The size of flavor changing effects induced by the symmetry breaking sector

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Abstract

It has recently been shown that strong interactions underlying electroweak symmetry breaking will induce four-fermion amplitudes proportional to $m_t^2$, which in turn will influence a variety of flavor changing processes. We argue that the size of these effects are likely to be far below the current experimental bounds.

The corrections induced by a new strong sector underlying electroweak symmetry breaking are conveniently encoded in an effective chiral Lagrangian. Recently attention has focused on a particular term in this effective Lagrangian which induces corrections proportional to $m_t^2$ in charged current interactions [1]. Integrating out the $t$ quark then yields interesting effects in the down-type quark sector, inducing corrections to $R_b$ and $B_d^0 - \bar{B}_d^0$ mixing [1], and various rare $B$ and $K$ decays [2]. All these effects are correlated since they are related to one parameter in the effective Lagrangian [2].

In this note we will provide an estimate of the size of the new parameter, expressed in terms of the number of new fermion doublets in some underlying theory of electroweak symmetry breaking. Such an estimate in the case of the $S$ parameter proved useful to constrain technicolor theories. The $S$ parameter is related to a term in an effective Lagrangian with coefficient $L_{10} = -S/16\pi$ which is completely analogous to the $L_{10}^{QCD}$ term appearing at order $p^4$ in the low energy QCD chiral Lagrangian [3]. Since $L_{10}^{QCD}$ is a measured quantity, an estimate for $S$ is thereby obtained [4] for QCD-like technicolor theories.

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The situation is somewhat different for the parameter of interest here, which is
the coefficient of another term appearing at order $p^4$. In the QCD case this coefficient
is not a measurable quantity since the term can be removed by the equations of
motion. Fortunately in the QCD case there are quark models which model the chiral
symmetry breaking and which quite successfully reproduce the values of all ten mea-
sured parameters, $L_1$–$L_{10}$. Such models can then be expected to provide a reasonable
estimate of the new parameter, which we will refer to as $L_{11}$ (corresponding to $a_{11}$ in
\cite{1} and $\alpha_{11}$ in \cite{2}).

A naive quark model may be based on the nonlinear sigma model where effects
of a single quark loop are considered. This does a fair job of reproducing those $L_i$’s
which happen to correspond to convergent loop integrals \cite{3}. A more sophisticated
quark-based approach leads to the extraction of all ten parameters \cite{3}. Most con-
venient for our purposes is the gauged nonlocal constituent (GNC) quark model \cite{7},
which incorporates the momentum dependence expected for dynamically generated
fermion masses. In QCD the mass function is known to fall as the square of the mo-
mentum (up to logarithms) for large momentum. The GNC model can incorporate
such momentum dependence in an manner which preserves the local chiral symme-
tries of the underlying theory. The mass function then naturally regulates the loop
integrals, and successful values for $L_1$–$L_{10}$ are obtained. Here we are interested in $L_{11}$
and its sensitivity to the choice of mass function.

We will focus on the following two terms in the chiral Lagrangian,

$$L_\chi \ni L_{10}\Tr(U^\dagger L_{\mu\nu}UR^{\mu\nu}) + L_{11}\Tr((D_\mu V^\mu)^2), \quad (1)$$

with

$$U(x) = e^{i\pi(x)^a\sigma_a/v}, \quad V_\mu = (D_\mu U)U^\dagger,$$

$$D_\mu U = \partial_\mu U + iL_\mu U - iUR_\mu,$$

$$D_\mu V_\nu = \partial_\mu V_\nu + i[L_\mu, V_\nu],$$

$$L_{\mu\nu} = \partial_\mu L_\nu - \partial_\nu L_\mu - [L_\mu, L_\nu],$$

$$L_\mu = (V_\mu^a - A_\mu^a)a^a, \quad R_\mu = (V_\mu^a + A_\mu^a)a^a.$$

By identifying $V_\mu^a - A_\mu^a$ with weak gauge fields $gW_\mu^a/2$ and using the equation of
motion for $W_\mu$ the $L_{11}$ term can be transformed into a sum of 4-quark operators,
which include the following charged current interaction \cite{1}:

$$-\frac{8L_{11}m_t^2}{v^4} \sum_{i,j=d,s,b} V_{\bar t_i}^a V_{\bar t_j}^a q_{\bar t_i}^L \bar R t^R q_{\bar t_j}^L \quad (2)$$
$V_{ts}$ are the CKM matrix elements.

The chiral symmetry breaking physics in the underlying theory will produce finite contributions to the $L_{10}$ and $L_{11}$ coefficients. These coefficients become running couplings within the effective theory, and the finite values we are referring to correspond to the values renormalized at the chiral symmetry breaking scale. These values may be determined by choosing some convenient amplitude and using that to match the effective theory onto the underlying theory (or at least a model of the underlying theory).

