Concrete structures serving in cold and wet regions usually suffer frost damage and thus have sever deterioration. Many researches have been conducted to reveal the damaging mechanism and damaged mechanical properties of concrete under the effect of frost action. It has been widely known that the strength and stiffness of frost damaged concrete without using air-entraining agent decrease under room temperature. However, there will be a different story if the frost-damaged concrete is saturated and loaded under freezing temperature. Water existing in pores and cracks will freeze into ice, which provides additional strengthening effects. This paper presents a multi-scale modeling and simulation work on the static and fatigue behaviors of frost damaged concrete with consideration of such ice-strengthening effects. The micro-mesoscale damaging and strengthening effects induced by ice formation are modeled and integrated into the mesoscale analytical approach – Rigid Body Spring Model, and the macroscale static and fatigue behaviors are simulated. It is found that the freezing temperature has a positive (strengthening) effect on the static strength, while it has a negative effect on the fatigue life for both intact and frost-damaged concrete. Test is also conducted with available experimental evidence to validate the developed approach. Satisfactory correlation is found through the comparison between simulation and experiment.

1. Introduction

Concrete structures built in cold and humid areas are suffering the freezing and thawing environment, which leads the structural performance deteriorates. The degradation could be roughly attributed to two reasons: one is the degradation of concrete materials and the other one is the loss of bond between concrete and reinforcement (Wang et al. 2020a).

To find out the deterioration of concrete materials, many researches have been conducted in terms of modeling, simulation and experimentation. Modeling works of frost damage were mainly based on the thermodynamic equilibrium of moisture in porous media and the hydraulic pressure caused by the volume expansion when water frozen into ice (Powers 1949; Coussy and Monteiro 2008; Sun and Scherer 2010). Gong et al. (2015a) integrated both mechanisms and proposed a comprehensive model accounting for the internal pressure induced by ice formation. These models well explained the damaging mechanisms in microscale poro-mechanics while it was still difficult to relate them with the macroscale mechanical properties of frost damaged concrete. Mesoscale simulation approach offered an option to link the different scales and achieve the multi-scale analysis. With upscaling the microscale internal pressure model due to ice formation into the non-linear mesoscale constitutive behaviors of porous element with considering the loading (freezing) and unloading (thawing), macro-scale concrete deformation during the temperature cycles and the mechanical properties after frost damage were successfully simulated and verified (Gong et al. 2015b). Besides, huge amount of experimental results could be found with respect to the evaluation of mechanical properties of frost damaged concrete. These tests could be categorized in terms of loading types including uniaxial compression (Hasan et al. 2004; Hanjari et al. 2011), flexural tension (Hasan et al. 2002), splitting tension (Hanjari et al. 2011), biaxial compression (Shang and Song 2006) and fatigue compression (Hasan et al. 2008). In addition, some important factors were investigated, such as whether adopting the air-entraining agent (AEA) (Hasan et al. 2004; Zhou et al. 2008) or stirrups (Duan et al. 2011) to alleviate the frost damage level. Furthermore, some empirical models to calculate the macroscale mechanical behaviors of frost damaged concrete have been developed according to the experimental results (Hasan et al. 2004, 2008; Zhou et al. 2008; Duan et al. 2011). However, all these works were limited to the evaluation of mechanical behaviors of concrete under thawing temperature. In other words, only the damaging effects (cracks) were considered. However, the concrete structures are serving under not only thawing temperature but also freezing temperature. Under freezing temperature, there is a different story for the concrete since water in the pores and cracks would freeze into ice, which could offer additional load carry-

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ing capacity in addition to concrete itself (Wang et al. 2020b, 2021). In other words, both strengthening and damaging effects might exist for concrete when it is loaded under freezing temperature.

Therefore, the authors have been conducting a series of simulation and experiment studies on the mechanical properties of concrete under the effect of frost damage, which aims to cover as much cases such as intact/damaged concrete, with/without ice-strengthened pores and meso-cracks, static/fatigue loading, as shown in Fig. 1: With aid of the development of Rigid Body Spring Model (Nagai et al. 2004; Matsumoto et al. 2008) and verified by various experiments, the mesoscale simulation platform is provided for conducting this series of studies (first line in Fig. 1). As the starting step, the decayed static and fatigue properties of frost damaged concrete was evaluated experimentally by Hasan et al. (2004, 2008) and numerically by Gong et al. (2017a) (third line in Fig. 1). Afterwards, the static behaviors of concrete with icing effect in pores was numerically proceeded by Gong et al. (2017b, 2018) and validated by Lee et al. (1988a, 1988b) (second line in Fig. 1). Followed by these studies, the static behaviors of concrete with icing effects in both pores and meso-cracks was investigated numerically and experimentally by Wang et al. (2021) (fourth line in Fig. 1). Up to now, the enhanced static properties were found in terms of the ice-strengthening effects in pores and meso-cracks. However, what is the fatigue response of intact and damaged concrete in such case still remains unknown (red boxes in second and fourth lines in Fig. 1).

