Revisiting sticking property of submillimetre-sized aggregates

Sota Arakawa\textsuperscript{1,2}\textsuperscript{⋆}

\textsuperscript{1}Department of Earth and Planetary Sciences, Tokyo Institute of Technology, Meguro, Tokyo, 152-8551, Japan.
\textsuperscript{2}Division of Science, National Astronomical Observatory of Japan, Mitaka, Tokyo, 181-8588, Japan.

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ABSTRACT
Understanding the physical properties of dust aggregates is of great importance in planetary science. In this study, we revisited the sticking property of submillimetre-sized aggregates. We revealed that the “effective surface energy” model used in previous studies underestimates the critical pulling force needed to separate two sticking aggregates. We also derived a new and simple model of the critical pulling force based on the canonical theory of two contacting spheres. Our findings indicate that we do not need to consider the “effective surface energy” of dust aggregates when discussing the physical properties of loose agglomerates of submillimetre-sized aggregates.

Key words: comets: general – planets and satellites: formation – protoplanetary discs

1 INTRODUCTION
The formation of planetesimals is initiated by the collisional growth of small dust grains and aggregates in protoplanetary disks. The collision velocities are small enough for initially \( \mu \)-m-sized dust grains to be able to stick together following a collision, and thereby, larger dust aggregates are formed. However, as the aggregates grow, the collision velocities increase, which makes it more difficult for more dust particles to stick to the aggregates under the conditions in the solar nebula (e.g., Blum & Wurm 2008). Therefore, revealing the transition between sticking and bouncing regimes for colliding dust aggregates is of great importance in the context of planet formation.

The suborbital particle and aggregation experiment was carried on the REXUS 12 suborbital rocket (see Brisset et al. 2013). The sticking properties of submillimetre-sized aggregates were measured by Brisset et al. (2016). They observed a growth of dust aggregates and the formation and fragmentation of clusters of up to a few millimetres in size. The transition from bouncing to sticking collisions happened at collision velocities of around 10 cm s\(^{-1}\) for dust aggregates composed of monodisperse monomer grains of \( \approx 1 \) \( \mu \)-m in size (Brisset et al. 2016).

The experimental results of Brisset et al. (2016) were interpreted using the dust aggregate model proposed by Weidling et al. (2012). Weidling et al. (2012) developed a microgravity experiment and carried out experiments of free collisions between dust aggregates with diameters of 0.5–2 mm. They also attempted to develop a dust aggregate model to explain their experimental results by introducing an “effective surface energy” of millimetre-sized dust aggregates (see Section 3.1).

The effective surface energy model (Weidling et al. 2012) has been widely used in recent studies of loose agglomerates of dust aggregates (e.g., Gundlach & Blum 2012; Blum et al. 2017; Hu et al. 2019). This is because recent numerical simulations and laboratory experiments suggest that planetesimals formed in the gaseous solar nebula have a pebble-pile structure (e.g., Blum 2018). In the solar nebula, millimetre- and centimetre-sized dust aggregates called “pebbles” may be formed via collisional growth and compaction (e.g., Güttler et al. 2010). These pebbles can clump together through streaming instabilities, and form gravitationally bound “pebble clouds” (e.g., Carrera et al. 2015; Yang et al. 2017). Pebbles in a cloud will undergo dissipative mutual collisions (e.g., Wahlberg Jansson & Johansen 2014), and finally turn into pebble-pile planetesimals, i.e., loose agglomerates of pebbles.

However, the physical background of the effective surface energy model is unclear. In this study, we revisit the sticking properties of submillimetre-sized aggregates studied by Brisset et al. (2016). We reveal that the effective surface energy model underestimates the critical pulling force needed to separate two sticking aggregates (see Section 4). We also derive a simple new model of the critical pulling force based on the canonical theory of two contacting spheres (Johnson et al. 1971) in Section 3.2.

2 CRITICAL PULLING FORCE NEEDED TO SEPARATE TWO STICKING MONOMERS
First, we review the critical pulling force needed to separate two spherical monomer grains, which forms the basis of the theory of the critical pulling force needed to separate two sticking aggregates. Based on the JKR theory (Johnson, Kendall & Roberts 1971) for elastic, deformable spheres, the critical pulling force needed to separate two sticking spheres, \( F_{\text{crit}} \), is given by

\[
F_{\text{crit}} = \frac{3}{2} \pi r y r,
\]
where $\gamma$ is the material surface energy and $r$ is the monomer radius (see also Wada et al. 2007). In this study, we assume that all monomer particles have the same radius of $r$. It has been experimentally confirmed that the above equation of $F_{\text{crit}}$ is applicable to micron-sized SiO$_2$ spherical particles (e.g., Heim et al. 1999).

