Remarks on Universal Quantum Computer

Yu Shi

Cavendish Laboratory, University of Cambridge, Cambridge CB3 0HE, United Kingdom

Abstract

According to Deutsch, a universal quantum Turing machine (UQTM) is able to perform, in repeating a fixed unitary transformation on the total system, an arbitrary unitary transformation on an arbitrary data state, by including a program as another part of the input state. We note that if such a UQTM really exists, with the program state dependent on the data state, and if the prescribed halting scheme is indeed valid, then there would be no entanglement between the halt qubit and other qubits, as pointed out by Myers. If, however, the program is required to be independent of the data, the concerned entanglement appears, and is problematic no matter whether the halt qubit is monitored or not. We also note that for a deterministic programmable quantum gate array, as discussed by Nielsen and Chuang, if the program is allowed to depend on the data state, then its existence has not been ruled out. On the other hand, if UQTM exists, it can be simulated by repeating the operation of a fixed gate array. However, more importantly, we observe that it is actually still open whether Deutsch’s UQTM exists and whether a crucial concatenation scheme, of which the halting scheme is a special case, is valid.

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Quantum computing is under intensive study [1]. Most of the researches follow the approach of quantum computational network or circuit [2]. In the present Letter, we are concerned with quantum Turing machine (QTM) [3,4] and the so-called universal, or programmable, quantum computer. In classical computation, Turing machine is employed in stating the Church-Turing thesis, which is the foundation of the theory of computability: Every computable function can be computed by a universal Turing machine. In other words, all finitely realized computing machines can be simulated by a single machine called “universal Turing machine”. The remarkable fact that one uses a single computer for various different computational tasks is a consequence of the Church-Turing thesis. In the founding paper of quantum computation [3], Deutsch quantized Turing machine. Furthermore, it was claimed that there exists a universal QTM (UQTM), which was defined to be a QTM for which there always exists a program as a part of the input state, which effects a unitary transformation on an arbitrary number of qubits, herein called data qubits, arbitrarily close to any desired one.

The construction of a QTM includes an additional halt qubit which signals whether the computation has completed. Recently Myers pointed out that there can be an entanglement between the halt qubit and the computational qubits, hence monitoring the halt qubit spoils the computation [5]. Here we note that this situation does not occur if UQTM exists and the prescribed halting scheme is valid, and if the program state is dependent on the data state. However, it does occur if one further requires that the program state is independent of the data state, while the execution time depends on the data state. Moreover, the entanglement between the halt qubit and the computational qubits is a problem no matter whether the halt qubit is monitored or not.

Deutsch’s UQTM is not the only model of universal quantum computer. An interesting and perhaps more practical model is the programmable quantum gate array discussed by Nielson and Chuang [6]. Unfortunately it was immediately shown that it does not exist if it
is required to operate deterministically. We note that the disproof is nullified if the program state is data-dependent. Hence this model may be more interesting than previously thought. We also note that if UQTM exists, one may use a fixed gate array to simulate each step of the UQTM, and use the output of each run of the gate array as the input for the next run.

However, the more important message we would like to convey is that in Deutsch’s construction of UQTM, the proof of the existence of a program which effects a unitary transformation on the data qubits, arbitrarily close to any desired one, has not actually been accomplished, and that the concatenation scheme, of which the halting scheme is a special case, is also not yet proved to be valid.

2. QUANTUM TURING MACHINE

As a generalization of classical Turing machine, a QTM consists of a finite processor consisting of $N$ qubits $\mathbf{n} = \{n_i\} (i = 0, \cdots, N-1)$, and an infinite tape consisting of an infinite sequence of qubits $\mathbf{m} = \{m_i\} (i = \cdots, -1, 0, 1, \cdots)$, of which only a finite portion is ever used. The currently scanned qubit on the tape, i.e. the position of the head, is specified by $x$. Thus the state of a QTM is a unit vector in the Hilbert space spanned by the basis states

$$| x \rangle | \mathbf{n} \rangle | \mathbf{m} \rangle,$$

where $| \mathbf{n} \rangle \equiv |n_0, n_1, \cdots, n_{N-1} \rangle$, $| \mathbf{m} \rangle \equiv | \cdots, m_{-1}, m_0, m_1, \cdots \rangle$. The dynamics is summarized by a constant unitary operator $U$ whose only nonzero matrix elements are $\langle x \pm 1; \mathbf{n}; m_x', m_y \neq x | U | x; \mathbf{n}; m_x, m_y \neq x \rangle$. Each different choice of $U$ defines a different QTM. To signal whether the computation is completed, another qubit $n_h$ is added, which is initialized to 0 and is expected to flip to 1 when the computation is completed. Therefore one may observe $n_h$ to know whether the computation has been completed. The evolution of the QTM state can be written as

$$| \Psi(sT) \rangle = U^s | \Psi(0) \rangle,$$
where $|\Psi(0)\rangle$ is the initial state, $s$ is the number of steps, $T$ is the time duration of each step.

