LQG propagator: III. The new vertex

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Abstract

In the first article of this series, we pointed out a difficulty in the attempt to derive the low-energy behavior of the graviton two-point function, from the loop-quantum-gravity dynamics defined by the Barrett–Crane vertex amplitude. Here we show that this difficulty disappears when using the corrected vertex amplitude recently introduced in the literature. In particular, we show that the asymptotic analysis of the new vertex amplitude recently performed by Barrett, Fairbairn and others, implies that the vertex has precisely the asymptotic structure that, in the second article of this series, was indicated as the key necessary condition for overcoming the difficulty.

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1. Introduction

A technique for computing \(n\)-point functions in a background-independent context has been introduced in [1–3] and developed in [4]. Using this technique, we have found in the first paper of this series [5] that the definition of the dynamics of loop quantum gravity defined by the Barrett–Crane (BC) spinfoam vertex [6] fails to give the correct graviton propagator in the large-distance limit. This result has prompted a lively search for an appropriate correction of the BC vertex [7, 9–12]. The search has yielded an alternative vertex, given by the square of an \(SU(2)\) Wigner 15\(j\) symbol, contracted with certain natural fusion coefficients [7, 8]. The vertex can be defined for general values of the Immirzi parameter \(\gamma\) [10] and can be extended to the Lorentzian case [9, 10]. The same vertex has also been derived [12] using the coherent state techniques introduced by Livine and Speziale [11]. For \(\gamma < 1\) the two techniques yield exactly the same theory—the theory we consider here.

In the second article of this series [13], we argued that the correct graviton propagator in the large-distance limit could be obtained only if the vertex had a certain asymptotic form.
The asymptotic analysis of the new vertex amplitude has been recently performed by John Barrett, Winston Fairbairn, and their collaborators [14]. Here we show that the result of the Barrett–Fairbairn analysis implies that the new vertex has precisely the asymptotic form guessed in the second article of this series, and therefore it resolves the difficulty of the old Barrett–Crane vertex.

This paper is not self-contained. It is based on the two previous papers of this series [5, 13], where all relevant definitions are given. For an introduction to the formalism, see [3]; for a general introduction to background independent loop quantum gravity [15], see [16, 17].

2. Conditions on the vertex asymptotics

The quantity on which we focus is the (non-gauge-invariant) euclidean propagator $G^{\mu\nu\rho\sigma}(x, y) = \langle 0 | h^{\mu\nu} (x) h^{\rho\sigma} (y) | 0 \rangle$, where $| 0 \rangle$ is a vacuum state peaked on the flat euclidean metric $\delta^{\mu\nu}$, and $h^{\mu\nu}(x)$ is the difference between the gravitational quantum field and its euclidean background value $\delta^{\mu\nu}$. Let $L$ be the distance between $x$ and $y$ (in the flat euclidean metric). Choose a regular 4-simplex with two boundary tetrahedra $n$ and $m$ centered at the points $x$ and $y$; the indices $i, j, k, l, m, n, \ldots = 1, \ldots, 5$ label the five tetrahedra bounding the 4-simplex. Define $G^{(n,m)}_L(x, y) = G^{\mu\nu\rho\sigma}(x, y) (n^\rho_n)_\mu (n^\sigma_n)_\rho (n^\mu_m)_\rho (n^\nu_m)_\rho$, where $n^\mu_n$ is the normal 1-form to the triangle bounding the tetrahedra $n$ and $m$ in the hyperplane defined by the tetrahedron $m$ (with $| n \rangle$ equal to the area of the triangle). Clearly, knowing $G^{(n,m)}_L(L)$ is the same as knowing $G^{\mu\nu\rho\sigma}(x, y)$. Following [1–3], $G^{(n,m)}_L(L)$ can be computed in a background independent context as the scalar product

$$G^{(n,m)}_L = \langle W | (E_n^i \cdot E_n^j - n^i_n \cdot n^j_n) (E_m^i \cdot E_m^j - n^i_m \cdot n^j_m) | \Psi_q \rangle.$$  \hspace{1cm} (1)

Here $\langle W |$ is the boundary functional, which can be intuitively understood as the path integral of the Einstein–Hilbert action on a finite spacetime region $R$, with given boundary configuration. The operator $E_n^i$ is the triad operator at the point $n$, contracted with $n^i_n$. $| \Psi_q \rangle$ is a state on the boundary of $R$, peaked on a given classical boundary (intrinsic and extrinsic) geometry $q$. Fixing such a boundary geometry is equivalent to fixing a background metric $g$ in the interior, where $g$ is the solution of the Einstein equations with boundary data $q$. The existence of such a background metric is part of the definition of the propagator: the propagator is indeed a measure of the fluctuations around a given background. The (intrinsic and extrinsic) boundary geometry chosen in [1–5] is that of the boundary of a regular 4-simplex, immersed in $R^4$.

