Effect of ball bearing misalignment on dynamic characteristics of rotor system

Baogang Wen¹, Meiling Wang², Qingkai Han³, Changxin Yu⁴

1. School of Mechanical Engineering and Automation, Dalian Polytechnic University, Dalian, 116034, PR China, wbg_dlut@163.com
2. College of Locomotive and Rolling stock Engineering, Dalian Jiaotong University, Dalian, 116028, PR China, meilingcc@163.com
3. School of Mechanical Engineering and Automation, Northeastern University, Shenyang, 116024, PR China, qk.han@hotmail.com
4. Wafangdian Bearing Group Corp, Wafangdian, 116300, China, yucx@zwz-bearing.com

* Corresponding author: Tel.: +86-411-84106065; E-mail address: meilingcc@163.com

Abstract. A theoretical model with 5 degrees of freedom (DOF) is derived for a rigid rotor system supported by two ball bearings considering the changes of the bearing stiffness due to misalignment. And then radial and axial vibrations of the rotor system and its bearing reactions under different misaligned offsets are investigated with theoretical and numerical analysis. Then, the effects of misalignment on the vibrations of rotor and the bearing, the variation of stiffness and reaction characteristics of the misaligned ball bearing are predicted theoretically. It is concluded that angular misalignment may cause axial vibration and parallel misalignment can bring out extra moment loads for misaligned bearing.

Keywords. Rotor system; bearing misalignment; bearing stiffness; theoretical model

1. Introduction

Bearing is one of the important basic parts in the major equipment, such as aero-engine, turbo-machinery and so on[1, 2]. Its performance will directly affect the rotor dynamics, even the security of the whole equipment[3-5]. Misalignment of bearing is present because of improper assembly and thermal distortion, resulting in abnormal vibration and excessive preload[6]. However, perfect alignment cannot be obtained. The need for understanding the phenomena is important to practical engineers for the purpose of trouble shooting. There are two types of misalignment: coupling misalignment and bearing misalignment. Recently, the majority of the misalignment studies is about coupling misalignment, discussed the effects of its structure[7, 8] and type on the vibration of connected rotor system[9, 10], and evaluated the 2× vibration response as to be the characteristic feature of misalignment[11-15]. Hussain and Redmond[16, 17] pointed coupling misalignment is a source of both torsional and lateral excitations, of which angular misalignment may generate purely static forces and displacements. References[18, 19] showed that super-harmonic components are the most remarkable in the vibration spectrum for a hyper-static shaft-line with rigid coupling misalignment.
As to the bearing misalignment, no much work has been reported, and the corresponding dynamic mechanism is still unclear and insufficient. In this paper, a dynamic model of a misaligned rigid rotor system with 5 DOF is deduced based on Lagrangian theory, in which the variation of bearing stiffness due to the un-concentricity of supports is taken into account. The effects of misalignment are analyzed theoretically on the vibration of the rotor and bearing, the stiffness and reaction force of the misaligned bearing.

2. Motion equations of rigid rotor system with misaligned ball bearing

As the rigid rotor system shown in figure 1(a) with misaligned bearing, an angular tilt is present for the shaft and offset are available for its right bearing B2, which refer to the angular and parallel misalignment, respectively. In addition, the upward point of bearing B1 may yaw around its initial position (Letting the deflection angle). A moving coordinate system O2X2Y2Z2 is introduced to illustrate the misaligned rotor system. At any time t, the relationship between OXYZ, and O2X2Y2Z2 is shown in figure 1(b).

![Figure1.Misaligned rotor system](image)

The equations of motion of misaligned rotor system are obtained following the Lagrangian theory, in which assumptions adopted are as follows: (1) the rotor is rigid and represented by one concentrated mass point; (2) bending and axial vibrations without torsional vibration of the rotor system are considered; (3) variable stiffness of misaligned bearing are considered; (4) shaft rotating speed is constant.

