Perturbation in Warm Chromo-Natural Inflation

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We examined the chromo-natural inflation in the context of warm inflation. The dynamical equations of this model are obtained. We studied the cosmological perturbation theory in this model. The sources of density fluctuations in this model are mainly the thermal fluctuations of the inflaton field like general warm inflationary model. Finally, cosmological observables, namely, the spectral index and tensor to scalar ratio are calculated. It is found that the cosmological observables are consistent with observational data and the tensor to scalar ratio is smaller than that in the chromo-natural inflation.

1 Introduction

The idea of inflation was first proposed by Alan Guth [1] in 1981 to solve the problems in the big bang model, notably the horizon, flatness and horizon problem. Further, many inflationary theory has been developed by Linde [2][3][4][5], Albrecht [6] and many other scientists. It is the most successful theory so far for explaining the large scale structure of the universe. During inflation, a scalar field coupled to background metric slowly rolls down its potential. Inflationary dynamics were realized in two different methods, one is cold inflation and another is warm inflation. In cold inflation, the scalar field is assumed to be isolated during inflation and has no interaction with the other fields. So, there is no possibility for radiation to be produced during inflation, thus leading to a thermodynamically supercooled phase of the Universe during inflation. So, a reheating period is needed to be invoked at the end of inflation to fill the universe with radiation. In the other picture, termed warm inflation [7][8][9][10], the inflaton field is supposed to be coupled to several other fields during inflationary period. So, in warm inflation radiation production can simultaneously occur with the expansion of the universe and there is a smooth transition from the inflationary phase to radiation dominated phase without the necessity of introduction of any separate reheating phase. This can be achieved by introducing a dissipation term $\Gamma$ in the equation of motion of the inflaton field.

A successful inflationary model must provide sufficiently long period of inflation to solve the horizon problem. To satisfy these constraints, the potential for the inflaton must be very flat. Natural inflation is one of the viable model [11] in which shift symmetry protects the inflaton potential from quantum corrections which assures a flat inflaton potential. But Natural inflation requires a large axion decay constant, $f \sim M_P$ [12] which appears to be difficult to realize within the framework of string theory.

Later in 2012, Adshead et al [13] proposed an axionic inflationary model which has a sub-Planckian axionic decay constant. There in addition to the axionic field, they considered a collection of non-abelian gauge fields having a rotationally invariant vacuum expectation value. Slow roll inflation in the model has been achieved by the efficient transfer of axionic energy into classical gauge fields, rather than from dissipation via Hubble friction. This model is known Chromo- Natural Inflation. In this paper, we have extended Chromo-Natural inflation to a warm inflationary scenario, termed as ‘Warm Chromo-Natural Inflation’. Studying Chromo-natural inflation in the context of warm inflation is more natural and proper as the coupling between the axion and the non-abelian gauge field can cause the production of thermal bath during inflation period which has been neglected in chromo-natural inflation. Secondly, the condition of the flat potential required is somewhat relaxed in warm inflation. As a result inflation can last long enough even if the potential is not very flat.

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This paper is organized as follows: In section 2, we briefly introduce the basic dynamics of the inflaton field in the warm chromo-natural inflation. Then in section 3, we will study the scalar perturbations of metric and the thermal fluctuations of the inflaton field which are different from that in the usual chromo-natural inflation. After that the spectra of scalar perturbations and tensor perturbations are studied in section 4. And finally in section 5, we draw the conclusion.

## 2 Dynamics of the warm chromo-natural inflation

We consider the action that contains the gauge field and the pseudo scalar axion field [13],[14]:

$$S_{\text{chromo}} = \int d^4x \sqrt{-g} \left[ -\frac{M_p^2}{2} R - \frac{1}{4} F_{\mu\nu}^a F^{\mu\nu}_a - \frac{1}{2} (\partial \chi)^2 - \mu^4 \left( 1 + \cos \left( \frac{\chi}{f} \right) \right) + \frac{\lambda}{8f} \epsilon^{\mu\nu\rho\sigma} F_{\mu\nu}^a F^{a}_{\rho\sigma} \right]$$  \hspace{1cm} (1)

