Anisotropic Cyclic Universe in $F(X) - V(\phi)$ model

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June 17, 2014

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Abstract

We investigate the cosmology of a class of model with noncanonical scalar field and matter in an anisotropy background. We find fixed points and their stability which constraints equation of state parameter for the matter. This is done after expressing the Einstein equations in terms of dimensionless variables. Similarly we define a set of suitable dynamical variables for studying bouncing solutions. The condition for nonsingular bounce is obtained. Here we show, numerically, that solution that of a cyclic universe exist for the certain form of kinetic term of noncanonical scalar field and the time period of cycle depends on the kinetic term of noncanonical scalar field. In certain case we find that a cyclic universe can be approximated to be a single bouncing scenario for the entire evolution of the dynamical variables. Resembling an eternal bouncing scenario, this model may also be free from BKL instability. Also, the universe isotropize in the expansion phase and this isotropization can be delayed to some extent by tuning the the parameter of the model.

1 Introduction

There are two scenarios exist in the literature which address the shortcomings of the standard model of cosmology. First one is Inflation which is most popular in the cosmology community. Though the Inflation solves most of the problems faced by standard model of cosmology, the issues with initial singularity still remains to be unresolved. The second one comes under Bouncing models where initial singularity problem can be resolved by constructing a non singular bouncing model. The most promising models in this category are matter bounce models [1] and Ekpyrotic models [2]. For a review on these scenarios refer to [3] and [4].

A severe problem with bouncing cosmologies is that instability develops due to the growth of anisotropic stress during contracting phase because this stress goes as sixth inverse power of scale factor. This is called the BKL instability [5]. Originally Ekpyrotic scenarios were developed to cure this instability [6]. This requires a matter field with equation of state parameter $w$ greater than 1. Then energy density of this matter field dominates over anisotropic stress so that this instability is eliminated.

Just with one canonical scalar field it may be impossible to build a consistent Ekpyrotic bouncing model [2]. The Ekpyrotic scenario given in [2] is not able to produce correct scale invariant power spectrum without adding another scalar field. However the scale invariant spectrum can be achieved by non trivial matching of curvature perturbation across the bounce [7]. In order to solve this problem a New Ekpyrotic scenario developed where a second scalar field is added to this model which is an isocurvature mode to start with and converts to adiabatic during the evolution [8]. BKL instability also develops in this scenario [10]. The ”New Ekpyrotic” scenario can produce nonsingular bounce if the second scalar field is a ghost condensate [9].

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However this nonsingular bouncing model is not free from unstable growth of curvature perturbation and anisotropy [10]. However this is not the end of the story. There are other nonsingular bouncing models which can be free from these instabilities as shown in ref. [11].

In this paper we restrict to a noncanonical scalar field with a general function of kinetic term $F(X)$, where $X = -1/2\partial^\mu\phi\partial_\mu\phi$. These theories are originally motivated to provide a large tensor to scalar perturbation in a inflationary setting [12–15]. Dark energy with a general kinetic term $F(X)$ is modeled first in ref. [16]. For other variants of models of dark energy in this context refer to [17]. Other works related to unifying dark matter, dark energy and/or inflation for noncanonical scalar field models are studied [18–21]. In order to study cosmology we study the equations of dynamical variables in analogy with the approach given in ref. [22]. The motivation to use such a noncanonical scalar field is to construct nonsingular bouncing models. The phase space analysis of a cosmological model with scalar field Lagrangian $F(X) - V(\phi)$ and matter for an FRW background done in ref. [23]. The condition for nonsingular bounce is also discussed in ref. [23].

Here we study the cosmology of an anisotropic universe filled with noncanonical scalar field with a Lagrangian of the form $F(X) - V(\phi)$ and matter. In section 2 we set up dynamical equation for our cosmological model with a noncanonical scalar field and matter. Fixed points and their stability for the dynamical equations are discussed in section 3. The stability of fixed points are also shown numerically for a chosen set of parameters. Dynamical equations for a different set of dynamical variables for bouncing cosmology are derived in section 4. Conditions for existence of nonsingular bouncing solution is explained in subsection 4.1. The solution that resembling a cyclic universe is discussed in detail in subsection 4.2. The presence of such a solution is shown numerically and analysed. Our conclusion is given in section 5.

