Lepton polarization effects in $\Lambda_b \rightarrow \Lambda \ell^+ \ell^-$ decay in family non–universal $Z'$ model

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Abstract

Possible manifestation of the family non–universal $Z'$ boson effects in lepton polarization in rare, exclusive baryonic $\Lambda_b \rightarrow \Lambda \ell^+ \ell^-$ decay is examined. It is observed that the double lepton polarizations $P_{TT}$ and $P_{NN}$ are sensitive to the $Z'$ contribution. Moreover, it is found that the zero position of the polarized forward–backward asymmetry $A_{FB}^{LL}$ is shifted to the left compared to the standard model prediction. Therefore, determination of the zero value of $A_{FB}^{LL}$ is quite an efficient tool for establishing new $Z'$ boson, but also in discriminating various scenarios of the considered family non–universal $Z'$ model.

PACS numbers: 13.30.–a, 14.20.Mr

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1 Introduction

Investigation of the rare decays described by the $b \rightarrow s(d)$ transitions represents one of the main directions of high energy physics. The attractive property of these decays is that they are forbidden at tree level in the Standard Model (SM) and appear only at loop level. Therefore these decays are quite promising for checking gauge structure of the theory at quantum level. These decays are also excellent candidates in search of new physics beyond the SM.

Rare decays in the $B$–meson sector described by $b \rightarrow s(d)$ transitions have been studied theoretically (see for example [1] and references therein) and experimentally in detail (see for example [2]).

Exclusive $\Lambda_b \rightarrow \Lambda \ell^+\ell^-$, $\Lambda_b \rightarrow \Lambda \gamma$ decays in baryonic sector, which are described by $b \rightarrow s$ transition are also very interesting. The main advantage of these baryonic decays is that, unlike mesonic decays, they can give information about the helicity structure of the effective Hamiltonian [3].

The baryonic decays $\Lambda_b \rightarrow \Lambda \ell^+\ell^-$, $\Lambda_b \rightarrow \Lambda \gamma$, $\Lambda_b \rightarrow \Lambda \bar{\nu}\nu$ induced by the flavor changing neutral current (FCNC) are studied comprehensively in many works [2, 4–11]. The first step in experimental investigation of rare baryonic decays has recently been taken by the CDF Collaboration, and they announced the observation of the baryonic rare $\Lambda_b \rightarrow \Lambda \mu^+\mu^-$ decay. LHC–b Collaboration is planning to study this decay in the near future [13]. The experimental observation of this decay has stimulated researches for a more refined theoretical analysis of this subject.

As has already been noted, rare decays induced by $b \rightarrow s$ transition are quite promising for checking prediction of the SM and searching new physics beyond the SM. In this sense, the physical observables like branching ratio, forward–backward asymmetry $A_{FB}$, single and double lepton polarization effects, polarized forward–backward asymmetry are very useful.

Recently we have studied the rare $\Lambda_b \rightarrow \Lambda \ell^+\ell^-$ decay within non–universal $Z'$ model [14]. The sensitivities of the branching ratio, forward–backward asymmetry, and asymmetry parameters due to the polarization of the $\Lambda$ and $\Lambda_b$ baryons, on $Z'$ model parameters are investigated in detail.

In the present work we perform an analysis of the single and double lepton polarization effects, and polarized forward–backward asymmetries in the framework of the non–universal $Z'$ model developed in [15]. It should also be noted here that, so far, the effects of non–universal $Z'$ model in the $B$–meson sector have been studied in many works [16–18].

The outline of the paper is as follows. In section 2 we present the effective Hamiltonian responsible for the $b \rightarrow s\ell^+\ell^-$ transition. In this section we also present the matrix element for the $\Lambda_b \rightarrow \Lambda \ell^+\ell^-$ decay, and expressions of the polarized forward–backward asymmetries in the $Z'$ model. In section 3 the numerical results of these physical observables are given.

2 Theoretical framework

Neglecting doubly Cabibbo–suppressed contribution, the effective Hamiltonian responsible for the $b \rightarrow s\ell^+\ell^-$ transition at $\mu = O(m_b)$ scale is given as [19] (see also the first reference
in [1]),

$$H_{eff} = -\frac{4G_F}{\sqrt{2}} V_{tb} V_{ts}^* \sum_{i=1}^{10} C_i(\mu) O_i(\mu) . \quad (1)$$

