The cosmic recombination history in light of EDGES measurements of the cosmic dawn 21-cm signal

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The recent EDGES measurements of the global 21-cm signal from the cosmic dawn suggest that the kinetic temperature of the inter-galactic medium (IGM) might be significantly lower compared to its expected value. The colder IGM directly affects the hydrogen recombination of the universe during the cosmic dawn and dark ages by enhancing the rate of recombinations. Here, we study and quantify the impact of the colder IGM scenario on the recombination history of the universe in the context of DM-baryonic interaction model which is widely used to explain the EDGES 21-cm signal. We find that, in general, the hydrogen ionisation fraction gets suppressed during the dark ages and cosmic dawn and the suppression gradually increases at lower redshifts. However, accurate estimation of the ionisation fraction requires knowledge of the entire thermal history of the IGM, from the thermal decoupling of hydrogen gas and the CMBR to the cosmic dawn. It is possible that two separate scenarios which predict very similar HI differential temperature during the cosmic dawn and are consistent with the EDGES 21-cm signal might have very different IGM temperature during the dark ages. The evolutions of the ionization fraction in these two scenarios are quite different. This prohibits us to accurately calculate the ionisation fraction during the cosmic dawn using the EDGES 21-cm signal alone. We find that the changes in the ionisation fraction w.r.t the standard scenario at redshift $z \sim 17$ could be anything between $\sim 0\%$ to $\sim 36\%$. This uncertainty remains even for a more precise measurement of the 21-cm signal from the cosmic dawn. However, the IGM temperature measured at two widely separated epochs should be able to constrain the ionisation fraction more accurately.

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Introduction. Recently, the EDGES has reported a detection of the global HI 21-cm signal from the cosmic dawn $\dagger$. The measured signal, if confirmed, is found to be around two times deeper compared to the predicted signal from the standard model of cosmology and astrophysical processes. Several fundamentally different explanations have been proposed in order to explain this unusually deep signal $\ddagger$. There are also concerns about modelling the foregrounds and unaccounted systematics which can lead to significantly different interpretations of the EDGES measurements $\S$. However, a great deal of efforts have been put up to understand the 21-cm signal reported by the EDGES. One of the most explored explanations considers interactions between the cold dark matter and baryonic gas (hereafter DM-b interaction) $\|$ which allows the heat energy to flow from the relatively warm baryonic gas to the colder dark matter. Consequently, the baryonic gas cools much faster compared to the standard adiabatic cooling rate $\|$ and could explain the EDGES signal. This possibility opens up a new and promising avenue to constrain the DM properties such as the mass of dark matter particles and the interaction cross section. A large number of studies focus on this aspect and put tight constraints on the DM properties $\|$.$\|$.$\%$.

The ‘colder inter galactic medium (IGM)’ scenario has direct impact on the hydrogen cosmic recombination history of the universe. The hydrogen recombinations in the universe start very early at redshift $z \sim 1500$ and the universe becomes largely neutral by redshift $z \sim 1000$. However, a very small fraction ($\sim 5\%$) of hydrogen still remains in ionised form at redshift $z \sim 1000$ and it gradually becomes neutral over the redshift range $1000 > z \gtrsim 20$. The residual electron plays an important role in determining the IGM temperature and the HI differential brightness temperature during the dark ages and cosmic dawn. The hydrogen recombination rate which determines the residual electron fraction (or the hydrogen ionisation fraction) increases in the colder IGM scenario. This results in faster recombination and lower ionisation fraction.

Detailed understanding of the cosmic recombination history is of immense importance and a lot of efforts, since late 60s, has been invested for its accurate calculations $\|$.$\|$.$\%$. State of the art codes have been developed to keep the errors in estimating the hydrogen ionization fraction at $\sim 1\%$ level $\|$.$\%$. Accurate knowledge of the ionisation fraction during the cosmic dawn and dark ages is crucial for understanding the roles of the cosmic magnetic field on the thermal history of the IGM $\|$ and the large scale structure formation $\|$,$\|$,$\%$, formation of molecules in the early universe $\|$,$\%$, contributions of ionisation fluctuations to the total fluctuations in HI 21-cm field $\%$.

