Leptogenesis in Supersymmetric Hybrid Inflation

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Abstract

The nonsupersymmetric as well as the supersymmetric hybrid inflationary model is reviewed. The scenario of baryogenesis via a primordial leptogenesis is discussed and the role of the nonperturbative electroweak sphaleron effects is analyzed in detail. A supersymmetric model based on a left-right symmetric gauge group, which ‘naturally’ leads to hybrid inflation, is presented. The $\mu$ problem is solved, in this model, and the baryon asymmetry of the universe is produced through leptogenesis. For masses of $\nu_\mu$, $\nu_\tau$ from the small angle MSW resolution of the solar neutrino problem and SuperKamiokande, maximal $\nu_\mu - \nu_\tau$ mixing can be achieved. The required values of the relevant parameters are, however, quite small.

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I. INTRODUCTION

The hybrid inflationary scenario [1], which can reproduce the measurements of the cosmic background explorer (COBE) [2] with more or less ‘natural’ values of the relevant coupling constants, is almost automatically realized [3,4] in supersymmetric grand unified theories (GUTs). In particular, a moderate extension of the minimal supersymmetric standard model (MSSM) based on a left-right symmetric gauge group provides [5] a ‘natural’ framework for the implementation of hybrid inflation. The $\mu$ problem of MSSM can be easily resolved [6] in the context of this model by coupling the inflaton system to the electroweak higgs superfields.

At the end of inflation, the inflaton (oscillating system) predominantly decays into electroweak higgs superfields, thereby ‘reheating’ the universe. However, its subdominant decay mode to right handed neutrinos leads [7], via their subsequent decay, to the production of a primordial lepton asymmetry in the universe. Nonperturbative electroweak sphaleron effects, which violate baryon and lepton number, then partially transform this asymmetry to the observed baryon asymmetry of the universe (BAU).

We analyze the consequences of this baryogenesis mechanism on $\nu_\mu - \nu_\tau$ mixing. We find [7] that, for masses of $\nu_\mu$, $\nu_\tau$ which are consistent with the small angle MSW resolution of the solar neutrino problem and the recent results of the SuperKamiokande experiment [8], maximal $\nu_\mu - \nu_\tau$ mixing can be achieved. The required values of the relevant parameters are, however, quite small.

In Sec.II, we review the nonsupersymmetric (Sec.IIA) as well as the supersymmetric (Sec.IIB) version of the hybrid inflationary scenario. In Sec.III, we discuss baryogenesis through a primordial leptogenesis. In particular, Sec.IIIA is devoted to the generation of the primordial lepton number. The topologically nontrivial structure of the vacuum in gauge theories and the resulting nonperturbative baryon and lepton number violating phenomena in the standard model are analyzed in Sec.IIIB. The rate of these phenomena at finite temperatures is calculated by employing electroweak sphalerons and the final BAU is estimated. Finally, in Sec.IV the supersymmetric model based on a left-right symmetric gauge group is presented. In particular, the solution of the $\mu$ problem (Sec.IVA), inflation (Sec.IVB) and leptogenesis (Sec.IVC) are sketched.
II. HYBRID INFLATION

A. The non Supersymmetric Version

The most important disadvantage of most inflationary scenarios was that they needed extremely small coupling constants in order to reproduce the results of COBE \cite{2}. This difficulty was overcome some years ago by Linde \cite{1} who proposed, in the context of nonsupersymmetric GUTs, a clever inflationary scenario known as hybrid inflation. The idea was to use two real scalar fields $\chi$ and $\sigma$ instead of one that was normally used. The field $\chi$ provides the vacuum energy which drives inflation while $\sigma$ is the slowly varying field during inflation. The main advantage of this scenario is that it can reproduce the observed temperature fluctuations of the cosmic background radiation (CBR) with ‘natural’ values of the parameters in contrast to previous realizations of inflation (like the ‘new’ \cite{3} or ‘chaotic’ \cite{10} inflationary scenarios). The potential utilized by Linde is

$$V(\chi, \sigma) = \kappa^2 \left( M^2 - \frac{\chi^2}{4} \right) + \frac{\lambda \chi^2 \sigma^2}{4} + \frac{m^2 \sigma^2}{2},$$

(1)

where $\kappa$, $\lambda$ are dimensionless positive coupling constants and $M$, $m$ mass parameters. The vacua lie at $\langle \chi \rangle = \pm 2M$, $\langle \sigma \rangle = 0$. Putting $m=0$, for the moment, we observe that the potential possesses an exactly flat direction at $\chi = 0$ with $V(\chi = 0, \sigma) = \kappa^2 M^4$. The mass squared of the field $\chi$ along this flat direction is given by $m_\chi^2 = -\kappa^2 M^2 + \frac{1}{2} \lambda^2 \sigma^2$ and remains nonnegative for $\sigma \geq \sigma_c = \sqrt{2} \kappa M / \lambda$. This means that, at $\chi = 0$ and $\sigma \geq \sigma_c$, we obtain a valley of minima with flat bottom. Reintroducing the mass parameter $m$ in Eq.(1), we observe that this valley acquires a nonzero slope. A region of the universe, where $\chi$ and $\sigma$ happen to be almost uniform with negligible kinetic energies and with values close to the bottom of the valley of minima, follows this valley in its subsequent evolution and undergoes inflation. The quadrupole anisotropy of CBR produced during this inflation can be estimated to be

$$\left( \frac{\delta T}{T} \right)_Q \approx \left( \frac{16\pi}{45} \right)^{1/2} \frac{\lambda \kappa^2 M^5}{M_P^3 m^2},$$

(2)

where $M_P = 1.22 \times 10^{19}$GeV is the Planck scale. The COBE \cite{2} result, $(\delta T/T)_Q \approx 6.6 \times 10^{-6}$, can then be reproduced with $M \approx 2.86 \times 10^{16}$ GeV, the supersymmetric
GUT vacuum expectation value (vev), and \( m \approx 1.3 \kappa \sqrt{\lambda} \times 10^{15} \text{ GeV} \sim 10^{12} \text{ GeV} \) for \( \kappa, \lambda \sim 10^{-2} \). Inflation terminates abruptly at \( \sigma = \sigma_c \) and is followed by a ‘waterfall’, i.e., a sudden entrance into an oscillatory phase about a global minimum. Since the system can fall into either of the two available global minima with equal probability, topological defects are copiously produced if they are predicted by the particular particle physics model one is considering.