3. UNIVERSAL QUANTUM TURING MACHINE

For a UQTM, we may write its state as $|Q_{x,n}|n_h\rangle|\mathcal{D}\rangle|\mathcal{P}\rangle|\Sigma\rangle$, where $|Q_{x,n}\rangle$ is the state of the processor, including the head position, $|n_h\rangle$ is the halt qubit, $|\mathcal{D}\rangle$ is the state of the data register, $|\mathcal{P}\rangle$ is the program state. $|\mathcal{D}\rangle$ and $|\mathcal{P}\rangle$ are both parts of the tape, $|\Sigma\rangle$ is the rest part of the tape, not affected during the computation. Deutsch said that there is a UQTM, for which there exists a program that effects a unitary transformation, arbitrarily close to any desired unitary transformation, on a finite number of data qubits $\Box$. In more mathematical details, this can be expressed as the following. There exists a special QTM, i.e. a special $U$, for an arbitrary unitary transformation $\mathcal{U}$ and an arbitrary accuracy $\epsilon$, there is always a program state $|\mathcal{P}(\mathcal{D},\mathcal{U},\epsilon)\rangle$ and a whole number $s(\mathcal{D},\mathcal{U},\epsilon)$, such that

$$U^{s(\mathcal{D},\mathcal{U},\epsilon)}|Q_{x,n}\rangle|\mathcal{D}\rangle|\mathcal{P}(\mathcal{D},\mathcal{U},\epsilon)\rangle|\Sigma\rangle = |Q_{x,n}'\rangle|\mathcal{D}'\rangle|\mathcal{P}'(\mathcal{D},\mathcal{U},\epsilon)\rangle|\Sigma\rangle,$$

where $|\mathcal{D}'\rangle$ is arbitrarily close to $|\mathcal{U}\mathcal{D}\rangle$, i.e., $||\mathcal{D}'\rangle - |\mathcal{U}\mathcal{D}\rangle||^2 < \epsilon$. In the most general case, the program $\mathcal{P}$ depends on the desired transformation $\mathcal{U}$ and the accuracy $\epsilon$, as well as the data state $\mathcal{D}$. $\mathcal{P}'$ is the state of the program register after $s$ steps, $Q$ and $Q'$ are the states of the processor at the initial time and after $s$ steps, respectively. For the time being, as in Deutsch’s discussion on this issue, we do not consider the halt qubit, which will be specifically discussed in Section 5.

4. IS THERE A UQTM?

Deutsch gave a proof for the above claim. A key step is the following inductive proof of the existence of a program which accurately evolves an arbitrary $L$-bit data state $|\mathcal{D}\rangle = |\psi_{1\sim L}\rangle$
to $|0_{1-L}\rangle$, i.e., with which $|D\rangle$ is evolved to $|D'\rangle$ which is arbitrarily close to $|0_{1-L}\rangle$. Here $|0_{i-j}\rangle$ denotes $|0_i\rangle|0_{i+1}\rangle\cdots|0_j\rangle$. Write

$$|\psi_{1-L}\rangle = c_0|0_1\rangle|\psi^0_{2-L}\rangle + c_1|1_1\rangle|\psi^1_{2-L}\rangle.$$  \hfill (3)

By inductive hypothesis there exist programs $\rho_0$ and $\rho_1$ which accurately evolve $|\psi^0_{2-L}\rangle$ and $|\psi^1_{2-L}\rangle$, respectively, into $|0_{2-L}\rangle$. Therefore there exists a program $\rho$ with the following effect:

If qubit no. 1 is 0, execute $\rho_0$, otherwise execute $\rho_1$. Thus (3) is converted to

$$(c_0|0_1\rangle + c_1|1_1\rangle)|0_{2-L}\rangle,$$ \hfill (4)

which can be evolved accurately to $|0_{1-L}\rangle$ by a one-bit transformation of the qubit no. 1, using the program for one-bit transformations, the existence of which had been claimed earlier, using the so-called concatenation, which will be commented on in the next section.

