On The $pp \rightarrow pp\eta(\eta')$ Reactions Near Threshold

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Abstract

The production rate for $\eta'$ in $pp \rightarrow pp\eta'$ at rest is calculated in a covariant one boson exchange model, previously applied to study $\pi^0$ and $\eta$ production in NN collisions. The transition amplitudes for the elementary $BN \rightarrow \eta'N$ processes with $B$ being the meson exchanged ($B = \pi, \sigma, \eta, \rho, \omega$ and $\delta$) are taken to be the sum of s and u channels with a nucleon in the intermediate states, and a $\delta$ meson pole in a t-channel. The couplings of the $\eta'$ to hadrons are a factor 0.437 weaker than the respective $\eta$-hadron couplings, as suggested by a quark model and a singlet-octet mixing angle $\theta = -23^\circ$. The model reproduces near threshold cross sections for the quasielastic processes $\pi^-p \rightarrow n\eta(\eta')$ and $pp \rightarrow pp\eta(\eta')$ reactions.

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1 Introduction

The production of $\eta'$ mesons in proton-proton collisions near threshold has been reported recently by the SPESIII group at Saturne[1] and by COSY-Jülich[2]. The $\eta'$ is observed as a missing mass peak by detecting both final protons in a magnetic spectrometer. The total cross section of the $pp \rightarrow ppp'\eta'$ reaction is found to be a factor $\approx (m_\pi/m_{\eta'})^2$ smaller than for the corresponding $pp \rightarrow ppp\pi^0$, indicating a similar production mechanisms for $\eta'$ and $\pi^0$. The observation that the $\eta$ production rate is nearly as large as $\pi^0$ production rate is attributed to a dominant contribution from resonant production via virtual excitation of the $N^*$ (1535 MeV) S11 nucleon isobar.

Various phenomenological one-boson-exchange (OBE) models for these processes have achieved impressive descriptions of extensive data from Saclay[3, 4, 5], Indiana[6], and Uppsala [7, 8, 9, 10, 11]. Particularly, a relativistic covariant OBE model, reproduces consistently the cross section data (both scale and energy dependence) for the $pp \rightarrow ppp\pi^0$ and $pp \rightarrow ppp\eta$ reactions[12, 13]. The main objective of the present note is to consider $\eta'$ production by applying a slightly generalized model. As in Ref.[12, 13] we assume a reaction mechanism as depicted in Fig. 1, where a virtual boson $B$ ($B=\pi, \eta, \sigma, \rho, \omega, \delta$) created on one of the incoming protons is converted into a pseudoscalar meson $P$ ($\pi, \eta, \eta'$) on the second via a $BN \rightarrow PN$ conversion process. The half off mass-shell amplitudes for these conversion processes, hereafter denoted by $T_{BN \rightarrow PN}$, are taken to be the sum of three pole terms corresponding to $s$, $u$, and $t$-channels (see Fig. 2). The latter accounts for meson production occurring on internal lines such as $\sigma$ and $\delta$ meson lines, and requires knowledge about three meson legs vertices like $\sigma\pi\pi$, $\delta\eta\pi$ and $\delta\eta'\pi$ vertices. Such mechanisms are considered in Ref. [13], where an effective $\sigma$-meson in a $t$ channel, strongly enhanced due to offshellness, found to play a prominent role for $\pi^0$ production in the $pp \rightarrow ppp\pi^0$ reaction at threshold. For $\eta'$ production most relevant is a $\delta\pi$ exchange. Generally, both nucleon and nucleon isobar excitations intermediate states may contribute to diagrams 2a and 2b. Such is the case for $\eta$ production, where production via exciation, propagation and subsequent decay of the $N^*$ (1535 MeV) S11 isobar into a $\eta N$ pair play a significant role.
in the process. We shall demonstrate below that cross section data for $\eta'$ production can be explained without a resonant production term.

