Resource Allocation for Multi-Cell IRS-Aided NOMA Networks

Wanli Ni, Xiao Liu, Yuanwei Liu, Hui Tian and Yue Chen

Abstract

This paper proposes a novel framework of resource allocation in multi-cell intelligent reflecting surface (IRS) aided non-orthogonal multiple access (NOMA) networks, where an IRS is deployed to enhance the wireless service. The problem of joint user association, subchannel assignment, power allocation, phase shifts design, and decoding order determination is formulated for maximizing the achievable sum rate. The challenging mixed-integer non-linear problem is decomposed into an optimization subproblem (P1) with continuous variables and a matching subproblem (P2) with integer variables. In an effort to tackle the non-convex optimization problem (P1), iterative algorithms are proposed for allocating transmission power, designing reflection matrix, and determining decoding order by invoking relaxation methods such as convex upper bound substitution, successive convex approximation, and semidefinite relaxation. In terms of the combinational problem (P2), swap matching-based algorithms are developed for achieving a two-sided exchange-stable state among users, BSs and subchannels. Numerical results demonstrate that: i) the sum rate of NOMA networks is capable of being enhanced with the aid of the IRS; ii) the proposed algorithms for multi-cell IRS-aided NOMA networks can enjoy 28% and 21% higher energy efficiency than the conventional NOMA and OMA schemes; iii) the trade-off between spectrum and energy efficiency can be tuned by judiciously selecting the location of the IRS.

Index Terms

Intelligent reflecting surface, multi-cell non-orthogonal multiple access, resource allocation, three-dimensional matching.

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I. INTRODUCTION

The ever-increasing deployment of wireless devices have placed unprecedented requirements on spectrum, energy and cost efficiency for the forthcoming 5G/beyond networks. By modifying the amplitude and phase of reflective signals, the software-controlled intelligent reflecting surfaces (IRSs) can reconfigure the wireless channels between transmitters and receivers [2]. This remarkable feature of IRSs can be utilized to enhance the performance of wireless communication networks from various aspects such as coverage extension, secrecy improvement, and fairness guarantee. Compared to the conventional active relays supporting massive multiple-input multiple-output (MIMO) [3] or millimeter wave (mm-Wave) communication [4], decode or amplify are not requested in the IRS-aided wireless networks due to the reason that the IRS is equipped with a large number of passive reflecting elements. Thus, both hardware cost and energy consumption of the IRS-aided wireless networks are lower than the conventional amplify-and-forward (AF) and decode-and-forward (DF) schemes [5]. Meanwhile, IRSs are capable of operating in a full-duplex and noise-free manner, which leads to improved spectrum efficiency. Given the aforementioned advantages of IRSs, they are recognized as promising candidates for signal enhancement, energy saving and cost reduction in the next-generation wireless networks.

Recently, by simultaneously transmitting the superimposed signal to multiple users on the same frequency, non-orthogonal multiple access (NOMA) scheme has been deemed as a promising technique for enhancing the network performance in terms of throughput and connectivity [6]. In sharp contrast to the conventional orthogonal multiple access (OMA) schemes, the signal for different user is distinguished in the power domain, and the successive interference cancellation (SIC) approach is adopted to decode their desired informations at the receivers [7]. Therefore, it is of great significance for NOMA networks to jointly optimize the power allocation and decoding order to improve the spectrum and energy efficiency [8], as well as reduce interferences. More particularly, for the multi-cell NOMA networks with large-scale devices, the co-channel interference makes the resource allocation problem among base stations (BSs) coupled and correlated [9], which leads to a challenging optimization problem. Given these challenges, it is particularly important to jointly design user scheduling and resource allocation for performance improvement in the multi-cell NOMA networks.

Inspired by the advantages of both IRSs and NOMA, it is valuable and imperative to integrate them together to further improve the spectrum and energy efficiency, coverage and connectivity,
due to the following profits and reasons:

- Firstly, the interference can be suppressed by applying IRSs into multi-cell NOMA networks and properly designing the reflection matrix of IRSs. The desired signal can be enhanced by IRSs, which leads to improved system throughput and reduced energy consumption.
- Secondly, for the cell-edge NOMA users that suffers high signal attenuation, IRSs can be deployed to passively relay the intended signal in a low-cost way, and thus the coverage of NOMA networks is extended. Namely, IRSs are beneficial to provide better service for these cell-edge users with poor signal strength.
- Thirdly, the SIC decoding performance will be significantly degraded when users’ original channels are not aligned, then the decoding order of users can be effectively tuned by adjusting IRSs to reconfigure the propagation environment. Therefore, IRSs are also profitable to optimize the user pairing and connectivity.

A. Related Works

1) Resource Allocation in NOMA Networks: In order to avoid the exponential complexity brought by the interaction between inter-cell interference and SIC decoding, many research contributions focus on the simplified single-cell NOMA networks [10]–[15]. With the aim of improving the energy efficiency, Fang et. al [11] utilized the difference of convex (DC) programming to solve the power allocation problem, and a suboptimal matching algorithm was developed for subchannel assignment. To strike a balance between the system throughput and user fairness, Liu et. al [12] proposed a dynamic power allocation algorithm to maximize the weighted sum-rate. The impact of user pairing on sum rate and outage probability was investigated in [14], where both numerical and analytical results demonstrated that NOMA can provide better performance than conventional OMA by exploiting the distinctive channel conditions among users. the authors of [15] proposed a joint power allocation and receive beamforming algorithm to maximize the fairness-based system utility under imperfect channel state information (CSI) feedback.

Due to the coupled resource allocation and user pairing problem in the multi-cell NOMA networks [9], [16]–[20], it is non-trivial to optimize them jointly. For the purpose of maximizing the energy efficiency, low-complexity algorithms were developed in [17] to solve the resource allocation problem by adopting the matching theory and DC programming. Furthermore, to reduce the overheads brought by the information exchange among BSs, Fu et. al [18] designed
a fully distributed power control algorithm to minimize the total transmission power at the transmitters. Taking both user fairness and spectrum efficiency into consideration, Zhao *et al.* [20] adopted the matching game and SCA methods to iteratively update spectrum allocation and power control results.

2) **IRS-Aided Wireless Communication Networks:** The majority of existing research contributions on IRS-aided wireless networks focus on the theoretical analysis [21]–[25] and performance optimization in terms of the system throughput [26], [27], energy efficiency [28]–[31], and user fairness [32]. By considering the perfect and imperfect SIC decoding of the IRS-aided NOMA network, the authors of [21]–[23] derived the closed-form expressions for the outage probability and ergodic rate. Due to the hardware limitations of IRSs in practice, the impacts of finite-resolution amplitude and phase shifts on outage probability and achievable data rate were analyzed in [24] and [25], respectively. By considering the ergodic and delay-limited capacity for IRS-aided OMA and NOMA networks, Mu *et al.* [26] jointly optimized the phase shifts and resource allocation to maximize the average sum rate of all users by invoking the Lagrange duality method. With the objective to minimize the transmission power at the access point, Wu *et al.* [30] proposed both optimal and suboptimal algorithms to design the active and passive beamforming alternately in both the single-user and multi-user cases. Based on the second-order-cone programming and semidefinite relaxation, Xie *et al.* [32] maximized the received minimal signal-to-interference-plus-noise ratio (SINR) to guarantee user fairness in IRS-aided multiple-input single-output (MISO) networks.

**B. Motivations and Contributions**

Inspired by the aforementioned benefits of both IRSs and NOMA, the IRS-aided NOMA transmission scheme can be regarded as an innovative and promising candidate for the next-generation networks. Although some research contributions on IRS-aided NOMA networks have addressed the challenging transmission power and reflection beamforming optimization problem iteratively, the system models are limited to single-cell and/or single-carrier setups. The motivations and challenges of this paper are summarized as follows:

- Currently, there is still a paucity of research contributions on investigating the IRS-aided multi-cell NOMA networks with multiple subchannels, especially for the user association and resource allocation problem with the mutual SIC decoding constraints and individual quality of service (QoS) constraints.
• So far, it is still a challenging issue to maximize the achievable sum rate by jointly designing the transmission power, reflection matrix, and decoding order, while guaranteeing the QoS requirements of all users within the available power budget.

• Moreover, the combinational optimization with respect to (w.r.t.) the user association and subchannel assignment is NP-hard. The complexity of exhaustive search is exponential, and it is non-trivial to obtain an optimal scheme in the polynomial-time complexity.

In order to tackle the aforementioned challenges, in this paper, we formulate the resource allocation problem for the IRS-aided multi-cell multi-subchannel NOMA networks to maximize the achievable sum rate under co-channel interference. Compared to the single-cell NOMA networks, the decoding order optimization of the multi-cell NOMA networks becomes more complex even without integrating IRSs into the networks, which makes the signal processing and performance improvement more complicated. Against the aforementioned background, the main contributions of this paper are summarized as follows:

1) We propose a novel framework of resource allocation in the multi-cell NOMA network for enhancing the spectrum efficiency with the aid of a single IRS. We formulate the sum-rate maximization problem subject to the SIC decoding conditions, QoS requirements, and maximum power constraints by jointly optimizing the decoding order, transmission power, reflection matrix, user association, and subchannel assignment. We analyze that the formulated problem is a mixed-integer non-linear programming (MINLP) problem, which is NP-hard and is non-trivial to solve directly.

