Within the context of standard cosmology, an accelerating universe requires the presence of a third ‘dark’ component of energy, beyond matter and radiation. The available data, however, are still deemed insufficient to distinguish between an evolving dark energy component and the simplest model of a time-independent cosmological constant. In this paper, we examine the cosmological expansion in terms of observer-dependent coordinates, in addition to the more conventional co-moving coordinates. This procedure explicitly reveals the role played by the radius $R_h$ of our cosmic horizon in the interrogation of the data. (In Rindler’s notation, $R_h$ coincides with the ‘event horizon’ in the case of de Sitter, but changes in time for other cosmologies that also contain matter and/or radiation.) With this approach, we show that the interpretation of dark energy as a cosmological constant is clearly disfavored by the observations. Within the framework of standard Friedman-Robertson-Walker cosmology, we derive an equation describing the evolution of $R_h$, and solve it using the WMAP and Type Ia supernova data. In particular, we consider the meaning of the observed equality (or near equality) $R_h(t_0) \approx c t_0$, where $t_0$ is the age of the Universe. This empirical result is far from trivial, for a cosmological constant would drive $R_h(t)$ towards $c t$ (where $t$ is the cosmic time) only once—and that would have to occur right now. Though we are not here espousing any particular alternative model of dark energy, for comparison we also consider scenarios in which dark energy is given by scaling solutions, which simultaneously eliminate several conundrums in the standard model, including the ‘coincidence’ and ‘flatness’ problems, and account very well for the fact that $R_h(t_0) \approx c t_0$.

Keywords: cosmology; dark energy; gravitation.

1. Introduction

Over the past decade, Type Ia supernovae have been used successfully as standard candles to facilitate the acquisition of several important cosmological parameters. On the basis of this work, it is now widely believed that the Universe’s expansion is accelerating.\cite{1,2} In standard cosmology, built on the assumption of spatial homogeneity and isotropy, such an expansion requires the existence of a third form of energy, beyond the basic admixture of (visible and dark) matter and radiation.
One may see this directly from the (cosmological) Friedman-Robertson-Walker (FRW) differential equations of motion, usually written as

\[ H^2 = \left( \frac{\dot{a}}{a} \right)^2 = \frac{8\pi G}{3c^2} \rho - \frac{k c^2}{a^2}, \quad (1) \]

\[ \frac{\ddot{a}}{a} = -\frac{4\pi G}{3c^2} (\rho + 3p), \quad (2) \]

\[ \dot{\rho} = -3H(\rho + p), \quad (3) \]

in which an overdot denotes a derivative with respect to cosmic time \( t \), and \( \rho \) and \( p \) represent, respectively, the total energy density and total pressure. In these expressions, \( a(t) \) is the expansion factor, and \((r, \theta, \phi)\) are the coordinates in the comoving frame. The constant \( k \) is +1 for a closed universe, 0 for a flat universe, and −1 for an open universe.

Following convention, we write the equation of state as \( p = \omega \rho \). A quick inspection of Eq. (2) shows that an accelerated expansion (\( \ddot{a} > 0 \)) requires \( \omega < -1/3 \). Thus, neither radiation (\( \rho_r \), with \( \omega_r = 1/3 \)), nor (visible and dark) matter (\( \rho_m \), with \( \omega_m \approx 0 \)) can satisfy this condition, leading to the supposition that a third ‘dark’ component \( \rho_d \) (with \( \omega_d < -1/3 \)) of the energy density \( \rho \) must be present. In principle, each of these contributions to \( \rho \) evolves according to its own dependence on \( a(t) \).

Over the past few years, complementary measurements\(^3\) of the cosmic microwave background (CMB) radiation have indicated that the Universe is flat (i.e., \( k = 0 \)), so \( \rho \) is at (or very near) the “critical” density \( \rho_c = \frac{3c^2 H^2}{8\pi G} \). But among the many peculiarities of the standard model is the inference, based on current observations, that \( \rho_d \) must itself be of order \( \rho_c \). Dark energy is often thought to be the manifestation of a cosmological constant, \( \Lambda \), though no reasonable explanation has yet been offered as to why such a fixed, universal density ought to exist at this scale. It is well known that if \( \Lambda \) is associated with the energy of the vacuum in quantum theory, it should have a scale representative of phase transitions in the early Universe—many, many orders of magnitude larger than \( \rho_c \).

