Intuitionistic fuzzy semi d-ideal spectrum

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Abstract. The main purpose of this paper is to study the spectrum of intuitionistic fuzzy semi d-ideal in d-algebra, and the relationship between the topological properties and the algebraic properties of the Spectrum of d-algebra \(X\) with respecting to connectedness and separation axioms .

1. Introduction

BCK-algebra is a classe of abstract algebras introduced by Y. Imai and K. Iseki [9,15] . A d-algebra is a useful generalization of BCK-algebra was introduced by J. Negger and H. S. Kim [7]. J. Negger , Y. B. Jun and H. S. Kim [8] discussed ideal theory in d-algebra. After the introduction of intuitionistic fuzzy set by Atanassov in 1986 [10], there was a number of generalizations of this concept. This concept was generalizations for fuzzy set concept which was introduced by Zadeh in 1965 [11]. In [14] Y. B. Jun, J. Neggars and H. S. Kim apply the ideal theory in fuzzy d-ideals of d-algebras . H. K. Abdullah and A. K. Hasan introduce the notation of semi d-ideal of d-algebra in [5]. Y. B. Jun , H. S. Kim and D.S. Yoo in [13] introduced the notion of intuitionistic fuzzy d-algebra. A. K. Hasan introduce the notion of intuitionistic fuzzy semi d-ideals of d-algebra in [1] . Ali K. Hasan and Osamah A. Shaheed introduce the notion of intuitionistic fuzzy prime semi d-ideals of d-algebra in [2], and in this paper we study the spectrum of intuitionistic fuzzy semi d-ideal in d-algebra, and the relationship between the topological properties and the algebraic properties of the d-algebra \(X\). Also we consider strongly connected and separated properties .

2. Background

This section contains some basic about intuitionistic fuzzy set and the ordinary and intuitionistic fuzzy concepts about semi d-ideal and prime semi d-ideal in d-algebra, with some theorems and propositions.

Definition (2.1) : [7] A d-algebra is any non-empty set \(X\) with a binary operation \(\ast\) and a constant \(0\) which satisfies that:
I. \(a \ast a = 0\)
II. \(0 \ast a = 0\)
III. If \(a \ast b = b \ast a = 0\) then \(a = b\ \forall a, b \in X\).

We will refer to \(a \ast b\) by \(ab\), and it is said to be commutative if \(a(ab) = b(ba)\) for all \(a, b \in X\), and \(b(ba)\) is denoted by \((a \land b)\). Every set \(X\) in the following is a d-algebra

Definition (2.2) :[5] A semi d-ideal of a d-algebra \(X\) is a non empty subset \(J\) of \(X\) satisfies i) \(a, b \in J\) imply \(ab \in J\) , ii) \(ab \in J\) and \(b \in J\) imply \(a \in J\) , for all \(a, b \in X\)

Definition(2.3) : [4] In a commutative d-algebra \(X\), a semi d-ideal \(I\) is said to be prime if \(a \land b \in I\) implies \(a \in I\) or \(b \in I\) , for all \(a, b \in X\).

Definition (2.4) [10] : An IFS " intuitionistic fuzzy set " \(A\) in a set \(X\) is an object having the form \(A = \{< a, \alpha_A(a), \beta_A(a) >: a \in X\}\) , such that \(\alpha_A: X \rightarrow [0,1]\) and \(\beta_A: X \rightarrow [0,1]\) denoted the degree of
membership (namely \(\alpha_A(a)\)) and the degree of non membership (namely \(\beta_A(a)\)) for any elements \(a \in X\) to the set \(A\), and \(0 \leq \alpha_A(a) + \beta_A(a) \leq 1\), for all \(a \in X\). To simplicity, we shall use \(A = \{ < a, \alpha_A(a), \beta_A(a) > : a \in X \}\).

