted with the isotropic (3-dim.) lines of force between electric charges](image)

Fig. 4. The squeezed (1-dim.) lines of force between colour charges contrasted with the isotropic (3-dim.) lines of force between electric charges.

Consequently the number of colour lines of force intercepted and the resulting force is constant, i.e. the potential increases linearly with distance

\[ V_s = \alpha_s r. \quad (4) \]

Thus the quarks are perpetually confined inside the hadrons as it would cost an infinite amount of energy to split them apart. In contrast the isotropic distribution of the electric lines of force implies that the number intercepted and hence the resulting force decreases like \(1/r^2\), i.e.

\[ V_E = \frac{\alpha}{r}. \quad (5) \]

Finally the weak interaction is mediated by massive vector particles, the charged \(W^\pm\) and the neutral \(Z^0\) bosons, which couple to all the quarks and leptons. The former couples to each pair of quarks and leptons listed above with a universal coupling strength \(\alpha_W\), since they all belong to the doublet representation of \(SU(2)\) (i.e. carry the same gauge charge). This is illustrated in Fig. 2c. Note the \(W\) mass term appearing in the boson propagator,
represented by the denominator of the amplitude. The Fourier transform of this quantity gives the weak potential

$$V_W = \frac{\alpha_W}{r^2} e^{-rM_W},$$

(6)
i.e. the weak interaction is restricted to a short range $\sim 1/M_W$. One can understand this easily from the uncertainty principle (1), since the exchange of a massive $W$ boson implies a transient energy nonconservation $\Delta E = M_W c^2$, corresponding to a range $\Delta x = \hbar/M_W c$. Fig. 2c represents the charged current neutrino scattering processes

$$\nu_e \mu^- \rightarrow e^- \nu_\mu, \quad \nu_e d \rightarrow e^- u$$

(7)
as well as the corresponding decay processes

$$\mu^- \rightarrow \nu_\mu e^- \bar{\nu}_e, \quad d \rightarrow u e^- \bar{\nu}_e$$

(8)
since an incoming particle is equivalent to an outgoing antiparticle. The first process is the familiar muon beta decay while the second is the basic process underlying neutron beta decay.

The weak and the electromagnetic interactions have been successfully unified into a $SU(2) \times U(1)$ gauge theory by Glashow, Salam and Weinberg for which they were awarded the 1979 Nobel Prize. According to this theory the weak and the electromagnetic interactions have similar coupling strengths, i.e.

$$\alpha_W = \alpha / \sin^2 \theta_W,$$

(9)
where $\sin^2 \theta_W$ represents the mixing between the two gauge groups. This parameter can be determined from the relative rates of charged and neutral current ($W^\pm$ and $Z^0$ exchange) neutrino scattering, giving

$$\sin^2 \theta_W \simeq 1/4, \text{i.e. } \alpha_W \simeq 1/32.$$  

(10)
Thus the weak interaction is in fact somewhat stronger than the electromagnetic. Its relative weakness in low energy processes like $\mu$ decay ($Q^2 \ll M_W^2$) is due to the additional $M_W^2$ term in the boson propagator, reflecting its short range. This suppression is a transient phenomenon, which goes away at a high energy scale ($Q^2 \gg M_W^2$), where the rates of weak and EM interactions become similar. A direct prediction of this theory is the mass of the $W$ boson from the $\mu$ decay amplitude $\alpha_W/M_W^2$ or the equivalent Fermi coupling $G_F$. The observed $\mu$ decay rate gives

$$G_F \equiv \frac{\pi \alpha_W}{\sqrt{2} M_W^2} = 1.17 \times 10^{-5} \text{ GeV}^{-2}.$$  

(11)
From (10) and (11) we get

$$M_W \simeq 80 \text{ GeV and } M_Z = M_W / \cos \theta_W \simeq 91 \text{ GeV}.$$  

