The new “MUON G-2” result and supersymmetry

Manimala Chakraborti1,a, Sven Heinemeyer2,3,4,b, Ipsita Saha5,c

1 Astrocent, Nicolaus Copernicus Astronomical Center of the Polish Academy of Sciences, ul. Rektorska 4, 00-614 Warsaw, Poland
2 Instituto de Física Teórica (UAM/CSIC), Universidad Autónoma de Madrid, Cantoblanco, 28049 Madrid, Spain
3 Campus of International Excellence UAM+CSIC, Cantoblanco, 28049 Madrid, Spain
4 Instituto de Física de Cantabria (CSIC-UC), 39005 Santander, Spain
5 Kavli IPMU (WPI), UTIAS, University of Tokyo, Kashiwa, Chiba 277-8583, Japan

Received: 7 May 2021 / Accepted: 2 December 2021 / Published online: 17 December 2021
© The Author(s) 2021

Abstract The electroweak (EW) sector of the Minimal Supersymmetric Standard Model (MSSM), with the lightest neutralino as Dark Matter (DM) candidate, can account for a variety of experimental data. This includes the DM content of the universe, DM direct detection limits, EW SUSY searches at the LHC and in particular the so far persistent 3 $\sigma$ discrepancy between the experimental result for the anomalous magnetic moment of the muon, $(g - 2)_\mu$, and its Standard Model (SM) prediction. The recently published “MUON G-2” result is within 0.8 $\sigma$ in agreement with the older BNL result on $(g - 2)_\mu$. The combination of the two results was given as $a_\mu^{exp} = (11659206.1 \pm 4.1) \times 10^{-10}$, yielding a new deviation from the SM prediction of $\Delta a_\mu = (25.1 \pm 5.9) \times 10^{-10}$, corresponding to 4.2 $\sigma$. Using this improved bound we update the results presented in Chakraborti et al. (Eur Phys J C 80(10):984, 2020) and set new upper limits on the allowed parameters space of the EW sector of the MSSM. We find that with the new $(g - 2)_\mu$ result the upper limits on the (next-to-) lightest SUSY particle are in the same ballpark as previously, yielding updated upper limits on these masses of $\sim 750$ GeV. In this way, a clear target is confirmed for future (HL-)LHC EW searches, as well as for future high-energy $e^+e^-$ colliders, such as the ILC or CLIC.

1 Introduction

Searches for Beyond the Standard Model (BSM) particles are performed directly, such as at the LHC, or indirectly in low-energy experiments and via astrophysical measurements. Among the BSM theories under consideration the Minimal Supersymmetric Standard Model (MSSM) [1–4] is one of the leading candidates. Supersymmetry (SUSY) predicts two scalar partners for all SM fermions as well as fermionic partners to all SM bosons. Contrary to the case of the SM, in the MSSM two Higgs doublets are required. This results in five physical Higgs bosons instead of the single Higgs boson in the SM. These are the light and heavy $CP$-even Higgs bosons, $h$ and $H$, the $CP$-odd Higgs boson, $A$, and the charged Higgs bosons, $H^\pm$. The neutral SUSY partners of the (neutral) Higgs and electroweak (EW) gauge bosons gives rise to the four neutralinos, $\tilde{\chi}^0_{1,2,3,4}$. The corresponding charged SUSY partners are the charginos, $\tilde{\chi}^{\pm}_{1,2}$. The SUSY partners of the SM leptons and quarks are the scalar leptons and quarks (sleptons, squarks), respectively.

In Ref. [5] we performed an analysis taking into account all relevant data for the EW sector of the MSSM, assuming that the lightest SUSY particle (LSP) is given by the lightest neutralino, $\tilde{\chi}^0_1$, that makes up the full Dark Matter (DM) content of the universe [6,7].1 The experimental results comprised the direct searches at the LHC [9,10], the DM relic abundance [11], the DM direct detection (DD) experiments [12–14] and in particular the (then current) deviation of the anomalous magnetic moment of the muon [15,16].

Three different scenarios were analyzed, classified by the mechanism that brings the LSP relic density into agreement with the measured values. The scenarios differ by the Next-to-LSP (NLSP), where we investigated $\tilde{\chi}_1^\pm$-coannihilation, $\tilde{\tau}$-coannihilation with either “left-” or “right-handed” sleptons close in mass to the LSP, Case-L and Case-R, respectively. Using the then current bounds we found upper limits on the LSP masses. While for $\tilde{\chi}_1^\pm$-coannihilation this is $\sim 570$ GeV, for $\tilde{\tau}$-coannihilation Case-L $\sim 540$ GeV and for Case-R values up to $\sim 520$ GeV are allowed. Similarly, upper limits to masses of the coannihilating SUSY

1 More recently in Ref. [8] we updated the analysis using the DM relic density only as an upper bound.
particles are found as, \( m_{\tilde{H}^+} \lesssim 610 \text{ GeV}, m_{\tilde{l}_e} \lesssim 550 \text{ GeV}, \) \( m_{\tilde{l}_e} \lesssim 590 \text{ GeV} \). For the latter, in the \( \tilde{t}^-\)-coannihilation case-R, the upper limit on the lighter \( \tilde{t} \) is even lower, \( m_{\tilde{t}_2} \lesssim 530 \text{ GeV} \). The then current \( (g-2)_\mu \) constraint also yields limits on the rest of the EW spectrum, although much loser bounds were found. Previous articles with similar, but in general less advanced studies can be found in Refs. [17–36].

