Fermionic Heisenberg Model for Spin Glasses with BCS Pairing Interaction

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In the present paper we have analysed a fermionic infinite-ranged quantum Heisenberg spin glass \((s = 1/2)\) with a BCS coupling in real space in the presence of an applied magnetic field. This model has been obtained by tracing out the conducting fermions in a superconducting alloy. The magnetic field is applied in the resulting effective model. The problem is formulated in the path integral formalism where the spin variables are defined as bilinear combinations of the Grassmann fields. The static approximation is used to treat both the pairing and the spin terms together with the replica symmetry ansatz. Henceforth, the problem can be reduced to a one site problem. The field in the \(z\) direction, \(H_z\), separates the order parameters in two groups: parallel and transverse to it. We have obtained a phase diagram in \(T–g\) space with zero transverse spin-glass ordering, \(g\) being the strength of the pairing interaction. It has been possible to locate the transition temperature between the normal paramagnetic phase (NP) and the phase where there is a long range order corresponding to formation of pairs (PAIR). The transition ends at the temperature \(T_f\), the transition temperature between the NP phase and the spin glass (SG) phase. \(T_f\) decreases for stronger fields allowing us to calculate the NP–PAIR line transition even at low temperatures. The NP–PAIR transition line has a complex dependence with \(g\) and \(H_z\), having a tricritical point depending on \(H_z\) from where second order transitions occur for higher values of \(g\) and first order transitions occur for lower values of \(g\).

I. INTRODUCTION

The interplay between a magnetic order and superconductivity is a current issue. Particularly, spin glass ordering has been reported in many physical systems that includes high-\(T_c\) superconductors, heavy fermions, and conventional superconductors doped with magnetic impurities like \(Gd_xTh_{1-x}RU_2\). These conventional superconductors can be modeled by a s-d exchange interaction between the localized magnetic impurities and the conducting fermions which have conventional BCS interaction (Nass et al.\(^6\)). The main interest of Nass et al. was to find the density of states in the presence of magnetic impurities. However, relatively little consideration has been given to the phase transition problem between spin glass and a superconductive order\(^7–9\).

Nevertheless, from a wider perspective the role of quantum fluctuations in disordered strongly interacting systems still needs to be fully understood. Theoretical studies in recent years have shown plenty of interesting results. For instance, the presence of a non-Fermi liquid behavior near \(T = 0\) transition between a metallic paramagnetic and a metallic spin glass in a model introduced to study the competition between Kondo effect and RKKY interaction\(^1\). In addition, the random quantum Heisenberg with generalized SU(M) spin\(^2\) produces a spin liquid ground state in the large M limit with a local dynamic susceptibility that is identical to a marginal Fermi Liquid.

Recently, a model has been introduced to study the phase transition between the spin glass ordering and the BCS pairing among fermions of opposite spins\(^3\). This model has been obtained from the model of Nass et al.\(^4\) by tracing out the conducting fermionic degrees of freedom by perturbation expansion. The remaining effective problem could be solved in its infinite ranged Ising version. A rich phase diagram has been found in the \(T–g\) plane \((g = \text{the strength of the pairing interaction})\) for fixed \(J\) (variance of the spin random coupling \(J_{ij}\)) with a normal paramagnetic phase (NP), a spin glass phase (SG) and a pairing phase (PAIR) where there is formation of pairs in the sites. The first two phases (NP and SG) are separated by a second order line at the spin glass transition temperature \(T_f\). However, the line \(g = g_c(T)\) between the NP and PAIR phases is more complex. It is a second order line which ends at a tricritical point \((T_{tc}, g_{tc})\), where a first order line starts. Below the spin glass transition temperature \(T_f\), the last two phases (SG and PAIR) are separated by the same first order line transition.

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The purpose of the present study is to investigate the infinite ranged Heisenberg version of the model given by Ref. 12 in presence of a magnetic field for the half filling case. This many fermion problem is formulated by defining the SU(2) spin variables as a bilinear combination of Grassmann fields and computing the partition function through the path integral formalism. The disorder in the coupling $J_{ij}$ is treated by standard replica formalism. For this representation of the spin variable, the Fock space ($s = 1/2$) has four quantum states per site but only two of them are magnetic. As a consequence, the sensitivity of the site to magnetic coupling is dependent on the occupation number which is controlled by quantum statistics. When the strength of the BCS coupling is increased, forcing the double occupation, the number of sites non-sensitive to the magnetic coupling increases. Therefore, this mechanism could suppress magnetic ordering such as the spin glass phase. Actually, that is what happens in the Ising model even at the half filling situation.