(12)
Discovery of the Fundamental Particles

As mentioned earlier, the up and down quarks are the constituents of proton and neutron. Together with the electron they constitute all the visible matter around us. The heavier quarks and charged leptons all decay into the lighter ones via interactions analogous to the muon decay. So they are not freely occurring in nature. But they can be produced in laboratory or cosmic ray experiments. The muon and the strange quark were discovered in cosmic ray experiments in the late forties, the latter in the form of $K$ meson. Next to come were the neutrinos. Although practically massless and stable the neutrinos are hard to detect because they interact only weakly with matter. The $\nu_e$ was discovered in atomic reactor experiment in 1956, for which Reines got the Nobel Prize in 1995. The $\nu_\mu$ was discovered in the Brookhaven proton synchrotron in 1962, for which Lederman and Steinberger got the Nobel Prize in 1988. The first cosmic ray observation of neutrino came in 1965, when the $\nu_\mu$ was detected in the Kolar Gold Field experiment.

The rest of the particles have all been discovered during the last 25 years, thanks to the advent of the electron-positron and the antiproton-proton colliders. First came the windfall of the seventies with a quick succession of discoveries mainly at the $e^+e^-$ colliders: charm quark (1974), Tau lepton (1975), bottom quark (1977) and the gluon (1979). This was followed by the discovery of $W$ and $Z$ bosons (1983) and finally the top quark (1995) at the $\bar{p}p$ colliders. In view of the crucial role played by the colliders in these discoveries a brief discussion on them is in order.

The $e^+e^-$ and $\bar{p}p$ Colliders

These are synchrotron machines, where the particle and antiparticle beams are simultaneously accelerated in the same vacuum pipe using the same set of bending magnets and accelerating field. Thanks to their equal mass and opposite charge the particle and antiparticle beams go around in identical orbits on top of one another throughout the course of acceleration (Fig. 5a). On completion of the acceleration mode, the two beams are made to collide almost head on by flipping a magnetic switch (Fig. 5b). In this mode the two beams continue to collide repeatedly at the collision points. Indeed, the machine spends a major part of its running time in the collision mode.

![Fig. 5. The $e^+e^-$ (or $\bar{p}p$) collider.](image)

The advantage of a collider over a fixed target machine is an enormous gain of the Lorentz invariant energy $s$ (CM energy squared) at practically no extra cost. For, a collider (Fig.
6a) and a fixed target machine (Fig. 6b) correspond respectively to

\[ s = (2E)^2 \quad \text{and} \quad s = 2mE \]  \hspace{1cm} (13)

where \( E \) denotes the beam energy and \( m \) the target particle mass.

Thus a collider of beam energy \( E \) is equivalent to a fixed target machine of beam energy

\[ E' = 2E^2/m. \]  \hspace{1cm} (14)

Thus the TEVATRON \( \bar{p}p \) collider beam energy of 1 TeV (1000 GeV) is equivalent to a fixed target machine beam of 2000 TeV, since \( m_p \approx 1 \) GeV. For a \( e^+e^- \) collider, of course, the energy gain is another factor of a 1000 higher due to the smaller electron mass.

\( \bar{p}p \) vs. \( e^+e^- \) Collider – A proton (antiproton) is nothing but a beam of quarks (anti-quarks) and gluons. Thus the \( \bar{p}p \) collision can be used to study \( \bar{q}q \) interaction which can probe the same physics as the \( e^+e^- \) interaction (Fig. 7).

\[ \frac{s_{\bar{p}p}}{2} \sim \frac{s_{\bar{q}q}}{2} \sim (1/6)s_{\bar{p}p}^{1/2}, \]  \hspace{1cm} (15)

i.e. the energy of the \( \bar{p}p \) collider must be about 6 times as large as the \( e^+e^- \) collider to give the same basic interaction energy. This is a small price to pay, however, considering the

\footnote{The proton energy momentum is shared about equally between the quarks and the gluons; and since there are 3 quarks (\( uud \)), each one has a share of \( \sim 1/6 \) on the average.}
immensely higher synchrotron radiation loss for the $e^+e^-$ collider. The amount of synchrotron radiation loss per turn is

$$\Delta E \simeq \frac{4\pi e^2 E^4}{3 m^4 \rho},$$

(16)

where $e$, $m$ and $E$ are the particle charge, mass and energy and $\rho$ is the radius of the ring. This quantity is much larger for the $e^+e^-$ collider due to the small electron mass, which would mean a colossal energy loss and radiation damage. These are reduced by increasing the radius of the ring, resulting in a higher construction cost. The point is best illustrated by the following comparison between the $\bar{p}p$ collider (SPPS) and the $e^+e^-$ collider (LEP) at CERN, both designed for producing the $Z$ boson.

