On “dual” parametrizations of generalized parton distributions

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Abstract

We propose a parametrization for the generalized parton distributions (GPDs) which is based on representation of parton distributions as an infinite series of $t$-channel exchanges. The entire generalized parton distribution is given as an infinite sum over contributions of generalized light-cone distribution amplitudes in the $t$-channel. We also discuss the relations of the lowest Mellin moments of GPDs to basic mechanical characteristics of the nucleon as a compound system.

Introduction

After the first experimental data on deeply virtual Compton scattering (DVCS) \cite{1, 2, 3, 4} and hard exclusive meson production \cite{5, 6} have been presented, it is the right time to address the problem of the extraction of the twist-2 generalized parton distributions (GPDs) from the observables. One of the possibilities to do this is to introduce a parametrization of GPDs and then to extract the corresponding parameters from the data. The parametrization should respect all general constrains for GPDs (for reviews see \cite{7, 8, 9, 10}) and it should have clear physical interpretation.

To our best knowledge in all calculations of observables for hard exclusive processes the parametrization for GPDs based on double distributions (DDs) \cite{11, 12, 13} has been used. In this parametrization (supplemented by the contribution of the D-term \cite{14}) an important property of polynomiality of the Mellin moments \cite{13} of GPDs is satisfied automatically. The double distributions are modeled usually by the ansatz of the form \cite{13} (for details see next section):

$$
F(\beta, \alpha) = h(\alpha, \beta) \ q(\beta).
$$

(1)

Here $q(\beta)$ is usual quark distribution and the profile function $h(\alpha, \beta)$ popularly is chosen usually in the form \cite{13}:

$$
h(\alpha, \beta) = \frac{\Gamma (2b + 2)}{2^{2b+1}\Gamma^2 (b + 1)} \ \frac{[(1 - |\beta|)^2 - \alpha^2]^b}{(1 - |\beta|)^{2b+1}}.
$$

(2)
Here the parameter $b$ characterizes the strength of the $\xi$ dependence of the resulting GPDs. The advantage of the ansatz (1) is its simplicity, however this ansatz is too restrictive to have enough flexibility in modeling of GPDs. Also this form of DD does not commute with the QCD evolution, i.e. if we assume this ansatz at one normalization point than at higher normalization points it is generically impossible to put the resulting DD into the form given by eq. (1) [16].

In the present paper we suggest alternative way to parametrize GPDs based on the partial wave expansion of the GPDs. Before the discussion of the new parametrization we present in the next two sections the discussion of important property of polynomiality of Mellin moments and the relations of the lowest Mellin moments to the mechanical properties of the nucleon.

In this notes we shall restrict ourselves only to the singlet quark GPDs. The formulae relevant to the nonsinglet GPDs are collected in Appendix A. We also do not consider the gluon GPDs because their consideration is very close in spirit to that of the quark singlet GPDs. For GPDs we use notations of X. Ji (see e.g. [7]). Generically GPDs are functions of three variable (the fourth is log-dependence on the scale) $H(x, \xi, t)$, to simplify notations we shall omit the variable $t$ in the case when a quantity is assumed to be at $t = 0$ or when the $t$-dependence is irrelevant for the discussion.

Digression about polynomiality of Mellin moments and D-term

In this section we do not write explicitly the $t$ dependence of the GPDs as irrelevant for the present discussion but we always assume it. One of the non-trivial properties of the generalized parton distributions is the polynomiality of their Mellin moments. The polynomiality property means that [15]

$$
\int_{-1}^{1} dx \, x^N H(x, \xi) = h_0^{(N)} + h_2^{(N)} \xi^2 + \ldots + h_{N+1}^{(N)} \xi^{N+1},
$$

$$
\int_{-1}^{1} dx \, x^N E(x, \xi) = e_0^{(N)} + e_2^{(N)} \xi^2 + \ldots + e_{N+1}^{(N)} \xi^{N+1}.
$$

Due to the fact that the nucleon has spin 1/2, the coefficients in front of the highest power of $\xi$ for the functions $H$ and $E$ are related to each other [15]:

$$
e_{N+1}^{(N)} = -h_{N+1}^{(N)}.
$$

The polynomiality conditions (3) strongly restrict the class of functions of two variables $H(x, \xi)$ and $E(x, \xi)$. For example the conditions (3) imply that GPDs should satisfy the following integral constrains [9]:

$$
\int_{-1}^{1} \frac{dx}{x} \left[ H(x, \xi + xz) - H(x, \xi) \right] = - \int_{-1}^{1} \frac{dx}{x} \left[ E(x, \xi + xz) - E(x, \xi) \right] = z \sum_{n=0}^{\infty} h_{n+1}^{(n)} z^n. \ (5)
$$

Note that the skewedness parameter $\xi$ enters the lhs of this equation, whereas the rhs of the equation is $\xi$-independent. Therefore this $\xi$-independence of the above integrals is a criterion of whether functions $H(x, \xi)$, $E(x, \xi)$ satisfy the polynomiality conditions
Simultaneously these integrals are generating functions for the highest coefficients \( h_{N+1}^{(N)} \). Also the condition (3) shows that there are nontrivial functional relations between functions \( H(x, \xi) \) and \( E(x, \xi) \), more on this see below.

An elegant possibility to implement the polynomiality conditions (3) for the GPDs is to use the double distributions [11, 12, 13]. In this case the generalized distributions are obtained as a one-dimensional section of the two-variable double distributions \( F(\beta, \alpha) \) and \( K(\beta, \alpha) \):

\[
H(x, \xi) = \int_{-1}^{1} d\beta \int_{-1+|\beta|}^{1-|\beta|} d\alpha \delta(x - \beta - \alpha \xi) F(\beta, \alpha) ,
\]

and an analogous formula for the GPD \( E(x, \xi) \):

\[
E(x, \xi) = \int_{-1}^{1} d\beta \int_{-1+|\beta|}^{1-|\beta|} d\alpha \delta(x - \beta - \alpha \xi) K(\beta, \alpha) .
\]

[Recently in refs. [17, 18] the inversion formula has been derived.

