Structural-analytical mesomechanics of a solid body with reversible martensitic transformations

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Abstract. In the given article the structurally-analytical mesomechanics for environments with martensitic mechanism of mass transfer is developed. Wave equations are derived to describe the deformation process initiated by martensitic transformations at meso and macro-scale levels.

1. Introduction
This article presents the analysis of structural and mechanical aspects of the problem of structural and analytical mesomechanics of multilevel media with reversible phase transformations of martensitic type and physical justification of the need to use the effective field method as a tool to take into account the interaction of deformation processes, the evolution of damage and reversible martensitic transformations at various scale and structural levels. Particular attention is paid to the factors of inheritance of defective structures in reversible transformations of austenite to martensite, as well as the development of a hierarchy scenario of martensitic transformations and plastic deformation. This scenario allows us to provide a holistic perception of the methodology of structural and analytical mesomechanics.

2. Analysis of structural and mechanical aspects of the problem
The task of creating the theory of deformation of materials with shape memory effect (SME) is to build a model of a deformable solid body, which should take into account its complex and multi-level structural organization, various deformation mechanisms, including dislocation slip, diffusion, martensitic transformations and evolution of the material damage on the one hand and generating their thermal effects and mechanical stresses on the other.

The task of solid state physics in this regard is to analyze the micro-scale level and build models of mechanical behavior of crystals, which take into account specific physical mechanisms and their influence on the corresponding parameters of the governing equations, taking into account the structural organization of crystals. The physics of plasticity and fracture allows to analyze crystallography and thermodynamics of phase transformations, to describe the laws of motion of structural imperfections in a loaded solid body, using the methodology of the theory of defects, in particular, the apparatus of the theory of dislocations. Significant progress has been made in the study of dislocation plasticity of crystals. A large array of experimental data obtained using a well-developed experimental technique gives a broad idea of the mechanism of formation of elementary acts and laws of plasticity. Here not only the structural and physical mechanisms of inelastic deformation processes are understood but also...
effective methods of calculation are created. At the same time, the physical theory of plasticity of crystals did not reach engineering level, having kept the value only for an explanation and the description of elementary acts of deformation and destruction or close to them. In the framework of this approach the basic mechanisms of the motion of defects on the microscale level and the quality of the interpretation of many laws of macrodeformation and macroresults are studied. Nevertheless, the analysis of the stress-strain state of the macroscopic system as a whole is beyond the capabilities of the microscopic approach of the defect theory [1–3].

The mechanics of the deformed solid provides for the creation of analytical models that allow to predict the mechanical behavior of real macroscopic products. In this case, the problem is solved by means of phenomenological hypotheses formulated on the basis of experimental data on the mechanical behavior of macroscopic samples under given loading programs and involving the basic laws of mechanics: dynamic equilibrium equations, continuity equations for deformations, balance conditions for temperature and defining equations of state of phenomenological nature. The calculation of the stress-strain state of a solid body performed within the framework of the considered approach doesn’t take into account the architecture of the internal structure of the material, the processes of martensitic transformations of crystal lattices and real deformation mechanisms. As a result, the theory of defects and phase transformations usually is not used in the models of solid mechanics.

It should be noted that the mechanics of plasticity of materials (experiencing dislocation not elasticity) is widespread in engineering practice and has a good analytical base. However, its substantial progress is very modest. Being openly phenomenological, it describes only the regularities, on which the analytical relation is calibrated. The predictive power of the equations of plasticity mechanics in relation to complex methods of mechanical, thermal and other methods of influence on the material is unsatisfactory. It should be noted that the fundamental principles of plasticity mechanics used in the derivation of defining relations, such as the Drukker and Odquist postulates, the hypothesis of the existence of yield surfaces, a single deformation curve, established at the time on the basis of the analysis of experimental study of the behavior of objects such as iron or copper, do not stand up to criticism with respect to a number of new materials, in particular to materials with shape memory effect or to complex (non-trivial) modes of deformation. For example, in titanium nickelide (the most well-known representative of the material having EPF), deformation hardening can be determined not by the length of the deformation path, as in steel, but depends on the final value of the deformation. In the same object the deformation can initiate the release, not the absorption of energy, etc. [1, 3].

