Regional gravity field determination from satellite altimetry data in the Black Sea and Azov Sea area

A.N. Marchenko, Z.R. Tartachynska, A.N. Yakimovich

National University “Lviv Polytechnic”, Lviv

Gravity anomalies in the Black sea area were determined using satellite altimetry observations of the Sea Surface Heights over this region. The satellite data applied in the analysis include ERS-1, ERS-2, and TOPEX/POSEIDON altimetry from 1992 to 2001. The solutions for gravity anomalies at $(3' \times 3')$ grid points are evaluated by the Tikhonov regularization method to provide estimations based on kernel functions, which are described by singular point harmonic functions. Comparison with independent solution KMS01 of the Danish geodetic institute was performed.

1. INTRODUCTION

Recent altimetry satellites give more and more accurate determination of the Sea Surface Heights (SSH), which in combination with an ocean circulation model can be considered as a direct determination of the geoid undulations in different ocean areas. Appropriate data sets from various altimetry satellites are collected also for such relatively small internal marine regions of the globe as Black sea and Sea of Azov. Thus, the inversion of SSH data set into the gravity anomalies $\Delta g$ becomes possible within the mentioned internal marine areas, assuming corrected SSH data as “measured” geoid undulations $N$ (see, for instance, [2, 5]).

This paper represents further development of the recent results from [5] and will focus on the recovery of the gravity anomalies $\Delta g$ from the combination of ERS-1, ERS-2, and TOPEX/POSEIDON altimetry (SSH) in the Black and Azov sea area by the Tikhonov regularization method.

The following data sets corrected by AVISO for different geophysical and instrumental effects are used (Table 1):

- subset 1 represents 36836 values of SSH taken for the period from October 1992 to June 1996 of the ERS-1 mission;
- subset 2 represents 122973 values of SSH taken for the period from April 1995 to September 2001 of the ERS-2 mission;
Table 1. Distribution of satellite altimetry data in the Black Sea and Sea of Azov area

| Satellite       | Period               | Number of corrected SSH |
|-----------------|----------------------|-------------------------|
| ERS1            | 10.04.1994 – 09.03.1995 | 17921                   |
| ERS1            | 24.03.1995 – 02.06.1996 | 18915                   |
| ERS1            | Total = 36836         |                         |
| ERS2            | 24.04.1995 – 06.06.1996 | 16564                   |
| ERS2            | 06.06.1996 – 01.09.1997 | 19207                   |
| ERS2            | 01.09.1997 – 26.10.1998 | 25471                   |
| ERS2            | 26.10.1998 – 11.10.1999 | 22076                   |
| ERS2            | 11.10.1999 – 25.09.2000 | 20863                   |
| ERS2            | 25.09.2000 – 10.09.2001 | 18792                   |
| ERS2            | Total = 122973        |                         |
| TOPEX/POSEIDON  | 02.10.1992 – 04.10.1993 | 18649                   |
| TOPEX/POSEIDON  | 04.10.1993 – 16.10.1994 | 18485                   |
| TOPEX/POSEIDON  | 16.10.1994 – 08.10.1995 | 19128                   |
| TOPEX/POSEIDON  | 09.10.1995 – 09.10.1996 | 18076                   |
| TOPEX/POSEIDON  | 09.10.1996 – 11.10.1997 | 18183                   |
| TOPEX/POSEIDON  | 11.10.1997 – 13.10.1998 | 17906                   |
| TOPEX/POSEIDON  | 13.10.1998 – 05.10.1999 | 17310                   |
| TOPEX/POSEIDON  | 06.10.1999 – 06.10.2000 | 18460                   |
| TOPEX/POSEIDON  | 07.10.2000 – 08.10.2001 | 15050                   |
| TOPEX/POSEIDON  | Total = 161247        |                         |

• subset 3 represents 161247 values of SSH taken for the period from October 1992 to October 2001 of the TOPEX/POSEIDON mission.

2. METHOD

As before [5] the traditional “remove–restore” procedure was used to get the initial information $\delta N$ for further determination of the gravity anomalies $\Delta g$:

$$\delta N = SSH - N_{EGM96},$$  \hspace{1cm} (1)

where $SSH$ are the corrected Sea Surface Heights, assumed to be coincided with the geoid height $N$; $N_{EGM96}$ is the long wavelength part of $N$ adopted according to EGM96 gravity field model (360, 360).

