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Modeling coupling dynamics between the transmission, intervention of COVID-19 and economic development

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A B S T R A C T

Current explosive outbreak of the novel coronavirus (COVID-19) pandemic is posing serious threats to public health and economics around the world. To clarify the coupling mechanism between this disease and economic development, a new dynamical system is established by ordinary differential equations (ODEs). It is theoretically proved that the basic reproduction number is a nonlinear combination of parameters regarding disease transmission, intervention and economy effect, which totally determines the stability of the disease-free and endemic equilibria. Further analyses indicate the existence of interaction and mutual restraint among transmissibility, quarantine and economics, in which (1) COVID-19 would cause a long-term impact on halting economic progress; (2) strong coupling of COVID-19 and economics would easily trigger disease outbreak, cause more human infections and alleviate the intervention effects of quarantine; and (3) there exists optimal strategy of time-varying quarantine for disease control and economic development. It is highlighted that adaptive isolation (rather than constant isolation) of at-risk population (rather than random individuals) is highly effective in reducing morbidity at the cost of least economic loss.

Introduction

In late December 2019, a novel coronavirus disease (COVID-19) suddenly emerged in Hubei Province, China, and rapidly spread to other countries [1]. As of 20 July 2021, nearly all countries are hit by COVID-19, which results in over 0.19 billion confirmed cases, including about 4.1 million deaths. WHO claimed that the pandemic is still accelerating, and its impacts on society and economics may last for decades [2].

A series of unprecedented intervention measures have been taken to prevent the outbreak and transmission of COVID-19. Non-pharmaceutical interventions mainly focus on social distancing, including travel restrictions, patient isolation, closure of schools and workplaces, suspension of public transportation, and wearing masks. Such strategies are playing critical roles in reducing the morbidity and mortality [3]. However, it should be noted that these interventions would directly/indirectly hinder economic development [4]. Data in Fig. 1 indicates that there exists association between COVID-19 infection and economics, in which richer countries would yield higher incidence, and those countries with higher incidence would cause larger shrink of GDP per capita (GDPC) in 2020. Specifically, Hubei’s GDP shrank by 39.2% in the first quarter of 2020 compared with the previous one, possibly due the implementation of various intervention measures. It is estimated that the reduction in GDP range from 4% under alert level 1 up to 37% under level 4 in New Zealand [5]. Hence, in facing COVID-19, it is of great significance to clarify the correlation patterns between economics and public health, and further weigh the dynamic balance between them, which can help to improve the effectiveness of mitigating disease burden and reducing economic loss.

Recently, epidemiological models were incorporated into economic patterns allowed for understanding the interplay between economic growth and epidemic ecology [6–9]. By doing so, it is found significant empirical link between tuberculosis and economic production systems [6]. Further research indicated that the health safety nets can help to break cycles of poverty and disease [7]; and a large influx of capital is successful in escaping the disease-driven poverty trap, but increasing health spending alone is not [8]. These studies offer certain technological approaches to capture the coupling mechanisms between infectious disease and economics. Existing studies mainly focus on the dynamical connection of poverty toward tuberculosis and malaria [6,10]. In reality, many disease are dynamically coupled with economics, such as HIV/AIDS and measles, especially the current COVID-19 pandemic.

On the other hand, since the massive outbreak of COVID-19, many mathematical, computational and statistical models were continually
established via ordinary differential equation (ODEs) \[11,12\], difference equations \[3\], agent-based system \[13\] and so on. By using the models to simulate the transmission process, recent studies mainly focus on predicting propagation trends \[11\], tracing transmission dynamics \[12\], estimating infection risk \[13\], and evaluating intervention effects \[3,14\]. Existing studies on the relation between COVID-19 and economics mainly stays in descriptive or analytical analysis \[15, 16\]. Typical works include quantitatively analyzing the impact of the COVID-19 pandemic in low- and middle-income countries \[15\], and evaluating the monthly economic losses in Wuhan under the lockdown against COVID-19 by using compartment model and Input-Output model \[16\]. However, there is a lack of mechanism framework to model the coupling dynamics between the transmission, intervention of COVID-19 and economic development.

This paper goes a further step to formulate and clarify the coupling patterns between COVID-19 transmission and economic development. First, a new coupled model is established by ODEs, which accounts for the interactive dynamics of (1) virus transmission in human population by contacts; (2) economy generation; (3) the intervention strategy against COVID-19; and (4) the mutual impacts of COVID-19 propagation and economic development. Second, the dynamical behaviors of the model are explored by using stability theory and numerical analysis. Third, the existence of optimal quarantine strategy is verified. Finally, the dynamical interaction between disease transmission/control and economic development is inferred. Based on that, strategies for controlling COVID-19 transmission and reducing economic loss are provided.

