Hyperon halo structure of C and B isotopes

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Abstract

We study the hypernuclei of C and B isotopes by Hartree-Fock model with Skyrme-type nucleon-nucleon and nucleon-hyperon interactions. The calculated Λ binding energies agree well with the available experiment data. We found halo structure in the hyperon 1p-state with extended wave function beyond nuclear surface in the light C and B isotopes. We also found the enhanced electric dipole transition between 1p- and 1s-hyperon states, which could be the evidence for this hyperon halo structure.

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I. INTRODUCTION

Since the halo structure of $^{11}$Li was observed in 1985 [1], the halo phenomena have been studied intensively experimentally and theoretically [2–5] in nuclei near and beyond the neutron and also proton drip lines. The halo nuclei are characterized by its extended density profile far beyond the nuclear surface region. Very much enhanced electric dipole transitions have been also observed in several halo nuclei as an unique phenomenon associated with the extended halo wave function [6]. As a theoretical model, for lighter nuclei such as $^6$He and $^{11}$Li, the framework of core+$n+n$ three-body model has been adopted often to describe so called ”Borromean system”, in which one nucleon+core system has never been bound, but only two nucleon+core system makes a bound nucleus [7, 8]. For sd-shell neutron-rich nuclei such as Ne isotope, some halo states have been found [9]. In addition, deformed structure with larger $\beta_2$ has been observed in these systems [10]. In the nuclei so far discussed, one or two nucleons will contribute to create the halo structure. When one goes to heavier nuclei, for instance, in neutron-rich Ca and Zr isotopes, theoretically in Refs. [11–16], giant halo nucleus is predicted, in which several neutrons contribute to make halo nuclei.

Let us consider hypernuclei consisting nuclei and a hyperon, especially a $\Lambda$ particle. Some authors pointed out that there were possibility to have halo states in lighter systems [17, 18]: In $^3\Lambda$H, the observed binding energy is 0.13 MeV with respect to deuteron+$\Lambda$ threshold, which is a very weakly bound state and then this system has a $\Lambda$ halo structure with respect to deuteron [17]. One of the present authors (E. H.) pointed out that neutron or proton densities in the ground state of $^6\Lambda$He, excited states of $^7\Lambda$He and $^7\Lambda$Li with isospin $T = 1$ have been enhanced with the framework of $^5\Lambda$He + $N+N$ three-body model [18]. Thus the study of halo structure in $\Lambda$ hypernuclei has been focused on lighter hypernuclei with $A \leq 7$. In this paper, we focus on the possibility to have a halo structure in more heavier $\Lambda$ hypernuclei such as Boron or Carbon isotopes with $A \geq 8$. Especially, in $^{13}\Lambda$C, we have observed data of 2 positive-parity and 2 negative-parity states: the ground state, $1/2^+_1$, and either of $3/2^+_1$ or $5/2^+_1$ positive-parity excited state, $3/2^-$ and $1/2^-$ negative-parity excited states. The dominate component of two negative-parity states is $^{12}\Lambda \otimes \Lambda (1p)$ configuration. Among these observed states, $3/2^-$ and $1/2^-$ states are important to extract the information on $\Lambda N$ spin-orbit force: they measured the spin-orbit splitting energy of $1/2^- - 3/2^-$ to be 0.15 MeV [19, 20]. Furthermore these states are weakly bound by about 1 MeV with respect
to $^{12}$C + Λ threshold. This means that we have a chance to find Λ halo structure in C isotopes. Therefore, in this paper, we focus on Λ halo states for this hypernuclear state. In addition, in C isotope, experimentally, a long isotope chain from $^9$C to $^{22}$C was observed. Considering this station, we study the ground states and the excited states ($C \otimes \Lambda(1p)$) of C hypernuclei systematically with Hartree-Fock model using nucleon-nucleon and nucleon-hyperon interactions and discuss on the halo structure of hypernuclei and the possibility to observe these halo structure by the calculation of $B(E1)$ from the Λ(1p) states to the ground state with Λ(1s).

