Parity Violation, the Neutron Radius of Lead, and Neutron Stars

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Abstract. The neutron radius of a heavy nucleus is a fundamental nuclear-structure observable that remains elusive. Progress in this arena has been limited by the exclusive use of hadronic probes that are hindered by large and controversial uncertainties in the reaction mechanism. The Parity Radius Experiment at the Jefferson Laboratory offers an attractive electro-weak alternative to the hadronic program and promises to measure the neutron radius of $^{208}$Pb accurately and model independently via parity-violating electron scattering. In this contribution we examine the far-reaching implications that such a determination will have in areas as diverse as nuclear structure, atomic parity violation, and astrophysics.

PACS. 21.10.Gv Mass and neutron distributions – 26.60.+c Nuclear matter aspects of neutron stars

1 Introduction

The nucleus of $^{208}$Pb is 18 order of magnitudes smaller and 55 orders of magnitude lighter than a neutron star. Yet remarkably, both the neutron radius of $^{208}$Pb as well as the radius of a neutron star depend critically on our (incomplete) knowledge of the equation of state of neutron-rich matter. The emergence of such a correlation among objects of such a disparate size is not difficult to understand. Heavy nuclei develop a neutron-rich skin as a result of a large neutron excess and a large Coulomb barrier that reduces the proton density at the surface of the nucleus. Thus the thickness of the neutron skin depends on the pressure that pushes neutrons out against surface tension [1]. It is this same pressure that supports a neutron star against gravitational collapse [2,3]. Thus models with thicker neutron skins often produce neutron stars with larger radii [4].

Attempts at mapping the neutron distribution of $^{208}$Pb, or of any other heavy nucleus, using hadronic probes have been met with limited success. Although highly mature and successful, the hadronic program will never attain the precision status that the electro-weak program enjoys. This is due to the large and controversial uncertainties in the reaction mechanism [5,6]. A particularly illustrative example of this situation is provided by the proton and neutron radii of $^{208}$Pb. While elastic electron scattering experiments have determined the charge radius of $^{208}$Pb to better than 0.001 fm [7], realistic estimates place the uncertainty in the neutron radius at about 0.2 fm [8].

The enormously successful parity violating program at the Jefferson Laboratory [9,10] provides an attractive electro-weak alternative to the hadronic program. Indeed, the Parity Radius Experiment (PREX) at the Jefferson Laboratory aims to measure the neutron radius of $^{208}$Pb accurately (to within 0.05 fm) and model independently via parity-violating electron scattering [8]. Parity violation at low momentum transfers is particularly sensitive to the neutron density because the $Z^0$ boson couples primarily to neutrons. Moreover, the parity-violating asymmetry, while small, can be interpreted with as much confidence as conventional electromagnetic scattering experiments. PREX will provide a unique observational constraint on the thickness of the neutron skin of a heavy nucleus. We note that since first proposed in 1999, many of the technical difficulties intrinsic to such a challenging experiment have been met [11]. For further details on the status of the experiment, see the contribution from Robert Michaels to these proceedings.

2 Formalism

The starting point for the calculation of both the properties of finite nuclei, their self-consistent linear response, and the structure and dynamics of neutron stars is based on a relativistic density functional. The underlying Lagrangian density includes an isodoublet nucleon field interacting via the exchange of two isoscalar mesons — a scalar and a vector — one isovector meson, and the photon. Details of this model may be found in Refs. [12,13]. In addition to meson-nucleon interactions, the Lagrangian density must be supplemented by nonlinear meson interactions that are responsible for a softening of the equation of state of symmetric nuclear matter at both normal and high densities [14]. Of particular relevance to the present contribution is an effective coupling constant (denoted by $\Lambda_v$) that induces isoscalar- isovector mixing and has been added to tune the poorly-known density dependence of the symmetry energy [15,16]. As a result of the strong correlation between the neutron radius of heavy nuclei and the
The neutron skin of heavy nuclei is also highly sensitive to changes in the asymmetry energy \[ \Sigma \] of neutron-rich matter \[1, 16] and the neutron skin of \[\Lambda\] is constant. Hence, the interacting Lagrangian density of Ref. \[14\] has been supplemented by the following term:

\[
\mathcal{L}_{\text{int}} = \Lambda_v \left( g_\rho^2 b_\mu \cdot b^\mu \right) \left( g_V^2 V_\mu V^\mu \right),
\]

where \( V^\mu \) and \( b^\mu \) denote isoscalar and isovector fields, respectively.

