Nonequilibrium finite-frequency noise of a resonance-level quantum dot close to a dissipative quantum phase transition: Functional Renormalization Group approaches

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We calculate the finite-frequency current noise of a nonequilibrium resonance-level quantum dot close to a dissipative quantum phase transition of the Kosterlitz-Thouless (KT) type between a de-localized phase for weak dissipation and a localized phase for strong dissipation. The resonance-level is coupled to two spinless fermionic baths with a finite bias voltage and an Ohmic boson bath representing the dissipative environment. The system is equivalent to an effective anisotropic Kondo model out of equilibrium. To compute the finite-frequency noise, we combine two recently developed Functional Renormalization Group (FRG) approaches in Refs. [17, 22] and in Ref. [23]. The nonequilibrium current noise at zero-temperature and finite frequencies shows a singular dip in the de-localized phase for the magnitude of frequencies equal to the bias voltage; while the dip is smeared out as the system moves to the localized phase. The corresponding peak-to-dip crossover is observed in the AC conductance for the magnitude of frequencies equal to the bias voltage. The relevance and applications of our results for the experiments and for tunnelings between Fractional Quantum Hall Edge (FQHE) states and chiral Luttinger liquids are discussed.

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Introduction

Quantum phase transitions (QPTs) due to competing quantum ground states in strongly correlated systems have been extensively investigated over the past decades. Near the transitions, exotic quantum critical properties are realized. Meanwhile, quantum transport in quantum dots has attracted much attention in recent years due to both the advance in nano-technology and the novel nonequilibrium effects in these devices. The well-known Kondo effect plays an important role in determining the charge transport in these systems. Recently, there has been growing interest in quantum phase transitions associated with the Kondo breakdown when an additional coupling to the quantum dots competes with the Kondo effect.

Nevertheless, much of the attention has been focused on equilibrium properties; while much less is known on the nonequilibrium properties. The bias voltage plays a very different role as the temperature $T$ in equilibrium systems as the voltage-induced decoherence behaves very differently from the decoherence at finite temperature, leading to exotic transport properties near the quantum phase transition compared to that in equilibrium at finite temperatures. Very recently, QPTs have been extended to nonequilibrium nanosystems. A generic example is the transport through an Ohmic dissipative resonance-level (spinless quantum dot) at a finite bias voltage where dissipative bosonic bath (noise) coming from the environment in the leads.

The system is equivalent to an effective anisotropic Kondo model; in equilibrium it gives rise to quantum phase transition of the Kosterlitz-Thouless type in transport between a conducting (de-localized) phase where resonant tunneling dominates and an insulating (localized) phase where the dissipation prevails.

The nonequilibrium current and conductance near the above de-localized-to-localized quantum phase transition have been addressed in Ref. [13]. Nevertheless, further insight on the nonequilibrium transport near the quantum phase transition can be obtained from the current fluctuations (or noise). The zero frequency shot noise has been used to probe the fractional charge of quasiparticle excitations in FQHE state tunnelings [14]. However, more useful information can be found in the finite-frequency (FF) current noise, which can be used to probe the crossover between different quantum statistics of the quasiparticles [15]. Recently, there has been theoretical studies on the FF current noise of a nonequilibrium Kondo dot [23, 25]. So far, these studies have not been extended to the nonequilibrium FF current noise of a dissipative quantum dot.

Based on recent developments shown in Refs. [13, 16], and in Ref. [23], we study in this paper the nonequilibrium FF current noise at zero temperature near the de-localized-to-localized quantum phase transition of a dissipative resonance-level quantum dot. To address this issue, we apply the Functional Renormalization Group (FRG) approach in Refs. [17, 22] and the real-time FRG...
approach recently developed in Ref. [24]. From our numerical solutions of the nonequilibrium RG scaling equations, as the system moves from the de-localized to the localized phase, we find the smearing of the dips in current noise spectrum for frequencies $\omega \approx \pm V$; while we find a peak-to-dip crossover in the AC conductance for $\omega \approx \pm V$. These features are detectable in experiments and can serve (in additional to the conductance) as alternative signatures of the QPT in the dissipative resonance-level quantum dot.