Where $F_{\mu\nu}^a = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a - \hat{g} f^{abc} A_\mu^b A_\nu^c$, $\chi$ and $A_\mu^a$ are the axion and the $SU(2)$ gauge fields respectively. Here, $\hat{g} = c = M_P = 1$ (in natural units). Last term in the action is the interaction term called the Chern-Simon term which describes the interaction between the axionic field and the gauge field. Here, $\hat{g}$ is the gauge coupling constant, $f$ is the axion decay constant, $\mu$ is the mass scale of the theory, $\lambda$ is the gauge parameter and $f^{abc}$ is the structure constant of the gauge group with normalization condition $f^{123} = 1$. The background metric is the standard FRW metric, $ds^2 = -dt^2 + a(t)^2 \delta_{ij} dx^i dx^j$

When the mass of the fluctuation is much heavier than the Hubble scale, the linear fluctuation of the gauge field which introduces a perturbative instability will disappear. In this condition gauge field can be integrated out in a single field effective action which involves only the axion field and the new effective action takes the form:

$$S = \int d^4x \sqrt{-g} \left[ -\frac{M_p^2}{2} R + P(X, \chi) \right]$$  \hspace{1cm} (2)

The term $\frac{1}{4\Lambda^4} (\partial \chi)^4$ captures the effect of the gauge field in the single field effective theory with, $\Lambda^4 = \frac{8f^4\hat{g}^2}{\chi f}$

The above effective action can be expressed as:

$$S = \int d^4x \sqrt{-g} \left[ -\frac{M_p^2}{2} R + P(X, \chi) \right]$$  \hspace{1cm} (3)

Where $P(X, \chi)$ is the matter Lagrangian density and has the expression

$$P(X, \chi) = X + \frac{X^2}{\Lambda^4} - V(\chi)$$  \hspace{1cm} (4)

with $X = -\frac{1}{2} (\partial \chi)^2$ and $V(\chi) = \mu^4 \left( 1 + \cos \left( \frac{\chi}{f} \right) \right)$ is the potential of the axionic field $\chi$.

Now the energy-momentum tensor of the inflaton field

$$T_{\mu\nu} = (\rho_\chi + p_\chi) u_\mu u_\nu + p_\chi g_{\mu\nu}$$  \hspace{1cm} (5)

Where $\rho_\chi$, $p_\chi$ and $u$ are the energy density, pressure and 4-velocity of the inflaton field. Also, the energy-momentum tensor of the inflaton field can be obtained by varying the Lagrangian of the inflaton field with respect to metric which can be expressed as

$$T_{\mu\nu} = P_X \partial_\mu \chi \partial_\nu \chi + g_{\mu\nu} P(X, \chi)$$  \hspace{1cm} (6)

$P_X$ denotes derivative of $P$ with respect to $X$. Comparing equation (4),(5) and (6) we find

$$\rho_\chi = X + \frac{3X^2}{\Lambda^4} + V(\chi)$$  \hspace{1cm} (7)
Here pressure term $p_\chi$ is same as the matter Lagrangian density $P$. Now, the basic dynamical equations i.e., Friedmann equation and the equation of motion of the inflaton field are \[H^2 = \frac{\rho}{3M_p^2}\]

\[\dot{\rho}_\chi + 3H(\rho_\chi + P) = -\Gamma \dot{\chi}^2\]

Where $\Gamma$ is the dissipation coefficient in warm inflation which describe the decay of the inflaton field into radiation. Now, considering a homogeneous inflaton field, $X = \frac{\dot{\chi}^2}{2}$, the equation of motion can be obtained by substituting equations (7) and (8), in equation (10),

\[\ddot{\chi} \left( 1 + \frac{3\dot{\chi}^2}{\Lambda^4} \right) + 3H\dot{\chi} \left( 1 + \frac{\dot{\chi}^2}{\Lambda^4} + \Gamma \dot{\chi} = \frac{\mu^4}{f} \sin \left( \frac{\chi}{f} \right) \right)\]

\[\ddot{\chi} \left( 1 + \frac{3\dot{\chi}^2}{\Lambda^4} \right) + 3H\dot{\chi} \left( 1 + \frac{\dot{\chi}^2}{\Lambda^4} + r \right) \frac{\mu^4}{f} \sin \left( \frac{\chi}{f} \right)\]

Where the dissipation rate is defined as $r = \frac{\Gamma}{3H}$. Now, in order for inflation to occur, the inflaton must be potential energy dominated and this approximation is called slow-roll approximation. The consistency of the slow-roll approximation is governed by a set of slow-roll parameters. So, the slow-roll parameters $\epsilon$ and $\eta$ and the speed of sound $c_s$ can be defined as

\[\epsilon = -(1 + r) \frac{\dot{H}}{H^2}\]

\[\eta = -(1 + r) \frac{\dot{\epsilon}}{\epsilon H}\]

