Molecule model
for kaonic nuclear cluster $\bar{K}NN$

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Abstract

We analyse the properties of the kaonic nuclear cluster (KNC) $\bar{K}NN$ with the structure $N \otimes (\bar{K}N)_{I=0}$, having the quantum numbers $I(J^\pi) = \frac{1}{2}(0^-)$, and treated as a quasi–bound hadronic molecule state. We describe the properties of the hadronic molecule, or the KNC $N \otimes (\bar{K}N)_{I=0}$, in terms of vibrational degrees of freedom with oscillator wave functions and chiral dynamics. These wave functions, having the meaning of trial wave functions of variational calculations, are parameterised by the frequency of oscillations of the $(\bar{K}N)_{I=0}$ pair, which is fixed in terms of the binding energy of the strange baryon resonance $\Lambda(1405)$, treated as a quasi–bound $(\bar{K}N)_{I=0}$ state. The binding energies $B_X$ and widths $\Gamma_X$ of the states $X = (\bar{K}N)_{I=0}$ and $X = \bar{K}NN$, respectively, are calculated in the heavy–baryon approximation by using chiral Lagrangians with meson–baryon derivative couplings invariant under chiral $SU(3) \times SU(3)$ symmetry at the tree–level approximation. The results are $B_{\bar{K}NN} = 40.2\text{ MeV}$ and $\Gamma_{\bar{K}NN} = \Gamma_{\bar{K}NN}^{(\pi)} + \Gamma_{\bar{K}NN}^{(\pi)} \sim (85 - 106)\text{ MeV}$ and, where $\Gamma_{\bar{K}NN}^{(\pi)} \sim 21\text{ MeV}$ and $\Gamma_{\bar{K}NN}^{(\pi)} \sim (64 - 86)\text{ MeV}$ are the widths of non–pionic $\bar{K}NN \rightarrow N\Lambda^0, N\Sigma$ and pionic $\bar{K}NN \rightarrow N\Sigma\pi$ decay modes, calculated for $B_{\bar{K}N} = 29\text{ MeV}$ and $\Gamma_{\bar{K}N} = (30 - 40)\text{ MeV}$, respectively.

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I. INTRODUCTION

The kaonic nuclear cluster (KNC) $\bar{K}NN$ (or $K^-pp$) with quantum numbers $I(J^\pi) = \frac{1}{2}(0^-)$ and the structure $N \otimes (\bar{K}N)_{I=0}$, which we denote as $\frac{2}{K}H$, has been predicted by Akaishi and Yamazaki [1]–[6] within a non–relativistic potential model approach [1]–[4] with the binding energy $B_{\frac{2}{K}H} = 48$ MeV and the width $\Gamma_{\frac{2}{K}H} = 61$ MeV, taking into account the pionic $\frac{2}{K}H \rightarrow p\Sigma\pi$ decay modes only [1]–[6]. According to Akaishi and Yamazaki [1]–[6], the formation of the quasi–bound $N \otimes (\bar{K}N)_{I=0}$ state occurs in the entrance channel through the isospin–singlet state $(\bar{K}N)_{I=0}$, identified with the $\Lambda(1405)$ hyperon, with the subsequent addition of one more proton. In such a scenario the KNC $\frac{2}{K}H$ acquires exactly the structure $N \otimes (\bar{K}N)_{I=0}$.

The theoretical analysis of the properties of the KNC $\frac{2}{K}H$ is focused on the calculation of the binding energy $B_{\frac{2}{K}H}$ and the partial widths $\Gamma_{\frac{2}{K}H}^{(\pi)}$ and $\Gamma_{\frac{2}{K}H}^{(\pi)}$ of all hadronic decay modes, which are the important two–body non–pionic $\frac{2}{K}H \rightarrow N\Lambda^0, N\Sigma$ decay modes and the three–body pionic $\frac{2}{K}H \rightarrow N\Sigma\pi$ decay modes, respectively. In the potential model approach by Akaishi and Yamazaki [1]–[6] as well as in other theoretical approaches for the description of the KNC $\frac{2}{K}H$ [7]–[13] such as relativistic mean field (RMF) models [7, 8], coupled–channel approach with chiral dynamics and variational analyses [9, 10], coupled–channel Faddeev equations [11, 12] and variational analyses [13] the widths $\Gamma_{\frac{2}{K}H}^{(\pi)}$ of pionic modes $\frac{2}{K}H \rightarrow N\Sigma\pi$ have been calculated only. Some estimates of the non–pionic decay modes of the KNC $\frac{2}{K}H$, carried out in the potential model approach and coupled–channel approach with chiral dynamics, predict the widths of order of $\Gamma_{\frac{2}{K}H}^{(\pi)} \sim 10$ MeV. Recent calculation of the transitions $\Lambda^*N \rightarrow N\Lambda^0$ and $\Lambda^*N \rightarrow N\Sigma$, where the $\Lambda(1405)$ resonance is denoted as $\Lambda^*$ [1]–[6], which has been carried out in [14] at threshold of the unbound $\Lambda^*N$ system, showed a total width of non–pionic decay modes of the unbound $\Lambda^*N$ system equal to $\Gamma_{\Lambda^*N}^{(\pi)} = 22$ MeV.

Since the decay mode $\frac{2}{K}H \rightarrow p\Lambda^0$ is used for the experimental observation of the KNC $\frac{2}{K}H$ [15, 16], an alternative approach allowing a calculation of the widths of all non–pionic and pionic decay modes for quasi–bound $\frac{2}{K}H$ state is needed.

The first announcement about the existence of the KNC $\frac{2}{K}H$ with the binding energy $B_{\frac{2}{K}H} = 115(7)$ MeV and the total width $\Gamma_{\frac{2}{K}H} = 67(14)$ MeV has been reported by the FINUDA Collaboration [15]. These data have been obtained from the stopped $K^-$–meson reactions on $^6Li$, $^7Li$ and $^{12}C$ by measuring the invariant-mass spectrum of back–to–back
emitted $p\Lambda^0$ pairs. However, the interpretation of these results as a KNC $^2\bar{K}H$ has been questioned and other explanations of the observed spectrum have been put forward [17].