Recently the “MUON G-2” collaboration published the results (referred to as “FNAL” result) of their Run 1 data, which is within 0.8 \( \sigma \) in agreement with the older BNL result on \( (g-2)_\mu \). We combine the two results, assuming that they are uncorrelated. We analyze the impact of the combination of the Run 1 FNAL data with the previous BNL result on the allowed MSSM parameter space. The results will be discussed in the context of the upcoming searches for EW particles at the HL-LHC. We will also comment on the discovery prospects for these particles at possible future \( e^+e^- \) colliders, such as the ILC [37,38] or CLIC [38–41].

2 The model and the experimental constraints

2.1 The model

A detailed description of the EW sector of the MSSM can be found in Ref. [5]. Here we just list the relevant input parameters and masses that are relevant for our analysis. Throughout this paper we assume that all parameters are real, i.e. we have no \( \mathcal{CP} \)-violation.

The masses and mixings of the charginos and neutralinos are determined by \( U(1)_Y \) and \( SU(2)_L \) gaugino masses \( M_1 \) and \( M_2 \), the Higgs mixing parameter \( \mu \) and \( \tan \beta \), the ratio of the two vacuum expectation values (vevs) of the two Higgs doublets of MSSM, \( \tan \beta = v_2/v_1 \). The four neutralino masses are given as \( m_{\tilde{\chi}_1^0} < m_{\tilde{\chi}_2^0} < m_{\tilde{\chi}_3^0} < m_{\tilde{\chi}_4^0} \). Similarly the two chargino-masses are denoted as \( m_{\tilde{\chi}_1^+} < m_{\tilde{\chi}_2^+} \).

As argued in Ref. [5] it is sufficient for our analysis to focus on positive values for \( M_1, M_2 \) and \( \mu \).

For the sleptons, as in Ref. [5], we choose common soft SUSY-breaking parameters for all three generations, \( m_{\tilde{l}_e} \) and \( m_{\tilde{l}_\mu} \). We take the trilinear coupling \( A_l \) (\( l = e, \mu, \tau \)) to be zero for all the three generations of leptons. In general we follow the convention that \( l_1 (l_2) \) has the large “left-handed” (“right-handed”) component. Besides the symbols equal for all three generations, \( m_{\tilde{l}_e} \) and \( m_{\tilde{l}_\mu} \), we also explicitly use the scalar electron, muon and tau masses, \( m_{\tilde{\tau}_1}, m_{\tilde{\tau}_2}, m_{\tilde{\mu}_1}, \) and \( m_{\tilde{\mu}_2} \).

2.2 Relevant constraints

The experimental result for \( a_\mu := (g-2)_\mu /2 \) was so far dominated by the measurements made at the Brookhaven National Laboratory (BNL) [44], resulting in a world average of [45]

\[
ad_\mu^{\text{exp-BNL}} = (11659209.1 \pm 6.3) \times 10^{-10},
\]

combining statistical and systematic uncertainties. The SM prediction of \( a_\mu \) is given by [46] (based on Refs. [15,16,47–64]),

\[
ad_\mu^{\text{SM}} = (11659181.0 \pm 4.3) \times 10^{-10}.
\]

Comparing this with the current experimental measurement in Eq. (1) results in a deviation of

\[
\Delta a_\mu^{\text{old}} = (28.1 \pm 7.6) \times 10^{-10},
\]

corresponding to a 3.7 \( \sigma \) discrepancy.

Recently, the “MUON G-2” collaboration [65] at Fermilab published their Run 1 data [66]

\[
ad_\mu^{\text{exp-FNAL}} = (11659204.0 \pm 5.4) \times 10^{-10},
\]

being within 0.8 \( \sigma \) well compatible with the previous experimental result in Eq. (1). The combined new world average was announced as

\[
ad_\mu^{\text{exp}} = (11659206.1 \pm 4.1) \times 10^{-10}.
\]

Compared with the SM prediction in Eq. (2), one arrives at a new deviation of

\[
\Delta a_\mu = (25.1 \pm 5.9) \times 10^{-10},
\]

corresponding to a 4.2 \( \sigma \) discrepancy. We use this limit as a cut at the \( \pm 2 \sigma \) level. Here one should note that the new lower 2 \( \sigma \) limit is similar to the old one, which leads to the

\[\text{\footnotesize In Ref. [5] a slightly different value was used, with a negligible effect on the results.}\]
expectation that the previously upper bounds, see Ref. [5], are confirmed.

Recently a new lattice calculation for the leading order hadronic vacuum polarization (LO HVP) contribution to $a^{\mu}_{\mu}$ [67] has been reported, which, however, was not used in the new theory world average, Eq. (2) [46]. Consequently, we also do not take this result into account, see also the discussions in Refs. [5,67–71]. On the other hand, we are also aware that our conclusions would change substantially if the result presented in [67] turned out to be correct.

In MSSM the main contribution to $(g - 2)_{\mu}$ at the one-loop level comes from diagrams involving $\tilde{\chi}^{\pm}_{1} - \tilde{\nu}$ and $\tilde{\chi}^{0}_{1} - \tilde{\mu}$ loops. In our analysis the MSSM contribution to $(g - 2)_{\mu}$ at two loop order is calculated using GM2Calc [72], implementing two-loop corrections from [73–75] (see also [76,77]).

All other constraints are taken into account exactly as in Ref. [5]. These comprise:

- Vacuum stability constraints:
  All points are checked to possess a stable and correct EW vacuum, e.g. avoiding charge and color breaking minima. This check is performed with the public code Evade [78,79].

- Constraints from the LHC:
  All relevant EW SUSY searches are taken into account, mostly via CheckMATE [80–82], where many analysis had to be implemented newly [5].

- Dark matter relic density constraints:
  We use the latest result from Planck [11]. The relic density in the MSSM is evaluated with MicrOMEGAs [83–86].

