On the Role of Squared Neutron Number in Reducing Nuclear Binding Energy in the Light of Electromagnetic, Weak and Nuclear Gravitational Constants – A Review

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Authors’ contributions

This work was carried out in collaboration between both authors. Author UVSS designed the study, performed the statistical analysis, wrote the protocol, wrote the first draft of the manuscript and managed the literature searches. Author SL managed the analyses of the study in all aspects. Both authors read and approved the final manuscript.

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ABSTRACT

With reference to authors recently proposed three virtual atomic gravitational constants and nuclear elementary charge, close to stable mass numbers, it is possible to show that, squared neutron number plays a major role in reducing nuclear binding energy. In this context, Z=30 onwards, ‘inverse of the strong coupling constant’, can be inferred as a representation of the maximum strength of nuclear interaction and 10.09 MeV can be considered as a characteristic nuclear binding energy coefficient. Coulombic energy coefficient being 0.695 MeV, semi empirical mass formula - volume, surface, asymmetric and pairing energy coefficients can be shown to be 15.29
Volume and Surface energy terms can be represented with \((A-A^{2/3})\times15.29\ \text{MeV}\). With reference to nuclear potential of 1.162 \text{MeV} and coulombic energy coefficient, close to stable mass numbers, nuclear binding energy can be fitted with two simple terms having an effective binding energy coefficient of \([10.09-(1.162+0.695)/2]=9.16\ \text{MeV}\). Nuclear binding energy can also be fitted with five terms having a single energy coefficient of 10.09 \text{MeV}. With further study, semi empirical mass formula can be simplified with respect to strong coupling constant.

Keywords: Three virtual atomic gravitational constants; nuclear elementary charge; nuclear stability; binding energy; squared neutron number; screened mass number.

1. INTRODUCTION

Considering neutrons and protons as microscopic molecules, the liquid drop model treats the atomic nucleus as a drop of incompressible nuclear fluid of very high density bound by strong nuclear force. The residual effect of the strong nuclear force plays a crucial role in understanding nuclear binding. Mathematical analysis of the model delivers a formula to predict the binding energy of any atomic nucleus in terms of its number of protons and neutrons with five different energy terms and five different energy coefficients. Energy coefficients of the formula are chosen in such a way to fit the wide range of nuclear binding energy data partly based on theory and partly based on empirical measurements. Hence ‘liquid drop formula’ is generally called as ‘Semi empirical mass formula (SEMF).’

In this context, authors would like to emphasize the fact that, physics and mathematics associated with fixing of the energy coefficients of SEMF are neither connected with residual strong nuclear force nor connected with strong coupling constant. Since nuclear force is mediated via quarks and gluons, it is necessary and compulsory to study the nuclear binding energy scheme in terms of nuclear coupling constants. In this direction, N. Ghahramany and team members have taken a great initiative in exploring the secrets of nuclear binding energy and magic numbers [7-11]. Very interesting point of their study is that - nuclear binding energy can be understood with two or three terms with single energy coefficient.

Now days a lot of progress is taking place in the fields of fluid mechanics at atomic and nano scales [12-17]. As the origin of SEMF was ‘Fluid Mechanics’, authors hope that, by considering a combined study on the residual nuclear force, ground sate quarks, strong coupling constant and atomic scale fluid mechanics, it may be possible to understand nuclear binding energy in a unified picture.

Objective of this paper is to review, simplify and establish the concepts proposed in authors’ recent papers and conference proceedings [18-38] pertaining to nuclear stability and binding energy connected with three virtual atomic gravitational constants.

The most desirable cases of any unified description are:

a) To implement gravity in microscopic physics and to estimate the magnitude of the Newtonian gravitational constant \(G_N\).

b) To simplify the complicated issues of known physics. (Understanding nuclear stability, nuclear binding energy, nuclear charge radii and neutron life time etc.)

c) To predict new effects, arising from a combination of the fields inherent in the unified description. (Understanding strong coupling constant, Fermi’s weak coupling constant and radiation constants etc.)

d) To develop a model of microscopic quantum gravity.

1.1 History of the Three Atomic Gravitational Constants

(1) Since 1974, K. Tennakone, Abdus Salam, C. Sivaram, K.P.Sinha, Dj. Sijacki, Y. Ne’eman, J.J. Perng, J. Strathdee, Usha Raut, V. de Sabbata, E. Recami, T.R. Mongan, Robert Oldershaw and S.G. Fedosin like many scientists proposed the existence of ‘Nuclear’ or ‘strong’ gravitational constant with a magnitude approximately \(10^{35}\) to \(10^{39}\) times the Newtonian gravitational constant. In this
context, one can see a detailed discussion by F. Akinto and Farida Tahir in their arXiv preprint [39].

(2) In 2010, 2011 and 2012, in a series of papers, authors proposed the existence of ‘electromagnetic’ gravitational constant [23,24,25]. In 2016 Franck Delplace also proposed its existence [14].

(3) In 2013, Roberto Onofrio proposed the existence of ‘weak’ gravitational constant [40].

(4) In 2016, Tüzemen, S. described a possible microscopic model for gravitational interaction [41].

1.2 To Estimate the Newtonian Gravitational Constant in a Theoretical Approach

According to Rosi et al. [42]: There is no definitive relationship indeed between G_N and the other fundamental constants and no theoretical prediction for its value to test the experimental results. Improving the knowledge of G_N has not only a pure metrological interest, but is also important for the key role that this fundamental constant plays in theories of gravitation, cosmology, particle physics, astrophysics, and geophysical models.

To estimate the value of G_N in a theoretical approach, authors would like to suggest the following points.

(1) Interaction constants are connected both with global phenomena of physics and with phenomena at small distances, such as quantum gravity. Therefore, the search for relations among the constants of the four types of interactions is important, relevant and necessary. At present, there exist no basic formulae or mechanisms using which one can develop at least models with ad hoc relations. In a unified approach, one can see a great initiative taken by J. E. Brandenburg [43]. It would be important to consider in detail such theories as microscopic quantum gravity and a combination of the fields inherent in the unified description of the four interactions.

(2) As there is a large gap in between nuclear and Planck scales, with currently believed notion of unification paradigm, it seems impossible to implement gravity in atomic, nuclear and particle physics.

(3) G_N is a man created empirical constant and is having no physical existence. Clearly speaking, it is not real but virtual. For understanding the secrets of large scale gravitational effects, scientists consider it as a physical constant.

(4) In the same way, each atomic interaction can be allowed to have its own virtual gravitational constant.

(5) With a combined study of the four gravitational constants, their magnitudes can be refined for a better fit and understanding of the nature.

1.3 Scope of This Work

(1) Current nuclear physical models and String theory models [44-46] are failing in implementing gravity in nuclear physics. In this context, authors proposed concepts can successfully be implemented in nuclear physics.

