Interaction spatial barrier distributions for the proton-halo system $^{8}\text{B}+^{58}\text{Ni}$

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Abstract. Radial barrier distributions for the proton-halo system $^{8}\text{B}+^{58}\text{Ni}$ are obtained by the use of Woods-Saxon polarization potentials calculated from a fit to elastic scattering angular distribution data. The strength, diffuseness and reduced radii of the polarization potentials, besides fitting the data, are such that reproduce the barrier height and position as extracted from a Wong’s fit to the universal behavior of reduced interaction cross sections for halo systems. It is found that, for the proton-halo system $^{8}\text{B}+^{58}\text{Ni}$, the interaction barrier height $V_I$ and the interaction barrier radius $R_I$ result correspondingly lower and larger than the Coulomb nominal barrier height $V_B$ and barrier position $R_B$ as expected from Barrier Penetration Models and double folding density dependent interactions such as the São Paulo Potential. It is also shown that, the determined energy dependent polarization potentials, besides satisfying the dispersion relation, present a behavior consistent with the Breakup Threshold Anomaly.

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1. Introduction

Recently, elastic scattering measurements[1] of $^{8}\text{B}$ and $^{7}\text{Be}$ on a $^{58}\text{Ni}$ target were shown to provide strong evidence for proton-halo effects on $^{8}\text{B}$ reactions. It was established that a characteristic feature expected for any halo state, such as $^{8}\text{B}$ or $^{6}\text{He}$, an underlying decoupling between the core and the valence nucleons occurs. That is, the $^{8}\text{B}$ ($^{6}\text{He}$) reaction cross section could be entirely accounted for, by breakup (transfer) of the halo state ($p$ or $2n$) plus reactions that occur with the $^{7}\text{Be}$ ($^{4}\text{He}$) core[2, 3, 4]. It was also pointed out that, in the space of reduced cross sections and reduced energies, the interaction cross sections for $^{8}\text{B}$ (proton halo) and $^{6}\text{He}$ (neutron halo) with different targets, lie on the same trajectory. Similarly, “normal” weakly bound (non halo) and tightly bound systems follow characteristic paths. That is, in the space of reduced cross sections defined by,

$$
\sigma_{\text{red}} = \frac{\sigma_R}{\eta}, \quad \eta = \left(\frac{A_P}{3} + \frac{A_T}{3}\right)^2,
$$

(1)

where $A_P$ and $A_T$ are the mass numbers of the projectile and target nuclei, and $\sigma_R$ is the reaction cross section, and reduced energies given by,

$$
E_{\text{red}} = \frac{E_{\text{cm}}}{\xi}, \quad \xi = \frac{Z_PZ_T}{\eta^{1/2}},
$$

(2)
Figure 1. Reduced total reaction cross sections for a number of systems. Data for \( ^8\)B+\(^{58}\)Ni, \( ^7\)Be+\(^{58}\)Ni, \( ^6\)Li+\(^{58}\)Ni from Ref.[1], \( ^6\)Li+\(^{64}\)Zn, \( ^9\)Be+\(^{64}\)Zn, \( ^{16}\)O+\(^{64}\)Zn from Ref.[5], \( ^6\)He+\(^{64}\)Zn from Ref.[6], \( ^6\)He+\(^{59}\)Co from Ref.[7], \( ^6\)He+\(^{209}\)Bi from Ref.[8], and \( ^6\)He+\(^{27}\)Al from Ref.[9].

where \( Z_P \) and \( Z_T \) are the charges of the nuclei. Reactions with several types of projectiles describe different behaviors regardless of the nuclear structure, binding energies or reaction mechanisms. This feature is shown in Fig.1, for a number of systems. It is seen that for an energy range from \( 0.68 \text{ MeV} < E_{\text{red}} < 2.1 \text{ MeV} \), the trajectory described by the halo systems \( ^8\)B+\(^{58}\)Ni and \( ^6\)He+\(^{209}\)Bi lies above that for “normal” weakly bound and, this in turn, is above that for the tightly bound system \( ^{16}\)O+\(^{64}\)Zn. Thus, it could be established that, if the trajectories of Fig.1 should be characterized by the mean value of the projectile binding energies, then these trajectories lie progressively lower as the binding energy increases. Also shown in Fig.1 are Wong model fits illustrated by the curves to the three different trajectories as described by Kolata et al.[2]. The Wong model[10] gives a simple expression for the interaction cross section as a function of the energy \( E_{\text{c.m.}} \) in terms of the s-wave interaction barrier. It depends on three parameters, namely the barrier height \( V_0 \), the barrier position \( R_0 \) and the curvature \( \bar{\hbar}\omega_0 \), i.e.,

