Ekpyrotic Loop Quantum Cosmology

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We consider the ekpyrotic paradigm in the context of loop quantum cosmology. In loop quantum cosmology the classical big-bang singularity is resolved due to quantum gravity effects, and so the contracting ekpyrotic branch of the universe and its later expanding phase are connected by a smooth bounce. Thus, it is possible to explicitly determine the evolution of scalar perturbations, from the contracting ekpyrotic phase through the bounce to the post-bounce expanding epoch. The possibilities of having either one or two scalar fields have been suggested for the ekpyrotic universe, and both cases will be considered here. In the case of a single scalar field, the constant mode of the curvature perturbations after the bounce is found to have a blue spectrum. On the other hand, for the two scalar field ekpyrotic model where scale-invariant entropy perturbations source additional terms in the curvature perturbations, the power spectrum in the post-bounce expanding cosmology is shown to be nearly scale-invariant and so agrees with observations.

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I. INTRODUCTION

The ekpyrotic paradigm is an alternative to inflation where scale-invariant perturbations are generated during a contracting pre-big-bang phase, and it has been argued that these perturbations will travel through some transition to an expanding Friedmann-Lemaître-Robertson-Walker (FLRW) space-time and provide the seeds for structure formation [1–3]. For the most up-to-date discussion on the observational status of the ekpyrotic model, see [4].

While the ekpyrotic paradigm is motivated by string theory and sees the big-bang singularity as the collision of two parallel branes, one of which is the four-dimensional space-time we observe [3]; in much of the work studying the ekpyrotic universe an effective four-dimensional theory has been used where there are typically one or two scalar fields representing the relevant higher dimensional degrees of freedom that impact upon the dynamics of the lower-dimensional brane.

Depending on the effective four-dimensional model used, the scalar perturbations responsible for structure formation are generated by one of two methods. In the case where there is one scalar field with a particular negative exponential potential, the growing mode of the gauge-invariant Bardeen potential becomes scale-invariant during the contracting phase [2]. On the other hand, when there are two (or more) scalar fields, then the entropy perturbations of the scalar fields grow and also become scale-invariant during the contracting phase. Then, it is possible for these entropy fluctuations to generate scale-invariant curvature perturbations [6, 8].

In both scenarios, it is necessary to show that the scale-invariance obtained in the contracting branch survives to the expanding branch in order for the predictions of the theory to match observations. This is a difficult task, since it is expected that quantum gravity effects will become important at the transition point and the expectations one may have coming from general relativity could be incorrect.

Because of this problem, there has been a lot attention paid to various “matching conditions” between the contracting and expanding branches in order to determine the form the perturbations will have in the expanding universe. Depending on the specific prescription for the matching conditions, different conclusions are reached. For example, for the case where there is a single scalar field, some matching conditions argue that the perturbations in the expanding branch should be scale-invariant [2], while others give a blue spectrum [10–12].

In order to avoid the ambiguity present in the choice of matching conditions, it is possible to use the presence of a dynamical attractor in the contracting phase of the ekpyrotic universe to motivate the approximation that the observationally relevant perturbations remain frozen during the transition [13]. If this is the case, then in the single scalar field realization of the ekpyrotic universe the curvature perturbations have a blue spectrum. Of course, it is necessary to understand precisely how the transition occurs in order to determine whether the approximations are valid or not.

An approach which addresses this last problem is to model the transition as a nonsingular bounce by using a ghost condensate [14, 15], and then it can be shown that for the two-field ekpyrotic paradigm the perturbations in the expanding branch are scale-invariant. (The ghost condensate can cause a bounce and avoid the classical big-bang singularity as it violates the null energy condition.) However, here the equations of general relativity are used at all times including the bounce, and so it is not clear whether some important quantum gravity effects may be missed in the ghost condensate ekpyrotic universes.

In this paper, we propose an alternative: rather than using a ghost condensate to cause the bounce, we will instead consider modifications to the gravitational dynam-
ics due to quantum gravity effects. This will be done by working in the framework of loop quantum cosmology, where the singularity is generically resolved and replaced by a bounce \[10\]. Using the effective equations of loop quantum cosmology (LQC), it will be possible to explicitly evolve the perturbations through the bounce and determine the spectrum of the scalar perturbations in the expanding branch. The homogeneous sector of the ekpyrotic universe has already been studied in LQC, for the cases of the flat FLRW space-time \[17–19\] and the Bianchi I model \[20\]. For general reviews of loop quantum cosmology, see e.g. \[21–23\].

The outline of the paper is as follows: in Sec. II we begin by building on the results of \[17, 18\] and show how the ekpyrotic universe can easily be incorporated in LQC. Then, we will describe the prescription used in order to determine the evolution of the perturbations, from the distant past of the contracting branch, through the quantum-gravity-dominated bounce, to a sector of the expanding branch where quantum gravity effects are negligible and the equations of general relativity are once again valid.

This procedure will be carried out in Sec. III for the case where there is one scalar field, and in Sec. IV for the two-field ekpyrotic model, where entropy perturbations source additional terms in the curvature perturbations. As we shall see, in the expanding branch, the curvature perturbations in the single field model have a blue spectrum, while in the two field model they are scale-invariant. Thus, it is only the two scalar field model of ekpyrotic loop quantum cosmology that is in agreement with observations. We conclude with a brief discussion in Sec. V.

II. THE EKPYROTIC UNIVERSE IN LOOP QUANTUM COSMOLOGY

A. Loop Quantum Cosmology

In loop quantum cosmology, cosmological space-times are quantized by taking holonomies and areas to be the fundamental geometrical operators, just as in loop quantum gravity. Furthermore, the same techniques and mathematical tools are used in order to define the Hamiltonian constraint operator. While several cosmologies have been studied in LQC, including Bianchi and Gowdy space-times, here we will focus on the flat FLRW cosmology, as it is this background that is relevant for the ekpyrotic universe\[1\].

The LQC dynamics of the flat FLRW space-time have been studied in some detail now, and the most important result is the resolution of the classical big-bang singularity. Working in the Schrödinger picture, states never become singular (assuming that they are initially non-singular) and undergo a bounce that bridges contracting and expanding branches of the cosmology \[24, 25\].

