Multi-Carrier Differential Trellis-Coded Modulation/Demodulation Employing Multiple Differential Detection with Channel Prediction

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Abstract—This paper proposes a multi-carrier (MC) modulation scheme employing differential trellis-coded modulation (DTCM) and multiple differential detection with channel prediction in order to cope with doubly-selective fading at the good required signal-to-noise power ratio (SNR), where doubly-selective channels correspond to severe time/frequency-selective channels. For frequency-selective fading, MC modulation schemes are effective. For time-selective fading, per-survivor processing (PSP-MDD) with channel prediction is effective. However, channel prediction degrades the required SNR for PSP-MDD. In order to cope with doubly-selective fading and improve the required SNR, this paper proposes MC-DTCM employing PSP-MDD with channel prediction. Finally, computer simulation results show that the proposed scheme can improve both the performance for doubly-selective fading and the required SNR compared with the conventional PSP-MDD with channel prediction.

I. INTRODUCTION

For mobile radio communications, doubly-selective fading is a serious problem, where doubly-selective fading corresponds to severe time/frequency-selective fading. Large-cell wireless high-speed train radio communication systems suffer from severe doubly-selective fading [1]. Train radio communications systems for point-to-multipoint (P-MP) communications with the same frequency band simultaneously receive the signals from the adjacent base stations, resulting in an artificial delay spread. Thus, this communication system suffers from a severe doubly-selective propagation environment due to both larger delay spread and higher mobile speed.

To cope with doubly-selective fading, it is necessary to cope with both time-selective fading and frequency-selective fading. For frequency-selective fading, multi-carrier (MC) modulation schemes, e.g., orthogonal frequency division multiplexing (OFDM), are good solutions. However, MC modulation schemes cause severe time-selective fading due to their larger symbol duration. For time-selective fading, the following two schemes are good solutions:

- coherent detection (CD) with pilot-symbols;
- joint detection (JD) without pilot-symbol.

CD detects data based on the estimated channel impulse responses (CIRs) by means of pilot-symbols. In this scheme, the transmission frequency efficiency degrades due to an increase in the frequency of pilot-symbols on fast time-varying channels. JD, e.g., multiple differential detection employing per-survivor processing (PSP-MDD) [2], [3], can detect data without pilot-symbol. Therefore, PSP-MDD is effective for time-selective fading because the transmission frequency efficiency does not degrade on fast time-varying channels. However, PSP-MDD with channel prediction degrades the required signal-to-noise power ratio (SNR).

This paper proposes MC differential trellis-coded modulation (DTCM) employing PSP-MDD with channel prediction in order to improve both the performance for doubly-selective fading and the required SNR. This paper employs multiple single carrier (MSC) modulation scheme [4] for MC modulation scheme. Finally, computer simulation results show that the proposed scheme can improve both the performance for doubly-selective fading and the required SNR compared with the conventional PSP-MDD with channel prediction.

II. COMMUNICATION SYSTEM MODEL

![Diagram](image)

Fig. 1 shows a block diagram of the single-input multiple-output (SIMO) communication system under consideration with \( N_R \) receive antennas. \( a_k \) denotes sequence \( a \) at symbol time \( k \), and \( a_k[q] \) denotes sequence \( a \) at symbol time \( k \) and the \( q \)th \( (q = 1, 2, \ldots, N_R) \) receive antenna. Each
encoder/modulator generates the transmitted modulation sequence \( x_k \) according to the information sequence \( b_k \) \((b_k \in \{0, 1, \ldots, 2^m - 1\})\), i.e., \( m \) bit/symbol). The transmitted modulation sequence \( x_k \) is corrupted by additive white Gaussian noise (AWGN) \( w_k[q] \), resulting in the following received sequence \( r_k[q] \):

\[
r_k[q] = h_k[q]x_k + w_k[q],
\]

where \( h_k[q] \) denotes CIRs on the SIMO channel. The decoder and demodulator estimate the information sequence \( \hat{b}_k \) according to the received sequence \( r_k[q] \).

