Design of optimal control strategies for a supply chain with competing manufacturers under consignment contract

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ABSTRACT
In this paper, we discuss the effects of competition among the manufacturers on both the equilibrium strategies and the performance of supply chain under consignment contract based on optimal control theory. The retailer is responsible for the improvement of store-assistance to reduce consumer returns, and two manufacturers sell their products to end customers through the retailer and control the retail price. The aim of this paper is to derive and compare the equilibrium strategies under the different game environments. Accordingly, it is illustrated that the cooperation between retailer and manufactures can lead to a higher retail prices and the store-assistance service investment, moreover, a higher competition also could result in a higher store-assistance service investment. Furthermore, the supply chain in the non-cooperative environment can be coordinated by designing a modified cost-sharing contract, where all members can benefit from this coordination. Finally, numerical simulations are presented to illustrate the impact from the competition among the manufacturers on the profit and contract of supply chain.

1. Introduction
Over the past decades, the consignment contract with revenue sharing has been widely used in the Internet commerce and the mobile applications industry (Tal, Tatyana, & Yael, 2015; Li, Zhu, & Huang, 2009). For example, as the famous e-commerce platform, the Amazon not only sells the products directly to consumers, but also invites other sellers to sell their products through ‘Amazon Marketplace’. In this platform, the sellers can decide the inventories and prices of their products, and Amazon charges the commission according to certain percentage of the selling price when the products have been sold (Wang, Jiang, & Shen, 2004). Accordingly, the consignment contract has attracted an increasing research attention from researchers due to its efficiency in improving the performance of the supply chain (Adida & Ratisootorn, 2011; Chen, Lin, & Cheng, 2010; Hu & Li, 2012; Ru & Wang, 2010). However, it is worthwhile to mention that most of the existing results have never addressed the impact from consumer behaviour onto the decisions of the supply chain under consignment contract. In fact, as a common phenomenon, the consumer return exists frequently in a variety of business activities and indeed brings a negative impact on seller’s interests (Chen & Bell, 2009, 2011). Therefore, there is a need to focus on the problem of the optimal control strategies for supply chain subject to consumer return.

From the static perspective, the performance problems have been widely discussed for whole supply chain under consignment contract (Hu, Li, & Govinda, 2014; Ru & Wang, 2010; Wang et al., 2004). For example, in Ru & Wang (2010), the optimal channel decisions (pricing and inventory) and the performance have been discussed for supply chains under two regimes (the vendor-managed consignment inventory and the retailer-managed consignment inventory), where it has illustrated that the performance of supply chain is higher in the vendor-managed consignment inventory scenario. Subsequently, based on the results in Ru & Wang (2010), the customer returns behaviour has been considered in Hu et al. (2014) and the impact from the customer return policy onto the performance of channel has been revealed, where it has shown that the vendor-managed consignment inventory still is beneficial to the performance of supply chain irrespective of the customer returns. On the other hand, some effort has been devoted to discuss the consignment contract from the dynamic perspective (Chen et al., 2010; Lu, Zhang, & Tang, 2017). Accordingly, a great number of analysis methods have been provided for dynamics systems via the optimal control method (Ding, Wang,
Shen, & Dong, 2015; Hunt & Gerber, 2017; Li, Dong, Han, Hou, & Li, 2017; Li, Shen, Liu, & Alsaaadi, 2016; Liu, Liu, & Alsaaadi, 2016; Liu, Liu, Obaid, & Abbas, 2016; Peng, Wang, Gao, & Wang, 2017; Wang, Ding, Dong, & Shu, 2013; Yuan, Yuan, Wang, Guo, & Yang, 2017). In Lu et al. (2017), both the wholesale price contract and consignment contract have been compared by considering the effect of dynamic advertising, and it has revealed that the consignment contract is always more desirable from the perspectives of the retailer and the entire channel. Moreover, the manufacturer prefers the consignment contract when the retailer’s advertising effort is very effective (Lu et al., 2017).

