Dynamic Sliding Mode Control based on Fractional calculus subject to uncertain delay based chaotic pneumatic robot

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ABSTRACT

This paper considers the chattering problem of sliding mode control while delay in robot manipulator caused chaos in such electromechanical systems. Fractional calculus as a powerful theorem to produce a novel sliding mode; which has a dynamic essence; is used for chattering elimination. To realize the control of a class of chaotic systems in master-slave configuration this novel fractional dynamic sliding mode control scheme is presented and examined on delay based chaotic robot in joint and workspace. Also the stability of the closed-loop system is guaranteed by Lyapunov stability theory. Beside these, delayed robot motions are sorted out for qualitative and quantificational study. Finally, the numerical simulation example illustrates the feasibility of the proposed control method.

1. Introduction

Inherent insensitivity characteristic of Sliding Mode Control (SMC) [1, 2] to dynamical system uncertainty makes it as an acceptable controller. Although, its known chattering problem even makes wear and tear in actuators where lead to combine whichever techniques with this controller for attenuating of chattering, such as boundary layer, fuzzy, observer, relay control gain [3-6], but do not confirm sliding mode condition exactly. Dynamic SMC (DSMC) [7-12]; a generalized controller of SMC is a powerful chattering reduction and is based on injecting dynamics into the major system. Also, fractional DSMC (FDSMC) which is established on fractional derivatives, is propounded for a class of chaotic system in a master-slave configuration and particularly presented on a robot manipulator when delay exists as an inseparable content of actual systems which leads to chaotic motions. Fractional derivatives [13-17] by its tuning order vs. traditional integer types were well examined in many controllers either in theoretical and industrial applications [18-23], furthermore became more appropriate in system modeling [24, 25]. Chaos [26-31], can be achieved by time delays, external inputs, system parameters and controllers in two-link robot manipulators [32-35], which have much more important role in many different situations. To realize; task and joint space of chaotic dead time two-link robot motions [32] and qualitative and quantificational characteristics such as phase attractor, time series, maximum Lyapunov exponent, bifurcation diagram with respect to dead time and Poincaré map are investigated. Finally fractional proposed controller eventuates chattering elimination and tracks the same periodic two-link manipulator in a finite time even in existence of high uncertainty. This paper is organized as follows: next section prepares some preliminaries of fractional calculus. In section 3 fractional dynamic sliding mode controller is proposed. The dynamic of two-link robot and their control method studies in section 4. Moreover, numerical simulation results are presented in section 5 to show the effectiveness of the proposed method. Finally, in section 6 the paper is concluded.

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2. Preliminaries

Fractional differintegration’s set foot in control compass, also consistency of Caputo definition among the other fractional derivative’s definitions with engineering applications, is caused to devote this section to the basic definition of fractional calculus and its properties (Table 1). The Riemann–Liouville (RL) and the Caputo are the most popular fractional derivatives definitions.

**RL definition:**

\[ a D^q_t f(t) = \frac{1}{\Gamma(n-q)} \int_{t}^{\infty} f^\prime(\tau) (\tau-t)^{n-q-1} d\tau, \quad (n-1) \leq q < n \]

(1)

**Caputo definition:**

\[ a D^q_t f(t) = \frac{1}{\Gamma(n-q)} \int_{t}^{\infty} f^{(n)}(\tau) (\tau-t)^{n-q-1} d\tau, \quad (n-1) \leq q < n \]

(2)

Where \( D^q_t \) denotes the \( q \)th order of differintegral operator, \( a \) and \( t \) are the limits of operation, \( \Gamma(x) \) is the well-known Gamma function and \( n \) is the first integer not less than \( q \), i.e. \( n - 1 < q < n \).

| Caputo Definition | Riemann-Liouville Definition |
|-------------------|-----------------------------|
| \( \zeta D^q_t A = 0 \) can be zero if \( A = \text{cte} \) | \( \zeta D^q_t \frac{d}{dt} f(t) = \zeta D^q_t f(t) \) can be zero if \( f^{(k)}(0) = 0 \) for \( k = 0, 1, 2, \ldots, n-1 \) and \( \lambda < \) |
| \( \frac{d}{dt} f(t) = \zeta D^q_t f(t) \) for \( \alpha = \beta \) | \( \frac{d}{dt} f(t) = \zeta D^q_t f(t) \) for \( \alpha = \beta \) |
| \( \zeta D^q_t f(t) = \zeta D^q_t f(t) \) for \( \alpha = \beta \) | \( \zeta D^q_t f(t) = \zeta D^q_t f(t) \) for \( \alpha = \beta \) |
| \( \frac{d}{dt} f(t) = \zeta D^q_t f(t) \) for \( \alpha = \beta \) | \( \frac{d}{dt} f(t) = \zeta D^q_t f(t) \) for \( \alpha = \beta \) |