In the replica formalism for the quantum spin glass, the replica diagonal order parameter has to be solved coupled with the non-diagonal one (the spin glass order parameter) and the remaining order parameters of the problem. In consequence, the diagonal component is no longer constrained to unity and has an important role in the problem. In fact, the location of $T_f$ can be obtained through this component. This is the essential point: the location of other transitions of the problem are also dependent of this diagonal component even when the non-diagonal component is zero.

Another important aspect of these problems is that the order parameter (the spin-spin correlation function) are time dependent. Bray and More13 in their pioneering work, investigated the role of the quantum fluctuations in the Heisenberg spin glass model by using the static approximation. They formulated the problem in the Feynman’s path integral formalism where the spins variables have a dependence on fictitious time $\tau$. The static approximation neglects the time dependence giving an upper bound for the thermodynamic potential and the location of the transition points. For temperatures above the $T_f$, this approximation is expected to be exact together with the replica symmetric solution. This same problem has been studied with a formalism where the spin variables were represented by bilinear combinations of Grassmann fields. The static approximation was used for the spin glass order parameter, while for the susceptibility (associated to the replica diagonal order parameter) was applied an instantaneous approximation. This approach has found a non-zero spin glass transition temperature for $s = 1/2$. Goldschmidt and Pik-Yin La14 used the same treatment as Bray and More to obtain a phase diagram for the infinite ranged quantum Heisenberg and XY spin glass in an external magnetic field. Due to the magnetic field, the order parameters were separated in two groups: transversal to the field and parallel to it. This approach allowed to locate the spin glass transition transversal to the field and investigate the role of the tunneling in the transition. For instance, the results showed that the spin glass ($s = 1/2$) transition temperature $T_f$ is depressed for the Heisenberg model as the field strength is increased. Nevertheless, the tunneling was not strong enough to produce a transition at $T = 0$.

In the present fermionic formulation, we have combined the methods of Refs. 14 and 15 by applying a field over the fermionic spin variables, taking the transversal spin glass order parameter as null and using the static approximation for the remaining order parameters. In these circumstances, the magnetic field also has depressed the transversal spin glass transition temperature. This approach has allowed us to study the transition line between NP phase and the PAIR phase (we use here the same terminology of Ref. 12) even in lower temperature. This transition line ends at the transversal spin glass transition temperature which moves down as the field strength enhances.

This paper is organized as follows. In section II, we present our model and develop it to obtain the Gran Canonical Potential and the saddle point equations for the order parameters in the half filling case. In section III, we analyse the transition line between the NP and PAIR phases for several values of the magnetic field. The results show that as the magnetic field increases, larger values of the pairing interaction strength are necessary to produce a transition. We also locate the transition line between the normal paramagnetic phase and the spin glass phase. Discussions and concluding remarks are presented in the last section.

II. GENERAL FORMULATION

We consider the following Hamiltonian in the presence of a magnetic field $H_z$:

$$\overline{H} = H - \mu N = - \sum_{ij} J_{ij} \left[ \sigma_i^+ \sigma_j^- + \frac{1}{2} \left( \sigma_i^- \sigma_j^+ + \sigma_i^+ \sigma_j^- \right) \right] - \mu \sum_i \sum_{s=\uparrow,\downarrow} c_i^\dagger c_i - \frac{g}{N} \sum_{i,j} c_i^\dagger c_i^\dagger c_i c_j - H_z \sum_i \sigma_i^z \tag{1}$$

where $\sigma_i^\pm = \sigma_i^x \pm \sigma_i^z$, the chemical potential is given by $\mu$ and $c_i^\dagger$ ($c_i$) are fermions creation (destruction) operators ($s = \uparrow$ or $\downarrow$ means the spin projection). The spin operators are represented as bilinear combination of fermionic operators:
\[ \sigma_i^+ = \frac{1}{2} [c_{it}^\dagger c_{it} - c_{it}^\dagger c_{it}^\dagger] \]
\[ \sigma_i^- = c_{it}^\dagger c_{it} \quad . \]

The random coupling \( J_{ij} \) in Eq. (1) are infinite ranged with a Gaussian distribution with mean and variance given by:
\[ \langle J_{ij} \rangle_{ca} = 0 \]
\[ \langle J_{ij}^2 \rangle_{ca} = 4J^2/N \quad (4) \]

where \( N \) is the number of sites.