**Definition (2.5)**: [3] Let \(X, Y\) be d-algebra and let \(f : X \rightarrow Y\) be a homomorphism mapping, and \(C\) be IFS in \(X\) we define IFS, \(f(C)\) in \(Y\), by

\[
\begin{align*}
    & f(C)_y = \{ \alpha_{f(C)}(b), \beta_{f(C)}(b) \} \quad \text{where} \quad \alpha_{f(C)}(b) = \\
    & \sup \{ \alpha_C(a) \mid a \in X, f(a) = b \} \quad \text{if} \quad f^{-1}(b) \neq \emptyset , \quad \text{and} \\
    & 0 \quad \text{otherwise}, \\
    & \beta_{f(C)}(b) = \begin{cases} 
    \inf \beta_C(a) & a \in X, f(a) = b \\
    0 & \text{otherwise}
    \end{cases} \quad \text{if} \quad f^{-1}(b) \neq \emptyset \quad \text{for each} \quad b \in Y.
\end{align*}
\]

**Definition (2.6)**: [6] Let \(\mu, \nu \in [0,1]\) such that \(\mu + \nu \leq 1\). An intuitionistic fuzzy point \(x_{(\mu,\nu)}\) is defined to be an IFS in \(X\), define by \(x_{(\mu,\nu)}(y) = \begin{cases} 
    (\mu, \nu) & \text{if} \quad y = x \\
    (0,1) & \text{if} \quad y \neq x
    \end{cases} \quad \text{for all} \quad y \in X\), and \(x_{(\mu,\nu)} \in A\) if and only if \(\alpha \leq \mu(x)\) and \(\beta \geq \nu(x)\).

**Notation (2.7)**: Let \(A\) be an IFS of a d-algebra \(X\). We denote a level cut set \(A_\alpha\), by \(A_\alpha = \{ x \in X : \alpha_A(x) = \alpha_0\} \).

**Definition (2.8)**: [3] The IFS \(\bar{0}\) and \(\bar{1}\) in \(X\) are define as \(\overline{0} = \{ (x, 0.1), x \in X \}\) and \(\overline{1} = \{ (x, 1.0), x \in X \}\), where \(1\) and \(0\) represent the constant maps sending every element of \(X\) to \(1\) and \(0\), respectively.

**Definition (2.9)**: [1] An intuitionistic fuzzy semi d-ideal of \(X\), "shortly IFSd – ideal", is an IFS \(D = \{ \alpha_D, \beta_D \} \) in \(X\) satisfies the following inequalities:

\[
\begin{align*}
    & (\text{IFSd}_1) \quad \alpha_D(a) \geq \min(\alpha_D(ab), \alpha_D(b)), \quad (\text{IFSd}_2) \quad \beta_D(a) \leq \max(\beta_D(ab), \beta_D(b)) \\
    & (\text{IFSd}_3) \quad \alpha_D(ab) \geq \min(\alpha_D(a), \alpha_D(b)), \quad \text{and } (\text{IFSd}_4) \quad \beta_D(ab) \leq \max(\beta_D(a), \beta_D(b)), \quad \text{for all} \quad a, b \in X.
\end{align*}
\]

**Definition (2.10)**: [2] An IFSd – ideal \(D = \{ \alpha_D, \beta_D \} \) of \(X\) is an intuitionistic fuzzy prime semi d-ideal "shortly IFSd – ideal " in \(X\) if it is satisfies \((\text{IFSd}_1) \) \(\alpha_D(a, b) \leq \max(\alpha_D(a), \alpha_D(b))\) \((\text{IFSd}_2)\) \(\beta_D(a \land b) \geq \min(\beta_D(a), \beta_D(b))\), for all \(a, b \in X\).

**Theorem (2.11)**: [2] If \(D = \{ \alpha_D, \beta_D \} \) is an IFSd – ideal, then the set \(A_\alpha = \{ a \in X : \alpha_D(a) = \alpha_D(0) \text{ and } \beta_D(a) = \beta_D(0) \}\) is a prime semi d-ideals.

**Definition (2.12)**: [2] A non-constant intuitionistic fuzzy ideal \(A\) of a d-algebra \(X\) is called an intuitionistic fuzzy maximal semi d-ideal if for any intuitionistic fuzzy semi d-ideal \(B\) of \(X\), if \(\text{if } A \subseteq B\), then either \(B_\alpha = A_\alpha\) or \(B_\beta = X\).

**Theorem (2.13)**: [2] Let \(A\) be an intuitionistic fuzzy maximal semi d-ideal of a d-algebra \(X\), then \(A_\alpha\) is a maximal semi d-ideal of \(X\).

