Students’ Thinking Processes on Quadrilateral Concept: A Case of Providing Non-Examples

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Abstract. Characteristic of someone understand the concepts is the ability to give the examples and non-examples and the reasons. The thinking process in determining the example and non-examples of a concept will be different because of its cognitive style. This is an explorative study which qualitatively analyze the thinking processes of field independent (FI) and field dependent (FD) students in providing non examples of quadrilateral concepts. The results of the study indicate that: 1) The essential characteristics of each shape in the quadrilateral as the basis for the FI participant to determine the non-examples, so that the shape which does not meet those characteristics are selected as non-examples. The selected first non-example tends to be similar with the grouped shapes. 2) The essential characteristics of each shape in the quadrilateral as the basis for the FD participant to determine the non-examples, so the shapes have striking differences are chosen as non-examples to make her be easier to give the reasons. The determination of the essential characteristics of a quadrilateral is important in determining non-example of quadrilateral. Based on the selection of examples and non-examples of quadrilateral will facilitate to determine the relationship of many concepts in the quadrilateral.

1. Introduction
The main task of teaching mathematics in school is to know the thinking process of students in learning mathematics and how mathematical knowledge is interpreted in the mind. Suryabrata [1] argues that in the process of thinking appear mental activities to form understanding, form opinions and make conclusions. One of the goals given a math lesson at the school level is that students understand the concept of mathematics. One indicator of understanding the concept of mathematics is being able to provide an example and a non-example of a concept. Sjogren [2] argues that the concepts can be obtained through three stages: (a) categorization stage, (b) eliminating inappropriate categories and combining appropriate categories to form a concept, and (c) concluding a concept so as to give examples and non-examples of concepts with its categorization.

In understanding a concept, students have different ways. This is appropriate with Tayler [3] who states that: “dimensions of individual differences, among others such as intelligence, reasoning ability, creativity, cognitive style and gender”. Where as Slameto [4] is of the opinion that the cognitive style is the way a person create perceptions and compiles information (stimulus) that comes from the surrounding environment that is influenced by the experiences that have been owned. Based on psychological aspects, Witkin [5] grouped cognitive styles into two, are field independent (FI) and field dependent (FD) types.
Furthermore Adelina [6] and Witkin [5] stated that: “cognitive styles are the characteristic, self consistent modes of functioning which individuals show in their perceptual and intellectual activities”, “cognitive style represent a person’s typical modes of perceiving, remembering, thinking and problem solving”.

Based on the Rahaju’s research [7], showed that there are differences in the thinking process of the FI and FD in understanding concept of quadrilateral. While one of indicator to understand a concept is capability to give examples and non-examples of a concept. In general in the process of learning mathematics, teachers only provide examples or ask students to give examples of a concept. Therefore researcher was interested in conducting a research that aims to explore thinking process of students in determining non-examples of the quadrilateral concept.

2. Method
This research aims to explore the thinking processes of students in determining non-examples of quadrilateral based on cognitive styles. The instrument used to categorize students’ cognitive styles is GEFT (group embedded figure test), an instrument developed by Witkin [8]. At GEFT, participants were asked to thicken a simple figure found on complex figure. Tracing the thinking process of participants in determining non-examples of quadrilateral was done through the task given to the participants. In this task, participants were asked to express how they were gave non-examples of a concept in quadrilateral. Their way can be expressed with think aloud. To obtain more in-depth and accurate data about participants' thinking processes, this study conducted task-based interviews. The participants in this research was two students grade VII public junior schools in Surabaya, Indonesia with cognitive different style, namely field independent (FI) and field dependent (FD). The control variables chosen in this research were equivalent mathematical abilities and had the same gender.

3. Result and Discussion
The search for the thought processes done by both the participants in determining non-examples of quadrilateral, served according to the order of the concept of selected by both the participants.

