How light can the Higgs be?

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It is widely believed that, for a given Top mass, the Higgs mass has a lower bound: if $m_{Higgs}$ is too small, the Higgs vacuum is unstable due to Top dynamics. From vacuum instability, the state-of-the-art calculation of the lower bound is close to the current experimental limit. Using non-perturbative simulations and large $N$ calculations, we show that the vacuum is in fact never unstable. Instead, we investigate the existence of a new lower bound, based on the intrinsic cut-off of this trivial theory.

1. INTRODUCTION

It is generally accepted that the Higgs sector of the Standard Model is trivial. A quantum field theory is defined by a set of bare parameters and a regulator with some cut-off $\Lambda$. Renormalized quantities are calculated in terms of the bare parameters, then the cut-off is sent to infinity. In a trivial theory, the renormalized couplings vanish as $\Lambda \rightarrow \infty$ for any choice of bare parameters. To have a non-trivial interacting theory, the cut-off must remain finite. Hence, the Higgs sector is an effective theory which is only valid at energy scales below the cut-off $\Lambda$.

Fig. 1 shows the current phenomenological upper and lower bounds for $m_{Higgs}$ as a function of $\Lambda$, the threshold of new physics [1]. For a given cut-off, the upper bound is the largest $m_{Higgs}$ that can be generated by any choice of bare parameters. Lattice simulations without Top dynamics have found that $m_{Higgs} \simeq 650$ GeV when the bare Higgs coupling $\lambda \rightarrow \infty$ and the momentum cut-off $\Lambda = \pi/a$ is a few TeV [2,3], where $a$ is the lattice spacing. For the cut-off effects to be acceptably small, we require the correlation length $\xi/a = 1/(m_{Higgs}a) \geq 2$ so that e.g. violation of rotational symmetry in particle scattering is less than a few % [3].

The phenomenological lower bound for $m_{Higgs}$ is based on the belief that, for a given Top quark mass, the Higgs mass cannot be too small, otherwise the Higgs field effective potential $V_{eff}(\phi)$ is unstable. The one-loop contributions to the renormalized effective potential from Higgs and Top loops are of the form $\lambda R \phi^4 \ln(\phi_R/v_R)$ and $-y_R^2 \phi^4 \ln(\phi_R/v_R)$ respectively, where $v_R$ is the renormalized vacuum expectation value of $\phi_R$, and $\lambda_R$ and $y_R$ are the renormalized Higgs and Yukawa couplings. If $y_R^2 \gg \lambda_R$, the negative Top contribution dominates, $V_{eff} \rightarrow -\infty$ as $\phi_R \rightarrow \infty$ and the potential no longer has its absolute minimum at $v_R$. For a given $m_{Higgs}$ and $m_{Top}$ (i.e. $\lambda_R$ and $y_R$), renormalized perturbation theory is valid only if the vacuum is stable, defining an energy scale $\Lambda$ where new physics must occur.

The state-of-the-art calculation of the lower bound from vacuum instability gives $m_{Higgs} \geq
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86 GeV for $\Lambda = 10$ TeV and $m_{\text{Top}} = 175$ GeV, with an uncertainty of less than 5 GeV [4]. As the current experimental limit is $m_{\text{Higgs}} \geq 114.3$ GeV, this is a very relevant and important statement for the validity of the Standard Model. However, we claim that there is no vacuum instability in the theory, making the validity of the current lower bound uncertain.

2. THERE IS NO INSTABILITY

It was first shown in [5] how to measure the constraint effective potential $V_{\text{eff}}$ non-perturbatively via lattice simulations. In particular, we examine a model of a single component real scalar field coupled to two copies of staggered fermions, corresponding to 8 degenerate continuum fermion flavors. The bare parameters are the Higgs mass $m$ and the Higgs and Yukawa couplings $\lambda$ and $y$. The scalar field is coupled locally to the fermions. We use the leapfrog Hybrid Monte Carlo algorithm with step-size $\Delta t = 0.01$ and trajectory length $N_t \Delta t \geq 1$. Our statistics are $10^4$ trajectories, with a separate simulation required at every value of the scalar field to measure $V_{\text{eff}}$. Further details will be presented in [6].

