Propagation of acoustic waves in multifractional polydisperse gas suspension

D A Gubaidullin¹, E A Teregulova ¹

¹ Institute of Mechanics and Engineering, Kazan Science Center, Russian Academy of Sciences, 2/31 Lobachevsky str., Kazan 420111, Russia

E-Mail: teregulova@inbox.ru

Abstract. The propagation of acoustic waves in multifractional polydisperse gas suspension is studied. A mathematical model is presented, the dispersion equation is obtained, dispersion curves are calculated. The influence of the particle size and the parameters of the dispersed phase for multifractional gas mixture with ice particles, aluminum and sand on dissipation and dispersion of sound waves is analyzed.

1. Introduction

The acoustics and wave dynamics of multiphase media is of great theoretical and practical interest, due to the wide spread of such media in nature and use them in practice. The study of non-stationary wave processes is one of the urgent problems of mechanics of multiphase media. Basic models of wave dynamics of dispersed media and a number of results in this area are presented in [1]. Work [2] is devoted to the problems of the investigation of two-phase flows with solid particles, droplets and bubbles. In monography [3] a brief overview of the results on the study of acoustic disturbances in monodisperse gas mixtures without phase transformations is given. Influence of polydisperse composition of the gas mixture on the distribution of monochromatic disturbances in one-component mixtures of gas and particles, or vapor with droplets is studied in [4]. Peculiarities of the propagation of monochromatic waves in two-component polydisperse mixtures of gas and vapor and liquid droplets are investigated in [5]. Propagation of spherical and cylindrical waves of small amplitude in polydispersed fogs with phase transformations is considered in [6]. A general dispersion relation of the wave number to the oscillation frequency and thermal properties of the phases is obtained. The anomalous effect of nonmonotonic dependence of dissipation of weak harmonic and pulse disturbances on the mass concentration of droplets in monodisperse aerosols with heat and mass transfer is studied in [7]. Fairly complete presentation of the propagation linear theory of flat disturbances in mono-and polydisperse two-phase gas-vapor-liquid droplet mixtures is given in [8]. The propagation of acoustic waves of different geometry in two- fraction gas mixtures with particles of different sizes and materials, without taking phase transformations into account is studied in [9]. Peculiarities of plane, cylindrical and spherical waves of small amplitude in the gas-vapor-droplet mixtures with solid particles are analysed in [10], [11] and [12]. In this paper the
propagation of plane, cylindrical, and spherical waves of small amplitude in multifractional mixtures of gas with an arbitrary number of solid particles of different sizes and substances with significantly different thermal properties is studied.

2. Basic equations and dispersion equation.

Similarly to [8], the linearized system of equations of disturbed motion of multifractional polydisperse gas suspension, in a coordinate system relative to which the undisturbed medium is at rest, can be written as

\[
\frac{\partial p'_i}{\partial t} + \rho_{10} \left( \frac{\partial v'_i}{\partial r} + \theta v'_i \right) = 0, \quad \frac{\partial p''_i}{\partial t} + \rho_{10}' \left( \frac{\partial v''_i}{\partial r} + \theta v''_i \right) = 0, \quad (j = 2, N, i = 1, M)
\]

\[
\rho_{10} \frac{\partial v'_i}{\partial t} + \rho_{10}' \frac{\partial v'_i}{\partial r} + \sum_{j=2}^{N} \sum_{i=1}^{M} n'_{ij} \left( 6 \pi \alpha_{ij} \mu_i \left( v'_i - v''_i \right) + 6 \left( a'_i \right)^2 \frac{\partial}{\partial \tau} \left( v'_i - v''_i \right) \right) = 0.
\]

\[
\rho_{10}' \frac{\partial v''_i}{\partial t} = n''_{ij} \left( 6 \pi \alpha_{ij} \mu_i \left( v'_i - v''_i \right) + 6 \left( a'_i \right)^2 \frac{\partial}{\partial \tau} \left( v'_i - v''_i \right) \right), \quad (j = 2, N, i = 1, M)
\]

\[
\rho_{10} \frac{\partial T''_i}{\partial t} = \alpha_{10} \frac{\partial p'{'}_i}{\partial t} - \sum_{j=2}^{N} \sum_{i=1}^{M} 2 \pi n'_{ij} a'_i \lambda_i \nu_{ij} \left( T''_i - T''_{ij} \right), \quad (j = 2, N, i = 1, M)
\]

\[
\lambda_i \nu_{ij} \left( T''_i - T''_{ij} \right) + \lambda'_i \nu'_{ij} \left( T''_i - T''_{ij} \right) = 0, \quad (j = 2, N, i = 1, M)
\]

\[
p'_i = \frac{C_i^2}{\gamma_i \alpha_{10}^2} + \frac{p_T}{T_0} T''_i.
\]

