Spin flip of neutrinos with magnetic moment in core-collapse supernova

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Abstract

Neutrino with magnetic moment can experience a chirality flip while scattering off charged particles. This effect may lead to important consequences for the dynamics and the neutrino signal of the core-collapse supernova. It is known that if neutrino is a Dirac fermion, then $\nu_L \rightarrow \nu_R$ transition, induced by the chirality flip, leads to the emission of right-handed neutrinos, which are sterile (almost do not interact with matter). The typical energies of these sterile neutrinos are rather high, $E \sim (100 - 200)$ MeV. Neutrino spin precession in the magnetic field either inside the collapsing star or in the interstellar space may lead to the backward transition, $\nu_R \rightarrow \nu_L$. Both possibilities are known to be interesting. In the former case high-energy neutrinos can deliver additional energy to the supernova envelope, which can help the supernova to explode (Dar’s scenario of supernova explosion). In the latter case high-energy neutrinos may be detected simultaneously with the "normal" supernova neutrino signal, which would be a smoking gun for the Dirac neutrino magnetic moment. We report the results of the calculation of the supernova right-handed neutrino luminosity up to 250 ms after bounce, based on a dynamical model of the collapse. They allow to refine the estimates of the energy injected in the supernova envelope in the Dar’s scenario. Also the sensitivity of water Cherenkov detectors to the Dirac neutrino magnetic moment is estimated. For $\mu_{\nu_{\text{Dirac}}} = 10^{-13} \mu_B$ Super-Kamiokande is expected to detect at least few high-energy events from a galactic supernova explosion.

Also we briefly discuss the case of Majorana neutrino magnetic moment. It is pointed out that in the inner supernova core spin flips may quickly equilibrate electron neutrinos with non-electron antineutrinos if $\mu_{\nu_{\text{Majorana}}} \gtrsim 10^{-12} \mu_B$. This may lead to various consequences for supernova physics.

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1 Introduction

A straightforward way to account for neutrino masses is to introduce three singlet right-handed neutrinos (one per generation) in addition to three left-handed neutrinos of the Standard Model (SM). This allows to generate neutrino masses in the same way as up-quark masses, i.e. through the standard Higgs mechanism. Neutrinos, as quarks and charged leptons, are Dirac fermions in this case. Neutrino-Higgs vertexes are the only tree level vertexes which include right-handed neutrinos. Tiny neutrino masses imply tiny couplings with Higgs, therefore right-handed neutrinos interact extremely weakly with matter. In other words, they appear to be nearly sterile. In fact, measurements of the invisible Z-boson decay width (see e.g. [1]) and cosmological considerations (see e.g. [2]) tell us that there are only three active neutrino species, which are $\nu_eL$, $\nu_\mu L$, $\nu_\tau L$ (along with their antiparticles), and therefore right-handed Dirac neutrinos should be nearly sterile in any extension of the SM (with any additional particles and interactions).

Neutrinos may acquire magnetic moments through the loop diagrams. In the minimal extension of the SM (only three right-handed neutrinos added) magnetic moment of the neutrino mass eigenstate $\nu_i$ is proportional to its mass $m_i$ and reads (see [3],[4])

$$\mu_i = \frac{3eG_Fm_i}{8\sqrt{2}\pi^2} = 3.2 \cdot 10^{-19} \frac{m_i}{1\text{ eV}} \mu_B. \quad (1)$$

This value seems to be too small to produce any observable effect. However, a number of extensions of the SM exist in which neutrino magnetic moments are orders of magnitude larger (see one of the pioneering papers [5] and a review [6] with further references therein).

Historically a considerable interest to the possibility of large neutrino magnetic moment was caused by a proposition to explain the Solar neutrino deficit through the neutrino spin precession in the magnetic field of the Sun. The idea was first presented in 1971 [7], and elaborated on in a set of papers [8]-[12] in 1986-1987. Later, however, the neutrino flavor mixing was established to be a correct solution of the Solar neutrino problem; as for the spin precession hypothesis, neutrino magnetic moment values which it implied were disfavored by the astrophysical constraints (which are discussed below).

