Identification of defects in a ferromagnet by solving the inverse problem of magnetostatics

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Abstract. For many industries, the problem of detecting and classifying surface defects by a magnetic method is very important. Wherein the mutual configuration of magnetic converters and the object under study can be shown on the example of a device for in-pipe inspection of large-diameter pipes of gas pipelines. In this case, the magnetic transducers are located in the device inside the pipe, while the defects are located on the outer surface of the pipe. The data obtained from magnetic Hall transducers, measuring the magnetic field of scattering over the entire surface of the pipe, allows us to solve the inverse problem of magnetostatics on the restoration of magnetic fields to their sources. Due to the fact, that the pipes are made of a magnetically soft ferromagnet, it becomes possible to solve the problem of restoring the geometry of the surface corrosion defect in the metal. The proposed work contains an algorithm for reconstructing the geometry of surface defects of corrosion of an arbitrary shape from the measured magnetic scattering field over the metal surface opposite to surface, which contained the defect.

1. Introduction

Before, the magnetic method is used to determine only “problem” zones, i.e. pipe areas with abnormal magnetic fields above them, due to the presence of defects of the loss of metal continuity in these areas. Inspection of the “problem” zones of the pipe requires additional time and additional funding, since for the final determination of the possibility of further pipe operation it is necessary to resort to additional studies using, for example, ultrasound.

However, it's well known that in magnetically soft materials the corresponding magnetic field lines with a fine accuracy coincide with the edge surfaces of a ferromagnet. Since the materials for the manufacturing of pipes for main pipelines are magnetically soft ferromagnets, the geometrical parameters of surface defects of corrosion of an arbitrary shape can be determined with great accuracy by a magnetic method. Specifically, by measuring the magnetic field inside the pipe, on some distance from its surface and after that by solving the problem of reconstructing of magnetic field components toward to its sources, i.e. defects.

2. Formulation of the problem

Consider a flat metal plate (magnetically soft ferromagnet), which is a large diameter pipe wall (such pipes are used by main gas pipelines). The diameter of the pipe is large enough to ignore the effects associated with the curvature of the wall surface (figure 1). We assume that the wall is flat. $\mu$ is the...
relative magnetic permeability of a ferromagnet. As a rule, for magnetically soft ferromagnets, \( \mu \) takes on numerical values from several hundred to tens of thousands of units.

Suppose that the X axis is directed horizontally from left to right. Z axis is directed vertically from bottom to top (figure 1). By the magnetic field measured at level Zm, it is necessary to find the geometrical parameters of the corrosion defect.

It should be noted that the normal component of the magnetic field strength experiences a jump in \( \mu \) times in the direction of increasing when moving from the metal to the air.

In the metal itself, despite the fact that at the metal-air boundary in the defect-free surface we have zero values of the normal component of magnetic field strength. The derivative of the normal component of magnetic field strength on this boundary, according to Maxwell’s equations, is not zero, which leads to an increase of it, when restoring magnetic field components down to its source, for example, by the finite difference method.

![Figure 1. The geometry of the problem.](image)

3. Algorithm of shape recovery of corrosion defects

We will solve the inverse problem in four stages.

At the first stage, we will restore the components of the magnetic field strength in the air from the level of measurements, up to the border with the metal.

The procedure for solving such problems is connected with the solution of integral equations [1–6], or using grid methods, which are described in detail in [7]. Grid methods are based on solving differential equations in finite differences. We used the finite difference method [7, 8], because this method allows obtaining solutions for the three-dimensional case, taking into account the nonlinear properties of the ferromagnet [9-11].

Let us write Maxwell’s equations (1) in the formulation of the initial value Cauchy problem

\[
\text{div} (\vec{B}) = 0 \quad \text{and} \quad \text{rot} (\vec{H}) = 0,
\]

where \( \vec{B} \) is magnetic induction vector, \( \vec{H} \) is magnetic field strength vector.