Many authors have attempted to circumvent these difficulties by proposing alternative forms of dark energy, including Quintessence,\(^4,5\) which represents an evolving canonical scalar field with an inflation-inducing potential, a Chameleon field\(^6,7,8\) in which the scalar field couples to the baryon energy density and varies from solar system to cosmological scales, and modified gravity, arising out of both string motivated, or General Relativity modified actions,\(^9,10,11\) which introduce large length scale corrections modifying the late time evolution of the Universe. The actual number of suggested remedies is far greater than this small, illustrative sample.

Nonetheless, though many in the cosmology community suspect that some sort of dynamics is responsible for the appearance of dark energy, until now the sensitivity of current observations has been deemed insufficient\(^7\) to distinguish between...
an evolving dark energy component and the simplest model of a time-independent cosmological constant $\Lambda$. This conclusion, however, appears to be premature, given that constraints on the universe’s expansion arising from the observed behavior of our cosmic horizon has not yet been fully folded into the interrogation of the current data. The purpose of this paper is to demonstrate that a closer scrutiny of the available measurements, if proven to be reliable, can in fact already delineate between evolving and constant dark energy theories, and that a simple cosmological constant $\Lambda$, characterized by a fixed $\omega_d \equiv \omega_{\Lambda} = -1$, is disfavored by the observations.

2. The Cosmic Horizon

In an earlier paper, we introduced a transformation of the Robertson-Walker (RW) metric (from which Eqs. 1, 2, and 3 are derived) into a new set of (observer-dependent) coordinates $(cT, R, \theta, \phi)$, where $R \equiv a(t)r$ and $T(R)$ is the time (in the observer’s frame) corresponding to the radius $R$. The cosmic time $t$ coincides with $T$ only at the origin, i.e., $T(0) = t$. For all other radii, $T$ is dilated relative to $t$ from the effects of curvature induced by the mass-energy content of the universe. Ironically, de Sitter’s own metric was first written in terms of these observer-dependent coordinates,

$$\frac{2GM(R_h)}{c^2} = R_h. \quad (6)$$

In this expression, $M(R_h) = \frac{4\pi}{3} R_h^3 \rho / c^2$ is the enclosed mass at $R_h$. In terms of $\rho$, we find that

$$R_h = \left(\frac{3c^4}{8\pi G \rho}\right)^{1/2} \quad (7)$$

or, more simply, $R_h = c / H_0$ in a flat universe. Not surprisingly, this is the radius at which a sphere encloses sufficient mass-energy to create an infinite redshift (i.e., $T(R) \rightarrow \infty$ as $R \rightarrow R_h$) as seen by an observer at the origin of the coordinates $(cT, R, \theta, \phi)$.

When the Robertson-Walker metric is written in terms of $(cT, R, \theta, \phi)$, the presence of $R_h$ alters the intervals of time we measure (using the clocks fixed to our origin) progressively more and more as $R \rightarrow R_h$. And since the gravitational time dilation becomes infinite at $R_h$, it is physically impossible for us to see any process
occurring beyond this radius. Light emitted beyond $R_h$ is infinitely redshifted by the time it reaches us and it therefore carries no signal.

This is precisely the reason why the recent observations have a profound impact on our view of the cosmos. The Hubble Space Telescope Key Project on the extragalactic distance scale has measured the Hubble constant $H$ with unprecedented accuracy, yielding a current value $H_0 \equiv H(t_0) = 71 \pm 6 \text{ km s}^{-1} \text{ Mpc}^{-1}$. (For $H$ and $t$, we will use subscript “0” to denote cosmological values pertaining to the current epoch.) With this $H_0$, we infer that $\rho(t_0) = \rho_c \approx 9 \times 10^{-9} \text{ ergs cm}^{-3}$.