It is easy to check that the GPDs obtained by reduction from the double distributions satisfy the polynomiality conditions (3) but always with \( h_{N+1}^{(N)} = e_{N+1}^{(N)} = 0 \), i.e. the highest power of \( \xi \) is absent. In other words the parametrization of GPDs in terms of double distributions is not complete. It can be completed adding the so-called D-term to eq. (3) [14]:

\[
H(x, \xi) = \int_{-1}^{1} d\beta \int_{-1+|\beta|}^{1-|\beta|} d\alpha \delta(x - \beta - \alpha \xi) F(\beta, \alpha) + \theta \left[ 1 - \frac{x^2}{\xi^2} \right] D \left( \frac{x}{\xi} \right) ,
\]

\[
E(x, \xi) = \int_{-1}^{1} d\beta \int_{-1+|\beta|}^{1-|\beta|} d\alpha \delta(x - \beta - \alpha \xi) K(\beta, \alpha) - \theta \left[ 1 - \frac{x^2}{\xi^2} \right] D \left( \frac{x}{\xi} \right) .
\]

Here \( D(z) \) is odd function having a support \(-1 \leq z \leq 1\). In the Mellin moments the D-term generates the highest power of \( \xi \):

\[
h_{N+1}^{(N)} = -e_{N+1}^{(N)} = \int_{-1}^{1} dz z^N D(z) .
\]

Note that for both GPDs \( H(x, \xi) \) and \( E(x, \xi) \) the D-term is the same, it contributes to both functions with opposite signs.

The D-term evolves with the change of the renormalization scale according to ERBL [19, 20] evolution equation. Hence it is useful to decompose the D-term in Gegenbauer series (eigenfunctions of the LO ERBL evolution equation):

\[
D(z) = (1 - z^2) \left[ d_1 C_1^{3/2}(z) + d_3 C_3^{3/2}(z) + d_5 C_5^{3/2}(z) + ... \right] ,
\]

\[
D_g(z) = \frac{3}{4}(1 - z^2) \left[ d_1^G C_1^{5/2}(z) + d_3^G C_2^{5/2}(z) + d_5^G C_4^{5/2}(z) + ... \right] .
\]
GPDs and “mechanical properties” of the nucleon

The $x$-moments of the GPDs $H$ and $E$ play a special role as they are related to the form factors of the symmetric energy momentum tensor. The nucleon matrix element of the traceless part of the symmetric energy momentum tensor is characterized by three scalar form factors [21, 15]. These form factors describe the mechanical structure of the spin−$\frac{1}{2}$ system–nucleon. At zero momentum transfer these three form factors can be related to three basic static mechanical characteristics of the composite system – nucleon. Two of them are energy-momentum and angular momentum carried by partons. The third one is related to the constants $d_1(0)$ and $d_1^G(0)$ in expansion of the D-term, see eqs. (11,12). These constants can be interpreted as the characteristic of the spatial distribution of the stress tensor or as of the spatial distribution of “forces” experienced by various species of partons inside of the nucleon. At $t = 0$ we can express the $x$-moments of the GPDs as follows [15]:

\[
\int_{-\frac{1}{2}}^{\frac{1}{2}} dx \ x (H(x, \xi) + E(x, \xi)) = 2J^Q ,
\]

\[
\int_{-\frac{1}{2}}^{\frac{1}{2}} dx \ H(x, \xi) = M^Q_2 + \frac{4}{5} d_1 \xi^2.
\]

Here $J^Q$ is a fraction of the nucleon angular momentum carried by all quarks, $d_1$ is the first Gegenbauer coefficient in the expansion of the D-term (11) and $M^Q_2$ is a momentum fraction carried by quarks and antiquarks in the nucleon:

\[
M^Q_2 = \sum_q \int_0^{1} dx \ x (q(x) + \bar{q}(x)) .
\]

Analogous expressions one can write for gluon GPDs. To see physics interpretation of nucleon “mechanical” characteristics $J^Q, M^Q_2$ and $d_1$ let us write their relations to the static energy-momentum tensor of the nucleon. In the Breit frame (in which the energy transfer $\Delta^0 = 0$) and non-relativistic limit ($\vec{\Delta}^2 \ll m_N^2$) we can define static energy-momentum tensor of the nucleon as (see e.g. [22]):

\[
T^Q_{\mu\nu}(\vec{r}) = \frac{1}{2m_N} \int \frac{d^3\Delta}{(2\pi)^3} e^{i\vec{\Delta}\cdot\vec{r}} \langle p', S | \hat{T}^Q_{\mu\nu}(0) | p, S \rangle ,
\]

where $\hat{T}^Q_{\mu\nu}(0)$ is the QCD operator of the symmetric energy momentum tensor of quarks, and $S$ is the polarization vector of the nucleon. Now it is easy to express the constants $J^Q, M^Q_2$ and $d_1$ in the following way:

\[
M^Q_2 = \frac{1}{m_N} \int d^3r T^Q_{00}(\vec{r})
\]

\[
J^Q = \int d^3r \ \varepsilon^{ijk} S_i r_j T^Q_{0k}(\vec{r}).
\]

*An analogous equation and the discussion below can be trivially extended for gluons as well
These relations illustrate the interpretation of $M_Q^2$ and $J_Q^2$ as fractions of energy-momentum and angular momentum of the nucleon carried by quarks and antiquarks. This follows from the fact that $T_{QQ}^Q(\vec{r})$ and $T_{QG}^Q(\vec{r})$ can be interpreted as a spatial distribution of energy and momentum carried by quarks correspondingly.

The constant $d_1$ can be expressed in terms of the stress tensor $T_{ij}(\vec{r})$:

$$d_1 = -\frac{m_N}{2} \int d^3r \ T_{ij}^Q(\vec{r}) \left( r^i r^j - \frac{1}{3} \delta^{ij} r^2 \right).$$

(16)

If one would consider the nucleon as a continuous medium than $T_{ij}^Q(\vec{r})$ would characterize the force experienced by quarks in an infinitesimal volume at distance $\vec{r}$ from the centre of the nucleon. More detailed distribution of forces one can obtain from the $t$-dependence of $d_1(t)$ because of relation

$$\int \frac{d^3 \Delta}{(2\pi)^3} e^{-i\vec{r} \cdot \vec{\Delta}} d_1(\Delta^2) \propto T_{ij}^Q(\vec{r}) \left( r^i r^j - \frac{1}{3} \delta^{ij} r^2 \right).$$

(17)

Obviously the sums $M_Q^2 + M_G^2 = 1$, $J_Q^2 + J_G^2 = 1/2$ and $d_1 + d_G^2 = d$ are scale independent. Two first sums corresponds to the total momentum and total angular momentum of the nucleon. The third constant $d$ using eq. (16) for the conserved total energy-momentum tensor can be represented in the form:

$$d = \frac{5m_N}{9} \int d^3r \ r^2 p(r),$$

(18)

where we introduced the following parametrization of the total (quarks+gluons) static stress tensor:

$$T_{ij}(\vec{r}) = s(r) \frac{T_{ij}^Q(r)}{r^2} + p(r) \delta_{ij}.$$

(19)

The function $s(r)$ and $p(r)$ are related to each other by conservation of the total energy-momentum tensor. The function $p(r)$ can be interpreted as the radial distribution of the “pressure” inside the nucleon. We note also that the $t$-dependence of the GPDs provides us with more detailed spatial images of the nucleon, see e.g. [23, 24, 25].

