Study on decays of $Z_c(4020)$ and $Z_c(3900)$ into $h_c + \pi$

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Abstract

At the invariant mass spectrum of $h_c\pi^\pm$ a new resonance $Z_c(4020)$ has been observed, however the previously confirmed $Z_c(3900)$ does not show up at this channel. In this paper we assume that $Z_c(3900)$ and $Z_c(4020)$ are molecular states of $D\bar{D}^*(D^*\bar{D})$ and $D^*\bar{D}^*$ respectively, then we calculate the transition rates of $Z_c(3900) \to h_c + \pi$ and $Z_c(4020) \to h_c + \pi$ in the light front model. Our results show that the partial width of $Z_c(3900) \to h_c + \pi$ is only three times smaller than that of $Z_c(4020) \to h_c + \pi$. $Z_c(4020)$ seems to be a molecular state, so if $Z_c(3900)$ is also a molecular state it should be observed in the portal $e^+e^- \to h_c\pi^\pm$ as long as the database is sufficiently large, by contrary if the future more precise measurements still cannot find $Z_c(3900)$ at $h_c\pi^\pm$ channel, the molecular assignment to $Z_c(3900)$ should be ruled out.

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I. INTRODUCTION

Since discovery of the exotic XYZ particles and as well as the pentaquarks, to determine their inner structure and relevant physics composes a challenge to our understanding of the basic principles, especially the non-perturbative QCD effects. Gaining knowledge on their inner structure can only be realized through analyzing their production and decays behaviors, absolutely, it is indirect, but efficient. In 2013 the BES collaboration observed a new resonance $Z_c(4020)$ at the $h_c\pi^\pm$ invariant mass spectrum by studying the process $e^+e^-\rightarrow h_c\pi^+\pi^-$ with the center-of-mass energies from 3.90 GeV to 4.42 GeV\cite{[1]}. Its mass and width are $(4022.9 \pm 0.8 \pm 2.7)\text{MeV}$ and $(7.9 \pm 2.7 \pm 2.6)\text{MeV}$. Recently the neutral charmonium-like partner of $Z_c(4020)$ has also been experimentally observed\cite{[2]}. In 2013 $Z_c(3900)$ was measured at the invariant mass spectrum of $J/\psi\pi^\pm$ with the mass and width being $(3.899 \pm 3.6 \pm 4.9)\text{GeV}$ and $(46 \pm 10 \pm 10)\text{MeV}$ respectively\cite{[3–5]}. Since the new resonances $Z_c(4020)$ and $Z_c(3900)$ are charged, they cannot be charmonia, but their masses and decay modes imply that they are hidden charm states, namely should be exotic states with a $c\bar{c}q\bar{q}'$ structure. The authors of Ref.\cite{[6–9]} considered that the two resonances should be studied in a unique theoretical framework due to their similarity. It is suggested that the two resonances could be molecular states\cite{[9–13]} whereas some other authors regard them as tetraquark\cite{[8]}, a mixture of the two structures\cite{[14]} or a cusp structure\cite{[15]}. The key point is whether one can use an effective way to confirm their structures. No doubt, it must be done through combing careful theoretical studies and precise measurements in the coming experiments.

Even though the masses of the two resonances are close, but their widths are quite apart, especially at present no significant $Z_c(3900)$ signal has been observed at the $h_c\pi^\pm$ mass spectrum through the process $e^+e^-\rightarrow h_c\pi^+\pi^-\text{[1]}$. Its absence may imply that the two resonances might be different, but do we have an evidence to make a conclusion? If they are of different inner structures, their decay modes should be different, i.e. different structures would lead to different decay rates for the same channel which can be tested by more precise measurements. Theoretically assigning a special structure to any of $Z_c(3900)$ and $Z_c(4020)$, one can predict its decay rate in an appointed channel and then the data would tell if the assignment is valid or should be negated. That is the strategy of this work.

In our early paper\cite{[16]}, we explored some strong decays of $Z_c(3900)$ and $Z_c(4020)$ which were assumed to be molecular states of $D\bar{D}^*(D^*\bar{D})$ and $D^*\bar{D}^*$ and the achieved numerical results are satisfactorily consistent with experimental observations. In this paper we are going to study the strong decays $Z_c(3900)\rightarrow h_c\pi$ and $Z_c(4020)\rightarrow h_c\pi$ with the same method.

In order to explore the decays of a molecular state\cite{[16]}, we extended the light front quark model (LFQM) which was thoroughly studied in literature\cite{[17–27]}. In this situation the constituents are two mesons instead of a quark and an antiquark in the light front frame. In the case of covariant form the constituents are off-shell. The effective interactions between the two constituent mesons are adopted following the literature\cite{[28–33]}, where by fitting relevant processes, the effective coupling constants have been obtained. Using the method
FIG. 1: Strong decays of molecular states (two diagrams where $h_c$ and $\pi$ in the final states are switched are omitted).

given in Ref. [16], we deduce the corresponding form factors and estimate the decay widths of $Z_c(3900) \to h_c \pi$ and $Z_c(4020) \to h_c \pi$ while both $Z_c(3900)$ and $Z_c(4020)$ are assumed to be molecular states. In fact there exist three degenerate S-wave bound states of $D^* \bar{D}^*$ whose quantum numbers are respectively $0^+$, $1^+$ and $2^+$. In our work we evaluate the decay rates of the $D^* \bar{D}^*$ molecules which can be either of the three quantum states.