Then the prediction of the residual gravity anomalies $\delta \Delta g_P$ and the residual geoid heights $\delta N_P$ was estimated at some point $P$ (inside the studying area) applying the regularization method

$$\delta \Delta g_P = C_{\delta \Delta g, \delta N}(C + \alpha C_{nn})^{-1}l,$$  \hspace{1cm} (2)

$$\delta N_P = C_{\delta N, \delta N}(C + \alpha C_{nn})^{-1}l,$$  \hspace{1cm} (3)

where $l$ is the $q$-vector consisting in this case of the components $\delta N_i$ ($i = 1, 2, \ldots, q$); $q$ is the number of the observations $\delta N_i$; $C$ is the $(q \times q)$-covariance matrix of the residual geoid height $\delta N$; $C_{\delta \Delta g, \delta N}$ is the $(1 \times q)$-cross-covariance matrix between $\delta \Delta g$ and $\delta N$; $C_{\delta N, \delta N}$ is the $(1 \times q)$-auto-covariance matrix of $\delta N$; $C_{nn}$ is the $(q \times q)$-covariance matrix of the measurements noise $n$; $\alpha$ is the Tikhonov regularization parameter [5–7].

Having the values (1) at some set of scattered points and the above covariance matrices, the residual gravity anomalies $\Delta g$ and the residual geoid heights $\delta N$ are estimated straightforward at chosen grid points by the regularization method. After solving this basic problem the predicted gravity anomalies $\Delta g$ and geoid undulations $N$ were restored at the same grid by means of EGM96 gravity field model

$$\Delta g = \Delta g_{EGM96} + \delta \Delta g,$$  \hspace{1cm} (4)

$$N = N_{EGM96} + \delta N,$$  \hspace{1cm} (5)

For further use of the relationships (2), (3) the following problems have to be solved:

1. The construction of the analytical covariance function $K(P; Q)$ of the anomalous potential $T$. 

12 Marchenko A.N., Tartachynska Z.R., Yakimovich A.N.
2. The choice of a suitable method for the computation of the regularization parameter $\alpha$

The analytical covariance function or reproducing kernel $K(P, Q)$, described only by singular point harmonic functions [3, 4], is chosen in the following way

$$K_n(P, Q) = \left(\frac{GM}{R}\right)^2 \beta_n \sigma^{n+1} \tilde{v}_n, \quad \sigma = \frac{R_B^2}{r_P r_Q}$$  \hspace{1cm} (6)

where $R$ is the Earth’s mean radius; $R_B$ is the Bjerhammar’s sphere radius; $r_P$ and $r_Q$ are the geocentric distances to the external points $P$ and $Q$; $GM$ is the product of the gravitational constant $G$ and the

Fig. 1. Accuracy of the geoid prediction from ERS-1, ERS-2, and TOPEX/POSEIDON altimetry. Contour interval: 0.01 m

Fig. 2. Accuracy of the gravity anomalies inversion from ERS-1, ERS-2, and TOPEX/POSEIDON altimetry. Contour interval: 1 mGal
planet’s mass $M$; $v_n$ is the dimensionless potential of radial multipole of the degree $n$; $\beta_n$ represents some dimensionless coefficient. Expressions for the analytical auto-covariance function of geoid heights and cross-covariance function between gravity anomalies and geoid undulations (based on the covariance propagation) can be found in [3].

Note now that the traditional determination of the regularization parameter $\alpha$ in (2) or (3) according to [7, 8] requires in the frame of a special iterative process the inversion of matrixes with a dimension
Table 2. Statistics of the predicted residual geoid heights $\delta N$ and gravity anomalies $\delta \Delta g$ at grid points ($3' \times 3'$)

| Statistics       | $N$, m | $\delta \Delta g$, mGal |
|------------------|--------|-------------------------|
| Minimum          | -2.48  | -81.72                  |
| Maximum          | 1.92   | 78.29                   |
| Mean             | -0.57  | -7.42                   |
| Standard deviation | 0.56   | 15.57                   |

Table 3. Statistics of the geoid heights and gravity anomalies restored at grid points ($3' \times 3'$) and their accuracy estimation

| Statistics       | $N$, m | $\sigma N$, m | $\Delta g$, mGal | $\sigma \Delta g$, mGal |
|------------------|--------|---------------|------------------|-------------------------|
| Minimum          | 11.99  | 0.02          | -121.26          | 3.16                    |
| Maximum          | 40.13  | 0.17          | 102.59           | 10.56                   |
| Mean             | 23.13  | 0.04          | -17.52           | 4.48                    |
| Standard deviation | 6.63   | 37.57         |                  |                         |

Table 4. Comparison of the predicted ($3' \times 3'$) geoid heights and ($3' \times 3'$) KMS2001 SSH

| Statistic       | $N - N_{\text{KMS}}$, m |
|-----------------|--------------------------|
| Minimum         | -2.31                    |
| Maximum         | 1.49                     |
| Mean            | -0.37                    |
| St. deviation   | 0.51                     |

equal to the number $q$ of observations. So, when a number of observations are large we come to a time consuming procedure. As before [5] to avoid this difficulty another possible value of $\alpha$ is used

$$\alpha = 1 + \sqrt{1 + \text{Trace}(\mathbf{C}_{nn}) / \text{Trace}(\mathbf{C} \mathbf{C}_{nn})}$$

leading to the estimation of $\alpha$ prior to matrix inversion in (2) and (3).

