Functional Renormalization Group analysis of a Tensorial Group Field Theory on $\mathbb{R}^3$

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Abstract – We study a model of Tensorial Group Field Theory (TGFT) on $\mathbb{R}^3$ from the point of view of the Functional Renormalization Group (FRG). This is the first attempt to apply a renormalization procedure to a TGFT model defined over a non-compact group manifold. IR divergences (with respect to the metric on $\mathbb{R}$) coming from the non-compactness of the group are regularised via compactification, and a thermodynamic limit is then taken. We identify then IR and UV fixed points of the RG flow and find strong hints of a phase transition of the TGFT system from a symmetric to a broken or condensate phase in the IR.

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Introduction. – Group Field Theories [1–4] (GFTs) are a particular class of quantum field theories with fields defined over a group manifold and characterised by combinatorially non-local interaction terms. This combinatorial non-locality makes the Feynman diagrams of the theory stranded diagrams dual to cellular complexes (simplicial complexes in the simplest constructions) [1,3]. GFTs were born from an attempt to generalise matrix models [5] of 2$d$ gravity to higher dimensions in the form of tensor models [6–8]. These models were soon enriched with group theoretic data in such a way that the Feynman amplitudes of specific GFT models coincide with state sum models of topological field theory [8,9]. A connection between GFTs and Loop Quantum Gravity (LQG) [10,11] was immediately pointed out [12] at the level of quantum states, and it was later shown [13] that GFTs provide a (formal) complete definition of spin foams models, a covariant formulation of LQG. For GFTs (and spin foam models) endowed with a discrete geometric interpretation (which requires appropriate group theoretic data and suitable choices of dynamics) there is also a direct link with simplicial quantum gravity path integrals, generically manifest in the flux representation of the GFT Feynman amplitudes [14], and in their semiclassical analysis, where one recovers the Regge action [15].

One key open issue of all of the above quantum gravity approaches, and GFTs in particular, is the emergence of a continuous geometry out of their discrete and quantum pre-geometric structures, and of General Relativity as an effective description of their (collective) dynamics. One suggested scenario for the emergence of spacetime and geometry out of such quantum gravity models involves a phase transition from a pre-geometric phase to a geometric phase, which may be further identified to a condensate phase [16,17] (a similar idea was proposed also in a loop quantum gravity context in [18]). Indeed, GFT condensate states seem to possess an effective dynamics with a cosmological interpretation [19]. The GFT formalism offers the possibility to address this issue with standard quantum field theory tools. Indeed, an important line of recent developments has concerned the renormalization of GFT models, since the renormalization group is the key tool to address both the quantum consistency of field theory dynamics and its continuum limit, aimed at a precise mapping of the phase diagram. GFT renormalization is also one of the two main strategies to define the renormalization of spin foam models, the other being through a generalised lattice gauge theory approach [20]. Most work has concerned a particular class of GFTs, called Tensorial Group Field Theories (TGFTs) [21–30], which incorporate...
recent advances in the statistical analysis of colored tensor models [31–34]. In the TGFT framework, fields are endowed with tensorial transformation properties under the action of the group manifold. A large set of models proves to be perturbatively renormalizable and asymptotically free. However, understanding the continuum limit, the phase diagram and phase transitions of the same models requires a non-perturbative analysis.

This being the goal, the Functional Renormalization Group (FRG) is an efficient framework to reach it [35–37].

The FRG approach has been applied first to matrix models [38–40] (with the double scaling critical point re-interpreted as a fixed point in the RG flow). More recently, the FRG framework has been adapted for the first time to TGFTs in [41], and applied to a rank-3 TGFT defined over a compact $U(1)$ group manifold. The $\beta$-functions define a non-autonomous system in the cut-off $N$. Then, the authors studied two regimes of the cut-off, the large and small $N$, where an autonomous system of RG equations is obtained. The notion of UV or IR “fixed points” is then only loosely (i.e. asymptotically) defined, as the existence of a trajectory from a UV to an IR fixed point becomes more difficult to ascertain. This is not surprising nor problematic per se, and it simply signals the presence of an additional scale, the size of the group manifold. In fact, the same feature is found in different contexts like quantum field theory at finite temperature, on non-commutative manifolds and on a curved spacetimes (see [42] and references therein). Still, in the approximation of large size of the group manifold, hints of a phase transition from a symmetric to a broken phase were found\(^1\).