(2) Nuclear charge radii, nucleon magnetic moments, nuclear stability, nuclear binding energy, magic proton numbers [5,6,34], nucleons kinetic energy [35] and atomic radii can be understood in terms of gravity. Super heavy elements can also be studied in this direction.

(3) Hadronic mass spectrum and melting points of quarks can be understood [36].

(4) Strong coupling constant, Fermi’s weak coupling constant, Newtonian gravitational constant and Avogadro number can be studied in a unified manner [37,38].

(5) Astrophysical mass units like Chandrasekhar mass limit [47] and neutron star mass limit [48,49] can be understood.

(6) Recently observed astrophysical emission line of 3.5 keV [38,50,51] can be understood.

1.4 Four Basic Semi Empirical Reference Relations

With reference to our recent publications and conference presentations [18-38], authors propose the following set of four semi empirical ‘reference’ relations. Let,

Electromagnetic gravitational constant = G_e
Nuclear gravitational constant = G_n
Weak gravitational constant = G_w

G_N = G_E + G_N - G_W
Working on deriving developed many interesting relations and experiments. In a verifiable approach authors considered as a standard reference for future estimated absolute theoretical value of relations can be using which in near future, an absolute set of help in producing a variety of such relations by played by the four gravitational constants seems Even though our approach is speculative, role played by the four gravitational constants seems to be fairly natural. This kind of approach may help in producing a variety of such relations by using which in near future, an absolute set of relations can be developed. Proceeding further, estimated absolute theoretical value of $G_s$ can be considered as a standard reference for future experiments. In a verifiable approach authors developed many interesting relations and working on deriving them from basic principles.

2. THREE SIMPLE ASSUMPTIONS PERTAINING TO NUCLEAR PHYSICS

1) There exists a strong elementary charge in such a way that,

$$
\frac{e_s}{e} \equiv \left( \frac{G_s m_p^2}{\hbar c} \right) \equiv \left( \frac{G_s m_p^2}{G_v G_N} \right)^{1/3} m_p^2
$$

(1)

$$
\frac{G_s}{m_p} \equiv 2 \pi \left( \frac{4 \pi \hbar c G_s m_p^2}{e^2} \right) \equiv \left( \frac{G_s m_p^2}{\hbar c} \right)
$$

(2)

$$
G_s \equiv \left[ (G_s m_s^2) (G_s m_s^2) \right] \left( \frac{2G_s m_s^2}{e^2} \right) \equiv \frac{4G_s \hbar c}{e^2}
$$

(3)

$$
G_s \equiv \left( \frac{m_p}{m_c} \right)^{1/0}
$$

(4)

Based on relation (1), magnitudes of $(G_s, G_v)$ can be estimated. Based on relation (2), magnitude of $G_s$ can be estimated. Based on relation (3), magnitudes of $(G_s, G_v)$ can be estimated [40,52]. Again, based on relation (4), $G_s$ can be estimated. Estimated values seem to be:

| $G_s$ | $G_v$ | $G_w$ | $G_N$ | $G_F$ |
|-------|-------|-------|-------|-------|
| $2.374335 \times 10^{-10}$ m$^3$ kg$^{-1}$ sec$^{-2}$ | $3.329561 \times 10^{-28}$ m$^3$ kg$^{-1}$ sec$^{-2}$ | $2.909745 \times 10^{-22}$ m$^3$ kg$^{-1}$ sec$^{-2}$ | $6.679855 \times 10^{-11}$ m$^3$ kg$^{-1}$ sec$^{-2}$ | $6.44021 \times 10^{-62}$ J m$^3$ |

Even though our approach is speculative, role played by the four gravitational constants seems to be fairly natural. This kind of approach may help in producing a variety of such relations by using which in near future, an absolute set of relations can be developed. Proceeding further, estimated absolute theoretical value of $G_s$ can be considered as a standard reference for future experiments. In a verifiable approach authors developed many interesting relations and working on deriving them from basic principles.

2) Strong coupling constant [52] can be expressed with,

$$
\alpha_s \equiv \left( \frac{\alpha_s}{\alpha_s} \right) \equiv \left( \frac{\alpha_s}{\alpha_s} \right) \equiv \frac{G_s m_s^2}{G_s m_s^2}
$$

(8)

3) Nuclear charge radius can be addressed with,

$$
R_0 \equiv \frac{2G_s m_p}{e^2}
$$

(9)

Based on relations (5) to (9),

| $e_s$ | $\alpha_s$ | $R_0$ |
|-------|------------|-------|
| $2.9463591 e$ | $0.1151937$ | $1.23929 \times 10^{-15}$ m |

3. UNDERSTANDING PROTON-NEUTRON STABILITY WITH THREE ATOMIC GRAVITATIONAL CONSTANTS

Let,

$$
\frac{G_s m_p m_c}{\hbar c} \equiv \frac{G_s m_p m_c}{G_s G_N} \equiv \frac{G_s^2}{G_s G_N} \equiv \frac{G_s^2}{G_s G_N}
$$

(10)

Nuclear beta stability line can be addressed with a relation of the form [relation 8 of Ref.3],

$$
A_s \equiv 2Z + s(2Z)^2 \equiv 2Z + (4s)Z^2 \equiv 2Z + kZ^2 \equiv Z(2 + kZ)
$$

(11)
where \( (4s) \equiv k \equiv 0.0064185 \)

By considering a factor like \( 2 + \sqrt{k} \), likely possible range of \( A_s \) can be addressed with,

\[
\begin{align*}
(A_s)_{\text{lower}} & \equiv Z[(2 \pm 0.08) + kZ] \\
(A_s)_{\text{mean}} & \equiv Z(1.92 + kZ) \\
(A_s)_{\text{upper}} & \equiv Z(2.08 + kZ)
\end{align*}
\]

(12)

Table 1. Likely possible range of \( A_s \) for \( Z=5 \) to \( 115 \)

| Proton number | \( (A_s)_{\text{lower}} \) | \( (A_s)_{\text{mean}} \) | \( (A_s)_{\text{upper}} \) |
|---------------|-----------------|-----------------|-----------------|
| 5             | 10              | 10              | 11              |
| 15            | 30              | 31              | 33              |
| 25            | 52              | 54              | 56              |
| 35            | 75              | 78              | 81              |
| 45            | 99              | 103             | 107             |
| 55            | 125             | 129             | 134             |
| 65            | 152             | 157             | 162             |
| 75            | 180             | 186             | 192             |
| 85            | 210             | 216             | 223             |
| 95            | 240             | 248             | 256             |
| 105           | 272             | 281             | 289             |
| 115           | 306             | 315             | 324             |

Interesting point to be noted is that, for \( Z=112, 113 \) and \( 114 \), estimated lower stable mass numbers are 296, 299 and 302 respectively. Corresponding neutron numbers are 184, 186 and 188. These neutron numbers are very close to the currently believed shell closure at \( N=184 \). It needs further study [53]. See Table 1.