In this paper, we explore the impact that the colder IGM scenario has on the recombination history of the universe, particularly during the cosmic dawn and dark ages. We study this in the context of the DM-b inter-
interaction model introduced in [9][10]. It is shown that interactions between the DM and baryons along with the standard heating and cooling processes can explain the EDGES 21-cm profile [2][3]. We calculate the thermal history of the IGM for all possible combinations of the DM-b interaction model parameters (mass of the dark matter particle and the interaction cross section) and compare with the EDGES 21-cm signal. We then focus on the changes in the hydrogen ionisation fraction from the standard scenario, i.e., without the interaction, for model parameters which are consistent with the EDGES 21-cm signal. Next, we discuss the prospects of more precise measurements of the cosmic dawn 21-cm signal in the accurate estimation of the ionisation fraction. We also discuss roles of various physical processes that determine the thermal and ionisation history during the cosmic dawn and dark ages.

The paper is organised as follows. In the next two sections, we set up the basic equations for calculating the global 21-cm signal and briefly discuss the EDGES measurements respectively. Next, we present the equations for calculating the evolution of the IGM, dark matter temperature and hydrogen ionisation fraction in the context of DM-b interaction. We present our results on the thermal history and study the impact of the colder IGM on the ionisation history in the subsequent section. Finally, we summarise and discuss our results. We use cosmological parameters $\Omega_{m0} = 0.3$, $\Omega_{\Lambda} = 0.7$, $\Omega_{b0} = 0.0486$, $h = 0.667$ which are consistent with measurements by the PLANCK experiment [25].

Global HI 21-cm signal. The global HI 21-cm differential brightness temperature at redshift $z$ can be written as [26]

$$ T_b \approx 27 x_{\text{HI}} \left( \frac{\Omega_{b0} h^2}{0.023} \right) \left( \frac{0.15}{\Omega_{m0} h^2} \right) \left( \frac{1 + z}{10} \right)^{0.5} \left( \frac{T_s - T_\gamma}{T_s} \right), $$

where $T_\gamma$ and $x_{\text{HI}}$ are the cosmic microwave background radiation (CMBR) temperature and hydrogen neutral fraction respectively. The spin temperature $T_s$ is a measure of the ratio of ground state HI atoms in the triplet and singlet states. It is determined by three physical processes, 1. radiative coupling with the background CMBR, 2. coupling with the gas kinetic temperature $T_g$ through collisions and, 3. coupling with $T_\gamma$ through Ly-\alpha photons (also known as Wouthuysen - Field coupling). Therefore, the spin temperature, in general, can be represented as [27]

$$ T_s^{-1} = \left[ \frac{(x_c + x_\alpha) T_g^{-1} + T_\gamma^{-1}}{1 + x_c + x_\alpha} \right], $$

where $x_c$ and $x_\alpha$ are the collisional and Ly-\alpha coupling coefficients respectively [26]. In the standard picture the collisional coupling starts to dominate over the other two processes at redshifts $z \sim 200$. During the cosmic dawn, the first generation of stars start to emit radiation including the Ly-\alpha photons. This makes the Ly-\alpha coupling strong and, as a consequence, the spin temperature is expected to be coupled to the gas kinetic temperature, i.e., $T_s \approx T_g$ at redshift $z \lesssim 20$.

EDGES measurements and colder IGM. The EDGES has reported the first detection of the global HI 21-cm differential brightness temperature $T_b$ in the redshift range of $15 \lesssim z \lesssim 22$ [11]. The measured signal is found to match a flattened gaussian function centred around redshift $z = 17.2$ and has a best fit amplitude of $-500 \text{ mK}$. It is, therefore, likely that the Ly-\alpha coupling becomes very strong, i.e., $T_s \approx T_g$ by redshift $z \sim 17$. The experiment also reported that the amplitude of the signal should be between $-300 \text{ mK}$ and $-1000 \text{ mK}$ (with 99% confidence) if the uncertainties due to thermal and systematic noise are considered. This suggests (from eq. [1]) that the IGM temperature at redshift $z = 17.2$ should lie between 1.76 K and 5.4 K (with 99% confidence). However, according to the known standard cosmology and astrophysical processes the IGM temperature at redshift $z = 17.2$ should be around 7 K which is significantly higher than that found by the EDGES.

