Updated Measurement of the CKM Angle $\alpha$ Using $B^0 \to \rho^+ \rho^-$
Decays

The BABAR Collaboration

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Abstract

We present results from an analysis of $B^0 \to \rho^+ \rho^-$ using 316 $fb^{-1}$ of $\Upsilon(4S) \to B\overline{B}$ decays observed with the BABAR detector at the PEP-II asymmetric-energy B Factory at SLAC. We measure the $B^0 \to \rho^+ \rho^-$ branching fraction, longitudinal polarization fraction $f_L$, and the $CP$-violating parameters $S_{\text{long}}$ and $C_{\text{long}}$:

$$\mathcal{B}(B^0 \to \rho^+ \rho^-) = (23.5 \pm 2.2(\text{stat}) \pm 4.1(\text{syst})) \times 10^{-6},$$

$$f_L = 0.977 \pm 0.024(\text{stat})^{+0.015}_{-0.013}(\text{syst}),$$

$$S_{\text{long}} = -0.19 \pm 0.21(\text{stat})^{+0.05}_{-0.07}(\text{syst}),$$

$$C_{\text{long}} = -0.07 \pm 0.15(\text{stat}) \pm 0.06(\text{syst}).$$

Using an isospin analysis of $B \to \rho\rho$ decays we determine the angle $\alpha$ of the unitarity triangle. One of the two solutions, $\alpha = [74, 117]^\circ$ at 68% CL, is compatible with the standard model. All results presented here are preliminary.

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Stanford Linear Accelerator Center, Stanford University, Stanford, CA 94309

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The BABAR Collaboration,

B. Aubert, R. Barate, M. Bona, D. Boutigny, F. Couderc, Y. Karyotakis, J. P. Lees, V. Poireau, V. Tisserand, A. Zghiche

Laboratoire de Physique des Particules, IN2P3/CNRS et Université de Savoie, F-74941 Annecy-Le-Vieux, France

E. Grauges

Universitat de Barcelona, Facultat de Fisica, Departament ECM, E-08028 Barcelona, Spain

A. Palano

Università di Bari, Dipartimento di Fisica and INFN, I-70126 Bari, Italy

J. C. Chen, N. D. Qi, G. Rong, P. Wang, Y. S. Zhu

Institute of High Energy Physics, Beijing 100039, China

G. Eigen, I. Ofte, B. Stugu

University of Bergen, Institute of Physics, N-5007 Bergen, Norway

G. S. Abrams, M. Battaglia, D. N. Brown, J. Button-Shafer, R. N. Cahn, E. Charles, M. S. Gill, Y. Groysman, R. G. Jacobsen, J. A. Kadyk, L. T. Kerth, Yu. G. Kolomensky, G. Kukartsev, G. Lynch, L. M. Mir, T. J. Orimoto, M. Pripstein, N. A. Roe, M. T. Ronan, W. A. Wenzel

Lawrence Berkeley National Laboratory and University of California, Berkeley, California 94720, USA

P. del Amo Sanchez, M. Barrett, K. E. Ford, A. J. Hart, T. J. Harrison, C. M. Hawkes, S. E. Morgan, A. T. Watson

University of Birmingham, Birmingham, B15 2TT, United Kingdom

T. Held, H. Koch, B. Lewandowski, M. Pelizaeus, K. Peters, T. Schroeder, M. Steinke

Ruhr Universität Bochum, Institut für Experimentalphysik 1, D-44780 Bochum, Germany

J. T. Boyd, J. P. Burke, W. N. Cottingham, D. Walker

University of Bristol, Bristol BS8 1TL, United Kingdom

D. J. Asgeirsson, T. Cuhadar-Donszelmann, B. G. Fulsom, C. Hearty, N. S. Knecht, T. S. Mattison, J. A. McKenna

University of British Columbia, Vancouver, British Columbia, Canada V6T 1Z1

A. Khan, P. Kyberd, M. Saleem, D. J. Sherwood, L. Teodorescu

Brunel University, Uxbridge, Middlesex UB8 3PH, United Kingdom

V. E. Blinov, A. D. Bukin, V. P. Druzhinin, V. B. Golubev, A. P. Omuchin, S. I. Serednyakov, Yu. I. Skovpen, E. P. Solodov, K. Yu Todyshev

Budker Institute of Nuclear Physics, Novosibirsk 630090, Russia

D. S. Best, M. Bondioli, M. Bruinsma, M. Chao, S. Curry, I. Eschrich, D. Kirkby, A. J. Lankford, P. Lund, M. Mandelkern, R. K. Monnies, W. Roethel, D. P. Stoker

University of California at Irvine, Irvine, California 92697, USA

S. Abachi, C. Buchanan

University of California at Los Angeles, Los Angeles, California 90024, USA
A. Buzzo, R. Capra, R. Contri, M. Lo Vetere, M. M. Macri, M. R. Monge, S. Passaggio, C. Patrignani, E. Robutti, A. Santroni, S. Tosi

Università di Genova, Dipartimento di Fisica and INFN, I-16146 Genova, Italy

G. Brandenburg, K. S. Chaisanguanthum, M. Morii, J. Wu

Harvard University, Cambridge, Massachusetts 02138, USA

R. S. Dubitzky, J. Marks, S. Schenk, U. Uwer

Universität Heidelberg, Physikalisches Institut, Philosophenweg 12, D-69120 Heidelberg, Germany

D. J. Bard, W. Bhimji, D. A. Bowerman, P. D. Dauncey, U. Egede, R. L. Flack, J. A. Nash, M. B. Nikolich, W. Panduro Vazquez

