Stability of strange quark matter: model dependence

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Abstract

The minimum energy per baryon number of strange quark matter is studied, as a function of the strangeness fraction, in the MIT bag model and in two different versions of the Color Dielectric Model: a comparison is made with the hyperon masses having the same strangeness fraction, and coherently calculated within both models. Calculations are carried out in mean field approximation, with one gluon exchange corrections. The results allow to discuss the model dependence of the stability of strangelets: they can be stable in the MIT bag model and in the double minimum version of the Color Dielectric Model, while the single minimum version of the Color Dielectric Model excludes this possibility.

1 Introduction

The production of strange quark matter and/or hypermatter in central heavy ion collisions has been suggested long ago, either in the form of multi-hypernuclear objects (strange hadronic matter) \( \bar{s}sN \), or strangelets (strange multiquark droplets) \( \bar{s}s \bar{s}s \). The formation of the latter would be rather appealing, since it would be an unambiguous signature that a deconfined, strangeness rich state of quark gluon plasma has been created during the reaction. The investigation of the strangelet stability is therefore of primary importance for their detection in heavy ion experiments.

The idea is that, even if no strangeness is present in the initial state of the collision, and no net strangeness is expected after the reaction, nevertheless a large number of \( ss \) pairs can be produced in a single central event; the antiquarks \( \bar{s} \) are
then able to rapidly combine with the abundantly available $u$ and $d$ quarks to form antikaons that immediately leave the fireball region, which becomes strangeness rich matter. The hadronization process is then of fundamental importance: the copious formation of strange particles cannot be considered as a reliable signature of QGP formation, since kaons and hyperons can be produced in hadronic reactions as well [9]. If, on the contrary, after the formation of the deconfined plasma, this strangeness rich matter could coalesce into colorless multiquark states, the so-called strangelets, this would be an unambiguous signature of QGP formation; this process might be favoured by a rapid QGP cooling due to the prompt anti-kaon (and also pion) emission from the surface of the fireball.

Up to now, the stability of strangelets has been only investigated within the MIT bag model [10], also including $O(\alpha_s)$ corrections to the properties of bulk strange matter: according to this pioneering work, heavy, slightly positively charged, strangelets could be more stable than ordinary nuclei. A detailed calculation of strangelet properties within the MIT bag model, including shell effects and all the hadronic decay channels has been performed by J. Schaffner et al. [11]: a valley of stability clearly appears for $A_B = 5 \div 16$ with charge fraction $Z/A$ between 0 and $-0.5$. On the other hand, strangelets having a larger mass should be positively charged according to the results of Ref. [12].

In ref. [13] the authors confront the predictions about the stability of strangelets within the MIT bag model and the Color Dielectric Model (CDM): the equilibrium energy of the strange matter is compared with the masses of hyperons having the same strangeness fraction, and coherently calculated within both models. The main goal is to find out whether and to which extent the stability of strange matter and/or strangelets depends on the model employed to describe the confined system of quarks. The present contribution is largely based on the results of ref. [13], with a special focus on the model dependence of the strangelets stability. We consider homogeneous quark matter made up of $u$, $d$ and $s$ quarks, without imposing chemical equilibrium on the density of the strange quarks. Rather, we assume that there exists in the system a definite strange fraction $R_s = \rho_s/\rho$, $\rho$ being the total baryon density of quarks and $\rho_s$ the baryonic density of strange quarks. This is coherent with the hypothesis that, during a high energy collision between heavy ions, this state of matter, if formed at all, can only survive for a very short time, so that it has no time to reach $\beta$ equilibrium; hence the minimal energy per baryon number can be studied as a function of the strange fraction $R_s$. We also consider the effect induced by the introduction of perturbative gluons in both models. Since electromagnetic interaction has been neglected, the minimum of the energy corresponds to an equal number of $u$ and $d$ quarks.

We consider, for simplicity, an infinite and homogeneous system, but strangelets are indeed finite objects, and therefore one should remember that the energy of the infinite system appears to be a lower limit with respect to the envelop of strangelet
energies versus strangeness fraction: the latter was nicely illustrated by Schaffner et al. [11] calculating the strangelet masses within the MIT bag model with shell mode filling. We simply recall that surface effects, which we do not consider, would increase the energy curves of bulk matter, typically of 50-100 MeV: hence, if hyperons should turn out to be more stable than strange matter, then this would exclude also the stability of strangelets. If, on the contrary, strange matter is more stable, then this provides only an indication in favour of stable strangelets unless the mass gap between the two states is large enough.