Johnson et al. (1971) stressed that the critical pulling force needed to separate two sticking spheres, $F_{\text{crit}}$, is independent of the Young’s modulus of the monomers, $E$. Therefore, $F_{\text{crit}}$ is independent of the contact radius, $a_c$. The contact radius of two sticking monomer grains is given by

$$a_c = \left[ \frac{9\pi\gamma(1 - \nu^2)}{2Er} \right]^{1/3} r,$$

where $\nu$ is Poisson’s ratio (e.g., Johnson et al. 1971; Wada et al. 2007). The material properties of SiO$_2$ grains are listed in Dominik & Tielens (1997): $\gamma = 25$ mJ m$^{-2}$, $E = 54$ GPa, and $\nu = 0.17$.

### 3 CRITICAL PULLING FORCE NEEDED TO SEPARATE TWO STICKING AGGREGATES

In Section 3, we introduce two models for the critical pulling force needed to separate two sticking aggregates, namely, the “effective surface energy” model (Weidling et al. 2012) and the monomer–monomer contact model (Section 3.2, this study). The critical pulling force for separating two sticking submillimetre-sized aggregates, $F_c$, was experimentally obtained by Brisset et al. (2016). In their experiments, the statistical threshold velocity between the sticking and bouncing regimes of aggregates was measured. Then, the critical pulling force $F_c$ was evaluated based on the threshold velocity between the sticking and bouncing regimes (see Brisset et al. 2016, for details).

#### 3.1 Effective surface energy model

Here, we briefly summarize the “effective surface energy” model. Weidling et al. (2012) introduced the effective surface energy, $\gamma_{\text{eff}}$, as a combination of the material surface energy, $\gamma$, the filling factor of aggregates, $\phi$, and the Hertz factor (i.e., the ratio between the contact surface and the cross section of two monomer grains), $a_c^2/r^2$. The effective surface energy is given by

$$\gamma_{\text{eff}} = \gamma + \frac{\phi a_c^2}{r^2}.$$

In the framework of the effective surface energy model, the critical pulling force is given by

$$F_{\text{c}}^{\text{ESE}} = N \cdot \frac{3}{2} \pi \gamma_{\text{eff}} r_{\text{agg}},$$

where $N$ is the number of inter-aggregate connections per aggregate and $r_{\text{agg}}$ is the aggregate radius. When a dust cluster consists of two aggregates, the number of inter-aggregate connections per aggregate is $N = 1$. The upper limit of $N$ is 12, which is the maximum coordination number for the packing of monodisperse dust aggregates. Brisset et al. (2016) obtained $N \approx 2.5$ from snapshot images of dust clusters during their experiment.

The filling factor of dust aggregates used in Brisset et al. (2016) is $\phi = 0.37$, in which case the critical pulling force is given by

$$F_{\text{c}}^{\text{ESE}} = N \cdot 1.5 \times 10^{-9} \left( \frac{\gamma}{25 \text{ mJ m}^{-2}} \right)^{5/3} \left( \frac{r_{\text{agg}}}{1 \mu \text{m}} \right)^{-2/3} \left( \frac{r_{\text{agg}}}{0.1 \text{ mm}} \right) \text{N}.$$

However, the experimental results suggest that $F_{\text{c}} \approx 10^{-7}$ N for submillimetre-sized aggregates (see Figure 2). As the maximum coordination number for monodisperse dust aggregates in a dust cluster is $N = 12$, we conclude that the effective surface energy model (Weidling et al. 2012) cannot explain the critical pulling force of submillimetre-sized aggregates.

#### 3.2 Monomer–monomer contact model

Here, we propose a new simple model of $F_{\text{c}}$ for two sticking aggregates. Figure 1 shows a schematic illustration of two sticking aggregates. When there is no external pressure, the number of inter-aggregate contacts of monomers, $n$, is $n \sim 1$. Then, the critical pulling force of two sticking aggregates is given by

$$F_{\text{c}}^\text{MMC} = n \cdot \frac{3}{2} \pi \gamma r,$$

which is equal to the $F_{\text{crit}}$ of two sticking monomers when we assume $n = 1$. We found that the monomer–monomer contact model can explain the experimental results of Brisset et al. (2016).

### 4 RESULTS

Figure 2 shows the experimental results of $F_c$ for various aggregate radii $r_{\text{agg}}$. The monomers used in Brisset et al. (2016) are 1 mm-sized SiO$_2$ grains, and we set $r = 1$ mm in our calculations of $F_{\text{c}}^{\text{MMC}}$ and $F_{\text{c}}^{\text{ESE}}$.

