“Long memory investigation during demonetization in India”

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Abstract

Long-range dependence (LRD) in financial markets remains a key factor in determining whether there is market memory, herding traces, or a bubble in the economy. Usually referred to as ‘Long Memory’, LRD has remained a key parameter even today since the mid-1970s. In November 2016, a sudden and drastic demonetization measure took place in the Indian market, aimed at curbing money laundering and terrorist funding. This study is an attempt to identify market behavior using long-range dependence during those few days in demonetization. Besides, it tries to identify nascent traces of bubble and embedded herding during that time. Auto Regressive Fractionally Integrated Moving Average (ARFIMA) is used for three consecutive days around the event. Tick-by-tick data from CNX Nifty High Frequency Trading (CNX Nifty HFT) is used for three consecutive days around demonetization (approximately, 5000 data points from morning trading sessions on each of the three days). The results show a clear and profound presence of herd behavior in all three data sets. The herd intensity remained similar, indicating a unique mixture of both ‘Noah Effect’ and ‘Joseph Effect’, proving a clear regime switch. However, the results on the event day show stable and prominent herding. Mandelbrot’s specified effects were tested on an uncertain and sudden financial event in India and proved to function perfectly.

INTRODUCTION

When there is a non-negligible link between an existing data point and its past counterparts (even extreme past), it is usually referred to as long memory or long-range dependence on a stationary time series. A stationary process (with finite variance) is said to have long-range dependence (LRD) if its autocorrelation function experiences a decay as a power of the lag; however, the decay is extremely slow. In a plausible understanding, the past is closely related to the present. Moreover, the connection strength refuses to die down, despite being farfetched (10-25 years). Structural events in economics, such as ‘demonetization’, should ideally put an end to such a continuous connection, as it is surrounded by volatility. Therefore, it would be quite intriguing to delve into such an economic (structural) event to search for a specific finding. Whether LRD is still present, despite such a heavy structural event, accompanied by volatility of all kinds, remains the main research question.

Before moving on the economic scenario in India at the time of demonetization and delving deeper, let’s take a quick look at the fundamental and proven ‘stylized facts’ of long-range dependence (LRD) or long memory (LM). Stylized facts are empirically proven patterns and dimensions over a long time period across various datasets:
1. Excess volatility would be found in asset returns.
2. It would possess heavy or fat tails with high kurtosis.
3. Autocorrelations of asset returns are insignificant.
4. High volatility zones would be clustered together.
5. Volatility and volume would be positively correlated.

Coming to the economic situation with demonetization, one can find chaos and random market behavior from all sides. Dead of night of November 8, 2016 was marked by an apparent audacious policy (pronounced by certain quarters) to withdraw all 500 and 1,000 denominations of notes from the basket of available currencies. Demonetizing 86% of all currency notes suddenly came as a shock to many, as India’s cash-based economy (parallel economy) in some accounts was larger than possibly the visible economy. The Government officially announced three reasons for this extreme event:

1. Sucking liquidity from the system with a substantial amount of counterfeit currency notes in circulation.
2. This counterfeit currency was used for drug trafficking and terrorism, thus restricting both those activities.
3. To empty the storage of unaccounted wealth, indicated by law enforcement agencies.

Whatever the reason, it was a shock to the economy. Most market analysts confirmed that the bellwether CNX Nifty 50 would have plummeted and an enormous amount of negative bubble caused by the herd behavior would surface. Events like demonetization are termed as fat-tailed or heavy-tailed events, defying the Gaussian distribution. Usually, the index movements in such conditions are not stationary. On the contrary, they are mean-shifting processes. However, the increments of the index could well be stationary in nature, and, most probably, within a stable Lévy process. A pure SS process or even a mixed SS process (with Joseph and Noah effects) would indicate the LRD condition. In simple terms, the index would show long memory and self-similar movements, which in turn would confirm both ‘Herd behavior’ and ‘Bubble’ (irrespective of its direction).

