AN ADAPTIVE DICTIONARY LEARNING APPROACH FOR MODELING DYNAMICAL TEXTURES

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ABSTRACT
Video representation is an important and challenging task in the computer vision community. In this paper, we assume that image frames of a moving scene can be modeled as a Linear Dynamical System. We propose a sparse coding framework, named adaptive video dictionary learning (AVDL), to model a video adaptively. The developed framework is able to capture the dynamics of a moving scene by exploring both sparse properties and the temporal correlations of consecutive video frames. The proposed method is compared with state of the art video processing methods on several benchmark data sequences, which exhibit appearance changes and heavy occlusions.

Index Terms— Dynamic textures modeling, sparse representation, dictionary learning, linear dynamical systems.

1. INTRODUCTION

Temporal or dynamic textures (DT) are image sequences that exhibit spatially repetitive and certain stationarity properties in time. This kind of sequences are typically videos of processes, such as moving water, smoke, swaying trees, moving clouds, or a flag blowing in the wind. Study and analysis of DT is important in several applications such as video segmentation [1], video recognition [2], and DT synthesizing [3].

One classical approach is to model dynamic scenes via the optical flow [4]. However, such methods require a certain degree of motion smoothness and parametric motion models [1]. Non-smoothness, discontinuities, and noise inherence to rapidly varying, non-stationary DTs (e.g. fire) pose a challenge to develop optical flow based algorithms. Another technique, called particle filter [5], models the dynamical course of DTs as a Markov process. A reasonable assumption in DT modeling is that each observation is correlated to an underlying latent variable, or “state”, and then derive the parameter transition operator between these states.

Some approaches directly view each observation as a state, and then focus on transitions between the observations in the time domain. For instance, the work in [6] treats this transition as an associated probability problem, and other methods construct a spatio-temporal autoregressive model (STAR) or position affine operator for this transition [7, 8].

Differently, feature-based models capture the intrinsic law and underlying structures of the data by projecting the original data onto a low-dimensional feature space via feature extracted techniques, such as principal component analysis (PCA). G. Doretto et al. [2, 3] model the evolution of the dynamic textured scenes as a linear dynamical system (LDS) under a Gaussian noise assumption. As a popular method in dynamic textures, LDS and its derivative algorithms have been successfully used for various dynamic texture applications [3, 2]. However, constraints are imposed on the types of motion and noise that can be modeled in LDS. For instance, it is sensitive to input variations due to various noise. Especially, it is vulnerable to non-Gaussian noise, such as missing data or occlusion of the dynamic scenes. Moreover, stability is also a challenging problem for LDS [9].

To tackle these challenges, the approach taken here is to explore an alternative method to model the DTs...
by appealing to the principle of sparsity. Instead of using the Principle Components (PCs) as the transition “states” in LDS, sparse coefficients over a learned dictionary are imposed as the underlying “states”. In this way, the dynamical process of DTs exhibits a transition course of corresponding sparse events. These sparse events can be obtained via a recent technique on linear decomposition of data, called dictionary learning [10, 11]. Formally, these sparse representations $x \in \mathbb{R}^k$ to a signal $y \in \mathbb{R}^m$, can be written as

$$y = Dx$$

where $D \in \mathbb{R}^{m \times k}$ is a dictionary, and $x$ is sparse, i.e. most of its entries are zero or small in magnitude. That is, the signal $y$ can be sparsely represented only using a few elements from some dictionary $D$.

In this work, we start with a brief review of the dynamic texture model from the viewpoint of convex $\ell_2$ optimization, and then deduce a combined regression associated with several regularizations for a joint estimation course of corresponding sparse events. These sparse “states” in LDS, sparse coefficients over a learned dictionary $\mathcal{D}$, together with the hidden states, given the sequence of observations $Y$.

