Normal glow discharge between curvilinear electrodes

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Abstract. A diffusion-drift calculation model of a normal glow discharge between curvilinear electrodes is presented. The verification of the created computational model has been performed using the example of a normal glow discharge between flat electrodes. Results of numerical simulation are presented of normal glow discharge between electrodes in the form of spherical segments at pressure $p = 5$Torr and emf of power supply in the region of $1000 \div 1750$ V.

1. Introduction

A normal steady-state glow discharge (NGD) is investigated in the laboratory between flat (disk) electrodes spaced a few centimeters apart. In the normal glow discharge a column of the gas-discharge plasma should be located far from the boundaries of the electrodes. Such a discharge burns in the self-organizing mode of transverse dimensions, and the current density at the cathode is predicted with good accuracy by the one-dimensional theory of Engel and Steenbeck [1-3]

$$V_n = \frac{3B}{A} \ln \left(1 + \frac{1}{\gamma}\right), \text{V},$$

$$\frac{j}{\rho^2} = 5.92 \times 10^{-14} \frac{AB^2 \mu_1 p (1 + \gamma)}{\ln (1 + \gamma^{-1})}, \text{A/(cm}^2 \times \text{Torr)},$$

$$d_n p = \frac{3.78}{A} \ln (1 + \gamma^{-1}), \text{cm} \times \text{Torr},$$

where: $V_n$, $j_n$, $d_n$ are the voltage drop on the gas-discharge gap, current density, thickness of the cathode layer of a normal glow discharge; $\rho$ is the pressure in Torr; $\mu_1$ is the mobility of ions; $\gamma$ is the coefficient of secondary electron emission. It is assumed that the coefficient of impact ionization of molecules by electrons $\alpha$ is approximated as follows
\[ \alpha = \alpha \left( \frac{E}{p} \right) = Ap \exp \left( -\frac{B}{E/p} \right), \text{ cm}^{-1}, \]

where \( A, B \) are the approximating coefficients; \( E \) is the electric field strength.

The increase in the total current through the gas-discharge gap (typical values from units to tens of mA) in such a glow discharge leads to an increase in the transverse dimensions of the current column, but the current density in the near-axis region remains practically unchanged. This phenomenon is called the law of normal current density \([1, 2]\) and has been studied in classical physics of gas discharge for more than 100 years \([3]\).

Since the 80s of the last century, the symmetrical normal glow discharge is the object of study by methods of computational physics \([4–6]\). Numerical simulation of this type of discharge proved to be very useful for the development of computational methods and algorithms for mathematical modeling. Subsequently, the problems were investigated of calculating the heating of a neutral gas, the dynamics of a glow discharge in a transverse and longitudinal magnetic field with induction up to \( \sim 1.0 \) T, taking into account the physical kinetics of excitation of \( \text{N}_2 \) vibrational states \([2, 7]\).

Localization of the discharge column of a normal glow discharge between flat electrodes is achieved in a computational experiment by setting the initial conditions. In a real physical experiment, it is not easy to achieve a stationary state far from the electrode boundaries \([8]\).

The subject of this paper is a computational study of the structure of a normal glow discharge between convex electrodes. Schematic of the task is shown in figure 1, as well as an external electric circuit consisting of a constant-current power supply with an emf \( \varepsilon \) and ohmic resistance \( R_0 \). The discharge gap is placed in a vacuum chamber. The investigated range of molecular nitrogen pressures is \( p = 1 \div 10 \text{ Torr} \).

![Figure 1. Schematic of the task.](image)

2. **Equations for the diffusion-drift model in curvilinear geometry**

The equations of the diffusion-drift model used to simulate a normal glow discharge are obtained from the system of equations for a multi-fluid and a multi-temperature partially ionized gas mixture (a detailed derivation of these equations is given in \([2]\))

\[
\text{div}E = -\varepsilon \left( n_e - n_{e0} \right) \tag{1} \\
\frac{\partial n}{\partial t} + \text{div} \Gamma_e = \alpha |\Gamma| - \beta n_n e \tag{2}
\]
\[ \frac{\partial n_e}{\partial t} + \text{div} \Gamma_e = \alpha |\Gamma| - \beta n_e n_i, \]  

(3)

where \( n_e, n_i \) are the concentrations of ions and electrons; \( \mathbf{E} \) is the vector of electric field strength with electric potential \( \varphi \)

\[ \mathbf{E} = - \nabla \varphi, \]  

(4)

