Two-point functions of SU(2)-subsector and length-two operators in dCFT

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Abstract

We consider a particular set of two-point functions in the setting of $N = 4$ SYM with a defect, dual to the fuzzy-funnel solution for the probe D5-D3-brane system. The two-point functions in focus involve a single trace operator in the SU(2)-subsector of arbitrary length and a length-two operator built out of any scalars. By interpreting the contractions as a spin-chain operator, simple expressions were found for the leading contribution to the two-point functions, mapping them to earlier known formulas for the one-point functions in this setting.

1. Introduction

Integrable structures in $N = 4$ SYM have been explored extensively since they were first noted in [1] and have provided a useful tool for both deeper field theoretic understanding and numerous tests of the AdS/CFT correspondence. For a pedagogical overview of the first decade, see [2]. Among other directions, the work has lead on to look for, and to employ, surviving integrability in similar theories, departing in different ways from $N = 4$ SYM. One particular branch of this focus is the study of various CFTs with defects (dCFTs).

The setting for these notes is $N = 4$ SYM with a codimension-one defect residing at the coordinate value $z = 0$. The theory is the field theory dual of the probe D5-D3-brane system in $AdS_5 \times S^5$, in which the probe-D5-brane has a three-dimensional intersection (the defect) with a stack of $N$ D3-branes. We will study the dual of the so called fuzzy-funnel solution[3–6], in which a background gauge field has $k$ units of flux through an $S^2$-part of the D5-brane geometry, meaning that $k$ D3-branes dissolve into the D5-brane. These parameters appear on the field theory side as the rank $N$ of the gauge group which is broken down to $N - k$ by the defect.

The dCFT action is built out of the regular $N = 4$ SYM field content plus additional fields constrained to the three dimensional defect. These additional fields interact both within themselves and with the bulk fields. However, only the six scalars from $N = 4$ SYM will play a role within these notes.

The defect breaks the 4D conformal symmetry down to those transformations that leave the boundary intact (i.e. that map $z = 0$ onto itself). Its presence thus changes many of the general statements about CFTs, such as allowing for non-vanishing one-point functions and two-point functions between operators of different conformal dimensions. These new features were first studied in [7, 8] and within the described setting, they have been the topic of a series of recent works. Tree-level one-point functions in the SU(2)- and SU(3)-subsectors where considered in [9–11] while bulk propagators and loop corrections to the one-point functions where worked out in [12–14]. Two-point functions were very recently addressed in [15] and earlier in [16].

The underlaying idea of all this business is to interpret single-trace operators as states in a spin-chain and employ the Bethe ansatz from within this context. The one-point functions were in this spirit found to be expressible in a compact determinant formula, making use of a special spin-chain state, called the Matrix Product State (MPS), and Gaudin norm for Bethe states. The end result for the tree-level one-point functions of operators

\[ \langle O_L \rangle_{tree} = \frac{2^{L-1}}{z^L} C_2(u) \prod_{j=1}^{L-1} j! \prod_{j=1}^{\frac{N}{2}} u_j^2 \left( u_j^2 + \frac{4}{k} \right) \]

under the condition that both the length $L$ and the number of excitations $M$ are even and that the set of $M$ Bethe rapidities has the special form $u = \{ u_1, -u_1, u_2, -u_2, \ldots \}$. The parameter $k$ can be any positive integer and

\[ C_2(u) = 2 \left( \frac{2\pi^2}{\lambda} \right)^{L} \prod_j \frac{1}{\lambda} \left[ \frac{u_j^2 + \frac{4}{k}}{u_j^2 + \frac{4}{k}} \frac{\det G^{-1}}{\det G} \right]^{\frac{1}{2}} \]

1meaning the region $z > 0$

2Wilson loops in these settings with a defect have also attracted attention, see e.g. [17–19].
where \( G^\pm \) are \( \frac{M}{2} \times \frac{M}{2} \) matrices with matrix elements

\[
G_{jk}^\pm = \left( \frac{L}{u_j + u_k} \right) \delta_{jk} \pm \frac{2}{u_j - u_k}
\]

within which, in turn,

\[
K_{jk}^\pm = \frac{2}{1 + (u_j - u_k)} \pm \frac{2}{1 + (u_j + u_k)}.
\]

The expression for \( C_2 \) was obtained from the spin-chain overlap

\[
C_2 = \left( \frac{8\pi^2}{\alpha} \right)^{L/2} \frac{1}{\sqrt{L}} \langle \text{MPS}\vert \Psi \rangle
\]

which is the form we will mostly refer to here. \( \vert \Psi \rangle \) is the spin-chain Bethe state corresponding to the operator \( O_L \); the MPS will be defined below in equation (2).

