Quantum-Gravity phenomenology and high energy particle propagation

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Quantum-gravity effects may introduce relevant consequences for the propagation and interaction of high energy cosmic rays particles. Assuming the space-time foamy structure results in an intrinsic uncertainty of energy and momentum of particles, we show how low energy (under GZK) observations can provide strong constraints on the role of the fluctuating space-time structure.

1. Introduction

Already few years after the first theoretical study of the possible absorption of UHE protons on the low energy thermal photon background \cite{Kirzhnits} the experimental evidence of ultra GZK cosmic rays was under debate. This situation was the starting point for some speculative work on possible new physics to explain an unexpected CR spectrum above $\sim 10^{19}$ eV. To our knowledge the first authors to recognize such possibility where Kirzhnits and Chechin in 1971 \cite{Kirzhnits}. In their pioneering work in fact they wrote “Primary protons with energy above $5 \cdot 10^{19}$ eV are expected to be strongly slowed down by the interaction with the background thermal radiation. However, no break is observed in the CR spectrum in this region. It is of course premature in this circumstances...” and the key observation was “The point is that the primary protons have a uniquely large Lorentz factor $\gamma > 510^{10}$ larger by many order of magnitudes than in any other experiment.”. With these premises they proposed a modified theory to introduce small violations in the dispersion relation of particles at sufficiently high energies in such a way to account for the absence of the so-called GZK feature in the spectrum.

More than 30 years after the experimental situation is still unclear: present operating experiments are AGASA \cite{AGASA} and HiRes \cite{HiRes}, and they do not provide strong evidence either in favor or against the detection of the GZK feature \cite{HiRes}. A substantial increase in the statistics of events, as expected with the Auger project \cite{Auger} is expected to clarify the scenario in one or two more years.

2. From Quantum Gravity to Lorentz invariance breaking

The theoretical approach to possible violations to standard (i.e. Lorentz and Poincaré invariant) physics changed substantially in last years. It is not any more a simple exotic possibility to interpret unclear experimental data but there is a growing feeling that such violations will be a necessary ingredient to properly describe phenomena at very small distance scales.

The most ambitious program tries to merge the
typical quantum behavior and the General Relativity in a well defined theory of Quantum Gravity (QG). Several attempts to construct a model for QG have been done. They basically share a new interpretation for space-time: it is no more a given background for physical objects but, more properly, a derived concept itself. It is expected to be characterized by a typical length scale (the Planck length $l_p \approx 10^{-33}$ cm is a natural candidate) where its structure becomes dominated by quantum fluctuations and basically undefined (at least for our present capabilities to describe it). This attempts include Loop QG, some string-based model and the space-time foam approach (the latter is indeed the older one and traces back to ’60s [7]).

Apart from those first principles constructions other models have been proposed as effective theories that should try to catch some of the possible new QG physics at small but still super Planckian length scales. They basically include the large family of models based on Quantum deformed Poincaré Groups as well as the Doubly Special Relativity construction(s).

All these approaches are very different and they predict some modification of basic physical principles. The following is a non-exhaustive cumulative list of the different possibilities. The first is the possibility of modification of Poincaré and Lorentz symmetries. Depending on the specific model they can still be exactly realized as well as explicitly broken (introducing a preferred reference frame) or kept but in a deformed way. Also the energy-momentum relation can be modified including extra terms that can be of fixed or stochastic nature. Generally a new invariant physical scale ($l_p$ or the Planck energy $E_p$) is introduced and this scale can (eventually) coexist with the standard invariant: the low energy light velocity $c$. Other possible effects are indetermination in position and/or momentum measurements due to the fluctuating nature of the space-time structure and the appearance of new non-linear composition laws for energy and momentum of multiparticle states i.e. $P_{tot} \neq \sum_i P_i$.

Many of these possibilities have been investigated trying to find possible experimental signatures for new physics even at energy scales much smaller the $10^{28}$ eV that correspond to the Plank energy.

