Calculation of Electromagnetic Thrust for Long Primary Double Sided Linear Induction Motors with Secondary of Arbitrary Length

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Abstract. The expressions of the air gap flux density and secondary current density of long primary double-sided linear induction motor (LPDLIM) with arbitrary secondary length are derived by the numerical method, which with the secondary as the moving reference frame. The effects of the forward and backward components of end effects on the normal travelling wave and the secondary current density are analyzed respectively, and their temporal and spatial distributions are presented. And then the electromagnetic thrust and its ripple expression considering the longitudinal end effect of the air gap magnetic field and the components of the secondary current density are solved. The slip characteristics distribution of each electromagnetic thrust and thrust ripple components, and the contribution of end effect to resultant thrust and force ripple of the motor with arbitrary secondary length are investigated. Finally, the thrust characteristics of the LPDLIM with arbitrary length secondary are verified by the results of the FEM model. The calculation shows that the appropriate secondary length can reduce the thrust ripple of the LPDLIM.

1. Introduction

The long primary double-sided linear induction motor (LPDLIM) has a simple secondary structure and remarkable reliability, which is convenient for acceleration and braking. It is suitable for applications where the load is accelerated to a predetermined speed in a short time and a short distance, such as collision test platform, UAV ejection, electromagnetic launch system of aircraft carrier [1-3], etc. In [4], the design equation is derived and used to design double sided linear induction motor. The slotless LPDLIM [5], multi-phase motor [6], and multi-stator motor sharing one secondary [7] are proposed for electromagnetic launch. Different from the diversity of the secondary structure of the short primary single-sided linear induction motor [8], the secondary of these machines consists only of the conductive sheet (typically aluminium or copper). In [9], the influence of design parameters, such as secondary thickness and conductivity, etc., on end effect related attenuation constant in LPDLIM are studied. The ladder-slit plate secondary is designed for double sided linear induction motor [10]. The ladder-slit type secondary for LPDLIM eliminates the second transverse end effects, standardizes eddy current path, and improves the ratio of thrust to power [11].

Since the secondary of LPDLIM is a metal conductive plate of finite length, the secondary length can be any value greater than or less than the primary length. However, the length of secondary is usually the same length as one segmented primary, or selected as integer multiple pole pitch. While the influence of the secondary length on the thrust performance of the LPDLIM has not been studied.
The finite length secondary of LPDLIM produces the longitudinal end effect, and the end effect not only produces thrust, but also produces thrust ripple [12]. When the secondary length of the LPDLIM is any length (not the integer multiple of the pole pitch), the electromagnetic thrust generated by the normal travelling wave contains a certain amplitude of force ripple, which is different from that of the traditional secondary with the integer multiple of the pole pitch, only contains steady thrust. It is possible that the force ripple caused by the normal travelling wave magnetic field of the secondary with any length may offset part of the thrust ripple caused by the longitudinal end effect, thus reducing the output ripple of the motor.

In this paper, in order to calculate the steady thrust and thrust ripple of LPDLIM with any length secondary, an analytical calculation model of the motor with the secondary as the motion reference was established. The analytical expressions of the air gap flux density and the secondary surface current density when the LPDLIM with any length secondary is driven by a constant current source are deduced, and the distribution characteristics of the air gap flux density and the surface current density of the secondary are analysed. The electromagnetic thrust and its ripple expression considering the longitudinal end effect of the air gap magnetic field and the components of the secondary current density are solved, and the effect of each thrust component on the resultant thrust is analysed. Finally, the distribution of resultant thrust and ripple with the secondary length considering the end effect is studied. The comparison between the results of analytical calculation and that of finite element calculation shows that the analytical expressions are reliable. The calculation shows that the appropriate secondary length can reduce the thrust ripple of the LPDLIM.

2. Air Gap Flux Density and Current Density of Secondary

2.1. Expression of Air Gap Flux Density
In order to realize the modular design and avoid the pulsating MMF when the relative position of the primary and secondary changes due to the half-filled staggered double-layer winding, single-layer windings are more suitable for long primary motors. The cross section diagram of the LPDLIM with arbitrary length secondary and single layer stator winding is shown in figure 1. Some assumptions are made to simplify the analysis as [12], and the mathematical model of LPDLIM with the secondary as the motion reference is shown in figure 2.

