Dynamic Sliding Mode Control of Gyroscopic Balancing Vehicles Based on Disturbance Observer

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Abstract. Gyroscopic balancing vehicle is a new means of transportation with flexible structure and environmental friendliness, which has broad application prospects. However, it is also a typical system of nonlinear characteristics, static instability and parameter uncertainty, making its balance control problem difficult to dealing with. Based on the Lagrange equation of the dynamics model for single-frame CMG, a dynamic sliding mode controller with disturbance observer is designed to solve the severe chattering in conventional sliding mode method by compensating the parameter variation and external disturbance. The simulation results show that the introduced method effectively suppresses the chattering phenomenon, and the dynamic performance of the system is improved at the same time, which further enhances the robustness and application range of the gyroscopic balancing vehicle.

1. Introduction
As a kind of clean, convenient and safe means of transportation, the balancing vehicle has been widely developed and used in recent years [1, 2]. One of the important types is the gyroscopic balancing vehicle based on control moment gyroscope (CMG). The CMG is widely used in aerospace field as a control actuator, and it can generate a gyro moment to balance the interference of gravity and external torque, which makes the vehicle have better safety and stability. At the same time, gyroscopic balancing vehicle is also a typical system of static instability, nonlinear time-varying characteristics and parameters uncertainty, so that the research of related problems is full of challenges, especially the control of the rolling stabilization under still situation, which has great theoretical and practical value.

Louis Brennan firstly applied CMG to balance control and designed a gyro-stabilized monorail in 1905 [3], however, due to accuracy of sensors and system structure limitations it was quickly replaced by regular vehicles; With the development of micro devices for control and measurement, as well as the needs of modern living and environments, New progress has been made in the study of monorail vehicles. Beznos et al [4], introduced the system composition and principle of the gyroscopic self-balancing vehicle they developed themselves; Temitope Akinlua et al. [5] designed a dualing-CMG model with a scissor structure, and used PID algorithm to realize its stabilization; In 2014, Harun Yetkin et al. [6] installed the CMG on the bottom of a self-balancing bicycle, and designed a sliding mode variable structure controller achieving good dynamic performance, but there is still a problem that the system is chattering severely near the equilibrium point.
In order to further enhance the stability of the balancing vehicle and reduce the chattering phenomenon, this paper improves the conventional sliding mode method based on the existing model. The dynamic switching function is introduced to transfer the discontinuity term to the first derivative of the control input, and the dynamic sliding mode control law which is essentially continuous in time is obtained. Furthermore, a dynamic sliding mode controller based on disturbance observer is designed to compensate the parameter variation and external disturbance items of system. Through the analysis of simulation results, we can see that the disturbance signal can be well tracked, and the chattering is effectively suppressed with overshoot and adjustment time of the tilt angle reduced.

The outline of this paper is as follows. In Section 2, the composition and principle of gyroscopic balancing vehicle is introduced, and the dynamic equations are derived. In Section 3, a dynamic sliding mode controller based on disturbance observer is designed to improve the conventional sliding mode control method. Section 4 shows the results of simulation and make a conclusion after comparative analysis.

2. Model of Gyroscopic Balancing Vehicle

2.1. Structure and Principle

As shown in Figure 1, the vehicle adopts a monorail structure with two wheels laying front and rear, which has obvious static instability characteristic. The rear of the vehicle is equipped with a single-frame CMG as the actuator generating a gyro moment to maintain the balance. In the middle part are the control unit with a NI my RIO control board as core device and the measuring unit for the measurement of tilt angle. The front part of the vehicle is the power supply unit, and there is a safety isolating frame locating on the outermost side.

![Figure 1. Structure of Self-balancing Vehicle](image)

When the gyroscopic balancing vehicle produces a certain tilt angle, the rotation motor drives the flywheel to rotate at a high speed, and the precession motor applies a torque to make the flywheel precess relative to the vehicle body while rotating. According to the gyro moment effect, the external moment in the direction of precession axis will also generate a reaction torque to the vehicle body, the value of which can be calculated by equation (1)

\[ M = -H \times \omega \] (1)

And the direction is both orthogonal to the rotation axis and the precession axis, as shown in Figure 2.
2.2. *Dynamic Equations*

The balancing vehicle can be regarded as a rigid system composed of a car body, a high-speed rotating flywheel and a gyro frame [7]. Point $O$ is the contact point between the rear wheel and the ground, $O_b$ is the massive center of the car body. Considering that the flywheel only has an angular velocity $\omega$ about $Y_{gf}$ axis relative to the gyro frame, so that their common centroid can be defined as $O_{gf}$; The tilt angle of car body around the $X$-axis is defined as $\theta$, the angular velocity is $\dot{\theta}$, and the precession angle of flywheel and gyro frame around the $Z_b$ axis is defined as $\alpha$, the precession angular velocity is $\dot{\alpha}$; The vertical distance from $O_b$ and $O_{gf}$ to the ground is respectively $d_b$ and $d_{gf}$; The mass of the car body is $m_b$, the moment of inertia along the $X$ axis is $I_b$, the total mass of flywheel and gyro frame is $m_{gf}$, the polar moment of inertia is $I_{pg}$, $I_{pf}$, and the radial moment of inertia is $I_{rf}$, $I_{rg}$. The model in coordinate system is shown in figure 5.

\[
T = \frac{1}{2} (I_b + m_b d_b^2 + m_{gf} d_{gf}^2) \dot{\theta}^2 + \frac{1}{2} I_{pf} \omega^2 + \frac{1}{2} (I_{rf} + I_{rg}) (\dot{\alpha}^2 + \dot{\theta}^2 \cos^2 \alpha) + \frac{1}{2} (I_{pf} + I_{pg}) \dot{\theta}^2 \sin^2 \alpha
\]

According to above assumptions and parameter settings, the kinetic energy of the system can be obtained

\[
T = T_b + T_g + T_f = \frac{1}{2} m_b v_b^2 + \frac{1}{2} m_{gf} v_{gf}^2 + \frac{1}{2} I_b \dot{\theta}^2 + \frac{1}{2} I_{pf} \omega^2 + \frac{1}{2} I_{pg} (\dot{\theta} \sin \alpha)^2 + \frac{1}{2} I_{pf} (\dot{\theta} \cos \alpha)^2
\]
As gravity is the only powerful force of system, then the potential energy of the system is

$$V = m_b g d_b \cos \theta + m_{gf} g d_{gf} \cos \theta$$  \hspace{1cm} (3)

The rolling motion of the body and the precession of flywheel and gyro frame are selected as the two general coordinates of system, according to the Lagrange equation

$$\frac{d}{dt} \frac{\partial T}{\partial q_j} - \frac{\partial T}{\partial q_j} + \frac{\partial V}{\partial q_j} = Q_j \quad (j = 1, 2, ..., k)$$

For the general coordinate $\theta$, the non-potential general moment is

$$Q_\theta = -I_{pf} \omega \dot{\theta} \cos \alpha$$

While $q_1 = \theta$, the equation of tilt angle can be derived as

$$\dot{\theta} \left[ l_b + m_b d_b^2 + m_{gf} d_{gf}^2 + l_p \sin^2 \alpha + l_r \cos^2 \alpha \right] + 2 \sin \alpha \cos \alpha \left( l_p - l_r \right) \dot{\theta} \dot{\alpha} - g \left( m_b d_b + m_{gf} d_{gf} \right) \sin \theta = -I_{pf} \omega \dot{\theta} \cos \alpha$$  \hspace{1cm} (4)

For the general coordinate $\alpha$, the non-potential general moment is

$$Q_\alpha = K_m i - I_{pf} \omega \dot{\theta} \cos \alpha$$

Where $K_m$ is the torque amplification factor of precession motor. While $q_2 = \alpha$, the equation of precession angle can be derived as

$$\ddot{\alpha} l_r - \dot{\theta}^2 (l_p - l_r) \sin \alpha \cos \alpha = K_m i - I_{pf} \omega \dot{\theta} \cos \alpha$$  \hspace{1cm} (5)

At last, the linearization model of gyroscopic balancing vehicle system near the equilibrium point is

$$\begin{cases}
\dot{\theta} \left[ l_b + m_b d_b^2 + m_{gf} d_{gf}^2 + l_p \right] - \\
g \left( m_b d_b + m_{gf} d_{gf} \right) \theta + I_{pf} \omega \dot{\alpha} = 0 \\
\ddot{\alpha} - K_m i - I_{pf} \omega \dot{\theta} = 0
\end{cases}$$  \hspace{1cm} (6)

