V + A and V − A Correlators at Large $N_C$:
From OPE to Resonance Theory *

J. J. Sanz-Cillero

Groupe Physique Théorique, IPN Orsay
Université Paris-Sud XI, 91406 Orsay, France
E-mail: cillero@ipno.in2p3.fr

The spin–1 correlators are analysed in this talk through a large $N_C$ resonance theory. The matching to perturbative QCD and the first terms in the OPE constrains the hadronic parameters. A further sum-rule analysis shows the wider range of validity of the resonance description, which can help to discern the proper structure of the QCD mass spectrum.

1 Introduction

The purely perturbative Quantum Chromodynamics calculations (pQCD) and its Operator Product Expansion (OPE) are essential tools to describe the strong interactions [1]. However, they stop being valid as we enter into the non-perturbative QCD regime and one needs to consider alternative descriptions keeping, nevertheless, the agreement to OPE at high energies. A resonance theory with infinite narrow–width resonances has been shown to fulfill these requirements in the large $N_C$ limit [2], being $N_C$ the number of colours. Within this framework, we will consider a matching to pQCD and OPE up to $O(\alpha_s)$.

In this talk, we analyse the two–point Green-functions,

$$\langle q^\mu q^\nu - q^2 g^{\mu\nu} \rangle \Pi(q^2)_{XY} = i \int dx^4 e^{i q x} \langle T \{ J_X^\mu(x) J_Y^\nu(0) \} \rangle,$$

(1)

with $J_X^\mu$ and $J_Y^\nu$ denoting either a vector or an axial-vector current. We actually analyse the $V−A$ and $V+A$ combinations, respectively $\Pi_{LR} = \Pi_{VV} - \Pi_{AA}$ and $\Pi_{LL} = \Pi_{VV} + \Pi_{AA}$. Only the light quarks $u/d/s$ are considered and within the chiral and large $N_C$ limits.

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Several structures \{M^2_n\} for the resonance mass spectrum are explored in Ref. [3]. Here we refer just to the Regge-like models and the spectrum of the 5D–holographic models.

2 Fixing the resonance parameter through pQCD and OPE

In the deep euclidean region \(Q^2 = -q^2 \gg \Lambda_{QCD}^2\), the correlators are given by the OPE power series [1],

\[
\Pi^{OPE}_{LL} = \Pi^{pQCD}_{LL} + \sum_{m=2}^{\infty} \frac{\langle O^2_{2m}\rangle}{Q^{2m}}, \quad \Pi^{OPE}_{LR} = \sum_{m=3}^{\infty} \frac{\langle O^2_{(2m+1)}\rangle}{Q^{2m}}.
\]

The \(O(\alpha_s^2)\) corrections are dropped in this work so the \(1/Q^{2m}\) coefficients are just the constant condensates \(\langle O^2_{2m}\rangle\).

On the other hand, the correlators are provided at large \(N_C\) by the infinite series,

\[
\Pi^{NC \to \infty}_{LL} = \frac{F \pi}{Q^2} + \sum_{j=1}^{\infty} \frac{F^2}{M_j^2 + Q^2}, \quad \Pi^{NC \to \infty}_{LR} = -\frac{F^2}{Q^2} + \sum_{j=1}^{\infty} (-1)^{j+1} \frac{F^2}{M_j^2 + Q^2},
\]

where a resonance spectrum with alternating parity is assumed, being the first one, \(j = 1\), the vector \(\rho(770)\). The masses are ordered on increasing order (\(M_1 \leq M_2 \leq \ldots\)).

2.1 Step 1: matching pQCD

In order to recover the leading contribution in \(\Pi_{LL}\), provided by pQCD at \(Q^2 \gg \Lambda_{QCD}^2\), one needs to impose the constraint [3, 4]:

\[
F^2_j = \delta M_j^2 \left[ \frac{1}{\pi} \text{Im} \Pi^{pQCD}_{LL}(M_j^2) + O\left(\frac{\Lambda_{QCD}^2}{M_j^2}\right) \right],
\]

with \(\delta M_j^2 = M_{j+1}^2 - M_j^2\). The logarithmic behaviour from \(\frac{1}{\pi} \text{Im} \Pi^{pQCD}_{LL}(M_j^2)\) ensures the dominant log dependence \(\Pi^{NC \to \infty}_{LL} \simeq \Pi^{pQCD}_{LL}\) at high \(Q^2\). The \(O\left(\frac{\Lambda_{QCD}^2}{M_j^2}\right)\) corrections are suppressed for large masses and they are here neglected for \(M_j^2 > 2\) GeV^2.