2 Einstein Equations in Bianchi I

Our starting action is given by

$$S = \int d^4x \sqrt{-g} \left[\frac{1}{2}R + F(X) - V(\phi)\right]$$

As our prime interest is to see how anisotropy of the spacetime behaves as the universe evolves, while transiting from a contracting phase to an expanding one through bounce in particular, we choose one of the simplest homogenous but anisotropic spacetime called Bianchi I with a planar symmetry. The line element of Bianchi I with planar symmetry is:

$$ds^2 = -dt^2 + a^2(t)(dx^2 + b^2(t)(dy^2 + dz^2)).$$

We define average hubble parameter $H$ and $h$

$$H = \frac{1}{3} (H_a + 2H_b), \quad h = \frac{H_b - H_a}{\sqrt{3}},$$

In terms of averaged Hubble parameter and shear $h$, the Einsteins equation take the following form

$$\frac{dH}{dt} = -H^2 - \frac{2}{3}h^2 - \frac{1}{6}(\rho + 3p),$$

$$\frac{dh}{dt} = -3hH,$$

$$H^2 = \frac{\rho}{3} + \frac{h^2}{3},$$

where $\rho = \rho_\phi + \rho_m$ and $p = p_\phi + p_m$. 

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The energy density and pressure of the scalar field is found to be

\[ \rho_\phi = 2XF_X - F + V, \]
\[ p_\phi = F(X) - V(\phi), \]  \tag{5}

where \( \rho_m \) and \( p_m \) are density and pressure of matter.

Equation of motion in the presence of scalar field and matter are

\[ \frac{dH}{dt} = -H^2 + \frac{2}{3}h^2 - \frac{1}{6}(2XF_X - F + V + \rho_m + 3(F - V) + 3p_m) \]  \tag{6}

\[ \frac{dh}{dt} = -3hH, \]
\[ H^2 = \frac{2XF_X - F}{3} + \frac{V}{3} + \frac{h^2}{3} + \frac{\rho_m}{3}, \]  \tag{7}

In order to do a fixed point analysis we need to write the above equations in terms of dimensionless variables. Now we define the following dimensionless quantities as

\[ x = \sqrt{\frac{2XF_X - F}{\sqrt{3}H}}, \]
\[ y = \sqrt{\frac{V}{\sqrt{3}H}}, \]
\[ z = \frac{h}{\sqrt{3}H}, \]
\[ \Omega_m = \frac{\rho_m}{3H^2} \]  \tag{8}

and which obeys the modified Friedmann constraint equation as,

\[ 1 = x^2 + y^2 + z^2 + \Omega_m. \]  \tag{9}

We also define the following parameters for our fixed point analysis

\[ \rho_k = 2XF_X - F, \]
\[ w_k = \frac{-F}{2XF_x - F}, \]
\[ \sigma = -\frac{1}{\sqrt{3}\rho_k} \frac{d\log V}{dt}, \]
\[ w_\phi = \frac{p_\phi}{\rho_\phi} = \frac{w_kx^2 - y^2}{x^2 + y^2}, \]  \tag{10}

where \( \rho_k \) is the kinetic part of the energy density \( \rho_\phi \), \( w_k \) is the ratio of kinetic part of the pressure \( p_\phi \) to the \( \rho_k \) and \( \sigma \) is the auxiliary variable which depends on the variation of potential with time.

As there is no interaction between the component of the matter sector, we get a continuity equation for each of them. Expressing the continuity equation for \( \rho_{phi} \) them in terms of dimensionless time variable \( N \) \((dN = d\log a)\), we have

\[ \frac{d}{dN}(2XF_X - F + V) + 6XF_X = 0. \]  \tag{11}

In order to get the evolution, we differentiate \( x, y \) and \( z \) w.r.t. \( N \), we have the following set of equations:
\[
\frac{dx}{dN} = \frac{2XF_{XX} + F_X}{2\sqrt{3}(2XF_X - F)H} \left( \frac{dx}{dN} \right) - x \frac{\dot{H}}{H^2},
\]
\[
\frac{dy}{dN} = -\frac{3}{2} \sigma y x - y \frac{\dot{H}}{H^2},
\]
\[
\frac{dz}{dN} = -3z - z \frac{\dot{H}}{H^2},
\]
\[
(12)
\]

