A Cautionary Note on Gamma Ray Burst Nearest Neighbor Statistics

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ABSTRACT

In this letter we explore the suggestion of Quashnock and Lamb (1993) that nearest neighbor correlations among gamma ray burst positions indicate the possibility of burst repetitions within various burst sub-classes. With the aid of Monte Carlo calculations we compare the observed nearest neighbor distributions with those expected from an isotropic source population weighted by the published BATSE exposure map. The significance of the results are assessed via the Kolmogorov-Smirnov (K-S) test, as well as by a comparison to Monte Carlo simulations. The K-S results are in basic agreement with those of Quashnock and Lamb. However, as Narayan and Piran (1993) point out, and the Monte Carlo calculations confirm, the K-S test overestimates the significance of the observed distributions. We compare the sensitivity of these results to both the definitions of the assumed burst sub-classes and the burst positional errors. Of the two, the positional errors are more significant and indicate that the results of Quashnock and Lamb may be due to systematic errors, rather than any intrinsic correlation among the burst positions. Monte Carlo simulations also show that with the current systematic errors, the nearest neighbor statistic is not very sensitive to moderate repetition rates. Until the BATSE statistical and systematic errors are fully understood, the burst nearest neighbor correlations cannot be claimed to be significant evidence for burst repetitions.

Subject Headings: gamma rays: bursts — methods: statistical

1. Introduction

Currently the three most favored scenarios for the location of gamma ray bursts are that they originate in the Oort cloud, an extended galactic halo, or at cosmological distances (for recent reviews, see Blaes 1993, Hartmann 1994). If bursts are found to repeat from the same location in the sky, this would have profound implications for which of these three models is correct. For example, it is difficult to imagine an Oort cloud model that allows repetitions, whereas a galactic halo model could allow for several events from a single neutron star (see the aforementioned reviews for references to specific models). In the cosmological model, burst repetitions can be produced by the effects of gravitational lensing, although these should be relatively rare events (Paczyński 1986; Mao 1992, 1993; and others).

One way to search for repetitions and other anisotropies is to perform a nearest neighbor analysis and search for excesses at small angles on the order of the BATSE statistical and systematic errors. That is, one can ask the question: “For a randomly chosen burst, what is the probability that its nearest neighbor lies within an angle $\theta$, and how does that probability compare with what is expected from an isotropic, non-repeating distribution?” Quashnock and Lamb (1993) (QL93 hereafter) have addressed this question and claim to find a burst excess at small angular scales. Here we investigate the rigorousness of this result and try to determine whether the observed deviations from the expected distributions can be attributed to an intrinsic property of the sources.
2. Methods

As QL93 point out, if \((N + 1)\) gamma ray bursts are drawn from an isotropic distribution, then

\[
P(y) = 1 - \exp(-Ny/2)
\]

is the probability that any given burst has at least one neighbor that lies within an angle \(\theta\) such that \(1 - \cos \theta \leq y\). This formula does not take into account the detector response, therefore we use a Monte Carlo approach instead. Random samples of \((N + 1)\) bursts are drawn from an isotropic distribution weighted by the detector response (calculated from a curve fit to the azimuthally averaged BATSE exposure map; Fishman et al. 1992). Several hundred of these random samples are averaged together to yield the theoretical distributions.

The theoretical distributions are compared to distributions calculated from the publicly available burst data set (Fishman et al. 1992) which currently contains positions for 260 bursts. Following QL93, the bursts are divided into two categories, “Type I” and “Type II”, based upon a measure of their variability (cf. Lamb et al. 1993; Lamb and Graziani 1993a,b). Defining

\[
V = (\langle C^{64} \rangle_{max}/\langle C^{1024} \rangle_{max}, \text{ where } \langle C^{64} \rangle_{max} \text{ and } \langle C^{1024} \rangle_{max}\text{ are respectively the maximum count rate in 64 ms and 1024 ms, Type I bursts have } \log V \leq -0.8, \text{ and Type II bursts have } \log V > -0.8.
\]

Type I is found to contain 160 bursts, while Type II is found to contain 44 bursts. (\(V\) is not a well defined quantity for all 260 bursts.) This is not the only basis for subdividing the bursts (cf. Kouveliotou et al. 1993), and variations are explored in this work. Specifically, “Type Ia” will be defined to have \(\log V \leq -0.7\) and “Type IIa” will be defined to have \(\log V > -0.7\). Type Ia contains 167 bursts and Type IIa contains 37 bursts. “Type Ir” and “Type IIr” are based upon a completely random subdivision of those bursts with a defined \(V\). 160 bursts are assigned to Type Ir and 44 bursts are assigned to Type IIr. These other subdivisions are defined in order to explore the sensitivity of the results to the classifications. All three varieties of Type I bursts are further subdivided into three roughly equal groups – Faint, Medium, and Bright – ordered by their maximum count rate in 1024 ms.

