Robust Tracking Control of Variable Stiffness Joint Based on Feedback Linearization and Disturbance Observer With Estimation Error Compensation

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ABSTRACT The variable stiffness joint (VSJ) has the characteristics of independent and controllable position and stiffness. The variable stiffness characteristics and inherent flexibility make the VSJ suitable to be used as the actuation joint of the physical human-robot interaction application robot, so as to improve the task adaptability of the robot and physical human-robot interaction safety. The VSJ based on equivalent lever mechanism has the advantages of low energy consumption in stiffness adjustment, so there are many researches on this type of VSJ. The tracking control of output link angular position and joint output stiffness are two basic control targets of the VSJ. For the system dynamic model of the VSJ based on equivalent lever mechanism, considering the unknown parametric perturbations, the unknown friction torques acting on the drive units, the unknown external disturbance acting on the output link and the control input saturation constraints, a robust tracking controller based on feedback linearization, disturbance observer with anti-windup measures, sliding mode control and estimation error compensator is designed to improve the tracking control accuracy of the position and stiffness of the VSJ. The simultaneous tracking control of position and stiffness of the VSJ can be achieved by the designed controller, and the simulation results show the effectiveness and robustness of the proposed controller. Moreover, the simulation results show that the proposed estimation error compensator for the disturbance observer with fixed preset observation gain can effectively reduce the system output tracking error and improve the anti-disturbance characteristics of the controller.

INDEX TERMS Variable stiffness joint, feedback linearization, disturbance observer, sliding mode control, estimation error compensator, robust tracking control.

I. INTRODUCTION

In the current research field of the flexible robot, variable stiffness joint (VSJ) is a kind of flexible robot actuation joint with independent and controllable characteristics of position and stiffness [1]. The inherent flexibility (affected by the internal elastic elements of the joint) and the adjustability of the joint output stiffness make the VSJ suitable for physical human-robot interaction applications such as the robotic-assistive devices [2], [3], exoskeleton robot [4], prosthetic joint [5], wearable devices [6], [7], etc. When the VSJ is used as the actuated joint of the rehabilitation training robot, the therapist can adjust the joint stiffness of the robot according to the patients’ rehabilitation training process. When the VSJ is used as the prosthesis joint, the joint output stiffness of the VSJ can be adjusted according to the road condition and walking frequency to improve the user’s comfort and road adaptability. When the VSJ is used as the actuating joint of the cooperative robot, the positioning accuracy of the end of the manipulator can be improved by increasing the joint stiffness, and the compliance can be increased by reducing the joint stiffness. Therefore, the VSJ can enhance the task adaptability and physical interaction security of the human-robot interaction application robot. At present, the studies on
VSJ mainly focus on mechanical design [8], [9], energy optimization [10], tracking control and its application in multi-DOF robot, etc. In the studies of the VSJs, the tracking control performance of the position and stiffness has an important influence on the actuation characteristics of the VSJ. The angular position tracking control is related to the positioning accuracy of the output link of the VSJ, while the joint stiffness tracking control is related to the compliance of the VSJ in the process of task execution. The tracking control of the position and stiffness of the VSJ has important research significance and application value.

In the existing many types of VSJs, the VSJ based on the equivalent lever mechanism has the advantages of low energy consumption for joint stiffness adjustment, so there are many studies and applications on this type of VSJ. The research status of tracking control of the VSJ based on equivalent lever mechanism is described as follows. For the position and stiffness tracking control of the AwAS (Actuator with Adjustable Stiffness) [11], the proportional derivative (PD) controller with an additional active damping term is used for the tracking control of the output link position, and a conventional PD controller is used to regulate the joint output stiffness. The proportional-integral-derivative (PID) controller used to realize the simultaneous position and stiffness tracking control of the HVSA (Hybrid Variable Stiffness Actuator) [12]. The PD controllers are used for the tracking control of the position and stiffness of the AwAS-II simultaneously [13]. The PD controllers were implemented for both position and stiffness tracking control of the CompAct-VSA [14]. The PD feedback plus feedforward controllers are used for both position & stiffness regulation of the SVSA [15] (Serial Variable Stiffness Actuator) and the SVSA-II [16]. A gain scheduling control strategy based on linear quadratic regulators (LQR) is proposed to regulate both stiffness and position of the VSJ based on equivalent lever mechanism, and the AwAS [11] has been used to verify the effectiveness of the control strategy [17]. A gain-scheduled torque controller based on the linear quadratic Gaussian (LQG) is designed for torque and stiffness control of the MeRIA (Mechanical-Rotary Variable Impedance Actuator) [18].

The above control strategies do not focus on the robust tracking control of the position and stiffness of the VSJs based on the equivalent lever mechanisms. The PID controller [12] using integral term to improve the robustness of tracking control may have unexpected transient response performance. At present, some robust tracking control schemes for the VSJ based on the equivalent lever mechanism have also been proposed. A neural network adaptive control strategy based on feedback linearization is proposed for the position and stiffness tracking of the SVSA, and the feasibility of the proposed approach is verified by simulation results [19]. A robust tracking controller based on feedback linearization, disturbance observer (DOB) and sliding mode control is designed for the simultaneous tracking control of the position and stiffness of the AwAS-II, and the effectiveness of the controller has been illustrated by simulation results [20].

A robust tracking control scheme based on linear extended state observer with estimation error compensation is proposed for the tracking control of the antagonistic VSJ based on equivalent nonlinear torsion spring and the serial VSA based on the equivalent lever mechanism, and the effectiveness of the proposed controller is verified by simulation results [21]. A DOB based composite controller is developed to achieve the tracking control of the stiffness and position of the SVSA, and the effectiveness of the controller is experimentally verified [22], [23]. A DOB based adaptive neural network control is proposed for the output link position tracking control of the robotic systems with joints driven by VSJs [24]. A state feedback controller which guarantees prescribed performance of the tracking errors has been designed for the variable stiffness actuated robots, and the simulation studies based on the CompAct-VSA show the robustness of the designed controller [25].

The control strategies mentioned above aim at the tracking control of the VSJ based on the equivalent lever mechanism. At present, some control schemes for tracking control of other types of VSJs have also been proposed. The feedback linearization control law has been designed for the accurate tracking of desired link and stiffness trajectories of the robot manipulator driven by the antagonistic VSJ [26]. A quasi-finite-time tracking control law based on mapping filtered forwarding technique has been designed for the position tracking control of the antagonistic VSJ [27]. The command-filtered backstepping controller [28] for the multi-joint variable stiffness robots has been designed and used for the position tracking control of the DLR Hand Arm System. The nonlinear model predictive controller [29] has been used for the position tracking control of the two link planar robot driven by antagonistic VSJs. A feedforward motion controller has been designed for the stiffness control of the VSJ developed using variable radius gear transmission inspired by biological musculoskeletal system [30].

Although there are many control strategies for the tracking control of the VSJs, they are all aimed at different models, research objectives and applications. Many control strategies [11]–[18] focus on the studies of the control characteristics and applications of the VSJs, but not on the robustness of the position and stiffness tracking control of the VSJ. Although the DOBs have been used in the tracking control of the VSJs to improve the robustness of tracking control, the DOBs in these control schemes [20], [22]–[24] have fixed preset observation gain, and the estimation errors of the DOBs are not considered in the design of controllers. There are also some controllers for the position tracking control of multi-DOF robot system driven by the VSJ based on equivalent lever mechanism [24], [25], but these controllers do not focus on the implementation of simultaneous position and stiffness tracking control of the VSJ. Other existing control schemes [26]–[29] are aimed at the position tracking control of the antagonistic VSJ and the multi-DOF robot driven by the antagonistic VSJs. The tracking control performance of the position and stiffness has an important
influence on the actuation performance of the VSJ based on the equivalent lever mechanism. The design of robust high-precision tracking controller for the VSJ based on equivalent lever mechanism is of great significance to improve its actuation performance, and the designed controller can also be extended to the tracking control of antagonistic VSJs. Therefore, it is necessary to further study the robust tracking controller which can realize the simultaneous position and stiffness tracking control of the VSJs, and take measures to further improve the robust tracking accuracy of the system outputs (i.e., the output link position and joint output stiffness).

In the robust tracking control schemes of the VSJs [20], [22]–[24] or non VSJ systems [31]–[35], the DOB is a widely used technique to improve the robustness of system tracking control. In this article, considering the possible model parameter perturbations, the unknown friction torques acting on the driving units, the unknown external disturbance acting on the output link and the control input saturation constraints in the system dynamics of the VSJ based on the equivalent lever mechanism, a novel robust tracking control scheme is designed to realize the simultaneous position and stiffness tracking control of this type of VSJ. Moreover, the designed controller takes the anti-windup measures for the traditional DOB to reduce the influence of input saturation constraint on the disturbance estimation, and the novel estimation error compensators are designed to compensate the estimation error of the traditional widely used DOB with fixed preset gain [20], [22]–[24], [31]–[35] to improve the tracking accuracy and anti-disturbance characteristics of the controller.

The main contributions of this article are presented as follows.

1. A novel robust tracking controller based on feedback linearization, disturbance observer with anti-windup measures, sliding mode control and estimation error compensator is designed to achieve simultaneous position and stiffness tracking control of the VSJ based on the equivalent lever mechanism. Compared with the existing tracking control schemes for the VSJs [11]–[20], [22]–[29], this controller is dedicated to improving the tracking control accuracy and anti-disturbance characteristics of the position and stiffness tracking control of the VSJ based on the equivalent lever mechanism. By using feedback linearization and nonlinear coordinate transformation, the nonlinear system dynamic model of the VSJ based on equivalent lever mechanism is transformed into a linear system model with lumped disturbances and input saturation constraints. Then, the DOB with anti-windup measures is used to estimate the lumped disturbances in the linear system model. Subsequently, in order to compensate the estimation error of the traditional DOB with fixed preset gain, the estimation error compensator is designed. Finally, a novel composite tracking controller is designed to achieve simultaneous position and stiffness tracking control of the VSJ based on the equivalent lever mechanism, and the semi-global ultimate uniformly bounded stability of the closed-loop system is proved based on the stability analysis of the candidate Lyapunov function.

The guidance of controller parameter setting is given. Simulation results show the robustness and effectiveness of the controller. The simulation comparison results reveal the effectiveness of the novel estimation error compensation measures for the traditional DOB with fixed preset observation gain in reducing the system output tracking errors and improving the anti-disturbance performance of the controller.

2. The designed single input single output (SISO) estimation error compensator is inspired by the design idea of the SISO disturbance observer [34], and the two have the same structural form. However, unlike the traditional SISO DOB, the input of the designed estimation error compensator includes the tracking error and the estimation error of the DOB. The estimation values provided by the estimation error compensators are used in the controller to improve the tracking accuracy and anti-disturbance characteristics. As far as I know, this is the first estimation error compensator designed based on the structure of the SISO DOB. This design greatly expands the application of the DOB and is a flexible application of the SISO DOB. By combining the traditional DOB with fixed preset observation gain [31]–[35] and the novel estimation error compensator, the tracking control accuracy and anti-disturbance characteristics of the controller can be significantly improved. The designed SISO estimation error compensator has only one gain parameter and is easy to adjust. The simulation results show that the designed estimation error compensator for the traditional DOB with fixed preset observation gain can improve the tracking accuracy and anti-disturbance characteristics of the controller. Moreover, the design idea of the estimation error compensator can also be applied to the tracking control of other systems [31]–[35] to improve the tracking accuracy.

3. Although the existing VSJs based on the equivalent lever mechanisms [4], [11], [13]–[16] have different mechanical implementation schemes of variable stiffness mechanisms, this type of VSJs all have the same structural form of system dynamics model [36]. Therefore, the designed controller will be suitable for the tracking control of this type of VSJ, and will have good applicability in different VSJs based on equivalent lever mechanism. Moreover, the designed controller will also be suitable for the simultaneous tracking control of the position and stiffness of the antagonistic VSJ [1], [26]–[28]. The controller designed in this article provides a novel control scheme with good applicability for the robust tracking control of the position and stiffness of the VSJ.

