Ponderomotive forces due to electron modes in unmagnetized plasmas described by kappa distribution functions

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Abstract
The Washimi and Karpman ponderomotive interaction due to electron wave propagation is investigated for low-temperature unmagnetized plasmas described by an isotropic kappa distribution. We perform a brief analysis of the influence of the kappa distribution in the dispersion relations for a low-temperature plasma expansion at the lowest order in which the thermal effects are appreciable without considering the damping characteristics of the wave. The spatial and temporal factors of the ponderomotive force are obtained as a function of the wavenumber, the spectral index $\kappa$ and the ratio between the plasma thermal velocity and the speed of light. Our results show that for unmagnetized plasmas non-thermal effects are negligible due to the spatial ponderomotive force when non-relativistic thermal velocities are considered. However, for unmagnetized plasmas, the temporal factor of the ponderomotive force appears only due to the presence of suprathermal particles, with a clear dependence on the $\kappa$ index. We also analyze the role of the non-thermal effect in the induced Washimi and Karpman ponderomotive magnetization and the total power radiated associated with it. Furthermore, we show that the magnitude of the slowly varying induced ponderomotive magnetic field increases as the plasma moves away from thermal equilibrium.

Keywords: ponderomotive force, kappa distribution, non-linear effects

(Some figures may appear in colour only in the online journal)

1. Introduction
Ponderomotive forces are time-averaged nonlinear forces caused by high-frequency electromagnetic waves in plasmas. Despite their complexity, the analysis of these forces facilitates the study of phenomena associated with electromagnetic waves and plasma interactions by simplifying their dynamics. The exact behavior of a dynamical system using the Lorentz force is very complex, and we are usually more interested in the average dynamic of the system (Lundin and Guglielmi 2007). Hence, the study of ponderomotive forces has been of great importance in the understanding of plasma phenomena in different environments. The ponderomotive force has been investigated in laser phenomena where it can lead to the appearance of self-focusing (Karpman and Washimi 1977, Washimi 1989, Rezapour et al 2018, Gupta et al 2022).

It is known that the ponderomotive force can be deduced using a fluid formalism, or by an approach that uses the space-time derivatives of the stress tensor of the medium.
In addition, the inclusion of ponderomotive forces in plasmas can act as a generator of slowly varying magnetic fields (Washimi and Watanabe 1977). These effects have been extensively investigated recently, particularly for quantum plasmas, due to their importance in the magnetic field generation in laser-matter interaction and in dense plasmas of astrophysical compact objects (Na and Jung 2009, Shukla et al 2010, Jamil et al 2019).

Relative to space physics phenomena where the plasma constantly interacts with waves, the ponderomotive force plays an essential role in the physics of electromagnetic ultra-low frequency (ULF) waves in the terrestrial magnetosphere. Indeed, considering ponderomotive force analysis allows us to model the interaction of the plasma with the waves that, in conjunction with the particles, are responsible for the transfer of energy, mass and momentum into the Earth’s magnetosphere from the solar wind. Therefore, the ponderomotive forces are partially responsible for phenomena such as the acceleration of particles in the polar regions or the redistribution of plasma in the magnetosphere (Guglielmi and Lundin 2001, Lundin and Guglielmi 2007, NeKRasov and Feygin 2012, 2014). Due to the contribution of ponderomotive forces to the understanding of the dynamics of the near-Earth space environment, a method has been proposed recently to verify these phenomena experimentally (Guglielmi and Feygin 2018).

On the other hand, it has been observed that in near-Earth space plasma the particle velocity distributions exhibit suprathermal tails that are well described by the family of kappa distributions (Lazar and Fichtner 2021). These distributions depend on the spectral index $\kappa$ and can be understood as a power-law generalization from which the Maxwellian distribution is recovered as a limiting case when $\kappa$ tends to infinity. However, a detailed analysis of the effect of suprathermal particles on the ponderomotive interaction of electromagnetic waves with non-thermal plasmas has not been investigated yet. As explained above, in the near-Earth space environment, it has been observed that plasmas are mainly described by kappa distributions (Espinoza et al 2018, Eyelade et al 2021). Therefore, to better understand the dynamics of our space environment, the impact of the non-thermal nature of the plasma on its interaction with the waves that propagate in the Earth’s atmosphere must be evaluated.