![Figure 1. Cross section of LPDLIM with secondary of arbitrary length.](image1)

![Figure 2. Mathematical model of LPDLIM with secondary of arbitrary length.](image2)

The general expressions of the air gap flux density with secondary as motion reference for LPDLIM are derived by solving the electromagnetic equations. By introducing the corresponding boundary conditions, the expression of the air gap flux density in the coupling region is obtained as equation (1), which is different from [9]. Where \( s \) is the slip rate, \( \omega \) is the angular frequency of supply, \( L_2 \) is the secondary length, \( \mu_0 \) is vacuum permeability, \( \delta \) is the equivalent electromagnetic air gap length, \( m_1 \) is phase number, \( W_1 \) is the number of series turns per phase, \( k_{w1} \) is the fundamental coefficient of winding, \( I_1 \) is the RMS value of primary phase current, \( p \) is the pole pairs; \( \tau \) is the pole pitch, \( \sigma_e \) is the effective surface conductivity of secondary, \( f \) is the current frequency, \( J_1 \) is the surface current density for the primary, \( G \) is defined as the goodness factor of the motor, \( \phi_{1f} = \arctan(1/sG) \) and \( k = \pi/\tau \).
For LPDLIM with arbitrary length secondary, the air gap flux density in coupling region I consists of three parts with the same slip angular frequency $s\omega$. The first item $B_{y1}$ of equation (1) is the normal travelling wave, the second item $B_{y2}$ is referred to as the forward travelling wave and the last item $B_{y3}$ is the backward travelling wave. It can be seen that the forward travelling wave travels in the opposite direction compared with the backward travelling wave in figure 3 and figure 4, and they are symmetrically distributed about the secondary centre in the longitudinal direction. Different from the [9] that the effect of backward travelling wave can be negligible, the backward wave of air gap flux density with secondary as motion reference cannot be ignored. From the backward travelling wave that if the secondary length $L_2$ is the integral multiple pole pitch, i.e. $2\pi r$, the $(-kL_2)$ will not have any effect on the backward wave, which can be ignored, otherwise, it cannot be ignored.

$$
\begin{align*}
B_{y} &= B_{y1} + B_{y2} + B_{y3} \\
B_{y1} &= J_1 \frac{\mu_0}{k \delta} \left(1 + s^2 G^2 \right)^{-1/2} \cos(s\omega - kx + \phi_{F1}) \\
B_{y2} &= -J_1 \frac{\mu_0}{k \delta} \left(1 + s^2 G^2 \right)^{-1/2} sG e^{-ax} \sin(s\omega - ax + \phi_{F1}) \\
B_{y3} &= -J_1 \frac{\mu_0}{k \delta} \left(1 + s^2 G^2 \right)^{-1/2} sG e^{a(x-L_2)} \sin(s\omega - kL_2 + \alpha(x-L_2) + \phi_{F1})
\end{align*}
$$

0 < x < L_2

Figure 3. The forward travelling wave.  
Figure 4. The backward travelling wave.  
Figure 5. The normal travelling wave.  
Figure 6. The resultant air-gap flux density.

For rotary machines, only the normal travelling wave in the air gap. However, due to the discontinuous of the secondary, there are two end effect travelling waves in the air gap magnetic field. These two travelling waves are superimposed upon the normal traveling waves (figure 5), which results in the distortion of the resultant air gap flux density distribution, as in figure 6, especially at the entry and exit end of the secondary, thus affecting the performance of the motor.

2.2. Surface Current Density of Secondary

The derived expression of secondary surface current density is shown in equation (2), and the $\phi_{F2}$ = $\arctan[(1-sG)/(1+sG)]$. The secondary surface current density is composed of three parts, just like the expression of air gap flux density. The temporal and spatial distribution of the normal surface secondary current density is sinusoidal. The forward and backward current density caused by the end effect makes the secondary density deform in the spatial distribution, especially at the entry end and exit end of the secondary, as shown in figure 7 and figure 8. Similar to the distribution of the air gap density, the forward surface current density component attenuates along $x$-direction and whose
attenuation constant is 1/\(\alpha\), and the backward current density component attenuates along the negative direction of \(x\) and whose attenuation constant is also 1/\(\alpha\). For any length secondary \((L_2=2\pi r)\), the envelope of the longitudinal distribution of the secondary surface current density at different slip is shown in figure 8. The influence of the backward component of the secondary surface current density on the resultant surface current density is significantly greater than that of the forward component. The forward and backward components have little effect on the surface current density distribution in the middle of the secondary.