Imperial College London, London, SW7 2AZ, United Kingdom

P. K. Behera, X. Chai, M. J. Charles, U. Mallik, N. T. Meyer, V. Ziegler

University of Iowa, Iowa City, Iowa 52242, USA

J. Cochran, H. B. Crawley, L. Dong, V. Eyges, W. T. Meyer, S. Prell, E. I. Rosenberg, A. E. Rubin

Iowa State University, Ames, Iowa 50011-3160, USA

A. V. Gritsan

Johns Hopkins University, Baltimore, Maryland 21218, USA

A. G. Denig, M. Fritsch, G. Schott

Universität Karlsruhe, Institut für Experimentelle Kernphysik, D-76021 Karlsruhe, Germany

N. Arnaud, M. Davier, G. Grosdidier, A. Höcker, F. Le Diberder, V. Lepeltier, A. M. Lutz, A. Oyanguren, S. Pruvot, S. Rodier, P. Roudeau, M. H. Schune, A. Stocchi, W. F. Wang, G. Wormser

Laboratoire de l’Accélérateur Linéaire, IN2P3/CNRS et Université Paris-Sud 11, Centre Scientifique d’Orsay, B.P. 34, F-91898 ORSAY Cedex, France

C. H. Cheng, D. J. Lange, D. M. Wright

Lawrence Livermore National Laboratory, Livermore, California 94550, USA

C. A. Chavez, I. J. Forster, J. R. Fry, E. Gabathuler, R. Gamet, K. A. George, D. E. Hutchcroft, D. J. Payne, K. C. Schofield, C. Touramanis

University of Liverpool, Liverpool L69 7ZE, United Kingdom

A. J. Bevan, F. Di Lodovico, W. Menges, R. Sacco

Queen Mary, University of London, E1 4NS, United Kingdom

G. Cowan, H. U. Flaecher, D. A. Hopkins, P. S. Jackson, T. R. McMahon, S. Ricciardi, F. Salvatore, A. C. Wren

University of London, Royal Holloway and Bedford New College, Egham, Surrey TW20 0EX, United Kingdom

D. N. Brown, C. L. Davis

University of Louisville, Louisville, Kentucky 40292, USA
Bakel, M. Weaver, A. J. R. Weinstein, W. J. Wisniewski, M. Wittgen, D. H. Wright, A. K. Yarritu, K. Yi, C. C. Young

*Stanford Linear Accelerator Center, Stanford, California 94309, USA*

P. R. Burchat, A. J. Edwards, S. A. Majewski, B. A. Petersen, C. Roat, L. Wilden

*Stanford University, Stanford, California 94305-4060, USA*

S. Ahmed, M. S. Alam, R. Bula, J. A. Ernst, V. Jain, B. Pan, M. A. Saeed, F. R. Wappler, S. B. Zain

*State University of New York, Albany, New York 12222, USA*

W. Bugg, M. Krishnamurthy, S. M. Spanier

*University of Tennessee, Knoxville, Tennessee 37996, USA*

R. Eckmann, J. L. Ritchie, A. Satpathy, C. J. Schilling, R. F. Schwitters

*University of Texas at Austin, Austin, Texas 78712, USA*

J. M. Izen, X. C. Lou, S. Ye

*University of Texas at Dallas, Richardson, Texas 75083, USA*

F. Bianchi, F. Gallo, D. Gamba

*Università di Torino, Dipartimento di Fisica Sperimentale and INFN, I-10125 Torino, Italy*

M. Bomben, L. Bosisio, C. Cartaro, F. Cossutti, G. Della Ricca, S. Dittongo, L. Lanceri, L. Vitale

*Università di Trieste, Dipartimento di Fisica and INFN, I-34127 Trieste, Italy*

V. Azzolini, N. Lopez-March, F. Martinez-Vidal

*IFIC, Universitat de Valencia-CSIC, E-46071 Valencia, Spain*

Sw. Banerjee, B. Bhuyan, C. M. Brown, D. Fortin, K. Hamano, R. Kowalewski, I. M. Nugent, J. M. Roney, R. J. Sobie

*University of Victoria, Victoria, British Columbia, Canada V8W 3P6*

J. J. Back, P. F. Harrison, T. E. Latham, G. B. Mohanty, M. Pappagallo

*Department of Physics, University of Warwick, Coventry CV4 7AL, United Kingdom*

H. R. Band, X. Chen, B. Cheng, S. Dasu, M. Datta, K. T. Flood, J. J. Hollar, P. E. Kutter, B. Mellado, A. Mihalyi, Y. Pan, M. Pierini, R. Prepost, S. L. Wu, Z. Yu

*University of Wisconsin, Madison, Wisconsin 53706, USA*

H. Neal

*Yale University, New Haven, Connecticut 06511, USA*
1 INTRODUCTION

In the standard model, charge conjugation-parity (CP) violating effects in the B-meson system arise from a single phase in the Cabibbo-Kobayashi-Maskawa (CKM) quark-mixing matrix [1]. Interference between the direct decay and decay after $B^0 \overline{B}^0$ mixing in $B^0 \rightarrow \rho^+\rho^-, \rho^+\pi^+, \pi^+\pi^-$ results in a time-dependent decay-rate asymmetry that is sensitive to the angle $\alpha \equiv \arg [-V_{td}^* V_{tb}/V_{ud}^* V_{ub}]$ in the unitarity triangle of the CKM matrix. These decays mainly proceed through a $b \rightarrow u\overline{d} d$ tree diagram. The presence of penguin loop contributions introduces additional phases that shift the experimentally measurable parameter $\alpha_{\text{eff}}$ away from the value of $\alpha$. Figure 1 shows the leading order tree and gluonic penguin loop contributions to this decay. Measurements of the $B^+ \rightarrow \rho^+ \rho^0$ [2] and $B^0 \rightarrow \rho^0 \rho^0$ [3] branching fractions show that the penguin contribution in $B \rightarrow \rho\rho$ is smaller than the leading order tree diagram [3]. Results from $B \rightarrow \pi\pi$ decays [4, 5] and $B^0 \rightarrow \rho^\pm \pi^\mp$ [6, 7] are discussed elsewhere [8].