Much of the model success to explain meson production data in NN collisions depends on how well the half off mass shell amplitudes for the elementary conversion processes $BN \rightarrow PN$ are calculated. Unfortunately, there are no direct measurements which link directly to the off mass shell behavior of these amplitudes. In comparison with the $pp \rightarrow pp\eta$ reaction, the momentum transfer in $pp \rightarrow pp\eta'$ is nearly twice as large and it would be of interest to verify that the off shell behavior presumed reproduces $\eta'$ production rate as well. There are some near threshold cross section data for the $\pi N \rightarrow N\eta(\eta')$ reactions\cite{14}. Here as for production in NN collisions the cross sections depart by more than an order of magnitude\cite{14}. It is to be shown that taken on mass shell, the amplitudes $T_{\pi N \rightarrow N\eta(\eta')}$ reproduce the cross sections for the quasieelastic processes $\pi^0p \rightarrow p\eta$ and $\pi^0p \rightarrow p\eta'$ at their respective thresholds.

2 Theoretical perspective

To account for the production of $\eta'$ meson, we slightly generalize the Lagrangian of Refs. \cite{12,13}, by including a Lagrangian density,

$$L_{\eta'NN} = \frac{f_{\eta'NN}}{m_{\eta'}} \bar{N} \gamma^5 \gamma^\mu \partial_\mu \eta' N ,$$  \hspace{1cm} (1)

where $N$ and $\eta'$ represent the fields of a nucleon and an $\eta'$ meson.

Furthermore, in order to allow $\eta'$ formation on an internal $\delta$-meson line we write the $\delta\eta'\pi$ vertex in a rather general form\cite{13},

$$V_{\delta\eta'\pi}(k^2, q^2, (k - q)^2) = m_\delta \left( g_{0\eta'} + g_{1\eta'} \frac{k^2}{m_\delta^2} + g_{2\eta'} \frac{q^2}{m_\delta^2} + g_{3\eta'} \frac{(k - q)^2}{m_\delta^2} \right) .$$ \hspace{1cm} (2)

This expression is similar to the $\delta\eta\pi$ vertex used in Ref. \cite{13} for $\pi^0$ (or $\eta$) production, where the vertex constants $g_{i\eta'}; i = 0 - 3$ were deduced from the partial decay width for $\delta \rightarrow \pi\eta$, and by applying the three Adler’s consistency conditions to the $T_{\pi^0p \rightarrow \eta p}$ amplitude\cite{13}.

There is no direct experimental information about the strength of the $\eta'$ couplings. However, being the heaviest member of the ground state pseudoscalar meson nonet,
the \( \eta' \) couplings can be related to those of the \( \eta \). In the quark model the \( \eta NN \) and \( \eta' NN \) couplings are\[15\],

\[
g_{\eta NN} = g_8 \cos \theta - g_1 \sin \theta , \\
g_{\eta' NN} = g_8 \sin \theta + g_1 \cos \theta ,
\]

with \( \theta, g_1, g_8 \) being the mixing angle, singlet and octet couplings, respectively. By making the simplifying assumption \( g_8 = g_1 \), and taking a linear mixing angle\[15\], \( \theta = -23^\circ \), one obtains,

\[
g_{\eta' NN} / g_{\eta NN} = 0.437 .
\]

Similarly, the \( \delta \eta \pi \) and \( \delta \eta' \pi \) vertex constants scale also according to Eq. 4. With the relevant \( \eta \) couplings taken from Ref.\[13\] and disregarding uncertainties in \( \theta \) one obtains

\[
g_{\eta' NN} = 2.68, \quad g_{0\eta'} = 0.02 \pm 0.01, \quad g_{1\eta'} = -2.6 \pm 0.20, \\
g_{2\eta'} = -1.51 \pm 0.12, \quad g_{3\eta'} = 1.44 \pm 0.11 .
\]

The \( \eta \) meson couples rather strongly to the \( N^* (1535 \text{ MeV}) \) S11 and to a lesser extent to the \( N^* (1710 \text{ MeV}) \) P11 resonances. Particularly, the resonance mass almost coincides with the mass of a \( \eta N \) pair so that contributions from graphs 2a and 2b with nucleon isobar excitations become prominent near the \( \eta \) production threshold. There are no evidence for strong couplings of the \( \eta' \) to baryon resonances. The decays of the \( N^* (1535 \text{ MeV}) \) S11 and \( N^* (1710 \text{ MeV}) \) P11 resonances into a free \( \eta' N \) pair are not accessible either. It is then reasonable to assume that resonant \( \eta' \) production terms are small and can be neglected. With this in mind and following Refs.\[12, 13\] the amplitudes for the \( \pi^0 p \to \eta p \) and \( \pi^0 p \to \eta' p \) are,

