Feasibility study of Muon Tomography application in a non-invasive representation of tumulus

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Overview

➢ Study of the effectiveness of muon tomography in a geological structure of smaller scale and of archaeological interest

➢ A MicroMegas telescope will be placed near Appolonia’s tumulus (EKATY programme)

➢ Simulation of the tumulus geometry
  Scanning of its internal structure by measuring the flux deficit which determines the integrated 2D density in the direction of observation (Transmission muography)

Combination of several 2D projections (by moving the muon detector or by surrounding the object with several instruments) → 3D information
Presentation Outline

• **Muon Tomography**

• Simulation
  ➢ Tumulus simulation
  ➢ Test for different materials
  ➢ Object inside the tumulus
  ➢ Geant4-MatLab Generator
  ➢ Telescope and real tumulus

• Back-projection method
  ➢ Telescope below the tumulus
  ➢ Telescope at the side of the tumulus

• Conclusions and future work
Muon Tomography

- A technique that uses cosmic muons to generate three-dimensional images of volumes

- **Useful Information**
  - Energy loss (Muon Radiography)
  - Multiple Coulomb Scattering (Muon Scattering Tomography)

- **Fields applied**
  - Geology
  - Archaeology (Large hidden chamber in the Great Pyramid of Giza)
  - Security (tracking of dangerous cargo)

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"Discovery of a big void in Khufu's Pyramid by observation of cosmic-ray muons", Kunihiro Morishima et al.

Large area detectors of Decision Sciences® enable scanning of commercial trucks.

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**Simulation**

**Tumulus simulation**

- **Geant4**
- **Uniform initial position at yz plane**
- **Initial momentum at +x**
- **Monoenergetic muons of 4GeV**
- **Material dirt \( \rho=1.2\, \text{g/cm}^3 \)**

Composition [The Engineer ToolBox]

Al(9%), Ca(3%), Fe (5%)/ Mg (0.6%), K(2%), Si (29%), Na (2.4%)

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**Dimensions**

- Top radius: 4m
- Bottom radius: 8m
- Height: 8m

smaller dimensions for first tests
Simulation
Test for different materials

- Ethanol ($\rho=0.78\text{g/cm}^3$)
- Concrete ($\rho=2.3\text{g/cm}^3$)
- Marble ($\rho=2.8\text{g/cm}^3$)

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Simulation
Object inside the tumulus

- **Dimensions**
  - Inner radius: 1.5m
  - Outer radius: 2.5m
  - Height: 5m

- **Material** → Marble
  1. Orientation parallel to z axis
  2. 90° rotation around y axis and repositioning to touch the ground
Simulation
Geant4-MatLab Generator

➢ Simulate cosmic muon distributions
➢ Generate user-defined histograms for use with the Geant4 General Particle Source
➢ Phenomenological model & statistical algorithms
➢ Implementation by postdoctoral researcher Georgios Tsiledakis

MATLAB distributions
• Energy range 1-60 GeV
• Zenith angle range 0–90°

Chatzidakis, Stylianos. (2015). A Geant4-MATLAB Muon Generator for Monte-Carlo Simulations
Simulation
Telescope and real tumulus

Real tumulus dimensions
- Base diameter 92m
- Top diameter 32m
- Height 17m

Muon telescope
- 4 detection planes 45x45cm, spaced by 20cm (L=60cm)
- Very thin planes of vacuum
- Top detection plane 3m below tumulus base

Cut in the tracks
- Projection of the initial muon track
- Accepted area ±3m from the detector centre
- Artificially kill the tracks that will not end to the accepted area

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Back-Projection method

➢ Specify the size and the location of the hidden object

➢ Favorable for the imaging of hidden objects surrounded by a large amount of different density material

➢ Not point-like muon telescope compared to the object under investigation

➢ Comparable distance between detector-object to the detector size

➢ Both the direction of the muon tracks and their impact point on the detector are useful

"A projective reconstruction method of underground or hidden structures using atmospheric muon absorption data", L. Bonechi et al
Back-Projection method

➢ Back-project muon tracks in vertical planes to the telescope axis
➢ Pitch (bin size) of back-planes: \( p(x) = \delta_\theta x + p_0 \)
➢ Two-dimensional histograms \((y/p, z/p)\)
➢ Subtract tumulus-with-monument histogram from uniform-tumulus histogram
➢ Width-to-pitch → rms of each projection
➢ Minimum of width-to-pitch ratio → object location
➢ FWHM at minimum → object dimensions

“A projective reconstruction method of underground or hidden structures using atmospheric muon absorption data”, L. Bonechi et al
Back-projection method
Telescope below the tumulus