- Direct detection constraints of Dark matter:
  We employ the constraint on the spin-independent DM scattering cross-section $\sigma_p^{SI}$ from XENON1T [12] experiment, evaluating the theoretical prediction for $\sigma_p^{SI}$ using MicrOMEGAs [83–86]. A combination with other DD experiments would yield only very slightly stronger limits, with a negligible impact on our results.

3 Parameter scan

We scan the relevant MSSM parameter space to obtain lower and upper limits on the relevant neutralino, chargino and slepton masses. As detailed in Ref. [5] three scan regions cover the relevant parameter space:

(A) bino/wino DM with $\tilde{\chi}^{\pm}_{1}$-coannihilation

\[
100 \text{ GeV} \leq M_1 \leq 1 \text{ TeV}, \quad M_1 \leq M_2 \leq 1.1M_1, \\
1.1M_1 \leq \mu \leq 10M_1, \quad 5 \leq \tan \beta \leq 60, \\
100 \text{ GeV} \leq m_{\tilde{t}_L} \leq 1 \text{ TeV}, \quad m_{\tilde{t}_R} = m_{\tilde{t}_L}. \tag{7}
\]

(B) bino DM with $\tilde{\chi}_{1}^{\pm}$-coannihilation

(B1) Case-L: SU(2)-doublet

\[
100 \text{ GeV} \leq M_1 \leq 1 \text{ TeV}, \quad M_1 \leq M_2 \leq 10M_1, \\
1.1M_1 \leq \mu \leq 10M_1, \quad 5 \leq \tan \beta \leq 60, \\
M_1 \text{ GeV} \leq m_{\tilde{t}_L} \leq 1.2M_1, \quad M_1 \leq m_{\tilde{t}_R} \leq 10M_1. \tag{8}
\]

(B2) Case-R: SU(2)-singlet

\[
100 \text{ GeV} \leq M_1 \leq 1 \text{ TeV}, \quad M_1 \leq M_2 \leq 10M_1, \\
1.1M_1 \leq \mu \leq 10M_1, \quad 5 \leq \tan \beta \leq 60, \\
M_1 \text{ GeV} \leq m_{\tilde{t}_R} \leq 1.2M_1, \quad M_1 \leq m_{\tilde{t}_L} \leq 10M_1. \tag{9}
\]

In all three scans we choose flat priors of the parameter space and generate $O(10^7)$ points. In order to obtain reliable upper limits on the various EW SUSY masses, we performed additional smaller scans targeting the highest allowed mass ranges. This will be visible in some of the plots as areas with higher point density. We stress that the point density itself does not have any physical meaning, but only enable us to reliably obtain the upper mass limits.

$M_A$ has also been set to be above the TeV scale. Consequently, we do not include explicitly the possibility of $A$-pole annihilation, with $M_A \sim 2m_{\tilde{t}^0}$. As we will briefly discuss below the combination of direct heavy Higgs-boson searches with the other experimental requirements constrain this possibility substantially. Similarly, we do not consider $h$- or $Z$-pole annihilation (see, e.g., Ref. [36]), as such a light neutralino sector likely overshoots the $(g - 2)_{\mu}$ contribution, see the discussion in Ref. [5].

Analysis flow

The data sample is generated by scanning randomly over the input parameter range mentioned above, using a flat prior for all parameters. We use SuSpect [87] as spectrum and SLHA file generator. The points are required to satisfy the $\tilde{\chi}^{\pm}_{1}$ mass limit from LEP [88]. The SLHA output files from SuSpect are then passed as input to GM2Calc and MicrOMEGAs for the calculation of $(g - 2)_{\mu}$ and the DM observables, respectively. The parameter points that satisfy the new $(g - 2)_{\mu}$ constraint, Eq. (6), the DM relic density and the direct detection constraints and the vacuum stability constraints as checked with Evade are then taken to the final step to be checked against the latest LHC constraints implemented in CheckMATE, as described in detail in Ref. [5]. The branching ratios of the relevant SUSY particles are computed using SDECAY [89] and given as input to CheckMATE.
4 Results

We present the results of the allowed parameter ranges in the three scenarios defined above. We follow the analysis flow as described above and denote the points surviving certain constraints with different colors:

- grey (round): all scan points.
- green (round): all points that are in agreement with \((g-2)_\mu\), taking into account the new limit as given in Eq. (6).
- blue (triangle): points that additionally give the correct relic density, see Sect. 2.2.
- cyan (diamond): points that additionally pass the DD constraints, see Sect. 2.2.
- red (star): points that additionally pass the LHC constraints, see Sect. 2.2.

4.1 Results in the three DM scenarios

We start in Fig. 1 with the results in the \(\tilde{\chi}_1^\pm\)-coannihilation scenario. In the \(m_{\tilde{\chi}_1^0}-m_{\tilde{\ell}_l}\) plane, shown in the upper left plot, by definition of \(\tilde{\chi}_1^\pm\)-coannihilation the points are clustered in the diagonal of the plane. One observes a clear upper limit on the (green) points allowed by the new \((g-2)_\mu\) result slightly below 700 GeV, which is similar to the previously obtained one in Ref. [5]. Applying the CDM constraints slightly reduce the upper limit to about 650 GeV, again similar as for the old \((g-2)_\mu\) result. Applying the LHC constraints, corresponding to the “surviving” red points (stars), does not yield a further reduction from above, but cuts always only points in the lower mass range. Thus, the new experimental data set an upper as well as a lower bound, yielding a clear search target for the upcoming LHC runs, and in particular for future \(e^+e^-\) colliders, as will be briefly discussed in Sect. 4.2.