1. LITERATURE REVIEW

Financial time series are often found to be with plenty of stylized facts. These may or may not be proven but are necessarily observed during empirical analysis. Most of the asset returns were found to have higher level of kurtosis, ensuring the presence of a heavy-tailed distribution, which is colloquially mentioned as a ‘fat tail’. Despite the static properties of the fundamental economic parameters, asset returns often showcase traces of exuberance, which is extremely difficult to explain. Autocorrelation mostly is found to be insignificant, other than high frequency markets (probably, due to micro-structure effects). Volatility in asset prices seems to follow queer clusters, and two clusters are often distanced by a relatively low volatility period. As famously said by Mandelbrot, “… large changes tend to be followed by large changes, of either sign, and, small changes tend to be followed by small changes.” Volume and volatility are often found to be in sync (positively correlated) and they even follow the same long memory pattern.

Benoit Mandelbrot was the first person who recognized the importance of fractional Brownian motion and gave it this name. His Ph.D. scholar Murad S. Taqqu documented a first-hand account of the history behind this and some other developments. Benoit Mandelbrot also played a significant role in the field of long memory. Long memory is important as it allows one to predict future data based on a limited amount of data observed in the past. Graves, Gramacy, Watkins, and Franzke (2017) documented Mandelbrot’s work in this field. In a genuine search, they furthered and explained various approaches used to study long memory in different fields. Baillic (1996) explored different econometric works on long memory processes, fractional integration and their applications. The researcher reviewed some definitions of long memory and described the population
characteristics of various long memory processes. Jan Beran reviewed the definition of LRD from a long period (from 1992 to 2010), different models that can be used to generate LRD, how statistical inferences can be drawn for both stationary and non-stationary models used for this purpose and how to differentiate LRD models from the other models that simply mimic such a behavior.

Usually, a strange coordination is observed in this regard. Self-similarity (SS) and long-range dependence (LRD) come together. Conventionally, slow decay of an ACF function indicates the presence of LRD. This theory, generated around 1963, was a mere stylized fact back then. Criticized heavily in 1991, it was finally accepted partially around the mid-2000s. The first criticism of LRD, generated out of higher values of Hurst exponent, came to the foray when Bhattacharya showed that high ‘Hurst Exponent’ (Mandelbrot’s rechristened method) did not necessarily mean LRD and herd behavior. They were furthered by mathematically proving that short memory, when disturbed by a non-linear monotonic trend, often took shape of a pseudo long memory. Robust resistance came from Andrew W. Lo, when he slated to prove that the Hurst-Mandelbrot measure of rescaled range statistics did prove short-range dependence (SRD) but not LRD. He blamed non-stationarity of the log return data for such spurious LRD. Theories are germinated and amended over a period of time. Similar feat happened here as well. Robinson showed that the premise of ‘null limit theory’, used by Andrew Lo for his modified rescaling, remained non-standard. He furthered it by quoting that ‘sensitivity to LRD cannot be overlooked’. Mandelbrot also proved these apparently strong rebuttals to be potentially flawed. Moreover, if there is any sudden change from a persistent pattern to antipersistent pattern, it resembles the Biblical story of Noah. Hence, Mandelbrot coined the ‘Noah Effect’ to identify the transient phase change in market memory, which also represents ‘heavy tails’. In addition, the continuation of the same trend has been referred to as the ‘Joseph Effect’, linking ancient thoughts of seven years of good harvesting followed by seven years of draught. In a sense, Mandelbrot meant that the ‘Joseph Effect’ signified long memory or long-range dependence.