DM-baryonic interaction, IGM temperature and recombination history. In order to calculate the accurate recombination history, we need to know the full thermal history of the IGM. The evolution of the IGM kinetic temperature, in the backdrop of the DM-b interaction, can be calculated using following equation [10].

$$ \frac{dT_g}{dz} = \frac{2T_g}{1 + z} - \frac{32 \sigma_T \sigma_{SB} T_0^4}{3 m_e c^2 H_0 \sqrt{\Omega_{m0}}} (T_\gamma - T_g) (1 + z)^{3/2} \frac{x}{1 + x} - \frac{2}{3k_B H(z)(1 + z)} \dot{Q}_b $$

(3)

The first, second and third term in the rhs correspond to the cooling due to adiabatic expansion of the universe, heating due to heat flow from the CMBR through its interaction with free electrons, and cooling/heating due to interactions between the baryonic IGM and dark matter respectively. $\sigma_T$, $\sigma_{SB}$ are the Thomson scattering cross-section, Stefan Boltzmann constant respectively. $T_0$ is the CMBR temperature at present. $\dot{Q}_b$ is the heating/cooling rate of baryons. The evolution of the dark matter temperature $T_\chi$ is similar to the above except the fact that the dark matter does not interact with the CMBR. Hence we can write

$$ \frac{dT_\chi}{dz} = \frac{2T_\chi}{1 + z} - \frac{2}{3k_B H(z)(1 + z)} \dot{Q}_\chi $$

(4)

where $\dot{Q}_\chi$ is the heating rate of dark matter. We model both the quantities $\dot{Q}_b$ and $\dot{Q}_\chi$ as prescribed in [10] (see equation [9] and subsequent texts of this paper for...
The equation governing the heating rate of the dark matter $Q_\chi$ can be obtained by flipping $\chi \leftrightarrow b$ in the above equation. The interaction cross-section has been parameterised as $\sigma = \sigma_0 (v/c)^{-4}$. This kind of scaling of the cross section with the velocity $v$ is very effective in transferring the heat energy from baryonic gas to the dark matter without significantly affecting other episodes of the cosmic evolution. The milli-charged dark matter model follows this type of scaling with the velocity and is a promising candidate for explaining the EDGES 21-cm signal [3]. The first term in the rhs makes sure that heat energy flows from the warmer fluid (in our case the baryon) to the colder fluid (the dark matter). We see that the heating rate is proportional to $(T_b - T_\gamma)$, i.e., the temperature difference between the two fluids. This will try to make the temperatures of the two fluids equal. The second term accounts for the heating caused due to friction between the dark matter and baryonic fluids. The dark matter and baryonic fluid flow at two different velocities which produces friction between the two and that heats up both the fluids. Note that this heating depends on the relative velocity between the two fluids $V_{\chi b}$ and drag $D(V_{\chi b})$ which can be calculated using the equations

$$\frac{dV_{\chi b}}{dz} = \frac{V_{\chi b}}{1 + z} \frac{\rho_m \sigma_0 c^4}{m_b + m_\chi} \frac{1}{2} \frac{D(V_{\chi b})}{V_{\chi b}^2}$$

and

$$D(V_{\chi b}) = \frac{4 \pi m_b m_\chi}{m_b + m_\chi} \frac{V_{\chi b}^2}{H(z)(1 + z)}$$

where $\rho_\chi$, $\rho_m$ are the energy densities of the dark matter and the total matter respectively, $m_b$ and $m_\chi$ are the masses of the baryonic and dark matter particles respectively. ‘$r$’ is defined as $r = \frac{V_{\chi b}}{u_{th}}$, where $u_{th} = c \sqrt{k_B T_b/m_b + T_\chi/m_\chi}$. The function $F(r)$ is defined as

$$F(r) = erf\left(\frac{r}{\sqrt{2}}\right) - \frac{1}{\sqrt{\pi}} r e^{-r^2/2}.$$

$F(0) = 0$ and $F(r) \to 1$ when $r \to \infty$.

