Nonleptonic $\Lambda_b$ decays to $D_s(2317)$, $D_s(2460)$ and other final states in Factorization.

Alakabha Datta$^1$, Harry J. Lipkin$^2$ and Patrick J. O’Donnell$^3$

$^1$Department of Physics, University of Toronto
60 St George Street, Toronto, ON, Canada M5S 1A7.

$^2$Department of Particle Physics,
Weizmann Institute,
Rehovot 76100, Israel
and
School of Physics and Astronomy,
Raymond and Beverly Sackler Faculty of Exact Sciences
Tel-Aviv University,
Tel-Aviv, Israel
and
High Energy Physics Division
Argonne National Laboratory
Argonne, IL 60439-4815, USA

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Abstract

We consider nonleptonic Cabibbo–allowed Λ_b decays in the factorization approximation. We calculate nonleptonic decays of the type \( \Lambda_b \rightarrow \Lambda_c P \) and \( \Lambda_b \rightarrow \Lambda_c V \) relative to \( B_d^0 \rightarrow D^+ P \) and \( B_d^0 \rightarrow D^+ V \) where we include among the pseudoscalar states(P) and the vector states(V) the newly discovered \( D_s \) resonances, \( D_s(2317) \) and \( D_s(2460) \). In the ratio of \( \Lambda_b \) decays to \( D_s(2317) \) and \( D_s(2460) \) relative to the \( B_d^0 \) decays to these states, the poorly known decay constants of \( D_s(2317) \) and \( D_s(2460) \) cancel leading to predictions that can shed light on the nature of these new states. In general, we predict the \( \Lambda_b \) decays to be larger than the corresponding \( B_d^0 \) decays and in particular we find the branching ratio for \( \Lambda_b \rightarrow \Lambda_c D_s(2460) \) can be between four to five times the branching ratio for \( B_d^0 \rightarrow D^+ D_s(2460) \). This enhancement of \( \Lambda_b \) branching ratios follows primarily from the fact that more partial waves contribute in \( \Lambda_b \) decays than in \( B_d^0 \) decays. Our predictions are largely independent of model calculations of hadronic inputs like form factors and decay constants.
1 Introduction

Nonleptonic decays are widely used to obtain information about the elements of the CKM matrix in the Standard Model (SM), as well as to obtain insights about the non-perturbative aspects of QCD. Nonleptonic decays in the $B$ and $\Lambda_b$ systems are interesting since the heavy mass of the $b$ quark relative to the scale of soft non-perturbative physics allows for simplifications and makes tractable the difficult problem of calculating nonleptonic decays.

The nonleptonic decays of the $\Lambda_b$ baryon have received relatively less attention than those of the $B$ meson. In the $\Lambda_b$ baryon, the spin of the baryon is carried by the $b$ quark with the light diquark in a spin– and isospin–singlet state. This fact plays an important role in $\Lambda_b$ decays\textsuperscript{[1]} and leads to simplification of the non-perturbative dynamics involved in these decays. Because of this spin correlation between the the $b$ and the $\Lambda_b$ polarized $\Lambda_b$ decays can provide important information about the weak interaction of the $b$ quark \textsuperscript{[2]}. Nonleptonic $\Lambda_b$ decays can therefore be used to test the SM and to obtain insights into non-perturbative QCD.

In this work we consider Cabibbo–allowed $\Lambda_b$ decays. Cabibbo–allowed $\Lambda_b$ and $B$ decays are usually calculated using factorization. We will also concentrate only on the factorizable part and discuss briefly nonfactorizable effects later in this section.

The factorizable amplitude is expressed in terms of form factors and decay constants. However the form factors and, barring a few cases, the decay constants are unknown hadronic inputs. Therefore, predictions for nonleptonic $\Lambda_b$ decays, depend on model calculation of form factors and decay constants and can have a wide range even within the factorization assumption \textsuperscript{[3]}. Our purpose in this paper is to obtain predictions for $\Lambda_b$ decays, within factorization, using the heavy $m_b$ limit and using experimental inputs.

The method we use is the following: instead of directly calculating the $\Lambda_b$ decays we consider instead the ratio of Cabibbo–allowed $\Lambda_b$ decays relative to the corresponding Cabibbo–allowed $B$ decays. The branching ratios for the $\Lambda_b$ decays can then be obtained by simply using the experimental numbers for the Cabibbo–allowed $B$ decays. One obvious advantage of considering such ratios is that the dependence on decay constants drop out in the ratio. Furthermore, in the heavy $m_b$ limit, these ratios can be expressed as ratios of squared form factors. In the heavy $m_b$ limit all form factors can be related to one single form factor and a dimensional constant representing the effective mass of the light degrees of freedom in the $\Lambda_b$ baryon. These ratios of form factors are obtained using a mild assumption about the $q^2$ behavior of the form factors and the measurement of $BR[\Lambda_b \to \Lambda_c \pi^-]/BR[\bar{B}_d \to D^+ \pi^-]$. Our predictions turn out to be minimally dependent on hadronic inputs like form factors and decay constants.

Another advantage of calculating ratios of branching ratios is that some of the nonfactorizable amplitudes cancel in the ratio. To see how this happens consider the decays $\Lambda_b \to \Lambda_c P$ and $\bar{B}_d \to D^+P$. Now the underlying quark transition is $b \to cP$. The corrections to factorization can arise from gluon emmision between the $b$ or the $c$ quark and the quark constituents of $P$. However these corrections are the same for $\Lambda_b$ and $\bar{B}_d$ decays and so cancel in the ratio of their amplitudes. Gluon emmissions involving the spectator quark in $\bar{B}_d$ and the spectator diquark in $\Lambda_b$ may
also be similar given the fact that the diquark in $\Lambda_b$ belongs in the $\overline{3}$ under color $SU(3)_c$ as does the spectator anti-quark in $B_d^0$ and so both the spectators may have similar color interactions. Furthermore, within factorization, the small perturbative corrections to the form factors will also cancel.