It is worth mentioning that few results have addressed the decision problem for dynamic supply chain with the consumer behaviour under consignment contract. When the consumers purchase the products, they may not make sure whether the products are meeting their needs and expectations or not and the returns indeed exist commonly. In order to reduce the purchasing risk of consumers and encourage them to buy, many sellers have provided certain return policies to the consumers. Accordingly, the topic of consumer return has attracted considerable attention due to its wide existence in business (Chen & Bell, 2013; Hu et al., 2014; Mostard & Teunter, 2006). To be specific, the pricing problems have been investigated under an assumption that the returns quantity is proportional to the sales quantity (Chen & Bell, 2009, 2012). Unlike the above assumption, some effective methods have been given based on the consumer’s decision process (Huang, Gu, Ching, & Siu, 2014; Wu, Chen, & Yu, 2016). For example, the returned ratio of consumer has been characterized in Su (2009) by means of the consumer’s post-purchase evaluation, and the performances of supply chain under different return policy have been compared. In addition, new coordination method has been proposed in Xiao, Shi, & Yang (2010) by distinguishing the unsold and returned products. In business, the customer returns are affected by many factors, such as the product quality, return deadline, and so on Xu, Li, Govindan, & Xu (2015); Yoo (2014). In Ofek, Katona, & Sarvary (2011), it has mentioned that the higher store assistance service can effectively reduce the returns rate. Subsequently, by considering the store-assistance level, the problems of the information revelation mechanism, consumer returns policy and the choice of channel structure have been discussed in (Shi & Xiao, 2015, 2016; Xiao & Shi, 2016). Similarly, we also consider the function of the store-assistance service level, but mainly focus on the dynamic evolution feature of the store-assistance service level.

Motivated by the above discussions, the aim of this paper is to discuss the optimal control strategies for supply chain subject to consumer return under consignment contract. To be specific, we propose the optimal control models to study the problems of pricing and investing concerning on reducing returned rate. Firstly, the optimal pricing and optimal store-assistance service investments are proposed for cooperative and non-cooperative game scenarios. Subsequently, based on the optimal control strategies under two game scenarios, the modified cost-sharing contract is designed to coordinate the supply chain. Finally, we examine the effect resulted from parameter variations on the optimal control strategies and coordination contract. The main contribution of this paper can be summarized as follows: (1) the dynamic evolution of store-assistance service level is considered and the effect of store-assistance service level on the return rate is revealed; (2) the optimal control problem of supply chain with manufacturer’s competition is investigated from the dynamic perspective; and (3) new coordination contract is designed for the supply chain by modifying traditional cost-sharing contract, which can achieve the coordination of the supply chain in the dynamic setting.

2. Problem description and basic model

We consider a supply chain system consisting of a single retailer $R$ and two manufacturers $M_i \ (i = 1, 2)$, in which the manufacturer $M_i$ produces the product $i$ and sell to the end consumer through a common retailer’s consignment platform. In order to encourage the consumer to buy the products and increase the sales quantity, two manufacturers provide a full refund policy to their consumers. By considering the impact of consumer return, the retailer should set a fixed revenue share for per unit sold $\phi$ to manufacturers based on the actual sales quantity. It should be noted that the consumer returns not only negatively affect the profits of manufacturers but also affect the profit of retailer. In order to get more benefits, the retailer can reduce the return rate through a variety of store-assistance service (e.g. offering bigger space to display of products, installing special equipments to enhance the consumer experience).

Firstly, the dynamics of store-assistance service level is characterized in mathematical way. Let the store-assistance service level be represented by a state variable $S(t)$ and the retailer’s store-assistance service investment over time $t$ be denoted by $u(t) \geq 0$. Then, the service level evolves over time described by the following equation:

$$\dot{S}(t) = u(t) - \delta S(t),$$

which means that the store-assistance service level can be improved through the retailer’s investment. In (1), the parameter $\delta$ represents the decay rate of store-assistance service level over time due to the out-dated service style or the increasing product attributes. As in
Shi & Xiao (2016), we assume that the cost of the store-assistance service investment is quadratic and increasing, that is

\[ C(u(t)) = u^2(t). \] (2)

Since the store-assistance service level can reduce the consumer returns, we assume that the return rate of products Q(t) is described as follows:

\[ Q(t) = Q_{\text{max}}(1 - \lambda S(t)), \] (3)

where \( Q_{\text{max}} \) (\( Q_{\text{max}} \in [0, 1] \)) is a constant and represents the maximum value of return rate. Also, \( \lambda > 0 \) reflects the effectiveness of the store-assistance service level.

