Diffraction phenomenology with massive gluons:
some recent developments

E.G.S. Luna

Instituto de Física e Matemática, Universidade
Federal de Pelotas, 96010-900, Pelotas, RS, Brazil

Abstract

In this talk we introduce the main features of a QCD-based model in which the coupling $\alpha_s$ is constrained by an infrared mass scale. We show recent applications of this model to hadron-hadron collisions, gap survival probability calculations, and soft gluon resummation techniques. These results indicate a smooth transition from nonperturbative to perturbative behaviour of the QCD.
I. INTRODUCTION

At high energies the soft and the semihard components of the scattering amplitude are closely related \[1, 2\], and it becomes important to distinguish between semihard gluons, which participate in hard parton-parton scattering, and soft gluons, emitted in any given parton-parton QCD radiation process. A class of models based on QCD incorporate soft and semihard processes in the treatment of high-energy hadronic interactions using a formulation compatible with analyticity and unitarity constraints \[3–7\]. In this talk we present a QCD-based model \[7\] in which the coupling \(\alpha_s\) is constrained by the value of the so called “dynamical gluon mass”, whose existence is strongly supported by recent QCD lattice simulations. This frozen coupling, obtained by means of the pinch technique in order to derive a gauge invariant Schwinger-Dyson equation for the gluon propagator \[8, 9\], has been adopted in many phenomenological studies \[7, 10–17\]. More specifically, we discuss some recent applications of the model to hadron-hadron collisions, gap survival probability calculations, and soft gluon resummation techniques. These results indicate a smooth transition from nonperturbative to perturbative behaviour of the QCD.

II. HADRON-HADRON COLLISIONS

In the QCD-based (or “mini-jet”) models the increase of the total cross sections is associated with semihard scatterings of partons in the hadrons \[3, 7\]. The energy dependence of the cross sections is driven especially by gluon-gluon scattering processes, where the behaviour of the gluon distribution function at small \(x\) exhibits the power law \(g(x, Q^2) \sim x^{-J}\). Since gluon-gluon subprocesses are potentially divergent at small transferred momenta, it becomes important to regulate this behaviour by introducing a mass scale which separates the perturbative from the non-perturbative QCD region. In the so called “DGM” model (Dynamical Gluon Mass model) \[7\], the dynamical gluon mass, as well as the infrared finite coupling constant associated to it \[18\], are the natural regulators for the cross sections calculations. In this eikonal model, the total cross section, the ratio \(\rho\) of the real to the imaginary part of the forward scattering amplitude, and the differential elastic scattering cross section are given by

\[
\sigma_{tot}(s) = 4\pi \int_0^\infty b db \left[ 1 - e^{-\chi(b,s)} \cos \chi_R(b, s) \right],
\]  

(1)
\[ \rho(s) = \frac{\text{Re}\{i \int b \, db \left[ 1 - e^{i \chi(b,s)} \right]\}}{\text{Im}\{i \int b \, db \left[ 1 - e^{i \chi(b,s)} \right]\}}, \]  

(2)

and

\[ \frac{d\sigma_{\text{el}}}{dt}(s,t) = \frac{1}{2\pi} \left| \int b \, db \left[ 1 - e^{i \chi(b,s)} \right] J_0(qb) \right|^2, \]  

(3)

respectively, where \( s \) is the square of the total CM energy, and \( \chi(b,s) = \chi_R(b,s) + i \chi_I(b,s) \) is the (complex) eikonal function, which is written as a combination of an even and odd eikonal terms related by crossing symmetry. In terms of the proton-proton (\( pp \)) and antiproton-proton (\( \bar{p}p \)) scatterings, this combination reads \( \chi_{pp}^\ddagger(b,s) = \chi^+(b,s) \pm \chi^-(b,s) \). The even eikonal is written as the sum of gluon-gluon, quark-gluon, and quark-quark contributions:

\[ \chi^+(b,s) = \chi_{qq}(b,s) + \chi_{qg}(b,s) + \chi_{gg}(b,s) = i[\sigma_{qq}(s)W(b;\mu_{qq}) + \sigma_{qg}(s)W(b;\mu_{qg}) + \sigma_{gg}(s)W(b;\mu_{gg})], \]  

(4)

where \( W(b;\mu) \) is the overlap function at impact parameter space and \( \sigma_{ij}(s) \) are the elementary subprocess cross sections of colliding quarks and gluons (\( i,j = q,g \)). The odd eikonal \( \chi^-(b,s) \), that accounts for the difference between \( pp \) and \( \bar{p}p \) channels, is parametrized as

\[ \chi^-(b,s) = C^- \Sigma \frac{m_g}{\sqrt{s}} e^{i\pi/4} W(b;\mu^-), \]  