Nonleptonic decays involving the newly discovered $D_s(2317)$ \cite{11} and $D_s(2460)$ \cite{12} states are of particular interest. It was shown in Ref. \cite{6} that nonleptonic $B_d^0$ decays involving these states can provide clues to the true nature of these states which is still not known \cite{13, 14, 15, 16}. It will be therefore interesting to see if these new $D_s$ resonances show up in the $\Lambda_b$ decays and how the rates for these decays compare to the $B_d^0$ decays involving the new $D_s$ states. As shown in Ref. \cite{6} nonleptonic $B_d^0$ decays to $D_s(2317)$ and $D_s(2460)$ involve the poorly known decay constants of these new states. However, in the ratio of $\Lambda_b$ decays to $D_s(2317)$ and $D_s(2460)$ relative to the $B_d^0$ decays to these states the decay constants of $D_s(2317)$ and $D_s(2460)$ cancel, leading to robust predictions that can shed additional light on the nature of these new states.

1.1 Masses and Form Factors

This assertion that the diquark in $\Lambda_b$ belongs in the $\overline{3}$ under color $SU(3)_c$ as does the spectator anti-quark in $B_d^0$ leads to similar color interactions has been dramatically confirmed by relations between hadron masses based on a simple QCD-based argument which goes beyond simple models for the spectator diquarks and the spectator anti-quark \cite{11, 12}.

The hadrons under consideration all consist of a quark, denoted by $q_i$, of any flavor $i$ and “light quark brown muck” having either the quantum numbers of a $\overline{3}$ color diquark denoted by $ud$ or a $\overline{3}$ color anti-quark denoted by $\bar{u}$. While we use the notations $ud$ and $\bar{u}$ for these light quark states, they can apply to any more complicated light quark configuration containing the same quantum numbers.

Consider the following four states of a quark of flavor $i$ bound to a $ud$ or $\bar{u}$ configuration. These are the pseudoscalar and vector mesons

$$|P_i\rangle = |q_i \bar{u}\rangle_{S=0}; \quad |V_i\rangle = |q_i \bar{u}\rangle_{S=1}$$ (1)

and the isoscalar and isovector baryons with spins $1/2$ and $3/2$, respectively

$$|B^0_i\rangle = |q_i (ud)_{I=0}\rangle_{S=1/2}; \quad |B^1_i\rangle = |q_i (ud)_{I=1}\rangle_{S=3/2}$$ (2)

Interesting mass relations between these hadrons were obtained\cite{14} from the following QCD-motivated assumptions:

1. The effective mass of any constituent in a hadron depends on the hadron wave function only via the color–electric field seen by the constituent. The color–electric fields are very simply related in these hadrons.

2. The color–electric field seen by the light quark systems $ud$ and $\bar{u}$ are independent of the flavor of the quark $q_i$.

3. The color–electric field seen by the quark $q_i$ is independent of whether the color $\overline{3}$ light quark system is a $ud$ diquark or a $\bar{u}$ anti-quark.
4. The color–magnetic interaction between the quark $q_i$ and the spin-zero diquark vanishes in the baryon state $|B^o_i\rangle$.

5. The color–magnetic contribution to the meson mass cancels out in the linear combination of masses\cite{11}

$$\tilde{M}_i = \frac{3M(V_i) + M(P_i)}{4} \quad (3)$$

6. The hyperfine splitting between the meson states $|P_i\rangle$ and $|V_i\rangle$ is inversely proportional to the effective mass $m_i^{eff}$ of the quark of flavor $i$ and similarly for the hyperfine splitting between the baryon states $|B^o_i\rangle$ and $|B^1_i\rangle$. However this cancels out in the combination of Eq. 3.

These immediately give for any two quark flavors $i$ and $j$,

$$\tilde{M}_i - \tilde{M}_j = M(B^o_i) - M(B^o_j) \equiv m_i^{eff} - m_j^{eff} \quad (4)$$

and

$$\frac{M(V_i) - M(P_i)}{M(V_j) - M(P_j)} = \frac{M(B^1_i) - M(B^o_i)}{M(B^1_j) - M(B^o_j)} \equiv \frac{m_j^{eff}}{m_i^{eff}} \quad (5)$$

Eq. 4 and Eq. 5 give all the mass relations between mesons and baryons previously obtained\cite{11, 13, 14, 15, 16} from the Sakharov-Zeldovich model\cite{17} improved by DeRujula, Georgi and Glashow\cite{18}. In particular we note that the change in baryon masses when the $b$ quark in a $\Lambda_b$ is changed into a $c$ quark to make a $\Lambda_c$,

$$\langle m_b^{eff} - m_c^{eff} \rangle_{\text{bar}} = M(\Lambda_b) - M(\Lambda_c) = 3339 \text{ MeV} \quad (6)$$

is exactly equal to the change in meson masses when the $b$ quark in a $B$ meson is changed into a $c$ quark to make a $D$ when the appropriate average of pseudoscalar and vector mesons is taken to cancel out the hyperfine interaction.

$$\langle m_b^{eff} - m_c^{eff} \rangle_{\text{mes}} = \frac{3(M_B^* - M_D^*) + M_B - M_D}{4} = 3342 \text{ MeV}. \quad (7)$$

The fact that the change in the hadron mass produced by the quark transition $b \rightarrow c$ is the same when the quark is bound to a $ud$ diquark and to a $\bar{u}\bar{d}$ anti-quark suggests that the diquark and anti-quark are spectators in the transition and will also effect the transition $b \rightarrow c$ in the same way when it is produced by the emission of a $W$ in a weak decay.

We now note that rearranging Eq. 6 gives the dimensional constant $\bar{\Lambda}$ defined in\cite{19} to represent the effective mass of the light degrees of freedom in the $\Lambda_b$ and $\Lambda_c$ baryon

$$\bar{\Lambda} = m_{\Lambda_b} - m_b = m_{\Lambda_c} - m_c \quad (8)$$

The value $\bar{\Lambda} = 575 \text{ MeV}$ was estimated in Ref.\cite{11} using quark masses that fit both meson and baryon masses.
1.2 Nonfactorization

Although we will use a factorization assumption there is a question of the correctness of such an assumption and what corrections would enter from nonfactorization. Nonfactorizable effects are known to be important for hyperon and charmed-baryon nonleptonic decays [20, 21, 22]. An unambiguous signal for the presence of nonfactorizable effects in \( \Lambda_b \) decays would be the observation of the decay \( \Lambda_b \to \Sigma_c P \) or \( \Lambda_b \to \Sigma_c V \). This is because, for the factorizable contribution, the light diquark in the \( \Lambda_b \) baryon remains inert during the weak decay. Thus, since the light diquark is an isosinglet, and since strong interactions conserve isospin to a very good approximation, the above \( \Lambda_b \) decays are forbidden within the factorization assumption [1].

