Wave-tilt characteristics of TE and TM-mode Waves

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Received on August 20th, 1979

ABSTRACT

Theoretical analysis have been carried out to investigate the effect of subsurface electrical parameters such as: resistivity and dielectric constant; and angle of incidence of electromagnetic waves on the amplitude and phase of the wave-tilt over a homogeneous earth model over a frequency range of 10^2 - 10^8 Hz. Numerical analysis have been performed for a range of incident angle and for a range of electrical parameters. For better knowledge of subsurface features, need for wave-tilt measurement in TE and TM-mode waves have been discussed.

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RIASSUNTO

Sono state eseguite analisi teoriche per studiare gli effetti di parametri elettrici di sottosuperficie come: resistività, costante dielettrica e angolo di incidenza delle onde elettromagnetiche sull'ampiezza e la fase (*) Dipartimento di Geofisica, Università Banaras Hindu, (Varanasi - 221005, INDIA.)
INTRODUCTION

The measurement of wave-tilt as a diagnostic technique for exploring the earth's subsurface properties dates back to the beginning of the twentieth century (Zenneck, 1907; Hack, 1908). It was shown that the electric vector of Zenneck surface wave traced out a narrow ellipse which was tilted slightly forward. The theory of wave-tilt was further formulated for air-earth interface by Sommerfield (1926) and Norton (1937). The potentiality of investigation of earth's subsurface properties has been studied by Barlow and Fernando (1956), Wait (1957) and Furutsu (1959). Thereafter a number of investigations on the characteristics and mode of measurement of wave-tilt have been reported in the literature (Wait, 1962; King, 1968, 1969, 1974, 1976; Lytle et al., 1976; Sinha, 1977). The increasing potentiality and popularity of the method has lead to its successful applications for resistivity mapping (McNeill and Hoekstra, 1973; Hoekstra and McNeill, 1973; Hoekstra et al., 1974, 1975; Arcone, 1977, 1978; Arcone et al., 1978; Hoekstra, 1978; Arcone, 1979 and Thiel, 1979).

The analytical study of wave-tilt amplitude and phase angle of radiowave reflected at grazing incidence from a given earth's surface model plays a very important role in the interpretation of experimental data. The master curves showing the theoretical response can be used to invert the measured wave-tilt data and obtain the electrical parameters of earth's surface.

The potentiality of the wave-tilt measurements in TM and TE-modes have been commented upon by King, 1974; Lytle and Lager, 1975; and Thorson, 1975. The main objective of the study presented in this paper is to decipher the characteristic wave-tilt features in both the TM and TE-modes and bring out comparative potentialities of measurements in the two modes.
Wave-tilt in TM-mode on the homogeneous earth's model in terms of normalized plane wave surface impedance is written as (King, 1969):

\[ W_e = \frac{E_x}{E_z} = -A(\theta) \cos \phi \]

where

\[ A(\theta) = \frac{1}{\sqrt{2}} \left[ 1 - \frac{\cos \theta}{\cos \phi} \right] \]

normalized plane wave surface impedance

\[ Z_r = \frac{\mu_m}{\omega \varepsilon_0} \] complex refractive index

\[ \varepsilon_r = \text{dielectric constant of the medium} \]

\[ \sigma = \text{conductivity of the medium} \]

\[ \omega = \text{angular frequency} \]

\[ \phi \] the angle of the incident wave makes when it is reflected from the earth's model

On grazing incidence, \( \phi \to 0 \) and the wave-tilt on the earth's model in terms of normalized plane wave surface impedance is written from equation [1] as

\[ W_e = \frac{E_x}{E_z} = A(0) \cos \phi \]

where

\[ A(0) = \frac{1}{\sqrt{2}} \left[ 1 - \frac{1}{\cos \phi} \right] \]

normalized plane wave surface impedance

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\[ W_e = \frac{E_x}{E_z} = A(0) \cos \phi \]
For TE-mode waves, wave-tilt is defined as (Lytle, 1976)

\[ \psi_z = \frac{H_z}{H} = \frac{\frac{1}{2} H_z}{H} \cdot \cos \theta = 0 \quad \text{(3)} \]

Substituting for \( z \) and writing

\[ v = \frac{1}{M} \]

as before, we obtain an equation for TE-mode which analogous to equation (2)

\[ W_z = \frac{H_z}{H} \quad \text{at} \quad 0 = 0 \quad \text{(4)} \]