The expressions of the local operators \( O_i(\mu) \) can be found in [19] and the first reference in [1]. The Wilson coefficients are calculated in numerous works (see for example [20] and the references therein). The matrix element for the \( b \to s\ell^+\ell^- \) transition in SM is given by,

$$M = \frac{G_F \alpha_{em}}{2 \sqrt{2} \pi} V_{tb} V_{ts}^* \left[ C_9^{eff} s_\gamma \mu (1 - \gamma_5) b \bar{\ell} \gamma_\mu \ell + C_{10} s_\gamma \mu (1 - \gamma_5) b \bar{\ell} \gamma_\mu \gamma_5 \ell \right. - 2m_b C_7 s_\sigma \mu \nu \frac{q}{q^2} (1 + \gamma_5) b \bar{\ell} \gamma_\mu \ell \left. \right], \quad (2)$$

where \( G_F \) is the Fermi constant, \( \alpha_{em} \) is the fine structure constant, \( C_9^{eff}, C_{10} \) and \( C_7 \) are the relevant Wilson coefficients. \( V_{ij} \) are the elements of Kobayashi–Maskawa matrix.

The family non–universal \( Z' \) model considered in this work could lead to FCNC at tree level, as well as to the appearance of new weak phases. Appearance of FCNS at tree level can be attributed to the non–diagonal chiral coupling matrix. Assuming that the couplings of right–handed quarks with \( Z' \) boson are flavor diagonal, and neglecting \( Z - Z' \) mixing, the \( Z' \) part of the effective Hamiltonian is given by,

$$H_{eff}^{Z'} = \frac{2G_F}{\sqrt{2}} V_{tb} V_{ts}^* \left[ B_L^b B_L^{\ell\ell} s_\gamma \mu (1 - \gamma_5) b \bar{\ell} \gamma_\mu \ell + B_L^b B_R^{\ell\ell} s_\gamma \mu (1 + \gamma_5) b \bar{\ell} \gamma_\mu \ell \right] , \quad (3)$$

which can be rewritten as,

$$H_{eff}^{Z'} = -\frac{4G_F}{\sqrt{2}} V_{tb} V_{ts}^* \left( C_9^{Z'} O_9 + C_{10}^{Z'} O_{10} \right) , \quad (4)$$

where

$$C_9^{Z'} = -\frac{g_2^2}{e^2} \frac{B_L^b B_L^{\ell\ell}}{V_{tb} V_{ts}^*} S_{LL}^{LR} ,$$

$$C_{10}^{Z'} = \frac{g_2^2}{e^2} \frac{B_L^b B_R^{\ell\ell}}{V_{tb} V_{ts}^*} D_{LL}^{LR} , \quad (5)$$

and,

$$S_{LL}^{LR} = \left( B_L^{\ell\ell} + B_R^{\ell\ell} \right) ,$$

$$D_{LL}^{LR} = \left( B_L^{\ell\ell} - B_R^{\ell\ell} \right) . \quad (6)$$

The off–diagonal element \( B_L^{s_b} \) might contain a new phase, and therefore can be written as \( |B_L^{s_b}| e^{i\varphi} \).
The essential point of this model is that $Z'$ contribution does not lead to the appearance of any new operators that exist in the SM, and its contribution modifies the Wilson coefficients $C_9$ and $C_{10}$. As a result, in order to take $Z'$ effects into account it is enough to make the following replacements in Eq. (2),

$$C_{9}^{\text{eff}} \rightarrow C_{9}^{\text{eff}} - \frac{4\pi}{\alpha_S} (28.82) \frac{B_{tb}^L}{V_{tb} V_{ts}^*} \frac{C_{\ell \ell}^{LR}}{\ell \ell} = C_{9}^{\text{tot}} \ ,$$

$$C_{10} \rightarrow C_{10} + \frac{4\pi}{\alpha_S} (28.82) \frac{B_{tb}^L}{V_{tb} V_{ts}^*} D_{\ell \ell}^{LR} = C_{10}^{\text{tot}} \ .$$ (7)

Our next task is to obtain the amplitude of the exclusive $\Lambda_b \rightarrow \Lambda \ell^+ \ell^-$ decay. For this purpose we sandwich Eq. (2) between initial and final baryon states. Obviously, we need to determine the matrix elements,

$$\langle \Lambda(p) | \bar{s} \gamma_{\mu}(1 - \gamma_5)b| \Lambda_b(p_B) \rangle \ ,$$

$$\langle \Lambda(p) | \bar{s} i \sigma_{\mu \nu} q''(1 + \gamma_5)b| \Lambda_b(p_B) \rangle \ .$$

