A Computational Method of Active Earth Pressure from Finite Soil Body

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Nowadays, Coulomb and Rankine earth pressure theories have been widely applied to solve the earth pressure on a retaining structure. However, both of the theories established on the basis of the semi-infinite space assumption are not suitable for calculating the earth pressure from finite soil body. Therefore, this paper focuses on a theoretical study about the active earth pressure from finite soil body. Firstly, a common calculation model of finite soil body is established according to the results of previous studies. And then, based on Coulomb's theory and the wedge element method, an analytical solution of the unit active earth pressure from finite soil body is deduced without an assumption of its linear distribution in advance. Meanwhile, formulas of the active earth pressure strength coefficient and the application point of the resultant force are also deduced. Finally, the influence of parameters such as the frictional angle between the retaining wall back and backfill, slope angle of backfill, dip angle of the retaining wall back, the frictional angle between backfill and rock slope, and uniformly applied load on the backfill surface on the distribution of the unit active earth pressure and the application point of the resultant force is analyzed in detail.

1. Introduction

With wide applications of retaining walls in engineering, there have been more and more researches on the retaining walls. The methods to determine the active pressure on a retaining wall constitute a classical topic of soil mechanics [1]. Many scholars have studied this topic, and Coulomb and Rankine earth pressure theory are the most classical theories [2–4]. However, both of the theories are established on the basis of semi-infinite space assumption, which does not conform to actual conditions of many projects. When the retaining walls built in mountainous areas are often close to rock slope, which leads backfill behind the wall to form finite soil body, it is not suitable to calculate the earth pressure from finite soil body by Coulomb's and Rankine's theory. There are more and more problems about finite soil body in the design and construction of actual projects, but only a small number of scholars have conducted a preliminary study on them. According to the previous soil arch theory, Take and Valsangkar do an extensive series of centrifuge model tests to evaluate the use of flexible subminiature pressure cells and measure lateral earth pressures behind retaining walls of narrow backfill width [5]. Kniss et al. investigate the earth pressure against walls in narrow spaces using the finite element method [6]. Yang and Liu present finite element analyses of earth pressures in narrow retaining walls for both at-rest and active conditions, and the predicted data show a favorable agreement with measured data from centrifuge tests [7]. Fan and Fang present a numerical study on the behaviour of active earth pressures behind a rigid retaining wall with limited backfill space of various geometries [8]. On the basis of the limit equilibrium method with planar slip surfaces, Greco presents an analytical method to obtain a solution for the active thrust exerted by backfill of narrow width on gravity retaining walls [9]. Greco presents a limit equilibrium method, for calculating the active thrust on fascia retaining walls, where common methods cannot be used owing to the narrowness of the backfill [10].
However, the above achievements are limited and insufficient. Therefore, based on Coulomb's theory and wedge element method, analytical solutions of the unit active earth pressure from finite soil body and the application point of the resultant force are deduced. Finally, the influence of parameters on the distribution of the unit active earth pressure from finite soil body and the application point of the resultant force is analyzed in detail.

2. Method of Analysis

A retaining wall model of finite soil body is shown in Figure 1. It is a common retaining wall near rock slope, which leads backfill behind the wall to form finite soil body. The retaining wall is a rigid retaining wall of height $H$ and inclination $\alpha$ with the vertical. The cohesionless backfill surface is inclined at an angle $\beta$ with the horizontal. $q_0$ is the uniform surcharge acting on the ground surface. Rock slope is inclined at an angle $\theta_0$ with the horizontal.

2.1. Inclination $\theta$ of the Sliding Surface. According to Coulomb's theory based on semi-infinite space assumption, the inclination $\theta_1$ of the Coulomb sliding surface is what makes the earth pressure reach the maximum, and it can be obtained when the Coulomb active earth pressure coefficient reaches the maximum as well. For the situation of finite soil body in Figure 1, the inclination $\theta$ of the sliding surface is determined as follows: when $\theta_0$ is bigger than $\theta_1$, backfill slides along the rockslope surface and the inclination $\theta$ of the sliding surface is taken equal to $\theta_0$. When $\theta_0$ is smaller than $\theta_1$, the sliding surface is within backfill and the inclination $\theta$ of the sliding surface is taken equal to $\theta_1$. $\theta_1$ can be acquired from the following equation.

