Entropy and initial conditions in cosmology

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Abstract: I discuss the Boltzmann-Penrose question of why the initial conditions for cosmology have low entropy. The modern version of Boltzmann’s answer to this question, due to Dyson, Kleban and Susskind, seems to imply that the typical intelligent observer arises through thermal fluctuation, rather than cosmology and evolution. I investigate whether this can be resolved within the string landscape. I end with a review of the suggestion that Holographic Cosmology provides a simpler answer to the problem. This paper is a revision of unpublished work from the spring of 2006, combined with my talk at the Madrid conference on String theory and Cosmology, Nov 2006.
1. Introduction

The Second Law of Thermodynamics is one of our most robust and profound physical principles. Its origins in the combination of the laws of mechanics and probability were, in large measure, understood in the 19th century, though quantum mechanics simplifies some of the considerations. While a completely rigorous derivation still eludes us, the modern physicist’s understanding of the way in which this principle should be applied to laboratory and astrophysical systems is adequate for almost all purposes.

Ever since the discovery of the Second Law, physicists have been faced with the question of why the universe began in a low entropy state, so that we can see the Second Law in operation. Roger Penrose has emphasized this question in the context...
of modern cosmology[1]. He has also rejected the claims of inflation theorists that the problem is resolved or avoided by inflation. I agree with Penrose on this point and will discuss it further below.

The increase in entropy that we observe in laboratory systems is a consequence of fluctuations out of equilibrium (sometimes forced on the system by actions of the experimenter). A system in equilibrium has already maximized its entropy and sees no further entropy increase until a fluctuation puts it into a low entropy state. Penrose has emphasized for many years, that the questions of why the universe began in a low entropy state, and what the nature of that state is are among the most important puzzles in cosmology. It is believed by many physicists, though I have not been able to find a precise reference, that Boltzmann himself proposed that the answer to this question is the same as that for systems in the laboratory[3]. The universe is a finite system and the low entropy beginning was just one of the inevitable random fluctuations that any finite system undergoes.

A modern version of Boltzmann’s explanation was recently proposed as a straw man in a paper by Dyson, Kleban and Susskind[2] (DKS). The main purpose of these authors was to criticize the proposal that the origin of low entropy initial conditions is a fluctuation. I will review their work extensively below and criticize their conclusions about what a typical history in their model looks like. My criticism depends on a particular point of view about quantum gravity, which I will detail in the appropriate place. However, I will show that although the DKS model does not suffer from the problems described by its authors, it does suffer from the problem of Boltzmann’s Brain (BB)\(^1\). That is, if we make the hypothesis that intelligent observers are isolated physical systems, whose state can be described by a very small subset of the degrees of freedom of the universe, then, in the DKS model, it is much more probable to produce a single intelligent observer by thermal fluctuation, than it is to observe the cosmological evolution they propose as the history of the universe. We will call such thermally produced intelligences, Boltzmann Brains. Here are references to the recent rash of papers on this subject[4].

The String Landscape proposes an explanation of the part of the history of the universe that we can observe, which is very similar to that of the DKS model. It will also contain the phenomenon of thermally produced intelligence, unless all meta-stable dS states in the Landscape, which are capable of supporting life,\(^2\), decay rapidly into

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\(^1\)This was pointed out to me by R. Bousso.

\(^2\)There is not the slightest possibility that we will be able to determine whether a given model of particle physics supports the kind of complex organization that we call intelligence, unless the low energy effective Lagrangian at nuclear physics scales is very close to that of the standard model. In this paper life will always mean the kind of life we know exists in the real world.
negative cosmological constant Big Crunches. The Landscape differs from the DKS model in that it does not really give an explanation of how the universe gets into the low entropy state from which it tunnels into the basin of attraction in which we find ourselves. Thus, at its present state of development, it does not address the question posed by the Second Law. Nonetheless, many landscape enthusiasts are nervous about the possibility that Boltzmann’s Brain, isolated in a spaceship, is the typical form that intelligence takes in the Landscape. Indeed the results of [5] suggest that the landscape contains many approximately supersymmetric dS states, whose decay is extremely slow. Certain methods of counting probabilities in the landscape tell us that the landscape is dominated by the progeny of such states. In order to completely remove the spectre of BB’s without a deeper understanding of initial conditions in the landscape, one would have to show that none of these nearly SUSic states has low energy physics like that of our own world.

After discussing these abstruse questions, and counting how many angels there are on a pin head (a p-brane?) , I will take up another explanation of low entropy initial conditions. Holographic cosmology was invented[6] in order to find a proper description of the Big Bang. It was based on the observation of Fischler and Susskind[8] that the only FRW cosmology compatible with the holographic principle at very early times has equation of state \( p = \rho^3 \). Assuming an extensive entropy, this implies that the entropy density \( \sigma \propto \sqrt{\rho} \), which is what one obtains from a collection of black holes at distances of order their Schwarzschild radii. In order for this configuration to remain steady in an expanding universe, the black holes have to continually merge so that each horizon volume is filled with a black hole. We called this the dense black hole fluid. It is not clear what one means by such a configuration in general relativity. However, in [9] the authors constructed an explicit quantum mechanical model, satisfying a set of plausible rules for quantum cosmology, and obeying the scaling laws of the flat FRW cosmology with \( p = \rho \). The space-time geometry was an emergent quantity, derived from the rules of the quantum system. By contrast, space-time topology was fixed by an \textit{a priori} lattice. The model also had the properties one expects of a dense black hole fluid. The entropy in each horizon volume was maximized and the energy was that of a black hole filling the horizon.

In the penultimate section of this paper, I will describe holographic cosmology in a bit more detail, and indicate how it addresses the problem of low entropy initial conditions. In the conclusions I will compare the various approaches to this problem and suggest avenues for further progress.

\[^3\text{Generic cosmologies with less stiff matter, violate the holographic principle prior to some time } t_0. \text{ It is interesting that given the current conditions in our universe, backward extrapolation of ordinary cosmology gives } t_0 \text{ of order the Planck time.}\]
2. DKS *redux*

2.1 The DKS model

As noted above, when we apply the Second Law of Thermodynamics to cosmology, we encounter a puzzle. Much folklore among inflation theorists is devoted to the claim that inflation resolves this problem and simultaneously hides the precise nature of the initial state from our view. Penrose rejects this claim, and I agree with him, though my reasons do not coincide with his. The initial inflationary patch contains only a small number of degrees of freedom that can be described by effective field theory\(^4\). Most of the degrees of freedom of the observable universe are not well described by quantum field theory until a large number of e-folds occur. The most sophisticated argument for the conventional discussion of these degrees of freedom starts from the *assumption* that they were in the ground state of some slowly varying Hamiltonian, which approaches the conventional Hamiltonian of field theory in the inflationary background, co-moving mode by mode, as the physical size of each mode crosses the Planck scale. There are many ad hoc assumptions in this treatment, but it is clear that the low entropy initial condition is *put in* by assuming that the system was in its ground state. The excited states of every large quantum system that we know, are highly degenerate, and the adiabatic theorem simply does not apply to generic initial conditions chosen as a linear combination of highly degenerate states. Thus, most conventional discussions of inflation assume a very special state for a huge number of degrees of freedom at a time when we do not have a reliable dynamical description of these variables. In this sense, inflation *does not* solve the problem of homogeneity and isotropy of the early universe.

A few years ago, DKS [2] proposed another approach to the problem of low entropy initial conditions. It was based almost entirely on semi-classical notions, sprinkled with a few grains of holographic wisdom. These authors concluded that their model predicts phenomena, which are not verified observationally (we will review their arguments below). Indeed the main intent of their paper was to show that the hypothesis that the beginning of the universe is a fluctuation in a finite entropy system was incompatible with observation. The purpose of the present section is to show that a proper understanding of the quantum dynamics of de Sitter space, removes the objections of Dyson and her collaborators. Indeed, I would claim that, apart from the problem of Boltzmann Brains, their model presents us with a solution of the problem of initial conditions, which can be investigated with semi-classical tools.

The basic idea of the proposal of [2] was that the universe we live in is a de Sitter space with c.c. Λ. This system has finite entropy, and thus Poincare recurrences.

\(^4\)To be more precise, we should talk about a causal diamond of an observer in the initial inflationary patch, and apply the covariant entropy bound[7].
DKS proposed that the cosmology we observe is the result of such a recurrence, in which the universe entered a very low entropy state. For reasons which will become apparent, I will discuss a particular version of this model, which has a meta-stable, low entropy excitation modeled by a subsidiary minimum of the effective potential with c.c. \( \Lambda_1 \gg \Lambda_5 \). The process of tunneling to the low entropy minimum and back is well described in the semiclassical approximation, by a Coleman De Luccia (CDL)\[10\] instanton. The tunneling rates calculated from this instanton indeed satisfy the law of detailed balance\[34\] and are a compelling piece of evidence for a model of quantum de Sitter space as a finite system with Poincare recurrences\(^6\).

The quantum mechanics is supposed to be that of a system of a finite number of states. It has a static Hamiltonian, \( H \), which is the quantum representation of the static time-like Killing vector of a given observer’s causal diamond. All of the eigenstates of \( H \) resemble the lower dS vacuum macroscopically. Particle excitations of the lower minimum are assumed to be identical to the low energy particle spectrum\(^7\) in the real world (so there are other low energy fields coupled to the inflaton). One way to get a universe that resembles our own is to have a fluctuation into the very low entropy false vacuum state, followed by rapid (on the time scale of the true vacuum Hubble constant), Coleman-De Luccia (CDL) tunneling back to the true vacuum. \[11\] argue that after such a tunneling event, the only way to get galaxies is to have a sufficiently flat potential in the basin of attraction of the true minimum, so that one gets at least 58 e-folds of inflation. Field theoretic naturalness suggests that we are unlikely to have many more e-folds than this\[12\], and that the inflaton potential is of the form \( \mu^4 v(\phi/m_P) \), with Planck scale couplings to ordinary matter. For potentials of this form, the normalization of primordial inflationary fluctuations scales like \( Q \sim \mu^2/m_P^2 \). The reheat temperature is of order \( \mu^3/m_P^2 \) and the growth of the scale factor between inflation and reheating is \( (m_P/\mu)^{3/2} \). We must have \( \mu \ll m_P \) in order to use this effective field theory description, while if \( \mu \) is too small the universe will collapse into inhomogeneous clumps of Bose condensate (or black holes) because many scales will go non-linear before radiation domination begins. The value of \( \mu \) that reproduces the observed CMB fluctuations avoids both of these disasters.