Also, we can express the speed of sound $c_s$ and slow-roll parameters $\epsilon$ and $\eta$ in terms of $X$ as

\[c_s^2 = \frac{P_X}{P_x + 2XP_{XX}}\]

\[\epsilon = (1 + r) \frac{XP_X}{M_p^2H^2}\]

\[\eta = (1 + r) \frac{6XP_X}{2XP_x - P}\]

And the slow roll parameters have to satisfy the conditions, $\epsilon < 1$, $\eta < 1$ during inflationary phase.

Now, in the slow roll regime, $\rho \simeq V(\chi)$, then the Friedmann equation and the equation of motion of the axion field take the form :

\[H^2 = \frac{V}{3M_p^2}\]

\[3H\dot{\chi}(1 + \frac{\dot{\chi}^2}{\Lambda^4} + r) = \frac{\mu^4}{f} \sin \left( \frac{\chi}{f} \right)\]

The slow-roll hierarchy scaling (SRHS) demands the condition $X >> \Lambda^4$ and with this condition, the
expression for the slow-roll parameters and the speed of sound can be expressed as:

\[
\epsilon = (1 + r) \frac{2X^2}{H^2 M_p^2 \Lambda^4} = (1 + r) \frac{\dot{\chi}^4}{2H^2 M_p^2 \Lambda^4} 
\]  

(20)

\[
\eta = (1 + r) \frac{12X^2}{3X^2 + \Lambda^4 V(\chi)} = (1 + r) \frac{12\dot{\chi}^4}{3\dot{\chi}^4 + 4\Lambda^4 \mu^4 (1 + \cos(\chi/2))} 
\]  

(22)

\[
\eta = (1 + r) \frac{12\dot{\chi}^4}{3\dot{\chi}^4 + 4\Lambda^4 \mu^4 (1 + \cos(\chi/2))} 
\]  

(23)

\[
c_s^2 = \frac{1}{3} 
\]  

(24)

The number of e-foldings when the inflation field \( \chi \) rolls from its value \( \chi_i \) to \( \chi_f \) is

\[
N = \int H dt = \int_{\chi_i}^{\chi_f} \frac{H}{\dot{\chi}} d\chi 
\]  

(25)

Then we solve equation (19) for \( \dot{\chi} \). There are three solutions of this equation. Out of the three solutions, two turn out to be complex which we ignore and only one is real solution which is given by

\[
\dot{\chi} = 0.74 \left( \sqrt{\frac{\mu^4 \Lambda^4 M_p^2 \sin^2 \left( \frac{0.5\chi}{f} \right)}{f^2}} + \frac{0.015 \Lambda^2 \Gamma^3 M_p^3 \sec^3 \left( \frac{0.5\chi}{f} \right)}{\mu^6} - \left\{ \frac{\mu^2 \Lambda^4 M_p \sin \left( \frac{0.5\chi}{f} \right)}{f} \right\} + \left\{ \frac{\mu^2 \Lambda^4 M_p \sin \left( \frac{0.5\chi}{f} \right)}{f} \right\} + \left\{ \frac{\mu^2 \Lambda^4 M_p \sin \left( \frac{0.5\chi}{f} \right)}{f} \right\} \right)^{1/3} 
\]  

(26)

With this real solution we will do all the calculations. Now, substituting \( \dot{\chi} \) in equations (21) and (23), we get the slow-roll parameters as

\[
\epsilon = \frac{3}{2\mu^4 \Lambda^4 \left( \cos \left( \frac{\chi}{f} \right) + 1 \right)} \left( \frac{M_p \Gamma}{\sqrt{3} \sqrt{\mu^4 \left( \cos \left( \frac{\chi}{f} \right) + 1 \right)}} + 1 \right) \left\{ \frac{\mu^2 \Lambda^4 M_p \sin \left( \frac{0.5\chi}{f} \right)}{f} \right\} + \frac{\mu^2 \Lambda^4 M_p \sin \left( \frac{0.5\chi}{f} \right)}{f} 
\]

(27)
Inflation ends when either of the two parameters becomes of the order of unity. From the figure 1, it is evident that \( \eta \) reaches unity earlier than \( \epsilon \). So, the equation \( \eta = 1 \) will fix the final field value i.e. \( \chi_f \).

