On Dependent Elements of Semiprime Rings

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Abstract
In this paper we study and investigate concerning dependent elements of semiprime rings and prime rings $R$ by using generalized derivation and derivation, when $R$ admits to satisfy some conditions, we give some results about that.

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1. Introduction and preliminaries
This research has been motivated by the work of J.Vukman and I.Kosi-Ulbl[16]. Some researchers have studied the notion of free action on operator algebras, Murray and von Neumann [13] and von Neumann [14] introduced the notion of free action on abelian von Neumann algebras and used it for the construction of certain factors (see M.A.Chaudhry and M.S.Samman[5], F.Ali and M.A.Chaudhry[2] and Dixmier[8]). Kallman[11] generalized the notion of free action of automorphisms of von Neumann algebras, not necessarily abelian, by using implicitly the dependent elements of an automorphism. Choda, Kashahara and Nakamoto [6] generalized the concept of freely acting automorphisms to $C^*$-algebras by introducing dependent elements associated to automorphisms, where $C^*$-algebra is a Banach algebra with an anti-automorphic involution $^*$ which satisfies (i) $(x^*)^*=x$, (ii) $x^*y^*=(yx)^*$, (iii) $x^*y^*=(x+y)^*$ , (iv) $(cx)^*=\overline{c}x^*$ where $\overline{c}$ is the complex conjugate of $c$, and whose norm satisfies $\|y\|\|xx^*\|\leq\|x\|^2$. Several other authors have studied dependent elements on operator algebras. A brief account of dependent elements in $W^*$-algebras has also appeared in the book of Stratila [15]. It is well-known that all $C^*$-algebras and von Neumann algebras are semiprime rings; in particular, a von Neumann algebra is prime if and only if its center consists of scalar multiples of identity. Thus a natural extension of the notions of dependent elements of mappings and free actions on $C^*$-algebras and von Neumann algebras is the study of these notions in the context of semiprime rings and prime rings. Laradji and Thaheem [12] initiated a study of dependent elements of endomorphisms of semiprime rings and generalized a number of results of H.Choda, I.Kasahara, R.Nakamoto[6] to semiprime rings. Vukman and Kosi-Ulbl[16] and Vukman [17] have made further study of dependent elements of various mappings related to automorphisms, derivations, $(\alpha, \beta)$-derivations and generalized derivations of semiprime rings. The main focus of the authors of J.Vukman, I.Kosi-Ulbl [16]
and [17] has been to identify various freely acting mappings related to these mappings, on semiprime and prime rings. The theory of centralizers (also called multipliers) of C*-algebras and Banach algebras is well established (see C.A. Akemann, G.K. Pedersen, J. Tomiyama [1] and P. Ara, M. Mathieu [3]). Zalar [19] and Vukman and Kosi-Ubl [18] have studied centralizers in the general framework of semiprime rings. Throughout, R will stand for associative ring with center Z(R). As usual, the commutator xy−yx will be denoted by [x, y]. We shall use the basic commutator identities [xy, z] = [x, z]y + x[y, z] and [x, yz] = [x, y]z + y[x, z]. A ring R is said to be n-torsion free, where n ≠ 0 is an integer, if whenever nx = 0, with x ∈ R, then x = 0. Recall that a ring R is prime if aRb = (0) implies that either a = 0 or b = 0, and is semiprime if aRa = (0) implies a = 0. A prime ring is semiprime but the converse is not true in general. An additive mapping d: R → R is called a derivation provided d(xy) = d(x)y + xd(y) holds for all pairs x, y ∈ R. An additive mapping d: R → R is called centralizing (commuting) if [d(x), x] ∈ Z(R) ([d(x), x] = 0) for all x ∈ R. By Zalar [19], an additive mapping T: R → R is called a left (right) centralizer if T(xy) = T(x)y (T(xy) = xT(y)) for all x, y ∈ R. If a ∈ R, then La(x) = ax and Ra(x) = xa (x ∈ R) define a left centralizer and a right centralizer of R, respectively. An additive mapping T: R → R is called a centralizer if T(xy) = T(x)y = xT(y) for all x, y ∈ R. Let β be an automorphism of a ring R. An additive mapping d: R → R is called an β-derivation if d(xy) = d(x)y + β(x)d(y) holds for all x, y ∈ R. Note that the mapping, d = I, where I denotes the identity mapping on R, is an β-derivation. Of course, the concept of an β-derivation generalizes the concept of a derivation, since any I-derivation is a derivation. β-derivations are further generalized as (α, β)-derivations. Let α, β be automorphisms of R, then an additive mapping d: R → R is called an (α, β)-derivation if d(xy) = d(x)α(y) + β(x)d(y) holds for all pairs x, y ∈ R. β-derivations and (α, β)-derivations have been applied in various situations, in particular, in the solution of some functional equations. An additive mapping T of a ring R into itself is called a generalized derivation, with the associated derivation d, if there exists a derivation d of R such that T(xy) = T(x)y + xd(y) for all x, y ∈ R. The concept of a generalized derivation covers both the concepts of a derivation and of a left centralizer provided T = d and d = 0, respectively (see B. Hvala [10]). Following A. Laradji, A. B. Thaheem [12], an element a ∈ R is called a dependent element of a mapping T: R → R if T(x)a = ax holds for all x ∈ R. A mapping T: R → R is called a free action or (act freely) on R if zero is the only dependent element of T. It is shown in [12] that in a semiprime ring R there are no non zero nilpotent dependent elements of a mapping T: R → R. For a mapping T: R → R, D(F) denotes the collection of all dependent elements of F. For other ring theoretic notions used but not defined here we refer the reader to I. N. Herstein [9]. In this paper we study and investigate a a dependent elements on a semiprime ring and prime ring R, we give some results about that. We will use the following result in the sequel.

