Soft-Fermion-Pole Mechanism to Single Spin Asymmetry in Hadronic Pion Production

Yuji Koike and Tetsuya Tomita

Department of Physics, Niigata University, Ikarashi, Niigata 950-2181, Japan

Abstract. Single spin asymmetry (SSA) is a twist-3 observable in the collinear factorization approach. We present a twist-3 single-spin-dependent cross section formula for the pion production in $pp$-collision, $p^\uparrow p \rightarrow \pi X$, relevant to RHIC experiment. In particular, we calculate the soft-fermion-pole (SFP) contribution to the cross section from the quark-gluon correlation functions. We show that its effect can be as large as the soft-gluon-pole (SGP) contribution owing to the large SFP partonic hard cross section, even though the derivative of the SFP function does not participate in the cross section.

Keywords: Single spin asymmetry, Twist-3, Soft-fermion-pole

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The observed large single spin asymmetry (SSA) in $p^\uparrow p \rightarrow hX \ (h = \pi, K \ etc)$ cannot be explained by the usual parton model and perturbative QCD. In the framework of collinear factorization, SSA is a twist-3 observable and can be described in terms of the twist-3 quark-gluon-correlation correlation functions in the nucleon and/or pion $[1]$-$[9]$. Among various components of the twist-3 cross sections, a large contribution is expected to come from the twist-3 distribution functions in the transversely polarized nucleon. ( See $[4]$, however.) There are two independent twist-3 distributions $G_F^a(x_1,x_2)$ and $\tilde{G}_F^a(x_1,x_2)$ which are defined from a light-cone correlation function in the nucleon $\sim \langle \bar{\psi} \gamma^\mu F_\mu^a \psi^a \rangle$ where $\psi^a$ is the quark field of flavor $a$ and $F_\mu^a$ is the gluon’s field strength. For the explicit definition of $\{ G_F^a(x_1,x_2), \tilde{G}_F^a(x_1,x_2) \}$ and the relation between different expressions for the twist-3 distribution functions, see $[5, 7]$. Due to the naively $T$-odd nature of SSA, the relevant cross section occurs as a pole contribution of an internal propagator in the partonic hard part. For $p^\uparrow p \rightarrow \pi X$, two kinds of poles contribute: soft-gluon-pole (SGP) leading to $x_1 = x_2$ and soft-fermion-pole (SFP) leading to $x_i = 0 \ (i = 1 \ or \ 2)$. Thus the contribution to the cross section from these functions can be schematically written as

$$\Delta \sigma^{tw3} = \left( G_F(x,x) - x \frac{dG_F(x,x)}{dx} \right) \otimes f(x') \otimes D(z) \otimes \hat{\sigma}_{SGP}$$
$$+ \ G_F(0,x) \otimes f(x') \otimes D(z) \otimes \hat{\sigma}_{SFP} + \tilde{G}_F(0,x) \otimes f(x') \otimes D(z) \otimes \hat{\sigma}'_{SFP},$$

(1)

where we have used the fact that the SGP contribution appears in the combination of $G_F(x,x) - x \frac{dG_F(x,x)}{dx}$ $[6, 9]$. Note only $G_F(x,x)$ contributes through the SGP, since $\tilde{G}_F(x,x) = 0$ due to the anti-symmetric nature of $\tilde{G}_F(x_1,x_2)$ under $x_1 \leftrightarrow x_2$. The origin of this combination and the connection of $\hat{\sigma}_{SGP}$ to the twist-2 cross section was clarified in $[9, 8]$. Previous analyses $[2, 6]$ focussed on the SGP contribution (first term of (1)).
assuming that this one is a dominant contribution. However, there is no clue that the SFP functions $G_F(0,x)$ and $\tilde{G}_F(0,x)$ themselves are small, and its importance depends on the behavior of the partonic hard cross sections $\hat{\sigma}_{\text{SFP}}$ and $\hat{\sigma}_{\text{SFP}}'$.

