Supporting Information

Friction Dynamics of Foams under Nonlinear Motion

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1. The size of the bubbles in the friction interface. Figure S1 shows the particle size distributions of the bubbles before friction evaluation. Using a microscope (Hozan Tool Ind. Co., Ltd., Osaka, Japan), the state of the bubbles before friction was observed at a magnification ratio of 3, and the particle size of the bubbles was determined by measuring the Feret diameter. In the case of foam 1 containing no thickener, the peak of the particle size distribution was broad, and bubbles of approximately 0.41 mm were observed. Conversely, in the case of bubbles 2 and 3 containing thickener, the peak of the particle size distribution was sharp, and many bubbles of 0.06–0.11 mm were observed. Figure S2 shows the average particle size of bubbles before friction evaluation of each normal force. No significant change in average particle size was observed when the normal force was changed. However, in the case of foam 1, the variation in average particle size was larger than that in the two foams. The result suggests that the addition of the thickener, cationized cellulose, stabilizes the foam. The average particle sizes of foams 1–3 were 0.16 ± 0.12, 0.11 ± 0.10, and 0.08 ± 0.05 mm, respectively.

2. The friction properties of foams under sinusoidal motion. Table S1 shows the friction parameters obtained in the friction evaluation of foams under sinusoidal motion. The parameters include average friction force, average friction coefficient, delay time δ, a, n, and R².

3. The friction properties of surfactant aqueous solutions under sinusoidal motion. Figure S3 shows the relationship between the average friction force and the normal force on the friction evaluation of surfactant aqueous solutions. Figure S4 shows the delay time δ for the first cycle. Figure S5 shows the dependence of normal force and angular velocity on the number of reciprocations where an unstable pattern was observed. Table S2 shows the friction parameters obtained in the friction evaluation of surfactant aqueous solution under sinusoidal motion. The parameters include average friction force, average friction coefficient, delay time δ, a, n, and R².

4. The friction properties of foam containing only SDS. Figure S6 shows the relationship between the average friction force and the normal force. The average friction forces of foam containing 20 wt% SDS at ω = 2.1 rad s⁻¹ were 0.04–0.05 N and similar to those of foam 1. The average friction force was analyzed on the basis of the power law. a and n were 0.049 and 0.140, respectively. Figure S7 shows the delay times, δ, for the first cycle. The delay times at ω = 2.1 rad s⁻¹ were 0.016–0.026. Figure S8 shows the dependence of normal force on the number of reciprocations where an unstable pattern was observed. In the case of foam containing 20 wt% SDS, the number of conditions in which the hydrodynamic stable pattern appeared was less than that of foam 1. This result suggests that the addition of myristic acid improves the durability of the foam. It was reported that the fatty acid leads to a significant decrease in surface tension and increase in surface modulus [Mitrinova, Z. et al. Surface and foam properties of SLES + CAPB + fatty acid mixtures: Effect of pH for C12–C16 acids. Coll. Surf. A 2013, 438(5), 186-198.]. Table S3 shows the friction parameters obtained in the friction evaluation of foam containing only 20 wt% SDS under sinusoidal motion. The parameters include average friction force, average friction coefficient, delay time δ, a, n, and R².
Figure S1 The occurrence frequency (rectangle) and the cumulative frequency (cross) for the particle size distribution; (a) foam 1, (b) foam 2, and (c) foam 3.

Figure S2 The average particle size of bubbles before friction evaluation for each normal force; (○) foam 1, (△) foam 2, and (◇) foam 3.
Table S1 Values of average friction force, average friction coefficient, delay time $\delta$, $a$, $n$, and $R^2$ for friction of foams under sinusoidal motion.

