SCALAR-TENSOR DARK ENERGY MODELS

R. GANNOUJI, D. POLARSKI, A. RANQUET

Lab. de Physique Théorique et Astroparticules, CNRS
Université Montpellier II, 34095 Montpellier Cedex 05, France

A. A. STAROBINSKY

Landau Institute for Theoretical Physics, Moscow, 119334, Russia

We present here some recent results concerning scalar-tensor Dark Energy models. These models are very interesting in many respects: they allow for a consistent phantom phase, the growth of matter perturbations is modified. Using a systematic expansion of the theory at low redshifts, we relate the possibility to have phantom like DE to solar system constraints.

1. Introduction

The late-time accelerated expansion of the universe is a major challenge for cosmology. The component producing this acceleration accounts for about two thirds of the total energy density. While this has gradually become a building block of our present understanding, the nature of Dark Energy (DE) still remains mysterious.1–3 The simplest solution is a cosmological constant \( \Lambda \). A major contender is Quintessence, a minimally coupled scalar field (with canonical kinetic term). We will consider scalar-tensor (ST) DE models, a more elaborate alternative involving a new physical degree of freedom, the scalar partner \( \phi \) of the graviton responsible for a modification of gravity.4–6 It is not clear yet whether some modification of gravity is required or even preferred in order to explain the bulk of data. The increasing accuracy of the data, should allow to severely constrain the various viable models. ST DE models allow for phantom DE, \( w_{DE} < -1 \), moreover the equation for the growth of matter perturbations is modified.4 We will review here results concerning their low \( z \) behaviour, in particular how the DE equation of state is related to solar system constraints.9

2. Scalar-tensor DE models

We consider the microscopic Lagrangian density in the Jordan frame

\[
L = \frac{1}{2} \left( F(\Phi) \, R - Z(\Phi) \, g^{\mu\nu} \partial_\mu \Phi \partial_\nu \Phi \right) - U(\Phi) + L_m(g_{\mu\nu}) . \tag{1}
\]

We define what we mean by the energy density \( \rho_{DE} \) and the pressure \( p_{DE} \) by writing the gravitational equations in the following Einsteinian form:

\[
3F_0 \, H^2 = \rho_m + \rho_{DE} \tag{2}
\]

\[
-2F_0 \, \dot{H} = \rho_m + \rho_{DE} + p_{DE} . \tag{3}
\]

This can be seen as the Einsteinian form, with constant \( G_0 = G_N(t_0) = F_0^{-1} \), of the gravitational equations of ST gravity. With these definitions, the usual conservation
equation applies, and the equation of state parameter \( w_{DE} \equiv \frac{p_{DE}}{\rho_{DE}} \) plays its usual role. Using (2,3), one gets \( w_{DE}(z) \) from the observations through

\[
\frac{dh^2}{dz} = 3 \Omega_{m,0} (1 + z)^2 + 2 \Omega_{k,0} (1 + z) .
\]

if we allow for a nonzero spatial curvature and \( \Omega_m \equiv \frac{\rho_m}{3H_0^2} \).

Looking at the equations above, everything looks the same as in GR, ST gravity is hidden in the definitions of \( \rho_{DE}, p_{DE} \), and the various \( \Omega \)'s. The condition for DE to be of the phantom type, \( w_{DE} < -1 \), reads

\[
\frac{dh^2}{dz} < 3 \Omega_{m,0} (1 + z)^2 + 2 \Omega_{k,0} (1 + z) .
\]

in the presence of spatial curvature. As first emphasized, the weak energy condition for DE can be violated in scalar-tensor gravity (see also 8).

3. General low \( z \) expansion of the theory

We investigate now the low \( z \) behaviour of the model and the possibility to have phantom boundary crossing in a recent epoch. For each solution \( H(z), \Phi(z) \), the basic microscopic functions \( F(\Phi) \) and \( U(\Phi) \) can be expressed as functions of \( z \) and expanded into Taylor series in \( z \):

\[
\frac{F(z)}{F_0} = 1 + F_1 z + F_2 z^2 + ... > 0 , \tag{6}
\]

\[
\frac{U(z)}{3H_0^2} \equiv \Omega_{U,0} u = \Omega_{U,0} + u_1 z + u_2 z^2 + ... . \tag{7}
\]

From (6,7), all other expansions can be derived, in particular:

\[
w_{DE}(z) = w_0 + w_1 z + w_2 z^2 + ... , \tag{8}
\]

\[
H_0^{-1} \frac{\dot{G}_{eff}}{G_{eff}} = g_0 + g_1 z + g_2 z^2 + .... . \tag{9}
\]

A viable ST gravity model must be very close to General Relativity, viz.

