Efficiency analysis of Muskingum-Cunge method and kinematic wave method on the stream routing (Study case: upper Ciliwung watershed, Indonesia)

D Prawira 1, H Soeryantono 1, E Anggraheni 1 and D Sutjiningsih 1
1Department of Civil Engineering, Universitas Indonesia, Depok, Indonesia
darmaprawira108@gmail.com, dwita@eng.ui.ac.id

Abstract. Flood routing is needed to determine the distribution of flood peak time and magnitude of peak flooding that occurs. Flood routing can be done on the river channel using the transform function principle. There are two methods of flood routing in reach that have significant differences based on general concepts and number of data requirements needed, namely Muskingum-Cunge method and Kinematic Wave method. This study aims to evaluate the efficiency of two flood routing methods, Muskingum-Cunge method and Kinematic Wave method using HEC-Geo HMS, the efficiency of each routing method is obtained from comparison of simulation’s hydrograph peak discharge. Results with the observation’s hydrograph peak discharge. Results of the comparisons show that Kinematic Wave method give a large peak discharge which is closer to the observation than simulation results with Muskingum-Cunge method. However, difference of result from two methods are not too far from observation, although Kinematic Wave method uses more detailed data. Based on these results, it can be concluded that Muskingum-Cunge method is more efficient than Kinematic Wave method in conducting flood routing because it provides a hydrograph that is close enough to fewer data requirements.

1. Introduction
Flood routing is a procedure to determine time and magnitude of a flow at a point from a hydrograph that is known or assumed at one or more points upstream. Flood routing can be done on river channel using transform function [1]. The transform function is a principle that focuses on transformation function to get output based on input given.

If viewed from the number of data requirements needed and the general concepts, there are two transform function methods that have significant differences, namely the Muskingum-Cunge method and the Kinematic Wave method. Muskingum-Cunge method performs a lump-sum from storage value in a certain time range. This method focuses on the relationship between storage and discharge in a channel cross section. Meanwhile, Kinematic-wave method estimates unsteady flow equation by ignoring pressure [2]. As a result of the more detailed needs, so a statement arises that Kinematic Wave method gives more accurate results than Muskingum-Cunge method of conducting flood routing. However, many data requirements effect to a longer data search process and require greater costs.

Based on these comparisons, the question arises, namely how much flood routing efficiency uses the Muskingum-Cunge method compared to routing floods using the Kinematic Wave method. In this journal, the study aimed to evaluate the efficiency of flood routing using the Muskingum-Cunge method.
rather than routing the flood of the Kinematic Wave method in simulating existing data hydrographs. The efficiency of each routing method is obtained from comparison of simulation’s hydrograph peak discharge results with the observation’s hydrograph peak discharge. Observation’s data are obtained from recording water level data in Katulampa Weir. Katulampa Weir is an outlet of the Upper Ciliwung watershed, Indonesia.

2. Research Methodology

The study was conducted using HEC-HMS to make a flood-routing model through reach using Muskingum-Cunge method and Kinematic wave method. Each method has different data needs. The results of modelling in HEC-HMS are hydrographs. This study located at Ciliwung Watershed. Ciliwung river is the main river that affecting Jakarta. The section that is used as test location is last reach to Katulampa’s Weir as we called Upper Ciliwung Watershed. This sub watershed has rapid infrastructure development that causes the hydrological regime change [3]. The observation point taken as existing hydrograph data recording point is outlet of Upper Ciliwung watershed, namely Katulampa’s Weir. This study used secondary data to modelling the watershed. As for data needed like river maps, topographical maps, soil type maps, land use maps, rainfall data and river cross sections.

The first step is by delineating the Upper Ciliwung watershed using HEC-Geo HMS based on DEM maps and rivers maps. By using HEC-Geo HMS, data preparation is done in the form of slope, river length, and many more. Results of modelling on HEC-Geo HMS will be exported into HEC-HMS. In process of flood routing, rain data is needed based on rainfall-station data that affects the watershed. The rainfall data entered must be in accordance with the rain that occurred at the time of the observation. Therefore, a daily rainfall calculation is carried out. Rainfall data are entered during the modelling stage at HEC-HMS.