These expressions (1) mean that the following relations between the magnetic field components hold true:
\frac{\partial H_x}{\partial x} = \frac{\partial H_z}{\partial z} \quad \text{and} \quad \frac{\partial H_x}{\partial z} = \frac{\partial H_z}{\partial x} \tag{2}

The initial conditions for the Cauchy problem are the magnetic field components measured at the level \(Z_m\). By solving the system of Eqs. (2) in finite differences, we arrive at (\(Z_m = z_0\))

\[ H_x(x, z_0 - \Delta z) = H_x(x, z_0) - \frac{\partial H_z}{\partial x} \Delta z; \tag{3} \]

\[ H_z(x, z_0 - \Delta z) = H_z(x, z_0) + \frac{\partial H_x}{\partial x} \Delta z, \tag{4} \]

where the derivatives \(\frac{\partial H_z(x, z_0)}{\partial x}\) and \(\frac{\partial H_x(x, z_0)}{\partial x}\) are approximated by finite differences from the results of the measurement at the level \(z_0\).

After calculating the field components \(H_x(x, z_0 - \Delta z)\) and \(H_z(x, z_0 - \Delta z)\) at the level \((z_0 - \Delta z)\), the field components are calculated in the same manner at the level \((z_0 - \Delta z) - \Delta z\) and so on all the way to the defect-free surface of the ferromagnet in the “air”: \(H_x(x, +0)\) and \(H_z(x, +0)\).

The algorithm was tested using the results of solving the direct problem. Defects were simulated and the direct problem was solved (see figure 1) using the FEMM software package. The external longitudinal field \(H_0\) directed along the OX axis is defined by conditions that are standard for the FEMM program package and are set on the outer boundary of the computational domain. The obtained results were used as initial data for the inverse problem. The results of juxtaposing the “measured” and reconstructed fields are shown in figures 2 and 3.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure2.png}
\caption{1 - \(H_x(x, z_0)\) is “measured” field; 2 - \(H_x(x, +0)\) is obtained as a result of solving a direct problem using the FEMM software package; 3 - \(H_z(x, +0)\) is obtained by the method described above in this chapter.}
\end{figure}
At the second stage, the components of magnetic field strength vector and the components of magnetic induction vector were determined, at the air-metall border, but already inside the ferromagnetic plate, based on the field reconstructed at the defect-free plate surface. This can be easily done since the tangential component of magnetic field strength is continuous at the ferromagnet surface and the normal component of magnetic induction is continuous, too. Thus, inside the ferromagnetic plate at the level \( z = -0 \), we obtain

\[
H_x(x,-0) = H_x(x,+0), \quad H_z(x,-0) = H_z(x,+0)/\mu_{\text{ferromagnet}}
\]

At the third stage, we again take advantage of the algorithm already described in the first step, with allowance for formulas (3) and (4). The initial conditions for the Cauchy problem are now the computed components of magnetic field at the defect-free plate surface in the ferromagnet: \( H_x(x,-0) \) and \( H_z(x,-0) \). The components of magnetic field in the plate will thus be reconstructed toward the defect surface. We may solve the Cauchy problem from the defect-free metal surface toward the opposing surface, while assuming that we stay within the domain occupied by the ferromagnet.

Since in magnetically soft materials the fluxes of force lines of the magnetic stray fields make up only an insignificant part in comparison with the fluxes flowing in the pipe wall even in the area of defects, such force lines almost perfectly describe the geometry of the defect.

Exceptions are only defects such as cracks, which are not considered in this paper.

According to the results of calculating the components of the magnetic induction vector, we construct the magnetic field lines of force. Magnetic line of force, together with the left and right edges of the computational domain and the “upper” defect-free ferromagnet surface, contains a region with a border on which the boundary conditions for the normal component of the magnetic induction vector were set everywhere. These boundary conditions are satisfied to initial conditions, that is, the measured magnetic field.

**Figure 3.** 1 - \( H_x(x,z_0) \) is “measured” field; 2 - \( H_x(x,+0) \) is obtained as a result of solving a direct problem using the FEMM software package; 3 - \( H_x(x,+0) \) is obtained by the method described above in this chapter.
Choosing a line of force, on the thickness of the corresponding thickness of the plate under study, we obtain the geometry of the defect that satisfies the initial "measured" values of the magnetic field components.

