Energy-time entanglement of quasi-particles in solid-state devices

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We present a proposal for the experimental observation of energy-time entanglement of quasi-particles in mesoscopic physics. This type of entanglement arises whenever correlated particles are produced at the same time and this time is uncertain in the sense of quantum uncertainty, as has been largely used in photonics. We discuss its feasibility for electron-hole pairs. In particular, we argue that the recently fabricated 2DEG-DHG junctions, irradiated with a continuous laser, behave as “entanglers” for energy-time entanglement.

Entanglement lies at the heart of quantum mechanics, whose astonishing features come mainly from it [1]. Interest on entanglement has grown, since it was recognized as a resource needed to perform tasks that would classically be impossible [2]. Correlative to the notion of entanglement is the notion of sub-systems: algebraically, $C^2 \otimes C^2$ is equivalent to $C^4$, so, if one cannot address separately the two sub-systems, one cannot investigate entanglement. This is the reason why, although entangled states arise in every sub-field of quantum physics (e.g. the eigenstates of total momentum), it is usually an experimental challenge to achieve control over entanglement. After the spectacular results of photonics [3], entanglement has recently demonstrated in other physical systems [4]. There is a growing list of proposals aimed at the observation of entanglement in solid-state physics, using quantum dots, Josephson junctions and other devices. In this Letter we focus on quasi-particles in mesoscopic devices.

Coherent transport of quasi-particles in semiconductors has been widely demonstrated, so one can envisage to demonstrate entanglement. A few years ago, Burkard and coworkers noticed that electron-electron entanglement in spin could be detected by measuring correlation in the current noise [5]. In Ref. [6], the scheme was completed with a proposal for an “entangler”, that is, for a source of spin-entangled electrons: a Cooper pair from a superconducting material. Recently, these ideas have been extended to entanglement in spatial degrees of freedom, for two electrons generated as Cooper pairs [7] and for electron-hole pairs in edge states [8].

In this Letter we take a different approach, that works in principle for any kind of particles produced in pairs. The basic idea is the following: two particles $a$ and $b$ are produced at the same, but uncertain, time. This quantum uncertainty is within the coherence time of the source. The latter is typically a photon from a laser beam, called the pump photon, whose well-determined energy is shared between the two produced particles. Hence the energy and the time of creation of each particle are uncertain, but the sum of the energies and the difference of the times are well-determined. This form of entanglement is known as energy-time entanglement.

To observe a signature of this entanglement for photon pairs, Franson proposed in 1989 a very convenient interferometer [9], sketched in Fig. 1. Each particle is sent through unbalanced interferometers, with the same difference between the long (L) and the short (S) arm: $L_a - S_a = L_b - S_b = \Delta L$. If $\Delta L$ is larger than the single-particle coherence lengths $\ell_{a,b}^c$, no single-particle interferences will be observed. However, the coherence length $\ell_{c}$ of the pair is usually much larger than $\ell_{a,b}^c$. Let then

$$\ell_c > \Delta L > \ell_{a,b}^c. \quad (1)$$

In this case, the alternatives ”both particles have taken the long arm” (LL) and ”both particles have taken the short arm” (SS) are indistinguishable and exhibit interference fringes; while the two other alternatives, LS and SL, are distinguishable because one particle clearly arrives before the other one to its detector. Thus, in the runs in which both particles are detected at the same time, an interference pattern is observed that is due to the entanglement in energy-time.

The photonic setup. We start by briefly reviewing the photonic setup, that has already been the object of successful experiments e.g. [10]. A non-linear crystal is pumped by a cw laser with coherence time $\tau_c$. This coherence time is defined as usual: the state of the laser light is a coherent beam with a fluctuating phase $\phi(t)$, such that $\phi(t + \tau) - \phi(t)$ is a stationary Gaussian process of mean value $\langle \phi \rangle = 0$ and of variance $\langle \phi^2 \rangle = 2\tau/\tau_c$. A non-linear, purely quantum-mechanical process (parametric down-conversion) takes place, that produces a field in two initially empty modes $a$ and $b$, whose wave-vectors and polarizations are determined by energy and momentum conservation. If the intensity of the pump is weak enough, the field in $a$ and $b$ consists essentially of a large vacuum component (that we neglect) plus a two-photon component. In each mode $a$ or $b$, we shall write $|1,0\rangle$ (resp. $|0,1\rangle$) for one photon propagating along a horizontal (resp. vertical) direction in Fig. 1. The state of the down-converted field can be written as a superposition of two-photon fields produced at any time $t$:

$$|\Psi\rangle = \sqrt{A} \int dt e^{i\phi(t)} |1,0\rangle_a |1,0\rangle_b , \quad (2)$$
where $A$ is proportional to the power of the laser and the efficiency of the down-conversion process. The states $|1_t, 0⟩_{a,b}$ can be seen as an over-complete set; the overlap $⟨1_t, 0|1_t’, 0⟩_{a,b}$ decreases rapidly as a function of $|t - t'|/τ^a,b$, where $τ^a,b$ are the single-photon coherence time. As we discussed, in our experiment this time is much shorter than the other times involved ($Δt$, $τ_c$).