3.1. Research Results FI Subject

S: (In giving of the reasons to select non-example of square, subject saw that figure)

The kite has 4 sides of different length, the rectangle has 4 sides of different length and 2 pairs of the same length, the trapezoid has 4 sides of different length

S: The first time, I think is a shape that have no four sides of equal length. Rectangel and square easily seen difference does it make which is to long different sides

Figure 3.1 Task based interview non-example of square

Based on the results in figure 3.1, it shows that the FI in selecting non-examples square focus on long edge attributes and suggests that the FI concludes the characteristic "four sides of equal length" is an essential characteristic for a square. So the non-examples selected are shapes that does not have these characteristics.

S: (When determine reasoning non-example of rectangle (parallelogram and trapezoid), subject look at the figures in the previous question while making some scratch)

The square because the four sides are the same length, parallelogram because the sides are not perpendicular, the trapezium because it has only one pair of parallel sides
S: The first time I think to find non-example of rectangle is quadrilateral who have no characteristic rectangle. The essential characteristic of the rectangle is to have 2 pairs of sides of different lengths, all angles are right and 2 pairs of parallel sides. I choose square, because most easy the difference with rectangle, where the long sides. Square had four sides of equal length while rectangle have two pairs sides of the unequal length.

**Figure 3.2** Task based interview non-example of rectangle

Based on the results in figure 3.2, it shows that FI which is the essential characteristic of the rectangle is "having 2 pairs of sides of different lengths, right angles and 2 pairs of parallel sides". So the non-examples of the rectangle selected are square, parallelogram, and trapezoid. The non-example selected are the shape that does not have one of the essential characteristics of a rectangle, but a square is an incorrect non-example because FI is still affected by the rectangular shape obtained during the math learning in elementary school, which has two pairs of sides of different lengths.

S: (In giving of the reasons to select non-example of parallelogram (trapezoid and kite), subject look at the figures in the previous question)

S: The first time I think to find non-example of parallelogram is quadrilateral who have no characteristic parallelogram. The essential characteristic of the parallelogram is to have 2 pairs of sides of different lengths, and 2 pairs of parallel sides. I choose rhombus, because most easy the difference with parallelogram, where the long sides. Rhombus had four sides of equal length while parallelogram have two pairs of side of the unequal length.

**Figure 3.3** Task based interview non-example of parallelogram

Based on the results in Figure 3.3, it shows that the essential characteristics of the parallelogram are "having 2 pairs of sides of different lengths, and 2 pairs of parallel sides". So the FI chooses non-examples of the parallelogram are rhombus, trapezium and kite. The rhombus is an incorrect non-example of the parallelogram, because FI is still affected by the parallelogram shape obtained during the math learning in elementary school, which has 2 pairs of sides of different lengths.

S: (In giving of the reasons to select non-example of kite (rhombus and parallelogram), subject look at the figures in the previous question)

S: The essential characteristic of the kite are "having 2 pairs of sides adjacent to different lengths and pairs of opposite corners equal.

**Figure 3.4** Task based interview non-example of kite

Based on the results in Figure 3.4, it shows that the FI chose non-example of kite is a shape that does not have one of the essential characteristics are rhombus, parallelogram and square. The non-examples of kite selection are an incorrect, because the essential characteristics of the kites concluded by FI are less precise. The fault is also caused by the FI is still affected by the form of kite that he studied in elementary school.

S: The essential characteristic of rhombus are "having four sides the same length, each other the angles is opposite and the diagonals perpendicular to each other
Based on the results in Figure 3.5, it shows that the FI selects non-examples of rhombus are shapes that do not have any of the essential characteristics, are rectangles, parallelograms and kites. The square is an incorrect non-example because FI participant is still affected by the rhombus shape obtained during the math learning in elementary school, which has acute angle and obtuse angle.

S: (In giving of the reasons to select non-example of rhombus, subject look at the figures in the previous question)

**Figure 3.5** Task based interview non-example of rhombus

Based on the results in Figure 3.6, it shows that the FI selects non-examples of trapezoid are shapes that have two pairs of parallel sides, since the trapezoid essential characteristics is to have exactly a pair of parallel sides.