Now, to write our equation of motion completely in terms of the newly defined variables and parameters given in Eq. 8 and 10, we substitute the term \(\frac{\dot{H}}{H^2}\) and \(\frac{dX}{dN}\) by using the Raychaudhuri equation (Eq. 6) and the continuity equation (Eq. 11). Thus writing,
\[
\frac{\dot{H}}{H^2} = -\frac{3}{2} \left( w_k - w_m \right) x^2 + (1 + w_m)(1 - y^2) + (1 - w_m)z^2 \right],
\]
\[
(13)
\]
\[
\frac{dX}{dN} = \frac{1}{2XF_{XX} + F_X} \left[ \frac{\sigma\sqrt{3}\rho_k}{H} - 6XF_X \right],
\]
\[
(14)
\]

and substituting them into Eq. 12 we get
\[
\frac{dx}{dN} = \frac{3}{2} \left( \sigma y^2 - x(w_k + 1) \right),
\]
\[
\frac{dy}{dN} = \frac{3}{2} \sigma y x \left( (1 + w_m)(1 - y^2) + x^2(w_k - w_m) + (1 - w_m)z^2 \right),
\]
\[
\frac{dz}{dN} = -3z + \frac{3}{2} z \left( (w_k - w_m)x^2 + (1 + w_m)(1 - y^2) + (1 - w_m)z^2 \right)
\]
\[
(15)
\]

In the next section, we do a fixed point analysis and check the stability of fixed points. We also confirm stable fixed points by the evolution of the dynamical variables x, y and z.

3 Fixed Point Analysis

In this section we do a fixed point analysis of our system of dynamical equation (Eq.15) in order to extract the qualitative information about the nature of solution. Fixed points are calculated by taking the first derivative of the dynamical variables to zero. The stability of a fixed point is determined from the behaviour of a small perturbation around that fixed point.

We get the set of fixed points \(x_c, y_c\) and \(z_c\) by solving the following set of equations simultaneously (where the subscript c denotes fixed points). Now, if we define the slopes of the dynamical variables x, y, z as \(f(x, y, z)\), \(g(x, y, z)\) and \(h(x, y, z)\) then the set of equations we need to solve to get the fixed point is
\[
f(x, y, z) \equiv \frac{dx}{dN} = 0,
\]
\[
g(x, y, z) \equiv \frac{dy}{dN} = 0,
\]
\[
h(x, y, z) \equiv \frac{dz}{dN} = 0,
\]
\[
(16)
\]
Fixed Points \((x_c, y_c, z_c)\) | Stability Conditions
--- | ---
\((0,0,0)\) | Stable for \(w_m < -1\)
\((1,0,0)\) | Not Stable
\((0,1,0)\) | Stable for \(w_m > -1\)

Table 1: Stability Analysis of fixed points for \(z = 0\) with \(w_k = 1\).

where,

\[
f(x, y, z) = \frac{3}{2}[\sigma y^2 - (w_k + 1)x] + \frac{3}{2}[(w_k - w_m)x^2 + (1 + w_m)(1 - y^2) + (1 - w_m)z^2],
\]

\[
g(x, y, z) = -\frac{3}{2}\sigma y x + \frac{3}{2}y [(w_k - w_m)x^2 + (1 + w_m)(1 - y^2) + (1 - w_m)z^2],
\]

\[
h(x, y, z) = -3z + \frac{3}{2}z[(w_k - w_m)x^2 + (1 + w_m)(1 - y^2) + (1 - w_m)z^2].
\] (17)

The corresponding fixed point for \(\Omega_m\) can be found using the constraint equation.

\[
1 = x^2 + y^2 + z^2 + \Omega_m.
\] (18)

The stability or unstability of the fixed points can be examined by perturbing around fixed points and from the evolution of their perturbations. Now, if \((x_c, y_c, z_c)\) is a fixed point and \(\delta x = x - x_c, \delta y = y - y_c\) and \(\delta z = z - z_c\) be the respective perturbation around it, then the evolution of the perturbation is determined by

\[
\begin{align*}
\delta \dot{x} &= \dot{x} = f(x_c + \delta x, y_c + \delta y, z_c + \delta z), \\
\delta \dot{y} &= \dot{y} = g(x_c + \delta x, y_c + \delta y, z_c + \delta z), \\
\delta \dot{z} &= \dot{z} = h(x_c + \delta x, y_c + \delta y, z_c + \delta z).
\end{align*}
\] (19)

