Proton structure function.
Soft and hard Pomerons *

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Abstract

Regge models for proton structure function with and without a hard Pomeron contribution are compared with all available data in the region \( W > 3 \text{ GeV}, Q^2 \leq 3000 \text{ GeV}^2 \) and \( x < 0.75 \). It is shown that the data do not support a hard Pomeron term in \( \gamma^* p \) amplitude. Moreover, the data support the idea that the soft Pomeron, either is a double pole with \( \alpha_P(0) = 1 \) in the angular momentum \( j \)-plane, or is a simple pole with \( \alpha_P(0) = 1 + \epsilon \) where \( \epsilon \ll 1 \).

The Regge approach to Deep Inelastic Scattering (DIS) is based on several facts and assumptions such as the similarity of photon and hadrons at high energies, the one-photon approximation, the optical theorem, the relation between elastic forward \( \gamma p \) amplitude and structure functions (SF). The most important assumptions for the Regge description of DIS are the analyticity of \( \gamma p \) amplitude and the properties of a dominating contribution, namely, of a Pomeron. One should have in mind that the very reasonable assumption about analyticity is only an assumption. The unitarity restrictions on the amplitude and on the related observable quantities are not proved in the case of DIS while they are often supposed to be valid, at least at \( Q^2 \sim 0 \) and \( x \rightarrow 0 \). The universality of the Pomeron and of the Reggeons are crucial ingredients in Regge approach being especially important if it is applied to DIS. This means that the Reggeons are the same in pure hadronic and in lepton-hadronic processes, their trajectories can not depend on the individual properties of interacting particles.

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The starting point in Regge approach to DIS is a choice of a model for the Pomeron. There are two main possibilities used in hadron physics as well as in DIS. Both of them take into account the fact that the total cross-sections are rising with energy. The first one assumes that the input Pomeron behaves as a simple \( j \)-pole and that its trajectory has an intercept \( \alpha_P(0) = 1 + \Delta > 1 \). Such a Pomeron leads to \( \sigma_{\text{tot}} \propto s^\Delta \), when \( s \to \infty \), violating the unitarity restriction \( \sigma_{\text{tot}} \leq C\ln^2(s/m^2) \). In order to restore unitarity, the Pomeron must be unitarized. An other possibility to describe the growing total cross-sections is to assume that the Pomeron is a more hard singularity than a simple pole. For instance, it can be a double pole (then, \( \sigma_{\text{tot}} \propto \ln s \)) or a triple pole (then, \( \sigma_{\text{tot}} \propto \ln^2 s \)). Such a Pomeron does not violate the Froissart-Martin bound.

Both types of Pomeron models have been developed and compared to the available data of the total cross-sections of nucleon-nucleon, meson-nucleon, photon-proton and photon-photon interactions \([1–6]\). The main lessons of these investigations are the following.

1. A better description of the data on total cross-sections and \( \rho \)-ratios of real to imaginary part of the forward scattering amplitudes is achieved in Pomeron models of the second type. Moreover the Dipole Pomeron (or double \( j \)-pole with \( \alpha_P(0) = 1 \)) model gives the minimal \( \chi^2 \)\([1–3,6]\).

2. If a simple pole Pomeron contribution, \( C(-is/s_0)\Delta \) is generalized \([1,3]\) into \( C_0 + C_1(-is/s_0)\Delta \), then fitting the data leads to \( \Delta \sim 0.001 \) (\( s_0 \) is fixed at 1 GeV\(^2\)). Practically this "supercritical" Pomeron reduces to the Dipole Pomeron because \( s^\Delta \approx 1 + \Delta\ln s \) at \( \Delta \ll 1 \) \([1,3]\).

