Two-photon imaging and quantum holography

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Recenty, there has been a constant focus of interest in quantum limited imaging [1, 2, 3, 4] and lithography [5, 6, 7, 8]. In the former of the applications, the quantum correlation between the amplitude fluctuations of an appropriately entangled multimode state of light is used to beat the standard quantum limit of spatial resolution. The latter application capitalizes on the short de Broglie wavelength of entangled multiphoton states and offers, in principle, a spatial resolution that is independent of the classical wavelength of the electromagnetic field [8].

In a recent paper by Abouraddy et al. [9], a method has been proposed where an entangled photon pair is used to image an object. The setup is depicted in Fig. 1. Each of the photons in a photon pair is sent through an object. The objects labeled 1 and 2 are described by the impulse response operators \( h_1 \) and \( h_2 \), respectively. The claim by Abouraddy et al. is that, if the source emits an entangled two-photon multimode state, and if one of the photons emitted is detected by a “bucket detector” [1, 2], information about a test object is obtained that cannot be obtained with any classical source of light. Abouraddy et al. have proposed two schemes, in the first, called distributed quantum imaging, the test object, that we want to characterize, is described by \( h_1 \), while \( h_2 \) is a (known) reference object [9]. The authors have also suggested a scheme where the roles of the objects 1 and 2 are reversed [10], so that the test object is described by \( h_2 \) and the photon that goes through it is detected by a bucket detector, while the photon going through the reference object, now described by \( h_1 \), is detected by a detector array. This imaging method was named quantum holography.

The claim by Abouraddy et al. has been put in doubt by a recent analysis, and an experiment, by Bennink et al. [11]. Their analysis showed that if the test object in a quantum holographic setup is lossless, no information about the test object is obtained. In their experiment they used a lossy test object and a correlated classical source of light. Yet, using what Bennink et al. called “‘two-photon’ coincidence imaging” [12], the authors’ quotation marks indicate that the detection method mimics what Abouraddy et al. call quantum holography, but that classically correlated states were used instead of two-photon states. They showed that the method proposed in that paper, with “bucket detection” of one of the photons, will give identical results for entangled states as for appropriately prepared classically correlated states.

It has been claimed that “the use of entangled photons in an imaging system can exhibit effects that cannot be mimicked by any other two-photon source, whatever strength of the correlations between the two photons” [A. F. Abouraddy, B. E. A. Saleh, A. V. Sergienko, and M. C. Teich, Phys. Rev. Lett. 87, 123602 (2001)]. While we believe that the cited statement is true, we show that the method proposed in that paper, with “bucket detection” of one of the photons, will give identical results for entangled states as for appropriately prepared classically correlated states.

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interaction with object 1, the output state becomes

\[ \hat{h}_1 \sum_{q'} (1_{q'}|\hat{\rho}|1_{q'}) \hat{h}_1^\dagger = \sum_{q'} (1_{q'}|\hat{h}_1 \hat{\rho} \hat{h}_1^\dagger|1_{q'}) , \]

(3)

where the equality follows from the fact that we have assumed that \( \hat{h}_1 \) does not operate on the primed vectors. It may be prudent, at this point, to establish that the specific pair of mode sets will lead to a unique transformation \( \hat{h}_1 \). On the detector side it is convenient to work in the eigenmodes of the detectors. Then, the probability \( p(h) \) of a photon detection in mode (detector) \( q \) is

\[ p_1(q) = \sum_{q'} \langle 1_{q'}, 1_{q'}|\hat{h}_1 \hat{\rho} \hat{h}_1^\dagger|1_{q'}, 1_{q'} \rangle. \]

(4)

If one defines \( \gamma_1(i, j) = \sum_{k'} \varphi(i, k')\varphi^*(j, k') \), the coefficients \( \gamma_1(i, j) \) turn out to be the density operator coefficients \( \langle 1_i|\text{Tr}_2(\hat{\rho})|1_j \rangle \), so that

\[ \text{Tr}_2(\hat{\rho}) = \sum_i \sum_j \gamma_1(i, j)|1_i\rangle \langle 1_j|. \]

(5)

By inserting the resolution of the identity twice into Eq. (4), one arrives at

\[ p_1(q) = \sum_i \sum_j \gamma_1(i, j) h_1(q, i) h_1^\dagger(q, j). \]

(6)

where \( h_1(q, i) \equiv (1_q|\hat{h}_1|1_i) \) and \( h_1^\dagger(q, j) \equiv ((1_q|\hat{h}_1^\dagger|1_j)^\dagger \). The expression above is the direct discrete-mode equivalent to Eq. (8) in Ref. [9]. If one permutes the indices \( q \) and \( q' \), and lets the index \( 1 \rightarrow 2 \) in Eqs. (4) and (5), one obtains the expressions for \( p_2(q') \).