**Lemma 1 [4, Lemma 4]**

Let R be a 2-torsion free semiprime ring and let a, b ∈ R. If for all x ∈ R, the relation axb + bxa = 0 holds, then axb = bxa = 0 is fulfilled for all x ∈ R.

**2. The main results**

**Theorem 2.1**

Let R be a semiprime ring and let D and G be derivations of R into itself, then the mapping x → D(x) + G²(x) for all x ∈ R is a free action.

**Proof:** We have

F(x) = ax for all x ∈ R.

Where F(x) stands for D(x) + G²(x) (1)

and [17] has been to identify various freely acting mappings related to these mappings, on semiprime and prime rings. The theory of centralizers (also called multipliers) of C*-algebras and Banach algebras is well established (see C.A. Akemann, G.K. Pedersen, J. Tomiyama [1] and P. Ara, M. Mathieu [3]). Zalar [19] and Vukman and Kosi-Ubl [18] have studied centralizers in the general framework of semiprime rings. Throughout, R will stand for associative ring with center Z(R). As usual, the commutator xy−yx will be denoted by [x, y]. We shall use the basic commutator identities [xy, z] = x[y, z] + [x, z]y and [x, yz] = [x, y]z + y[x, z]. A ring R is said to be n-torsion free, where n ≠ 0 is an integer, if whenever nx = 0, with x ∈ R, then x = 0. Recall that a ring R is prime if aRb = (0) implies that either a = 0 or b = 0, and is semiprime if aRa = (0) implies a = 0. A prime ring is semiprime but the converse is not true in general. An additive mapping d: R → R is called a derivation provided d(xy) = d(x)y + xd(y) holds for all pairs x, y ∈ R. An additive mapping d: R → R is called centralizing (commuting) if [d(x), x] ∈ Z(R) ([d(x), x] = 0) for all x ∈ R. By Zalar [19], an additive mapping T: R → R is called a left (right) centralizer if T(xy) = T(x)y (T(xy) = xT(y)) for all x, y ∈ R. If a ∈ R, then La(x) = ax and Ra(x) = xa (x ∈ R) define a left centralizer and a right centralizer of R, respectively. An additive mapping T: R → R is called a centralizer if T(xy) = T(x)y = xT(y) for all x, y ∈ R. Let β be an automorphism of a ring R. An additive mapping d: R → R is called a β-derivation if d(xy) = d(x)y + β(x)d(y) holds for all x, y ∈ R. Note that the mapping, d = I, where I denotes the identity mapping on R, is an β-derivation. Of course, the concept of a β-derivation generalizes the concept of a derivation, since any I-derivation is a derivation. β-derivations are further generalized as (α, β)-derivations. Let α, β be automorphisms of R, then an additive mapping d: R → R is called an (α, β)-derivation if d(xy) = d(x)α(y) + β(x)d(y) holds for all pairs x, y ∈ R. β-derivations and (α, β)-derivations have been applied in various situations, in particular, in the solution of some functional equations. An additive mapping T of a ring R into itself is called a generalized derivation, with the associated derivation d, if there exists a derivation d of R such that T(xy) = T(x)y + xd(y) for all x, y ∈ R. The concept of a generalized derivation covers both the concepts of a derivation and of a left centralizer provided T = d and d = 0, respectively (see B. Hvala [10]). Following A. Laradji, A. B. Thaheem [12], an element a ∈ R is called a dependent element of a mapping T: R → R if T(x)a = ax holds for all x ∈ R. A mapping T: R → R is called a free action or (act freely) on R if zero is the only dependent element of T. It is shown in [12] that in a semiprime ring R there are no non zero nilpotent dependent elements of a mapping T: R → R. For a mapping T: R → R, D(F) denotes the collection of all dependent elements of F. For other ring theoretic notions used but not defined here we refer the reader to I. N. Herstein [9]. In this paper we study and investigate a a dependent elements on a semiprime ring and prime ring R, we give some results about that. We will use the following result in the sequel.

