Non-anticommutative N=2 Supersymmetric SU(2) Gauge Theory

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Abstract

We calculate the component Lagrangian of the four-dimensional non-anticommutative (with a singlet deformation parameter) and fully N=2 supersymmetric gauge field theory with the simple gauge group $SU(2)$. We find that the deformed (classical) scalar potential is unbounded from below, in contrast to the undeformed case.

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1 Introduction

Supersymmetric gauge field theories in Non-AntiCommutative (NAC) superspace \[1\] recently became the area of intensive study, see e.g., ref. \[2\]. The motivation is two-fold, at least. Firstly, those NAC-deformed field theories naturally arise from superstrings in certain supergravity backgrounds and, second, they are natural extensions of the ordinary supersymmetric gauge field theories (formulated in the standard (anti)commutative superspace).

Here we always assume that merely a chiral part of the fermionic superspace coordinates becomes NAC, whereas the other superspace coordinates still (anti)commute (in some basis). This is only possible when the anti-chiral fermionic coordinates (\(\bar{\theta}\)) are not complex conjugates to the chiral ones, \(\bar{\theta} \neq (\theta)^*\), which is the case in Euclidean or Atiyah-Ward spacetimes with the signature \((4,0)\) and \((2,2)\), respectively. The Euclidean signature is relevant to instantons and superstrings \[2\], whereas the Atiyah-Ward signature is relevant to the critical N=2 string models \[3\] or the supersymmetric self-dual gauge field theories \[4\].

Extended supersymmetry offers more opportunities depending upon how much of supersymmetry one wants to preserve, as well as which NAC deformation (e.g., a singlet or a non-singlet) and which operators (the supercovariant derivatives or the supersymmetry generators) one wants to employ in the Moyal-Weyl star product \[1, 5\]. The \(N = (1,1)\) (or just \(N = 2\)) extended supersymmetry is special since it allows one to choose a singlet NAC deformation that preserves all the fundamental symmetries \[6, 7\]. Indeed, the most general nilpotent deformation of \(N = (1,1) = 2 \times (1/2, 1/2)\) supersymmetry is given by

\[
\{\theta^\alpha_i, \theta^\beta_j\}_* = \delta^{(\alpha\beta)}_{(ij)} C^{(\alpha\beta)} + 2iP \epsilon^{\alpha\beta} \epsilon_{ij} \quad \text{(no sum!)} ,
\]

where \(\alpha, \beta = 1, 2\) are chiral spinor indices, \(i, j = 1, 2\) are the indices of the internal R-symmetry group \(SU(2)_R\), while \(C^{\alpha\beta}\) and \(P\) are some constants. Taking only a singlet deformation to be non-vanishing, \(P \neq 0\), and using the chiral supercovariant \(N=2\) superspace derivatives \(D_{i\alpha}\) in the Moyal-Weyl star product,

\[
A \star B = A \exp \left(iP \epsilon^{\alpha\beta} \epsilon_{ij} \overset{\leftarrow}{D}_{i\alpha} \overset{\rightarrow}{D}_{j\beta}\right) B ,
\]

allows one to keep manifest \(N=2\) supersymmetry, Lorentz invariance and R-invariance, as well as (undeformed) gauge invariance (after some non-linear field redefinition) \[6, 7\]. The star product (1.2) matching those conditions is unique, while it requires \(N = 2\).
We choose flat Euclidean spacetime for definiteness, but continue to use the notation common to N=2 superspace with Minkowski spacetime signature, as it is becoming increasingly customary in the current literature (see ref. [8] for details about our notation). Our NAC N=2 superspace with the coordinates $(x^m, \theta^i, \bar{\theta}^\alpha)$ is defined by eq. (1.1), with $C^{\alpha\beta} = 0$ and $P \neq 0$, as the only non-trivial (anti)commutator amongst the N=2 superspace coordinates. This choice preserves most fundamental features of N=2 supersymmetry, such as G-analyticity [6].

