CPTM reversal symmetry, cosmological constant problem and wormholes

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Abstract

We discuss the consequences of the charge, parity, time and mass (CPTM) reversal symmetry for the problems of the vacuum energy density and value of the cosmological constant. The results obtained are based on the framework with the separation of extended space-time of the interest on the different regions connected by the CPTM symmetry with the action of the theory valid for the full space-time and symmetrical with respect to the CPTM symmetry transformations. The value of the cosmological constant in this model is defined by the graviton interaction terms between the different parts of the space-time. It is proposed that the constant’s value depends on the form and geometry of the wormholes which glue the separated parts of the extended solution of Einstein equations determining in turn it’s classical geometry.

1 Introduction

In this note we consider the consequences of the discrete reversal charge, parity, time and mass (CPTM) symmetry in application to the field theory and general relativity. The proposed symmetry relates the different parts (manifolds) of the extended solution of Einstein equations preserving the same form of the metric $g$. Namely, for the two manifolds, for example, with coordinates $x$ and $\tilde{x}$, the condition $g_{\mu\nu}(x) = g_{\mu\nu}(\tilde{x})$ must be satisfied by the CPTM symmetry transform:

$$ q \rightarrow -\tilde{q}, r \rightarrow -\tilde{r}, t \rightarrow -\tilde{t}, m_{\text{grav}} \rightarrow -\tilde{m}_{\text{grav}}; \quad \tilde{q}, \tilde{r}, \tilde{t}, \tilde{m}_{\text{grav}} > 0. \quad (1) $$

The easiest way to clarify this construction is to consider the different and separated parts of the extended solution of Einstein equations defined in the light cone coordinated $u, v$ or corresponding Kruskal-Szekeres coordinates. The CPTM transform in this case inverses the sign of these coordinates and relates the different regions of the extended solution preserving the form of the metric unchanged, see [1] for the case of Schwarzschild’s spacetime and the similar description of the Reissner-Nordström space-time in [11] for example.

The framework, therefore, consists of different manifolds with the gravitational masses of different signs in, see details in [11]. The motivation of the introduction of the negative mass in the different cosmological models is very clear. In any scenario, see [3][4] for example, the presence of some kind of repulsive gravitation forces in our Universe helps with explanation of the existence of dark energy and dark matter in the models, see also [5][9] and references therein. We can define also the negative mass particles in the another part of the Universe with the sign of the gravitational mass of them is changed from positive to negative in comparison to the gravitational mass of all kind of the particles in our Universe. Important that the gravitation properties of this matter is also described by Einstein equations after the discrete symmetry transformations applied, see [3] or [4] and [10] for the examples of the discrete symmetries applications in the case of the quantum and classical systems.
In this formulation the proposed approach can be considered as some version of the Multiverse where, nevertheless, the number of the separated worlds is not arbitrary but defined by the type of the extended solution of the Einstein equations, from two in the Schwarzschild to infinity in the Reissner-Nordstr"om’s extended solution, see for example [11]. Further, for the simplicity, we consider only two different manifolds related by CPTM symmetry, the generalization for the case of another manifold’s number or for the case when exist additional complex topological structures, see [11–13] for example. Instead, on the classical level, we require an existing of the two non-related equivalent systems of Einstein equations in the each part of the extended solution separately, which due the CPTM symmetry can be written as one system of equations valid in the extended space-time.

Therefore, we consider a connection between the manifolds established by the gravitons exchange. The natural candidate for this manifold’s gluing is the kind of the foam of wormholes which belongs to the same manifold but to the two at least. In this case the framework contains two or more non-manifolds which “talk” each with other by the gravitons non-local correlators, the similar construction can be written as one system of equations valid in the extended space-time.