- Iron and marble compact cylinder
- Diameter 1.5m, Height 1.5m
- Its side touches the ground
- Energy range 4-100GeV, Zenith angle 0-50°
- 34x34m generation plane, 1m above tumulus
- 1 billion initial muons
- Initial pitch 5mm → $\delta_\theta = 8.3\text{mrad} \ [\text{atan}(p/L)]$

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Back-projection method
Telescope below the tumulus

Width-to-pitch = Rms  \quad \text{Width} = \text{FWHM} \times \text{pitch}

Iron

Marble

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Back-Projection method
Telescope at the side of the tumulus

➢ Telescope centre at (48.0, -8.75)m below the ground level (z=-8.5m)
➢ 80° rotation around y
➢ Telescope “sees” the tumulus base

➢ Initial muon distributions
  i. Energy: 15-30GeV
  ii. Zenith: 70-88°
  iii. Azimuth: 78-102°

➢ Additional arrangement
  i. Tracks calculated at x=46.5m (the back-projection starts from the same plane)

➢ 300x40m generation plane at (-150,0,9.55)m
Back-projection method
Telescope at the side of the tumulus

- Two boxes at x=34m & x=39m
- Large box → Hollow box of 3m side and 0.5m width
  Small box → Compact box of 1.5m side
- Initial pitch $p=3\text{mm} \rightarrow \delta \theta = 5\text{mrad}$
- Materials: Marble & Iron (alternately)
- Monument width → $4\sigma$ (instead of FWHM)
Back-Projection method
Telescope at the side of the tumulus

- Small box closer
- Large box closer

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Back-projection method
Telescope at the side of the tumulus

- Large box at x=34m, small box at x=39m
- $p=5\text{mm} \rightarrow \delta_\theta=8.33\text{mrad}$
- Fit width-to-pitch ratio with a second order polynomial function in the region of the minimum
- Fit width distribution with a linear function in the same range

Minimum of fitted function: $d=7.015\text{m}$ (real distance $d=6.75\text{m}$)
Width at $d=7.015\text{m}$: $1.9602\text{m}$ (real box width $1.5\text{m}$)

Minimum of fitted function: $d=7.72488\text{m}$ (real distance $d=6.75\text{m}$)
Width at $d=7.72488\text{m}$: $1.59423\text{m}$ (real box width $1.5\text{m}$)
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Conclusions and future work

- Muon tomography could be applied in the scanning of tumuli
- By back-projecting the muon tracks the hidden structure becomes observable under certain conditions
- The structure is properly localized when its distance from the telescope and its size are comparable to the telescope dimensions (Structure 3-4 times the telescope size & a few meters away)
- Considering setting up a second detection apparatus in the experiment would provide more information
Thank you for your time!!
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Back-up slides
Muon angular distribution

- Directional Intensity $I_i(\theta, \phi)$ of particles of a given kind $i$: The number of particles $dN_i$, incident upon an element of area $dA$, per unit time $dt$, within an element of solid angle $d\Omega$

  $I_v = I(0^\circ)$ vertical intensity

- Flux $J_i$: Number of particles of a given kind $i$, traversing in a downward sense a horizontal element of an area $dA$, per unit time $dt$

\[
I_i(\theta, \phi) = \frac{dN_i}{dA \ dt \ d\Omega} \quad [\text{cm}^{-2}\text{s}^{-1}\text{sr}^{-1}]
\]

\[
J_i = \int I(\theta, \phi) \cos(\theta) \ d\Omega \quad [\text{cm}^{-2}\text{s}^{-1}]
\]

\[
I(\theta) = I(0^\circ) \cos^n(\theta)
\]
MatLab code
Inverse transformation

- Inverse transformation

The inverse transformation involves inversion of the Cumulative Distribution Function (CDF) to obtain random samples \( X = F^{-1}(U) \), where \( U \) is the uniform probability density function.

1) Invert CDF \( F_X \) to obtain the expression \( F_X^{-1}(U) \)
2) Generate random numbers uniformly distributed \( U(0,1) \)
3) Obtain random variable \( X \) from \( X = F_X^{-1}(U) \)

The inverse transform takes a uniformly generated random number and transforms it to a random observation \( x \) distributed as \( F_X \)

The muon angular distribution follows a squared cosine distribution, where \( A \) is the normalization constant

\[
f_{\Theta}(\theta) = A \cos^2(\theta)
\]

Correcting for the solid angle effect, where \( C \) is the normalization constant

\[
f_{\Theta}(\theta) = 2C \pi \sin(\theta) \cos^2(\theta)
\]

Calculating the CDF

\[
F_{\Theta}(\theta) = \int_0^\theta 2C \pi \sin(\theta) \cos^2(\theta) d\theta = 1 - \cos^3(\theta)
\]