Core-collapse supernova was soon realized to be another astrophysical object for which neutrino magnetic moment could be important. In the beginning of the year 1987 A. Dar proposed a scenario of supernova explosion based on the two-stage $\nu_L \rightarrow \nu_R \rightarrow \nu_L$ transition of Dirac neutrinos, where the first stage could occur in the supernova core due to the electromagnetic scattering of neutrinos on charged particles, and the second one – in the supernova envelope due to the neutrino spin precession in the magnetic field of the star [13]. The detection of neutrinos from a nearby supernova SN1987A on February 23,
Laboratory experiment GEMMA  \[28\] \( \mu_\nu < 5.8 \cdot 10^{-11} \mu_B \) 90% CL

Cooling rates of white dwarfs  \[25\] \( \mu_\nu \lesssim 10^{-11} \mu_B \)

Cooling rates of red giants (see e.g.  \[26\] ) \( \mu_\nu \lesssim 3 \cdot 10^{-12} \mu_B \)

Supernova energy losses  \[30\] \( \mu_\nu \lesssim (1.1 - 2.7) \cdot 10^{-12} \mu_B \)

Absence of high-energy events in the SN1987A neutrino signal  \[16\] \( \mu_\nu \lesssim 10^{-12} \mu_B \)

Table 1: Some bounds on the neutrino magnetic moment. The first three bounds apply to both Dirac and Majorana magnetic moments. The last two bounds apply only to Dirac magnetic moment.

1987 triggered a bunch of papers \[14\]-\[17\] (see also a related paper \[18\]), which idea was closely related to the Dar’s one. Namely, the supernova core was regarded as a source of right-handed sterile neutrinos, \( \nu_R \), as in \[13\], but the \( \nu_R \to \nu_L \) transition was assumed to occur in the interstellar magnetic field. It was shown that this could lead to the registration of high-energy neutrino events in terrestrial detectors simultaneously with the ordinary supernova neutrino signal. The absence of such events and the estimation of the supernova core cooling rate due to the sterile neutrino emission was used to put stringent bounds on the neutrino magnetic moment (see Table 1). Different aspects of the role of neutrino magnetic moment in the supernova explosion and neutrino emission were further investigated in \[19\]-\[22\]. In particular, it was pointed out in \[19\]|\[20\] that a resonant \( \nu_R \to \nu_L \) transition could proceed inside a supernova, which could greatly facilitate the explosion through the Dar’s mechanism and under certain circumstances to annul the supernova bounds on the neutrino magnetic moment.

The neutrino magnetic moment could also play a role in the cooling of stars through the plasmon decay in two neutrinos. This was first pointed out as early as 1963 \[23\]; in this work first astrophysical bound on \( \mu_\nu \) was derived from the cooling rate of the Sun. Later cooling of different types of stars, He burning stars (see e.g \[24\]), white dwarfs (see e.g. \[25\]) and red giants (see e.g. \[26\]) in particular, was studied to put bounds on the neutrino magnetic moment.

The neutrino magnetic moment should slightly change the neutrino-electron scattering cross-section. This fact underlies the laboratory experiments which search for the neutrino magnetic moment. Borexino \[27\], GEMMA \[28\] and MUNU \[29\] experiments currently provide the best limits. The most relevant limits on the neutrino magnetic moment are summarized in Table 1.

In the present paper we calculate the supernova core luminosity in right-handed Dirac neutrinos up to 250 ms after core bounce. We use a dynamical supernova model in contrast with all previous studies \[13\]-\[17\], \[31\], \[33\], \[34\]|\[3\] Also we use an accurate expression for the spin-flip rate \[31\]|\[32\] in contrast with the early studies \[13\]-\[17\] and with \[33\]|\[34\]. We implement our result to refine the estimate of the energy injected in the supernova envelope in the Dar’s scenario. Also we calculate the expected number of high-energy neutrino events in a water Cherenkov detector for a galactic supernova explosion. The

\[\text{In a recent paper \[30\] dynamical supernova models are employed too.}\]
above-mentioned advantages of the present study allow us to substantially diminish some of the uncertainties which existed previously.

