From Strings Theory to the Dark Matter in Galaxies

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Starting from the effective action of the low energy limit of Strings theory, I find an exact solution of the field equations which geodesics behave exactly as the trajectories of stars around a spiral galaxy. Here dark matter is of dilatonic origin. It is remarkable that the energy density of this space-time is the same as the used by astronomers to model galaxy stability. Some remarks about a universe dominated by dilatons are pointed out.

Till some years ago it was believed that in our era, matter is made of leptons, quarks and gauge bosons. Only beyond the Standard Model predict other exotic particles which can exist at very higher energies, maybe near of the origin of the universe. But the discovery of the existence of a great amount of dark matter in galaxies and galaxy clusters could change our building of how is matter constructed, furthermore it could be possible that we do not know of what 90% of the matter of the universe is made. Let me explain in some lines this affirmation. Since the discovery of Zwicky and Smith of the necessity of a great amount of wanting matter in the Coma and Virgo clusters in 1933 in order that these clusters remain stable, the astronomers have discovered that a great amount of luminous matter is absent in the galaxies in order to understand their stability and age (for a better explanation of the dark matter problem in galaxies, see the G. Raffelt contribution in this volume). Astronomers have discovered even a greater amount of wanting luminous matter in most of the galaxy clusters, since these clusters have also shown to be very stable. In terms of the reason $\Omega_x = \rho_x/\rho_{crit}$ between a $x$ matter specie and the critical density $\rho_{crit}$ which makes the universe flat, we can express the contribution of the luminous matter in the universe by $\Omega_{obs} = (0.003 \pm 0.002)/h$ (see for example [2]), which depends on the value of the Hubble constant $h$ in units of 100 km/sec/ Mpc. If we consider the matter needed in the halos of the galaxies in order to conserve their stability, the mass density of the universe is $\Omega_{halos} \sim 0.05$, and considering the matter needed in order to have stability of the galaxies clusters, the density of the universe grows to $\Omega_{close} = 0.25 \pm 0.10$. These two last densities do not depend on the value of the Hubble constant. Neutrinos can contribute to the total density of the universe, nevertheless due to their recently discovered low mass, their contribution cannot be much greater than the luminous one (see the contribution of R. Peccei in this volume).

Our actual understanding of the universe is sustained by the Standard Model of cosmology, namely the Freedman-Robertson-Weaker (FRW) cosmological model. The predictions of the FRW model are supported for important observations; the universe expansion, the microwave background and the observations in the early elements composition in the era of nucleosynthesis. All these three predictions are supported for a extraordinary coincidences with observations, therefore it could be very difficult to construct another cosmological model with so nice features. Remarkable is the fact that the theoretical predictions of nucleosynthesis do not permit a great amount of baryonic matter. If the value of the Hubble constant $h = 1$, the permitted values for the baryonic density implies $0.06 < \Omega_{baryon} < 0.02$. If the Hubble constant $h = 0.4$ these values could increase to $0.05 < \Omega_{baryon} < 0.12$ (see for example [3]). In any case this limits do not permit sufficient baryons for explaining the needed matter in clusters. This fact implies that there must exist exotic matter in the universe, i.e., there is a great amount of non-baryonic matter in the cosmos and we do not know its nature. There are many hypothesis about the nature of this exotic matter. In this lines I want to explain one of this hypothesis, namely; the scalar field as dark matter in galaxies and in the universe [4].

The FRW model contains some problems related with the origin of the universe, in the quantum mechanic era of the universe. Some of the most important problems of the FRW model are; the horizon, the flatness problem, galaxy formation, etc. Some of these problems can be resolved using an inflationary model of the universe, i.e., introducing a scalar field by hand into the Einstein field equations. This procedure is preferred by theoretical physicist because it is elegant and simple. In general the inflationary model implies that $\Omega = 1$, where most of the matter is due to the scalar field. It is quit remarkable that all the actual most important unification theories,
like the Standard Model of particles, the Kaluza-Klein and the Strings theories predict the existence of scalar fields. Scalar fields are needed in order to maintain consistency in the respective theory. Therefore the question arrays; is it possible that the wanting matter could have a scalar nature? In this lines I will show that this seems to be the case, this fact puts the scalar fields as a good candidate to be the dark matter in the universe. I will start supposing that scalar fields are the dark matter in spiral galaxies. Observational data show that the galaxies are composed by almost 90% of dark matter. Nevertheless the halo contains a larger amount of dark matter, because otherwise the observed dynamics of particles in the halo is not consistent with the predictions of Newtonian theory, which explains well the dynamics of the luminous sector of the galaxy. So we can suppose that luminous matter does not contribute in a very important way to the total energy density of the halo of the galaxy at least in the mentioned region, instead the scalar matter will be the main contributor to it. Luminous matter in galaxies possess a Newtonian behavior, we expect that only gravitational interactions are important in them. So, we can perfectly neglect all the other interactions, I will suppose that only gravitation and scalar interactions are present. So, the model I am dealing with will be given by the gravitational interaction modified by a scalar field and a scalar potential. Then, I start with the effective low energy action of Strings theories with cosmological constant \( \Lambda \) in the Einstein frame:

\[
S = \int d^4x \sqrt{-g} \left( \frac{R}{\kappa_0} + 2(\nabla \Phi)^2 + e^{-2\phi} \Lambda \right),
\]

where \( R \) is the scalar curvature, \( \Phi \) is the scalar field, \( \kappa_0 = \frac{8\pi G}{c^4} \) and \( \sqrt{-g} \) is the determinant of the metric. I have carried out a conformal transformation in order to have a more simple form of the field equations. Action (3) actually states that an exponential potential appears in a natural way in this theory.

On the other hand, the exact symmetry of the halo is stills unknown, but it is reasonable to suppose that the halo is symmetric with respect to the rotation axis of the galaxy. Here I let the symmetry of the halo as general as I can, so I choose it to be axial symmetric. Furthermore, the rotation of the galaxy do not affect the motion of test particles around the galaxy, dragging effects in the halo of the galaxy should be too small to affect the tests particles (stars) traveling around the galaxy. Hence, in the region of interest we can suppose the space-time to be static, given that the circular velocity of stars (like the sun) of about 230 Km/s seems not to be affected by the rotation of the galaxy and we can consider a time reversal symmetry of the space-time. The most general static and axial symmetric metric compatible with this action, written in the Papapetrou form is

\[
ds^2 = \frac{1}{f} e^{2\kappa}(dz d\bar{z}) + W^2 d\phi^2 - f c^2 dt^2,
\]

where \( z := \rho + i\zeta \) and \( \bar{z} := \rho - i\zeta \) and the functions \( f, W \) and \( k \) depend only on \( \rho \) and \( \zeta \). This metric represents the symmetries posted above.

An exact solution of the field equations derived from the action (4) in Boyer-Lindquist coordinates

\[
ds^2 = 1 + \frac{b^2 \cos^2 \theta}{r^2} \left( \frac{dt^2}{f_0 r_0} + r^2 d\theta^2 \right) + \frac{r^2 + b^2 \sin^2 \theta}{f_0 r_0} d\phi^2 - f_0 c^2 r^2 + b^2 \sin^2 \theta \frac{dt^2}{r_0}
\]

The effective energy density \( \mu_{DM} \) of (3) is given by the expression

\[
\mu_{DM} = \frac{1}{2} V(\Phi) = \frac{2 f_0 r_0}{\kappa_0 (r^2 + b^2 \sin^2 \theta)}
\]

The energy density (4) coincides with that required for a galaxy to explain the rotation curves of test particles in its halo, but in our model, this energy density is produced by the scalar field and the scalar field potential, that is, this dark matter is produced by a \( \Phi \) particle.

In what follows I study the circular trajectories of a test particle on the equatorial plane taking the space-time (3) as the background. The motion equation of a test particle in the space-time (3) can be derived from the Lagrangian

\[
\mathcal{L} = \frac{1}{f} \left( \frac{d\rho}{dt} \right)^2 + \left( \frac{d\zeta}{dt} \right)^2 + W^2 \left( \frac{d\phi}{dt} \right)^2 - f c^2 \left( \frac{dt}{d\tau} \right)^2.
\]

This Lagrangian contains two constants of motion, the angular momentum per unit of mass \( B \) and the total energy of the test particle \( A \). In terms of the metric components and the test particle velocity \( v = (\dot{\rho}, \dot{\zeta}, \dot{\phi}) \) I obtain \( A^2 = c^4 f^2/(f - \frac{v^2}{c^2}) \). For a circular trajectory at the equatorial plane \( \zeta = \rho = 0 \) the equation of motion is \( B^2 f/W^2 - A^2/c^2 f = -c^2 \). This last equation determines the circular trajectories of the stars of the galaxy. Using these equations I obtain an expression for \( B \) in terms of \( v^2 \), \( B^2 = v^2/(f - \frac{v^2}{c^2})W^2/f \sim v^2 W^2/f^2 \), since \( v^2 \ll c^2 \). From this equation one concludes that for our solution \( B^2 = f_0^2 B^2 \), i.e.

\[
v_{DM} = f_0 B,
\]

where I call \( v \rightarrow v_{DM} \) the circular velocity due to the dark matter alone.

