Validity of the Semiclassical Approximation
and Back-Reaction

Sukanta Bose,∗ Leonard Parker,† and Yoav Peleg‡

Department of Physics
University of Wisconsin-Milwaukee, P.O.Box 413
Milwaukee, Wisconsin 53201, USA

Abstract

Studying two-dimensional evaporating dilatonic black holes, we show that
the semiclassical approximation, based on the background field approach, is
valid everywhere in regions of weak curvature (including the horizon), as long
as one takes into account the effects of back-reaction of the Hawking radiation
on the background geometry.

∗Electronic address: bose@csd.uwm.edu
†Electronic address: leonard@cosmos.phys.uwm.edu
‡Electronic address: yoav@csd.uwm.edu
The suggestion that quantum gravitational effect may play a role in the evaporation of black holes [1] received some attention recently [2]. Since we do not have a full theory of quantum gravity it is not clear how to define a criterion according to which we can check the validity of the semiclassical approximation. 't Hooft suggests that the resolution of the information puzzle (in a unitary framework) is one of the conditions that a quantum theory of gravity should satisfy, and if the present semiclassical theory implies a non-unitary evolution, it cannot be consistent with the full theory of quantum gravity. This may very well be the case, but some try to go even further and show that the standard semiclassical approximation (based on the background field approach) breaks down in regions of weak curvature, just outside the horizon of a black hole [3,4]. Though in Ref. [3] quantum matter fields forming a black hole (and consequently emitting Hawking radiation) was considered, the back-reaction of the quantum radiation on the geometry was ignored. We show in this work that when we take into account the back-reaction, we avoid the conclusions of [3]. Namely, we show in the framework of two-dimensional (2D) dilaton Gravity (in which we can solve the semiclassical equations including the back-reaction) that the semiclassical approximation is valid as long as one includes the back-reaction. We therefore conclude that, at least in 2D dilaton Gravity, the breakdown of the background field method in regions of weak curvature is an artifact of the neglect of the back-reaction.

In this work we use the same criterion as in [3] according to which we check the validity of the semiclassical approximation. One considers two almost identical classical space-times with a classical dilaton field, which may differ only at the Planck scale. Then the evolution of a quantum scalar (matter) field on these two classical backgrounds is considered. By taking identical space-like hypersurfaces (i.e., hypersurfaces with the same internal geometry) in both space-times, one can compare the evolution of the two quantum states in the two dif-

\[1\] For the hypersurfaces that we consider the shift vector does not necessarily vanish at spatial infinity, and one can have different ADM masses for the same intrinsic geometry.
ferent space-times. For example one can take two identical hypersurfaces at asymptotic past infinity, and start with a vacuum state on both hypersurfaces. Since the initial hypersurfaces are identical one can calculate the scalar product of the two states (using the Klein-Gordon scalar product defined on the hypersurface). In the collapsing black hole geometry one finds that the scalar product of the two vacua is one. Namely, the two past asymptotic vacua are the same. Then (using the Schrödinger picture) one evolves the two states, each in its space-time, to late-time identical hypersurfaces (in the two different space-times). It was shown in [3] that (neglecting the back-reaction) for very special late-time space-like hypersurfaces, called the S-hypersurfaces, (ones that intersect the infalling matter near the horizon and also capture a significant amount of Hawking radiation), the scalar product between the two states is almost zero, even for spacetimes with a mass difference of the order of \( \Delta M \sim \exp(-M/M_P) \), where \( M_P \) is the Planck mass, or its 2D analog. Since in this case \( \Delta M \ll M_P \), they conclude that the semiclassical approximation is not valid. We are going to show in this work that even for the special hypersurfaces considered in [3], if we include the back-reaction, the scalar product is not zero, but almost one even for \( \Delta M \sim M_P \). This suggests that the semiclassical approximation is valid in regions of weak curvature as long as we include the back-reaction.