The distribution of the lighter slepton mass (where it should be kept in mind that we have chosen the same masses for all three generations, see Sect. 2.1) is presented in the \(m_{\tilde{\chi}_1^0}-m_{\tilde{\ell}_l}\) plane, shown in the upper right plot of Fig. 1. The \((g-2)_\mu\) constraint places important constraints in this mass plane, since both types of masses enter into the contributing SUSY diagrams, see Sect. 2.2. The constraint is satisfied in a triangular region with its tip around \((m_{\tilde{\chi}_1^0}, m_{\tilde{\ell}_l}) \sim (700 \text{ GeV}, 800 \text{ GeV})\), compatible with the old limits. This is slightly reduced to \(\sim (650 \text{ GeV}, 700 \text{ GeV})\) when the DM constraints are taken in to account, as can be seen in the distribution of the blue, cyan and red points (triangle/diamond/star). The LHC constraints cut out lower slepton masses, going up to \(m_{\tilde{\ell}_l} \lesssim 500 \text{ GeV}\), as well as part of the very low \(m_{\tilde{\chi}_1^0}\) points nearly independent of \(m_{\tilde{\ell}_l}\). Details on these cuts can be found in Ref. [5]. Clearly visible in the highest \(m_{\tilde{\chi}_1^0}\) range as well as in the high \(m_{\tilde{\ell}_l}\) region is the higher point density due to the additional scan as described in Sect. 3.

We finish our analysis of the \(\tilde{\chi}_1^\pm\)-coannihilation case with the \(m_{\tilde{\chi}_1^0}-\tan \beta\) plane presented in the lower plot of Fig. 1. The \((g-2)_\mu\) constraint is fulfilled in a triangular region with largest neutralino masses allowed for the largest \(\tan \beta\) values (where we stopped our scan at \(\tan \beta = 60\)). In agreement with the previous plots, the largest values for the lightest neutralino masses allowed by all the constraints are \(\sim 650 \text{ GeV}\), compatible with the old \((g-2)_\mu\) limit [5]. The LHC constraints cut out points at low \(m_{\tilde{\chi}_1^0}\) nearly independent of \(\tan \beta\). Again, the areas with higher point density are plainly visible in the plot (see above). In this plot we also show as a black line the current bound from the LHC searches for heavy neutral Higgs bosons [90] in the channel \(pp \to H/A \to \tau\tau\) in the \(M_{125}(\tilde{\chi})\) benchmark scenario (based on the search data published in Ref. [91] using 139 fb\(^{-1}\)). The black line corresponds to \(m_{\tilde{\chi}_1^0} = M_{1/2}, i.e. roughly to the requirement for A-pole annihilation, where points above the black lines are experimentally excluded. There are very few points passing the current \((g-2)_\mu\) constraint below the black A-pole line, reaching up to \(m_{\tilde{\chi}_1^0} \sim 250 \text{ GeV}\), for which the A-pole annihilation could provide the correct DM relic density. It can be expected that with the improved limits as given in [91] this possibility is further restricted. These effects make the possibility of A-pole annihilation in this scenario marginal. The final parameter constrained in this scenario is the Higgs-mixing parameter \(\mu\). Here in particular, the DD bounds play an important role. Following the analysis in Ref. [5] we find a lower limit of \(\mu/M_1 \gtrsim 1.5\) in the \(\tilde{\chi}_1^\pm\)-coannihilation scenario.

We now turn to the case of \(\tilde{\chi}^\pm\)-coannihilation Case-L, as shown in Fig. 2. We start with the \(m_{\tilde{\chi}_1^0}-m_{\tilde{\ell}_l}\) plane in the upper left plot. By definition of the scenario, the points are located along the diagonal of the plane. The new constraint from \((g-2)_\mu\) puts an upper bound of \(\sim 650 \text{ GeV}\) on the masses, which is in the same ballpark as for the old \((g-2)_\mu\) results [5]. Including the DM and LHC constraints, the bound is nearly saturated, i.e. we find an upper limit of \(\sim 650 \text{ GeV}\).\(^3\) As in the case of \(\tilde{\chi}_1^\pm\)-coannihilation, the LHC constraints cut away only low mass points. The corresponding implications for the searches at future colliders are briefly discussed in Sect. 4.2.

In upper right plot of Fig. 2 we show the results in the \(m_{\tilde{\chi}_1^0}-m_{\tilde{\ell}_l}\) plane. The \((g-2)_\mu\) limits on \(m_{\tilde{\chi}_1^0}\) become slightly

\(^3\) We thank T. Stefaniak for providing us the limit, using the latest version of HiggsBounds [92–96].

\(^4\) This limit is \(\sim 100 \text{ GeV}\) higher than what was found in Ref. [5]. The reason for this increase is the following: a bug in the GM2calc-v1.5.0 had been detected, which was then fixed in the updated version v1.7.5. The bug-fixing leads to a significant deviation in \((g-2)_\mu\) value in this scenario (but not in the others), which results in a large deviation in the upper limits on the EW masses.
Fig. 1 Results in the $\tilde{\chi}_1^\pm$-coannihilation scenario: $m_{\tilde{\chi}_1^\pm}-m_{\tilde{\mu}_1}$ plane (upper left), $m_{\tilde{\chi}_1^0}-m_{\tilde{\mu}_1}$ plane (upper right), $m_{\tilde{\chi}_1^0}\tan \beta$ plane (lower plot); for the color coding: see text.

stronger for larger chargino masses, and upper limits on the chargino mass are set at $\sim 1.5$ TeV. This limit is reduced by $\sim 100$ GeV by the inclusion of the DM bounds. The LHC limits cut away a lower wedge going up to $m_{\tilde{\chi}_1^\pm} \lesssim 600$ GeV. Very low chargino masses are only found in the “accidentally” compressed region of $m_{\tilde{\chi}_1^\pm} \approx m_{\tilde{\chi}_1^0}$. Similar to the $\tilde{\mu}_1$-coannihilation scenario, a higher point density can be observed for larger $m_{\tilde{\chi}_1^0}$ due to the additional scanning, see Sect. 3.