\[
\sigma = (R_0^2\hbar\omega_0/2E_{\text{c.m.}})\ln\{1 + \exp[2\pi(E_{\text{c.m.}} - V_0)/\hbar\omega_0]\},
\]

where the interaction barrier radius \( R_0 \) is defined by \( R_0 = r_0b(\bar{A}_P^{1/3} + \bar{A}_T^{1/3}) \). Thus, the Wong model fit for a given system consists of finding the parameters that best adjust the experimental reaction cross section data. In terms of reduced parameters, Eq.(3) can be written as[4],

\[
\sigma_{\text{red}} = (r_0^2\epsilon_0/2E_{\text{red}})\ln\{1 + \exp[2\pi(1/\epsilon_0)(E_{\text{red}} - V_{\text{red}})]\},
\]

where the reduced curvature, barrier position and barrier height are respectively given by,

\[
\epsilon_0 = \hbar\omega_0/\xi, \quad r_0b = R_0\eta^{-1/2}, \quad V_{\text{red}} = V_0/\xi,
\]

where \( \xi \) and \( \eta \) are given by Eqs.(1) and (2). As discussed in Ref.[4], the Wong-model fit for halo-systems shown by the dashed-line of Fig.1 is achieved with the reduced parameters, \( V_{\text{red}} = 0.78 \pm 0.014 \text{ MeV} \), \( r_0b = 1.79 \pm 0.04 \text{ fm} \) and \( \epsilon_0 = 0.43 \pm 0.06 \text{ MeV} \). It is very interesting to point out that, these reduced parameters were determined by fitting experimental reduced reaction cross section data of the neutron-halo projectile \( ^6\)He with a variety of targets, measured by different authors and by different technics. It is remarkable that the experimental reduced reaction cross section data for the proton-halo \( ^8\)B with \(^{58}\)Ni extracted from the measurements
of Ref.[1], is also well described by the same dashed-curve of Fig.1. Thus, from the “universal” halo-reduced parameters just obtained, i.e., \( V_{\text{red}}, r_{\text{ob}} \) and \( \epsilon_0 \), the corresponding Wong-model parameters of Eq.(3), for the interaction barrier of the \(^8\text{B}+\text{\textit{Ni}}\) system are,

\[
V_I = 18.6 \pm 0.33 \text{ MeV}, \quad R_I = 10.5 \pm 0.23 \text{ fm}, \quad \hbar \omega_I = 10.25 \pm 0.2 \text{ MeV}.
\]  

(6)

It is interesting to notice that, for this nuclear system, the interaction barrier height \( V_I \) and barrier position \( R_I \) are correspondingly lower and larger, than the Coulomb barrier height and barrier position, as determined by Barrier Penetration Models (BPM)[11] or double-folding density dependent potentials like the São Paulo Potential (SPP)[12, 13], that predict \( V_B = 20.8 \text{ MeV} \) and \( R_B = 8.89 \text{ fm} \). However, it is important to notice, that when the polarization potential is included, the interaction barrier can be modified by the action of this potential. A possible explanation is twofold, namely, the low threshold energy of \(^8\text{B} \) (0.137 MeV) that facilitates the production of breakup reactions \((^8\text{B} \rightarrow p^+\text{Be})\) and thus lowers the barrier. And the large radial density distribution of the halo structure of \(^8\text{B} \), whose effect could be of “pushing” the nuclear potential to larger distances and thus the barrier. These two effects can be due to the action of the polarization potential, that accounts for couplings between the elastic channel and the different absorption mechanisms, i.e., fusion, breakup and inelastic reactions. It is the purpose of the present work to calculate the interaction barrier \( V_I \) and barrier position \( R_I \), by calculating the effect that the polarization potential \( V_{\text{pol}} \) has on the barrier. This potential has real and imaginary parts, \( V_{\text{pol}} = V_{\text{real}} + iW_{\text{pol}} \), where the real part \( V_{\text{pol}} \) results from virtual excitations to the non-elastic channels and \( W_{\text{pol}} \) describes the loss of incident flux. The energy dependent real and imaginary potentials \( V_{\text{pol}}(E) \) and \( W_{\text{pol}}(E) \) are linked by the dispersion relation[14, 15], and should be such that comply with the Breakup Threshold Anomaly (BTA) as proved be the case for many weakly bound systems.