This version of the DKS model thus invokes both old and slow roll inflation. In

\(^5\)Although this version was not emphasized in the DKS paper, it featured prominently in conversations I had with Lenny Susskind, in which he explained the DKS model.

\(^6\)It also provides evidence for the spectrum of the static dS Hamiltonian that I will describe below. The law of detailed balance involves entropies rather than free energies, which makes sense if all of the states are at energies below the dS temperature.

\(^7\)Here spectrum means the spectrum of the operator \( P_0 \), to be introduced below, not that of the operator \( H \).
doing so, it avoids the criticism of generic inflation models that we reviewed above. The period spent in the meta-stable state with c.c. $\Lambda_1 \gg \Lambda$ explains why we begin with low entropy and also why the initial conditions for cosmology are homogeneous and isotropic.

It is thus plausible that a model like this\textsuperscript{8} can reproduce a universe like our own. The authors of [2] do not dispute this. Rather, they suggest that as we cycle through the infinite set of Poincare recurrences\textsuperscript{9} that this finite system can undergo, there are many more histories which contained very unlikely events (and thus \emph{e.g.} have a current CMB temperature so high that conventional extrapolation would predict no surviving primordial nuclei besides hydrogen). Implicit in this paradox is the claim that the time evolution we call cosmology is the same as that defined by the static patch time in dS space. In particular, different recurrences are supposed to correspond to different cosmological histories. The claim of [2] is that, even if we restrict ourselves to histories in which carbon based life could evolve, the typical history has more entropy than our own.

2.2 Io credo: a digression

Any investigation of recurrences in this system has to deal with the interpretation of the quantum theory of eternal inflation. From the global point of view, this has led to the problem of the proper measure for counting bubbles of true and false vacuum. The answers one obtains depend crucially on the choice of measure, which is surely determined by the correct theory of quantum gravity. At the present time, for cosmological space-times, the nature of the correct theory is a matter of religious conjecture rather than scientific or mathematical fact. DKS implicitly used a measure similar to

\textsuperscript{8}With the value of $\mu$ either determined by a unique microphysical theory or chosen from a landscape by the sort of environmental selection adumbrated above.

\textsuperscript{9}In a previous paper[13] we criticized this prediction because it involves properties of the mathematical universe, which can never be tested \textit{in principle}. According to DKS, their mathematical model contains one history that resembles our universe and predicts everything we see, but also makes predictions about events that are in principle unobservable, to any observer which experiences that history. If one takes the point of view that the role of theoretical physics is simply to make mathematical models that explain and predict what we can observe, then there is little difference between this situation and one in which we have many different mathematical models, one of which describes what we see, while the others don’t. In the current paper, I accept the assumption that one must average over Poincare recurrences, but argue that a proper understanding of how observable cosmology emerges from the model, is consistent with the idea that the model predicts the universe we see in a unique way. The arguments of this paper are related to those of [13] because they concentrate on the observations of localized observers, rather than the overwhelmingly larger set of states on the cosmological horizon. In [16] I argued that there are many horizon Hamiltonians which reproduce the same local physics, within the maximum precision that local measurements can attain.
that proposed (later) by Bousso et. al.[14], and I will follow them. I note that the problem of Boltzmann’s Brain, discussed below, may be resolved by other choices of measure[15].

In such a situation, one must begin by announcing one’s own religious credo:

- String theory provides us with many examples of consistent quantum theories of space-time with asymptotically flat and low curvature asymptotically AdS boundary conditions. All of these theories are exactly consistent with the ordinary laws of quantum mechanics.

- These theories provide evidence for the holographic principle.

- A strong version of the holographic principle is that the Hilbert space corresponding to a finite area causal diamond is finite dimensional. In particular, the quantum theory of a stable dS space would have a finite number of states. It immediately follows that the mathematical quantum theory of such finite regions is ambiguous: many mathematical quantum theories will give the same results, within the precision allowed by the inevitable quantum fluctuations in measuring devices constructed from a finite dimensional quantum system.

- Classical space-times with different asymptotics correspond to quantum theories with different Hamiltonians. Therefore, different classical solutions of the same low energy field equations may not live in the same quantum theory. In general, the Hamiltonian for most space-times will be time dependent.

- The correct quantum description of a theory of quantum gravity may involve several different Hamiltonians. Typically, some of these are more fundamental, and the others are emergent descriptions of a subset of degrees of freedom of the system, over time scales where the fundamental Hamiltonians do not evolve those degrees of freedom.

The first two of these principles are widely believed in the string theory community. The third was postulated by the author and W. Fischler. The covariant entropy bound refers to the entropy of some maximally excited density matrix. In a generic space-time, with no asymptotic Killing vectors, there is no natural choice besides the maximally uncertain density matrix, whose entropy counts the logarithm of the total number of states.

The strongest evidence for the fourth principle comes from the AdS/CFT correspondence, and from comparing its description of Anti-de Sitter space, with the rather
different descriptions of asymptotically flat, and linear dilaton space-times. More generally, it follows from the fact that quantum theories are differentiated by their high energy spectra, and that the spectrum of black holes depends on the space-time asymptotics in a crucial way. Semi-classical quantization via the Wheeler-DeWitt equation also leads to a time dependent Hamiltonian which depends on the classical background. Finally, if we accept the third principle, so that dS space has a finite number of states, then the quantum theories of dS spaces with different values of the c.c. are not equivalent to each other. Note however, that in this case, it might make sense to think of a dS space with large c.c. as a subsystem of one with small c.c. This is the premise of the double well DKS model.

The last principle is exemplified by the description of in-falling observers for black holes in asymptotically flat or AdS space-time. Although the technical details remain to be worked out, a plausible description is the following: Consider, e.g. the synchronous, Novikov, coordinate system, with initial time slice coinciding with the $t = 0$ slice of Schwarzschild coordinates. The quantum field theory Hilbert space on this slice, breaks up as a tensor product of interior and exterior degrees of freedom. We restrict attention to initial states that do not lead to a significant distortion of the background geometry (by conventional semi-classical estimates). This becomes a more and more restrictive condition if we consider interior states, which become localized near the singularity. Thus, we should consider the Novikov Hamiltonian an emergent Hamiltonian for a small subsystem, which is only relevant for a limited period of time.

This should be contrasted with the Schwarzschild Hamiltonian, which is an exact description of the system, as seen by asymptotic observers\textsuperscript{10}. The two Hamiltonians do not commute with each other. In the local field theory approximation, their failure to commute comes from regions near the horizon of the black hole. From the external point of view, not much happens over Schwarzschild time periods of order the in-fall time to the singularity. The only interesting evolution during this period is that experienced by the interior Novikov degrees of freedom, under the Novikov Hamiltonian. On time scales much longer than this, the only valid description is that of the external observer. The interior thermalizes at the Hawking temperature, and begins to radiate\textsuperscript{11}. There is no good description of the interior in terms of field theory, and no meaning to the word \textit{interior} in the exact description\textsuperscript{12}. The interior Novikov Hamiltonian is an emergent quantity, describing a subsystem for a limited period of time.

\textsuperscript{10}In the asymptotically flat case there is no exact local time evolution, only a scattering matrix.

\textsuperscript{11}We are imagining a black hole formed from gravitational collapse, rather than an eternal one.

\textsuperscript{12}Rather, certain aspects of the interior geometry can be read from asymptotic expansions of exact answers, and have the limited meaning usual to asymptotic expansions\textsuperscript{17}. 

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2.3 Un-digression

I now want to argue that a similar situation exists in dS space. Here the exact description of the system is associated with the static dS Killing vector in a particular horizon volume. The emergent Hamiltonian is the time dependent Hamiltonian describing cosmology, as well as the approximately Poincare invariant dynamics of local physics, to which the cosmological Hamiltonian converges at late times. I claim that neither of them is the static Hamiltonian of dS space, and that even in the DKS model, the cosmological evolution that is identified with the real world does not follow from the dynamics of the static Hamiltonian. In fact, there are strong indications that our cosmological time cannot be identified with static dS time, except in the far future.

In [16], I attempted to find a quantum theory that would account for all known semiclassical results of quantum field theory in dS space. I concluded that the system we call dS space must be described by two Hamiltonians, acting in a Hilbert space whose dimension is approximately the exponential of the Gibbons-Hawking entropy $S_{GH}$. The static $H$ has a spectrum with spacing $e^{-S_{GH}}/2\pi R$, where $c$ is a constant of order one, and $R$ the dS radius. $cT_{GH}$ is the maximal eigenvalue of this Hamiltonian.

There are two arguments for this spectrum. One comes from an examination of CDL tunneling rates and was mentioned in a footnote. The other is that every excitation of dS space decays to the dS vacuum. Since the vacuum shows no macroscopic evidence of even the macroscopically large energies of black holes it is reasonable to assume that black hole masses are not related to eigenvalues of the static Hamiltonian. Furthermore, the dS vacuum is an equilibrium state with a unique temperature. A generic state (with the flat probability distribution on the unit sphere in Hilbert space) of a random Hamiltonian obeying such a bound, approaches equilibrium at a fixed temperature $T_{GH}$, as the number of states in the system is taken to infinity. The constant $c$ is independent of the number of states. Fluctuations in the temperature in the ensemble of all states in Hilbert space, are of order $e^{-\frac{1}{2}S_{GH}}$. Thus, the assumption of a finite dimensional Hilbert space with a Hamiltonian bounded by something of order the dS temperature, explains the results of QFT in curved space for static dS space, as well as CDL transition rates between dS minima.

