Supergravity brane worlds and tachyon potentials

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We study massless and massive graviton modes that bind on thick branes which are supergravity domain walls solutions in \( D \)-dimensional supergravity theories where only the supergravity multiplet and the scalar supermultiplet are turned on. The domain walls are bulk solutions provided by tachyon potentials. Such domain walls are regarded as BPS branes of one lower dimension that are formed due to tachyon potentials on a non-BPS D-brane.

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I. INTRODUCTION

The localization of gravity on brane worlds is a very active matter of investigation in modern physics that was initiated in \cite{1}. We study here background of \( p \)-branes created by scalar fields of the bosonic sector of a \( D \)-dimensional supergravity theory. We shall mainly explore the localization of the graviton on thick \( p \)-branes \cite{3,2} supported by these scalars. These thick \( p \)-branes are domain wall solutions extended in \( p = D - 2 \) dimensions.

As one knows, thick branes can be studied without adding any scalar field, by smoothing out singular brane solutions \cite{4}. However, we are interested in a supergravity theory with the contents relevant to realize brane/string cosmology in arbitrary \( D \)-dimensions where scalar fields can play important role. We recall that such scalars play the role of tachyon fields whose dynamics are truncated up to first order on the derivatives. We choose some particular potentials in order to have connection with modern cosmology. One of the many issues found in cosmology is the accelerated expansion of our universe \cite{5,6,7}, and this may have to do with dark energy. In view of this problem new proposals of cosmological scenarios as, for instance, quintessence \cite{8,9} and tachyon cosmology \cite{5,6} have been considered in the literature.

In the present paper we focus mainly on the scalar contents of tachyon cosmology in a supergravity theory. We consider here two different tachyon potentials which we call type I and type II. The tachyon potential of type I that one uses here is a potential in the bulk, for which the global minima are at infinity \cite{10,11}. The tachyon potential of type II is unbounded from below but there are BPS solutions, i.e., domain wall solutions, with finite energy, connecting inflection points of the potential \cite{12}. These inflection points defines an interval in which the potential engenders the properties of being positive definite, with global minima at the boundaries of the interval. The tachyon potentials of type I and type II that we study are connected by a deformation function \cite{12}. They correspond to different and relevant cosmological scenarios. For models which minima are at infinity the tachyon matter is pressureless and may be considered a cold dark matter candidate \cite{13}. On the other hand, for potentials that present minima at finite critical points the tachyon matter has negative pressure and may be considered a candidate for quintessence \cite{14}. Finally we should mention that in spite of the fact that the tachyon potential of type II considered here in general breaks the supersymmetry, one sees that one half of the supersymmetry is preserved in the solitonic sector, i.e., in the sector where fields configurations are in the finite interval and also where the domain walls (thick \( p \)-branes) can form.

The paper is organized as follows. In Sec. II we introduce the \( D \)-dimensional supergravity theory. In Sec. III we discuss the issue of graviton mode localization on \( p \)-branes. The properties of the graviton localization for the backgrounds created by the aforementioned potentials are discussed in Sec. IV and in Sec. V. We end the paper in Sec. VI where we present our final considerations.

II. SUPERGRAVITY ACTION IN ARBITRARY \( D \)-DIMENSIONS

We consider a supergravity Lagrangian for arbitrary spacetimes in \( D \)-dimensions (\( D > 3 \)) invariant under supersymmetry transformations, at least to lowest order in the fermions, of the form \cite{14,15}

\[
S = \frac{1}{\kappa^{D-2}} \int d^D x \left[ R - \kappa^{D-2} g^{MN} \partial_M \phi \partial_N \phi + \bar{\psi}_M \Gamma^{MNP} \nabla_N \psi_P + \kappa^{D-2} \bar{\chi}_M \Gamma^M \nabla_i \chi^i + M(\phi) \kappa^{D-2} \bar{\chi}_M \nabla_i \chi^i - \kappa^{D-2} V(\phi) \right] + \frac{1}{2} \kappa^{D-2} \partial_N \phi (\bar{\psi}_M \Gamma^N \chi^i + \bar{\chi}_M \Gamma^N \psi^i_M) + W_2 \kappa^{D-2} (\bar{\psi}_M \Gamma^M \chi^i - \bar{\chi}_M \Gamma^M \psi^i_M) \right]
- \kappa^{D-2} (\bar{\psi}_M \Gamma^M \psi^i_M) \right]
- \kappa^{D-2} (\bar{\psi}_M \Gamma^M \psi^i_M) \right]
+ (\text{higher order fermi terms}),
\]

