Investigating the agricultural losses due to climate variability: An application of conditional
value-at-risk approach

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ABSTRACT

The agricultural sector is directly affected by climate variables. The presence of climate variability causes a considerable risk to agricultural productivities. Thus, risk management is an alternative to reduce risks, including optimizing the allocation of farmland and choosing crop insurance for a specific planting date. The purpose of this study is to investigate the agricultural risk management through risk measure of climate variability using the Conditional Value-at-Risk (CVaR) in rice production. This paper investigated several possible considerations of agricultural insurance premiums based on losses climate index. We concluded that the climate index insurance policy is the best choice that farmers can choose for each planting date, the higher the significance value considered, the more the value of Value-at-Risk and Conditional Value-at-Risk.

1. Introduction

Agriculture is a sector that has high risk, such as production risk which is influenced by climate variables (Azad et al., 2013; Ochieng et al., 2016; Pasaribu & Sudiyanto, 2016). Climate variability directly influences the uncertainty of agricultural production, which correlates with losses of agricultural production (Yoshioka, 2017; Caruso et al., 2014). Thus, a solution is needed to reduce risk. One effort that can be done is by optimizing risk management. Risk management that can be carried out consists of allocating agricultural land to a type of crop and selecting agricultural insurance products on the planting date. Agricultural insurance by sharing risk is an essential step in reducing the risk of production losses (Yoshioka, 2017; Pasaribu & Sudiyanto, 2016). Indonesia is one of the agrarian countries that has implemented agricultural insurance to protect losses caused by disasters, such as floods, droughts, pests, and diseases (Pasaribu and Sudiyanto, 2016; Pasaribu, 2010). In addition to loss insurance, Kawanishi and Mimura (2015) have researched to investigate the feasibility of climate index insurance in Indonesia, by comparing the correlation coefficients to agricultural productivity. Cabrera et al. (2006) conducted a study to reduce agricultural risk by choosing agricultural insurance products to increase the stability of agricultural income. Liu et al. (2007) helped farmers make decisions in purchasing agricultural insurance products to reduce the risk of climate variables and product selling prices by applying Conditional Value-at-Risk (CVaR) to control the level of risk aversion. Liu et al. (2007) model optimization problems to minimize farmer loss expectations. Cabrera et al. (2009) illustrate the potential for synergies and conflicts of interest between farmers and insurance companies in the selection of optimal agricultural insurance. The optimization model for farmers is modelled by maximizing profits by considering a constant coefficient for risk aversion, while the optimization model for farmers aims to minimize the expectation of loss by assessing the risk of Conditional Value-at-Risk (CVaR). Wang and Huang (2016) developed an insurance contract to
maximize expected utility by considering Value-at-Risk and Conditional Value-at-Risk constraints. Conditional Value-at-Risk (CVaR) is a well-known risk measure used in various optimization problems. CVaR with a discrete sample approach (scenario-based or data-based CVaR) can be reformulated as linear programming under the linear function of loss and return (Krokhmal et al. 2002). Tong et al. (2010) consider semi-smooth, i.e. solving the nonsmooth problem with numerical methods for CVaR. This study refers to the model proposed by Liu et al. (2007), i.e minimizing farmers’ loss expectations by considering the climate variable Conditional Value-at-Risk (CVaR) risk measure.

2. Materials and Methods

2.1 Climate Variable Risk Measure

In quantitative risk management, risk measures are used to determine special orders between certain position positions and random results. Climate risk plays a vital role in agriculture. The risk is influenced by several things, including primary risk, price uncertainty and the effect of diversification (Berg & Schmitz, 2007). Value-at-Risk (VaR) is a measure of risk that has become popular in risk management. However, VaR has several disadvantages, including VaR, which is not subadditive in the case of general distribution; this causes VaR is not convex (Artzner et al., 1999). VaR can have several local extremes for discrete distribution, and VaR is only a percentile of the distribution of losses, so there is not much to say about the nature of the extreme losses that go beyond it (Zhu & Fukushima, 2009). To overcome this problem, Rockafellar and Uryasev (2002) introduce a further risk measure known as Conditional Value-at-Risk (CVaR). The CVaR risk measure is similar to the Value-at-Risk (VaR) risk measure, which is a percentile of the distribution of losses. Using the same level of confidence, VaR is the lower limit for CVaR. In optimization applications, CVaR is superior to VaR (Rockafellar & Uryasev, 2000). Conditional Value-at-Risk (CVaR) is defined as the average tail distribution that exceeds the same level of confidence, VaR is the lower limit for CVaR. In optimization applications, CVaR is superior to VaR (Rockafellar & Uryasev, 2000). Definition Conditional Value-at-Risk (CVaR) (Quaranta & Zaffaroni, 2008).