| Machine | Energy ($s^{1/2}$) | Radius ($\rho$) | cost       |
|---------|-------------------|-----------------|------------|
| SPPS ($\bar{p}p$) | 600 GeV | 1 Km | $300 million |
| LEP ($e^+e^-$) | 100 GeV | 5 Km | $1000 million |

Of the total construction cost of the $\bar{p}p$ collider, $200 million went into building the fixed target super proton synchrotron (SPS) and only about a $100 million in converting it into a $\bar{p}p$ collider.

On the other hand, the $e^+e^-$ collider has an enormous advantage over $\bar{p}p$ as a tool for precise and detailed investigation. This is because one can tune the $e^+e^-$ energy to a desired particle mass (e.g. $M_Z$), which cannot evidently be done with the quark-antiquark energy. Thus one could produce about a million of $Z \rightarrow e^+e^-$ events per year at LEP while the annual yield of such events at the SPPS was only about a few dozen. Moreover the $e^+e^-$ collider signals are far cleaner than the $\bar{p}p$ collider since one has to contend with the debris from the remaining quarks and gluons in the latter case.

In short the $\bar{p}p$ collider is more suitable for surveying a new energy domain, being comparatively less expensive, while the $e^+e^-$ is better suited for intensive follow-up investigation. The following table gives a list of the important colliders starting from the early seventies.

First came the Stanford $e^+e^-$ collider (SPEAR) with a CM energy similar to that of the fixed-target proton synchrotron at Brookhaven. In fact the charm quark was simultaneously discovered at both these machines in 1974, for which Richter and Ting got the 1975 Nobel Prize. The tau lepton was also discovered at SPEAR the following year, for which Pearl got the Nobel Prize in 1995. The bottom quark was discovered in the fixed-target proton synchrotron at Fermilab in 1977; and soon followed by the study of its detailed properties at the $e^+e^-$ colliders at Hamburg (DORIS) and Cornell (CESR). Then the construction of a more energetic $e^+e^-$ collider (PETRA) at Hamburg resulted in the discovery of gluon in 1979.
| Machine | Location | Beam | Energy (GeV) | Radius | Highlight |
|---------|----------|------|-------------|--------|-----------|
| SPEAR  | Stanford | $e^+e^-$ | 3 + 3       |        | $c, \tau$ |
| DORIS  | Hamburg | $''$ | 5 + 5       |        | $b$       |
| CESR   | Cornell | $''$ | 8 + 8       | 125 m  | $''$      |
| PEP    | Stanford | $''$ | 18 + 18     |        | $-$       |
| PETRA  | Hamburg | $''$ | 22 + 22     | 300 m  | $g$       |
| TRISTAN| Japan    | $''$ | 30 + 30     |        | $-$       |
| SPPS   | CERN    | $\bar{p}p$ | 300 + 300 | 1 Km   | $W, Z$   |
| TEVATRON| Fermilab| $''$ | 1000 + 1000 |        | $t$       |
| SLC    | Stanford | $e^+e^-$ | 50 + 50     | $-$    | $Z$       |
| LEP-I  | CERN    | $''$ | $''$        | 5 Km   | $Z$       |
| (LEP-II)| $''$ | 100 + 100 | $''$       | $W$    |           |
| HERA   | Hamburg | $ep$ | 30 + 800    | 1 Km   | $-$       |
| LHC    | CERN    | $pp$ | 7,000 + 7,000 | 5 Km  | Higgs? SUSY? |
| (LEP Tunnel) | CERN | $pp$ | 7,000 + 7,000 | 5 Km  | Higgs? SUSY? |
| NLC    | ?       | $e^+e^-$ | 500 + 500?  | $-$    | $''$      |