4. Unified Energy Coefficients of Semi Empirical Mass Formula (SEMF)

Let,

A characteristic nuclear binding energy coefficient be expressed as,

\[
B_r \equiv \frac{e^2}{8\pi\epsilon(G_m/c^2)} \equiv \left( \frac{1}{\alpha_r} \right) \left( \frac{e^2}{4\pi\epsilon R_r} \right) \approx 10.09 \text{ MeV}
\]

(13)

With reference to a new factor of the form,

\[
\ln \left( \frac{e^2}{4\pi\epsilon G/m \cdot m} \right) \approx 1.515,
\]

(1) Volume or surface energy coefficient can be expressed as \( a_v \equiv a_s \equiv 1.515 \times 10.09 \approx 15.29 \text{ MeV} \).

(2) Asymmetric energy coefficient can be expressed as \( a_a \equiv 1.515 a_v \equiv 1.515 \times 15.29 \approx 23.16 \text{ MeV} \).

(3) Pairing energy coefficient can be expressed as \( a_p \equiv B_r \equiv 10.09 \text{ MeV} \).

(4) 10.09 MeV, 15.29 MeV and 23.16 MeV seem to follow a geometric series with a geometric ratio, 1.515.

(5) For \( Z \geq 10 \), by considering coulombic energy coefficient as \( a_c \equiv 0.695 \text{ MeV} \), nuclear binding energy [1-6] can be estimated with,

\[
B_r \equiv \left( A^2 - A^{10} - 1 \right) * 15.29 - \left( \frac{Z}{A^{10}} \right) * 0.695 - \left( \frac{A - 2Z}{A} \right) * 23.16 \pm \left( \frac{10.09}{\sqrt{A}} \right) \text{ MeV}
\]

(14)

Data estimated with relation (14) can be compared with the standard relation [3],

\[
B_r \equiv \left( A^2 - A^{15.78} - (A^{10} - 18.34) \right) - \left( \frac{Z}{A^{10}} \right) * 0.71 - \left( \frac{A - 2Z}{A} \right) * 23.21 \pm \left( \frac{12.0}{\sqrt{A}} \right) \text{ MeV}
\]

(15)

For \( Z=50 \), starting from \( A = (100 \text{ to } 150) \), error in estimated binding energy seems to increase from -1.66 MeV to 1.63 MeV respectively.

4.1 Observations Pertaining to Term1 to Term2 of Relation (14):

(1) Ratio of (Term1-Term2)/10.09 MeV is a straight line and slope is practically constant for \( Z = 10 \) to 100.
5. UNDERSTANDING NUCLEAR BINDING ENERGY WITH SINGLE AND UNIFIED ENERGY COEFFICIENT

A. New Integrated Model

Based on the new integrated model proposed by N. Ghahramany et al. [10,11]

$$B(Z,N) = \left\{ A - \left( \frac{(N^2 - Z^2)}{3Z} + \delta(N - Z) \right) \right\} \frac{m_c^2}{\gamma}$$

(16)

where, $\gamma = $ Adjusting coefficient $\approx$ (90 to 100).

if $N \neq Z$, $\delta(N - Z) = 0$ and if $N = Z$, $\delta(N - Z) = 1$.

Readers are encouraged to see references there in [10] for derivation part and other details pertaining to the estimation of the adjusting coefficient (90 to 100) [11]. Points to be noted are: close to the beta stability line, $\left[ \frac{N^2 - Z^2}{3Z} \right]$ takes care of the combined effects of coulombic and asymmetric effects and nuclear binding energy can be addressed with a single energy coefficient.

B. Unified Approach-1

Interesting points to be noted are:

1) $Z \geq 30$ seems to represent a characteristic reference number in understanding nuclear binding of light and heavy atomic nuclides.

2) With reference to electromagnetic interaction and based on proton number,

$$B_{\text{effective}} \approx \frac{e^2}{8\pi \epsilon \left( G \frac{m_e}{c^2} \right)} = \left\{ \frac{e^2}{4\pi \epsilon R_e} \right\} \approx 8.928 \text{ MeV}$$

$$B_{\text{effective}} \approx \frac{e^2}{8\pi \epsilon \left( G \frac{m_e}{c^2} \right)} - \frac{3}{5} \left\{ \frac{e^2}{4\pi \epsilon R_e} \right\} \approx 9.395 \text{ MeV}$$

(18)

where $\left( \frac{e^2}{4\pi \epsilon R_e} \right) \approx 1.162 \text{ MeV}$ and $\frac{3}{5} \left( \frac{e^2}{4\pi \epsilon R_e} \right) \approx 0.695 \text{ MeV}$ can be considered as repulsive nuclear binding energy coefficients. To fit the data authors consider,

$$B_{\text{effective}} \approx \frac{8.928 + 9.395}{2} \approx 9.16$$

(19)
Based on the above relations and close to the stable mass numbers of \( Z = 2 \) to 118, with a common energy coefficient of 9.16 MeV, authors would like to suggest the following two terms for fitting and understanding nuclear binding energy.

First term helps in increasing the binding energy and can be considered as,

\[
T_1 \cong \eta \times A \times 9.16 \text{ MeV}
\]

where
\[
\begin{align*}
\eta &\cong \left( \frac{Z}{30} \right)^{0.08} \quad \text{for } Z < 30 \\
\eta &\cong 1 \quad \text{for } Z \geq 30
\end{align*}
\]

Second term helps in decreasing the binding energy and can be considered as,

\[
T_2 \cong \eta \left[ \left( \frac{k}{\ln(30)} \right) N^2 + \frac{1}{2} \right] \times 9.16 \text{ MeV}
\]

\[
\cong \eta \left( 0.00189 N^2 + \frac{1}{2} \right) \times 9.16 \text{ MeV}
\]

Considering light atomic nuclides, authors introduced the numerical factor \( \frac{1}{2} \). It needs further study.

Thus, close to stable atomic nuclides, authors introduced the numerical factor \( \frac{1}{2} \). It needs further study. Thus, close to stable mass numbers, binding energy can be fitted with,

\[
(B.A) \cong T_1 + T_2
\]

\[
\cong \eta \left[ A - \left( 0.00189 N^2 + \frac{1}{2} \right) \right] \times 9.16 \text{ MeV}
\]

\[
\cong \left( \frac{Z}{30} \right)^{0.08} \left[ A - \left( 0.00189 N^2 + \frac{1}{2} \right) \right] \times 9.16 \text{ MeV} \quad \text{for } Z < 30
\]

\[
\cong \left[ A - \left( 0.00189 N^2 + \frac{1}{2} \right) \right] \times 9.16 \text{ MeV} \quad \text{for } Z \geq 30
\]