The vector \( \mathbf{q} = \{ x, y, z, \theta_y, \theta_z \}^T \) of concentrated mass point D in the fixed coordinate system OXYZ is chosen as the generalized displacement vector, including three translational DOF and two rotational DOF around Y- and Z- axes. From the perspective of the rigidity assumption, rotational displacements of points on rotor are all \( \theta_y, \theta_z \).

2.1 Kinetic and potential energy

The kinetic energy of the rotor system shown in figure 1 is easily computed as a sum of the translational kinetic energy \( T_t \) and rotational kinetic energy \( T_r \),

\[
T_t = \frac{1}{2} m (\ddot{x}^2 + \dot{y}^2 + \dot{z}^2) = \frac{1}{2} m \left[ (\dot{x} + \alpha \dot{e} \Omega \sin(\Omega t + \Psi_0))^2 + (\dot{y} - \epsilon \dot{e} \Omega \sin(\Omega t + \Psi_0))^2 + (\dot{z} + \epsilon \dot{e} \Omega \cos(\Omega t + \Psi_0))^2 \right] 
\]

\[
T_r = \frac{1}{2} \left[ J_p (\dot{\theta}_y + \dot{\theta}_z)^2 + J_d [(-\dot{\theta}_y \sin(\Omega t + \theta_y) \cos(\Omega t))^2 + (\dot{\theta}_z \cos(\Omega t + \theta_y) \sin(\Omega t))^2 \right] 
\]

where \( m, J_p, J_d \) are the mass, polar moment of inertia and diameter moment of inertia respectively, \( \Omega \) is the rotating speed of rotor, \( e \) and \( \Psi_0 \) are the eccentricity and initial phase angle.
The total potential energy of the rotor system is a sum of potential energy of the two supports $U_1$ and $U_2$, both of which are

$$U_1 = \frac{1}{2} q_{b1}^T K q_{b1}, \quad U_2 = \frac{1}{2} q_{b2}^T K q_{b2}$$  \hspace{1cm} (3)

Where $K$ is a $5 \times 5$ stiffness matrix of the bearing, and its diagonal elements can be represented by $K_1-K_5$ if ignoring the cross stiffness coefficients. $q_{b1}$ and $q_{b2}$ are the generalized displacement vectors at the two support points, respectively. In the misaligned condition, the relationships among $q_{b1}$, $q_{b2}$ and $q$ can be expressed as

$$q_{b1} = R B_1 R^T q + S_{b1}, \quad q_{b2} = R B_2 R^T q + S_{b2}$$  \hspace{1cm} (4)

where $R = \begin{bmatrix} 1 & -\alpha & 0 & 0 & 0 \\ \alpha & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$, $R^T$ is the transpose matrix of $R$, $B_1 = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & -a \\ 0 & 0 & 1 & a & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$, $B_2 = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & b \\ 0 & 0 & 1 & -b & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$, $S_{b1} = \begin{bmatrix} 0 \\ -\alpha^2 \delta \cos \phi \\ 0 \\ -a \delta \sin \phi \\ a \delta \cos \phi \end{bmatrix}$, $S_{b2} = \begin{bmatrix} 0 \\ -\alpha^2 \delta \cos \phi \\ 0 \\ b \delta \sin \phi \\ -b \delta \cos \phi \end{bmatrix}$, $a$ and $b$ are the distances between D and B1, B2 respectively.

2.2 Stiffness matrix of the misaligned bearing

As to the rotor system shown in figure 1, an angular tilt $\alpha$ and offset $\delta$ are present for the misaligned bearing bringing out extra preloads. So the displacements of the misaligned bearing (in figure 2) can be represented by the axial and radial displacements $\delta^x$, $\delta$ and angular displacement $\alpha$ respectively.