In this framework, the $q^+ = 0$ condition is applied i.e. $q^2 < 0$, it means that the final mesons are not on-shell, thus the obtained form factors are space-like. Then one needs to extrapolate analytically the form factors from the un-physical space-like region to the time-like region to reach the physical ones. With the form factors we calculate the corresponding decay widths. The numerical results will provide us with information about the structures of $Z_c(3900)$ and $Z_c(4020)$.

After the introduction we derive the form factors for transitions $Z_c(3900) \to h_c \pi$ and $Z_c(4020) \to h_c \pi$ in section II. Then we numerically evaluate the relevant form factors and decay widths in section III. In the last section we discuss the numerical results and draw our conclusion. Some details about the approach are collected in the Appendix.

II. THE STRONG DECAYS $Z_c(3900) \to h_c + \pi$

In this section we calculate the strong decay rate of $Z_c(3900) \to h_c + \pi$, while assuming $Z_c(3900)$ as a $1^+ \, D \bar{D}^*$ molecular state, in the light-front model. Since the success of applying the method [16], we have reason to believe that this framework also works in this case. The configuration of the $D \bar{D}^*$ molecular state is $\frac{1}{\sqrt{2}} (D \bar{D}^* + \bar{D} D^*)$. The Feynman diagrams for $Z_c(3900)$ decaying into $h_c \pi$ by exchanging $D$ or $D^* \pi$ mesons are shown in Fig. 1.

Following Ref. [24], the hadronic matrix element corresponding to the diagrams in Fig. 1 is written as

$$A_{11} = i \frac{1}{(2\pi)^4} \int d^4p_1 \left[ \frac{H_{A_{d1}} S_{d1}^{(a)} + H_{A_{d1}} S_{d1}^{(b)}}{N_1 N_1' N_2} \right] \epsilon_1^d \epsilon_1^a$$

with

$$S_{d1}^{(a)} = -i g_{D^*_h D^* h_c} \frac{g_{\mu_1' \mu_2} g_{\rho_1 \rho_2}}{2} \epsilon_{\alpha \mu \nu \rho} T^a_{\rho_1} (p_1^c - q^c) g_{\nu' \rho'} F(m_1, p_1) F(m_2, p_2) F^2(m_{D^*}, q'),$$

$$S_{d1}^{(b)} = -i g_{D^*_h D^* h_c} \frac{g_{\mu_1' \mu_2} g_{\rho_1 \rho_2}}{2} \epsilon_{\alpha \mu \nu \rho} T^a_{\rho_1} (p_1^c - q^c) g_{\nu' \rho'} F(m_1, p_1) F(m_2, p_2) F^2(m_{D^*}, q'),$$

$$S_{d1}^{(c)} = -i g_{D^*_h D^* h_c} \frac{g_{\mu_1' \mu_2} g_{\rho_1 \rho_2}}{2} \epsilon_{\alpha \mu \nu \rho} T^a_{\rho_1} (p_1^c - q^c) g_{\nu' \rho'} F(m_1, p_1) F(m_2, p_2) F^2(m_{D^*}, q').$$

3
\[ S_{d\alpha}^{1(b)} = i \frac{g_{\mu\nu}}{\sqrt{2}} g_{\alpha\beta} g^{\mu\beta} (p_{1\nu} + q_{\nu}) P^\mu \varepsilon_{\omega \delta \mu \nu} F(m_1, p_1) F(m_2, p_2) F^2(m_{D^*}, q'), \] (2)

\[ N_1 = p_1^2 - m_1^2 + ie, \]
\[ N_1' = q^2 - m_1^2 + ie \text{ and } N_2 = p_2^2 - m_2^2 + ie. \]

A form factor \( F(m_i, p^2) = \frac{(m_i + A^2 - m_1^2)}{(m_i + A^2 - p^2)} \) is introduced to compensate the off-shell effect caused by the intermediate meson of mass \( m_i \) and momentum \( p \). \( H_{A10} \) and \( H_{A01} \) are vertex functions which include the normalized wavefunctions of the decaying mesons with the assigned quantum numbers and are invariant in the four-dimension. In fact, for the practical computation their exact forms are not necessary, because after integrating over \( dp_1^- \) the integral is reduced into a three-dimensional integration, and \( H_{A10} \) (or \( H_{A01} \)) would be replaced by \( h_{A10} \) (\( h_{A01} \)) whose explicit form(s) is calculable. In the light-front frame the momentum \( p_i \) is decomposed into its components as \((p_i^-, p_i^+, p_{i\perp})\) and integrating out \( p_1^- \) with the methods given in Ref. [2] one has

\[ \int d^4 p_1 \frac{H_A S_{d\alpha}}{N_1 N_1' N_2} \varepsilon_1 \varepsilon_\alpha \rightarrow -i \pi \int dx_1 d^2 p_1 \frac{h_A S_{d\alpha}}{x_2 N_1 N_1'} \varepsilon_1 \varepsilon_\alpha, \] (3)

with

\[ \hat{N}_1 = x_1(M^2 - M_0^2), \]
\[ \hat{N}_1' = x_2 q^2 - x_1 M_0^2 + x_1 M'^2 + 2 p_{1\perp} \cdot q, \]
\[ h_A = \sqrt{x_1 x_2 (M^2 - M_0^2) h'} \]

where \( M \) and \( M' \) are the masses of initial and final mesons. The factor \( \sqrt{x_1 x_2 (M^2 - M_0^2)} \) in the expression of \( h_A \) was introduced [24] and an additional normalization factor \( \sqrt{m_1 m_2} \) appears corresponding to the boson constituents in the molecular state. The explicit expressions of the effective form factors \( h'_A \) are collected in the Appendix.