Here we have considered three different number of e-foldings \( (N = 50, 60, 70) \) before the end of inflation for fixing initial field value \( \chi_i \) at which all the observables will be computed. The final field values and the corresponding initial field values for \( \mu = 0.5, f = 0.05, \Lambda = 0.00025 \), \( \Gamma = 1 \) and \( N = 50, 60 \) and 70 are presented below.

Table 1: The final field values and the corresponding initial field values for \( N = 50, 60 \) and 70;

| \( N \) | \( \chi_f \) | \( \chi_i \) |
|-------|----------|----------|
| 50    | 0.1408   | 0.1183   |
| 60    | 0.1408   | 0.1153   |
| 70    | 0.1408   | 0.1124   |
3 Perturbations of the metric and inflaton field

We consider the scalar perturbations of the FRLW metric in the longitudinal gauge which can be expressed as

\[ ds^2 = -(1 + 2\Phi)dt^2 + a^2(t)(1 - 2\Psi)\delta_{ij}dx^i dx^j \]  

(29)

Here \( a(t) \) is the scale factor, \( \Phi(t, x) \) and \( \Psi(t, x) \) are the metric perturbations, also called the Bardeen potentials [18]. As there is no anisotropic stress arise in our system, the Einstein equations imply that the metric potentials are equal i.e. \( \Phi = \Psi \). Now, perturbed Einstein equations in momentum space are

\[-3H(\dot{\Phi} + H\Phi) - \frac{k^2}{a^2}\Phi = 4\pi G\delta\rho \]

(30)

\[ \dot{\Phi} + H\Phi = 4\pi G\delta\sigma \]

(31)

\[ \ddot{\Phi} + 4H\dot{\Phi} + (2\dot{H} + 3H^2)\Phi = 4\pi G\delta p \]

(32)

The quantities \( \delta\rho \), \( \delta\sigma \) and \( \delta p \) are the scalar quantities representing the linear perturbation in the energy density, the momentum flux and the pressure, respectively and can be expressed as

\[ \delta\rho = \dot{\chi}\delta\chi + \frac{3}{\Lambda^2}\chi^3\delta\chi - \Phi\chi^2 - \frac{3}{\Lambda^2}\Phi\chi^4 + V\chi \delta\chi \]

(33)

\[ \delta\sigma = \dot{\chi}\delta\chi \]

(34)

\[ \delta p = \dot{\chi}\delta\chi + \frac{1}{\Lambda^2}\chi^3\delta\chi - \Phi\chi^2 - \frac{1}{\Lambda^2}\Phi\chi^4 - V\chi \delta\chi \]

(35)

Now we calculate the perturbed equation of the inflaton field in the longitudinal gauge. The main components of the universe during warm inflation are the inflaton field and radiation. The total energy momentum \( T_{\mu}^{\nu} = T_{\mu}^{\nu(\gamma)} + T^{\mu(\chi)}_{\nu} \), constituted by radiation and inflaton field is conserved or \( T_{\nu;\mu}^{\mu} = 0 \). And each satisfies [19]

\[ T_{\nu;\mu}^{\mu(\gamma)} = Q_{\nu}^{\gamma} \]

(36)

\[ T_{\nu;\mu}^{\mu(\chi)} = Q_{\nu}^{\chi} \]

(37)
where $Q_\nu^{(\gamma)}$ and $Q_\nu^{(\chi)}$ describe the interaction between radiation and inflaton field. Because the total energy momentum is conserved, the interaction terms $Q_\nu^{(\gamma)}$ and $Q_\nu^{(\chi)}$ satisfy the relation, $Q_\nu^{(\gamma)} + Q_\nu^{(\chi)} = 0$. From [19], the interaction terms can be expressed as

$$Q_\nu^{(\gamma)} = -Q_\nu^{(\chi)} = \Gamma u^\mu(\chi(x,t))\nabla_\nu\chi(x,t)$$ (38)

where, $u^\mu = -\frac{\partial\chi}{\sqrt{2}}$ is the four velocity of the inflaton field. Now, the linear perturbation of the interaction terms is

