Loop Corrections to Cosmological Perturbations in Multi-field Inflationary Models

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ABSTRACT: We investigate one-loop quantum corrections to the power spectrum of adiabatic perturbation from entropy modes/adiabatic mode cross-interactions in multiple DBI inflationary models. We find that due to the non-canonical kinetic term in DBI models, the loop corrections are enhanced by slow-varying parameter $\epsilon$ and small sound speed $c_s$. Thus, in general the loop-corrections in multi-DBI models can be large. Moreover, we find that the loop-corrections from adiabatic/entropy cross-interaction vertices are IR finite.

KEYWORDS: Cosmological perturbation theory, Inflation, Cosmology of theories beyond the SM, Physics of the early universe.
1. Introduction

Current observational data support standard ΛCDM model greatly [1], in which primordial perturbations which are assumed to be responsible for Cosmic Microwave Background anisotropies and large-scale structure formation are generated from quantum fluctuations and stretched to superhorizon scales during inflation (see e.g. [2] for a review). Standard single-field slow-roll inflationary models predict almost scale-invariant, Gaussian and adiabatic primordial fluctuation.

Actually, inflation itself is not a single model, but rather a theoretical framework. From the point of view of power spectrum, many inflation models are “degenerate”. Power spectrum, or strictly speaking, tree-level two-point correlation functions of cosmological perturbation do not give us an unique theory of inflation. In fact, phenomena beyond linear-order have been extensively investigated over the past several years.

The most significant progress beyond power spectrum is the investigation of statistical non-gaussianities of CMB anisotropies and primordial fluctuations (see e.g. [4] for a nice review). From the field theoretical point
of view, non-gaussianity describes the interactions of perturbations, which will cause non-vanishing higher-order correlation functions. Such interactions are mandatory in any realistic inflationary models, which come from both the non-linear nature of gravity and the self-interactions of inflation field(s). In standard slow-roll inflation scenario, however, the non-gaussianities have been proved too small to be detected [3], even with PLANCK. Thus, any detection of non-gaussianities would not only rule out the simplest inflation models but also give us valuable insight of fundamental physics of inflation. Various models have been investigated to generate large non-gaussianities by introducing more complicated kinetic terms [5, 6, 7, 8] or more fields [9, 10, 11, 12, 13, 14, 15, 16, 18, 19, 22, 23].

Interactions not only cause non-vanishing higher-order correlation functions which are responsible for non-gaussianities of primordial perturbations, they also cause quantum loop-corrections. It is possible that such loop corrections may be large in their own right. Especially, we could expect large loop corrections in models which have also large non-gaussianities, since both of them describe the interactions among perturbations. Moreover, by collecting signatures of both quantum loop corrections and non-gaussianities, we will obtain a more sensitive test of the physics of inflation.

Early estimations of loop effects in cosmological perturbations were pioneered by Mukhanov, Abramo and Brandenberger [24, 25], Abramo and Woodard [26], and Unruh [27]. Subsequently, loop effects have been investigated by many authors [28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49]. Recently, stimulated by works of S. Weinberg [50, 51], loop corrections to primordial fluctuations have been re-investigated by several authors [52, 53, 54, 55, 56, 57, 58, 59]. It has been found that cosmological loop corrections seem to have some infrared divergences, which scale as $\ln(k\ell)$, where $\ell^{-1}$ is some IR comoving momentum cut-off [52, 53, 54, 55]. Other discussions on cosmological loop corrections and the infrared divergences can be found in [63, 64, 65, 66, 67, 68, 69]. The presence of IR divergences implies that loop-contributions may potentially give large corrections to tree-level results, although it was argued in [63, 64, 65] that these IR divergent corrections do not contribute in quantities that are directly observable.

Actually, now there are two types of so-called “loop corrections” in the literatures. One type is the quantum loops (q-loop) which originate from the interactions among fields and are evaluated using standard quantum field theory, while the other type is the classical loops (c-loop) which arise mathematically from some “loop-type integrals” due to the non-linear map from $\delta\phi$ to $\zeta$ [60, 61, 62]. In this work, we focus on the quantum loops. However, it is well-known that the observational variable is the curvature perturbation $\zeta$ and there has non-linear relation between $\zeta$ and $\delta$ during superhorizon evolution (e.g. in terms of the “$\delta N$-formalism” [73, 74, 75]). Thus, these “loop” effects also have to be treated seriously, and especially they can be significant, if the higher-order derivatives of e-folding number $N$ with respect to the inflaton fields are extremely large. While in [62] it was shown that the loop corrections for $\zeta$ are always suppressed during inflation, if one assume that the fields follow a classical background trajectory.

Up to now, one-loop corrections to the power spectrum of the cosmological perturbations have been investigated in single scalar-field loops [52, 53, 54, 55, 50, 51], concerning effects from many light scalar fields [57], and from graviton loops [56]. Actually, it has been recognized that it is hard to generate large “local-type” non-gaussianities in single-field models, unless we abandon the slow-roll-type conditions. Thus, the investigation of multiple field models has particular importance. In multiple field inflationary models, perturbations can be decomposed instantaneously into one adiabatic mode and several entropy modes [20] (see [21] for a review). There are interactions between adiabatic mode and entropy modes.

In this work, we investigate the one-loop corrections to adiabatic power spectrum due to the interactions between entropy mode and adiabatic mode, at the horizon exiting. We find that due to the non-canonical kinetic term in DBI models, the loop corrections are enhanced by slow-varying parameter $\epsilon$ and small sound speed
$c_s$, as the enhancement of non-gaussianities in such models. Thus, in general, the loop-corrections in multi-DBI models can be large. Moreover, we find that the loop-corrections from adiabatic/entropy cross-interaction vertices are IR finite. This is because in the limit of speed sound $c_s$, the leading-order three-point interactions are totally “derivatively interacted”. While the derivatives (both with respect to time and space) give momentum factors which cancel the momentum factors in the denominator from the tree-level correlation functions to make the loop-integrals IR safe.

Quantum cosmological correlation functions are evaluated by using “in-in formalism” (also dubbed as “Schwinger-Keldysh formalism”) [70, 71, 72] (also see e.g. [50] for a nice review), which we also give a brief review in Appendix A. In [58] the application of “in-in” formalism on cosmological perturbation, especially on loop corrections, has been investigated in details.

This paper is organized as follows. In section 2, firstly we briefly review the linear perturbation of multiple-DBI models, then we describe the third-order perturbation action, especially the three-point adiabatic/entropy modes cross-interaction vertices. In section 3, we calculate the one-loop corrections to the adiabatic power spectrum from the adiabatic/entropy modes cross-interaction vertices, by using the “in-in” formalism. Finally, we make a conclusion and discuss the related issues.

In this paper, we set $\hbar = c = 8\pi G = 1$.

2. Basic Setup

Perturbation theory of general multiple field modes with non-canonical kinetic terms up to third-order action has been calculated by several authors [10, 11, 12, 13, 14, 15, 16, 17, 22]. In this work, we focus on multi-field DBI models [10, 11, 12, 13, 14, 15, 16]. In this section, we briefly review previous known results and setup our conventions.

2.1 The model and the background

In this work, we focus on two-field DBI inflationary models, with action of the form

$$S = \int d^4x \left( \frac{1}{2} R + \mathcal{L}^\phi \right),$$

with

$$\mathcal{L}^\phi \equiv -\frac{1}{\sqrt{f(\phi^I)}} \left( \sqrt{1 - 2fX + 2f^2(X^2 - X_{IJ}X^{IJ})} - V(\phi^I) \right),$$

where

$$X^{IJ} \equiv -\frac{1}{2} \partial_\mu \phi^I \partial^\mu \phi^J,$$

and $X \equiv G_{IJ}X^{IJ}$.

The slow-rolling parameter is defined as usual

$$\epsilon = -\frac{\dot{H}}{H^2}. \quad (2.4)$$

\textsuperscript{1}In general, the full DBI-type action takes the form

$$\mathcal{L} = -\frac{1}{\sqrt{D}} \left( \sqrt{D} - 1 \right) - V,$$

with

$$D = \det (\delta_{IJ} - 2fX_{IJ}) \equiv 1 - 2fX + 4f^2X^2_{IJ}X^{IJ} - 8f^3X^{IJK}X^{JK} + 16f^4X^{IJKL}X^{IJKL}.$$  

However, it is shown that higher-order terms proportional to $f^3$ and $f^4$ do not contribute to the calculation of power spectra and leading order bispectra. Thus, in this work we neglect these terms for simplicity. One can refer [10, 11, 12, 13, 14, 15, 16] for detailed investigations of multi-DBI inflation models.
The background “inflaton velocity” $\dot{\sigma}$ can be related with $\epsilon$ as
\[
\dot{\sigma} \equiv \sqrt{2X} = H \sqrt{2\epsilon c_s},
\] (2.5)
where $c_s$ plays the role of the propagating speed of the perturbation. In DBI inflation model,
\[
c_s^2 \equiv 1 - 2fX,
\] (2.6)
where $X$ is evaluated with the background quantities.