What is more is that states that are initially sharply peaked remain sharply peaked throughout their evolution \[24\]. Thus, it is possible to speak of a “mean geometry” for such a state at all times, including at the bounce point. Furthermore, the dynamics of the mean geometry is given by a relatively simple set of modified Friedmann equations, called the LQC effective equations. For the flat FLRW space-time, the effective equations are \[26\]

\[
H^2 = \frac{8\pi G}{3} \rho \left(1 - \frac{\rho}{\rho_c}\right),
\]

\[
\dot{H} = -4\pi G (\rho + P) \left(1 - \frac{2\rho}{\rho_c}\right),
\]

\[
\dot{\rho} + 3H (\rho + P) = 0.
\]

Here \(H = \dot{a}/a\) is the Hubble rate, \(\rho\) and \(P\) are respectively the energy density and pressure of the matter field, and the dots represent derivatives with respect to the proper time \(t\). The critical energy density \(\rho_c \sim \rho_{\text{Pl}}\) encodes the modifications to the Friedmann equations, and it is easy to see how its presence causes a bounce to occur.

While these equations only hold for semi-classical states (i.e., states that satisfy the scalar constraint and are initially sharply peaked), in this paper we are interested in a universe which has a nice classical limit, both before the bounce in the contracting branch, and in the expanding branch after the bounce. In order to have a nice classical limit, the states should be sharply peaked and therefore we will restrict our attention to semi-classical states for the remainder of this paper. So, it is sufficient to work with the effective equations in this setting.

B. The Ekpyrotic Universe

The potential for the scalar field in the ekpyrotic model is somewhat complicated, but during the contracting phase of the universe, the section of the potential that is relevant for that portion of the gravitational dynamics has the form of a negative exponential and is usually parametrized as

\[
V(\varphi) = -V_o e^{-\sqrt{16\pi G/\rho} \varphi},
\]

with \(0 < p \ll 1\). One of the nice properties of this potential is that it admits a solution where the scalar field mimics an ultrastiff fluid with a constant equation of state \(P = \omega \rho\), with \(\omega \gg 1\), and this solution is precisely the one of interest in the ekpyrotic universe.
However, this is a result that comes from the classical Friedmann equations, and no longer holds if one uses the LQC-corrected Friedmann equations instead. In order to allow a solution with the same $\omega$ in LQC, a slightly different potential must be used, namely \[ \text{(27)} \]

$$V(\varphi) = \frac{-V_0 e^{-\sqrt{16\pi G/p} \varphi}}{\left(1 + \frac{3pV_o}{4\rho_c(1-3p)} e^{-\sqrt{16\pi G/p} \varphi}\right)^2}. \tag{5}$$

Note that the potentials \( \text{(4)} \) and \( \text{(5)} \) agree so long as quantum gravity effects remain negligible: the potentials only differ once the bounce begins.

Using the usual relations \( \rho = \dot{\varphi}^2/2 + V(\varphi) \) and \( P = \dot{\varphi}^2/2 - V(\varphi) \) together with the LQC effective equations, it is easy to see that there exists a solution where

$$P = \omega \rho, \quad \omega = \frac{2}{3p} - 1. \tag{6}$$

This is the ekpyrotic solution that we shall consider here. Since \( p \) is very small, it follows that \( \omega \gg 1 \) and so this is an ultrastiff perfect fluid.

We will use this potential for the ekpyrotic scalar field, during the collapse and the bounce. This choice means that the matter field is essentially a perfect fluid with a constant equation of state, and this will significantly simplify the calculations. Although we assume a transition to a radiation-dominated era soon after the bounce (but after quantum gravity effects have become small), we will only study the dynamics up to a time before this transition.

It is worth pointing out that in many ekpyrotic models the form of the potential changes just before the bounce, contrary to what we assume here; we leave the possibility of allowing different potentials for future work. However, we do expect that the main predictions obtained in this paper, including the value of the scalar index, will not be affected by a change in the potential during (or just before) the bounce. We discuss this point further in Sec. IV D.

For the ekpyrotic solution \( \text{(4)} \) from the potential \( \text{(5)} \), the scale factor is given by

$$a(t) = (a_o t^2 + 1)^{p/2}, \quad a_o = \frac{8\pi G \rho_c}{3p^2}. \tag{7}$$

where \( \rho_c \) is the critical energy density. The overall scaling freedom in the scale factor has been fixed by setting \( a(0) = 1 \).

It is easy to obtain the classical limit for either the contracting or expanding branch, which correspond to \( t \ll -1/\sqrt{a_o} \) and \( t \gg 1/\sqrt{a_o} \) respectively. We will combine both cases simply by considering \( |t| \gg 1/\sqrt{a_o} \), in which case the scale factor is given by

$$a(t) = a_o^{p/2} |t|^p. \tag{8}$$

It is important to keep in mind the role played by the absolute values around \( t \) when derivatives are calculated in the classical limit. For example, the Hubble rate is given by \( H = \dot{a}/a = \text{sgn}(t)p/|t| = p/t \).

Some other useful relations are those between the proper time \( t \) and the conformal time \( \eta \),

\[ \text{(9)} \]

$$d\eta = \frac{dt}{a(t)}$$

given by

$$|\eta| = \frac{|t|^{1-p}}{a_o^{p/2}(1 - p)}, \tag{10}$$

and the scale factor as a function of conformal time is

$$a(\eta) = a_o^{p/(1-p)} \left| (1 - p)|\eta| \right|^{p/(1-p)}, \tag{11}$$

again in the classical limit of \( |t| \gg 1/\sqrt{a_o} \).

### C. With Two Scalar Fields

Many realisations of the ekpyrotic universe use two scalar fields and it is possible to do this in LQC as well. As before, it is desired to have the scalar fields, together, mimic a perfect fluid with a constant equation of state.

In order to do this, it is necessary to modify the potential \( \text{(5)} \) in the following way. If there are two scalar fields \( \varphi_1, \varphi_2 \), with potentials \( V_1(\varphi_1) \) and \( V_2(\varphi_2) \), then taking the potentials

\[ \text{(12)} \]

$$V_1(\varphi_1) = \frac{-V_0 e^{-\sqrt{16\pi G/q_1} \varphi_1}}{\left(1 + \frac{3p^2V_o}{4q_1\rho_c(1-3p)} e^{-\sqrt{16\pi G/q_1} \varphi_1}\right)^2}, \tag{12}$$

\[ \text{(13)} \]

$$V_2(\varphi_2) = \frac{-\tilde{V}_0 e^{-\sqrt{16\pi G/q_2} \varphi_2}}{\left(1 + \frac{3p^2V_o}{4q_2\rho_c(1-3p)} e^{-\sqrt{16\pi G/q_2} \varphi_2}\right)^2}, \tag{13}$$