QL93 compare the data distributions to the theoretical distributions with the aid of the Kolmogorov-Smirnov (K-S) test (Press et al. 1986), which is a measure of the maximum deviation of the measured distribution from the theoretical distribution. This deviation is assigned a “\(Q\)” value, which is a measure of the likelihood that the two distributions are the same. However, as Narayan and Piran (1993) recently have shown, the K-S test actually overestimates the true significance of an overabundance of nearest neighbors. This is because the K-S test assumes uncorrelated data points, whereas nearest neighbors tend to become correlated as their separations decrease. A more proper statistic can be calculated from Monte Carlo simulations of the maximum deviations between the measured distributions and the theoretical distributions. (In this paper the significance of the deviations is estimated from 10,000 Monte Carlo simulations of each theoretical distribution.) The \(Q\) from the K-S test \(Q_{KS}\) are always smaller than the \(Q\) from the Monte Carlo
simulations \( (Q_M) \), and to a good approximation \( Q_M \sim Q_{KS}^{0.7} \).

The above statistics do not account for experimental errors or uncertainty. First, there is the worry that systematic errors may actually produce a false signal. Second, whereas one expects averaging the burst positions over the error boxes to degrade the signal’s significance, the measured significance should be near the “mean” significance. A mean significance near unity and a measured significance several standard deviations away from this mean could indicate several possibilities, such as systematic errors producing a false signal, or a very unusual chance occurrence of a false signal. At the very least such an occurrence indicates that the errors need to be studied in detail.

The public data set contains one sigma error bars that give the radius, in degrees, of a circle with the same area as the calculated BATSE one sigma error ellipse (Fishman et al. 1992). In this work the error box is taken to be gaussianly distributed in right ascension and declination, with the one sigma square having the same area as the larger of the BATSE error ellipse or a circle of radius 4° (the BATSE systematic error). Each burst data set is then varied with these errors in mind, and the mean and standard deviation of \( \log Q_M \) are calculated for each burst sub-class.

3. Results

The results of the calculations are presented in Table 1. With the proper Monte Carlo estimation of the significance, only the All Type I & II and All Bursts sub-classes deviate significantly from the expected distribution \( (Q_M = 2.4 \times 10^{-3}, 8.5 \times 10^{-3}, \text{respectively}) \). Bright Type I and Ia and the Faint & Bright Type Ir sub-classes are marginally significant \( (Q_M \sim 1 - 3 \times 10^{-2}) \). The \( Q_{KS} \) values, however, are in rough agreement with those of QL93 (who take slightly different subdivisions of the Type I class). In agreement with QL93, the maximum deviations between the measured and theoretical distributions tend to be burst excesses that occur at an angular scale of 4°–5°, the BATSE systematic error. All varieties of Bright Type I bursts, however, have deficits at angles near 20°. Plots of \( P(y) \) for two of the measured and theoretical distributions are presented in Figures 1a,b.

It is questionable if the Type I – Type II division plays any role in determining the deviations from the expected distributions. None of the individual Type I or Type II sub-classes shows significant deviations on its own. In addition, the third most significant measurement (Bright Type Ia) was based on a different definition than that of QL93, and the fourth most significant measurement (Faint & Bright Type Ir) was based on a random subdivision. There is the worry that given enough “reasonable” subdivisions, some fraction of them must show “significant” deviations.

