COUPLED QUINTESSENCE AND CMB

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We revise the stability of the tracking solutions and briefly review the potentials of quintessence models. We discuss the evolution of linear perturbations for $V(\phi) = V_0 \exp(\lambda \phi^2/2)$ potential in which the scalar field is non-minimally coupled to cold dark matter. We consider the effects of this coupling on both cosmic microwave background temperature anisotropies and matter perturbations. We find that the phenomenology of this model is consistent with current observations up to the coupling power $n_c \leq 0.01$ while adopting the current parameters measured by WMAP, $\Omega_{\phi}^{(0)} = 0.76$, $\Omega_{cdm}^{(0)} = 0.191$, $\Omega_b^{(0)} = 0.049$, and $h = 0.70$. Upcoming cosmic microwave background observations continuing to focus on resolving the higher peaks may put strong constraints on the strength of the coupling.

1. Introduction

If we treat Type Ia supernovae (SNe Ia) as standardized candles, then the Hubble diagram of them shows that the expansion of the Universe is currently accelerating. Combining measurements of the acoustic peaks in the angular power spectrum of the cosmic microwave background (CMB) anisotropy and the matter power spectrum of large scale structure (LSS) which is inferred from galaxy redshift surveys like the Sloan Digital Sky Survey (SDSS) and the 2-degree Field Galaxy Redshift Survey (2dFGRS) has also confirmed that a component with negative pressure (dark energy) should be added to the matter component to make up the critical density today.

A quintessence field is a dynamical scalar field leading to a time dependent equation of state (EOS), $\omega_{\phi}$. The possibility that a scalar field at early cosmological times follows an attractor-type solution and tracks the evolution of the visible matter-energy density has been explored. This may help alleviate the severe fine-tuning associated with the cosmological constant problem. However we need to investigate the tracking condition and its stability at the matter dominated epoch carefully.
Are there experimental ways of checking for the existence or absence of dark energy in the form of quintessence? There are several different observational effects of matter coupling to the scalar field on CMB spectra and matter power spectrum compared to the minimally coupled models. And this can be used to check the existence of quintessence.

This paper is organized as follows. We briefly investigate the condition and the stability of tracker solution and review the potentials of quintessence models in the next section. In Sec. III, we show the coupling effects on CMB and matter power spectrum. We conclude in the last section.

2. Quintessence Models and Tacker Solutions

Many models of quintessence have a tracker behavior, which solves the “coincidence problem” (i.e. initial condition). In these models, the quintessence field has a density which closely tracks (but is less than) the radiation density until matter-radiation equality, which triggers quintessence to start behaving as dark energy, eventually dominating the Universe. This naturally sets the low scale of the dark energy. However the present small value of dark energy density still cannot be solved with quintessence (“fine-tuning problem”).

Since the energy density of the scalar field generally decreases more slowly than the matter energy density, it appears that the ratio of the two densities must be set to a special, infinitesimal value in the early Universe in order to have the two densities nearly coincide today. To avoid this initial conditions problem we focus on tracker fields.

2.1. Tracking Condition

We rely details of this section on the reference due to the shortage of space. We introduce new quantity $\theta$ which is related to the ratio of the kinetic energy and the potential energy of the scalar field:

$$2\theta = \ln \frac{KE}{PE} = \ln \frac{1 + \omega_\phi}{1 - \omega_\phi}$$

(1)

$\theta$ can have any value and especially positive $\theta$ means the kinetic energy dominated era and negative $\theta$ indicates the potential energy dominated one. Now we can define EOS as the function of this new quantity $\theta$:

$$\omega_\phi = \tanh \theta$$

(2)
The “tracker equation” \((\Gamma \equiv V''V/(V')^2)\) can be expressed by \(\theta\);

\[
\Gamma = 1 + \frac{3}{2} \left( \frac{\omega_r - \omega_\phi}{1 + \omega_\phi} \right) \frac{1 - \omega_\phi}{(1 + \omega_\phi)} - \frac{1}{2(1 + \omega_\phi)} \frac{\tilde{\theta}}{(3 + \tilde{\theta})} - \frac{1}{(1 + \omega_\phi)(3 + \tilde{\theta})^2}
\]

(3)

where \(\omega_r\) is EOS of the radiation, \(A = a_{eq}/(a + a_{eq})\), \(a_{eq}\) is the scale factor when the energy density of the radiation and that of the matter become equal, and tilde means the derivative with respect to \(x = \ln a = -\ln(1 + z)\). This equation looks like quite different from the well known tracker equation. But when we choose the early Universe constraints (\(A \simeq 1\) and \(\Omega_\phi \simeq 0\)) we can get the well known tracker equation.