The system of equations (1) for the parameter \( \theta = 0 \) describes plane waves in a Cartesian coordinate system, when \( \theta = 1 \) - cylindrical waves in a cylindrical coordinate system, when \( \theta = 2 \) - spherical waves in a spherical coordinate system. Here and further, variables with subscript 1 refer to the carrier phase, with indexes \( j = 2, N, i = 1, M \) to the particles of the dispersed phase of the radius \( a'_i \). Index 0 corresponds to the initial undisturbed state. Dashes above are used to denote disturbed parameters. Here and further \( \rho \) - reduced density, \( \rho' \) - true density, \( v \) - speed, \( \alpha \) - volume content, \( p \) - pressure, \( \mu_i \) - coefficient of the dynamic viscosity of the carrier medium, \( T_i \) - the temperature of the carrier phase, \( T''_{ij} \) - the temperature in the surface \( \Sigma \) - layer of a particle of the radius \( a'_i \), \( T''_i \) - temperature of solid particles of the radius \( a'_i \), \( \nu_{ij} \) - the dimensionless (Nusselt number) coefficient of the carrier phase heat transfer with the border of the interface of particles of the radius \( a'_i \), \( \nu'_{ij} \) - the dimensionless (Nusselt number) coefficient of the heat transfer of particles of the radius \( a'_i \) with the border of the interface, \( \lambda \) - coefficient of thermal conductivity.

The system of equations (1) is closed and can be used to investigate the propagation of acoustic disturbances in multifractional polydisperse gas suspension in flat, spherical and cylindrical cases.

Let us introduce into the system of equations (1) the phase velocity potentials and explore the solutions of the resulting system of equations in the form of progressive waves for disturbances

\[
\eta' = A_i \psi
\]

where

\[
\psi = \exp \left[ i (K, r - \omega t) \right] \quad \text{for flat disturbances}
\]
\[ \psi = H_0^{(1)}(Kr) \exp(-i\omega t) \] - for cylindrical disturbances

\[ \psi = \frac{1}{r} \exp\left[i(Kr - i\omega t)\right] \] - for spherical disturbances

\[ K_r = K + iK_{\sigma}, \quad C_p = \frac{\omega}{K}, \quad \sigma = 2\pi \frac{K_{\sigma}}{K}. \]

Here \( A_0 \) - the oscillation amplitude of parameters disturbances, \( K_r \) - the complex wave number, \( i \) - the imaginary unit, \( C_p \) - the phase velocity, \( K_{\sigma} \) - the linear attenuation coefficient, \( \sigma \) - the attenuation decrement on the wave length, \( H_0^{(1)}(z) \) - Hankel function, which is a combination of Bessel functions of the first and second kind of order zero \( J_0(z) \) and \( Y_0(z) \) \( \left(H_0^{(1)}(z) = J_0(z) + iY_0(z)\right) \).

Substituting the velocity potentials and solutions of the form (2) in the system of equations (1), we obtain the following system of linear algebraic equations and from conditions of the existence of nontrivial solution of a system of linear algebraic equations the following dispersion equation is obtained

\[ \left( \frac{C_iK_{\sigma}}{\omega} \right)^2 = V(\omega)D(\omega), \]  

where

\[ V(\omega) = 1 + \sum_{i=2}^{M} \sum_{j=1}^{N_i} \frac{m_j}{1 - i\omega \tau_{ij}}, \]

\[ D(\omega) = 1 + (\gamma_1 - 1) \frac{\sum_{i=2}^{M} \sum_{j=1}^{N_i} \frac{m_j}{1 - i\omega \tau_{ij}}}{1 + \sum_{i=2}^{M} \sum_{j=1}^{N_i} \frac{m_j}{1 - i\omega \tau_{ij}}}, \quad \tau_{ij}^* = \tau_{ij}^* + \frac{m_j}{c_{pi}} \tau_{ij}^*. \]

