Multi-objective optimization of multilayer passive magnetic shield based on genetic algorithm

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The performance of a multilayer magnetic shield directly affects and limits the sensitivity improvement of an atomic magnetometer. To better meet the requirements of spin-exchange relaxation free atomic magnetometer for the environmental magnetic field, the magnetic shield should be optimized. At present, the optimizations have focused only on a single objective, such as the axial shielding factor. However, the importance of other goals should not be neglected. In this paper, multiobjective optimization of the shield is carried out to obtain a better comprehensive performance. First, according to the structural characteristics of the multilayer shield, a multiobjective optimization model is established. Then, a multiobjective genetic algorithm is utilized to optimize the shield. After optimization, a Pareto optimal solution set is obtained. Furthermore, depending on the desired design requirements, two sets of optimal combinations of target values and variable parameters are selected, based on the fuzzy comprehensive evaluation method and the lowest magnetic noise. This method can obtain a balance between different optimization objectives and effectively improve the comprehensive performance of the shield.

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I. INTRODUCTION

Atomic magnetometer is a kind of weak magnetic sensor that senses magnetic fields using the polarized atomic ensemble, which experiences Larmor precession in the applied magnetic field. An atomic magnetometer working in the spin-exchange relaxation-free mode is called spin-exchange relaxation free (SERF) atomic magnetometer. Since it was first proposed in 2002, the SERF magnetometer has been widely researched by many research institutions and scholars (both domestic and foreign). In 2010, a superhigh sensitivity of 0.16 fT/Hz was obtained. Theoretically, the potential sensitivity is below 0.01 fT/Hz. A SERF magnetometer has important advantages such as high measurement accuracy and easy miniaturization. Therefore, it is widely used in many fields, such as magnetic measurement of very weak substance molecules, fundamental physics, and brain science. With the development of technology, chip-level SERF magnetometers have appeared, and gradually, they are becoming the future development trend. The key to achieving ultrahigh sensitivity is to make the shielding system attenuate the external stray magnetic field to a very weak level (below 10 nT). Besides, the chip-level atomic magnetometer requires a smaller size shield. The performance of the passive magnetic shield directly affects and limits further improvements in the sensitivity. Therefore, it is necessary to optimize the multilayer shield.

The optimum design of a magnetic shield mainly focuses on theoretical analyses and finite element simulation. In 2005, Paperno et al. used a numerical calculation method to optimize the interlayer gap of a two-layer magnetic shield. Burt and Ekstrom studied the effects of clearance, end cover shape, end cover opening, and clearance between the mating surfaces on the axial shielding factor through numerical simulation. Sergeant et al. optimized the active and passive shields of an induction heating system using a genetic algorithm. Sergeant et al. compared three methods of calculating and evaluating the shielding effectiveness of a cylindrical ferromagnetic shield, namely, analytical method, finite element
method, and neural network.\textsuperscript{23} Although there are intelligence algorithms such as the genetic algorithm, particle swarm optimization and neural network play an important role in developing optimal designs in many industries and fields.\textsuperscript{21–24} However, there are a few literature on the optimization of the magnetic shield by using these intelligent algorithms. Besides, the existing literature mainly involves single objective optimization while lacking multiobjective and multiparameter optimization of the magnetic shield.

A multiobjective optimization based on the genetic algorithm is proposed to optimize a four-layer shield. First, a multiobjective optimization model is established. The optimization objectives are “the highest shielding factor,” “the smallest volume,” and “the lowest magnetic noise.” Then, the multiobjective genetic algorithm of the MATLAB toolbox is utilized to solve the problem. Finally, a series of Pareto optimal solutions and the corresponding design variables are obtained. The set of optimization objective solutions can be selected based on the design requirements.

II. SHIELDING PRINCIPLE

A. Shielding theory

When evaluating the shielding performance, it is preferable to use the shielding factor as a figure of merit. The shielding factor $S$ characterizes the ability of the multilayer screen to attenuate external unwanted magnetic fields. It can be defined as follows:

$$ S = \frac{B_o}{B_i}, $$

where $B_o$ is the ambient magnetic field and $B_i$ is the remaining magnetic field at the same position in the shield.