The estimate which is based on the calculation of GPDs in the chiral quark soliton model [27] at a low normalization point $\mu \approx 0.6$ GeV, gives [30] rather large and negative value of $d_1 \approx -4.0$. The negative values of this constant has a deep relation to the spontaneous breaking of the chiral symmetry in QCD, see [26, 30, 31].

Partial wave decomposition of the GPDs

Here we discuss the decomposition of GPDs in $t$-channel partial waves. Such kind of decomposition is useful for understanding of physical mechanism contributing to generalized
parton distributions. In a sense we attempt to model GPDs not by “deforming” ("skewing") the forward parton distributions but rather representing the GPDs as an infinite sum of the distribution amplitudes in the t-channel. Using the analytical continuation of the t-channel exchange representation we shall try to relate two “dual” approaches to the GPDs.

The partial wave decomposition in the t-channel for singlet GPDs $H(x, \xi, t)$ and $E(x, \xi, t)$ can be written as the following formal series [26]:

$$H(x, \xi, t) = \sum_{n=1}^{\infty} \sum_{l=0}^{n-1} B_{nl}(t) \theta \left(1 - x^2 \xi^2 \right) \left(1 - x^2 \xi^2 \right) C_n^{3/2} \left(\frac{x}{\xi} \right) P_l \left(\frac{1}{\xi} \right), \quad (20)$$

$$E(x, \xi, t) = \sum_{n=1}^{\infty} \sum_{l=0}^{n-1} C_{nl}(t) \theta \left(1 - x^2 \xi^2 \right) \left(1 - x^2 \xi^2 \right) C_n^{3/2} \left(\frac{x}{\xi} \right) P_l \left(\frac{1}{\xi} \right). \quad (21)$$

Here the index $l$ corresponds to an exchange in the t–channel with the orbital momentum $l$, because in the hard regime the t-channel scattering angle $\theta_t$ is related to $\xi$ via $\cos \theta_t = 1/\xi$. The coefficients $B_{nl}(t)$ and $C_{nl}(t)$ are generalized form factors. Note that for the fixed power of the Gegenbauer polynomial $n$ the orbital momentum can reach maximally the value $l = n + 1$, this is the consequence of the Lorentz invariance [26]. The formal series (20) corresponds to analytical continuation (crossing) of the corresponding expansion for the generalized distribution amplitudes [41] entering the description of hard processes like $\gamma^* \gamma \rightarrow h \bar{h}$, see discussion in refs. [26, 18].

Below we shall discuss mostly the GPD $H(x, \xi, t)$ the expressions for $E(x, \xi, t)$ are obtained by a trivial generalization. The form factors $B_{nn+1}(t = 0)$ at zero momentum transfer are fixed in terms of the Mellin moments of usual singlet quark distributions [26]

$$B_{nn+1}(t = 0) = \frac{2n+3}{2(n+2)} \int_0^1 dx \ x^n \left(q(x) + \bar{q}(x)\right). \quad (22)$$

The other coefficients $B_{nl}$ with $l \leq n − 1$ are not related to the forward distribution and thus are “truly non-forward” quantities.

Each term in the sum (20) has a support $-\xi < x < \xi$, however this does not imply that the GPD $H(x, \xi, t)$ has the same support because generically the sum (20) is divergent at fixed $x$ and $\xi$. In the formal solution (21) each individual term of the series gives the contribution to the amplitude which behaves as $1/\xi^l$ at small $\xi$ what would imply the violation of unitarity for the hard exclusive reactions. This “disaster” is avoided through the fact that the series is divergent and one can not calculate the asymptotic behaviour term by term. This situation is similar to duality idea: the entire generalized and usual quark distributions are given by infinite sum over t–channel exchanges.

The formal representation (21) can be equivalently rewritten as the convergent series:

$$H(x, \xi, t) = (1 - x^2) \sum_{n=1}^{\infty} A_n(\xi, t) C_n^{3/2}(x), \quad (23)$$

† Actually this formula can be effectively used for numerical construction of GPDs from the coefficients $B_{nl}$.
where

\[
A_n(\xi, t) = -\frac{2n + 3}{(n+1)(n+2)} \sum_{\substack{p=1 \\ \text{odd}}}^{n} \xi \frac{(p+1)(p+2)}{2p+3} \sum_{\substack{l=0 \\ \text{even}}}^{p+1} B_{pl}(t) P_l \left(\frac{1}{\xi}\right). \tag{24}
\]

Here \(R_{np}(\xi)\) are polynomials in \(\xi\) of the order \(n\) introduced in ref. [28, 29]:

\[
R_{np}(\xi) = (-1)^{n+p} \frac{\Gamma \left(\frac{3}{2} - \frac{p+1}{2}\right)}{\Gamma \left(\frac{p+1}{2} + 1\right) \Gamma \left(\frac{3}{2} + p\right)} \xi^{p} F_{1} \left(\frac{p}{2} - \frac{n}{2} - \frac{3}{2} + \frac{n}{2} + \frac{p}{2} + \frac{5}{2}; p; \xi^{2}\right). \tag{25}
\]

In the series (23) at small \(\xi\) the terms with \(l = n+1\) are dominant, so that in the limit \(\xi \to 0\) the forward distribution is recovered from eq. (23). Corrections to the forward limit of order \(\xi^{2}\) at fixed \(x\) is completely determined by orbital momenta of \(l = n+1\) and \(l = n-1\). We see that the smaller skewedness \(\xi\) the less important the deviations of the orbital momentum \(l\) from its maximal value of \(n+1\). We shall use this fact in next section to perform summation over orbital momenta.

