Wake in anisotropic Quark-Gluon Plasma

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We calculate the wake in charge density and the wake potential due to the passage of a fast parton in an anisotropic quark gluon plasma (AQGP). For the sake of simplicity small $\xi$ (anisotropic parameter) limit has been considered. When the velocity ($v$) of the jet is parallel to the anisotropy direction ($\hat{n}$) and remains below the phase velocity ($v_p$), the wake in induced charge density shows a little oscillatory behavior in AQGP, contrary to the isotropic case. With the jet velocity greater than the phase velocity, the oscillatory behavior increases with $\xi$. Also for $v > v_p$, one observes a clear modification of the cone-like structure in presence of anisotropy. For the parallel direction in the backward region, the depth of the wake potential decreases with the increase of $\xi$ for $v < v_p$ and the potential becomes modified Coulomb-like for higher values of $\xi$. In the forward region, the potential remains modified Coulomb-like with the change in magnitude for nonzero $\xi$, for both $v > v_p$ and $v < v_p$. In the perpendicular direction, the wake potential is symmetric in the forward and backward regions. With the increase of $\xi$, the depth of negative minimum is moving away from the origin irrespective of the jet velocity. On the other hand when the jet velocity is perpendicular to the anisotropy direction, we find significant changes in the case of both wake charge density and potential in comparison to the isotropic case. For nonzero $\xi$, the oscillatory nature of the color charge wake is reduced at $v > v_p$. Also the oscillatory behavior of the wake potential along the direction of motion of the parton is attenuated in the backward direction for anisotropic plasma at parton velocity $v > v_p$. In the presence of anisotropy, for $v < v_p$, the screening potential along the perpendicular direction of the parton is transformed from the Lennard-Jones type to a modified Coulomb-like potential.

PACS numbers: 25.75.-q, 12.38.Mh

Keywords: wake, quark-gluon plasma, anisotropy

I. INTRODUCTION

One of the primary goals of ultrarelativistic heavy-ion collision experiments at BNL RHIC and at CERN LHC is to create and explore the novel state of matter in which quarks and gluons are deconfined, commonly known as quark-gluon plasma (QGP). In the early stage of heavy-ion collision, high $p_T$ partons produced by the hard scatterings will travel through the hot and dense medium and lose energy, mainly by radiative processes. As a consequence high $p_T$ hadrons produced due to parton fragmentation are suppressed. This phenomenon is the so-called jet quenching, because in the direction of propagation of the jet one observes a decrease of high energy hadrons and increase in the soft hadrons \[1-9\]. Moreover the experimental azimuthal dihadron distribution at RHIC shows a double peak structure in the away side \[10, 11\] for the intermediate $p_T$ particles. The wake induced by the jets are proposed as one of the possible explanations for the formation of double peak structure in two-particle correlation \[12, 13\]. However, recent studies show that the observation of double peak structure could be because of initial energy density fluctuations as has been argued in Refs. \[14, 15\]. Nevertheless, the passage of jets through the plasma leads to the formation of wakes \[16, 20\]. Mach cones \[12, 21, 22\] and Cerenkov radiation \[23-25\], which may be observable as collective excitation and shock wave in heavy-ion collisions.

When a test charge moves in the plasma, it creates a screening potential. The screening potential for a heavy quark-antiquark pair is strongly anisotropic and loses forward-backward symmetry with respect to the direction of the charged particle \[26\]. First, Ruppert and Muller \[16\] have investigated that when a jet propagates through the medium, a wake of current and charge density is induced which can be studied within the framework of linear response theory in two different scenarios: (i) a weakly coupled quark-gluon plasma described by hard thermal loop (HTL) perturbation theory and (ii) a strongly coupled QGP, which has the properties of a quantum liquid. The result shows the wake in both the induced charge and current density due to the screening effect of the moving parton. On the other hand, in the quantum liquid scenario, the wake exhibits an oscillatory behavior when the charge parton moves very fast. Later, Chakraborty et al. \[17\] also found the oscillatory behavior of the induced charge wake in the backward direction at
large parton speed using the high temperature approximation. In a collisional quark-gluon plasma, it is observed that the wake properties change significantly compared to the collisionless case [18]. Recently, Jiang et al. [19, 20] have investigated the color response wake in the viscous QGP with the HTL resummation technique. It is shown that the increase of the shear viscosity enhances the oscillation of the induced charge density as well as the wake potential.

