A symmetric capacity-constrained differentiated oligopoly model for the United States pediatric vaccine market with linear demand

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The United States pediatric vaccine market is examined using Bertrand–Edgeworth–Chamberlin price competition. The proposed game captures interactions between symmetric, capacity-constrained manufacturers in a differentiated, single-product market with linear demand. Results indicate that a unique pure strategy equilibrium exists in the case where the capacities of the manufacturers are at their extreme. For the capacity region where no pure strategy equilibrium exists, there exists a mixed strategy equilibrium where the distribution function, its support, and the expected profit of the manufacturers are characterized. Three game instances are introduced to model the United States pediatric vaccine market. In each instance, the manufacturers are assumed to have equal capacity in producing vaccines. Vaccines are differentiated based upon the number of reported adverse medical events for that vaccine. Using a game-theoretic model, equilibrium prices are computed for each monovalent vaccine. Results indicate that the equilibrium prices for monovalent vaccines are lower than the federal contract prices. The numerical results provide both a lower and upper bound for the vaccine equilibrium prices in the public sector, based on the capacity of the vaccine manufacturers. Results illustrate the importance of several model parameters such as market demand and vaccine adverse events on the equilibrium prices. Supplementary materials are available for this article. Go to the publisher's online edition of IIE Transactions for datasets, additional tables, detailed proofs, etc.

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1. Introduction

Oligopoly theory analyzes the determination of prices in markets with a limited number of competitors. Central issues in oligopoly theory, such as the solution concept, defined as rules that predict how each competitor selects prices or production quantities, and the existence and uniqueness of the equilibria, are raised by Cournot (1838), Bertrand (1883), and Edgeworth (1925). Their analyses are seen as precursors to the development of modern game theory tenets by Von Neumann and Morgenstern (1944) and Nash (1950). Other significant contributions to oligopoly theory include Hotelling (1929), Chamberlin (1993), and Robinson (1933), each of whom emphasize the impact of product differentiation.

The Cournot and Bertrand models differ based on the strategic variable of interest for each manufacturer. Cournot (1838) introduces a competition framework in which manufacturers independently select production quantities, which lead to a price using the supply and demand interactions in the market (Vives, 1999). An equilibrium occurs when each manufacturer selects a quantity level that maximizes their profits, given the quantity level chosen by the other manufacturers (Vives, 1999); this is called a Nash equilibrium (Nash, 1950), where the strategic variables are the production quantities. Bertrand (1883) introduces a competition framework in which manufacturers independently select the price of their product. The market prices lead to a production quantity using the supply and demand interactions; this is called a Nash equilibrium (Nash, 1950), where the strategic variables are the prices of the products (Vives, 1999). The Bertrand framework considers three main assumptions (Tirole, 1988). First, Bertrand assumes that each manufacturer can entirely fill the market demand and, hence, the manufacturers are not capacity-constrained. Second, Bertrand assumes that the
manufacturers’ products are absolutely substitutable and, hence, no product differentiation exists in the model. Third, Bertrand assumes that the manufacturers compete with each other only once and, hence, their competition is static.

Edgeworth (1925) introduces the Bertrand–Edgeworth competition model, which explores price competition in a duopoly with capacity constraints. In this model, manufacturers compete on price though no manufacturer is obligated to supply all of the demand at the predetermined price (i.e., the first assumption of the Bertrand framework is relaxed; Vives (1999)). In Bertrand–Edgeworth competition, the existence of equilibria can be guaranteed exclusively in mixed strategies (Vives, 1999). Hotelling (1929), Chamberlin (1933), and Robinson (1933) study product differentiation, where manufacturers compete on price. Whereas Hotelling explores heterogeneity in terms of location, Chamberlin and Robinson independently develop the monopolistic competition model (Vives, 1999). In Bertrand–Edgeworth–Chamberlin competition, capacity-constrained manufacturers compete on price over the sale of differentiated products (i.e., the first and second assumptions of the Bertrand framework are relaxed).

The analysis in this article focuses on analyzing the United States pediatric vaccine market using Bertrand–Edgeworth–Chamberlin competition, in which the price competition among symmetric capacity-constrained manufacturers is studied. Here, the term symmetric capacity-constrained manufacturers refers to manufacturers with equal production capacity. The assumption of equal production capacity among manufacturers facilitates the tractability of the equilibria. Nash equilibrium is the solution concept applied to study the formulated game (Nash, 1950), which is the key solution concept in oligopoly pricing models (Vives, 1999). Both pure and mixed strategy equilibrium are considered in analyzing the pediatric vaccine market.

A review of the existing game-theory literature motivates the need for a model that captures oligopoly price competition among capacity-constrained manufacturers producing differentiated products. At first, studies analyzing the price competition among capacity-constrained manufacturers only examine homogeneous products (i.e., Bertrand–Edgeworth competition). Bertrand–Edgeworth competition is built upon the Bertrand price competition and the Edgeworth model (Edgeworth, 1925), which acknowledges the effect of production capacity constraints. Levitan and Shubik (1972) examine a duopolistic homogeneous product market in which two manufacturers compete, with price as the strategic variable, and are limited by equal capacity constraints. Kreps and Scheinkman (1983) characterize the equilibria in a duopoly capacity and price game where the capacity of the two manufacturers are not necessarily equal. Kreps and Scheinkman (1983) assume that homogeneous goods are produced at a constant and identical unit variable cost up to some fixed capacity. They also assume the demand to be non-increasing and concave. Osborne and Pitchnik (1986) consider the same assumptions of the Kreps–Scheinkman model and consider non-concave demand. Deneckere and Kovenock (1996) consider the same assumptions of the Kreps–Scheinkman model and consider differences in unit cost among the manufacturers. Analyzing equilibria under oligopoly, Vives (1986) considers the case where manufacturers are limited by equal capacities, with concave demand and constant and identical unit costs. Francesco and Salvadori (2009) and Hirata (2009) analyze a triopoly with concave demand, where the former authors discuss a complete representation of mixed strategy equilibria. No results have been reported in the literature that characterize the mixed strategy equilibria in an oligopolistic Bertrand–Edgeworth competition with asymmetric capacity-constrained manufacturers (i.e., manufacturers with unequal capacities).