We see from eq. [3] and [6] that the IGM temperature $T_b$ depends on the relative velocity $V_{\chi b}$. As a consequence the global HI 21-cm differential brightness temperature $T_b$ and the ionisation fraction $x$ also become $V_{\chi b}$ dependent. The initial value $V_{\chi b,0}$ follows the probability distribution $P(V_{\chi b,0}) = e^{-\frac{(V_{\chi b,0}/\sigma_{rms})^2}{2(2\pi V_{\chi b,0}/\sigma_{rms})^2}}$. We calculate the velocity averaged differential brightness temperature using

$$\langle T_b(z) \rangle = \int d^3 V_{\chi b} T_b(V_{\chi b}) P(V_{\chi b}).$$

Similarly, we calculate the velocity averaged IGM temperature $(T_b(z))$ and ionisation fraction $(x)$. $V_{\chi b,0}$ is assumed to be 29 km/s at the initial redshift $z = 1010$ [28].

**Results.** We solve eqs. [3] [4] [5] and [7] simultaneously after setting the initial conditions at redshift $z = 10$. At the starting redshift $z = 1010$, we assume $T_b = T_\gamma$ and $T_\chi = 0$. We obtain the initial ionisation fraction $x(z = 1010) = 0.055$ from the RECFAST code [30]. As discussed above the initial relative velocity $V_{\chi b,0}$ follows a Gaussian probability distribution with the rms of $\sim 29$ km/s and we average over the relative velocity to obtain various temperatures and the ionisation fraction.
Figure 1. This plot shows the evolution of IGM temperature, dark matter temperature for scenarios where the DM-b interactions are included and compares with the standard scenario without the DM-b interaction (black-solid line). The red-dashed lines and blue-dash-dotted lines correspond to $(m_\chi/\text{GeV}, \sigma_{45}) = (0.4, 100)$ and $(0.4, 0.2)$ respectively. The upper and lower lines in each set represent the IGM and dark matter temperature. The HI differential brightness temperature $T_b$ predicted using these two sets of parameters are consistent with the EDGES 21-cm profile measurements at redshift $z = 17.2$. The black-dotted line shows the CMBR temperature. The shaded region shows the redshift range covered by the EDGES measurements.

- Evolution of IGM temperature. First, we discuss our results on how the DM-b interaction alters the evolution of the IGM temperature from the standard predictions. This helps us to understand the impact of colder IGM on the recombination history. Fig. 1 shows the evolution of the IGM and the dark matter temperature for two sets of dark matter mass $m_\chi$ and the interaction cross section $\sigma_{45}$ $(\sigma_{45} = \sigma_{0,10}^{10^{-45} m_\chi^2})$ which are $(0.4 \text{ GeV}, 100)$ and $(0.4 \text{ GeV}, 0.2)$ respectively. It also shows the IGM temperature (solid black line) for the standard scenario which does not include the DM-b interaction. The two competitive interactions, i.e., between baryons and the CMBR and between baryons and the DM, together determine the resultant IGM temperature. The interaction between the CMBR and baryons tries to keep the IGM temperature same as the CMBR temperature. On the other hand, the DM-b interaction tries to bring the dark matter and IGM temperatures to some other thermal equilibrium with temperature $T_{eq}$ which is lower than the CMBR temperature. For a large value of the DM-b cross-section $\sigma_{45}$, the DM and baryon reach to a thermal equilibrium much faster at higher redshift. We see in Fig. 1 that the thermal equilibrium between the dark matter and baryon is reached by redshift $z \sim 200$ for $\sigma_{45} = 100$. The large value of $\sigma_{45}$ also helps the IGM to decouple from the CMBR earlier at redshift around $z \sim 500$. After attaining the equilibrium the dark matter and baryonic gas remain thermally coupled for the rest of the redshift range we explore and the equilibrium temperature scales as $(1 + z)^2$. The early decoupling of the IGM temperature from the CMBR helps it to cool faster and explain the EDGES results. For smaller cross section, the equilibrium between the DM and baryons is reached later. Here, the CMBR-baryon interaction dominates over the DM-b interaction and the evolution of IGM temperature follows the standard model upto very late. However, at later times the CMBR-baryon interaction becomes weaker and the DM-b interaction takes over. Consequently, the IGM temperature decouples from the CMBR temperature and both the IGM and dark matter temperature approach to each other. In this phase the IGM temperature falls very rapidly as redshift decreases which we see from Fig. 1 from curves corresponding to $\sigma_{45} = 0.2$ and $m_\chi = 0.4 \text{ GeV}$. Although, in this case, the IGM temperature follows the standard prediction up to redshift $z \sim 40$, it drops rapidly after that. We note that this parameter set too predicts colder IGM at redshifts $z \sim 17$ and explain the EDGES 21-cm signal measurements. We find that the differential brightness temperature $T_b$, in the first and second cases are $-724$ mK and $-668$ mK at redshift $z = 17.2$ respectively, both of which are consistent with the EDGES measurements. However, we see that the thermal history of the IGM ac-
Figure 3. This shows the percentage change in the ionisation fraction w.r.t. standard scenario at redshift $z = 17.2$ for all possible combinations of $(m_{\chi}, \sigma_{45})$ which predict the differential brightness temperature $T_b$ in the range between $-300$ mK and $-1000$ mK, allowed by the EDGES 21-cm profile. The two vertical lines show the changes in the ionisation fraction $x$ if $T_b$ is restricted between $-450$ mK and $-550$ mK.

