Superfluidity with dressed nucleons

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Abstract

The gap equation with dressed propagators is solved in symmetric nuclear matter. Nucleon self-energies are obtained within the self-consistent in medium $T$ matrix approximation. The off-shell gap equation is compared to an effective quasiparticle gap equation with reduced interaction. At normal density, we find a reduction of the superfluid gap from 6.5MeV to 0.45MeV when self-energy effects are included.

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The importance of pairing correlation in nuclear systems was realized very early [1]. In finite nuclei, pairing effects are known in the mass systematics of nuclei and the properties of deformed nuclei. In extended systems, nuclear pairing is expected to occur in the dense matter inside the neutron stars [2, 3]. Quantitative description of pairing correlations starting from bare nucleon-nucleon interactions is still limited. In medium effects, going beyond simple gap equation with free interaction, are important [4, 5, 6, 7, 8, 9, 10, 11, 12]. The use of the induced interaction in the gap equation was studied for neutron matter leading to a significant reduction of the pairing strength [4, 5, 6, 7]. Another effect is due to the dressing of nucleons in an interacting system, leading to a modification of the density of states and of the effective energy gap [8, 9, 10]. From the Bardeen-Cooper-Schrieffer result [13], it is obvious that self-energy effects modifying the effective mass change the value of the superfluid gap. Furthermore, the reduced quasiparticle strength at the Fermi surface leads to a modification of the effective pairing interaction [8, 9, 10].

Recently self-consistent in medium $T$ matrix was calculated for nuclear matter [14, 15, 16, 17, 18, 19]. In this approach ladder diagrams with dressed particle-particle and hole-hole propagators are summed

\[
\langle p| T(P, \Omega) | p' \rangle = V(p, p') + \int \frac{d\omega_1}{2\pi} \int \frac{d\omega_2}{2\pi} \int \frac{d^3q}{(2\pi)^3} V(p, q) \frac{1 - f(\omega_1) - f(\omega_2)}{\Omega - \omega_1 - \omega_2 + i\epsilon} \times A(p_1, \omega_1) A(p_2, \omega_2) \langle q | T(P, \Omega) | p' \rangle
\]

(1)

where \( p_{1,2} = P/2 \pm q \), \( f(\omega) \) is the Fermi distribution, and

\[
A(p, \omega) = \frac{-2 \text{Im} \Sigma(p, \omega)}{\omega - p^2/2m - \text{Re} \Sigma(p, \omega)^2 + \text{Im} \Sigma(p, \omega)^2}
\]

(2)

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is the self-consistent spectral function of the nucleon. The imaginary part of the corresponding retarded self-energy can be obtained from

\[
\text{Im} \Sigma(p, \omega) = \int \frac{d\omega_1}{2\pi} \int \frac{d^3k}{(2\pi)^3} A(k, \omega_1) \langle (p - k)/2 | \text{Im} T(p + k, \omega + \omega_1) | (p - k)/2 \rangle_A [f(\omega_1) + b(\omega + \omega_1)],
\]

(3)

\(b(\omega)\) is the Bose distribution. The real part of the self-energy is related to Im\(\Sigma\) by a dispersion relation

\[
\text{Re} \Sigma(p, \omega) = \Sigma_{HF}(p) + P \int \frac{d\omega'}{\pi} \frac{\text{Im} \Sigma(p, \omega')}{\omega' - \omega}
\]

(4)

with \(\Sigma_{HF}(p)\) the Hartree-Fock self-energy. Eqs. (1), (3), (2), and (4) are solved iteratively. Numerical calculations with off-shell propagators are very complex; recently, the self-consistent T-matrix approximation scheme was solved for a realistic interaction with several partial waves [20]. It is the aim of the present work to analyze the effects of the self-consistent dressing of fermion propagators on nucleon superfluidity for such a realistic interaction. For the case of the symmetric nuclear matter, the dominant pairing correlations appear in the \(^3S_1 - ^3D_1\) partial wave with the value of the superfluid gap of the order of several MeV's [21, 22, 23]. Such a large value of the superfluid gap is not seen in nuclear mass systematics for \(N \simeq Z\) nuclei. Although the existence of nuclear pairing in the \(^1S_0\) channel is well established in the data, the calculation of the corresponding pairing force from the bare interaction is not available. The strength of the \(^1S_0\) pairing force in a simple mean-field approach is small, unlike for the deuteron channel. Pairing gaps observed in finite nuclei result from bulk and surface pairing interactions [11, 12]. The first mechanism should be dominant for the \(^3S_1 - ^3D_1\) channel if the large value of the BCS gap is taken, which would result in an unrealistically large value of the proton-neutron gap. No such paradox exist for the \(^1S_0\) channel, where the BCS gap in nuclear matter is small, which means that the pairing force in finite nuclei has (at least partly) a different origin. It is also important to clarify the role of dominant pairing correlations for nuclear matter calculations, especially in view of recent suggestions that normal nuclear matter is not superfluid [17, 23, 24] due to self-energy corrections. In the following, we demonstrate explicitly that self-energy corrections reduce the value of the superfluid gap to a small, but nonzero, value in symmetric nuclear matter at saturation density \(\rho_0\).

