Evaluation of the wheel slip control quality during braking with the ABS system acting

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Abstract. The paper presents the method of assessing the degree of the wheel slip utilization during braking in relation to the slip adopted as optimal in the analyzed braking process.

In an ideal ABS system, the wheel slip should be equal to the slip assumed to be optimal under the given braking conditions. In a real, pulsed system ABS, wheel slip changes in a wide range of values.

The quantitative assessment of the correct operation of the real ABS/ESP system can be made on the basis of indicators showing the degree of approximation of the actual slip of the wheel to the value assumed to be optimal due to the wheel's adhesion to the roadway.

In the paper has been analyzed the wheel slip utilization as the difference of the actual circumferential speed of the wheel and the speed of the wheel at which the slip is optimal in the given braking conditions, referenced to a specific braking time:

\[ \Delta L = v_k - v_{k, opt} \]

The calculation of this indicator requires the measurement of the car's speed and wheel speed and the determination of the optimal slip value on a given surface. The paper analyzes the impact of the optimal slip value on the assessment of ABS performance with various control algorithms (Independent Regulation IR, Select Low SL).

The article presents the results of tests of the slip utilization index in various braking conditions and for brakes with damaged mechanical components.

1. Introduction
Development of ABS systems requires the adoption of a quality criterion for these systems. Criterion should be objective and as simple as possible for implementation.

The task of the ABS/ESP system is to maintain the stability of the vehicle's motion and steerability during braking and overcoming corners, and to optimally use the adhesion of the wheels to the road surface. Execution of these requirements is closely related to the wheel slip of the front and rear wheels of the car.

During the homologation tests, the assessment of adhesion utilization is carried out in road conditions, determining the coefficient of adhesion utilization during straight braking [1]:

\[ e_{de} = \frac{z_{max}}{k_{dr}} \]  

where:
- \( z_{max} \) - the ratio of the average braking deceleration when decreasing the vehicle speed from 40 to 20 km/h with the ABS system working, to the acceleration \( g \),
- \( k_{dr} \) - coefficient of adhesion utilization of the road surface on which the test is carried out, determined during braking the wheels of one axle, at the border of blocking the wheels, with the ABS system off.
The utilization of wheel grip to the roadway is closely related to the slip of the tire in relation to the road:

\[ s_k = \frac{v_s - v_k}{v_s} 100\% \]  

where:

\( v_s \) - longitudinal speed of the wheel axis,
\( v_k \) - circumferential speed of the wheel.

Optimal use of adhesion, while maintaining the stability of the motion during braking requires keeping the longitudinal slip of the wheel within a certain narrow range of values between two vertical lines, Fig. 1. Increasing the longitudinal slip results in a quick reduction of lateral grip, thus limiting the possibility of overcoming corners.

![Figure 1](image)

**Figure 1.** The dependence of the longitudinal adhesion coefficient \( \mu_x \) and transverse \( \mu_y \) of the tire to the slip \( s_k \) [8].

The paper presents the method of assessing the degree of use of wheel slip in relation to the slip adopted as optimal in the braking process.

2. **Analysis of the wheel slip utilization during braking**

The ABS control systems work impulsively It causes that the wheel slip changes within a wide range of values, Fig. 4. In the ideal system, this slip should maintain a value close to the slip adopted as optimal under the given braking conditions.

The quantification of the correct operation of the actual ABS / ESP system can be made on the basis of indicators showing the difference between the actual wheel slip and the value assumed to be optimal due to the wheel's adhesion to the roadway.

In the case of ABS, the optimum slip value means the slip, at which function \( d\mu/ds \) (derivative of the coefficient of adhesion relative to wheel slip) achieves a certain minimum value during the rise period of the coefficient of adhesion.

Wheel slip control algorithms are constantly developing [4, 6, 7]. In ABS systems, the control program aims to maintain such a circumferential wheel delay so that the slip does not exceed the value at which the coefficient of adhesion reaches its maximum. In ESP systems, the control algorithm adjusts the values of longitudinal forces (brake caliper pressure) to the current values of lateral forces, so that longitudinal slip does not exceed the limit value adopted in the control algorithm.

As a parameter of the ABS system performance, the difference between the actual circumferential speed of the wheel \( v_k \) and the optimal wheel speed \( v_{kopt} \) can be taken, Fig. 2. If the analysis refers to a specific braking time \( \Delta t = t_2 - t_1 \), the parameter \( \Delta L \) can be determined as:

\[ \Delta L = \sum_{i=1}^{n} \Delta L_i = \int_{t_1}^{t_2} v_{kopt} - v_k \, dt \]  

where:
\( v_k \) - circumferential speed of the wheel,

\( v_{kopt} \) - the speed of the wheel, at which the wheel reaches the slip adopted as optimal,

\( t_1 \div t_2 \) - ABS system operation time accepted for analysis.

