A numerical simulation of the backward Raman amplifying in plasma

Wang Hong-Yu\textsuperscript{1,2}, Huang Zu-Qia\textsuperscript{1}

\textit{Institute of Low Energy Nuclear Physics, Beijing Normal University, Beijing, 100875, China}

\textit{Department of Physics, Anshan Normal University, Anshan, 114005, China}

Abstract

This paper describes a numerical simulation method for the interaction between laser pulses and low density plasmas based on hydrodynamic approximation. We investigate Backward Raman Amplifying (BRA) experiments and their variants. The numerical results are in good agreement with experiments.

\textit{Key words:} plasma simulation, laser plasma interactions, backward Raman amplifier

1 introduction

Recently, the concept of laser amplifying by backward raman scattering (Backward Raman Amplifiers, BRA) in plasma was presented\textsuperscript{[1],[2],[3],[4]}. In which a short seed pulse is amplified by a counter-propagating long pumping pulse when the resonance relation $\omega_{\text{seed}} + \omega_{\text{plasma}} = \omega_{\text{pump}}$ is satisfied. In the non-linear regime, the seed pulse is strongly amplified and compressed temporally. Theoretically, unfocused intensities of $10^{17}$ W/cm$^2$ in 50 fs pulses are accessible by this technique.

Up to now two schemes of BRA have been presented: the amplifier in non-linear regime and transient regime. The basic concepts of the first scheme are based on the $\pi$-pulse solutions of laser-plasma coupled equations\textsuperscript{[2]}. When the amplitudes of the plasma wave and the seed pulse are both quite large, the pumping pulse’s energy is absorbed by the front of the seed pulse. So the front edge of the seed pulse is amplified while the back edge will be reduced for the energy feedback into the exhausted pumping pulse. In this way, the seed pulse...
is amplified and compressed. Finally, the seed pulse evolves to a narrow wave train which is called a $\pi$ pulse.

The scheme of transient BRA is based on the matching of the plasma’s density gradient and the pumping pulse’s frequency chirp \[4\][5]. In the amplifying process, the strict matching of the two pulses and the plasma frequency is always needed. If we let the pumping pulse chirp and the plasma density have a gradient, and set the plasma frequency gradient just be a half of pumping chirp gradient, the seed pulse will be amplified only at a fixed position of itself. By passing a plasma of sufficient length, the amplifying take place and a ultra-fast laser pulse formed.

Each scheme is very difficult to be realized. For the limit of plasma technology, the density of gas-jet plasma is limited for about $10^{19} cm^{-3}$ now. To get distinct amplification, the seed pulse must travel several millimeters in plasma. The amplification require rigorous match of the frequencies. Keeping the uniformity of the plasma is a great challenge in experiments. In the transient scheme, we need even longer plasma length to compensate the lower efficiency.

However, some good results of amplifying experiments have been posted recently[6][7]. Especially, the experiments informed that the non-linear BRA scheme was possible. In the experiments by W. Cheng et al, the seed pulses were amplified to one order higher than pumping pulse and the amplifying ratio was several thousands.

Consequently, the numerical simulation of BRA becomes a valuable problem in computational plasma physics. However, compared with the common plasma simulations there are some special difficulties because of the long-distance travelling wave[8][9][10]. First, the computing time and memory needed is very large. Second, the investigation requires low-error simulation technology for the low increase rates of Raman effects. For example, we need a great number of particles to get rid of the numerical wave-breaking and numerical heating in Particle-In-Cell method[11]. Besides, we need a high-order difference scheme of the electromagnetic field equation to overcome the difficulties of numerical dispersion while applying it to the long-distance traveling wave. So the simulations of BRA are mainly focused on two regime now: 1) averaging the effects of electromagnetic fields to reduce the space-time steps, such as the APIC method[12]. 2) using the amplitudes coupled equations [13],[14],[15] or linearized plasma wave equations to get approximately analysis.

An attracting idea is applying a moving window which following the seed pulse to the computation. This technology is convient to investigate the interactions between laser and low-density plasma. In the corona region, the plasma density is about $\frac{1}{10}$ to $\frac{1}{100}$. The group velocity of laser pulse is very close to light speed. So the calculated window is shorter than $\frac{1}{10}$ of the total interaction region.
There is a simulation method which applies a moving window on the standard PIC simulation (Moving window PIC Simulation, MWPIC) [16]. However, MWPIC needs the space-time step match (\(dx = cdt\)) to avoid the interpolation error. The step setting is unstable at 2-dimensions and 3-dimensions explicit differential schemes. That means we need some implicit differential schemes. The latter technology are still developing now. In addition, MWPIC need a very large number of particles to get acceptable precision too.