A NAC-deformed (non-abelian) supersymmetric gauge field theory can also be rewritten to the usual form, with the standard gauge transformations of field components, i.e. as some kind of effective action, after certain (non-linear) field redefinition, known as the Seiberg-Witten map (cf. ref. [9]). In the case of the $P$-deformed N=2 super-Yang-Mills theory such (non-abelian) map was calculated in ref. [6] with the following result for the effective anti-chiral N=2 superfield strength $\mathcal{W}$:

$$\mathcal{W}_{NAC} = \frac{\mathcal{W}}{1 + P\mathcal{W}} ,$$

(1.3)

where $\mathcal{W}$ is the standard (Lie algebra-valued) N=2 anti-chiral superfield strength. The effective N=2 superspace action reads

$$S_{NAC} = -\frac{1}{2} \int d^4x \, d^4\bar{\theta} \, \text{Tr} \, \mathcal{W}_{NAC}^2 \equiv -\frac{1}{2} \int d^4x \, d^4\bar{\theta} \, \text{Tr} \, f(\mathcal{W}) ,$$

(1.4)

whose structure function $f(\mathcal{W})$ is thus given by [6]

$$f(\mathcal{W}) = \left( \frac{\mathcal{W}}{1 + P\mathcal{W}} \right)^2 .$$

(1.5)

It is non-trivial to calculate eq. (1.4) in components because of the need to perform the (non-abelian) group-theoretical trace (the Lagrangian is no longer quadratic in $\mathcal{W}$!). In this Letter we consider only the simplest non-abelian gauge group $SU(2)$. Some partial results in the $SU(3)$ case will be reported elsewhere [10]. The component action of the $P$-singlet NAC-deformed N=2 supersymmetric $U(1)$ gauge field theory is fully straightforward to calculate from eqs. (1.4) and (1.5) — see refs. [6, 7].

Our paper is organized as follows. In sect. 2 we perform the $SU(2)$ group-theoretical trace in eq. (1.4) and find yet another effective function of the colorless variable $\text{Tr}(\mathcal{W}^2)$ that governs the component action. In sect. 3 we give the full component action of the $P$-deformed N=2 supersymmetric $SU(2)$ gauge field theory. In sect. 4 we focus on the scalar potential of the deformed theory. Sect. 5 is our conclusion.
2 Calculation of the $SU(2)$ trace

The anti-hermitian $SU(2)$ matrices in the adjoint (vector) representation are

$$ T^1 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{pmatrix}, \quad T^2 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ -1 & 0 & 0 \end{pmatrix}, \quad T^3 = \begin{pmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}. \quad (2.1) $$

Their matrix elements are given by the $SU(2)$ structure constants, $(T^a)^{bc} = -\varepsilon^{abc}$, where $\varepsilon^{abc}$ is the totally antisymmetric Levi-Civita symbol, $\varepsilon^{123} = 1$. The matrices (2.1) obey the $SU(2)$ Lie algebra, $[T^a, T^b] = \varepsilon^{abc} T^c$, where $a, b, \ldots = 1, 2, 3$.

The $SU(2)$ trace in eqs. (1.4) and (1.5) is given by

$$ \text{Tr} \left( \frac{W^a T^a}{1 + P W^a T^a} \right)^2 = \text{Tr} \left[ W^a T^a (1 + P W^b T^b)^{-1} W^c T^c (1 + P W^d T^d)^{-1} \right] $$

$$ = \text{Tr} \left[ W^a T^a \sum_{n=0}^{\infty} (-)^n (P W^b T^b)^n W^c T^c \sum_{m=0}^{\infty} (-)^m (P W^d T^d)^m \right] $$

$$ = \text{Tr} \sum_{m,n=0}^{\infty} (-)^{n+m} P^{n+m} (W^a T^a)^{n+m+2} $$

$$ = \sum_{n=0}^{\infty} (-)^n (n+1) P^n \text{Tr}(W^a T^a)^{n+2}. \quad (2.2) $$

It is straightforward to calculate $(m \geq 1)$

$$ \text{Tr}(W^a T^a)^{2m} = 2(-)^m (W^a W^a)^m \quad \text{and} \quad \text{Tr}(W^a T^a)^{2m+1} = 0. \quad (2.3) $$