For the lifetime assessment of RC structures (such as road/bridge decks) in cold area, the load (e.g., passing traffic) are acting both under freezing temperatures, such as winter in yearly cycle and nighttime in daily cycle, and thawing temperature, such as sum-...
mer and daytime. Neglecting such effect would cause underestimation of static behaviors of the damaged concrete, which seems so far so good since it’s conservative (Wang et al. 2020b, 2021). But for RC road/bridge decks, the fatigue response arouses more interests among civil engineers. Thus, it is very important to figure out both damaging and strengthening effects in order for the more precise assessment of both material and structural performance, with respected to not only static but also fatigue loading case. Then numerically it requires to integrate the constitutive law of frost damage law, ice strengthening law and fatigue law in RBSM, which will be conducted in current work.

This paper focuses on the multi-scale modeling and simulation of the static and fatigue behaviors of concrete under both thawing and freezing temperatures. Microscale and mesoscale ice-strengthening effects are integrated with the damaging effects and implemented to the mesoscale simulation application - Rigid Body Spring Model (Nagai et al. 2004). Following the simulation of freezing-thawing cycles (FTC), different equilibrium is judged depending on the loading temperature. Afterwards, the external static or fatigue loading is applied to the concrete, and the macroscale mechanical properties is simulated and analyzed. It should be noted that the current study discusses the pure concrete without AEA or stirrups. For validation, experiment of static loading test has also been conducted with several available experimental evidences of fatigue loading test. The simulation and experiment results were compared and discussed where good agreement is found. Furthermore, evaluations could be hopefully conducted with respected to the deterioration of bond behaviors and structural performances of frost damaged reinforced concrete components under freezing temperature following the scheme in previous studies (Wang et al. 2020a).

2. Basic modeling

2.1 Meso-scale scheme with RBSM

As introduced, the impacts of ice formation take place in different scales thus a multi-scale approach is needed. Following the scale separation by Gong et al. (2018), the damaging and strengthening effects are explained.

At microscale level, the objective element is composed of cement hydrates and pores filled with air, water and ice. When ice forms in the pore, the internal pressure takes place according to the pore-mechanics which makes the micro-cracks initiate. Meanwhile, the clusters of cement hydrates with pores filled with ice will gain higher strength and stiffness. These are the microscale damaging and strengthening effects induced by ice formation. Although the microscale mechanical actions are rather local events according to the pore size and saturation, since these effects will be upscaled and smeared to the mesoscale constitutive stress-strain relationships (springs in Rigid Body Spring Model), microscale elements are treated as poro-elastic media. Due to the scale selection of RBSM, the damage and other mechanistic events start from mesoscale level, which might dismiss the fact that frost damage initiates from very fine microscale level. But it is still very useful to use such integrated/smeared micro-meso simplification to conduct the meso-macro simulation with far less computation cost, comparing to directly simulate the macroscale behaviors from the cement hydrates and pore structures. In details, the pore pressure is transferred into the equivalent volumetric stress applied in the meso-springs in RBSM. At the same time, the strength and stiffness of the meso-springs are enlarged by considering the ice-filled pores.