Upto first order the evolution equations for these pertubation are

\[
\begin{pmatrix}
\delta \dot{x} \\
\delta \dot{y} \\
\delta \dot{z}
\end{pmatrix} = \mathbf{A}
\begin{pmatrix}
\delta x \\
\delta y \\
\delta z
\end{pmatrix}
\] (20)

where the matrix is

\[
\mathbf{A} = \begin{pmatrix}
\frac{\partial f}{\partial x} & \frac{\partial f}{\partial y} & \frac{\partial f}{\partial z} \\
\frac{\partial g}{\partial x} & \frac{\partial g}{\partial y} & \frac{\partial g}{\partial z} \\
\frac{\partial h}{\partial x} & \frac{\partial h}{\partial y} & \frac{\partial h}{\partial z}
\end{pmatrix}
\] (21)
Stability Conditions

is the Jacobian matrix and is evaluated at the fixed point \((x_c, y_c, z_c)\) and hence each entry of \(A\) is a number. The solution of this system of equation can be found by diagonalizing the matrix \(A\). And a non trivial solution exist only when the determinant \(|A - \lambda I|\) is zero. Thus, solving this cubic equation in \(\lambda\) we would get all the eigen values of the system corresponding to each fixed points.

Here we consider a particular case where \(F(X)\) and \(V(\phi)\) are
\[
F(X) = X^a \quad \text{and} \quad V(\phi) = V_0 \exp(-\lambda\phi).
\]
In this section we restrict our analysis for a canonical scalar field \(F(X) = X\) which gives \(w_k = 1\) and \(\sigma = 0.82\) for \(\lambda = 1\). Now we divide the set of fixed points into two categories one for which \(z_c = 0\) and another for \(z_c \neq 0, z_c = 0\) corresponds to a scenario where universe isotropizes at late times where a nonzero value of \(z_c\) gives rise to an anisotropic universe at late times. The stability of the fixed points are summarized in the table 1 and 2 for both \(z_c = 0\) and \(z_c \neq 0\) respectively. It can be seen that the stability of the fixed points depend on the value of \(\omega_m\).

The stability of fixed point \((x_c, y_c, z_c) = (0, 0, 0)\) is shown in Fig.1 for \(w_m = -5\). In this case our universe eventually evolves into a state dominated by matter at late times. However equation of state of the matter violates all the energy conditions [24]. The fixed point \((x_c, y_c, z_c) = (0, 1, 0)\) is also stable for \(w_m > -1\) as plotted in Fig.2. In this case we have taken \(w_m = 1\). Our universe is dominated by potential of scalar field at late times.

We get two more fixed points for \(z_c \neq 0\) which are
\[
(\frac{2}{\sigma}, \frac{2w_k}{\sigma^2}, \sqrt{1 + \left(\frac{1 + w_m}{1 - w_m}\right)(\frac{2w_k}{\sigma^2}) - \left(\frac{w_k - w_m}{1 - w_m}\right)(\frac{2}{\sigma^2})}) \quad \text{for } w_m \neq 1
\]
\[
(\frac{2}{\sigma}, \frac{2w_k}{\sigma^2}, z_c) \quad \text{for } w_m = 1 \quad \text{and} \quad w_k = \frac{\sigma^2 - \sqrt{\sigma^2 - 8\sigma^2}}{4}.
\]
Stability analysis for these two fixed points are beyond the scope of this paper. This will be reported elsewhere.

4 Condition for Nonsingular Bounce

In this section we look for non singular bouncing solution where the scale factor has a minimum value (nonzero) at the bounce. We evolve a new set of dimensionless variables which is suitable for studying nonsingular bounce and the behaviour of anisotropy near the bounce. In this case the first derivative of the scale factor is zero and second derivative of the scale factor is positive during the bounce. The Hubble parameter transits from a negative value (a contracting phase) to a positive value (an expanding phase).