For the above reasons, we developed a numerical simulation technique based on hydrodynamic approximation. In this technique, we only use the cold-plasma approximation which is often used in theoretical researches. The numerical trials indicate that the technique is suitable for the traveling wave problems in plasmas and can give results in concordance with experiments. Besides, the technique has a clear physical picture and the results can be easily analyzed.

We will discuss the BRA equations and the simulation technology in section 2 and section 3, then analyse the numerical results in section 4.

2 the basic equation of BRA

In the typical instance of BRA, we can introduce the cold-plasma approximation and the transverse canonical momentum conservation [17]:

\[
\vec{v} = \vec{u} + \frac{e\vec{A}}{mc} = \vec{u} + c\vec{a},
\]

where \(\vec{u}\) is the longitude velocity and \(\vec{A}\) is the vector potential at Coulomb gauge while \(\vec{a}\) is the normalized vector potential: \(\vec{a} = \frac{e\vec{A}}{mc}\). Because the electrons' velocities are far less than the light speed, we can ignore the relativistic factor \(\gamma\).

The plasma hydrodynamic equation can be written as: (We ignore the thermal pressure here. We can recover it to the rhs. of the second equation whenever needed)

\[
\frac{\partial n}{\partial t} + \nabla(n\vec{v}) = 0 \tag{1}
\]

\[
\frac{\partial \vec{v}}{\partial t} + (\vec{v} \cdot \nabla)\vec{v} = -\frac{e\vec{E}}{m} - \frac{e(\vec{v} \times B)}{mc} \tag{2}
\]

Supposing the plasma flow are non-vertex before the laser action and applying the cold-plasma approximation, we get the plasma hydrodynamic equation:
The electromagnetic field satisfies the wave equation and Poisson equation:

\[
(\frac{1}{c^2} \frac{\partial^2}{\partial t^2} - \nabla^2)\vec{a} = -\frac{4\pi n e^2}{mc^2}\vec{a}
\]  

(5)

\[
\nabla^2 \phi = \frac{4\pi e^2}{mc^2}(n - n_i)
\]  

(6)

Applying the coordinate transformation

\[
\xi = x - ct
\]

(7)

\[
\eta = t
\]

(8)

we get the wave equation in the new variables:

\[
\left(\frac{1}{c^2} \frac{\partial^2}{\partial \eta^2} - \frac{2}{c} \frac{\partial}{\partial \xi}\frac{\partial}{\partial \eta} - \frac{\partial^2}{\partial \xi^2} - \frac{\partial^2}{\partial \zeta^2}\right)\vec{a} = -\frac{4\pi n e^2}{mc^2}\vec{a}
\]  

(9)

and the Poisson equation has no change.

To solve the wave equation numerically, we introduce the medium variable \(\vec{\psi}\):

\[
\left(\frac{1}{c} \frac{\partial}{\partial \eta} - 2\frac{\partial}{\partial \xi}\right)\vec{a} = \vec{\psi}
\]

Finally, we get a pair of first order differential equations:

\[
\frac{\partial \vec{\psi}}{\partial \eta} = -\frac{4\pi n e^2}{mc^2}\vec{a} + \nabla^2_\perp \vec{a}
\]

\[
\frac{1}{c} \frac{\partial \vec{a}}{\partial \eta} = \vec{\psi} + 2\frac{\partial \vec{a}}{\partial \xi}
\]  

(10)

With the same transformation, the hydrodynamic equations became
\[
\frac{\partial n}{\partial t} + (\beta_x - 1) \frac{\partial n}{\partial x} + \beta_y \frac{\partial n}{\partial y} = -n \left[ \frac{\partial (\beta_x - 1)}{\partial x} + \frac{\partial \beta_y}{\partial y} \right]
\]

\[
\frac{\partial \beta_x}{\partial t} + \left[ (\beta_x - 1) \frac{\partial}{\partial x} + u_y \frac{\partial}{\partial y} \right] (\beta_x - 1) = \frac{\partial \phi}{\partial x} - \frac{1}{2} \frac{\partial \vec{a}^2}{\partial x}
\]

\[
\frac{\partial \beta_y}{\partial t} - \left[ \frac{\partial}{\partial x} + \beta_y \frac{\partial}{\partial y} \right] \beta_y = \frac{\partial \phi}{\partial y} - \frac{1}{2} \frac{\partial \vec{a}^2}{\partial y}
\]

(11)

The equations (6),(10),(11) are the basic equations to describe the laser-plasma interactions in BRA.

