Toppling failure analysis of rock slopes based on a three-dimensional discontinuous deformation analysis method

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Abstract. The present study focused on the study of the toppling failure of rock slopes using a three-dimensional discontinuous deformation analysis (3D DDA) method. The failure conditions and their related impact factors of toppling rock slopes were derived by building the corresponding mathematical models based on the limit equilibrium method (LEM), and the deficiency of the existed LEM for analysing the toppling system was pointed out. Several numerical examples, such as a single rock block of different dimensions, classical toppling rock slope model, and multiple rock blocks of different distributions, were adopted to investigate the toppling failure mechanisms under different conditions. The results showed that the 3D DDA is a very powerful numerical tool to study the failure mechanisms of toppling rock slopes and simulate the failure process of various toppling rock slopes with different distributions of structural planes inside rock mass. The controversial safety factor of the slope stability was avoided in the 3D DDA numerical simulation. Through the 3D DDA results, some more real phenomena of slope failure could also be observed and explained. For example, the slope block toppling tended to be toppling-sliding rather than pure toppling. In addition, the general laws of toppling slope failure were presented.

1. Introduction
Toppling tends to be a typical failure pattern of rock slopes, which have discontinuous properties as the joints and faults are inside the rock mass. Goodman and Bray [1] proposed a meaningful tool to study the toppling failure of a rock slope based on the limit equilibrium method (LEM), i.e. G-B method, and the toppling failure analysis was described by Hoek and Bray [2] systematically. From that time forth, many theories, analytical methods and numerical methods of studying toppling rock slopes have been developed [3-7]. However, most of the theories and methods need the limit equilibrium analysis and the safety factor must be involved, but the definition of the safety factor for toppling is controversial.

As a discontinuum-based method, discontinuous deformation analysis (DDA) proposed by Shi [8] owns the rigorous contact judgment criterion and unified format for the consideration of not only the translation, rotation and deformation of an individual rock block, but also the movement forms such as sliding and opening along block boundaries, and has the advantages of many other numerical methods [9]. Thus, it is very suitable for the simulation of rock mechanics and rock engineering [10], such as rockfalls [11-12], landslides [13-14], and underground caverns [15-16]. These applications demonstrate the high accuracy of DDA in simulating large displacement and deformation of rock mass. In essence, the external topography and internal structural plane distribution of rock slopes have three-dimensional characteristics [17]. Of course, the slope toppling failure belongs to a three-dimensional problem. For
example, the joint configurations, block shapes, boundary conditions, loading conditions, block displacement, and rock mass deformation are three-dimensional [18]. Therefore, the toppling failure needs to be studied by considering the three-dimensional spatial geometry of rock slopes.

In the present study, the three-dimensional (3D) DDA method [19] is adopted to study the toppling failure of rock slopes. The rest of the paper is organized as follows. In Section 2, the conditions and some impact factors of toppling rock slopes are derived by establishing mathematical models. In Section 3, the fundamental theory of 3D DDA is introduced. The accuracy of the 3D DDA simulation is verified, and the toppling failure mechanism is studied using 3D numerical examples under different conditions in Section 4, and in Section 5, conclusions are drawn.

2. Analysis of block systems

2.1. A single rock block
The simplest model is a single rock block resting on an inclined plane with the dip angle $\theta$ (Figure 1(a)). The height and width of the block are $h$ and $t$, respectively. The block weight is $G$, and the cohesion is not considered. Assuming the internal friction angle is $\varphi$. Only considering the block toppling and taking the moment around the lower left corner point $O$ (i.e., rotation point), the toppling condition is $\theta > \arctan(t/h)$. Conversely, if $\theta < \arctan(t/h)$, the block may be sliding or stable without toppling. Based on the geometrical analysis, it can be seen from Figure 1(a) that toppling occurs as long as the weight vector lies outside the block base. In addition, the block slides if $\theta > \varphi$. The toppling or sliding conditions of a single block on an inclined plane are summarized in Figure 1(b).

