Quantization of singular systems with second order Lagrangians *

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Abstract

The path integral formulation of singular systems with second order Lagrangians is studied by using the canonical path integral formulation method. The path integral quantization of Podolsky electrodynamics is studied.

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1 Introduction

The study of constrained systems with higher order Lagrangians has been applied in many relevant physical problems. Poddolsky’s electrodynamics [1] and a relativistic particle with curvature and torsion in three dimensional space-time [2] are some examples.

The treatment for theories with higher order Lagrangians has been first developed by Ostrogradski [3] and leads to obtain the Euler and the Hamilton’s equations of motion.

The Lagrangian formulation of these theories require the configuration space formed by \( n \) generalized coordinates \( q_i, \dot{q}_i \) and \( \ddot{q}_i \). The Euler Lagrangian equations of motion, which are obtained from

\[
S = \int L(q_i, \dot{q}_i, \ddot{q}_i) dt,
\]

using the Hamilton’s principle, are given by:

\[
\frac{\partial L}{\partial q_i} - \frac{d}{dt} \left( \frac{\partial L}{\partial \dot{q}_i} \right) + \frac{d^2}{dt^2} \left( \frac{\partial L}{\partial \ddot{q}_i} \right) = 0.
\]

The passage from the Lagrangian approach to the Hamiltonian approach is achieved by introducing the generalized momenta \( (p_i, \pi_i) \) conjugated to the generalized coordinates \( (q_i, \dot{q}_i) \) respectively as

\[
p_i = \frac{\partial L}{\partial \dot{q}_i} - \frac{d}{dt} \left( \frac{\partial L}{\partial \ddot{q}_i} \right),
\]

\[
\pi_i = \frac{\partial L}{\partial \ddot{q}_i}.
\]

The phase-space will then be spanned by the canonical variables \( (q_i, p_i) \) and \( (\bar{q}_i, \pi_i) \), where \( \bar{q}_i = \dot{q}_i \).

However, a valid phase space is formed if the rank of the Hessian matrix

\[
\frac{\partial^2 L}{\partial \bar{q}_i \partial \bar{q}_j}, \quad i, j = 1, ..., n,
\]

is \( n \). Systems which have this property are called regular and their treatments are found in a standard mechanics books. Systems which have the rank less than \( n \) are called singular systems.
Now we will give two formulations to investigate singular systems which are: Dirac’s method and the canonical path integral formulation [6-11]

2 Dirac method

The well-known method to investigate the Hamiltonian formulation of constrained systems was initiated by Dirac [4,5]. In his formulation one defines the total Hamiltonian as

\[ H_T = H_c + \nu_\alpha H'_\alpha, \quad \alpha = 1, \ldots, m < 2(n - 1), \] (6)

where \( H_c \) being the canonical Hamiltonian and determined as

\[ H_c = p_i \dot{q}_i + \pi_i \dot{\bar{q}}_i - L, \] (7)

\( \nu_\alpha \) are unknown coefficients.

Due to the singular nature of the Hessian, we have \( \alpha \) functionally independent relations of the form

\[ H'_\alpha(q_i, p_i, \bar{q}_i, \pi_i) \approx 0. \] (8)

Consistency conditions

\[ H'_\alpha = \{ H'_\alpha, H_c \} + \nu_\mu \{ H'_\alpha, H'_\beta \} \approx 0, \] (9)

leads to the secondary constraints. Repeating this procedure as many times as needed, one arrives at a final set of constraints or / and specifies some of \( \nu_\alpha \). Such constraints are divided into two types: first-class constraints which have vanishing Poisson brackets with all other constraints and second-class constraints which have non-vanishing Poisson brackets. As there is an even number of second-class constraints, these can be used to eliminate conjugate pair of \( (p's, q's) \) and \( (\pi's, \bar{q}'s) \) from the theory by expressing them as functions of the remaining \( (p's, q's) \) and \( (\pi's, \bar{q}'s) \). The Dirac Hamiltonian for the remaining variables is then the canonical Hamiltonian plus all the independent first class constraints \( \Psi_\lambda \). So that the total Hamiltonian is defined as

\[ H_T = H_c + \nu_\lambda \Psi_\lambda. \] (10)
Since first-class constraints are the generators of gauge transformations, this will lead to the gauge freedom. In other words, the equations of motion are still degenerate and depend on the functional arbitrariness. Besides, some $\nu_\lambda$ are still undermined. To remove this arbitrariness, one has to impose external gauge fixing conditions for each first class constraints.

