Damage localization in composite plates that exploits edge-reflected Lamb waves

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Abstract. Lamb waves can travel a relatively long distance in the plate-like structures. Reflection and scattering may occur when they arrive at an obstacle, and the associated wave packets may contain the information about the obstacle and the whole path they travel through. Those extra reflections change the direction once they meet an obstacle, which enlarges the monitoring zone and enriches the inspecting directions. The motivation and objective of this paper is to exploit the information included in these reflections for damage identification. Firstly, dispersion compensation is applied to the residual between the measured signal and the pre-recorded baseline. Thus the propagation distance of each scattered wave packet could be estimated. On this basis, the sparse reconstruction algorithm is modified to adapt to both direct-scattered waves and indirect-scattered waves for damage identification. Two experimental examples are conducted on CFRP laminates, where damage is correctly identified with the minimum sensor array consisting of one transmitter and one receiver.

1. Introduction
Lamb waves have the advantages of long-distance propagation, high-damage sensitivity and well-founded theories, making them a promising tool for SHM [1-3]. Especially, the development of piezoelectric ceramics and wafer transducers which could be surface mounted or embedded leave-in-place on the structures, give the possibility to develop SHM systems for damage detection in composite panels without affecting their mechanical performance. On this basis, many researchers focused on reducing the number of sensor elements through optimal design of array geometries, e.g., rectangle, triangle, circle, concentric circles and the Mills cross [4-8]. Besides, there are some researches achieve the same purpose through advanced signal processing, such as the compressed sensing technique [9]. However, all these studies are based on the Born approximation where Lamb waves only scattered once from the damage, and thus the information included in the extra reflections are abandoned.

Generally, the practical structures always contain some discontinuous, such as edges, stiffened panels, reveted-lap joints and adhesive butt joints, which may influence the propagation directions of Lamb waves [10]. These extra echoes provide many more sensing rays and bring more information about the region of interest [11,12]. Such multipath scattered signals can be exploited to improve damage identification performance while reduce the number of transducers at the mean time [13-15]. To achieve that, a damage localization method is proposed based on the concept of sparse
reconstruction, which uses both the direct scattered Lamb waves and the edge-reflected wave packets. In essence, making use of these wave packets is equal to introducing virtual transducers into the actual sensor array. Benefitting from that, the minimum sensor array which consists of one transmitter and one receiver is enough to achieve damage localization.

The rest of paper is organized as follows. Section II gives a brief review of multipath scattering of Lamb waves. Then the steps of the proposed method are elaborated in section III. In section IV, two experimental examples are introduced to demonstrate the effectiveness of the method. Finally, section V gives the conclusions.

2. Review of multipath scattering
Suppose an excitation signal \( F(\omega) \) is launched at the structure where the location of the source is \( s=[s_1, s_2]^T \) and the position of the receiver is \( r=[r_1, r_2]^T \), the response \( U(\omega; s, r) \) may satisfy,

\[
0 \leq 0 < \omega_0 \leq \omega < \omega_n \leq \omega_1 \leq \omega_2 < \omega_{\text{ref}} \leq \omega_{\text{max}}
\]

with

\[
d_{\text{ref}} + d \leq d_{\text{ref}} + \omega_{\text{max}}
\]

Here, \( d \) is the distance from the source to the receiver, \( d_{\text{ref}} \) is a reference, and \( k(\omega) \) is the wavenumber of a Lamb wave mode.

Assume a scatterer sits at \( u \), the frequency-domain field \( U^s \) satisfies,

\[
U^s(\omega; s, r) = \sum_{n=1}^{N} \psi(\omega; \theta^u, \theta^w) \sum_{m=1}^{M} U^s(\omega; s, r_m) \delta(\omega - \omega_m)
\]

where \( \psi(\omega; \theta^u, \theta^w) \) is a multivariate function which denotes the scattering pattern, i.e., the inputs \( \theta^u = \theta^u(\omega) = \angle(u - s) \) and \( \theta^w = \angle(r - u) \) are the incoming and outgoing angles.

Without loss of generality, besides the damages, other scatterers in the structure also arise scattering of Lamb waves. In this case, the scattered field \( U^s \) is the sum of the fields scattered from each scatterer,

\[
U^s(\omega; s, r) = \sum_{n=1}^{N} U^s(\omega; s, r_{nM}) \delta(\omega - \omega_n)
\]

where \( \omega_n \) is the total number of scatterers, and \( N-M \) is the number of damages.

3. Multipath Lamb wave imaging
The damage localization method consists of two main steps, including composition of a virtual sensor array, and damage imaging via sparse reconstruction, which are elaborated as follows.

The obstacle in the structure (e.g., edge) acts as a new source, producing Lamb waves propagating in the specimen. Those edge-reflected Lamb waves travel and meet the damage, and then further scatter. For illustration, figure 1 gives the wave paths associated with multiple scattering between the damage and two adjacent edges, from the viewpoint of geometric acoustics.
Assuming that damage is absent or present at only a small number of discrete locations (for a structure in operation), which enables the use of sparse reconstruction techniques to determine the locations of possible damage sites [16,17].