The problem of learning the LDS (1) can be considered as a coupled linear regression problem [9]. Let us denote $X = [x_0, \ldots, x_n] \in \mathbb{R}^{k \times (n+1)}$, $X_0 = [x_0, \ldots, x_{n-1}] \in \mathbb{R}^{k \times n}$, and $X_1 = [x_1, \ldots, x_n] \in \mathbb{R}^{k \times n}$. The system dynamics and modeling matrix are expected to be caught by solving the following minimization problem,

$$\min_{A,D,X} \|X_1 - AX_0\|_F^2 \quad \text{s.t.} \quad \|Y - DX\|_F^2 \leq \varepsilon, \quad (2)$$

where $\varepsilon$ is a small positive constant. In our approach, we assume that all observations $y_i$ admit a sparse representation with respect to an unknown dictionary $D \in \mathbb{R}^{m \times k}$, i.e.

$$y_i = Dx_i, \quad \text{for all } i = 0, 1, \ldots, n, \quad (3)$$

where $x_i \in \mathbb{R}^k$ is sparse. Without loss of generality, we further assume that all columns of the dictionary $D$ have unit norm. We then define the set

$$\mathcal{S}(m,k) := \{D \in \mathbb{R}^{m \times k} | \text{ddiag}(D^T D) = I_k\}, \quad (4)$$

where ddiag($Z$) is the diagonal matrix whose entries on the diagonal are those of $Z$, $I_k$ denotes the identity matrix. The set $\mathcal{S}(m,k)$ is the product of $k$ unit spheres, and is hence a $k(m - 1)$ dimensional smooth manifold. Finally, by adopting the common sparse coding framework to problem (2), we have the following minimization problem

$$\min_{A,D,X} \|X_1 - AX_0\|_F^2 + \mu_1\|Y - DX\|_F^2 + \mu_2\|X\|_1, \quad (5)$$

where $D \in \mathcal{S}(m,k)$, $\| \cdot \|_F$ denotes the Frobenius norm of matrices, and $\| \cdot \|_1$ is the $\ell_1$ norm, which measures the overall sparsity of a matrix. The parameter $\mu_2 > 0$ weights the sparsity measurement against the residual errors.

2. ADAPTIVE VIDEO DICTIONARY LEARNING

In this section, we start with a brief introduction to the linear dynamical systems (LDS) model and develop an adaptive dictionary learning framework for sparse coding.

2.1. Linear Dynamical Systems

Let us denote a given sequence of $(n + 1)$ frames by $Y := [y_0, \ldots, y_n] \in \mathbb{R}^{m \times (n+1)}$, where the time is indexed by $i = 0, 1, \ldots, n$. The evolution of a LDS is often described by the following two equations

$$\begin{cases}
    x_{i+1} = Ax_i + w_i \\
y_i = Dx_i + v_i,
\end{cases} \quad (1)$$

where $y_i \in \mathbb{R}^m$, $x_i \in \mathbb{R}^k$, $w_i \in \mathbb{R}^k$ and $v_i \in \mathbb{R}^m$ denote the observation, its hidden state or feature, state noise, and observation noise, respectively. The system is described by the dynamics matrix $A \in \mathbb{R}^{k \times k}$, and the modeling matrix $D \in \mathbb{R}^{m \times k}$. Here we are interested in estimating the system parameters $A$ and $D$, together with the hidden states, given the sequence of observations $Y$.
2.2. A Dictionary Learning Model for Dynamical Scene

Solving the minimization problem as stated in Eq. (5) is a very challenging task. In this work, we employ an idea similar to subspace identification methods [9], which treat the state as a function of \((A, D)\). Here, we confine ourselves to the sparse solution of an elastic-net problem, which is proposed in [12], as

\[
x^* := \arg\min_{x \in \mathbb{R}^k} \frac{1}{2} \|y - Dx\|^2_2 + \lambda_1 \|x\|_1 + \frac{\lambda_2}{2} \|x\|^2_2,
\]

where \(\lambda_1 > 0\) and \(\lambda_2 > 0\) are regularization parameters, which play an important role in ensuring stability and uniqueness of the solutions. Let us define the set of indices of the non-zero entries of the solution \(x^* = [x_1^*, \ldots, x_k^*]^T \in \mathbb{R}^k\) as

\[
\Lambda := \{i \in \{1, \ldots, k\} | x_i^* \neq 0\}.
\]

