Hair formation in the background of noncommutative reflecting stars

Yan Peng\textsuperscript{1}, Qiyuan Pan\textsuperscript{2,†}, Shuangxi Yi\textsuperscript{3,‡}

\textsuperscript{1} School of Mathematical Sciences, Qufu Normal University, Qufu, Shandong 273165, China
\textsuperscript{2} Key Laboratory of Low Dimensional Quantum Structures and Quantum Control of Ministry of Education, Synergetic Innovation Center for Quantum Effects and Applications, and Department of Physics, Hunan Normal University, Changsha, Hunan 410081, China
\textsuperscript{3} School of Physics and Physical Engineering, Qufu Normal University, Qufu, Shandong 273165, China

Abstract

We investigate scalar condensations around noncommutative compact reflecting stars. We find that the neutral noncommutative reflecting star cannot support the existence of scalar field hairs. In the charged noncommutative reflecting star spacetime, we provide upper bounds for star radii. Above the bound, scalar fields cannot exist outside the star. In contrast, when the star radius is below the bound, we show that the scalar field can condense. We also obtain the largest radii of scalar hairy reflecting stars.

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I. INTRODUCTION

The classical no scalar hair theorem provides insights into physical properties of black holes [1–4]. According to this theorem, asymptotically flat black holes cannot support static massive scalar fields, for references please refer to [5–17] and reviews can be found in [18, 19]. It was usually believed that these no scalar hair behaviors are due to the existence of black hole horizons, which can inevitably absorb matter and radiation fields.

Whether the no scalar hair behavior exists in the horizonless spacetime is a question to be answered. Recently, it was found that the no scalar hair behavior appears in the background of asymptotically flat horizonless neutral compact reflecting stars [20]. Moreover, it was shown that the massless scalar field nonminimal coupled to the gravity cannot condense around the asymptotically flat neutral compact reflecting star [21]. When extending the discussion to spacetimes with a positive cosmological constant, it was proved that massive scalar, vector and tensor hairs all die out outside asymptotically dS neutral reflecting stars [22].

However, the no hair theorem obtained in the regular neural gravity could be challenged in the charged spacetime. In the case of regular charged reflecting shell, it was analytically found that the scalar hair can form outside the shell when the shell radius is below an upper bound [23–25]. It was also shown that regular charged reflecting stars can support the existence of the scalar hair if the star radius is below an upper bound [26–30]. We mention that there are usually radii bounds, above which the scalar field cannot condense outside the objects [25–30].

Recently, the noncommutative gravity has attracted a lot of attentions based on the belief that the noncommutativity will appear at the Planck scale, where usual semiclassical approaches fail. The area law of noncommutative black holes has been studied [31–37]. Another important motivation to study noncommutative theories is it’s natural emergence in string theory and some surprising consequences [38–44]. As we know, the discussions on no hair theorem in regular gravities were carried out in the commutative spacetimes. So it is interesting to extend the discussion to noncommutative backgrounds. And it is also meaningful to construct regular hairy configurations in the noncommutative gravity.

The rest of this work is organized as follows. In section II, we construct the gravity system composed of a scalar field and a charged reflecting star in the noncommutative geometry. In part A of section III, we analytically obtain an upper bound for the charged scalar hairy star radius. In part B of section III, we numerically study scalar hairy star solutions. The last section is devoted to the conclusion.
II. THE GRAVITY SYSTEM IN THE NONCOMMUTATIVE GEOMETRY

The idea of noncommutative spacetime was firstly introduced by Snyder [45] and such a noncommutative structure naturally emerges in string theory [46]. There are mainly two ways to obtain the noncommutative quantum field theory: the Weyl-Wigner-Moyal star product approach and the coordinate coherent state approach [47]. Most recently, inspired by the coordinate coherent state approach, P. Nicolini and other authors obtained noncommutative black hole metrics [35–37], where the noncommutativity is introduced by writing down the Gaussian distribution of mass and charge densities in the form

$$\rho^{(M)}_\theta = \frac{M}{(4\pi\theta)^{\frac{3}{2}}} \exp\left(-\frac{r^2}{4\theta}\right), \quad (1)$$

$$\rho^{(Q)}_\theta = \frac{Q}{(4\pi\theta)^{\frac{3}{2}}} \exp\left(-\frac{r^2}{4\theta}\right). \quad (2)$$