This article is organized as follows. Section 2 describes the mechanism transmission structure of the VSJ based on the equivalent lever mechanism. Section 3 gives the system dynamics model of the VSJ based on the equivalent lever mechanism. Section 4 presents the problem formulation for the tracking control of the VSJ based on the equivalent lever mechanism. Subsequently, the controller design and stability analysis are formulated in Section 5. Simulation studies are described in Section 6. Finally, conclusions and directions for future work are presented in Section 7.
II. MECHANISM TRANSMISSION STRUCTURE OF THE VARIABLE STIFFNESS JOINT BASED ON EQUIVALENT LEVER MECHANISM

Although the existing VSJs based on the equivalent lever mechanisms have different implementation schemes of lever pivot position adjustment mechanisms [4], [8], [9], [11]–[18], [37], this type of VSJs all have the mechanical transmission structure design of series configuration form, and the mechanism transmission scheme can be unified and simplified as shown in Figure 1. The description of Figure 1 is stated as follows. The position drive unit (i.e., position control motor + retarder) is used to drive the mounting base (i.e., the position control base). The stiffness adjustment mechanism and the nonlinear elastic element are installed on the position control base. The position driving unit, the elastic unit and the output link are arranged in series configuration. An equivalent nonlinear elastic element with adjustable stiffness is connected between the position control base and the output link. The stiffness control unit (i.e., stiffness control motor + retarder + transmission mechanism) is installed on the position control base and rotates synchronously with it. When the output shaft of the stiffness control motor rotates, the pivot position of the equivalent lever mechanism can be adjusted, and then the stiffness of the equivalent elastic element can be adjusted.

III. DYNAMICS MODEL OF VARIABLE STIFFNESS JOINT BASED ON THE EQUIVALENT LEVER MECHANISM

At present, there are mainly three types of mechanical implementation schemes of variable stiffness mechanism based on equivalent lever mechanism. The first type of variable stiffness mechanism is to change the position of the spring on the lever arm, while keeping the position of the pivot and the position of the force acting point unchanged, for example, the AwAS [11] and the HVSA [12]. The second type of variable stiffness mechanism implementation scheme is to change the position of the force acting point on the lever arm, while maintaining the pivot position and spring position unchanged. The third type of variable stiffness mechanism is to change the position of the pivot along the direction of the lever arm, while keeping the position of the spring and the force acting point unchanged, for example, the AwAS-II [13], the CompAct-VSA [14], the SVSA [15], the SVSA-II [16], the MeRIA [18] and the vsaUT-II [37].

Compared with the former two types of variable stiffness implementation scheme, the third type of variable stiffness mechanism implementation scheme can achieve a wider range of adjustable stiffness and a smaller energy consumption of stiffness adjustment. Therefore, the VSJ based on equivalent lever mechanism more uses the third type of variable stiffness scheme [13]–[18], [37]. Although the VSJs based on equivalent lever mechanism have different variable stiffness mechanism implementation schemes, this type of VSJs have the same series configuration transmission structure form, as shown in Figure 2, and the same system dynamics structure form, as shown in (1) [36]. The parameter definitions of the system dynamics (1) is given by Table 1.

\[
M\ddot{q} + D_p \dot{q} + \tau_e + E_g \sin(q) = \tau_{ext} \\
J_p \ddot{\theta}_p + D_p \dot{\theta}_p - \tau_e = \tau_p \\
J_s \ddot{\theta}_s + D_s \dot{\theta}_s + \tau_r = \tau_s
\]

(1)

Where:

| Symbol | Description |
|--------|-------------|
| $M$    | Equivalent inertia of the output components assembly of the VSJ |
| $J_p$  | Equivalent inertia of the position control unit |
| $J_s$  | Equivalent inertia of the stiffness control unit |
| $D_p$  | Equivalent friction damping coefficient of the output link assembly of the VSJ |
| $D_r$  | Equivalent friction damping coefficient of the position control driving unit of the VSJ |
| $D_s$  | Equivalent friction damping coefficient of the stiffness control driving unit of the VSJ |
| $\tau_e$ | Elastic actuating torque of the VSJ |
| $\tau_r$ | Coupling reaction torque on stiffness control unit due to the elastic transmission |
| $E_g=mgd$ | Gravity effect of the output link assembly |
| $m$ | Equivalent mass of the output link assembly |
| $d$ | Distance from the rotating axis of the output components assembly to its center of mass |
| $\tau_{ext}$ | Unknown external disturbance applied to the output link of the VSJ |
| $\tau_p$ | Control input torque provided by position drive unit |
| $\tau_r$ | Control input torque provided by stiffness drive unit |
| $q$ | Angular position of the output link of the VSJ |
| $\theta_p$ | Angle position of the position control driving unit |
| $\theta_s$ | Angle position of the stiffness control driving unit |
### IV. PROBLEM FORMULATION

Although the system dynamics model of the VSJ based on the equivalent lever mechanism is known, the accurate system dynamics model is difficult to obtain. Model parameter perturbations, dynamics model uncertainties and unknown disturbances are unavoidable in the system dynamics model of the VSJ. The controller designed based on the precise system model may not achieve good tracking performance. Considering the parametric uncertainties, unknown friction torques (i.e., \(\tau_p\) and \(\tau_s\)), unknown external disturbance (i.e., \(\tau_{ext}\)) and control input saturation constraints in the system dynamics model (1), the actual system dynamics model is given by (2). In equation (2), the \(M_i, D_2, E_g, J_{pt}, D_{pt}, J_{st}\) and \(D_q\) are the actual system model parameters. The "\(\Delta\)" represents the difference between the nominal values and the true values of the system model parameter, as shown in (3).

The control input saturation models sat \((\tau_i)\) \((i = p, s)\) are described in (4). The \(\tau_i\) \((i = p, s)\) is the control input torque. The \(\tau_{i, max}\) and the \(\tau_{i, min}\) are the known bounds of the control input torque \(\tau_i\), respectively.

\[
M_i\ddot{q} + D_i\dot{q} + \tau_e + E_g \sin(q) = \tau_{ext}
\]

\[
\begin{align*}
J_{pt}\ddot{\theta}_p + D_{pt}\dot{\theta}_p - \tau_p &= \text{sat}\left(\tau_p\right) \\
J_{st}\ddot{\theta}_s + D_{st}\dot{\theta}_s + \tau_s &= \text{sat}\left(\tau_s\right)
\end{align*}
\]

\[
\Delta M = M_i - M; \Delta J_p = J_{pt} - J_p; \Delta J_s = J_{st} - J_s
\]

\[
\begin{align*}
\Delta D_2 &= D_q - D_2; \Delta D_1 &= D_1 - D_q; \Delta D_i &= D_i - D_{pt}; \Delta D_i &= D_i - D_{st}; \Delta E_g &= E_g - E_g; \Delta \tau_e &= \tau_e - \tau_e; \Delta \tau_s &= \tau_s - \tau_s
\end{align*}
\]

\[
sat(\tau_i) = \begin{cases} 
\tau_{i, max}; & \text{if } \tau_i \geq \tau_{i, max} \\
\tau_i; & \text{if } \tau_{i, min} < \tau_i < \tau_{i, max} \\
\tau_{i, min}; & \text{if } \tau_i \leq \tau_{i, min}
\end{cases}
\]

For the actual system dynamics (2), the system dynamics model uncertainties, the unknown parameter perturbations and the unknown disturbances are regarded as the equivalent disturbances of the system model. The system dynamics (2) can be further transformed into a model with equivalent disturbances, as shown in equation (5), and the equivalent disturbances in the system (5) are given by equation (6).

\[
\begin{align*}
\ddot{q} &= -\frac{D_q}{M}\dot{q} - \frac{\tau_e}{M} + \frac{E_g}{M} \sin(q) + d_{wq} \\
\dot{\theta}_p &= -\frac{D_{pt}}{J_p}\dot{\theta}_p - \frac{\tau_p}{J_p} + \text{sat}\left(\tau_p\right) + d_{wp} \\
\dot{\theta}_s &= -\frac{D_{st}}{J_s}\dot{\theta}_s - \frac{\tau_s}{J_s} + \text{sat}\left(\tau_s\right) + d_{ws}
\end{align*}
\]

\[
\begin{align*}
d_{wq} &= -\frac{\Delta M}{M}\ddot{q} - \frac{\Delta D_2}{M}\dot{q} - \frac{\Delta \tau_e}{M} + \frac{\Delta E_g}{M} \sin(q) + \frac{\tau_{ext}}{M} \\
d_{wp} &= -\frac{\Delta D_1}{J_p}\dot{\theta}_p - \frac{\Delta D_{pt}}{J_p}\dot{\theta}_p + \frac{\Delta \tau_p}{J_p} - \frac{\tau_{p}}{J_p} \\
d_{ws} &= -\frac{\Delta D_2}{J_s}\dot{\theta}_s - \frac{\Delta D_{st}}{J_s}\dot{\theta}_s + \frac{\Delta \tau_s}{J_s} - \frac{\tau_{s}}{J_s}
\end{align*}
\]

The calculation process of the \(d_{wq}\) is given by equations (7)-(11) to show the calculation method of the equivalent disturbances in the system (5).

\[
\begin{align*}
\ddot{q} &= \frac{M}{M_i}\left(\frac{D_q}{M}\dot{q} + \frac{\tau_e}{M} - \frac{E_g \sin(q)}{M} + \frac{\tau_{ext}}{M}\right) \\
\ddot{q} &= \frac{M}{M_i}\left(-\frac{D_q}{M} - \frac{\tau_e}{M} + \frac{E_g \sin(q)}{M} + \frac{\tau_{ext}}{M}\right)
\end{align*}
\]

\[
\ddot{q} - \ddot{q} + \frac{M_i}{M}\ddot{q} = \frac{D_q}{M}\dot{q} + \frac{\tau_e}{M} - \frac{E_g \sin(q)}{M} + \frac{\tau_{ext}}{M}
\]

\[
\ddot{q} - \ddot{q} + \frac{M_i}{M}\ddot{q} = \frac{D_q}{M}\dot{q} + \frac{\tau_e}{M} - \frac{E_g \sin(q)}{M} + \frac{\tau_{ext}}{M}
\]

\[
\ddot{q} - \ddot{q} + \frac{M_i}{M}\ddot{q} = \frac{D_q}{M}\dot{q} + \frac{\tau_e}{M} - \frac{E_g \sin(q)}{M} + \frac{\tau_{ext}}{M}
\]

In order to establish the state space model of the VSJ based on the equivalent lever mechanism, we make the following vector definitions. The \(y \in R^2\) is the system output vector. The sat \((\tau)\) \(\in R^2\) is the system constrained control input vector. The \(x = [x_1, x_2, x_3, x_4, x_5, x_6]^T = [q, \dot{q}, \theta_p, \dot{\theta}_p, \theta_s, \dot{\theta}_s]_T\) is the system state vector. The \(d_w \in R^6\) is the equivalent disturbance vector. The state space model corresponding to the system dynamics model (5) is shown in equations (12)-(16).