For this calculation, we use the Skyrme-Hartree-Fock model [21], which is commonly adopted for the description of the gross properties of the nuclei in a broad region of mass table. The original Skyrme model has no strangeness degree of freedom. In 1981, Rayet introduced the Skyrme-type ΛN interaction to describe the hypernuclei within the Skyrme model [22]. Since then, many Skyrme-type ΛN interactions were proposed based on realistic hyperon-nucleon interactions, stimulated by many hypernuclear data [23–30]. With these interactions, the hypernuclear structures have been investigated extensively [31–34]. But most of these investigations did not include the ΛN spin-orbit interaction, since it was expected to be rather small. In this paper, we will adopt the Skyrme-type ΛN interaction [26] obtained by the $G$–matrix calculation from the one-boson-exchange potential with a reduced ΛN spin-orbit coupling strength which can reproduce the spin-orbit splitting of the 1p states in $^{13}_\Lambda$C [19]. The method is also applied to neighboring Boron isotopes to discuss p-wave halo structure of a Λ hyperon. These studies are performed for the first time with this framework.

Organization of the present paper is as follows: In Section II, the Method is explained. The results are discussed in Sec. III and finally we summarize in Sec. IV.

II. THEORETICAL FRAMEWORK

Hypernuclei of C and B isotopes are studied by using HF model with Skyrme-type NN and NA interactions. The model is extended to describe systematically from light to heavy hypernuclei including the hyperon degree of freedom. In the Skyrme model, the two-body NN interaction [35] reads,

$$v_{NN}(r_1 - r_2) = t_0 (1 + x_0 P_\sigma) \delta(r_1 - r_2) + \frac{1}{2}t_1 (1 + x_1 P_\sigma) \left[k'^2 \delta(r_1 - r_2) + \delta(r_1 - r_2)k^2\right]$$

$$+ t_2 (1 + x_2 P_\sigma) k' \cdot \delta(r_1 - r_2)k + iW_0(\sigma_1 + \sigma_2) \cdot k' \delta(r_1 - r_2) \times k, \quad (1)$$
where \( \mathbf{k} = (\mathbf{\nabla}_1 - \mathbf{\nabla}_2)/2i \) is the relative momentum operator acting on the wave functions on the right and \( \mathbf{k}' = -(\mathbf{\nabla}_1 - \mathbf{\nabla}_2)/2i \) acting on the left, \( P_\sigma = (1 + \sigma_1 \cdot \sigma_2)/2 \) is the spin-exchange operator. The effective density-dependent \( N \), with an effective density-dependent \( \Lambda \) interaction, is also introduced as 

\[
v_{\text{den}-NN}(\mathbf{r}_1, \mathbf{r}_2, \mathbf{r}_3) = \frac{1}{6} t_3 (1 + x_3 P_\sigma) \delta(\mathbf{r}_1 - \mathbf{r}_2) \rho^\alpha \left( \frac{\mathbf{r}_1 + \mathbf{r}_2}{2} \right),
\]

where \( \alpha \) is the power of density dependence. The Skyrme-type three-body force is equivalent to the interaction (2) with choices of \( x_3 = 1 \) and \( \alpha = 1 \) for HF calculations.

The Skyrme-like two-body \( \Lambda \), with an effective density-dependent \( \Lambda \) interaction is taken as [26]

\[
v_{\Lambda N}(\mathbf{r}_A - \mathbf{r}_N) = t_0^\Lambda (1 + x_0^\Lambda P_\sigma) \delta(\mathbf{r}_A - \mathbf{r}_N) + \frac{1}{2} t_1^\Lambda \left[ \mathbf{k}'^2 \delta(\mathbf{r}_A - \mathbf{r}_N) + \delta(\mathbf{r}_A - \mathbf{r}_N) \mathbf{k}' \right]
+ t_2^\Lambda \mathbf{k}' \delta(\mathbf{r}_A - \mathbf{r}_N) \cdot \mathbf{k} + i W_0^\Lambda \mathbf{k}' \delta(\mathbf{r}_A - \mathbf{r}_N) \cdot (\mathbf{\sigma}_N + \mathbf{\sigma}_A) \times \mathbf{k}
\]

with an effective density-dependent \( \Lambda \) force

\[
v_{\text{den}-\Lambda N}(\mathbf{r}_A, \mathbf{r}_N, \rho) = \frac{3}{8} t_3^\Lambda (1 + x_3^\Lambda P_\sigma) \delta(\mathbf{r}_A - \mathbf{r}_N) \rho^\gamma \left( \frac{\mathbf{r}_A + \mathbf{r}_N}{2} \right),
\]

where \( \gamma \) is the power of density dependence.