The above-mentioned examples, the number of which can be increased, rightly raise the question of the causes of the situation and possible solutions to the problem. We will analyze the denoted issues.

The reasons for the absence of the physical theory of plasticity on the engineering level are quite obvious. In addition to microstructural acts of plasticity, the laws of which are well studied by analyzing the behavior of single dislocations or their simplest formations, large-scale processes play an important role. In complex ensembles of defective formations, powerful collective effects come into power. This leads to the fact that the properties of the ensemble are not identical to the properties of single dislocations that make up the ensemble. Strong inside ensemble interactions create a complex collective properties. In large-scale ensembles, the properties of self-organization of the structure can come to the fore, which in terms of synergetics should be considered as dissipative [2, 3]. The material undergoes a variety of kinetic phase transitions, the control parameter of which is not only temperature, but also other variables, for example, the total density of dislocations. Moreover, in complex structures, in addition to translational plasticity, rotational plasticity is inevitably excited and characteristic turbulence occurs. One more large-scale level is involved in the process. As the analysis of experimental data shows, in real highly plastic objects of loading is accompanied by mass transfer of matter at several structural and large-scale interacting levels [2].

This analysis allows us to draw the most important conclusion that the consistent physical consideration of the problem of plasticity requires the correct consideration of numerous ways to implement the elementary act not only on the lower deformation floor but also on each of the subsequent scales of structures, their properties and laws of evolution as well as the nature of the inter-level mutual
influence and interaction between structures of one type. It is obvious that macroscopic properties are formed at all stages of the mass transfer process and cannot be reduced to only one of them [1–4].

It is necessary to formulate such a theory of deformation that would be based on strict physical postulates, it means taking into account real physical processes and at the same time would allow to solve practical problems of engineering nature. Numerous attempts to build such a theory have been made long time ago but the hope to move directly from the micro-scale to the macro-scale area with the help of various methods of orientation and statistical averaging have not been successful [2].

As it is noted in [2, 4], the current situation is determined by two fundamental circumstances: first, a consistent and correct description of the evolution of the complex stochastic distribution of martensitic lamellae and their ensembles faces with insurmountable mathematical difficulties and secondly, self-organization of dislocation ensembles and martensitic lamellae leads to a new quality – in a continuous environment there is a move of larger than the dislocation of meso defects [2]. For the transition from the microstructural level to the macroscopic level, it is necessary to take into account the contribution of the evolution of the intermediate meso structural level where the move of the corresponding meso defects dominates, providing the formation of translational-rotational deformation modes.

The formulation of a general algorithm for constructing a model of physical mesomechanics to describe the deformation of a solid body with any internal structure, taking into account the possibility of transformation of the structure during the phase transformations and for arbitrary modes of its loading is an important problem. This article is devoted to one of the possible solutions to this problem based on the development of methods of structural and analytical mesomechanics [4] for materials with shape memory effect.

3. The script hierarchy of scale and structural levels

Deformation of a loaded object deformed by initiation of reversible martensitic reactions can be represented as a multilevel relaxation process. We will briefly consider the main stages of the origin and development of mass transfer processes in reversible phase transformations of martensitic nature. For convenience, figure 1 shows the scheme of scale and structural levels, adopted during the construction of the model.

According to the data of physical studies, the first loss of shear stability occurs at the micro level, in the local areas of the crystal, where the nuclei of the martensitic phase occur. Microconcentrators of stresses arising at the structural inhomogeneities cause martensitic transformation of the crystal lattice in certain crystallographic planes and directions. Such local structural transformation appears as the emergence and development of martensitic nuclei in the austenitic matrix. The indicated stage is shown in figure 1A, where the shaded areas are schematically represented by martensite nuclei. Since the influence of microconcentrators is short-range, the resulting distortion transformations with a good approximation can be characterized by translational modes of deformation.