Astroparticle physics is a privileged arena for such studies both for the availability of very energetic particles and for the possibility to consider their motion along large (cosmological) distances. Among the others the large distance propagation of photons with energy dependent velocity and modifications induced in the standard synchrotron radiation emission process have been considered to put limits on possible Lorentz Invariance (LI) breaking [10][11][12].

Another interesting possibility to test such models is to consider physical processes with a kinematic energy threshold, which is in turn very sensitive to the smallest violations of LI. This is the case for UHECRs and TeV gamma rays. UHECRs are expected to suffer severe energy losses due to photopion production off the photons of the cosmic microwave background (CMB), and this should suppress the flux of particles at the Earth at energies above $\sim 10^{20}$ eV, the so called GZK feature. Super-TeV energy photons from sources at cosmological distances are expected to undergo electron-positron production in interactions with low energy photons of the far infra red background (FIRB) and CMB.

In both cases a very large $\gamma$ factor is involved in moving from the laboratory to the center of mass reference frame. The sharply defined thresholds can be substantially shifted (or even disappear) if a small LI breaking term is introduced giving potential for investigation in this field. The new phenomena, if present, should show up in modification of expected UHECRs spectrum.

Some authors [13][14][15] have invoked possible violations of LI as a plausible explanation to some puzzling observations related to the detection of ultra high energy cosmic rays (UHECRs) with energy above the GZK feature, and to the unexpected shape of the spectrum of photons with super-TeV energy from sources at cosmological distances.

Both types of observations have in fact many uncertainties, either coming from limited statistics of very rare events, or from accuracy issues in the energy determination of the detected particles, and most likely the solution to the alleged
puzzles will come from more accurate observations rather than by a violation of fundamental symmetries.

For this reason, from the very beginning we proposed [16] that cosmic ray observations should be used as an ideal tool to constrain the minuscule violations of LI, rather than as evidence for the need to violate LI.

We adopt some reasonable choice to parametrize the LI violations predicted by QG models, consider the theoretical consequences and compare with experimental data. If the features in the spectrum related to the processes thresholds are indeed found this will provide limits on LI violation scale. If such features are absent this will allow us to reject some models but, for the moment, not to prove the existence of LI breaking new phenomena.

3. Fixed violations

The recipes for the violations of LI generally consist of requiring an explicit modification of the dispersion relation of high energy particles. This modification is an effective way to describe their propagation in the “vacuum”, now affected by quantum gravity (QG) phenomena. This effect is generally parametrized by introducing a mass scale \( M \), expected to be of the order of the Planck mass, that sets the scale for QG to become effective.

Without referring to any specific model, we write a modified dispersion relation obeying the following postulates:

1) violations are universal, i.e. do not depend on particle type;
2) violations preserve rotational invariance;
3) violations are an high energy phenomenon, vanishing at low momenta.

With these conditions we write the following expression:

\[
E^2 - p^2 = m^2 + p^2 f(p/M) \tag{1}
\]

This deformed dispersion relation has been proposed by several authors [17,18,19,20] and is the most popular in the literature.

Just for completeness we note that another possibility compatible with the dimensional analysis exists: it refers to the so called conformal models of LI breaking and was considered by Kirzhnits and Chechin in their paper. It accounts to introduce the extra (respect to the standard case) term proportional to the particle mass squared instead that to \( p^2 \). When considering thresholds modifications this last possibility gives no detectable effects for UHECRs propagation if \( M \) is the Planck mass.

The standard way to proceed is to expand the last term in rhs of (1) and this, at lowest order, gives a term of the form

\[
\pm (p/M)^\alpha \tag{2}
\]

where \( \alpha \) is model dependent. To get a quick result and some physical insight we can argue that, for massive particles, the above extra term in dispersion relation becomes relevant for the kinematics of particle interactions when its modulus is comparable with the particle squared mass. For the protons (i.e. for the GZK case) we get immediately the following numbers for the critical momentum \( p_c \) where we may expect changes (in the following formula we fix \( M \) to the Planck mass value):

\[
\begin{align*}
\alpha = 1 & \rightarrow p_c = (m_p^2 M^2)^{1/3} \simeq 10^{15} eV << M \\
\alpha = 2 & \rightarrow p_c = (m_p^2 M^2)^{1/4} \simeq 10^{18} eV << M
\end{align*}
\]

In both case we see that the value of \( p_c \) is much smaller than the Planck mass scale. This gives a first indication that if we modify the dispersion relation with terms related to some scale (the Planck mass in our case), the resulting particle kinematics can indeed be sensitive to such changes already at much lower energy scales. In other words possibly we do not need Planck scale experiments to detect effects related to new physics at Planck scale.