The ponderomotive force not only depends on the spatial and temporal variations of the wave amplitude, but also strongly depends on the properties of the medium and its interaction with the electromagnetic waves described by its dielectric tensor. Therefore, this nonlinear effect principally stands on the distribution of the particles that describe the plasma. Hence, the non-thermal effects of the plasma described by the kappa distribution can significantly affect the behavior of the ponderomotive force and thus the phenomena associated with it. This can lead to great impact in space physics where, as mentioned above, various phenomena related to ponderomotive forces occur in plasmas described by kappa distributions. In fact, we know from the work of Kim and Jung (2009) that the non-thermal effect plays a significant role in the temporal term of the ponderomotive force for electrostatic waves, where it is shown that the Washimi and Karpman induced magnetization decreases because of the non-thermal effects. However, to the best of our knowledge, the impact of non-thermal effects due to kappa distribution in the ponderomotive force due to electromagnetic waves interacting with plasmas has not been studied. The purpose of this study is to investigate the influence of the kappa distribution on the Washimi and Karpman ponderomotive force for electron waves in low-temperature unmagnetized plasmas. This study can be useful to generalize the results given by Hora (1969b), Gupta and Kumar (2021). We include these effects by using the dielectric tensor for kappa distributions in the finite but low-temperature approximation without considering the damping characteristics of the wave, and we compare the magnitude of the term that accompanies the variation of the wave amplitude for the kappa and Maxwellian distributions. Hence, this study can also be useful to understand the implications of the plasma temperature on the ponderomotive force interaction.

This article is organized as follows: in section 2, we include the non-thermal effects in the kinetic dielectric tensor, and we obtain asymptotic expressions for low temperatures. We also give a brief analysis of the influence of the kappa parameter on the transverse and longitudinal dispersion relations. Then, in section 3, we include the dielectric tensor in the spatial and temporal terms of the ponderomotive force, and we deduce its expressions. Later, we analyze, for each term of the ponderomotive force and wave mode, the influence of the non-thermal effects, and compare our results with the thermal (Maxwellian) case. In section 4, we use this analysis to study the Washimi and Karpman ponderomotive induced magnetic field for electromagnetic waves. Finally, in section 5, we summarize the main conclusions of this study.

### 2. Dispersion relation for thermal and non-thermal plasmas with low temperature

In this study, we consider a kappa distribution for isotropic 3D plasmas, with its corresponding dispersion function (Summers and Thorne 1991, Hellberg and Mace 2002, Hau et al 2009, Viñas et al 2015, Lazar et al 2016, Livadiotis 2017, Moya et al 2020).

$$f_{\kappa s}(v) = \frac{n_s}{\pi^{3/2} \alpha_s^3 \kappa^{3/2}} \frac{\Gamma(\kappa + 1)}{\Gamma(\kappa - 1/2)} \left(1 + \frac{v^2}{\kappa \alpha_s^2}ight)^{-(\kappa + 1)}, \quad (1)$$

where $f_{\kappa s}$ is the kappa distribution for the species $s$, $n_s$ is their number density, $\alpha_s = \sqrt{2k_B T_s/m_s}$ is their thermal velocity, $k_B$ is the Boltzmann constant, $m_s$ is the mass, $T_s$ is the temperature and $\Gamma$ is the Gamma function.

In this section, we analyze the dispersion relation for high-frequency waves propagating through low-temperature plasmas described by a kappa distribution (1), with no background magnetic field. We consider only the dynamics of electrons with a static background of ions to achieve quasi-neutrality. Thus, we neglect the contribution of the ions to the dispersion relation. A detailed deduction of the dielectric tensor and the
dispersion relation for a non-relativistic magnetized plasma modeled by isotropic kappa distributions can be found in Mace (1996). The dispersion relation for kappa distributed plasmas has also been investigated in a variety of different contexts (Hellberg et al. 2009, Pierrard and Lazar 2010, Kourakis et al. 2012, Lazar et al. 2018). The unmagnetized dispersion relation for transverse and longitudinal modes with respect to the direction of wave propagation are given respectively by,

$$\frac{\varepsilon^2 k^2}{\omega^2} = 1 + \sum_s \frac{\omega_{ps}^2}{\omega k \alpha_s} Z_{s,M} \left( \frac{\omega}{k \alpha_s} \right),$$

(2)

$$1 - \sum_s \left( \frac{\omega_{ps}^2}{k^2 \alpha_s^2} \right) Z_{s,M} \left( \frac{\omega}{k \alpha_s} \right) = 0,$$

(3)

where $\omega$ is the frequency, $k$ is the wave number, $c$ is the velocity of light, $Z_{s,M}$ is the generalized plasma dispersion function (Hellberg and Mace 2002), $\omega_{ps}$ is the plasma frequency for the species $s$ and $Z_{s,M}(\omega) = dZ_{s,M}(\omega)/d\omega$ is the derivative of the generalized plasma dispersion function.