Although the main subject of the paper is neutrino spin flip due to the Dirac magnetic moment, we also briefly comment upon the case of Majorana magnetic moment. We point out that in this case spin flips may effectively convert electron neutrinos to non-electron antineutrinos inside the inner supernova core. This may lead to various consequences for supernova physics.

2 Right-handed Dirac neutrino emission from supernova core

Although numerous studies have failed to reproduce an explosion of a core-collapse supernova, there exists a commonly accepted general picture of the collapse, see e.g. review [35]. When the mass of the iron core of a massive star reaches the Chandrasekhar limit, the infall phase of the collapse starts. The core contracts due to the gravitational attraction. Some fraction of electrons is converted to electron neutrinos through the inverse beta processes. When the density of the inner part of the core reaches the nuclear density value, \( \sim 3 \times 10^{14} \) g/cm\(^3\), the infalling matter of the outer core bounces from it. A shock wave is created; it propagates outwards increasing the temperature up to tens of MeV and dissociating heavy nuclei into nucleons.

When the densities and temperatures reach extreme values, a fraction of left-handed neutrinos experience spin flips in collisions with charged particles [13]. After the bounce, mainly protons, neutrons and leptons constitute the core, therefore spin flips on electrons and protons play the major role:

\[
\nu_L + e \rightarrow \nu_R + e
\]
\[
\nu_L + p \rightarrow \nu_R + p.
\] (2)

During a short period of time in the end of the infall (few milliseconds), when the density is high, but the temperature is low, a coherent spin flip scattering on nuclei may dominate [17]. We believe that its contribution to the total (integrated over hundreds of milliseconds after bounce) right-handed neutrino output is relatively small. Therefore we do not take it into account in the present work.

The rate of the emission of right-handed neutrinos from a supernova core reads

\[
\frac{dN_{\nu_R}}{dEdt} = \int d^3r \frac{dn_{\nu_R}}{dEdt}(E, n_e(r, t), n_\nu(r, t), T(r, t)).
\] (3)

Here \( dn_{\nu_R}/dEdt \) is a spin flip rate, i.e. the number of right-handed neutrinos with energy \( E \) emitted per unit energy interval per unite time from unite volume of supernova matter with temperature \( T(r, t) \), electron and neutrino number densities \( n_e(r, t) \) and \( n_\nu(r, t) \) correspondingly. The integration is performed over the volume of the supernova core. Note that in the first hundreds of milliseconds of the collapse only electron neutrinos are numerous inside the core; \( \mu \)- and \( \tau \)-neutrinos, as well as antineutrinos of all flavors, are nearly absent. Therefore it is left-handed electron neutrinos which experience spin
Figure 1: Left: supernova core temperature in MeVs. Right: logarithm of supernova core density in g/cm$^3$. The mass coordinate here and in what follows is measured in Sun mass units. Time here and in what follows is time after bounce. Bounce occurs at 230 ms after the beginning of the infall.

Figure 2: Electron fraction (left) and neutrino fraction (right) inside the supernova core.
flips and turn to right-handed neutrinos. Thus $n_{\nu}(r, t)$ is the number density of electron neutrinos.

Two major ingredients are necessary to perform the calculation of $\frac{dN_{\nu}}{dEdt}$. They are, firstly, the spin-flip rate $\frac{dn_{\nu}}{dEdt}$ as a function of supernova matter parameters, $T$, $n_e$ and $n_{\nu}$, and, secondly, the supernova matter parameters themselves as functions of time and coordinate, $T = T(r, t)$, $n_e = n_e(r, t)$ and $n_{\nu} = n_{\nu}(r, t)$.