We shall consider the two-point function

$$\int e^{i qx} \langle V_{\mu a}(x)V_{\nu b}(0) - A_{\mu a}(x)A_{\nu b}(0) \rangle dx = i \Gamma_{\mu \nu}(q^2) \delta_{ab}$$

where the only contributions from the order $p^4$ chiral Lagrangian are from $L_{10}$ and $L_{11}$.

$$\Gamma_{\mu \nu}(q^2) = 16 L_{10} q^2 g_{\mu \nu} + 16(L_{11} - L_{10}) q_\mu q_\nu$$

The naive quark loop yields

$$L_{10}^N = L_{11}^N = - \frac{N_d}{96\pi^2},$$

where $N_d$ is the number of fermion doublets contributing in the loop.

The couplings of the gauge fields to the fermions in the GNC model are the following, where $\Sigma(p^2)$ is the dynamical fermion mass function.

$$\Gamma_{\mu a}^V(p_1, q, p_2 = p_1 + q) = i \gamma^\mu \sigma^a - i G(p_2, p_1)(p_1 + p_2)^\mu \sigma^a$$

$$\Gamma_{\mu a}^A = i \gamma^\mu \gamma_5 \sigma^a$$

$$\Gamma_{\nu V}^{\mu ab}(p_1, q_1, q_2, p_2 = p_1 + q_1 + q_2) = -i[G(p_2, p_1)g_{\mu \nu} \sigma^b \sigma^a$$

$$+ \frac{G(p_2, p_1) - G(p_1 + q_1, p_1)}{(p_2 + p_1 + q_1) \cdot q_2}(2p_1 + q_1)^\mu (p_2 + p_1 + q_1)^\nu \sigma^b \sigma^a]$$

$$- i[(q_1, \mu, a) \leftrightarrow (q_2, \nu, b)]$$

$$\Gamma_{\nu a}^{ab} = 0$$

$$G(p_2, p_1) = \frac{\Sigma(-p_2^2) - \Sigma(-p_1^2)}{p_2^2 - p_1^2}$$

Thus as a consequence of gauge invariance one diagram will have the two vector fields attached at the same point on the loop. To study the effect of the momentum dependence in $\Sigma(p^2)$ we will consider the following one parameter family of mass functions satisfying $\Sigma(m^2) = m$.

$$\Sigma(p^2) = \frac{(1 + A)m^3}{p^2 + Am^2}$$
For $A \to \infty$, the naive quark loop result is obtained, whereas QCD is better modeled by a value of $A$ closer to unity.

We plot $L_{10}^{GNC}/L_{10}^N$ and $L_{11}^{GNC}/L_{11}^N$ in Fig. (1) as a function of $A$. When $A = 1$ we see that $L_{10}^{GNC}$ is more than twice as large as the naive quark loop result, which brings it into line with the measured QCD value \[^7\]. $L_{11}^{GNC}$ on the other hand shows surprisingly little sensitivity to $A$, and its value for all $A$ is close to $L_{11}^N$.\[^4\] We also checked other functional forms for $\Sigma(p^2)$ and found that $L_{11}^{GNC}$ is generally quite insensitive to the momentum dependence of the fermion mass function.

In \[^1\] and \[^2\] it was argued that the experimental constraints from $R_b$ and $B_d^0 - \overline{B_d^0}$ mixing imply that $|L_{11}| < .1$. But we now see that this limit is far above what we could reasonably expect. Our results imply that the limit is saturated for $N_d \approx 90$, which is clearly impossible given the constraint on $N_d$ from $S$. To put it another way, if we assume that a heavy fourth family ($N_d = 4$) is still allowed by the constraint on $S$, then the contribution to $L_{11}$ is only $-0.004$. The shifts in the various rare $B$ and $K$ decay modes from standard model values as described \[^2\] would then be less than 5%.

While our models of strong interactions are admittedly simple-minded, they strongly suggest that the effects of $L_{11}$ will be very difficult to observe given the constraints on $S$. We conclude that if flavor changing effects beyond the standard model are seen, they will likely have more to do with the physics responsible for quark and lepton masses rather than the physics responsible for electroweak symmetry breaking.

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\[^1\]These results agree with those presented in \[^6\], given that the $L_{11}$ term in those references is defined differently so that it appears in \[^6\] with the opposite sign.
Figure 1: $L_{10}^{GNC}$ and $L_{11}^{GNC}$ are coefficients of terms in the chiral Lagrangian (1) as determined by the gauged nonlocal constituent quark model. The naive quark model gives $L_{10}^N = L_{11}^N = -N_d/96\pi^2$. The parameter $A$ appears in the fermion mass function in (11).

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