In this work, we consider a rank-3 TGFT with fields defined on the non-compact manifold $\mathbb{R}^3$, and endowed with a Laplacian kinetic term. It possesses no additional scale, thus it is expected to have proper fixed points. This model is of interest as a toy model for quantum gravity, due to the combinatorics of its Feynman diagrams, and as a (much) simplified version of Lorentzian (T)GFTs for $4d$ quantum gravity, also based on a non-compact group manifold. It is a useful step towards a renormalization analysis of more realistic models, hopefully providing useful hints of what to expect for them. One certainly generic feature is that the non-compact manifold introduces IR divergences, that we properly address through a careful definition of a thermodynamic limit for TGFTs, in this FRG context. In this limit, we recover an autonomous system of $\beta$-functions of the coupling constants, and we can then identify the UV and IR fixed points of the RG flow. We also find evidence for a phase transition from a symmetric to a broken (or condensed) phase.

**The model.** We consider a rank-3 TGFT for a complex field defined over $\mathbb{R}^3$ endowed with a specific $\delta^3$ interaction called “melonic”\(^2\) [31], shown in fig. 1. In general, rank-3 melonic interactions correspond to peculiar triangulations of the 3-sphere and are the most dominant objects in the large cut-off $N$ limit [21,33,34] (in both simple tensor models and topological GFTs, but this result is expected to extend to a wider class of models). Written in momentum space\(^2\), the classical action of the model reads

$$
S[\phi, \overline{\phi}] = \int_{\mathbb{R}^3} \, d^3p \, \phi_{123} \left( \sum_s p_s^2 + \mu \right) \phi_{123} + \frac{\lambda}{2} \int_{\mathbb{R}^6} \, d^6p \, \left[ \phi_{123} \phi_{123} + \text{sym} \{1, 2, 3\} \right],
$$

where we used the notation $\phi_{123} = \phi(p_1, p_2, p_3)$ for the field modes and “sym” indicates that we include all the interactions obtained by symmetrization over the color labels (see fig. 1). The kinetic term is defined by a sum of Laplacians acting on the field indices and a mass term with coupling $\mu$. It is immediate to see that the action is built using generalised traces over field indices convoluted with the kinetic and interaction kernels and, once exponentiated, defines a quantum theory through a Gaussian field measure of covariance $(\sum_s p_s^2 + \mu)^{-1}$.

**FRG equations for tensorial models.** The FRG approach [35–37] rephrases the problem of integrating out the high modes of a theory, as one of solving a differential equation, the FRG equation. Being non-perturbative in nature, the FRG allows in principle to deal with the full set of quantum fluctuations of the model and to study its critical behavior.

The implementation of the FRG method in TGFT [41] follows closely the usual one [35–37], with special attention paid to the fact that we are dealing with a convolution of tensors and thus with peculiar non-local interactions. We start by decoupling the field modes as typical in the Wilsonian approach to renormalization, by adding to the action a mass-like regulator term $\Delta S_k = \text{Tr}(\overline{\phi} \cdot R_k \cdot \phi)$ depending on the IR cut-off $k$, that splits the modes into high modes ($|p| > k$) and low modes ($|p| < k$). The scale-dependent quantum theory will then be defined through a partition function where high modes are integrated out:

$$
Z_k[J, \overline{J}] = \int d\phi d\overline{\phi} e^{-S[\phi, \overline{\phi}] - \Delta S_k[\phi, \overline{\phi}] + \text{Tr}[\overline{\phi} J \cdot \phi] + \text{Tr}[\overline{\phi} \overline{J} \cdot \overline{\phi}]},
$$

\(^2\)Progress towards the a better characterisation of the phase diagram in quartic tensor models has also been recently achieved [43,44], with evidence of spontaneous symmetry breaking.

\(^1\)We adopt the standard QFT terminology for field modes, but no standard physical interpretation should be associated to them. The same remark applies to our use of the terms “UV” and “IR”.