Serial correlations, reduced with time, are often observed in real data. This proves to be problematic, as many statistical conclusions are drawn assuming data independence. Beran (1992) reviewed different studies undertaken in the field of LRD to address such issues. Booth and Tse (1995) found that the Treasury and Eurodollar futures did not possess long memory. The results were similar to those observed in some earlier studies. However, the researchers also observed the presence of a slow mean reversion. This was at variance with the observations in earlier studies. They also found that the Treasury and Eurodollar futures markets were fractionally cointegrated after the crash, but not in the time preceding it. Jacobsen (1996) explored the LRD phenomenon for Italy, Germany, Netherlands, UK, France, US, and Japan. For the study, the author used the range statistics introduced by Andrew Lo in 1991, which corrected the range statistics introduced by Hurst in 1951. It was concluded that the return series for indices in the countries that they studied did not exhibit LRD. Though there was some evidence of LRD for Germany and Italy, they showed that this was likely caused by short-range dependence. Thus, it was found that many LRD measures are biased on account of short-range dependence. Walter Willinger et al. (1999) found that Andrew Lo’s conclusion about lack of LRD in stock price returns was much weaker than the previously thought, which showed that his rebuttal of Mandelbrot’s theory was on shaky ground. They found evidences of LRD in stock price returns, but they regarded even their own work as inconclusive due to a low $H$-value of 0.6. Lobato and Savin (1998) checked for the presence of LRD in daily stock returns and their squares. They found that one was likely to arrive at incorrect conclusions on account of non-stationarity and aggregation. They tried addressing these problems by dividing the period into sub-periods and by using the returns for individual stocks. They found no evidences of LRD in returns, but the squared returns showed LRD. Breidt, Crato, and de Lima (1998) proved the superiority of LMSV (long-memory stochastic volatility) models over the short-memory volatility models that had been in vague till then. They developed their LMSV model by including an ARFIMA process in the standard models.
Potters, Cont, and Bouchaud (1998) showed that though the Black-Scholes formula failed to account for ‘fat tails’ and correlations in the scale of asset fluctuations, financial markets corrected these limitations of the formula. They reached this conclusion by studying the actual option prices prevalent in the markets. Thus, they concluded, that financial markets act as efficient adaptive systems. Ray and Tsay (2000) studied the daily stock returns of S&P 500 companies and found evidences of strong persistence in volatility (Ray & Tsay, 2000). They suggested that stocks of companies in the same business sector exhibited higher LRD as compared to that of companies grouped only on the basis of size. Beran and Feng (2002) proposed SEMIFAR (semiparametric fractional autoregressive) models that could be used to check if a time series contained a stationary memory component (short or long), a difference stationary component, and/or a deterministic trend component. Weron (2002) compared various methods to check for the presence of LRD, using an R/S analysis, Detrended Fluctuation Analysis (DFA) and periodogram regression methods. DFA was found to provide the most accurate results. Christodoulou-Volos and Siokis (2006) checked for the presence of LRD, using semiparametric methods in a sample of 30+ stock index returns. They found LRD in around 65% of the series. Thus, they concluded that stock markets were fairly predictable, and an efficient market hypothesis did not hold true. They found that the impact of the 2008 financial crisis started ebbing in February 2009. Shin Kim (2016) used the fractional Lévy process to develop a discrete time option pricing model and found that, as compared to other less volatile markets, LRD in the S&P500 index option market was much stronger for the more volatile market created in the aftermath of the collapse of Lehman Brothers. Masoliver and Perello (2006) developed a linear “Ornstein-Uhlenbeck stochastic volatility model” to account for two very kinds of correlations in the same time series – volatility autocorrelations that have a very long-range memory and the asymmetric return-volatility correlations that have much shorter memory. They found that except for the crash days, their results were valid for the 100 years of the Dow Jones index daily returns spanning the time period from 1900 to 2000.

2. METHODS

Literature clearly indicates different methodologies to estimate long-range dependence (LRD) in time series data such as ARFIMA, Hurst exponent, Rescaled Range Method, Whittle Estimator, Regression Method based on the periodogram, Differencing the Variance Method, Absolute Moments Method, and Detrended Fluctuation Analysis. Three separate research teams, namely, Beran (1994), Palma (2007), and Box, Jenkins, and Reinsel (2008), estimated and reviewed long memory (LRD) models. In 2004, two eminent researchers in Northwestern University, Illinois, stated that Spectral analysis, Detrended Fluctuation Analysis (DFA) or Rescaled range analysis (R/S analysis) could not differentiate between LRD and SRD but these classical methods could be relevant to quantify long memory in the time series. They furthered their work in 2004 to 2005 and suggested the presence of LRD in time series calculation based on ARFIMA (auto-regressive fractionally integrated moving average) modeling.