Figure 4. This contour plot shows percentage change in the ionisation fraction $x$ w.r.t. the standard scenario at redshift $z = 17.2$. The white region is excluded by the EDGES measurements at 99% confidence level as the HI differential brightness temperature $T_b$ is either lower than $-1000$ mK or higher than $-300$ mK in the region.

According to the later parameter set is quite different from the first. The first one predicts colder IGM for a longer period of the cosmic time whereas the IGM is colder for a shorter period in the second case. The ‘drag heating’ term heats up both the IGM and the dark matter irrespective of their individual temperatures. We notice that this helps the IGM temperature to get coupled with the dark matter temperature earlier.

Recombination history. We now focus on the impact of the colder IGM on the evolution of ionisation fraction, i.e., the recombination history. The upper panel of Fig. 2 shows the evolution of the ionisation fraction for the same two sets of $m_{\chi}$ and $\sigma_{45}$ used above. As expected, both scenarios, which are consistent with the EDGES 21-cm signal, predict lower ionisation fraction compared to that predicted in the standard scenario. However, we notice, in the lower panel, that the percentage changes in the ionisation fraction $x$ at redshift $z \sim 17.2$ are $\sim 27\%$ and $\sim 2.1\%$ respectively although the differential brightness temperatures $T_b$ are very similar ($-724$ mK and $-668$ mK respectively) in these two scenarios. This apparent ambiguity can be explained if we look at the evolution of the IGM temperature in these two scenarios presented in Fig. 4. The IGM temperature $T_g$ for the first case $(m_{\chi} = 0.4, \sigma_{45} = 100)$ deviates from the standard scenario much earlier and remains deviated for the rest of the redshift range of interest. This helps the recombination rate to remain higher for an extended period of the cosmic time and, as a result, the ionisation fraction is considerably lower. On the other hand, for the second scenario $(m_{\chi} = 0.4, \sigma_{45} = 0.2)$ the evolution of the IGM temperature is very similar to the standard scenario and starts to deviate from it only at redshift $z \sim 40$. This is because the CMBR-baryon interaction dominates over the DM-b interaction up to redshift $z \sim 40$. Consequently, the recombination rate is very similar to the standard scenario and starts to increase at redshifts $z \lesssim 40$. In this scenario the recombination rate becomes higher only for a shorter period of the cosmic time and, therefore, the deviation of the ionisation fraction from the standard scenario prediction is less. We, therefore, conclude that very different thermal histories of the IGM can be consistent with the EDGES 21-cm signal. However, the impact of them on the ionisation history could vary considerably. This prohibits us to accurately estimate the ionisation fraction during the cosmic dawn and dark ages. Further, we see (Fig. 2) that the effect is quite significant even at redshift as high as $z \sim 80$.