The calculation of this indicator requires the measurement of the car's speed and wheel speed and the adoption of a certain value of the optimal slip.

![Figure 2. Graphical interpretation of the \( \Delta L \) index (sum of surfaces \( \Delta L_i \)).](image)

The \( \Delta L \) indicator is shown graphically in Fig. 2 as the sum of the area of \( \Delta L_i \) fields. For the ideal anti-blocking system, the value of \( \Delta L \) for straight braking should aim at zero. The fields below the \( v_{kopt} \) line indicate that the optimum slip has been exceeded and that the transverse force transmission is limited. The fields above the \( v_{kopt} \) line indicate that the slip is less than optimal and reduced coefficient of longitudinal adhesion in relation to the possible value, i.e. braking distance is longer.

From the dependence (2), it follows that \( v_k = v_s (1-s_k) \) and \( v_{kopt} = v_s (1-s_{kopt}) \). After inserting into (3), we can calculate the parameter \( \Delta L \):

\[
\Delta L = \int_{t_1}^{t_2} [s_k - s_{opt}] v_s \, dt
\]

where: \( s_{opt} \) - wheel slip on the surface on which braking takes place, adopted as optimal.

The calculation of the \( \Delta L \) index requires setting the optimal slip value, which should keep the wheel on the given surface. It depends on the type of surface, the properties of the tires, and changes with the speed of the tested wheel [8, 9]. Therefore, the \( s_{opt} \) value should be determined for specific conditions of braking tests, or as a limit slip adopted in a given ABS control algorithm. Such a fixed value of the \( s_{opt} \) allows to evaluate the algorithms of wheel slip control for individual IR control, for select low SL principle and others. However, for the purpose of evaluating the operation of the ABS valves, or for diagnostics, the mean slip or modal slip value can be used as a reference value. This adoption makes easier the diagnosis procedure, but limits the ability to evaluate the system only to control of the operation of ABS valves.

The more universal indicator of the evaluation of the ABS system for individual wheels is the coefficient \( k \), expressed as follows:

\[
k = 1 - \frac{\Delta L}{L_{kopt}} = 1 - \frac{\int_{t_1}^{t_2} [v_k - s_{opt}] v_s \, dt}{\int_{t_1}^{t_2} v_{kopt} v_s \, dt} = 1 - \frac{\int_{t_1}^{t_2} [v_{kopt} - v_k] \, dt}{\int_{t_1}^{t_2} v_{kopt} \, dt}
\]

where: \( L_{kopt} \) - the road which the wheel would have traveled in time \( t_1 \div t_2 \), with a slip of the \( s_{opt} \), Fig.2,
When the wheel slip $s_k = s_{opt}$, the value of coefficient $k = 1$. If the wheel is locked, $s_k = 1$, and the coefficient $k = 0$.

The tests on the high speed drum stand have shown that when braking with an efficient ABS, the $k$-factor achieves $k > 0.8$ (for front wheels).

The coefficient $k$ determines the use of the wheel slip in relation to the slip adopted as optimal. However, it does not allow to determine the tendency of the braking process. This is due to the fact that $k$ assumes the same values for the slip $s_k = s_{opt} + \Delta s$ and $s_k = s_{opt} - \Delta s$. In order to obtain information on the scope of the real slip in relation to the one optimal, the expression (6) should be calculated:

$$\Delta L' = \int_{t_1}^{t_2} (s_k - s_{opt}) v dt = \int_{t_1}^{t_2} (v_{\text{kp}} - v_k) dt$$

(6)

This is the difference between the $\sum \Delta L_i$ fields above and below the $v_{\text{kp}}$ speed curve (Fig. 2). If $\Delta L' > 0$, it means that the vehicle has been braked with a slip greater than the $s_{opt}$, i.e. driving with reduced lateral force transmission. If $\Delta L' < 0$, there was a tendency to not use optimal slip and maximum road adherence.

3. Test results

The tests of the slip utilization coefficient were performed in the stand conditions on a high speed drum stand and in road conditions. In the stand conditions, the tests were performed on smooth, dry steel drums, with speed about 50 km/h. In road conditions on wet asphalt road. The car was equipped with 4-channel ABS, summer and winter tires. The value of this coefficient for the braking system with mechanical damage to the caliper of one of the wheels was also examined.