**B. The Supersymmetric Version**

The hybrid inflationary scenario is ‘tailor made’ for application to supersymmetric GUTs except that the mass of \( \sigma, m \), is unacceptably large for supersymmetry, where all scalar fields acquire masses of order \( m_{3/2} \sim 1 \text{ TeV} \) (the gravitino mass) from soft supersymmetry breaking. To see this, consider a supersymmetric GUT with a (semi-simple) gauge group \( G \) of rank \( \geq 5 \) with \( G \rightarrow G_S \) (the standard model gauge group) at a scale \( M \sim 10^{16} \text{ GeV} \). The spectrum of the theory below \( M \) is assumed to coincide with the MSSM spectrum plus standard model singlets so that the successful predictions for \( \alpha_s, \sin^2 \theta_W \) are retained. The theory may also possess global symmetries. The breaking of \( G \) is achieved through the superpotential

\[
W = \kappa S (\phi \bar{\phi} - M^2),
\]

where \( \phi, \bar{\phi} \) is a conjugate pair of standard model singlet left handed superfields which belong to nontrivial representations of \( G \) and reduce its rank by their vevs and \( S \) is a gauge singlet left handed superfield. The coupling constant \( \kappa \) and the mass parameter \( M \) can be made positive by phase redefinitions. This superpotential is the most general renormalizable superpotential consistent with a \( U(1) \) R-symmetry under which \( W \rightarrow e^{i \theta} W, \ S \rightarrow e^{i \theta} S, \ \phi \bar{\phi} \rightarrow \phi \bar{\phi} \) and gives the potential

\[
V = \kappa^2 | M^2 - \phi \bar{\phi} |^2 + \kappa^2 | S |^2 (| \phi |^2 + | \bar{\phi} |^2) + \text{D terms}. \tag{4}
\]

Restricting ourselves to the D flat direction \( \phi = \bar{\phi}^* \) which contains the supersymmetric minima and performing appropriate gauge and R-transformations, we can bring \( S, \phi, \bar{\phi} \) on the real axis, i.e., \( S \equiv \sigma / \sqrt{2}, \ \phi = \bar{\phi} \equiv \chi / 2 \), where \( \sigma, \chi \) are normalized real scalar
fields. The potential then takes the form in Eq. (1) with \( \kappa = \lambda \) and \( m = 0 \) and, thus, Linde’s potential for hybrid inflation is almost obtainable from supersymmetric GUTs but without the mass term of \( \sigma \) which is, however, of crucial importance since it provides the slope of the valley of minima necessary for inflation.

One way to obtain a valley of minima useful for inflation is \([11]\) to replace the trilinear term in \( W \) in Eq. (3) by the next order nonrenormalizable coupling. Another way, which we will adopt here, is \([4]\) to keep the renormalizable superpotential in Eq. (3) and use the radiative corrections along the inflationary valley \((\phi = \bar{\phi} = 0, S > S_c \equiv M)\). In fact, due to the mass splitting in the supermultiplets \( \phi, \bar{\phi} \) caused by the supersymmetry breaking ‘vacuum’ energy density \( \kappa^2 M^4 \) along this valley, there are important radiative corrections. At one-loop, the inflationary potential is given \([4,5]\) by

\[
V_{\text{eff}}(S) = \kappa^2 M^4 \left[ 1 + \frac{\kappa^2}{32\pi^2} \left( 2 \ln \left( \frac{\kappa^2 S^2}{\Lambda^2} \right) + \left( \frac{S^2}{S_c^2} - 1 \right)^2 \ln \left( 1 - \frac{S^2}{S^2_c} \right) + \left( \frac{S^2}{S_c^2} + 1 \right)^2 \ln \left( 1 + \frac{S^2}{S^2_c} \right) \right) \right],
\]

(5)

where \( \Lambda \) is a suitable mass renormalization scale. For \( S \) sufficiently larger than \( S_c \),

\[
V_{\text{eff}}(S) = \kappa^2 M^4 \left[ 1 + \frac{\kappa^2}{16\pi^2} \left( \ln \left( \frac{\kappa^2 S^2}{\Lambda^2} \right) + \frac{3}{2} - \frac{S^4_c}{12S^4} + \cdots \right) \right].
\]

(6)

Using this effective potential, one finds that the cosmic microwave quadrupole anisotropy

\[
\left( \frac{\delta T}{T} \right)_Q \approx 8\pi \left( \frac{N_Q}{45} \right)^{1/2} \frac{x_Q}{y_Q} \left( \frac{M}{M_P} \right)^2.
\]

(7)

Here, \( N_Q \) is the number of e-foldings experienced by the universe between the time the quadrupole scale exited the inflationary horizon and the end of inflation and \( y_Q = x_Q(1-7/(12x_Q^2)+\cdots) \) with \( x_Q = S_Q/M, S_Q \) being the value of the scalar field \( S \) when the scale which evolved to the present horizon size crossed outside the de Sitter (inflationary) horizon. Also, from Eq. (3), one finds

\[
\kappa \approx \frac{8\pi^{3/2}}{\sqrt{N_Q}} y_Q \frac{M}{M_P}.
\]

(8)

The inflationary phase ends as \( S \) approaches \( S_c \) from above. Writing \( S = xS_c \), \( x = 1 \) corresponds to the phase transition from \( G \) to \( G_S \) which, as it turns out, more or
less coincides with the end of the inflationary phase as one deduces from the slow roll conditions \[4,12\]. Indeed, the 50 – 60 e-foldings needed for the inflationary scenario can be realized even with small values of \(x_Q\). For definiteness, we will take \(x_Q \approx 2\). From COBE \[2\] one then obtains \(M \approx 5.5 \times 10^{15} \text{ GeV}\) and \(\kappa \approx 4.5 \times 10^{-3}\) for \(N_Q \approx 56\). Moreover, the primordial density fluctuation spectral index \(n \approx 0.98\). We see that the relevant part of inflation takes place at \(S \sim 10^{16} \text{ GeV}\). An interesting consequence of this is \[3,5,13\] that the supergravity corrections can be negligible.

In conclusion, it is important to note that the superpotential \(W\) in Eq.(3) leads to hybrid inflation in a ‘natural’ way. This means that a) there is no need of very small coupling constants, b) \(W\) is the most general renormalizable superpotential allowed by the gauge and R- symmetries, c) supersymmetry guarantees that the radiative corrections do not invalidate inflation, but rather provide a slope along the inflationary trajectory which drives the inflaton towards the supersymmetric vacua, and d) supergravity corrections can be brought under control so as to leave inflation intact.