The expression of the Coulomb active earth pressure coefficient is shown as

$$K_a = \frac{\sin(\theta_1 - \varphi) \cos(\alpha - \theta_1) \cos(\alpha - \beta)}{\cos(\theta_1 - \alpha - \varphi - \delta) \cos^2 \alpha \sin(\theta_1 - \beta)}, \quad (1)$$

where $\varphi$ is the internal friction angle of backfill and $\delta$ is the frictional angle between the back of the wall and backfill. To obtain the critical value of $\theta_1$ which yields the maximum value of $K_a$, $dK_a/d\theta$ is set equal to 0.

2.2. Calculation Model. According to Coulomb's theory, the earth pressure against a retaining wall is due to the thrust exerted by a sliding soil body between the back of the wall and the sliding surface. The sliding soil body is shown as Figure 2. $\theta$ is the inclination of the sliding surface, and its value which is equal to $\theta_0$ or $\theta_1$ is determined as the above-mentioned method. ABD, a part of the sliding soil body at a depth $y$ above the heel, is taken as an isolated unit for analysis and discussion. Its plane AD is parallel with the ground surface. Forces analysis on the part wedge ABD of the thrust wedge is shown in Figure 3. $G(y)$ is the weight of wedge ABD, and $q(y)$ is the uniform vertical pressure on the top of wedge ABD. $P_a(y)$ is the force on the wall, namely, the active earth pressure, and the angle between $P_a(y)$ and the
normal of the wall back is the frictional angle \( \delta \) between the retaining wall back and backfill. \( R(y) \) is the force on sliding surface. When the inclination \( \theta \) of the sliding surface is equal to \( \theta_1 \), the angle between \( R(y) \) and the normal of the sliding surface is the internal frictional angle \( \varphi \) of backfill. When the inclination \( \theta \) of the sliding surface is equal to \( \theta_0 \), the angle between \( R(y) \) and the normal of the sliding surface is the frictional angle \( \delta_r \) between backfill and rock slope. If there is no test data about \( \delta_r \), \( \delta_r \) is set to be 0.33 times as \( \varphi \) [11].

2.3. Establishment of Fundamental Equations. Figure 4 shows the force polygon considered in wedge ABD. \( Q(y) \) is the resultant force acting on plane AD. According to sine theorem,

\[
\frac{R(y)}{\cos(\alpha + \delta)} = \frac{P_a(y)}{\sin(\theta - \varphi)} = \frac{G(y) + Q(y)}{\sin(\alpha + \delta + \varphi - \theta)}. \tag{2}
\]

Using the formula of Coulomb’s active earth pressure,

\[
P_a(y) = \frac{1}{2} K_a y^2 + K_a q(y) \frac{\cos \alpha \cdot \cos \beta}{\cos(\alpha - \beta)} y, \tag{3}
\]

where \( y \) is the unit weight of backfill. \( K_a \) is the active earth pressure coefficient, which is written as

\[
K_a = \frac{\sin(\theta - \varphi) \cos(\alpha - \theta) \cos(\alpha - \beta)}{\cos(\theta - \alpha - \varphi - \delta) \cos^2 \alpha \sin(\theta - \beta)}. \tag{4}
\]

Introducing two variables \( p_a(y) \) and \( r(y) \), \( p_a(y) \) is the unit active earth pressure, and \( r(y) \) is the unit force on failure surface. There are two equations shown as

\[
\begin{align*}
P_a(y) &= \int_0^y p_a(y) \, dy, \\
R(y) &= \int_0^{y_o} r(y) \, dy. \\
\end{align*} \tag{5}
\]

Putting (3) into the first equation of (5) and, meanwhile, taking a derivative from both sides of the equation with respect to \( y \), it can be obtained that

\[
p_a(y) = K_a y y + K_a \left[ q(y) + q'(y) y \right] \frac{\cos \alpha \cdot \cos \beta}{\cos(\alpha - \beta)} \tag{6}
\]

Let \( p_a(y) = K q(y) \),

\[
q'(y) = \left( \frac{K}{K_a} \frac{\cos(\alpha - \beta)}{\cos \alpha \cdot \cos \beta} - 1 \right) \frac{q(y)}{y} \tag{7}
\]

where \( K \) is the unit active earth pressure coefficient. Putting (7) into (6), it can be obtained that

\[
q'(y) = \left( \frac{K}{K_a} \frac{\cos(\alpha - \beta)}{\cos \alpha \cdot \cos \beta} - 1 \right) \frac{q(y)}{y} \tag{8}
\]