**Definition (2.14)**: [2] Let \(A\) be an IFS of \(X\). Then the least IFSd – ideal of \(X\) containing \(A\) is called the IFSd – ideal of \(X\) generated by \(A\) and is denoted by \(\langle A \rangle\).

### 3. Topological spectrum

In this section we introduce the spectrum of d-algebra and we discuss the relationship between some algebraic and topological properties of d-algebra.

**Notation (3.1)**:

(i) \(\chi = \{ P, P \text{ is IFSd} – \text{ideal of } X \}\).

(ii) \(V(A) = \{ P \in \chi, A \subseteq P, \text{wher } A \text{ is an IFSd} – \text{ideal of } X \}\).

(iii) \(\chi(A) = \chi \setminus V(A) \text{ the complement of } V(A) \text{ in } X\).

**Lemma (3.2)**: Let \(A\) and \(B\) be IFSd – ideal. If \(A \subseteq B\), then \(V(B) \subseteq V(A)\).

**proof**: Let \(P \in V(B)\) that implies \(B \subseteq P\), and so \(A \subseteq B \subseteq P\) that mean \(P \in V(A)\).

**proposition (3.3)**: If \(P\) is a smallest IFSd – ideal containing \(A\), then \(V(A) = V(P)\).

**proof**: It is clear that \(V(P) \subseteq V(A)\) by lemma (3.2). Now let \(P_1 \in V(A)\), so \(A \subseteq P_1\), but \(P\) is a smallest IFSd – ideal containing \(A\), so \(P \subseteq P_1\), then \(P_1 \in V(A)\). Thus \(V(A) = V(P)\).

**Proposition (3.4)**: Let \(A\) be an IFSd – ideal, then \(V(A) = V(A)\).

**proof**: Let \(P \in V(A)\) that implies \(A \subseteq P\), and so \(\langle A \rangle \subseteq P\). Hence \(P \in V(A)\).
Conversely, let \( P \in V(A) \), then \((A) \subseteq P\), note that so \( A \subseteq (A) \subseteq P\), we get \( P \in V(A) \). Therefore \( V((A)) = V(A) \).

**Proposition (3.5)**: Let \( A \) and \( B \) be two \( IFSd - ideal \), then \( V(A \cup B) \subseteq V(A) \cup V(B) \).

Proof: Since \( A \subseteq A \cup B \) and \( B \subseteq A \cup B \), so \( V(A \cup B) \subseteq V(A) \) and \( V(A \cup B) \subseteq V(B) \). Thus \( V(A \cup B) \subseteq V(A) \cup V(B) \).

**Definition (3.6)**: For an \( IFSd - ideal \) \( A \) of \( X \). The prime radical \( rad(A) \) of \( A \) is the intersection of all \( IFSd - ideals \) of \( X \) containing \( A \). In case there is no such \( IFSd - ideal \), then \( rad(A) = \emptyset \).

**Proposition (3.7)**: Let \( A \) be an \( IFSd - ideal \), then
- \( i) \) \( A \subseteq rad(A) \)
- \( ii) \) \( rad(rad(A)) = rad(A) \)
- \( iii) \) If \( A \) \( IFSd - ideal \), then \( rad(A) = A \)
- \( iv) \) If \( A \subseteq B \), then \( rad(A) \subseteq rad(B) \).

Proof:
- \( i) \) It is clear that \( A \subseteq \cap \{ P_i, P \in A, \forall i \in \Lambda \} \).
- \( ii) \) We can easily show that \( \cap \{ P_i, rad(A) \subseteq P \} = \cap \{ P_i, P \subseteq \hat{P} \} \) for all \( i \in \Lambda \), so \( A \subseteq P \), then \( rad(rad(A)) = rad(A) \).
- \( iii) \) Since \( A \) is an \( IFSd - ideal \), then \( \cap P_i = A \) for all \( i \in \Lambda \) this mean \( rad(A) = A \).
- \( iv) \) It is clear.

**Proposition (3.8)**: For any \( IFSd - ideal \) \( A \) and \( B \) the following are hold
- \( i) \) \( V(A) = V(rad(A)) \)
- \( ii) \) \( V(A) = V(B) \) if and only if \( rad(A) = rad(B) \).