Since we calculate the transition in the \( q^+ = 0 \) plane the zero mode contributions which come from the residues of virtual pair creation processes, are not involved. To include the contributions, \( p_{1\mu} p_{1\nu} \) and \( p_{1\mu} p_{1\nu} \) in \( S_{d\alpha} \) must be replaced by the appropriate expressions as discussed in Ref. [24]

\[ p_{1\mu} \rightarrow P_{\mu} A_{1}^{(1)} + q_{\mu} A_{2}^{(1)}, \]
\[ p_{1\mu} p_{1\nu} \rightarrow g_{\mu\nu} A_{1}^{(2)} + P_{\mu} P_{\nu} A_{2}^{(2)} + (P_{\mu} q_{\nu} + q_{\mu} P_{\nu}) A_{3}^{(2)} + q_{\mu} q_{\nu} A_{4}^{(2)} \] (4)

where \( P = P' + P'' \) and \( q = P' - P'' \) with \( P' \) and \( P'' \) denote the momenta of the concerned mesons in the initial and final states respectively.

For example, after the replacement \( S_{d\alpha}^{1(a)} \) turns into

\[ S_{d\alpha}^{1(a)} = -i g_{h_c D^* D^*} g_{D^* D^*} g_{\alpha \beta} g_{\mu \beta} (p_{1\nu} + q_{\nu}) P^\mu \varepsilon_{\omega \delta \mu \nu} F(m_1, p_1) F(m_2, p_2) F^2(m_{D^*}, q'), \]
\[ = i g_{h_c D^* D^*} g_{D^* D^*} (A_{1}^{(1)} - A_{3}^{(2)}) P_{\alpha} q_{\delta} \varepsilon_{\alpha \delta c d o} F(m_1, p_1) F(m_2, p_2) F^2(m_{D^*}, q'), \] (5)
Some notations such as $A^{(i)}$ and $M_0'$ can be found in Ref.[24]. With the replacement, $h_A \tilde{S}_{da}$ is decomposed into

$$i F_1 P'_aq_b \varepsilon_{abda}, \quad (6)$$

with

$$F_1 = \sqrt{2} g_{h_c D^* D_s} h_{A_{03}} \left( A^{(1)}_1 - A^{(2)}_3 \right) F(m_1, p_1) F(m_2, p_2) F^2(m_{D^*}, q')$$

$$+ \frac{g_{h_c D^* D_s}}{\sqrt{2}} h_{A_{10}} \left( A^{(1)}_1 + A^{(2)}_2 + 1 \right) F(m_1, p_1) F(m_2, p_2) F^2(m_D, q'). \quad (7)$$

For convenience of derivation, let us introduce a new form factor which is defined as following

$$f_1(m_1, m_2) = \frac{1}{16 \pi^3} \int dx_2 d^2 p_\perp \frac{F_1}{x_2 N_1 N'_1}. \quad (8)$$

Then the amplitude is written in terms of $f_1(m_1, m_2)$ as

$$A_{11} = i f_1(m_1, m_2) P'_aq_b \varepsilon_{abda} \varepsilon_{1}^d \varepsilon^a. \quad (9)$$

The contributions from the Feynman diagrams by switching around $h_c$ and $\pi$ in the final states (in Fig.1) can be formulated by simply exchanging $m_1$ and $m_2$ in the expression $f_1(m_1, m_2)$. Then the total amplitude is

$$A_1 = i [f_1(m_1, m_2) + f_1(m_2, m_1)] P'_aq_b \varepsilon_{abda} \varepsilon_{1}^d \varepsilon^a = ig_1 P'_aq_b \varepsilon_{abda} \varepsilon_{1}^d \varepsilon^a, \quad (10)$$

and the factor $g_1$ will be numerically evaluated in next section.

### III. THE STRONG DECAY $Z_c(4020) \rightarrow h_c + \pi$

Similar to what we have done for $Z_c(3900)$, we calculate the decay rate of $Z_c(4020) \rightarrow h_c \pi$ by respectively supposing $Z_c(4020)$ as $0^+, 1^+$ and $2^+ D^*\bar{D}^*$ molecular states. The Feynman diagrams are shown in Fig.2.

#### A. $Z_c(4020)$ as a $0^+$ molecular state

In terms of the vertex function given in the appendix, the hadronic matrix element is

$$A_{21} = i \frac{1}{(2\pi)^4} \int d^4 p_1 \frac{H_{A_{03}}}{N_1 N'_1 N_2} \left( S_d^{2(a)} + S_d^{2(b)} \right) \varepsilon_{1}^d, \quad (11)$$

where

$$S_d^{2(a)} = i g_{h_c D^* D_s} g_{e D D_s} g_{\mu \nu} g_{\mu' \nu'} (2q_{\mu'} - p_{\mu'}) g^{\mu' \nu'} g_{\nu' d} F(m_1, p_1) F(m_2, p_2) F^2(m_D, q'),$$

$$S_d^{2(b)} = i g_{h_c D^* D_s} g_{e D D_s} g_{\mu \nu} g_{\mu' \nu'} (2q_{\mu'} - p_{\mu'}) g^{\mu' \nu'} g_{\nu' d} F(m_1, p_1) F(m_2, p_2) F^2(m_D, q').$$
and \( S_d^{(b)} = -ig_{h_c\pi} g_{\pi D^*} g_{D^* D^*} \varepsilon_{\omega \mu \rho \nu} p_1^{\mu} q^{\rho} g^{\sigma \nu} P^{\mu \nu} \varepsilon_{\omega} f_{d c \nu} F(m_1, p_1) F(m_2, p_2) F^2(m_{D^*}, q'). \)

