Asynchronous Finite-Time $H_{\infty}$ Control for Networked Switched Control Systems via Mode-Dependent Dynamic State-Feedback

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Abstract. In this paper, we studied asynchronous finite-time $H_{\infty}$ control for a class of Networked Switched Control Systems (NSCSs) with time-delays and time-varying disturbances. The concepts of stability and boundedness of NSCSs are given by 2 assumptions and 5 definitions at first. Then Linear Matrix Inequalities (LMIs), Average Dwell Time (ADT) and Multiple Lyapunov Functions (MLFs) are used to guarantee that the system is finite-time with a prescribed $H_{\infty}$ performance with asynchronously switched control. Moreover, a set of mode-dependent dynamic state feedback controllers are obtained to solve the finite-time stabilization problem. At last, two examples are given to confirm the efficiency of methods.

Introduction

Switched systems is a class of hybrid systems which consists of subsystems and switching law specifying the switching between the subsystems [1-2]. Due to their important value in theoretical development and practical applications, switched systems has attracted lots of attention during the last decades. In the paper [3], the authors focused on the stability analysis for switched linear systems under arbitrary switching, and described conditions for asymptotic stability of switched linear systems. Du H B, Lin X Z, Li S H [4] studied stabilization problems of a class of switched linear systems with time-varying exogenous disturbances with finite-time boundedness and designed a class of switching signals and stabilizing controllers which solved the finite-time stabilization problem. In a recent paper, Hao Liu, Yi Shen and Xudong Zhao [5] given a set of mode-dependent dynamic state feedback controllers by matrix inequalities, Average Dwell Time (ADT) and Multiple Lyapunov Functions (MLFs) which solved the finite-time stabilization problem. However, these articles simply studied the switched systems and didn’t take the advantages of switched systems into the most popular theories of current applications.

With the fast development of computer technology, communication researches about Network Control Systems (NCSs) are more and more deeply [6-8]. It is the integration of computer technology and the control theory. On the more practical side, more and more switched systems are connected with network, by connecting NCSs and Switched Systems, we have switched systems deployed over communication networks, and we also referred to as a “Networked Switched Control Systems” (NSCSs). NSCSs has attracted lots of attention during the past years [9-12]. However, no attention has paid to asynchronously finite-time $H_{\infty}$ control for NSCSs with time-delays and time-varying exogenous disturbances. Thus, to the best of the authors’ knowledge, we focus on this complex control problem.

The main contributions of this paper are given as follows. First, by using ADT and MLFs methods, some sufficient conditions which can guarantee that the NSCSs closed-loop system is finite are given. Unlike the results in [5], we consider the situation where independent delays are used in both state and control input. Then, the asynchronous finite-feedback $H_{\infty}$ control problem for NSCSs is solved by designing a set of mode-dependent dynamic state-feedback controllers and finding a set of switching signals satisfied a certain ADT condition.
Problem Formulation

Consider a class of the NSCSs described by

\[
\dot{x}(t) = A_\sigma x(t - \tau) + B_\sigma u(t - \tau) + G_\sigma \omega(t), t \in \mathbb{R}^n
\]  

(1)

\[
z(t) = E_\sigma x(t - \tau) + H_\sigma \omega(t)
\]  

(2)

Where \(x(t - \tau) \in \mathbb{R}^n\) is the state, \(u(t - \tau) \in \mathbb{R}^m\) is the control input, \(z(t) \in \mathbb{R}^q\) is the controlled output and \(\omega(t) \in \mathbb{R}^p\) is the exogenous disturbance. \(\tau\) is time-delays of Networked Switched Control Systems. \(\sigma\) is a piecewise constant function of time, which is so-called switching signal and takes its values in a finite set \(\xi = \{1, \ldots, N\}\), where \(N\) is the number of subsystems. Given a switching time sequence \(0 < \tau_1 < \tau_2 < \ldots\), \(\sigma\) is continuous from the right everywhere. When \(t \in [\tau_k, \tau_{k+1})\), the \(\sigma(t_i)\)th subsystems is activated and thus the trajectory \(x(t)\) of NSCSs (1) is the trajectory of the \(\sigma(t_i)\)th subsystem. \(A_i, B_i, G_i, E_i, H_i\) are constant real matrices for \(i \in \xi\).

