Results on Binary Linear Codes With Minimum Distance 8 and 10

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Abstract—All codes with minimum distance 8 and codimension up to 14 and all codes with minimum distance 10 and codimension up to 18 are classified. Nonexistence of codes with parameters [33,18,8] and [33,14,10] is proved. This leads to 8 new exact bounds for binary linear codes. Primarily two algorithms considering the dual codes are used, namely extension of dual codes with a proper coordinate, and a fast algorithm for finding a maximum clique in a graph, which is modified to find a maximum set of vectors with the right dependency structure.

I. INTRODUCTION

Let $F_2^n$ denote the $n$-dimensional vector space over the field $F_2$ and let the inner product $\langle \cdot, \cdot \rangle : F_2^n \times F_2^n \rightarrow F_2$ be defined in the natural way as $\langle u, v \rangle = \sum_{i=1}^{n} u_i v_i$, where addition is in $F_2$. The Hamming distance between two vectors in $F_2^n$ is defined as the number of coordinates in which they differ, and the weight $\text{wt}(v)$ of a vector $v \in F_2^n$ is the number of the nonzero coordinates of $v$. A linear binary $[n, k, d]$ code $C$ is a $k$-dimensional subspace of $F_2^n$ with minimum distance $d = \min \{ \text{wt}(c) : c \in C, c \neq 0 \}$. A generator matrix $G$ for an $[n, k] = [n, k, d \geq 1]$ code is any matrix whose rows form a basis for the code. The orthogonal complement $C^\perp$ of $C$ in $F_2^n$ is called the dual code of $C$ and is an $[n, n-k, d^\perp]$ code, where $d^\perp$ is called the dual distance of $C$ and $n-k$ the codimension. A generator matrix $H$ for the dual code is called a parity check matrix of the code $C$. In this paper we will say that a code $C$ is an $[n, k, d^\perp]$ code if it is an $[n, k, d]$ code with dual distance $d^\perp$. Further, two binary linear codes $C_1$ and $C_2$ are said to be equivalent if there is a permutation of coordinates which sends $C_1$ to $C_2$. Throughout this paper all codes are assumed to be binary.

A central problem in coding theory is that of optimizing one of the parameters $n$, $k$, and $d$ for given values of the other two. Usually this optimization is related to the following functions: $n_2(k, d)$ - the minimum length of linear codes for given minimum distance $d$ and dimension $k$ and $d_2(n, k)$ the largest value of $d$ for which a binary $[n, k]$ code exists. Codes with parameters $[n_2(k, d), k, d]$ and $[n, k, d_2(n, k)]$ are called optimal. There are many reasons to study optimal codes. These codes are interesting not only for detection and correction of errors. Some of them have rich algebraic and combinatorial structure. The problems of optimality are strongly connected and can be considered as packing problem in statistics and in finite projective spaces [10]. Unfortunately, all these problems are, as many others in coding theory, computationally difficult. Tables with bounds and exact values for $d_2(n, k)$ are given in [6] and [9].

Another application of optimal codes is directly related to the design method of cryptographic Boolean functions suggested by Kurosawa and Satoh [11]. In this case the optimal linear codes have to be with largest possible dual distance. More precisely one have to study the function $N(d, d^\perp)$ as the minimal $n$ such that there exists a linear binary code of length $n$ with minimum distance $d$ and dual distance $d^\perp$. The investigation of $N(d, d^\perp)$ seems to be much harder than the investigation of $n_2(k, d)$. There are some general bounds (see [12]) but these bounds can be reached only for a few values of $d$ and $d^\perp$. In a previous work, we studied $N(d, d^\perp)$ for $d^\perp \leq d \leq 12$ by computer using the package Q-EXTENSION [2]. With this package we attempted to construct generator matrices and classify codes with fixed parameters. Practically, we had no success in the cases where the dual distance was more than 6. In the case of codes with fixed minimum distance larger than 2, it is quite natural to look at the duals of the codes with needed properties and to the parity check matrices. In other words, extending an $[n-1, k-1, d]$ code $C_1$ to an $[n, k, d]$ code $C_2$ can be considered as extending an $[n-1, n-k]^d$ to an $[n, n-k]^d$ with one coordinate. This approach helps us to develop two different methods, which are much more effective when we study $N(d, d^\perp)$, and also $n_2(k, d)$, for $d = 8$ and $d = 10$. For small dimensions, it is convenient to use a brute force algorithm that takes generator matrices of all inequivalent $[n, k, d]^d$ codes as input, extends them in all possible ways and then checks the constructed codes for minimum and dual distance and for equivalence. As a result we get all inequivalent $[n+1, k, \geq d]^d$ codes. This method is of course impossible to use for larger dimensions $k$, because of the large number of possible extensions. To avoid this problem we use a second method for larger dimensions adopting a strategy for bounding of the search space similar to the strategy for finding a maximum clique in a graph suggested in [13].