At mesoscale level, the discrete numerical application - Rigid Body Spring Model is adopted. The concept was brought up by Kawai (1978) where the elements are rigid and connected with springs accounting for the deformation. Compared with the continuum finite element method, RBSM has the advantages on simulation and visualization of the cracking issues. Bolander and Saito (1998) introduced the method in fracture analysis of reinforced concrete components. In 2004, Nagai et al. introduced the physical separation of mortar and coarse aggregate elements which made the RBSM a mesoscale approach. The mesoscale properties including elastic modulus and Poisson’s ratio are calibrated according to the macroscale material test. The normal and shear stiffness of the mesoscale springs connecting the rigid elements are calculated as the weighted average value of the adjacent elements. A normal distribution of tensile strength is applied to the normal springs of mortar based on the target macroscale strength, where element with higher strength is more homogeneous. While the shear strength of mortar is calculated with respected to the normal stress, which is originally proposed by Nagai et al. (2004). For the interface transition zone between mortar and coarse aggregate (ITZ), the objective normal strength is calculated according to the water to cement ratio. Similarly, as the mortar springs, normal distribution is adopted to the normal springs of ITZ. The shear strength of ITZ is calculated following the friction model. Coarse aggregate element is assumed elastic with large normal/shear stiffness and strength. The internal pressure caused by ice formation has been upscaled from microscale and applied on the mesoscale springs. Meanwhile, the strengthening effects of ice filled pores (both microscale and mesoscale) have been converted to the enlarged strength and stiffness of the springs. Then expansion takes place with meso-crack initiation up to the equilibrium between the porous media and ice. When temperature rises and ice melts into water, the expansive deformation recovers but not to the original status with an unrecoverable meso-crack remaining deformation. Such meso-crack accounts for the mesoscale damaging effect under whether thawing or freezing temperature. For latter case (freezing temperature), when water fills in the meso-crack and freezes
into ice, an additional load-carrying capacity is contributed which is defined as the mesoscale ice-strengthening effect. The detailed modeling of micro-mesoscale damaging and strengthening effects will be presented in section 2.2.

After all the micro-mesoscale damaging and strengthening effects have been integrated with RBSM approach, the simulation could be conducted. The whole simulation is divided into two parts: in the first step, the freezing and thawing cycles are applied to the concrete models and the deformation is calculated; in the second step, the external load are applied under either thawing temperature (without ice strengthening effect) or freezing temperature (with ice strengthening effect) to examine the mechanical properties of concrete.

2.2 Micro-mesoscale Modeling

The microscale models include the pore pressure induced by ice formation and the porous mortar components filled with ice. For the pore pressure, the model proposed by Gong et al. (2015a) is adopted where the effective internal pressure is determined by hydraulic pressure \( p_h \), cryosuction pressure \( p_l \) and crystallization pressure \( p_c \), see Eq. (1).

\[
\sigma_u = B \left( p_h + p_l + p_c \right),
\]

\[
p_b = \frac{0.09 \psi_c}{\psi_c / K_c + \psi_l / K_l} \cdot f(S_r, \varphi_{in}, \ldots)
\]

\[
p_l = \psi_l \cdot \Delta S_{p_l} \cdot \Delta T
\]

\[
p_c = -\psi_c \cdot (1 - \lambda) \cdot \Delta S_{p_c} \cdot \Delta T
\]

where \( \psi \) is the volume fraction with subscript \( c \) and \( l \) standing for ice and water; \( K \) is the bulk modulus (8.8 GPa and 2.2 GPa for ice and water, respectively); \( S_r \) is the saturation degree; \( \varphi_{in} \) is the air content; \( \Delta T \) is the temperature variation; \( \Delta S_{p_l} \approx 1.2 \text{ J/cm}^3\text{K} \) is the molar entropy of fusion; \( \lambda \) is the pore shape factor (Sun and Scherer 2010); \( B \) is the Biot coefficient where \( B \approx 2\varphi/(1+\varphi) \) and \( \varphi \) is the total porosity (Sun and Scherer 2010). The elastic modulus of ice \( (k_w) \) is calculated by Eq. (2).

\[
k_w = \frac{3B}{\varphi \psi_c / K_c + \psi_l / K_l}
\]

For the ice-filled micro and meso pores, the bulk and shear modulus could be calculated according to the volume fraction of each component as suggested by Gong et al. (2018), see Eq. (3).

\[
E_{el} = \frac{9K_{hom}G_{hom}}{3K_{hom} + G_{hom}}
\]

\[
v_{el} = \frac{1}{2} \left( 1 - \frac{1}{1/3 + K_{hom} / G_{hom}} \right)
\]

\[
K_{hom} = \sum_{r=1}^{m} f_r K_r P_r / \sum_{r=1}^{m} f_r P_r
\]

\[
G_{hom} = \sum_{r=1}^{m} f_r G_r Q_r / \sum_{r=1}^{m} G_r Q_r
\]

in which \( E_{el} \) and \( v_{el} \) are the enlarged elastic modulus and Poisson’s ratio; \( f_r \) is the volume fraction of component \( r \); \( P_r \) and \( Q_r \) are defined as the compressibility and shear compliance. With the enlarged elastic modulus and Poisson’s ratio, the strengthened stiffness of springs can be determined as indicated in Fig. 3 (\( k_n \) to \( k_{el} \)).