On the basis of preliminary tests, for the calculation of the coefficient $k$ in the stand tests, the value of $s_{opt} = 18\%$ was assumed, and in road tests $s_{opt} = 10\%$. The peripheral speed of the drums was assumed as speed $v_s$. In road tests, the speed $v_s$ was measured by the Correvit optical system.

Exemplary results of braking tests on the drum stand with activated ABS system are shown in Fig. 3. The results of calculations of the slip coefficient $k$ and average slip values $\vec{s}$ are presented in Tab. 1 and 2. The coefficient $k$ was calculated for $s_{opt} = 18\%$.

![Figure 3](image-url)

**Figure. 3.** Speed and wheel slip on the drum stand. $v_s$ - peripheral speed of drums ("speed of the car"), $v_{kl}, v_{kp}$ - peripheral speed of left and right wheel, $s_{kl}, s_{kp}$ - wheel slip - left and right wheel, a), b) summer tires, c), d) winter tires.
Table 1. The coefficient of optimal slip utilization \( k \), the average slip value \( \bar{s} \). Tests on the drum stand, dry steel surface, summer tires.

| Wheel | \( k \) [%] – average of 3 measurements | Standard dev. \( k \) [%] | \( \bar{s} \) [%] – average of 3 measurements | Standard dev. \( \bar{s} \) [%] |
|-------|---------------------------------|-----------------|---------------------------------|-----------------|
| FL    | 90,0                            | 0,7             | 16,8                            | 0,2             |
| FR    | 90,5                            | 1,0             | 20,0                            | 0,5             |
| RL    | 85,3                            | 1,0             | 8,4                             | 1,4             |
| RR    | 85,3                            | 0,4             | 13,1                            | 0,7             |

Table 2. The coefficient of optimal slip utilization \( k \), the average slip value \( \bar{s} \). Tests on the drum stand, dry steel surface, winter tires.

| Wheel | \( k \) [%] – average of 3 measurements | Standard dev. \( k \) [%] | \( \bar{s} \) [%] – average of 3 measurements | Standard dev. \( \bar{s} \) [%] |
|-------|---------------------------------|-----------------|---------------------------------|-----------------|
| FL    | 90,4                            | 0,2             | 19,2                            | 1,0             |
| FR    | 90,6                            | 0,4             | 22,1                            | 0,8             |
| RL    | 86,9                            | 0,1             | 14,2                            | 0,7             |
| RR    | 84,0                            | 0,7             | 13,5                            | 0,9             |

The exemplary results of road tests with activated ABS system for wheels of the right side of the car are shown in Fig. 4. The results of calculations of the slip coefficient \( k \) and average slip values \( \bar{s} \) are presented in Tab. 3 and 4. The coefficient \( k \) was calculated for \( s_{opt} = 10\% \) and for comparative purposes for \( s_{opt} = 18\% \) (as for the drum stand).

![Graph](image1.png)

**Figure 4.** Speed and wheel slip in road conditions. \( v_s \) – car speed, \( v_{kp}, v_{kt} \) – peripheral speed of front and rear wheels, \( s_{kp}, s_{kt} \) – slip of the front and rear wheels, a) summer tires on wet asphalt, b) summer tires on dry asphalt, c), d) winter tires on wet asphalt.

The braking shown in Fig. 4b) is more intensive in comparison to the others. This braking took place on the dry asphalt.
Table. 3. The coefficient of the optimal slip utilization $k$, the average slip value $\bar{s}$. Road conditions, wet asphalt surface, summer tires.

| Wheel | $k_{10}$ [%] average of 3 measurements | Standard dev. $k$ | $\bar{s}$ [%] | Standard dev. $\bar{s}$ | $k_{18}$ [%] |
|-------|----------------------------------------|------------------|----------------|--------------------------|-------------|
| FR    | 92.6                                   | 1.1              | 11.3           | 0.9                      | 85.4        |
| RR    | 92.0                                   | 0.5              | 6.7            | 1.5                      | 81.7        |

Table. 4. The coefficient of the optimal slip utilization $k$, the average slip value $\bar{s}$. Road conditions, wet asphalt surface, winter tires.

| Wheel | $k_{10}$ [%] average of 3 measurements | Standard dev. $k$ | $\bar{s}$ [%] | Standard dev. $\bar{s}$ | $k_{18}$ [%] |
|-------|----------------------------------------|------------------|----------------|--------------------------|-------------|
| FR    | 91.8                                   | 0.7              | 7.8            | 0.7                      | 85.4        |
| RR    | 91.4                                   | 0.1              | 6.9            | 0.5                      | 83.6        |

The presented results indicate that in stand tests on the high-speed drum stand, the coefficient of optimal slip utilization $k$ was higher for the front wheels than for the rear wheels. The average slip was much larger for the front wheels than for the rear wheels. The utilization of slip for summer and winter tires was similar. Average slip was greater for winter tires than for summer tires.

