A fast, primary Coulomb blockade thermometer

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We have measured the third derivative of the current-voltage characteristics, \( \frac{d^3I}{dV^3} \), in a two-dimensional array of small tunnel junctions using a lock-in amplifier. We show that this derivative is zero at a voltage which scales linearly with the temperature and depends only on the temperature and natural constants, thus providing a primary thermometer. We demonstrate a measurement method which extracts the zero crossing voltage directly using a feedback circuit. This method requires only one voltage measurement, which makes it substantially faster than the original Coulomb blockade thermometry method.

Coulomb blockade thermometry (CBT) is a primary thermometry method which is suitable for cryogenic temperatures in the range 20 mK - 30 K. It is based on the properties of the Coulomb blockade in one- or two-dimensional arrays of tunnel junctions at temperatures where the charging energy \( E_C < k_B T \). Here \( E_C = e^2/2C_{\text{eff}} \), where \( C_{\text{eff}} \) is the effective capacitance of the tunnel junctions. In the CBT method, the first derivative of the current-voltage characteristics (IV-curve) is measured and from the properties of this curve the temperature can be extracted, using only natural constants and a calculable prefactor. The major advantage of the CBT method is the simple electrical measurement and the insensitivity to magnetic field.

In this letter we present an alternative approach which has the advantage of measurement speed. Using the same type of tunnel junction array we can measure the third derivative of the IV-curve. The third derivative has a zero crossing at a voltage which is (to the first order in \( E_C \)) zero crossing voltage directly using a feedback circuit. This method requires only one voltage measurement, which makes it substantially faster than the original Coulomb blockade thermometry method.

\[ \frac{d^3I}{dV^3} = -\frac{M e}{R_T C_{\text{eff}}} \left( \frac{e}{Nk_B T} \right)^3 g'' \left( \frac{eV}{Nk_B T} \right). \]  

Here \( N \) and \( M \) are the number of tunnel junctions in series and in parallel (in the case of a two-dimensional array) respectively, \( R_T \) is the tunnelling resistance of one junction at voltages well above the Coulomb blockade and \( g(x) \) is defined by Pekola et al. and can be written

\[ g(x) = \frac{(x/2) \coth(x/2) - 1}{2 \sinh^2(x/2)} \] (2)

The function \( g''(x) \) becomes

\[ g''(x) = \frac{((x/2) \coth(x/2) - 1)(3 \coth^2(x/2) - 2)}{2 \sinh^2(x/2)} \] (3)

Eq. (2) is valid in the limit \( E_C < k_B T \) and \( R_T \gg R_K = h/e^2 \approx 25.8 \text{kΩ} \). Lower temperatures and lower tunnelling resistances cause deviations which can be calculated theoretically. In this paper we have taken into account the effects due to low temperature. The deviations due to low resistance were not considered here, but are estimated to be less than 1% in the measurements presented here.

We can calculate the voltage \( V_0 \) at the zero-crossing of Eq. (4) numerically and the result is

\[ eV_0 = \pm 2.144 Nk_B T. \] (4)

At low temperatures (where \( k_B T \) approaches \( E_C \)) higher order corrections should be included. This gives a correction to the zero crossing in Eq. (5), which is independent of temperature:

\[ eV_0 = 2.144 Nk_B T - 0.465 NE_C. \] (5)

The sample we have measured was a two-dimensional array of 256 × 256 tunnel junctions and each junction had an effective capacitance of 2.2 fF and a tunnelling resistance of 17 kΩ. The array was fabricated using standard shadow evaporation of aluminium and in situ oxidation. The measurements were carried out by applying a DC voltage and an additional AC excitation (123 Hz) to
the Taylor series we can write higher derivatives. Including fifth and seventh orders in amplitude, comparable to $V$ values). The sample in series with a resistor $R_b$ with a resistance of 20 kΩ. The voltage over the resistor was measured with a Stanford SR830 lock-in amplifier which locked to the third harmonic (369 Hz) of the excitation AC voltage. This signal $\delta V_{3\omega}$ is proportional to the third derivative of the IV-curve. The excitation voltage $\delta V_{\omega}$ over the sample was measured with another lock-in amplifier which locked to the basic frequency (123 Hz) and the DC voltage was measured with a voltmeter. Additional low- and high-pass filtering was used to improve the measurements.

We used a pumped $^4$He cryostat equipped with a vacuum regulator which kept the bath at a constant pressure, and therefore constant temperature, during the measurements.