There are two simulations in this research namely the Muskingum-Cunge method as the first method and the Kinematic Wave method as the second method. The simulation hydrograph from two methods was compared with observational hydrograph data. Hydrograph comparison is done in terms of visuals (difference in peak discharge and duration of flooding, and prediction of hydrological model using Nash-Sutcliffe test). After the comparison, an evaluation of efficiency of flood routing can be carried out with both methods.

2.1. Flood routing method in reach

2.1.1. Muskingum-Cunge method

The Muskingum method is a hydrological routing method that solved by mass conservation laws and the relationship of discharge with storage [1]. This method models flood storage volumes in river channels with a combination of wedge storage and prism storage. Wedge storage is a storage change that occurs due to changes in inflow and outflow when a flood wave occurs in a river cross section. Whereas, prism storage is a constant volume of storage from a cross section along the river channel. During periods of increased flooding, wedge storage is positive and added to prism storage. It is better, during the decline in floods, wedge storage is negative and reduces the prism storage value.

Wedge storage is assumed as $KX(I - Q)$ and prism storage is assumed as $KQ$. So, it can be seen that the total storage equation of a river channel cross section for the Muskingum method is as follows:

$$S = KX + KX(I - Q)$$

(1)

The change in storage over time interval $\Delta t$ are as follows:

$$S_{j+1} - S_j = K \left[ (X_{i+1} + (1 - X)Q_{i+1} + 1) \cdot [X_{i+1} + (1 - X)Q_{i+1}] \right] \cdot$$

(2)

The change in storage can also be expressed:

$$S_{j+1} - S_j = \frac{(I_j + I_{i+1})}{2} \Delta t - \frac{(Q_{j+1} - Q_j)}{2} \Delta t$$

(3)
Combining (2) and (3) and simplifying gives:

\[ Q_{i+1}^{j+1} = C_1 Q_i^{j+1} + C_2 Q_i^j + C_3 Q_{i+1}^j. \]  

where i and j are sequence of n. The values of C1, C2, and C3 are routing parameters which can be calculated with the following equation:

\[ C_i = \frac{\Delta t - 2k\Delta x}{2K(1 - k) + \Delta t}. \]

\[ C_2 = \frac{\Delta t + 2k\Delta x}{2K(1 - k) + \Delta t}. \]

\[ C_3 = \frac{2k(1 - k) - \Delta t}{2K(1 - k) + \Delta t}. \]

Note that C1 + C2 + C3 = 1.

Cunge determines the value of K and X according to the Kinematic Wave equation where the value of K and X can be formulated as follows:

\[ K = \frac{\Delta t}{c_i} = \frac{\Delta x}{dQ/dA}. \]

and

\[ X = \frac{1}{2} \left( 1 - \frac{Q}{Bc_i S_{b\Delta x}} \right). \]

where Ck as celerity, and B as the channel width.

2.1.2. Kinematic wave method.
Kinematic Wave method model comes from the Saint-Venant equation. The Saint-Venant equation uses continuity equations and momentum equations. The general continuity equation states that inflow minus outflow equals the rate of change in storage [4]. The continuity equation is

\[ \frac{\partial A}{\partial t} + \frac{\partial Q}{\partial x} = q \]

where q is the lateral inflow rate per unit channel length and A is the channel cross-sectional area. For overland flow model, the momentum equation can be expressed as follows:

\[ S_f = S_0 - \frac{\partial y}{\partial x} - \frac{v}{g} \frac{\partial v}{\partial x} - \frac{1}{g} \frac{\partial v}{\partial t}. \]

For shallow flow, bottom slope and the energy gradient are approximately equal and acceleration effects are negligible, so the momentum equation simplifies to:

\[ S_0 = S_f \]

where So is base slope of the channel and Sf is slope of friction.