The results for the $\tilde{\mu}_1$-coannihilation Case-L in the $m_{\tilde{\chi}_1^0}-m_{\tilde{\mu}_2}$ plane are presented in the lower plot of Fig. 2. The overall picture is similar to the $\tilde{\chi}_1^\pm$-coannihilation case shown above in Fig. 1 i.e. larger LSP masses are allowed for larger tan $\beta$ values. On the other hand, the combination of small $m_{\tilde{\mu}_2}$ and large tan $\beta$ leads to a too large contribution to $c_\mu^{\text{SUSY}}$ and is thus excluded. As in Fig. 1, we also show the limits from $H/A$ searches at the LHC, where we set (as above) $m_{\tilde{\mu}_2} = M_A/2$, i.e. roughly to the requirement for A-pole annihilation, where points above the black lines are experimentally excluded. In this case for the current $(g-2)_\mu$ limit, only very few points passing the $(g-2)_\mu$ constraint “survive” below the black line, i.e. they are potential candidates for A-pole annihilation. The masses reach up to $\sim 260$ GeV. Together with the already stronger bounds on $H/A \to \tau \tau$ [91] this does not fully exclude A-pole annihilation, but leaves it as a rather remote possibility. The limits on $\mu/M_1$ (not shown) in the $\tilde{\tau}_1$-coannihilation Case-L are again mainly driven by the DD-experiments. Given both CDM constraints and the LHC constraints, the smallest $\mu/M_1$ value we find is 1.7.

We now turn to our third scenario, $\tilde{\tau}_1$-coannihilation Case-R, where in the scan we require the “right-handed” sleptons to be close in mass with the LSP. It should be kept in mind that in our notation we do not mass-order the sleptons: for negligible mixing as it is given for selectrons and smuons the “left-handed” (“right-handed”) slepton corresponds to $\tilde{e}_2$ ($\tilde{\tau}_2$). The results are displayed in Fig. 3, where in all four plots the higher point densities due to the additional scans is plainly visible (see Sect. 3). We start our discussion with the $m_{\tilde{\tau}_1^0}-m_{\tilde{\mu}_2}$ plane shown in the upper left plot. By definition of the scenario the points are concentrated on the diagonal. The new $(g-2)_\mu$ bound yields an upper limit on the LSP of $\sim 700$ GeV, in agreement with the previous results [5]. The new $(g-2)_\mu$ bound also places an upper limit on $m_{\tilde{\mu}_2}$ (which is close in mass to the $\tilde{e}_2$ and $\tilde{\tau}_2$) of $\sim 800$ GeV, again in the same ballpark as for the old $(g-2)_\mu$ result. Including the CDM and LHC constraints, these limits reduce to $\sim 650$ GeV for the LSP, and correspondingly to $\sim 750$ GeV.
for $m_{\tilde{\mu}_2}$ and $\sim 650$ GeV for $m_{\tilde{\tau}_2}$. The LHC constraints cut out some, but not all lower-mass points. The upper limits obtained here show a slight increase w.r.t. Ref. [5], which can be attributed to the improved scanning in the higher mass range.

The distribution of the heavier slepton is displayed in the $m_{\tilde{\chi}_0^1}-m_{\tilde{\mu}_1}$ plane in the upper right plot of Fig. 3. Although the “left-handed” sleptons are allowed to be much heavier, the $(g-2)_{\mu}$ constraint imposes an upper limit of $\sim 1000$ GeV, about the same as in Ref. [5]. This effect is discussed in detail in Ref. [5]. The DM constraints lead to an upper limit of $\sim 850$ GeV. The LHC constraints to not yield a further relevant reduction in this case, which cut away only lower mass points and set a lower limit of $\sim 400$ GeV for the heavier sleptons in the Case-R.

In the lower left plot of Fig. 3 we show the results in the $m_{\tilde{\chi}_0^1}-m_{\tilde{\tau}_1}$ plane. As in the Case-L the $(g-2)_{\mu}$ limits on $m_{\tilde{\tau}_1}$ become slightly stronger for larger chargino masses. The upper limits on the chargino mass, however, are substantially stronger as in the Case-L. They are, also taking into account the DM and LHC constraints reached at $\sim 950$ GeV using the new $(g-2)_{\mu}$ result, similar to the limits for the old $(g-2)_{\mu}$ result [5].

For the $\tilde{\tau}^\pm$-coannihilation Case-L: $m_{\tilde{\tau}_2}-m_{\tilde{\tau}_1}$ plane (upper left), $m_{\tilde{\tau}_2}-m_{\tilde{\chi}_1}$ plane (upper right), $m_{\tilde{\tau}_2}$-tan $\beta$ plane (lower plot); for the color coding: see text.