with \( 0 < q_i \ll 1 \) and

$$p = q_1 + q_2, \tag{14}$$

there exists a solution to the LQC Friedmann equations with

\[ \text{(15)} \]

$$\varphi_1(t) = \sqrt{\frac{q_1}{4\pi G}} \ln \left[ -\frac{2pGV_o}{q_1(1-3p)} \left( t + \sqrt{t^2 + \frac{1}{a_o}} \right) \right], \tag{15}$$

\[ \text{(16)} \]

$$\varphi_2(t) = \sqrt{\frac{q_2}{4\pi G}} \ln \left[ -\frac{2pGV_o}{q_2(1-3p)} \left( t + \sqrt{t^2 + \frac{1}{a_o}} \right) \right], \tag{16}$$

where \( a_o = 8\pi G \rho_c/3p^2 \), as before. For this solution, the scale factor is given by \( \text{(4)} \) and thus we see that this combination of the two scalar fields mimic a perfect fluid with a constant equation of state \( \text{(5)} \), just as in the previous section.
So, by having two scalar fields with the specific potentials given here, it is possible to have exactly the same ekpyrotic solution for the background gravitational degrees of freedom as in the single scalar field case.

Finally, from this discussion it is clear how to generalize this procedure in order to allow additional scalar fields in the ekpyrotic LQC universe; the potentials \( V_i(\varphi_i) \) should be of the same form as (12), where \( p \) is redefined as \( p = \sum_i q_i \).

### D. Scalar Perturbations

One of the main reasons to consider the ekpyrotic universe in the context of LQC is that it is now known how to go beyond homogeneous space-times and study linear perturbations on a flat FLRW background in LQC. The two approaches developed so far are lattice loop quantum cosmology \([28, 30]\), where the separate universes framework \([29, 31]\) is adapted to the setting of loop quantum cosmology, and also a hybrid approach where the perturbations are quantized à la Fock on the homogeneous loop quantum cosmology background space-time \([31, 32]\).

In addition, effective equations for the linear perturbations—including the effects coming from the holonomy corrections that cause the bounce in the homogeneous models—have been obtained in two independent ways, namely, derived from the Hamiltonian constraint in lattice loop quantum cosmology \([28, 33]\) and obtained by using the anomaly freedom algorithm \([34, 36]\). Using this Mukhanov-Sasaki-like LQC effective equation on an ekpyrotic background, it will be possible to track the evolution of the perturbations through the bounce and determine their precise form in the expanding branch.

As mentioned above, for the homogeneous and isotropic FLRW cosmologies, it has been shown that states that are initially sharply peaked remain sharply peaked throughout their evolution determined by the Hamiltonian constraint operator. So, if one is interested in the mean geometry of the space-time rather than quantum fluctuations, the effective equations for the background variables describe their evolution to a high degree of accuracy. However, the full quantum dynamics of linear perturbations have not yet been studied in sufficient detail in order to determine whether, for sharply peaked states, the effective equations again provide a good approximation for the expectation values of operators corresponding to linear perturbations.

Nonetheless, it does not seem unreasonable to hope that, at least for linear perturbations, the effective equations can be useful beyond the setting of homogeneous space-times. Therefore, in this paper we shall restrict our attention to states that are sharply peaked, and use the effective equations to determine the dynamics of the background geometry and also of the propagation of linear perturbations on it. This will allow us to determine the mean geometry of the space-time, including perturbations, at all times.

In principle, it is possible to solve the effective equations of LQC for all times and thus immediately determine the mean geometry of the entire space-time. However, in practice the effective equations for the perturbations are difficult to solve exactly and so we will instead use the following procedure:

1. The mean background geometry has already been determined, in Sec. II B if there is one scalar field, and in Sec. II C if there are two scalar fields.

2. We will start in the portion of the contracting branch where quantum gravity effects are negligible. The perturbations visible today in the cosmic microwave background are of a sufficiently long wavelength that as the universe contracts, they will all exit the Hubble radius before any LQC effects become important.

Therefore, the standard equations of classical cosmological perturbation theory can be used in order determine the long wavelength behaviour of the perturbations in the classical, contracting branch once the modes exit the Hubble radius, but before quantum gravity effects become important.

3. The LQC effective equations for the perturbations simplify significantly in the long wavelength limit. Therefore, it will be a relatively straightforward task to determine the solution to the LQC-corrected differential equations in the long wavelength limit, up to the constant prefactors of the two independent solutions to the differential equation.

This solution will hold at all times, before, during and after the bounce, for long wavelengths. Therefore, in the classical limit before the bounce, the LQC solution must agree with the classical solution obtained in the previous step, and this condition can be used in order to determine the two unknown prefactors in the LQC solution.

4. After this matching condition is imposed, the solution for long wavelength modes is known at all times, including in the expanding branch after the bounce, and so it is possible to check whether the resulting perturbations are scale-invariant or not.

These steps will allow us to determine the precise spectrum of the curvature perturbations in the expanding branch of ekpyrotic loop quantum cosmology.

Finally, before beginning this procedure, a few comments on the validity and robustness of the LQC effective equations are in order. Although there are some concerns about the validity of the effective Friedmann equations \([1, 2] \) and \([3] \)—especially in situations where quantum back-reaction would be important \([37]\)—so far numerical studies have shown that the effective equations provide an excellent approximation to the dynamics of expectation values for sharply peaked states in all homogeneous LQC space-times studied so far, including for
certain classes of cyclic models similar to the ekpyrotic universe [19].

The effective equations for the perturbations are obtained in the same manner as the Friedmann effective equations. However, as the full LQC quantum dynamics of the perturbations have not yet been analyzed in sufficient detail, it is not known whether effective equations give as good of an approximation to the quantum dynamics of perturbations as they do for homogeneous space-times. Nonetheless, in the separate universe paradigm long wavelength perturbations are viewed as a patchwork of separate FLRW cosmologies [28–30]; and since the effective equations are valid for FLRW space-times, the separate universe approach suggests that the effective equations for perturbations will likely be an excellent approximation to the quantum theory, at least for long wavelength modes (which are the modes of interest during the bounce when LQC effects are important).

III. THE SINGLE SCALAR FIELD MODEL

The dynamics of scalar perturbations in the contracting branch of the single scalar field ekpyrotic universe have been studied in some detail, and there is by now a wealth of literature on the subject. In particular, it is well known that the perturbations in the scalar field \( \phi \) are scale-invariant [1]; and although the comoving curvature perturbation has not yet been analyzed in sufficient detail, it is not known whether effective equations exist for homogeneous space-times.