\( \Gamma_e, \Gamma_i \) are the vectors of electron and ion flux densities,

\[ \Gamma_e = -n_e \mu_e \mathbf{E} - D_e \nabla n_e, \quad \Gamma_i = -n_i \mu_i \mathbf{E} + D_i \nabla n_i, \]  

(5)

\( \mu_e, \mu_i \) are the mobilities of electrons and ions; \( D_e, D_i \) are the diffusion coefficients of electrons and ions; \( \alpha, \beta \) are the coefficient of impact ionization of a neutral gas by electrons (the 1st Townsend coefficient) and the recombination coefficient; \( \varepsilon^* = 4\pi \varepsilon = 1.81 \times 10^{-6} \text{ V} \times \text{cm} \).

Boundary conditions for the solution of the boundary value problem (1)–(3) are

\[ x = 0: \quad \Gamma_{e,x} = \gamma \Gamma_{i,x}, \quad \frac{\partial n_e}{\partial x} = 0, \quad \varphi = 0, \]

\[ x = H: \quad \frac{\partial n_e}{\partial x} = 0, \quad n_i = 0, \quad \varphi = V, \]

\[ r = 0: \quad \frac{\partial n_e}{\partial r} = \frac{\partial n_i}{\partial r} = \frac{\partial \varphi}{\partial r} = 0, \]

\[ r = R_e: \quad \frac{\partial n_e}{\partial r} = \frac{\partial n_i}{\partial r} = \frac{\partial \varphi}{\partial r} = 0, \]  

(6)

where \( V \) is the voltage drop at the electrodes, determined from the electric circuit equation

\[ \varepsilon - V \frac{1}{eR_0} = 2\pi \int_0^R \Gamma_{e,n_e} r \, dr \]  

(7)

For curvilinear electrodes, the integral in (7) should be calculated along the surface of the electrode with projection of the electron flux density \( \Gamma_{e,n_e} \) on the local normal to the surface \( n_{s,\varphi} \).

Condition (7) determines the electric current through the anode. For a stationary glow discharge, the total electric current through the gas-discharge gap is a constant value and can be determined from the following equation:

\[ \varepsilon - V \frac{1}{eR_0} = 2\pi \int_0^R \Gamma_{e,n_e} r \, dr, \]

where \( n_{s,\varphi} \) is the local normal to surface of cathode.

We also note that when analyzing the transient processes of establishing a stationary state, one should take into account the effect of accumulation of charges on electrodes (see [4] for more details).

The closing relations for the problem being solved are given for molecular nitrogen [2]

\[ \mu_e \rho = 4.4 \times 10^5, \quad \mu_i \rho = 1.45 \times 10^3 \text{ Torr} \times \text{cm}^2/\text{V} \times \text{s}, \quad \beta = 2 \times 10^{-7}, \text{ cm}^3/\text{s}, \]

\[ \frac{\alpha}{\rho} = A \exp \left( - \frac{B}{E/\rho} \right), 1/(\text{cm} \times \text{Torr}), \quad A = 12, \quad (\text{cm} \times \text{Torr})^{-1}, \quad B = 342, \quad V/(\text{cm} \times \text{Torr}), \]
\[ D_x = \mu_T \varepsilon, \quad D_\sigma = \mu_T \varepsilon, \quad T = 1 \text{eV}, \quad T_\sigma = 0.026 \text{eV} \]

The construction of a computational model of a normal glow discharge in the computational domain with curvilinear boundaries requires the introduction of a curvilinear coordinate system \((\xi, \eta)\) connected by single-valued transformations with the original cylindrical coordinate system \((x, r)\). The method of transforming the solved system of equations that is adapted to curvilinear surfaces is given in [10] (Ch.14). Below these transformations are given for the system of the Poisson equation and equations of the diffusion-drift model.

3. **Transformation of the Poisson equation for the electric potential and the continuity equations for electrons and ions**

The initial Poisson equation in an orthogonal cylindrical coordinate system is formulated as following:

\[
\frac{\partial}{\partial x} \left( \frac{\partial \varphi}{\partial x} \right) + \frac{\partial}{\partial r} \left( \frac{\partial \varphi}{\partial r} \right) + \frac{\partial}{\partial \varphi} (\varphi) = \Phi (r, x), \tag{8}
\]

where \(\Phi (r, x) = 4\pi e (n_e - n_i), \chi = 0\) for plane discharge, \(\chi = 1\) for axisymmetric discharge.