Let’s consider the two Minkowski spaces and consequences of the CPTM symmetry for the case of the ordinary free scalar fields there, applying Eq. (1) transforms to the scalar field we obtain:

\[ CPTM(\phi(x)) = CPTM \left( \int \frac{d^3 k}{2(2\pi)^3 \omega_k} \left( a(k) e^{-i k \cdot x} + a^*(k) e^{i k \cdot x} \right) \right) = \tilde{\phi}(\tilde{x}) = \]

\[ = \int \frac{d^3 k}{2(2\pi)^3 \omega_k} \left( a(k) e^{i k \cdot \tilde{x}} + a^*(k) e^{-i k \cdot \tilde{x}} \right) = \int \frac{d^3 k}{2(2\pi)^3 \omega_k} \left( \tilde{a}(k) e^{-i k \cdot \tilde{x}} + \tilde{a}^*(k) e^{i k \cdot \tilde{x}} \right) \] (2)

that provides:

\[ a(k) \rightarrow \tilde{a}^*(k) , \quad a^*(k) \rightarrow \tilde{a}(k) , \quad \omega(k) = \sqrt{m^2 + k^2} \rightarrow \omega(k) = \sqrt{m^2 + k^2} . \] (3)

Additionally, the application of this symmetry transformation leads to the following modification of the commutation relations between the creation and annihilation operators of the scalar field in the second manifold:

\[ CPTM \left( [a(k), a^*(k')] = 2(2\pi)^3 \omega_k \delta^3_{kk'} \right) \rightarrow [\tilde{a}(k), \tilde{a}^*(k')] = -2(2\pi)^3 \omega_k \delta^3_{kk'} . \] (4)

The energy-momentum tensor of the scalar field is symmetrical under the action of CPTM symmetry and we write the Hamiltonian for the free scalar field valid in related by the CPTM symmetry regions of the Schwarzschild space-time, for example, as

\[ H = \int \frac{d^3 k}{2(2\pi)^3 \omega_k} \frac{\omega_k}{2} \left( a^*(k) a(k) + a(k) a^*(k) + \tilde{a}^*(k) \tilde{a}(k) + \tilde{a}(k) \tilde{a}^*(k) \right) = \]

\[ = \int \frac{d^3 k}{2(2\pi)^3 \omega_k} \frac{\omega_k}{2} \left( a^*(k) a(k) + \tilde{a}^*(k) \tilde{a}(k) \right) . \] (5)
Assuming that there is a mutual vacuum state for these regions of the Schwarzschild space-time or C and C’ regions of the Reissner-Nordström space-time, see Appendix A, we obtain that

\[ <0 | H | 0 > = 0 \] (6)

precisely, in some extend this mechanism is similar to proposed in [10], see also [17]. The same holds as well for the case of charged scalar field, for the complex scalar field we also obtain the precise zero vacuum density value doing similarly to done in Eq. (2), Eq. (3) and Eq. (4) with

\[
CPTM(\phi(x)) = CPTM \left( \int \frac{d^3 k}{2(2\pi)^3 \omega_k} \left( a(k) e^{-i k \cdot x} + b^*(k) e^{i k \cdot x} \right) \right) = \hat{\phi}(\tilde{x}) = \int \frac{d^3 k}{2(2\pi)^3 \omega_k} \left( \tilde{a}(k) e^{-i k \cdot \tilde{x}} + \tilde{b}^*(k) e^{i k \cdot \tilde{x}} \right) \] (7)

and

\[ a(k) \rightarrow \tilde{b}^*(k), \quad b^*(k) \rightarrow \tilde{a}(k), \quad [\tilde{a}(k), \tilde{a}^*(k')] = -2 (2\pi)^3 \omega_k \delta^3_{k k'}, \quad [\tilde{b}(k), \tilde{b}^*(k')] = -2 (2\pi)^3 \omega_k \delta^3_{k k'} . \] (8)

Additionally, the CPTM transforms provide for the overall charge operator of the charged scalar field:

\[ Q \propto a^* a - b^* b + \tilde{a}^* \tilde{a} - \tilde{b}^* \tilde{b} = 0 \] (9)

as we expect for the overall charge of the regions related by the discrete C transform. We put attention, therefore, that in the given example the vacuum energy density is precisely zero at the classical level provided by the CPTM symmetry request, see discussion in [17].