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Fig. 1: Colored symmetric interaction terms.
where $J$ is a complex tensor playing the role of a source and $\text{Tr}(\mathcal{F}) := \int_{\mathbb{R}^3} d^3 k \bar{\varphi}_{123}$. After a Legendre transform, we identify a scale-dependent effective action which encodes the full information about the quantum theory:

$$\Gamma_k[\varphi, \bar{\varphi}] = \sup_{\mathcal{F}} \left[ \text{Tr}(\mathcal{F}) + \text{Tr}(\mathcal{J} \varphi) - W_k[\mathcal{J}, \bar{\mathcal{J}}] - \Delta S_k[\varphi, \bar{\varphi}] \right],$$

where $\varphi = \langle \phi \rangle$ and $W_k[\mathcal{J}, \bar{\mathcal{J}}] = \log Z_k[\mathcal{J}, \bar{\mathcal{J}}]$. The term $\Delta S_k$ is also chosen to be compatible with the choice of initial conditions for the FRG differential equation, encoding the scaling of effective action to the bare one in the UV: $\Gamma_k[\varphi, \bar{\varphi}] \to S[\varphi, \bar{\varphi}]$, where $\Lambda$ plays the role of the UV cut-off. Introducing the logarithmic scale $t = \log k$ and $\Gamma_k^{(2)} := \partial_\varphi \log \Gamma_k$, the Wetterich equation for tensorial GFT models has the form [41]:

$$\partial_t \Gamma_k = \text{Tr}(\partial_t R_k \cdot [\Gamma_k^{(2)} + R_k])^{-1}.$$

Evaluating (4) requires an infinite volume regularisation. In the local field theory case, infinite volume divergences are cured by passing to constant field modes or taking a thermodynamic limit [35,36]. Because of the non-locality of the interactions, the use of constant field modes is misleading (one cannot neglect the fact that different $\delta^4$ terms in the TGFT case do not have the same combinatorics), thus we perform a thermodynamic limit.

**Truncation scheme.** In order to be able to perform practical computations, we need to adopt a truncation scheme for the effective action. Of course performing a truncation means losing the exact nature of the Wetterich equation. As in the usual local QFT case, further analysis with enlarged truncations to include more trace invariants and check the convergence of the results is needed to confirm our present conclusions.

We choose to truncate $\Gamma_k$ to the quadratic term in the derivative of the fields and to order four in the fields, thus obtaining a form similar to the action itself:

$$\Gamma_k[\varphi, \bar{\varphi}] = \int_{\mathbb{R}^3} d^3 p \bar{\varphi}_{123} \left( Z_k \sum_{s} p_s^2 + \mu_k \right) \varphi_{123} + \frac{\lambda_k}{2} \int_{\mathbb{R}^3} d^3 p d^3 p' \left[ \bar{\varphi}_{123} \bar{\varphi}_{1'2'3'} \bar{\varphi}_{1'2'3'} + \text{sym}\{1,2,3\} \right].$$

This is consistent with the UV initial condition.

From (5), the 2-point one-particle irreducible (1PI) Green function expresses as $\Gamma_k^{(2)}(p, p') = (Z_k \sum_{s} p_s^2 + \mu_k) \delta(p - p') + F_k(p, p')$, where

$$F_k(p, p') = \lambda_k \left[ \int dq_1 \bar{\varphi}_{q_1} \bar{\varphi}_{q_1} \bar{\varphi}_{q_1} \delta(p_1 - p_1') \right] + \int dq_2 dq_3 \bar{\varphi}_{q_2} \bar{\varphi}_{q_2} \bar{\varphi}_{q_2} \delta(p_2 - p_2') \delta(p_3 - p_3') + \text{sym}\{1,2,3\}.$$

The regulator function is chosen as [45]: $R_k(p, p') = \delta(p - p') Z_k(k^2 - \sum_{s=1}^4 p_s^2) \theta(k^2 - \sum_{s=1}^3 p_s^2)$, where $\theta$ stands for the Heaviside step function. This is a standard choice and it satisfies all basic requirements, namely $R_k = 0$, $\forall p$, so that $Z_k[0, \mathcal{J}] = Z[\mathcal{J}]$, $R_k = \propto \Lambda^2$, $\forall p$ such that $|p| < k$, to approximately freeze the propagation of modes with norm smaller than $k$; $R_k(|p| > k) = 0$, so that high modes are unaffected by the regulator. In addition, this choice allows the analytic evaluation of spectral sums.