RESULTS AND DISCUSSIONS

Effect of angle of incidence, resistivity and dielectric constant

Numerical computations using equations (1) and (3) show

the effect of angle of incidence, resistivity and dielectric constant

on the amplitude and phase of wave-tilt for both TM and TE-

modes. In Fig. 1 variation of the amplitude of the wave-tilt

with angle of incidence \( 0^\circ \rightarrow 90^\circ \) for the resistive values 10, 100, 1000, 10000, 100000 Ohm-meter at frequency \( 10^4 \) Hz have been

shown. It is observed that the magnitude of wave-tilt is larger

in TE-mode than TM-mode. It is seen that with the increase of

incidence, the magnitude of wave-tilt also increases in both the

modes. It is further seen that for TM-mode, the amplitude of

wave-tilt increases with increase in resistivity of the ground

whereas reverse holds good for TE-mode. The phase of the

wave-tilt variation with angle of incidence for resistivity value


Fig. 1 - Variation of the amplitude of wave-tilt with angle of incidence.
We have a chart showing the variation of phase of wave-tilt with angle of incidence. It is observed that with the change in angle of incidence in both TE and TM-modes, phase of wave-tilt is not affected at lower frequencies (10^4 Hz) whereas at higher frequencies (10^5 and 10^6 Hz), the phase of wave-tilt is seen to decrease for TM-modes but increase for TE-modes with a small increase in angle of incidence.

The amplitude and phase angle of wave-tilt versus frequency are also illustrated in the chart.
Wave tilt characteristics of TE and TM modes for a resistivity value of $10^2$ Ohm-m have been computed for TM and TE-mode waves and are shown in Fig. [3] and [4]. The amplitude of wave tilt for TM mode shows a linear increase with frequency whereas for TE-mode, a reverse trend is seen (Fig. 3). The amplitude of wave tilt for TM and TE-mode attains higher values for a resistivity value of $10^2$ Ohm-meter.
At higher angle of incidence, being much larger at $\theta = 90^\circ$ as compared to $\theta = 0^\circ$ and $45^\circ$ which are relatively close to each other.

The curve for $\theta = 90^\circ$ for wave-tilt of TE-mode waves is not shown in the Fig. 3, because it is larger than 10. At lower values of resistivity, the displacement current becomes much larger and the curve for $\theta = 90^\circ$ is larger than 10.

The phase of wave-tilt and their variation with frequency for TM and TE-modes are almost mirror image of each other, with positive values for TM-mode and negative for TE-mode. The phase of TE and TM-mode waves are not affected by the change in the angle of incidence (Fig. 4). The change of dielectric constant considerably influences the phase of wave-tilt. It is seen that for $\varepsilon = 1$, the phase is 45° unaffected by frequency, whereas...
Fig. 5 - Variation of the amplitude of wave-tilt with frequency for resistivity $10^4$ Ohm-m.

For $\theta = 45^\circ$, phase changes from $45^\circ$ at $10^4$ Hz to about $20^\circ$ at $10^6$ Hz.

Figs. 5 and 6 show the amplitude and phase angle of TM and TE modes, respectively, at dielectric constant and frequency combinations similar to those presented earlier. The effect of dielectric constant variation becomes distinguishable at higher values of surface resistivity. The amplitude of wave-tilt for TM-mode decreases with increase in dielectric constant, whereas it increases for TE-mode for all angles of incidence. The wave-tilt of TE-mode is higher for $\theta = 90^\circ$ compared to all other angles. The phase curves (Fig. 6) for TE-mode are mirror image of TM-mode and show a decreasing trend in frequency.

For $\theta = 45^\circ$, phase changes from $45^\circ$ at $10^4$ Hz to about $20^\circ$ at $10^6$ Hz.
phase angle with increasing frequency. For \( n = 3 \), the phase angles for TM and TE mode are 0° for the wave having free \( n = 0 \) and \( -90° \) respectively and gradually decrease to zero degree phase angle at 10 Hz. For dielectric constant values 81, the phase angles are lowest showing 7° to 9° at 10 Hz and gradually decrease to zero degree phase angle at about 10 Hz. It is observed that the phase angle increases with increasing frequency and decreases with increasing angle of incidence. The phase angles for TM mode are higher for lower angle of incidence. For higher resistivity of the earth's surface, the phase angles are higher for lower angle of incidence. The variation in amplitude and phase of TM and TE mode waves do not show significant variations in amplitude and phase of TM and TE mode waves. The variation of phase angle with frequency for different \( n \) values is shown in Fig. 4. Variations of amplitude with angle of incidence and dielectric constant of the earth's surface do not show significant variations in amplitude and phase of TM and TE mode waves.
WAVE-SILT CHARACTERISTICS OF TEC.

Fig. 7 - Variation of the amplitude of waves with frequency for resistivity 10$^6$ Ohm-m.

