Scale-free avalanche dynamics in the stock market

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Abstract

Self-organized criticality has been claimed to play an important role in many natural and social systems. In the present work we empirically investigate the relevance of this theory to stock-market dynamics. Avalanches in stock-market indices are identified using a multi-scale wavelet-filtering analysis designed to remove Gaussian noise from the index. Here new methods are developed to identify the optimal filtering parameters which maximize the noise removal. The filtered time series is reconstructed and compared with the original time series. A statistical analysis of both high-frequency Nasdaq E-mini Futures and daily Dow Jones data is performed. The results of this new analysis confirm earlier results revealing a robust power law behaviour in the probability distribution function of the sizes, duration and laminar times between avalanches. This power law behavior holds the potential to be established as a stylized fact of stock market indices in general. While the memory process, implied by the power law distribution of the laminar times, is not consistent with classical models for self-organized criticality, we note that a power-law distribution of the laminar times cannot be used to rule out self-organized critical behaviour.

Key words: Complex Systems, Econophysics, Self-Organized Criticality, Wavelets

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1. Introduction

Attracted by several analogies with the dynamics of natural systems, physicists, especially during the last decade, have attempted to understand the mechanism behind stock-market dynamics by applying techniques and ideas developed in their respective fields [1].

In this context, possible connections between self-organized criticality (SOC) and the stock market, or economics in general, have been investigated theoretically [2,3,4,5,6]. The theory of SOC, originally proposed in the late eighty’s by Bak, Tang and Wiesenfeld (BTW) [7] to explain the ubiquity of power laws in Nature, is claimed to be relevant in several different areas of physics as well as biological and social sciences [4,8]. The key concept of SOC is that complex systems — i.e. systems constituted by many non-linear interacting elements — although obeying different microscopic physics, may naturally evolve toward a critical state where, in analogy with physical systems near the phase transition, they can be characterized by power laws. The critical state is an ensemble of metastable configurations and the system evolves from one to an-
other via an avalanche-like dynamics [8].

The classical example of a system exhibiting SOC behaviour is the 2D sandpile model [7,8,4]. Here the cells of a grid are randomly filled, by an external driver, with “sand”. When the gradient between two adjacent cells exceeds a certain threshold a redistribution of the sand occurs, leading to more instabilities and further redistributions. The benchmark of this system, indeed of all systems exhibiting SOC, is that the distribution of the avalanche sizes, their duration and the energy released, obey power laws. As such, they are scale-free.

In the present work we search for imprints of SOC in the stock market by studying the statistics of the coherent periods (that is, periods of high volatility), or avalanches, which characterize its evolution. We analyze the tick-by-tick behaviour of the Nasdaq E-mini Futures (NQ) index, \( P(t) \), from 21/6/1999 to 19/6/2002 for a total of \( 2^{19} \) data. In particular, we study the logarithmic returns of this index, which are defined as \( r(t) = \ln \left[ \frac{P(t)}{P(t-1)} \right] \). Possible differences between daily and high frequency data have also been taken into consideration through the analysis of the Dow Jones daily closures (DJ) from 2/2/1939 to 13/4/2004, for a total of \( 10^{14} \) data.

This work extends our earlier work on this subject [9] by introducing new criteria to optimize the filtering of the time series essential to separating quiescent and avalanche dynamics. The properties of the time series reconstructed from the filtered returns are also examined. The issue regarding the presence of SOC in the stock market is of not only of theoretical importance, since it would lead to improvements in financial modeling, but could also enhance the predictive power [10] of Econophysics.

In the next section we present the analysis methodology while in Sec. 3 the results of the analysis are presented. Discussions and conclusions are contained in the last section.

2. Avalanche Identification via Wavelets

The logarithmic returns of stock indices rarely display intervals of genuinely quiescent periods, yet such periods are vital to the quantitative identification of avalanche dynamics. As such, noise must be filtered from the time series. Ideally, only Gaussian noise, associated with the efficient phases of the market where the movements can be well approximated by a random walk [1], is to be filtered from the time-series returns. Such dynamics have no memory and contrast the avalanche dynamics, i.e. anomalous periods characterized by large fluctuations, that we aim to analyze.

Naively, one might simply set a threshold for the logarithmic returns, below which the index is deemed to be laminar. However, a simple threshold method is not appropriate, as it would include in the filtering some non-Gaussian returns at small scales that are relevant in our analysis.

This difficulty is illustrated in Fig. 1 (Top) where the probability distribution function (PDF) for the returns of the NQ index, filtered using a fixed threshold of \( r_{th} = 5 \) standard deviations is shown by the open squares. In this case broad wings, related to events that do not follow Gaussian statistics, are clearly evident.

