Glueball-baryon interactions in holographic QCD

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Abstract

Studying the Witten-Sakai-Sugimoto model with type IIA string theory, we find the glueball-baryon interaction is predicted in this model. The glueball is identified as the 11D gravitational waves or graviton described by the M5-brane supergravity solution. Employing the relation of M-theory and type IIA string theory, glueball is also 10D gravitational perturbations which are the excited modes by close strings in the bulk of this model. On the other hand, baryon is identified as a D4-brane wrapped on $S^4$ which is named as baryon vertex, so the glueball-baryon interaction is nothing but the close string/baryon vertex interaction in this model. Since the baryon vertex could be equivalently treated as the instanton configurations on the flavor brane, we identify the glueball-baryon interaction as “graviton-instanton” interaction in order to describe it quantitatively by the quantum mechanical system for the collective modes of baryons. So the effective Hamiltonian can be obtained by considering the gravitational perturbations in the flavor brane action. With this Hamiltonian, the amplitudes and the selection rules of the glueball-baryon interaction can be analytically calculated in the strong coupling limit. We show our calculations explicitly in two characteristic situations which are “scalar and tensor glueball interacting with baryons”. Although there is a long way to go, our work provides a holographic way to understand the interactions of baryons in hadronic physics and nuclear physics by the underlying string theory.

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1 Introduction

The underlying fundamental theory QCD for nuclear physics and particle physics has achieved great successes. However, nuclear physics remains one of the most difficult and intriguing branches of high energy physics because physicists are still unable to analytically predict the behavior of nuclei or even a single proton. The key problem is that the behavior in the strong-coupling regime of QCD is less clear theoretically. Fortunately, gauge/gravity (gauge/string) duality (See, e.g., [1, 2, 3, 4, 5] for a review) has become a revolutionary and powerful tool for studying the strongly coupled quantum field theory. Particularly, the Witten-Sakai-Sugimoto (WSS) model [6, 7, 8], as one of the most famous models, has been proposed to holographically study the non-perturbative QCD for a long time [9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19]. Therefore, in this paper, we are going to extend the previous works to study the interactions in holographic QCD.

The holographic glueball-meson interaction has been studied in [20, 21, 22, 23] by naturally considering the gravitational waves or graviton in the bulk of this model. Since the gravitational waves or graviton signals the glueball states holographically and mesons are excited by the open string on the flavor branes, the close/open string (on the flavorbrane) interaction is definitely interpreted as glueball-meson interaction. And the effective action could be derived by taking account of the gravitational perturbation in the flavor brane action.

On the other hand, in the WSS model, baryon could be identified as a D4′-brane wrapped on $S^4$, which is named as “baryon vertex” [24, 25]. The D4′-brane has to attach the ends of $N_c$ fundamental strings since the $S^4$ is supported by $N_c$ units of a R-R flux in the supergravity (SUGRA) solution. Such a D4′-brane is realized as a small instanton configuration in the world-volume theory of the flavor branes in this model. Basically, the baryon states could be obtained by quantizing the baryon vertex. In the strong coupling limit (i.e. the 't’Hooft coupling constant $\lambda \gg 1$), the two-flavor case (i.e. $N_f = 2$) has been studied in [9] and it turns out that baryons can be described by the $SU(2)$ Belavin-Polyakov-Schwarz-Tyupkin (BPST) instanton solution with a $U(1)$ potential in the world-volume theory of the flavor branes. And employing the soliton picture, baryon states could be obtained by a holographic quantum mechanical system for collective modes, see also Appendix B.

Accordingly, there must be close string/D4′-brane interaction if the baryon vertex is taken into account, which could be interpreted as the glueball-baryon interaction in this model. Thus we will explore whether or not it is able to describe this interaction by the quantum mechanical system (B-7). So the main contents of this paper are: First, we find that there must be the glueball-baryon interaction in this holographic model. Second, we use the holographic quantum mechanical system in [9] (or Appendix B) to describe the the glueball-baryon interaction quantitatively in the $\lambda \gg 1$ limit. Since the analytical instanton configuration with generic numbers

\footnote{In order to distinguish from $N_c$ D4-branes who are responsible for the background geometry, we denote the baryon vertex as “D4′-brane” in this paper.}
of the flavors is not known, only the two-flavor case (i.e. $N_f = 2$) [9] is considered in this paper.

The outline of this paper is very simple. In Section 2, we discuss the glueball-baryon interaction in this model and how to describe it quantitatively in the strong coupling limit. Section 3 is the Summary and discussion. Since there are many papers and lectures about the WSS model (such as [6, 14]), we will not review this model systematically. Only the relevant parts of this model are collected in Appendix A and B on which our discussions and calculations are based. Appendix C shows some details of the calculation in our manuscript.

2 The equivalent description of the glueball-baryon interactions

In this section, we will explore how the “close string interacting with baryon vertex” can be interpreted as the “glueball interacting with baryon” and how to describe it by the holographic quantum mechanical system in [9] (or in Appendix B). First of all, let us take a look at the most general aspects about the interaction of the graviton (close string) in this model.

As a gravity theory, it is very natural to consider the gravitational waves (or graviton) in the bulk of this D4/D8 system (WSS model). According to [20, 21, 22, 23], such a gravitational perturbation signals the glueball states and can definitely interact with the open string on the flavor branes. Thus such close/open string interactions have been holographically interpreted as “glueball-meson interactions” or “glueball decays to the mesons” because mesons are excited by the open strings on the flavor branes in this model. Interestingly, once the baryon vertex is taken into account, the interaction between baryon vertex and graviton (or gravitational waves) must occur since the graviton is excited by the close string in the bulk. Hence there must be close string/baryon vertex interaction which could be interpreted as glueball-baryon interaction in this model. It provides a holographic way for understanding and can be treated as a parallel mechanism to “glueball-meson interaction” proposed in [20, 21, 22, 23].

Basically, the “glueball-baryon interaction” in this model is nothing but the close/open string (D-brane, baryon vertex) interaction. However, it is not easy to quantitatively describe the close/open string or close string/D-brane interaction by the underlying string theory in a generic spacetime, in order to describe glueball interacting with baryons. Fortunately, according to [6, 9, 10, 12, 13, 14, 24, 25], baryon vertex is equivalently described by the instanton configuration in world-volume of the D8/D8-branes with the BPST solution [B-2]. Therefore, the “glueball-baryon interaction” could be identified as the “gravitons (or gravitational waves) interacting with instantons” in the world-volume theory of the flavor branes.