The third term in Eq. (1) represents a BCS pairing interaction in real space. This pairing mechanism in fermionic problems already has been proposed, although, in different contexts\[15\]. The remaining terms describe a fermionic Heisenberg spin glass in the presence of a field. Our interest in this fermionic problem is to analyse the phase transitions at mean field level as firstly introduced by Sherrington and Kirkpatrick\[16\] for classical spin glass, reducing the initial many site problem to a one site problem. In order to obtain that, the partition function is expressed in terms of functional integrals using the anti-commuting Grassmann variables \( \phi_{ix}(\tau) \) and \( \phi_{ix}(\tau) \) (\( \tau \) is a complex time):
\[ Z = \int D(\phi^* \phi) \exp[A_0 + A_{pair} + A_{SG} + A_{field}] \quad (5) \]

where the action \( A_0 \) in Eq. (4) is the free part of the total action, \( A_{pair} \) is the pairing part and \( A_{SG} \) is the spin part.

Using time Fourier transformed quantities, the free part is given by
\[ A_0 = \sum_i \sum_\omega (i\omega + \beta \mu) \left[ \phi_{it}^\dagger (\omega) \phi_{it} (\omega) + \phi_{it}^\dagger (\omega) \phi_{it} (\omega) \right] \quad (6) \]

and the \( A_{field} \) is
\[ A_{field} = \frac{\beta H_s}{2} \sum_i \sum_\omega \left[ \phi_{it}^\dagger (\omega) \phi_{it} (\omega) - \phi_{it}^\dagger (\omega) \phi_{it} (\omega) \right] \quad (7) \]

while the pairing part of the action in the static approximation becomes
\[ A_{pair} \approx A_{pair}^{st} = \frac{\beta g}{N} \sum_{ij} \rho_i (0) \rho_j (0) \quad (8) \]

where \( \beta = \frac{1}{T} \), \( T \) is the temperature and
\[ \rho_i (0) = \sum_\omega \phi_{it} (\omega) \phi_{it} (\omega) \quad (9) \]

with Matsubara’s frequencies \( \omega = (2m + 1)\pi/T \), \( m = 0, \pm 1, \ldots \)

Indeed, functional integral techniques within the static approximation have succeeded in studying phase transitions in conventional BCS superconductivity\[17\] and in the presence of metal transition impurities\[16\]. It has been possible to build a mean field theory in momentum space by using a particle-hole transformation. Thus, the problem became exactly resolved.

The spin part of the action is written as
\[ A_H = \sum_{ij\nu} \frac{\beta J_{ij}}{2} \left\{ S_i^\dagger (\nu) S_i^\dagger (\nu) + \frac{1}{2} [S_i^\dagger (\nu) S_j^\dagger (\nu) + S_i^\dagger (\nu) S_j^\dagger (\nu)] \right\} \quad (10) \]

where the spins variables are expressed in terms of Grassmann fields:
\[ S_i^\dagger (\nu) = \frac{1}{2} \sum_\omega \left[ \phi_{it}^\dagger (\omega + \nu) \phi_{it} (\omega) - \phi_{it}^\dagger (\omega + \nu) \phi_{it} (\omega) \right] \quad (11) \]
\[ S_i^+ (\nu) = \sum_\omega \phi_{it}^\dagger (\omega + \nu) \phi_{it} (\omega) \]
\[ S_i^- (\nu) = \sum_\omega \phi_{it} (\omega + \nu) \phi_{it} (\omega) \quad (12) \]
where $\nu = 2\pi n$ in Eq. (12).