From the Thouless criterion on the self-consistent T matrix at finite temperature, it is known that self-consistent dressing of nucleons reduces significantly the critical temperature [18]. Calculation performed in the normal phase of cold nuclear matter suggest a strong suppression of pairing correlations due nucleon dressing [16, 18]. However, no explicit calculation of superfluid properties with dressed propagators and realistic interaction is available. In Ref. [2] the gap equation with off-shell propagators

\[
\Delta(p) = \int \frac{d\omega d\omega' d^3k}{(2\pi)^5} A(k, \omega) A_0(k, \omega) \frac{1 - f(\omega) - f(\omega')}{-\omega - \omega'} V(p, k) \Delta(k)
\]

(5)

was solved for a simple interaction. In the above equation \(A_0(p, \omega)\) denotes the spectral function of the nucleon, including the diagonal self-energy \(\Sigma(p, \omega)\) (obtained in the T matrix approximation) and the off-diagonal self-energy \(\Delta(p)\) obtained from Eq. (3) itself; \(A(p, \omega)\) is the spectral function of the nucleon dressed with the diagonal self-energy only [2]. It was found that the superfluid gap for dressed nucleons is reduced by a factor \(2 - 3\) in comparison to the result of the quasiparticle gap equation.
Self-energy effects can be effectively taken into account in a quasiparticle gap equation \( \Delta(p) = Z_p \Delta(p) \) \( (6) \). The first consequence of dispersive self-energy corrections to the propagator is that the superfluid energy gap is not the off-diagonal self-energy \( \Delta(p) \) but \( \hat{\Delta}(p) = Z_p \Delta(p) \) \( (6) \) where

\[
Z_p = \left( 1 - \frac{\partial \Sigma(p, \omega_p)}{\partial \omega} \bigg|_{\omega=\omega_p} \right)^{-1} \quad .
\]

It means that if the position of the quasiparticle peak in \( A(p, \omega) \) is given by:

\[
\omega_p = \frac{p^2}{m} + \text{Re} \Sigma(p, \omega_p) ,
\]

(8)

the poles of the propagator in the superfluid are located approximately at

\[
\omega = \pm E_p = \pm \sqrt{\left( \omega_p - \mu \right)^2 + \hat{\Delta}(p)^2} = \pm \sqrt{\left( \omega_p - \mu \right)^2 + Z_p^2 \Delta(p)^2} .
\]

(9)