One way to estimate the size of nonfactorizable corrections is to use the pole model. In this model, one assumes that the nonfactorizable decay amplitude receives contributions primarily from one-particle intermediate states, and that these contributions then show up as simple poles in the decay amplitude. Estimates of such pole diagrams in \( \Lambda_b \) decays have been found to be small and so are neglected in our analysis [3]. Note that these pole diagrams arise only through weak interactions involving the spectator quark and so small estimates of the pole diagram confirms the assumption of small spectator interaction in \( \Lambda_b(\bar{B}_0^0) \) decays [23].

In the next section we discuss \( \Lambda_b \to \Lambda_c P \) decays while in the following section we discuss \( \Lambda_b \to \Lambda_c V \) decays. Finally we present our summary.

2 Color allowed \( \Lambda_b \) decays

2.1 \( \Lambda_b \to \Lambda_c P \)

We begin our analysis by studying the nonleptonic decay \( \Lambda_b \to \Lambda_c P \). The general form for this amplitude can be written as

\[
\mathcal{M}_P = A(\Lambda_b \to \Lambda_c P) = i\bar{u}_{\Lambda_c}(a + b\gamma_5)u_{\Lambda_b}.
\]

(9)

In the rest frame of the parent baryon, the decay amplitude reduces to

\[
A(\Lambda_b \to \Lambda_c P) = i\chi^\dagger_{\Lambda_c}(S + P\hat{p})\chi_{\Lambda_b},
\]

(10)

where \( \hat{p} \) is the unit vector along the direction of the daughter baryon momentum, and the \( S \) and \( P \) wave amplitudes are given by \( S = \sqrt{2m_{\Lambda_b}(E_{\Lambda_c} + m_{\Lambda_c})a} \) and \( P = -\sqrt{2m_{\Lambda_b}(E_{\Lambda_c} - m_{\Lambda_c})b} \), where \( E_{\Lambda_c} \) and \( m_{\Lambda_c} \) are, respectively, the energy and mass of the final-state baryon \( \Lambda_c \). The decay rate is then given by

\[
\Gamma = \frac{|\hat{p}|}{8\pi m_{\Lambda_b}} (|S|^2 + |P|^2),
\]

(11)

where \( |\hat{p}| \) is the magnitude of the momentum of the decay products in the rest frame of the \( \Lambda_b \).

We will use factorization in order to estimate various nonleptonic amplitudes. The starting point is the SM effective Hamiltonian for hadronic \( B \) decays [24]:

\[
H_{eff}^q = \frac{G_F}{\sqrt{2}} [V_{ub}V_{uq}^* (c_1 O_1^q + c_2 O_2^q) - \sum_{i=3}^{10} V_{tb}V_{iq}^* c_i^q O_i^q] + h.c.,
\]

(12)
where

\[ O_1^q = \bar{q}_\alpha \gamma_\mu L c \gamma^\mu \bar{c} L b_\alpha \quad , \quad O_2^q = \bar{q}_\alpha \gamma_\mu L c \bar{c} \gamma^\mu L b \quad , \]
\[ O_{3(5)}^q = \bar{q}_\gamma \mu L b \sum_{q'} q' \gamma^\mu L(R) q' \quad , \quad O_{4(6)}^q = \bar{q}_\alpha \gamma_\mu L b_\beta \sum_{q'} q'_5 \gamma^\mu L(R) q'_\alpha \quad , \]
\[ O_{7(9)}^q = \frac{3}{2} \bar{q}_\gamma \mu L b \sum_{q'} e_q q' \gamma^\mu R(L) q' \quad , \quad O_{8(10)}^q = \frac{3}{2} \bar{q}_\alpha \gamma_\mu L b_\beta \sum_{q'} e_q q'_5 \gamma^\mu R(L) q'_\alpha \quad . \]

In the above, \( q \) can be either a \( d \) or an \( s \) quark, depending on whether the decay is a \( \Delta S = 0 \) or a \( \Delta S = -1 \) process, \( q' = d, u, s \) or \( c \), with \( e_q \) the corresponding electric charge, and \( R(L) = 1 \pm \gamma_5 \). The values of the Wilson coefficients \( c_i \) can be found in Ref. [25]:

We now apply the effective Hamiltonian to specific exclusive \( \Lambda_b \) and \( B \) decays. We will focus on those processes for which factorization is expected to be a good approximation, namely color–allowed decays.

We begin with \( \Lambda_b \to \Lambda_c \pi^- \) and \( \bar{B}^0 \to D^+ \pi^- \) which is a \( b \to c \bar{d} \bar{u} \) transition. Factorization allows us to write

\[
\begin{align*}
A(\Lambda_b \to \Lambda_c \pi^-) & = i f_\pi q^\mu \langle \Lambda_c | \bar{c} \gamma_\mu (1 - \gamma_5) b | \Lambda_b \rangle X_\pi \quad , \\
A(\bar{B}^0 \to D^+ \pi^-) & = i f_\pi q^\mu \langle D^+ | \bar{c} \gamma_\mu (1 - \gamma_5) b | \bar{B}^0 \rangle X_\pi \quad , \\
X_\pi & = \frac{G_F}{\sqrt{2}} V_{cb} V_{ub}^* a_2 \quad .
\end{align*}
\]

The pseudoscalar decay constant \( f_\pi \) is defined as

\[ i f_\pi q^\mu = \langle \pi | \bar{d} \gamma^\mu (1 - \gamma_5) u | 0 \rangle \quad , \]

and \( a_2 = c_2 + c_1 / N_c \).