With this idea, let us consider a gravitational perturbation in the bulk geometry since the glueball states are signaled by the graviton in this model, i.e. replace the metric as,

$$g_{MN} \rightarrow g_{MN}^{(0)} + h_{MN},$$

(2.1)

where $g_{MN}^{(0)}$ is the background metric (A-1) and $h_{MN}$ is a perturbative tensor which satisfies...
$h_{MN} \ll g_{MN}^{(0)}$. Then, we consider the interaction (coupling) between graviton (glueball) and instantons in the world-volume of the flavor branes. The dynamic in the world-volume theory of the flavor branes is described by the Yang-Mills action (A-5) plus the Chern-Simons (CS) action (A-8). Since the CS action (A-8) is independent of the metric, it remains to be (A-8) even if (2.1) is imposed. However the Yang-Mills action (A-5) contains additional terms which depend on $h_{MN}$ as,

$$S_{YM} = S_{YM}^{(0)} + S_{YM}^{(1)} + O(h_{MN}^2),$$

$$S_{YM}^{(0)} = -\frac{1}{4}(2\pi\alpha')^2 T_8 \int_{D8/D8} d^4x dU d\Omega_4 e^{-\Phi(0)} \sqrt{-\det g_{ab}^{(0)}} \Tr \left[ g^{(0)ac} g^{(0)bd} F_{ab} F_{cd} \right],$$

$$S_{YM}^{(1)} = \frac{1}{4}(2\pi\alpha')^2 T_8 \int_{D8/D8} d^4x dU d\Omega_4 e^{-\Phi(0)} (1 - \delta \Phi) \sqrt{-\det g_{ab}^{(0)}} \times \Tr \left[ \left( h^{ac} g^{(0)bd} + h^{bd} g^{(0)ac} \right) F_{ab} F_{cd} \right] \quad (2.2)$$

Notice that only the linear perturbation of $h_{MN}$ is considered in (2.2). So if $h_{MN}$ is the mode of gravitational waves propagating in the bulk, it must depend on time which means $h_{MN} = h_{MN}(t)$. Furthermore, because the baryon states are given by the quantum mechanical system (Appendix B), so once we evaluate the potential term (B-7) in the moduli space by using (B-6) with (2.2), it implies there must be an additionally time-dependent term $H(t)$ to the Hamiltonian (B-7). Therefore, the transition amplitude can be calculated by the standard technique of time-dependent perturbation in quantum mechanics in order to describe the glueball-baryon interaction quantitatively. So let us evaluate the perturbed Hamiltonian $H(t)$ and the transition amplitude explicitly by taking account of two characteristic situations in the following subsections.

### 2.1 Interactions with scalar glueball

In this section, let us consider the “scalar glueball interacting with baryons”. In order to evaluate the perturbed Hamiltonian, we need to write the explicit formulas of the gravitational perturbation first. In this model, the scalar glueball can be described by the gravitational polarization [21, 22, 23]. So let us consider the gravitational polarization in 11D SUGRA because the WSS model is based on type IIA SUGRA which can be reduced from M5-brane solution of 11D SUGRA for M-theory (See [26, 27] for a complete review). For scalar glueball, the gravitational polarization in 11D SUGRA takes the following forms,
\[ H_{44} = -\frac{r^2}{L^2} F (r) H_E (r) G_E (x), \]
\[ H_{\mu\nu} = \frac{r^2}{L^2} H_E (r) \left[ \frac{1}{4} \eta_{\mu\nu} - \left( \frac{1}{4} + \frac{3r_{KK}^6}{5r^6 - 2r_{KK}^6} \right) \frac{\partial_\mu \partial_\nu}{M_E^2} \right] G_E (x), \]
\[ H_{55} = \frac{r^2}{4L^2} H_E (r) G_E (x), \]
\[ H_{rr} = -\frac{L^2}{r^2} \frac{1}{F (r)} \frac{3r_{KK}^6}{5r^6 - 2r_{KK}^6} H_E (r) G_E (x), \]
\[ H_{r\mu} = \frac{90r^7 r_{KK}^6}{M_E^2 L^2 (5r^6 - 2r_{KK}^6)} H_E (r) \partial_\mu G_E (x), \]
\[ (2.3) \]

We use \( G_{AB}, H_{AB} \) to represent the 11D metric and gravitational polarizations in order to distinguish 10D metric \( g_{MN} \) and perturbation \( h_{MN} \). The 11 coordinates correspond to \( \{ x^\mu, x^4, x^5, r, \Omega_4 \} \) and \( x = \{ x^\mu \} \) in our convention. The function \( F (r) \) and the relation between 11D \(( r, r_{KK}, L) \) and 10D variables \(( U, z, U_{KK}, R) \) are given as,
\[ F (r) = 1 - \frac{r_{KK}^6}{r^6}, \quad U = \frac{r^2}{2L}, \quad 1 + \frac{z^2}{U_{KK}^2} = \frac{r^6}{r_{KK}^6}, \quad U = \frac{U^3}{U_{KK}^3}, \quad L = 2R. \]
\[ (2.4) \]

Since the near-horizon solution of M5-branes in 11D is \( AdS_7 \times S^4 \), the 11D metric satisfies the equations of motion from the following action with the integration on \( S^4 \),
\[ S_{11D} = \frac{1}{2\kappa_{11}^2} \left( \frac{L}{2} \right)^4 V_4 \int d^7 x \sqrt{- \det G} \left( R_{11D} + \frac{30}{L^2} \right). \]
\[ (2.5) \]

Imposing the near-horizon solution of M5-branes with \( (2.3) \) to \( (2.5) \), we obtain the eigenvalue equation for \( H_E (r) \) as,
\[ \frac{1}{r^3} \frac{d}{dr} \left[ r^5 F (r) \frac{d}{dr} H_E (r) \right] + \left[ \frac{432r^2 r_{KK}^{12}}{5r^6 - 2r_{KK}^6} + L^4 M_E^2 \right] H_E (r) = 0, \]
\[ (2.6) \]

and the kinetic action of the function \( G_E (x) \) which is,
\[ S_{G_E (x)} = C_E \int d^4 x dx^4 dx^5 \frac{1}{2} \left[ (\partial_\mu G_E)^2 + M_E^2 G_E^2 \right], \]
\[ (2.7) \]

with
\[ C_E = \int_{r_{KK}}^{\infty} dr \frac{r^3}{L^3} \frac{5}{8} H_E^2 (r). \]
\[ (2.8) \]

