Numerical Problem of Thermo-Consolidation of Porous Media with Rheological Skeleton Depending on the Variable Ambient Temperature

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Abstract. The study presents the results of the numerical calculations of the temperature impact on the two phase medium deformation described with the usage of Biot equations of consolidations with the rheological Kelvin-Voigt skeleton at the example of thermo-consolidation of flotation waste landfill “Żelazny Most” under influence of its dead weight, filtration and temperature gradient. 3D geometrical model of the landfill is based on the geometric measurements of the area and its neighbourhood. A starting point for the calculations was the calculated water table of the underground waters in the landfill area. The data referring to the effective parameters of the model were partially obtained via laboratory tests of the materials coming from the landfill. The rest of the data was taken from the literature related to the mediums with similar characteristics. The results of the stress state in the landfill allow to define the sensitivity of the model’s parameters in the temperature changes.

1. Introduction

For the construction of soil structures are increasingly used unconventional materials. For this group of materials also includes mining waste. The problem of the use of mining waste is of particular importance at the time when the hydrotechnical construction through its impressive size, occupies a large area of space, without being subject inert environment. “Żelazny Most” is just an example of this type of construction engineering and one of the largest hydrotechnical objects in the world. The rate of production of the mines of KGHM-in, requires ongoing assessment of the strength parameters and keep permanent deformation characteristics of waste built into the embankment. KGHM Mine production rate now, and thus the rate of landfill grow, requires ongoing assessment of many crucial parameters such as: strength parameters, deformation characteristics of waste built into the embankments and the control of the static water level inside the embankments. Thus, the current development and exploitation of the “Żelazny Most” site even just in the geotechnical meaning is a very complex issue, which justifies the need to build more accurate rheological models simulating the processes taking place there.

Rheological properties of soils and rocks are the subject of numerous publications and research. In these works, for the description the processes are used rheological models of continuum mechanics. Taking as a starting point a model of a multiphase resort, assuming that the solid phase has a hydraulically connected pores or micro-activities which allow to flow filtration of liquid and/or gas
mathematical model of the porous medium creeping defined as a multicomponent, two-phase body was introduced for the first time by Maurice Biot [1, 2]. This model has been analyzed, in a very general way, based on the theory of asymptotic homogenization by method of periodic structures, where the slightly compressible fluid fills the pores of two-phase medium by Auriault [3], Bensoussan, Lions Papanicolau Bensoussan [4], Auriault i Sanchez Palencia [5] and with statistical methods by Kröner [6] Rubinstein i Torquato [7]. The mathematical model for the case where the gas fills pores of the medium, based on asymptotic homogenization method was presented by Auriault, Strzelecki, Bauer and He [8]. One of the fundamental principles of porous media mechanics is a postulate of the continuity of the porous medium. Those models are used for describing the complex mediums processes with discontinuous skeleton consisting of soil particles of different size and shape. The models that describes the creep processes of this type of soils accurately reflects the real process of deformation of cohesive soils. This issue was described Bartlewska [9] in her dissertation as well as by Bartlewska and Strzelecki in [10, 11]. The authors show that in creep process of this kind of resorts an important role in addition to the volume and form compressibility has viscosity of the skeleton. The effect of temperature field in the case of adiabatic processes in the deformation process of two-phase medium composed of elastic skeleton and weakly compressible fluid was described by Coussy. Based on the thermodynamics of irreversible processes he developed a mathematical model of thermal consolidation in work [12] and Kowalski et al in [13].

The presented paper is a continuation of numerical researches of the model of the flotation waste landfill “Żelazny Most” [14] and reflection [15] on the effect of temperature on the deformation process. The authors present a three-dimensional model which is extended with thermal parameters. The proposed model includes a generalization of Biot's equations for any non-isothermal processes, considering the rheological characteristics of the skeleton. The authors analyze the process of changing the displacement of the crown of flotation waste landfill “Żelazny Most” in time, depending on the temperature variation based on annual distribution of temperature ambient building represented by a sine function. In the first variant considered the impact of the dead load and the temperature gradient inside the landfill and variable temperatures during the process of deformation, the second option assumes a constant temperature. Comparison of the experiments’ results allowed to formulate conclusions on the effect of temperature as a result of seasonal changes in the deformation process of the flotation waste landfill “Żelazny Most”.

Administratively flotation waste landfill “Żelazny Most” is located in Lower Silesia in the Legnicko–Głogowski Copper District (LGOM), in two districts: Lubin and Polkowice. This flotation waste landfill was built in 1974, and its operation and the simultaneous expansion begun in 1977. The total length of barriers surrounding the landfill is 14.3 km and the total area - 1394 ha. The annual volume of waste deposited to the flotation ranged from 20 to 26 million tonnes, of which almost 75% is to further the superstructure and only 25% is in the process of disposal.