$$\delta Q_\nu^{(\gamma)} = \delta \Gamma u^\mu(\chi(x,t))\nabla_\nu\chi(x,t) + \Gamma \delta u^\mu(\chi(x,t))\nabla_\nu\chi(x,t) + \Gamma u^\mu(\chi(x,t))\nabla_\nu\delta\chi(x,t) \nabla_\nu\chi(x,t)$$

$$\delta Q_\nu^{(\chi)} = \delta \Gamma u^\mu(\chi(x,t))\nabla_\nu\chi(x,t) + \Gamma \delta u^\mu(\chi(x,t))\nabla_\nu\chi(x,t) + \Gamma u^\mu(\chi(x,t))\nabla_\nu\delta\chi(x,t) \nabla_\nu\chi(x,t)$$

$$-\delta Q_0^{(\gamma)} = -\delta \Gamma^0 = \delta \Gamma \dot{\chi}^2 + 2\Gamma \dot{\chi} \delta \chi - \Gamma \dot{\chi} \Phi$$

$$\delta Q_i^{(\gamma)} = -\delta \Gamma_i = \Gamma \dot{\chi_i}$$

Using equations (33)-(42), we can derive the perturbed equation of the inflaton field in the momentum space

$$\left(1 + \frac{3}{A^4} \chi^2 \right) \dot{\delta\chi} + \left[ 3H \left(1 + \frac{3}{A^4} \chi^2 \right) + \Gamma \right] \delta\chi + \left[ V_{,\chi} + \frac{k^2}{a^2} \left( \frac{\chi^2}{A^4} + 1 \right) \right] \dot{\delta\chi}$$

$$= 2 \left[ 2 + \frac{3}{A^4} \chi^2 \right] \dot{\chi}^2 + \left( \frac{6}{A^4} \chi \dot{\chi} - 6H \dot{\chi}^3 - \Gamma \dot{\chi} \right) \Phi - \delta \Gamma \dot{\chi} - 2\Phi V_{,\chi}$$

(43)

Now, we will consider the thermal fluctuations of the inflaton field in the spatially flat gauge and assume that $\Gamma = \Gamma(\chi)$. The perturbed FRW metric in this gauge is

$$ds^2 = -(1 + 2A)dt^2 + 2a(t)\delta_iBdt dx^i + a^2(t)\delta_{ij}dx^i dx^j$$

(44)

Where $A(x,t)$ and $B(x,t)$ are scalar perturbations of the metric. We make the splitting, $\chi(x,t) = \chi(t) + \delta\chi(x,t)$, Where $\delta\chi(x,t)$ is the linear response due to the thermal stochastic noise $\xi(x,t)$. Thus the perturbed equation of inflaton field (43) in the spatial flat gauge becomes

$$\left(1 + \frac{3}{A^4} \chi^2 \right) \dot{\delta\chi_k} + \left[ 3H \left(1 + \frac{3}{A^4} \chi^2 \right) + \Gamma \right] \delta\chi_k + \left[ V_{,\chi} + \frac{k^2}{a^2} \left( \frac{\chi^2}{A^4} + 1 \right) \right] \dot{\delta\chi_k}$$

$$= \left(1 + \frac{3}{A^4} \chi^2 \right) \dot{\chi} A + \left( \frac{6}{A^4} \chi \dot{\chi} - 6H \dot{\chi}^3 - \Gamma \dot{\chi} \right) A + \left[ \frac{k^2}{a} \left( \frac{\chi^2}{A^4} + 1 \right) \right] \dot{\chi} B - \delta \Gamma \dot{\chi} - 2AV_{,\chi}$$

(45)

The metric terms on the right-hand-side obey the energy and momentum constraints

$$3H^2 A + \frac{k^2}{a} H B = -4\pi G \delta\rho$$

(46)

$$HA = 4\pi G \delta\sigma$$

(47)

Introducing the thermal stochastic noise source $\xi_k(t)$ in the perturbed inflaton equation, we get

$$\left(1 + \frac{3}{A^4} \chi^2 \right) \dot{\delta\chi_k}(t) + \left[ 3H \left(1 + \frac{3}{A^4} \chi^2 \right) + \Gamma \right] \delta\chi_k(t) + \left[ \frac{k^2}{a^2} \left( \frac{\chi^2}{A^4} + 1 \right) \right] \delta\chi_k(t) = \xi_k(t)$$

(48)
\[ P, X c_s^{-2} \ddot{\delta \chi}_k(t) + 3H(P, X c_s^{-2} + r)\dot{\delta \chi}_k(t) + P, X \frac{k^2}{a^2} \dot{\delta \chi}_k(t) = \xi_k(t) \]  