| Angular rate / rad s$^{-1}$ | Normal force / N | Average friction force / N | Average friction coefficient / - | $\delta$ / - | $a$ | $n$ | $R^2$ |
|-----------------------------|------------------|---------------------------|-------------------------------|----------|-----|-----|------|
| 0.1                         | 0.20             | 0.01 ± 0.00               | 0.06 ± 0.01                   | 0.030 ± 0.009 |
|                             | 0.39             | 0.02 ± 0.01               | 0.04 ± 0.02                   | 0.018 ± 0.019 |
|                             | 0.59             | 0.01 ± 0.00               | 0.02 ± 0.00                   | 0.023 ± 0.014 |
|                             | 0.98             | 0.01 ± 0.00               | 0.01 ± 0.00                   | 0.019 ± 0.019 |
|                             | 1.47             | 0.02 ± 0.01               | 0.01 ± 0.01                   | 0.021 ± 0.015 |
| 1.0                         | 0.20             | 0.03 ± 0.00               | 0.15 ± 0.00                   | 0.005 ± 0.003 |
|                             | 0.39             | 0.03 ± 0.00               | 0.07 ± 0.00                   | 0.006 ± 0.004 |
|                             | 0.59             | 0.03 ± 0.00               | 0.05 ± 0.00                   | 0.007 ± 0.004 |
|                             | 0.98             | 0.03 ± 0.00               | 0.03 ± 0.00                   | 0.009 ± 0.004 |
|                             | 1.47             | 0.04 ± 0.00               | 0.03 ± 0.00                   | 0.011 ± 0.007 |
| 2.1                         | 0.20             | 0.04 ± 0.00               | 0.22 ± 0.01                   | 0.011 ± 0.004 |
|                             | 0.39             | 0.04 ± 0.00               | 0.11 ± 0.01                   | 0.004 ± 0.004 |
|                             | 0.59             | 0.04 ± 0.00               | 0.08 ± 0.00                   | 0.005 ± 0.003 |
|                             | 0.98             | 0.05 ± 0.00               | 0.05 ± 0.00                   | 0.005 ± 0.003 |
|                             | 1.47             | 0.05 ± 0.01               | 0.03 ± 0.00                   | 0.004 ± 0.003 |
| 3.0                         | 0.20             | 0.01 ± 0.00               | 0.06 ± 0.00                   | 0.032 ± 0.008 |
|                             | 0.39             | 0.01 ± 0.00               | 0.03 ± 0.00                   | 0.032 ± 0.012 |
|                             | 0.59             | 0.01 ± 0.00               | 0.02 ± 0.00                   | 0.030 ± 0.009 |
|                             | 0.98             | 0.01 ± 0.00               | 0.01 ± 0.00                   | 0.023 ± 0.016 |
|                             | 1.47             | 0.01 ± 0.00               | 0.01 ± 0.00                   | 0.026 ± 0.014 |
| 4.0                         | 0.20             | 0.05 ± 0.00               | 0.24 ± 0.00                   | 0.017 ± 0.002 |
|                             | 0.39             | 0.05 ± 0.00               | 0.13 ± 0.01                   | 0.017 ± 0.002 |
|                             | 0.59             | 0.05 ± 0.00               | 0.08 ± 0.00                   | 0.016 ± 0.002 |
|                             | 0.98             | 0.05 ± 0.00               | 0.05 ± 0.00                   | 0.016 ± 0.002 |
|                             | 1.47             | 0.05 ± 0.00               | 0.03 ± 0.00                   | 0.016 ± 0.003 |
| 5.0                         | 0.20             | 0.06 ± 0.00               | 0.32 ± 0.01                   | 0.016 ± 0.001 |
|                             | 0.39             | 0.06 ± 0.00               | 0.16 ± 0.00                   | 0.012 ± 0.003 |
|                             | 0.59             | 0.07 ± 0.00               | 0.12 ± 0.00                   | 0.013 ± 0.003 |
|                             | 0.98             | 0.07 ± 0.00               | 0.07 ± 0.00                   | 0.013 ± 0.003 |
|                             | 1.47             | 0.07 ± 0.00               | 0.05 ± 0.00                   | 0.013 ± 0.003 |
| 6.0                         | 0.20             | 0.02 ± 0.00               | 0.07 ± 0.01                   | 0.016 ± 0.010 |
|                             | 0.39             | 0.02 ± 0.00               | 0.04 ± 0.00                   | 0.012 ± 0.003 |
|                             | 0.59             | 0.02 ± 0.00               | 0.03 ± 0.00                   | 0.007 ± 0.009 |
|                             | 0.98             | 0.02 ± 0.00               | 0.02 ± 0.00                   | 0.016 ± 0.006 |
|                             | 1.47             | 0.02 ± 0.00               | 0.02 ± 0.00                   | 0.012 ± 0.008 |
| 7.0                         | 0.20             | 0.06 ± 0.00               | 0.27 ± 0.01                   | 0.016 ± 0.004 |
|                             | 0.39             | 0.07 ± 0.00               | 0.18 ± 0.01                   | 0.014 ± 0.002 |
|                             | 0.59             | 0.08 ± 0.00               | 0.14 ± 0.01                   | 0.013 ± 0.002 |
|                             | 0.98             | 0.08 ± 0.00               | 0.08 ± 0.00                   | 0.013 ± 0.002 |
|                             | 1.47             | 0.08 ± 0.00               | 0.06 ± 0.00                   | 0.013 ± 0.003 |
| 8.0                         | 0.20             | 0.07 ± 0.00               | 0.35 ± 0.01                   | 0.014 ± 0.002 |
|                             | 0.39             | 0.10 ± 0.01               | 0.25 ± 0.01                   | 0.011 ± 0.002 |
|                             | 0.59             | 0.11 ± 0.00               | 0.18 ± 0.01                   | 0.012 ± 0.003 |
|                             | 0.98             | 0.12 ± 0.00               | 0.12 ± 0.00                   | 0.011 ± 0.002 |
|                             | 1.47             | 0.12 ± 0.00               | 0.08 ± 0.00                   | 0.010 ± 0.002 |
Table S2 Values of average friction force, average friction coefficient, delay time $\delta$, $a$, $n$, and $R^2$ for the friction of surfactant aqueous solutions under sinusoidal motion.