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**Proof:** We have

F(x) = ax for all x ∈ R.

Where F(x) stands for D(x) + G²(x) (1)
Replacing \( x \) by \( xy \) with some routine calculation, we obtain
\[
F(xy) = F(x)y + xF(y) + 2D(x)D(y) \quad \text{for all } x, y \in R. \tag{2}
\]
In (1) putting \( xa \) for \( x \) with using (2), we get
\[
F(x)a^2 + xF(a)a + 2D(x)D(a)a = axa \quad \text{for all } x \in R. \tag{3}
\]
According to (1), we reduced (3) to
\[
2D(x)D(a)a + xa^2 + xa^2 = 0 \quad \text{for all } x \in R. \tag{4}
\]
Replacing \( x \) by \( y \) in (4) with using (4), we obtain
\[
2D(y)xD(a)a = 0 \quad \text{for all } x, y \in R. \tag{5}
\]
Left–multiplying (4) by \( D(y) \) and applying (5), we obtain
\[
D(y)xa^2 = 0 \quad \text{for all } x, y \in R.
\]
Replacing \( y \) by \( D(a) \) and \( y \) by \( a \), we get
\[
D(a)^3 a^2 = 0. \tag{6}
\]
Right–multiplying (4) by \( a \) with replacing \( x \) by \( a \) and using (6), we obtain
\[
a^4 = 0. \quad \text{which means that also } a = 0. \quad \text{Thus our mapping is free action.}
\]

**Theorem 2.2**

Let \( R \) be a prime ring, \( \psi : R \to R \) be a generalized derivation and \( a \in R \) be an element dependent on \( \psi \), then either \( a \in Z(R) \) or \( \psi(x) = x \) for all \( x \in R \).

**Proof:** We have the relation
\[
\Psi(x)a = ax \quad \text{for all } x \in R. \tag{7}
\]
Replacing \( x \) by \( xy \) in (7), we obtain
\[
(\psi(y)x + xd(y))a = axy \quad \text{for all } x, y \in R. \tag{8}
\]
According to the fact that \( \Psi \) can be written in form \( \Psi = d + T \), where \( T \) is a left centralizer, replacing \( d(y)a \) by \( \psi(y)a - T(y)a \) in (8), which gives according to (7).
\[
\Psi(x)ya + [x,a]y - xT(y)a = 0 \quad \text{for all } x, y \in R. \tag{9}
\]
Replacing \( y \) by \( \psi(n) \) in (9), we obtain
\[
\Psi(x)y\psi(x)a + [x,a]y\psi(x) - xT(y)\psi(x)a = 0 \quad \text{for all } x, y \in R. \tag{10}
\]
Again since \( T \) is left centralizer, then (10) become
\[
\psi(x)y\psi(x)(a + [x,a]y\psi(x) - xT(y)\psi(x)a = 0 \quad \text{for all } x, y \in R. \tag{11}
\]
According to (7), (11) reduces to
\[
\psi(x)yax + [x,a]y\psi(x) - xT(y)ax = 0 \quad \text{for all } x, y \in R. \tag{12}
\]
Right–multiplying (9) by \( x \) gives
\[
\psi(x)yax + [x,a]yT(y)ax = 0 \quad \text{for all } x \in R. \tag{13}
\]
Subtracting (12) and (13), we obtain
\[
[x,a]y(\psi(x) - x) = 0 \quad \text{for all } x, y \in R. \quad \text{Since } R \text{ is prime ring, we obtain either } [x,a] = 0 \quad \text{for all } x \in R, \text{which leads to } a \in Z(R) \quad \text{or } \psi(x) = x \quad \text{for all } x \in R.
\]