2 Strangelets in the MIT bag model

2.1 MIT bag without gluons

We consider first the simplest version of the MIT bag, not including one gluon exchange corrections; therefore the model has only two parameters: the vacuum pressure $B$ and the strange quark mass $m_s$. In order to discuss the various possible scenarios, we have used a wide range for both parameters:

$$\begin{align*}
B &= 60, 100, 150 \text{ MeV/fm}^3; \\
m_s &= 100, 200, 300 \text{ MeV}.
\end{align*}$$

The single flavor contribution to the energy density of the system is given by:

$$\epsilon_f = 6 \int \frac{d\vec{k}}{(2\pi)^3} E_f(k) \theta(k_{F_f} - k),$$

where $E_f(k) = \sqrt{k^2 + m_f^2}$ and $k_{F_f}$ is the Fermi momentum of flavor $f$. It can be analytically expressed, for $u, d$ (massless) and $s$ quarks, respectively:

$$\begin{align*}
\epsilon_{u,d} &= \frac{3}{(2\pi)^2} k_{F_{u,d}}^4, \\
\epsilon_s &= \frac{3}{8\pi^2} \left[ m_s^4 \ln \left( \frac{m_s}{k_{F_s} + \sqrt{k_{F_s}^2 + m_s^2}} \right) + k_{F_s} \sqrt{k_{F_s}^2 + m_s^2} \left( 2k_{F_s}^2 + m_s^2 \right) \right].
\end{align*}$$

The total energy density of our system turns then out to be:

$$\epsilon_{tot} = 2\epsilon_{u,d} + \epsilon_s + B.$$
\[ k_{F_s} = \left( 3\pi^2 \rho_s \right)^{1/3} \]
\[ k_{F_{u,d}} = \left( \frac{3\pi^2}{2} \rho (1 - R_s) \right)^{1/3}, \]

In the above, \( \rho \) is the total baryon number density in the system \((\rho = A_B/V)\), and the color degeneracy and baryon number \(1/3\) of the quarks have been taken into account. From the above formulas we calculate the energy per baryon number to be:

\[ \frac{E_{\text{tot}}}{A_B} = \frac{\epsilon_{\text{tot}}}{\rho}. \] (6)

In Fig. 1 the results of the minimal energy per baryon \((\text{E})\) corresponding to \(B = 60, 100, 150 \text{ MeV/fm}^3\) are shown as a function of \(R_s\). For each value of \(B\) we explore three different values of the strange mass, \(m_s = 100, 200, 300 \text{ MeV}\), and we compare these results with the experimental nucleon and hyperon masses (full circles). We have also evaluated, according to formula (3.6) of Ref. [14], the baryonic masses which are obtained within the same model employed for bulk strange matter, using the same sets of bag parameter and strange quark mass. As it appears from the figure, the three lines corresponding to the different values of \(m_s\) are much lower than the experimental hyperon masses for \(B = 60 \text{ MeV/fm}^3\) and \(B = 100 \text{ MeV/fm}^3\), while this is not the case for \(B = 150 \text{ MeV/fm}^3\) and \(m_s = 300 \text{ MeV}\); however, if we compare the energy of strange matter with the corresponding theoretical masses of the various hyperons, we find that strange matter is always lower in energy, and thus more stable. We can therefore conclude that the MIT bag model without perturbative gluon corrections allows the existence of strangelets.

### 2.2 MIT bag model with perturbative gluons

We consider now the effects of introducing in the calculation perturbative corrections due to the exchange of gluons. At first order in \(\alpha_s\), two contributions to the energy can be considered, the direct and the exchange one. Since the system is globally colorless the direct term vanishes, while the exchange one gives the following contribution to the energy density of quarks of flavor \(f\) \([10]\):

\[ \epsilon_f^{\text{OGE}} = -\frac{\alpha_s}{\pi^2} m_f^4 \left\{ x_f^4 - \frac{3}{2} \ln \left( \frac{x_f + \eta_f}{\eta_f} \right) - x_f \eta_f \right\}^2 + \frac{3}{2} \ln^2 \left( \frac{1}{\eta_f} \right) - 3 \ln \left( \frac{\mu}{m_f \eta_f} \right) \left[ \eta_f x_f - \ln \left( x_f + \eta_f \right) \right] \]. (7)