The total energy functional can be separated into two parts,

\[
E = \int d\mathbf{r}(\mathcal{H}_N + \mathcal{H}_\Lambda),
\]

where \( \mathcal{H}_N \) is the Hamiltonian density only related with the nucleons, and \( \mathcal{H}_\Lambda \) is the one with \( \Lambda \) hyperon degree of freedom. The nucleon Hamiltonian density \( \mathcal{H}_N \) can be written as

\[
\mathcal{H}_N = \frac{\hbar^2}{2m_N} \tau_N + \frac{1}{2} t_0 \left( 1 + \frac{1}{2} x_0 \right) \rho_N^2 - \frac{1}{2} t_0 \left( x_0 + \frac{1}{2} \right) (\rho_n^2 + \rho_p^2)
+ \frac{1}{4} \left[ t_1 \left( 1 + \frac{1}{2} x_1 \right) + t_2 \left( 1 + \frac{1}{2} x_2 \right) \right] \rho_N \tau_N + \frac{1}{4} \left[ -t_1 \left( \frac{1}{2} + x_1 \right) + t_2 \left( \frac{1}{2} + x_2 \right) \right] (\rho_n \tau_n + \rho_p \tau_p)
+ \frac{1}{16} \left[ 3 t_1 \left( 1 + \frac{1}{2} x_1 \right) - t_2 \left( 1 + \frac{1}{2} x_2 \right) \right] (\nabla \rho_N)^2
- \frac{1}{16} \left[ 3 t_1 \left( \frac{1}{2} + x_1 \right) + t_2 \left( \frac{1}{2} + x_2 \right) \right] [(\nabla \rho_n)^2 + (\nabla \rho_p)^2]
+ \frac{1}{16} \left[ (t_1 - t_2) (J_n^2 + J_p^2) - (t_1 x_1 + t_2 x_2) J_N^2 \right]
+ \frac{1}{12} t_3 \left( 1 + \frac{1}{2} x_3 \right) \rho_N^{\alpha+2} - \frac{1}{12} t_3 \left( \frac{1}{2} + x_3 \right) \rho_N^\alpha (\rho_n^2 + \rho_p^2)
+ \frac{1}{2} W_0 (\nabla \rho_N \cdot \mathbf{J}_N + \nabla \rho_n \cdot \mathbf{J}_n + \nabla \rho_p \cdot \mathbf{J}_p) + \mathcal{H}_{\text{coul}}.
\]
In Eq. (6) and the following, we define the baryon density \( \rho_B(r) = \sum_{i,\sigma} n_i |\phi_{i,B}(r, \sigma)|^2 \),

the kinetic energy density

\[
\tau_B(r) = \sum_{i,\sigma} n_i |\nabla \phi_{i,B}(r, \sigma)|^2,
\]

and the spin density

\[
J_B(r) = -i \sum_{i,\sigma,\sigma'} n_i \phi_{i,B}^*(r, \sigma) [\nabla \times \sigma \phi_{i,B}(r, \sigma')],
\]

where \( \phi_{i,B}(r, \sigma) \) is the wave function of the single-particle state, and \( n_i \) is the corresponding occupation number, which is defined by \( n_i = v_i^2(2j+1) \). The occupation probability \( v_i^2 \) of the single-particle state \( i \) will be determined by either HFB, BCS or the filling approximation depending on the model. In Eq. (6), the nucleon total densities are defined as \( \rho_N = \rho_n + \rho_p \), \( \tau_N = \tau_n + \tau_p \), and \( J_N = J_n + J_p \).