The charm, bottom and tau production via Fig. 7b results in back to back pairs of particle and antiparticle, as illustrated in Fig. 8. The typical life time of these particles are $\sim 10^{-12}$ sec, corresponding to a range of $c\tau \sim 300\mu m(0.3 mm)$ at relativistic energies. This is adequate to identify these particles before their decay using high resolution silicon vertex detectors. The production of gluon can be inferred from the observation of 3-jet events as one of the quark-antiquark pair produced via Fig. 7b radiates an energetic gluon. How are the coloured quarks and gluons able to come out of the confinement region? This is possible because quantum mechanical vacuum contains many quarks and gluons, with the uncertainty principle accounting for their mass and kinetic energy. Each of the produced particles picks up some of these extra quarks and gluons to come out as a colourless cluster of hadrons, sharing its original momentum. There is only a limited momentum spread due
to the intrinsic momenta of these ‘vacuum particles’ – i.e. $\Delta p \sim 0.2 \text{ GeV}$ corresponding to a confinement range of $\sim 1 \text{ fm}$. Thus the produced quarks and gluons can each be recognised as a narrow and energetic jet of hadrons.

Fig. 8.

Next came the CERN $\bar{p}p$ collider, with a higher CM energy of 100 GeV, even after accounting for the above mentioned factor of 6. This resulted in the discovery of $W$ and $Z$ bosons in 1983, for which Rubbia got the Nobel Prize the following year. The construction of the Fermilab $\bar{p}p$ collider (TEVATRON) increased this effective CM energy further to $\sim 300 \text{ GeV}$ and resulted in the discovery of the top quark in 1995. Being very heavy, these particles decay almost at the instant of their creation. Nonetheless they can be recognised by the unmistakable imprints they leave behind in their decay products, as illustrated above for the $W$ and $Z$ decays (Fig. 8). The huge energy released in the $W \to \ell \nu$ decay often results in a hard lepton, carrying a transverse momentum ($p_T$) $\sim M_W/2 \sim 40 \text{ GeV}$, with an apparent $p_T$-inbalance (missing-$p_T$) as the neutrino escapes detection. Similarly the $Z \to \ell^+\ell^-$ decay results in a pair of azimuthally back-to-back hard leptons, carrying a $p_T \sim M_Z/2 \sim 45 \text{ GeV}$ each. The top quark event is more complex, since they are produced in pair and each undergoes a 3-body decay via $W$ as shown in Fig. 9. This results in a hard isolated lepton and a number of hard quark jets. The lepton isolation and the jet hardness criteria have been successfully exploited to extract the top quark signal from the background.
The last batch of machines are the Stanford linear collider (SLC) and the large electron-positron collider (LEP). They have helped to bridge the energy gap between the $e^+e^-$ and the $\bar{p}p$ colliders and made a detailed study of $Z$ and $W$ bosons. Moreover a $ep$ collider (HERA) at Hamburg is probing deeper into the structure of the proton by extending the $ep$ scattering experiment of Fig. 1 to higher energies.

Finally a large hadron collider (LHC), being built in the LEP tunnel at a cost of 3-4 billion dollars, is expected to be complete by 2005. It will have a total CM energy of 14 TeV, i.e. an effective energy of 2 TeV (2000 GeV). Thus it will push up the energy frontier by almost an order of magnitude. The reason for going to these higher energies is that, although we have seen all the basic constituents of matter and the carriers of their interactions, the picture is not complete yet. As we shall see below, a consistent theory of their masses requires the presence of a host of new particles in the mass range of a few hundred GeV, which await discovery at the LHC. Indeed there is already an active proposal for a detailed follow-up investigation of this energy range with a $e^+e^-$ machine, generically called the next linear collider (NLC), although one does not know yet when and where.