\[
\begin{align*}
\dot{x} &= f(x) + g(x) sat(u) + d_w \\
y &= h(x) = \begin{bmatrix} h_1(x) \\
h_2(x) \end{bmatrix} = \begin{bmatrix} q \\
k_{eq} \end{bmatrix}
\end{align*}
\]

\[
f(x) = \begin{bmatrix} x_2 \\
x_4 \end{bmatrix} \begin{bmatrix} -\frac{D_q}{M} x_2 - \frac{\tau_e}{M} - \frac{E_g \sin(x_1)}{M} \\
x_4 \end{bmatrix} \begin{bmatrix} -\frac{D_q}{M} x_2 - \frac{\tau_e}{M} - \frac{E_g \sin(x_1)}{M} \\
x_4 \end{bmatrix}
\]

\[
\begin{bmatrix} x_6 \\
k_{eq} \end{bmatrix} = \begin{bmatrix} \frac{D_q}{M} x_4 + \frac{\tau_e}{M} - \frac{E_g \sin(x_1)}{M} \\
x_6 \end{bmatrix}
\]
The input saturation models \(\text{sat}(\tau_i) = \text{sat}(u_i)\) \((i = p, s)\) are described in equations (14) and (15). The \(u_{i,\max}\) and \(u_{i,\min}\) are the known bounds of the control input \(u_i\) \((i = p, s)\) to be designed. The \(\Delta u_i\) \((i = p, s)\) is the input-output differences of the control input saturation models, respectively.

\[
\text{sat}(u_i) = \begin{cases} 
    u_{i,\max} & \text{if } u_i \geq u_{i,\max} \\
    u_i & \text{if } u_{i,\min} < u_i < u_{i,\max} \\
    u_{i,\min} & \text{if } u_i \leq u_{i,\min}
\end{cases}
\]

\[
\text{sat}(u) = \begin{bmatrix} u_p + \Delta u_p \\ u_s + \Delta u_s \end{bmatrix}
\]

The disturbance vector \(d_w \in \mathbb{R}^6\) is given by (16).

\[
\begin{bmatrix}
    d_{w1} \\
    d_{w2} \\
    d_{w3} \\
    d_{w4} \\
    d_{w5} \\
    d_{w6}
\end{bmatrix} = \\
\begin{bmatrix}
    -\frac{\Delta M}{M} x_2 - \frac{\Delta D_q}{M} x_2 - \frac{\Delta \tau_\epsilon}{M} - \frac{\Delta E_p}{M} \sin(x_1) + \frac{\tau_{\text{ext}}}{M} \\
    -\frac{\Delta J_p}{J_p} x_4 - \frac{\Delta D_p}{J_p} x_4 + \frac{\Delta \tau_\epsilon}{J_p} - \frac{\tau_{p}}{J_p} \\
    -\frac{\Delta J_s}{J_s} x_6 - \frac{\Delta D_s}{J_s} x_6 - \frac{\Delta \tau_s}{J_s} - \frac{\tau_{s}}{J_s}
\end{bmatrix}
\]

In the simulation of this article, in order to select a representative system dynamics model of the VSJ based on equivalent lever mechanism without losing generality, the CompAct-VSA [14] developed at the Italian Institute of Technology (IIT) is selected as the study object for tracking control simulation. For the CompAct-VSA model, the elastic actuating torque \(\tau_\epsilon\), the coupling reaction torque \(\tau_r\) and the joint output stiffness \(k_{qj}\) can be expressed as

\[
\begin{align*}
\phi &= q - \theta_p = x_1 - x_3; \\
\tau_\epsilon &= \frac{2k_s \delta_1^2 \Delta^2 \phi}{(\Delta - \delta_1)^2}; \\
\tau_r &= \frac{2k_s n^2 \theta_1 \phi^2 \Delta^3}{(\Delta - n \delta_1)^3} = \frac{2k_s n^2 x_5 \phi^2 \Delta^3}{(\Delta - n x_5)^3}; \\
k_{qj} &= \frac{2k_s \delta_1^2 \Delta^2}{(\Delta - \delta_1)^2}
\end{align*}
\]

where the \(\phi = q - \theta_p\) is the joint angular deflection due to the elastic transmission. The maximum angular deflection of the CompAct-VSA is constrained to be \(\pm 0.35\) rad by a pair of mechanical locks. The parameters of the CompAct-VSA [14] are defined as follows: the \(\delta_1\) is the distance from the pivot point of the equivalent lever mechanism to the center of rotation of the joint; the \(n (0.006 \text{ m/rad})\) is the transmission rate of the rack and pinion mechanism in the variable stiffness mechanism; the \(k_s (10000 \text{ N/m})\) is the equivalent spring rate; the \(\Delta (0.015 \text{ m})\) is the length of the lever arm. Other nominal parameters for the CompAct-VSA system model are \(D_p = 10.2763 \text{ N-m/s/rad}, D_s = 0.1316 \text{ N-m/s/rad}, D_q = 0.2 \text{ N-m/s/rad}\) (assumed value), \(M = 0.072 \text{ kg-m}^2, J_p = 0.1055 \text{ kg-m}^2, J_s = 0.000795 \text{ kg-m}^2, d = 0.3\) m (assumed distance from the rotating axis of the output link to its center of mass). \(m = 2 \text{ kg}\) (assumed output link mass).

The main control objective is as follows. For the system dynamics model of the VSJ based on the equivalent lever mechanism with parametric uncertainties, unknown friction torques, unknown external disturbance and control input saturation constraints, we design a robust tracking controller to realize the simultaneous robust tracking control of the position and stiffness of the VSJ to the desired trajectories. In the design process of the controller, we introduce a novel estimation error compensator for the traditional DOB with fixed preset observation gain to improve the tracking control accuracy and anti-disturbance performance of the controller.

**V. CONTROLLER DESIGN**

In this section, a novel robust tracking controller based on feedback linearization (FL), disturbance observer (DOB), sliding mode control (SMC) and disturbance estimation error compensator (DEEC) is designed. The controller is used to realize the simultaneous tracking control of the position and stiffness of the VSJ based on the equivalent lever mechanism. The design process of the controller is described as follows. Firstly, by using the feedback linearization and coordinate transformation, the nonlinear state space model (12) is transformed into a linear system model with lumped disturbances and input saturation constraints, as shown in (19). Subsequently, the DOB is used to estimate the lumped disturbances in the linear system model. By transforming the nonlinear state space model (12) into the linear system model (19), the parameter adjustment of the DOB is more convenient, and the linear system model (19) is more conducive to the controller design. In order to reduce the influence of the control input saturation constraint on the lumped disturbance estimation, the DOB adopts an anti-windup measures, that is, directly using the saturation control input instead of the calculated control input [20], [31]. The DOB with fixed preset observation gain inevitably has estimation error. Especially when the desired tracking trajectories change or the system disturbances change suddenly. In order to improve the tracking accuracy and the anti-disturbance characteristics of the controller, a robust tracking controller based on FL, DOB, SMC and DEEC is designed. Based on the stability analysis of candidate Lyapunov functions, the semi-global uniformly bounded stability of the closed-loop system is proved. The setting rules of the controller parameters are discussed.

**A. FEEDBACK LINEARIZATION OF THE SYSTEM MODEL**

By utilizing the coordinate transformations as shown (18), the nonlinear state space model (12) can be transformed...
into a linear system model (19), which includes the lumped disturbance vector \( d_l \in \mathbb{R}^6 \) and the saturated control input vector \( \text{sat}(v) \in \mathbb{R}^2 \). In equation (20), the \( \text{sat}(v) \in \mathbb{R}^2 \) is the control law vector with saturation constraints, and the \( v \in \mathbb{R}^6 \) is the control law vector to be designed, and the \( \Delta v \in \mathbb{R}^2 \) is the difference between the control law with saturation constraints and the control law to be designed.

\[
\xi = \begin{bmatrix} \xi_q \\ \xi_k \end{bmatrix}; \quad \xi_q = \begin{bmatrix} \xi_{q1} \\ \xi_{q2} \\ \xi_{q3} \\ \xi_{q4} \end{bmatrix} = \begin{bmatrix} h_1(x) \\ L_f h_1(x) \\ L_f^2 h_1(x) \\ L_f^3 h_1(x) \end{bmatrix};
\]

\[
\xi_k = \begin{bmatrix} \xi_{kej_1} \\ \xi_{kej_2} \end{bmatrix} = \begin{bmatrix} h_2(x) \\ L_f h_2(x) \end{bmatrix};
\]

\[
{\dot{\xi}} = A\xi + B\text{sat}(v) + d_l
\]

\[
y_o = C\xi
\]

\[
\text{sat}(v) = \begin{bmatrix} \text{sat}(v_q) \\ \text{sat}(v_{kej}) \end{bmatrix} = v + \Delta v = \begin{bmatrix} v_q \\ v_{kej} \end{bmatrix} + \begin{bmatrix} \Delta v_q \\ \Delta v_{kej} \end{bmatrix}
\]

Based on the differential geometry theory and nonlinear coordinate transformation, the relationship between the saturation control input \( u(s) \in \mathbb{R}^2 \) and the new saturation control input \( \text{sat}(v) \in \mathbb{R}^2 \) is given by equations (21)-(23).

\[
\begin{bmatrix} \text{sat}(v_q) \\ \text{sat}(v_{kej}) \end{bmatrix} = G(x) \begin{bmatrix} u_p \\ u_s \end{bmatrix} + \begin{bmatrix} L_f h_1(x) \\ L_f^2 h_1(x) \\ L_f h_2(x) \\ L_f^2 h_2(x) \end{bmatrix} \begin{bmatrix} \text{sat}(u_p) \\ \text{sat}(u_s) \end{bmatrix}
\]

\[
\begin{bmatrix} \Delta v_q \\ \Delta v_{kej} \end{bmatrix} = G(x) \begin{bmatrix} \Delta u_p \\ \Delta u_s \end{bmatrix}
\]

For the linear system (19), the state matrix \( A \), the input matrix \( B \), and the output matrix \( C \) are given by equation (24).

\[
A = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}; \quad B = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}; \quad C = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}
\]

For the CompAct-VSA system, the determinant of the decoupling matrix \( G(x) \) is given by equation (25).

\[
\det(G(x)) = \frac{8\Delta^3 k_2 n^3 x_3^3}{J_p J_s M (\Delta - nx_5)^5} = \frac{8\Delta^3 k_2 n^3}{J_p J_s M (\Delta - \delta_1)^5}
\]

In the tracking control simulation of this article corresponding to the CompAct-VSA [14], the joint output stiffness reference trajectory is always defined between 0 and infinity. Invoking the expression of the joint stiffness of the CompAct-VSA shown in equation (17), it can be concluded that the determinant of the decoupling matrix \( G(x) \) is always different from zero, and the decoupling matrix \( G(x) \) is always nonsingular. For the CompAct-VSA, the full static feedback linearization can be achieved since the decoupling matrix \( G(x) \) is nonsingular and the sum of the relative degrees of the outputs of the system is equal to the state dimension of the system. Therefore, by using feedback linearization, the simultaneous position and stiffness tracking control of the VSJ based on the equivalent lever mechanism can be realized. In addition, the feasibility of applying the feedback linearization method to the system model of the VSJ based on the equivalent lever mechanism has been verified before [19], [22], [23], although the involution conditions are not checked before using the feedback linearization method.

**B. DESIGN OF DISTURBANCE OBSERVER WITH ANTI-WINDUP MEASURE**

In order to estimate the lumped disturbance vector \( d_l = [d_{l1}, d_{l2}, d_{l3}, d_{l4}, d_{l5}, d_{l6}]^T \in \mathbb{R}^6 \) in the linear system model (19), the DOB can be expressed as

\[
\dot{p} = -L_d (p + L_d \dot{p}) - L_d (A\xi + B\text{sat}(v))
\]

where \( \dot{d}_l \in \mathbb{R}^6 \) is the estimation of the \( d_l \in \mathbb{R}^6 \), and the \( p \in \mathbb{R}^6 \) is the auxiliary internal state vector of the DOB, and \( L_d \) is the disturbance observer gain matrix to be designed. It should be noted that for the DOB shown in equation (26), in order to reduce the influence of control input saturation constraint on disturbance estimation, the saturation control input \( \text{sat}(v) \) is directly used as the control input of the disturbance observer rather than the calculated control input \( v \).