2) In order to tackle the non-linear optimization problem of joint power allocation, reflection matrix design and decoding order determination, we first adopt relaxation methods such as convex upper bound substitution and SCA to transform the non-convex constraints into convex ones, which can be solved by suboptimal solutions with polynomial time complexity. Afterwards, we invoke the semidefinite relaxation (SDP) and Gaussian randomization methods to handle the rank-one constraint. Finally, the decoding order is obtained according to the combined channel gains arranged in ascending order.

3) In an effort to solve the three-dimensional (3D) matching problem among users, BSs and subchannels, we first reformulate the decomposed two-dimensional (2D) subproblems into many-to-many (one) matching games that have peer effects but lack substitutability. Then, based on the swap operation, we develop two efficient matching algorithms to
achieve a two-sided exchange-stable state among the involved players. Finally, we analyze the stability, convergence, complexity and optimality of the proposed algorithms from a theoretical perspective.

4) We demonstrate that the proposed resource allocation algorithms outperform the benchmarks in terms of sum rate and energy efficiency, while NOMA is capable of achieving a better performance than conventional OMA. Additionally, the performance of NOMA networks can be further improved with the aid of the IRS.

The rest of this paper is organized as follows. First, the system model of the IRS-aided multi-cell NOMA network is given in Section II. Then, the sum-rate maximization problem is solved in Section III and IV. Finally, numerical simulations are presented in Section V, which is followed by conclusions and future works in Section VI.

II. SYSTEM MODEL AND PROBLEM FORMULATION

As illustrated in Fig. 1 we consider an IRS-aided multi-cell NOMA transmission scenario, where an IRS is deployed for enhancing wireless service from \( J \) single-antenna BSs to \( I \) single-antenna cellular users, while \( \mathcal{I} = \{1, 2, \ldots, I\} \) and \( \mathcal{J} = \{1, 2, \ldots, J\} \). It is assumed that each cell is served by one BS, and each cellular user has to be associated with one BS. The IRS is equipped with \( M \) passive reflecting elements, denoted by \( \mathcal{M} = \{1, 2, \ldots, M\} \). The diagonal reflection matrix of IRS is denoted by \( \Theta = \text{diag}\{\lambda_1 e^{j\theta_1}, \lambda_2 e^{j\theta_2}, \ldots, \lambda_M e^{j\theta_M}\} \), where \( \lambda_m \in [0, 1] \)
and \( \theta_m \in [0, 2\pi] \) denote the reflection amplitude\(^1\) and phase shift of the \( m \)-th element equipped on the IRS, respectively. The total bandwidth \( W \) is divided into \( K \) subchannels, denoted by \( \mathcal{K} = \{1, 2, \ldots, K\} \), and all subchannels can be reused among BSs to improve the spectrum efficiency. In an effort to reduce the decoding complexity of SIC procedure at the receiver, we assume that the number of paired NOMA users, simultaneously sharing the available spectrum in each cell, is no more than \( A_{\text{max}} \), while \( A_{\text{max}} \geq 2 \).

A. IRS-Aided NOMA Transmission

In the NOMA downlink transmission, let \( \alpha_{ij} \in \{0, 1\} \) and \( \beta_{jk} \in \{0, 1\} \) denote the user association indicator and subchannel assignment factor, respectively. Specifically, we have \( \alpha_{ij} = 1 \) if the \( i \)-th user is associated with the \( j \)-th BS, otherwise \( \alpha_{ij} = 0 \). Furthermore, we have \( \beta_{jk} = 1 \) if the \( k \)-th subchannel is assigned to the \( j \)-th BS, otherwise \( \beta_{jk} = 0 \). Hence, the \( i \)-th user will be served by the \( j \)-th BS on the \( k \)-th subchannel if and only if \( \alpha_{ij} \beta_{jk} = 1 \), otherwise \( \alpha_{ij} \beta_{jk} = 0 \). Then, the superimposed signal, \( x_{jk} \), broadcasted by the \( j \)-th BS on the \( k \)-th subchannel can be given by

\[
x_{jk} = \alpha_{ij} \beta_{jk} \sqrt{p_{ijk}} x_{ijk} + \sum_{t=1, t \neq i}^{I} \alpha_{tj} \beta_{jk} \sqrt{p_{tjk}} x_{tjk},
\]

where \( x_{ijk} \) and \( p_{ijk} \) denote the signal and power transmitted by BS \( j \) on subchannel \( k \) for user \( i \), respectively.

Considering the intra-cell and inter-cell interference on the \( k \)-th subchannel, the received signal of user \( i \) associated with BS \( j \) on subchannel \( k \) is expressed as

\[
y_{ijk} = (h_{ijk} + g_{ik}^H \Theta f_{jk}) \alpha_{ij} \beta_{jk} \sqrt{p_{ijk}} x_{ijk} + (h_{ijk} + g_{ik}^H \Theta f_{jk}) \sum_{t=1, t \neq i}^{I} \alpha_{tj} \beta_{jk} \sqrt{p_{tjk}} x_{tjk} + \\
+ \sum_{s=1, s \neq j}^{J} (h_{isk} + g_{isk}^H \Theta f_{sk}) \sum_{i=1}^{I} \alpha_{is} \beta_{sk} \sqrt{p_{isk}} x_{isk} + z_{ijk},
\]

where \( h_{ijk} \) denotes the Rayleigh fading channel between BS \( j \) and user \( i \) on subchannel \( k \), \( f_{jk} \in \mathbb{C}^{M \times 1} \) represents the Rician fading channel between BS \( j \) and IRS on subchannel \( k \),

\(^1\)Without loss of generality, we set \( \lambda_m = 1, \forall m \) to simplify the analysis in the rate-centric communication networks, where the IRS is usually deployed to enhance the amplitude of the reflective signals.
\(g_{ik} \in \mathbb{C}^{M \times 1}\) formulates the Rayleigh fading channel between IRS and user \(i\) on subchannel \(k\), and \(z_{ijk}\) indicates the additive white Gaussian noise (AWGN) with zero mean and variance \(\sigma^2\), i.e., \(z_{ijk} \sim \mathcal{CN}(0, \sigma^2)\).

**B. SIC Decoding**

We denote the SIC decoding order as \(\pi_{jk}(i)\) for the user \(i\) associated with BS \(j\) on subchannel \(k\). Specifically, we have \(\pi_{jk}(i) = n\) if the message of user \(i\) is the \(n\)-th signal to be decoded at the receiver, namely, user \(i\) first decodes the signals of all the previous \((n - 1)\) users, and then successively subtracts their signals to decode its own desired signal. For example, two users \(i\) and \(\tilde{i}\) associated with BS \(j\) on subchannel \(k\), satisfying \(\pi_{jk}(i) \leq \pi_{jk}(\tilde{i})\), user \(\tilde{i}\) is capable of successfully canceling interference from the superposition signal of user \(i\) with the aid of SIC.

Let \(H_{ijk} = h_{ijk} + g_{ijk}^H \Theta_{jk}\) denote the combined channel gain, and \(P_{ijk} = \alpha_{ij}\beta_{jk}p_{ijk}\) represents the power allocation. Then, the decoding order constraints for guaranteeing success SIC can be formulated as the equation (3).

\[
\frac{|H_{ijk}|^2 P_{ijk}}{\sum_{\pi_{jk}(i) > \pi_{jk}(i)} P_{ijk} + \sum_{s \neq j} |H_{isk}|^2 \sum_i P_{isk} + \sigma^2} \geq \frac{|H_{ijk}|^2 P_{ijk}}{\sum_{\pi_{jk}(i) > \pi_{jk}(i)} P_{ijk} + \sum_{s \neq j} |H_{isk}|^2 \sum_i P_{isk} + \sigma^2} \geq 0 \tag{3}
\]

It indicates that the achievable SINR of user \(\tilde{i}\) to decode user \(i\) is no less than that of user \(i\).