![Figure 2. Sketch of misaligned ball bearing](image-url)
Under combined loads condition, the rolling bearing stiffness matrix (ignoring cross stiffness) $K_{lim}$ with 5 degrees of freedom [20, 21] is

$$
K_{lim} = \begin{bmatrix}
K_{11} & 0 & 0 & 0 & 0 \\
0 & K_{22} & 0 & 0 & 0 \\
0 & 0 & K_{33} & 0 & 0 \\
0 & 0 & 0 & K_{44} & 0 \\
0 & 0 & 0 & 0 & K_{55}
\end{bmatrix}
$$

(5)

Where $K_{11} \sim K_{55}$ are bearing translational stiffness coefficients in three directions and rotational coefficients around $Y$- and $Z$- axes, respectively.

2.3 Motion equations

The total kinetic energy of the rotor system is a sum of translational kinetic energy and rotational kinetic energy, namely, $T=T_1+T_2$. Total potential energy is a sum of elastic potential energy at two elastic supports, namely, $U=U_1+U_2$. By introducing these into Lagrangian equation and performing the relevant derivatives, the equations of motion of misaligned rotor system with 5-DOF are readily obtained

$$
M\ddot{q} + G\dot{q} + K_{eg}q = Q
$$

(6)

Where, $M, G, K_{eg}, Q$ are mass matrix, gyroscopic matrix, stiffness matrix, generalized excitation force vector respectively.

$$
K_{eg} = RB_i^7R^7KR_iR_i^7 + RB_i^7R^7KR_iR_i^7
$$

(7)

In addition, by differentiating Lagrangian equation on, an algebraic equation is obtained to determine the angular position of the support point, which is related to $y, \theta_1, \theta_2$, namely
3. Results and discussions

In order to investigate the influences of misalignment on the stiffness, vibration and reaction forces of bearings, numerical simulations are conducted with the fourth-order variable step Runge-Kutta method, based on the dynamic model of misaligned rotor system (Eqs. (6), (9)) in the above section. The initial calculation conditions in simulations are as follows: the rotating speed is constant as 2400r/min (40Hz). The basic parameters of rotor system and the angular ball bearing of type 71906 are shown in Tables 1-2.

\[
2\alpha^2 K_3 y \sin \phi + (b-a)(\alpha^2 K_4 + K_3)i \sin \phi + (b-a)K_4 \theta \cos \phi + \cdots
\]

\[+(a^2 + b^2)\delta(K_4 - K_3) \sin \phi \cos \phi = 0 \tag{9}\]

| Parameters                          | Value   |
|-------------------------------------|---------|
| Length of rotor                     | 0.480m  |
| Diameter of rotor                    | 0.036m  |
| Young modulus of rotor               | 2.06×10^{11}Pa |
| Poisson ratio of rotor               | 0.3     |
| Mass of rotor                       | 6Kg     |
| Imbalance of rotor                   | 6×10^{-3}kg.m |
| Density of rotor                     | 7.85×10^{3}kg/m³ |

| Parameters                          | Value   |
|-------------------------------------|---------|
| Diameter of inner ring Di           | 0.030 m |
| Diameter of outer ring Do           | 0.047m  |
| Width                               | 0.009m  |
| Diameter of roller                  | 0.004 m |
| Contact angle α0                    | 15°     |
| No. of roller m                     | 16      |
| Load-deformation index n            | 3/2     |
| Load deformation coefficient Kn     | N/m^6   |
| Axial preload                       | 500N    |

As to the rotor with a length of 480mm and maximum elevation adjustment of 3mm on one side, the maximum angle misalignment amount obtained is about 0.00625rad (0.35°). Thus, the \(\delta = 0\text{mm}, \alpha = 0\text{–}1\degree\) angular misalignment varying case is selected for the further simulation analysis. And also, the \(\alpha = 0\degree, \delta = 0\text{–}1\text{mm}\) parallel misalignment varying condition is adopted to grasp the effects of misalignment.