Input coordinate system \((\xi, \eta)\) must be uniquely linked to the original coordinate system \((x, r)\), therefore (8) can be rewritten in the form (hereinafter \(y = r\))

\[
\frac{\partial}{\partial \xi} \left( \frac{\partial \varphi}{\partial \xi} \right) + \frac{\partial}{\partial \eta} \left( \frac{\partial \varphi}{\partial \eta} \right) + \frac{\partial}{\partial \xi} \left( \frac{\partial \varphi}{\partial \xi} \right) + \frac{\partial}{\partial \eta} \left( \frac{\partial \varphi}{\partial \eta} \right) + \frac{\partial}{\partial \xi} \left( \frac{\partial \varphi}{\partial \xi} \right) + \frac{\partial}{\partial \eta} \left( \frac{\partial \varphi}{\partial \eta} \right) + \frac{\partial}{\partial \xi} \left( \frac{\partial \varphi}{\partial \xi} \right) + \frac{\partial}{\partial \eta} \left( \frac{\partial \varphi}{\partial \eta} \right) + \frac{\partial}{\partial \xi} \left( \frac{\partial \varphi}{\partial \xi} \right) + \frac{\partial}{\partial \eta} \left( \frac{\partial \varphi}{\partial \eta} \right) + \frac{\partial}{\partial \xi} \left( \frac{\partial \varphi}{\partial \xi} \right) + \frac{\partial}{\partial \eta} \left( \frac{\partial \varphi}{\partial \eta} \right) + \frac{\partial}{\partial \xi} \left( \frac{\partial \varphi}{\partial \xi} \right) + \frac{\partial}{\partial \eta} \left( \frac{\partial \varphi}{\partial \eta} \right) + \frac{\partial}{\partial \xi} \left( \frac{\partial \varphi}{\partial \xi} \right) + \frac{\partial}{\partial \eta} \left( \frac{\partial \varphi}{\partial \eta} \right) + \frac{\partial}{\partial \xi} \left( \frac{\partial \varphi}{\partial \xi} \right) + \frac{\partial}{\partial \eta} \left( \frac{\partial \varphi}{\partial \eta} \right) = \Phi (x, y), \tag{9}
\]

Division on Jacobian of the transformation \(J\) and differentiation by parts allows us to obtain the equation

\[
\frac{\partial}{\partial \xi} \left( \frac{\xi^\prime \partial \varphi}{\xi^\prime} \right) - \frac{\partial}{\partial \xi} \left( \frac{\xi \partial \varphi}{\xi} \right) + \frac{\partial}{\partial \eta} \left( \frac{\eta^\prime \partial \varphi}{\eta^\prime} \right) + \frac{\partial}{\partial \eta} \left( \frac{\eta \partial \varphi}{\eta} \right) = \Phi (x, y), \tag{10}
\]

Taking into account the following relations between components of the Jacobian of the transformation

\[
y_\eta = \frac{x_\eta}{J}, \quad -y_\xi = \frac{x_\eta}{J}, \quad -x_\eta = \frac{x_\eta}{J}, \quad x_\xi = \frac{x_\xi}{J}, \tag{11}
\]
notice, that
\[-\varepsilon_x \frac{\partial \varphi}{\partial \xi} y_{\eta\xi} - \eta_x \frac{\partial \varphi}{\partial \eta} y_{\eta\xi} + \xi_x \frac{\partial \varphi}{\partial \xi} y_{\eta\xi} + \xi_x \frac{\partial \varphi}{\partial \eta} y_{\eta\xi} + \eta_x \frac{\partial \varphi}{\partial \eta} y_{\eta\xi} + \eta_x \frac{\partial \varphi}{\partial \xi} y_{\eta\xi} +
\]
\[+ \xi_x \frac{\partial \varphi}{\partial \eta} x_{\eta\xi} + \eta_x \frac{\partial \varphi}{\partial \xi} x_{\eta\xi} - \xi_x \frac{\partial \varphi}{\partial \xi} x_{\eta\xi} - \eta_x \frac{\partial \varphi}{\partial \eta} x_{\eta\xi} \equiv 0, \quad (12)\]