The classical Ricci flat bulk metric $g$ determined by these boundary data is obviously the flat metric, thus allowing a comparison with the free-graviton propagator of the theory linearized around flat space. It is convenient to write

$$| \Psi_{(n,m)}^{(i)} \rangle = (E_n^i \cdot E_n^j - n^i_n \cdot n^j_n) (E_m^i \cdot E_m^j - n^i_m \cdot n^j_m) | \Psi_q \rangle$$  \hspace{1cm} (2)

so that

$$G^{(n,m)}_L = \langle W | \Psi_{(n,m)}^{(i)} \rangle.$$  \hspace{1cm} (3)

We are interested in the value of (3) on a triangulation formed by a single 4-simplex, or, equivalently, to first order in the group-field-theory [18] expansion parameter, and in the limit in which the boundary surface (whose size is determined by $q$) is large. On the physical interpretation of this approximation, see [7]. To first order, the leading contribution to $W$ has support only on spin networks with a 4-simplex graph. If $j = (j_{nm})$ and $k = (k_n)$, are,
respectively, the ten spins and the five (spins of the virtual links labeling the five) intertwiners that color this graph, then in this approximation (3) reads

\[ G^{j,k}_{q,n,m} = \sum_{j,k} W(j, k) \Psi^{j,k}_{q,n,m}(j, k). \]  

To this order, \( W \) is just determined by the amplitude of a single vertex (up to some normalization factors that are irrelevant here). Since \( \Psi^{j,k}_{q,n,m}(j, k) \) is peaked on large values \( j_{nm} \) and \( k_n = k_0 \) of \( j \) and \( k \), the propagator depends only on the asymptotic (large \( j \) and large \( k \)) behavior of the vertex, or, more precisely, on the behavior of \( W(j_0 + \delta j, k_0 + \delta k) \) for large \((j_0, k_0)\) and small \((\delta j_{nm}, \delta k_n) = (j_{nm} - j_0, k_n - k_0).\)

In the second paper of this series [13], we showed that if the vertex \( W \) has a certain asymptotic structure, described below, then the boundary state can be appropriately chosen to give the correct propagator in the large distance limit. Expanding \( W \) to second order in the fluctuations, we write

\[ W(j_0 + \delta j, k_0 + \delta k) \sim N e^{iG(\delta j_{nm}, \delta k_n)} e^{i\phi_{nm} j_{nm} + i\phi_{kn} k_n} + \text{c.c.} \]  

Here \( N \) is a slowly varying function, \( G \) is a quadratic form in its 15 arguments, which scales as \( 1/j_0 \). The key quantity for us is the 15d vector \((\phi_{nm}, \phi_{kn})\) that determines the first-order variation of \( W \) around \((j_0, k_0)\): it is the frequency of the rapidly oscillating phase factor around this point. The result of [13] is that if \( W \) has the form (5), with ‘appropriate’ values of \((\phi_{nm}, \phi_{kn})\) of the phases, then we obtain the correct graviton propagator. ‘Appropriate’ means here that these phases must cancel corresponding phases in the boundary state.

To explain this point in detail, let us pause one moment and consider the general situation in quantum mechanics. In general, in quantum mechanics, the frequency of this rapidly oscillating factor codes the classical equations of motion, and therefore gives the semiclassical limit of the dynamics. For instance, the propagator of a free particle in a time \( t \) between two points \( x_0 \) and \( y_0 \) behaves like

\[ W(t, x_0 + \delta x, y_0 + \delta y) \equiv \langle x_0 + \delta x | e^{itH} | y_0 + \delta y \rangle \sim e^{ip_x \delta x - p_y \delta y}, \]  

where the frequency of the oscillation is precisely the momentum of the classical trajectory going from \( x_0 \) to \( y_0 \), that is

\[ p_x = p_y = m \frac{y_0 - x_0}{t}. \]  