In order to study bouncing we define a new set of dynamical variables as the previous definition of variables \(x, y\) and \(z\) are ill-defined at the bouncing. The new set of dynamical variables are
\[
\tilde{x} = \frac{\sqrt{3}H}{\sqrt{|p_k|}}, \tilde{y} = \frac{\sqrt{|V|}}{\sqrt{|p_k|}} \text{sign}(V), \tilde{z} = \frac{h}{\sqrt{|p_k|}}, \tilde{\Omega}_m = \frac{\rho_m}{|p_k|},
\]
\(\tilde{z}\) are given by,

| Fixed Points \((x_c, y_c, z_c)\) | Stability Conditions |
|-------------------------------|----------------------|
| \((0,0,1)\) with \(w_m \neq 1\) | not stable |
| \((0,0,z_c)\) with \(w_m \neq 1\) | not stable |
| \((\bar{x}_c, \sqrt{1 + \left(\frac{w_k - w_m}{1 - w_m}\right)\bar{x}_c^2})\) for \(w_m \neq 1\) | not stable |
| \((\bar{x}_c, \bar{z}_c)\) for \(w_m = 1\) | not stable |

Table 2: Stability Analysis of fixed points for \(z \neq 0\) with \(w_k = 1\)
Figure 1: Evolution of the dynamical variables $x$ (top left), $y$ (top right), $z$ (bottom left), and $\Omega$ (bottom right) for the fixed point $(x_c, y_c, z_c) = (0, 0, 0)$ with the values of parameters $w_k = 1.0$, $w_m = -5.0$ and $\sigma = 0.82$
Figure 2: Evolution of the dynamical variables $x$ (top left), $y$ (top right), $z$ (bottom left), and $\Omega$ (bottom right) for the fixed point $(x_c, y_c, z_c) = (0, 1, 0)$ with the values of parameters $w_k = 1.0$, $w_m = 1.0$ and $\sigma = 0.82$
\[ \frac{d\tilde{x}}{dN} = -\frac{3}{2} \left[ (w_k - w_m) \text{sign}(\rho_k) + (1 + w_m)(\tilde{z}^2 - \tilde{y}|\tilde{g}|) + (1 - w_m)\tilde{z}^2 \right] + \frac{3}{2} \tilde{x} [(w_k + 1)\tilde{x} - \sigma\tilde{y}|\tilde{g}|\text{sign}(\rho_k)], \]

\[ \frac{d\tilde{y}}{dN} = \frac{3}{2}\tilde{y}[ -\sigma + (w_k + 1)\tilde{x} - \sigma\tilde{y}|\tilde{g}|\text{sign}(\rho_k)], \]

\[ \frac{d\tilde{z}}{dN} = -3\tilde{z}\tilde{x} + 3\tilde{z}\tilde{x}(1 + w_k) - 3\tilde{z}\tilde{y}|\tilde{g}|\text{sign}(\rho_k), \]

\[ \frac{d\tilde{\Omega}_m}{dN} = -3(1 + w_m)\tilde{x}\tilde{\Omega}_m - \tilde{\Omega}_m [3\sigma\tilde{y}|\tilde{g}|\text{sign}(\rho_k) - 3\tilde{x}(1 + w_k)] \]

(23)

and the constraint equation is

\[ \tilde{x}^2 - \tilde{y}|\tilde{g}| - \tilde{z}^2 - \tilde{\Omega}_m = 1 \times \text{sign}(\rho_k). \]

(24)

The equation for parameter \( \sigma' \) is given by

\[ \frac{d\sigma'}{dN} = -3\sigma'^2 (\Gamma - 1) + \frac{3\sigma'(2\Xi(w_k + 1) + w_k - 1)}{2(\sigma' + 1)(w_k + 1)} \left[ (w_k + 1)\tilde{x} - \sigma'\tilde{y}^2 \right] \]

(25)

where \( \Xi = \frac{XF}{X^2} \) and \( \Gamma = \frac{V\phi}{\phi_x} \). \( \sigma' \) is same as the parameter \( \sigma \) defined in Eq.(10).