By simple operations, the inequality (3) can be reformulated as the equation (4).

\[
\Delta_{jk}(i, \tilde{i}) = |H_{ijk}|^2 \left(\sum_{s \neq j} |H_{isk}|^2 \sum_i P_{isk} + \sigma^2\right) - |H_{ijk}|^2 \left(\sum_{s \neq j} |H_{isk}|^2 \sum_i P_{isk} + \sigma^2\right) \geq 0 \tag{4}
\]

Accordingly, the received SINR of user \(i\) associated with BS \(j\) on subchannel \(k\) is given by

\[
\text{SINR}_{ijk} = \frac{|H_{ijk}|^2 P_{ijk}}{I_{\text{intra}} + I_{\text{inter}} + \sigma^2}, \tag{5}
\]

where \(I_{\text{intra}} = |H_{ijk}|^2 \sum_{\pi_{jk}(i) > \pi_{jk}(i)} P_{ijk}\) and \(I_{\text{inter}} = \sum_{s=1}^{J} |H_{isk}|^2 \sum_{i=1}^{I} P_{isk}\) are the intra-cell and inter-cell interference, respectively. Therefore, the corresponding achievable downlink data rate of user \(i\) associated with BS \(j\) on subchannel \(k\) is calculated as

\[
R_{ijk} = \frac{W}{K} \log_2 \left(1 + \frac{|H_{ijk}|^2 P_{ijk}}{I_{\text{intra}} + I_{\text{inter}} + \sigma^2}\right). \tag{6}
\]
C. Problem Formulation

By jointly designing user association, subchannel assignment, reflection matrix, power allocation, and decoding order in the IRS-aided multi-cell NOMA network with multiple subchannels, the objective of this paper is to maximize the sum rate of users subject to the SIC decoding constraints, the QoS requirements, and the maximum power constraints, etc. Hence, the optimization problem can be formulated as

\[
\text{var} \{ \alpha_{ij}, \beta_{jk}, \Theta, p_{ijk}, \pi_{jk}(i) \mid \forall i, j, k \}, \quad (7a)
\]

\[
\max \sum_{i=1}^{I} \sum_{j=1}^{J} \sum_{k=1}^{K} R_{ijk}, \quad (7b)
\]

\[
\text{s.t.} \quad \Delta_{jk}(i, \tilde{i}) \geq 0, \text{ if } \pi_{jk}(i) \leq \pi_{jk}(\tilde{i}), \quad (7c)
\]

\[
\sum_{j=1}^{J} \sum_{k=1}^{K} R_{ijk} \geq R_{\text{min}}, \forall i, \quad (7d)
\]

\[
\sum_{i=1}^{I} \sum_{k=1}^{K} P_{ijk} \leq P_{\text{max}}, \forall j, \quad (7e)
\]

\[
\sum_{j=1}^{J} \alpha_{ij} = 1, \forall i, \quad (7f)
\]

\[
2 \leq \sum_{i=1}^{I} \alpha_{ij} \leq A_{\text{max}}, \forall j, \quad (7g)
\]

\[
\sum_{k=1}^{K} \beta_{jk} \geq 1, \forall j, \quad (7h)
\]

\[
\sum_{j=1}^{J} \beta_{jk} \geq 1, \forall k, \quad (7i)
\]

\[
\alpha_{ij}, \beta_{jk} \in \{0, 1\}, \forall i, j, k, \quad (7j)
\]

\[
\theta_m \in [0, 2\pi], \forall m, \quad (7k)
\]

\[
p_{ijk} \geq 0, \forall i, j, k, \quad (7l)
\]

\[
\pi_{jk} \in \Omega, \forall j, k, \quad (7m)
\]

where the optimization variables are given in (7a), \( R_{\text{min}} \) is the minimum data rate required by each user, \( P_{\text{max}} \) is the maximum transmission power provided by each BS, and \( \Omega \) is the set of all possible SIC decoding orders. Constraint (7c) ensures that the SIC decoding can be conducted successfully at the receiver. Constraint (7d) guarantees that the QoS requirement of each user is satisfied. Constraint (7e) denotes the available transmission power for each BS. Constraint (7f) represents that each user is associated with one BS. Constraint (7g) indicates that the number of NOMA users multiplexed in each cell is no less than two, and no more than \( A_{\text{max}} \). Constraints (7h)-(7i) describe that each BS is assigned with at least one subchannel, and vice
versa. Constraints (7j)-(7m) are invoked for restricting the indication factors, reflection matrix, transmission power, and decoding order, respectively.

Due to the existence of integer variables $\alpha_{ij}$, $\beta_{jk}$ and the continuous variables $p_{ijk}$, $\Theta$, as well as their highly coupling at the non-convex objective function and constraints, it can be observed that the sum-rate maximization problem (7) is a MINLP problem, which is NP-hard \cite{13} and is non-trivial to solve optimally by common standard optimization approaches. Additionally, the exhaustive search is not feasible, since the computational complexity grows exponentially over the total number of variables. Therefore, it is essential to transform problem (7) into some tractable convex subproblems.

III. Joint Optimization of Power, Reflection and Decoding Order

Given user association, subchannel assignment, and decoding order, we first aim to solve the problem of power allocation and reflection matrix design, which can be expressed as

$$\max_{p, \Theta} \sum_i \sum_j \sum_k R_{ijk},$$  \hspace{1cm} (8a)

$$\text{s.t.} \quad (7c), (7d), (7e), (7k), (7l),$$  \hspace{1cm} (8b)

where $p = \{p_{ijk} | \forall i, j, k\}$ is the power allocation profile. In inequality (7c), due to the inter-cell interference $I_{\text{inter}}^{ij}$ and $I_{\text{inter}}^{\tilde{i}j}$ for user $i$ and $\tilde{i}$, respectively, it is intractable to solve this non-linear and non-convex problem (8) by standard convex optimization approaches. Since each BS aims for maximizing their sum rate, they are expected to allocate as much power as possible to their associated users. Thus, we assume that the inter-cell interference $I_{\text{inter}}^{ij}$ and $I_{\text{inter}}^{\tilde{i}j}$ for user $i$ and $\tilde{i}$ in the same cell are approximately equal to the preset threshold $I_{\text{th}}$, then $\Delta_{jk}(i, \tilde{i})$ is simplified as $|H_{ijk}|^2 \geq |H_{ij\tilde{k}}|^2$. Moreover, due to the non-concavity of $R_{ijk}$, we introduce an auxiliary variable set $\gamma = \{\gamma_{ijk} | \text{SINR}_{ijk} \geq \gamma_{ijk}, \forall i, j, k\}$, and thus the problem (8) can be reformulated as

$$\max_{p, \Theta, \gamma} \sum_i \sum_j \sum_k \frac{W}{K} \log_2 (1 + \gamma_{ijk}),$$  \hspace{1cm} (9a)

$$\text{s.t.} \quad |H_{ijk}|^2 \geq |H_{ij\tilde{k}}|^2, \text{ if } \pi_{jk}(i) \leq \pi_{jk}(\tilde{i}),$$  \hspace{1cm} (9b)

$$\sum_j \sum_k \frac{W}{K} \log_2 (1 + \gamma_{ijk}) \geq R_{\text{min}}, \forall i,$$  \hspace{1cm} (9c)

$$\text{SINR}_{ijk} \geq \gamma_{ijk}, \forall i, j, k,$$  \hspace{1cm} (9d)

(7c), (7k), (7l).  \hspace{1cm} (9e)
A. Power Allocation

Given reflection matrix, the power allocation subproblem is given by
\[
\max_{p, \gamma} \sum_i \sum_j \sum_k W K \log_2 (1 + \gamma_{ijk}),
\]
\[\text{s.t.} \quad (7e), (7l), (9c), (9d). \tag{10a} \]

Notice that all constraints in problem (10) are convex excluding the constraint (9d), which can be recast as
\[
p_{ijk} \geq \gamma_{ijk} \hat{p}_{ijk} + \gamma_{ijk} \xi_{ijk}, \tag{11}\]
where \(\hat{p}_{ijk} = \sum_{l, j > \pi_{jk}(i)} p_{ijl} \) and \(\xi_{ijk} = \frac{I_n + \sigma^2}{|H_{ijk}|^2}\). It is worth noting that the product term \(\gamma_{ijk} \hat{p}_{ijk}\) is non-convex in the defined domain, \(\gamma_{ijk} \geq 0, \hat{p}_{ijk} \geq 0\), and thus inequality (11) is not a convex constraint. Therefore, it is necessary to transform the constraint (11) into a convex one.

Let \(f(\gamma_{ijk}, \hat{p}_{ijk}) = \gamma_{ijk} \hat{p}_{ijk}\), while \(\gamma_{ijk}, \hat{p}_{ijk} \geq 0\). By replacing \(f(\gamma_{ijk}, \hat{p}_{ijk})\) with its convex upper bound (CUB), the resulting constraint becomes convex. To this end, we define the following function
\[
g(\gamma_{ijk}, \hat{p}_{ijk}, \lambda_{ijk}) = \frac{\lambda_{ijk}}{2} \gamma_{ijk}^2 + \frac{1}{2\lambda_{ijk}} \hat{p}_{ijk}^2, \tag{12}\]
where \(\Lambda = \{\lambda_{ijk} | \forall i, j, k\}\) is a coefficient set. It can be proved that (12) is a convex function, and \(g(\gamma_{ijk}, \hat{p}_{ijk}, \lambda_{ijk}) \geq f(\gamma_{ijk}, \hat{p}_{ijk})\) is satisfied for all \(\lambda_{ijk} > 0\). Moreover, it can be derived that the equation will turn to equality when \(\lambda_{ijk} = \hat{p}_{ijk}/\gamma_{ijk}\). Hence, by replacing \(f(\gamma_{ijk}, \hat{p}_{ijk})\) with its convex upper bound \(g(\gamma_{ijk}, \hat{p}_{ijk}, \lambda_{ijk})\), constraint (11) is transformed as the following second-order cone constraint:
\[
p_{ijk} \geq \frac{\lambda_{ijk}}{2} \gamma_{ijk}^2 + \frac{1}{2\lambda_{ijk}} \hat{p}_{ijk}^2 + \gamma_{ijk} \xi_{ijk}. \tag{13}\]

Next, by replacing (9d) with its approximate constraint (13), it can be observed that both the objective function and all constraints in (10) become convex, and hence the Karush-Kuhn-Tucker (KKT) solution of (10) can be iteratively updated until convergence by optimally solving its convex approximation problem with CVX. The details of the proposed CUB-based power allocation algorithm with adjustable convergence accuracy, \(\epsilon\), are summarized in Algorithm 1, where the fixed coefficient \(\gamma_{ijk}\) in the \(n_1\)-th iteration can be updated by
\[
\lambda_{ijk}^{(n_1)} := \frac{\hat{p}_{ijk}^{(n_1-1)}}{\gamma_{ijk}^{(n_1-1)}}. \tag{14}\]
Algorithm 1 CUB-Based Algorithm for Power Allocation

1: **Initialize** $p^{(0)}$, $\gamma^{(0)}$, the tolerance $\epsilon$, maximum iteration number $N_1$, and set current iteration number as $n_1 = 0$.

2: Compute utility $U^{(0)} = \sum_i \sum_j \sum_k W K \log_2 (1 + \gamma^{(0)}_{ijk})$;

3: repeat

4: With given $p^{(n_1)}$ and $\gamma^{(n_1)}$, update $\lambda^{(n_1+1)}$ by (14);

5: With given $\lambda^{(n_1+1)}$, obtain $p^{(n_1+1)}$ and $\gamma^{(n_1+1)}$ by solving the substituted problem of (10);

6: With given $\gamma^{(n_1+1)}$, compute $U^{(n_1+1)} = U(\gamma^{(n_1+1)})$;

7: Update $n_1 := n_1 + 1$;

8: until $|U^{(n_1)} - U^{(n_1-1)}| < \epsilon$ or $n_1 > N_1$;

9: **Output** the converged solutions $p^*$ and $\gamma^*$;

B. Reflection Matrix Design

With the converged results $p^*$ and $\gamma^*$ derived from Algorithm 1, the problem (9) is simplified to the following feasibility-check subproblem

\[
\text{find } \Theta, \quad \text{s.t.} \quad (7k), (9b), (9d). \quad (15a)
\]

For notational convenience, we define $\rho_{ijk} = \text{diag}(\rho_{ik}^H) f_{jk}$ and $\nu = [\nu_1, \nu_2, \ldots, \nu_M]^H$, where $\nu_m = e^{j\theta_m}$. Thus, $|h_{ijk} + \nu^H \rho_{ijk}| = |h_{ijk} + \nu^H \rho_{ijk}|$. Meanwhile, we denote the real and imaginary parts of $H_{ijk}$ as $x_{ijk}$ and $y_{ijk}$, respectively, such that $x_{ijk}^2 + y_{ijk}^2 = |h_{ijk} + \nu^H \rho_{ijk}|^2$. Then, the feasibility-check problem (15) is rewritten as

\[
\text{find } \nu, \quad \text{s.t.} \quad x_{ijk}^2 + y_{ijk}^2 \geq x_{ijk}^2 + y_{ijk}^2, \quad \text{if } \pi_{jk}(i) \leq \pi_{jk}(\tilde{i}), \quad (16a)
\]

\[
x_{ijk}^2 + y_{ijk}^2 \geq (x_{ijk}^2 + y_{ijk}^2) \phi_{ijk} + \xi_{ijk}, \quad (16b)
\]

\[
|\nu_m| \leq 1, \quad \forall m, \quad (16c)
\]

\[
x_{ijk} = \text{real} \left( h_{ijk} + \nu^H \rho_{ijk} \right), \quad (16d)
\]

\[
y_{ijk} = \text{imag} \left( h_{ijk} + \nu^H \rho_{ijk} \right), \quad (16e)
\]
where $\varphi_{ijk} = \frac{\gamma_{ijk}P_{ijk}}{p_{ijk}}$ and $\xi_{ijk} = \frac{\gamma_{ijk}(I_n + \sigma^2)}{p_{ijk}}$. Owing to the fact that both (16b) and (16c) are non-convex quadratic constraints, the problem (16) is non-trivial to be solved directly. Thus, we invoke the SCA to replace $x_{ijk}^2 + y_{ijk}^2$ with its first-order Taylor approximation and iteratively solve the resulting problem until it converges to a KKT solution within the preset accuracy. Toward this end, the lower-bound approximation for $x_{ijk}^2 + y_{ijk}^2$ is given by

$$
\tau(x_{ijk}, y_{ijk}) = \bar{x}_{ijk}^2 + \bar{y}_{ijk}^2 + 2\bar{x}_{ijk}(x_{ijk} - \bar{x}_{ijk}) + 2\bar{y}_{ijk}(y_{ijk} - \bar{y}_{ijk}),
$$

(17)

where $\{(\bar{x}_{ijk}, \bar{y}_{ijk})|\forall i, j, k\}$ is a set of feasible solution of (16), and they can be updated in the $n_2$-th iteration by

$$
\bar{x}_{ijk}^{(n_2)} := \text{real} \left( h_{ijk} + \nu^{(n_2-1)} P_{ijk}^{(n_2-1)} \right),
$$

(18)

$$
\bar{y}_{ijk}^{(n_2)} := \text{imag} \left( h_{ijk} + \nu^{(n_2-1)} P_{ijk}^{(n_2-1)} \right).
$$

(19)

Afterwards, by replacing the expressions on the left-side of (16b) and (16c) with their first-order Taylor approximation, it can be observed that the substituted problem of (16) becomes a convex one, which also can be optimally solved with the existing convex optimization approaches such as CVX. The details of the proposed SCA-based algorithm for reflection matrix design are summarized in Algorithm 2.

**Algorithm 2** SCA-Based Algorithm for Reflection Matrix

1: Initialize $\nu^{(0)}$, the maximum iteration number $N_2$, and set the current iteration number as $n_2 = 0$.
2: repeat
3: With given $\nu^{(n_2)}$, update $\{(\bar{x}_{ijk}^{(n_2+1)}, \bar{y}_{ijk}^{(n_2+1)})|\forall i, j, k\}$ using (18) and (19);
4: With given $\{(\bar{x}_{ijk}^{(n_2+1)}, \bar{y}_{ijk}^{(n_2+1)})|\forall i, j, k\}$, obtain $\nu^{(n_2+1)}$ by solving the approximated problem of (16);
5: Update $n_2 := n_2 + 1$;
6: until $\nu^{(n_2)}$ converges or $n_2 > N_2$;
7: Output the converged reflection matrix $\nu^*$;
C. Decoding Order Determination

The decoding order in each cell depends on the combined channel gains experienced by users clustered in the cell. Due to the same phase shifts applied for all users with different channels, the combined channel gains of different users cannot be maximized at the same time. Thus, we alternatively maximize the sum of all combined channel gains, which is recalculated as

$$\max_\Theta \sum_i \sum_j \sum_k |H_{ijk}|^2,$$

subject to $\theta_m \in [0, 2\pi], \forall m.$ (20b)

1) Semidefinite Relaxation: Define $\Upsilon_{ijk} = \text{real}(\nu^H \rho_{ijk})$ and $\Gamma_{ijk} = \rho_{ijk} \rho_{ijk}^H,$ then it can be noticed that

$$|h_{ijk} + \nu^H \rho_{ijk}|^2 = |h_{ijk}|^2 + 2h_{ijk} \Upsilon_{ijk} + \nu^H \Gamma_{ijk} \nu.$$ (21)

Therefore, we have

$$|H_{ijk}|^2 = \tilde{\nu}^H C_{ijk} \tilde{\nu} + |h_{ijk}|^2,$$ (22)

where

$$C_{ijk} = \begin{bmatrix} \Gamma_{ijk} & h_{ijk} \rho_{ijk} \\ h_{ijk} \rho_{ijk}^H & 0 \end{bmatrix} \quad \text{and} \quad \tilde{\nu} = \begin{bmatrix} \nu \\ 1 \end{bmatrix}.$$ (23)

Furthermore, we define $V = \tilde{\nu} \tilde{\nu}^H,$ while $V \succeq 0$ and $\text{rank}(V) = 1.$ Then we have $\tilde{\nu}^H C_{ijk} \tilde{\nu} = \text{tr}(C_{ijk} V),$ and the problem (20) is equivalently reformulated as

$$\max_V \sum_i \sum_j \sum_k \text{tr}(C_{ijk} V) + |h_{ijk}|^2,$$

subject to $V_{m,m} \leq 1, \forall m.$ (24b)

$$V_{M+1,M+1} = 1,$$ (24c)

$$V \succeq 0,$$ (24d)

$$\text{rank}(V) = 1.$$ (24e)

Although the rank-one constraint is still non-convex, the semidefinite relaxation (SDR) can be applied to relax (24) to a convex SDP problem, and thus the optimal $V^*$ can be obtained by solving the relaxed problem with CVX. Finally, with $V^* = \tilde{\nu}^* \tilde{\nu}^{*H},$ the optimal reflection matrix $\Theta^*$ is obtained. Based on the converged results $p^*$ and $\Theta^*$, if the combined channel gains experienced by any two users $(i, \tilde{i})$ associated with BS $j$ on subchannel $k$ can be arranged as $H_{ijk} \leq H_{\tilde{ijk}},$ then the decoding order is given by $\pi_{jk}(i) \leq \pi_{jk}(\tilde{i}).$ However, if $\text{rank}(V) \neq 1,$ then
objective value obtained from the relaxed problem is only an upper bound of (20). In this case, the Gaussian randomization (GR) method has to be invoked to construct a rank-one solution based on the higher-rank solution of the relaxed problem.

2) Gaussian Randomization: If \( \text{rank}(\mathbf{V}) = 1 \), the optimal reflection matrix \( \mathbf{\Theta}^* \) can be derived by calculating the eigenvalue and eigenvector of \( \mathbf{V} \). When \( \text{rank}(\mathbf{V}) \neq 1 \), the GR method is adopted, and the eigenvalue decomposition of \( \mathbf{V} \) is defined as

\[
\mathbf{V} = \mathbf{U} \mathbf{\Sigma} \mathbf{U}^H,
\]

where \( \mathbf{U} = [e_1, e_2, \ldots, e_{M+1}] \) is a unitary matrix of eigenvectors, and \( \mathbf{\Sigma} = \text{diag}\{\varpi_1, \varpi_2, \ldots, \varpi_{M+1}\} \) is a diagonal matrix of eigenvalues.

Then, we generate two independent normally distributed random vectors \( \mathbf{x} \in \mathbb{R}^{(M+1)\times 1} \) and \( \mathbf{y} \in \mathbb{R}^{(M+1)\times 1} \) with zero mean and covariance matrix \( \frac{1}{2} \mathbf{I}_{M+1} \). Let \( N \) denote the maximum generation of candidate random vectors, and the Gaussian random vector in the \( n \)-th generation is given by

\[
\mathbf{r}_n = \mathbf{x} + \mathbf{y} \sqrt{-1}, \quad n = 1, 2, \ldots, N.
\]

Based on the generated Gaussian random vector \( \mathbf{r}_n \in \mathcal{CN}(0, \mathbf{I}_{M+1}) \) in the complex plane, we can obtain a suboptimal solution to (24), denoting as

\[
\tilde{\mathbf{\nu}}_n = \mathbf{U} \mathbf{\Sigma}^{1/2} \mathbf{r}_n, \quad n = 1, 2, \ldots, N.
\]

Next, the candidate reflection matrix can be expressed as

\[
\mathbf{\Theta}_n = \text{diag}\left\{e^{j \text{arg}\{\tilde{\mathbf{\nu}}_n[m]\}} \mid \forall m \in \mathcal{M}\right\},
\]

where \( \tilde{\mathbf{\nu}}_n[m] \) denotes the \( m \)-th elements of \( \tilde{\mathbf{\nu}}_n \). With the obtained candidate set of reflection matrix \( \{\mathbf{\Theta}_n | n = 1, 2, \ldots, N\} \), we can find the optimal one that maximizes the combined channel gains of all users, i.e.,

\[
n^* = \arg \max_n \sum_i \sum_j \sum_k |h_{ijk} + \mathbf{g}_{ik}^H \mathbf{\Theta}_n \mathbf{f}_{jk}|^2.
\]

Based on the above derivation, the GR-based algorithm for decoding order is summarized in Algorithm 3.
Algorithm 3 GR-Based Algorithm for Decoding Order

1. **Initialize** the maximum generation of candidate random vector as $N_3$, solve the relaxed SDP problem of (24) and obtain an optimal solution $V$.

2. **if** rank($V$) = 1 **then**
   3. Calculate the eigenvalue $\varpi$ and eigenvector $u$ of $V$;
   4. Update $\Theta^* := \text{diag}\{\sqrt{\varpi}u\}$;

5. **else**
   6. Obtain the eigenvalue decomposition using (25);
   7. **for** $n_3 = 1, 2, \ldots, N_3$ **do**
     8. Generate a Gaussian random vector $r_{n_3}$ using (26);
     9. Obtain a candidate solution $\Theta_{n_3}$ using (27) and (28);
   10. **end for**
   11. Find the optimal $\Theta^* := \Theta_{n_3}$ according to (29);
   12. **end if**

13. With the optimal $\Theta^*$, calculate all combined channel gains $\{|h_{ijk} + g_{ik}\Theta^*f_{jk}|^2 | \forall j,k\}$ and rank them in ascending order for each BS $j$ on subchannel $k$;

14. **Output** the optimal decoding order $\pi^*_{jk}$, $\forall j,k$.

---

D. Convergence and Complexity Analysis

1) **Convergence:** In Algorithm 1 denote $p^{(n_1)}$ and $\gamma^{(n_1)}$ as the solution to problem (10) obtained in the $n_1$-th iteration, where the utility value is given by $U^{(n_1)} = U(\gamma^{(n_1)})$. Then, the coefficient set $\lambda^{(n_1+1)}$ can be updated by (14). Note that the utility value in Algorithm 1 only depends on $\gamma$, such that

$$U(\gamma^{(n_1)}) = U(p^{(n_1)}, \gamma^{(n_1)}) = U(p^{(n_1)}, \gamma^{(n_1)}, \lambda^{(n_1+1)}).$$

(30)

By substituting $\lambda^{(n_1+1)}$ into the problem (10), we can obtain $p^{(n_1+1)}$ and $\gamma^{(n_1+1)}$ by solving the resulting problem once again, and thus we have

$$U(p^{(n_1)}, \gamma^{(n_1)}, \lambda^{(n_1+1)}) \leq U(p^{(n_1+1)}, \gamma^{(n_1+1)}, \lambda^{(n_1+1)}).$$

(31)

Similar to (30), the utility value only depends on $\gamma$. Thus, the following equation is satisfied:

$$U(\gamma^{(n_1+1)}) = U(p^{(n_1+1)}, \gamma^{(n_1+1)}) = U(p^{(n_1+1)}, \gamma^{(n_1+1)}, \lambda^{(n_1+2)})$$

$$= U(p^{(n_1+1)}, \gamma^{(n_1+1)}, \lambda^{(n_1+1)}).$$

(32)
Therefore, combining (30) − (32), it can be observed that the utility value of problem (10) is non-decreasing over iterations, which can be expressed as

\[ U^{(n_1)} = U(\gamma^{(n_1)}) \leq U(\gamma^{(n_1+1)}) = U^{(n_1+1)}. \]  

Finally, due to the fact that the system bandwidth and available transmission power are limited in practice, the utility value (i.e., achievable sum rate) has an upper bound. Hence, Algorithm 1 is guaranteed to converge as long as the value of \( N_1 \) is set large enough. The convergence proofs of Algorithm 2 and 3 are omitted here for brevity, due to their similar derivations.

2) Complexity: When the substituted convex problems of (10) and (16) are solved by CVX, the interior point method is adopted. In Algorithm 1, the dimension of variables to be solved is \( 2IJK \). Thus, the complexity of Algorithm 1 can be expressed as \( O(N_1(2IJK)^3) \), where \( N_1 \) is the maximal iteration number for finding the converged power allocation strategy. Similarly, the complexity of Algorithm 2 is bounded by \( O(N_2(M + 2IJK)^3) \), where \( N_2 \) is the maximal iteration number for checking the feasibility of reflection matrix. In Algorithm 3, the complexity for solving the relaxed SDP problem of (24) is \( O((M + 1)^6) \). Meanwhile, define \( N_3 \) as the maximal number of the generated Gaussian random vectors, and denote \( T_{GR} \) as the complexity of performing one Gaussian random. Thereby, the complexity of Algorithm 3 can be expressed as \( O((M + 1)^6 + N_3T_{GR}) \) in the worst case.

IV. MATCHING THEORY FOR USER ASSOCIATION AND SUBCHANNEL ASSIGNMENT

In this section, we focus on the user association and subchannel assignment problem in (7) with fixed power allocation and reflection matrix, which can be expressed as

\[
\begin{align*}
\text{max} \quad & \sum_{i=1}^{I} \sum_{j=1}^{J} \sum_{k=1}^{K} R_{ijk}, \\
\text{s.t.} \quad & (7f) - (7j),
\end{align*}
\]

where \( \alpha = \{\alpha_{ij} | \forall i, j\} \) denotes the user association profile and \( \beta = \{\beta_{jk} | \forall j, k\} \) represents the subchannel assignment profile. It can be observed that (34) is a 3D matching problem involving three finite and disjoint sets (i.e., user set \( \mathcal{I} \), BS set \( \mathcal{J} \), and subchannel set \( \mathcal{K} \)), which is proved to be NP-hard for obtaining the optimal solution. In order to address this challenging issue, we decompose the 3D matching problem (34) into two 2D matching problems, namely, the user association problem and subchannel assignment problem. The former problem is to cluster all
users into multiple disjoint user groups, and the latter problem is to assign all subchannels into multiple subchannel sets. More expectantly, in the user association problem, the users in each group form a cell served by one BS through the NOMA transmission. Thus, it is a many-to-one matching problem. In the subchannel assignment problem, one subchannel can be reused by multiple BSs and multi-subchannel can be assigned to one BS, which is a many-to-many matching problem.

A. Matching Problem Formulation

Before solving the aforementioned two matching problems, we give the following remarks and definitions for ease of exposition.

Remark 1: The above mentioned 2D matching problems is a many-to-many (one) matching problem with peer effects.

Proof: On the one hand, in the user association problem, owing to the feature of multiplexing power domain NOMA, the achievable data rate of any user \( i \) associated with BS \( j \) over all subchannels is related to other paired users sharing the same subchannel. As a result, each BS should take into account the internal relationship of the associated users when it selects a certain user to match with. It is the intra-cell interference that makes the problem of user association a many-to-one matching problem with peer effects. On the other hand, in the subchannel assignment problem, owing to the reuse of subchannels among different cells, the sum rate of each BS \( j \) over subchannel \( k \) is affected by other BS assigned with the same subchannel. Thus, the preference of each BS not only depends on the subchannel it matches with, but also depends on other BSs that match with the same subchannel. Therefore, the individual BS preference depends on other peers, and it is the inter-cell interference that makes the problem of subchannel assignment a many-to-many matching problem with peer effects, which completes the proof.

Definition 1 (2D Matching): A matching \( \mu \) is a function from the set \( \mathcal{E} \cup \mathcal{W} \) to the set of all subsets of \( \mathcal{E} \cup \mathcal{W} \) such that 1) \( \mu(e) \subseteq \mathcal{W} \) and \( |\mu(e)| = \ell_w \), \( \forall e \in \mathcal{E} \); 2) \( \mu(w) \subseteq \mathcal{E} \) and \( |\mu(w)| = \ell_e \), \( \forall w \in \mathcal{W} \); 3) \( \mu(e) \subseteq \mathcal{W} \) if and only if \( \mu(w) \subseteq \mathcal{E} \); 4) \( e \in \mu(w) \) if and only if \( w \in \mu(e) \); where \( \mathcal{E} = \{e_1, \ldots, e_n\} \) and \( \mathcal{W} = \{w_1, \ldots, w_u\} \) are two finite and disjoint player sets, \( \ell_w \) and \( \ell_e \) are two positive integers.

Note that the above condition 1) implies that each player \( e \in \mathcal{E} \) can be matched with \( \ell_w \) players in \( \mathcal{W} \). Similarly, condition 2) means that each player \( w \in \mathcal{W} \) can be matched with
Condition 3) indicates that the mapping of player \( e \in \mathcal{E} \) is the subset of \( \mathcal{W} \), and vice versa. Condition 4) represents that if player \( e \in \mathcal{E} \) matched with \( w \in \mathcal{W} \), then player \( w \in \mathcal{W} \) is also matched with \( e \in \mathcal{E} \). It is worth noting that when \( \ell_w \geq 2 \) and \( \ell_e \geq 2 \), then one can obtain the definition of many-to-many matching. When \( \ell_w \geq 2 \) and \( \ell_e = 1 \), it becomes a many-to-one matching.

**Remark 2:** The formulated many-to-many (one) matching problem is lack of the property of substitutability.

**Proof:** Given player set \( \mathcal{E} \) and \( \mathcal{W} \), each player \( e \in \mathcal{E} \) can determine which subset of \( \mathcal{W} \) it is most likely to match with. This is called the choice set of \( e \) in \( \mathcal{W} \), denoted by \( C_e(\mathcal{W}) = \mathcal{W}' \). That is, the player \( e \) prefers \( \mathcal{W}' \) to any subset of \( \mathcal{W} \), which can be expressed as

\[
\forall \mathcal{W}'' \subset \mathcal{W}, \mathcal{W}'' \neq \mathcal{W}' \Rightarrow \mathcal{W}' \succ_e \mathcal{W}''.
\]

(35)

For any set \( \mathcal{W} \) that contains \( w \) and \( w' \), the preference of \( e \) over sets of \( \mathcal{W} \) has the property of substitutability if and only if \( w \in C_e(\mathcal{W}) \) and \( w \in C_e(\mathcal{W}\setminus\{w'\}) \). It means that when a player \( e \in \mathcal{E} \) has the property of substitutability, it regards the players in the choice set \( C_e(\mathcal{W}) \) as alternatives rather than complements, even if a player \( w' \in \mathcal{W} \) in the choice set rejects it, its selection of other players in the choice set will not be affected. Nevertheless, on the one hand, due to the intra-cell interference from the user pairing in the user association problem, the achievable data rate of user \( i \) associated with BS \( j \) may change after its paired user \( i' \) is unmatched with BS \( j \). Thus, user \( i \) may not be in the preferred set of BS \( j \) any more, which implies that the formulated many-to-one user association problem does not have the property of substitutability. On the other hand, due to the inter-cell interference among BSs assigned with the same subchannels, the achievable rate of subchannel \( k \) with BS \( j \) may change after \( j' \) is unmatched with \( k \). Hence, BS \( j \) may not be in the preferred set of subchannel \( k \) any more, which indicates that the formulated many-to-many subchannel assignment problem does not have the property of substitutability as well, which completes the proof.

During the matching process, each player \( e \in \mathcal{E} \) has a transitive and strict preference list w.r.t. its interests over the set of \( \mathcal{W} \), and vice versa. We use \( w_1 \succ_e w_2 \) to denote that player \( e \) strictly prefers \( w_1 \) to \( w_2 \). If \( w_2 \succ_e w_3 \) is satisfied at the same time, then we have \( w_1 \succ_e w_3 \). Due to the existence of peer effects and non-substitutability in the formulated many-to-many (one) matching problem, the preference lists of players vary continuously over the matching process, which makes the matching mechanisms complex to design. Given a matching function \( \mu \), and
assume that \( \mu(e) = w \) and \( \mu(e') = w' \). Then, in order to handle the peer effects and ensure exchange stability, we define the swap matching as

\[
\mu_e' = \left\{ \mu \setminus \{(e, w), (e', w')\} \cup \{(e, w'), (e', w)\} \right\},
\]

where players \( e \) and \( e' \) exchange their matched players \( w \) and \( w' \) while keeping all other matching states the same. Based on the swap operation in (36), we define the concept of swap-blocking pair as follows.

**Definition 2 (Swap-Blocking Pair):** A pair of players \((e, e')\) is called a swap-blocking pair in \( \mu \) if and only if 1) \( \forall q \in \{e, e', w, w'\}, U_q(\mu_e') \geq U_q(\mu) \); 2) \( \exists q \in \{e, e', w, w'\}, \text{ such that } U_q(\mu_e') > U_q(\mu) \); where \( U_q(\mu) \) denotes the utility of player \( q \) under matching \( \mu \).

The aforementioned condition 1) shows that the utilities of all involved players should not be decreased after the swap operation. Condition 2) implies that at least one of the involved player’s utilities is increased after the swap operation. What is worth mentioning is that the matching \( \mu \) is two-sided exchange-stable if and only if there does not exist a swap-blocking pair. Otherwise, the swap matching \( \mu_e' \) in a swap-blocking pair would be approved, and the achievable utilities of the involved players will not decrease and at least one player’s utility will increase after the swap operation.

**B. Many-to-One Matching for User Association**

In the many-to-one matching problem of user association, we define the preference of each user \( i \) associated with BS \( j \) as

\[
U_{ij} = \sum_{k \in \mathcal{K}} \frac{W}{K} \log_2 (1 + \gamma_{ijk}).
\]

If user \( i \) can achieve a higher data rate when being associated with BS \( j \) compared to that of being associated with BS \( j' \), i.e., user \( i \) prefers to be associated with the BS \( j \) in matching \( \mu \) rather than the BS \( j' \) in matching \( \mu' \), then we have

\[
(j, \mu) \succ_i (j', \mu') \iff U_{ij}(\mu) > U_{ij}(\mu').
\]

Similarly, the preference of each BS \( j \) associated with a set of users \( \mu(j) \) can be given by

\[
U_j = \sum_{i \in \mu(j)} \sum_{k \in \mathcal{K}} \frac{W}{K} \log_2 (1 + \gamma_{ijk}).
\]
For any two subsets of users \( I_1 = \mu(j) \) and \( I_2 = \mu'(j) \) while \( I_1 \neq I_2 \), if BS \( j \) obtain get a higher data rate when being associated with \( I_1 \) than that of being associated to \( I_2 \), i.e., BS \( j \) prefers the user subset \( I_1 \) in matching \( \mu \) to the user subset \( I_2 \) in matching \( \mu' \), then we have

\[
(I_1, \mu) \succ_j (I_2, \mu') \iff U_j(\mu) > U_j(\mu').
\]  

(40)

According to (38) and (40), the preference lists of all users and BSs are constructed. Subsequently, each user proposes to the most preferred BS that has never rejected them before. Then, each BS accepts the most preferred users and rejects the others. Finally, the initial matching state between users and BSs is obtained when there is no unmatched user. After that, each user tries to search for another user to form a swap-blocking pair and swaps their matching states based on (36), which terminates when no swap-blocking pair exists. In summary, the many-to-one matching for user association is described in Algorithm 4.

**Algorithm 4 Many-to-One Matching for User Association**

1. **Initialize** the User-BS matching state as \( \Phi_1 \).
2. **repeat**
3. For every user \( i \in \Phi_1 \), it searches for another user \( i' \in \Phi_1 \setminus \Phi_1(\mu(i)) \) to check whether \((i, i')\) is a swap-blocking pair;
4. **if** \((i, i')\) is a swap-blocking pair **then**
5. Update \( \mu := \mu_{i'} \);
6. **else**
7. Keep the current matching state;
8. **end if**
9. **until** No swap-blocking pair can be constructed.
10. **Output** the stable User-BS matching \( \mu^* \) and its corresponding utility \( U_1 = U(\mu^*) \).

---

C. Many-to-Many Matching for Subchannel Assignment

Analogously, in the many-to-many matching problem of subchannel assignment, the preference of each BS \( j \) assigned with subchannel \( k \) is defined as

\[
U_{jk} = \sum_{i \in I} \frac{W}{K} \log_2 (1 + \gamma_{ijk}).
\]  

(41)
If BS $j$ can achieve a higher data rate when being assigned with subchannel $k$ compared to that of being assigned with subchannel $k'$, i.e., BS $j$ prefers to the subchannel $k$ in matching $\mu$ rather than the subchannel $k'$ in matching $\mu'$, then we have

$$(k, \mu) \succ_j (k', \mu') \Leftrightarrow U_{jk}(\mu) > U_{jk'}(\mu'). \quad (42)$$

Similarly, the preference of each subchannel $k$ on a set of BSs $\mu (k)$ can be given by

$$U_k = \sum_{i \in I} \sum_{j \in \mu (k)} \frac{W}{K} \log_2 (1 + \gamma_{ijk}). \quad (43)$$

For any two subsets of BSs $J_1 = \mu (k)$ and $J_2 = \mu' (k)$ while $J_1 \neq J_2$, if subchannel $k$ can get a higher data rate when being assigned to $J_1$ than $J_2$, i.e., subchannel $k$ prefers to the BS subset $J_1$ in matching $\mu$ rather than the BS subset $J_2$ in matching $\mu'$, then we have

$$(J_1, \mu) \succ_k (J_2, \mu') \Leftrightarrow U_k(\mu) > U_k(\mu'). \quad (44)$$

First, the preference lists of all (User-BS) units and subchannels are established according to (42) and (44). Then, an initial matching state can be generated by adopting the aforementioned method in Section IV-B. Finally, the search process is executed based on (36), which terminates until there exists no swap-blocking pair. The many-to-many matching for subchannel assignment is described in Algorithm 5.

**Algorithm 5 Many-to-Many Matching for Subchannel Assignment**

1: **Initialize** the (User,BS)-Subchannel matching state as $\Phi_2$.

2: **repeat**

3: For every (User,BS) $j \in \Phi_2$, it searches for another (User,BS) $j' \in \Phi_2 \setminus \Phi_2(\mu (j))$, and let $U = \{U_1\}$;

4: For a given $j$, calculate the candidate $U_{j}^{j'}$ for the swapping pair $(j, j')$;

5: **if** $(j, j')$ is a swap-blocking pair **then**

6: Update $U := U \cup \{U_{j}^{j'}\}$;

7: **end if**

8: Find $j'^* = \arg \max_{j'} U$;

9: Update $\bar{\mu} := \bar{\mu}_{j'^*}$, and set $U_2 = U_{j'^*}$;

10: **until** No swap-blocking pair can be constructed.

11: **Output** the stable (User,BS)-Subchannel matching $\bar{\mu}^*$.
D. Property Analysis

The properties in terms of stability, convergence, complexity and optimality of the proposed matching-based algorithms are analyzed in the following propositions.

Proposition 1 (Stability): The final matching $\mu^*$ and $\bar{\mu}^*$ derived from Algorithm 4 and 5 are both two-sided exchange-stable matching.

Proof: This Proposition can be proved by contradiction. Assume that there exists a blocking pair $(i, i')$ in the final matching $\mu^*$ satisfying that $\forall q \in \{i, i', \mu(i), \mu(i')\}$, $U_q((\mu^*)_{i'}^i) \geq U_q(\mu^*)$ and $\exists q \in \{i, i', \mu(i), \mu(i')\}$ such that $U_q((\mu^*)_{i'}^i) > U_q(\mu^*)$. According to step 2 to step 9 in Algorithm 4, the swap operation continues until there exists no swap-blocking pair. That is to say, $\mu^*$ is not the final matching, which contradicts our initial assumption and the proposition is proved. As a result, it can be concluded that the proposed algorithm reaches a two-sided exchange stability in the end. The proof for $\bar{\mu}^*$ in Algorithm 5 can be derived similarly, which is omitted here for brevity.

Proposition 2 (Convergence): Both Algorithm 4 and 5 converge to a two-sided exchange-stable matching within a limited number of iterations.

Proof: Given a matching function $\mu$ for the user association problem, suppose that $\mu(i) = j$, $\mu(i') = j'$, while $(i, i')$ is a swap-blocking pair. According to Definition 2, at least one of the utilities of BS $j$ and $j'$ increases after the swap operation. Thus, there are three cases: i) $U_j(\mu_{i'}^{i}) > U_j(\mu)$ and $U_{j'}(\mu_{i'}^{i}) > U_{j'}(\mu)$; ii) $U_j(\mu_{i'}^{i}) = U_j(\mu)$ and $U_{j'}(\mu_{i'}^{i}) > U_{j'}(\mu)$; iii) $U_j(\mu_{i'}^{i}) > U_j(\mu)$ and $U_{j'}(\mu_{i'}^{i}) = U_{j'}(\mu)$. It can be observed that the utilities of the involved BSs are non-decreasing, and the achievable sum rate of each BS has an upper bound due to the limited system bandwidth and transmission power constraint in practice. Therefore, the number of iterations of Algorithm 4 is limited, and it converges to a two-sided exchange-stable matching when there exists no swap-blocking pair that can further improve any player’s utility. The convergence proof for Algorithm 5 can be derived similarly, which is omitted here for brevity.

Proposition 3 (Complexity): The computational complexity of Algorithm 4 and 5 is upper bounded by $O(IJ^2 + A_{\text{max}}IN_{it})$ and $O(J^2K^2(\bar{N}_{it} + 1))$, respectively.

Proof: The complexity of the proposed matching-based algorithms depends on the initialization and swap process. In Algorithm 4, the initialization process requires each user to propose to one BS and each BS can accept or reject the proposal based on its preference. The complexity of constructing the initial User-BS matching state is $O(IJ^2)$ in the worst case. For the swap process in Algorithm 4, there are no more than $A_{\text{max}}$ users in each cell can perform the swap
operation with other \((J - 1)\) unassociated BSs, and thus the maximum swap operation number for each user is \(A_{\text{max}}(J - 1)\). Let \(N_{\text{it}}\) denote the number of total iteration when there is no swap-blocking pair. Thus, the complexity of swap operation is \(O(A_{\text{max}}IJN_{\text{it}})\). Overall, the complexity of Algorithm 4 can be calculated as \(O(IJ^2 + A_{\text{max}}IJN_{\text{it}})\).

In Algorithm 5, each (User-BS) unit can propose to multiple subchannels and each subchannel decides to accept or reject the proposal based on its preference. The complexity of the initialization process in Algorithm 5 is \(O(J^2K^2)\) in the worst case. For the swap process in Algorithm 5 each (User-BS) unit can perform the swap operation with other \((J - 1)\) units for a given subchannel, each subchannel can perform the swap operation with other \((K - 1)\) subchannels for a given (User-BS) unit. Let \(\bar{N}_{\text{it}}\) denote the number of total iteration when there is no swap-blocking pair. Thus, the complexity of swap operation is \(O(J(J - 1)K(K - 1)\bar{N}_{\text{it}})\). Overall, the complexity of Algorithm 5 can be calculated as \(O(J^2K^2(\bar{N}_{\text{it}} + 1))\), which completes the proof.

**Proposition 4 (Optimality):** All local optimal utilities of Algorithm 4 and 5 correspond to a two-sided exchange stable matching, but not vice versa.

**Proof:** This Proposition can be proved by contradiction. Suppose that the converged utility \(U_1 = U(\mu)\) of Algorithm 4 is a local optimal value. If \(\mu\) is not a two-sided exchange stable matching, it means that we can find a swap-blocking pair to further improve the utilities of users and/or BSs, which contradicts our initial assumption that the utility \(U_1 = U(\mu)\) is a local optimum. Therefore, it can be concluded that \(\mu\) is a two-sided exchange stable matching. However, not all two-sided exchange stable matchings \(\mu\) correspond to a local optimum of utility. This can be explained by the following example: given a stable matching \(\mu\), and assume that \(j = \mu(i), j' = \mu(i')\). It can be observed that \((i, i')\) is not a swap-blocking pair when \(\mu\) is a stable matching. Thus, BS \(j\) will not approve a swap operation with BS \(j'\), due to the fact that none of BSs’ utilities is improved after the swap operation. But, user \(i\) and \(i'\) will reap a lot of benefits if this swap operation is accepted, which may further improve the utility of BSs. The proof for Algorithm 5 can be derived similarly, which is omitted here for brevity.