3 numerical simulation method

The electromagnetic equations (6),(10) describe the electromagnetic wave transmitting in plasma. In BRA scheme, the transverse dynamic effects are mainly slow processes such as self-focusing. We can use the periodic boundary conditions in the transverse directions, apply FFT calculation to eliminate the transverse derivatives and use direct elimination technology in the longitudinal direction. So we will focus on the 1-dimension scheme at following discussion. However, the generalization to multi-dimensions is straightforward.

The elimination of Poisson equation is quite simple. In our moving window, the right boundary of calculating region are always at the front of seed pulse. We ignore the interactions between the pumping pulse and the origin plasmas for the uniform of the pumping pulse, so all laser-plasma interaction can not affect the region in the right side of the boundary. Then we can assume that there are no electrostatic fields in the right region of seed pulse: \( \vec{E} = 0 \) then \( \phi_i = \phi_{i+1} = \ldots = \text{const.} \) The differential scheme is

\[
\phi_{i-1} = 4\pi e^2 \frac{\delta x^2}{mc^2}(n_i - N_i) - \phi_{i+1} + 2\phi_i
\]

(12)

To get the numerical solution of the wave equation, we use a generation of R.Liu’s leapfrog-upwind scheme[18]:

\[
\frac{1}{2} \left[ \frac{m}{\delta t} \left( a_{i-1}^{n+1} - a_i^n \right) + (a_i^n - a_{i-1}^{n-1}) \right] = -2c \frac{1 - m}{2} \left( a_{i+1}^n - a_i^n \right) + m \left( a_i^n - a_{i-1}^n \right) + \frac{1 - m}{2} \left( a_{i-1}^n - a_{i-2}^n \right)
\]

\[
= \frac{1}{2} c \left( \psi_i^n + \psi_{i-1}^n \right)
\]

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Where

\[ \kappa = \frac{2c \delta t}{\delta x} \]  
(13)

\[ m = \frac{1}{6}(5 + 3 \kappa - 2 \kappa^2) \]  
(14)

\[ a_i^{n+1} = a_i^{n-1} - (1 - 2m\kappa)(a_{i+1}^{n} - a_i^{n}) \]
\[ + (1 - m)\kappa(a_{i+2}^{n} - a_{i+1}^{n}) + (1 - m)\kappa(a_i^{n} - a_{i-1}^{n}) \]
\[ + c\delta t(\psi_{i+1}^{n} + \psi_{i}^{n}) \]  
(15)

The hydrodynamic equation (11) can be solved with leap-frog method too. However, we must apply the FCT technology [19] to avoid the numerical oscillation while introduce little numerical damping. The Flux Corrects are used in all leap-frog steps by rewritten to convection equations:

\[ \frac{\partial U}{\partial t} + \frac{\partial F}{\partial x} = S \]  
(16)

F means the convect terms and S means source terms, then the leap-frog scheme is:

\[ U_i^{n+1} = U_i^{n-1} - 2\frac{\Delta t}{\Delta x}(F_{i+1/2}^{n} - F_{i-1/2}^{n}) + 2\Delta tS_i^{n} \]  
(17)

To use FCT technology, first we calculate the low-order flux of plasma density (there is no source term in the continuum equation):

\[ F_{i+1/2}^{low} = \overline{v}_i * n_i^{DC} * \delta t + \eta(n_{i+1} - n_{i}) \]  
(18)

\[ \overline{v}_i = \frac{1}{2}(v_{i+1} + v_{i}) \]  
(19)

\[ if(\overline{v} > 0) \]
\[ w_i^{DC} = w_{i+1}^{k-1} \]  
(20)

\[ else \]
\[ w_i^{DC} = w_{i+1}^{k-1} \]  
(21)

\[ w = n, v \]  
(22)

\[ \eta \] is a coefficient about \( \frac{1}{24} \) to \( \frac{1}{8} \). The smaller \( \eta \) means less error but the more numerical oscillation. Then the high-order flux is:

\[ F_{i+1/2}^{high} = \frac{7}{12}(f_{i+1} + f_{i}) - \frac{1}{12}(f_{i+2} + f_{i-1}) \]  
(23)
We don’t introduce the correct of the velocity’s flux because it will cause much damping in the simulation. So the velocity flux can use the same scheme as the high-order density flux.