2.2 Brief review of linear perturbation

In multi-field inflation models, perturbations can be instantaneously decomposed into one adiabatic mode and several entropy modes [20]. Restricting to two-field case, we introduce the basis $\{e_\sigma, e_s\}$ with $e_I^\sigma \equiv \dot{\phi}^I / \dot{\sigma}$, where $\dot{\sigma}$ is the background inflaton velocity defined in (2.5), and $e_I^s$ is orthogonal to $e_I^\sigma$. Then we introduce the decomposition
\[
Q^I \equiv \delta \phi^I = Q_\sigma e_I^\sigma + Q_s e_I^s, \quad I = 1, 2.
\] (2.7)
$Q_\sigma$ and $Q_s$ are adiabatic mode and entropy mode respectively.

After the adiabatic/entropy modes decomposition, the second-order perturbation action for two-field DBI inflation model up to leading-order was derived in [10, 11, 14] in spatially-flat gauge:
\[
S^{(2)} = \int d\eta d^3x \frac{a^2}{2c_s^3} \left\{ [(Q_\sigma')^2 - c_s^2(\partial_i Q_\sigma)^2] + c_s^2 [(Q_s')^2 - c_s^2(\partial_i Q_s)^2] \right\},
\] (2.8)
where $H \equiv \dot{a}/a$ is the Hubble parameter, $\eta$ is the comoving time, and a prime denotes the derivative with respect to $\eta$. In deriving (2.8), we assume that $c_s$ is approximately constant and neglect coupling between adiabatic and entropy modes which may give significant effect in some cases, and keep only the leading order terms. Note that in general multi-field models, adiabatic mode and entropy modes can propagate with different (and arbitrary) speeds of sound, which was first pointed out in [12, 13] (see also [14, 15, 16, 17, 22]). Multi-field DBI model is a special case, where adiabatic mode and entropy modes propagate with the same speed of sound $c_s$.

In canonical quantization procedure, we write
\[
Q_\sigma(\eta, k) = u_k(\eta)a_k + u_k^*(\eta)a_{-k}^\dagger, \\
Q_s(\eta, k) = v_k(\eta)a_k + v_k^*(\eta)a_{-k}^\dagger.
\] (2.9)
In de Sitter approximation $a(\eta) = -1/H\eta$, the mode solutions can be easily solved
\[
u_k(\eta) = \frac{H}{c_s \sqrt{2k^3}} (1 + ic_s k\eta) e^{-ic_s k\eta},
\] (2.10)
Note that this choice of mode functions corresponds to the Bunch-Davis vacuum. Now it is straightforward to write down the tree-level two-point correlation functions (see Appendix A for more details),
\[
\langle 0\left| Q_\sigma(\eta, k_1)Q_\sigma(\eta', k_2) \right| 0 \rangle = (2\pi)^3 \delta^3(k_1 + k_2) G_{k_1}(\eta, \eta'),
\] (2.11)
\[
\langle 0\left| Q_s(\eta, k_1)Q_s(\eta', k_2) \right| 0 \rangle = (2\pi)^3 \delta^3(k_1 + k_2) F_{k_1}(\eta, \eta').
\]
the leading-order of slow-varying parameters and in the limit of small terms. Note that there is no simplification.

The third-order perturbation action of two-field DBI modelswas derived in [10, 11] (see also [12, 13, 14]). In

$$S_3 = \int d\eta d^3x \frac{a}{2c_s^2} \left\{ c_s^2 \left[ (Q_{\sigma})^3 - c_s^2(Q_{\sigma}')(\nabla Q_{\sigma})^2 \right] + Q_{\sigma}'(Q_s)^2 + c_s^2 Q_{\sigma}'(\nabla Q_s)^2 - 2c_s^2 Q_{\sigma}'\nabla Q_s \nabla Q_{\sigma} \right\} .$$

(2.14)

The first line in (2.8) is $Q_{\sigma}$ three-point self-interaction terms, while the second line is $Q_{\sigma}$-$Q_s$ cross-interaction terms. Note that there is no $Q_s$ self-interaction term.

In this work, we focus on the cross-interaction terms involving entropy mode and adiabatic mode. The interacting Hamiltonian at third-order is given by $H_i = -L_{(3)}$, where $L_{(3)}$ is the Lagrangian read from (2.14). There are three cross-interaction vertices,

$$H_i = H_{ia} + H_{ib} + H_{ic},$$

(2.15)

where subscript “i” denotes interacting part of the Hamiltonian, and

$$H_{ia}(\eta) = \frac{g}{\eta} \int \prod_{i=1}^3 \frac{d^3k_i}{(2\pi)^3} (2\pi)^3 \delta^3(k_{123}) Q_{\sigma}'(\eta, k_1) Q_s(\eta, k_2) Q_s'(\eta, k_3),$$

$$H_{ib}(\eta) = -\frac{g c_s^2}{\eta} \int \prod_{i=1}^3 \frac{d^3k_i}{(2\pi)^3} (2\pi)^3 \delta^3(k_{123}) (k_2 \cdot k_3) Q_{\sigma}'(\eta, k_1) Q_s(\eta, k_2) Q_s(\eta, k_3),$$

$$H_{ic}(\eta) = 2g c_s^2 \int \prod_{i=1}^3 \frac{d^3k_i}{(2\pi)^3} (2\pi)^3 \delta^3(k_{123}) (k_1 \cdot k_2) Q_{\sigma}(\eta, k_1) Q_s(\eta, k_2) Q_s'(\eta, k_3),$$

(2.16)

where we have changed into momentum space and

$$g \equiv \frac{1}{2H^2 c_s^3 \sqrt{2c_s}}.$$  

(2.17)

\footnote{In the literatures on “in-in formalism”, it used to define “two” types of Green’s functions: $G^c(\eta, \eta') \equiv u_k(\eta)u^*(\eta')$ and $G^s(\eta, \eta') \equiv u_k(\eta)u(\eta)$, keeping $\eta > \eta'$. It is easy to see that, mathematically, $G^c(\eta, \eta') \equiv (G^c(\eta, \eta'))^* \equiv G^s(\eta', \eta)$. Thus, if we relax the artificial restriction $\eta > \eta'$, there is no need to keep track of this odd notation. In this paper, we use $G_k(\eta, \eta')$ for simplicity.}
which plays the role of an “effective coupling constant” and we assume to be approximately constant during inflation. In deriving (2.16) we have used de Sitter approximation \(a = -1/H\eta\), and \(k_{123} \equiv k_1 + k_2 + k_3\) for simplicity.

From (2.16) it can be seen that the three types of three-point interaction vertices are all “derivative interactions”, as shown in Fig.1. This is different from ordinary field theory where fields are locally coupled. Moreover, as we will see, the derivatives bring momentum factors, which will cancel the momentum factors in the denominator come from the two-point correlation functions, and increase the finiteness in IR. Especially, in this work we will show that loop corrections arising from the above three-point vertices are all IR finite.

![Diagrammatic representations of three types of adiabatic-entropy three-point cross-interaction vertices. A red dot denotes temporal derivative while a blue dot denotes spatial derivative, or in fourier space, a momentum factor.](image)

There is another issue should be emphasized. In standard slow-roll inflation models, the “effective couplings” of the interaction vertices are greatly suppressed by slow-roll parameters (this is also why primordial non-gaussianities are much small in these models) and thus contributions to the correlation functions from multi-vertices diagrams are suppressed and can be neglected at least in the leading-order. However, this is no longer the case for \(c_s \ll 1\) models, where the strength of the interactions are much larger. This can be seen directly from (2.17), the effective coupling is enhanced by small \(c_s\). For example, in [23], trispectrum from exchanging scalar modes were investigated, it was shown that in \(c_s \ll 1\) models, in general the contribution from two-vertices diagrams cannot be neglected in comparison with one-vertices diagrams. The essential idea is that, contributions from diagrams with more numbers of vertices are of the same importance in comparison with contributions from diagrams with less numbers of vertices, even in the leading-order. This is also one of the motivations of this work.

### 3. One-loop Entropy Mode Corrections

Correlation functions in cosmological context are calculated by using “in-in” formalism [70, 71, 72] (see Appendix A for a brief review). In this work, we focus on the one-loop contributions arising from the three-point vertices, thus we need

\[
\langle Q_\sigma(\eta_*, k_1)Q_\sigma(\eta_*, k_2)\rangle_{1-\text{loop}} = -2 \text{Re} \left[ \int_{-\infty}^{\eta_*} d\eta_1 \int_{-\infty}^{\eta_1} d\eta_2 \langle 0 | Q_\sigma(\eta_*, k_1)Q_\sigma(\eta_*, k_2)H_1(\eta_1)H_1(\eta_2)| 0 \rangle \right] \\
+ \int_{-\infty}^{\eta_*} d\eta_1 \int_{-\infty}^{\eta_1} d\eta_2 \langle 0 | H_1(\eta_1)Q_\sigma(\eta_*, k_1)Q_\sigma(\eta_*, k_2)H_1(\eta_2)| 0 \rangle ,
\]

where we have written explicitly the lower bound for time-integrals\(^3\): \(-\infty^\pm \equiv -\infty(1 \mp i\epsilon)\) (see Appendix A for details). Note that all quantities on the right-hand side in (3.1) are “interaction-picture” quantities. In this paper we choose \(H_i\) involving three-point cross-interactions of \(Q_\sigma\) and \(Q_s\) which is given by (2.15) and (2.16).