This static Hamiltonian is the one that governs the long time scale recurrences of dS space. In a static time $t \ll R$, a wave function $|\psi> = \sum a_n |E_n>$ evolves very little. Thus such a Hamiltonian cannot describe the evolution of a universe which changes significantly over time scales short compared to the Hubble scale associated with the c.c.. It is also clear that its eigenvalues can have nothing to do with ordinary particle masses or energy levels of nuclei and atoms. Instead, one proposes an alternative Hamiltonian $P_0$, whose eigenvalues are related to particle masses and other energies
we measure. $P_0$ is called the Poincare Hamiltonian. Considerations based on the holographic principle suggest a commutation relation, which is a finite dimensional approximation to

$$[H, P_0] = \frac{1}{R} P_0.$$  

(2.1)

Low lying eigenspaces of $P_0$ are thus approximately stable under the $H$ time evolution. In order that the thermal density matrix

$$\rho = Z^{-1} e^{-H/T_{GH}}$$

look approximately like the thermal density matrix

$$\rho_{EFT} = Z^{-1} e^{P_0/T_{GH}},$$

one must postulate that Poincare eigenvalues much less than $RM_P^2$ are equal to the entropy deficit of the corresponding eigenspace, relative to the dS vacuum. For eigenvalues $\gg M_P$, one can calculate the entropy deficit from the black hole entropy formula, and it indeed agrees with this expectation if we identify the black hole mass parameter with the eigenvalue of $P_0$.

The operator $P_0$ removes the vast degeneracy of the spectrum of $H$, for a small subset of states of the system. The Hilbert space breaks up as

$$\mathcal{H} = \bigoplus \mathcal{H}_{p_0} \otimes \mathcal{K}_{p_0},$$

where $P_0$ is the difference between the GH entropy and the logarithm of the dimension of $\mathcal{K}_{p_0}$. The dimension of $\mathcal{H}_{p_0}$ is the degeneracy of the eigenvalue $p_0$ from the point of view of ordinary physics. For black holes, this dimension is the exponential of one quarter of the area of the black hole horizon, while the entropy of $\mathcal{K}_{p_0}$ is that of the cosmological horizon in the presence of the black hole. The tensor split of the Hilbert space described above is thus the split into degrees of freedom localized near the observer, and those on its cosmological horizon. It is important to recall that the entropy in localized excitations is bounded by something of order $(RM_P)^{3/2} \ll (RM_P)^2$, the entropy of dS space. $P_0$ encodes the information relevant for local measurements. It is useful on time-scales short compared to the decay time of localized excitations. A commutation relation between $H$ and $P_0$ roughly like that given in 2.1 follows from this description of the spectra of the two operators.

On long enough time scales, where time is defined in terms of the evolution under $H$, none of the eigenstates of $P_0$ is stable. For large $p_0$ the decay time is of order $p_0^3 M_P^4$. For some of the lower lying eigenstates it is exponential in the dS radius. This is the time one has to wait for a thermal fluctuation to destroy a localized object which is not a black hole.
localized systems decay, and their decay products disappear through the horizon \(^{14}\) the dynamics described by evolution under \(P_0\) is no longer relevant for those degrees of freedom. Eventually, all localized excitations disappear (until next time) and the very slow time evolution generated by \(H\) is the asymptotic dynamics of the system.

The Hamiltonian \(P_0\) is thus an emergent feature of the dynamics of de Sitter space. Once a localized object has appeared, through a fluctuation, the dynamics of \(H\) does not change it at all, over time scales \(< R\), and in some cases much longer time scales, though still short compared to the recurrence time \(e^{\pi(RM_P)^2}\). For these short times the Hamiltonian \(P_0\) evolves the localized degrees of freedom, by resolving the huge degeneracy of \(H\). The Hamiltonian \(P_0\) was introduced in order to understand how a Poincare invariant theory describing particle physics could emerge from the theory of dS space in the large \(R\) limit. In a cosmology which is only future asymptotically dS, the analog of \(P_0\) is a time dependent \(P_0(t)\). For the semi-classical cosmology of DKS, one would compute it approximately by studying quantum field theory in the future Lorentzian CDL bubble, with boundary conditions determined by analytic continuation from the Euclidean section. There are important corrections to the classical background during the period of reheating, when oscillations of the scalar field can decay into radiation.

What is important however is that once we have established the correct background classical geometry, we must restrict attention to states of the QFT which do not have significant back-reaction on the geometry. The Hilbert space of degrees of freedom on which \(P_0(t)\) acts, consists only of those small fluctuations. Other classical homogeneous background geometries, which do not evolve from the CDL initial conditions are not part of the quantum theory, according to religious principle number 4 of the previous subsection.

The time dependent Hamiltonian, \(P_0(t)\), of this system will, for this subset of states, approach the Poincare Hamiltonian \(P_0\) as cosmological time gets large. On a longer time scale - that of the disappearance of all localized degrees of freedom through catastrophic thermal fluctuations followed by Hubble flow of the debris out through the horizon - the slow dynamics of the static Hamiltonian \(H\) becomes relevant.

The Poincare recurrences discussed in [2], are a feature of the \(H\) dynamics. The Hamiltonian \(P_0\) and its time dependent cosmological cousin, do not have such recurrences. Indeed they are only relevant over time scales at most of order \(e^{c(RM_P)^{3/2}}\), with \(c\) a constant of order 1. Let us try to see whether we can reproduce the paradox of [2] with our new understanding of the dynamics. Most of the time (H evolution) our system resembles the dead dS vacuum. Interesting observers can only exist in the period

\(^{14}\)The phrase *decay products* includes galaxies that are not gravitationally bound to us.
following a low entropy fluctuation. One such fluctuation is the CDL tunneling which produces a system which resembles a dS vacuum state with c.c. $\Lambda_1$. Inflation rapidly wipes out any non-vacuum excitations of this state, and the reverse tunneling event automatically produces an approximately homogeneous and isotropic FRW universe. For a long time on real cosmological scales, the evolution according to the Hamiltonian $H$ is irrelevant, because the bound on the spectrum of $H$ tells us that the wave function has not changed. Instead, the dynamics of cosmological observers is governed by $P_0(t)$. It is extremely important to note that all of conventional cosmology takes place during this period. The dynamics described by $H$ becomes relevant only when all of the localized excitations produced by the tunneling event, have decayed or exited through the cosmological horizon\textsuperscript{15}.

One should remember that the dynamics described by $P_0(t)$ only involves a small subset of the degrees of freedom of the system. The entropy of the relevant localized degrees of freedom is bounded by $c(RM_\rho)^{3/2}$. For the rest of the degrees of freedom, only the time evolution defined by $H$ is relevant. Not much happens to them on the time scale defined by cosmology. However, as things begin to go out of the cosmological horizon of the large dS space, fewer and fewer degrees of freedom of the original cosmological fluctuation participate in the $P_0(t) \rightarrow P_0$ dynamics. Thus, the time evolution which describes cosmology is an emergent property of the semiclassical low entropy fluctuation. Eventually, on time scales $\gg R$, when we return to the equilibrium dS vacuum, there are no degrees of freedom to which to apply the $P_0(t)$ Hamiltonian and only the $H$ evolution is relevant.

Future recurrences of the events which led to semi-classical cosmology will not lead a substantially different history. First of all, by the assumption that the process went through tunneling, the entropy of possible initial states is just that of the small radius de Sitter space. Secondly, traces of the initial state will be wiped out by false vacuum inflation. Finally the slow roll inflation in the basin of attraction of the true minimum will, for appropriate values of $\mu$, smooth out initial inhomogeneities and leave only the slow roll inflationary fluctuations to be seen in the CMB. For a small number of e-foldings, such as we expect from field theoretically natural potentials, there may be evidence of the initial conditions left at the largest cosmological scales.

There is no reason to suspect that recurrences can lead to situations like a cosmology with higher CMB temperature, as envisioned by DKS. Cosmology is time evolution under the emergent Hamiltonian $P_0(t)$, which is defined in terms of a particular semi-

\textsuperscript{15}This is an exaggeration. Some localized excitations decay only because an unlikely thermal fluctuation destroys them. In principle the microscopic description of this fluctuation is governed by the $H$ dynamics, though for all practical purposes it may be that it can be described by the thermal physics of $P_0$. 

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classical fluctuation of the dS background. There are no CDL instantons which describe an alternative cosmological history differing only in the temperature of the CMB. The dominant quantum/thermal fluctuations around the dS background produced localized excitations, and we have already seen that localized fluctuations around the CDL background do not have a significant effect on cosmology. If we try to extrapolate generic late time dS fluctuations back to the past, we find a Big Bang, rather than the CDL cosmology[18]. This cosmology is not only out of semi-classical control, but unstable. If we make small changes in the initial conditions near the Big Bang, we will produce a universe which does not have an inhabitable future. There is of course no argument that actual recurrences in the time evolution generated by $H$ will actually lead to any of these semi-classical cosmologies.

To summarize, in a system evolving under the dS Hamiltonian $H$, if the potential characterizing the low energy degrees of freedom has a subsidiary minimum with larger c.c., then the CDL instanton gives us evidence for transition to a state which looks like an evolving cosmology. The time evolution in this cosmology is not that of $H$, but of an emergent time dependent Hamiltonian $P_0(t)$, which gives an approximate description of a small subset of localized degrees of freedom. The time scales for $P_0(t)$ include some which are much shorter than $R$ and none which are larger than $e^{c(RM_p)^{3/2}}$.

Since the $P_0(t)$ evolution that describes cosmology is completely decoupled from the recurrences, we no longer have any plausible evidence, let alone a compelling argument, that the system has the kind of anthropically allowed, but observationally forbidden histories that DKS imagined$^{17}$. Every recurrence of cosmological history that we can study semiclassically resembles every other in all but the finest details. It is possible that we can actually observe some of those details in the future, but virtually certain that we have not yet done so.