The real and imaginary parts of the polarization potential are chosen to have Woods-Saxon volumetric geometrical shapes with the same diffuseness and reduced radii parameters. The potential parameters, i.e., the diffusiveness \( a_{\text{pol}} \), reduced radius \( r_{\text{pol}} \) and the strengths \( V_{0_{\text{pol}}} \) and \( W_{0_{\text{pol}}} \) are determined from fittings to elastic scattering angular distributions of Ref.[1] at the energies \( E_{\text{c.m.}} = 18.17, 20.56, 22.23, 23.9 \) and \( 25.75 \text{ MeV} \) from which reaction cross sections are extracted. The real part of the polarization potential \( V_{\text{pol}} \) should be such that, when added to the bare potential \( V_{\text{bare}} \) of the type SPP of Ref.[12, 13], correctly gives the interaction barrier height and barrier position. It is expected that the calculated values of the parameters \( a_{\text{pol}} \) and \( r_{\text{pol}} \) can be related to the large radial extension of the halo projectile \(^8\text{B} \) and be such that push the barrier to higher distances as predicted by the Wong model.

In the next section, the basic features of the model used are presented, section 3 is dedicated to the results of the calculations and finally a summary is given.

2. Brief Model Description.

The Hamiltonian \( H \) for the nuclear system is of the form,

\[
H_a = T_a + V_a,
\]  

(7)

where the distorted wave \( \chi_a^{(+)} \) in the incident elastic channel \( a \) satisfies the expression,

\[
(T_a + V_a)\chi_a^{(+)} = E_a\chi_a^{(+)}.
\]  

(8)

The potential \( V_a \) is defined by,

\[
V(r,E) = V_{\text{cout}}(r) - V_{\text{bare}}(r,E) - V_{\text{pol}}(r,E),
\]  

(9)
where the subindex $a$ has been suppressed. Here, $V_{\text{bare}}(r, E)$ is a double-folding density dependent nuclear potential (São Paulo Potential).

$$V_{\text{bare}}(r, E) = V_{\text{SPP}}(r, E) = V_F(r)e^{-4v^2/c^2}.$$  \hspace{1cm} (10)

where,

$$v^2(r, E) = \frac{2}{\mu} [E - V_{\text{Coul}}(r) + V_{\text{bare}}(r, E) + V_{\text{pol}}(r, E)],$$  \hspace{1cm} (11)

is the relative velocity of the colliding nuclei and,

$$V_F(r) = \int \rho(r_1)\rho(r_2)V_0\delta(r - r_1 + r_2)dr_1dr_2.$$  \hspace{1cm} (12)

An extensive systematics of nuclear densities of the SPP has been proposed by L.C. Chamon et al.,[12, 13], that is, within the two-parameter Fermi distribution approach to describe the densities, the radii of the distributions are given by,

$$R_{\text{density}} = 1.31A^{1/3} - 0.84,$$  \hspace{1cm} (13)

where $A$ is the number of nucleons in the nucleus. In a zero-range approximation the strength $V_0 = -456$ MeV-fm$^3$ is assumed. Also, within the systematics, the matter diffuseness $a$ of the density distributions has an average value $a = 0.56$ fm throughout the periodic table. The normalization of the double-folding potential $e^{-4v^2/c^2}$ arising from the Pauli non-locality due to exchange of the nucleons vanishes at near barrier energies, and therefore the São Paulo potential becomes an usual double-folding potential.