Assumption 1. The trajectory \(x(t)\) is everywhere continuous, i.e., the state of the NSCSs does not jump at the switching instants.

Assumption 2. For a given constant \(T \), the exogenous disturbance \(\omega(t)\) is time-varying and satisfies the constraint \(\int_{t_0}^{t} \omega(t)\omega(t)dt \leq \alpha, \alpha \geq 0\).

Definition 1[13]. For a switching signal \(\sigma\) and any \(t_0 < t_1 < t_2\), let \(N_\sigma(t_1, t_2)\) be the switching numbers of \(\omega(t)\) over the interval \([t_1, t_2]\). If \(N_\sigma(t_1, t_2) \leq \alpha_0 + (t_2 - \tau)/\tau_{\alpha}\) holds for \(N_0 \geq 0, \tau_{\alpha} > 0\), then \(\tau_{\alpha}\) and \(N_0\) are called the average dwell time and the chatter bound, respectively.

Controller Design

In this paper, the following mode-dependent dynamic state-feedback controllers with order \(n_i\) are considered

\[
\dot{\tilde{x}}(t) = A_{c,\sigma} \tilde{x}(t - \tau) + B_{c,\sigma} x(t - \tau), \tilde{x}(0) = 0
\]  

(3)

\[
u(t) = C_{c,\sigma} \tilde{x}(t - \tau) + D_{c,\sigma} x(t - \tau)
\]  

(4)

Where \(A_{c,\sigma}, B_{c,\sigma}, C_{c,\sigma}, D_{c,\sigma}\) are the matrices to be determined. Denoting

\[
\bar{R}_{c,\sigma} = \begin{bmatrix} B_{c,\sigma} & A_{c,\sigma} \\ D_{c,\sigma} & C_{c,\sigma} \end{bmatrix}, \bar{x} = \begin{bmatrix} x \\ x_c \end{bmatrix}
\]  

(5)

The problem can be cast as the search of a robust static state feedback control gain \(\bar{R}_{c,\sigma} \in \mathbb{R}^{(n_c + p)(n_c + n)}\) for the augmented NSCSs

\[
\dot{\tilde{x}} = (\bar{A}_{\sigma} + \bar{B}_{\sigma} \bar{R}_{c,\sigma}) \tilde{x}(t - \tau) + \bar{G}_{\sigma} \omega
\]  

(6)
\[
\omega(t) = E_\sigma \ddot{x}(t) + \bar{H}_\sigma \omega
\]

(7)

where
\[
\begin{bmatrix}
\bar{A}_\sigma \\
\bar{B}_\sigma \\
\bar{C}_\sigma
\end{bmatrix}
= \begin{bmatrix}
0 & 0 & 0 \\
I & 0 & 0 \\
0 & 0 & 0
\end{bmatrix},
\begin{bmatrix}
\bar{E}_\sigma \\
\bar{F}_\sigma \\
\bar{G}_\sigma
\end{bmatrix}
= \begin{bmatrix}
0 & 0 & 0 \\
E_\sigma & 0 & 0 \\
0 & 0 & H_\sigma
\end{bmatrix}.
\]

Assume the time delay of switched controllers to system modes is \(\tau_1\), then \(\bar{K}_{c,\sigma}\) will become
\[
\bar{K}_{c,\sigma(t-\tau_1)} = \begin{bmatrix}
B_{c,\sigma(t-\tau_1)} & A_{c,\sigma(t-\tau_1)} \\
J_{c,\sigma(t-\tau_1)} & C_{c,\sigma(t-\tau_1)}
\end{bmatrix},
\]

(8)

And the resulting closed-loop system is given by
\[
\dot{x} = (\bar{A}_\sigma + \bar{B}_\sigma \bar{K}_{c,\sigma(t-\tau_1)}) \ddot{x} + \bar{C}_\sigma \omega,
\]

(9)
\[
z = E_\sigma \ddot{x} + \bar{H}_\sigma \omega.
\]

(10)

Without loss of generality, we assume that the maximal delay of asynchronous switching is \(\tau_{1,\text{max}}\), which depends on the running time of the unmatched controller, is known in advance.