For the VSJ based on the equivalent lever mechanism, the disturbance observer gain matrix can be expressed as \( L_d = \text{diag} \{L_{d1}, L_{d2}, L_{d3}, L_{d4}, L_{d5}, L_{d6} \} \). For the purpose of the controller design and the stability analysis, the following assumptions are presented.

**Assumption 1:** The reference trajectories and the derivatives of the reference trajectories are all bounded.

**Assumption 2:** The disturbance estimation error vector \( e_{dl} = [ed_{l1}, ed_{l2}, ed_{l3}, ed_{l4}, ed_{l5}, ed_{l6}]^T = \dot{d}_l - d_l - \dot{d}_l \) is bounded.

By invoking the equations (19) and (26), the estimation error dynamics can be formulated as

\[
\dot{e}_{dl} = \dot{\hat{d}}_l - \dot{d}_l = \dot{d}_l - L_d (p + L_d \hat{p}) - L_d (A\hat{\xi} + B\text{sat}(v))
\]

\[
= \dot{d}_l - L_d (p + L_d \hat{p})
\]

\[
\dot{d}_l - L_d e_{dl}
\]

(27)
Based on the equation (27), the dynamics of the estimated lumped disturbances can be derived from (27) that

\[ \dot{d}_q = L_d e_{d1} \]

\[ \Rightarrow \dot{d}_q = L_{d11} e_{d11}; \dot{d}_q = L_{d22} e_{d12}; \dot{d}_q = L_{d33} e_{d13} \]

\[ \dot{d}_q = L_{d44} e_{d14}; \dot{d}_q = L_{d55} e_{d15}; \dot{d}_q = L_{d66} e_{d16}. \] (28)

Remark 1: The estimation error dynamics system (27) is asymptotically stable if the \(-L_d\) is Hurwitz, and the \(\dot{d}_q\) is bounded, and the \(\lim_{t \to \infty} \dot{d}_q = 0\). Although asymptotic tracking of the DOB can only be obtained based on the assumption that \(\lim_{t \to \infty} \dot{d}_q = 0\), it has been pointed out that the estimation values obtained by the DOB can also track fast time-varying disturbances with bounded errors as long as the DOB dynamics are designed to be sufficiently fast [32], [33], [35].

C. COMPOSITE CONTROL LAW DESIGN

In Section 5.2, the DOB (26) is added with anti-windup measure to reduce the influence of input saturation constraints on disturbance estimation, so as to improve the tracking accuracy of the position and stiffness of the VSJ. On the other hand, increasing the gain of the DOB is also conducive to improving the output tracking accuracy of the VSJ system. However, the DOB with fixed preset observation gain inevitably has estimation error, which will reduce the output tracking accuracy of the system, and the measures to reduce the estimation error by increasing the observation gain of the DOB may cause system instability.

For the commonly used DOB with fixed preset observation gain [20], [22]–[24], [31]–[35] as shown in equation (26), a novel robust tracking control scheme based on FL, DOB, SMC and DEEC is designed in this section to improve the tracking accuracy of the position and stiffness of the VSJ based on the equivalent lever mechanism. In the process of controller design, the nonlinear state space model (12) can be transformed into the linear system model (19) by feedback linearization, which is convenient for disturbance estimation and controller design. The DOB (26) is used to estimate the lumped disturbances in the linearized model (19). The newly designed estimation error compensator is used to estimate the estimation error of the DOB to improve the tracking accuracy of the VSJ. The purpose of using the sliding mode control is to prove the semi-global ultimately uniformly bounded stability of the closed-loop system and give the guidance of the controller parameter settings.

The controller design based on FL, DOB, SMC and DEEC is described as follows.

Step 1: Based on the equations (19) and (26), the output link position tracking error \(e_{q_i}\) \((i = 1, 2, 3, 4)\) and the joint output stiffness tracking error \(e_{\xi_j}\) \((j = 1, 2)\) of the VSJ based on equivalent lever mechanism can be expressed as:

\[ e_{q1} = \xi_{q1} - q_d \]

\[ e_{q2} = \xi_{q2} - q_d^{(1)} + \hat{d}_{l1} \]

\[ e_{q3} = \xi_{q3} - q_d^{(2)} + \hat{d}_{l2} \]

\[ e_{q4} = \xi_{q4} - q_d^{(3)} + \hat{d}_{l3} \]

\[ e_{\xi1} = \xi_{\xi1} - k_{\xi d} \]

\[ e_{\xi2} = \xi_{\xi2} - k_{\xi d}^{(1)} + \hat{d}_{l5}. \] (29)

By invoking (18), (19), (20), (26)–(29) and calculating the first derivative of the \(e_{q1}\) with respect to time \(t\), the expression as shown in (30) can be obtained, and the \(\delta_{q1}\) represents the estimation error of the DOB with fixed preset observation gain.

\[ \dot{e}_{q1} = \xi_{q2} + d_{l1} - q_d^{(1)} = e_{q2} + e_{d11} = e_{q2} + \delta_{q1}; \quad \delta_{q1} = e_{d11} \] (30)

The estimation error compensator shown in equation (31) is used to estimate the estimation error \(\delta_{q1}\) and obtain the estimated value \(\hat{\delta}_{q1}\). The new estimation error is defined as \(\delta_{q1} = \delta_{q1} - \hat{\delta}_{q1}\). The \(\lambda_{q1}\) is the output of the estimation error compensator, and the \(\gamma_{q1} > 0\) is the gain to be designed for the estimated error compensator (31).

\[ \dot{\lambda}_{q1} = -\gamma_{q1} \lambda_{q1} - \gamma_{q1} \left[ (e_{q2}) + \gamma_{q1} e_{q1} \right] \]

\[ \hat{\delta}_{q1} = \lambda_{q1} + \gamma_{q1} e_{q1}; \quad \lambda_{q1} (0) = 0; \quad \delta_{q1} (0) = 0 \] (31)

Step 2: By calculating the first derivative of \(e_{q2}\) with respect to time \(t\), the following expression can be obtained:

\[ \dot{e}_{q2} = e_{q3} + e_{d2} + L_{d11} e_{d11} = e_{q3} + \delta_{q2}; \]

\[ \delta_{q2} = e_{d2} + L_{d11} e_{d11} \] (32)

where the \(\delta_{q2}\) is the estimation error of the DOB.

The disturbance estimation error compensator (33) is used to estimate the estimation error \(\delta_{q2}\) and obtain the estimated value \(\hat{\delta}_{q2}\). The new estimation error is defined as \(\delta_{q2} = \delta_{q2} - \hat{\delta}_{q2}\). The \(\lambda_{q2}\) is the output of the estimation error compensator, and the \(\gamma_{q2} > 0\) is the gain to be designed for the estimated error compensator (33).

\[ \dot{\lambda}_{q2} = -\gamma_{q2} \lambda_{q2} - \gamma_{q2} \left[ (e_{q3}) + \gamma_{q2} e_{q2} \right] \]

\[ \hat{\delta}_{q2} = \lambda_{q2} + \gamma_{q2} e_{q2}; \quad \lambda_{q2} (0) = 0; \quad \delta_{q2} (0) = 0 \] (33)

Step 3: The first derivative of the \(e_{q3}\) with respect to time \(t\) is depicted by

\[ \dot{e}_{q3} = e_{q4} + e_{d3} + L_{d22} e_{d12} = e_{q4} + \delta_{q3}; \]

\[ \delta_{q3} = e_{d3} + L_{d22} e_{d12} \] (34)

where the \(\delta_{q3}\) is the estimation error of the DOB with fixed preset observation gain.

The disturbance estimation error compensator (34) is used to estimate the estimation error \(\delta_{q3}\) and obtain the estimated value \(\hat{\delta}_{q3}\). The \(\hat{\delta}_{q3} = \delta_{q3} - \hat{\delta}_{q3}\) is the new estimation error, and the \(\lambda_{q3}\) is the output of the estimation error compensator, and the \(\gamma_{q3} > 0\) is the gain to be designed for the estimated error compensator (34).

\[ \dot{\lambda}_{q3} = -\gamma_{q3} \lambda_{q3} - \gamma_{q3} \left[ (e_{q4}) + \gamma_{q3} e_{q3} \right] \]

\[ \hat{\delta}_{q3} = \lambda_{q3} + \gamma_{q3} e_{q3}; \quad \lambda_{q3} (0) = 0; \quad \delta_{q3} (0) = 0 \] (35)
Step 4: The first derivative of the $e_{q4}$ with respect to time $t$ is given by
\[
\dot{e}_{q4} = v_q + \Delta v_q + \hat{d}_{l4} - q_d^{(4)} + \delta_{q4} = e_{d4l} + Ld_{33} e_{d3} \tag{36}
\]
where the $\delta_{q4}$ is the estimation error of the DOB.

The estimation error $\delta_{q4}$ can be estimated by the disturbance estimation error compensator (37) and obtain the estimated value $\hat{\delta}_{q4}$. The $\delta_{q4} = \hat{\delta}_{q4} - \delta_{q4}$ is the new estimation error, and the $\lambda_{q4}$ is the output of the estimation error compensator, and the $\gamma_{q4} > 0$ is the gain to be designed.

\[
\begin{align*}
\dot{\lambda}_{q4} &= -\gamma_{q4} \lambda_{q4} - \gamma_{q4} \left[ (sat (v_q) + \hat{d}_{l4} - q_d^{(4)}) + \gamma_{q4} e_{q4} \right] \\
\hat{\delta}_{q4} &= \lambda_{q4} + \gamma_{q4} e_{q4}; \lambda_{q4} (0) = 0; \hat{\delta}_{q4} (0) = 0
\end{align*} \tag{37}
\]

Step 5: A sliding mode surface $\zeta_q$ for the position tracking errors $e_{qi}$ $(i = 1, 2, 3, 4)$ is designed as
\[
\zeta_q = c_{q1} e_{q1} + c_{q2} e_{q2} + c_{q3} e_{q3} + c_{q4} \tag{38}
\]
where the gain parameters $c_{qi}$ $(i = 1, 2, 3)$ have to be designed such that the polynomial
\[
p(s) = s^3 + c_{q3}s^2 + c_{q2}s + c_{q1} = 0 \tag{39}
\]
is Hurwitz.

The first derivative of the sliding mode surface $\dot{\zeta}_q$ with respect to time $t$ is obtained as:
\[
\begin{align*}
\dot{\zeta}_q &= c_{q1} (e_{q2} + \hat{\delta}_{q1}) + c_{q2} (e_{q3} + \hat{\delta}_{q2}) + c_{q3} (e_{q4} + \hat{\delta}_{q3}) \\
&\quad + sat (v_q) + \hat{d}_{l4} - q_d^{(4)} + \delta_{q4} \qquad \quad \quad \\
\hat{\delta}_{q4} &= \dot{c}_{q1} \zeta_q + c_{q2} \hat{\delta}_{q2} + c_{q3} \hat{\delta}_{q3} + \dot{\delta}_{q4}
\end{align*} \tag{40}
\]
where the $\delta_{q4}$ is the estimation error.

The disturbance estimation error compensator (41) is used to estimate the estimation error $\delta_{vyq}$ and obtain the estimated value $\hat{\delta}_{vyq}$. The output of the estimation error compensator (41) is $\lambda_{vyq}$ and the $\gamma_{vyq} > 0$ is the gain parameter to be designed. The new estimation error is defined by $\delta_{vyq} = \delta_{vyq} - \hat{\delta}_{vyq}$.

\[
\begin{align*}
\dot{\lambda}_{vyq} &= -\gamma_{vyq} \lambda_{vyq} - \gamma_{vyq} \left[ \left( c_{q1} (e_{q2} + \hat{\delta}_{q1}) + c_{q2} (e_{q3} + \hat{\delta}_{q2}) \\
&\quad + c_{q3} (e_{q4} + \hat{\delta}_{q3}) + sat (v_q) + \hat{d}_{l4} - q_d^{(4)} + \delta_{q4} \right) + \gamma_{vyq} \zeta_q \right] \\
\hat{\delta}_{vyq} &= \lambda_{vyq} + \gamma_{vyq} \zeta_q; \lambda_{vyq} (0) = 0; \hat{\delta}_{vyq} (0) = 0
\end{align*} \tag{41}
\]

Step 6: Invoking (29), the first derivative of the joint stiffness tracking error $e_{k1}$ with respect to time $t$ is given by
\[
\begin{align*}
\dot{e}_{k1} &= \xi_{kej2} - k_{ejd}^{(1)} + \hat{d}_{l5} + \hat{d}_{l5} = e_{k2} + \delta_{k1} \\
\delta_{k1} &= e_{d5}
\end{align*} \tag{42}
\]
where the $\delta_{k1}$ represents the estimation error of the DOB.