### A. Proposed DTC8PSK

Fig. 2 shows a block diagram of the proposed DTC8PSK 

![Block diagram of the proposed DTC8PSK](#)

Fig. 2 shows a block diagram of the proposed DTC8PSK \((m = 2)\). Ungerboeck encoder \([5]–[8]\) (coding rate \( R = 2/3 \)) generates \( b_0^k, b_1^k, \) and \( b_2^k \) according to the information sequence \( b_k \). \( b_0^k, b_1^k, \) and \( b_2^k \) are converted to the 8PSK modulated sequence \( u_k \) as follows:

\[
u_k = \exp \left( j \frac{2\pi}{8} b_k \right),
\]

A differential encoder generates the transmitted modulation sequence \( x_k \) as follows:

\[
x_k = u_k x_{k-1}.
\]

This paper discusses that DTC8PSK has good performance on fast time-varying channels. DTC8PSK can correct burst errors within the constraint length \( K \) thanks to the convolutional encoder. On fast time-varying channels, the signal-fade occurs frequently, and the fade-duration gets shorter. DTC8PSK can correct errors due to the short signal-fade of fast time-varying fading when the fade-duration is within the constraint length \( K \). Therefore, DTC8PSK has good performance regarding fast time-varying fading with shorter fade-duration. In addition, DTC8PSK can improve the performance of fast time-varying fading by increasing the constraint length \( K \). Ungerboeck proposed an encoder with the constraint length \( K = 3 \) in \([5]\), while encoders with the constraint length \( K = 4 \) or more seem to be unclarified. Thus, this paper proposed an encoder with the constraint length \( K = 5 \) in Fig. 3. This encoder maximizes the minimum free distance.

### III. Multiple Differential Detection with Channel Prediction

This section discusses PSP-MDD based on the Viterbi algorithm (VA) for a demodulator. Let us define the state \( S_k \) and the branch \( S_k/S_{k-1} \) at \( k \) as follows:

\[
S_k = \hat{b}_k \hat{b}_{k-1} \ldots \hat{b}_{k-v+1},
\]

\[
S_k/S_{k-1} = b_k \hat{b}_{k-1} \ldots \hat{b}_{k-v},
\]

where \( \hat{b}_k \) denotes a candidate of the information sequence, and \( V \) denotes the memory length of the VA. The branch metrics \( \Gamma_k \) and path metrics \( H_k \) are as follows:

\[
\Gamma_k[S_k/S_{k-1}/S_{k-v}^S] = \sum_{q=1}^{N_R} r_k[q] - \sum_{n=1}^{N} v_n r_{k-n}[q] \prod_{i=0}^{n-1} \hat{u}_{k-i} \right|^2,
\]

\[
H_k[S_k/S_{k-1}] = H_k[S_{k-1}/S_{k-2}] + \Gamma_k[S_k/S_{k-1}/S_{k-2}^S],
\]

\[
H_k[S_k/S_{k-1}^S] = \min_{\{S_{k-1} \rightarrow S_k\}} H_k[S_k/S_{k-1}].
\]

\( S_k/S_{k-1}^S \) and \( S_k/S_{k-1}/S_{k-2}^S \) are the surviving paths connected to the state \( S_k \) and the branch \( S_{k-1}/S_{k-2} \), respectively. \( \{S_{k-1} \rightarrow S_k\} \) denotes all possible candidates from the state \( S_{k-1} \) to the state \( S_k \). \( \hat{u}_k \) denotes the candidates of the information sequence for \( \hat{b}_k \). \( v_n \) denotes the weighting coefficients for the MDD, and \( N \) denotes the number of observation symbols. Ref. \([9]\) proposes the method to estimate the CIRs by averaging the previous symbols. This method can suppress fluctuation due to noise, while it suffers from tracking delay on fast time-varying channels. Ref. \([10]\) proposes the method to improve tracking capability with channel prediction. This method employs the weighting coefficients, \( v_n \), as follows:

\[
v_n = (-1)^n \left( \begin{array}{c} N \\ n \end{array} \right).
\]

This paper defines the order of channel prediction as \( L \) \((L = N - 1)\). Channel prediction can improve tracking capability by larger \( L \), while it suffers from the larger fluctuation due to
noise and degrades the required SNR. The memory length $V$ of the proposed VA is represented as follows:

$$V = K + L - 1.$$  \hspace{1cm} (10)

Thus, the computational complexity exponentially grows with $K$ and $L$.