Let \( x \in X \subseteq \mathbb{R}^n \) be a decision vector, \( y \in Y \subseteq \mathbb{R}^m \) is the future value of several variables and \( \forall x \) is denoted by \( \psi (x, \cdot) \). The loss distribution function \( z = f(x, y) \),

\[
\psi (x, \alpha) = P \{ y \mid f(x, y) \leq \alpha \},
\]

If given \( \alpha > 0 \), then the \( \alpha-CVaR \) of the loss associated with \( x \) is the average of the \( \alpha \) – tail distribution of the loss function, this means that the average distribution function \( \psi_a (x, \cdot) \) is defined as follows:

\[
\psi_a (x, \alpha) = \begin{cases} 0 & \text{jika } a < a_a (x), \\ \psi (x, a) - \alpha & \text{jika } a \geq a_a (x), \end{cases}
\]

and \( a_a (x) \) is \( \alpha-CVaR \) of the loss associated with \( x \). Conditional Value at Risk is a coherent measure of risk in general. Furthermore, the above formulation makes it possible to minimize CVaR using linear programming methods. Rockafellar and Uryasev introduce simple assistive functions concerning variables \( \alpha \in \mathbb{R} \)

\[
F_{\beta} (x, \alpha) = \alpha + \frac{1}{1 - \beta} \int \left[ f(x, u) - \alpha \right]^{-} p(u) du \text{ where } \left[ t \right] = \max \{ t, 0 \}.
\]

So that \( CVaR_{\beta} (x) = \min_{\alpha} F_{\beta} (x, \alpha) \).

Using the assumption that the random vector distribution \( u \) is not known precisely, the density function is only known to belong to the distribution of a set \( \mathcal{P} \) of course.

**Theorem Fundamental Minimization Formulation** (Rockaffelar & Uryasev, 2000).

Let \( F_{\beta} (x, \alpha) \) be convex, continuous and differentiable. \( CVaR_{\beta} (x) \) of the loss distribution with \( x \in X \) is as follows,

\[
CVaR_{\beta} (x) = \min_{\alpha \in \mathbb{R}} F_{\beta} (x, \alpha),
\]

where \( \alpha \) reaches the minimum, that is \( A_{\beta} (x) = \arg \min_{\alpha \in \mathbb{R}} F_{\beta} (x, \alpha) \) is not empty, closed and limited. \( VaR_{\beta} (x) \) of the loss distribution is

\[
VaR_{\beta} (x) = \text{Left endpoint of } A_{\beta} (x).
\]

In particular, there is always

\[
VaR_{\beta} (x) \in \arg \min_{\alpha \in \mathbb{R}} F_{\beta} (x, \alpha) \quad \text{and} \quad CVaR_{\beta} (x) = F_{\beta} \left( x, VaR_{\beta} (x) \right).
\]
2.2 Conditional Value-at-Risk Optimization Based on Scenario

Let \( f(x, \xi) \) be a loss function, with \( x \in \mathcal{X} \subseteq \mathbb{R}^n \) decision vector and \( y \in \mathbb{R}^m \) as random vector. In general, four models under Conditional Value-at-Risk (CVaR) exist i.e., minimization, maximization with constraints, minimization with return constraint, and maximization composed of return and CVaR measure (Tong et al., 2010). Liu et al. (2007) completed an optimization model of agricultural management to minimize the expected loss function by considering CVaR constraints. In this paper, the optimization model aims to minimize the risk of CVaR, with the objective and constraint functions referring to Liu et al. (2007). The objective function of the optimization model is to minimize the size of the Conditional Value-at-Risk (CVaR) risk with the loss function referring to Liu et al. (2007) and Jain et al. (2019). The first constraint assumes the amount of land allocated for the date of planting the type of agriculture is equal to the total area of land available. The second obstacle assumes that farmers can only buy one type of insurance policy for each crop. Agricultural risk management solutions are obtained from the optimization model, such as land allocation for plants \( k \) on the date of planting \( kd \) and choose the optimal insurance policy for plants \( k \) on planting dates \( kd \).