| Angular rate / rad s$^{-1}$ | Normal force / N | Average friction force / N | Average friction coefficient / - | $\delta$ / - | $a$ | $n$ | $R^2$ |
|-----------------------------|------------------|-----------------------------|-----------------------------|-------------|-----|-----|-------|
| 0.20 | 0.04 ± 0.00 | 0.20 ± 0.02 | 0.005 ± 0.002 | 0.39 | 0.10 ± 0.00 | 0.25 ± 0.01 | 0.013 ± 0.001 |
| 0.59 | 0.13 ± 0.00 | 0.23 ± 0.01 | 0.013 ± 0.004 | 0.98 | 0.22 ± 0.01 | 0.23 ± 0.01 | 0.016 ± 0.000 |
| 1.47 | 0.35 ± 0.01 | 0.23 ± 0.01 | 0.018 ± 0.000 |
| 0.20 | 0.05 ± 0.00 | 0.26 ± 0.00 | 0.004 ± 0.002 | 0.39 | 0.10 ± 0.00 | 0.25 ± 0.01 | 0.013 ± 0.000 |
| 0.59 | 0.15 ± 0.00 | 0.24 ± 0.00 | 0.012 ± 0.001 | 0.98 | 0.24 ± 0.01 | 0.24 ± 0.01 | 0.014 ± 0.001 |
| 1.47 | 0.36 ± 0.01 | 0.24 ± 0.01 | 0.015 ± 0.001 |
| 0.20 | 0.02 ± 0.00 | 0.10 ± 0.01 | 0.007 ± 0.007 | 0.39 | 0.10 ± 0.02 | 0.24 ± 0.04 | 0.017 ± 0.014 |
| 0.59 | 0.09 ± 0.00 | 0.15 ± 0.00 | 0.009 ± 0.000 | 0.98 | 0.17 ± 0.01 | 0.17 ± 0.01 | 0.011 ± 0.000 |
| 1.47 | 0.18 ± 0.06 | 0.12 ± 0.04 | 0.009 ± 0.005 |
| 0.20 | 0.05 ± 0.01 | 0.26 ± 0.03 | 0.038 ± 0.055 | 0.39 | 0.04 ± 0.04 | 0.10 ± 0.10 | 0.064 ± 0.089 |
| 0.59 | 0.03 ± 0.02 | 0.05 ± 0.03 | 0.007 ± 0.005 | 0.98 | 0.17 ± 0.08 | 0.18 ± 0.09 | 0.014 ± 0.004 |
| 1.47 | 0.34 ± 0.04 | 0.23 ± 0.03 | 0.018 ± 0.002 |
| 0.20 | 0.07 ± 0.03 | 0.34 ± 0.15 | 0.013 ± 0.001 | 0.39 | 0.18 ± 0.02 | 0.47 ± 0.05 | 0.015 ± 0.002 |
| 0.59 | 0.13 ± 0.01 | 0.22 ± 0.01 | 0.013 ± 0.001 | 0.98 | 0.16 ± 0.06 | 0.16 ± 0.06 | 0.014 ± 0.001 |
| 1.47 | 0.25 ± 0.04 | 0.17 ± 0.02 | 0.015 ± 0.001 |