Now, the vector and axial-vector matrix elements between the \( \Lambda_b \) and \( \Lambda_c \) baryons can be written in the general form

\[
\begin{align*}
\langle \Lambda_c | \bar{c} \gamma^\mu b | \Lambda_b \rangle & = \bar{u}_{\Lambda_c} \left[ f_1 \gamma^\mu + i f_2 \sigma^{\mu\nu} q_\nu + \frac{f_3}{m_{\Lambda_b}} q^\mu \right] u_{\Lambda_b} \\
\langle \Lambda_c | \bar{c} \gamma^\mu \gamma_5 b | \Lambda_b \rangle & = \bar{u}_{\Lambda_c} \left[ g_1 \gamma^\mu + i g_2 \sigma^{\mu\nu} q_\nu + \frac{g_3}{m_{\Lambda_b}} q^\mu \right] \gamma_5 u_{\Lambda_b} \quad ,
\end{align*}
\]

where the \( f_i \) and \( g_i \) are Lorentz-invariant form factors. Heavy-quark symmetry imposes constraints on these form factors. In our approach we will only consider the \( b \) as heavy and consider corrections up to order \( 1/m_c \). In the \( m_b \to \infty \) limit (but with \( 1/m_c \) corrections), one obtains the relations [19]

\[
\begin{align*}
f_1 & = g_1 = \left[ 1 + \frac{\Lambda}{2m_{\Lambda_c}} (1 - \frac{\Lambda}{m_{\Lambda_c}} \frac{\omega}{(\omega + 1)}) \right] \xi_B(\omega) + \frac{\eta(\omega)}{2m_c} \\
f_2 & = g_2 = g_3 = - \frac{\Lambda}{2m_{\Lambda_c} (\omega + 1)} (1 - \frac{\Lambda}{m_{\Lambda_c}}) \xi_B(\omega) \quad .
\end{align*}
\]

5
where $\xi_B(\omega)$ is the Isgur-Wise function for $\Lambda_b \to \Lambda_c$ transition, $\bar{\Lambda}$ is defined in Eq. (8), $\eta(\omega)$ represents the correction from the kinetic energy of heavy quark in the baryon and

$$\omega = \frac{m_{\Lambda_b}^2 + m_{\Lambda_c}^2 - q^2}{2m_{\Lambda_b}m_{\Lambda_c}}.$$ 

We point out that it is not necessary to estimate the quantity $\eta(\omega)$ for our calculation as we only use the relation $f_1 = g_1$. Estimates of $\eta(\omega)$ are found to be negligible\cite{26} and so we will set $\eta(\omega) = 0$.

The dimensional constant $\bar{\Lambda}$ representing the effective mass of the light degrees of freedom in the $\Lambda_b$ and $\Lambda_c$ baryon is estimated from Ref. \cite{11} with $\bar{\Lambda} = 575$ MeV. Now from Eq. 17 we see that in the $m_c \to \infty$ limit only the form factors $f_1$ and $g_1$ are non zero and the form factors $f_2$, $g_2$, $f_3$ and $g_3$ are suppressed by $O(1/m_c)$. We will use this fact later on in our calculations. Using Eq. 14 and Eq. 17, the amplitudes $a$ and $b$ of Eq. 9 can be written as

$$a_\pi = f_\pi X_\pi \left[ (m_{\Lambda_b} - m_{\Lambda_c})f_1(q^2 = m_{\pi}^2) + f_3 \frac{m_{\pi}^2}{m_{\Lambda_b}} \right],$$

$$b_\pi = f_\pi X_\pi \left[ (m_{\Lambda_b} + m_{\Lambda_c})g_1(q^2 = m_{\pi}^2) - g_3 \frac{m_{\pi}^2}{m_{\Lambda_b}} \right]. \quad (18)$$

In Eq. 18 we can drop the suppressed contributions from the $f_3(g_3)$ form factors and using the HQET relation $f_1 = g_1$ the quantities $a_\pi$ and $b_\pi$ can be expressed in terms of only one form factor. The $S$ and $P$ wave amplitudes are then written as,

$$S = f_\pi X_\pi \left[ (m_{\Lambda_b}^2 - m_{\Lambda_c}^2) \right] \sqrt{1 - \frac{m_{\pi}^2}{(M_{\Lambda_b} + M_{\Lambda_c})^2} f_1(q^2 = m_{\pi}^2)}$$

$$P = f_\pi X_\pi \left[ (m_{\Lambda_b}^2 - m_{\Lambda_c}^2) \right] \sqrt{1 - \frac{m_{\pi}^2}{(M_{\Lambda_b} - M_{\Lambda_c})^2} f_1(q^2 = m_{\pi}^2)} \quad (19)$$

The vector and axial-vector matrix elements between the $\bar{B}^0$ and $D^+$ mesons can be written in terms of form factors\cite{27}

$$< D^+(p_D)|J_\mu|\bar{B}_d^0 >= \left[ (p_B + p_D)_\mu - \frac{m_B^2 - m_D^2}{q^2} q_\mu \right] F_1(q^2) + \frac{m_B^2 - m_D^2}{q^2} q_\mu F_0(q^2) \quad (20)$$

where $q = p_B - p_D$. From Eq. 13 one then obtains

$$A(\bar{B}_d^0 \to D^+ \pi^-) = f_\pi X_\pi (m_B^2 - m_D^2) F_0(q^2 = m_{\pi}^2) \quad (21)$$

Note that the form of this amplitude is similar to the one in Eq. 19 with the important difference that for the $\Lambda_b$ decays there are two partial waves allowed by angular momentum conservation.
We are interested here in the ratio
\[ R_\pi = \frac{BR[\Lambda_b \to \Lambda_c \pi^-]}{BR[B_d^0 \to D^+ \pi^-]} \]  
(22)

We can define similar ratios \( R_K, R_{D_s}, R_D \) and \( R_{D_s(2317)} \). In passing we note that it is useful to also consider ratios of nonleptonic to semileptonic decays
\[ SL_{\Lambda_b} = \frac{\Gamma[\Lambda_b \to \Lambda_c M]}{d\Gamma[\Lambda_b \to \Lambda_c l\nu]/d\omega} \]
\[ SL_{B_d^{\text{ip}}} = \frac{\Gamma[B_d^{\text{ip}} \to D M]}{d\Gamma[B_d^{\text{ip}} \to D l\nu]/d\omega} \]
\[ SL_{\Lambda_b B_d^{\text{ip}}} = \frac{\Gamma[\Lambda_b \to \Lambda_c l\nu]}{d\Gamma[B_d^{\text{ip}} \to D l\nu]/d\omega} \]  
(23)

where \( M \) is a \( P \) or a \( V \) meson. The semileptonic \( \Lambda_b \to \Lambda_c l\nu \) decay distribution \( 26 \) as well as the nonleptonic \( \Lambda_b \to \Lambda_c M \) transition in factorization can be expressed in terms of the \( \Lambda_b \to \Lambda_c \) form factors in Eq. 16. Now using Eq. 17 and the estimate of \( \bar{\Lambda} \) the quantity \( SL_{\Lambda_b} \) is independent of form factors and can therefore be used to check for the validity of factorization in \( \Lambda_b \to \Lambda_c M \) transitions. One can use the ratio \( SL_{B_d^{\text{ip}}} \) to check for factorization in \( B_0 d \) decays. However the structure of the \( 1/m_{c,b} \) corrections are not so simple here \( 28 \). Finally the ratio \( SL_{\Lambda_b B_d^{\text{ip}}} \) can be used to express the ratio of \( \Lambda_b \to \Lambda_c \) form factor and \( B_0 d \to D \) form factor as a function of \( \omega \).