### 3 Results

We start this section by displaying in Fig. 1 the impact of adding the empirical coupling constant \( \Lambda_v \) on both the charge and point-neutron densities of \( {^{208}\text{Pb}} \). An important constraint that must be satisfied by the addition of any new coupling constant into the Lagrangian is that the success of the effective model in reproducing well-determined ground-state observables is maintained. Figure 1 indicates that this is indeed the case for the charge density — an observable largely insensitive to the value of \( \Lambda_v \). In contrast, the changes in \( \Lambda_v \) depicted in the figure yield a significant reduction in the value of neutron skin of \( {^{208}\text{Pb}} \): from \( R_n - R_p = 0.28 \text{ fm} \) to \( R_n - R_p = 0.20 \text{ fm} \). Note that the neutron skin \( R_n - R_p \) is defined as the difference between the (point) root-mean-square neutron and proton radii.

![Fig. 1. Proton (charge) and neutron (point) densities for \( {^{208}\text{Pb}} \) using a variety of values for the isoscalar-isovector coupling constant \( \Lambda_v \).](image)

3.1 Atomic Parity Violation

Due to the widespread interest of the PAVI06 audience on atomic parity violation, we add a brief discussion that the impact of a 1% measurement of the neutron radius in \( {^{208}\text{Pb}} \) could have on the neutron radius of those heavy nuclei that have been identified as promising candidates to the atomic parity violation program. These include Barium, Dysprosium, Ytterbium, and Francium. For more in depth discussions on atomic parity violation, see the contributions to these proceedings by Profs. Derevianko, Lintz, Budker, Tsigutkin, Gwinner, and Sanguinetti.

In large part the choice of suitable atomic systems is the existence of very close (nearly degenerate) levels of opposite parity that enhance significantly the parity violating amplitudes. Unfortunately, parity-violating matrix elements are contaminated by uncertainties in both atomic and nuclear structure. A fruitful experimental strategy for removing the sensitivity to the atomic theory is to measure ratios of parity violation observables along an isotopic chain. This leaves nuclear-structure uncertainties, in the form of differences in neutron radii, as the limiting factor in the search for physics beyond the standard model \[17, 18, 19\]. All three elements, Barium, Dysprosium, and Ytterbium, have long chains of naturally occurring isotopes. While the experimental strategy demands a precise knowledge of neutron radii along the complete isotopic chain, we only correlate here (as means of illustration) the neutron radius of \( {^{208}\text{Pb}} \) to the neutron radius of a member of the isotopic chain having a closed neutron shell (or subshell).

That is, we focus exclusively on: \( {^{138}\text{Ba}}(Z = 56; N = 82), {^{158}\text{Dy}}(Z = 66; N = 92) \), and \( {^{176}\text{Yb}}(Z = 70; N = 106) \).

The neutron skins of \( {^{138}\text{Ba}}, {^{158}\text{Dy}}, \) and \( {^{176}\text{Yb}} \), are correlated to the corresponding neutron skin of \( {^{208}\text{Pb}} \) in the three panels of Fig. 2. We observe a tight linear correlation that is largely model independent. The linear regression coefficients (slope \( m \) and intercept \( b \)) have been enclosed in parenthesis \[20\]. A theoretical spread of approximately 0.2 to 0.3 fm in the neutron radius of \( {^{208}\text{Pb}} \) was estimated in Refs. \[16, 17\]. Most of this spread is driven by difference between relativistic and nonrelativistic models, which has recently been attributed to the poorly known density dependence of the symmetry energy \[21, 22\]. With the culmination of the the Parity Radius Experiment at the Jefferson Laboratory \[11\], the theoretical spread will be replaced by a genuine experimental error that is five times smaller, that is, \( \Delta R_n({^{208}\text{Pb}}) \approx 0.056 \text{ fm} \). This 1% measurement of the neutron radius in \( {^{208}\text{Pb}} \) translates into a neutron radius uncertainty of \( \Delta R_n({^{138}\text{Ba}}) \approx 0.045 \text{ fm} \), \( \Delta R_n({^{158}\text{Dy}}) \approx 0.034 \text{ fm} \), and \( \Delta R_n({^{176}\text{Yb}}) \approx 0.052 \text{ fm} \), respectively. The results presented above employ a nuclear-structure model that lacks both deformation and pairing correlations — effects that may be important for the nuclei considered in atomic parity violating experiments. Thus, the present calculations were limited to the study of that single member of each isotopic chain having a closed neutron shell. This shortcoming was overcome in Ref. \[23\] through the inclusion of both nuclear deformation and pairing correlations. Further, in anticipation of future experiments on high \( Z \) atoms — where the accuracy in the measurements of atomic parity violating effects may be significantly improved — nuclear structure corrections to the weak charge in Francium isotopes were also computed. Insofar as the neutron radius of heavy nuclei may be re-
Fig. 2. Skin-skin correlations for three heavy nuclei of possible relevance to atomic parity violation ($^{138}\text{Ba}$, $^{158}\text{Dy}$, and $^{176}\text{Yb}$) as a function of the neutron skin of $^{208}\text{Pb}$ for three models predicting a different density dependence for the symmetry energy. Quantities in parenthesis represent linear regression coefficients (slope and intercept).