### 3 Cosmological constant through the gravity’s modified action

Now we consider the two regions of the full space-time connected by CPTM symmetry with the possible presence of the scalar fields separately in the each region. We introduce the partition function which preserves the discussed symmetry in the two separated parts of the space-time:

\[ Z = Z_0^{-1} \int Dg_{\mu
u} D\phi(x) D\tilde{\phi}(\tilde{x}) e^{i S[g, \phi(x), \tilde{\phi}(\tilde{x})]} \] (10)

with \((c = \hbar = 1)\)

\[ S = -\frac{1}{16 \pi G} \int d\Omega \sqrt{-g} R + \int d^4 x \sqrt{-g} L(\phi(x)) + \int d^4 \tilde{x} \sqrt{-g} L(\tilde{\phi}(\tilde{x})) + S_{int}(g, \phi(x), \tilde{\phi}(\tilde{x})) . \] (11)

Here the gravitational filed is defined everywhere in the space-time related by the CPTM transform, i.e.

\[ d\Omega \sqrt{-g} = d^4 x \sqrt{-g(x)} + d^4 \tilde{x} \sqrt{-g(\tilde{x})} , \] (12)

whereas for the scalar fields we wrote the Lagrangians separately in the each region because of Eq. (11) difference in the commutation relations and consequent difference of the corresponding Green’s functions. Firstly we try to guess the possible form of the interaction part of the Eq. (11) action. We request that this term will preserve the deserved symmetry of the problem and that the interaction between the fields is carried out only through the gravitons, i.e. by the fluctuations around any classical metric. The simplest possible variant of the interaction term can be introduced in the form of the sum of source terms for each scalar field separately:

\[ S_{\phi int} = \int d^4 x \sqrt{-g(x)} \int d^4 \tilde{x} \sqrt{-g(\tilde{x})} \xi_{\phi}(\tilde{x}, x) \phi(x) + \int d^4 x \sqrt{-g(x)} \int d^4 \tilde{x} \sqrt{-g(\tilde{x})} \int d^4 x \sqrt{-g(x)} \xi_{\tilde{\phi}}(x, \tilde{x}) \tilde{\phi}(\tilde{x}) , \] (13)
Additional interaction term in Eq. (11) can be introduced for the pure gravitational interactions between the manifolds, it can be written as:

\[ S_{g_{\text{int}}} = \int d^4x \sqrt{-g(x)} \int d^4\tilde{x} \sqrt{-g(\tilde{x})} \xi_g(x, \tilde{x}). \quad (14) \]

Consequently we rewrite Eq. (10) for the pure gravitational action as:

\[ Z_g = Z_{0g}^{-1} \int Dg_{\mu\nu}(x) Dg_{\mu\nu}(\tilde{x}) e^{i S_g[g(x), g(\tilde{x})]} \quad (15) \]

with

\[ S_g = -\frac{1}{16\pi G} \int d^4x \sqrt{-g} R - \frac{c^3}{16\pi G} \int d^4\tilde{x} \sqrt{-g} R + \int d^4x \sqrt{-g(x)} \int d^4\tilde{x} \sqrt{-g(\tilde{x})} \xi_g(x, \tilde{x}). \quad (16) \]

The equations of motion for the gravitational field in the each region have the same form and look as

\[ R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R + 8\pi G g_{\mu\nu} \int d^4\tilde{x} \sqrt{-g(\tilde{x})} \xi_g(x, \tilde{x}) = 0 \quad (17) \]

plus the equation with \( x \to \tilde{x} \) replace. The equations provide the “matter” terms in the expressions even in the case of absent of the real matter, but the role of the \( \xi \) function is still unclear here. So, performing further an integration with respect to the \( g(\tilde{x}) \) field, we obtain a modified partition function averaged over the second part of the full space-time:

\[ \bar{Z}_g = \bar{Z}_{0g}^{-1} \int Dg_{\mu\nu}(x) e^{i \bar{S}_g[g(x)]} \quad (18) \]

where

\[ \bar{S}_g[g(x)] = -\frac{1}{16\pi G} \int d^4x \sqrt{-g} R + \int d^4x \sqrt{-g} \langle \xi_g(x) \rangle. \quad (19) \]

Here, as usual, \( \langle \xi_g(x) \rangle \) means the averaging of the interaction filed with respect to \( g(\tilde{x}) \), the bare effective gravitational action \( \Gamma[g(\tilde{x})] \) is canceled here by the corresponding \( Z_{0g}^{-1} \) constant. The resulting equations of motion read as

\[ R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R + 8\pi G g_{\mu\nu} \langle \xi_g(x) \rangle = 0. \quad (20) \]