The market demand for product \( i \) is assumed to be linearly dependent on the retail prices and given by

\[ D_i(t) = \alpha - \beta p_i(t) + \beta p_{3-i}(t), \] (4)

where \( \alpha \) is the market capacity, and \( \beta \) depicts the competition intensity between two products. Specifically, the consumer who returns the product will stop buying these products, then the actual sales of the product \( i \) can be given by \( D_i(t)(1 - Q(t)) \) via (3) and (4).

Throughout this paper, we assume that the production cost of manufacturers \( M_i \) is equal to 0. Given an infinite time horizon and a positive discount rate \( \rho \), the objective functions of the retailer, two manufacturers and the supply chain are respectively expressed as

\[
J_R = \int_0^{+\infty} e^{-\rho t} \left[ \sum_{i=1}^{2} (\phi p_i(t) (\alpha - p_i(t) + \beta p_{3-i}(t)) \right. \\
\left. \times \ (1 - Q(t))) - u^2(t) \right] dt,
\] (5)

\[
J_{M_i} = \int_0^{+\infty} e^{-\rho t} [(1 - \phi) p_i(t) (\alpha - p_i(t) + \beta p_{3-i}(t)) \right. \\
\left. \times \ (1 - Q(t))) ] dt,
\] (6)

\[
J_C = \int_0^{+\infty} e^{-\rho t} \left[ \sum_{i=1}^{2} (p_i(t) (\alpha - p_i(t) + \beta p_{3-i}(t)) \\
\times \ (1 - Q(t))) - u^2(t) \right] dt.
\] (7)

### 3. The optimal control strategies under the non-cooperative game

Under the non-cooperative game, the retailer and two manufacturers maximize their own profits respectively. We suppose that the retailer’s revenue share for per unit sold \( \phi \) (\( 0 < \phi < 1 \)) is exogenous, moreover, the manufacturers and retailer play the Nash game. In particular, the retailer \( R \) determines the store-assistance service investment \( u(t) \) and the manufacturer \( M_i \) determine the retail price \( p_i(t) \) simultaneously.

Now, we are ready to solve and obtain the optimal control strategies of supply chain under the non-cooperative game.

**Theorem 3.1:** Under the non-cooperative game, the optimal control strategies of supply chain members are given by

\[
p_{i*}^R = \frac{\alpha}{2 - \beta'}, \quad u^{*} = \frac{\phi \lambda Q_{\text{max}} \alpha^2}{(\rho + \delta)(2 - \beta')},
\] (8)

Furthermore, the optimal store-assistance service level and the optimal profit functions for supply chain members satisfy

\[
S^d(t) = S_0 e^{-\delta t} + \frac{\phi \lambda Q_{\text{max}} \alpha^2}{\delta(\rho + \delta)(2 - \beta')}(1 - e^{-\delta t}),
\] (9)

\[
J_{M_i}^* = \frac{2 \phi(1 - Q_{\text{max}}) \alpha^2}{\rho(2 - \beta')^2} + \frac{2 \phi \lambda Q_{\text{max}} S_0 \alpha^2}{(\rho + \delta)(2 - \beta')^2} \\
+ \frac{\phi^2 \lambda^2 Q_{\text{max}} \alpha^4}{\rho(\rho + \delta)^2(2 - \beta')^4}.
\] (10)