![Figure 1. A single rock block on an inclined plane. (a) Mathematical model. (b) Stability conditions.](image)

2.2. Multiple rock blocks
The toppling failure modes always involves rotation, sliding, bending, and fracture of the dipping rock blocks, which are caused by the action of external forces (e.g., gravity and earthquake load) and the interaction forces between blocks (e.g., friction and extrusion force). Here, a block system composed of multiple rock blocks on an inclined and stepped base plane (ISBP) is firstly considered (Figure 2). The
dip angles of the inclined plane and stepped columns base are $\alpha$ and $\beta$, respectively. The deformation of the block system includes sliding, toppling, and stability, and it is distributed in three parts, zones I, II, and III, which represent the sliding, toppling, and stable zones, respectively. In the sliding zone, a block slides under gravity, friction, and the interaction forces between blocks. In the toppling zone, a block topples under gravity and moments as the interaction forces between blocks. There is no deformation failure in the stable zone.

Figure 3(a) and (b) shows an arbitrary toppling block $i$ below and above the top line of the slope, respectively. Block $i$ has width $t$, height $h_i$, and friction angle $\phi$. Blocks $i-1$, $i$, and $i+1$ are assumed as the blocks along the stepped base from the toe to the top of the slope. Block $i$ is under gravity $G$. $N_{i-1}$, $Q_{i-1}$, $N_{i+1}$, and $Q_{i+1}$ from block $i-1$ and block $i+1$ act on block $i$. The analysis of forces for block $i$ below the top line of the slope can be shown in Figure 3(a), then,

$$N_{i+1} = \frac{N_{i+1}(h_i - t - \tan \phi) + G(h_i \sin \gamma - t \cos \gamma)}{h_i - t - \tan \gamma}$$

(1)

The analysis of forces for block $i$ above the top line of the slope can be shown in Figure 3(b), then,

$$N_{i+1} = \frac{N_{i+1}(h_i - t - \tan \gamma - t \tan \phi) + G(h_i \sin \gamma - t \cos \gamma)}{h_i - t - \tan \gamma}$$

(2)

The analysis of forces for an arbitrary block $i$ in the sliding zone can be shown in Figure 3(c). The mechanical and geometrical parameters of this zone are the same as those in the toppling zone, then,

$$N_{i+1} = \frac{N_{i+1}(1 - \tan^2 \phi) + G(\sin \gamma - \cos \gamma \tan \phi)}{1 - \tan^2 \phi}$$

(3)

From the models in Sections 2.1 and 2.2, the main impact factors in the analysis of toppling failure of rock slopes can be summarized as follows: (1) the width $t$ and height $h$ of the blocks; (2) the dip angle $\alpha$ of the inclined plane; (3) the dip angle $\beta$ of the stepped base; (4) the friction angle $\phi$; and (5) the block weight $G$. Of course, referring to other studies [17], some additional impact factors are included, such as the strength of rocks, the initial structure stress field, the distribution and dipping direction of the discontinuous structural planes, and the external forces, e.g., weathering unloading, excavation disturbance, earthquake action, and groundwater.

**Figure 3.** Arbitrary block $i$ in the toppling and sliding zones. (a) Toppling block $i$ below the top line. (b) Toppling block $i$ above the top line. (c) Sliding block $i$.

3. Fundamentals of the 3D DDA method

3.1. Simultaneous equilibrium equations of 3D DDA

In the 3D DDA method, the block system is formed by the contacts between blocks and by the displacement constraints on individual blocks. The 3D DDA block system obeys the principle of minimum total potential energy, which is the summation of all potential energy sources including (1) elastic stresses, (2) initial constant stresses, (3) body loading, (4) point loading, (5) inertia forces, (6)
displacement constraints, and (7) contact forces of the block system. Assuming the block system is composed of \( n \) blocks, the simultaneous equilibrium equations can be derived by minimizing the total potential energy at the end of each time step and are written in matrix form:

\[
\begin{pmatrix}
K_{11} & K_{12} & \cdots & K_{1n} \\
K_{21} & K_{22} & \cdots & K_{2n} \\
\vdots & \vdots & \ddots & \vdots \\
K_{n1} & K_{n2} & \cdots & K_{nn}
\end{pmatrix}
\begin{pmatrix}
D_1 \\
D_2 \\
\vdots \\
D_n
\end{pmatrix}
=
\begin{pmatrix}
F_1 \\
F_2 \\
\vdots \\
F_n
\end{pmatrix}
\]