Model Hamiltonian

The starting point is a spin-polarized quantum dot coupled to two Fermi-liquid leads subjected to noisy Ohmic environment, which coupled capacitively to the quantum dot [13]. The noisy environment here consists of a collection of harmonic oscillators with theOhmic correlation[25]:

$$G_\phi(\omega) = \frac{\alpha}{\pi} \delta(\omega - \omega_0) = \frac{2\pi R}{\hbar e^2} \approx 25.8k\Omega$$

with $R$ being the circuit resistance and $R_K = 2\pi R/\hbar e^2 \approx 25.8k\Omega$ being the quantum resistance. For a dissipative resonant level (spinless quantum dot) model, the quantum phase transition separating the conducting and insulating phase for the level is solely driven by dissipation. Our Hamiltonian is given by:

$$H = \sum_{k,i=1,2} \left( \epsilon_k - \mu_i \right) c_{ki}^\dagger c_{ki} + t_i c_{ki}^\dagger d + h.c. + \sum_r \lambda_r (d_1^\dagger d_1 - 1/2) b_r^\dagger b_r + \sum_r \omega_r b_r^\dagger b_r + h(d_1^\dagger d_1 - 1/2),$$

where $\epsilon_k$ is the energy of the level, $\mu_i = \pm V/2$ is the chemical potential (bias voltage) applied on the lead $i$, while $h$ is the energy level of the dot (we restrict ourselves here to the case where $h = 0$ for simplicity). We assume that the electron spins have been polarized by a strong magnetic field. Here, $b^\dagger$ are the boson operators of the dissipative bath with an ohmic spectral density [6]:

$$J(\omega) = \sum_\alpha \lambda_\alpha^2 \gamma(\omega - \omega_\alpha) = \alpha \omega$$

with $\alpha$ being the strength of the dissipative boson bath.

We map our model to an equivalent anisotropic Kondo model with the effective left $L$ and right $R$ Fermi-liquid leads [13]. The effective Kondo model takes the form:

$$H_K = \sum_{k,\gamma=L,R,\sigma=\uparrow,\downarrow} [\epsilon_k - \mu_\gamma] c_{k\gamma}^\dagger c_{k\gamma} + (J_1^1 s_{\gamma L}^R S^- + J_1^2 s_{\gamma L}^R S^- + h.c.) + \sum_{\gamma=L,R} J_2 s_{\gamma L}^R S^Z + h S_z,$$

where $c_{kL(R)\sigma}^\dagger$ is the electron operator of the effective lead $L(R)$, with spin $\sigma$. Here, the spin operators are related to the electron operators on the dot by: $S^+=d^\dagger$, $S^-=d$, and $S^Z=d^\dagger d-1/2=n_d-1/2$ where $n_d=d^\dagger d$ describes the charge occupancy of the level. The spin operators for electrons in the effective leads are $s_{\gamma\beta}^\dagger = \sum_{\alpha,\delta,k,k} 1/2 c_{\alpha k\gamma}^\dagger c_{\beta k\delta}^\dagger$, the transverse and longitudinal Kondo couplings are given by $J_1^1 \propto t_1^2(2)$ and $J_2 \propto \frac{1}{2}(1-1/\sqrt{2\alpha^*})$ respectively, and the effective bias voltage is $\mu_\gamma = \pm \sqrt{2}\alpha^*/\sqrt{1/(2\alpha^*)}$, where $1/\alpha^* = 1 + \alpha$. Note that $\mu_\gamma \rightarrow \pm V/2$ near the transition ($\alpha^* \rightarrow 2$ or $\alpha \rightarrow 1$) where the above mapping is exact. The spin operator of the quantum dot in the Kondo model $S_f^\dagger$ can also be expressed in terms of spinful pseudofermion operator $f_\sigma$: $S_f^\dagger = \sum_{\alpha=\pm x,y,z} f_\alpha^\dagger f_\alpha$. In the Kondo limit where only the singly occupied fermion states are physically relevant, a projection onto the singly occupied states is necessary in the pseudofermion representation, which can be achieved by introducing the Lagrange multiplier $\lambda$ so that $Q = \sum_{\gamma} f_1^\dagger f_1 = 1$. For simplicity, we assume symmetric hopping $t_1 = t_2$, and therefore symmetric Kondo couplings $J_1^1 = J_1^2 = J_1$. The dimensionless (bare) Kondo couplings under RG are defined as: $g_1^0 = n(0)J_1^1 = n(0)J_1^2 = g_1^0 = n(0)J_z$.