Carrying out the integration and making the required replacements, we have

\[
\begin{align*}
\hat{h}_{A_0} (\hat{S}_d^{(a)} + \hat{S}_d^{(b)}) &= iF_2 q_d, \quad (12)
\end{align*}
\]

with

\[
\begin{align*}
F_2 &= g_{\psi DD^*} g_{\pi DD^*} h_{A_0} \left( 2 - A_1^{(1)} - A_2^{(1)} \right) F(m_1, p_1) F(m_2, p_2) F^2(m_{D^*}, q') \\
&\quad - 4 g_{\psi DD^*} g_{\pi DD^*} h_{A_0} \left( A_1^{(2)} + A_3^{(2)} \right) M^{\mu \nu} F(m_1, p_1) F(m_2, p_2) F^2(m_{D^*}, q'). \quad (13)
\end{align*}
\]

Simultaneously, we have derived the form factor

\[
\begin{align*}
f_2(m_1, m_2) &= \frac{1}{16\pi^3} \int d^4 p_2 d^2 p_\perp \frac{F_2}{x_2 N_1 N'_1}. \quad (14)
\end{align*}
\]

With this form factor the transition amplitude is obtained as

\[
\begin{align*}
\mathcal{A}_{21} &= if_2(m_1, m_2) q \cdot \epsilon_1. \quad (15)
\end{align*}
\]

Similarly, the amplitude corresponding the Feynman diagrams where the mesons in the final state are switched around, can be easily obtained by exchanging \( m_1 \) and \( m_2 \). The total amplitude is

\[
\begin{align*}
\mathcal{A}_2 &= i[f_2(m_1, m_2) + f_2(m_2, m_1)] q \cdot \epsilon_1 \\
&= ig_2 q \cdot \epsilon_1. \quad (16)
\end{align*}
\]

**B. \( Z_c(4020) \) as a \( 1^+ \) molecular state**

For the \( 1^+ \) state, the hadronic matrix element would be different from the case where \( Z_c(4020) \) is assumed to be a \( 0^+ \) meson. Now the hadronic matrix element is written as

\[
\begin{align*}
\mathcal{A}_{31} &= i \frac{1}{(2\pi)^4} \int d^4 p_1 \frac{H_{A_1} N_1 N'_1 N_2}{N_1 N'_1 N_2} (S_{d a}^{2(a)} + S_{d a}^{2(b)}) \epsilon_1^d \epsilon_1^e, \quad (17)
\end{align*}
\]
where

\[ S_{d\alpha}^{2(a)} = ig_{h_cDD^*}g_{\pi DD^*} \varepsilon_{\mu\nu\alpha\beta}g^{\mu\nu}(2q_{\mu'} - p_{1\mu'}) P^\beta g^{\nu\rho'} g_{\rho'd} F(m_1, p_1) F(m_2, p_2) F^2(m_D, q'), \]

and

\[ S_{d\alpha}^{2(b)} = -ig_{h_cDD^*}g_{\pi DD^*} \varepsilon_{\mu\nu\alpha\beta}g^{\mu\nu} P^\beta \varepsilon_{\omega\mu'\rho\alpha} P_1^\omega q^\rho g^{ac} P^{af} \varepsilon_{f d\sigma} F(m_1, p_1) F(m_2, p_2) F^2(m_{D*}, q'). \]

After integrating over the momentum, we have

\[ h_{A_1}(S_{d\alpha}^{2(a)} + S_{d\alpha}^{2(b)}) = iF_3 p'_a q_b \bar{\varepsilon}_{abcdef}, \]  

with

\[ F_3 = g_{h_cDD^*}g_{\pi DD^*} f_{A_1} \left( A_2^{(1)} - A_1^{(1)} - 2 \right) F(m_1, p_1) F(m_2, p_2) F^2(m_D, q') \]

\[ + g_{h_cDD^*}g_{\pi DD^*} f_{A_1} \left( A_1^{(1)} + A_3^{(2)} \right) (M'^2 + M''^2 - q^2) F(m_1, p_1) F(m_2, p_2) F^2(m_{D^*}, q'). \]

The form factor is

\[ f_3(m_1, m_2) = \frac{1}{16\pi^3} \int dx_2 d^2 p_{1/2} \frac{F_3}{x_2 N_1 N_1'}, \]

which will be numerically evaluated. With these form factors the transition amplitude is obtained as

\[ A_{31} = if_3(m_1, m_2) P'_a q_b \bar{\varepsilon}_{abcdef} \epsilon^d \epsilon^c. \]

Including the contributions of the Feynman diagrams where we switch around \( h_c \) and \( \pi \) in the final states, the amplitude is

\[ A_3 = if_3(m_1, m_2) \langle P' a | q_b \bar{\varepsilon}_{abcdef} \rangle | \epsilon^d \epsilon^c. \]