Proof:
- \( i) \) Since \( A \subseteq rad(A) \), then \( V(rad(A)) \subseteq V(A) \). Now let \( P \in V(A) \), thus \( A \subseteq P \), so \( rad(A) \cap \{ \hat{P} \in Spec(X): A \subseteq \hat{P} \} \), this imply that \( rad(A) \subseteq P \). Thus \( P \in V(rad(A)) \), then \( V(A) \subseteq V(rad(A)) \). Hence \( V(A) = V(rad(A)) \).
- \( ii) \) It is clear.

**Proposition (3.9)**: If \( f \) is a \( d \)-morphism from \( X \) to \( \hat{X} \), then \( f(x_{(\mu,\nu)}) = (f(x))_{(\mu,\nu)} \), for all \( x \in X \) and for all \( \mu, \nu \in \{ 0, 1 \} \) such that \( \mu + \nu \leq 1 \).

Proof: Let \( y \in \hat{X} \) be any element, then \( f(x_{(\mu,\nu)})(y) = \{ \alpha_f(x_{(\mu,\nu)})(y), \beta_f(x_{(\mu,\nu)})(y) \} \), where
\[
\alpha_f(x_{(\mu,\nu)})(y) = \sup \{ \alpha_{x_{(\mu,\nu)}}(p), f(p) = y \} = \begin{cases} \mu ; & \text{if } p = x, y = f(x) \\ 0 ; & \text{otherwise} \end{cases} = \alpha_{(f(x))_{(\mu,\nu)}}(y),
\]
\[
\beta_f(x_{(\mu,\nu)})(y) = \inf \{ \beta_{x_{(\mu,\nu)}}(p), f(p) = y \} = \begin{cases} \nu ; & \text{if } p = x, y = f(x) \\ 0 ; & \text{otherwise} \end{cases} = \beta_{(f(x))_{(\mu,\nu)}}(y).
\]

Hence \( f(x_{(\mu,\nu)}) = (f(x))_{(\mu,\nu)} \).

**Definition (3.10)**: Let \( A \) and \( B \) are \( IFS \) we will define \( A.B = \{ \langle a, \alpha_{A,B}(a), \beta_{A,B}(a) \rangle : a \in X \} =< \alpha_A, \alpha_B, \beta_A, \beta_B > \).

**Theorem (3.11)**: Let \( T = \{ \chi(A), A \ is \ IFSd - ideal \ in \ X \} \). Then \( T \) is a topology on \( X \).

Proof: Since \( V(\emptyset) = X \) and \( V(\overline{X}) = \emptyset \), so that \( \chi(\emptyset) = \emptyset \) and \( \chi(\overline{X}) = X \), and that implies \( \emptyset, X \in T \).

Next let \( A_1 \) and \( A_2 \) be any two \( IFSd - ideal \). Then let \( B \in V(A_1 \cup A_2) \) that mean \( A_1 \subseteq B \) or \( A_2 \subseteq B \) then \( A_1 \cap A_2 \subseteq B \), so \( B \in V(A_1 \cap A_2) \), and if \( B \in V(A_1 \cap A_2) \) we get that \( A_1 \cap A_2 \subseteq B \) and that's mean \( A_1, A_2 \subseteq B \) then \( A_1 \subseteq B \) or \( A_2 \subseteq B \) and thus \( B \in V(A_1 \cup A_2) \). Hence \( V(A_1) \cup V(A_2) = V(A_1 \cup A_2) \), so \( \chi(A_1) \cap \chi(A_2) = \chi(A_1 \cap A_2) \),
and that mean
\[ \chi(A_1) \cap \chi(A_2) = \chi(A_1 \cap A_2) \]. This show that \( T \) closed under finite intersection.

Finally, let \( \{ A_i, i \in \Lambda \} \) be any family of IF5d – ideal of \( X \) it can be easily confirm that \( \bigcup \{ V(A_i), i \in \Lambda \} = V((\bigcup \{ A_i, i \in \Lambda \})) \). In other words, \( \bigcup_{i \in \Lambda} \chi(A_i) = \chi((\bigcup_{i \in \Lambda} A_i)) \). Hence \( T \) is closed under arbitrary unions. Thus \( T \) is a topology on \( X \).