The hamiltonian density with \( \Lambda \) can be written as

\[
\mathcal{H}_{\Lambda} = \frac{\hbar^2}{2m}\tau_{\Lambda} + t_{0}\mathcal{A} + \left( 1 + \frac{1}{2}x_0^2 \right) \rho_{\Lambda} + \frac{1}{2} \left( t_1^A + t_2^A \right) \left( \tau_{\Lambda} \rho_N + \tau_N \rho_{\Lambda} \right) + \frac{1}{8} \left( 3t_1^A - t_2^A \right) \nabla \rho_{\Lambda} \cdot \nabla \rho_N + \frac{1}{2} W_0^A \left( \nabla \rho_N \cdot J_{\Lambda} + \nabla \rho_{\Lambda} \cdot J_N \right) + \frac{3}{8} t_3^A \left( 1 + \frac{1}{2}x_3 \right) \rho_N^2 + \rho_{\Lambda}^2
\]

As a first step, we assume the spherical symmetry for the hypernucleus, and the pairing correlation is not considered explicitly, but the filling approximation is adopted for the occupation probability \( v_i^2 \) from the bottom of potential to the Fermi energy in order. The single-particle wave function for nucleons and \( \Lambda \) can be written as

\[
\phi_{i,B}(r, \sigma) = \frac{R_{i,B}(r)}{r} Y_{ijm}(\hat{r}, \sigma), \quad i = (nljm) \quad \text{and} \quad B = (n, p, \Lambda),
\]

where \( R_{i,B}(r) \) is the radial wave function, and \( Y_{ijm}(\hat{r}, \sigma) \) is the vector spherical harmonics.

A parameter set SkM* [36] is chosen as the Skyrme \( NN \) interaction. With this parameter set, the \( J^2 \) terms in the nucleon hamiltonian density \( \mathcal{H}_N \) are neglected for parameter fitting procedure. We examine also other Skyrme parameter sets SLy4, SIII and SkO', but the present results are essentially not changed by the other Skyrme parameter sets. The \( \Lambda N \) interaction is chosen as the 'set V' in Ref. [26] which is fitted to the \( \Lambda \) potential energy in
nuclear matter obtained by the $G$-matrix calculation from the one-boson-exchange potential. In particular, this parameter set included the $\Lambda N$ spin-orbit interaction $W_0^\Lambda = 62$ MeV fm$^5$. However, we found the obtained spin-orbit splitting of the $1p$ states in $^{13}\Lambda C$ is too large compared to the experiment data $0.152$ MeV [19]. Therefore, we use a reduced value $W_0^\Lambda = 4.7$ MeV fm$^5$ instead, and obtain a realistic spin-orbit splitting $0.155$ MeV of $1p$ states in $^{13}\Lambda C$.

The center of mass correction is considered simply by multiplying the factor $1 - m_N/(Am_N + m_\Lambda)$ and $1 - m_\Lambda/(Am_N + m_\Lambda)$ in front of the mass terms $\hbar^2/2m_N$ and $\hbar^2/2m_\Lambda$ respectively. The binding energy of $\Lambda$ particle can be calculated by

$$B_\Lambda = E_A - E_{A+1}^\Lambda,$$ (12)

where $E_A$ is the total energy of the nucleus with $A$ nucleons, and $E_{A+1}^\Lambda$ is the total energy of the hypernucleus with one additional $\Lambda$. 
III. RESULTS AND DISCUSSIONS

A. Hypernuclei of C isotopes

We first discuss C-isotopes since the spin-orbit splitting of hyperon states was observed only in $^{13}_ΛC$. The HF results of hypernuclei of C isotopes from $^{9}_ΛC$ to $^{23}_ΛC$ are tabulated in Table I. As it is expected, the binding energy of $Λ(1s)$-state increases for heavier C isotopes since the $NΛ$ potential is deeper for the heavier isotopes. The agreements between the calculated and experimental $B_Λ$ are surprisingly good as shown in the last column of Table I. The $^{13}_ΛC$ was also observed by $^{13}_C(π^+,K^+)^{13}_ΛC$ reaction [37]. The binding energy was determined as $B_Λ = 11.38 ± 0.05$(stat)$±0.36$(syst) MeV, which agrees well with the emulsion data value, $B_Λ = 11.69 ± 0.12$ MeV. The $Λ(1p)$-state was also observed at $E_x = 9.73 ± 0.14$ MeV, which is somewhat lower than HF values in Table I. In ref. [20], the excitation energies of the $Λ(1p_{1/2})$ and $Λ(1p_{3/2})$ states were obtained as $E_x = 10.982 ± 0.031$(stat)$±0.056$(syst) MeV and $E_x = 10.830 ± 0.031$(stat)$±0.056$(syst) MeV, respectively. The HF calculated values are $E_x=11.345$ and $11.190$ MeV for $Λ(1p_{1/2})$ and $Λ(1p_{3/2})$ states and show reasonable agreement. The spin-orbit interaction of $NΛ$ is taken as much smaller value than the the $G$-matrix results. The HF gives the spin-orbit splitting of $p$–hyperon orbits in $^{13}_ΛC$, 

