Application of mixed geometric interpolants for modeling the strength characteristics of steel fiber concrete

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Abstract. The paper proposes an approach to modeling geometric multifactorial processes and phenomena using mixed geometrical interpolants, for example of the steel fiber concrete strength characteristics’ simulation. A key role is played by the formation of the geometric model, as developed in detail in the article. Analytical description of the geometric model obtained by means of algebraic curves (that pass through the predetermined point in advance) is realized by means of the BN-calculus mathematical formalism.

1. Introduction

The development of multidimensional interpolation techniques is an important scientific task, which have a great practical importance. One of such methods include multidimensional interpolation geometric theory proposed in papers [1-3], which is a special case of a linear-valued [4] and a parabolic [5] multidimensional interpolation. Applying the theory of geometric interpolation is widely used in the simulation: composite building materials [6], surfaces shells engineering constructions irregular shape [7], topographic surfaces [8] heat and mass transfer multifactorial process [9, 10] and other. Analytical description of the geometric theory of multivariate interpolation is made in the framework of the mathematical apparatus of the BN-calculus [11, 12].

The underlying concept of multidimensional interpolation geometric theory is geometric interpolant which is a multidimensional space geometric object, passing through the predetermined point in advance, the coordinates of which correspond to the initial experimental and statistical information. To construct the geometric interpolant is necessary to form a geometric model of the process in the form of a geometric circuit, which comprises an interconnected plurality of fixed and mobile (current) points to describe it analytically using of algebraic curves arcs passing through a predetermined point in advance [13].

The tree of geometric model process is generated based on fixed points of the chain (in general multidimensional), the coordinates of which correspond to the original experimental statistical information. An important aspect is the process of formation points’ chain corresponding to the original information. Traditionally, this chain is regular and irregular. The simplest and most widely common case is to use a regular grid of points, for which the geometric interpolant may be formed on a rectangular plane, or using its generalization to multidimensional space. In this case, use of the arc curves extending through the same number of points, each of the flat sections of the modelling object. However, the experimental data does not always have the necessary properties to form a regular chain of points. In this case, it becomes necessary to use the geometric mixed interpolants for which each
section application can then call of a different number of fixed points, which will be held reference lines of the modelled geometric object.

2. The formation of a geometric model thee for mixed geometrical interpolant

Consider the example of the process of forming a geometric model’s tree for mixed geometric interpolant. As the input data for modeling we use the experimental results of physical and mechanical properties of high steel fiber concrete [14] obtained by brief heating (Table. 1).

| The relative strength characteristics of steel fiber concrete | Reinforcement percentage, $\mu$, % | Scaling factor, $\lambda$ | at a temperature $t$, °C |
|------------------------------------------------------------|-----------------------------------|------------------------|------------------------|
| $\frac{R_{sb}}{R_b}$ | 0 | 30 | 1,00 | 0,9 | 1,00 | 0,95 |
| | | 16 | 1,11 | – | – |
| | 0,6 | 30 | 1,11 | 1,00 | 1,14 | 1,06 |
| | | 16 | 1,19 | 1,00 | – | 1,11 |
| | 2,5 | 30 | 1,21 | 1,06 | – | 1,16 |
| | | 16 | 1,3 | 1,15 | – | 1,21 |

In this example the response is a function of the relative strength $\frac{R_{sb}}{R_b}$, which is affected by the percentage of reinforcement $\mu$, the scale factor $\lambda$ and the heating temperature $t$. With this in mind, we choose to simulate a 4-dimensional rectangular coordinate system with the axes of $\mu$, $\lambda$, $t$ and $\frac{R_{sb}}{R_b}$. From this we obtain the coordinates 18 fixed reference points shown in Table 2.

| Points | Coordinates | $\frac{R_{sb}}{R_b}$ |
|--------|-------------|---------------------|
| $M_{1,1,1}$ | 0 | 30 | 20 | 1,00 |
| $M_{1,1,2}$ | 0 | 30 | 90 | 0,9 |
| $M_{1,1,3}$ | 0 | 30 | 90 | 1,00 |
| $M_{1,1,4}$ | 0 | 30 | 200 | 0,95 |
| $M_{1,2,1}$ | 0 | 16 | 20 | 1,11 |
| $M_{2,1,1}$ | 0,6 | 30 | 20 | 1,11 |
| $M_{2,1,2}$ | 0,6 | 30 | 90 | 1,00 |
| $M_{2,1,3}$ | 0,6 | 30 | 150 | 1,14 |
| $M_{2,1,4}$ | 0,6 | 30 | 200 | 1,06 |
| $M_{2,2,1}$ | 0,6 | 16 | 20 | 1,19 |
| $M_{2,2,2}$ | 0,6 | 16 | 90 | 1,11 |
| $M_{2,2,3}$ | 0,6 | 16 | 90 | 1,11 |
| $M_{2,2,4}$ | 0,6 | 16 | 200 | 1,11 |