The dielectric tensor for unmagnetized plasmas can be deduced using kinetic theory by linearly perturbing the Vlasov equation. In this way, we get the same result as Mace (1996) when setting the background magnetic field equal to zero. Namely,

$$\varepsilon_\perp(k, \omega) = 1 + \sum_s \frac{\omega_{ps}^2}{\omega k \alpha_s} Z_{s,M} \left( \frac{\omega}{k \alpha_s} \right),$$

(4)

$$\varepsilon_\parallel(k, \omega) = 1 - \sum_s \frac{\omega_{ps}^2}{k^2 \alpha_s^2} Z_{s,M} \left( \frac{\omega}{k \alpha_s} \right),$$

(5)

where $\varepsilon_\perp$ and $\varepsilon_\parallel$ are the dielectric tensor components for the transverse and longitudinal modes, respectively. We notice that if we include in these expressions the dispersion relations (2) and (3) corresponding to the transverse and longitudinal case, we obtain that $\varepsilon_\perp = k^2 c^2/\omega^2$ and $\varepsilon_\parallel = 0$, which is another way of expressing the dispersion relations. However, to calculate the temporal part of the ponderomotive force (see below), we must compute the partial derivative in the expressions (4) and (5) before using the dispersion relations.

We consider low-temperature plasmas so that the phase speed of the waves under study is much larger than the electron thermal speed ($\omega/k \alpha_s \gg 1$). Under this approximation, we make use of the asymptotic expansion of the generalized plasma dispersion function for large arguments, and truncate the series at the lowest order in which the effect of the kappa parameter appears so that the temperature is included in the dielectric tensor. In this way, we expand in $k \alpha_s/\omega$ terms at second order for the transverse component of the dielectric tensor and at third order for the longitudinal component of the dielectric tensor. As a first approach to the study of the non-thermal effects on the ponderomotive force we do not consider the imaginary terms, i.e. the damping characteristics of the wave in the expansion of the generalized plasma dispersion function, which we will leave for future research. Accordingly, we use the following asymptotic expression (Hellberg and Mace 2002):

$$Z_\perp^\prime(\zeta) \approx -\frac{1}{\zeta} \left( 1 + \frac{\kappa}{2 \kappa - 3} \frac{1}{\xi^2} + \frac{3 \kappa^2}{(2 \kappa - 5)(2 \kappa - 3)} \xi^2 \right) + 15 \frac{1}{(2 \kappa - 7)(2 \kappa - 5)(2 \kappa - 3)} \xi^3 + \cdots,$$

(6)

By considering only the electron species, we have the approximated dielectric tensor components for electron waves (Chen 1984) at finite and low temperature for transverse and longitudinal modes, respectively, as follows:

$$\varepsilon_\perp \approx 1 - \frac{1}{\bar{\omega}^2} \left( 1 + \frac{3}{2} \left( \frac{\alpha_e^2}{c^2} \right) \frac{\kappa}{\kappa - 3/2} \frac{k^2}{\bar{\omega}^2} \right),$$

(7)

$$\varepsilon_\parallel \approx 1 - \frac{1}{\bar{\omega}^2} \left( 1 + \frac{3}{2} \left( \frac{\alpha_e^2}{c^2} \right) \frac{\kappa}{\kappa - 3/2} \frac{k^2}{\bar{\omega}^2} \right),$$

(8)

where $\bar{\omega} = \omega/\omega_{pe}$ and $k = kc/\omega_{pe}$ are normalized frequency and wavenumbers. Note that in equation (8) we recover the result obtained by Ziebell et al. for Langmuir waves with a plasma distributed by an isotropic type 1 kappa distribution (Ziebell et al. 2017). We neglect ions as long as the ion-plasma frequency is very low compared to the wave frequency $\omega_{pe} \ll \bar{\omega}$, which is commonly the case due to their large mass. Then, if we include the dispersion relation in the dielectric components, we have that $\varepsilon_\perp = k^2 \omega^2$ and $\varepsilon_\parallel = 0$. Thus, considering that $\alpha_e \ll c$, and we include the dispersion relation replacing $k^2/\omega^2$ with $\varepsilon_\perp$ in (7), we can solve the transverse dielectric component as a function of the frequency:

$$\varepsilon_\perp \approx \left( 1 + \frac{1}{\omega^2} \right) \left( 1 - \frac{1}{2} \left( \frac{\alpha_e^2}{c^2} \right) \frac{\kappa}{\kappa - 3/2} \frac{1}{\omega^2} \right).$$

(9)