therefore
\[
\frac{\partial}{\partial \xi} \left[ \left( \frac{\xi^2 + \xi^2}{J} \right) \frac{\partial \varphi}{\partial \xi} \right] + \frac{\partial}{\partial \xi} \left[ \left( \frac{\xi^2 + \xi^2}{J} \right) \frac{\partial \varphi}{\partial \eta} \right] + \frac{\partial}{\partial \eta} \left[ \left( \frac{\xi^2 + \xi^2}{J} \right) \frac{\partial \varphi}{\partial \xi} \right] + \frac{\partial}{\partial \eta} \left[ \left( \frac{\xi^2 + \xi^2}{J} \right) \frac{\partial \varphi}{\partial \eta} \right] +
\]
\[+ \frac{\partial}{\partial \eta} \left[ \left( \frac{\xi^2 + \xi^2}{J} \right) \varphi \right] + \chi \left( \varphi, \frac{\partial \varphi}{\partial \xi} + \eta, \frac{\partial \varphi}{\partial \eta} \right) = \Phi \quad (13)\]

The continuity equations for the electrons and ions (2) and (3) are similar, so consider the transformation of only the equation for determining the electron concentrations. In equation (2), we use the relation for the electron flux density (5), as well as the relation between the electric field strength and the electric potential (4). We rewrite (2) in the variables \((\xi, \eta)\), then divide it by Jacobian \(J\) and using partial differentiation, receive
\[
\frac{\partial}{\partial t} \left( \frac{n}{J} \right) + \frac{\partial}{\partial \xi} \left( n, \mu, \xi, \frac{\partial \varphi}{\partial \xi} \right) + \frac{\partial}{\partial \xi} \left( n, \mu, \xi, \frac{\partial \varphi}{\partial \eta} \right) + \frac{\partial}{\partial \eta} \left( n, \mu, \xi, \frac{\partial \varphi}{\partial \xi} \right) + \frac{\partial}{\partial \eta} \left( n, \mu, \xi, \frac{\partial \varphi}{\partial \eta} \right) +
\]
\[+ \frac{\partial}{\partial \eta} \left( n, \mu, \xi, \varphi \right) + \frac{\partial}{\partial \eta} \left( n, \mu, \xi, \varphi \right) + \frac{\partial}{\partial \eta} \left( n, \mu, \xi, \varphi \right) + \frac{\partial}{\partial \eta} \left( n, \mu, \xi, \varphi \right) +
\]
\[+ \frac{\partial}{\partial \eta} \left( n, \mu, \xi, \varphi \right) + \frac{\partial}{\partial \eta} \left( n, \mu, \xi, \varphi \right) + \frac{\partial}{\partial \eta} \left( n, \mu, \xi, \varphi \right) + \frac{\partial}{\partial \eta} \left( n, \mu, \xi, \varphi \right) =
\]
\[= \frac{\partial}{\partial \xi} \left( \frac{\xi^2}{J} \frac{\partial n}{\partial \xi} \right) + \frac{\partial}{\partial \eta} \left( \frac{\xi^2}{J} \frac{\partial n}{\partial \eta} \right) + D_x \left( \frac{\xi^2}{J} \frac{\partial n}{\partial \xi} + \eta \frac{\partial n}{\partial \eta} \right) +
\]
\[+ \frac{\partial}{\partial \eta} \left( \frac{\xi^2}{J} \frac{\partial n}{\partial \xi} + \eta \frac{\partial n}{\partial \eta} \right) + D_x \left( \frac{\xi^2}{J} \frac{\partial n}{\partial \xi} + \eta \frac{\partial n}{\partial \eta} \right) +
\]
\[+ \frac{\partial}{\partial \eta} \left( \frac{\xi^2}{J} \frac{\partial n}{\partial \xi} + \eta \frac{\partial n}{\partial \eta} \right) + D_x \left( \frac{\xi^2}{J} \frac{\partial n}{\partial \xi} + \eta \frac{\partial n}{\partial \eta} \right) +
\]
By virtue of relations (11), this equation is simplified as follows

$$\frac{\partial n}{\partial t} + \frac{\partial }{\partial \xi} \left( n \mu_e \frac{\xi_e^2 + \xi_\eta^2}{J} \frac{\partial \varphi}{\partial \xi} + \frac{\partial }{\partial \eta} \left( n \mu_e \frac{\eta_e^2 + \eta_\eta^2}{J} \frac{\partial \varphi}{\partial \eta} \right) \right) +$$

