Assessing curve number uncertainty for green roofs in a stochastic environment

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Abstract. Curve number (CN) is well-known by hydrologists for estimating rainfall induced runoff from a catchment. It can also be used as an indicator for measuring the impact of engineering or non-engineering measures on the runoff production in a catchment. In this study, a method is presented to quantify the uncertainty of CN for hydrologic performance of a green roof system. Latin hypercube sampling approach, coupled with the antithetic variate technique, is used to achieve efficient and accurate quantification of the uncertainty features of CN for a green roof system. Elements in green roofs subject to uncertainty considered are rainfall characteristics (i.e. amount and inter-event dry period), soil-plant-climate factors (i.e. field capacity, wilting point, interception, evapotranspiration rate), and model error in SCS Ia-S relation. Numerical study shows that model error in SCS Ia-S relation has the dominant effect on the uncertainty features of CN for green roof performance.

1. Introduction

Curve number (CN) method is a widely used empirical approach for calculating rainfall induced runoff volume from a catchment. The CN represents the combined effect of land use, soil type, plant cover, and soil moisture conditions on runoff generation. Numerous researchers have developed CN values for different land use types according to their runoff characteristics for estimating catchment runoff [1-3]. Alternatively, CN can be used as an indicator for measuring the effectiveness of engineering or non-engineering measures on the runoff production in a catchment. For example, urbanization in a catchment results in higher runoff production, which would be reflected by a higher value of CN. On the other hand, the effectiveness of installing runoff retardation/retention facilities (e.g., detention basins) and/or implementing low impact development (LID) measures (e.g., green roofs, rain gardens, infiltration trenches, etc.) in runoff reduction can be reflected by lower values of CN for the catchment.

Yoo et al [4], according to Carter and Rasmussen [5], analyzed the effect of several best management practices (BMPs) for urban runoff reduction and proposed a methodology for quantifying runoff reduction in terms of CN value. Sin et al [6] used the method of Yoo et al [4] proposed to quantify the effect of LID facilities. The results of reduction in volume by Yoo et al [4] were compared with the results from SWMM 5.0 and both were similar. According to the above research, CN could be a viable indicator in estimating the effectiveness of a green roof (GR) system. However, rainfall amount, inter-event dry period, and model parameters of a GR system all have certain degrees of uncertainty. Hence, the calculated runoff volume and the corresponding CN are also subject to
uncertainty.

In this study, uncertainties of model random rainfall properties and parameters of a GR system are considered to quantify the uncertainty features of \( CN \) for assessing the hydrologic performance of a green roof system. In particular, special algorithms of Monte Carlo simulation (MCS), namely, Latin hypercube sampling (LHS) and antithetic variate (AV) technique, are implemented in a coupled fashion to obtain a fast and accurate quantification of the uncertainty of \( CN \) of a GR system.

2. Rainfall-runoff relation for a simplified GR system
A simple hydrologic model for a GR system used by Zhang and Guo is adopted herein [7]. The model follows water balance principle in which retention capacity of a GR system in defined by

\[
R_c = S_l + S_c + (\theta_f - \theta_i)h
\]

where \( R_c \) = retention capacity of the GR system (mm); \( S_l \) = interception (mm); \( S_c \) = storage capacity of storage layer (mm); \( \theta_f \) = field capacity of growing medium; \( \theta_i \) = initial soil moisture content of growing medium at each dry-rain cycle; and \( h \) = depth of growing medium (mm).

In the above hydrologic model, by further assuming that the growing medium is to maintain above the wilting point of the plants, \( \theta_w \), the maximum retention capacity for the GR system, \( R_{c,max} \), can be obtained with \( \theta_i = \theta_w \) by

\[
R_{c,max} = S_l + S_c + (\theta_f - \theta_w)h
\]

in which \( \theta_w \) = soil moisture content of the growing medium at wilting point.

The initial soil moisture content at the beginning of a rainfall event depends on the length of dry period, \( b \), proceeding the current rainfall event, evapotranspiration rate, \( E_a \), and evapotranspirable water content, \( W_i \), as

\[
\theta_i = \begin{cases} 
\frac{W_i - E_ab}{h}, & b \leq \frac{W_i}{E_a} \\
\theta_w, & b > \frac{W_i}{E_a}
\end{cases}
\]

Combining equations (1)-(3), the retention capacity of a GR system at the beginning of a rainfall event can be determined as

\[
R_c = \begin{cases} 
R_{c,max} + E_a b - W_i, & b \leq \frac{W_i}{E_a} \\
R_{c,max}, & b > \frac{W_i}{E_a}
\end{cases}
\]