However, an important stylized fact of financial returns – the intermittency of financial returns [1] – can be used to identify an appropriate filtering scheme. Already, physicists have drawn analogies with the well known phenomenon of intermittency in the spatial velocity fluctuations of hydrodynamic flows [11,12,13]. Both systems display broad tails in the probability distribution function [1] and a non-linear multifractal spectrum [12] as a result of this feature. The empirical analogies between turbulence and the stock market suggest the existence of a temporal information cascade for the latter [12]. This is equivalent to say that various traders require different information according to their specific strategies. In this way, different time scales become involved in the trading process.

In the present work we use a wavelet method in order to study multi-scale market dynamics. The wavelet transform is a relatively new tool for the study of intermittent and multifractal signals [14]. This approach enables one to decompose the signal in terms of scale and time units and so to separate its coherent parts – i.e. the bursty periods related to the tails of the PDF – from the noise-like background. This enables an independent study of the
The wavelet transform (WT) is defined as the scalar product of the analyzed signal, \( f(t) \), at scale \( \lambda \) and time \( t \), with a real or complex “mother wavelet”, \( \psi(t) \). In the discrete wavelet transform (DWT) case, used herein, this reads:

\[
W_T f(t) = \frac{1}{\sqrt{\lambda}} \int f(u) \psi(\frac{u-t}{\lambda}) \, du \tag{1}
\]

\[
= 2^{-j/2} \int f(u) \psi(2^{-j}u - n) \, du,
\]

where the mother wavelet is scaled using a dyadic set. One chooses \( \lambda = 2^j \), for \( j = 0, ..., L - 1 \), where \( \lambda \) is the scale of the wavelet and \( L \) is the number of scales involved, and the temporal coefficients are separated by multiples of \( \lambda \) for each dyadic scale, \( t = n2^j \), with \( n \) being the index of the coefficient at the \( j \)th scale.

The wavelet coefficients are a measure of the correlation between the original signal, \( f(t) \), and the mother wavelet, \( \psi(t) \), at scale \( j \) and time \( n \). In the analysis presented in the next section, we use the Daubechies–4 wavelet as the orthonormal basis [16]. However, tests performed with different sets do not show any qualitative difference in the results.

The utility of the wavelet transform in the study of turbulent signals lies in the fact that the large amplitude wavelet coefficients are related to the extreme events corresponding to the tails of the PDF, while the laminar or quiescent periods are related to the coefficients with smaller amplitude [17]. In this way, it is possible to define a criterion whereby one can filter the time series of the coefficients depending on the specific needs. In our case, we adopt the method used in Ref. [17] and originally proposed by Katul et al. [18]. In this method wavelet coefficients that exceed a fixed threshold are set to zero, according to

\[
\tilde{W}_{j,n} = \begin{cases} 
W_{j,n} & \text{if } W^2_{j,n} < C \langle W^2_{j,n} \rangle_n, \\
0 & \text{otherwise.}
\end{cases} \tag{2}
\]

Here \( \langle ... \rangle_n \) denotes the average over the time parameter \( n \) at a certain scale \( j \) and \( C \) is the threshold coefficient. In this way only the dynamics associated with the efficient phases of the market where

![Fig. 1. (Top) Comparison between the PDF of the original time series for the NQ index and its wavelet filtered version for \( C = 1 \). A Gaussian distribution is plotted for visual comparison. The simple threshold, \( r_{th} = 5 \), method for filtering is also shown. In this case it is clear that we do not remove just Gaussian noise, but also coherent events that can be relevant for the analysis. (Bottom) A window of the time series of the residuals obtained by subtracting filtered time series from the original time series. Avalanches of high volatility contrast periods of genuinely quiescent behavior. All the data in the plots have been standardized, \( r(t) \rightarrow (r(t) - \langle r \rangle)/\sigma(r) \), where \( \langle ... \rangle \) and \( \sigma(r) \) are, respectively, the average and the standard deviation during the period under study.]

The movements can be well approximated by a random walk [1] are preserved.

Once we have filtered the wavelet coefficients \( \tilde{W}_{j,n} \), an inverse wavelet transform is performed, obtaining what should approximate Gaussian noise. The PDF of this filtered time series is shown, along with the original PDF in Fig. 1 (Top). It is evident how the distribution of the filtered signal matches perfectly a Gaussian distribution.

In the same figure (Bottom), we also show the logarithmic returns, \( R(t) \), of the original time series after the filtered time series has been subtracted. Truly quiescent periods are now evident, contrasting the the bursty periods, or avalanches, which we aim to study.