In 2009, a French scientific research group in Université de Montpellier, demonstrated ARFIMA in a plausible manner. Torre, Delignières and Lemoine (2009) from Université de Montpellier demonstrated ARFIMA as an effective and important model that could be used to determine long memory in the data series. Wagenmakers, Farrell, and Ratcliff (2004) proposed an ARMA (1,1) and an ARFIMA (1, d, 1) model. Further, Wagenmakers, Farrell, and Ratcliff (2005) proposed ARFIMA modeling to determine long-range dependence by fitting 18 models. Nine of these models were ARMA (p, q) models, p and q varying from 0 to 2. The other nine models were related to ARFIMA (p, d, q) models by inclusion of a fractional-differencing parameter (d) representing persistent serial correlations. They showed that the ARFIMA model was better than transient ARMA models. Box and Jenkins (1970) introduced ARMA (p, q) or ARIMA(p, d, q) (auto-regressive, integrated, moving average) models that could capture only the short-range dependence property in time series. In ARIMA (p, d, q), d was confined to the standard integer values. Granger and Joyeux (1980) showed that the same model could rep-
resent the long-range dependence process by taking fractional values for differencing parameter \(d\), thereby obtaining an ARFIMA model. Finally, the ARFIMA parameters can be estimated using exact maximum likelihood (EML). Ercan, Kavvas, and Abbasov (2013) described the ARFIMA models as the generalization of the linear stationary ARMA and linear non-stationary ARIMA models. Moreover, differencing parameter \(d\) allows determining the intensity of long-range correlations in the time series. Further, \(d\) is related to the spectral exponent \(b\) by the simple equation \(b = 2d\).

The usual AR and MA lags \(d\) are the fractional Brownian motion coefficient \((0, 2)\). Since the assumption is a Gaussian distribution, white noise should therefore be present, \(d < 0.5\), in such circumstances.

\[ H = \frac{2d + 1}{2}. \]

\(H\) represents the Hurst exponent of the time series and can be any real number in the range \(0 < H < 1\). If \(d = 0\), then \(H = 0.5\); then there is no LRD. As a result, ARFIMA \((p, d, q)\) becomes ARMA \((p, q)\). Under revised condition, \(0 < d < 0.5\), Hurst exponent becomes \(0.5 < H < 1\), then the series is stationary and clearly shows the presence of LRD. If \(d = 0.5\), then \(H = 1\); then the series is rather mean-shifting. Torre, Delignières, and Lemoine (2007) tested and suggested a wide range of Hurst exponents \((H)\) from 0.1 to 0.9. Error estimates play a key role in studies like this. In 1978, Schwarz introduced a Bayesian information criterion (BIC) as an error estimation tool, which was represented by \(\text{BIC} = -2 \log L + K \log N\), where, \(N\) number of observations, \(K = p + d + q\) (number of free parameters + one added parameter), \(L = \text{maximum likelihood of the model}\). The second term represents a different penalty for parsimony. This penalty term considers both the sample size and the number of free parameters. Wagenmakers in 2005 and Torre, Delignières, and Lemoine in 2007 identified that AIC or Akaike information criterion issuing an easier penalty on the errors was essential for complex ABM models (Agent Based Models), whereas, BIC was conclusive with a heavy penalty on errors in relatively straight forward time series-based models.

Conceptually, a slow decay of autocorrelation means a strong link between the cardinal time series and its immediate lag. Interestingly, even if time tends to infinity, the relationship remains fairly strong. Logically, this cannot be a mean-reverting process, but a mean shifting process. Mean reverting process, or a stationary process, is predictable in nature when compared to the mean shifting or non-stationary process. Let’s suppose that \(H = 0.3\), this means that the process is chaotic and anti-persistent in nature. Thus, it does not have any self-similar traits. However, \(a = 3.33\), meaning that the process is not a stable Lévy process, as \(a = 1\). Ideally, the upper limit of \(a\) is supposed to be ‘2’ to qualify as a stable Lévy process. In a diametrically opposite scenario, when \(H = 0.75\) means it is self-similar and persistent with a clear pattern, the value of \(a\) is 1.33 (inside the upper limit). Hence, a persistent self-similar (SS) process with LRD has an a stable Lévy process. Rather willingly, the higher values of \(H\) are linked to risky and momentum play, where emotional crowd behavior sweeps even the most rational investor. The mid value, i.e. \(H = 0.5\), ideally depicts a chaotic, random and highly arbitraged market. A kind of a true reflection of an ideal Brownian motion. For a low value of \(H\), it adopts fractional values, hence, some of those interdependent increments are considered as fractional Brownian motion.

Auto Regressive model (AR) indicates an organic approach, where lags of the same time series hold the key to predict the future.

\[ Y_t = \alpha + \sum_{i=1}^{p} \Pi_i Y_{t-i} + \varepsilon_t. \]  

Equation (1) represents an Auto regressive model of order \(p\), where, \(\Pi_i\ldots\Pi_p\) are coefficients of lagged values of a dependent variable, \(\alpha\) is a constant, \(\varepsilon_t\) pure white noise error term. The model is assumed to be stationary.

