All-electrical detection of spin Hall effect in semiconductors

Markus Ehlert*,1, Cheng Song1,2, Mariusz Ciorga1, Thomas Hupfauer1, Junichi Shiogai1,3, Martin Utz1, Dieter Schuh1, Dominique Bougeard1, and Dieter Weiss*,1

1 Institute for Experimental and Applied Physics, University of Regensburg, 93040 Regensburg, Germany
2 Key Laboratory of Advanced Materials (MOE), School of Materials Science and Engineering, Tsinghua University, 100084 Beijing, P. R. China
3 Department of Materials Science, Tohoku University, 980-8579 Sendai, Japan

Received 15 September 2013, revised 19 December 2013, accepted 23 December 2013
Published online 10 February 2014

Keywords gallium arsenide, nonlocal resistance, spin Hall effect, spin injection

* Corresponding author: e-mail markus.ehlert@physik.uni-regensburg.de, Phone: +49 941 943 3198, Fax: +49 941 943 3196
** e-mail dieter.weiss@physik.uni-regensburg.de

This is an open access article under the terms of the Creative Commons Attribution License, which permits use, distribution and reproduction in any medium, provided the original work is properly cited.

Since the prediction of the spin Hall effect more than 40 years ago, significant progress was made in theoretical description as well as in experimental observation, especially in the last decade. In this article, we present three different concepts and measurement geometries for all-electrical detection of the direct and the inverse spin Hall effect in semiconductors. Based on experiments with n- and p-doped GaAs microstructures, we describe our experimental approaches and methods to experimentally identify the spin Hall effect and compare our results to previous experiments and theoretical considerations.

1 Introduction The spin Hall effect (SHE) [1], initially predicted in 1971 by theorists D’yakonov and Perel [2, 3], describes the generation of a spin current perpendicular to a spin-unpolarized charge current in the presence of spin–orbit coupling, which is also referred to as direct spin Hall effect (DSHE). The inverse spin Hall effect (ISHE) [1], that is when a spin current induces a transverse charge current, is mathematically equivalent to the SHE due to Onsager symmetry relations. In the original proposal [2, 3] an impurity mechanism was suggested, where spin-dependent Mott scattering on unpolarized impurities leads to a spatial separation of charge carriers with opposite spins [1, 4]. More recent studies [5, 6] also consider the existence of an intrinsic mechanism related to band structure properties. Accompanied by numerous theoretical studies (e.g., [7, 8]), a number of spin Hall devices has been realized so far in various materials (see [9]). However, most of the reported experiments on semiconductors rely on optical techniques (e.g., [10–15]) or on ferromagnetic (FM) resonance [16], which are employed for either detecting a spin imbalance (DSHE experiments) or generating a spin current (ISHE experiments). A fully electrical realization of SHE semiconductor devices requires to control electrical spin injection and detection, and has been reported only very recently [17–19]. In this feature article, we will give an overview on the all-electrical detection of SHE in semiconductor devices, since the SHE offers plenty of potential applications in the large field of semiconductor spintronics, e.g., using the SHE as a spin source (DSHE) or as a spin-detector (ISHE). In the following, we will describe three different types of SHE device concepts in detail.

The first presented device allows for measurements of the direct spin Hall effect. A spin-unpolarized charge current flows through a n-GaAs channel and induces, due to DSHE, a transverse spin current. Hence, spins accumulate at the...
edges of the channel and are probed by spin-sensitive contacts [19]. We then proceed with inverse spin Hall effect experiments. In the first ISHE device, we used spin-injecting contacts to generate a spin current, which, via ISHE, should lead to a measurable charge imbalance in a Hall bar geometry [18, 20]. The second device consists of the so-called H-bar geometry, where an electric current is driven in one leg of the H-shaped structure. This generates, due to DSHE, a transverse spin current, which flows along the connection between both legs of the “H”. Due to ISHE, this spin current induces a charge imbalance in the second leg of the structure [21], as it was experimentally observed by Brüne et al. [22] in HgTe quantum wells.

2 Detection of direct spin Hall effect in GaAs

2.1 Overview In this first section, we present experiments [19] where we probed the direct spin Hall effect in n-doped GaAs devices with (Ga,Mn)As/GaAs Esaki diodes. An important prerequisite for the appearance of the SHE is the presence of spin–orbit fields, which in bulk GaAs are mediated through crystal bulk inversion asymmetry (BIA). In our case, the SHE is of extrinsic nature, this means it is mediated through spin-dependent scattering of charge carriers at impurities [23, 24]. The device geometry, which we used in our experiments, is shown in Fig. 1a. We pass a spin-unpolarized current \( j_x \) through a bulk n-GaAs transport channel. By means of DSHE, charge carriers with opposite spins are deflected in opposite directions, thus giving rise to a spin current \( j_y \) perpendicular to \( j_x \). The direction in which the spins are partially polarized is perpendicular to the plane formed by \( j_x \) and \( j_y \). The generated spin current leads then to a spin accumulation of opposite sign at the channel boundaries, which we probed with FM (Ga,Mn)As/GaAs Esaki diode structures [25–27] placed above the transport channel. The Esaki diodes provide high spin detection efficiency [28, 29], allowing for efficient detection of the low-level spin polarization induced by the DSHE.

Due to the spin-charge coupling occurring in the FM material [30], a voltage drop across the contact can be measured, which is proportional to the spin accumulation underneath. Since FM electrodes can detect spin components solely parallel to their own magnetization axis \( M \) (which in our case lies in the plane of the sample, along the \( \pm x \)-direction) and the SHE-induced spin polarization is aligned out-of-plane, we applied an external magnetic field \( B_y \) to induce Hanle spin precession of the out-of-plane spin component. In consequence, the spins acquire an in-plane component, which is then detected by the FM diodes. From our data, we calculated the value of the spin Hall conductivity \( \sigma_{SH} = j_y / E_x \) (\( E_x \) is the electric field along the channel), for which we also investigated the dependence on channel conductivity. This allowed us to determine the contribution of skew-scattering and side-jump to the total spin Hall conductivity. We also compare our results to theoretical predictions [23] and experiments with metallic FM contacts [17].

2.2 Experimental devices We fabricated our devices from a single wafer, which was grown by molecular-beam epitaxy (MBE) on (001) GaAs substrates. Similar wafers were already used for spin injection experiments in Ref. [29]. The wafer consists (in order of growth) of a 1000 nm thick n-type transport channel (\( n \approx 2 \times 10^{16} \) cm\(^{-3}\)), a 15 nm thin \( n \rightarrow n^+ \) GaAs transition layer (\( n^+ \approx 5 \times 10^{18} \) cm\(^{-3}\)), 8 nm \( n^+ \)-GaAs, 2.2 nm low-temperature (LT)-grown...
Al$_{0.36}$Ga$_{0.64}$As, which serves as diffusion barrier, and a 15 nm thick LT-grown layer of Ga$_{0.95}$Mn$_{0.05}$As. The Esaki diode, which we used for spin-detection, is formed by the highly doped (Ga,Mn)As/GaAs pn-junction [25]. Without breaking vacuum, we transferred the wafer into an attached metal-MBE chamber, where 2 nm [14 monolayers (MLs)] of Fe were epitaxially grown at room temperature. The wafer was finally covered with 4 nm (20 MLs) of Au to prevent oxidation of Fe.