Now we can solve the Eq. (20) as it is. As usual the introduced term can be considered as a density of the vacuum energy:

\[ \langle \xi_g(x) \rangle \propto \rho_{\text{vac}} \quad (21) \]

which is equal to zero at the classical level, see Eq. (6). Identifying this contribution with the cosmological constant

\[ \Lambda_g = 8\pi G \langle \xi_g \rangle = \text{const}, \quad (22) \]

we also conclude that the constant is a dynamical variable which depends on the overall evolution of the manifolds and which is small due it’s non-classical origin. There is also a possibility of the fine tuned condition satisfied:

\[ R = 16\pi G \langle \xi_g(x) \rangle > 0 \quad (23) \]

that determines the \( \langle \xi_g(x) \rangle \) function and provides vacuum solutions of Einstein equations. This equality is a problematic one, of course, because in the derivations of the equations above we assumed that the \( \langle \xi_g(x) \rangle \) does not depend on the metric and in general we do not know which dynamic

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1We also can assume the version of the interaction term with \( \phi \to \phi^n \) change and corresponding redefinition of the the dimension of \( \xi \) function, but we consider the expression as the simplest type of the source term preserving \( n = 1 \) value.
mechanism will provide this equality. Nevertheless, if we assume that the \( <\xi_g(x)>\) does depend on the metric fields we will obtain instead Eq. (20) a similar equation:

\[
R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R - 8\pi G T_{g\mu\nu} = 0.
\]

(24)

Here the energy-momentum tensor is defined with the help of \( <\xi_g(x)>\) function by the receipts of \[18\]. As mentioned above, we see that the dynamical change of the curvature and metric in the model is possible on the base of the pure geometry without any additional fields. The usual vacuum solutions of the equations are achieved by request

\[
T_{g\mu\nu} = 0
\]

(25)

that provides the following equation for \( <\xi_g(x)>\):

\[
\frac{\partial \sqrt{-g} <\xi_g(x)>}{\partial g^{\mu\nu}} - \frac{\partial}{\partial x^\rho} \frac{\partial \sqrt{-g} <\xi_g(x)>}{\partial g^{\mu\nu}} \frac{\partial}{\partial g_{\mu\nu}} = 0,
\]

(26)

see \[18\], the Eq. (6) here will be satisfied at classical level as well of course. For a non-zero value of \( T_{\mu\nu}\), the corresponding solutions will be vacuum solution as well, there is no any matter or scalar fields present in Eq. (24).

Now we can add to our action the Eq. (13) part of the interaction of the gravitation field with the scalar ones. In this case we can subsequently average the corresponding parts of Eq. (11) with respect to the \( \phi \) and \( g(\tilde{x}) \) fields. The partition function will acquire the following form therefore:

\[
Z_{g\phi} = Z_{0\phi}^{-1} \int Dg_{\mu\nu}(x) D\phi(x) e^{i S_{g\phi}[g(x), \phi(x)]}
\]

(27)

with mostly general action

\[
S_{g\phi} = -\frac{1}{16\pi G} \int d^4 x \sqrt{-g} R + \int d^4 x \sqrt{-g} L(\phi(x)) + \int d^4 x \sqrt{-g} <\xi_g(g(x), \phi(x)) > +
\]

\[
+ \int d^4 x \sqrt{-g} <\xi_\phi(g(x), \phi(x)) > .
\]

(28)

The contributions of the last two terms in Eq. (28) we can combine into the joint energy-momentum tensor writing the equations of motion as

\[
R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R - 8\pi G T_{g\mu\nu} = 8\pi G T_{\phi\mu\nu},
\]

(29)

which reproduce the Eq. (24) at the limit of zero scalar field.