### 1.1. The goal of the present notes

These notes consider the leading contribution, in the 't Hooft coupling \( \lambda \), to the specific two-point function \( \langle O_L O_2 \rangle_{\text{constr.}} \), where

- both \( O_L \) and \( O_2 \) are single-trace scalar operators of length \( L \) and 2, respectively, and
- \( O_L \) is restricted to the SU(2)-subsector while \( O_2 \) can be built out of any pair of scalars.

We do this by interpreting the contraction as a spin-chain operator \( Q \) acting on the Bethe state corresponding to \( O_L \), whence re-expressing the two-point function in terms of the previously known one-point functions.

### 2. The particular two-point functions

We define the complex scalar fields as

\[
Z = \phi_1 + i\phi_4, \quad X = \phi_2 + i\phi_4, \quad W = \phi_3 + i\phi_6, \quad \overline{Z} = \phi_1 - i\phi_4, \quad \overline{X} = \phi_2 - i\phi_4, \quad \overline{W} = \phi_3 - i\phi_6,
\]

which in the dual fuzzy-funnel solution each has the non-zero classical expectation value

\[
\phi_i^1 = \frac{1}{z} t_I \otimes \theta_{i(N-k)} , \quad I = 1, 2, 3; \quad \phi_j^0 = 0 , \quad J = 4, 5, 6,
\]

where \( \{ t_I, t_2, t_3 \} \) forms a \( k \times k \) unitary representation of SU(2) and the \( \theta_{i(N-k)} \) pads the rest of the matrix to the full dimensions \( N \times N \).

For definiteness, we choose \( Z \sim \uparrow \) and \( X \sim \downarrow \) as the SU(2)-subsector.

We now set out to calculate

\[
\langle O_L O_2 \rangle_{\text{constr.}} = \sum_{i=1}^{L} \text{Tr} (X_i \cdots X_i) \text{Tr} (Y_1 Y_2) + (Y_1 \leftrightarrow Y_2), \quad i_e = \uparrow, \downarrow
\]

where \( X_I = Z, X_1 = X, Y_{1,2} \) can be any complex scalar and the coefficients \( \Psi^{\pm \pm} \) of \( O_L \) are chosen such that they map to a Bethe state \( \vert \Psi \rangle \) in the spin-chain picture.

We will express it by help of the MPS, which is the following state in the spin-chain Hilbert space:

\[
\langle \text{MPS} \rangle = \text{Tr} \left[ \left( \langle \uparrow \left| t_1 \right\rangle + \langle \downarrow \left| t_2 \right\rangle \right)^{d_L} \right], \quad (2)
\]

where the trace is over the resulting product of \( t \)'s.

### 2.1. Scalar propagators

The defect mixes the scalar propagator in both color and flavor indices, explained in detail in [13]. However, since the contracted fields are multiplied by classical fields from both sides we will only need the upper \( (k \times k) \)-block. The propagator diagonalization involves a decomposition of these components in terms of fuzzy spherical harmonics \( \hat{Y}_{\ell}^m \) :

\[
[\phi]^{s_1}_{s_2} = \sum_{s=1}^{k} \phi_{s, m} [\hat{Y}_{\ell}^m]^{s_1}_{s_2}, \quad s_{1,2} = 1, \ldots, k.
\]

Translating back to the \( s \)-indices, the relevant propagators for \( I, J = 1, 2, 3 \) read

\[
\langle [\phi_I]^{s_1}_{s_2} [\phi_J]^{s_3}_{s_4} \rangle = \delta_{IJ} \sum_{\ell, m} [\hat{Y}^{s_1}_{\ell} m]^{s_2}_{s_3} [\hat{Y}^{s_3}_{\ell} t]^{s_4}_{s_2} K^I_{\ell}(x,y) - \epsilon_{ijk} \sum_{\ell, m, m'} [\hat{Y}^s_{\ell} m]^{s_1}_{s_2} [\hat{Y}^s_{\ell} m']^{s_3}_{s_4} K^{i} \ell_{(2\ell+1)}(x,y)
\]

where \( K^{i} \ell_{(2\ell+1)} \) is in the \( (2\ell + 1) \)-dimensional representation. The remaining scalars \( I, J = 4, 5, 6 \) have the diagonal propagator

\[
\langle [\phi_I]^{s_1}_{s_2} [\phi_J]^{s_3}_{s_4} \rangle = \delta_{IJ} \sum_{\ell, m = \ell}^{\ell} [\hat{Y}^{s_1}_{\ell} m]^{s_2}_{s_3} [\hat{Y}^{s_3}_{\ell} m]^{s_4}_{s_2} K^{m = \ell}(x,y) + \ell K^{m = \ell+2}(x,y) .
\]