Effective field theorists will object that there are other solutions of the low energy field equations, which describe the misanthropic cosmologies of DKS. Although singu-

\[16\] This is the time scale over which localized measurements become impossible. No localized device is free from quantum fluctuations of its pointers on any longer time scale. In reality, it is very improbable for any localized system to survive a time longer than $e^{2\pi M R}$, where $M$ is its Poincare eigenvalue. Within a few times $R$, the evolution of $P_0(t)$ is applicable only to the small subset of the (currently) visible universe that is gravitationally bound to the observer. If this system has mass $M$, it can collapse into a black hole, and decay to the dS vacuum in a time of order $\frac{M^3}{M^3}$, or survive until a thermal dS fluctuation annihilates it in a time of order $e^{2\pi R M}$. In any case, \textbf{LONG} before the recurrence time, $e^{c(RM_p)^{3/2}}$, the cosmological Hamiltonian has “de-emerged”. There are no longer any degrees of freedom to which this time evolution applies.

\[17\] There is one kind of recurrence of anthropically allowed, but observationally forbidden histories which is not ruled out by this argument. We will deal with it below in the section entitled \textit{Boltzmann’s Brain}. 

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larities occur in the backward extrapolation of most modifications of the CDL geometry, there are plenty of other homogeneous isotropic solutions of the field equations. I have argued long and hard and to no avail[20] that different solutions of low energy effective field equations are not necessarily part of the same quantum theory of gravity. This was religious principle 4 of the previous subsection.

In the present context I believe we have strong evidence that small quantum fluctuations around the dS and CDL backgrounds, and black hole states in the dS background are really excitations of the quantum Hamiltonian $H$. Note however that most of the black hole solutions are not allowed during the cosmological evolution in the CDL background. Only those black holes which form as a consequence of evolution from the non-singular CDL background will appear. Large black holes that do not form as a consequence of cosmological evolution, can only appear as thermal fluctuations. This will occur after the normal cosmological history has ended, and the system returns to its dS equilibrium. Thus, our system really has 3 different interesting Hamiltonians, $H$, $P_0$, and $P_0(t)$. $P_0$ describes the Lorentz invariant particle physics which emerges in the $R \to \infty$ limit, and finite $R$ corrections to it like SUSY breaking[21]. $P_0(t)$ describes the DKS cosmology. At late times, low eigenvalues of $P_0$ and $P_0(t)$ (in the sense of the adiabatic theorem) coincide, but $P_0(t)$ has no analog of the large black hole eigenstates of $P_0$. $H$ describes the long time evolution of the system and is the only full description of it.

My claim is that we have no reason to assume that other classical solutions of the low energy field equations have anything to do with our quantum system, until someone shows us a calculation at least as compelling as that of CDL for computing transition probabilities between empty dS space and these other semiclassical geometries. These transition probabilities should satisfy a law of detailed balance compatible with the entropic characteristics of the system.

The idea that evolution operators in quantum gravity, are tailored to a specific classical background, is familiar from the resolution of the Problem of Time[19] in the Wheeler-DeWitt approach. The solution space of the hyperbolic Wheeler-DeWitt equation does not possess a positive definite metric. Expansion around a specific classical solution reduces the equation to a parabolic time dependent Schrodinger equation, which does. Thus, different classical solutions of Einstein’s equations lead to different quantum theories. Some of these may be the same theory as viewed by different classical observers, but there is no reason to expect that to be a general feature, since it is not even true in the semi-classical approximation. What is peculiar in the present context is that DKS thought they were dealing with such a fixed time evolution, under the static Hamiltonian $H$. However, the characteristics of the spectrum of $H$, which are necessary to explain the global semi-classical properties of the causal patch of dS
space, imply that it cannot be identified with ordinary energy, or with the cosmological evolution operator $P_0(t)$. The observers that experience cosmological evolution, as described by $P_0(t)$ are built out of localized degrees of freedom. They cannot experience recurrences.

We have also argued that every recurrent appearance of this sort of localized observer will evolve in practically the same manner. The DKS cosmology thus provides a rationale for a low entropy beginning of the universe as a fluctuation. As a bonus, it suggests that the entire history of the universe can be described by semi-classical physics. There is no singular Big Bang.

2.4 Landscapism

So far, the considerations of this paper apply to all potential landscapes with potentials above the great divide\[22\]. The CDL transition probabilities out of the dS minimum with smallest c.c. are all suppressed by an entropy factor, and it is consistent to postulate a system with a finite number of states. It should be noted that in such a system it is possible for anthropic considerations to trump purely dynamical expectations for how long the system spends in each of its meta-stable states. The authors of [11] have emphasized that in systems of this type, galaxy formation requires both a small c.c. and a period of slow roll inflation. Thus, although CDL probabilities predict that the system will spend the vast bulk of its time in the basin of attraction of the minimum with lowest positive c.c., this may not be a region in which galaxies can form. Most galaxies will be found in cosmologies based on a CDL instanton whose Lorentzian section lives in the basin of attraction with smallest c.c., subject to the constraint that there be enough e-folds of inflation\[18\].

Unfortunately, we cannot apply these considerations to the only landscape that has any sort of theoretical underpinning, The Landscape of String Theory. The string theory landscape certainly contains terminal vacua\[19\], which are asymptotically SUSic and do not allow for recurrences. Furthermore, it is likely that the string theory landscape lies below the great divide: that is, it is replete with non-supersymmetric, small c.c. dS minima which tunnel rapidly to a Big Crunch. This is also inconsistent with a model having a finite entropy.

Thus, although the model of DKS can be analyzed in a predominantly semi-classical manner, and, within the semi-classical approximation, provides a real solution to Pen-

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18The real situation is of course much more complicated. Different minima will have different low energy degrees of freedom. It is possible that some minima will not have appropriate matter from which to form galaxies.

19To use Boussonesque\[26\] language, if not his equations.
rose’s puzzle about initial conditions, it is not clear that it is a semi-classical approximation to a real model of quantum gravity.

3. Boltzmann’s brain

There is one kind of “anthropically allowed” but observationally forbidden history that is not ruled out by our previous considerations. This is the situation colloquially known as Boltzmann’s brain. Our system cycles through all of its allowed states, populating them statistically. Suppose that we consider a state which looks like the true vacuum dS space, marred only by a fluctuation consisting of Boltzmann’s brain, replete with all of its memories. The probability for this can be estimated by computing the probability for nucleating a black hole with the same mass as the brain, and then dividing by the number of localized states of the black hole. This gives

\[ P_{\text{brain}} \simeq e^{-2 \pi m_b R} e^{-\pi (m_b / M_{\text{Pl}})^2}. \]

The probability is small, but much larger than the CDL probability to produce the DKS cosmology. The claim is that this implies that the dominant form of intelligent life in the DKS model is a brain spontaneously nucleated in empty dS space with all of its memories of the CDL instanton cosmology. Of course, such a brain would immediately explode and/or die of oxygen starvation but we can solve this by nucleating a support system for the brain, still with exponentially higher probability than the CDL process. The real prediction is that the dominant form of intelligent life in the DKS universe is a form created spontaneously with knowledge of a spurious history, which lives just long enough to realize that its memories are faulty.

Pursuit of this line of reasoning leads us into two opposite directions, both somewhat speculative. The first is the theoretical bound on the mass and radius of the smallest intelligent system that will ever be constructed in our cosmological history\(^{20}\). This definitely includes artificial intelligences that our descendants might someday create. Certainly, unless we declare \textit{a priori} that silicon based machines will never have human scale intelligence, the smallest viable version of Boltzmann’s brain will be many orders of magnitude smaller and lighter than Boltzmann’s own, as well as much more robust. R. Bousso\(^{21}\) has suggested \(m_b R_b > 10^{25}\) as a bound on the minimum mass and size of a brain.

\(^{20}\)Like all anthropic questions, we must narrow ourselves to local physics and chemistry like our own in order to do any calculation at all. We will probably never have the theoretical power to determine whether intelligent life would have been possible with other low energy gauge groups and representation content.

\(^{21}\)Private communication, later repudiated.
The other direction to take is to speculate on the possibility that the minimal “support system” for a brain is actually very large. Our immediate reaction to the proposal for spontaneous nucleation of Boltzmann’s brain is that it is ridiculous. It is ridiculous because we know about the complex and tortuous history that went in to making the original version. We had to make galaxies and the galaxies had to make the right kind of stars and planets and there had to be a planet at the right distance from the Sun and it had to undergo both long periods of relative stasis as well as short catastrophic periods of dramatic environmental change. And even then we don’t know how much of an accident intelligent life was, because we really only have one data point.

In fact, I believe this intuitive reaction has some validity. What we call the state of the brain or the state of a galaxy in our universe is in fact not a quantum state of those subsystems of the universe, but an entangled state of those subsystems with other degrees of freedom in the universe. In this sense, the past is encoded in the current state of Boltzmann’s brain, which cannot be properly described in terms of any wave function for the degrees of freedom in the brain itself.

One can imagine constructing a mathematical argument which would turn this observation into a refutation of the claim that Boltzmann’s brain and its memories can be created by a thermal fluctuation, because the brain would not function without the phase correlations with the rest of the universe that encode its past. I have not yet done so. It is likely that even this appeal to past history might not be sufficient to resolve the problem. The most important quantum correlations between different parts of the universe are those encoded in Newtonian forces. A. Aguirre has pointed out to me that in simulations of galaxy formation, it seems sufficient to restrict attention to a region of size (10 megaparsecs)³ surrounding our galaxy (Boltzmann’s local group). That is, a region of this size will evolve like our own surroundings did, for 10 - 15 billion years, even if it is embedded in flat space. The rest of the universe doesn’t seem to matter. The thermal probability for producing such a region in precisely its current state, is small, but still larger than the CDL probability, for producing the full history of the DKS cosmology as a fluctuation. Thus, while the records of past history encoded in the actual correlations between Boltzmann’s brain and the rest of the universe (some of which we feel in particular as Newtonian forces) might help to explain our conundrum, we have to do more work to show that the most probable way to obtain our local group of galaxies as a fluctuation in the DKS model is to follow the full course of CDL evolution. These observations might resolve the string landscape

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²²Lest the reader think that I am advocating some kind of weird New Age effect of quantum mechanics on consciousness, I would ask her/him to recall e.g. that the classical force between static sources is, in quantum field theory, encoded in just such a phase correlation in the wave function describing the two sources.
version of the Boltzmann’s brain paradox, to which we now turn.

