Seeing Sound Waves in the Early Universe

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Temperature and polarization power spectra of the cosmic microwave background can provide essentially incontrovertible evidence for coherent acoustic oscillations in the early universe. A simple model calculation demonstrates explicitly how polarization couples to velocities at the surface of last scatter and is nearly independent of gravitational or density perturbations. For coherent acoustic oscillations, peaks in the temperature and polarization power spectra are precisely interleaved. If observed, such a signal would provide strong support for initial density perturbations on scales larger than the horizon, and thus for inflation.

A hallmark of inflation-type cosmological models is density perturbations on all scales, including those larger than the causal horizon. Prior to recombination, a given Fourier mode density perturbation begins oscillating as an acoustic wave once the horizon overtakes its wavelength. Since all modes with a given wavelength begin evolving simultaneously, the resulting acoustic oscillations are phase-coherent, leading to the familiar acoustic peaks (often termed, misleadingly, “Doppler peaks”) in the temperature power spectrum of the cosmic microwave background. These peaks give sufficient structure to the power spectrum to enable precision determination of cosmological parameters in this simple and general class of models.

While a measured temperature power spectrum which matches a particular inflation-type model would be quite strong evidence in favor of the model, it would still constitute only indirect evidence for acoustic oscillations, requiring a model fit. Even more importantly, if the observed power spectrum has apparent acoustic peaks but is not well-fit by any simple model, the nature and source of these temperature power spectrum features would be open to question.

This Letter points out that polarization of the microwave background, when combined with temperature information, provides an intuitively clear and compelling physical signature of coherent acoustic oscillations in the early universe.

Figure 1 displays the temperature and polarization power spectra and the cross-correlation between temperature and polarization for a typical inflationary model. The multipole moment corresponds to an angle on the sky and to a flat-universe comoving length scale at the surface of last scattering which is imaged by the microwave background. The peak structure in the spectra result from coherent acoustic oscillations. Microwave background polarization couples almost exclusively to velocity perturbations and not to density perturbations, while the peaks in the temperature power spectrum arise almost completely from density perturbations. The main point of this paper is that for coherent acoustic oscillations, the alternating temperature and polarization peaks are precisely interleaved. The acoustic modes have the same initial phase but different oscillation frequencies depending on the wavelength of the mode. Thus at the surface of last scattering, the phase of acoustic oscillations varies smoothly with wavelength, resulting in alternating temperature (density) and polarization (velocity) peaks as a function of . As an additional check, the extrema in the cross correlation must fall between the polarization and temperature peaks. The remainder of this paper is devoted to a quantitative derivation of this signature.

Three basic effects must be understood: (1) Polarization is generated through Thomson scattering, and only the quadrupole moment of an incoming unpolarized radiation field can be scattered into a polarized outgoing field; (2) The radiation field just before decoupling has only monopole and dipole angular dependence, and a non-zero quadrupole is generated through free-streaming of the radiation; (3) The specific form of the acoustic modes fixes the spatial dependence of the initial monopole and dipole radiation fields. Combining these three ingredients results in the acoustic signature in question. Each of these three points will be considered in turn.

Polarization in the microwave background is generated through polarization-dependent Thomson scattering. Consider Thomson scattering of an incoming monochromatic electromagnetic plane wave by an electron. The total scattering cross-section is given by

\[ \frac{d\sigma}{d\Omega} = \frac{3\sigma_T}{8\pi} |\hat{\epsilon}' \cdot \hat{\epsilon}|^2 \]  \hspace{1cm} (1)

where \( \sigma_T \) is the total Thomson cross section and the vectors \( \hat{\epsilon} \) and \( \hat{\epsilon}' \) are unit vectors in the planes perpendicular to the propagation directions which are aligned with the outgoing and incoming linear polarization, respectively. Unpolarized radiation may be considered as the superposition of two plane waves with equal amplitudes and phases but orthogonal polarizations. A straightforward calculation gives the net polarization produced by the scattering of an unpolarized radiation field of intensity \( I(\theta, \phi) \) incident on a small volume containing an electron as
FIG. 1. The power spectra for temperature fluctuations (top), polarization (center) and temperature-polarization cross-correlation (bottom) for a typical inflationary model. The oscillations remain in phase up to $l = 3000$. 
\[ Q(\hat{z}) - iU(\hat{z}) = \frac{3\sigma_T}{16\pi\sigma_B} \int d\Omega \sin^2 \theta e^{i2\phi} I'(\Omega), \]  

