Theoretical phase diagram of bilayer $\nu = 1$ quantum Hall system

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Abstract. We theoretically study the bilayer $\nu = 1$ quantum Hall system under an in-plane magnetic field. When the in-plane field becomes strong, interlayer coherence in the system is suppressed by the field and the ground state is expected to show a phase transition from a commensurate state to an incommensurate one. We investigate a phase diagram of this system by analyzing the mode-softening properties of the random-phase-approximation excitation spectrum. The diagram indicates another phase between these ones and shows a qualitatively good agreement with the one obtained by a recent experiment.

1. Introduction
Over the decade the bilayer quantum Hall (QH) system at total Landau level filling $\nu = 1$ has attracted much attention because it is expected to have spontaneous interlayer phase coherence [1]. Interlayer coherence in the bilayer $\nu = 1$ QH system is suppressed by applied in-plane magnetic field and then the ground state is considered to show a phase transition from a commensurate state to an incommensurate one [2]. Very recently, a bump in longitudinal resistance was observed in the vicinity of this transition point, and it was also claimed that another phase can be realized between these two phases [3, 4]. In this paper, we show a new theoretical phase diagram based on the random-phase-approximation (RPA) approach [5] and show that the phase diagram qualitatively agrees with the one claimed in the recent experimental study.

2. Theoretical Formulation
We consider a two-dimensional electron system on two parallel layers between which interlayer tunneling of electron exists [1]. When a strong magnetic field is applied perpendicularly to the layers, the two-dimensional kinetic energy is quantized to a series of Landau levels. For simplicity, we consider only the lowest Landau level and assume that total electron spin is polarized because of large Zeeman energy. The layer degrees of freedom (which layer is occupied by an electron) are described in terms of pseudospin, $\uparrow/\downarrow$. In the presence of interlayer tunneling, single-particle states are split into bonding and anti-bonding combinations of each-layer state and the energy of bonding states is lower than that of anti-bonding ones. When applied the magnetic field has
an in-plane component, the creation operators of bonding and anti-bonding states are given by
\[
bq_j = \frac{1}{\sqrt{\mathcal{N}_L}} \left( c_{j,\uparrow}^\dagger + e^{iQjk}\epsilon_{j,\downarrow}^\dagger \right), \quad \alpha_j = \frac{1}{\sqrt{\mathcal{N}_L}} \left( c_{j,\downarrow}^\dagger - e^{iQjk}\epsilon_{j,\uparrow}^\dagger \right),
\]
where \(c_{j,\uparrow}^\dagger\) (\(c_{j,\downarrow}^\dagger\)) creates an electron with pseudospin \(\uparrow\) (\(\downarrow\)) in the \(j\)-th (\(k\)-th) orbit in the lowest Landau level, \(Q = edB_0/c\hbar\), and \(x_j\) is the \(x\) coordinate of the guiding center of the \(j\)-th orbit in the Landau gauge \(\mathbf{A}(\mathbf{r}) = (0, B_{\perp}x - B_0z, 0)\). Here \(B_{\parallel}\) and \(B_{\perp}\) are in-plane and perpendicular components of the applied magnetic field, respectively, and \(d\) is the interlayer distance. The energy difference between bonding and anti-bonding states is given by \(\Delta = \Delta_{\text{SAS}} \exp(-Ql_B^2/4)\), where \(l_B \equiv \sqrt{\hbar/eB_0}\) and \(\Delta_{\text{SAS}}\) is the tunneling gap in the absence of in-plane field. Since we consider only the case of the Landau level filling \(\nu = 1\), the perpendicular component \(B_{\perp}\) is fixed. Thus \(Ql_B = B_{\parallel}/B_{\perp} \times d/l_B\) can be used as a parameter giving the strength of in-plane field.

