Autonomous wheeled mobile robot maneuvering in constraint environment. Trajectory tracking quality criteria

D V Shabanov, A V Kozlovich, R R Valiev and O V Kochneva

Peter the Great St. Petersburg Polytechnic University, 29, Polytekhnicheskaya st., St. Petersburg, 195251, Russia

E-mail: dsb956@yandex.ru

Abstract. In this article the problem of control system synthesis for differential drive wheeled mobile robots operating within constraint environment is considered. The proposed approaches of solving trajectory tracking problem take into account the geometric features of both the robot and the transported cargo. The collision avoidance problem is solved at the controller level, without any changes to the planned trajectory. Three approaches are proposed: minimizing the distance to the trajectory, minimizing the distance to the reference position, and maximizing the distance to obstacles. In these approaches the constant analysis of position of each point of robot’s body outline contour is used. Negative influence of the position of these points on qualitative trajectory pass is represented in vector form. For each of the approaches formalized tracking quality criteria are determined, methods of implementation of control systems and controllers taking into account these criteria are proposed. The part of the control signal responsible for taking into account geometry of the robot and avoiding collisions is defined via the force-moment vector representation. Results of simulation modeling prove the efficiency of the proposed approaches. A comparison of the proposed approaches is presented. Potential ways of more efficient use of these approaches are given.

1. Introduction

It is hard to imagine automation in any industry spheres without transport systems. It can be a factory transport system based on automated guided vehicles [1] or transport part of a complex device autonomously fulfilling some technological operations [2]. Automation in dangerous places, such as mines and tunnels is especially relevant [3]. Places like this have a special feature – constraint environment, which requires an individual approach to solving maneuvering problem.

Also, it is necessary to take into account that the cargo carried by transport robots may have different dimensions and it can go beyond the robot’s dimensions. The same factory robot can carry both a long 6-meter profile and a wide metal sheet.

Most studies of obstacle avoidance by wheeled mobile robots (WMR) focus on changing the planned trajectory (for example, [4, 5, 6, 7]). These techniques cannot be used if the only path is a narrow corridor due to disadvantages in the trajectory tracking algorithms. Also, they don’t take into account the robot’s geometric features and simplify it to the simplest geometric bodies – circles and points, or use one dimension of the safe distance from the robot’s center point.

This article presents approaches to solving the trajectory tracking problem taking into account geometric features of the robot and the cargo. The effectiveness and features of the proposed approaches are shown.
2. Disadvantage of the classical WMR control system

Let's illustrate the existing disadvantage of the classical WMR control system. Consider a special case: there is a rectangular differential wheeled robot and a narrow corridor on its way. Classical trajectory tracking algorithms use deviation from the reference position as arguments for the feedback regulator function [8, 9] and as a trajectory tracking quality criterion. It can be represented by two components:

- normal distance from the robot's "center" point to the trajectory ($e_n$);
- difference between the robot's angular position and the angle of trajectory tangent ($\Delta \phi$).

The coefficients that prioritize these criteria are set only once.

The robot passes through the narrow corridor with some initial deviation from the trajectory (Figure 1a). If the robot tries to get rid of this deviation (Figure 1b), it will begin to turn and crash into the wall. So, reducing the angular error has a higher priority.

![Figure 1. Robot motion in the narrow corridor: a – current position; b – attempt to minimize the deviation](image)

![Figure 2. Robot motion at the entrance of the narrow corridor: a – current position; b – attempt to maintain angular orientation; c – attempt to minimize the deviation](image)
However, at the entrance to the narrow passage, it is more important to reduce the distance to the trajectory than to save the correct angular position (Figure 2). It shows the need to change the coefficients depending on the robot’s position relative to obstacles.

It is worth paying attention to the fact that if the robot’s geometry changes, then the priority of the controller arguments also changes. For example, if the robot had a shorter length, the collision would not have occurred in case shown in Figure 1b. Therefore, it is necessary to take into account changes in the robot’s geometry at trajectory tracking.

3. Object of study – WMR
The object of research in this study is a differential drive wheeled mobile robot. This type of kinematics is one of the most popular due to its high maneuverability, low cost and high speed of movement.

In developing the control system, a “unicycle” mathematical model is used because of its simplicity and usability. The WMR’s position in a plane is described by the vector $\mathbf{q}(t) = [x(t), y(t), \phi(t)]^T$ (Figure 3). During simulations, we must take into account some things that make the robot deviate from the trajectory: feedback sensor accuracy and delay, inertial forces and wheel slip. Therefore, simulations use a “Differential Drive Model” (Figure 4) [1].

4. Approaches to solving the problem; trajectory tracking quality criteria
To design the trajectory tracking controller based on the unicycle model, we use the control system described in the first part of [10]. The control signal $\mathbf{u}$ consists of the robot’s linear and angular velocities. Also, $\mathbf{u}$ is defined by two components: control signal sufficient to follow the trajectory in an ideal situation (feedforward) $\mathbf{u}_{ff}$ and control signal that seeks to reduce the errors (feedback) $\mathbf{u}_{fb}$. The last one is implemented on the basis of minimizing the robot’s deviation from its reference position, which is described by $\mathbf{q}_r = [x_r, y_r, \phi_r]^T$. This deviation is described by the matrix $\mathbf{e}$.

$$
\mathbf{u} = \begin{bmatrix} v \\ \omega \end{bmatrix} = \mathbf{u}_{ff} + \mathbf{u}_{fb} = \begin{bmatrix} v_{ff} \cos e_3 \\ \omega_{ff} \end{bmatrix} + \begin{bmatrix} \omega_{fb} \end{bmatrix}
$$
(1)

$$
\mathbf{u}_{fb} = \begin{bmatrix} k_1 & 0 & 0 \\ 0 & k_2 & k_3 \end{bmatrix} \cdot \mathbf{e}
$$
(2)

$$
\mathbf{e} = \begin{bmatrix} e_1 \\ e_2 \\ e_3 \end{bmatrix} = \begin{bmatrix} \cos \phi & \sin \phi & 0 \\ -\sin \phi & \cos \phi & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} x - x_r \\ y - y_r \\ \phi - \phi_r \end{bmatrix}
$$
(3)

To solve the problem, it is necessary to add one more summand to the control signal $\mathbf{u}$, which makes adjustments to avoid collisions: $\mathbf{u}_{oa}$. We seek to provide the necessary cruising speed $v = v_{ff}$ and spend minimum time to complete the trajectory. Therefore, only the angular velocity $\omega$ is used to adjust the robot’s position. At the same time, due to the fact that collisions are more critical than the robot delay, the equations (1-3) are amended. The control signal $\omega(t)$ will lead the robot not to the
position where the robot has to be at the moment $t$, but to the nearest position on the trajectory (Figure 5). Ideologically, this is similar to solving the path following problem [11, 12, 13]. Now, some of the previously described functions will become functions of the completed part of the trajectory in the time equivalent $\tau(t)$ (4, 5). This "completed part of the trajectory" is equal to the value of the time when the robot should have been at the nearest trajectory point. Following the above, we have:

$$u(t) = \begin{bmatrix} v_{\|}(\tau(t)) \\ \omega_{\|}(\tau(t)) \end{bmatrix} + \begin{bmatrix} 0 \\ \omega_{0\theta}(e(t)) \end{bmatrix} + \begin{bmatrix} 0 \\ \omega_{0\phi}(t) \end{bmatrix}$$  \quad (4)$$

$$e(t) = \begin{bmatrix} \cos \phi(t) & \sin \phi(t) & 0 \\ -\sin \phi(t) & \cos \phi(t) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_{\|}(\tau(t)) - x(t) \\ y_{\|}(\tau(t)) - y(t) \\ \phi_{\|}(\tau(t)) - \phi(t) \end{bmatrix}$$  \quad (5)$$

**Figure 5.** Current reference position  
**Figure 6.** Equivalent force and torque

The function $\omega_{oa}(t)$ is defined via the force-moment representation. All virtual forces that affect the robot contour points are represented by the equivalent force-moment system. In general, the control signal $\omega_{oa}$ is defined as:

$$\omega_{oa} = k_{Mp}M_p + k_{Fp}F_p,$$  \quad (6)$$

where $k_{Mp}$ and $k_{Fp}$ – controller coefficients, $M_p$ – value of equivalent torque; $F_p$ - value of equivalent force’s $(F_p)$ projection on the robot wheels axis (Figure 6).

This is similar to the obstacle avoidance method described in [10, 11], but we also have the equivalent torque, because the WMR has not been simplified to a physical point. Determining the $M_p$ and $F_p$ values depending on the type of approach are described below.

PD controller is not used in the equation 6 due to the fact that feedback sensor has a low data refresh rate.