The inverse transformation

\[
\theta = \arccos \left( \sqrt[3]{1 - F_{\Theta}(U)} \right)
\]

100000 zenith angles were randomly selected using the inverse transformation

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Chatzidakis, Stylianos. (2015). A Geant4-MATLAB Muon Generator for Monte-Carlo Simulations

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Back-projection method

➢ Signal tracks lying within the four lines $r_1$, $r_2$, $r_3$ and $r_4$

➢ Space to the right of the detector ($x>0$) → two zones: zone “1” limited by red lines, zone “2” limited by blue lines

➢ Width-to-pitch of the signal at distance $d$ approximated as the angular aperture $\lambda(d)$

➢ The structure of side $L$ has to be contained within the aperture of the hodoscope accepted angle $h$

Pitch: $p(x) = \delta_0 x + p_0$

- $x$: distance of the back-plane from the detector
- $p_0$: Spatial resolution of the single detection plane
- $\delta_0$: Angular resolution of the telescope
Width and Errors

- Width-to-pitch $\rightarrow$ rms of each projection
- Monument Width $\rightarrow$ FWHM*pitch

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Error in width-to-pitch $\rightarrow$ rms error
Error in width $\rightarrow$ 2.356*(rms error)*p

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Tumulus simulation
Secondary particles

- 300000 primary muons
- The secondary particles are of very low energy and their effect will not be observable
- They are ignored in the proceeding analysis

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Tumulus simulation
Object inside the tumulus

- Sphere of 1.5m radius in the centre
- Visualization of 20 muons generated at a constant position
- Muons are completely stopped by 3m of iron
- Less ionizations inside the void sphere
Tumulus simulation
Object inside the tumulus

- **Air** ($\rho=0.0012\text{g/cm}^3$)
  - Final muon position (Sphere of Air)
  - Kinetic energy at exit (Sphere of Air)

- **Concrete** ($\rho=2.3\text{g/cm}^3$)
  - Final muon position (Sphere of Concrete)
  - Kinetic energy at exit (Sphere of Concrete)

- **Marble** ($\rho=2.8\text{g/cm}^3$)
  - Final muon position (Sphere of Marble)
  - Kinetic energy at exit (Sphere of Marble)
Tumulus simulation
Sphere inside the tumulus

- Without sphere
- Sphere with air
- Sphere of marble

- Y projected angle vs Z projected angle with respect to the x axis at exit
- Only muons that traversed the tumulus
- Multiple Scattering is more important for materials of larger density
  (the different density material is relatively small → No apparent differentiation → Clearer image with more statistics)

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Back-projection method
Telescope below the tumulus

- Iron hollow box
- Outer side 3m, Inner side 2m
- Box base 6.5m from top detection plane

- \( p = 5.729 \text{cm} \)
- Projection width \( \sim 50 \text{cm}/p \)
- Box width 2.865m
Back-Projection method
Telescope at the side of the tumulus

- Telescope centre at (48,0,-8)m
- 80° rotation around y
- Telescope “sees” the tumulus base

- Initial muon distributions
  i. Energy: 15-50GeV
  ii. Zenith: 80-90°
  iii. Azimuth: 85-95°

- 400x20m generation plane at (-400,0,9)m

- Additional arrangements
  i. Tracks calculated at x=46.5m (the back-projection starts from the same plane)
  ii. Reject tracks with θ_z<80° & θ_z>90°
  iii. Initial pitch 1mm → δ_θ=1.67mrad
Back-projection method
Telescope at the side of the tumulus

- “Thin” box: 3m outer side, 2m inner side
- “Thick” box: 3m outer side, 0.5m inner side
- Its base touches the ground
- Marble
- Box at x=0
- 4 billion initial muons
Back-projection method
Telescope at the side of the tumulus

- Box at \( x = 30 \text{m} \)
Back-projection method
Telescope at the side of the tumulus

- Box at x=30m
- Energy range 20-30GeV
- Zenith angle range 85-90°
- “Thick” marble box
- 2 billion initial muons
Back-Projection method
Telescope at the side of the tumulus

- Telescope centre at (48,0,-8.75)m below the ground level (z=-8.5m)
- Box at x=39m
- Initial muon distributions
  i. Energy: 15-30GeV
  ii. Zenith: 70-88°
  iii. Azimuth: 78-102°
- 300x40m generation plane at (-150,0,9.55)m
Back-projection method
Telescope at the side of the tumulus

- Box at x=39m
- “Thick” marble box
- Two pitch values:
  - $p=3\text{mm} \rightarrow \delta_\theta=5\text{mrad}$
  - $p=5\text{mm} \rightarrow \delta_\theta=8.33\text{mrad}$
- Monument width $\rightarrow 4\sigma$ (instead of FWHM)