**Proposition 2.3**

Let \( R \) be a 2-torsion semiprime ring and let \( a, b \in R \) be fixed elements. Suppose that \( c \in R \) is an element dependent on the mapping \( x \to xa + bx \), then \( ac = ca \).

**Proof:** We will assume that \( a \neq 0 \) since there is nothing to prove in case \( a = 0 \) and \( b = 0 \), we have
\[
(xa + bx)c = cx \quad \text{for all } x \in R. \tag{14}
\]
Replacing \( x \) by \( xy \), we obtain
\[
(xya + bxy)c = cxy \quad \text{for all } x, y \in R. \tag{15}
\]
According to (14) then (15) reduces to
\[
(xya + bxy)c = (xa + bx)cy \quad \text{for all } x, y \in R. \quad \text{Then}
\]
x(yac – acy) + bx(yc – cy) = 0 \quad \text{for all } x, y \in R. \quad \text{Then}
x[a, c] + [x, a]c + bx[y, c] = 0 \quad \text{for all } x, y \in R.
Replacing \( y \) by \( c \), we get
\[
x[c, a]c = 0 \quad \text{for all } x \in R. \quad \text{Then}
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R[c, a]c = 0 Since R is semiprime, we get
[c, a]c = 0. Then
\[ [c, a][c, r]_c + [c, a]r_c = 0 \] for all \( r \in R \).
\[ [c, a][c, r]_c + [c, a]r_c = 0 \] for all \( r \in R \).
Right-multiplying (16) by \( r \), we obtain
\[ [c, a]r_c = 0 \] for all \( r \in R \).
Subtracting (17) and (18), we get
\[ [c, a][c, r]_c + [c, a]r_c = 0 \] for all \( r \in R \).
Since \( R \) is 2-torsion free with replacing \( r \) by \( ra \), we obtain
\[ [c, a]r[c, a] = 0 \] for all \( r \in R \).
Then
\[ [c, a]R[c, a] = 0 \]. Since \( R \) s semiprime ring, then \( ca = ac \). The proof of the theorem is complete.

**Theorem 2.4**

Let \( R \) be a prime ring and let \( a, b \in R \) be fixed elements. Suppose that \( c \in R \) is an element dependent on the mapping \( x \to axb \), then \( ac \in Z(R) \) or \( bc \in Z(R) \).

**Proof:** We will assume that \( a \neq 0 \) and \( b \neq 0 \), since there is nothing to prove in case \( a = 0 \) or \( b = 0 \). We have
\[ (axb)c = cx \] for all \( x \in R \).

Let \( x = xy \) in (19), we obtain
\[ (axyb)c = cxy \] for all \( x, y \in R \).

According to (19) one can replace \( cx \) by \( (axb) \) in (20), we get
\[ ax[bc, y] = 0 \] for all \( x, y \in R \).
Replacing \( x \) by \( cy \) in the above relation, then we have
\[ acyx[bc, y] = 0 \] for all \( x, y \in R \).

Again in (21) replacing \( x \) by \( cx \) with left-multiplying by \( y \), we get
\[ yacx[bc, y] = 0 \] for all \( x, y \in R \).

Subtracting (22) and (23), we obtain
\[ [ac, y][bc, y] = 0 \] for all \( x, y \in R \).
Then
\[ [ac, y]R[bc, y] = 0 \]. Since \( R \) is prime, we get.
either \( ac \in Z(R) \) or \( bc \in Z(R) \), the proof of the theorem is complete.

**Theorem 2.5**

Let \( R \) be a noncommutative 2-torsion free semiprime ring with cancellation property and \( a, b \in R \) be fixed elements. Suppose that \( c \in Z(R) \), is an element dependent on the mapping \( x \to axb + bxa \) then \( a \in Z(R) \).

**Proof:** Similarly, in Theorem 2.4, we will assume that \( a \neq 0 \) and \( b \neq 0 \). We have the relation
\[ (axb+bxa)c = cx \] for all \( x \in R \).
Replacing \( x \) by \( xy \) in (24), we get
\[ (axyb+bxya)c = cxy \] for all \( x, y \in R \).
Right-multiplying (24) by \( y \), we get
\[ (axb+bxa)c = cxy \] for all \( x, y \in R \).
Subtracting (24) from (25), we obtain
\[ ax[y, bc] + bx[y, ac] = 0 \] for all \( x, y \in R \).
Replacing \( x \) by \( cx \) in above relation, we get
\[ ax[y, bc] + bx[y, ac] = 0 \] for all \( x, y \in R \).
Left-multiplying by \( y \) with replacing, by \( yx \), we obtain
\[ yacx[y, bc] + ybcx[y, ac] = 0 \] for all \( x, y \in R \).
Subtracting (29) and (28), we get
\[ [y, ac]x[y, bc] + [y, bc]x[y, ac] = 0 \] for all \( x, y \in R \).

Suppose that \( ac \) non belong to \( Z(R) \), we have \( [y, ac] \neq 0 \) for some \( y \in R \).
Then from (30) with Lemma 1, we obtain \( [y, ac] = 0 \), thus (27), reduces to \( bx[y, ac] = 0 \) for all \( x, y \in R \), by using the cancellation property on \( b \), we obtain that \( [y, ac] = 0 \), contrary to assumption. We have, therefore, \( ac \in Z(R) \).
According to (27), we get $ax \cdot [y, bc] = 0$ for all $x, y \in R$, whence it follows that $bc \in Z(R)$, now we have $ac \in Z(R)$ and $bc \in Z(R)$, therefore, according to (24), we obtain
\[(ab + ba)c = c\] (31)
Right-multiplying (31) by $((ab + ba)c - c)$ with using $R$ is semiprime, we get.
\[(ab + ba)c = c\] (32)
Then $((ab + ba)c, r) = [c, r]$.

\[(ab + ba)[c, r] + [(ab + ba), r]c = [c, r]\]
for all $r \in R$.
Replacing $r$ by $c$, above relation reduces to
\[[ab + ba], c]\]
for all $c \in R$. By using the cancellation property on $[(ab + bc), c]$, we obtain $c \in Z(R)$.
The proof of the theorem is complete.

**Theorem 2.6**

Let $R$ be a noncommutative semiprime ring with extended centroid $C$ and cancellation property, let $a, b \in R$ be fixed elements the mapping $x \rightarrow axb - bxa$ is a free action.

**Proof:** We assume that $a \neq 0$ and $b \neq 0$, with that $a$ and $b$ are $C$, independent, otherwise, the mapping $x \rightarrow axb - bxa$ would be zero. Then, we have the following relation.
\[(ab - bxa)c = cx\] (33)
Replacing $x$ by $xy$ in the above relation, we obtain
\[(axb - bxya)c = cxy\] (34)
Right-multiplication of (33) by $y$, we get
\[(axb - bxa)c = cxy\] (35)
Subtracting (34) and (35), we obtain
\[ax[y, bc] - bx[y, ac] = 0\] (36)
Replacing $x$ by $cx$, we get
\[acx[y, bc] - bcx[y, ac] = 0\] (37)
Left-multiplying (37) by $y$, we get
\[yacx[y, bc] - ybcx[y, ac] = 0\] (38)
In (37) replacing $x$ by $yx$, we obtain
\[acyx[y, bc] - bcyx[y, ac] = 0\] (39)
Subtracting (39) and (38), we obtain
\[[y, bc] = \lambda y[y, ac]\] (40)
Holds for some $\lambda \in C$. According to (40) one can replace $[y, bc]$ by $\lambda y[y, ac]$ in (36), we obtain
\[(b - \lambda y)x[y, ac] = 0\] for all $x, y \in R$.
Replacing $x$ by $cx$, we obtain
\[(b - \lambda y)cxc[y, ac] = 0\] (41)
Using the cancellation property on $[y, ac]$ in (41), we obtain
\[(b - \lambda y)cxc = 0\] (42)
Again using the cancellation property on $(b - \lambda y)$ in (42) with using $R$ is semiprime, we obtain $c = 0$, which completes the proof of the theorem.