We finish our analysis of the $\tilde{\tau}^\pm$-coannihilation Case-R with the results in the $m_{\tilde{\chi}_0^1}$-tan $\beta$ plane, presented in the lower right plot of Fig. 3. The overall picture is similar to the previous cases shown above. Larger LSP masses are allowed for larger tan $\beta$ values. On the other hand, the combination of small $m_{\tilde{\tau}_1}$ and very large tan $\beta$ values, tan $\beta \gtrsim 40$ leads to stau masses below the LSP mass, which we exclude from the CDM constraints. The LHC searches mainly affect parameter points with tan $\beta \lesssim 20$. Larger tan $\beta$ values induce a larger mixing in the third slepton generation, enhancing the probability for charginos to decay via staus and thus evading the LHC constraints. As above we also show the limits from $H/A$ searches at the LHC, where we set $m_{\tilde{\chi}_1} = M_A/2$, i.e. roughly to the requirement for $A$-pole annihilation, where points above the black lines are experimentally excluded. Comparing Case-R and Case-L, in the former case, slightly more points are passing the new $(g-2)_{\mu}$ constraint below the black line, i.e. are potential candidates for $A$-pole annihilation. The masses reach only up to $\sim 200$ GeV, similar to the case of the old $(g-2)_{\mu}$ result. Together with the already stronger bounds on $H/A \to \tau \tau$ [91] this leaves $A$-pole annihilation as a quite remote possibility in this scenario.
The limits on $\mu / M_1$ (not shown) in the $\tilde{l}^\pm$-coannihilation Case-R are as before mainly driven by the DD-experiments. Given both CDM constraints and the LHC constraints, the smallest $\mu / M_1$ value we find is 1.7.

4.2 Implications for future colliders

In Ref. [5] we had evaluated the prospects for EW searches at the HL-LHC [97] and at a hypothetical future $e^+e^−$ collider such as ILC [37,38] or CLIC [38–41]. The prospects for BSM phenomenology at the HL-LHC have been summarized in [97] for a 14 TeV run with 3 ab$^{-1}$ of integrated luminosity. For the direct production of charginos and neutralinos through EW interaction, the projected 95% exclusion reach as well as a $5\sigma$ discovery reach have been presented. Following the discussion in Ref. [5] we conclude that via these searches the updated $(g − 2)_μ$ limit together with DM constraints can conclusively probe “almost” the entire allowed parameter region of $\tilde{l}^\pm$-coannihilation Case-R scenario and a significant part of the same parameter space for Case-L scenario at the HL-LHC. On the other hand, the analysis for compressed higgsino-like spectra at the HL-LHC, see the discussion in Ref. [8], may exclude $m_{\tilde{\chi}^0_2} \sim m_{\tilde{\chi}^\pm_1} \sim 350$ GeV with mass gap as low as 2 GeV for $m_{\tilde{\chi}^\pm_1}$ around 100 GeV. Hence, a substantial parameter region can be curbed for the $\tilde{\chi}^\pm_1$-coannihilation case in the absence of a signal in the compressed scenario analysis with soft leptons at the final state. However, higher energies in $pp$ collisions or an $e^+e^−$ collider with energies up to $\sqrt{s} \sim 1.5$ TeV will be needed to probe this scenario completely [98,99]).

Direct production of EW particles at $e^+e^−$ colliders clearly profits from a higher center-of-mass energy, $\sqrt{s}$. Consequently, we focus here on the two proposals for linear $e^+e^−$ colliders, ILC [37,38] and CLIC [38–41], which can reach energies up to 1 TeV, and 3 TeV, respectively. In Ref. [5] we had evaluated the cross-sections for the various SUSY pair production modes (based on Refs. [100,101]) for the energies currently foreseen in the run plans of the two colliders. Taking into account the results for the cross-sections evaluated in Ref. [5], one can conclude that the new accuracy on $(g − 2)_μ$, yielding similar upper limits on EW SUSY particles, guarantees the discovery at the higher-energy stages of the ILC and/or CLIC. This holds in particular for the LSP and the NLSP. The improved $(g − 2)_μ$ constraint, confirming the deviation of $\alpha^{\text{Exp}}_μ$ from the SM prediction, clearly strengthens the case for future $e^+e^−$ colliders.
As discussed in Sect. 3 we have not considered the possibility of $Z$ or $h$ pole annihilation to find agreement of the relic DM density with the other experimental measurements. However, it should be noted that in this context an LSP with $M_1 \sim m_{\tilde{\chi}^0_1}$, $M_2/2$ or $M_1/2$ would yield a detectable cross-section $e^+e^- \rightarrow \tilde{\chi}^0_1 \tilde{\chi}^0_1 \gamma$ in any future high-energy $e^+e^-$ collider. Furthermore, depending on the values of $M_2$ and $\mu$, this scenario likely yields other clearly detectable EW-SUSY production cross-sections at future $e^+e^-$ colliders. We leave this possibility for future studies.

On the other hand, the possibility of $A$-pole annihilation was briefly discussed for all three scenarios. While it appears a rather remote possibility, it cannot be fully excluded by our analysis. However, overall an upper limit for this scenario on $m_{\tilde{\chi}^0_1}$ of $\sim 260$ GeV can be set. While not as low as in the case of $Z$ or $h$-pole annihilation, this would still offer good prospects for future $e^+e^-$ colliders. We leave also this possibility for future studies.

**5 Conclusions**

The electroweak (EW) sector of the MSSM, consisting of charginos, neutralinos and scalar leptons can account for a variety of experimental data: the CDM relic abundance with the lightest neutralino, $\tilde{\chi}^0_1$ as LSP, the bounds from DD experiments as well as from direct searches at the LHC. Most importantly, the EW sector of the MSSM can account for the long-standing discrepancy of $(g-2)_\mu$. The new result for the Run 1 data of the “MUON G-2” experiment confirmed the deviation from the SM prediction found previously.

Under the assumption that the previous experimental result on $(g-2)_\mu$ is uncorrelated with the new “MUON G-2” result we combined the data and obtained a new deviation from the SM prediction of $\Delta a_\mu = (25.1 \pm 5.9) \times 10^{-10}$, corresponding to a $4.2 \sigma$ discrepancy. We used this limit as a cut at the $\pm 2 \sigma$ level.