![Fig. 3 Static constitutive law of normal spring (Wang et al. 2021).](image-url)
As the key point, the constitutive stress-strain relationships of normal and shear springs with static and fatigue loading under both thawing and freezing temperatures are explained here. Detailed information of modeling for static behavior could be found in Wang et al. (2020b, 2020c). In RBSM, normal spring is defined elastic with infinite compressive strength and specific tensile strength, see the black dashed curve in Fig. 3(b). Ueda et al. (2009) introduced the concept of focal compressive strain ($\varepsilon_{pa}$) which the unloading curve would point to and the residual strain ($\varepsilon_{pf}$) remained accordingly to account for the damage. As shown in Fig. 3(b), the ice strengthening effect of ice-filled pores is indicated as the switch of black dashed curve to the red solid curve where both the stiffness ($k_2$ to $k_{n2}$) and strength ($f_{elem}$ to $f_{\text{ice}}$) are enlarged. When ice forms under freezing temperature and assuming that ice formation takes place in very short time, the porous matrix suffers the expansive pressure ($\sigma_0$) and the ice particle suffers the compressive pressure ($-\sigma_0$) simultaneously. Therefore, the system includes the interactive components of porous body (naming mortar spring) and ice (naming ice spring), see the red and blue springs shown in Fig. 3(a). In Fig. 3(b), the behavior of ice is plotted as the blue thick curve with its own coordinate system. The mutual action caused by ice formation results in the tensile deformation of mortar spring and release of ice spring. Such interaction stops when the equilibrium is reached where absolute value of stress is the same for mortar and ice, see point 1 in Fig. 3(b).

Two cases are discussed from here. Case 1 is applying external load under freezing temperature where the ice-strengthening effect in meso-crack exists. In such case, the external load is acting on the combined system of porous body and ice. Thus, the constitutive stress-strain curve follows the combination (red curve for mortar spring and blue thick curve for ice), as indicated by the purple dash-dot curve in Fig. 3(b). Case 2 is to increase the temperature with ice melting and then applying the external load. In this case, unloading follows the model proposed by Ueda et al. (2009), and residual deformation remains after thawing process stops, see point 2 in Fig. 3(b). The constitutive relationship follows the porous matrix (black dashed curve) and the deteriorated tensile strength of normal spring becomes $f_{\text{d}}$. Comparing two cases, the impact of ice-strengthening effect in meso-cracks on the normal springs is clearly yielded by the strength gap ($\Delta_n$). It could be easily found that the contribution is attributed to the tensile strength of ice ($f_{\text{ice}}$). Through such modeling, the macroscale mechanical properties are simulated, which is also expected to obtain an enhanced strength due to this icing effect.

The story of shear behavior is illustrated in Fig. 4. Similarly, the shear stiffness ($k_s$ to $k_{s1}$) and strength ($\tau_{\text{max}2}$ to $\tau_{\text{max1}}$) of mortar spring is enlarged with consideration of the ice-filled pores. The shear stiffness and strength of ice ($\tau_{\text{int}}$) is directly considered by adding its contribution to the shear strength of mortar. In addition, the shear strength of mortar is also determined by the crack width of normal spring of mortar, as shown in Fig. 4(b). Through comparing the shear strength under freezing (solid blue curve) and thawing (dashed black curve) temperature in Fig. 4(b), the impact of ice-strengthening effect in meso-cracks on shear springs is also clearly indicated. Accordingly, the macroscale mechanical prop-

![Fig. 4 Static constitutive law of shear spring (Wang et al. 2021).](image-url)
properties of concrete are expected to increase through such modeling, which will be presented in the meso-macro simulation. Besides, it should be noted that aggregate is assumed to suffer no frost damage due to its rather small porosity. For mortar and ITZ, the fact that their different pore structures may cause different ice contents at same temperature and different internal pressures (Eq. (1)) is also neglected for simplification. In other words, the internal pressure induced by ice formation is assumed same for mortar and ITZ springs.