\[
\begin{align*}
J_2 &= J_{21} + J_{22} + J_{23} \\
J_{21} &= -\frac{s\omega\sigma}{k} \left( 1 + s^2 G^2 \right)^{1/2} \cos(s\omega t - kx + \phi_{F1}) \\
J_{22} &= \frac{s\omega\sigma}{k} \left( 1 + s^2 G^2 \right)^{1/2} \sqrt{sG} e^{-\alpha x} \cos(s\omega t - \alpha x + \phi_{F2}) \\
J_{23} &= -\frac{s\omega\sigma}{k} \left( 1 + s^2 G^2 \right)^{-1/2} \sqrt{sG} e^{\alpha(x-L_2)} \cos(s\omega t + \alpha x - kL_2 - \alpha L_2 + \phi_{F2}) \\
\end{align*}
\]

\(0 < x < L_2\) \hspace{1cm} (2)

Figure 7. Secondary surface current density distribution.

Figure 8. Surface current density distribution with different slip.

3. Thrust Characteristics Calculation

3.1. Expression of Thrust

Because the length of the secondary is not necessarily the same as that of the primary \((2\pi r)\), the thrust expression calculated by the integration of primary surface current density and air gap flux density cannot accurately reflect the thrust characteristics. Therefore, the product integral of surface current density and air gap flux density is used to calculate the thrust characteristics of LPDLIM with any length secondary as equation (3), where \(2a\) is the stack thickness of primary core.

\[
F = -2a \int_0^{L_2} j_2 \cdot B_s dx = \sum_{i=1}^{3} \sum_{j=1}^{3} \left( F_{emij} + F_{epij} \right) \hspace{1cm} (3)
\]

The \(i, j\) in equation (3) represents the interaction between the normal, forward and backward components of air gap flux density and secondary surface current density respectively. According to the expressions of air gap flux density and secondary surface current density, the expression of thrust has 9 parts. Each thrust \(F_{emij}\) includes steady thrust \(F_{emij}\) and thrust ripple \(F_{epij}\) (the item with \(2s\omega t\)). Among them, the steady fundamental thrust \(F_{em11}\) generated by the normal travelling wave and the normal component of secondary surface current density is as shown in equation (4). And other thrusts and ripples can be expressed by this item as equation (5) to (13), where \(A_1 = [(k+\alpha)^2 + \alpha^2]^{1/2}, A_2 = [(k-\alpha)^2 + \alpha^2]^{1/2}, \phi_{F3} = \arctan ((k+\alpha)/\alpha), \) and \(\phi_{F4} = \arctan ((k-\alpha)/\alpha)\).

\[
F_{em11} = \frac{\mu_0 J_1^2}{2k}\frac{2a}{1+s^2 G^2} sGL_2 = f_{em11}L_2 \hspace{1cm} (4)
\]