2. Consolidation process equations
The below presented mathematical equations of thermal consolidation process for the Biot body with rheological skeleton have been derived based on the fundamental laws of Newtonian mechanics for continuous center and thermodynamics of irreversible processes. The starting point are the initial assumptions of the theory of two-phase mediums composed from elastic-viscous compressible skeleton and a viscous fluid filling out the pores of this center. Assumptions are described in detail in [9].

Let define \( \Omega \) is the space defining element \( \mathcal{V} \) filled out with two-phase medium and limited by the surface \( S \). Vector \( \vec{n} \) is a unit vector normal to the surface \( S \) facing the outside of the element \( \Omega \).

If the \( \vec{v}^l \) and \( \vec{v}^s \) means respectively the vectors of the speed of liquid and skeleton then \( \vec{v}' = \vec{v}^l - \vec{v}^s \) specifies relative speed of the filtration flow of the liquid through the porous medium.
If the $\rho_s$ and $\rho_l$ stands for specific density accordingly of the skeleton and fluid, we can make volumetric density of the skeleton $\rho_s = (1-f)\rho_s$ and of the fluid $\rho_l = f\rho_l$ where $\rho$ will be volumetric density of the two-phase medium equal in value to the total sum of $\rho_s + \rho_l$. The value of $\bar{\rho}$ will mean density of the liquid flowing through the surface $S$: $\bar{\rho} = f_s\rho_l$.

The continuity equation of the two phases center is:

$$\int_S \rho v'_i n_i dS + \int_S \bar{\rho} v'_i n_i dS + \int_{\Omega} \frac{\partial \bar{\rho}}{\partial t} d\Omega = 0$$

(1)

Taking under consideration in the above captioned equation the Gauss Ostrogradski theorem, we can write down the above equation in the form of a local relationship:

$$\frac{D^s \rho}{Dt} + \rho \dot{\varepsilon} = -[\bar{\rho} v'_i]_i$$

(2)

Where $\frac{D^s}{Dt} = \frac{\partial}{\partial t} + v'_i \frac{\partial}{\partial x_i}$ is the material derivative.

The equation of the continuity of the fluid flow through the skeleton of medium is given by:

$$\int_S \bar{\rho} v'_i n_i dS + \int_S \bar{\rho} v'_i n_i dS + \int_{\Omega} \frac{\partial \bar{\rho}}{\partial t} d\Omega$$

(3)

Considering the Gauss Ostrogradski theorem we can write down the above captioned equation in the form of a local relationship:

$$\frac{D^s \bar{\rho}}{Dt} + \bar{\rho}(\dot{\varepsilon} - \dot{\varepsilon'}) = -v'_i [\bar{\rho}]_i$$

(4)

The $\dot{\varepsilon}$ and $\dot{\varepsilon'}$ mean accordingly the rate of change of dilatation fluid and soil skeleton. The equations of movement of the solid phase medium are:

$$\int_S \sigma_{ij} n_j dS + \int_{\Omega} b v'_i d\Omega + \int_{\Omega} (\rho - \bar{\rho}) X'_i d\Omega = \int_{\Omega} (\rho_{11} v'_i + \rho_{12} v'_i) d\Omega$$

(5)

where $b$ is the filtration coefficient of viscous resistance, and $\rho_{11} + \rho_{12} = \rho_s > 0$, $\rho_{12} < 0$. Local relationship which defines the equation of laminar motion of skeleton for quasi – static problems is reduced to the form:

$$\sigma_{ij} + X'_i (\rho - \bar{\rho}) = -b v'_i$$

(6)

The equations of motion of the liquid phase in the case of laminar motion is expressed by the formula:
\[
\int \sigma_i dS - \int b v_i d\Omega + \int X_i \rho d\Omega = \int \left( \rho_{12} \dot{v}_i + \rho_{22} \dot{v}_i \right) d\Omega
\]

(7)

where \( \rho_{12} + \rho_{22} = \rho > 0 \). Local relationship which defines the equation of laminar motion of fluid for the case of quasi-static issues is reduced to the form:

\[
\sigma_i + X_i \bar{\rho} = b v_i
\]

(8)

where \( X_i = -\delta_{ij} g \) is the gravitational acceleration in the right-hand frame of reference, \( \sigma_i \) the components of the stress tensor in the skeleton related to the total cross-sectional area, \( \sigma = -p\cdot f \) the diffuse tension of a liquid filling the center of a porous.