These matrix elements are parametrized in terms of the form factors as follows,

$$\langle \Lambda(p) | \bar{s} \gamma_{\mu}(1 - \gamma_5)b| \Lambda_b(p_B) \rangle = \bar{u}_\Lambda(p) \left[ f_1 (q^2) \gamma_{\mu} + i f_2 (q^2) \sigma_{\mu \nu} q'' \right. $$

$$+ \left. f_3 (q^2) q_\mu - g_1 (q^2) \gamma_5 \gamma_{\mu} - i g_2 (q^2) \sigma_{\mu \nu} \gamma_5 q'' - g_3 (q^2) \gamma_5 q_\mu \right] u_{\Lambda_b}(p_B) \ ,$$ (8)

$$\langle \Lambda(p) | \bar{s} i \sigma_{\mu \nu} q''(1 + \gamma_5)b| \Lambda_b(p_B) \rangle = \bar{u}_\Lambda(p) \left[ i f_2^T (q^2) \gamma_{\mu} + i f_2^T (q^2) \sigma_{\mu \nu} q'' \right. $$

$$+ \left. f_3^T (q^2) q_\mu + g_1^T (q^2) \gamma_5 \gamma_{\mu} + i g_2^T (q^2) \sigma_{\mu \nu} \gamma_5 q'' + g_3^T (q^2) \gamma_5 q_\mu \right] u_{\Lambda_b}(p_B) \ ,$$ (9)

where $q^2 = (p_B - p_\Lambda)^2$ and $f_i$, $g_i$, $f_i^T$, $g_i^T$ are the form factors responsible for the $\Lambda_b \rightarrow \Lambda$ transition.

Using Eqs. (7)–(9), one can easily obtain the matrix element of the $\Lambda_b \rightarrow \Lambda \ell^+ \ell^-$ decay which is given by

$$M = \frac{G_F \alpha_{em}}{4\sqrt{2}\pi} V_{tb} V_{ts}^* \left\{ \bar{\ell} \gamma_{\mu} \ell \bar{u}_\Lambda(p) \left[ A_1 \gamma_{\mu}(1 + \gamma_5) + B_1 \gamma_{\mu}(1 - \gamma_5) + i \sigma_{\mu \nu} q'' \left( A_2 (1 + \gamma_5) \right. \right. $$

$$\left. \left. + B_2 (1 - \gamma_5) \right) + q_\mu \left( A_3 (1 + \gamma_5) + B_3 (1 - \gamma_5) \right) \right] u_{\Lambda_b}(p_B) $$

$$+ \bar{\ell} \gamma_{\mu} \gamma_5 \ell \bar{u}_\Lambda(p) \left[ D_1 \gamma_{\mu}(1 + \gamma_5) + E_1 \gamma_{\mu}(1 - \gamma_5) + i \sigma_{\mu \nu} q'' \left( D_2 (1 + \gamma_5) + E_2 (1 - \gamma_5) \right) \right. $$

$$\left. \left. + q_\mu \left( D_3 (1 + \gamma_5) + E_3 (1 - \gamma_5) \right) \right] u_{\Lambda_b}(p_B) \right\} \ ,$$ (10)

where

$$A_1 = - \frac{2m_b}{q^2} C_7 (f_1^T + g_1^T) + C_{9}^{\text{tot}} (f_1 - g_1) \ ,$$

$$A_2 = A_1 (1 \rightarrow 2) \ , \ A_3 = A_1 (1 \rightarrow 3) \ ,$$

$$B_i = A_i (g_i \rightarrow -g_i, g_i^T \rightarrow -g_i^T) \ ,$$

$$D_i = C_{10}^{\text{tot}} (f_1 - g_1) \ , \ D_2 \rightarrow D_4 (1 \rightarrow 2) \ , \ D_3 \rightarrow D_4 (1 \rightarrow 3) \ ,$$

$$E_i = D_i (g_i \rightarrow -g_i) \ .$$

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The matrix element for the $\Lambda_b \to \Lambda \ell^+ \ell^-$ decay given in Eq. (10) is the starting for us for all further discussion. In order to calculate the double lepton polarization effects, we introduce the orthogonal unit vectors $s_{i}^{\pm \mu}$ in the rest frame of leptons,

\[
\begin{align*}
    s_{L}^{-\mu} &= (0, \vec{e}_{L}) = \left(0, \frac{\vec{p}_{-}}{|\vec{p}_{-}|}\right), \\
    s_{N}^{-\mu} &= (0, \vec{e}_{N}) = \left(0, \frac{\vec{p}_{\Lambda} \times \vec{p}_{-}}{|\vec{p}_{\Lambda} \times \vec{p}_{-}|}\right), \\
    s_{T}^{-\mu} &= (0, \vec{e}_{T}) = (0, \vec{e}_{L} \times \vec{e}_{N}).
\end{align*}
\]