In our random-phase-approximation (RPA) approach [5], the state fully occupying the bonding subband is used as a reference state and a pair of an electron in anti-bonding subband and a hole in bonding one is treated as a boson excitation. Under the periodic boundary condition, the boson creation operator for wave vector \(\mathbf{q}\) is given by
\[
\beta_{\mathbf{q}}^\dagger = \frac{1}{\sqrt{\mathcal{N}_L}} \sum_{j_1, j_2} \delta (j_1 - j_2) \frac{q_y L_y}{2\pi} \cos(q_x x_j) \alpha_{j_1}^\dagger \beta_{j_2},
\]
where \(\delta(j_1, j_2) = 1\) when \(j_1 = j_2\) (mod \(N_L\)) and zero otherwise, \(L_y\) (\(L_x\)) is the length of the layers along the \(y\) (\(x\)) direction, and \(N_L = L_x L_y/2\pi l_B^2\) is the Landau level degeneracy. In this approach, one can construct the RPA Hamiltonian which has a bilinear form of the boson operators as
\[
\mathcal{H}_{\text{RPA}} = \sum_{\mathbf{q} \in \mathbb{B.Z.}} \left[ \epsilon(\mathbf{q}) + \lambda(\mathbf{q}) \right] \beta_{\mathbf{q}}^\dagger \beta_{\mathbf{q}} + \frac{1}{2} \lambda(\mathbf{q}) \left( e^{i\phi_{\mathbf{q}}} q_y q_y - e^{i\phi_{\mathbf{q}}}/\mathbf{q} \right)_{\mathbf{q} \rightarrow \mathbf{-q}} + \text{h.c.},
\]
\[
\epsilon(\mathbf{q}) = \Delta + \frac{1}{L_x L_y} \sum_{\mathbf{k} \neq 0} V_{\text{E}}(\mathbf{k}) \cos(Qk_y l_B^2) \exp(-k^2 l_B^2/2) \left[ 1 - \cos[(\mathbf{k} \wedge \mathbf{q}) l_B^2] \right],
\]
\[
\lambda(\mathbf{q}) = \frac{N_L}{L_x L_y} \sum_{\mathbf{k} \neq 0} \frac{V_{\text{A}}(\mathbf{k}) - V_{\text{E}}(\mathbf{k}) \cos(Qk_y l_B^2)}{2} \exp(-k^2 l_B^2/2) \delta \left( \frac{L_x k_x}{2\pi}, \frac{L_y k_y}{2\pi} \right) \delta \left( \frac{L_x q_x}{2\pi}, \frac{L_y q_y}{2\pi} \right) - \frac{1}{L_x L_y} \sum_{\mathbf{k} \neq 0} \frac{V_{\text{A}}(\mathbf{k}) - V_{\text{E}}(\mathbf{k}) \cos(Qk_y l_B^2)}{2} \exp(-k^2 l_B^2/2) \cos[(\mathbf{k} \wedge \mathbf{q}) l_B^2],
\]
where \(\mathbf{k} \wedge \mathbf{q} = k_x q_y - k_y q_x\), \(V_{\text{A}}(\mathbf{k}) = 2\pi e^2/\epsilon k\) and \(V_{\text{E}}(\mathbf{k}) = V_{\text{A}}(\mathbf{k}) e^{-kd}\) are the Fourier transforms of intralayer and interlayer Coulomb interactions (\(\epsilon\) dielectric constant of host material), respectively, and a wave vector \(\mathbf{q}\) in Brillouin zone (B.Z.) is given by \(q_x = 2\pi n_x/L_x\) and \(q_y = 2\pi n_y/L_y\) with integers \(n_x\) and \(n_y\) satisfying \(-N_L/2 < n_x, n_y \leq N_L/2\). The RPA excitation spectrum can be obtained by the following Bogoliubov transformation:
\[
\mathcal{H}_{\text{RPA}} = \sum_{\mathbf{q} \in \mathbb{B.Z.}} \omega(\mathbf{q}) \gamma_{\mathbf{q}}^\dagger \gamma_{\mathbf{q}} + \text{const.},
\]
\[
\gamma_{\mathbf{q}}^\dagger = \cosh \left( \frac{\phi_{\mathbf{q}}}{2} \right) \beta_{\mathbf{q}}^\dagger + e^{i\phi_{\mathbf{q}}} q_y q_y \sinh \left( \frac{\phi_{\mathbf{q}}}{2} \right) \beta_{\mathbf{-q}},
\]
\[
\omega(\mathbf{q}) = \sqrt{\epsilon(\mathbf{q}) \left[ \epsilon(\mathbf{q}) + 2\lambda(\mathbf{q}) \right]},
\]
where \(\tanh \phi_{\mathbf{q}} = \lambda(\mathbf{q})/|\epsilon(\mathbf{q}) + \lambda(\mathbf{q})|\). Our phase diagram is obtained by analyzing the mode-softening properties of the spectrum \(\omega(\mathbf{q})\).
Figure 1. Lowest-lying excitation spectrum as a function of wave vector $q$ for $d/l_B = 1.0$ and $QL_B = 0.0$ (a), $d/l_B = 2.0$ and $QL_B = 0.0$ (b), $d/l_B = 0.2$ and $QL_B = 1.5$ (c), and $d/l_B = 1.0$ and $QL_B = 1.5$ (d) in the bilayer $\nu = 1$ quantum Hall system with $\Delta S_{\text{SAS}}/(e^2/\epsilon l_B) = 0.1$ and Landau-level degeneracy $N_L = 32$. Closed squares linked with dashed line (closed circles linked with solid line) show the excitation spectrum along the $q_x$ ($q_y$) direction.