2.2. Rainfall distribution
To obtain the rainfall data the average of watershed can be calculated by polygon thiessen method. Polygon thiessen method is calculated based on the equation as follows:

\[ \bar{P} = \frac{A_1P_1 + A_2P_2 + A_3P_3 + \cdots + A_nP_n}{A} = \sum A_iP_i. \]

where \( \bar{P} \) is the regional average rainfall, \( n \) is number of observer station, \( P_i \) is rainfall of each observer station, and \( A_i \) is influence area from each observer station [5].
If there are no rain data obtained within 24 hours from results of the recording, the synthetic rainfall pattern is used to obtain rain data. This study uses the distribution of rain in West Java shown in Table 1.

| Rain pattern (hourly) | Hourly Percentage (%) 1 2 3 4 5 6 7 8 | Interval (hour/pattern) |
|----------------------|----------------------------------------|------------------------|
| 3                    | 68 24 8                               | 1                      |
| 4                    | 26 61 10 3                            | 1                      |
| 5                    | 11 54 28 6 1                         | 1                      |
| 6                    | 12 54 24 6 3 1                       | 1                      |
| 7                    | 50,5 25,5 12,6 6,5 3,4 1,2 0,3 1      |                        |
| 8                    | 12,3 50,2 4,4 7,7 21,5 2,4 1,2 0,3 1  |                        |

### 2.3. Model calibration

Model calibration in this research visualizes the simulation hydrograph of the observed hydrograph. Visual comparison of hydrographs is based on three aspects, namely rising time, peak discharge and flood time. When the time rises, it is compared to the length of time when a new flood occurs and reach a peak discharge. At peak discharge, the difference of peak discharge will be compared between the simulation and the existing results. At time of the flood, the length of time needed will be compared to the flood from the new start until it approaches the base flow.

The Nash-Sutcliffe test is used for calibration. The efficiency coefficient of the Nash-Sutcliffe model is used to assess the predictive power of the hydrological model. The most important statistical indicator in determining the reliability of the model is $R^2$. $R^2$ shows the efficiency value of the hydrological model with its existing conditions. The $R^2$ indicator value is calculated based on equation (12) below:

$$R^2 = 1 - \frac{\sum (Q_0 - Q_m)^2}{\sum (Q_0 - \bar{Q})^2}$$  \hspace{1cm} (12)

where $Q_0$ is the simulation discharge (m$^3$/s), $Q_m$ is the observational discharge (m$^3$/s), and $\bar{Q}$ is the average simulation discharge (m$^3$/s) [6]. The statistical indicator is to evaluate the performance of the model in terms of comparing the results of the model with the observed data. The optimal value for $R^2$ approaches one. The formulation of the $R^2$ equation is based on the Nash-Sutcliffe model efficiency indicator. Nash-Sutcliffe test has a range between 0 and 1. Nash-Sutcliffe test has several criteria shown in Table 2.

| Nash-Sutcliffe Efficiency (NSE) | Interpretation     |
|--------------------------------|--------------------|
| NSE > 0,75                     | Good               |
| 0,36 < NSE < 0,75              | Satisfied          |
| NSE > 0,36                     | Not Satisfied      |

### 3. Result and Discussion

#### 3.1. Property identification of Upper Ciliwung watershed

Model 1 uses the Muskingum-Cunge method, while model 2 uses the Kinematic Wave method. Upper Ciliwung watershed consists of seven sub-catchments and three reach. The seven sub-watersheds are W80, W90, W100, W110, W120, W130, and W140 sub-basins. Meanwhile, the three reach are R10, R30 and R40 (Figure 1).