Using the asymptotic expressions for the generalized dispersion function (6), the analytical solutions for the dispersion relation for the transverse and longitudinal modes are respectively given by,

$$\bar{\omega}_{\alpha, \perp}^2 = \frac{1}{2} \left( \frac{k^2 + 1}{k^2 + 1} \right) + \frac{1}{2} \sqrt{\left( \frac{k^2 + 1}{k^2 + 1} \right)^2 + 2 \left( \frac{\alpha_e^2}{c^2} \right)^2 \frac{\kappa}{\kappa - 3/2} k^2},$$

(10)

$$\bar{\omega}_{\alpha, \parallel}^2 = \frac{1}{2} \left( \frac{1}{k^2 + 1} \right) \left( 1 + 6 \left( \frac{\alpha_e^2}{c^2} \right)^2 \frac{\kappa}{\kappa - 3/2} k^2 \right),$$

(11)

where we have included the subscript $\kappa$ to make explicit its dependence on kappa, and the subscript $\perp$ ($\parallel$) to indicate the solution for transverse (longitudinal) waves. From the dispersion relations (10) and (11) it can be seen that unlike the electromagnetic waves the electrostatic wave propagation occurs only due to the thermal effect that is contained in the thermal
velocity. It is therefore to be expected that the effect of the spectral index $\kappa$ will be more pronounced for the longitudinal modes than for the transverse modes. In both cases, when $\kappa \to \infty$, we recover classical Maxwellian results, which we denote by the subscript $M$ in terms of the frequency and the ponderomotive force (see below).

Figure 1(a) represents the relative difference of the frequencies for the transverse waves by comparing the kappa and Maxwellian distributions $|\bar{\omega}_{\perp}/\bar{\omega}_{M\perp} - 1|$ as a function of the velocity ratio $\alpha_v/c$ for various values of $\kappa$. As can be seen in this figure, the scaled frequency in the transverse dispersion relation varies with the kappa parameter at the order of $\sim 10^{-3}$ or less relative to the Maxwellian case for non-relativistic thermal velocity values ($\alpha_v/c < 10^{-2}$). This is different to the longitudinal mode, which varies at the order of $\sim 10^{-2}$ with respect to the Maxwellian case, as can be seen in figure 2, which represents the solution for the dispersion relation for longitudinal waves as a function of the scaling frequency $k$ for various values of $\kappa$. It can be seen in equation (7) that the term that contains the kappa parameter is multiplied by $\alpha_v^2/c^2$, so for the transverse waves in non-relativistic plasmas the thermal effects are not significant unless we have a plasma very far from thermal equilibrium with $\kappa \sim 3/2$.

Figure 1(b) represents the relative difference of the frequencies for the transverse waves $|\bar{\omega}_{\perp}/\bar{\omega}_{M\perp} - 1|$ by comparing the kappa and Maxwellian distribution results as a function of the scaled wavenumber $k$ for various values of the spectral index $\kappa$. As can be seen in this figure for lower wavenumbers different from zero, the relative difference between the scaled frequency for the kappa distribution and Maxwellian distributions tends to a maximum, which we demonstrate analytically that occurs at $k = 1$. For non-relativistic plasmas with $\alpha_v/c \sim 10^{-2}$ and $\kappa < 6$ this maximum is of the order of $10^{-5}$. We note that the maximum occurs when the wavelength $\lambda$ is equal to the inertial length $c(2\pi/\omega_p\alpha_v)$ regardless of the thermal velocity or the $\kappa$ parameter.

3. Ponderomotive force

The ponderomotive force is a non-linear phenomenon induced by the interaction of a high-frequency field in a slow timescale motion with the plasma (Kentwell and Jones 1987). The purpose of this study is to investigate how the non-thermal effect described by the kappa distribution impacts the non-linear slow timescale interaction of unmagnetized plasmas with high-frequency electromagnetic fields. In this discussion, we use the Washimi and Karpman ponderomotive force formalism (Karpman and Shagalov 1982) and previous results for the dispersion relations for Lorentzian cold plasmas.

In the absence of a background magnetic field the total Washimi–Karpman ponderomotive force $\mathbf{f}_{ik} = \mathbf{f}_{ik} + \mathbf{f}_{i} = \phi \mathbf{E} + \mathbf{f}_{i}$, due to the electromagnetic field $\mathbf{E}(r,t) = (1/2)[\mathbf{E}(r,t)e^{i(k \cdot r - \omega t)} + \mathbf{E}^*(r,t)e^{-i(k \cdot r - \omega t)}]$, is described by a force $\mathbf{f}_{i}$, associated with the spatial variation of the electric field amplitude $|\mathbf{E}|$.
In this case, the medium has no effect on the ponderomotive force for longitudinal waves, as described by Karpman and Washimi (1977):

$$f_{(s)} = \frac{1}{8\pi} (\varepsilon - 1) \nabla |E|^2,$$

where $\varepsilon$ is the component of the dielectric tensor for the transverse and longitudinal modes, as appropriate. If we also assume that the magnitude of the electric field varies slowly in our time and space scales, and we discard second time derivatives ($\partial|E|/\partial x, \partial|E|/\partial t \ll 1$ and $\partial|E|^2/\partial x \partial t \ll 1$), it can be deduced that the temporal variation part of the ponderomotive force becomes (Washimi and Karpman 1976),

$$f_{(s)} = \frac{k}{16\pi \omega^2} \frac{\partial \omega^2 (\varepsilon - 1)}{\partial \omega} \frac{\partial |E|^2}{\partial t}.$$