Moving Average Model (MA)

\[ Y_t = \gamma + \sum_{i=1}^{q} \psi_i \varepsilon_{t-i} + \varepsilon_t. \]  

Equation (2) represents a moving average model of order \(q\), where \(1\ldots q\) are the moving average parameters, \(\gamma\) represents the constant (expectation of \(X\)) and \(\varepsilon_t\) represents the white noise error.
ARIMA Model

Equations (1) and (2) can be generalized as follows:

\[
(1 - \sum_{i=1}^{p} \Pi_iB^i)(1 - B^d)(Y_t - \gamma) = \left(1 + \sum_{i=1}^{q} \psi_iB^i\right)\epsilon_t.
\]

ARIMA model can capture only the short-range dependence property. In this case, \(d\) is limited only to integer values.

ARFIMA Model

Many authors demonstrated that the long-range dependence property can be captured by using the ARIMA model with a fractional difference operator rather confined to integer values. Then, the general ARIMA\((p, d, q)\) can be shown as AFRIMA\((p, d, q)\) as follows:

\[
\Phi(B)(1 - B^d)Y_t = \Theta(B)\epsilon_t,
\]

where \(\Phi(B)\) and \(\Theta(B)\) are autoregressive and moving average operators, \(d\) is the fractional difference parameter, and \(\epsilon_t\) is the white noise error term. 8,000 tick-by-tick CNX Nifty HFT data points for each morning (between 9:15am to 12:45noon) were initially considered. However, observation points were reduced due to duplicate time stamping. 5,631 observations as of November 8th, 5,331 observations as of November 9th, and 4,990 observations as of November 10th were used.

3. RESULTS

Figures 1, 2 and 3 depicts the decay of autocorrelation function over 90 lags. These depict LRD clearly as the decay rates are quite slow. Slow rate of decay in autocorrelation function empirically proves presence of LRD. In these cases, ACF came down from 0.8 to 0.65 after 90 lags (refer Figure 1), 0.6 to 0.35 after 90 lags (refer Figure 2), 0.6 to 0.4 after 90 lags (refer Figure 3).

Table 1. Various zones of Hurst exponent \((H)\)

| Hurst exponent \((H)\) | Interpretation |
|------------------------|----------------|
| \(0.5 < H < 1\)       | Persistent with clear shape, evident proof of profound herd as it approaches ‘1’, fat tails, black noise detected |
| \(H = 0.5\)            | Random Walk, completely stochastic in nature |
| \(0 < H < 0.5\)        | Non-persistent in pattern, evident proof of ‘No’ herd as it approaches ‘0’, fat tails, pink noise detected |

![Figure 1. ACF graph for November 10, 2016 depicting slow ACF decay indicating long memory](http://dx.doi.org/10.21511/imfi.17(2).2020.23)
4. DISCUSSION

Robustness measures (see Table 2) indicate that Day (0)'s calculations are slightly more accurate than those of Day (-1) and Day (+1). However, all three are in close proximity in terms of results and their robustness. The most intriguing observation from Table 2 indicates very strong herd behavior even in the morning of November 8, 2016. The announcement came in the very same day late in the evening. This indicates information asymmetry and information cascading, despite being kept secret. Ideally, economic and structural events like these are kept secret, with the exception of a few.

Table 2. Hurst exponent calculated by ARFIMA with its goodness of fit for consecutive three days around the event

| Day   | H      | d      | SE      | AIC     | BIC     | LL     |
|-------|--------|--------|---------|---------|---------|--------|
| –1(8/11/16) | 0.846484 | 0.3464837 | 0.00762627 | –53998 | –53978 | 27002 |
| 0 (9/11/16) | 0.822964 | 0.3229635 | 0.00801748 | –64562 | –64542 | 32284 |
| +1(10/11/16) | 0.999323 | 0.4993231 | 6.31585e-07 | –56726 | –56706 | 28366 |