Fixing a gauge is not always an easy task, which make one be careful when applying Dirac’s method.

Now we would like to give the path integral formulation using the canonical path integral method and demonstrate the fact that the gauge fixing problem is solved naturally if this method is used.

3 The canonical path integral formalism for second order Lagrangians

Recently the canonical method [12-14] has been developed to investigate singular systems using the Caratheodory’s equivalent Lagrangian method and the equations of motion are obtained as total differential equations in many variables.

Now we will give a brief review of the Caratheodory’s equivalent Lagrangian method. Let us consider a Lagrangian $L(q_i, \dot{q}_i, \ddot{q}_i, t)$, we can obtain a completely equivalent one by

$$L' = L(q_i, \dot{q}_i, \ddot{q}_i, t) - \frac{dS(q_i, \dot{q}_i, t)}{dt},$$

such a function $S(q_i, \dot{q}_i, t)$ must satisfy

$$\frac{\partial S}{\partial t} = -H_0,$$

$$H_0 = p_i \dot{q}_i + \pi_i \ddot{q}_i - L,$$

$$p_i = \frac{\partial S}{\partial q_i},$$

$$\pi_i = \frac{\partial S}{\partial \dot{q}_i}.$$  

These are the fundamental equations of equivalence Lagrangian method.
If the rank of the Hess matrix \( \frac{\partial^2 L}{\partial \dot{q}_i \partial \dot{q}_j} \) is \( n-R \), \( R < n \), then the generalized momenta conjugated to the generalized coordinates \( \bar{q}_i \) are defined as

\[
\pi_a = \frac{\partial L}{\partial \dot{\bar{q}}_a}, \quad a = R + 1, \ldots, n,
\]

\[
\pi_\alpha = \frac{\partial L}{\partial \dot{\bar{q}}_\alpha}, \quad \alpha = 1, \ldots, R.
\]

Since the rank of the hess matrix \( \frac{\partial^2 L}{\partial \dot{q}_i \partial \dot{q}_j} \) is \( n-R \), then one can solve the \( n-R \) accelerations \( \dot{\bar{q}}_a \) in terms of coordinates \( (q_i, \bar{q}_i) \), the momenta \( \pi_a \) and \( \dot{\bar{q}}_\alpha \) as follows

\[
\dot{\bar{q}}_a = w_a(q_i, \bar{q}_i, \pi_a, \dot{\bar{q}}_\alpha).
\]

Substituting (18) in (17) one has

\[
\pi_\alpha = \frac{\partial L}{\partial \dot{\bar{q}}_\alpha} \bigg|_{\dot{\bar{q}}_a = w_a(q_i, \bar{q}_i, \pi_a, \dot{\bar{q}}_\alpha)} = -H^\pi_\alpha(q_i, \bar{q}_i, p_b, \pi_a).
\]

On the other hand, from equation (3) if the rank of the Hess matrix \( \frac{\partial^2 L}{\partial \bar{q}_i \partial \bar{q}_j} \) is \( n-R \), we can obtain a similar expression for the momenta \( p_\gamma \):

\[
p_\gamma = -H^p_\gamma(q_i, \bar{q}_b, p_b, \pi_a), \quad \gamma = 1, \ldots, r, \quad b = r + 1, \ldots, n.
\]