\(^3\)One can refer a recent work [58], for a detailed discussion on the application of “in-in” formalism on cosmological correlation functions, and the issue of the lower bound of time integrals.
In the following, we neglect the subscript “i” for simplicity. Denote
\[
\langle Q_\sigma(\eta_1, k_1)Q_\sigma(\eta_2, k_2)\rangle_{\alpha\beta i} = -2 \text{Re} \left[ \int_{-\infty}^{\eta_1} d\eta_1 \int_{-\infty}^{\eta_2} d\eta_2 \langle 0 | Q_\sigma(\eta_1, k_1)Q_\sigma(\eta_2, k_2) H_\alpha(\eta_1) H_\beta(\eta_2) | 0 \rangle \right],
\]
\[
\langle Q_\sigma(\eta_1, k_1)Q_\sigma(\eta_2, k_2)\rangle_{\alpha\beta} = \int_{-\infty}^{\eta_1} d\eta_1 \int_{-\infty}^{\eta_2} d\eta_2 \langle 0 | H_\alpha(\eta_1) Q_\sigma(\eta_1, k_1)Q_\sigma(\eta_2, k_2) H_\beta(\eta_2) | 0 \rangle,
\]
where $H_\alpha$’s are given by (2.15) and (2.16). Thus, the final loop-corrections read,
\[
\langle Q_\sigma(\eta_1, k_1)Q_\sigma(\eta_2, k_2)\rangle_{1\text{-loop}} = \sum_{\alpha,\beta=\alpha,b,c}^2 \sum_{i=1}^2 \langle Q_\sigma(\eta_1, k_1)Q_\sigma(\eta_2, k_2)\rangle_{\alpha\beta i}.
\]

It is useful to note that $\langle \cdots \rangle_{\alpha\beta} = \langle \cdots \rangle_{\beta\alpha}$.

### 3.1 Calculating the Loop Corrections

Now we investigate the loop corrections. The calculation is straightforward but rather lengthy. In this section we show the explicit steps. Readers who are interested only in the final loop contributions can go to the next subsection directly.

#### 3.1.1 Diagonal contributions

From (3.1), the first integral reads,
\[
\langle Q_\sigma(\eta_1, k_1)Q_\sigma(\eta_2, k_2)\rangle_{\alpha\beta 1} = -2 \text{Re} \left[ \int_{-\infty}^{\eta_1} d\eta_1 \int_{-\infty}^{\eta_2} d\eta_2 \langle 0 | Q_\sigma(\eta_1, k_1)Q_\sigma(\eta_2, k_2) H_\alpha(\eta_1) H_\beta(\eta_2) | 0 \rangle \right].
\]

The integral in the squared-bracket in (3.4) is
\[
(2\pi)^3 \delta^3 (k_1 + k_2) 4 g^2 \int_{-\infty}^{\eta_1} d\eta_1 \int_{-\infty}^{\eta_2} d\eta_2 \frac{1}{\eta_1 \eta_2} \frac{d}{d\eta_1} G_{k_1}(\eta_1) \frac{d}{d\eta_2} G_{k_1}(\eta_2)
\]
\[
\times \int \frac{d^3 p}{(2\pi)^3} \frac{d^2}{d\eta_1 d\eta_2} F_p(\eta_1, \eta_2) \frac{d^2}{d\eta_1 d\eta_2} F_{|p-k_1|}(\eta_1, \eta_2).
\]

Actually, this expression can be read from the “Feynman-type” diagramatic representation of the contribution, as shown in Fig.2. The main differences from usual field theory is that, here the perturbations are mostly “derivatively-interacted”, rather than in usual field theory the interactions are described by local products of fields. Moreover, in cosmological context, we are interested in the expectation values of some arbitrary time rather than scattering amplitudes. This is why the so-called “in-in” formalism has to be taken. The only thing we should note is the order among three “time” — $\eta_1$, $\eta_2$ and $\eta_2$ between $\langle 0 |$ and $| 0 \rangle$. More precisely, in calculating the first integral of (3.2), the order is (from left to right) $\eta_1$, $\eta_2$, while in the second integral in (3.2), the order is $\eta_1$, $\eta_2$. Keeping these things in mind, we are able to write the above explicit expressions from Feynman-type diagrams directly.

Now, from (2.10) and (2.12), we have
\[
\frac{d}{d\eta_1} G_{k_1}(\eta_1) \frac{d}{d\eta_2} G_{k_1}(\eta_2) = \frac{c_4 H^4}{4k_1^2} \eta_1 \eta_2 \left(1 + i c_6 k_1 \eta_1 \right)^2 e^{i c_6 k_1 (\eta_1 + \eta_2 - 2\eta_i)},
\]
\[
\frac{d^2}{d\eta_1 d\eta_2} F_p(\eta_1, \eta_2) \frac{d^2}{d\eta_1 d\eta_2} F_{|p-k_1|}(\eta_1, \eta_2) = \frac{c_4 H^4}{4} \eta_1 \eta_2^2 \left| p + k_2 \right| e^{-i c_6 (p+|p+k_2|)(\eta_1-\eta_2)}.
\]
Thus, (3.5) can be written as
\[
(2\pi)^3 \delta^3(k_1 + k_2) g_a^2 c_s^8 H^8 \frac{c_s^8 H^8}{4k_1^3} (1 + i c_s k_1 \eta^*_a)^2 e^{-2i c_s k_1 \eta^*_a} \int \frac{d^3 p}{(2\pi)^3} \ p \ |p - k_1| \ x \ \int_{-\infty}^{\eta^*} d\eta_1 \ \eta_1^2 e^{ic_s(k_1 - p - |p - k_1|)\eta_1} \ \int_{-\infty}^{\eta^*} d\eta_2 \ \eta_2^2 e^{ic_s(k_1 + p + |p - k_1|)\eta_2},
\]
\[
(3.7)
\]

The lower bound of both time integrals are \(-\infty\), where the integrants highly oscillate. However, we have to remind ourselves that the lower bound should be chosen carefully in order to choose the true vacuum (see Appendix A for a detailed discussion on this problem and the issue of “choosing vacuum”). Actually, here the lower bound of \(\eta_1\) integral and \(\eta_2\) integral is the same, \(-\infty^+ \equiv -\infty(1 - i\epsilon)\). Thus, the time integrals can be evaluated without any ambiguity. For example, the integral w.r.t. \(\eta_2\) is evaluated as
\[
\int_{-\infty}^{\eta_1} d\eta_2 \ \eta_2^2 e^{ic_s(k_1 + p + |p - k_1|)\eta_2} = \frac{(2i + 2c_s K \eta_1 - i c_s^2 K^2 \eta_1^2) e^{ic_s K \eta_1}}{c_s^3 K^3},
\]
\[
(3.8)
\]

where we write \(K \equiv k_1 + p + |p - k_1|\) for simplicity. Thus, after performing the time integrals and taking the real part, we finally get
\[
\langle Q_\sigma(\eta^*_a, k_1) Q_\sigma(\eta^*_a, k_2) \rangle_{aa} = -(2\pi)^3 \ \delta^3(k_1 + k_2) g_a^2 c_s^8 H^8 \frac{c_s^8 H^8}{16 k_1^3} \ \int \frac{d^3 p}{(2\pi)^3} \ p \ |p - k_1| \ \left( a_1 K^2 + a_2 K \tilde{K} + a_3 \tilde{K}^2 \right),
\]
\[
(3.9)
\]

where we have defined \(\tilde{K} \equiv k_1 - p - |p - k_1|\), and
\[
a_1 \equiv 2 \ (2x^6_e - 5x^2_e - 5),
\]
\[
a_2 \equiv - (2x^4_e + 5x^2_e + 5),
\]
\[
a_3 \equiv - (2x^4_e + x^2_e + 1),
\]
\[
(3.10)
\]

with \(x_e \equiv -c_s k_1 \eta^*_a\) for short, note that \(K + \tilde{K} = 2 k_1\).

The next step is to evaluate the momentum loop-integral:
\[
I(k) \equiv \int \frac{d^3 p}{(2\pi)^3} \ p \ |p - k_1| \ \left( a_1 K^2 + a_2 K \tilde{K} + a_3 \tilde{K}^2 \right),
\]
\[
(3.11)
\]

remember that \(K \equiv k_1 + p + |p - k_1|\) and \(\tilde{K} \equiv k_1 - p - |p - k_1|\). Note that the integral (3.11) is IR finite, thus we do not need to introduce infrared cutoff \(\ell^{-1}\) here. It will be shown in this work that loop corrections from the three-point vertices from (2.16) are all IR finite, due to the “derivatively-coupling” of the perturbations. The
Integrand in (3.11) has the structure $f(k_1, p, |p - k_1|)$, in Appendix B, we describe the method to evaluate such type of integrals. In particular, here the function $f(k_1, p, |p - k_1|)$ is

$$f(k_1, p, |p - k_1|) = \frac{p |p - k_1|}{K_3} \left( a_1 K^2 + a_2 K \tilde{K} + a_3 \tilde{K}^2 \right).$$

After a straightforward but tedious calculation, we get

$$\langle Q_\sigma(\eta_1, k_1)Q_\sigma(\eta_2, k_2) \rangle_{aa1} = (2\pi)^3 \delta^3(k_1 + k_2) P_\sigma(k_1) \frac{H^2}{\pi^2 \epsilon C^2} (c_1 \ln k + \alpha_1), \quad (3.12)$$

where

$$c_1 = \frac{-4 - 4x_+^2 - 25x_+^4 + x_+^6}{1920}, \quad (3.13)$$

and $\alpha_1$ denotes the finite part of the momentum integral together with the scheme-dependent UV renormalization constant, which can be found in Appendix C.