III. BARYOGENESIS VIA LEPTOGENESIS

A. Primordial Leptogenesis

In most hybrid inflationary models, it is not convenient to produce the observed BAU in the customary way, i.e., through the decay of color \(3, \bar{3}\) fields \((g, g^c)\). Some of the reasons are the following: i) For theories where leptons and quarks belong to different representations of the unifying gauge group \(G\) (which is the case, for example, for \(G = G_{LR} \equiv SU(3)_c \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}\) or \(SU(3)_c \times SU(3)_L \times SU(3)_R\)), the baryon number can be made almost exactly conserved by imposing an appropriate discrete symmetry. In particular, for \(G = G_{LR}\), we can impose \[14\] a discrete symmetry under which \(q \rightarrow -q, q^c \rightarrow -q^c, \bar{q} \rightarrow -\bar{q}, \bar{q}^c \rightarrow -\bar{q}^c\) and all other superfields remain invariant \((q, q^c, \bar{q}, \bar{q}^c\) are superfields with the quantum numbers of the quarks, antiquarks and their conjugates respectively). ii) For theories where such a discrete symmetry is absent, we could, in principle, use as inflaton a pair of conjugate standard model singlet superfields \(N, \bar{N}\) which decay into \(g, g^c\). For \(G = SU(3)_c \times SU(3)_L \times SU(3)_R\),
$N$ (\(\bar{N}\)) could be the standard model singlet component of the \((1, \bar{3}, 3)\) superfields with zero $U(1)_{B-L}$ charge. But this is again unacceptable since the breaking of $SU(3)_c \times SU(3)_L \times SU(3)_R$ by the vevs of $N$, $\bar{N}$ predicts [15] magnetic monopoles which can then be copiously produced after inflation. Also the gravitino constraint [16] on the ‘reheat’ temperature, $T_r \lesssim 10^9$ GeV, implies $m_\sigma \lesssim 10^{10}$ GeV (from the coupling ($m_\sigma/\langle N\rangle)Ngg^c$) leading to strong deviation from MSSM and possibly proton decay.

So it is preferable to produce first a primordial lepton asymmetry [17] which can then be partially turned into the observed baryon asymmetry of the universe by the nonperturbative sphaleron effects [18] of the electroweak sector. In the particular model based on $G_{LR}$ which we will consider later, this is the only way to produce the BAU since the inflaton decays into higgs superfields and right handed neutrinos. The subsequent decay of right handed neutrinos into ordinary higgs particles (higgsinos) and light leptons (sleptons) can produce the primordial lepton asymmetry. It is important, though, to ensure that this primordial lepton asymmetry is not erased [19] by lepton number violating $2 \rightarrow 2$ scattering processes such as $ll \rightarrow h^{(1)}h^{(1)*}$ or $lh^{(1)} \rightarrow \bar{h}^{(1)}$ (\(l\) represents a lepton doublet and \(h^{(1)}\) the higgs superfield which couples to the up type quarks) at all temperatures between $T_r$ and 100 GeV. This is automatically satisfied since the primordial lepton asymmetry is protected [20] by supersymmetry at temperatures between $T_r$ and $T \sim 10^7$ GeV, and for $T \lesssim 10^7$ GeV, these $2 \rightarrow 2$ scattering processes are well out of equilibrium provided [20] $m_\nu \lesssim 10$ eV, which readily holds in our case (see below).

The lepton asymmetry produced by the out-of-equilibrium decay ($M_{\nu^c} \gg T_r$) of the right handed neutrinos $\nu^c_i$, which emerged from the inflaton decay, is [17]

\[
\frac{n_L}{s} \approx -\frac{3}{16\pi m_{\text{infl}}} \frac{T_r}{m_{\text{infl}}} \sum_{l \neq i} g(r_{ii}) \frac{\text{Im}(U M^{D'} M^{D'*} U^\dagger)^\dagger}{(\langle h^{(1)} \rangle)^2 (U M^{D'} M^{D'*} U^\dagger)_{ii}},
\]

(9)

where $n_L$ and $s$ are the lepton number and entropy densities, $m_{\text{infl}}$ in the inflaton mass, $M^{D'}$ is the diagonal ‘Dirac’ mass matrix, $U$ a unitary transformation so that $UM^{D'}$ is the ‘Dirac’ mass matrix in the basis where the ‘Majorana’ mass matrix of $\nu^c$’s is diagonal and $|\langle h^{(1)} \rangle| \approx 174$ GeV for large $\tan \beta$. The function

\[
g(r_{ii}) = r_{ii} \ln \left(\frac{1 + r_{ii}^2}{r_{ii}^2}\right), \quad r_{ii} = \frac{M_i}{M},
\]

(10)
with \( g(r) \sim 1/r \) as \( r \to \infty \). Here we have taken into account the following prefactors: i) At ‘reheat’, \( n_{\text{infl}}m_{\text{infl}} = (\pi^2/30)g_*T_r^4 \) (\( n_{\text{infl}} \) is the inflaton number density and \( g_* \) the effective number of massless degrees of freedom) which together with the relation \( s = (2\pi^2/45)g_*T_r^3 \) implies that \( n_{\text{infl}}/s = (3/4)(T_r/m_{\text{infl}}) \). ii) Since each inflaton decays into two \( \nu^c \)'s, their number density \( n_{\nu^c} = 2n_{\text{infl}} \) which then gives \( n_{\nu^c}/s = (3/2)(T_r/m_{\text{infl}}) \). iii) Supersymmetry gives an extra factor of two.

**B. Sphaleron Effects**

To see how the primordial lepton asymmetry partially turns into the observed BAU, we must first discuss the nonperturbative baryon (\( B^- \)) and lepton (\( L^- \)) number violation [21] in the standard model. Consider the electroweak gauge symmetry \( SU(2)_L \times U(1)_Y \) in the limit where the Weinberg angle \( \theta_W = 0 \) and concentrate on \( SU(2)_L \) (inclusion of \( \theta_W \neq 0 \) does not alter the conclusions). Also, for the moment, ignore the fermions and higgs fields so as to have a pure \( SU(2)_L \) gauge theory. This theory has [22] infinitely many classical vacua which are topologically distinct and are characterized by a ‘winding number’ \( n \in \mathbb{Z} \). In the ‘temporal gauge’ \( (A_0 = 0) \), the remaining gauge freedom consists of time independent transformations and the vacuum corresponds to a pure gauge

\[
A_i = \frac{i}{g} \partial_i g(\bar{x}) g^{-1}(\bar{x}) ,
\]

where \( g \) is the \( SU(2)_L \) gauge coupling constant, \( \bar{x} \) belongs to ordinary 3-space, \( i = 1,2,3 \), \( g(\bar{x}) \in SU(2)_L \), and \( g(\bar{x}) \to 1 \) as \( |\bar{x}| \to \infty \). Thus, the 3-space compactifies to a sphere \( S^3 \) and \( g(\bar{x}) \) defines a map: \( S^3 \to SU(2)_L \) (with the \( SU(2)_L \) group being topologically equivalent to \( S^3 \)). These maps are classified into homotopy classes constituting the third homotopy group of \( S^3 \), \( \pi_3(S^3) \), and are characterized by a ‘winding number’

\[
n = \int d^3x \; \epsilon^{ijk} \text{tr} \left( \partial_i g(\bar{x})^{-1}(\bar{x}) \partial_j g(\bar{x})^{-1}(\bar{x}) \partial_k g(\bar{x})^{-1}(\bar{x}) \right) .
\]

The corresponding vacua are denoted as \( |n\rangle, n \in \mathbb{Z} \).