In all the above phenomenological treatments of the plasma the system is assumed to be isotropic in momentum space. However, in the very early stage of heavy-ion collision, due to rapid longitudinal expansion of the matter along the beam axis the system becomes colder along the longitudinal direction than the transverse direction, leading to \( \langle p_L^2 \rangle < \langle p_T^2 \rangle \). Consequently, the plasma is anisotropic in momentum space. To characterize this anisotropy many signals have been proposed [25, 26]. We, in this work, will study the wake induced by a jet propagating through anisotropic quark-gluon plasma. The dielectric response function contains all the information of the chromoelectromagnetic properties of the plasma. In the presence of momentum-space anisotropy, the distribution functions of the quark and gluon are modified and it will affect the dielectric function. Moreover, due to anisotropic momentum distribution at very early stage of the heavy-ion collision, the collective modes of the AQGP may be unstable [27, 34]. These unstable modes grow exponentially in time, which leads to a more rapid thermalization and isotropization of the soft modes in QGP. Such process may play a significant role in the dynamical properties of the QGP. For example, the induced charge density and wake potential will be influenced by the anisotropic effect of the plasma as we shall see in the following.

The current work is organized as follows: In section 2 we briefly recall the necessary ingredients to calculate the induced color charge density and screening potential in QGP. Section 3 is devoted for the evaluation of dielectric tensor in AQGP. In section 4 and 5 the wake in charge density and potential will be described along with the numerical results. Finally, we conclude in section 6.

II. BASIC EQUATIONS

The covariant form of the Maxwell equation is

\[
\partial_\mu F^{\mu\nu}(K) = J^{\nu}_{ind}(K) + J^{\nu}_{ext}(K),
\]

where the total current is composed of the induced current \( J^{\nu}_{ind} \) and the external current \( J^{\nu}_{ext} \) introduced from the external sources. In the linear approximation, the equation of the gauge field is represented by the induced current in terms of the self-energy [27, 33]:

\[
J^{\mu}_{ind}(K) = \Pi^{\mu\nu}(K) A_\nu(K).
\]

where according to the semiclassical transport theory, the hard thermal loop resummed gluon self-energy can be written as [27]

\[
\Pi^{\mu\nu}(K) = g^2 \int \frac{d^4p}{(2\pi)^4} P^\mu \frac{\partial f(p)}{\partial p^\nu} \left( g^{\beta\gamma} - \frac{P^\beta K^\gamma}{K^2} + i\epsilon \right).
\]

One can easily show that the polarization tensor is symmetric, \( \Pi^{\mu\nu}(K) = \Pi^{\nu\mu}(K) \) and transverse, \( K^\mu \Pi^{\mu\nu}(K) = 0 \). In deriving the above equation it is assumed that the momentum is soft, \( k \sim gT \ll T \) and the magnitude of the field fluctuations is \( A \sim \sqrt{gT} \). By combining Eqs. (1) and (2) the external current can be related to the gauge field as

\[
[K^2 g^{\mu\nu} - K^\mu K^\nu + \Pi^{\mu\nu}(K)] A_\nu(K) = -J^{\mu}_{ext}(K).
\]

In the temporal axial gauge, where \( A_0 = 0 \), the above equation becomes,

\[
[(k^2 - \omega^2) \delta^{ij} - k^i k^j + \Pi^{ij}(K)] E^j = [\Delta^{-1}(K)]^{ij} E^j(K) = i\omega J^{i}_{ext}(K).
\]

The above expression can also be written in terms of chromodielectric tensor \( e^{ij}(K) \) as

\[
[k^2 \delta^{ij} - k^i k^j - \omega^2 \epsilon^{ij}(k, \omega)] E^j = i\omega J^{i}_{ext}(k, \omega).
\]

The poles of the propagator \( \Delta^{ij}(K) \) give the dispersion relation for the waves in the medium. When an external fast parton passes through QGP, it will disturb the plasma and create induced color charge density [33]. The total color charge density is given as

\[
\rho^{a}_{tot}(k, \omega) = \rho^{a}_{ext}(k, \omega) + \rho^{a}_{ind}(k, \omega),
\]
where $a$ represents the color index and $\rho_{ext}^a(k, \omega)$ is the external color charge density. The total color charge density is linearly related to $\rho_{ext}^a$ through dielectric response function $\epsilon(k, \omega)$, so that we may write

$$\rho_{tot}^a(k, \omega) = \frac{\rho_{ext}^a(k, \omega)}{\epsilon(k, \omega)}.$$  

