Decay Anisotropy of $e^+e^-$ Sources from $pN$ and $pd$ collisions *

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Abstract

A full calculation of lepton-pair angular characteristics is carried out for $e^+e^-$ pairs created in $pp$, $pn$ and $pd$ collisions at intermediate energies. It is demonstrated that the proposed new observable, the dilepton decay anisotropy, quite sensitively changes for different sources and may be useful for their disentangling. The relevance of the dilepton decay anisotropy is shown in the context of a puzzling energy behavior for the ratio of the lepton yield from $pd$ to $pp$ reactions as observed at the BEVALAC.

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As is known since a long time dileptons are quite attractive probes since they provide almost direct information on hot and dense nuclear matter during its evolution in heavy-ion collisions at BEVALAC/SIS and SPS energies [1]. The information carried out by leptons may tell us not only about the interaction dynamics of colliding nuclei, but also on properties of hadrons in the nuclear environment or on a possible phase transition of hadrons into a quark-gluon plasma.

However, there are a lot of hadronic sources for dileptons because the electromagnetic field couples to all charges and magnetic moments. In particular, in hadron-hadron collisions, the $e^+e^-$ pairs are created due to the electromagnetic decay of time-like virtual photons which can result from the bremsstrahlung process or from the decay of baryonic and mesonic resonances including the direct conversion of vector mesons into virtual photons in accordance with the vector dominance hypothesis. In the nuclear medium, the properties of these sources may be modified and it is thus very desirable to have experimental observables which would allow to disentangle the various channels of dilepton production.

For a decomposition of $e^+e^-$ sources contributing to the dilepton invariant mass spectra it seems to be natural to start from the study of elementary nucleon-nucleon collisions and then to move successively to nucleon-deuteron and more complicated systems. A step in this direction has been done in Ref. [2] where the $pd/pp$ dielectron ratio was measured in the 1−5 GeV energy range. In contrast to naive expectations, this $pd/pp$ ratio displays a puzzling beam-energy dependence which is still a matter of debate. There are several interpretations of this effect: the interferences of different channels [3], the contribution of the inelasticity effects [4], and of the subthreshold $\eta$-meson production [5]. Even by considering the new and more precise data on the transverse momentum distributions of lepton pairs [2] the puzzle could not be resolved unambiguously.

Recently, we have proposed to use lepton pair angular distributions for a distinction between different sources [6, 7]. Indeed, the coupling of a virtual photon to hadrons induces a dynamical spin alignment of both the resonances and the virtual photons. One thus can expect that the angular distribution of a lepton will be anisotropic with respect to the direction of the dilepton (i.e. virtual photon) emission. It has been shown that due to the spin alignment of the virtual photon and the spins of colliding or decaying hadrons, this lepton decay anisotropy turns out to be quite sensitive to the specific production
Following our previous work [3, 4] we focus in this letter on the study of the dilepton angular anisotropy as a way to disentangle various $e^+e^-$ sources in $pN$ and $pd$ reactions. Alongside with an extended computation of the Dalitz decay channel for pseudoscalar mesons, we present the first full results for nucleon-nucleon collisions at energies of a few GeV. By a simple extension of these calculations to the nucleon-deuteron case we will investigate if the anisotropy effect can help in solving the $pd/pp$ dilepton ratio puzzle.

As in [3, 4] we choose the polar $\theta$ and azimuthal $\varphi$ angles of the momentum $\vec{l}$ of a created electron with respect to the momentum $\vec{q}$ of a virtual photon to characterize the decay anisotropy, where $\vec{l}_-,\vec{l}_+$ are measured in the rest frame of this virtual photon, i.e. $\vec{q} \equiv \vec{l}_- + \vec{l}_+ = 0$. For comparing the shape of the angular distributions for different channels, the differential cross section for dilepton production in the channel $i$ may be presented in the form:

$$S_i(M, \theta) \equiv \frac{d\sigma_i}{dMd\cos\theta} = A_i(1 + B_i \cos^2 \theta),$$

where $M$ is the invariant mass of a lepton pair ($M^2 = q_0^2 - \vec{q}^2$) and $A_i$ is defined by the normalization to unity of the angular distribution of $\vec{q}$. The decay anisotropy coefficient for channel $i$, i.e. $B_i$, then is given by

$$B_i = \frac{S_i(M, \theta = 0^\circ)}{S_i(M, \theta = 90^\circ)} - 1. \quad (2)$$

Since the coefficient $B_i$ is sensitive to the spin structure of the interacting hadrons, it is in general a function of $M$ and the masses of the hadrons involved in the reaction.