Neutrino spin flip is due to the exchange of a photon between a charged fermion and a neutrino with magnetic moment $\mu_{\nu}$. The cross-section of these reactions is proportional to $\mu_{\nu}^2$. As the process occurs in the extremely hot and dense plasma of a supernova core, photon dispersion in medium should be taken into account. Early studies [13]-[16] relied on the simplified expressions for the spin flip rate. An accurate expression, which is used in the present paper, was obtained in [31][32]. It is rather bulky; therefore we do not quote it and refer the reader to the original papers.

To obtain supernova matter state parameters as functions of coordinate and time, we employ a one-dimensional astrophysical code "Boom" [37]. A collapse of a 1.5$M_\odot$ iron core is numerically simulated. The bounce occurs in 230 ms after the beginning of the infall. The simulation ends at 250 ms after bounce. Thus it covers almost 0.5 seconds of the collapse. The profiles of temperature, density, electron and neutrino fractions $Y_e$ and $Y_{\nu}$ correspondingly are presented at Fig.1 and Fig.2. The time is countered from the moment of the bounce.

Earlier another mechanism of the neutrino spin flip in supernova was discussed. It is based on the mismatch between chirality and helicity states of massive Dirac neutrinos (see e.g. the detailed paper [36] and references therein). However current upper bounds on neutrino masses ensure that this mechanism can not contribute significantly to the neutrino spin flip rate in supernova.
Figure 4: Spectrum of the emitted right-handed neutrinos for $\mu_\nu = 10^{-13}\mu_B$ and for typical supernova matter parameters, $T_{\text{eff}} \simeq 10$ Mev, $\eta_{e\text{ eff}} \simeq 250$ Mev and $\eta_{\nu\text{ eff}} \simeq 170$ MeV.

As was mentioned above, only $t \gtrsim 0$ are relevant for the right-handed neutrino emission, as the supernova matter is not sufficiently dense and hot before bounce. The emission of the supernova matter in right-handed neutrinos (energy per gram per second) is plotted at Fig.3 (a reference value $\mu_\nu = 10^{-13}\mu_B$ is used here and in what follows). One can estimate the effective parameters of the emitting matter: $M_{\text{eff}} \simeq 0.6M_\odot$, $T_{\text{eff}} \simeq 10$ MeV, $\rho_{\text{eff}} \sim 10^{14}$ g/cm$^3$, $\eta_{e\text{ eff}} \simeq 250$ MeV and $\eta_{\nu\text{ eff}} \simeq 170$ MeV, chemical potentials $\eta_e$ and $\eta_\mu$ being related to electron and neutrino number densities by (see e.g. [38])

$$n_e = \frac{1}{3\pi^2}(\eta_e^3 + \pi^2\eta_e T^2)$$

$$n_\nu = \frac{1}{6\pi^2}(\eta_\nu^3 + \pi^2\eta_\nu T^2).$$

These parameters are somewhat different from what was usually assumed in the previous works [16][31][33]. In particular, we emphasize that only the inner supernova core contributes significantly to the $\nu_R$ emission. The evident reasons is that both neutrino and electron chemical potentials are high inside the inner core, but fall drastically in the outer core.

The spectrum of the emitted right-handed neutrinos for the effective supernova matter parameters is plotted at Fig.4. It is peaked at $\sim 130$ MeV, which is several times larger than typical energies of “ordinary” neutrinos thermally emitted from the neutrino sphere.

Finally, the luminosity of the whole supernova in right-handed neutrinos as a function of time is presented at Fig.5. Two curves at this figure represent the uncertainty of our result due to the ignorance of the exact conditions inside the supernova core. We checked the reliability of the employed code ”Boom” by comparing its results with the results reported in [39]. In this paper the results of the simulations are presented for two possible equations of state (EOS) of nuclear matter. Boom results for densities and temperatures are in a good agreement with the results from [39] for a more stiff EOS. However, a softer
Figure 5: Total luminosity of the supernova core in right-handed neutrinos for $\mu_\nu = 10^{-13}\mu_B$. The upper and the lower curves correspond to a more soft and more stiff EOS of nuclear matter accordingly.