3. Results.

As an example the mixture of air and particles of aluminum of the radius \( r_a = 10^{-6} \) m, sand of the radius \( r_s = 8 \times 10^{-6} \) m and ice of the radius \( r_l = 10^{-5} \) m. Fig. 1,2 illustrate the effect of inclusions mass content on the dependences of the relative speed of sound and attenuation decrement on the wavelength on the dimensionless oscillation frequency \( \omega \tau_{vs} \) respectively. The calculated curves are built using the dispersion relation (10). The graphs show that with increasing mass content of particles the sound speed dispersion and dissipation of the waves increase. Accounting of the three-fraction composition and difference of thermal parameters of fractions lead to the appearance of characteristic inflexions for the dependence of the relative speed of sound in the frequency field inversely proportional to the characteristic relaxation times of the phase velocities \( \tau_{va}, \tau_{vb} \) and \( \tau_{vl} \) (Fig. 3). As shown in Fig. 4, the difference between the sizes of inclusions and thermal parameters of particles fractions leads to appearance of three maxima for the dependence of attenuation decrement on the wavelength at the characteristic values \( \omega \tau_{va}, \omega \tau_{vb} \) and \( \omega \tau_{vs} = 1 \).
Figure 1. Dependence of the relative speed of sound on the dimensionless oscillation frequency for a mixture of air with particles of ice, aluminum and sand with different mass content of dispersed phase (I: \( m = 0.2 \), II: \( m = 0.4 \), III: \( m = 0.6 \)).

Figure 2. The attenuation decrement dependence on the wavelength on the dimensionless oscillation frequency for a mixture of air with particles of ice, aluminum and sand with different mass content of dispersed phase (I: \( m = 0.2 \), II: \( m = 0.4 \), III: \( m = 0.6 \)).

4. Conclusion.
This paper presents a closed system of linear differential equations of motion for multifractional polydisperse gas suspension. A dispersion equation, determining the propagation of plane, spherical and cylindrical disturbances of small amplitude derived. The dispersion curves are calculated. The
influence of the parameters of the dispersed phase for three-fraction gas mixture with aluminum particles, carbon black, and ice on the dissipation and dispersion of sound waves is analysed. It is found that the difference of thermal properties and inclusions sizes significantly affects on the dynamics of weak waves in multifractional polydisperse gas suspension and it must be considered in the development of methods of acoustic diagnostics of considered environments.

References
[1] Nigmatulin R I 1991 Dynamics of Multiphase Media vol 1,2 (Washington: Hemisphere)
[2] Varaksin A Yu 2013 Fluid dynamics and thermal physics of two-phase flows: Problems and achievements High Temperature 51(3) 421-455
[3] Temkin S 2005 Suspension acoustics: An introduction to the physics of suspension (Cambridge: Cambridge University Press) p 398
[4] Gumerov N A and Ivandaev A I 1988 Sound propagation in polydisperse gas suspensions Journal of Applied Mechanics and Technical Physics 5 115-124.
[5] Gubaidullin D A and Ivandaev A I 1993 Effect of polydispersion on sound propagation in gas mixtures with vapor and liquid drops Journal of Applied Mechanics and Technical Physics 4 75-83
[6] Gubaidullin D A 2003 Spherical and Cylindrical Low-Amplitude Waves in Polydisperse Fogs with Phase Transitions Fluid Dynamics 5 85-94
[7] Nigmatulin R I, Ivandaev A I and Gubaidullin D A 1991 Effect of the non-monotonic dependence of the dissipation of sound on the concentration of drops in a suspension in gas Dokl. USSR Academy of Sciences 316(3) 601-605
[8] Gubaidulln D A 1998 Dynamics of twophase gas-vapor-droplet media (Kazan: Kazan mathematical society) p 153
[9] Gubaidullin D A, Nikiforov A A and Utkina E A 2009 Propagation of acoustic waves to two-fractional gas mixtures with particles of different materials and sizes Izvestiya vysshikh uchebnykh zaavedenii. Problemi Energetiki 1-2 25-33.
[10] Gubaidullin D A, Nikiforov A A and Utkina E A 2011 Effect of the phase transformations on propagation of acoustic waves in two-fraction mixtures of gas with vapor, droplets and solid particles of different materials and sizes High Temperature 49(6) 942-947
[11] Gubaidullin D A, Nikiforov A A and Utkina E A 2011 Acoustic waves in two-fraction mixtures of gas with vapor, droplets and solid particles of different materials and sizes in the presence of phase transitions Fluid Dynamics 1 95-103
[12] Nikiforov A A, Utkina E A and Gafiyatov R N 2011 Acoustic disturbances in vapor-gas-liquid systems Vestnik of Lobachevsky State University of Nizhni Novgorod 4(3) 1017-1018