The tunneling amplitude from the vacuum \( |n_-\rangle \) at \( t = -\infty \) to the vacuum \( |n_+\rangle \) at \( t = +\infty \) is given by the functional integral

\[
\langle n_+ | n_- \rangle = \int (dA) \; e^{-S(A)}
\]
over all gauge field configurations satisfying the appropriate boundary conditions at \( t = \pm \infty \). Performing a Wick rotation, \( x_0 \equiv t \rightarrow -ix_4 \), we can go to Euclidean spacetime. Any Euclidean field configuration with finite action is characterized by an integer topological number known as the Pontryagin number

\[
q = \frac{g^2}{16\pi^2} \int d^4x \, \text{tr} \left( F^{\mu\nu} \tilde{F}_{\mu\nu} \right),
\]

(14)

with \( \mu, \nu = 1,2,3,4 \) and \( \tilde{F}_{\mu\nu} = \frac{1}{2} \epsilon_{\mu\nu\lambda\rho} F^{\lambda\rho} \) being the dual field strength. But \( \text{tr}(F^{\mu\nu} \tilde{F}_{\mu\nu}) = \partial^\mu J_\mu \), where \( J_\mu \) is the ‘Chern-Simons current’ given by

\[
J_\mu = \epsilon_{\mu \alpha \beta} \text{tr} \left( A^\nu F^{\alpha \beta} - \frac{2}{3} g A^\nu A^\alpha A^\beta \right).
\]

(15)

In the ‘temporal gauge’ \((A_0 = 0)\),

\[
q = \frac{g^2}{16\pi^2} \int d^4x \, \partial^\mu J_\mu = \frac{g^2}{16\pi^2} \Delta \int d^3x \, J_0
\]

\[
= \frac{1}{24\pi^2} \Delta x = \pm \infty \int d^3x \, \epsilon^{ijk} \text{tr} \left( \partial_i g g^{-1} \partial_j g g^{-1} \partial_k g g^{-1} \right) = n_+ - n_-. \tag{16}
\]

This means that the Euclidean field configurations which interpolate between the vacua \(| n_+ \rangle, | n_- \rangle\) at \( x_4 = \pm \infty \) have Pontryagin number \( q = n_+ - n_- \) and the path integral in Eq.(13) should be performed over all these field configurations.

For a given \( q \), there is a lower bound on \( S(A) \),

\[
S(A) \geq \frac{8\pi^2}{g^2} | q | ,
\]

(17)

which is saturated if and only if \( F_{\mu\nu} = \pm \tilde{F}_{\mu\nu} \), i.e, if the configuration is self-dual or self-antidual. For \( q=1 \), the self-dual classical solution is called instanton \[23\] and is given by (in the ‘singular’ gauge)

\[
A_{a\mu}(x) = \frac{2\rho^2}{g(x-z)^2} \frac{\eta_{a\mu}(x-z)^\nu}{(x-z)^2 + \rho^2},
\]

(18)

where \( \eta_{a\mu} (a=1,2,3; \mu,\nu = 1,2,3,4) \) are the t’ Hooft symbols with \( \eta_{aij} = \epsilon_{aij} \) \((i,j = 1,2,3)\), \( \eta_{a4i} = -\delta_{ai}, \eta_{a4i} = \delta_{ai} \) and \( \eta_{a44} = 0 \). The instanton depends on four Euclidean coordinates \( z_\mu \) (its position) and its scale (or size) \( \rho \). Two successive vacua \(| n \rangle, | n+1 \rangle\) are separated
by a potential barrier of height $\propto \rho^{-1}$. The Euclidean action of the interpolating instanton is always equal to $8\pi^2/g^2$, but the height of the barrier can be made arbitrarily small since the size $\rho$ of the instanton can be taken arbitrarily large.

We now reintroduce the fermions into the theory and observe [21] that the $B$- and $L$-number currents carry anomalies, i.e.,

$$\partial_\mu J_B^\mu = \partial_\mu J_L^\mu = -n_g \frac{g^2}{16\pi^2} \text{tr}(F_{\mu\nu}F^{\mu\nu}), \quad (19)$$

where $n_g$ is the number of generations. It is then obvious that the tunneling from $|n_-\rangle$ to $|n_+\rangle$ is accompanied by a change of the $B$- and $L$- numbers, $\Delta B = \Delta L = -n_g q = -n_g(n_+ - n_-)$. Note that i) $\Delta(B - L) = 0$, and ii) for $q=1$, $\Delta B = \Delta L = -3$ which means that we have the annihilation of one lepton per family and one quark per family and color (12-point function).

We, finally, reintroduce the Weinberg-Salam higgs doublet $h$ with its vev given by

$$<h> = \frac{v}{\sqrt{2}} \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \quad v \approx 246 \text{ GeV}. \quad (20)$$

It is then easy to see that the instanton ceases to exist as an exact solution. It is replaced by the so called ‘restricted instanton’ [24] which is an approximate solution for $\rho \ll v^{-1}$.

For $|x - z| \ll \rho$, the gauge field configuration of the ‘restricted instanton’ essentially coincides with that of the instanton and the higgs field is

$$h(x) \approx \frac{v}{\sqrt{2}} \left( \frac{(x - z)^2}{(x - z)^2 + \rho^2} \right)^{1/2} \begin{pmatrix} 0 \\ 1 \end{pmatrix}. \quad (21)$$

For $|x - z| \gg \rho$, the gauge and higgs fields decay to a pure gauge and the vev in Eq.(20) respectively. The action of the ‘restricted instanton’ is $S_{ri} = (8\pi^2/g^2) + \pi^2 v^2 \rho^2 + \cdots$, which implies that the contribution of big size ‘restricted instantons’ to the path integral in Eq.(13) is suppressed. This justifies $a \ posteriori$ the fact that we restricted ourselves to approximate instanton solutions with $\rho \ll v^{-1}$.