**Proof:** To derive the equilibrium in the non-cooperative game, we first characterize the retail prices of two products. The optimization problem of manufacturer \( M_i \) is:

\[
\max_{p_i > 0} J_{M_i} = \int_0^{+\infty} e^{-\rho t} [(1 - \phi) p_i(t) (\alpha - p_i(t) + \beta p_{3-i}(t)) \\
\times \ (1 - Q(t))) ] dt \\
\text{s.t.} \ S(t) = u(t) - \delta S(t), \quad S(0) = S_0.
\]

To solve the established optimization model, we introduce the costate variable \( \mu_{M_i} = \mu_{M_i}(t) \) to construct the following current-value Hamiltonian for the manufacturer \( M_i \):

\[
H_{M_i} = (1 - \phi) p_i(t) (\alpha - p_i(t) + \beta p_{3-i}(t))(1 - Q(t)) \\
+ \mu_{M_i}(t)(u(t) - \delta S(t)).
\] (11)
Substituting (3) into (13), we have
\[ H_M = (1 - \phi)p_i(t)(\alpha - p_i(t) + \beta p_{3-i}(t))(1 - Q_{\text{max}}) + \lambda Q_{\text{max}} S(t) + \mu_M(t)(u(t) - \delta S(t)). \] (14)

Applying the necessary conditions for maximum principle, we obtain
\[ \frac{\partial H_M}{\partial p_i} = (1 - Q(t))(-2p_i + \alpha + \beta p_{3-i}) = 0, \quad (15) \]
\[ \frac{\partial H_R}{\partial S} = \rho \mu_M - \mu_R, \quad (16) \]

On the other hand, the retailer’s optimization problem is
\[ \max_{u > 0} J_R = \int_0^{+\infty} e^{-\rho t} \left[ \sum_{i=1}^{2} (\phi p_i(t)(\alpha - p_i(t) + \beta p_{3-i}(t))(1 - Q(t))) - u^2(t) + \mu_R(t)(u(t) - \delta S(t)) \right] dt \]
s.t. \( \dot{S}(t) = u(t) - \delta S(t) \), \( S(0) = S_0 \),
and the current-value Hamiltonian for the retailer is given by
\[ H_R = \sum_{i=1}^{2} (\phi p_i(t)(\alpha - p_i(t) + \beta p_{3-i}(t))(1 - Q(t))) - u^2(t) + \mu_R(t)(u(t) - \delta S(t)), \quad (17) \]
where \( \mu_R(t) \) is a costate variable. Applying the necessary conditions of maximum principle, we obtain
\[ \frac{\partial H_R}{\partial u} = 0, \quad (18) \]
\[ \frac{\partial H_M}{\partial S} = \rho \mu_R - \mu_R. \quad (19) \]

Solving the above equations (15), (16), (18) and (19), one has:
\[ p_i = \frac{\alpha}{2 - \beta}, \quad (20) \]
\[ u = \frac{1}{2 \mu_R}, \quad (21) \]
\[ \mu_R(t) = c e^{(\phi + \delta)t} + \frac{2 \phi \lambda Q_{\text{max}} \alpha^2}{(\rho + \delta)(2 - \beta)^2}. \quad (22) \]

where \( c \) is a constant to be determined. The transversality condition \( \lim_{t \to \infty} e^{-\rho t} \mu_R(t) = 0 \) imply \( c = 0 \). Therefore, the optimal store-assistance service investment of retailer under the non-cooperative game is given as follows
\[ u(t) = \frac{1}{2} \mu_R = \frac{\phi \lambda Q_{\text{max}} \alpha^2}{(\rho + \delta)(2 - \beta)^2}. \quad (23) \]