$$\frac{\partial }{\partial \xi} \left( n \mu_e \frac{\xi_e \xi_\eta + \xi_\xi \eta_e}{J} \frac{\partial \varphi}{\partial \eta} \right) + \frac{\partial }{\partial \eta} \left( n \mu_e \frac{\xi_e \eta_\eta + \xi_\eta \eta_e}{J} \frac{\partial \varphi}{\partial \eta} \right) + n \mu_e \frac{\xi_e \eta_\eta + \xi_\eta \eta_e}{J} \frac{\partial \varphi}{\partial \eta}$$

$$= \frac{\partial }{\partial \xi} \left( D_e \frac{\xi_e^2 + \xi_\eta^2}{J} \frac{\partial n}{\partial \xi} \right) + \frac{\partial }{\partial \eta} \left( D_e \frac{\eta_e^2 + \eta_\eta^2}{J} \frac{\partial n}{\partial \eta} \right) +$$

$$\frac{\partial }{\partial \xi} \left( D_e \frac{\xi_e \xi_\eta + \xi_\xi \eta_e}{J} \frac{\partial n}{\partial \eta} \right) + \frac{\partial }{\partial \eta} \left( D_e \frac{\xi_e \eta_\eta + \xi_\eta \eta_e}{J} \frac{\partial n}{\partial \eta} \right) +$$

$$\frac{\beta}{J} n \frac{\partial n}{\partial \eta}$$

(15)

To simplify this equation, we introduce the notation

$$a = \frac{\eta_e^2 + \eta_\eta^2}{J}, \quad b = \frac{\xi_e^2 + \xi_\eta^2}{J}, \quad g = \frac{\xi_e \eta_\eta + \xi_\eta \eta_e}{J}$$

(16)

After this, the equation of continuity for electrons in the new coordinate system takes the form

$$\frac{\partial }{\partial t} \left( \frac{n}{J} \right) + \frac{\partial }{\partial \xi} \left( n \mu_e b \frac{\partial \varphi}{\partial \xi} + \frac{\partial }{\partial \eta} \left( n \mu_e a \frac{\partial \varphi}{\partial \eta} \right) \right) +$$

$$\frac{\partial }{\partial \eta} \left( n \mu_e g \frac{\partial \varphi}{\partial \eta} \right) + \frac{\partial }{\partial \eta} \left( \frac{\partial n}{\partial \eta} \right) =$$

$$\frac{\partial }{\partial \xi} \left( D_e b \frac{\partial n}{\partial \xi} \right) + \frac{\partial }{\partial \eta} \left( D_e a \frac{\partial n}{\partial \eta} \right) + \frac{\partial }{\partial \xi} \left( D_e g \frac{\partial n}{\partial \xi} \right) +$$

$$\frac{\partial }{\partial \eta} \left( D_e g \frac{\partial n}{\partial \eta} \right) + \frac{\beta}{J} n \frac{\partial n}{\partial \eta}$$

(17)

We note that, in spite of the considerable complication of the equation in introduced coordinate system (\(\xi, \eta\)), the construction of the finite-difference scheme for (17) proves to be a simpler problem than for the initial equation (2). For the purpose of constructing a finite-difference scheme, we use the grid template shown in figure 2. To obtain the finite-difference relations by the finite-volume method the integrating of equation (9) is used on the volume selected in figure 2.
\[ \text{Vol}_{i,j} = \int_{\eta_{i-1}}^{\eta_{i+1}} \int_{\xi_{j-1}}^{\xi_{j+1}} \{ \} \, d\xi = p_j q_j, \]  
\[ p_i = \frac{1}{2} \left( \eta_{i-1} - \eta_{i+1} \right), \quad q_j = \frac{1}{2} \left( \xi_{j-1} - \xi_{j+1} \right) \]  

where \( \xi \) and \( \eta \) are the coordinates in the grid, and \( p \) and \( q \) are the weights.

\[ \text{Figure 2. Difference grid template.} \]

This leads to the five-point finite-difference equation of the following form

\[ A_{i,j} U_{i-1,j} + B_{i,j} U_{i+1,j} - C_{i,j} U_{i,j} + \overline{A}_{i,j} U_{i,j-1} + \overline{B}_{i,j} U_{i,j+1} + F_{i,j} = 0 \]  
\[ \text{where} \]

\[ U_{i,j} = \varphi_{i,j}, \]

\[ A_{i,j} = \frac{a_{i,j/2} + g_{i,j/2}}{p_j p_i}, \quad B_{i,j} = \frac{a_{i+1,j/2} + g_{i+1,j/2}}{p_j p_i} + \chi \left( \frac{x}{y} \right)_{i,j} \frac{1}{p_i}, \]