Definition 2 (Finite-time Stable). Given a positive definite matrix \(R\), three positive constants \(c_1, c_2, T_r\), with \(c_1 < c_2\), and a switching signal \(\sigma\), the NSCSs (1) with \(\dot{u}(t) \equiv 0\) and \(\dot{\omega}(t) \equiv 0\) is said to be finite-time stable with respect to \((c_1, c_2, T_r, R, \sigma)\), if
\[
\int_0^T x_0(t) R x_0(t) dt < c_1 \Rightarrow \int_0^T x(t) R x(t) dt < c_2 \forall t \in [0, T_r].
\]

Definition 3 (Finite-time boundedness). Given a positive definite matrix \(R\), four positive constants \(c_1, c_2, d, T_r\), with \(c_1 < c_2\), and a switching signal \(\sigma\), the NSCSs (1) with \(\dot{u}(t) \equiv 0\) is said to be finite-time bounded with respect to \((c_1, c_2, d, T_r, R, \sigma)\), if
\[
\int_0^T \dot{x}(t) \dot{R} \dot{x}(t) dt \leq d.
\]

Definition 4 (Finite-time \(H_\infty\) performance). Given a positive definite matrix \(R\), three positive constants \(c_2, T_r\) and a switching signal \(\sigma\), the NSCSs (1) with \(\dot{u}(t) \equiv 0\) is said to have finite-time \(H_\infty\) performance with respect to \((c_1, c_2, d, T_r, \gamma, R, \sigma)\), if the system is finite-time bounded and the following inequality is satisfied
\[
\int_0^T z^T(s) z(s) ds < \gamma^2 \int_0^T \omega^T(s) \omega(s) ds
\]

(11)

where \(\gamma > 0\) is a prescribed scalar and \(\dot{\omega}(t)\) satisfies the Assumption 2.

Definition 5 (Finite-time \(H_\infty\) Control). The NSCSs (1) is said to be finite-time stabilizable with \(H_\infty\) disturbance attenuation level \(\gamma\), if there exists a control input with time-delays \(\dot{u}(t - \tau), \forall t \in [0, T_r]\), such that
(a) The closed-loop system is finite-time bounded.
(b) Under the Zero-initial condition, the controlled output \(z\) satisfies inequality (11).
The purpose of this paper is to design a mode-dependent dynamic state-feedback controller and a set of switching signals with ADT such that the closed-loop system is bounded and has finite-time $H_{\infty}$ performance.

Theorem 1. For any $(i, j) \in \xi \times \xi$, let $\gamma_{i,j} = R_{1}^{1/2} R_{1}^{1/2}$, $\gamma_{2,j} = R_{2}^{1/2} R_{2}^{1/2}$ and assume exist matrices $Y, \tilde{Y}$, and constants $B \geq \alpha > 0$, $\gamma > 0$, such that

$$\begin{bmatrix}
\Theta_{i,j} & G_{i}\gamma_{i}
\end{bmatrix} < 0$$

$$\begin{bmatrix}
\Theta_{i,j} & G_{i}\gamma_{i}
\end{bmatrix} < 0$$

$$\frac{c_{i}}{\lambda_{i}} e^{\alpha \tau_{e}} + \frac{\gamma \lambda_{i}}{\lambda_{i}} e^{\beta \tau_{e}} < \frac{c_{i}}{\lambda_{i}}$$

Where

$$V_{i,j} = \begin{bmatrix}
Y_{i,j} & 0
\end{bmatrix} \Theta_{i,j} = Y_{i,j}^{T} A_{i}^{c} + A_{j}^{c} Y_{i,j} + B_{i}^{c} M_{j} + M_{j}^{T} B_{i}^{c} - \alpha \lambda_{i} \Theta_{i,j} = Y_{i,j}^{T} A_{i}^{c} + A_{j}^{c} Y_{i,j} + B_{i}^{c} M_{j} + M_{j}^{T} B_{i}^{c} - \beta Y_{i,j}$$

If the average dwell time of switching signal $\sigma$ satisfies

$$\tau_{a} > \tau_{a}^{*} = \frac{T_{f}(\alpha + \ln \mu)}{1 \ln \frac{C_{i}}{\lambda_{i}} + \frac{\gamma \lambda_{i}}{\lambda_{i}} e^{(\beta-\alpha) \tau_{e}} - \alpha T_{f}}$$