The unit vectors for the polarizations of $\ell^+$ lepton can be obtained from Eq. (11) by making the replacement $\vec{p}_{-} \to \vec{p}_{+}$. Here, $\vec{p}_{-}(\vec{p}_{+})$ and $\vec{p}_{\Lambda}$ are the three momenta of the $\ell^-(\ell^+)$ lepton and $\Lambda$ baryon in the center of mass frame (CM) of the lepton pair. Transformation of the unit vector $s_{i}^{\pm \mu}$ from rest frame to CM of the leptons can done by Lorentz boosting. It should be noted here that, in performing Lorentz boosts transversal and normal components are unchanged, and only longitudinal component $s_{L}^{-\mu}$ is transformed. As a result we get,

\[
\frac{d\Gamma(s_{i}^{-}, s_{j}^{+})}{ds} = \frac{d\Gamma(-s_{i}^{-}, -s_{j}^{+})}{ds} - \frac{d\Gamma(s_{i}^{-}, s_{j}^{+})}{ds} - \frac{d\Gamma(-s_{i}^{-}, -s_{j}^{+})}{ds} - \frac{d\Gamma(s_{i}^{-}, -s_{j}^{+})}{ds} + \frac{d\Gamma(-s_{i}^{-}, s_{j}^{+})}{ds} + \frac{d\Gamma(-s_{i}^{-}, -s_{j}^{+})}{ds} + \frac{d\Gamma(s_{i}^{-}, -s_{j}^{+})}{ds} + \frac{d\Gamma(-s_{i}^{-}, s_{j}^{+})}{ds}.
\]

Now we are ready to define the double lepton polarizations. Following [21] we define double and single lepton polarizations in the following way,

\[
\begin{align*}
P_{ij}(q^{2}) &= \frac{d\Gamma(s_{i}^{-}, s_{j}^{+})}{ds} - \frac{d\Gamma(-s_{i}^{-}, -s_{j}^{+})}{ds} - \frac{d\Gamma(s_{i}^{-}, -s_{j}^{+})}{ds} + \frac{d\Gamma(-s_{i}^{-}, s_{j}^{+})}{ds}, \\
\end{align*}
\]

\[
\begin{align*}
P_{i}(q^{2}) &= \frac{d\Gamma(s_{i}^{-}, s_{j}^{+})}{ds} - \frac{d\Gamma(-s_{i}^{-})}{ds}.
\end{align*}
\]

The first (second) subindex of $P_{ij}$ represents polarization of lepton (anti–lepton).

In this work we also investigate the polarized forward–backward asymmetries, which are defined as,

\[
\begin{align*}
\mathcal{A}_{FB}^{ij}(s) &= \left(\frac{d\Gamma(s)}{ds}\right)^{-1} \left\{ \int_{0}^{1} dz \int_{-1}^{0} dz \right\} \left\{ \left[ \frac{d^{2}\Gamma(s, s_{i}^{-}, s_{j}^{+})}{dsdz} - \frac{d^{2}\Gamma(s, -s_{i}^{-}, -s_{j}^{+})}{dsdz} \right] \right.
\end{align*}
\]

\[
\begin{align*}
&- \left[ \frac{d^{2}\Gamma(s, s_{i}^{-}, -s_{j}^{+})}{dsdz} - \frac{d^{2}\Gamma(s, -s_{i}^{-}, s_{j}^{+})}{dsdz} \right] \right\}, \\
&= \mathcal{A}_{FB}(s_{i}^{-}, s_{j}^{+}) + \mathcal{A}_{FB}(s_{i}^{-}, -s_{j}^{+}) - \mathcal{A}_{FB}(-s_{i}^{-}, s_{j}^{+}) \\
&+ \mathcal{A}_{FB}(-s_{i}^{-}, -s_{j}^{+}) + \mathcal{A}_{FB}(s_{i}^{-}, s_{j}^{+}) - \mathcal{A}_{FB}(s_{i}^{-}, -s_{j}^{+}) + \mathcal{A}_{FB}(-s_{i}^{-}, -s_{j}^{+}) + \mathcal{A}_{FB}(-s_{i}^{-}, s_{j}^{+}).
\end{align*}
\]