Obviously, \( (2.7) \) shows why \( (2.3) \) signals scalar glueball field. Then we have to translate 11D gravitational polarization \( (2.3) \) into 10D WSS model in order to evaluate the Hamiltonian for collective modes. Employing the dimensional reduction as \([26, 27]\), the components of 10D \( h_{MN} \) are collected by subtracting \( g_{MN}^{(0)} \). As a result, they are,
\( h_{\mu\nu} = \left( \frac{U}{R} \right)^{3/2} \left[ \frac{R}{2U} H_{55}\eta_{\mu\nu} + \frac{R}{U} H_{\mu\nu} \right], \)

\( h_{44} = \left( \frac{U}{R} \right)^{1/2} \left[ H_{44} + \frac{1}{2} f(U) H_{55} \right], \)

\( h_{zz} = \frac{4R^{3/2}U_{KK}}{9U^{5/2}} \left( \frac{R}{2U} H_{55} + \frac{U_{KK}^2}{RU^2} H_{rr} \right), \)

\( h_{z\mu} = \frac{2U_{KK}^2}{3U^2} H_{r\mu}, \quad h_{\Omega\Omega} = \frac{R^{5/2}}{2U^{1/2}} H_{55}, \)

(2.9)

with the dilaton,

\( e^{4\Phi/3} = \frac{U}{R} \left( 1 + \frac{R}{U} H_{55} \right). \)

(2.10)

For the reader convenience, we give the explicit form of the equation (2.6) in the \( z \) coordinate, which is,

\[
0 = H_E''(z) + \frac{U_{KK}^2 + 3z^2}{z(U_{KK}^2 + z^2)} H_E'(z) + \frac{432U_{KK}^{13/3}(U_{KK}^2 + z^2)^{1/3}}{9U_{KK}^{1/3}(U_{KK}^2 + z^2)^{4/3}} (3U_{KK}^2 + 5z^2)^2 H_E(z).
\]

(2.11)

While (2.11) is difficult to solve, we have to search for a solution for \( H_E \) in order to evaluate the perturbed Hamiltonian for collective modes. Since only the \( O(\lambda^0) \) of the Hamiltonian (B-5) (13-7) is the concern in our paper, we need to solve (2.11) up to \( O(\lambda^{-1}) \). Rescale (2.11) as (B-1), we obtain the following equation (derivatives are w.r.t. \( z \)),

\[
H_E''(z) + \left( \frac{1}{z} + \frac{2z}{\lambda} \right) H_E'(z) + \frac{16 + 3M_E^2}{3\lambda} H_E(z) + O(\lambda^{-2}) = 0.
\]

(2.12)

In order to compare our calculations with [9], we have employed the unit of \( M_{KK} = U_{KK} = 1 \) so that \( R^3 = 9/4 \). The (2.12) is easily to solve in terms of hypergeometric function and Meijer \( G \) function which is,

\[
H_E(z) = C_1 \text{HypergeometricF1} \left[ \frac{4}{3} + \frac{M_E^2}{4}, 1, -\frac{z^2}{\lambda} \right]
\]

\[
+ C_2 \text{MeijerG} \left[ \{#\}, -\frac{1}{3}, \frac{M_E^2}{4}, \{0, 0, \{#\}\}, \frac{z^2}{\lambda} \right],
\]

(2.13)

where \( C_1, C_2 \) are two integration constants. Since the background is the bubble solution of D4-branes, the metric in our model must be regular everywhere. Accordingly, we have to set

\(^3\)"z" is the rescaled coordinate defined as in (B-1).
$C_2 = 0$ because Meijer G function diverges at $U = U_{KK} = 1$. On the other hand, $C_1$ has to consistently satisfy $C_1 \ll 1$, because $H_E$ appearing in (2.9) should also be the perturbation to the background metric $g_{MN}^{(0)}$. Therefore the solution of $H_E$ is only valid up to $O(\lambda^{-1})$ according to (2.12). Hence, we have the solution of $H_E$ in the large $\lambda$ expansion,

$$H_E (z) \simeq C_1 - C_1 \left( \frac{16 + 3 M_E^2}{12 \lambda} \right) z^2 + O(\lambda^{-2}).$$

(2.14)

Next, we will evaluate the perturbed Hamiltonian with (2.9) additional to (B-7). Using (B-6) and (2.2),

$$S_{YM}^{(0)} + S_{YM}^{(1)} + S_{CS} = - \int dt \left[ U^{(0)} (X^\alpha) - H (t, X^\alpha) \right].$$

(2.15)

$U^{(0)} (X^\alpha)$ is obtained by evaluating $S_{YM}^{(0)} + S_{CS}$ which is the exact forms of the potential in (B-7). Therefore we need to evaluate $S_{YM}^{(1)}$ in order to obtain the perturbed Hamiltonian $H (t, X^\alpha)$ in (2.15) and the procedures are as follows,

1. We decompose the $U (2)$ gauge field in $S_{YM}^{(1)}$ (2.2) as (A-9) and use (B-2) to represent the instanton (baryon) in the world-volume of D8/D8-branes.

2. Insert (2.9) into $S_{YM}^{(1)}$ (2.2), rescale the obtained formula of $S_{YM}^{(1)}$ by imposing (B-1) and then expand the result up to $O(\lambda^{-1})$.

3. Finally, we use (2.15) to evaluate the perturbed Hamiltonian for the collective modes up to $O(\lambda^{-1})$.

While the above procedures are quite straightforward, the calculation is very messy. So let us give the resultant formula here.

The perturbed Hamiltonian can be written as,

$$H_{Scalar} (t, X^\alpha) = \mathcal{A} \kappa \int d^3x dz d\Omega_4 K (t, x, z, X^\alpha),$$

(2.16)

where the function $K (t, x, z, X^\alpha)$ is given in (C-6) in the unit of $U_{KK} = M_{KK} = 1$. Hence the Hamiltonian is calculated as,

$$H_{Scalar} (t, X^\alpha) = \mathcal{A} \kappa C_1 \cos (\omega t) \left\{ \frac{9}{2} \pi^2 \left( 2 - \frac{5 \omega^2}{M_E^2} \right) + \left[ \left( \frac{40 k^2 - 18 M_E^2}{16 M_E^2} + \frac{420 \omega^2}{16} \right) \left( 2 Z^2 + \rho^2 \right) \pi^2 + \left( \frac{9 M_E^2}{1280 M_E^2 \pi^2 \rho^2} \right) + 9 k^2 \rho^2 \pi^2 \left( \frac{5 \omega^2}{8 M_E^2} - \frac{1}{4} \right) \right] \lambda^{-1} + O(\lambda^{-2}) \right\}. \tag{2.17}$$