In addition to the issue of capacity constraint, price competition among product-differentiated manufacturers has recently gained interest among game theorists. Chamberlin (1993) and Robinson (1933) independently discuss imperfect competition with product differentiation initiated from a demand system. The current article follows the Chamberlin–Robinson path and considers the product differentiation introduced in the demand system. The type of demand system is therefore essential. Benassy (1989) studies the Bertrand–Edgeworth–Chamberlin model with a general demand system, in which it is proven that a pure strategy equilibrium does not exist if the degree of product differentiation is sufficiently large. Canoy (1996) studies the same model as a parameterized duopoly in which a pure strategy equilibrium does not exist if the products are sufficiently similar. However, Benassy (1989) and Canoy (1996) do not analyze the characterization of a mixed strategy equilibrium. Sinitsyn (2007) takes a step forward and analyzes the equilibria (in both pure and mixed strategy) in a duopolistic Bertrand–Edgeworth–Chamberlin competition with logit demand for both symmetric and asymmetric manufacturers. However, logit demand is not applicable to the United States pediatric vaccine market as it incorporates the independence of irrelevant alternatives property, which is often violated in vaccine markets (Davis and Wilson, 2005). Furthermore, whereas Sinitsyn analyzes a duopoly setting, the current article generalizes the problem to an oligopoly setting.

No results have been reported in the literature to characterize and study the existence of equilibria in pure and mixed strategies for the Bertrand–Edgeworth–Chamberlin competition in an oligopoly with quadratic utility and linear demand. This article explores equilibria in the model of a price-setting oligopoly with manufacturers having finite, equal capacities and producing differentiated products. The manufacturers are assumed to follow a linear demand. This article proves that a unique pure strategy equilibrium exists if the capacities of the manufacturers are at their extreme. For the other capacity values, the mixed strategy equilibrium is completely characterized.
In the United States pediatric vaccine market, a small number of manufacturing companies manufacture vaccines. Public sector vaccine prices result from an annual negotiation process between the Centers for Disease Control and Prevention (CDC) and manufacturer representatives; prices are then fixed for a period of 1 year. The equilibrium vaccine prices are of interest. Therefore, Bertrand competition is an appropriate framework for modeling the United States pediatric vaccine market (Robbins et al., 2014). Moreover, vaccine manufacturers may have limited capacity for producing each vaccine. The analysis presented in this article treats the pricing strategies of the United States public sector pediatric vaccine market using a Bertrand–Edgeworth–Chamberlin competition framework. Three vaccine manufacturers compete with each other over the sale of monovalent vaccines. This article focuses on diseases for which there are competing vaccines produced by different vaccine manufacturers. The competing vaccines are slightly differentiated in terms of their medically adverse events; the number of reported medically adverse events are used to differentiate the competing vaccines.

Pediatric vaccine market pricing has not been broadly studied in the literature. Robbins et al. (2014) analyze the United States pediatric vaccine market by applying a Bertrand oligopoly pricing model, which determines oligopolistic interactions between manufacturers in a homogeneous multiple product market. This article differs from that paper by analyzing the United States pediatric vaccine manufacturing market using Bertrand–Edgeworth–Chamberlin competition, which allows for both capacity constraints and product differentiation in the price game. Using this model, the equilibrium prices of each vaccine, in both pure and mixed strategies, are computed. The results indicate that the equilibrium prices for monovalent vaccines are generally lower than the federal contract prices. The numerical results provide a lower bound and an upper bound for the vaccines’ equilibrium prices in the public sector. The results illustrate the importance of several model parameters such as the degree of product differentiation, the number of manufacturers, and the market demand on the equilibrium prices. The analysis highlights the importance of degree of product differentiation on equilibrium price. If the total capacity of the vaccine manufacturers is limited, equilibrium prices increase as the degree of product differentiation increases. On the other hand, if the vaccine manufacturers are able to meet the entire market demand, equilibrium prices increase up to a point and then decrease as the degree of product differentiation increases. The results presented in this study should appeal to the pediatric healthcare community, including federal government officials (who negotiate the vaccine prices with vaccine manufacturers) and vaccine manufacturers (who are seeking effective pricing strategies).

The article is organized as follows: Section 2 outlines the model formulation and states the necessary model assumptions. Section 3 presents a description of the proposed model and discusses the Nash equilibrium existence results. Section 4 describes the United States pediatric vaccine market, indicates the application of the game proposed in Section 3 to this market, reports the results, and provides a sensitivity analysis on the equilibrium prices. Section 5 provides a discussion on the results, presents the limitations of the study, and provides concluding comments and directions for future research.

2. Game formulation

This section describes the oligopoly pricing model for characterizing oligopolistic interaction between capacity-constrained symmetric manufacturers in a differentiated multiple products market. Consider $n$ capacity-constrained manufacturers facing a competitive situation, producing differentiated products. Let $q_i = D_i(p)$ denote the demand for product $i$, where $p = (p_1, p_2, \ldots, p_n)$ is the price (of the products) vector ($p \in \mathbb{R}^n$). The following assumption (from Vives (1999)) concerning demand is maintained.

Assumption 1. For any product $i$, $D_i(\cdot)$ is smooth (i.e., $D_i(\cdot)$ has continuous derivatives) whenever positive, where the Jacobian of $D(\cdot)$ is negative definite. The Jacobian of the demand system is needed in order to obtain the inverse demand functions.

Let $U(q)$ denote the utility function on $q$, where $q = (q_1, q_2, \ldots, q_n)$ is the quantity (of the products) vector ($q \in \mathbb{R}^n$). The following optimization problem of a representative consumer captures the demand system

$$\max_{q} \{U(q) - pq\}.$$ 

Suppose that $U$ is quadratic and strictly concave, given by

$$U(q) = \alpha \sum_{i=1}^{n} q_i - 1/2 \left( \beta \sum_{i=1}^{n} q_i^2 + 2\gamma \sum_{i,j=1, j \neq i}^{n} q_i q_j \right),$$

where $\beta > \gamma > 0$ and $\alpha > 0$. The quadratic utility function is selected, as it yields linear inverse demand functions for differentiated products (by taking the derivative of $U(q)$ with respect to $q_i, i = 1, 2, \ldots, n$) given by

$$\frac{\partial U(q)}{\partial q_i} = \alpha - \beta q_i - \gamma \sum_{j \neq i} q_j = P_i(q).$$

for the quantity values for which the prices are positive. Assuming $\beta = 1$ for simplicity, then the inverse and direct demands, respectively are given by