Equation (49) is called the Langevin equation which describes the behavior of a scalar field interacting with radiation. Here we have used the relations \( c_s^2 = \frac{1 + 3r^2}{1 + r^2} \) and \( P, X = 1 + \frac{r^2}{a^2} \). Now, in the slow roll regime, the term \( \ddot{\delta \chi} \) can be neglected. Thus the Langevin equation becomes

\[ 3H(P, X c_s^{-2} + r)\dot{\delta \chi}_k(t) + P, X \frac{k^2}{a^2} \delta \chi_k(t) = \xi_k(t) \]  

The solution of the equation (50) is

\[ \delta \chi_k(t) = \exp \left[ -\frac{t - t_0}{\tau(\chi_0)} \right] \frac{1}{3H(P, X c_s^{-2} + r)} \int_{t_0}^{t} \exp \left[ \frac{t' - t_0}{\tau(\chi_0)} \right] \xi_k(t') dt' + \delta \chi_k(t_0) \]  

Where, \( \tau(\chi) = \frac{3H(P, X c_s^{-2} + r)}{P, X \frac{k^2}{a^2}} \) and \( k_p \) is the physical wave number. From the above equation, it is found that when \( k_p \) is larger, \( \tau(\chi) \) will be smaller and as a result relaxation rate will be faster. If \( k_p \) for one mode is sufficiently large to relax within Hubble time, the mode will be thermal. When \( k_p \) of one mode of \( \delta \phi_k \) is smaller than the freeze-out physical wave number \( k_F \), the mode will not thermalize during a Hubble time [20]. So, the freeze-out physical wave number \( k_F \) can be obtained as

\[ k_F = \sqrt{\frac{3H^2}{c_s^2}} \left( 1 + \frac{r c_s^2}{P, X} \right) \]  

In general, the thermal fluctuations of the inflaton field \( \delta \chi_k \) in warm inflation can be expressed as

\[ \delta \chi_k^2 = \frac{k_F T}{2\pi^2} \]  

From equation (52), the thermal fluctuations of the inflaton field becomes

\[ \delta \chi_k^2 = \frac{HT}{2\pi^2} \sqrt{\frac{3}{c_s^2}} \left( 1 + \frac{r c_s^2}{P, X} \right) \]  

4 Perturbation Spectra

The power spectrum for the scalar fluctuations in warm inflation is \( P = \left( \frac{H}{\chi} \right)^2 \delta \chi^2 \). Thus, the spectrum of scalar perturbations in the warm chromo-natural inflation:

\[ P_k^c = \frac{H^3 T}{4\pi^2 X} \sqrt{\frac{3}{c_s^2}} \left( 1 + \frac{r c_s^2}{P, X} \right) \]  

\[ = \frac{T \left( \mu^4 \left( 1 + \cos \left( \frac{\chi}{\mu} \right) \right) \right)^{3/2}}{2\sqrt{3\pi^2 \chi^2 M_P^3}} \sqrt{\frac{9 \chi^2 \left( \mu^4 \left( 1 + \cos \left( \frac{\chi}{\mu} \right) \right) \right) \sqrt{3\Gamma A^4 M_P}}{2\sqrt{3\pi^2 \chi^2 M_P^3}}} \]  

And the power spectrum for the tensor perturbation can be expressed as
\[ P_h^k = \frac{2V}{3\pi^2 M_P^4} \]  
\[ = \frac{2\mu^4}{3\pi^2 M_P^4} \left(1 + \cos \left(\frac{\chi}{f}\right)\right) \]  
\[ (57) \]
\[ (58) \]

which has the same form as in cold inflation. Now, the two important cosmological observables, namely, the spectral index \( n_s \) and tensor to scalar ratio \( R \) can be defined as

\[ n_s - 1 = \frac{d\ln P_k^\zeta}{d\ln k} \]  
\[ (59) \]
\[ R = \frac{P_h^k}{P_k^\zeta} \]  
\[ (60) \]

We carry out the numerical analysis and obtain the values of the cosmological observables for different number of e-foldings which are consistent with PLANCK result. Also we fix the range of model parameters where the cosmological observables lie within the PLANCK bound. We present our results in the following tables 2, 3 and 4 for \( N = 50, 60 \) and 70 respectively.

