MULTIPLICITY AND $p_t$ CORRELATIONS
IN RELATIVISTIC NUCLEAR COLLISIONS

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Abstract
The theoretical description of the correlations between observables in two separated rapidity intervals for AA-interactions at high energies is presented. In the case with the real nucleon distribution density of colliding nuclei the MC calculations of the long-range correlation functions at different values of impact parameter are done. For $n$–$n$ and $p_t$–$n$ correlations it is shown that the impact parameter fluctuations at a level of a few fermi, unavoidable in the experiment, significantly change the magnitude of correlation coefficients. The rise of $p_t$–$n$ and especially $p_t$–$p_t$ correlation coefficients is found when one passes from SPS to RHIC and LHC energies.

1 String fusion model (SFM)
The colour string model [1, 2] originating from Gribov-Regge approach is being widely applied for the description of the soft part of the multiparticle production in hadronic and nuclear interactions at high energies. In this model at first stage of hadronic interaction the formation of the extended objects - the quark-gluon strings - takes place. At second stage the hadronization of these strings produces the observed hadrons. In the original version the strings evolve independently and the observed spectra are just the sum of individual string spectra. However in the case of nuclear collision, with growing energy and atomic number of colliding nuclei, the number of strings grows and one has to take into account the interaction between them.

One of possible approaches to the problem is the colour string fusion model [3]. The model is based on a simple observation that due to final transverse dimensions of strings they inevitably have to start to overlap with the rise of their density in transverse plane. At that the interaction of string colour fields takes place, which changes the process of their fragmentation into hadrons as compared with the fragmentation of independent strings. So we have one more interesting nonlinear phenomenon in nuclear interactions at high energies - the field of physics the investigations in which were initiated by pioneer works of academician A.M. Baldin [4].

It was shown [3, 5, 6] that the string fusion phenomenon considerably damps the charged particle multiplicity and simultaneously increase their mean $p_t$ value as compared with the case of independent strings. In accordance with a general Schwinger idea [7] and the following papers [8, 9] (colour ropes model) two possible versions of string fusion mechanism were suggested.

The first version [5] of the model assumes that the colour fields are summing up only locally in the area of overlaps of strings in the transverse plane. So we will refer to this
case as a local fusion or overlaps. In this case one has

\[ \langle n \rangle_k = \mu_0 \frac{S_k}{\sigma_0} \sqrt{k} \quad \langle p_t^2 \rangle_k = p^2 \sqrt{k} \]  \hspace{1cm} (1)

Here \( \langle n \rangle_k \) is the average multiplicity of charged particles originated from the area \( S_k \), where \( k \) strings are overlapping, and \( \langle p_t^2 \rangle_k \) is the same for their squared transverse momentum. The \( \mu_0 \) and \( p^2 \) are the average multiplicity and squared transverse momentum of charged particles produced from a decay of one single string, and \( \sigma_0 \) is its transverse area.

In the second version \([10]\) of the model one assumes that the colour fields are summing up globally - over total area of each cluster in the transverse plane - into one average colour field. This case corresponds to the summing of the source colour charges. We will refer to this case as a global fusion or clusters. In this case we have

\[ \langle n \rangle_{cl} = \mu_0 \frac{S_{cl}}{\sigma_0} \sqrt{k_{cl}} \quad \langle p_t^2 \rangle_{cl} = p^2 \sqrt{k_{cl}} \quad k_{cl} = \frac{N_{str}}{S_{cl}} \sigma_0 \]  \hspace{1cm} (2)

Here \( \langle n \rangle_{cl} \) is the average multiplicity of charged particles originated from the cluster of the area \( S_{cl} \) and \( \langle p_t^2 \rangle_{cl} \) is the same for their squared transverse momentum. The \( N_{str} \) is the number of strings forming the cluster.

Note that in two limit cases both versions give the same results.

For \( N \) non-overlapping strings we have in the local version: \( k = 1, S_1 = N\sigma_0, \langle n \rangle = \langle n \rangle_1 = N\mu_0 \) and \( \langle p_t^2 \rangle = \langle p_t^2 \rangle_1 = p^2 \). In the global version in this case we have \( N \) clusters each formed by only one string, so \( k_{cl} = 1, \langle n \rangle = N \langle n \rangle_{cl} = N\mu_0 \) and \( \langle p_t^2 \rangle = \langle p_t^2 \rangle_{cl} = p^2 \).