$\kappa$ is given in (B-8) and more details of the calculation for (2.17) are given in the Appendix C.
\( A \) is a constant independent on \( \lambda \). \( k \) and \( \omega \) is the 3-momentum and the frequency of the glueball field \( G_E (t, x) \). Notice that (2.17) is suitable to be a perturbation since \( C_1 \) has to satisfy \( C_1 \ll 1 \). Hence, with the quantum mechanical system of baryons (B-7), the average transition amplitude \( \mathcal{M} \) and the probability of transition \( \mathcal{P} \) can be evaluated by the standard technique in the quantum mechanics with a time-dependent perturbation, which is,

\[
\mathcal{P}_{i \rightarrow f} = \left| \int_0^t \langle H (t', X^\alpha) e^{-iE_i t'} \rangle dt' \right|^2,
\]

\[
= \left| \int_0^t e^{i(E_i t - \omega) t'} \mathcal{M} (i \rightarrow j) dt' \right|,
\]

where \( E_{ij} = E (l', n'_\rho, n'_z) - E (l, n_\rho, n_z) \) is defined by (B-9). For simplification, let us consider the case of small \( k \) limit i.e. \( k \rightarrow 0 \) which means the glueball field, as an external field, is homogeneous. In this limit, it implies \( \omega \simeq M_E \) since the “classical glueball field \( G_E (t, x) \)” means the onshell condition \( \omega^2 - k^2 = M_E^2 \) has to be satisfied. Thus in small \( k \) limit, we can simplify (2.17) as,

\[
H_{Scalar} (t, X^\alpha) = A \kappa C_1 \cos (\omega t) \left\{ -27 \frac{\pi^2}{2} + \left[ -\frac{81}{1280 a^2 \pi^2 \rho^2} + \frac{27 M_E^2 + 468}{16} \pi^2 (2Z^2 + \rho^2) \right] \lambda^{-1} \right. 
\]

\[
+ O (\lambda^{-2}) \right\},
\]

(2.19)

By analyzing the eigenfunctions (B-9) of the Hamiltonian (B-7), we find the following selection rules,

\[
\begin{cases} 
\tilde{l}' = \tilde{l} (l' = l) \\
n'_z = n_z
\end{cases} \quad \text{or} \quad \begin{cases} 
\tilde{l}' = \tilde{l} (l' = l) \\
n'_z = n_z \pm 2
\end{cases}.
\]

(2.20)

Working out (2.18), it is easy to find another constraint of the transition which is \( \omega = E_{ij} \). Interestingly, our holographical quantum mechanical system is very similar as the atomic spectrum of hydrogen. The “holographic baryon interacting with glueball” behaves similarly as the “electron interacting with photon” in the hydrogen atomic. Both of them can be described by the quantum mechanics which means the baryon (electron) is described by the quantum mechanics while the glueball (photon), as a classically external field, is described by the classical gravity theory (classical electrodynamics) respectively.

Furthermore, let us examine whether or not the transition procedures, in this quantum mechanical system with the constraints and selection rule discussed above, are really possible.

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5In the unit of \( M_{KK} = U_{KK} = 1 \), \( A \) should be \( A = \frac{243}{512 \pi^2} \).

6Since our theory is symmetrically rotated in the 3d \( x^i \)-space, we assume the momentum \( k \) of the glueball field \( G (t, x) \) is along \( x^3 \) direction.
to occur. We consider the low energy (small momentum) limit as the most simple case which is \( k \to 0 \), so that \( \omega \approx M_E \). Since \( M_E \) represents the mass spectrum of the scalar glueball in (2.11), it reads with the WKB approximation [20] (\( \beta = 2\pi \) in the unit of \( M_{KK} = U_{KK} = 1 \)),

\[
M_E (j) \simeq \frac{8.12}{\beta} \sqrt{j \left( j + \frac{5}{2} \right)}.
\]  

(2.21)

Consequently, we find the following transitions,

\[
\frac{E (l = 1, 3; n_\rho = 3; n_z = 0, 1, 2, 3) - E (l = 1, 3; n_\rho = 0; n_z = 0, 1, 2, 3)}{M_E (j = 1)} \simeq 1.013, \quad (2.22)
\]

and

\[
\frac{E (l = 1, 3; n_\rho = 5; n_z = 0, 1, 2, 3) - E (l = 1, 3; n_\rho = 0; n_z = 0, 1, 2, 3)}{M_E (j = 2)} \simeq 1.053, \quad (2.23)
\]

are possible to occur according to the above selection rules and constraint. Notice that the WSS model is a low-energy effective theory for baryons or mesons, so it may not be very consistent to consider the high energy states of baryons in this model. Using (2.17) and (2.18), the transition amplitude corresponding to (2.22) (2.23) can be calculated, respectively, as,

\[
\begin{align*}
\mathcal{M} (n_\rho = 3 \to n'_\rho = 0) & \bigg|_{l = l', n_z = n'_z} = \frac{(27M_E^2 - 270\omega^2 - 135k^2) \, \mathcal{A} \kappa C_1}{44800M_E^2 a^2\pi^2 m_0^4 \omega^4 \lambda} \\
& \simeq -\frac{243 \mathcal{A} \kappa C_1}{44800a^2\pi^2 m_0^4 \omega^4 \lambda} + \mathcal{O} (k^2), \\
\mathcal{M} (n_\rho = 5 \to n'_\rho = 0) & \bigg|_{l = l', n_z = n'_z} = \frac{(9M_E^2 - 90\omega^2 - 45k^2) \, \mathcal{A} \kappa C_1}{53760M_E^2 a^2\pi^2 m_0^4 \omega^4 \lambda} \\
& \simeq -\frac{81 \mathcal{A} \kappa C_1}{63760a^2\pi^2 m_0^4 \omega^4 \lambda} + \mathcal{O} (k^2).
\end{align*}
\]  

(2.24)

2.2 Interactions with tensor glueball

Let us consider another special example for the interaction with tensor glueball. In the bulk, the 11D gravitational polarization of the tensor glueball could be simply chosen as [23],

\[
H_{11} = -H_{22} = -\frac{r^2}{L^2} H_T (r) G_T (x),
\]  

(2.25)

where the equation of motion for the radial function \( H_T \) is,

\[
\frac{1}{r^3} \frac{d}{dr} \left[ r \left( r^6 - r_{KK}^6 \right) \frac{d}{dr} H_T (r) \right] + L^4 M_T^2 H_T (r) = 0.
\]  

(2.26)
While (2.25) has to be reduced into 10D metric, it satisfies the traceless condition,

\[ h_{11} = -h_{22}. \] (2.27)

Inserting (2.27) into (2.2), we can immediately find that the perturbed Hamiltonian of the collective coordinates is vanished. However, the perturbed Hamiltonian from the tensor glueball should be \( H_{\text{Tensor}}(t, X^\alpha) \sim \mathcal{O}(H^2_{AB}) \). Since the gravitational polarization (2.25) is solved by the linear gravity perturbation, it would be inconsistent to consider the contribution from \( \mathcal{O}(H^2_{AB}) \) to the Hamiltonian of the collective coordinates.