The Hamiltonian \( H_0 \) is defined as

\[
H_0 = p_b \bar{q}_b + \bar{q}_\gamma p_\gamma \bigg|_{p_\beta = -H^p_\beta} + \pi_a w_a
\]

\[
+ \bar{q}_\alpha \pi_\alpha \bigg|_{p_\beta = -H^p_\beta} - L(q_i, \dot{\bar{q}}_i, \dot{\bar{q}}_\alpha, \dot{\bar{q}}_a = w_a), \quad \epsilon = 1, \ldots, r.
\]

Relabeling the coordinates \( t \) and \( q_\gamma \) as \( t_0 \) and \( t_\gamma \) respectively, and \( \bar{q}_\alpha \) will be called \( t_\alpha \). defining the momenta \( P_0 \) as

\[
P_0 = \frac{\partial S}{\partial \dot{t}},
\]

then the set of Hamilton Jacobi partial differential equations [HJPDE] is expressed as
\begin{align*}
H_0' &= P_0 + H_0(t_0, t_\gamma, t_\alpha; q_b, \bar{q}_a; p_b = \frac{\partial S}{\partial q_b}; \pi_a = \frac{\partial S}{\partial \bar{q}_a}) = 0, \\ H_\gamma' &= p_\gamma + H_\gamma'(t_0, t_\gamma, t_\alpha; q_b, \bar{q}_a; p_b = \frac{\partial S}{\partial q_b}; \pi_a = \frac{\partial S}{\partial \bar{q}_a}) = 0, \\ H_\alpha' &= \pi_\alpha + H_\alpha'(t_0, t_\gamma, t_\alpha; q_b, \bar{q}_a; p_b = \frac{\partial S}{\partial q_b}; \pi_a = \frac{\partial S}{\partial \bar{q}_a}) = 0,
\end{align*}

The equations of motion are obtained as total differential equations in many variables as follows:

\begin{align*}
dq_i &= \frac{\partial H_0'}{\partial p_i} dt_0 + \frac{\partial H_\gamma'}{\partial p_i} dt_\gamma + \frac{\partial H_\alpha'}{\partial p_i} dt_\alpha, \\
d\bar{q}_i &= \frac{\partial H_0'}{\partial \bar{q}_i} dt_0 + \frac{\partial H_\gamma'}{\partial \bar{q}_i} dt_\gamma + \frac{\partial H_\alpha'}{\partial \bar{q}_i} dt_\alpha, \\
dp_i &= -\frac{\partial H_0'}{\partial q_i} dt_0 - \frac{\partial H_\gamma'}{\partial q_i} dt_\gamma - \frac{\partial H_\alpha'}{\partial q_i} dt_\alpha, \\
d\pi_i &= -\frac{\partial H_0'}{\partial \bar{q}_i} dt_0 - \frac{\partial H_\gamma'}{\partial \bar{q}_i} dt_\gamma - \frac{\partial H_\alpha'}{\partial \bar{q}_i} dt_\alpha, \\
dP_0 &= -\frac{\partial H_0'}{\partial t_0} dt_0 - \frac{\partial H_\gamma'}{\partial t_0} dt_\gamma - \frac{\partial H_\alpha'}{\partial t_0} dt_\alpha, \\
dZ &= (-H_0 + p_b \frac{\partial H_0'}{\partial p_b} + \pi_a \frac{\partial H_0'}{\partial \pi_a}) dt_0 \\
&\quad+ (-H_\gamma + p_b \frac{\partial H_\gamma'}{\partial p_b} + \pi_a \frac{\partial H_\gamma'}{\partial \pi_a}) dt_\gamma \\
&\quad+ (-H_\alpha + p_b \frac{\partial H_\alpha'}{\partial p_b} + \pi_a \frac{\partial H_\alpha'}{\partial \pi_a}) dt_\alpha.
\end{align*}