Recent experimental data [16] on the analysis of the exclusive $K^+\bar{K}N$ missing mass and $p\Lambda^0$ invariant mass spectra of the final state in the $pp \rightarrow K^+\Lambda^0p$ reaction at the incident proton kinetic energy $T_p = 2.85$ GeV showed the quasi–bound state $^2\bar{K}H$ with the binding energy $B_{^2\bar{K}H} = 103(6)$ MeV and the width $\Gamma_{^2\bar{K}H} = 118(13)$ MeV. The important distinction of such a quasi–bound state is its two–body decays into the $p\Lambda^0$ channel, enhanced at high momentum transfer and predicted by the relevant reaction theory [1]–[6], showing a creation of a compact object. Thus, for the analysis of such a quasi–bound state one cannot use the coupled–channel Faddeev equation approach [11, 12], which is unable to describe the two–body decay channels. In turn, a description of the experimental data by the DISTO Collaboration within other theoretical approaches [7]–[10, 13] seems to be also questionable.

The KNC $^2\bar{K}H$ with the structure $N \otimes (\bar{K}N)_{I=0}$ we propose to treat as a kaonic molecule [2]. In the center of mass frame molecular states are defined by rotational and vibrational degrees of freedom [18]. Since the angular momentum of the KNC $^2\bar{K}H$ as well as of the KNC $^1\bar{K}H$, the quasi–bound $(\bar{K}N)_{I=0}$ state, are equal to zero, the properties of these states are defined by the vibrational degrees of freedom only, which we describe by trial harmonic oscillator wave functions. Such a choice can be also justified by a short–range character of forces producing quasi–bound KNCs. We recall that the harmonic oscillator wave functions are used also in the shell–model of nuclei [19] for the description of strongly bound nuclear systems. In addition, trial Gaussian wave functions in the coordinate representation have been used in [10] for the variational calculation of the parameters of the KNC $^2\bar{K}H$ with a potential, induced by chiral dynamics with $SU(3)$ coupled–channels technique.

The calculation of the parameters of the quasi–bound states $^n\bar{K}H$ with $n = 1, 2, \ldots$ we propose to carry out by using a $T$–matrix, defined by chiral Lagrangians with derivative meson–baryon couplings invariant under chiral $SU(3) \times SU(3)$ symmetry [20–22], which are used for the analysis of low–energy strong interactions.

According to quantum field theoretic description of decays of particles and nuclear states [23], the width of the decay modes $^n\bar{K}H \rightarrow X$ of the nuclear state $^n\bar{K}H$ is defined by the decay amplitudes $M(^n\bar{K}H \rightarrow X)$ as

$$\Gamma_{^n\bar{K}H} = \frac{1}{2M_{^n\bar{K}H}^2} \sum_X (2\pi)^4 \delta^{(4)}(k_X - k_{^n\bar{K}H}) |M(^n\bar{K}H \rightarrow X)|^2 = \lim_{V,T \rightarrow \infty} \sum_X \frac{|\langle X|T|^2^n\bar{K}H(\bar{0})\rangle|^2}{2M_{^n\bar{K}H}^2VT}, \quad (I.1)$$
where we have used a relation \( \langle X|\mathcal{T}_K n \mathcal{H}(\vec{0}) = (2\pi)^4 \delta^{(4)}(k_X - k_{n \mathcal{H}})M(\mathcal{T}_K n \mathcal{H} \to X) \). Then, \( \sum_X \) assumes a summation over all allowed decay channels \( n \mathcal{H} \to X \) and an integration over phase volumes of the final \( X \)-states, \( |n \mathcal{H}(\vec{0}) \rangle \) is the wave function of the KNC \( n \mathcal{H} \) in the momentum and particle number representation, the \( \mathcal{T} \)-matrix is defined by the chiral Lagrangians with derivative meson–baryon couplings invariant under non–linear chiral \( SU(3) \times SU(3) \) transformations \([20, 21]\); \( V \mathcal{T} = (2\pi)^4 \delta^{(4)}(0) \) is a space–time volume \([23]\).

Taking into account the unitarity condition for the \( \mathcal{T} \)-matrix \( \mathcal{T} - \mathcal{T}^\dagger = i\mathcal{T}\mathcal{T}^\dagger \) we can generalise Eq.(I.1) as follows

\[
B_{n \mathcal{H}} + i \frac{\Gamma_{n \mathcal{H}}}{2} = \lim_{V,T \to \infty} \frac{\langle n \mathcal{H}(\vec{0})|\mathcal{T}|n \mathcal{H}(\vec{0}) \rangle}{2M_{n \mathcal{H}}VT},
\]

(I.2)

where \( B_{n \mathcal{H}} \) is related to the energy shift of the quantum state \( n \mathcal{H} \) in the center of mass frame as \( B_{n \mathcal{H}} = -\epsilon_{n \mathcal{H}} \). We identify it with the binding energy of the quantum state \( n \mathcal{H} \). In such a definition the binding energy \( B_{n \mathcal{H}} \) and the width \( \Gamma_{n \mathcal{H}} \) of the KNC \( n \mathcal{H} \) are determined by the same low–energy strong interactions. This agrees with the definition of the binding energy and the width of strongly coupled particles and nuclear systems within the optical potential approach \([24]\).

The width \( \Gamma_{2 \mathcal{H}} \) of the KNC \( 2 \mathcal{H} \) is the sum of non–pionic decay modes \( 2 \mathcal{H} \to NY \), where \( NY = N\Lambda^0, N\Sigma^0 \) and \( N\Sigma^+ \), and pionic decay modes \( 2 \mathcal{H} \to NY \pi \), where \( NY \pi = N\Sigma \pi \) and \( N\Lambda^0 \pi \). It is given by

\[
\Gamma_{2 \mathcal{H}} = \frac{\Gamma_{2 \mathcal{H}}^{(\pi)}}{2} + \sum_{NY} \Gamma(2 \mathcal{H} \to NY) + \sum_{NY \pi} \Gamma(2 \mathcal{H} \to NY \pi).
\]

(I.3)

In turn, the width \( \Gamma_{1 \mathcal{H}} \) of the KNC \( 1 \mathcal{H} \) is defined by the \( 1 \mathcal{H} \to \Sigma \pi \) decay modes only as \( \Gamma_{1 \mathcal{H}} = \Gamma(1 \mathcal{H} \to \Sigma \pi) \).