Optimization model is written as follows:

\[
\begin{align*}
\text{min} & \quad CVaR_\beta(f(x,y)) \\
\text{s.t.} & \quad \sum_{d_k=1}^D x_{d_k} = q_k, \\
& \quad \sum_{i=1}^K \lambda_{ik} = 1, \quad k = 1, \ldots, K \\
& \quad x_{d_k} \geq 0, \quad d_k = 1, \ldots, D \\
& \quad \lambda_{ik} = \begin{cases} 1, & \text{If farmer choose the polis } i \text{ for plant } k \\ 0, & \text{Otherwise} \end{cases}
\end{align*}
\]

(5)

For the loss functions we have:

\[
\begin{align*}
f(x,y) &= \sum_{k=1}^K \left[ c_k q_k - \left( \sum_{d_{k,s}} x_{d_{k,s}} y_{d_{k,s}} \right) p_{ik} + \sum_{i=1}^I \lambda_{ik} \left( r_{ik} q_{ik} - \left( \sum_{d_{k,s}} x_{d_{k,s}} \left( y^*_{d_{k,s}} - y_{d_{k,s}} \right) \right) p^*_i \right) \right];
\end{align*}
\]

(6)

where \( c_k \) shows the cost of crop production \( k \) per ha, \( q_k \) states crop land allocation \( k \), \( r_{ik} \) denotes insurance policy premiums \( i \) for plants \( k \) per ha, \( p_{ik} \) represents agricultural market prices \( k \) per kg with scenario \( s \), \( p^*_i \) is price chosen for agricultural standards \( k \), \( y_{d_{k,s}} \) is crop yields \( k \) per ha for planting date \( d_{k,s} \), and \( y^*_{d_{k,s}} \) is crop yield \( k \) per hectare insured by policy \( i \). Conditional Value-at-Risk (CVaR) is challenging to resolve because of its convoluted and implicit version. Rockafellar and Uryasev (2000) introduce assistive functions that are continuous and convex as follows:

\[
F^*_\beta(x, \alpha) = \alpha + \frac{1}{S(1-\beta)} \sum_{s=1}^S \left[ f(x, y_s) - \alpha \right]^+, \quad (7)
\]

where

\[
CVaR_\beta(x,y) = \min_{\alpha} \quad F^*_\beta(x, \alpha) = \min_{\alpha} \left( \alpha + \frac{1}{S(1-\beta)} \sum_{s=1}^S \left[ f(x, y_s) - \alpha \right]^+ \right).
\]

Then the objective function becomes

\[
\min \quad CVaR_\beta(f(x,y)) = \min_x F^*_\beta(x, y_s) = \min_{x,\alpha} \left( \alpha + \frac{1}{S(1-\beta)} \sum_{s=1}^S \left[ f(x, y_s) - \alpha \right]^+ \right)
\]

So the optimization model (5) is equivalent to

\[
\begin{align*}
\text{min} & \quad CVaR_\beta(f(x,y)) = \min_{x,\alpha} \left( \alpha + \frac{1}{S(1-\beta)} \sum_{s=1}^S \left[ f(x, y_s) - \alpha \right]^+ \right) \\
\text{s.t.} & \quad \sum_{d_k=1}^D x_{d_k} = q_k, \\
& \quad \sum_{i=1}^K \lambda_{ik} = 1, \quad k = 1, \ldots, K \\
& \quad x_{d_k} \geq 0, \quad d_k = 1, \ldots, D \\
& \quad \lambda_{ik} = \begin{cases} 1, & \text{If farmer choose the polis } i \text{ for plant } k \\ 0, & \text{Otherwise} \end{cases}
\end{align*}
\]