Let us re-state the above proof in more mathematical details. The inductive hypothesis is the following. There is a program state $|P_0\rangle$ such that

$$U^{s_0}|Q_{x,n}|\phi_1|\psi^0_{2-L}\rangle|P_0\rangle|\Sigma\rangle = |Q_{x,n}'|\phi_1|D^0\rangle|P_0'\rangle|\Sigma\rangle,$$ \hfill (5)

while there is a program state $|P_1\rangle$ such that

$$U^{s_1}|Q_{x,n}|\phi_1|\psi^1_{2-L}\rangle|P_1\rangle|\Sigma\rangle = |Q_{x,n}'|\phi_1|D^1\rangle|P_1'\rangle|\Sigma\rangle,$$ \hfill (6)

where $|D^0\rangle$ and $|D^1\rangle$ are both arbitrarily close to $|0_{2-L}\rangle$, $|\phi_1\rangle$ is a state of the qubit no. 1, $s_0$ and $s_1$ are execution times of the two programs, respectively.

The existence of the program $\rho$ with the effect that if qubit no. 1 is 0, execute $\rho_0$, otherwise execute $\rho_1$ means that there is a program state $|P\rangle$ such that $|0_1\rangle|P\rangle$ leads to the same effect on $|0_{2-L}\rangle$ as $|P_0\rangle$, while $|1_1\rangle|P\rangle$ leads to the same effect on $|0_{2-L}\rangle$ as $|P_1\rangle$. This can be written as
\[ U^{s_0}|Q_x,n\rangle|0_1\rangle|\psi^0_{2\sim L}\rangle|P\rangle|\Sigma\rangle = |Q'^{0'}_{x,n}\rangle|0_1\rangle|D^{0'}\rangle|P^{0'}\rangle|\Sigma\rangle, \quad (7) \]
\[ U^{s_1}|Q_x,n\rangle|1_1\rangle|\psi^1_{2\sim L}\rangle|P\rangle|\Sigma\rangle = |Q'^{1'}_{x,n}\rangle|1_1\rangle|D^{1'}\rangle|P^{1'}\rangle|\Sigma\rangle, \quad (8) \]

where \( s_0, s_1, |D^{0'}\rangle \) and \( |D^{1'}\rangle \) are the same as those in (5) and (6). Or, in a more relaxed fashion,

\[ U^{s_0'}|Q_x,n\rangle|0_1\rangle|\psi^0_{2\sim L}\rangle|P\rangle|\Sigma\rangle = |Q''^{0''}_{x,n}\rangle|0_1\rangle|D^{0''}\rangle|P^{0''}\rangle|\Sigma\rangle, \quad (9) \]
\[ U^{s_1'}|Q_x,n\rangle|1_1\rangle|\psi^1_{2\sim L}\rangle|P\rangle|\Sigma\rangle = |Q''^{1''}_{x,n}\rangle|1_1\rangle|D^{1''}\rangle|P^{1''}\rangle|\Sigma\rangle, \quad (10) \]

where \( |D^{0''}\rangle \) and \( |D^{1''}\rangle \) are both arbitrarily close to \( |0_{2\sim L}\rangle \). In general, \( s_0 \neq s_1, s_0' \neq s_1' \).

Now we can see two problems. First, there is no reason why the existence of such a \( |P\rangle \) automatically follows the assumption given in (5) and (6). Second, even if there exists such a \( |P\rangle \), it still does not follow that the decision-making construction can be realized simultaneously in two branches of a superposed quantum state, i.e.

\[ U^s|Q_x,n\rangle \left( c_0|0_1\rangle|\psi^0_{2\sim L}\rangle + c_1|1_1\rangle|\psi^1_{2\sim L}\rangle \right) |P\rangle|\Sigma\rangle = |Q''^{'}_{x,n}\rangle \left( c_0|0_1\rangle + c_1|1_1\rangle \right) |D^{'}\rangle|P^{'}\rangle|\Sigma\rangle, \quad (11) \]

for a certain \( s \) and \( |D^{'}\rangle \) arbitrarily close to \( |0_{2\sim L}\rangle \). What can be known is only that if (5) and (8) are valid with \( s_0=s_1 \), or if (9) and (10) are valid with \( s_0'=s_1' \), then one can obtain (11) by linearly superposing (7) and (8), or (9) and (10).