Using the same convention and notations used in [7], for the double lepton polarizations we get,
\[ P_{LL} = \frac{16m_A^4}{3\Delta} \text{Re} \left\{ -6m_{\Lambda_b} \sqrt{\hat{\rho}_\Lambda} (1 - \hat{\rho}_\Lambda + \hat{s}) \left[ \hat{s}(1 + v^2)(A_1A_2^* + B_1B_2^*) - 4\hat{m}_\ell^2(D_1D_3^* + E_1E_3^*) \right] \right. \\
+ 6m_{\Lambda_b}(1 - \hat{\rho}_\Lambda - \hat{s}) \left[ \hat{s}(1 + v^2)(A_1B_2^* + A_2B_1^*) + 4\hat{m}_\ell^2(D_1E_3^* + D_3E_1^*) \right] \\
+ 12\sqrt{\hat{\rho}_\Lambda} \hat{s}(1 + v^2) \left( A_1B_1^* + D_1E_1^* + m_{\Lambda_b}^2 \hat{s}A_2B_2^* \right) \\
+ 12m_{\Lambda_b} \hat{m}_\ell^2 \hat{s}(1 + \hat{\rho}_\Lambda - \hat{s}) \left( |D_3|^2 + |E_3|^2 \right) \\
- (1 + v^2) \left[ 1 + \hat{\rho}_\Lambda^2 - \hat{\rho}_\Lambda(2 - \hat{s}) + \hat{s}(1 - 2\hat{s}) \right] \left( |A_1|^2 + |B_1|^2 \right) \\
- \left[ (5v^2 - 3)(1 - \hat{\rho}_\Lambda)^2 + 4\hat{m}_\ell^2(1 + \hat{\rho}_\Lambda) + 2\hat{s}(1 + 8\hat{m}_\ell^2 + \hat{\rho}_\Lambda) - 4\hat{s}^2 \right] \left( |D_1|^2 + |E_1|^2 \right) \\
- m_{\Lambda_b}^2 \hat{s}(1 + v^2) \left[ 2 + 2\hat{\rho}_\Lambda^2 - \hat{s}(1 + \hat{s}) - \hat{\rho}_\Lambda(4 + \hat{s}) \right] \left( |A_2|^2 + |B_2|^2 \right) \\
- 2m_{\Lambda_b}^2 \hat{s}v^2 \left[ 2(1 + \hat{\rho}_\Lambda^2) - \hat{s}(1 + \hat{s}) - \hat{\rho}_\Lambda(4 + \hat{s}) \right] \left( |D_2|^2 + |E_2|^2 \right) \\
+ 12m_{\Lambda_b} \hat{s}(1 - \hat{\rho}_\Lambda - \hat{s}) v^2(\hat{D}_1\hat{E}_2^* + \hat{D}_2\hat{E}_1^*) \\
- 12m_{\Lambda_b} \sqrt{\hat{\rho}_\Lambda} \hat{s}(1 - \hat{\rho}_\Lambda + \hat{s}) v^2(\hat{D}_1\hat{D}_2^* + \hat{E}_1\hat{E}_2^*) \\
+ 24m_{\Lambda_b}^2 \sqrt{\hat{\rho}_\Lambda} \hat{s}^2(\hat{D}_2\hat{D}_3^* + \hat{D}_3\hat{D}_1^*) \} , \tag{15} \]

\[ P_{LN} = -P_{NL} = \frac{16\pi m_A^4 \hat{m}_\ell \sqrt{\lambda v}}{\Delta \sqrt{\hat{s}}} \text{Im} \left\{ (1 - \hat{\rho}_\Lambda)(A_1^*D_1 + B_1^*E_1) \right. \\
+ m_{\Lambda_b} \hat{s}(A_1^*E_3^* - A_2^*E_1^* + B_1^*D_3^* - B_2^*D_1^*) \\
+ m_{\Lambda_b} \sqrt{\hat{\rho}_\Lambda} \hat{s}(A_1^*D_3^* + A_2^*D_1^* + B_1^*E_3^* + B_2^*E_1^*) - m_{\Lambda_b}^2 \hat{s}^2(B_1^*E_3^* + A_2^*D_3^*) \} , \tag{16} \]

\[ P_{LT} = \frac{16\pi m_A^4 \hat{m}_\ell \sqrt{\lambda v}}{\Delta \sqrt{\hat{s}}} \text{Re} \left\{ (1 - \hat{\rho}_\Lambda) \left( |D_1|^2 + |E_1|^2 \right) - \hat{s} \left( A_1^*D_1^* - B_1^*E_1^* \right) \right. \\
- m_{\Lambda_b} \hat{s} \left[ B_1D_2^* + (A_2 + D_2 + D_3)E_1^* - A_1E_2^* - (B_2 - E_2 + E_3)D_1^* \right] \\
+ m_{\Lambda_b}^2 \hat{s}(1 - \hat{\rho}_\Lambda)(A_2D_2^* - B_2E_2^*) \\
+ m_{\Lambda_b} \sqrt{\hat{\rho}_\Lambda} \hat{s} \left[ A_1D_2^* + (A_2 + D_2 + D_3)D_1^* - B_1E_2^* - (B_2 - E_2 - E_3)E_1^* \right] \\
- m_{\Lambda_b}^2 \hat{s}^2(2D_2^*E_3^* + E_2^*E_3^*) \} , \tag{17} \]