In the first part of this section, for the sake of completeness and also to establish notation, we will briefly review some of these results. Then, in the next two parts we will use the LQC Mukhanov-Sasaki equation to evolve the perturbations through the bounce and determine their spectrum in the expanding branch.

A. The Contracting Branch

We will use the gauge-invariant Mukhanov-Sasaki variable \( v \) in order to study the evolution of perturbations. It is defined as

\[
v = a \left( \delta \varphi (\mathbf{x}, t) + \frac{\dot{\varphi}}{H} \Phi \right),
\]

where \( \delta \varphi (\mathbf{x}, t) \) is the gauge-invariant scalar field perturbation and \( \Phi \) is the gauge-invariant Bardeen potential. For a review of cosmological perturbation theory, see e.g. [39, 40].

When there is only a single matter field, as is the case here, the equations of motion for \( v \) are

\[
v'' - \nabla^2 v - \frac{v''}{z} v = 0,
\]

where primes denote derivatives with respect to the conformal time, and

\[
z = a \frac{\dot{\varphi}}{H} = \frac{a}{\sqrt{4 \pi G p}},
\]

where the second equality holds due to the definitions of \( \rho \) and \( P \) for a scalar field together with (18).

Therefore, in Fourier space this equation becomes

\[
\nu''(k) + \left( k^2 - \frac{p(2p-1)}{(p-1)^2} \Omega^2 \right) \nu(k) = 0,
\]

and the solution is

\[
\nu(k) = \frac{\pi \sqrt{G p}}{4 \sqrt{\eta}} H(1) (-k \eta) \quad n = \frac{1 - 3p}{2(1 - p)},
\]

where \( H(1) (x) \) is the Hankel function. The prefactor of \( \sqrt{\pi} \theta / 4 \) ensures that initially (at time \( \eta \rightarrow -\infty \)), \( v \) are quantum vacuum fluctuations.

The comoving curvature perturbation \( R \) is related to the Mukhanov-Sasaki variable by

\[
R = \frac{v}{z} = \frac{\pi \sqrt{G p}}{[\sqrt{a_0} (1 - p)]^{(1-p)/2}} k (-k \eta) \quad (21)
\]

When the modes exit the Hubble radius, \( -k \eta \ll 1 \) and then the asymptotic form of the Hankel function can be used. This shows that for modes that have exited the Hubble radius,

\[
R_k = \frac{\pi \sqrt{G p}}{2^n \Gamma (n+1) [\sqrt{a_0} (1 - p)]^{p/(1-p)} k^{n} (-k \eta)^{2n} - \frac{i 2^n \Gamma (n) \sqrt{G p}}{[\sqrt{a_0} (1 - p)]^{p/(1-p)} k^{-n}}.
\]

Since \( 0 < p \ll 1 \), we have \( n \approx \frac{1}{2} - p \), and thus both modes of \( R \) have a blue spectrum.

From \( R \) we can calculate the Bardeen potential \( \Phi \) via the relation [39]

\[
- k^2 \Phi_k = 4 \pi G a_0 \frac{\phi^2}{H} \frac{R_k}{R_k'} = \frac{R_k'}{(1 - p) \eta}.
\]

Using \( d [x^H H_0^1 (x)]/dx = x^H H_{1-1} (x) \), we find that

\[
\Phi_k = \frac{- \pi \sqrt{G p}}{k [a_0^{p/2} (1 - p)]^{1/(1-p)}} H_{-1}^{(1)} (-k \eta).
\]

Again, for modes that are outside the Hubble radius, the asymptotics of the Hankel function can be used in order to determine the \( k \)-dependence of the perturbations,

\[
\Phi_k = \frac{\pi \sqrt{G p}}{2^{n-1} \Gamma (n) [a_0^{p/2} (1 - p)]^{1/(1-p)}} k^{-(2-n)} (-\eta)^{2n-2} - i \frac{2^{n-1} \Gamma (n - 1) \sqrt{G p}}{[a_0^{p/2} (1 - p)]^{1/(1-p)}} k^{-n}.
\]
The first mode is growing as the space-time contracts and is almost scale-invariant with a small red tilt, while the second mode is constant and proportional to \( k^{-1/2+p} \). Recalling that for small \( p, n \approx \frac{1}{2} - p \), the power spectrum of \( \Phi \) is easily determined. Keeping only the growing mode, which is also the dominant term for long wavelengths,

\[
\Delta_\Phi^2(k) = \frac{k^3}{2\pi^2}\Phi_k^2 \sim \frac{k^{-2p}}{(-\eta)^{1+2p}},
\]

and it is clear that the Bardeen potential is scale-invariant with a slight red tilt due to the \(-2p\) in the exponent.

Now the question is whether this mode survives the bounce and if it is observationally relevant in the expanding branch of the ekpyrotic universe. We expect the constant modes in \( \Phi \) to determine the spectrum of the observationally relevant modes in the ekpyrotic universe. We expect the constant mode of \( \Phi \) is indeed relevant, there should appear a new term in the curvature perturbation \( R \) as well. In order to determine the spectrum of the observationally relevant modes in the expanding branch, we will use the effective equations of LQC to evolve the perturbations through the bounce.

**B. The Bounce**

The LQC effective equation for the Mukhanov-Sasaki variable defined in (17) is

\[
v'' = \left(1 - \frac{2\rho}{\rho_c}\right)\nabla^2 v - \frac{z''}{z}v = 0,
\]

where \( z \) is given by the first equality appearing in (19). Note however that the second equality no longer holds as the Friedmann equations relating \( H \) and \( \ddot{\phi} \) are modified in LQC. Thus, once corrections from LQC are included,

\[
z(t) = \frac{1}{4\pi G}\sqrt{\frac{3\rho}{2\rho_c}}\frac{a(t)^{(p+1)/p}}{t}. \tag{29}
\]

For long wavelength modes, the second term in (28) is negligible and then the solution to the differential equation for each Fourier mode is

\[
v_k = A_1z(\eta) + A_2z(\eta)\int_0^\eta \frac{d\eta}{z(\eta)^2}, \tag{30}
\]

where \( A_1 \) and \( A_2 \) are constants that must be fixed by matching this solution with the classical solution in a regime when both solutions are valid.