\[ \overline{A}_{i,j} = \frac{b_{i,j-1/2} + g_{i,j-1/2}}{q_j q_i}, \quad \overline{B}_{i,j} = \frac{b_{i+1,j-1/2} + g_{i+1,j-1/2}}{q_j q_i} + \chi \left( \frac{x}{y} \right)_{i,j} \frac{1}{q_j}, \]

\[ F_{i,j} = \frac{\chi^*}{J_{i,j}} \left( N_e - N_0 \right) N_0 L^2 \]  
\[ \text{For the convenience of numerical implementation, the functions and variables are dimensionless as follows} \]

\[ N_e = \frac{n_e}{N_0}, \quad N_r = \frac{n_r}{N_0}, \quad U = \frac{\varphi}{\epsilon}, \quad X = \frac{x}{L}, \quad Y = \frac{y}{L} \]

where \( N_0 \) is the characteristic concentration of electrons in the positive column of the glow discharge (\( N_0 = 10^9 \text{ cm}^{-3} \)); \( L \) is the distance between the electrodes.

In (20), (21) we used the variables

\[ a_{i,j/2} = \frac{1}{2} \left( a_{i,j} + a_{i+1,j} \right), \quad b_{i,j+1/2} = \frac{1}{2} \left( b_{i,j} + b_{i,j+1} \right). \]
$$g_{i,i+2/2,j} = \frac{1}{2}(g_{i,j} + g_{i+2,j}), \quad g_{i,j+2/2} = \frac{1}{2}(g_{i,j} + g_{i,j+1}),$$

$$p^+_i = \eta_{i+1} - \eta_i, \quad p^-_i = \eta_i - \eta_{i-1}, \quad q^+_j = \xi_{j+1} - \xi_j, \quad q^-_j = \xi_j - \xi_{j-1}$$

When obtaining finite-difference equations for electron concentrations, the continuity equation (17) is integrated according to the volume shaded in figure 2 and over a time interval $\tau = t^{p+1} - t^p$

$$Vol_{i,p} = \int_i d\eta \int_{\xi_{i+1/2}}^{\xi_{i+1/2}} \frac{d\xi}{n \rho J} = \tau p q^p$$

This leads to a five-point equation of the form (20), where

$$U_{i,j} = n_{e,i,j},$$

$$A_{i,j} = \frac{R^e_i}{p_i} + \left[D_e\left(a + g\right)\right]_{i+1/2,j}, \quad B_{i,j} = -\frac{R^e_i}{p_i} + \left[D_e\left(a + g\right)\right]_{i-1/2,j} + \frac{\eta D_e}{yJ} \left(\eta n D_e\right)_{i,j},$$

$$C_{i,j} = \frac{1}{\tau J_{i,j}} + A_{i,j} + B_{i,j} + \frac{\eta D_e}{yJ} \left(\eta n D_e\right)_{i,j} + \frac{\mu D_e}{2q_i}, \quad \frac{\mu D_e}{2q_j}, \quad \frac{\mu D_e}{2q_j},$$

$$\bar{A}_{i,j} = \frac{S^e_i}{q_j} + \left[D_e\left(b + g\right)\right]_{i+1/2,j}, \quad \frac{\bar{B}_{i,j} = -S^e_i}{q_j} + \left[D_e\left(b + g\right)\right]_{i-1/2,j} + \frac{\xi D_e}{yJ} \left(\eta n D_e\right)_{i,j},$$

$$F_{i,j} = \frac{U_{i,j}^p}{\tau J_{i,j}} + \phi_\varepsilon, \quad \phi_\varepsilon = \left(\frac{a}{J} \left|\Gamma_1 - \frac{p}{J} \eta \eta\right|\right) \frac{\mu \varepsilon}{\varepsilon \mu_r N_0} \left(\xi D_e\right)_{i,j}$$

(22)

Here we use the following notation for the projection of the electron velocity in the coordinate system $(\xi, \eta)$

$$R_e = V_e + Z_e, \quad S_e = V_e + W_e, \quad V_e = \mu \eta a \frac{\phi}{\eta}, \quad U_e = \mu \eta b \frac{\phi}{\eta}, \quad Z_e = \mu \eta q \frac{\phi}{\eta}, \quad W_e = \mu \eta q \frac{\phi}{\eta}$$

Note that the velocity components $V_e$ and $U_e$ correspond to the drift velocities along the coordinate lines, and the velocity components $Z_e$ and $W_e$ correspond to the contribution of the corresponding cross components. The finite-difference equation for ions is determined by the same relationships with the difference that

$$V_i = -\mu a \frac{\phi}{\eta}, \quad U_i = -\mu b \frac{\phi}{\eta}, \quad Z_i = -\mu q \frac{\phi}{\eta}, \quad W_i = -\mu g \frac{\phi}{\eta}$$

in accordance with the definition of charged particle flux densities (5).