Having in mind the above circumstances and the quite good description of the data on proton SF within the Soft Dipole Pomeron model \([7]\) we verify the validity of the above property (2) in DIS when \( Q^2 \neq 0 \). At the same time using a common data set we compare models for SF with and without a hard Pomeron contribution. This last component was introduced \([8]\) to describe a steep rise of SF when \( Q^2 \) is large and \( x \to 0 \). In our opinion however, adding a hard Pomeron component contradicts the idea of Pomeron universality because it is not observed in pure hadron processes \([4–6]\) and in \( \gamma p \) interaction at \( Q^2 = 0 \). Finally, we suggest a new model for the proton SF that does not violate unitarity restrictions for cross-section but at high \( Q^2 \) mimics the contribution of a hard Pomeron with \( \Delta \approx 0.4 \).

1 Models for proton SF at \( x \ll 1 \)

**Donnachie-Landshoff Soft+Hard Pomeron (SHP) model**

The model is constructed as sum of two Pomeron components, a hard one \( F_{\text{hard}} \) and a soft one \( F_{\text{soft}} \). The contribution of an \( f(a_2) \)-Reggeon is necessary.
to describe the structure function and the $\gamma p$ total cross-section at low energies.

$$F_2(x, Q^2) = F_{\text{hard}} + F_{\text{soft}} + F_f .$$  \hfill (1)

We have considered the version published by Donnachie and Landshoff (D-L) in [8].

$$F_{\text{hard}} = C_h \left( \frac{Q^2}{Q^2 + Q_h^2} \right)^{1+\epsilon_h} \left( 1 + \frac{Q^2}{Q^2 + Q_h^2} \right)^{\frac{1}{2}} \left( \frac{1}{x} \right)^{\epsilon_h},$$  \hfill (2)

$$F_{\text{soft}} = C_s \left( \frac{Q^2}{Q^2 + Q_s^2} \right)^{1+\epsilon_s} \left( \frac{1}{x} \right)^{\epsilon_s},$$  \hfill (3)

$$F_f = C_f \left( \frac{Q^2}{Q^2 + Q_f^2} \right)^{\alpha_f} \left( \frac{1}{x} \right)^{\alpha_f - 1}.$$  \hfill (4)

One can easily obtain from (1)-(4) the expression for $\gamma p$ total cross-section

$$\sigma_T(W) = \left. \frac{4\pi^2 \alpha}{Q^2} F_2(x, Q^2) \right|_{Q^2=0} = 4\pi^2 \alpha \sum_{i=h,s,f} \frac{C_i}{(Q_i^2)^{1+\epsilon_i}} (W^2 - m_p^2)^{\epsilon_i}.  \hfill (5)$$

where $\epsilon_f = \alpha_f - 1$.

**Soft Dipole Pomeron (SDP) model**

This model constructed in [7] makes use of the optical theorem and of the relation between elastic forward $\gamma^* p$ amplitude and structure function:

$$\sigma_{T}^{\gamma^* p} = 8\pi^3 m A(W^2, Q^2; t = 0) = \frac{4\pi^2 \alpha}{Q^2(1 - x)} \left( 1 + \frac{4m_p^2 x^2}{Q^2} \right) F_2(x, Q^2);$$  \hfill (6)

$$A(W^2, t = 0; Q^2) = P_0 + P_1 + f,$$  \hfill (7)

where

$$f = iG_f(Q^2)(-iW^2/m_p^2)^{\alpha_f - 1}(1 - x)^{B_f(Q^2)}$$  \hfill (8)

$$P_0 = iG_0(Q^2)(1 - x)^{B_0(Q^2)}, \quad P_1 = iG_1(Q^2) \ln \left( -i \frac{W^2}{m_p^2} \right) (1 - x)^{B_1(Q^2)}$$  \hfill (9)

$$G_i(Q^2) = \frac{C_i}{(1 + Q^2/Q_i^2)^{D_i(Q^2)}}, \quad D_i(Q^2) = d_{i\infty} + \frac{d_{i0} - d_{i\infty}}{1 + Q^2/Q_{i0}^2}.$$  \hfill (10)

$$B_i(Q^2) = b_{i\infty} + \frac{b_{i0} - b_{i\infty}}{1 + Q^2/Q_{i0}^2}.  \hfill (11)$$
We would like to emphasize that here the Pomeron is a double $j$-pole, the trajectory of which being $Q^2$-independent with an intercept $\alpha_P(0) = 1$. The intercept of $f$-reggeon was fixed at 0.785 as determined [3] from the fit to total cross-sections.