$$P_i(q) = \alpha - q_i - \gamma \sum_{j \neq i} q_j,$$

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and

\[ D_i(p) = a - bp_i + c \sum_{j \neq i} p_j, \quad (2) \]

where \( a = \alpha/(1 + (n - 1)\gamma), \) \( b = (1 + (n - 2)\gamma)/(1 + (n - 1)\gamma), \) and \( c = \gamma/(1 + (n - 1)\gamma)(1 - \gamma), \) provided that \( a - bp_i + c \sum_{j \neq i} p_j > 0. \) Note that \( a, b, \) and \( c \) are functions of \( \gamma. \) The parameter \( \gamma \) captures the degree of product differentiation, ranging from zero (for independent products) to one (for perfect substitutes). According to Chamberlin (1993), the nature of the product differentiation may be either based on the attributes of the products such as trademarks and packaging or circumstances related to their sale. In studying the United States pediatric vaccine market, vaccines are differentiated based on the total medical adverse events reported for each vaccine, which is a characteristic of the products themselves. The manufacturers are capacity-constrained with equal capacity \( k. \)

Comments regarding the assumption of linear demand are warranted. Ideally, stipulation of a demand system should be based upon consumer theory and supported by empirical indication (Farahat and Perakis, 2010). Manufacturers often learn about the market demand function through price experiments (Silvestre, 1977; Bonanno and Zeeman, 1985). Such price experiments are performed in a narrow range to avoid losing existing customers. As Silvestre (1977) and Bonanno and Zeeman (1985) claim, manufacturers perform price experiments to obtain local, linear approximations of demand. In the United States pediatric vaccine market, each year the change in the price of a vaccine produced by a manufacturer is small (Centers for Disease Control and Prevention, 2014a, 2014b). Therefore, a linear demand function is a reasonable approximation in the narrow range of prices considered here. Furthermore, given the shortcomings of other demand specifications, such as logit demand, Farahat and Perakis (2010) suggest the use of a linear demand function when examining markets with differentiated products. The assumption of a linear demand function enables the complete characterization of pure and mixed strategy equilibria in the oligopolistic Bertrand–Edgeworth–Chamberlin competition.

To complete the description of the game, denote the profit of each manufacturer \( i \in \{1, 2, \ldots, n\} \) as

\[ \pi_i = P_i(q_i)q_i = p_iD_i(p), \quad (3) \]

which holds assuming that the production costs for all of the manufacturers are constant and equal to zero. This assumption is reasonable in the United States pediatric vaccine market, since vaccine research and development costs are large and actual production costs per vaccine dose are relatively small (Robbins et al., 2014). This analysis ignores the sunk costs of research and development and only considers the actual vaccine production cost, which is small and therefore assumed to be zero. The game \( \Gamma \) is given by

\[ \Gamma = (n, \gamma, (D_i(p))_{i \in \{1, \ldots, n\}}, (\pi_i)_{i \in \{1, \ldots, n\}}). \quad (4) \]

3. Equilibria analysis

This section discusses the game-theoretic solution to \( \Gamma. \) In the study of games, the level of cooperation between the manufacturers impacts the solution concepts (Myerson, 1999). In a market where having legal contracts is not applicable, the manufacturers make decisions that influence each other’s profits (Vives, 1999). In these markets, noncooperative game theory is a suitable approach for analyzing the game (Robbins et al., 2014). In this study, it is assumed that no cooperation is permitted between the manufacturers.

A Nash equilibrium is a set of prices for which no manufacturer has an incentive to unilaterally deviate from its own price to receive a benefit from deviating. A pure strategy Nash equilibrium describes exactly how a manufacturer will act in a game. A mixed strategy Nash equilibrium assigns a probability to each pure strategy. An important area of research is determining the existence of and the computation of Nash equilibria (Robbins et al., 2014). Having a pure strategy equilibrium in a game is desirable since it describes the way a manufacturer will act in a game. However, a pure strategy equilibrium does not always exist. According to Nash’s theorem (Nash, 1950), every finite game has a mixed strategy equilibrium. Therefore, when a pure strategy equilibrium does not exist, a mixed strategy equilibrium is sought. Section 3.1 introduces Bertrand–Edgeworth–Chamberlin competition and seeks the associated pure and mixed strategy equilibria. The Nash equilibria in the quantity (Cournot) and price (Bertrand) competition are defined first.

Assuming that the marginal production costs for all of the manufacturers are constant and equal to zero, Cournot and Bertrand equilibria are unique. Cournot–Chamberlin and Bertrand–Chamberlin equilibria are given as follows (Vives, 1999):

\[ q^C = \frac{a}{2 + (n - 1)\gamma}, \quad i = 1, 2, \ldots, n. \quad (5) \]

The term \( q^C \) is called the Cournot–Chamberlin equilibrium.

\[ p^B = \frac{a}{2b - (n - 1)c}, \quad i = 1, 2, \ldots, n. \quad (6) \]

The term \( p^B \) is called the Bertrand–Chamberlin equilibrium.

3.1. Bertrand–Edgeworth–Chamberlin competition

Bertrand–Edgeworth competition illustrates a particular type of competition between manufacturers, where each
manufacturer has a fixed capacity constraint that limits its amount of production. In Bertrand–Edgeworth competition, the existence of equilibria is guaranteed only in mixed strategies (Vives, 1999). Chamberlin (1993) emphasizes the impact of product differentiation in the study of oligopoly theory by introducing the monopolistic competition model. In such a model, each manufacturer faces a downward-sloping demand. As in Dixit (1979), Singh and Vives (1984), and Ross (1992), the Chamberlinian product differentiation is employed by considering the linear demand system (2). Theorems 1 and 2 completely characterize both pure and mixed strategy equilibria in Bertrand–Edgeworth–Chamberlin competition with symmetric capacity-constrained manufacturers in an oligopoly market with linear demand, which is original in the literature. The regions where pure strategy equilibria exist are characterized in Theorem 1. The proofs of Theorems 1 and 2 are given in the online supplement.