(8)

It is clearly seen that the dielectric function provides a direct measure of the screening of external charge density due to the induced color charge density in the plasma [36]. The dielectric function can be calculated from the dielectric tensor using the following relation

$$\epsilon(k, \omega) = \frac{k_i \epsilon_{ij}(k, \omega) k_j}{k^2}.$$  

(9)

In an isotropic and homogeneous medium, dielectric response function is related to the longitudinal dielectric tensor which is independent of the direction of $k$ i.e. $\epsilon(k, \omega) = \epsilon_L(k, \omega)$. In the general case, the induced color charge density is explicitly written as

$$\rho_{ind}^a(k, \omega) = -\left\{1 - \frac{1}{\epsilon(k, \omega)}\right\} \rho_{ext}^a(k, \omega).$$  

(10)

The wake potential induced by the fast parton is determined from the Poisson equation:

$$\Phi^a(k, \omega) = \rho_{ext}^a(k, \omega) k^2 \epsilon(k, \omega).$$  

(11)

Also one can relate the external current to the induced current in the linear response theory by

$$J_{ind}^i(k, \omega) = -\Pi_{ij}(k, \omega) \Delta^j_{ext}(k, \omega) J_{ext}^k.$$  

(12)

Now we consider a charge particle, $Q^a$ moving with a constant velocity $v$ and interacting with the anisotropic plasma. The external current and charge density associated due to the test charge particle can be written as [16, 17]:

$$J_{ext}^a = 2\pi Q^a v \delta(\omega - k.v),$$  

$$\rho_{ext}^a = 2\pi Q^a \delta(\omega - k.v).$$  

(13)

The delta function indicates that the value of $\omega$ is real and the velocity of the charge particle is restricted between $0 < v < 1$ which is also known as the Cerenkov condition for the moving parton in a medium. Therefore the plasmon modes are determined in the spacelike region of the $\omega - k$ plane.

### III. SELF-ENERGY IN ANISOTROPIC PLASMA

In this section, we shall briefly discuss how to calculate the dielectric function $\epsilon(k, \omega)$ in an anisotropic media. The hard-loop gluon polarization tensor within the Valsov approximation is given by [27, 35]

$$\Pi^{ij}(K) = -g^2 \int \frac{d^3p}{(2\pi)^3} p^i \partial^j f(p) \left(\delta^{jl} + \frac{v^j k^l}{K.V + i\epsilon}\right),$$  

(14)

where $f(p)$ is the distribution function which is completely arbitrary. We assume that the phase-space distribution for the anisotropic plasma is given by the following ansatz [27, 34]:

$$f(p) = f_\xi(p) = N(\xi)f_{iso}(\sqrt{p^2 + \xi(p.n)^2}).$$  

(15)

$N(\xi)$ is a normalization constant, $\hat{n}$ the is direction of anisotropy which is along the beam direction, $\xi$ is a parameter which represents the strength of anisotropy and $f_{iso}$ is an arbitrary isotropic distribution function. Using the above ansatz one can simplify Eq. (14) to

$$\Pi^{ij}(K) = m_D^2 \sqrt{1 + \xi} \int \frac{d\Omega}{(4\pi)} v^i \left(\delta^{jl} + \frac{v^j k^l}{K.V + i\epsilon}\right).$$  

(16)
where $m_D$ is the Debye mass, represented by

$$m_D^2 = -\frac{g^2}{2\pi^2} \int_0^\infty dp p^2 \frac{df_{iso}(p^2)}{dp}$$  \hspace{1cm} (17)$$

Because of the anisotropy direction, we therefore, need to construct a tensorial basis to represent the self-energy which depends on not only the momentum $k^i$ but also the anisotropy vector $n^i$, with $n^2 = 1$. Using the proper tensor basis\[27\] one can decompose the self-energy into four structure functions as:

$$\Pi^{ij}(k) = \alpha A^{ij} + \beta B^{ij} + \gamma C^{ij} + \delta D^{ij}$$  \hspace{1cm} (18)$$

$\alpha$, $\beta$, $\gamma$ and $\delta$ are determined by the following contractions:

$$k^i \Pi^{ij} k^j = k^2 \beta, \quad \hat{n}^i \Pi^{ij} k^j = \hat{n}^2 k^2 \delta,$$

$$\hat{n}^i \Pi^{ij} \hat{n}^j = \hat{n}^2 (\alpha + \gamma), \quad Tr\Pi^{ij} = 2(\alpha + \beta + \gamma).$$