\[ P_{TL} = \frac{16\pi m_A^4 \hat{m}_\ell \sqrt{\lambda v}}{\Delta \sqrt{\hat{s}}} \text{Re} \left\{ (1 - \hat{\rho}_\Lambda) \left( |D_1|^2 + |E_1|^2 \right) + \hat{s} \left( A_1^*D_1^* - B_1^*E_1^* \right) \right. \\
+ m_{\Lambda_b} \hat{s} \left[ B_1D_2^* + (A_2 + D_2 + D_3)E_1^* - A_1E_2^* - (B_2 + E_2 - E_3)D_1^* \right] \\
- m_{\Lambda_b}^2 \hat{s}(1 - \hat{\rho}_\Lambda)(A_2D_2^* - B_2E_2^*) \\
- m_{\Lambda_b} \sqrt{\hat{\rho}_\Lambda} \hat{s} \left[ A_1D_2^* + (A_2 - D_2 - D_3)D_1^* - B_1E_2^* - (B_2 + E_2 + E_3)E_1^* \right] \\
- m_{\Lambda_b}^2 \hat{s}^2(2D_2^*E_3^* + E_2^*E_3^*) \} , \tag{18} \]
\[
P_{NT} = -P_{TN} = \frac{64 m_{\Lambda_0}^4 \lambda v}{3\Delta} \text{Im}\left\{ (A_1 D_1^* + B_1 E_1^*) + m_{\Lambda_0}^2 \hat{s}(A_2^* D_2 + B_2^* E_2) \right\}, \tag{19}\]

\[
P_{NN} = \frac{32 m_{\Lambda_0}^4}{3s\Delta} \text{Re}\left\{ 24\hat{m}_\ell^2 \sqrt{\hat{r}_\Lambda} \hat{s}(A_1 B_1^* + D_1 E_1^*) - 12 m_{\Lambda_0} \hat{m}_\ell^2 \sqrt{\hat{r}_\Lambda} \hat{s}(1 - \hat{r}_\Lambda + \hat{s})(A_1 A_2^* + B_1 B_2^*) \right. \]
\[
+ 6 m_{\Lambda_0} \hat{m}_\ell^2 \hat{s}\left[ m_{\Lambda_0} \hat{s}(1 + \hat{r}_\Lambda - \hat{s}) \left( |D_3|^2 + |E_3|^2 \right) + 2\sqrt{\hat{r}_\Lambda}(1 - \hat{r}_\Lambda + \hat{s})(D_1 D_3^* + E_1 E_3^*) \right] \]
\[
- 12 m_{\Lambda_0} \hat{m}_\ell^2 \hat{s}(1 - \hat{r}_\Lambda - \hat{s})(A_1 B_2^* + A_2 B_1^* + D_1 E_3^* + D_3 E_1^*) \]
\[
- [\lambda \hat{s} + 2 m_{\ell}^2 (1 + \hat{r}_\Lambda - 2\hat{r}_\Lambda + \hat{r}_\Lambda \hat{s} + \hat{s} - 2\hat{s}^2)] \left( |A_1|^2 + |B_1|^2 - |D_1|^2 - |E_1|^2 \right) \]
\[
+ 24 m_{\Lambda_0}^2 \hat{m}_\ell^2 \sqrt{\hat{r}_\Lambda} \hat{s}^2 (A_2 B_2^* + D_3 E_3^*) - m_{\Lambda_0}^2 \lambda \hat{s}^2 v^2 \left( |D_2|^2 + |E_2|^2 \right) \]
\[
+ m_{\Lambda_0}^2 \hat{s}\{\lambda \hat{s} - 2 m_{\ell}^2 [2(1 + \hat{r}_\Lambda^2) - \hat{s}(1 + \hat{s}) - \hat{r}_\Lambda(4 + \hat{s})] \left( |A_2|^2 + |B_2|^2 \right) \} \tag{20}, \]