(5)

where \( m_g \) is the dynamical gluon mass and the parameters \( C^- \) and \( \mu^- \) are constants to be fitted. The factor \( \Sigma \) is defined as \( \Sigma = \frac{9\pi\alpha_s^2(0)}{m^2_g} \), with the dynamical coupling constant \( \bar{\alpha}_s \) set at its frozen infrared value. The gluon-gluon eikonal contribution, that dominates at high energy and determines the asymptotic behaviour of the total cross sections, is written as \( \chi_{gg}(b,s) \equiv \sigma_{gg}^{\text{DPT}}(s)W(b;\mu_{gg}) \), where

\[ \sigma_{gg}^{\text{DPT}}(s) = C' \int_{4m^2_g/s}^{1} d\tau F_{gg}(\tau) \hat{\sigma}_{gg}^{\text{DPT}}(\hat{s}). \]  

(6)

Here \( F_{gg}(\tau) \) is the convoluted structure function for pair \( gg \), \( \hat{\sigma}_{gg}^{\text{DPT}}(\hat{s}) \) is the subprocess cross section and \( C' \) is a fitting parameter. In the above expression it is introduced the energy threshold \( \hat{s} \geq 4m^2_g \) for the final state gluons, assuming that these are screened gluons \([8,9]\). In the expression (6) the gluon-gluon subprocess cross section \( \hat{\sigma}_{gg}^{\text{DPT}}(\hat{s}) \) is calculated using a procedure dictated by the dynamical perturbation theory (DPT) \([19]\). It means that the effects of the dynamical gluon mass in the propagators and vertices are retained, and the sum of polarizations is performed for massless (free-field) gluons. As a result, since the
dynamical masses go to zero at large momenta, the elementary cross sections of perturbative QCD in the high-energy limit are recovered \[7\]. The result of the fit to $\sigma_{\text{tot}}$ for both $pp$ and $\bar{p}p$ channels is displayed in Figs. 1, together with the experimental data.

III. SURVIVAL PROBABILITY OF LARGE RAPIDITY GAPS

The study of the survival probability $\langle |S|^2 \rangle$ of large rapidity gaps (LRG) is currently a subject of intense theoretical and experimental interest. Its importance lies in the fact that systematic analyses of LRG open the possibility of extracting New Physics from hard diffractive processes. On the theoretical side, its significance is due to the reliance of the $\langle |S|^2 \rangle$ calculation on subtle QCD methods. Since rapidity gaps can occur in the case of Higgs boson production via fusion of electroweak bosons, we have focused on $WW \rightarrow H$ fusion processes \[14\]. The inclusive differential Higgs boson production cross section via $W$ fusion is given by

$$\frac{d\sigma_{\text{prod}}}{d^2b} = \sigma_{WW \rightarrow H} W(b; \mu_W),$$ \hspace{2cm} (7)

where $W(b; \mu_W)$ is the overlap function at impact parameter space of the $W$ bosons. This function represents the effective density of the overlapping $W$ boson distributions in the colliding hadrons. The cross section for producing the Higgs boson and having a large rapidity gap is given by

$$\frac{d\sigma_{\text{LRG}}}{d^2b} = \sigma_{WW \rightarrow H} W(b; \mu_W) P(b, s),$$ \hspace{2cm} (8)
FIG. 2. The survival probability $\langle |S|^2 \rangle$ (central values) as a function of the dynamical gluon mass $m_g$.

where $P(b,s)$ is the probability that the two initial hadrons have not undergone a inelastic scattering at the parton level. In the DGM model this probability is given by $P(b,s) = e^{-2\chi_I(b,s)}$, where the imaginary part $\chi_I(b,s)$ of the eikonal function receives contributions of parton-parton interactions. Therefore, the factor $P(b,s)$ suppresses the contribution to the Higgs boson cross section where the two initial hadrons overlap and there is soft rescatterings of the spectator partons. Hence in our model we can write down the survival factor $\langle |S|^2 \rangle$ for Higgs production via $W$ fusion as

$$\langle |S|^2 \rangle = \frac{\int d^2\vec{b} \sigma_{WW \rightarrow H}(b; \mu_W) e^{-2\chi_I(b,s)}}{\int d^2\vec{b} \sigma_{WW \rightarrow H}(b; \mu_W)} = \int d^2\vec{b} W(b; \mu_W) e^{-2\chi_I(b,s)},$$

where we have used the normalization condition $\int d^2\vec{b} W(b; \mu_W) = 1$. The sensitivity of the survival probability $\langle |S|^2 \rangle$ (for $pp$ collisions) to the gluon dynamical mass is shown in Figure 1 for some CM energies. The $\langle |S|^2 \rangle$ value decreases with the increase of the energy of the incoming hadrons, in line with the available experimental data for LRG.