For fatigue behaviors, the cyclic constitutive laws of normal and shear springs are proposed in the previous study by the authors (Gong et al. 2017a). For normal spring, the unloading and reloading curves are not exactly same to simulate the cyclic degradation. Thus, the shifting of focal compressive strain ($\varepsilon_{pa}$ to $\varepsilon_{pa}'$) has been introduced to model such cyclic degradation, see Fig. 5. The fatigue deterioration parameter $\varepsilon_{pa}$ is determined on the stress level of previous cycle, as Eq. (4).

$$\varepsilon_{pa}' = \varepsilon_{pa} \cdot \left(1 + c_s \left(\frac{\sigma_{up}}{f_i} \right)^{\alpha} \left(\frac{\sigma_{up} - \sigma_{lw}}{f_i}\right)^{-1}\right)^{\beta}$$

In Eq. (5), $\sigma_{up}$ and $\sigma_{lw}$ are the upper and lower bound of normal stress in the previous cycle; $f_i$ is the tensile strength of normal spring; $c_n$ and $\alpha$ are parameters for mortar or ITZ which reflect the effect of unloading and reloading on the stress reduction. Both parameters have been calibrated pertaining to the experimental data (Hasan et al. 2008; Gong et al. 2017). In each fatigue cycle, $\varepsilon_{pa}$ is adopted in the unloading cycle to determine the unrecoverable residual strain ($\varepsilon_{pa}'$). While the changing value of focal strain $\varepsilon_{pa}'$ is used in the reloading cycle. It is noted that with increasing plastic strain and deceasing of upper bound of normal stress, the value of $\varepsilon_{pa}'$ is getting closer to $\varepsilon_{pa}$ and the cyclic degradation becomes smaller. For shear spring, similar stress drop is introduced, as shown in Fig. 6. The shear strength will be affected by the unloading and reloading history of shear spring, as Eq. (5).

$$\tau_{\text{max}}' = \tau_{\text{max}} \cdot \prod_{x} \left(1 - c_s \left(\frac{\tau_{up}}{f_i}\right)^{\beta} \left(\frac{\tau_{up} - \tau_{lw}}{f_i}\right)^{-1}\right)$$

In Eq. (5), $\tau_{up}$ and $\tau_{lw}$ are the upper and lower bound of shear stress; similarly, $c_n$ and $\beta$ are parameters for mortar or ITZ (Gong et al. 2017a); $\tau_{\text{max}}$ is the shear strength before modification as shown in Fig. 4(b), which has already taken the consideration of impact by the crack width of corresponding normal spring. It could be emphasized that in Eqs. (4) and (5), the fatigue deterioration parameters for normal and shear behaviors are normalized in terms of tensile strength ($f_i$) of the corresponding normal spring. Considering that ice in meso-cracks may absorb energy and melt with temperature rise when suffering from the fatigue load, the tensile strength is calculated with only including the strengthening effect of ice-filled pores ($f_i$) but no effect of ice-filled cracks ($f_{crack}$) is considered.

### 3. Multi-scale simulation and experiment

With the developed application, simulation was conducted with respect to frost damage level (with/without), loading temperature (thawing/freezing), loading type (compression/tension) and dynamic condition (static/fatigue). The model was tagged with “C/T/P/N-0/75” and “P/N-0.9/0.85/0.8” for static and fatigue cases, respectively. For static loading; “C/T” represented that the loading type was whether compression or tension; “0/75” stood for the intact or damaged specimen where 0 or 75 FT cycles had been applied; “P/N” showed that the loading temperature was whether positive (thawing) or negative (freezing); For fatigue loading: only compression fatigue was adopted, and the number “0.9/0.85/0.8” showed the normalized stress level. Each series had three models with different arrangement of coarse aggregate to calculate the average value. Two types of model with different dimension were created for the static loading case in terms of loading type: 200 x 100 x 100 mm$^3$ prism was prepared for compressive simulation “C”, while 100 x 100 x 100 mm$^3$ cube was prepared for splitting tensile simulation “T”. For fatigue loading case, the model was unified to be 200 x 100 x 100 mm$^3$ prism with respected to the validation test (Koda et al. 2015, 2018). The volume fraction of coarse aggregate was around 37% and mesh size was 2-3 mm. Targeted
compressive strength were 20 MPa and 30 MPa for static and fatigue cases, which were also adopted to calculate other material inputs used for the simulation. As suggested by previous studies, the material inputs could be determined with targeted macroscale compressive strength, such as water-cement ratio, averaged tensile strength of springs, averaged elastic modulus of mortar (Kosaka et al. 1975), averaged tensile strength and pure shear strength of ITZ (Nagai et al. 2004), and porosity of mortar (Gong et al. 2017b). Other material properties including the elastic modulus of coarse aggregate (50 GPa), Poisson’s ratio of mortar (0.2) and aggregate (0.25), and frictional angle of shear criterion of ITZ (35°) were all set to be constant. As key factors in the ice-strengthening model explained in section 2.2, the strengthening parameter \( f_{\text{int}} \) was assumed to be the tensile strength of ice. Several researchers had reported that the tensile strength of ice varied from 0.7-3.1 MPa depending on the temperature, strain rates, tested volumes and ice grain sizes (Willson and Horeth 1948; Currier and Schulson 1982; Petrovic 2003). An average value of 2.23 MPa under -25°C was adopted for the tensile strength of ice in current study, as suggested by Meng and Guo (2015). For the enhanced shear strength \( \tau_{\text{int}} \), the criterion developed by Nagai et al. (2004) for mortar in 2D RBSM was adopted for ice as well. Once the strength between ice and mortar \( \tau_{\text{int}} \) could be determined, see Eq. (6).