Table 3. Hurst exponent calculated by ARFIMA with its goodness of fit intraday on November 9, 2016

| Day   | H      | d      | SE      | AIC     | BIC     | LL     |
|-------|--------|--------|---------|---------|---------|--------|
| P1(9/11/16) | 0.82361 | 0.3236069 | 0.01084896 | –39395 | –39377 | 19700 |
| P2 (9/11/16) | 0.78882 | 0.2888253 | 0.01382389 | –24913 | –24893 | 12459 |
The information cascading effect proved that market either received the news in advance or expected something similar to break out. Understandably, the day (0) shows a profound herd. Herd continued to stay strong even the very next morning of trading. This is a major structural event in the Indian currency market, so sudden changes in herd behavior can be rational. This may be well qualifying as the biblical Noah effect, as described by Mandelbrot. In economics, such a shift is known as a regime switch or change-points. Here, it means a persistent volatility regime to non-persistent volatility regime (Noah Effect). Joseph Effect indicates the continuation of the same regime. Study found Joseph effect on the contrary in 8th and 9th November instead (Shin Kim, 2016; Willinger, Taqqu, & Teverovsky, 1999). The biblical Joseph effect described by Mandelbrot indicates holding a long-term trend rather than a trend change in a short period. Noah effect occurs the very next day, i.e. on November 10, 2016. Hurst exponent witnessed a staggering 21.5% jump from November 9th morning trade values. This proves an economic regime switch very clearly. Theoretically, a self-similar (SS) process is usually a mixture of both Noah and Joseph effects; hence, these observations validate the theoretical premise once again. This pattern during demonetization resembles a pure self-similar model (SS), where the pattern will take a clue or two from its own past movements and move accordingly.

Table 3 depicts an in-depth analysis of the day of demonetization in India. There was a prominent spike around 10:30 AM in the dataset of CNX Nifty HFT. P1 represents a zone from 9 am to 10:29 am, and P2 represents a zone from 10:31 am to 1 pm. Further, the bubble and herd traces reduced marginally between these zones. Substantial effects are not observed, as Hurst exponent lowered by about 4.25% in intraday trading. This was a relatively smaller regime switch. This study reaffirms the same theoretical premise that was proved back in 1960s by Mandelbrot and reiterated by Rama Cont in 2001 that LRD and SS go hand in hand.

CONCLUSION

Structural events are usually associated with a phase of unavoidable spree of volatility around them. Volatility can be of two types. It will not produce herd behavior if its direction changes too frequently. On the contrary, it will produce profound herd behavior or mindless following (forming a clear pattern) if the volatility is unidirectional in most parts of the observation. This study finds the second instance in a profound nature. CNX Nifty HFT or high frequency showed clear pattern of strange information asymmetry. Whether it was a sudden jump of herd behavior (in either direction) indicating the Noah Effect, or a smooth cyclical pattern of herd behavior indicating the Joseph Effect, both were found with equal vigor during demonetization in India. Information asymmetry, unlikely for such a structural event, indicated the information cascading effect. The market vehemently showcased long memory on those three eventful days with varying degrees. Another interesting idea that emerged out of this study is to be presence of a Self-Similar process (SS). The market may not have been stationary in nature as a whole, but its growth was found to be stationary. A similar pattern can be observed for both LRD and SS processes in this study. In those three days, the market showed long memory and behaved like a near-perfect SS process. Often ignored as a ‘Stylized Fact’, SS proved to play a crucial role, since it indicated that Noah and Joseph effects were mixed effects in the Indian market, leading to the possible bubble formation (both positive and negative). It can be found from an economic perspective that certain parts of observations strictly followed a regime switch or a common change-points (Noah to Joseph or the other way round); however, it remained in one economic regime in the other.

In a nutshell, CNX Nifty HFT morning tick-by-tick trade values in three consecutive days in and around demonetization in India confirmed long memory (LRD) and self-similar (SS) behavior, which indicated profound herd behavior and confirmed the bubble formation. This study empirically demonstrated the senseless following of market participants forming a herd during the demonetization effect. Herd intensity has changed, but in a narrow range. Economic patterns, such as regime switch or change-points, were tracked by mathematical constructs and proven.
AUTHOR CONTRIBUTIONS

Conceptualization: Bikramaditya Ghosh, Aniruddha Oak.
Data curation: Bikramaditya Ghosh, Saleema J. S., Manu K. S.
Formal analysis: Bikramaditya Ghosh.
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Methodology: Manu K. S.
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Visualization: Saleema J. S.
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Writing – review & editing: Bikramaditya Ghosh, Aniruddha Oak, Sangeetha R.

DECLARATION

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