where \( \kappa = 1/M_* \) is the \( D \)-dimensional Planck length, \( M_* \) is the fundamental \( D \)-dimensional Planck scale, \( e = \)
\[
\det e^A_M = |\det g_{MN}|^{1/2}, \text{ with a mostly plus signature} \quad (- + + + +), \quad \text{and} \\
V(\phi) = 4(D - 2)^2 \left[ \left( \frac{\partial W}{\partial \phi} \right)^2 - \kappa^{D-2} \left( \frac{D - 1}{D - 2} \right) W^2 \right], \quad (2) \\
W_2 = (D - 2) \frac{\partial W}{\partial \phi}., \quad (3) \\
M(\phi) = 2(D - 2) \frac{\partial^2 W}{\partial \phi^2} - (D - 2) \kappa^{D-2}W. \quad (4)
\]

\( W \) is the superpotential, the pair of graviton and gravitino fields \((e^A, \psi^A_M)\) corresponds to the supergravity multiplet whereas the pair of scalar and spin-1/2 fermion fields \((\phi, \chi^i)\) forms a scalar supermultiplet. We use \(\Gamma_M\) to represent Dirac matrices. We include an internal spinor index \(i\) for generality. The action \(1\) is invariant under the following supersymmetry transformations:

\[
\delta e^A_M = - \bar{\epsilon}_i \Gamma^A \psi^A_M + c.c., \quad \delta \phi = i \epsilon_i + c.c., \quad \delta \psi^A_M = \nabla_M e^i + \kappa^{D-2}W \Gamma_M e^i, \quad \delta \chi^i = \left( \frac{1}{2} \Gamma^M \partial_M \phi + W_2 \right) e^i, \quad (8)
\]

where \(e^i\) is a local supersymmetry parameter and

\[
\nabla_M e^i = \partial_M e^i + \frac{1}{4} \omega^A_M \Gamma_{AB} e^i, \quad \Gamma_{AB} = \frac{1}{2} \{\Gamma_A, \Gamma_B\}. \quad (9)
\]

A similar consideration but exploring other issues concerning localization of gravity on brane in supergravity has also been considered in \([10, 11, 18]\).

For our purposes we employ the “generalized” Randall-Sundrum metric:

\[
ds^2_D = e^{A(y)} \eta_{\mu\nu} dx^\mu dx^\nu + dy^2, \quad (10)
\]

where \(\mu, \nu = 0, 1, 2, ..., D - 2\) are indices on the \((D - 2)\)-brane. For this metric the Killing spinor equations \(\delta \psi^A_M = 0\) and \(\delta \chi^i = 0\) provide us the following equations:

\[
\partial_A A = \mp 2 \kappa^{D-2}W, \quad (11) \\
\partial_y \phi = \pm 2(D - 2) \frac{\partial W}{\partial \phi}. \quad (12)
\]

Notice that we are assuming the scalar field \(\phi\) and the spinor field \(e^i\) only depend on the transverse coordinate \(y\). These equations describe BPS solutions since they come from the fermion supersymmetry transformations. Such solutions preserve only half of the supersymmetries.

III. GRAVITON MODES ON \((D - 2)\)-BRANES

The wave function of graviton modes due to linearized gravity equation of motion in an arbitrary number of dimensions \((D > 3)\) is given by \([4, 13, 24]\)

\[
\partial_M (\sqrt{-g} g_{MN} \partial_N \Phi) = 0, \quad (13)
\]

where \(\Phi\) describe the wave function of the graviton on non-compact coordinates. \(M, N = 0, 1, 2, ..., D - 1\). This can be found by taking into account only the traceless transverse (TT) sector of the linear perturbation, in which the gravity equations of motion do not couple to matter fields.

Let us consider \(\Phi = h(y)M(x^\mu)\) into \([13]\) and the fact that \(\square_{D-1} = m^2 M\), where \(\square_{D-1}\) is the flat Laplacian on the tangent frame. Thus the wave function for the graviton through the transverse coordinate \(y\) reads

\[
\partial_y (\sqrt{-g} g^{\mu\nu} \partial_\mu h(y)) = -m^2 |g| h(y). \quad (14)
\]

This is our starting point to investigate both zero and massive graviton modes on the branes.