The Grand Canonical Potential can be obtained through the replica method by

$$\Omega = -\frac{1}{\beta} \lim_{n \to 0} \frac{1}{n} \left( \frac{Z^n}{n} \right) - 1 . \quad (13)$$

The problem can be linearized and reduced to one site problem with the introduction of the auxiliary complex fields

for the pairing part $\eta_\alpha(0)$, $\eta^*_\alpha(0)$ ($\eta_\alpha(0) = \eta R_\alpha(0) + i \eta I_\alpha(0)$) and for the spin part $R_{aa}^{tt}(\nu, \nu') = [R_{aa}^{tt}(\nu, -\nu')]^*$ and

$$Q_{\alpha\beta}^{tt}(\nu, \nu') = [Q_{\alpha\beta}^{tt}(\nu, -\nu')]^*$$

where $\alpha$ and $\beta$ are replica index and $t = x, y, z$. Thus the configurational averaged replicated partition function is

$$\langle Z^n \rangle_{\alpha\beta} = \frac{1}{N^n} \int D(\eta^*) D(Q^*) D(R^*) \exp \left\{ -\frac{N}{8\beta^2 J^2} \left[ \sum_\alpha \sum_{t < t'} \left| R_{aa}^{tt}(\nu, \nu') \right|^2 + \sum_{\alpha \neq \beta} \sum_{t < t'} \left| Q_{\alpha\beta}^{tt}(\nu, \nu') \right|^2 + \frac{N\beta g}{4} \sum_\alpha |\eta_\alpha(0)|^2 + \ln(\Lambda) \right] \right\} \quad (14)$$

where $\Lambda = \left( \frac{2\pi}{N\beta^2 J^2} \right) \left( \frac{\pi}{N\beta g} \right)$.

The function $\Lambda$ in Eq. (13) is

$$\Lambda = \int D(\phi^* \phi) \exp \left\{ A_0 + \frac{\beta g}{2} \sum_\alpha \left[ \eta_\alpha(0) \rho^*(0) + \eta_\alpha^*(0) \rho(0) \right] + \beta H_z \sum_{i\alpha} S^z_{i\alpha}(0) \right. \right.$$  

$$+ \left. \left[ \sum_{i\alpha} \sum_{t \neq t'} R_{aa}^{tt}(\nu, \nu') S_{i\alpha}^t(\nu) S_{i\alpha}^t(\nu) + \sum_{i\alpha} \sum_{t < t'} Q_{\alpha\beta}^{tt}(\nu, \nu') S_{i\alpha}^t(\nu) S_{i\beta}^t(\nu) \right] \right\} . \quad (15)$$

Within the static approximation, together with the replica symmetric ansatz, the auxiliary fields of the spin glass part are giving by the following relations.

$$Q_{\alpha\beta}^{tt}(\nu, \nu') = 0 \quad t \neq t'$$

$$R_{aa}^{xx}(\nu, \nu') = 4\beta^2 J^2 R_0 \delta_{\nu, \nu'} \delta_{\nu, 0}$$

$$R_{aa}^{yy}(\nu, \nu') = R_{aa}^{yy}(\nu, \nu') = 4\beta^2 J^2 R_0 \delta_{\nu, \nu'} \delta_{\nu, 0}$$

$$Q_{\alpha\beta}^{tt}(\nu, \nu') = Q_{\alpha\beta}^{tt}(\nu, \nu') = 4\beta^2 J^2 Q_0 \delta_{\nu, \nu'} \delta_{\nu, 0}$$

$$Q_{\alpha\beta}^{zz}(\nu, \nu') = 4\beta^2 J^2 Q_0 \delta_{\nu, \nu'} \delta_{\nu, 0}$$

where $R$, $R_0$ and $Q_0$ are now real parameters.

We also take the spin glass order parameter transversal to the field $Q = 0$ to work in a temperature region where both static approximation and replica symmetric ansatz are reliable. The sum over the replica index gives again quadratic forms which are also linearized by new auxiliary fields. Thus, the one site $\Lambda$ function becomes

$$\Lambda = \int_{-\infty}^{+\infty} Dw \left\{ \int_{-\infty}^{+\infty} Du_x \int_{-\infty}^{+\infty} Du_y \int_{-\infty}^{+\infty} Du_v \int_{-\infty}^{+\infty} Du_\nu \right. \right.$$  

$$\left. \left[ D(\phi^* \phi) \exp\left[ A_0 + A_{pair} + \beta \vec{h} \cdot \vec{S} \right] \right] \right\} \quad (17)$$

where $Dv = \frac{e^{-v^2/2}}{\sqrt{2\pi}} dv$ and

$$\vec{h} \cdot \vec{S} = h_x S_x + \frac{h_+ S_+ + h_- S_-}{2}$$

$$h_\pm = h_x \pm ih_y . \quad (18)$$
The field $\vec{h}$ in the equation above is defined as:

$$h_z = J \left[ v \sqrt{2(R_z - Q_z)} + w \sqrt{2Q_z} + \frac{H_z}{2J} \right]$$