We then expand the Wetterich equation in powers of $\varphi$, up to the third order, discarding vacuum terms, to obtain the beta functions.

**Thermodynamic limit.** In order to regularise volume divergences, we perform a lattice regularisation in the $p$-space, which follows from a compactification in the direct space, according to the conventions of [46]. Defining the model (1) over a lattice $D^* = \{2\pi \mathbb{Z})/L\}^3 = \{2\mathbb{Z})^3\}$, of spacing $\ell^3$ proportional to the volume of the direct space, the Fourier transform becomes a Fourier series and, for any function $f(p)$, we have $\int_{\mathbb{R}^3} d^3 p f(p) = \ell^3 \sum_{p \in D^*} f(p)$. We define the delta distribution in $D^*$ as: $\delta_{D^*}(p, q) = \delta(p, q)/\ell^3$, with $\delta(p, q)$ the Kronecker delta. As a result, we have: $\delta_{D^*}(p, p') = \delta_{D^*}(p, p') = \ell^3/\ell^3 = 1/\ell^3$.

The effective action is then computed to be

$$\Gamma_k[\varphi, \bar{\varphi}; l] = \ell^3 \sum_{p \in D^*} \bar{\varphi}_{123} \left( Z_k \sum_{s} p_s^2 + \mu_k \right) \varphi_{123} + \frac{\ell^3 \lambda_k}{2} \sum_{p, p' \in D^*} \left[ \bar{\varphi}_{123} \bar{\varphi}_{1'2'3'} \bar{\varphi}_{1'2'3'} + \text{sym}\{1,2,3\} \right],$$

where $\varphi(p) = \int_{\mathbb{R}^3} d^3 x e^{-i \lambda \sum_{s=1}^3 p_s x_s} \varphi(x_1, x_2, x_3)$. The dependence of the system on the volume of the direct space is now explicit, and we can tune this dependence in order to consistently remove all the divergences, and be left with the physical $\beta$-functions. The continuous description is recovered in the thermodynamic limit $l \to 0$.

**$\beta$-functions.** The IR regularisation of the system of $\beta$-functions is direct: we need to extract from the coupling constants an explicit dependence on the volume of the direct space, in addition to their scaling with the momentum cut-off. After a lengthy but straightforward calculation, the set of $\beta$-functions reads

$$\beta(Z_k) = \frac{\lambda_k}{(Z_k k^2 + \mu_k)^2} \left[ 2Z_k \left( \frac{2}{\ell^2} + \frac{2}{\ell} \right) + \partial_t \frac{\ell^2}{Z_k} \right],$$

$$\beta(\mu_k) = -\frac{3 \lambda_k}{(Z_k k^2 + \mu_k)^3} \left[ 2Z_k \left( \frac{4}{\ell^2} + \frac{2}{\ell} \right) + \partial_t \frac{4}{Z_k} \right],$$

$$\beta(\lambda_k) = \frac{2 \lambda_k^2}{(Z_k k^2 + \mu_k)^3} \left[ 2Z_k \left( \frac{4}{\ell^2} + \frac{10}{\ell} \right) + \partial_t \frac{2}{Z_k} \right],$$

We drop here the symbol $\lim_{\ell \to 0}$ to simplify the notation.

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To make sense of the infinite volume limit, we need to eliminate the dependence of the system on (the negative powers of) $l$. Extracting the dimensions of the coupling constants using only one parameter ($k$ or $l$), one obtains a set of $\beta$-functions which are either trivial or still divergent in the limit. This peculiar feature can be attributed to the non-local combinatorics of the TQFT interactions, which requires a departure from conventional procedures in local QFTs in inducing non-homogeneous powers in both $k$ and $l$. In turn, this prevents the possibility of removing an overall volume factor, as in local QFT, and modifies the ordinary scaling dimensions of the couplings. Indeed, we find necessary to adopt the general prescription

$$Z_k = Z(k^{2\lambda-\chi}, \mu_k = \bar{\mu}_k Z(k^{2\lambda-\chi}, \lambda_k = \bar{\lambda}_k Z_0^2 k^\sigma),$$