Recalling the definition of the comoving curvature perturbation variable \( R \), and using (9),

\[
R_k = A_1 + A_2\int a(t)\frac{\sqrt{\rho}}{z(t)^2}dt \tag{31}
\]

\[
= A_1 + A_2\frac{3\rho_o\rho^3}{2\rho_c}t \left[ 2F_1\left(\frac{1}{2}, \frac{3}{2}, \frac{3}{2}, -a_0 t^2\right) - 2F_1\left(\frac{1}{2}, 1 + \frac{3}{2}, \frac{3}{2}, -a_0 t^2\right) \right] + \alpha A_2, \tag{32}
\]

where \( 2F_1 \) are hypergeometric functions and \( \alpha \) is a constant of integration. A nonzero \( \alpha \) simply defines the constant mode by \( A_1 \to A_1 + \alpha A_2 \), and we will take advantage of this freedom in order to simplify the matching that determines \( A_1 \) and \( A_2 \). Of course, the physics is independent of \( \alpha \) in the sense that the matching uniquely determines \( A_1 + \alpha A_2 \), but a particular choice of \( \alpha \) will shorten the calculations.

The solution (32) can be trusted for any mode whose conformal wavelength is larger than \( \sqrt{\eta/\eta'} \). Since in the classical limit \( z/\eta' \sim r_H^2 \) (with \( r_H \) being the conformal Hubble radius), this solution holds for modes outside of the Hubble radius during the classical contracting phase.

Therefore, this solution can be matched to (23) by taking the classical limit of \( t \ll -1/\sqrt{\eta} \), in which case

\[
R_k = A_1 + A_2\left[ \frac{3\rho^3 a_0^{1-3p/2} e^{-i\pi p} (-t)^{1-3p}}{2(1-3p)\rho_c} + \alpha \right]
\]

\[
+ \frac{3\rho^3}{4\rho_c}\left( \frac{(\Gamma(1+3p)/2 - \Gamma(-1-3p)/2)}{\Gamma(1+3p)}/ \Gamma(1+3p) \right), \tag{33}
\]

This expression can be simplified by setting

\[
\alpha = -\frac{3\rho^3}{4\rho_c}\left( \frac{(\Gamma(1+3p)/2 - \Gamma(-1-3p)/2)}{\Gamma(1+3p)}/ \Gamma(1+3p) \right), \tag{34}
\]

and since in the classical limit the relation (31) can be trusted, we find that the matching of this solution with the classical one in the regime where they both hold implies that

\[
A_1 = -\frac{i2^p\Gamma(n)\sqrt{G\rho_p}}{a_o^{p/2}}k^{-n}, \tag{35}
\]

\[
A_2 = \frac{2^{1-n}\pi\sqrt{G\rho_p}}{3\Gamma(n+1)p^3a_o^{(2-p)/2}}k^n. \tag{36}
\]

The expressions given here hold for small \( p \). It is a straightforward calculation to determine the exact numerical factors to all orders of \( p \), but the precise form of the prefactors will not be important here. Indeed, the most important point here is that \( A_1 \sim k^{-n} \) and \( A_2 \sim k^n \).

**C. The Expanding Branch**

In the expanding branch after quantum gravity effects become negligible, the standard equations of motion for linear cosmological perturbations coming from general relativity are valid, so the differential equation (15) can be used and therefore the solution for \( R_k \), in the expanding branch, is

\[
R_k = \eta^n\left[ B_1J_n(\eta k) + B_2Y_n(\eta k) \right]. \tag{37}
\]
where the prefactors must be determined by matching this solution in the long wavelength limit with the LQC solution in the late time limit.

The long wavelength limit of (37) is simply given by

\[ R_k = \frac{B_1}{2^n \Gamma(n+1)} k^n \eta^{2n} - \frac{B_2 \Gamma(n)}{\pi} k^{-n}, \]  

(38)

and the late time solution for the LQC solution (32) is

\[ R_k = A_1 + A_2 \left[ \frac{3p^3 a_0^{1-3p} \gamma_1 - 3p}{2(1-3p) \rho_c} - 2\alpha \right]. \]  

(39)

Thus, by matching the prefactors, we find that we must have

\[ B_1 = \frac{\sqrt{\pi} \hbar}{2}, \quad B_2 = \frac{\sqrt{\pi} \hbar}{2} \left( 1 - \frac{2\alpha A_2}{A_1} \right). \]  

(40)

Therefore, the first mode of (38) is proportional to \( k^n \) while the second has two contributions, one proportional to \( k^n \) as well and the other proportional to \( k^{-n} \).

As in the ekpyrotic scenario \( n \approx \frac{1}{3} - p \) and \( p \) is small, it is clear that neither of the modes of the curvature variable \( R \) contain a scale-invariant term. While this is not a surprising result (no term in the curvature perturbations was scale-invariant in the contracting branch), it does indicate that the constant mode of the Bardeen potential should also contain two contributions, proportional to \( k^n \) and \( k^{-n} \), neither being scale-invariant.

This expectation can be checked explicitly by using the relation (24) in order to relate the curvature perturbation in (37) to the Bardeen potential. Dropping the numerical part of the overall prefactor [which is the same as in Eq. (20)], this gives

\[ \Phi_k \sim \frac{\eta^{n-1}}{k} H_{n-1}(k\eta) - \frac{2\alpha A_2}{A_1} Y_{n-1}(k\eta). \]  

(41)

In the long wavelength limit, the Bessel functions can be expanded giving

\[ \Phi_k \sim k^{n-2n} \eta^{2n-2} + k^{-n} + k^n, \]  

(42)

where we have only kept the \( k \) and \( \eta \) dependence of each of the terms. (Recall that \( \alpha A_2 / A_1 \sim k^{2n} \).)

Since \( n \approx \frac{1}{3} - p \), we see that only the first term is scale-invariant. However, since this mode decays rapidly as the universe expands, it is not relevant for structure formation. Rather, the dominant contribution to the power spectrum of the Bardeen potential at late times is

\[ \Delta_k^2(k) \sim k^{3-2n} \sim k^{2p}, \]  

(43)

giving a blue spectrum with a scalar index of \( n_s = 3 + 2p \), which is in perfect agreement with the expectations coming from the form of \( R \) after the bounce.

It is worth pointing out that the \( k^n \) contribution appearing in the constant mode of \( \Phi \) (and also \( R \)) comes from the scale-invariant growing mode of \( \Phi \) in the contracting branch. The reason that this term is not scale-invariant in the constant mode is that it is, in effect, multiplied by \( k^2 \), just as suggested by the matching conditions used in (10).

Thus, the results obtained here by evolving the perturbations through the bounce and in (10) by using the constant energy density matching conditions given in (41, 42) are in agreement, at least for predictions of the scalar index \( n_s \).