**Mass Problem (Higgs Mechanism)**

By far the most serious problem with the Standard Model is how to give mass to the weak gauge bosons (as well as the quarks and leptons) without breaking the gauge symmetry of the Lagrangian, which is essential for a renormalisable field theory. Let us illustrate this with the simpler example of a $U(1)$ gauge theory, describing the EM interaction of a charged scalar (spin 0) field $\phi$. The corresponding Lagrangian is

$$\mathcal{L}_{EM} = (\partial_\mu - ieA_\mu)\phi^* (\partial_\mu + ieA_\mu)\phi - \left[\mu^2 \phi^* \phi - \lambda (\phi^* \phi)^2\right] - \frac{1}{4}F_{\mu\nu}F^{\mu\nu},$$
\[ F_{\mu \nu} = \partial_{\mu} A_{\nu} - \partial_{\nu} A_{\mu}, \]  

where \(A_{\mu}\) is the EM field (photon) and \(F_{\mu \nu}\) the corresponding field tensor. Since \(\partial_{\mu}\) is the momentum operator the last term of the Lagrangian represents the gauge kinetic energy. The middle term represents the scalar mass and self-interaction, while the first one represents scalar kinetic energy and gauge interaction. It is clear that the scalar mass and self-interaction terms are invariant under the phase transformation
\[ \phi \rightarrow e^{i \alpha(x)} \phi, \]

which is called gauge transformation for historical reason. But \(\partial_{\mu} \phi\) and the resulting scalar kinetic energy term are evidently not invariant under this phase (gauge) transformation. However the invariance of the Lagrangian under this gauge transformation on \(\phi\) is restored if one makes a simultaneous transformation on \(A_{\mu}\), i.e.
\[ A_{\mu} \rightarrow A_{\mu} + \frac{1}{e} \partial_{\mu} \alpha(x). \]

Thus it is a remarkable property of gauge theory that the symmetry of the Lagrangian under the phase transformation of a charged particle field implies its interaction dynamics. But the symmetry of the Lagrangian would be lost once we add a mass term for the gauge field, \(M^2 A_{\mu} A_{\mu}\), which is clearly not invariant under (19). While the photon has no mass the analogous gauge bosons for weak interaction, \(W\) and \(Z\), are massive. So the question is how to add such mass terms without destroying the gauge symmetry of the Lagrangian.

The clue is provided by the observation that the scalar mass term, \(\mu^2 \phi^* \phi\), is gauge symmetric. This is exploited to give mass to \(W\) and \(Z\) (as well as the quarks and leptons) through backdoor. They acquire mass by absorbing scalar particles, somewhat similar to a snake acquiring mass by swallowing a rabbit. This is the famous Higgs mechanism. One starts with a scalar field of imaginary mass (negative \(\mu^2\)). The corresponding scalar potential
\[ V = \mu^2 \phi^* \phi + \lambda (\phi^* \phi)^2 \]

is shown in the 3-dimensional plot below (Fig. 10a), as a function of the complex field.

Fig. 10. The scalar potential \(V\) as a function of the complex scalar field with (a) negative \(\mu^2\) and (b) positive \(\mu^2\), resembling the bottom of a wine bottle and the top of a wine glass respectively.
Starting from zero it becomes negative at small $|\phi|$ where the quadratic term wins and positive at large $|\phi|$ where the quartic term takes over. The minimum occurs at a finite value of $|\phi|$, i.e. at

$$v = \sqrt{-\mu^2/2\lambda}.$$  \hfill (21)

Thus the ground state (vacuum state) corresponds to this finite value of $|\phi|$. 

Note that the potential of Fig. 10a looks like the bottom of a wine bottle. It is a useful analogy since the shape of the bottle maps the potential energy. A drop of wine left in the bottle will not sit at the origin but drop off to the rim because it corresponds to minimum potential energy (ground state). This may be contrasted with the potential with positive $\mu^2$, shown in Fig. 10b, which looks like the top of a wine glass. In this case the drop of wine will rest at the origin because it is the state of minimum potential energy. One can shake the drop a little and study its motion perturbatively as a small oscillation problem around the origin, which is the position of stable equilibrium. One can do the same thing in the previous case (Fig. 10a) except that the perturbative expansion has to be done not around the origin but a point on the rim, because the latter corresponds to minimum potential energy (stable equilibrium).