The paper is organised as follows. In Section 2 we construct the wave functions of the quasi–bound \((\mathcal{K}N)_{I=0}\) and \( N \otimes (\mathcal{K}N)_{I=0} \) states. Since angular momenta of these states are zero, we describe them in terms of vibrational degrees of freedom by the oscillator wave functions \([18, 19]\) (see also \([10]\)). We identify the quasi–bound \((\mathcal{K}N)_{I=0}\) state with the strange baryon \( \Lambda(1405) \), denoted below as \( \Lambda^* \). We give the analytical expressions for the binding energy and the width of the quasi–bound \((\mathcal{K}N)_{I=0}\) state in terms of the frequency \( \Omega_{\Lambda^*} \) of a motion of the \( \mathcal{K} \) meson relative to the nucleon \( N \) and the analytical expression for the binding energy of the quasi–bound \( N \otimes (\mathcal{K}N)_{I=0} \) state in terms of the frequencies \( \Omega_{\Lambda^*} \) and \( \Omega_{\Lambda^*N} \), the latter defining the motion of the \((\mathcal{K}N)_{I=0}\) system relative to the nucleon \( N \). Thus,
the model contains three input parameters. They are the frequencies $\Omega_{\Lambda^*}$ and $\Omega_{\Lambda^*N}$ and the coupling constant $g_{\Lambda^*}$, which is used to fit the partial width of the $\Lambda^*$ state to the values $\Gamma_{\Lambda^*} = (30 - 40)$ MeV \[14\]. The number of the input parameters can be reduced to $\Omega_{\Lambda^*}$ and $g_{\Lambda^*}$, which can be obtained from the fit of the binding energy and width of the KNC $\frac{1}{k}H$ only. For this aim we assume that the stiffnesses of harmonic oscillator potentials, keeping the pairs $(\bar{K}N)_{I=0}$ and $N \otimes (\bar{K}N)_{I=0}$ bound, are equal. In this case the frequencies $\Omega_{\Lambda^*}$ and $\Omega_{\Lambda^*N}$ become related by $\mu_{\Lambda^*} \Omega_{\Lambda^*}^2 = \mu_{\Lambda^*N} \Omega_{\Lambda^*N}^2$, where $\mu_{\Lambda^*} = m_K m_N / (m_N + m_K) = 324$ MeV and $\mu_{\Lambda^*N} = m_N (m_N + m_K) / (2m_N + m_K) = 568$ MeV are reduced masses of the $\bar{K}N$ and $N(\bar{K}N)$ pairs, respectively. In Sections 3 we give the analytical expressions for the amplitudes and widths of the non–pionic $N \otimes (\bar{K}N)_{I=0} \to NY$ and pionic $N \otimes (\bar{K}N)_{I=0} \to NY\pi$ decay modes of the quasi–bound $N \otimes (\bar{K}N)_{I=0}$ state. We make the calculations of the binding energies and widths of the KNCs $\frac{1}{k}H$ and $\frac{2}{k}H$ at the tree–level approximation. In this case the binding energies $B_{1\bar{K}H}$ and $B_{2\bar{K}H}$ of the KNCs $\frac{1}{k}H$ and $\frac{2}{k}H$ are induced by the Weinberg–Tomozawa interactions only. Such a dominance of the Weinberg–Tomozawa low–energy strong interactions agrees well with the coupled–channel approach, based on chiral dynamics \[9, 10, 26\]. We denote them as $B_{1\bar{K}H}^{WT}$ and $B_{2\bar{K}H}^{WT}$. The masses of the KNCs $\frac{1}{k}H$ and $\frac{2}{k}H$ are given by $M_{1\bar{K}H} = m_N + m_K - B_{1\bar{K}H}^{WT}$ and $M_{2\bar{K}H} = m_N + m_K - B_{2\bar{K}H}^{WT}$, respectively. In Section 4 we give numerical values of the obtained binding energies and widths by using for the analysis of the $\Lambda^*$ hyperon the following prediction for its mass $m_{\Lambda^*} = 1405$ MeV and widths $\Gamma_{\Lambda^*} = 30$ MeV, obtained from the experimental data on stopped–$K^-$ meson absorption in a deuteron target \[25\], and $\Gamma_{\Lambda^*} = 40$ MeV, used by Akaishi and Yamazaki in their original work \[1\]. In Conclusion we discuss the obtained results.

II. QUASI–BOUND $(\bar{K}N)_{I=0}$ AND $N \otimes (\bar{K}N)_{I=0}$ STATES

A. Wave function of quasi–bound $(\bar{K}N)_{I=0}$ state

The harmonic oscillator wave function of the KNC $\frac{1}{k}H$ in the momentum representation can be taken in the form \[18\]

$$\Phi_{1\bar{K}H}(\vec{q}) = \left(\frac{4\pi}{\mu_{\Lambda^*} \Omega_{\Lambda^*}}\right)^{3/4} \exp\left(-\frac{\vec{q}^2}{2\mu_{\Lambda^*} \Omega_{\Lambda^*}}\right),$$

(II.1)

where $\Omega_{\Lambda^*}$ is the frequency of relative motion of the $\bar{K}N$ pair. In terms of the stiffness parameter $k$ the frequency $\Omega_{\Lambda^*}$ is defined by $\Omega_{\Lambda^*} = \sqrt{k/\mu_{\Lambda^*}}$, where $\mu_{\Lambda^*} = m_K m_N / (m_K +$
\(m_N = 324\) MeV is the reduced mass of the \(KN\) pair, calculated for \(m_N = 940\) MeV and \(m_K = 494\) MeV.