In this paper, under the assumption that the $\tilde{\chi}^0_1$ provides the full DM relic abundance we analyzed which mass ranges of neutralinos, charginos and sleptons are in agreement with all relevant experimental data: the new limit for $(g-2)_\mu$, the relic density bounds, the DD experimental bounds, as well as the LHC searches for EW SUSY particles. These results present an update of Ref. [5], where the previous $(g-2)_\mu$ result had been used (as well as a hypothetical “MUON G-2” result).

We analyzed three scenarios, depending on the mechanism that brings the relic density in agreement with the experimental data: $\tilde{\chi}^\pm_1$-coannihilation, $\tilde{\chi}^\pm_2$-coannihilation with the mass of the “left-handed” (“right-handed”) slepton close to $m_{\tilde{\chi}^0_1}$, Case-L (Case-R). We find in all three cases a clear upper limit on $m_{\tilde{\chi}^0_1}$. We find that the upper limits on the LSP mass are roughly reproduced to be of $\sim 650$ GeV in all three scenarios, confirming the collider targets w.r.t. the old $(g-2)_\mu$. Similarly, the upper limits on the NLSP masses are confirmed to about 750 GeV in the three cases that we have explored, again compatible with the previous $(g-2)_\mu$ result.

For the HL-LHC we have briefly discussed the prospects to cover the parameter regions that are preferred by the new $(g-2)_\mu$ result. In particular the $\tilde{\chi}^\pm$-coannihilation Case-R can be conclusively tested at the HL-LHC, while the other scenarios are only partially covered. Concerning future high(er) energy $e^+e^-$ colliders, ILC and CLIC, one can conclude that the new accuracy on $(g-2)_\mu$ confirms the upper limits on EW SUSY, and it can be expected that at least some particles can be discovered at the higher-energy stages of the ILC and/or CLIC. This holds in particular for the LSP and the NLSP. Therefore, the new $(g-2)_\mu$ constraint, confirming the deviation of $a_\mu$ from the SM prediction, strongly motivates the need of future $e^+e^-$ colliders.

**Acknowledgements** I.S. thanks S. Matsumoto for the cluster facility. The work of I.S. is supported by World Premier International Research Center Initiative (WPI), MEXT, Japan. The work of S.H. is supported in part by the MEINCOP Spain under contract PID2019-110058GB-C21 and in part by the AEI through the grant IFT Centro de Excelencia Severo Ochoa SEV-2016-0597. The work of M.C. is supported by the project AstroCeNT: Particle Astrophysics Science and Technology Centre, carried out within the International Research Agendas programme of the Foundation for Polish Science financed by the European Union under the European Regional Development Fund.

**Data Availability Statement** This manuscript has no associated data or the data will not be deposited. [Authors’ comment: There is no additional data or the data is already included in the manuscript.]

**Open Access** This article is licensed under a Creative Commons Attribution 4.0 International License, which permits use, sharing, adaptation, distribution and reproduction in any medium or format, as long as you give appropriate credit to the original author(s) and the source, provide a link to the Creative Commons licence, and indicate if changes were made. The images or other third party material in this article are included in the article’s Creative Commons licence, unless indicated otherwise in a credit line to the material. If material is not included in the article’s Creative Commons licence and your intended use is not permitted by statutory regulation or exceeds the permitted use, you will need to obtain permission directly from the copyright holder. To view a copy of this licence, visit http://creativecommons.org/licenses/by/4.0/.

**Funded by SCOAP³.**

**References**

1. H. Nilles, Phys. Rep. 110, 1 (1984)
2. R. Barbieri, Riv. Nuovo Cim. 11, 1 (1988)
3. H. Haber, G. Kane, Phys. Rep. 117, 75 (1985)
4. J. Gunion, H. Haber, Nucl. Phys. B 272, 1 (1986)
5. M. Chakrabarti, S. Heinemeyer, I. Saha, Eur. Phys. J. C 80(10), 984 (2020). arXiv:2006.15157 [hep-ph]
6. H. Goldberg, Phys. Rev. Lett. 50, 1419 (1983)
67. S. Borsanyi, Z. Fodor, J.N. Guenther, C. Hoelbling, S.D. Katz, L. Lellouch, T. Lippert, K. Miura, L. Parato, K.K. Szabo, et al. Leading hadronic contribution to the muon magnetic moment from lattice QCD. Nature 593(7857), 51–55 (2021). https://doi.org/10.1038/s41586-021-03418-1, arXiv:2002.12347 [hep-lat]

68. C. Lehner, A.S. Meyer, Consistency of hadronic vacuum polarization between lattice QCD and the R-ratio. Phys. Rev. D 101, 074515 (2020). https://doi.org/10.1103/PhysRevD.101.074515, arXiv:2003.04177 [hep-lat]

69. A. Crivellin, M. Hoferichter, C.A. Manzari, M. Montull, Hadronic vacuum polarization: \((g - 2)_\mu\) versus global electroweak fits. Phys. Rev. Lett. 125(9), 091801 (2020). https://doi.org/10.1103/PhysRevLett.125.091801, arXiv:2003.04886 [hep-ph]

70. A. Keshavarzi, W.J. Marciano, M. Passera, A. Sirlin, Muon \(g - 2\) and \(\Delta\alpha\) connection. Phys. Rev. D 102(3), 033002 (2020). https://doi.org/10.1103/PhysRevD.102.033002, arXiv:2006.12666 [hep-ph]

71. E. de Rafael, Constraints between \(\Delta\alpha\text{had}(M_Z^2)\) and \((g - 2)_\mu\) in \(\gamma\text{HVPE}\). Phys. Rev. D 102(5), 056025 (2020). https://doi.org/10.1103/PhysRevD.102.056025, arXiv:2006.13880 [hep-ph]