3.1 Boltzmann’s brain in the string landscape

Susskind has emphasized that one of the attractive features of the string landscape is that it seems to avoid the problems of the DKS model, because it does not have general recurrences. That is, in the string landscape, meta-stable dS vacua decay into terminal vacua where no life can exist, on a time scale no longer than the recurrence time. Thus the kind of misanthropic cosmological histories envisaged by DKS cannot occur in the landscape.

However, we have seen that cosmologies with a slightly higher CMB temperature are not really a problem in the DKS model. The real problem is Boltzmann’s brain. If we ignore the correlations between the state of the brain and the rest of the universe, which we alluded to in the last paragraphs of the previous subsection, then Boltzmann’s brain might be a problem for the landscape. Using Bousso’s estimates for the minimal size and weight of any brain, the probability of thermal fluctuations producing a brain is $e^{-10^{25}}$. CDL decay probabilities for moduli fields with potentials below the Great Divide are of order $e^{-\left(\frac{\mu}{m_P}\right)^4}$, where $\mu$ is the typical scale of the potential. These will be larger than the brain production probability only if $\mu > 10^{-6}m_P$. Note that, in order to produce thermal brains, we only need one vacuum with low energy physics like our own, and cosmological horizon size $> 1$ cm. (Bousso’s estimate of the minimal size of a reasonable brain), which violates this bound on $\mu$ (or the analogous bound for Brown-Teitelboim tunneling).

At the level of validity of all arguments about the String Landscape, it is fairly certain[5], that there are approximately SUSic dS vacua in the landscape, which lie above the great divide. These have all decay probabilities of order $e^{-\pi(RM_P)^2}$, much smaller than the probabilities for producing BBs. Thus, in order for the landscape to completely evade the problem of BBs, one must establish that none of these nearly SUSic vacua have low energy physics resembling our own. This seems fairly unlikely, given the reasonable success that string theorists have had in finding supersymmetric states with more or less the standard model in various sorts of perturbative string vacua. Thus, it seems rather unlikely that the landscape will completely avoid confronting recurrences of the great Austrian physicist.

In the landscape, in contrast to the DKS model, it is harder to estimate the probability of producing complex intelligence by the historically favored route, so even if the landscape produces Boltzmann brains, we don’t know with certainty that they are the most probable form of intelligence. This is a direct consequence of the fact that the landscape does not deal with the problem of low entropy initial conditions at all. It is reputed to have an infinite number of quantum states, so the Boltzmann/DKS
argument does not apply. Discussions of cosmology in the landscape generally begin with a CDL tunneling event, but no explanation is given of why the universe ever got into the low entropy state from which this tunneling proceeds.

In the next section we will explore a completely different mode of explaining low entropy initial conditions.

4. Holographic cosmology

4.1 The roots of holographic cosmology

Holographic cosmology does not fit into the paradigm of quantum gravity as a Feynman path integral over space time metrics. It imposes a fixed causal structure and a fixed foliation on space-time as a matter of principle. This is because it focuses on the physics that can in principle be measured by a single observer. The “relativity” of general relativity is enforced by considering a whole collection of observers, with partially overlapping degrees of freedom, and insisting that they all give a consistent description of the physics in the overlapping regions.

In quantum mechanics, an observer is a large system with many semiclassical degrees of freedom that can serve as pointer observables. Our mathematical understanding of such systems is based on local field theory (perhaps with a cutoff), and the pointers are averages of local fields over volumes much larger than the cutoff scale. Ideal quantum measurements refer to the limit of infinitely large pointers. The existence of such observers in localized regions of space-time is at odds with the holographic principle, and with our understanding of semi-classical general relativity. In GR we expect that if we put too much mass into a region, we form a black hole. The holographic principle formalizes this expectation as a bound on the entropy of certain space time regions. Consider in particular a causal diamond, the intersection of the interior of the backward light cone of a point P with the interior of the future light cone of a point Q in the causal past of P. We can think of these as two points along the worldline of a time-like observer, and we can build up the world line itself in terms of a nested sequence of causal diamonds. In classical GR, the causal diamond represents the entire region on which an observer traveling from Q to P can do experiments.

The covariant entropy bound[7] is a conjectured bound on the entropy of the causal diamond, by (one fourth of) the area in Planck units of the maximal area spacelike surface on the boundary of the diamond. For sufficiently small time-like interval between P and Q, this is always finite. The bound can be proved from Einstein’s equations with additional assumptions bounding entropy density by energy density[27]. However, it is clearly a statement about quantum gravity and cannot be proven without a theory of
quantum gravity. In this respect, string theory is not much help. String theory does not talk about local physics. In asymptotically flat space-time the only causal diamond that appears in string theory is the conformal boundary. In asymptotically AdS space-time one can give unambiguous meaning only to the causal diamonds of points P and Q whose time-like separation is large enough for the causal diamond to intersect the boundary.$^{23}$

Our idea has been to take the holographic principle and the entropy bound as the defining property of quantum gravity. To do this, one has to decide which quantum density matrix the bound refers to. Fischler and I argued that the only general answer could be the maximally uncertain one: the covariant entropy bound for a causal diamond refers to the logarithm of the dimension of the Hilbert space which describes experiments that can be performed by the observer traveling between Q and P. Other, more restrictive density matrices would only be natural if there were a preferred operator, like the Hamiltonian, in our system. The message of general covariance is that there should be no such preferred operators except for infinite systems. Energy and other symmetry generators are only defined in terms of asymptotic diffeomorphisms acting on the boundary of an infinite space-time.

The idea that the theory of a finite causal diamond has a finite number of states, immediately implies that that theory is ambiguous. A quantum theory with a finite number of states cannot perform measurements on itself with arbitrary accuracy. If the a priori quantum uncertainty in the results of measurements is irreducibly finite, then many different mathematical quantum theories will predict the same answers within the unavoidable measurement error. This is what I mean by the statement that the quantum theory of a finite area causal diamond is intrinsically ambiguous. We call a choice of a particular mathematical quantum theory for a sequence of causal diamonds a choice of a quantum observer. For asymptotically large sequences of causal diamonds, with the property that the asymptotic limit is well described by quantum field theory in curved space-time (this is a restriction on the allowed asymptotic states of the system), this notion approaches the notion of observer in classical GR and ordinary quantum theory. That is, under such conditions, we know that there will exist large quantum subsystems with many semi-classical observables. In this limit, we can easily compare the results obtained by different causally connected observers$^{24}$ because their semi-classical observables commute with each other. The usual equivalence relations imposed by GR should apply, but only in the limit.

$^{23}$More localized measurements in AdS/CFT are related to ambiguous choices of UV cutoffs on the boundary field theory.

$^{24}$The proper definition of what we mean by different observers will be given in a moment.
4.2 Executive summary

Before proceeding to a long discussion of holographic cosmology, I want to summarize the results I intend to demonstrate. This should be useful for the small number of readers who have followed the details of previous papers on holographic cosmology, and the much larger group, which has no interest in those details.

Holographic cosmology is an attempt to study generic initial conditions at a Big Bang singularity in terms of quantum mechanical models built to satisfy the holographic principle. The claim is that generic initial conditions lead to a state which can be colloquially described by saying that the particle horizon of each observer is, at all times, filled with a maximal size black hole. Averaging over many horizon volumes, this defines a homogeneous, isotropic, flat universe, with equation of state $p = \rho$. This system is called the dense black hole fluid. No real observers can exist in such a universe, because all of its degrees of freedom are always in equilibrium.

The real universe is supposed to arise as a lower entropy initial configuration, which has maximal entropy among all those which avoid collapse into a dense black hole fluid state. We argued that these initial conditions make a transition to a nearly homogeneous dilute black hole gas at a critical value of the particle horizon size, $M$. The inhomogeneous fluctuations are exactly scale invariant over a range of scales whose physical size ranges from the Planck length to $M$ at the time of the transition. Their amplitude is small, but neither its exact value, nor the value for $M$, can be calculated at present.

These initial conditions are such that inflation is relatively probable if the low energy effective field theory has a field with a relatively flat potential. Only around 20 e-folds of inflation are needed to explain the correlations in the CMB, in a causal manner. The observable fluctuations in the CMB could be generated either in the $p = \rho$ era or during the inflationary era. If experimental indications that the fluctuations are not exactly scale invariant hold up, then the CMB fluctuations must come from inflation. The energy scale during inflation in holographic cosmology is less than $10^{-3}M^{-2}$ in Planck units, and $M$ itself is likely to be $>> 1$. Inflationary fluctuations in such a low scale model can be consistent with the data only in certain kinds of hybrid inflation models. Such models usually require either fine tuning or supersymmetry. Also, since the scale of inflation is low, the ratio of tensor to scalar modes is small.

We have not been able to find a significant source of primordial gravitational waves in the pre-inflationary physics of the holographic model, so one would tentatively conclude that holographic cosmology predicts no observable primordial gravitational waves. The final state of a holographic cosmology is a stable de Sitter space. Since the normal region, which evolves to this dS space is a low entropy fluctuation of the $p = \rho$
fluid, purely statistical arguments favor the largest value of the cosmological constant. This requires the smallest deviation from the maximal entropy configuration. However, if we want the normal region to contain galaxies, the cosmological constant is bounded from above[28]. The connection between the value of the c.c. and the scale of SUSY breaking[21] may provide an even stronger upper bound[29]. The current model[29] of low energy physics based on these ideas also gives a lower bound on $\Lambda$. Thus it is at least possible that we will end up predicting that the value of the c.c. is determined by a combination of statistical arguments and the requirement that certain gross features of low energy physics are reproduced by our model.

