A Comparative Study on the Derivation of Unit Hydrograph for Dhadhar River Basin
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ABSTRACT

Several techniques are available for the development of the unit hydrograph. But most of these traditional methods require manual fitting of the unit hydrograph through few points, which does not guarantee the area under the unit hydrograph to be unity. More over most of the stations are ungauged, due to which it becomes difficult to develop the unit hydrograph. So in order to overcome these problems, two methods have been considered in this study for the development of the unit hydrograph for Dhadhar river basin. They are the “two parameter Gamma distribution” and “three parameter Beta distribution”, both of which are based on Probability Distribution Functions (pdfs). The unit hydrograph developed by the two parameter Gamma distribution match well with the one developed by CWC method, but the unit hydrograph developed by the three parameter Beta distribution does not match well with the one developed by the CWC method. From the unit hydrograph, runoff hydrograph is convoluted for the year 2018. For this the hourly rainfall are generated from daily rainfall values by disaggregation. But on plotting, the simulated discharge hydrograph is found to be greater than the observed discharge. This may be due to non-incorporation of the inflow outflow processes of many hydraulic structures such as dams, irrigation schemes etc., existing in the basin in the model study.

Keywords : Discharge Hydrograph, Unit Hydrograph, Gamma Distribution, Beta Distribution, CWC Method.

I. INTRODUCTION

The beginning of rainfall runoff modelling for solving engineering problems dates back to the second half of the nineteenth century. Even today it is very important in activities such as design of hydraulic structures in watershed, flood control and management etc. But in the absence of runoff data, as in case of ungauged stations, it becomes necessary to obtain runoff data with the help of set up models. Historically researchers have relied on conventional techniques; however these techniques do not ensure the area under the graph to be unity. Hence to develop a runoff data for a rainfall event, L. K. Sherman introduced the concept of “unit hydrograph” in 1932. This was the first attempt to predict a unit hydrograph instead of just peak flow rate and time to peak. Later many researchers attempted to improve the unit hydrograph shape by incorporating more complexities in the model. Thus unit hydrograph is the most widely accepted tool for hydrological analysis and synthesis. However as most of the sites are ungauged, the idea of Synthetic Unit hydrograph (SUH) came into existence. The term ‘synthetic’ in SUH denotes that the unit hydrograph has been derived from watershed characteristics and not from the rainfall runoff data. Moreover the traditional methods for developing SUH required manual fitting of the points which is subjective and do not assure the area under the graph to be unity. The empirical equations used in the traditional methods also involve certain constants which vary over wide range. Due to
the similarity in shape of statistical distribution and a conventional unit hydrograph, several attempts have been made to use pdfs for the development of unit hydrograph. The Gamma distribution and Beta distribution are such pdfs and they not only ensure the area under the graph to be unity but also provide a smooth shape for the unit hydrograph.

**Nomenclature**

- **A** Area of the watershed in km²
- **b** The scale parameter (in hours) in Beta distribution
- **B (a, b)** The Beta function of a and b, where a and b are parameters
- **CWC** Central Water Commission
- **IUH** Instantaneous Unit Hydrograph
- **K** Equal storage coefficient
- **n** Number of linear reservoirs
- **p** Non-dimensional shape parameters in Beta distribution
- **pdf** Probability Distribution Function
- **q** Depth of runoff per unit of time per unit of effective rainfall
- **qₚ** Peak discharge of unit hydrograph per unit area in cumecs per km²
- **Qₚ** Peak discharge in cumecs
- **r** Non-dimensional shape parameters in Beta distribution
- **Rᵢ** Rainfall event
- **SUH** Synthetic unit hydrograph
- **t** Time in hours
- **Tᵢ** Base width of the unit hydrograph in hours
- **tₚ** Time to peak discharge in hours
- **W₅₀** Width of the unit hydrograph measured at 50% peak discharge ordinate in hours
- **W₇₅** Width of the unit hydrograph measured at 75% peak discharge ordinate in hours
- **W₉₅₀** Width of the rising side of the unit hydrograph measured at 50% of peak discharge ordinate in hours
- **W₉₇₅** Width of the rising side of unit hydrograph measured at 75% of peak discharge in hours
- **α** Dimensionless parameter in Beta distribution
- **β** Dimensionless parameter in Gamma and Beta distribution
- **γ** Dimensionless parameter in Beta distribution
- **Γ** Gamma function

**II. OBJECTIVES**

The main objective of the study is to derive the unit hydrograph for the Dhadhar river basin by the Gamma distribution and Beta distribution, and compare them. For this, first it is required to determine the peak discharge (qₚ), time to peak discharge (tₚ) and base width of the unit hydrograph (Tᵢ) by the CWC method. Then compare the unit hydrograph developed by the gamma and beta distribution. Using this unit hydrograph, the discharge hydrograph is convoluted for the year 2018.