**Theorem 1.** In an oligopoly with linear demand and n manufacturers, demand given by (2), and manufacturers facing equal capacity constraints \( k_i = k \), for \( i = 1, 2, \ldots, n \), a unique pure strategy equilibrium exists if either:

(a) \( k \geq k(\gamma) \), in which case a Bertrand–Chamberlin equilibrium exists, where

\[
k(\gamma) = \alpha \left[ \frac{1}{\gamma(n-1)} - \frac{1}{\gamma(n-1)} - \frac{2(1 + (n-2)\gamma)(1 - \gamma)^{1/2}}{(1 + (n-1)\gamma)^{1/2}(2 + 2(n-2)\gamma - (n-1)\gamma)} \right],
\]

or

(b) \( k \leq q^C \), in which case the equilibrium, called the competitive equilibrium, is given by \( P(k) \).

Dasgupta and Maskin (1986) show that in the region \( q^C < k < k(\gamma) \) there exists a mixed strategy equilibrium with a continuous distribution, with support on a price interval \([p, \bar{p}]\) (i.e., a set of points that are the members of the distribution). The mixed strategy equilibrium is characterized by Theorem 2.

**Theorem 2.** In an oligopoly with linear demand and n manufacturers, demand given by (2), and manufacturers facing equal capacity constraints \( k_i = k \), for \( i = 1, 2, \ldots, n \), if \( q^C < k < k(\gamma) \), there exists a mixed strategy equilibrium where each manufacturer sets prices according to a continuous distribution function \( \phi \) with support \([p, \bar{p}]\) and expected profit \( \bar{\pi} \). The distribution function, its support, and the expected profit are given by

\[
\bar{\pi} = \arg \max_p \left\{ p(\alpha - \gamma(n-1)k - p) \right\},
\]

\[
\bar{\pi} = \bar{p}(\alpha - \gamma(n-1)k - \bar{p}),
\]

\[
p = \bar{\pi}/k,
\]

\[
\phi(p) = \left( \frac{k - \bar{\pi}/p}{k(\gamma(n-1)+1) - \alpha + p} \right)^{(1/(n-1))}.
\]

Note that \( \phi(p) \) is the probability that a manufacturer sets its price for a product to be less than \( p \); i.e., the cumulative distribution function (CDF).

4. The United States pediatric vaccine market

This section provides a description of the United States public sector pediatric vaccine market and discusses the equilibria in \( \Gamma \) by applying it to this market based on federal contract prices of 2010. This explores the result of a Bertrand–Edgeworth–Chamberlin competition on the prices of the competing vaccines. The 2010 vaccine prices are used in this study since the most recent vaccine demand data available is for this year.

4.1. Market description

There are a limited number of vaccine manufacturers in the United States that contribute to the manufacture and distribution of pediatric vaccines (Douglas et al., 2008). Pediatric vaccines in the United States are manufactured privately and there exists a competition between vaccine manufacturers in gaining higher profits. Three manufacturing companies (GlaxoSmithKline plc, Sanofi Pasteur, and Merck & Co., Inc.) manufacture all of the monovalent competitive vaccines (defined as the vaccines with one similar antigen, which are produced by more than one manufacturer and can be administered in the same time period; Robbins et al., 2010), which are the vaccines of interest in this paper. The focus of this article is on three competitive antigens that protect against the following diseases: diphtheria, tetanus, and pertussis (DTaP), Haemophilus influenzae type b (Hib), and hepatitis B (HepB). Note that IPV monovalent vaccine is not included in this study as it is manufactured by only one vaccine manufacturer and hence does not engage in any competition.

Other than vaccine manufacturers, there are several other stakeholders in the United States pediatric vaccine market. The CDC is the main public health organization responsible for immunization in the United States. Once the Food and Drug Administration (FDA) approves vaccines for sale in the United States, the Advisory Committee on Immunization Practices (ACIP), an advisory body to the CDC, may recommend including the vaccines in the Recommended Childhood Immunization Schedule (RCIS; see Centers for Disease Control and Prevention (2012)). The
Table 1. Competitive vaccines used in the game theoretic model (2010)

| (I) Vaccine       | (II) Manufacturer   | (III) 2010 federal contract prices (public sector) ($) | (IV) 2010 private sector prices ($) |
|-------------------|---------------------|---------------------------------------------------------|-------------------------------------|
| DTaP              | GlaxoSmithKline     | 14.25                                                   | 21.44                               |
| DTaP              | Sanofi Pasteur      | 13.25                                                   | 23.055                              |
| HepB              | GlaxoSmithKline     | 10.25                                                   | 21.37                               |
| HepB              | Merck               | 10.25                                                   | 23.20                               |
| Hib               | Merck               | 11.511                                                  | 22.77                               |
| Hib               | Sanofi Pasteur      | 8.83                                                    | 23.606                              |
| Hib               | GlaxoSmithKline     | 8.66                                                    | 22.83                               |

RCIS is a sequence and timing of required pediatric vaccines to protect children from several diseases (Robbins et al., 2010). The prices of the vaccines that are sold in the public sector are set as a result of negotiations between the CDC and the vaccine manufacturers. The state and local government officials (e.g., local public health departments) then purchase the vaccines for the immunization needs of the children living in their administrative areas of responsibility. Pediatric vaccines purchased at the public sector price represent approximately 57% of the total pediatric purchases by volume in the United States (Orenstein et al., 2005). Since more than half of all pediatric vaccines produced are purchased by state and local government officials through CDC-negotiated contracts, the CDC has negotiating power with manufacturers (Coleman et al., 2005).

In the game-theoretic analysis of the United States public sector pediatric vaccine market introduced in Section 4.2, the CDC is not modeled as a stakeholder; the model includes the vaccine manufacturers, as the underlying game is a price competition between vaccine manufacturers. The results of the model, which are the equilibrium prices for monovalent pediatric vaccines, may provide insights for the CDC while negotiating vaccine prices with vaccine manufacturers.

Table 1 serves as a summary of the information regarding the 2010 United States monovalent pediatric vaccines, which are used in this analysis. Column I indicates the set of pediatric vaccines analyzed in this study, along with their registered trademark names. Column II (from CDC (2011)) indicates the manufacturer of each vaccine. Column III indicates the public sector vaccine prices (from CDC (2011)). These prices result from negotiations between the CDC and vaccine manufacturers. Column IV indicates the private sector vaccine prices, which is used to further provide additional explanation on vaccine pricing in the United States.