C. \( Z_c(4020) \) as a 2\(^+\) molecular state

Then as we suppose \( Z_c(4020) \) is a 2\(^+\) molecule, the hadronic matrix element is

\[ A_{41} = i \frac{1}{(2\pi)^4} \int d^4 p_1 \frac{H_{A_1}}{N_1 N_2} (S_{d\mu}^{2(a)} + S_{d\mu}^{2(b)}) \epsilon_1^d \epsilon^\mu, \]

where

\[ S_{d\alpha}^{2(a)} = ig_{h_cDD^*}g_{\pi DD^*} g^{\mu\nu}(2q_{\mu'} - p_{1\mu'}) g^{\nu\rho'} g_{\rho'd} F(m_1, p_1) F(m_2, p_2) F^2(m_D, q'), \]

and \( S_{d\alpha}^{2(b)} = -ig_{h_cDD^*}g_{\pi DD^*} \varepsilon_{\mu\nu\alpha\beta} g^{\mu\nu} \varepsilon_{\omega\mu'\rho\alpha} P_1^\omega q^\rho g^{ac} P^{af} \varepsilon_{f d\sigma} F(m_1, p_1) F(m_2, p_2) F^2(m_{D^*}, q'). \]

Carrying out the integration, one has

\[ h_{A_1}(S_{d\alpha}^{2(a)} + S_{d\alpha}^{2(b)}) = i(F_4 q_4 g_{d\nu} + F_5 q_5 g_{d\mu} + F_6 q_6 g_{d\mu}), \]
with

\[ F_4 = g_{h_c DD^*} g_{s DD^*} h_{A_1} \left( 2 + A_4^{(1)} - A_2^{(1)} \right) \mathcal{F}(m_1, p_1) \mathcal{F}(m_2, p_2) \mathcal{F}^2(m_D, q') \]

\[ F_5 = 2g_{h_c D^* D^*} g_{s D^* D^*} h_{A_1} \left( A_4^{(1)} + A_2^{(2)} \right) \frac{(M^2 + M'^2 - q'^2)}{2} \mathcal{F}(m_1, p_1) \mathcal{F}(m_2, p_2) \mathcal{F}^2(m_{D^*}, q') \]

\[ F_6 = 2g_{h_c D^* D^*} g_{s D^* D^*} h_{A_1} \left( A_4^{(1)} + A_2^{(2)} \right) \mathcal{F}(m_1, p_1) \mathcal{F}(m_2, p_2) \mathcal{F}^2(m_{D^*}, q'). \]  \( (25) \)

The new form factors are defined as following

\[ f_a(m_1, m_2) = \frac{1}{16\pi^3} \int dx_2 d^2 p_1 \frac{F_a}{x_2 \vec{N}_1 \vec{N}'_1}, \]  \( (26) \)

where the subscript \( a \) denotes 4,5 and 6. Substituting these form factors into the formulae, the transition amplitude is obtained as

\[ \mathcal{A}_{11} = i[f_4(m_1, m_2)q_\mu g_{d\nu} + f_5(m_1, m_2)q_\mu g_{d\mu} + f_6(m_1, m_2)q_\mu q_\mu g_{d\mu}]\epsilon_1^{\mu\nu}. \]  \( (27) \)

Similarly, as all the contributions are incorporated, the total amplitude is

\[ \mathcal{A}_4 = i[(f_4(m_1, m_2) + f_4(m_2, m_1)]q_\mu g_{d\nu} + [f_5(m_1, m_2) + f_5(m_2, m_1)]q_\mu g_{d\mu} + [f_6(m_1, m_2)q_\mu q_\mu + f_6(m_2, m_1)q_\mu q_\mu g_{d\mu}]\epsilon_1^{\mu\nu} \]

\[ = i[g_4 q_\mu g_{d\nu} + g_5 q_\mu g_{d\mu} + g_6 q_\mu q_\mu g_{d\mu}]\epsilon_1^{\mu\nu}. \]  \( (28) \)

### IV. NUMERICAL RESULTS

In this section we present our predictions on the decay rates of \( Z_c(3900) \rightarrow h_c \pi \) and \( Z_c(4020) \rightarrow h_c \pi \) along with all the input parameters. First we need to calculate the corresponding form factors which we deduced in last section. Those formulas involve some parameters which need to be priori fixed. We use 3.899 GeV as the mass of \( Z_c(3900) \) and the mass of \( Z_c(4020) \) is determined to be 4.02 GeV. The masses of the involved mesons are set as \( m_{h_c} = 3.525 \) GeV, \( m_\pi = 0.139 \) GeV, \( m_D = 1.869 \) GeV and \( m_{D^*} = 2.007 \) GeV according to the data book \( [34] \). The coupling constants \( g_{s DD^*} = 8.8 \) and \( g_{s D^* D^*} = 9.08 \) GeV\(^{-1} \) are adopted according to Refs. \( [28, 29] \). At present one cannot fix the couplings \( h_c DD^* \) and \( h_c D^* D^* \) from data yet. However there exists a simple, but approximate relation \( m_D g_{h_c DD^*} = g_{h_c D^* D^*} \) which is in analog to the case about the couplings \( \psi D^{(*)} D^{(*)} \) \( [31, 32] \), so only one undetermined parameter remains. Since the values of the most coupling constants are of order \( O(1) \), we set \( g_{h_c D^* D^*} = 1 \) as a reasonable choice. If one could fix \( g_{h_c D^* D^*} \) later, he just needs to multiply a number to the corresponding form factor and it does not affect our final conclusion. The cutoff parameter \( \Lambda \) in the vertex \( \mathcal{F} \) was suggested to be set as 0.88 GeV to 1.1 GeV \( [32] \). In our calculation we use 0.88 GeV and 1.1 GeV respectively to study the effect on the results. The parameter \( \beta \) in the wavefunction is not very certain until now. In Ref. \( [16] \) we estimated its value and decided that it is close to or slightly smaller than 0.631 GeV\(^{-1} \) \( [35] \), and it is the \( \beta \) number for the wavefunction of \( J/\psi \).
Since the form factors are derived in the reference frame of \( q^+ = 0 \) (\( q^2 < 0 \)) i.e. in the space-like region, we need to extend them to the time-like region by means of the normal procedure provided in literatures. In Ref. [24] a three-parameter form factor was suggested as