**Remark (3.12):** The topological space \((X, T)\) defined in theorem (3.11) is called the intuitionistic fuzzy prime semi d-ideal spectrum of \( d \)-algebra and is denoted by IFPSd – Spec\((X)\) or for convenience \( \chi \).

**Notations (3.13):**

1. We will denoted for all \( x \in X \) and \( \mu, \nu \in [0, 1] \) such that \( \mu + \nu \leq 1 \), then
2. Let \( A \) be an IF5S of the \( X \). Put \( (A) = ((\alpha_0, \beta_0), (\alpha_1, \beta_1), \ldots, (\alpha_n, \beta_n)) \), where \( \alpha_i, \beta_i \in [0, 1] \) such that \( \alpha_i + \beta_i \leq 1 \) for all \( i = 0, 1, 2, \ldots, n \).

**Theorem (3.14):** Let \( x, y \in X \) and \( \mu, \nu \in [0, 1] \) such that \( \mu + \nu \leq 1 \), then

i) \( \chi(x(\mu, \nu)) \cap \chi(y(\mu, \nu)) = \chi(xy(\mu, \nu)) \)

ii) \( \chi(x(\mu, \nu)) = \emptyset \) if and only if \( x \) is \( \bar{x} \).

iii) \( \chi(x(\mu, \nu)) = \chi \) if and only if \( x \) is \( \bar{x} \) in \( X \).

**proof:**

i) If \( P \in \chi(x(\mu, \nu)) \cap \chi(y(\mu, \nu)) \) and \( P \in \chi(x(\mu, \nu)) \), then \( P \in \chi(x(\mu, \nu)) \), that means \( x(\mu, \nu) \notin P \) and \( y(\mu, \nu) \notin P \), and that implies \( \alpha_p(x) < \mu, \beta_p(x) > \nu \) and \( \alpha_p(y) < \mu, \beta_p(y) > \nu \). Thus \( \mu > \alpha_p(x) = \alpha_p(y) = \alpha_p(xy) \) and \( v = \beta_p(x) = \beta_p(y) = \beta_p(xy) \), since \( P = \{ x \in X : \mu_p(x) = 1, \beta_p(x) = 0 \} \) is a prime semi d-ideal of \( X \) and \( \bigwedge(P) = \{(0,1), (\mu, \nu)\} \) implies that \( \alpha_p(a) = \alpha_p(b) \) and \( \beta_p(a) = \beta_p(b) \) for all \( a, b \in X \setminus P \) and \( x, y, xy \notin P \). Then \( xy(\mu, \nu) \notin P \), which means that \( P \in \chi(xy(\mu, \nu)) \). The proof of (i) is complete, since all the implication can be reversed.

ii) Let \( J \) be any prim semi d-ideal of \( d \)-algebra \( X \) and let \( X_J \) be the intuitionistic fuzzy characteristic function of \( J \). It is follows that \( X_J \subseteq X \). Next if \( \chi(x(\mu, \nu)) \notin X \), then \( V(x(\mu, \nu)) = X \), which implies that \( x(\mu, \nu) \notin X \), and therefore \( \alpha_{x_J}(x) = 1 \) and \( \beta_{x_J}(x) = 0 \), so \( x \in J \). Thus \( x \in \bigcap\{J: J \text{ is prime semi } d- \text{ ideal in } X\} \). Hence \( x \) is \( \bar{x} \). Conversely, assume that \( x \) is \( \bar{x} \). Let \( A \in X \), then \( A \) is prim semi d-ideal of \( X \), and \( x \in A \), therefore \( \alpha_A(x) = 1, \beta_A(x) = 0 \). Hence \( \mu = \alpha_{x(\mu, \nu)}(x) \leq \alpha_A(x) \) and \( \nu = \beta_{x(\mu, \nu)}(x) \geq \beta_A(x) \), where \( x(\mu, \nu) \notin A \) for all \( A \in X \). Thus \( V(x(\mu, \nu)) = X \), i.e. \( \chi(x(\mu, \nu)) = \emptyset \).

iii) Let \( J \) be any prim semi d-ideal of \( d \)-algebra \( X \) and let \( X_J \) be the intuitionistic fuzzy characteristic function of \( J \). Now if \( \chi(x(\mu, \nu)) \notin X \), then \( V(x(\mu, \nu)) = \emptyset \), which implies that \( x(\mu, \nu) \notin X \), and therefore \( \alpha_{x_J}(x) < \mu \) and \( \beta_{x_J}(x) > \nu \), so \( x \notin J \). Thus \( x \notin \bigcap\{J: J \text{ is prime semi } d- \text{ ideal in } X\} \). Hence \( x \) is \( \bar{x} \). The converse in the conversive way.