(19)

$$h_+ = J \sqrt{2Ru_+}$$

$$h_- = J \sqrt{2Ru_-}$$

The resulting problem obtained in Eq. (17) is equivalent to fermions in the presence of internal fields $g\eta/2$, $g\eta^*/2$ and $\vec{h}$. The two first ones are the pairing fields related to the long range order where there is pairing formation in the sites. The later is a random gaussian field related to the replica components of the spin part of the auxiliary fields introduced in Eq. (14), namely, the non-diagonal $Q_z$ and diagonal $R_z$ which are parallel to $H_z$, and diagonal $R$ which is transversal to $H_z$.

In order to construct a mean field theory which can be solved, we introduce the following matrices:

$$\Psi(\omega) = \begin{bmatrix} \phi_\uparrow(\omega) \\ \phi_\downarrow(\omega) \\ \phi_\uparrow(-\omega) \\ \phi_\downarrow(-\omega) \end{bmatrix}$$

(20)

$$\Psi^\dagger(\omega) = \begin{bmatrix} \phi_\uparrow^*(\omega) & \phi_\downarrow^*(\omega) & \phi_\downarrow(-\omega) & \phi_\uparrow(-\omega) \end{bmatrix}$$

(21)

and

$$G^{-1}_{ss}(\omega) = \begin{bmatrix} G^{-1}_{\uparrow\uparrow}(\omega) & \beta h_- & \Delta & 0 \\ \beta h_+ & G^{-1}_{\downarrow\downarrow}(\omega) & 0 & -\Delta \\ \Delta^* & 0 & -G^{-1}_{\downarrow\downarrow}(-\omega) & -\beta h_- \\ 0 & -\Delta^* & -\beta h_+ & -G^{-1}_{\downarrow\downarrow}(-\omega) \end{bmatrix}.$$  

(22)

In the equation above $G^{-1}_{ss}(\omega) = \sum_{s=\pm}(i\omega + \beta\mu + s\beta h_z)$ and $\Delta = \frac{\beta g}{2}\eta$. Hence, for the $\Lambda$ function in Eq. (17) we obtain

$$\Lambda = \int_{-\infty}^{+\infty} Dw \left\{ \int_{-\infty}^{+\infty} Du_x \int_{-\infty}^{+\infty} Du_y \int_{-\infty}^{+\infty} Dv[I(w, u_x, u_y, v)] \right\}^n$$

(23)

where

$$I(w, u_x, u_y, v) = \int D(\phi^* \phi) \exp \left[ \frac{1}{2} \sum_\omega \Psi^\dagger(\omega)G^{-1}(\omega)\Psi(\omega) \right].$$  

(24)

The functional integral in Eq. (24) can now be performed. The sum over Matsubara’s frequencies in the resulting expression should be done as in Ref. 12. The matrix formalism introduced in Eqs. (20), (21) and (22) produces a particle-hole transformation in the fermions of spin down. Therefore, the Grand Canonical Potential can be found from Eqs. (10) and (23) as

$$\Omega = 4\beta J^2 R^2 + 2\beta J^2 R_z - 2\beta J^2 Q_z + \frac{g}{4} |\eta|^2 - \frac{1}{\beta} \int_{-\infty}^{+\infty} Dw \ln(I_\beta)$$

(25)

where

$$I_\beta = \int_0^{+\infty} uDu \int_{-\infty}^{+\infty} Dv \left[ \cosh(\beta \mu') + \cosh(\beta |\vec{h}|) \right]$$

(26)

and

$$\beta \mu' = \sqrt{\beta \mu^2 + \Delta^2}$$

(27)
with \( \vec{h} \) already defined in Eq. (19).