From either (20) or (23) we can easily obtain the partial wave decomposition of the Mellin moments [26]:

\[
\int_{-1}^{1} dx \, x^{N} H(x, \xi, t) = \xi^{N+1} \sum_{\substack{n=1 \\ \text{odd}}}^{N} \sum_{\substack{p=0 \\ \text{even}}}^{n-1} B_{np}(t) P_{l} \left(\frac{1}{\xi}\right) \frac{\Gamma \left(\frac{3}{2}\right) \Gamma(N+1)(n+1)(n+2)}{2^{\nu} \Gamma \left(\frac{N-n+1}{2}\right) \Gamma \left(\frac{N+n+5}{2}\right)}, \tag{26}
\]

which obviously satisfies the polynomiality condition (3). For the lowest \(N = 1\) moment we have:

\[
\int_{-1}^{1} dx \, x \, H(x, \xi, t) = \frac{6}{5} \left[ B_{12}(t) - \frac{1}{3} (B_{12}(t) - 2B_{10}(t)) \xi^{2} \right], \tag{27}
\]

\[
\int_{-1}^{1} dx \, x \, E(x, \xi, t) = \frac{6}{5} \left[ C_{12}(t) - \frac{1}{3} (C_{12}(t) - 2C_{10}(t)) \xi^{2} \right], \tag{28}
\]

which with help of relations (24) and (13) gives at \(t = 0\):

\[
\int_{-1}^{1} dx \, x \, H(x, \xi) = M_{2}^{Q} - \frac{1}{3} \left( M_{2}^{Q} - \frac{12}{5} B_{10}(0) \right) \xi^{2}, \tag{29}
\]

\[
\int_{-1}^{1} dx \, x \, E(x, \xi) = 2J^{Q} - M_{2}^{Q} - \frac{1}{3} \left( 2J^{Q} - M_{2}^{Q} - \frac{12}{5} C_{10}(0) \right) \xi^{2}. \tag{30}
\]

From this expression we can easily obtain the value of the first Gegenbauer coefficient of the D-term \(d_1\) at \(t = 0\):

\[
d_1 = -\frac{5}{12} M_{2}^{Q} + B_{10}(0). \tag{31}
\]

Equivalently we can extract the \(d_1\) using the fact that the D-term is the same (up to the sign) for \(H\) and \(E\):

\[
d_1 = \frac{5}{12} \left( 2J^{Q} - M_{2}^{Q} \right) - C_{10}(0). \tag{32}
\]
Comparing eq. (31) with (32) we obtain new representation for the Ji’s sum rule in terms of S-wave exchanges:

\[ J^Q = \frac{6}{5} (B_{10}(0) + C_{10}(0)) , \] (33)

which should be contrasted with the original formulation of this sum rule done in terms of D-wave exchanges:

\[ J^Q = \frac{3}{5} (B_{12}(0) + C_{12}(0)) . \] (34)

We see that the \( J^Q \) can be extracted from the S-wave exchange in the t-channel. This is not surprising because between functions \( H(x, \xi) \) and \( E(x, \xi) \) there are nontrivial functional relations, see e.g. eq. (5). Generically for the \( N \)-th Gegenbauer coefficient of the D-term we have two equivalent representations

\[ d^N(t) = \sum_{l=0}^{N+1} \text{even} \ B_{Nl}(t) \frac{(-1)^{l/2} \Gamma \left( \frac{l}{2} + \frac{1}{2} \right)}{\Gamma \left( \frac{l}{2} + 1 \right) \Gamma \left( \frac{1}{2} \right)} , \] (35)

\[ d^N(t) = -\sum_{l=0}^{N+1} \text{even} \ C_{Nl}(t) \frac{(-1)^{l/2} \Gamma \left( \frac{l}{2} + \frac{1}{2} \right)}{\Gamma \left( \frac{l}{2} + 1 \right) \Gamma \left( \frac{1}{2} \right)} . \] (36)

This implies the following relations between coefficients \( B_{Nl}(t) \) and \( C_{Nl}(t) \):

\[ \sum_{l=0}^{N+1} \text{even} \ B_{Nl}(t) \frac{(-1)^{l/2} \Gamma \left( \frac{l}{2} + \frac{1}{2} \right)}{\Gamma \left( \frac{l}{2} + 1 \right) \Gamma \left( \frac{1}{2} \right)} = -\sum_{l=0}^{N+1} \text{even} \ C_{Nl}(t) \frac{(-1)^{l/2} \Gamma \left( \frac{l}{2} + \frac{1}{2} \right)}{\Gamma \left( \frac{l}{2} + 1 \right) \Gamma \left( \frac{1}{2} \right)} . \] (37)

Such kind of relations is useful for modeling of GPDs.

The expression of the \( J^Q \) in terms of S-wave t-channel exchanges (33) is useful because it allows us to analyze \( J^Q \) using various models and methods of low energy physics.

### Summing up the partial waves

The partial wave decomposition of GPDs discussed in previous section can be used to write a parametrization of the GPDs in terms of “forward like” functions. To do this we introduce a set of functions whose Mellin moments generate the coefficients \( B_{nl}(t) \) and \( C_{nl}(t) \)

\[ B_{n+1-k}(t) = \int_0^1 dx \ x^n \ Q_k(x, t) , \] (38)

\[ C_{n+1-k}(t) = \int_0^1 dx \ x^n \ R_k(x, t) , \] (39)

where index \( k \) (always even number!) characterizes the deviation of the orbital momentum from the maximally possible value of \( l = n + 1 \). Note that the LO evolution of the functions \( Q_k(x, t) \) and \( R_k(x, t) \) is governed by the DGLAP evolution equation. Since
$B_{nn+1}(0)$ is fixed by the Mellin moments of forward distributions, see eq. (22), the function $Q_0(x, t = 0)$ is related to forward distributions:

\[
Q_0(x) = \left[ (q(x) + \bar{q}(x)) - \frac{x}{2} \int_x^1 \frac{dz}{z^2} (q(z) + \bar{q}(z)) \right].
\] (40)

We sequence of functions $Q_0(x, t), Q_2(x, t), Q_4(x, t), \ldots$ is introduced in such a way that the higher functions (with high index $k$) are more and more suppressed for small skewedness parameter $\xi$.