Now, assume that both photons in the photon pair are measured. The output state after the interaction with the two objects becomes \( \hat{h}_1 \hat{h}_2 \hat{\rho} \hat{h}_2^\dagger \hat{h}_1^\dagger \). Note that, since \( \hat{h}_1 \) and \( \hat{h}_2 \) operate in different vector spaces, they commute. The probability of registering a correlated detection event between detector \( q \) and detector \( q' \) can be expressed as

\[ p(q, q') = \langle 1_q, 1_{q'}|\hat{h}_1 \hat{h}_2 \hat{\rho} \hat{h}_2^\dagger \hat{h}_1^\dagger|1_q, 1_{q'} \rangle. \]

(7)

A “bucket detector” is a multimode detector where all the modes propagating through an object are measured jointly. Hence, the information about the location of the detected photon (or equivalently, in what mode the photon was detected) is “erased.” A schematic of such a detector is depicted in Fig. 2.

![Figure 2](source.png)

**FIG. 2:** Schematic drawing of “bucket detection” of the photon emitted into the primed modes corresponding to the lower detector array in the figure.

By inserting the resolution of the identity twice into Eq. (4), one arrives at

\[ p_1(q) = \sum_{q'} \langle 1_q, 1_{q'}|\hat{h}_1 \hat{h}_2 \hat{\rho} \hat{h}_2^\dagger \hat{h}_1^\dagger|1_q, 1_{q'} \rangle. \]

(8)

If we assume that the state from the source is entangled such that \( \varphi(i, j) = \varphi(i)\delta_{ij'} \), Eq. (8) can be rewritten as

\[ p_1(q) = \sum_i \sum_{q'} \varphi(i)\varphi^*(j) h_1(q, i) h_1^\dagger(q, j) h_2(q', i) h_2^\dagger(q', j). \]

(9)

Following Ref. [9], we also introduce the parameter

\[ g_2(k', l') = \sum_{q'} h_2(q', k') h_2^\dagger(q', l'). \]

Replacing the sum
over $q'$ in Eq. (11) with this expression, we can write

$$\tilde{p}_1(q) = \sum_{i} \sum_{j} \varphi(i) \varphi^*(j) g_2(i, j) h_1(q, i) h_1^*(q, j).$$

(10)

This is the discrete counterpart of Eq. (12c) in Ref. [1]. Comparing expressions (10) and (8). Abouraddy et al. claim that “based on classical probability analysis one would intuitively expect that $p_1(q)$ would be equal to $\tilde{p}_1(q)$. This is not always the case, however.” From a strictly mathematical point of view it is evident that $p_1(q)$ and $\tilde{p}_1(q)$ are not equal for general functions $h_1(q, i)$ and $h_2(q', i')$. However, we shall now prove that if the reference object is lossless, the two expressions are equal.

Assume, therefore, that object 2, described by $h_2$ is lossless. This immediately implies that the operator $\hat{h}_2$ is unitary. Using this fact, we can write the expression for $p_1(q)$ as

$$p_1(q) = \sum_{q'} \langle 1_q, 1_{q'} | \hat{h}_1 \hat{h}_2 \hat{h}_2^\dagger \hat{h}_2^\dagger | 1_q, 1_{q'} \rangle.$$  

(11)

Furthermore, since $\hat{h}_1$ and $\hat{h}_2$ commute, we can recast this equation as

$$p_1(q) = \sum_{q'} \langle 1_q, 1_{q'} | \hat{h}_2 \hat{h}_1 \hat{h}_2 \hat{h}_1^\dagger | 1_q, 1_{q'} \rangle.$$  

(12)