Here $\theta$ is the parameter used to describe the noncommutativity of the spacetime. And the mass $M$ and charge $Q$ diffuse throughout a region of linear size $\sqrt{\theta}$. When the object size is below the characteristic length $\sqrt{\theta}$, noncommutativity becomes non-negligible. And in the case of $\theta \to 0$, the model returns to the commutative one.

We can search for solutions of Einstein’s equations with the mass and charge distribution (1) and (2). The metric of the spherically symmetric noncommutative compact star is [35–37]

$$ds^2 = -g(r)dt^2 + g^{-1}dr^2 + r^2(d\varphi^2 + \sin^2\varphi d\phi^2). \quad (3)$$

The solution is

$$g(r) = 1 - \frac{4M}{\sqrt{\theta}} \gamma\left(\frac{3}{2}, \frac{r^2}{4\theta}\right) + \frac{Q^2}{\pi^2} \left[\gamma\left(\frac{1}{2}, \frac{r^2}{4\theta}\right) - \frac{r}{\sqrt{\theta}} \gamma\left(\frac{1}{2}, \frac{r^2}{2\theta}\right) + \sqrt{\frac{2}{\theta}} \sqrt{\gamma\left(\frac{3}{2}, \frac{r^2}{4\theta}\right)}\right],$$

where $\gamma$ is the incomplete gamma function in the form $\gamma(n, z) = \int_0^z t^{n-1}e^{-t}dt$. Here $M$ and $Q$ are interpreted as the star mass and star charge respectively. We also define $r_s$ as the regular star radius and there is $g(r) > 0$ for $r \geq r_s$.

We study scalar condensations in the noncommutative charged reflecting star background. And the Lagrange density with scalar fields coupled to the Maxwell field reads

$$\mathcal{L} = -\frac{1}{4} F^{MN} F_{MN} - |\nabla \psi - qA_{\mu}\psi|^2 - m^2\psi^2. \quad (4)$$

Here $\psi(r)$ is the scalar field and $A_{\mu}$ corresponds to the Maxwell field. We also label $q$ and $m$ as scalar field charge and scalar field mass respectively.

We assume that the Maxwell field has only the nonzero $t$ component in the form $A_t = \phi(r)dt$. Then the equation of the Maxwell field is [48–53]

$$\phi'' + \frac{2}{r} \phi' - \frac{g^2\psi^2(r)\phi(r)}{2g(r)} = 0. \quad (5)$$
In this work, we neglect the scalar field’s backreaction on the charged star. So the third term of equation (5) disappears and the electric potential is not affected by the non-commutativity. We take the electric potential in the form \( A_\ell = -\frac{Q}{r} \) and the scalar field equation is

\[
\psi'' + \left(\frac{2}{r} + \frac{g'}{g}\right)\psi' + \left(\frac{\ell^2 Q^2}{r^2 g^2} - \frac{m^2}{g}\right)\psi = 0. \tag{6}
\]

We need to impose boundary conditions to solve the equation (6). At the star surface, we take the scalar reflecting condition that the scalar field vanishes. In the region far from the star, the general solutions behave in the form \( \psi \sim A \cdot \frac{1}{r} e^{-mr} + B \cdot \frac{1}{r} e^{mr} \), where A and B are integral constants. We set \( B = 0 \) in order to obtain the physical solution satisfying \( \psi(\infty) = 0 \). So boundary conditions can be put as

\[
\psi(r_s) = 0, \quad \psi(\infty) = 0. \tag{7}
\]

With relations (6), (7) and the Hod’s method in [20], it is easy to check that massive scalar fields cannot exist outside neutral noncommutative reflecting stars. In the next section, we turn to study scalar condensations in the background of charged noncommutative reflecting stars.