\[
P_{TT} = \frac{32 m_{\Lambda_0}^4}{3s\Delta} \text{Re}\left\{ - 24\hat{m}_\ell^2 \sqrt{\hat{r}_\Lambda} \hat{s}(A_1 B_1^* + D_1 E_1^*) - 12 m_{\Lambda_0} \hat{m}_\ell^2 \sqrt{\hat{r}_\Lambda} \hat{s}(1 - \hat{r}_\Lambda + \hat{s})(D_1 D_3^* + E_1 E_3^*) \right. \]
\[
- 24 m_{\Lambda_0}^2 \hat{m}_\ell^2 \sqrt{\hat{r}_\Lambda} \hat{s}^2 (A_2 B_2^* + D_3 E_3^*) \]
\[
- 6 m_{\Lambda_0} \hat{m}_\ell^2 \hat{s}\left[ m_{\Lambda_0} \hat{s}(1 + \hat{r}_\Lambda - \hat{s}) \left( |D_3|^2 + |E_3|^2 \right) - 2\sqrt{\hat{r}_\Lambda}(1 - \hat{r}_\Lambda + \hat{s})(A_1 A_2^* + B_1 B_2^*) \right] \]
\[
- 12 m_{\Lambda_0} \hat{m}_\ell^2 \hat{s}(1 - \hat{r}_\Lambda - \hat{s})(A_1 B_2^* + A_2 B_1^* + D_1 E_3^* + D_3 E_1^*) \]
\[
- [\lambda \hat{s} - 2 m_{\ell}^2 (1 + \hat{r}_\Lambda - 2\hat{r}_\Lambda + \hat{r}_\Lambda \hat{s} + \hat{s} - 2\hat{s}^2)] \left( |A_1|^2 + |B_1|^2 \right) \]
\[
+ m_{\Lambda_0}^2 \hat{s}\{\lambda \hat{s} + 2 m_{\ell}^2 [4(1 - \hat{r}_\Lambda^2) - 2\hat{s}(1 + \hat{r}_\Lambda) - 2\hat{s}^2] \left( |A_2|^2 + |B_2|^2 \right) \}
\[
+ \{\lambda \hat{s} - 2 m_{\ell}^2 [5(1 - \hat{r}_\Lambda^2) - 7\hat{s}(1 + \hat{r}_\Lambda) + 2\hat{s}^2] \left( |D_1|^2 + |E_1|^2 \right) \}
\[
- m_{\Lambda_0}^2 \lambda \hat{s}^2 v^2 \left( |D_2|^2 + |E_2|^2 \right) \right\}. \tag{21}\]

Using the definition of single lepton polarization we find,

\[
P_{TL}^\tau = \frac{64 m_{\Lambda_0}^4 \hat{s} v}{\Delta} \left\{ \pm \sqrt{\hat{r}_\Lambda} \left( 2 \text{Re}[A_1^* E_1 + B_1^* D_1] - m_{\Lambda_0} (1 - \hat{r}_\Lambda + \hat{s}) \text{Re}[A_1^* D_2 + A_2^* D_1] \right) \right. \]
\[
\mp m_{\Lambda_0} \sqrt{\hat{r}_\Lambda} (1 - \hat{r}_\Lambda + \hat{s}) \text{Re}[B_1^* E_2 + B_2^* E_1] \pm 2 m_{\Lambda_0} \hat{s} \sqrt{\hat{r}_\Lambda} \text{Re}[A_1^* E_2 + B_2^* D_2] \]
\[
\mp m_{\Lambda_0} (1 - \hat{r}_\Lambda - \hat{s}) \text{Re}[A_1^* E_2 + A_2^* E_1 + B_1^* D_2 + B_2^* D_1] \]
\[
\mp \frac{1}{3s} [1 + \hat{r}_\Lambda + \hat{r}_\Lambda (\hat{s} - 2) + \hat{s}(1 - 2\hat{s})] \text{Re}[A_1^* D_1 + B_1^* E_1] \]
\[
\mp \frac{1}{3} m_{\Lambda_0}^2 [2 + \hat{r}_\Lambda (2\hat{r}_\Lambda - 4 - \hat{s}) - \hat{s}(1 + \hat{s})] \text{Re}[A_2^* D_2 + B_2^* E_2] \right\}, \tag{22}\]

\[
P_{TT}^\tau = \frac{16 \pi m_{\Lambda_0}^3 \hat{m}_\ell \sqrt{\hat{s} \hat{\lambda}}}{\Delta} \left\{ - \left( |A_1|^2 - |B_1|^2 \right) + 2 m_{\Lambda_0} \text{Re}[A_1^* B_2 - A_2^* B_1] \right. \]
\[
\mp m_{\Lambda_0} \text{Re}[A_1^* E_3 - A_2^* E_1 + B_1^* D_3 - B_2^* D_1] + m_{\Lambda_0}^2 (1 - \hat{r}_\Lambda) \left( |A_2|^2 - |B_2|^2 \right) \right\}. \tag{23}\]
In all expressions the quantities \( A^i \) and \( A^j \) are defined as \( A^i \) and \( A^j \) correspond to the lepton and anti-lepton polarizations, respectively, and \( \Delta \) is determined from the differential decay rate,

\[
\frac{d\Gamma}{d\hat{s}} = \frac{G_F^2 \alpha_{em}^2}{8192\pi^5} |V_{tb}V_{ts}^*|^2 v \sqrt{\lambda(1, \hat{r}_A, \hat{s})}\Delta .
\]

In all expressions the quantities \( \lambda(1, \hat{r}_A, \hat{s}), \hat{s}, \hat{r}_A, \hat{m}_\ell \) and \( v \) are defined as \( \lambda(1, \hat{r}_A, \hat{s}) = 1 + \hat{r}_A^2 + \hat{s} - 2\hat{r}_A - 2\hat{s} - 2\hat{r}_A\hat{s}, \hat{s} = q^2/\hat{m}_\ell^2, \hat{r}_A = m_A/m_{\Lambda_\ell}, \hat{m}_\ell = m_\ell/m_{\Lambda_\ell}, \) and \( v = \sqrt{1 - \frac{4\hat{m}_\ell^2}{\hat{s}}} \).
3 Numerical analysis