3.1 Changes of the stiffness of the misaligned bearing

An angular tilt and offset are available for misaligned bearing, resulting uneven and additional displacement loads, which will bring out the variation of stiffness of the misaligned bearing. An error ratio of bearing stiffness is defined to illustrate the variation,

\[
err_K = \frac{\Delta K}{K_{\text{norm}}} \times 100\% \tag{10}\]

Where \(\Delta K\) is additional stiffness caused by misalignment, \(K_{\text{norm}}\) is the stiffness under normal state. Based on the proposed misaligned bearing model (Eq. (5)), variation of bearing stiffness is discussed under two cases mentioned above. The proposed error ratios of three translational stiffness coefficients of the bearing are altering along misalignment, as shown in Fig.3.
As seen from figure 3, misalignment has a strong influence on the stiffness coefficients of bearing. Because the misalignment can directly result in uneven and additional displacement loads for misaligned bearing.

3.2 Vibration analysis of the misaligned rotor system
With the angular misalignment varying from 0° to 1° and the parallel misalignment varying from 0 mm to 1 mm, the radial vibrations of the rotor and misaligned bearing are analyzed. The vibration response amplitudes of the rotor and bearing with angular and parallel misalignment are obtained, as shown in figure 4.
Figure 4. Variation in amplitudes of vibration responses for the rotor (a-b) and misaligned bearing (c-d) with angular and parallel misalignment.

For both two cases, the radial vibration amplitudes of the rotor and misaligned bearing decrease slightly with increasing misalignment in the range of variation of $\alpha = 0\text{ to } 1^\circ$, $\delta = 0\text{ to } 1\text{ mm}$, but the variation is not very evident.

3.3 Bearing reaction forces due to misalignment

The generalized reaction load $F$ about the misaligned bearing can be easily obtained

$$F = K q_b$$  \hspace{1cm} (11)

Where $K$ is a $5 \times 5$ stiffness matrix of the bearing, as mentioned above. $q_b$ denotes the generalized displacement vectors at the support point with 5 DOF. Accordingly, the generalized reaction load $F$ can be written as $\{F_x, F_y, F_z, M_y, M_z\}$, in which $F_x$ denotes the axial force. And the force $F_y$ and the moment $M_z$ are in $Y$-direction, the force $F_z$ and $M_y$ are in $Z$-direction. The changes of the generalized reaction forces of the misaligned bearing along with the misalignment increments are shown in Fig.5.
As shown in Figure 5, the radial reaction forces $F_y$, $F_z$ and the moments $M_y$, $M_z$ of the misaligned rotor system do not change significantly, but the axial force $F_x$ greatly increases, as the angular misalignment increasing in the range of variation of $\alpha = 0 \degree$ to $1 \degree$. However, when the parallel misalignment varying from 0mm to 1mm, the translational forces $F_x$, $F_y$, $F_z$ and the moment $M_y$, $M_z$, decrease slightly, and the moment $M_z$ around Z-axis increase sharply. It is, therefore, deemed angular misalignment as the cause of axial vibration and parallel misalignment as extra moment loads for misaligned bearing.
4. Conclusions
The dynamic model of a misaligned rigid rotor system with 5 DOF is deduced by means of Lagrangian approach. The effects of misalignment are investigated numerically on the vibration responses of rotor and bearing, the stiffness and reaction force of the misaligned bearing. The obtained results on such a rigid rotor system with misaligned ball bearing reveal that the vibration of the rotor system seems not to be sensitive to the misalignment, which has a strong influence on the stiffness coefficients of bearing. The transverse vibration amplitudes of rotor and bearing will decrease along the misaligned value, but the variations are not evident. And the axial and radial stiffness coefficient increases greatly with the value of angular and parallel misalignment. But the reaction loads change seriously attributed to the misaligned bearing. The angular misalignment can bring out extra axial force, and the parallel misalignment can give rise to additional moment loads.

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Conflict of interest
The Authors declare that there is no conflict of interest.

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