The model labeled with “75” was applied with FTC boundary for frost damage, where no thermal gradient was assumed thus uniform expansion took place. For the static loading case, temperature cycle changed between 20 and -25°C during FTC and the loading temperature was kept -25°C and 20°C for case “N” (positive) and “P” (negative), respectively. While as for the fatigue loading case, the loading temperature was kept -20°C. All the temperatures (for FTC or loading) were set according to the experiments (Koda et al. 2015, 2018). During the external loading, displacement control was adopted for static loading while load control was adopted for fatigue loading, where the horizontal and rotational directions were restraint and prescribed displacement/load was applied in vertical direction. It should be noted that normalized stress level was adopted for the fatigue loading. In other words, the applied stress was calculated according to the static strength of the corresponding model.

\[
\tau_{\text{int}} = \pm \left(0.11 f_{\text{int}}^{-0.1} (-\sigma_{\text{a}} + f_{\text{int}})^{0.6} + f_{\text{int}}^{0.6}\right) \quad (6)
\]

To validate the proposed multi-scale approach, experiment of static loading test was conducted with some available data of fatigue loading test (Wang et al. 2020b, 2021; Koda et al. 2015, 2018). Concrete prisms and cubes were casted with water-cement ratio of 0.61. No air-entraining agent was added to the concrete mixture in order to assure sufficient deterioration. River sand and gravel with surface dried density of 2.67 and 2.63 g/cm³ were used for the determination of porosity of mortar and volume fraction of coarse aggregates (Gong et al. 2017b), and the maximum grain size were 2.5 and 20 mm, which were also inputs for the simulation program. Three specimens were prepared for each case to take the average value. The specimens were cured in water for 28 days after casting to reach fully saturation for freezing-thawing test (water occupied the capillary pores and big gel pores). FTC environment was applied to certain specimens following the ASTM C666/C666M (2015) standard, in which the specimens were submerged in water during temperature variations. Therefore, the water supply was also sufficient during the frost damage process, so water could fully occupy the cracked space as well. Each temperature cycle took around 6 hours with temperature varying between 20 and -25°C. After FTC conditioning, all the specimens were stored in water to keep the fully saturation state before moving to environmental chambers with room temperature (submerged in water for “P” specimens) and freezing temperature (located in freezing air for “N” specimens). Since the static loading apparatus located under room temperature, temperature rise happened for “N” specimens (which supposed to be loaded under certain freezing temperature). As a result, the storing temperature for “N” specimens was slightly lower (-26.5°C) than the targeted loading temperature (-25°C).

With simple calculation of thermal transfer following the governing Eq. (7), the temperature rise would be around 2°C within 5 minutes’ static loading period, which could satisfy the requirement of loading under -25°C. In Eq. (7), \( Q_i \) is the energy exchanged at \( i^{th} \) concrete-air surface in unit time, \( h \) is the surface heat transfer coefficient (5 J/(s · m² · K) for air), \( \Delta T_{c-a} \) is the temperature difference between concrete and ambient air (around 45 K at beginning), \( A_i \) is the area of surface of concrete, \( t \) is the time, \( C \) is the heat capacity (around 1000 J/(kg · K) for concrete) and \( m \) is the mass of concrete specimen.

\[
\frac{\partial T}{\partial t} = \frac{\sum Q_i}{C \cdot m}, \quad Q_i = h \cdot \Delta T_{c-a} \cdot A_i \quad (7)
\]

The apparatus for static loading was so far good enough since the loading time was short. However, special apparatus was needed for the freezing-fatigue case to keep constant temperature, since the cyclic loading took much longer time. Some experimental data was available where the loading frame was set inside the temperature chamber (Koda et al. 2015, 2018). In those studies, intact concrete cylinders were successfully tested with compressive fatigue loading under freezing temperatures, which could be the verification of simulation approach.
4. Results and discussions