The structure functions depend on $\omega$, $k$, $\xi$, and the angle between the anisotropy vector and the momentum($\theta$). In isotropic limit($\xi \rightarrow 0$) the structure functions $\alpha$ and $\beta$ are directly related to the isotropic transverse and longitudinal self-energies respectively and other structure functions vanish \[27\]. To get the analytical expression one can calculate the structure functions in the small limit. To linear order in $\xi$ we have \[27\]

$$\alpha = \Pi_T(z) + \xi \left[ z^2 \left( 3 + 5 \cos 2\theta \right) m_D^2 \frac{1}{6} \left( 1 + \cos 2\theta \right) m_D^2 + \frac{1}{4} \Pi_T(z) \left( (1 + 3 \cos 2\theta) - z^2 (3 + 5 \cos 2\theta) \right) \right],$$

$$\beta = z^2 \left[ \Pi_L(z) + \xi \left[ \frac{1}{6} \left( 1 + 3 \cos 2\theta \right) m_D^2 + \Pi_L(z) \left( \cos 2\theta - \frac{z^2}{2} (1 + 3 \cos 2\theta) \right) \right] \right],$$

$$\gamma = \frac{\xi}{3} \left( 3 \Pi_T(z) - m_D^2 \right) (z^2 - 1) \sin^2 \theta,$$

$$\delta = \frac{\xi}{3k} \left[ 4 z^2 m_D^2 + 3 \Pi_T(z) (1 - 4 z^2) \right] \cos \theta,$$  \hspace{1cm} (20)$$

with

$$\Pi_T(K) = \frac{m_D^2}{2} \left[ 1 - \frac{1}{2} \left( z - \frac{1}{z} \right) \left( \ln \left| \frac{z + 1}{z - 1} \right| - i\pi \Theta(1 - z^2) \right) \right],$$

$$\Pi_L(K) = m_D^2 \left[ 2 \left( \ln \left| \frac{z + 1}{z - 1} \right| - i\pi \Theta(1 - z^2) \right) - 1 \right],$$

where $z = \frac{\omega}{k}$. The dielectric tensor and the self-energy are related by the following relation:

$$\epsilon^{ij} = \delta^{ij} - \frac{\Pi^{ij}}{\omega^2}$$  \hspace{1cm} (22)$$

Using Eqs.\[19\] and \[18\] together with Eq.\[22\], one can find the dielectric function in terms of structure functions in AQGP.

IV. INDUCED CHARGE DENSITY

In the presence of the test charge particle, the induced charge density and the wake potential depend on the velocity of the test charge and the distribution of the background particle \[37\]. When a charge particle is introduced into a plasma at rest, it acquires a shielding cloud. As a result, the induced charge distribution is spherically symmetric. When a charge particle is moving with a fixed velocity in the plasma, the induced charge distribution no longer remains symmetric. It deforms the screening cloud from a spherical shape to an ellipsoidal one. Therefore the induce charge distribution loses forward and backward symmetry.

We assume that in the AQGP, the average phase velocity is $v_p$. According to Cerenkov condition there will be two important scenarios which occur due to the interaction of the particle with the plasmon wave: First, the modes with low speed(less than the plasmon phase velocity) can be excited, but the particles moving slightly slower than the wave, will be accelerated while the charge particle moving faster than the wave will decrease its average velocity \[30\].
The slowly moving particle absorbs energy from the wave and the faster particle transfers its extra energy to the wave. The absorption and emission of energy result in a wake in the induced charge density as well as in the potential. Second, when the charge particle moving with a speed greater than the average phase velocity \( v_p \), the modes are excited and they may not be damped. The excited modes can generate Cerenkov-like radiation and Mach stem which leads to oscillation both in induced charge density and in wake the potential. It is well known that a parton moving with a supersonic speed in a plasma produces a Mach region which is of conical shape with an opening angle with respect to the direction of the particle propagation given by the expression

\[
\theta_M = \arcsin\left(\frac{c_s}{v}\right),
\]

where \( c_s \) is the sound velocity. The cone of the front of the shock wave has the angle \( \theta_f = \pi/2 - \theta_M \). Therefore, the cone of the particles should be produced, when the parton is moving with a supersonic speed in a plasma.