\[
\Gamma \simeq 1 + \frac{3}{2} \left( \frac{\omega_r - \omega_\phi}{1 + \omega_\phi} \right) \frac{1}{(3 + \tilde{\theta})} - \frac{1}{2(1 + \omega_\phi)} \frac{\tilde{\theta}}{(3 + \tilde{\theta})} - \frac{1}{(1 + \omega_\phi)(3 + \tilde{\theta})^2}
\]

(4)

When we can ignore the change of \(\theta\) (i.e. when \(\omega_\phi\) is almost constant), we can get the tracking solution. From the above tracking equation we can check this condition.

\[
\Gamma \simeq 1 + \frac{1}{2} \left( \frac{A\omega_r - \omega_\phi}{1 + \omega_\phi} \right) (1 - \Omega_\phi)
\]

(5)

This equation can be rearranged to see the behavior of the EOS as following.

\[
\omega_\phi \simeq \omega_r A(1 - \Omega_\phi) - 2(\Gamma - 1) / (1 - \Omega_\phi) + 2(\Gamma - 1)
\]

(6)

To investigate this equation more carefully, we define the new quantities.

\[
Q = (1 - \Omega_\phi)
\]

(7)

\[
F = (\Gamma - 1)
\]

(8)

where \(Q\) shows the energy information of the Universe and \(F\) depends on the form of the given potential. With these we can rewrite the equation (6).

\[
\omega_\phi \simeq \omega_r \frac{Q}{Q + 2F} A - \frac{2F}{Q + 2F}
\]

(9)

In the reference 5, this equation is expressed as:

\[
\omega_\phi \simeq \frac{\omega_r - 2F}{1 + 2F}
\]

(10)

\(^a\text{Where we put the present value of scale factor, } a^{(0)} \text{ as one.}\)
This equation (10) can be true only when \( Q, A \approx 1 \), which can be satisfied at the early Universe and not at the late one. So with this equation, it is not proper to check the evolution of tracking solutions at late Universe. Instead we should use the equation (9) to check the evolution of the tracking solutions. Before checking the properties of this equation, we should notice that \( Q \) is always positive and has the interval as \( 0 \leq Q \leq 1 \). \( F \) can be positive or negative based on the given shape of the potential.

2.2. Stability Of Tracker Solution

We need to check that solutions with \( \omega_\phi \), which is not equal to the tracker solution value (\( \omega_0 \)) can converge to the tracker ones (i.e. Are tracker solutions stable?). To check this we need to check the small deviation (\( \delta \omega \)) of the tracker solution of EOS.

\[
\omega_\phi = \omega_0 + \delta \omega
\]

If we insert this into the tracker equation (3), then we have following.

\[
\delta \omega + \frac{3}{2} \left[ (A_\omega r - \omega_0)(1 - \Omega_\phi) + (1 - \omega_0) \right] \delta \omega + \frac{9}{2} (1 - \omega_0) \left[ (1 + A_\omega r)(1 - \Omega_\phi) \right] \delta \omega = 0
\]

where we use the tracking condition (5). The general solution to this nonlinear differential equation cannot be obtained analytically. But this equation can be simplified as follow in the early Universe constraints.

\[
\delta \omega + \frac{3}{2} \left[ (1 + \omega r) - 2 \omega_0 \right] \delta \omega + \frac{9}{2} (1 - \omega_0)(1 + \omega r) \delta \omega \approx 0
\]

The solution of this equation is

\[
\delta \omega \propto a^{\gamma_1}
\]

where

\[
\gamma_1 = -\frac{3}{2} \left[ \frac{1}{2} (1 + \omega r) - \omega_0 \right] \pm \frac{i}{2} \sqrt{18(1 + \omega r)(1 - \omega_0) - 9 \left[ \frac{1}{2} (1 + \omega r) - \omega_0 \right]^2}
\]

The real part of this is negative for \( \omega_0 \) less than \( 2/3 \). So \( \delta \omega \) will decays exponentially and solution reaches to the tracking one. In addition to this it also oscillates due to the second term. For the late Universe case, we can change this equation. In the late Universe \( A \) goes to zero and \( \Omega_\phi \) is not zero. With these we can modify the general equation (12).

\[
\delta \omega + \frac{3}{2} \left[ (1 + \Omega_\phi \omega_0) - 2 \omega_0 \right] \delta \omega + \frac{9}{2} (1 - \omega_0)(1 - \Omega_\phi) \delta \omega \approx 0
\]
Table 1. Quintessence models.

