Genetic Algorithm Based LQR Control for AGV Path Tracking Problem

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Abstract. In order to study path tracking of AGV, the lateral dynamic model of AGV is established, and the state equation of the system is obtained. In order to make the state equation better adapt to the LQR controller, a lateral error integral is added to the controller. Then an improved state equation is obtained. The energy function is constructed and the LQR controller is designed. In order to find the optimal value of the weight Q and R in the LQR controller, Genetic Algorithm (GA) is introduced so as to realize the optimization of the LQR controller. The simulation results show that compared with the empirical method to determine the Q and R of LQR control, the LQR control method optimized by GA can effectively reduce the overshoot of the system, improve the convergence speed and stability of the system, and obtain a better path tracking effect.

Keywords: LQR controller, Genetic Algorithm, path tracking, AGV motion control.

1. Introduction

The object of this research is AGV (Automated Guided Vehicle), which is a wheeled mobile robot. Because of its flexible path layout in the storage environment, high operating efficiency, easy integration with other systems, and convenient management, it has become the preferred solution for automated storage [1, 2]. Motion control, environment perception, decision-making and planning are also known as the three major technologies of vehicle intelligence [3], so path tracking control has always been a research focus of AGV-related technologies.

Currently, the most widely used algorithms for path tracking include PID control, LQR control and model predictive control (MPC) [4]. Among them, LQR control is widely used because it can obtain higher control performance at a lower cost. Therefore, this paper selects the LQR controller to carry out the path tracking control of the AGV. At present, the basic state space equation established based on the dynamic model cannot adapt to the LQR control because of the extra road curvature term, and the weight values Q and R in the traditional LQR control are determined based on engineering experience, and there is no specific scientific theoretical basis. Therefore, better control performance cannot be obtained. In this regard, the error integral term is first introduced to eliminate the influence of the road curvature term. Secondly, genetic algorithm is introduced to optimize the Q and R parameters with lateral error, yaw angle error and front wheel angle as the optimization objectives. The genetic algorithm has the characteristics of fast global optimization. It can find the best Q and R values according to the different
requirements of the system. The optimized Q and R parameters make the LQR control obtain better control performance.

2. AGV Lateral Dynamics Modeling

2.1. Basic State Space Equation

The tire characteristics are not considered in the kinematics model of AGV, and the model is too simplified, especially when the AGV runs at high speed, it will produce large errors, so the dynamic model of AGV will be established in this article. Because the left and right wheels of the AGV are symmetrical, the dynamic model can be simplified to a two-degree-of-freedom "bicycle model" that only considers the lateral displacement and yaw angle of the AGV [5].

![Fig.1 Lateral dynamics modeling - "bicycle model"](image)

As shown in Figure 1, the AGV modeling parameters are as follows:

- **X, Y**: Inertial coordinate frame.
- **x, y**: AGV Local coordinate frame.
- **l_f**, **l_r**: Longitudinal distance between front/rear wheel and center of gravity (c.g.).
- **α_f**, **α_r**: Front/Rear wheel slip angle.
- **V_f**, **V_r**: Front/Rear wheel velocity.
- **F_f**, **F_r**: Lateral tire force (on front and rear tires).
- **v_c**: Velocity at the c.g. of a vehicle.
- **β**: Vehicle slip angle at the c.g. of a vehicle, i.e., the angle between velocity at the c.g. and the axis of the car.
- **φ**: Yaw, i.e., the angle between the axis of the car and x axis.
- **θ**: Heading angle, the angle between velocity at the c.g. and x axis, \( θ = φ + β \), because \( β \) is so small that it can be ignored, \( θ ≈ φ \).
- **δ**: Steering angle.

Assuming that the longitudinal velocity of the AGV is constant, the relationship between the lateral displacement \( y \), the yaw angle \( φ \) and the steering angle \( δ \) can be established to obtain the system's lateral dynamic equation, and the front wheel angle \( δ \) is the input of the system, the lateral dynamic equation is shown as:
The meaning of each parameter in (1) is as follows:

$C_{af}, C_{ar}$: Front/Rear tire cornering stiffness.

$m$: Mass of AGV.

$I_{z}$: Moment of inertia of AGV.

$V_x$: Longitudinal speed of AGV.

$y, \dot{y}$: Lateral displacement and lateral velocity of AGV.

$\phi, \dot{\phi}$: Yaw and Yaw rate of AGV.