For the decays \( \Lambda_b \to \Lambda_c(\pi^-, K^-) \) there are no penguin contributions. However, for the decays \( \Lambda_b \to \Lambda_c(D_s^-, D^-, D_s(2317)) \) there are penguin contributions and the penguin operators affect the \( \Lambda_b \) and \( B \) decays differently \( 2 \). For the decay \( \Lambda_b \to \Lambda_c D_s^- \) we obtain
\[ A(\Lambda_b \to \Lambda_c D_s^-) = i f_{D_s} q^\mu \langle \Lambda_b | \bar{c} \gamma_\mu (1 - \gamma_5) b | \Lambda_b \rangle X_K + i f_{D_s} q^\mu \langle \Lambda_c | \bar{c} \gamma_\mu (1 + \gamma_5) b | \Lambda_b \rangle Y_K , \]
(24)

where
\[ X_{D_s} = \frac{G_F}{\sqrt{2}} \left[ V_{cb} V_{cs} a_2 - \sum_{q=u,c,t} V_{qb} V_{qs}^* (a_q^0 + a_q^{10}) \right] , \]
\[ Y_{D_s} = - \frac{G_F}{\sqrt{2}} \left[ \sum_{q=u,c,t} V_{qb} V_{qs}^* (a_q^0 + a_q^8) \right] \chi_{D_s} , \]  
(25)

with
\[ \chi_{D_s} = \frac{2 m_{D_s}^2}{(m_s + m_c)(m_b - m_c)} \]  
(26)

and for even “i”, \( a_i = a_i + a_{i-1}/N_c \).

In the above equations we have used
\[ i f_{D_s} q^\mu = \langle D_s | \bar{s} \gamma_\mu (1 - \gamma_5) c | 0 \rangle \]  
(27)
where \( q^\mu \equiv p^\mu_{L_b} - p^\mu_{\Lambda_c} = p^\mu_{D_s} \) is the four-momentum transfer. One can then show that

\[
\langle D_s^- \mid \bar{s}(1 \pm \gamma_5)c \mid 0 \rangle = -\frac{f_{D_s} m_{D_s}^2}{m_s + m_c} \left[ \langle \Lambda_c \mid \bar{c}(1 \pm \gamma_5)b \mid \Lambda_b \rangle = \frac{q^\mu}{(m_b - m_c)} \langle \Lambda_c \mid \bar{c} \gamma_\mu (1 \mp \gamma_5)b \mid \Lambda_b \rangle \right].
\]  

This then leads to

\[
a_{D_s} = f_{D_s}(X_{D_s} + Y_{D_s}) \left[ (m_{b_L} - m_{\Lambda_c}) f_1 + f_3 \frac{m_{D_s}^2}{m_{\Lambda_b}} \right],
\]

\[
b_{D_s} = f_{D_s}(X_{D_s} - Y_{D_s}) \left[ (m_{b_L} + m_{\Lambda_c}) g_1 - g_3 \frac{m_{D_s}^2}{m_{\Lambda_b}} \right].
\]  

and

\[
S = f_{D_s}(X_{D_s} + Y_{D_s}) \left[ (m_{b_L}^2 - m_{\Lambda_c}^2) \right] \sqrt{1 - \frac{m_{D_s}^2}{(m_{\Lambda_b} + m_{\Lambda_c})^2}} f_1(q^2 = m_{D_s}^2),
\]

\[
P = f_{D_s}(X_{D_s} - Y_{D_s}) \left[ (m_{b_L}^2 - m_{\Lambda_c}^2) \right] \sqrt{1 - \frac{m_{D_s}^2}{(m_{\Lambda_b} - m_{\Lambda_c})^2}} f_1(q^2 = m_{D_s}^2).
\]  

The corresponding \( B \) decay, \( B^0_d \to D^+D_s^- \), is

\[
A(B^0_d \to D^+D_s^-) = f_{D_s}(X_{D_s} + Y_{D_s})(m_{B_d}^2 - m_{D_s}^2) F_0(q^2 = m_{D_s}^2)
\]  

Similar expressions can be written for the pair of decays \( \Lambda_b \to \Lambda_c D^- \) and \( B^0_d \to D^+D^- \) with obvious changes. Note that from Eq. 25 and Eq. 26 the quantity \( Y_{D_s} \) or \( \chi_{D_s} \) is formally suppressed by \( 1/m_b \) though with a large coefficient. Taking the effective quark masses \( m_b = 5.050 \text{ GeV} \), \( m_c = 1.710 \text{ GeV} \), and \( m_s = 0.602 \text{ GeV} \) we find \( \chi_{D_s} \sim 1 \), which shows the effect of the large coefficient. However, to simplify our discussion we will neglect \( Y_{D_s} \). Given the fact that the penguins are smaller than the tree amplitude, the error from the neglect of \( Y_{D_s} \) is of the same order as the sub-leading \( 1/m_b \) effects which we have neglected. We should point out that for CP violating studies the quantities \( X_{D_s} \) and \( Y_{D_s} \) play an important role. However here we are interested in decay rates only and not CP-violating observables.