## V. Numerical Results

We consider that there are 6 users, 3 BSs and 3 subchannels in the IRS-aided NOMA network. Specifically, in the 3D Cartesian coordinates, the location of user \(i\) is denoted by \((x_i, y_i, z_i) = (50i, 30, 0), i = 1, 2, \ldots, 6\), the location of BS \(j\) is denoted by \((x_j, y_j, z_j) = (100j, 0, 20),\)
j = 1, 2, 3, and the location of the IRS is denoted by \((x_{\text{IRS}}, y_{\text{IRS}}, z_{\text{IRS}}) = (200, 50, 20)\). We assume that the path loss model is given by \(L(d) = \varsigma_0(d)^{-a}\), where \(\varsigma_0 = -30\) dB is the path loss at the reference distance of 1 meter, \(d\) denotes the link distance, and \(a\) is the path loss exponent. Specifically, the path loss exponent of the BS-user, IRS-user and BS-IRS links are set as 3.2, 2.6, and 2.2, respectively. The small-scale fading model is given by \(F = \sqrt{\frac{\kappa}{1+\kappa}} F_{\text{LoS}} + \sqrt{\frac{1}{1+\kappa}} F_{\text{NLoS}}\), where \(\kappa = 2\) is the Rician factor for the BS-IRS link, \(F_{\text{LoS}}\) denotes the deterministic line-of-sight (LoS) channel component with \(|F_{\text{LoS}}| = 1\) and \(F_{\text{NLoS}}\) is random non-line-of-sight (NLoS) channel component that follows the Rayleigh distribution with parameter \(\iota = 1\). In particular, the small-scale fading \(F\) is simplified to Rayleigh fading when \(\kappa = 0\), which is applicable for the BS-user and IRS-user links. Then, the channel gain equals to the small-scale fading multiplied by the square root of the path loss. Moreover, the number of reflecting elements is set as \(M = 100\), the system bandwidth is assumed to be \(W = 3\) MHz. The noise power is \(\sigma^2 = -80\) dBm, and the minimum rate requirement of each user is assumed to be \(R_{\text{min}} = 500\) Kbps. The maximum transmission power of each BS is set as \(P_{\text{max}} = 23\) dBm, unless otherwise stated.

In order to validate the effectiveness of our proposed algorithms for the IRS-aided multi-cell NOMA network with multiple subchannels, the following three schemes are considered as benchmarks: 1) OMA without IRS: Frequency reuse and time division multiple access (TDMA) are considered in a multi-cell OMA network, where an BS communicates with at most one user in each time slot; 2) OMA with IRS: Compared to scheme 1, the only difference in scheme 2 is that there is one passive IRS with finite reflecting elements whose reflection matrix can be adjusted to intelligently reconfigure the wireless communication environment; 3) NOMA without IRS: All frequency can be reused adjacent cells, and the SIC approach is applied in each cell to decoding the intended signal of each user. Furthermore, we simulate 2000 trials, and all results are averaged over independent channel realization.

A. Performance Analysis of Achievable Sum Rate

Fig. 2 illustrates the empirical CDF performance of our proposed algorithms for the IRS-aided multi-cell NOMA network and the benchmark schemes. It can be observed that the NOMA schemes enjoys a significant performance gain than OMA schemes, which is mainly because NOMA allows multiple users to simultaneously reuse the same subchannel, and thus can obtain a higher spectrum efficiency. In particular, IRS-aided NOMA/OMA networks can achieve a better performance than the conventional NOMA/OMA schemes without IRS in terms of achievable
Fig. 2. Empirical CDF of sum rate.

Fig. 3. Sum rate versus the maximum transmission power.

sum rate, which demonstrates that the IRS is capable of enhancing the system performance by proactively modifying wireless channels between transmitters and receivers.

Fig. 3 compares the impact of the maximum transmission power of each BS on the achievable sum rate under four schemes. Similar observations are achieved in Fig. 2. NOMA schemes outperform the OMA schemes, and the performance gain becomes more significant when the IRS is leveraged. It can be observed that the achievable sum rate of all the four schemes increase when the maximum transmission power increases. It can be noticed that the lower the $P_{\text{max}}$ value is, the larger the slope of the sum rate curves will be. Thus, different from the approximately linear growth at a low $P_{\text{max}}$, the sum rate curves increase more slowly at a high $P_{\text{max}}$ due to the existence of intra-cell and inter-cell interference. Its worth pointing that the performance of NOMA/OMA schemes would reach their peak as the maximum transmission power increases to a certain threshold, and more reflecting elements have to be equipped on the IRS to further eliminate interference and improve performance.

Fig. 4 characterizes the achievable sum rate versus the locations of all users in diverse situations. When the coordinates of BSs and IRS are fixed, moving users farther away from all BSs by increasing their y-axis coordinates will lead to a lower sum rate. In this case, although users will be closer to the IRS, the increased signal attenuation is the dominant factor compared to power gain provided by the IRS. Specifically, it is worth noting that when $y_i \geq 40$ the OMA schemes with IRS outperform the conventional NOMA without IRS, this is because the power gain compensated by the IRS greater than the performance gap between OMA and NOMA.
More particularly, when $y_i = 60$ and $z_{IRS} = 10$, it can be obtained that the multi-cell IRS-aided NOMA and OMA networks are capable of providing up to 35.4% and 22.7% higher sum rate than the conventional NOMA and OMA schemes, respectively.

**B. Performance Analysis of System Energy Efficiency**

Energy efficiency of the IRS-aided multi-cell NOMA networks with multiple subchannels is defined as the ratio of the achievable sum rate over the total power consumption on all subchannels. Fig. 5 shows the impact of the maximum transmission power of each BS on energy efficiency. On the one hand, it can be observed that the trend in this figure is opposite to that in Fig. 3 where the energy efficiency decreases as the maximum transmission power increases. The reason is that the objective of maximizing sum-rate requires all available power at the BS, which is different to energy efficiency maximization. Hence, the proposed algorithms for sum-rate maximization objective lead to the decrease of energy efficiency. Indeed, once the QoS constraints are met, the energy efficiency becomes better when using full transmission power for a lower $P_{\text{max}}$. This is due to the fact that the interferences experienced by users are weak for low $P_{\text{max}}$, and the available transmission power can be fully utilized. On the contrary, for large $P_{\text{max}}$, the increased interferences deteriorate the energy efficiency rapidly. On the other hand, it shows that the larger the $P_{\text{max}}$ value is, the lower the slope of the energy efficiency curves will be. This is due to the fact that less performance gain will be obtained in terms of sum rate, when the BS’s maximum transmission power continues to increase.
Fig. 6. Energy efficiency versus reflecting elements.

Fig. 7. Energy efficiency versus the location of the IRS.

Fig. 6 plots the energy efficiency versus the number of reflecting elements at different heights. The trend can be sketched that the larger the $M$ value is, the larger energy efficiency will be obtained in the IRS-aided networks. Furthermore, compared to the benchmark schemes without IRS, the performance gains of energy efficiency in the IRS-aided NOMA networks are larger than that in the OMA schemes with IRS. This is because IRS can be used to suspend interferences and enhance desired signals at the same time in the NOMA networks, while it can only play a role in enhancing the signals in the considered OMA schemes. Particularly, when $M = 140$ and $z_{IRS} = 10$, it can be obtained that the IRS-aided NOMA and OMA networks are capable of enjoying 13.8% and 11.6% higher energy efficiency than the conventional NOMA and OMA schemes, respectively. Thus, it is a direct consequence of the fact that better performance can be achieved by employing a large number of reflecting elements to alleviate interferences and enhance the desired signals in the multi-cell IRS-aided NOMA networks.

Fig. 7 demonstrates the impact of the location of the IRS on energy efficiency. As it can be observed, when the locations of all BSs and users are fixed, the increase of y-axis coordinate of IRS leads to the degradation of the energy efficiency. This is due to the fact that the larger the BS-IRS and IRS-user distances are, the larger path loss will be experienced the reflective channel, and the smaller power gain will be brought by the IRS. Besides, when the height of the IRS drops, there is a slight performance improvement at the cost of coverage. In other words, there exists a trade-off between the sum rate and coverage area when deploying the IRS into wireless networks. Similar observations are achieved in Fig. 6, the IRS allows the available
power in the NOMA-based networks to be used more efficiently. Concretely, when $y_{\text{IRS}} = 25$ and $z_{\text{IRS}} = 10$, it can be obtained that the IRS-aided NOMA and OMA networks are capable of enjoying 27.8% and 21.2% higher energy efficiency than the conventional NOMA and OMA schemes, respectively.

VI. CONCLUSIONS AND FUTURE WORKS

In this paper, we investigated the sum-rate maximization problem in the IRS-aided multi-cell NOMA network, which was formulated as a MINLP problem. Then, relaxation methods were invoked to transform the intractable subproblems into convex ones, and efficient algorithms were designed to solve these challenging subproblems iteratively. Next, in order to achieve a two-sided exchange-stable state among users, BSs and subchannels, swap matching-based algorithms were proposed. Finally, numerical results under various settings demonstrated that through proactively reconfiguring the wireless communication environment, the IRS is capable of enhancing the system performance. Additionally, the proposed algorithms can significantly improve both the system throughput and energy efficiency.

Based on the algorithms designed in this paper, it is a promising direction to investigate a general algorithm for multi-IRS aided multi-cell NOMA networks under imperfect CSI feedback. Enlightened by our observations from the simulation results, an extension of the contributions in this paper is to find the optimal Pareto-front of multiple objectives, such as the system throughput, energy efficiency, and coverage, etc. In an effort to provide valuable guidance for the practical design and deployment of the finite-resolution IRS, the impacts of limited phase shifts are worthy of further discussion and analysis.

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