Fig. 1. Binding energy per nucleon close to stable mass numbers of \( Z = 2 \) to 118.
| Proton number | Estimated mass number close to stable mass number | Neutron number | Estimated Binding energy (MeV) | SEMF binding energy | Error (MeV) |
|--------------|-----------------------------------------------|---------------|-------------------------------|---------------------|-------------|
| 2            | 4                                             | 2             | 25.76                         | 22.01               | -3.75       |
| 3            | 6                                             | 3             | 41.77                         | 26.88               | -14.90      |
| 4            | 8                                             | 4             | 58.24                         | 52.86               | -5.37       |
| 5            | 10                                            | 5             | 75.02                         | 62.29               | -12.74      |
| 6            | 12                                            | 6             | 92.07                         | 87.39               | -4.67       |
| 7            | 14                                            | 7             | 109.32                        | 98.81               | -10.51      |
| 8            | 16                                            | 8             | 126.74                        | 123.25              | -3.49       |
| 9            | 19                                            | 10            | 152.33                        | 148.85              | -3.48       |
| 10           | 21                                            | 11            | 170.06                        | 167.52              | -2.55       |
| 11           | 23                                            | 12            | 187.91                        | 186.14              | -1.76       |
| 12           | 25                                            | 13            | 205.84                        | 204.72              | -1.13       |
| 13           | 27                                            | 14            | 223.86                        | 222.22              | -0.64       |
| 14           | 29                                            | 15            | 241.96                        | 241.65              | -0.31       |
| 15           | 31                                            | 16            | 260.12                        | 259.98              | -0.14       |
| 16           | 34                                            | 18            | 286.48                        | 290.77              | 4.29        |
| 17           | 36                                            | 19            | 304.77                        | 305.06              | 0.29        |
| 18           | 38                                            | 20            | 323.11                        | 327.23              | 4.12        |
| 19           | 40                                            | 21            | 341.49                        | 341.47              | 341.49      |
| 20           | 43                                            | 23            | 368.02                        | 371.57              | 3.55        |
| 21           | 45                                            | 24            | 386.47                        | 389.59              | 3.12        |
| 22           | 47                                            | 25            | 404.96                        | 407.47              | 2.51        |
| 23           | 49                                            | 26            | 423.47                        | 425.20              | 1.72        |
| 24           | 52                                            | 28            | 450.08                        | 454.57              | 4.50        |
| 25           | 54                                            | 29            | 468.63                        | 468.89              | 0.25        |
| 26           | 56                                            | 30            | 487.21                        | 489.58              | 2.37        |
| 27           | 59                                            | 32            | 513.81                        | 515.20              | 1.40        |
| 28           | 61                                            | 33            | 532.40                        | 532.52              | 0.11        |
| 29           | 63                                            | 34            | 551.02                        | 549.67              | -1.35       |
| 30           | 66                                            | 36            | 577.57                        | 577.93              | 0.35        |
| 31           | 68                                            | 37            | 594.63                        | 591.98              | -2.65       |
| 32           | 71                                            | 39            | 619.48                        | 619.81              | 0.32        |
| 33           | 73                                            | 40            | 636.44                        | 636.62              | 0.19        |
| 34           | 75                                            | 41            | 653.36                        | 653.27              | -0.09       |
| 35           | 78                                            | 43            | 677.93                        | 677.88              | -0.05       |
| 36           | 80                                            | 44            | 694.75                        | 697.05              | 2.30        |
| 37           | 83                                            | 46            | 719.11                        | 721.32              | 2.20        |
| 38           | 85                                            | 47            | 735.83                        | 737.59              | 1.76        |
| 39           | 88                                            | 49            | 759.99                        | 761.58              | 1.59        |
| 40           | 90                                            | 50            | 776.59                        | 780.20              | 3.60        |
| 41           | 93                                            | 52            | 800.55                        | 803.86              | 3.33        |
| 42           | 95                                            | 53            | 817.05                        | 819.75              | 2.70        |
| 43           | 98                                            | 55            | 840.80                        | 843.16              | 2.37        |
| 44           | 100                                           | 56            | 857.20                        | 861.24              | 4.05        |
| 45           | 103                                           | 58            | 880.74                        | 884.37              | 3.63        |
| 46           | 106                                           | 60            | 904.14                        | 909.61              | 5.47        |
| 47           | 108                                           | 61            | 920.36                        | 922.70              | 2.34        |
| 48           | 111                                           | 63            | 943.56                        | 947.65              | 4.09        |
| 49           | 113                                           | 64            | 959.68                        | 962.85              | 3.17        |
| 50           | 116                                           | 66            | 982.66                        | 987.48              | 4.81        |
| 51           | 119                                           | 68            | 1005.51                       | 1009.66             | 4.15        |
| 52           | 121                                           | 69            | 1021.46                       | 1024.59             | 3.13        |
| 53           | 123                                           | 70            | 1037.38                       | 1039.35             | 1.97        |
| Proton number | Estimated mass number close to stable mass number | Neutron number | Estimated Binding energy (MeV) | SEMF binding energy | Error (MeV) |
|---------------|-----------------------------------------------|---------------|-------------------------------|---------------------|-------------|
| 54            | 127                                           | 73            | 1066.60                       | 1070.45             | 3.85        |
| 55            | 129                                           | 74            | 1082.38                       | 1085.10             | 2.72        |
| 56            | 132                                           | 76            | 1104.67                       | 1108.72             | 4.05        |
| 57            | 135                                           | 78            | 1126.83                       | 1130.06             | 3.23        |
| 58            | 138                                           | 80            | 1148.84                       | 1153.27             | 4.43        |
| 59            | 140                                           | 81            | 1164.38                       | 1165.55             | 1.17        |
| 60            | 143                                           | 83            | 1186.19                       | 1188.52             | 2.33        |
| 61            | 146                                           | 85            | 1207.86                       | 1209.31             | 1.45        |
| 62            | 149                                           | 87            | 1229.39                       | 1231.90             | 2.51        |
| 63            | 151                                           | 88            | 1244.69                       | 1245.86             | 1.18        |
| 64            | 154                                           | 90            | 1266.01                       | 1268.20             | 2.19        |
| 65            | 157                                           | 92            | 1287.20                       | 1288.44             | 1.25        |
| 66            | 160                                           | 94            | 1308.24                       | 1310.44             | 2.19        |
| 67            | 163                                           | 96            | 1329.15                       | 1330.38             | 1.22        |
| 68            | 166                                           | 98            | 1349.93                       | 1352.04             | 2.12        |
| 69            | 169                                           | 100           | 1370.56                       | 1371.69             | 1.13        |
| 70            | 171                                           | 101           | 1385.40                       | 1385.09             | -0.32       |
| 71            | 174                                           | 103           | 1405.83                       | 1404.54             | -1.29       |
| 72            | 177                                           | 105           | 1426.12                       | 1425.66             | -0.45       |
| 73            | 180                                           | 107           | 1446.27                       | 1444.84             | -1.43       |
| 74            | 183                                           | 109           | 1466.28                       | 1465.66             | -0.61       |
| 75            | 186                                           | 111           | 1486.15                       | 1484.57             | -1.58       |
| 76            | 189                                           | 113           | 1505.88                       | 1505.10             | -0.79       |
| 77            | 192                                           | 115           | 1525.48                       | 1523.74             | -1.74       |
| 78            | 195                                           | 117           | 1544.94                       | 1543.98             | -0.95       |
| 79            | 198                                           | 119           | 1564.25                       | 1562.37             | -1.88       |
| 80            | 201                                           | 121           | 1583.44                       | 1582.34             | -1.10       |
| 81            | 204                                           | 123           | 1602.48                       | 1600.48             | -2.00       |
| 82            | 207                                           | 125           | 1621.38                       | 1620.17             | -2.12       |
| 83            | 210                                           | 127           | 1640.15                       | 1638.07             | -2.08       |
| 84            | 213                                           | 129           | 1658.77                       | 1657.49             | -2.82       |
| 85            | 216                                           | 131           | 1677.26                       | 1675.15             | -2.11       |
| 86            | 219                                           | 133           | 1695.61                       | 1694.32             | -1.30       |
| 87            | 223                                           | 136           | 1718.30                       | 1718.61             | 0.30        |
| 88            | 226                                           | 138           | 1736.31                       | 1737.47             | 1.16        |
| 89            | 229                                           | 140           | 1754.17                       | 1754.62             | 0.45        |
| 90            | 232                                           | 142           | 1771.90                       | 1773.24             | 1.34        |
| 91            | 235                                           | 144           | 1789.49                       | 1790.16             | 0.67        |
| 92            | 238                                           | 146           | 1806.94                       | 1808.53             | 1.59        |
| 93            | 242                                           | 149           | 1828.28                       | 1830.19             | 1.90        |
| 94            | 245                                           | 151           | 1845.39                       | 1848.29             | 2.90        |
| 95            | 248                                           | 153           | 1862.36                       | 1864.75             | 2.40        |
| 96            | 251                                           | 155           | 1879.18                       | 1882.62             | 3.44        |
| 97            | 254                                           | 157           | 1905.88                       | 1908.87             | 2.99        |
| 98            | 258                                           | 160           | 1916.07                       | 1922.72             | 6.65        |
| 99            | 261                                           | 162           | 1932.42                       | 1938.72             | 6.31        |
| 100           | 264                                           | 164           | 1948.62                       | 1956.10             | 7.48        |
| 101           | 267                                           | 166           | 1964.69                       | 1971.90             | 7.20        |
| 102           | 271                                           | 169           | 1983.96                       | 1993.59             | 9.64        |
| 103           | 274                                           | 171           | 1999.68                       | 2009.17             | 9.49        |
| 104           | 277                                           | 173           | 2015.26                       | 2026.08             | 10.82       |
| 105           | 281                                           | 176           | 2033.80                       | 2047.28             | 13.48       |
| 106           | 284                                           | 178           | 2049.04                       | 2063.96             | 14.92       |
| 107           | 287                                           | 180           | 2064.14                       | 2079.10             | 14.96       |
Table 3. Isotopic binding energy of $Z=20$