| Points | Coordinates | $\frac{R_{sb}}{R_b}$ |
|--------|-------------|---------------------|
| $M_{3,1,1}$ | 2,5 | 30 | 20 | 1,21 |
| $M_{3,1,2}$ | 2,5 | 30 | 90 | 1,06 |
| $M_{3,2,1}$ | 2,5 | 16 | 90 | 1,15 |
| $M_{3,2,4}$ | 2,5 | 16 | 200 | 1,21 |
Based on initial data, presented in Table 2, it is possible to offer several options for interpolation. In our opinion, it is the most suitable interpolation with the construction of three ruled surfaces, the first of which is a cone (Fig. 1). Based on this, we will divide the data in Table 1 for 3 parts depending on the percentage of reinforcement. Then the first - response conical surface at $\mu = 0$ will pass through the line $M_{1,1,1}, M_{1,1,2}, M_{1,1,3}, M_{1,1,4}$ and the point $M_{1,1,2} = M_{1,2,1}$. The second (if $\mu = 0.6$) and the third (if $\mu = 2.5$) linear surface defined by pairs of lines, $M_{2,1,1}, M_{2,1,2}, M_{2,1,3}, M_{2,1,4}$, $M_{2,2,1}, M_{2,2,2}, M_{2,2,4}$ and $M_{3,1,1}, M_{3,1,2}, M_{3,1,4}$, $M_{3,2,1}, M_{3,2,2}, M_{3,2,4}$ accordingly.

![Figure 1. The tree of geometric model of the steel fiber concretes relative strength.](image)

We combine all three linear response surfaces in 3-parametric response hypersurface via forming line passing through the current 3 point: $M_1$, $M_2$ and $M_3$, that are simultaneously forming the corresponding response surfaces. Thus, the geometric model of steel fiber concrete relative strength will 3-parametric response hypersurface, which appropriate to 4-dimensional space and passing through the 18 fixed starting points.

3. Analytical description of the geometric model tree of steel fiber concrete relative strength

In accordance with the tree of the geometric model, a third-order line passing through 4 points predefined [13]:

$$M_{1,1} = M_{1,1,1} \left( \bar{u}^3 - 2.5\bar{u}^2u + \bar{u}^2 \right) + M_{1,1,2} \left( 9\bar{u}^2u - 4.5\bar{u}u^2 \right) +$$

$$+ M_{1,1,3} \left( -4.5\bar{u}^3u + 9\bar{u}u^3 \right) + M_{1,1,4} \left( \bar{u}^3u - 2.5\bar{u}u^2 + u^3 \right).$$

where $M_{1,1}$ – current arc point of the 3rd order curve.

$M_{1,1,1}, M_{1,1,2}, M_{1,1,3}, M_{1,1,4}$ – are fixed points through which the arc of a 3rd order curve passes (Fig. 1 and Table 2);

$\bar{u}$ – current parameter, which varies from 0 to 1;

$\bar{u} = 1 - u$ – padding to 1.

Note that all polynomial coefficients are obtained by modifying the curves based on Bernstein polynomials and are fixed for a curve of a certain order. In more detail, with their analytical definition in point form, one can familiarize oneself with [2, 6, 8, 13]. With the value of the parameter $u = 0$, we get the starting point $M_{1,1,1}$, with $u = \frac{1}{3}$ – $M_{1,1,2}$, with $u = \frac{2}{3}$ – $M_{1,1,3}$ and with $u = 1$ – $M_{1,1,4}$. A feature of this equation is that for any coordinates of the point $M_{1,1,1}, M_{1,1,2}, M_{1,1,3}, M_{1,1,4}$, the curve
will still pass through these points. The only condition is the sequence of points, which cannot be violated, since it is consistent with a change in the current parameter $u$.