Simplest illustration of a possible values of the regularization parameter $\alpha$ given by (7) can be made under several assumptions. First one, geodetic measurements of one kind only are considered. Second one, the matrix $\mathbf{C}_{nn}$ can be represented as $\mathbf{C}_{nn} = d \mathbf{I}$, where $d$ is the variance of a noise and $\mathbf{I}$ is the unite matrix. Third one, the matrix $\mathbf{C}$ can be described by the Dirac delta function and can be written as $\mathbf{C} = C_0 \mathbf{I}$, where $C_0$ is the variance of a studying field. With these assumptions the expression for the regularization parameter corresponded to (7) can be found as

$$\alpha = 1,$$

$$\alpha = 1 + \sqrt{1 + \frac{C_0}{d}}.$$
In fact, the first root (8) corresponds in (2) and (3) to the least-squares collocation solution. The second root (9) corresponds to the relationship (7) under the adopted assumptions and can serve for the illustration of a possible dependence of \( \alpha \) on the given \( C_0 \) and \( d \).

Note again that the formulae (7) and (9) represent only possible upper limit of \( \alpha \), which requires a further improvement of the considered estimation of \( \alpha \).

3. RESULTS AND CONCLUSIONS

Removing the contribution of the geopotential model EGM96 (360,360) from altimetry data (SSH) the residual geoid heights \( \delta N \) were adopted as initial information. Then the empirical covariance function (ECF) of the residual geoid heights \( \delta N = \delta SSH \) was constructed and approximated by the analytical covariance function (ACF) based on the radial multipoles potentials or ACF of the so-called point singularities [4]. As a result, the optimal degree \( n = 1 \) in the formula (6) was chosen from ECF approximation that corresponds to the dipole kernel function (Poisson kernel without zero degree harmonics). The optimal ACF has the following essential parameters: (a) the variance of field - \( \text{var}(N) = 0.259 \, m^2 \); (b) the correlation length \( \xi = 0.449^\circ \); (c) the curvature parameter \( \chi = 4.088 \). Note that this optimal ACF has the same parameters as constructed by [5] in the preliminary solution despite of essentially larger number of observations SSH.

According to the expressions (2), (3) and (7), the prediction of the residual gravity anomalies \( \delta \Delta g \) and the residual geoid heights \( \delta N \) was done by the regularization method at the adopted 20770 grid points with the resolution \((3^\prime \times 3^\prime)\). Statistics of the estimated \( \delta N \) and \( \delta \Delta g \) and their accuracy are shown in the Table 2.

Accuracy distributions are shown in Fig. 1, 2, 3, 4 illustrate the geoid heights and gravity anomalies, respectively, based on the 321056 SSH from ERS-1, ERS-2, and TOPEX/POSEIDON altimetry and predicted by the regularization method.

Table 4 and Fig. 5 illustrates the comparison of the constructed above geoid solution from ERS-1, ERS-2, and TOPEX/POSEIDON altimetry and \( (3^\prime \times 3^\prime) \) KMS2001 SSH derived from ERS-1, GEOSAT and TOPEX/POSEIDON data. Fig. 5 reflects also the adopted in KMS approach [1] of the “piecewise processing” within every \( 1^\circ \times 5^\circ \) chosen rectangular cell because data processing in this study was made for the whole geographical region without any separation to cells.

1. Andersen O.B., Knudsen P. Geodetic marine gravity field from the ERS-1 and Geosat geodetic mission altimetry // Journal of Geophysical Research. — 1998. — 103, No C4. — P. 8129–8137.
2. Küçükoğlu A. Determination of the Gravity Field from Satellite Altimetry Data in the Black Sea / Paper presented at the IAG 2001 Scientific Assembly. Budapest, Hungary, 2001.
3. Marchenko A.N. Parameterization of the Earth’s Gravity Field. Astronomical and Geodetic Society. Lviv, Ukraine, 1998.
4. Marchenko A.N., Lelgemann D. A classification of reproducing kernels according to their functional and physical significance // IGeS Bulletin. Milan. — 1998. — No 8. — P. 49–52.
5. Marchenko A.N., Tartachynska Z.R. Gravity anomalies in the Black sea area derived from the inversion of GEOSAT, TOPEX/POSEIDON and ERS-2 altimetry // Bolletino di Geodesia e Scienze Affini. — 2003. — ANNO LXII, n.1. — P. 50–62.
6. Moritz H. Advanced Physical Geodesy. — Wichmann, Karlsruhe, 1980.
7. Neyman Yu.M. Variational method of Physical Geodesy. — Moscow: Nedra, 1979 (in Russian).
8. Tikhonov A.N., Arsenin V.Y. Methods of solution of ill-posed problem. — Moscow: Nauka, 1974 (in Russian).

Received 15.07.2003

Marchenko A.N., Tartachynska Z.R., Yakimovich A.N.