Finally, it is also worth pointing out that in another bouncing cosmology, the matter bounce scenario, the scalar index \( n_s \) obtained from LQC effective equations and the constant energy density matching conditions agree once again, even though there is a nontrivial transfer from the growing mode in the contracting phase to the constant mode in the expanding phase (43, 44). (Note however that the predicted amplitudes of the perturbations do not agree.) It is possible that this correspondence may be true in general, and that the constant energy density matching conditions might capture some of the physics occurring during the bounce in LQC.

### IV. THE TWO SCALAR FIELD MODEL

In the two scalar field model of the ekpyrotic universe, the entropic perturbations become scale-invariant in the contracting branch (6, 4, 14, 12). Then, the (scale-invariant) entropy perturbations can source (scale-invariant) curvature perturbations.

We begin this section by reviewing the dynamics of entropy perturbations and recall results from (6, 4, 14, 15) that show that, for the ekpyrotic solution, entropic perturbations are scale-invariant for modes outside the Hubble radius. Then, we will consider a specific case in order to show how the entropic perturbations can generate curvature perturbations, and evolve these curvature perturbations through the LQC bounce.

#### A. Entropy Perturbations in the Contracting Branch

As there are two scalar fields \( \varphi_1 \) and \( \varphi_2 \), there are also two independent perturbations \( \delta \varphi_1 \) and \( \delta \varphi_2 \). It is of course possible to study the perturbations of the matter degrees of freedom using these variables, but it is more convenient to rewrite these perturbations in terms of adiabatic and entropic perturbations.

It will be useful to briefly review some basic information regarding entropy perturbations; for more details see e.g. (6, 15). For the case where there are two scalar fields, a simple way to rewrite the perturbations \( \delta \varphi_1 \) and \( \delta \varphi_2 \) in terms of adiabatic and entropic perturbations is available if the trajectory of the scalar fields through their configuration space \((\varphi_1(t), \varphi_2(t))\) is known. The key ingredient is the angle \( \theta \) describing the direction of the motion of
the scalar fields in their configuration space,
\[ \cos \theta = \frac{\dot{\phi}_1}{\sqrt{\dot{\phi}_1^2 + \dot{\phi}_2^2}}, \quad \sin \theta = \frac{\dot{\phi}_2}{\sqrt{\dot{\phi}_1^2 + \dot{\phi}_2^2}} \] (44)

For the ekpyrotic solution given in (15) and (16),
\[ \theta = \arctan \sqrt{\frac{q_2}{q_1}}. \] (45)

Then, it is possible to use the angle \( \theta \) in order to split the perturbations \( (\delta \varphi_1, \delta \varphi_2) \) into the part tangential to the configuration space trajectory of the scalar fields, giving the adiabatic perturbation
\[ \delta \sigma = \cos \theta \delta \varphi_1 + \sin \theta \delta \varphi_2; \] (46)
while the gauge-invariant entropic perturbation is orthogonal to the configuration space trajectory of the scalar fields,
\[ \delta s = -\sin \theta \delta \varphi_1 + \cos \theta \delta \varphi_2. \] (47)

The adiabatic perturbations will have the same behaviour as \( \delta \varphi \) in the single field case [45] and so the adiabatic perturbations alone would give the same results as the model studied in Sec. III.

Therefore, it is more interesting to focus on the entropic perturbations, which we shall show contain a growing scale-invariant mode in the contracting space-time. Since entropic modes appear as a source in the equations of motion for the curvature perturbation \( \mathcal{R} \), if they are scale-invariant they can generate scale-invariant curvature perturbations.

In order to see that the growing mode of the entropic perturbations is scale-invariant, we start with the equation of motion for the entropic perturbations in Fourier space [45]
\[ \dot{\delta s}_k + 3H \delta s_k + \left( \frac{k^2}{a^2} + V_{ss} + 3\dot{\theta}^2 \right) \delta s_k = \frac{k^2}{2\pi G a^2 \dot{\sigma}} \Phi_k, \] (48)

where we have defined
\[ V_{ss} = \cos^2 \theta \frac{d^2 V_1}{d \varphi_1^2} + \sin^2 \theta \frac{d^2 V_2}{d \varphi_2^2}. \] (49)

(Note that there is an additional term appearing in \( V_{ss} \) if there are any interactions between \( \varphi_1 \) and \( \varphi_2 \).

For the ekpyrotic solution given by (7), (15) and (16), \( \dot{\theta} = 0 \) and
\[ V_{ss} = \frac{-2(1 - 3p)}{t^2 + 1/a_o} \left( 1 - \frac{3/2}{a_o t^2 + 1} \right) \approx -\frac{2}{t^2} (1 - 3p), \] (50)
where the second equality provides an excellent approximation to \( V_{ss} \) so long as quantum gravity effects are negligible.

Using \( \dot{\theta} = 0 \) and Eq. (50), the differential equation (48) becomes
\[ (a \delta s_k)'' + \left( k^2 \frac{1}{\eta^2} \left[ 2 - 3p + p^2 \right] \right) a \delta s_k = 0. \] (51)
The solution to this differential equation is similar to the one for the Mukhanov-Sasaki equation,
\[ a \delta s_k = \frac{\sqrt{\pi \hbar}}{2} \sqrt{-\eta H_{\tilde{n}}^{(1)} (-k\eta)}, \] (52)
with the difference that now
\[ \tilde{n} = \frac{9}{4} - 3p + p^2 \approx \frac{3}{2} - p, \] (53)
where the second equality holds for small \( p \). As before, the overall prefactor has been chosen so that at early times \( \eta \to -\infty \) one obtains quantum vacuum fluctuations.

Recalling that \( a(\eta) \sim (\eta)^{p/(1-p)} \) in the contracting branch, and using the asymptotic expansion of the Hankel function, the long wavelength limit of \( \delta s_k \) is given by
\[ \delta s_k = \sqrt{\frac{\pi \hbar}{2}} a_o^{-p/2} \left[ \frac{k^{3/2-p} (\eta)^{2-2p}}{3} - i \frac{k^{-3/2+p}}{\sqrt{\pi (-\eta)}} \right], \] (54)
where we have dropped terms of the order \( p \) in the prefactors, and of the order \( p^2 \) in the exponents. This shows that the growing mode is almost scale-invariant, with a slight blue tilt.

Here there is a blue tilt because of the choice of the potentials we have worked with. It is easy to choose slightly different potentials and get a slightly different tilt. In particular, it is relatively easy to obtain a red tilt that agrees with observations [8, 14, 16].