EOS leads to larger densities and temperatures \cite{39}, which leads to the increase of the luminosity. Moreover, the degree of deleptonization predicted by Boom is larger than such in \cite{39}. As a result, the mass of the inner core predicted by Boom is smaller than such in \cite{39}. As the inner core gives the main contribution to the right-handed neutrino emission, the luminosity calculated on the basis of Boom results may occur to be underestimated.\footnote{We thank N. V. Mikheev for attracting our attention to this source of uncertainty in our calculations.}

In order to take into account all the above uncertainties of the astrophysical nature, we use the following procedure. We increase the Boom densities and temperatures “by hand” by 60% and 40% correspondingly, keeping in the integral \eqref{3} the spherical volume elements, $d^3r = 4\pi r^2 dr$, unchanged. This automatically increases the mass of emitting matter by 60%. Basing on results of \cite{39}, we expect that such procedure leads to a reasonable estimate of the uncertainty associated with the ignorance of the parameters of the supernova core. The resulting luminosity is represented by the upper curve at Fig \ref{fig:5}.

One can see that the luminosity grows abruptly at bounce and stays almost constant during at least 250 ms, being equal to $(0.5 - 1.8) \cdot 10^{50}$ erg/s for $\mu_\nu = 10^{-13}\mu_B$. Note that the luminosity does not show any signatures of decrease at $t = 250$ ms – the greatest time accessible to date in our numerical simulation. We expect the total amount of energy emitted during the collapse in right-handed neutrinos to be a factor of (2-4) greater than the energy emitted in the first 250 ms after bounce. This means, in particular, that in order to inject $10^{51}$ ergs in the supernova envelop according to the Dar’s mechanism, the neutrino magnetic moment should equal $(2 - 6) \cdot 10^{-13}\mu_B$. This estimate is in agreement with the estimate presented in \cite{40}. Remind that the constraints on the Dirac magnetic moment presented in two last lines of Table \ref{table:1} may be, in general, invalid if the exploding star possesses strong magnetic field, which is required for the Dar’s mechanism to work.

It should be noted that we do not take into account the back reaction of the right-
Table 2: Previously reported and present results for the supernova core luminosity in right-handed neutrinos during several hundreds of milliseconds after bounce. The approximate coincidence between the present result and the result of [16] is accidental. Spin flip rate used in the present work is larger than in [16], while conditions inside the supernova core employed in the present work are less favorable for right-handed neutrino emission compared to those in [16]. The difference between the present result and the result of [31] stems solely from the difference in the supernova core models.

In order for right-handed neutrinos to reveal themselves in the detectors, they should be converted to left-handed ones due to the spin precession in the interstellar magnetic field. If this occurs, then high-energy (with a spectrum peaked at (100-150) MeV, see Fig. 4) neutrinos may be registered in the detectors simultaneously with the ordinary neutrino signal (with a spectrum exponentially suppressed for energies greater than ~70 MeV). We emphasize that we do not study the ordinary supernova neutrino signal. We are interested in high-energy neutrinos only.
The number of neutrino events with energies greater than $E_0$ in a water Cherenkov detector reads

$$N = \kappa \cdot \int_{E_0}^{\infty} dE \frac{M_{\text{H}_2\text{O}}}{m_{\text{H}_2\text{O}}} \frac{\sigma(E)}{4\pi D^2} \int dt \frac{dN_{\nu_R}}{dEdt}. \quad (5)$$

Here

$\kappa$ is the fraction of $\nu_{eR}$ converted to $\nu_{eL}$ in the interstellar space on the way from the supernova to the Earth,

$M_{\text{H}_2\text{O}}$ and $m_{\text{H}_2\text{O}}$ are the fiducial mass of water and the mass of the H$_2$O molecule, correspondingly,

$D$ is the distance from the supernova (we use a reference value $D = 10$ kpc in what follows),

$\sigma(E)$ is the cross section of the reaction $\nu_e + O \rightarrow e^- + F$ for the neutrino with energy $E$, through which the electron neutrinos are registered in water detectors. We employ the cross section presented in [41].