Using the components of the metric \([14]\) into the equation \([14]\) we have

\[
\frac{1}{2} (D - 1) \partial_\nu A \partial_\mu h(y) + \partial_\mu h(y) = -m^2 e^{-A(y)} h(y), \quad (15)
\]

which can be written as a Schrödinger like equation by changing the metric \([10]\) to a conformally flat metric as

\[
ds^2_D = e^{A(z)} (\eta_{\mu\nu} dx^\mu dx^\nu + dz^2), \quad (16)
\]

and also by considering the following changes of variables:

\(h(y) = \psi(z) e^{-A(y)/(D-2)/4}\) and \(z(y) = \int e^{-A(y)/2} dy\). In this way we can write \([15]\) as a Schrödinger like equation

\[- \partial_z^2 \psi(z) + V(z) \psi(z) = m^2 \psi(z) \quad (17)
\]

where the potential \(V(z)\) is given in the general form

\[
V(z) = \frac{(D - 2)^2}{16} (\partial_z A)^2 + \frac{D - 2}{4} \partial^2 A. \quad (18)
\]

Here we notice that the potential depends only on the geometrical variable \(A(z)\) up to constant factors. It is easy to check that for \(D = 5\) and \(A(y) = -2k|y|\) one reproduces the Randall-Sundrum \([1]\) potential.

IV. EXPlicit EXAMPLES OF TACHYon POTENTIALS

We shall look for soliton solutions provided by potentials of tachyons that live on unstable D-branes in superstring and bosonic string theory \([13, 22, 23]\) — for D-brane/anti-D-brane systems, the tachyon field should be complex, and this will not be considered here. The tachyon dynamics on the world-volume of a \(p\)-brane is described by the action \([24, 25]\)

\[
S = - \int d^{p+1} x V(T) \sqrt{1 + \eta^{\mu\nu} \partial_\mu T \partial_\nu T}, \quad (19)
\]

where \(V(T)\) is the tachyon potential, which should have the general property of being a positive definite function.
with stable minima at $|T| \to \pm \infty$. Expanding this action up to quadratic first derivative terms gives

$$S = -\int d^{p+1}x V(T) \left(1 + \frac{1}{2} \eta^\mu{}\!{}^\nu \partial_\mu T \partial_\nu T + \ldots \right)$$

$$= \int d^{p+1}x \left( -\frac{1}{2} \eta^\mu{}\!{}^\nu \partial_\mu \phi \partial_\nu \phi - V(\phi) + \ldots \right), \quad (20)$$

where in the second equation above we assumed that $V(T) = \phi^2$ — here $\phi_T$ stands for $\partial_\mu \phi_T$ — such that we can replace $\phi_T \partial_\mu T$ to $\partial_\mu \phi$. The tachyon dynamics is now described by the action for the scalar field $\phi$. This action can be viewed as the scalar sector of the action $\mathcal{A}$. The scalar field here is related to the former tachyon field $T$ and transforms as $\phi = f(T)$.

We now focus on two special examples that allow to form domain walls solutions.

The model type I: $\phi = (1/\lambda) \arcsinh (\lambda T)$. This produces a tachyon potential

$$V(T(\phi)) = \frac{1}{1 + \lambda^2 T^2} = \operatorname{sech}^2(\lambda \phi), \quad (21)$$

Notice that the potential above has the property of having global minima at $T(\phi) = \pm \infty$. Potentials of this type are related to the massless limit of supersymmetric QCD. In this limit, supersymmetric QCD coupled to supergravity might play the role of the hidden sector of supergravity theories $\mathcal{G}$. These potentials have been employed in models of cosmology $\mathcal{C}$.

The model type II: $\phi = T / \sqrt{T + T^2}$. This produces a tachyon potential of the form

$$V(T(\phi)) = \frac{1}{1 + \phi^2} \frac{1}{(1 - \phi^2)^3} = (1 - \phi^2)^3, \quad (22)$$

We see here a different asymptotic behavior, since the above potential is unbounded below. However, the two inflection points at $\phi = \pm 1$ identify the interval $-1 \leq \phi \leq 1$, where BPS solution exists $\mathcal{B}$. Thus, if one restricts the field to the interval $\phi \in [-1, 1]$, the potential gets the required profile.