Given such precision, it is now possible to accurately calculate the radius $R_h$. From the Hubble measurement of $\rho(t_0)$, we infer that $R_h \approx 13.5$ billion light-years; this is the maximum distance out to which measurements of the cosmic parameters may be made at the present time. At first glance, it may seem that it had to be this way, since the age $t_0$ of the universe is also known to be 13.7 billion years. But in fact, the FRW equations predict that $R_h$ should not be equal to $ct_0$, unless $\omega$ has a very special value.

Let us consider how the radius $R_h$ evolves with the universal expansion. Clearly, in a de Sitter universe\textsuperscript{14} with a constant $\rho$ (proportional to $\Lambda$), $R_h$ is fixed forever. But for any universe with $\omega \neq -1$, $R_h$ must be a function of time. From the definition of $R_h$ and Eqn. (3), it is easy to see that

$$\dot{R}_h = \frac{3}{2} (1 + \omega) c, \quad (8)$$

a remarkably simple expression that nonetheless leads to several important conclusions regarding our cosmological measurements. We will use it here to distinguish between constant and evolving dark energy theories. Notice, for example, that $\dot{R}_h = c$ only for the special case $\omega = -1/3$, which is not consistent with the currently favored $\Lambda$CDM model of cosmology.

Take $t$ to be some time in the distant past (so that $t \ll t_0$). Then, integrating Eqn. (8) from $t$ to $t_0$, we find that

$$R_h(t_0) - R_h(t) = \frac{3}{2} (1 + \langle \omega \rangle) c t_0, \quad (9)$$

where

$$\langle \omega \rangle \equiv \frac{1}{t_0} \int_t^{t_0} \omega \, dt \quad (10)$$

is the time-averaged value of $\omega$ from $t$ to the present time.

Now, for any $\langle \omega \rangle > -1$, $\rho$ drops as the universe expands (i.e., as $a(t)$ increases with time), and since $R_h \sim \rho^{-1/2}$, clearly $R_h(t) \ll R_h(t_0)$. Therefore,

$$R(t_0) \approx \frac{3}{2} (1 + \langle \omega \rangle) c t_0. \quad (11)$$

\textsuperscript{a}In this situation, $R_h$ coincides with the event horizon defined by Rindler.\textsuperscript{16}
The reason we can use the behavior of \( R_h \) as the universe expands to probe the nature of dark energy is that the latter directly impacts the value of \( \langle \omega \rangle \). A consideration of how the cosmic horizon \( R_h \) evolves with time can therefore reveal whether or not dark energy is dynamically generated. Indeed, we shall see shortly that the current observations, together with Eqn. (11), are already quite sufficient for us to differentiate between the various models.

Before we do that, however, we can already see from this expression that there may be a serious problem with our interpretation of \( t_0 \) in the standard model of cosmology. From WMAP observations,\(^3\) we infer that the age \( t_0 \) of the universe is \( \approx 13.7 \) billion years. Since \( R_h \approx 13.5 \) billion light-years, this can only occur if \( \langle \omega \rangle \leq -\frac{1}{3} \). Of course, this means that the existence of dark energy (with such an equation of state) is required by the WMAP and Hubble observations alone, independently of the Type Ia supernova data. But an analysis of the latter (see \( \S \) 4 below, particularly Figs. 5 and 8) reveals that the value \( \langle \omega \rangle = -\frac{1}{3} \) is apparently ruled out, so in fact \( \langle \omega \rangle < -\frac{1}{3} \).

But this means that \( R_h \neq c t_0 \); in fact, \( R_h \) must be less than \( c t_0 \), which in turns suggests that the universe is older than we think. Unless \( \omega = -\frac{1}{3} \), what we infer to be the time since the Big Bang, is instead the “horizon” time \( t_h \equiv R_h/c \), which must be shorter than \( t_0 \). This may seem absurd at first, but we must remember that any events occurring beyond \( R_h \) are not visible to us yet (or ever, depending on whether or not \( R_h \) is constant). In addition, \( T(R) \) becomes progressively more dilated as we view events taking place closer and closer to \( R_h \), so that we see the universe as it appeared just after the Big Bang when \( R \approx R_h \), regardless of when the Big Bang actually occurred, as long as \( t_0 \geq R_h/c \).