The quasiparticle strength at the poles \( \pm E_p \) of the superfluid propagator can be expressed, to a very good accuracy, by the quasiparticle renormalization strength \( Z_p \) of the pole of the normal propagator \( \Delta(p) \). The quasiparticle effective gap equation with dressed propagators is \( \Delta(p) = - \int \frac{d^3k}{(2\pi)^3} \frac{Z_k^2 V(p, k)}{2E_k} \left[ 1 - \frac{2f(E_k)}{2E_k} \right] \Delta(k) \). \( \Delta(p) = - \int \frac{d^3k}{(2\pi)^3} \frac{Z_k^2 V(p, k)}{2E_k} \left[ 1 - \frac{2f(E_k)}{2E_k} \right] \Delta(k) \). \( \Delta(p) = - \int \frac{d^3k}{(2\pi)^3} \frac{Z_k^2 V(p, k)}{2E_k} \left[ 1 - \frac{2f(E_k)}{2E_k} \right] \Delta(k) \). \( \Delta(p) = - \int \frac{d^3k}{(2\pi)^3} \frac{Z_k^2 V(p, k)}{2E_k} \left[ 1 - \frac{2f(E_k)}{2E_k} \right] \Delta(k) \). \( \Delta(p) = - \int \frac{d^3k}{(2\pi)^3} \frac{Z_k^2 V(p, k)}{2E_k} \left[ 1 - \frac{2f(E_k)}{2E_k} \right] \Delta(k) \). \( \Delta(p) = - \int \frac{d^3k}{(2\pi)^3} \frac{Z_k^2 V(p, k)}{2E_k} \left[ 1 - \frac{2f(E_k)}{2E_k} \right] \Delta(k) \). \( \Delta(p) = - \int \frac{d^3k}{(2\pi)^3} \frac{Z_k^2 V(p, k)}{2E_k} \left[ 1 - \frac{2f(E_k)}{2E_k} \right] \Delta(k) \). \( \Delta(p) = - \int \frac{d^3k}{(2\pi)^3} \frac{Z_k^2 V(p, k)}{2E_k} \left[ 1 - \frac{2f(E_k)}{2E_k} \right] \Delta(k) \). \( \Delta(p) = - \int \frac{d^3k}{(2\pi)^3} \frac{Z_k^2 V(p, k)}{2E_k} \left[ 1 - \frac{2f(E_k)}{2E_k} \right] \Delta(k) \). \( \Delta(p) = - \int \frac{d^3k}{(2\pi)^3} \frac{Z_k^2 V(p, k)}{2E_k} \left[ 1 - \frac{2f(E_k)}{2E_k} \right] \Delta(k) \). \( \Delta(p) = - \int \frac{d^3k}{(2\pi)^3} \frac{Z_k^2 V(p, k)}{2E_k} \left[ 1 - \frac{2f(E_k)}{2E_k} \right] \Delta(k) \). \( \Delta(p) = - \int \frac{d^3k}{(2\pi)^3} \frac{Z_k^2 V(p, k)}{2E_k} \left[ 1 - \frac{2f(E_k)}{2E_k} \right] \Delta(k) \). \( \Delta(p) = - \int \frac{d^3k}{(2\pi)^3} \frac{Z_k^2 V(p, k)}{2E_k} \left[ 1 - \frac{2f(E_k)}{2E_k} \right] \Delta(k) \). \( \Delta(p) = - \int \frac{d^3k}{(2\pi)^3} \frac{Z_k^2 V(p, k)}{2E_k} \left[ 1 - \frac{2f(E_k)}{2E_k} \right] \Delta(k) \). \( \Delta(p) = - \int \frac{d^3k}{(2\pi)^3} \frac{Z_k^2 V(p, k)}{2E_k} \left[ 1 - \frac{2f(E_k)}{2E_k} \right] \Delta(k) \). \( \Delta(p) = - \int \frac{d^3k}{(2\pi)^3} \frac{Z_k^2 V(p, k)}{2E_k} \left[ 1 - \frac{2f(E_k)}{2E_k} \right] \Delta(k) \). \( \Delta(p) = - \int \frac{d^3k}{(2\pi)^3} \frac{Z_k^2 V(p, k)}{2E_k} \left[ 1 - \frac{2f(E_k)}{2E_k} \right] \Delta(k) \). \( \Delta(p) = - \int \frac{d^3k}{(2\pi)^3} \frac{Z_k^2 V(p, k)}{2E_k} \left[ 1 - \frac{2f(E_k)}{2E_k} \right] \Delta(k) \). \( \Delta(p) = - \int \frac{d^3k}{(2\pi)^3} \frac{Z_k^2 V(p, k)}{2E_k} \left[ 1 - \frac{2f(E_k)}{2E_k} \right] \Delta(k) \). \( \Delta(p) = - \int \frac{d^3k}{(2\pi)^3} \frac{Z_k^2 V(p, k)}{2E_k} \left[ 1 - \frac{2f(E_k)}{2E_k} \right] \Delta(k) \). \( \Delta(p) = - \int \frac{d^3k}{(2\pi)^3} \frac{Z_k^2 V(p, k)}{2E_k} \left[ 1 - \frac{2f(E_k)}{2E_k} \right] \Delta(k) \). \( \Delta(p) = - \int \frac{d^3k}{(2\pi)^3} \frac{Z_k^2 V(p, k)}{2E_k} \left[ 1 - \frac{2f(E_k)}{2E_k} \right] \Delta(k) \). \( \Delta(p) = - \int \frac{d^3k}{(2\pi)^3} \frac{Z_k^2 V(p, k)}{2E_k} \left[ 1 - \frac{2f(E_k)}{2E_k} \right] \Delta(k) \). \( \Delta(p) = - \int \frac{d^3k}{(2\pi)^3} \frac{Z_k^2 V(p, k)}{2E_k} \left[ 1 - \frac{2f(E_k)}{2E_k} \right] \Delta(k) \). \( \Delta(p) = - \int \frac{d^3k}{(2\pi)^3} \frac{Z_k^2 V(p, k)}{2E_k} \left[ 1 - \frac{2f(E_k)}{2E_k} \right] \Delta(k) \). \( \Delta(p) = - \int \frac{d^3k}{(2\pi)^3} \frac{Z_k^2 V(p, k)}{2E_k} \left[ 1 - \frac{2f(E_k)}{2E_k} \right] \Delta(k) \). \( \Delta(p) = - \int \frac{d^3k}{(2\pi)^3} \frac{Z_k^2 V(p, k)}{2E_k} \left[ 1 - \frac{2f(E_k)}{2E_k} \right] \Delta(k) \). \( \Delta(p) = - \int \frac{d^3k}{(2\pi)^3} \frac{Z_k^2 V(p, k)}{2E_k} \left[ 1 - \frac{2f(E_k)}{2E_k} \right] \Delta(k) \). \( \Delta(p) = - \int \frac{d^3k}{(2\pi)^3} \frac{Z_k^2 V(p, k)}{2E_k} \left[ 1 - \frac{2f(E_k)}{2E_k} \right] \Delta(k) \). \( \Delta(p) = - \int \frac{d^3k}{(2\pi)^3} \frac{Z_k^2 V(p, k)}{2E_k} \left[ 1 - \frac{2f(E_k)}{2E_k} \right] \Delta(k) \). \( \Delta(p) = - \int \frac{d^3k}{(2\pi)^3} \frac{Z_k^2 V(p, k)}{2E_k} \left[ 1 - \frac{2f(E_k)}{2E_k} \right] \Delta(k) \). \( \Delta(p) = - \int \frac{d^3k}{(2\pi)^3} \frac{Z_k^2 V(p, k)}{2E_k} \left[ 1 - \frac{2f(E_k)}{2E_k} \right] \Delta(k) \). \( \Delta(p) = - \int \frac{d^3k}{(2\pi)^3} \frac{Z_k^2 V(p, k)}{2E_k} \left[ 1 - \frac{2f(E_k)}{2E_k} \right] \Delta(k) \). \( \Delta(p) = - \int \frac{d^3k}{(2\pi)^3} \frac{Z_k^2 V(p, k)}{2E_k} \left[ 1 - \frac{2f(E_k)}{2E_k} \right] \Delta(k) \).
self-energy. The largest value of the superfluid gap comes from the gap equation with the bare interaction \( \Delta(p_F) \), where the sole influence of the nuclear medium comes through the modification of single-particle energies. On the other hand, the gap equation with dressed propagators \( \hat{\Delta}(p_F) \) leads to much smaller values of the superfluid gap. We find the maximal value of the superfluid gap \( \hat{\Delta}(p_F) \) is \( \approx 2.2 \text{MeV} \) at the saturation density \( \rho_0 \). At the saturation density \( \hat{\Delta}(p_F) = 0.45 \text{MeV} \). The last value is significantly smaller than the corresponding BCS gap \( \Delta(p_F) = 6.5 \text{MeV} \). The reasons are twofold. First, the effective energy gap is \( Z \Delta(p) \) instead of \( \Delta(p) \) due to the dispersive self-energy \( \Delta \). Second, the spectral functions for fully dressed nucleons \( (A_s(p, \omega)) \) and \( A_s(p, \omega) \) give an effective reduction of the density of states at the Fermi surface \( \Delta \). Both effects are taken into account in the effective gap equation with reduced interactions \( \Delta \). Indeed, the superfluid gap obtained from the effective gap equation \( \Delta \) is close to the solution of the full equation \( \Delta \).