Finally, substituting \( p_i \) and \( u \) into (1), (5) and (6) respectively, the optimal store-assistance service level and the optimal profit functions for supply chain members are given as follows:
\[ S^d(t) = S_0 e^{-\beta t} + \frac{\phi \lambda Q_{\text{max}} \alpha^2}{\delta(\rho + \delta)(2 - \beta)^2} (1 - e^{-\beta t}), \quad (24) \]
\[ J^R_{\text{opt}} = \frac{2 \phi (1 - Q_{\text{max}}) \alpha^2}{(\rho + \delta)^2} + \frac{2 \phi \lambda Q_{\text{max}} S_0 \alpha^2}{(\rho + \delta)^2(2 - \beta)^2} + \frac{\phi(1 - \phi) \lambda Q_{\text{max}} S_0 \alpha^2}{(\rho + \delta)^2(2 - \beta)^2} + \frac{\phi(1 - \phi) \lambda Q_{\text{max}} S_0 \alpha^2}{(\rho + \delta)^2(2 - \beta)^2}. \quad (25) \]

\[ J^M_{\text{opt}} = \frac{(1 - \phi)(1 - Q_{\text{max}}) \alpha^2}{(\rho + \delta)^2} + \frac{(1 - \phi) \lambda Q_{\text{max}} S_0 \alpha^2}{(\rho + \delta)^2(2 - \beta)^2} \]

\[ + \frac{\phi(1 - \phi) \lambda Q_{\text{max}} S_0 \alpha^2}{(\rho + \delta)^2(2 - \beta)^2}. \quad (26) \]

**Remark 3.1:** From (10), we can see that the store-assistance service level \( S^d(t) \) will tend to be the steady state \( S^\infty = \frac{\phi \lambda Q_{\text{max}} \alpha^2}{\delta(\rho + \delta)(2 - \beta)^2} \) when \( t \to \infty \). Theorem 3.1 shows that both the store-assistance service investment \( u^d \) and the steady state \( S^\infty \) are increasing functions with respect to the retailer’s revenue share for per unit sold \( \phi \). When setting high revenue share for per unit sold, the retailer should invest more in store-assistance, which result in higher store-assistance service level. An increasing competition intensity \( \beta \) leads to a higher store-assistance investment and store-assistance service level in non-cooperative game, which leads to higher profit (i.e. \( \partial H_R/\partial \beta > 0, \partial H_M/\partial \beta > 0 \)). Similarly, a higher \( \lambda \) also leads to a higher store-assistance investment and store-assistance service level, which brings the more profit (i.e. \( \partial H_R/\partial \lambda > 0, \partial H_M/\partial \lambda > 0 \)).

4. The optimal control strategies under the non-cooperative game

In the cooperative game environment, all members of supply chain are integrated as a whole. The objective is to set the optimal control strategies \( (p_i(t), u(t)) \) in view of maximizing the profit of whole supply chain.
\textbf{Theorem 4.1:} In the cooperative game, the optimal control strategies of supply chain systems are given by

\begin{equation}
\rho_i^* = \frac{\alpha}{2(1 - \beta)},
\end{equation}

\begin{equation}
u^* = \frac{\lambda Q_{\text{max}} \alpha^2}{4(1 - \beta)(\rho + \delta)}.
\end{equation}

Furthermore, the optimal store-assistance service level and optimal profit of the total supply chain are:

\begin{equation}
S^c(t) = S_0 e^{-\delta t} + \frac{\lambda Q_{\text{max}} \alpha^2}{4\delta(1 - \beta)(\rho + \delta)}(1 - e^{-\delta t}),
\end{equation}

\begin{equation}
\bar{J}^c = \frac{(1 - Q_{\text{max}}) \alpha^2}{2\rho(1 - \beta)} + \frac{\lambda Q_{\text{max}} \delta \alpha^2}{2(1 - \beta)(\rho + \delta)} + \frac{\lambda^2 Q_{\text{max}}^2 \alpha^4}{16 \rho(1 - \beta)^2(\rho + \delta)^2}.
\end{equation}

\textbf{Proof:} The optimization problem of supply chain systems is a standard optimal control problem as follows:

\[
\max_{\rho > 0 \mu > 0} J_C = \int_0^{+\infty} e^{-\rho t} \left[ \sum_{i=1}^{2} (p_i(t)(\alpha - p_i(t) + \beta p_{3-i}(t)(1 - Q(t))) - u^2(t) - \mu_C(t)(u(t) - \delta S(t)),
\right.
\]

\[
\left. \text{s.t. } \dot{S}(t) = u(t) - \delta S(t), \quad S(0) = S_0. \right]
\]