where \( \sigma_B \) is the cross-sectional area of the small test volume, the \( z \)-axis is taken as the scattering direction, and \( Q \) and \( U \) are the Stokes parameters describing linear polarization. Expanding the incident radiation field in spherical harmonics,

\[ I'(\theta, \phi) = \sum_{lm} a_{lm} Y_{lm}(\theta, \phi), \]

gives the outgoing Stokes parameters in terms of the multipole coefficients as

\[ Q(\hat{z}) - iU(\hat{z}) = \frac{3\sigma_T}{4\pi\sigma_B} \sqrt{\frac{2\pi}{15}} a_{22}. \]

Thus polarization is generated along the outgoing \( z \)-axis provided that the \( a_{22} \) quadrupole moment of the incoming radiation is non-zero. To determine the outgoing polarization in an arbitrary direction \( \hat{n} \) making an angle \( \beta \) with the \( z \)-axis, the same physical incoming field must be multipole expanded in a coordinate system rotated through the Euler angle \( \beta \). If the incoming radiation field is independent of \( \phi \), as it will be for individual Fourier components of a density perturbation, then

\[ Q(\hat{n}) - iU(\hat{n}) = \frac{3\sigma_T}{16\pi\sigma_B} \sqrt{\frac{4\pi}{5}} a_{20} \sin^2 \beta \]

which can be derived using explicit expressions for the \( l = 2 \) components of the rotation matrix. In other words, an azimuthally-symmetric radiation field will generate a polarized scattered field if it has a non-zero \( a_{20} \) multipole component, and the magnitude of the scattered polarization will be proportional to \( \sin^2 \beta \). Since the incoming field is real, \( a_{20} \) will be real, \( U = 0 \), and the polarization orientation will be along longitudes of the coordinate system \([4]\).

Moving on to the second part of the calculation, where does an incident quadrupole radiation field come from? Before decoupling, the radiation is tightly coupled to the baryons in the universe by rapid Thomson scattering and the radiation field possesses only monopole (from density or gravitational potential perturbations) and dipole (from a Doppler shift) angular components; thus the radiation is unpolarized. A quadrupole is subsequently produced at the radiation field begins at time \( \eta_* \), then the solution at a later time is simply \( \Theta(k, \mu, \eta) = \Theta(k, \mu, \eta_*) \exp(-ik\mu(\eta - \eta_*)) \). Reexpressing the \( \mu \) dependence as a multipole expansion,

\[ \Theta(k, \mu, \eta) = \sum_{l=0}^{\infty} (-i)^l \Theta_l(k, \eta) P_l(\mu), \]

the free streaming becomes

\[ \Theta_l(k, \eta) = (2l + 1)[\Theta_0(k, \eta_*) j_l(k\eta - k\eta_*) + \Theta_1(k, \eta_*) j_l'(k\eta - k\eta_*)], \]

where \( j_l \) is the usual spherical Bessel function.

We are interested in the behavior of the free streaming in the at times near decoupling; at later times, the number density of free electrons which can Thomson scatter has dropped to negligible levels and no further polarization can be produced. The physical length scales of interest for microwave background fluctuations will be larger than the thickness of the last scattering surface, so \( k(\eta - \eta_*) \) will be small compared to unity. For small arguments \( x \ll 1 \), \( j_l(x)/j_l'(x) \sim x/l \), which implies that if the monopole and dipole radiation components are initially of comparable size, free streaming through the region of polarization generation with thickness \( \Delta \) will generate a quadrupole component from the dipole which is a factor of \( 2/(k\Delta) \) larger than the quadrupole component from the monopole. In other words, on length scales large compared to the thickness of the surface of last scattering, the quadrupole moment and thus the polarization couples much more strongly to the velocity of the baryon-photon fluid than to the density. Note that on smaller scales with \( k\Delta \gtrsim 1 \), the polarization can couple more strongly to either the velocity or the density, depending
on the scale, but for standard recombination these scales are always small enough that the microwave background fluctuations are strongly diffusion damped.