**Table 2**: Possible values of the observables for different values of model parameters \( f, \mu \) and \( \Lambda \) (in units of \( M_P \)) with \( \Gamma = 1 \) and \( N = 50 \)

| \( \mu \) | \( f \) | \( \Lambda \) | \( n_s \) | \( R \) |
|---|---|---|---|---|
| 0.5 | 0.05 | 0.00025 | 0.967878 | \( (3.39 \times 10^{-6}) \frac{H}{T} \) |
| 0.5 | 0.1 | 0.0005 | 0.967896 | \( (1.35 \times 10^{-5}) \frac{H}{T} \) |
| 0.5 | 0.5 | 0.0025 | 0.967819 | \( (0.00034) \frac{H}{T} \) |
| 0.5 | 1 | 0.005 | 0.967795 | \( (0.0013) \frac{H}{T} \) |

**Table 3**: Possible values of the observables for different values of model parameters \( f, \mu \) and \( \Lambda \) (in units of \( M_P \)) with \( \Gamma = 1 \) and \( N = 60 \)

| \( \mu \) | \( f \) | \( \Lambda \) | \( n_s \) | \( R \) |
|---|---|---|---|---|
| 0.5 | 0.05 | 0.00025 | 0.972526 | \( (2.9 \times 10^{-6}) \frac{H}{T} \) |
| 0.5 | 0.1 | 0.0005 | 0.972554 | \( (1.16 \times 10^{-5}) \frac{H}{T} \) |
| 0.5 | 0.5 | 0.0025 | 0.972492 | \( (0.00029) \frac{H}{T} \) |
| 0.5 | 1 | 0.005 | 0.972453 | \( (0.0011) \frac{H}{T} \) |

**Table 4**: Possible values of the observables for different values of model parameters \( f, \mu \) and \( \Lambda \) (in units of \( M_P \)) with \( \Gamma = 1 \) and \( N = 70 \)

| \( \mu \) | \( f \) | \( \Lambda \) | \( n_s \) | \( R \) |
|---|---|---|---|---|
| 0.5 | 0.05 | 0.00025 | 0.976044 | \( (2.55 \times 10^{-6}) \frac{H}{T} \) |
| 0.5 | 0.1 | 0.0005 | 0.976045 | \( (1.02 \times 10^{-5}) \frac{H}{T} \) |
| 0.5 | 0.5 | 0.0025 | 0.976403 | \( (0.00025) \frac{H}{T} \) |
| 0.5 | 1 | 0.005 | 0.975987 | \( (0.001) \frac{H}{T} \) |
5 Conclusions

The theory of cosmological inflation has drawn a considerable amount of attention in recent years due to its remarkable predictions about the early universe. In this work, we studied chromo-natural inflation in the context of warm inflation. The axion-gauge field coupling in warm chromo-natural inflation creates an damping effect on inflaton as in warm inflation. So, warm chromo-natural inflation also follows the basic idea of warm inflation that the particle production occurs during inflation.

We calculated the perturbation spectra, tensor-to-scalar ratio and spectral index. It is found that power spectrum for the scalar fluctuations is larger than that in chromo-natural inflation. Here we have three model parameters namely $\mu$, $f$ and $\Lambda$ and it is seen that to have 50 or 60 or 70 e-foldings, we do not have much freedom to choose $\mu$. So we avoided the small changes in $\mu$ and kept it fixed. We obtain parameter space for which cosmological observables are in range. We find that the tensor to scalar ratio $r$ increases as $N$ decreases as in the normal chromo-natural inflation. Since $T > H$ during inflationary period, the values of tensor-to-scalar ratio $R$ in warm chromo-natural inflation lie below the upper limit of the observable result and are found to be less than that in normal chromo-natural inflation. It is also observed that for 50 and 60 e-foldings, the values of spectral index $n_s$ consistent with PLANCK observations for planckian as well as sub-planckian values of the axion decay constant and for 70 e-foldings, this value is slightly greater than the PLANCK data, where in chromo-natural inflation, for 50 e-foldings, the value of spectral index is less than the value predicted by PLANCK.

In this study, we consider the linear coupling of the scalar field with the Chern-Simon term. To study warm chromo-natural inflation further, one can take sinusoidal coupling of scalar field with the Chern-Simon term in the action. Also further study may be carried out by taking into account the temperature dependency of the dissipative term.

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