In the process of deformation the density of the forming martensitic crystals increases, the variants of martensitic transformations that do not coincide with the favorably oriented directions in the field of external stresses due to the loading trajectories in the stress space are involved.

The mentioned circumstances cause the need for self-organization of martensitic domains in accordance with the given boundary conditions, that means with the directions of the principal axes of the strain tensor at each moment of the deformation process. As a result, there are self-encoded groups of martensitic crystals forming dissipative structures within the initial structure of the material, in accordance with figure 1b. Such dissipative structures contribute to the appearance of translational-rotational deformation modes according to the shift+rotation scheme and are classified as mesoscale level 1.

At the considered scale and structural level, the hydrodynamic characteristics of the substance, depending on the history of thermomechanical action and the type of stress-strain state, are clearly manifested [3, 4].
Figure 1. Scheme of scale and structural levels of a deformable object made of materials with a martensitic mechanism of mass transfer, the shaded areas represent the nuclei of the martensitic crystals: a – micro, b – meso-1, c – meso-2, d – macro-1.

With the development of martensitic reactions, the density of structural imperfections of a larger scale level increases and, at a certain critical value of meso concentrators, when the distortion of the transformation at the meso-1 level reaches the limit value, the shear stability is lost in the local zones of the next mesostructural level, meso-2, figure 1c. Thus, structural changes in areas of considerable length and in arbitrary directions, not necessarily coinciding with crystallographic variants of shifts at martensitic transformations, become possible. Extended band-pass deformation structures appear, they originate on meso stress concentrators and spread over long distances through many structural elements regardless of the crystallographic orientation. At this scale level, the stochastic features of the formation of the translational-rotational field of the deformable material are of particular importance [3, 4].

Self-organization of complex stochastic processes in various volumes of the large-scale level meso-2 leads to the formation of a non-trivial tensor properties at the scale level of particulate substances, i.e., at the level of macro-1, figure 1d. Inhomogeneous distribution of macrodeformation in the loaded actual
product requires analysis of the stress-strain state in the framework of solution of the boundary task of mechanics of deformable solids, that means the consideration of the large-scale macro-2, figure 1.

It should be noted that the scheme of structural and scale levels of the deformation process given in figure 1 under the conditions of phase transformations of martensitic type, is very conditional, however, it allows to properly represent the sequence and the main stages of modeling, which in the following sections are formulated in the form of the corresponding connected defining relations. A brief overview of mathematical models of martensitic transformations initiated by phase deformations and structural stresses is given further.

4. Model of martensitic transformations process at the micro-scale level
Characteristic averaging volume \( V_0 \gg a^3 \), where \( a \) - is the lattice parameter [2].

Micro-level deformation \( \beta^{ik}_{\Phi} \) in martensitic reactions is identified with distortion of transformation \( D_\Phi \). The kinetics of the accumulation of the martensitic phase (\( \Phi \)) is shown in the figure 2 [2, 3]:

![Figure 2](image-url)

\[
\Phi = \tilde{T}^* \left\{ H(1 - \Phi_S)H(-\tilde{T}^*)H[M_H - \Phi(M_H - M_K)] - T^* \left[ M_H - M_K \right]^{-1} + \right. \\
\left. + H(\Phi)H(\tilde{T}^*)H[M_H - \Phi(A_K - A_H)A_K - A_H]^{-1} \right\}; \]

\[
\beta^{ik}_{\Phi} = \beta^{21}_{ik} (\dot{\delta}^{i3}_{k3} + \dot{\delta}^{i1}_{k3}) \beta^{31}_{ik} = D_{31} \Phi; \]

\[
D_{ik} = D_{31} (\dot{\delta}^{i3}_{k1} + \dot{\delta}^{i1}_{k3}) \tau^{ik} = \tilde{T} - \frac{T_0}{q_0} D_{ik} \tau^{ik}; \]