In $B \rightarrow \rho\rho$ decays, a spin 0 particle decays into two spin 1 particles (as shown in Fig. 2). Subsequently each $\rho$ meson decays into a $\pi\pi$ pair: $\rho^\pm \rightarrow \pi^\pm\pi^0$ and $\rho^0 \rightarrow \pi^+\pi^-$. As a result, the CP analysis of $B^0 \rightarrow \rho^+\rho^-$ is complicated by the presence of one mode with longitudinal polarization and two modes with transverse polarization. The longitudinal mode is CP even, while the transverse modes contain CP-even and CP-odd states. The decay is observed to be dominated by the longitudinal polarization [9], with a fraction $f_L$ defined as the fraction of the helicity zero state in the decay. Integrating over the angle between the $\rho$ decay planes $\phi$, the angular decay rate is

$$d^2\Gamma/d\cos\theta_1 d\cos\theta_2 = \frac{9}{4} \left[ f_L \cos^2\theta_1 \cos^2\theta_2 + \frac{1}{4}(1-f_L) \sin^2\theta_1 \sin^2\theta_2 \right],$$

where $\theta_{i=1,2}$ are the angles between the $\pi^0$ momentum and the direction opposite to that of the $B^0$ in the $\rho$ rest frame.

In this paper we present an update of previous BABAR measurements of the $B^0 \rightarrow \rho^+\rho^-$ branching fraction, longitudinal polarization fraction, CP-violating parameters and measurement of the CKM angle $\alpha$, previously reported in Ref. [9]. There are several theoretical predictions of the branching fraction and $f_L$ in $B^0 \rightarrow \rho^+\rho^-$ decays [10]. The branching fraction of $B^0 \rightarrow \rho^\pm \pi^\mp$ can be used to provide constraints on calculations of the branching fractions and charge asymmetries in $B \rightarrow \pi\pi$ and $K\pi$ decays [11].
The analysis reported here uses 316 fb$^{-1}$ of data, which is significantly more than our previous branching fraction result (82 fb$^{-1}$) or $f_L$ and CP-violating parameter results (210 fb$^{-1}$). We have changed the selection requirements in order to reduce background with only a modest reduction in signal efficiency. The new improved probability density-function (PDF) models better account for the correlations between variables entering the maximum likelihood (ML) fit, and result in a reduced systematic uncertainty of the final results.

2 THE BaBar DETECTOR AND DATASET

The data used in this analysis were collected with the BaBar detector at the PEP-II asymmetric-energy B Factory at SLAC. This represents a total integrated luminosity of 316 fb$^{-1}$ taken at the $\Upsilon$(4S) resonance (on-peak), corresponding to a sample of $(347 \pm 3.9) \times 10^6 B\overline{B}$ pairs. An additional 27.2 fb$^{-1}$ of data, collected at approximately 40 MeV below the $\Upsilon$(4S) resonance (off-peak), were used to study background from non-resonant (continuum) $e^+e^- \rightarrow q\overline{q}$ events, where $q = u,d,s,c$.

The detector is described in detail elsewhere [12]. Surrounding the interaction point is a five double-sided layer silicon vertex tracker (SVT) which measures the impact parameters of charged particle tracks in both the plane transverse to, and along the beam direction. A 40 layer drift chamber (DCH) surrounds the SVT and provides measurements of the momenta for charged particles. Both the SVT and the DCH operate in the magnetic field of a 1.5 T solenoid. Charged hadron identification is achieved through measurements of particle energy-loss in the tracking system and the Cherenkov angle obtained from a detector of internally reflected Cherenkov light. A CsI(Tl) electromagnetic calorimeter provides photon detection, electron identification, and $\pi^0$ reconstruction. Finally, the instrumented flux return of the magnet allows discrimination of muons from pions. We use the GEANT4 [13] software to simulate interactions of particles traversing the BaBar detector.
3 EVENT SELECTION

We reconstruct $B^0 \to \rho^+ \rho^-$ candidates ($B_{\text{rec}}$) from combinations of two charged tracks and two $\pi^0$ candidates. We require that both tracks have particle identification information inconsistent with the electron, kaon, and proton hypotheses. The $\pi^0$ candidates are formed from pairs of photons, each of which has a measured energy greater than 50 MeV. The reconstructed $\pi^0$ mass must satisfy $0.10 < m_{\gamma\gamma} < 0.16 \text{ GeV}/c^2$. The mass of the $\rho$ candidates must satisfy $0.5 < m_{\pi^\pm \pi^0} < 1.0 \text{ GeV}/c^2$.

Continuum events are the dominant background which is reduced by requiring that $|\cos(TB, TR)|$ be less than 0.8, where $|\cos(TB, TR)|$ is the absolute value of the cosine of the angle between the $B_{\text{rec}}$ thrust axis and that of the rest of the event (ROE) calculated in the CM frame. To distinguish signal from continuum we use a neural network ($\mathcal{N}$) combining the following eight discriminating variables.