\[
T_{\pi^0 p \to \eta p} = -ig_{\eta NN} \cdot g_{\eta NN} \cdot f_\eta(k) f_\pi(q) \tilde{u}(p') \left[ \frac{1}{M_R - \sqrt{s} + i\Gamma / 2} + \frac{1}{M_R - \sqrt{u} + i\Gamma / 2} \right] u(p) + i\frac{g_{\eta NN} f_\eta(q-k)}{(q-k)^2 - m_\eta^2} V_{\eta \pi}(k, q) \tilde{u}(p') u(p) ,
\]

\[
T_{\pi^0 p \to \eta' p} = i\frac{g_8}{(q-k)^2 - m_\delta^2} V_{\eta \pi}(k, q) \tilde{u}(p') u(p) ,
\]
\[ T_{\pi^0 p \to \eta' p} = \frac{i2Mf_{\pi NN}2Mf_{\eta' NN}}{m_\pi m_{\eta'}}f_{\eta'}(q)f_\pi(k)\bar{u}(p') \left[ k \left( \frac{1}{s-M^2} - \frac{1}{u-M^2} \right) - \frac{1}{M} \right] u(p) + \]

\[ ig_{\delta NN}f_\delta(q-k)\eta'\pi(k; q)\bar{u}(p')u(p), \]  

where \( M_R, \Gamma \) represent the mass and width of the \( N^* (1535 \text{ MeV}) \) S11 resonance; \( M \) and \( m_B \) the mass of a nucleon and a boson \( B \); \( f_B \) a source form factor parametrized in the usual form \( [16] \),

\[ f_b = \frac{\Lambda^2_B - m^2_b}{\Lambda^2_B - q^2}. \]

It is easy to trace in the expressions above the contribution from s, u and t channels. The first two terms in Eq. 6 describe resonant production via nucleon isobar excitations, while the last term in both of these expressions stands for contributions from a \( \delta \) meson t pole. The other meson conversion amplitudes and numerical details of the calculations are given in Refs. \([12, 13]\) and shall not be repeated here.

We may now use Eqns. 6 and 7 to evaluate the cross sections for the quasielastic processes \( \pi^0 p \to \eta p \) and \( \pi^0 p \to \eta' p \). At threshold these expressions predict for the amplitude squared:

\[ |f(\pi^0 p \to \eta p)|^2 = (15 \pm 7) \mu b/sr \] and \( |f(\pi^0 p \to \eta' p)|^2 = (346 \pm 40) \mu b/sr \) which are consistent with the experimental values \( [14] \) \( |f(\pi^0 p \to \eta' p)|^2 = 10 \pm 1 \mu b/sr \) and \( |f(\pi^0 p \to \eta p)|^2 = 365 \pm 30 \mu b/sr \), respectively. Predictions for \( |f|^2 \) are drawn in Figs. 3-4 versus the energy available in the center of mass (CM) system. Due to mutual cancellation, the contribution from both of the s and u nucleon pole terms (drawn separately as dot-dashed curve) is negligibly small for both processes. The t pole terms are also of the same size though playing a different role in the two cases. For the \( \pi^0 p \to \eta p \) reaction the resonant production exceeds by far any of the other contributions. The t pole term is relatively weak, becoming noticeable only through interference with the strong resonant production term. In case of the \( \pi^0 p \to \eta' p \) however, there is no strong resonance term and the t pole contribution determines the cross section almost solely.

We now turn to consider the cross sections for the \( pp \to pp\eta \) and \( pp \to pp\eta' \) reactions. Let us call

\[ \Pi_j = \frac{p_j}{E_j + M}, \]

where \( p_j \) and \( E_j \) are three-momentum and total energy of the j-th nucleon. For the
incoming particles in the CM system, $\Pi_1 = -\Pi_2 = \Pi$. The total energy square and the energy available in the CM system are respectively, $s = (p_1 + p_2)^2$ and $Q = \sqrt{s} - 2M - m_{\eta'}$. There is only one isovector amplitude which determines the reaction cross section at rest, corresponding to a $^{33}P_0 \rightarrow ^{31}S_0$ transition in the two nucleon system. We write this amplitude in the form

$$M_{11}(pp \rightarrow pp\eta') = M_\pi + M_\eta + M_\sigma + M_\delta + M_\rho + M_\omega,$$  \hspace{1cm} (10)