The shielding performance mainly depends on the shielding material, structure, and size parameters. In the process of production, assembly, and transportation, the magnetic shields may be affected by stress, which will cause degradation in the shielding performance. Therefore, demagnetization is required before using the magnetic shield. Degaussing has a significant influence on the shielding performance. For a cylindrical multilayer magnetic shield, the shielding factors mainly include the transverse shielding factor $S_{tot}^T$ and axial shielding factor $S_{tot}^A$. The formula for the shielding factor of a multilayer concentric shield with no opening can be described as:

$$ S_{tot}^T = 1 + \sum_{i=1}^{n} S_i^T + \sum_{i=1}^{n} \sum_{j=1}^{n} S_i^T S_j^T \left( 1 - \left( \frac{R_j}{R_i} \right)^2 \right) + \cdots $$

$$ + \sum_{i=1}^{n-1} \sum_{j=1}^{n-1} \sum_{k=1}^{n} S_i^T S_j^T S_k^T \left( 1 - \left( \frac{R_k}{R_i} \right)^2 \right) \left( 1 - \left( \frac{R_j}{R_i} \right)^2 \right) + \cdots $$

$$ + \prod_{i=1}^{n} S_i^T \left( 1 - \left( \frac{R_i}{R_{i+1}} \right)^2 \right), $$

and

$$ S_{tot}^A = 1 + \sum_{i=1}^{n} S_i^A + \sum_{i=1}^{n} \sum_{j=1}^{n} S_i^A S_j^A \left( 1 - \left( \frac{L_j}{L_i} \right) \right) + \cdots $$

$$ + \sum_{i=1}^{n-1} \sum_{j=1}^{n-1} \sum_{k=1}^{n} S_i^A S_j^A S_k^A \left( 1 - \left( \frac{L_k}{L_i} \right) \right) \left( 1 - \left( \frac{L_j}{L_i} \right) \right) + \cdots $$

$$ + \prod_{i=1}^{n} S_i^A \left( 1 - \left( \frac{L_i}{L_{i+1}} \right) \right). $$

In Eqs. (2) and (3), $S_i^T$, $S_j^T$, $S_k^T$, and $S_l^T$ are the $i$-layer, $j$-layer, $k$-layer, and $n$-layer transverse shielding factors. $S_i^A$, $S_j^A$, $S_k^A$, and $S_l^A$ are the $i$-th layer, $j$-th layer, $k$-th layer, and $n$-th layer axial shielding factors. $L_i$ and $R_i$ are the average length and radius of $i$-layer, respectively. The $i$-layer transverse shielding factor $S_i^T$ is defined as follows:

$$ S_i^T = \frac{u_i t_i}{2 R_i}, $$

where $u_i$ is the relative permeability of $i$-layer, $t_i$ is the thickness of $i$-layer. The $i$-layer axial shielding factor $S_i^A$ is expressed as

$$ S_i^A = (1 + 4 N_i S_i^T) / (1 + R_i / L_i), $$

where $N_i$ is the demagnetizing factor, which is defined as follows:

$$ N_i = \frac{1}{R_i^2 - 1} \left( \frac{p_i}{\sqrt{(p_i^2 - 1)}} \right) \ln \left[ p_i + \sqrt{(p_i^2 - 1)} \right] - 1, $$

where $p_i = L_i / (2 R_i)$ is the $i$-layer aspect ratio.

B. Geometric model

Figure 1 shows the three-dimensional model and sectional dimension map of the designed shield, respectively. Design variables and size parameters are listed in Table 1. There are six open holes on the shielding assemblies, with a diameter of 25 mm, to provide an access path for the fluxgate probe. Based on the size parameters, the volume $V$ of the four layer shield can be expressed as follows:

$$ V = \sum_{i=1}^{4} \pi \times \left( \left( R_i + \frac{t_i}{2} \right)^2 \times (L_i + t_i) - \left( R_i - \frac{t_i}{2} \right)^2 \times (L_i - t_i) \right), $$

where $R_i$ and $L_i$ are defined as follows:

$$ R_i = D / 2 + (i - 1) \times G_r + t_i / 2 + \sum_{i=1}^{n} t_i, $$

and

$$ L_i = L + 2 \times (i - 1) \times G_d + t_i + 2 \times \sum_{i=1}^{n} t_i. $$

Besides, Johnson magnetic noise of the innermost layer shield limits further improvements in sensitivity. Therefore, the noise