4.1 Static loading

The experiment and simulation results of static compressive behaviors were plotted in Fig. 7. Figures 7(a) and 7(b) showed the stress-relative strain curves by experiment (Exp) and simulation (Sim) where all the three specimens were shown. Figures 7(c) and 7(d) showed the compressive strengths in terms of absolute and normalized values, respectively. The relative strengths were calculated in terms of the averaged strength of specimen C-P-0 ($f_{c-n}/f_{c-aveCP0}$, $n$ is standing for any specific specimen). Relative strain was calculated based on the averaged peak strain of intact concrete under thawing temperature (C-P-0). And the normalized strength was also calculated in terms of the averaged intact strength under room temperature (C-P-0). Satisfactory agreement could be found from the comparison in Fig. 7. It yielded that due to the damaging effect, the damaged concrete always had lower strength than the intact concrete, no matter under thawing temperature or freezing temperature. However, due to the ice-strengthening effect, the compressive strength was several times higher under freezing temperature for both intact and damaged concrete. In Fig. 7(d), the previous simulation results were also plotted where only the effect of ice-filled pores was taken into consideration (no effect of ice-filled meso-cracks). It could be found that the current model with integrating the ice-filled meso-cracks was closer to the experimental data. Besides, a previous study by Gong et al. (2018) reported that the even under freezing temperature, the strength of frost damaged concrete could not go beyond the strength of intact concrete under room temperature, in case that only the ice-filled pores effect was integrated. However, current study indicated that the freezing strength of damaged concrete could be higher than the strength of intact concrete in thawing temperature by both simulation and experiment. Besides the good correlation, it found that the strength of intact concrete (C-N-0) under freezing temperature was underestimated in the simulation compared with experiment, see Fig. 7(b). This is attribute to that the effect of ice-strengthened pores in Eq. (3) was determined based on the pore structures in room temperature. However, pores would become bigger with damage initiation under freezing, which caused larger ice volume fraction at the given freezing temperature. In other words, the impact of ice-strengthened pores was underestimated in the simulation. This phenomenon became less predominant with accumulation of damage (C-N-75), since more pores became meso-cracks and the impact of ice-strengthened meso-cracks gradually became predominant. The simulation results were still believed to be acceptable since the difference for C-N-0 was around 10%. The post-peak deformation and crack failure pat-
terns by both experiment and simulation were also illustrated in Fig. 8 (Wang et al. 2021). The red short lines meant the corresponding normal spring had reached its maximum crack width (0.03 mm) and became ineffective. Under room temperature, compared with the typical X-shape failure of the intact concrete, damaged concrete showed more distributed cracks, which were induced by the frost action and thus the post-peak behavior was more ductile. It should be noted that experimental results indicated that the concrete (both intact and damaged) was rather brittle at post-peak stage under the freezing temperature. This was not well simulated by the current multi-scale approach, which may attribute to the softening model of normal springs of ice in meso-cracks, such as the maximum crack width of ice in Fig. 3. This would be improved in the future work, but the simulation results were still acceptable since the main interest was strength at the current stage.

The results of splitting tension were plotted in Figs. 9 and 10, with respected to the tensile strength, post-peak deformation and failure pattern. The strain of splitting tension test was defined as the loading deformation over the height of specimen (100 mm). Same as the compression test, the normalized strain was adopted for splitting tension, which was calculated based on the averaged peak strain of intact specimen under thawing temperature (T-P-0). Similarly to the compression strength, the splitting tensile strength of frost damaged concrete was always lower than the intact concrete, no matter under thawing or freezing temperature. In addition to the good correlation between experiment and

Fig. 8 Post-peak deformation and failure pattern – compression (Wang et al. 2021).

Fig. 9 Tensile behaviors (a)stress-strain under thawing temperature (b)stress-strain under freezing temperature (c)absolute strength (d)relative strength.
As the weakest part in concrete, meso-cracks initiated in the ITZ first, which was well simulated as shown in Fig. 10 (Wang et al. 2021). Although a major crack in the middle of specimen was observed for all cases in the experiment, there was still difference in the post-peak behaviors: intact concrete was still connected when the maximum load was reached while the damaged concrete immediately broke into two pieces when the major crack initiated.