(4)

where each \( K_{ij} \) in the coefficient matrix given by Equation (4) is a 12 \times 12 sub-matrix. The diagonal entries \( K_{ii} \), which depend on the material properties of block \( i \), represent the sum of the contributing sub-matrices for the \( i \)-th block. The non-diagonal entries \( K_{ij} \) represent the sum of the contributing sub-matrices of the interaction between blocks \( i \) and \( j \), where \( i \neq j \) is defined by the contacts between blocks \( i \) and \( j \). \( D_i \) and \( F_i \) are 12 \times 1 sub-matrices, where \( D_i \) represents the 12 deformation variables of block \( i \), and \( F_i \) represents the loading on block \( i \) distributed to \( D_i \). The detailed derivation of the formulae for 3D DDA is in Shi’s conference paper [19].

3.2. Contact treatment

The 3D contacts include seven forms: vertex-to-vertex (V-V), vertex-to-edge (V-E), vertex-to-face (V-F), crossing edge-to-edge (E-E), parallel E-E, edge-to-face (E-F), and face-to-face (F-F). The last three contacts can be translated into the first four fundamental contacts, i.e., V-V, V-E, V-F, and crossing E-E, or their combinations. To tackle the complicated contacts between two general blocks (\( A \) and \( B \)), Shi [20] introduced a new concept, “the entrance block”, associated with an operation of point sets. Thus, the contact conditions between blocks \( A \) and \( B \) can be simplified as the relation between a reference point \( a_0 \) on block \( A \) and an entrance block \( E(A, B) \) defined by Equation (5). 

\[
E(A, B) = \bigcup_{a \in A, b \in B} (b - a + a_0) = B - A + a_0
\]

(5)

Every point of \( E(A, B) \) is a position of \( a_0 \) when blocks \( A \) and \( B \) have common points. All complex blocks can be regarded as the union of convex blocks, and \( E(A, B) \) is convex if blocks \( A \) and \( B \) are convex. The boundary of \( E(A, B) \) is proved to be a cover system [20], with each cover corresponding to a possible contact. As the 3D DDA follows short time step size and the step displacements are small enough, all contacts are independent and local in computation. At the beginning of each time step, the contact polygons are computed. From the relative position of the reference point \( a_0 \) and the contact polygons, the closed contact point pairs are found. All closed contact point pairs are defined by the contact polygons, and they control the movements and deformations throughout this time step.

\[\text{Figure 4. Three-dimensional contact model.}\]

For each time step, no inter-penetration and no tension must be satisfied by iteratively adding or subtracting the contact spring in all contacts and ensuring the criteria of calculation convergence (i.e., open–close iteration). Figure 4 shows the relative positions between a 3D solid angle of block \( A \) and an entrance plane of block \( B \). There are three possible states in each contact, i.e., open, sliding, and close, which are determined by the Mohr-Coulomb failure criterion. If \( d_n \geq 0 \), the contact state is “open”, and
no spring or friction is employed. If \( d_n < 0 \) and \( k_d s > k_d a \tan \phi + c A \), the contact state is “sliding”, and a normal spring and a pair of frictions are employed. If \( d_n < 0 \) and \( k_d s \leq k_d a \tan \phi + c A \), the contact state is “close”, and a normal spring and a shear spring are employed. Here, \( \phi \) and \( c \) are the internal friction angle and cohesion, respectively, and \( A \) is the contact area. The inter-penetration distances in normal and shear directions are \( d_n \) and \( d_s \), respectively. The stiffnesses of the normal and shear springs are \( k_n \) and \( k_s \), respectively.