Using the methods developed in refs. [31, 32] we can derive the following integral transformation relating GPDs $H(x, \xi, t)$ and functions $Q_k(y, t)$:

\[
H(x, \xi, t) = \sum_{l=0}^{\infty} \left\{ \frac{\xi^k}{2} \left[ H^{(k)}(x, \xi, t) - H^{(k)}(-x, \xi, t) \right] \right. \\
+ \left. \left. \left( 1 - \frac{x^2}{\xi^2} \right) \theta(\xi - |x|) \sum_{l=1 \text{ odd}}^{k-3} C_{k-l-2}^{3/2} \left( \frac{x}{\xi} \right) P_l \left( \frac{1}{\xi} \right) \int_0^1 dy y^{k-l-2} Q_k(y, t) \right\},
\] (41)

where the function $H^{(k)}(x, \xi, t)$ is given as the following integral transformation:

\[
H^{(k)}(x, \xi, t) = \theta (x > \xi) \frac{1}{\pi} \int_{y_0}^1 dy y \left[ \left( 1 - y \frac{\partial}{\partial y} \right) Q_k(y, t) \right] \int_{s_1}^{s_2} ds \frac{x_s^{1-k}}{\sqrt{x_s^2 - 2x_s - \xi^2}} \\
+ \theta (x < \xi) \frac{1}{\pi} \int_0^1 dy y \left[ \left( 1 - y \frac{\partial}{\partial y} \right) Q_k(y, t) \right] \int_{s_1}^{s_3} ds \frac{x_s^{1-k}}{\sqrt{x_s^2 - 2x_s - \xi^2}} \] (42)

Here $x_s = 2 \frac{\xi - y}{(1 + x^2)y}$ and integration limits $s_1, s_2, s_3$ and $y_0$ are given by the following expressions:

\[
s_1 = \frac{1}{y\xi} \left[ 1 - \sqrt{1 - \xi^2} - \sqrt{2 \left( 1 - xy \right) \left( 1 - \sqrt{1 - \xi^2} - \xi^2 \left( 1 - y^2 \right) \right)} \right],
\]

\[
s_2 = \frac{1}{y\xi} \left[ 1 - \sqrt{1 - \xi^2} + \sqrt{2 \left( 1 - xy \right) \left( 1 - \sqrt{1 - \xi^2} - \xi^2 \left( 1 - y^2 \right) \right)} \right],
\]

\[
s_3 = \frac{1}{y\xi} \left[ 1 + \sqrt{1 - \xi^2} - \sqrt{2 \left( 1 - xy \right) \left( 1 + \sqrt{1 - \xi^2} - \xi^2 \left( 1 - y^2 \right) \right)} \right],
\]

\[
y_0 = \frac{1}{\xi^2} \left[ x \left( 1 - \sqrt{1 - \xi^2} \right) + \sqrt{\left( 1 - \sqrt{1 - \xi^2} \right) \left( 1 - \xi^2 \right) \left( 2 \left( 1 - \sqrt{1 - \xi^2} \right) - \xi^2 \right)} \right].
\] (43)

Actually the integral transformation (42) can be written compactly as:

\footnote{The discussion below is applied to the functions $R_k(x, t)$ as well}

\footnote{Remember that here we discuss only the singlet ($C = +1$) GPDs, collection of formulas for nonsinglet ($C = -1$) GPDs is given in the Appendix A}
\[ H^{(k)}(x, \xi, t) = \frac{1}{\pi} \int_0^1 \frac{dy}{y} \left[ \left( 1 - y \frac{\partial}{\partial y} \right) Q_k(y, t) \right] \int ds \frac{x_s^{1-k}}{x_s^2 - 2x_s - \xi^2} \theta(x_s^2 - 2x_s - \xi^2). \] (44)

Derivation of eqs. (42, 44) is given in the Appendix B. Also we note that the numerical realization of these equations is stable and fast, it takes fraction of a second on a PC to compute GPDs for given function \( Q_k(x, t) \).

The simple analytical application of these integral transformations is to estimate the small-\( \xi \), Regge-type, behaviour of the imaginary part of the leading order amplitude proportional to \( \sum_k H^{(k)}(\xi, \xi) \). Taking the power-like parametrization for the function \( Q_k(x) \) in the region of small \( y - Q_k(y) \sim y^{-\alpha} \), we get (for \( \alpha > k - 1/2 \))

\[ \xi^k H^{(k)}(\xi, \xi) \sim \frac{1}{\pi} \left( \frac{\xi}{2} \right)^{k-\alpha} \frac{\Gamma(1/2)\Gamma(\alpha - k + 1/2)}{\Gamma(\alpha - k + 1)}, \] (45)

which is in an agreement with the result of Ref. [31, 32] for \( k = 0 \). Also we see that for the case when the \( \alpha \) does not increase strongly with increasing of \( k \) the leading term is determined by the functions \( Q_0(x) \) which is completely fixed by the forward distributions.

The original idea of Refs. [32, 31] to relate the GPDs to functions which are evolved according to usual DGLAP equation, as pointed out in Refs. [34, 16], implies nontrivial (\( \xi \) dependent) constrains for the support of the “effective forward functions”. This makes the parametrization of these functions almost impractical. Our construction, given by eq. (42), seems to be free of this problem.