- The coefficients, $L_0, L_2$, where these are split into sums over the ROE for neutral and charged particles. The coefficients are defined as:

  $$L_0 = \sum_{\text{ROE}} p_j$$  \hspace{1cm} (2)

  $$L_2 = \sum_{\text{ROE}} p_j |\cos(\psi_j)|^2$$  \hspace{1cm} (3)

  where $p_j$ is the particle momentum and $\psi_j$ is the angle of the particle direction relative to the thrust axis of the $B$ candidate in the center-of-mass (CM) frame.

- $|\cos(B, Z)|$, the absolute value of the cosine of the angle between the direction of the $B$ and $z$ axis in the CM frame, where the $z$ axis is along the electron beam direction.

- $|\cos(TB, TR)|$ as previously defined.

- $|\cos(TB, Z)|$, the absolute value of the cosine of the angle between the $B$ thrust and the $z$ axis.

- the sum of the transverse momentum $p_t$ of the ROE relative to the $z$ axis.

Continuum backgrounds dominate near $|\cos \theta_i| = 1$, and backgrounds from $B$ decays tend to concentrate at negative values of $\cos \theta_i$. We reduce these backgrounds with the requirement $-0.90 < \cos \theta_i < 0.98$.

Signal events are identified kinematically using two variables, the difference $\Delta E$ between the CM energy of the $B$ candidate $E_B$ and $\sqrt{s}/2$

$$\Delta E = E_B - \sqrt{s}/2,$$  \hspace{1cm} (4)

and the beam-energy-substituted mass

$$m_{ES} = \sqrt{(s/2 + p_i \cdot p_B)^2/E_i^2 - p_B^2},$$  \hspace{1cm} (5)

where $\sqrt{s}$ is the total CM energy. The $B$ momentum $p_B$ and four-momentum of the initial state $(E_i, p_i)$ are measured in the laboratory frame. We accept candidates that satisfy $5.25 < m_{ES} < 5.29 \text{ GeV}/c^2$ and $-0.12 < \Delta E < 0.15 \text{ GeV}$. An asymmetric $\Delta E$ selection is used in order to reduce background from higher-multiplicity $B$ decays.
When multiple $B$ candidates are formed, we select the one that minimizes the sum of $(m_{\gamma\gamma} - m_{\pi^0})^2$ where $m_{\pi^0}$ is the $\pi^0$ mass reported in [14]. In 0.3% the same $\pi^0$ mesons are used by multiple $B$ candidates. We randomly select the candidate entering the fit for such events.

In order to study the time-dependent decay-rate asymmetry one needs to measure the proper-time difference $\Delta t$ between the two $B$ decays in the event, and to determine the flavor of the other $B$ meson ($B_{\text{tag}}$). We calculate $\Delta t$ from the measured separation $\Delta z$ between the $B_{\text{rec}}$ and $B_{\text{tag}}$ decay vertices [15]. We determine the $B_{\text{rec}}$ vertex from the two charged-pion tracks in its decay. The $B_{\text{tag}}$ decay vertex is obtained by fitting the other tracks in the event, with constraints from the $B_{\text{rec}}$ momentum and the beam-spot location. The RMS resolution on $\Delta t$ is 1.1 ps. We only use events that satisfy $|\Delta t| < 15$ ps and have an error on $\Delta t$ of less than 2.5 ps. The flavor of the $B_{\text{tag}}$ meson is determined with a multivariate technique [16]. The performance of this algorithm is summarised in Table 1 for the seven mutually-exclusive tag categories. Events are categorised according to decreasing signal purity and increasing mistag probability $\omega$. The categories assigned correspond to events with leptons, kaons and pions in the decay products of $B_{\text{tag}}$. The untagged category of events are dominated by continuum background, have a mistag probability of 50%, and are not used in this analysis.

| Category ($c$) | $\epsilon_c$ | $\omega_c$ | $\Delta \omega_c$ |
|---------------|--------------|------------|-------------------|
| Lepton        | 0.080 ± 0.001| 0.032 ± 0.004| 0.002 ± 0.008     |
| Kaon 1        | 0.114 ± 0.001| 0.053 ± 0.005| -0.012 ± 0.009    |
| Kaon 2        | 0.176 ± 0.002| 0.152 ± 0.005| -0.015 ± 0.008    |
| Kaon-Pion     | 0.135 ± 0.002| 0.233 ± 0.006| -0.018 ± 0.010    |
| Pion          | 0.137 ± 0.002| 0.327 ± 0.007| 0.059 ± 0.010     |
| Other         | 0.095 ± 0.001| 0.412 ± 0.008| 0.042 ± 0.001     |
| Untagged      | 0.266 ± 0.002| 0.500 ± 0.000| 0.000 ± 0.000     |

Mis-reconstructed signal candidates or self-cross-feed (SCF) signal may pass the selection requirements even if one or more of the pions assigned to the $\rho^+\rho^-$ state belongs to the other $B$ in the event. These SCF candidates constitute 50.7% (27.9%) of the accepted longitudinally (transversely) polarised signal. The majority of SCF events have both charged pions from the $\rho^+\rho^-$ final state, and unbiased $CP$ information. These correct track SCF events are denoted by CT SCF. There is a SCF component (13.8% of the signal) where at least one track in $B_{\text{rec}}$ is from the ROE. These wrong track (WT) events have biased $CP$ information, and are treated separately for the $CP$ result. The PDF describing WT events is used only to determine the signal yield and polarization. A systematic error is assigned to the $CP$ results from this type of signal event. The total selection efficiency for longitudinally (transversely) polarised signal is 7.7% (10.5%).