The purpose of this report is to present the SFP cross section for $p\uparrow p \to \pi X$ and to study its impact compared with the SGP contribution, assuming that the SFP functions $G_F(0,x)$ and $\tilde{G}_F(0,x)$ have the same order of magnitude as the SGP function $G_F(x,x)$.

For the calculation of the SFP contribution to $p\uparrow(p,S_\perp) + p(p') \to \pi(P_h) + X$, we followed the formalism in [7]. With the Mandelstam variables for this process, $S = (p + p')^2$, $T = (p - P_h)^2$ and $U = (p' - P_h)^2$, the SFP contribution can be written as [10]:

$$E_h \frac{d^3 \hat{\sigma}_{\text{SFP}}}{dp_h^3} = \frac{\alpha_s^2 M_N \pi}{2} e^{-p_h S_\perp} \sum_{a,b,c} \int_{z_{\text{min}}}^{1} \frac{dz}{z^3} \int_{x_{\text{min}}}^{1} \frac{dx}{x'} \int \frac{dx}{x} \frac{1}{x'S + T/z} \times \delta \left( x - \frac{-x'U/z}{x'S + T/z} \right) \left[ \sum_{a,b,c} \left( G_F^a(0,x) + \tilde{G}_F^a(0,x) \right) \right] \times \left\{ q^b(x') \left( D^c(z) \hat{\sigma}_{ab\to c} + D^c(z) \hat{\sigma}_{ab\to c} \right) + q^b(x') \left( D^c(z) \hat{\sigma}_{ab\to g} + D^c(z) \hat{\sigma}_{ab\to g} \right) \right\} + \sum_{a,b} \left( \tilde{G}_F^a(0,x) + \tilde{G}_F^a(0,x) \right) \left( q^b(x') D^c(z) \hat{\sigma}_{ab\to g} + q^b(x') D^c(z) \hat{\sigma}_{ab\to g} \right) + \sum_{a,c} \left( G_F^a(0,x) + \tilde{G}_F^a(0,x) \right) G(x') \left( D^c(z) \hat{\sigma}_{ag\to c} + D^c(z) \hat{\sigma}_{ag\to c} \right) + \sum_{a} \left( G_F^a(0,x) + \tilde{G}_F^a(0,x) \right) D^c(z) \hat{\sigma}_{ag\to g} \right], \tag{2}$$

where the summation $\sum_{a,b,c}$ implies that the sum of $a$ is over all quark and anti-quark flavors ($a = u, d, s, \bar{u}, \bar{d}, \bar{s}, \cdots$) and the sum of $b$ and $c$ is restricted onto quark flavors when $a$ is a quark and onto anti-quark flavors when $a$ is an anti-quark. (For an anti-quark $b$, $\bar{b}$ denotes quark flavor.) The integration region in the convolution formula is specified by $x'_{\text{min}} = -\frac{T}{S + U/z}$ and $z_{\text{min}} = -\frac{T + U}{S}$. $G(x')$ represents the gluon distribution and $D^c(z)$ is the gluon fragmentation function for the pion. $\hat{\sigma}_{ab\to c}$ etc represents partonic hard cross sections where $c$ is the flavor of the parton fragmenting into the pion. They are functions of the Mandelstam variables in the parton level; $\hat{s} = (xp + x'p')^2 = xx'S$, $\hat{t} = (xp - P_h/z)^2 = \frac{x'U}{z}$ and $\hat{\bar{u}} = (x'p' - P_h/z)^2 = \frac{\bar{x}U}{z}$ and are given as follows ($N = 3$ is the number of color):

$$\hat{\sigma}_{ab\to c} = -\frac{(N^2 \hat{s} + 2 \hat{t})(\hat{s}^2 + \hat{u}^2)}{N^2 \hat{t}^3 \hat{u}} \delta_{ac} + \frac{(N^2 \hat{t} + \hat{u} - \hat{s}) \hat{\delta}_{ab} \delta_{ac}}{N^3 \hat{t}^2 \hat{u}},$$