The leading order amplitude of hard exclusive reactions is expressed in terms of the following elementary amplitude:

\[ A(\xi, t) = \int_0^1 dx \frac{1}{\xi + i0} + \frac{1}{\xi - i0} \left[ H(x, \xi, t) \right]. \] (46)

Now we can express the amplitudes in terms of “forward-like” functions \( Q_k(x) \). For this we substitute the formal series representation for GPDs (20) into eq. (46) and obtain the partial wave decomposition of the amplitude in the \( t \)-channel:

\[ A(\xi, t) = -2 \sum_{n=1}^{\infty} \sum_{l=0}^{n+1, even} B_{nl}(t) P_l \left( \frac{1}{\xi} \right). \] (47)

Substituting into this equation the expression for the coefficients \( B_{nl}(t) \) in terms of functions \( Q_k(x, t) \) (39), we can sum up the partial waves with the result:

\[
A(\xi, t) = -\int_0^1 dx \sum_{k=0}^{\infty} x^k Q_k(x, t) \left[ \frac{1}{\sqrt{1 - \frac{2x}{\xi} + x^2}} + \frac{1}{\sqrt{1 + \frac{2x}{\xi} + x^2}} - 2 \delta_{k0} \right] \] (48)

\footnote{For the case of \( \alpha < k - 1/2 \) we obtain that \( H^{(k)}(\xi, \xi) \sim \sqrt{\xi} \) with the coefficient which depends also on behaviour of the function \( Q_k(y) \) at large \( y \).}

\footnote{We restrict ourselves to the singlet or even signature amplitudes only. For the nonsinglet (odd signature) amplitude see Appendix A.}
Using this relation we can write down explicit expressions for the real and imaginary parts of the amplitudes:

\[
\begin{align*}
\text{Im } A(\xi, t) &= -\int_{1-\sqrt{1-\xi^2}}^{1} \frac{dx}{x} \sum_{k=0}^{\infty} x^k Q_k(x, t) \left[ \frac{1}{\sqrt{1 - \frac{2x}{\xi} - x^2}} \right] , \\
\text{Re } A(\xi, t) &= -\int_{0}^{1-\sqrt{1-\xi^2}} \frac{dx}{x} \sum_{k=0}^{\infty} x^k Q_k(x, t) \left[ \frac{1}{\sqrt{1 - \frac{2x}{\xi} + x^2}} + \frac{1}{\sqrt{1 + \frac{2x}{\xi} + x^2}} - 2 \delta_{k0} \right] \\
&\quad - \int_{1-\sqrt{1-\xi^2}}^{1} \frac{dx}{x} \sum_{k=0}^{\infty} x^k Q_k(x, t) \left[ \frac{1}{\sqrt{1 + \frac{2x}{\xi} + x^2}} - 2 \delta_{k0} \right].
\end{align*}
\] (49)

These expressions would allow us to analyze the experimental data on hard exclusive reactions in terms of “forward-like” functions \(Q_k(x, t)\) and \(R_k(x, t)\). Among these functions only \(Q_0(x)\) is fixed in terms of forward distributions, see eq. (40). Note also that the smaller \(\xi\) the more suppressed the contribution of the functions \(Q_k(x, t)\) and \(R_k(x, t)\) with higher \(k\) to the amplitude. We also note that from point of view of practical applications the numerical calculations of the amplitude according our formulae (49) is more stable and faster than calculations of more singular integral (46). From expression (49) for the imaginary part of the amplitude (corresponding to \(-\pi H(\xi, \xi, t)\)) we see that for its calculation we need to know the functions \(Q_k(x)\) (and hence forward parton distributions) for \(x \geq (1 - \sqrt{1 - \xi^2})/\xi > \xi/2\). In this way the problem (discussed in refs. [35]) that in the parametrization of GPDs in terms of double distributions one needs to sample forward distributions down to very small (unmeasured) values of \(x\) is solved. In our parametrization we see explicitly that the contribution to the amplitude of very small \(x\) is strongly suppressed as it should be from physics point of view.

From the expressions (49) one can easily obtain the behaviour of the amplitude at small \(\xi\). Indeed, let us assume that at small \(x\) we have \(Q_k(x, t) \sim x^{-\alpha_k}\), then the contribution of each individual function \(Q_k(x, t)\) to the amplitude has the form:

\[
\begin{align*}
\text{Im } A^{(k)}(\xi, t) &\sim -\left(\frac{\xi}{2}\right)^{k-\alpha_k} \frac{\Gamma(1/2) \Gamma(\alpha_k - k + 1/2)}{\Gamma(\alpha_k - k + 1)} , \\
\text{Re } A^{(k)}(\xi, t) &\sim \text{Im } A^{(k)}(\xi, t) \tan \left( \frac{\pi(\alpha_k - k - 1)}{2} \right).
\end{align*}
\] (50) (51)

The first equation coincides with the small \(\xi\) behaviour of \(-\pi H^{(k)}(\xi, \xi, t)\) (see eq. (45)) as it should be. The second equation reproduces the result of general dispersion relations for the even signature scattering amplitudes at high energies [33]. Also we see that if \(\alpha_k\) does not grow drastically with \(k\), the leading contribution comes from the function \(Q_0(x)\) which is fixed in terms of the forward parton distributions.

To derive eqs. (50,51) we note that at small values of the skewedness parameter \(\xi\) the expressions for the real and imaginary part of the amplitude (49) can be rewritten for the case \(\alpha_k - k > 1/2\) as (after rescaling of the integration variables \(2x/\xi \to x\):
Performing integrals in the above expressions we obviously obtain eqs. (50, 51).

Using eqs. (49) one can try to fit experimental data on hard exclusive processes adopting a simple parametrization of the functions $Q_k(x, t)$ (for $k \geq 2$) borrowed from analysis of the DIS data:

$$Q_k(x, t) = N_k \frac{1}{x^{\alpha_k}} (1 - x)^{\beta_k} \left(1 + \gamma_k \sqrt{x} + \delta_k x\right),$$

and analogous form for the functions $R_k(x)$. Where, in principle, all parameters $\alpha_k, \beta_k$, etc. are functions of the momentum transfer squared $t$. Introduced parameters for $k = 0, 2$ at $t = 0$ can be related to the basic mechanical characteristics of the nucleon–$M_Q^2, J_Q$ and $d_1$ by:

$$\int_0^1 dx x R_0(x) = \frac{5}{6} (2J_Q - M_Q^2) - d_1.$$
of experimental data. The most interesting “mechanical” properties of the nucleon are contained in the lowest functions for \( k = 0, 2 \). In this section we discuss possible minimal set of the “forward-like” functions. Clearly the choice with only \( k = 0 \) is too restrictive, in particular, such a choice would imply that \( H(x, \xi, t) = -E(x, \xi, t) \) which is too strong constrain. One may try to fit the data with functions \( Q_0(x, t) \) and \( Q_2(x, t) \) (and \( R_0(x, t), R_2(x, t) \)) one of those is fixed by the forward parton distributions. In the model with only two types of generating functions \( Q_0(x, t) \) and \( Q_2(x, t) \) (and \( R_0(x, t), R_2(x, t) \)) one can easily derive from the constrain \( (37) \) the functional relation for the function \( R_2(x, t) \):

\[
R_2(x, t) = -Q_2(x, t) + Q_0(x, t) + R_0(x, t) - \int_x^1 \frac{dz}{z} \left[ Q_0(z, t) + R_0(z, t) \right].
\]  

(56)

We see that in such model one has only two new “forward-like” functions \( Q_2(x, t) \) and \( R_0(x, t) \), \( Q_0(x, t) \) is essentially fixed by the forward distributions, see eq. \( (40) \)†† and \( R_2(x, t) \) by the relation \( (56) \).