Taking the moment equilibrium equation around the B point on the wedge ABD, it can be obtained that

\[
\int_0^y p_a(y) \cdot \cos \delta \cdot y \cos \varphi \cdot \cos(\alpha - \beta) \, dy + \frac{y \cdot \cos(\theta - \alpha) \cdot \cos \beta}{\cos \alpha \cdot \sin(\theta - \beta)} \cdot q(y) \cdot \frac{1}{2} \frac{\cos(\theta - \alpha)}{\cos \alpha \cdot \sin(\theta - \beta)} - \frac{\sin \alpha}{\cos \beta \cdot \cos \alpha}
\]

\[
\cdot \frac{\cos \beta \cdot y + \frac{y^2}{2 \cos^2 \alpha}}{\cos(\theta - \alpha) \cdot \cos(\alpha - \beta) \cdot \frac{2}{\sin(\theta - \beta)}} - \frac{1}{2} \left( \frac{\cos(\theta - \alpha)}{\cos \alpha \cdot \sin(\theta - \beta)} - \frac{\sin \alpha}{\cos \beta \cdot \cos \alpha} \right) \cdot \cos \beta
\]

\[
\cdot \frac{y}{\sin \theta} dy.
\]

Putting (2), the second equation of (5), and (7) into (9) and taking a derivative from both sides of the equation with respect to \( y \), it can be obtained that

\[
q'(y) = \left[ A_1 K - A_2 K - 2 \right] \frac{q(y)}{y} \frac{\cos(\alpha - \beta)}{\cos \alpha \cdot \cos \beta} \cdot y, \tag{10}
\]

where

\[
A_1 = \frac{\cos(\alpha + \delta) \cdot \cos \varphi \cdot \cos(\alpha - \beta)}{\sin(\theta - \varphi) \cdot \cos(\theta - \alpha) \cdot \cos^2 \beta \cdot \left[ \frac{1}{2} \frac{\cos(\theta - \alpha)}{\cos \alpha \cdot \sin(\theta - \beta)} - \frac{\sin \alpha}{\cos \beta \cdot \cos \alpha} \right]},
\]

\[
A_2 = \frac{\cos \delta \cdot \sin(\theta - \beta)}{\cos(\theta - \alpha) \cdot \cos^2 \beta \cdot \left[ \frac{1}{2} \frac{\cos(\theta - \alpha)}{\cos \alpha \cdot \sin(\theta - \beta)} - \frac{\sin \alpha}{\cos \beta \cdot \cos \alpha} \right]}.
\]
2.4. Solution of the Basic Equation. According to (8) with (10),
the expression of the unit active earth pressure coefficient $K$
can be obtained:

$$K = \frac{1}{A_1 - A_2 - A_3},$$

where $A_3 = \cos(\alpha - \beta)/(\cos \alpha \cdot \cos \beta) \cdot 1/K_a$.
Letting $\lambda = K/K_a \cdot \cos(\alpha - \beta)/(\cos \alpha \cdot \cos \beta) - 1$ and solving
(8), it can be obtained that

$$q(y) = C y^{\lambda} + \frac{\cos(\alpha - \beta)}{(\lambda - 1) \cos \alpha \cdot \cos \beta} \cdot y^\gamma,$$

where $C$ is an integration constant; it can be determined
by the boundary condition that $q(y) = q_0$ when $y = H$. It can
be written as

$$C = q_0 H^{-\lambda} - \frac{\cos(\alpha - \beta)}{(\lambda - 1) \cos \alpha \cdot \cos \beta} \cdot \gamma \cdot H^{1-\lambda}.$$  

According to (7), the unit active earth pressure can be
obtained as

$$P_a(y) = K \left[ q_0 H^{-\lambda} - \frac{\cos(\alpha - \beta)}{(\lambda - 1) \cos \alpha \cdot \cos \beta} \cdot y^\gamma \right]$$

$$\cdot y^\lambda + K \frac{\cos(\alpha - \beta)}{(\lambda - 1) \cos \alpha \cdot \cos \beta} \cdot \gamma H.$$  

2.5. Application Point of the Resultant Earth Pressure. The
height $H_p$ of application point of the resultant earth pressure
from the wall bottom is

$$H_p = \left. \int_0^H y P_a(y) dy \right|_{y=0} = \frac{2}{3} \left( \frac{\lambda + 1}{\lambda + 2} \right)$$

$$\cdot 3q_0 + \left( \cos(\alpha - \beta) / (\cos \alpha \cdot \cos \beta) \right) yH$$

$$\cdot 2q_0 + \left( \cos(\alpha - \beta) / (\cos \alpha \cdot \cos \beta) \right) yH.$$

3. Parametric Study

In this section, when the inclination of the sliding surface
is $\theta_0$, the influence of dip angle $\alpha$ of the retaining wall
back, slope angle $\beta$ of backfill, the frictional angle $\delta$
backfill and rock slope, and uniformly applied load $q_0$
on the backfill surface on the distribution of the unit
active earth pressure and the application point of the resultant
force is discussed as follows. $\theta_0$, the inclination of the rock
slope, is assumed to be $75^\circ$. Another condition can refer to
[12].