$$\Delta ε(Λ(1p_{1/2}) - Λ(1p_{3/2})) = 0.155\text{MeV},$$  \hspace{1cm} (13)$$

while the experimental value is $\Delta ε(Λ(1p_{1/2}) - Λ(1p_{3/2})) = 0.152\text{MeV}$. It is interesting to notice that the adopted spin-orbit coupling strength in $NΛ$ channel $W^A_0 = 4.7$ MeVfm$^5$ is more than a factor 20 smaller than the $NN$ spin-orbit coupling strength $W_0 = 120$ MeVfm$^5$. The spin-orbit splitting is almost constant in C isotopes, i.e., from $Δε(Λ(p_{1/2}) - Λ(p_{3/2})) = 0.155\text{MeV}$ in $^{13}_ΛC$ to $Δε(Λ(p_{1/2}) - Λ(p_{3/2})) = 0.140\text{MeV}$ in $^{23}_ΛC$.

The rms radii of $Λ(1s)$- and $Λ(1p)$-orbits in C isotopes are listed in Table I. The $Λ(1p)$-orbits are either quasi-bound (resonance) states or loosely-bound states. Especially, the rms radii of $p$-orbits show a peculiar halo nature in $^{12}_ΛC$ and $^{13}_ΛC$ similar to the halo state in nuclei such as $^{11}Li$ and $^{11}Be$. The wave functions of $Λ(1s_{1/2})$- and $Λ(1p_{1/2})$-states in $^{13}_ΛC$ are drawn in Fig. I. The enhancement of r.m.s. radii of $Λ(1p)$-state is about 60% compared with the $Λ(1s)$-orbit. Thus we can conclude to find the $Λ(1p)$ halo structure in $^{13}_ΛC$. For $^{12}_ΛC$ and $^{14}_ΛC$ hypernuclei, the $Λ(1p)$ states have very small binding energies and show the similar halo structure to that of $^{13}_ΛC$. 

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The total binding energies and matter r.m.s. radii \( r_{\text{rms}}^A \) of C isotopes are tabulated in Table II. The HF result gives reasonable binding energies in nuclei near \(^{12}\text{C}\), where deformation effect might play a minor role. On the other hand, in the lighter and heavier isotopes, the deformation effect will contribute to increase the total binding energies, which remains to be studied. The mass radii of C-isotopes are observed by heavy-ion reactions \(^{38,39}\) and listed in Table II. The calculated results reproduce reasonably well the experiment values except the neutron halo nuclei \(^{19}\text{C}\) and \(^{22}\text{C}\). The r.m.s. radii of cores of corresponding hypernuclei are also listed as \( r_{\text{rms}}^{\text{core}A} \). In comparison between \( r_{\text{rms}}^A \) and \( r_{\text{rms}}^{\text{core}A} \), we can point out shrinkage or expansion effect of core nucleus in hypernucleus. For \( \Lambda(1s) \) hyperon case, we can see small shrinkage effect of the core, 0.05 – 0.02 fm, from light to heavy C isotopes. For \( \Lambda(1p) \) hyperon case, it is interesting to see an expansion effect of the core for nuclei \( A \leq 13 \), but quantitatively it is even smaller than the shrinkage effect of \( \Lambda(1s) \) hyperon in the same nucleus.