Our Hall bar devices were designed by optical lithography, chemically assisted ion beam etching and wet chemical etching. Electron beam lithography was used to pattern the Fe/(Ga,Mn)As spin-detecting contacts along the easy axis of the Fe ([110] direction). The thin Fe layer makes the spin-detecting contacts magnetically harder, assuring that the magnetization stayed aligned along the contacts, when we applied a perpendicular magnetic field $B_x$ to induce spin precession. After defining the spin-detecting contacts, we etched away the top layers to confine the transport exclusively to the low-doped 1000 nm thick n-GaAs channel. The contacts are connected to big bonding pads via Ti/Au paths, which are isolated from the conducting n-GaAs channel by a 50 nm thick layer of Al$_2$O$_3$, deposited by atomic layer deposition (ALD). Figure 1a shows a micrograph image of one of our devices, where a pair of 2.5 $\mu$m wide spin probes (contacts a and b) with a distance $L$ (measured from the center of the contact to the edge of the channel) is placed between a Hall cross (contacts c and d), which we used to monitor the Hall induced background. We designed our devices in such a way, that we could simultaneously perform measurements on structures with the even components of the data, since the contribution of an antisymmetric behavior near $B_x = 0$ T, on the other hand the signal changes its sign for opposite magnetizations ($+x$ and $-x$-direction). This becomes especially obvious in comparison with the simultaneously monitored Hall background $V_{ab}$ (Inset of Fig. 1b), which lacks both features. To eliminate the different background contributions to the raw signal $V_{ab}$, we first subtracted the ordinary Hall background $V_{cd}$. The remaining background was removed by subtracting the curves obtained for two different parallel configurations of the magnetization ($+x$ and $-x$-direction), since the background for both curves is spin-independent (i.e., does not depend on magnetization). We finally removed the even components of the data, since the contribution of an imperfect cancellation of magnetic fringe fields of the FM contacts (which point along the $\pm z$-direction) is even with respect to $B_x$ [17]. Figure 1c shows spin Hall curves for two opposite current directions, for which we removed the background as described above. It can clearly be seen that the sign of the spin Hall signal is changed by reversal of current direction, which is fully consistent with the theory of SHE. We fitted the curves (symbols) with standard Hanle effect equations [28, 31] (solid lines) for the case of perpendicular relative orientation of spins (which in our case originate from DSHE) and the spin-detecting contact. We took the final size of the contacts into account by integrating the spin Hall signal over their width.

Figure 2a–c shows SHE curves (symbols), after the background was removed, together with the corresponding Hanle fits (solid lines) as a function of $B_y$ for different distances $d$ at $T = 4.2$ K and a current density $j_x = 1.7 \times 10^4$ A cm$^{-2}$. The corresponding plot of $\Delta V_{SH}(L)$ versus distance $L$ (Fig. 2d) reveals a spin diffusion length $\lambda_{sd} = 8.5$ $\mu$m. From the Hanle fits, we obtained a similar value for $\lambda_{sd}$ and a spin relaxation time of $\tau_s = 3.5$ ns. The latter value is much smaller than expected for GaAs with such a doping level [32]. Here, we assume that the reason is one of the contacts as spin injector, whereas the other contact of the pair was employed as spin-detecting contact.

### 2.3 Experiments

The spin Hall measurements were performed according to the following procedure. First, we swept an in-plane magnetic field $B_x$ (i.e., along the easy axis of the Fe) to the saturation value of $+0.5$ T and then back to zero field to align the magnetization of the FM contacts in the $+x$-direction. An in-plane magnetic field $B_y$ was then swept from zero to $+0.5$ T to induce a precession of the out-of-plane spin component (which hence acquires an in-plane component). This procedure was then repeated with $B_y$ swept from zero to $-0.5$ T. Typical results of our measurements are shown in Fig. 1b, where we plot the dependence of the voltage $V_{ab}$ (measured between a pair of contacts) on the applied external magnetic field $B_y$ for different initial orientation of the magnetizations ($\pm x$-direction).

Although the raw curves (Fig. 1b) contain a roughly linear background from different contributions, they clearly exhibit spin-related features. On the one hand, one can notice an antisymmetric behavior near $B_y = 0$ T, on the other hand the signal changes its sign for opposite magnetizations ($+x$ and $-x$-direction). This becomes especially obvious in comparison with the simultaneously monitored Hall background $V_{ab}$ (Inset of Fig. 1b), which lacks both features. To eliminate the different background contributions to the raw signal $V_{ab}$, we first subtracted the ordinary Hall background $V_{cd}$. The remaining background was removed by subtracting the curves obtained for two different parallel configurations of the magnetization ($+x$ and $-x$-direction), since the background for both curves is spin-independent (i.e., does not depend on magnetization). We finally removed the even components of the data, since the contribution of an imperfect cancellation of magnetic fringe fields of the FM contacts (which point along the $\pm z$-direction) is even with respect to $B_x$ [17]. Figure 1c shows spin Hall curves for two opposite current directions, for which we removed the background as described above. It can clearly be seen that the sign of the spin Hall signal is changed by reversal of current direction, which is fully consistent with the theory of SHE. We fitted the curves (symbols) with standard Hanle effect equations [28, 31] (solid lines) for the case of perpendicular relative orientation of spins (which in our case originate from DSHE) and the spin-detecting contact. We took the final size of the contacts into account by integrating the spin Hall signal over their width.
the high electric field $E_x$ in the conductive channel ($\sim 120 \text{ V cm}^{-1}$ for measurements shown in Fig. 2), which is expected to drastically decrease spin lifetime above the donor impact ionization threshold of $\sim 10 \text{ V cm}^{-1}$ [33].

We now calculate the spin density polarization $P_n = (n^l - n^\perp)/(n^l + n^\perp)$, which is directly related to $V_{\text{SH}}$ through the expression [30]

$$P_n = \frac{e V_{\text{SH}} g_s(E_F)}{2P} = \frac{e V_{\text{SH}}}{2P} h^2 (3\pi^2 n)^{2/3}. \quad (1)$$

Here, $g_s(E_F)$ is the density of states at the Fermi energy and $m^*$ is the effective mass of GaAs. From our measurements we obtained $V_{\text{SH}}(0) = 83 \text{ \mu V}$ (this is the voltage which would be detected if the electrodes were directly placed at the edges of the channel) and thus a spin polarization of $P_n(0) \approx 3\%$. This is roughly double the value of the spin polarization obtained for higher doped n-GaAs [17], however, our results show that the SHE is a low-level polarization effect.