The thrusts generated by normal travelling wave are shown in equation (5) - (7).
\[ F_{e11} = F_{em11} + F_{ep11} = F_{em11} \left\{ 1 + 1/4k \left[ \sin(2s\omega t + 2\phi_1) - \sin(2s\omega t - 2kL_2 + 2\phi_1) \right] \right\} \]  
\[ F_{e12} = F_{em12} + F_{ep12} = -f_{em11} \sqrt{s}G \left\{ A_2 \cos(-\phi_{f1} + \phi_{f2} + \phi_4) + A_4 \cos(2s\omega t + \phi_{f1} + \phi_{f2} - \phi_{f3}) \right\} \]  
\[ F_{e13} = F_{em13} + F_{ep13} = f_{em11} \sqrt{s}G \left\{ A_1 \cos(-\phi_{f1} + \phi_{f2} - \phi_{f3}) + A_2 \cos(2s\omega t - 2kL_2 + \phi_{f1} + \phi_{f2} + \phi_4) \right\} \]  
\[ F_{e21} = F_{em21} + F_{ep21} = -f_{em11}sG \left\{ A_2 \sin(\phi_{f4}) + A_4 \sin(2s\omega t + 2\phi_{f1} - \phi_{f3}) \right\} \]  
\[ F_{e22} = F_{em22} + F_{ep22} = f_{em11} \frac{sG}{2k} \left\{ \sqrt{2} \left[ \sin(\phi_{f1} - \phi_{f2}) \right] \left[ 1 - e^{-2\alpha L_2} \right] + \sin(2s\omega t + \phi_{f1} + \phi_{f2} - \pi/4) \right\} \]  
\[ F_{e23} = F_{em23} + F_{ep23} = -f_{em11}sGe^{-\alpha L_2} \left\{ \sqrt{2k} \left[ \cos((k - \alpha) L_2 + \phi_{f1} - \phi_{f2}) - \cos((k + \alpha) L_2 + \phi_{f1} - \phi_{f2}) \right] \right\} + \sqrt{s}G \left[ L_2 \cdot \sin(2s\omega t - (k - \alpha) L_2 + \phi_{f1} + \phi_{f2}) \right] \]  
\[ F_{e31} = F_{em31} + F_{ep31} = -f_{em11}sG \left\{ A_1 \sin(\phi_{f3}) + A_2 \sin(2s\omega t - 2kL_2 + 2\phi_{f1} + \phi_4) \right\} \]  
\[ F_{e32} = F_{em32} + F_{ep32} = f_{em11}sGe^{-\alpha L_2} \left\{ \sqrt{2k} \left[ \cos(-((k + \alpha) L_2 + \phi_{f1} - \phi_{f2}) \right] \right\} + \sqrt{s}G \left[ L_2 \cdot \sin(2s\omega t - (k + \alpha) L_2 + \phi_{f1} + \phi_{f2}) \right] \]  
\[ F_{e33} = F_{em33} + F_{ep33} = -f_{em11} \frac{sG}{2k} \left\{ \sqrt{2} \left(1 - e^{-2\alpha L_2}\right) \sin(\phi_{f1} - \phi_{f2}) + \sin(2s\omega t - 2kL_2 + \phi_{f1} + \phi_{f2} - \pi/4) \right\} \]  

The thrust generated by forward travelling wave are shown in equation (8) - (10).  
While the thrusts generated by backward travelling wave as (11) - (13).  
The thrust ripples of LPDLIM with any length secondary does not produce time average thrust, and the frequency of force ripples are twice the slip supply frequency. The terms containing exp(-\alpha L_2) in the above thrust expressions are ignored, and their contribution to the total thrust can be ignored due to their values are much smaller than the other thrust components without exp(-\alpha L_2).

3.2. Steady Thrust Characteristics

The parameters of LPDLIM are shown in Table A1, when the secondary length is 0.48m (not the integral multiple of pole pitch or tooth pitch), the characteristic curve of each end effect electromagnetic force component and the normal thrust without considering the end effect are shown in figures 9-13. The electromagnetic forces \( F_{em12} \) and \( F_{em13} \) in figure 9 produced by normal travelling wave and the forward and backward components of secondary current density, and the force \( F_{em33} \) in figure 11 generated by backward travelling wave and backward secondary current density, are opposite to the direction of fundamental thrust \( F_{em11} \), so they are braking forces. The force \( F_{em23} \) in figure 10 and \( F_{em32} \) in figure 11 calculated from the air gap magnetic field and secondary current density with opposite attenuation direction are much smaller than other force components, which are about 0 N, and their contribution to the resultant force is negligible. The force \( F_{em22} \) in figure 10 due to forward travelling wave and forward secondary current density, and the force \( F_{em33} \) in figure 11 caused by the backward travelling wave and normal secondary current density are in the same direction as \( F_{em11} \), which increase the output thrust.

The force \( F_{em21} \) is braking force or thrust force depends on the value of \( k-\alpha \), when the slip is low, it appears as the braking force. The end effect force, that is, except for \( F_{em11} \), is the sum of thrust components including the forward or backward air gap flux density or secondary current density, as shown in figure 12. The end effect force is mainly the braking force, which reduces the thrust \( F_{em11} \), but it increases the locked thrust of the motor when the slip is 1, as the figures 12 and 13. When the motor operates in no-load conditions, the end effect does not exist and the end effect force is 0 N.
3.3. Force Ripple Characteristics
When the secondary length is any value that is not the integral multiple pole pitch, the normal traveling wave interacts with the normal secondary current density to produce a certain amplitude of thrust ripple \(F_{ep11}\), as shown in figure 14, and when the secondary length is the integer multiple pole pitch, the value is 0N. It can also be seen that, in addition to generating thrust ripple with the normal secondary current density component, the normal traveling wave also interacts with the forward and backward current density components to produce thrust ripples \(F_{ep12}\) and \(F_{ep13}\).