Now we study the color charge density induced by the fast parton in an anisotropic plasma. By substituting Eq. (13) into Eq. (10) and transforming into \( r - t \) space, the induced charge density becomes,

\[
\rho_{\text{ind}}^a(r, t) = 2\pi Q^2 \int \frac{d^2 k}{(2\pi)^2} \int \frac{d\omega}{2\pi} \exp(i(k \cdot r - \omega t)) \left( \frac{1}{\epsilon(\mathbf{k}, \omega)} - 1 \right) \delta(\omega - k \cdot \mathbf{v}).
\]

We assume that the parton is moving along the z-direction which is the beam direction i.e. \( \mathbf{v} \parallel \mathbf{n} \). We use spherical coordinates system for \( \mathbf{k} \) i.e. \( \mathbf{k} = (k \sin \theta \cos \phi, k \sin \theta \sin \phi, k \cos \theta) \) and cylindrical coordinates for \( \mathbf{r} = (\rho, \theta, z) \). The induced charge density can be written as

\[
\rho_{\text{ind}}^a(r, t) = \frac{Q^2 m_D^3}{2\pi^2} \int_0^\infty dk \int_0^1 d\chi \int_0^{\pi}(d\phi \int_0^{\pi} d\omega \left[ \sin \left( \frac{\text{Re} \omega}{\Delta} \right) - 1 \right]) \left[ \cos \left( \frac{\text{Re} \omega}{\Delta} \right) \right] \left[ \text{Im} \left( \frac{\mathbf{k} \cdot \mathbf{v}}{\Delta} \right) \right] \bigg|_{\omega = k \cdot \mathbf{v}},
\]

where \( \chi \) is represented as \( \cos \theta \), \( J_0 \) is the zeroth-order Bessel function, \( \Gamma = k(\chi(z - vt)m_D \) and \( \Delta = (\text{Re} \omega(\mathbf{k}, \omega))^2 + (\text{Im} \omega(\mathbf{k}, \omega))^2 \). To get the above equation we use the simple transformation \( \omega \rightarrow \omega m_D \) and \( k \rightarrow km_D \). It is seen that the charge density \( \rho_{\text{ind}}^a \) is proportional to \( m_D^3 \). We now discuss the numerical result of the induced color charge density with two different speeds of the fast parton: One is below the average phase velocity \( v = 0.55 \) and other is greater than it \( (v = 0.99) \).

In Figs. (1a-1f) we display the contour plot of the scaled equicharge lines for isotropic and anisotropic plasma. The contour plot of the equicharge lines shows a sign flip along the direction of the moving parton in Fig. 1. The left(right) panel in Fig. 1 shows the contour plot of the induced color charge density in isotropic and anisotropic plasma for two different anisotropic parameter \( \xi \) with parton velocity \( v = 0.55(0.99) \). It is clearly seen that the equicharge lines are modified in anisotropic plasma for parton velocity \( v = 0.55 \). With the increase of strength of the anisotropy we found that the positive charge lines appear alternately in the backward space which leads to small oscillatory behavior of the color charge wake(see in Fig. (1e)).

When a charge particle moves faster than the average speed of the plasmon, the induced charge density forms a cone like structure as shown in the right panel of Fig. (1f). It is clearly noticed that the color charge wake is significantly different from when the parton velocity is \( v = 0.55 \). It is also seen that the induced charge density is oscillatory in nature. The supersonic nature of the parton leads to formation of the Mach cone and the plasmon modes could emit a Cerenkov-like radiation which spatially limits the disturbances in the induced charge density [39]. Due to the effect of anisotropy, the color charge wake is modified significantly and the oscillatory behavior is more pronounced than the isotropic case and it is also seen that the oscillatory nature increases with the increase of the anisotropic parameter \( \xi \). In the backward space \((z - vt) < 0\), induced color charge density is very much more sensitive to the anisotropic plasma than that in the forward space \((z - vt) > 0\). If we consider charge particle moves in a plane, the structure of the contour plot is the same as Fig. 1, the only difference is that its makes an angle within the \( \rho - z \) plane.

