Misalignments Computation of a Two-Mirror Optical System based on Modified Merit Function Regression Method

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Abstract. The computer-aided alignment (CAA) is a key technology to align a precision optical system, which is generally a difficult and laborious work. However, many methods proposed has a weak point in that they cannot calculate the misalignments accurately if there are figure error in mirrors. In order to solve this problem, a new modified merit function regression (MFR) is developed, which can compute the misalignments even if the effect of figure error. Misalignments computation based on the traditional and modified MFR are tested in a typical two-mirror optical system, experimental results shows the correctness of modified MFR and its high calculation accuracy.

1. Introduction

Assembly precision optical systems is generally a difficult and time-consuming work, especially in all-reflecting optical systems. In the 1980s, in order to solve the problem, Ira M. Egdall [1], an optical engineer in Itek Company, proposed a novel method that uses the computer program to calculate the assembly errors of optical systems and provide the effective instruction to optical system alignment, which is the first idea of computer-aided alignment (CAA). After decades of research and development, many theories and misalignment calculation methods about how to assemble optical system quickly and exactly have been proposed, such as sensitivity table method (STM) [2], merit function regression (MFR) [3] and differential wavefront sampling (DWS) [4].

STM and MRF are the most widely used methods of CAA. STM is based on the linearity between misalignments and wavefront coefficients, which is easily too established in practice and function well in three-mirror optical systems and off-axis optical systems. STM is a kind of CAA that can compute the misalignments by establish sensitivity table equations between them and wavefront coefficients. Zhang Xuemin [5] aligns an off-axis RC reflective optical system correctly and gets a good wave aberration of 0.04λ by STM. Zhao Xiting [6] also test it in a three-mirror optical system and finds it’s a good tool to align optical systems. However, some researches show that misalignments computation based on STM are not accurately, this is because that the wavefront coefficients are not linear with the misalignments in some optical systems, such as co-axial two-mirror optical systems especially in...
central field of view. In order to overcome the weakness of STM, Kim [3] proposed a new reverse-optimization algorithm which uses the merit function (MF) regression instead of the sensitivity table. It is verified in a co-axis Cassegrain type collimator, the final result shows a reduction of the mean system rms wavefront error (WFE) from 0.283λ to 0.194λ (λ=632.8nm). Luo Miao [7] also applies MFR for alignment a diameter 250mm off-axis Cassegrain system and gets a good performance which produces a measured wavefront error 0.0405λ at λ=632.8nm. However, all those works are based on the condition of assembly mirrors are perfect, the influence of figure error are ignored. The misalignments calculation can’t be accurate and can’t be accepted in real assembly progress while the mirrors are not perfect and exist figure error.

In this paper, aiming to compute the misalignments of optical systems accurately while existing figure error, a modified MFR method is proposed. Firstly, the theory of MFR method is analyzed, its characteristics and shortcomings are pointed out, and then the modified MFR method is introduced; secondly, the design of Hilbert telescope, which is a typical two-mirror optical system, is introduced and the effects of figure error are analyzed; thirdly, some analysis and misalignments computation are conducted to compare the two MFR methods. Finally, we will summarize and conclude the paper.

2. Computer-aided alignment algorithms

In an ideal optical system, the design imaging equality is near diffraction limited, but it is susceptible and can be easily influenced by assembly error. The misalignment of optical element is the key reason that because the decline of system performance, it will be take a long time to align a complex optical system. Researchers find that the relations between misalignments and wave front coefficients are regular and can be utilized for assembly instruction. Based on the relations, CAA algorithms are aimed to find the misalignment value to instruct optical system assembly and improve the system performance.

2.1. Traditional MFR method

Like the sensitivity table method which need to establish the approximate mathematical equations, the MFR method establishes the merit function (MF) in commercial optical design software, such as ZEMAX and CODE V. Then an algorithm built-in software is used to minimize the MF value to derive the best-fit parameters, which are the misalignment values of system.

The MF, commonly used in optical modeling software, is defined as follows:

\[
MF^2 = \frac{\sum W_i (V_i - T_i)^2}{\sum W_i}
\]  

(1)

Where \( W_i \) is the weighting factor, \( V_i \) and \( T_i \) are the current and target values of chosen parameter that usually are the wave front aberrations. For uniform the expression, the wave front aberrations of optical system are expressed with Fringe Zernike coefficients that are commonly used in commercial optical design software and optical practice.