4.2 Fatigue loading
For the intact concrete, the stress-strain relationships and strain-step curves by simulation were plotted in Figs. 11, 12 and 13 for stress level 0.9, 0.85 and 0.8, respectively. It should be noted that the initial strain of N-series caused by freezing process was manually deleted for better comparison with P-series whose initial strain was zero. The curves of one representative model
were plotted among the three models with different aggregate arrangement. Besides, the comparison of S-N curve between experiment and simulation in terms of intact concrete suffering fatigue load under room and freezing temperatures was plotted in Fig. 14. The fatigue life was shown in logarithmic scale. It was indicated that the fatigue life of intact concrete was shorter under freezing temperature than that under room temperature by both experiment and simulation. This was different from the phenomenon of static loading, in which the freezing strength was higher than the thawing strength. As explained at the end of section 2.2, the fatigue constitutive models were not including the ice-strengthening effect in meso-cracks to account for the ice-melting effect during fatigue loading, as Eqs. (4) and (5). However, the applied fatigue load with same stress level was larger under freezing temperature since the static strength under freezing temperature was higher. Thus, the fatigue life was shorter for concrete under freezing temperature. In other words, the fatigue deterioration impacts were larger for the case of freezing temperature.

Beside the same tendency shown by experiment and simulation, there was slight disagreement for the thawing case (P), where experiment showed less fatigue life than the simulation at same stress level. This was because the specimens P-0.7 in the test was not exactly dry but with certain water saturation degree (N-0.7 was fully saturated). It was well known that the water motion in porous medium would lead to additional pore pressure under fatigue loading, which could accelerate the fatigue failure (Biot 1963; Maekawa et al. 2013). But such water motion effect was not considered in the current multi-scale simulation approach. Besides, the material inputs in the simulation were calculated according to the compressive strength as explained in Chapter 3. Such water motion effect would be further studied and integrated with the multi-scale application.

In addition, the fatigue simulation with the frost damaged concrete was also conducted. 100 FTC (20°C ~ -20°C) was applied for the concrete and then the fatigue loading was applied under both thawing (20°C) and freezing temperatures (-20°C). The simulated stress ratio-fatigue life results were plotted together with that of sound concrete, as shown in Fig. 15. P and N meant the intact concrete while P-100 and N-100 stood for the frost damaged concrete which suffered 100 FTC. Despite some scattering of each data for the same case, the linear regressed S-N curve clearly showed that the fatigue life of frost damaged concrete was shorter than that of intact concrete, which was applicable under both thawing and freezing temperatures. The fatigue deterioration under room temperature due to frost damage had been successfully simulated and validated by previous research (Gong et al. 2017a; Hasan et al. 2008). In this study, the simulation results provided further information that the ice-strengthening effects might only contribute to the static strength but not to the fatigue strength. Oppositely, the freezing temperature may cause earlier fatigue failure for both intact and damaged concrete with the same stress ratio. Furthermore, from the current simulation, there seemed to be more negative impacts on the fatigue life of concrete by the freezing temperature than by frost damage, if comparing the results of N and P-100 in Fig. 15. But unfortunately, no available test data could be found pertaining to the fatigue test on frost-damaged concrete under freezing temperature. Therefore, it is meaningful to conduct such test in the future for validation and modification of the current numerical approach.

5. Conclusions

In this paper, the damaging and strengthening effects by ice formation were modeled and integrated with developing a multi-scale numerical application for concrete material. Several conclusions could be draw:

1. The ice damaging and strengthening effects at different scale were clarified and implemented into the Rigid Body Spring Model. Especially, the ice-strengthening effect in meso-cracks were verified for the static loading case by both experiment and simulation. Since ice inside meso-cracks could offer extra load-carrying capacity, the compressive and tensile static strength of concrete under freezing
temperature were several times higher than the strength under thawing temperature.

2. Different from the static loading case, longer fatigue life with same stress ratio was observed for the specimen under thawing temperature than that under freezing temperature. This phenomenon was well explained and modeled in the current multi-scale approach. Therefore, sufficient attention should be paid when evaluating the fatigue behavior of concrete under freezing temperature.

3. This work assumed the simple case of fully saturation degree while further studies with different saturation degrees would be conducted, since the ice-strengthening effect was reported to be relevant to the moisture contents in the porous medium. The water motion effect may lead to some uncorrelations, which needs to be integrated into the program. Furthermore, experiments of freezing fatigue test on the damaged concrete with meso-cracks are needed in the future so that to validate and modify the current numerical approach.

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