Coefficient $\kappa$ deserves some special attention. Two effects should be taken into account while calculating it, the neutrino spin precession in the interstellar magnetic field and the neutrino flavor mixing. We elaborate on it in the Appendix. It is normally somewhat less than $1/2$ for $\mu_\nu \gtrsim 10^{-13}\mu_B$. For our estimates we take it to be $0.3$ (see the Appendix).

We find that the first 250 ms after bounce should provide (3-13) high-energy events in Super-Kamiokande (with fiducial mass equal to 22 kt) if $\mu_\nu = 10^{-13}\mu_B$. This confirms the rough estimate presented in [34]. One can see that Super-Kamiokande is well sensitive to the Dirac neutrino magnetic moment of order of $10^{-13}\mu_B$.

4 Spin flips of neutrinos with Majorana neutrino magnetic moment inside the inner supernova core

If neutrinos are Majorana fermions, spin flips do not produce sterile neutrino species. Instead they convert electron neutrinos into non-electron antineutrinos. Remind that in the standard picture of the collapse electron neutrinos inside the supernova core form a highly degenerate fermion gas, while other neutrino and antineutrino species are nearly absent. The reason for this is that standard model interactions conserve lepton number and lepton flavor. Majorana neutrino spin flips violate both conservation laws. As a result, for a sufficiently large Majorana transition moments, muon and tau antineutrinos may become as numerous as electron neutrinos. In principle, this may alter both the supernova dynamics and the supernova neutrino signal. The impact of the decrease of the electron neutrino degeneracy on the shock dynamics is considered in [42][43]. In [34] it is pointed out that $\nu_e \rightarrow \bar{\nu}_\mu, \bar{\nu}_\tau$ transition due to spin flips of neutrinos with Majorana magnetic moment in collisions with charged particles may change the flavor composition of the supernova neutrino output. In particular, if the composition of the neutronization burst could be altered, which would drastically increase its observability in water Cherenkov detectors.

Finally, we note that the emergence of non-electron antineutrinos inside the...
core in addition to electron neutrinos should fasten the cooling rate of the core due to the neutrino diffusion, which may influence both supernova dynamics and neutrino signal.

All these effects imply that considerable fraction of electron neutrinos experience spin flips. Therefore, the back reaction of spin flips on the supernova core evolution should be taken into account in order to study these effects in detail. We do not perform such analysis in the present paper. However we make a rough estimate of the Majorana magnetic moment value necessary to equilibrate electron neutrinos with non-electron antineutrinos in the inner core at a timescale of 1 ms (this is a relevant timescale for shock propagation and neutronization burst). For the typical parameters of the supernova core matter presented in Section 2 (before eq.(4)) we find this value to be $\mu_\nu \simeq 10^{-12}\mu_B$. This value is not yet constrained by terrestrial experiments and astrophysical considerations. Therefore it is interesting to thoroughly explore the impact of the Majorana neutrino magnetic moment on the physics of core-collapse supernova.

5 Conclusions

We have performed a calculation of the luminosity of the supernova core in right-handed neutrinos in the first half of a second of a collapse, assuming that neutrinos are Dirac fermions with magnetic moment. It is shown that the luminosity grows abruptly at bounce and stays almost constant during at least 250 ms, being equal to $(0.5 - 1.8) \cdot 10^{50}$ erg/s for $\mu_\nu = 10^{-13}\mu_B$. We expect that it is significant also for larger times; further work is necessary to demonstrate this explicitly. However, already obtained results allow to conclude that Super-Kamiokande may register at least few high-energy neutrino events if $\mu_\nu$ is of order of $10^{-13}\mu_B$.

