Generic causal determination of thermal model and heat input by using backward simulator

Y Hiranaka¹, S Miura¹ and T Taketa¹
¹ Yamagata University, Yonezawa, Yamagata 992-8510, Japan
E-mail: zioi@yz.yamagata-u.ac.jp

Abstract. Generally, it is difficult to estimate model parameters for time varying output system especially when the input is unknown. One way to estimate them is to repeat simulations to search for the case where the outputs match by changing model parameters and input signals. However, such method is time consuming. Our previous studies show that the backward simulation with an appropriate model can estimate the input signal. Therefore, it is efficient only to change model parameters in the backward simulation if we can properly assume an appropriate model for the system. This paper describes a method using a generic causal model for determination of a thermal model to match the measured temperature change. Such simulator is implemented by using Node-RED. The performance of the simulator is shown for the cases of ideal resolution data, limited resolution data, and real measured data.

1. Introduction
In the paper [1], the thermal room model was created for the temperature measurement and the heat input was estimated by the backward simulation [2-4]. While the parameters of the thermal model [5, 6] were estimated coarsely to match the model equations and searched finely by changing the model parameters, it was still difficult to find the best parameters for a perfect match in the case where we had no exact knowledge of heat input.

However, if we know the model, the input time sequence can be derived by backward simulation applied to the measured output. Although it cannot derive correct inputs if the model is inaccurate. To obtain correct or near correct inputs, it is desirable to generate a model based only on reliable principles. Specifically, we consider a causal model as most reliable, which includes time order relation and conservation laws such as energy conservation. We created a backward simulator to realize it and show its effectiveness.

2. Generic causal model and simulations
2.1. Heat conservation and transfer model
According to the causality law, any change must be caused by some source of change. Energy conservation is a derivation of it. There exists an energy supplier or absorber if an energy storage changes its contents. Assuming a storage of energy quantity u(t), energy supply x(t) and energy loss v(t), the following basic equation must hold

\[ u(t+1) = u(t) + x(t) - v(t), \]

where \( t \) denotes time.

The measured temperature change of a small room (in Figure 8 of the paper [1]) can be represented by a three heat storage model of the vicinity of the heater \( u_1 \), entire room \( u_2 \) and environment \( u_3 \) with one heat input \( x(t) \) and one temperature output \( y(t) \) (Figure 2). Assuming that the amount of heat transfer...
is proportional to the amount of heat, it can be expressed as follows, using the heat transfer coefficients $k_{12}$, $k_{21}$, $k_{23}$, $k_{32}$,

$$v_{12}(t) = k_{12}u_1(t), \quad v_{21}(t) = k_{21}u_2(t), \quad v_{23}(t) = k_{23}u_3(t), \quad v_{32}(t) = k_{32}u_3(t).$$

Figure 3. Three heat storage model with temperature output at the second storage.

### 2.2. Forward simulation

If we limit our thermal system within one heat input and one temperature output, any thermal model can be simplified as shown in Figure 3. The time change can be expressed as



where $C_2$ is the heat capacity of the second storage. If we set $x(t)$ as shown in Figure 4 and $k_{12} = 0.2$, $k_{21} = 0.05$, $k_{23} = 0.02$, $k_{32} = 0.001$, the temperature $y(t)$ is obtained as shown in Figure 5.

Figure 4. Test heat input for forward simulation.

Figure 5. Forward simulated temperature change corresponding to Figure 4.

### 2.3. Backward simulation

The backward simulation is to trace the causal order backward as in Figure 6. The heat storage $u_2(t)$ is determined directly from the temperature $y(t)$, and $u_3(t)$, $u_1(t)$, and $x(t)$ are obtained by

$$u_3(t) = \frac{u_3(t+1) - k_{32}y_2(t)}{k_{32}}, \quad y_1(t) = \frac{u_3(t) - (1-k_{21}k_{23}y_2(t))}{k_{32}},$$

$$x(t) = u_1(t+1) - (1-k_{12}u_1(t)) - k_{21}u_2(t).$$

Figure 6. Backward casual trace corresponding to Figure 3.
The heat capacities $C_1$, $C_3$ of the storages $u_1$ and $u_3$ have the relations

$$\frac{C_1}{C_2} = \frac{k_{21}}{k_{12}},$$

$$\frac{C_3}{C_2} = \frac{k_{23}}{k_{32}}.$$ 

As values other than $y(t)$ can be arbitrarily multiplied by a constant, $C_1 / C_2$ and $C_3 / C_2$ can be arbitrarily determined. They were set to 1/4 and 1/20, respectively, to match the forward simulation, specifically $k_{21} = k_{12}/4$ and $k_{32} = k_{23}/20$.

Since $u_1$, $u_2$, $x$ are energy values, they are nonnegative. If one of them is negative in the backward simulation, it means that the simulation failed, which indicates the combination of simulation parameters $k_{12}$, $k_{23}$, $k_{20}$, $k_{32}$ are inappropriate. In order to allow some error, we set 0.00001 as allowance error, and it is judged as nonconformance when it becomes less than -0.00001.