Then there exist a set of mode-dependent dynamic state-feedback controllers with asynchronous delay $\tau_{1}^{\max}$ and NSCSs time-delays $\tau$ such that the corresponding NSCSs system (9) is finite-time bounded with respect to $(c_{i}, \gamma_{i}, d_{i}, k_{i}, \sigma)$, where $\mu = \lambda_{2} / \lambda_{1}$, $\lambda_{1} = \min_{i \in \xi} (\lambda_{\min}(\gamma_{i}))$, $\lambda_{2} = \max_{i \in \xi} (\lambda_{\min}(\gamma_{i})), \lambda_{3} = \min_{i \in \xi} (\lambda_{\min}(Y_{1})), \lambda_{4} = (\beta - \alpha) \tau_{1}^{\max}$. Moreover, the matrices $\tilde{K}_{c,j}$ are given by

$$\tilde{K}_{c,j} = M_{j} Y_{1}^{-1}$$

**Proof.** Consider the NSCSs controller (4) with the asynchronous case, and the resulting closed-loop system is given as

$$\dot{\tilde{x}} = \begin{cases}
\tilde{A}_{i,j} \tilde{x}(t - \tau) + \tilde{G}_{i} \omega(t), & \forall \ t \in [t_{k}, t_{k} + \tau_{1}^{\max})
\tilde{A}_{j,i} \tilde{x}(t - \tau) + \tilde{G}_{i} \omega(t), & \forall \ t \in [t_{k} + \tau_{1}^{\max}, t_{k+1})
\end{cases}$$

where $\tilde{A}_{i,j} = \tilde{A}_{i} + \tilde{B}_{i} \tilde{K}_{c,j}, \tilde{A}_{j,i} = \tilde{A}_{j} + \tilde{B}_{j} \tilde{K}_{c,i}$.

Use the following Lyapunov functional candidate
\[ V'_i(x) = \ddot{\chi}^T P_i \ddot{\chi} + x_i^T P_{i,i} x_i + x_i^T P_{i,i} x_i, \forall \sigma(t) = i \in \xi \]  

(18)

where \( P_{k,i} > 0, P_{i,i} > 0, P_{i,i} = Y_i^{-1} \), \( P_{2,i} = Y_i^{-1} \) satisfy conditions (12,13) with (17,18), we have

\[ \dot{V}_i(t) - \beta \dot{\alpha}(t) = \left( \ddot{\chi}^T (t - \tau) \dot{\alpha}(t) \right) \left[ \ddot{\chi}^T (t - \tau) \dot{\alpha}(t) \right], \]

where \( t \in [t_k, t_k + \tau_{1_{\text{max}}}] \),

\[ \Sigma_{ij} = \left[ \begin{array}{ccc} A_{ii} & P_{i,i} & P_i \bar{G}_i \end{array} \right] \]

(19)

where \( t \in [t_k + \tau_{1_{\text{max}}}, t_{k+1}] \), we assume

\[ L = \left[ \begin{array}{ccc} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{array} \right] > 0, \]  

(20)

with (12) and for \( t \in [t_k + \tau_{1_{\text{max}}}, t_{k+1}] \) we have

\[ \dot{V}_i(t) - \alpha V_i(t) = \left[ \ddot{\chi}^T (t - \tau) \dot{\alpha}(t) \right] \Sigma_{ij} \left[ \ddot{\chi}^T (t - \tau) \dot{\alpha}(t) \right] < \gamma \omega \omega^{-1} \chi \]

Finally

\[ \dot{V}_i(t) < \alpha V_i(t) + \gamma \omega \omega^{-1} \chi, \forall t \in [t_k + \tau_{1_{\text{max}}}, t_{k+1}] \]

Similarly we can confirm

\[ V_i(t) < \beta V_i(t) + \gamma \omega \omega^{-1} \chi, \forall t \in [t_k, t_{k+1}] \]

(21)