We study a modified theory of 2D dilaton gravity, described by the action [5]

\[
S_{\text{mod}} = \frac{1}{2\pi} \int d^2 x \sqrt{-g(x)} \left[ (e^{-2\phi} - \kappa \phi)R(x) \right. \\
\left. + (4e^{-2\phi} + \kappa)(\nabla \phi)^2 + 4\lambda^2 e^{-2\phi} - \frac{1}{2} \sum_{i=1}^{N} (\nabla f_i)^2 \right] \\
- \frac{\kappa}{8\pi} \int d^2 x \sqrt{-g(x)} \int d^2 x' \sqrt{-g(x')} R(x) G(x, x') R(x'),
\]

where \( R(x) \) is the 2D Ricci scalar, \( \phi \) is the dilaton field, \( f_i \) are \( N \) massless scalar fields, \( \kappa = N\hbar/12 \), and \( G(x, x') \) is an appropriate Green’s function for \( \nabla^2 \). The effective action (1) describes the full quantum theory in the large \( N \) limit, in which case the fluctuations of \( \phi \) and \( g_{\mu\nu} \) can be neglected. Namely, the semiclassical approximation is exact at the large \( N \) limit. For finite \( N \), the action (1) is a semiclassical approximation to the full quantum
theory. Nevertheless it includes the back-reaction of the quantum scalar fields on the classical metric and dilaton fields. In this work we do not take the large $N$ limit, and so the action is only a semiclassical effective action, and we assume that it is a good approximation as long as the curvature and the coupling are small.

One can derive the effective action (1) by first fixing the diffeomorphism gauge and then quantizing the reduced system [2]. After choosing an appropriate initial quantum state, one gets the action (1) as the semiclassical effective action [6].

It is convenient to use null coordinates and conformal gauge, $g_{++} = -e^{2\rho}/2$, and $g_{--} = g_{--} = 0$. In Kruskal coordinates, $x^{\pm}$, for which $\phi(x^{+}, x^{-}) = \rho(x^{+}, x^{-})$, the evaporating black hole solution of (1) is [5]

$$e^{-2\phi} = e^{-2\rho} = -\lambda^2 x^+ x^- - \frac{M}{\lambda x^0_0} (x^+ - x^+_0) \Theta(x^+ - x^+_0) - \frac{\kappa}{4} \ln(-\lambda^2 x^+ x^-).$$  \hfill (2)

This solution describes the formation and subsequent evaporation of a black hole which forms by an infalling shock wave [5] with mass $M$. In Fig. 1 we show its Penrose diagram.

Before the shock wave the solution is a static vacuum solution with no Hawking radiation,

$$\left(e^{-2\phi}\right)_{x^+ < x^+_0} = -\lambda^2 x^+ x^- - \frac{\kappa}{4} \ln(-\lambda^2 x^+ x^-) = \exp(2\lambda y) - \frac{\kappa \lambda y}{2},$$  \hfill (3)

where $y = (y^+ - y^-)/2$, and $\lambda y^{\pm} = \pm \ln(\pm \lambda x^{\pm})$ are the asymptotically flat coordinates in the static region $x^+ < x^+_0$. The solution (3) includes a region of strong coupling $e^{2\phi} \gg 1$. In order to avoid that region in which the semiclassical approximation is not valid we impose reflection boundary condition on a timelike hypersurface $y = y_b = \text{const}$, such that $\exp[2\phi(y_b)] \ll 1$, and consider the solution only in the weak coupling region $y > y_b$.

After the shock wave we have an evaporating black hole space-time,

$$\left(e^{-2\phi}\right)_{x^+ > x^+_0} = -\lambda^2 x^+ \left(x^- + \frac{M}{\lambda^3 x^+_0}\right) - \frac{\kappa}{4} \ln(-\lambda^2 x^+ x^-) + \frac{M}{\lambda}.$$  \hfill (4)

In this region the asymptotically flat coordinates are $\lambda \sigma^+ = \ln(\lambda x^+)$ and $\lambda \sigma^- = -\ln\{-[\lambda x^- + M/(\lambda^2 x^+_0)]\}$. The black hole singularity is initially hidden behind a timelike
apparent horizon. As the black hole evaporates by emitting Hawking radiation, the singularity meets the shrinking horizon in finite retarded time to become naked, at $x^- = x^-_{int}$, see Fig. 1.