**III. STUDY AREA AND DATA REQUIRED**

The study area considered is Dhadhar river basin (72° 30’ to 73° 45’N and 21° 45’ and 22° 45’E), located in the state of Gujarat, India. With a length of 142km. The river originates from Pavagadh Hill and meets to bay of Kambhat. The watershed has a total area of 3423 km². The watershed caters many irrigation projects. The data required for the study are the toposheet of the study area, SRTM data, and satellite imagery, land use map, daily/hourly rainfall and discharge data. The daily rainfall data is obtained for the year 2018.

**IV. THEORETICAL BACKGROUND**

**A. Central Water Commission (CWC) Method**

The CWC method is used for the determination of qₚ, tₚ and Tᵢ. For this, first the equivalent slope is to be computed, for which first the longest stream is identified and then the points are identified where bed level changes considerably. Using these, the equivalent slope is computed by using the formula of CWC method. Then qₚ, tₚ and Tᵢ are computed, followed by the computation of other parameters which include W₅₀, W₇₅, W₉₅₀ and W₉₇₅ which are
required for plotting the unit hydrograph. All this are computed by using the formula in the CWC method.

B. Two Parameter Gamma Distribution Method

On the basis of the concept of \( n \)-linear reservoirs having equal storage coefficient \( K \), Nash and Dooge (1959) developed the IUH in the form of gamma function as

\[
q = \frac{1}{K T n} (t / k)^{n-1} e^{-t/k} \tag{1}
\]

Where \( n \) and \( K \) determine the shape of the IUH. Equation (1) is used for the derivation of SUH from parameters \( n \) and \( K \). In this, the parameter \( K \) is derived from

\[
K = \frac{t_p}{n-1} \tag{2}
\]

The value of \( n \) is computed on the basis of the dimensionless parameter \( \beta \), which is given by

\[
\beta = \frac{q_p}{t_p} \tag{3}
\]

Then on the basis of \( \beta \) values, the formula for \( n \) is given in (4) and (5)

\[
n = 5.53 \beta^{1.75} + 1.04 \quad ; \quad 0.01 < \beta < 0.35 \tag{4}
\]

\[
n = 6.29 \beta^{1.998} + 1.157 \quad ; \quad \beta < 0.35 \tag{5}
\]

Thus the two parameters of the gamma distribution that determine the shape of the unit hydrograph are \( n \) and \( K \), which can be computed by using \( q_p \) and \( t_p \). So by knowing the value of \( q_p \) and \( t_p \), the hydrograph can be developed by using two parameter gamma distribution.

C. Three Parameter Beta Distribution Method

Haktanir and Sezen (1990) worked on finding the suitability of three parameter Betadistribution as Synthetic Unit Hydrograph. The probability distribution function of beta distribution is given as

\[
f (x) = \left( \frac{1}{B (a, b)} \right) x^{a-1} (1 - x)^{b-1} \tag{6}
\]

\[
B (a, b) = \int_0^1 t^{a-1} (1-t)^{b-1} dt \tag{7}
\]

Equation (6) can be converted into a three parameter distribution as

\[
Q = \left[ \frac{t (r - t) (b - t) (p - t)}{B (r, p - r) b (p - r)} \right] \tag{8}
\]

Where \( r, p, b, q \) and \( t \) have already been defined in nomenclature. The advantage of the equation is that the area under the curve described by (8) for limit \( t = 0 \) and \( t = \) time base of the hydrograph is one unit. So the three parameters in this case are \( p, r \) and \( b \). But for the determination of \( p, r \) and \( b \), it is required to define three other non-dimensional groups, which are \( \alpha, \beta \) and \( \gamma \), which are given by the equation

\[
\alpha = \frac{T_B}{t_p} \tag{9}
\]

\[
\beta = \frac{q_p}{t_p} \tag{10}
\]

\[
\Gamma = \frac{q_p}{T_B} \tag{11}
\]

The non-dimensional parameter \( \beta \), is to be taken as a form factor which quantifies the hydrograph peakness and influences the hydrograph shape. The non-dimensional parameter \( \gamma \) can be related to the hydrograph area and the non-dimensional parameter \( \alpha \) is similar to the scale parameter since it influences the skewness of the unit hydrograph. Using \( \alpha, \beta \) and \( \gamma \), the three parameters \( p, r \) and \( b \) are obtained as

\[
P = 2.5355 \times (0.0481 + 2.961 \beta^{1.875}) \times (\alpha^{0.954}) - 10.72777 \beta \tag{12}
\]

\[
\Gamma = 1 - [(2 - p)/\alpha] \tag{13}
\]

\[
b = T_B \tag{14}
\]

Thus by using known \( q_p, t_p \) and \( T_B \), the parameters of the beta distribution can be estimated, and hence the complete shape of the unit hydrograph can be derived for any ungauged catchment.