When $H = 4.0$ m, $r = 18$ Kn/m$^3$, and $\theta_0 = 75^\circ$,
the active earth pressure distribution and the height $H_p$ of
application point of the resultant earth pressure with change
in $\delta r$, in $\delta$, in $\alpha$, in $\beta$, and in $q_0$ are shown in Figures 5–14,
respectively. As shown in Figure 5, the active earth pressure
increases as $\delta r$ increases near the heel of the wall; the active
earth pressure increases as $\delta r$ decreases in other places. As
shown in Figure 7, the active earth pressure decreases as $\delta
increases near the heel of the wall; the active earth pressure
increases as $\delta$ increases in other places. From Figures 9 and 11, it can be seen that the active earth pressure increases as $\alpha$ and $\beta$ increase, respectively. As shown in Figure 13, the active earth pressure increases in proportion as $q_0$ increases. From Figures 6, 10, and 12, it can be seen that the height $H_p$ of application point of resultant earth pressure decreases as $\delta r$, $\alpha$, and $\beta$ increase, respectively. From Figures 8 and 14, it can be seen that the height $H_p$ of application point of the resultant earth pressure decreases as $\delta$ and $q_0$ increase, respectively.

From Figures 5, 7, 9, and 11, it is obvious that there is a critical value existing in dip angle $\alpha$ of the retaining wall back, slope angle $\beta$ of backfill, the frictional angle $\delta$ between the retaining wall back and backfill, and the frictional angle $\delta r$ between backfill and rock slope. The critical values divide the earth pressure distribution curves into two types. One type is a crooked nonlinear curve and another type is close to a line [12].
4. Conclusions

(1) Based on Coulomb’s theory and wedge element method, an analytical solution of the unit active earth pressure from finite soil body is deduced. Meanwhile, the formula of the application point of the resultant force is also deduced. When these formulas are applied, we firstly estimate whether backfill slides along rock slope or the sliding surface happens inside of backfill. And then whether the angle between $R(y)$ and the normal of the sliding surface is the internal frictional angle $\varphi$ of backfill, or the frictional angle $\delta_1$, between backfill and rock slope is determined.

(2) By parametric study, it is found that the frictional angle between the retaining wall back and backfill, slope angle of backfill, dip angle of the retaining wall back, the frictional angle between backfill and rock slope, and uniformly applied load on the backfill surface have great influence on the active earth pressure distribution and the application point of the resultant earth pressure.

(3) According to the method of this paper, we can study the earth pressure from finite clayey soil body in the future.

Symbols

- $H$: Height of retaining wall (Figure 1)
- $\alpha$: Angle that backfill wall interface makes with the vertical (Figure 1)
- $\beta$: Angle that backfill surface makes with the horizontal (Figure 1)
- $q_0$: Uniform surcharge on the ground surface (Figure 1)
- $\theta_0$: Angle that the backfill rock interface makes with the horizontal (Figure 1)
- $\theta$: Angle that the failure surface makes with the horizontal
- $K_a$: Coulomb active earth pressure coefficient as defined in (1)
- $\theta_1$: Angle obtained according to the Coulomb active earth pressure coefficient in (1)
- $\varphi$: Internal friction angle of backfill
- $\delta$: Frictional angle between the wall and backfill
- $\delta_1$: Frictional angle between backfill and rock slope
- $\gamma$: Unit weight of backfill
- $K$: Unit active earth pressure coefficient as defined in (12)
- $G(y)$: Weight of wedge ABD (Figure 3)
- $P_a(y)$: Active earth pressure on plane AD of wedge ABD (Figure 3)
- $R(y)$: Force on failure surface BD of wedge ABD (Figure 3)
- $q(y)$: Uniform vertical pressure on plane AD of wedge ABD (Figure 3)
- $Q(y)$: Resultant force on plane AD of wedge ABD
\( r(y) \): Unit force on failure surface BD of wedge ABD

\( A_1, A_2, A_3, \lambda \): Terms as defined in Sections 2.2 and 2.3

\( H_p \): Height of application point of the resultant earth pressure from the wall bottom as shown in (16).

**Conflicts of Interest**

The authors declare that there are no conflicts of interest regarding the publication of this paper.

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