The set of equations (27-32) is integrable if \([14]\)

\begin{align*}
dH_0' &= 0, \\
dH_\gamma' &= 0, \\
dH_\alpha' &= 0.
\end{align*}
If conditions (33-35) are not satisfied identically, one considers them as new constraints and again tests the consistency conditions. Thus, repeating this procedure one may obtain a set of conditions. Hence, the canonical formulation leads us to obtain the set of canonical phase-space coordinates as follows

\[
q_b \equiv q_b(t_0, t_\gamma, t_\alpha), \quad p_b \equiv p_b(t_0, t_\gamma, t_\alpha), \quad b = r + 1, \ldots, n, \quad \gamma = 1, \ldots, r, \quad \alpha = 1, \ldots, R.
\]

(36)

\[
\bar{q}_a \equiv \bar{q}_a(t_0, t_\gamma, t_\alpha), \quad \pi_a \equiv \pi_a(t_0, t_\gamma, t_\alpha), \quad a = R + 1, \ldots, n, \quad \gamma = 1, \ldots, r, \quad \alpha = 1, \ldots, R.
\]

(37)

Besides the canonical action integral is obtained in terms of the canonical coordinates. \(H'_0, H'_p\) and \(H'_a\) can be interpreted as the infinitesimal generators of canonical transformations given by parameters \(t_0, t_\gamma\) and \(t_\alpha\) respectively. In this case, the path integral representation may be written as [9-13]

\[
\langle q_b, \bar{q}_a, t_\gamma, t_\alpha | q'_b, \bar{q}'_a, t'_\gamma, t'_\alpha \rangle = \int \prod_{b=1}^{r} dq^b \prod_{a=1}^{R} dp^b \prod_{a=1}^{R} d\bar{q}^a \prod_{a=1}^{R} d\pi^a \times \\
\exp \left\{ i \int_{t_\gamma, t_\alpha}^{t'_\gamma, t'_\alpha} \left( -H_0 + p_b \frac{\partial H'_0}{\partial p_b} + \pi_a \frac{\partial H'_0}{\partial \pi_a} \right) dt_0 \\
+ \left( -H'_p + p_b \frac{\partial H'_p}{\partial p_b} + \pi_a \frac{\partial H'_p}{\partial \pi_a} \right) dt_\gamma \\
+ \left( -H'_a + p_b \frac{\partial H'_a}{\partial p_b} + \pi_a \frac{\partial H'_a}{\partial \pi_a} \right) dt_\alpha \right\}.
\]

(38)

4 Example

In this section we will consider a singular system with a Lagrangian density depend on the dynamical field variables: \(\mathcal{L} = \mathcal{L}(\psi, \partial_\mu \psi, \partial_\mu \partial_\nu \psi)\). One can obtain the Euler Lagrange equations of motion as follows

\[
\frac{\partial \mathcal{L}}{\partial \psi} - \frac{\partial}{\partial \mu} \left( \frac{\partial \mathcal{L}}{\partial (\partial_\mu \psi)} \right) + \partial_\mu \partial_\nu \left( \frac{\partial \mathcal{L}}{\partial (\partial_\mu \partial_\nu \psi)} \right) = 0, \quad \mu, \nu = 0, 1, 2, 3.
\]

(39)
The momenta conjugated to $\dot{\psi}$ and $\ddot{\psi}$ are:

\[
p = \frac{\partial L}{\partial \dot{\psi}} - 2\partial_k \left( \frac{\partial L}{\partial (\partial_k \psi)} \right) - \partial_0 \left( \frac{\partial L}{\partial \psi} \right), \quad k = 1, 2, 3, \tag{40}
\]

\[
\pi = \frac{\partial L}{\partial \ddot{\psi}}. \tag{41}
\]