A nonsingular bounce is attained whenever the universe passes from a contracting phase to an expanding phase through a minimum value of the average scale factor \( A(t) = (ab^2)^{1/3} \) but not zero. Mathematically, it satisfies

\[ (H)_b \equiv \frac{1}{A_b(t)} \left( \frac{dA(t)}{dt} \right)_b = 0, \]

(26)

where subscript \( b \) denotes value of the variable at the bounce, and

\[ \left( \frac{d^2A(t)}{dt^2} \right)_b > 0 \]

(27)

for minimum to occur. This implies

\[ \left( \frac{dH}{dt} \right)_b = \left( \frac{\ddot{A}}{A} \right)_b - \left( \frac{\dot{A}}{A} \right)_b^2 > 0 \]

(28)

Now, writing the above conditions in terms of dynamical variables for bouncing, we get \( \tilde{x}_b = 0 \) and \( \left( \frac{d\tilde{x}}{dN} \right)_b > 0 \) which translates to the following equations.

\[ \left( \frac{d\tilde{x}}{dN} \right)_b = -\frac{3}{2} \left[ (w_k - w_m)\text{sign}(\rho_k) + (1 + w_m)(\tilde{z}^2 - \tilde{y}|\tilde{g}|) + (1 - w_m)\tilde{z}^2 \right] > 0 \]

(29)

and

\[ \left( \frac{\tilde{y}|\tilde{g}|}{(1 - w_m)} - \frac{\tilde{z}^2}{(1 + w_m)} \right)_b > 1 \times \text{sign}(\rho_k) \frac{(w_k - w_m)}{1 - w_m^2}. \]

(30)

At the bounce we then obtain the following relation among dynamical variable as

\[ \left( \tilde{x}^2 - \tilde{y}|\tilde{g}| - \tilde{z}^2 - \tilde{\Omega}_m \right)_b = -\tilde{y}|\tilde{g}| - \tilde{z}^2 - \tilde{\Omega}_m = 1 \times \text{sign}(\rho_k). \]

(31)

In the next section we show that our solution gives rise to a cyclic universe where at each bounce Eq. 29 30 31 are satisfied.
4.1 Cyclic Universe

It would be interesting to get a solution where our universe bounces eternally many times. In a cyclic universe there is no horizon problem. In such a scenario where the universe passes from contraction to expansion phases through bounces and repeats the process after reaching a minimum value of the scale factor at the bounce. In other words, it oscillates between contraction to expansion phase through a nonsingular bounce.

In this section, we attain a cyclic bouncing universe by invoking a power law $F(X) = X^\eta$ kinetic term with $\eta > 2$ together with an exponential potential $V(\phi) = V_0 \exp(-\lambda \phi)$. The set of bouncing equations are evolved for an initially contracting phase with the set of parameters $w_m = 1/3$, sign of $\rho_k = +ve$, sign of $y = +ve$, $\Gamma = 1$ and $\Xi = \eta - 1$. Then dynamical variables are evolved for three different values of $\eta = 3, 6, 9$ denoted by the red, green and blue respectively in all the plots. Here we have taken care of the evolution of $\sigma'$ which remains almost constant around bounce. It can be observed from the fig.3 and 4 that $\sigma'$ blows up at late times because potential vanishes at those times.

Starting the evolution with a contracting phase the Fig. 5 shows the evolution of $\tilde{x}$ for the values of different $\eta$. It shows the oscillation of the universe from contracting phase to expanding phase through a non-singular bounce. The value of the exponent $\eta$ has a direct impact on the time period of oscillation which is time elapsed between two consecutive bounces. Indeed, the time period increases with the increase in value of the exponent $\eta$. Subsequently universe stays for a longer time both in contraction and expansion taking values $\tilde{x} = -1$ and $\tilde{x} = 1$ respectively for higher values of $\eta$ and hence a model of this kind with a large value of the $\eta$ can be approximated as a single Bounce scenario for the entire evolution of the universe. During this period kinetic part of scalar field dominates the evolution. We don’t find any bounce for $\eta \leq 2$ for our choice of values of parameters. As we can see from the Fig. 6, potential doesn’t play any major role in our cyclic universe as it goes to zero after 4-5 e-folds of evolution.