![FIG. 1. (a) Three-dimensional model and (b) sectional dimension map of the designed four-layer shield.](image-url)
TABLE I. Initial parameter variable values.

| Parameter definition                     | Design variable | Initial value (mm) |
|------------------------------------------|-----------------|-------------------|
| Innermost inner diameter                 | D               | 152               |
| Innermost inner length                   | L               | 337               |
| Axial gap between layers                 | Ga              | 9.5               |
| Radial gap between layers                | Gr              | 20                |
| First layer thickness                    | t₁              | 1.0               |
| Second layer thickness                   | t₂              | 1.0               |
| Third layer thickness                    | t₃              | 1.0               |
| Fourth layer thickness                   | t₄              | 1.5               |

A key objective is chosen as the main objective, and the other objectives, variables, and constraints are chosen as the optimisation variables. The transverse $S_{nax}^T$ and axial shielding $S_{nax}^A$ factors, Johnson magnetic noise, and shield volume are used as optimization objectives,

$$\begin{align*}
\max f_1 &= S_{nax}^T(D, L, Ga, Gr, t_1 \ldots t_4) \\
\max f_2 &= S_{nax}^A(D, L, Ga, Gr, t_1 \ldots t_4) \\
\min f_3 &= V(D, L, Ga, Gr, t_1 \ldots t_4) \\
\min f_4 &= \delta B_{cur}(D, L, Ga, Gr, t_1 \ldots t_4).
\end{align*}$$

Parameters $D, L, Ga, Gr, t_1, t_2, t_3,$ and $t_4$ are chosen as the optimization variables (Table II). The ranges of the design variables are as follows (unit: mm):

\begin{align*}
122 \leq D &\leq 182 \\
307 \leq L &\leq 367 \\
2 < Ga &< 20 \\
2 < Gr &< 20 \\
0.05 \leq t_i &\leq 1 \\
1 \leq t_2, t_3, t_4 &\leq 2.
\end{align*}

The constraint condition is summarized as follows: $L \geq 1.5 \cdot D$.

B. Optimization algorithm and results

The genetic algorithm is a stochastic intelligent search method based on Darwin’s natural selection rule “survival of the fittest” and genetic mechanisms. The gamultiobj function of Matlab has been added to the objective functions of the shielding factor. The multiobjective genetic algorithm toolbox is utilized to optimize the shield. The population size is set to 400, and the function tolerance is $1 \times 10^{-5}$. Others use default values. After 1600 iterations, a series of Pareto optimal solutions are obtained by the multiobjective genetic algorithm. The solution sets of the four optimization objectives are shown in Figs. 2(a)–2(d). Depending on the desired design requirements, the corresponding combinations of optimization objective solutions can be obtained.

C. Fuzzy comprehensive evaluation

Fuzzy comprehensive evaluation (FCE) is used to evaluate the performance of the multilayer shield. The fuzzy decision model consists of four elements, namely, factor set $U$, evaluation set $E$, fuzzy evaluation matrix $R$, and weight set $W$. First, 24 groups of schemes are selected from these Pareto optimal solutions. Then, the 1–9 scale method of the analytic hierarchy process (AHP) is used to determine the comprehensive evaluation matrix. The fuzzy decision model established can be expressed as $U = \{u_1, u_2, \ldots, u_{24}\}$, where $u_1$–$u_{24}$ are the selected 24 solutions. $E = \{E_{nax}, E_{nax}^T, V, \delta B_{cur}\}$. $R = \{r_{ij}\}_{1 \times 24}$ is a fuzzy map from $U$ to $E$, $r_{ij}$ represents the subordination degree of the $j$-th scheme to the $i$-th factor. The membership degree of each factor is determined by the following two formulas: $r_{ij} = x_j/x_{max}$ and $r_{ij} = 1 - x_j/x_{max}$, where $x_j$ is the value of scheme $j$ to factor $i$. $x_{max}$ is the maximum factor $i$. After calculation, the fuzzy evaluation matrix $R$ is demonstrated as follows:

\[
R = \begin{bmatrix}
0.05 & 0.37 & 0.37 & 0.07 & 0.17 & 0.24 & 0.73 & 0.05 \\
0.31 & 0.52 & 0.46 & 0.37 & 0.35 & 0.43 & 0.64 & 0.34 \\
0.19 & 0.34 & 0 & 0.21 & 0.32 & 0.27 & 0.11 & 0.19 \\
0.36 & 0.06 & 0.13 & 0.33 & 0.19 & 0.17 & 0.08 & 0.35 \\
0.32 & 0.81 & 0.12 & 0.60 & 0.81 & 0.24 & 0.65 & 1 \\
0.39 & 0.72 & 0.61 & 0.71 & 0.80 & 0.40 & 0.45 & 0.63 \\
0.35 & 0.23 & 0.08 & 0.29 & 0.22 & 0.37 & 0.35 & 0.16 \\
0.10 & 0.02 & 0.33 & 0 & 0.001 & 0.11 & 0.03 & 0.01 \\
0.30 & 0.30 & 0.31 & 0.49 & 0.57 & 0.25 & 0.55 & 0.78 \\
0.67 & 0.49 & 0.42 & 0.52 & 0.62 & 1 & 0.77 & 0.81 \\
0.13 & 0.34 & 0.24 & 0.12 & 0.15 & 0.21 & 0.16 & 0.30 \\
0.09 & 0.06 & 0.07 & 0.12 & 0.18 & 0.04 & 0.04 & 0.001
\end{bmatrix}
\]
FIG. 2. The solution sets of the four optimization objectives: (a) transverse shielding factor, (b) axial shielding factor, (c) volume, and (d) Johnson magnetic noise.

To determine the weight coefficients of each factor, comparison matrix $A$ is established by the 1–9 scale method. Then, the weight coefficient vector $W$ is calculated by the geometric average method and then normalized. $A$ and $W$ are both demonstrated in Table III. To verify the rationality of the weight coefficient distribution, it is necessary to check the consistency of the judgment matrix $A$. The maximum eigenvalue formula $\lambda_{\text{max}}$ is defined as

$$\lambda_{\text{max}} = \frac{1}{n} \sum_{i=1}^{n} \frac{(A W)_{ii}}{W_i}$$

when $n = 4$ and $\lambda_{\text{max}} = 4.1170$. The consistency ratio CR is expressed as

$$CR = \frac{CI}{RI} = \frac{\lambda_{\text{max}} - n}{(n - 1)RI}$$

where CI is a consistency index, RI is the average consistency index, and when $n = 4$, RI is 0.893; therefore, CR = 0.0437. Due to CR = 0.0437 < 0.1, the consistency of the judgment matrix is satisfactory.

Finally, evaluation results are calculated by the weighted average model, based on known $R$ and $W$. $B = W \cdot R = \begin{pmatrix} 0.256 & 0.200 & 0.196 & 0.250 & 0.209 & 0.215 & 0.286 & 0.252 \\ 0.203 & 0.291 & 0.261 & 0.231 & 0.284 & 0.191 & 0.254 & 0.323 \\ 0.182 & 0.180 & 0.173 & 0.240 & 0.303 & 0.168 & 0.229 & 0.286 \end{pmatrix}$.

According to the principle of maximum membership, the sixteenth set of parameters is optimal. Besides giving more attention to magnetic noise, the optimal combination objectives, with the lowest magnetic noise, relatively high axial shielding factor, and small volume, are selected. The initial and optimized parameters based on the FCE method and the lowest magnetic noise are shown in Table IV.