Model formulation

Based on the compartmental principle and COVID-19 epidemiology, the following assumptions are proposed for model formulation.

(1) During the outbreak of COVID-19 infection, human population is divided into five subpopulations which refer to those people at susceptible, latent, infected, cured or recovered status. The susceptible refers to healthy individuals who may become infected through contact with individuals that can transmit the disease. The latent represent the class of individuals who have been infected and are capable of transmitting the disease, corresponding to the time before the onset of symptoms. The infectious individuals are those with apparent symptoms and can transmit the disease, but they have not been detected and received treatment. The cured represent the infectious individuals who have been detected and isolated (could not infect others), and are under treatment. The recovered represent the individuals who have acquired immune due to recovery and no longer involve the transmission. The proportion of population at the corresponding states at time \( t \) are represented by \( s(t), l(t), i(t), c(t) \) and \( r(t) \), respectively.

(2) The specific transmission process is as follows: susceptible individuals can be infected by contacting latent and infectious individuals, and then go through an incubation period \( 1/\gamma \). They receive treatment after an infected period \( 1/\delta \) and finally recover. It is assumed that the total density of human is constant as unit, in which birth rate equals death rate \( b \). For disease control, parts of healthy people at susceptible, latent and infectious states are quarantined/isolated, the proportions of which are denoted by \( q_1, q_2, \) and \( q_3 \), respectively.

(3) The economic condition is measured by GDPC and its development in time is formulated by Logistic model: \( g'(t) = \theta g(t)(\eta g - g(t)) \), where \( g(t) \) represents the GDPC at time \( t \) and other parameters can be seen in Table 1. This model can describe the S-curve of the structural changes, which have been widely employed to model economic growth process \[17,18\]. It also performs well in fitting the monthly GDPC in China, in the absence of COVID-19 infection (see Fig. 1 C).

(4) The dynamics of COVID-19 and economics are inexorably linked. First, economic growth can improve working environment and healthy living conditions, and make treatment more standardized and perfect. Hence high GDPC can potentially reduce infectivity and speed up recovery. Second, lower incidence rate of COVID-19 would cause less damage to social-economic environment, and more people can work normally, which can potentially promote economics. Particularly, since COVID-19 patients (latent, infectious and treated) and quarantine people could not work, GDPC growth will be constrained by COVID-19 epidemic and its development only depends on susceptible and recovered people. Taking such effect into account, the Logistic model of GDPC growth is modifies as the first equation in model (1), which is employed from the income formula proposed in \[7\]. Referring to the modeling framework proposed in \[6–9\], the coupling dynamics are further described as follows. The infectivity and recovery rate are nonlinear saturation functions of GDPC \[7\], with \( k_i \) \((i = 1, 2, 3)\) as the coupling weight between economics and infectivity/treatment (see model (1)). Specifically, they refer to the half-saturation constants, in which \( k_i \) is the amount of GDPC necessary to reduce the infectivity from latent (infectious) individuals by half, and \( k_i \) represents the GDPC level necessary for an individual to be in a half-recovery state \[9\].

Based on the above descriptions, the transfer flowchart is shown in Fig. 2, and the corresponding dynamical process is simulated by the following ODEs:

\[
\begin{align*}
\frac{ds'}{dt} &= \theta g \left[ g_0 ((1 - q_1)s + r) - g \right], \\
\frac{dl'}{dt} &= b - (1 - q_1)s \left[ 1 - q_2 \left( \frac{\eta r k_1}{g + k_1} + (1 - q_3) \left( \frac{\delta r k_2}{g + k_2} \right) \right) \right] - bs, \\
\frac{di'}{dt} &= (1 - q_1)s \left[ 1 - q_2 \left( \frac{\eta r k_1}{g + k_1} + (1 - q_3) \left( \frac{\delta r k_2}{g + k_2} \right) \right) \right] - \eta l - bl, \\
\frac{dc'}{dt} &= \eta l - \delta i - hi, \\
\frac{dr'}{dt} &= \eta g - \delta c - br, \\
\end{align*}
\]

where the description of the model parameters is presented in Table 1. It should be noted that referring to the infectivity formulated in \[11\], the original reproduction number (i.e., \( R_0 \)) is incorporated into the model, which equals the product of effective contact rates and time duration staying in the corresponding state.
The Jacobi matrix of system (1) at locally asymptotically stable.