### III. SCALAR FIELD CONDENSATIONS OUTSIDE NONCOMMUTATIVE CHARGED REFLECTING STARS

#### A. Upper bounds for radii of noncommutative hairy reflecting stars

Introducing a new function \( \tilde{\psi} = \sqrt{r}\psi \), we can express the equation (6) as

\[
r^2 \tilde{\psi}'' + \left(r + \frac{r^2 g'}{g}\right)\tilde{\psi}' + \left(-\frac{1}{4} - \frac{rg'}{2g} + \frac{q^2 Q^2}{g^2} - \frac{m^2 r^2}{g}\right)\tilde{\psi} = 0. \tag{8}
\]

The boundary conditions are

\[
\tilde{\psi}(r_s) = 0, \quad \tilde{\psi}(\infty) = 0. \tag{9}
\]

Then the function \( \tilde{\psi} \) must possess at least one extremum point \( r = r_{\text{peak}} \) above the star radius \( r_s \). At this extremum point, the following characteristic relations hold

\[
\{ \tilde{\psi}' = 0 \quad \text{and} \quad \tilde{\psi}\tilde{\psi}' \leq 0 \} \quad \text{for} \quad r = r_{\text{peak}}. \tag{10}
\]

From (8) and (10), we deduce the inequality

\[
-\frac{1}{4} - \frac{rg'}{2g} + \frac{q^2 Q^2}{g^2} - \frac{m^2 r^2}{g} \geq 0 \quad \text{for} \quad r = r_{\text{peak}}. \tag{11}
\]
It can be transformed into
\[ m^2 r^2 g \leq q^2 Q^2 - \frac{r g g'}{2} - \frac{1}{4} g^2 \quad \text{for} \quad r = r_{\text{peak}}. \] (12)

With the relation \( \gamma(s + 1, z) = s \gamma(s, z) - x^4 e^{-z} \), the metric function \( g(r) \) can be expressed as
\[ g(r) = 1 - \frac{2M}{\sqrt{4\pi r}} \left[ \gamma(\frac{1}{2}, \frac{r^2}{4\theta}) - \frac{r}{\sqrt{\theta}} e^{-\frac{r^2}{4\theta}}\right] + \frac{Q^2}{\pi r^2} \left[ \gamma(\frac{1}{2}, \frac{r^2}{4\theta}) + \frac{r}{\sqrt{2\theta}} \gamma(\frac{1}{2}, \frac{r^2}{4\theta}) - \frac{r}{\sqrt{2\theta}} \frac{r^2}{4\theta} e^{-\frac{r^2}{4\theta}} \right]. \] (13)

Then we have
\[ g' = \frac{2M}{\sqrt{4\pi r}} \gamma(\frac{1}{2}, \frac{r^2}{4\theta}) - \frac{2Q^2}{\pi r^2} \gamma(\frac{1}{2}, \frac{r^2}{4\theta}) + \frac{r^2}{2\theta \sqrt{\theta}} e^{-\frac{r^2}{4\theta}} \right] \]
\[ \frac{2M}{\sqrt{4\pi r}} \gamma(\frac{1}{2}, \frac{r^2}{4\theta}) - \frac{2Q^2}{\pi r^2} \gamma(\frac{1}{2}, \frac{r^2}{4\theta}) + \frac{r^2}{2\theta \sqrt{\theta}} e^{-\frac{r^2}{4\theta}} + \frac{1}{\sqrt{2\theta}} \gamma(\frac{1}{2}, \frac{r^2}{4\theta}) \]
\[ - \frac{2M}{\sqrt{4\pi r}} \gamma(\frac{1}{2}, \frac{r^2}{4\theta}) + \frac{r^2}{2\theta \sqrt{\theta}} e^{-\frac{r^2}{4\theta}} \right] \]
\[ \frac{r^2}{2\theta \sqrt{\theta}} e^{-\frac{r^2}{4\theta}} + \frac{1}{\sqrt{2\theta}} \gamma(\frac{1}{2}, \frac{r^2}{4\theta}) \] (14)