Next we consider the case when the parton moves perpendicular to the anisotropy direction in which case the induced charge density can be written as

\[
\rho_{\text{ind}}^a(r, t) = \frac{Q^2 m_D^3}{2\pi^2} \int_0^\infty dk \int_0^1 d\chi \int_0^{2\pi} d\phi \frac{1}{2\pi} \left[ \cos \Omega \left( \frac{\text{Re} \omega(\mathbf{k}, \omega)}{\Delta} \right) - 1 \right] \left[ \sin \Omega \left( \frac{\text{Im} \omega(\mathbf{k}, \omega)}{\Delta} \right) \right] \bigg|_{\omega = k \cdot \mathbf{v}},
\]

with \( \Omega = k(\chi + (\rho - vt) - \cos \phi)m_D \). Numerical evaluation of the above equation leads to the contour plots of the induced charge density shown in Fig. (2a-2d). The left(right) panels show the contour plots of the induced charge density for the parton velocity is \( v = 0.55(0.99) \). Because of the effect of anisotropy, one observes a clear modification of the induced charge density. When \( v = 0.99 \), i.e. \( v \) is larger than the phase velocity \( v_p \), the number of induced
FIG. 1: (Color online) Left Panel: The plot shows equicharge line with parton velocity $v = 0.55$ for different $\xi (0, 0.4, 0.8)$. Right Panel: Same as left panel with parton velocity $v = 0.99$. 
charged lines that appear alternately in the backward space is reduced for the anisotropic plasma in comparison to the isotropic plasma. Therefore the anisotropy reduces the oscillatory behavior of the induced color charge density, when the parton moves in the transverse plane. This, as we shall see later, will lead to less oscillatory wake potential. Note that these observations are just opposite to the case when the parton moves parallel to the anisotropy direction.

V. WAKE POTENTIAL IN AQGP

By combining Eqs. (11) and (13), the wake potential in \( r - t \) space due to the motion of a charge parton can be written as \[16, 17\]

\[
\Phi^a(r, t) = 2\pi Q^a \int \frac{d^3 k}{(2\pi)^3} \int \frac{d\omega}{2\pi} \exp\{i(k.r - \omega t)\} \frac{1}{k^2 \epsilon(\omega, k)} \delta(\omega - k.v). \tag{27}
\]

Using similar coordinate system as before, the screening potential turns into

\[
\Phi^a(r, t) = \frac{Q^a m_D}{2\pi^2} \int_0^\infty dk \int_0^1 d\chi J_0(k\rho \sqrt{1 - \chi^2 m_D}) \left[ \cos \frac{Re\epsilon(\omega, k)}{\Delta} + \sin \frac{Im\epsilon(\omega, k)}{\Delta} \right] \bigg|_{\omega = k.v}. \tag{28}
\]

The above equation shows that the wake potential is proportional to the Debye mass. We solve the wake potential for two spatial cases, (i) along the parallel direction of the fast parton, i.e. \( r \parallel v \) and also \( \rho = 0 \) and (ii) perpendicular...
much more different than in the case of \( v \) compared to the case when \( v \) direction. It is clearly visible that the depth of negative minimum is increased and it is shifted towards the origin both isotropic and anisotropic plasma. Such oscillation of the wake potential is clearly reflected only in the backward potential. Moreover, with the increase of \( \xi \) negative minimum for different \( \xi \) is more pronounced and it extends to a large distance. But, there is no significant change of the absolute value of the wake potential in (28) is also solved for two spatial cases: (i) along the direction of the moving parton i.e. \( r \parallel v \) and (ii) the moving charge particle. When \( v \) wake potential is found to be the Lennard-Jones type due to the deformed screening charge cloud in the presence of \( v \) space, the wake potential for isotropic plasma decreases with the increase of \( \xi \). The structure of the wake potential is symmetric in backward and forward directions, no matter what the speed is. The left panel shows the wake potential along the direction of the moving color charge. In the backward direction, the screening potential is a modified Coulomb potential. When the parton moves faster than \( v = 0.99 \) away from the origin for both the jet velocities considered here.

FIG. 3: (Color online) Left panel: Scaled wake potential along the motion of the fast parton i.e. \( z \)-axis for different \( \xi \) with two different parton velocity \( v = 0.55 \) and \( v = 0.99 \). Right panel: same as left panel but perpendicular to direction of motion of the parton.

to direction of the parton, i.e. \( r \perp v \). The potential for the first case is obtained as

\[
\Phi^a(r, t) = \frac{Q^a m_D}{2\pi^2} \int_0^\infty dk \int_0^1 d\chi \left[ \cos \Gamma \frac{\text{Re}(\omega, k)}{\Delta} + \sin \Gamma \frac{\text{Im}(\omega, k)}{\Delta} \right] |_{\omega = k, v},
\]

whereas that for the perpendicular case is

\[
\Phi^a_\perp(r, t) = \frac{Q^a m_D}{2\pi^2} \int_0^\infty dk \int_0^1 d\chi J_0(k\rho\sqrt{1 - \chi^2 m_D}) \left[ \cos \Gamma' \frac{\text{Re}(\omega, k)}{\Delta} - \sin \Gamma' \frac{\text{Im}(\omega, k)}{\Delta} \right] |_{\omega = k, v},
\]

with \( \Gamma' = k\chi vtm_D \).