Stability in the numerical integration of finite-difference equations of continuity of electrons and ions is provided by the following relationships:

$$R_{p,L}^e = \frac{1}{2}(R_{p,L} + |R_{p,L}|), \quad S_{p,L}^e = \frac{1}{2}(S_{p,L} + |S_{p,L}|)$$

and also the one-sided derivatives of the drift and diffusion terms in (21) and (22), proportional to $1/(yJ)$. 

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4. Results of numerical modelling of a normal glow discharge

Based on the calculation model described in the previous section, a computer code was created to calculate the two-dimensional structure of a normal glow discharge. Trial test calculations were performed using the example of a well-studied normal glow discharge in molecular nitrogen between plane electrodes at $p = 5$ Torr, $\varepsilon = 1000$ V, $R_0 = 300$ kΩ, $\gamma = 0.33$, $L = 2$ cm [2]. Two-dimensional fields of concentrations of ions and electrons for axisymmetric ($\chi = 1$) and flat ($\chi = 0$) glow discharges are shown in figure 3. The data presented are in good agreement with the results of calculations [2]. The calculations were performed on a $71 \times 71$ grid. In these figures, the cathode and anodic layers are well identified, as well as the quasi-neutral positive column.

![Figure 3](image)

**Figure 3.** Distribution of the concentrations of ions ($a, b$) and electrons ($c, d$) in an axisymmetric ($a, c$) and flat ($b, d$) discharges between plane electrodes at $p = 5$ Torr, $\varepsilon = 1000$ V, $R_0 = 300$ kΩ, $\gamma = 0.33$, $L = 2$ cm.

Figures 4 and 5 show the axial distributions of electron and ion concentrations along the axis of cylindrical and planar discharges, as well as ionization and recombination rates. The latter are normalized by $N_0\mu_e e/L^2$, having the dimension 1/(cm$^3$·s). The concentrations of ions in the cathode layer for axisymmetric and flat discharges are close, and the concentration of charged particles in the positive column of the axisymmetric discharge is somewhat higher. It is seen from figure 5 that the cathode and anode layers are the main regions of generation of charged particles.
Calculations of an axisymmetric normal glow discharge line between curvilinear electrodes were performed for similar conditions discussed above, but in the range \( \varepsilon = 1000 \text{ to } 1750 \text{ V} \). The radius of curvature of the cathode and the anode were \( R_c = 5 \text{ cm} \) and \( R_a = 10 \text{ cm} \), respectively. Configuration of the computational domain with the number of nodes \( 141 \times 141 \) is shown in figure 1.

![Figure 4](image1.png)

**Figure 4.** The distribution of electron and ion concentrations along the symmetry axis (a) and the symmetry plane (b) for the conditions in the discharge in figure 3.

![Figure 5](image2.png)

**Figure 5.** Distribution of ionization and recombination rates along the symmetry axis (a) and the symmetry plane (b) for the conditions in the discharge in figure 3.

Figures 6 and 7 show the fields of ion and electron concentrations, electric potential and reduced field \( E/p \) in discharges at \( \varepsilon = 1000 \text{ V} \) and \( \varepsilon = 1750 \text{ V} \), respectively. With increasing \( \varepsilon \), the transverse dimensions of the glow discharge column increase.

Also, as in the case of a discharge between plane electrodes, the cathode layer (region of increased ion concentration) is characterized by a potential well of the electric potential (figures 6, c and 7, c) and increased values of the reduced field (figures 6, d and 7, d).
The axial distributions of the concentrations of charged particles in the two glow discharges, the distribution of the electric potential $\phi$ of the axial component of the electric field and the reduced field are shown in figures 8–9. Figures 10, $a$, $b$ show the rates of ionization and recombination. We note the main differences from the normal glow discharge between flat electrodes. In the axial distribution of electron and ion concentrations, in a positive column, a local maximum appears.