The above models are relevant to cosmology, where each one plays a different role $\mathcal{R}$. For the purposes of this paper, we are mainly concerned with the fact that these potentials have “minima” at finite or infinite critical points. They allow for domain walls solutions that can be regarded as BPS branes of one lower dimension — see $\mathcal{B}$ and references therein.

Now we shall look for domain walls solutions and their gravitational localization properties. As a first example we shall consider the model type I. We introduce a superpotential with which we are able to find soliton solutions that connect the superpotential critical points at infinity. Let us consider the general superpotential

$$W(\phi) = \frac{\mu}{\lambda} \arctan \sinh(\lambda \phi). \quad (23)$$

By using $\mathcal{B}$ we can see that $\mathcal{C}$ produces the potential

$$V(\phi) = 4(D - 2)^2 \left\{ \frac{\lambda^2}{\lambda^2} \operatorname{sech}^2(\lambda \phi) \right\} - \kappa (D - 2) \left( \frac{\kappa}{D - 2} \right) \frac{\lambda^2}{\lambda^4} \arctan^2[\sinh(\lambda \phi)] \right\} \quad (24)$$

The first term in $\mathcal{C}$ comes from the derivative

$$\left. \frac{\partial W}{\partial \phi} \right|_{\mu} = \frac{\mu}{\lambda} \operatorname{sech}(\lambda \phi) \quad (25)$$

and should appear even in global supersymmetry $\mathcal{G}$, i.e., when $\kappa = 0$. The potential $\mathcal{C}$ (for $\kappa = 0$) up to constant factors is of the type given in $\mathcal{C}$.

We can deal with the system of equations $\mathcal{E}$ and $\mathcal{S}$ easily because they are decoupled. The soliton solution can be easily found by using $\mathcal{S}$:

$$\phi(\xi) = \pm \frac{1}{\lambda} \arcsinh \xi, \quad (26)$$

where we have considered $\xi = 2(D - 2) \mu$. These solutions asymptote to infinity. Now we apply the positive solution to the equation that describes the geometry $\mathcal{X}$ — we have set here $\kappa = 1$ for simplicity. By using the superpotential $\mathcal{C}$, we find

$$A(\xi) = 2 \left[ \ln (1 + \xi^2)^{1/2} - \xi \arctan \xi \right]. \quad (27)$$

We set $2\lambda^2 (D - 2) = 1$. It is not difficult to see that the warp factor $e^{A(\xi)}$ falls off to zero as $|\xi|$ goes to infinity.

We now search for the conformal “$z$-coordinate” in order for to write down the potential $\mathcal{P}$ in the Schrödinger like equation $\mathcal{S}$. We need to perform the integral $z(y) = \int e^{-A(y)/2} dy$, i.e.,

$$z(y) = \frac{1}{2(D - 2) \mu} \int d\xi \frac{[\arctan \xi - \ln (1 + \xi^2)^{1/2}]}{\frac{D - 3}{2}}. \quad (28)$$

For very large $\mu$, the thin wall limit, the integral in $\mathcal{S}$ turns out to have approximately the simple form

$$\int \exp [\left( |\xi| \right) d\xi = \operatorname{sgn}(\xi) \exp [\left( |\xi| \right)].$$

By continuity in the parameter $\mu$, this means in the thick wall regime we can approximate such integration by the function $\sinh(\xi)$, by smoothing out back the functions $\operatorname{sgn}(\xi) \to \tanh(\xi)$ and $|\xi| \to \ln \cosh(\xi)$.

All we have assumed above suffices to guarantee integrability and inversion in $\mathcal{S}$. In this sense, up to a constant we have set to zero, we have the following:

$$z(y) = \frac{\sinh \left( \frac{\alpha \mu y}{\alpha \mu} \right)}{\alpha \mu} \quad \text{or} \quad y(z) = \frac{\arcsinh \left( \frac{\alpha \mu z}{\alpha \mu} \right)}{\alpha \mu}, \quad (29)$$

where $\alpha = 2(D - 2)$. The approximation $\mathcal{S}$ can actually be confirmed numerically, since the numerical integral of $\mathcal{S}$ has a “$\sinh$” type profile.
The potential \([13]\) is now written explicitly as 

\[
V(z) = \frac{1}{16} \frac{\alpha^4 \mu^2 \arctan^2[\arcsinh(\alpha \mu z)]}{1 + \alpha^2 \mu^2 z^2} + \frac{1}{4} \frac{\alpha^4 \mu^3 \arctan[\arcsinh(\alpha \mu z)] z}{(1 + \alpha^2 \mu^2 z^2)^{3/2}} - \frac{1}{4} \frac{\alpha^3 \mu^2}{(1 + \alpha^2 \mu^2 z^2)[\arcsinh^2(\alpha \mu z) + 1]}.
\]