Then, the runoff volume generated from a GR system by a rainfall event is obtainable as [7]

\[
\nu_{rg} = \begin{cases} 
0, & \left[ v \leq R_{c,max}, b > \frac{W_i}{E_a} \right] \text{ or } \left[ v \leq R_{c,max} - W_i + E_ab, b \leq \frac{W_i}{E_a} \right] \\
v + W_i - R_{c,max} - E_ab, & \left[ v > R_{c,max} - W_i + E_ab, b \leq \frac{W_i}{E_a} \right] \\
v - R_{c,max}, & \left[ v > R_{c,max}, b > \frac{W_i}{E_a} \right]
\end{cases}
\]

in which \( v \) = rainfall depth; and \( \nu_{rg} \) = runoff volume from the GR system.

3. CN representation of GR performance
One of the important hydrologic performances of a GR system lies in its runoff volume reduction [7]. In the Natural Resources Conservation Service (NRCS) method, the relation between event-based rainfall and runoff can be represented by the following equation [8].
where $V = \text{rainfall depth;}$ $V_{rg} = \text{runoff depth;}$ $I_a = \text{initial abstraction;}$ and $S = \text{storage parameter representing potential maximum retention of the catchment.}$

In equation (6), $I_a$ and $S$ are related through the well-known empirical equation, i.e., $I_a = 0.2S$, obtained from fitting the data shown in figure 1. According to SCS assumed functional form, the relation between $I_a$ and $S$ can be written as

$$I_a = \lambda S \epsilon$$

where $\lambda = \text{the initial abstraction rate [9];}$ and $\epsilon = \text{the model error term.}$ By fitting data in figure 1 using the least square criterion, one obtains $\lambda = 0.18$ (dash line) which is slightly lower than commonly used value of $\lambda = 0.2$ (solid line).

Substituting equation (7) into equation (6) yields:

$$V_{rg} = \begin{cases} 
\frac{(V - I_a)^2}{(V - I_a) + S}, & \text{for } V_{rg} > I_a \\
0, & \text{for } V_{rg} \leq I_a
\end{cases}$$

and the storage parameter $S$ can be expressed in terms of other variables as:

$$S = \frac{\epsilon \lambda (2V - V_{rg}) + V_{rg} \pm \sqrt{V_{rg}^2 (\epsilon \lambda - 1)^2 + 4 \lambda V_{rg} V \epsilon}}{2 \lambda^2 \epsilon^2}$$

Since the curve number ($CN$) is related to storage parameter $S$ as:

$$CN = \frac{1000}{10 + S} \quad \text{(with } S \text{ in in.) or } CN = \frac{25400}{254 + S} \quad \text{(with } S \text{ in mm)}$$

Combining equation (9) with equation (10), $CN$ value can be directly calculated from GR rainfall-
runoff data (in mm) as

\[ CN = \frac{25400}{254 + \varepsilon \lambda (2V - V_g) + Q \pm \sqrt{V_{rg}^2 (\lambda e - 1)^2 + 4 \lambda V_g V e}} \]  

(11)

4. Uncertainties in GR systems

In practice, hydrologic performance of a GR system often is assessed by using representative models with different complexities. These models, in general, involve uncertainties that can be categorized into two types: aleatory and epistemic [10,11]. Aleatory uncertainty arises from the inherent randomness of the processes involved whereas epistemic uncertainty is attributed mainly by lack of perfect knowledge of the process represented by the model and/or parameters. The presence of uncertainties in GR models results in uncertain model outputs.

In this study, for the GR model under consideration the model inputs subject to aleatory uncertainty are rainfall amount \( (V) \) and inter-event dry period \( (B) \). As for the epistemic uncertainty, they include model parameters characterizing climate \( \left( E_a \right) \), soils \( \left( \theta_f, \delta_f \right) \), plants \( \left( S_l, \theta_w \right) \), and the conceptual model constant \( c = W_l / R_{c,max} \). In addition to the uncertainties involved in the GR model mentioned above, there exists a model error \( (\varepsilon) \) in describing the functional relation between \( I_a \) and \( S \) in equation (7). This model error \( (\varepsilon) \) is the uncertainty of epistemic type whose statistical features can be estimated by regression analysis. In this study, the regression model describing the \( I_a - S \) relation was under logarithmic scale as

\[ \ln(I_a) = \ln(\lambda S) + \varepsilon' \]  

(12)

Since \( \varepsilon' \) generally is assumed to have a normal distribution, the model error, \( \varepsilon \), then has a log-normal distribution with the mean and variance, respectively, as

\[ \mu_{\varepsilon} = \exp[\mu_{\varepsilon'} + 0.5 \sigma_{\varepsilon'}^2]; \sigma_{\varepsilon}^2 = \mu_{\varepsilon}^2 [\exp(\sigma_{\varepsilon'}^2) - 1] \]  