The corresponding current-value Hamiltonian is given by:

\[
H_C = \sum_{i=1}^{2} (p_i(t)(\alpha - p_i(t) + \beta p_{3-i}(t)(1 - Q(t))) - u^2(t) + \mu_C(t)(u(t) - \delta S(t)),
\]

where \(\mu_C(t)\) is a costate variable. The necessary conditions for optimal strategies are given by

\[
\frac{\partial H_C}{\partial p_i} = 0,
\]

\[
\frac{\partial H_C}{\partial u} = 0,
\]

\[
\frac{\partial H_C}{\partial S} = \rho \mu_C - \mu_C.
\]

It follows from (32) and (33) that

\begin{equation}
\rho_i = \frac{\alpha}{2(1 - \beta)},
\end{equation}

\begin{equation}
u^* = \frac{\lambda Q_{\text{max}} \alpha^2}{4(1 - \beta)(\rho + \delta)}.
\end{equation}

Substituting \(p_i\) into equation (34) and solving the differential equation yield

\[
\mu_C = (\rho + \delta) \mu_C - \frac{\lambda Q_{\text{max}} \alpha^2}{2(1 - \beta)(\rho + \delta)}.
\]

Similar to proof of Theorem 3.1, the optimal store-assistance service investment is:

\[
\nu^* = \frac{\lambda Q_{\text{max}} \alpha^2}{4(1 - \beta)(\rho + \delta)}.
\]

The optimal store-assistance service level and optimal profit of supply chain are:

\begin{equation}
S^c(t) = S_0 e^{-\delta t} + \frac{\lambda Q_{\text{max}} \alpha^2}{4\delta(1 - \beta)(\rho + \delta)}(1 - e^{-\delta t}),
\end{equation}

\begin{equation}
\bar{J}^c = \frac{(1 - Q_{\text{max}}) \alpha^2}{2\rho(1 - \beta)} + \frac{\lambda Q_{\text{max}} \delta \alpha^2}{2(1 - \beta)(\rho + \delta)} + \frac{\lambda^2 Q_{\text{max}}^2 \alpha^4}{16 \rho(1 - \beta)^2(\rho + \delta)^2}.
\end{equation}

\[
\]

\textbf{Remark 4.1:} It follows from Theorems 3.1 and 4.1 that \(S^c(t) < S^c(t)\). In addition, \(S^c(t)\) is the weighted sum of \(S_0\) and \(\lambda Q_{\text{max}} \alpha^2 / 4\delta(1 - \beta)(\rho + \delta)\). Hence, \(1 - \lambda S(t) \geq 0\) holds (i.e. \(\lambda S(t) \leq 1\)) if \(\lambda S^c(t) \leq 1\). Therefore, we assume the parameter \(\lambda\) satisfies the condition \(\lambda \leq \min[1/S_0, 2\sqrt{\delta(1 - \beta)(\rho + \delta)/Q_{\text{max}} \alpha^2}]\). This assumption is frequently used in the economics literature, that is, the effectiveness on reducing the return rate of the store-assistance service level will not too high.

Obviously, the changes of the optimal control strategies in the cooperative game are similar to these in the non-cooperative game, i.e. when the effectiveness of service level is higher, the retailer will invest more in store-assistance service investment. Meanwhile, the higher competition intensity will induce more store-assistance service level.

\textbf{Proposition 4.1:} Comparing with optimal control strategies and optimal profits in the cooperative and non-cooperative games, we have

\[
\rho_i^* > \rho_i^* \quad \text{and} \quad u^* > u^*, \quad \bar{J}^c > \bar{J}^c + \bar{J}^c + \bar{J}^c.
\]

\textbf{Remark 4.2:} Proposition 4.1 shows that both the retail price and store-assistance service investment are higher in the cooperative game, which means that the total
profit is higher in the cooperative game. Indeed, the non-cooperation jeopardizes the performance of the whole channel. Hence, we need to design the appropriate contract so as to improve the performance of supply chain in the non-cooperative game.

