Kinematic and Static Analysis and Experimental Study for Dual-arm Continuum Robot

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Abstract. The continuum robot, which is different from the traditional rigid robot, consists of a flexible structure, and becomes a research hotspot in the field of robots, because of its good flexibility and adaptability in the complex environment. According to the advantages of the continuum robot, it could be applied in medical, exploration and agricultural operations. This paper proposed a cable-driven dual-arm continuum robot with three-degree-of-freedom. The kinematics of posture parameters and driving parameters, as well as the statics, was established. And the dual-arm coordinated reachable work space was described according to the principle of constant curvature deformation, on the basis of which, the theoretical motion model of the continuum robot was simulated using MATLAB, and experiments of the posture of the dual-arm continuum robot was conducted on the prototype to verify the accuracy of the theoretical kinematics and statics model.

1. Introduction
Robots can be devided into three categories which are discrete robots, squat robots, and continuum robots according to the structure format[1]. The comlexity of the work environment robots faced with is increasing with the development of science and technology. The continuum robot is becoming the hotpot of the robot field in recent years with its advantages of flexibility and self-adaptivity. The continuum robots are inspired by bionics[2-5]. And the flexible characteristic of continuum robots stems from their elastic parts, such as elastic rods[6-7], TiNi wires[8] and concentric tubes[9-10], etc. This paper presented a dual-arm continuum robot which possesses 3 DOFs(degree of freedom), of which each arm is driven by 3 wires installed symetrically, and the flexible shell is adopted to support the posture. On the basis of a single arm, the theory of wire driving kinematics and statics are established according to principle of constant curvature deformation. The correctness and feasibility of the theoretical model shall be verified by theoretical and experimental analysis.

As the Figure 1 a) shows, driving wires, base, segments and flexible shells consist of the main body of the continuum robot, among which the flexible shell is made up of flexible materials. And the flexible, which supports the posture of the continuum robot and keeps it bending with constant curvature, is designed as a hollow shape. As is show in Figure 1 b), a single arm of the continuum manipulator, of which the length is 300mm and diameter is 100mm, is composed of 3 flexible shells with the same length, each flexible shell is connected by a segment. There are three holes, which is designed for installing the driving wires, in the flexible shell of which the radius of graduated circle is 40mm. And the elastic deformation of the driving wire as tiny as can be neglected.
The stretching, bending and rotating movement of the continuum can be achieved by driving the 3 driving wires with different motion amount.

2. Kinematics

According to the design, the motion form and principle of the two continuum manipulators are the same. The kinematics research can be based on one of them, and then the corresponding coordinate conversion can be obtained. Figure 2 shows the bending of the continuum manipulator in \(Oxz\) plane.

\[
\begin{bmatrix}
    x \\
    y \\
    z
\end{bmatrix} = \frac{L}{\theta} \begin{bmatrix}
    (1 - \cos \theta) \cdot \cos \varphi \\
    (1 - \cos \theta) \cdot \sin \varphi \\
    \sin \theta
\end{bmatrix}
\]  

(2.1)

Equation (2.1) is obtained under the condition that the initial state of the continuum manipulator is coincide with \(z\) axis. Therefore, coordinate transformation is needed according to the structure, and the homogeneous transformation is:

\[
T = \text{Rot}(y, \frac{\pi}{4})[x, y, z, 1]^T
\]  

(2.2)
The coordinate of \( A \) is abtaibed:

\[
\begin{bmatrix}
x_1 \\
y_1 \\
z_1 
\end{bmatrix} = \frac{\sqrt{2}L}{2\theta} \begin{bmatrix}
1 - \cos \theta \sin \varphi + \sin \theta \\
\sqrt{2} (1 - \sin \theta \sin \varphi) \\
\sin \theta - (1 - \cos \theta) \cdot \sin \varphi
\end{bmatrix}
\]  
(2.3)

Simplify the continuum manipulator in Figure 1 into the form in Figure 3. \( A \) and \( B \) are the free-ends of the two continuum manipulators respectively. The homogeneous transformation from \( A \) to \( B \) is:

\[
T_i = \text{Trans}(0,0,0) \cdot \text{Rot}(-\varphi, -\theta, 0) \cdot [x_i, y_i, z_i, 1]^T
\]  
(2.4)

The coordinate of \( B \) is abtaibed:

\[
\begin{bmatrix}
x_2 \\
y_2 \\
z_2 
\end{bmatrix} = \frac{\sqrt{2}L}{2\theta} \begin{bmatrix}
1 - \cos \theta \sin \varphi - \sqrt{2}a \\
\sqrt{2} (1 - \cos \theta \sin \varphi) \\
(1 - \cos \theta) \cdot \sin \varphi + \sin \theta
\end{bmatrix}
\]  
(2.5)

And the posture of the continuum manipulator is:

\[
\begin{cases}
x = \frac{L}{\theta} \cos \varphi + \frac{L}{\theta} \\
y = \frac{L}{\theta} \sin \varphi \\
z = \frac{L}{\theta} \sin \alpha \\
\alpha = [\pi - \theta, \pi]
\end{cases}
\]  
(2.6)

According to equations (2.4) and (2.5), the work space of the robot can be abtaibed which is shown in Figure 4.