The results of the Monte Carlo simulations show that all of the burst sub-classes, with the exceptions of Faint Type Ia and the Faint Type Ir, have measured \( \log Q_M \) that are less than the mean \( (\log Q_M) \). Furthermore, most of the means are within 1.2 standard deviations \( (\sigma_{\log Q_M}) \) of \( \log Q_M = 0 \), with only the Bright Type I sub-classes falling more than 2 standard deviations from 0. Histograms for the distribution of \( \log Q_M \) for two sub-classes are presented in Figures 1a,b. In general, all of the sub-classes have \( \log Q_M \) histograms that are reasonably consistent with no
deviations from an isotropic, non-repeating distribution.

This result is somewhat surprising in light of the apparent significance of the $Q_M$ for the All Type I & II and All Bursts sub-classes. Figure 1b shows that the measured log $Q_M$ for the former sub-class falls on the tail of the histogram (4.6 $\sigma_{\text{log } Q_M}$ from the mean). In addition, the measured log $Q_M$ for All Bursts falls 2.6 $\sigma_{\text{log } Q_M}$ from the mean, and the measured log $Q_M$ for the various Faint & Bright Type I sub-classes fall 2 $\sigma_{\text{log } Q_M}$ from the mean. This is an unlikely situation if the errors are solely statistical. Without detailed knowledge of the shapes of the BATSE error boxes, however, any preferential clustering or declustering cannot be modeled. Of the most significant measurements, only the various Bright Type I bursts have measured log $Q_M$ that fall within 2 $\sigma_{\text{log } Q_M}$ of the mean. However, these measurements show burst deficits at $\sim 20^\circ$ rather than burst excesses at $\sim 4^\circ$, and they are only marginally significant.

As an illustration of the above points, Figure 2 presents the results of a Monte Carlo simulation where it was assumed that each burst has a 20% probability of repeating once within the data set. Bursts were generated and checked for repetition until a total of 204 bursts was reached. This process was repeated for 100 data sets. A 4$^\circ$ systematic error (distributed as described above) was applied to each set and the logarithm of the significance value was calculated ($\text{log } Q_R$ in Figure 2). This was repeated 100 times. Each of these simulations in turn had a further 4$^\circ$ systematic error applied and the mean logarithm of the significance value was calculated ($\text{log } Q_{RR}$ in Figure 2). This was done to simulate the posterior averaging over error boxes that we performed on the measured distributions. Figure 2 plots log $Q_R$ vs. log $Q_{RR}$ for these 10,000 simulations. There are two things to note here. First, even with a 20% chance of repetition, the average log $Q_R$ is greater than $-2$, indicating that the nearest neighbor test is not very sensitive in light of the systematic errors. Second, the measured log $Q_M$ (corresponding to log $Q_{RR}$) are unusually close to 0. Only 2 out of 10,000 simulations have both log $Q_R < \text{log } Q_M$ and log $Q_{RR} > \text{log } Q_M$, where the measured values are for the 204 Type I & II Bursts with only the 4$^\circ$ systematic error applied. Again, the measurements that are most consistent with the distribution are the results for the various Bright Type I sub-classes, which show burst deficits not excesses.

4. Conclusions

We have reexamined the analysis of Quashnock and Lamb (1993) in order to determine whether or not the measured gamma ray burst nearest neighbor distributions differ significantly from the theoretical distributions. Following QL93, the bursts were subdivided into several sub-classes which deviated from the theoretical distributions with marginal significance. It is unclear to what extent these significances were effected by the definition of the sub-classes.

Varying the data by the larger of the systematic or published errors showed that the logarithm of the measured significance values, log $Q_M$, were typically lower than their means. The smallest (“most significant”) measured log $Q_M$ were seen to be $\sim 2 - 4.6$ standard deviations away from this mean. This is an unlikely situation for a purely statistical error, however, it might be possible
to explain with a systematic error. The most “robust” results (Bright Type I Bursts) showed burst deficits at 20° rather than burst excesses at 4°.

Monte Carlo simulations that include both single and double application of the systematic errors show the extent to which the experimental results are unusual. They also show that the nearest neighbor statistic is not very sensitive when 20% or fewer of the bursts repeat. We end by reiterating the warning that without an understanding of the burst positional errors, the burst nearest neighbor correlations cannot be claimed to be significant evidence for burst repetitions or clustering.

