Best-of-Both-Worlds Fair-Share Allocations

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Abstract

We consider the problem of fair allocation of indivisible items among \( n \) agents with additive valuations, when agents have equal entitlements to the goods, and there are no transfers. Best-of-Both-Worlds (BoBW) fairness mechanisms aim to give all agents both an ex-ante guarantee (such as getting the proportional share in expectation) and an ex-post guarantee. Prior BoBW results have focused on ex-post guarantees that are based on the “up to one item” paradigm, such as envy-free up to one item (EF1). In this work we attempt to give every agent a high value ex-post, and specifically, a constant fraction of her maximin share (MMS). The up to one item paradigm fails to give such a guarantee, and it is not difficult to present examples in which previous BoBW mechanisms give some agent only a \( \frac{1}{n} \) fraction of her MMS.

Our main result is a deterministic polynomial-time algorithm that computes a distribution over allocations that is ex-ante proportional, and ex-post, every allocation gives every agent at least her proportional share up to one item, and more importantly, at least half of her MMS. Moreover, this last ex-post guarantee holds even with respect to a more demanding notion of a share, introduced in this paper, that we refer to as the truncated proportional share (TPS). Our guarantees are nearly best possible, in the sense that one cannot guarantee agents more than their proportional share ex-ante, and one cannot guarantee agents more than a \( \frac{n}{2n-1} \) fraction of their TPS ex-post.

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