Now let us move to the calculation of (aa2) contribution. The expression can also be read from fig.2:

$$\langle Q_\sigma(k_1, \eta_1)Q_\sigma(k_2, \eta_2) \rangle_{aa2} = (2\pi)^3 \delta^3(k_1 + k_2) 4 g^2 \int_{-\infty}^{\eta_1} d\eta_1 \int_{-\infty}^{\eta_2} d\eta_2 \frac{1}{\eta_1 \eta_2} \int \frac{d^3p}{(2\pi)^3} \frac{d}{d\eta_1} G_{k_1}(\eta_1, \eta_2) \frac{d}{d\eta_2} G_{k_1}(\eta_1, \eta_2) \quad (3.14)$$

It is useful to note that, the only difference of integrand from (aa1)-contribution is that, here in fig.2, the order of time is $\eta_1 - \eta_2 - \eta_2$. That is, what we need to do is to simply change the first Green’s function in (aa1)-case as following

$$\frac{d}{d\eta_1} G_{k_1}(\eta_1, \eta_1) \quad \Rightarrow \quad \frac{d}{d\eta_1} G_{k_1}(\eta_1, \eta_2),$$

to get the above result. Following the same strategy as above, we finally get

$$\langle Q_\sigma(k_1, \eta_1)Q_\sigma(k_2, \eta_2) \rangle_{aa2} = (2\pi)^3 \delta^3(k_1 + k_2) P_\sigma(k_1) \frac{H^2}{\pi^2 \epsilon C^2} (c_2 \ln k_1 + \alpha_2), \quad (3.15)$$

with

$$c_2 = \frac{1}{384} x_+^4 (1 + x_+^2), \quad (3.16)$$

and $\alpha_2$ can be found in Appendix C.

Collecting (3.12) and (3.15) together, we get the contribution from (aa)-term

$$\langle Q_\sigma(k_1, \eta_1)Q_\sigma(k_2, \eta_2) \rangle_{aa} = (2\pi)^3 \delta^3(k_1 + k_2) P_\sigma(k_1) \frac{H^2}{\pi^2 \epsilon C^2} (c_{aa} \ln k_1 + \alpha_{aa}), \quad (3.17)$$

with $P_\sigma(k_1)$ is the (tree-level) power spectrum of adiabatic mode $Q_\sigma$, and

$$c_{aa} = c_1 + c_2 = \frac{1}{960} (-2 - 2x_+^2 - 10x_+^4 + 3x_+^6). \quad (3.18)$$

The contributions from (bb)-term can be read from Fig.3.

$$\langle Q_\sigma(k_1, \eta_1)Q_\sigma(k_2, \eta_2) \rangle_{bb1}$$

$$= -(2\pi)^3 \delta^3(k_1 + k_2) 8 g^2 c_4^2 \times \text{Re} \int_{-\infty}^{\eta_1} d\eta_1 \int_{-\infty}^{\eta_2} d\eta_2 \frac{1}{\eta_1 \eta_2} \int \frac{d^3p}{(2\pi)^3} |p \cdot (p - k_1)|^2 \quad (3.19)$$

$$\times \frac{d}{d\eta_1} G_{k_1}(\eta_1, \eta_1) \frac{d}{d\eta_2} G_{k_1}(\eta_1, \eta_2) F_p(\eta_1, \eta_2) F_{|p-k_1|}(\eta_1, \eta_2).$$

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Figure 3: Diagrammatic representation of the loop corrections from (bb)-term. As before, a “red dot” denotes derivative with respect to comoving time associated with the vertex, a “blue dot” denotes a momentum factor associated with the line momentum. We may freely label the momentum flows into a vertex a “+” sign and momentum flows out of a vertex a “−” sign. The momentum factors of the same vertex are “scalar-producted”, e.g., the left vertex gives a factor $-p \cdot (p - k_1)$.

And

$$\langle Q_\sigma(k_1, \eta_*) Q_\sigma(k_2, \eta_*) \rangle_{bb} = (2\pi)^3 \delta^3(k_1 + k_2) 4 g^2 c_s^4 \int_{-\infty}^{\eta_*} d\eta_1 \int_{-\infty}^{\eta_*} d\eta_2 \frac{1}{\eta_1 \eta_2} \int \frac{d^3p}{(2\pi)^3} [p \cdot (p - k_1)]^2$$

$$\times \frac{d}{d\eta_1} G_{k_1}(\eta_1, \eta_*) \frac{d}{d\eta_2} G_{k_1}(\eta_*, \eta_2) F_p(\eta_1, \eta_2) F_{p - k_1}(\eta_1, \eta_2)$$

(3.20)

After lengthy but straightforward calculation, we get

$$\langle Q_\sigma(k_1, \eta_*) Q_\sigma(k_2, \eta_*) \rangle_{bb} = (2\pi)^3 \delta^3(k_1 + k_2) P_\sigma(k_1) \frac{H^2}{\pi^2 c_s^4} (c_{bb} \ln k_1 + \alpha_{bb})$$

(3.21)

with

$$c_{bb} = \frac{-273 + 27x_*^2 + 90x_*^4 - 8x_*^6}{3840}$$

(3.22)

again $\alpha_{bb}$ is the finite part of the momentum integral and can be found in Appendix C.

Contributions from (cc)-term can be read from Fig.4.

Figure 4: Diagrammatic representation of the loop corrections from (cc)-contribution. As before, a red dot denotes derivative with respect to comoving time associated with the vertex, a blue dot denotes a momentum factor associated with the line momentum. Note that there are two types of diagrams from (cc)-contributions.
In this work, we consider the one-loop two vertices contributions. Besides the loop contributions from the same vertices, there are contributions involving two different vertices, as shown in Fig. 5. As described in the previous section, their explicit expressions can be read from these Feynman-type diagrammatic representations.

\[
\langle Q_\sigma(k_1, \eta_s)Q_\sigma(k_2, \eta_s) \rangle_{cc1} = -(2\pi)^3 \delta^3(k_1 + k_2) 16 g^2 c_s^4 \times \text{Re} \int_{-\infty}^{\eta_s} d\eta_1 \int_{-\infty}^{\eta_1} d\eta_2 \frac{1}{\eta_1 \eta_2} \int \frac{d^3p}{(2\pi)^3} G_{k_1}(\eta_s, \eta_1) G_{k_1}(\eta_s, \eta_2) \times \left\{ (p \cdot k_1)^2 F_p(\eta_1, \eta_2) \frac{d^2}{d\eta_1 d\eta_2} F_{[p-k_1]}(\eta_1, \eta_2) - (p \cdot k_1) [k_1 \cdot (p - k_1)] \frac{d}{d\eta_1} F_p(\eta_1, \eta_2) \frac{d}{d\eta_2} F_{[p-k_1]}(\eta_1, \eta_2) \right\} . \tag{3.23}
\]

And

\[
\langle Q_\sigma(k_1, \eta_s)Q_\sigma(k_2, \eta_s) \rangle_{cc2} = (2\pi)^3 \delta^3(k_1 + k_2) 8 g^2 c_s^4 \int_{-\infty}^{\eta_s} d\eta_1 \int_{-\infty}^{\eta_1} d\eta_2 \frac{1}{\eta_1 \eta_2} \int \frac{d^3p}{(2\pi)^3} G_{k_1}(\eta_s, \eta_1) G_{k_1}(\eta_s, \eta_2) \times \left\{ (p \cdot k_1)^2 F_p(\eta_1, \eta_2) \frac{d^2}{d\eta_1 d\eta_2} F_{[p-k_1]}(\eta_1, \eta_2) - (p \cdot k_1) [k_1 \cdot (p - k_1)] \frac{d}{d\eta_1} F_p(\eta_1, \eta_2) \frac{d}{d\eta_2} F_{[p-k_1]}(\eta_1, \eta_2) \right\} . \tag{3.24}
\]

Finally we get

\[
\langle Q_\sigma(k_1, \eta_s)Q_\sigma(k_2, \eta_s) \rangle_{cc} = (2\pi)^3 \delta^3(k_1 + k_2) P_\sigma(k_1) \frac{H^2}{\pi^2 c_s^2} (c_{cc} \ln k_1 + \alpha_{cc}) \tag{3.25}
\]

with

\[
c_{cc} = \frac{1}{480} \left[ -14 + x_s^2 \left( -7 + x_s^2 \right) \left( 2 + x_s^2 \right) \right], \tag{3.26}
\]

and \( \alpha_{cc} \) can be found in Appendix C.

### 3.1.2 Off-diagonal contributions

In this work, we consider the one-loop two vertices contributions. Besides the loop contributions from the same vertices, there are contributions involving two different vertices, as shown in Fig. 5. As described in the previous section, their explicit expressions can be read from these Feynman-type diagrammatic representations.