4. Numerical examples

4.1. A single rock block

As shown in Figure 5, a single rock block \( i \) is allowed to move under gravity from its resting state along a fixed block \( j \) with an inclined angle \( \theta = 30^\circ \). \( t_0 \), \( t \), and \( h \) are the block thickness, width, and height, respectively. The thickness and width of block \( i \) are assumed to be \( t_0 = 0.1 \) m and \( t = 0.1 \) m, respectively. The width/height ratios are \( t/h = 0.4 \), \( 0.5 \), \( 0.57735 \), and \( 0.8 \). Five friction angles, i.e., \( \theta = 25^\circ \), \( 28^\circ \), \( 30^\circ \), \( 31^\circ \), and \( 33^\circ \), are included. The involved mechanical properties and calculation parameters are as follows. Unit weight is \( 25.0 \) kN/m\(^3\), gravitational acceleration is \( 10.0 \) m/s\(^2\), cohesion is \( 0.0 \) N/m\(^2\), time step size is \( 0.005 \) s, and normal and shear spring stiffnesses are \( 2.0 \times 10^5 \) MPa and \( 1.0 \times 10^4 \) MPa, respectively. To investigate the impact of the \( \phi \) and \( t/h \) on the toppling of a single rock block, the displacements of the bottom-left corner \( B \) on block \( i \) are recorded and the displacement-time curves are drawn in Figures 6 and 7, respectively. From the 3D DDA results, we can see that:

1. When \( t/h < \tan \theta \) the block has two failure forms: a) if \( \theta < \phi \), the sliding and toppling occur simultaneously; b) if \( \phi > \theta \), the block topples only. When \( t/h > \tan \theta \) the block has two failure forms: a) if \( \phi < \theta \), the block slides only; b) if \( \phi > \theta \), the block is stable. Certainly, these behaviors are consistent with the conclusions obtained in Section 2.1. Figures 6(a) and 7(d) plot the displacement-time curves of the pure sliding conditions, and the 3D DDA results agree well with the analytical sliding solution (i.e., \( g (\sin \theta - \cos \theta \tan \phi) t^2 / 2 \)).

2. If the friction angle is a certain value, the higher the \( t/h \) value is, the later the block topples. For example, when \( \phi < \theta \) (e.g., \( \phi = 25^\circ \) and \( 28^\circ \)), the sliding displacement happens while the block topples, but the displacement-time curves are not a parabola. This is because the contact between blocks \( i \) and \( j \) is no longer the initial F-F contact as block \( i \) rotates in the process, i.e., the block topples with sliding. When \( \phi > \theta \) (e.g., \( \phi = 31^\circ \) and \( 33^\circ \)), the sliding displacement also happens while the block topples, and it is not just toppling owing to the change of the friction with the contact transformation, although the sliding displacement is small.

3. When the friction angles and width/height ratios reach some values, e.g., \( \phi > 28^\circ \) and \( t/h \leq 0.57735 \), the displacements of point \( B \) sharply increase before it completely topples. Then, block \( i \) slides, vibrating up and down along the inclined plane. Finally, block \( i \) only slides, but the corresponding displacement-time curves are not a parabola. The possible reason is that the friction is affected by the large initial velocity originated from the toppling process.

4. When \( t/h = \tan \theta \), block \( i \) topples, and thus, this instability condition can be classified into the right half part of \( \theta = \arctan(t/h) \) in Figure 1(b). When \( \phi = \theta \), if \( t/h < \tan \theta \), block \( i \) topples; and with \( t/h > \tan \theta \), block \( i \) is stable. Thus, this instability condition can be classified into the top left part of \( \theta = \phi \) in Figure 1(b). Therefore, this section expands the instability conditions in Section 2.1.

**Figure 5.** Geometrical model of a single rock block for 3D DDA.
4.2. Classical toppling rock slope

A rock slope shown in Figure 8(a), with 92.5 m height and 56.6° slope angle, is cut by 60° anti-inclined structural planes into a rock block system composed of 16 blocks, which are on an ISBP, and block 10 is at the top line (with 4° elevation angle) of the slope [2]. To perform the 3D DDA analysis, the 2D model is extended to 3D, and the rock slope thickness is assumed to be \( t_0 = 10.0 \) m. The geometrical and physical parameters are listed in Table 1. The failure modes under different friction angles, such as \( \varphi = 18°, 30°, 31°, 38.15°, 40°, 44°, \) and \( 45° \), are obtained by 3D DDA, and the representative failure mode under \( \varphi = 38.15° \) is shown in Figure 8(b). The failure modes under different friction angles are summarized and discussed as follows:

1. When \( \varphi < 30° \) and \( \varphi \) is small, the sliding failure of the block system occurs, because the friction that the bottom of all blocks provides is not enough to overcome the sliding force. For example, with \( \varphi = 18° \), the whole block system slides along the slope. In addition to the sliding, the toppling also happens with the increase of the friction angle even if \( \varphi < 30° \). The possible reason is that when \( \varphi \) is close to the slope angle, the shorter blocks are extruded by the higher blocks, and the sliding with toppling occurs, because there is E-F contact between them.