Now, we proceed with extracting important parameters of the SHE, namely the spin Hall conductivity $\sigma_{\text{SH}}$ and the spin Hall angle $\alpha_{\text{SH}} = \sigma_{\text{SH}}/\sigma_{xx}$. The SHE induced spin current $j_s$ creates a spin accumulation $\mu_s(0) = j_s \lambda_s \sigma_{xx}$ at the edges of the channel, which would lead, through spin–charge coupling, to a voltage $V_{\text{SH}}(0) = 2P \mu_s(0)$ between lower and upper edge. By using the definitions of spin current $j_s = \sigma_{\text{SH}} E_x$ and electrical current $j = \sigma_{xx} E_x$, we derive

$$\sigma_{\text{SH}} = \frac{V_{\text{SH}}(0)}{2P j_s \lambda_s}. \quad (2)$$

For a current density $j_s = 1.7 \times 10^3 \text{ A cm}^{-2}$, we obtained from our measurements $V_{\text{SH}}(0) = 83 \text{ \mu V}$, $\lambda_s = 8.5 \text{ \mu m}$ and $\sigma_{xx} = 1370 \Omega^{-1} \text{ m}^{-1}$. With Eq. (2), we calculate $\sigma_{\text{SH}} \approx 1.1 \Omega^{-1} \text{ m}^{-1}$ and $\sigma_{xx} = 8 \times 10^{-4}$. Both values are very consistent with previous experiments [17] and theoretical predictions [23]. This strongly suggests that the experimentally determined magnetic field dependence of the FM electrode signal is induced by the SHE.

### 2.4 Bias and temperature dependence
According to theory [23], the spin Hall conductivity $\sigma_{\text{SH}}$ in n-GaAs is given by

$$\sigma_{\text{SH}} \approx \frac{\gamma}{2} \sigma_{xx} + \sigma_{SJ}. \quad (3)$$

The first term in the above equation is due to skew-scattering, where $\gamma$ is the so-called skewness parameter. The second component, $\sigma_{SJ}$, describes the side-jump contribution, which is independent of $\sigma_{xx}$ and is only determined by density $n$ and spin–orbit interaction parameter $\lambda_{so}$ as [23]

$$\sigma_{SJ} = \frac{-2n e^2 \lambda_{so}}{h}. \quad (4)$$

Hence, the total spin Hall conductivity $\sigma_{\text{SH}}$ can be varied with the electrical conductivity $\sigma_{xx}$, which allows us to determine the skewness $\gamma$ and the side-jump contribution $\sigma_{SJ}$ from Eq. (3). Since the mobility depends on the applied electric field $E_x$, the conductivity $\sigma_{xx}$ can be tuned by changing the current density $j_s$ [17, 34]. We performed bias dependent measurements at $T = 4.2 \text{ K}$ for current densities in the range of $j_s = 3.3 \times 10^{-2} - 3.3 \times 10^{3} \text{ A cm}^{-2}$ ($\sigma_{xx} \approx 1000 \Omega^{-1} \text{ m}^{-1}$ to $1600 \Omega^{-1} \text{ m}^{-1}$). In Figure 3a–c, we show spin Hall signals (symbols) for three different values of the current density together with Hanle fits (solid lines), from which we extract the spin Hall conductivity. Given the measured dependence of $\sigma_{\text{SH}}$ on $\sigma_{xx}$, we performed a linear fit of our data and derived from Eq. (3) the skewness parameter $\gamma \approx 4 \times 10^{-4}$ and the side-jump contribution $\sigma_{SJ} \approx 0.6 \Omega^{-1} \text{ m}^{-1}$. Here, the deduced value for $\gamma$ is a half of the value $\gamma \approx 1/900$, which was calculated by Engel et al. [23], and is approximately one order of magnitude smaller than the value obtained by Garlid et al. [17] in higher conductive n-GaAs channels. The value of the side-jump contribution differs by $\sim 1 \Omega^{-1} \text{ m}^{-1}$ from the one which we calculated with $n \approx 2 \times 10^{16} \text{ cm}^{-3}$ from Eq. (4) (the fact that the experimental value has a positive sign is an artifact of linear extrapolation). Altogether, the discrepancy between theory and our experiments is approximately one order of magnitude smaller than the one reported by Garlid et al. [17]. However, one has to note that Engel et al. [23] performed their calculations for n-GaAs channels with parameters similar to ours, what could explain why our results are closer to their theoretical predictions than those in Ref. [17].

Figure 3d shows the dependence of the spin Hall conductivity on the channel conductivity $\sigma_{xx}$. Beside our
own experimental data (closed symbols), we also included data, which were previously reported by Garlid et al. [17] (open symbols). Due to our low channel doping, we can provide data for conductivities up to \( \sim 1600 \, \Omega^{-1} \cdot \text{m}^{-1} \), whereas Garlid et al. [17] performed experiments at \( T = 30 \, \text{K} \) for channel conductivities above \( \sim 2500 \, \Omega^{-1} \cdot \text{m}^{-1} \). If one compares experimental data for \( \sigma_{xx} > 3000 \, \Omega^{-1} \cdot \text{m}^{-1} \) with the corresponding values for \( \sigma_{xx} < 1600 \, \Omega^{-1} \cdot \text{m}^{-1} \), a large deviation of the skewness parameter (which is proportional to the slope of the extrapolated line) can be noticed. However, one clearly sees that in the range of \( \sim 2500–3000 \, \Omega^{-1} \cdot \text{m}^{-1} \) the corresponding data points deviate substantially from the extrapolated line. Values of \( \gamma \) and \( \sigma_{sj} \) extracted from this region of \( \sigma_{xx} \) (interpolation with dashed line) seem to fit better with our results. We conclude, that the spin Hall conductivity can be well described by Eq. (3), however, one cannot treat both skewness parameter \( \gamma \) and side-jump contribution \( \sigma_{SJ} \) as fully independent on the channel conductivity \( \sigma_{xx} \). Here, we speculate on the existence of two (or more) regimes of \( \sigma_{xx} \), in which different sets of parameters \( \gamma \) and \( \sigma_{SJ} \) determine the spin Hall conductivity \( \sigma_{SH} \).