4.2. Game-theoretic analysis of 2010 public sector pediatric vaccine market

Three instances of $\Gamma$ are formulated for the competitive monovalent vaccines DTaP, HepB, Hib: $\Gamma_{DTaP}$ for the DTaP monovalent vaccines, $\Gamma_{HepB}$ for the HepB monovalent vaccines, and $\Gamma_{Hib}$ for the Hib monovalent vaccines. Each $\Gamma$ instance gives the equilibrium prices for the monovalent vaccines DTaP, HepB, and Hib, respectively.

Three different statistics are used to determine the demand function for vaccines: the number of children completing the RCIS annually, the number of children fully immunized with the vaccines purchased at the public sector, and the vaccination coverage rate. According to a National Vital Statistics Report, in 2010, the number of births in the United States was approximately 4000000 (Martin et al., 2012). The number of children less than age 5 years who immigrated to the United States in 2010 is negligible compared with the birth cohort and hence is not included in this study (U.S. Census Bureau, 2010). Using this value as an upper bound for demand, 57% of these children received vaccines, purchased at the public sector prices (Orenstein et al., 2005). The 2010 National Immunization Survey (NIS) provides vaccine coverage rates for children up to 36 months of age (CDC, 2010). To find the market demand for each vaccine, the expected value of the number of doses given to each child is found using the 2010 NIS data. This number is multiplied by the birth cohort of 2010 to find the total number of doses given to all the children.

To capture the demand in the public sector, the number of doses given to the birth cohort is multiplied by 0.57. Table 2 indicates the market demand (up to two significant digits) for the vaccines used in the games. The demand function used is Equation (2), where the demand provided by the public sector is given by $D_i(p)$, for $i = 1, 2, \ldots, n$. The vaccine prices are reported in Table 1. To find $b$ and $c$ in (2), the degree of product differentiation $\gamma$ is required. In this study, vaccines are differentiated based on the total medically adverse events reported for each vaccine. The demand

Table 2. Demand provided by the public sector

| Vaccine       | Demand (public sector) x 10^6 |
|---------------|-------------------------------|
| DTaP monovalents | 3.9                            |
| HepB monovalents | 3.8                            |
| Hib monovalents   | 5.6                            |
Table 3. Number of medical adverse events and vaccine reporting rates

| Vaccine | Manufacturer         | Total number of adverse events (2010) | Number of doses distributed (×10⁶) (2010) | Reporting rate per 100,000 vaccine doses |
|---------|----------------------|--------------------------------------|------------------------------------------|-----------------------------------------|
| DTaP    | Sanofi Pasteur       | 181                                  | 3.4                                      | 5.32                                    |
| DTaP    | GlaxoSmithKline      | 353                                  | 3.4                                      | 10.38                                   |
| HepB    | GlaxoSmithKline      | 264                                  | 3.3                                      | 8.00                                    |
| HepB    | Merck                | 220                                  | 3.3                                      | 6.67                                    |
| Hib     | Merck                | 154                                  | 1.3                                      | 12.03                                   |
| Hib     | Sanofi Pasteur       | 648                                  | 8.3                                      | 7.77                                    |
| Hib     | GlaxoSmithKline      | 92                                   | 0.260                                    | 35.38                                   |

intercept $a$ in (2) is then computed for the vaccines in each $\Gamma$ instance.

To capture the degree of product differentiation $\gamma$, the (national) Vaccine Adverse Event Reporting System (VAERS) database is reviewed. VAERS is a passive surveillance system originated in 1990, co-managed by the CDC and the FDA, to which adverse events after administration of any vaccine are reported by the patients, healthcare providers, and vaccine manufacturers (Niu et al., 1998; CDC, 2003). The adverse events reported to VAERS may or may not be causally related to the vaccine, and therefore a segment of it may be coincidental (Niu et al., 1996). Despite the limitations associated with VAERS, such as reporting biases and statistical limitations (i.e., VAERS fails to acquire data on the number of vaccine doses administered), it is the most aggregate database available regarding vaccine adverse events and hence is used in this study. The events reported to VAERS are organized by severity, from death to nonserious events. Note that the competing vaccines may be differentiated based on factors other than the number of medical adverse events. Factors such as brand loyalty, formulary inertia, and special medical advantages of a vaccine further differentiate the competing vaccines. However, due to lack of data in quantifying such factors, this study considers medical adverse events as the sole factor, which differentiates the competing vaccines.

Table 3 presents the total number of adverse events for monovalent vaccines reported in 2010 according to VAERS. To compute the vaccine-specific reporting rates for each vaccine, the number of vaccine doses administered in the United States in 2010 is computed for each vaccine using the NIS data (CDC, 2010) (see Table 3). The number of vaccine doses administered is not available through NIS for DTaP and HepB vaccines by type. In these cases, it is assumed that each manufacturer distributes the same number of vaccine doses. This assumption is relaxed in Section 4.4.

The vaccine-specific reporting rates for each vaccine type is computed for the number of adverse events reported per 100,000 vaccine doses distributed (Niu et al., 1998; CDC, 2003) (see Table 3). The degree of product differentiation $\gamma$ is defined as the Relative Reporting Rate (see Niu et al. (1998) for details) among two or more vaccines, which is computed by dividing the smallest vaccine reporting rate by the sum of the reporting rates for all the vaccines.

Table 5 indicates that for $\Gamma_{DTaP}$, $\Gamma_{HepB}$, and $\Gamma_{Hib}$, a pure strategy equilibrium exists for $k = 1.1a$, which suggests that the equilibrium price for the DTaP, HepB, and Hib monovalent vaccines in 2010 was $10.39, $9.78, and $10.39. This results in the total production capacity of the vaccines in one game instance to be approximately 10% higher than the total market demand for those vaccines. Table 4 provides the relevant information on the model parameters for the three $\Gamma$ instances. A base value, which is used to calculate the equilibrium price, is reported for each parameter. The low and high values reported for the parameters are used to perform a sensitivity analysis (see Section 4.4).