This matching relation breaks down for the resonances laying by the non-perturbative regime of QCD. The lightest states, \(\pi\), \(\rho(770)\) and \(a_1(1260)\), need to be considered apart
into “non-perturbative” sub-series

\[
\Delta \Pi_{LL}^{\text{non-p.}} = \frac{F_\pi^2}{Q^2} + \frac{F_\rho^2}{M_\rho^2 + Q^2} + \frac{F_{a_1}^2}{M_{a_1}^2 + Q^2}, \quad \Delta \Pi_{LR}^{\text{non-p.}} = -\frac{F_\pi^2}{Q^2} + \frac{F_\rho^2}{M_\rho^2 + Q^2} - \frac{F_{a_1}^2}{M_{a_1}^2 + Q^2},
\]

The couplings \( F_\pi, F_\rho \) and the mass \( M_{a_1} \) are left as free parameters, whereas \( F_\pi = 92.4 \) MeV and \( M_\rho = 0.77 \) GeV are taken as inputs.

An asymptotic structure is assumed for the masses with \( M_n^2 \gtrsim 2 \) GeV \( (n \geq 3) \). In this talk we refer just to the Regge spectrum [4, 5, 6], with equal squared mass interspacing, \( M_n^2 = \Lambda^2 + n \delta M^2 \), and the 5D–spectrum [7, 8], with equal mass interspacing, \( M_n = \Lambda + n \delta M \). The parameters are set such that \( M_3 = M_\rho' \simeq 1.45 \) GeV and \( M_4 = M_{a_1} \simeq 1.64 \) GeV [9]. This fixes the couplings \( \{ F_n^2 \}_{n \geq 3} \) through Eq. (4) and, hence, the “perturbative” contribution \( \Delta \Pi^{\text{pert.}} = \Pi^{NC \to \infty} - \Delta \Pi^{\text{non-p.}} \).

### 2.2 Step 2: matching the leading OPE power behaviour

The second step is to match power behaviour of the first non-trivial operator in the OPE, this is, to demand

\[
\langle O_{(2)}^{LL} \rangle = \langle O_{(2)}^{LR} \rangle = \langle O_{(4)}^{LR} \rangle = 0, \quad \text{with} \quad \langle O_{(2m)} \rangle = \Delta \langle O_{(2m)} \rangle^{\text{non-p.}} + \Delta \langle O_{(2m)} \rangle^{\text{pert.}}.
\]

The contributions \( \Delta \langle O_{(2m)} \rangle^{\text{non-p.}} \) from \( \Delta \Pi^{\text{non-p.}} \) are obtained by expanding Eq. (5) in powers of \( M_j^2/Q^2 \). The contributions \( \Delta \langle O_{(2m)} \rangle^{\text{pert.}} \) come from the “perturbative” sub-series \( \Delta \Pi^{\text{pert.}} \). Taking \( \Pi^{\text{pert.}} \) and the asymptotic spectrum \( \{ M_n^2 \}_{n \geq 3} \) as inputs, one gets \( \{ F_n^2 \}_{n \geq 3} \) through Eq. (4) and finds that the combination \( \left( \Delta \Pi^{\text{pert.}} - \Pi^{\text{pert.}} \right) \) shows the power structure \( \sum_{m=1}^{\infty} \frac{\Delta \langle O_{(2m)} \rangle^{\text{pert.}}}{Q^{2m}} \). The trivial expansion in powers of \( M_j^2/Q^2 \) is not valid here since there is always an infinite number of states with \( M_j^2 > Q^2 \). The first coefficients \( \Delta \langle O_{(2m)} \rangle^{\text{pert.}} \) are recovered through a numerical analysis in the range \( Q^2 = 2 - 6 \) GeV\(^2 \), together with a consistent estimate of their uncertainties [3].