To generate the damage image, the dictionary matrix $A \in \mathbb{R}^{l\times d}$ and the measurement vector $y \in \mathbb{R}^d$ need be constructed firstly. Since the scattering pattern for each scatterer, $\psi_j(\omega; \theta^n, \theta^m)$, is unknown, it is nearly impossible to construct the multiple scattering model. In this case, an alternative construction procedure is given as follows.

The area of interest is divided into $J$ pixels, and the location of the $j$th pixel is indicated as $u_j$. Especially, a nonzero value of $x\_j$ represents that a scatterer is present at $u_j$.

Subsequently, the linear mapping dispersion removal algorithm [18,19] is employed, which replaces the nonlinear wavenumber $k(\omega)$ with the first-order series expansion, i.e.,

$$\omega_0 = \omega + \sum_{n=1}^{\infty} \frac{\partial k}{\partial \omega}(\omega_0) \frac{\delta \omega}{\omega_0}$$

where $\omega_0$ is the excitation central frequency. As a result, the multiple wave packets corresponding to multipath scattering are compressed to a nondispersive form.

To remove the effects of scattering patterns, $\psi_j(\omega; \theta^n, \theta^m)$, the compensated residual signal is simplified to consist of several nondispersive wave packets which sit at $x = d_\gamma$ and have the same waveform as the original excitation [20]. Therefore, the vector $y$ may be constructed as [in the form of a function of distance, $y(x)$],

$$y(x) = \sum_{j=1}^{J} a_{\gamma,j} g_x$$

where

$$H[.]$$

Here, $H[.]$ indicates the Hilbert transform, and the distance-domain signal $f(x)$ is obtained from the time-domain excitation signal $f(t)$. In equation (8), the amplitudes of all multipath scattered wave packets are identically set to be that of $g(x)$, which means that the amplitude term of $\psi_j(\omega; \theta^n, \theta^m)$ is simplified as a constant, i.e., 1. Besides, the phase term of $\psi_j(\omega; \theta^n, \theta^m)$ is also neglected, as each wave packet is replaced by the envelope of the excitation signal, i.e., $g(x)$.

Accordingly, the multiple scattering model could also be constructed with the nondispersive excitation signal, $g(x)$. Let $A_j$ be the dictionary for the $j$th scattering path, and $a_{\gamma,j}$ the $j$th column of $A_j$, which corresponds to the case that a scatterer locates at $u_j$. Thus,

$$y(x) = \sum_{j=1}^{J} a_{\gamma,j} g_x$$

Here, $s_j$ and $r_j$ are the coordinates of the transmitter and receiver (either virtual or actual one), respectively. The multiple scattering path is equal to the direct scattering path among the transmitter, damage and receiver (see figure 1).

Once the dictionary matrix $A$ and the vector $y$ are given, it is desirable to formulate their relation matching the form $y = Ax + e$. However, it is noted that the scattering path of each wave packet in $U^m$
cannot be traced unless the damage location is known in advance. In other words, it is an arduous task to obtain the vector $\mathbf{y}$ for the $\gamma$th scattering path (i.e., with the dictionary matrix $\mathbf{A}_\gamma$).

Alternatively, we could form the formulation $\mathbf{y} = \mathbf{Ax} + \mathbf{e}$ with the substitutions,

$$\mathbf{A} = \sum_{\gamma=1}^{P} \mathbf{A}_\gamma$$

since the vector $\mathbf{y}$ consists of components of all individual scattering paths, i.e.,

$$\mathbf{y} = \sum_{\gamma=1}^{P} \mathbf{y}_\gamma$$

where $P$ is the number of scattering paths.

Subsequently, sparse reconstruction could be applied to solve above linear inverse problems based on sparsity assumption. Here, the basis pursuit denoising (BPDN) is utilized [20], and $\mathbf{x}$ could be solved as,

$$\arg \min \lambda + \frac{1}{2} \| \mathbf{x} \|_2^2 + \frac{1}{2} \| \mathbf{y} - \mathbf{Ax} \|_2^2$$

where $\lambda$ is a user-specified regularization parameter, which balances the accuracy ($2$-norm term) and sparsity ($1$-norm term).

4. Experimental Investigation

The specimen takes a 2 mm-thick 16 plies CFRP laminates $[+45^\circ/-45^\circ/0^\circ/90^\circ]_2$. The basis is 7901 epoxy resin and the fiber takes T300 carbon fiber. The size is 690 mm$\times$690 mm. Firstly, Lamb wave testing is applied to the healthy specimen to obtain the reference signal. Then, simulated damage is introduced through bonding an added mass (15 mm$\times$15 mm$\times$4 mm) to the plate [21,22]. Lamb wave testing is applied again, and thus the response captures the information of damage. Finally, the residual signal is obtained as the response captured in the damaged case and the reference signal are directly subtracted.