### 3 Summary and discussion

In this paper, we consider the linearly gravitational perturbation in the bulk of the Witten-Sakai-Sugimoto model. According to [20, 21, 22, 23], such gravitational perturbations signal the glueball states. On the other hand, baryon can be identified as wrapped D-brane which is named as the “baryon vertex” as [24, 25]. So in the viewpoints of the string theory, there must be the glueball-baryon interaction if the baryon vertex is taken into account. Therefore the glueball-baryon interaction is nothing but the close string/D-brane (baryon vertex) interaction in this model. Since baryons can be treated as instanton configurations in the world-volume of the flavor branes, we identify the glueball-baryon interaction as “graviton-instanton” interaction as an equivalent description. With the BPST instanton configuration, we find the perturbed Hamiltonian for the collective modes of the baryons could be evaluated quantitatively in the strong coupling limit. Hence the amplitude and the selection rules of the glueball-baryon interaction can be accordingly calculated. In order to quantitatively clarify our idea, we show our methods in two characteristic situations which are “scalar and tensor glueball interacting with baryons”. Particularly, the perturbed Hamiltonian of “tensor glueball-baryon” interaction is vanished in the linearly gravitational perturbation which means it should be non-linear interaction in the gravity side.

Our work should be an application of strongly Maldacena’s conjecture since we have considered the quantum effect (graviton) in the gravity side. So our conclusions may also be suitable with finite \( N_c \) and \( \lambda \). Moreover, if combining [20, 21, 22, 23] with our work, it shows the complete glueball-meson-baryon interaction. Although these holographic approaches are a little different from traditional theories, they show us an analytical way to study the strongly coupled interactions in hadronic physics and nuclear physics by the string theory.

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Appendix A: The geometry of the Witten-Sakai-Sugimoto model

In the WSS model, there are \( N_c \) coincident D4-branes representing “colors” of QCD, wrapped on a supersymmetry breaking compact circle. The background geometry produced by these D4-branes is described by 10-dimensional type IIA supergravity in the near horizon limit. The metric reads \[ ds^2 = \left( \frac{U}{R} \right)^{3/2} \left[ \eta_{\mu\nu} dx^\mu dx^\nu + f(U) (dx^4)^2 \right] + \left( \frac{R}{U} \right)^{3/2} \left[ \frac{dU^2}{f(U)} + U^2 d\Omega_4^2 \right], \] which is the bubble geometry of the D4-brane solution. And the dilaton, Romand-Romand 4-form field, the function \( f(U) \) are given as,

\[
e^\phi = e^{\Phi - \Phi_0} = g_s \left( \frac{U}{R} \right)^{3/4}, \quad F_4 = dC_3 = \frac{2\pi N_c}{V_4} \epsilon_4, \quad f(U) = 1 - \frac{U_{KK}}{U^3}, \tag{A-2}
\]

where \( x^\mu, \mu = 0, 1, 2, 3 \) and \( x^4 \) are the directions which the D4-branes are extended along. \( U \) is the coordinate of the holographic radius and \( U_{KK} \) is the coordinate radius of the bottom of the bubble. The relation between \( R \) and the string coupling \( g_s \) with string length \( l_s \) is given as \( R^3 = \pi g_s N_c l_s^3 \). Respectively, \( d\Omega_4^2 \), \( \epsilon_4 \) and \( V_4 = 8\pi^2/3 \) are the line element, the volume form and the volume of an \( S^4 \) with unit radius. We have used \( x^4 \) to denote the periodic direction where the D4-branes are wrapped on as \( x^4 \sim x^4 + \delta x^4 \) with \( \delta x^4 = \frac{4\pi R^{3/2}}{U_{KK}^{1/2}} \). Accordingly, the Kaluza-Klein mass can be defined as \( M_{KK} = \frac{2\pi}{\delta x^4} = \frac{3}{2} U_{KK}^{1/2}/R^{3/2} \). Hence the parameters \( R, \ U_{KK}, \ g_s \) can be expressed in terms of QCD variables \( g_{YM}, \ M_{KK}, \ l_s \) as,

\[
R^3 = \pi g_s N_c l_s^3, \quad U_{KK} = \frac{2}{9} g_{YM}^2 N_c M_{KK} l_s^2, \quad g_s = \frac{1}{2\pi} \frac{g_{YM}^2}{M_{KK} l_s}. \tag{A-3}
\]

Additionally, the “flavors” of QCD could be introduced into this model by embedding a stack of \( N_f \) D8 and anti-D8 branes (D8/\overline{D8}-branes) as probes into the background \[ A-1 \]. The dynamic of the flavor branes is described by the following action,

\[
S_{D8/D\overline{8}} = S_{DBI} + S_{WZ}, \tag{A-4}
\]

The first term in \[ A-4 \] is the Dirac-Born-Infield (DBI) action and the second term is the Wess-Zumino (WZ) action. The DBI action of D8/\overline{D8}-branes in this model can be expanded in small field strength. Keeping only \( O(F^2) \), we get the Yang-Mills action for the dual field theory on the flavor branes, which is \[ 11 \].

\[ F \] is the dimensionlessful gauge field strength which is defined as \( F = 2\pi\alpha'F \).
\[ S_{\text{DBI}} \simeq S_{YM} + \mathcal{O}(F^4), \]
\[ S_{YM} = -\frac{1}{4} (2\pi \alpha')^2 T_8 \int_{D8/D8} d^4 x d U d \Omega_4 e^{-\Phi} \sqrt{-\det g_{ab} \text{Tr} \left[ g^{ac} g^{bd} F_{ab} F_{cd} \right]} . \]  

On the other hand, since only \( C_3 \) in non-vanished (A-2), the relevant term in WZ action is,

\[ S_{\text{WZ}} = \frac{1}{3!} \mu_8 (2\pi \alpha')^3 \int_{D8} C_3 \wedge \text{Tr} F^3 \]
\[ = \frac{1}{3!} \mu_8 (2\pi \alpha')^3 \int_{D8} dC_3 \omega_5(A), \]  

where \( \omega_5(A) \) is Chern-Simons 5-form given as,

\[ \omega_5(A) = \text{Tr} \left( A F^2 - \frac{i}{2} A^3 F - \frac{1}{10} A^5 \right). \] 