Fig. 2 shows results for the derivative of the constraint effective potential $dV_{\text{eff}}(\phi) / d\phi$ for a particular choice of bare parameters with $y^2 \gg \lambda$. There is no sign of any instability at large values of the scalar field. One-loop bare perturbation theory is in excellent agreement with the non-perturbative simulations, and when $\phi$ is large, even tree-level perturbation theory is very close to the full result.

Renormalized perturbation theory also agrees with the non-perturbative determination of $V_{\text{eff}}$ when $\phi$ is small. However, when $\phi$ is large, the renormalization method breaks down because the cut-off terms $O(\phi^2 / \Lambda^2)$ are not small. The effective potential appears unstable in renormalized perturbation theory because these large positive cut-off effects are neglected. The effective potential is always stable, as shown by numerical simulations and cut-off-dependent bare perturbation theory. This picture is confirmed in a calculation of the effective potential using a Pauli-Villars regulator, in the limit of a large number of fermion flavors [6].

3. CORRECT LOWER BOUND

In the same model of a single component real scalar field coupled to staggered fermions, we show how to compute the correct $m_{\text{Higgs}}$ lower bound (see [7] for some previous studies of Higgs–Top systems). We choose the bare parameters $\lambda, y$ and $m$ to be in the Higgs phase of the theory, close to the critical surface which separates the Higgs and the symmetric phases. For a particular choice of bare parameters, we measure $v_Ra$, the renormalized vev in lattice spacing units, using $v_R = 246$ GeV to convert the momentum cut-off $\Lambda = \pi/a$ into physical units.

We measure $m_{\text{Higgs}}$ and the wave-function renormalization factor $Z_\phi$ from the scalar field
propagator in momentum space $G_\phi(p^2)$

$$G_\phi^{-1}(p^2) = (m_{\text{Higgs}}^2 + p^2)/Z_\phi,$$

where $p^2 = \sum_\mu 4\sin^2(p_\mu/2)$. We measure $m_{\text{Top}}$ from the zero momentum piece of the fermion propagator. All simulations were performed in $8^3 \times 16$ volumes, again using the HMC algorithm, varying the step-size $\Delta t$ to achieve acceptance rates of more than 90%. Our statistics are $10^4$ trajectories for each choice of bare parameters.

In Fig. 3, we show a typical measurement of $G_\phi^{-1}(p^2)$ giving $m_{\text{Higgs}} a = 0.310(3)$ and $Z_\phi = 0.971(2)$.

In Fig. 4 we plot a summary of the simulations. The solid lines are measurements with $\Lambda$ fixed in physical units, varying the bare parameters to stay at a fixed distance from the critical surface. For a given $\Lambda$ and $m_{\text{Top}}$, the smallest Higgs mass is generated when $\lambda \to 0$. This is completely analogous to the upper bound, where $m_{\text{Higgs}}$ is largest when $\lambda \to \infty$. To keep the cut-off effects acceptably small, we require both the scalar and fermion correlations lengths $\xi/a \geq 2$. The dashed line in Fig. 4 corresponds to $\xi/a = 2$, to the left of the dashed line is the allowed region of small cut-off effects. From this, we can extract the lower bound. For example, in this model of 8 degenerate fermions, the smallest Higgs mass that can be generated for $m_{\text{Top}} = 175$ GeV and $\Lambda = 2$ TeV is $m_{\text{Higgs}} \simeq 230$ GeV.

4. SUMMARY

The apparent instability of the Higgs vacuum when coupled to a heavy Top quark is due to the breakdown of renormalized perturbation theory when the cut-off effects are large. This places the current phenomenological $m_{\text{Higgs}}$ lower bound in doubt. The correct lower bound in a particular cut-off scheme can be found by measuring the smallest Higgs mass that can be generated while simultaneously keeping the cut-off effects acceptably small. A similar calculation for a realistic Higgs–Top system is a very timely and important project for the lattice community.

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