\[
\tau^{ik} = \beta^{31}_{ik} (\dot{\delta}^{i3}_{i3} \dot{\delta}^{i3}_{k3} + \dot{\delta}^{i1}_{i3} \dot{\delta}^{i3}_{k3}) \tau^{ik} = \tau^{ik} - \psi^{ik}; \tau^{ik} = \alpha_{pi3qk} \sigma_{pq} \psi^{ik} = \alpha_{pi3qk} \rho_{pq} (1) \]

In the formula (1) \( T^* \) - effective temperature, \( D_\Phi \) - tensor distortion of the phase origin, \( T_0 \) - the temperature of thermodynamic balance; \( q_0 \) - thermal effect of reaction; \( \tau_\delta \) - voltage induced by external loads; \( \psi^{ik} \) - structural strains; \( \dot{\delta}^{i3}_{k3} \) - Kronecker delta, \( H \) - Heaviside function, \( \Phi_S \) - the total phase at the macro level; \( M_H, M_K, A_H, A_K \) - characteristic temperatures; \( \alpha_{ik} \) - direction cosines that transform local basis in the global

5. Model of the process of martensitic transformations at the mesoscale level \( V_{m1} \) MESO-1
\( (V_0 << V_{m1} << V_{m2}) \), where \( V_{m2} \) - characteristic volume meso-2 [2-5].

Deformation at the meso-level -1 \( L^{\Phi}_{ik} \) in martensitic reactions it is identified with meso-1 distortion \( B^{\Phi}_{ik} \). The governing equations in the considered scale and structural levels obey the conservation equations of hydrodynamic type. The modification of these equations is represented by the proportion (2).

\[
B^{\Phi}_{ik} = B^{\Phi}_{31} \delta^{i3}_{k1}, B^{\Phi}_{31} = -\nabla \Omega - \tilde{T} \beta + \sigma \beta, L^{\Phi}_{ik} = L^{\Phi}_{31} \delta^{i3}_{k1}, L^{\Phi}_{31} = B^{\Phi}_{31} \Phi^{0}_{m1} ; \]
\[ \phi_{m1}^{0} = \int_{0}^{t} \int_{-D}^{D} \int \psi(S,G) \phi(S,G) dS \cdot G dS \cdot J dS; \]

\[ \langle \dot{\beta}_{31}^{\phi} \rangle = \int_{-D}^{D} \int_{-D}^{D} \int \psi(S,G) \phi(S,G) \dot{\beta}_{31}^{\phi} dS \cdot G dS \cdot J dS; \]

\[ I_{\beta} = A_{1\Phi} \bar{V}_{\Omega} \langle \dot{\beta}_{31}^{\phi} \rangle; \]

\[ \langle \dot{\beta}_{31}^{\phi} \rangle = \int_{0}^{t} \langle \dot{\beta}_{31}^{\phi} \rangle dS; \]

\[ \sigma_{\beta} = K_{1\Phi} \langle \dot{\beta}_{31}^{\phi} \rangle + I_{l\Phi} (\bar{V}_{\Omega} \times \bar{a}_{\Phi}) \cdot \bar{e}_{n} \]

\[ B \bar{a}_{\Phi} = \langle \dot{\beta}_{31}^{\phi} \rangle \langle \dot{e}_{i} \rangle; I_{1\Phi} = \eta_{1\Phi} \left( \frac{\text{mod} \bar{V}_{\Omega} \langle \dot{\beta}_{31}^{\phi} \rangle}{\sqrt{2} \Omega \langle \dot{\beta}_{31}^{\phi} \rangle} \right) \]