\[
g(q^2) = \frac{g(0)}{1 - a \left( \frac{q^2}{M_{Zc}^2} \right) + b \left( \frac{q^2}{M_{Zc}^2} \right)^2}.
\]  

(29)

| \( g \) | \( g(0) \) | \( a \) | \( b \) |
|---|---|---|---|
| \( g_1 \) | -0.253 | 2.72 | 4.60 |
| \( g_2 \) | 0.364 | 2.75 | 4.70 |
| \( g_3 \) | -0.129 | 2.74 | 3.25 |
| \( g_4 \) | -0.243 | 3.24 | 7.01 |
| \( g_5 \) | -0.486 | 2.82 | 4.88 |
| \( g_6 \) | -0.0341 | 2.82 | 4.88 |

TABLE I: The three-parameter form factors with (\( \Lambda = 0.88 \) GeV, \( \beta = 0.631 \) GeV\(^{-1} \)).

| decay mode (\( \Lambda = 0.88 \) GeV) | width (GeV) | decay mode (\( \Lambda = 1.1 \) GeV) | width (GeV) |
|---|---|---|---|
| \( Z_c(3900) \rightarrow h_c\pi \) | \( 5.85 \times 10^{-5} \) | \( Z_c(3900) \rightarrow h_c\pi \) | \( 8.91 \times 10^{-5} \) |
| \( Z_c(4020)(0^+) \rightarrow h_c\pi \) | \( 1.49 \times 10^{-4} \) | \( Z_c(4020)(0^+) \rightarrow h_c\pi \) | \( 2.36 \times 10^{-4} \) |
| \( Z_c(4020)(1^+) \rightarrow h_c\pi \) | \( 1.51 \times 10^{-4} \) | \( Z_c(4020)(1^+) \rightarrow h_c\pi \) | \( 2.34 \times 10^{-4} \) |
| \( Z_c(4020)(2^+) \rightarrow h_c\pi \) | \( 1.54 \times 10^{-4} \) | \( Z_c(4020)(2^+) \rightarrow h_c\pi \) | \( 2.38 \times 10^{-4} \) |

TABLE II: The decay widths of some modes (\( \beta = 0.631 \) GeV\(^{-1} \)).

The resultant form factors are listed in table I and the corresponding decay widths are presented in table II. The molecular states of \( D^* \bar{D}^* \) can be in three different quantum states, thus the Lorentz structures of their decay amplitudes for \( Z_c \rightarrow h_c\pi \) are different and the values of the corresponding form factors should also be different. However we find that the decay widths of all those states are very close to each other, and it is easy to understand because the three states with different spin assignments are degenerate. One can also note, \( \Gamma(Z_c(4020) \rightarrow h_c\pi) \) is three times larger than \( \Gamma(Z_c(3900) \rightarrow h_c\pi) \) for different parameter \( \Lambda \).

In our calculation, we notice that the model parameter \( \beta \) can affect the numerical results to a certain degree. We illustrate the dependence of \( \Gamma(Z_c(3900) \rightarrow h_c\pi) \) and \( \Gamma(Z_c(4020) \rightarrow h_c\pi) \) on \( \beta \) in Fig.3 and depict the dependence of the ratio of \( \Gamma(Z_c(4020) \rightarrow h_c\pi)/\Gamma(Z_c(3900) \rightarrow h_c\pi) \) on \( \beta \) in Fig.4. Lines A and B in Fig.3 correspond to \( Z_c(3900) \) and \( Z_c(4020) \) respectively. It is also noted that the ratio \( \Gamma(Z_c(4020) \rightarrow h_c\pi)/\Gamma(Z_c(3900) \rightarrow h_c\pi) \approx 2.5 \) does not vary much as \( \beta \) changes.
V. CONCLUSION AND DISCUSSIONS

In this work, supposing $Z_c(3900)$ and $Z_c(4020)$ to be $D\bar{D}^*$ and $D^*\bar{D}^*$ molecular states, we calculate the decay rates of $Z_c(3900) \to h_c\pi$ and $Z_c(4020) \to h_c\pi$ respectively in the light front model. It is noted that for the $D^*\bar{D}^*$ system there are three degenerate states whose quantum numbers are $0^+$, $1^+$ and $2^+$ with orbital angular momentum $L = 0$. Thus we calculate the decay rates of the molecular state $D^*\bar{D}^*$ of different quantum numbers in this work. Using the effective interactions we calculate the corresponding form factors for the decays $Z_c(3900) \to h_c\pi$ and $Z_c(4020) \to h_c\pi$. Our numerical results show $\Gamma(Z_c(4020)(0^+) \to h_c\pi)$, $\Gamma(Z_c(4020)(1^+) \to h_c\pi)$ and $\Gamma(Z_c(4020)(2^+) \to h_c\pi)$ are indeed close to each other. By the results one would think that $Z_c(4020)$ behaves as a molecular state.