Since we are going to discuss the two-flavor case i.e. \( N_f = 2 \), the explicit form of (A-6) after integrating out \( dC_3 \) can be written as,

\[ S_{\text{WZ}} = \frac{N_c}{24\pi^2} \epsilon_{mnpq} \int d^4 x d z \left[ \frac{3}{8} \hat{A}_0 \text{Tr}(F_{mn} F_{pq}) - \frac{3}{2} \hat{A}_m \text{Tr}(\partial_0 A_n F_{pq}) + \frac{3}{4} \hat{F}_{mn} \text{Tr}(A_0 F_{pq}) \right. \]
\[ + \frac{1}{16} \hat{A}_0 \hat{F}_{mn} \hat{F}_{pq} - \frac{1}{4} \hat{A}_m \hat{F}_{0n} \hat{F}_{pq} + (\text{total derivatives}) \left. \right] \equiv S_{\text{CS}}. \]  

where the \( U(2) \) gauge field \( A \) has been decomposed into its \( U(1) \) and \( SU(2) \) part as \(^8\)

\[ A = A^i \tau^i - \frac{1}{2} \sqrt{2N_f} \hat{A} \times 1_{N_f \times N_f}. \] 

Notice that (A-8) is expressed in the \( z \) coordinate with transformation \( U^3 = U_{KK}^3 + U_{KK} z^2 \) and the index is defined as \( m,n,p,q = 1,2,3,z \) in the above equation. \( \tau^i \)'s are the Pauli matrices. Hence we have used the Yang-Mill action (A-5) plus Chern-Simons action (A-8) to govern the low energy dynamics on the flavored D8/\( D8 \)-branes in this paper.

Appendix B: Baryon as instanton in the Witten-Sakai-Sugimoto model

In the WSS model, baryon has been provided by a \( D4' \)-brane wrapped on \( S^4 \), which is named as “baryon vertex”. In the world-volume theory of the flavor branes, the coordinates \( x^M \) and

\(^8\)We have used “\( \hat{\cdot} \)” to represent the Abelian part of the gauge field while the non-Abelian part is expressed without a “\( \hat{\cdot} \)”.
the $U(2)$ gauge field $A_m$ need to be rescaled as $[9]$ in order to obtain the variables independent of $\lambda$,

$$x^m = \lambda^{-1/2}x^m, \quad x^0 = x^0,$$

$$A_0(t,x) = A_0(t,x), \quad A_m(t,x) = \lambda^{1/2}A_m(t,x),$$

$$F_{0m}(t,x) = \lambda^{1/2}F_{0m}(t,x), \quad F_{mn}(t,x) = \lambda F_{mn}(t,x),$$

in the expansion of $\lambda^{-1}$. Then by solving the equations of motion the resultant non-vanished components of the gauge field take the following forms,

$$A_0 = \frac{1}{8\pi^2 a_1} \frac{1}{\xi^2} \left[ 1 - \frac{\rho^4}{(\rho^2 + \xi^2)^2} \right],$$

$$F_{ij} = Q(\xi,\rho) \epsilon_{ijk} \tau^k,$$

$$F_{zi} = Q(\xi,\rho) \delta_{ij} \tau^j,$$

$$Q(\xi,\rho) = \frac{2\rho^2}{(\xi^2 + \rho^2)^2}, \quad a = \frac{1}{216\pi^3},$$

where

$$\xi^2 = (\vec{x} - \vec{X})^2 + (z - Z)^2.$$ 

In (B-2), we have used same convention as $[9]$, so that $\vec{x} = \{x^i\}$, $i = 1, 2, 3$ represents the 3-spatial coordinates where the baryons or instantons live and $\rho$ represents its size. According to $[9]$ (See $[28]$ for a complete review), the baryon spectrum could be obtained by a quantum mechanical system for the collective coordinates in a moduli space of one instanton. Since we are working in the strong coupling limit (i.e. $\lambda \gg 1$), the contribution of $O(\lambda^{-1})$ could be neglected. Accordingly the moduli space takes the following topology,

$$\mathcal{M} = \mathbb{R}^4 \times \mathbb{R}^4/\mathbb{Z}_2.$$ 

(B-4)

The first $\mathbb{R}^4$ corresponds to the position of the instanton which is parameterized by the collective coordinates $\vec{X}, Z$ and $\mathbb{R}^4/\mathbb{Z}_2$ is parameterized by the size $\rho$ and the $SU(2)$ orientation of the instanton. $\mathbb{R}^4/\mathbb{Z}_2$ can be parametrized by $y_I$, $I = 1, 2, 3, 4$ and the size of the instanton corresponds to the radial coordinate i.e. $\rho = \sqrt{y_1^2 + \cdots + y_4^2}$. The $SU(2)$ orientation is parameterized by $a_I = \frac{y_I}{\rho}$ with the normalized constraint $\sum_{i=1}^4 a_I^2 = 1$. It has been turned out that the Lagrangian of the collective coordinates in such a moduli space is given as,

$$L = \frac{mX}{2} g_{\alpha\beta} X^\alpha X^\beta - U(X^\alpha) + O(\lambda^{-1}).$$ 

(B-5)

Such a parameterization is also used in $[11, 29, 31, 30]$. 

13
The first term in (B-5) is the line element of the moduli space which corresponds to the kinetic term in the Lagrangian while the second term corresponds to the potential of this quantum mechanical system. Notice that we have used $X^{\alpha} = \left( \hat{X}, \ Z, \ y_{I} \right)$, and $m_{X} = 8\pi^{2}aN_{c}$. The potential term $U(X^{\alpha})$ in (B-5) could be calculated by employing the soliton picture as \[9, 11, 28, 29, 30, 31\], which takes the following form,

$$S_{\text{onshell}} \simeq S_{\text{onshell}}^{YM + CS} = - \int dt U(X^{\alpha}). \quad (B-6)$$