5. The design of coordination contract

Motivated by the coordination method in Ryan, Sun, & Zhao (2013), a modified cost-sharing contract is provided to coordinate the supply chain and improve the performance of the supply chain in the non-cooperative game. Specifically, contract provisions are structured as follows: (i) the retailer announces revenue share for per unit sold \( \phi \), and the manufacturer \( M_i \) should undertake the fraction \( \frac{1}{n}(1 - \phi) \) of the store-assistance service investment; (ii) the manufacturer \( M_1 \) and manufacturer \( M_2 \) jointly make their decisions in order to maximize their joint profits, meanwhile, the retailer controls the store-assistance service investment by maximizing its own profit. Under this contract, the profit function of the retailer is expressed as

\[
J_{R}^{mc} = \int_0^{+\infty} e^{-\mu t} \left[ \sum_{i=1}^{2} (\phi p_i(t) (\alpha - p_i(t)) + \beta p_{3-i}(t)) 
\times (1 - Q(t)) - \psi u(t)^2 \right] dt,
\]

where \( \mu \) is a costate variable. Applying the necessary conditions for maximum principle, we obtain,

\[
\begin{align*}
\frac{\partial H_{R}^{mc}}{\partial \mu} &= -2\psi u + \mu_{R}^{mc} = 0, \\
\frac{\partial H_{R}^{mc}}{\partial S} &= \rho \mu_{R}^{mc} - \dot{\mu}_{R}^{mc}.
\end{align*}
\]

Similar to proof of Theorem 3.1, the optimal store-assistance service investment under the modified cost-sharing contract is:

\[
u_{mc} = \frac{\phi \lambda Q_{max} \alpha^2}{4\psi (1 - \beta)(\rho + \delta)}.
\]

Equation (43) implies

\[
\rho_{mc}^{lmc} = \rho_{mc}^{lmax} = \frac{\alpha}{2(1 - \beta)}.
\]

Furthermore, the current-value Hamiltonian of retailer is given by

\[
\begin{align*}
\dot{H}_{R}^{mc} &= \frac{\phi \alpha^2}{2(1 - \beta)} (1 - Q(t)) - \psi \dot{u}(t)^2 + \mu_{R}^{mc}(t) \\
&\times (u(t) - \delta S(t)),
\end{align*}
\]

where \( \mu_{R}^{mc}(t) \) is a costate variable. Applying the necessary conditions for maximum principle, we obtain,

\[
\begin{align*}
\frac{\partial H_{R}^{mc}}{\partial \mu} &= -2\psi u + \mu_{R}^{mc} = 0, \\
\frac{\partial H_{R}^{mc}}{\partial S} &= \rho \mu_{R}^{mc} - \dot{\mu}_{R}^{mc}.
\end{align*}
\]

**Remark 5.1:** When the supply chain is coordinated, the profits of supply chain members can be rewritten as \( J_{R}^{mc} = \phi J_{C}^{mc} \) and \( J_{M}^{mc} = \frac{1}{n}(1 - \phi) J_{C}^{mc} \). In order to ensure that all members of supply chain are involved in this coordination contract, the following conditions should be satisfied simultaneously: \( J_{R}^{mc} > J_{R}^{mc} \) and \( J_{M}^{mc} > J_{M}^{mc} \), that is, when \( \phi \in [J_{R}^{mc} / J_{C}^{mc}, \min(J_{C}^{mc} - 2J_{M}^{mc}, J_{C}^{mc} / J_{C}^{mc})] \), the modified cost-sharing contract is Pareto-improving.

6. A numerical example

Based on the results proposed in the above sections, a simulation is given here to gain more managerial insights.