\[ \mathbf{J}(\mathbf{x}) = \begin{bmatrix} -\theta g_2(1 - q_1) & \theta g_1(1 - q_2) & 0 & 0 & \theta g_0(1 - q_1) \\ 0 & -\theta g_1(1 - q_2) & 0 & 0 & \theta g_2(1 - q_1) \\ 0 & -\theta g_2(1 - q_1) & -C & -\delta & -D \\ 0 & 0 & -\theta g_1(1 - q_2) & -\delta & -D \\ 0 & 0 & 0 & -\theta g_0(1 - q_1) & -\delta \end{bmatrix} \]

where

\[ C = \frac{\eta r_1 k_1 (1 - q_1) (1 - q_2)}{g_0 (1 - q_1) + k_1}, \quad D = \frac{\delta r_2 k_2 (1 - q_1) (1 - q_2)}{g_0 (1 - q_1) + k_2}, \]

\[ E = \frac{g_0 (1 - q_1)}{k_1} \]

The characteristic equation of \( DF(0) \) is

\[ (\lambda + \theta g_2(1 - q_1)) (\lambda + E + b) (\lambda + b) (\lambda^2 + a_1 \lambda + a_2) = 0 \]

where \( a_1 = \delta + b + q + b - C \) and \( a_2 = (\eta + b - C)(\delta + b) - D = (\delta + b)(q + b)(1 - R_0) \). Since the eigenvalues \( \lambda_1 = -\theta g_2(1 - q_1) \) and \( \lambda_2 = -E - b \) and \( \lambda_3 = -b \) are negative, it only needs to consider the solution of \( F(\lambda) = \lambda^2 + a_1 \lambda + a_2 = 0 \). When \( R_0 < 1 \), it follows that \( a_1 > 0 \) and \( a_2 > 0 \). According to Hurwitz criterion, \( F(\lambda) \) has two eigenvalues of negative real part. Hence \( e_2 \) is locally asymptotically stable when \( R_0 < 1 \).

**Theorem 2.** When \( R_0 < 1 \), the disease-free equilibrium \( e_2 \) of system (1) is globally asymptotically stable.

**Proof.** A Lyapunov function is defined as

\[ V = \frac{(\delta + b) C + \eta D}{\eta + b(\delta + b)} + \frac{D}{\delta + b} \]

For \( R_0 < 1 \), the derivatives of \( V \) along model (1) is

\[ V = (C + D) \left[ \frac{(\delta + b) C + \eta D}{\eta + b(\delta + b)} - s \right] \leq (C + D)(R_0 - 1) \leq 0 \]

Since \( V = 0 \) only if \( l = i = 0 \) or \( R_0 = 1 \), and the maximum invariant set in \( \{(g, s, l, i, c, r) : V = 0\} \) is the singleton \( \{e_2\} \), it follows from LaSalle’s invariance principle that \( e_2 \) is globally stable.

**Theorem 3.** When \( R_0 > 1 \), there exists an endemic equilibrium \( P^* \) in system (1), which is locally asymptotically stable.
Theorem 4. There exists an optimal quarantine \((q_1^*, q_2^*, q_3^*)\) such that (4) holds, subject to the control system (1) of state variables with nonnegative initial conditions.

The above theorem indicates that there exists optimal quarantine strategy under which maximal reduction of human infections and best economic development can be realized simultaneously. The following analysis will demonstrate that such strategy refers to adaptive quarantine.

Quantifying coupling dynamics

The model framework is activated by specific parameters with biological background, which allows for further digging the dynamical interactions between COVID-19 transmission and economic development. In what following, the evolutions of model variables are simulated with different coupling weights and quarantine strategies. The initial conditions are assumed to be \(g(0) = 5908\) (real data in China in 2019), \(s(0) = 0.9999\), \(t(0) = 0.001\) and \(c(0) = r(0) = 0\).

Fig. 3 shows the effects of coupling weight (between COVID-19 and economics) on the progress of disease and GDPc, in case of no quarantine. It is observed that (1) when \(R_0 < 1\), the incidence declines directly and rapidly, and in this case, GDPc is scarcely influenced by human infection and the coupling weights do not work; (2) when \(R_0 > 1\), followed by a sharp rise in early three months, the incidence begins to decrease and then keeps stable in very low level; (3) when \(R_0 > 1\), GDPc progress is halted during the epidemic period and then increases gradually at relatively lower level, compared with disease-free case; (4) when \(R_0 > 1\), larger coupling weights (bigger \(k_i\)) would cause higher morbidity and more significant reduction in GDPc, (5) treatment weight coefficient of GDP \((k_i)\) could not work on incidence rate, but it can hinder economic growth due to longer treatment cycle. It is highlighted that the outbreak of COVID-19 causes damage to economic growth, and it will take over ten years for economy to recover.