Fig. 3 describes the wake potential in two specific directions. In these figures, the scaled parameter \( \Phi^a_0 \) is given by \( \frac{2\pi^2}{m_D} \Phi^a \). The left panel shows the wake potential along the direction of the moving color charge. In the backward space, the wake potential for isotropic plasma decreases with the increase of \( z - vt \) and exhibits a negative minimum when \( v = 0.55 \) i.e. in the backward direction, the wake potential is a Lennard-Jones potential type which has a short range repulsive part as well as a long range attractive part [17]. With the increase of the anisotropic parameter \( \xi \) the depth of the negative minimum decreases for \( v = 0.55 \). The position of the negative minimum is same for \( \xi = 0 \) and \( \xi = 0.5 \). But for \( \xi = 0.8 \), the wake potential increases towards the origin, and their are no minima and behaves like a modified Coulomb potential. When the parton moves faster than \( v_p \), i.e. when \( v = 0.99 \), the wake potential is much more different than in the case of \( v = 0.55 \). For \( v = 0.99 \), the wake potential shows an oscillatory behavior in both isotropic and anisotropic plasma. Such oscillation of the wake potential is clearly reflected only in the backward direction. It is clearly visible that the depth of negative minimum is increased and it is shifted towards the origin compared to the case when \( v = 0.55 \). Due to the anisotropic effect of the plasma, the oscillation of the wake potential is more pronounced and it extends to a large distance. But, there is no significant change of the absolute value of the negative minimum for different \( \xi (0.5, 0.8) \). In the forward direction, the screening potential is a modified Coulomb potential. Moreover, with the increase of \( \xi \) the potential increases for both \( v = 0.55 \) and \( v = 0.99 \). The right panel of Fig. 3 describes the wake potential along the perpendicular direction of the moving parton. It can be seen that the wake potential is symmetric in backward and forward directions, no matter what the speed is. The structure of the wake potential is found to be the Lennard-Jones type due to the deformed screening charge cloud in the presence of the moving charge particle. When \( v = 0.55 \), the value of negative minimum is increased with an increase of \( \xi \), but in the case of \( v = 0.99 \), it decreases with \( \xi \). However, with the increase of \( \xi \) the depth of negative minimum is moving away from the origin for both the jet velocities considered here.

We now discuss the wake potential when the parton moves perpendicular to the anisotropy direction. The wake potential in (29) is also solved for two spatial cases: (i) along the direction of the moving parton i.e. \( r \parallel v \) and (ii)
perpendicular direction of the parton, i.e. \( \mathbf{r} \perp \mathbf{v} \). The wake potential for the parallel case can be written as

\[
\Phi_{\parallel}^i(r,t) = \frac{Q^a D m_D}{2 \pi^2} \int_0^\infty dk \int_0^{\pi} d\chi \int_0^{2\pi} d\phi \left[ \cos \Omega' \frac{\Re\epsilon(k)}{\Delta} + \sin \Omega' \frac{\Im\epsilon(k)}{\Delta} \right]_{\omega = k \cdot \mathbf{v}},
\]

where \( \Omega' = k(\rho - vt) \sqrt{1 - \chi^2 \cos \phi} m_D \). For the perpendicular case it is given by,

\[
\Phi_{\perp}^i(r,t) = \frac{Q^a D m_D}{2 \pi^2} \int_0^\infty dk \int_0^{\pi} d\chi \int_0^{2\pi} d\phi \left[ \cos \Omega'' \frac{\Re\epsilon(k)}{\Delta} + \sin \Omega'' \frac{\Im\epsilon(k)}{\Delta} \right]_{\omega = k \cdot \mathbf{v}},
\]