The state (2) can be seen as a continuous version of the maximally entangled state of two $d$-dimensional quantum systems, indexed by the parameter $t$. Franson’s setup is a way of partially detecting this entanglement, by projection onto a two-dimensional subspace and post-selection. The evolution of mode $a$ in the unbalanced interferometer — the beam-splitters are 50-50 couplers — is

$$|1_t, 0⟩_a \rightarrow |1_t, 0⟩_a + i|0, 1⟩_a + ie^{iα}|0, 1+Δt⟩_a - e^{iα}|1_t+Δt, 0⟩_a$$

(3)

where we have omitted a global factor $|^⟩$ and have re-defined the origin of time to take into account the propagation from the source. The evolution of mode $b$ is identical, with a phase $β$ instead of $α$.

The two-photon state at the detection stage is obtained by replacing the evolved state into (2); it is a sum of sixteen basic kets. We focus on a pair of detectors, say the two detectors labelled $+$ in Fig. 1. This means that we project onto the four kets of the form $|1, 0⟩_a|1, 0⟩_b$, that we write for conciseness $|1; 1⟩$:

$$|Ψ_{++}⟩ \simeq \int dt e^{iφ(t)} \left[ |1_t; 1⟩ + e^{i(α+β)}|1_t+Δt; 1_t+Δt⟩ + e^{iβ}|1; 1⟩ + e^{iα}|1_t+Δt; 1⟩ \right].$$

(4)

The two first terms correspond to the cases where the two photons arrive at the same time in the detectors (paths SS and LL in Fig. 1); because of the invariance through the form of the parameters, these two cases are indistinguishable, and interfere. We can re-write these two first terms as

$$|Ψ_{++}⟩ \simeq \int dt e^{iφ(t)} + e^{i(α+β+φ(t-Δt))}|1_t; 1⟩.$$  

(5)

The third term is the case where photon $b$ is delayed by $Δt$ with respect to photon $a$ (path SL), and the fourth term is the opposite case (path LS). These last two cases are in principle distinguishable from the others, so they don’t contribute to any interference [11]. It is a different matter of course to distinguish them in practice. So, a priori we have to consider two possible outcomes of the experiment: if one can select only the interfering cases, the detection rate is [12]

$$R(++) = ||Ψ_{++}||^2 \propto 1 + e^{-Δt/τ} \cos(α + β);$$

(6)

recalling that the visibility $V$ is defined by $R \propto 1 + V \cos θ$ for a sinusoidal fringe, we find $V = e^{-Δt/τ} \approx 1$. If one cannot select only the interfering cases, the visibility of the observed interference fringes will be reduced down to $V \approx 1/2$, since one has

$$R(++) = ||Ψ_{++}||^2 \propto 2 + e^{-Δt/τ} \cos(α + β).$$

(7)

In optics, for typical coherence times and jitters of the detectors, one can select only those cases where the photons arrive at the same time; that’s how $V \approx 1$ has been reached and the Bell inequality could be violated [10].

The proposal: overview. We can now turn to the main goal of this paper: a proposal for the production and detection of energy-time entangled quasi-particles in mesoscopic physics [13]. Specifically, we consider electron-hole pairs produced in semiconductor junctions illuminated by a laser. A low intensity cw laser with coherence time $τ_c$ illuminates a junction, producing electron-hole pairs. When the electron and the hole do not recombine, they will be accelerated out of the junction in opposite directions. Once produced, each quasi-particle travels in a semi-conductor structure, tailored for single-mode coherent transport of the electron [14] or the hole [15]; typically, a two-dimensional electron or hole gas, noted respectively 2DEG and 2DHG. The unbalanced Mach-Zehnder interferometer is engineered in the semiconductor, in the form of an asymmetric loop, where the phase between the two arms can be varied using the Aharonov-Bohm effect [16]. The two paths are then recombined and split again, each ending in a detector. The rest of the paper is devoted to a detailed analysis of the three parts of the setup: entangler (preparation), interferometer (evolution), and detectors (measurement).