The used point equation is a symbolic record, which, through coordinate-wise calculation, reduces to a system of parametric equations. Given the coordinate system selected with the axes $\mu$, $\lambda$, $t$ receive $\frac{R_{\text{sb}}}{R_b}$:

\[
\begin{align*}
\mu_{1,1} &= \mu_{1,1,1} (\bar{u}^2 - 2.5\bar{u}^2 u + \bar{u}^3) + \mu_{1,1,2} (9\bar{u}^2 u - 4.5\bar{u}^3) + \\
&+ \mu_{1,1,3} (-4.5\bar{u}^2 u + 9\bar{u}u^2) + \mu_{1,1,4} (\bar{u}^2 u - 2.5\bar{u}u^2 + u^3), \\
\lambda_{1,1} &= \lambda_{1,1,1} (\bar{u}^2 - 2.5\bar{u}^2 u + \bar{u}^3) + \lambda_{1,1,2} (9\bar{u}^2 u - 4.5\bar{u}u^2) + \\
&+ \lambda_{1,1,3} (-4.5\bar{u}^2 u + 9\bar{u}u^2) + \lambda_{1,1,4} (\bar{u}^2 u - 2.5\bar{u}u^2 + u^3), \\
t_{1,1} &= t_{1,1,1} (\bar{u}^2 - 2.5\bar{u}^2 u + \bar{u}^3) + t_{1,1,2} (9\bar{u}^2 u - 4.5\bar{u}u^2) + \\
&+ t_{1,1,3} (-4.5\bar{u}^2 u + 9\bar{u}u^2) + t_{1,1,4} (\bar{u}^2 u - 2.5\bar{u}u^2 + u^3), \\
\frac{R_{\text{sb}}}{R_b}_{,1,1} &= \left( \frac{R_{\text{sb}}}{R_b} \right)_{,1,1,1} (\bar{u}^3 - 2.5\bar{u}^2 u + \bar{u}^3) + \left( \frac{R_{\text{sb}}}{R_b} \right)_{,1,1,2} (9\bar{u}^2 u - 4.5\bar{u}^3) + \\
&+ \left( \frac{R_{\text{sb}}}{R_b} \right)_{,1,1,3} (-4.5\bar{u}^2 u + 9\bar{u}u^2) + \left( \frac{R_{\text{sb}}}{R_b} \right)_{,1,1,4} (\bar{u}^2 u - 2.5\bar{u}u^2 + u^3).
\end{align*}
\]

where are all the values $\mu_{i,j,k}$, $\lambda_{i,j,k}$, $t_{i,j,k}$ and $\left( \frac{R_{\text{sb}}}{R_b} \right)_{i,j,k}$ are the coordinates of the starting points and are taken in accordance with table 2:

\[
\begin{align*}
\mu_{1,1} &= 0, \\
\lambda_{1,1} &= 30, \\
t_{1,1} &= 45u^2 + 225u + 20, \\
\left( \frac{R_{\text{sb}}}{R_b} \right)_{1,1} &= -1,575u^3 + 2,475u^2 - 0,95u + 1.
\end{align*}
\]

Similarly, any point equation can be represented in the form of a system of parametric equations, the number of which depends on the dimension of the space in which the desired geometric object is determined. However, it makes sense to perform coordinate-wise calculation not of intermediate point equations, but of the final point equation of a 3-parameter response hypersurface, taking into account all coordinate values of the source points presented in Table 2.

\[M_i = M_{i,1}\bar{v} + M_{i,2}\nu,\]

where $M_i$ – is the current point of the conical surface; 
$\nu$ – current parameter, which varies from 0 to 1; 
$\bar{v} = 1 - \nu$ – padding to 1.

Given that the second row of Table 1 contains only one value of the heating temperature, which the geometric representation is only one point, will form a conical surface by rolling simplex dot equation is described by the equation generator:

\[M_i = M_{i,1}\bar{v} + M_{i,2}\nu.\]

The second ruled surface defined by support lines of the 3rd and 2nd order, which pass through 4 and 3 in advance given points [13]. Generating line is movable linear simplex of one-dimensional space:
\[ M_{2,1} = M_{2,1,1}(\bar{a}^3 - 2,5\bar{a}^2u + \bar{u}u^2) + M_{2,1,2}(9\bar{a}^2u - 4,5\bar{a}u^2) + \]
\[ + M_{2,1,3}(-4,5\bar{a}^2u + 9\bar{a}u^3) + M_{2,1,4}(\bar{a}^2u - 2,5\bar{a}u^2 + u^3); \]
\[ M_{2,2} = M_{2,2,1}\bar{a}(1 - 2u) + 4\bar{a}uM_{2,2,2} + M_{2,2,4}u(2u - 1); \]
\[ M_2 = M_{2,1}\bar{v} + M_{2,2}v. \]

where \( M_2 \) – current point of the 2nd ruled surface

Similarly, the second and third linear surface are defined via parabolic arcs of 2nd order, passing through three points specified in advance [13]:
\[ M_{3,1} = M_{3,1,1}\bar{a}(1 - 2u) + 4\bar{a}uM_{3,1,2} + M_{3,1,4}u(2u - 1); \]
\[ M_{3,2} = M_{3,2,1}\bar{a}(1 - 2u) + 4\bar{a}uM_{3,2,2} + M_{3,2,4}u(2u - 1); \]
\[ M_3 = M_{3,1}\bar{v} + M_{3,2}v. \]

where \( M_3 \) – current point of a ruled parabolic surface.