![Diagram](image)

**Figure 5:** Diagrammatic representations of off-diagonal contributions. From left to right are contributions from (ab), (ac) and (bc) combinations respectively. We do not show the (ba), (ca) and (cb) contributions explicitly.

Contributions from (ab)-term are

\[
\langle Q_\sigma(k_1, \eta_s)Q_\sigma(k_2, \eta_s) \rangle_{ab1} = -(2\pi)^3 \delta^3(k_1 + k_2) 8 g^2 c_s^4 \times \text{Re} \int_{-\infty}^{\eta_s} d\eta_1 \int_{-\infty}^{\eta_1} d\eta_2 \frac{1}{\eta_1 \eta_2} \int \frac{d^3p}{(2\pi)^3} [p \cdot (p - k_1)] \times \frac{d}{d\eta_1} G_{k_1}(\eta_s, \eta_1) \frac{d}{d\eta_2} G_{k_1}(\eta_s, \eta_2) \frac{d}{d\eta_1} F_p(\eta_1, \eta_2) \frac{d}{d\eta_1} F_{[p-k_1]}(\eta_1, \eta_2) . \tag{3.27}
\]
And
\[
\langle Q_\sigma(k_1, \eta_s)Q_\sigma(k_2, \eta_s)\rangle_{ab2} = (2\pi)^3 \delta^3(k_1 + k_2) 4g^2c_s^2 \int_{-\infty}^{\eta_s} d\eta_1 \int_{-\infty}^{\eta_s} d\eta_2 \frac{1}{\eta_1\eta_2} \int \frac{d^3p}{(2\pi)^3} [p \cdot (p - k_1)]
\]
\[
\times \frac{d}{d\eta_1}G_{k_1}(\eta_1, \eta_s)G_{k_1}(\eta_s, \eta_2) \frac{d}{d\eta_1}F_p(\eta_1, \eta_2) \frac{d^3p}{d\eta_2}F_p|_{p - k_1}(\eta_1, \eta_2).
\]
(3.28)

The (ba) contributions can be also read from Fig.5 which we do not write down explicitly. Finally, we denote the contributions from (ab) and (ba) combinations as:
\[
\langle Q_\sigma(k_1, \eta_s)Q_\sigma(k_2, \eta_s)\rangle_{ab} = (2\pi)^3 \delta^3(k_1 + k_2) P_\sigma(k_1) \frac{H^2}{\pi^2 c_s^5} (c_{ab} \ln k_1 + \alpha_{ab})
\]
(3.29)

with
\[
c_{ab} = \frac{4x_s^6 - 10x_s^4 - 11x_s^2 + 49}{3840}.
\]
(3.30)

Contributions from (ac)-term are:
\[
\langle Q_\sigma(k_1, \eta_s)Q_\sigma(k_2, \eta_s)\rangle_{ac1} = (2\pi)^3 \delta^3(k_1 + k_2) 16g^2c_s^2 \times \text{Re} \int_{-\infty}^{\eta_s} d\eta_1 \int_{-\infty}^{\eta_s} d\eta_2 \frac{1}{\eta_1\eta_2} \int \frac{d^3p}{(2\pi)^3} [p \cdot k_1]
\]
\[
\times \frac{d}{d\eta_1}G_{k_1}(\eta_s, \eta_1)G_{k_1}(\eta_s, \eta_2) \frac{d}{d\eta_1}F_p(\eta_1, \eta_2) \frac{d^2}{d\eta_1d\eta_2}F_p|_{p - k_1}(\eta_1, \eta_2).
\]
(3.31)

And
\[
\langle Q_\sigma(k_1, \eta_s)Q_\sigma(k_2, \eta_s)\rangle_{ac2} = -(2\pi)^3 \delta^3(k_1 + k_2) 8g^2c_s^2 \int_{-\infty}^{\eta_s} d\eta_1 \int_{-\infty}^{\eta_s} d\eta_2 \frac{1}{\eta_1\eta_2} \int \frac{d^3p}{(2\pi)^3} [p \cdot k_1]
\]
\[
\times \frac{d}{d\eta_1}G_{k_1}(\eta_s, \eta_1)G_{k_1}(\eta_s, \eta_2) \frac{d}{d\eta_1}F_p(\eta_1, \eta_2) \frac{d^2}{d\eta_1d\eta_2}F_p|_{p - k_1}(\eta_1, \eta_2).
\]
(3.32)

Similarly, we denote the contributions from (ac) and (ca) as
\[
\langle Q_\sigma(k_1, \eta_s)Q_\sigma(k_2, \eta_s)\rangle_{ac} = (2\pi)^3 \delta^3(k_1 + k_2) P_\sigma(k_1) \frac{H^2}{\pi^2 c_s^5} (c_{ac} \ln k_1 + \alpha_{ac})
\]
(3.33)

with
\[
c_{ac} = \frac{-28x_s^6 + 6x_s^4 - 23x_s^2 - 23}{3840}.
\]
(3.34)

Contributions from (bc)-term are:
\[
\langle Q_\sigma(k_1, \eta_s)Q_\sigma(k_2, \eta_s)\rangle_{bc1} = (2\pi)^3 \delta^3(k_1 + k_2) 16g^2c_s^4 \times \text{Re} \int_{-\infty}^{\eta_s} d\eta_1 \int_{-\infty}^{\eta_s} d\eta_2 \frac{1}{\eta_1\eta_2} \int \frac{d^3p}{(2\pi)^3} [p \cdot (p - k_1)] (p \cdot k_1)
\]
\[
\times \frac{d}{d\eta_1}G_{k_1}(\eta_s, \eta_1)G_{k_1}(\eta_s, \eta_2) \frac{d}{d\eta_1}F_p(\eta_1, \eta_2) \frac{d^3p}{d\eta_2}F_p|_{p - k_1}(\eta_1, \eta_2).
\]
(3.35)

And
\[
\langle Q_\sigma(k_1, \eta_s)Q_\sigma(k_2, \eta_s)\rangle_{bc2} = -(2\pi)^3 \delta^3(k_1 + k_2) 8g^2c_s^4 \int_{-\infty}^{\eta_s} d\eta_1 \int_{-\infty}^{\eta_s} d\eta_2 \frac{1}{\eta_1\eta_2} \int \frac{d^3p}{(2\pi)^3} [p \cdot (p - k_1)] (p \cdot k_1)
\]
\[
\times \frac{d}{d\eta_1}G_{k_1}(\eta_s, \eta_1)G_{k_1}(\eta_s, \eta_2) \frac{d}{d\eta_1}F_p(\eta_1, \eta_2) \frac{d^3p}{d\eta_2}F_p|_{p - k_1}(\eta_1, \eta_2).
\]
(3.36)
While the total contributions from (bc) and (cb) terms are,

\[ \langle Q_\sigma(k_1, \eta^*_s)Q_\sigma(k_2, \eta^*_s) \rangle_{bc} = (2\pi)^3 \delta^3(k_1 + k_2) P_\sigma(k_1) \frac{H^2}{\pi^2 c^2_s} (c_{bc} \ln k_1 + \alpha_{bc}) \]  

(3.37)

with

\[ c_{bc} = \frac{12x_s^6 + 66x_s^4 + 147x_s^2 + 227}{3840}. \]  

(3.38)

### 3.2 Final Results

Collect (3.17)-(3.37) together, the final one-loop contributions from entropy mode to the adiabatic power spectrum is

\[ \langle Q_\sigma(\eta^*_s, k_1)Q_\sigma(\eta^*_s, k_2) \rangle_{1\text{-loop}} = (2\pi)^3 \delta^3(k_1 + k_2) P_\sigma(k_1) \frac{H^2}{\pi^2 c^2_s} (c \ln k_1 + \alpha), \]  

(3.39)

with

\[ c = \frac{1}{960} (18x_s^4 + 5x_s^2 - 35). \]  

(3.40)

Note that the above result should be supplemented with a scheme-dependent UV renormalization constant. This result is consistent with previous analysis [50, 51, 52, 53, 54, 55, 56, 57, 58, 59]. In this work, the loop corrections show no IR divergences. This is mainly due to that in \( c_s \ll 1 \) limit, the leading-order interaction vertices are dominated by “derivatively-coupled” ones, which enhance the IR safety. Moreover, our result agrees with the bound obtained in [76, 77, 78], where perturbation theory breaks down.

### 4. Conclusion and Discussion

In this work, we calculate the one-loop corrections to the power spectrum of adiabatic mode from three-point adiabatic/entropy cross-interactions, at the time of horizon exiting. As mentioned before, in \( c_s \ll 1 \) models, there is enhancement of the effective couplings for interaction vertices, thus contributions (higher-order correlations functions and their loop-corrections) from multi-vertices diagrams are in general cannot be neglected. Precisely, one may expected the two-vertices one-loop contributions and the one-vertex one-loop contributions are of the same order. We find that, as the enhancement of non-gaussianities in models with non-canonical kinetic terms due to the enhancement of small speed of sound \( c_s \), the loop corrections are also enhanced by both the slow-roll parameter \( \epsilon \) and \( c_s \), more precisely, of order \( \sim \frac{H^2}{c^2_s} \). Thus in general, the loop corrections can be large. Moreover, we find that the loop contributions are IR finite. Although in this work we shown these “large” loop effect through explicit calculations, they can be expected by a simple counting of the order of magnitude of the interaction vertices. The point is that, in relatively large parameter region we need to take into account the higher order loop corrections when we discuss models with small \( c_s \), which may be important in the investigation of large non-Gaussianities.