Now turn to the problem of Boltzmann’s brain in holographic cosmology. The stable dS endpoint of holographic cosmology will certainly produce Boltzmann brains, if such objects can exist. However, in contrast to the DKS model, ordinary cosmology is not an extremely low entropy fluctuation of our system. The system has a fixed initial condition (the Big Bang) and we will try to argue that a cosmology like our own is the most probable result of initial conditions which do not produce a dense black hole fluid. This argument depends on the claim that there is a theory of stable dS space for every value of the c.c. $\ll$ the Planck scale, and that the limiting theory for vanishing c.c. is exactly supersymmetric and has a compact moduli space. Since there are no examples of such isolated SUSic theories in which we can do reliable calculations (the state of the art is [23]), it is safe to say that the possible theories of stable dS space are much less numerous than the points in the hypothetical String Landscape. It is even reasonable to assume that there is only a unique such theory, which one would of course hypothesize to have low energy physics coinciding with that in the real world. If this is the case, then production of intelligent life by evolution is a natural consequence of our model\textsuperscript{25}. By contrast, Boltzmann brains are low probability events, which produce short lived intelligence. It is difficult, though perhaps not impossible, to imagine constructing devices during the evolutionary era of the model (which we identify with our own), which have high enough resolution to detect small Boltzmann brains, but whose recording devices have quantum fluctuations sufficiently small to be ignored for times of order the waiting time to produce a Boltzmann brain. Thus, in holographic cosmology, Boltzmann brains are in principle observable by the much more probable intelligent organisms (our descendants?) produced by the inevitable cosmological evolution of the model. They are freaks of nature and pose no philosophical conundrums.

The issue of \textit{Poincare Recurrences} has also been raised, as a criticism of models which, like holographic cosmology, terminate in an asymptotically de Sitter system, with a finite number of states[2]. In such systems, the asymptotic de Sitter Hamiltonian

\textsuperscript{25}and might one day be proved to be a consequence of the model with high probability.
has Poincare recurrences. In holographic cosmology, the universe we observe, over the
time that we currently characterize as its entire history, is described by an emergent
time dependent Hamiltonian. The entropy of such states scales like \((RM_P)^{3/2}\) as the dS
radius goes to infinity. As noted above, the fundamental claim of holographic cosmology
is that normal semiclassical cosmological evolution, is a high probability result of those
initial conditions that do not produce horizon filling black holes at every cosmological
time. In this view, normal cosmological evolution is not described by the asymptotic dS
Hamiltonian, and that history will never recur. Poincare recurrences may produce some
of the states encountered along this history, but will not evolve them in the same way
that the time dependent Hamiltonian does. Thus, in this model, Poincare recurrences
will never reproduce the cosmological evolution of intelligence that we believe occurred
in the world we observe.

Finally, in the case of Poincare recurrences, general arguments [13] show that no
machine built out of local quantum field theory degrees of freedom\(^{26}\) can remain classical
long enough to observe a Poincare recurrence. This means that, unlike Boltzmann
brains, there is in principle no operational way to test for the existence of Poincare
recurrences. Mathematically, this means that we can build many models of the universe
of holographic cosmology, all of which make the same predictions for observations
about local physics and the evolutionary part of cosmic history, but which differ in
their predictions for the behavior of the system over a Poincare recurrence time. It
seems most reasonable to declare all such descriptions to be gauge equivalent, and the
mathematics of Poincare recurrences to refer to gauge degrees of freedom.

4.3 Holographic cosmology: the details

We want to construct a microscopic theory, which is purely quantum mechanical, and
mimics the causal structure and equivalence relations for observers in general relativity
or quantum field theory in curved space time. We define an observer by a nested
sequence of Hilbert spaces \(\mathcal{H}(t) = \mathcal{K} \otimes \mathcal{H}(t-1)\). These represent the degrees of freedom
accessible to the observer in a nested sequence of causal diamonds, which eventually
engulf its entire history. In the cosmological context, the only one we will discuss in
this paper, we should think of these spaces as associated with a nested sequence of
causal diamonds, with their past tips lying on the same point on the Big Bang surface.
The growth of the Hilbert space with \(t\) refers to the growth of the particle horizon of
the observer, as its trajectory gets further from the Big Bang. The finite dimensional
Hilbert space \(\mathcal{K}\), refers to the degrees of freedom in a single pixel of the observer’s

\(^{26}\)Unless one can build complicated machines using the classical degrees of freedom of black holes, in
contradiction to the No Hair theorem, this means no machines that can exist in any quantum theory
of gravity.
holographic screen, and we will discuss its structure in a moment. As the particle horizon grows, more pixels are added to the screen.

The dynamics in each of these Hilbert spaces is given by a sequence of unitary operators $U(t, k)$ with $1 \leq k \leq t$, obeying the consistency conditions

$$U(t, k) = U(s, k) \otimes V(t, s, k),$$

whenever $t \geq s \geq k$. As the tensor product indicates, $V(s, t, k)$ operates only on that tensor factor of the Hilbert space $\mathcal{H}(t)$, which is complementary to $\mathcal{H}(s)$. In words: as the particle horizon expands the observer sees new degrees of freedom, which couple to those already within its purview. The consistency condition guarantees that the past as seen by an observer at a future time is not changed from that seen by the earlier observers.

A full quantum cosmology is defined by a spatial lattice of such observers. Only the topology of the lattice has meaning. Geometry of the space-time is supposed to emerge as a coarse grained approximation in the limit of large causal diamonds. Together with the time lattice of each observer, the space-lattice defines a discrete topological space-time. It has a causal structure, and a global foliation by space-like surfaces. The time function, is a monotonically increasing function of the area of the causal diamond whose future tip lies on each space-like surface. The full causal structure is defined by specifying, for each pair of Hilbert spaces $\mathcal{H}(t, x)$ and $\mathcal{H}(s, y)$ an overlap Hilbert space $\mathcal{O}(t, s, x, y)$, which is a tensor factor of each. The overlaps are constrained so that one gets the same result for $\mathcal{O}(t, s, x, y)$ by moving along any path between the points. Furthermore, for nearest neighbor spatial lattice points $\mathcal{O}(t, t, x, x + \hat{n}) = \mathcal{K}$. Finally, the dimension of $\mathcal{O}(t, t, x, y)$ should decrease monotonically with the inverse length of the shortest lattice path between $x$ and $y$. Two points at fixed time are causally disconnected when $\mathcal{O}$ is one dimensional.

On the overlap Hilbert space, we must also insist that the unitary evolution operators defined by the individual observers agree with each other\textsuperscript{27} This is an incredibly complicated condition, and we think of it as the dynamical principle of holographic space-time. Different solutions to it will be the allowed quantum cosmologies of this formalism.

Here is the only solution to it that we have found. Let $\mathcal{O}(t, t, x, y) = \mathcal{H}(t - N, x)$ where $N$ is the number of steps in the shortest lattice path between $x$ and $y$. Other overlaps are computed by combining this rule with the rule for overlaps along the trajectory of a given observer. Since $\mathcal{H}(t, x)$ is the same Hilbert space for every $x$, this

\textsuperscript{27}Perhaps only up to unitary conjugation: $U(t, k, x) = WU(t, k, y)W^\dagger$ on $\mathcal{O}(t, t, x, y)$. The model we will study has $W = 1$. 

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rule can give consistent dynamics if (and only if) we choose the same sequence of unitary operators for the observer at each \( x \). Thus, our formalism for quantum cosmology gives a consistent result in this case only if the cosmology is spatially homogeneous. Similarly, for large \( k \), the locus of points that are \( k \) lattice steps away from a given point fills out a sphere in the Euclidean space in which the lattice is embedded, as long as the lattice has the topology of Euclidean space. Thus for large \( t \), the distance at which two observers become causally disconnected becomes spherically symmetric. Our cosmology is homogeneous, isotropic, and flat.

To understand the rest of the dynamics we must discuss a point we have omitted, the nature of the quantum variables which describe cosmology. Consider the holographic screen of a causal diamond. It is a space-like surface, which records information from the interior of the diamond when a massless particle penetrates it. A pixel on the screen, is an infinitesimal area element. We would like to specify the orientation of this element in space-time, and the direction of the null vector penetrating it. In ancient times, Cartan and Penrose pointed out that this information is precisely encoded in a pure spinor, namely a Dirac spinor which satisfies

\[
\bar{\psi} \gamma^\mu \psi \gamma^\rho \psi = 0.
\]

The null vector is \( \bar{\psi} \gamma^\mu \psi \). The non-vanishing components of the other Dirac bilinears all lie in a hyperplane transverse to this null vector, which defines the direction of the holographic pixel. The non-vanishing components of the spinor transform as a spinor of the \( SO(d-2) \) tangent space group of the screen. Thus, we can think of the orientational information in a holographic screen as encoded in an element of the spinor bundle over the screen. We can quantize the pixel orientation by postulating commutation relations

\[
[S_a(m), S_b(n)]_+ = \delta_{mn} \delta_{ab},
\]

for the non-vanishing real components of the spinor. We choose anti-commutators in order to have a finite dimensional Hilbert space for each pixel\(^{28}\). This is the most general rule covariant under the \( SO(d-2) \) tangent space group of the holographic screen, if we restrict each pixel to carry only information about the orientation of the screen in the non-compact dimensions of space-time. Compact factors in the space-time will lead to more general pixel algebras, and the theory of these is only beginning to be worked out\(^{[31]}\).

It is remarkable that the algebra of a pixel on the holographic screen is the same as that of a massless superparticle with fixed momentum. This suggests that for very

\(^{28}\)One might have expected the degrees of freedom of independent pixels to commute. We have made everything anti-commute by exploiting the \((-1)^F\) gauge invariance of the formalism\(^{[30]}\).
large causal diamonds, our formalism should describe scattering states of massless superparticles, and certain examples have been worked out[32], which show that this is kinematically correct. For consistency, the dynamics must also be supersymmetric.

Returning to our cosmological model, we identify the Hilbert space $\mathcal{K}$ with the irreducible representation of the single pixel algebra. We define a class of random Hamiltonians as follows: First choose a random fermion bilinear

$$H_2 \equiv \sum S_a(n)A_{mn}S_a(m).$$

For a large number of pixels, the spectrum of $H_2$ approaches that of the massless Dirac equation in $1 + 1$ dimensions, with a cutoff. We can now add higher order terms, as long as they are irrelevant perturbations in the infrared of this $1 + 1$ dimensional field theory. The spectral properties of all of these Hamiltonians are the same, except for a small number of states near the cutoff.