It is noticed that the resultant $\Gamma(Z_c(3900) \to h_c\pi)$ is only three times smaller than $\Gamma(Z_c(4020) \to h_c\pi)$ for various values of $\Lambda$ and $\beta$.

Considering the total width, even though the branching ratio of $\Gamma(Z_c(3900) \to h_c\pi)$ is
slightly small, we still have a remarkable opportunity to observe \( Z_c(3900) \) in this channel. If \( Z_c(3900) \) and \( Z_c(4020) \) are \( D\bar{D}^* \) and \( D^*\bar{D}^* \) molecular states, we should observe the \( Z_c(3900) \) peak at the invariant mass spectrum of \( e^+e^- \rightarrow h_c\pi \). No doubt, since this portal has not been “seen” at BES III data so far, the reason may be attributed to the relatively small database at present. Thus with more data accumulating to a reasonable stack, the experimental exploration of \( Z_c(3900) \rightarrow h_c\pi \) will eventually reach a conclusion, namely a peak at 3900 MeV shows up or does not. Namely, it does appear, one can celebrate the assumption that \( Z_c(3900) \) is indeed a molecular state of \( D\bar{D}^*(D^*\bar{D}) \) to be valid, or at least it possess a large fraction of molecular state. By contrary, if there is still no the signal of \( Z_c(3900) \) to be observed at \( h_c\pi \) invariant mass spectrum, the the proposal that \( Z_c(3900) \) were a \( D\bar{D}^* \) molecular state would not be favored or ruled out.

Even though in our calculation the coupling constant \( g_{h_cD^*\bar{D}^*} \) is not well determined, so that the estimated widths are not precise. However the ratio \( \Gamma(Z_c(3900) \rightarrow h_c\pi)/\Gamma(Z_c(4020) \rightarrow h_c\pi) \) does not depend on the coupling. Therefore, our scheme for judging whether \( Z_c(3900) \) is a molecular state is still working. A relevant question arises: what is the inner structure of \( Z_c(3900) \) if it is not a molecule? In Ref.\[36\] the authors study some strong decays of \( Z_c(3900) \) by assuming it to be a tetraquark with the QCD sum rules, but unfortunately the channel of \( Z_c(3900) \rightarrow h_c\pi \) was not discussed in their work. In our next work we will explore some strong decays of \( Z_c(3900) \) as a tetraquark especially including \( Z_c(3900) \rightarrow h_c\pi \) in the light front model, and will show the partial width of this channel should indeed be small.

Since \( Z_c(3900) \) was found from the final state \( J/\psi\pi \), it is natural to suggest that one should detect if \( Z_c(4020) \) shows up in the invariant mass spectrum of \( J/\psi\pi \). Postulating both \( Z_c(3900) \) and \( Z_c(4020) \) to be molecular states we find \( \Gamma(Z_c(4020) \rightarrow J/\psi\pi) \) is five times larger than \( \Gamma(Z_c(3900) \rightarrow J/\psi\pi) \)[16]. Thus we suggest our experimental colleagues to adjust the center-mass-energy to produce a larger database for \( Z_c(4020) \) to measure the corresponding decay rate. It will be an ideal scheme to determine the identity of both \( Z_c(3900) \) and \( Z_c(4020) \).

Moreover, at the invariant mass spectrum of \( D^*\bar{D}^* \), another resonance \( Z_c(4025) \) was measured with a mass of \( (4026.3 \pm 2.6 \pm 3.7)\text{MeV} \) and width \( (24.8 \pm 5.6 \pm 7.7)\text{MeV} \)[37]. Its peak heavily overlaps with that of \( Z_c(4020) \), and the deviation is within 1.5\( \sigma \), therefore it seems that \( Z_c(4020) \) and \( Z_c(4025) \) might be degenerate, even more, they are the same state, but the measurement errors cause a misidentification. Thus in the future work it is our task to identify them as two different resonances whose masses are close or just degenerate states or the same one.
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Appendix A: the vertex function of molecular state

Supposing \( Z_c(3900) \) and \( Z_c(4030) \) are molecular states which consists of \( D \) and \( \bar{D}^* \) and \( D^* \) respectively. The wavefunction of a molecular state with total spin \( J \) and momentum \( P \) is \[|X(P, J, J_z)⟩ = \int \{d^3\bar{p}_1\} \{d^3\bar{p}_2\} \frac{2(2\pi)^3}{(2\pi)^3} \delta^3(\bar{p} - \bar{p}_1 - \bar{p}_2) \times \sum_{\lambda_1} \Psi^{SS_1}(\bar{p}_1, \bar{p}_2, \lambda_1, \lambda_2) F \left| D^+(p_1, \lambda_1) \bar{D}^*(p_2, \lambda_2) \right⟩. \tag{A1}\]