The spacetime dependent factors are

\[
K^{I}_{\ell}(x,y) = \frac{1}{2\ell + 1} K^{m = \ell}(x,y) + \frac{\ell}{2\ell + 1} K^{m = \ell+2}(x,y),
\]

\[
K^{I}_{\ell}(x,y) = \frac{1}{2\ell + 1} \left( K^{m = \ell}(x,y) - K^{m = \ell+2}(x,y) \right).
\]

\( K^{m_{\ell}} \) is related to the scalar propagator in AdS and reads

\[
K^{m_{\ell}}(x,y) = \frac{8\pi}{z} \sum_{l=1}^{d_L} \int \frac{d^3k}{(2\pi)^3} e^{i(k \cdot x - y)} L_{\ell}(\bar{t}_k | \bar{x}_y) K_{\ell}(\bar{t}_k | \bar{x}_y),
\]

in which \( I \) and \( K \) are modified Bessel functions with \( \chi^2(\chi) \) the smaller (larger) of \( x_3 \) and \( y_3 \), and lastly where \( v = \sqrt{m^2 + \frac{1}{4}} \).

We will from now on suppress all spacetime dependence.

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3 See appendices in [13, 20]. We use the normalization of [13].
3. The contraction as a spin-chain operator

With the expressions of the propagators, we can now view the contraction in equation (1) as a $(k \times k)$-matrix

$$[T_{x_i,y_j}]^{s_1,s_2} = \left\langle [x_i]^{s_1} [y_j]^{s_2} \right\rangle [y_j]^{s_2} = [x_i]^{s_1} [y_j]^{s_2}$$

replacing the field at site $i$ in the first trace while absorbing the second trace completely.

It turns out that this matrix always is proportional to either $t_1$, $t_2$, or $t_3$. To see this, first use that the fuzzy spherical harmonics are tensor operators, such that

$$\sum_{m} \tilde{t}_{m} \hat{T}_m \hat{t}_{m+1} = \hat{t}_k \hat{t}_{m+1} = \hat{t}_{m+1} \hat{t}_k = m \hat{t}_m.$$

Then use the orthogonality of the fuzzy spherical harmonics in the trace by decomposing the $t$ in $Y_2$ as

$$t_j = d_j (\hat{t}_1^{-1} + (-1)^j \hat{Y}_1), \quad j = 1, 2,$$

$$t_3 = \sqrt{2} d_3 \hat{t}_1,$$

Together, these factors in $T$ then conspire to always give $t_j$’s for any considered scalar combination. What is left can thus be interpreted as a one-point function of a slightly modified $O_L$. As such, we can write the two-point function (1) as an operator insertion

$$\langle \text{MPS}|Q_{y_1,y_2}|\Psi \rangle$$
in the spin-chain picture, acting on the Bethe state corresponding to $O_L$.

3.1. The spin-chain operator $Q_{y_1,y_2}$

$T$’s dependence on the involved scalars can be compactly written when expanded in terms of the real scalars:

$$T_{lijk} = \delta_{l1} \delta_{i1} \delta_{j1} \delta_{k1} + (\delta_{l1} \delta_{k2} - \delta_{i1} \delta_{j2}) K_{l1}^{2} + \delta_{ij} K_{m}^{2} \delta_{k2}^{2} L_{ijk},$$

$I, J, K = 1, \ldots, 6$ and where the $\delta^4$ ($\delta^6$) is only non-zero for indices 1, 2 and 3 (4, 5, and 6). Taking into account both the sums in the two-point function (1), we can then write the contractions in the spin-chain picture as

$$Q_{y_1,y_2}|\Psi \rangle = \sum_{l=1}^{L} \bigotimes_{i=1}^{\ell} Q_{y_1,y_2}^{i} \bigotimes_{i=1}^{\ell} |\Psi \rangle,$$

i.e. a linear combination of the spin-chain operators $[\mathbf{1}^{\delta_4}, S^+, S^-, S^3]^{\delta_6}$.