In the formulae (2) \( B_{ik}^{\phi} \) - transformation distortion on meso-1; vector \( I_{\beta} \) characterizes the density of the distortion flow on meso-1; parameter \( \sigma_{\beta} \) describes the production of an internal source of distortion \( B_{31}^{\phi} \) due to the initiation in meso amount \( V_{m1} \) statistical ensemble of velocity distortions of the martensitic transformation (\( \langle \dot{\beta}_{31}^{\phi} \rangle \)) and the circulation flow of the deformation field (component \( l_{1}(\bar{V}_{\Omega} \times a_{\Phi}) \cdot \bar{e}_{n} \)). Parameter \( I_{l} \) characterizes the size of the emerging vortex; \( K_{1\Phi}, A_{1\Phi}, \eta_{1\Phi} \) - the constants of a substance; \( \bar{V}_{\Omega} \) - the operator "nabla" for the orientation of the coordinate system.

6. Model of the process of martensitic transformations at the mesoscale level \( V_{m2} \) Meso-2
\( (V_{m2}<<V_{m2}), \) where \( V_{m2} \) - characteristic volume meso-2 [3-7].

Distortion rate \( \phi_{ik}^{\phi} \) and the intensity of structural strains \( \dot{q}_{ik}^{\phi} \) at the considered scale level we will describe a system of integro-differential equations that takes into account the condition of preserving the continuity of the environment. Within the development of the model the continuity condition of the environment means the need to harmonize the sliding velocity of the considered bands and shifts in the random meso bands of deformation. Using a technique developed in statistical mechanics of continuum, the following analytical relations can be formulated for meso-2:

\[ \phi_{ik}^{\phi} = \phi_{31}^{\phi} \delta_{i3} \delta_{k1}; \]

\[ \dot{q}_{31}^{\phi} (\Omega_{i}) = \int f(\Omega_{i}) R(\Omega_{i}, \Omega_{i}) \frac{1}{2} \left( \alpha_{m3}(\Omega_{i}) \alpha_{n1}(\Omega_{i}) + \alpha_{m1}(\Omega_{i}) \alpha_{n3}(\Omega_{i}) \right) \bar{D}(\phi_{mn}^{\phi}) \Omega_{i}; \]

\[ \phi_{31}^{\phi} = \int f(\Omega_{am}) A_{m}(\Omega_{am}, \Omega_{am}) \frac{1}{2} \left( \alpha_{m3}(\Omega_{am}) \alpha_{n1}(\Omega_{am}) + \alpha_{m1}(\Omega_{am}) \alpha_{n3}(\Omega_{am}) \right) \times \]

\[ \times \bar{D}(\sigma_{mn}^{*}) d\Omega_{am} + \int f(\Omega_{ma}) M_{A}(\Omega_{ma}, \Omega_{ma}) \frac{1}{2} \left( \alpha_{m3}(\Omega_{ma}) \alpha_{n1}(\Omega_{ma}) + \alpha_{m1}(\Omega_{ma}) \alpha_{n3}(\Omega_{ma}) \right) \times \]

\[ \times \bar{D}(\varepsilon_{mn}^{*}) d\Omega_{ma}; \]

\[ \bar{D}(\sigma_{ik}^{*}) = \sigma_{ik}^{*} / \sigma_{i}^{*} \sigma_{ik}^{*} = \sigma_{ik} - \rho_{ik}^{*} \sigma_{i}^{*} = \frac{1}{\sqrt{2}} \left( \sigma_{ik}^{*} \sigma_{ik}^{*} \right)^{1/2}; \]
\[ \overline{D}(\tilde{\Phi}_{ik}) = \tilde{\epsilon}_{ik}^\Phi / \tilde{\epsilon}_{ik}^\Phi : \tilde{\epsilon}_{ik}^\Phi = \frac{1}{\sqrt{2}} (\tilde{\epsilon}_{ik}^\Phi \cdot \tilde{\epsilon}_{ik}^\Phi)^{1/2} \] (3)