3.2 Neutron Star Structure

Neutron stars contain a non-uniform crust above a uniform liquid mantle (or outer core). See Fig. 3, courtesy of Dany Page, for an accurate rendition of the expected structure of a neutron star. To compute various properties of neutron stars, the equation of state for the uniform liquid phase is assumed to consist of neutrons, protons, electrons, and muons in beta equilibrium. Further, it is assumed that this description remains valid in the high-density inner core. Thus, the very interesting possibility of transitions to various exotic phases, such as meson condensates, hyperonic matter, and/or quark matter, are not considered here. In the opposite domain, namely, at the lower densities of the crust, the uniform system becomes unstable against density fluctuations. That is, at these densities it becomes energetically favorable for the system to separate into regions of high- and low-density matter. In this non-uniform region the system is speculated to consist of a variety of complex and exotic structures, such as spheres, cylinders, rods, plates, etc. — collectively dubbed as nuclear pasta [24,25]. While microscopic calculations of the nuclear pasta are now becoming available [26,27,28,29], it is premature to incorporate them in our calculation. Hence, after determining the transition density from the uniform liquid mantle to the non-uniform solid crust via an RPA stability analysis [15], a simple polytropic equation of state is used to interpolate between the outer crust [30] and the uniform liquid [31].

Results for the transition density from the uniform liquid mantle to the non-uniform solid crust as a function of the neutron skin in $^{208}\text{Pb}$ are displayed in Fig 4. Various models are used to show the nearly model-independent relation between these two seemingly distinct observables. The figure displays an inverse correlation between the neutron-skin and the transition density found in Ref [15]. This correlation suggests that models with a stiff equation of state predict a low transition density, as it becomes energetically unfavorable to separate nuclear matter into regions of high and low densities. Finally, this “data-to-data” relation illustrates how an accurate and model-independent determination of the neutron skin of $^{208}\text{Pb}$ at the Jefferson Laboratory — assumed here purely on theoretical biases to be $R_n - R_p = 0.20$ fm — would determine an important neutron-star observable.
Having constructed several accurately calibrated equation of states, such as NL3 \[32\] together with its softer versions \[15\] — and FSUGold \[33\], we now examine their predictions for the structure of a “canonical” 1.4 solar-mass neutron star \((M = 1.4 \, M_{\odot})\). To do so, we solve the Tolman-Oppenheimer-Volkoff (TOV) equations, a set of equations appropriate for the structure of spherically-symmetric neutron stars in hydrostatic equilibrium. These equations are a generalization of Newton’s equation for hydrostatic equilibrium supplemented by three corrections from general relativity \[34\]. Incorporating these correction terms is critical, as typical escape velocities from neutron stars are of the order of half the speed of light. The TOV equations with their associated boundary conditions are still incomplete without the provision of an equation of state, i.e., a relation between the pressure and the energy density. Indeed, if general relativity is assumed valid — a very modest and safe assumption — the only physics that the structure of neutron stars is sensitive to is the equation of state of neutron-rich matter.