Hence, eq. (2.2) is equal to

$$ 2 \sum_{n=0}^{\infty} (-)^{n+1}(2n+1) P^{2n} (W^a W^a)^{n+1} = \frac{2}{P^2} \sum_{n=1}^{\infty} (-)^n (2n-1)(P^2 W^a W^a)^n $$

$$ = \frac{2}{P^2} \sum_{n=1}^{\infty} n(\frac{1}{P^2 W^a W^a})^n - \frac{2}{P^2} \sum_{n=1}^{\infty} (-P^2 W^a W^a)^n $$

$$ = \frac{(P^2 W^a W^a)}{(1 + P^2 W^a W^a)^2} + \frac{P^2 W^a W^a}{(1 + P^2 W^a W^a)^2} \quad (2.4) $$

In the limit $P \to 0$ we obtain the usual (undeformed) $SU(2)$-based $N=2$ super-Yang-Mills theory. Having introduced the gauge coupling constant $g_{\text{YM}}$ explicitly and rescaled the action by the factor of $1/g^2_{\text{YM}}$, we can also consider another limit, $P \neq 0$ but $g_{\text{YM}} \to 0$, that gives rise to the undeformed (free and abelian) $N=2$ gauge theory.
3 The Lagrangian in components

The standard (undeformed) $N=2$ gauge superfield strength $\mathcal{W}$ is defined by the anticommutator of two gauge- and super-covariant spinor derivatives in $N=2$ superspace,

$$\{ \mathcal{D}_\alpha^i, \mathcal{D}_\beta^j \} = -2\varepsilon^{ij} \varepsilon_{\alpha\beta} \mathcal{W},$$

(3.1)

and it obeys the $N=2$ superfield Bianchi identities,

$$\mathcal{D}_{ia} \mathcal{W} = 0 \quad \text{and} \quad \overline{\mathcal{D}}_{ij} \mathcal{W} = \mathcal{D}_{ij} \mathcal{W}.$$  

(3.2)

The non-abelian $N=2$ superfield $\mathcal{W}$ is thus a covariantly anti-chiral $N=2$ superfield, being not an $N=2$ anti-chiral one. However, the composite ‘colorless’ $N=2$ superfield $\mathcal{W}^2 \equiv \mathcal{W}^a \mathcal{W}^a$ is $N=2$ anti-chiral, $\mathcal{D}_{ia} \mathcal{W}^2 = 0$, so that it can be expanded with respect to the anticommuting $N=2$ superspace variables $\overline{\theta}_{ia}$ (in the anti-chiral $N=2$ basis) as follows (cf. ref. [7]):

$$\mathcal{W}^2 = U + V_{ai} \overline{\theta}_{ia} + X_{ij} \overline{\theta}_{ij} + Y_{\alpha\beta} \overline{\theta}_{\alpha\beta} + Z_{ia} \overline{(\overline{\theta}_{ia})}_{ia} + L \overline{\theta}_{ia},$$

(3.3)

where we have introduced its (composite) field components $(U, V_{ai}, X_{ij}, Y_{\alpha\beta}, Z_{ia}, L)$.

We define the covariant field components of the $N=2$ superfield $\mathcal{W}$ by covariant differentiation of $\mathcal{W}$,

$$| \mathcal{W} | = \overline{\phi}, \quad \overline{\mathcal{D}}_{ia} | \mathcal{W} | = \overline{\lambda}_{ia}, \quad \overline{\mathcal{D}}_{ij} | \mathcal{W} | = \mathcal{D}_{ij}, \quad \overline{\mathcal{D}}_{\alpha\beta} | \mathcal{W} | = F_{\alpha\beta},$$

(3.4)

where $|$ denotes the leading ($\theta$- and $\overline{\theta}$-independent) component of an $N=2$ superfield. In particular, $F_{\alpha\beta} = (\sigma^{mn})_{\alpha\beta} F_{mn}^a$ is the anti-self-dual part of the Yang-Mills field strength $F_{mn}^a$, the chiral spinors (gaugino) $\overline{\lambda}_{ia}$ transform as a doublet under $SU(2)_R$ and as a triplet under $SU(2)$, the scalars (higgs) $\overline{\phi}$ form a triplet under $SU(2)$, whereas the $SU(2)_R \times SU(2)$ double triplet $D_{ij}^a = D_{ji}^a$ are the auxiliary fields.