where $M_B$ represents the contribution from the exchange of a boson B. Following Ref.[12],

$$M_\pi = iG_{\pi NN}g_{\pi NN}g_{\eta'NN}\Sigma_P$$
$$- if_{\pi NN}\left(\frac{2M}{m_\pi}\right)\left(\frac{1}{m_\pi^2 - q^2}\right)f_\pi(q)g_{\delta NN}\left(\frac{f_\delta(q)}{m_\delta - q^2}\right)V_{\delta \eta'}(k^2 = m_{\eta'}^2; q^2), \hspace{1cm} (11)$$

$$M_\sigma = iG_{\sigma NN}g_{\sigma NN}g_{\eta'NN}\Sigma_S,$$ \hspace{1cm} (12)

$$M_\eta = iG_{\eta NN}g_{\eta NN}g_{\eta'NN}\Sigma_P,$$ \hspace{1cm} (13)

$$M_\rho = G_{\rho NN}g_{\rho NN}g_{\eta'NN}(\Sigma_{\rho}^{(1)} + 2\Sigma_{\rho}^{(2)}), \hspace{1cm} (14)$$

$$M_\omega = G_{\omega NN}g_{\omega NN}g_{\eta'NN}(\Sigma_{\omega}^{(1)} + 2\Sigma_{\omega}^{(2)}), \hspace{1cm} (15)$$

$$M_\delta = iG_{\delta NN}g_{\delta NN}g_{\eta'NN}\Sigma_S,$$ \hspace{1cm} (16)

where

$$G_{BNN} = g_{BNN} \frac{E + M}{M} \frac{1}{M(m_{\eta'} + Q) + m_B^2}f_B^2(-M[m_{\eta'} + Q])\Pi,$$ \hspace{1cm} (17)

$$\Sigma_S = \frac{1}{M} \left(1 - \frac{5m_{\eta'}}{2M + m_{\eta'}}\right), \hspace{1cm} (18)$$

$$\Sigma_P = \left(\frac{m_{\eta'}}{2M}\right)^2 \left[1 + \frac{2(E + M)}{m_{\eta'}}\Pi \cdot \Pi\right] \frac{1}{2M + m_\eta}, \hspace{1cm} (19)$$

$$\Sigma_{\rho}^{(1)} = i\frac{m_{\eta'}}{4M^2}$$
$$\left\{(1 + \kappa\Pi \cdot \Pi) \left[2 - (1 - \frac{\kappa}{2})\frac{m_{\eta'}}{2M + m_{\eta'}}\right] + \frac{\kappa m_{\eta'}^2}{4M(2M + m_{\eta'})}\right\}, \hspace{1cm} (20)$$

$$\Sigma_{\rho}^{(2)} = i\kappa(1 + \kappa)\frac{m_{\eta'}}{2M}$$
$$\left\{-\frac{E + M}{M}\Pi \cdot \Pi \left(1 - \frac{m_{\eta'}}{2M + m_{\eta'}}\right) + \frac{m_{\eta'}^2}{4M(2M + m_{\eta'})}\right\}, \hspace{1cm} (21)$$

$$\Sigma_{\omega}^{(1)} = i\frac{m_{\eta'}}{4M^2} \left(2 - \frac{m_{\eta'}}{2M + m_{\eta'}}\right), \hspace{1cm} (22)$$

$$\Sigma_{\omega}^{(2)} = 0.$$

6
All amplitudes are the sum of s and u nucleon pole terms, except \( M_\pi \), Eqn. \[1\] which includes also a \( \delta \) meson t pole term.

The various exchange contributions are shown in Fig. 5 versus the energy available in the CM system. The solid line is the transition amplitude obtained with the relative phases of all different exchange contributions set to be +1. Clearly, strong cancellations amongst these lower the transition amplitude to below \( M_\pi \). Most important are the \( \sigma \) and \( \rho \) exchanges with ratios \( M_\sigma : M_\rho : M_\omega : M_\pi : M_\delta : M_\eta \approx 14 : 10 : 8 : 7 : 2.5 : 1 \). The ratios quoted above differ considerably from the ones reported in Ref. \[12\] for the \( \eta \) production, where the reaction proceeds mainly via nucleon isobar excitations and the relative importance of various exchanges is determined by s+u nucleon isobar pole terms. Also the kinematic \( \Sigma _B \) factors in Eqns. \[12, 13\] vary strongly with the mass of the meson produced, giving rise to quite different ratios for the s+u nucleon pole terms. At threshold the vertices \( V_{\delta \eta \pi } \left( k^2 = m_\eta ^2 \right) \) and \( V_{\delta \eta ' \pi } \left( k^2 = m_\eta ' ^2 \right) \) are small and practically do not affect the calculated cross sections.