The disturbance estimation error compensator (43) is used to estimate the estimation error $\delta_{k1}$ and obtain the estimated value $\hat{\delta}_{k1}$. The $\hat{\delta}_{k1} = \delta_{k1} - \hat{\delta}_{k1}$ is the new estimation error.

The $\lambda_{k1}$ is the output of the estimation error compensator (43). The $\gamma_{k1} > 0$ is the gain of the estimation error compensator to be designed.

\[
\begin{align*}
\dot{\lambda}_{k1} &= -\gamma_{k1} \lambda_{k1} - \gamma_{k1} \left[ (e_{k2} + \gamma_{k1} e_{k1}) \right] \\
\hat{\delta}_{k1} &= \lambda_{k1} + \gamma_{k1} e_{k1}; \lambda_{k1} (0) = 0; \hat{\delta}_{k1} (0) = 0
\end{align*} \tag{43}
\]

Step 7: Invoking (29), the first derivative of the $e_{k2}$ with respect to time $t$ is given by
\[
\begin{align*}
\dot{e}_{k2} &= sat (v_{kej}) - k_{ejd}^{(2)} + \hat{d}_{l6} + \delta_{k2} \\
\hat{\delta}_{k2} &= e_{d6l} + Ld_{55} e_{d5}
\end{align*} \tag{44}
\]
where the $\hat{\delta}_{k2}$ represents the estimation error. The disturbance estimation error compensator (45) is used to estimate the $\hat{\delta}_{k2}$ and obtain the estimated value $\hat{\delta}_{k2}$. The $\delta_{k2} = \delta_{k2} - \hat{\delta}_{k2}$ is the new estimation error. The $\lambda_{k2}$ is the output of the estimation error compensator and the $\gamma_{k2} > 0$ is the gain of the estimation error compensator to be designed.

\[
\begin{align*}
\dot{\lambda}_{k2} &= -\gamma_{k2} \lambda_{k2} - \gamma_{k2} \left[ (sat (v_{kej}) - k_{ejd}^{(2)} + \hat{d}_{l6} + \gamma_{k2} e_{k2}) \right] \\
\hat{\delta}_{k2} &= \lambda_{k2} + \gamma_{k2} e_{k2}; \lambda_{k2} (0) = 0; \hat{\delta}_{k2} (0) = 0
\end{align*} \tag{45}
\]

Step 8: A sliding mode surface $\zeta_k$ for the joint stiffness tracking errors $e_{kj}$ $(j = 1, 2)$ is designed as
\[
\zeta_k = c_{k1} e_{k1} + e_{k2} \tag{46}
\]
where the $c_{k1} > 0$ is the gain parameter to be designed. By calculating the first derivative of the sliding mode surface $\zeta_k$ with respect to time $t$, we can obtain
\[
\begin{align*}
\dot{\gamma}_{k} &= c_{k1} (e_{k2} + \delta_{k1}) + sat (v_{kej}) - k_{ejd}^{(2)} + \hat{d}_{l6} + \hat{\delta}_{k2} + \delta_{vk} \\
\delta_{vk} &= c_{k1} \delta_{k1} + \hat{\delta}_{k2}
\end{align*} \tag{47}
\]
where the $\delta_{vk}$ is the estimation error of the DOB.

The estimation error compensator (48) is used to estimate the estimation error $\delta_{vk}$ and obtain the estimated value $\hat{\delta}_{vk}$. The output of the estimation error compensator (41) is $\lambda_{vk}$ and $\gamma_{vk} > 0$ is the gain parameter to be designed. The new estimation error is represented as $\delta_{vk} = \delta_{vk} - \hat{\delta}_{vk}$.

\[
\begin{align*}
\dot{\lambda}_{vk} &= -\gamma_{vk} \lambda_{vk} \\
\hat{\delta}_{vk} &= \lambda_{vk} + \gamma_{vk} \zeta_k; \lambda_{vk} (0) = 0; \hat{\delta}_{vk} (0) = 0
\end{align*} \tag{48}
\]

Step 9: The control laws $v_q$ and $v_{kej}$ are given by:
\[
\begin{align*}
v_q &= -c_{q1} (e_{q2} + \hat{\delta}_{q1}) - c_{q2} (e_{q3} + \hat{\delta}_{q2}) - c_{q3} (e_{q4} + \hat{\delta}_{q3}) \\
&\quad - \hat{d}_{l4} + q_d^{(4)} - \hat{\delta}_{vyq} - \mu_k v_k \\
v_{kej} &= -c_{k1} (e_{k2} + \hat{\delta}_{k1}) + k_{ejd}^{(2)} - \hat{d}_{l6} - \hat{\delta}_{k2} - \hat{\delta}_{vk} - \mu_k v_k
\end{align*} \tag{49}
\]
Therefore, the control inputs of the VSJ based on the equivalent lever mechanism are designed as

\[
\begin{bmatrix}
    u_p \\
    u_s
\end{bmatrix} = \left( \begin{bmatrix}
    L_{pp} L_{h1}^T (x) & L_{gq} L_{h1}^T (x) \\
    L_{pp} L_{h2}^T (x) & L_{gq} L_{h2}^T (x)
\end{bmatrix} \right) \begin{bmatrix}
    v_q - L_p^T h_1 (x) \\
    v_{kej} - L_p^T h_2 (x)
\end{bmatrix}
\]

\[
\Rightarrow \begin{bmatrix}
    sat (u_p) \\
    sat (u_s)
\end{bmatrix}
\]

(50)

The schematic diagram of the proposed controller is shown in Figure 3.

**D. SYSTEM STABILITY ANALYSIS**

In this section, under the action of the designed controller, the semi-global ultimate uniformly bounded stability analysis of the closed-loop system based on the candidate Lyapunov function is given. Invoking the equations (29)–(49), the following correlation calculations can be obtained.

\[
\ddot{\delta}_{q1} = \delta_{q1} (\gamma_{q1} - \gamma_{gq} \delta_{q1}) \leq -\gamma_{q1} \left| \delta_{q1} \right|^2 + \left| \delta_{q1} \right| \left| \delta_{q1} \right| \leq -\frac{\gamma_{q1}}{2} \left| \delta_{q1} \right|^2 + \frac{1}{2\gamma_{q1}} \left| \delta_{q1} \right|^2
\]

(51)

\[
\ddot{\delta}_{q2} = \delta_{q2} (\gamma_{q2} - \gamma_{gq} \delta_{q2}) \leq -\gamma_{q2} \left| \delta_{q2} \right|^2 + \left| \delta_{q2} \right| \left| \delta_{q2} \right| \leq -\frac{\gamma_{q2}}{2} \left| \delta_{q2} \right|^2 + \frac{1}{2\gamma_{q2}} \left| \delta_{q2} \right|^2
\]

(52)

\[
\ddot{\delta}_{q3} = \delta_{q3} (\gamma_{q3} - \gamma_{gq} \delta_{q3}) \leq -\gamma_{q3} \left| \delta_{q3} \right|^2 + \left| \delta_{q3} \right| \left| \delta_{q3} \right| \leq -\frac{\gamma_{q3}}{2} \left| \delta_{q3} \right|^2 + \frac{1}{2\gamma_{q3}} \left| \delta_{q3} \right|^2
\]

(53)

\[
\ddot{\delta}_{p} = \delta_{p} (\gamma_{p} - \gamma_{gp} \delta_{p}) \leq -\gamma_{p} \left| \delta_{p} \right|^2 + \left| \delta_{p} \right| \left| \delta_{p} \right| \leq -\frac{\gamma_{p}}{2} \left| \delta_{p} \right|^2 + \frac{1}{2\gamma_{p}} \left| \delta_{p} \right|^2
\]

(54)

\[
\ddot{\delta}_{vq} = \delta_{vq} (\gamma_{vq} - \gamma_{gq} \delta_{vq}) \leq -\gamma_{vq} \left| \delta_{vq} \right|^2 + \left| \delta_{vq} \right| \left| \delta_{vq} \right| \leq -\frac{\gamma_{vq}}{2} \left| \delta_{vq} \right|^2 + \frac{1}{2\gamma_{vq}} \left| \delta_{vq} \right|^2
\]

(55)

\[
\ddot{\zeta}_{q} = \zeta_{q} (\Delta v_q + \delta_{vq} - \mu_{q} \delta_{q}) \leq -\mu_{q} \left| \zeta_{q} \right|^2 + \left( \frac{\mu_{q}}{4} \left| \zeta_{q} \right|^2 + \left| \zeta_{q} \right| \left| \Delta v_q \right| \right)
\]

(56)

\[
\ddot{\delta}_{k1} = \delta_{k1} (\Delta v_{k1} + \delta_{v_{k1}}) \leq -\gamma_{k1} \left| \delta_{k1} \right|^2 + \left| \delta_{k1} \right| \left| \delta_{k1} \right| \leq -\frac{\gamma_{k1}}{2} \left| \delta_{k1} \right|^2 + \frac{1}{2\gamma_{k1}} \left| \delta_{k1} \right|^2
\]

(57)

\[
\ddot{\delta}_{k2} = \delta_{k2} (\Delta v_{k2} + \delta_{v_{k2}}) \leq -\gamma_{k2} \left| \delta_{k2} \right|^2 + \left| \delta_{k2} \right| \left| \delta_{k2} \right| \leq -\frac{\gamma_{k2}}{2} \left| \delta_{k2} \right|^2 + \frac{1}{2\gamma_{k2}} \left| \delta_{k2} \right|^2
\]

(58)
where \( Q_\gamma \) and \( Q_k \) are given by equation (63), and the controller parameter \( r_F \) is set as shown in equation (64).

\[
\begin{align*}
Q_\gamma &= \frac{1}{2\gamma_1} |\delta_{q1}|^2 + \frac{1}{2\gamma_2} |\delta_{q2}|^2 + \frac{1}{2\gamma_3} |\delta_{q3}|^2 + \frac{1}{2\gamma_4} |\delta_{q4}|^2 \\
&+ \frac{1}{2\gamma_v} |\delta_v|^2 + \frac{1}{\mu_k} |\Delta v_k|^2 \\
Q_k &= \frac{1}{2\gamma_k} |\delta_{k1}|^2 + \frac{1}{2\gamma_k} |\delta_{k2}|^2 + \frac{1}{2\gamma_v} |\delta_v|^2 + \frac{1}{\mu_k} |\Delta v_k|^2 \\
\gamma_v &= \frac{1}{2\mu_k} > 0; \quad \gamma_k &= \frac{1}{2\mu_k} > 0 \\
r_F &= 2 \min \left\{ \frac{\gamma_1}{2}; \frac{\gamma_2}{2}; \frac{\gamma_3}{2}; \frac{\gamma_4}{2}; \frac{\gamma_v}{2}; \frac{1}{\mu_k}; \frac{1}{\mu_k} \right\}
\end{align*}
\]

Finally, the inequality (65) can be obtained according to the boundedness theorem.

\[
V(t) \leq V(0) e^{-r_F t} + \frac{Q_{\gamma}}{r_F} \left( 1 - e^{-r_F t} \right) \Rightarrow \lim_{t \to \infty} V(t) \leq \frac{Q}{r_F}; \quad r_F > 0
\]

**E. DISCUSSION ON THE CONTROLLER PARAMETER SETTINGS**

The newly proposed disturbance estimation error compensator in this article has the same structure as the commonly used DOB with single input and single output. However, the designed estimation error compensator is different from the typical single input single output DOB. The control input of the estimation error compensator includes the tracking error signal and estimation error, and it can provide an estimate value for the estimated error of the DOB. The estimation values provided by the disturbance estimation error compensators are added to the control law, thus improving the tracking accuracy of the system outputs.