After quantization, the Hamiltonian corresponding to (B-5) for the collective coordinates is given as,

$$H = M_{0} + H_{y} + H_{Z} + O(\lambda^{-1}),$$

$$H_{y} = -\frac{1}{2m_{y}} \sum_{I=1}^{4} \frac{\partial^{2}}{\partial y_{I}^{2}} + \frac{1}{2} m_{y} \omega_{y}^{2} \rho^{2} + \frac{Q}{\rho^{2}},$$

$$H_{Z} = -\frac{1}{2m_{Z}} \frac{\partial^{2}}{\partial Z^{2}} + \frac{1}{2} m_{Z} \omega_{Z}^{2} Z^{2}, \quad (B-7)$$

where \[10\]

$$M_{0} = 8\pi^{2}\kappa, \quad \omega_{y}^{2} = \frac{2}{3}, \quad \omega_{\rho}^{2} = \frac{1}{6}, \quad Q = \frac{N_{c}}{40\pi^{2}a}, \quad \kappa = \frac{\lambda N_{c}}{216\pi^{3}}. \quad (B-8)$$

The eigenfunctions and eigenvalues of (B-7) can be easily evaluated by solving its Schrödinger equation, respectively they are \[11\]

$$\psi(y_{I}) = R(\rho)T^{(l)}(a_{I}), \quad R(\rho) = e^{-\frac{m_{y} \omega_{y}^{2}}{2} \rho \hat{\rho}} \text{Hypergeometric}_{1} F_{1} \left( -n_{\rho}, \tilde{l} + 2; m_{y} \omega_{y}^{2} \rho^{2} \right),$$

$$E(l, n_{\rho}, n_{z}) = \omega_{\rho} \left( \tilde{l} + 2n_{\rho} + 2 \right) = \sqrt{\frac{(l+1)^{2}}{6} + \frac{2}{15} N_{c}^{2} + \frac{2(n_{\rho} + n_{z}) + 2}{\sqrt{6}}}. \quad (B-9)$$

Notice that $T^{(l)}(a_{I})$ is the function of the spherical part which satisfies $\nabla^{2}_{S^{3}}T^{(l)} = -l(l+2)T^{(l)}$ since $H_{y}$ could be rewritten with the radial coordinate $\rho$,

$$H_{y} = -\frac{1}{2m_{y}} \left[ \frac{1}{\rho^{3}} \partial_{\rho} (\rho^{3} \partial_{\rho}) + \frac{1}{\rho^{2}} \left( \nabla^{2}_{S^{3}} - 2m_{y} Q \right) \right] + \frac{1}{2} m_{y} \omega_{y}^{2} \rho^{2}. \quad (B-10)$$

And we have used the quantum numbers $n_{z}, n_{\rho}, \tilde{l}$ to denote the eigenvectors of the combined quantum system $H_{y} + H_{Z}$ as $|n_{z}, n_{\rho}, \tilde{l}\rangle$ in this paper.

\[10\]Eqs. (B-7) - (B-10) are expressed in the unit of $M_{KK} = U_{KK} = 1$.

\[31\]The relation of $l$ and $\tilde{l}$ is $\tilde{l} = -1 + \sqrt{(l+1)^{2} + 2m_{y} Q}$ and the quantum number of the angle momentum can be represented by either $l$ or $\tilde{l}$. 
Appendix C: Some calculations about the perturbed Hamiltonian

The perturbed Hamiltonian (2.17) can be computed by (2.2) (2.15) or (B-6), equivalently,

$$-H_{Scalar}(t, X^\alpha) = -\frac{1}{4}(2\pi\alpha')^2 T_8 \int_{D_8/D_8} d^4x d^4\Omega_4 e^{-\Phi} \sqrt{-\det g_{ab} \text{Tr}[g^{ac}g^{bd}F_{ab}F_{cd}]} -$$

$$\left\{-\frac{1}{4}(2\pi\alpha')^2 T_8 \int_{D_8/D_8} d^4x d^4\Omega_4 e^{-\Phi^{(0)}} \sqrt{-\det g_{ab}^{(0)} \text{Tr}[g^{(0)ac}g^{(0)bd}F_{ab}F_{cd}]} \right\}. \quad (C-1)$$

With the linear perturbation of gravity, we have,

$$g^{ab} = g^{(0)ab} - h^{ab}, \quad h^{ab} = g^{(0)ac}g^{(0)bd}h_{cd}. \quad (C-2)$$

Therefore all the functions in (C-1) have been given in (2.3) (2.14) (A-1) (B-2). Rescale the formulas in (C-1) as (B-1), we can obtained the following result by direct computation,

$$H_{Scalar}(t, X^\alpha) = \frac{1}{4}(2\pi\alpha')^2 T_8 \int_{D_8/D_8} d^3x d^3\Omega_4 \mathcal{K}(t, x, z, X^\alpha), \quad (C-3)$$

where

$$\mathcal{K}(t, x, z, X^\alpha) = C_1 \text{Tr} \left\{ \frac{9Q^2 \delta_{ij} \tau^i \tau^j}{8M_F^2} (2M_F^2 - 5\omega^2) G_E(t, x) + \frac{45k\omega Q (\hat{F}_{02}\tau^1 - \hat{F}_{01}\tau^2 + \hat{F}_{0z}\tau^3)}{8M_F^2} \times G_E(x) \lambda^{-1/2} \right\}$$

$$\times \left[ -\frac{9}{64M_F^2} \left( (5\hat{F}_{02}^2 - 5\hat{F}_{03}^2) k^2 + (7\hat{F}_{0z}^2 - 3\hat{F}_{03}^2) M_F^2 \right) + (5\hat{F}_{03}^2 + 5\hat{F}_{0z}^2) \omega^2 + \left( \hat{F}_{01}^2 + \hat{F}_{02}^2 \right) (5k^2 - 3M_F^2 + 5\omega^2) \right] G_E(x)$$

$$- 15i\omega z Q \hat{F}_{0i}\tau^i M_F^2 F_E(t, x) + \frac{3z^2Q^2}{32M_F^2} \left( 40k^2 \left( (\tau^1)^2 + (\tau^2)^2 - (\tau^3)^2 \right) \right) G_E(t, x) \lambda^{-1}$$

$$- \left( (\tau^1)^2 + (\tau^2)^2 + (\tau^3)^2 \right) \times \left( 6M_F^4 - 140\omega^2 - M_F^2 (16 + 15\omega^2) \right) G_E(t, x) \right\}. \quad (C-6)$$