4.3. Results

According to Theorem 1, if $k \geq k(\gamma)$ the Bertrand–Chamberlin equilibrium exists and if $k \leq q^C$ the competitive equilibrium exists. If $q^C < k < k(\gamma)$, then by Theorem 2, there exists a mixed strategy equilibrium where each manufacturer randomizes the price according to the continuous distribution function $\phi$ with support $[p, \bar{p}]$. Table 5 presents the equilibrium prices, along with the profits generated for each manufacturer for the three $\Gamma$ instances indicated in Table 4, when the production capacity is 10% higher than the market demand. Based on the value of the capacity and Theorem 1, in all three $\Gamma$ instances, firms are not capacity constrained and the Bertrand–Chamberlin equilibrium exists.
Table 4. Model parameters for game instances

| Description                        | Parameter (Base value, low value, high value) | Source       |
|------------------------------------|-----------------------------------------------|--------------|
| Number of manufacturers            | \( n \)                                        | (2, 2, 3) CDC (2011) |
| Demand function intercept          | \( a \)                                        | (1.95 \times 10^6, 1 \times 10^6, 3 \times 10^6) Calculated |
| Degree of product differentiation  | \( \gamma \)                                    | (0.34, 0.1, 0.9) Calculated |
| Production capacity of each manufacturer | \( k \)                                      | (1.1a, 0.1a, 1.3a)* Jacobson et al. (2006) |

Table 5. Equilibrium prices for the public sector

| \( k \) | Equilibrium price ($) | Profits ($ \times 10^6) |
|---------|-----------------------|-------------------------|
| 1.1a    | 10.39                 | 12.20                   |
| 1.1a    | 9.78                  | 12.25                   |
| 1.1a    | 10.46                 | 11.32                   |

*Note: The range for the capacity may vary based on the existence of a specific type of equilibrium (see Fig. 1).
Table 6. Equilibrium prices for the whole market

| k      | $\Gamma_{DTaP}$ | $\Gamma_{HepB}$ | $\Gamma_{Hib}$ |
|--------|------------------|-----------------|----------------|
| 1.1a   | 18.11            | 16.98           | 18.16          |

States pediatric vaccine market, as only 57% of the birth cohort who receive their vaccines in the public sector are considered in the analysis. However, vaccine manufacturers are producing the same vaccines for both the public and private sectors of the vaccine market. Therefore, a second case can be constructed in which the equilibrium price of each vaccine is computed for the whole market demand using the same methodology established for the original analysis (see Table 6). In this case it is assumed that the vaccine manufacturers are capable of meeting the whole market demand. The difference between the prices reported in Table 6 and the prices reported in Table 5 are $7.7, $7.2, and $7.7, for DTaP, HepB, and Hib monovalent vaccines, respectively. These differences represent the price reductions resulted from the negotiations between the CDC and vaccine manufacturers. The prices in Table 6 provides a lower bound for the private sector prices. Compared with Table 1, one can see that the 2010 private sector prices, as expected, are all larger than the prices reported in Table 6.

A third case can be constructed in which the whole market demand is considered but the capacity of the vaccine manufacturers is equal to 57% of the market (only the public sector). This case results in the equilibrium prices reported in Table 7. These prices are the maximum prices that the CDC may negotiate for the vaccines distributed in the public sector provided that the vaccine manufacturers can only meet the demand of the public sector. Therefore, this analysis provides a lower bound (see Table 5) and an upper bound (see Table 7) for the vaccine prices in the public sector of the pediatric vaccine market.

Table 7. Maximum equilibrium prices negotiated by the CDC (with $k = 0.57a$)

|                  | $\Gamma_{DTaP}$ Competitive equilibrium | $\Gamma_{HepB}$ Competitive equilibrium | $\Gamma_{Hib}$ Mixed strategy equilibrium |
|------------------|----------------------------------------|----------------------------------------|------------------------------------------|
| $\Gamma_{DTaP}$  | 19.59                                  | 20.57                                  | [18.16, 18.50]                           |

Figures 2(a) and 2(b) indicate how the region of existence of each type of equilibrium changes with the degree of product differentiation and market demand, respectively. Increasing the degree of product differentiation (i.e., vaccines become more similar) and the market demand result in the expansion of competitive equilibrium and mixed strategy equilibrium regions and the contraction of the Bertrand–Chamberlin equilibrium region. This result (regarding the degree of product differentiation) is consistent with Shapley and Shubik (1969)). This means that by increasing the degree of product differentiation (Fig. 2(a)) and the market demand (Fig. 2(b)) the lower bound for the equilibrium vaccine prices exists for a smaller region of the capacities.

The effect of other model parameters (e.g., number of manufacturers, market demand, and degree of product differentiation) on the equilibrium prices can also be discussed. Since based on the value of the production capacity either a pure strategy equilibrium (Bertrand–Chamberlin equilibrium or competitive equilibrium) or a mixed strategy equilibrium exists, the effect of each model parameter is studied on $p^B$ (Bertrand–Chamberlin equilibrium), $P(k)$ (competitive equilibrium), $\nu$ and $\bar{\nu}$ (infimum and supremum of the distribution function support for mixed-strategy equilibrium, respectively) separately. These analyses are for DTaP monovalent vaccines only. However, similar results hold for the HepB and Hib monovalent vaccines.
From Fig. 3(a), if the number of manufacturers increases, the equilibrium price (regardless of type) decreases. Figure 3(b) indicates that by increasing market demand, the equilibrium price also increases. However, the rate of increase in the price is larger when the manufacturers are significantly capacity constrained. From Fig. 3(c), if the total capacity of the manufacturers is less than the market demand, the equilibrium prices (either competitive equilibrium or supremum and infimum of the distribution support) increase as the degree of product differentiation increases. On the other hand, if the total capacity of the manufacturers is more than the market demand (Bertrand–Chamberlin equilibrium exists), equilibrium prices increase up to a point and then decrease as the degree of product differentiation increases. The maximum equilibrium price occurs when $\gamma = 0.3$. In this case, the equilibrium price tends to decrease as vaccines become more similar, since customers become more neutral about the choice of vaccine to purchase. Figure 3(c) further indicates that when the vaccines are indistinguishable in terms of the number of adverse events ($\gamma = 1$), the value of the production capacity significantly influences the equilibrium price. However, when the vaccines are highly differentiated in terms of the number of adverse events ($\gamma = 0.1$), the equilibrium prices are nearly identical, regardless of the equilibrium type. Therefore, the type of equilibrium becomes more significant the more alike the vaccines are. This observation is consistent with Singh and Vives (1984).