In the particle number and momentum representation the wave function of the \(KNC^1_{\bar{K}H}\) reads
\[
|1_{\bar{K}H}(\vec{k}, \sigma)\rangle = \sqrt{2E_{\bar{K}H}(\vec{k})} \int \frac{d^3k_1}{(2\pi)^3} \frac{d^3k_2}{(2\pi)^3} \frac{(2\pi)^3\delta(3)(\vec{k} - \vec{k}_1 - \vec{k}_2)}{\sqrt{2E_K(\vec{k}_1)2E_N(\vec{k}_2)}} \times \Phi_{1_{\bar{K}H}}\left(\frac{m_N\vec{k}_1 - m_K\vec{k}_2}{m_K + m_N}\right) \frac{1}{\sqrt{2}} \sum_{j=1,2} c_{\bar{K}H}^\dagger(\vec{k}_1) a_{\bar{N}H}^\dagger(\vec{k}_2, \sigma)|0\rangle,
\]
where \(\sigma = \pm \frac{1}{2}\) is a polarisation and \(j\) is the isospin index. The creation and annihilation operators obey standard relativistic covariant commutation and anti–commutation relations [23]. The wave function (II.2) has a standard relativistic covariant normalisation [23].

In our approach the binding energy and width of the \(KNC^1_{\bar{K}H}\) are defined by Eq.(I.2), where the main contributions to the binding energies of quasi–bound \(1_{\bar{K}H}\) and \(2_{\bar{K}H}\) states are caused by the Weinberg–Tomozawa low–energy strong interactions, which produce the necessary attractions in the \((\bar{K}N)_{I=0}\) and \(N \otimes (\bar{K}N)_{I=0}\) systems allowing to treat them as quasi–bound \(1_{\bar{K}H}\) and \(2_{\bar{K}H}\) states. This agrees well with the coupled–channel approach, based on chiral dynamics [9, 10, 26]. We perform the calculation of the binding energy for the Weinberg–Tomozawa \((\bar{K}N)_{I=0} \rightarrow (\bar{K}N)_{I=0}\) interaction and the width for the Weinberg–Tomozawa–like \((\bar{K}N)_{I=0} \rightarrow (\Sigma \pi)_{I=0}\) interaction with a constant \(g_{\Lambda^*}\), which we obtain from the fit of the width of the \(KNC^1_{\bar{K}H}\). Such a procedure of the calculation of the binding energy and width of the \(KNC^1_{\bar{K}H}\) is equivalent to the fit of the optical potential in the potential model approach by Akaishi and Yamazaki [1]–[6]. Then, we use these interactions for the calculation of the binding energy and widths of the \(KNC^2_{\bar{K}H}\).

The calculation of the matrix elements of the \(T\)–matrix in the non–relativistic and heavy–baryon approximation gives the following analytical expressions for the binding energy and the width of the \(KNC^1_{\bar{K}H}\)
\[
B_{1_{\bar{K}H}}^{WT} = \frac{3}{4} \frac{F_\pi^2}{F_\pi^2} \int d^3q \Phi_{1_{\bar{K}H}}(\vec{q})^2 = \frac{3}{4} \frac{1}{F_\pi^2} \left(\frac{\mu_{\Lambda^*} \Omega_{\Lambda^*}}{\pi}\right)^{3/2},
\]
\[
\Gamma_{1_{\bar{K}H}} = g_{\Lambda^*}^2 \frac{3}{8\pi} \frac{m_K m_\Sigma}{M_{1_{\bar{K}H}}} \frac{k_{\Sigma\pi}}{F_\pi^2} B_{1_{\bar{K}H}},
\]
where \(g_{\Lambda^*}\) is a coupling constant of the \(1_{\bar{K}H} \rightarrow \Sigma\pi\) decays, \(M_{1_{\bar{K}H}} = m_K + m_p - B_{1_{\bar{K}H}}^{WT}\) is the mass of the quasi–bound \(1_{\bar{K}H}\) state, \(k_{\Sigma\pi} = 147\) MeV is a relative momentum of the \(\Sigma\pi\) pair.
and \( m_\Sigma = 1193 \text{ MeV} \) and \( m_\pi = 140 \text{ MeV} \) are masses of the \( \Sigma \)–hyperon and the \( \pi \)–meson, respectively. The numerical values of \( B_{k_H}^{WT} \) and \( \Gamma_{k_H} \) are discussed in Section 4.

**B. Wave function of quasi–bound \( N \otimes (\bar{K}N)_{I=0} \) state**

The vibrational degrees of freedom of the molecule \( N \otimes (\bar{K}N)_{I=0} \) are defined by the frequency \( \Omega_{\Lambda}\ast \), the oscillations of the \( \bar{K}N \) pair, and the frequency \( \Omega_{\Lambda\ast N} \), the oscillations of the nucleon relative to the \( \bar{K}N \) pair. The oscillator wave function of the KNC \( ^2_{\bar{K}}H \), determined by \( \Phi_{\Lambda\ast}(\vec{k},\vec{k}_N) = \Phi_{\Lambda\ast N}(\vec{k}_N) \), is equal to

\[
|\Phi_{\Lambda\ast}(\vec{k},\vec{k}_N)\rangle = \left(\frac{4\pi}{\mu_{\Lambda\ast}}\right)^{3/4} \left(\frac{4\pi}{\mu_{\Lambda\ast N}}\right)^{3/4} \exp\left(-\frac{\vec{k}^2}{2\mu_{\Lambda\ast}} - \frac{\vec{k}_N^2}{2\mu_{\Lambda\ast N}}\right),
\]

where \( \Omega_{\Lambda\ast N} = \Omega_{\Lambda\ast} \sqrt{\mu_{\Lambda\ast}/\mu_{\Lambda\ast N}} \) due to equal stiffnesses by assumption.