The total differential cross section for proton–nucleon collisions can be represented as a sum of differential cross sections for all channels:

$$\frac{d\sigma^{pN}}{dMd\cos\theta} = \sum_{i=\text{channel}} \frac{d\sigma_i}{dMd\cos\theta} = A(pN)(1 + B^{pN}\cos^2 \theta). \quad (3)$$

For the total anisotropy coefficient we then get

$$B^{pN} = \sum_{i=\text{channel}} <B_i^{pN}>, \quad <B_i^{pN}> = \frac{\int d\sigma_i dM \cdot \frac{B_i}{1 + \frac{1}{3}B_i}}{\sum_i d\sigma_i dM \cdot \frac{1}{1 + \frac{1}{3}B_i}}. \quad (4)$$

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where the special weighting factors originate in the necessary angle-integrations. Thus, the anisotropy coefficient \( B^{pN} \) is the sum of the “weighted” anisotropy coefficients \( \langle B_i \rangle \) for each channel \( i \) obtained by means of the convolution of \( B_i \) with the corresponding invariant mass distribution.

In a preceding study \[7\] the anisotropy coefficients were calculated for the bremsstrahlung and \( \Delta \)–Dalitz decay channels within a microscopic One-Boson-Exchange (OBE) model \[3\]. We will use the latter results in the present investigation without any modification. Note that we take into account the interference terms between bremsstrahlung and \( \Delta \)–channels for proton–nucleon and proton–proton interactions. As a result, the contributions of these two channels to \( (1) \) are reduced to a single coherent term.

For the Dalitz decay of an \( \eta \)-meson in Ref. \[6\] a first simple estimate \( B_\eta = 1 \) had been made appropriate for an \( \eta \)-meson with vanishing momentum in the center-of-mass system (cms) of the colliding hadrons. In this case the direction of a virtual photon \( \vec{q}_\eta \) is not influenced by the \( \eta \)-meson momentum \( \vec{P}_\eta \) which simplifies the kinematical considerations. In extension to \[6\] we now include the full dynamics, \( \vec{P}_\eta \neq 0 \), where the direction of the dilepton momentum in the cms of the colliding nucleons, \( \vec{q} \), does not coincide with that in the eta rest frame \( \vec{q}_\eta \); as a consequence the anisotropy coefficient becomes a function of the invariant mass \( M \) and the collision energy \( T_{lab} \).

To take this functional dependence into account the \( \eta \)–channel for the lepton differential cross section \( (1) \) is written in the following form:

\[
S_\eta(M, \theta) = \int d\vec{P}_\eta \frac{d\sigma_\eta}{d\vec{P}_\eta} |T(M, \theta, \vec{P}_\eta)|^2,
\]

where the averaged transition matrix element of the \( \eta \)–decay is given by

\[
|T(M, \theta, \vec{P}_\eta)|^2 \sim \int dV \frac{1}{M^4} \left( \epsilon_{\mu\alpha\beta\gamma} q_\beta q_1 \gamma \epsilon_{\nu\alpha\rho\sigma} q_\rho q_1 \sigma \right) \cdot L_{\mu\nu},
\]

and \( q, q_1 \) are the four momenta of a virtual and real photon, respectively. The integral is taken over the available phase-space volume. The lepton tensor \( L_{\mu\nu} \) is defined as

\[
L_{\mu\nu} = \text{Tr} \hat{l}_-\gamma_\mu \hat{l}_+\gamma_\nu,
\]

where \( \hat{l} = \gamma_\sigma l^\sigma \), \( l_- \), \( l_+ \) are the four momenta of the leptons.