The result arranges itself in the two cases $Y_1^{cl} = Y_2^{cl}$ and $Y_1^{cl} \neq Y_2^{cl}$, for which

$$Q_{y_1} = \left( \begin{array}{cc} c^+ & 0 \\ 0 & c^- \end{array} \right), \quad Q_{y_2} = \left( \begin{array}{cc} 0 & c^- \\ c^+ & 0 \end{array} \right),$$

and the various coefficients $c$ implicitly depend on $Y_1, Y_2$. They are listed in Appendix A.

• Case $Y_1^{cl} = Y_2^{cl}$. The action of $Q_a$ is trivial on any Bethe state. Still denoting the total number of spin-down excitations as $M$, we immediately get

$$Q_a|\Psi \rangle = (c^+(L - M) + c^+ M)|\Psi \rangle.$$

Combining this with the one-point function formula implies

$$\langle O_L O_a \rangle_{\text{contr.}} = (c^+(L - M) + c^+ M) \langle O_L \rangle_{\text{tree}}.$$

As an example, the Konishi operator has the two-point function $2K^{m=4} L \langle O_L \rangle_{\text{tree}}$ with any SU(2)-subsector operator.

• Case $Y_1^{cl} \neq Y_2^{cl}$. In this case we have the spin-flipping operator

$$Q_a = c^+ S^+ + c^+ S^-.$$

Its action simplifies significantly when acting on a Bethe state. First of all, Bethe states with non-zero momenta are highest weight states implying that $S^+ |\Psi \rangle = 0$. Secondly, we have that

$$S^- |\Psi_M \rangle = \lim_{\rho \rightarrow 0} |\Psi_M \rangle,$$

meaning that acting on a Bethe state with the lowering operator creates a new Bethe state with one more excitation but with the corresponding momentum $p_{M+1} = 0$. All other momenta are the same. These states are called (Bethe) descendants.

It was shown in [9] that only states with $L$ and $M$ both even can have a non-zero overlap with the MPS. Furthermore, by studying the action of $Q_3$, the third conserved charge in the integrable hierarchy, it was proven that only unpaired7 states yield finite overlaps. This is true since $Q_3 |\Psi_{M} \rangle = 0$ and because $Q_3$ is non-zero on states that are not invariant under parity.

That $Q_a$ alters the number of excitations now makes it possible to have non-zero overlaps with states with odd $M$. However, since

$$[Q_3, S^+] = 0,$$

the requirement of an unpaired state is still imposed. Hence, the only possible way for the overlap

$$\langle \text{MPS}|Q_a|\Psi_M \rangle$$
to be non-vanishing is that is $M$ is odd and that the Bethe state is a descendant.

The general expression for such a state is

$$|\Psi_{M=L+n} \rangle = (S^-)^n |\Psi_0 \rangle, \quad n \text{ odd}.$$
We find
\[
\langle O_{L,M+n} O_\sigma \rangle_{1,\text{contr.}} = \left( c^n n(L - 2M - n + 1) C_{L,M,n}^+ + c^{-n} C_{L,M,n}^- \right) \langle O_{L,M} \rangle_{\text{tree}}
\]
with
\[
C_{L,M,n}^\pm = \frac{(n \mp 1)!}{n!} \left( \frac{L - 2M - n + 1}{2} \right) n!(L - 2M)!
\]

(3)

3.2. Remark on \( T \propto t_3 \)

When one of \( Y_1 \) or \( Y_2 \) is either \( W \) or \( \overline{W} \), \( T \) is proportional to \( t_3 \) and the corresponding \( Q_{ij}^{(0)} \) is no longer a proper spin-chain operator. Insisting on a spin-chain interpretation would describe it as a flip of site \( l + 1 \) followed by a removal of the site \( l \), thus shrinking the length \( L \) by one. \( Q_{ij}^{(0)} \) always appears preceded by a projection \( \Pi_{l(1)} \) on either spin-up or spin-down, depending on the \( Y \) which does not involve \( W(\overline{W}) \). It is straightforward to show by explicit calculation that
\[
\langle \text{MPS}_{L-1} \rangle \sum_{\text{basis vectors}} \langle 1 \otimes \cdots \otimes Q_{ij}^{(0)} \Pi_{l(1)} \otimes \cdots \otimes 1 \rangle_L = 0
\]
for any basis vector \( | 1 \rangle_L \) of length \( L \).