Considering \( \beta \geq \gamma > 0 \), by integrating (19-20) for \( t \in [t_k, t_{k+1}] \), we can have

\[ V_{\alpha}(x, \dot{x}) = e^{\gamma(t - t_k)} e^{\beta \int_{t_k}^t - \sigma_i(t) \left[ V_{\sigma_i}(x, \dot{x}, (t - \tau), \sigma_i) + \int_{t_k}^t e^{\beta \int_{t_k}^s - \sigma_i(t) \omega \omega^{-1} \chi} ds \right] ds} \]

Then, according to the definitions of \( \lambda_i \) and \( \lambda_2 \) for any \( i, j \in \xi, i \neq j \) and \( x \in \mathbb{R}^n \), we have

\[ \ddot{\chi}^T (t - \tau) \bar{F}_i \ddot{\chi} (t - \tau) \leq \frac{1}{\lambda_i} (\ddot{\chi}^T (t - \tau) \bar{F}_i \ddot{\chi} (t - \tau)) \]

(22)

\[ \ddot{\chi}^T (t - \tau) \bar{F}_i \ddot{\chi} (t - \tau) \leq \frac{1}{\lambda_i} (\ddot{\chi}^T (t - \tau) \bar{F}_i \ddot{\chi} (t - \tau)) \]

(23)

According to Assumption 1, \( \tilde{x}_{i,j}(t - \tau) = \ddot{x}_i(t - \tau) \). Then

\[ V_{\sigma_i}(\tilde{x}_{i,j}(t - \tau)) \leq \mu V_{\sigma_i}(\tilde{x}_{i,j}(t - \tau)) \]

(24)

For any \( t \in (0, T_r) \), noticing that \( \mu \geq 1, N_0(t) \leq N_0 + t / \tau_{1_{\text{a}}} \leq T_r / \tau_{1_{\text{a}}} \) and according to (21) and (24), we have

\[ V_{\sigma_i}(\tilde{x}_{i,j}(t - \tau)) \leq e^{\gamma (t - t_k) \omega \omega^{-1}} [V_{\sigma_i}(\tilde{x}_{i,j}(t - \tau)) + \frac{\gamma}{\lambda_i} e^{\beta T_r} ] \]

(25)
On the other hand,
\[ V_{\sigma(t)}(t - \tau) \geq \frac{1}{\lambda_2} x^T (t - \tau) R x (t - \tau) \] (26)
\[ V_{\sigma(0)}(0) \leq \frac{1}{\lambda_1} x^T (0) R x (0) \] (27)

With (25-27), we have
\[ x^T (t - \tau) R x (t - \tau) < \lambda_2 e^{(\sigma + \eta_2)(t - \tau)} \left( \frac{C_1}{\lambda_2} e^{\alpha t} + \frac{\gamma d}{\lambda_2} e^{\beta t} \right) \] (28)

We assume condition (14) is satisfied, we obtain

\[ \frac{T_f}{\tau_{ia}} \leq \frac{\ln \left( \frac{c_{2}}{\lambda_2} \right) - \ln \left( \frac{c_1}{\lambda_1} + \frac{\gamma d}{\lambda_2} e^{(\sigma - \eta_2)\tau_f} \right) - \alpha T_f}{\alpha + \ln \mu} \] (29)

Leads to
\[ e^{(\sigma + \ln \mu)(t - \tau_f)} \leq \frac{c_2}{\lambda_2 (c_1 e^{\alpha T_f} / \lambda_1 + \gamma d e^{\beta T_f} / \lambda_2)} \] (30)

With (28-30) we can have
\[ x^T (t - \tau) R x (t - \tau) < c_2 \] (31)

As the trajectory of the switched system (9) is continuous at instant \( T_f \), we can conclude that (31) holds for all \( t \in [0, T_f] \). According to Definition3, system (9) is finite-time bounded.