For \( N \) totally overlapped strings we have in the local version: \( k = N, S_N = \sigma_0, \langle n \rangle = \langle n \rangle_N = \sqrt{N} \mu_0 \) and \( \langle p_t^2 \rangle = \langle p_t^2 \rangle_N = \sqrt{N}p^2 \). In the global version in this case we have one cluster of the area \( S_{cl} = \sigma_0 \) formed by \( N \) string, so \( k_{cl} = N, \langle n \rangle = \langle n \rangle_{cl} = \sqrt{N} \mu_0 \) and \( \langle p_t^2 \rangle = \langle p_t^2 \rangle_{cl} = \sqrt{N}p^2 \).

So in both versions of the model when we pass from \( N \) non-overlapping strings to \( N \) totally overlapped strings the average multiplicity decreases from \( \langle n \rangle = N\mu_0 \) to \( \langle n \rangle = \sqrt{N} \mu_0 \) and the mean \( p_t^2 \) increases from \( \langle p_t^2 \rangle = p^2 \) to \( \langle p_t^2 \rangle = \sqrt{N}p^2 \).

## 2 Cellular analog of SFM

To simplify calculations in the case of real nucleus-nucleus collisions a simple cellular model originating from the string fusion model was proposed \([11]\). In the framework of the cellular analog along with the calculation simplifications the asymptotics of correlation coefficients at large and small string densities can be found analytically in the idealized case with the homogeneous string distribution, which enables to use these asymptotics later for the control of the Monte-Carlo (MC) algorithms.

Two versions of the cellular model as the original SFM can be formulated - with local and global string fusion. In this model we divide all transverse (impact parameter) plane into sells of order of the transverse string size \( \sigma_0 \).

In the version with local fusion the assumption of the model is that if the number of strings belonging to the \( ij \)-th cell is \( k_{ij} \), then they form higher colour string, which emits in average \( \mu_0 \sqrt{k_{ij}} \) particles with mean \( p_t^2 \) equal to \( p^2 \sqrt{k_{ij}} \) (compare with \([11]\)). Note that zero ”occupation numbers” \( k_{ij} = 0 \) are also admitted.
In the version with global fusion at first we define the neighbour cells as the cells with a common link. Then we define the cluster as the set of neighbour cells with non zero occupation numbers \( k_{ij} \neq 0 \). After that we can apply the same formulae of the global fusion \( \text{(2)} \) as in the original SFM, where \( N_{str}^{cl} \) is the number of strings in the cluster and \( S_{cl}/\sigma_0 \) is the number of cells in the cluster.

From event to event the number of strings \( k_{ij} \) in the \( ij \)-th cell will fluctuate around some average value - \( \overline{k}_{ij} \). Clear that in the case of real nuclear collisions these average values \( \overline{k}_{ij} \) will be different for different cells. They will depend on the position \( (s_{ij}) \) of the \( ij \)-th cell in the impact parameter plane \( (s \) is two dimensional vector in the transverse plane). In the case of nucleus-nucleus \( AB \)-collision at some fixed value of impact parameter \( b \) one can find this average local density of primary strings \( \overline{k}_{ij} \) in the point \( s_{ij} \) using nuclear profile functions \( T_{A}(s_{ij} + b/2) \) and \( T_{B}(s_{ij} - b/2) \).

In MC approach knowing the \( \overline{k}_{ij} \) one can generate some configuration \( C \equiv \{k_{ij}\} \). To get the physical answer for one given event (configuration \( C \)) we have to sum the contributions from different cells in accordance with local or global algorithm (see above), which corresponds to the integration over \( s \) in transverse plane. Then we have to sum over events (over different configurations \( C \)). Note that as the event-by-event fluctuations of the impact parameter at a level of a few fermi are inevitable in the experiment one has to include the impact parameter \( b \) into definition of configuration \( C \equiv \{b, k_{ij}\} \).

### 3 Long-range correlations

The idea [5, 6, 12] to use the study of long-range correlations in nuclear collisions for observation of the colour string fusion phenomenon based on the consideration that the quark-gluon string is an extended object which fragmentation gives the contribution to wide rapidity range. This can be an origin of the long-range correlations in rapidity space between observables in two different and separated rapidity intervals. Usually in an experiment they choose these two separated rapidity intervals in different hemispheres of the emission of secondary particles one in the forward and another in the backward in the center mass system. So sometimes these long-range rapidity correlations are referred as the forward-backward correlations (FBC).