And there is also the relation
\[ \gamma(\frac{1}{2}, \frac{r^2}{4\theta}) - \gamma(\frac{1}{2}, \frac{r^2}{2\theta}) = \frac{\partial \gamma}{\partial z} \bigg|_{z = -\frac{r^2}{4\theta}} \left( \frac{r^2}{40} - \frac{r^2}{2\theta} \right) = \frac{2\sqrt{\theta}}{r} e^{-\frac{r^2}{4\theta}} \left( \frac{r^2}{4\theta} \right) \] (15)

with \( r^2 \in [r^2, 2r^2] \).

We mention that \( \gamma\left(\frac{1}{2}, \frac{r^2}{4\theta}\right) \) increases as a function of \( r \) with values of \( \gamma\left(\frac{1}{2}, \frac{r^2}{4\theta}\right) \) in the range \([0, \sqrt{\pi}]\). According to (14), (15) and the fact that \( e^{-\frac{r^2}{4\theta}} \) decreases very quickly, \( g' \) asymptotically behaves as
\[ g' \approx \frac{2M}{\sqrt{4\pi r}} \gamma(\frac{1}{2}, \frac{r^2}{4\theta}) - \frac{2Q^2}{\pi r^2} \gamma(\frac{1}{2}, \frac{r^2}{4\theta}) \approx \frac{2M}{r^2} - \frac{2Q^2}{r^3} \] (16)
in the large \( r \) region.

The dominant terms of (14) satisfy
\[ \frac{2M}{\sqrt{4\pi r}} \gamma(\frac{1}{2}, \frac{r^2}{4\theta}) \geq \frac{3M}{2} \quad \text{for} \quad \frac{r^2}{4\theta} \geq 1; \] (17)
\[ -\frac{2Q^2}{\pi r^2} \gamma(\frac{1}{2}, \frac{r^2}{4\theta}) \geq -\frac{2Q^2}{r^3}. \] (18)

Other terms in (14) satisfy
\[ \frac{2M}{\sqrt{4\pi r}} \left[ \frac{r}{\sqrt{\theta}} e^{-\frac{r^2}{4\theta}} \right] = -\frac{2M}{r^2} \left[ \frac{1}{\sqrt{2\theta} \sqrt{\pi}} e^{-\frac{r^2}{4\theta}} \right] \geq -\frac{2M}{2\theta} \frac{1}{8} \quad \text{for} \quad \frac{r^2}{4\theta} \geq 6; \] (19)
\[ -\frac{2M}{\sqrt{4\pi r}} \left[ \frac{2\sqrt{\theta}}{r} e^{-\frac{2r^2}{4\theta}} + \frac{r^2}{2\theta \sqrt{\theta}} e^{-\frac{r^2}{4\theta}} \right] = -\frac{2M}{r^2} \left( \frac{1}{\sqrt{2\theta}} \frac{r}{\sqrt{\pi}} e^{-\frac{2r^2}{4\theta}} + \frac{4}{\pi \sqrt{8\theta} \sqrt{\theta}} e^{-\frac{r^2}{4\theta}} \right) \geq -\frac{2M}{2\theta} \frac{1}{8} \quad \text{for} \quad \frac{r^2}{4\theta} \geq 6; \] (20)
\[ \frac{Q^2}{\pi r^2} \left[ \frac{r}{\sqrt{2\theta}} e^{-\frac{r^2}{4\theta}} \right] = -\frac{Q^2}{r^3} \left[ \frac{1}{2\theta} e^{-\frac{r^2}{4\theta}} \right] \geq -\frac{Q^2}{r^3} \frac{1}{3} \quad \text{for} \quad \frac{r^2}{4\theta} \geq 6; \] (21)
\[
\frac{Q^2}{\pi r^2} \left[ \frac{1}{\sqrt{2\theta}} \gamma\left(\frac{1}{2}, \frac{r^2}{4\theta}\right) - 1 \right] = \frac{Q^2}{\pi r^2} \left[ \frac{2\sqrt{\theta}}{r_\star} e^{-\frac{r^2}{4\theta}} - \frac{r^2}{4\theta} \right] = -1 \frac{Q^2}{\pi r^2} \frac{2\sqrt{\theta}}{r_\star} e^{-\frac{r^2}{4\theta}}
\]
\[
\geq -1 \frac{Q^2}{r^3} \frac{2\sqrt{\theta}}{4\theta} \quad \text{for } \frac{r^2}{4\theta} \geq 3; \quad (22)
\]