Figure 9. Force of normal traveling wave.

Figure 10. Force of forward traveling wave.

Figure 11. Force of forward travelling wave.

Figure 12. End effect force.

Figure 13. Thrust with and without end effect.

Figure 14. Ripple of normal travelling wave.

Figure 15. Ripple of forward travelling wave.

Figure 16. Ripple of backward travelling wave.
Similar to the steady thrust, the thrust ripple $F_{ep23}$ and $F_{ep32}$, which are caused by the interaction of traveling wave and the secondary current density with the opposite attenuation direction, respectively, i.e. $F_{ep23}$ is produced by the forward travelling wave and backward current density component, and $F_{ep32}$ is produced by backward travelling wave and forward current density component, have a ripple amplitude of approximately 0N, and basically have no contribution to the total thrust ripple.

From the figures 14-16, among the thrust ripple components, the amplitude of thrust ripple generated by the backward component and the normal component is larger than that of other thrust ripple, that is, the ripple of $F_{ep13}$ is caused by the backward current density component and the normal travelling wave, and the $F_{ep31}$ caused by the backward travelling wave and normal current density component, have greater amplitudes than other thrust ripple components, which are the main source of thrust ripple.

4. Comparative Verification of Thrust Characteristics
The finite element (FEM) simulation software ANSYS was used to establish a two-dimensional transient field model of the LPDLIM, and the electromagnetic thrust and thrust ripple characteristic curves when the secondary length was any value were calculated.

4.1. Calculation of Electromagnetic Thrust Verification
When the secondary length is not the integer multiple of the pole pitch, take the secondary length of 0.48m as an example, the power frequency is 50Hz and the current is 6A, the thrust-speed characteristic curve calculated by the analytical method is shown in figure 17, the thrust data calculated by finite element simulation confirms the correctness of the derived analytical expression of electromagnetic thrust.

When the supply frequency is 50Hz and the speed is 5.28m/s, the electromagnetic thrust calculated by the two methods varies with the secondary length as shown in figure 18. It can be seen from equation (6) to (13) that the amplitude of end effect force is independent of the secondary length, so it is basically not affected by the secondary length, and the resultant thrust increases linearly with the secondary length. In the figure, when $L_2=396$mm, the effective pole number of the motor is 6, while $L_2=528$mm, the effective pole number is 8.

Figure 17. Thrust-speed characteristic curve.

Figure 18. The resultant thrust distribution with secondary length.

Figure 19. Thrust ripple-speed characteristic curve.

Figure 20. The resultant thrust ripple distribution with secondary length.
4.2. Calculation of Electromagnetic Thrust Ripple Verification

The thrust ripple characteristic curve under the same conditions is shown in figure 19. The thrust ripple-speed characteristic curve is similar to the thrust characteristic curve. In the stable operation area of the motor, the thrust ripple increases as the speed decreases. When the power frequency and speed are constant, the thrust ripple value is a sine function of the secondary length, and its period is one pole pitch, as in figure 20. It can be seen that when the length is appropriate, the fundamental thrust ripple $F_{ep11}$ can offset a part of the end effect thrust ripples, so that smaller total thrust ripple can be obtained than that of LPDLIM with a secondary length of the integer multiple pole pitch.

5. Conclusion

The expression of the secondary surface current density in the secondary coordinate system are derived, and which is used to calculate the thrust characteristics of the LPDLIM with arbitrary length secondary. The calculation results show that when the secondary length is any value, the end effect will not only produce effective thrust, but also a certain thrust ripple. The end effect force is the braking force that hinders the secondary advancement in the stable working area, and increases the starting thrust when it is blocked. The appropriate secondary length can effectively reduce the force ripple, which is not necessarily the integer multiple pole pitch.

6. Appendices

| Table A1. The parameters of LPDLIM. |
|-------------------------------------|
| Item | Value | Item | Value |
| Pole pairs | 4 | Secondary length(mm) | 480 |
| Slots per pole per phase | 2 | Secondary thickness(mm) | 4 |
| Primary stack thickness(mm) | 60 | Mechanical air gap length(mm) | 2 |
| Pole pitch(mm) | 66 | Secondary conductivity(S/m) | 38E6 |

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