In the momentum and particle number representation the wave function of the KNC \( ^2_{\bar{K}}H \) with the structure \( N \otimes (\bar{K}N)_{I=0} \) reads

\[
|\Phi_{\Lambda\ast}(\vec{k},\vec{k}_N)\rangle = \sqrt{2E_{\bar{K}}^2(\vec{k})} \int \frac{d^3k_1}{(2\pi)^3} \frac{d^3k_2}{(2\pi)^3} \frac{d^3k_3}{(2\pi)^3} \Phi_{\bar{K}H}(\vec{k},\vec{k}_1,\vec{k}_2,\vec{k}_3) \Phi_{\Lambda\ast N}(\vec{k}_1,\vec{k}_2,\vec{k}_3) \sqrt{2E_N(k_1)2E_N(k_2)2E_\bar{K}(k_3)}
\times \frac{1}{\sqrt{2}} \sum_i a^\dagger_{Nj}(\vec{k}_1,\vec{k}_2,\vec{k}_3) a^\dagger_{N_i}(\vec{k}_1,\vec{k}_2,\vec{k}_3) |0\rangle,
\]

where we have denoted

\[
\Phi_{\bar{K}H}(\vec{k},\vec{k}_1,\vec{k}_2,\vec{k}_3) = (2\pi)^3 \delta^{(3)}(\vec{k} - \vec{k}_1 - \vec{k}_2 - \vec{k}_3) \Phi_{\Lambda\ast}(m_K\vec{k}_2 - m_N\vec{k}_3)
\times \Phi_{\Lambda\ast N}(\vec{k}_1,\vec{k}_2,\vec{k}_3).
\]

The wave function (II.4) has a standard relativistic covariant normalisation [23]. The wave functions \( \Phi_{\Lambda\ast} \) and \( \Phi_{\Lambda\ast N} \) describe the two–body \((\bar{K}N)_{I=0}\) and three–body \(N(\bar{K}N)_{I=0}\) correlations of the \( N \otimes (\bar{K}N)_{I=0} \) molecule, respectively. Of course, the wave function Eq.(II.4) takes into account also the two–body \(NN\) correlations.

**C. Binding energy of \( N \otimes (\bar{K}N)_{I=0} \) state**

The binding energy of the KNC \( ^2_{\bar{K}}H \) is defined by the attractive low–energy Weinberg–Tomozawa interactions with isospin \( I = 0 \) and \( I = 1 \). The contribution of the Weinberg–Tomozawa interaction with isospin \( I = 1 \) appears due to \(NN\) exchange interaction only.
FIG. 1: The Feynman diagrams, defining the amplitude of the reaction \( N(\bar{K}N)_{I=0} \to N\Lambda^0 \) in the molecule model of the KNC \( \frac{2}{3}K \).

The binding energy of the KNC \( \frac{2}{3}K \) is defined by

\[
(B^{WT}_{\frac{2}{3}K})_{j'j} = (B^{WT,I=0}_{\frac{2}{3}K})_{j'j} + (B^{WT,I=1}_{\frac{2}{3}K})_{j'j}.
\]

The result of the calculation of the corresponding matrix elements of the \( T \)-matrix, obtained in the heavy–baryon approximation, is

\[
(B^{WT}_{\frac{2}{3}K})_{j'j} = \delta_{j'j} \frac{3}{4} \frac{1}{F_\pi^2} \left\{ \left( \frac{\mu_{\Lambda^*}\Omega_{\Lambda^*}}{\pi} \right)^{3/2} + \frac{1}{2} \left( \frac{\mu_{\Lambda^*}\Omega_{\Lambda^*}}{\pi} \right)^{3/2} \left( 1 + \frac{\mu_{\Lambda^*}\Omega_{\Lambda^*}}{\mu^2_{\Lambda^*} m^2_K} \right)^{3/2} \right\}.
\]

The contribution of the second term in (II.8) is caused by the \( NN \) exchange interaction and Weinberg–Tomozawa interactions with isospin \( I = 0 \) and \( I = 1 \), respectively. It makes up of about 24.7% of the binding energy.

### III. DECAY MODES OF \( N \otimes (\bar{K}N)_{I=0} \) STATE

#### A. Non–pionic decay modes of \( N \otimes (\bar{K}N)_{I=0} \) state. Decay \( \frac{2}{3}K \to N\Lambda^0 \)

The amplitude of the \( N(\bar{K}N)_{I=0} \to N\Lambda^0 \) reaction, defining the non–pionic decay mode \( \frac{2}{3}K \to N\Lambda^0 \), is determined by the Feynman diagrams in Fig.1. The analytical expression, obtained in the heavy–baryon approximation, takes the form

\[
M(\frac{2}{3}K \to Nj'\Lambda^0) = \delta_{j'j} i \left[ \frac{9}{32} \frac{g_{\pi NN}}{F^2_\pi} \right] \sqrt{2M^2_{\frac{2}{3}K}m_K} \Psi_{\frac{2}{3}K}(0) \\
\times \left\{ \frac{4}{3} - 2\alpha_D \right\} \frac{1}{\sqrt{3}} \frac{1}{m^2_K + 2T_{N\Lambda^0}m_N} - \frac{3 - 4\alpha_D}{\sqrt{3}} \frac{1}{m^2_\eta + 2T_Nm_N} + \frac{1}{\sqrt{3}} \frac{1}{m^2_\pi + 2T_Nm_N} \right\} \\
\times \frac{M^2_{\frac{2}{3}K} + m_{\Lambda^0} + m_N}{\sqrt{2m_{\Lambda^0}m_N}} \varphi^\dagger_{\Lambda^0}(\vec{\sigma} \cdot \vec{k}_{N\Lambda^0}) \chi_N = \sqrt{2M^2_{\frac{2}{3}K} \mathcal{M}(\frac{2}{3}K \to Nj'\Lambda^0),}
\]

where \( \varphi_{\Lambda^0}(\sigma_{\Lambda^0}) \) and \( \chi_N(\sigma_N) \) are spinorial wave functions of the \( \Lambda^0 \)–hyperon and the nucleon, respectively, \( g_{\pi NN} = g_{AMN}/F_\pi = 13.3 \) is the \( \pi NN \) coupling constant \( \text{[28]} \), \( k_{N\Lambda^0} \) is
a momentum of a relative motion of the $N\Lambda^0$ pair, $T_{\Lambda^0}$ and $T_N$ are kinetic energies of the baryons in the final $N\Lambda^0$ state. Then, $M_{K^H} = 2m_N + m_K - B^{WT}_{K^H}$ is the mass of the KNC $^2\bar{K}H$ and $\Psi_{K^H}(0)$ is the coordinate wave function of the KNC $^2\bar{K}H$ at the origin

$$\Psi_{K^H}(0) = \Psi_{A^*}(0)\Psi_{\Lambda^* N}(0) = \left(\frac{\mu_{A^*} \Omega_{A^*} \mu_{\Lambda^* N} \Omega_{\Lambda^* N}}{\pi}\right)^{3/4}.$$  \hspace{1cm} (III.2)