B. Generating Curvature Perturbations

The dynamics of the curvature perturbation \( \mathcal{R} \) is affected by the presence of entropic perturbations, and so it is possible for a scale-invariant entropy perturbation to generate scale-invariant curvature perturbations.

If the Bardeen potential and the entropic perturbations are known, then the evolution of the curvature perturbation is given by (15)
\[ \mathcal{R}_k = \frac{k^2}{a^2} \cdot \frac{H}{H} \Phi_k + \frac{2H \dot{\theta}}{\sqrt{\dot{\varphi}_1^2 + \dot{\varphi}_2^2}} \delta s_k, \] (55)
and in the long wavelength limit, the first term is negligible. Note however that in the solution we have been considering so far, \( \dot{\theta} = 0 \) and so in this case the entropic perturbations cannot source the curvature perturbations.

However, by modifying the potential so that one of the scalar fields suddenly changes directions in their configuration space, it is possible to generate new terms in the
curvature perturbation. Here we will review one such scenario that has been proposed in order to show how this is possible. We shall merely outline the procedure studied in detail in [8, 14]. For alternative scenarios, see [47, 48].

So far we have been working with negative exponential potentials (plus some minor modifications to simplify LQC calculations). Let us now assume that the potential $V_2(\varphi_2)$ has the form where part of it is accurately described by [49], but then the potential rises sharply and flattens out. (We assume that $V_1(\varphi_1)$ remains unchanged.) If the form of the potential is chosen carefully, then it is possible to obtain a solution $\dot{\varphi}_2 = 0$ after $\varphi_2$ climbs the potential, and this solution will be conserved in time if the potential $V_2(\varphi_2)$ becomes flat after its sharp rise.

In this scenario, the new value of $\theta = 0$ shows that there must have been a $\dot{\theta} \neq 0$ at some point. If we assume that the potential rises very sharply, it is possible to approximate the change in $\theta$ by a delta function [14],

$$\dot{\theta} = -\left(\frac{\arctan\sqrt{q_2}}{q_1}\right) \delta(t - t_i),$$

where $t_i$ is the time when $\varphi_2$ hits the sudden rise in the potential. Of course, as $\varphi_2$ starts to rise in the potential, the dynamics of the background will be affected, so $H$, $\dot{\varphi}_1^2 + \dot{\varphi}_2^2$ and $\delta s$ will change. However, if we assume that the change in $\theta$ is instantaneous, we can approximate these quantities by their values just before $\varphi_2$ starts to go up the sharp rise in the potential.

As an aside, note that while a sharp change in $\theta$ simplifies the calculation, a more gradual transition in the value of $\theta$ results in a larger amplitude of the terms in $\mathcal{R}_k$ generated by the entropy perturbations [49].

In this limit, it is easy to integrate [50], giving

$$\mathcal{R}_k(t) = \mathcal{R}_k(t) - 8\pi G \sqrt{p} \left(\arctan\sqrt{q_2}\right) t_i \delta s_k(t_i),$$

where $\mathcal{R}_k$ represents the terms that were already present in the curvature perturbation variable. Recall that the curvature perturbation, in a contracting ekpyrotic cosmology, has a constant mode as well as a decaying mode as seen in Sec. IIIA. Since the new terms generated by the entropy perturbations are evaluated at the time $t_i$, these new terms do not evolve after this time and so contribute to the constant mode.

Therefore, the curvature perturbation in this scenario is schematically given by

$$\mathcal{R}_k \sim k^{\frac{1}{2} - p} (-\bar{\eta})^{1{-2p}} + k^{-\frac{1}{2} + p} + k^{\frac{3}{2} - p} + k^{-\frac{3}{2} + p},$$

where the first two terms come from $\dot{\mathcal{R}}_k$ (see Sec. IIIA) and the last two terms come from the entropy perturbations. There are three contributions to the constant mode, and in the long wavelength limit, the dominant contribution is almost scale-invariant.

An important point is that while the amplitude of the new terms in $\mathcal{R}_k$ sourced by the entropy perturbations depends quite strongly on the specifics of how $\dot{\theta} \neq 0$, the $k$-dependence of the two new terms, namely $k^{-3/2 + p}$ and $k^{3/2 - p}$, is robust so long as the $k$-dependence of $\delta s_k$ does not change while $\dot{\theta} \neq 0$.

Finally, note that since the prefactors to the terms generated due to the entropy perturbations highly depend on $t_i$ —not only due to the time-dependence of the prefactors in (58), but also the time-dependence in $\delta s_k$ itself—the amplitude of the scale-invariant mode is also highly dependent on $t_i$.

C. The Bounce

Using the LQC effective equations, it is possible to evolve the curvature perturbations through the bounce. Note that in general relativity, in the absence of entropy perturbations and for modes that are outside the Hubble radius, $\mathcal{R}' = 0$. However, this is not the case in LQC and so it is necessary to solve the equations of motion for $\mathcal{R}$ even in the absence of entropy perturbations.

Here we will assume that the entropy perturbations do not play a role once the scale-invariant perturbations have been generated, and then the long wavelength solution (31) to the LQC effective equation for $\mathcal{R}$,

$$\mathcal{R}_k = C_1 + C_2 \int \frac{\bar{\eta}}{z(\bar{\eta})^2} d\bar{\eta},$$

is valid.

Now, since at this point $\varphi_2$ has climbed the potential, the scalar fields no longer mimic an ultrastiff perfect fluid and so the exact dynamics of the background are not known without specifying the entire form of the potential $V_2(\varphi_2)$ (and also $V_1(\varphi_1)$ if that potential is to be modified as well), and then determining the dynamics of the background metric by using the LQC effective Friedmann equations. Because of this, it is not possible to determine the exact values of $C_1$ and $C_2$ without further input.

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2. At first it seems as though such an approximation must fail for both $\dot{\varphi}_1^2 + \dot{\varphi}_2^2$ and $\delta s$, as they each contain contributions from $\varphi_2$ which changes rapidly as $\varphi_2$ climbs the potential. However, it can be shown that the first term must vary continuously and $\delta s$ changes by at most a factor of order unity (see [14] for details), and so this approximation is valid. Note however that this approximation fails in the treatment of non-Gaussianities [49].