Next, we determine the point equation of the parabolic curve’s response of the 3-parameter hypersurface:
\[ M = M_{2,1}\bar{r}(1 - 2\bar{w}) + 4\bar{r}\bar{w}M_{2,2} + M_3w(2w - 1). \]

where \( M \) – current point of a 3-parameter response hypersurface;
\( \bar{w} \) – current parameter, which varies from 0 to 1;
\( \bar{r} = 1 - \bar{w} \) – padding to 1.

The result is a computational algorithm for modeling the desired response 3-parameter hypersurface, represented as a sequence of point equations.

It should be noted that the movement of the current point on the reference lines is coordinated by single parameter \( u \) when forming the ruled surfaces, and the current motion point forming the same surfaces that are simultaneously supporting for response hypersurface – by a single parameter \( v \). The motion of the current point which forming a response hypersurface determined by parameter \( w \). All three parameters within the investigated range of values vary between 0 and 1. Thus, we obtain a 3-point description of the parametric geometrical interpolant, which is a 3-parameter response hypersurface, passing through 18 fixed base points.

In order to move from the symbolic point equation to parametric one, you have to make coordinatewise calculation. Geometric sense of coordinatewise calculation is that the resulting hypersurface, attributable to 4-dimensional space, projecting subsequent to the four 1-dimensional spaces, which is the axis of the selected coordinate system. As a result, given the coordinates of reference points shown in Table 2, we obtain the following system of 4 parametric equations after some transformation:
\[
\begin{align*}
R_{ab} &= a_1u^3 + b_1u^2 + c_1u + d_1, \\
R_b &= a_2u^2 + b_2u + 20, \\
\lambda &= -14v + 30, \\
\mu &= 2,6\bar{w}^2 - 0.1\bar{w},
\end{align*}
\]

where \( a_1 = (-5,31v + 5,31)w^2 + (3,735v - 3,735)w + 1,575v - 1,575; \)
\[ b_1 = (5.45v - 7.01)w^2 + (-3.055v + 5.035)w - 2.475v + 2.475; \]
\[ c_1 = 1.7w^2 + (-0.91v - 1.3)w - 0.95 + 0.95v; \]
\[ d_1 = (-0.02 + 0.08v)w^2 + (0.23 - 0.1v)w + 1 + 0.11v; \]
\[ a_2 = (-410v + 250)w^2 + (365v - 125)w + 45v - 45; \]
\[ b_2 = (50v - 250)w^2 + (175v + 125)w + 225 - 225v. \]

If we consider table 1 with the initial data for modeling as an experiment planning matrix, then it can be argued that the obtained computational algorithm in the form of a sequence of point equations allows us to obtain a process corresponding to the chosen experiment matrix, regardless of the specific values given in table 1. Thus, the obtained model without making any changes can be used not only for modeling the strength characteristics of steel fiber concrete, but also for constructing other models using multidimensional interpolation. If you change the initial values in table 1, you can immediately get a new system of parametric equations for the analytical description of the 3-factor process.

4. Conclusion

In the example of the geometric modeling of steel fiber concrete strength characteristics are stated the main principles of the construction and deposits of mixed geometric analytical description interpolants of multivariate affine space. Compared with the model, which was obtained in [14] using multidimensional approximation the interpolation model turned out to be smoother and with fewer extreme points in the interval between the nodal points. Thus, the curvature of the obtained interpolation model more closely matches to the nature of the process. However, the lack of source data during the experiment for zero percent reinforcement and a scale factor equal to 16, leads to the fact that instead of the reference line, the reference point, is used by forming a conical surface. Therefore, when values \( \mu = 0 \) and \( \lambda = 16 \) for any values of the temperature value of the relative strength will always be equal to 1.11 (the coordinate of the cone vertex), which is not entirely true. A possible way out of this situation is to build the geometric averaged interpolator comprising the averaged values of several different geometric patterns that is the prospect of further research on the geometric theory of multidimensional interpolation.

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