Another method that applies to all proposed approaches is a predictive method. It is often used in control systems, including control systems for wheeled robots [7, 14, 15]. In this case the coefficient deviation simulation is used. The robot continuously calculates the simulations of its further movement with a different set of coefficients. The best set of selected coefficients is based on the value of the cost function, which should include one of the proposed quality criteria $Q_{oa}$. The main requirement for the function $Q_{oa}(t)$ is the maximum possible continuity, because this is necessary for using local minimum search algorithms.

For more unified mathematical descriptions, the term "Point’s deviation vector” (PDV) is introduced. “Point’s deviation vector” is a vector located in the plane of robot’s motion that characterizes the negative effect of the current position of the robot’s geometric point on the quality and safety of passing the current part of the trajectory. For ease of perception, PDVs are drawn starting from the current point itself.
Only points located on the outer contour of the robot can collide, so only their PDVs are taken into account. Similarly, only the outer contour of the obstacles is considered.

Approaches to solving the problem are classified by PDV endpoints location.

4.1. Minimization deviations from the trajectory line (MDTL)
The concept is that the WMR is trying to take a position as close as possible to the path line. That is the center of the robot’s geometric shape that moves along the trajectory instead of the selected “central” point. The location of obstacles is not considered in any way. The end point of the PDV is located at the nearest trajectory point (Figure 7). This approach focuses on accounting for changes in the robot’s geometry and its cargo and requires responsible design of trajectories.

The length of the maximum PDV is used as the tracking quality criterion $Q_{oa}$:

$$Q_{MDTL} = e_{p \text{max}}.$$  \hspace{1cm} (7)

All vectors are forces that defined the control signal to compensate for the deviation. All these forces are replaced by system of equivalent force $F_p$ and the equivalent torque $M_p$. The longest ones of the vectors $e_{pi}$ should be more important for the control system than other. Therefore, the power function is used:

$$F_p = \sum_{i=0}^{n} e_{pi} \cdot e_{pi}^m, \quad M_p = \sum_{i=0}^{n} (e_{pi} \cdot e_{pi}^m) \cdot r_{i},$$  \hspace{1cm} (8)

where $r_{i}$ – lever arm vector of the force $e_{pi}$.

![Figure 7. PDVs in the MDTL approach](image1)

![Figure 8. PDVs in the MDRP approach](image2)

Figure 7. PDVs in the MDTL approach

Figure 8. PDVs in the MDRP approach

The control signal $\omega_{oa}$ is completely determined by the equations (6) and (8). This approach takes into account the deviation from the trajectory. Therefore, the signal $\omega_{oa}$ is an alternative to the $\omega_{fb}$, not its complement. In other words, the value of $\omega_{fb}$ must be equal to zero.

4.2. Minimization deviations from the reference robot position (MDRP)

This approach is similar to following the robot's reference position. The difference is that the deviation of each geometric point of the robot contour is taken into account, not just the deviation of the “center” point and the angular error. It allows to create the corrective signal in real time, taking into account the current geometry of the robot with the cargo. In this case, the end point of the PDV is located directly at the corresponding geometric point of the reference virtual robot (Figure 8).

Equations (7) and (8) are also relevant for this approach.
4.3. Maximization the distance to obstacles (MDO)

The WMR “stays away” from all obstacles. The end point of the PDV is located on the points of the obstacle contour. Due to the fact that the distance is measured not from the “center” point, changes in the geometry of the robot with the cargo are taken into account. This approach also has the intentional deviation of the robot from the trajectory, even if this is not necessary for the successful passage of trajectory.

For long term planning of obstacle avoidance it is necessary to have PDVs $e_{pj}$ between each point of the robot contour and each point of the obstacle contour. In order to reduce the count of computing operations, only obstacle points that are closer than a fixed distance $R$ from the robot are considered. Further, the set of PDVs $e_{pj}$ are replaced by one vector $e_{pi}$ for each point (Figures 9, 10). To increase the priority of the most problematic vectors, the power function is used:

$$e_{pi} = \frac{1}{k} \sum_{j=0}^{k} e_{pj} - e_{pj}$$

In this approach, the quality criterion is inversely related to the length of vector $e_{pi}$, because reducing the length of the vector means that the robot is approaching an obstacle. Therefore, in the equation (9) the power of vector’s length has a negative sign. Also, for easier use in the force-moment representation, the vector $e_{pi}$ has the direction opposite to $e_{pj}$. The length of the maximum vector $e_{pi}$ is used as the tracking quality criterion: $Q_{MDO} = e_{pi}$ max. The equivalent force and equivalent torque are calculated as follows:

$$F_p = \sum_{i=0}^{n} e_{pi}$$
$$M_p = \sum_{i=0}^{n} e_{pi} \cdot r$$

Figure 9. Replacing vectors $e_{pi}$ with one $e_{pi}$

Figure 10. Vectors $e_{pi}$ in the MDRP approach

5. Simulation results

Simulation was used to evaluate the performance of the proposed methods. The following problem was solved: a rectangular robot follows a trajectory, which passes through a narrow corridor (Figure 11). Corridor width – 1.72 meters; width of the robot is 6 meters. During the simulation, feedback sensor inaccuracy and delay, inertial forces, wheel slip and motor torque limitations are taken into account.
Figure 11. Initial condition

Four controllers are compared. Controllers’ coefficients are calculated by optimization according to the values of the corresponding criteria $Q_{oa}$ along the entire path length. For the classical controller that does not contain $\omega_{oa}$, the normal deviation of the robot from the trajectory ($e_n$) is used as a tracking quality criterion. The simulation results are shown in the figures 12 – 15. These graphs show the values of tracking quality criteria as the robot moves along the trajectory. The graphs of each controllers are hardly to distinguish at this scale. Therefore, only the graphs of the criteria corresponding to their type of controller are shown. More detailed results are shown in the table 1. The first part of the table shows the maximum values of the criteria at curved trajectory section; second part of the table - at the trajectory section with obstacle (starting with getting the robot's front point in the gap between the obstacles).
Table 1. Simulation results

| Controller type | Curved section | Section with obstacle |
|-----------------|----------------|-----------------------|
|                 | $e_n$ (10^{-3} m) | $\Delta \phi$ (10^{-3} rad) | $Q_{\text{MDTL}}$ | $Q_{\text{MDRP}}$ | $Q_{\text{MDO}}$ | $d_{\text{min}}$ (10^{-3} m) |
| $\omega_{oa} = 0$ | 60 | 34 | 1.124 | 0.102 | 45 | 30 | 0.878 | 0.081 | 3.129 | collision |
| MDTL            | 62 | 27 | 1.033 | 0.072 | 50 | 90 | 0.866 | 0.066 | 1.139 | 11 |
| MDRP            | 98 | 34 | 1.205 | 0.143 | 26 | 90 | 0.857 | 0.059 | 0.595 | 20 |
| MDO             | 173 | 71 | 1.289 | 0.242 | 24 | 12 | 0.848 | 0.05 | 0.475 | 45 |

$^a$ Min distance to the obstacle

Also, simulations were performed with the offset of the robot’s external contour for the MDTL and MDO controllers. The MDTL controller is able to easily adapt to changes in robot geometry. The MDO controller performs a collision-free trajectory only if the robot can enter the corridor with its front part, because up to that point $\omega_{oa} = 0$.

The simulation results prove the efficiency of the proposed approaches for solving the problem, but do not show their full potential and disadvantages.

6. Conclusion

The proposed maneuvering approaches in constraint environment can be effective, but their features (Table 2) must be taken into account. Some disadvantages of the proposed approaches can be avoided by using complex trajectory planning algorithms.

Table 2. Comparison of approaches

| Advantages | MDTL | MDRP | MDO |
|------------|------|------|-----|
| Wide range of geometric configurations | + | - | + |
| Significant deviation from the trajectory is excluded | - | + | - |
| The position of obstacles may change after planning the trajectory | - | + | - |
| Need small computing power of the robot | + | + | - |
| No need for detailed trajectory design | + | + | - |

It is useful to note that this article did not consider the stability of the system in a situation where the width of the robot exceeds its length. Also, there are approaches that may be more effective than the proposed approaches in their original form:

- using PD-controller (with a good feedback sensor interpolator);
- using the predictive method based on the proposed quality criteria;
- using classic controllers configured according to the proposed quality criteria;
- using synthesis of the several proposed approaches (for example, MDTL and MDO).

The proposed approaches can improve the efficiency of using WMR in the automation of technological processes. Taking into account the geometry of the robot and the cargo reduces the probability of collisions and increases the average speed of movement. Using these approaches expands the range of tasks that can be solved using mobile robots.

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