In this paper we present two algorithms which can be used for constructing of linear codes with fixed dual distance. We give classification results for all codes with minimum distance 8 and codimension up to 14 and minimum distance 10 and codimension up to 18. We would like to refer to [7] and [8] for a detailed bibliography of works which study linear codes with minimum distance 8 and 10.
II. PRELIMINARIES

In this Section we give some properties of linear codes and the relations with their dual codes which help us to design the construction algorithms.

The first proposition considers the even linear codes, namely the linear binary codes which consist only of even weight vectors.

**Proposition 1** If \( d \geq 2 \) is even and a linear \([n, k, d]\) code exists, then there exists an even \([n, k, d]\) code.

*Proof:* If \( C \) is a linear \([n, k, d]\) code, the code produced by puncturing \( C \) in one coordinate has parameters \([n - 1, k, d] \) or \([n - 1, k, d - 1]\]. Adding a parity check bit to all codewords, we obtain an even \([n, k, d]\) code.

Practically, we use the following corollary:

**Corollary 2** If \( d \geq 2 \) is even and even linear \([n, k, d]\) codes do not exist, then no \([n, k, d]\) code exists.

Later on we give the definition and some properties of residual codes.

**Definition 1** The residual code \( \text{Res}(C, c) \) with respect to a codeword \( c \in C \) is the restriction of \( C \) to the zero coordinates of \( c \).

A lower bound on the minimum distance of the residual code is given by

**Theorem 3** (Lemma 3.9) Suppose \( C \) is a binary \([n, k, d]\) code and suppose \( c \in C \) has weight \( w \), where \( d > w/2 \). Then \( \text{Res}(C, c) \) is an \([n - w, k - 1, d']\) code with \( d' \geq d - w + \lceil w/2 \rceil \), and on the dual distance by

**Proposition 4** Suppose \( C \) is a binary \([n, k, d]\) code with dual distance \( d^\perp \), \( c \in C \), and the dimension of \( \text{Res}(C, c) \) is \( k - 1 \). Then the dual distance of \( \text{Res}(C, c) \) is at least \( d^\perp \).

There is also a well known elementary relationship between the minimum distance of a linear code and the parity check matrix.

**Proposition 5** A linear code has minimum distance \( d \) if and only if its parity check matrix has \( d \) linearly dependent columns but no set of \( d - 1 \) linearly dependent columns.

The next proposition gives a connection between weights of the rows of generator matrices and columns of parity check matrices.

**Proposition 6** Any even linear code \( C \) has a parity check matrix whose columns have odd weights.

*Proof:* Let \( G = [I_k | P] \) be a generator matrix for \( C \) in standard form. Then every row of \( P \) has odd weight. And accordingly the parity check matrix \( H = [P^T | I_{n-k}] \) has only odd weight columns. The sum of all rows of \( H \) gives the all-ones vector.

**Theorem 4** A linear code has minimum distance \( d \) if and only if its parity check matrix has \( d \) linearly dependent columns but no set of \( d - 1 \) linearly dependent columns.

**Definition 2** Let \( d^\perp \geq 3 \). Then \( L(k, d^\perp) \) is the maximum length \( n \) such that a binary \([n, k] d^\perp\) code exists.

The next theorem holds.

**Theorem 5** Let \( C \) be a binary linear \([n, k, d]\) code. Then \( d \geq n - L(k - 1, d^\perp) \).

*Proof:* Let \( C \) be a code with parameters \([n, k, d]\) and \( c_q \) be a codeword in \( C \) of weight \( d \). If \( d < n - L(k - 1, d^\perp) \) then the residual code \( \text{Res}(C, c_q) \) has dual distance at least \( d^\perp \), length \( n - d > L(k - 1, d - 1) \) and dimension \( k - 1 \). This is impossible because \( L(k, d^\perp) \geq L(k, d^\perp) \) for \( d^\perp \geq 4 \).

In fact, the function \( L(k, d^\perp) \) has been already investigated in another setting for dual distance greater than or equal to 4 in connection with studies on \( \kappa \)-caps in projective geometries since it is known that a \( \kappa \)-cap in \( PG(k - 1, q) \) is equivalent to a projective \( q \)-ary \([n = \kappa, k] d^\perp \) code with \( d^\perp \geq 4 \). And \( L(k, d^\perp) \) have thus been considered in connection with the function

\[ \mu_q(N, q) = \text{the maximum value of } \kappa \text{ such that there exist a } \kappa \text{-cap in } PG(N, q), \]

where \( q \) is the order of the underlying Galois field, in our case 2. More information on this can be found in the survey by Hirschfeld and Storme [10].