The time series of logarithmic prices is reconstructed from the residuals in Fig. 2 and is contrasted with the one reconstructed from the filtered Gaussianly distributed returns. Note how, in the latter case, the time series is completely indepen-
Fig. 2. The Dow Jones time series is superimposed with the time series reconstructed from the filtered returns and the residual returns remaining after the filtered returns are subtracted from the original returns. The price behaviour generated by the “efficient” or filtered returns is largely independent of the observed price. The filtering parameter, in this case, is $C = 1$.

To this point, the filtering parameter, $C$, has been constrained to 1, thus preserving coefficients that are less than the average coefficient at a particular scale. However, one might wonder if it is possible to tune this parameter to maximally remove the uninteresting Gaussian noise from the original signal.

Fig. 3 illustrates the extent to which the filtered signal is Gaussian as a function of the filter parameter $C$ for the NQ and DJ indices. A sample of Gaussian noise is also included for contrast. An optimal value of $C \sim 1$ is found, optimally filtering the original market time series to the level of noise.

3. Data Analysis

Once we have isolated and removed noise from the time series we are able to perform a reliable statistical analysis on the avalanches of the residual returns, Fig. 1 (Bottom). In particular, we define an avalanche as the periods of the residual returns in which the volatility, $v(t) \equiv |r(t)|$, is above a small threshold, typically two orders of magnitude smaller than the characteristic return.

A parallel between avalanches in the classical sandpile models (BTW models) exhibiting SOC [7] and the previously defined coherent events in the stock market is straightforward. In order to test the relation between the two, we make use of some properties of the BTW models. In particular, we use the fact that the avalanche size distribution and the avalanche duration are distributed according to power laws, while the laminar, or waiting times between avalanches are exponentially distributed, reflecting the lack of any temporal correlation between them [19,20]. This is equivalent to stating that the triggering process has no memory.

Similar to the dissipated energy in a turbulent flow, we define an avalanche size, $V$, in the market context as the integrated value of the squared volatility over each coherent event of the residual returns. The duration, $D$, is defined as the inter-
val of time between the beginning and the end of a coherent event, while the laminar time, \( L \), is the time elapsing between the end of an event and the beginning of the next.

The results for the statistical analysis of the optimally-filtered NQ and DJ indices are shown in Figs. 4, 5 and 6 for the avalanche size, duration and laminar times, respectively. A power law relation is clearly evident for all three quantities investigated.

The data analyzed herein display a distribution of laminar times different from the BTW model of the classical sandpile. As explained previously, the BTW model shows an exponential distribution for \( L \), derived from a Poisson process with no memory [19,20]. The power law distribution found here implies the existence of temporal correlations between coherent events. However this correlation may have its origin in the driver of the market, contrasting the random driver of the classical sandpile.

4. Discussion and Conclusion

We have investigated the possible relations between the theory of self-organized criticality and the stock market. The existence of a SOC state for the latter would be of great theoretical importance, as this would impose constraints on the dynamics, as implied by the presence of a bounded attractor in the state space. Moreover, it would be possible to build new predictive schemes based on this framework [10].

After a multiscale wavelet filtering, an avalanche-like dynamics has been revealed in two samples
of market data. The avalanches are characterized by a scale-free behaviour in the size, duration and laminar times. The power laws in the avalanche size and duration are a characteristic feature of a critical underlying dynamics in the system.

However, the power law behavior in the laminar time distribution implies a memory process in the triggering driver that is absent in the classical BTW models, where an exponential behavior is expected. Remarkably, the same features have been also observed in other physical contexts [19,17,21,22,23].

The problem of temporal correlation in the avalanches of real systems, has raised debates in the physics community, questioning the practical applicability of the SOC framework [24]. Motivated by this issue, several numerical studies have been devoted to including temporal correlations in SOC models [25,26,27]. A power-law distribution in the laminar times has been achieved, for example, by substituting the random driver with a chaotic one [28,29]. Alternatively, it has been shown that non-conservative systems, as for the case of the stock market, could be in a near-SOC state where dissipation induces temporal correlations in the avalanches while the power law dynamics persist for the size and duration [30,31].

In conclusion, a definitive relation between SOC theory and the stock market has not been found. Rather, we have shown that a memory process is related with periods of high activity. The memory could result from some kind of dissipation of information, similar to turbulence, or have its origin in a chaotic driver applied to the self-organized critical system. While a combination of the two processes can also be possible, it is the latter property that prevents one from ruling out the possibility that the stock market is indeed in a SOC state [29].

Similar power-law behaviour has been found in the ASX index for the Australian market [9] and different single stock time series. If this power-law behaviour is confirmed by further studies, this should be considered as a stylized fact of stock market dynamics.

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