4 MAXIMUM LIKELIHOOD FIT

We obtain a sample of 33902 events that enter an unbinned extended ML fit. These events are dominated by backgrounds from $q\bar{q}$ (81.4%) and $B\bar{B}$ (16.6%) events. The remaining 2% of events are signal. We distinguish between the following components in the fit: (i) correctly reconstructed
signal; (ii) SCF signal, split into CT and WT parts; (iii) charm $B^\pm$ background ($b \to c$); (iv) charm $B^0$ background ($b \to c$); (v) charmless $B^0$ backgrounds; (vi) charmless $B^\pm$ backgrounds; and (vii) continuum background. The dominant $B$ backgrounds come from categories (iii) and (iv). The dominant charmless backgrounds are summarised in Table 2. The branching fractions of $B^\pm \to (a_1\pi)^\pm$ decays have been estimated using the measurements of $B^0 \to a_1^\pm \pi^\mp$ [17].

Table 2: Dominant charmless backgrounds with assumed or measured branching fraction and the number of selected events in the data sample.

| Decay        | Branching fraction (10^{-6}) | Number of events |
|--------------|------------------------------|-----------------|
| $a_1^+\pi^+$ | $39.7 \pm 3.7$ [17]          | $94 \pm 9$     |
| $\rho^0\rho^+$ (long) | $19.1 \pm 3.5$ [2]          | $70 \pm 13$    |
| $a_1^+\pi^0$  | $20 \pm 20$                | $55 \pm 55$    |
| $a_0^0\pi^+$  | $20 \pm 20$                | $45 \pm 45$    |
| $a_1^-\rho^+$ (long) | $24 \pm 2.5$ [18]          | $40 \pm 4$     |
| $\rho^+\pi^0$  | $30 \pm 30$ [19]            | $40 \pm 40$    |

We allow the charm and the non-resonant $B^0 \to \rho^+\pi^-\pi^0$ yields to vary in the fit to data. The remaining background yields are fixed to their expected values determined from measured branching fractions where available [20].

The likelihood function incorporates the following previously defined eight variables to distinguish signal from background: $m_{ES}$, $\Delta E$, $\Delta t$, $\mathcal{N}$, and the $m_{\pi^+\pi^-}$, and $\cos \theta_i$ values of the two $\rho$ mesons. For each of the aforementioned components we construct a PDF that is the product of one-dimensional PDFs for each of the variables. The PDFs for all of the components are combined to give the likelihood function used in the fit. Table 3 summarises the PDF shapes used for different components of the signal.

The $B$ background $\Delta E$ distributions are described by $3^{rd}$ order polynomials, except for non-resonant $B^0 \to \rho^+\pi^-\pi^0$ which uses a non-parametric (NP) PDF. The $m_{\rho}$ distributions for true $\rho$ mesons are parameterised using a relativistic Breit-Wigner, and the fake $m_{\rho}$ (combinatorial $\pi^+\pi^-\pi^0$) distribution is described using $3^{rd}$ order polynomials. The $m_{ES}$ distribution of $b \to c$ backgrounds is described using a phase-space-motivated distribution [21]. All remaining background shapes are described using NP PDFs.

The continuum distribution for $m_{ES}$ is described by a phase-space-motivated distribution [21]. The $\Delta E$ and $\mathcal{N}$ shapes are modeled with $3^{rd}$ and $4^{th}$ order polynomials, respectively. The parameters of the $m_{ES}$, $\Delta E$ and $\mathcal{N}$ shapes are allowed to vary in the fit to data. The continuum $m_{\pi^+\pi^-}$ distribution is described using a relativistic Breit-Wigner and a $3^{rd}$ order polynomial PDF for true and fake $\rho$ contributions, respectively. The $\cos \theta_i$ distribution is described by a $3^{rd}$ order polynomial. The parameters of the $m_{\pi^+\pi^-}$ and $\cos \theta_i$ distributions are obtained from a fit to the off-peak data.

The signal decay-rate distribution for both polarizations $f_+(f_-)$ for $B_{tag}=B^0 (B^0)$ is given by

$$f_{\pm}(\Delta t) = \frac{e^{-|\Delta t|/\tau}}{4\tau} [1 \pm S \sin(\Delta m_d \Delta t) \mp C \cos(\Delta m_d \Delta t)] \quad (6)$$

where $\tau$ is the mean $B^0$ lifetime, $\Delta m_d$ is the $B^0\overline{B}^0$ mixing frequency, and $S = S_{\text{long}}$ or $S_{\text{tran}}$ and $C = C_{\text{long}}$ or $C_{\text{tran}}$ are the CP-asymmetry parameters for the longitudinally and transversely polarized
Table 3: PDFs used to parameterize signal distributions. The abbreviations used are as follows:
long—longitudinal signal Monte Carlo (MC) simulated data; tran—transverse signal MC simulated data; true—true signal (or in the case of background, true ρ meson); SCF—SCF signal; pN—polynomial of order N; G—a Gaussian distribution; CB—a Gaussian with an exponential tail which takes the form of [22]; NP—a non-parametric PDF defined as smoothed histogram of MC simulated data; RBW—a relativistic Breit-Wigner; SigDT—a distribution of the form of Eq. 6, convoluted with a triple Gaussian resolution function. Where indicated, the PDF denoted by pN acc. refers to the physical distribution multiplied by a polynomial acceptance function of order N.