| 0.20 | 0.05 ± 0.01 | 0.26 ± 0.03 | 0.054 ± 0.059 | 0.39 | 0.10 ± 0.05 | 0.25 ± 0.14 | 0.052 ± 0.068 |
| 0.59 | 0.17 ± 0.01 | 0.29 ± 0.02 | 0.011 ± 0.003 | 0.98 | 0.16 ± 0.02 | 0.16 ± 0.02 | 0.010 ± 0.002 |
| 1.47 | 0.26 ± 0.01 | 0.17 ± 0.00 | 0.012 ± 0.001 |
| 0.20 | 0.01 ± 0.00 | 0.03 ± 0.00 | 0.032 ± 0.014 | 0.39 | 0.01 ± 0.00 | 0.02 ± 0.00 | 0.061 ± 0.037 |
| 0.59 | 0.01 ± 0.00 | 0.02 ± 0.01 | 0.056 ± 0.027 | 0.98 | 0.02 ± 0.00 | 0.02 ± 0.00 | 0.048 ± 0.030 |
| 1.47 | 0.03 ± 0.01 | 0.02 ± 0.00 | 0.042 ± 0.041 |
| 0.20 | 0.02 ± 0.00 | 0.08 ± 0.00 | 0.029 ± 0.013 | 0.39 | 0.03 ± 0.00 | 0.07 ± 0.00 | 0.019 ± 0.006 |
| 0.59 | 0.03 ± 0.00 | 0.06 ± 0.00 | 0.018 ± 0.004 | 0.98 | 0.04 ± 0.00 | 0.05 ± 0.00 | 0.022 ± 0.011 |
| 1.47 | 0.06 ± 0.00 | 0.04 ± 0.00 | 0.022 ± 0.010 |
| 0.20 | 0.03 ± 0.00 | 0.12 ± 0.01 | 0.015 ± 0.005 | 0.39 | 0.04 ± 0.00 | 0.10 ± 0.00 | 0.015 ± 0.005 |
| 0.59 | 0.05 ± 0.00 | 0.08 ± 0.00 | 0.012 ± 0.003 | 0.98 | 0.07 ± 0.00 | 0.07 ± 0.00 | 0.013 ± 0.005 |
| 1.47 | 0.08 ± 0.00 | 0.05 ± 0.00 | 0.013 ± 0.005 |
Figure S3 The average friction force at each normal force: $\omega = 0.1$ (rhombus), 1.0 (triangle), 2.1 rad s$^{-1}$ (circle). (a)–(c) Surfactant aqueous solutions 1, 2, and 3.
Figure S4 The delay time, $\delta$, at each normal force: $\omega = 0.1$ (rhombus), 1.0 (triangle), 2.1 rad s$^{-1}$ (circle). (a)–(c) Surfactant aqueous solutions 1, 2, and 3.
Figure S5 Dependence of normal force and angular velocity on the number of reciprocations when unstable pattern was observed. (a)–(c) Surfactant aqueous solutions 1, 2, and 3.
Figure S6 The average friction force at each normal force: 2.1 rad s$^{-1}$ (circle). Lubricant, foam containing 20 wt% SDS.