| Proton number | Mass number | Neutron number | Estimated binding energy (MeV) | SEMF binding energy | Error (MeV) |
|---------------|-------------|----------------|-------------------------------|---------------------|-------------|
| 20            | 40          | 20             | 343.58                        | 339.70              | -3.88       |
| 20            | 41          | 21             | 351.76                        | 350.10              | -1.65       |
| 20            | 42          | 22             | 359.91                        | 363.19              | 3.28        |
| 20            | 43          | 23             | 368.02                        | 371.57              | 3.55        |
| 20            | 44          | 24             | 376.10                        | 382.69              | 6.59        |
| 20            | 45          | 25             | 384.15                        | 389.32              | 5.18        |
| 20            | 46          | 26             | 392.16                        | 398.73              | 6.57        |
| 20            | 47          | 27             | 400.14                        | 403.85              | 3.70        |
| 20            | 48          | 28             | 408.09                        | 411.76              | 3.67        |
| 20            | 49          | 29             | 416.00                        | 415.54              | -0.46       |
| 20            | 50          | 30             | 423.88                        | 422.13              | -1.75       |

Table 4. Isotopic binding energy of $Z=28$

| Proton number | Mass number | Neutron number | Estimated binding energy (MeV) | SEMF binding energy | Error (MeV) |
|---------------|-------------|----------------|-------------------------------|---------------------|-------------|
| 28            | 58          | 30             | 508.33                        | 501.75              | -6.58       |
| 28            | 59          | 31             | 516.39                        | 511.65              | -4.74       |
| 28            | 60          | 32             | 524.41                        | 523.97              | -0.44       |
| 28            | 61          | 33             | 532.40                        | 532.52              | 0.11        |
| 28            | 62          | 34             | 540.36                        | 543.50              | 3.13        |
| 28            | 63          | 35             | 548.28                        | 550.82              | 2.53        |
| 28            | 64          | 36             | 556.17                        | 560.58              | 4.41        |
| 28            | 65          | 37             | 564.03                        | 566.79              | 2.76        |
| 28            | 66          | 38             | 571.85                        | 575.45              | 3.60        |
| 28            | 67          | 39             | 579.63                        | 580.65              | 1.01        |
| 28            | 68          | 40             | 587.38                        | 588.30              | 0.91        |
| 28            | 69          | 41             | 595.10                        | 592.57              | -2.53       |
| 28            | 70          | 42             | 602.78                        | 599.30              | -3.48       |
| 28            | 71          | 43             | 610.43                        | 602.74              | -7.70       |
| 28            | 72          | 44             | 618.05                        | 608.62              | -9.43       |

Estimated binding energy can be compared with the standard relation (15). See Fig. 1. Dotted red curve plotted with relations (17) to (22) can be compared with the green curve plotted with the standard semi empirical mass formula (SEMF). For medium and heavy atomic nuclides, it is excellent. It seems that some correction is required for light and super heavy atoms. See Table 2 for the estimated data close to stable mass numbers.
### Table 5. Isotopic binding energy of Z=40

| Proton number | Mass number | Neutron number | Estimated binding energy (MeV) | SEMF binding energy | Error (MeV) |
|---------------|-------------|----------------|-------------------------------|---------------------|-------------|
| 40            | 86          | 46             | 746.59                        | 740.40              | -6.19       |
| 40            | 87          | 47             | 754.15                        | 749.73              | -4.41       |
| 40            | 88          | 48             | 761.66                        | 761.18              | -0.48       |
| 40            | 89          | 49             | 769.15                        | 769.63              | 0.48        |
| 40            | 90          | 50             | 776.59                        | 780.20              | 3.60        |
| 40            | 91          | 51             | 784.01                        | 787.83              | 3.82        |
| 40            | 92          | 52             | 791.39                        | 797.57              | 6.19        |
| 40            | 93          | 53             | 798.73                        | 804.43              | 5.70        |
| 40            | 94          | 54             | 806.04                        | 813.41              | 7.37        |
| 40            | 95          | 55             | 813.32                        | 819.55              | 6.23        |
| 40            | 96          | 56             | 820.56                        | 827.80              | 7.24        |
| 40            | 97          | 57             | 827.76                        | 833.27              | 5.50        |
| 40            | 98          | 58             | 834.94                        | 840.84              | 5.91        |
| 40            | 99          | 59             | 842.07                        | 845.67              | 3.60        |
| 40            | 100         | 60             | 849.18                        | 852.61              | 3.44        |
| 40            | 101         | 61             | 856.24                        | 856.85              | 0.61        |
| 40            | 102         | 62             | 863.28                        | 863.18              | -0.09       |
| 40            | 103         | 63             | 870.28                        | 866.86              | -3.41       |
| 40            | 104         | 64             | 877.24                        | 872.63              | -4.61       |
| 40            | 105         | 65             | 884.17                        | 875.78              | -8.39       |
| 40            | 106         | 66             | 891.06                        | 881.02              | -10.05      |