In principle one can study three types of such long-range correlations:

- \( n-n \) - the correlation between multiplicities of charged particles in these rapidity intervals,
- \( p_t-p_t \) - the correlation between transverse momenta in these intervals and
- \( p_t-n \) - the correlation between the transverse momentum in one rapidity interval and the multiplicity of charged particles in another interval.

Usually to describe these correlations numerically one studies the average value \( \langle B \rangle_F \) of one dynamical variable \( B \) in the backward rapidity window \( \Delta y_B \), as a function of another dynamical variable \( F \) in the forward rapidity window \( \Delta y_F \). Here \( \langle ... \rangle_F \) denotes averaging over events having a fixed value of the variable \( F \) in the forward rapidity window. The \( \langle ... \rangle \) denotes averaging over all events. So we find the correlation function \( \langle B \rangle_F = f(F) \).

It’s naturally then to define the correlation coefficient as the response of \( \langle B \rangle_F \) on the variations of the variable \( F \) in the vicinity of its average value \( \langle F \rangle \). At that useful also to
go to the relative variables, i.e. to measure a deviation of $F$ from its average value $\langle F \rangle$ in units of $\langle F \rangle$, and the same for $B$. So it’s reasonable to define a correlation coefficient $b_{B-F}$ for correlation between observables $B$ and $F$ in backward and forward rapidity windows in the following way:

$$b_{B-F} \equiv \frac{\langle F \rangle}{\langle B \rangle} \frac{d\langle B \rangle_F}{dF} \bigg|_{F=\langle F \rangle}$$

As the dynamical variables we use the multiplicity of charged particles ($n$), produced in the given event in the given rapidity window, and the event(!) mean value of their transverse momentum ($p_t$), i.e. the sum of the transverse momentum magnitudes of all charged particles, produced in the given event in the given rapidity window ($\Delta y$), divided by the number of these particles ($n$):

$$p_t \equiv \frac{1}{n} \sum_{i=1}^{n} |p_{ti}|, \quad \text{where} \quad y_i \in \Delta y; \quad i = 1, \ldots, n.$$  \hspace{1cm} (4)

So we can define three correlation coefficients:

$$b_{n-n} \equiv \frac{\langle n_F \rangle}{\langle n_B \rangle} \frac{d\langle n_B \rangle_{n_F}}{dn_F} \bigg|_{n_F=\langle n_F \rangle} \quad b_{p_t-n} \equiv \frac{\langle p_{tF} \rangle}{\langle p_{tB} \rangle} \frac{d\langle p_{tB} \rangle_{p_{tF}}}{dp_{tF}} \bigg|_{p_{tF}=\langle p_{tF} \rangle} \quad b_{p_t-n} \equiv \frac{\langle n_F \rangle}{\langle p_{tB} \rangle} \frac{d\langle p_{tB} \rangle_{n_F}}{dn_F} \bigg|_{n_F=\langle n_F \rangle}$$

Here $n_B$, $n_F$ are the multiplicities and $p_{tB}$, $p_{tF}$ are the event(!) mean transverse momentum of the charged particles, produced in the given event correspondingly in the backward ($\Delta y_B$) and forward ($\Delta y_F$) rapidity windows.

## 4 Results of the calculations

In Figs.1-3 the results of the MC calculations of these correlation coefficients are presented for nucleus-nucleus collisions at different values of the centrality. In all figures (○) and (●) denote the results of calculations in the framework of the original SFM (with the taking into account the real geometry of merging strings) for its local (overlaps) and global (clusters) versions correspondingly. The (□) and (■) denote the results of calculations in the framework of the cellular analog of SFM for its local and global (clusters) versions. All presented results are for the forward rapidity window of 2 unit length ($\Delta y_F = 2$). The lines are only to guide the eye.