with \( \Omega'' = k(z\chi - vt) \sqrt{1 - \chi^2 \cos \phi} m_D \). In Fig. 4, the scaled wake potentials are shown for both parallel and perpendicular directions of the parton i.e. when \( \mathbf{v} \parallel \mathbf{r} \) and \( \mathbf{v} \perp \mathbf{r} \). The left panel shows screening potential along the parallel direction of the moving color charge. For both the velocity limits considered here, in the forward (backward) direction, the behavior of the wake potential is more like a modified Coulomb (Lennard-Jones) potential as in the case when \( \mathbf{v} \parallel \hat{\mathbf{n}} \). For \( v = 0.99 \), the wake potential shows an oscillatory behavior in an isotropic plasma but in anisotropic case, oscillatory structure of the wake potential is smeared out for \( \xi = 0.5 \) and \( \mathbf{v} \perp \hat{\mathbf{n}} \). It is clearly seen that the depth of the negative minimum is increased in the case of anisotropic plasma for both the parton velocities considered here. Moreover, in the anisotropic case the depth of the negative minimum is moving away from the origin.

The contribution of the wake potential in the perpendicular direction of the moving parton is shown in the right panel in Fig. 4. When the parton moves with the velocity \( v = 0.55 \), the anisotropy modifies the structure of the wake potential significantly; i.e. it becomes a modified coulomb potential instead of Lennard-Jones potential. For \( v = 0.99 \), the wake potential is a Lennard-Jones potential type but the depth of the minimum decreases in anisotropic plasma.

VI. SUMMARY

In this work, we have investigated the wake in charge density as well as the wake potential induced by a fast parton propagating through the anisotropic quark-gluon plasma expected to be formed in heavy-ion collisions. For the sake of simplicity small \( \xi \) expansion \((0 < \xi < 1)\) of the polarization tensor has been considered. The anisotropic effect modifies the dielectric tensor of the QGP, as a consequence, it influences the distribution of the induced color charge and the screening potential. To calculate the induced charge density of AQGP, two different velocities of the parton have been used for both the induced charge density and the wake potential. When the parton moves parallel to the anisotropy direction with a speed \( v = 0.55 \), the anisotropic effect at \( \xi = 0.8 \) makes a small oscillation
of the induced charge density in contrast to the isotropic case. For a larger speed $v = 0.99$, which is greater than the average plasmon speed, the effect of the anisotropy on the charge density becomes more remarkable. We also show that with the increase of $\xi$, the distribution of the induced charge density is much more oscillatory in nature. For the case of the wake potential, we focus on the parallel and perpendicular directions of the fast parton. Along the parallel direction of parton, the wake potential shows a decrease of negative minimum in presence of anisotropy in the backward direction and the potential at $\xi = 0.8$ behaves as a modified Coulomb-like potential for $v = 0.55$. For $v = 0.99$, the wake potential exhibits an oscillatory behavior in the backward direction and it is amplified with the increase of the strength of the anisotropy but no appreciable change of the negative minimum is observed. The numerical analysis of the wake potential along the perpendicular direction of moving parton leads to Lennard-Jones type potential for both $v = 0.55$ and $v = 0.99$. Furthermore, we have investigated the case when $v \perp \hat{n}$. Results show that the anisotropy minimizes the oscillatory strength of the induced charged density, when the parton moves with the velocity $v = 0.99$. As a consequence we do not find any oscillatory nature of the wake potential in the backward direction at $\xi = 0.5$, when the parton moves along the parallel direction with $v = 0.99$. For $\xi = 0.5$, the depth of the negative minimum is increased and also shifts away from the origin for both the parton velocities considered here. In the perpendicular direction of the moving parton with velocity $v = 0.55$ and $\xi = 0.5$, the screening potential behaves like modified Coulomb potential instead of a Lenard-Jones potential, contrary to the case when $v \parallel \hat{n}$.

We end by making the following comments. The effect of collision on the collective modes in anisotropic QGP has been investigated by using BGK collisional kernel where the wave vector is parallel to the anisotropy direction [40]. The reason for this is that the growth of the unstable modes is the highest in such case. It is found that the inclusion of collision slows down the growth rate of the unstable modes. Similar effects can also be incorporated in the present calculation as has been done in Ref. [18] for the isotropic case where it has been shown that the wakes are significantly modified. It is thus expected that inclusion of collision in the case of AQGP might lead to further modifications, such as, for both the velocity limits considered here, we expect more oscillatory behavior when $v \parallel n$ and almost non-oscillatory nature in the wake potential in the backward direction for $v > v_p$ when $v \perp \hat{n}$. Work on this issue is in progress [41]. It is to be noted that we have considered here the small $\xi$ case. The extension of the present calculation for arbitrary $\xi$ is worth investigating.

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