(13)

5. Uncertainty quantification of CN for green roofs

In quantifying the overall uncertainty of model outputs, it requires knowing the distributional properties of each input or parameter with uncertainty [12]. In this study, according to the statistical properties of model inputs \( (V \ and \ B) \) and model parameters \( (E_a, \theta_f, \delta_f, S_l, c, \varepsilon) \) and assume all model inputs/parameters having no correlation, the uncertainty features of \( CN \) corresponding to the hydrologic performance of a GR system can be quantified. Due to the complexity of functional relationship between \( CN \) and model inputs/parameters of a GR system, approximated methods for uncertainty quantification of \( CN \) are feasible. Various approximated methods can be applied to quantify uncertainty features of \( CN \) of a GR system from those of the inputs and parameters of the system. They include, but not limited to, the probabilistic point estimation (PPE) methods as well as Monte-Carlo simulation (MCS) and its variations. According to a preliminary analysis (not reported herein due to page limit), PPE methods were found not too accurate. Therefore, MCS is implemented to assess the uncertainty of \( CN \) of a GR system. In particular, a special form of MCS, called Latin hypercube sampling (LHS) is applied.

By MCS, large number of representative model inputs \( (V, B) \) and parameters \( (E_a, \theta_f, \delta_w, S_l, c, \varepsilon) \) are synthesized to compute the corresponding values of runoff from equation (5) and then the corresponding values of \( CN \) from equation (11). From the randomly generated values of \( CN \), one can assess its uncertainty features. However, due to high nonlinearity of rainfall-runoff-CN relation for the GR system, i.e., equation (11), the number of MCS repetitions by simple random sampling would have to be high to arrive stable and accurate estimations of uncertainty features of \( CN \). In the cases, the computation intensiveness of MCS could be reduced by using a more computationally efficient LHS
in conjunction with the antithetic variate (AV) technique.

6. Application
For illustration, the data presented in Zhang and Guo [7] for a probabilistic GR model are adopted herein in which aleatory uncertainty is associated with rainfall depth and inter-event dry period. The study adopts the statistical properties of the two rainfall characteristics (listed in table 1(a)) obtained by Zhang and Guo [7] from analyzing rainfall data (1960-2006) at Detroit Airport, Michigan. The GR system parameters in equation (5) (i.e. $E_a, \theta_f, \theta_w, S_l, c$) associated with epistemic uncertainties are assumed to be independent variables having uniform distribution with bounds listed in table 1(b). The mean values of model parameters are adopted from Zhang and Guo [7]. The ranges of variation of the GR model parameters were obtained through literature review by You et al [13]. As for the uncertainty features associated with the $I_a$-$S$ relation, the data shown figure 1 were analyzed by fitting $\ln(I_a) = \ln(0.2S) + \varepsilon'$ through data points

### Table 1. Statistical properties of uncertain factors in the example GR system (adopted from [7]). (a) Model inputs subject to aleatory uncertainty and (b) GR model parameters subject to epistemic uncertainty.

| Factor                          | Mean Value | Distribution |
|---------------------------------|------------|--------------|
| Rainfall event amount, $V$      | $\mu_V = 14.35$mm (Exponential) | |
| Dry period between rainfall events, $B$ | $\mu_B = 97.95$hr (Exponential) | |
| ET rate, $E_a$ (mm/hr)          | $0.11 \pm 25\%$ (Uniform) | |
| Field capacity, $\theta_f$      | $0.232 \pm 15\%$ (Uniform) | |
| Wilting point, $\theta_w$       | $0.116 \pm 20\%$ (Uniform) | |
| Interception loss, $S_l$ (mm)   | $2 \pm 30\%$ (Uniform) | |
| Multiplier constant, $c$        | $0.5$ to $0.9$ (Uniform) | |

In figure 1 and assuming a log-normal distribution for $\varepsilon = \exp(\varepsilon')$, one can determine that $\mu_\varepsilon = 1.49$ and standard deviation $\sigma_\varepsilon = 1.96$.

Note that the uncertainty in $CN$ for GR systems is affected by those of rainfall (aleatory), runoff (both aleatory and epistemic), and error in $I_a$-$S$ relation (epistemic). In this study, effects of different types of uncertainty on $CN$ representation in rainfall retention of a GR system are examined. Specifically, four scenarios of uncertainty are listed in table 2. The uncertainties considered in scenario-(I) & (III) will limit their effects only on GR runoff uncertainty whereas scenario-(II) & (IV) will affect the uncertainty of $CN$.