Fig. 4 illustrates the impacts of coupling weight and quarantine/isolation on the basic reproduction number \(R_0\). It is observed that low infectivity weights of GDPc \((k_1\) and \(k_2)\) and large quarantine/isolation rates \((q_i\), \(i = 1, 2, 3)\) can drive rapid rising of \(R_0\), and the latter is more effective in case of weak coupling weight. When \(k_1\) and \(k_2\) increase from 0 to 12,000, \(R_0\) will increase from 0 to 2.1 if \(q_1 = q_2 = q_3 = 0\), and \(R_0\) will increase from 0 to 1 if \(q_1 = q_2 = q_3 = 0.37\), suggesting the key role of quarantine in preventing COVID-19 transmission. When the coupling weight \(k_1 = k_2 = k_3 = 4000, 6000\) and 8000, to reduce \(R_0 < 1\), it requires separately the following proportions of people to be quarantined/isolated: (1) about 12%, 23% and 31% of individuals at susceptible, latent and infected states; or (2) about 18%, 33%, and 42% of infectious individuals; or (3) about 38%, 52%, and
Fig. 3. Time evolutions of GDPC and human infections with different coupling weights (denoted by $k_i, i = 1, 2, 3$, which varies from 600 to 12,000) between economies and COVID-19. The quarantining/isolation rates are $q_1 = q_2 = q_3 = 0$. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

Fig. 4. The impacts of quarantining/isolation (denoted by $q_1, q_2, q_3$) and coupling weight (denoted by $k_1, k_2, k_3$) between economy and COVID-19 on the basic reproduction number $R_0$, in which $q_1 = 0$ in (B) and $k_1 = k_2 = 6000$ in (C).

60% of susceptible people. By comparing figures A and B, it seems that quarantining susceptible people has small effect on inhibiting the raise of $R_0$, which just reduces the mean value of $R_0$ by 20.5% in all situations.

Fig. 5 displays the restrictive relationships of COVID-19 and economics in case of implementing quarantine measures with different strategies. It is found that both of constant and adaptive isolation can work on COVID-19 control and economic condition, in which some typical patterns are observed: (1) adaptive isolation is more effective in decreasing morbidity, but it also prolongs the epidemic in very low prevalence rate, and it may lead to second wave of human infections; (2) compared with adaptive quarantine, constant quarantine of susceptible individuals would cause larger damage to economic growth and release less infection burden; (3) compared with no quarantine, quarantining susceptible individuals would cause much more economic loss; and (4) adaptive isolation of those people in latent and infected states has minimum impact on GDPC.

Finally, sensitivity index of $R_0$ to model parameters is calculated by using formula $\frac{\partial R_0}{\partial \delta}$ (for parameters $\delta$) [22]. As show in Fig. 6, it is found that the original reproduction number ($r_i$), and the weighting coefficients of economics on infectivity ($k_1$ and $k_1$) are positively correlated with $R_0$, indicating that releasing the coupling weight between COVID-19 and economics, and especially cutting down contacts with infected people can effectively reduce infection risk. Furthermore, the maximum GDPC in the absence of disease ($g_0$) and the isolation rates ($q_i$) are negatively associated with $R_0$, indicating that economic development and quarantine strategy can significantly reduce infection risk. Here $R_0$ is most sensitive to $r_2$ and $g_0$, indicating their critical roles in determining the transmission threshold.

Discussion

The current COVID-19 pandemic is far more than a global health crisis, but a huge challenge to societies and economics, which is likely to increase poverty and inequalities at a global scale [22]. The International Labour Organization predicts that the pandemic will trigger a 60% decline in earning for 1.6 billion informal works, and half of the world is trying to survive without any form of social protection. Hence, assessing the impacts of the COVID-19 pandemic on societies and economics is fundamental to inform and tailor the authoritative responses to recover from the crisis [23]. This paper is an attempt to clarify the coupling patterns between the transmission, intervention of COVID-19 and economic development.