Theorem 2. For any \((i, j) \in \mathcal{S} \times \mathcal{S}, i \neq j \), let \( \bar{F}_{i,i} = R^{\gamma}F_{i,i}R^{\gamma}, \bar{F}_{i,j} = R^{\gamma}F_{i,j}R^{\gamma}, \bar{F}_{j,i} = R^{\gamma}F_{j,i}R^{\gamma} \) and assume exist \( Y_{i,i} > 0, Y_{j,j} > 0, M_2 \) and constants \( \beta > 0, \gamma > 0 \), such that

\[
\begin{bmatrix}
\Theta_{ii} & \bar{F}_{i,i} + Y_{i,i} \bar{F}_{i,i} & Y_{i,i} \\
* & -\gamma^2 I + H_{i,i}^T H_{i,i} & 0 \\
* & * & -I
\end{bmatrix} < 0
\] (32)

\[
\begin{bmatrix}
\Theta_{jj} & \bar{F}_{j,j} + Y_{j,j} \bar{F}_{j,j} & Y_{j,j} \\
* & -\gamma^2 I + H_{j,j}^T H_{j,j} & 0 \\
* & * & -I
\end{bmatrix} < 0
\] (33)

\[ \gamma^2 d < \frac{C_0}{\lambda_2} e^{-\beta T_f} \] (34)
Where

\[
Y_i = \begin{bmatrix}
Y_{i1} & 0 \\
0 & Y_{i2}
\end{bmatrix}, \quad \Theta_{ij} = Y_{ij}^T + \bar{A}_{ij} Y_{ij} + \bar{B}_{ij} M_i + M_i^T \bar{B}_{ij}^T - \alpha_i Y_i^T \quad \Theta_{ij} = Y_{ij}^T + \bar{A}_{ij} Y_{ij} + \bar{B}_{ij} M_j + M_j^T \bar{B}_{ij}^T - \beta Y_j^T.
\]

If the average dwell time of switching signal \(\sigma\) satisfies

\[
\tau_{1,a} > \tau_{1,a}^* = \max\{ \frac{r_r (\bar{\sigma} + \ln \mu)}{\ln (\frac{c_2}{\lambda_i}) - \ln (\gamma^T d) - \beta T_f} - \bar{\sigma} + \ln \mu \}.
\]

Then there exist a set of controllers with delay \(\tau_{1,\text{max}}\) and NSCSs time-delays \(\tau\) such that the corresponding NSCSs closed-loop system is finite-time stabilizable with \(H_{\infty}\) performance \(\bar{\gamma}\) with respect to \((0, c_2, d, T_f, \bar{\gamma}, R, \sigma)\), where \(\lambda_1 = \min_{\xi}(\lambda_{\min}(Y_i))\), \(\lambda_2 = \max_{\xi}(\lambda_{\max}(Y_i))\), \(\mu = \lambda_2 / \lambda_1\), \(\bar{\sigma} = (\beta - \alpha) \tau_{1,\text{max}}\), \(\bar{\gamma} = \rho^{\beta T_f}\). Moreover, the matrices \(K_{ij}\) are given by (16).

Proof. Note that

\[
\begin{bmatrix}
\bar{E}_i^T \\
\bar{H}_i^T
\end{bmatrix}
\begin{bmatrix}
\bar{E}_i \\
\bar{H}_i
\end{bmatrix} = \begin{bmatrix}
\bar{E}_i^T \bar{E}_i & \bar{E}_i^T \bar{H}_i \\
\bar{H}_i^T \bar{E}_i & \bar{H}_i^T \bar{H}_i
\end{bmatrix} \geq 0,
\]

then with (32-33) that

\[
\begin{bmatrix}
\Theta_{ij} & \bar{G}_i \\
* & -\gamma^T I
\end{bmatrix} < 0,
\]

\[
\begin{bmatrix}
\Theta_{ij} & \bar{G}_i \\
* & -\gamma^T I
\end{bmatrix} < 0.
\]

It can confirm from Theorem 1 that conditions (34), (35), (37) and (38) can guarantee that the system (9) is finite-time bounded with respect to \((0, c_2, d, T_f, \bar{\gamma}, R, \sigma)\) by setting \(\bar{W}_i = I\) and \(c_1 = 0\). According to (19),(20),(25) and noting the zero initial condition, we can get the following inequalities by assume \(\Psi(s) = \rho^{\omega (\Theta_{ij}) - \omega(\Theta_{ij})} \Psi(s)\) and substituting \(\omega(s)\) with \(\Psi(s)\)

\[
0 \leq V(t) < \int_0^t \rho^{\beta(t-s)} [\mu e^{\sigma}] \Psi(s) ds
\]