In conclusion, in [3], although UQTM was defined, its existence was not really proved.

We do not think that this issue has been solved in a later work on quantum Turing machine [4], which Myers regards as constrained to special cases. Besides, it seems to us that in several occasions, the constructions of QTM in [4] rely on a so-called synchronization theorem, which is a property of classical computation. But a quantum process is not a combination of independent classical processes.
5. CONCATENATION AND HALTING SCHEME

In Deutsch’s construction, a crucial element is concatenation. The concatenation of two programs is a program whose effect is that of the first program followed by the second one. It was taken for granted that if the two programs are valid, their concatenation also exists. For example, it was claimed that after obtaining (4), the system automatically switches to realizing the effect of the program on one-bit rotation.

Suppose there is a program $|P_a⟩$ which effects $U_a$ on a data state $|D⟩$, and a program $|P_b⟩$ which effects $U_b$ on $|U_aD⟩$. This means

$$U^{sa}|Q_x.n⟩|D⟩|P_a⟩|Σ⟩$$

$$= |Q^a_x,n⟩|D⟩|P'_a⟩|Σ⟩, \quad (12)$$

with $|D_a⟩ \approx |U_aD⟩$, and

$$U^{sb}|Q_x.n⟩|U_aD⟩|P_b⟩|Σ⟩$$

$$= |Q^b_x,n⟩|D⟩|P'_b⟩|Σ⟩, \quad (13)$$

with $|D_b⟩ \approx |U_bU_aD⟩$. But this does not automatically imply the existence of a program, denoted as $|P_{ab}⟩$, which has the effect

$$U^{sab}|Q_x.n⟩|D⟩|P_{ab}⟩|Σ⟩$$

$$= |Q^{ab}_x,n⟩|D⟩|P'_{ab}⟩|Σ⟩, \quad (14)$$

for a certain $s_{ab}$ and with $|D_{ab}⟩ \approx |U_bU_aD⟩$.

Therefore the validity of concatenation scheme is not proved.

The halting scheme is a special case of concatenation scheme. It is based on the following tacit assumption: if there is a program which effects $U$ on the data state $|D⟩$, then there is a program which effects $|1_h⟩⟨0_h| \otimes U$ on $|0_h⟩|D⟩$. The validity of this assumption is not proved.
Furthermore, the halting scheme also requires that the halt qubit is not flipped before and after the computation is completed, i.e. it flips only once, when the computation is completed. This is a more stringent requirement, the possibility of which is not proved.

If such a halting scheme really exists, then in Eq. (4), and in each equation from (5) to (14), one may add $|0_h\rangle$ on the left-hand-side and $|1_h\rangle$ on the right-hand-side.

6. MYERS’ PROBLEM

Recently, Myers argued that there can be an entanglement between halt qubit and the other qubits, thereby a measurement on $n_h$ spoils the computation, as follows [5]. Suppose two computations, which start respectively from basis states $|A\rangle|0_h\rangle$ and $|B\rangle|0_h\rangle$, are evolved after $N_A$ and $N_B$ steps to the desired states $|A'\rangle|1_h\rangle$ and $|B'\rangle|1_h\rangle$, respectively. If $N_B > N_A$, then for a computation starting from $(|A\rangle + |B\rangle)|0_h\rangle$, after $N$ steps, with $N_A < N < N_B$, the state is something like $|A''\rangle|1_h\rangle + |B''\rangle|0_h\rangle$, because of linearity of the evolution. In general $|A''\rangle \neq |B''\rangle$. Because the computation time is unknown, if one measures $n_h$ after $N$ steps, with $N_A < N < N_B$, the computation is spoiled since the state will be reduced to either $|A''\rangle|0_h\rangle$ or $|B''\rangle|1_h\rangle$. Myers regarded this as a conflict between being universal and being fully quantum.