where $[Z_0] = [\bar{\mu}_k] = [\bar{\lambda}_k] = 0$, $[\varphi] = -5/2$ and $\xi + \sigma = 4$. These dimensions are fixed by requiring that $[\lambda_k] = 0$ with $[p] = 1$. Now, defining the anomalous dimension $\eta_k = \partial_k \log Z_k$, the dimensionless $\beta$-functions read

$$\eta_k = \frac{\lambda_k 4 \kappa^a}{l^2 k^4 - 2 \pi (1 + \mu_k)^2} \left( \eta_k - \chi \right) \left( \frac{\pi k^2}{2 l^2} + \frac{4 k^3}{3 l} \right) + \left( \frac{k^2}{l^2} + 2 \frac{k^3}{l} \right) + \chi,$$

$$\beta(\bar{\mu}_k) = -\frac{3 l^2 k^4}{4 l^2 k^6 - 2 \pi (1 + \mu_k)^2} \left( \eta_k - \chi \right) \left( \frac{\pi k^2}{2 l^2} + \frac{4 k^3}{3 l} \right) + 2 \left( \frac{k^2}{l^2} + 2 \frac{k^3}{l} \right) - \eta_k \bar{\mu}_k - (2 - \chi) \bar{\mu}_k,$$

$$\beta(\bar{\lambda}_k) = \frac{2 \lambda_k 4 \kappa^a}{l^2 k^4 - 2 \pi (1 + \mu_k)^2} \left( \eta_k - \chi \right) \left( \frac{\pi k^2}{2 l^2} + \frac{20 k^3}{3 l} + 2k^3 \right) + 2 \left( \frac{k^2}{l^2} + 10 \frac{k^3}{l} + 2k^2 \right) - 2 \eta_k \bar{\lambda}_k - \sigma \bar{\lambda}_k.$$

The system (10) of $\beta$-functions is non-autonomous in the IR cut-off $k$ as long as the parameter $l$ is kept finite. This feature is due to the peculiar combinatorics of the tensorial vertices which span the 1PI 2-point functions with different volume contributions. The fact that the set of $\beta$-functions of a TQFT over a compact group manifold is non-autonomous is consistent with the analysis of standard field theories on compact (and curved) manifolds [42].

To regularize the divergences due to the non-compact manifold we need to impose the following condition on the parameters $\chi$ and $\xi$: $\xi - 2 \chi - 2 = 0$. Choosing $\chi = 0$, $\xi = 2$, (which is equivalent to absorb $\chi$ in $\eta_k$) which entails $\sigma = 2$, we fix all scaling dimensions. The resulting system of differential equations for the theory is:

$$\eta_k = \frac{\pi \bar{\lambda}_k}{(1 + \mu_k)^2} (\eta_k + 2),$$

$$\beta(\bar{\mu}_k) = -\frac{3 \pi \bar{\lambda}_k}{(1 + \mu_k)^2} \left( \eta_k + 2 \right) - \eta_k \bar{\mu}_k - 2 \bar{\mu}_k,$$

$$\beta(\bar{\lambda}_k) = \frac{2 \pi \bar{\lambda}_k}{(1 + \mu_k)^2} (\eta_k + 4) - 2 \eta_k \bar{\lambda}_k - 2 \bar{\lambda}_k,$$

which is the starting point of our computation of the RG flow. As expected, absent any remaining fixed external scale, the system is now autonomous.

**The RG flow.** – Proceeding with the standard analysis, we first determine the fixed points and then study the linearised system around them to determine the critical exponents of the model. From the non-linear nature of the $\beta$-functions, we have a singularity at $\bar{\mu}_k = -1$ and $\bar{\lambda}_k = (1 + \bar{\mu}_k)^2/\pi$. In a neighbourhood of those singularities, we do not trust the linear approximation and, being interested mainly in the sector of the theory connected with the Gaussian fixed point (i.e. to the perturbative regime of the theory), we will not study the flow around points beyond the singularity. By numerical evaluation, we find a Gaussian fixed point (GFP) and three non-Gaussian (NGFP) fixed points in the plane $(\bar{\mu}_k, \bar{\lambda}_k)$. We discard one of them because it lies beyond the singularity. The others correspond to