The operators $U(t,k)$ have the form

$$e^{i[H_1(t,k)+H_2(t,k)]},$$

where for each $(t,k)$, $H_{1,2}$ are chosen independently from the distribution of random Hamiltonians described above. They are subject to the restriction that $H_1$ is built only from those $S_a(n)$ variables that act in $\mathcal{H}(k)$, while $H_2$ depends only on those which act on its tensor complement in $\mathcal{H}(t)$. We have left the $x$ labels off these operators, because our overlap rules specify that the same random choices are made at each $x$. As far as we can tell, there is no other way to solve all of our dynamical consistency conditions.

We define the overlap rule that $O(t,t,x,y) = \mathcal{H}(t-P)$. The dynamics then satisfies all the overlap conditions if the sequence of Hamiltonians seen by each observer is the same. Given random choices for individual observers, there seems to be no other way to satisfy consistency. Thus, our emergent spatial geometry is forced to be homogeneous by the overlap consistency conditions. This overlap rule also defines an isotropic spatial metric. Indeed, the overlap rules in general define a causality distance on the spatial lattice of observers. The causal distance between two points at time $t$ is the minimal number of lattice steps that have to be taken before the overlap Hilbert space becomes trivial. Our overlap rule defines a causality distance which is blind to direction on the lattice. Thus, the set of points that are a given causality distance, $R$ from the origin is the locus of endpoints of self avoiding lattice walks of $R$ steps. For large $R$ this describes a sphere in Euclidean space, for any lattice with the topology of Euclidean space. Once we have reached the causal boundary at fixed $t$, we continue to define the distance in terms of the minimal number of lattice steps. This definition is flat as well as being homogeneous and isotropic.
I will not go into the details here[6], but one can also use the scaling laws of the fermion system at large $N$, to show that the horizon expands as one would expect for a flat FRW space with equation of state $p = \rho$. Furthermore, the conformal Killing vector of this emergent geometry defines an exact symmetry of the quantum system in the large $N$ limit. It is the asymptotic scaling symmetry of free 1 + 1 dimensional fermions perturbed by irrelevant operators.

The energy density in a fixed horizon volume is defined in terms of the fermion system and the total entropy is of order $N$. These relations tell us that the entropy and energy of a horizon volume satisfy the black hole entropy formula, justifying our assertion that we have constructed a mathematical model of the dense black hole fluid.

4.4 A realistic cosmology

The mathematical model sketched above is a completely consistent quantum cosmology, but not one you would want to live in. All degrees of freedom inside a horizon volume are, at all times, in equilibrium. There can be no particles, no observers separate from the cosmological background - nothing of interest. The coarse grained description of the model as FRW geometry can be derived as indicated above, but is somewhat misleading. This is not a geometry in which anything can propagate.

In this subsection I will describe the heuristic model of more realistic cosmology, which Fischler and I have developed. I emphasize that there is at present no real mathematical model of this more general system. The basic idea is to imagine a normal region as a defect in the $p = \rho$ background, where observers on some compact region of the lattice see a more normal FRW geometry. We take it to be radiation dominated, with the thought that the region originally had a black hole in it (to maximize the entropy of the initial conditions) but that black hole was too small to merge with the surroundings as the particle horizon increased. Instead, a region of empty space is created, and the black hole evaporates into it, producing a radiation gas. This picture is irrelevant to what follows, and should be viewed only as motivation.

Consider first a spherical region of FRW geometry with equation of state $p = w\rho$ with $w < 1$, embedded in a $p = \rho$ background. Applying the Israel junction condition to the interface, we find that the coordinate size of the $w < 1$ region must shrink. Only in the case $w = -1$ can we obtain a stable configuration of non-decreasing physical size. Einstein’s equations have solutions corresponding to black holes embedded in any FRW background. The horizon of the hole is a marginally trapped surface of arbitrary size. If we excise the interior of the hole and replace it with the static coordinate patch of dS space with dS and Schwarzschild radii equal, then we satisfy the longitudinal part of the Israel condition. The surface stress tensor implied by the transverse condition, obeys the weak energy condition. We conclude that a spherical ball of radiation or
matter dominated universe can only survive if it asymptotes to dS space and initially occupies a huge coordinate volume, much larger than our current horizon size. This seems like a very improbable initial condition.

In fact, there are surely initial configurations which are more probable than this, which can also evolve to an asymptotically dS space with a large radius. Let us make a “tinkertoy”, connecting together a bunch of balls of normal region, each of fairly small radius (\(< e.g. 100\)) in Planck units. The balls are glued together into a tinkertoy (Fig. 1) entirely contained in a ball on the lattice whose coordinate radius is that of the asymptotic dS horizon. In describing these balls, we are using the coarse grained FRW geometry which emerges from the dense black hole fluid. The balls describe regions in that background geometry (which ultimately means a subclass of observer Hilbert spaces associated with a compact region on the lattice) in which the dynamics is not what we described in the previous section, but resembles that of a radiation dominated FRW universe.

Initially, the tinkertoy takes up a small fraction of the physical volume of the compact region of the lattice corresponding to the asymptotic dS horizon. We can define an equal area time slicing on the inhomogeneous geometry consisting of tinkertoy plus \( p = \rho \) background. At each time, we insist that the area of the causal diamond that goes all the way back to the Big Bang, is the same at every lattice point. In terms of the Hilbert spaces \( \mathcal{H}(t, x) \), we are just enforcing the already enunciated rule that the dimension of all Hilbert spaces at time \( t \) is the same.

It is an easy exercise to show that on equal area time slices, the volume of space in the \( p = 1/3\rho \) regions grows more rapidly than that in the dense black hole fluid. Thus, after some time, a better picture of the geometry is given by Fig 2. In this figure we see a large volume normal region, interspersed with small volumes in which all the degrees of freedom are in equilibrium. It does not take much imagination to guess that a proper description of this regime in cosmic history is a dilute black hole gas. We say that the cosmology has made a transition between the dense black hole fluid and the dilute black hole gas. The horizon size at the time of this transition defines an average black hole mass \( M \), and the energy density is of order \( \frac{1}{M^2} \) in Planck units.

If we want to find the most probable initial condition which leads to this transition, we should look for the tinkertoy which takes up the smallest fraction of the initial value. This is because initials conditions in a normal region are much more constrained than those in the dense black hole fluid, which saturates the covariant entropy bound at every moment. Thus, we expect \( M \) to be a very large number, but we cannot yet calculate it from first principles. Note that before this transition it does not make sense to describe the evolution of the universe in terms of low energy effective field theory. The real criterion for the validity of effective field theory in a space-time region
is that the state of the system is very far from saturating the entropy bound. The dense black hole fluid gives us examples of low energy density configurations which cannot be described by effective field theory. There is a coarse grained space-time description, but the excitations of this space-time are not particles traveling through it, but the low entropy tinkertoys we have just discussed\textsuperscript{29}.

The tinkertoy configuration evolves to an inhomogeneous state of the dilute black hole gas, whose inhomogeneities are in principle determined (statistically) by the most probable choice of tinkertoys that actually leads to the transition. We cannot of course figure out what this is, but one thing is clear. The fluctuations in black hole masses around the average value $M$ must be quite small, and so must the fluctuations in their velocities around a uniform cosmic velocity. Indeed, if this were not so, then the black holes would quickly collide and merge. The system would recollapse to the dense black hole fluid. While this is qualitatively obvious, we have not yet been able to turn it into a quantitative bound on the size of the fluctuations.

The statistics of these fluctuations also depends on what the most probable configuration for the tinkertoys is and we do not yet have a grasp of that. However, the exact scale invariance of the $p = \rho$ system enables us to show that the two point function of these fluctuations satisfies Harrison-Zeldovitch scaling in a range of scales whose physical size runs from the Planck scale to the size $M$ of the horizon at the transition to the dilute black hole gas phase. The fluctuations outside this range are cut off. Recent observations seem to suggest that such exact scale invariance is ruled out for fluctuations in the CMB in the largest few orders of magnitude of scales on the sky.

Another problem with the model so far is that the correlation length of these fluctuations cannot be as big as that of the correlations in the CMB unless we have a period of inflation. This is the real horizon problem. Only inflation can explain correlations in the fluctuations on our current particle horizon scale. We have seen that approximate homogeneity and isotropy follows from the properties of the dense black hole fluid, combined with the requirement that the normal regions do not recollapse into the $p = \rho$ phase. Note however that we only need 10-20 e-folds of inflation to explain the observed correlations in the CMB.

After the phase transition to the dilute black hole gas, we can begin to use effective field theory to describe the dynamics of the universe. The detailed microstates of the black holes are not well described by this formalism, but their coarse grained thermodynamics is. This effective field theory might be fairly unique, or it might be

\textsuperscript{29}There is something in this discussion reminiscent of duality transformations in classical statistical mechanics. In the Ising model, for example, the maximal entropy high temperature state is described as a frozen state of dual variables. In some sense, our tinkertoys are the dual variables with which to define excitations of the $p = \rho$ system.
part of a landscape. However, our cosmology terminates in a stable dS space with a finite number of states\textsuperscript{30}. Thus, the potential on the Landscape should be above the Great Divide\textsuperscript{[22]} with regard to transitions out of the dS minimum with lowest c.c. At any rate, we will assume that the potential on the space of low energy scalar fields allows for inflation.

Recall that for potentials which vary in field space over field intervals of order the reduced Planck scale $m_P$, we only need a $\frac{1}{N_e}$ coincidence to have $N_e$ e-folds of inflation. Furthermore potentials of this form are technically natural if all the couplings of the relevant scalar fields are suppressed by $m_P$. Finally, the initial conditions for the field must be fairly homogeneous, in order to avoid recollapse into the dense black hole fluid. It is reasonably probable that the initial field configuration will be sufficiently homogeneous for inflation to begin, as long as the inflationary energy density is well below $1/M^2$, so that we can be sure that we are in a regime described by low energy effective field theory. This probably constrains the potential to be such that no measurable tensor fluctuations will be generated during inflation. We\textsuperscript{[33]} have searched for other mechanisms, which could generate a spectrum of primordial gravitational waves in holographic cosmology, but have not found any so far. Thus, one could tentatively conclude that holographic cosmology predicts a vanishing tensor to scalar ratio. The most important conclusion is that holographic cosmology provides an explanation of the homogeneous initial conditions necessary for inflation to work.