\[
\frac{Q^2}{\pi r^2} \left[ -\frac{\sqrt{2\theta} e^{-\frac{r^2}{4\theta}}}{\theta} \right] = -\frac{Q^2}{r^3} \frac{4\sqrt{2\theta} r^2 e^{-\frac{r^2}{4\theta}}}{4\theta} \geq -\frac{Q^2}{r^3} \frac{1}{3} \quad \text{for } \frac{r^2}{4\theta} \geq 6; \quad (23)
\]

\[
-\frac{2Q^2}{\pi r^3} \left[ r \frac{1}{\sqrt{2\theta}} \gamma\left(\frac{1}{2}, \frac{r^2}{4\theta}\right) - \frac{r^2}{\sqrt{2\theta}} \gamma\left(\frac{1}{2}, \frac{r^2}{2\theta}\right) \right] \geq 0;
\]
\[
-\frac{2Q^2}{\pi r^3} \left[ -\frac{r^2}{\sqrt{2\theta}} e^{-\frac{r^2}{4\theta}} \right] \geq 0;
\]

\[
-\frac{2M}{\sqrt{\pi r}} \left[ -\frac{1}{\sqrt{\theta}} e^{-\frac{r^2}{4\theta}} \right] \geq 0;
\]

\[
\frac{Q^2}{\pi r^2} \left[ 2\gamma\left(\frac{1}{2}, \frac{r^2}{4\theta}\right) \frac{2\sqrt{\theta}}{r} e^{-\frac{r^2}{4\theta}} \right] \geq 0;
\]

\[
\frac{Q^2}{\pi r^2} \left[ r \frac{2\sqrt{\theta}}{r} e^{-\frac{r^2}{4\theta}} \right] \geq 0;
\]

\[
\frac{Q^2}{\pi r^2} \left[ \frac{r^3}{2\sqrt{2\theta}^2} e^{-\frac{r^2}{4\theta}} \right] \geq 0.
\]

According to (14-29), the following relation

\[
g' > \frac{M}{r^2} - \frac{3Q^2}{r^3} = \frac{M}{r^2} (1 - \frac{3Q^2}{Mr}) > 0
\]

holds for \( r > \frac{3Q^2}{M} \) and \( \frac{r^2}{4\theta} \geq 6 \).

In the case of \( \frac{r^2}{4\theta} \geq 6 \), we have

\[
-\frac{4\gamma\left(\frac{3}{2}, \frac{r^2}{2\theta}\right)}{\sqrt{\pi}} \geq -3,
\]

\[
\frac{1}{\sqrt{2\theta}} \gamma\left(\frac{1}{2}, \frac{r^2}{4\theta}\right) - \frac{r}{\sqrt{2\theta}} \gamma\left(\frac{1}{2}, \frac{r^2}{2\theta}\right) + \sqrt{\frac{2}{\theta}} \gamma\left(\frac{3}{2}, \frac{r^2}{4\theta}\right) \geq \frac{1}{2}.
\]