We found that the effective surface energy model underestimates the critical pulling force by order(s) of magnitude, even if we...
set $N = 12$, which is the theoretical upper limit of the number of inter-aggregate connections per aggregate. In contrast, the predictions of $F_c$ of the monomer–monomer contact model are consistent with the experimental results of Brisset et al. (2016), especially for the case of $n = 1$. We can imagine that separations of sticking monomers are not necessarily simultaneous, and that the lower limit of $F_c$ needed to separate two sticking aggregates is equal to $F_{\text{crit}}$, that is, the critical pulling force needed to separate two sticking monomers.

We acknowledge that there is a large uncertainty in the estimate of the material surface energy of SiO$_2$ grains. Kimura et al. (2015) and Steinpilz et al. (2019) claimed that the surface energy of hydrophilic amorphous silica grains depends on the surface chemistry (i.e., the water content) of the grains; the surface energy of dry samples ($\sim 200$ mJ m$^{-2}$) is one order of magnitude higher than that of wet samples ($\sim 25$ mJ m$^{-2}$). If the material surface energy is $\gamma \sim 200$ mJ m$^{-2}$, the effective surface energy model may reproduce the experimental results of Brisset et al. (2016). However, we note that spherical monomer grains used in their experiments are amorphous silica samples manufactured by Micromod Partikeltechnologie GmbH (see Kothé et al. 2013), and several layers of water molecules might be present on the grain surface when they did not heat samples to remove surface water before experiments (Steinpilz et al. 2019). Therefore, we assumed that the material surface energy of SiO$_2$ grains used in their experiments is $\gamma \sim 25$ mJ m$^{-2}$.

5 DISCUSSION

It is known that the adhesion force between elastic solids is affected by their surface roughness, and the roughness usually reduces the adhesion force (e.g., Fuller & Tabor 1975). The significant reduction of the adhesion force of contacting dust aggregates in comparison with smooth spheres is also experimentally confirmed (e.g., Weidling et al. 2012; Brisset et al. 2016; Demirci et al. 2019). These facts might be the motivation of Weidling et al. (2012) to introduce the effective surface energy model.

Fuller & Tabor (1975) revealed that the total adhesion force is obtained by applying the JKR theory to each individual asperity, as we assumed in the monomer–monomer contact model. The number of asperities is identical to $n$ of Equation (6). Their model predicts that the adhesion force depends on the ratio between $\sigma$ and $\delta$, where $\sigma$ is the deviation of asperity heights and $\delta$ is the elastic displacement. Fuller & Tabor (1975) showed that the total adhesion force increases with decreasing the ratio, $\sigma/\delta$ (see Figure 6 of Fuller & Tabor 1975). For the case of two sticking dust aggregates, the deviation of asperity heights may be approximately given by the monomer radius, $\sigma \sim r$. On the other hand, the elastic displacement of dust aggregates should depend on the effective Young’s modulus of dust aggregates and the external force applied to contacting aggregates (e.g., Weidling et al. 2012). Then $\delta$ increases with increasing applied force. In addition, the increase of the applied force also enlarges the contact radius of two sticking aggregates. Therefore, the total adhesion force increases with increasing applied force because the number of asperities, $n$, increases.

We note that Blum et al. (2014) measured the tensile strength of loose agglomerates of pebbles, and they found that the tensile strength depends on the applied compression. We will discuss the tensile strength of loose agglomerates of pebbles in future study.

6 CONCLUSION

Understanding the sticking property of submillimetre-sized aggregates is of great importance in planetary science. Using the REXUS 12 suborbital rocket, Brisset et al. (2016) performed a microgravity experiment of collisional sticking and fragmentation of submillimetre-sized aggregates of 1 mm-sized SiO$_2$ grains.

Brisset et al. (2016) reported the critical pulling force needed to separate two sticking aggregates, $F_c$, and we compared two theoretical models, the effective surface energy model (e.g., Weidling et al. 2012) and the monomer–monomer contact model (Section 3.2, this study), with the experimental results.

We found that the effective surface energy model underestimates the critical pulling force by order(s) of magnitude, while the monomer–monomer contact model is consistent with the experimental results (see Figure 2). Therefore, we do not need to consider the “effective surface energy” of pebbles (millimetre- and centimetre-sized dust aggregates) when evaluating the physical properties of pebble-pile planetesimals.

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DATA AVAILABILITY

The data underlying this article will be shared on reasonable request to the corresponding author.
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