4.1. Case I

In this example, both transducers sit adjacent to the lower horizontal edge, with coordinates of (150,45) mm and (450,45) mm, respectively. The added mass sits at (196,136) mm. Figure 2 gives the specimen with an added mass and the distribution of transducers. The excitation signal takes a classical toneburst centered at 50 kHz. Under that excitation, $A0$ is dominant and $S0$ mode is nearly invisible. Thus, dispersion removal is applied to the residual signal using the dispersion characteristic of $A0$ mode.
From figure 3, it can be seen that even though the incident wave and the reflected wave from the nearest edge are not eliminated, the amplitude of these remaining waves is obviously smaller than that of the damage scattered waves (the waveform enclosed by the dashed rectangle).

Figure 3. The compensated residual signal and its envelope.

Figure 4. The vector $y$ constructed from the envelope of the compensated residual signal.

Figure 4 displays the envelope of the compensated residual signal as the blue solid line, where the peaks of the three wave packets sit at 356 mm, 397 mm, 426 mm, respectively. The vector $y$ is then obtained from equation (8), and also displayed in figure 4 as the red dashed line. As both transducers sit adjacent to the lower horizontal edge, the virtual transmitter and virtual receiver stand at $(150, -45)$ mm and $(450, -45)$ mm, respectively. In this case, the virtual sensor network consists of three possible transmitter-receiver pairs, i.e., AT-AR pair, AT-VR pair and VT-AT pair.

The dictionary matrix $A_{γ}$ for the $γ$th transmitter-receiver pair could then be constructed from equation (10). On this basis, matrix $A$ is obtained from equation (11). As the regularization parameter $λ$ takes $\max|2A^Ty|$, the sparse reconstruction is achieved through the basis pursuit denoising algorithm [23]. The image constructed via sparse reconstruction is shown in figure 5, with the area enclosed by a rectangle is zoomed up and displayed at the top right-hand corner. The identified damage sits at $(198,134)$ mm, only 2.8 mm away from the actual one (marked by ‘Δ’). It demonstrates that through exploiting multipath scattering, damage detection could be achieved by even a single transmitter-receiver pair.

Figure 5. Image constructed via sparse reconstruction using the virtual sensor array, and zoomed version of the sparse imaging results enclosed by the rectangle.

4.2. Case II
In this experiment, the transmitter sits at (50,120) mm, while the receiver stands at (350,45) mm. The added mass sits at the same position as in Case I, i.e., (196,136) mm. Figure 6 gives the specimen with an added mass and the distribution of transducers. The excitation signal takes the same toneburst as in case I.

Since multipath scattering is considered, the residual signal may consist of multiple wave packets [equation (6)]. Through dispersion removal, the spreading due to dispersion could be significantly reduced. Figure 7 displays the envelope of the compensated residual signal as the blue line, where the peaks of the three wave packets sit at 328 mm, 387 mm, 427 mm, respectively. Subsequently, the vector $\mathbf{y}$ is obtained from equation (8), i.e., the red dashed line in figure 7.

The reflection at the left vertical edge mirrors a virtual transmitter at (-50,120) mm, and that at the lower horizontal edge adds a virtual receiver at (350,-45) mm. Hence, the virtual sensor array consists of two transmitters and two receivers. If single scattering approximation and double scattering approximation (scattered by two objects) are considered, the sensor network has three possible transmitter-receiver pairs, i.e., AT-AR pair, AT-VR pair and VT-AR pair, see figure 1. The resulting image is displayed in figure 8, and the zoomed version of the area enclosed by a rectangle is given at the top right-hand corner. The identified damage sits at (195,140) mm, which coincides with the actual one (marked by ‘Δ’) well. Thus it demonstrates the effectiveness of exploiting multipath scattering for damage detection.
5. Conclusions
This paper extends damage identification from the Born approximation to multiple scattering approximation. It could help reducing the number of sensor elements and the cost of the SHM system. Some conclusions are obtained as follows.

(1) The scatterers in the structure behave like new sources which produce Lamb waves propagating through different directions. Those waves may interact with damage and thus can be used for damage identification. Through exploiting that information, the specimen could be monitored by the minimum sensor array.

(2) In the sparse reconstruction algorithm, the dictionary matrix $A$ and the measured vector $y$ is constructed without characterizing and predicting Lamb wave behavior at a scatterer. Besides, the dictionary matrix corresponding to different transmitter-receiver pairs $A_{\gamma}$ shares the same measured vector $y$. Hence, it is available under multiple scattering circumstance.

(3) In theory, the proposed method is also available for multi-site damage detection. However, it may face the challenge that the wave packets from multipath scattering of multi-site damage are quite difficult to be separately identified.

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