CNNs on Surfaces using Rotation-Equivariant Features

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Gifs: https://rubenwiersma.nl/hsn

Paper presentation by Thomas Lew, 02/09/2022
Applying CNNs to 3D deep learning

2D data (images)

https://sthalles.github.io/deep_segmentation_network/

Krizhevsky et al, 2012

3D data (triangle meshes)
Applying CNNs to 3D deep learning

**2D data** (images)

- MNIST
- https://sthalles.github.io/deep_segmentation_network/

**3D data** (triangle meshes)

- Rotation-invariant vector features
- Rotation-equivariant vector features

Krizhevsky et al, 2012
CNNs on meshes / charting approaches

Graph CNNs

MeshCNN [Hanocka et al, 2019]
Charting approaches (CNNs: 2D to 3D)

2D

```
1 x1 1 x0 1 x1 0 0
0 x0 1 x1 1 0
0 x1 0 x0 1 1
0 0 1 1 0
0 1 1 0 0
```

Image

Convolved Feature

3D

https://danielnouri.org/notes/2014/12/17/using-convolutional-neural-nets-to-detect-facial-keypoints-tutorial/
Charting approaches

a) define a kernel on $\mathbb{R}^2$

![Gaussian kernel on R2](https://www.youtube.com/watch?v=kg1wRBGUYqk)

Different kernels are possible, see

- [Poulenard and Ovsjanikov 2018]
- [Boscaini et al. 2016]
- [Monti et al. 2017]

b) apply kernel to tangent plane

$$T_pS \cong \mathbb{R}^2$$

e.g., with exponential map, see GeodesicCNN [Masci et al 2015]
The Vector Heat Method

N. Sharp, Y. Soliman, and K. Crane, ACM Trans. Graph. 38(3), 2019

triangle mesh

Allows efficiently computing **tangent spaces** on meshes and **parallel transport** maps
Charting approaches: limitations

Rotation ambiguity

Convolving

*how to move the filter along surface manifold without introducing rotations?*
Proposed approach

- Features expressed as complex vectors $Xe^{i\theta}$
- Use circular harmonics (harmonic networks: learn radial and angular parameters)
  - rotational-equivariant kernels
- Propose convolutional filters that apply to surfaces
  - Idea: circular harmonics + parallel transport
## Circular harmonics

Circular harmonic filters

\[ W_m(r, \theta, R, \beta) = R(r)e^{i(m\theta+\beta)} \]

**Rotation equivariance**

\[ [W_m \ast x^\phi](p) = e^{im\phi}[W_m \ast x^0](p) \]

- \((r, \theta)\) Polar coordinates
- \(R: \mathbb{R}^+ \rightarrow \mathbb{R}\) Radial profile
- \(\beta\) Phase offset
- \(m \in \mathbb{Z}\) Rotation order

### Input

- **Convolve**

### Rotational invariant

- **Example \(m=0\)**

### Rotational equivariant

- **Example \(m=1\)**
Circular harmonics => Harmonic Nets

\[
[W_m \star x^\phi](p) = e^{im\phi} [W_m \star x^0](p)
\]

[Poulenard and Guibas 2021] uses “spherical” harmonics instead, since 3D pointcloud
Parallel Transport (exponential map)

\[ P_{j \rightarrow i}(x_j) = e^{i\phi_{ij}} x_j \]
Convolution on a surface

\[ x^{(l+1)}_i = \sum_j w_j \left( R(r_{ij}) e^{im\theta_{ij}+\beta} P_{j\rightarrow i}(x^{(l)}_j) \right) \]
Convolution on a surface

\[ x^{(\ell+1)}_i = \sum_j w_j \left( R(r_{ij}) e^{i(m\theta_{ij} + \beta)} P_{j \rightarrow i}(x^{(\ell)}_j) \right) \]

- Integration weights (*depend on mesh*)
- Circular harmonics (H-Net) (*rotation equivariant*)
- Parallel transport
ReLU

\[ X e^{i\theta} \mapsto \text{ReLU}(X + b) e^{i\theta} \]

Only the magnitude (radius) of \( X e^{i\theta} \) is changed
Model architecture

deep U-ResNet architecture from [Poulenard and Ovsjanikov 2018]
Dataset and metrics

- shape classification: SHREC dataset [Lian et al. 2011],
- correspondence: FAUST dataset [Bogo et al. 2014]
- shape segmentation: human segmentation dataset [Maron et al. 2017]
HSNs perform better than state-of-the-art

| Method      | Accuracy |
|-------------|----------|
| HSN (ours)  | 96.1%    |
| MeshCNN     | 91.0%    |
| GWCNN       | 90.3%    |
| GI          | 88.6%    |
| MDGCNN      | 82.2%    |
| GCNN        | 73.9%    |
| SG          | 62.6%    |
| ACNN        | 60.8%    |
| SN          | 52.7%    |

| Method      | # Features | Accuracy |
|-------------|------------|----------|
| HSN (ours)  | 3          | 91.14%   |
| MeshCNN     | 5          | 92.30%   |
| SNGC        | 3          | 91.02%   |
| PointNet++  | 3          | 90.77%   |
| MDGCNN      | 64         | 89.47%   |
| Toric Cover | 26         | 88.00%   |
| DynGraphCNN | 64         | 86.40%   |
| GCNN        | 64         | 86.40%   |
| ACNN        | 3          | 83.66%   |
Features visualization + ablation

Table 4. Results of HSN tested on shape segmentation for multiple configurations.

| Method             | Streams ($M = \ldots$) | Accuracy  |
|--------------------|------------------------|-----------|
| HSN                | 0, 1                   | 91.14%    |
| HSN                | 0                      | 88.74%    |
| HSN (parameters $\times 4$) | 0            | 87.25%    |
| HSN (pc aligned)   | 0, 1                   | 86.22%    |

Fig. 13. Architecture for classification of Rotated MNIST.

Fig. 14. Validation accuracy per training epoch several configurations of HSN on shape segmentation.
Conclusion

• Proposed convolutional filters that apply to surfaces
  ○ Idea: circular harmonic kernels + parallel transport
    Rotational invariant/equivariant depending on filter order M

• Better performance and requires less parameters than other approaches

• Next:
  ○ using the learned features / representations for other tasks
  ○ extensions to pointclouds
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