Using the values of the particle masses as well as the lifetimes of the \( \Lambda_b \) and \( B^0_d \) we obtain

\[
R_\pi = 1.73 \frac{f_1^2(q^2 = m_{\pi}^2)}{F_0^2(q^2 = m_{\pi}^2)}
\]  

Now using Eq. 32 and experimental information on \( R_\pi \) allows us to extract the form factor ratio

\[
r(q^2 = m_\pi^2) = f_1^2(q^2 = m_\pi^2)/F_0(q^2 = m_\pi^2).
\]

There has been a preliminary measurement of \( \Lambda_b \to \Lambda_c \pi^- \) by CDF with the branching ratio \((6.0 \pm 1.0(\text{stat}) \pm 0.8(\text{syst}) \pm 2.1(\text{BR})) \times 10^{-3} \). Using the PDG value...
for $\overline{B}_d \to D^+\pi^-$ which is $(2.76 \pm 0.25) \times 10^{-3}$ and taking the central value of the measurements we obtain $R_\pi \approx 2.17$. This then leads to, using Eq. 32

$$\frac{f_1(q^2 = m_\pi^2)}{F_0(q^2 = m_\pi^2)} = 1.12.$$  \hspace{1cm} (33)

In the heavy $m_c$ and $m_b$ limit we can relate the form factors $f_1$ and $F_0$ to the Isgur-Wise functions for $\Lambda_b \to \Lambda_c$ and $B \to D$ transition, $\xi_B(\omega_B)$ and $\xi_M(\omega_M)$

$$f_1(m_\pi^2) \approx f_1(0) = \xi_B(\omega_B^{\max})$$

$$F_0(m_\pi^2) \approx F_0(0) = F_1(0) = \frac{m_B + m_D}{2\sqrt{m_B m_D}}\xi_M(\omega_M^{\max})$$  \hspace{1cm} (34)

This then leads to

$$\xi_B(\omega_B^{\max}) = 1.4\xi_M(\omega_M^{\max})$$  \hspace{1cm} (35)

In the heavy $m_c$ and $m_b$ limit $\omega_B = \omega_M$. However for actual masses $\omega_B^{\max} = 1.458$ and $\omega_M^{\max} = 1.588$ which indicates that $m_c \to \infty$ is not a very good limit. Keeping in mind that $\xi_{B,M}(\omega = 1) = 1$, Eq. 35 indicates that the baryon Isgur-Wise function falls off slower than the mesonic counterpart.

To make predictions for the ratio $R_P$ for the other decays we would need the ratio of form factors $r(q^2 = m_P^2) = f_2(q^2 = m_P^2)/F_0(q^2 = m_P^2)$. This requires a dynamical input which will be our only assumption for the calculation of the decays besides factorization.

We assume a general parameterization of the form factors for the region of $q^2$ that we are interested in

$$f_1(q^2) = f_1(0)\eta_B(\frac{q^2}{M_{B^*}^2})$$

$$F_0(q^2) = F_0(0)\eta_M(\frac{q^2}{M_{M^*}^2})$$

where $M_{B^*}$ and $M_{M^*}$ are some heavy masses that scale as $m_b$. In other words the difference $M_{B^*} - M_{M^*}$ vanishes as $m_b \to \infty$. Furthermore $\eta_{B,M}(0) = 1$ by definition. Assuming $q^2$ to be smaller than $M_{B^*}^2$ and $M_{M^*}^2$ we can write

$$\eta_B(\frac{q^2}{M_{B^*}^2}) = 1 + \alpha_B \frac{q^2}{M_{B^*}^2} + ..$$

$$\eta_M(\frac{q^2}{M_{B^*}^2}) = 1 + \alpha_M \frac{q^2}{M_{B^*}^2} + ..$$  \hspace{1cm} (36)

and so

$$\frac{\eta_B(\frac{q^2}{M_{B^*}^2})}{\eta_M(\frac{q^2}{M_{B^*}^2})} = 1 + \alpha_B \frac{q^2}{M_{B^*}^2} - \alpha_M \frac{q^2}{M_{B^*}^2} + ..$$  \hspace{1cm} (37)

Note that the often used pole model for form factors is just one example of the general parameterization in Eq. 36 where $M_{B^*}$ and $M_{M^*}$ can be identified with the excited baryon and meson states.
Table 1: Table for $R_P = \frac{BR[\Lambda_b \to \Lambda_c P^{-}]}{BR[B_d^{0} \to D^{+} P^{-}]}$ with experimental input

| $R_P$   | Theory | Experiment |
|---------|--------|------------|
| $R_\pi$ | 2.17   | 2.17 [30]  |
| $R_K$   | 2.14   | --         |
| $R_D$   | 1.74   | --         |
| $R_{Ds}$| 1.75   | --         |
| $R_{Ds(2317)}$ | 1.58 | --         |

Now the largest $q^2$ we will be interested in is $q^2 \sim 4 GeV^2$ and so taking $M_{M^*, B^*}$ around 5-6 GeV we expect the second term in Eq. 36 to be around 10-15 %. Furthermore, we expect $\alpha_B$ and $\alpha_M$ to be of the same sign as the form factors increase with $q^2$. This implies further cancellation in the second term in Eq. 37, and so to a good approximation

$$\frac{\eta_B(q^2)}{\eta_M(q^2)} = 1$$

(38)

So in the heavy $m_b$ limit we can write for the form factor ratio $r$

$$r(q^2) \approx r(q^2 = 0) \approx r(q^2 = m_\pi^2)$$

(39)

Hence the measurement of $r(q^2 = m_\pi^2)$ allows us to make predictions for other decays which are presented in Table.

### 2.2 $\Lambda_b \to \Lambda_c V$

We now turn to the decays $\Lambda_b \to \Lambda_c V$ where $V = \rho, K^* a_1, D^*, D_{s}^*, D_s(2460)$ The general decay amplitude can be written as [20]

$$M_V = \text{Amp}(\Lambda_b \to \Lambda_c V) = \bar{u}_{\Lambda_b} \varepsilon_\mu^* \left[ \frac{p_{\Lambda_b}^\mu + p_{\Lambda_c}^\mu (a + b \gamma_5)}{m_{\Lambda_b}} + \gamma^\mu (x + y \gamma_5) \right] u_{\Lambda_b},$$

(40)

where $\varepsilon_\mu^*$ is the polarization of the vector meson. In the rest frame of the $\Lambda_b$, we can write $p_V = (E_V, 0, 0, |\hat{p}|)$ and $p_{\Lambda_c} = (E_{\Lambda_c}, 0, 0, -|\hat{p}|)$ and Eq. 40 can be reduced to [20]

$$M_V = \chi_f^\dagger [S \bar{\sigma} + P_1 \hat{p} + iP_2 \hat{p} \times \bar{\sigma} + D(\bar{\sigma} \cdot \hat{p})] \cdot \bar{\chi}_i$$