Then the Frenet coordinate system [6] is introduced to eliminate $\dot{y}$ and $\dot{\phi}$ to obtain the relationship between the lateral error $e_y$, the yaw error $e_\phi$ (the error is defined as shown in Fig. 2) and the steering angle $\delta$, then the basic state space equation is obtained as shown in (2):

$$
\dot{\mathbf{y}} = 
\begin{bmatrix}
0 & 1 & 0 & L \\
0 & a_{22} & a_{23} & a_{24} \\
0 & 0 & 0 & 1 \\
0 & a_{42} & a_{43} & a_{44}
\end{bmatrix}
\begin{bmatrix}
e_y \\
\dot{e}_y \\
\phi \\
\dot{\phi}
\end{bmatrix}
+ 
\begin{bmatrix}
0 \\
0 \\
0 \\
0
\end{bmatrix}
\begin{bmatrix}
\frac{2C_{af}f + 2C_{ar}r}{mV_x} \\
-\frac{2C_{af}f - 2C_{ar}r}{mV_x} \\
-V_x - \frac{2C_{af}f - 2C_{ar}r}{mV_x} \\
-\frac{2l_f^2C_{af} + 2l_r^2C_{ar}}{l_zV_x}
\end{bmatrix}
\begin{bmatrix}
\delta
\end{bmatrix}
$$

The value of the matrix element is:

$$
a_{24} = -\frac{2C_{af}f - 2C_{ar}r}{mV_x},
a_{42} = -\frac{2C_{af}f - 2C_{ar}r}{l_zV_x},
$$

$$
a_{43} = -a_{42}V_x, a_{44} = -\frac{2l_f^2C_{af} + 2l_r^2C_{ar}}{l_zV_x},
$$

$$
b_{21} = \frac{2C_{af}}{mV_x}, b_{41} = \frac{2C_{af}f}{l_z}
$$

Fig. 2 Definition of error
In (2) L is the distance to the forward point, because the corner calculated at the current moment cannot meet the road conditions at the next moment, resulting in a driving error. Therefore, this error can be eliminated by adding forward point prediction.

$e$ is the error matrix, $u$ is the system input, i.e., the steering angle, $\phi_d$ is the desired yaw rate from the road, as shown in (4):

$$
e = \begin{bmatrix} e_y \\ \dot{e}_y \\ e_{\phi} \\ \dot{e}_{\phi} \\ \phi - \phi_d \\
\end{bmatrix} = \begin{bmatrix} e_x \\ v_y + v_x e_{\phi} \\ \phi - \phi_d \\
\end{bmatrix}$$

The meaning of each parameter in (4) is as follows:

- $e_y, \dot{e}_y$: Lateral error and lateral error rate at the forward point.
- $v_y$: Lateral speed of AGV.
- $\phi_d$: Desired yaw from the road.
- $\phi, \dot{\phi}$: Yaw error and yaw error rate.
- $C_R$: Road curvature.

Therefore, the basic state space equation of the system can be abbreviated as:

$$\dot{e} = Ae + Bu + D\phi_d$$

(6)

$A$ and $B$ in (6) are the coefficient matrices of state space.

### 2.2. Improved State Space Equation

Because the system using the LQR controller needs to meet a requirement, i.e., the state space equation of the system needs to meet the following form:

$$\dot{x} = Ax + Bu$$

(7)

However, the basic state space equation obtained in (6) has a third term in comparison with (7), i.e., the desired yaw rate from the road $\phi_d$. Therefore, the error matrix is redesigned and the lateral error integral term is introduced:

$$
e = \begin{bmatrix} e_y \\ \dot{e}_y \\ e_{\phi} \\ \dot{e}_{\phi} \\ \phi - \phi_d \\
\end{bmatrix} = \begin{bmatrix} e_x \\ v_y + v_x e_{\phi} \\ \phi - \phi_d \\
\end{bmatrix}$$

(8)

The added lateral error integral term can eliminate the interference caused by the road curvature, so that the road interference $\omega$ can be ignored, so the improved state space equation can better adapt to the LQR control.
3. GA Based LQR Control

3.1. LQR Controller Design

LQR is a linear quadratic regulator. LQR can obtain the optimal control law of state linear feedback, which is easy to form a closed-loop optimal control. Because it can use lower cost to obtain better control performance [8], this paper selects LQR control to study the path tracking of AGV, as shown in Fig 3.

![Fig.3 LQR control block diagram](image)

Carsim is a simulation software specifically for vehicle dynamics. It is mainly used to predict and simulate the handling stability, braking, ride comfort, power and economy of the entire vehicle. At the same time, it is widely used in the development of modern vehicle control systems. Therefore, this experiment is based on the joint simulation of Carsim and Simulink. In order to calculate the coefficient matrix $A$ and $B$ in the LQR control, firstly, it is necessary to calculate the wheel cornering stiffness, i.e., the ratio of the cornering force of the wheel to the cornering angle.

When LQR control is set in Q and R parameters, if Q is larger, the performance of the control algorithm is better, but stability is sacrificed. If R is larger, the control process is smoother, and the front wheel angle will not change drastically, ensuring the system Security, but it is easy to cause poor tracking effect. The weight matrix Q and the weight parameter R determined by experience are only approximately optimal, so a genetic algorithm with better global search ability is introduced to solve the optimization problem.

In Carsim, the tire related parameters of the vehicle model include the ratio of the cornering force received by the tire to its cornering angle, as shown in Fig 4.

![Fig.4 The relationship curve between the cornering force and the cornering angle](image)
The cornering stiffness is calculated by interpolation: 
\[ C_{\alpha f} = -26563 N/\text{rad}, \quad C_{\alpha r} = -24413 N/\text{rad} \]
Then the coefficient matrix \( A \) and \( B \) are calculated by (3) according to other AGV vehicle parameters.