![FIG. 1: The square of single \( \Lambda \) wave function \( R_{r,\Lambda}^2 \) of \( \Lambda(1s1/2) \) and \( \Lambda(1p1/2) \) states in the hypernucleus \(^{13}\Lambda\text{C}\).](image)

B. Hypernuclei in B isotopes

The calculated results of hypernuclei of B isotopes are listed in Table III. Compared with the experimental value of \( B_\Lambda(\text{exp}) \sim 11.4 \) MeV in \(^{12}\Lambda\text{B}\) \(^{46}\), the calculated result is quite
reasonable to be $B_\Lambda$(HF)=10.8 MeV. The potential depth is becoming deeper for heavier isotopes and the binding energy of $\Lambda(1s_{1/2})$-orbit increases from 8.97 MeV in $^{10}\Lambda B$ to 14.50 MeV in $^{22}\Lambda B$. The halo structure of 1p-orbits can be also seen in light $B$ isotopes, especially in $^{12}\Lambda B$ and $^{13}\Lambda B$. The wave functions of $\Lambda(1s_{1/2})$- and $\Lambda(1p_{1/2})$-states in $^{13}\Lambda B$ are drawn in Fig. 2. The wave functions in $^{13}\Lambda B$ are essentially identical to those of $^{13}C$. Th spin-orbit splittings in $B$ isotopes show a similar feature to that in $C$ isotopes; $\Delta \varepsilon(\Lambda(1p_{1/2}) - \Lambda(1p_{3/2})) = 0.138$ MeV in $^{13}\Lambda B$ and $\Delta \varepsilon(\Lambda(1p_{1/2}) - \Lambda(1p_{3/2})) = 0.129$ MeV for a heavier isotope $^{22}\Lambda B$. Two $\Lambda(1p)$-shell states were also observed in Ref. 40, as $J^\pi = (1_1^+ \text{ or } 2_1^+)$ and $(2_2^+ \text{ or } 3_1^+)$ states, which are considered as coupling states of $3/2^-$ ground state of $^{11}B$ and $\Lambda(1p_{3/2})$ or $\Lambda(1p_{1/2})$ states. Since the spin-spin interaction of $N\Lambda$ is not included in the present HF calculations, we cannot predict precisely the energy splitting of $1^+, 2^+$ and $3^+$ states. However, the HF excitation energies of $\Lambda(1p)$ states $E_x \sim 11.1$ MeV are reasonable compared with the experimental excitation energies $E_x(\text{exp})=10.24 \pm 0.05$ and $10.99 \pm 0.03$ MeV for $J^\pi = (1_1^+ \text{ or } 2_1^+)$ and $(2_2^+ \text{ or } 3_1^+)$ states, respectively.

\begin{figure}[h]
\centering
\includegraphics[width=0.8\textwidth]{figure2.png}
\caption{The square of single $\Lambda$ wave function $R_{i,\Lambda}^2$ of $\Lambda(1s_{1/2})$ and $\Lambda(1p_{1/2})$ states in the hypernucleus $^{13}_\Lambda B$.}
\end{figure}

C. Electric dipole transition in Hypernuclei

We will study the electric dipole transition between hyperon 1p- and 1s-state. Electromagnetic transitions may provide precise information of hyperon wave functions in quanti-
tative manner. Suppose the hypernucleus is initially in the excited state, e.g., \( \Lambda \) is in the 1p orbit, it will decay to the ground state 1s orbit. This \( E1 \) transition has the reduced transition probability

\[
B(E1; J_i \rightarrow J_f) = \frac{3e^2_\Lambda}{4\pi} (f|r|i)^2 (2j_f + 1) \left( \frac{j_f}{2} \ 0 \ \frac{j_i}{2} \right)^2
\]

where \( e_\Lambda \) is the effective charge for \( \Lambda \) hyperon and the integration \( (f|r|i) \) can be calculated by the radial wave functions of the initial and final single-\( \Lambda \) state as

\[
\langle f|r|i \rangle = \int_0^\infty R_{f,\Lambda}(r)R_{i,\Lambda}(r)dr.
\]

Since hyperons \( \Lambda \) have no electric charges, the effective charge in Eq. (14) is given as

\[
e^{(E1)}_\Lambda = -Z\Lambda e/(AM_N + M_\Lambda)
\]

due to the recoil of the core nucleus [42].