The first and the second terms in the amplitude (III.1) are defined by the $K-$meson and $\eta-$meson exchange diagrams in Fig. 1, respectively, and Weinberg–Tomozawa interaction with isospin $I = 0$. In turn, the contribution of the $\pi-$meson exchanges is fully caused by the Weinberg–Tomozawa interaction with isospin $I = 1$ and the $NN$ exchange interaction. The structure $[\varphi_{\Lambda^0}^*(\vec{\sigma} \cdot \vec{F}_{N\Lambda^0}) \chi_N]$ testifies that in the $^2\bar{K}H \rightarrow N\Lambda^0$ decay the $N\Lambda^0$ pair is in the $^3P_0$ state, whereas the KNC $^2\bar{K}H$ is in the $^1S_0$ state. The partial width of the decay $^2\bar{K}H \rightarrow N\Lambda^0$ amounts to

$$\Gamma(^2\bar{K}H \rightarrow N\Lambda^0) = \frac{1}{4\pi} \sum_{\sigma_{\Lambda^0}, \sigma_N = \pm \frac{1}{2}} |\mathcal{M}(^2\bar{K}H \rightarrow N\Lambda^0)|^2 \frac{|\vec{F}_{N\Lambda^0}|}{M_{K^H}},$$  \hspace{1cm} (III.3)

where we have summed over polarisations of the particles in the final state. The numerical value of $\Gamma(^2\bar{K}H \rightarrow N\Lambda^0)$ is discussed in Section 4.

\section*{B. Non–pionic decay modes of $N \otimes (\bar{K}N)_{I=0}$ state. Decay $^2\bar{K}H \rightarrow N\Sigma$}

The amplitudes of the $N(\bar{K}N)_{I=0} \rightarrow N\Sigma$ reactions, defining the non–pionic decay modes $^2\bar{K}H \rightarrow N\Sigma$, are determined by the Feynman diagrams in Fig. 2. The analytical expression, obtained in the heavy–baryon approximation, takes the form

$$M(^2\bar{K}H_j \rightarrow N_j \Sigma^a) = -i (\tau^a)_{jj'} \left[ \frac{9}{32} \frac{g_{\pi NN}}{F_\pi^2} \right] \sqrt{2M_{K^H}^2 m_K} \Psi_{K^H}(0)$$

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{feynman_diagram}
\caption{The Feynman diagrams, defining the amplitudes of the reactions $N(\bar{K}N)_{I=0} \rightarrow N\Sigma$ in the molecule model of the KNC $^2\bar{K}H$.}
\end{figure}
FIG. 3: The Feynman diagram of the decay mode $^2\bar{K}\ H \to N\Sigma\pi$ of the KNC $^2\bar{K}\ H$.

\[
\times \left\{ (2\alpha_D - 1) \left( g_{\Lambda^*} - \frac{1}{9} \right) \frac{1}{m_K^2 + 2T_\Sigma m_N} - \left( \frac{1}{\sqrt{3}} + \frac{2}{9} \right) \frac{1}{m_\pi^2 + 2T_N m_N} - \frac{3 - 4\alpha_D}{9} \frac{1}{m_\eta^2 + 2T_N m_N} \right\} \frac{M_{\bar{K}\ H} + m_\Sigma + m_N}{\sqrt{2m_\Sigma 2m_N}} \left[ \varphi_{\Sigma}^\dagger (\vec{\sigma} \cdot \vec{k}_{N\Sigma}) \chi_N \right] = \sqrt{2M_{\bar{K}\ H}} M_{(\bar{K}\ H)} j \to N_j' \Sigma^a), \quad (\text{III.4})
\]

where $g_{\Lambda^*}$ is the coupling constant of the $^1\bar{K}\ H \to \Sigma\pi$ decays. The contributions of the Weinberg–Tomozawa interactions with isospin $I = 1$ survive due to the $NN$ exchange interactions for the $\bar{K}$, $\pi$ and $\eta$ meson exchanges. The partial width of the $^2\bar{K}\ H \to N\Sigma^0$ decay amounts to

\[
\Gamma(\bar{K}\ H \to N\Sigma^0) = \frac{1}{4\pi} \sum_{\sigma_{\Lambda^0}, \sigma_N = \pm 1/2} |M(\bar{K}\ H \to N\Sigma)|^2 \frac{|\bar{K}_{N\Sigma}|}{M_{\bar{K}\ H}}, \quad (\text{III.5})
\]

The partial widths of the decay modes $^2\bar{K}\ H \to N\Sigma^0$ and $^2\bar{K}\ H \to N\Sigma^+$ are related by $\Gamma(\bar{K}\ H \to N\Sigma^+) = 2\Gamma(\bar{K}\ H \to N\Sigma^0)$. The numerical values are discussed in Section 4.

C. Pionic decay modes of $N \otimes (\bar{K}N)_{I=0}$ state

Since the $\bar{K}N$ pair in the KNC $^2\bar{K}\ H$ is in the isospin–singlet state, the dominant pionic three–body decay modes are $^2\bar{K}\ H \to N\Sigma\pi$. In the tree–approximation the Feynman diagram of the decay modes $^2\bar{K}\ H \to N\Sigma\pi$ is shown in Fig. 3. The amplitude of the $^2\bar{K}\ H \to N\Sigma\pi$ decay is equal to

\[
M(\bar{K}\ H_j \to N_j' \Sigma^a \pi^b) = -\delta_{jj'} \delta^{ab} \frac{1}{2F_\pi^2} \left( g_{\Lambda^*} \frac{1}{2F_\pi^2} \sqrt{6M_{\bar{K}\ H} m_\Sigma m_N m_K} \Psi_{\Lambda^*}(0) \Phi_{\Lambda^* N}(0) \right) \times \mathcal{M}(\Omega_{\Lambda^*}, k_N^2) \sigma_N, \sigma_\Sigma), \quad (\text{III.6})
\]