By imposing the OPE constraints in Eq. (4), \( F_\rho, M_{a_1} \) and \( F_{a_1} \) become fixed and, hence, the correlators \( \Pi^{NC \to \infty} \) result fully determined.
3 Discerning between resonance models: sum–rule analysis

The matching to pQCD and the leading OPE operators can be easily reached for any resonance model by simply splitting the correlator into a “perturbative” and a “non-perturbative” part (respectively $\Delta \Pi^{pert.}$ and $\Delta \Pi^{non-pert.}$), and imposing the constraints in Eq. (4) and (6).

What I would like to remark in this talk is that the analysis must be taken one more step forward; the correlators $\Pi^{NC \to \infty}$ carry extra information which can –and must– be actually exploited. Deeper analyses may help to discern between the different large $N_C$ resonance models present in the bibliography [4, 5, 6, 7, 8].

In Ref. [3], a set of sum-rules specially sensitive to the resonance parameters was proposed: in order to compare the theoretical description to the experimental data we defined the moments

$$B^{(k)}(Q^2) = (-1)^{k-1} \sqrt{\frac{2k-1}{2}} \int_0^\infty \frac{2 Q^2 \, dt}{(Q^2 + t)^2} \, P_{k-1} \left[ \frac{t - Q^2}{t + Q^2} \right] \cdot \frac{1}{\pi} \text{Im}\Pi(t), \quad (7)$$

with $Q^2 > 0, \, k = 1, 2, ...$ and where the $P_k[x]$ are the Legendre Polynomials ($P_0[x] = 1, \, P_1[x] = x, ...$). As the order $k$ grows the Legendre Polynomials pinch the intermediate region $t \sim Q^2$ in the integral whereas both the low and high energy extremes–where we
rely on the accurate experimental data \[11\] and QCD duality, respectively—are enhanced. On the experimental side, we considered the \(\tau\)-decay spectral functions up to \(t = 3 \text{ GeV}^2\) and pQCD beyond \[3\]. On the large \(N_C\) side, these sum-rules are absolutely convergent for any \(k\), avoiding any problem of convergence in the resonance series. Moreover, when the pion pole is removed, \(\text{Im}\Pi(t)\) are bounded functions, and then the moments also result bounded and obey the behaviour \(B^{(k)}(Q^2) \frac{k \to \infty}{\to 0} 0\) \[3\].

These moments are related to a combination of derivatives of the correlator in the euclidean at \(t = -Q^2\) so, \textit{a priori}, they can be computed through OPE \[3\]. However, one can see in Fig. (1) how the OPE is able to reproduce just the very first moments for \(Q^2 = 3 \text{ GeV}^2\), breaking down afterwards. Indeed, it is quite a complicated task to separate the contributions from the anomalous dimensions and higher dimension condensates. It is no wonder the current controversy about the value of high dimension condensates \[10\].

By construction, the resonance descriptions reproduce pQCD and the first terms of the OPE. Furthermore, one can see in Fig. (1) that they are able to reproduce the experiment up to much higher moments. At this point, one must be aware of the presence of next-to-leading order effects in \(1/N_C\) due the non-zero meson widths. Although the estimate of the subleading corrections is still under investigation \[12\], one finds that the size of the required corrections is much more important for the 5D–models than for the Regge–like mass spectrum, what seems to favour the latter as the proper one. Nevertheless, further studies on alternative metrics that could solve this feature of the current 5D–holographic models are really looked forward \[7\].

4 Conclusions

In this talk we have performed a matching of a large \(N_C\) resonance description to pQCD and the first terms of the OPE. The matching to pQCD in Eq. \[4\] points out that imposing the asymptotic freedom behaviour leads to a lack of control on the lightest multiplet parameters; the resonance series must be split into a “perturbative”, \(\Delta\Pi^{pert}\), and a “non-perturbative” sub-series, \(\Delta\Pi^{non-pert}\).

The second thing to remark is that, once performed the matching to pQCD and OPE, there is still extra information which admits being compared to phenomenology. Through the set of proposed sum-rules \[3\], one can see that the large \(N_C\) resonance theories are able to described a wider range of moments than OPE. This sort of studies can help to discern the proper structure of the hadronic mass spectrum of QCD. Although the experimental uncertainties are of the percent level, an estimated of the size of subleading
corrections in $1/N_C$ is crucial if one wants to perform a phenomenological analysis \cite{12}, one needs still to estimate the size of the subleading corrections in $1/N_C$ \cite{12}, in order to distinguish between resonance models.

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