We denote the spacetime (2) by $\mathcal{M}$. One can consider another spacetime described by (4) but with a different mass, $\bar{M} = M + \Delta M$. Since the Kruskal coordinates for which $\phi = \rho$ depend on the solution and therefore on the ADM mass of the spacetime, the Kruskal coordinates for $\bar{M}$ will be denoted by $\bar{x}^\pm$. We denote the spacetime (2) with $M$ and $x^\pm$ by $\mathcal{M}$. In the region $x^+ < x^+_0$ ($\bar{x}^+ < x^+_0$), the space-times $\mathcal{M}$ and $\bar{\mathcal{M}}$ are the same, so we are interested in the region $x^+ > x^+_0$. In this region we have evaporating black holes, one with mass $M$ in $\mathcal{M}$ and the other with mass $\bar{M}$ in $\bar{\mathcal{M}}$. Without loss of generality, we take $\lambda x^+_0 = 1$. A necessary condition for the validity of the semiclassical approximation is that $M >> \kappa \lambda$ ($\kappa \lambda$ may be considered as the 2D analog of the Planck mass, $M_P[7]$).

The last term on the right-hand-side of Eq. (2) is due to the back reaction. If we set $\kappa = 0$ in (2) we get

$$e^{-2\phi} = e^{-2\rho} = -\lambda^2 x^+ x^- - \frac{M}{\lambda x^+_0} (x^+ - x^+_0) \Theta(x^+ - x^+_0), \quad (5)$$

which is exactly the classical CGHS solution [8], that was considered in Ref. [3]. The CGHS solution (3) is the one with no back reaction, and so taking the limit $\kappa \to 0$ in our semiclassical solution is equivalent to ignoring the back-reaction. We would like to stress that the solution (3) does not describe a non-unitary evolution, because the black hole in (3) does not evaporate away. It is the solution (2) that may describe a non-unitary evolution [4], and according to 't Hooft one should examine the validity of the semiclassical approximation that lead to the solution (2) and not (3).

We follow [3] and calculate the scalar product of two initial vacuum states in $\mathcal{M}$ and $\bar{\mathcal{M}}$. First we calculate it on past asymptotic hypersurfaces and then on future S-hypersurfaces. We do the calculations for a finite $\kappa$. When $\kappa = 0$ we recover the results of [3]. As we noted above, this limit corresponds to ignoring the back-reaction which is inconsistent with the quantum Hawking radiation. By expressing the results explicitly in terms of $\kappa$, we find that
the scalar product is almost one for any Planck scale perturbations.

We define the space-like hypersurfaces, $\Sigma$, (which are one dimensional in our 2D theory) by means of their intrinsic geometry, which can be determined by giving the dilaton field $\phi$, and its first derivative $d\phi/ds$, where $s$ is the proper distance along the space-like hypersurface $\Sigma$. The two different semiclassical solutions are described by (4) with different masses, $M$ and $\tilde{M} \equiv M + \Delta M$. Let $\tilde{\Sigma}$ be the spacelike hypersurface in $\tilde{\mathcal{M}}$ having the same intrinsic geometry as $\Sigma$ in $\mathcal{M}$. We denote this equivalence relation by $\Sigma = \tilde{\Sigma}$. The 1D hypersurfaces $\Sigma$ and $\tilde{\Sigma}$ can be written in the form

$$x^- = x^+_{\Sigma}(x^+) \quad , \quad \Sigma\text{-hypersurface in } \mathcal{M}$$

$$\tilde{x}^- = \tilde{x}^+_{\Sigma}(\tilde{x}^+) \quad , \quad \tilde{\Sigma}\text{-hypersurface in } \tilde{\mathcal{M}}.$$  

Let us define

$$f(\lambda x^+) \equiv \lambda x^+_{\Sigma}(x^+) + \frac{M}{\lambda} \quad \text{and} \quad \tilde{f}(\lambda \tilde{x}^+) \equiv \lambda \tilde{x}^+_{\Sigma}(\tilde{x}^+) + \frac{\tilde{M}}{\lambda} \quad (8)$$