For \( S_\beta = \sinh(\beta|\vec{h}|)/(2\beta|\vec{h}|) \) and \( \theta = h_z/J \), the functions \( R, Q_z, R_z \) and \( |\eta| \) (from now on we write \( \eta \) instead of \( |\eta| \)) can be determined from the saddle point equations that follow from Eq. (27):

\[
R = \frac{1}{4} \int_{-\infty}^{+\infty} Dw \frac{u^3 Du}{I_\beta} \int_{-\infty}^{+\infty} \frac{v Du}{I_\beta} \int_{-\infty}^{+\infty} Dw S_\beta
\]

(28)

\[
R_z = \frac{1}{2\sqrt{(R_z - Q_z)}} \int_{-\infty}^{+\infty} Dw \frac{u Du}{I_\beta} \int_{-\infty}^{+\infty} v Du S_\beta \theta
\]

(29)

\[
Q_z = \frac{\beta^2 J^2}{2} \int_{-\infty}^{+\infty} Dw \left[ \frac{u Du}{I_\beta} \int_{-\infty}^{+\infty} Dw S_\beta \theta \right]^2 \]

(30)

\[
\eta = \int_{-\infty}^{+\infty} Dw \frac{\sinh(\beta \mu')}{I_\beta}.
\]

(31)

We have solved numerically the set of Eqs. (28), (29), (30) and (31) for the situation where there is one fermion per site (in average) which corresponds to \( \mu = 0 \) due to the particle-hole transformation. The validity range in temperature for this theory is the region where \( T > T_f \) implying that it is necessary to locate the \( T_f \) in this problem. This can be done by expanding the Grand Canonical Potential in Eq. (46) in powers of \( Q_{\alpha\beta}^2 \) up to second order. The four spin correlation function that appears in the coefficient of the quadratic term can be related to the parameter \( R \) in the static approximation, where the averages are computed with an action where the auxiliary fields are given by Eq. (16) and \( Q = 0 \). Thus the temperature \( T_f \) can be obtained from the condition that the coefficient of the quadratic term vanishes, so we get

\[
1 = 4\beta_f JR
\]

(32)

along the set of Eqs. (28)-(31).

### III. PHASE DIAGRAM AND TRICRITICAL POINT

The numerical solution of the equations (28)-(31) has allowed us to locate the transition temperature between the NP phase (\( \eta = 0 \)) and the PAIR phase (\( \eta \neq 0 \)) as a function of the pairing interaction strength \( g \) and the magnetic field \( H_z \) (see Fig. (1)). The nature of the transition line has a complex dependence on both quantities.

For instance, if \( H_z=0 \), for high \( T \) and small \( g \) there is no long range order \( \eta = 0 \) (\( Q \) is always zero) that corresponds to the NP phase. On the other hand, for high \( T \) and high \( g \) the parameter \( \eta \) is nonzero. In this situation there is pair formation on the sites which corresponds to the PAIR phase. The transition between NP and PAIR phases is a second order type, which means that the order parameter \( \eta \) goes continuously to zero as \( g \) decreases. That is shown in Fig. (2). However, if \( g \) is decreased below a particular value \( g = g_{tc} \) (which corresponds to the temperature \( T = T_{tc} \)), the order parameter starts to display a discontinuous behavior indicating that the nature of the transition line has changed to a first order one. In the absence of the magnetic field the behavior of the other parameters are \( R=R_z \) and \( Q_z = 0 \).

If the magnetic field is turned on, the transition NP-PAIR only exists for larger values of the pairing strength \( g \) and the field tends to destroy the PAIR phase (Fig. (1)). The tricritical point \( (g_{tc}, T_{tc}) \) is moved up and, as a consequence, the first order transition line exists over a larger interval of the pairing strength \( g \).

The \( (g_{tc}, T_{tc}) \) point has been confirmed as a tricritical point by the expansion of the Grand Canonical Potential in powers of the order parameter \( \eta \) which defines the symmetry of the pairing phase:

\[
\beta \Omega = \beta \Omega_0 + A_1 \eta^2 + A_2 \eta^4
\]

(33)

where

\[
A_1 = \frac{\beta^2 g^2}{2} \int_{-\infty}^{+\infty} Dw \frac{1}{\int_{-\infty}^{+\infty} u e^{-u^2} Du \int_{-\infty}^{+\infty} Dw [1 + \cosh(\beta |\vec{h}|)]} - 1
\]

(34)
\[ A_2 = \frac{\beta^4 g^4}{64} \int_{-\infty}^{+\infty} Dw \left( 1 - \frac{1}{3} \int_0^{+\infty} du \int_{-\infty}^{+\infty} Dv [1 + \cosh(\beta |\vec{h}|)] \right) \cdot \left( \int_{-\infty}^{+\infty} Dw \int_{-\infty}^{+\infty} Dv [1 + \cosh(\beta |\vec{h}|)] \right)^2 \].