In the formulae (3) \( \overline{D}(\tilde{\sigma}_{ik}^\Phi) \) – guiding tensors of effective stresses \( \tilde{\sigma}_{ik}^\Phi \) and phase deformation rates \( \tilde{\epsilon}_{ik}^\Phi \); \( f(\Omega_i) \) and \( f(\Omega'_i) \) – density distribution by orientation \( \{\Omega_i\} \) and \( \{\Omega'_i\} \); \( A_m(\Omega_{am}, \Omega'_{am}) \) \( M_a(\Omega_{ma}, \Omega'_{ma}) \) and \( R(\Omega_i, \Omega'_i) \) – impact functions that make sense of structural pliability, structural shape memory and structural relaxation, respectively; \( q_{31}^\Phi(\Omega_i) \) – structural stress intensities \( \rho_{31} \); \( \{\Omega_i\} \) – areas of orientation space where shifts occur; \( \{\Omega'_i\} \) – spheres of impact [2].

7. Model of the process of martensitic transformations at the macro-scale level V Macro level-1 \( (V_{m2}<<V<<R^3) \), where \( R \) – the characteristic size of macro-production.

At the macro level, in the results of orientation averaging, the defining relations are obtained in the traditional form adopted in the mechanics of a deformable solid [2–7].

\[ \Phi_\Sigma = \int f(\Omega_0)\Phi_{m1}^0(\Omega_0) d\Omega_0; \]

\[ \tilde{\epsilon}_{ik}^\Phi \{V_0\} = \frac{1}{2} \int f(\Omega_0)(\alpha_{i3}\alpha_{k1} + \alpha_{i1}\alpha_{k3})(\beta_{31}(\Omega_0)) d\Omega_0; \]

\[ \tilde{\epsilon}_{ik}^\Phi \{V_m1\} = \frac{1}{2} \int f(\Omega_m)(\alpha_{i3}\alpha_{k1} + \alpha_{i1}\alpha_{k3})\tilde{\epsilon}_{31}(\Omega_m) d\Omega_m; \]

\[ \tilde{\epsilon}_{ik}^\Phi \{V_m2\} = \frac{1}{2} \int f(\Omega_i)(\alpha_{i3}(\Omega_i)\alpha_{k1}(\Omega_i) + \alpha_{i1}(\Omega_i)\alpha_{k3}(\Omega_i))\tilde{\epsilon}_{31}^\Phi d\Omega_i. \]

\[ \tilde{\rho}_{ik}(i) = \frac{1}{2} \int f(\Omega_i)(\alpha_{i3}(\Omega_i)\alpha_{k1}(\Omega_i) + \alpha_{i1}(\Omega_i)\alpha_{k3}(\Omega_i))\tilde{\epsilon}_{31}^\Phi d\Omega_i. \]

\[ \tilde{\epsilon}_{ik}^\Phi \{V_m2\} = \Delta_{ikpq}\sigma_{pq}^\Phi + M_{ikpq}\tilde{\epsilon}_{pq}^\Phi; \]

\[ \sigma_{ik}^\Phi = \sigma_{ik} - \rho_{ik}; \tilde{\rho}_{ik} = R_{ikpq}\tilde{\epsilon}_{pq}. \] (5)

Here, \( \tilde{\epsilon}_{ik}^\Phi \{V_0\}, \tilde{\epsilon}_{ik}^\Phi \{V_m1\}, \tilde{\epsilon}_{ik}^\Phi \{V_m2\} \) – macroscopic strain tensors characterizing the contribution of mass transfer processes at micro, meso-1 and meso-2 structural and scale levels, respectively. The correlations (1)-(5) are discussed in more detail in [4].

8. Mesomechanics model taking into account the wave character of deformation

As it is known [4], the peculiarity of the meso bands motion in the deformable solid is the complex nature of the interaction of effective shear stresses with the subsystem of structural imperfections, which in the given boundary conditions (determined by the direction of the main axes of the stress state in the stress space at "soft" loading trajectories or corresponding directions in the deformation space under rigid loading conditions) leads to the wave nature of the plastic flow spread.