In order to express the traditional MFR method, a schematic diagram is concluded as follow figure 1 and it carries out the following tasks:
Figure 1. Schematic diagram of the traditional MFR method.

1. Acquire the Zernike coefficients from wave front sensors in actual system assembly process, which represents the misaligned system wave front error (WFE).
2. Assign the Zernike coefficients in designed system, which represents the WFE in current alignment status.
3. Run the auto-optimization algorithm (e.g. damped least square technique [8]) embedded in the software to minimize the MF, which varies the alignment parameters to make approaches as closely as possible.
4. Get the misalignments when MF is minimized to the threshold that usually be set zero and use them to instruct assembly in practice.

2.2 Modified MRF method

In many optical systems, the mirrors can be manufactured in a near perfect status, so the traditional MFR can function well in those cases. While in some infrared and near-infrared optical systems, their imaging performance is not easily disturbed by figure error. Even if the figure error reaches up to 1~2 \( \lambda \) (\( \lambda = 632.8 \text{nm} \)), systems can still function normally in working waveband. However, it will have a great influence in the process of assembly, and it cannot be ignored in real assembly process.

In order to consider the influence of figure error, we modify the traditional MFR method, and MF in modified MFR method can be written as follows:

\[
MF^2 = \sum_i W_i (V_i + F_i - T_i)^2 \]

Where \( F_i \) is the WFE that introduced by figure error.

Unlike the MF in the traditional MFR method, MF defined here introduces the effect of figure error.

The new schematic diagram can be modified as follow figure 2.
The main difference between the two MFR methods is that figure error measured in practice should be added into the design system firstly, then the following steps are nearly the same with the steps mentioned before. There are many methods that can add figure error into the commercial optical design software. Considering the figure error measured are generally express as Fringe Zernike coefficients, we adopt the Zernike Fringe Phase surface to introduce figure error, which has a substrate shape identical to the Standard surface plus additional phase terms defined by the Zernike Fringe coefficients.

3. Hilbert optics and performance analysis

The influence of figure error is similar in all co-axis two-mirror optical systems, so a typical two-mirror system Hilbert telescope is adopted in this paper. Hilbert optics consist of a concave hyperboloid primary mirror (PM) and a convex hyperboloid secondary mirror (SM) (See Table 1, Figure 3(a)). The RMS radius of spot diagram in designed optical is 1.344um and is less than the airy radius 6.628um (Figure 3(b)), which shows that the designed system can reach up to diffraction limited and satisfy the design requirement.

Table 1. Optical Parameters for the Hilbert optics.

| Surface | Type | Conic constant | Radius | Thickness | Semi-Diameter |
|---------|------|----------------|--------|-----------|---------------|
| PM      | Conic| -1.06          | -1291.2| -485.5    | 152.4         |
| SM      | Conic| -3.3095        | -425   | 649.2     | 43.05         |
| Image   | Plane| 0              | Infinity| -         | -             |

Radius and thickness are in mm.

Figure 3. (a) The layout of Hilbert optics (b) spot diagram of Hilbert optics ($\lambda=632.8$nm).
As the purposes of optical system are different, so the requirements of mirrors fabrication are also various. Here, we introduce a figure error in PM and research its effect. Astigmatism deformation is adopted to add to ideal system as it is dominant in mirrors error. The introduced figure error is present in Table 2, and the impact on Hilbert optics at two different wavelengths are presented in Table 3.

Table 2. Introduced surface error on PM.

| Type     | $C_{5\text{PM}}$ | $C_{6\text{PM}}$ |
|----------|------------------|------------------|
| value    | 1.05             | 0.45             |

Fringe Zernike coefficients are in $\lambda$=632.8nm.

Table 3. Spot diagram characteristic of Hilbert optics with and without figure error.

| Wavelength | 0.6328 | 10 |
|------------|--------|----|
| Figure error | Without | With | Without | With |
| Airy radius | 6.628 | 6.628 | 104.7 | 104.7 |
| RMS radius | 1.344 | 18.782 | 1.344 | 18.782 |
| Diffraction limit | YES | NO | YES | YES |

All units are in um.

From the spot diagram analysis conclude in Table 3, it is obvious that optical system performance shows absolute different result in the visible and infrared light under the influence of the same figure error. When no figure error, Hilbert optics shows great performance in two different wavelengths and the rms radiuses of spot diagram are the same that is 1.344um. But it also is become worse and increase to 18.782um in two different wavelengths when PM exists figure error. By contrast, Airy spot can’t be affected by figure error and can only be determined by system structure parameters, which is 6.628um at $\lambda$=632.8nm and 104.7um at $\lambda$=10um. Due the effect of figure error, the RMS radius becomes much greater than Airy radius at $\lambda$=632.8nm, which shows the performance is declined and is not diffraction limit. On the contrary, it’s still has good performance even with the existence of figure error at infrared waveband.