(8)
The CVaR formula contains the maximum function \( f(x, y) - \alpha \), this causes computational difficulties (Tong et al., 2010). The maximum function can be completed by introducing an auxiliary variable \( z_s \) and adding additional constraints, so the optimization model (8) can be written as follows

\[
\min_{x} \quad CVaR_\beta(f(x, y)) = \min_{x, \alpha} \left( \alpha + \frac{1}{S(1 - \beta)} \sum_{s=1}^{S} z_s \right)
\]

s.t.
\[
\sum_{d_j=1}^{D} x_{d_j} = q_k, \\
\sum_{i=1}^{I} \lambda_{ik} = 1, \quad k = 1, \ldots, K \\
x_{d_k} \geq 0, \quad d_k = 1, \ldots, D \\
z_s \geq f(x, y_s) - \alpha, \quad s = 1, \ldots, S \\
\lambda_{ik} = \begin{cases} 1, & \text{If farmer choose the polis } i \text{ for plant } k \\ 0, & \text{Otherwise} \end{cases}
\]

3. Results and Discussion

This section discusses the application of the CVaR optimization model for risk management of lowland rice farming in Cirebon, Indonesia. The risk considered is the variability of climate variables on the productivity of lowland rice. Productivity is obtained by using the regression equation, as follows:

\[
Y = -0.451X_1 - 0.556X_2 - 0.589X_3 + 0.016X_4, \quad (10)
\]

where \( Y \) is rice productivity, \( X_1 \) represents velocity wind, \( X_2 \) denotes the maximum temperature, \( X_3 \) is for minimum temperature, and \( X_4 \) is considered for rainfall.

![RICE PRODUCTION](image)

Fig. 1. Rice production - estimated and actual value-based

The regression model (10) has a \( R^2 \) value of 0.991, indicating that the climate variable influences 99.1% of rice productivity. The coefficient of wind speed, maximum temperature, minimum temperature, and rainfall is negative, meaning that each of these variables has increased. The productivity of lowland rice tends to decrease, and vice versa. While the rainfall coefficient is positive, meaning that if rainfall has increased, the productivity of lowland rice tends to increase as well. Planting dates are divided into two, namely semester 1 (January-June) and semester 2 (July-August). There are two policies considered in this numerical experiment, namely insurance based on losses and climate index insurance. For example, the cost of producing lowland rice per ha is IDR \( c = 6,000,000 \), and the allocation of land for lowland rice is \( q = 1 \) ha. There are two insurance policies, namely loss insurance and climate index insurance, with premiums of each policy...
being 180,000 and 194,000. In this numerical experiment, 11 annual scenarios were used from 2008 to 2018. Average market prices, semester 1 rice productivity, and semester 2 rice productivity are presented in Table 1.

Table 1
Average market prices, paddy productivity semester 1 and semester 2 in 2008-2018

| Scenario | Average of Market Price | Paddy Productivity Semester 1 | Paddy Productivity Semester 2 |
|----------|-------------------------|-------------------------------|-------------------------------|
| 1        | 2792                    | 5.845                         | 5.870                         |
| 2        | 2997                    | 6.035                         | 6.074                         |
| 3        | 3532                    | 6.075                         | 6.088                         |
| 4        | 4030                    | 6.145                         | 6.191                         |
| 5        | 4457                    | 6.266                         | 6.332                         |
| 6        | 4594                    | 6.375                         | 6.375                         |
| 7        | 4748                    | 6.422                         | 6.461                         |
| 8        | 5280                    | 6.462                         | 6.510                         |
| 9        | 5458                    | 6.638                         | 6.693                         |
| 10       | 5500                    | 6.718                         | 6.723                         |
| 11       | 5501                    | 6.746                         | 6.796                         |

Based on these data, the optimization model (9) can be rewritten as follows:

$$\min_{x, \alpha} \alpha + \frac{1}{11(1-\beta)} \sum_{s=1}^{11} z_s$$

s.t.

$$x_1 + x_2 = 5,$$

$$\lambda_1 + \lambda_2 = 1,$$

$$x_1, x_2 \geq 0,$$

$$z_s \geq f(x, y_{x_s}) - \alpha, \quad s = 1, \ldots, 11$$

$$\lambda_1, \lambda_2 \in [0, 1]$$

(10)

Next, the optimization model (10) is solved by several different and premium simulations. When \( r_1 = r_2 = 180000 \), two optimal solutions are obtained based on several levels of significance. Land allocation for this situation is presented in Table 2, as follows.