We now consider possible explanations for the observed behavior of \( \sigma_{SH} \). For our bias dependent experiments, we tuned the conductivity \( \sigma_{xx} \) by applying different bias currents. This gives rise to different electric fields \( E_x \) in the channel, which in low doped GaAs can influence both mobility, due to its effect on mean electron energy, and carrier concentration \( n \) in the sample, due to impact ionization of donors [34]. According to Eq. (4), the latter could generally lead to a change of the side-jump contribution and thus to a change of the total spin Hall conductivity. However, ordinary Hall measurements did not show a dependence of \( n \) on \( E_x \), neither for our samples, nor for the devices presented in Refs. [17, 35]. In Ref. [23], the skewness parameter \( \gamma \) is treated as independent of the electric field, at least up to \( \sim 200 \, \text{V cm}^{-1} \). As both sets of data shown in Fig. 3d were obtained for approximately the same range of electric field \( E_x \) (\( \sim 30–200 \, \text{V cm}^{-1} \) for our data and \( \sim 5–200 \, \text{V cm}^{-1} \) for Ref. [17]) one should either rethink this assumption or search for some other effects at work. Further experiments on samples with different doping densities are needed in order to get a better understanding of the behavior of spin Hall conductivity in GaAs.

We also performed temperature dependent studies of the spin Hall signal in the range of \( T = 4.2–80 \, \text{K} \) for a constant current density \( j_x = 1.7 \times 10^5 \, \text{A cm}^{-2} \). Since the channel conductivity \( \sigma_{xx} \) increased with temperature, we expect with Eq. (3) an increase of the spin Hall conductivity, which was indeed observed. The spin Hall signal decreases with increasing temperature mainly as a result of decreasing \( \tau_s \). Above \( T = 70 \, \text{K} \), the signal was no longer observable, which is consistent with spin injection experiments on the same wafer [29].

### 2.5 Results

In the preceding section, we presented all-electrical measurements of direct spin Hall effect in lightly doped n-GaAs channels, where we used Esaki diodes as FM spin detectors. The high spin detection efficiency of the Esaki diodes allowed for spin Hall measurements with relatively large amplitudes of the signal, e.g., compared to experiments in higher conductive channels with Fe/GaAs Schottky diodes as spin sensitive contacts [17]. Our experiments revealed spin Hall conductivities that are consistent with those calculated by Engel et al. [23] and that are smaller than those presented in Ref. [17]. Combined results of these two experiments clearly show that both skewness and side-jump contribution to the total spin Hall conductivity can be treated as independent on the channel conductivity, as it was predicted by theory [23], only in a certain regime, and may have different values in different ranges of conductivity.

### 3 Detection of inverse spin Hall effect in p-GaAs spin injection devices

#### 3.1 Overview

In the following, we focus on experiments to detect the inverse spin Hall effect. We report on ISHE experiments in p-type GaAs, where we used Fe/GaAs spin injection devices [18, 36] with multiterminal Hall bar structures. The SEM image in Fig. 4a shows the layout of one of our samples. A series of Hall crosses is located at different distances \( L \) from the Fe electrodes, which were employed for injecting a spin-polarized current into the conductive p-doped GaAs channel.

In the presence of spin–orbit fields, the scattering of charge carriers is spin-dependent, leading to a spatial separation of spin-up and spin-down charge carriers. Since...
the injected charge current is spin-polarized, the number of electrons scattered to the right and the left side of the channel boundaries \((\pm y\text{-direction})\) is not equal. This gives rise to a net charge imbalance, which is then nonlocally detected at a Hall cross (see Fig. 4a, e.g., contacts e and f) [18, 20, 37, 38]. Since this generation of a Hall voltage is described by the Hall cross (see Fig. 4a, e.g., contacts e and f) [18, 20, 37, 38].

All experimental devices were fabricated from a single wafer, which was grown by MBE on a (001) GaAs substrate. It consists (in order of growth) of a GaAs buffer layer and a GaAs/(Al,Ga)As superlattice, 150 nm of undoped GaAs, a conductive 150 nm p\(^{+}\)-doped \((1 \times 10^{18} \text{ cm}^{-3})\) GaAs transport channel, a 15 nm highly p\(^{++}\)-doped \((5 \times 10^{18} \text{ cm}^{-3})\) GaAs layer and 60 nm of Fe. The wafer was finally covered with 40 nm of Au to protect Fe from oxidization.

3.2 Experiments For ISHE measurements, we injected a spin-polarized current by applying a constant dc current \(I_{\text{ab}} = 50 \mu\text{A}\) (larger electrode) or \(I_{\text{cb}} = 50 \mu\text{A}\) (smaller electrode). Only spins oriented along the \(z\)-axis (out-of-plane) contribute to a measurable spin-Hall-induced charge imbalance at the Hall crosses. The orientation of the injected spins is determined by the orientation of magnetization of the Fe electrode (which initially lies in the plane of sample). They both can be controlled by an external magnetic field \(B\), which points out-of-plane (see Fig. 4a). The resistance \(R_{\text{NL}}\), at each Hall cross is expected to increase with \(B\), because the \(z\)-component of both magnetization of the Fe electrode and the polarization of the injected spins also increases with \(B\) [20, 37].

Figure 4b shows nonlocal Hall resistances \(R_{\text{NL}}\) data as a function of \(B\). The \(R_{\text{NL}}\) curves, which were obtained at \(T = 1.8\ \text{K}\) for different distances \(L\), resemble ISHE shapes with a linear dependence on \(B\) for magnetic fields up to \(B = \pm 1.5\ \text{T}\). At \(B = \pm 1.75\ \text{T}\), which is a typical value for the saturation field \((B_{S})\) of Fe in the perpendicular magnetic field [20], a saturation plateau can be found. However, in contrast to typical ISHE curves [20, 37, 38], \(R_{\text{NL}}\) does not saturate for higher magnetic fields, but changes its slope from negative to positive as \(B\) is further increased. The total magnitude \(\Delta R_{\text{NL}}\), determined at the saturation field \(B = B_{S}\), decays exponentially with increasing \(L\) and no signal was observed for \(L\) larger than 22 \(\mu\text{m}\). From the corresponding plot (not shown here) of \(\Delta R_{\text{NL}}\) versus distance \(L\), we calculate \(\lambda = 3.56 \mu\text{m}\) as decay length.

Furthermore, we investigated the nonlocal resistance \(R_{\text{NL}}\) as a function of the tilting angle \(\theta\) between \(B\) and the normal of the \(xy\) plane of the sample (see Fig. 4a). Data for \(L = 2 \mu\text{m}\) (Fig. 5a) indicate that the saturation field \(B_{S}\) and the magnitude \(\Delta R_{\text{NL}}\) are strongly dependent on the out-of-plane component of \(B\), because \(\Delta R_{\text{NL}}\) vanishes when \(B\) is applied in the plane of the sample \((\theta = 90^{\circ})\). The inset of Fig. 5a shows that the angular dependence of \(B_{S}\) (normalized by the value of \(B_{S}\) at \(\theta = 0^\circ\)) when \(B\) is oriented out-of-plane) can be fitted with a \(1/\cos \theta\) law, consistent with previous ISHE experiments [37].