Notice that (C-6) is written in the unit of $U_{KK} = M_{KK} = 1$, so that $R^3 = 9/4$. The explicit formula of the glueball field $G_E(t, x)$ is needed in order to work out (2.17). The most simple way is to solve its classical equation of motion from the action (2.7). So we choose the real solution for $G_E(t, x)$ since it also appears in the perturbed metric (2.9) of the bulk geometry, therefore we have
\[ G_E(t, x) = \frac{e^{-ik_\mu x^\mu} + e^{ik_\mu x^\mu}}{2} = \cos (kx^3 - \omega t) \]

\[ = \cos \left( \frac{kx^3}{\lambda^{1/2}} - \omega t \right) \tag{C-7} \]

so that the derivatives of \( G_E(t, x) \) in (2.3) can be calculated as,

\[ \partial_\mu G_E(t, x) = \frac{ik_\mu (e^{ik_\mu x^\mu} - e^{-ik_\mu x^\mu})}{2} \equiv ik_\mu F_E(t, x), \tag{C-8} \]

Notice that since the system is rotationally symmetric in \( x^i \)-space, we have assumed that the momentum \( k \) in (C-7) (C-8) has only one component along \( x^3 \) direction. In order to further simplify (C-6), we calculate the following integrals appearing in (C-3),

1) \[ \int_{-\infty}^{+\infty} d^3xd\mathbf{z}Q(x, \mathbf{z})^2 G_E(t, x) = \frac{1}{3} k^2 \pi^2 \rho^2 BesselK \left[ 2, \frac{k\rho}{\lambda^{1/2}} \right] \cos (\omega t) \equiv I_1 \cos (\omega t) \]

\[ \simeq \left[ \frac{2}{3} \pi^2 - \frac{1}{6} \pi^2 \rho^2 k^2}{\lambda} + O \left( \lambda^{-2} \right) \cos (\omega t), \tag{C-9} \]

2) \[ \int_{-\infty}^{+\infty} d^3xd\mathbf{z}^2 Q(x, \mathbf{z})^2 G_E(t, x) \]

\[ = \frac{1}{9} \pi^2 \rho^2 \left\{ \frac{k^2}{\lambda} (3Z^2 + \rho^2) BesselK \left[ 2, \frac{k\rho}{\lambda^{1/2}} \right] \right. \]

\[ + 2 MeijerG \left[ \left\{ -\frac{1}{2}, \# \right\}, \left\{ 0, 1 \right\}, \left\{ \frac{1}{2} \right\}, \frac{k^2 \rho^2}{4 \lambda} \right] \right\} \cos (\omega t) \equiv I_2 \cos (\omega t) \]

\[ \simeq \left[ \frac{1}{3} \pi^2 (2Z^2 + \rho^2) + \frac{1}{36} \pi^2 \rho^2 \left( -6Z^2 - 5\rho^2 + 6\gamma \rho^2 + 6\rho^2 \log \frac{k}{\lambda^{1/2}} + 3\rho^2 \log \frac{\rho^2}{4} \right) \right. \]

\[ - 3 \rho^2 PolyGamma \left[ 0, \frac{3}{2} \right] + 3 \rho^2 PolyGamma \left[ 0, \frac{5}{2} \right] \frac{k^2}{\lambda} \left. + O \left( \lambda^{-2} \right) \right] \cos (\omega t), \tag{C-10} \]

3) \[ \int_{-\infty}^{+\infty} d^3xd\mathbf{z} \hat{F}^2_{03} G_E(t, x) \]

\[ = \frac{1}{11520a^2 \pi^2 \rho^2} \left\{ 4 MeijerG \left[ \left\{ -\frac{5}{2}, \# \right\}, \left\{ 0, 1 \right\}, \left\{ \frac{1}{2} \right\}, \frac{k^2 \rho^2}{4 \lambda} \right] \right. \]

\[ + 18 MeijerG \left[ \left\{ -\frac{3}{2}, \# \right\}, \left\{ 0, 2 \right\}, \left\{ \frac{1}{2} \right\}, \frac{k^2 \rho^2}{4 \lambda} \right] \right\} \cos (\omega t) \equiv I_3 \cos (\omega t) \]

\[ \simeq \left[ \frac{1}{80a^2 \pi^2 \rho^2} + \frac{1}{8960a^2 \pi^2} \left( -101 + 70\gamma + 70 \log \frac{k}{\lambda^{1/2}} + 35 \log \frac{\rho^2}{4} - 35 PolyGamma \left[ 0, \frac{3}{2} \right] \right. \right. \]

\[ + 35 PolyGamma \left[ 0, \frac{9}{2} \right] \frac{k^2}{\lambda} + O \left( \lambda^{-2} \right) \right] \cos (\omega t), \tag{C-11} \]
IV) \[ \int_{-\infty}^{+\infty} d^3x d^2\bar{z} F_{01,02,03}^2 G_E (t, x) \]
\[ = \frac{1}{11520 \omega \pi^2 \rho^2} \left\{ \frac{3}{4} \frac{k \rho}{\lambda^{1/2}} BesselK \left( 3, \frac{k \rho}{\lambda^{1/2}} \right) + 16 \text{MeijerG} \left[ \left\{ \frac{3}{2}, \# \right\}; \{0, 1\}, \{ \frac{1}{2} \}; \frac{k^2 \rho^2}{4 \lambda} \right] \right\} \cos(\omega t) \equiv \mathcal{I}_4 \cos(\omega t) \]
\[ \approx \left\{ \frac{1}{80 \omega \pi^2 \rho^2} + \frac{1}{11520 \omega \pi^2 \rho^2} \left[ -49 + 30 \gamma + 30 \log \frac{k \lambda}{\nu} + 15 \log \frac{\rho^2}{4} - 15 \text{PolyGamma} \left[ 0, \frac{3}{2} \right] \right. \right\} \cos(\omega t), \quad \text{(C-12)} \]

where \( \gamma \) is the Euler-Gamma constant. Using (C-6), (C-9) - (C-12) and \( \text{Tr} \{ \tau^i \} = 0 \), we can obtain,

\[ H_{\text{Scalar}} (t, X^a) = A_\kappa C_1 \cos(\omega t) \left\{ \right. \]
\[ \left. \begin{array}{c}
\frac{27}{4M_E^2} (2M_E^2 - 5\omega^2) \mathcal{I}_1 \\
+ \left[ \frac{120k^2 - 9 (6M_E^2 - 140\omega^2 - M_E^2 (16 + 15\omega^2))}{16M_E^2} \right] \mathcal{I}_2 \\
- \frac{9 (M_E^2 + 15\omega^2 + 15k^2)}{32M_E^2} \lambda^{-1} + \mathcal{O}(\lambda^{-2}) \end{array} \right\} \cos(\omega t), \quad \text{(C-13)} \]

Inserting the expansion of large \( \lambda \) of \( \mathcal{I}_{1,2,3,4} \), then (2.17) can be obtained.

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