2. When \( \varphi \geq 30° \), the block system on the slope can be divided into a sliding zone at the slope toe, a toppling zone in the middle, and a stable zone on the slope top, as Goodman and Bray described [1].
However, the number of blocks in the different zones varies with the change in strength parameters. For example, with $\varphi = 30^\circ$, the sliding zone has four blocks, i.e., blocks 1 to 4, and the stable zone has three blocks, i.e., blocks 14 to 16. With $\varphi = 40^\circ$, the sliding zone has three blocks, i.e., blocks 1 to 3, and the stable zone has three blocks, i.e., blocks 14 to 16.

(3) When $\varphi = 38.15^\circ$, the 3D DDA results are in good agreement with the G-B method at the beginning of the block system failure, i.e., blocks 1 to 3, blocks 4 to 13, and blocks 14 to 16 are sliding, toppling, and stable, respectively (Figure 8(b)). However, with the increase in the sliding displacement of the blocks in the sliding zone, the rotation angles of the toppling blocks quickly increase and rotate with large angular accelerations. Moreover, the rotational torque is generated because of the large friction between the bottom of sliding block 3 and its base plane, and the high thrust exerted on block 3 from block 4 as E-F contact is gradually formed between them. Therefore, the blocks in the sliding zone may turn into toppling, after initial sliding during the block system deformation.

| Parameters | Values | Parameters | Values |
|------------|--------|------------|--------|
| $a_1$ (m)  | 5.0    | Gravitational acceleration (m/s$^2$) | 10.0   |
| $a_2$ (m)  | 5.2    | Cohesion (N/m$^2$)         | 0.0    |
| $b$ (m)    | 1.0    | Normal spring stiffness (MPa)  | $8.0 \times 10^5$ |
| $\Delta x$ (m) | 10.0 | Shear spring stiffness (MPa) | $4.0 \times 10^5$ |
| Unit weight (kN/m$^3$) | 25.0 | Time step (s) | 0.0005 |

Figure 8(c) shows the displacement-time curves of measured point A on block 10 under different friction angles $\varphi$. When $\varphi = 18^\circ$, the curve is a parabola before 0.5 s, because the bottom of all blocks cannot provide enough friction to keep them stable and the overall slope is sliding. When $\varphi = 30^\circ$ and $31^\circ$, large displacements of the blocks happen, excepting the stable blocks 14 to 16, and tend to be stable after 3.325 s and 3.340 s, respectively. When $\varphi = 38.15^\circ$, $40^\circ$, and $44^\circ$, the block system quickly tends to be stable after small displacements. The displacements of point A under different friction angles at 1 s, 3 s, and 5 s are listed in Table 2. The time and displacements to ultimate stability decrease with the increase in the friction angle. For instance, the displacement reaches 0.0019 m at just 0.2355 s and the block system attains a limit equilibrium state when $\varphi = 45^\circ$, which is larger than the $\varphi = 38.15^\circ$ in the G-B method. Two possible reasons for this difference are as follows. (1) The toppling failure involves the sliding of the pivot edge of the block rotation rather than pure toppling (i.e., toppling-sliding), and the E-E and E-F contacts are unstable, whereas the G-B method overestimates the stability capacity of the toppling slope. (2) The 3D DDA is a dynamic method and the velocity of the current time step inherits the previous time step in the iterative computation, which is different from the G-B method. From the perspective of the solution, the inertia force term is added to the total stiffness matrix. In fact, the toppling failure is a dynamic procedure, and the 3D DDA is a more meaningful tool to analyze toppling failure compared with the G-B method. In addition, unlike the G-B method, the controversial safety factor of the slope stability is avoided in the 3D DDA numerical simulation.