| Component            | m_E | ΔE  | N   | Δt  | cos θ_p | m_ρ |
|----------------------|-----|-----|-----|-----|---------|-----|
| true (long)          | CB  | CB+G| NP  | SigDT | p4 acc. | N.A.|
| true (tran)          | CB+G| CB+G| NP  | SigDT | p4 acc. | N.A.|
| CT SCF (long) (TT)   | CB  | p2+G| NP  | SigDT | p6 acc. | N.P |
| CT SCF (long) (TF/FT)| CB+G| p2  | NP  | SigDT | p6 acc. | N.P |
| CT SCF (long) (FF)   | CB+G| p1  | NP  | SigDT | p6 acc. | N.P |
| WT SCF (long) (TF/FT)| CB+G| p2  | NP  | SigDT | p6 acc. | N.P |
| WT SCF (long) (FF)   | CB+G| p1  | NP  | SigDT | p6 acc. | N.P |
| SCF (tran)           | CB+G| GG  | NP  | SigDT | NP      | NP  |

Signal. The parameters S and C describe B-mixing induced and direct CP violation, respectively. S and C for the longitudinally polarized WT signal are set to zero. The Δt PDF takes into account incorrect tags and is convolved with the resolution function described below. Since f_L is approximately 1, the fit has no sensitivity to either S_tr or C_tr. We set these parameters to zero and we vary them in the evaluation of systematic uncertainties.

The signal Δt resolution function consists of three Gaussians (≈ 85% core, ≈ 14% tail, ≈ 1% outliers), and takes into account the per-event error on Δt from the vertex fit. The resolution function parameters are obtained from a large sample of fully reconstructed hadronic B decays [15]. For WT SCF we replace the B-meson lifetime by an effective lifetime obtained from Monte Carlo (MC) simulation to account for the difference in the resolution. The nominal Δt distribution for the B backgrounds is a NP PDF representation of the MC samples; in the study of systematic errors we replace this model with the one used for signal. The resolution for continuum background is described by the sum of three Gaussian distributions whose parameters are determined from data.

5 RESULTS

From the extended maximum likelihood fit described above, we obtain the following results

\[
N(\text{signal}) = 615 \pm 57(\text{stat}),
\]
\[
f_L = 0.977 \pm 0.024(\text{stat}),
\]
\[
S_{\text{long}} = -0.19 \pm 0.21(\text{stat}),
\]
\[
C_{\text{long}} = -0.07 \pm 0.15(\text{stat}),
\]

after correction for a +28 event fit bias on the signal yield and a correction for a −0.007 fit bias on f_L. We discuss the origin of these fit biases in Section 6. The B^0 \rightarrow \rho^\pm \pi^\mp \pi^0 background
yield obtained from the fit is $9.2 \pm 53.6$ events. Figure 3 shows distributions of $m_{ES}$, $\Delta E$, $\cos \theta_i$ and $m_{\pi^\pm \pi^0}$ for the highest purity tagged events with a loose requirement on $N$. The plot of $m_{ES}$ contains 15.6% of the signal and 1.1% of the background. For the other plots there is an added constraint that $m_{ES} > 5.27$ GeV/$c^2$; these requirements retain 13.9% of the signal and 0.4% of the background. Figure 4 shows the $\Delta t$ distribution for $B^0$ and $\bar{B}^0$ tagged events. The time-dependent decay-rate asymmetry

$$a_{CP}(\Delta t) = \frac{N(\Delta t) - \overline{N}(\Delta t)}{N(\Delta t) + \overline{N}(\Delta t)}$$

is also shown, where $N$ ($\overline{N}$) is the decay-rate for $B^0$ ($\bar{B}^0$) tagged events.

![Graphs showing distributions for highest purity tagged events](image)

Figure 3: The distributions for the highest purity tagged events for the variables $m_{ES}$ (a), $\Delta E$ (b), cosine of the $\rho$ helicity angle (c) and $m_{\pi^\pm \pi^0}$ (d). The dashed lines are the sum of backgrounds, and the solid lines are the full PDF.

### 6 SYSTEMATIC ERROR STUDIES

Table 4 lists the possible sources of systematic uncertainties on the values of the signal yield, $f_L$, $S_{long}$ and $C_{long}$ that have been studied. The systematic uncertainties, listed in Table 4, are briefly discussed in the following.

- The uncertainty from PDF parameterisation is obtained by varying PDF shape parameters by $\pm 1\sigma$, in turn. The deviations obtained are added in quadrature to give the quoted uncertainty from the PDF parameterisation.

- The systematic uncertainty on the yield from the fraction of SCF events is obtained taking the difference between the nominal result and the number of events fitted (after correcting for the fraction of SCF events in MC simulated data) when using the true signal PDFs to extract...
Figure 4: The $\Delta t$ distribution for a sample of events enriched in signal for (a) $B^0$ and (b) $B^0$ tagged events. The dotted lines are the sum of backgrounds, the dashed lines are the $q\bar{q}$ background and the solid lines are the sum of signal and backgrounds. The time-dependent $CP$ asymmetry (see text) is shown in (c), where the curve is the measured asymmetry.

the yield. The uncertainty on the other signal observables comes from varying the fraction of SCF events by 5% per $\pi^0$ (10% total), which is twice the observed data/MC difference seen in our control sample of $B^0 \to D^- \rho^+$ events.

- The uncertainty on the $m_{ES}$ and $\Delta E$ widths is obtained from the observed shifts relative to our nominal result, when allowing these parameters to vary independently in the fit to data.

- We vary the $B$ background normalisation within expectations for each background in turn. The deviations obtained when doing this are added in quadrature to give the quoted uncertainty from this source.

- As the branching fractions of some of the $B$ backgrounds are not well known, we assign an additional uncertainty coming from the maximum shifts obtained when allowing each of the fixed backgrounds to vary in turn in the fit to data.
• Additional uncertainties on the CP results come from possible CP violation in the B background. We use existing experimental constraints where possible, otherwise we allow for a CP asymmetry up to 10% in B decays to final states with charm, and up to 50% in B decays to charmless final states.