Then using (4) and (8), the equations $\phi(x^+, x^-_{\Sigma}) = \phi(\tilde{x}^+, \tilde{x}^-_{\Sigma})$, and $d\phi(x^+, x^-_{\Sigma})/ds = d\phi(\tilde{x}^+, \tilde{x}^-_{\Sigma})/d\tilde{s}$, (which imply that $\Sigma = \tilde{\Sigma}$), become

$$\frac{M}{\lambda} - \lambda x^+ f - \frac{\kappa}{4} \ln[\lambda x^+ (f - M/\lambda)] = \frac{\tilde{M}}{\lambda} - \lambda \tilde{x}^+ \tilde{f} - \frac{\kappa}{4} \ln[\lambda \tilde{x}^+ (\tilde{f} - \tilde{M}/\lambda)] \quad (9)$$

$$f + \lambda x^+ f' + \frac{\kappa}{4} \left( \frac{1}{\lambda x^+} + \frac{f'}{f-M/\lambda} \right) \sqrt{-f'} = \tilde{f} + \lambda \tilde{x}^+ \tilde{f}' + \frac{\kappa}{4} \left( \frac{1}{\lambda \tilde{x}^+} + \frac{\tilde{f}'}{\tilde{f}-\tilde{M}/\lambda} \right) \sqrt{-\tilde{f}'} \quad (10)$$

where $f = f(\lambda x^+)$, $\tilde{f} = \tilde{f}(\lambda \tilde{x}^+)$, and $'$ denotes derivative with respect to the argument of the function. Eqs. (9) and (10) reduced to the corresponding expressions in [3] when the back-reaction is ignored, (i.e., when $\kappa = 0$). We must find the solutions of (4) and (10), $\tilde{x}^+ = \tilde{x}^+(x^+)$ and $\tilde{x}^- = \tilde{x}^-(x^-)$, in order to determine the scalar product of the two vacua.

Consider first past asymptotic space-like hypersurfaces (like the $\Sigma^0$-hypersurface in Fig. 1) in $\mathcal{M}$ or $\tilde{\mathcal{M}}$. A past asymptotic hypersurface in $\mathcal{M}$ can be given by the equation

$$f(\lambda x^+) = -A^2 \lambda x^+ \quad \text{the } \Sigma^0 \text{ hypersurface,} \quad (11)$$
where \( A^2 \) is a constant satisfying \( A^2 \gg M/\lambda \) in order for the \( \Sigma^0 \)-hypersurface to cross the infalling null matter long before the apparent horizon is formed. The hypersurface (II) corresponds to a fixed asymptotic time, namely, \( t = (\sigma^+ + \sigma^-)/2 = -(2\lambda)^{-1} \ln[-f(\lambda x^+)/x^+] = -\lambda^{-1} \ln(|A|) = \text{const.} \) For the \( \Sigma^0 \) hypersurface (II) one can show that in Eqs. (II) and (III) \( \kappa \) appears only in the combination \( \kappa/A^2 \). Since \( A^2 \gg M/\lambda \) and \( M/\lambda \gg \kappa \), one can neglect terms of order \( \kappa/A^2 \). Namely, the effect of back-reaction is negligible on the \( \Sigma^0 \) hypersurface. This is expected since the Hawking radiation (and so the back-reaction) starts when the apparent horizon is formed, to the far future of \( \Sigma^0 \). Neglecting \( \kappa/A^2 \) one can easily solve (II) and (III) exactly (see [3]) to get

\[
\lambda \bar{x}^+ = \lambda x^+ + \sqrt{\frac{\Delta M}{\lambda A^2}}, \quad \lambda \bar{x}^- = \lambda x^- + \sqrt{\frac{A^2 \Delta M}{\lambda}} - \frac{\Delta M}{\lambda}. \tag{12}
\]

We see that as \( \Sigma^0 \) approaches \( \Im^- \), i.e., as \( \frac{M/\lambda}{A^2} \rightarrow 0 \), we have \( \bar{x}^+ = x^+ \). So on \( \Im^- \) and \( \tilde{\Im}^- \) the vacua that are defined with respect to the modes \( \exp(-ik_+ \sigma^+) \) and \( \exp(-\tilde{k}_+ \tilde{\sigma}^+) \) are the same, because \( \tilde{\sigma}^+ = \lambda^{-1} \ln(\lambda \bar{x}^+) = \lambda^{-1} \ln(\lambda x^+) = \sigma^+ \).