The entangler. A standard bulk p-n junction is most probably not an entangler: because of the massive damping, the motion of the quasi-particles in the device is in principle diffusive rather than ballistic, implying that quantum phase coherence is lost even before reaching the interferometers. Even if this description was too pessimistic, one should bother about the interfaces between the bulk and the 2D-gases. Fortunately, there is an elegant way of by-passing both obstacles: by creating a junction between a 2DEG and a 2DHG, the source is just the interface, the interferometer can be engineered in the same materials and the whole motion can be ballistic. The first 2DEG-2DHG junction has been fabricated very recently in AlGnAs/GaAs heterostructures [17] according to the scheme that we reproduce in Fig. 2. The full understanding of the physics of such junctions and the optimization of the parameters will need further work [18]. But there is no fundamental objection to considering a 2DEG-2DHG irradiated by a laser behave as an entangler to generate electron-hole pairs entangled in energy-time. Even more, this goal may be a strong motivation to boost technical improvements. Finally note that p-n junctions have also been fabricated in another material exhibiting ballistic transport, namely carbon nanotubes [19]. These can also be candidates as entanglers.
The interferometer. From now on, we shall use for numerical estimates typical values for electrons, extracted from Ref. [20]. One must not forget that holes often have smaller mobilities in these structures, so the figures may not apply to one half of the interferometer — but the principles of the analysis do apply. In the forthcoming discussion, we shift when convenient from "lengths" \( \ell \) to "times" \( \tau \), the link being provided by \( \ell = v_F \tau \) where \( v_F \) is the Fermi velocity in a 2DEG or 2DHG, typically \( v_F \approx 3 \times 10^7 \text{ cm/s} \). In the optical experiment, we introduced the requirement (1) on \( \Delta L \) for the Franson interferometer to show two-particle interferences. In the present proposal, it is trivial to have the coherence length of the pair \( \ell_c \) exceed all the other meaningful lengths, since \( \ell_c \) is determined by the coherence time of the pump laser, and cw lasers easily have a coherence time of tens of nanoseconds. However, here we must meet an additional constraint due to the role of the environment. When photons essentially do not couple to the environment, electrons and holes propagating in semi-conductors in-

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analogous optical visibility as (11), in the simplest case [23], through \( \langle \rho^0_S | \rho^0_L \rangle = L_{S}^{(L_e+S_h)}/L_{S}^{e} \) for both \( x = e, h \).

The detectors. A convenient detection scheme should use the observables of mesoscopic physics, that are current-correlation measurements [24]. We can consider that the detector for the electron, resp. the hole, is a metal reservoir biased with a voltage \(-V\), resp. \(+V\). If the temperature is small enough \((kT << |eV|)\), no electron (hole) can be injected from the reservoirs into the semiconductor. The situation becomes then analog to those studied in Refs. [5–8]: entanglement can be detected by measuring the zero-frequency current cross-correlator. Since this detection scheme is not time-resolved, it leads to (7) with the correction (11). We want to conclude by addressing the question of time-

resolves immediately that condition (8) holds if and only if \( (\Delta \epsilon)_{\max} << \epsilon_F \), and this can in principle be achieved by decreasing \( \phi' \) (junction engineering) or \( w \) (laser focusing).

Condition (8) being possible, the calculation of the two-particle interference pattern goes along the same lines as for the optical implementation [22], adding the presence of different environments. Following the approach of Stern et al. [23], we write \( \langle \xi^e_S | \xi^e_L \rangle \) and \( \langle \xi^h_S | \xi^h_L \rangle \) the four environments associated to the paths, where \( L, S \) stand for Long and Short, and \( e, h \) stand for electron and hole. The evolution of the electron state, replacing (3), is then

\[
|1_t,0 \rangle_e \rightarrow \left( |1_t,0 \rangle_e + i|0,1_t \rangle_e \right) |\xi^e_S \rangle + \langle \xi^e_S | \langle \xi^e_L | \right) \left( |e^i\alpha|0,1_t+\Delta t \rangle_e - e^{i\alpha}|1_t+\Delta t,0 \rangle_e \right) |\xi^e_L \rangle \tag{9}
\]

and a similar evolution for the hole. In the interfering terms \( \langle \Phi_{+} | \), the term into brackets in eq. (5) is replaced by

\[
e^{i\varphi(t)}|\xi^h_S \rangle + e^{i(\alpha+\beta t-\varphi(\Delta t))}|\xi^h_L \rangle \tag{10}
\]

and finally the visibility is reduced with respect to the analogous optical visibility as

\[
V_{e-h} = V_{opt} \left| \left( \langle \xi^e_S | \xi^e_L \rangle \right) \right| \left. \left( \langle \xi^h_S | \xi^h_L \rangle \right) \right| . \tag{11}
\]

The phase-relaxation length that we introduced above is related to the expressions in (11), in the simplest case [23], through \( \langle \xi^e_S | \xi^e_L \rangle = e^{-\langle L_e+S_h \rangle/L_{S}^{e}} \) for both \( x = e, h \).