| Potential | Reference | Properties |
|-----------|-----------|------------|
| $V_0 \exp (-\lambda \phi)$ | Ratra & Peebles (1988), Wetterich (1988) | $\omega = \lambda^2 / 3 - 1$ |
| $V_0 / \phi^\alpha, \alpha > 0$ | Ratra & Peebles (1988) | $\lambda > 5.5 - 4.5, \Omega < 0.1 - 0.15$ |
| $m^2 \phi^2, \lambda \phi^4$ | Frieman et al (1995) | $\Omega_m \geq 0.2, \omega < -0.8$ |
| $V_0(\exp M_p / \phi - 1)$ | Zlatev, Wang & Steinhardt (1999) | $\alpha \geq 11, \omega \simeq -0.82$ |
| $V_0 (\lambda \phi - 1)^p$ | Sahni & Wang (2000) | $p < 1/2, \omega < -1/3$ |
| $V_0 \sinh^{-\alpha} (\lambda \phi)$ | Sahni & Starobinsky (2000) | early time: inverse power |
| $V_0(e^{\alpha \kappa \phi} + e^{\beta \kappa \phi})$ | Barreiro, Copeland & Nunes (2000) | late time: exponential |
| $V_0[(\phi - B)^\alpha + A e^{-\lambda \phi}]$ | Albrecht & Skordis (2000) | $\omega \sim -1$ |
| $V_0\exp[\lambda \phi / M_p]^2$ | Lee, Olive, & Pospelov (2004) | $\omega \sim -1$ |
| $V_0 \cosh[\lambda \phi / M_p]$ | Lee, Olive, & Pospelov (2004) | $\omega \sim -1$ |

We can repeat the similar step to find the solution of this equation if we assume that $\Omega_\phi$ is almost constant.

$$
\delta \omega \propto a^{7/2}
$$

where

$$
\gamma_2 = \frac{3}{2}\left[\frac{1}{2}(1+\Omega_\phi \omega_0) - \omega_0\right] + \frac{1}{2} \sqrt{18(1+\Omega_\phi)(1-\omega_0) - 9\left[\frac{1}{2}(1+\Omega_\phi \omega_0) - \omega_0\right]^2}
$$

The real part of this solution can be negative if $\omega_0$ satisfies following.

$$
\omega_0 < \frac{1}{(2 - \Omega_\phi)}
$$

where $1 \leq (2 - \Omega_\phi) \leq 2$ for the entire history of Universe.

2.3. Quintessence Potentials

We display the potentials of the quintessence models in Table 1. Any detail of each model can be found in each reference.
3. Coupled Quintessence

The general equation for the interaction of a light scalar field $\varphi$ with matter is,

$$S_\varphi = \int d^4x \sqrt{-g} \left\{ \frac{M^2}{2} [\partial^\mu \varphi \partial_\mu \varphi - R] - V(\varphi) - \frac{B F_i(\varphi)}{4} F^{(i)\mu \nu} F_{\mu \nu} \right\} + \sum_j [\bar{\psi}_j i \gamma_5 \gamma^\mu \psi_j - B_j(\varphi) m_j \bar{\psi}_j \gamma_5 \psi_j].$$

The coupling gives rise to the additional mass and source terms of the evolution equations for CDM and scalar field perturbations. This also affects the perturbation of radiation indirectly through the background bulk and the metric perturbations $^7, ^{17}$. The value of the energy density contrast of the CDM ($\Omega_c$) is increased in the past when the coupling is increased. We specify the potential and the coupling as in the reference $^7, ^{16}$.

$$V(\varphi) = V_0 \exp\left( \frac{\lambda \varphi^2}{2} \right), \quad \exp[B_c(\varphi)] = \left( \frac{b_c + V(\varphi)/V_0}{1 + b_c} \right)^{n_c}$$