| Table 2. Displacements (m) of measured point A under different friction angles at different moments. |
| Time | $\varphi = 18^\circ$ | $\varphi = 30^\circ$ | $\varphi = 31^\circ$ | $\varphi = 38.15^\circ$ | $\varphi = 40^\circ$ | $\varphi = 44^\circ$ | $\varphi = 45^\circ$ |
|------|---------------------|---------------------|---------------------|---------------------|---------------------|---------------------|---------------------|
| $t = 1$ s | 0.6984 | 0.5667 | 0.5550 | 0.2845 | 0.2385 | 0.0816 | 0.0019 |
| $t = 3$ s | 5.3064 | 3.0866 | 2.9603 | 2.1250 | 2.0145 | 1.7905 | 0.0019 |
| $t = 5$ s | 8.6229 | 3.1966 | 3.0971 | 2.2015 | 2.0319 | 2.0248 | 0.0019 |
4.3. Rock block systems on IFBP

4.3.1. Non-staggered distribution. Based on Section 4.2, the toppling of the rock block system resting on an inclined and flat base plane (IFBP) is analyzed in this section. The slope angle is 30°, and the internal friction angle is assumed to be 38.15°. The numbering, geometrical parameters, and physical parameters of the blocks are the same as those of the classical model. The 3D DDA slope model and its deformation are shown in Figure 9(a), indicating that the classification of deformational zones is consistent with that in the classical model, i.e., sliding, toppling, and stable zones.

Figure 9(b) shows the comparison of the configurations of the block system before and after the slope deformation. Apparently, the blocks have significant sliding displacements and the main contact between blocks is F-F in the toppling zone, which may pose some difficulties for the analytical method to study the toppling failure of this system. Table 3 provides the number of blocks in the sliding, toppling, and stable zones under different friction angles. The block system tends to be stable when φ = 50°, which is 5° larger than the friction angle with which the blocks tend to be stable on the ISBP. The possible reason for this difference is that toppling-sliding happens and the sliding displacements of all blocks, except the stable blocks, are larger than for the blocks on the ISBP. Thus, it can be hard for the friction between blocks and between the block bottom and base plane to resist the toppling and sliding forces, because of the considerable inertia forces of the blocks. In addition, the states of some blocks may change over time. With the increase in the block displacements, e.g., the sliding turns into toppling or the toppling turns into stability, owing to the transformation of the contact forms of some blocks or the action of the inertia forces, which cause the change of interaction between blocks.

The displacements of the bottom-left corner C on block 10 under different friction angles, such as φ = 25°, 29°, and 38.15°, are also measured, and the initial displacement versus time (only 0–0.5 s) curves are shown in Figure 9(c). When φ is small (e.g., φ = 25°), the gap between the displacements of measured
point C on the ISBP and IFBP becomes more and more remarkable with the extension of time, but the displacement versus time curves tend to be consistent as $\phi$ increases (e.g., $\phi = 38.15^\circ$), and the displacement values are kept small. It may be difficult for the LEM to analyze rock blocks on the IFBP, because of the indefinite moment arm lengths caused by F-F contacts between the blocks and the block rotation accompanied by large sliding displacement. However, it may have reference value to study this type of the slope using the LEM, like in the classical slope when $\phi$ is large, because the block sliding displacement is small.

Table 3. The number of rock blocks in different zones under different friction angles.

| Friction angles ($^\circ$) | 0–20 | 21–23 | 24–26 | 27–29 | 30–32 | 33–35 | 36–49 | 50 |
|---------------------------|------|-------|-------|-------|-------|-------|-------|----|
| Sliding zone (toe + top)  | 16+0 | 5+4   | 4+4   | 4+3   | 3+0   | 3+0   | 2+0   | 0+0|
| Toppling zone             | 0    | 7     | 8     | 9     | 11    | 10    | 11    | 0  |
| Stable zone               | 0    | 0     | 0     | 0     | 2     | 3     | 3     | 16 |