### Table III. Comparison matrix $A$ and normalized weight coefficient vector $W$.

| Performance index | $S_{\text{hot}}^1$ | $S_{\text{hot}}^2$ | $S_T^1$ | $S_T^2$ | V | $\delta B_{\text{curr}}$ | W |
|------------------|-------------------|-------------------|---------|---------|---|-------------------|---|
| $S_{\text{hot}}$ | 1                 | 5                 | 3       | 1/3     | 0.2634 |                   |   |
| $S_T$            | 1/5               | 1                 | 1/3     | 1/7     | 0.0550 |                   |   |
| V                | 1/3               | 3                 | 1       | 1/5     | 0.1178 |                   |   |
| $\delta B_{\text{curr}}$ | 3                 | 7                 | 5       | 1       | 0.5638 |                   |   |

### Table IV. Initial and optimized parameters.

| Design variable | Initial value (mm) | Optimized 1 (mm) | Optimized 2 (mm) |
|-----------------|--------------------|------------------|------------------|
| $D$             | 152                | 134.6            | 182              |
| $L$             | 337                | 317.5            | 318.4            |
| $Ga$            | 9.5                | 14.5             | 14               |
| $Gr$            | 20                 | 15.3             | 7.5              |
| $t_1$           | 1.0                | 0.66             | 0.5              |
| $t_2$           | 1.0                | 1.9              | 1.9              |
| $t_3$           | 1.0                | 1.9              | 1.3              |
| $t_4$           | 1.5                | 1.9              | 2.0              |
The corresponding objective values before and after optimization are listed in Table V. Compared with the calculated values before optimization, the first group of optimization results shows that the axial and transverse shielding factor improved from $3.3 \times 10^4$ to $8.2 \times 10^6$ and $5.0 \times 10^6$ to $2.2 \times 10^7$, respectively. The magnetic noise is 0.92 of the original value. For the second group of optimization results, the magnetic noise is only 0.6 of the original value and the axial shielding factor improved slightly from $3.3 \times 10^4$ to $3.8 \times 10^4$. Although the transverse shielding factor drops to half of its original level, it has little effect on the sensitivity. There is little difference in volume between the two groups but slightly increased compared with the volume before optimization. The two optimization solutions have their own advantages and can be made a binary choice according to the actual demand.

Besides, other optimization algorithms, such as multiobjective particle swarm and NSGA-II, are also used to optimize the shield with the optimization software iSIGHT combined with MATLAB. Using the same ranges of the design variables and constraint conditions, the magnetic noise should be less than 12 fT/Hz and the volume should be less than 0.0020 m$^3$. These results validate the effectiveness of the multiobjective genetic algorithm.

| Design objectives | $S_{\text{init}}^A$ | $S_{\text{init}}^B$ | $S_{\text{init}}^C$ | $\delta B_{\text{cur}}$ |
|-------------------|-------------------|-------------------|-------------------|-------------------|
| Initiate value    | $1.1 \times 10^4$ | $5.0 \times 10^4$ | $0.0016 \text{ m}^3$ | $12.9 \text{ fT/Hz}^{0.5}$ |
| Optimization 1    | $7.5 \times 10^4$ | $2.2 \times 10^5$ | $0.0019 \text{ m}^3$ | $11.9 \text{ fT/Hz}^{0.5}$ |
| Increased by a factor | 6.8            | 4.4              | 1.2              | 0.9              |
| Optimization 2    | $4.1 \times 10^4$ | $1.1 \times 10^5$ | $0.0019 \text{ m}^3$ | $7.7 \text{ fT/Hz}^{0.5}$ |
| Increased by a factor | 3.7            | 0.2              | 1.2              | 0.6              |

**IV. CONCLUSION**

This paper describes the optimization of a multilayer ferromagnetic shield using a multiobjective genetic algorithm. Within a given range of parameters and desired demand, a Pareto optimal solution set has been obtained by optimization. Then, the Pareto set is evaluated with the FCE method after a preliminary selection. According to the desired demand, the magnetic noise is very important to the sensitivity. Therefore, two sets of optimal combinations of target values and variable parameters are selected, based on the FCE method and the lowest magnetic noise. The optimization method is universal, as long as the desired objectives and certain constraints are set well, the corresponding Pareto solution set can be obtained. It is of great significance for the optimum design of the magnetic shield.

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