Gluonic Higgs Scalar, Abelianization and Monopoles in QCD – Similarity and Difference between QCD in the MA Gauge and the NAH Theory

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We study the similarity and the difference between QCD in the maximally abelian (MA) gauge and the nonabelian Higgs (NAH) theory by introducing the “gluonic Higgs scalar field” $\vec{\phi}(x)$ corresponding to the “color-direction” of the nonabelian gauge connection. The infrared-relevant gluonic mode in QCD can be extracted by the projection along the color-direction $\vec{\phi}(x)$ like the NAH theory. This projection is manifestly gauge-invariant, and is mathematically equivalent to the ordinary MA projection. Since $\vec{\phi}(x)$ obeys the adjoint gauge transformation and is diagonalized in the MA gauge, $\vec{\phi}(x)$ behaves as the Higgs scalar in the NAH theory, and its hedgehog singularity provides the magnetic monopole in the MA gauge like the NAH theory. We observe this direct correspondence between the monopole appearing in the MA gauge and the hedgehog singularity of $\vec{\phi}(x)$ in lattice QCD, when the gluon field is continuous as in the SU($N_c$) Landau gauge. In spite of several similarities, QCD in the MA gauge largely differs from the NAH theory in the two points: one is infrared monopole condensation, and the other is infrared enhancement of the abelian correlation due to monopole condensation.

1. QCD in the MA Gauge and Dual Superconductor Theory for Confinement

To understand the confinement mechanism is one of the most difficult problems remaining in the particle physics \cite{1}. Quark confinement is characterized by one-dimensional squeezing of the color-electric flux with the string tension $\sigma \simeq 1$GeV/fm, which is the universal key quantity in QCD \cite{2}. On the confinement mechanism, based on the electromagnetic duality, Nambu \cite{3} first proposed the dual superconductor theory, where the one-dimensional squeezing of the color-electric flux occurs by the dual Meissner effect due to condensation of bosonic color-magnetic monopoles. But, there are two large gaps between QCD and the dual superconductor theory.

1. The dual superconductor theory is based on the abelian gauge theory subject to the Maxwell-type equations, where electro-magnetic duality is manifest, while QCD is a nonabelian gauge theory.

2. The dual superconductor theory requires color-magnetic monopole condensation as the key concept, while QCD does not have color-magnetic monopoles as the elementary degrees of freedom.
Figure 1. The gluonic Higgs scalar field $\phi(x) = \phi^a(x) T^a$ in the SU(2) Landau gauge in SU(2) lattice QCD with $\beta = 2.4$ and $16^4$. The arrow denotes the SU(2) color direction, $(\phi^1(x), \phi^2(x), \phi^3(x))$. The monopoles (dots) in the MA gauge appear at the hedgehog singularities of the gluonic Higgs scalar $\phi(x)$.

These gaps may be filled simultaneously by taking maximally abelian (MA) gauge fixing, which reduces QCD to an abelian gauge theory including color-magnetic monopoles.

In Euclidean QCD, the MA gauge is defined so as to minimize the “total amount” of the off-diagonal gluon amplitude,

$$R_{\text{off}}[A_\mu(\cdot)] \equiv \int d^4x \tr \left\{ [\hat{D}_\mu, \vec{H}][\hat{D}_\mu, \vec{H}]^\dagger \right\}$$

by the SU($N_c$) gauge transformation $\Omega(x) \in \text{SU}(N_c)$ so as to minimize

$$R[\bar{\phi}(\cdot)] \equiv \int d^4x \tr \left\{ [\hat{D}_\mu, \vec{\phi}(x)][\hat{D}_\mu, \vec{\phi}(x)]^\dagger \right\}$$

in the Euclidean metric. The gluonic Higgs scalar $\vec{\phi}(x)$ physically corresponds to the “color-direction” of the nonabelian gauge connection $\hat{D}_\mu$ averaged over $\mu$ at each $x$.

2. The similarity between QCD in the MA gauge and the Nonabelian Higgs theory: Appearance of Magnetic Monopoles and Infrared Abelianization

QCD in the MA gauge is similar to the NAH theory in terms of appearance of magnetic monopoles. To clarify the similarity between QCD in the MA gauge and the NAH theory, we introduce the “gluonic Higgs scalar field” $\vec{\phi}(x)$ as a function of the gluon-field configuration $\{A_\mu(x)\}$. For arbitrary given gluon configuration $\{A_\mu(x)\}$, we define

$$\vec{\phi}(x) \equiv \Omega(x)\vec{H}\Omega^\dagger(x)$$

with $\Omega(x) \in \text{SU}(N_c)$ so as to minimize

in the Euclidean metric. The gluonic Higgs scalar $\vec{\phi}(x)$ physically corresponds to the “color-direction” of the nonabelian gauge connection $\hat{D}_\mu$ averaged over $\mu$ at each $x$.

Similar to the covariant derivative $\hat{D}_\mu$, the gluonic Higgs scalar $\vec{\phi}(x)$ obeys the adjoint gauge transformation as $\vec{\phi}(x) \rightarrow V^\dagger(x)\vec{\phi}(x)V(x)$, and is diagonalized in the MA gauge.
Therefore, \( \tilde{\phi}(x) \) behaves as the Higgs scalar in the NAH theory, and the hedgehog singularity of \( \tilde{\phi}(x) \) provides the magnetic monopole in the MA gauge [5, 6, 7, 8], as a direct analogue of the appearance of the 't Hooft-Polyakov monopole in the SU(\( N \)) NAH theory.

Actually in lattice QCD, we observe this direct correspondence between the monopole appearing in the MA gauge and the hedgehog singularity of \( \tilde{\phi}(x) \) [5, 6, 7, 8], when the gluon field is continuous as in the SU(\( N_c \)) Landau gauge, as shown in Fig. 1. (In the SU(\( N_c \)) Landau gauge, the gauge field is maximally continuous, so that the correspondence between the hedgehog singularity and the monopole position become manifest. However, in the original random gauge on lattice, such correspondence cannot be observed.)

Note here that the adjoint gauge-transformation property of \( \tilde{\phi}(x) \) is essential on the correspondence between the hedgehog singularity of \( \tilde{\phi}(x) \) and the monopole singularity appearing in the MA gauge. For instance, consider the other operator composed by the link-variable \( U_\mu(s) \),

\[
X(s) \equiv \sum_{\mu=1}^{4} \{ U_\mu(s) \tau_3 U_\mu^\dagger(s) + U_\mu^\dagger(s - \hat{\mu}) \tau_3 U_\mu(s - \hat{\mu}) \},
\]

which corresponds to \( X(x) = [\hat{D}_\mu, [\hat{D}_\mu, \tau_3]] \) in the continuum limit. In SU(2) QCD, \( X(s) \) is also diagonalized in the MA gauge, but it does not obey the adjoint transformation by the gauge transformation, so that there is no simple correspondence between the hedgehog singularity of \( X(s) \) and the monopole position in the MA gauge.

Next, let us consider the massive behavior of off-diagonal gluons and resulting infrared abelianization in QCD in the MA gauge. Like the off-diagonal (charged) gauge fields in the NAH theory, the off-diagonal gluons behave as massive vector fields with a large mass of about 1GeV in QCD in the MA gauge with the abelian Landau gauge [4, 6, 7, 8], as shown in Fig. 2. Therefore, through the projection along \( \tilde{\phi}(x) \), one can extract the abelian U(1)\(^{N_c-1} \) sub-gauge-manifold which is close to the original SU(\( N_c \)) gauge manifold. This projection is manifestly gauge-invariant and is mathematically equivalent to the ordinary MA projection. In fact, the infrared-relevant gluonic mode in QCD can be extracted by the projection along the color-direction \( \tilde{\phi}(x) \) like the NAH theory [7, 8], with the similar argument to infrared abelian dominance in the MA gauge.
Thus, we have the similarities between QCD in the MA gauge and the NAH theory on the appearance of magnetic monopoles, the massive behavior of off-diagonal gluons and infrared abelianization.

3. Difference between QCD in the MA gauge and the NAH theory: Infrared Monopole Condensation and Infrared Enhancement of Abelian Correlation

So far, we have shown the similarities between QCD in the MA gauge and the NAH theory. Of course, these theories are essentially different on the points how the gauge symmetry is broken:

1. While the spontaneous gauge-symmetry breaking occurs in the NAH theory, the MA gauge is brought as a gauge fixing in QCD.

2. While the NAH theory has the Higgs scalar as an elementary field, the gluonic Higgs scalar $\vec{\phi}(x)$ is a composite field of gluons.

Except for these trivial differences, QCD in the MA gauge largely differs from the NAH theory on the following two points: one is infrared monopole condensation, and the other is infrared enhancement of the abelian correlation due to monopole condensation.

1. Infrared monopole condensation occurs in QCD in the MA gauge, while the magnetic monopole appears as an ordinary massive particle in the NAH theory.

2. Infrared enhancement of abelian correlation is caused by monopole condensation, and provides the linear potential at large distances, which leads to the quark confinement, while the NAH theory only provides the Coulomb potential at large distances.

We conjecture that infrared monopole condensation occurs as a result of the large quantum fluctuation of gluon fields in the infrared region, reflecting the asymptotic freedom. In fact, the gluonic Higgs field $\vec{\phi}(x)$ corresponding to the color-direction of the nonabelian gauge connection $\vec{D}_\mu$ is expected to be largely fluctuated in QCD at a large scale, and this fluctuation would lead to stochastic monopole excitations which may be interpreted as infrared monopole condensation.

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