(41)

where $\hat{p}$ is a unit vector in the direction of the vector meson momentum. The amplitudes for the three helicity states of the vector meson can be written as

$$\mathcal{M}(+1) = \frac{P_2 - S}{\sqrt{2}} \chi_f^\dagger \left[ \bar{\sigma} \cdot (\varepsilon_1 + i \varepsilon_2) \right] \chi_i$$

$$\mathcal{M}(-1) = \frac{P_2 + S}{\sqrt{2}} \chi_f^\dagger \left[ \bar{\sigma} \cdot (\varepsilon_1 - i \varepsilon_2) \right] \chi_i$$

$$\mathcal{M}(0) = \frac{E_V}{m_V} \chi_f^\dagger [(S + D) \bar{\sigma} \cdot \hat{p} + P_1] \chi_i$$

(42)
In terms of the quantities defined in Eq. 40 we then have

\[
S = -\sqrt{2m_{\Lambda_b}(E_{\Lambda_c} + m_{\Lambda_c})y}
\]

\[
P_1 = \sqrt{2m_{\Lambda_b}(E_{\Lambda_c} + m_{\Lambda_c})} \frac{p}{E_V} \left[ \frac{m_{\Lambda_b} + m_{\Lambda_c}x + 2a}{E_{\Lambda_c} + m_{\Lambda_c}} \right]
\]

\[
P_2 = -\sqrt{2m_{\Lambda_b}(E_{\Lambda_c} + m_{\Lambda_c})} \frac{px}{E_{\Lambda_c} + m_{\Lambda_c}} \left[ 2b - y \right]
\]

(43)

We note from Eq. 42 that for light \( V \), \( E_V \sim m_{\Lambda_b} \) and so as \( m_b \to \infty \) the amplitude with longitudinally polarized \( V \) dominates. Hence in this limit only two combinations of partial waves contribute. We also note that the longitudinal amplitude \( \mathcal{M}(0) \) is of the same form as Eq. 10 for \( \Lambda_b \to \Lambda_c P \). Hence in the \( m_b \to \infty \) and a light \( V \) limit we can write the decay rate for \( \Lambda_b \to \Lambda_c V \), following Eq. 11, as

\[
\Gamma_{V0} = \frac{|\vec{p}|}{8\pi m_{\Lambda_b}^2} \left[ (|S + D|^2 + |P_1|^2) \frac{E_V^2}{m_V^2} \right],
\]

(44)

The complete expression for the decay rate with finite \( m_b \) and \( V \) not necessarily light is given by

\[
\Gamma_V = \frac{|\vec{p}|}{8\pi m_{\Lambda_b}^2} \left[ (|S + D|^2 + |P|^2) \frac{E_V^2}{m_V^2} + (|S|^2 + |P_2|^2) \right],
\]

(45)

where \( |\vec{p}| \) is the magnitude of the momentum of the decay products in the rest frame of the \( \Lambda_b \).

We use factorization to calculate the coefficients \( a, b, x \) and \( y \) in Eq. 40 for various decays. Consider first the decay \( \Lambda_b \to \Lambda_c \rho \). We define the decay constant \( g_\rho \) as

\[
m_\rho g_\rho \varepsilon_\mu^* = \langle \rho | \bar{c} \gamma_\mu u | 0 \rangle.
\]

(46)

and so we obtain

\[
A(\Lambda_b \to \Lambda_c \rho) = m_\rho g_\rho \left\{ \varepsilon_\mu^* \langle \Lambda_c | \bar{c} \gamma_\mu (1 - \gamma_5) b | \Lambda_b \rangle X_\rho \right\}
\]

\[
X_\rho = \frac{G_F}{\sqrt{2}} V_{cb} V_{ud}^* a_2
\]

(47)

with \( a_2 = c_2 + c_1/N_c \), and \( a, b, x \) and \( y \) in Eq. 40 given by

\[
a_\rho = m_\rho g_\rho f_2 X_\rho,
\]

\[
b_\rho = -m_\rho g_\rho g_2 X_\rho,
\]

\[
x_\rho = m_\rho g_\rho [f_1 - \frac{m_{\Lambda_c} + m_{\Lambda_b}}{m_{\Lambda_b}} f_2] X_\rho,
\]

\[
y_\rho = -m_\rho g_\rho [g_1 + \frac{m_{\Lambda_b} - m_{\Lambda_c}}{m_{\Lambda_b}} g_2] X_\rho.
\]

(48)
For the general decay $\Lambda_b \to \Lambda_c V$ the quantities $a$, $b$, $x$ and $y$ have the same form as Eq. [48] and we can then write

$$S + D = 2m_V g_V m_{\Lambda_b} \left[ f_1 + f_2 \frac{m_V^2}{(m_{\Lambda_b} - m_{\Lambda_c})m_{\Lambda_b}} \right] \frac{K_1}{K_2}$$

$$P_1 = 2m_V g_V m_{\Lambda_b} \left[ f_1 - f_2 \frac{m_V^2}{(m_{\Lambda_b} + m_{\Lambda_c})m_{\Lambda_b}} \right] \frac{m_{\Lambda_b}^2 - m_{\Lambda_c}^2}{m_{\Lambda_b}^2 + m_{\Lambda_c}^2} \frac{K_4}{K_1}$$

$$P_2 = -m_V g_V m_{\Lambda_b} \left[ f_1 - f_2 \frac{m_{\Lambda_b} - m_{\Lambda_c}}{m_{\Lambda_b}} \right] \frac{K_4}{K_1}$$

$$S = m_V g_V m_{\Lambda_b} \left[ f_1 + f_2 \frac{m_{\Lambda_b} + m_{\Lambda_c}}{m_{\Lambda_b}} \right]$$

where

$$K_1 = \sqrt{1 - \frac{m_V^2}{(m_{\Lambda_b} + m_{\Lambda_c})^2}}$$

$$K_2 = 1 + \frac{m_V^2}{(m_{\Lambda_b} - m_{\Lambda_c})^2}$$

$$K_3 = 1 - \frac{m_V^2}{(m_{\Lambda_b} + m_{\Lambda_c})^2}$$

$$K_4 = \sqrt{1 - \frac{2m_V^2 (m_{\Lambda_b} + m_{\Lambda_c})}{(m_{\Lambda_b} - m_{\Lambda_c})^2}} \left[ 1 - \frac{m_V^2}{2(m_{\Lambda_b}^2 + m_{\Lambda_c}^2)} \right]$$