The height of the potential barrier between the vacua $|n\rangle$, $|n + 1\rangle$ cannot be now arbitrarily small. This can be understood by observing that the static energy of the ‘restricted instanton’ at $x_4 = z_4$ ($\lambda$ is the higgs self-coupling),

$$E_b(\rho) \approx \frac{3\pi^2}{g^2} \frac{1}{\rho} + \frac{3}{8} \pi^2 v^2 \rho^2 + \frac{\lambda}{4} \pi^2 v^4 \rho^3, \quad (22)$$
is minimized for
\[
\rho_{\min} = \frac{\sqrt{2}}{g v} \left( \frac{\lambda}{g^2} \right)^{-1/2} \left( \left( \frac{1}{64} + \frac{\lambda}{g^2} \right)^{1/2} - \frac{1}{8} \right)^{1/2} \sim M_W^{-1},
\] (23)
and, thus, the minimal height of the potential barrier turns out to be \( E_{\text{min}} \sim M_W / \alpha_W \) (\( M_W \) is the weak mass scale and \( \alpha_W = g^2 / 4\pi \)). The static solution which corresponds to the top (saddle point) of this potential barrier is called sphaleron [23] and is given by
\[
\bar{A} = v \left( \frac{f(\xi)}{\xi} \right) \hat{r} \times \bar{\tau}, \quad h = \frac{v}{\sqrt{2}} t(\xi) \hat{r} \cdot \bar{\tau} \left( \begin{array}{c} 0 \\ 1 \end{array} \right),
\] (24)
where \( \xi = 2 M_W r, \hat{r} \) is the radial unit vector in ordinary 3-space and the 3-vector \( \bar{\tau} \) consists of the Pauli matrices. The functions \( f(\xi), t(\xi) \), which can be determined numerically, tend to zero as \( \xi \to 0 \) and to 1 as \( \xi \to \infty \). The mass (static energy) of the sphaleron solution is estimated to be
\[
E_{\text{sph}} = \frac{2 M_W}{\alpha_W} k, \quad 1.5 \leq k \leq 2.7, \quad \text{for } 0 \leq \lambda \leq \infty,
\] (25)
and lies between 10 and 15 TeV.

At zero temperature the tunneling from \( | n \rangle \) to \( | n + 1 \rangle \) is utterly suppressed [21] by the factor \( \exp(-8\pi^2/g^2) \). At high temperatures, however, thermal fluctuations over the potential barrier are frequent and this transition can occur [18] with an appreciable rate. For \( M_W \lesssim T \lesssim T_c \) (\( T_c \) is the critical temperature of the electroweak transition), this rate can be calculated [18] by expanding around the sphaleron (saddle point) solution and turns out to be
\[
\Gamma \approx 10^4 n_g \frac{v(T)^9}{T^8} \exp(-E_{\text{sph}}(T)/T).
\] (26)
Assuming that the electroweak phase transition is a second order one, \( v(T) \) and \( E_{\text{sph}}(T) \) vary \( (1 - T^2/T_c^2)^{1/2} \). One can then show that \( \Gamma \gg H \) (\( H \) is the Hubble parameter) for temperatures \( T \) between \( \sim 200 \) GeV and \( \sim T_c \). Furthermore, for temperatures above \( T_c \), where the sphaleron solution ceases to exist, it was argued [18] that we still have \( \Gamma \gg H \). The overall conclusion is that nonperturbative \( B \)- and \( L \)-number violating processes are in equilibrium in the universe for cosmic temperatures \( T \gtrsim 200 \) GeV. Remember that \( B - L \) is conserved by these processes.
Given a primordial $L$-number density, one can calculate the resulting $n_B/s$ ($n_B$ is the $B$-number density). In MSSM, the $SU(2)_L$ instantons produce the effective operator (in symbolic form)

$$O_2 = (qqql)^{ns} (\bar{h}^{(1)} \bar{h}^{(2)}) \bar{W}^4,$$

and the $SU(3)_c$ instantons the operator

$$O_3 = (qqu^c d^c)^{ns} \tilde{g}^6,$$

where $q, l$ are the quark, lepton $SU(2)_L$ doublets respectively, $u^c, d^c$ the up, down type antiquark $SU(2)_L$ singlets respectively, $h^{(1)}, h^{(2)}$ the higgses which couple to up, down type quarks respectively, $g, W$ the gluons and $W$ bosons and tilde represents their superpartners. We will assume that these interactions together with the usual MSSM interactions are in equilibrium at high temperatures. The equilibrium number density of ultrarelativistic particles $\Delta n \equiv n_{\text{part}} - n_{\text{antipart}}$ is given by

$$\frac{\Delta n}{s} = \frac{15g}{4\pi^2 g_s} \left( \frac{\mu}{T} \right)^2 \epsilon,$$

where $g$ is the number of internal degrees of freedom of the particle under consideration, $\mu$ its chemical potential and $\epsilon = 2$ or $1$ for bosons or fermions. For each interaction in equilibrium, the algebraic sum of the chemical potentials of the particles involved is zero. Solving these constraints, we end up with only two independent chemical potentials, $\mu_q$ and $\mu_{\tilde{g}}$, and the $B$- and $L$- asymmetries are expressed in terms of them:

$$\frac{n_B}{s} = \frac{30}{4\pi^2 g_s T} (6n_g \mu_q - (4n_g - 9)\mu_{\tilde{g}}),$$

$$\frac{n_L}{s} = \frac{45}{4\pi^2 g_s T} \left( \frac{n_g (14n_g + 9)}{1 + 2n_g} \mu_q + \Omega(n_g)\mu_{\tilde{g}} \right),$$

where $\Omega(n_g)$ is a known function. Now soft supersymmetry breaking couplings come in equilibrium at $T \approx 10^7$ GeV since their rate $\Gamma_S \approx m_{3/2}^2/T \approx H \approx 30 T^2/M_P$. In particular, the nonvanishing gaugino mass implies $\mu_{\tilde{g}} = 0$ and Eqs.(30) give

$$\frac{n_B}{s} = \frac{4(1 + 2n_g)}{22n_g + 13} \frac{n_{B-L}}{s}.$$

Equating $n_{B-L}/s$ with the primordial $n_L/s$, we have $n_B/s = (-28/79)(n_L/s)$, for $n_g = 3$. 

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IV. THE ‘LEFT-RIGHT’ MODEL

We will now study in detail a moderate extension of MSSM based on the left-right symmetric gauge group $G_{LR}$ which provides a suitable framework for hybrid inflation. The inflaton is associated with the breaking of $SU(2)_R$ and consists of a gauge singlet and a pair of $SU(2)_R$ doublets. The $\mu$ problem is resolved by introducing a trilinear superpotential coupling of the gauge singlet inflaton to the electroweak higgs doublets. In the presence of gravity-mediated supersymmetry breaking, this gauge singlet acquires a vev and, thus, generates, via its coupling to the higgses, the $\mu$ term.

The inflaton system, after the end of inflation, predominantly decays into higgs superfields and ‘reheats’ the universe. Moreover, its subdominant decay into right handed neutrinos provides a mechanism for baryogenesis via leptogenesis. For $\nu_\mu$, $\nu_\tau$ masses from the small angle MSW resolution of the solar neutrino puzzle and the recent results of the SuperKamiokande experiment, maximal $\nu_\mu - \nu_\tau$ mixing can be achieved.