4. Misalignments computation simulation

To research the influence of figure error on misalignments computation, Monte Carlo simulation is performed to compare the computation accuracy between the modified MFR method and traditional MFR method.

In the simulation process, we fist artificially introduce a set of figure errors on the PM ($C_{5\text{PM}}$ & $C_{6\text{PM}}$) and the lateral misalignments of the SM (XDE/YDE/ADE/BDE) into optical model, and then can be obtain the resulting Zernike aberration coefficients. Finally, we utilize the two MFR methods to compute misalignments of SM. XDE, YDE, ADE and BDE represent the decenter and tilt of second mirror along x axis, y axis respectively.

Four different cases will be discussed in the Monte Carlo simulations as show in Table 4, which different in figure error added in PM. In case 1, there are nearly no figure error added, and from case 2 to case 4, the amplitude of figure error from 0.01 increase to 0.5. All misalignments introduced are vary in range ±0.5 mm(°).
Table 4. Six different cases considered in Monte Carlo simulations.

| Type   | $C^\text{PM}_5$ & $C^\text{PM}_6$ |
|--------|-----------------|
| Case 1 | [-0.00001, 0.00001] |
| Case 2 | [-0.01, 0.01] |
| Case 3 | [-0.1, 0.1] |
| Case 4 | [-0.5, 0.5] |

Fringe Zernike coefficients are in $\lambda=632.8\text{nm}$

For each case, 100 perturbation status including figure error and misalignments will be randomly generated following a uniform distribution and introduced into the optical simulation software. The root mean square deviation (RMSD) between the introduced and computed values is adopted to evaluating the computation accuracy of the two methods, which is expressed as

$$RMSD = \sqrt{\frac{1}{N} \sum_{i=1}^{N} \left( x^i_{\text{int}} - x^i_{\text{com}} \right)^2}$$

(3)

Where $x^i_{\text{com}}$ and $x^i_{\text{int}}$ represent the computed and introduced set of values for $i^{th}$ perturbation parameter, respectively.

The computation results for four different cases are presented in table 5.

Table 5. Simulation results of the four cases.

| Type   | XDE    | YDE    | ADE    | BDE    |
|--------|--------|--------|--------|--------|
| Case 1 | MFR1   | 1.59e-5| 1.79e-5| 7.27e-6| 6.47e-6|
|        | MFR2   | 9.20e-10| 1.01e-9| 4.16e-10| 2.39e-10|
| Case 2 | MFR1   | 1.9e-2| 1.7e-2| 0.7e-2| 0.8e-2|
|        | MFR2   | 8.22e-10| 9.54e-10| 3.64e-10| 2.81e-10|
| Case 3 | MFR1   | 0.15  | 0.18  | 0.07  | 0.06  |
|        | MFR2   | 8.47e-10| 1.14e-9| 3.68e-10| 2.43e-10|
| Case 4 | MFR1   | 0.75  | 1.26  | 0.51  | 0.31  |
|        | MFR2   | 5.79e-10| 1.46e-9| 5.60e-10| 2.84e-10|

MFR1 represent the traditional MFR method, and the MFR2 represent the modified MFR method.

In case 1, the calculation accuracy expressed near same, because there are nearly without figure error. We can know that the tradition MFR can be acceptable in a no figure error case, which demonstrates the correctness of the two MFR methods. However, from the computation results for case 2 to case 4, RMSD of MRF1 are increased rapidly along the figure error increases, which shows that figure error can have a great influence in calculation based on traditional MFR. We also can know that it is not instructive to use traditional MFR to computer the misalignments if the mirrors are not perfect. By contrast, from the results in four cased, it is obviously that the modified MFR method can keep a very high calculation accuracy, while performance of the tradition MFR is not satisfying.

5. Conclusion

In this study, based on the traditional MFR method, we present a modified MFR method by considering the influence of figure error, which can improve the calculation accuracy of misalignment. In an analysis of spot diagram in visible and infrared waveband, it is revealed that the mirrors don’t need to be controlled within a high-level precision in infrared optical system. Finally, a misalignment computation of 400 times Monte Carlo simulation is conducted to verify that the computation accuracy of modified MFR method.
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