We explore the entire $(m_{\chi}, \sigma_{45})$ parameter space and present our results on the changes in $x$ in Fig. 3 and 4. Fig. 3 shows the percentage change in the ionisation fraction w.r.t. the standard scenario at redshift $z = 17.2$ for all possible combinations of $(m_{\chi}, \sigma_{45})$ which predict the differential brightness temperature $T_b$ in the range between $-300$ mK and $-1000$ mK which is allowed by the EDGES measurements. We see that the suppression in the ionisation fraction can be anything from $\sim 0\%$ upto $\sim 36\%$. This large uncertainty in estimating $x$ is reduced only by a small amount even if we restrict $T_b$ in between $-450$ mK and $-550$ mK. This suggests that precise estimation of the ionisation fraction seems difficult even for more accurate measurements of the global 21-cm signal from the cosmic dawn. The uncertainty in estimating the ionisation fraction due to a wide range of possibili-
ties of the IGM thermal histories also remains at higher redshifts. Fig. 3 shows the changes in the ionisation fraction in the \((m_\chi, \sigma_{45})\) parameter space. It highlights regions where the \(T_b\) is within \(-300\) mK and \(-1000\) mK. We see that large cross sections suppress the ionisation fraction more whereas smaller cross sections have very little impact on the ionisation fraction.

We further note in Fig. 1 that the IGM temperatures \(T_g\) at redshift \(z \sim 50\) are quite different for the two scenarios although they predict similar \(T_b\) during cosmic dawn. We, therefore, propose that the IGM temperature measured at two widely separated epochs (e.g. dark ages and cosmic dawn) can be used to better constrain the evolution of the IGM temperature and accurately estimate the ionisation fraction.

**Summary and Discussion.** Recent measurements of the global 21-cm signal from the cosmic dawn by the EDGES suggest that the IGM can be significantly colder at redshift \(z \sim 17\) compared to its expected value. The ‘colder IGM’ scenario enhances the recombination (of neutral Hydrogen) rate and affects the recombination history of the universe. We study this, in detail, in the context of DM-b interaction model which is a promising way to explain the EDGES detection.

We find that the hydrogen ionisation fraction gets suppressed for all possible combinations of the DM-b model parameters \((m_\chi, \sigma_{45})\) which are consistent with EDGES measurements. Although the suppression is stronger during the cosmic dawn, we see that the effect is significant even at higher redshifts during the dark ages. However, the actual amount of suppression in the ionisation fraction during the cosmic dawn and dark ages depends on the entire thermal history of the IGM, from the thermal decoupling of hydrogen gas and the CMBR to the cosmic dawn. It is possible that two scenarios which predict very similar HI differential brightness temperature at redshifts where the EDGES measured the 21-cm signal have completely different IGM and HI differential brightness temperature at higher redshifts. Consequently, the ionisation history is also different in these scenarios. We explore the entire parameter space \((m_\chi, \sigma_{45})\) of the DM-b interaction model and find that the suppression in the ionisation fraction at redshift \(z \sim 17\), w.r.t the standard scenario, i.e., without the DM-b interaction, could be anything between \(\sim 0\%\) to \(\sim 36\%\) for scenarios which are consistent with the EDGES measurements. We also see that this large uncertainty in estimating the ionisation fraction remains even for a more accurate measurement of 21-cm signal from the cosmic dawn. However, IGM temperature measured at two widely separated epochs such as during the cosmic dawn and dark ages may be able constrain the evolution of the IGM temperature and ionisation fraction more accurately.

The suppressed ionisation fraction for a considerable duration of the recombination history has several implications. First, it affects the formation of molecules in the early universe [23] and might have indirect influence on the early star formations. Second, the impact of the magnetic field on the early structure formation and universe’s thermal history prior to the cosmic dawn could change. Further, the contribution of spatial fluctuations in the ionisation fraction to the total fluctuations in the HI 21-cm signal during the dark ages [24] is likely to get affected. A thorough and detailed investigation is required to assess the impact of the suppressed ionisation fraction on all these important astrophysical observables.

We would like to mention that results presented here, in principle, might change if we include other effects such as dark matter annihilation [13], magnetic field [29] along with the DM-b interaction or any other phenomena which can alter the IGM temperature. However, given the EDGES constraints on the HI differential brightness temperature and the fact that the DM-b model considered here brackets extreme cases, we do not expect the results to change considerably. Finally, there are also a few fundamentally different explanations for the unusually strong signal such as the excess radio background, axion induced cooling [4–6] which have very little effects on the thermal history of the IGM, and consequently on the recombination history.

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