For the simulation purpose, the following parameter values are set in the example: \( \alpha = 1, \beta = 0.2, \delta = 0.05, Q_{max} = 0.3, \rho = 0.1, \lambda = 0.06, S_0 = 0.15 \) and \( \phi = 0.5 \). Define \( \phi_1 = J_{R}^{mc} / J_{C}^{mc}, \phi_2 = \min(J_{C}^{mc} - 2J_{M}^{mc}, J_{C}^{mc} / J_{C}^{mc}) \) and the gap of profits \( \Delta J = J_{C}^{mc} - J_{R}^{mc} - J_{M}^{mc} - J_{M}^{mc} \).

According to the Theorems 3.1, 4.1 and Remark 5.1, we have Figures 1–7. As shown in Figure 1, the optimal store-assistance service level in the cooperative game setting is higher than one in non-cooperative game setting due to the fact that the investment of store-assistance service
**Figure 1.** The dynamic change of store-assistance service level.

**Figure 2.** The impact of $\beta$ on the profits.

**Figure 3.** The impact of $\beta$ on the gap of profits.

**Figure 4.** The impact of $\lambda$ on the profits.

**Figure 5.** The impact of $\lambda$ on the gap of profits.

**Figure 6.** The impact of $\beta$ on the contract.
The impact of $\lambda$ on the contract. Level is higher in the cooperative game scenario. Accordingly, the return rate in the cooperative game is lower than one in non-cooperative game.

Figures 2 and 3 show that the profits of supply chain in both cooperative and non-cooperative setting increase with the competition intensity $\beta$. Moreover, the gaps of the profits among two settings enlarge with the increasing of competition intensity $\beta$. It means that the higher competition intensity $\beta$ aggravate double marginalization, the design of channel coordination mechanism is necessary to reduce the impact of double marginalization. Figures 4 and 5 show the the changes of profits and gaps between the profits with effectiveness of store-assistance service level $\lambda$. Similar to the previous conclusion, regardless of the cooperation and non-cooperation, the increasing in $\lambda$ will lead to more channel profits. This result indicates that, the retailer should focus more on increasing of the service effectiveness. Moreover, the gaps of profits slowly enlarge among two games when $\lambda$ increases.

Figure 6 shows that the plane is divided by two curves $\phi_1$ and $\phi_2$ into three regions. The region above curve $\phi_1$ ensures that retailer is better off, and the one below curve $\phi_2$ means that manufacturers are better off. It should be mentioned that, when the retailer’s revenue share for per unit sold $\phi$ locates between $\phi_1$ and $\phi_2$ (i.e. W-W region), the modified cost-sharing contract can coordinate the supply chain well and both members are better off. Specifically, when the competition intensity $\beta$ increases, the W-W region becomes large. This implies that a larger competition intensity will provide the retailer a greater degree of flexibility to coordinate the supply chain. Figure 7 shows that the increasing trend of $\phi_1$ is often not obvious when the effectiveness of store-assistance service level $\lambda$ increases, but $\phi_2$ becomes smaller clearly when the effectiveness of store-assistance service level $\lambda$ increases, which results in a larger W-W region.

7. Conclusions

In this paper, the optimal strategies of pricing and store-service investment have been presented by applying the optimal control theory. Both the cooperative scenario and the non-cooperative scenario have been considered and the coordination problem has been discusses by using a modified cost-sharing contract. Finally, the effect of competition intensity and effectiveness of the store-assistance service level on the optimal profits of supply chain and the channel performance have revealed. In particular, we have obtained the following results: (1) the optimal store assistance service investment is higher in the cooperative setting, which results in a lower return rate in the cooperative setting; (2) the modified cost-sharing contract can effectively improve the double marginalization; and (3) both competition intensity and effectiveness of the store-assistance service level increase the degree of flexibility to coordinative contract. Future research topics include the extension of the proposed strategies to a dynamic supply chain model with endogenous revenue share and make the decision-making with hope to improve the performance of channel when consumer return affects the demand. In addition, we aim to consider the related applications of proposed method based on the background as in Grema & Cao (2017) and Tavakol & Binazadeh (2017).

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