To find the relation between the third derivative and $\delta V_{3\omega}$ we can make a Taylor expansion of the IV-curve to the third order and arrive at the formula

$$\frac{d^3 I}{dV^3} = \frac{24}{R_b} \frac{\delta V_{3\omega}}{\delta V_{\omega}^3},$$

(6)

where $\delta V_{3\omega}$ and $\delta V_{\omega}$ are voltage amplitudes (not rms values).

This method requires a relatively large excitation amplitude, comparable to $V_0$, which introduces errors due to higher derivatives. Including fifth and seventh orders in the Taylor series we can write

$$\frac{24}{R_b} \frac{\delta V_{3\omega}}{\delta V_{\omega}^3} = \frac{d^3 I}{dV^3} + \frac{\delta V_{3\omega}^5}{16} \frac{d^5 I}{dV^5} + \frac{\delta V_{3\omega}^7}{640} \frac{d^7 I}{dV^7}.$$  

(7)

Knowing the amplitude of the excitation $\delta V_{\omega}$, the errors due to the higher order terms can be calculated and be corrected for. However, right at the zero crossing of the third derivative the higher order terms are quite small and therefore a relatively large $\delta V_{\omega}$ can be used without introducing large errors.

Fig. 1 shows a measurement of $d^3 I/dV^3$ at three different temperatures. The shape of the curves follow the expected $g''(x)$ behaviour, and the zero crossing follows Eq. 6 within a few percent. Fig. 2 shows the temperature calculated from the zero crossing plotted against the temperature calculated from the $^4$He vapour pressure $^3$ from 1.6 K to 4.2 K.

To test the principal advantage of this measuring method compared to the one used by Pekola et al. we set up a feedback loop, illustrated in Fig. 3. We used a DC output voltage from the lock-in amplifier, which was proportional to the $\delta V_{3\omega}$ signal amplitude, as an error signal to a PID regulator. The output of the regulator was used as the DC bias voltage and added to the AC excitation provided by the lock-in amplifier. The voltage over a resistor in series with the array was applied to the input of the lock-in amplifier, which was set to extract the third harmonic of the excitation frequency. After we adjusted the PID parameters to proper values, the DC voltage over the array stabilised at the voltage $V_0$, as defined by Eq. 6.

As a demonstration that the voltage follows the temperature as expected, we took a time trace of this voltage while we adjusted the temperature in steps, by changing the bath pressure, and the result is the graph in Fig. 4, where the voltage $V_0$ is converted to temperature using Eq. 6. The temperature steps are evident in the figure, and agrees well with the temperature calculated from the $^4$He vapour pressure, except for the lowest step. The disagreement at this step is probably due to higher derivatives and higher order corrections to Eq. 6. At the beginning of the fourth step, at 600 s in Fig. 4, the $P$ gain of the feedback was too large and the signal started to oscillate, but after reducing the gain (at around 680 s in the graph) the signal was stable again. Note that the relatively slow time response is not due to the thermometer or the measurement, but rather due to the time it takes to pump down the pressure in the $^4$He bath.

While Fig. 3 shows that this method is working, the precision is not very impressive in this first experiment,

\footnote{A PID regulator (Proportional, Integrating and Derivating) is a general feedback circuit with three adjustable parameters, which can be used with a wide range of applications.}
with fluctuations up to 10%. However, there are several ways to improve the method. The use of a lock-in amplifier picks up the very weak third harmonic signal below the main excitation signal and the noise, but by notching out the basic frequency before the input we can increase the dynamic range of the amplifier, and get a cleaner signal. The PID parameters can also be better optimised to the measurement, and continuously adjusted.

Looking at Eq. 1, it is obvious that we can make another improvement by increasing $M$, i.e. the number of parallel junctions in the array. The signal amplitude increases linearly with $M$. This speaks in favour of using 2D arrays with this measurement method. Note that we do not gain anything by decreasing $N$, the number of junctions in series, because in order to avoid higher order derivatives to affect the measurement we need to decrease the excitation amplitude $\delta V_\omega$ by the same amount. Even though it was not done in this experiment, it would be natural to let $\delta V_\omega$ be proportional to the temperature, to compensate for the strong signal dependence on temperature ($\sim T^{-3}$).

In conclusion, we have measured the third derivative of the IV-curve of a two-dimensional array of tunnel junctions. We show, theoretically and experimentally, that the zero crossing of this curve scales linearly with temperature, to the first order, and provides a primary temperature measurement. We also demonstrate the use of a feedback loop to create a fast primary thermometer. The feedback loop is possible because only one measurement is needed to get at a quantity which is proportional to the temperature.

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