This phenomenon has some important consequences that may resolve several long-standing conflicts in cosmology. Through our analysis below, we will gain a better understanding of how it works, which will permit us to calculate \( t_0 \) more precisely.

### 3. The Cosmological Constant

The standard model of cosmology contains a mixture of cold dark matter and a cosmological constant with an energy density fixed at the current value, \( \rho_d(t) \equiv \rho_{\Lambda}(t) \approx 0.7 \rho_c(t_0) \), and an equation of state with \( \omega_d \equiv \omega_{\Lambda} = -1 \). Known as \( \Lambda \)CDM, this model has been reasonably successful in accounting for large scale structure, the cosmic microwave background fluctuations, and several other observed cosmological properties.\(^3,17,18\)

But let us now see whether \( \Lambda \)CDM is also consistent with our understanding of \( R_h \). Putting \( \rho = \rho_m(t) + \rho_{\Lambda} \), where \( \rho_m \) is the time-dependent matter energy density, we may integrate Eqn. (8) for a \( \Lambda \)CDM cosmology, starting at the present time \( t_0 \), and going backwards towards the era when radiation dominated \( \rho \) (somewhere around 100,000 years after the Big Bang). Fig. 1 shows the run of \( R_h/c t \) as a function of time, along with the time-averaged \( \omega \) given in Eqn. (10), to be distinguished from
Fig. 1. Plot of the horizon radius $R_h$ in units of $ct$, and $\langle \omega \rangle$, the equation of state parameter $\omega \equiv p/\rho$ averaged over time from $t$ to $t_0$. The asymptotic value of $\langle \omega \rangle$ (called $\langle \omega \rangle_\infty$ in the text) for $t \to 0$ is approximately $-0.31$. These results are from a calculation of the universe’s expansion in a $Λ$CDM cosmology, with matter energy density $\rho_m(t_0) = 0.3\rho_c(t_0)$ and a cosmological constant $\rho_\Lambda = 0.7\rho_c(t_0)$, with $\omega_\Lambda = -1$.

Fig. 2. Plot of the matter energy density parameter $Ω_m \equiv \rho_m(t)/\rho_c(t)$, the cosmological constant energy density parameter $Ω_\Lambda \equiv \rho_\Lambda(t)/\rho_c(t)$, and the expansion factor $a(t)$, as functions of cosmic time $t$ for the same $Λ$CDM cosmology as that shown in Fig. 1. Since the energy density associated with $\Lambda$ is constant, $Ω_m \to 1$ as $t \to 0$. A notable (and well-known) peculiarity of this cosmology is the so-called “coincidence problem,” also dubbed because $Ω_\Lambda$ is approximately equal to $Ω_m$ only in the current epoch.

The asymptotic value $\langle \omega \rangle_\infty$, which is the equation-of-state parameter $\omega$ averaged over the entire universal expansion, from $t = 0$ to the present. The present epoch is indicated by a vertical dotted line. The calculation begins at the present time $t_0$, with the initial value $R_h = (3/2)(1 + \langle \omega \rangle_\infty)ct_0$, where $\langle \omega \rangle_\infty$ is obtained by an
iterative convergence of the solution to Eqn. (8). In order to have $R_h = ct_0$ at the present time, $\langle \omega \rangle_\infty$ must be ($\approx -0.31$). In $\Lambda$CDM, the matter density increases towards the Big Bang, but $\rho_\Lambda$ is constant, so the impact of $\omega_d$ on the solution vanishes as $t \to 0$ (see the dashed curve in Fig. 1). Thus, as expected, $R_h/ct \to 3/2$ early in the Universe’s development. This is the correct behavior within the framework of the Friedman-Robertson-Walker cosmology.

What is rather striking about this result is that in $\Lambda$CDM, $R_h(t)$ approaches $ct$ only once in the entire history of the Universe—and this is only because we have imposed this requirement as an initial condition on our solution. There are many peculiarities in the standard model, some of which we will encounter shortly, but the unrealistic coincidence that $R_h$ should approach $ct_0$ only at the present moment must certainly rank at—or near—the top of this list.