Constitutive relationships for Biot body with a rheological skeleton of Kelvin-Voigt for the adiabatic processes are presented:

\[
\begin{align*}
\sigma_{ij} &= 2N\varepsilon_{ij} + M\varepsilon_{ij} + 2NT_{ab} \ddot{\varepsilon}_{ij} + \left( A T_{ab} + N T_{ab} \right) \varepsilon_{ij} \dot{\varepsilon}_{ij} + \frac{Q}{R} \sigma \delta_{ij} + P_1 (T - T_0) \delta_{ij} \\
\sigma &= Q \varepsilon + R Q + d(T - T_0)
\end{align*}
\]

(9)

where \( N \) is the shear module of the skeleton, \( A \) - module volume deformations of the skeleton, \( Q \) - volumetric strain rate effect on the tension of the liquid in the shell or vice versa, the volumetric strain rate effect on the skeleton of stress in the liquid, \( R \) - module of volume deformations of the liquid filling out the pores of the body Biota. The parameter \( M \) is expressed by: \( M = A - \frac{Q^2}{R} \)

Constant d is given by: \( d = -[3Q r^i + r^i R] \)

where \( r^i \) and \( r^j \) presents accordingly the linear expansion of the skeleton and the volumetric expansion of the liquid, constant \( P_1 \) is calculated by the formula:

\[
P_1 = -\frac{T(3K r^i + Qr^j)}{\lambda}
\]

where \( \lambda \) is the heat transfer coefficient of soil, \( T_a, T_b \) are the parameters of the skeleton expressed by the formulas: \( T_a = \frac{\eta^*}{N} i \quad T_b = \frac{\lambda^*}{A} \)

\( \eta^*, \lambda^* \) are the shear and volume viscosity of soil skeleton.

The simultaneous equations of the linear theory of thermal consolidation in the movements of the skeleton and the function of stress in the liquid, the filtration flow equation and the heat conduction
equation for the Biot body with a rheological Kelvin-Voigt skeleton consists of five differential equations

\[
\begin{align*}
N\Psi_k \nabla^2 u_i + (A\Psi_L - \frac{Q^2}{R} + N\Psi_k)\varepsilon_{ii} + \frac{H}{R} \sigma_{ii} - \rho g \delta_{ij} &= -P_i T_i, \\
k R \frac{\varepsilon_{ii} \sigma}{f_o \rho g} &= T_0 \left[ \sigma - H \varepsilon + P_4 T \right], \\
\lambda \nabla^2 T &= T_0 \left[ P_2 \varepsilon - P_1 \sigma + P_5 T \right].
\end{align*}
\]

(10)

where \( \Psi_k = 1 + T_a \frac{\partial}{\partial t}, \Psi_L = 1 + T_b \frac{\partial}{\partial t} \) - are differential operators, \( k \) is the coefficient of filtration of fluid through a porous medium, \( g \) - gravitational acceleration, and the coefficients \( P_2, P_1, P_4 \) and \( P_5 \) are given by equations:

\[
P_2 = 3r^i (K - \frac{HQ}{R}) - Rr^i, \quad P_3 = 3r^i \frac{Q}{R} + r^i, \quad P_4 = R P_3,
\]

\[
P_5 = \frac{(3Q r^i + r^i R)^2}{R} + \frac{(\tilde{\rho}_s + \tilde{\rho}_w) c_v}{T}
\]

\( c_v \) is the specific heat at constant volume. The above presented set of equations is starting point for the issue resolved in a study.

3. Construction of numerical model of thermo-consolidation for issues triaxial.

Process of thermo-consolidation of the porous medium is described by the system of equations (10) based of which the three dimensional model of the medium was created. The calculations were conducted with the usage of the Flex PDE V.6. Professional program [16]. To generate the numerical model of 3D consolidation of the landfill and the surrounding area based on the analytical mode of Biot with the rheological Kelvin-Voigt skeleton, the physical and strength parameters received during laboratory tests were used. The range of basic research included a standard soil analysis as in the studies [9]. To determine oedometric compressibility modulus the oedometric tests was made for four loads. The value of the primary compressibility modulus can be defined as \( Mo = 1662.9 \) kPa. For calibration (to obtain the effective parameters of Biot model with the Kelvin-Voigt skeleton), the statistical methods presented in the study [9] were used. Three parameters were calibrated as the initial analysis of the traceability of the model was conducted at the model equations. The strength parameters were received based on the research in the direct shear apparatus and presented in the study [9]. The remaining effective model’s parameters, including heat parameters, were taken from the literature.