Since backward simulation goes back in time, the last values of $u_1$, $u_2$ and $u_3$ are needed initially. However $u_1$ and $u_2$ do not depend on the initial values in the backward simulation. As for $u_3$, we have to set the initial value $u_{3last}$ and there is a possibility of the range from zero to $k_{23}/k_{32}u_2$. The range was divided into 100 and simulated individually. Also, $k_{12}$ and $k_{23}$ are changed in the range from 0 to 1 divided by $ndiv$, which is increased to obtain higher resolution results.

3. Simulator implementation

The simulator was implemented by using Node-RED [7], which enables quick programming, easy debugging and web visualization. By sequentially changing $k_{12}$, $k_{23}$, $u_{3last}$, we check whether the simulation is successful, which means that the parameters $k_{12}$, $k_{23}$ and the heat input sequence $x(t)$ that match the temperature data exist in the non-negative range of $u_1$, $u_3$ and $x$.

4. Results

4.1. Test data with ideal resolution

Figure 7 shows the success area for the test data (Figure 5, 32-bit precision floating point, 100 time points). The bottom horizontal line corresponds roughly to the minimum value of $k_{23}$ and the lower left diagonal line corresponds to the value of $k_{12} + k_{23}$, which determines the slowest decline of the temperature change $y(t)$. Therefore, the maximum of $k_{12}$ is the value at the bending point. The upper limit of $k_{23}$ is near $k_{12}$ because $k_{12}$ and $k_{23}$ are interchangeable on the model in Figure 3.

![Figure 7](image)

**Figure 7.** Conforming parameter range for unlimited resolution data for $ndiv=1024$.

![Figure 8](image)

**Figure 8.** Backward simulated heat input results for the points indicated in Figure 7.
4.2. Test data with limited resolution

In actual measurements, the measurement accuracy is limited. Therefore, we created data from the above test data by truncating to the second decimal place. Figure 9 shows the results for that data, which is similar to Figure 7, while the difference is seen in the lower left. As the small changes in the trailing decay points in Figure 5 are truncated, small values of $k_{23}$ also fall within the conforming range. From the figure, it is possible to estimate $k_{12}$, while it indicates the necessity of countermeasure against the quantization error to obtain good estimation of $k_{23}$.

![Figure 9. Conforming parameter range for the limited resolution data for ndiv=256.](image)

Figure 9. Conforming parameter range for the limited resolution data for ndiv=256.

4.3. Real measurement data

The result of the backward simulation for the measured data (in Figure 8 of the paper [1]) is shown in Figure 10, which shows only the lower limit parameter as the limit of the upper side is 1.0 for all ndiv. For noisy data, parameter $k_{23}$ would be large in order to cope with rapid changes of the noise. If the resolution of the backward simulation is low, the conforming range is wide because parameter $k_{23}$ does not need to care about noise smaller than the resolution. Figure 10 indicates that $k_{12}$ is around 0.4. For obtaining $k_{23}$, noise countermeasure is necessary.

![Figure 10. Conforming parameter range for the real data for ndiv=64, 128, 256.](image)

Figure 10. Conforming parameter range for the real data for ndiv=64, 128, 256.

5. Discussion

It was shown that the backward simulation effectively estimate at least one model parameter $k_{12}$ for the cases of test data and real measurement data. Accuracy improvement is necessary to estimate $k_{23}$ and verify the result for real data. Since the necessary parameters are $k_{12}$ and $k_{23}$ only, calculation time would be shortened if the values are examined only at the bending point of Figures 7, 9 and 10.

As the energy conservation is a linear law, similar results can be obtained even if the measured values obey a nonlinear expression by converting into a linear preservation amount. Node-RED took about 30ms for one turn of the backward simulation with time sequence of 100 points. It is necessary to create a speed-oriented simulator program for quick processing of large combination of parameters.

6. Conclusion

The backward simulation is a practical and promising model estimation method which can derive both model parameters and input sequence. It is not necessary to determine a mathematical model if the model can be expressed in a generic causal model. We are considering applications to targets other than one-input-one-output model as well as applications to natural phenomena other than heat transfer.

References

[1] Hirakawa Y, Miura S and Taketa T 2017 Hybrid backward simulator for determining causal heater state with resolution improvement of measured temperature data through model conformation ACTA IMEKO 6 13-19
[2] Huang C and Wang H 2009 Backward Simulation with Multiple Objectives Control Proc. IMECS
[3] Hirakawa Y and Taketa T 2012 Designing backward range simulator for system diagnoses Proc. XX IMEKO World Congress
[4] Oguni K 2011 Inverse problem and instrumentation (Tokyo: Ohmsha)
[5] Beck J, Blackwell B and Clair Jr C 1985 Inverse Heat Conduction Ill-posed Problems (John Wiley & Sons)
[6] Jarny Y and Maillet D, Linear Inverse Heat Conduction Problem – Two Basic Examples, http://www.sfl.asso.fr/Local/sfl/dir/user-3775/documents/actes/Metis_School/Lectures%26Tutorials-Texts/Text-L10-Jarny.pdf
[7] Node-RED https://nodered.org/