Here:

\[ x_f = \frac{k_{F_f}}{m_f} \]
and $\mu$ is a renormalization scale, for which we choose the value $\mu = 313$ MeV, according to Ref. [10]. For sake of illustration, we adopted two different values for $\alpha_s$, a small perturbative value ($\alpha_s = 0.5$), which is in line with the choices and motivations of Fahri and Jaffe [10], and the canonical value which was employed by DeGrand et al. [14] ($\alpha_s = 2.2$), to reproduce the hyperon masses. The corresponding results are illustrated in Figs. 2 and 3. From Fig. 2 we can see that, even after the inclusion of perturbative gluons, strangelets are more stable than hyperons for almost all values of the model parameters. However, when we use the stronger coupling of Fig. 3 the stability of strange matter (and hence strangelets) becomes questionable, particularly for low values of the strange mass $m_s$. Only for $m_s = 300$ MeV the theoretical masses of hyperons always lie above the energy of bulk matter (not so the experimental masses).

From this analysis we can conclude (in agreement with previous findings) that, apart from rather extreme choices of the model parameters, metastable strangelets can exist in the MIT bag model.

3 Strangelets in the Color Dielectric Model

The Color Dielectric Model provides absolute confinement of quarks through their interaction with a scalar field $\chi$ which represents a multi–gluon state and produces a density dependent constituent mass (see for example the review articles [13, 14, 17]).

The typical Lagrangian of the CDM reads:

\[
L = \sum_{f=u,d,s} \bar{\psi}_f i\gamma^\mu (\partial_\mu - ig_s \frac{\lambda^a}{2} A_\mu^a) \psi_f - \frac{gf_\pi}{\chi} \sum_{f=u,d} \bar{\psi}_f \psi_f - m_s(\chi) \bar{\psi}_s \psi_s + \frac{1}{2} (\nabla_\mu \chi)^2 - U(\chi) - \frac{1}{4} \kappa(\chi) F^a_{\mu\nu} F^{a\mu\nu},
\]

(8)

where $\psi_f$ are the quark fields, $A_\mu^a$ is the (effective) gluon field, $F^{a}_{\mu\nu}$ its strength tensor and $\chi$ is the color dielectric field; $g_s$ is the strong (colour) coupling ($g_s^2/4\pi = \alpha_s$).

The $u$ and $d$ quark mass terms arise as a consequence of their interaction with the $\chi$–field and read:

\[
m_{u,d} = \frac{gf_\pi}{\chi},
\]

(9)

where $g$ is a parameter of the model and $f_\pi$ the pion decay constant, which is fixed to its experimental value, $f_\pi = 93$ MeV. For the strange quark mass we consider two different versions of the 3–flavors CDM, namely a scaling model, with

\[
m_s = \frac{gf_\pi}{\chi},
\]

(10)
and a non-scaling model, with a constant shift of the $s$-mass with respect to the $u,d$ one:

$$m_s = \frac{g f_\pi}{\chi} + \Delta m \equiv m_{u,d} + \Delta m.$$  \hspace{1cm} (11)

In the above $g'$ (or $\Delta m$) is another parameter of the model.

Concerning the color dielectric field, there exist in the literature several options, both for its coupling to the gluon tensor and for the potential $U(\chi)$. We adopt here both the single minimum (SM), quadratic potential:

$$U_{SM}(\chi) = \frac{1}{2} M^2 \chi^2,$$  \hspace{1cm} (12)

which introduces the third parameter of the model, $M$ (the mass of the glueball), and the double minimum (DM), quartic potential:

$$U_{DM}(\chi) = \left(\frac{1}{2} M^2 \frac{3B}{\chi_0^2} \chi^4 + \left(\frac{4B}{\chi_0^3} - \frac{M^2}{\chi_0}\right) \chi^3 + \frac{1}{2} M^2 \chi^2 \right).$$  \hspace{1cm} (13)

The latter introduces an extra parameter, the bag pressure $B$, while the parameter $\chi_0$ is used to make the ratio $\chi/\chi_0$ dimensionless. The color–dielectric function, $\kappa(\chi)$, is usually assumed to be a quadratic or quartic function of $\chi$: we will use both options and hence we set:

$$\kappa(\chi) = \left(\frac{\chi}{\chi_0}\right)^\beta,$$  \hspace{1cm} with $\beta = 2, 4$.  \hspace{1cm} (14)

The field equations are solved in the mean field approximation and neglecting the gluon fields: the latter are subsequently taken into account as a perturbation. The unperturbed (i.e. without gluon contribution) energy density reads:

$$\epsilon_0 = \sum_{f=u,d,s} \frac{3}{8\pi^2} \left\{ m_f^4 \ln \left( \frac{m_f}{k_{F_f} + \sqrt{k_{F_f}^2 + m_f^2}} \right) + k_{F_f} \sqrt{k_{F_f}^2 + m_f^2} \left( 2k_{F_f}^2 + m_f^2 \right) \right\} + U(\bar{\chi}),$$  \hspace{1cm} (15)

the quark masses being given by eqs. (8) and (10) [or (11)] with $\chi = \bar{\chi}$.