On the basis of the described in the study [9] three dimensional numerical model of the geological structure and for the filtration flow, numerical model of creeping of the reservoir “Żelazny Most” was built. The numerical model of the thermo-consolidation was created for the vertical cross-surface at the line north-south through the geometrical gravity center of the landfill. The entire area under computer simulation of the consolidation process is built from the smaller areas with different parameters of the sediments stored within. This is why the entire area was divided into smaller areas with some well-known parameters. In this way, 5 sub-regions were separated what is visible at the Figure 2 that shows
the initial network of the finite elements. The displacements of the specific layers of the landfill were observed under the influence of its death weight and the temperature gradient between the lower surface and the landfill’ surrounding environment, including the existing drainage systems of the waste landfill. Two variants, differing with the value of the temperature at the upper surface were taken. It was assumed the time of the experiment 30 years and the results for the two last years of the simulation were presented. The lower surface of the landfill was assumed to be permeable for the heat; T=27°C, while the sides and the upper surface was defined as isolated. At the upper surface of the sample, the horizontal components of the skeleton’s displacement vector were assumed equal to zero and the value of the water pressure equal to the atmospheric pressure.

3.1. Variant with the constant ambient temperature

In the first variant the constant temperature was assumed equal to the average yearly temperature observed in the landfill surrounding, $T= 9,18 \, ^\circ C$

3.2. Variant with the variable ambient temperature

In the second variant the analysis of the impact of the temperature changes at the deformation process of the landfill was presented. After analyzing the average monthly temperatures in the surrounding of “Żelazny Most” with the usage of the linear optimization method, to the given average temperatures the sinusoid was adjusted based on the minimization of the squared error (Figure 1) and receiving the boundary equation (11) describing the volatility of the temperatures in the examined period of time.

$$T(t) = 11,42 \times \sin(1992e^{-7t} - 0,825) + 9,18$$  \hfill (11)

The results were generated for the first year of the experiment and then after 10, 20 and 30 years. It was observed the impact of the volatility of the temperatures at the landfill’s deformation process in time.

![Figure 1](image)

**Figure 1.** Sinusoid adjustment (calculated) to the average temperature values (Measured).

4. Results of numerical calculations of thermo-consolidation

The Flex PDE system has module automatically generating the finite elements network that can be modified during the calculation process that the pre-defined accuracy of the calculation is met at each stage of the calculation process. The second changing element is, for transient processes, and such a process we have here, a time step $dt$. The program starts with starting time step $dt = 10^{-5}$ s and it ends with $dt = 2,06 \times 10^8 s$. At Figure 2 the distribution of the finite elements in the starting calculation phase is presented.
Figure 2. Finite element mesh generated by the software in the contaminated scale phase in contaminated scale height.

Figures 3a) and 3b) presents the distribution of the temperature in the landfill from June and December respectively. The months with the extreme average temperatures were chosen purposely. Remembering about the constant temperature of the upper layer of the landfill equal to 27 °C, we can observe a more-less regular distribution of the temperature at the edges of the landfill in both cases.

However, the higher temperatures in the center of the landfill are observed in the warm month (June). The analysis of the cross-section shows clearly the chaos in the June simulation case. The high ambient temperature causes that the low temperature is kept inside of the landfill (Figure 3a). At the Figure 3c the distribution of the temperature in the variant with the constant ambient temperature is presented. The distribution is rather regular. The highest temperature is in the lower layer, the lowest – in the landfill’s crown.

Figure 3. Distribution of the temperature inside landfill a) June b) December c) constant ambient

The Figures 4 and 5 present the distribution of the vertical displacements. In the variant with the variable ambient temperature we observe in June the highest values of the displacement of the landfill’s center down and at the same time the biggest values of the displacement of the landfill’s sides up. In December the differences between falling landfill’s center and the rising edges are much smaller. In the benchmarking variant with the constant yearly ambient temperature (Figures 4c and 5c) we observe same tendency: the landfill’s center moves down while the edges up- but the differences between the subsiding the landfill’s center and its edges are smaller. It can be assumed that the distribution of the subsides is more regular in the constant ambient temperature. It is visible that with the higher ambient temperature the landfill deformations are bigger.
The graphs from the Figures 6 and 7 shows the temperature changes within the time for the specific point with the coordinates $z = 110, z = 130, z = 150, z = 170$ for both variants (with the variable and constant ambient temperature), for the last two simulations. It is clearly seen that in the landfill center (Figure 6) as well as at the edges (Figure 7) the biggest temperature changes are observed around the upper layer of the landfill while the temperature at its bottom oscillate around values closed to the variant with the constant ambient temperature.