The designed estimation error compensator has only one gain parameter, which is easy to adjust. However, we still discuss the parameter settings of the estimation error compensators. For the nonlinear state space model (12), when \( \tau_{ext} \) is time-varying disturbance, the \( d_{w2} \) shown in (16) will be the time-varying disturbance, and the lumped disturbances \( d_{l2}, d_{l3}, d_{l4} \) and \( d_{l6} \) will also be time-varying since they all contain \( d_{w2} \). In general, the larger the value of the lumped disturbance \( d_{lj} \) \( (j = 2, 3, 4, 6) \), the larger the estimated error of the \( d_{lj} \) \( (j = 2, 3, 4, 6) \) provided by the DOB, especially when the system disturbance suddenly occurs. Considering the disturbance estimation error descriptions in (66), when setting the gain parameters of the estimation error compensator (i.e., \( \gamma_{q1}, \gamma_{q2}, \gamma_{q3}, \gamma_{q4}, \gamma_{q5}, \gamma_{k1}, \gamma_{k2}, \gamma_v \)), the corresponding adjustment shall be made according to the estimated value to be estimated. For example, \( \gamma_{q1} \) can take small value, while the values of \( \gamma_{q2}, \gamma_{q3}, \gamma_{q4} \), and \( \gamma_{q4} \) can increase sequentially. It should be noted that the gain of the disturbance estimation error compensator should not be too large to prevent the sharp fluctuations of the control input.

\[
\begin{align*}
\delta_{q1} &= e_{d1} \\
\delta_{q2} &= e_{d2} + L_{d11} e_{d1} \\
\delta_{q3} &= e_{d3} + L_{d22} e_{d2} \\
\delta_{q4} &= e_{d4} + L_{d33} e_{d3} \\
\delta_{vq} &= c_{q1} \delta_{q1} + c_{q2} \delta_{q2} + c_{q3} \delta_{q3} + \tilde{\delta}_{q4} \\
\delta_{k1} &= e_{d5} \\
\delta_{k2} &= e_{d6} + L_{d55} e_{d5} \\
\delta_{vk} &= c_{k1} \delta_{k1} + \delta_{k2}
\end{align*}
\]

For the DOB shown in (26), increasing the observation gain \( L_d \) can reduce the estimation error and improve the tracking accuracy. However, when the observation gain increases to a certain extent, increasing the observation gain continuously has little effect on improving the tracking accuracy of the system output. Moreover, too large DOB observation gain may affect the stability of the system. Therefore, the observation gain \( L_d \) should be weighed between disturbance estimation and control performance.

According to (65), the convergence rate of the \( V(t) \) is mainly determined by \( r_F \). The larger the parameter \( r_F \) is, the faster the convergence rate of tracking error will be. It can be seen from (65) that setting larger \( \gamma_{q1}, \gamma_{q2}, \gamma_{q3}, \gamma_{q4}, \gamma_{q5}, \gamma_{k1}, \gamma_{k2}, \gamma_v, \mu_k \) and \( \mu_k \) will lead to larger parameter \( r_F \), thus obtaining faster convergence rate of the tracking error. However, the setting of these controller parameters also requires a trade-off between the control input response performance and the tracking control performance.

The system model of the VSJ based on equivalent lever mechanism is highly nonlinear subject to saturation...
ensure that of parameter adjustment of the controller, it is necessary to the controller in the form of theory. However, in the process tracking errors and anti-disturbance characteristics. There-
gain) of each component of the controller will affect the compensator gain, feedback linearization gain, sliding mode controller with complex parameter regulation rules, and the disturbances. The designed controller is also a nonlinear constraints, the parametric uncertainties and the unknown external disturbance acting on the output link are considered in the tracking control simulation. The predefined system disturbances (i.e., the unknown parameter perturbations, the unknown friction torques, and the unknown external disturbance) are shown in Table 2. In order to demonstrate the effectiveness and robustness of the proposed FL+DOB+SMC+DEEC controller, the predefined system disturbances shown in Table 2 are set to be different in four time periods.

VI. SIMULATION
In this Section, taking the CompAct-VSA [14] as an example, considering the parametric uncertainties, unknown friction torques, unknown external disturbance and control input saturation constraints in its system dynamics model, the simulations based on the designed controller are carried out. The purpose of simulation is to show the effectiveness and robustness of the designed controller, especially the proposed estimation error compensation measures for the traditional DOB with fixed preset observation gain can effectively reduce the system output tracking errors and improve the disturbance rejection performance of the controller. The simulation software used in this Section is MATLAB.

In order to show the effectiveness of the proposed estimation error compensator in reducing the system output tracking error, the FL+DOB+SMC controller (i.e., the tracking controller based on feedback linearization (FL), disturbance observer (DOB) and sliding mode control (SMC)) is used to compare with the FL+DOB+SMC+DEEC controller (i.e., the tracking controller based on feedback linearization (FL), disturbance observer (DOB), sliding mode control (SMC) and disturbance estimation error compensation (DEEC)) designed in this article. It should be noted that the FL+DOB+SMC controller shown in (67) is only lack of estimation error compensation measures compared with the FL+DOB+SMC+DEEC controller.

\[
\begin{align*}
\dot{v}_q &= -c_q \hat{e}_q - c_{q2} \hat{e}_q - c_{q3} \hat{e}_q + \hat{d}_1 + \hat{q}_d^{(4)} - \mu_q \xi_q \\
\dot{v}_{kej} &= -c_{kej} k_{e2} + k_{e2} - \hat{d}_1 - \mu_k \xi_k 
\end{align*}
\]

(67)

For the system model of the CompAct-VSA [14], the possible model parameter perturbations, the unknown friction torques acting on the driving units and the unknown external disturbance acting on the output link are considered in the tracking control simulation. The predefined system disturbances (i.e., the unknown parameter perturbations, the unknown friction torques, and the unknown external disturbance) are shown in Table 2.

| Predefined disturbances imposed on CompAct-VSA. |
|-----------------------------------------------|
| 0–4s | 4s–10s | 10s–17s | 17s–20s |
| \( \Delta D_q \) | \( \Delta D_q \) | \( \Delta D_q \) | \( \Delta D_q \) |
| \( \Delta D_d \) | \( \Delta D_d \) | \( \Delta D_d \) | \( \Delta D_d \) |
| \( \Delta D_s \) | \( \Delta D_s \) | \( \Delta D_s \) | \( \Delta D_s \) |
| \( \Delta M \) | \( \Delta M \) | \( \Delta M \) | \( \Delta M \) |
| \( \Delta I_q \) | \( \Delta I_q \) | \( \Delta I_q \) | \( \Delta I_q \) |
| \( \Delta I_s \) | \( \Delta I_s \) | \( \Delta I_s \) | \( \Delta I_s \) |
| \( \Delta \tau_e \) | \( \Delta \tau_e \) | \( \Delta \tau_e \) | \( \Delta \tau_e \) |
| \( \tau_{ext} \) | \( \tau_{ext} \) | \( \tau_{ext} \) | \( \tau_{ext} \) |
| \( \tau_f \) | \( \tau_f \) | \( \tau_f \) | \( \tau_f \) |
| \( \tau_b \) | \( \tau_b \) | \( \tau_b \) | \( \tau_b \) |

(68)

For the system model of the CompAct-VSA [14], the possible model parameter perturbations, the unknown friction torques acting on the driving units and the unknown external disturbance acting on the output link are considered in the tracking control simulation. The predefined system disturbances (i.e., the unknown parameter perturbations, the unknown friction torques, and the unknown external disturbance) are shown in Table 2. In order to demonstrate the effectiveness and robustness of the proposed FL+DOB+SMC+DEEC controller, the predefined system disturbances shown in Table 2 are set to be different in four time periods.

A. COMPARISON AND ANALYSIS OF THE EFFECT OF ESTIMATION ERROR COMPENSATION ON TRACKING PERFORMANCE BASED ON SIMULATION SETTING 1
For the robust tracking control of the VSJ based on equivalent lever mechanism, a control scheme based on FL and DOB has been designed [22], [23]. In order to compare with the controller designed in this article, a tracking controller based on FL and the DOB with anti-windup measure is given by

\[
\begin{bmatrix}
  u_p \\
  u_y
\end{bmatrix} = G^{-1} (x) \times \begin{bmatrix}
  q_d^{(4)} + k_{p4} (q_d^{(3)} - \xi_q - \hat{d}_1) + k_{q3} (q_d^{(2)} - \hat{d}_2) \\
  + k_{q2} (q_d^{(1)} - \hat{d}_1) + k_{q1} (q_d - \xi_q) - \hat{d}_4 - L_{q}^{h1} (x) \\
  - \hat{d}_{16} - L_h^2 h_2 (x)
\end{bmatrix}
\]

(68)
is Hurwitz.

\[
\begin{align*}
\dot{s}^4 + k_q s^3 + k_\alpha s^2 + k_q s + k_d &= 0 \\
\dot{s}^2 + k_\beta s + k_d &= 0
\end{align*}
\]

The comparative simulation comparison studies between the developed FL+DOB+SMC+DEEC controller and the FL+DOB controller are carried out as illustrated in Figure 4. In the simulation, the initial system states of the CompAct-VSA are defined as \(x(0) = [0, 0, 0, 0, 1.8, 0] \) and the initial joint output stiffness of the CompAct-VSA is \( k_{ej0} = 29.7551 \) Nm/ rad. The desired trajectories for the output link angular position tracking control and the joint output stiffness tracking control is given by

\[
\begin{align*}
q_d &= 1.5 + 1.5 \sin(t) × (t > 3s) + 1 × (t > 7s) - 1.5 \sin(t) × (t > 15s) \\
k_{ejd} &= k_{ej0} + 10 + 5 × (t > 8.5s) + 2 \sin(t) × (t > 12.5s) - 2 \sin(t) × (t > 18s)
\end{align*}
\]

Note that 1.5×sin(t)(t > 3s) = 1.5×sin(t) when time \( t > 3s \). The control input constraints of the CompAct-VSA system are chosen as \( sat(u_p) ∈ [−40N \cdot m, 40N \cdot m] \) and \( sat(u_c) ∈ [−15N \cdot m, 15N \cdot m] \) in the simulation. The gain settings of the different controllers corresponding to the simulation setting 1 are shown in Table 3. The tracking performance comparisons for the simulation setting 1 are given by Table 4, and the total integration time is selected as \( T = 20s \).

As shown in Figure 4, the FL+DOB controller with setting 1, the FL+DOB controller with setting 2, the FL+DOB with setting 3 and the FL+DOB+SMC+DEEC controller are used to simulation comparisons. During the period of \( t = 4~5s, t = 10~11s, t = 17~18s \), the obvious tracking error of output link angular position can be observed, which is caused by the sudden change of predefined system disturbances. During \( t = 4~5s \) and \( t = 17~18s \), the tracking error of joint stiffness can also be observed, which is also caused by the change of predefined system disturbances. The fast attenuation of the position tracking error and the joint stiffness tracking error shows that the DOB can quickly estimate lumped disturbances and the controller has good disturbance compensation ability.