IV. DOUBLY-SELECTIVE FADING AND MSC MODULATION SCHEME

This section discusses the parameters of doubly-selective fading and the MSC modulation scheme. The main parameter of frequency-selective fading is the maximum delay interval normalized by the symbol duration, $\tau_D/T$, and the main parameter of time-selective fading is the maximum Doppler frequency normalized by the symbol rate, $f_D T$, where $T$ denotes the symbol duration. $\tau_D/T$ and $f_D T$ are as follows:

$$\tau_D/T = \frac{d}{c T}, \hspace{1cm} (11)$$

$$f_D T = \frac{v f_c}{c} T, \hspace{1cm} (12)$$

where $d$ is the maximum difference of path-difference, $c$ is the light speed, $v$ is the mobile speed, and $f_c$ is the carrier frequency. The MC modulation scheme can control the trade-off between frequency selectivity and the time selectivity with respect to $T$. This paper employs the MSC modulation scheme as the MC modulation scheme. Fig. 4 shows the structure of the MSC modulation scheme in the frequency domain.

In Fig. 4, $\alpha$ is the roll-off factor, $\delta$ is the frequency interval of subcarriers, and $N_{SC}$ is the number of subcarriers. The MSC modulation scheme transmits multiple subcarriers with a narrow signal bandwidth. Thus, $T$ is dependent on $N_{SC}$ of the MSC modulation scheme. This paper defines $T_0$ as the symbol duration $T$ in the case that $N_{SC} = 1$.

Figs. 5 and 6 show the single carrier (SC) modulation scheme and 4SC modulation scheme in the frequency and time domains, respectively, where $f_0 (=1/T_0)$ is the symbol rate in the case that $N_{SC} = 1$.

Fig. 6 shows that the degradation due to the maximum delay interval on the 4SC modulation scheme is one-quarter that on the SC modulation scheme. Therefore, the MSC modulation scheme can mitigate the frequency selectivity to $1/N$ thanks to the $N$ subcarriers. On the other hand, the larger $T_0$ increases time selectivity to $N$ times. The above-mentioned relation is as follows:

$$\tau_D/T = \frac{1}{N}\tau_D/T_0, \hspace{1cm} (13)$$

$$f_D T = N f_D T_0. \hspace{1cm} (14)$$

V. COMPUTER SIMULATION

This section compares the bit error rate (BER) and block error rate (BLER) performances of the proposed DTC8PSK and the conventional differential quadrature phase shift keying (DQPSK) by computer simulation. This paper assumes that the transmit/receive filter is 20% root cosine roll-off, $m = 2$, and the demodulators are PSP-MDD.

A. COMPARISON ON STATIC AND TIME-VARYING CHANNELS

This subsection confirms that the proposed DTC8PSK has the better required SNR on static channels without intersymbol interference (ISI) and better tracking capability on independent Rayleigh fading channels without ISI compared with the conventional scheme. Fig. 7 shows BER performance as a function of $E_b/N_0$ on static channels without ISI, where $N_R$ is 1. From Fig. 7, we can obtain the following results:

- the proposed DTC8PSK has the better BER performance at a high SNR compared with the conventional scheme;
- larger $L$ results in the degradation of the required SNR due to the fluctuation of noise caused by channel prediction;
- the proposed DTC8PSK can improve the required SNR because of the larger constraint length $K$;
- the proposed DTC8PSK ($L = 1$, $K = 5$) improves the required SNR by around 3 dB at BER of $10^{-4}$ compared with the conventional scheme ($L = 1$).
BER of $10^{-4}$ compared with the conventional scheme;
- $L = 2$ or more slightly improves tracking capability;
- the proposed DTC8PSK can improve tracking capability by the larger constraint length $K$;
- from the viewpoint of tracking capability, the proposed DTC8PSK ($K = 5$, $L = 1$) can improve $f_DT$ of around 4% compared with the conventional scheme ($L = 1$).

From the results in Figs. 7 and 8, the proposed DTC8PSK ($K = 5$) improves both the required SNR and tracking capability compared with the proposed DTC8PSK ($K = 3$). In the following subsection, computer simulation assumes that $L$ is 1 for both schemes, and $K$ is 5 for the proposed DTC8PSK.