$V_{\text{pol}}(r, E)$ of Eq.(9), is defined by $V_{\text{pol}}(r, E) = V_{\text{pol}}(r, E) + iW_{\text{pol}}(r, E)$, where the real and imaginary potentials $V_{\text{pol}}$ and $W_{\text{pol}}$, are Woods-Saxon volume potentials with the same reduced radii $r_{VW}$ and diffuseness $a_{VW}$ parameters, i.e.,

$$V(W)_{\text{pol}}(r, E) = \frac{V(W)_{\text{pol}, 0}(E)}{1 + \exp[(r - R_{VW})/a_{VW}]}.$$  \hspace{1cm} (14)

where $R_{VW} = r_{VW}(A_p^{1/3} + A_T^{1/3})$. The potential strengths $V_{\text{pol}, 0}(E)$, $W_{\text{pol}, 0}(E)$, the reduced radius $r_{VW}$ and the diffuseness $a_{VW}$ are determined during the fitting of elastic scattering angular distributions for $^8\text{B}$ and $^{58}\text{Ni}$. It is expected that the polarization potentials also satisfy the dispersion relation[14, 15],

$$V_{\text{pol}}(R_{sa}, E) = \frac{1}{\pi} \frac{P}{\int_0^{\infty} W_{\text{pol}}(R_{sa}, E') dE'},$$  \hspace{1cm} (15)

at the strong absorption radius $R_{sa}$. Finally, once the parameters of the polarization potentials are obtained, the interaction barrier $V_I$ will be given by;

$$V_I(r, E) = V_{\text{Coul}}(r) - V_{\text{bare}}(r, E) - V_{\text{pol}}(r, E).$$  \hspace{1cm} (16)

3. Results.

The bare potential of Eq.(10) is a parameter-free potential, that is, for a given nuclear system, it depends only on the incident energy. Nuclear matter densities are calculated with matter radii and diffuseness determined by the systematics. The Coulomb barrier calculated with the bare potential $V_{\text{bare}}$ gives a barrier height and barrier position of approximately 20.8 MeV and 8.89 fm respectively for all the energies for which elastic scattering angular distributions were measured[1]. In order to "push" the interaction barrier to larger distances as required by Eq.(6),
Figure 2. Elastic scattering angular distributions as obtained from the analysis of elastic scattering angular distributions of $^8\text{B}+^{58}\text{Ni}$. Data from Ref.[1].

A large reduced radius parameter $r_{VW}$ of the polarization potentials $V_{\text{pol}}$ and $W_{\text{pol}}$ is needed. The values of the parameters of Eqs.(14), resulting from a $\chi^2$-analysis of elastic angular distributions of $^8\text{B}+^{58}\text{Ni}$ are listed in Table 1 and the results are shown in Fig.2. Table 1, also shows the calculated $\sigma_{R,\text{the}}$ and experimental $\sigma_{R,\text{exp}}$ reaction cross section as extracted from the angular distributions.

| $E_{\text{c.m.}}$ | $W_{\text{pol},0}$ | $V_{\text{pol},0}$ | $a_{VW}$ | $r_{VW}$ | $\chi^2/N$ | $\sigma_{R,\text{exp}}$ | $\sigma_{R,\text{the}}$ |
|---------------|----------------|----------------|---------|---------|----------|----------------|----------------|
| 18.17 MeV     | 89.1           | 12.1           | 0.32    | 1.67    | 0.10     | 198 $\pm$ 10  | 198.6          |
| 20.56 MeV     | 31.7           | 10.2           | 0.26    | 1.67    | 0.54     | 365 $\pm$ 20  | 367.8          |
| 22.23 MeV     | 18.3           | 8.3            | 0.25    | 1.66    | 0.40     | 515 $\pm$ 20  | 518.9          |
| 23.90 MeV     | 18.1           | 4.1            | 0.30    | 1.66    | 0.73     | 827 $\pm$ 10  | 825.2          |
| 25.75 MeV     | 18.0           | 5.6            | 0.33    | 1.66    | 0.71     | 1007 $\pm$ 40 | 1004.6         |

From eqs.(14) and (16), the interaction barrier can now be calculated for the different energies. In Fig.3, $V_I(r, E) = V_{\text{Coul}}(r) - V_{\text{bare}}(r, E) - V_{\text{pol}}(r, E)$ is shown for $E_{\text{c.m.}} = 18.17$ MeV. As seen, due to $V_{\text{pol}}$, the interaction barrier height and position are much closer to the values of Eq.(6), obtained from reduced cross sections. Fig.4, shows the results for the other energies, for which similar values are obtained.

It is important to point out, that $V_{\text{pol}}$ includes the effects on the barrier, from all the possible reaction channels, that is, the attractive polarization effect due to couplings to the fusion channel, the attractive effect from inelastic couplings and the repulsive contribution from breakup processes. However, all of these absorption couplings, contribute to the total reaction as attractive potentials (minus sign in Eq.(16)). Of course, the fusion barrier, should include the separate attractive or repulsive effects from the different polarization potentials with their respective signs.