### Table 6. Isotopic binding energy of Z=50

| Proton number | Mass number | Neutron number | Estimated binding energy (MeV) | SEMF binding energy | Error (MeV) |
|---------------|-------------|----------------|-------------------------------|---------------------|-------------|
| 50            | 110         | 60             | 940.78                        | 931.76              | -9.02       |
| 50            | 111         | 61             | 947.84                        | 940.74              | -7.11       |
| 50            | 112         | 62             | 954.88                        | 951.65              | -3.23       |
| 50            | 113         | 63             | 961.88                        | 959.96              | -1.92       |
| 50            | 114         | 64             | 968.84                        | 970.20              | 1.36        |
| 50            | 115         | 65             | 975.77                        | 977.88              | 2.11        |
| 50            | 116         | 66             | 982.66                        | 987.48              | 4.81        |
| 50            | 117         | 67             | 989.52                        | 994.55              | 5.03        |
| 50            | 118         | 68             | 996.35                        | 1003.55             | 7.20        |
| 50            | 119         | 69             | 1003.14                       | 1010.05             | 6.91        |
| 50            | 120         | 70             | 1009.90                       | 1018.47             | 8.57        |
| 50            | 121         | 71             | 1016.62                       | 1024.43             | 7.81        |
| 50            | 122         | 72             | 1023.31                       | 1032.30             | 9.00        |
| 50            | 123         | 73             | 1029.96                       | 1037.75             | 7.79        |
| 50            | 124         | 74             | 1036.58                       | 1045.10             | 8.52        |
| 50            | 125         | 75             | 1043.16                       | 1050.05             | 6.89        |
| 50            | 126         | 76             | 1049.71                       | 1056.91             | 7.19        |
| 50            | 127         | 77             | 1056.23                       | 1061.39             | 5.16        |
| 50            | 128         | 78             | 1062.71                       | 1067.77             | 5.07        |
| 50            | 129         | 79             | 1069.15                       | 1071.81             | 2.66        |
| 50            | 130         | 80             | 1075.56                       | 1077.74             | 2.18        |
| 50            | 131         | 81             | 1081.94                       | 1081.35             | -0.59       |
| 50            | 132         | 82             | 1088.28                       | 1086.85             | -1.43       |
| 50            | 133         | 83             | 1094.59                       | 1090.06             | -4.53       |
| 50            | 134         | 84             | 1100.86                       | 1095.15             | -5.72       |
| 50            | 135         | 85             | 1107.10                       | 1097.96             | -9.14       |
Table 7. Isotopic binding energy of Z=66

| Proton number | Mass number | Neutron number | Estimated binding energy (MeV) | SEMF binding energy | Error (MeV) |
|---------------|-------------|----------------|--------------------------------|---------------------|-------------|
| 66            | 154         | 88             | 1272.17                        | 1262.97             | -9.20       |
| 66            | 155         | 89             | 1278.26                        | 1270.46             | -7.81       |
| 66            | 156         | 90             | 1284.33                        | 1279.65             | -4.68       |
| 66            | 157         | 91             | 1290.36                        | 1286.71             | -3.65       |
| 66            | 158         | 92             | 1296.36                        | 1295.47             | -0.89       |
| 66            | 159         | 93             | 1302.32                        | 1302.10             | -0.21       |
| 66            | 160         | 94             | 1308.24                        | 1310.44             | 2.19        |
| 66            | 161         | 95             | 1314.14                        | 1316.67             | 2.54        |
| 66            | 162         | 96             | 1319.99                        | 1324.60             | 4.61        |
| 66            | 163         | 97             | 1325.82                        | 1330.45             | 4.63        |
| 66            | 164         | 98             | 1331.61                        | 1337.98             | 6.38        |
| 66            | 165         | 99             | 1337.36                        | 1343.46             | 6.10        |
| 66            | 166         | 100            | 1343.08                        | 1350.61             | 7.53        |
| 66            | 167         | 101            | 1348.76                        | 1355.73             | 6.96        |
| 66            | 168         | 102            | 1354.41                        | 1362.52             | 8.10        |
| 66            | 169         | 103            | 1360.03                        | 1367.28             | 7.25        |
| 66            | 170         | 104            | 1365.61                        | 1373.72             | 8.11        |
| 66            | 171         | 105            | 1371.16                        | 1378.15             | 7.00        |
| 66            | 172         | 106            | 1376.67                        | 1384.25             | 7.58        |
| 66            | 173         | 107            | 1382.15                        | 1388.36             | 6.21        |
| 66            | 174         | 108            | 1387.59                        | 1394.13             | 6.54        |
| 66            | 175         | 109            | 1393.00                        | 1397.92             | 4.93        |
| 66            | 176         | 110            | 1398.37                        | 1403.38             | 5.01        |
| 66            | 177         | 111            | 1403.71                        | 1406.87             | 3.16        |
| 66            | 178         | 112            | 1409.01                        | 1412.02             | 3.01        |
| 66            | 179         | 113            | 1414.28                        | 1415.22             | 0.94        |
| 66            | 180         | 114            | 1419.52                        | 1420.07             | 0.56        |
| 66            | 181         | 115            | 1424.72                        | 1422.99             | -1.72       |
| 66            | 182         | 116            | 1429.88                        | 1427.56             | -2.33       |
| 66            | 183         | 117            | 1435.02                        | 1430.21             | -4.81       |
| 66            | 184         | 118            | 1440.11                        | 1434.49             | -5.62       |
| 66            | 185         | 119            | 1445.17                        | 1436.88             | -8.30       |
| 66            | 186         | 120            | 1450.20                        | 1440.90             | -9.31       |

From the above Table 2 or Fig. 1, proposed relation (22) can be validated. With reference to unification paradigm, authors new approach seems to be more informative than the recent works of Ghahramany et al [10,11]. Advantage of relation (22) is that it constitutes only one energy coefficient and two simple terms. On applying the proposed relations (17) to (22) to \(A_{S} \) and \(A_{L}\), authors noticed significant errors. See Tables 3 to 9 for the estimated isotopic binding energy of Z = 20, 28, 40, 50, 66, 82 and 100 respectively.