Then there is \( g > 1 - \frac{3M}{r} + \frac{Q^2}{r^2} \) for \( \frac{r^2}{4\theta} \geq 6 \). We obtain an lower bound of the metric function as

\[
g > 1 - \frac{3M}{r} + \frac{Q^2}{r^2} = \frac{1}{2} + \frac{1}{2} (1 - \frac{6M}{r} + \frac{Q^2}{r^2}) > \frac{1}{2}
\]

on conditions \( r > 3M + \sqrt{9M^2 - Q^2} \) and \( \frac{r^2}{4\theta} \geq 6 \).
We assume that the star radius satisfies $r_s^2 \geq 6$, $r_s > \frac{3Q^2}{M}$ and $r_s > 3M + \sqrt{9M^2 - Q^2}$, otherwise we will arrive at an upper bound

$$mr_s \leq \max \left\{ 2m \sqrt{6\theta}, \frac{3mQ^2}{M}, 3mM + m \sqrt{9M^2 - Q^2} \right\}. \quad (34)$$

According to (12), (30), (33) and $r_{\text{peak}} \geq r_s$, there is

$$\frac{1}{2}m^2 r_s^2 \leq \frac{1}{2}m^2 r^2 \leq m^2 r^2 g \leq q^2 Q^2 - \frac{rg^2}{2} - \frac{1}{4}g^2 \leq q^2 Q^2 - \frac{1}{16} \quad \text{for} \quad r = r_{\text{peak}}. \quad (35)$$

Then we have

$$m^2 r_s^2 \leq 2q^2 Q^2 - \frac{1}{8}. \quad (36)$$

In all, our analysis shows that the hairy star radius is below the bounds (34) or (36). With dimensionless quantities according to the symmetry (38), we obtain upper bounds for hairy star radii as

$$mr_s \leq \max \left\{ 2m \sqrt{6\theta}, \frac{3mQ^2}{M}, 3mM + m \sqrt{9M^2 - Q^2}, \sqrt{2q^2 Q^2 - \frac{1}{8}} \right\}. \quad (37)$$

Above this bound, the static scalar field cannot condense. Below the bound, we will numerically obtain hairy reflecting star solutions in the next part.

We mention that there are also upper bounds on the size of hairy black holes. According to the no short hair conjecture, the hairy black hole has an upper bound for the horizon $r_H < \frac{2}{3}(\eta)^{-1}$, where $\eta$ is the field mass \[54\]. In fact, the numerical results in \[55\] also support the idea that big black holes tend to have no massive hair. Similar to black hole theories, we find that big reflecting star cannot support the existence massive scalar hair. So it is natural that there is a maximal radius for the hairy star. In the following, we will numerically search for the maximal radius.

B. Scalar field configurations supported by noncommutative charged reflecting stars

We firstly show the metric solution $g(r)$ with different values of the noncommutative parameter $\theta$ in Fig. 1. It can be seen from the panels that the noncommutative spacetime is regular at $r = 0$ and the metric behaves like Schwarzschild geometry at $r \to \infty$, in accordance with results in \[35\]. Our results also imply that in cases of very small $\theta$, the metric almost coincides with the Schwarzschild geometry even at the points $r \approx 0$, but the metric always keeps regular at the center with $g(0) = 1$. We point out that the regular star radius should be imposed above the outmost would-be horizon of the metric.
FIG. 1: (Color online) We plot $g(r)$ as a function of the coordinate $r$ with $q = 2$, $Q = 4$ and $M = 5$. In the left panel, the blue curves from top to bottom correspond to various $\theta$ as $\theta = 100$, $\theta = 50$, $\theta = 10$, $\theta = 5$, $\theta = 3$ and $\theta = 0.5$. The blue curve in the right panel is the case of $\theta = 0.1$. And the red lines represent Schwarzschild geometries with $\theta = 0$. 

In the following, we numerically obtain regular configurations composed of scalar fields and noncommutative charged reflecting stars. We take $m = 1$ according to the symmetry

$$r \rightarrow kr, \quad m \rightarrow m/k, \quad M \rightarrow kM, \quad Q \rightarrow kQ, \quad q \rightarrow q/k, \quad \theta \rightarrow k^2 \theta. \quad (38)$$

Around the star surface, the scalar field behaves in the form $\psi = \psi_0(r-r_s) + \cdots$. With the symmetry $\psi \rightarrow k\psi$ of equation (6), we can set $\psi_0 = 1$ without loss of generality. Then we numerically search for the proper $r_s$, where the corresponding scalar field satisfies the boundary condition (7) at the infinity.