\[
\omega_{BD,0} = \frac{6(\Omega_{DE,0} - \Omega_{U,0} - F_1)}{F_1^2} = \frac{\Delta^2}{F_1^2} > 4 \times 10^4 , \tag{10}
\]

with \( \Delta^2 \equiv 6(\Omega_{DE,0} - \Omega_{U,0} - F_1) \). Therefore, we must have \( |F_1| < 1 \) and \( \Delta^2 \approx 6(\Omega_{DE,0} - \Omega_{U,0}) > 0 \). Moreover, for positive \( U, \Delta^2 < 6\Omega_{DE,0} < 5 \)

\[
|F_1| < \left( \frac{5}{\omega_{BD,0}} \right)^{1/2} \lesssim 10^{-2} . \tag{11}
\]

It can be shown that the condition \( |F_1| \ll 1 \) is sufficient to ensure here that solar system constraints are satisfied.\textsuperscript{9}

We now specialize to the case \( |F_1| \ll 1 \) yet assuming that other \( F_i \) are not as small. Then all expansions simplify considerably and we have in particular,

\[
1 + w_0 \simeq \frac{2F_2 + 6(\Omega_{DE,0} - \Omega_{U,0})}{3\Omega_{DE,0}} . \tag{12}
\]
From (12), the necessary condition to have phantom DE today reads

\[
\frac{d^2 F}{d\Phi^2} \bigg|_0 = \frac{F_2}{3 (\Omega_{DE,0} - \Omega_{U,0})} < -1.
\]

(13)

Hence \( F_2 < 0 \) is necessary for phantom DE, because \( \Omega_{DE,0} - \Omega_{U,0} > 0 \) from \( \Delta^2 > 0 \).

In addition significant phantom DE requires \(|F_2| \sim 1\). If \(|F_1| \sim |F_2| \ll 1\), the present phantomness is very small.

It is actually possible to invert all expansions and to obtain all coefficients in function of the post-Newtonian parameters \( \gamma, \beta \) and \( g_0 \). The following results are finally obtained

\[
F_1 = g_0 \frac{\gamma - 1}{\gamma - 1 - 4(\beta - 1)}
\]

(14)

\[
F_2 = -2 g_0^2 \frac{\beta - 1}{[\gamma - 1 - 4(\beta - 1)]^2}
\]

(15)

\[
\Omega_{DE,0} - \Omega_{U,0} = -\frac{1}{6} g_0^2 \frac{\gamma - 1}{[\gamma - 1 - 4(\beta - 1)]^2}
\]

(16)

\[
1 + w_{DE,0} = -\frac{1}{3} g_0^2 \frac{4(\beta - 1) + \gamma - 1}{\Omega_{DE,0} [\gamma - 1 - 4(\beta - 1)]^2}
\]

(17)

The best present bounds are \( \gamma_{PN} - 1 = (2.1 \pm 2.3) \cdot 10^{-5} \), \( \beta_{PN} - 1 = (0 \pm 1) \cdot 10^{-4} \). \( \frac{\dot{H}_{at,0}}{\rho_{at,0}} = (-0.2\pm 0.5) \cdot 10^{-13} \text{y}^{-1} \). Though possible in principle, the interesting possibility to test phantomness in the solar system is very hard while its amount depends critically on the small quantity \( g_0^2 \). In this respect cosmological data are certainly better suited, a conclusion reminiscent of that reached in\(^6\) concerning the viability of ST DE models with vanishing potential.

References

1. V. Sahni and A. A. Starobinsky, Int. J. Mod. Phys. D 9, 373 (2000).
2. T. Padmanabhan, Phys. Rep. 380, 235 (2003).
3. V. Sahni, astro-ph/0502032 (2005); E. J. Copeland, M. Sami and S. Tsujikawa, Int. J. Mod. Phys. D 15, 1753 (2006).
4. B. Boisseau, G. Esposito-Farèse, D. Polarski and A. A. Starobinsky, Phys. Rev. Lett. 85, 2236 (2000).
5. Y. Fujii, Phys. Rev. D 62, 044011 (2000); N. Bartolo and M. Pietroni, Phys. Rev. D 61 023518 (2000); F. Perrotta, C. Baccigalupi and S. Matarrese, Phys. Rev. D 61, 023507 (2000).
6. G. Esposito-Farèse and D. Polarski, Phys. Rev. D 63, 063504 (2001).
7. D. Polarski and A. Ranquet, Phys. Lett. B 627, 1 (2005).
8. D. Torres, Phys. Rev. D 66, 043522 (2002).
9. R. Gannouji, D. Polarski, A. Ranquet, A. A. Starobinsky, JCAP 0609, 016 (2006).
10. J. Martin, C. Schimd and J.-P. Uzan, Phys. Rev. Lett. 96, 061303 (2006).