There are several issues to be addressed. Firstly, the perturbations of inflaton fields and thus the correlation functions themselves are not observable directly. What is responsible for the CMB anisotropies and LSS is the curvature perturbation \( \zeta \). Thus we should combine the perturbations of inflaton fields to yield the perturbation of curvature perturbation long after horizon exiting, e.g. by using \( \delta N \) formalism [73, 74, 75], as in [55]. However, it is well-known that in multiple field inflation models, the curvature perturbation is not conserved on superhorizon scales, or in other words, there are also cross-correlations among different modes after horizon exiting. Due to these reasons, the complete analysis of loop corrections to the curvature perturbation in multi-field models is a complicate task which needs more subsequent work.

In this work, we focus on the three-point adiabatic/entropy cross-interactions. While from (2.14), in general, there are adiabatic mode self-interactions. Moreover, in multiple-DBI models, these various interaction-vertices
give the same order of contributions, both to non-gaussianities and loop corrections. A complete analysis of loop corrections including contributions from adiabatic loops and also the relations with observables (e.g. the curvature perturbation and the CMB anisotropies) are also needed.

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A. A Brief Review of “In-in” Formalism

A.1 Preliminaries

The “in-in formalism” (also dubbed as “Schwinger-Keldysh formalism”, or “Closed-time path formalism”) \[70, 71, 72\] is a perturbative approach for solving the evolution of expectation values over a finite time interval. It is therefore ideally suited not only to backgrounds which do not admit an S-matrix description, such as inflationary backgrounds.

In QFT applied in the calculation of S-matrix in particle physics, the goal is to determine the amplitude for a state in the far past \(|\psi\rangle\) to become some state \(|\psi'\rangle\) in the far future,

\[
\langle \psi' | S | \psi \rangle = \langle \psi' (+\infty) | \psi (-\infty) \rangle.
\]

Here, conditions are imposed on the fields at both very early and very late times. This can be done because that in Minkowski spacetime, the states are assumed to be non-interacting at far past and at far future, and thus are usually taken to be the free vacuum, i.e., the vacuum of the free Hamiltonian \(H_0\). The free vacuum are assumed to be in “one-to-one” correspondence with the true vacuum of the whole interacting theory, as we adiabatically turn on and turn off the interactions between \(t = -\infty\) and \(t = +\infty\).

While the physical situation we are considering here is quite different. Instead of specifying the asymptotic conditions both in the far past and far future, we develop a given state forward in time from a specified initial time, which can be chosen as the beginning of inflation. In cosmological context, the initial state are usually chosen as free vacuum, such as Bunch-Davis vacuum, since at very early times when perturbation modes are deep inside the Hubble horizon, according to the equivalence principle, the interaction-picture fields should have the same form as in Minkowski spacetime.

A.2 “In” vacuum

The Hamiltonian can be split into a free part and an interacting part: \(H = H_0 + H_i\). The time-evolution operator in the interacting picture is well-known

\[
U(\eta_2, \eta_1) = T \exp \left( -i \int_{\eta_1}^{\eta_2} dt' H_i(\eta') \right),
\]  

(A.1)

where subscript “I” denotes interaction-picture quantities, \(T\) is the time-ordering operator. Our present goal is to relate the interacting vacuum at arbitrary time \(|\Omega_t(t)\rangle\) with free vacuum \(|0_i\rangle\) (e.g., Bunch-Davis vacuum).
The trick is standard. First we may expand $|\Omega_1(\eta)\rangle$ in terms of eigenstates of free Hamiltonian $H_0$, $|\Omega_1(\eta)\rangle = \sum_n |n\rangle \langle n| \Omega_1(\eta)\rangle$, then we evolve $|\Omega_1(\eta)\rangle$ by using (A.1)

$$|\Omega_1(\eta_2)\rangle = U(\eta_2, \eta_1) |\Omega_1(\eta_1)\rangle = |0\rangle \langle 0| \Omega_1 + \sum_{n \geq 1} e^{+iE_n(\eta_2-\eta_1)} |n\rangle \langle n| \Omega_1(\eta_1)\rangle. \quad (A.2)$$

From (A.2), we can immediately see that, if we choose $\eta_2 = -\infty(1 - i\epsilon)$, all excited states in (A.2) are suppressed. Thus we relate interacting vacuum at $\eta = -\infty(1 - i\epsilon)$ with free vacuum $|0\rangle$ as

$$|\Omega_1(-\infty(1 - i\epsilon))\rangle = |0\rangle \langle 0| \Omega_1$$

Thus, the interacting vacuum at arbitrary time $\eta$ is given by

$$|\text{VAC, in}\rangle \equiv |\Omega_1(\eta)\rangle = U(\eta, -\infty(1 - i\epsilon)) |\Omega_1(-\infty(1 - i\epsilon))\rangle = T \exp \left( -i \int_{-\infty(1 - i\epsilon)}^{\eta} d\eta' H_{il}(\eta') \right) |0\rangle \langle 0| \Omega_1.$$ \quad (A.4)

### A.3 Expectation values in “in-in” formalism

The expectation value of operator $\hat{O}(\eta)$ at arbitrary time $\eta$ is evaluated as

$$\langle \hat{O}(\eta) \rangle = \langle \text{VAC, in}\rangle |\hat{O}(\eta)| \langle \text{VAC, in}\rangle = \left\langle 0 | T \exp \left( i \int_{-\infty(1 + i\epsilon)}^{\eta} d\eta' H_{il}(\eta') \right) \hat{O}_l(\eta) T \exp \left( -i \int_{-\infty(1 - i\epsilon)}^{\eta} d\eta' H_{il}(\eta') \right) |0\rangle \right\rangle, \quad (A.5)$$

where $T$ is the anti-time-ordering operator.

For simplicity, we denote $-\infty(1 - i\epsilon) \equiv -\infty^+$ and $-\infty(1 + i\epsilon) \equiv -\infty^-$, since, e.g., $-\infty^+$ has a positive imaginary part. Now let us focus on the time-order in (A.5). In standard S-matrix calculations, operators between $|0\rangle$ and $|0\rangle$ are automatically time-ordered. While in (A.5), from right to left, the time starts from infinite past, or $-\infty^+$ precisely, to some arbitrary time $\eta$ when the expectation value is evaluated, then back to $-\infty^-$ again. This time-contour, which is shown in Fig.6, forms a closed-time path, so “in-in” formalism is sometimes called “closed-time path” (CTP) formalism.

![Figure 6: Closed-time path in “in-in” formalism.](image)

### A.4 Perturbation theory

The starting point of perturbation theory is the free theory two-point correlation functions. In canonical quantization procedure, we write a scalar field as

$$\phi_k(\eta) = u(k, \eta)a_k + u^*(k, \eta)a_k^\dagger,$$ \quad (A.6)

where $u(k, \eta)$ is the mode function for $\phi_k(\eta)$ (in practice, $u_k(\eta)$ and $u_k^*(\eta)$ are two linear-independent solutions of equation of motion for $\phi_k(\eta)$, which are Wroskian normalized and satisfy some initial or asymptotic conditions ).
The free two-point function takes the form
\[ \langle 0 | \phi_{k_1}(\eta_1) \phi_{k_2}(\eta_2) | 0 \rangle \equiv (2\pi)^3 \delta^3(k_1 + k_2) G_{k_1}(\eta_1, \eta_2), \]  
(A.7)

with
\[ G_{k_1}(\eta_1, \eta_2) \equiv u_{k_1}(\eta_1) u^*_{k_1}(\eta_2). \]
(A.8)

In this work, we take (A.7) and (A.8) as the starting point.

Now Taylor expansion of (A.5) gives

- 0th-order
  \[ \left\langle \hat{O}(\eta) \right\rangle^{(0)} = \left\langle 0 | \hat{O}(\eta) | 0 \right\rangle. \]
  (A.9)

- 1st-order (one interaction vertex)
  \[ \left\langle \hat{O}(\eta) \right\rangle^{(1)} = 2 \text{Re} \left[ -i \int_{-\infty}^{\eta} d\eta' \left\langle 0 | \hat{O}(t) H_{\eta}(t') | 0 \right\rangle \right]. \]
  (A.10)

- 2nd-order (two interaction vertices)
  \[ \left\langle \hat{O}(\eta) \right\rangle^{(2)} = -2 \text{Re} \left[ \int_{-\infty}^{\eta} d\eta' \int_{-\infty}^{\eta''} d\eta'' \left\langle 0 | \hat{O}(\eta) H_{\eta}(\eta') H_{\eta''}(\eta'') | 0 \right\rangle \right]. \]
  (A.11)

Here in this work, since we are considering one-loop contributions from three-point interaction vertices, (A.11) is needed.