III. COMPUTATIONAL ALGORITHMS

We use mainly two algorithms for the extension of codes. If the dimension of the considered codes with given dual distance is small we can find the next column of the generator matrix of the new code relatively easy because we can represent any generator matrix \( G \) in the packing form as a vector \( G_0 \) of \( n \) computer words. This algorithm, named BRUTEFORCE, can be described with the following steps:

**Algorithm BRUTEFORCE:**

INPUT: \( C_{inp} \) - Set of all inequivalent \([n, k] d^\perp \) codes represented by their generator matrices in packing form.

OUTPUT: \( C_{out} \) - Set of all inequivalent \([n, k + 1] d^\perp \) codes.

\[ \text{var } a \text{:array}[1..2^k - 1] \text{ of integer}; \]
In the beginning, \( C_{\text{out}} \) is the empty set. For any code \( C_r \) in \( C_{\text{inp}} \) with generator matrix in packing form \( G_r \) do the following:

1. Set \( a[i] := 1 \) for any \( i \).
2. Find all linear combinations \( b \) of up to \( d^\bot - 2 \) column vectors of \( G_r \) and set \( a[b] := 0 \).
3. For all \( j \) such that \( a[j] = 1 \) extend \( G_r \) with one coordinate, equal to \( j \), to \( G'_r \). If there are no codes in \( C_{\text{out}} \) equivalent to \( C'_r \) (generated by \( G'_r \)) do \( C_{\text{out}} := C_{\text{out}} \cup C'_r \).

The big advantage of this algorithm is given by Step 2. In that step, all possible solutions for the \((n+1)\)th column of the generator matrices are determined with approximately with \( \sum_{i=1}^{d^\bot} \binom{n}{i} \) operations. Actually, to find all vector solutions for the \((n+1)\)th column, we take all \( k \)-dimensional vectors and delete those which are not solutions. We find all sums of less than \( d^\bot - 2 \) columns of the known part of the generator matrix. Each sum gives us one vector which is not a solution and have to be deleted. All remaining vectors are solutions.

In Step 3, we use canonical representation of the objects. The main priority of the canonical representation is that the equivalence (isomorphism) test is reduced to check of coincidence of the canonical representations of the structures. In the case of many inequivalent codes, the computational time for comparing is growing fast. A technique for surmounting this problem is worked out. We split the set of inequivalent codes into a big amount of cells according to a proper invariant.

To explain the next algorithm, we need the following definition.

**Definition 3** Let \( E \) be a set of \( k \)-dimensional vectors.

1. We call \( E \) \( p \)-proper if all subsets of \( p \) vectors of \( E \) are linearly independent.
2. Let \( M \) be a \( k \times n \) matrix. The set \( E \) is called \( p \)-proper with respect to \( M \) if \( E \cup \{ \text{columns in } M \} \) is a \( p \)-proper set.

Observe that, by Proposition 5, the columns of a parity check matrix for an \([n,k,d]\) code form a \((d-1)\)-proper set.

We consider the following problem: How to find a set of \( t = d - 1 \) binary vectors which have a certain property, i.e., \((d^\bot-1)\)-proper subset of the set of all possible binary vectors with respect to a fixed generator matrix. To attempt to solve this problem in reasonable time, we adopt an idea suggested by Östergård in [13] for finding a maximum clique in a graph in the algorithm EXTEND.

Let \( C \) be an \([n,k,d]\) code with a generator matrix \( G \) in the form

\[
G = \begin{bmatrix}
00 \ldots 0 & 1 \\
R_{\text{res}}(C) & X
\end{bmatrix}
\]

where \( R_{\text{res}}(C) \) is a generator matrix of the residual \([n-d,k-1, \geq d^\bot] \geq d^\bot \) code. Given that we know all such inequivalent generator matrices the problem is reduced to finding all \((d^\bot-1)\)-proper sets \( X \) with respect to \( A \) of \( d-1 \) binary vectors on the form \((1, x_2, \ldots, x_k)^T\).