**3.2. Research Results FD Subject**

Based on the results in Figure 3.7, it shows that according to FD which is the essential characteristic of the rectangle are "having 2 pairs of sides of different lengths, and four right angles". She drew the first non-examples, to make it easier to give the reason for the non-example selection. Non-examples selected are shapes that has a striking difference with the rectangle. Square is an incorrect non-example because she is still affected by the rectangular is a shape always having different side lengths as she were studied in elementary school.

**Figure 3.7** Task based interview non-example of rectangle

rectangle because its sides are not equal in length, trapezoid has no right angle, triangle of angle number 180°.

S: First what I think to find non-example is find differences easy. Square most easily the difference with rectangle, because all sides of equal length. Trapezoid not had four right angle. Triangle only

**Figure 3.8** Task based interview non-example of square
Based on the results in Figure 3.8, it shows that according to the FD which is the essential characteristic of the square are "having four sides of equal length and four right angles". She drew the first non-examples, to make it easier to give the reason for the non-examples selection. The non-example selected is a shape has a striking difference with a square, is that does not have any of the essential characteristics.

Figure 3.9 Task based interview non-example of

(The subject of drawing non-example of parallelogram without saw the previous figure. She was giving the reasons for the selection with each non-example look at an figure has already been prepared earlier)

S: I think to find non-example is find differences easy with parallelogram. Triangle only have three sides. Square have diagonal that intersect perpendicular, while a parallelogram not

Figure 3.10 Task based interview non-example of rhombus

(The subject of drawing non-example of kite about previous figure. The subject of giving the reasons for the selection with each non-example look at an figure has already been prepared earlier)

S: First what I think to find non-example is find differences easy with rhombus, has no right angle. Square and rectangle have right angle and kite has no four sides of equal length

Figure 3.11 Task based interview non-example of

(She wrote the reasons in general first, then drew the non-examples. She gives the reason for each non-sample has chosen by looking at the figure has created)

Based on the results in Figure 3.10, it shows that according to the FD which is the essential characteristic of the rhombus are "do not have right angle and have four sides of equal length". She drew the first non-examples, to make it easier to give the reason for the non-examples selection. The non-example selected is a shape has right angle or has no four sides of equal length. The square is an incorrect non-example because she is still affected by the rhombus is a shape always having acute angle and obtuse angle as she were studied in elementary school.

Based on the results in Figure 3.11, it shows that the FD which is the essential characteristic of the kite are "do not have right angle and have two pairs of sides are unequal length". She drew the first non-examples, to make it easier to give the reason for the non-examples selection. The non-example selected is a shape have right angle or have diagonals do not intersect perpendicular. The reason given for the parallelogram
chosen as a non-example of kite does not correspond to the essential characteristics, but rather she sees
the diagonal intersection. This shows that she is more focused on the figure of the kite.

(She wrote the reasons in general first, then drew the non-examples. She gives the reason for each non-
example has chosen by looking at the figure has created)

based the shape and sides are different

S: To make it easier to give reasons for the selection non-examples
of trapezoid, I drew the example first. I think to find non-
example is find differences easy with trapezoid, it has a pair
of parallel sides. Square and rectangle have two pairs of parallel
sides and kite has no pair of parallel sides

Figure 3.12 Task based interview non-example of

Based on the results in Figure 3.12, it shows that according to the FD which is the essential characteristic
of the trapezoid is “has a pair of parallel sides”. She drew the first non-examples, to make it easier to give
the reason for the non-examples selection. The non-example selected is a shape have no parallel sides or
have two pairs of parallel side

3.3. Discussion

Based on the results (Picture 3.1 – 3.6) showed the FI thinking process in determining the non-example of
a shape in a quadrilateral was by looking to essential characteristics of each shape. So to determine a non-
example of a shape in a quadrilateral, she chose a shape has no one of the essential characteristics of the
shape. To ensure the truth of non-example selection reasons, she considered the quadrilateral figures was
contained in her previous task. Based on her reasons of the non-example selection performed, it showed
that attributes of side length and the measure of angle are very important. She concerned with the
parallelism attribute when determining non-examples of trapezoid, since an essential characteristic of the
trapezoid is it has a pair of parallel sides. She chose the first non-example, it is the similar shapes of the
ones has been grouped, whereas in the second and third of non-examples she tent to choose shapes that
have striking differences. There are mistakes that she conducted in selecting non-examples of rectangles
and parallelogram. This happened because she was affected by the rectangular and the parallelogram
shapes always have two pairs of sides with different lengths. The mistake indicated that she was still
influenced by familiar shapes at the time of math learning in elementary school, even though she has
already known the essential characteristics of each shape.