To see how the semiclassical limit works, consider sandwiching the propagator between two wave packets centered in \( x_0 \) and \( y_0 \), respectively, and with oscillating phases having values \( P_x \) and \( P_y \), respectively. Then the amplitude is suppressed by the rapidly oscillating phase factor unless \( P_x = m(y_0 - x_0)/t \) and \( P_y = m(y_0 - x_0)/t \). That is, a semiclassical wave packet propagates from \( x_0 \) to \( y_0 \) in a time \( t \) only if it starts (and ends) with the correct momentum (velocity). If we propagate further for an addition time, destructive interference is only allowed along the classical trajectory.

More precisely, the classical equations of motion are normally interpreted as giving \( y_0 \) and \( p_y \) given \( x_0 \) and \( p_x \); but they can equally be interpreted as giving the momenta \( p_x \) and \( p_y \) given \( x_0 \) and \( y_0 \). This second interpretation is the one that generalizes naturally in a general covariant context [16]. Under this second interpretation, the classical equations of motion determine the momenta (and hence the gluing conditions for paths) given initial and final positions (or, more in general, given boundary configuration variables). The semiclassical limit of quantum mechanics determines these momenta as the phases of the propagator. Thus, the value of these phases codes the classical limit of the theory. Another way of seeing the same point is to observe that in the semiclassical approximation the propagator is given by the exponential of
the classical action along a given classical solution of the equations of motion; that is, by the Hamilton function [16]. When varying a boundary configuration variable, the variation of the Hamilton function gives the momenta.

In expression (5), the 15 factors \( \phi = (\phi_{nm}, \phi_n) \) determine the semiclassical behavior of the theory near the configuration \((j_0, k_0)\). These are determined by the geometry on which the boundary state is peaked (like the momenta \( p_x, p_y \) are determined by \( x_0 \) and \( y_0 \) in (7)). In the case we are considering, the boundary state is chosen to be peaked on the geometry of a regular 4-simplex in a flat spacetime.

The phase factor of the \( j_{nm} \) variables in the boundary state gives the mean value of the variable canonically conjugate to the variables \( j_{nm} \). The variables \( j_{nm} \) represent the areas of the ten faces in the 4-simplex. The variables canonically conjugate to these were identified in [2] as the momentum variables conjugate to the triangle areas, namely the 4d dihedral angle between adjacent tetrahedra, as is the case in Regge calculus. For a regular 4-simplex, this gives

\[
\cos \phi_{nm} = -\frac{1}{4}. \tag{8}
\]

Remarkably, the BC vertex has precisely this phase factor.

The situation with the angles \( \phi_n \) is more delicate and crucial. The intertwiner \( k_n \) is the quantum number of a dihedral angle \( \beta \) between two chosen faces of the tetrahedron \( n \). The semiclassical boundary state will peak it on some value \( k_0 \). But the semiclassical state must peak also the value of the quantum number of the dihedral angle between any two other faces of the same tetrahedron \( n \), and these are quantities that do not commute with \( \beta \) [23]. The only possibility for having all angles peaked is to have a properly chosen semiclassical state. The phase dependence of the state on \( k_n \) determines where the other angles are peaked (like the phase dependence of a wave packet depends where the packet is peaked in momentum space). In [24] it was shown that in order to have all angles peaked on the proper values for a regular tetrahedron, the value of this phase must be

\[
\phi_n = -\frac{\pi}{2}. \tag{9}
\]

(For a general boundary geometry, the values (8) generalize to appropriate functions of this geometry [25].) The BC vertex does not have this phase dependence, and this was the key detail that made the derivation of the propagator impossible in [5]. In [13] we observed that this phase dependence is needed for having a boundary state yielding the correct propagator. A hypothetical vertex characterized by an asymptotic behavior around large \( j_0, k_0 \) being like the BC vertex (namely asymptotic to the Regge action) but also having such a phase dependence was therefore guessed in [13], and shown to yield the correct propagator with an appropriate boundary state.

Here we show that the new vertex has precisely this asymptotic structure, with the phase (9). This is a consequence of the asymptotic analysis of the vertex, recently completed by Barrett and Fairbairn et al, and which we summarize below.