Figure 6. The temperature changes in $^\circ C$ at the defined heights of the landfill for the point $x=2500$ $y=2500$ for the last two years of both simulations. The values of the average temperature at the certain level without taking under consideration the temperature was drew with dashed line.
Figure 7. The temperature changes in °C at the defined heights of the landfill for the point $x=2500 \, y=4000$ for the last two years of both simulations. The values of the average temperature at the certain level without taking under consideration the temperature was drew with dashed line.

The graphs presented at the figure 8 and 9 show the displacement in time for both: the center of the landfill (Figure 8) and for its edges (Figure 9) for the variable temperature (solid line) and for the variant with the constant ambient temperature (dashed line). For the landfill’s center the biggest displacements are observed in the upper surface of the landfill, the biggest values are observed for the warm moths and the values significantly differ from the subsides in the constant ambient temperature, also in the lower part of the landfill ($z=110$) the impact of the temperature at the subsiding process is clearly visible. However, in this case the values are much lower and close to the value obtained in the constant ambient temperature.

Figure 8. The displacement changes at the defined heights of the landfill for the point $x=2500 \, y=2500$ for the last two years of both simulations. The values of the average temperature at the certain level without taking under consideration the temperature was drew with dashed line.

The biggest discrepancies between both variants are seen at the Figure 9 showing the subsiding of the chosen point at edge of the landfill. The subsides value at the constant temperature (dashed line) differs a lot form the subsides at the variable temperature especially at the lower landfill surface that moves down while the upper landfill’s surface clearly moves up. We can than observe the significant impact of the temperature at the process of the vertical displacement and the landfill’s deformation at its different levels.
Figure 9. The displacement changes at the defined depths of the landfill for the point x=2500 y=2500 for the last two years of both simulations. The values of the average temperature at the certain level without taking under consideration the temperature was drew with dashed line.

5. Conclusions

In the study the results of the numerical calculations of the impact of the temperature on the displacement of two phase medium described with the Biot equation of consolidation with the Kelvin-Voigt rheological skeleton were presented at the example of thermo-consolidation of the flotation waste landfill “Żelazny Most” under the influence of its dead weight filtration and temperature gradient. 3D geometrical model of the landfill is based at the geodetic measurements of the area and neighbourhood. The presented study is a continuation of the numerical studies at the model of flotation waste landfill “Żelazny Most” and the deliberations about the impact of the temperature and the deformation process. The authors present 3D model at the same time extending it by the heat parameters. The numerical resolution assumed the analysis of the displacement of the specific layers of the landfill exposed to the influence of the dead weight and temperature gradient between the upper surface of the landfill and the environment (neighbourhood). It was assumed the time of the experiment of 30 years and the results were presented for the last 2 years of simulation. The obtained results of the temperature and the displacements in the “Żelazny Most” landfill depending on the environment temperature shows its impact the deformation process of the landfill. The experiment shows that with the higher temperature of the environment comes the biggest deformations are both - inside and outside of the landfill. The biggest temperature changes apply to the upper layers of the landfill that are the most susceptible to the subsidence. However what needs to be highlighted is the fact the subsidence is irregular, while in the warm months it is the center of the landfill that subside the most and its edges are moving in the opposite directions (Figure 5). In the cold months the differences are so evident and the values of the displacements are smaller. It is worth to see the discrepancy between the results of the experiments at the constant and variable temperature variants. The course of the value of the subsides differ significantly taking the most similar values in the lower layers of the landfill (Figure 7). The most significant difference between the examined layers can be observed at the example of displacement of the single place at the edge (border) of dam (Figure 8). The absolute values of the subsides may differ from the ones measured during the exploitation of the reservoir. It may be caused by the values of the constants that were used for the calculations. We did not manage at the current stage of the experiment, to define of all the effective parameters of the specific layers of the landfill and the background of flotation waste landfill. Automatically, some of the values were taken directly from the literature. Besides the qualitative differences in the value of the subsides the model shows well the natural process that are happening in the environment.

The presented resolution of the creeping process as a function of time of the “Żelazny Most” landfill shows the possibility of the practical usage of mathematical model of consolidation of the Biot bodies
with the rheological skeleton of Kelvin-Voigt. The inclusion of the temperature at the creeping process may have in important impact at the definition of the areas potentially threatened by the loss the stability and at the same time – at the further designing of the “Żelazny Most” landfill.

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