One may be tempted to think of this result as another manifestation of the so-called “coincidence problem” in $\Lambda$CDM cosmology, arising from the peculiar near-simultaneous convergence of $\rho_m$ and $\rho_\Lambda$ towards $\rho_c$ in the present epoch. But these are definitely not the same phenomenon. Whether or not $\rho_m$ and $\rho_\Lambda$ are similar, the requirement that $R_h/ct \to 1$ only at $t_0$ implies a very special evolution of these quantities as functions of cosmic time $t$ (see Fig. 2). This odd behavior casts doubt on the viability of $\Lambda$CDM cosmology as the correct description of the Universe.

4. Dynamical Dark Energy

Given the broad range of alternative theories of dark energy that are still considered to be viable, it is beyond the scope of this paper to exhaustively study all dynamical scenarios. Instead, we shall focus on a class of solutions with particular importance to cosmology—those in which the energy density of the scalar field mimics the background fluid energy density. Cosmological models in this category are known as “scaling solutions,” characterized by the relation

$$\frac{\rho_d(t)}{\rho_m(t)} = \frac{\rho_d(t_0)}{\rho_m(t_0)} \approx 2.33$$

(some of the early papers on this topic include Refs. 18–25).

By far the simplest cosmology we can imagine in this class is that for which $\omega = -1/3$, corresponding to $\omega_d \approx -1/2$ (within the errors). This model is conceptually very attractive, but does not appear to be fully consistent with Type Ia supernova data, so either our interpretation of current observations is wrong or the Universe just doesn’t work this way. But it’s worth our while spending some time with it because it will help us understand the models that follow it.

To begin with, we see immediately from Eqn. (8) (and illustrated in Figs. 3 and 4) that when $\omega = -1/3$, we have $R_h(t) = ct$. Thus, for a scaling solution satisfying Eqn. (12), this value of $\omega$ solves three of the major problems in standard cosmology: first, it explains why $R_h(t_0)$ should be equal to $ct_0$ (because these quantities are always equal). Second, it removes the inexplicable coincidence that $\rho_d$ and $\rho_m$ should be comparable to each other only in the present epoch (since they are always
Fig. 3. Same as Fig. 1, except for a “scaling solution” in which $\rho_d \propto \rho_m$ and $\langle \omega \rangle = -1/3$. The corresponding dark-energy equation-of-state parameter is $\omega_d \approx -0.48$. This is sufficiently close to $-0.5$, that it may actually correspond to this value within the errors associated with the measurement of $\rho_m(t_0)/\rho_c(t_0)$ and $\rho_d(t_0)/\rho_c(t_0)$. Note that for this cosmology, $R_h/ct$ is always exactly one. A universe such as this would do away with the otherwise inexplicable coincidence that $R_h(t_0) = ct_0$ (since it has this value for all $t$), but as we shall see in Fig. 5, it does not appear to be entirely consistent with Type Ia supernova data.

Fig. 4. Same as Fig. 2, except for a “scaling solution” in which $\rho_d \propto \rho_m$ and $\langle \omega \rangle = -1/3$. The corresponding dark-energy equation-of-state parameter is $\omega_d \approx -0.48$. Comparable to each other. Third, it does away with the so-called flatness problem. To see this, let us return momentarily to Eqn. (1) and rewrite it as follows:

$$H^2 = \left( \frac{c}{R_h} \right)^2 \left( 1 - \frac{kR_h^2}{a^2} \right).$$

(13)

Whether or not the Universe is asymptotically flat hinges on the behavior of $R_h/a$.\end{quote}
as $t \to \infty$. But from the definition of $R_h$ (Eqn. 7), we infer that

$$\frac{d}{dt} \ln R_h = 3(1 + \omega) \frac{d}{dt} \ln a .$$

(14)

Thus, if $\omega = -1/3$,

$$\frac{d}{dt} \ln R_h = \frac{d}{dt} \ln a ,$$

(15)

and so

$$H = \text{constant} \times \left( \frac{c}{R_h} \right) ,$$

(16)

which is the equation for a flat universe. (We caution, however, that in deriving this result, we have used Eqns. 8 and 11, which implicitly assume a flat universe, together with Eqn. 13 which is more general. If a scaling solution proves to be the likely representation of dark energy, then a demonstration of flatness would need to be shown more rigorously.) We also learn from Eqn. (2) that in this universe, $\dot{a} = 0$. The Universe is coasting, but not because it is empty, as in the Milne cosmology, but rather because the change in pressure as it expands is just right to balance the change in its energy density.