The strategy is therefore to consider a vacuum state on \( \Im^- \) and \( \tilde{\Im}^- \), to evolve it to a future hypersurface, \( \Sigma^f \), once embedded in \( \mathcal{M} \) and then embedded in \( \tilde{\mathcal{M}} \). In \( \mathcal{M} \) the vacuum state corresponds to the modes \( \exp[-ik_+ \ln(\lambda x^+)/\lambda] \), while in \( \tilde{\mathcal{M}} \) it corresponds to the modes \( \exp[-\tilde{k}_+ \ln(\lambda \bar{x}^+)/\lambda] \). In order to calculate the scalar product on \( \Sigma^f \), we need to know \( \bar{x}^+ \) as a function of \( x^+ \) on \( \Sigma^f \). Unlike on \( \Sigma^0 \), on \( \Sigma^f \) we do not necessarily have \( \bar{x}^+ = x^+ \). We have to solve (III) and (IV) on \( \Sigma^f \) and find \( \bar{x}^+ = \bar{x}^+(x^+) \).

The future space-like hypersurfaces that we consider are the S-hypersurfaces described in [3]. Those can be written in \( \mathcal{M} \) as

\[
f(\lambda x^+) = -\alpha^2 \lambda x^+ - 2\alpha \sqrt{\frac{M}{\lambda}} \quad \text{the S-hypersurfaces,} \tag{13}
\]

where \( \alpha \) is a very small constant. An example for a S-hypersurface is shown in Fig. 1. Let \( \gamma M \) be the amount of Hawking radiation that is captured by the S-hypersurface, where \( \gamma \) is a constant between one and zero, then one can use (III) to show that \( \alpha \) is of the order of

\[
\alpha \sim \sqrt{\frac{M}{\lambda}} \exp \left(-\frac{4\gamma M}{\kappa \lambda} \right). \tag{14}
\]
Since $M/\kappa \lambda >> 1$ we see that $\alpha$ is exponentially small (i.e., non-perturbatively small in terms of $\kappa \lambda / M$).

Since by construction for $M = \tilde{M}$ we have $\bar{x}^\pm = x^\pm$, we expand $\Delta x^\pm = \bar{x}^\pm - x^\pm$ in powers of $\Delta M / M$. We assume that the leading term is of order $\Delta M / M$, which will turn out to be self-consistent. We therefore write

$$
\lambda \bar{x}^+ = \lambda(x^+ + \Delta x^+) \equiv \lambda x^+ + \mathcal{E}^+ (\lambda x^+) \frac{\Delta M}{M} + \mathcal{O}\left(\frac{\Delta M}{M}\right)^2
$$

(15)

$$
\lambda \bar{x}^- = \lambda(x^- + \Delta x^-) \equiv \lambda x^- + \mathcal{E}^- (\lambda x^-) \frac{\Delta M}{M} + \mathcal{O}\left(\frac{\Delta M}{M}\right)^2,
$$

(16)

where $\mathcal{E}^\pm$ are functions to be determined. We assume that for $\Delta M / M << 1$ the term of order $(\Delta M / M)^2$ can be neglected.

Using (15), (16) and (13), Eqs. (9) and (10) become

$$
\left(\lambda x^+ - \frac{\kappa}{4(\alpha^2 \lambda x^+ + 2\alpha \sqrt{M/\lambda} + M/\lambda)}\right) \mathcal{E}^- =
$$

$$
= \left(\alpha^2 \lambda x^+ + 2\alpha \sqrt{M/\lambda} - \frac{\kappa}{4\lambda x^+}\right) \mathcal{E}^+ - (\lambda x^+ - 1) \frac{M}{\lambda}
$$