3. Note that while the time dependence of the first, decaying mode may change when the field $\varphi_2$ climbs the potential, this mode will continue to be time-dependent and its $k$-dependence will not change.
Nonetheless, it is possible to determine the $k$-dependence of these terms in the following manner. Recall that in Sec. III B it was shown how by choosing an appropriate constant of integration, it is possible to relate $C_1$ only to the constant mode of $\mathcal{R}$ in the contracting phase and $C_2$ only with the time-dependent mode of $\mathcal{R}$; this procedure gives

$$C_1 \sim k^{-\frac{3}{2}+p} + k^{\frac{3}{2}-p} + k^{-\frac{3}{2}+p}, \quad C_2 \sim k^{\frac{3}{2}-p}. \quad (60)$$

On the other side of the bounce, the equations of general relativity can be trusted once more, and the curvature perturbation will contain the two standard modes, of which the constant mode is the one relevant for structure formation.

From the solution (59), it is clear that at least the three terms in $C_1$ will contribute to the constant mode, and generically the term in $C_2$ will contribute as well (as it did in Sec. III C). Therefore, after the bounce the constant mode of the curvature perturbation will typically contain the four following terms (and always the first three terms),

$$\mathcal{R}_k \sim k^{-\frac{1}{2}+p} + k^{\frac{3}{2}-p} + k^{-\frac{3}{2}+p} + k^{\frac{1}{2}-p}, \quad (61)$$

and the dominant contribution at long wavelengths is almost scale-invariant, so

$$\Delta_R^2(k) \sim k^{2p}. \quad (62)$$

Note that the amplitudes of each term in the constant mode depend on the specifics of the pre-bounce dynamics. Furthermore, the background dynamics during the bounce only affect the amplitude of the last term, which comes from the decaying mode in the contracting branch.

What can be seen here is that while $\mathcal{R}' \neq 0$ in LQC — even for adiabatic perturbations that are outside the Hubble radius — the terms already appearing in the constant mode of $\mathcal{R}$ are conserved. Typically, however there are additional terms that are generated during the bounce. Note that as the new terms will have a different dependence on $k$ (except perhaps in some pathological cases) the new terms cannot affect the amplitudes of the already existing modes or cancel them out.

In the ekpyrotic universe, the new term added during the bounce has a blue spectrum, so it is not relevant from an observational point of view and can be ignored. Thus, the almost scale-invariant term dominates the scalar power spectrum in ekpyrotic LQC.

D. More General Potentials

Clearly, the discussion so far in this section has only considered a very specific set of potentials and it may appear that a significant amount of fine-tuning — both in the choice of the potentials, and in the initial conditions so that $\varphi_2$ starts climbing its potential at the right time — is necessary in order to obtain a scale-invariant spectrum of curvature perturbations.

However, these choices have been made in order to simplify the calculations so that the derivation is as clear and transparent as possible. It is of course possible to consider different potentials, as well as different procedures that generate a nonzero $\theta$, and some of these choices will no doubt require less fine-tuning.

Let us clarify which ingredients are necessary in order to obtain scale-invariant curvature perturbations in ekpyrotic loop quantum cosmology:

1. The first ingredient is a contracting phase where the matter degrees of freedom mimic a perfect fluid with a very large (constant) equation of state. If there are at least two matter fields, and the entropic perturbations begin as quantum vacuum fluctuations, then the entropic perturbations will acquire a scale-invariant spectrum.

2. Then, the (scale-invariant) entropy perturbations can source new (scale-invariant) terms in the curvature perturbations. If the matter fields are scalars, this occurs if there is a sharp turn in the trajectory of the scalar fields in their configuration space.

3. Using the LQC effective equations, and assuming that the entropy perturbations no longer affect the dynamics of the curvature perturbations, it is possible to evolve the perturbations through the bounce. It then follows that the constant mode of the curvature perturbation in the expanding branch contains an almost scale-invariant term, and it is this term that is the dominant one for long wavelengths.

There are two further comments that are necessary. First, the amplitude and tilt of the almost scale-invariant perturbations strongly depend on the specifics of (i) the background dynamics of the contracting phase and (ii) the manner in which $\theta$ changes. In particular, there are rather simple examples that give predictions consistent with the observations of the cosmic microwave background [8, 14, 46]. In any case, there is a lot of freedom in these choices, and an important point is that these choices can be studied entirely in the classical limit, as both the tilt and the amplitude remain constant through the bounce (so long as the entropy perturbations decouple from the curvature perturbations during the bounce).

The second comment concerns this last caveat: entropy perturbations have not yet been studied in LQC. For this reason, in this paper we have assumed that $\delta s$ sources $\mathcal{R}$ before quantum gravity effects become important, and also that the two decouple during the bounce. In order to go beyond these assumptions it will be necessary to determine the LQC effective equations in the presence of entropy perturbations. It seems likely that allowing a coupling between the curvature and entropy perturbations during the bounce will not affect any qualitative predictions, but this should be checked.
V. CONCLUSIONS

In this paper, we have determined the spectrum of curvature perturbations in ekpyrotic loop quantum cosmology, for both the single scalar field and the two scalar field model. While the case of a single field is not viable as it predicts a blue spectrum, in the two scalar field scenario—where entropy perturbations generate additional contributions to the curvature perturbations—the resulting power spectrum is indeed scale-invariant and so is compatible with observations.

In this work, particular potentials for the scalar fields have been chosen in order to simplify the calculations. It is of course possible to consider a larger family of potentials than the ones used here, at the expense of complicating some of the calculations. Indeed, it will be necessary to do just this in order to obtain perturbations with a slight red tilt to their scale-invariance, as the potentials considered here give a slight blue tilt.

This is not very problematic however, as some potentials that do give a red tilt are already known [8, 14, 46]. In addition, as argued in Sec. IV D any contributions to the constant mode of the curvature perturbation $R$ (the mode that is observationally relevant) before the bounce will survive the bounce. There may be additional contributions to the constant mode of $R$, but so long as these are small compared to $k^{-3/2}$ in the long wavelength limit, the almost scale-invariant term will dominate. Furthermore, the amplitude of this mode is entirely determined by the pre-bounce physics, where the quantum gravity effects are negligible and the standard cosmological perturbation equations can be used. Therefore, in ekpyrotic loop quantum cosmology it is possible to determine the amplitude of the curvature perturbations and their departure from scale-invariance for a specific set of potentials purely from the classical equations of motion (so long as the entropic perturbations source the curvature perturbations before quantum gravity effects become important) and so the results obtained for a larger family of potentials given in [8, 14, 46] hold in LQC as well.

On the other hand, if the generation of curvature perturbations from the entropy perturbations occurs at a time when LQC effects are important, then there may be some additional corrections to the resulting scalar power spectrum that have not been considered here. An investigation of this possibility is left for future work.

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