$$\hat{\sigma}_{ab\to \bar{c}} = 0,$$

$$\hat{\sigma}_{\bar{a}b\to c} = \frac{(N^2 \hat{t} + 2 \hat{s})(\hat{s}^2 + \hat{u}^2)}{N^2 \hat{t}^3 \hat{u}} \delta_{ac} + \frac{(N^2 \hat{t} + 2 \hat{s})(\hat{t}^2 + \hat{u}^2)}{N^2 \hat{t}^2 \hat{u}^2} \delta_{ab} - \frac{(N^2 - 1) \hat{u}^2}{N^3 \hat{t}^2} \delta_{ab} \delta_{ac},$$

$$\hat{\sigma}_{\bar{a}b\to \bar{c}} = \frac{-(N^2 \hat{t} + 2 \hat{s})(\hat{t}^2 + \hat{u}^2)}{N^2 \hat{t}^2 \hat{u}^2} \delta_{ab} + \frac{-N^2 \hat{s} + \hat{t} - \hat{u}}{N^3 \hat{u}^2} \delta_{ab} \delta_{ac}, \tag{3}$$
so that the SGP contribution reproduces the unless the SFP function itself is small.

\[
\delta_{ab\rightarrow g} = \frac{(N^2 \hat{s} + 2\hat{t})(\hat{s}^2 + \hat{u}^2)}{N^2 \hat{s} \hat{t} \hat{u}} + \frac{-1}{N^3 \hat{s} \hat{t} \hat{u}} (N^2 (\hat{s}^3 + 3\hat{s}^2 \hat{u} - 2\hat{t} \hat{s}^3) + \hat{s}^3 - \hat{s}^2 \hat{u}) \delta_{ab},
\]

\[
\delta_{ab\rightarrow g} = \frac{-(N^2 \hat{u} + 2\hat{t})(\hat{s}^2 + \hat{u}^2)}{N^2 \hat{s} \hat{t} \hat{u}} + \left\{ \frac{1}{N^3} \left( \frac{\hat{u}}{\hat{s} \hat{t} \hat{u}} + 1 \right) \right. \\
\left. + \frac{1}{N} (\frac{\hat{s} + \hat{s} \hat{t} + \hat{t}^2}{\hat{s} \hat{u}^2} - \frac{\hat{u}}{\hat{t}^2}) + \frac{N(\hat{u} - \hat{t}^3)(\hat{t}^2 + \hat{u}^2)}{\hat{s} \hat{t}^3 \hat{u}^2} \right\} \delta_{ab},
\]

\[
\delta_{ag\rightarrow e} = \frac{\hat{s} + 2\hat{t} - N^2 \hat{s}}{N^2 (N^2 - 1) \hat{s} \hat{t} \hat{u}} \delta_{ac} + \frac{-(N^2 \hat{t} + 2\hat{s})(\hat{t}^2 + \hat{u}^2)}{N^2 (N^2 - 1) \hat{s} \hat{t} \hat{u}},
\]

\[
\hat{\delta}_{ag\rightarrow g} = \frac{-N^2}{(N^2 - 1) \hat{s} \hat{t} \hat{u}^2} \left( 4\hat{s}^6 + 11\hat{s}^5 \hat{t} + 19\hat{s}^4 \hat{t}^2 + 22\hat{s}^3 \hat{t}^3 + 19\hat{s}^2 \hat{t}^4 + 11\hat{s} \hat{t}^5 + 4\hat{t}^6 \right) \\
+ \frac{1}{N^2 (N^2 - 1) \hat{s} \hat{t} \hat{u}^2} \left\{ -\hat{s} \hat{t} \hat{u}^2 + N^2 (\hat{s}^4 + \hat{s}^3 \hat{t} + 2\hat{s}^2 \hat{t}^2 + \hat{s} \hat{t}^3 + \hat{t}^4) \right\}.
\]

A remarkable feature of (2) is that the partonic hard cross sections for \( GF(0, x) \) and \( \tilde{G}_F(0, x) \) are the same, even though each diagram gives different contributions for the two functions \[10\]. Accordingly, they appear in the combination of \( GF^d(0, x) + \tilde{G}_F^d(0, x) \) in (2). We also note that some terms in the hard cross section (the first terms in \( \delta_{ab\rightarrow c}, \delta_{ab\rightarrow g}, \delta_{ag\rightarrow c}, \delta_{ag\rightarrow g} \) etc) accompany the large color factors compared to the SGP cross section derived in \[6\], and show the steep rising behavior in the forward direction (i.e. large \( S/T \)). This suggests that the SFP contribution gives rise to the nonnegligible contribution to the asymmetry.