Let \( V^* = \{(1, x_2, x_3, \ldots, x_k) : x_i \in \mathbb{F}_2\} \)

*Remark:* If the dual code is even we may, by Proposition 6, reduce the search space to the set of odd-weight binary vectors. Delete from \( V^* \) all linear combinations of \( d^\bot-2 \), or less, vectors from \( A \). The remaining set

\[
V = \{v_1, v_2, \ldots, v_N\}
\]

is the search space for our search strategy. Now, for each integer \( 1 \leq i \leq N \), let

\[
V_i = \{v_i, v_{i+1}, \ldots, v_N\}
\]

and let \( r \) be the \( N \)-tuple, defined by \( r[i] = \min\{s, t\}, 1 \leq i \leq N \), where \( s \) is the size of the largest \((d^\bot-1)\)-proper subset of \( V_i \). First we consider \((d^\bot-1)\)-proper subsets with respect to \( A \) of \( V_N \) that contain the vector \( v_N \), this obviously is \( \{v_N\} \), and we record the size of the largest proper subset found up to now in the tuple \( r \), so \( r[N] = 1 \).

In the \( i \)-th step we consider \((d^\bot-1)\)-proper subsets with respect to \( A \) in \( V_i \) containing \( v_i \) and record the minimum between size of the largest proper subset found up to now and \( t \) in \( r[i] \) (row *** in the algorithm).

The tuple \( r \) for the already calculated steps enables the pruning strategy for the search. Since we are looking for a proper subset of size \( t = d - 1 \) and if the vector \( v_i \) is to be the \((\text{size})\)st vector in the subset and size + \( r[i] \) < \( t \), then we can prune the search (row * in the algorithm). When the search terminates, the size of the largest \((d^\bot-1)\)-proper subset with respect to \( A \) of \( V \) or \( t \) (if \( t < s \)) will then be recorded in \( r[1] \).

In Step \( \text{size} \), \( \text{size} > 1 \), we choose all \( k \) dimensional vectors with first coordinate 1 which are not linearly dependent with \( d^\bot-2 \) column vectors of the constructed until now part of the generator matrix. Our idea is with one pass to find all proper vectors (all elements of \( U_{\text{size}} \)) using all column vectors from the generator matrix obtained until this step using \( U_{\text{size}-1} \) (all proper vectors from previous step). To find all proper vectors \( U_{\text{size}} \) we take \( U_{\text{size}-1} \) and delete those which are not proper, with respect to the already constructed part (row ** in the algorithm). We find all sums of less than \( d^\bot-2 \) columns. Each sum gives us one vector which is not \((d^\bot-1)\)-proper to the current step and have to be deleted. All remaining vectors are proper. To improve the speed of the algorithm, we pack each column in a computer word and use the bit operation XOR for computer words. The presented algorithm is much faster than the algorithm in [5].

**IV. RESULTS**

In this section, we present obtained result for codes with dual distance 8 and 10 using the above algorithms. With algorithm Bruteforce we construct all codes with length \( n \leq 28 \), dimension \( k \leq 14 \) and dual distance at least 8, and all codes with dimension up to 18 and dual distance at least 10. The summarized results for the number of inequivalent codes for given parameters are presented in the tables below. The stars in some cells mean that for the corresponding
parameters $n$ and $k$ there are codes with dual distance greater than the considered one. In the remaining cases, the number of inequivalent codes of length $n$ and dimension $k$, given in the table, coincide with the number of optimal codes with minimum distance 8 (respectively 10) which have dimension $n-k$ and length not larger than $n$. We can use the numbers in the tables to determine the exact number of inequivalent optimal codes with length $n' = n$ and dimension $k' = n-k$ in some of the cases. For the cells without *, the number of inequivalent optimal \( [n', k'] \) codes is equal to the number of inequivalent \([n, k]\) codes minus the number of inequivalent \([n-1, k-1]\) codes. The calculations took about 72 hours in contemporary PC. The number of all inequivalent codes with dimension 15 and dual distance 8, and dimension 19 and dual distance 9, grows exponentially, so we could not calculate all cases. That is why we consider the problems for existence of codes with parameters \([33, 18, 8]\) and \([33, 15, 10]\).

The existence of a \([33, 18, 8]\) code leads to the existence of a \([33, 18, 8]\) even code and \([32, 17, 8]\) even code (from the properties of shortened codes and Lemma 6) and its dual code \(C_{32}^2\) with parameters \([32, 15, 10]\). We know that \(L(14, 8) = 28\) (see Table 1) and \(d \leq 8\) from the tables for bounds of linear codes [9]. Theorem 8 gives us that the minimum distance \(d\) of \(C_{32}^1\) has to be \(4 \leq d \leq 8\). Using the algorithm EXTEND and already constructed even codes with parameters \([28, 14, 8]\), \([27, 14, 7]\), …, \([24, 14, d \geq 4]\), we obtain that there are exactly two inequivalent even codes \(C_{32}^1\) and \(C_{32}^2\) with generator matrices \(G_{32}^1\) and \(G_{32}^2\) and weight enumerators:

\[
1 + 124z^8 + 1152z^{10} + 3584z^{12} + 6016z^{14} + 11014z^{16} + 6016z^{18} + 3584z^{20} + 1152z^{22} + 124z^{24} + z^{32}
\]

and

\[
1 + 116z^8 + 1216z^{10} + 3360z^{12} + 6464z^{14} + 10454z^{16} + 6464z^{18} + 3360z^{20} + 1216z^{22} + 116z^{24} + z^{32}.
\]