B. Comparison on Doubly-Selective Channels

This section confirms that the proposed MSC-DTC8PSK improves the performance of doubly-selective fading and the required SNR on doubly-selective channels compared with the conventional MSC-DQPSK. The simulation parameters are as follows:
- channels are independent 2-path Rayleigh fading;
- the maximum Doppler frequency normalized by the symbol rate on the SC modulation scheme, $f_D T_0$, is 2.5% ($f_D T$ is 5.0% and 10.0% for the 2SC and 4SC modulation schemes, respectively);
- the desired-to-undesired signal power ratio (DUR) is 6 dB;
- the error-correcting code is the Reed–Solomon (RS) code RS(192, 168, 8);
- $N_R$ is 4.

From the results in Figs. 7 and 8, the proposed DTC8PSK ($K = 5$) improves both the required SNR and tracking capability compared with the conventional DQPSK scheme. In the following subsection, computer simulation assumes that $L$ is 1 for both schemes, and $K$ is 5 for the proposed DTC8PSK.

Fig. 8 shows BER performance as a function of $f_D T$ at average $E_b/N_0$ of 25 dB on independent flat Rayleigh fading channels without ISI ($N_R = 4$).

Fig. 8 shows BER performance as a function of $f_D T$ at average $E_b/N_0$ of 25 dB on independent flat Rayleigh fading channels, where $N_R$ is 4. From Fig. 8, we can obtain the following results:
- the proposed DTC8PSK has better tracking capability at $f_D T = 10^{-3}$ compared with the conventional scheme;
Fig. 9 shows BLER performance as a function of $\tau_D/T_0$ at average $E_s/N_0$ of 25 dB. From Fig. 9, we can obtain the following results:

- for a larger number of subcarriers, both schemes can mitigate degradation due to the delay spread;
- the proposed 4SC-DTC8PSK can achieve BLER of $10^{-6}$ up to $\tau_D/T_0$ of 19/16;
- for $\tau_D/T_0$ of 1/4, the proposed 4SC-DTC8PSK improves the performance at BLER of $10^{-6}$ compared with the conventional scheme.

Fig. 10 shows BLER performance as a function of average $E_s/N_0$ at $\tau_D/T_0$ of 3/4 and $f_D T_0$ of 2.5%. From Fig. 10, we can obtain the following results:

- both the SC modulation scheme and the 2SC modulation scheme cannot achieve BLER of $10^{-6}$ at a relatively high average $E_s/N_0$, while the 4SC modulation scheme can achieve BLER of $10^{-6}$ at a relatively high average $E_s/N_0$;
- from the pointview of the required SNR, the proposed 4SC-DTC8PSK improves performance gain by around 3 dB compared with the conventional 4SC-DQPSK.

Thus, the results in Figs. 9 and 10 show that the proposed MSC-DTC8PSK can improve both the performance for doubly-selective fading and the required SNR compared with the conventional scheme.

VI. CONCLUSION

This paper has proposed MSC-DTCM employing PSP-MDD with channel prediction in order to improve both the performance of doubly-selective fading and the required SNR. The proposed DTCM combines TCM with differential encoding and can improve both the required SNR and the performance on fast time-varying channels. Finally, computer simulation results have confirmed that the proposed MSC-DTC8PSK can improve both the performance of doubly-selective fading and the required SNR compared with the conventional MSC-DQPSK. In particular, assuming $f_D T_0$ of 2.5%, computer simulation results have confirmed that the proposed 4SC-DTC8PSK can achieve BLER of $10^{-6}$ up to $\tau_D/T_0$ of 19/16. In addition, assuming $\tau_D/T_0$ of 3/4 and $f_D T_0$ of 2.5%, computer simulation results have confirmed that the proposed 4SC-DTC8PSK can improve in the required SNR by around 3 dB at BLER of $10^{-6}$ compared with the conventional 4SC-DQPSK.

This paper has not investigated the following two items in detail:

- the suppression of peak to average power (PAPR) by the MSC modulation scheme;
- the comparison between the MSC modulation scheme and OFDM.

These topics are also under investigation by the authors.

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