The Threshold Anomaly present in reactions with tightly bound nuclei, is deduced by the correlated behavior between the energy dependent potentials $V_{\text{pol}}(r, E)$ and $W_{\text{pol}}(r, E)$. That is, as the collision energy between the nuclei is decreased towards the Coulomb barrier $V_B$, the absorptive imaginary potential $W_{\text{pol}}(r, E)$ also decreases, a fact that is related to the closure
Figure 3. Barrier height and position at $E_{c.m.} = 18.17$ MeV as obtained with the total nuclear potential, $V_I(r, E) = V_{Coul}(r) - V_{bare}(r, E) - V_{pol}(r, E)$.

Figure 4. Barrier height and position at $E_{c.m.} = 20.56, 22.23, 23.9$ and 25.75 MeV with corresponding $V_I(r, E) = V_{Coul}(r) - V_{bare}(r, E) - V_{pol}(r, E)$.

of reaction channels. On the other side, the real potential, $V_{pol}(r, E)$ shows an increasing-decreasing behavior (bell-shape) just around the barrier, producing a more attractive potential, when added to the bare nuclear potential. Thus, the net effect of $V_{pol}$ is to lower the Coulomb barrier and consequently fusion is enhanced. Recent studies show that this is not the case, when weakly bound nuclei are involved[16, 17, 18]. Instead, weakly bound projectiles, with low threshold energies produce strong couplings between the breakup/transfer and elastic channels. That is, for energies around and below the Coulomb barrier, reactions with weakly bound nuclei show strong direct reaction yields. For this reason the imaginary potential $W_{pol}(r, E)$ can not sharply decrease, as it does for tightly bound nuclei. The real polarization potential $V_{pol}(E)$ should include contributions from polarization effects coming from all the possible reaction mechanisms. That is, the attractive polarization effects from fusion and inelastic couplings that lower the fusion barrier and, the repulsive contribution from couplings to the breakup channel. Hence, the former effects enhance fusion while the latter suppresses it. For this reason the energy behavior of the net $V_{pol}(E)$ has not the bell-shape form around the Coulomb barrier as
for the case of tightly bound nuclei. Instead, as the energy goes below the Coulomb barrier, \( V_{\text{pol}}(E) \) becomes dominated by the increasing repulsive contribution from breakup effects. This correlated behavior between the real and imaginary polarization potentials is known as the Breakup Threshold Anomaly (BTA)\[19\]. It is expected that the BTA also be present for halo systems as is the case for \(^8\)B\(^+\)\(^{58}\)Ni as seen in Fig.5. Here, \( V_{\text{pol}}(E) \) and \( W_{\text{pol}}(E) \) are the strengths of the polarization potentials evaluated at the strong absorption radius \( R_{\text{sa}} \). It is clear that \( W_{\text{pol}}(E) \) does not decrease even for energies lower than the Coulomb barrier, indicating the reaction channels remain open. On the other hand, the increasing strength of \( V_{\text{pol}}(E) \) manifests that the rising of the barrier due to the breakup couplings are stronger as the energy is lowered.

In summary, the total reaction barrier height and position of the system \(^8\)B\(^+\)\(^{58}\)Ni is calculated by means of Woods-Saxon polarization potentials whose parameters are calculated by a \( \chi^2 \)-analysis of elastic scattering data. It has been found that, when these polarization potentials are added to a bare double-folding density dependent nuclear potential (São Paulo Potential), the barrier height and position of the total nuclear potential agree with the values extracted from the Wong’s fit to reduced total cross sections and energies for halo systems. The effect of the polarization potentials is to displace the interaction barrier to a higher position from the usual Coulomb barrier position. This fact should be related to the spatial extension of the proton-halo structure in \(^8\)B that pushes the interaction barrier to larger distances. On the other hand, the interaction barrier height (18.6 MeV) is smaller than the corresponding Coulomb barrier height (20.8 MeV). This result should be connected to the small breakup threshold energy of \(^8\)B (0.137 MeV), that causes breakup reactions \(^8\)B \( \rightarrow \) \(^7\)Be + \( p \) to begin at large distances and to continue being appreciable even at small collision energies. This last fact, is also observed in the presence of the Breakup Threshold Anomaly in the reaction \(^8\)B\(^+\)\(^{58}\)Ni.

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