It is the same thing in perturbative field theory. Perturbative expansions must be done around a point of minimum energy (stable equilibrium). So one must translate the origin to this point and do perturbation theory with the redefined field

$$h(x) = \phi(x) - v,$$  \hfill (22)

which represents the physical scalar (called the Higgs boson). Substituting (22) in (17) we find several pleasant surprises. The first term of the Lagrangian gives a quadratic term in $A^\mu$, $e^2v^2A_\mu A_\mu$, corresponding to a mass of the gauge boson

$$M = ev.$$  \hfill (23)

Moreover the middle term leads to a real mass for the physical scalar,

$$M_h = \sqrt{-\mu^2} = (\sqrt{2}\lambda)v = \frac{\sqrt{2}\lambda}{e}M.$$  \hfill (24)

Thus the Higgs boson mass is equal to the mass of the gauge boson times the ratio of the self-coupling to the gauge coupling. Although we do not know the value of the self-coupling $\lambda$, the validity of perturbation theory implies $\lambda \lesssim 1$. This means that the Higgs boson mass is expected to be in the same ball park as $M_W$, i.e. within a few hundred GeV. Hence it is a prime candidate for discovery at the LHC.

In order to appreciate the Higgs mechanism let us look back at Fig. 10. The choice of negative $\mu^2$ meant that the ground state has moved from the origin to a finite value of $|\phi|$ (i.e. a point on the rim). While the former was invariant under phase transformation, the latter point is not. Thus the phase (gauge) symmetry of the Lagrangian is not shared by the ground state (vacuum). This phenomenon is known as spontaneous symmetry breaking. There are many examples of this in physics, the most familiar one being that of magnetism. As we cool a Ferromagnet below a critical temperature the electron spins get aligned with one
another because that corresponds to ground state of energy. Thus the Lagrangian possesses
a rotational symmetry, but not the ground state. The same thing happens in the Higgs
mechanism, except that the rotation is in the phase space of the complex field $\phi$ instead
of the ordinary space. The breaking of the phase (gauge) symmetry by the ground state
enables the gauge boson to acquire mass. At the same time the fact that this symmetry is
retained by the Lagrangian enables the renormalisation theory to go through. Needless to
say that this last feature is very important because it ensures a systematic cancellation of
infinites, without which there would be no predictive field theory. The renormalisability
of the Electroweak gauge theory in the presence of spontaneous symmetry breaking was shown
to be valid by t’Hooft in the early seventies, for which he got the Nobel Prize this year along
with his teacher, Veltman.

**Hierarchy Problem (Supersymmetry)**

The Higgs solution to the mass problem is not the full story because it leads to the
so called hierarchy problem – i.e. how to control the Higgs boson mass in the desired
range of $\sim 10^2$ GeV. This is because the scalar particle mass has quadratically divergent
quantum corrections unlike the fermion or gauge boson masses. The quantum corrections
arise from the interaction of the particle with those present in the quantum mechanical
vacuum – i.e. it is analogous to the effect of medium on a particle mass in classical mechanics.
This is illustrated by the Feynman diagrams of Fig. 11 below, showing the radiative loop
contributions to the Higgs mass coming from its quartic self-interaction of (17) and its
interaction with a fermion pair. Thanks to the uncertainty principle, the momentum $k$ of
the vacuum particles can be arbitrarily large.

![Fig. 11. Radiative loop contributions to Higgs boson mass coming from its (a) quartic
self-coupling and (b) coupling to fermion-antifermion pair.](image)

Integrating the boson propagator factor of Fig. 11a, $1/(k^2 - M^2)$, over this 4-momentum
gives

$$\int \frac{d^4k}{k^2 - M^2} \sim \int \frac{k^3 dk}{k^2 - M^2} \sim \int k dk \sim k^2 \bigg|_{0}^{\Lambda},$$