However, the difference scheme of plasma density does not preserve positivity. To get rid of the difficulties of negative electron density, we apply a limiter of density flux: the out-flux density can not exceed the present electron density of each point. The physics means of this limiter are clear.

4 numerical results and the analysis

W.Cheng’s experiment results[7] were the best experiment validate of BRA scheme. In this experiment, the author amplified the seed pulse in a gas-jet plasma. In the plasma with 2mm effective length, the intensities of seed pulses were amplified several thousands times and the length of seed pulses was compressed from 550fs (origin) to about 150fs. The recorded intensity reached $1.7 \times 10^{15} W/cm^2$ which was higher than pump intensity by more than 1 order in magnitude. The author recorded the broadened (800 fs) and compressed (<200fs) output pulses both in the experiment. Which informed that the amplifying process had been in non-linear region.

We investigated the process numerically. As the experiments setting up, we set the seed pulse length to 550 fs and set the pumping pulse length to 10 ps. The plasma’s length was about 2 mm and the plasma’s density was $1.1 \times 10^{19} cm^{-3}$. The wavelength of pumping pulse was 0.8 $\mu m$ and the wavelength of seed pulse was set to matching with the plasma and pumping pulse.

At the peak top of the pumping pulse, we let the two pulses collide each other. For the uncertainty of the collide points and the plasma temperatures, the simulating results varied a little. When the interaction finished, the seed pulse evolved to narrow peek which FWHM about 150fs – 170fs and peak intensity about $1.5 \times 10^{15} W/cm^2 – 1.9 \times 10^{15} W/cm^2$. The typical simulate results were shown at figure 1. We can find that the $\pi$-pulse shape easily. Which means the non-linear amplifying region was reached. Considered the uncertainly of the plasma temperature and length, we can find good agreement between the simulations and the experiments.

From the simulation results, we can find the evolving process of seed pulse. Let the seed pulse collide with different length pumping pulse, we can image the evolving of the seed pulse: it was broadened to about 750 fs and compressed later. At the end, it evolved to a pulse shorter than 160 fs.

The typical results are shown at figure 2.
The simulating process shows the non-linear character of plasma wave. When the amplitude of plasma wave is quite large, the wave’s shape are different from sine wave, the typical wave form are shown in figure 3. As we see, the difference are quite evidence in our results. In fact, the effective $\delta n/n$ can exceed 1 when the amplifier reaches the exhaust region. This phenomena made an adding stability of amplifier from wave-breaking.

However, our results show that when the amplifying length reach to 2mm, the plasma wave amplitude was close to cold plasma wave-breaking amplitude.
figure 3: The typical wave-shape of density oscillating when the amplitudes become large.

Which means, if we want to make an practical amplifier ,we must increase the plasma density to get higher stabilities. Although It was very difficult to get dense and uniform plasma in experiments, we simulated the BRA process in a quite dense plasma \( (n = 0.04n_{cr}) \). The results were coarse in precision but show that the amplifying processes were much more stable and easier to reach the nonlinear-regime in a more dense plasma. The higher increase rates of raman effects and higher stability compensated the lower conversion effects in the more dense plasmas. The results of our simulation are shown in figure 4. In this density, we need shorter than 1mm plasmas to generate obviously amplified intensities. It means that the increasing of uniform plasma density should be the most important improvement at future experiments.

Our method can be easily improved to include the effects as pumping heat, plasma un-uniformity, inverse-Bremsstrahlung, or even linear landau damping, tunnel-ionized (by phenomena) effects et al. The investigation of these effects in BRA are proceeding now.

5 conclusion

We introduced a hydrodynamic simulation method which was suitable for the interaction of lasers and corona plasmas. The method is convenient for the simulation of ultra-fast laser’s transmitting. We simulated the Backward Raman Amplifiers experiments numerically, and the results show good agreement
The pulse shape of the seed after being amplified by different length pumping pulse; the unit is the natural of plasma.

with experiments. The following study will appear in another paper.

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