Note that to compute the partial derivative of $\varepsilon$ in the temporal term of the ponderomotive force, we have to use equation (7) instead of (9). Now let us analyze the factors $f_{(s)} = (1/8\pi)(\varepsilon - 1)$ and $f_{(s)} = (k/16\pi \omega^2) [\partial \omega^2 (\varepsilon - 1)/\partial \omega]$ that accompany the spatial and temporal variations of the magnitude of the electric field in the ponderomotive force for unmagnetized plasmas of electron species described by kappa distributions.

### 3.1 Spatial ponderomotive force factor for longitudinal waves

In this case, the medium has no effect on the ponderomotive force, since $\varepsilon || = 0$ so is given by

$$f_{(s)}^\parallel = \frac{1}{8\pi} \nabla |E|^2,$$

where the superscript $\parallel$ indicates that we are considering the ponderomotive force for longitudinal waves. We notice that this force is always in the direction of the decreasing amplitude of the wave.

### 3.2 Spatial ponderomotive force factor for transverse waves

If we include the dispersion relation in the dielectric component of the ponderomotive force, using equations (9) and (12), we can deduce that the factor $f_{(s)}^\perp$ that accompanies the spatial variation in the ponderomotive force for the kappa distribution is given by,

$$f_{(s)}^\perp = -\frac{1}{8\pi \bar{\omega}_{\perp}^2} - \frac{1}{16\pi} \left( \frac{\alpha_e^2}{c^2} \right) \left( \frac{\kappa}{\kappa - 3/2} \right) \frac{1}{\bar{\omega}_{\perp}^2} \left( 1 - \frac{1}{\bar{\omega}_{\perp}^2} \right),$$

where the super-index $\kappa$ indicates its dependence in the kappa parameter, $\perp$ indicates that we are considering the ponderomotive force for transverse waves, and $\bar{\omega}_{\perp}$ corresponds to the solution for the dispersion relation for the parameter $\kappa$ for the transverse waves (10). In addition, for the Maxwellian ponderomotive force (when $\kappa \to \infty$) we will denote the super-index $M$. Here, we note that if we neglect our finite temperature correction, i.e. we put $\alpha_e = 0$ in equation (15), then we recover the classical ponderomotive force for electromagnetic waves with spatial inhomogeneities (Washimi and Karpman 1976).

Figure 3 shows the relative difference of the kappa and Maxwellian ponderomotive spatial factors for transverse waves $f_{(s)}^\perp / f_{(s)}^{M\perp} - 1$ as a function of the scaled wave number. It is seen that the magnitude of the ponderomotive force for the kappa distributions is greater compared to the Maxwellian distribution case. Moreover, when the scaled wavenumber decreases, both magnitudes are equal so that the non-thermal effect is canceled. We also show that for $\bar{k} \to \infty$ the rate between the kappa and Maxwellian ponderomotive spatial factors tends to an asymptotic value. Figure 3(b) represents the relative difference of the kappa and Maxwellian ponderomotive spatial factors for transverse waves as a function of $\alpha_e/c$. It can be seen in these figures that the non-thermal effect on the ponderomotive force increases with the thermal velocity scaled with the velocity of light. In this case, we are dealing with non-relativistic thermal velocities, so that the effect...
of the spectral index $\kappa$ is negligible, having a relative difference between the ponderomotive forces of an order less than or equal to $\sim 10^{-4}$ for $\kappa > 2$ and $\alpha_e/c < 10^{-2}$. This result is reasonable considering the previous discussion of the dispersion relation where it was concluded that the thermal effect is not appreciable for non-relativistic thermal velocities.

In the limit of large wavenumbers (which serves as an upper bound), we have that the ratio for the kappa and Maxwellian ponderomotive spatial terms tends to 

$$\left[1 + \frac{1}{2} \left(\frac{\alpha_e^2}{c^2}\right) \left(\frac{\kappa}{\kappa - 3/2}\right) \right] / \left[1 + \frac{1}{2} \left(\frac{\alpha_e^2}{c^2}\right)\right] > 1,$$

where it can be clearly seen that, when $\alpha_e^2/c^2 \ll 1$, the effect of the kappa parameter is canceled. In addition, we can calculate that for $\kappa > 2$ and $\alpha_e/c = 10^{-2}$ the spatial term of the ponderomotive force for Lorentzian plasmas is larger than for Maxwellian plasmas for less than 0.015%.