**A.5 Positive/negative-fields notation**

There is a formulation of “in-in formalism” in terms of “doubled fields” in the literatures, mostly applied in the path-integral formulation. The reason is that, the closed-time path (CTP) is imagined to circle the time axis in the complex \( \eta \)-plane (see fig.6). Accordingly, we distinguish operator values from the upper and lower branches by labeling a “+” on the upper increasing-time contour and a “−” on the lower decreasing-time contour. Then it is convenient to introduce a new “time-path-ordering operator” \( T_p \), which orders operator along the “CTP-path”, as shown in Fig.6. Obviously, \( T_p \) acts on “+” fields as time-ordering operator \( T \), and acts “−” fields as anti-time-ordering operator \( \tilde{T} \). Thus (A.5) can be recast into a more convenient and familiar form
\[ \left\langle \hat{O}(\eta) \right\rangle \equiv \left\langle 0 | T_p \exp \left( \int_{-\infty}^{\eta} d\eta' H^{-}_{\eta'}(\eta') \right) \hat{O}(\eta) \exp \left( -i \int_{-\infty}^{\eta} d\eta' H^{+}_{\eta'}(\eta') \right) | 0 \right\rangle \]
\[ = \left\langle 0 | T_p \hat{O}(\eta) \exp \left( -i \int_{-\infty}^{\eta} d\eta' \left[ H^{+}_{\eta'}(\eta') - H^{-}_{\eta'}(\eta') \right] \right) | 0 \right\rangle. \]
(A.12)

Now, since we have two types of fields, the contractions between different pairs yields four kinds of correlation functions,
\[ \left\langle 0 | T_p \phi^\pm_{k_1}(\eta_1) \phi^\pm_{k_2}(\eta_2) | 0 \right\rangle \equiv (2\pi)^3 \delta^3(k_1 + k_2) G^{\pm\pm}_{k}(\eta_1, \eta_2), \]
(A.13)

where
\[ G^{++}_{k}(\eta_1, \eta_2) = G^{\pm}_{k}(\eta_1, \eta_2) \theta(\eta_1 - \eta_2) + G^{\pm}_{k}(\eta_1, \eta_2) \theta(\eta_2 - \eta_1), \]
\[ G^{+-}_{k}(\eta_1, \eta_2) = G^{\pm}_{k}(\eta_1, \eta_2) \theta(\eta_1 - \eta_2) + G^{\pm}_{k}(\eta_1, \eta_2) \theta(\eta_2 - \eta_1), \]
\[ G^{\pm-}_{k}(\eta_1, \eta_2) = G^{\pm}_{k}(\eta_1, \eta_2), \]
\[ G^{--}_{k}(\eta_1, \eta_2) = G^{\pm}_{k}(\eta_1, \eta_2), \]
(A.14)
and
\[
G^>_k(\eta_1,\eta_2) \equiv u_k(\eta_1)u^*_k(\eta_2), \\
G^<_k(\eta_1,\eta_2) \equiv u^*_k(\eta_1)u_k(\eta_2) \equiv (G^>_k(\eta_1,\eta_2))^* \equiv G^>_k(\eta_2,\eta_1),
\]
(A.15)
are the vacuum Wightman functions. Now the perturbation calculations are standard, after expanding the T\_\text{p}-ordered operator exponential in (A.12).

The "doubled-field" notation is a convenient "book-marker" in path-integral formulation of "in-in formalism" in order to put everything into "one" exponential, while this notation itself is not necessary in principle. Especially, in operator formalism, there is no need to take this notation at all (see also [58] for a discussion on this point).

B. Momentum Integral

In this work, the momentum loop-integrals have the structure as
\[
I(k) \equiv \int \frac{d^3p}{(2\pi)^3} f(k,p,|p-k|)
\]
(B.1)
where \(k\) is the momentum of external line, and \(p\) is the loop momentum. In this appendix, we briefly describe the methodology for evaluating such type of integrals.

In general, one may use spherical coordinates \((p,\theta,\phi)\) to calculate (B.1). However, it is more convenient to introduce a new variable
\[
q \equiv |p-k| = \sqrt{k^2 + p^2 - 2kp \cos \theta},
\]
(B.2)
and change coordinate \((p,\theta,\phi)\) into \((p,q,\phi)\). Note that this can be done only when \(k \neq 0\). The integral measure is simply \(d^3p = \frac{d^2q}{k} dq \, d\phi\). Thus, the original integral (B.1) becomes
\[
I(k) = \frac{1}{(2\pi)^2 k} \int_0^{+\infty} dp \int_{p-k}^{p+k} dq \, pq f(k,p,q).
\]
(B.3)
The integral region for \(p\) and \(q\) is shown in Fig.7.

![Figure 7: Integral region for variables \(p\) and \(q\).](image.png)

One may use (B.3) as the starting point. However, it can be seen directly from Fig.7 that sometimes it is more convenient to use "rotated" variables
\[
x \equiv q + p, \quad y \equiv q - p, \\
p = \frac{1}{2}(x - y), \quad q = \frac{1}{2}(x + y),
\]
(B.4)
with integral measure \( dpdq = \frac{1}{2} dx dy \). Finally, the integral can be put into the form
\[
I(k) = \frac{1}{8(2\pi)^2 k} \int_{-k}^{k} dx \int dx \int dy \left( x^2 - y^2 \right) f \left( k, \frac{1}{2}(x-y), \frac{1}{2}(x+y) \right).
\]  

(B.5)

C. Finite parts

For clarity, we collect the finite constant parts of the loop integrals.

\[
\alpha_1 = -59 - 256 \ln 2 - (59 + 256 \ln 2)x_+^2 + (464 - 1600 \ln 2)x_+^4 + (-30 + 64 \ln 2)x_+^6
\]
\[\frac{122880}{122880},\]

\[
\alpha_2 = (1 + x_+^2) \left( 331 + 30(-21 + 40 \ln 2)x_+^4 \right)
\]
\[\frac{1843200}{1843200},\]

\[
\alpha_{aa} = 439 - 3840 \ln 2 + (439 - 3840 \ln 2)x_+^2 - 120(-37 + 160 \ln 2)x_+^4 + 90(-33 + 64 \ln 2)x_+^6
\]
\[\frac{614400}{614400},\]

\[
\alpha_{bb} = -2687 - 43680 \ln 2 + (34913 + 4320 \ln 2)x_+^2 + 360(-47 + 40 \ln 2)x_+^4 + (1410 - 1280 \ln 2)x_+^6
\]
\[\frac{614400}{614400},\]

\[
\alpha_{cc} = 5647 - 6720 \ln 2 + x_+^2 \left( 5647 - 6720 \ln 2 + 60x_+^2 \left( 23 - 40 \ln 2 + 8 \ln 2 \right) \right)
\]
\[\frac{230400}{230400},\]

\[
\alpha_{ab} = 3x_+^2 \left( 30x_+^2 \left( 11 + 32 \ln 2 \right) - 452 - 80 \ln 2 \right) - 11161 - 2640 \ln 2) + 4439 + 11760 \ln 2
\]
\[\frac{921600}{921600},\]

\[
\alpha_{ac} = -120x_+^6(28 \ln 2 - 11) + 20x_+^4(21 + 36 \ln 2) + x_+^2(601 - 2760 \ln 2) + 601 - 2760 \ln 2
\]
\[\frac{460800}{460800},\]

\[
\alpha_{bc} = 120x_+^6(12 \ln 2 - 19) + 60x_+^4(77 + 132 \ln 2) + x_+^2(17640 \ln 2 - 4849) - 1249 + 27240 \ln 2
\]
\[\frac{460800}{460800}.\]

And
\[
\alpha = x_+^2 \left( -6x_+^2 \left( x_+^2 + 22 - 18 \ln 2 \right) + 347 + 30 \ln 2 \right) + 137 - 210 \ln 2
\]
\[\frac{5760}{5760}.
\]  

(C.1)

References

[1] E. Komatsu et al. [WMAP Collaboration], Astrophys. J. Suppl. 180, 330 (2009)

[2] D. H. Lyth and A. Riotto, Phys. Rept. 314, 1 (1999) [arXiv:hep-ph/9807278].

[3] J. M. Maldacena, JHEP 0305, 013 (2003) [arXiv:astro-ph/0210603].

[4] N. Bartolo, E. Komatsu, S. Matarrese and A. Riotto, Phys. Rept. 402, 103 (2004) [arXiv:astro-ph/0406398].

[5] D. Seery and J. E. Lidsey, JCAP 0506, 003 (2005) [arXiv:astro-ph/0503692].

[6] X. Chen, M. x. Huang, S. Kachru and G. Shiu, JCAP 0701, 002 (2007) [arXiv:hep-th/0605045].

[7] X. Chen, M. x. Huang and G. Shiu, Phys. Rev. D 74, 121301 (2006) [arXiv:hep-th/0610235].

[8] F. Arroja and K. Koyama, Phys. Rev. D 77, 083517 (2008) [arXiv:0802.1167 [hep-th]].

[9] D. Seery and J. E. Lidsey, JCAP 0509, 011 (2005) [arXiv:astro-ph/0506056].

[10] D. Langlois, S. Renaux-Petel, D. A. Steer and T. Tanaka, Phys. Rev. D 78, 063523 (2008) [arXiv:0806.0336 [hep-th]].