In this paper we do not present our numerical results for GPDs in the “minimal model” just discussed, the corresponding results we shall present elsewhere. Our main aim here was to set the theoretical framework for modeling of GPDs.

**Conclusions and outlook**

We analyzed the representation of the generalized parton distributions (GPDs) as the infinite sum over distribution amplitudes in the \( t \)-channel. Such representation is close in its spirit to the duality idea–the entire parton distribution is obtained as an infinite sum over distribution amplitudes in the \( t \)-channel. On basis of this dual representation we suggested a parametrization of the GPDs which satisfy automatically all general constraints–forward limit, polynomiality, etc. In this parametrization the GPDs and the leading order amplitudes are expressed in terms of “forward-like” functions \( Q_k(x, t) \) (\( R_k(x, t) \)). The dependence of these functions on the scale in the leading order is governed by the DGLAP evolution equation. The index \( k \) which enumerates the set of functions \( Q_k(x, t) \) (\( R_k(x, t) \)) describes the deviation of the orbital momentum \( l \) in the \( t \)-channel from its maximal value of \( l = n + 1 \) at fixed conformal spin \( n \) of the twist-2 operator. The basic mechanical characteristics of the nucleon–momentum and angular momentum fractions carried by quarks and additionally the radial distribution of forces experienced by quarks in the nucleon–are contained in the lowest functions \( Q_0(x, t), Q_2(x, t) \) (\( R_0(x, t), R_2(x, t) \)).

With our parametrization of GPDs still there are several points which should be worked out further:

- To prove that suggested parametrization covers all possible forms of GPDs and derive the inversion formula expressing functions \( Q_k \) and \( R_k \) through GPDs \( H \) and \( E \).
- To extend the analysis to the next to leading order. On NLO evolution of GPDs see, e.g. ref. \([12]\).††

†† An analogous expression can be also written for \( R_0(x) \) where in rhs the role of forward distribution is played by \( E(x, \xi = 0) \).
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Appendix A

In the main text we discussed only the singlet \((C = +1)\) GPDs which are reduced to 
\[ q(x) + \bar{q}(x) \] in the forward limit. In this Appendix we collect formulae for the nonsiglet \((C = -1)\) GPDs which in the forward limit correspond to 
\[ q(x) - \bar{q}(x) \]. Below we simply list the corresponding formulae, skipping the discussion.

**Partial wave decomposition:**

\[
H(x, \xi, t) = \sum_{n=0}^{\infty} \sum_{l=1, \text{ odd}}^{n+1} B_{nl}(t) \theta \left(1 - \frac{x^2}{\xi^2}\right) \left(1 - \frac{x^2}{\xi^2}\right) C_{n/2} \left(\frac{x}{\xi}\right) P_l \left(\frac{1}{\xi}\right), \quad (57)
\]

\[
E(x, \xi, t) = \sum_{n=0}^{\infty} \sum_{l=1, \text{ odd}}^{n+1} C_{nl}(t) \theta \left(1 - \frac{x^2}{\xi^2}\right) \left(1 - \frac{x^2}{\xi^2}\right) C_{n/2} \left(\frac{x}{\xi}\right) P_l \left(\frac{1}{\xi}\right). \quad (58)
\]

**Mellin moments:**

\[
\int_{-1}^{1} dx \, x^n H(x, \xi, t) = \xi^{N+1} \sum_{n=0}^{N} \sum_{l=0, \text{ odd}}^{n+1} B_{nl}(t) \, P_l \left(\frac{1}{\xi}\right) \frac{\Gamma \left(\frac{3}{2}\right)}{2^n \Gamma \left(N+n+\frac{5}{2}\right)} \frac{\Gamma \left(N+n+\frac{5}{2}\right)}{\Gamma \left(N+n+1\right)}. \quad (59)
\]

The lowest \(x^0\) Mellin moments of GPDs are related to the electromagnetic form factors of the nucleon

\[
\int_{-1}^{1} dx \, H(x, \xi, t) = F_1(t) = \frac{2}{3} B_{01}(t),
\]

and

\[
\int_{-1}^{1} dx \, E(x, \xi, t) = F_2(t) = \frac{2}{3} C_{01}(t),
\]

Generating functions \(Q_k\) and \(R_k\) are introduced in the same way as in eqs. \((39)\) only in the nonsinglet case the number \(n\) is even. The function \(Q_0(x, t = 0)\) is related to the nonsinglet forward quark distribution:

\[
Q_0(x) = \left[(q(x) - \bar{q}(x)) - \frac{x}{2} \int_x^1 \frac{dz}{z^2} (q(z) - \bar{q}(z))\right]. \quad (60)
\]
Integral transformations $Q_k \rightarrow H$.

\[
H(x, \xi, t) = \sum_{k=0 \text{ even}}^{\infty} \left\{ \frac{\xi^k}{2} \right\} \left[ H^{(k)}(x, \xi, t) + H^{(k)}(-x, \xi, t) \right] + \left( 1 - \frac{x^2}{\xi^2} \right) \theta(|x| - \xi) \sum_{l=0 \text{ even}}^{k-2} C_{3/2}^{k-l-2} \left( \frac{x}{\xi} \right) P_l \left( \frac{1}{\xi} \right) \int_{0}^{1} dy y^{k-l-2} Q_k(y, t) \right\},
\]

with $H^{(k)}$ given by eq. (42).