A. The $\mu$ Problem

The breaking of $SU(2)_R \times U(1)_{B-L}$ is achieved by the renormalizable superpotential

$$W = \kappa S(l^c \bar{l}^c - M^2) ,$$

where $S$ is a gauge singlet chiral superfield and $l^c, \bar{l}^c$ is a conjugate pair of $SU(2)_R$ doublet chiral superfields which acquire superheavy vevs of magnitude $M$. The parameters $\kappa$ and $M$ can be made positive by phase redefinitions.

The $\mu$ problem can be resolved by introducing the extra superpotential coupling

$$\delta W = \lambda S h^2 = \lambda S e^{ij} h^{(1)}_i h^{(2)}_j ,$$

where the chiral electroweak higgs superfield $h = (h^{(1)}, h^{(2)})$ belongs to a $(1, 2, 2)_0$ representation of $G_{LR}$ and $\lambda$ can again be made positive. The scalar potential which results from the terms in Eqs. (32) and (33) is (for canonical Kähler potential):

$$V = |\kappa l^c \bar{l}^c + \lambda h^2 - \kappa M^2|^2 + (m^2_{3/2} + \kappa |l^c|^2 + \kappa^2 |\bar{l}^c|^2 + \lambda^2 |h^2|^2)|S|^2 + m^2_{3/2}(|\bar{l}^c|^2 + |l^c|^2 + |h|^2) + (A m_{3/2} S (\kappa l^c \bar{l}^c + \lambda h^2 - \kappa M^2) + 2\kappa m_{3/2} M^2 S + \text{h.c.}) ,$$

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where $m_{3/2}$ is the universal scalar mass (gravitino mass) and $A$ the universal coefficient of the trilinear soft terms. For exact supersymmetry ($m_{3/2} \to 0$), the vacua are at

$$S = 0, \quad \kappa l^c \bar{c} + \lambda h^2 = \kappa M^2, \quad \lambda^{(1)} = e^{i\theta} \epsilon_{ij} h^{(2)j*},$$

where the last two conditions arise from the requirement of D flatness. We see that there is a twofold degeneracy of the vacuum which is lifted by supersymmetry breaking. We get two degenerate (up to $m_{3/2}^4$) ground states ($\kappa \neq \lambda$): the desirable (‘good’) vacuum at $h = 0$ and $l^c \bar{c} = M^2$ and the undesirable (‘bad’) one at $h \neq 0$ and $l^c \bar{c} = 0$. They are separated by a potential barrier of order $M^2 m_{3/2}^2$.

To leading order in supersymmetry breaking, the term of the potential $V$ in Eq.(34) proportional to $A$ vanishes, but a destabilizing tadpole term for $S$ remains:

$$2\kappa m_{3/2} M^2 S + \text{h.c.} .$$

This term together with the mass term of $S$ (evaluated at the ‘good’ vacuum) give $\langle S \rangle \approx -m_{3/2}/\kappa$ which, substituted in Eq.(33), generates a $\mu$ term with

$$\mu = \lambda \langle S \rangle \approx -\frac{\lambda}{\kappa} m_{3/2} .$$

Thus, coupling $S$ to the higgses can lead to the resolution of the $\mu$ problem.

The model can be extended to include matter fields too. The superpotential has the most general form respecting the $G_{LR}$ gauge symmetry and a global $U(1)$ R-symmetry. Baryon number is automatically implied by this R-symmetry to all orders in the superpotential, thereby guaranteeing the stability of proton.

**B. The Inflationary Trajectory**

The model has a built-in inflationary trajectory parametrized by $|S|$, $|S| > S_c = M$ for $\lambda > \kappa$ (see below). All other fields vanish on this trajectory. The $F_S$ term is constant providing a constant tree level vacuum energy density $\kappa^2 M^4$, which is responsible for inflation. One-loop radiative corrections (from the mass splitting in the supermultiplets $l^c, \bar{l}^c$ and $h$) generate a logarithmic slope along the inflationary trajectory which drives the inflaton toward the minimum. For $|S| \leq S_c = M$, the $l^c, \bar{l}^c$
components become tachyonic and the system evolves towards the ‘good’ supersymmetric minimum at \( h = 0, \bar{l}c = \bar{l}c = M \) (for \( \kappa > \lambda \), \( h \) is destabilized first and the system would have evolved towards the ‘bad’ minimum at \( h \neq 0, \bar{l}c = \bar{l}c = 0 \)). For all values of the parameters considered here, inflation continues at least till \( |S| \) approaches the instability at \( |S| = S_c \) as one deduces from the slow roll conditions \([12]\). The cosmic microwave quadrupole anisotropy can be calculated \([4]\) by standard methods and turns out to be

\[
\left( \frac{\delta T}{T} \right)_Q \approx \frac{32\pi^{5/2}}{3\sqrt{5}} \left( \frac{M}{M_P} \right)^3 \kappa^{-1}x_Q^{-1} \Lambda(x_Q)^{-1},
\]

(38)

\[
\Lambda(x) = \left( \frac{\lambda}{\kappa} \right)^3 \left[ \left( \frac{\lambda}{\kappa} x^2 - 1 \right) \ln \left( 1 - \frac{\kappa}{\lambda} x^{-2} \right) + \left( \frac{\lambda}{\kappa} x^2 + 1 \right) \ln \left( 1 + \frac{\kappa}{\lambda} x^{-2} \right) \right] + (x^2 - 1) \ln(1 - x^{-2}) + (x^2 + 1) \ln(1 + x^{-2}),
\]

(39)

with \( x = |S|/S_c \) and \( S_Q \) being the value of \( |S| \) when the present horizon scale crossed outside the inflationary horizon. (Notice that here we had to replace the contribution to the effective potential in Eq.(5) from the \( \phi, \bar{\phi} \) supermultiplets of the model in Sec.II B by the contribution from the \( l^c, \bar{l}c \) and \( h \) supermultiplets.) The number of e-foldings experienced by the universe between the time the quadrupole scale exited the horizon and the end of inflation is

\[
N_Q \approx 32\pi^3 \left( \frac{M}{M_P} \right)^2 \kappa^{-2} \int_1^{x_Q^2} \frac{dx^2}{x^2} \Lambda(x)^{-1}.
\]

(40)

The spectral index of density perturbations turns out to be very close to unity.