### Table 2. Scenarios of considering different types of uncertainty.

| Scenario | (I) | (II) | (III) | (IV) |
|----------|-----|------|-------|------|
| Uncertainties considered | $V$, $B$ | $V$, $B$, $\varepsilon$ | $V$, $B$, GR Para. | $V$, $B$, GR Para., $\varepsilon$. |

In the numerical investigations, uncertainty features of random variables in each scenario are defined in table 1(a) and 1(b). For those factors whose uncertainties not considered in different scenarios, the mean values are used. Through LHS approach, embedded with the AV technique, 1000 samples were generated for calculating the corresponding value of $CN$. The uncertainty features (i.e. mean, standard deviation, and skew coefficient) of $CN$ of the example GR system with the growing medium depth under different considerations of uncertainty type are shown in figures 2(a)-2(c).

Figure 2(a) shows that the mean value of $CN$, $E(CN)$, decreases with the growing medium depth for all scenarios of uncertainty type considered. This is expected because the GR rainfall retention
capacity increases with the growing medium depth which leads to a decrease in runoff production. Two interesting behaviors of $E(CN)$ can be observed from figure 2(a). One is that the inclusion of GR parameter uncertainties results in slight increase in $E(CN)$. The other behavior, which is more eye-catching, is a distinct separation of curves into two groups depending on whether the model error term in SCS $I_{a}$-$S$ relation is included or not. The consideration of model error $\varepsilon$ in $I_{a}$-$S$ relation results in a significant drop in $E(CN)$ value as shown in figure 2(a). The discrepancy in $E(CN)$ between the two groups increases with growing medium depth. This result shows that the model error in $I_{a}$-$S$ functional relation has a dominant effect on the results of quantifying the mean value of $CN$.

**Figure 2.** Variation of statistical properties of $CN$ with the GR growing medium depth under different considerations of uncertainty types.

**Figure 3.** Histograms of sampled $CN$ under different growing medium depths and considerations of uncertainty type.
With regard to the standard deviation of $CN$ shown in figure 2(b), it is observed that $STD(CN)$ increases with growing medium depth. The model error $\varepsilon$ in $I_a$-$S$ relation has significant impact on heightening the value of $STD(CN)$ by twice as much. Although the inclusion of epistemic uncertainties associated with the GR system results in higher value of $STD(CN)$ over the scenario-(I) and (II), the incremental amount is not significant.

On the skew coefficient of $CN$, it is observed the model error $\varepsilon$ in SCS $I_a$-$S$ relation can also exert significant influence on $SKW(CN)$. Without considering model error $\varepsilon$, the value of $SKW(CN)$ varies from slightly lower than $-1.0$ to $\approx -1.0$. However, by incorporating model error $\varepsilon$ the skew coefficient of $CN$ becomes significantly more negatively skewed from $-4.0$ to $-2.0$.

Figures 3(a) and 3(b) illustrate the histograms of 1000 LHS generated samples of $CN$. From the figures, one can appreciate the effect of considering different types of uncertainty in the GR system on the overall random feature of $CN$. In particular, the effect of model error $\varepsilon$ term in $I_a$-$S$ relation on the uncertainty of $CN$ can be clearly displayed.

7. Summary and conclusions

Curve number ($CN$) method is a well-known approach that has long history being used by hydrologists for estimating rainfall induced runoff volume from a catchment. Also, $CN$ can be used as an indicator for measuring the effectiveness of engineering and/or non-engineering measures on the runoff production in a catchment. In this study, a method is presented to quantify the uncertainty features of $CN$ for assessing the hydrologic performance of a GR system. Latin hypercube sampling approach, coupled with the antithetic variate technique, is used to achieve efficient and accurate quantification of the uncertainty features of $CN$ for a GR system. The statistical properties of $CN$ such as the mean, standard deviation, and skew coefficient are estimated by considering uncertainties of model inputs ($V$ and $B$) and model parameters ($E_a$, $\theta_f$, $\theta_w$, $S_l$, $c$) of in a GR model, as well as the uncertainty associated with the well-known $I_a=0.2S$ relation in the SCS $CN$ method.

From the numerical application, it is found that the mean $CN$ decreases as the growing medium depth increases for all four scenarios of uncertainty type considered. The inclusion of GR parameter uncertainties results in slight increase in mean $CN$. As for the standard deviation and skew coefficient of $CN$, they increase with growing medium depth. Among the various types of input/parameter uncertainty affecting that of $CN$, it is interesting to find that, in quantifying all statistical features of $CN$ for a GR system (i.e. mean, standard deviation, skew coefficient), the epistemic uncertainty due to model error in SCS $I_a$-$S$ relation has the dominant effect on the results whereas those from GR model parameters are relatively insignificant.

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