As shown in Table 3 and Table 4, compared with the system output response curves corresponding to the FL+DOB controller with setting 1, the system output response curves corresponding to the FL+DOB controller with setting 2 have smaller tracking errors by increasing the DOB gains. Moreover, compared with the FL+DOB controller with setting 2, the FL+DOB controller with setting 3 can achieve the smaller tracking errors by increasing the control gains. However, for the FL+DOB controller, although increasing the control gains and DOB gains can effectively reduce the system output tracking errors, the excessive controller gains may cause the system control input to have unexpected response performance, for example, as shown in Figure 4(c) and Figure 4(d), the control input response curves have the sharp fluctuations at about \( t = 3s \) and \( t = 15s \). As shown in Table 3, Table 4, Figure 4(a) and Figure 4(e), under the relatively small observation gain settings, the system with the FL+DOB+SMC+DEEC controller has smaller tracking errors and smaller control cost, and the control input response curves of the system is relatively smoother. The comparisons of the tracking control performances between the FL+DOB controller [22], [23] and the FL+DOB+SMC+DEEC controller show that the estimation error compensation measures not only effectively reduce the tracking errors of the system outputs, but also have no significant unexpected impact on the control input response characteristics. The simulation results show that FL+DOB+SMC+DEEC controller has better tracking control performance among these controllers.

### Table 3. Controller parameter setting corresponding to the simulation setting 1.

| Controller type | Controller gain parameter setting |
|-----------------|----------------------------------|
| FL+DOB with setting 1 | \( k_{e1} = 5625, k_{e2} = 13500, k_{e3} = 1350, k_{e4} = 60, k_{e5} = 225, k_{e6} = 30 \) |
| FL+DOB with setting 2 | \( k_{e1} = 5625, k_{e2} = 13500, k_{e3} = 1350, k_{e4} = 60, k_{e5} = 225, k_{e6} = 30 \) |
| FL+DOB with setting 3 | \( k_{e1} = 104976, k_{e2} = 23328, k_{e3} = 1944, k_{e4} = 72, k_{e5} = 625, k_{e6} = 50 \) |
| FL+DOB+SMC+DEEC | \( c_{em1} = 2197, c_{em2} = 507, c_{em3} = 39, c_{em4} = 60, \mu_{e1} = 10, \mu_{e2} = 15, \gamma_{e1} = 30, \gamma_{e2} = 60, \gamma_{e3} = 90, \gamma_{e4} = 120, \gamma_{e5} = 2, \gamma_{e6} = 5, \gamma_{e7} = 10 \) |

### B. COMPARISON AND ANALYSIS OF THE EFFECT OF ESTIMATION ERROR COMPENSATION ON TRACKING PERFORMANCE BASED ON SIMULATION SETTING 2

In order to further show the effectiveness of the designed estimation error compensation measures in improving the tracking accuracy of the system output, the simulation comparisons are shown in Figure 5 and the controller parameter settings are shown in Table 5. Compared with the FL+DOB+SMC+DEEC controller with setting 1, the FL+DOB+SMC controller with setting 1 only lack the estimated error compensators, and the gain parameters of the two controllers are the same. The control gains of the FL+DOB+SMC controller with setting 1 and the FL+DOB+SMC controller with setting 2 are the same, but the DOB gains of the FL+DOB+SMC controller with setting 2 are set to be greater than that of the FL+DOB+SMC controller with setting 1. For the FL+DOB+SMC+DEEC controller with setting 1 and the FL+DOB+SMC+DEEC controller with setting 2, their DOB gains and control gains are the same, but the estimation error compensator gains of the FL+DOB+SMC+DEEC controller with setting 2 is larger than that of the FL+DOB+SMC+DEEC controller with setting 1. In this
FIGURE 4. Comparison of response curves of the Compact-VSA system corresponding to the simulation setting 1. Note that the predefined disturbances imposed on the CompAct-VSA system dynamics model is shown in Table 2.
Section, the tracking reference trajectories are set to the same as that in simulation setting 1.

As shown in Figure 5 (a) and Table 5, the tracking errors of the system output under the action of the FL+DOB+SMC controller with setting 1 is relatively large due to the sudden change of the predefined system disturbances and the relatively small DOB observation gain setting of the controller. For the FL+DOB+SMC controller with setting 1 and the FL+DOB+SMC+DEEC controller with setting 1, although they have the same DOB gain settings, the tracking control performance of the FL+DOB+SMC+DEEC controller with setting 1 is significantly better than that of the FL+DOB+SMC controller with setting 1. This simulation comparison shows the effectiveness of the estimation error compensators in reducing the tracking errors of the system outputs and improving the anti-disturbance performance of the controller.

For the FL+DOB+SMC controller, the comparison of the tracking control performances between the FL+DOB+SMC controller with setting 1 and the FL+DOB+SMC controller with setting 2 shows that the anti-disturbance performance of the system output can be improved by increasing the DOB observation gain greatly. However, it should be noted that excessive DOB observation gain setting may cause instability of the system. Moreover, the DOB with large fixed preset observation gain will still have estimation error when the system disturbances change greatly or the reference trajectories change greatly.

For the designed FL+DOB+SMC+DEEC controller, the comparisons of the tracking performances between the FL+DOB+SMC+DEEC controller with setting 1 and the FL+DOB+SMC+DEEC controller with setting 2 show that the anti-disturbance characteristic of system output can be improved by increasing the gains of the estimation error compensators. For example, during the period of $t = 4s \sim 5.5s$ and $t = 17s \sim 17.5s$, the joint output stiffness tracking error corresponding to the FL+DOB+SMC+DEEC

| Controller type | Controller gain parameter setting |
|-----------------|-----------------------------------|
| FL+DOB+SMC with setting 1 | $L^o=\text{diag}(0, 80, 80, 80, 0, 80)$, $c_f=2197, c_f=507, c_f=39, c_f=60$, $\mu_f=10, \mu_f=10$ |
| FL+DOB+SMC with setting 2 | $L^o=\text{diag}(0, 300, 300, 300, 0, 300)$, $c_f=2197, c_f=507, c_f=39, c_f=60$, $\mu_f=10, \mu_f=10$ |
| FL+DOB+SMC+DEEC with setting 1 | $L^o=\text{diag}(0, 80, 80, 80, 0, 80)$, $c_f=2197, c_f=507, c_f=39, c_f=60$, $\mu_f=10, \mu_f=10, \gamma_f=15, \gamma_f=30, \gamma_f=60$, $\gamma_f=90, \gamma_f=120$, $\gamma_f=5, \gamma_f=10, \gamma_f=20$ |
| FL+DOB+SMC+DEEC with setting 2 | $L^o=\text{diag}(0, 80, 80, 80, 0, 80)$, $c_f=2197, c_f=507, c_f=39, c_f=60$, $\mu_f=10, \mu_f=10, \gamma_f=15, \gamma_f=30, \gamma_f=60$, $\gamma_f=90, \gamma_f=120$, $\gamma_f=10, \gamma_f=20, \gamma_f=40$ |
FIGURE 5. Comparison of response curves of the Compact-VSA corresponding to the simulation setting 2. Note that the predefined disturbances imposed on the CompAct-VSA system dynamics model is shown in Table 2.
FIGURE 6. (Continued.) Comparison of response curves of the Compact-VSA corresponding to the simulation setting 2. Note that the predefined disturbances imposed on the CompAct-VSA system dynamics model is shown in Table 2.

TABLE 6. Comparison of tracking performances of the Compact-VSA system corresponding to the simulation setting 2.

|               | FL+DOB+SMC with setting 1 | FL+DOB+SMC with setting 2 | FL+DOB+SMC+DEEC with setting 1 | FL+DOB+SMC+DEEC with setting 2 |
|---------------|----------------------------|---------------------------|-------------------------------|-------------------------------|
| IAEQ = \int_0^T \| q - q_d \| dt | 8.4916                     | 2.8632                     | 1.8438                        | 1.8404                        |
| IAKE = \int_0^T \| k_e - k_{ad} \| dt | 10.6040                    | 4.4232                     | 3.2618                        | 1.9288                        |
| SIAE = IAEQ + IAKE | 19.0956                   | 7.2864                     | 5.1055                        | 3.7692                        |
| IAUP = \int_0^T \| \text{sat}(u_p) \| dt | 440.3875                   | 291.9017                   | 273.3366                      | 273.343                       |
| IAUS = \int_0^T \| \text{sat}(u_s) \| dt | 67.2204                    | 56.3542                    | 54.9158                       | 54.9171                       |
| SIAU = IAUP + IAUS | 507.6079                  | 348.2559                   | 328.2524                      | 328.2604                      |

controller with setting 2 is smaller than that corresponding to the FL+DOB+SMC+DEEC controller with setting 1. It should be noted that the position tracking response curves corresponding to the FL+DOB+SMC+DEEC controller with setting 1 and the FL+DOB+SMC+DEEC controller with setting 2 are almost coincident due to the gain settings of the estimation error compensators. Compared with the output tracking errors corresponding to the FL+DOB+SMC controller with setting 2, the FL+DOB+SMC+DEEC controller with setting 2 achieves smaller tracking errors and less control cost with smaller DOB observation gain settings. This simulation comparison shows that the estimation error compensators designed for the DOB are effective in improving the tracking performance of the system output, and the adoption of the estimation error compensators do not have much impact on the response performance of the system control input.

C. COMPARISON AND ANALYSIS OF THE EFFECT OF ESTIMATION ERROR COMPENSATION ON TRACKING PERFORMANCE BASED ON SIMULATION SETTING 3

In order to reduce the complexity of the controller and further verify the effectiveness of the estimation error compensation measures in improving the tracking accuracy and disturbance suppression performance of the controller, the following settings are made in this simulation of this Section, i.e., the viscous friction damping coefficients (i.e., $D_q, D_p, D_s$) and the gravity effect parameter of the output link (i.e., $E_g = mgd$) are directly set to zero in the controller. Although the viscous friction damping coefficients and the output link gravity effect parameter are directly set to zero in the controller calculation, the viscous friction damping coefficients and the output link gravity effect parameter of the system dynamics model in the simulation are not zero. It should be noted that this setting will improve the calculation efficiency of the controller and the real-time performance of the tracking control, but it will reduce the tracking accuracy due to the more uncertainties of the system model parameters. As shown in Table 7, the label (ss) in the controller indicates that the controller has special simulation settings, i.e. $D_q = D_p = D_s = mgd = 0$ are set in the calculation of the controller. This means that the $L_2^2 h_1(x), L_2^2 h_2(x), G(x)$ shown in (50) and the $\xi \in \mathbb{R}^6$ shown in (18) used in the calculation of the controller no longer contain the $D_q, D_p, D_s$ and $E_g = mgd$.

In this Section, the desired tracking trajectories for the output link angular position and the joint output stiffness of
FIGURE 6. Comparison of response curves of the Compact-VSA system corresponding to the non time-varying external disturbance.

For the simulation shown in Figure 6, the parameters of the FL+DOB+SMC+DEEC (ss) controller are set to be the same as those of the FL+DOB+SMC+DEEC controller used in Section 6.1 and Section 6.2. For comparison purposes, the parameters of the FL+DOB+SMC (ss) controller are set to be the same as those of the FL+DOB+SMC+DEEC (ss) controller. In the simulation shown in Figure 6, the predefined non time-varying external disturbance, the other
FIGURE 7. Comparison of response curves of the Compact-VSA corresponding to the simulation setting 3. Note that the external disturbance imposed on the Compact-VSA is reset to time-varying disturbance, i.e. \(\tau_{\text{ext}}(t) = (2 + 2.5\sin(3t) + 1.5\sin(4t)) N \cdot m = \) during \(t = 0\sim4s,\) \(\tau_{\text{ext}}(5 + 3\sin(5t)) N \cdot m = \) during \(t = 4s\sim10s,\) \(\tau_{\text{ext}}7.5 N \cdot m = \) during \(t = 10s\sim17s\) and \(\tau_{\text{ext}}(3.5 + 2\sin(4t) + 2\sin(5t)) N \cdot m = \) during \(t = 17s\sim20s.\) The other predefined disturbances and parameter perturbations in the Compact-VSA still refer to Table 2.