V. METHODOLOGY

D. Determination Of \( q_p, t_p \) And \( T_B \) By The CWC Method

In this, first it is required to determine \( q_p \), \( t_p \) and \( T_B \) by the CWC method. For this first the watershed is to be delineated, which is done by processing DEM in the GIS platform, by considering Amod discharge station...
as an outlet point. Then the longest stream is identified, followed by the identification of those points where bed level changes occur. The equivalent slope is then computed as explained in section 4. Using this, \(q_p\), \(t_p\) and \(T_8\) is obtained. Further \(W_{50}\), \(W_{75}\), \(WR_{50}\) and \(WR_{75}\) are computed and using all these points, unit hydrograph is developed by the CWC method.

**E. Derivation of the Unit Hydrograph by Gamma Distribution**

Using the value of \(q_p\) and \(t_p\) which is already computed, the parameters of the Gamma distribution, that is, \(n\), \(K\) and \(\beta\) are estimated using (2) and (3), (4) and (5), as explained in section 4. But prior to plotting of the unit hydrograph, the value of gamma \(n\) is determined using the Sterling's equation as shown below.

\[\Gamma = (e^{-n}) \times n^{(n-0.5)} \times 2\pi^{0.5} \times [1 + (1/12n) + (1/288n^2) - (139/518n^3) - (571/2488320n^4)]^{0.5} \times e^{(3n/2)} \times [1 + (1/12n) + (1/288n^2) - (139/518n^3) - (571/2488320n^4)]^{0.5}\]

(15)

Once the parameters of the gamma distribution are estimated, then \(q\) is developed by varying the value of 't' in (1). Finally, the discharge \((Q)\) in cumecs is computed by (16) and the unit hydrograph is plotted.

\[Q = 2.776 \times q \times A\]

(16)

**F. Derivation of the Unit Hydrograph by Beta Distribution**

The steps in beta distribution are same as the gamma distribution, with the only difference being in parameter estimation. The parameters \(p\), \(r\) and \(b\) of beta distribution are estimated using (12-14) from section 4, where \(TB\), which is already determined by the CWC method is used, in addition to \(q_p\) and \(t_p\). But prior to plotting of the unit hydrograph, the value of \(B(r, p-r)\) is determined by using the formula

\[B (r, p-r) = [\Gamma (r) \times \Gamma (p-r)] / \Gamma (p)\]

(17)

Where \(\Gamma (r)\), \(\Gamma (p-r)\) and \(\Gamma (p)\) are estimated using Sterling's formula. Then \(q\) is developed by varying the value of 't' in (8). Finally, the discharge \((Q)\) in cumecs is given by (16) and unit hydrograph is plotted.

**G. Determination of Average Hourly Rainfall**

For the determination of average hourly rainfall, first the average daily rainfall is computed by using ‘Theissen polygon’ option in ArcGIS. For this 14 rain gauge stations are considered and the theissen polygon is drawn. Then this average daily rainfall is disaggregated to average hourly rainfall by using the CWC method.

**H. Convolution**

The discharge hydrograph is then obtained by multiplying the ordinates of the unit hydrograph with the rainfall. Rain events are selected and labeled as \(R_1\), \(R_2\), and \(R_3\) etc. Runoff from the storm event \(R_1\) on area \(A_2\) arrives at the outlet at the same time as the rainfall event \(R_2\) on \(A_1\). Hence

\[Q_t = R_1 \times A_1 + R_{i-1} \times A_2 + R_{i-2} \times A_3 + \ldots + R_1 \times A_i\]

(18)

**VI. RESULTS AND DISCUSSIONS**

**I. Delineation of Watershed**

First the watershed is delineated using Vadodara discharge station as the outlet point. The delineated watershed is shown in fig 1. (a) And the slope determination identified is shown in fig 1. (b).
Then the peak discharge, time to peak discharge and other parameters which are required for the development of the unit hydrograph by the CWC method are computed as shown in table I.