(17)

and

$$
\left(-\alpha \sqrt{\frac{M}{\lambda}} + \frac{\kappa}{8\lambda x^+} - \frac{\kappa \alpha^2}{8(\alpha^2 \lambda x^+ + 2\alpha \sqrt{M/\lambda} + M/\lambda)}\right) \left[(\mathcal{E}^+)' - (\mathcal{E}^-)'ight] =
$$

$$
= \left(\frac{\kappa}{4(\alpha^2 \lambda x^+)^2} + \alpha^2\right) \mathcal{E}^+ - \left(1 + \frac{\kappa \alpha^2}{4(\alpha^2 \lambda x^+ + 2\alpha \sqrt{M/\lambda} + M/\lambda)^2}\right) \mathcal{E}^- + \frac{M}{\lambda}.
$$

(18)

Since $\alpha$ and $\kappa$ are both small compared to one, we solve (17) and (18) by expanding in powers of $\kappa$ and $\alpha$. The solutions are

$$
\mathcal{E}^+ = \frac{-M/\lambda}{2\alpha \sqrt{M/\lambda} - \kappa/(4\lambda x^+)} \left(1 - \frac{\lambda \kappa}{4M} + \mathcal{O}(\kappa^2, \alpha^2)\right)
$$

(19)

$$
\mathcal{E}^- = \frac{-M}{\lambda} \left(1 + \frac{\alpha^2}{2\alpha \sqrt{M/\lambda} - \kappa/(4\lambda x^+)} + \mathcal{O}(\kappa^2, \alpha^2)\right).
$$

(20)

Setting $\kappa = 0$ gives
\[(\mathcal{E}^+)^{\kappa=0} = -\frac{\sqrt{M/\lambda}}{2\alpha}, \quad (\mathcal{E}^-)^{\kappa=0} = -\frac{M}{\lambda} \left( 1 + \frac{\alpha}{2\sqrt{M/\lambda}} \right) \]\n
ignoring back-reaction,

\[(21)\]

which are exactly the results of [3], as expected. Since \(\alpha\) is exponentially small (see (14)),

the shift in \(x^+\), i.e., \(\mathcal{E}^+\), is very large in (21). This leads to a breakdown of the semiclassical approximation. However, one should remember that in order to get (21) we assume that \(\kappa\) is much smaller than \(\alpha\sqrt{M/\lambda}\). Is this consistent with (14)? Using \(M/(\kappa\lambda) \gg 1\) and (14) we see that

\[
\sqrt{\frac{M}{\lambda}} \alpha \sim \frac{M}{\lambda} \exp \left( -\frac{4\gamma M}{\kappa\lambda} \right) << \frac{M\kappa\lambda}{\lambda M} = \kappa,
\]

\[(22)\]

where \(X \gg Y\) means that \(X\) is non-perturbatively larger than \(Y\) (i.e., there exist a constant \(c < 1\), such that \((Y/X) < c^n\), for any \(n \in \mathbb{N}\)). We see that for the S-hypersurfaces \(\kappa\) is non-perturbatively larger than \(\alpha\sqrt{M/\lambda}\), and therefore we cannot take \(\kappa = 0\) in (19) and (20). On the other hand, we definitely can take \(\alpha = 0\), and get

\[(\mathcal{E}^+)^{\alpha=0} = \frac{4Mx^+}{\kappa} \left( 1 - \frac{\lambda\kappa}{4M} \right), \quad (\mathcal{E}^-)^{\alpha=0} = -\frac{M}{\lambda}\]

including back-reaction,

\[(23)\]

or for \(\bar{x}^\pm\)

\[
\lambda\bar{x}^+ \simeq \lambda x^+ \left( 1 + \frac{\Delta M}{\kappa\lambda/4} - \frac{\Delta M}{M} \right), \quad \lambda\bar{x}^- \simeq \lambda x^- - \frac{\Delta M}{\lambda}.
\]

\[(24)\]

From (24) we see that the relations between the asymptotically flat coordinates \(\bar{\sigma}^\pm\) and \(\sigma^\pm\) are approximately linear on the S-hypersurface. In particular,

\[
\bar{\sigma}^+ = \lambda^{-1} \ln(\lambda\bar{x}^+) \simeq \sigma^+ + \lambda^{-1} \ln(1 + 4\Delta M/\kappa\lambda).
\]

\[(25)\]