C. Unified approach-2

Based on the above data, believing in the workability of the number 0.00189 and to improve the accuracy in estimation of binding energy of isotopes, authors developed the following 5 term expression with single energy coefficient. Physics behind it can be understood in the following way.

Energy coefficient being 10.09 MeV, nuclear binding energy:

1. Increases with increasing mass number. (Term-1)
2. Decreases with increasing radius. (Term-2)
3. Decreases with the ratio of proton number to neutron number. (Term-3)
4. Decreases with \(A/\sqrt{ZN}\) where proportionality coefficient is 0.00189. (Term-4).
5. Stable mass number plays a key role in estimating the isotopic binding energy of Z. (Term-5)
Table 8. Isotopic binding energy of Z=82

| Proton number | Mass number | Neutron number | Estimated binding energy (MeV) | SEMF binding energy | Error (MeV) |
|---------------|-------------|----------------|---------------------------------|--------------------|-------------|
| 82            | 202         | 120            | 1596.76                         | 1587.37            | -9.39       |
| 82            | 203         | 121            | 1601.76                         | 1593.56            | -8.20       |
| 82            | 204         | 122            | 1606.71                         | 1601.28            | -5.44       |
| 82            | 205         | 123            | 1611.64                         | 1607.16            | -4.47       |
| 82            | 206         | 124            | 1616.53                         | 1614.58            | -1.95       |
| 82            | 207         | 125            | 1621.38                         | 1620.17            | -1.21       |
| 82            | 208         | 126            | 1626.20                         | 1627.29            | 1.08        |
| 82            | 209         | 127            | 1630.99                         | 1632.59            | 1.60        |
| 82            | 210         | 128            | 1635.74                         | 1639.42            | 3.68        |
| 82            | 211         | 129            | 1640.45                         | 1644.45            | 3.99        |
| 82            | 212         | 130            | 1645.14                         | 1650.99            | 5.86        |
| 82            | 213         | 131            | 1649.78                         | 1655.75            | 5.97        |
| 82            | 214         | 132            | 1654.40                         | 1662.03            | 7.63        |
| 82            | 215         | 133            | 1658.97                         | 1666.53            | 7.55        |
| 82            | 216         | 134            | 1663.52                         | 1672.53            | 9.01        |
| 82            | 217         | 135            | 1668.03                         | 1676.78            | 8.75        |
| 82            | 218         | 136            | 1672.50                         | 1682.52            | 10.02       |
| 82            | 219         | 137            | 1676.94                         | 1686.52            | 9.58        |
| 82            | 220         | 138            | 1681.35                         | 1692.02            | 10.67       |
| 82            | 221         | 139            | 1685.72                         | 1695.77            | 10.06       |
| 82            | 222         | 140            | 1690.05                         | 1701.03            | 10.97       |
| 82            | 223         | 141            | 1694.35                         | 1704.55            | 10.19       |
| 82            | 224         | 142            | 1698.62                         | 1709.56            | 10.94       |
| 82            | 225         | 143            | 1702.85                         | 1712.86            | 10.00       |
| 82            | 226         | 144            | 1707.05                         | 1717.64            | 10.59       |
| 82            | 227         | 145            | 1711.21                         | 1720.71            | 9.49        |
| 82            | 228         | 146            | 1715.34                         | 1725.26            | 9.92        |
| 82            | 229         | 147            | 1719.44                         | 1728.12            | 8.68        |
| 82            | 230         | 148            | 1723.50                         | 1732.46            | 8.96        |
| 82            | 231         | 149            | 1727.52                         | 1735.10            | 7.58        |
| 82            | 232         | 150            | 1731.51                         | 1739.22            | 7.71        |
| 82            | 233         | 151            | 1735.47                         | 1741.67            | 6.20        |
| 82            | 234         | 152            | 1739.39                         | 1745.58            | 6.19        |
| 82            | 235         | 153            | 1743.28                         | 1747.82            | 4.54        |
| 82            | 236         | 154            | 1747.13                         | 1751.53            | 4.40        |
| 82            | 237         | 155            | 1750.94                         | 1753.58            | 2.63        |
| 82            | 238         | 156            | 1754.73                         | 1757.08            | 2.36        |
| 82            | 239         | 157            | 1758.48                         | 1758.94            | 0.47        |
| 82            | 240         | 158            | 1762.19                         | 1762.26            | 0.07        |
| 82            | 241         | 159            | 1765.87                         | 1763.93            | -1.94       |
| 82            | 242         | 160            | 1769.51                         | 1767.06            | -2.45       |
| 82            | 243         | 161            | 1773.12                         | 1768.55            | -4.57       |
| 82            | 244         | 162            | 1776.70                         | 1771.49            | -5.20       |
| 82            | 245         | 163            | 1780.24                         | 1772.81            | -7.43       |
| 82            | 246         | 164            | 1783.74                         | 1775.57            | -8.17       |
| 82            | 247         | 165            | 1787.22                         | 1776.72            | -10.50      |

Based on these points, for \((Z \approx 3 \text{ to } 118)\),
\begin{align}
B_d & \equiv \left[ A - A^{1/3} - \frac{Z}{N} - \frac{kA\sqrt{ZN}}{3.4} - \frac{(A_s - A)^2}{A_s} \right] \times 10.09 \text{ MeV} \\
& \equiv \left[ (1 - 0.00189\sqrt{ZN}) A - A^{1/3} - \frac{Z}{N} - \frac{(A_s - A)^2}{A_s} \right] \times 10.09 \text{ MeV} 
\end{align}

Close to the stable mass number,
\begin{align}
B_d & \equiv \left[ A - A_s^{1/3} - \frac{Z}{N_s} - \frac{kA_s\sqrt{ZN_s}}{3.4} \right] \times 10.09 \text{ MeV} \\
& \equiv \left[ (1 - 0.00189\sqrt{ZN_s}) A_s - A_s^{1/3} - \frac{Z}{N_s} \right] \times 10.09 \text{ MeV} 
\end{align}

Note points:
1) First three terms play a key role in estimating the binding energy of light atomic nuclides.
2) Term-1 and Term-4, both can be clubbed into a single term as, \( A(1 - 0.00189\sqrt{ZN}) \) and can be called as “Screened mass number”. The coefficient \( \frac{k}{\ln(30)} \approx \frac{k}{3.4} \approx 0.00189 \) can be called as 'Mass number screening factor'.
3) Rather than the mass number, binding energy can be assumed to be proportional to the screened mass number.