Based on the results (Picture 3.7 – 3.12) showed the FD thinking process in determining the non-
examples of a quadrilateral was by looking to essential characteristics of each shape. So to determine a non-
examples of a quadrilateral shape, she chose a shape has a very different to others. She gave non-
examples for each shape in quadrilateral in the form of figures, then she gave a general reason for the non-
examples selection. Through the figures she can create easily providing the reasons for choosing any non-
examples. She tent to chose triangles as non-examples in quadrilateral because they have many different
side. Based on the reason of the given non-example selection, it showed that side length, measure of
angle and number of sides are the important attributes. She took parallelism attribute when selecting a
non-example of trapezoid, since it has a pair of parallel sides is an essential characteristic of trapezoid. She
chose the first non-example was square or rectangle because their characteristic is very familiar. While the
second and third non-examples, she tend to choose shape that have striking differences with the grouped
shape. There is a mistake in selecting non-examples rectangle and parallelogram, because she still assume
that rectangle or parallelogram have two pairs of sides of different lengths.

By observing the thought processes of the two participants in determining non-examples of
quadrilateral, it appears that they do the thinking steps through the formation of understanding by looking
at the essential characteristics of a shape, so they can make the conclusion that the non-example of a shape
in a quadrilateral, it is a shape that does not have one or more essential characteristics. Once confirmed there are one or more essential characteristics that are not met, they can conclude that a shape is a non-example[1]. Looking at the way of both participants to determine the non-example of quadrilateral, there are striking differences. This can be seen when specifying non-examples, the FI can directly provide the reason for the non-examples selection, whereas the FD to provide the reason for the non-example selection must be through drawing the non-example. By looking at the non-example figure her creates, the FD can easily see that the figure non-example does not have essential characteristic a shape in quadrilateral. The way of the FI in giving the non-examples of quadrilateral show that she tends to be consistently not easily influenced by changing the situation. While the FD shows tend to be consistently easily influenced by changes in the situation. The conditions of both participants are in accordance with Adelina's [6] and Witkin's [5].

To learn the quadrilateral comprehensively, it is necessary to consider the relationship between the shapes in it. The non-example selection of a quadrilateral has an important to determining the relationship between the shapes in the quadrilateral. If the non-example selection of each shape in a quadrilateral is done correctly, then there will be no mistake in determining the relationship in quadrilateral. For example an FD made a mistake in selecting a square as a non-example of rectangular, she would be able to result in no relationship between square and rectangle. This indicates that the wrong non-example selection will result in an error in determining the relationship of two shapes.

4. Conclusion

The thinking process of the FI and FD in determining non-examples of each shape in quadrilateral is done an understanding based on the essential characteristics of quadrilateral. Based on these characteristics, the participants holds that a non-example selection of a quadrilateral is a shape which does not have any of these essential characteristics of each shape in quadrilateral. So they make the conclusion that the non-examples of quadrilateral which they chooses has one or more characteristic difference. The FI participant give two kinds of the non-examples of a quadrilateral: the first non-example of quadrilateral which a shape has some similarities and the next non-example is a shape which has a striking difference. The FI provide a direct reason for the non-example selected. While the FD participant in giving non-examples of a quadrilateral, she tends to choose triangle or square or rectangle as the first non-example. She chose a shape has a striking difference with a shape has been grouped as the next non-example. The non-example selection begins with a generally accepted reason (for all non-examples selected) and continues with non-example drawing. The FD give the reasons for each non-example selection after viewing the figure it has created.

5. References

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