4.3.2. Staggered distribution. The staggered distribution of rock blocks is studied using the same model as that in Figure 9 (a), but the staggered distances between the adjacent blocks are $d = 1$ m, 2 m, 3 m, and 4 m. Figure 10 plots the displacement-time curves of measured point P under different friction angles, e.g., $\phi = 25^\circ$, 30°, and 38.15°, and different staggered distances, e.g., $d = 0$ m, 1 m, 2 m, and 3 m ($d = 0$ m represents non-staggered distribution). When the friction angle is small ($\phi = 25^\circ$ or 30°) (Figure 10(a) and (b)), the magnitude and variation trends of the displacements of point P under different staggered distances tend to be consistent, i.e., the staggered distance has a minimal impact on the toppling failure of the block system when the friction angle is small. When the friction angle becomes large (Figure 10(c)), the block system is easy to be stable with the increase of the staggered distance. The above phenomena are found in the case of $d \leq 3$ m (e.g., $dt_0 = 0.3$), but the lateral toppling happens when $d > 3$ m (e.g., $dt_0 > 0.3$). Take $d = 4$ m as an example, the failure states of the block system under different friction angles are shown in Figure 11. When the friction angle is less than the slope angle ($\phi = 25^\circ$), the failure is mainly along the slope. When the friction angle is close or equal to the slope angle ($\phi = 30^\circ$), the failure includes both sliding and lateral toppling by a big margin. When the friction angle is large ($\phi = 38.15^\circ$), the initial deformation is smaller but the substantial lateral toppling happens after a period of time. It is noteworthy that there is no lateral movement in initial toppling, but the lateral motion can be generated during the movement of rock blocks.

![Figure 10](image-url) Displacement-time curves of measured point P under different friction angles. (a) $\phi = 25^\circ$. (b) $\phi = 30^\circ$. (c) $\phi = 38.15^\circ$.

To further investigate the failure differences between the non-staggered and staggered distributions of blocks, two multi-row models (non-staggered and staggered) are considered. A non-staggered model composed of multiple identical blocks with square bottom and its geometrical parameters are shown in Figure 12. The staggered model is the same as the model in Figure 12, besides the staggered distance $d = 5$ m. Table 4 lists the comparison of the failure states between the non-staggered and staggered
distributions. The non-staggered distribution is not stable although the friction angle is much larger than
the slope angle, and there is no obvious lateral instability in the failure process; while the lateral
displacements of the blocks always happen in the failure process of the staggered distribution, which is
easy to stabilize with the increase of the friction angle. Friction angles making the non-staggered and
staggered distributions stable are 60° and 45°, respectively. The staggered distribution is more beneficial
to the block system stability than the non-staggered distribution, but its lateral instability when the
friction angle is small may be dangerous.

Table 4. Comparison of the failure states between the non-staggered and staggered distributions.

| Friction angles | Non-staggered distribution | Staggered distribution |
|-----------------|----------------------------|------------------------|
| $\phi = 30^\circ$ | $t = 3$ s | $t = 6$ s | $t = 3$ s | $t = 6$ s |
| $\phi = 35^\circ$ | ![Image](image1.png) | ![Image](image2.png) | ![Image](image3.png) | ![Image](image4.png) |
| $\phi = 40^\circ$ | ![Image](image5.png) | ![Image](image6.png) | ![Image](image7.png) | ![Image](image8.png) |

Figure 11. Failure states of the rock block system with $d = 4$ m. (a) $\phi = 25^\circ$, $t = 2.5$ s. (b) $\phi = 30^\circ$, $t = 6.05$ s. (c) $\phi = 38.15^\circ$, $t = 11.35$ s.
5. Conclusions
The toppling failure of the rock slopes is successfully researched by the 3D DDA method, which has high accuracy in discontinuous computation. By observing the entire movement process and recording the displacements of key points under different conditions, the failure (i.e., toppling or sliding) or stability of the rock block system can be easily judged, and the controversial safety factor can be avoided. Normally, the so-called toppling is toppling-sliding rather than pure toppling.