\textbf{Table 9. Isotopic binding energy of } Z = 100 \\

| Proton number | Mass number | Neutron number | Estimated binding energy (MeV) | SEMF binding energy | Error (MeV) |
|---------------|-------------|----------------|-------------------------------|---------------------|------------|
| 100           | 256         | 156            | 1919.61                       | 1909.69             | -9.92      |
| 100           | 257         | 157            | 1923.36                       | 1915.13             | -8.22      |
| 100           | 258         | 158            | 1927.07                       | 1921.96             | -5.11      |
| 100           | 259         | 159            | 1930.75                       | 1927.18             | -3.57      |
| 100           | 260         | 160            | 1934.39                       | 1933.78             | -0.61      |
| 100           | 261         | 161            | 1938.00                       | 1938.78             | 0.78       |
| 100           | 262         | 162            | 1941.58                       | 1945.16             | 3.58       |
| 100           | 263         | 163            | 1945.12                       | 1949.94             | 4.82       |
| 100           | 264         | 164            | 1948.62                       | 1956.10             | 7.48       |
| 100           | 265         | 165            | 1952.10                       | 1960.68             | 8.58       |

See Fig. 2. Dashed red curve plotted with relations (11) and (24) can be compared with the green curve plotted with the standard relation (15). For light, medium and heavy atomic nuclides, fit is reasonable.

See Figs. 3 to 11 for the estimated isotopic binding energy of Z = 20, 30, 40, 50, 60, 70, 80, 90, and 100 respectively. Dotted blue curve represents the estimated binding energy with relations (11) and (23). Green curve represents the binding energy estimated with standard relation (15).

Based on these Figs. 2 to 11, it is possible to say that,

1) Relations (23) and (24) can also be given some priority in understanding nuclear binding energy scheme.
2) For \( N < Z \) and \( N \approx Z \) estimated binding energy seems to be increasing compared to SEMF estimation.
3) For \( A >> A_s \), estimated binding energy seems to be decreasing compared to SEMF estimation.
Fig. 2. Binding energy per nucleon close to stable mass numbers of $Z = 3$ to $118$.

Fig. 3. Isotopic binding energy of $Z=20$.

Fig. 4. Isotopic binding energy of $Z=30$. 
Fig. 5. Isotopic binding energy of $Z=40$

Fig. 6. Isotopic binding energy of $Z=50$

Fig. 7. Isotopic binding energy of $Z=60$
Fig. 8. Isotopic binding energy of Z=70

Fig. 9. Isotopic binding energy of Z=80

Fig. 10. Isotopic binding energy of Z=90
4) Fine tuning seems to be required in the terms \( \left( \frac{Z}{N} \right) \) and \( \left( \frac{A_s - A}{A_s} \right) \) of relation (23).

6. UNDERSTANDING NEUTRON LIFE TIME WITH ELECTROMAGNETIC AND WEAK GRAVITATIONAL CONSTANTS

One of the key objectives of any unified description is to simplify or eliminate the complicated issues of known physics. In this context, in a quantitative approach, authors noticed that, electromagnetic and weak gravitational constants play a crucial role in understanding and estimating neutron life time \([22,54]\). The following strange relation can be given some consideration.

\[
t_n \equiv \left( \frac{G_e}{G_w} \right) \left( \frac{G^2 m_n^3}{(m_n - m_p)c^3} \right) \equiv \left( \frac{G^2 m_n^2}{G_w(m_n - m_p)c^3} \right) \approx 874.94 \text{ sec} \tag{25}
\]

Plausible point to be noted is that, relativistic mass of neutron seems to play a crucial role in understanding the increasing neutron life time. It can be understood with,

\[
t_n \propto \frac{m_n^2}{\left[ 1 - (v^2/c^2) \right]} \quad \text{and} \quad t_n \approx \frac{874.94 \text{ sec}}{\left[ 1 - (v^2/c^2) \right]} \tag{26}
\]

7. NUCLEAR CHARGE RADIUS

As per the current literature \([55]\), nuclear charge radii can be expressed with the following formulae.

\[
R \equiv \left[ 1 + 0.015 \left( \frac{N - (N/Z)}{Z} \right) \right] \times 1.245 \text{ fm} \tag{27}
\]

\[
R \equiv \left[ 1 - 0.349 \left( \frac{N - Z}{N} \right) \right] \times 1.262 \text{ fm} \tag{28}
\]

\[
R \equiv \left[ 1 - 0.182 \left( \frac{N - Z}{A} \right) \right] + \frac{1.652}{A} \times 0.966 \text{ fm} \tag{29}
\]

Our earlier proposed relation \([26]\) is,

\[
R_{(Z,A)} \equiv \left[ \frac{Z^{1/3}}{a} + \frac{(A - Z)^{1/3}}{a} \right] \left( \frac{G_c m_p}{c^3} \right) \tag{30}
\]

Based on these relations and by considering the charge radii of stable atomic nuclides, \(R_s\) and \(G_s\) can be fitted.
8. RESULTS AND DISCUSSION

Based on the data presented in Tables 1 to 9 and Figs. 1 to 11, authors would like to suggest that,

1) From the proposed relations, authors recent and earlier works and from Ghahramany’s integrated nuclear model [10,11], it is very clear that, nuclear binding energy can be understood with a single unified energy coefficient.

2) Close to stable mass numbers, squared neutron number plays a major role in reducing major part of nuclear binding energy.

3) The number, $\frac{k}{3.4} \approx 0.00189$ seems to play a very interesting role in estimating neutron-proton stability. Hence it can also be considered as a characteristic result oriented number in the context of understanding nuclear stability.

4) The number, $\frac{k}{3.4} \approx 0.00189$ seems to play a very interesting role in estimating the major reduction part of nuclear binding energy. Hence it can also be considered as a characteristic result oriented number in the context of understanding nuclear binding energy.

5) Authors are working on understanding the physics connected with $Z = 30$. Application point of view, it is quite interesting to note that relation (23) of the proposed unified approach-2 is quite different from relation (22) of unified approach-1.

6) Based on the relations (17) to (24) and by modifying the terms, $\left(\frac{Z}{N}\right)$ and $\left(\frac{(A_s-A)}{A_s}\right)^2$

binding energy for $(A \ll A_s)$ and $(A \gg A_s)$, can be understood and semi empirical mass formula can be modified into a much more simple form.

9. CONCLUSION

1) Understanding nuclear binding energy with a single energy coefficient and two simple terms in terms of fundamental interactions is a very challenging task. In this context, authors tried their level best in presenting a very simple and effective semi empirical formula with one unique energy coefficient. It needs further investigation.

2) Current unification paradigm is failing in developing a ‘practical unification procedure’. Even though our approach is speculative, role played by the four gravitational constants seems to be fairly natural. This kind of approach may help in producing a variety of such relations by using which in near future, an absolute set of relations can be developed. Proceeding further, estimated absolute theoretical value of $G_s$ can be considered as a standard reference for future experiments.

3) By implementing four such gravitational constants in String theory models, it may be possible to explore the hidden unified physics. With further study, a practical model of materialistic quantum gravity can be developed and magnitude of the Newtonian gravitational constant can be estimated in a theoretical approach bound to Fermi scale.

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COMPETING INTERESTS

Authors have declared that no competing interests exist.

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