(35)

The condition \( A_1 = 0 = A_2 \) (together with the equations for \( R, R_z \) and \( Q_z \)) gives the precise location of the tricritical point according to the known criteria. This location is shown in Fig. (1). Tricritical points have been already found in fermionic spin glasses for an Ising model with a pairing interaction in the half filling situation as well for an Ising model with charge fluctuation.

The behavior of the parameters \( R, R_z \) and \( Q_z \) also changes in the presence of the magnetic field. Fig. (2) shows the results for \( H_z = 0.00 \) where \( Q_z = 0 \) and \( R = R_z \). Figs. (3), (4) and (5) show the results, respectively, for \( H_z/J/(8)^{1/2} = 0.25, 0.50 \) and 0.75 where one can see that the parameter \( Q_z \) is no longer null while \( R \neq R_z \).

IV. CONCLUSIONS

In this work we have investigated how the mechanism responsible for the BCS pairing formation in real space (PAIR phase) can be affected for a spin glass diagonal replica symmetric order parameter through a long ranged Heinsenberg model with a pairing interaction in presence of magnetic field in \( \tau \) direction. This model could be treated at mean field level within the static approximation and the replica symmetry ansatz, always with no transversal (to the field) spin glass ordering in order to remain in a region where the static approximation and replica symmetry are reliable. So, the initial problem has been reduced to one site problem with two different effective fields applied on, which are \( \eta (\eta^*) \) and \( \vec{h} \). The former is a long range internal field related to a symmetry breaking which produces a pairing long range order and appears in the problem combined with the chemical potential to give an effective chemical potential. For the half filling case, just the pairing field survives. The later is a random gaussian field related to \( R, R_z \) and \( Q_z \) as defined in Sec. II. As we can see in Eqs. (28)–(31), the effective fields have to be solved coupled. That would be the mechanism through which the magnetic part of the problem always affects the PAIR phase, even above of the spin glass transition.

By solving numerically the Eqs. (28)–(31), it is been possible to construct the phase diagram for several values of \( H_z \) (Fig. (1)). For high temperatures, it has been found a crossover from a continuous pair breaking symmetry to a sharp one where the PAIR order parameter \( \eta \) displays a discontinuous behavior as shown in Figs. (3)–(5). The presence of the field \( H_z \) creates an anisotropy which can be seen clearly in the behavior of the \( R, R_z \) and \( Q_z \), shown in Figs. (3)–(5). Therefore, the region in the phase diagram where there is a first order transition becomes larger. This behavior could be verified by positioning the tricritical point using the Eqs. (34) and (35).

We remark that the our fermionic spin representation constitutes an important difference from Ref. [15]. The consequences of different representations for the spins are clear if one compares our equations for the order parameters with those obtained in Ref. [15]. We also point out that the transition line between SG–PAIR is not attainable from the theory used in this work.

Lastly, we have worked within the scope of the static approximation. The dynamic is absent in this formulation and certainly it is responsible for important effects at very low temperatures. Nevertheless, the static approximation is an upper bound of the theory from where the dynamic should be properly included. That would be subject for future work.

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FIG. 1: Phase diagram as a function of temperature and pairing coupling $g/J$ for several values of $H^*$ where $H^* = H_z/J/(8)^{1/2}$. Solid lines indicate second order transitions while dotted lines indicate first order transitions. Tricritical points are shown as filled squares. In the lower right corner it is shown the relation between $H^*$ and the NP–SG transition temperature $T_f$ relative to $T_f(H^* = 0.00)$. The transition line between SG–PAIR is not attainable from the present approach.
FIG. 2. Dependence of the order parameters $\eta$, $R$, $R_z$ and $Q_z$ for $H^* = 0.00$ as a function of $g/J$ for several values of temperature where $T^* = T/J$. For high values of $g/J$ the transition is clearly a continuous one as opposed to a discontinuous one for lower values of $g/J$. For $H^* = 0.00$, $Q_z$ is always null.
FIG. 3. Same as Fig. 2 for $H^* = 0.25$. 

Figure 4

FIG. 4. Same as Fig. 3 for $H^* = 0.50$. 
Figure 5

H^* = 0.75

FIG. 5. Same as Fig. 3 for $H^* = 0.75$. 

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