(25)

where $\Lambda$ is the cut-off scale of the theory. Similarly integrating the product of the fermion
and anti-fermion propagators in Fig. 11b gives

$$\int \frac{d^4k}{(k - m)^2} \sim k^2 \bigg|_{0}^{\Lambda},$$

(26)
where $\mathbf{k} = k_\mu \gamma^\mu$, i.e. the invariant product of the 4-momentum with the Dirac matrices. It may be noted here that an analogous radiative contribution from fermion loop is also present for the photon field $A_\mu$. However there is mutual cancellation between the divergent contributions to different spin states of the photon via the so called gauge condition. In other words it is the gauge symmetry that protects the gauge boson masses from divergent quantum corrections. Similarly the fermion masses are protected by chiral symmetry. In the absence of any protecting symmetry the quadratically divergent quantum corrections from (25) and (26) would push up the output Higgs boson mass to the cutoff scale $\Lambda$. The cutoff scale of the Electroweak theory is where it encounters new interactions – typically the GUT scale of $10^{16}$ GeV or Plank scale of $10^{19}$ GeV. The former represents the energy scale where the strong and Electroweak interactions are presumably unified as per Grand Unified Theory, while the latter represents the energy where gravitational interaction becomes strong and can no longer be neglected. So the question is how to control the Higgs boson mass in the desired range of $\sim 10^2$ GeV, which is tiny compared to these cutoff scales.

That the scalar mass is not protected by any symmetry was of course used in the last section to give mass to gauge bosons and fermions via Higgs mechanism. The hierarchy problem encountered now is the flip side of the same coin. The most attractive solution is to invoke a protecting symmetry – i.e. the supersymmetry (SUSY), which is a symmetry between fermions and bosons. As per SUSY all the fermions of the Standard Models have bosonic Superpartners and vice versa. They are listed below along with their spin, where the Superpartners are indicated by tilde.

| quarks & leptons S | Gauge bosons S | Higgs S |
|-------------------|----------------|--------|
| $q, \ell$ 1/2     | $\gamma, g, W, Z$ 1 | $h$ 0 |
| $\tilde{q}, \tilde{\ell}$ 0 | $\tilde{\gamma}, \tilde{g}, \tilde{W}, \tilde{Z}$ 1/2 | $\tilde{h}$ 1/2 |

SUSY ensures cancellation of quadratically divergent contributions between superpartners, e.g. between the higgs loop of Fig. 11a and the corresponding fermionic ($\mathbf{h}$) loop of Fig. 11b. For the cancellation to occur to the desired accuracy of $\sim 10^2$ GeV the mass difference between the superpartners must be restricted to this scale. Thus one expects a host of new particles in the mass range of a few hundred GeV, which can be discovered at the LHC.

**Conclusion**

The Higgs and the SUSY particles are the minimal set of missing pieces, required to complete the picture of particle physics. Therefore the search for these particles is at the forefront of present and proposed research programmes in this field. A comprehensive search for these particles up to the predicted mass limit of $\sim 1000$ GeV will be possible at the LHC, which should go on stream in 2005. Hopefully the LHC will discover these particles and complete the picture a la the Minimal Supersymmetric Standard Model or else provide valuable clues to an alternate picture.

It should be emphasised of course that, while the Supersymmetric Standard Model represents a complete and self-consistent theory, it is far from being the ultimate theory. The
ultimate goal of particle physics is the unification of all interactions. This has inspired theorists to propose more ambitious theories unifying all the three gauge interactions (Grand Unified Theory) and even roping in gravity (Superstring Theory). As indicated above, however, the energy scales of these unifications are many orders of magnitude higher than what can be realised at present or any foreseeable future experiment. Thus it is possible and in the light of our past experience very likely that nature has many surprises in store for us in the intervening energy range; and the shape of the ultimate theory (if any) would have little resemblance to those being postulated today.
(a) $\frac{C_q^2 \alpha_s}{Q^2}$  
(b) $\frac{e_q e_f \alpha}{Q^2}$  
(c) $\frac{\alpha_w}{Q^2 - M_w^2}$
\[ e^- \rightarrow u \quad e^+ \quad \bar{e} \rightarrow d \quad \bar{e}^+ \]

charm  
tag leptons  
bottom  
gluon

\[ \bar{p} \rightarrow \bar{d} \quad p \quad \bar{u} \rightarrow u \quad \bar{u} \rightarrow \nu_e, \nu_\mu, \nu_\tau \]

\( W \) & \( Z \) bosons