which implies that

\[
\int_0^t \rho^{\beta(t-s)} [\mu e^{\sigma}] \Psi(s) ds < \gamma \int_0^t \rho^{\beta(t-s)} [\mu e^{\sigma}] \omega(s) \Psi(s) ds
\]

for \(\mu \geq 1\), we can have

\[
\int_0^t \rho^{\beta(t-s)} [\mu e^{\sigma}] \Psi(s) ds > \int_0^t z^T(s) z(s) ds
\]
\[
\int_0^\tau \gamma^2 e^{\beta(t-s)} [\mu e^{\tau_s} \gamma_s] \alpha(s) \omega(s) ds < \gamma^2 e^{\beta t} [\mu e^{\tau_t} \gamma_t] \int_0^\tau \omega(s) \alpha(s) ds
\]  \tag{42}

with (35), we get \( \tau_{1a} \geq (\alpha + 1n \mu) / \beta \). Then

\[
0 \leq N_\sigma(0, t) \leq t / \tau_{1a} \leq t\beta / (\alpha + 1n \mu)
\]  \tag{43}

Substituting (43) into (42) and noting (41), we can have

\[
\int_0^\tau z^T(s) z(s) ds < \gamma^2 e^{\beta t} \int_0^\tau \omega(s) \alpha(s) ds
\]  \tag{44}

setting \( t = T_f \), we can get

\[
\int_0^T z^T(s) z(s) ds < \widetilde{\gamma} \int_0^T \omega(s) \alpha(s) ds
\]  \tag{45}

where \( \widetilde{\gamma} = \gamma e^{\beta T_f} \). The proof is completed.

**Numerical Examples**

In this section, we give two examples to confirm the efficiency of the methods. Example 1.

Consider the NSCSs given by (1), where

\[
A_1 = \begin{bmatrix} -0.9 & 0.1 \\ 1.9 & 0.4 \end{bmatrix}, \quad A_2 = \begin{bmatrix} 0.6 & -2 \\ -0.5 & -1 \end{bmatrix}, \quad B_1 = \begin{bmatrix} 1.1 & 0.1 \\ 1 & 1.4 \end{bmatrix}, \quad B_2 = \begin{bmatrix} 2.4 & 0.3 \\ 0 & 1.8 \end{bmatrix}, \quad G_1 = \begin{bmatrix} 0.9 & 0 \\ -0.5 & 0.1 \end{bmatrix}, \quad G_2 = \begin{bmatrix} -0.1 & 0 \\ -0.1 & 0.3 \end{bmatrix},
\]

\[
\alpha(t) = \begin{bmatrix} -0.4 \cos(5t + 3) \\ 0.2 \sin(3t) \end{bmatrix}, \quad \chi_0 = \begin{bmatrix} -0.4 \\ 0.7 \end{bmatrix}.
\]

The parameters are given as: \( \alpha = 0.05, \beta = 0.1, \gamma = 0.1, c_1 = 1, c_2 = 20, T_f = 10 \) and \( R = I \). We have \( \int_0^T \omega^T(s) \alpha(s) ds = 0.9974 \), We choose \( d = 1 \).

By applying Theorem 1 and solving corresponding matrix inequalities (12-13) for \( \tau_{1_{\max}} = 0.5 \) and \( n_c = 3 \), we can obtain the following feasible solutions

\[
\Psi_1 = \begin{bmatrix} 1.960 & 0.4673 \\ 0.4673 & 4.2198 \end{bmatrix}, \quad \Psi_2 = \begin{bmatrix} 3.5642 & 0.5610 \\ 0.5610 & 2.7567 \end{bmatrix}
\]

With the given \( W \) and according to Theorem 1, where

\[
Y_1 = \begin{bmatrix} 26.4609 & 0.0000 \\ 0.0000 & 26.4609 \end{bmatrix}, \quad Y_2 = \begin{bmatrix} 23.1492 & 0.0000 \\ 0.0000 & 23.1492 \end{bmatrix}
\]
And for any switching signal \( \sigma \) with average dwell time \( \tau_{1a} > \tau_{1a}^* = 1.3913s \), switched system (1) is finite-time bounded with respect to \((1, 20, 10, I, \sigma)\).