**Theorem (3.15):** The sub-family \( \{\chi(x(\mu, \nu)), x \in X \) and \( \mu, \nu \in (0, 1) \) such that \( \mu + \nu \leq 1 \) of \( \chi \) is a base for \( T \).
proof: Let $\chi(A) \in T$, and let $B \in \chi(A)$, then $\alpha_{B}(x) < \alpha_{A}(x)$ and $\beta_{B}(x) > \beta_{A}(x)$ for some $x \in X$.

Let $\alpha_{A}(x) = \mu$ and $\beta_{A}(x) = v$, then $x_{(\mu,v)} \notin A$ and so $A \notin \chi(x_{(\mu,v)})$. Now $V(A) \subseteq V(x_{(\mu,v)})$.

because if $P \in V(A)$, then $\alpha_{P}(x) \geq \alpha_{A}(x) = \mu = \alpha_{x_{(\mu,v)}}(x)$, and $\beta_{P}(x) \leq \beta_{A}(x) = v = \beta_{x_{(\mu,v)}}(x)$.

So that $x_{(\mu,v)} \notin P$ and thus $P \in V(x_{(\mu,v)})$. Hence $\chi(x_{(\mu,v)}) \subseteq \chi(A)$. Thus $B \in \chi(x_{(\mu,v)}) \subseteq \chi(A)$.

And this complete the proof.

**Theorem (3.16):** Spec(X) is disconnected if and only if there exist two $IFSd$-ideal A, B such that $\text{rad}(A \cup B) = \text{rad}(1)$ and $\text{rad}(A \cap B) = \text{rad}(0)$.

proof: Let Spec(X) be disconnected, then there exist two $IFSd$-ideal $A, B$ in $X$ such that $\chi(A) \neq \emptyset$, $\chi(B) \neq \emptyset$, $\chi(A) \cap \chi(B) = \emptyset$, $\chi(A) \cup \chi(B) = \text{spec}(X)$. That is mean $\chi(A) \cap \chi(B) = \chi(\emptyset)$ and $\chi(A) \cup \chi(B) = \chi(\overline{1})$. Thus $\chi(A \cap B) = \chi(\emptyset)$ and $\chi(A \cup B) = \chi(\overline{1})$. So by proposition (3.10)(ii) we get $\text{rad}(A \cap B) = \text{rad}(\emptyset)$ and $\text{rad}(A \cup B) = \text{rad}(\overline{1})$.

Recall that a subset $A$ of a topological space $X$ is called strongly connected (s-connected) when we get for any open subset $U$ and $V$ of $X$, if $A \subseteq U \cup V$, then $A \subseteq U$ or $A \subseteq V$. [12]

**Theorem (3.17):** Any subset of Spec(X) is S-connected.

proof: Let $\emptyset$ be a collection of an $IFSd$-ideal of Spec(X), and let $C, D$ be an $IFSd$-ideal in X. Since $\emptyset \subseteq \chi(C) \cup \chi(D) \subseteq \chi(C \cup D)$. Then by proposition (3.5) we get that $\emptyset \subseteq \chi(C)$ or $\emptyset \subseteq \chi(D)$ and this complete the proof.

**Theorem (3.18):** Spec(X) is a $T_{0}$ - space.

Proof: Let $A, B \in X$ and $A \neq B$. Then either $A \not\subseteq B$ or $B \not\subseteq A$. Let $A \not\subseteq B$ then $B \notin V(A)$, but $A \in V(A)$, then $B \in X(A)$, and $A \notin X(A)$. Now let $B \not\subseteq A$ similarly we can get $A \in X(B)$ but $B \notin X(B)$. It follow that Spec(X) is a $T_{0}$-space.