We take as a generalized coordinate the shear component of the distortion tensor \( \Phi_{31}^\Phi \) at the meso-2 scale level [4]. It is necessary to enter the tensor parameter to construct the wave equation \( \Delta_{\Omega} \Phi_{31}^\Phi \delta_{i3} \delta_{k1} \) (\( \Delta_{\Omega} \)– Laplace operator). However, it is more appropriate instead \( \Delta_{\Omega} \Phi_{31}^\Phi \delta_{i3} \delta_{k1} \) to consider the
appropriate analogue, by component \( B_{31}^\Phi \) [4], as a parameter \( \Delta \Omega B_{31}^\Phi \delta_{33} \delta k_1 \). This technique is inconsistent for the transfer processes described at the same scale level. In our case of multilevel analysis, this approach is natural and methodically justified, since it allows us to organize additional coherence of the processes of evolution of deformation structures at the micro-, meso- and macro-scale levels. Taking into account the abovementioned, we introduce a tensor parameter \( d_{ik} \) in the form:

\[
d_{ik} = \Delta \Omega B_{31}^\Phi \delta_{33} \delta k_1 = d_{31} \delta_{33} \delta k_1
\]  

The wave equation for the shear strain component at the meso structural level of meso-2 can be represented by the following equation:

\[m_2 \dot{\gamma}_{31}^\Phi (\Omega) + m_1 \dot{\gamma}_{31}^\Phi (\Omega) + m_0 \dot{\gamma}_{31}^\Phi (\Omega) = A_B d_{31} (\Omega) + \]

\[+ \int f(\Omega) A(\Omega, \Omega) \frac{1}{2} [\alpha_m^3 (\Omega) \alpha_{nl} (\Omega) + \alpha_{ml} (\Omega) \alpha_{n3} (\Omega)] \Phi_{mn}^* \delta_{am} + \]

\[+ \int f(\Omega') M_A (\Omega, \Omega') \frac{1}{2} [\alpha_m^3 (\Omega') \alpha_{nl} (\Omega') + \alpha_{ml} (\Omega') \alpha_{n3} (\Omega')] \Phi_{mn}^* \delta_{ma}
\]  

Here \( \Delta \Omega \) – Laplace operator for hydrodynamic orientation space \( \{ \Omega_2 \} \), \( m_0, m_1, m_2, A_B \) – the material constants. Availability is dependent on the orientation of coordinates and time, the components of the right side of the equation (7) indicates a strong attenuation of the waves of plastic flow in a structurally heterogeneous environment. The specificity of the initiation of meso structure vibrations in the deformation process allows us to characterize it as an active excitable environment.

The available experimental data [4] convincingly testify about the auto wave nature of shifts of meso bands of deformation at the meso structural level. Equation (7) is a natural generalization of equation (3) and takes into account the wave nature of the deformation process at the meso structural level.

Taking into account the description of auto wave processes at the scale level of macro-1, leads to the appearance in the equation for the calculation of the components of the tensor of inelastic deformation \( \epsilon_{ik} (V_{m2}) \) of derivatives, both of the first and second order, which naturally describes the oscillating nature of the spread of the traveling pulse in the active elastoplastic environment.

Having performed the orientation averaging of the relations (7) of the mesoscale level, we will obtain the desired wave equations at the macro-scale level in the form:

\[m_2 \dot{\epsilon}_{ik} (V_{m2}) + m_1 \dot{\epsilon}_{ik} (V_{m2}) + m_0 \dot{\epsilon}_{ik} (V_{m2}) = A_B D_{ik} (V_{m2}) + A_{iknm}^\Phi \sigma_{mn}^* + M_{iknm}^\Phi \phi_{mn}
\]  

Here parameter \( D_{ik} (V_{m2}) \) is calculated with a formula:

\[D_{ik} (V_{m2}) = \frac{1}{2} \int f(\Omega_2) [\alpha_{ik} (\Omega) \alpha_k (\Omega) + \alpha_{il} (\Omega) \alpha_{k3} (\Omega)] i_{31} (\Omega) d\Omega
\]  