All told, these are quite impressive accomplishments for such a simple model, and yet, it doesn’t appear to be fully consistent with Type Ia supernova data. It is quite straightforward to demonstrate this with the evolutionary profiles shown in Figs. 3 and 4. The comoving coordinate distance from some time $t$ in the past to the present is $\Delta r = \int_t^0 c \, dt / a(t)$. With $k = 0$, the luminosity distance $d_L$ is $(1 + z) \Delta r$, where the redshift $z$ is given by $(1 + z) = 1/a$, in terms of the expansion factor $a(t)$ plotted in Fig. 4.

The data in Fig. 5 are taken from the “gold” sample with coverage in redshift from 0 to $\sim 1.8$. The distance modulus is $5 \log d_L(z) + \Delta$, where $\Delta \approx 25$. The dashed curve in this plot represents the fit based on the scaling solution shown in Figs. 3 and 4, with a Hubble constant $H_0 = 71 \text{ km s}^{-1} \text{ Mpc}^{-1}$ ($\Delta$ is used as a free optimization parameter in each of Figs. 5 and 8). The “best” match corresponds to an unacceptable reduced $\chi^2$ of 1.11 with $180 - 1 = 179$ d.o.f.

Interestingly, if we were to find a slight systematic error in the distance modulus for the events at $z > 1$, which for some reason has led to a fractional over-estimation in their distance (or, conversely, a systematic error that has lead to an under-estimation of the distance modulus for the nearby explosions), the fit would improve significantly. So our tentative conclusion right now must be that, although an elegant scaling solution with $\omega = -1/3$ provides a much better explanation than $\Lambda$CDM for the observed coincidence $R_h \approx c t_0$, it is nonetheless still not fully consistent with the supernova data.

Fortunately, many of the attractive features of an $\omega = -1/3$ cosmology are preserved in the case where $\omega_d = -2/3$, corresponding to a scaling solution with $\omega \approx -1/2$. This model fits the supernova data quite strikingly, but it comes at an additional cost—we would have to accept the fact that the universe is somewhat
Fig. 5. Plot of the observed distance modulus versus redshift for well-measured distance Type Ia supernovae. The dashed curve shows the theoretical distribution of magnitude versus redshift for the “scaling solution” in which $\rho_d \propto \rho_m$ and $\langle \omega \rangle = -1/3$ (see Figs. 3 and 4). The corresponding dark-energy equation-of-state parameter is $\omega_d \approx -0.48$. The best fit corresponds to a reduced $\chi^2 \approx 1.11$ for $180 - 1 = 179$ d.o.f., which is not an adequate representation of the Type Ia supernova data.

older (by a few billion years) than we now believe. Actually, this situation is unavoidable for any cosmology with $\omega < -1/3$ because of the relation between $R_h$ and $c t_0$ in Eqn. (11). This conclusion may seem incompatible with the WMAP and HST data, but is actually fully consistent with them, though our interpretation of the currently inferred “age” of 13.7 billion years would need to be revised along the following lines.

Light reaching us from beyond the cosmic horizon (at radius $R_h$) is infinitely redshifted, so only phenomena occurring within a time $t_h \equiv R_h/c$ of the present can produce a measurable signal in our instruments. Of course, there is no a priori reason why the horizon time $t_h$ must coincide with the time $t_0$ elapsed since the Big Bang. We can say with certainty that $t_0$ cannot be smaller than $t_h$, for otherwise $R_h > c t_0$, which is inconsistent with the observations. However, a situation with $t_0 > t_h$ simply means that the Universe has been expanding longer than the time it has taken the observed CMB to reach us from $R_h$. As noted earlier, this situation cannot be avoided for any scenario in which $\omega < -1/3$. The case $\omega = -1/3$ is special because then $R_h = c t_0$.