**IV. RESUMMATION OF SOFT GLUON RADIATION**

It is possible to apply the soft resummation mechanism to the calculation of overlap functions in which the coupling $\alpha_s$ is constrained by the value of the dynamical gluon mass. The main steps of this calculation can be summarized as follow: in QED the soft photon
resummation in the energy-momentum $K_\mu$ can be obtained order by order as \[3, 20\]

\[d^4 P(K) = d^4 K \int \frac{d^4 x}{(2\pi)^4} e^{i K \cdot x - h(x, E)}, \quad \text{where} \]

\[h(x, E) = \int_0^E d^3 \tilde{n}(k)[1 - e^{-ik \cdot x}] = \int_0^E \frac{d^3 k}{2k_0} |j_\mu(k, \{p_i\})|^2; \quad (11)\]

here $d^4 P(K)$ is the four-dimensional probability distribution for soft massless quanta emitted by a semiclassical source, $E$ is the maximal energy allowed to single photon emission and $\{p_i\}$ is the momenta of the emitting fields.

For strong interactions the resummed transverse momentum distribution is given by

\[d^2 P(K_\perp) = d^2 K_\perp \int \frac{d^2 b}{(2\pi)^2} e^{-i K_\perp \cdot b - h(b, E)}; \quad (12)\]

in QCD applications $h(b, E)$ can be written as \[21, 22\]

\[h(b, E) = \frac{16}{3} \int_0^E \frac{dk_t}{k_t} \frac{\alpha_s(k_t^2)}{\pi} \ln \left(\frac{2E}{k_t}\right) [1 - J_0(k_t b)], \quad (13)\]

with the integral of the function $h(b, E)$, which describes the relative transverse momentum distribution induced by soft gluon emission from a pair of initially collinear partons, downed to infrared momentum modes.

The attenuation of the rise of the total cross sections comes from soft gluon $k_t$-emission. These emissions break collinearity between the colliding partons, hence changing the overlap function $W(b, s)$ (matter distribution), given by

\[W(b, s) = N \int d^2 K_\perp e^{-i K_\perp \cdot b} \frac{d^2 P(K_\perp)}{d^2 K_\perp} = W_0(s) e^{-h(b, q_{\text{max}})}. \quad (14)\]

Note that the high-energy dependence of the total hadronic cross section (large $b$ value) is intrinsically related to the small $k_t$ value of $\alpha_s$. Thus the behaviour of $\alpha_s$ in the infrared limit plays a central role.

In order to calculate $W(b, s)$ we have adopted the frozen-strong coupling $\tilde{\alpha}_s$ obtained by Cornwall \[8, 9\],

\[\tilde{\alpha}_s(k_t^2) = \frac{4\pi}{\beta_0 \ln \left[(k_t^2 + 4M_g^2(k_t^2))/\Lambda^2\right]}, \quad \text{where} \]

\[M_g^2(k_t^2) = m_g^2 \left[\frac{\ln \left(\frac{k_t^2 + 4m_g^2}{A^2}\right)}{\ln \left(\frac{4m_g^2}{A^2}\right)}\right]^{-12/11}. \quad (16)\]
FIG. 3. The overlap functions calculated by means of $\bar{\alpha}_s$ and $\alpha_s^{RCCH}$.

We have compared our result with those obtained using the usual phenomenological prescription to the strong coupling based on Richardson potential [3, 20]:

$$\alpha_s^{RCCH}(k_t^2) = \frac{4\pi}{\beta_0} \ln \left[ 1 + \frac{p}{(k_t^2/\Lambda^2)^p} \right].$$ (17)

The overlap functions $W^{DGM}(b, s)$ and $W^{RCCH}(b, s)$, calculated using the expressions (15) and (17), respectively, are displayed in the Fig. 3.

V. CONCLUSION

The frozen coupling $\bar{\alpha}_s$ avoids small $k_t$ divergences in partonic subprocesses. Thus it provides an useful phenomenological tool to the study of high-energy strong interactions. The general picture indicates a smooth transition from nonperturbative to perturbative behaviour of the QCD. Interestingly enough, by means of a triple-Regge analysis which explicitly accounts for absorptive corrections, this smooth transition can also be inferred from recent determinations of the bare triple-Pomeron vertex [23, 24].

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