Now we will consider the Podolsky electrodynamics as a constrained system with second order Lagrangian. The Lagrangian density for such a system is given as

\[
L = -\frac{1}{4} F_{\mu\nu} F_{\mu\nu} + a^2 \partial_\lambda F^{\alpha\lambda} \partial_\rho F^\rho_\alpha, \quad \mu, \nu, \alpha, \rho = 0, 1, 2, 3, \tag{42}
\]

where

\[
F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu, \tag{43}
\]

and the metric convention $g_{\mu\nu} = \text{diag}(+1, -1, -1, -1)$. With the dynamical variables chosen as $A^{\mu}$ and $\bar{A}^{\mu} = \dot{A}^{\mu}$, the conjugated momenta are obtained as

\[
p_\mu = -F_{0\mu} - 2a^2 (\partial_k \partial_\lambda F^{0\lambda} \delta^k_\mu - \partial_0 \partial_\lambda F_{\mu}^\lambda), \tag{44}
\]

\[
\pi_\mu = 2a^2 (\partial_\lambda F^{0\lambda} \delta^0_\mu - \partial_\lambda F_{\mu}^\lambda). \tag{45}
\]

The primary constraints are

\[
H_1' = \pi_0 = 0, \tag{46}
\]

\[
H_2' = p_0 - \partial^k \pi_k = 0. \tag{47}
\]

The expressible velocities $\dot{A}^i$ are obtained as

\[
\dot{A}^i = \frac{1}{2a^2} \pi^i + \partial_k F^{ik} + \partial^i \bar{A}_0. \tag{48}
\]

The canonical Hamiltonian is given by

\[
H_0 = \int d^3x (p_\mu \bar{A}^\mu + \pi_\mu \dot{A}^\mu - L). \tag{49}
\]
Making use of equation (48), we get

\[
H_0 = \int d^3x [\bar{A}_0 \partial^i \pi_i + p_i \bar{A}^i + \frac{1}{4a^2} \pi_i \pi^i \partial_k F^{ik} + \pi_i \partial_k F^{ik} + \pi_i \partial^i \bar{A}_0 + \frac{1}{4} F_{\mu \nu} F^{\mu \nu} + \frac{1}{2} (\bar{A}_i - \partial_i A_0)(\bar{A}^i - \partial^i A^0) - a^2 (\partial_k \bar{A}^k - \partial_k \partial^k A_0)(\partial_i \bar{A}^i - \partial_i \partial^i A_0)].
\]  

Equations (46),(47) and (50) lead us to obtain the set of Hamilton-Jacobi partial differential equations [HJPDE] as follows

\[
H'_0 = P_0 + H_0; \quad P_0 = \frac{\partial S}{\partial t},
\]

\[
H'_1 = \pi_0 = 0, \quad \pi_0 = \frac{\partial S}{\partial \bar{A}_0},
\]

\[
H'_2 = p_0 - \partial^k \pi_k = 0. \quad p_0 = \frac{\partial S}{\partial A_0},
\]

The equations of motion are obtained as total differential equations in many variables as follows

\[
d\bar{A}^i = \frac{\partial H'_0}{\partial p_i} dt + \frac{\partial H'_1}{\partial p_i} d\bar{A}^0 + \frac{\partial H'_2}{\partial p_i} dA^0, \]

\[
d\bar{A}^i = \bar{A}^i dt,
\]

\[
d\bar{A}^i = \frac{\partial H'_0}{\partial \pi_i} dt + \frac{\partial H'_1}{\partial \pi_i} d\bar{A}^0 + \frac{\partial H'_2}{\partial \pi_i} dA^0,
\]

\[
= \left( \frac{1}{2a^2} \bar{A}_i + \partial_k F^{ik} + \partial^i \bar{A}_0 \right),
\]

\[
dp^i = -\frac{\partial H'_0}{\partial A_i} dt - \frac{\partial H'_1}{\partial A_i} d\bar{A}^0 - \frac{\partial H'_2}{\partial A_i} dA^0,
\]

\[
= (-\partial^i \partial^k \pi_k + \partial_i \partial^k \pi^i - \partial_k F^{ki}) dt,
\]

\[
dp^0 = -\frac{\partial H'_0}{\partial A_0} dt - \frac{\partial H'_1}{\partial A_0} d\bar{A}^0 - \frac{\partial H'_2}{\partial A_0} dA^0,
\]

\[
= (-\partial_0 F^{0i} - 2a^2 \partial^i \partial^j \partial_k F^{kj}) dt,
\]