Select $x = \begin{bmatrix} \theta \\ \dot{\theta} \\ \alpha \\ \dot{\alpha} \end{bmatrix}$ as state variable, $y = \begin{bmatrix} \theta \\ \alpha \end{bmatrix}$ as output variable, and $u = i$ as control input, so that the linear state space expression derived from the dynamics model is

$$\dot{x} = \begin{bmatrix} \dot{\theta} \\ \dot{\theta} \\ \dot{\alpha} \\ \dot{\alpha} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & - \frac{g}{l_p} & \frac{g}{l_p} \\ 0 & 0 & 0 & 1 \\ 0 & \frac{g}{l_r} & 0 & 0 \end{bmatrix} \begin{bmatrix} \theta \\ \dot{\theta} \\ \alpha \\ \dot{\alpha} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ \frac{K_m}{l_r} \end{bmatrix} i = Ax + Bu$$

$$y = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} \theta \\ \dot{\theta} \\ \alpha \\ \dot{\alpha} \end{bmatrix} = Cx$$  \hspace{1cm} (7)
Here, $G = g(m_bd_b + m_gfd_gf)$, $M = I_b + m_bd_b^2 + m_gfd_gf^2 + I_r, H = I_pf\omega$.

3. Design of Improved Sliding Mode Controller

3.1. Dynamic sliding mode control

From a theoretical point of view, the variable structure control of sliding mode can design the sliding mode arbitrarily according to the requirements, so that the motion of sliding mode is independent of the internal parameter changes and external disturbances, and the system has stronger robustness than the general continuous one [8]. Assuming that the switching function has an ideal switching characteristic, the control input is not limited, and there is no accuracy error in measurement information, then sliding mode of the system will always be smooth and gradually converges to the equilibrium position without chattering. However, these assumptions in a realistic system are unlikely to be fully established, especially for sampling systems, resulting in saw-tooth waves on the sliding surface due to the hysteresis of time and space. Therefore, we change the traditional selection methods of switching function that only depends on the state variable but does not consider the control input, and the first derivative of the control input is introduced into the switching function by the differential link. So the discontinuous terms in control input are eliminated, and we can obtain a dynamic sliding mode control law that is substantially continuous in time.

For state space equations of discrete-time system

$$x(k+1) = Ax(k) + Bu(k)$$

(8)

Set the expected value of its state variable $R = [0 \ 0 \ 0 \ 0]^T$, then the tracking error is

$$e(k) = R(k) - x(k)$$

Define the switch function as

$$s(k) = Ce(k) + Du(k)$$

(9)

Here $D > 0$, $C = [C(1) \ C(2) \ C(3) \ C(4)]^T$. When $s = 0$, $Du(k) = -Ce(k)$, $C < 0$ is required in order to form a negative feedback making the system tend to be steady, and the first derivative of the switching function is

$$s(k+1) - s(k) = C[R(k + 1) - R(k) - x(k + 1) + x(k)] + Du(k + 1) - u(k)$$

(10)

Besides, exponential approach law (11) is chosen to satisfy the arrival condition and speed up the convergence of sliding mode motion.

$$s(k+1) - s(k) = -\epsilon T \text{sgn}(s(k)) - \eta Ts(k)$$

(11)

From formula (10) and (11), the equations of dynamic sliding mode control can be derived

$$u(k + 1) - u(k) = D^{-1}[-C(x(k + 1) - x(k)) - \epsilon T \text{sgn}(s(k)) - \eta Ts(k)]$$

After deformation it is

$$u(k) = D^{-1} \left[-C(x(k) - x(0)) - \frac{\epsilon T}{k} \text{sgn}(s(k)) - \frac{\eta T}{k} s(k) \right]$$

(12)
3.2. Dynamic sliding mode control based on disturbance observer

The invariance of sliding mode is an important reason that variable structure control has been widely concerned since the 1980s, and its realization mainly relies on large gain coefficient of the control switching term. However, the large gain coefficient may reduce robustness of system, causing strong chattering when there is large external interference. A disturbance observer is established in this paper, which can observe and compensate the external disturbance, and the gain coefficient of switching term in the sliding mode controller is appropriately reduced. At the same time, the influence of the unmodeled dynamics as well as the chattering phenomenon is effectively improved, making the system more robust.