4 Wormholes and \( \xi \) function

In the previous section we did not specify how to derive the \( \xi_g \) and \( \xi_\phi \) functions, the only assumption there made was about their non-classical origin and their zero value at the classical level. This condition must be satisfied not only at the case of the flat Minkowski space but also at the case of an arbitrary topology of the manifolds simply by request of CPTM symmetry and request of the preserving of the form of classical Einstein equations. The functions we introduced are non-local, i.e. they have an usual form of some non-local vertices, which must be different from the usual propagators or vertices defined on the manifold’s bulk. Therefore, we propose to consider these functions indeed as some gravitons correlators defined, as usual, with ”legs” placed on the separated manifolds. We can build such correlators taking as example the foam of wormholes of \[19\]. Namely, we can consider the patch of the new wormhole type which ends are separated from the bulk of the manifolds by some boundaries
and which connects the manifolds with CPTM symmetry introduced. This wormhole must respect the
Eq. (1) symmetry and we can consider the fluctuations of the gravitational field (gravitons) around
the wormhole’s classic geometry:
\[ g_{\mu\nu} = g_{0\mu\nu} + h_{\mu\nu}(x) + h_{\mu\nu}(\tilde{x}), \quad h = g^{0\nu}_{\mu} h_{\mu\nu}. \] (30)

Adding to the wormhole’s action the cosmological term contribution, we can construct an effective
action of the following form which describes the interaction between the two manifolds:
\[ \Gamma_w = \sum_{l,k=1} h_{\mu_1\nu_1}(x) \cdots h_{\mu_k\nu_k}(x) V_{kl}^{\mu_1\nu_1 \cdots \mu_k\nu_k; \rho_1 \cdots \rho_k \sigma_l}(x_1 \cdots x_k; \tilde{x}_1 \cdots \tilde{x}_l) h_{\rho_1\sigma_1}(\tilde{x}) \cdots h_{\rho_k\sigma_k}(\tilde{x}) + \]
\[ + h_{\mu\nu}(x) V_{10 \mu\nu}^{\mu\nu} + V_{01}^{\rho\sigma} h_{\rho\sigma}(\tilde{x}) \] (31)

with \( V_{kl} \) as effective vertices of the theory which connect the different manifolds. This effective action
can be smoothly continued to the actions of the main manifolds by the methods of the reggeized action
approach, see [20] and [21] for example, providing the matching between the geometry of the patch
and geometries of the manifolds. Now, integrating the linear and quadratic terms of the effective
action, we obtain:
\[ \xi_g(x, \tilde{x}) = \frac{1}{l_p^2} < h(x) h(\tilde{x}) > \approx \frac{1}{l_p^2} \frac{1}{d^2(x, \tilde{x})}, \quad d(x, \tilde{x}) = \int_x^{\tilde{x}} g^{\mu\nu}_{\mu\nu} dx^\mu dx^\nu, \] (32)

where the metric \( g^{\mu\nu}_{\mu\nu} \) is a proposed wormhole metric which connects the basic manifolds of the interests
related by CPTM symmetry. There are two important points we need to clarify. The first one is that
the non-local vertices we introduced are the vertices of the gravitons interactions, there are no other
types of the perturbative non-local vertices in the present action, see [15] for another construction
of the vertices with nevertheless similar final expression. The second point is that exists another
additional effective action expression which describes the gravitons wormholes of different type which
connect the different point of the same manifold and which are not restricted by the CPTM symmetry
requests, we do not discuss these contributions here.

In general, basing on the principles of QFT, we can consider more complex contributions to the
interacting term of Eq. (11). There are arbitrary correlators with more than two external points, their
contributions in the case of pure gravity can be written with the help of the Eq. (31) effective action
expression:
\[ S^{MN}_g \propto \int d^4x \sqrt{-g(x)} \int d^4\tilde{x} \sqrt{-g(\tilde{x})} \cdots \int d^4\tilde{x} \sqrt{-g(\tilde{x})} \xi_g(x_1, \cdots, x_M, \tilde{x}_1, \cdots, \tilde{x}_N) \] (33)

where
\[ \xi_g(x_1, \cdots, x_M, \tilde{x}_1, \cdots, \tilde{x}_N) \propto V_{MN} \] (34)

with correspondingly contracted indexes of the effective vertices. In the case with the scalar fields
included, the effective action will acquire the following form (we write it in the short simplified version):
\[ \Gamma_w = \sum_{k,l,m,n} h(x)^k(x) \phi^n(x) V_{klmn} h(\tilde{x})^l \phi^n(\tilde{x}) \] (35)