The amplitude becomes more pronounced in higher frequency range but do not show any variation with frequency. Computations carried out in figures 3 and 6 are repeated at a still higher resistivity value of 10$^6$ Ohm-m and results are shown in figures 7 and 8. The amplitude does not show any variation with frequency and remains virtually same at all the frequencies. The effect of dielectric constant is more pronounced showing appreciable amplitude differences at the same angle of incidence. The phase angle values are further reduced. It changes from 45° at 10$^2$ Hz to about zero degree at about 10$^5$ Hz. For dielectric constant value 81, phase for all angles of incidence is close to zero degree at higher frequencies (greater than 10$^5$ Hz).
Using equations [2] and [4] we have computed the wave-tilt magnitude for various practical resistivity values of the earth's surface. The variation of magnitudes of \( W \) and \( W^c \) in the frequency range \( 10^2 \) - \( 10^7 \) Hz are shown in Fig. 3. Two sets of limiting dielectric constant values 3 and 81 have been chosen with a view to demonstrate the wave-tilt response from earth's surface and water surface. The variations for intermediate values of dielectric constants can be easily inferred from these curves. The \( W^c \) curves for resistivity values 10\(^2\), 10\(^4\) and 10\(^6\) Ohm-m show an increasing magnitude of the wave-tilt with increasing frequencies. At higher frequencies the wave-tilt becomes saturated and do not show any increase with increasing frequencies, especially between \( 10^6 \) - \( 10^7 \) Hz as shown in Fig. 3.

![Figure 3: Variation of phase of wave-tilt with frequency for resistivity 10 Ohm-m.](image)
The phase angle variation of wave tilt with frequency for some surface parameters has been shown in Fig. 10. The phase angle of wave tilt decreases almost linearly in certain frequency range nearly between $10^3$ to $10^4$ Hz for resistivity $10^2$ Ohm-m and dielectric constant 3. For dielectric constant 81, this frequency shifts to lower frequency range of $10^2$—$10^3$ Hz. Similar decrease in the phase angle variation is also seen for resistivity values $10^4$ and $10^6$ Ohm-m and the phase angle change for dielectric constant value 3 and 81 are seen to considerably large. The corresponding values of magnitude and phase angle of wave tilt $W_m$ have been computed and shown in Fig. 9 and 10 for the sake of comparison. It is shown in Fig. 9 that the magnitude of $W_m$ is almost two orders magnitude higher as compared to $W_e$ for dielectric constant 3. The effect of higher...
values of dielectric constant $\varepsilon$ is seen only at higher frequency where the solid and dotted curves are seen to fork showing higher magnitude for dielectric constant $\varepsilon$. The magnitude of $\varepsilon$ for resistivity value $10^2$ Ohm-m varies from 0 to 100 in the frequency range $10^{-4}$ to $10^{-2}$ Hz. For higher values of resistivity $10^4$ Ohm-m the wave-tilt response shifts to lower frequency. The phase angle variations for TE-mode waves are shown in Fig. 10. For a given value of surface resistivity, the phase angle of $\varepsilon$, decreases from $45^\circ$ to $0^\circ$, whereas the phase angle of $\varepsilon$, decreases from $0^\circ$ to $45^\circ$. The phase angle variation for $\varepsilon$, is between $-45^\circ$ to $0^\circ$, whereas for $\varepsilon$, the phase angle varies between $0^\circ$ to $45^\circ$. For a given value of surface resistivity, the phase angle of $\varepsilon$, decreases from $45^\circ$ to $0^\circ$, whereas the phase angle of $\varepsilon$, decreases from $0^\circ$ to $45^\circ$.
Wave-Silt Characteristics of TE Mode.

In certain frequency range, for resistivity values $10^2$, $10^4$ and $10^6$ Ohm-m, the frequency range of maximum sensitivity are $10^6$ Hz and beyond, $10^4$—$10^7$ Hz and $10^2$—$10^4$ Hz respectively. The change in dielectric produces a significant change in the phase angle in the entire frequency range.

CONCLUSION

The study of amplitude and phase of wave-tilt of TE and TM-modes have been carried out. The amplitude and phase of wave-tilt of TE-mode waves have been found to vary significantly with various parameters of the earth’s subsurface. The amplitude and phase of waves of TM-mode maxima and vary rapidly in the lower frequency region, whereas for TM-mode wave is known to maxima and change rapidly in the higher frequency region. Studies of dependence of various parameters on the amplitude and phase can be used as diagnostic technique for resistivity survey of the subsurface. The present analysis clearly shows the complementary nature of characteristics in the two modes and shows that the simultaneous measurement of wave-tilt in both TE and TM-modes are capable of giving better information in the wider frequency range.

ACKNOWLEDGEMENT

The author is grateful to CSIR, New Delhi for financial support. The author is grateful to Dr. T. Lai and Dr. J. Singh for their inspiration and stimulating discussions.
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