None of these two codes can be extended to a code with parameters \([33, 15, 8]\). This leads to

**Theorem 9** Codes with parameters \([33, 18, 8]\) do not exist and \(n_2(18, 8) = 34\).

It follows that codes with parameters \([32, 18, 7]\) and \([33, 19, 7]\) do not exist and the minimum distance of a putative \([33, 19, d]\) code has to have \(d < 7\). From Theorem 8 and value of \(L(18, 10) = 28\) we have that \(d\) has to be 5 or 6.

There are exactly 30481 codes with parameters \([27, 18]\) and 11 codes with parameters \([26, 18]\) and the dual codes of all these codes are even. But none can be extended to a code with parameters \([33, 19, d]\). This leads us to the conclusion

**Theorem 10** Codes with parameters \([33, 14, 10]\) do not exist and \(n_2(14, 10) = 34\).

The calculations for nonexistence of codes with parameters \([33, 18, 8]\) and \([33, 14, 10]\) took about 4 weeks in a contemporary PC.

From Theorem 9 and Theorem 10 and tables for bounds of codes [9], we have:

**Corollary 11** \(n_2(19, 8) = 35, n_2(20, 8) = 36, n_2(21, 8) = 37, n_2(22, 8) = 38, n_2(15, 10) = 35\) and \(n_2(16, 10) = 36\).

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Algorithm EXTEND (Input: \( V, t \)):

```java
var U: array of sets; max,i:integer;
found:boolean; X:set; r:array of integer;

Procedure ext(X;set; size:integer);
var i:integer;
{
  if size = t then { print(X); exit; }
  if \(|U[|size - 1]| = 0 \} then
    if (size>max) and (size < t) then
      { max:=size;
        if max < t then found:=true;
      }
    exit;
  }
  while \(|U[|size - 1]| < t \} do
    if \(((size + \max) and (size < t)) then
      { max:=size;
        if max < t then found:=true;
      }
    exit;
    \{|U[|size - 1]| <= \max \}
    \{|U[|size - 1]| < t \} or
    \{|U[|size - 1]| \} \{v_j \in U[|size - 1]| \}
    \{|v_j \in U[|size - 1]| \} \{X ∪ v_j \} \in P\};
    if size < t then ext(X ∪ v_j, size + 1);
    if found = true then exit;
  }
}

Procedure Main;
{ max:=0;
  for i := |V| downto 1 do
    { found:=false;
      \( X := \{v_i\}; \)
      \( U[0] := \{v_j : v_j \in V, j > i \} \cup \{X \cup v_j \} \in P\};
      ext(X, 1);
      ** r[i]:=max; ***
    }
}
```

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TABLE 2 - Classification results for \([n,k]_{d+1 \geq 10}\) codes

| \(n \setminus k\) | 18  | 17  | 16  | 15  | 14  |
|---------------------|-----|-----|-----|-----|-----|
| 15                  | 0   | 0   | 0   | 0   | 1** |
| 16                  | 0   | 0   | 0   | 1*  | 6*  |
| 16                  | 0   | 0   | 1*  | 7*  | 3   |
| 17                  | 0   | 1*  | 8*  | 7   | 0   |
| 18                  | 1*  | 9*  | 14  | 1   | 0   |
| 19                  | 10* | 24* | 7   | 0   | 0   |
| 20                  | 38* | 29* | 3   | 0   | 0   |
| 21                  | 90* | 30* | 2   | 0   | 0   |
| 22                  | 237*| 39  | 0   | 0   | 0   |
| 23                  | 1037*| 29  | 0   | 0   | 0   |
| 24                  | 1114 | 6    | 0   | 0   | 0   |
| 25                  | 188572 | 0   | 0   | 0   | 0   |
| 26                  | 563960 | 0   | 0   | 0   | 0   |
| 27                  | 30481 | 0   | 0   | 0   | 0   |
| 28                  | 11   | 0   | 0   | 0   | 0   |
| 29                  | 0    | 0   | 0   | 0   | 0   |

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