**Proposition 2.7**

Let $R$ be a prime ring, $\sigma$ and $\beta$ be automorphisms of $R$, then the mapping $\sigma + \beta$ is free action or $\sigma = \beta$.

**Proof:** We have the relation
\[(\sigma(x) + \beta(x))a = ax\] for all $x \in R$. (43)
Let $x$ be $xy$ in the above relation, we obtain
\[(\sigma(x)\sigma(y) + \beta(x)\beta(y))a = axy\] for all $x, y \in R$. (44)
According to (43) above relation (44) gives
\[(\sigma(x) + \beta(x))\beta(y)a = (\sigma(x) + \beta(x))axy\] for all $x, y \in R$. (45)
Again according to (43) a bobe relation (45) gives
(σ(x)σ(y)+β(x)β(y))a=(σ(x)+β(x)(σ(y)+β(y)))a for all x,y∈R.

Which reduce to

σ(x)β(y)a+β(x)σ(y)a=0 for all x,y∈R. (46)

Replacing x by xz in (46) we obtain

σ(z)σ(x)β(y)a+β(z)β(x)σ(y)a=0 for all x,y,z∈R. (47)

Left-multiplying (46) by σ(z) gives

σ(z)σ(x)β(y)a+σ(z)β(x)σ(y)a=0 for all x,y,z∈R. (48)

Subtracting (47) from (48), we obtain

(σ(z)–β(z))(β(x)σ(y)a)=0 for all x,y,z∈R. We have

(σ(z)–β(z))(xya)=0 for all x,y,z∈R. Then

σ(z)–β(z)Rya=0. Since R is prime ring. Then

either σ(z)–β(z)=0 for all z∈R, which implies

σ=β or ya=0 for all y∈R. By the primeness of R, we obtain

a=0, which completes the proof of the theorem.

**Theorem 2.8**

Let R be a prime ring and let ψ:R→R be a non-zero (σ,β)-derivation, then ψ is a free action.

**Proof:** We have the relation ψ(x)a=ax for all x∈R. (49)

Replacing x by xy, we obtain

Ψ(x)σ(y)a+β(x)ψ(y)a=axy for all x,y∈R.

According to (49) one can replace ψ(y)a by ay above relation, which gives

Ψ(x)σ(y)a+(β(x)φa–ax)ya=0 for all x,y,z∈R. (50)

Replacing y by yz in (50) we obtain

Ψ(x)σ(y)σ(z)a+β(x)a–ax)yz=0 for all x,y,z∈R. (51)

Right-multiplying (50) by z, we get

Ψ(x)σ(y)(az)+(β(x)a–ax)yz=0 for all x,y,z∈R. (52)

Subtracting (52) from (51), we get

Ψ(x)σ(y)(σ(z)a–az)=0 for all x,y,z∈R. In other words, we have

Ψ(x)σ(y)(σ(z)a–az)=0 for all x,y,z∈R. Then

Ψ(x)R(σ(z)a–az)=0. Since R is prime and ψ is non-zero, we obtain

σ(z)a=az for all z∈R. (53)

Since σ is automorphism of R, then by other words from (53) we have

za=az for all z∈R. (54)

Also, since B is automorphism of R, then from (50), we obtain

Ψ(x)σ(y)a+(x–a)ax)=0 for all x,y,z∈R. (55)

Apply (54) in above relation, we obtain

Ψ(x)σ(y)a=0 for all x,y∈R. By other words we have

Ψ(x)ya=0 for all x,y∈R, then

Ψ(x)Rα=0. By the primeness of R and ψ is non-zero of R, we obtain

a=0, the proof of the theorem is complete.

**Corollary 2.9**

Let R be a prime ring and let σ and β be automorphisms of R. the mappings σ–β and aβ–βa, where a∈R is a fixed element, are free actions on R.

**Proof:** According to Theorem 2.8, there is nothing to prove, since the mappings σ–β and aβ–βa are (σ,σ)-derivations.

**Corollary 2.10**

Let R be a prime ring, let ψ: R→R be a non-zero derivation, and let σ be an automorphism of R, the mapping x→ψ(σ(x)), x→σ(ψ(x)), x→ψ(σ(x)+σ(ψ(x))) and x→ψ(σ(x)–σ(ψ(x))) are free actions.

**Proof:** A special case of Theorem 2.8, since all mappings are (σ,σ)-derivations.
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