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Table 1: Burst Correlation Significance Values

| Data Set                  | (# bursts) | $Q_{KS}$     | $Q_M$     | $\log Q_M$ | $\sigma_{\log Q_M}$ |
|---------------------------|------------|--------------|-----------|-------------|----------------------|
| Faint “Type I”            | (53)       | $3.6 \times 10^{-2}$ | 0.12      | $-0.54$     | 0.47                 |
| Middle “Type I”           | (54)       | $6.2 \times 10^{-2}$ | 0.16      | $-0.46$     | 0.40                 |
| Bright “Type I”           | (53)       | $4.6 \times 10^{-3}$ | $2.5 \times 10^{-2}$ | $-1.25$     | 0.57                 |
| “Type II”                 | (44)       | 0.21         | 0.40      | $-0.42$     | 0.38                 |
| Faint & Bright “Type I”   | (106)      | $1.4 \times 10^{-2}$ | $5.8 \times 10^{-2}$ | $-0.46$     | 0.38                 |
| Faint “Type Ia”           | (56)       | 0.20         | 0.39      | $-0.49$     | 0.43                 |
| Middle “Type Ia”          | (55)       | 0.11         | 0.26      | $-0.58$     | 0.47                 |
| Bright “Type Ia”          | (56)       | $1.6 \times 10^{-3}$ | $1.2 \times 10^{-2}$ | $-1.32$     | 0.58                 |
| “Type IIa”                | (37)       | 0.15         | 0.31      | $-0.40$     | 0.38                 |
| Faint & Bright “Type Ia”  | (112)      | $1.4 \times 10^{-2}$ | $5.5 \times 10^{-2}$ | $-0.45$     | 0.38                 |
| Faint “Type Ir”           | (53)       | 0.26         | 0.47      | $-0.45$     | 0.40                 |
| Middle “Type Ir”          | (54)       | 0.11         | 0.24      | $-0.50$     | 0.41                 |
| Bright “Type Ir”          | (53)       | $1.1 \times 10^{-2}$ | $4.8 \times 10^{-2}$ | $-1.23$     | 0.62                 |
| “Type IIr”                | (44)       | 0.20         | 0.39      | $-0.40$     | 0.35                 |
| Faint & Bright “Type Ir”  | (106)      | $3.5 \times 10^{-3}$ | $2.3 \times 10^{-2}$ | $-0.45$     | 0.40                 |
| All “Type I & II”         | (204)      | $9.7 \times 10^{-5}$ | $2.4 \times 10^{-3}$ | $-0.51$     | 0.46                 |
| All Bursts                | (260)      | $9.7 \times 10^{-4}$ | $8.5 \times 10^{-3}$ | $-0.61$     | 0.50                 |

Table 1: The significance values that each burst sub-class with the given # of bursts is drawn from an isotropic distribution weighted by the BATSE exposure map. $Q_{KS}$ are estimated from the Kolmogorov-Smirnov test, and the $Q_M$ are estimated by comparison to 10,000 Monte Carlo simulations. The mean of the logarithm of this value ($\log Q_M$) and the standard deviation of the logarithm ($\sigma_{\log Q_M}$), are based upon 1000 Monte Carlo calculations that include the burst positional uncertainty.
Figure Captions

Figure 1a – Top: Measured cumulative distribution, $P(y)$, for the 53 brightest Type I bursts, and Monte Carlo calculation for the theoretical distribution, $P(y)$, for 53 bursts drawn from an isotropic distribution weighted by the BATSE exposure map.

Bottom: Histogram of $\log Q_M$, the logarithm of the significance value, calculated for 1000 realizations of the Bright Type I data that included the statistical and systematic errors in the burst positions.

Figure 1b – Same as in Figure 1a, except now for the 204 bursts that represent the combined All Type I & II Burst sub-class.

Figure 2 – Logarithm of significance value for data sets with once applied errors ($\log Q_R$) vs. the mean logarithm of significance for sets with twice applied errors ($\log Q_{RR}$). 100 burst data sets were generated where each burst had a 20% chance of repeating once. Bursts were generated until a total of 204 bursts was reached. 100 values of $\log Q_R$ vs. $\log Q_{RR}$ were generated for each data set. The solid diamond corresponds to the mean values for the 10,000 runs, and the solid line corresponds to the best fit straight line. The open circles correspond to the experimental values of $\log Q_M$ vs. $\log Q_M$ for the most significant burst sub-classes.