In the previous section we present the expressions for double and single lepton polarizations in family non–universal $Z'$ model. We now proceed with the numerical analysis of these physical observables. In addition to the input parameters in the SM, the considered version of the family non–universal $Z'$ model contains four new parameters, namely, $|B_{sb}^L|$, $\varphi_s^L$, $B_{\ell\ell}^L$, $B_{\ell\ell}^R$. The constraints to these parameters coming from the analysis of present experimental data in the $B$ meson sector are studied in detail in the literature [22].

The values of the new input parameters appearing in family non–universal $Z'$ model are given in Table 1, in which S1 and S2 correspond to UT–fit Collaboration result [23].

| Parameter | Value |
|-----------|-------|
| $|B_{sb}^L| \times 10^{-3}$ | 1.09 ± 0.22 |
| $\varphi_s^L$ | $-72 \pm 7$ |
| $S_{\mu\mu}^L \times 10^{-2}$ | $-2.80 \pm 3.90$ |
| $D_{\mu\mu}^R \times 10^{-2}$ | $-6.70 \pm 2.60$ |

Table 1: The values of four input parameters appearing in family non–universal $Z'$ model.

We have studied the sensitivities of single single and double lepton polarizations on input parameters of family non–universal $Z'$ model. We can summarize the result of our analysis as follows:

- $P_L$ decreases maximally %5 in both scenarios compared to the SM prediction.
- The values of $P_T$ and $P_N$ practically do not change. Therefore we can conclude that single lepton polarization effects are not so efficient for establishing new physics in the framework of family non–universal $Z'$ model.

As a result of the analysis of double lepton polarization we obtain that:

- Predictions for $P_{LL}$, $P_{LT}$, $P_{TL}$ do coincide for both SM and family non–universal $Z'$ model.

- Double lepton polarizations $P_{NN}$ and $P_{TT}$ are quite sensitive to the parameters of $Z'$ model. We present the $q^2$ dependence of $P_{NN}$ and $P_{TT}$ in Figs. (1) and (2), respectively. We observe from these figures that, in the region $3 \text{GeV}^2 \leq q^2 \leq 15 \text{GeV}^2$ there occurs considerable difference between the predictions of the SM and family non–universal $Z'$ model. Especially, the predictions of S1 scenario for $P_{NN}$ and $P_{TT}$ shows larger discrepancy compared to S2.

- In Fig. (3) we present the dependence of the polarized forward–backward asymmetry $A_{LL}$ on $q^2$ in the SM and family non–universal $Z'$ model. It follows from this figure that the zero position of $A_{LL}$ is shifted to left compared to the prediction of the SM. Therefore determination of the zero position of $A_{LL}$ can give invaluable information, not only about the existence of new physics, but also about the discrimination of the scenarios S1 and S2.
We have also analysed the remaining forward–backward asymmetries $A_{FB}^L$, $A_{FB}^N$, $A_{FB}^{LT}$, $A_{FB}^{NL}$, $A_{FB}^{LT}$ and $A_{FB}^{NN}$ and obtained that the contribution of new $Z'$ bosons to these asymmetries are negligibly small.

As the concluding remark we can summarize our analysis as follows. Contributions of family non–universal $Z'$ model to the single and double lepton polarizations, as well as polarized forward–backward asymmetry $A_{LL}$ in rare, exclusive baryonic $Λ_b \rightarrow Λ \ell^+ \ell^−$ decay is studied. It is obtain that $P_{NN}$ and $P_{TT}$ are quite sensitive to the $Z'$ boson contributions. Moreover, it is found that zero position of the forward–backward asymmetry $A_{LL}$ is shifted to left compared to the SM case. Determination of the value of zero position of $A_{LL}$ is also a very important information for the scenarios under consideration. The results we obtain can all be checked in future planned LHC–b experiments.
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Figure captions

Fig. (1) The dependence of the double–lepton polarization asymmetry $P_{LL}$ on $q^2$ for the $\Lambda_b \rightarrow \Lambda\mu^+\mu^-$ decay.

Fig. (2) The same as in Fig. (1), but for the double–lepton polarization asymmetry $P_{TT}$.

Fig. (3) The dependence of the double–lepton polarization asymmetry $A^{LL}_{FB}$ on $q^2$ for the $\Lambda_b \rightarrow \Lambda\mu^+\mu^-$ decay.
Figure 1:

Figure 2:
Figure 3: