Measurements on the reality of the wavefunction

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Quantum mechanics is an outstandingly successful description of nature, underpinning fields from biology through chemistry to physics. At its heart is the quantum wavefunction, the central tool for describing quantum systems. Yet it is still unclear what the wavefunction actually is: does it merely represent our limited knowledge of a system, or is it in direct correspondence to reality? Recent no-go theorems argued that if there was any objective reality, then the wavefunction must be real. However, that conclusion relied on debatable assumptions. Here we follow a different approach without these assumptions and experimentally bound the degree to which knowledge interpretations can explain quantum phenomena. Using single photons, we find that no knowledge interpretation can fully explain the limited distinguishability of non-orthogonal quantum states in three and four dimensions. Assuming that a notion of objective reality exists, our results thus strengthen the view that the wavefunction should directly correspond to this reality.

Albert Einstein’s famous question ‘Do you really believe the moon exists only when you look at it?’ encapsulates a century-old debate over the measurement problem and the very nature of the quantum wavefunction1. Not all scientists—including for example Quantum Bayesianists2–4—believe that our observations of the physical world can be entirely derived from an underlying objective reality. If one does, however, want to maintain a realist position at the quantum level, a question naturally arises: does the wavefunction directly correspond to the underlying reality, or does it only represent our partial knowledge about some real physical property of a quantum system?

There are compelling reasons to subscribe to the latter, epistemic view5. For instance it resolves seemingly inconvenient phenomena such as Schrödinger’s ‘poor cat being alive and dead at the same time: in the epistemic interpretation the cat could well be definitely alive or definitely dead, and the superposition state would merely represent our ignorance about this real physical state of affairs before the measurement6. Another example is the indistinguishability of non-orthogonal quantum states, which could be explained by a lack of information about some real physical property associated with those states. It has long been an open question whether the epistemic view, together with the assumption of an underlying objective reality, is compatible with quantum measurement statistics.

Let us formalize this question. Adopting the view that some underlying objective reality exists and explains quantum predictions, we shall denote the ‘real state of affairs’ that completely specifies a given physical system by \( \lambda \). The preparation of a system in a quantum state \( |\psi\rangle \) may not determine \( \lambda \) uniquely; instead, it determines a classical distribution \( \mu_\psi \) over the set of \( \lambda \) states, describing the probability that the preparation results in a specific state \( \lambda \). Quantum measurement statistics on \( |\psi\rangle \) are then assumed to be recovered after averaging over \( \lambda \), with the probability distribution \( \mu_\psi \), which is assumed to be independent of the measurement being carried out.

A model that reproduces quantum predictions within the above framework is called an ontological model6. The states \( \lambda \) are called ontic states—or, historically, hidden variables—whereas the distributions \( \mu_\psi \) are called epistemic states. Specific examples of ontological models include those that involve hidden variables complementing a real wavefunction, as was famously suggested by Einstein, Podolsky and Rosen8 to address the alleged ‘incompleteness’ of quantum mechanics. Another example was formulated by John Bell in his celebrated theorem9, which shows the incompatibility of quantum mechanics with local hidden variables10.

Here we are not concerned with locality; we are instead interested in the correspondence between the wavefunction that describes the quantum state of a single, indivisible quantum system and its possible ontic states.

If the quantum state \( \psi \) is uniquely determined by the ontic state, it is itself an element of reality and the model is called \( \psi \)-ontic. In this case the epistemic states—that is, probability distributions—\( \mu_\psi \) and \( \mu_\phi \) corresponding to any two distinct pure states \( |\psi\rangle \) and \( |\phi\rangle \) (Fig. 1a) must be disjoint (up to sets of measure zero, Fig. 1b). In all other cases the wavefunction has to be treated as a representation of the limited knowledge about the real state of the system—a so-called \( \psi \)-epistemic model. In such models the epistemic states of two distinct pure quantum states might overlap—that is, two different wavefunctions might be compatible with the same ontic state (Fig. 1c).

A breakthrough in the study of these models was recently made by Pusey, Barrett and Rudolph in ref. 11. They showed a no-go theorem suggesting that \( \psi \)-epistemic models were not compatible with quantum mechanics. However, their theorem and related ones that followed12–14 crucially relied on further, rather problematic assumptions beyond the basic ontological model framework. For example, Pusey et al. assume that independently prepared systems have independent physical states11. This requirement has been challenged15 as being analogous to Bell’s local causality, which is already ruled out by Bell’s theorem16.

Although completely ruling out \( \psi \)-epistemic models is impossible without such further assumptions13,17, one can severely constrain them by bounding the degree to which they can explain quantum phenomena. In particular, it was shown theoretically19–21 that the limited distinguishability of non-orthogonal quantum states cannot be fully explained by overlapping probability distributions in \( \psi \)-epistemic models for systems of dimension

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Figure 1 | Ontological models for quantum theory. a, Pure quantum states $|\psi\rangle$ and $|\phi\rangle$ correspond to unit vectors in a $d$-dimensional Hilbert space. b, c, In ontological models every quantum state $|\psi\rangle$ is associated with a probability distribution $\mu_\psi$ over the set of ontic states $\lambda$. b, In a $\psi$-ontic model, the distributions are disjoint for any pair of non-identical pure quantum states, such that the state itself can be regarded as an ontic element of the objective reality. c, In $\psi$-epistemic models, the probability distributions can overlap and the quantum state is not uniquely determined by the underlying ontic state $\lambda$.

Figure 2 | Scheme for probing the reality of the wavefunction. a, A $d$-dimensional system is prepared in a state from the set $\{|\psi_i\rangle\}$ and then subjected to measurements $\{M_{\psi_i}\}$. b, Experimental implementation. Pairs of single photons are created via spontaneous parametric down-conversion (SPDC) in a periodically poled potassium titanyl phosphate (KTIOPO$_4$) crystal pumped by a 410 nm diode laser$^{30}$ (purple arrow). The heralded signal photon is prepared in the initial state $|H\rangle$ by means of a Glan–Taylor polarizer (GT). The subsequent half-wave plate (HWP) defines the relative amplitudes and phase of the initially populated modes $|1\rangle$ and $|2\rangle$. A calcite beam displacer (BD) separates the orthogonal polarization components and a set of HWPs is used to adjust the relative amplitudes and phases of all the basis states $|0\rangle$, $|1\rangle$, $|2\rangle$ (and $|3\rangle$ for the ququart). The same set-up in reverse is used to perform the measurements $\{M_{\phi_i}\}$. Using only one output port of the final analysing polarizer and one single-photon detector (APD) ensures maximal fidelity of the measurement process. Furthermore, although additional quarter-wave plates (QWPs) could be used to access the full (complex) state space, this is not necessary for the present experiment. Hence, the QWPs were not used to allow for higher accuracy.

larger than two. Here we demonstrate this experimentally on indivisible quantum systems—single photons—in three and four dimensions. We conclusively rule out the possibility that quantum indistinguishability can be fully explained in this form in any $\psi$-epistemic model that reproduces our observed statistics. We further establish experimental bounds on how much such models can explain. Our implementation relies on a new result$^{21}$, which generalizes the proof of ref. 19.

A $\psi$-epistemic model could explain the limited distinguishability of a pair of non-orthogonal quantum states $|\psi\rangle$, $|\phi\rangle$ as resulting from their two different preparation procedures sometimes producing the same ontic state $\lambda$. Without resorting to further mechanisms—for example, involving limitations on measurement precision—such an explanation is fully satisfactory only if the distinguishability of two states is fully explained by the classical overlap of $\mu_\phi$ and $\mu_\psi$. In particular, the probability of successfully distinguishing two
quantum states using optimal quantum measurements must be the same as that of distinguishing the two corresponding epistemic states, given access to the ontic states. These probabilities are given by $1 - \omega_0(\{\phi_i\}, |\psi\rangle)/2$ and $1 - \omega_0(\{\mu_j\}, |\sigma\rangle)/2$, respectively, where $\omega_0(\{\phi\}, |\psi\rangle) = 1 - \sqrt{1 - |\langle \psi | \phi \rangle|^2}$ is the quantum overlap of the two states and $\omega_0(\{\mu\}, |\sigma\rangle) = \min_{|\sigma\rangle} |\langle \mu | \sigma \rangle|^2$ is the classical overlap of the probability distributions.

Note that $0 \leq \omega_0 \leq \omega_0 \leq 1$ in general; a model that satisfies $\omega_0(\{\phi\}, |\psi\rangle) = \omega_0(\{\mu\}, |\sigma\rangle)$ for all states $|\phi\rangle, |\psi\rangle$—that is, for which the two above-mentioned success probabilities are equal and all the indistinguishability is thus explained by the overlapping probability distributions—is called maximally $\psi$-epistemic.

Maximally $\psi$-epistemic models can, however, not reproduce all quantum measurement statistics. As detailed in ref. 21, one approach to demonstrate this is to prepare a set of $n + 1$ quantum states $\{|\psi_j\rangle\}_{j=0}^n$ with $n \geq 3$, and for each triplet of states $\{|\psi_0\rangle, |\psi_1\rangle, |\psi_2\rangle\}$ perform a measurement $M_{j,k,l}$ with three outcomes $(m_j, m_k, m_l)$. Denoting the respective outcome probabilities $P_{M_{j,k,l}}(m_j | |\psi_0\rangle)$, $P_{M_{j,k,l}}(m_j | |\psi_1\rangle)$ and $P_{M_{j,k,l}}(m_j | |\psi_2\rangle)$ are 0, the following inequality has to be satisfied by maximally $\psi$-epistemic models:

$$S(|\psi_0\rangle, M_{j,k,l}) = \frac{1 + \sum_{i=0}^2 \sum_{j=0}^n P_{M_{j,k,l}}(m_j | |\psi_i\rangle)}{\sum_j \omega_0(|\psi_0\rangle, |\psi_j\rangle)} \geq 1$$

where $1 \leq j < k < n \leq n$ and $1 \leq j \leq n$, respectively. Quantum mechanics predicts that this inequality cannot be violated and any observed value of $S < 1$ rules out maximally $\psi$-epistemic models.

The denominator in $S$ is a measure of the quantum overlaps of the chosen states and the numerator represents an upper bound on the classical overlaps of the corresponding probability distributions in a $\psi$-epistemic model that reproduces the measurement statistics. The parameter $S$ represents an upper bound on the smallest ratio between the classical and quantum overlaps $\kappa(|\psi_0\rangle, |\psi_1\rangle) = \omega_0(\{\mu_j\}, |\sigma\rangle)/\omega_0(|\psi_0\rangle, |\psi_1\rangle)$ for all states $|\psi_j\rangle$ ($1 \leq j \leq n$) under consideration. For non-maximal models the right-hand side of inequality (1) is then replaced with $\kappa = \min |\psi_0\rangle, |\psi_1\rangle$, which constrains the degree to which the indistinguishability of non-orthogonal quantum states can be explained through overlapping probability distributions in $\psi$-epistemic models.

In our experiment we violate inequality (1) using single-photon qutrit ($d = 3$) and ququart ($d = 4$) states. These photons are created in pairs in a spontaneous parametric down-conversion process, and one photon is used as a trigger to herald the presence of a signal photon in the experiment. To access higher dimensions we dual encode our signal photon in the polarization and path degrees of freedom (see Fig. 2). The computational basis states are $|0\rangle = |H\rangle$, $|1\rangle = |V\rangle$, $|2\rangle = |H\rangle$, $|3\rangle = |V\rangle$, where the index denotes the spatial mode and $H$ and $V$ correspond to horizontal and vertical polarization, respectively. The photon polarization was manipulated with half-wave plates and polarization beam splitters. The path degree of freedom was controlled through an interferometer formed by half-wave plates and two beam splitters. Using this set-up we are able to prepare and measure arbitrary states of the form $|a\rangle|0\rangle + |\beta\rangle|1\rangle + |\gamma\rangle|2\rangle + |\delta\rangle|3\rangle$, where $|\alpha\rangle, |\beta\rangle, |\gamma\rangle, |\delta\rangle \in \mathbb{R}$. In the qutrit case the state $|3\rangle$ was not populated.

Although the derivation of inequality (1) makes no assumptions on the measurements, it does require accurate preparation of the states $|\psi_j\rangle$, as these are used to calculate the quantum overlaps $\omega_0$. To this end, every optical element in the experimental set-up was carefully calibrated and characterized. For increased precision and elimination of any systematic bias in the measurement, only one output port of the set-up was used and the three outcomes $m_0, m_1, m_2$ of each measurement $M_{j,k,l}$ were obtained from individual, consecutive measurements. This design might, however, be susceptible to imperfections in the optical components and one has to ensure that the normalized sum of the measurement operators for the three outcomes is as close as possible to the identity operator. Using the calibration data for our optical elements we bounded the average drop in measurement fidelity caused by these imperfections to $0.0007 \pm 0.0002$. The probabilities $P_{M_{j,k,l}}(m_j | |\psi_i\rangle)$ were estimated from the normalized single-photon count rates. On average, we obtained $2 \times 10^6$ coincidence counts per measurement, with an integration time of 10 s per setting.

In principle, inequality (1) can be violated to an arbitrary degree for $d \geq 4$ — that is $S$ can range arbitrarily close to 0, if one uses a sufficiently large number of states. However, increasing $n$ also requires a quadratic increase in the number of measurements on these states, which rapidly becomes infeasible in practice. Nevertheless, a violation is already possible with as few as $n = 3 + 3 + 1$ states and is becoming more pronounced as $n$ increases. We chose to prepare up to $n = 9$ qutrit states and $n = 15$ ququart states (see Methods), and performed a series of four independent measurements for each configuration of $d$ and $n$.

For $n = 9$ qutrit states, we obtain $S = 0.9184 \pm 0.002$, violating inequality (1) by more than 45 s.d. For the higher-dimensional ququart and $n = 10$ states we experimentally established $S = 0.690 \pm 0.001$, achieving a violation by more than 250 s.d. Figure 3 shows the scaling of $S$ with the number of states and dimensions. The individual measurements agree well with the expected performance of the set-up (red-shaded rectangles representing $1 \sigma$ regions). The advantage of using higher-dimensional systems is illustrated in the inset of Fig. 3, where we compare the cases $d = 3$ and $d = 4$ for $n = 3, 4, 5$.

As one can see in Fig. 3, the gap between experimental and theoretical $S$-values increases with $n$, owing to the compound error resulting from the quadratic increase of the number of required
measurements. Eventually, a further increase of \( n \) does not yield lower \( S \)-values. In our case the optimal trade-off is reached for \( n = 10 \), and despite slightly lower experimental \( S \)-values no more significant violation of inequality (1) could be achieved for \( n > 10 \). This is a consequence of the extensive number of measurements and long measurement times, which cause larger fluctuations and relative errors in the data. Although long-term drifts of the interferometer mainly affect the numerical value of the obtained \( S \), rather than its statistical significance, they might be avoided by using active stabilization. In Fig. 4 we show projections of the prepared states for the exemplary case of \( n = 7 \) onto the real subspace orthogonal to \( | \psi_0 \rangle = |0 \rangle \). The deviations of the measured probabilities \( P_{ \text{meas} } (m_i | \psi_j \rangle \) from the expected values according to the Born rule, for one measurement in this series, ordered as they appear in equation (1), are shown in Fig. 4b and their distribution in Fig. 4c. These data were used to calculate error bars and the expected performance of the set-up. A detailed description of error handling is given in the Methods. Although the overlap ratio \( S \) quantifies how much of the indistinguishability can be accounted for by overlapping epistemic states, it does not indicate how much overlap there was to begin with. We discuss this issue in the Supplementary Discussion.

Crucially, our theoretical derivation and conclusions do not require any assumptions beyond the ontological model framework—such as preparation independence \(^{11,22} \), symmetry \(^{15} \) or continuity \(^{14,24} \)—allowing us in particular to rule out a strictly larger class of \( \psi \)-epistemic models than the experiment of ref. 22. We do, however, rely on fair sampling \(^{10,25} \)—the physically reasonable assumption that the detected events are a fair representation of the overall ensemble—to account for optical loss and inefficient detection. In the Supplementary Discussion, we estimate that, with the states and measurements used in this work, an average detection efficiency above \( \sim 98\% \) would be required to rule out maximally \( \psi \)-epistemic models without relying on fair sampling. This is well above the efficiencies achieved at present in photonics, but might be achievable in other architectures such as trapped ions or superconductors—where, however, precise control of qudits has yet to be demonstrated. Note also that it would be necessary to monitor all potential measurement outputs simultaneously\(^{26} \) to reach efficiencies close to 100%.

Recall that the ontological model framework covers all interpretations of the quantum wavefunction in which there is an observer-independent, objective reality underlying quantum mechanics. Within these realist interpretations our results...
conclusively rule out the most compelling $\psi$-epistemic models, namely those that fully explain quantum indistinguishability by overlapping probability distributions. They further exclude a range of non-maximal models, characterized by a minimal ratio of classical-to-quantum overlap larger than $\kappa_0 = 0.690 \pm 0.001$ for the states we used. A mechanism which can explain the residual indistinguishability of quantum states in the range of non-maximal models we have not ruled out—and indeed explains all of it in $\psi$-ontic models—is that measurements may reveal only coarse-grained information about ontic states. It is hence highly desirable to further lower the experimental bounds on $S$ with future technological improvements, rendering the $\psi$-epistemic explanation in terms of overlap increasingly untenable. Tightening this constraint might allow clarification of the necessary role of coarse-graining in realist explanations of quantum indistinguishability.

Our work puts strong limitations, beyond those imposed by Bell’s theorem, on possible realist interpretations of quantum theory. If we want to hold on to objective reality, our results strongly suggest that one should adopt a $\psi$-ontic interpretation of quantum mechanics such as the Bohmian pilot-wave interpretation, or the many-worlds interpretation. Alternatively, we may have to consider interpretations outside the scope of the ontological model framework, by allowing, for instance, retrocausality—so that the epistemic states could depend on the measurement they are subjected to—or abandoning any notion of objective physical properties. This latter approach, which follows Bohr’s position as opposed to Einstein’s, is favoured by interpretations such as Quantum Bayesianism.

Methods

Choosing states and measurements in three and four dimensions. There is no known analytical form for the optimal states and measurements for a given $n$. We obtained $n = 3, 4$ and 5 qudit states and $n = 3, 4, \ldots, 15$ ququart states numerically. To make the non-convex optimization routine more tractable, and to allow a precise experimental implementation, we restricted the searched state space to real vectors. All states and measurements are listed in the Supplementary Methods. These states might not be optimal, considering the full (complex) state space allows in principle a larger violation of inequality (1). Note, however, that this would come with an increased experimental complexity and additional possible error sources, which may limit the advantage of using complex states.

Error handling. Violating inequality (1) requires high experimental precision, which also makes a careful error analysis essential. To obtain confidence intervals for $S$, we need to estimate the error in the quantities $P_{\psi_{j},m}(\psi_{j})$ and $\alpha_{\psi_{j},m}(\psi_{j},\psi_{j})$ in (1).

The first can be obtained directly from the measured single-photon count rates, which are Poissonian distributed. Even in the most robust scenario we found, $n = 5$, the maximal permissible average deviation from the predicted probabilities $P_{\psi_{j},m}(\psi_{j})$ is $\varepsilon_0 = 0.005$ for $d = 3$ and $\varepsilon_0 = 0.008$ for $d = 4$. Our data showed an average deviation per measurement of $\varepsilon = 0.001 \pm 0.002$. In Fig. 4, we show how this error affects our measured quantities for the exemplary case $n = 7$.

For the quantum overlaps we must account for experimentally unavoidable imperfection in the state preparation. We thus performed careful classical calibration, independent of the experiment, of every individual optical component, using a strong laser beam and a low-noise photodiode. From this data we calculate a 1σ range of the systematic error in each $\alpha_{\psi_{j},m}(\psi_{j},\psi_{j})$ of the order of $10^{-3}$. We compared this estimate with results from quantum state and process tomography. Tomography does not distinguish imperfections in preparation and measurement and it tends to overestimate the actual error. We obtain an average fidelity and purity of the prepared quantum states of $\mathcal{F} = 0.998 \pm 0.002$ and $\mathcal{T} = 0.998 \pm 0.003$, respectively. We can also use this data to obtain an estimate of about 0.02 for the standard deviation in $\alpha_{\psi_{j},m}$.

With an appropriate generalization of $\alpha_{\psi_{j},m}$ this analysis could be extended to mixed quantum states. However, for our experiment this was not strictly necessary. Our down-conversion source was pumped at low power to limit the probability of creating more than one photon pair within the timing resolution of the single-photon detectors to $10^{-5}$ per photon pair, and we are therefore dealing with single photons of very high intrinsic purity.
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**Author contributions**

A.F., A.G.W., C.B., E.C. and M.R. conceived the study. A.F., M.R. and B.D. designed the experiment. C.B. provided the lists of states and measurements to be used. M.R. and B.D. performed the experiment, collected and analysed the data. All authors contributed to writing the paper.

**Additional information**

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**Competing financial interests**

The authors declare no competing financial interests.