9
\[ d\pi^i = -\frac{\partial H'_0}{\partial A_i^0} dt - \frac{\partial H'_1}{\partial A_i^0} dA^0 - \frac{\partial H'_2}{\partial A_i^0} dA^0, \quad (62) \]

\[ = (-p^i - F^{0i} - 2a^2 \partial^i \partial_k F^{0k}) dt, \quad (63) \]

\[ d\pi^0 = -\frac{\partial H'_0}{\partial A_0^0} dt - \frac{\partial H'_1}{\partial A_0^0} dA^0 - \frac{\partial H'_2}{\partial A_0^0} dA^0, \quad (64) \]

\[ = (-\partial_k p^k) dt, \quad (65) \]

\[ dP^0 = -\frac{\partial H'_0}{\partial t} dt - \frac{\partial H'_1}{\partial t} dA^0 - \frac{\partial H'_2}{\partial t} dA^0 = 0. \quad (66) \]

To check whether the set of equations (54-66) is integrable or not, let us consider the total variations of (51-53). In fact

\[ dH'_0 = H'_3 dA_0 = (\partial^k p_k) dA_0 = 0, \quad (67) \]

\[ dH'_0 = -H'_3 dt = (-\partial^k p_k) dt = 0, \quad (68) \]

\[ dH'_0 = H'_3 dt = (\partial^k p_k) dt = 0. \quad (69) \]

The total variation of \( H'_3 \) is identically zero. Hence, the equations of motion are integrable and the canonical phase space coordinates \((A^i, p^i, \bar{A}^i, \pi^i)\) are obtained in terms of parameters \((t, A^0, \bar{A}^0)\). Besides the canonical action integral is obtained in terms of the canonical variables as

\[ dz = \int d^3x \left[ \frac{1}{2a^2} \pi_i \pi^i - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \frac{1}{2} (\bar{A}_i - \partial_i A_0)(\bar{A}_i - \partial^i A^0) - a^2 (\partial_k \bar{A}^k - \partial_k \partial^k A_0)(\partial_i \bar{A}^i - \partial_i \partial^i A_0) \right] dt. \quad (70) \]

Making use of equation (38) and equation (70), the path integral for the Podolsky electrodynamics is given as

\[ \langle A^i, \bar{A}^i, t, A^0, \bar{A}^0|A'^i, \bar{A}'^i, t', A'^0, \bar{A}'^0 \rangle = \int \prod_{i=1}^3 dA^i \; dp^i \; d\bar{A}^i \; d\bar{\pi}^i \times \]

\[ \exp i \left\{ \int d^3x \left[ \frac{1}{2a^2} \pi_i \pi^i - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} \right. \right. \]

\[ - \frac{1}{2} (\bar{A}_i - \partial_i A_0)(\bar{A}_i - \partial^i A^0) \]

\[ - a^2 (\partial_k \bar{A}^k - \partial_k \partial^k A_0)(\partial_i \bar{A}^i - \partial_i \partial^i A_0) \times \]

\[ (\partial_i \bar{A}^i - \partial_i \partial^i A_0) \right\} dt. \quad (71) \]
The path integral representation (71) is an integration over the canonical phase-space coordinates \((A^i, p^i, \bar{A}^i, \pi^i)\).

5 conclusion

We have obtained the path integral for singular systems with second order Lagrangians. For the Podolsky electrodynamics example since the integrability conditions \(dH'_0 = 0, dH'_1 = 0\) and \(dH'_2 = 0\) are satisfied the canonical phase-space coordinates \((A^i, p^i, \bar{A}^i, \pi^i)\) are obtained in terms of parameters \((t, A^0, \bar{A}^0)\) and the path integral (71) is obtained directly as an integration over the canonical phase-space coordinates without using any gauge fixing conditions. The Faddeev’s Popov [15,16] method treatment for this model needs gauge fixing conditions to arrive at the result (71). The generalization of the present work for Lagrangians of order higher than two is given in reference [11].

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