For the \( \eta \) production cross section \( d\sigma_\eta/d\vec{P}_\eta \) we use the phase-space oriented expression:

\[
\frac{d\sigma_\eta}{d\vec{P}_\eta} \approx \frac{1}{E_\eta} \frac{\sqrt{M_x^2 - 4m_N^2}}{M_x} T_\eta^2(P_\eta),
\]

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with \( M_z^2 = (p_a + p_b - P_\eta)^2 = s - 2\sqrt{s}E_\eta + m_\eta^2 \). Here \( s = (p_a + p_b)^2 \) is the center-of-mass energy squared and \( p_a, p_b, P_\eta \) are the four-momenta of the colliding nucleons and the \( \eta \)-meson, respectively, while \( E_\eta \) is the energy of the \( \eta \)-meson in the cms of the colliding nucleons. It is noteworthy that the particular case considered in Ref. [6] corresponds formally to the substitution \( d\sigma/\tilde{P}_\eta \sim \delta(\tilde{P}_\eta) \). The modification of the phase-space formula (8) is included in the term \( T_\eta^2(P_\eta) \), which is the production matrix element squared. Here we use a simple parametrization of the calculations presented in Ref. [9], where the \( \eta \)-production cross section in nucleon–nucleon collisions was calculated on the basis of an effective OBE model.

Results of our numerical calculations for \( B_\eta \) at initial energies from the \( \eta \) production threshold to about 5 GeV are shown in Fig. 1. We find that the anisotropy coefficient indeed depends on both the initial energy \( E \) and the dilepton mass \( M \). \( B_\eta \) approaches 1 only close to threshold or for \( M \to 0 \). This behavior is quite natural because at the threshold energy we have \( |\tilde{P}_\eta| \approx 0 \); for low mass dileptons their velocity is close to the velocity of light and thus the difference between the two directions discussed above becomes negligible. If the invariant mass of a virtual photon is close to its kinematical limit (\( M \to m_\eta \)), the anisotropy coefficient approaches a lower limit of about \( 0.1 \div 0.25 \) for bombarding energies from 1.26 to 5 GeV.

In Fig. 2 the dilepton invariant mass distributions \( d\sigma/dM \) from \( pp \) and \( pn \) interactions are shown at bombarding energies of 1.26 GeV and 2.1 GeV, which are the necessary ingredients for calculating the weighted anisotropy coefficients, Eq. (9). In Fig. 2 the “\( \eta \)” denotes the contribution of the \( \eta \)–channel the “\( \Delta \)” labels the contribution of the \( \Delta \)–resonance term, while “\( Br \)” denotes the bremsstrahlung channel. The dashed curves are the sum of bremsstrahlung and \( \Delta \)–channel with interference.

Using the \( pN \) cross sections for \( \Delta \)–production and bremsstrahlung channels from Ref. [3], we are now in the position to present the first estimate for the weighted coefficients \( \langle B_i(M) \rangle \) for \( pp \) and \( pn \) collisions at the energies of 1.26 and 2.1 GeV (Fig. 3). The notation is the same as in Fig. 2. The solid line denoted by “\( all \)” represents the sum of bremsstrahlung, \( \Delta \)–channel with interference and \( \eta \)–decay source. At the energy of 1.26 GeV the anisotropy is determined by the \( \Delta \)–decay since its contribution is dominating at this energy (cf. Fig. 2). The large contribution of the \( \eta \)–source at 2.1 GeV leads to the specific structure in \( \langle B_i(M) \rangle \) for \( M \leq 0.3 \) GeV.
As mentioned above, a possible explanation for the experimental $pd/pp$ dilepton ratio is related to the contribution of leptons from $\eta$–decay \cite{5}. Whereas the energy threshold of $\eta$–production in $pN$ interactions is about 1.26 GeV, in the proton-deuteron case the effective threshold is lower due to the Fermi motion of the constituent nucleons in the deuteron. Thus at projectile energies below 1.26 GeV the contribution from the $\eta$–Dalitz decay to the dilepton spectrum may be significant for the $pd$ case, but negligible for $pp$. Such an enhancement of the $\eta$–yield in $pd$ compared to $pp$ reactions has been also seen experimentally by the PINOT collaboration \cite{10}. In line with this argument we expect larger values for the $pd/pp$ dilepton ratio at low energy, and according to the calculations above one can expect that subthreshold $\eta$–production may manifest itself also in the angular characteristics (anisotropy coefficients) of the created dileptons.