where $k_N$ is a 3–momentum of the nucleon, $\sigma_\Sigma = \pm 1/2$ and $\sigma_N = \pm 1/2$ are polarisations of the $\Sigma$–hyperon and the nucleon in the final state, respectively, $\Psi_{\Lambda^*}(0) = (\mu_{\Lambda^*} \Omega_{\Lambda^*}/\pi)^{3/4}$
and \( \Phi_{\Lambda^*N}(0) = (4\pi/\mu_{\Lambda^*N}\Omega_{\Lambda^*N})^{3/4} \) and \( \mathcal{M}(\Omega_{\Lambda^*}, k^2_N)_{\sigma_N, \sigma_{\Sigma}} \) is equal to

\[
\mathcal{M}(\Omega_{\Lambda^*}, k^2_N)_{\sigma_N, \sigma_{\Sigma}} = \left\{ \Phi_{\Lambda^*N}(\vec{k}_N) + \frac{1}{B_{K^*}^2 - B_{K^*}^1 + \vec{k}_N^2/2\mu_{\Lambda^*N}} \times \frac{3}{4} \frac{1}{F_\pi} \int \frac{d^3q}{(2\pi)^3} \Phi_{\Lambda^*}(\vec{k} + \frac{\mu_{\Lambda^*}}{m_K}(\vec{q} + \vec{k}_N)) \Phi_{\Lambda^*N}(\vec{q}) \right\} \delta_{\sigma_N, \pm \frac{1}{2}} \delta_{\sigma_{\Sigma}, -\frac{1}{2}}. \tag{III.7}
\]

The amplitude of the \( ^2\bar{K}H \rightarrow N\Sigma\pi \) decay is defined by the Weinberg–Tomozawa–like interaction, responsible for the \( ^1\bar{K}H \rightarrow \Sigma\pi \) decay, and the Weinberg–Tomozawa interactions \( (\bar{K}N)_{I=0} \rightarrow (\bar{K}N)_{I=0} \) and \( (\bar{K}N)_{I=1} \rightarrow (\bar{K}N)_{I=1} \) with isospin \( I = 0 \) and \( I = 1 \), respectively.

The contribution of the Weinberg–Tomozawa interaction \( (\bar{K}N)_{I=1} \rightarrow (\bar{K}N)_{I=1} \) with isospin \( I = 1 \) appears due to the \( NN \) exchange interaction. The partial width of the \( ^2\bar{K}H \rightarrow N\Sigma\pi \) decays is

\[
\Gamma(^2\bar{K}H \rightarrow N\Sigma\pi) = \frac{g_{\Lambda^*}^2}{4 F_\pi^4} |\Psi_{\Lambda^*}(0)|^2 |\Phi_{\Lambda^*N}(0)|^2 M^2_{\bar{K}H} m_{\Sigma} m_{N} m_{K} f_{N\Sigma\pi}(\Omega_{\Lambda^*}), \tag{III.8}
\]

where \( f_{N\Sigma\pi}(\Omega_{\Lambda^*}) \), caused by the contribution of the phase volume of the final \( N\Sigma\pi \) state, is defined by

\[
f_{\rho\Sigma\pi}(\Omega_{\Lambda^*}) = \frac{1}{128\pi^3 M^4_{\bar{K}H}} \int_{(m_{\Sigma} + m_\pi)^2}^{(M^2_{\bar{K}H} - m_{\Sigma})^2} \frac{ds}{s} \sqrt{(s - (m_{\Sigma} + m_\pi)^2)(s - (m_{\Sigma} - m_\pi)^2)} \times \sum_{\sigma_N = \pm \frac{1}{2}} \sum_{\sigma_{\Sigma} = \pm \frac{1}{2}} |\mathcal{M}(\Omega_{\Lambda^*}, k^2_N)_{\sigma_N, \sigma_{\Sigma}}|^2. \tag{III.9}
\]

The numerical value of the partial width of the pionic decay modes we discuss in Section 4.

### IV. NUMERICAL VALUES OF BINDING ENERGY AND WIDTHS OF \( ^2\bar{K}H \)

In this section we give numerical values of the binding energy and partial widths of the KNC \( ^2\bar{K}H \) for the KNC \( ^1\bar{K}H \) with mass \( M_{\bar{K}H} = 1405 \text{ MeV} \) and widths \( \Gamma_{\bar{K}H} = 30 \text{ MeV} \).
Molecule model  | Potential Model
---|---
$M_{\bar{K}H} = 1405$ MeV | $M_{\bar{K}H} = 1405$ MeV | $M_{\bar{K}H} = 1405$ MeV
$B_{\bar{K}H}^{WT}$ | 29.0 MeV | 29.0 MeV | 27.0 MeV
$\Gamma_{\bar{K}H}^1$ | 30.0 MeV | 40.0 MeV | 40.0 MeV
$\Omega_{\Lambda^*}$ | 46.3 MeV | 46.3 MeV |
$g_{\Lambda^*}$ | 1.095 | 1.265 |
$\Omega_{\Lambda^*N}$ | 35.0 MeV | 35.0 MeV |
$B_{\bar{K}H}^{WT}$ | 40.2 MeV | 40.2 MeV | 48 MeV
$\Gamma(\bar{K}_H^2 \rightarrow N\Lambda^0)$ | 14.6 MeV | 14.6 MeV |
$\Gamma(\bar{K}_H^2 \rightarrow N\Sigma^0)$ | 2.2 MeV | 2.0 MeV |
$\Gamma(\bar{K}_H^2 \rightarrow N\Sigma^+)$ | 4.4 MeV | 4.0 MeV |
$\Gamma^{(\pi)}_{\bar{K}H^2}$ | 21.2 MeV | 20.6 MeV | $\approx 12$ MeV |
$\Gamma^{(\pi)}_{\bar{K}H^2}$ | 64.0 MeV | 85.5 MeV | 61 MeV |
$\Gamma_{\bar{K}H^2} = \Gamma^{(\pi)}_{\bar{K}H^2} + \Gamma^{(\pi)}_{\bar{K}H^2}$ | 85.2 MeV | 106.1 MeV | $\approx 73$ MeV |

**TABLE I:** The binding energies and widths of the KNC $^2\bar{K}H$

and $\Gamma_{\bar{K}H}^1 = 40$ MeV, predicted by potential model [25] and [1], respectively. In Table I we summarise the results of our molecule model and the potential model.