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Figure 4. Work space of the continuum robot

### 3. Statics

Figure 5 shows the posture and mechanical analysis of the continuum manipulator, the method of dividing the continuum manipulator into multi-segments is adopted, therefore the coordinate of the free-end of each segment is:

\[
\begin{bmatrix}
x_i \\
y_i 
\end{bmatrix} = r_i \cdot \begin{bmatrix}
1 - \cos \theta_i \\
\sin \theta_i
\end{bmatrix}
\]  
(3.1)

The homogeneous transformation from \( \{O_{i+1}\} \) to \( \{O_i\} \) is:
\[ o_i^T = \begin{bmatrix} \cos \theta_i & -\sin \theta_i & x_i \\ \sin \theta_i & \cos \theta_i & y_i \\ 0 & 0 & 1 \end{bmatrix} \]  

(3.2)

\[ \text{Trans} = \begin{bmatrix} 1 & 0 & r_i (1 - \cos \theta) \\ 0 & 1 & r_i \sin \theta \\ 0 & 0 & 1 \end{bmatrix} \]  

(3.4)

Expressed the force in \( \{O_i\} \):

\[ F_{0i}^{O_i} = o_i^T \times F_{0i}^{o_i} = (\prod_{i=1}^{n} o_i^T)^{-1} \times F_{0i}^{o_i} \]  

(3.5)

The moment of the force:

\[ M_{ri} = \overline{O_iO_n} \times F_{0i}^{O_i} \]  

(3.6)

\[ \overline{O_iO_n} = [r_i (1 - \cos \theta), r_i \sin \theta, 0]^T \]  

(3.7)

\[ M_{ri} = \frac{(EI)_i}{r_i} \]  

(3.8)

Thus the coordinate of the free-end of each segment can be calculated by equations from (3.1) to (3.8), by which the posture of the continuum manipulator can be obtained.

4. Experiment

4.1 Experiment platform introduction
The experiment platform includes a checkerboard as calibration scale, several standard weight for providing external load, and a high speed camera for taking pictures of postures of the continuum manipulator, see Figure 6.
4.2 Experiment for kinematics
In the experiment, let the bending angle be $\pi/6$, $\pi/3$, $\pi/2$ respectively, and the postures are attained as shown in Figure 7, the errors of each experiment are listed in Table 1 as well.

![Figure 6. The experiment platform](image)

![Figure 7. Contrast between theoretical and experimental posture of kinematics](image)

| Bending angle | $\pi/6$ | $\pi/3$ | $\pi/2$ |
|---------------|--------|--------|--------|
| Error         | 1.7%   | 3.4%   | 5.8%   |

4.3 Experiment for statics
In the experiment, let the bending angle be $\pi/6$, $\pi/3$, $\pi/2$ respectively, then the external load from 1N to 5N is applied to each experiment. And the postures are attained as shown in Figure 8, the errors of each experiment are listed in Table 2 as well.

![Figure 8. Contrast between theoretical and experimental posture of statics](image)
Table 2. Maximum error of each experiment

| Bending angle | External load |
|---------------|---------------|
|               | 1N            | 2N            | 3N            | 4N            | 5N            |
| π/6           | 0.888%        | 0.746%        | 1.533%        | 0.667%        | 1.080%        |
| π/3           | 0.631%        | 2.055%        | 1.546%        | 2.407%        | 2.119%        |
| π/2           | 1.081%        | 3.060%        | 3.686%        | 1.099%        | 1.354%        |

From the verification experiments above, as for the kinematics, the errors, which increase with the increment of bending angle, are no more than 5.8%. As for statics, the theory fits well with the results of experiments, and all the errors, which are no more than 3.686%, change with no obvious trend.

5. Conclusions

This paper presented a continuum robot that can achieve dual-arm motion. A mathematical model based on the principle of constant curvature is proposed to establish the basic deformation motion of the robot, which can achieve precise control, as well as statics of applying external load at the free-end. And experimental platform of three-degree-of-freedom flexible continuum robot was built for verification experiments. According to the verification experiments aimed at the kinematics and statics, maximum errors of them are less than 5.8% and 3.686% respectively.

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