In Fig.1 we present the $b_{n-n}$ correlation coefficient for AuAu collisions at RHIC energy and in Fig.2 we present the $b_{p_t-n}$ correlation coefficient for PbPb collisions at LHC energy. In both figures the calculations are fulfilled three times:

1) at fixed values of impact parameter ($db = 0$),
2) with impact parameter fluctuations within 1 fm window ($db = 1$),
3) with impact parameter fluctuations within the whole class of centrality ($db = \text{class}$) (for LHC by convention this value is taken to be equal 3 fm, $db = 3$)

We see that the impact parameter fluctuations at a level of a few fermi significantly change the magnitude of correlation coefficients. Note also that all results obtained in the framework of the
Figure 1: The $b_{n-n}$ correlation coefficient for AuAu collisions at $\sqrt{s} = 130$ GeV as a function of the impact parameter $b$ for three choices of impact parameter window $db$ (see text).

original SFM and its cellular analog for their local and global versions practically coincide, except for the $p_t$-$n$ correlation at fixed value of impact parameter at LHC energy, where the correlation coefficient $b_{p_t-n}$ is very small.

In Fig.3 the energy dependence of the $b_{p_t-p_t}$ correlation coefficient is presented. The calculations are made for the 1 fm impact parameter window ($db = 1$). In this case we see the considerable rise of $p_t-p_t$ correlation coefficient from SPS to LHC energies.

5 Conclusion

In the case with the real nucleon distribution density of colliding nuclei the MC calculations of the long-range correlation functions at different values of impact parameter are done. For $n-n$ and $p_t-n$ correlations it is shown that the impact parameter fluctuations at a level of a few fermi, unavoidable in the experiment, significantly change the magnitude
of correlation coefficients for all centrality classes as compared to ones calculated earlier at the fixed values of impact parameter [13].

It is shown also, that for the $p_t-p_t$ correlation the event-by-event correlation between event mean values of transverse momenta of the particles emitted in two different rapidity intervals does not decrease to zero with the increase of the number of strings in contrast with the correlation between the transverse momenta of single particles produced in these two rapidity windows which was studied earlier [13].

The rise of $p_t-n$ and especially $p_t-p_t$ correlation coefficients is found when one passes from SPS to RHIC and LHC energies.

**Acknowledgments**

The authors would like to thank M.A. Braun and G.A. Feofilov for numerous valuable encouraging discussions. The work has been partially supported by the Russian Foundation for Fundamental Research under Grant No. 05-02-17399-a.
Figure 3: The $b_{p_t-p_t}$ correlation coefficient as a function of the impact parameter $b$ (at $db = 1$ fm) for three choices of the initial energy: $\sqrt{s} = 17, 130$ and $5500$ GeV.

References

[1] A. Capella, U.P. Sukhatme, C.–I. Tan and J. Tran Thanh Van, Phys. Lett. B81, 68 (1979); Phys. Rep. 236, 225 (1994).

[2] A.B. Kaidalov, Phys. Lett. 116B, 459 (1982); A.B. Kaidalov and K.A. Ter-Martirosyan, Phys. Lett. 117B, 247 (1982).

[3] M.A. Braun and C. Pajares, Phys. Lett. B287, 154 (1992); Nucl. Phys. B390, 542, 549 (1993).

[4] A.M. Baldin et al., Preprint JINR P9-5442, Dubna, 1970; A.M. Baldin et al., Preprint JINR P1-5819, Dubna, 1971; A.M. Baldin, Investigations with relativistic nuclei, MIFI, Moscow, 1975.

[5] M.A. Braun and C. Pajares, Eur. Phys. J. C16, 349 (2000).
[6] M.A. Braun and C. Pajares, Phys. Rev. Lett. 85, 4864 (2000).

[7] J. Schwinger, Phys. Rev. 82, 664 (1951).

[8] T.S.Biro, H.B.Nielsen and J.Knoll, Nucl. Phys. B245, 449 (1984).

[9] A.Bialas and W.Czyz, Nucl. Phys. B267, 242 (1986).

[10] M.A. Braun F. del Moral and C. Pajares, Phys. Rev. C65, 024907 (2002).

[11] V.V. Vechernin and R.S. Kolevatov, hep-ph/0304295; hep-ph/0305136 (Russian versions will appear in the journal "Vestnik SPbU").

[12] N.S. Amelin, N. Armesto, M.A. Braun, E.G. Ferreiro and C. Pajares, Phys. Rev. Lett. 73, 2813 (1994).

[13] M.A. Braun, R.S. Kolevatov, C. Pajares and V.V. Vechernin, Eur. Phys. J. 32, 535 (2004).