The process of identifying the Ciliwung Watershed is carried out using HEC-Geo HMS. Results of identification are used as data in running HEC-HMS. The watershed properties sought is the area of each sub-watershed, reach length and slope, and Curve Number value for each sub-watershed. (Table 3 and Table 4).
Figure 1. Sub-areas and reach of kelapa gading watershed

Table 3. Sub-areas properties data

| Sub-area Name | A (Km²) | Curve Number |
|---------------|---------|--------------|
| W80           | 17.731  | 83           |
| W90           | 26.036  | 76           |
| W100          | 49.531  | 79           |
| W110          | 16.683  | 80           |
| W120          | 1.231   | 83           |
| W130          | 22.271  | 78           |
| W140          | 16.971  | 77           |

Table 4. Reach properties data

| Reach Name | Length (m)  | Slope (%) |
|------------|-------------|-----------|
| R10        | 3933.847    | 0.02593   |
| R30        | 1277.203    | 0.03837   |
| R40        | 2694.670    | 0.03562   |

3.2. Observational hydrograph
Simulation results will be compared against observational data. Observation data that is compared is one day's discharge. Data selected is the discharge data on April 13th, 2017 at 07.00 to April 14th, 2017 at 07.00. This data was chosen because it has highest discharge in 2017. (Figure 2). Based on observation’s hydrograph, it is known that peak discharge occurred in the 14th hour with the amount of discharge valued at 153,625 m³/s. The time needed to reach peak of flood, which is for two hours and starts at the 12th hour and there is a flood for 12 hours.
3.3. Analysis of rain load

Rain load is calculated based on hourly rainfall data from several rain stations that affect Upper Ciliwung watershed. The rain load given in the simulation is regional rainfall data on the day according to time of observation. There are three rain stations that affect this watershed. The three rain stations are Gadog rain station, Cilember rain station, and Gunung Mas rain station. The extent of the influence of each rain station on the Upper Ciliwung watershed can be seen in Table 5. The selected rainfall data is rainfall data on April 13th, 2017 for the three rain stations. The amount of daily rainfall data selected and the results of calculations with the area of influence are shown in Table 5.

Table 5. Result of regional rainfall

| Name of rainfall station | Influence percentage (%) | Rainfall Data (mm) | Result of influence area (mm) |
|-------------------------|--------------------------|--------------------|-----------------------------|
| Gadog                   | 16.93                    | 14.3               | 2.4                         |
| Cilember                | 32.06                    | 41                 | 13.1                        |
| Gunung Mas              | 51.01                    | 26                 | 13.3                        |
| Daily Rainfall          |                          |                    | 28.8                        |

After obtaining regional rainfall in the watershed, weighting is carried out on daily rainfall to obtain rainy days on that day. Hourly rain distribution is calculated based on the distribution of rain patterns in West Java shown in Table 1. Duration of rain distribution chosen based on observational hydrographs in Figure 2. Based on changes in the hydrographic discharge of the observation, it can be predicted that rain occurs for 3 hours. The distribution of rainfall for 3 hours on April 13th 2017 at 07:00 until April 14th 2017 at 07:00 is shown in Table 6.
Table 6. Result of hourly rain distribution

| Date            | Rain distribution of percentage | Rain distribution of Upper Ciliwung watershed |
|-----------------|---------------------------------|-----------------------------------------------|
| 13Apr2017, 07:00| 0%                              | 0                                             |
| 13Apr2017, 08:00| 0%                              | 0                                             |
| 13Apr2017, 09:00| 0%                              | 0                                             |
| 13Apr2017, 10:00| 0%                              | 0                                             |
| 13Apr2017, 11:00| 0%                              | 0                                             |
| 13Apr2017, 12:00| 0%                              | 0                                             |
| 13Apr2017, 13:00| 0%                              | 0                                             |
| 13Apr2017, 14:00| 0%                              | 0                                             |
| 13Apr2017, 15:00| 0%                              | 0                                             |
| 13Apr2017, 16:00| 0%                              | 0                                             |
| 13Apr2017, 17:00| 0%                              | 0                                             |
| 13Apr2017, 18:00| 68% 19.6                        |                                               |
| 13Apr2017, 19:00| 24% 6.92                        |                                               |
| 13Apr2017, 20:00| 8% 2.31                         |                                               |
| 13Apr2017, 21:00| 0%                              | 0                                             |
| 13Apr2017, 22:00| 0%                              | 0                                             |
| 13Apr2017, 23:00| 0%                              | 0                                             |
| 14Apr2017, 00:00| 0%                              | 0                                             |
| 14Apr2017, 01:00| 0%                              | 0                                             |
| 14Apr2017, 02:00| 0%                              | 0                                             |
| 14Apr2017, 03:00| 0%                              | 0                                             |
| 14Apr2017, 04:00| 0%                              | 0                                             |
| 14Apr2017, 05:00| 0%                              | 0                                             |
| 14Apr2017, 06:00| 0%                              | 0                                             |
| 14Apr2017, 07:00| 0%                              | 0                                             |