Since the trace operation is basis invariant, the trace $\text{Tr}_2$ of any operator $\hat{O}$ can be expressed either $\sum_{q'} \sum_{n_{q'}} \langle n_{q'} | \hat{O} | n_{q'} \rangle$ or $\sum_{q'} \sum_{n_{q'}} \langle n_{q'} | \hat{h}_2 \hat{h}_2^\dagger | n_{q'} \rangle$, where $\hat{h}_2$ is an arbitrary, unitary operator. Hence, from Eq. (12), we get

$$p_1(q) = \sum_{q'} \langle 1_q, 1_{q'} | \hat{h}_2 \hat{h}_1 \hat{h}_2 \hat{h}_1^\dagger | 1_q, 1_{q'} \rangle = \tilde{p}_1(q).$$

(13)

This relation implies that Eqs. (8) and (11) are identical if the object 2 (the reference object) is lossless, irrespective of $\hat{\rho}$ and $\hat{h}_1$. Hence, in this case, nothing is gained by using distributed quantum imaging over single photon (per necessity, uncorrelated) imaging. The single-photon state $\text{Tr}_2(\hat{\rho})$ will give the same detection statistics as $\hat{\rho}$.

Let us now consider the case where the test object is described by $\hat{h}_2$, that is, it is the modes through the test object that are detected with the “bucket detector.” This imaging principle is called “quantum holography” [10] or “two-photon coincidence imaging” [11]. Abouraddy et al. [11] claimed that it is possible to use entanglement between the states in the primed and unprimed modes to read out holographic information about the test object $\hat{h}_2$ even if the photons traversing it are detected with a bucket detector, such as a sphere coated on the inside with a photosensitive film that surrounds it. However, Bennink et al. showed that if the test object is lossless, then the bucket detector always clicks, effectively tracing out all information about the test object. Hence, quantum holography of such test objects does not work.

However, assume that the test object (object 2) is lossy. It is clear that in this case the reference object (object 1) should be lossless, as losses will only introduce additional randomness in the measurement. Since $\hat{h}_1$ and $\hat{h}_2$ commute, this randomness corresponds to loss of information about $\hat{h}_2$. If the reference object is lossless, it is rather obvious that if a two-photon (possibly) entangled state $|\Psi\rangle$ is used, its quantum holography detection statistics will be exactly mimicked by the classically correlated state

$$p' = \sum_i \hat{h}_1^\dagger |1_i\rangle \langle 1_i | \hat{h}_1 \otimes |1_i\rangle \hat{\rho} \hat{h}_1^\dagger |1_i\rangle.$$  

(14)

Hence, it seems to us that entangled-state holography with bucket detection of any object is of limited use, since identical results can be obtained using classical states. This was what Bennink et al. [11] demonstrated experimentally. The statistical distribution $\tilde{P}_1(q)$ in their experiment very clearly brought out the information encoded in the mask although all the photons interacting with the (lossy) mask were detected by a bucket detector.

Above, we have shown that distributed quantum imaging and quantum holography with a lossless reference object (but, in general, a lossy test object) will not offer anything imaging with classical states cannot provide. Bennink et al. [11] proved that quantum holography of a lossless test object also can be mimicked by classical states. Now, we will treat the case when both object 1 and 2 are lossy. The standard way of including losses is to extend the two original sets of modes with auxiliary modes. We shall do so by assuming that the respective detector eigenmodes constitute only a subset of the primed and unprimed modes. To be realistic we can assume that only the photons in $N'$ of the unprimed (primed) modes are detected, and that these modes are labeled 1, 2, ..., $N'$ (1', 2', ..., $N'$). The probability of detecting a photon in mode $q$ then becomes

$$P_1(q) = \sum_{q'=1}^{N'} \langle 1_q, 1_{q'} | \hat{h}_1 \hat{h}_2 \hat{h}_2^\dagger \hat{h}_1^\dagger | 1_q 1_{q'} \rangle$$

$$+ \sum_{q' > N'} \langle 1_q, 1_{q'} | \hat{h}_1 \hat{h}_2 \hat{h}_2^\dagger \hat{h}_1^\dagger | 1_q, 1_{q'} \rangle$$

$$= \tilde{P}_1(q) + P_{1,0}(q).$$

(15)

From Eq. (15), we see that photodetection probability $P_1(q)$ is simply the sum over all the detection events $q'$ of the two-photon joint probability distribution $\tilde{P}_1(q) = \sum_{q'=1}^{N'} P(q, q')$ and the probability of detecting one photon in mode $q$ and no photon in the modes $q' = 1', 2', ..., N'$. This is just what one expects intuitively. For lossy reference objects, $P_1(q)$ differs from $\tilde{P}_1(q)$.