### TABLE I
PARAMETERS ESTIMATED BY CWC METHOD

| Parameter                                | Value obtained               |
|------------------------------------------|------------------------------|
| Peak Discharge ($q_p$)                   | 0.081m$^3$/s/km$^2$          |
| Time of Peak Discharge ($t_p$)            | 23.58 hrs                    |
| Base width of unit hydrograph ($T_b$)     | 75.15 hrs                    |
| $W_{50}$                                  | 26.69 hrs                    |
| $W_{75}$                                  | 14.02 hrs                    |
| $W_{R50}$                                 | 9.33 hrs.                    |

### J. Results of Gamma & Beta distribution
After the determination $q_p$, $t_p$, and $T_b$ by the CWC method, the various parameters of the Gamma and Beta distribution are computed as shown in table 2 and table 3 respectively.

#### TABLE II
PARAMETERS ESTIMATED BY GAMMA DISTRIBUTION

| Parameter   | Value obtained |
|-------------|----------------|
| $\beta$     | 0.68           |
| $n$         | 3.816          |
| $K$         | 8.37 hours     |
| Gamma n ($\Gamma_n$) | 7.24          |

#### TABLE III
PARAMETERS ESTIMATED BY BETA DISTRIBUTION

| Parameter   | Value obtained |
|-------------|----------------|
| $\beta$     | 0.68           |
| $\alpha$    | 3.186          |
| $p$         | 4.268          |
| $r$         | 1.711          |
| $b$         | 75.15 hours    |
| $B (r, p-r)$ | 0.148          |

Substituting these parameters in (1) for gamma distribution and (8) for beta distribution, the equation for $q$ reduces to the form (19) and (20) respectively.
Then by varying the value of $t$, the value of $q$ are obtained. Finally discharge ($Q$) is obtained by using (16). The unit hydrograph thus developed for gamma and beta distribution is shown in fig 2. (a) and fig 2.(b) respectively.

\[ Q = (0.0184) \times (t / 8.37)^{2.816} \times e^{-t / 8.37} \quad (19) \]

\[ Q = \left[ t^{0.711} \times (75.15-t)^{1.557} \right] / 200567.33 \quad (20) \]

**K. Comparison of the methods**

The unit hydrograph developed by all the methods are as shown in figure 3.

**Fig.3. Comparison of unit hydrograph developed by different methods;**

In case of beta distribution, the peak discharge is under estimated. However the value for base width of the unit hydrograph ($T_B$) is taken as a parameter for unit hydrograph derivation. As the unit hydrograph developed by the CWC method and gamma distribution match with each other, they can be considered as good methods for developing unit hydrograph.

**I. Generation of Discharge Hydrograph**

After the generation of unit hydrograph, the discharge hydrograph is simulated for the year 2018 by convolution. A comparison of actual discharge hydrograph with the simulated one for the period July to September 2018 is shown in fig 4. This is done by taking the hourly rainfall derived from daily data by disaggregation.
Figure: 4. Comparison of actual discharge hydrograph with simulated discharge hydrograph

It is found that the simulated values are much higher than the observed values. Dhadhar basin has large areal extent of order of 3423 km², which caters many dams and irrigation structures. The inflow-outflow Processes of these hydraulic structures are not accounted in the model study due to non-availability of data. So these may be the reasons for over estimation of simulated discharge.

VII. CONCLUSION

The unit hydrograph is developed by the gamma distribution and beta distribution, both of which depend on the watershed characteristics rather than the rainfall-runoff data. Some of the results observed in the study are:

1. The unit hydrograph developed by the gamma distribution is matching with the one developed by the CWC method.
2. The unit hydrograph developed by the beta distribution is flat. This is attributed to the fact that in beta distribution, the base width of the unit hydrograph is taken as a parameter for the derivation of unit hydrograph and any error in the estimation of base width can cause corresponding change in peak so as to adjust the unit runoff volume.
3. The simulated discharge however does not match with the observed discharge. This can be attributed to various physical characteristics pertaining to the study area. Firstly, the Dhadhar river basin is spread over an area of 3423 km².

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Cite this article as:
Prerna Sutariya, "A Comparative Study on the Derivation of Unit Hydrograph for Dhadhar River Basin", International Journal of Scientific Research in Science, Engineering and Technology (IJSRSET), Online ISSN : 2394-4099, Print ISSN : 2395-1990, Volume 6 Issue 4, pp. 36-43, July-August 2019. Available at doi : https://doi.org/10.32628/IJSRSET196378
Journal URL : http://ijsrset.com/IJSRSET196378