Our predictions for the total cross sections are shown in Fig. 6, along with the data from Refs. \[1, 2, 4, 5, 9, 7, 9, 10\]. The curves shown are corrected for final state interactions according to the procedure described in Ref. \[12\]. To be consistent with the OBE picture of the NN interactions \[16\] we have disregarded \( \eta ' \) exchange contributions. Such contributions scale like \( \left( g_{\eta ' NN} / g_{\eta NN} \right) ^2 \) and practically do not influence the cross sections presented here for the \( pp \rightarrow pp\eta (\eta ') \) reactions.

3 Summary

In summary we have applied a covariant OBE model to calculate cross sections for the \( \pi ^0 p \rightarrow p\eta ' (\eta ) \) and \( pp \rightarrow pp\eta ' (\eta ) \) reactions. In marked difference with the \( \eta \), we could reproduce near threshold cross sections for \( \pi ^0 p \rightarrow \eta ' p \) and \( pp \rightarrow pp\eta ' \) without a resonant production via an intermediate baryon resonance. As in previous studies \[12, 13\] the model is based on the OBE picture of the NN force \[16\] and accounts for relativistic effects, energy dependence and nonlocality of the hadronic interactions. The calculations reported here and in Refs. \[12, 13\] for \( pp \rightarrow pp\pi ^0 (\eta ) \) provide a consistent description for pseudoscalar meson production in NN collisions. All calculations are carried out with the same formalism and meson-nucleon couplings as obtained by
Machleidt[10] from fitting NN scattering data. The model success to explain data for these processes depends on how well the half off mass shell amplitude for the conversion processes, $BN \rightarrow PN$ are calculated. As there are no data available which link directly to the off mass shell behavior of these amplitudes, this feature of the model needs still further verifications. Yet it is encouraging that such a simple model reproduce the production rate for as light as a pion and as heavy as the $\eta'$ meson.

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Figure 1: The primary production mechanism for the $NN \rightarrow NNP$ reaction. A boson $B$ created on nucleon 1 (momentum $p_1$), is converted into $\eta'$ (momentum $k$) on nucleon 2 (momentum $p_2$).
Figure 2: Feynman diagrams contributing to the conversion process $BN \rightarrow \eta'N$. For $\eta$ production, because of the strong coupling to the $N^*$ ($1535$ MeV) nucleon isobar we include contributions from both nucleon and nucleon isobar excitation in the intermediate states. Graph (e) describes a $\delta$-meson pole in a $t$ channel.
Figure 3: The $\pi^0 p \rightarrow p\eta$ conversion amplitude: the curves labeled by $r$, $s+u$ and $t$ represent contributions from $s+u$ nucleon isobar pole terms (resonance production), $s+u$ nucleon pole terms and $\delta$-meson pole in a $t$ channel, respectively (see text). The $s+u$ nucleon pole contributions (dash-dotted curve) are to be evaluated using the scale on the right, while all other contributions use the scale on the left.
Figure 4: The $\pi^0 p \rightarrow p \eta'$ conversion amplitude: the curves labeled by s+u and t represent contributions s+u nucleon pole terms and $\delta$-meson pole in a t channel, respectively (see text). The t pole term contribution dominates the process and is not resolved from the total amplitude (solid line). The s+u nucleon pole contributions (dash-dotted curve) are to be evaluated using the scale on the right, while the t pole contribution and the total amplitude and total amplitude use the scale on the left.
Figure 5: Partial exchange amplitude for the $pp \rightarrow pp\eta'$ reaction. The contribution from the $\delta$-meson pole is included in $M_\pi$. 
Figure 6: Energy integrated cross sections for the $pp \rightarrow pp\eta$ reaction versus the energy available in the CM system. All curves include FSI corrections via the approximation of Ref. [12]. The calculated cross section for the $pp \rightarrow pp\eta$ with the $\delta$ pole term included (solid line) is practically identical with prediction reported in Ref. [12].