To this end, a new mathematical model is developed by using a deterministic system with biological significance. Differing from existing studies related to COVID-19 which aims at either mathematical/numerical analysis on spread dynamics or analytical analysis on economic impacts [1,3–5,11,12], our model formulates the essential interactions between disease prevalence and economic development,
Fig. 5. Time evolutions of GDPC and human infections with different intervention strategies with $k_1 = k_2 = k_3 = 6000$. The adaptive quarantine/isolation rates in (A), (B) and (C),(D) at time $t$ are defined as $q(t) = \min\{6c(t-1), 1\}$, and $q(t) = \min\{10c(t-1), 1\}$, respectively. For comparability, the constant quarantine/isolation rate equals the mean value of $q(t)$. The color legend is the same in the four figures.

Fig. 6. Sensitivity indices of model parameters to the basic reproduction number. which allows for clarifying the dynamical mechanism behind the incidence data and economic situations. Specifically, the proposed model incorporates the critical parts necessary for describing the integrated dynamics: (1) the contact-based infection and the essential transmission of the virus among humans are simulated by a compartmental susceptible–latent–infected–treated–recovered model via ODEs, and the modeling technology is similar to recent studies [3,11,12,14]; (2) the intrinsic production of GDP is portrayed by a Logistic equation adopted from [7]; (3) similarly to the modeling framework in [11], the isolation strategy is quantified by restricting the pool of active population; and (4) economics affects COVID-19 transmission by modulating the infectivity and recovery, while COVID-19 reacts upon economics by constraining its development, in which coupling expressions are referred to the technology established in [6–9]. Hence, the proposed model is a combination update of recent works, which covers more dynamical details connecting disease transmission, individual quarantine and economic growth.

By using the theory of next generation matrix, Lyapunov function and Jacobi matrix, the basic reproduction number $R_0$ is formulated and is proved to be a sharp threshold to determine the outbreak of the disease. When $R_0 < 1$, no matter how many individuals are initially infected, COVID-19 disease will be quickly eliminated, and economics suffer little damage. Otherwise, when $R_0 > 1$, the disease will become endemic and will cause large economic loss. Sensitivity analysis reveals that the value of $R_0$ greatly depends on intrinsic transmissibility (e.g., infection rate, incubation period and infected period), coupling weight, and intervention strategies (e.g., early detection, early treatment and quarantine). Yet this number is independent of the rate of economic growth.

Further results indicate that there is a mutual restraining patterns between COVID-19 transmission and economic growth. Tight coupling correlation between economics and COVID-19 would easily trigger disease outbreak and raise morbidity, which potentially hold back economic progress, resulting in a poverty trap. The possible reasons are that high incidence rate leads to less labor force and less economic output. In this case, providing financial assistance for attacking areas or patients can release the connection between infection and economy loss, which can help to escape the poverty trap stemmed from COVID-19 disease.

Particularly, it is found that quarantine measures can avail to contain the propagation of COVID-19, but also slow down economic development in different degrees. Specifically, quarantining healthy individuals would dramatically retard economics and only release limited infection burden. Compared with constant quarantine and adaptive quarantine toward random population, adaptive quarantine toward people in latent and infected states is the best strategy, which can reduce most human infections and bring a minimal barrier to economic development. Doing so requires timely detection and isolation of those persons at risk (e.g., the people who once contact with patient or has been to infected regions), and guarantees that healthy persons can work in safe environment. Such strategies are currently adopted in China and many other regions. Moreover, numerical results demonstrate that it would take dozen years for economics to recover from a half-year infection of COVID-19, indicating a long-term impact of COVID-19 on halting economic progress. The obtained results can provide scientific reference for COVID-19 epidemic control and policy formulation.

The following limitation in this paper is worthy of further improvement and discussion: the proposed model is a simplification of practical issue, which does not take into account all potential factors, such as human mobility, asymptomatic infection and vaccination. It is also not
employed to fit real surveillance data, in which the biological parameters are extracted from existing study. The model can be generalized to incorporate more information in reality, such as vaccination and economic measures, and to fit the data of incidence and GDP, which can help to further clarify the coupling patterns between disease and economics, and provide more scientific clues to disease prevention and economic development.

CRediT authorship contribution statement

Zhaowang Zhang: Formal analysis, Validation, Funding acquisition, Writing – original draft. Lingming Kong: Formal analysis, Funding acquisition, Writing – review & editing. Huiliang Lin: Investigation, Data curation, Funding acquisition, Writing – review & editing. Guanghu Zhu: Supervision, Methodology, Validation, Funding acquisition, Writing – review & editing.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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