Now, denote the probability of not detecting the photon in the primed modes $P_0 = \sum_q P_{1,0}(q)$ and consider,
Assume that \( \hat{\rho} \) is a two-photon, four-mode, entangled state. Consider the two-photon, four-mode, entangled state and the fact that although the marginal probability distributions did not yield the joint probability distribution of a two-photon state, [13], where a diffraction pattern could be read out through a classical source, the advantage with this imaging method is lost if “bucket detection” of one of the photons is employed, at least if the reference object is lossless (and this ideal case is what one should strive for, since it gives the optimal information about the test object). To capitalize on the unique properties of two-photon imaging, the detection joint probability distribution needs to be retained.

In summary, we have shown that although entangled two-photon imaging may bring out effects that cannot be mimicked by any classical source, the advantage with this imaging method is lost if “bucket detection” of one of the photons is employed, at least if the reference object is lossless (and this ideal case is what one should strive for, since it gives the optimal information about the test object). To capitalize on the unique properties of two-photon imaging, the detection joint probability distribution needs to be retained.

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\[ \sum_{j' = 1}^{N'} \langle 1_{j'} | \hat{h}_2 | 1_{j'} \rangle \hat{h}_2 \left( | 1_{1'} \rangle \langle 1_{1'} | + \frac{P_0 | 1_{M'} \rangle \langle 1_{M'} |}{1 - P_0} \right) \hat{h}_2, \]  

where \( M' > N' \). It is quite obvious that this state, that lacks any correlation between the photon in the primed and unprimed modes, will yield an identical photodetection statistics distribution \( \hat{P}_1(q) \) to the corresponding distribution of the state \( \hat{\rho} \) (that may be entangled). Note that this is true irrespective of \( \hat{\rho}, \hat{h}_1, \) and \( \hat{h}_2 \). The result depends critically on the fact that \( \hat{h}_2 \) is an operation local in the primed modes. However, this is, in general, not a physically accessible state since it may contain excitation in modes with \( M' > N' \) (the modes corresponding to the loss). If only the modes \( q' = 1', \ldots, N' \) can be excited, then one cannot, in general, mimic the photodetection statistics of the two lossy objects illuminated by an two-photon entangled state, with a classically correlated state. Since we have shown that in the case when the reference object is lossless, entanglement combined with bucket detection offers no improvement for neither two-photon imaging nor for quantum holography, we conjecture that if bucket detection of one of the photons is used, entangled two-photon imaging never offers any advantage over (classically) correlated-photon imaging.

An effect similar to “quantum holography,” but not to be confused with it, has been demonstrated in Refs. [12, 13], where a diffraction pattern could be read out through the joint probability distribution of a two-photon state, although the marginal probability distributions did not show any diffraction pattern. This is an effect due to entanglement and the fact that bucket detection was not used. To model such an effect in the simplest possible fashion, consider the two-photon, four-mode, entangled state

\[ |\Psi\rangle = \frac{1}{2} ( | 1_1, 1_{1'} \rangle + | 1_1, 2_{2'} \rangle + | 1_2, 1_{1'} \rangle - | 1_2, 2_{2'} \rangle). \]  

Assume that \( \hat{h}_1 = \hat{1}_1 \) and \( h_2(1, 1) = h_2(1, 2) = h_2(2, 1) = -h_2(2, 2) = 1/\sqrt{2} \). One can then readily show that \( \rho(1, 1') = \rho(2, 2') = 1/2 \) and \( \rho(1, 2') = \rho(2, 1') = 0 \). That is, which one of the (two) detectors behind each object that will “click” is totally unpredictable. The statistics of either detector pair alone do not contain any information about object 2. This is so regardless if one simply ignores the outcome of the other detector pair, or if one uses a bucket detector (that, in this case, always will “click”). Due to the input-state entanglement, there is, however, a perfect correlation between the joint detection events. The correlation statistics hence reveal (some) information about the object.

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