Also we point out that if neutrinos are Majorana fermions with magnetic moment around $10^{-12}\mu_B$, then their spin flips inside the inner core may equilibrate electron neutrinos with non-electron antineutrinos. This may affect the supernova dynamics and the supernova neutrino signal.

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Appendix: neutrino spin precession and flavor transformation in the interstellar space

Here we estimate the coefficient $\kappa$.

The interaction of the neutrino magnetic moment with the magnetic field $B$ leads to the neutrino spin precession (or, in other words, to $\nu_R \leftrightarrow \nu_L$ oscillations), which is described (in the ultra-relativistic case) by

$$i\frac{d}{dx} \nu(x) = (E + \mu \sigma B_\perp(x)) \nu(x).$$

(6)

Here $\nu(x) = \begin{pmatrix} \nu_L(x) \\ \nu_R(x) \end{pmatrix}$, $E$ is neutrino energy, $\sigma$ is a vector constructed from Pauli matrices, and $B_\perp(x)$ is a component of $B$ normal to the neutrino momentum. If for every $x$ magnetic field $B_\perp(x)$ lies in the same plane, then the phase of oscillations is given by

$$\phi = \int \mu B(x) dx.$$  

(7)

The oscillation probability, $P(\nu_R \rightarrow \nu_L) = \sin^2 \phi$, may be easily calculated for this case. For the constant magnetic field one gets

$$\phi = \mu \nu B_\perp x = 0.9 \left( \frac{\mu}{10^{-13} \mu_B} \right) \left( \frac{B_\perp}{\mu G} \right) \left( \frac{x}{10 \text{ kpc}} \right).$$

(8)

Galactic magnetic field has a complicated structure (see, for example, [50], [51]). Its typical strength is not less than $1 \mu$G, and probably somewhat larger. It can be represented as the sum of regular (large-scale) and random (small-scale) components. Length scales of the random component are much smaller than 1 kpc, therefore, according to (8), this component is irrelevant for our purposes.\(^6\) Length scales of the regular component are of order of 1 kpc. In the galactic disk regular magnetic field is directed along the spiral arms, clockwise or counterclockwise depending on the spiral arm. There is a number of galactic magnetic field models (see the above mentioned references). We use a phenomenological model described in [50]. It fits well the data extracted from observations of 350 pulsars. Also it reproduces the main qualitative features of the radial dependence of the magnetic field in the inner galaxy, i.e. two field reversals at $\sim 4.5$ and $\sim 6.5$ kpc from the center of the galaxy, as well as the characteristic strength of the magnetic field.

\(^6\) Strictly speaking, relevance of the random magnetic field to the spin rotation is determined by $\gamma = \mu^2 < B^2 > L_c x$ \(^52\), where $L_c$ is a field variation length scale. Taking $\mu = 10^{-12} \mu_B$, $x = 10$ kpc and $B \sim 1 \mu$Gs, $L_c \sim 10$ pc (see [51] and reference therein), one obtains $\gamma \sim 0.1 \ll 1$. This means that the effect of the random magnetic field on the spin precession may be neglected.
We consider a frequently discussed case of a supernova exploding in the inner part of the disk of our galaxy, \( D = 10 \text{ kpc} \) away from the Solar system. For simplicity we assume that it is situated on the line which connects Solar system and galactic center. The radial dependence of \( B_\perp(r) \) reads \( [50] \)

\[
B_\perp(r) = \begin{cases} 
0.9 \mu G & 0 \text{ kpc} < r \leq 2 \text{ kpc} \\
3.8 \mu G & 2 \text{ kpc} < r \leq 3 \text{ kpc} \\
3.1 \mu G & 3 \text{ kpc} < r \leq 4 \text{ kpc} \\
-2.2 \mu G & 4 \text{ kpc} < r \leq 5 \text{ kpc} \\
-1.9 \mu G & 5 \text{ kpc} < r \leq 6 \text{ kpc} \\
1.9 \mu G & 6 \text{ kpc} < r \leq 7 \text{ kpc} \\
2.5 \mu G & 7 \text{ kpc} < r \leq 8 \text{ kpc}, 
\end{cases}
\]

(9)

Here \( r \) is the galactocentric distance. The distance from the Sun to the galactic center is taken to be 7.2 kpc in \([50]\).