In the LQR controller, the energy function is defined as [9]:
\[ J = \int_0^T (x^T Q x + u^2 R) dt \tag{10} \]

Q and R in (10) are the weight matrix and weight parameter, respectively, where Q is a 5*5 diagonal matrix, and each value of the diagonal matrix reflects the importance of the corresponding variable in the state matrix. R reflects the importance of the input u. In engineering practice, the values of Q and R parameters are often determined based on experience as follow[3]:
\[
Q = \begin{bmatrix}
10 & 0 & 0 & 0 & 0 \\
0 & 10 & 0 & 0 & 0 \\
0 & 0 & 10 & 0 & 0 \\
0 & 0 & 0 & 10 & 0 \\
0 & 0 & 0 & 0 & 10
\end{bmatrix}, \quad R = 1
\]

The value of gain K calculated by matlab is:
\[ K = [3.162 \quad 5.657 \quad 1.940 \quad 23.085 \quad 0.048] \]

3.2. Genetic Algorithm Optimizes Q and R Parameters
When LQR control is set in Q and R parameters, if Q is larger, the performance of the control algorithm is better, but stability is sacrificed. If R is larger, the control process is smoother, and the front wheel angle will not change drastically, ensuring the system Security, but it is easy to cause poor tracking effect. The weight matrix Q and the weight parameter R determined by experience are only approximately optimal, so a genetic algorithm with better global search ability is introduced to solve the optimization problem.

Set Q and R as:
\[
Q = \begin{bmatrix}
a & 0 & 0 & 0 & 0 \\
0 & b & 0 & 0 & 0 \\
0 & 0 & c & 0 & 0 \\
0 & 0 & 0 & d & 0 \\
0 & 0 & 0 & 0 & e
\end{bmatrix}, \quad R = 1
\]

The specific optimization steps are as follows:
Step 1: Randomly generate an initial population within the set constraint range;
Step 2: Assign the individuals in the initial population to the state weight matrix \( Q \), use the LQR controller to find the optimal control quantity \( K \), enter the vehicle model, and get the fitness value;
Step 3: According to the fitness value, keep some individuals with small fitness values, and form a new population with the excellent individuals obtained through cross mutation;
Step 4: Repeat steps 2 to 4 until the number of iterations is met.

Because only the lateral error, the yaw error and the steering angle are concerned in the whole tracking process, the objective function of the genetic function is defined as:
\[ J_{GA} = \sum_{t=1}^{N} e_{y_t}^2 + e_{\phi_t}^2 + u_t^2 \tag{11} \]

Set the value range of the two parameters of the matrix to [0,20], set the number of initial population individuals to 50, and the number of iterations to 50. The optimal individual fitness obtained in the optimization process is shown in Fig. 5.
The optimal individual value $Q$ found by genetic algorithm and its corresponding matrix $K$ are:

$$Q = \begin{bmatrix}
9.02 & 0 & 0 & 0 & 0 \\
0 & 8.71 & 0 & 0 & 0 \\
0 & 0 & 7.30 & 0 & 0 \\
0 & 0 & 0 & 8.16 & 0 \\
0 & 0 & 0 & 0 & 6.43
\end{bmatrix}$$

$$K = [3.003 \quad 5.217 \quad 1.537 \quad 23.156 \quad 0.031]$$

The quadratic function $y = \frac{x^2}{300}$ is used as the desired path, and the empirically determined $Q$ matrix and the $Q$ matrix optimized by the genetic algorithm are used to control the AGV. The simulation time is set to 60s. The comparison results before and after GA optimization are shown in Fig. 6, Fig. 7, and Fig. 8.
Fig. 8 Steering angle comparison

The simulation shows that the maximum lateral error after optimization based on GA is 15.7mm, and it is 45.0mm before optimization, the maximum error is reduced by 65%. The maximum yaw error is 0.019°, and it is 0.018° before optimization. The change is extremely small, so it can be ignored. It can be seen from Fig. 6 that the optimized system can converge faster, and from Fig. 8 it can be seen that the steering angle has smaller fluctuations than before optimization, which means the control system has better stability.

4. Summary
Through the above research, it can be concluded that after introducing the error integral term in the error state equation, the LQR control based on genetic algorithm optimization has a better control effect on the path tracking of the AGV. Compared with the empirical method, the maximum lateral error of the AGV system can be reduced by 65%, which means the control accuracy of the system has been improved. At the same time, the convergence speed and stability of the system have also been improved. Therefore, compared with the traditional LQR control method, this method has better control performance in the AGV path tracking. After the text edit has been completed, the paper is ready for the template. Duplicate the template file by using the Save As command, and use the naming convention prescribed by your conference for the name of your paper. In this newly created file, highlight all of the contents and import your prepared text file. You are now ready to style your paper; use the scroll down window on the left of the MS Word Formatting toolbar.

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