3.1. CMB

Now, we investigate the effects of non-minimal coupling of a scalar field to the CDM on the CMB power spectrum. Firstly, the Newtonian potential at late times changes more rapidly as the coupling increases. This leads to an enhanced ISW effect. Thus we have a relatively larger $C_\ell$ at large scales (i.e. small $\ell$). Thus, if the CMB power spectrum normalized by COBE, then we will have smaller quadrupole $^{18}$. This is shown in the first panel of Figure 1. One thing that should be emphasized is that we use different parameters for the $\Lambda$CDM and the coupled quintessence models to match the amplitude of the first CMB anisotropy peak. The parameter used for the quintessence model is indicated in Figure 1 (i.e. $\Omega_\varphi^{(0)} = 0.76, \Omega_m^{(0)} = 0.191, \Omega_b^{(0)} = 0.049$, and $h = 0.7$, where $h$ is the present Hubble parameter in the unit of $100$km$s^{-1}$Mpc$^{-1}$). However, these parameters are well inside the 1 $\sigma$ region given by the WMAP data. We use the WMAP parameters for the $\Lambda$CDM model (i.e. $\Omega_\varphi^{(0)} = 0.73, \Omega_m^{(0)} = 0.23, \Omega_b^{(0)} = 0.04$, and $h = 0.72$) $^b$. In both models we use the same spectral index $n_s = 1$. The heights of the acoustic peaks at small scales (i.e large $\ell$) can be affected by the

$^b$Our data prefers WMAP 3 year data to WMAP 1 year one.
following two factors. One is the fact that the scaling of the CDM energy density deviates from that of the baryon energy density. Therefore for the given CDM and baryon energy densities today, the energy density contrast of baryons at decoupling ($\Omega_b^{(ls)}$) is getting lower as the coupling is being increased. This suppresses the amplitude of compressional (odd number) peaks while enhancing rarefaction (even number) peaks. The other is that for models normalized by COBE, which approximately fixes the spectrum at $\ell \sim 10$, the angular amplitude at small scales is suppressed in the coupled quintessence. This is shown in the second panel of Figure 1. The third peak in this model is smaller than that in the ΛCDM model.

Figure 1. (a) CMB large-scale anisotropy power spectra of ΛCDM (solid line), minimally coupled $n_c = 0$ (dotted line), and non-minimally coupled $n_c = 10^{-3}, 10^{-2}$ (dashed, dash-dotted line respectively) quintessence models. (b) Same spectra for the entire scales.

3.2. Matter Power Spectrum

The coupling of quintessence to the CDM can change the shape of matter power spectrum because the location of the turnover corresponds to the scale that entered the Hubble radius when the Universe became matter-dominated. This shift on the scale of matter and radiation equality is indicated

$$a_{eq} \simeq \frac{\rho_r^{(0)}}{\rho_c^{(0)}} \exp[B_c(\phi_0) - B_c(\phi_{eq})],$$

(22)

where $\rho_r^{(0)}$ and $\rho_c^{(0)}$ are the present values of the energy densities of radiation and CDM respectively, and the approximation comes from the fact that
the present energy density of CDM is bigger than that of baryons ($\rho_c^{(0)} > \rho_b^{(0)}$). This is indicated in Figure 2. Increasing the coupling shifts the epoch of matter-radiation equality further from the present, thereby moving the turnover in the power spectrum to smaller scale. If we define $k_{eq}$ as the wavenumber of the mode which enters the horizon at radiation-matter equality, then we will obtain

$$k_{eq} = \frac{2\pi}{\eta_{eq}}.$$  \hfill (23)

Figure 2. Matter power spectra for the models using the same parameters in Figure 1.

4. Conclusions

We have investigated the tracking condition of the quintessence models and their stability. We have shown that it is necessary to distinguish the tracking condition at the matter dominated epoch and at the radiation dominated one.

We have considered the CMB anisotropy spectrum and the matter power spectrum for the non-minimally coupled models. Additional mass and source terms in the Boltzmann equations induced by the coupling give the
rapid changes of the Newtonian potential $\Phi$ and enhance the ISW effect in the CMB power spectrum. The modification of the evolution of the CDM, $\rho_c = \rho_c^0 a^{-3+\xi}$, changes the energy density contrast of the CDM at early epoch. We have adopted the current cosmological parameters measured by WMAP within 1$\sigma$ level. With the COBE normalization and the WMAP data we have found the constraint of the coupling $n_c \leq 0.01$. The locations and the heights of the CMB anisotropy peaks have been changed due to the coupling. Especially, there is a significant difference for the heights of the second and the third peaks among the models. Thus upcoming observations continuing to focus on resolving the higher peaks may constrain the strength of the coupling. The suppression of the amplitudes of the matter power spectra could be lifted by a bias factor. However, a detailed fitting is beyond the scope of this paper. The turnover scale of the matter power spectrum may be also used to constrain the strength of the coupling $n_c$.

Acknowledgments

We thank ICGA7 organizing committee, J. M. Nester, C-M. Chen, and J-P. Hsu for their hospitality and for organizing such a nice meeting.

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