(30)

This potential has the asymptotic behavior: \(V(z = \pm \infty) = 0\). This means that the potential provides no mass gap to separate the graviton zero mode from KK (massive) modes. At \(z = 0\) the potential has a minimum with depth \(V(0) = -(1/4) \alpha^4 \mu^2\). In fact, this is a volcano type potential, which tends to the singular one found in the Randall-Sundrum scenario as \(\mu \to \infty[1]\).

The analysis for the tachyon potential of the model type II is completely analogous to the previous case. The superpotential is chosen to be of the form 

\[
W = \frac{3 \mu}{8} \left[ \frac{3}{3} \frac{2 \phi (a^2 - \phi^2)^{3/2}}{a^2} + \phi (a^2 - \phi^2)^{1/2} \right] + a^2 \arctan \left( \frac{\phi}{\sqrt{a^2 - \phi^2}} \right),
\]

(31)

which is restricted to the interval \(-a < \phi < a\) and produces the potential 

\[
V(\phi) = 4(D - 2)^2 \left\{ \frac{\mu^2 (a^2 - \phi^2)^{3/2}}{a^4} - \kappa D - 2 \left( \frac{D - 1}{D - 2} \right) \right\} \times
\]

\[
\frac{9 \mu^2}{64} \left[ \frac{2 \phi (a^2 - \phi^2)^{3/2}}{a^2} + \phi (a^2 - \phi^2)^{1/2} \right] + a^2 \arctan \left( \frac{\phi}{\sqrt{a^2 - \phi^2}} \right)^2.
\]

(32)

If gravitational effects are not included in the theory, i.e. \(\kappa = 0\), the remaining term in \([33]\), which has also been found in global supersymmetry \([12]\), is of the type in the Eq. \([22]\). This term is in fact related to the remaining term of the potential \([24]\), for \(\kappa = 0\), by applying a function of deformation \(f(\phi) = a \tanh(\lambda \phi)\) as has been shown in \([12]\). Precisely, we mean that given the tachyon potential of the model type II \(V_{\kappa = 0}(\phi) \sim (\mu^2/a^4)(a^2 - \phi^2)^3\) one can obtain the tachyon potential of the model type II \(\tilde{V}_{\kappa = 0}(\phi) \sim (\mu^2/\lambda^2)\sec^2(\lambda \phi)\) by just using the transformation 

\[
\tilde{V}_{\kappa = 0}(\phi) = \frac{V_{\kappa = 0}[f(\phi)]}{[f'(\phi)]^2}.
\]

(33)

As we mentioned in the introduction, the tachyon potential of the model type II preserves half of supersymmetries in the solitonic sector, where there are BPS solutions defined in the interval \(-a < \phi < a\).

We use the equation \([12]\) and the tachyon superpotential of the model type II \([31]\) to obtain the following solution 

\[
\phi = \pm a \frac{\xi}{\sqrt{\xi^2 + 1}}.
\]

(34)

Recall that we are considering \(\xi = 2(D - 2) \mu y\). This is a topological soliton solution connecting two critical points of the superpotential \(W\) which are inflections points of \(V_{\kappa = 0}(\phi)\). This solution, just as the domain wall solution found in the previous case, is regarded here as a “thick” \(p\)-brane, where \(y\) represents the only transverse coordinate in \(D = p + 2\) dimensions.

We use equation \([11]\), setting \(\kappa = 1\) for simplicity, in order to find the warp factor on this background. We get 

\[
A(\xi) = 2 \left[ \frac{1/3}{\xi^2 + 1} - \xi \arctan \frac{\xi}{\sqrt{4(D - 2) 3}} \right],
\]

(35)

where we have set \(a = 4 \sqrt{(D - 2)/3}\). Notice that \(e^{A(\xi)}\) falls off to zero as \(|\xi|\) goes to infinity. The same arguments used before can be applied to find that here the conformal \(z\)-coordinate is also given by the equation \([23]\).