We now calculate the spin Hall angle \(\alpha_{\text{SH}}\) by using the expression for the spin Hall resistance \(\Delta R_{\text{ISHE}}\), which is given by [20]

\[
\Delta R_{\text{ISHE}} = \alpha_{\text{SH}} \left( \frac{(\sigma_{xS})^{-1}}{t_{\text{GaAs}}} \right) P \exp \left( \frac{-L}{\lambda} \right).
\]

Here, \(t_{\text{GaAs}} = 150\ \text{nm}\) is the thickness of the conductive GaAs layer, \(\sigma_{xS} = 3 \times 10^{2} \Omega^{-1} \text{m}^{-1}\) is the electrical conductivity and \(P\) is the effective current spin polarization, where \(P \approx 20\%\) is a typical value for Fe/GaAs systems [36, 39]. Using these values we derive from Eq. (5) a spin Hall angle \(\alpha_{\text{SH}} = 2.3 \times 10^{-1}\). The calculated value for the spin Hall angle is significantly larger compared to theoretical predictions for p-GaAs [24] and to electrical [17–19] and optical experiments in n-type GaAs [10], which typically yield values for the spin Hall angle in the order of
measurements for a much smaller current density (discuss possible origins of the
revealed a dependence is obviously not related to the ISHE, we brie
41]. Since our system consists of two layers with different
doping, we explored also the effect of this sandwich structure
GaAs transport channel, when a constant dc current
unpolarized dc current \( I_{bd} \) through the Hall bar by using non-magnetic contacts (contacts b and d, see
etching process, the curve becomes more linear, suggesting
occurrence of a ISHE-like signal.

\( \alpha_{SH} = 10^{-3} - 10^{-4} \). Since our results deviate by two orders of magnitude, we doubt that the measured nonlocal signal is solely induced by the ISHE.

In order to reveal the origin of the \( B \)-dependence of the \( R_{\text{NL}} \) curves, we performed control experiments to examine the spin-independent background. First, we passed a spin-
unpolarized dc current \( I_{bd} = 50 \mu \text{A} \) through the Hall bar using non-magnetic contacts (contacts b and d, see Fig. 4a) and measured the local resistance \( R_{\text{local}} \) in a perpendicular magnetic field. Measurement data (not shown here) for several distances \( L \) revealed a \( B \)-dependence of \( R_{\text{local}} \) with similar features and a comparable magnitude as in the case of a spin-polarized current (Fig. 4b).

Furthermore, we fabricated control samples, where the Fe electrodes were replaced by a non-magnetic material (Au). This assured that no spin-polarized current was flowing through the p-doped GaAs transport channel, when a constant dc current \( I_{ab} = 50 \mu \text{A} \) or \( I_{ab} = 50 \mu \text{A} \) was applied. Here, \( R_{\text{NL}} \) also revealed a \( B \)-dependence (not shown here) comparable to the data presented in Fig. 4b.

Since the main source of the observed magnetic field dependence is obviously not related to the ISHE, we briefly discuss possible origins of the \( R_{\text{NL}} \) curves. We can exclude a significant contribution of thermal diffusion effects to the measured signal, since the current density is generally quite small in our experiments (\( j_{bd} = 3.3 \times 10^{3} \text{ A cm}^{-2} \)). Moreover, measurements for a much smaller current density (\( j_{bd} = 6.6 \text{ A cm}^{-2} \)) revealed \( R_{\text{NL}} \) curves comparable to those shown in Fig. 4b. For our samples, it is likely that a current distribution forms near the vicinity of the Fe electrodes and spreads into the Hall bar, as it is depicted in Fig. 5b. A small fraction of the charge carriers could then move toward a voltage lead of the Hall cross and induce a nonlocal charge imbalance [40, 41]. Since our system consists of two layers with different doping, we explored also the effect of this sandwich structure on \( R_{\text{NL}} \). For this purpose, we etched away the p\(^{++}\)-doped GaAs top layer and confined the charge carrier transport exclusively to the 150 nm thick p\(^+\)-doped transport channel.

Since the modified samples were highly resistive at \( T = 1.8 \text{ K} \), we carried out our measurements at \( T = 30 \text{ K} \). The resulting \( R_{\text{NL}} \) signal is shown in Fig. 5c. After the etching process, the curve becomes more linear, suggesting that transport in the highly doped top layer plays a significant role in mimicking an ISHE-like signature.

### 3.3 Results

The presented experiments with p-doped GaAs spin injection devices reveal ISHE-like features. On the one hand \( R_{\text{NL}} \) shows the characteristic \( B \)-dependence (Fig. 4b), on the other hand a \( 1/\cos \theta \) angular dependence was found (Fig. 5a). However, our control experiments suggest that the generation of a spin-polarized current is not a necessary prerequisite for obtaining such magnetic field dependencies. Therefore, we conclude that the ISHE is not the origin for the magnetic field dependence of \( R_{\text{NL}} \). This finding is also supported by the huge value for the spin Hall angle \( \alpha_{SH} \), which is two orders of magnitude larger than expected. We assume that the origin of the nonlocal signal is due to current spreading in the vicinity of the Fe electrode; the differently doped p-layers support, as shown above, the occurrence of a ISHE-like signal.

Since the huge spin-independent background probably masks any ISHE signals, Olejněk et al. [18] introduced a novel measurement setup for ISHE experiments to circumvent this problem. Ultrathin Fe/GaAs spin injection contacts with a strong in-plane magnetic anisotropy were employed to inject a spin-polarized current into n-doped (5 \( \times 10^{16} \text{ cm}^{-2} \)) GaAs channels, similar to our geometry. After setting the magnetization of the Fe electrode along the in-plane magnetic easy axis (e.g., +y-direction), a perpendicular in-plane magnetic field, which points along the magnetic hard axis of the electrode, was applied to induce Hanle spin precession. The spins, which are initially oriented in the plane of the sample along the easy axis of the electrode, hence acquire an out-of-plane component (z-direction). Due to ISHE a net charge imbalance can be detected nonlocally at a Hall cross. The

---

**Figure 5** (a) Angular dependence of the nonlocal resistance \( R_{\text{NL}} \) (\( T = 1.8 \text{ K}, L = 2 \mu \text{m} \)), where \( \theta \) is the tilting angle between \( B \) and the normal of the \( xy \) plane of the sample (\( B \) is oriented in the plane of the sample for \( \theta = 90^\circ \) and out-of-plane for \( \theta = 0^\circ \)). (b) Schematic explanation for the measured \( B \)-dependence of the \( R_{\text{NL}} \) curves: Formation of a current distribution around the FM contact. (c) Nonlocal resistance \( R_{\text{NL}} \) at \( T = 30 \text{ K} \) as a function of the out-of-plane magnetic field \( B \) before and after etching away the p\(^{++}\)-doped GaAs top layer.
measurement procedure was then repeated with reversed in-plane magnetization of the Fe electrode (e.g., \(-y\)-direction). Since the ordinary Hall effect and other contributions to the nonlocal signal are independent from the magnetization of the Fe electrode and thus from the orientation of the injected spins, the pure ISHE induced signal was obtained by subtracting both measurement data sets. Additionally, a modulation of the ISHE signal could be realized by applying a drift current along the transport channel. The additional drift velocity either pushes the electrons toward the Hall cross (amplification of the ISHE signal) or back to the injection electrode (suppression of the ISHE signal).