**Modified two-Pomeron (Mod2P) model**

Modifying the SHP model we aimed to verify the phenomenon mentioned above for the total cross-sections of hadrons and real photons when adding a Pomeron component (simple $j$-pole with an intercept one) leads to a very small value of $\Delta = \alpha_P(0) - 1$. So we construct the model as a sum of two Pomeron terms (the first one has a unit intercept while the other one has $\alpha_P(0) > 1$). Comparing with SHP model the residues $C_i(Q^2)$ are chosen dimensionless and modified as shown below.

\[
F_2(x, Q^2) = F_0 + F_s + F_f, \quad (12)
\]

\[
F_0 = \frac{C_0 Q^2_0}{\Delta} \left( \frac{Q^2}{Q^2 + Q^2_0} \right) \left( 1 + \frac{Q^2}{Q^2_0} \right)^{d_0}, \quad (13)
\]

\[
F_s = \frac{C_s Q^2_s}{\Delta (m^2/Q^2_s)^\Delta} \left( \frac{Q^2}{Q^2 + Q^2_s} \right)^{1+\Delta} \left( \frac{1}{x} \right)^\Delta \left( 1 + \frac{Q^2}{Q^2_s} \right)^{d_s}, \quad (14)
\]

\[
F_f = \frac{C_f Q^2_f}{(m^2/Q^2_f)^{\alpha_f-1}} \left( \frac{Q^2}{Q^2 + Q^2_f} \right)^{\alpha_f} \left( \frac{1}{x} \right)^{\alpha_f-1} \left( 1 + \frac{Q^2}{Q^2_f} \right)^{d_f}, \quad (15)
\]

\[
\sigma_T(W) = 4\pi^2 \alpha \left\{ \frac{C_0}{\Delta} + \frac{C_s}{\Delta} \left( \frac{W^2}{m^2} - 1 \right)^\Delta + C_f \left( \frac{W^2}{m^2} - 1 \right)^{\alpha_f-1} \right\}. \quad (16)
\]

Parameter $\Delta = \alpha_P(0) - 1$ and intercept of $f$-reggeon are fixed from the fit to all total cross-sections [3], namely $\Delta = 0.001013$ anf $\alpha_f(0) = 0.7895$.

**Generalized Logarithmic Pomeron (GLP) model**

We have found in [9] a shortcoming of the SDP model, relative to the logarithmic derivative $B_x = \partial \ln F_2(x, Q^2)/\partial \ln (1/x)$ at large $Q^2$ and small $x$. Namely, in spite of a good $\chi^2$ in fitting the SF ($F_2$), theoretical curves for $B_x$ are systematically slightly lower than the data extracted from $F_2$. In our opinion, one reason might be an insufficiently fast growth of $F_2$ with $x$ at large $Q^2$, and small $x$ (the SDP model leads to a logarithmic behaviour in $1/x$) On the other side, essentially a faster growth of $F_2$ (and consequently of $B_x$) is, from a phenomenological point of view, a good feature of the D-L model. However as stressed above, the hard Pomeron component of this model contradicts the Pomeron universality and the data on total cross-sections. Moreover it leads (see below) to a worse $\chi^2$ than SDP does.

Thus, we have tried to construct a model that incorporates a slow rise of $\sigma_{tot}^p(W)$ and simultaneously a fast rise of $F_2(x, Q^2)$ at large $Q^2$ and small $x$. We propose below a model intended to link these desirable properties

\[
F_2(x, Q^2) = F_0 + F_s + F_f, \quad (17)
\]
\[ F_f = C_f \frac{Q^2}{(1+Q^2/Q_f^2)^{d_f}} \left( \frac{Q^2}{x m^2} \right)^{\alpha_f - 1} (1 - x)^{B_f(Q^2)}, \]  
(18)

\[ F_0 = C_0 \frac{Q^2}{(1+Q^2/Q_0^2)^{d_0}} (1 - x)^{B_0(Q^2)}, \]  
(19)