In equilibrium ($V = 0$), the above anisotropic Kondo model exhibits the Kosterlitz-Thouless (KT) transition.
from a de-localized phase with a finite conductance \( G \approx \frac{1}{4\pi} (e = h = 1) \) for \( g_0^L + g_0^R > 0 \) to a localized phase for \( g_0^L + g_0^R \leq 0 \) with vanishing conductance. The nonequilibrium transport near the KT transition has been shown to exhibit distinct profile from that in equilibrium [13]. For \( g_0^L \to 0 \) and as \( \alpha \to \alpha_c^- \), the Kondo temperature vanishes as \( \ln T_k \propto 1/(\alpha - \alpha_c) \) [9, 10, 13]. We will focus on in the following the the nonequilibrium noise at finite frequencies in the above Kondo model across the transition. We address this issue by combing two recently developed functional RG approaches in Ref. [23], and Refs. [17, 22], respectively.

III. Functional RG approaches to nonequilibrium FF current noise

First, via the above mapping, the current (in units of \( e = h = k_B = 1 \)) through the dissipative resonancel- level quantum dot described by the effective anisotropic Kondo model [13]. Following the real-time RG approach in Ref. [22] the Keldysh current operator through the left lead in the effective Kondo model, \( I_L^+(t) \), is given by:

\[
I_L^+(t) = \frac{e}{4} \sum_{\alpha,\beta} \int dt_1 dt_2 \sum L_{\alpha\beta}^+(t_1 - t, t - t_2) \times [s_{\alpha\beta}^+(t_1, t_2) S^-(t) + h.c.]
\]

with \( \alpha, \beta = L, R \), \( S_f(t) = f^{\dagger}(t) \bar{\sigma} f^\ast(t) \), \( s_{\alpha\beta}^+(t_1, t_2) = c_{\alpha}^{\dagger}(t_1) \sigma^+ c_{\beta}^<(t_2) \). Here, \( L_{\alpha\beta}(t_1, t- t_2) \) is the left current vertex function in the Fourier space, \( L_{\alpha\beta}(t_1, t- t_2) \) has a two-frequency structure: it keeps track of not only the times electrons enter \( (t_1) \) and leave \( (t_2) \) the dot, but also the time \( t \) which the current is measured [23]. The double-time structure of the current vertex function automatically satisfies the current conservation: \( I_L^+(t) = - I_R^-(t) \) [23].

The frequency-dependent current noise \( S(\omega) \) is computed via the second-order renormalized perturbation theory (see diagram in Fig. 1). Note that due to the double-time structure of the current vertex function \( L_{\alpha\beta}(t_1, t_2) \), in the Fourier (frequency) space, \( L_{\alpha\beta}(\epsilon + \omega, \epsilon) \) has a two-frequency structure; it depends on the incoming \((\epsilon + \omega)\) and outgoing \((\epsilon)\) frequencies of the electron (see Fig. 1). The result reads:

\[
S_c(\omega) = \sum_{\alpha,\beta} -2Re(D_{\alpha\beta}(\omega))
\]

where the correlator \( D_{\alpha\beta}(\omega) \) is computed by the diagram in Fig. 1.

\[
D_{\alpha\beta}(\omega) = \frac{d\Omega}{2\pi} \chi_{\alpha\beta}(\Omega, \omega) \chi_f(\Omega),
\]

\[
\chi_{\alpha\beta}(\Omega, \omega) = \frac{d\nu}{2\pi} \chi_f(\nu) \delta(\nu + \Omega)
\]

\[
\chi_f(\Omega) = \frac{d\nu}{2\pi} \chi_f(\nu) \delta(\nu + \Omega),
\]