**Theorem (3.19):** In Spec(X), $V(A) = \overline{A}$ for all $IFSd$-ideal in X

proof: It is clear that $\overline{A} \subseteq V(A)$, since $V(A)$ is closed set containing $A$. Now let $B \notin \overline{A}$, then there exist an open set $X \setminus V(C)$ containing $B$ but not $A$, therefore $C \not\subseteq B$, but $C \subseteq A$ and so $B \notin V(A)$. Thus $V(A) \subseteq \overline{A}$, and that complete the proof.

**Corollary (3.20):** $B \in \overline{A}$ if and only if $A \subseteq B$.

proof: it is follow directly from theorem (3.19).

**Theorem (3.21):** Let $Y = \{P \in X: \Lambda(P) = \{(0,1), (\mu, v)\}; \mu, v \in [0,1) \text{ such that } \mu + v \leq 1\}$, then $Y$ is $T_{1}$ if and only if every singleton element of $Y$ is an intuitionistic fuzzy maximal semi d-ideal of $X$.

proof: we need to show that the semi d-ideal $A_{1} = \{x \in X, \alpha_{A}(x) = 1, \beta_{A}(x) = 0\}$ is a maximal semi d-ideal. It is sufficient to show that there is no prime semi d-ideal of $X$ containing $A_{1}$. Let $J$ is a prime semi d-ideal containing $A_{1}$, consider an $IFSd$-ideal $B$ of $X$ defined by $\alpha_{B}(x) = \{1 \text{ if } x \in J \}$ and $\mu \text{ if } x \notin J$. 


\[
\beta_b(x) = \begin{cases} 
0 & \text{if } x \notin J \\
1 & \text{if } x \in J
\end{cases}
\]
where \( \mu + \nu \leq 1 \). Then \( B \in Y \) and \( A \) containing in \( B \). This contradiction the fact that \( V(A) \cap Y = \{A\} \).

Conversely, let \( A \) is an IFMSd-ideal then the ideal \( A_\ast = \{x \in X, \alpha_A(x) = 1, \beta_A(x) = 0\} \) is maximal, we claim that \( V(A) \cap Y = \{A\} \). Clearly \( \{A\} \subseteq V(A) \cap Y \). Now if \( B \in V(A) \cap Y \), then \( A \subseteq B \) and \( A_\ast \subseteq B_\ast \). This means that \( A_\ast = B_\ast \), since \( A_\ast \) is a maximal semi d-ideal. Hence \( B = A \), since \( \Lambda(A) = \Lambda(B) = \{(1,0),(\mu, \nu)\} \), therefore \( V(A) \cap Y = \{A\} \), consequently \( \{A\} \) is closed subset of \( Y \).

**Theorem (3.22)**: If every prime semi d-ideal in \( X \) is maximal, then the space \( IFPSd - Spec(X) \) is not Hausdorff.

**proof**: Let \( J \) be a prim semi d-ideal of \( X \), consider two IFPSd-ideals \( A, B \) of \( X \) defined by \( \alpha_A(x) = \begin{cases} 
1 & \text{if } x \in J \\
0 & \text{if } x \notin J
\end{cases} \)
and \( \beta_A(x) = \begin{cases} 
1 & \text{if } x \in J \\
0 & \text{if } x \notin J
\end{cases} \), \( \alpha_B(x) = \begin{cases} 
0 & \text{if } x \in J \\
1 & \text{if } x \notin J
\end{cases} \) and \( \beta_B(x) = \begin{cases} 
0 & \text{if } x \in J \\
1 & \text{if } x \notin J
\end{cases} \).
Let \( X(x_{(\mu, \nu)}) \) and \( X(y_{(\mu, \nu)}) \) be any two basic open set in \( X \) containing \( A \) and \( B \) respectively where \( x, y \in X \) and \( \mu + \nu \leq 1 \). Then \( x_{(\mu, \nu)} \subseteq A \) and \( y_{(\mu, \nu)} \subseteq B \), so \( x \notin A_\ast = J \) and \( y \notin B_\ast = J \). Since \( J \) is prime then \( xy \notin J \), then \( xy \) is not nilpotent and so by theorem "(3.14) (i) and (ii)" we have \( X(x_{(\mu, \nu)}) \cap X(y_{(\mu, \nu)}) = X(xy_{(\mu, \nu)}) \neq \emptyset \). Hence \( X \) is not Hausdorff.

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