Beyond $\epsilon_0$ we have perturbatively taken into account, to order $\alpha_s$, the exchange of gluons, whose contribution to the energy density of an infinite, color singlet system is the analogous of eq. (7), but with the quark masses defined by Eqs. (9) and (10) [or (11)], and with an effective strong coupling constant (dressed by the colour dielectric function), which reads:

$$\bar{\alpha}_s = \alpha_s \left( \frac{\chi_0}{\chi} \right)^\beta,$$  \hspace{1cm} (16)
as it can be deduced from the model Lagrangian. Eq. (7) only contains the exchange term of OGE, the direct one vanishing for infinite quark matter: at small baryonic densities the attractive electric contribution dominates the energy density; on the contrary the repulsive magnetic contribution becomes the dominant one at large densities.

Indeed the divergent behavior of the electric term for $\rho \to 0$ could prevent a perturbative treatment of OGE in this regime. We have overcome this difficulty by taking into account the Debye screening of the gluon propagator in the presence of a polarized medium. This can be achieved by replacing (16) with a new effective coupling:

$$
\alpha_s^{\text{eff}}(q) = \frac{q^2}{q^2 + \frac{1}{2} \sum_{f=u,d,s} 16\tilde{\alpha}_s m_f k_{F_f}^2 \Pi(q/k_{F_f})}, \quad (17)
$$

$\Pi(y)$ being the static limit of the polarization propagator [18]:

$$
\Pi(y) = \frac{1}{2} - \frac{1}{2y} \left( 1 - \frac{1}{4} y^2 \right) \ln \left| \frac{1 - \frac{1}{2} y}{1 + \frac{1}{2} y} \right|. \quad (18)
$$

Actually this expression should be utilized in the momentum dependent $V_{\text{OGE}}(\vec{q})$ and then integrated to obtain the new expression for $\epsilon_{\text{OGE}}$. For simplicity, since the $q$–integration is extended only up to $k_{F_f}$ and the function $\Pi(y)$ varies at most by 9% in the range $0 \leq y \leq 1$, we have adopted $q = k_{F_f}$.

### 3.1 Stability of strangelets in the CDM: I

We consider here the work by Aoki et al. [19]: these authors solve self-consistently the mean field equations for quarks, color dielectric field and gluons, starting from a CDM Lagrangian with the Double Minimum potential (13) for the color dielectric field. This model is known to produce an unrealistic, too large binding energy in infinite quark matter [20, 21]; concerning the color dielectric function, Aoki et al. choose $\beta = 2$; they employ both the scaling and non–scaling version of the model, with two different sets for the model parameters whose values are dictated by two different and extreme choices for the “bag” parameter $B$: $B^{1/4} = 0$ MeV, with two degenerate vacua, and a large bag pressure, $B^{1/4} = 103.5$ MeV. The latter value of $B$ is chosen to be as large as possible, but with the requirement that the two-phase picture must hold inside hadrons. In their calculation only the strange quark mass has to be considered as a truly free parameter, the remaining ones having been fixed in a previous work on the non–strange baryons [22].

We evaluate the minimum energy per baryon number using cases B and D (corresponding to $B \neq 0$) of the work of Aoki et al., both without and with the perturbative exchange of a gluon.
As we can see from Fig. 4, this version of the CDM seems to favour strangelets as a (meta)–stable form of matter. This is due to the fact that when the DM potential is used to study hadrons, i.e. confined objects, a large contribution to the hadronic mass is given by the space fluctuations of the fields. When this version of the model is used to describe infinite quark matter, these contributions vanish due to the homogeneity of the system. For this reason, deconfined matter is favoured in this version of the model, which would even imply spontaneous decay of ordinary nuclei or two flavor nuclear matter into quark matter. The effect of perturbative gluons in this model is very small, due to the rather strong Debye screening, which we have included. Whether or not we take into account gluon corrections, strange matter always appears to be more stable than baryons.