• The physics parameters $\tau = 1.530 \pm 0.009$ ps and $\Delta m_d = 0.507 \pm 0.005$ $\hbar$ ps [14] are varied within the quoted uncertainty.

• The tagging and mistag fractions for signal and the B backgrounds are corrected for data/MC differences observed in samples of fully reconstructed hadronic B decays. Each of the tagging and mistag parameters is varied in turn by the uncertainty from the correction. The deviations obtained when doing this are added in quadrature to give the quoted uncertainty from this source.

• Allowing for possible CP violation in the transverse polarization, and in the WT longitudinally polarised signal SCF events results in additional uncertainties on signal observables. We vary $S$ and $C$ by $\pm 0.5$ ($\pm 1.0$) for the transverse polarisation (WT SCF).

• Possible CP violation from interference in doubly Cabibbo-suppressed decays (DCSD) on the tag side of the event [23] contribute to systematic uncertainties on $S_{\text{long}}$ and $C_{\text{long}}$.

• We estimate the systematic error on our results coming from neglecting the interference between $B^0 \rightarrow \rho^+\rho^-$ and other $4\pi$ final states: $B \rightarrow a_1\pi$, $\rho\pi\pi^0$ and $B \rightarrow \pi\pi\pi^0\pi^0$. Strong phases are varied between $-180$ and $180$ degrees, and the CP content of the interfering amplitudes are varied between zero and maximum using uniform prior distributions, and the RMS deviation of the parameters from nominal is taken as the systematic error. We assume that the amplitudes for $B \rightarrow \rho\pi\pi^0$ and $\pi\pi\pi^0\pi^0$ are equal to the amplitude of $B \rightarrow a_1\pi$ when calculating this systematic uncertainty.

• As the PDFs used in the ML fit do not account for all of the correlations between discriminating variables used in the fit, the results have a small bias. We calculate the fit bias on the signal observables from ensembles of experiments obtained from samples of signal and charmless $B$ background MC simulated events combined with charm and $q\bar{q}$ background events simulated from the PDFs. The 28 event bias on signal yield and 0.007 bias on $f_L$ is corrected, and 100% of the calculated fit bias is assigned as a systematic uncertainty on all signal observables. We do not observe a significant bias on $S_{\text{long}}$ and $C_{\text{long}}$.

• Small imperfections in the knowledge of the geometry of the SVT over time can affect the measurement of $S_{\text{long}}$ and $C_{\text{long}}$. We vary the alignment according to the results obtained from the study of $e^+e^- \rightarrow e^+e^-, \mu^+\mu^-$ events in order to estimate the magnitude of this systematic error on our CP results.

The branching fraction has multiplicative systematic uncertainties from the reconstruction of $\pi^0$ mesons in the detector (6%), uncertainties in the reconstruction of charged particles (0.8%), and the discrimination of $\pi^\pm$ from other types of charged particles (1%). In addition to these uncertainties, there is a 1.1% uncertainty on the number of $B\bar{B}$ pairs in the data sample. Statistical uncertainties arising from the MC samples used in this analysis are negligible.
Table 4: Summary of additive systematic uncertainty contributions.

| Contribution                          | $\sigma(N_{signal})$ | $\sigma(f_L)$       | $\sigma(S_{long})$ | $\sigma(C_{long})$ |
|--------------------------------------|-----------------------|---------------------|---------------------|---------------------|
| PDF parameterisation                 | +16.7                 | +0.0082             | +0.0149             | +0.0300             |
|                                      | −30.2                 | −0.0064             | −0.0425             | −0.0306             |
| SCF fraction                         | 84.0                  | +0.0007             | +0.00235            | +0.0070             |
|                                      | −0.0011               | −0.00355            | −0.00683            |                     |
| $m_{ES}$ and $\Delta E$ width        | 22.9                  | 0.005               | 0.011               | 0.012               |
| $B$ background normalisation         | +16.0                 | +0.0033             | +0.0096             | +0.0024             |
|                                      | −17.2                 | −0.0038             | −0.0115             | −0.0015             |
| floating $B$ backgrounds             | 33.6                  | 0.004               | 0.033               | 0.006               |
| $CPV$ in $B$ background              | +3.3                  | +0.0006             | +0.0059             | +0.0118             |
|                                      | −2.0                  | −0.0016             | −0.0214             | −0.0115             |
| $\tau$                               | +0.1                  | +0.0000             | +0.0002             |                     |
|                                      | −0.4                  | −0.0002             | −0.0008             |                     |
| $\Delta m$                           | +0.0                  | +0.0000             | +0.0014             | +0.0018             |
|                                      | −0.2                  | −0.0002             | −0.0020             | −0.0012             |
| tagging and dilution                 | +2.6                  | +0.0029             | +0.0016             | +0.0068             |
| transverse polarisation $CPV$        | −8.1                  | −0.0021             | −0.0053             | −0.0054             |
| WT SCF $CPV$                         | +0.0                  | +0.0057             | +0.0125             | +0.0095             |
|                                      | −8.3                  | −0.0000             | −0.0152             | −0.0110             |
| DCSD decays                          | −                    | −                    | 0.012               | 0.037               |
| Interference                         | 14.8                  | 0.0036              | 0.023               | 0.022               |
| Fit Bias                             | 28                    | 0.007               | 0.002               | 0.022               |
| SVT Alignment                        | −                    | −                    | 0.0100              | 0.0055              |
| Total                                | +97                   | +0.015              | +0.05               | ±0.06               |
|                                      | −101                  | −0.013              | −0.07               |                     |
7  FINAL RESULTS