Table 2
Allocation of Rice Farming Land with various parameter \( \alpha \) values for \( r_1 = r_2 \)

| \( \alpha \) | 0.80 | 0.82 | 0.84 | 0.86 | 0.88 | 0.90 | 0.92 | 0.94 | 0.96 | 0.98 | 0.99 |
|--------------|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| land allocation of semester 1 (\( x_1 \)) | 1   | 1   | 1   | 1   | 1   | 1   | 1   | 1   | 1   | 1   | 1   |
| land allocation of semester 2 (\( x_2 \)) | 0   | 0   | 0   | 0   | 0   | 0   | 0   | 0   | 0   | 0   | 0   |

With some of these \( \alpha \) values, the farmer can choose between an insurance policy and a climate index insurance policy. Fig. 2 shows the Value-at-Risk (VaR) and Conditional Value-at-Risk (CVaR) values for the premiums of the two same policies.

![Optimization test with various parameter (a) and (r)](image-url)

Fig. 2. VaR and CVaR Value with various parameter \( \alpha \) value for \( r_1 = r_2 \)
If \( r_1 < r_2 \) where \( r_1 = 180000 \) and \( r_2 = 194000 \) then the optimal solution is obtained as follows:

**Table 3**

| \( \alpha \) | 0.80 | 0.82 | 0.84 | 0.86 | 0.88 | 0.90 | 0.92 | 0.94 | 0.96 | 0.98 | 0.99 |
|-------------|------|------|------|------|------|------|------|------|------|------|------|
| land allocation of semester 1 (\( x_1 \)) | 1    | 1    | 1    | 1    | 1    | 1    | 1    | 1    | 1    | 1    | 1    |
| land allocation of semester 2 (\( x_2 \)) | 0    | 0    | 0    | 0    | 0    | 0    | 0    | 0    | 0    | 0    | 0    |

With this significance value and land allocation in Table 3, it is generally obtained the optimal decision to choose a loss insurance policy on both planting dates. VaR and CVaR values are presented in Fig. 3.

![Fig. 3. VaR and CVaR Value with various parameter \( \alpha \) values for \( r_1 < r_2 \)](image)

If \( r_1 > r_2 \) where \( r_1 = 180000 \) and \( r_2 = 160000 \) then obtained land allocation solutions is as follows:

Considering several choices of significant value \( \alpha \) and land allocation, as presented in Table 3, the farmer must have a climate index insurance policy for each land allocation on both planting dates. VaR and CVaR values are presented in Fig. 4:

![Fig. 4. VaR and CVaR Value with various parameter \( \alpha \) values for \( r_1 > r_2 \)](image)

By considering the insurance premiums and climate index insurance, two conclusions are drawn and if the loss insurance premiums are more than or equal to the climate index insurance premiums, then the model chooses a climate index insurance policy, and vice versa. For Value-at-Risk (VaR) and Conditional Value-at-Risk (CVaR), the higher the value \( \alpha \), the value of both will increase. Under three circumstances, the value of insurance premiums, there are extreme values of VaR and CVaR at \( \alpha = 92 \) which shows the worst case of the loss function.
4. Conclusion

In this research, an optimization model for risk management of rice production was formed by considering the constraints of Conditional Value-at-Risk (CVaR) to minimize the size of the risk of the loss function. Risks considered in this model are lowland rice productivity using a regression model of the independent variable wind speed, maximum temperature, minimum temperature, and rainfall. By considering the several possible premium prices, this study has concluded that the climate index agricultural insurance is the best policy choice. Value-at-Risk and Conditional Value-at-Risk can be simultaneously obtained from the established model. The higher the significance value, the higher the value of VaR and CVaR.

Acknowledgments

Thanks to the Rector, Director of DRPMI, and the Dean of the Faculty of Mathematics and Natural Sciences, Padjadjaran University who have provided the Academic Leadership Grant (ALG) program under the coordination of Prof. Dr. Sudradjat Supian. Also, we would like to thanks for Universiti Malaysia Terengganu with research collaboration and the support provided for this research.

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