4 Detection of inverse spin Hall effect in H-bar shaped p-GaAs nanostructures

4.1 Overview In the last section of this article, we focus on the so-called H-bar geometry, which allows for measurements of the inverse spin Hall effect without the need to generate a spin polarized charge current. This setup was first described and studied theoretically by Hankiewicz et al. [21]. Corresponding measurements were reported by Brüne et al. [22] for HgTe microstructures and by Mihailovic et al. [42] for Au Hall bars. Kolwas et al. [43] carried out measurements in PbTe quantum wells and a similar setup was also used by Balakrishnan et al. [44] to experimentally determine the strength of spin–orbit coupling in graphene. However, reports for semiconductors are still rare.

We modified the original proposal [21] and employed for our experimental devices a double H-bar geometry. Figure 6a shows the corresponding design, where a charge current \(j_1\) is driven in the middle branch B. In the presence of spin–orbit fields, the scattering of charge carriers is spin-dependent, thus generating a spin current \(j_y\), perpendicular to \(j_1\) (DSHE). While spins accumulate at the channel boundaries, the spin current \(j_y\) can flow along the bridging channel, which connects all branches of the “H”. Scattering of charge carriers gives then rise to a charge current oriented perpendicular (\(y\)-direction) to the spin current \(j_y\) (ISHE). In consequence a net charge accumulation and hence a nonlocal resistance can be detected in the adjacent branches A and C. The concept of the double H-bar geometry allows for a validity check, because the charge accumulation, which is induced by the ISHE, is similar for both branches A and C, and depends on the polarity and magnitude of the applied charge current \(j_1\) [42].

The SEM image (Fig. 6b) shows one of our p\(^+\)-doped (2 \(\times\) 10\(^{18}\) cm\(^{-3}\)) GaAs nanostructures fabricated from a single wafer, which was grown by MBE on a (001) GaAs substrate. Four separate “H”-branches with a width of 350 nm are electrically connected by a bridging channel, which is \(w = 900\) nm wide. The distance between the centers of two adjacent branches is \(L = 700\) nm, whereas the height of the conduction channel measures \(t = 300\) nm.

4.2 Experiments All ISHE measurements were carried out at \(T = 50\) K. We passed a constant dc current \(I_B\) in the middle branch B and monitored simultaneously the local signal \(U_B\) and the nonlocal signals \(U_A\) and \(U_C\) (see Fig. 6a). Both nonlocal signals \(U_A\) and \(U_C\) have the same magnitude and sign and scale linearly with the applied current \(I_B\) in the range of \(I_B = \pm 100\) nA–10 \(\mu\)A (not shown here), furthermore \(U_A\) and \(U_C\) invert fully for opposite polarity of \(I_B\). This bias dependence of the nonlocal signals is consistent with the ISHE in a H-bar geometry. The spin Hall angle \(\alpha_{\text{SH}}\) can be calculated from the nonlocal spin Hall resistance \(R_{\text{NL}}\) [42, 45], which is given by

\[
R_{\text{NL}}^{\text{SH}} = \frac{1}{2} \alpha_{\text{SH}}^2 R_{\text{sq}} \left( \frac{w}{\lambda_{\text{sd}}} \right) \exp \left( -\frac{L}{\lambda_{\text{sd}}} \right). \tag{6}
\]

Here, \(R_{\text{sq}} = \rho t = 6 \times 10^3\) \(\Omega\) is the sheet resistance of our samples. We assume that the spin diffusion length is of order \(\lambda_{\text{sd}} = 1\) \(\mu\)m. For \(R_{\text{NL}}^{\text{SH}} = 1.3\) k\(\Omega\), we calculate \(\alpha_{\text{SH}} \geq 3.1 \times 10^{-1}\) as a lower limit for the spin Hall angle (note that \(\alpha_{\text{SH}}\) increases for larger values of \(\lambda_{\text{sd}}\)). The deduced value for \(\alpha_{\text{SH}}\) is then at least 2 orders of magnitude larger than values predicted by theory for p-GaAs [24] and from previous experiments in n-GaAs (\(\alpha_{\text{SH}} = 10^{-5}–10^{-4}\)) [10, 17–19]. Hence, it is very doubtful that the measured nonlocal signal is fully induced by the ISHE, despite the fact that the spin–orbit coupling is expected to be stronger in p-GaAs compared to n-GaAs [46].

In order to identify different contributions to the measured nonlocal signal, we performed measurements in an external

![Figure 6](image-url) (a) Sketch of a double H-bar geometry used for detection of ISHE. A charge current \(j_1\) is driven in the middle branch B. Induced by DSHE (1), a perpendicular spin current \(j_y\) flows along the connecting part of the “H”. Due to ISHE (2) a nonlocal charge imbalance can finally be detected in the adjacent branches A and C. (b) SEM images of one of our p-GaAs H-bar samples and of the central region of this device which consists of four “H”-branches (width 350 nm).
magnetic field. The spin polarization generated by means of
DSHE (see Fig. 6a) points along the ±z-direction. Therefore,
an in-plane magnetic field $B_x$ or $B_y$ along the ±x- or ±y-
direction should induce precession of the spins, what would
result in an oscillatory behavior of the nonlocal signal [45].
However, no such oscillations were observed in our
experiments (not shown here), suggesting that the ISHE is
not the origin of the signal. More information on its possible
origins was obtained from measurements in out-of-plane
magnetic field $B_z$ applied along the ±z-direction. No spin
precession, i.e., no change in the nonlocal signal, should be
observed if the signal stems from ISHE. However, if other
mechanisms than the ISHE contribute to the nonlocal signal, a
distinct magnetic field dependence of the nonlocal signals can
be expected. In these experiments, the magnetic field $B_z$
was swept in the range of ±10 T and all measurements were again
carried out at $T = 50$ K and for different values of the applied
current (±100 nA–10 μA). Typical data of local ($U_B$) and
nonlocal signals ($U_A$, $U_C$) for a current $I_B = ±5$ μA are
displayed in Fig. 7a and b.

Both local and nonlocal signals show distinct magnetic
field dependencies and depend on the polarity of $I_B$, but only
the local signal $U_B$ fully reverses with the latter. A strong and
approximately quadratic dependence of $U_B$ on $B_z$ is most
likely due to positive magnetoresistance (PMR), which can
be observed in non-magnetic materials. The magnitude of the
PMR yields ≈12% at $B_z = ±10$ T. For further analysis,
the nonlocal signals $U_A$ and $U_C$ are split up in symmetric and
antisymmetric parts with respect to $B_z$.