3. The asymptotic of the new vertex

We state here the result of the asymptotic form of the vertex found by Barrett and Fairbairn et al this is given in a different basis for the intertwiners: the overcomplete basis \( |n \rangle \) of the coherent state, introduced by Livine and Speziale [11]. Here \( n \) is a quadruplet of unit-length vectors in \( R^3 \) (whose geometrical interpretation is the normal of the faces of a tetrahedron). The matrix elements \( \langle k | n \rangle \) of this change of basis are defined in [11] and their asymptotic behavior has been studied in detail by one of us in [25].
The result of the asymptotic analysis of Barrett and Fairbairn is the following [14]. In the large \( j \) limit, the vertex defined in [10] behaves as follows:

\[
W(j, n) := (W[j, n] \approx N e^{-ij0} \mu(j, n) + \text{c.c.} \tag{10}
\]

Here \( n = (n_\alpha) \) is the family of the five quadruplets of \( \mathbb{R}^3 \) vectors forming the basis of the five intertwiner spaces. \( N \) is a slowly varying factor. \( \mu(j, n) \) is a factor that is strongly suppressed when \( (j, n) \) are not 'consistent'. Here 'consistent' means that there exist a 4-simplex imbedded in \( \mathbb{R}^4 \) whose triangles have the areas determined by the spins \( j \) and whose tetrahedra have the (3d) geometry determined by the normals \( n \). When \( (j, n) \) are consistent, \( S(j, n) \) is the Regge action [19] of such a geometrical 4-simplex (more precisely, the Dittrich–Speziale action [20]), divided by \( 8\pi h G_{\text{Newton}} \). This is obtained identifying \( J_{\text{int}} \) with \( 1/8\pi \gamma h G_{\text{Newton}} \) (\( \gamma \) is the Immirzi parameter) times the area of the boundary triangle—this being the proper identification in the theory, for large \( J_{\text{int}} \) [10]. This very remarkable result is obtained via a saddle point evaluation of the vertex amplitude defined in [10], written as an integral over copies of \( SU(2) \), with techniques derived from [21]; see also [22].

What we now need to show is that this result implies that \( W \) has the correct asymptotic form (5, 9). We can discard the c.c. term in both expressions (5) and (10), since we know from [2] that the boundary state selects only one of the two c.c. terms. What we need to do is simply to transform (10) to the intertwiner basis \( k \), that is, compute

\[
W(j, k) = \int d\mathbf{n} \langle W[j, n]|n, |k\rangle. \tag{11}
\]

We are only interested in this expression in the vicinity of \( (j_0, k_0) \). More precisely, we are specifically interested in the first-order variation of \( W \) when varying \( k \):

\[
W(j_0, k_0 + \delta k) = \int d\mathbf{n} \langle W[j_0, n]|n, |k_0 + \delta k\rangle. \tag{12}
\]

Since \( j_0 \) is large, we can use the Barrett–Fairbairn result (10). Inserting in the last equation gives

\[
W(j_0, k_0 + \delta k) = \int d\mathbf{n} N e^{-ij0}\mu(j_0, n)\langle n, |k_0 + \delta k\rangle. \tag{13}
\]

Now, the \( \mu(j_0, n) \) factor peaks the integral on those \( n \) that define a 4-simplex with all equal areas \( j_0 \). But the geometry of a four simplex is entirely determined by these ten areas (up to possible discrete degeneracy that we disregard here). Hence, \( n \) that contribute to the integral are only those characterizing regular tetrahedra. These still have a \( SU(2)^5 \) multiplicity, because being normal to the faces of a regular tetrahedron defines the quadruplet of vectors \( n_0 \) only up to a global \( SO(3) \) rotation, but this rotation does not affect (13) since \( S \) and the intertwiner states \( |k\rangle \) are \( SU(2) \) invariant. Hence, trivially integrating out this subgroup we can write

\[
W(j_0, k_0 + \delta k) = N e^{-ij0}\mu(j_0, n_0)\langle n_0, |k_0 + \delta k\rangle, \tag{14}
\]

where \( n_0 \) is an arbitrary set of five quadruplets of vectors, each normal to the faces of a regular 4-simplex.