The static LEM overestimates the anti-toppling capacity of the rock slopes, and the results of the 3D DDA calculation enrich the instability conditions. On an IFBP, the block system deformational zone is divided into sliding, toppling, and stable zones, which also happens on an ISBP, but the number of blocks distributed in these three zones is different. Meanwhile, the instability critical friction angle of the rock blocks on the IFBP is larger than that on the ISBP. The staggered distribution of the rock blocks is more favourable to slope stability and slope protection than the non-staggered distribution, but its lateral instability of the slope must be noticed.

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References
[1] Goodman R E and Bray J W 1976 Toppling of rock slopes. In: Proceedings of the specialty conference on rock engineering for foundations and slopes. Boulder, Colorado: American Society of Civil Engineers. pp 201–234.
[2] Hoek E and Bray J W 1977 Rock slope engineering. London: The Institution of Mining and Metallurgy.
[3] Bray J W and Goodman R E 1981 The theory of base friction models Int J Rock Mech Min Sci. 18 453–468
[4] Sagaseta C, Sanchez J M and Canizal J 2001 A general analytical solution for the required anchor force in rock slopes with toppling failure Int J Rock Mech Min Sci. 38 421–435
[5] Liu C H, Jaksa M B and Meyers A G. Toppling mechanisms of rock slopes considering stabilization from the underlying rock mass Int J Rock Mech Min Sci. 47 348-354
[6] Zhang G X, Zhao Y and Peng X C 2010 Simulation of toppling failure of rock slope by numerical manifold method Int J Comput Methods. 7 167–189
[7] Chen Z Y, Gong W J, Ma G W, Wang J, He L, Xing Y C and Xing J Y 2015 Comparisons between centrifuge and numerical modeling results for slope toppling failure Sci China Technol Sci. 58 1497–1508
[8] Shi G H 1988 Discontinuous deformation analysis: a new numerical model for the statics and dynamics of block systems [Ph.D. thesis]. University of California, Berkeley
[9] Jiang Q H and Yeung M R 2004 A model of point-to-face contact for three-dimensional discontinuous deformation analysis Rock Mech Rock Eng. 37 95–116
[10] Yagoda-Biran G and Hatzor Y H 2016 Benchmarking the numerical discontinuous deformation analysis method Comput Geotech. 71 30–46
[11] Chen G Q, Zheng L, Zhang Y B and Wu J 2013 Numerical simulation in rockfall analysis: A close comparison of 2-D and 3-D DDA Rock Mech Rock Eng. 46 527–541
[12] Wu J H, Ohnishi Y and Nishiyama S 2005 A development of the discontinuous deformation analysis for rock fall analysis Int J Numer Anal Methods Geomech. 29 971–988
[13] Peng X, Yu P, Zhang Y and Chen G 2018 Applying modified discontinuous deformation analysis to assess the dynamic response of sites containing discontinuities Eng Geol. 246 349–360
[14] Wang J M, Zhang Y B, Chen Y L, Wang Q D, Xiang C L, Fu H Y, Wang P, Zhao J and Zhao L 2021 Back-analysis of Donghekou landslide using improved DDA considering joint roughness degradation Landslides. 18 1925–1935
[15] Fu X, Sheng Q, Zhang Y and Chen J 2015 Investigations of the sequential excavation and reinforcement of an underground cavern complex using the discontinuous deformation analysis method Tunn Undergr Sp Tech. 50 79–93
[16] Zhang Y, Fu X and Sheng Q 2014 Modification of the discontinuous deformation analysis method and its application to seismic response analysis of large underground caverns Tunn Undergr Sp Tech. 40 241–250
[17] Liu G Y and Li J J 2019 A three-dimensional discontinuous deformation analysis method for investigating the effect of slope geometrical characteristics on rockfall behaviors Int J Comput Methods. 16 1850122
[18] Zhang H, Liu S G, Han Z, Zheng L, Zhang Y B, Wu Y Q, Li Y G and Wang W 2016 A new algorithm to identify contact types between arbitrarily shaped polyhedral blocks for three-dimensional discontinuous deformation analysis Comput Geotech. 80 1–15
[19] Shi G H 2001 Three-dimensional discontinuous deformation analysis. In: Proceedings of the 4th international conference on analysis of discontinuous deformation. Scotland. pp 1–21
[20] Shi G H 2015 Contact theory Sci China Technol Sci. 58 1–47