3.3. Temporal ponderomotive force factor for transverse waves

In this section, we analyze the non-thermal effect on the temporal factor of the ponderomotive force for transverse electron waves. Using the Washimi and Karpman expression of the temporal factor (13) and including the dispersion relation in the transversal dielectric component (7) after making the partial derivative, we find the expression for the temporal term of the ponderomotive force for the transverse waves:

$$f_{\omega \perp}^{\omega \perp} = \frac{1}{16\pi c} \left(\frac{\alpha_e^2}{c^2}\right) \left(\frac{\kappa}{\kappa - 3/2}\right) \frac{\bar{k}^3}{\omega_{\perp}^2}.$$  (16)

From the previous equation it follows that the temporal factor of the ponderomotive force is nonzero only when we consider the effects of the temperature in the dielectric tensor (Washimi and Karpman 1976). Hence, the temporal term of the Washimi and Karpman ponderomotive force for unmagnetized plasmas is a thermal effect. Figure 4 shows the ratio between the kappa and Maxwellian ponderomotive temporal factors for transverse waves as a function of $\bar{k}$. In this figure, it can be seen that the non-thermal effect on the temporal term of the ponderomotive force is very significant. It remains almost constant with respect to the scaled wavenumber. We also deduce from equation (16) that for any value of $\kappa$ the ratio between the forces does not vary significantly with thermal velocity. The effect of the kappa parameter is given mainly by the term $\kappa/(\kappa - 3/2)$. We know from previous dispersion relations that if $\alpha_e/c \ll 1$ then $\omega_{\perp}/\omega_{\perp M} \approx 1$. For non-relativistic velocities we can use the following approximation $f_{\omega \perp}^{\omega \perp} / f_{\omega \perp}^{\omega \perp M} \approx \kappa/(\kappa - 3/2) > 1$.

At first glance, we can see that this expression does not depend on the thermal velocity and only depends on the parameter $\kappa$. We calculate that for $\kappa < 6$ the temporal factor of the ponderomotive force for Lorentzian plasmas is larger than for Maxwellian plasmas by at least 33%. Hence, due to the thermal character of the ponderomotive temporal factor for unmagnetized plasmas, the non-thermal effect is strongly noticeable in the temporal ponderomotive term for transverse waves.

3.4. Temporal ponderomotive force factor for longitudinal waves

Here, we analyze the non-thermal effect on the temporal factor of the ponderomotive force for longitudinal electron waves. Using the Washimi and Karpman’s result for the temporal factor (13), and including the dispersion relation in the longitudinal dielectric component (8) after making the partial derivative, we find the expression for the temporal term of the ponderomotive force for longitudinal waves:

$$f_{\omega \parallel}^{\omega \parallel} = \frac{3}{16\pi c} \left(\frac{\alpha_e^2}{c^2}\right) \left(\frac{\kappa}{\kappa - 3/2}\right) \frac{\bar{k}^3}{\omega_{\parallel}^2},$$  (17)

where the $\parallel$ symbol stands for the ponderomotive force for longitudinal waves, while $\omega_{\parallel}$ corresponds to the solution for the dispersion relation for the parameter $\kappa$ for the longitudinal waves (11).

We show that this equation is very similar to the transverse wave ponderomotive temporal factor. Figure 5 represents the ratio between the kappa and Maxwellian ponderomotive temporal factors as a function of the scaled wave number $\bar{k}$. Hence, by including the longitudinal dispersion relation in the ponderomotive temporal factor, it can be seen in this figure that, for wavenumbers less than some value, the temporal factor is larger for Lorentzian plasmas than for Maxwellian plasmas. In addition, for small wavenumbers, its ratio tends to behave as $\kappa/(\kappa - 3/2) > 1$, which is the same behavior as in the transverse case. Nevertheless, because the longitudinal waves are more affected by the dispersion relation, the non-thermal effect varies with the wavenumber and tends to decrease as the wavenumber increases. For larger wavenumbers, the temporal factor is smaller for Lorentzian plasmas than for Maxwellian plasmas and tends to behave as $((\kappa - 3/2)/\kappa)^{1/2} < 1$.

However, we are not interested in such large wavenumbers, since in the fluid scale on which we are focusing, they lack relevance. We conclude that, as well as for transverse waves,
the non-thermal effect in the temporal ponderomotive term for longitudinal waves is also strongly noticeable.