We see in Fig. 6 that $R_h/c t$ is constant (say when $\omega_d = -2/3$), but at a value $(3/2)(1 + \langle \omega \rangle)$, where the time-averaged $\omega$ is now $\approx -0.47$. Thus, if $R_h$ is 13.5 billion light-years, $t_0$ must be approximately 16.9 billion years. This is simply a manifestation of the fact that anything that happened between the Big Bang and a horizon time $t_h$ ago would have produced infinitely redshifted signals at the present time, and is no longer observable. Thus, we can see only as far back as $t_h$, and if $\omega < -1/3$, we must therefore re-interpret the presently inferred “age” as the horizon
Constraints on Dark Energy

Fig. 6. Same as Fig. 1, except for a “scaling solution” in which \( \rho_d \propto \rho_m \) and \( \omega_d = -2/3 \). The time-averaged equation-of-state parameter is \( \langle \omega \rangle \approx -0.47 \). Thus, \( R_h \neq ct_0 \). Instead, \( t_0 \approx 16.9 \) billion years, approximately 3.4 billion years longer than the “horizon” time, \( t_h \equiv R_h/c \approx 13.5 \) billion years. This type of universe is not subject to the “coincidence” problem since \( R_h/c t \) is constant. It provides the best fit to the Type Ia supernova data (see Fig. 8).

time \( t_h \), not the Universe’s true age \( t_0 \). In Fig. 6, the distinction between \( t_h \) and \( t_0 \) is indicated primarily through the termination points of the \( R_h \) and \( \langle \omega \rangle \) curves, which extend past the vertical dotted line at \( t = t_h \).

Ironically, this unexpected result has several important consequences, such as offering an explanation for the early appearance of supermassive black holes\(^{29}\) (at a redshift \( > 6 \)), and the glaring deficit of dwarf halos in the local group.\(^{30}\) Both of these long-standing problems in cosmology would be resolved if the Universe were older. Supermassive black holes would have had much more time (\( 4–5 \) billion years) to form than current thinking allows (i.e., only \( \sim 800 \) million years), and dwarf halos would correspondingly have had more time to merge hierarchically, depleting the lower mass end of the distribution.

The matter and dark energy densities corresponding to the \( \omega_d = -2/3 \) scaling solution are shown as functions of cosmic time in Fig. 7, along with the evolution of the scale factor \( a(t) \). Here too, the “coincidence problem” does not exist, and the flatness problem is resolved since \( R_h/a \to 0 \) as a result of Eqn. (14), so that \( kR_h^2/a^2 \to 0 \) as \( t \to \infty \) in Eqn. (13). Thus, the constant in Eqn. (16) should be \( \approx 1 \) at late times, regardless of the value of \( k \). Very importantly, this model fits the Type Ia supernova data very well, as shown in Fig. 8. The best fit corresponds to \( \Delta = 25.26 \), with a reduced \( \chi^2 = 1.001 \) for \( 180 - 1 = 179 \) d.o.f.

5. Concluding Remarks

Our main goal in this study has been to examine what we can learn about the nature of dark energy from a consideration of the cosmic horizon \( R_h \) and its evo-
Fig. 7. Same as Fig. 2, except for a “scaling solution” in which $\rho_d \propto \rho_m$ and $\omega_d = -2/3$.

A principal outcome of this work is the realization that the so-called “coincidence” problem in the standard model is actually more severe than previously thought. We have found that in a ΛCDM universe, $R_h \to c t_0$ only once, and according to the observations, this must be happening right now. The unlikelihood of this occurrence is an indication that dark energy almost certainly is not due to a cosmological constant. Other issues that have already been discussed extensively in the literature, such as the fact that the vacuum energy in quantum theory should greatly exceed the required value of $\Lambda$, only make this argument even more compelling. Of course, this rejection of the cosmological-constant hypothesis then intensifies interest into the question of why we don’t see any vacuum energy at all, but this is beyond the scope of the present paper.