To see the impact of the SFP contribution, we have performed a numerical calculation of \( A_N \) for \( p^+p \rightarrow \pi X \), assuming that the SFP function is of the same order of magnitude as the SGP function. Kouvaris et al. \[6\] have parametrized the SGP function \( GF(x, x) \) so that the SGP contribution reproduces the \( A_N \) data obtained from RHIC and FNAL data. Their analysis shows that both data are reasonably well reproduced. In particular, they found that the derivative term in \[11\] brings dominant contribution compared to the nonderivative contribution. Here we adopt Fit(1) of \[6\] and assume \( GF^d(0, x) + \tilde{G}_F^d(0, x) = GF^d(x, x) \) (\( a = u, d \)). Fig. 1 shows \( A_N \) for the pion at \( \sqrt{S} = 200 \) GeV and \( P_{HT} = 1.5 \) GeV with and without SFP contribution. As is seen from the figure that the SFP contribution brings large effect in the positive \( x_F \) region, while its effect is negligible in the negative \( x_F \) region. From this figure it is clear that the SFP contribution can affect \( A_N \) significantly even though it does not receive enhancement by the derivative unlike SGP contribution, unless the SFP function itself is small.

To summarize, we have calculated the SFP contribution to the cross section for \( p^+p \rightarrow hX \) associated with the twist-3 quark-gluon correlation functions in the polarized
FIGURE 1. $A_N$ for $p^+p \rightarrow \pi X$ at $\sqrt{s} = 200$ GeV and $P_T = 1.5$ GeV. Solid, long-dashed, and dash-dot lines are, respectively, $A_N$ for $\pi^+$, $\pi^-$ and $\pi^0$ obtained with only the SGP contribution. Dotted, short-dashed, and dash-double-dot lines are, respectively, $A_N$ for $\pi^+$, $\pi^-$ and $\pi^0$ obtained with both SGP and SFP contributions.

nucleon, and have shown that its effect is significant and should be included in the analysis of $A_N$. For a more complete analysis, one needs to include the contribution from the triple-gluon twist-3 distribution function as well. Recent study [11] shows that $A_N$ for the open charm production has a potential to determine the function. We hope that the global analysis of all those data including all the QCD effects will clarify the origin of observed SSA.

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REFERENCES

1. J. Qiu and G. Sterman, Nucl. Phys. B378 (1992) 52.
2. J. Qiu and G. Sterman, Phys. Rev. D59 (1999) 014004.
3. Y. Kanazawa and Y. Koike, Phys. Lett. B478 (2000) 121; Phys. Lett. B490 (2000) 99.
4. Y. Koike, AIP Conf. Proc. 675 (2003) 449 [hep-ph/0210396]; Nucl. Phys. A721 (2003) 364.
5. H. Eguchi, Y. Koike and K. Tanaka, Nucl. Phys. B752 (2006) 1.
6. C. Kouvaris, J. W. Qiu, W. Vogelsang and F. Yuan, Phys. Rev. D 74, 114013 (2006).
7. H. Eguchi, Y. Koike and K. Tanaka, Nucl. Phys. B763 193 (2007).
8. Y. Koike and K. Tanaka, Phys. Lett. B 646, 232 (2007) [Erratum-ibid. B 668, 458 (2008)].
9. Y. Koike and K. Tanaka, Phys. Rev. D 76, 011502 (2007).
10. Y. Koike and T. Tomita, in preparation.
11. Z. B. Kang, J. W. Qiu, W. Vogelsang and F. Yuan, Phys. Rev. D 78, 114013 (2008).