4. Washimi and Karwan ponderomotive magnetization

According to the work of Washimi and Watanabe (1977), a slowly varying magnetic field \( \mathbf{B}_0 \) is generated by the ponderomotive force of the electromagnetic wave that is slowly varying in time. From the balance of the ponderomotive force with the slowly varying electromagnetic field in the electron-fluid equation of motion, it follows that the induced magnetic field is given by,

\[
\mathbf{B}_0(\mathbf{r}, t) = -\frac{e}{16\pi n_e \omega^2} \frac{\partial |\mathbf{E}|^2}{\partial \omega} \nabla \times (|\mathbf{E}|^2),
\]

where \( e \) is the electron charge. Kim and Jung have calculated the Washimi and Karwan ponderomotive magnetic fields for the non-thermal electrostatic case (Kim and Jung 2009). Hence, in this section, we perform this calculation for the electromagnetic case using the dielectric tensor approximation for transverse waves in low-temperature plasmas, by expanding by one order of magnitude more than in (7). This implies that \( \alpha_e/c \) appears in the ratio of the magnetizations. The dielectric tensor is now given by,

\[
\mathbf{\varepsilon}(\mathbf{k}, \omega) \approx 1 - \frac{1}{\omega^2} \left[ 1 + \frac{\alpha_e^2}{c^2} \left( \frac{\kappa}{\kappa - 3/2} \right) \frac{k^2}{\omega^2} \right] + \frac{3}{4} \frac{\alpha_e^4}{c^4} \left( \frac{\kappa^2}{\kappa - 5/2} \right) \left( \frac{\kappa}{\kappa - 3/2} \right) \frac{k^4}{\omega^4}.
\]

Inserting equation (19) into (18), the magnitude \( B_\kappa \) of the ponderomotive magnetic field for Lorentzian plasmas is obtained as follows:

\[
B_\kappa \approx \frac{c}{8\pi n_e \omega_p e} \left[ \frac{1}{2} \frac{\alpha_e^2}{c^2} \left( \frac{\kappa}{\kappa - 3/2} \right) \frac{k^2}{\omega^2} \right] + \frac{3}{2} \frac{\alpha_e^4}{c^4} \left( \frac{\kappa^2}{\kappa - 5/2} \right) \left( \frac{\kappa}{\kappa - 3/2} \right) \frac{k^4}{\omega^4}.
\]

where \( L \) is the scale length of the intensity of the field. Using this, we can calculate the scaled electron cyclotron frequency \( \bar{\omega}_{ce} = \omega_{ce}/\omega_p e \) generated by the induced magnetic field:

\[
\bar{\omega}_{ce} = \frac{1}{16\pi} \left( \frac{\alpha_e^4}{c^4} \right) \left( \frac{\kappa}{\kappa - 3/2} \right) \frac{k^4}{\omega^4} \left[ 1 + 3 \left( \frac{\alpha_e^2}{c^2} \right) \left( \frac{\kappa}{\kappa - 5/2} \right) \frac{k^2}{\omega^2} \right]
\]

\[
\times \frac{\lambda}{L} \left( \frac{e|\mathbf{E}|}{m_e \lambda_D \omega_{pe}} \right)^2 = \frac{\lambda}{L} \left( \frac{n_e}{\lambda_D \omega_{pe}} \right)^2,
\]

where we have defined the Washimi–Karpman ponderomotive magnetization \( M_p(\kappa, \bar{k}, \bar{\omega}) \) as in Kim and Jung (2009):

\[
M_p(\kappa, \bar{k}, \bar{\omega}) = \frac{1}{16\pi} \left( \frac{\alpha_e^4}{c^4} \right) \left( \frac{\kappa}{\kappa - 3/2} \right) \frac{k^4}{\omega^4} \left[ 1 + 3 \left( \frac{\alpha_e^2}{c^2} \right) \left( \frac{\kappa}{\kappa - 5/2} \right) \frac{k^2}{\omega^2} \right],
\]

with \( \lambda \) the wavelength of the wave, \( \lambda_D(= \alpha_e/\sqrt{2} \omega_p e) \) is the Debye length of the electrons and \( m_e |\mathbf{E}|/m_e \omega_{pe} \) is the quiver velocity (Kourakis and Shukla 2006). Then, analogously to Kim and Jung’s work, a non-thermal effect \( F_{NT} \) can be defined as the ratio of the magnetization for the kappa distribution and for the Maxwellian distribution (\( \kappa \to \infty \)):

\[
F_{NT} = \frac{M_p(\kappa, \bar{k}, \bar{\omega})}{M_p(\kappa \to \infty, \bar{k}, \bar{\omega})} = \left( \frac{\kappa}{\kappa - 3/2} \right) \left[ 1 + 3 \left( \frac{\alpha_e^2}{c^2} \right) \left( \frac{\kappa}{\kappa - 5/2} \right) \frac{k^2}{\omega^2} \right].
\]

In equation (23), it can be seen that the effect of the kappa distribution is mostly represented by the term \( \kappa/(\kappa - 3/2) \) for non-relativistic plasmas where \( \alpha_e/c \ll 1 \), implying that the non-thermal effect is enhanced for lower values of \( \kappa \), as expected.