For linear system with uncertain parameters

\[ x(k+1) = (A + \Delta A)x(k) + Bu(k) + f(k) \]

Where \( b > 0 \), assuming that the matching condition is met

\[ \Delta A = BA, \quad f = B \tilde{f} \]

Then the equation of original system is

\[ x(k+1) = Ax(k) + B[u(k) + d(k)] \]  \hspace{1cm} (13)

Here \( d(k) = \tilde{A}x(k) + \tilde{f}(k) \), taking the switching function of disturbance observer as

\[ s(k) = C^T e(k) \]

While \( s(k+1) = s(k) = 0 \), then

\[ s(k+1) = C^T e(k+1) \]
\[ = C^T x(k+1) - C^T R \]
\[ = C^T Ax(k) + C^T Bu(k) + C^T Bd(k) - C^T R \]
\[ = \eta Ts(k) - \varepsilon T \text{sgn}(s(k)) + C^T B \tilde{d}(k) = 0 \]  \hspace{1cm} (14)

Here \( \tilde{d}(k) = d(k) - \hat{d}(k) \), combining formula (13) and (14), the disturbance observer can be designed as

\[ \hat{d}(k) = d(k - 1) + (C^T B)^{-1} g T[s(k) - \eta s(k - 1) + \varepsilon \text{sgn}(s(k - 1))] \]  \hspace{1cm} (15)

Where \( g \), \( \eta \) and \( \varepsilon \) are all positive real numbers. The dynamic sliding mode controller has been designed in the section 2.1. On this basis, the dynamic sliding mode controller based on the disturbance observer can be obtained

\[ u(k) = -\hat{d}(k) + D^{-1} \left[ -C(x(k) - x(0)) - \sum_{i=4}^{k-1} \eta T s(k) - \sum_{i=4}^{k-1} \xi T s(k) \right] \]  \hspace{1cm} (16)

4. Simulation and Analysis

According to the parameters of balancing vehicle in Table 1, matrix \( A \) and \( B \) can be calculated as

\[ A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 27.41 & 0 & 0 & -0.74 \\ 0 & 0 & 0 & 1 \\ 0 & 38.06 & 0 & 0 \end{bmatrix} \quad B = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1.19 \end{bmatrix} \]
The vector of sliding mode is designed by Ackermann formula as \( \mathbf{C} = [-4.7673, 48.3564, -8.7401, 18.6534] \), adding white noise interference as the uncertainty of the system, and the controller parameters are defined as \( \mathbf{D} = [0.01, 0.8] \), \( \varepsilon = 0.01 \), \( \eta = 0.8 \), \( \zeta = 0.95 \), with sampling interval \( T = 0.02 \) s. \( \mathbf{x}(0) = [0.3, 0.5, 0] \) is set as the initial value to perform simulations in MATLAB according to the equations of control input in formula (12) and (16). Shown in Figures 4, 5 and 6 are the tilt angle controlling curve of vehicle body, the precession angle controlling curve of CMG and the control input changing curve respectively using conventional control method and dynamic control method based on disturbance observer. We can acknowledge that compensation of the disturbance observer makes the system more robust without serious chattering, and the adjustment time as well as overshoot of system is improved comparing with the conventional sliding mode control curve. Figure 7 shows the observation result of added white noise interference, proving the designed disturbance observer can track the interference signal in a sampling period and effectively solve the negative influence caused by uncertainty.

### Table 1. Parameters of Balancing Vehicle

| Parameter                  | Value                  | Unit          |
|----------------------------|------------------------|---------------|
| \([m_b, m_g, f]\)          | \([25.26, 16.96]\)     | kg            |
| \([d_g, d_b]\)             | \([36.56, 21.63]\)     | cm            |
| \([I_b, I_{pf}, I_{pg}, I_{ff}, I_{rg}]\) | \([7828.3, 856.70, 620.88, 436.55, 368.75]\) | kg \cdot cm² |
| \([K_m, \omega]\)         | \([0.062, 1880]\)      | [N \cdot m/A, r/min] |

5. **Conclusion**

The gyroscopic balancing vehicle is more flexible than traditional four-wheeled vehicles, and has an excellent performance in safety and stability. It consumes less energy as well as can adapt to rugged and narrow road conditions better. Based on the construction of dynamic model, this paper improves the
switching function of sliding mode for the problem that the conventional control method is not ideal for
the rolling stabilization. Moreover, a disturbance observer is used to compensate the uncertainty of
system. After comparing and analyzing the simulation results, it is obvious that the designed dynamic
sliding mode controller based on disturbance observer can effectively suppress the chattering
phenomenon occurring in the conventional sliding mode control. It also improves the performance
indicators such as adjustment time and overshoot of system, which further enhances the adaptive ability
and application range of the gyroscopic balancing vehicle.

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