From inflation to late acceleration: A new cosmological paradigm

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A new idea of deriving a cosmological term from an underlying theory has been proposed in order to explain the expansion history of the universe. We obtain the scale factor with this derived cosmological term and demonstrate that it reflects all the characteristics of the expanding universe in different era so as to result in a transition from inflation to late acceleration through intermediate decelerating phases by this single entity. We further discuss certain observational aspects of this paradigm.

Explaining the expansion history of the universe is one of the leading problems facing physicists over the years. Gravitational force, by its nature, is attractive. But both Supernovae [1] and CMB data [2] suggest that the present universe is accelerating. Hence there must be some yet-unknown entity that supplies an effective ‘anti-gravity’. Understanding the nature and evolution of this entity has drawn much attention from the gravity as well as field theory communities. So far the most promising candidate is the cosmological constant $\Lambda$ [3]. However, if cosmological constant is there, one needs extreme fine-tuning to match the observationally tiny value of $\rho_{\Lambda} \sim 10^{-124} GeV^4$ for recent time from any underlying theory, which is still an open issue. Subsequently, there are several alternative candidates. Examples include modifications of either the matter sector (quintessence, Kessence, Chaplygin gas etc. [4]) or the gravity sector (Brans-Dicke theory [5], $f(R)$ gravity [6], DGP [7] or generalized DGP-RS models [8]). Each one of them has its own merits and demerits, which makes the issue further wide open.

On the other hand, inflationary scenario [9] suggests that the universe had undergone a very fast accelerated expansion at early time, which requires a large cosmological constant. So, it appears nearly impossible to unify inflation and late acceleration under this common umbrella. Precisely, there is no prima facie solution to the problem: why should an universal constant take vastly different values in two different epochs? A possible way-out is that, maybe a variable cosmological term, whose value decays with time, can serve the purpose, with the observational bound to its present value $\Lambda_0 \sim H_0^2$ [10, 11, 12, 13, 14]. There is a spectrum of possible models by taking the cosmological term as a function of time, scale factor, Hubble parameter, deceleration parameter and so on [11, 12, 13, 14]. However, most of the models dealing with a variable cosmological term suffer from the drawback that they are merely phenomenologically motivated, which is somewhat ad-hoc, lacking any strong theoretical argument, thereby leading to serious debates like: What is the underlying theory that gives rise to its variable nature? Who governs the energy exchange between a variable cosmological term and matter (or any other cosmic entity)? Due to these serious drawbacks, the idea of a kinematical cosmological term did not appeal much of late and more popular models with dynamical quantities like inflaton and dark energy came to the limelight. The bottomline is that, even with these dynamical models, the issue is not yet fully resolved.

In this article, we follow a different route based on an underlying theory and demonstrate that this scenario indeed has great potentiality in addressing the issue of acceleration of the universe in different era from a common platform. The reader may consider it as an altogether new paradigm rather than the continuation of an old idea. We derive the form of the cosmological term from this theory, which rules out any ad-hoc behavior. This eventually answers to the question of energy exchange too. The dramatic consequence of this idea is that it reflects all the characteristics of the expansion history of the universe of all era by a single entity and demonstrates a transition from inflationary phase to late accelerating phase through radiation-dominated and matter-dominated era. We also show that $\Lambda$CDM falls as a subset of this framework, which leads to further observational consequences.

The Friedmann equations with a variable cosmological term are given by [11]

$$\left( \frac{\dot{a}}{a} \right)^2 = \frac{8\pi G}{3} \rho + \frac{\Lambda(t)}{3} - \frac{k}{a^2} \tag{1}$$
$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3} \left( \rho + 3p \right) + \frac{\Lambda(t)}{3} \tag{2}$$

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supplemented by an effective conservation equation
\[ \frac{d}{da} \left( \rho a^{3(1+w)} \right) = -\frac{\alpha^3(1+w)}{8\pi G} \frac{d\Lambda}{da} \]
for a barotropic fluid with equation of state \( w \). Further, for a spatially flat universe \( (k = 0) \), the above equations can be recast into a single second order differential equation (a special case of Riccati equation), by introducing a new variable \( a(t) = [x(t)]^{2/3(1+w)} \) as
\[ \frac{1}{\Lambda(t)} \frac{d^2 x}{dt^2} - \alpha^2 x = 0 \]
where \( \alpha = \sqrt{3(1+w)/2} \). The cosmological term we derive from our theory to have the form
\[ \Lambda(t) = \Lambda_0 e^{-t/t_0} \]
We stress here that, unlike most of the earlier models, this form of the cosmological term is not an ad-hoc choice, rather, is an outcome of an underlying theory. We shall, however, demonstrate this later in this article.

Let us now engage ourselves in analyzing the evolution of the scale factor for different era. First, for very early time \( t \to 0 \), the term \([7]\) is practically a constant \( \Lambda \sim \Lambda_c \), which is large enough so as to take care of inflationary expansion, so that Eq \([4]\) reduces to
\[ \frac{d^2 x}{dt^2} - \Lambda_0 \alpha^2 x = 0 \]
which has the solution
\[ x(t) = C_0 e^{\sqrt{\Lambda_0} \alpha t} \Rightarrow a(t) \propto e^{\sqrt{\Lambda_0} t} \]
under the substitution \( a(t) = [x(t)]^{2/3} \) and \( \alpha = \sqrt{3/2} \), revealing the evolution of the universe during inflation.

For post-inflation era, the value of the cosmological term is exponentially suppressed. In general, \( \forall t \to 0 \), Eq \([4]\), together with \([5]\) takes the form
\[ e^{t/t_0} \frac{d^2 x}{dt^2} - \Lambda_0 \alpha^2 x = 0 \]
which can be solved to give \( x(t) \) as follows
\[ x(t) = C_1 \text{Bessel}I_0 \left( 2\sqrt{\Lambda_0} \alpha e^{-t/2t_0} \right) + C_2 \text{Bessel}K_0 \left( 2\sqrt{\Lambda_0} \alpha e^{-t/2t_0} \right) \]
\((C_1, C_2 \) are arbitrary constants which can be determined from initial conditions) from where one can readily obtain the scale factor by substituting back the variable \( a(t) \). Here \( t_0 \) is the time at the onset of this behavior of the scale factor, which is essentially small and introduces a new parameter in the theory.

One might wonder whether this apparently complicated expression encodes proper information about the expanding universe. Below we answer to this question. First note that Bessel\(I_0\) and Bessel\(K_0\) show opposite behavior while varying with the argument. For a given form of the argument, one of them is a growing function whereas the other is a decaying one, a combination of which will naturally give an initially decelerating and later accelerating phase. FIG. 1 depicts the scale factor with the entire function \([11]\), which reveals this behavior. Note that the parameters satisfy the desired criteria that this form of \( K_0 \) dominates with time. The behavior of the scale factor will be further transparent if we analyze the limiting cases of those functions. For small argument limit, the functions are given by:
\[ I_0(y) \sim (y/2)^0 \Gamma(1) = 1 \text{ and } K_0(y) \sim -\ln y \]
so that we have
\[ x(t) = C_1 - C_2 \ln(2\sqrt{\Lambda_0} \alpha) + \frac{C_2}{2t_0} \]
which, of course, resonates with the cosmological evolution of the universe during decelerating phases.

What about the late accelerating phase? For large argument limit, the functions behave as:
\[ I_0(y) \sim (1/\sqrt{2\pi y})e^y \text{ and } K_0(y) \sim (\sqrt{\pi/2y})e^{-y} \]
so that the scale factor is given by
\[ a(t) \propto [\frac{C_1}{\sqrt{2\pi}} e^y + C_2 \sqrt{\pi/2} e^{-y}]^{2/3} y^{-1/3} \]
with \( y(t) = 2\sqrt{\Lambda_0} \alpha e^{-t/2t_0} \). This is clearly a generalization of the usual \( a(t) \propto [e^{\sqrt{2\pi t}} + e^{-\sqrt{2\pi t}}]^{2/3} = \]

\[ 0.5 \]
\[ 0.4 \]
\[ 0.3 \]
\[ 0.2 \]
\[ 0.1 \]
\[ 0 \]
\[ 1 \]
\[ 2 \]
\[ 3 \]
\[ 4 \]
\[ 5 \]
\[ t \]

**FIG. 1**: Plot of the variation of the scale factor with time for a specific choice of the parameters \( C_1 = 1, C_2 = 500, \sqrt{\Lambda_0} \alpha = 5 \). Here the horizontal axis represents time and the vertical axis the scale factor.
Thebrane are given by \[27, 28\].

In this scenario, the (4D) Friedmann equations on the ground. One can calculate quantities such as the Hub-

\[\text{acceleration probe } \bar{q} \] and confront them with observations. For example, the Hubble parameter is given by

\[H(t) = \frac{\dot{a}}{a} = \frac{2C_1I_0 + C_2K_0}{3C_1I_0 + C_2K_0} \frac{dy}{dt}\]

(12)

where a prime denotes a derivative w.r.t. \(y\). In large argument limit, \(I_0 \sim (1/\sqrt{2\pi y})e^{-y} \sim I_0\) and \(K_0 \sim -(\sqrt{\pi/2y})e^{-y} \sim -K_0\) and this form of \(K_0\) dominates at late time, so that Eq (12) is simplified to

\[H(t) \approx \sqrt{\frac{\Lambda}{3}} e^{-t/2\tau_0}\]

(13)

The exponential factor suppresses the cosmological term to a tiny value (clearly not exactly zero) during late time so that the observed bound to its present value \(\Lambda_0 \sim H_0^2\) is satisfied. Likewise, the other quantities can be derived and confronted with observations.

We shall now concentrate on a possible answer to the question about the origin of such a cosmological term. Below we demonstrate that there is indeed an underlying theory, namely, braneworld gravity, that gives rise to an entity behaving as a cosmological term of the form given in Eq (5) and, at the same time, takes care of the energy exchange issue.

In the general braneworld scenario, our 4D universe is a subspace (brane) of a higher dimensional Vaidya-

antide Sitter space (bulk), which exchanges energy with the brane \[18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28\]. To a brane-based (4D) observer, this energy exchange results in a non-conservation equation for brane matter \[18, 19, 20, 21, 22\].

\[\dot{\rho} + 3\frac{\dot{a}}{a}(\rho + p) = -2\psi\]

(14)

In this scenario, the (4D) Friedmann equations on the brane are given by \[27, 28\]

\[\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3} \left(\rho + \frac{\rho^2}{2\Lambda}\right) + \mathcal{E} - \frac{k}{a^2}\]

(15)

\[\frac{\ddot{a}}{a} = -\frac{4\pi G}{3} \left[\left(\rho + 3p\right) + \frac{(\rho^2 + 6pp)}{2\Lambda}\right] - \mathcal{E} - \frac{k^2}{3}\psi\]

(16)

where \(\mathcal{E}\) and \(\psi\) are contributions from bulk geometry (Weyl tensor) and bulk radiation field on the brane.

Note that we have written the above equations in the RS gauge, i.e., without introducing any arbitrary cosmological constant. In what follows we shall show that it rather appears from the underlying theory.

The quadratic terms here are from brane matter/ radiation sector and hence, can be dropped from the above equations since brane tension \(\lambda\) dominates over matter/ radiation right from the era they are formed. The quadratic terms are relevant only if one considers \(\rho\) to be inflaton field. We are not considering any inflaton either, and thus they are irrelevant during inflation too. Consequently, Eqs (15) and (16) go over to Eqs (1) and (2) respectively under the following identification of terms

\[\mathcal{E} = -\mathcal{E} - \frac{k^2}{3}\psi = \Lambda(t)\]

(17)

This leads to the most crucial feature of the scenario. It is well-known that for any decaying cosmological term model, formation of matter at the cost of the cosmological term is governed by Eq (3) giving \[10, 12\]

\[\dot{\rho} + 3\frac{\dot{a}}{a}(\rho + p) = -\dot{\Lambda}\]

(18)

Eq (18), together with Eq (14) and the expression for the radiation field \(\psi\) obtained from Eq (17), gives

\[\dot{\Lambda} = -\left(4/k^2\right)\Lambda\]

(19)

which has the solution of the form

\[\Lambda(t) = Ae^{-\left(4/k^2\right)t}\]

(20)

immediately leading to the exact form of \(\Lambda\) used in Eq (5), with \(A = \Lambda_1, I_0 = (k^2/4)\). This gives a possible explanation of why we started with such a behavior for \(\Lambda\), and subsequently, provides an underlying theory behind its origin. Nevertheless, as obvious from the discussions preceding Eq (14), the energy exchange is built in the theory, so that it gives a natural explanation of the decay of the cosmological term to the other cosmic entities via braneworlds.

Let us now summarize the key features of this article. The present article has the following significant contributions:

1. The exact solution for the scale factor has been obtained by using a cosmological term.
2. The form of the cosmological term has been derived from an underlying theory, which rules out ad hoc behavior and explains energy exchange too.
3. Transition from inflation to late acceleration, via radiation-dominated and matter-dominated era, has been demonstrated.
4. ΛCDM is found to be a subset of this framework and thus, all of its features are preserved.

5. Certain observational aspects have also been addressed. This framework opens up a spectrum of possible avenues to venture. For example, the effect of this decaying cosmological term on reheating can be investigated. Cosmological perturbations and behavior of power spectrum can also be studied. Another issue is to explore the observational sector, discussed to some extent in the present article, which may further comment on its acceptability as the driving entity for the expansion of the universe. We end up with this optimistic note that a details analysis of the observational sector may reveal its credentials in future.

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