From (7) and (9) one obtains the probability of \( \nu_R \rightarrow \nu_L \) oscillations:

\[
P_{\nu_R \rightarrow \nu_L} = \sin^2 \left( 1.1 \frac{\mu}{10^{-13} \mu_B} \right). \tag{10}
\]

Two cases should be distinguished. For \( \mu \gtrsim \mu_{th} \approx 10^{-13} \mu_B \) the probability oscillates rapidly with \( \mu \), phase being strongly dependent on the actual magnetic field along the line of sight to the supernova. This means, in fact, that for \( \mu \gtrsim \mu_{th} \) one should consider the phase \( \phi \) as a uniformly distributed random value. In this case \( P_{\nu_R \rightarrow \nu_L} \) is also a random value. Its expectation value is \( P_{av} = 0.5 \), and \( P_{\nu_R \rightarrow \nu_L} > 0.025 \) with 90\% probability.

Alternatively, if \( \mu \lesssim \mu_{th} \), sine may be approximated by its argument in eq.(10). In this case \( P_{\nu_R \rightarrow \nu_L} \) is proportional to \( \mu^2 \), magnetic field profile affecting only the coefficient of proportionality.

Although the exact value of the phase in eq.(10) depends on the factual magnetic field profile, the above described qualitative behavior of the probability \( P_{\nu_R \rightarrow \nu_L} \) with respect to \( \mu \) is common for any profile. The value of \( \mu_{th} \) is expected to be of order of \( 10^{-13} \mu_B \) for the considered case of a supernova in the inner part of the galactic disk.

One should also take into account flavor transformations along with the spin precession in the problem involved. Right-handed electron neutrinos, produced in a supernova, \( \nu_{eR} \), quickly decohere into the mixture of \( \nu_{1R} \) and \( \nu_{2R} \) (see e.g. [45]). The corresponding fractions in the mixture are equal to \( \cos^2 \theta_{12} \) and \( \sin^2 \theta_{12} \), \( \theta_{12} \approx 30^\circ \) being the mixing angle.\(^7\) In the interstellar medium \( \nu_{1R} \rightarrow \nu_{1L} \) and \( \nu_{2R} \rightarrow \nu_{2L} \) transitions occur with the probability \( P_{\nu_R \rightarrow \nu_L} \) as described above. Finally, in the detector \( \nu_{1L} \) and \( \nu_{2L} \) show themselves as electron neutrinos with probabilities \( \cos^2 \theta_{12} \) and \( \sin^2 \theta_{12} \) correspondingly. Combining all the probabilities together, one finds the fraction \( \kappa \) of right-handed electron neutrinos,

\[^7\] For simplicity, we assume that the vacuum mixing of right-handed neutrinos is equivalent to such for left-handed neutrinos. This allows to obtain a definite expression for the conversion coefficient \( \kappa \). However, this simplification influences only the mixing-angle prefactor in eq.(11), which is in any case generically of order of unity.
\( \nu_{eR} \), which are converted to left-handed electron neutrinos, \( \nu_{eL} \), in the interstellar space on the way from the supernova to the Earth:

\[
\kappa = (\cos^4 \theta_{12} + \sin^4 \theta_{12}) P_{\nu_R \rightarrow \nu_L} = (1 - 0.5 \sin^2 2\theta_{12}) P_{\nu_R \rightarrow \nu_L} \approx 0.6 P_{\nu_R \rightarrow \nu_L}.
\]

For \( \mu \geq \mu_{th} \) one finally obtains \( \kappa \approx 0.3 \)

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