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\]
where \( \hat{G} \) is the Green’s function in 2 \( \times \) 2 Keldysh space, and its lesser and greater Green’s function are related to its retarded, advanced, and Keldysh components by:

\[
G^\less = (G^K - G^R + G^A)/2
\]
\[
G^\greater = (G^K + G^R - G^A)/2
\]  

(7)
The lesser \((G^\less)\) and greater \((G^\greater)\) components of Green’s function of the conduction electron in the leads and of the quantum dot (impurity) are given by:

\[
G^\less_{L/R}(e) = iA_c(e)f_{\mu L/R}
\]
\[
G^\greater_{L/R}(e) = iA_c(e)(1 - f_{\mu L/R})
\]
\[
G^\less_{\sigma}(e) = 2\pi i\delta(e)n_{\sigma}(e) - 1,
\]
\[
G^\greater_{\sigma}(e) = 2\pi i\delta(e)(n_{\sigma}(e) - 1),
\]  

(8)

where \( A_c(e) = 2\pi N_0^2\Theta(D_0 - e) \) is the density of states of the leads, \( n_{\sigma}(e) = \int d\epsilon \delta(e - \epsilon) \) is the occupation number of the pseudofermion which obeys \( n_{\uparrow \downarrow}(e) = 1 \), \( n_{\downarrow \uparrow}(e) = 0 \) in the de-localized phase and \( n_{\uparrow \uparrow}(e) \rightarrow 0, n_{\downarrow \downarrow}(e) \rightarrow 0 \) in the localized phase. Here, the pseudofermion occupation number \( n_{\uparrow \downarrow} \) and the occupation number on the dot \( n_d \) are related via \(< n_{\uparrow \downarrow} - n_{\downarrow \uparrow} > = n_{\uparrow \downarrow} > = -1/2 \). The renormalized current vertex function \( L_{\alpha \beta}^{\uparrow \downarrow}(\omega_1, \omega_2) \) and the Kondo couplings \( g_{\perp}(\omega) \) are obtained from the nonequilibrium functional RG approaches in Refs. [23] and Refs. [21,22], respectively. Carrying out the calculations, the finite-frequency noise spectrum reads:

\[
S^\less(\omega) = \sum_{\alpha, \beta = L,R} \frac{3}{8} \int d\epsilon L_{\alpha \beta}^{\uparrow \downarrow}(\epsilon + \omega, \epsilon)L_{\beta \alpha}^{\downarrow \uparrow}(\epsilon, \epsilon + \omega) \times f_{\epsilon - \mu_\alpha}(1 - f_{\epsilon - \mu_\beta}),
\]

(9)

where \( f_{\epsilon - \mu_\alpha} \) is the Fermi function of the lead \( \alpha = L/R \) given by \( f_{\epsilon - \mu_\alpha} = 1/(1 + e^{(\epsilon - \mu_\alpha)/k_BT}) \). The symmetrized noise spectrum reads:

\[
S(\omega) = 1/2(S^\less(\omega) + S^\greater(\omega))
\]

(10)

with the relation between emission and absorption parts of the noise spectrum in frequency space \( S^\less(\omega) = S^\greater(-\omega) \) being used.

The above-mentioned frequency-dependent Kondo couplings \( g_{\perp}(\omega) \) and current vertex functions \( L_{\alpha \beta}^{\uparrow \downarrow}(\omega_1, \omega_2) \) are obtained via the FRG approaches, which can be devided into two parts. First, the RG scaling equations for the generalized frequency dependent nonequilibrium Kondo couplings in the effective anisotropic Kondo model are given by [17,22]:

\[
\frac{\partial g_{\perp}(\omega)}{\partial \ln D} = -\sum_{\beta = -1,1} \left[ g_{\uparrow \downarrow}(\beta V/2)^2 \Theta_{\omega + \delta \omega} \right]  
\]
\[
\frac{\partial g_{\perp}(\omega)}{\partial \ln D} = -\sum_{\beta = -1,1} g_{\perp}(\beta V/2) g_{\uparrow \downarrow}(\beta V/2) \Theta_{\omega + \delta \omega}(11)
\]