C. ‘Reheating’ and Leptogenesis

After reaching the instability at \( |S| = S_c \), the system continues \([23]\) inflating for another e-folding or so reducing its energy density by a factor of about \( 2 - 3 \). It then rapidly settles into a regular oscillatory phase about the vacuum. Parametric resonance is safely ignored in this case \([26]\). The inflaton (oscillating system) consists of the two complex scalar fields \( S \) and \( \theta = (\delta\phi + \delta\bar{\phi})/\sqrt{2} \), where \( \delta\phi = \phi - M, \delta\bar{\phi} = \bar{\phi} - M \), with mass \( m_{infl} = \sqrt{2}\kappa M \) (\( \phi, \bar{\phi} \) are the neutral components of \( l^c, \bar{l}c \)).
The scalar fields $S$ and $\theta$ predominantly decay into electroweak higgsinos and higgses respectively with a common decay width $\Gamma_h = (1/16\pi)\lambda^2 m_{\text{inf}}$, as one can easily deduce from the couplings in Eqs. (32) and (33). Note, however, that $\theta$ can also decay to right handed neutrinos $\nu^c$ through the nonrenormalizable superpotential term

$$M_{\nu^c} = \frac{\bar{\phi}\nu^c}{2M^2},$$

allowed by the gauge and R-symmetries of the model [5,6]. Here, $M_{\nu^c}$ denotes the Majorana mass of the relevant $\nu^c$. The scalar $\theta$ decays preferably into the heaviest $\nu^c$ with $M_{\nu^c} \leq m_{\text{inf}}/2$. The decay rate is given by

$$\Gamma_{\nu^c} \approx \frac{1}{16\pi} \kappa^2 m_{\text{inf}} \alpha^2(1 - \alpha^2)^{1/2},$$

where $0 \leq \alpha = 2M_{\nu^c}/m_{\text{inf}} \leq 1$. The subsequent decay of these $\nu^c$'s produces a primordial lepton number [17] which is then partially converted to the observed BAU through electroweak sphaleron effects.

The energy densities $\rho_S$, $\rho_\theta$, and $\rho_r$ of the oscillating fields $S$, $\theta$, and the 'new' radiation produced by their decay to higgsinos, higgses and $\nu^c$'s are controlled by the equations:

$$\dot{\rho}_S = -(3H + \Gamma_h)\rho_S, \quad \dot{\rho}_\theta = \rho_S(t)e^{-\Gamma_{\nu^c}(t-t_0)},$$

$$\dot{\rho}_r = -4H\rho_r + \Gamma_h\rho_S + (\Gamma_h + \Gamma_{\nu^c})\rho_\theta,$$

where

$$H = \frac{\sqrt{8\pi}}{\sqrt{3}M_P} (\rho_S + \rho_\theta + \rho_r)^{1/2}$$

is the Hubble parameter and overdots denote derivatives with respect to cosmic time $t$. The cosmic time at the onset of oscillations is taken $t_0 \approx 0$. The initial values of the various energy densities are taken to be $\rho_S(t_0) = \rho_\theta(t_0) \approx \kappa^2 M^4/6$, $\rho_r(t_0) = 0$. The 'reheat' temperature $T_r$ is calculated from the equation

$$\rho_S + \rho_\theta = \rho_r = \frac{\pi^2}{30} g_\ast T_r^4,$$

where the effective number of massless degrees of freedom is $g_\ast=228.75$ for MSSM.
The lepton number density \( n_L \) produced by the \( \nu^c \)'s satisfies the evolution equation:

\[
\dot{n}_L = -3Hn_L + 2\epsilon \Gamma_{\nu^c}n_\theta ,
\]

where \( \epsilon \) is the lepton number produced per decaying right handed neutrino and the factor of 2 in the second term of the rhs comes from the fact that we get two \( \nu^c \)'s for each decaying scalar \( \theta \) particle. The ‘asymptotic’ \( (t \to 0) \) lepton asymmetry turns out to be

\[
\frac{n_L(t)}{s(t)} \sim 3 \left( \frac{15}{8} \right)^{1/4} \pi^{-1/2} g_s^{-1/4} m_{infl}^{-1} \frac{e\Gamma_{\nu^c}}{\Gamma_h + \Gamma_{\nu^c}} \rho_r^{-3/4} \rho_se^{\Gamma_h t} .
\]

Assuming hierarchical light neutrino masses, we take \( m_{\nu_\mu} \approx 2.6 \times 10^{-3} \text{ eV} \) which is the central value of the \( \mu \)-neutrino mass coming from the small angle MSW resolution of the solar neutrino problem \[27\]. The \( \tau \)-neutrino mass is taken \( m_{\nu_\tau} \approx 7 \times 10^{-2} \text{ eV} \), the central value from SuperKamiokande \[8\]. Recent analysis \[28\] of the results of the CHOOZ experiment shows that the oscillations of solar and atmospheric neutrinos decouple. We thus concentrate on the two heaviest families ignoring the first one. Under these circumstances, the lepton number generated per decaying \( \nu^c \) is \[12,29\]

\[
\epsilon = \frac{1}{8\pi} g \left( \frac{M_3}{M_2} \right) \frac{c^2 s^2 \sin 2\delta (m_3^{D^2} - m_2^{2D^2})^2}{|\langle h^{(1)} \rangle|^2 (m_3^{D^2} s^2 + m_2^{D^2} c^2)} ,
\]

where \( g(r) = r \ln(1 + r^{-2}) \), \( |\langle h^{(1)} \rangle| \approx 174 \text{ GeV} \), \( c = \cos \theta \), \( s = \sin \theta \), and \( \theta \) \( (0 \leq \theta \leq \pi/2) \) and \( \delta \) \( (-\pi/2 \leq \delta < \pi/2) \) are the rotation angle and phase which diagonalize the Majorana mass matrix of \( \nu^c \)'s with eigenvalues \( M_2, M_3 \) \( (\geq 0) \). The ‘Dirac’ mass matrix of the neutrinos is considered diagonal with eigenvalues \( m_2^{D}, m_3^{D} \) \( (\geq 0) \).

For the range of parameters considered here, the scalar \( \theta \) decays into the second heaviest right handed neutrino with mass \( M_2 \) \( (< M_3) \) and, thus, \( M_{\nu^c} \) in Eqs.(11) and (12) should be identified with \( M_2 \). Moreover, \( M_3 \) turns out to be bigger than \( m_{infl}/2 \) as it should. We will denote the two positive eigenvalues of the light neutrino mass matrix by \( m_2 (=m_{\nu_\mu}) \), \( m_3 (=m_{\nu_\tau}) \) with \( m_2 \leq m_3 \). All the quantities here (masses, rotation angles and phases) are ‘asymptotic’ (defined at the grand unification scale \( M_{\text{GUT}} \)).