Figure 6 represents the non-thermal effect \( F_{NT}(\kappa, \bar{k}) \) (23) as a function of the scaled frequency \( \bar{k} \) for different values of \( \kappa \). It is found that the non-thermal effect decreases with an increase in the spectral index \( \kappa \). Therefore, the non-thermal effect of the kappa distribution enhances the induced magnetization due to the electromagnetic ponderomotive interactions in unmagnetized plasmas. We also show in this figure that the effect of the kappa distributions in the ponderomotive magnetization does not depend on the wavenumber because, for non-relativistic plasmas, this effect is mainly subject to the parameter \( \kappa/(\kappa - 3/2) \). It is also important to note that the non-thermal effect changes significantly the ponderomotive magnetization with a relative difference with the Maxwellian case of at least 30% for \( \kappa < 6 \), even if \( \alpha_e \to 0 \). In the non-relativistic limit, we calculate the total radiated power \( P \) averaged in one period produced by the gyromotion of the charges.
Lorentzian plasmas. Since the induced magnetization increases with the wavenumber, it is expected that the non-thermal ponderomotive force for unmagnetized plasmas. This is very relevant for both transverse and longitudinal wave propagation. This factor is also responsible for the generation of a slowly varying magnetic field in the ponderomotive interaction of the electromagnetic waves with the plasma, whose analysis can be used as a plasma diagnostic in non-thermal plasmas. It has been found that the non-thermal effect enhances the ponderomotive magnetization of electromagnetic waves. Consequently, it follows that because the total radiated power is proportional to the fourth power of the ponderomotive magnetization it should increase as the plasma moves away from the thermal equilibrium.

The above results will provide useful information for the analysis and interpretation of phenomena associated with the ponderomotive force due to wave propagation in unmagnetized non-thermal plasmas. It will also provide a useful basis from which to extend the analysis to the case of magnetized plasmas. This is very relevant in space physics phenomena where the presence of external magnetic fields is usually found, and where the spatial and temporal terms of the ponderomotive force are related respectively to the Miller and Abraham forces. Terms associated with other forces, such as magnetic moment pumping, are also added (Lundin and Guglielmi 2007). These forces for Alfvén and cyclotron waves appear in phenomena associated with the acceleration of ions in polar regions, auroral density cavities, the penetration of the solar wind in the magnetosphere, electromagnetic ULF waves in the terrestrial magnetosphere, among others. Hence, this formalism can be extended for external magnetic fields to contribute to the study of non-thermal effects in a variety of space physics phenomena, which we will leave for future investigation. As an example, we extend the procedure developed in this work to the study of Alfvén waves in order to include the effect of non-thermal plasmas in the first ionization potential (FIP) effect. The FIP effect corresponds to the difference in composition of the elements with FIP less than 10 eV between the photosphere and the solar corona (Feldman and Widing 2002). This can be explained as being due to the action of ponderomotive forces due to Alfvén waves propagating up through the chromosphere and either transmitting into or being reflected from coronal loops (Laming 2004, 2015). The study of the effect of the kappa distribution on these phenomena could be useful as a method to evaluate the distribution that exists in the solar corona and therefore to understand the origin of the non-thermal distributions that we can verify in the solar wind and that is still under discussion, see chapters 3 and 7 of Lazar and Fichtner (2021).

In summary, our results provide a useful basis to include thermal and non-thermal effects in phenomena associated with ponderomotive forces for unmagnetized plasmas. This also

![Figure 6. Non-thermal effect $F_{NT}(\hat{k}, \kappa)$, from equation (23), as a function of $\hat{k}$ for different values of $\kappa$ for $\alpha_e/c = 0.01$.](image)
sets the basis for extending it to magnetized plasmas. All the above calculations show the great impact of the non-thermal effects on the temporal factor of the ponderomotive force phenomena in plasmas described by kappa distributions. For future research, this work can be extended to analyze other versions of the kappa distribution (see e.g. Livadiotis 2017). With their application in the phenomena described above, new contributions to the understanding of the distribution of space plasmas can be achieved.

Data availability statement

No new data were created or analyzed in this study.

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