where \( g_{\perp}(\omega) = N(0)J_1^\perp(\omega) = N(0)J_2^\perp(\omega), g_{\perp}(\omega) = N(0)J_1^\perp(\omega) \) are dimensionless frequency-dependent Kondo couplings with \( N(0) \) being density of states per spin of the conduction electrons, \( \Theta_\omega = \Theta(D - |\omega + i\Gamma(\omega)|, D < D_0 \) is the running cutoff. Here, \( \Gamma(\omega) \) is the dynamical decoherence (dephasing) rate at finite bias which cuts off the RG flow [24]. It is obtained from the imaginary part of the pseudofermion self energy [17,21,22]:

\[
\Gamma(\omega) = \frac{\pi}{4} \int d\epsilon g_{\perp}(\epsilon + \omega)g_{\perp}(\epsilon)[f_{\epsilon + \omega}^L - f_{\epsilon + \omega}^R] + g_{\perp}(\epsilon + \omega)g_{\perp}(\epsilon)[f_{\epsilon + \omega}^L - f_{\epsilon + \omega}^R] + (L \rightarrow R).
\]

(12)

Note that in general there will be \( g_{\perp}^{LL/RR} \) and \( g_{\perp}^{RL/RL} \) terms in the scaling equations generated via RG procedures. However, as the initial (bare) values for these Kondo couplings \( g_{\perp}^{LL/RR} \) and \( g_{\perp}^{RL/RL} \) are zero, these terms are therefore negligible. We have solved the RG equations Eq. (11) subject to Eq. (12) self-consistently. The solutions for \( g_{\perp}(\omega), g_{\perp}(\omega) \) and \( \Gamma(\omega) \) close to the KT transition are shown in Refs. [13,17]. As the system goes from the de-localized to localized phase, the features in \( g_{\perp}(\omega) \) at \( \omega = \pm V/2 \) undergoes a crossover from symmetric two peaks to symmetric two dips, while the symmetric two peaks in \( g_{\perp}(\omega = \pm V/2) \) still remain peaks. The finite-frequency nonequilibrium decoherence rate \( \Gamma(\omega) \) monotonically increases with increasing \( \omega \), it shows logarithmic singularities at \( |\omega| = V \) in the de-localized phase [17]. As the system moves to the localized phase, the overall magnitude of \( \Gamma(\omega) \) decreases rapidly and the singular behaviors at \( \omega = \pm V \) get smeared out [17]. Note that, unlike the equilibrium RG at finite temperatures where RG flows are cutoff by temperature \( T \), here in nonequilibrium the RG flows will be cutoff by the decoherence rate \( \Gamma \), a much lower energy scale than \( V, \Gamma \ll V \). Moreover, at a fixed \( \omega = \omega_0, \Gamma(\omega_0, V) \) is a highly non-linear function in \( V \). (For example, at the KT transition, \( \Gamma(\omega_0, V = 0) \) \( \propto V^{-1/2|\omega_0|} \). The unconventional properties of \( \Gamma(\omega) \) lead to the distinct nonequilibrium conductance \((G(V, T = 0)) \) from that in equilibrium \((G(V, T = 0)) \).
vertex functions $L_{\alpha\beta}^{+}L^{\pm}(\omega_1,\omega_2)$ can be simplified as:

$$
\frac{dL_{\alpha\beta}(\omega_1, \omega_2)}{dnD} = \sum_{\gamma=L,R} L_{\alpha\gamma}(\omega_1, \omega_2) \Theta_\mu, (\omega_2) g_{\gamma\beta}(\omega_2)
+ g_{\alpha\gamma}(\omega_1) \Theta_\mu, (\omega_1) L_{\gamma\beta}(\omega_1, \omega_2)
$$