$$P_1 = (8.619, -47.049), \quad P_2 = 10^{-1}(-6.518, 0.096).$$

The stability matrix at the GFP has an eigenvalue with algebraic multiplicity 2 corresponding to the canonical scaling dimensions of the couplings: $\theta_{1,2}^G = 2$, but one single eigenvector $v = (1, 0)$, thus, considering that all the trajectories flow into the origin, the GFP must have a marginal direction in the UV. In a neighborhood of the non-Gaussian fixed points, we have

$$\theta_1^G \sim 0.351$$ for $v_1^+ \sim 10^{-1}(0.65, -9.98),$$

$$\theta_2^G \sim -2.548$$ for $v_1^- \sim 10^{-1}(-6.88, 7.26),$$

$$\theta_2^G \sim 10.066$$ for $v_2^+ \sim 10^{-1}(9.996, -0.269),$$

$$\theta_2^G \sim -1.988$$ for $v_2^- \sim 10^{-1}(9.878, 0.506).$$

The flow of the couplings between the two NGFPs and the Gaussian one are plotted in fig. 2. The origin is a UV sink for the flow; hence, the model is asymptotically free. As mentioned before, the absence of a second eigenvector for the stability matrix around the GFP requires an approximation beyond the linear order and is a signal of the presence of a marginal perturbation. By close inspection of the plots, confirmed by direct integration at second order of the system of $\beta$-functions, which can be performed for generic numerical constants/initial conditions, we infer that the behaviour of this direction is still UV attractive, i.e. that it corresponds to a marginally relevant direction.

Both the non-Gaussian fixed points have one relevant and one irrelevant direction. They are also characterised by the so-called “large river effect”. This effect shows a splitting of the space of coupling in two regions not connected by any RG trajectory. Thus, the irrelevant direction for the NGFP match the properties of a critical surface and suggests the presence of phase transitions in the model. In the $\bar{\lambda} > 0$ plane, the flow is similar to the one of standard local scalar field theory on $R^3$ in a neighbourhood of the Wilson-Fisher fixed point. That is: above
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Fig. 2: (Color online) RG flow of the model. Brown arrows are eigenperturbations of the non-Gaussian fixed points (in black), while the green ones are those of the Gaussian fixed point (in red). Arrows point in the UV direction. The thick black line is the singularity of the flow beyond which (shaded region) the flow cannot be trusted.

the critical surface, the IR limit of the RG trajectories brings the theory in a region where both $\mathbf{n}_k$ and $\mathbf{m}_k$ are positive, while below the irrelevant eigendirection for $P_1$, the mass parameter is driven to be negative in the IR, indicating spontaneous symmetry breaking. By comparison with the local case, one can guess that the order parameter will be related with the mean-field value $\langle \phi \rangle$. A proper characterization of the two phases is however missing, and a first step should be a mean-field analysis at the level of the classical equations of motion of the TGFT model. Next, a better parametrization of the potential $V(\phi) = \lambda(\phi^2 - |\phi_0|^2)^2$ in terms of the solution of the equation of motion $\phi_0$ could have led to a better understanding of the broken phase, away from the singularities introduced by the present choice of parameters. As a counterpart, the minimization of the non-local action given by (5) which must be sorted first, is non-trivial. In the sector $\mathbf{X} < 0$, the situation is rather peculiar. We might infer that $P_1$ has the same properties just discussed but reversed with respect to the critical surface. The symmetric phase where $\mathbf{n}_k$ and $\mathbf{m}_k$ have the same sign in the IR is below the irrelevant direction of the fixed point, while the broken phase lies above it. In this sense, we may have a phase transition also crossing the surface $\mathbf{X} = 0$, but this is not an irrelevant direction for any NGFP. This feature suggests that, in this case, we may have a first-order phase transition. Nevertheless, we must remember that the sector $\mathbf{X} < 0$ generates theories with a non-stable coupling. This sector must therefore be analysed in a different parametrisation.

Although drawing any general conclusion is still a bit premature at this stage of FRG analysis of TGFTs, we can however observe features shared by other recent studies on the same subject [41,47]. The existence of an IR non-Gaussian fixed point very similar to the Wilson-Fisher fixed point is common to all these studies. It is therefore tempting to claim that this fixed point is a universal property of $\phi^4$-type of TGFTs. Enlarging the truncation, we would like to see if this point is still present.