4.4. Sensitivity analysis

A one-way sensitivity analysis is performed to determine the effect of model parameters on the equilibrium price. The sensitivity analysis is for DTaP monovalent vaccines only. However, similar results hold for the HepB and Hib monovalent vaccines, unless otherwise stated. Each type of equilibrium price is studied separately. Each model parameter is allowed to take the low and high values indicated in Table 4. To guarantee the existence of each specific type of equilibrium, the base case of the production capacity changes in each case. All four model parameters influence the competitive equilibrium $P(k)$, where market demand has the greatest impact (Fig. 4(a)) (for $\Gamma_{Hib}$ (with three manufacturers), the degree of product differentiation has the greatest impact on the competitive equilibrium. Over the range of values estimated, the competitive equilibrium price reaches a minimum of $\$6.43$ and a maximum of $\$19.30$, compared with the base value of $\$12.54$ (with $k = 0.52a$). Figures 4(b) and 4(c) indicate the effect of the four model parameters on the infimum and supremum of the distribution support. The impact of the production capacity on $p$ is larger than the impact of the number of manufacturers. On the other hand, the impact of the number of manufacturers on $\bar{p}$ is larger than the production capacity. Market demand has the greatest impact on the mixed strategy equilibrium price. Over the range of values estimated, the infimum (supremum) of the distribution

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**Fig. 3.** Equilibrium prices as a function of (a) number of manufacturers; (b) demand function intercept; and (c) degree of product differentiation.
Fig. 4. Sensitivity of model parameters for DTaP monovalent vaccines to (a) competitive equilibrium; (b) infimum of the distribution support (mixed strategy equilibrium); (c) supremum of the distribution support (mixed strategy equilibrium); and (d) Bertrand–Chamberlin equilibrium.

support reaches a minimum of $5.63 ($5.71) and a maximum of $16.89 ($17.14), compared with the base value of $10.98 ($11.14) (with $k = 0.58a$). As the Bertrand–Chamberlin equilibrium does not change with production capacity, Fig. 4(d) shows the effect of market demand, degree of product differentiation, and number of manufacturers on $p^B$. Market demand has the greatest impact on the Bertrand–Chamberlin equilibrium price. Over the range of values estimated, the Bertrand–Chamberlin equilibrium price reaches a minimum of $5.33 and a maximum of $15.98, compared with the base value of $10.39 (with $k = 1.1a$).

In an oligopolistic Bertrand–Edgeworth–Chamberlin competition, the assumption of equal production capacity among manufacturers facilitates the characterization of the equilibria. The complete characterization of mixed strategy equilibrium under oligopoly, even with no product differentiation, has not been reported in the literature for asymmetric manufacturers (Francesco and Salvadori, 2009). Adding product differentiation into the model increases the complexity of the game. In analyzing the pricing strategies in the United States pediatric vaccine market, one can ask what if the production capacity of the vaccine manufacturers are not equal? In a follow-up study, Behzad and Jacobson (2014) answer this question by completely characterizing the pure and mixed strategy equilibria in the Bertrand–Edgeworth–Chamberlin competition for asymmetric capacity-constrained manufacturers producing differentiated products in a duopoly market with linear demand. The results show that the equilibria are similar to the case of symmetric manufacturers, in the sense that the pure strategy equilibrium exists if the production capacity of all manufacturers are at their extremes. Furthermore, for the capacity regions where no pure strategy equilibrium exists, Behzad and Jacobson (2014) provide the distribution functions of the mixed strategy equilibrium for both manufacturers, which is unique. In addition, Behzad and Jacobson (2014) apply the proposed game to the United States pediatric vaccine market. Similar to the case of the symmetric manufacturers, the numerical results indicate that the equilibrium prices of the vaccines are lower than the federal contract prices of those vaccines.

In Section 4.2, while calculating the degree of product differentiation, since the number of vaccine doses administered is not available through NIS for DTaP and HepB vaccines by type, it is assumed that each manufacturer distributes the same number of vaccine doses. Now assume that each vaccine manufacturer distributes a different number of vaccine doses for DTaP and HepB. Tables 8 and 9 indicate the degree of product differentiation and the vaccine equilibrium price for several distribution percentages of DTaP and HepB. Tables 8 and 9 indicate the degree of product differentiation and the vaccine equilibrium price for several distribution percentages of DTaP and HepB. Tables 8 and 9 indicate the degree of product differentiation and the vaccine equilibrium price for several distribution percentages of DTaP and HepB. The equilibrium prices in Tables 8 and 9 indicate the same trend as Fig. 3(c).
Table 8. Sensitivity analysis on the number of DTaP doses distributed by each manufacturer (with $k = 1.1a$)

| % of DTaP doses distributed by Sanofi Pasteur | % of DTaP doses distributed by GlaxoSmithKline | $\gamma$ | Equilibrium price ($) |
|---|---|---|---|
| 90 | 10 | 0.05 | 9.98 |
| 80 | 20 | 0.11 | 10.19 |
| 70 | 30 | 0.18 | 10.37 |
| 60 | 40 | 0.25 | 10.45 |
| 50 | 50 | 0.34 | 10.39 |
| 40 | 60 | 0.43 | 10.12 |
| 30 | 70 | 0.33 | 10.41 |
| 20 | 80 | 0.46 | 10.03 |
| 10 | 90 | 0.18 | 10.37 |

5. Discussion and conclusions

The game $\Gamma$ is a Bertrand–Edgeworth–Chamberlin price game for the analysis of a symmetric oligopoly market. The Nash equilibrium solution concept maintains a robust approach for examining and describing the pricing behavior of the manufacturers in all $\Gamma$ instances. The analysis indicates that a pure strategy equilibrium exists in the case where the capacity of the manufacturers is at their extreme. For the capacity region where no pure strategy equilibrium exists, there exists a mixed strategy equilibrium (Dasgupta and Maskin, 1986). Theorem 2 indicates the distribution function, its support, and the expected profit of the manufacturers for mixed strategy equilibrium.