3.4. Simulations of model hydrograph
Simulations were did on both models based on selected rain loads using HEC-HMS. Simulation results from both models are shown in Figure 3.

![Flood Hydrograph on 13-14 April 2017](image)

(a)

**Figure 3.** Flood hydrograph in outlet (a) Kinematic Wave method
From Figure 3, hydrograph of the simulation results using Kinematic Wave method has a simulated peak discharge of 150.6 m$^3$/s at the 14$^{th}$ hour. From the hydrograph of the simulation results, it can be seen that floods began to occur at the 11$^{th}$ hour and occurred for 13 hours. The time needed to reach peak discharge for three hours.

From Figure 3, hydrograph of the simulation results using Muskingum-Cunge method have simulated peak discharge of 150.5 m$^3$/s at the 14$^{th}$ hour. From simulation’s hydrograph, it can be seen that floods began to occur at the 11$^{th}$ hour and occurred for 13 hours. The time needed to reach the peak discharge for three hours.

3.5. Comparisons of model hydrograph and observational hydrograph
Simulation’s hydrograph of the two models were compared with the observational hydrograph in Katulampa’s Weir. Comparison is done by comparing visually and conducting Nash-Sutcliffe test. Visual comparison of hydrograph is based on three aspects, namely rising time, peak discharge and flood time, while Nash-Sutcliffe test is compared based on hourly discharge.

3.5.1. Visual comparison
Simulation’s hydrographs of both model and the observation hydrograph are juxtaposed in one graph in Figure 4 to find out the differences in the three forms of hydrograph. The comparison results are summarized in Table 7.

Based on Figure 4, it can be seen that from two hydrographs the simulation results have differences with the hydrograph of the observation. At the 1$^{st}$ to 11$^{th}$ hours, the two hydrographs of the simulation results have a large base flow which is quite similar to the hydrograph observation because the rain has not fallen on the watershed. After the 12$^{th}$ hour or after the rain falls into the watershed, the second form of hydrograph has begun to experience a difference because the size of the simulation discharge is greater than the observation discharge.

After reaching the peak discharge, the hydrograph of the simulation results using the Kinematic Wave method has a higher discharge than the observation hydrograph after reaching the peak discharge until the end of the running period. The discharge rate is above the base flow rate of the observation discharge. Whereas, the hydrograph discharge from the simulation results with the Muskingum-Cunge method initially had a higher discharge than the observation hydrograph, but at the 17th hour the amount
of the discharge from the simulation results was smaller than the observation. From the differences in the discharge values of the two simulations with respect to observations on the recession side, it can be seen that the hydrograph of the simulation results using the Kinematic Wave method is in an overestimate condition and means that the results have entered within the range of observational data. Meanwhile, the hydrograph of the simulation results using the Muskingum-Cunge method is underestimated and means that the results have not reached the range according to the observation data.