In Fig. 4, the pressure profile of a \(M = 1.4 \, M_{\odot}\) neutron star is displayed for various equations of states. The curves labeled as NL3 \[32\] — and FSUGold \[33\], we now examine their predictions for the structure of a “canonical” 1.4 solar-mass neutron star \((M = 1.4 \, M_{\odot})\). To do so, we solve the Tolman-Oppenheimer-Volkoff (TOV) equations, a set of equations appropriate for the structure of spherically-symmetric neutron stars in hydrostatic equilibrium. These equations are a generalization of Newton’s equation for hydrostatic equilibrium supplemented by three corrections from general relativity \[34\]. Incorporating these correction terms is critical, as typical escape velocities from neutron stars are of the order of half the speed of light. The TOV equations with their associated boundary conditions are still incomplete without the provision of an equation of state, i.e., a relation between the pressure and the energy density. Indeed, if general relativity is assumed valid — a very modest and safe assumption — the only physics that the structure of neutron stars is sensitive to is the equation of state of neutron-rich matter in beta equilibrium.

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In Fig. 5, the pressure profile of a \(M = 1.4 \, M_{\odot}\) neutron star is displayed for various equations of states. The curves labeled as NL3 \[32\] — and FSUGold \[33\], we now examine their predictions for the structure of a “canonical” 1.4 solar-mass neutron star \((M = 1.4 \, M_{\odot})\). To do so, we solve the Tolman-Oppenheimer-Volkoff (TOV) equations, a set of equations appropriate for the structure of spherically-symmetric neutron stars in hydrostatic equilibrium. These equations are a generalization of Newton’s equation for hydrostatic equilibrium supplemented by three corrections from general relativity \[34\]. Incorporating these correction terms is critical, as typical escape velocities from neutron stars are of the order of half the speed of light. The TOV equations with their associated boundary conditions are still incomplete without the provision of an equation of state, i.e., a relation between the pressure and the energy density. Indeed, if general relativity is assumed valid — a very modest and safe assumption — the only physics that the structure of neutron stars is sensitive to is the equation of state of neutron-rich matter in beta equilibrium.

The neutron radius of a heavy nucleus is a fundamental nuclear-structure observable that remains elusive, mainly due to our inability to use electro-weak probes to sample the neutron distribution. While a highly mature hadronic program has been used to map the neutron distribution, the clean extraction of the neutron radius has been marred by controversial uncertainties in the reaction mechanism. The established and successful parity-violating program at the Jefferson Laboratory provides an attractive electro-weak alternative to the hadronic program. The Parity Radius Experiment at the Jefferson Laboratory will take advantage of the strong coupling of the \(Z^0\) boson to neutrons to measure the neutron radius of \(^{208}\)Pb accurately and model independently. While the intrinsic achievement of this experiment is undeniable, in this contribution we...
have examined the far-reaching consequences that such a measurement could have over fields as diverse as atomic parity violation and astrophysics. First, a tight correlation was found between the neutron skin of $^{208}$Pb (the aim of the PREX experiment) and the neutron radius of a variety of elements (Barium, Dysprosium, and Ytterbium) of relevance to atomic parity violation program. Although the calculations presented here neglect both deformation and pairing correlations, we have argued — based on more sophisticated calculations — that being a bulk nuclear property, the neutron radius remains largely insensitive to these effects. Second, we demonstrated that PREX will also have a strong impact on various astrophysical observables. Indeed, a model-independent (or “data-to-data”) relation between the neutron skin of $^{208}$Pb and the transition density from the uniform liquid mantle to the non-uniform solid crust was established. This correlation emerged as a result of the similar composition of the neutron skin of a heavy nucleus and the crust of a neutron star: neutron-rich matter at similar densities. Further, we showed how the measurement of the neutron skin in $^{208}$Pb is strongly correlated to the radius of a $M=1.4 \, M_\odot$ neutron star. This result emerges as a direct consequence of the density dependence of the symmetry energy, as the same pressure that pushes neutrons out against gravity in a neutron star. Yet in contrast to the “data-to-data” relation described above, this correlation is model dependent — as the radius of the neutron star depends on both the low- and high-density components of the equation of state. Fortunately, earth- and space-based telescopes have started to place important constraints on the high-density component of the equation of state. Thus, we eagerly await observational results to constrain the high-density component of the equation of state. New telescopes operating at a variety of wavelengths are turning neutron stars from theoretical curiosities into powerful diagnostic tools. Significant advances in observational astronomy will soon yield the first combined measurement of mass-radius relations for a variety of neutron stars. These results — combined with the Parity Radius Experiment at the Jefferson Laboratory — will provide the most complete information to date on the long sought equation of state of neutron-rich matter.

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