In Fig. 2 are compared the kernels of the full gap equation \( \Delta \) and of the effective gap equation \( \Delta \)

\[
\frac{1}{2\langle E_k \rangle} = \int \frac{d\omega d\omega'}{(2\pi)^2} A(k, \omega) A_s(k, \omega) \left[ 1 - f(\omega) - f(\omega') \right] \omega + \omega' \tag{12}
\]

and of the effective gap equation \( \Delta \)

\[
\frac{1}{2E_k} = \frac{Z_k^2}{\sqrt{(\omega_k - \mu)^2 + \hat{\Delta}(k)^2}}. \tag{13}
\]

As expected, close to the Fermi momentum the kernel appearing in the gap equation with full spectral functions can be approximated by the renormalization of the quasiparticle poles in the quasiparticle gap equation. The momentum integration in the \(^3S_1 - ^3D_1\) gap equation cannot be restricted to the vicinity of the Fermi momentum only. Differences between the kernels of the gap equations at low momenta lead to small differences in the resulting gaps (Fig. 1). It must be stressed, however, that to a reasonable accuracy, the effective kernel \( \Delta \) is a good approximation of the kernel of the full gap equation \( \Delta \) and describes the mechanism of the reduction of the superfluid gap by nucleon dressing. It means that the background part of the spectral function does not modify significantly the effective interaction between quasiparticles in the gap equation.

Similar effects are visible in the temperature dependence of the superfluid gap. The largest gap and the largest critical temperature is obtained from the gap equation with the bare interaction (Fig. 3). The gap equation with dressed nucleons \( \Delta \) and the effective gap equation \( \Delta \) give much smaller values for the critical temperature. The single-particle potential is weakly dependent on the temperature \( T \), and the gap closure is due in all the approaches to the phase space factor \( \left[ 1 - f(\omega) - f(\omega') \right] \). At the critical temperature the effective gap equation \( \Delta \) corresponds to the Thouless condition for the quasiparticle \( T \) matrix with a reduced interaction \( Z_k^2 V(p, k) \).

We calculate the superfluid gap for the deuteron channel with off-shell propagators. The spectral functions serving as input for the gap equation are obtained in the self-consistent \( T \) matrix approximation. The most important result is the strong reduction of the superfluid energy gap in symmetric nuclear matter in comparison to results obtained with bare interactions. The superfluid gap is 0.45 MeV at the saturation density, when self-energy effects are included. The actual value of the superfluid gap in symmetric nuclear matter is influenced also by vertex corrections (the induced interaction) \[ 4, 5, 6, 7, 27, 28, 29 \].

The strong reduction of the gap that we find is important in two respects. First, such a small value of the superfluid energy gap in nuclear matter justifies standard approaches to
the nuclear matter problem which neglect the superfluid transition. Second, the small value of the neutron-proton pairing gap in nuclear matter is compatible with common inferred from the properties of finite nuclei. The value, or even the presence, of the a neutron-proton gap in finite nuclei is still a matter of debate [30, 31, 32, 33]; but a large value of the neutron-proton gap, as obtained from a simple BCS gap equation with bare interaction, is excluded by the data. Since we find a small value of the $^3S_1 - ^3D_1$ gap, the paradox is resolved. However, it is even more difficult to make definite predictions on the value of the energy gap in finite nuclei than in nuclear matter. Besides vertex corrections other mechanism of the pairing interactions in finite nuclei are possible, that cannot be treated in a local density approximation.

As a byproduct, we study the quasiparticle gap equation with effective interactions. We find a reasonable agreement with the results obtained with complete spectral functions. Therefore, the effective interaction $Z_k^2V(p, k)$ is a good starting point for the study of other many-body effects, such as the role of the induced interaction [4, 5, 6, 7, 27, 28, 29], for the pairing channel. In neutron matter a similar reduction of the superfluid gap is expected. The effect would be smaller since in medium dispersive effects are weaker [11, 10].

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Figure 1: The value of the superfluid energy gap at the Fermi momentum $\Delta_{p_F}Z_{p_F}$ as a function of the density for the BCS gap equation [Eq. (11)] (solid line), for the gap equation with dressed propagators [Eq. (5)] (dotted line), and for the effective gap equation [Eq. (10)] (dashed line).
Figure 2: The kernel of the full gap equation [Eq. (12)] (solid line) and of the effective gap equation [Eq. (13)] (dashed line), at \( \rho = .75\rho_0 \).
Figure 3: The value of the superfluid energy gap at the Fermi momentum $\Delta_{p_F}Z_{p_F}$ as a function of the temperature for the BCS gap equation [Eq. (11)] (solid line), for the gap equation with dressed propagators [Eq. (12)] (dotted line), and for the effective gap equation [Eq. (13)] (dashed line), at $\rho = 0.75\rho_0$. 