For the third part of the calculation, we need to know the mathematical form for an acoustic oscillation. As emphasized in Ref. [3], the photon-baryon density perturbation in the tight-coupling regime obeys the differential equation for a forced, damped harmonic oscillator with the damping coming from the expansion of the universe and the forcing from gravitational potential perturbations. The solution is of the form

\[ \Theta_0(k, \eta) = A_1(\eta) \cos(kr_s) + A_2(\eta) \sin(kr_s) \]  

(8)

where the amplitudes vary slowly in time and \( r_s \approx \eta/\sqrt{3} \) is the sound horizon. The velocity perturbation follows from the photon continuity equation \( \dot{\Theta}_0 = -k\Theta_1/3 \), again neglecting gravitational potential perturbations. A detailed consideration of boundary conditions reveals that initial isentropic density perturbations couple to the cosine harmonic in the small-scale limit, and this approximation is good even for the largest-wavelength acoustic oscillations [6]. Thus in an inflationary model, at the surface of last scattering, the photon monopole has a \( k \)-dependence of approximately \( \cos(k\eta_s/\sqrt{3}) \), while the dipole, which is the main contributor to the polarization, has a \( k \)-dependence of approximately \( \sin(k\eta_s/\sqrt{3}) \). For initial isocurvature perturbations, the density perturbations couple instead to the sine harmonic, but the photon monopole and dipole are still \( \pi/2 \) out of phase.

From this scale dependence, it is simple to read off the relative positions of the various acoustic peaks in the power spectra. In general, the amplitude of the velocity perturbations are suppressed by a factor of \( c_s \) with respect to the density perturbations, and the acoustic peaks in the power spectrum are dominated by the monopole temperature fluctuation. The temperature power spectrum is roughly the square of the temperature fluctuation, so acoustic peaks in the power spectrum will show up at scales where \( \cos^2(k\eta_s/\sqrt{3}) \) has its maxima. Likewise, the polarization coupling to the temperature dipole on scales larger than the thickness of the last scattering surface, and acoustic peaks in the polarization power spectrum will be present at scales where \( \sin^2(k\eta_s/\sqrt{3}) \) has its maxima, just the scales between the temperature peaks. Finally, the cross-correlation between the temperature and polarization will have extrema as \( -\cos(k\eta_s/\sqrt{3}) \sin(k\eta_s/\sqrt{3}) \) which fall between the temperature and polarization peaks. (The correlation between the polarization and the velocity contribution to the temperature averages to zero because of their different angular dependences.) All of these peak positions are evident in Fig. 1. The sign of the cross-correlation peaks can be used to deduce whether a temperature peak represents a compression or a rarefaction, which can be checked against the alternating peak-height signature if the universe has a large enough baryon fraction [5].

A combination of isentropic and isocurvature fluctuations shifts all acoustic phases by the same amount if the ratio of their amplitudes is independent of scale, thus leaving the acoustic signature intact. If the amplitude ratio depends on scale, the coherent acoustic oscillations could be modified, but fine tuning would be required to wash them out completely. Multi-field inflation models generically produce both isocurvature and isentropic perturbations [7], but the resulting microwave background power spectra are just beginning to be studied in detail [8].

Inflation unavoidably produces coherent acoustic oscillations due to isentropic perturbations on all scales. Defect models can in principle generate coherent oscillations, but the evolution of the defects must not destroy the phase coherence of a given oscillation mode [6] so the resulting models are rather artificial. One example is the large-\( N \) limit of the \( O(N)\) \( \sigma \)-model [10]. But despite freedom to choose five functions defining a general causal scaling solution for the evolution of a defect [11], it seems highly unlikely that an inflation-like set of \( C_l \) spectra can be closely matched [12]. Polarization, coupling only to velocities and not to density or gravitational fluctuations, provides a particularly stringent discriminator. It seems virtually impossible for any model with causally generated sources to reproduce the small first polarization acoustic peak (around \( l \approx 80 \) in a flat universe) [13].

So polarization provides a model-independent signature of coherent acoustic oscillations at the time of last scattering: acoustic peaks in the polarization power spectrum should fall almost precisely between the acoustic peaks in the temperature power spectrum, and the extrema of the temperature-polarization cross-correlation fall between the temperature and polarization acoustic peaks. This signature is the easiest polarization information to extract from the microwave background since it depends only on the largest-amplitude features of the polarization fluctuations. Hopefully the prospect of seeing sound waves in the early universe will provide further impetus to ongoing efforts to detect microwave background polarization.

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