We comment that the combination of the universality and being fully quantum does not require the whole quantum computer to evolve from any superposition; the arbitrary desired computation $U$ is effected only on the data state. It is the data state, instead of the state of the total system, that may start from any superposition. In the most general definition of the UQTM, each initial data state corresponds to a program state, which should contain the control of the halt qubit. When the initial data state is $|A\rangle + |B\rangle$, for example, the state of total system starts with $|Q_{x.n}|0\rangle|0_h\rangle(|A\rangle + |B\rangle)\mathcal{P}(|A\rangle + |B\rangle, U, \epsilon)$, rather than $|Q_{x.n}|0\rangle|0_h\rangle|A\rangle\mathcal{P}(|A\rangle, U, \epsilon) + |Q_{x.n}|0\rangle|0_h\rangle|B\rangle\mathcal{P}(|B\rangle, U, \epsilon)$. So if Deutsch’s UQTM really exists and the halting scheme is really valid, Myers’ problem would not occur.

However, Myers’ problem does occur if one makes a further requirement that the program
is only dependent on the desired transformation $U$ and the accuracy $\epsilon$, while independent of the initial data state. Note that this requirement alone does not require that there necessarily exists a program for any desired transformation $U$. In this case, to compute on a data state $|A\rangle + |B\rangle$, the system starts with $|Q_x,n\rangle|0\rangle_h(|A\rangle + |B\rangle)|P(U,\epsilon)\rangle$, which is equal to $|Q_x,n\rangle|0\rangle_h|A\rangle|P(U,\epsilon)\rangle + |Q_x,n\rangle|0\rangle_h|B\rangle|P(U,\epsilon)\rangle$, hence the entanglement between the halt qubit and the other qubits appears if the execution time is different for $|A\rangle$ and $|B\rangle$.

Obviously the problem would be avoided if the execution time is independent of the initial data state. However, this is unreasonable, since the number of the data qubits is arbitrary.

Our discussions above are from the point of view of whether Myers’ problem appears or not if Deutsch’s original claims are really valid. Ozawa considered this issue from a different angle [7,8]. Based on a re-formulation of halting scheme, he argued that Myers’ problem does not matter since monitoring halt qubit does not change the probability distribution of the output. As explained above, we are still concerned with whether UQTM exists and whether the original halting scheme is valid. Ozawa’s approach is not useful for this question [9]. If, on the other hand, Deutsch’s constructions of UQTM and the halting scheme are indeed valid, then Myers’ problem, as discussed by Ozawa, does not occur. If Deutsch’s UQTM exists and the halting scheme is valid, and if the program is required to be data-independent, Ozawa’s result still does not solve the problem. This is because in presence of the entanglement with the halt qubit, no matter whether the halt qubit is monitored or not, the computational state is effectively a mixed one and is not the the pure state designed in the algorithm. This is analogous to the situation in a double-slit experiment where the which-slit information destroys the interference [10]. We also note that the halting scheme does not necessarily require that the computational result, i.e. $|D'\rangle$ in Eq. (2), remains unchanged under the next execution of $U$ on the computer, although such a requirement would be a nice one. This is because in principle one could measure the halt qubit at each step, and stop the unitary evolution and make measurement immediately when the halt bit flips.

On the other hand, for any QTM, one may regard avoiding the entanglement between the halt qubit and computational qubits as a requirement on the dynamics $U$ validating this
QTM. One can thus see that in general a same computation with and without a halt qubit correspond to different $U$. For a particular quantum computer, it is specifically *devised* in the algorithm, which tells how to construct the computer, such that there is no need for a halt qubit. If and only if we consider universal quantum computer, the halting problem is inevitable.

**7. UNIVERSAL QUANTUM CIRCUIT**

In a quantum circuit, a gate array unitarily transforms a collection of qubits from an input state to an output state. In a particular, i.e. non-universal, quantum circuit, as the subject of most quantum computing researches, the desired unitary transformation is decomposed into operations of the gates. A few types, or even a single type, of gate, by repeated use in a repertoire, as prescribed by an algorithm, can generate the action of any unitary transformation $[[[]]]$. This is another, perhaps currently more popular, meaning of the word “universality”, different from what it means in our discussions in this paper.

The total effect of a quantum circuit is simply

$$G|\Psi\rangle_{input} = |\Psi\rangle_{output},$$

(15)

where $G$ is the unitary transformation, effected by the array of gates.

Nielsen and Chuang discussed a universal quantum circuit $[[[]]]$, which, with a fixed gate array, should perform an arbitrary unitary transformation $U$ on a data register $|D\rangle$, by encoding the program $|P(U)\rangle$ as another part of the input state. The evolution is

$$G(|D\rangle|P(U)\rangle) = |UD\rangle|P'(U)\rangle.$$  

(16)

However, it was immediately found that such a deterministic universal circuit does not exist, since the program states for distinct desired transformations are mutually orthogonal, consequently the number of program qubits is infinite for any finite number of data qubits.