One can see that the binding energy $B_{\bar{K}H}^2 = 40.2$ MeV and the width $\Gamma_{\bar{K}H}^2 = 85.2$ MeV of the KNC $^2\bar{K}H$, calculated for the KNC $^1\bar{K}H$ with mass $M_{\bar{K}H} = 1405$ MeV and the width $\Gamma_{\bar{K}H}^1 = 30$ MeV, agree well with the results, obtained in the potential model approach by Akaishi and Yamazaki. The width of the non–pionic decay modes $\Gamma^{(\pi)}_{\bar{K}H} \simeq 21$ MeV agrees well with the width $\Gamma^{(\pi)}_{\Lambda^*N} = 22$ MeV of the unbound $\Lambda^*N$ state, calculated recently in [14]. Our results agree also well with those obtained within the coupled–channel Faddeev equation approach [11, 12]. The agreement with other approaches [7]-[10, 13] is only qualitative.

**V. CONCLUSION**

We have investigated the properties of the simplest kaonic nuclear clusters (KNCs) $^1\bar{K}H$ and $^2\bar{K}H$ with the structures $(\bar{K}N)_{I=0}$ and $N \otimes (\bar{K}N)_{I=0}$, respectively, in the model, which we call “Molecule model for kaonic nuclear clusters”. It is based on the assumption that
KNCs are hadronic molecules [2]. In our model the calculation of the binding energies and the widths of KNCs is a kind of variational calculation with trial wave functions taken in the form of harmonic oscillator wave functions. Such a choice is justified as follows. It is known [18] that molecules are described by rotational and vibrational degrees of freedom. Since rotational degrees of freedom are absent for the KNC $\frac{1}{K}H$ and $\frac{2}{K}H$, their properties are determined by vibrational degrees of freedom only. The binding energy and the width of the KNCs $\frac{1}{K}H$ and $\frac{2}{K}H$ are defined by the diagonal elements $\langle \frac{1}{K}H | T | \frac{1}{K}H \rangle$ and $\langle \frac{2}{K}H | T | \frac{2}{K}H \rangle$ of the $T$–matrix, which are calculated by using chiral Lagrangians with derivative meson–baryon couplings invariant under chiral $SU(3) \times SU(3)$ symmetry at the tree–level approximation and in the heavy–baryon approximation. The dominant contributions to the binding energies and the widths come from the Weinberg–Tomozawa interactions. This agrees well with $SU(3)$ coupled–channel approach with chiral dynamics.

The KNC $(\bar{K}N)_{I=0}$ is identified with the hyperon $\Lambda(1405)$, the wave function of which is taken in the form of the trial harmonic oscillator wave function with a frequency $\Omega_{\Lambda^*}$, describing the relative motion or correlations of the $\bar{K}N$ pair in the state with $I = 0$. The wave function of the KNC $N \otimes (\bar{K}N)_{I=0}$ is defined by the frequencies $\Omega_{\Lambda^*}$ and $\Omega_{\Lambda^*N}$, where the latter describes a motion of a nucleon $N$ relative to the $\bar{K}N$ pair in the state with $I = 0$. They take into account two–body $\bar{K}N$ and three–body $N(\bar{K}N)$ correlations in the KNC $\frac{2}{K}H$, respectively, and, of course, the two–body $NN$ correlations.

The calculations of the binding energy and widths of the KNC $\frac{2}{K}H$ have been carried out for the low–lying KNC $\frac{1}{K}H$ with mass $M_{\frac{1}{K}H} = 1405$ MeV and widths $\Gamma_{\frac{1}{K}H} = (30 – 40)$ MeV, predicted by the potential model approach from the experimental data on the stopped–$K^-$ meson absorption in the deuteron target [25] and used by Akaishi and Yamazaki in their original work [1], respectively. Recently, the theoretical analysis of the contribution of the $\Lambda(1405)$ resonance to the cross sections for elastic and inelastic $K^-p$ scattering and elastic $\pi\Sigma \rightarrow \pi\Sigma$ scattering has been carried out in [29]. Following the results obtained in [29], one can conclude that in the pure $I = 0$ channel $\pi^0\Sigma^0 \rightarrow \pi^0\Sigma^0$ the maximum of the cross section is located around $m_{\Lambda^*} \simeq 1405$ MeV. It has been also found experimentally [30] that the shape and position of the $\Lambda(1405)$ distribution, reconstructed in the $\Sigma^0\pi^0$ channel, are consistent with mass $m_{\Lambda^*} \sim 1400$ MeV and width $\Gamma_{\Lambda^*} \sim 60$ MeV, agreeing well with the shape and position of the $\Lambda(1405)$ distribution, measured in the charge–exchange channels [31, 32].
We have shown that treating the Λ(1405) resonance as a quasi–bound (¯KN) with the mass, defined by \( M_{\bar{K}H} = m_K + m_N - B^{WT}_{\bar{K}H} \), and the widths \( \Gamma_{\Lambda^*} = (30 - 40) \text{ MeV} \), the molecule model for kaonic nuclear clusters describes the KNC \( \bar{K}H \) in a qualitative agreement with Akaishi and Yamazaki [2] and the results, obtained in the coupled–channel Faddeev equation approach [11, 12].

Our results for the widths of non–pionic decay modes \( \Gamma^{(\pi)}_{\bar{K}H} \simeq 21 \text{ MeV} \) agree well with the result \( \Gamma^{(\pi)}_{\Lambda^*N} = 22 \text{ MeV} \), obtained in [14]. Unlike [14] our analysis of the non–pionic decay modes \( \bar{K}H \to N\Lambda^0 \) and \( \bar{K}H \to N\Sigma \) takes into account the \( NN \) exchange interactions, which play an important role for the correct description of the properties of the KNC \( \bar{K}H \).

The explanation of the experimental data by the DISTO Collaboration \( B_{\bar{K}H} = 103(6) \text{ MeV} \) and \( \Gamma_{\bar{K}H} = 118(13) \text{ MeV} \) [16] in the molecule model for kaonic nuclear clusters is possible, but it goes beyond the description of the KNCs \( \bar{K}H \) at the tree–level approximation for the binding energies and the assumption of the equal stiffnesses of harmonic oscillator potentials. The analysis of the experimental data by the DISTO Collaboration in the molecule model for kaonic nuclear clusters we carry out in the forthcoming paper.

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