A detailed calculation of photopion and \( e^+ e^- \) threshold production for high energy protons and photons interacting with low energy background photons has been carried out [16]. In this calculation the conservation of total energy and momentum of incoming and outcoming particles is assumed.
If the total energy and momentum of multiparticle states are calculated as usual (just the sum of the contribution of each particle) and we assume that the scale parameter $M$ is the Planck mass we find that the GZK feature could be absent (the threshold goes to infinity) when we consider the minus sign in (2) or, for positive sign, shifted downward by five ($\alpha = 1$) or one ($\alpha = 2$) order of magnitude respect to the standard case.

The same calculation can be done in the framework of Doubly Special Relativity. In this case the theory is constructed in such a way that the relativity principle is still valid: no privileged reference system exists. The (non linear) deformed boost in momentum space require a change in the dispersion relation as the one previously considered but also a different definition of total energy and momentum in multiparticle states. For the DSR1 \cite{19} and DSR2 \cite{20} models we have \cite{21}:

$$E_{tot} = E_1 + E_2 - \frac{1}{2M}(p_1p_2 + p_2p_1) + O\left(\frac{1}{M^2}\right)$$

$$E_{tot} = E_1 + E_2 + \frac{1}{M}(E_1E_2 + E_2E_1) + O\left(\frac{1}{M^2}\right)$$

In this case basically no new particle processes (respect to the standard theory) are kinematically allowed and, for the GZK case, the momentum threshold is basically the same as in standard case \cite{22}.

In drawing conclusions from this kind of studies we have to keep in mind that there are two main problems. The first is the poor knowledge of UHECRs sources, a piece of information needed to predict the actual effect of photopion production on the cosmic ray spectrum and, at the end, necessary to correctly interpret the experimental data. The second is related to the limitation of approaches based uniquely on kinematic analysis: the present impossibility to include the dynamical effects of the full theory makes quantitative conclusions questionable (even if it seems reasonable to expect modifications to dynamics to be proportional to the energy scale divided by $M$ and hence highly suppressed for physics below GZK scale).

At the end, after a suitably parametrization of LI violating models and an analysis of interaction kinematics, we conclude that it is actually prema-

ture to explain GZK absence as a manifestation of LI violations. On the other side, once the experimental situation will be clarified, it will be possible to put strong limits on new physics i.e. on the breaking parameter $M$ considered as an independent quantity.

4. From fixed to stochastic violations

One can ask if the above is the only possibility to introduce some remnant of QG effects in low energy particle behavior. To answer this question we first note that it is generally believed that coordinate measurements cannot be performed with precision better some quantity related to the Planck distance (time): $\delta l \geq l_p(\eta l_p)^\eta$ (where $\eta = 0, 1/2, ...$ depending on the particular model), since such a measurement would result in the production of a black hole.

We consider the $\eta = 0$ case and argue that such indetermination can be seen as the result of a fluctuating metric tensor $g_{\mu\nu}$. The metric tensor, has felt by a propagating particle of wavelength $\lambda$, can be written as a “standard” term plus a variation $\delta g_{\mu\nu}$ that should account for the quantum fluctuations of space-time. If we stick again to the postulates of previous section we can write $\delta g_{\mu\nu} = h_{\mu\nu}(l_p/\lambda)$ where $h_{\mu\nu}$ is a rotationally symmetric tensor of order 1.