3. Fractional Dynamic Sliding Mode Control

3.1 System description

Consider a MIMO nonlinear differential system described by:

\[ \dot{x} = f(t,x,u) + \Delta f(t,x,u) + d_0(t) \]
\[ z = h(t,x,u) \]

Where \( x \in \mathbb{R}^m \) is the state vector, \( u \in \mathbb{R}^m \) is a control input vector, \( y \in \mathbb{R}^p \) is the output vector and \( f(t,x,u), g(t,x,u) \) are smooth nonlinear functions.

Defining

\[ d(t,x,u) = \Delta f(t,x,u) + d_0(t) \]

as a continuous-differential and \( \dot{d}(t,x,u) \) are limited functions.

3.2 Design of Fractional Dynamic Sliding Mode Control (FDSMC)

Consider the Eq. (3) as a following master system:

\[ x^{(\alpha)} = f(x,t) \]

(5)

And the following slave system with input controls:

\[ y^{(\beta)} = g(y,t) + b(y,t)u \]

(6)
Where
\[
0 < b_{\min} \leq b \leq b_{\max}, \quad 0 < a_{\min} \leq \dot{b} \leq a_{\max},
\]
\[
\dot{b} = (b_{\min} / b_{\max})^{1/2}, \quad \dot{\beta}^{-1} \leq b \dot{\beta}^{-1} \leq \dot{\beta}, \quad \dot{\beta} = (b_{\max} / b_{\min})^{1/2}
\]
and the tracking error is:
\[
e = y - x
\]

**First Step** (Existence Problem) designing a switching function which provides desirable system performance:
\[
S = \phi(t, \dot{\epsilon}, D^{k-1}\dot{\epsilon}, ..., D^{(n+1-k)}\dot{\epsilon}, u)
\]
\[
= k_1\dot{\epsilon} + k_2 D\dot{\epsilon} + k_3 D^2\dot{\epsilon} + ... + k_n D^{n+1-k}\dot{\epsilon}
\]
where \(k_i > 0, \ i = 1, 2, ..., n\).

**Second Step** (Reachability Problem) consists of obtaining a control law from that of the first order derivative of the control input, is contained within the system states and also control input, so that the dynamics of the system are added. Taking the time derivative of Eq. (9) could obtain:
\[
\dot{S} = \phi_t(t, \dot{\epsilon}, D\dot{\epsilon}, ..., D^{n+1-k}\dot{\epsilon}, u)
\]
\[
= k_1\ddot{\epsilon} + k_2 D\ddot{\epsilon} + k_3 D^2\ddot{\epsilon} + ... + k_n D^{n+1-k}\ddot{\epsilon}
\]

**Lemma:** The motion of the sliding mode is asymptotically stable, if the following condition is held \(\dot{S}S < 0\).

**Proof:** Consider the following Lyapunov candidate function:
\[
V = \frac{1}{2} S^2
\]
by the following condition:
\[
\dot{S} = -K \cdot sgn(S)
\]
where \(sgn(S)\) is signum function:
\[
sgn(S) = \begin{cases} 
  +1, & \text{if } S > 0, \\
  0, & \text{if } S = 0, \\
  -1, & \text{if } S < 0.
\end{cases}
\]
The time derivative of Eq. (11) is: \(\dot{V} = \dot{S}S = -K.sgn(S), \) since \(S.sgn(S) > 0 \) and \(K > 0\) we have \(\dot{V} = SS < 0\), hence \(V\) is negative definite, then \(\dot{S}(t)\) is toward the switching surface and sliding mode is asymptotically stable.