72. P. Athron et al., Eur. Phys. J. C 76(2), 62 (2016). arXiv:1510.08071 [hep-ph]

73. P. von Weitershausen, M. Schafer, H. Stockinger-Kim, D. Stockinger, Phys. Rev. D 81, 093004 (2010). arXiv:1003.5820 [hep-ph]

74. H. Fargnoli, C. Gnendiger, S. Paßehr, D. Stöckinger, H. Stöckinger-Kim, JHEP 1402, 070 (2014). arXiv:1311.1775 [hep-ph]

75. M. Bach, J.H. Park, D. Štöckinger, H. Stöckinger-Kim, JHEP 1510, 026 (2015). arXiv:1504.05500 [hep-ph]

76. S. Heinemeyer, D. Stockinger, G. Weiglein, Nucl. Phys. B 690, 62–80 (2004). arXiv:hep-ph/0312264

77. S. Heinemeyer, D. Stockinger, G. Weiglein, Nucl. Phys. B 699, 103–123 (2004). arXiv:hep-ph/0405255

78. W.G. Hollik, G. Weiglein, J. Witthödt, JHEP 03, 109 (2019). arXiv:1812.04644 [hep-ph]

79. P.M. Ferreira, M. Mühlleitner, R. Santos, G. Weiglein, J. Witthödt, JHEP 09, 006 (2019). arXiv:1905.10234 [hep-ph]

80. M. Drees, H. Dreiner, D. Schmeier, J. Tattersall, J.S. Kim, Comput. Phys. Commun. 187, 227–265 (2015). arXiv:1312.2591 [hep-ph]

81. J.S. Kim, D. Schmeier, J. Tattersall, K. Rolbiecki, Comput. Phys. Commun. 196, 535–562 (2015). arXiv:1503.01123 [hep-ph]

82. D. Dercks, N. Desai, J.S. Kim, K. Rolbiecki, J. Tattersall, T. Weber, Comput. Phys. Commun. 221, 383–418 (2017). arXiv:1611.09856 [hep-ph]

83. G. Belanger, F. Boudjema, A. Pukhov, A. Semenov, Comput. Phys. Commun. 149, 103–120 (2002). arXiv:hep-ph/0112278

84. G. Belanger, F. Boudjema, A. Pukhov, A. Semenov, Comput. Phys. Commun. 176, 367–382 (2007). arXiv:hep-ph/0607059

85. G. Belanger, F. Boudjema, A. Pukhov, A. Semenov, Comput. Phys. Commun. 177, 894–895 (2007). [84]

86. G. Belanger, F. Boudjema, A. Pukhov, A. Semenov, micrOMEGAs_3: A program for calculating dark matter observables. Comput. Phys. Commun. 185, 960–985 (2014). https://doi.org/10.1016/j.cpc.2013.10.016. arXiv:1305.0237 [hep-ph]

87. A. Djouadi, J.L. Kneur, G. Moulata, Comput. Phys. Commun. 176, 426 (2007). arXiv:hep-ph/0211331

88. Joint LEP2 SUSY Working Group, the ALEPH, DELPHI, L3 and OPAL Collaborations. http://lepsusy.web.cern.ch/lepsusy/

89. M. Muhlleitner, A. Djouadi, Y. Mambrini, Comput. Phys. Commun. 176, 383–418 (2017). arXiv:1611.09856 [hep-ph]

90. E. Bagnaschi et al., Eur. Phys. J. C 79(7), 617 (2019). arXiv:1808.07542 [hep-ph]

91. G. Aad et al. [ATLAS], Search for heavy Higgs bosons decaying into two tau leptons with the ATLAS detector using pp collisions at \(\sqrt{s} = 13\) TeV. Phys. Rev. Lett. 125(5), 051801 (2020). https://doi.org/10.1103/PhysRevLett.125.051801. arXiv:2002.12223 [hep-ex]

92. P. Bechtle, O. Brein, S. Heinemeyer, G. Weiglein, K.E. Williams, Comput. Phys. Commun. 181, 138–167 (2010). arXiv:0811.4169 [hep-ph]

93. P. Bechtle, O. Brein, S. Heinemeyer, G. Weiglein, K.E. Williams, Comput. Phys. Commun. 182, 2605–2631 (2011). arXiv:1102.1898 [hep-ph]

94. P. Bechtle, O. Brein, S. Heinemeyer, O. Stål, T. Stefaniak, G. Weiglein, K.E. Williams, Eur. Phys. J. C 74(3), 2693 (2014). arXiv:1311.0055 [hep-ph]

95. P. Bechtle, S. Heinemeyer, O. Stål, T. Stefaniak, G. Weiglein, Eur. Phys. J. C 75(9), 421 (2015). arXiv:1507.06706 [hep-ph]

96. P. Bechtle, D. Dercks, S. Heinemeyer, T. Klingl, T. Stefaniak, G. Weiglein, J. Witthödt, Eur. Phys. J. C 80(12), 1211 (2020). arXiv:2006.06007 [hep-ph]

97. X. Cid Vidal et al., CERN Yellow Rep. Monogr. 7, 585–865 (2019). arXiv:1812.07831 [hep-ph]

98. R.K. Ellis et al., arXiv:1910.11775 [hep-ex]

99. M. Berggren, arXiv:2003.12391 [hep-ph]

100. S. Heinemeyer, C. Schappacher, Eur. Phys. J. C 77(9), 649 (2017). arXiv:1704.07627 [hep-ph]

101. S. Heinemeyer, C. Schappacher, Eur. Phys. J. C 78(7), 536 (2018). arXiv:1803.10645 [hep-ph]