## 3. Results

Four types, (I)-(IV), of excitation spectra are obtained from Eq.(4) and their typical examples can be seen in Figure 1(a)-(d), respectively: (I) Excitation spectrum is gapful in every direction, (II) Mode softening occurs at $QL_B \simeq 1$ in every direction, (III) Mode softening occurs at $QL_B = 0$, (IV) Mode softening occurs at $q_x l_B \simeq 1$ and the spectrum is gapful along the $q_y$ direction. By investigating the excitation spectrum in Eq.(4) on the $d/l_B$ versus $QL_B$ ($\propto B$) plane, we obtain a phase diagram shown in Figure 2, where $d/l_B$ is the interlayer distance normalized by the magnetic length $l_B$ and $QL_B$ gives the in-plane field strength. The phases I-IV in Fig. 2 correspond to these types (I)-(IV) [i.e., excitation spectra in Fig. 1(a)-(d)], respectively.

In the phase I, there exists a finite gap in the excitation spectrum. The ground state is well described by the reference state fully occupying the bonding subband and a quantum Hall state can be stabilized by the energy gap. Since the reference state is the one obtained by diagonalising the tunneling term of the Hamiltonian, the ground state is almost dominated by interlayer tunneling effects. In the pseudospin language, the tunneling term is the Zeeman term describing the coupling between pseudospin and pseudo magnetic field. Thus each pseudospin is aligned along the direction of pseudo magnetic field in this phase and the ground state is a commensurate state on pseudospin order. It is noted that the mode softening around $QL_B \approx 1$ between the phases I and II is considered to be a transition from QH to no-QH state [1]. Thus the phase II is a no-QH phase.

In the phase III, a mode softening at $q = 0$ can be seen as shown in Fig. 1(c). This suggests that there exists a Goldstone mode in the phase. Here let us see pseudospin alignments in a coordinate system whose direction changes together with the one of pseudo magnetic field. In the commensurate state, each pseudospin is aligned along the direction of pseudo magnetic field and the translational symmetry is not broken in the coordinate system. On the other hand, an incommensurate phase has an spatial oscillation of pseudospin alignment in the coordinate system and the translational symmetry is broken. The Goldstone mode is considered to be related with this broken symmetry. Thus the transition from I to III is considered to be a commensurate-incommensurate transition [1].

The phase IV has not been found by the Hartree-Fock approach [1]. However, it exists over a
Figure 2. Theoretical phase diagram for $\Delta_{SAS}/(e^2/\epsilon_B) = 0.1$. An anisotropic phase IV does exist in addition to the phases I-III.

finite area on the $Ql_B$ versus $d/l_B$ plane. As seen in Fig. 1(d), this phase has anisotropic nature and the anisotropic one is consistent with a recent experimental observation [6].

4. Summary
We studied the excitation spectrum in the bilayer $\nu = 1$ quantum Hall system under an in-plane magnetic field by using the random-phase approximation. A new phase diagram was obtained by analyzing the mode-softening properties of the spectrum. In the phase diagram, there exists a new anisotropic phase over a finite area on the parameter plane in addition to the commensurate, incommensurate and no-QH phases.

Acknowledgments
This work was supported in part by Grants-in-Aid for Scientific Research (No. 14740181) and a 21st Century COE Program Grant of the International COE of Exploring New Science by Bridging Particle-Matter Hierarchy.

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