In Fig. 2, we plot scalar fields in the background of compact stars with $q = 2$, $Q = 4$, $M = 5$ and $\theta = 50$. With the star radius fixed at $r_s = 9.6719$, the scalar field approaches zero at the infinity. If we impose the star radius a little larger or a little smaller than $r_s$, the solutions asymptotically behave in the form $\psi \propto A \cdot \frac{1}{r}e^{-mr} + B \cdot \frac{1}{r}e^{mr}$ with nonzero $B$, which contradicts the infinity boundary condition (7). Our detailed calculations show that the hairy star radius is discrete.

FIG. 2: (Color online) We show $\psi$ as a function of the coordinate $r$ with $q = 2$, $Q = 4$, $M = 5$, $\theta = 50$ and the star radius at $r_s = 9.6719$.

Integrating the equation from $r_s = 9.6719$ to smaller radial coordinates, we find discrete points, which can be fixed as the hairy star radii. In Fig. 3, the discrete points are around $mr_s \approx 9.672$, 8.028, 6.867, 6.672, 6.472, 6.292, ...
below the upper bound $m_{r_s} \leq max \left \{ 2\sqrt{300}, \frac{48}{5}, 15 + \sqrt{9 \cdot 25 - 16}, \sqrt{128 - 18} \right \} = 2\sqrt{300} \approx 34.641$ according to (37). Similar to cases in commutative spacetimes [23–29], we find many discrete hairy star radii.

In the case of $q = 2, Q = 4, M = 5$ and $\theta = 50$, we numerically find that $m_{r_s} = 9.6719$ is the largest hairy star radius below the upper bound $m_{r_s} \leq 2\sqrt{300} \approx 34.641$ of (37). For every given set of parameters, we can obtain the largest hairy star radius labeled as $m_{R_s}$. We also study effects of the noncommutative parameter on the largest hairy star radius. With dimensionless quantities according to the symmetry (38), we plot $m_{R_s}$ as a function of $m^2 \theta$ with $q = 2, Q = 4, M = 5$ in Fig. 4. It can be seen from the picture that larger $m^2 \theta$ corresponds to smaller $m_{R_s}$.

The noncommutative physics is expected to be detected for a small size star $r_s \simeq \sqrt{\theta}$. However, the phenomenological impact of these results may be not visible since the presently accessible energy is $\sqrt{\theta} < 10^{-16}$ cm [35]. For large distance, the solution behaves very similar to the Schwarzschild metric. Recently, holographic superconductor models were constructed in the background of noncommutative AdS black holes [56, 57], which provided a novel way to investigate the role of noncommutative geometry through the AdS/CFT

FIG. 3: (Color online) We show various discrete radii with $q = 2, Q = 4, M = 5, \theta = 50$ and the largest hairy star radius at $r_s = 9.6719$.

FIG. 4: (Color online) We show the largest hairy star radius with $q = 2, Q = 4, M = 5$ and various noncommutative parameters.
duality.

IV. CONCLUSIONS

We studied static scalar field condensations outside noncommutative charged compact reflecting stars. We found that the scalar field cannot condense outside neutral noncommutative reflecting stars. And in the background of charged noncommutative reflecting compact stars, we provided upper bounds for the star radius as

\[ mr_s \leq \max \left\{ 2m \sqrt{\theta}, \frac{3mQ^2}{2M^2}, 3mM + m \sqrt{9M^2 - Q^2}, \sqrt{2q^2Q^2 - \frac{1}{8}} \right\}, \]

where \( m \) is the scalar field mass, \( q \) is the charge coupling parameter, \( M \) is the ADM mass, \( Q \) is the star charge and \( \theta \) is the noncommutative parameter. Above the bound, the scalar field cannot condense and below the bound, we obtained numerical solutions of scalar hairy reflecting stars. With detailed calculations, we found that the scalar hairy reflecting star radius is discrete. We also examined effects of the noncommutative parameter on the largest radius of the scalar hairy reflecting star.

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