In summary, we have applied the FRG analysis to a rank-3 TGFT defined over a non-compact group, $\mathbb{R}^3$. The issue of IR divergences has been addressed using a compactification of the manifold (lattice regularization in the momentum space) and then a thermodynamic limit. The system of $\beta$-functions, which is non-autonomous on the lattice, becomes autonomous in the non-compact limit using a particular concept of scaling dimensions of the couplings. We find both UV Gaussian and IR non-Gaussian fixed points. Our analysis suggests the existence of a phase transition from a symmetric to a broken phase, from positive to negative mass. In a GFT model with additional geometric data, and a proper simplicial gravity interpretation, a broken or condensate phase could be interpreted as a continuum geometric phase [17,19], and would support a phase transition scenario for the emergence of continuum spacetime and geometry from these GFT models. The model under consideration would therefore need to be enriched with such additional data to be more than an indirect support for such a scenario. Also in our model, in any case, a proper study of the broken phase, involving a change in parametrisation for the effective potential and a detailed study of the theory around the new ground state, solving the classical equation of motion of the model, in a saddle point approximation, would be needed to confirm conclusively the existence of a phase transition as envisaged.

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REFERENCES

[1] ORITI D., in Foundations of Space and Time, edited by ELLIS G. F. R., MURUGAN J. and WELTMAN A. (Cambridge University Press, Cambridge) 2012 (arXiv:1110.5606 [hep-th]).

[2] KRAJEWSKI T., PoS(QQGS 2011) (2011) 005 (arXiv:1210.6257 [gr-qc]).

[3] ORITI D., arXiv:0912.2441 [hep-th].

[4] ORITI D., arXiv:1310.7786 [gr-qc]; in Loop Quantum Gravity, edited by ASHTEKAR A. and PULLIN J. (World Scientific) 2015 (arXiv:1408.7112 [gr-qc]).

[5] DI FRANCESCO P., GINSPIRG P. H. and ZINN-JUSTIN J., Phys. Rep., 254 (1995) 1 (hep-th/9306153).

[6] AMBORN J., DURHUUS B. and JONSSON T., Mod. Phys. Lett. A, 6 (1991) 1133.

[7] GROSS M., Nucl. Phys. Proc. Suppl. A, 25 (1992) 144.

[8] BOULATOV D. V., Mod. Phys. Lett. A, 7 (1992) 1629 (hep-th/9202074).

[9] OOGURI H., Mod. Phys. Lett. A, 7 (1992) 2799 (hep-th/9205090).

[10] ROVELLI C., Quantum Gravity (Cambridge University Press, Cambridge) 2006.

[11] THIEMANN T., Modern Canonical Quantum General Relativity (Cambridge University Press, Cambridge) 2007.

[12] ROVELLI C., Phys. Rev. D, 48 (1993) 2702 (hep-th/9304164).

[13] REISENERBERGER M. P. and ROVELLI C., Class. Quantum Grav., 18 (2001) 121 (gr-qc/0002095).

[14] BARATIN A. and ORITI D., Phys. Rev. Lett., 105 (2010) 221302 (arXiv:1002.4723 [hep-th]); Phys. Rev. D, 85 (2012) 044003 (arXiv:1111.5842 [hep-th]).

[15] HAN M. X. and ZHANG M., Class. Quantum Grav., 29 (2012) 165004 (arXiv:1109.0500 [gr-qc]).

[16] ORITI D., PoS(QG-Ph) (2007) 030 (arXiv:0710.3276 [gr-qc]).

[17] ORITI D., Stud. Hist. Philos. Mod. Phys., 46 (2014) 186 (arXiv:1302.2849 [physics.hist-ph]).

[18] KOSLOWSKI T. A., arXiv:0709.3465 [gr-qc].

[19] GIELEN S., ORITI D. and SINDONI L., Phys. Rev. Lett., 111 (2013) 031301 (arXiv:1303.3576 [gr-qc]); JHEP, 06 (2014) 013 (arXiv:1311.1238 [gr-qc]).