The numerical analysis of the United States pediatric vaccine market provides several interesting observations. First, assuming that the vaccine manufacturers are capable of meeting the whole market demand, the federal vaccine prices negotiated in 2010 are higher than the equilibrium prices of the vaccines. This observation is intuitive as vaccine prices are clearly affected by several factors, which may have not been considered in the framework of Bertrand–Edgeworth–Chamberlin competition modeled in this analysis. As of 2010, there are six manufacturing companies producing vaccines for use in the United States (CDC, 2011), which is dramatically lower than the number of vaccine manufacturers who produced vaccines in the United States 30 years ago (35 manufacturing companies; (Prifti, 2010)). This trend is due to the limited profits and high research and development costs (Prifti, 2010). As sustaining high immunization levels is a vital societal need, meeting market demand is crucial. Therefore, the federal government, as the largest purchaser of pediatric vaccines, is required to provide adequate financial incentives for the vaccine manufacturers to remain in the market, while negotiating the vaccine prices for the public sector (see Robbins and Jacobson (2011)). Such incentives have not been included in this study and hence the equilibrium prices of the vaccines may not necessarily be equal to the negotiated prices. The two main roles of the CDC are negotiating lower prices for the vaccines and maintaining public health goals by meeting pediatric immunization needs (Coleman et al., 2005). This analysis mainly focuses on the first role of the CDC. The equilibrium prices could provide insights to the CDC while negotiating the vaccine prices as to what will arise if the vaccine manufacturers engage in Bertrand–Edgeworth–Chamberlin competition. Furthermore, some vaccines could not be licensed for sale in the United States despite the high research and development costs associated with them. According to vaccine manufacturers, the prices of the licensed vaccines are required to be negotiated to account for the research and development costs of the vaccines that are not licensed (Coleman et al., 2005). In addition, the vaccine production

Table 9. Sensitivity analysis on the number of HepB doses distributed by each manufacturer (with $k = 1.1a$)

| % of HepB doses distributed by GlaxoSmithKline | % of HepB doses distributed by Merck | $\gamma$ | Equilibrium price ($) |
|---|---|---|---|
| 90 | 10 | 0.12 | 9.96 |
| 80 | 20 | 0.23 | 10.17 |
| 70 | 30 | 0.34 | 10.12 |
| 60 | 40 | 0.44 | 9.82 |
| 50 | 50 | 0.45 | 9.78 |
| 40 | 60 | 0.36 | 10.08 |
| 30 | 70 | 0.26 | 10.18 |
| 20 | 80 | 0.17 | 10.08 |
| 10 | 90 | 0.08 | 9.83 |
costs and research and development costs are not included in this study. Therefore, the federal contract prices of the vaccines may not necessarily be equal to the equilibrium prices indicated in Table 5.

Second, this analysis provides a lower bound and an upper bound for the vaccine equilibrium prices in the public sector. If the capacity of each vaccine manufacturer is higher than the base case, 1.1a, the equilibrium price remains the same ($10.39 for DTaP, $9.78 for HepB, and $10.46 for Hib). Conversely, if the total capacity of the vaccine manufacturers is less than the market demand, the vaccine equilibrium price will be higher than the aforementioned equilibrium prices. The latter may occur when a vaccine shortage is in effect, which occurs when the production amount of a certain vaccine is not sufficient for the birth cohort. In these cases, the production capacity is limited and, hence, a competitive equilibrium or a mixed strategy equilibrium exists depending on the value of the production capacity. For example, in 2010 if the capacity of the two DTaP monovalent vaccines was each equal to $k = 0.4a$, the competitive equilibrium price would be $15.25. Furthermore, if the whole market demand is considered but the capacity of the vaccine manufacturers is equal to 57% of the market (only the public sector), the resulting equilibrium price provides an upper bound for the vaccine prices in the public sector of the pediatric vaccine market. In addition, this analysis provides a foundation for explaining the vaccine prices in the private sector by determining a lower bound for the private sector prices.

Third, the analysis highlights the importance of degree of product differentiation on equilibrium price. If the total capacity of the vaccine manufacturers is limited, equilibrium price (either competitive equilibrium or mixed strategy equilibrium) increases as the degree of product differentiation increases (i.e., vaccines become more similar). On the other hand, if the vaccine manufacturers are able to meet the entire market demand (Bertrand–Chamberlin equilibrium exists), equilibrium prices increase up to a point and then decrease as the degree of product differentiation increases. When the capacity is more than the market demand, the equilibrium prices tend to decrease as vaccines become more similar, since the customers are neutral about the choice of vaccine to purchase. Furthermore, the type of equilibrium becomes more significant the more similar the vaccines are.

Finally, given that the demand for a pediatric vaccine in a year is often proportional to the size of the birth cohort and hence is quite predictable (Prifti, 2010) and the number of adverse events for a specific vaccine does not fluctuate considerably over time, this analysis may provide insight for the CDC concerning vaccine equilibrium prices in the future. Whereas federal government officials pursue lower prices for vaccines, the vaccine manufacturers tend to set higher prices. The results presented in this study should appeal to the pediatric healthcare community, including federal government officials who negotiate the vaccine prices with vaccine manufacturers and vaccine manufacturers who are seeking effective pricing strategies.

The model presented in this study has several limitations. Due to the confidential and proprietary nature of any information regarding the production capacity of vaccines, the exact value of the production capacity of vaccine manufacturers is unknown. This study assumes an equal production capacity for each vaccine. Furthermore, a linear demand function is assumed in this study. These two assumptions enable the complete characterization of mixed strategy equilibrium for the proposed game. It may not be possible to draw general conclusions regarding the United States pediatric vaccine market until further analyses substantiate the validity of these assumptions. There are several factors that are not included in this study, due to a dearth of data. Factors other than medically adverse effects that further differentiate between vaccines such as brand loyalty and formulary inertia are not discussed. In addition, a metric that differentiates the Merck Hib vaccine based on its special advantage is not included in this study. Although the analysis cannot capture all factors that may influence the pediatric vaccine market, the results presented in this article compose an important step in understanding the pricing strategies in the United States pediatric vaccine market.

This study only considers monovalent pediatric vaccines. The same analysis for combination pediatric vaccines such as DTaP-HepB-IPV and DTaP-IPV/HIB is a fruitful area of future research. The equilibria analysis of a variation of the game presented in this article could be examined, in which Bertrand–Edgeworth–Chamberlin competition is applied to an asymmetric oligopoly market where the capacities of the manufacturers are not equal. Analyzing the repeated (multiple interaction) Bertrand–Edgeworth–Chamberlin competition is another area of future research. The game-theoretic approach introduced in this article provides a mathematical framework to analyze oligopolistic interactions in markets with the same general characteristics as the United States public sector pediatric vaccine market.

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Supplemental material

Supplemental data for this article can be accessed on the publisher’s website at www.tandfonline.com/uiie

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