The composites of eq. (3.3) in terms of the field components (3.4) read as follows:

$$U = \overline{\phi} \phi^a, \quad V_{ai} = 2 \overline{\lambda}_{ia} \phi^a, \quad X_{ij} = 2 \left( \overline{\phi}^a D_{ij}^a - \overline{\lambda}_{ia} \lambda_{ja}^a \right), \quad Y_{\alpha\beta} = 2 \left( \overline{\phi}^a F_{\alpha\beta}^a - \overline{\lambda}_{ia} \lambda_{ia}^a \right),$$

$$Z_{ia} = 4i \overline{\phi} \sigma^m \phi^a \mathcal{D}_{ia} \lambda_{ia}^a + \overline{\lambda}_{ia} D_{ij}^a - \overline{\lambda}_{ia}^a F_{\alpha\beta}^a,$$

(3.5a)

and

$$L = -2 \overline{\phi} \mathcal{D}_{ia} \mathcal{D}^a D_{mn} \phi^a - i \overline{\lambda}_{ia} (\sigma^m)_{ia} \mathcal{D}_{ia} \lambda_{ia}^a + \varepsilon^{abc} \lambda_{ia} \phi^b \lambda_{ia}^c + \varepsilon^{abc} \overline{\lambda}_{ia} \phi^b \overline{\lambda}_{ia}^c + \frac{1}{48} D_{ij}^a D_{aij} - \frac{1}{12} F^a_{mn} F_{mn}^a - \frac{1}{2} \overline{\phi} \phi^b \phi^c \phi^d \varepsilon_{abcdef} \varepsilon^{cdf},$$

(3.5b)
where $D_m$ are the usual gauge-covariant derivatives (in the adjoint), $F^{-}_{mn}$ is the antiself-dual part of $F_{mn}$. The last composite field $L$ is nothing but the usual $N=2$ super-Yang-Mills Lagrangian, $L = \mathcal{L}_{\text{SYM}}$.

The deformed $N=2$ gauge theory action in undeformed $N=2$ superspace is given by eqs. (1.4) and (1.5),

$$S = -\frac{1}{2} \text{Tr} \int d^4x_R d^4\bar{\theta} f(W) = -\frac{1}{2} \int d^4x_R \bar{D}^4 \text{Tr} f(W) = -\frac{1}{2} \int d^4x_R \text{Tr}(\bar{D}^4 f(W)) \ .$$

(3.6)

Here $\bar{D}^4$ is the gauge-covariant extension of $D^4$,

$$\bar{D}^4 = \frac{1}{4!} \varepsilon^{k m n} \bar{D}_k \bar{D}_l \bar{D}_m \bar{D}_n = \frac{1}{96} \left( \bar{D}_{ij} \bar{D}^i j - \bar{D}_{\alpha \beta} \bar{D}^\alpha \beta \right) - W^2 ,$$

(3.7)

where we have also used the composite indices, $\underline{i} = (i, \alpha)$, to introduce our definition of $\bar{D}^4$. It is most straightforward to compute the component Lagrangian specified by a (colorless) effective function $g(W^2)$ when using an identity

$$\int d^4\theta g(W^2) = g'(\bar{\phi}^2) L + g''(\bar{\phi}^2) \left[ -V_{i a} Y^a_{\alpha \beta} + 2X_{i j} X^{i j} - 2Y^a_{\alpha \beta} Y^a_{\alpha \beta} \right]$$

$$+ g'''(\bar{\phi}^2) \left[ -V_{i a} Y^a_{\beta} + V_{i a} V^a_{\alpha} X^{i j} \right]$$

$$+ g''''(\bar{\phi}^2) V^4 ,$$

(3.8)

where the primes denote differentiations with respect to $\bar{\phi}^2 \equiv \bar{\phi}^a \bar{\phi}^a$. We find now useful to introduce more book-keeping notation,

$$(\bar{\lambda}^2)_{ij} = \bar{\lambda}^a_{i \alpha} \bar{\lambda}^a_{j \beta} \quad \text{and} \quad (\bar{\lambda}^2)_{\alpha \beta} = \bar{\lambda}^a_{i \alpha} \bar{\lambda}^a_{i \beta} ,$$

$$(\bar{\lambda}^2)_{i a} = \bar{\lambda}^a_{i \alpha} \bar{\lambda}^a_{i \beta} \quad \text{and} \quad (\bar{\lambda}^2)_{a \beta} = \bar{\lambda}^a_{i \alpha} \bar{\lambda}^a_{i \beta} ,$$