The interferometer. From now on, we shall use for numerical estimates typical values for electrons, extracted from Ref. [20]. One must not forget that holes often have smaller mobilities in these structures, so the figures may not apply to one half of the interferometer — but the principles of the analysis do apply. In the forthcoming discussion, we shift when convenient from "lengths" \( \ell \) to "times" \( \tau \), the link being provided by \( \ell = v_F \tau \) where \( v_F \) is the Fermi velocity in a 2DEG or 2DHG, typically \( v_F \approx 3 \times 10^7 \text{ cm/s} \). In the optical experiment, we introduced the requirement (1) on \( \Delta L \) for the Franson interferometer to show two-particle interferences. In the present proposal, it is trivial to have the coherence length of the pair \( \ell_c \) exceed all the other meaningful lengths, since \( \ell_c \) is determined by the coherence time of the pump laser, and cw lasers easily have a coherence time of tens of nanoseconds. However, here we must meet an additional constraint due to the role of the environment. When photons essentially do not couple to the environment, electrons and holes propagating in semi-conductors interact strongly, especially with the other quasi-particles. Because of this coupling, some which-path information is transferred out of the system under study, whose coherence is thus decreased. So the size of each interferometer must not be too big: for both electrons and holes, \( L+S \) should be much smaller than \( L_F \), a phase-relaxation length that characterizes the coupling with the environment. Since (at least in principle) the S path can be made arbitrarily short we have \( L+S \approx \Delta L \), and we can summarize our present requirements as \( L_{e}^{e,h} > \Delta L_{e,h} \rightarrow \ell_{e,h} \). Confident in the precision of semiconductor manufacturing techniques, we admit that this requirement holds if and only if \( L_{e}^{e,h} \gg \ell_{e,h} \), that is, if

\[
\tau_{e,h}^{e} \gg \tau_{e,h}^{h} . \tag{8}
\]

We focus on the electron, the analog holds for the hole.

We want to show that (8) can be fulfilled if the electron is injected into the 2DEG close enough to the Fermi level. Refer to Fig. 2 (c). The electron is injected into the 2DEG with an energy \( E = \epsilon_F + \Delta \epsilon \), where \( \epsilon_F \) is the Fermi energy of the 2DEG, typically some 0.1 eV [20]. Since the main relaxation mechanism will be e-e inelastic scattering, the phase-relaxation time is \( \tau_{\varphi} \sim h\epsilon_F/\Delta \epsilon^2 [21] \). For instance, if the electron is injected into the 2DEG with \( \Delta \epsilon = 100\epsilon_F \sim 10^{-3} \text{ eV} \), then we obtain \( \tau_{\varphi} \approx 100 \text{ ps} \), in good agreement with the observed values of \( L_{\varphi}^{e,h} \), that are typically some 10 \( \mu \text{m} \) (see [15] and refs therein). The single-particle coherence time can be estimated by \( \tau_{c}^{e} \sim \hbar/\Delta E \), where \( \Delta E \) is the uncertainty in the electron kinetic energy. In Fig. 2 (c), one sees clearly that this uncertainty is determined by the relation between the steepness \( \phi' = \frac{dF}{dx}(x = 0) \) of the built-in potential and the width \( w \) of the laser spot. In particular, \( \Delta E \lesssim (\Delta \epsilon)_{\max} \approx c\phi'/w \), this being the largest value of \( \Delta \epsilon \). From the expressions of \( \tau_{\varphi}^{e} \) and \( \tau_{c}^{e} \), one de-

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principle achievable using single-electron transistors as detectors [25]. But very fast measurements introduce unwanted excitations that may hide the signal; moreover, \( \tau_{\text{meas}} > \tau_{e,h}^\text{c} \) must hold in order to detect the quasi-particles. Anyway, \( V \approx \frac{1}{\tau} \) would already be a fair demonstration of entanglement, because the origin of this reduced visibility in a Franson setup is well understood.

In conclusion, we have argued that energy-time entanglement of quasi-particles can be observed. The task is challenging, but the goal seems within reach with present-day technology. 2DEG-2DHG junctions are promising candidates as entanglers for energy-time entanglement of electron-hole pairs.

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