3.2 Stability of strangelets in the CDM: II

In this subsection we follow the approach of J. McGovern [23], using the model Lagrangian reported in eq. (8) with $\beta = 4$ in the color dielectric function. McGovern employs only the scaling model and the Single Minimum potential, with different values of the parameters. In this case the behavior of $\alpha_s^{eff}(\chi)$ is even more divergent, for small densities, than in the case $\beta = 2$ previously considered: hence the use of Debye screening in the effective strong coupling constant is mandatory. In Ref. [23] two different sets for the model parameters are used: they allow to satisfactorily reproduce the splittings between hyperon masses, but the absolute values of the masses themselves are generally too large. In Fig. 5 we show our results, comparing our curves with both the experimental and the theoretical masses. As we can see, also in this case the inclusion of perturbative gluons is rather irrelevant. The curves corresponding to strange matter are well above the experimental masses, and below the theoretical ones. Yet, if we take into account surface effects, which would increase our curves of about 50 ÷ 100 MeV, only for $R_s \simeq \frac{2}{3}$ strangelets are (marginally) allowed by the present calculation and a more refined one, taking into account surface energy contributions, is needed to clarify the situation. We notice that a larger strange quark mass ($g'/g = 1.89$) obviously excludes stable strangelets, while the smaller $m_s$ value ($g'/g = 1.37$) does not substantially alter the above considerations.

4 Conclusions

The aim of this contribution was to compare the predictions about strangelet stability within the MIT bag model and the Color Dielectric Model, and to draw conclusions about their model dependence: we have compared the curves corresponding to the minimum energy per baryon number to the mass of hyperons having the same
strangeness fraction and calculated within the same model and parameter values that we adopt in our calculations.

The analysis shows that the existence of (stable) strangelets is supported only by those models which entail a two-phase picture of hadrons, namely which maintain a false vacuum inside hadrons. This happens both in the MIT bag model, and in the Double Minimum version of the Color Dielectric Model. The Single Minimum version of the CDM does not allow the existence of strangelets, independently of the parameter sets used to perform the calculations.

The conclusions that we can draw indicate that the stability of strangelets depends rather crucially on the model employed; this fact can set serious challenges to the search for strangelets in heavy ion collisions.

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Figure 1: Minimal energy per baryon number in the MIT bag model, as a function of the strangeness fraction $R_s = \rho_s/\rho$, for various values of the model parameters. The continuous line corresponds to $m_s = 100$ MeV, the dashed line to $m_s = 200$ MeV and the dotted line to $m_s = 300$ MeV. Full circles correspond to experimental masses, the other points to the masses evaluated in the model, with $m_s = 100$ MeV (open triangles), $m_s = 200$ MeV (full triangles), $m_s = 300$ MeV (stars), respectively.

Figure 2: Minimal energy per baryon number in the MIT bag model, including the OGE potential with $\alpha_s = 0.5$, as a function of the strangeness fraction $R_s = \rho_s/\rho$. The continuous line corresponds to $m_s = 100$ MeV, the dashed line to $m_s = 200$ MeV and the dotted line to $m_s = 300$ MeV. Full circles represent the experimental masses, the other points refer to the masses evaluated in the model, with $m_s = 100$ MeV (open triangles), $m_s = 200$ MeV (full triangles), $m_s = 300$ MeV (stars), respectively.
Figure 3: The same as in Fig. 4, but for $\alpha_s = 2.2$.

Figure 4: Minimal energy per baryon number in the CDM, as a function of $R_s = \rho_s/\rho$, for the cases B (with and without gluons) and D (solid lines). Full circles are the experimental hyperon masses, while triangular dots are the masses calculated in Ref. [19]. In the first panel the curves corresponding to $g' = 106.6$ MeV (dashed line) and $g' = 85.7$ MeV (dotted line) are also presented, while in the third panel the curves corresponding to $\Delta m = 312$ MeV (dotted line) and $\Delta m = 112$ MeV (dashed line) are shown. The remaining parameters of the cases B and D, respectively, are kept unaltered.
Figure 5: Minimal energy per baryon number as a function of $R_s = \rho_s/\rho$ for the Single Minimum version of the CDM. The various panels correspond to: (a) parameter set I without gluons, (b) parameter set I with gluons, (c) parameter set II without gluons. Full circles are the experimental baryon masses, while triangular dots are the masses calculated in Ref. [23]. In the first and third panels the calculations obtained with $g'/g = 1.89$ (long-dashed lines) and $g'/g = 1.37$ (short-dashed lines) are also shown.