Study of the role of the magnetic configuration in a $\kappa$-$\varepsilon$ model for anomalous transport in tokamaks.

S. Baschetti, H. Bufferand, G. Ciraolo, P. Ghendrih

CEA Cadarache, Saint-Paul-lez-Durance 13108 France

A. Gallo

Aix-Marseille Univ., CNRS, Laboratoire PHIM, Marseille, France

E. Serre

Aix-Marseille Univ., Laboratoire M2P2, Marseille, France

the EUROfusion MST1 team

and the TCV team

See the author list of S Coda et al 2017 Nucl. Fusion 57 102011

Abstract

A reduced model for the anomalous transport of magnetically confined plasma in the edge and SOL, inspired by the Reynolds-Averaged Navier-Stokes (RANS) approach, is presented assuming diffusion as governing mechanism. An empirical equation is built for the turbulent kinetic energy and the system is closed via the scaling of global confinement, the experimental dependencies to machine parameters being recovered for the SOL width. The model is implemented in the transport code SolEdge2D and tested with respect to three magnetic configurations of TCV. Profiles of density, temperature and perpendicular heat flux are then compared to experimental data. Particular attention is devoted to the effect of the magnetic configuration on the SOL width and particle transport coefficient to different configurations.

Keywords: anomalous transport, reduced model, $\kappa$-$\varepsilon$, magnetic configuration
Introduction

For transport models, self-consistent calculation of effective radial transport coefficients and plasma mean fields is mandatory to embrace the need of routine-use simulations on ITER-size machines. Estimation of effective density and temperature profiles and decay lengths in the SOL are indeed key parameters in the plasma operation and also required inputs to couple properties of electromagnetic auxiliary systems. In this paper a reduced model for anomalous transport of magnetically confined plasma in the edge and SOL is presented and implemented numerically, being inspired by $\kappa$-$\varepsilon$ models widely used in the CFD (Computational Fluid Dynamics) community to address engineering problems of turbulence. Recalling the spirit of similar approaches [1], in the first section an empirical equation is introduced for the turbulent kinetic energy ("$\kappa$" in the CFD nomenclature), the latter being related to transport coefficients according to results of quasi-linear theory (QLT) for diffusive-like turbulent flux [2]. The $\kappa$ equation is not derived from first principles but is built in a semi-empirical fashion. It aims at describing main plasma instabilities such as interchange. A closure is required for turbulence saturation. In this contribution, as a first step, the closure has been adjusted to [3], [4] and [5]. In this way transport is self-consistently calculated and dependencies of the SOL width with respect to main machine parameters (especially $B_{pol}$) observed experimentally are recovered.

The $\kappa$-$\varepsilon$ model is integrated into 2D transport code SolEdge2D [6], which can self-consistently calculate mean fields of diverse plasma quantities (such as ion and electron density, temperature and velocity), for a wide range of tokamak magnetic geometries, by assuming axisymmetry with respect to the toroidal angle $\varphi$. In order to test such high-flexible model, one can investigate on the effect of varying the position of the outer divertor leg length $L_{div}$, motivated by the crucial role magnetic geometry can play in affecting the anomalous transport. Such kind of work is accomplished in this paper, where TCV (Tokamak à Configuration Variable) has been taken as study case due to the availability of experimental data concerning ohmic discharges with varying $L_{div}$ (section 2).

Preliminary results are discussed and compared to experimental findings (section 3), focusing on the main characteristics of outer divertor profiles of density, temperature and perpendicular heat flux. In section 4 the effects
of varying the magnetic geometry on the anomalous diffusivity maps are commented. Finally, conclusions and perspectives are outlined in section 5.

1. A model for large-scale turbulence in L-mode plasma

To describe the mechanisms underlying the anomalous transport in L-mode tokamak discharges and its effects on global confinement, a 2D model is introduced for the kinetic energy of plasma fluctuations, defined by $\kappa \equiv \frac{1}{2} \langle \tilde{u}^2 \rangle \ \text{[m}^2$/s$^2]$], where $\tilde{u}$ is the fluctuating component of plasma velocity field $u$. Here the average operator $\langle \cdot \rangle$ acts on small spatial and temporal scales associated with fluctuations. This definition is reminiscent of RANS models widely used in the CFD community, where a $\kappa$ equation is derived to close the equilibrium of the mean fields. In that framework the so-called "Boussinesq’s closure" is invoked to calculate the Reynolds stress tensor in the averaged Navier-Stokes equation through a bumping eddy viscosity which explicits the transport of mean flow due to fluctuations [7].

In the following we consider that $\kappa$ and other thermodynamic quantities are averaged over small scales and short times, so that the spatial scale is larger than the Larmor radius $\rho_L$ and the time evolution scale is slower than drift-wave time $\tau_d \sim a/c_s$. For $\kappa$ we assume the following general form of a transport equation:

$$\partial_t \kappa + \nabla \cdot (\kappa u_{||}) = \nabla \cdot (D_n \nabla \kappa) + \gamma \kappa - \varepsilon$$

(1)

In the left-hand-side the second term is the convective term, where $u_{||}$ is the velocity advecting $\kappa$ in the direction parallel to magnetic field lines.

In the right-hand-side there are a diffusion term, a source and a sink, respectively. The source is modeled to take into account the turbulent energy triggered by interchange instabilities, whose growth rate $\gamma [s^{-1}]$ has the following general form, inspired by [8] in the framework of linear stability analysis, and by [9]:

$$\gamma = \frac{c_s}{R} \sqrt{R^2 \frac{\nabla p_i \cdot \nabla B_\varphi}{p_i B_\varphi} - \Theta}$$

(2)

Here $c_s = \sqrt{(T_e + T_i)/m_D}$ is the thermal velocity in [m/s], $m_D$ the mass of deuterium and $R$ the major radius. In our case we set the amplitude coefficient $\gamma_0 = 1$ and the gradient threshold $\Theta = 0$. 

3
The sink is self-saturation of turbulent energy $\varepsilon = \Delta \omega \kappa^2 [m^2/s^3]$, a non-linear phenomenon whose growth rate is given by $\kappa \Delta \omega$. The dimension of $\Delta \omega$ is the inverse of diffusivity, $[s/m^2]$.

A diffusive description is assumed in the model for the anomalous transport, with $\lambda_{SOL} \simeq \sqrt{\chi_e L_\parallel/c_s \gamma_e}$, where $\chi_e$ is the transport coefficient, $\tau_\parallel = L_\parallel/c_s$ is the characteristic time scale of parallel dynamics and $\gamma_e$ is the sheath heat transmission coefficient for electrons. Given the aforementioned averaging over small scales, we focus on the equilibrium of large spatial scales and turn-over times of turbulence. In particular the former will scale with $\lambda_{SOL}$, instabilities gaining energy directly by the average motion of plasma, while the latter will scale with $\tau_D = R/c_s >> \omega_c^{-1}$, being $\omega_c$ the cyclotronic frequency and $\tau_D$ the characteristic time of turbulent structures.

Transport coefficients appearing in equations of plasma density, momentum and energy can be self-consistently calculated as functions of $\kappa$. The dependence is deduced from dimensional analysis, so we obtain:

$$\chi_\alpha \sim D_n = \tau_D \kappa \quad (3)$$

Linear dependence between transport coefficient and turbulent energy is also contemplated in QLT.

$\Delta \omega$ is the only free parameter of the system. In order to evaluate it, we first consider the equilibrium between the source and the sink in equation (1), namely $\gamma \kappa - \Delta \omega \kappa^2 = 0$, where the non-trivial solution is given by:

$$\kappa = \gamma / \Delta \omega \quad (4)$$

Then we recall that the characteristic time scale of plasma motion in perpendicular direction is not negligible with respect to the parallel one. Such competition is expressed by:

$$\frac{\chi_{SOL}^2 \gamma_e}{\chi_e} \simeq \frac{L_\parallel}{c_s} \quad (5)$$

Hence, provided equation (4), the SOL width can be estimated as:

$$\chi_{SOL}^2 \simeq \frac{L_\parallel}{\gamma_e c_s \chi_e} = \frac{L_\parallel R}{2 \gamma_e c_s^2} \left( \frac{\gamma}{\Delta \omega} \right) \quad (6)$$

The inverse relation returns the self-saturation coefficient:

$$\Delta \omega = \frac{L_\parallel R}{2 \gamma_e (\lambda_{SOL})^2 \gamma c_s^2} \quad (7)$$
where the closure comes into play by assuming the scaling law used in [3] and [5]. In particular, we set \( \lambda_{SOL} = \lambda_{L,SOL}^L = 4 \rho_L q_{cyl} \), where \( q_{cyl} = a B_c / R B_\theta \) is the safety factor and \( \rho_{L,s} = m_i v_i / q_i B \) the ion gyro-radius. Here \( "L" \) states that this is a \( L \)-mode study-case, so the reference scaling law is doubled to take into account the low-confinement mode, as stated in [4]. Equation (7) becomes:

\[
\Delta \omega = \frac{\pi A^2}{16 \gamma q_{cyl} \rho_s^2 \gamma c_s^2}
\]

with \( A = R/a \) the aspect ratio and \( \rho_s = \rho/a \) the normalized gyro-radius.

The approach underlying equation (4) is justified by the dimensional analysis of (1), given by (9) where quantities with star index (*) are dimensionless:

\[
\partial_t^* \kappa^* + \frac{c_0 \tau}{L_{||}} \nabla_{||} \cdot (\kappa^* u_{||}^*) = \frac{\tau D_0}{L_{SOL}^2} \nabla_{\perp}^* \cdot (D_\kappa^* \nabla_{\perp} \kappa^*) + \frac{\tau c_0}{R} \gamma^* \kappa^* - \varepsilon_0 \kappa^* - \varepsilon^* \tag{9}
\]

Here \( t = \tau t^* \) with \( \tau = R/c_0 \) and \( c_0 \) as reference thermal velocity, \( \nabla_{||} = \nabla_{||}^* / L_{||} \), \( \kappa = \kappa_0 \kappa^* \), \( \nabla_{\perp} = \nabla_{\perp}^* / \lambda_{SOL}^L \), \( D_\kappa = D_\kappa^* D_0 \) with \( D_0 = \lambda_{SOL}^L / \tau \) as reference diffusivity, \( \gamma = \gamma^* c_0 / R \) and \( \varepsilon = \varepsilon_0 \varepsilon^* \) with \( \varepsilon_0 = \kappa_0 / \tau \). In equation (9) it results that \( \nabla_{||} \cdot (\kappa u_{||}) \) and \( \nabla_{\perp} \cdot (D_\kappa \nabla_{\perp} \kappa) \) are lower order than the other terms: the order of magnitude of convection is \( \frac{\tau c_0}{R} = (2 \pi q_{cyl})^{-1} \) and the same is for the diffusion term due to equation (5).

When assuming ad-hoc constant transport coefficients for transport codes, equation (5) shows that the SOL width drops with increasing temperature at the separatrix, since \( \lambda_{SOL} \approx \sqrt{L_{I_{e,\perp}}} \approx T^{-1/4} \). When implementing the model, one recovers \( \lambda_{SOL} \propto \rho \propto \sqrt{T} \), see Figure 1, panel 4. One gets also the proper dependencies with respect to other main machine parameters (Figure 1, panels 1,2 and 3).

2. Numerical implementation of the model when varying the magnetic configuration

Equation 8 shows that \( \Delta \omega \) is a non-trivial function of the magnetic field (through the \( \frac{1}{q_{cyl} \rho_s} \) term). For given tokamak and discharge conditions, the scan of the model on the outer divertor leg \( L_{div} \) may represent a first step to test such closure and the model itself. TCV has been chosen as study case due to the availability of experimental data concerning lower single null
Figure 1: Scan of $\lambda_p$ compared to $\lambda_{SOL}$ with respect to main machine parameters.

(LSN), Ohmic, L-mode discharges with varying $L_{\text{div}}$ [10] [11] [12]. Indeed TCV benefits from a wide set of edge diagnostics: high resolution Thomson scattering (HRTS), reciprocating Langmuir probes (RLP) plunging at the outer mid-plane, wall-embedded Langmuir probes (LP) to monitor divertor plasma conditions and IR thermography system ([13],[14],[15]). The three magnetic configurations chosen for the study case are: for the short-leg ($L_{\text{div}} = 21$ cm) shot #51262, for the medium-leg ($L_{\text{div}} = 36$ cm) shot #51333, and for the long-leg ($L_{\text{div}} = 64$ cm) shot #51325. The following parameters are kept constant: major radius $R = 89$ cm, minor radius $a = 22$ cm, elongation $k = 1.4$, plasma current $I_p = 210$ kA, poloidal magnetic field at the outer midplane $B_{p,\text{omp}} = 0.18$ T.

The latter are used as input parameters in the 2D transport code SolEdge2D, where fluid equations of plasma density, momentum and energy are integrated with the full $\kappa$-$\varepsilon$ model, equation (1). Here the transport terms are included and they contribute in smoothening the solution, while the free parameter $\Delta \omega$ is calculated as in equation (8). Montecarlo code Eirene is also coupled to compute momentum and energy source terms due to interactions with neutrals (atoms, molecules) [16]. Simulations are run to estimate the distribution of plasma main fields at the equilibrium taking into account the effect of large-scale turbulence on the system, and when varying the outer divertor leg length.
3. Upstream plasma and outer target profiles remapped at the outer mid-plane

Outer mid-plane profiles obtained from simulation are first investigated when varying $L_{\text{div}}$. Since this procedure does not change the main plasma (input power, plasma current, gas puff, $B_{\text{pol}}$ are kept constant), density and temperature profiles are not expected to change meaningfully. This is indeed what is observed from simulation, see Figure 2 and supported by experimental evidence from the HRTS.

![Graph](image)

Figure 2: Outer mid-plane profiles of density (a) and electron temperature (b) obtained from simulating the three shots, compared to data available from HRTS. In the three study cases, a feed-back has been imposed on the gas puff to the plasma such that $n \approx 7 \cdot 10^{18} \text{ m}^{-3}$ at the separatrix. TCV data first discussed in Fig.3 of [10].

In figures 3, 4 and 5 profiles of $n_e$, $T_e$ and perpendicular heat flux $q_\perp$ are plotted with respect to the radial distance from the separatrix, provided that such quantities (both experimental and simulated) are remapped at the outer mid-plane along magnetic flux surfaces to allow the comparison of magnetic equilibria with different flux expansion.

In the right-hand side of these plots, normalized quantities are shown to help visualizing shape variations when changing $L_{\text{div}}$. Clearly, profiles width is not affected by $L_{\text{div}}$, in contrast to experimental data from LP and IR. This result is confirmed by plotting the perpendicular heat flux decay length $\lambda_q$ as a function of the divertor leg length (figure 6), where one can see that $\lambda_q$ is nearly constant around the value of 9 mm. In particular it is not retrieved the linear dependence observed in [10].
4. Self-consistent maps of plasma transport

2D maps of figure 7 show the distribution of the particle radial transport coefficient at the equilibrium. Along the outer divertor leg the anomalous
Figure 5: (a) Outer target profiles of perpendicular heat flux compared to IR data, as a function of radial distance from the outer mid-plane separatrix for short, medium and long outer divertor leg in green, blue and orange respectively. (b) Same profiles normalized with respect to the maximum value, compared to Eich fit (circles). TCV data first discussed in Fig. 8 of [11].

Figure 6: Perpendicular heat flux decay length at the outer target, remapped at the outer mid-plane as a function of the divertor leg length for short, medium and long configuration. (b) Normalization of profiles of figure 5.

Transport is significant and in good agreement with the distribution obtained in isothermal simulations from 1st principle turbulent code TOKAM3X [10] [17]. In particular, perpendicular transport turns to be enhanced at the outer mid-plane downwards close to the X-point (this is the so-called ballooning [18]) due to the ”bad” curvature of the LFS ($\nabla p \cdot \nabla B > 0$). In our simulations
the model, which cannot account on the calculation of fluctuations, has been able to capture also the anomalous transport in the common flux region, as also retrieved by TOKAM3X.

One may expect that, provided a major spreading of outer target profiles in the long-leg configuration, the divertor $\lambda_{SOL}$ increases with the divertor leg length, as actually retrieved in experiments. This is not recovered by the model, as confirmed by a nearly constant $\lambda_{SOL}$ for the three cases in figure 6, due to the closure used in the model, the self-saturation coefficient $\Delta \omega$ being calculated including the scaling law proposed in [3] and [5], which in principle is not expected to change with $L_{div}$. A derivation of such scaling from first principle turbulent fluid theory has been proposed in [19], where the scaling of $q_{cyl}$ with respect to $L_{||}/2\pi R$ could be envisaged as a next step to verify the linear dependence of $\lambda_{SOL}$ to $L_{div}$ seen in [10], at the cost of introducing a global quantity ($L_{||}$) into the model, which is local.

![Figure 7](image)

Figure 7: 2D maps of particle diffusivity for short-leg (a), medium-leg (b) and long-leg (c) magnetic configurations of the TCV study case reported in this work.

5. Conclusions

A $\kappa$-$\varepsilon$ model has been implemented numerically into transport code SolEdge2D and tested with respect to three magnetic configurations corresponding to three L-mode, ohmic discharges of TCV with different outer divertor leg lengths.
The model simulates the effects of time and space large-scale turbulence on mean fields by solving the transport equation of turbulent intensity $\kappa$. The closure is made by including the Eich’s and Scarabosio’s scaling law for the SOL width in the expression of the turbulence self-saturation coefficient, which is the free parameter.

Preliminary results show that the model recovers the anomalous transport not just at the outer mid-plane but also in the common flux region, as already retrieved in 1-st principle turbulent code TOKAM3X, while the perpendicular heat flux decay length is not affected by the scan on the outer divertor leg length, due to the closure of the model. Scaling of $q_{cyl}$ with respect to $L_{\|}/2 \pi R$ can be envisaged as the next step to verify the linear dependence of $\lambda_{SOL}$ to $L_{div}$, nevertheless that would introduce an integral quantity ($L_{\|}$) to the model, which is local. Other discharges from diverse machines are also expected to be tested with the model.

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