[20] BAHR B., DITTRICH B., HELLMANN F. and KAMIŃSKI W., Phys. Rev. D, 87 (2013) 044048 (arXiv:1208.3388); DITTRICH B., MARTÍN-BENTO M. and SCHNETTER E., New J. Phys., 15 (2013) 103004 (arXiv:1306.2987).

[21] BEN GELOUN J. and RIVASSEAU V., Commun. Math. Phys., 318 (2013) 09 (arXiv:1111.4997 [hep-th]).

[22] CARROZZA S., Tensorial Methods and Renormalization in Group Field Theories (Springer, New York) 2014 (arXiv:1310.3736 [hep-th]).

[23] BEN GELOUN J., Class. Quantum Grav., 29 (2012) 235011 (arXiv:1205.5513 [hep-th]).

[24] BEN GELOUN J. and SMARY D. O., Ann. Henri Poincaré, 14 (2013) 1599 (arXiv:1201.0176 [hep-th]).

[25] CARROZZA S., ORITI D. and RIVASSEAU V., Commun. Math. Phys., 327 (2014) 603 (arXiv:1207.6734 [hep-th]).

[26] SMARY D. O. and VIGNES-TOURNERET F., Commun. Math. Phys., 329 (2014) 545 (arXiv:1211.2618 [hep-th]).

[27] CARROZZA S., ORITI D. and RIVASSEAU R., Commun. Math. Phys., 330 (2014) 581 (arXiv:1303.6772 [hep-th]).

[28] BEN GELOUN J., Commun. Math. Phys., 332 (2014) 117 (arXiv:1306.1201 [hep-th]).

[29] CARROZZA S., Ann. Inst. Henri Poincaré D, Comb. Phys. Interact., 2 (2015) 49 (arXiv:1407.4615 [hep-th]).

[30] LAHOCHÉ V. and ORITI D., arXiv:1506.08393 [hep-th].

[31] BONZOM V., GURAU R., RIELLO A. and RIVASSEAU V., Nucl. Phys. B, 853 (2011) 174 (arXiv:1105.3122 [hep-th]).

[32] BONZOM V., GURAU R. and RIVASSEAU V., Phys. Lett. B, 711 (2012) 88 (arXiv:1108.6269 [hep-th]).

[33] GURAU R. and RYAN J. P., Ann. Henri Poincaré, 15 (2014) 2085 (arXiv:1302.4386 [math-ph]).

[34] BONZOM V., GURAU R. and RIVASSEAU V., Phys. Rev. D, 85 (2012) 084037 (arXiv:1202.3637 [hep-th]).

[35] DELAMOTTE B., Lect. Notes Phys., 852 (2012) 49 (arXiv:cond-mat/0702365).

[36] WETTERICH C., Phys. Lett. B, 301 (1993) 90.

[37] MORRIS T. R., Int. J. Mod. Phys. A, 9 (1994) 2411 (hep-ph/0005122).

[38] BAZÉN E. and ZINN-JUSTIN J., Phys. Lett. B, 288 (1992) 54 (hep-th/9206035).

[39] EICHHORN A. and KOSLOWSKI T., Phys. Rev. D, 88 (2013) 084016 (arXiv:1309.1690 [gr-qc]).

[40] EICHHORN A. and KOSLOWSKI T., Phys. Rev. D, 90 (2014) 10 (arXiv:1408.4127 [gr-qc]).

[41] BENEDETTI D., BEN GELOUN J. and ORITI D., JHEP, 03 (2015) 084 (arXiv:1411.3180 [hep-th]).

[42] BENEDETTI D., J. Stat. Mech. (2015) P01002 (arXiv:1403.6712 [cond-mat.stat-mech]).

[43] DELEPOUVE T. and GURAU R., JHEP, 06 (2015) 178 (arXiv:1504.05745 [hep-th]).

[44] BENEDETTI D. and GURAU R., arXiv:1506.08542 [hep-th].

[45] LITIM D. F., Phys. Rev. D, 64 (2001) 105007 (hep-th/0103195).

[46] SÅLMHOFER M., in Renormalization: An Introduction, edited by BALIAN R. et al. (Springer, Berlin) 1999.

[47] BENEDETTI D. and LAHOCHÉ V., arXiv:1508.06384.