![Diagram](image)

**Figure 6.** Distribution of concentrations of ions $N_i = n_i/N_0$ ($a$) and electrons $N_e = n_e/N_0$ ($b$), electric potential ($c$) and reduced field $EDP = E/p$ in $V/\text{cm}\times\text{Torr}$ ($d$) between segmental electrodes at $p = 5$ Torr, $\varepsilon = 1000$ V, $R_0 = 300$ k$\Omega$, $\gamma = 0.33$, $L = 2$ cm.

An analysis of the distributions of the rates of ionization and recombination (figure 10, $a$, $b$) shows that recombination prevails in the vicinity of this local maximum. The rate of ionization in the cathode layer is several times greater than the corresponding value for the discharge between plane electrodes (compare figures 10, $a$ and 5, $a$). With increasing $\varepsilon$, the concentration of ions and electrons in the positive column increases (figure 8, $a$, $c$).

An important result of the preliminary study of the structure of a normal glow discharge between curved electrodes was the demonstration of a satisfactory fulfillment of the law of the normal current.
density in figure 11. It can be seen that variation of $\varepsilon$ in the range of 1000÷1750 V resulted in an insignificant change in the current density at the cathode and anode.

Figure 7. Distribution of concentrations of ions $N_i = n_i/N_0$ (a) and electrons $N_e = n_e/N_0$ (b), electric potential (c) and reduced field $EDP = E/p$ in V/cm×Torr (d) between segmental electrodes at $p = 5$ Torr, $\varepsilon = 1750$ V, $R_0 = 300 \Omega$, $\gamma = 0.33$, $L = 2$ cm.
Figure 8. The distribution of electron and ion concentrations along the symmetry axis (a) and the symmetry plane (b) for $\varepsilon = 1000$ V (a) and $\varepsilon = 1750$ V (b).

Figure 9. The axial distributions of the electric potential $F_i = \phi$ (in V), the axial component of the electric field strength $E_x$, V/cm, and the reduced field $E/p$ (in V/cm×Torr) along the symmetry axis at $\varepsilon = 1000$ V (a) and $\varepsilon = 1750$ V (b).

Figure 10. Axial distributions of the rate of ionization and recombination at $\varepsilon = 1000$ V (a) and $\varepsilon = 1750$ V (b), referred to the value $N_0 \mu_e \varepsilon / L^2$. 
Figure 11. Current density at the cathode and anode as $\varepsilon$ varies in the range $\varepsilon = 1000 \div 1750$ V.

Conclusion
A diffusion-drift calculation model of a normal glow discharge between curvilinear electrodes is presented.

The verification of the created computational model is performed using the example of a normal glow discharge between flat electrodes.

Results of numerical simulation are presented of normal glow discharge between electrodes in the form of spherical segments with radii $R_c = 5\text{ cm}$ and $R_a = 10\text{ cm}$ at $p = 5$ Torr, $\varepsilon = 1000 \div 1750$ V, $R_0 = 300 \text{ k\Omega}$, $\gamma = 0.33$, distance between electrodes $L = 2\text{ cm}$.

A satisfactory fulfillment of the law of the normal current density for the investigated discharge is shown.

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References
[1] Raizer Yu P 1991 *Gas Discharge Physics* (Springer-Verlag) 449 p
[2] Surzhikov S T 2013 *Computational Physics of Electric Discharges in Gas Flows* Walter de Gruyter GmbH (Berlin/Boston) 428 p
[3] Engel A Von, Steenbeck M 1932 *Elektrische Gasentladungen* (Berlin:Springer)
[4] Gladush G G and Samokhin A A 1981 A numerical investigation of current lacing on the electrodes in a glow discharge *J. of Applied Mechanics and Technical Physics* 22 Issue 5 pp 608–615
[5] Petrucev A S, Surzhikov S T and Shang J S 2006 Chemical processes in air glow discharge for aerospace applications *AIAA Paper 06-1460* 11 p
[6] Petrucev A S, Surzhikov S T and Shang J S 2006 A two-dimensional model of glow discharge in view of vibrational excitation of molecular nitrogen *High Temperature* 44 (6) pp 814–822
[7] Surzhikov S T and Shang J S 2004 Two-component plasma model for two-dimensional glow discharge in magnetic field *J. of Computational Physics* 199 pp 437–464
[8] Kotov M A, Kozlov P V, Ruleva L B, Solodovnikov S I, Surzhikov S T and Tovstonog V A 2017 The spectral characteristic investigations of normal glow discharge *J. of Physics: Conf. Series* **815** 012026