Equations (8) include tensor parameters characterizing the evolution of the structure at the macro-scale level: \( A_{iknm}^\Phi \), \( M_{iknm}^\Phi \) and \( \epsilon_{mn}^\Phi \), \( \sigma_{mn}^* \). Here \( A_{iknm}^\Phi \) the kinetic coefficients of the structural compliance, the kinetic coefficients of the structural shape memory, \( \sigma_{mn}^* \) the effective stress. The indicated parameters are functionals depending on the history of loading in time and in the space of stresses (deformations), and are calculated by the method described in [4] in the derivation of the defining relations for the macro-scale level.
In the framework of this model, the basic functional and mechanical properties of materials with shape memory effect are predicted taking into account the wave character of deformations initiated by martensitic transformations.

To solve the practical problems of engineering mechanics [4], additional equations are required to take into account the spatial arrangement of volumes V (consideration of the scale level of macro-2), i.e. the traditional static and geometric equations of the boundary value problem of mechanics as well as the initial and boundary conditions for the corresponding variables.

9. **Formulation of the boundary task at the macro-scale level of macro-2**

Analytical relations (1) – (9) should be considered as defining equations for structurally inhomogeneous environment in macropoint, see figure 1d. To solve the practical problems of engineering mechanics, it is necessary, of course, to formulate additional equations that take into account the spatial arrangement of the volumes V1, that is the consideration of the scale level of macro, see figure 1e, dynamic and static properties of the environment, etc., as well as the initial and boundary conditions for the corresponding variables. The above mentioned is reduced to the formulation of the following equations of the boundary value problem of mechanics:

9.1. **The dynamic equation of balance for voltage:**

\[ \Delta_i \sigma_{ik} = \rho u_{ik} \]  

(10)

Where \( \rho \) – density of environment; \( u_{ik} \) – displacement vector;

9.2. **The condition of continuity for the deformation:**

\[ \varepsilon_{kst} \varepsilon_{qmt} \nabla_s \nabla_t \varepsilon_m = -\varepsilon_{kst} \varepsilon_{qmt} \nabla_s \nabla_t \varepsilon_m^H \]  

(11)

where \( \varepsilon_{kst} \) – Levi-Civita symbol; \( \varepsilon_{ik}^H = \varepsilon_{ik}^T + \varepsilon_{ik}^\Phi \);

9.3. **Balance condition for temperature:**

\[ T = \frac{K_{ik}}{\rho \cdot c} \nabla_i \nabla_k T - \frac{q_{0}}{\rho \cdot c} \Phi_{ik} \]  

(12)

where \( K_{ik} \) – tensor of thermal conductivity coefficients; \( c \) – specific heat. The second term in the right part (12), containing \( \Phi_{ik} \), takes into account the thermal effect of the martensitic transformation reaction. In the case of isotropic heat transfer at the macro level the thermal conductivity tensor is equal to:

\[ K_{ik} = k_0 \delta_{ik} \]

where \( k_0 \) – scalar coefficient of thermal conductivity.

For the final formulation of a specific boundary task, equations for voltage (10), deformations (11) and temperatures (12) must be supplemented with the corresponding boundary and initial conditions.

10. **Conclusion**

It is necessary to point out the following important aspects. In contrast to the traditional methodology of continuum mechanics, when the components of strain tensor are introduced as derivatives of the displacement field of points of the continuum, in this approach, they are specified through appropriate micro- and meso-characteristics, taking into account physical regularities in the formation of martensite crystals at the micro-level and processes of self-organization of the martensitic structures at the mesoscale level. In the physical sense, the formulated analytical relations take into account not only the translational and rotational nature of mass transfer at the mesoscopic level, but also the cross-effects of the interaction of elements of the medium of different scale levels.
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