To go from position indetermination $\delta l = l_p$ to momentum and/or energy fluctuation we need to assume the validity of some sort of QG-modified De Broglie relation (see for example \cite{23})

$$\delta p = \delta \left\{ \frac{1}{\lambda} \left[ 1 + f \left( \frac{l_p}{\lambda} \right) \right] \right\} \simeq p^2 l_p + O(l_p^2) \quad (3)$$

where $\lambda$ is again the particle wavelength. We think it is safe to assume the above relation to have some sense at least for energies small compared to $M$.

We can conclude that energy and momentum of particles are not constant during the propagation: their measurement would result in slightly different values in different space-time positions. If we consider each particle interaction equivalent to a $E,p$ measurement it results that kinematics becomes stochastic: a given process has some probability to be allowed even for four momentum
below the classical threshold value. So, in general, we can add a fluctuating term to $E, p$ and to the dispersion relation (the latter eventually adding a fixed extra term like the one considered in previous section) and write

$$E_i = \bar{E}_i + \alpha_i \frac{\bar{E}_i^2}{M}$$
$$p_i = \bar{p}_i + \beta_i \frac{\bar{p}_i^2}{M}$$

and

$$E_i^2 - p_i^2 = m_i^2 + \gamma_i \frac{\bar{E}_i^3}{M}$$

where $i$ labels the interacting particle, $\bar{p}$ and $\bar{E}$ are the mean (i.e. classical) values. $\alpha_i, \beta_i, \gamma_i$ are random variables expected to be of order one that we assume to be statistically independent since the typical interaction scale in the processes we are interested ($E \leq 10^{20}$ eV) is much larger than the QG scale $M$. The above expressions have been motivated in various ways in previous papers [24,25,26,27,28].

Assuming total energy-momentum conservation and standard composition laws we find that a proton with energy $E > 10^{15}$ eV has $\simeq 70\%$ probability to interact with a $\gamma_{CMB}$ thus loosing some of its energy. The result is insensitive to the detail of the fluctuation i.e. it is valid basically for any choice of the probability distribution function of the $\alpha, \beta, \gamma$ [27].

Apart from possible modification of $p - \gamma_{CMB}$ and $\gamma - \gamma_{FIRB}$ thresholds we also considered the effect of fluctuations on particle stability. Indeed, due to fluctuations, a particle propagating in vacuum acquires an energy dependent (fluctuating) mass, which may be responsible for kinematically forbidden decays to become kinematically allowed.

It is the case for photon decay $\gamma \rightarrow e^+e^-$, possible as soon as the photon has momentum $p > p_{th} \simeq 10^{13}$ eV, and proton decay $p \rightarrow p + \pi^0$, possible for $p > p_{th} \simeq 10^{15}$ eV. But the most dramatic effect would be the Vacuum Cerenkov radiation emission from charged particles $x \rightarrow x + \gamma$. In this case the threshold can be written as

$$p_{th} \simeq \left( \frac{m^2 M \omega}{\delta} \right)^\frac{1}{4}$$

where $m$ is the particle mass, $\omega$ is the photon energy and $\delta$ is some combination of the fluctuating coefficients of $E_i$. Clearly $p_{th} \rightarrow 0$ if $\omega \rightarrow 0$ and this will eventually result in a stability crisis for all charged particles [29].

This scenario is clearly experimentally excluded and we can go back trying to understand if the apparently natural fluctuation picture has to be discarded or some possible way out is still present. We first note that the presence of fixed modifications in the dispersion relation does not change the result.

A complete analysis unfortunately would imply some knowledge of the dynamical effects to answer questions like

1. if the particles were kinematically allowed to decay, and there were no fundamental symmetries able to prevent the decay, would it take place?
2. is the form adopted for the quantum fluctuations correct and if so, how general is it?
3. are the various fluctuating terms really independent?
4. if in fact the form adopted for the fluctuations is correct, how general and unavoidable is the consequence that (experimentally) unobserved decays should take place?

Another interesting point is to consider the combined effect of assuming different energy-momentum conservation laws, like the one in DSR models, and fluctuations [30].

There is no answer to this questions at the moment but we can say that the QG phenomenological models considered here can already be excluded.

In any case the positive conclusion is that it is clear that UHECRs are a powerful tool to test the LI breaking scenario either making QG phenomena detectable or severely constraining it.

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