where we make the following identifications: $g_{LR/RL}(\omega) \rightarrow g_{L,R}(\omega) \equiv g_{\alpha\alpha}(\omega) \rightarrow g_{L,R}(\omega) \equiv g_{\alpha\alpha}(\omega)$. Similarly, $L_{LR/RL}(\omega_1, \omega_2) \rightarrow L_{L,R}(\omega, \omega_2)$ refers to only the transverse component of the current vertex function $L_{\alpha\beta}(\omega_1, \omega_2)$; while $L_{LL/RR}(\omega_1, \omega_2) \rightarrow L_{L,R}(\omega_1, \omega_2)$ refers only to the longitudinal part of $L_{\alpha\alpha}$. Here, the frequency-dependent Kondo couplings $g_{L,R}(\omega)$ in Eq. 13 are obtained from the RG scaling equations Eq. 11. Note that the scaling functions $L_{\alpha\beta}(\omega_1, \omega_2)$ via Ref. 23 can also be expressed within the RG approach in Ref. 20 via a straightforward generalization by allowing for the two-frequency dependent vertex functions $L_{\alpha\beta}(\omega_1, \omega_2)$ where $\omega_{1(2)}$ refers to the incoming (outgoing) frequency.

We solved the self-consistent RG scaling equations Eq. 13 for the current vertex functions with the help of the solutions for the renormalized Kondo couplings via Eq. 11 and Eq. 12. The typical results at zero temperature are shown in Fig. 3 and Fig. 4; they exhibit the following symmetry: $L_{\alpha\beta}(\omega_1, \omega_2) = -L_{\beta\alpha}(\omega_2, \omega_1)$. Note that since the initial conditions for the current vertex function have the following structures: $L_{\alpha\alpha}(0,0) = 0$, $L_{L,R}^0 \neq 0$, we find $L_{\alpha\alpha}(\omega_1, \omega_2) \ll L_{L,R}(\omega_1, \omega_2)$. In the de-localized (Kondo) phase, a sharp peak is developed in $L_{L,R}(\omega_1, \omega_2)$ for $(\omega_1, \omega_2) = (V/2, -V/2)$; while as a small dip is formed for $(\omega_1, \omega_2) = (-V/2, V/2)$. Meanwhile, in general $L_{L,R}(\omega_1, \omega_2)$ is maximized at $\omega_{1(2)} = \pm V/2$ for fixed $\omega_{2(1)}$. This agrees perfectly with the result in Ref. 23. In the localized phase, however, we find the opposite: $L_{L,R}(\omega_1, \omega_2)$ develops a sharp dip at $(\omega_1, \omega_2) = (V/2, -V/2)$; and it is minimized $\omega_{1(2)} = \pm V/2$ for fixed $\omega_{2(1)}$. The peak-dip structure of the current vertex function $L_{\alpha\beta}$ plays a crucial role in determining the noise spectrum both in de-localized and in the localized phases.

Substituting the numerical solutions for $L_{\alpha\beta}(\omega_1, \omega_2)$ and $g_{\alpha\beta}(\omega)$ into Eq. 9 we get the zero-temperature FF noise $S(\omega)$. The results at zero temperature are shown in Fig. 5. First, the overall magnitude of $S(\omega)$ decreases rapidly as the system crossovers from the de-localized to the localized phase. This can be understood easily as the current decreases rapidly in the crossover, leading to a rapid decrease in the magnitude of noise. For $|\omega| > V$, $S(\omega)$ in both phases increases monotonically with increasing $\omega$ due to the increase of the photon emission at higher energies. For $|\omega| \leq V$, however, it changes from a peak to a dip centered at $\omega = 0$ as the system crossovers from de-localized to localized phase (see Fig. 5). At $|\omega| = V$, $S(\omega)$ exhibits a dip (minima) in the de-localized phase, a signature of the nonequilibrium Kondo effect; while as the system crossovers to the localized phase the dips are gradually smeared out and they change into a “kink”-like singular point at $\omega = \pm V$, connecting two curves between $\omega < V$ and $\omega > V$.

