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Overview document for:
A weight function theory of basis function interpolants and smoothers.

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I would like to thank the University of Canberra for their generous assistance.

Also, thanks to my Masters degree supervisors Dr Markus Hegland and Dr Steve Roberts of the CMA at the Australian National University.

Abstract. This document is a brief overview of two documents which continue to develop the weight function theory of basis function smoothers and interpolants. One document considers the zero order theory and one considers the positive order theory.

0.1. Change register.

05/Jul/2007  Created this document using the abstracts from the version 1 documents.
22/Oct/2007  Altered this document using the abstracts from the consolidated version 2 documents i.e. version 2 of arXiv:0708.0780 and version 2 of arXiv:0708.0795.
1. Overview

This work currently consists of two documents which continue to develop the Light weight
function theory of basis function smoothers and interpolants. One document considers the
zero order theory and one considers the positive order theory.

In brief, some important general features:

(1) Extends the positive order work of Light and Wayne [2], [3] and [1] to the zero order case and
extends the positive order case to tensor product weight functions.

For both the positive and zero order cases:

(2) A weight function is first defined and then used to define a continuous basis function and a data
function Hilbert space are defined using the Fourier transform. This technique is illustrated by
several examples.

(3) The standard minimal norm and seminorm interpolants are defined and pointwise orders of
convergence are derived on a bounded set.

(4) We define the well known variational non-parametric smoother which stabilizes the interpolant
using a smoothing parameter - I call this the Exact smoother. Orders of uniform pointwise
convergence are derived on a bounded open set.

(5) A scalable smoother is derived which I call the Approximate smoother. Orders of uniform
pointwise convergence are derived on a bounded open set.

(6) For the zero order case numeric examples are given which compare the theoretical and actual
errors w.r.t. the data function.

2. Zero order document (arXiv:0708.0780)

Here is a short description of the document (the abstract).

In this document I develop a weight function theory of zero order basis function interpolants and
smoothers.

In Chapter 1 the basis functions and data spaces are defined directly using weight functions. The
data spaces are used to formulate the variational problems which define the interpolants and smoothers
discussed in later chapters. The theory is illustrated using some standard examples of radial basis
functions and a class of weight functions I will call the tensor product extended B-splines.

In Chapter 2 the theory of Chapter 1 is used to prove the pointwise convergence of the minimal norm
basis function interpolant to its data function and to obtain orders of convergence. The data functions
are characterized locally as Sobolev-like spaces and the results of several numerical experiments using
the extended B-splines are presented.

In Chapter 3 a large class of tensor product weight functions will be introduced which I call the
central difference weight functions. These weight functions are closely related to the extended B-splines
and have similar properties. The theory of this document is then applied to these weight functions to
obtain interpolation convergence results. To understand the theory of interpolation and smoothing it is
not necessary to read this chapter.

In Chapter 4 a non-parametric variational smoothing problem will be studied using the theory of
this document with special interest in its order of pointwise convergence of the smoother to its data
function. This smoothing problem is the minimal norm interpolation problem stabilized by a smoothing
coefficient.

In Chapter 5 a non-parametric, scalable, variational smoothing problem will be studied, again with
special interest in its order of pointwise convergence to its data function. We discuss the SmoothOperator
software (freeware) package which implements the Approximate smoother algorithm. It has a full user
manual which describe several tutorials and data experiments.

3. Positive order document (arXiv:0708.0795)

Here is a short description of the document (the abstract).

In this document I develop a weight function theory of positive order basis function interpolants and
smoothers.

In Chapter 1 the basis functions and data spaces are defined directly using weight functions. The
data spaces are used to formulate the variational problems which define the interpolants and smoothers
discussed in later chapters. The theory is illustrated using some standard examples of radial basis functions and a class of weight functions I will call the tensor product extended B-splines.

**Chapter 2** shows how to prove functions are basis functions without using the awkward space of test functions $S_{0,n}$ which are infinitely smooth functions of rapid decrease with several zero-valued derivatives at the origin. Worked examples include several classes of well-known radial basis functions.

The goal of **Chapter 3** is to derive ‘modified’ inverse-Fourier transform formulas for the basis functions and the data functions and to use these formulas to obtain bounds for the rates of increase of these functions and their derivatives near infinity.

In **Chapter 4** we prove the existence and uniqueness of a solution to the minimal seminorm interpolation problem. We then derive orders for the pointwise convergence of the interpolant to its data function as the density of the data increases.

In **Chapter 5** a well-known non-parametric variational smoothing problem will be studied with special interest in the order of pointwise convergence of the smoother to its data function. This smoothing problem is the minimal norm interpolation problem stabilized by a smoothing coefficient.

In **Chapter 6** a non-parametric, scalable, variational smoothing problem will be studied, again with special interest in its order of pointwise convergence to its data function.
Bibliography

1. W. Light and H. Wayne, *Error estimates for approximation by radial basis functions*, Approximation theory, wavelets and applications (Maratea, 1994), NATO Adv. Sci. Inst. Ser. C Math. Phys. Sci., vol. 454, Kluwer Acad. Publ., Dordrecht, 1995, pp. 215–246.

2. ———, *On power functions and error estimates for radial basis function interpolation*, J. Approx. Th. 92 (1998), no. 2, 245–266.

3. ———, *Spaces of distributions, interpolation by translates of a basis function and error estimates*, Numer. Math. 81 (1999), no. 3, 415–450.