In a first order approximation the decay anisotropy coefficient for $pd$ collisions may be written as the sum of $B_i$ for $pp$ and $pn$ interactions. However, this approximation is not sufficient in the energy region close to the $\eta$–threshold. To take the contribution to the dilepton spectrum from near-threshold $\eta$–production into account we calculate the coefficient $B^{pd}$ as

$$B^{pd} = \sum_{k=p,n} < B^{kN}_{Br,\Delta} > + < B^{pd}_{\eta} >,$$

$$< B^{pd}_{\eta} > = \frac{d\sigma^{pd}_{\eta}}{dM} \cdot \frac{B^{pN}_{\eta}}{1 + \frac{1}{3} B^{pN}_{\eta}} - \frac{1}{\sum_{k=p,n} \frac{d\sigma^{kN}_{Br,\Delta}}{dM} \cdot \frac{1}{1 + \frac{1}{3} B^{kN}_{Br,\Delta}} + \frac{d\sigma^{pd}_{\eta}}{dM} \cdot \frac{1}{1 + \frac{1}{3} B^{pN}_{\eta}}}.$$

(9)

It should be stressed that in Eqs. (9), (10) we take into account the interference between bremsstrahlung and $\Delta$–channel for $pp$ and $pn$ separately.

Fig. 4 shows the resulting weighted anisotropy coefficients $< B_i(M) >$ for $pd$ collisions at 1.26 and 2.1 GeV. The cross section $d\sigma^{pd}/dM$ is taken from Ref. \cite{5} and the Paris deuteron wave function \cite{11} is used for describing the internal nucleon distribution in the deuteron. As expected, at threshold energy the total $< B_{all}(M) >$ coefficient for the $pd$ reaction differs substantially from that for $pp$ or $pn$ reactions due to the near threshold $\eta$–decay source. For higher energies (2.1 GeV) the shape of $< B_{all}(M) >$ is similar in all these cases. This energy-dependent effect is more pronounced in Fig. 5 where the ratio of the anisotropy coefficients for $pd$ to $pp$ reactions, $R_B = B^{pd}/B^{pp}$, is represented at
the energies of 1.26 and 2.1 GeV. The comparison of the solid (including the \( \eta \)--Dalitz decay) and dashed curves (without the \( \eta \)--channel) illustrates the relative importance of the \( \eta \)--decay contribution to the observable decay anisotropy of the dileptons.

Thus, summarizing, the calculated anisotropy coefficients for \( pp \), \( pn \) and \( pd \) reactions support our suggestion in Refs. [6, 7] that the dilepton decay anisotropy may serve as an additional observable to discriminate the dilepton sources by experimental means.

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Figure captions

Figure 1: The anisotropy coefficient $B_\eta$ for the $\eta$–channel at initial energies from 1.26 to 4.9 GeV.

Figure 2: The cross sections $d\sigma/dM$ for $pp$ and $pn$ interactions at 1.26 and 2.1 GeV bombarding energies. The “$\eta$” denotes the contribution of the $\eta$–channel, the “$\Delta$” labels the contribution of the $\Delta$–resonance term, while “$Br$” denotes the bremsstrahlung channel. The dashed curves are the sum of bremsstrahlung and $\Delta$–channel with interference.

Figure 3: The weighted anisotropy coefficients $< B_i(M) >$ for $pp$ and $pn$ collisions at 1.26 and 2.1 GeV. The notation is the same as in Fig. 2. The solid line denoted by “all” represents the sum of bremsstrahlung, $\Delta$–channel with interference and $\eta$–decay contributions.

Figure 4: The weighted anisotropy coefficients $< B_i(M) >$ for $pd$ interactions at 1.26 and 2.1 GeV. The notation is the same as in Fig. 3.

Figure 5: The ratio of anisotropy coefficients for $pd$ to $pp$ reactions at energies of 1.26 and 2.1 GeV. The solid lines correspond to calculations taking into account the $\eta$–decay contribution while the dashed lines do not include the $\eta$–Dalitz decay.
$B_\eta$ vs. $M$ (GeV) for different values of $E$ (GeV):

- $E = 4.9$ GeV
- $E = 2.1$ GeV
- $E = 1.6$ GeV
- $E = 1.4$ GeV
- $E = 1.3$ GeV
- $E = 1.26$ GeV

Fig. 1
Fig. 2
Fig. 3