We furthermore computed the nonequilibrium AC conductance at zero temperature\cite{22, 26}: $G_{AC}(\omega) = S^{\pm}(\omega) - S^{-}(\omega)$ across the transition. Note that $G(\omega = 0) = dI/dV$ corresponds to the nonequilibrium differential conductance. As shown in Fig. 6(a), in the de-localized phase the splitted peaks in $G_{AC}(\omega)$ at $\omega = \pm V$ are signatures of the Kondo resonance at

\[ FIG. 5: (Color online) S(\omega) at zero temperature versus \omega across the KT transition. The bias voltage is fixed at V = 0.32D_0. Inset: S(\omega) at zero temperature versus \omega normalized to S_0 = S(\omega = 0). Here, D_0 = 1 for all the figures. \]

\[ FIG. 6: (Color online) (a). The zero-temperature AC conductance G_{AC}(\omega) versus \omega across the KT transition. (b). G_{AC}(\omega) at zero temperature versus \omega normalized to G_0 \equiv G_{AC}(\omega = 0). The bias voltage is fixed at V = 0.32D_0. Here, D_0 = 1 for all the figures. \]
finite bias, and are consistent with the dips at seen in the noise spectrum. As the system moves to the localized phase, the overall magnitudes of \( G_{AC}(\omega) \) as well as the pronounced splitted Kondo peaks at \( \omega = \pm V \) get suppressed; they change into dips deep in the localized phase (see Fig. 4 (b)). In response to this change in the splitted Kondo peaks, the overall shape of \( G_{AC}(\omega \to 0) \) shows a dip-to-hump crossover near \( \omega = 0 \). Note that the suppression of the Kondo peaks for \( G_{AC}(\omega) \) at \( \omega = \pm V \) corresponds to the smearing of the dips at \( \omega = \pm V \) shown in the noise spectrum \( S(\omega) \) (see Fig. 3). The above evolution in the noise spectrum agree well with the nonequilibrium transport properties studied in Refs. [13, 16], and can be served as alternative signatures of the localized-de-localized transition in future experiments.

IV. Discussions and applications

We would like to make a few remarks here. First of all, though the current vertex function \( L_{\alpha\beta} \) looks similar to the Kondo coupling \( g_{\alpha\beta} \), they have different symmetries as the initial values for the Kondo couplings are \( g_{\alpha\beta}^0 = g_{\beta\alpha}^0 \); while the initial values for the current vertex functions are: \( L_{LL}^0 = L_{RR}^0 = 0 \), \( L_{LR}^0 = -L_{RL}^0 = ig_{LR}^0 \). Therefore, it is necessary to employ a two-frequency RG scheme for \( L_{\alpha\beta} \); while a one-frequency (either incoming or outgoing frequency) RG is sufficient for \( g_{\alpha\beta}(\omega) \). This also explains the necessity to employ two separated sets of RG scaling equations for \( g_{\alpha\beta}(\omega) \) and \( L_{\alpha\beta}(\omega_1, \omega_2) \), respectively.

Secondly, the FRG approaches in this work and in Ref. [23] lead to non-perturbative effects which are not captured by standard perturbation theory [24]. As a result, these two FRG approaches are able to correctly capture the logarithmic singularities in the nonequilibrium noise spectrum of a Kondo dot [23]. Furthermore, the RG scaling equations we use here for the current vertex function Eq. (1) have the same form as that in Ref. [23]. Nevertheless, the decoherence effect is included here self-consistently in the scaling equations for the Kondo couplings: while as it was absent in the approximated form in Ref. [23]. The inclusion of decoherence rate in the pseudo-fermion self-energy leads to more accurate results in both the Kondo couplings \( g_{\alpha\beta}(\omega) \) and the current vertex functions \( L_{\alpha\beta}(\omega_1, \omega_2) \) for \( \omega, \omega_1(2) \approx \pm V/2 \).

Thirdly, the functional RG approach by Rosch et. al. in Ref. [20] is a very efficient way to study the nonequilibrium transport in Kondo dot systems. However, the drawback of this approach regarding computing the current correlation in the one-frequency RG formalism is that it violates the current conservation [23]. Here, we fix this problem by employing the two-frequency RG scaling equations for the current vertex functions \( L_{\alpha\beta}(\omega_1, \omega_2) \).