4. Conclusion

We have studied the \( N = 4 \) SYM theory with a defect, dual to the probe D5-D3-brane system. Within this theory, the two-point function between a length \( L \) operator \( O_L \) in the SU(2)-subsector and any operator \( Q_{Y_1Y_2} \) of two scalars can, in the leading order, be written as a spin-chain operator insertion in the scalar product between a matrix product state (MPS) and the Bethe state \( |\Psi\rangle \) corresponding to the operator \( O_L \).

\[
\langle O_L O_{Y_1Y_2} \rangle_{1,\text{contr.}} \propto \langle \text{MPS} | Q_{Y_1Y_2} | \Psi \rangle.
\]

The operation of \( Q \) depends on the two fields \( Y_1, Y_2 \) but is simple for any choice of scalar fields:

- For \( Y_1 = Y_2 \) we get

\[
\langle O_L O_{Y_1Y_2} \rangle = (c^L L + c^{-L}(L - M)) \langle O_L \rangle_{\text{tree}}
\]

where both \( L \) and the number of excitations \( M \) need to be even and the Bethe state needs to be unpaired.

- For \( Y_1 \neq Y_2 \), the two-point function is zero for any \( O_L \) mapping to a highest weight Bethe state. For operators \( O_{L,M+n} \) mapping to (Bethe) descendants, however, the two-point function is non-vanishing, under the condition that \( n \) is odd and that the corresponding Bethe state descends from an unpaired state \( |\Psi_{L,M}\rangle \). The result is

\[
\langle O_{L,M+n} O_{Y_1Y_2} \rangle_{1,\text{contr.}} = \left( c^n n(L - 2M - n + 1) C_{L,M,n}^+ + c^{-n} C_{L,M,n}^- \right) \langle O_{L,M} \rangle_{\text{tree}}
\]

where the combinatorial factors \( C_{L,M,n}^\pm \) can be found in equation (3).

The coefficients \( c \) with various indices depend on \( Y_1, Y_2 \) and are all spacetime-dependent since they contain expressions of the propagator. See Appendix A below for the full list of coefficients.

These results hold for any \( k \).

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Appendix A. List of coefficients

Here follows the list of coefficients for the considered two-point functions, written in the form \( Q_{Y_1Y_2} : (c^L c^M) \):

\[
Q_{XX} = \begin{pmatrix}
\frac{1}{2} (2K_m^{a=0} - 3K_m^{a=2} + K_m^{a=6}) & 0 & -\frac{1}{2}(K_m^{a=0} - K_m^{a=6}) \\
0 & 0 & 0 \\
-\frac{1}{2}(K_m^{a=0} - K_m^{a=6}) & 0 & \frac{1}{2}(2K_m^{a=0} + K_m^{a=6})
\end{pmatrix}
\]

\[
Q_{XY} = \begin{pmatrix}
0 & K_m^{a=0} - K_m^{a=2} & 0 \\
K_m^{a=0} - K_m^{a=2} & 0 & 0 \\
0 & 0 & K_m^{a=0} + K_m^{a=2}
\end{pmatrix}
\]

\[
Q_{XW} = \begin{pmatrix}
0 & K_m^{a=0} + K_m^{a=2} & 0 \\
K_m^{a=0} + K_m^{a=2} & 0 & 0 \\
0 & 0 & K_m^{a=0} - K_m^{a=2}
\end{pmatrix}
\]

\[
Q_{YY} = \begin{pmatrix}
\frac{1}{2} (2K_m^{a=0} - K_m^{a=6}) & 0 & -\frac{1}{2}(2K_m^{a=0} + 3K_m^{a=2} + K_m^{a=6}) \\
0 & \frac{1}{2}(2K_m^{a=0} + 3K_m^{a=2} + K_m^{a=6}) & 0 \\
-\frac{1}{2}(2K_m^{a=0} + 3K_m^{a=2} + K_m^{a=6}) & 0 & \frac{1}{2}(2K_m^{a=0} + K_m^{a=6})
\end{pmatrix}
\]

\[
Q_{WW} = \frac{1}{2} (2K_m^{a=0} - K_m^{a=6}) \begin{pmatrix}
0 & 0 & -\frac{1}{2}(2K_m^{a=0} + 3K_m^{a=2} + K_m^{a=6}) \\
0 & \frac{1}{2}(2K_m^{a=0} + 3K_m^{a=2} + K_m^{a=6}) & 0 \\
-\frac{1}{2}(2K_m^{a=0} + K_m^{a=6}) & 0 & \frac{1}{2}(2K_m^{a=0} + K_m^{a=6})
\end{pmatrix}
\]

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