Then the solution \(x^*\) has a closed-form expression as

\[
x_y^*(D) := \left( D_A^T D_A - \lambda_2 I_m \right)^{-1} \left( D_A^T y - \lambda_1 s_A \right),
\]

where \(s_A \in \{-1\}^{\Lambda} \) carries the signs of \(x^*_A\), \(D_A\) is the subset of \(D\) in which the index of atoms (rows) fall into support \(\Lambda\). Furthermore, it is known that the solution \(x_y^*(D)\) as given in (8) is a locally twice differentiable function at \(D\). By an abuse of notation, we define

\[
X_0 : S(m, k) \to \mathbb{R}^{k \times n}
\]

\[
D \mapsto [x_{y_1}(D), \ldots, x_{y_{n-1}}(D)].
\]

In a similar way, \(X_1 : S(m, k) \to \mathbb{R}^{k \times n}\) is defined. Thus, the cost function reads as

\[
f : \mathbb{R}^{k \times k} \times S(m, k) \to \mathbb{R}
\]

\[
(A, D) \mapsto \frac{1}{2} \|X_1(D) - AX_0(D)\|^2_F.
\]

(10)

It is known that an LDS with the dynamic matrix \(A\) is said to be stable, if the largest eigenvalue of \(A\) is bounded by 1 [9]. Let \(\sigma\) be the largest eigenvalue of \(A\), then \(|\sigma| \leq \|A\|_F\). Thus, we enforce the small \(\sigma\) via imposing a penalty \(\|A\|_2^2\) on (10), and then end up with the cost function as

\[
\tilde{f} : \mathbb{R}^{k \times k} \times S(m, k) \to \mathbb{R}
\]

\[
(A, D) \mapsto f(A, D) + \frac{\mu}{2} \|A\|^2_F,
\]

(11)

2.3. Development of the Algorithm

In this section, we firstly derive a gradient descent algorithm to minimize \(\tilde{f}\) and then discuss some details of the choice of the parameters in the final implementation.

We start with the computation of the first derivative of the sparse solution of the elastic-net problem \(x_y^*(D)\) as given in (8). Given the tangent space of \(S(m, k)\) at \(D\) as

\[
T_D S(m, k) := \{ X \in \mathbb{R}^{m \times k} | \text{diag}(X^T D) = 0 \},
\]

the orthogonal projection of a matrix \(H \in \mathbb{R}^{m \times k}\) onto the tangent space \(T_D S(p, n)\) with respect to the inner product \(\langle X, Y \rangle = \text{tr}(X^T Y)\) is given by

\[
\Pi_D(H) := H - D \text{diag}(D^T H).
\]

(13)

Let us denote \(K := D_A^T D_A - \lambda_1 I_k\). The first derivative of \(x_y^*(D)\) in the direction \(H \in T_D S(m, k)\) is

\[
D x_y^*(D) H = K^{-1} H_A^T y - K^{-1} (D_A^T H_A 
+ H_A^T D_A) \cdot K^{-1} \left( D_A^T y - \lambda_1 s_A \right).
\]

(14)

By the product structure of \(\mathbb{R}^{k \times k} \times S(m, k)\), the Riemannian gradient of the function \(\tilde{f}\) is

\[
\nabla \tilde{f}(A, D) = \left( \nabla \tilde{f}(A), \Pi_D (\nabla \tilde{f}(D)) \right).
\]

(15)