We now need the asymptotic expression for the matrix elements of the change of basis. It is shown in [25] that when \( n \) are the vectors \( n_0 \) normal to the faces of a regular tetrahedron, the state \( |n\rangle \) can be shown to converge to the coherent tetrahedron state defined in [24]. This behaves like

\[
\langle k|n_0\rangle \sim e^{ij0}\delta^k \tag{15}
\]

for large spins. Inserting this we have

\[
W(j_0, k_0 + \delta k) \sim N e^{-ij0} e^{ij0}\delta^k, \tag{16}
\]

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The important result here is the appearance of the correct $\pi/2$ factor in the phase, which was missing in the BC vertex. This is precisely the phase (9) we were looking for. Thus, we knew that the vertex has the asymptotic behavior that was guessed in [13], in order to yield the graviton propagator. This is our main observation.

It is also instructive to analyze the first-order dependence on the spins. Keeping now the intertwiner fixed, we have

$$W(\mathbf{j}_0 + \delta \mathbf{j}, k_0) = \int d\mathbf{n} \langle W | \mathbf{j}_0 + \delta \mathbf{j}, \mathbf{n} \rangle \langle \mathbf{n}, | k_0 \rangle.$$  \hspace{1cm} (17)

Taking the factor $\mu$ of the saddle point approximation to behave like a delta function in the large $j$ limit in which we are, we obtain

$$W(\mathbf{j}_0 + \delta \mathbf{j}, k_0) = N e^{-i S(\mathbf{j}_0 + \delta \mathbf{j})} \mu(\mathbf{j}_0 + \delta \mathbf{j}, \mathbf{n}) \langle \mathbf{n}, | k_0 \rangle.$$  \hspace{1cm} (18)

where $S_{\text{Regge}}(\mathbf{j}) = S(\mathbf{j}, \mathbf{n}(\mathbf{j}))$ and $\mathbf{n}(\mathbf{j})$ are the $\mathbf{n}$ consistent with $\mathbf{j}$ in the neighborhood of $\mathbf{n}_0$. The angle $\Phi$ is determined in [25] as the twisting angle between two opposite edges in a tetrahedron. Expanding this gives $\Phi(\mathbf{n}(\mathbf{j}_0 + \delta \mathbf{j})) = \Phi(\mathbf{n}(\mathbf{j}_0)) + \Phi' \delta \mathbf{j} = \frac{\pi}{2} + \Phi' \delta \mathbf{j}$. The first-order variation of the $S_{\text{Regge}}$ action around a flat 4-simplex gives precisely the $\phi_{nm}$ phases satisfying (8). But these get corrected by the additional term $\Phi' \delta \mathbf{j}$. Therefore, in order to guarantee a cancellation of phases in $j$, the appropriate boundary state has to have an intertwiner dependence given by the exponential of $-i \Phi(\mathbf{n}(j)) k$. Such dependence coincides with the one discussed in [13] only to the zero order in $(j - j_0)$. If we expand to the next order, the full quadratic part of (5) is determined by the Hessian of the Regge action.

4. Conclusions

What we have shown here is only that the obstacle that prevented the Barrett–Crane vertex to yield the proper graviton propagator is resolved by the new vertex. Clearly it is now necessary to restart from scratch the calculation of the graviton propagator using the new vertex, and check that everything works properly. For this, it seems clear that the best basis to use in the intertwiner spaces is the Livine–Speziale coherent state basis. The disadvantage of using a badly over-complete basis is overcome by the advantage of having a basis that respects the symmetries of the 4-simplex, thus avoiding the complications due to the need to choose a pairing in picking a virtual-spin basis. The full calculation of the graviton two-point function using the Livine–Speziale basis is in course and will be presented elsewhere. Calculations of higher-$n$, $n$-point functions and higher-order terms of the propagator are also in course.

A number of issues need better clarification before we can say that we understand the low energy limit of loop quantum gravity. Among these are the role of gauge invariance [26] and finiteness [27]. Nevertheless, we see good reasons for optimism. The new vertex has been introduced in [7, 10] only as an attempt to give the intertwiners a dynamics. Whether this dynamics was correct at low-energy remained unclear during the last year, lacking the asymptotic analysis of the vertex. Remarkably, this analysis turns out to give precisely the intertwiner dependence that was previously indicated as the one hoped for.

5 The couple of edges to be considered is the one corresponding to the chosen recoupling basis. See [31] for a detailed derivation.

6 The coherent boundary state introduced in [31] implements automatically this requirement to all orders and provides a geometric interpretation for it.
After the completion of this work, a number of other papers confirming and much developing our result have appeared [28–31].

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