The controller is then solved out from \(\dot{S} = -K.sgn(S)\). First order derivative of the control law is as follows:
\[
\dot{u} = h(\dot{\epsilon}, \dot{\epsilon}, ..., D^{n}\dot{\epsilon}, u)
\]
\[
= -\dot{\beta}^{-1}(y,t)\left( k_1 \dot{\epsilon} + k_2 D\dot{\epsilon} + ... + \right.
\]
\[
\left. + \dot{\dot{\beta}}^{-1}(\dot{\epsilon} - \dot{\hat{g}}(y,t) + \hat{f}) + \dot{\hat{g}}(y,t) - \dot{\hat{f}}(x,t) + K D\dot{\epsilon}.sgn(S) \right)
\]
If the following control gain is selected by this sliding surface condition \(\dot{S}.sgn(S) \leq -\eta\) where \(\eta\) is a positive real constant, the stability of the system is satisfied:
\[
K_s \geq \dot{\beta}(k_1 \eta + (1 - \dot{\beta}^{-1})U)
\]
\[
+(\dot{\dot{\beta}}^{-1} - k_2 \dot{\dot{\beta}}^{-1})e^{(\eta)} + k_2(g_1 + g_2 + f_1 + f_2))
\]
by suppose that:
\[
\dot{u} = -\frac{k_1}{k_n} D\dot{\epsilon} + \frac{k_2}{k_n} \ddot{\epsilon} + ... + \dot{\dot{\beta}}^{-1}e^{(\eta)}, \quad |\dot{u}| < U
\]
\[
\Delta f = (\dot{\beta} - b)\dot{\beta}^{-1}\dot{f}, \quad \Delta \dot{f} = b\dot{\beta}^{-1}\dot{f} - \dot{f},
\]
\[
\Delta g = (-\dot{\dot{\beta}}^{-1} + b\dot{\dot{\beta}}^{-1}b^{-1})\dot{g}, \quad \Delta \dot{g} = \dot{g} - b\dot{\beta}^{-1}\dot{g}
\]
Finally, control law obtains as integral of Eq. (14).

4. **Application to control**

4.1 **Information about two-link pneumatic robot with a dead time**

Pneumatic robots serve a wide variety of industries, but long air tube which connects actuator and transducers of pneumatic robots is caused a dead time in the control section that even includes chaotic motion.
Fig. 1. shows a) a two-link pneumatic robot and b) its block diagram with PD control, where \( \dot{\theta}_d(t) = \frac{\pi}{4} \sin(0.5\pi t) \) is the desired value and the PD controller is \( K_p + K_d s \) \((K_p = K_d = 4)\).

The dynamics of the system are presented as \([33]:\)

\[
M(\theta(t))\ddot{\theta}(t) + H(\theta(t),\dot{\theta}(t)) + D\dot{\theta}(t) = \tau(t - L)
\]

where \( \theta(t) \) is the joint angle, \( M(\theta(t)) \) is the inertia matrix, \( H(\theta(t),\dot{\theta}(t)) \) is the centripetal and Coriolis torque, \( D \) is the viscous friction and \( \tau \) is input torque with a dead time \( L \).

Here are each element expressions:

\[
\theta(t) = [\theta_1(t), \dot{\theta}_1(t), \theta_2(t), \dot{\theta}_2(t)]^T
\]

\[
M(\theta(t)) = \begin{bmatrix}
J_1 + J_2 + 2\beta \cos\theta_2(t) & J_2 + \beta \cos\theta_2(t) \\
J_2 + \beta \cos\theta_2(t) & J_2
\end{bmatrix}
\]

\[
H(\theta(t),\dot{\theta}(t)) = \begin{bmatrix}
-\beta(2\dot{\theta}_1(t)\dot{\theta}_2(t) + \dot{\theta}_2(t)^2)\sin\theta_2(t) \\
\beta\dot{\theta}_1(t)^2 \sin\theta_2(t)
\end{bmatrix}
\]

\[
D = \begin{bmatrix}
D_1 & 0 \\
0 & D_2
\end{bmatrix}, \quad \tau(t - L) = \begin{bmatrix}
\tau_1(t - L) \\
\tau_2(t - L)
\end{bmatrix}
\]

\[
J_1 = I_1 + (m_i + 4m_2)l_1^2, \quad J_2 = I_2 + m_2l_2^2, \\
\beta = 2m_2l_2l_1, \quad I_1 = \frac{1}{3}m_1l_1^2, \quad I_2 = \frac{1}{3}m_2l_2^2, \\
D_1 = D_2 = 0.5 Nms, \quad l_1 = 0.25 m, \quad m_1 = m_2 = 1.0 kg
\]

where \( \theta_i, m_i \) are length and mass of link \( i (i=1,2) \), respectively. Some characteristics of the plant have been investigated such as the end effector motion in the workspace, time series, bifurcation diagram with respect to delay parameter and reconstitute attractor, the attractor and a Poincaré map shown in Figs. 2~4. Except bifurcation diagram, time delay is \( L=0.015 \) in all of the above figures and maximum Lyapunov exponent is equal to \( 0.041 \).