Many alternatives to a cosmological constant have been proposed over the past decade but, for the sake of simplicity, we have chosen in this paper to focus our attention on scaling solutions. The existence of such cosmologies has been discussed extensively in the literature, within the context of standard General Relativity, braneworlds (Randall-Sundrum and Gauss-Bonnett), and Cardassian scenarios, among others.\textsuperscript{31,32,33}

Our study has shown that scaling solutions fit the Type Ia supernova data at least as well as the basic ΛCDM cosmology, but they go farther in simultaneously solving several conundrums with the standard model. As long as the time-averaged value of $\omega$ is less than $-1/3$, they eliminate both the coincidence and flatness problems, possibly even obviating the need for a period of rapid inflation in the early universe.\textsuperscript{34,35}

But most importantly, as far as this study is concerned, scaling solutions account very well for the observed fact that $R_h \approx c t_0$. If $\langle \omega \rangle = -1/3$ exactly, then $R_h(t) = c t$ for all cosmic time, and therefore the fact that we see this condition in the present Universe is no coincidence at all. On the other hand, if $\langle \omega \rangle < -1/3$, scaling
Fig. 8. Same as Figure 5, except for a “scaling solution” in which \( \rho_d \propto \rho_m \) and \( \omega_d = -2/3 \). This type of universe provides the best fit to the Type Ia supernova data, with a reduced \( \chi^2 \approx 1.001 \) for \( 180 - 1 = 179 \) d.o.f. The best fit corresponds to \( \Delta = 25.26 \).

solutions fit the Type Ia supernova data even better, but then we have to accept the conclusion that the Universe is older than the horizon time \( t_h \equiv R_h/c \). According to our calculations, which produce a best fit to the supernova data for \( \langle \omega \rangle = -0.47 \) (corresponding to a dark energy equation of state with \( \omega_d = -2/3 \)), the age of the Universe should then be \( t_0 \approx 16.9 \) billion years. This may be surprising at first, but the fact of the matter is that such an age actually solves other major problems in cosmology, including the (too) early appearance of supermassive black holes, and the glaring deficit of dwarf halos in the local group of galaxies.

When thinking about a dynamical dark energy, it is worth recalling that scalar fields arise frequently in particle physics, including string theory, and any of these may be appropriate candidates for this mysterious new component of \( \rho \). Actually, though we have restricted our discussion to equations of state with \( \omega_d \geq -1 \), it may even turn out that a dark energy with \( \omega_d < -1 \) is providing the Universe’s acceleration. Such a field is usually referred to as a Phantom or a ghost. The simplest explanation for this form of dark energy would be a scalar field with a negative kinetic energy. However, Phantom fields are plagued by severe quantum instabilities, since their energy density is unbounded from below and the vacuum may acquire normal, positive energy fields. We have therefore not included theories with \( \omega_d < -1 \) in our analysis here, though a further consideration of their viability may be warranted as the data continue to improve.

On the observational front, the prospects for confirming or rejecting some of the ideas presented in this paper look very promising indeed. An eagerly anticipated mission, SNAP, will constrain the nature of dark energy in two ways. First, it will observe deeper Type Ia supernovae. Second, it will attempt to use weak gravitational lensing to probe foreground mass structures. If selected, SNAP should be launched...
by 2020. The Planck CMB satellite, an already funded mission, probably won’t have the sensitivity to measure any evolution in $\omega_d$, but it may be able to tell us whether or not $\omega_d = -1$.

Finally, we may be on the verge of uncovering a class of sources other than Type Ia supernovae to use for dark-energy exploration. Type Ia supernovae have greatly enhanced our ability to study the Universe’s expansion out to a redshift $\sim 2$. But this new class of sources may possibly extend this range to values as high as 5–10. According to Ref. 39, Gamma Ray Bursts (GRBs) have the potential to detect dark energy with a reasonable significance, particularly if there was an appreciable amount of it at early times, as suggested by scaling solutions. It is still too early to tell if GRBs are good standard candles, but since differences between $\Lambda$CDM and dynamical dark energy scenarios are more pronounced at early times (see Figs. 5 and 8), GRBs may in the long run turn out to be even more important than Type Ia supernovae in helping us learn about the true nature of this unexpected “third” form of energy.

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