Two remarks are in order. First, in the construction and the disproof of Nielson and Chuang, the program state is assumed to be dependent on the desired transformation $U$, but
independent of the data state. If this assumption is dropped, the proof of the orthogonality of program states for distinct desired transformations is nullified. Eq. (16) is changed to

\[ G (|D\rangle|P(U, D)) = |UD\rangle|P'(U, D)\].

(17)

For \(m\) data qubits, \(2^{m+1} - 1\) real numbers are needed to parametrize the data state, and \(2^m\) real numbers are needed to parametrize a unitary transformation on the data state. Hence \(2^m(2^{m+1} - 1)\) real numbers are needed to parametrize the program state, if the program is dependent on the data state. So a universal gate array with data-dependent program has not been ruled out. One may also allow a finite accuracy \(\epsilon\), i.e.,

\[ G (|D\rangle|P(U, D, \epsilon)) = |D'\rangle|P'(U, D, \epsilon)\],

(18)

with \(||D'\rangle - |UD\rangle|^2 < \epsilon\).

The second remark concerns the connection between the universal quantum circuit and the UQTM. If a UQTM exists, one can construct a fixed gate array such that its effect \(G\) corresponds to each step \(U\) of the UQTM. Then by repeating the operation of the same fixed gate array, one can simulate the operation of a UQTM. The qubits, simulating those of a UQTM, are specified as several registers, corresponding respectively to the processor state, the halt qubit, the data, the program, and maybe some other qubits. After each run of the same gate array, the output is taken as the input of the next run. The cycle continues until the halt qubit flips, and a measurement is then made on the data state. A flow chart is shown in Fig. 1. We may still call this construction a universal quantum circuit, though it acquires a different meaning from the conventional one.

8. SUMMARY

To summarize, we have discussed several issues regarding quantum Turing machine, universal quantum Turing machine, and programmable quantum gate array.

First, we note that for Deutsch’s construction of a universal quantum Turing machine, neither the claimed existence of a program effecting an arbitrary desired unitary transfor-
mation on the data state nor the validity of the concatenation scheme, of which the halting scheme is a special case, has really been proved. Second, we note that if Deutsch’s constructions of universal quantum Turing machine and halting scheme are really valid, then the entanglement between the halt qubit and computational qubits, as observed by Myers, can be avoided if the program is allowed to be dependent on the data state. If, however, the program is data-independent, the problem appears no matter whether the halt qubit is monitored or not. Finally, we note that in the programmable quantum gate array discussed by Nielson and Chuang, if the program state is allowed to be dependent on the data state, the disproof of the deterministic programmable gate array is nullified. Thus the existence of a quantum programmable gate array with data-dependent program has not been ruled out. Furthermore, if a universal quantum Turing machine exists, one can simulate it by running a fixed gate array in a cycling way. In conclusion, it is interesting to make further investigations on models of universal quantum computer with data-dependent programs.

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FIGURES

FIG. 1. Flow chart of using a fixed gate array to simulate the UQTM, if the latter exists. One run of the fixed gate array, $G$, corresponds to one step of the UQTM. The cycle continues till the halt qubit flips. $|D\rangle$ is the input data state, $U$ is an arbitrary desired unitary transformation on the data state, $\epsilon$ is an arbitrary accuracy, $|\mathcal{P}(D, U, \epsilon)\rangle$ is the corresponding program. $||D'\rangle - U|D\rangle|^2 < \epsilon$. $|Q(x, n)\rangle$ and $|Q'(x, n)\rangle$ are states of the processor, including the head position, of the simulated UQTM. $|\Sigma\rangle$ represents possible other qubits.

\[
|\Psi\rangle_{input} = |\Psi\rangle_0 = |Q(x, n)\rangle|0_h\rangle|D\rangle|\mathcal{P}(D, U, \epsilon)\rangle|\Sigma\rangle
\]

i ← i + 1, $|\Psi\rangle_i = G|\Psi\rangle_{i+1}$

Does $n_h = 1$?

\[
|\Psi\rangle_{output} = |Q'(x, n)\rangle|1_h\rangle|D'\rangle|\mathcal{P}'(D, U, \epsilon)\rangle|\Sigma\rangle
\]
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* Email: ys219@cam.ac.uk

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