![Fig. 1. a) a two-link pneumatic robot and b) its block diagram with PD control](image)

![Fig. 2. a) The end-effector motion in workspace and b) Time series of link(2) in joint-space; chaotic pneumatic robot with \( L=0.015 [sec] \)](image)
4.2 Control of plant with uncertainty

Control between two pneumatic robots by different time delays is demonstrated, by considering system defined in Eq. (17) with $L=0.005$[sec] as the master system, and the slave system with $L=0.015$[sec].

**Theorem 2**: if the control scheme satisfies:

$$
\dot{T}_s = M_s \frac{K_p}{K_f} D \dot{\theta}_s + K_p \dot{\theta}_s + \theta_m^{(3)} \\
-M_s(T_s - h_s) + M_s^{(3)} h_s + k_s D^3 \text{sign}(S)
$$

the error will be asymptotically stable, where $m$ denotes master and $s$ denotes slave and $\dot{\theta}(t) = \tau(t - L)$.

**Proof**: if we define the following surface:

$$
\dot{S} = K_s \dot{\theta}_s + K_p \frac{d}{dt} D \dot{\theta}_s + K_p \frac{d}{dt} D^3 \dot{\theta}_s
$$

applying the method in section 3 the dynamic control scheme results as (24).

In next section results without and with maximum uncertainty of 60 percent in slave system are exhibited.

5. Simulation results

Simulation results are shown in Figs. 5-9. Using MATLAB software and the following parameter value:

$K_s = 1, K_p = 2, K_f = 10, K_f = 0.1$ and $\lambda = 0.7$

\[ (25) \]
To evaluate the robustness of the control scheme, a system uncertainty term is inserted by maximum uncertainty of 60 percent in slave system:

\[ l_1 = l_2 = 0.15\text{m}, \quad m_2 = 0.4\text{kg}, \quad m_1 = 0.7\text{kg} \]  

(26)

Figs. 5~7 and Figs. 8, 9 show that results of control a pneumatic robot without and with uncertainty, respectively (The end-effector motion in the workspace, tracking error of link(2), sliding surfaces \( S \) and control inputs \( u \), the attractors and reconstitute attractor) via fractional dynamic sliding mode control (activated in \( t=0.1\text{sec} \)). Numerical simulations show torus behavior in a finite time and chattering free by the root-mean-square error of link (2) without and with uncertainty are \( 7.089\times10^{-5}, \ 1.675\times10^{-2} \), respectively and the maximum Lyapunov exponent is equal to \( 0.0032 \).

**Fig. 5.** Results of control between two pneumatic robots via fractional dynamic sliding mode control (activated in \( t=0.1\text{[sec]} \)); a) The end-effector motion in workspace and b) Tracking error of link(2)

**Fig. 6.** Results of (Sliding surfaces and Control inputs) control between two pneumatic robots via fractional dynamic sliding mode control (activated in \( t=0.1\text{[sec]} \))
Fig. 7. Results of control between two pneumatic robots via fractional dynamic sliding mode control (activated in t=0.1[sec])
   a) The attractor and b) Reconstitute attractor with \( \tau = 68 \) [sec]

Tracking error of the second link

Fig. 8. Results of control between two pneumatic robots with uncertainty via fractional dynamic sliding mode control (activated in t=0.1[sec])
   a) Tracking error of link(2) b) Control input

Fig. 9. Result of (sliding surface) control between two pneumatic robots with uncertainty via fractional dynamic sliding mode control (activated in t=0.1[sec])
6. Conclusion

In general, chattering problem and particularly a delay based chaotic pneumatic robot have been discussed. The fractional dynamic sliding mode has been proposed and examined in chaotic robot system in presence of maximum uncertainty of 60 percent. Numerical examples have been indicated the viability of control scheme.

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