Let us model the circular velocity profile of a spiral galaxy by the function

\[
v_{c}^2 = v^2(R_{opt}) \beta = \frac{1.97 x^{1.22}}{(a^2 + 0.78 x)^{1.43}}
\]

This result is an adaption of the King’s model to spiral galaxies.
which is the approximate model for the Universal Rotation Curves proposed by Persic et al. They find the profile of the dark matter velocity. The results obtained are shown in fig. 1 for some galaxies. We see that the circular velocity due to the luminous matter of a spiral galaxy is very important. In order to derive equation (1) from the effective action of the low energy limit of Strings theory, a scalar field interaction must be added to the original equations. This result could be the first contact of higher dimensional theories like the Kaluza-Klein or the Strings one with reality, furthermore, it could be the first trace to demonstrate the existence of extra dimensions in nature.

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FIG. 1. The circular velocity profiles of four galaxies. In this Black lines represents the total circular velocity \( v_c \), middle-gray is the contribution of the dark matter to the total velocity \( v_{DM} \) and the light-gray curves is the contribution of luminous matter \( v_L \); finally the dots represent the observational data. The units are in (Km/s) in the vertical axis and in (Kpc) in the horizontal one.

The crucial point for having the circular velocity \( v_{DM} = f_0 B \) is that \( f \sim W \) in the solution. But this fact remains unaltered after conformal transformations in the metric \( ds^2 = F(\Phi) ds^2 \), so that the circular velocity \( v_{DM} \) remains the same for all theories and frames related with metric by conformal transformations. This point is very important. In order to derive action from the effective action of the low energy limit of Strings theory, I have carried out a conformal transformation, so this result is valid also for this last action. But then the result should be valid for any theory conformally equivalent to action (1).

This result has some very interesting consequences. If this result is true, Scalar fields not only exist, but they represent 90% of the matter in the universe. This result and the inflationary models tell us that scalar fields are the most important part of matter in nature, they determine the structure of the universe. After the big bang, they inflated the universe; soon after they gave mass to the particles; later they concentrate maybe because of scalar field condensation, provoking that baryonic matter density fluctuations and forming stars, galaxies and galaxy clusters. Scalar fields can clarify why galaxies formed so soon after the recombination era, they condensed during the radiation era forming the arena that formed the galaxies. The question why nature use only the spin 1 and spin 2 fundamental interactions over the simplest spin 0 interactions becomes clear here. This result tells us that in fact nature have preferred the spin 0 interaction over the other two ones, scalar field interactions determine the cosmos structure. This result give also a limit for the validity of the Einstein’s equations, they are valid at local level; planets, stars, star-systems, but they are not more valid at galactic or cosmological level, a scalar field interaction must be added to the original equations. This result could be the first contact of higher dimensional theories like the Kaluza-Klein or the Strings one with reality, furthermore, it could be the first trace to demonstrate the existence of extra dimensions in nature.

References:

[1] Zwicky, F. Helv. Phys. Acta, 6, (1933), 110
[2] Schramm D. N. In “Nuclear and Particle Astrophysics”, ed. J. G. Hirsch and D. Page, Cambridge Contemporary Astrophysics, (1998).
[3] Matos, T. and Guzman, F. S. [gr-qc/9810028]
[4] G. Gibbons and K. Maeda. Nucl. Phys. B298, 741 (1988).
[5] Guzman, F. S., Matos, T. and Villegas B. H. [astro-ph/9811143]
[6] Persic, M. Salucci, P and Stel, S. MNRAS, 281, (1996), 27.
[7] K.G. Begeman, A.H. Broeils and R.H. Sanders, MNRAS, 249, (1991), 523-537.
[8] Vera C. Rubin, W. Kent Ford Jr., and Norbert Thonnard. Astrophys. Journal, 238, (1980), 471-487.
[9] T. Matos. Ann. Phys. (Leipzig) 46, (1989), 462-472. T. Matos. J. Math. Phys., 35, (1994), 1302-1321. T. Matos. Exact Solutions of G-invariant Chiral Equations. Math. Notes, 58, (1995), 1178-1182.
[10] T. Matos and F. S. Guzmán. Modeling Dilaton Stars with Arbitrary Electromagnetic Field. In Proceedings of the VIII Marcel Grossman Meeting, Jerusalem, Israel, Ed. by D. Ruffini et. al., Word Scientific Singapore, (1998), in press.
[11] T. Matos, F.S. Guzmán and H. Villegas B. in preparation 1999.