For an anisotropic nonsingular bouncing model, we require the value of shear to be finite at the bounce in particular and small compared to the value of $\tilde{x}$. The plot of anisotropy, Fig. 7, shows a cyclic behaviour of anisotropy which has maximum value at the bounce and vanishes quickly during the expanding phase. Anisotropy decreases as $\eta$ increases. This can also be noted from the Fig. 9, which implies an endless identical cycles of universe where anisotropy goes through the same process in each successive cycles, taking the same value at the corresponding point for $\eta = 3$. Thus isotropization in the expanding phase can be delayed to a small extent by tuning the controlling parameter to a greater value. Also the behaviour of anisotropy near bounce can be seen from Fig. 10. This feature may be important while constructing a more realistic non-singular bouncing scenario with a non-negligible shear and look for it’s signature in cosmic microwave background anisotropy. It is clear from Fig. 7 shear is subdominant at or near the bounce.

The sufficient condition for nonsingular bounce $\frac{d\tilde{x}}{dN} > 0$ is shown in Fig. 11. Also, a closed loop for $\frac{d\tilde{x}}{dN}V s\tilde{x}$ shown in Fig. 12, guarantees a undamped or steady oscillation with constant amplitude implying all the successive cycles to be exactly alike. Thus, it really does not matter if we shift our universe in time either towards left or right. If we start the evolution of equation with a positive value of $\tilde{x}$ instead of a negative one, we land up with the same pattern.

The feature of cyclic nature of our model is independent of initial conditions. Because the bouncing dynamical variables with a positive initial condition for $\tilde{x}$ with initially expanding phase, also shows the same behaviour of oscillatory bounce, which in a way confirm it to be an eternal bounce. The limit for eternal bounce is $w_k < 1/3$. 
Figure 3: $\tilde{\sigma}$ vs $\tilde{N}$. Here $\eta = 3$.

Figure 4: $\tilde{\sigma}$ vs $\tilde{H}$. Here $\eta = 3$. 

Figure 5: $\tilde{x}$ vs $\tilde{N}$. All the three case for $\eta = 3, 6, 9$ are denoted by the red, green and blue respectively.

Figure 6: $\tilde{y}$ vs $\tilde{N}$. All the three case for $\eta = 3, 6, 9$ are denoted by the red, green and blue respectively.
Figure 7: $\tilde{z}$ vs $\tilde{N}$. All the three case for $\eta = 3, 6, 9$ are denoted by the red, green and blue respectively.

Figure 8: $\tilde{\Omega}$ vs $\tilde{N}$. All the three case for $\eta = 3, 6, 9$ are denoted by the red, green and blue respectively.
Figure 9: $\tilde{z}$ vs $\tilde{x}$.

Figure 10: $\tilde{u}$ vs $\tilde{N}$.
Figure 11: $\frac{d\bar{x}}{d\bar{N}}$ vs $\bar{N}$.

Figure 12: $\frac{d\bar{\tilde{\omega}}}{d\bar{N}}$ vs $\bar{x}$. 
5 conclusion

We analyse an cosmological scenario with a noncanonical scalar field and matter. First we define a set of dynamical variables for an anisotropic universe. We study a fixed points and their stability for these dynamical variables. We get the range for equation of state of matter from the stability of fixed points. Then we check the stability of fixed points numerically by evolving the dynamical variable for a set of parameters.

Next we define another set of dynamical variables which is suitable for studying a bouncing universe. We derive the conditions for a nonsingular bounce. Finally, a multiple bounce scenario or a cyclic solution is possible in case of noncanonical scalar field and matter. This cyclic scenario is achieved when \( \eta > 2 \). Here we have taken \( w_m = 1/3 \). The effect of the higher value of \( \eta \) on time period of cyclic universe is studied in detail. The effect of increasing the value of \( \eta \) on the time period is noted and thus a multiple bounce can be approximated as a single bounce scenario for a large value of \( \eta \) as the time period approaches infinity. In addition to this, though for a small time, the istropization of the universe can be delayed in the expansion phase by tuning the parameter \( \eta \) to larger value. Thus, a large anisotropy can be possible for a few efolds with a scalar field but uncanonical kinetic term with higher power. Also initial anisotropy could be large enough that the anisotropic perturbation modes escape the horizon during the contraction and affect the scale invariant power spectrum. This analysis is kept for future. It would be interesting to such an dynamical system analysis for other noncanonical scalar field models with nonsingular bounce [11].

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