[11] D. Langlois, S. Renaux-Petel, D. A. Steer and T. Tanaka, Phys. Rev. Lett. 101, 061301 (2008) [arXiv:0804.3139 [hep-th]].
[12] D. A. Easson, R. Gregory, D. F. Mota, G. Tasinato and I. Zavala, JCAP 0802, 010 (2008) [arXiv:0709.2666 [hep-th]].
[13] M. x. Huang, G. Shiu and B. Underwood, Phys. Rev. D 77, 023511 (2008) [arXiv:0709.3299 [hep-th]].
[14] F. Arroja, S. Mizuno and K. Koyama, JCAP 0808, 015 (2008) [arXiv:0806.0619 [astro-ph]].
[15] Y. F. Cai and W. Xue, arXiv:0809.4134 [hep-th].
[16] Y. F. Cai and H. Y. Xia, arXiv:0904.0062 [hep-th].
[17] X. Ji and T. Wang, arXiv:0903.0379 [hep-th].
[18] Q. G. Huang, arXiv:0904.2649 [hep-th].
[19] Q. G. Huang, arXiv:0903.1542 [hep-th].
[20] C. Gordon, D. Wands, B. A. Bassett and R. Maartens, Phys. Rev. D 63, 023506 (2001) [arXiv:astro-ph/0009131].
[21] B. A. Bassett, S. Tsujikawa and D. Wands, Rev. Mod. Phys. 78, 537 (2006) [arXiv:astro-ph/0507632].
[22] Xian Gao, JCAP 0806, 029 (2008) [arXiv:0804.1055 [astro-ph]].
[23] Xian Gao and Bin Hu, arXiv:0903.1920 [astro-ph.CO].
[24] V. F. Mukhanov, L. R. W. Abramo and R. H. Brandenberger, Phys. Rev. Lett. 78, 1624 (1997) [arXiv:gr-qc/9709026].
[25] L. R. W. Abramo, R. H. Brandenberger and V. F. Mukhanov, Phys. Rev. D 56, 3248 (1997) [arXiv:gr-qc/9704037].
[26] L. R. W. Abramo and R. P. Woodard, Phys. Rev. D 60, 044010 (1999) [arXiv:astro-ph/9811430].
[27] W. Unruh, arXiv:astro-ph/9802323.
[28] V. K. Onemli and R. P. Woodard, Class. Quant. Grav. 19, 4607 (2002) [arXiv:gr-qc/0204065].
[29] T. Prokopec, O. Tornkvist and R. P. Woodard, Annals Phys. 303, 251 (2003) [arXiv:gr-qc/0205130].
[30] T. Prokopec and R. P. Woodard, Am. J. Phys. 72, 60 (2004) [arXiv:astro-ph/0303358].
[31] V. K. Onemli and R. P. Woodard, Phys. Rev. D 70, 107301 (2004) [arXiv:gr-qc/0406098].
[32] T. Brunier, V. K. Onemli and R. P. Woodard, Class. Quant. Grav. 22, 59 (2005) [arXiv:gr-qc/0408080].
[33] E. O. Kahya and R. P. Woodard, Phys. Rev. D 72, 104001 (2005) [arXiv:gr-qc/0508015].
[34] E. O. Kahya and R. P. Woodard, Phys. Rev. D 74, 084012 (2006) [arXiv:gr-qc/0608049].
[35] E. O. Kahya, V. K. Onemli and R. P. Woodard, arXiv:0904.4811 [gr-qc].
[36] R. H. Brandenberger, arXiv:hep-th/0210165.
[37] G. Geshnizjani and R. Brandenberger, JCAP 0504, 006 (2005) [arXiv:hep-th/0310265].
[38] R. Brandenberger and A. Mazumdar, JCAP 0408, 015 (2004) [arXiv:hep-th/0402205].
[39] R. H. Brandenberger and J. Martin, Phys. Rev. D 71, 023504 (2005) [arXiv:hep-th/0410223].
[40] R. H. Brandenberger and C. S. Lam, arXiv:hep-th/0407048.
[41] P. Martineau and R. H. Brandenberger, Phys. Rev. D 72, 023507 (2005) [arXiv:astro-ph/0505236].
[42] I. S. Gerstein, R. Jackiw, S. Weinberg and B. W. Lee, Phys. Rev. D 3, 2486 (1971).
[43] D. Boyanovsky, H. J. de Vega and N. G. Sanchez, Nucl. Phys. B 747, 25 (2006) [arXiv:astro-ph/0503669].
[44] D. Boyanovsky, H. J. de Vega and N. G. Sanchez, Phys. Rev. D 72, 103006 (2005) [arXiv:astro-ph/0507596].
[45] K. Chaicherdsakul, Phys. Rev. D 75, 063522 (2007) [arXiv:hep-th/0611352].
[46] B. Losic and W. G. Unruh, Phys. Rev. D 72, 123510 (2005) [arXiv:gr-qc/0510078].
[47] E. O. Kahya and V. K. Onemli, Phys. Rev. D 76, 043512 (2007) [arXiv:gr-qc/0612026].
[48] A. Bilandzic and T. Prokopec, Phys. Rev. D 76, 103507 (2007) [arXiv:0704.1905 [astro-ph]].
[49] S. P. Kim, Mod. Phys. Lett. A 22, 1921 (2007) [arXiv:astro-ph/0701399].
[50] S. Weinberg, Phys. Rev. D 72, 043514 (2005) [arXiv:hep-th/0506236].
[51] S. Weinberg, Phys. Rev. D 74, 023508 (2006) [arXiv:hep-th/0605244].
[52] M. S. Sloth, Nucl. Phys. B 748, 149 (2006) [arXiv:astro-ph/0604488].
[53] M. S. Sloth, Nucl. Phys. B 775, 78 (2007) [arXiv:hep-th/0612138].
[54] D. Seery, JCAP 0711, 025 (2007) [arXiv:0707.3377 [astro-ph]].
[55] D. Seery, JCAP 0802, 006 (2008) [arXiv:0707.3378 [astro-ph]].
[56] E. Dimastrogiovanni and N. Bartolo, JCAP 0811, 016 (2008) [arXiv:0807.2790 [astro-ph]].
[57] P. Adshead, R. Easther and E. A. Lim, arXiv:0809.4008 [hep-th].
[58] P. Adshead, R. Easther and E. A. Lim, arXiv:0904.4207 [hep-th].
[59] Y. Urakawa and K. i. Maeda, Phys. Rev. D 78, 064004 (2008) [arXiv:0801.0126 [hep-th]].
[60] H. R. S. Cogollo, Y. Rodriguez and C. A. Valenzuela-Toledo, JCAP 0808, 029 (2008) [arXiv:0806.1546 [astro-ph]].
[61] Y. Rodriguez and C. A. Valenzuela-Toledo, arXiv:0811.4092 [astro-ph].
[62] C. T. Byrnes, K. Y. Choi and L. M. H. Hall, JCAP 0902, 017 (2009) [arXiv:0812.0807 [astro-ph]].
[63] D. H. Lyth, JCAP 0712, 016 (2007) [arXiv:0707.0361 [astro-ph]].
[64] N. Bartolo, S. Matarrese, M. Pietroni, A. Riotto and D. Seery, JCAP 0801, 015 (2008) [arXiv:0711.4263 [astro-ph]].
[65] K. Enqvist, S. Nurmi, D. Podolsky and G. I. Rigopoulos, JCAP 0804, 025 (2008) [arXiv:0802.0395 [astro-ph]].
[66] A. Riotto and M. S. Sloth, JCAP 0804, 030 (2008) [arXiv:0801.1845 [hep-ph]].
[67] Y. Urakawa and T. Tanaka, arXiv:0902.3209 [hep-th].
[68] Y. Urakawa and T. Tanaka, arXiv:0904.4415 [hep-th].
[69] T. M. Janssen, S. P. Miao, T. Prokopec and R. P. Woodard, Class. Quant. Grav. 25, 245013 (2008) [arXiv:0808.2449 [gr-qc]].
[70] J. S. Schwinger, J. Math. Phys. 2 (1961) 407.
[71] E. Calzetta and B. L. Hu, Phys. Rev. D 35, 495 (1987).
[72] R. D. Jordan, Phys. Rev. D 33, 444 (1986).
[73] A. A. Starobinsky, JETP Lett. 42 (1985) 152 [Pisma Zh. Eksp. Teor. Fiz. 42 (1985) 124].
[74] M. Sasaki and E. D. Stewart, Prog. Theor. Phys. 95, 71 (1996) [arXiv:astro-ph/9507001].
[75] D. H. Lyth, K. A. Malik and M. Sasaki, JCAP 0505, 004 (2005) [arXiv:astro-ph/0411220].
[76] S. Shandera, arXiv:0812.0818 [astro-ph].
[77] L. Leblond and S. Shandera, JCAP 0808, 007 (2008) [arXiv:0802.2290 [hep-th]].
[78] C. Cheung, P. Creminelli, A. L. Fitzpatrick, J. Kaplan and L. Senatore, JHEP 0803, 014 (2008) [arXiv:0709.0293 [hep-th]].
[79] Xian Gao and Fanrong Xu, to appear