$$(\bar{\lambda}^2)_{a b} = \bar{\lambda}^a_{i \alpha} \bar{\lambda}^b_{j \beta} \quad \text{and} \quad (\bar{\lambda}^2)_{a b} = \bar{\lambda}^a_{i \alpha} \bar{\lambda}^b_{j \beta} ,$$

$$(\bar{\lambda}^4)_{a b c d} = \bar{\lambda}^d_{i 1} \bar{\lambda}^a_{i 2} \bar{\lambda}^b_{i 3} \bar{\lambda}^c_{i 4} \quad \text{and} \quad (\bar{\lambda}^4)_{a b} = \bar{\lambda}^d_{i 1} \bar{\lambda}^a_{i 2} \bar{\lambda}^b_{i 3} \bar{\lambda}^c_{i 4} ,$$

(3.9)

together with some related identities [8],

$$\bar{\lambda}^4 \equiv \frac{1}{12} (\bar{\lambda}^2)_{i j} (\bar{\lambda}^2)^{i j} = -\frac{1}{12} (\bar{\lambda}^2)_{\alpha \beta} (\bar{\lambda}^2)^{\alpha \beta} .$$

(3.10a)

and

$$(\bar{\lambda}^4)_{a b} \equiv \frac{1}{12} (\bar{\lambda}^2)_{i j} (\bar{\lambda}^2)^{i j} = -\frac{1}{12} (\bar{\lambda}^2)_{\alpha \beta} (\bar{\lambda}^2)^{\alpha \beta} .$$

(3.10b)

Equations (2.2), (2.4) and (3.8) imply that the full component Lagrangian $\mathcal{L}_{\text{deformed}}$ of the $P$-deformed $N=2$ supersymmetric $SU(2)$ gauge theory is governed by a single function,

$$F(\bar{\phi}^2) \equiv -\frac{1}{2} g'(\bar{\phi}^2) = \frac{1 - 3P^2 \bar{\phi}^2}{1 + P^2 \bar{\phi}^2} = 1 - 6P^2 \bar{\phi}^2 + O(P^4 \bar{\phi}^4) .$$

(3.11)
Putting everything together gives rise to our main result:

\[
\mathcal{L}_{\text{deformed SYM}} = F(\bar{\phi}^2)\mathcal{L}_{\text{SYM}} + 2F(\bar{\phi}^2)\left[-4i\bar{\phi}^a\bar{\phi}^b(\bar{\lambda}^a_i\tilde{\sigma}^m D_m \lambda^b) + \bar{\phi}^a(\bar{\lambda}^2)_{ij} D_{ij} \lambda^b \right.
\]
\[+ 8\bar{\phi}^a D_{ij}^a(\bar{\lambda}^2)_{ij} + 4\bar{\phi}^a \bar{\phi}^b D_{ij}^a D_{ij}^b - \bar{\phi}^a(\bar{\lambda}^2)_{ij} (\bar{\sigma}^{mn})_{ij} \bar{\phi}^b F_{mn} \]
\[-8\bar{\phi}^a F_{mn}^{a-mn}(\bar{\sigma}^{mn})_{ij}^{\alpha \beta} - 128\bar{\phi}^a \bar{\phi}^b F_{mn}^{a-mn} F_{mn}^{b-mn} + 96\lambda^4 \bigg]\]
\[+ 8F''(\bar{\phi}^2) \left[-\bar{\phi}^a \bar{\phi}^b \bar{\phi}^c (\bar{\lambda}^2)_{ij}^{ab} (\bar{\sigma}^{mn})_{ij}^{\alpha \beta} F^{c-n} + \bar{\phi}^a \bar{\phi}^b \bar{\phi}^c (\bar{\lambda}^2)_{ij}^{ab} D_{ij}^{cd} \right.
\[+ 24\bar{\phi}^a \bar{\phi}^b \bar{\phi}^c (\bar{\lambda}^4)_{abcd} \bigg] + 16F'''(\bar{\phi}^2) \bar{\phi}^a \bar{\phi}^b \bar{\phi}^c (\bar{\phi}^2)^2 D_{ij}^{abcd}, \tag{3.12}
\]

where the undeformed N=2 Lagrangian \( \mathcal{L}_{\text{SYM}} \) is given by eq. (3.5b).