Our results are

\[ \mathcal{B}(B^0 \to \rho^+ \rho^-) = (23.5 \pm 2.2\text{(stat)} \pm 4.1\text{(syst)}) \times 10^{-6}, \]
\[ f_L = 0.977 \pm 0.024\text{(stat)} \pm 0.015\text{(syst)}, \]
\[ S_{\text{long}} = -0.19 \pm 0.21\text{(stat)} \pm 0.05\text{(syst)}, \]
\[ C_{\text{long}} = -0.07 \pm 0.15\text{(stat)} \pm 0.06\text{(syst)}. \]

where the correlation between \( S_{\text{long}} \) and \( C_{\text{long}} \) is -0.058. This measurement corresponds to \( \alpha_{\text{eff}} = (95.5^{+6.9}_{-6.2})^\circ \), where \( S_{\text{long}} = \sqrt{1 - C_{\text{long}}^2} \sin 2\alpha_{\text{eff}}^{\rho \rho} \). The measured branching fraction, polarization, and \( CP \) parameters are in agreement with our earlier publication [9], with significantly improved precision.

We constrain the CKM angle \( \alpha \) from an isospin analysis [24] of \( B \to \rho \rho \). The inputs to the isospin analysis are the amplitudes of the \( CP \)-even longitudinal polarization of the \( \rho \rho \) final state, as well as the measured values of \( S_{\text{long}} \) and \( C_{\text{long}} \) for \( B^0 \to \rho^+ \rho^- \). We use the following numerical inputs in the isospin analysis:

- The average of the measurements of \( \mathcal{B}(B^0 \to \rho^+ \rho^-) \) and \( f_L \), presented here, and those reported in Ref. [25].
- The combined branching fraction and \( f_L \) for \( B \to \rho^+ \rho^0 \) from Ref. [2].
- The central value corresponding to the upper limit of \( \mathcal{B}(B \to \rho^0 \rho^0) \) from Ref. [3].
- \( S_{\text{long}} \) and \( C_{\text{long}} \) presented here.

We ignore possible \( I = 1 \) amplitudes [26] and electroweak penguins in this paper.

To interpret our results in terms of a constraint on \( \alpha \) from the isospin relations, we construct a \( \chi^2 \) that includes the measured quantities expressed as the lengths of the sides of the isospin triangles and we determine the minimum \( \chi^2_0 \). We have adopted a simulated experiment technique to compute the confidence level (CL) on \( \alpha \); our method is similar to the approach proposed in Ref. [27]. For each value of \( \alpha \), scanned between 0 and 180°, we determine the difference \( \Delta \chi^2_{\text{DATA}}(\alpha) \) between the minimum of \( \chi^2(\alpha) \) and \( \chi^2_0 \). We then generate MC experiments around the central values obtained from the fit to data with the given value of \( \alpha \) and we apply the same procedure. The fraction of these experiments in which \( \Delta \chi^2_{\text{MC}}(\alpha) \) is smaller than \( \Delta \chi^2_{\text{DATA}}(\alpha) \) is interpreted as the CL on \( \alpha \). Figure 5 shows 1 – CL for \( \alpha \) obtained from this method. Selecting the solution closest to the CKM combined fit average [28, 29] we find \( \alpha = [74, 117]^\circ \) at 68% CL, where the error is dominated by \(|\alpha_{\text{eff}} - \alpha|\) which is 18° at 68% CL. The constraint obtained on \( \alpha \) is less precise than the previous result in Ref. [9] for the following reasons: (i) the improved world average branching fraction for \( B^+ \to \rho^+ \rho^0 \) has come down, and (ii) the results of the latest search for \( B^0 \to \rho^0 \rho^0 \) show evidence for a signal. Both of these factors lead to an increase in the the penguin contribution to the total uncertainty on \( \alpha \) when using an isospin analysis.

It is possible to calculate a model-dependent constraint on \( \alpha \) using \( SU(3) \) as detailed in Ref. [30]. This model relates the penguin amplitude in \( B^+ \to K^*00 \rho^+ \) to the penguin amplitude in \( B^0 \to \rho^+ \rho^- \), and can be used to obtain a more precise, but model dependent constraint on \( \alpha \).
Figure 5: Confidence level on $\alpha$ obtained from the isospin analysis with the statistical method described in [28]. The dashed lines correspond to the 68% (top) and 90% (bottom) CL intervals.

8 SUMMARY

In summary we have improved the measurement of the branching fraction, $f_L$, and CP-violating parameters $S_{\text{long}}$ and $C_{\text{long}}$ in $B^0 \rightarrow \rho^+\rho^-$ using a data-sample of $(347 \pm 3.9) \times 10^6 B\overline{B}$ pairs. We do not observe mixing-induced or direct CP violation. We derive a model-independent measurement of the CKM angle $\alpha$. The results presented in this paper are consistent with previous results presented by BABAR [9] and Belle [25].

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$$A(m;m_0,\xi) = \frac{1}{N} \cdot m \sqrt{1 - (m/m_0)^2} \cdot \exp(\xi (1 - (m/m_0)^2)) \cdot \theta(m < m_0),$$

where $\theta(m < m_0) = 1$ and $\theta(m > m_0) = 0$.

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$$CB(m;m_0,\sigma,\alpha,n) = \frac{1}{N} \cdot \exp\left(-\frac{(m - m_0)^2}{2\sigma^2}\right), m > m_0 - \alpha\sigma$$

and

$$\frac{1}{N} \cdot \frac{(n/\alpha)^n \exp(-\alpha^2/2)}{((m_0 - m)/\sigma + n/\alpha - \alpha)^n}, m \leq m_0 - \alpha\sigma.$$  

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