The amount of inflation necessary to explain the data in this model is definitely smaller than in garden variety inflation models. Gross flatness, homogeneity and isotropy are explained by a non-inflationary mechanism. Inflation is only necessary in order to explain horizon spanning correlations, though it can also be the source of the CMB fluctuations. The necessary amount of inflation depends on the value of $M$, the scale of inflation, and the question of whether we want to explain observations in the CMB with inflationary fluctuations, or fluctuations generated during the $p = \rho$ era. Preliminary observations, which suggest that the spectrum is not exactly scale invariant, favor the inflationary fluctuations. In any case, we do not need more than 30 e-folds of inflation.

Since my purpose here is not to delve into the details of holographic cosmological models, I will stop, and reiterate the most important point of this section. Holographic cosmology provides an explanation for the apparent low entropy of cosmological initial conditions. The initial conditions are the highest entropy possible, consistent with

\textsuperscript{30}This is a probabilistic argument. The String Landscape assumes that the history of the universe has an infinite number of states that are well described by low energy effective field theory. Even if it exists, it would seem to be a less probable way for the universe to escape from the dense black hole fluid than the one we are proposing.
avoiding the collapse into a dense black hole fluid. We have provided qualitative arguments for this, but not a quantitative derivation of the degree of homogeneity we observe in the universe. As a bonus we predict that the final state of the universe is asymptotically dS with an \textit{a priori} distribution for the c.c., which favors large values.

The DKS model explains low entropy initial conditions as an unlikely fluctuation, and appears to founder on the bizarre phenomenon of Boltzmann’s brain. The model has a history which parallels the cosmology we have discovered by observation, but seems to predict that a typical observer will instead arise as a local thermal fluctuation. Holographic cosmology does not suffer from this problem. Its typical cosmological history goes through a conventional evolutionary production line for observers, with conditional\textsuperscript{31} probability one. The fact that it also predicts rare Boltzmann Brain events in the distant future is no more problematic than the fact that we have never seen all the air in the room spontaneously collect in a corner.

5. Conclusions
We have reanalyzed the model of DKS, which provides a reason for cosmological evolution to begin in a low entropy state. The basic mechanism for this is a Poincare recurrence of a low entropy fluctuation in a finite entropy system. This is an old idea, possibly going back to Boltzmann, but models of this type generally run into a paradox. If we imagine that cosmological evolution is the same unitary groupoid as the one which generates the recurrences, we can argue that the typical recurrence of cosmology, even when subjected to anthropic constraints, does not resemble the one we see.

We argued that the way out of this paradox comes in the recognition that the time dependent Hamiltonian, $P_0(t)$ which describes cosmology, is \textit{not the same} as the static Hamiltonian, $H$, which describes the recurrences. $P_0(t)$ is instead an emergent operator, describing the evolution of a small subset of the degrees of freedom of the system, over time periods much shorter than the recurrence time of $H$. Much of interesting cosmological history in fact occurs over times short compared to the asymptotic Hubble time, $R$. We argued that recurrences of events where $P_0(t)$ is a relevant description of a subset of the system all resemble each other apart from microscopic details, and the description of events in the very far future\textsuperscript{32}. The model simply does not contain cosmological histories with \textit{e.g.} different cosmic microwave background temperatures.

\textsuperscript{31}The only condition imposed is that the universe does not evolve forever as a dense black hole fluid.

\textsuperscript{32}In this model, most of the visible universe will go out of our horizon in a few times the current age of the universe. Even our gravitationally bound environment will eventually be destroyed by gravitational collapse or dS thermal fluctuations. It is likely that the history of these very late times, and particularly the effect of dS fluctuations, can recur in very different ways. Even if we should live so long we might not be able check whether the very late cosmic history we see is a typical member of the
The last statement is confusing to the effective field theorist, because at late times the CDL cosmology appears to allow such states. However, if we extrapolate this late time data back, we find a space-like singularity intervening between us and the coordinate singularity in the CDL solution (the bubble nucleation event). Thus, there is no way of connecting this data to any controlled fluctuation of the asymptotic dS vacuum state\textsuperscript{33}. The effective field theory in CDL cosmology is only valid for a small subset of the possible states in that field theory at late times. The other states of that effective theory are likely to have nothing to do with the correct description of any event in the full system. There are many fluctuations of the late time dS geometry which do not resemble possible events in the CDL cosmology, but there is no reason to imagine that they resemble anything close to it at all. For example, the late time dS space has black holes of size much larger than the size of the critical bubble. These are certainly produced as thermal fluctuations in the future, but have no description in terms of the degrees of freedom which follow the evolution of $P_0(t)$. A microscopic description of the events leading to the nucleation of such black holes involves the dynamics of the Hamiltonian $H$ in a crucial way, although we can estimate the probability of these events by doing the statistical mechanics of $P_0$ at the dS temperature.

A residual problem for DKS cosmologies is the thermal production of small intelligent systems. We argued that this could be a problem for the string landscape as well. It is likely that, if it exists, the landscape contains approximately SUSic dS spaces. If any of these have low energy physics resembling our own, then the string landscape will have Boltzmann brains. It is not possible at this time to decide whether these are the most probable form of intelligent life in the string landscape. This is directly connected to the fact that no one has addressed the problem of initial conditions and ensemble of all histories. Such a check would perforce involve knowledge of what has happened to parts of the universe outside our horizon. Our asymptotically dS universe has of order $(RM_p)^{1/2} \sim 10^{30}$ independent horizon volumes at late times, so we can expect to see coincidences of relative probability $10^{-30}$, if we only examine localized excitations in our own horizon volume.

\textsuperscript{33}Of course, if one were perverse one could insist on including all of these late time states as part of the evolution of the system, and proceed to rederive the DKS paradox. My point is that the model provides us with no evidence that these states have anything to do with fluctuations of a finite system. Indeed, the form of the CDL amplitudes, which obey the principle of detailed balance, suggests that the entropy of states which have anything to do with the actual tunneling event is much smaller than what one would estimate with cutoff field theory at late times. Cosmological evolution of these low entropy initial states, produces states which are indistinguishable (after a little coarse graining) from a much higher entropy ensemble. Most of the members of that late time ensemble also extrapolate back to a singularity, but we are not concerned with this because their prediction for the future are hard to distinguish from those of the initial low entropy ensemble. However, states which are distinguishable, like those with higher than expected CMB temperature, cannot be connected to any well established fluctuation of the system.
the explanation for their low entropy, in the string landscape context. The conventional wisdom is that we get into the basin of attraction of our minimum by CDL tunneling, which could provide an explanation for homogeneity, isotropy and slow roll inflation. However, unlike the DKS model, the landscape provides no way of estimating the probability of getting into the low entropy false dS vacuum which begins the current era of history of the universe.

Finally we reviewed the way in which Holographic Cosmology attempts to explain the low entropy of cosmological initial conditions, and the way in which it avoids the problem of Boltzmann brains. BBs exist, but are less probable ways to produce intelligence than that afforded by conventional cosmology and evolution. Indeed, there is a way to state our conclusions about holographic cosmology which is quite general. If cosmology is described by quantum mechanics with a time dependent Hamiltonian, then in order to avoid the problem of Boltzmann’s Brain we need the space of initial states to divide into two categories. In the first category, life (or perhaps just life of our type) is not possible. In the second, the universe undergoes a period of cosmological expansion with time dependent Hamiltonian and produces life via more or less standard cosmology and evolution. In the second category the far future evolution of the universe may produce a large number of Boltzmann brains, if the future is described by a stable dS space. This is, in my opinion, no more problematic than the fact that the ordinary laws of thermodynamics say that if a room exists long enough, all the air in it will collect in a corner. Every history in our model that supports Boltzmann brains also produces a prior period of evolved observers. This period never repeats as a thermal fluctuation, because the Hamiltonian describing the ultimate equilibrium state of the universe is not the time dependent Hamiltonian describing cosmology. This is similar to our discussion of the DKS model, but in the present class of model evolved life occurs with probability one.

Boltzmann Brains, and their support systems, can be observed by evolved observers, only if their size is much smaller than the cosmological horizon. Otherwise, the waiting time for producing the BB support system is longer than the maximal survival time of a local observer[13]. Thus, in this class of models, BBs are properly viewed as the kind of crazy fluctuation statistical mechanics teaches us to expect and discount. Any observer can tell whether it is a BB by making a relatively small number of observations. If our own observations are described by such a model, then it is certain that we are not BBs.

If one wants to believe the mathematical predictions of this class of models, over time scales much longer than the maximal survival time of an evolved local observer, then fluctuations might occur, which reproduce the current state of all localizable degrees of freedom in our universe. One could then choose to evolve that state with the time dependent Hamiltonian $P_0(t)$ starting at the present time and extending for
times short compared to $\frac{1}{R_{ds}}$. During this time period the actual evolution under $H$, which is prescribed by the definition of the model, does almost nothing to the state and so it makes sense to study an emergent Hamiltonian which describes variations over shorter time scales\textsuperscript{34}. This kind of BB would be hard to distinguish from ourselves, but it is enormously less probable than the smaller BBs. So, given a model of this type, we would conclude from our observations that we were either the original, evolved, observers, which are guaranteed to exist in any anthropically allowed history of the model, or an enormously improbable thermal fluctuation. I leave it to the reader to decide which explanation we would be likely to choose.

The idea that the second law of thermodynamics could be explained by assuming that our cosmological history was a fluctuation in a finite system probably originates with Boltzmann. His vast intelligence still haunts our considerations of this proposal. The other class of models described above, exemplified by holographic cosmology, provides a more satisfactory understanding of the apparent low entropy of cosmological initial conditions.

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