The potential \([33]\) can now be written explicitly as 

\[
V(\phi) = \frac{F_1^4 F_2^4}{4 F_2^4} - \frac{16}{3} \left( \frac{F_3^4}{(F_1^2 + 1)^4 F_2^4} + 2 F_4 F_3^4 \frac{z}{F_2^4} \right)
\]

(36)

where \(F_1 = \arcsinh[2(D - 2) \mu z]\), \(F_3 = D - 2\) and 

\[
F_4 = -2 \arctan(F_1) - 2 F_1 (3 F_3^2 + 1) \frac{2}{3 (F_1^2 + 1)^2}.
\]

This potential as the potential \([30]\) has the asymptotic behavior: \(V(z = \pm \infty) = 0\). This means that there is no mass gap. At \(z = 0\) the potential has a minimum with depth \(V(0) = -(16/3)(D - 2)^2 \mu^2 = -(2/3) \alpha^4 \mu^2\) — compare this with the depth in the previous case. In the following we shall look for solutions which have either zero or massive eigenvalues.

The graviton zero mode on the \((D - 2)\)-brane can be found easily since the Schrödinger equation \([17]\) with the general potential \([13]\) can be written as \(H \psi = m^2 \psi\), where the Hamiltonian operator is given by \(H = Q^2 Q\), with \(Q = -\partial_z + (1/4)(D - 2) \partial_z A\) — see \([3, 4, 21, 22]\) for further details on this issue. Since the operator \(H\) is positive definite, there are no normalizable modes with negative energy (mass), i.e., there is no tachyonic graviton mode. Thus the stability of the \((D - 2)\)-brane is ensured and the graviton zero mode is the lowest mode in the spectrum. The operator \(Q\) annihilates the zero mode \(\psi_0(z)\), i.e. \(Q \psi_0(z) = 0\). This implies that the graviton zero mode is 

\[
\psi_0(z) = N_0 e^{\frac{Q}{2} A(z)},
\]

(37)

where \(N_0\) is a normalization constant. Notice that the metric warp factor \(e^{A(z)}\) for both cases studied above
asymptotes to zero, even in the conformal $z$-coordinate, as $|z| \to \infty$, such that the graviton zero mode is normalizable, i.e. $\int_{\infty}^\infty |\psi_0(z)|^2dz < \infty$. We then conclude that the graviton zero mode binds to the $(D-2)$-brane located at $z = 0$. In addition to such a mode, the potentials suggest the existence of massive modes (KK modes). These modes can affect the Newtonian gravitational potential between two masses at the $(D-2)$-brane located at $z = 0$. In the next section we shall look for correction to the Newtonian potential. Due to the difficulty of integrability in massive modes, the calculations that follow involve only an asymptotic analysis.

V. CORRECTION TO THE NEWTONIAN POTENTIAL AT THE 3-BRANE

The four-dimensional gravitational potential is corrected by massive Kaluza-Klein modes. Since the KK modes correct the Newtonian potential, the correction should be very small, at least for the case of phenomenological interest, i.e. $D = 5$, in order to keep effectively the well known Newtonian potential in our four-dimensional universe, the 3-brane.

We can estimate the static Newtonian potential between two particles of mass $m_1$ and $m_2$ on the 3-brane by considering the exchange of the zero-mode and continuum KK mode propagators. The general form of this potential in five dimensions is given by

$$V(r) \sim G_N \frac{m_1m_2}{r} + M_5^{-3} \frac{m_1m_2}{r} \int_{m_0}^{\infty} dm e^{-mr} |\psi_m(0)|^2,$$  \hspace{1cm} (38)

where $M_5$ is the fundamental Planck scale in $D = 5$ dimensions. Notice we are integrating on the masses of the continuum modes. The lower limit $m_0$ in the integral is set to zero when there is no mass gap separating such continuum modes from the zero mode.