For 0\(^+\) molecular state of \( D^* \bar{D}^* \)

\[\Psi^{SS_1}(\bar{p}_1, \bar{p}_2, \lambda_1, \lambda_2) = C_0 \varphi(x, p_⊥) \epsilon_1(\lambda_1) \cdot \epsilon_2(\lambda_2) = h_{\epsilon_1}(\lambda_1) \cdot \epsilon_2(\lambda_2), \tag{A2}\]
for $1^+$ molecular state of $D^*D^*$

$$\Psi^{SS}(\tilde{p}_1, \tilde{p}_2, \lambda_1, \lambda_2) = C_1 \varphi(x, p_\perp) \varepsilon^{\mu\nu\alpha\beta} \epsilon_{1\mu}(\lambda_1) \epsilon_{2\nu}(\lambda_2) \epsilon_\alpha(J_z) P_\beta$$

$$= h'_C \epsilon^{\mu\nu\alpha\beta} \epsilon_{1\mu}(\lambda_1) \epsilon_{2\nu}(\lambda_2) \epsilon_\alpha(J_z) P_\beta, \quad (A3)$$

for $2^+$ molecular state of $D^*\bar{D}^*$

$$\Psi^{SS}(\tilde{p}_1, \tilde{p}_2, \lambda_1, \lambda_2) = C_2 \varphi(x, p_\perp) \epsilon_{1\mu}(\lambda_1) \epsilon_{2\nu}(\lambda_2) e^{\mu\nu}(J_z)$$

$$= h'_C \epsilon_{1\mu}(\lambda_1) \epsilon_{2\nu}(\lambda_2) e^{\mu\nu}(J_z), \quad (A4)$$

and for $1^+$ molecular state of $D\bar{D}^*$

$$\Psi^{SS}(\tilde{p}_1, \tilde{p}_2, \lambda_1, \lambda_2) = C_{01(10)} \varphi(x, p_\perp) \epsilon_{1\mu}(\lambda_1) \cdot \epsilon_\alpha(J_z)$$

$$= h'_C \epsilon_{1\mu}(\lambda_1) \cdot \epsilon_\alpha(J_z), \quad (A5)$$

where $C_{01}, C_{10}, C_0, C_1$ and $C_2$ are the normalization constants which can be fixed by normalizing the state $\underline{24}$

$$\langle X(P', J', J_z')|X(P, J, J_z)\rangle = 2(2\pi)^3 P^+ \delta^3(\tilde{P}' - \tilde{P}) \delta_{JJ'} \delta_{J_zJ_z'}, \quad (A6)$$

and let the normalization $\int \frac{dxdp_\perp}{2(2\pi)^3} \varphi_{P,L',L_z'}(x, p_\perp) \varphi_{L,L_z}(x, p_\perp) = \delta_{L,L'} \delta_{L_zL_z'}$ hold.

For example $C_0$ is fixed by calculating Eq. (A6) with the $0^+$ state

$$\int \frac{dxdp_\perp}{2(2\pi)^3} C_0^2 \epsilon_1^*(\lambda_1) \cdot \epsilon_2^*(\lambda_2) \epsilon_1(\lambda_1) \cdot \epsilon_2(\lambda_2) \varphi^*(x, p_\perp) \varphi(x, p_\perp) = 1, \quad (A7)$$

then $C_0 = \sqrt{\frac{2m_1 m_2}{M_0}}$. It is noted that $P^2 = M_0^2$, $p_1 \cdot P = \epsilon_1 M_0$, and $p_2 \cdot P = \epsilon_2 M_0$ are used as discussed in Ref. 24.

Similarly one can obtain

$$C_{01} = \sqrt{\frac{3m_1}{\epsilon_1^2 + 2m_1^2}}, \quad C_{10} = \sqrt{\frac{3m_2}{\epsilon_2^2 + 2m_2^2}},$$

$$C_1 = \frac{2\sqrt{3m_1 m_2}}{\sqrt{M^2(4\epsilon_1^2 m_2^2 - 4\epsilon_1 \epsilon_2(-M_0^2 + m_1^2 + m_2^2) + 4e_2^2 m_1^2 + 10m_1^2 m_2^2 - C_A)}},$$

$$C_2 = \sqrt{\frac{120m_1 m_2}{4\epsilon_1^2(4\epsilon_2^2 + 7m_2^2) + 4\epsilon_1 \epsilon_2(-M_0^2 + m_1^2 + m_2^2) + 28e_2^2 m_1^2 + 54m_1^2 m_2^2 + C_A}},$$

$$C_A = M_0^4 - 2M_0^2(m_1^2 + m_2^2) + m_1^4 + m_2^4.$$

and $\varphi = 4\left(\frac{\pi}{2\beta^2}\right)^3 \frac{\epsilon_1 \epsilon_2}{x_1 x_2 M_0} \exp\left(-\frac{p^2}{2\beta^2}\right)$. All other notations can be found in Ref. 20.
Appendix B: the effective vertices

the effective vertices can be found in \[28-32\],

\[
\begin{align*}
\mathcal{L}_{xDD^*} &= ig_{xDD^*}(D^\mu \partial_\mu \pi \bar{D} - \partial^\mu D\pi \bar{D}_\mu^* + \text{h.c.}), \\
\mathcal{L}_{\pi D^*D^*} &= -g_{\pi D^*D^*}\varepsilon^{\mu\nu\alpha\beta}\partial_\mu \bar{D}_\nu^*\pi \partial_\alpha D_\beta^*, \\
\mathcal{L}_{hc D^*D^*} &= -ig_{hc D^*D^*}\varepsilon^{\mu \nu \alpha \beta}\partial_\mu h_{c\nu}D_\alpha^* \bar{D}_\beta^*, \\
\mathcal{L}_{hc DD^*} &= g_{hc DD^*}h_{c}^\nu \bar{D} D_\nu^*.
\end{align*}
\]