that provides
\[ \xi_{\phi}(x, \tilde{x}) \propto V_{0,0,1} \] (36)

and the more complex terms in the action with scalar fields included, such as the following for example:
\[ S^{MN}_\phi \propto \int d^4x \sqrt{-g(x)} \int d^4\tilde{x} \sqrt{-g(\tilde{x})} \cdots \int d^4\tilde{x} \sqrt{-g(\tilde{x})} \xi_{\phi}(x_1, \cdots, x_M, \tilde{x}_1, \cdots, \tilde{x}_N) \phi(\tilde{x}_1) \cdots \phi(\tilde{x}_N) \] (37)
with

$$\xi_{\phi}(x_1, \ldots, x_M, \tilde{x}_1, \ldots \tilde{x}_N) \propto V_{0,0,N},$$

(38)

where the vertices symmetrical with respect to \(x\) and \(\tilde{x}\) must be included in the action as well. Integration of these terms must be done with respect to the all wormholes geometries in whole paired space-time manifolds, i.e. the final action’s interaction term will be complete sum of the terms of Eq. (33) and Eq. (37) types with all possible configurations and geometries of the wormholes connecting the manifolds of some basic \(g_{0,\mu
u}\) topology.

5 Conclusion

In this note we considered the application of the reversal CPTM symmetry of the extended space-time solutions of Einstein equations for the resolution of some problems of cosmology. By construction, the model proposed can be considered as a variant of Multiverse with different signs of gravitational mass, charge, radial coordinates and time direction in the separated parts of the extended space-time which are related by the transform. The immediate simplest consequences of the model is the zero value of the vacuum energy density and overall zero electrical charge on the classical level, see first Section of the note. In this extend the model is initially free from the problems of zero vacuum energy and baryon asymmetry, it describes maximally symmetrical Multiverse. The model has some similarities to two-time direction models proposed for the solution of the Universe’s low initial entropy value, see [22], CPT symmetric Universe model considered in [23] and models of [24].

Discussing the general action of the theory we note that it remains trivial if we do not introduce an interaction between the parts of the extended solution, see also [16]. On the classical level this interaction must be zero if we do not require to change the classical Einstein equations. An immediate result of the introduction of the interaction term and gluing of the different manifolds by the gravitons is that in the each separated manifold arises a term which play role of the cosmological constant in the Einstein equations even in absence of matter fields. Reformulating it stays that there is a dynamic classical evolution of the metric of each manifold in the form of Einstein equations with cosmological constant caused by the mutual interaction between the manifolds through the gravitons only. This interaction determines the classical topology of the separated manifolds and changes the value of the cosmological constant during the evolution. Important that constant’s non-zero value is small due it’s non-classical origin. In turn, the resolution of the cosmological constant problem here is different from the proposed in [15], in our model the cosmological constant is the result of the gravitons wormholes.

The number of the twins regions in the model depends on the basic bare geometry. There are only two regions in the Schwarzschild’s extended solution and infinitely many in the Reissner-Nordström extended solution of the classical equations for example. From this point of view, the cosmological constant depends on the basic geometry of the extended solution and forms and types of the proposed wormholes. The interesting task, therefore, is a direct calculation of the constant in Eq. (19) and/or Eq. (28) for the different geometries of interests. The properties of the graviton’s modes propagating through the proposed wormhole are also very interesting, the ”bridge” connects the manifolds with the different signs of the mass in. Therefore the problem of the stability of the wormholes is different from the discussed in [25]. Consequently there is an additional interesting question arises, this is a problem of the determination of the connected many-legs wormholes geometries and their classical metrics requested for the calculations. Namely, the N separated wormholes geometry exists and known, see [26] for example, but in general we need the geometry of connected N ends wormhole as well.

The last remark is about the properties of Eq. (29). The energy-momentum of the matter there, \(T_{\phi}\), contains also the contributions from the classical values of the \(\phi\) field. Through the graviton’s exchange processes, see [27] for example, we can therefore consider a creation of the negative mass particles by the gravitational field, that can be a source of the dark matter in our part of the Universe. Additional source of these particles can be a some quantum tunneling of them through/by proposed wormholes, it can be a very interesting problem to investigate as well.
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