It follows that the Bogoliubov coefficients, \(\beta_{k,\bar{k}}\), obtained from the Klein-Gordon scalar product, vanish: \(\beta_{k,\bar{k}} \propto (\exp(ik_+\bar{\sigma}^+), \exp(-ik_+\sigma^+)) = 0\). Hence the Fock space scalar product [10], \(\langle 0|0 \rangle = (\text{det}(1 + \beta^\dagger\beta))^{-1/2}\), between the vacua defined with respect to the modes \(\exp(-ik_+\sigma^+)\) and \(\exp(-i\bar{k}_+\bar{\sigma}^+)\) is 1. Our results are valid as long as \(\Delta x^\pm \leq x^\pm\), namely, when \(\Delta M \sim \kappa\lambda\). This is what we expect from the semiclassical approximation.
Since the scalar product of $|0\rangle$ and $|\bar{0}\rangle$ is 1 on $\mathcal{I}^−$ (or $\bar{\mathcal{I}}^−$), the quantum states $|0\rangle$ and $|\bar{0}\rangle$ are indistinguishable on $\mathcal{I}^−$. The fact that the scalar product remains nearly one even on late-time hypersurfaces (like the S-hypersurface) implies that the states remain indistinguishable throughout their evolution in the different backgrounds $\mathcal{M}$ and $\bar{\mathcal{M}}$. This shows that the quantum matter states are insensitive to Planck-scale changes in the background geometry. Therefore, the semiclassical approximation is valid everywhere in regions of weak coupling and curvature including the apparent horizon.

The semiclassical approximation evidently breaks down only near the naked singularity, the intersection point in Fig. 1. The dilaton field (and the curvature) diverges at that point, and from equations (9) and (10) one expects to get large shifts in the Kruskal coordinates, leading to a zero scalar product between the two vacua.

In passing, we note that one can find different initial conditions [5] in our effective theory (1) that yield exactly the same solutions as the ones given in Eq. (5). For those solutions the shifts in $x^±$ are exactly the ones found in [3], i.e., the ones given in Eq. (21). However, unlike the corresponding example considered in [3], one cannot conclude that the semiclassical approximation breaks down for this metric (5) in our model. This is because the solutions in (5) correspond in our effective theory to a different vacuum state of the scalar matter fields [5]. Unlike the collapsing black hole solutions (2) which correspond to a Schwarzschild vacuum, the solutions (5) correspond to the Kruskal vacuum. Namely, to the vacuum with respect to the modes $\exp(-ik_+x^+)$. The relations between the Kruskal coordinates given by (13), (16) and (21) are linear. Hence, the scalar product between the Kruskal vacua is 1, and not 0. Thus the semiclassical approximation remains valid even for the solutions (5) in our theory.

One can also study a larger class of exactly solvable 2D semiclassical models by considering area-preserving diffeomorphism invariant theories [11], where our model [5] arises as a special case. Since this larger class of 2D models that include the back-reaction can be explicitly solved, one can use our approach to check the validity of the semiclassical approximation in this larger class. On the other hand, since we do not have an explicit solution
for the four-dimensional (4D) evaporating black hole (including the back-reaction), it is not straight forward to extend our results to the more realistic 4D case. One may try to follow the approach of [12] by studying an effective 4D evaporating solution and comparing the results to those of [4] in which the semiclassical approximation is neglected. The generalization of our 2D results to 4D is the following conjecture:

*In four dimensional gravitational collapse to form an evaporating black hole, the semiclassical approximation is valid everywhere in regions of weak curvature (including the apparent horizon), as long as one takes into account the back-reaction (including that of the Hawking radiation) on the background geometry.*

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FIGURES

FIG. 1. Penrose diagram for the evaporating black hole. The lower dashed curve is the $\Sigma^0$ hypersurface, and the upper one is the S-hypersurface.
static solution

intersection point

singularity

end-state

apparent horizon

S-hypersurface

end-state

apparent horizon

S-hypersurface

strong coupling

static solution