The simulation results are given in Figs. 1-2. The lower bound of ADT \( \tau_{1a}^* \) under different \( c_2 \) can be obtained as shown in Fig.1. For a fixed constant \( c_1 \), \( \tau_{1a}^* \) is gradually reduced with the increase of \( c_2 \). For a given \( \alpha \), the relationship between ADT \( \tau_{1a}^* \) and \( \beta \) is shown in Fig.2. It can be seen from Fig.2 that \( \tau_{1a}^* \) is gradually increase of \( \beta \).

**Example 2.** Consider the NSCSs given by (1) and (2). Where
\[
E_1 = \begin{bmatrix} 0 & -0.2 & 0.3 \\ -0.2 & 0.5 & -0.1 \end{bmatrix}, \quad E_2 = \begin{bmatrix} 0.5 & -0.1 \\ -0.1 & 0.1 \end{bmatrix}, \quad H_1 = \begin{bmatrix} 0.1 & 0.2 \\ 0.2 & -0.1 \end{bmatrix}, \quad H_2 = \begin{bmatrix} 0.2 & -0.1 \end{bmatrix}
\]

Other system matrices are the same as Example 1, and the parameters are given as \( \alpha = 0.3 \), \( \beta = 0.4 \), \( \gamma^2 = 0.1 \), \( c_1 = 0 \), \( c_2 = 1.5 \), \( T_r = 10 \) and \( R = I \) and \( d = 1 \). By applying Theorem 2 and solving the corresponding matrix inequalities (32-33) for \( \tau_{1a}^* > 0.5 \) and \( n_c = 3 \), we can obtain the following feasible solutions
\[
Y_1 = \begin{bmatrix} 7.1972 & 0.0000 \\ 0.0000 & 7.1972 \end{bmatrix}, \quad Y_2 = \begin{bmatrix} 4.1987 & 0.0000 \\ 0.0000 & 4.1987 \end{bmatrix}
\]
\[
K_1 = \begin{bmatrix} -2.1241 & -28.4637 \\ 21.4416 & 19.1275 \end{bmatrix}, \quad K_2 = \begin{bmatrix} -1.6027 & -2.5008 \\ 4.8236 & -0.0828 \end{bmatrix}
\]

And the minimal average dwell time \( \tau_{1a} > \tau_{1a}^* = 1.4723s \).

**Figure 1.** Minimal ADT \( \tau_{1a}^* \) for different \( c_2 \).

**Figure 2.** Minimal ADT \( \tau_{1a}^* \) for different \( \beta \).

**Figure 3.** Minimal ADT \( \tau_{1a}^* \) for different \( \gamma^2 \).

**Figure 4.** Minimal ADT \( \tau_{1a}^* \) for different \( c_2 \).
The lower bound of ADT $\tau^*_{\alpha}$ under different $\gamma^2_c$ are shown in Fig. 3 and Fig. 4, respectively. For a fixed constant $c_1$, $\tau^*_{\alpha}$ is gradually increased with the increase of $\gamma^2_c$, and $\tau^*_{\alpha}$ is gradually increased with the increase of $c_1$ for given $\alpha$ and $\beta$. The state trajectory of the corresponding closed-loop switched system is given in Fig. 5. It can be easily obtained from Fig. 5 that the corresponding closed-loop system is finite-bounded with $H_\infty$ performance with respect to $(0, 1, 5, 1, 10, 17, 2640, I, \sigma)$.

**Conclusion**

In this paper, we discuss the problem of a class asynchronous finite-time NSCSs with time-delays and time-varying exogenous disturbances. A novel mode-dependent dynamic state-feedback controllers are proposed. It based on the asynchronous switched system theory, Multiple Lyapunov Functions and switched Average Dwell Time. At the same time we obtained a sufficient condition for the NSCSs to be guaranteed a prescribed performance. In addition, the finite-time stabilization problem has also been studied. At last, two examples are given to show the efficiency of the methods. However, this article does not take into account such as the uncertainty and packet dropout in NSCSs, and does not take into account the data-rate of the NSCSs either. A challenging and interesting future research topic is how to extend the results in this paper with these issues.

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