Figure S7 The delay time, $\delta$, at each normal force: 2.1 rad s$^{-1}$ (circle). Lubricant, foam containing 20 wt% SDS.

Figure S8 Dependence of normal force on the number of reciprocations when unstable pattern was observed. Lubricant, foam containing 20 wt% SDS.
Table S3 Values of average friction force, average friction coefficient, delay time $\delta$, $a$, $n$, and $R^2$ for friction of foam containing 20 wt% SDS under sinusoidal motion.

| Angular rate / rad s$^{-1}$ | Normal force / N | Average friction force / N | Average friction coefficient / - | $\delta$ / - | $a$ | $n$ | $R^2$ |
|----------------------------|-----------------|----------------------------|---------------------------------|-------------|-----|-----|-------|
| 20 wt% SDS 2.1             | 0.20            | 0.04 ± 0.00                | 0.23 ± 0.01                     | 0.016 ± 0.004 |     |     |       |
|                             | 0.39            | 0.04 ± 0.00                | 0.09 ± 0.00                     | 0.024 ± 0.006 |     |     |       |
|                             | 0.59            | 0.04 ± 0.00                | 0.07 ± 0.01                     | 0.021 ± 0.007 | 0.049 | 0.140 | 0.528 |
|                             | 0.98            | 0.05 ± 0.00                | 0.05 ± 0.00                     | 0.026 ± 0.011 |     |     |       |
|                             | 1.47            | 0.05 ± 0.00                | 0.04 ± 0.00                     | 0.016 ± 0.010 |     |     |       |

5. Examination of the temporal profiles of the friction and normal forces by Fourier transform.

Fine oscillations were observed in the temporal profiles of the friction and normal forces in Figure 2, as shown in the inset. The temporal profile of the normal force in Figure 2a, in particular, appears to be a somewhat chaotic pattern because it appears to be beating. Analytical results of the Fourier transform of the normal and friction forces were used to test its feasibility. As a result, it was discovered that the oscillation is primarily caused by foams, as shown below, with periodical changes in amplitude synchronized with the sinusoidal motion.

The analytical results of the Fourier transform of the normal and friction forces of foam 2 in Figure 2a are shown in Figures S9a and b, respectively. Peaks at around 50 and 80 Hz can be seen in the analytical results. Figures S9c and d, on the other hand, show these surfactant aqueous solution 2 without foams in Figure 2b. There are a few small peaks around 80 Hz and almost no peaks around 50 Hz. Because the peaks at around 80 Hz are also found in surfactant aqueous solution 2, it is possible to conclude that the peaks at around 50 Hz in Figures S9a and b are caused by foam friction and the peaks at around 80 Hz are caused by the noise generated by the sinusoidal motion friction evaluation system.
Figure S9 Analytical results (power spectrum) of the Fourier transform of the normal force (upper panels) and friction force (lower panels). (a) Normal force of foam \( \tilde{z} \), (b) friction force of foam \( \tilde{z} \), (c) normal force of surfactant aqueous solution \( \tilde{z} \), and (d) friction force of surfactant solution \( \tilde{z} \).

Consider why the peaks appear at around 50 Hz. In this case, we assume that foams (diameter: \( d \)) is packed between two plates and sheared along the homeward direction, with an average velocity of the motion of \( v_{\text{av}} \). Friction happens at the interface between bubbles. One of the situations is depicted in Figure S10a.

According to Figure S10a, the observed oscillation in friction could be interpreted as motion in a periodical potential (washboard-type potential) as depicted in Figure S10b. The frequency \( f \) is related to \( v_{\text{av}} \) and \( d \) as follows:

\[
d = \frac{v_{\text{av}}}{f}
\]  

(S1)

Using \( f = 50 \text{ Hz} \) and \( 20 \text{ mm s}^{-1} \), \( d \) is estimated to be 0.4 mm. Because the actual diameter of foams is in Figures 7b-1 and b-2 is around 0.3 mm, this could explain the observed peaks at 50 Hz. This discussion is valid if the time scale in deformation and recovery of foams in sinusoidal motion is longer than the period of the oscillation because oscillations at around 50 Hz were also observed around the sliding velocity \( \sim 0 \text{ mm s}^{-1} \) and do not depend on the sliding velocity. This would be an aspect of the viscoelastic properties of foams.

In addition to the above discussion, the amplitudes of the entire normal force profile in Figures 2a and b appear to be chaotic. The periodical changes in the amplitudes, on the other hand, were nearly
synchronized with the sliding velocity of the sinusoidal motion, and the amplitudes strongly correlate with the velocity. This would also reflect a different aspect of foam viscoelastic properties.

Figure S10 (a) Sheared foams sandwiched in the homeward direction by two plates. The upper and lower layers are represented by red and black, respectively. Friction is assumed to occur at the boundary between two layers. (b) Friction in tilted periodical potential (washboard-type potential) along the homeward direction. The red circle in (a) is the same as the red circle in (b).