Fourthly, we have solved the most general RG scaling equations for \( g_{1\pm}^0(\omega) \) and \( L_{1\pm}^0(\omega_1, \omega_2) \) [24,25]. We found indeed \( g_{1\pm}^0(\omega), g_{1\alpha}^0(\omega), L_{1\pm}^0(\omega_1, \omega_2), \) and \( L_{1\alpha}^0(\omega_1, \omega_2) \) are all negligible, which justifies the above simplications: \( g_{\alpha\beta}(\omega) \to g_{1\pm}^0(\omega), g_{\alpha\alpha}(\omega) \to g_{1\alpha}^0(\omega); L_{\alpha\beta}(\omega_1, \omega_2) \to L_{1\pm}^0(\omega_1, \omega_2), L_{\alpha\alpha}(\omega_1, \omega_2) \to L_{1\alpha}^0(\omega_1, \omega_2) \).

Finally, our results have direct relevance for the nonequilibrium noise spectrum in the tunneling between two chiral Luttinger liquid leads as well as between FQHE states [28]. The Hamiltonian of such system can be mapped onto an effective anisotropic Kondo model Eq. (2) with effective Kondo couplings \( g_{\alpha\alpha}^0 \propto t, g_{\alpha\beta}^0 \propto 1 - 1/\sqrt{2K} \) where \( K \) is the Luttinger parameter corresponding to a filling-factor \( \nu = 1/K \) in FQHE [28,29]. For example, \( \nu = 1/3 \) FQHE edge states corresponds to chiral Luttinger liquid with attractive interactions (\( K = 3 \)) [27,28], which falls into the de-localized phase in our model when considering the quasiparticle tunneling between two such states; and the localized phase in our model corresponds to tunneling between chiral Luttinger liquid leads with strong repulsive interactions (\( K \ll 1 \)). It is worthwhile mentioning that our results via functional RG approaches lead to non-perturbative results which are able to more accurately capture the finite-frequency noise compared to those via bare perturbation theory [27].

V. Conclusions

In conclusion, we have calculated the nonequilibrium finite-frequency current noise of a dissipative quantum dot close to localized-de-localized quantum phase transition via combining two recently developed Functional Renormalization Group approaches in Refs. [13, 22] and Ref. [23], respectively. The system is equivalent to the nonequilibrium anisotropic Kondo model. We formulated within the Kondo model the frequency-dependent nonequilibrium RG scaling equations for the Kondo couplings \( g_{\alpha\beta}(\omega) \) and current vertex functions \( L_{\alpha\beta}(\omega) \). The charge-flip decoherence rate, which cuts off the RG flows, is self-consistently taken into account. We have numerically solved the self-consistent scalings equations for the renormalized Kondo couplings and the current vertex functions. We find strong peaks (dips) for these functions at \( \omega = \pm V/2 \) are developed in the de-localized (localized) phase, respectively. Based on these solutions we compute the noise spectrum via second-order renormalized perturbation theory. In addition to the decrease in overall magnitudes, we find the smearing of the dip in the noise spectrum for \( \omega = \pm V \) as the system crossovers from the de-localized to localized phase, which comes as a direct consequence of the peak-dip structure in \( g_{\alpha\beta}(\omega) \) and \( L_{\alpha\beta}(\omega) \). The corresponding peak-to-dip crossover is seen in the AC conductance for \( \omega = \pm V \). These features can in principle
be detected in experiments and be served as alternative signatures of the de-localized-localized crossover in a dissipative resonance level far from equilibrium in addition to the current and conductance. Our results have direct relevance for the nonequilibrium current noise in tunneling between Fractional Quantum Hall Edge states.

Experimentally, though most of the transport measurements on the Kondo quantum dots are on the current and conductance, the low-frequency current noise measurements were reported very recently \[30\]. Meanwhile, further attempts have been made on the finite-frequency noise and the AC conductance measurements \[31\]. Therefore, it is reasonable to expect that the predictions we presented here may be detected in experiments in the near future.

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