**Figure 5.** Comparisons of model hydrograph and observational hydrograph

**Table 7.** Recapitulation of observational discharge and simulation discharge

| Point of observation | Peak discharge (m$^3$/s) | Time to peak (hour) | Flood time (hour) |
|----------------------|---------------------------|---------------------|------------------|
|                       | Q(observation) | Qr.w | Qm.c | Q(observation) | Qr.w | Qm.c | Q(observation) | Qr.w | Qm.c |
| Katulampa Weir        | 153.63         | 150.6 | 150.5 | 3.0           | 2.0   | 2.0   | 12.0           | 13.0   | 13.0 |

Based on Table 7 shows that the discharge simulation results are close enough to the observation discharge. From the two hydrographs of the simulation results, show the peak length of time and the same time of flooding between the two. The two hydrographs of the simulation results have a peak time that is one hour faster than the observation hydrograph. However, the duration of the flood is longer than one hour compared to the hydrograph of observation. Longer flood times are caused by the time needed to reach the peak faster. The large difference in peak discharge from the simulation results with observational discharge is not much different. The percentage of the large difference in peak discharge from the simulation results with the peak discharge observation of 1.97% for the simulation using the Kinematic Wave method and 2.03% for the simulation using the Muskingum-Cunge method.

Based on the value of the percentage difference, the peak discharge value from the watershed model simulation with Kinematic Wave method is closer than the other method. The value of the peak discharge from Kinematic Wave method simulation according to the initial statement that this method gives closer results than the results of Muskingum-Cunge method simulation. This is because Kinematic Wave method requires detailed cross-sectional data that is sized according to the field. Channel cross-section data is needed in calculating flow velocity and discharge so that the more detailed channel cross-section data, the more accurate flood routing is carried out. However, Muskingum-Cunge method yields results that are not much different and are still close enough to the observation discharge.
Therefore, just routing the flood using Muskingum-Cunge method is enough to provide results that are close enough with less data. Based on the results of visual comparison, the Muskingum-Cunge method is considered more efficient than the Kinematic Wave method. This statement is based on the results of the simulation that are close enough to observations with fewer data requirements.

3.5.2. Nash-Sutcliffe Test
Based on the results of the Nash-Sutcliffe test shown in Table 8, the $R^2$ value for the Kinematic Wave model is 0.773 and $R^2$ for the Muskingum-Cunge model is 0.833.

| Nash-Sutcliffe Test     | $R^2$ |
|------------------------|-------|
| Muskingum-Cunge Method | 0.833 |
| Kinematic Wave Method  | 0.773 |

Based on Table 2, it can be said that the two simulation results are in a good range so the level of accuracy of the simulation results on the observation discharge is quite good. From the two $R^2$ values, it is known that the $R^2$ value for the simulation results of the Kinematic Wave model is smaller than the $R^2$ value for the Muskingum-Cunge model simulation results. The Nash-Sutcliffe efficiency value shows the difference in the value of each debit data per hour from the discharge of the simulation results and observation discharge. The results of the Nash-Sutcliffe test showed that flood routing with the Muskingum-Cunge method was closer than the Kinematic Wave method when reviewing the hourly discharge. However, these two methods already have results that are close enough to do flood routing because they are categorized as good in the NSE test.

Based on the results of the Nash-Sutcliffe test, it is known that the Muskingum-Cunge method is more efficiently used in routing hourly debits with less data requirements than the Kinematic Wave method because it provides closer predictive power.

4. Conclusions
Based on comparison methods of simulation hydrograph, the result of two methods are quite close to observation. So, the method with less data show a more efficient method. Therefore, it can be stated that Muskingum-Cunge method (a method with less data requirements) is more efficient than Kinematic Wave method (a method that needs more detailed data) in simulating existing data recording hydrograph.

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Acknowledgement
The authors express their appreciation as the study was conducted using the research funding for indexed international publication “Hibah Penugasan Publikasi Internasional Terindeks 9 (PIT 9) with contract number NKB-0053/UN2.R3.1/HKP.05.00/2019” fiscal year 2019 granted by Universitas Indonesia.