## 4 Scalar potential

Perhaps, the most interesting part of the deformed Lagrangian (3.12) is its scalar potential

\[
V_{\text{deformed}} = -\frac{g_{\text{YM}}^2}{4} F(\bar{\phi}^2) \text{Tr} [\phi, \bar{\phi}]^2 \equiv F(\bar{\phi}^2) V_{\text{SYM}}, \tag{4.1}
\]

where we have explicitly introduced the gauge coupling constant, and the undeformed (non-abelian) N=2 super-Yang-Mills scalar potential \( V_{\text{SYM}} \). Equations (3.11) and (3.12) now imply

\[
V_{\text{deformed}} = \frac{1}{2} g_{\text{YM}}^2 F(\bar{\phi}^2) \varepsilon^{abf} \phi^a \bar{\phi}^b \varepsilon_{cdf} \phi^c \bar{\phi}^d
\]
\[= \frac{g_{\text{YM}}^2}{2(1 + P^2 \bar{\phi}^2)^3} \left[ \phi^2 \bar{\phi}^2 - (\phi^a \bar{\phi}^a)^2 \right]. \tag{4.2}
\]

When using the notation

\[
(\phi^a \bar{\phi}^a)^2 = \phi^2 \bar{\phi}^2 \cos^2 \vartheta \tag{4.3}
\]

we easily find

\[
V_{\text{deformed}}(\phi, \bar{\phi}) = \frac{1}{2} g_{\text{YM}}^2 \phi^2 \bar{\phi}^2 \sin^2 \vartheta \frac{1 - 3P^2 \bar{\phi}^2}{(1 + P^2 \bar{\phi}^2)^3}. \tag{4.4}
\]

The scalar potential \( V_{\text{SYM}} \) of the undeformed N=2 super-Yang-Mills theory is bounded from below (actually, non-negative), while the undeformed (and degenerate) classical vacua are given by solutions to the equation

\[
[\phi, \bar{\phi}] = 0. \tag{4.5}
\]
In the deformed case under consideration the fields \( \phi \) and \( \bar{\phi} \) are real and independent, while the \( P \)-deformation gives rise to the extra factor \( F(\bar{\phi}^2) \) in eqs. (4.1) and (4.4). Choosing \( P^2 < 0 \) gives rise to a singular scalar potential at \( \bar{\phi}^2 = -P^{-2} \). We choose \( P^2 > 0 \) to get a non-singular scalar potential at finite values of \( \phi \) and \( \bar{\phi} \).

A graph of the relevant function \( h(y) \equiv y(1 - 3P^2y)(1 + P^2y)^{-3} \) with \( y \equiv \bar{\phi}^2 \) is given in Fig. 1. The function \( h(y) \) is bounded from below and from above, as long as \( y \geq 0 \), with its maximum at \( y_1 = P^{-2}(4 - \sqrt{13})/3 \) and its minimum at \( y_2 = P^{-2}(4 + \sqrt{13})/3 \). Nevertheless, the full scalar potential is unbounded from below, just because the function \( h(y) \) can take negative values at some finite \( \bar{\phi} \) (including its value at the minimum), while one can still have \( \sin^2 \vartheta > 0 \) and \( \phi \to \infty \), which implies \( V \to -\infty \). Therefore, the classical \( P \)-deformed \( SU(2) \)-based N=2 supersymmetric gauge field theory does not have a stable vacuum.

5 Conclusion

It is worth noticing that the non-abelian \( SU(2) \)-based N=2 NAC Lagrangian found in this Letter is very different from the abelian \( U(1) \)-based Lagrangian with the same \( P \)-deformation and star product [6, 7], despite of the fact that both originate from the same Seiberg-Witten map (1.3). The non-abelian NAC Lagrangian in components is governed by another function – see eqs. (2.2) and (2.4).

Our considerations in this paper were entirely classical. It would be interesting to investigate the role of quantum corrections, both in quantum field theory and in string theory (e.g., by using geometrical engineering). It is particularly intriguing to know whether quantum corrections can stabilize the classical vacuum.
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