Here, the Euclidean gradient \(\nabla \tilde{f}(A)\) of \(\tilde{f}\) with respect to \(A\) is computed as

\[
\nabla \tilde{f}(A) = (A X_0(D) - X_1(D)) X_0(D) + \gamma A,
\]

with \(e_i\) being the \(i\)-th standard basis vector of \(\mathbb{R}^n\). Using the shorthand notation, \(r_i := D_A^T y_i - \lambda_1 s_{A, i}\), \(\Delta x_i := x_{y_i}(D) - A_{\Lambda, i} x_{y_{i-1}}(D)\), and \(q_i := r_i \Delta x_i^T\), the Euclidean gradient \(\nabla \tilde{f}(D)\) of \(\tilde{f}\) with respect to \(D\) is

\[
\nabla \tilde{f}(D) = \sum_{i=1}^n y_i (\Delta x_i)^T K_i^{-1} - D_{A, i} K^{-1}_i (q_i + q_i^T) \cdot K_i^{-1} - y_i (\Delta x_i)^T A_{\Lambda, i} (K_{i-1})^{-1} + D_{A, i-1} \cdot (K_{i-1})^{-1} (A_{\Lambda, i} q_{i-1} + q_{i-1} A_{\Lambda, i}^T (K_{i-1})^{-1}.
\]

(17)

For a gradient search iteration on manifolds, we employ the following smooth curve on \(S(m, k)\) through
$D \in S(m,k)$ in direction $H \in T_D S(m,k)$

$$\tau: (-\lambda, \lambda) \to S(m,k)$$

$$t \mapsto (D + tH)(\operatorname{d} \text{diag}((D + tH)^\top (D + tH)))^{-\frac{1}{2}}$$

(18)

with $\lambda > 0$. It essentially normalizes all columns of $D + tH$. For a detailed overview on optimization on matrix manifold, refer to [13].

**Algorithm 1: Adaptive Video Dictionary Learning**

1: Training data $Y$
2: Initialize the parameters $\lambda_1, \lambda_2, \gamma$, initial dictionary $D$, and initial transition matrix $A$.
3: for $i = 1, 2, \ldots, T$ do
4:  Sparse Coding Stage
5:  Use Lasso algorithm to compute $x$ via
6:  Compute the gradient of $f(A, D)$ according to
7:  Update the parameters $A$ and $D$
8:  return $A$ and $D$

Until now, we have computed the gradient of $\tilde{f}$ as defined in (11) with respect to its two arguments $D$ and $A$. An iterative scheme (such as the gradient descent method or conjugate gradient method) can be used to find the optimal $D$ and $A$, using the gradient expression above. The procedure displayed in Algorithm (1) is the version of AVDL based on gradient descent procedure. The learning rate $\rho_i$ can be computed via the well-known backtracking line search method, similar to [11].

Here, considering the high coherence among the temporal frames, we prefer non-redundant dictionary, that is, $k \ll m$ for the dictionary $D \in \mathbb{R}^{m \times k}$. For parameters $(\lambda_1, \lambda_2)$ in the elastic net, we put an emphasis on sparse solutions and choose $\lambda_2 \in (0, \frac{\lambda_1}{10})$, as proposed in [12].

**Fig. 1.** Reconstruction and synthesizing on the candle scene. (a), (b) are $(i = 1, 64, 128, 512, 1024)th$ frame of the corrupted data by Gaussian noisy and the reconstructed data using AVDL, respectively. (c) The top row is the synthesized sequence using LDS (128PCs), and the bottom row is the synthesized sequence using AVDL ($(i = 2, 1024, 3072, 5120, \ldots, 20480)th$ frame). (d) The top row is the sequence with missing data. The middle row the synthesized sequence using LDS, and the bottom row is the synthesized sequence using AVDL.

### 3. NUMERICAL EXPERIMENTS

We carry out a few experiments on natural image sequences data, and demonstrate the practicality of the proposed algorithm. Our test dataset comprises of videos from DynTex++ [14], and data from internet sources (for instance, YouTube). Firstly, we show the performance on reconstruction and synthesizing with a grayscale video of burning candle, which is corrupted by Gaussian noise or occlusion. This video has 1024 frames with size of $32 \times 32$, see figure [1]. The initial dictionary is $1024 \times 512$. After the acquisition of the dictionary $D$ and the transition $A$, the synthesized data can be generated easily by $x_{i+1} = Ax_i x_i^\top x_i (x_i^\top x_i)^{-1}$, or more precisely, using a convex formulation

$$\min_{x_{i+1}} \frac{1}{2} \|x_{i+1} - Ax_i\|_2^2 + \lambda \|x_{i+1}\|_1.$$
Table 1. Synthesizing results on sequence of burning candle.