First, we consider the antisymmetric parts of the nonlocal
signals (Fig. 8a), which are on the one hand independent of
the polarity of $I_B$ and on the other hand of opposite sign for
both branches A and C, respectively. These features exclude
the ISHE as origin, as it is illustrated in Fig. 6a. Furthermore,
the magnitude (determined at $B_z = 10$ T) scales quadratically
with the applied current $I_B$ (not shown here), suggesting a
thermally induced effect, which depends on the dissipated
power. Since the dimensions of our samples are very small
(cross-section ~0.1 μm²), it is likely that the middle
branch $B$ is heated when a current is passed through (e.g.,
$P_{\text{electrical}} = 22$ μW at $I_B = 5$ μA). In consequence a heat
current flows along the bridging channel, as illustrated in the
inset of Fig. 8a. The charge carriers are then deflected in the
perpendicular magnetic field $B_z$ due to the Lorentz force,
what gives rise to a measurable charge carrier accumulation
in the branches A and C [42]. This phenomenon, which is
known as Ettingshausen–Nernst effect [47], is independent
of the polarity of $I_B$ and was also reported for other SHE
experiments [37].

We now analyze the symmetric parts of nonlocal signals
(Fig. 8b), which have both the same polarity and invert for
opposite polarity of $I_B$. Since the symmetric part of both
nonlocal signals $U_A$ and $U_C$ scales quadratically with $B_z$,
we assume a correlation with the local signal $U_B$, which shows a
similar magnet field dependence (Fig. 7a). This can be
explained by diffusive transport, where the current density
spreads into the adjacent branches A and C and
induces a nonlocal charge imbalance of identical sign for both
branches.

Figure 7 (Inset) Measurement configuration: A current $I_B$ is driven
in the middle branch B while an out-of-plane magnetic field $B_z$
is swept in the range of ±10 T. We monitor both local signal $U_B$
and nonlocal signals $U_A$ and $U_C$. (a) Magnetic field dependence of
the local signal $U_B$ (monitored in the middle branch B) for a current
$I_B = ±5$ μA at $T = 50$ K. (b) Nonlocal signals $U_A$ and $U_C$ for the
same measurements.

Figure 8 (a) Antisymmetric part (with respect to $B_z$) of the
nonlocal signals $U_A$ and $U_C$ for $I_B = ±5$ μA (see Fig. 7b). (Inset)
Ettingshausen–Nernst effect: A heat current flows from the middle
branch B into the adjacent branches and induces a nonlocal charge
imbalance, which is of opposite sign for A and C. (b) Symmetric
part (with respect to $B_z$) of the nonlocal signals $U_A$ and $U_C$ for
$I_B = ±5$ μA. (Inset) van der Pauw theorem: In the diffusive regime
the current density spreads into the adjacent branches A and C
and induces a nonlocal charge imbalance of identical sign for both
branches.

$$R^C_{NL} = R_{nl} \exp\left(-\frac{\pi L}{w}\right).$$  (7)
For $L = 700 \text{ nm}$, we derive from Eq. (7) a nonlocal resistance $R_{NL} = 5.2 \text{ k}\Omega$, which is comparable to experimental data ($R_{NL} = 1.3 \text{ k}\Omega$, determined at $B_z = 0 \text{ T}$). Hence, the measured bias dependence of the nonlocal signals (without magnetic field), for which we calculated a huge value of the spin Hall angle, can mainly be explained by diffusive charge carrier transport. This effect could be reduced by fabricating samples with $w \ll L_d$ since this diminishes the contribution of diffusive transport to the nonlocal resistance by several orders of magnitude [45] (e.g., $R_{NL} \approx 1 \text{ \Omega }$ for $w = 200 \text{ nm}$).

4.3 Results Here, we reported on ISHE experiments in $p$-GaAs nanostructures with a double H-shaped geometry, where we passed a current $I_B$ through the middle branch of the double “H”. We could observe clear nonlocal signals in the adjacent branches which scaled with the current $I_B$, however, the calculated spin Hall angle was larger than expected. Measurements in an external out-of-plane magnetic field indicated that thermal effects (Ettingshausen–Nernst effect [47]) and diffusive transport (van der Pauw theorem [48]) dominate the measured nonlocal signals. The first effect is relevant due to the small dimensions of the samples, i.e., the width of the current-carrying branch, but can be easily identified, as it does not depend on the polarity of $I_B$. The diffusive transport strongly depends on the width $w$ of the bridging channel and can easily be confused with the ISHE, since it also scales with $I_B$. Hence, the unavoidable presence of these both effects requires careful analysis of the measured nonlocal signals, in order to determine the ISHE-related contribution.

5 Summary and outlook In this article, we presented three different device geometries and experiments for all-electrical detection of direct and inverse spin Hall effect in semiconductors. In the first section, we reported on our experiments [19], where we successfully probed the direct spin Hall effect in lightly doped n-GaAs by using FM Esaki diodes. Our experimental results are in accordance with previous experimental [17] and theoretical studies [23]. Detailed analysis allowed us to extract important parameters of the extrinsic SHE, what could help to improve theoretical descriptions. In the second part, we showed experiments on the detection of the inverse spin Hall effect in $p$-GaAs, where we used Fe/GaAs spin injection devices to generate a spin current. We observed a distinct magnetic field dependence of the nonlocal resistance, which revealed ISHE-like features. However, control experiments showed that the nonlocal resistance is unlikely to be induced by the ISHE. In the last section, we introduced the so-called H-bar geometry [21], which allows for inverse spin Hall effect experiments without the necessity of spin injection into a semiconductor. Here, we also observed clear nonlocal signals, which could not be unambiguously ascribed to the ISHE. We found instead that they are rather induced by diffusion effects. The main problem is that, especially in ISHE geometries, the spin-independent background tends to be much larger than the expected spin Hall signal.

Acknowledgements This work has been supported by the Deutsche Forschungsgemeinschaft (DFG) via SPP1285 and SFB689 projects. One of the authors, C.S., is grateful for the support of the Alexander von Humboldt Foundation.