So, at the fourth stage of the algorithm, we construct the magnetic lines of force, for example, magnetic induction lines of force, by using the recovered values of the components of the magnetic field.

Figure 4 shows magnetic induction lines of force constructed as a result of solving the inverse geometric problem of magnetostatics. The line of force corresponding to a plate thickness of 10 mm (constructed starting from a defect-free area at the thickness of 10 mm) is a solution to the inverse problem and accurately describes surface defects.

![Figure 4 Lines of force inside ferromagnet, reconstructed based on the results of solving the Cauchy problem.](image)

Similarly, the geometry of corrosion defects of arbitrary shape is reconstructed in the three-dimensional case, taking into account the nonlinear properties of the ferromagnet. For this, to the system of equations (1) it is necessary to add the material equations (5) and to have a numerical dependence of $\mu_{\text{ferromagnet}}$ on $|\vec{H}|$.

$$\vec{B} = \mu (|\vec{H}|)^* \vec{H}$$  \hspace{1cm} (5)

The system of Maxwell's equations, together with the material equations rewritten in finite differences as the initial Cauchy problem, will take the following form (6-11). For brevity, we omit the first two stages of the algorithm described above. Thus, we will assume that $Zm$ in equations (6-11) corresponds to -0. $Zm = -0.$
\[ B_z(x, y, Zm-\Delta z) = B_z(x, y, Zm) + (\frac{\partial B_x}{\partial x} + \frac{\partial B_y}{\partial y}) \Delta z \] (6)

\[ H_x(x, y, Zm-\Delta z) = H_x(x, y, Zm) - \frac{\partial H_z}{\partial x} \Delta z \] (7)

\[ H_y(x, y, Zm-\Delta z) = H_y(x, y, Zm) - \frac{\partial H_z}{\partial y} \Delta z \] (8)

\[ B_x(x, y, Zm-\Delta z) = \mu (H_x(x, y, Zm-\Delta z), H_y(x, y, Zm-\Delta z), H_z(x, y, Zm-\Delta z)) * H_x(x, y, Zm-\Delta z) \] (9)

\[ B_y(x, y, Zm-\Delta z) = \mu (H_x(x, y, Zm-\Delta z), H_y(x, y, Zm-\Delta z), H_z(x, y, Zm-\Delta z)) * H_y(x, y, Zm-\Delta z) \] (10)

\[ B_z(x, y, Zm-\Delta z) = \mu (H_x(x, y, Zm-\Delta z), H_y(x, y, Zm-\Delta z), H_z(x, y, Zm-\Delta z)) * H_z(x, y, Zm-\Delta z) \] (11)

The derivatives \( \frac{\partial B_x}{\partial x}, \frac{\partial B_y}{\partial y}, \frac{\partial H_z}{\partial x} \) and \( -\frac{\partial H_z}{\partial y} \) are approximated by finite-difference operators calculated at the ferromagnet surface on the metal side \( (Zm) \).

To reconstruct the three-dimensional geometry of the defects, taking into account the nonlinear properties of the ferromagnet, it is necessary to continue the algorithm described above.

The results of numerically solving the inverse geometric problem for a surface defect in the form of a cylinder are shown in figure 5. The geometrical parameters of the defect and the plate were as follows: plate thickness 17 mm, cylinder diameter 20 mm, cylinder height 5 mm. Figure 5 shows that magnetic induction lines of force with good accuracy has restored the shape of the defect.

Figure 5 Reconstruction of the geometry of three-dimensional defect by the magnetic induction lines.

4. Conclusions
The problem has been solved for loss of metal continuity defects (such as corrosion) located on the surface opposite to magnetic transducers. The fundamental possibility of solving the three-dimensional inverse geometric problem of magnetostatics with allowance for the nonlinear properties of a magnetically soft ferromagnet has been demonstrated for the first time.
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