Predefined unknown disturbances and unknown parameter perturbations in the Compact-VSA system can be referred to Table 2.
Comparison of response curves of the Compact-VSA corresponding to the simulation setting 3. Note that the external disturbance imposed on the Compact-VSA is reset to time-varying disturbance, i.e., \( \tau_{ext}(2 + 2.5 \sin(3t) + 1.5 \sin(4t)) \) N · m = during \( t = 0 \sim 4s \), \( \tau_{ext}(5 + 3 \sin(51)) \) N · m = during \( t = 4s \sim 10s \), \( \tau_{ext}(7.5 \) N · m = during \( t = 10s \sim 17s \) and \( \tau_{ext}(3.5 + 2 \sin(4t) + 2 \sin(51)) \) N · m = during \( t = 17s \sim 20s \). The other predefined disturbances and parameter perturbations in the Compact-VSA still refer to Table 2.

**FIGURE 7.** (Continued.) Comparison of response curves of the Compact-VSA corresponding to the simulation setting 3. Note that the external disturbance imposed on the Compact-VSA is reset to time-varying disturbance, i.e., \( \tau_{ext}(2 + 2.5 \sin(3t) + 1.5 \sin(4t)) \) N · m = during \( t = 0 \sim 4s \), \( \tau_{ext}(5 + 3 \sin(51)) \) N · m = during \( t = 4s \sim 10s \), \( \tau_{ext}(7.5 \) N · m = during \( t = 10s \sim 17s \) and \( \tau_{ext}(3.5 + 2 \sin(4t) + 2 \sin(51)) \) N · m = during \( t = 17s \sim 20s \). The other predefined disturbances and parameter perturbations in the Compact-VSA still refer to Table 2.

**TABLE 7.** Comparison of tracking performances of the Compact-VSA system corresponding to the non time-varying external disturbance (i.e., \( \tau_{ext} \)) and the special simulation settings (i.e., \( D_q = D_p = D_s = mgd = 0 \) are set in the calculation of the controller).

| FL+DOB+SMC (ss) | FL+DOB+SMC+DEEC (ss) |
|-----------------|------------------------|
| IAEQ = \( \int_0^T |q - q_d| dt \) | 8.3753 | 1.6406 |
| IAEK = \( \int_0^T |k_e - k_{ed}| dt \) | 14.3079 | 7.1603 |
| SIAE = IAEQ + IAEK | 22.6832 | 8.8069 |
| IAUP = \( \int_0^T |\text{sat}(u_p)| dt \) | 345.1722 | 162.3743 |
| IAUS = \( \int_0^T |\text{sat}(u_s)| dt \) | 68.9162 | 47.3967 |
| SIAU = IAUP + IAUS | 414.0884 | 209.7710 |

In order to further demonstrate the effectiveness of the proposed estimation error compensator in reducing the system output tracking errors and improving the anti-disturbance performance, the comparative simulation studies for the CompAct-VSA under the time-varying external disturbance and special simulation settings have also been conducted, as shown in Figure 7. The controller parameter settings in this simulation are shown in Table 8. As shown in Figure 7(a) and Table 9, the tracking error of the system output response curves corresponding to the FL+DOB+SMC (ss) controller is large because the friction damping coefficients and the gravity effect parameter in the controller are directly set to zero (i.e., \( D_q = D_p = D_s = E_{gd} = 0 \) and the time-varying external disturbance is imposed to the system. Compared with the system tracking response curves corresponding to the FL+DOB+SMC (ss) controller, the system output response curves corresponding to the

**TABLE 8.** Controller parameter setting corresponding to the time-varying external disturbance and the special simulation settings.

| Controller type | Controller gain parameter setting |
|-----------------|----------------------------------|
| FL+DOB+SMC (ss) | \( L_r = \text{diag}(0, 80, 80, 0, 80) \), \( c_v = 2197, c_p = 507, c_i = 39, c_k = 60 \), \( \mu_v = 10, \mu_p = 10 \) |
| FL+DOB+SMC+DEEC (ss) with setting 1 | \( L_r = \text{diag}(0, 80, 80, 0, 80) \), \( c_v = 2197, c_p = 507, c_i = 39, c_k = 60 \), \( \mu_v = 10, \mu_p = 10 \), \( \gamma_v = 15, \gamma_p = 30, \gamma_i = 60, \gamma_k = 90 \), \( \gamma_{v_0} = 120, \gamma_{p_0} = 2, \gamma_{i_0} = 5, \gamma_{k_0} = 10 \) |
| FL+DOB+SMC+DEEC (ss) with setting 2 | \( L_r = \text{diag}(0, 120, 120, 0, 120) \), \( c_v = 5832, c_p = 972, c_i = 54, c_k = 60 \), \( \mu_v = 10, \mu_p = 10 \), \( \gamma_v = 15, \gamma_p = 30, \gamma_i = 60, \gamma_k = 90 \), \( \gamma_{v_0} = 120, \gamma_{p_0} = 2, \gamma_{i_0} = 5, \gamma_{k_0} = 10 \) |
FL+DOB+SMC+DEEC (ss) controller with setting 1 have smaller tracking errors and smaller control effort consumptions. This comparison shows the effectiveness of the estimation error compensation measures in reducing the tracking errors of the controller and improving the anti-disturbance performance.

The simulation comparison results under the FL+DOB+SMC+DEEC (ss) controller with setting 1 and the FL+DOB+SMC+DEEC (ss) controller with setting 2 are also depicted in Figures 7(a) and Table 9. Due to the system is subjected to the time-varying external disturbance, the tracking error of the output link angular position can be observed during \( t = 4s \sim 10s \) and \( t = 17s \sim 20s \). The tracking error of the joint stiffness can also be observed during \( t = 4s \sim 5s \) and \( t = 17s \sim 18s \), which is caused by the change of predefined disturbances of the system. The comparison of the tracking control response curves of the FL+DOB+SMC+DEEC (ss) controller under different parameter settings shows that the tracking accuracy can be improved by increasing the observation gains and control gains properly. The simulation in Figure 7 shows the good robust tracking control ability of the designed controller to deal with large model uncertainties.

### VII. CONCLUSION AND FUTURE WORK

The VSJ is a kind of flexible actuator suitable for physical human-robot interaction applications such as rehabilitation training robot, exoskeleton robot, artificial limb and so on. When the VSJ is used as the actuation joint of the robot, its variable stiffness characteristics and the independent controllability of the joint position and stiffness will be conducive to improving the adaptability of the robot task and the safety of physical human-robot interaction. The VSJ with series configuration based on equivalent lever mechanism has the advantages of low energy consumption in joint stiffness adjustment, so there are many researches on this type of VSJs at present [1], [4], [7]–[9], [11]–[25], [37]. For the existing VSJs with series configuration based on the equivalent lever mechanisms [4], [11]–[16], [37], although they usually have different model parameters, elastic actuation torque functions and reaction torque functions, this type of VSJs all have the same structural type of the system dynamics model [36], as shown in equation (1). The dynamic model of the CompAct-VSA [14] developed by the Italian Institute of Technology is chosen as the study object in this article, and it is a representative VSJ in the type of VSJs with series configuration based on equivalent lever mechanism. The control scheme for the CompAct-VSA designed in this article will be suitable for the tracking control of the position and stiffness of the VSJ based on the equivalent lever mechanism. By adjusting the parameters of the controller, the designed controller will have good applicability. The control scheme proposed in this article aims to improve the tracking control accuracy of the output link angular position and joint output stiffness of the VSJ based on equivalent lever mechanism and the anti-disturbance characteristics of the tracking control. In order to improve the accuracy of robust tracking control for both position and stiffness of the VSJ based on the equivalent lever mechanism, a novel robust tracking control methods based on feedback linearization, disturbance observer with anti-windup measures, sliding mode control and the adaptive estimation error compensator are designed, and the feasibility, effectiveness and robustness of the designed controller have been verified by simulation studies. The main contributions and purposes of simulation comparison are presented as follows:

1. A novel robust tracking control scheme based on feedback linearization, disturbance observer with anti-windup measures, sliding mode control and adaptive estimation error compensator is designed to achieve the simultaneous position and stiffness tracking control of the VSA based on equivalent lever mechanism. Firstly, considering the parametric uncertainties, the unknown friction torques acting on the driving units, the unknown external disturbance acting on the output link and the control input saturation constraints that may exist in the system dynamics model of the CompAct-VSA [14], the nonlinear state space model with composite disturbances and input saturation constraints is obtained through the definitions of the state variables. Then, the nonlinear state space model is transformed into a two input-two output linear system model with lumped disturbances and input saturation constraints. Subsequently, the linear DOB with anti-windup measures is used to estimate the lumped disturbances in the
linear system model. Due to the dynamic model of the VSJ based on the equivalent lever mechanism has strong nonlinearity, it should be noted that the method of designing the nonlinear DOB to estimate the disturbance in the nonlinear state space model has the problems of difficult adjustment of gain parameters and poor disturbance estimation effect. Therefore, in this article, for the dynamic model of the VSJ based on the equivalent lever mechanism, the linear DOB with easily adjustable gain parameters is designed to estimate the lumped disturbance in the linear system model. Although the DOB with anti input saturation measures designed in this article has good disturbance estimation characteristics, it has fixed preset observation gain as most of the existing disturbance observers [31]–[35]. The DOB with fixed preset observation gain always has estimation error, especially when the reference trajectories change or the disturbances suddenly change. In order to reduce the estimation error of the DOB with fixed preset observation gain and improve the tracking accuracy and the anti-disturbance characteristics of the controller, the novel estimation error compensator for the traditional is proposed. Finally, a novel robust tracking control scheme based on feedback linearization, disturbance observer, sliding mode control and adaptive estimation error compensator is designed to improve the tracking accuracy and disturbance rejection performance in the tracking control of the VSJ based on the equivalent lever mechanism. The semi-global ultimate uniformly bounded stability of the closed-loop system is proved by the stability analysis based on the candidate Lyapunov function, and the guidances of controller parameter settings are also discussed. Comparison of simulation results show the effectiveness and robustness of the designed controller.

(2) The novel estimation error compensators are developed to estimate the estimation errors of the traditional DOB with fixed preset observation gains. The proposed estimation error compensator has the same structure as the DOB with single input and single output (SISO) [34], but the control input of the designed estimation error compensator contains the tracking error informations. Each disturbance estimation error compensator has only one gain parameter, which is easy to adjust. The estimation values provided by the estimation error compensators are added to the controller to improve the tracking control accuracy and disturbance suppression performance of the controller. The design idea of the estimation error compensator in this article still comes from the DOB, which shows that the SISO disturbance observer has good flexibility and extensibility in application. As far as our knowledge goes, the proposed disturbance estimation error compensator in this article is the first study to estimate the estimation error of traditional DOB based on the design idea of SISO disturbance observer. The simulation results show that the tracking accuracy, tracking speed and anti-disturbance characteristics of the controller are improved by combining the designed estimation error compensator, and the introduction of this measure does not make the response characteristics of the control input worse. The estimation error compensator designed in this article can be extended to the robust tracking control of other systems to improve the tracking accuracy, such as hypersonic vehicles [33], spacecraft [34], MAGLEV suspension system [35] and the traditional multi-DOF robot arm based on the rigid joint (i.e., motor+reducer+output link) etc.

The comprehensive simulation comparison results show the effectiveness and robustness of the designed controller, and the designed estimation error compensation measures significantly improve the tracking accuracy and anti-disturbance characteristics of the controller.

For the future work, firstly, a compact and modular VSJ experimental platform based on the joint stiffness adjustment principle of the CompAct-VSA should be designed. Then, the tracking control performance of the designed controller can be verified by real experiments, which provides a practical and effective robust control scheme for the tracking control of the VSJ based on the equivalent lever mechanism.

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