In the road tests, the utilization of optimal slip was greater than on the stand tests.

In all tests the slip of the rear wheels was smaller than the slip of the front wheels (e.g. Fig. 4 a), b). This is due to the fact that the rear wheels are responsible for maintaining the stability of the vehicle movement. The smaller longitudinal slip allows to get greater value of the adhesion coefficient in the transverse direction, Fig. 1.

In the tested ABS systems, the front wheels were controlled individually (IR), and the rear wheels were controlled according to the select low principle (SL). With different coefficients of adhesion of the right and left rear wheels to the road surface, the ABS system adjusts the slip of both rear wheels according to the parameters of the wheel rolling on the surface with a lower coefficient of adhesion. The effect of this type of control is less use of the grip of the wheel rolling on the surface with a higher coefficient of adhesion, and therefore smaller changes in its slip (right rear wheel in Fig. 6). The difference in the velocity and slip of the rear wheels, controlled according to the principle of select low SL, also occurs when their normal load is differentiated on the road, i.e. the asymmetric loading of the right and left side of the vehicle.

![Figure 5](image-url)  
**Figure. 5.** Speeds and slips of rear wheels on the surface of different grip: dry surface under the right wheel, wet under the left.

### 3.1. Testing the operation of the ABS system with a damaged brake system

The failure consisted in blocking the flow of brake fluid to the brake caliper. This caused difficult piston travel and delayed release the wheel. The results of braking with this damage are shown in Fig. 6, and
the results of calculations in tab. 5. You can notice a much higher amplitude of the wheel’s speed pulsation for the damaged braked wheel. The slip ratio $k$ decreased by approximately 35% in relation to the slip ratio for the efficient wheel. The average slip was about three times greater than the slip adopted as optimal ($s_{opt} = 18\%$). This test showed that the efficiency of mechanical components of the brakes has a decisive influence on the operation of ABS systems. Correct operation of the ESP system also requires full efficiency of mechanical components.

![Figure 6](image)

**Figure 6.** Speeds and slips of the front and rear wheels. Left front wheel brake caliper with piston partial locking.

**Table 5.** The coefficient of slip utilization $k$, the average slip value $\bar{s}$. The left front wheel brake is damaged.

| Wheel | $k_{10}\%$ average of 3 measurements | $\bar{s} \%$ |
|-------|-------------------------------------|--------------|
| FL    | 57,5                                | 52,1         |
| FR    | 89,0                                | 21,5         |
| RL    | 89,7                                | 18,5         |
| RR    | 85,6                                | 12           |

### 4. Conclusions

1. The assessment of the ABS system performance can be made on the basis of the calculation of the factors $\Delta L$, $k$, $\Delta L'$ showing the degree of approximation of the actual wheel slip to the value assumed to be optimal.
2. The application of the slip utilization coefficient $k$ for the evaluation of the operation of the ABS system in road conditions requires knowledge of the ABS control algorithm (reference speed and boundary slip). These conditions should be determined in consultation with the manufacturer of the program for ABS system. It is difficult to obtain these parameters. For this reason simpler method is proposal in point 5 of the conclusions.
3. Testing and analysis of the factor $k$ is applicable at the stage of programming the ABS controller. Its purpose is to minimize real slip deviations from the one adopted as optimal.
4. Determination of the slip utilization coefficient $k$ in relation to the optimal slip allows to assess the quality of the ABS operation.
5. The use of average slip $\bar{s}$ to calculate the coefficient $k$ means that the precision of wheel speed control by ABS valves is investigated, i.e. minimization of real slip deviations from average slip. This procedure is easier to implement and can be used in diagnostic tests of ABS systems but it does not allow to assess whether the ABS system ensures optimal road grip.
6. In the passenger cars under test, the rear wheels reached lower values of coefficient $k$ from the front wheels. Such a results should not be considered as a disability. This is due to the fact that the rear axle wheels are responsible for maintaining the stability of the vehicle movement. In the tested ABS systems they were controlled according to the principle (SL). The front wheels were controlled individually (IR).
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