| Instance | LDS, (PCs) | AVDL, $\gamma = 0.5$, (loops) |
|----------|------------|-------------------------------|
|          |            | Compression rate (%)          | $\sigma$ | $e_y$ | $e_x$ |
|          | 64         | 128                           | 256      | 1     | 50    | 100   | 200   | 400   |
|          | 6.25       | 12.50                         | 25.00    | 1.02  | 3.29  | 3.41  | 3.50  | 3.55  |
| $\sigma$| 0.9802     | 0.9833                        | 0.9849   | 1.78  | 1.06  | 0.9992| 0.9994| 0.9994|
| $e_y$    | $1.35 \times 10^5$ | $1.35 \times 10^5$ | $1.36 \times 10^3$ | 60.29 | 58.82 | 55.97 | 71.27 |
| $e_x$    | 101.58     | 135.88                        | 168.95   | 3.75 $\times 10^4$ | 171.99 | 75.52 | 61.96 | 46.18 |

$(e_x, e_y)$ are defined as $e_y = \sum_i \|y_i - Dx_i\|$, $e_x = \sum_i \|x_{i+1} - Ax_i\|$, and the largest eigenvalue of $A$ is denoted by $\sigma$. The compression rate for AVDL is sparsity of $x$ to $m \times (n + 1)$, and for LDS is number of PCs to $m$. Table 1 shows AVDL can obtain the stable dynamic matrix $A$ ($\sigma \leq 1$), smaller compression rate and smaller error $(e_x, e_y)$ of cost function (5), by increasing the numbers of main loops in Algorithm 1.

Figure 1 (a $\sim$ c) is the visual comparison between LDS and AVDL. AVDL performs well on denoising against corruption by Gaussian noise. In the case of occlusion in figure 1(d), random 50 frames of the 1024 burning candle video are corrupted by a $(6 \times 7)$ rectangle. The length of both synthesizing data is 1024, based on first frame of the burning candle. 87.01% of the synthesizing data from LDS are corrupted by this rectangle, but 9.47% for AVDL.

Table 2. DT recognition rates for videos with occlusion.

| Occlusion rate (%) | 0   | 5   | 15  | 30  |
|-------------------|-----|-----|-----|-----|
| LDS-NN (128PCs)   | 69.72 | 45.00 | 25.14 | 14.17 |
| AVDL-SRC          | 70.28 | 64.72 | 44.44 | 22.36 |

The second experiment is about scenes classification on DynTex++, which contains DTs from 36 classes. Each class has 100 subsequences of length 50 frames with $50 \times 50$ pixels. 20 videos are randomly chosen in each class and total 720 videos are used for our experiments. Classification for LDS is performed using the Martin distance with a nearest-neighbor classifier on its parameters pair $(A, D)$ [2]. Another classifier is AVDL associated with the sparse representation-based classifier (SRC) [15, 16], in which the class of a test sequence is determined by the smallest reconstruction error $e_y$ and transition error $e_x$. Table 2 provides the recognition results with increasing occlusion rates for test data. Compared to LDS with nearest-neighbor classifier (LDS-NN), Table 2 shows the proposed AVDL with SRC (AVDL-SRC) performs better while the test videos are corrupted by increasing occlusion.

4. CONCLUSIONS

This paper proposes an alternative method, called AVDL, to model the dynamic process of DTs. In AVDL, the sparse events over a dictionary are imposed as transition states. The proposed method show a robust performance for synthesizing, reconstruction and recognition on DTs corrupted by Gaussian noise. Especially, AVDL exhibits more powerful in the case of test data with non-Gaussian noise, such as occlusion. One possible future extension is to learn a dictionary for large scale DT sequences based on AVDL.

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