References

[1] J. E. Hirsch, Phys. Rev. Lett. 83, 1834 (1999).
[2] M. I. D’yakonov and V. I. Perel, JETP Lett. 13, 467 (1971).
[3] M. I. D’yakonov and V. I. Perel, Phys. Lett. A 35, 459 (1971).
[4] S. Zhang, Phys. Rev. Lett. 85, 393 (2000).
[5] S. Murakami, N. Nagaosa, and S.-C. Zhang, Science 301, 1348 (2003).
[6] J. Sinao, D. Culcer, Q. Niu, N. A. Sinitsyn, T. Jungwirth, and A. H. MacDonald, Phys. Rev. Lett. 92, 126603 (2004).
[7] J. Schliemann, Int. J. Mod. Phys. B 20, 1015 (2006).
[8] E. M. Hankiewicz and G. Vignale, J. Phys.: Condens. Matter 21, 253202 (2009).
[9] T. Jungwirth, J. Wunderlich, and K. Olejnık, Nature Mater. 11, 382 (2012).
[10] Y. K. Kato, R. C. Myers, A. C. Gossard, and D. D. Awschalom, Science 306, 1910 (2004).
[11] V. Sih, R. C. Myers, Y. K. Kato, W. H. Lau, A. C. Gossard, and D. D. Awschalom, Nature Phys. 1, 31 (2005).
[12] J. Wunderlich, B. Kastner, J. Sinao, and T. Jungwirth, Phys. Rev. Lett. 94, 047204 (2005).
[13] N. P. Stern, D. W. Steuermann, S. Mack, A. C. Gossard, and D. D. Awschalom, Nature Phys. 4, 843 (2008).
[14] S. Matsuzaka, Y. Ohno, and H. Ohno, Phys. Rev. B 80, 241305(R) (2009).
[15] J. Wunderlich, B.-G. Park, A. C. Irvine, L. P. Zárbo, E. Rozkotová, P. Nemec, V. Novák, J. Sinao, and T. Jungwirth, Science 330, 1801 (2010).
[16] K. Ando and E. Saitoh, Nature Commun. 3, 629 (2012).
[17] E. S. Garlid, Q. O. Hu, M. K. Chan, C. J. Palmstrøm, and P. A. Crowell, Phys. Rev. Lett. 105, 156602 (2010).
[18] K. Olejnık, J. Wunderlich, A. C. Irvine, R. P. Campion, V. P. Amin, J. Sinao, and T. Jungwirth, Phys. Rev. Lett. 109, 076601 (2012).
[19] M. Ehler, C. Song, M. Ciorga, M. Utz, D. Schuh, D. Bougeard, and D. Weiss, Phys. Rev. B 86, 205204 (2012).
[20] S. O. Valenzuela and M. Tinkham, Nature (London) 442, 176 (2006).
[21] E. M. Hankiewicz, L. W. Molenkamp, T. Jungwirth, and J. Sinao, Phys. Rev. B 70, 241301(R) (2004).
[22] C. Brüne, A. Roth, E. G. Novik, M. König, H. Buhmann, E. M. Hankiewicz, W. Hanke, J. Sinao, and L. W. Molenkamp, Nature Phys. 6, 448 (2010).
[23] H.-A. Engel, B. I. Halperin, and E. I. Rashba, Phys. Rev. Lett. 95, 166605 (2005).
[24] W. K. Tse and S. Das Sarma, Phys. Rev. Lett. 96, 056601 (2006).
[25] M. Kohda, Y. Ohno, K. Takamura, F. Matsukura, and H. Ohno, Jpn. J. Appl. Phys. Part 2 40, L1274 (2001).
[26] E. Johnston-Halperin, D. Lofgreen, R. K. Kawakami, D. K. Young, L. Coldren, A. C. Gossard, and D. D. Awschalom, Phys. Rev. B 65, 041306(R) (2002).
[27] P. Van Dorpe, Z. Liu, W. Van Roy, V. F. Motsnyi, M. Sawicki, G. Borghs, and J. De Boeck, Appl. Phys. Lett. 84, 3495 (2004).
[28] M. Ciorga, A. Einwanger, U. Wurstbauer, D. Schuh, W. Wegscheider, and D. Weiss, Phys. Rev. B 79, 165321 (2009).
[29] C. Song, M. Sperl, M. Utz, M. Ciorga, G. Woltersdorf, D. Schuh, D. Bougeard, C. H. Back, and D. Weiss, Phys. Rev. Lett. 107, 056601 (2011).
[30] M. Johnson and R. H. Silsbee, Phys. Rev. Lett. 55, 1790 (1985).
[31] J. Fabian, A. Matos-Abiague, C. Ertler, P. Stano, and I. Žutić, Acta Phys. Slov. 57, 565 (2007).
[32] I. Žutić, J. Fabian, and S. Das Sarma, Rev. Mod. Phys. 76, 323 (2004).
[33] M. Furis, D. L. Smith, and S. A. Crooker, Appl. Phys. Lett. 89, 102102 (2006).
[34] D. J. Oliver, Phys. Rev. 127, 1045 (1962).
[35] E. S. Garlid, PhD Thesis, University of Minnesota (2010).
[36] X. Lou, C. Adelmann, S. A. Crooker, E. S. Garlid, J. Zhang, K. S. Madhukar Reddy, S. D. Flexner, C. J. Palmström, and P. A. Crowell, Nature Phys. 3, 197 (2007).
[37] T. Seki, Y. Hasegawa, S. Mitani, S. Takahashi, H. Imamura, S. Maekawa, J. Nitta, and K. Takanashi, Nature Mater. 7, 125 (2008).
[38] T. Kimura, Y. Otani, T. Sato, S. Takahashi, and S. Maekawa, Phys. Rev. Lett. 98, 156601 (2007).
[39] R. Mallory, M. Yasar, G. Itskos, A. Petrou, G. Kioseoglou, A. T. Hanbicki, C. H. Li, O. M. J. van’t Erve, B. T. Jonker, M. Shen, and S. Saikin, Phys. Rev. B 73, 115308 (2006).
[40] H. Jou and A. M. Kriván, Superlattices Microstruct. 19, 030203 (1996).
[41] G. L. J. A. Rikken, J. A. M. M. van Haaren, W. van der Wel, A. P. van Gelder, H. van Kempen, P. Wyder, J. P. André, K. Ploog, and G. Weimann, Phys. Rev. B 37, 6181 (1988).
[42] G. Mihajlović, J. E. Pearson, M. A. Garcia, S. D. Bader, and A. Hoffmann, Phys. Rev. Lett. 103, 166601 (2009).
[43] K. A. Kolwas, G. Grabec, S. Trushkin, J. Wróbel, M. Aleszkiewicz, L. Cywi ski, T. Dietl, G. Springholz, and G. Bauer, Phys. Status Solidi B 250, 37 (2013).
[44] J. Balakrishnan, G. Kok Wai Koon, M. Jaiswal, A. H. Castro Neto, and B. Özyilmaz, Nature Phys. 9, 284 (2013).
[45] D. A. Abanin, A. V. Shytov, L. S. Levitov, and B. I. Halperin, Phys. Rev. B 79, 035304 (2009).
[46] E. M. Hankiewicz, J. Li, T. Jungwirth, Q. Niu, S.-Q. Shen, and J. Sinova, Phys. Rev. B 72, 155305 (2005).
[47] A. V. Ettingshausen and W. Nernst, Ann. Phys. Chem. 265, 343 (1886).
[48] L. J. van der Pauw, Philips Tech. Rev. 20, 220 (1958).