\[ F_s = C_s \frac{Q^2}{(1+Q^2/Q_s^2)^{d_s}} L(W^2, Q^2)(1 - x)^{B_s(Q^2)} \]  
(20)

where \( B_i(Q^2), \ i = 0, s, f \) are defined in accordance with (11) and

\[ L(W^2, Q^2) = \ell n \left[ 1 + \frac{a}{(1+Q^2/Q_{s l}^2)^{d_{s l}}} \left( \frac{Q^2}{x m^2} \right)^{\epsilon} \right]. \]  
(21)

At \( Q^2 = 0 \), we have \( L(W^2, 0) \approx \epsilon \ell n(W^2/m_p^2) \) when \( W^2/m_p^2 \gg 1 \). Thus, \( \sigma_{tot}^\gamma p(W) \propto \ell n W^2 \) at \( W^2 \gg m_p^2 \). For \( Q^2 \neq 0 \) the logarithmic factor (21) has the following behavior

\[ L(W^2, Q^2) \approx \begin{cases} 
\epsilon \ell n \left( \frac{W^2+Q^2}{m^2} - 1 \right), & \text{at } Q^2/Q_{s l}^2 \lesssim 1 \\
\frac{a}{(1+Q^2/Q_{s l}^2)^{d_{s l}}} \left( \frac{W^2+Q^2}{m^2} - 1 \right)^{\epsilon}, & Q^2/Q_{s l}^2 \gg 1
\end{cases} \]  
(22)

Thus the term (20) simulates a Pomeron contribution with intercept \( \alpha_P(0) = 1 + \epsilon \). We should emphasize that, in spite of its appearance, the GLP model cannot be treated as a model with a hard Pomeron, even when \( \epsilon \) issued from the fit (\( \epsilon \approx 0.32 \) [10]) is not small. For the \( f \)-reggeon intercept the fixed value 0.785 (as in SDP model) was used.

2 Comparison of the models with data

Fitting parameters of the models, we used all available data on \( \sigma_{tot}^\gamma p \) and \( F_2 \) in the region \( W \geq 3 \) GeV, \( Q^2 \leq 3000 \) GeV\(^2\) and \( x \leq 0.75 \). All details concerning the choice of data, corresponding references as well as the values of parameters for each model and figures illustrating the good agreement of the predictions with experimental points can be found in [10]. Here we present only the main results and conclusions.

The values of \( \chi^2/dof \) (dof means degree of freedom = number of experimental points – number of free parameters) representing a good indicator or a confidence level in the models are given in the Table for three cases. The two first fits (A and B) were made in the region of small \( x \) while the third fit (C) was performed for \( x \leq 0.75 \). When the models SDP and GLP were fitted at small \( x \) the factors \((1 - x)^{B_i(Q^2)}\) were set \( \equiv 1 \) in the expressions for SF. Intercepts are chosen as it is explained above.
Thus from a comparison of the models under interest with the data on structure functions at small \( x \) we can made the following conclusion: all structure function data at \( Q^2 \leq 3000 \text{ GeV}^2 \) and small \( x \) are described with a high quality without a hard pomeron. Moreover, these data support the idea that the soft Pomeron, either is a double pole with \( \alpha_P(0) = 1 \) in the angular momentum \( j \)-plane or is a simple pole having intercept \( \alpha_P(0) = 1 + \epsilon \) with a very small \( \epsilon \).

Table 1
The results of the fits of 4 Regge models to the small-\( x \) (in two regions of energy) and large-\( x \) SF data.

| Model                        | \( x \leq 0.07 \) | \( x \leq 0.75 \) |
|------------------------------|-------------------|-------------------|
| SHP (with hard Pomeron)      | 1.375             | -                 |
| SDP (no hard Pomeron)        | 0.945             | 0.976             |
| Mod2P (no hard Pomeron)      | 0.996             | 1.023             |
| GLP (no hard Pomeron)        | 0.941             | 0.9685            | 1.064             |

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