Amplitudes

The amplitude with $C = -1$ exchange in the $t$-channel (odd signature) defined as

\[
A(\xi, t) = \int_{0}^{1} dx H(x, \xi, t) \left[ \frac{1}{x - \xi + i0} - \frac{1}{x + \xi - i0} \right],
\]

can be expressed in terms of the nonsinglet functions $Q_k(x, t)$ as follows:

\[
\text{Re} \ A(\xi, t) = -\int_{1/\xi - \sqrt{1 - \xi^2}}^{1/\xi + \sqrt{1 - \xi^2}} \frac{dx}{x} \sum_{k=0}^{\infty} x^k Q_k(x, t) \left[ \frac{1}{\sqrt{1 - \frac{2\xi}{x} + x^2}} - \frac{1}{\sqrt{1 + \frac{2\xi}{x} + x^2}} \right] \\
+ \int_{1/\xi - \sqrt{1 - \xi^2}}^{1/\xi + \sqrt{1 - \xi^2}} \frac{dx}{x} \sum_{k=0}^{\infty} x^k Q_k(x, t) \frac{1}{\sqrt{1 + \frac{2\xi}{x} + x^2}},
\]

\[
\text{Im} \ A(\xi, t) = -\int_{1/\xi - \sqrt{1 - \xi^2}}^{1/\xi + \sqrt{1 - \xi^2}} \frac{dx}{x} \sum_{k=0}^{\infty} x^k Q_k(x, t) \left[ \frac{1}{\sqrt{\frac{2\xi}{x} - x^2 - 1}} \right].
\]

From these expressions we can derive analytically the results for the small $\xi$ behaviour of the amplitude for the case of $Q_k(x) \sim x^{-\alpha_k}$. The result is

\[
\text{Im} A^{(k)}(\xi, t) \sim -\left( \frac{\xi}{2} \right)^{k-\alpha_k} \frac{\Gamma(1/2)\Gamma(\alpha_k - k + 1/2)}{\Gamma(\alpha_k - k + 1)} \\
\text{Re} A^{(k)}(\xi, t) \sim \text{Im} A^{(k)}(\xi, t) \cot \left( \frac{\pi(\alpha_k - k - 1)}{2} \right).
\]

The first equation again coincides with the small $\xi$ behaviour of $-\pi H^{(k)}(\xi, \xi)$ as it should be. The second equation reproduces the well-known result of general dispersion relations for the odd signature scattering amplitudes at high energies.

Appendix B

Here we sketch the derivation of the eqs. (41,42,44). The basic relation we use is the following\(^{\dagger}\)

\(^{\dagger}\)We define the discontinuity as $\text{disc}_{z=x} f(z) = \frac{1}{\pi} [f(x - i0) - f(x + i0)]$. \]
disc_z = x \\
\frac{1}{y} \left( 1 + y \frac{\partial}{\partial y} \right) \int_{-1}^{1} ds \ z_s^{-N} = \theta \left( 1 - \frac{x^2}{\xi^2} \right) \left( 1 - \frac{x^2}{\xi^2} \right) \xi^{-N} y^{N-1} C_{N-1}^{3/2} \left( \frac{x}{\xi} \right). \quad (65)

Here

\[ z_s = 2 \frac{z - s \xi}{(1 - s^2)y}, \quad (66) \]

with \(0 < y < 1\). To prove eq. (65) we note that:

\[ \text{disc}_z z_s^{-N} = (-1)^{N-1} \frac{1}{\Gamma(N)} \delta^{(N-1)}(x_s), \quad (67) \]

with

\[ x_s = 2 \frac{x - s \xi}{(1 - s^2)y}. \quad (68) \]

This simple relation gives us

\[ \text{disc}_z \int_{-1}^{1} ds \ z_s^{-N} = (-1)^{N-1} \left( 1 - \frac{x^2}{\xi^2} \right) \frac{\theta}{2^N \xi^{N\Gamma(N)}} \left( \frac{\partial}{\partial s} \right)^{N-1} (1 - s^2) \Bigg|_{s=x/\xi}. \quad (69) \]

The step function in the above equation indicates that the zero of the \(\delta\)-function should be inside the interval \(-1 < s < 1\). Further with help of the identity:

\[ (1 + y \frac{\partial}{\partial y}) z_s^{-N} = (N + 1) z_s^{-N}, \quad (70) \]

and the Rodrigues formula for the Gegenbauer polynomials:

\[ (1 - x^2) C_{N-1}^{3/2} (x) = (-1)^{N-1} \frac{N + 1}{2^N \Gamma(N)} \left( \frac{\partial}{\partial x} \right)^{N-1} (1 - x^2)^N, \quad (71) \]

we arrive to the eq. (65).

Now we consider the following function:

\[ F^{(k)}(z, y) = \frac{1}{y} \left( 1 + y \frac{\partial}{\partial y} \right) \int_{-1}^{1} ds \ \xi^k z_s^{1-k} \left[ z_s^2 - 2 z_s + \xi^2 \right]^{-1/2}, \quad (72) \]

and let us compute its discontinuity with help of eq. (65) and generating function for the Legendre polynomials:

\[ \text{disc}_z F^{(k)}(z, y) = \left( 1 - \frac{x^2}{\xi^2} \right) \theta \left( 1 - \frac{x^2}{\xi^2} \right) \sum_i C_{k+l-1}^{3/2} \left( \frac{x}{\xi} \right) P_l \left( \frac{1}{\xi} \right) y^{k+l-1}. \quad (73) \]
In this expression we immediately recognize the integral kernel for the formal expansion
\[ H^{(k)}(x) = \left(1 - \frac{x^2}{\xi^2}\right) \theta\left(1 - \frac{x^2}{\xi^2}\right) \sum_l C_{k+l-1}^{3/2} \left(\frac{x}{\xi}\right) P_l\left(\frac{1}{\xi}\right) B_{k+l-1,l}, \]
with
\[ B_{k+l-1,l} = \frac{2}{3} \int dy \, y^{k+l-1} Q_k(y). \]

Now the trick is that we can compute the discontinuity of the function \( F^{(k)}(z, y) \) given by eq. (72) in a different way. Namely, we take contributions to the discontinuity from the cut \( 1 - \sqrt{1 - \xi^2} < z_s < 1 + \sqrt{1 - \xi^2} \) and from the poles at \( z_s = 0 \) for \( k \geq 2 \). The calculations are straightforward, the cut contribution gives us the expression (44), whereas the pole contribution provides us with the second line in eq. (41). Further, analyzing the solutions of the algebraic equation \( x_s^2 - 2x_s + \xi^2 = 0 \) we arrive at eq. (42).

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