In the light $V$ and $m_b \to \infty$ case $K_{1,2,3,4} \to 1$ and the dependence on the form factor $f_2$ drops out. Also, only the first two combination of partial waves, $S + D$ and $P_1$ contribute. In the heavy $m_b$ limit and identifying the light $V = \rho$, as an example, we can write, using the relations in Eq. [47] and dropping terms suppressed by $m_\rho^2/E_\rho^2$,

$$|M|^2(\Lambda_b \to \Lambda_c \rho^-) = (G_F \sqrt{2})^2 |V_{cb} V_{cd}^*|^2 a_s^2 m_\rho^2 f_\rho^2 f_1(m_\rho^2)^2 m_{\Lambda_b}^2 \frac{E_\rho^2}{m_\rho^2} \left[ 1 + \frac{(m_{\Lambda_b}^2 - m_{\Lambda_c}^2)^2}{(m_{\Lambda_b}^2 + m_{\Lambda_c}^2)} \right]$$

(51)

The corresponding expression for $\bar{B}^0 \to D^+ \rho^-$ is within the factorization assumption[31],

$$|M|^2(\bar{B}^0 \to D^+ \rho^-) = (G_F \sqrt{2})^2 |V_{cb} V_{cd}^*|^2 a_s^2 m_\rho^2 f_\rho^2 F_1(m_\rho^2)^2 m_{\Lambda_b}^2 \frac{E_\rho^2}{m_\rho^2}$$

(52)

From Eq. [52] we see, that unlike the pseudoscalar case, the form factor $F_1(q^2)$ appears. However, $F_0(q^2 = 0) = F_1(q^2 = 0)$ and for the values of $q^2$ we are interested in we will make the assumption $F_1(q^2) \approx F_0(q^2)$. We therefore obtain for the ratio of form factors

$$r(q^2) = \frac{f_1^2(q^2)}{F_0(q^2)} \approx \frac{f_2^2(q^2)}{F_1(q^2)} \approx r(q^2 = 0)$$

(53)
Table 2: Table for $R_V = \frac{BR(\Lambda_b \rightarrow \Lambda_c V^-)}{BR(B^0_d \rightarrow D^+ V^-)}$ for $f_2 = 0$

| $R_V$   | Theory($\Gamma_V$) | Theory($\Gamma_{V0}$) |
|--------|---------------------|------------------------|
| $R_\rho$ | 1.75                | 1.68                   |
| $R_{K^*}$ | 1.82                | 1.72                   |
| $R_{a_1}$ | 2.08                | 1.89                   |
| $R_{D^*}$ | 3.21                | 2.58                   |
| $R_{D_s^*}$ | 3.47               | 2.74                   |
| $R_{D_{s(2460)}}$ | 4.76               | 3.50                   |

We can now use the experimental input for $r(q^2 = m^2_\pi)$ from Eq. 32 to make predictions for the various $\Lambda_b \rightarrow \Lambda_c V$ decays.

It is clear from Eq. 49 that as $m_V$ gets larger the effect of the form factor $f_2$ becomes important and we have to introduce additional model dependency by requiring the value of $f_2$. However $f_2$ is suppressed by $1/m_c$ and so we will present our predictions in two cases. In the first case we shall take the $m_c \rightarrow \infty$ limit and so $f_2 = 0$. However, we will use the measured values of the various particle masses thereby including finite $m_c$ effects. Hence, the only assumption that we make here is that $m_c \rightarrow \infty$ is applicable only as far as the form factor $f_2$ is concerned. For the second case we estimate $f_2/f_1$ using Eq. 17 with $m_c = 1.710$, and $\bar{\Lambda} = 0.575$ GeV \[11\] and $\xi_B(\omega) \approx 1$.

We present our results in Table. 2 with $f_2 = 0$ while in Table. 3 we present results with $f_2 \neq 0$. The second column in Table. 2 and Table. 3 uses the full decay rate in Eq. 45 while column three uses the decay rate with only the longitudinal polarization as given in Eq. 44. From Table. 2 we make the following observations. When the vector meson $V$ is light then there is little difference between the entries in column two and column three indicating the dominance of the longitudinally polarized contribution. With higher $m_V$ the contributions from the transverse polarization components become important. The second observations is that, for light $V$, $R_V \leq 2$ as only two partial waves corresponding to the longitudinal vector polarization contribute. However, with increasing $m_V$ the various quantities $K_{1,2,3,4}$ become important and in particular the partial wave $P_1$ increases. The net effect is that, even with only the longitudinal vector polarization, the $\Lambda_b$ decay rate is more than the corresponding $B$ rate by more than a factor of two for charm final states. Finally we see that the branching ratio for $\Lambda_b \rightarrow \Lambda_c D_{s(2460)}$ is between four to 5 times times that of the corresponding $B$ mode. This is simply from the fact that more partial waves contribute in the $\Lambda_b$ decays and the fact that the $\Lambda_b \rightarrow \Lambda_c$ form factor is larger than the corresponding $B^0_d \rightarrow D^+$ form factor as suggested by experiment \[30\]. From Table. 3 we see that the effects of non zero $f_2$ from finite $m_c$ effects are rather small.
3 Summary

In this paper we have considered nonleptonic Cabibbo–allowed $\Lambda_b$ decays in the factorization approximation. We have discussed possible nonfactorizable effects and how experiments can be used to test look for them. We calculated decays of the type $\Lambda_b \to \Lambda_c P$ and $\Lambda_b \to \Lambda_c V$ relative to $B^0_d \to D^+ P$ and $B^0_d \to D^+ V$ where we included among the pseudoscalar states($P$) and the vector states($V$) the newly discovered $D_s$ resonances, $D_s(2317)$ and $D_s(2460)$. Using a preliminary measurement of the branching ratio for $\Lambda_b \to \Lambda_c \pi^-$ and a mild assumptions about the $q^2$ behavior of form factors we made predictions for several $\Lambda_b$ decays relative to the corresponding $B^0_d$ decays. In general we found the $\Lambda_b$ decays to be larger than the corresponding $B^0_d$ decays and in particular we found $\Lambda_b \to \Lambda_c D_s(2460)$ can be between four to five times $B^0_d \to D^+ D_s(2460)$. This enhancement of $\Lambda_b$ can be understood from the fact that more partial waves contribute in $\Lambda_b$ decays than in $B^0_d$ decays and the fact that the $\Lambda_b \to \Lambda_c$ form factor is larger than the corresponding $B^0_d \to D^+$ form factor.

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