The determinant and the trace invariance of the light neutrino mass matrix imply \[23\] two constraints on the (asymptotic) parameters which take the form:

\[
m_2 m_3 = \left( \frac{m_2^{D} m_3^{D}}{M_2 M_3} \right)^2 ,
\]
\[
m_2^2 + m_3^2 = \left( \frac{m_2^D}{M_2} \right)^2 + \left( \frac{m_3^D}{M_3} \right)^2 + \frac{2(m_3^D - m_2^D)^2 c^2 s^2 \cos 2\delta}{M_2 M_3}. \tag{51}
\]

The $\mu - \tau$ mixing angle $\theta_{23}$ ($= \theta_{\mu\tau}$) lies \[29\] in the range
\[
|\varphi - \theta^D| \leq \theta_{23} \leq \varphi + \theta^D \leq \pi/2,
\]
where $\varphi$ ($0 \leq \varphi \leq \pi/2$) is the rotation angle which diagonalizes the light neutrino mass matrix, and $\theta^D$ ($0 \leq \theta^D \leq \pi/2$) is the ‘Dirac’ (unphysical) mixing angle in the 2–3 lepton sector defined in the absence of the Majorana masses of the $\nu^c$'s.

Assuming approximate $SU(4)_c$ symmetry, we get the asymptotic (at $M_{GUT}$) relations:
\[
m_2^D \approx m_c, \quad m_3^D \approx m_t, \quad \sin \theta^D \approx |V_{cb}|. \tag{53}
\]

Renormalization effects, for MSSM spectrum and $\tan \beta \approx m_t/m_b$, are incorporated \[29\] by substituting in the above formulas the values: $m_2^D \approx 0.23$ GeV, $m_3^D \approx 116$ GeV and $\sin \theta^D \approx 0.03$. Also, $\tan^2 2\theta_{23}$ increases by about 40% from $M_{GUT}$ to $M_Z$.

We take a specific MSSM framework \[30\] where the three Yukawa couplings of the third generation unify ‘asymptotically’ and, thus, $\tan \beta \approx m_t/m_b$. We choose the universal scalar mass (gravitino mass) $m_{3/2} \approx 290$ GeV and the universal gaugino mass $M_{1/2} \approx 470$ GeV. These values correspond \[31\] to $m_t(m_t) \approx 166$ GeV and $m_A$ (the tree level CP-odd scalar higgs mass) = $M_Z$. The ratio $\lambda/\kappa$ is evaluated \[32\] from
\[
\frac{\lambda}{\kappa} = \frac{\mu}{m_{3/2}} \approx \frac{M_{1/2}}{m_{3/2}} \left(1 - \frac{Y_f}{Y_f} \right)^{-3/7} \approx 3.95, \tag{54}
\]
where $Y_t = h_t^2 \approx 0.91$ is the square of the top-quark Yukawa coupling and $Y_f \approx 1.04$ is the weak scale value of $Y_t$ corresponding to ‘infinite’ value at $M_{GUT}$.

Eqs.(38)-(40) can now be solved, for $(\delta T/T)_Q \approx 6.6 \times 10^{-6}$ from COBE, $N_Q \approx 50$ and any value of $x_Q > 1$. Eliminating $x_Q$, we obtain $M$ as a function of $\kappa$ depicted in Fig.1. The evolution Eqs.(43)-(45) are solved for each value of $\kappa$. The parameter $\alpha^2$ in Eq.(12) is taken equal to 2/3. This choice maximizes the decay width of the inflaton to

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νe ’s and, thus, the subsequently produced lepton asymmetry. The ‘reheat’ temperature is then calculated from Eq.(46) for each value of κ. The result is again depicted in Fig.1.

The mass of the second heaviest νe, into which the scalar θ decays partially, is given by 

\[ M_2 = M_{νe} = α_m_{infl}/2 \]

and \( M_3 \) is found from the ‘determinant’ condition in Eq.(50). The ‘trace’ condition in Eq.(51) is then solved for δ(θ) which is subsequently substituted in Eq.(49) for \( ϵ \). The leptonic asymmetry as a function of the angle θ can be found from Eq.(48). For each value of κ, there are two values of θ satisfying the low deuterium abundance constraint \( Ω_B h^2 \approx 0.025 \). (These values of θ turn out to be quite insensitive to the exact value of \( n_B/s \).) The corresponding \( ν_μ \) ’s are then found and the allowed region of the mixing angle \( θ_μτ \) in Eq.(52) is determined for each κ. Taking into account renomalization effects and superimposing all the permitted regions, we obtain the allowed range of \( \sin^2 2θ_μτ \) as a function of κ, shown in Fig.2. We observe that \( \sin^2 2θ_μτ \approx 0.8 \) (from SuperKamiokande [3]) corresponds to \( 1.2 \times 10^{-6} \approx \kappa \approx 3.4 \times 10^{-6} \) which is rather small. (Fortunately, supersymmetry protects it from radiative corrections.)

The corresponding values of \( M \) and \( T_r \) can be read from Fig.1. One finds that

\[ 1.4 \times 10^{15} \text{ GeV} \lesssim M \lesssim 2 \times 10^{15} \text{ GeV} \]
\[ 1.8 \times 10^7 \text{ GeV} \lesssim T_r \lesssim 8.7 \times 10^7 \text{ GeV} \]

We observe that \( M \) turns out to be somewhat smaller than the MSSM unification scale \( M_{GUT} \). (It is anticipated that \( G_{LR} \) is embedded in a grand unified theory.) The ‘reheat’ temperature, however, satisfies the gravitino constraint (\( T_r \lesssim 10^9 \text{ GeV} \)). Note that, for the values of the parameters chosen here, the lightest supersymmetric particle (LSP) is an almost pure bino with mass \( m_{LSP} \approx 0.43 M_{1/2} \approx 200 \text{ GeV} \) and can, in principle, provide the cold dark matter of the universe. On the contrary, there is no hot dark matter candidate, in the simplest scheme.

In conclusion, we have shown that, in a supersymmetric model based on a left-right symmetric gauge group and leading ‘naturally’ to hybrid inflation, the μ problem can be easily solved. The observed BAU is produced via a primordial leptogenesis. For masses of \( ν_μ, ν_τ \) from the small angle MSW resolution of the solar neutrino puzzle and SuperKamiokande, maximal \( ν_μ − ν_τ \) mixing can be achieved. The required values of the coupling constant κ are, however, quite small (\( \sim 10^{-6} \)).

This work is supported by E.U. under TMR contract No. ERBFMRX–CT96–0090.
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FIG. 1. The mass scale $M$ (solid line) and the reheat temperature $T_r$ (dashed line) as functions of $\kappa$. 
FIG. 2. The allowed region (bounded by the solid lines) in the $\kappa - \sin^2 2\theta_{\mu\tau}$ plane for $m_{\nu_\mu} \approx 2.6 \times 10^{-3}$ eV and $m_{\nu_\tau} \approx 7 \times 10^{-2}$ eV.