In Eq. (38) we see that in order to compute the correction of the Newtonian potential we need to specify the $\psi_m(z)$ functions for all possible $m$. In the models we are studying these functions cannot be found analytically in a closed form, therefore we only consider an asymptotic analysis of the problem. In this sense, we study the quantum mechanics problem for the potentials, by looking for solutions in the asymptotic limit $|z| \gg 1/\alpha \mu$. In such a limit the potentials and asymptote to

$$V(z) \sim \frac{\alpha'(\alpha' + 1)}{z^2},$$  \hspace{1cm} (39)

where $\alpha' = \alpha \pi/8$. We can now solve the quantum mechanics problem for these potentials. The solution is given by a linear combination of Bessel functions:

$$\psi_m(z) = a_m z^{1/2} Y_{\alpha'+1/2}(mz) + b_m z^{1/2} J_{\alpha'+1/2}(mz).$$  \hspace{1cm} (40)

After considering appropriate boundary conditions it is possible to estimate the coefficients $a_m$ and $b_m$, such that one can estimate the value of the wavefunction at $z = 0$. The leading term in $m$ is given by

$$\psi_m(0) \sim \left(\frac{m}{k}\right)^{\alpha'-1}.$$  \hspace{1cm} (41)

The factor of $k$ should agree with dimensional analysis. We can here approximate this factor to the inverse of width of the domain walls, i.e., $k \sim \mu$, which has dimension of mass.

We can now apply this result to Eq. (38) to find the correction to the Newtonian potential. Integrating over all masses by using, we find the four-dimensional gravitational potential corrected as

$$V(r) = G_N \frac{m_1m_2}{r} \left(1 + \frac{C}{(kr)^{2\alpha'-1}}\right),$$  \hspace{1cm} (42)

where $C$ is a dimensionless number and $G_N \sim k/M_5^2$. Notice that the correction is highly suppressed for $k$ around the fundamental five dimensional Planck scale and $r$ around the size tested with gravity. Since $\alpha' = \alpha \pi/8$, recalling that $\alpha = (2(D-2))/3$, we have the following situations: For the model type I, recall we have set $2\alpha'(D-2) = 1$, thus $\alpha' = \pi/8 \lambda^2$. On the other hand, for the model type II, recall we have set $a = 4\sqrt{(d-2)/3}$, thus $\alpha' = (3/64)\pi a^2$. At large distances, the Newtonian potential is not modified for $\alpha' > 1/2$. This implies the limits $0 < \lambda < \sqrt{\pi}/2$ and $a > (32/3\pi)^{1/2}$ on the parameters.

Notice that in both cases discussed above, the inverse power law of the correction is not always like $1/r^n$, where $n$ is an integer number. For arbitrary choice of both parameters $\lambda$ and $a$ we can have rational inverse power law. In this situation we are led to think of “fractal dimensions” inducing correction to the Newtonian potential. For instance if $\alpha = (1/6)3\sqrt{\pi}$, then the correction goes as $1/r^\delta$. However, for $\lambda$ slightly larger than this value, we have $1/r^p$, where $1 < p < 2$. On the other hand, for $a = 4\sqrt{2/\pi}$ we get the power law $1/r^2$, whereas for a slightly larger than this value, we find $1/r^p$, with $2 < p < 3$.

VI. CONCLUSIONS

In this paper we have investigated the localization of zero and massive graviton modes on thick branes supported by tachyon fields that live on the world-volume of non-BPS D-branes. The decay process of such objects due to tachyons ends up creating BPS branes of one lower dimension via tachyon fields — see and references therein. If we consider that the process happens in $D = 5$ dimensions, then the tachyon dynamics in the action can be regarded as the tachyon dynamics on the world-volume of a non-BPS D4-brane of superstring
theory. The brane of one lower dimension here is a BPS 3-brane and might be our four-dimensional universe.

In the quantum mechanics problem for the graviton spectrum on the BPS 3-brane, we have found volcano potentials for both models type I and type II. These potentials have just one bound state, the zero mode, and a continuum of Kaluza-Klein modes that correct the Newtonian potential in a highly suppressed way. With respect to the matter field excitations on the p-branes, it has already been shown \[\text{[15]}\] that there are no matter zero modes on p-branes solutions in the supergravity theory that we have considered. This is because there are pre-factors like \(e^{(D-n)A(z)}W(z)\), for \(n = 1, 3\), in the \((D - 1)\)-dimensional effective action that come from the integration of the matter zero modes. Such factors are zero due to exponential fall-off of the warp factor \(e^{A(z)}\). The investigation of matter massive modes is out of the scope of the paper. Such modes might reveal some new interesting physics and we leave this to be explored elsewhere.

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