The calculated \( B(E1) \) values are listed in Tables I for C isotopes and III for B isotopes, respectively. The values are larger in light isotopes than those in heavier nuclei because of the effective charge in Eq. (16). The \( B(E1 : 1p_{3/2} \rightarrow 1s_{1/2}) = 0.1036 \ e^2 \text{fm}^2 \) of hyperon configurations in \(^{13}_\Lambda\text{C}\) corresponds to 0.29\( B_W(E1) \), where \( B_W(E1) \) is the Weisskopf unit (single-particle unit) of electric dipole transition in \( A = 13 \) nucleus. The decay half life \( t_{1/2} \) is estimated as

\[
t_{1/2} = \frac{\ln 2}{T(E1)} = 2.99 \times 10^{-18} \text{ sec},
\]

where \( T \) is the decay rate,

\[
T(E1) = 1.59 \times 10^{15}(E_x)^3 B(E1) = 2.31 \times 10^{17} \text{ sec}^{-1}.
\]

The \( T(E1) \) is evaluated to be \( 1.505 \times 10^{17} \text{ sec}^{-1} \) for the transition \( (\Lambda(1p_{3/2}) \rightarrow \Lambda(1s_{1/2})) \) in \(^{13}_\Lambda\text{B}\) and the half life is estimated as \( t_{1/2} = 4.60 \times 10^{-18} \text{ sec}. \)

In halo nuclei without \( \Lambda \) degree of freedom, the largest \( B(E1) \) transition between discrete states is observed in \( 2s_{1/2} \rightarrow 1p_{1/2} \) transition in \(^{11}\text{Be}\) [43]; \( B(E1; 2s_{1/2} \rightarrow 1p_{1/2}) = 0.099 \pm 0.010 e^2 \text{fm}^2 = 0.31 \pm 0.03 B_W(E1) \), which is almost the same strength as \( B(E1 : \Lambda(1p_{3/2}) \rightarrow \Lambda(1s_{1/2})) \) of hyperon configurations in \(^{13}_\Lambda\text{C}\). Notice these \( B(E1) \) in halo nuclei (hypernuclei) are 2-3 order of magnitude larger than normal \( B(E1) \), which is less than \( 10^{-3} e^2 \text{fm}^2 \). The \( B(E1) \) strength of halo nuclei was studied also by the Coulomb breakup.
reactions, which measure the excitation from the halo state to the continuum. In these reactions, the $B(E1)$ value was found $B(E1: \text{exp}) = 1.05 \pm 0.06 e^2 \text{fm}^2$ in $^{11}\text{Be}$ \cite{44} and $B(E1: \text{exp}) = 0.71 \pm 0.07 e^2 \text{fm}^2$ in $^{19}\text{C}$ \cite{45}. Systematic measurements of electromagnetic transitions in $\Lambda(1p)$ states may give us a peculiar nuclear structure information including the characteristic features of hyperon halo wave functions.

We do not discuss details of the total binding energies of B isotopes since B-isotopes are odd-even or odd-odd nuclei so that spin-spin interaction might play an important role, which is missing in the present Skyrme EDF.

IV. SUMMARY AND FUTURE PERSPECTIVES

In this work, we calculated the $\Lambda$ single-particle states systematically in the C and B isotopes using the HF approach with the Skyrme-type $\Lambda N$ interaction derived from the $G -$matrix calculation of the one-boson-exchange potential. We tuned the strength of $\Lambda N$ spin-orbit interaction by fitting to the observed spin-orbit splitting data of $1/2^- - 3/2^-$ states in $^{13}\Lambda\text{C}$. The $\Lambda$ binding energies thus obtained agree with the available experiment data quite well for the C and B hypernuclei. In the light hypernuclei $^{12-14}\Lambda\text{C}$ and $^{12-14}\Lambda\text{B}$, we found very weakly bound excited $1p$ orbits for $\Lambda$ hyperon, which could have much extended density and large r.m.s radii compared with the ground $1s$ state. Furthermore, we calculated $B(E1)$ values. This halo structure may provide the enhanced $E1$ transition from the excited $1p$ states to the ground $1s$ state, which is a challenging open problem for the future experiment. On the other hand, with more neutrons, the $\Lambda$ levels become more deeply bound, so that the hyperon halo structure disappears.

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**TABLE I: Properties of single-$\Lambda$ states in hypernucleus $^{A}_{C}$**: single-particle energy $e_{s.p.}$, the binding energy $B_{\Lambda}$, the rms radius $r_{\text{rms}}^{\Lambda}$, $B(E1)$ value of the transition from the excited $\Lambda(p)$-state to the ground $\Lambda(s)$-state. Experimental data are taken from ref. [46, 47].

| Nucleus $^{A}_{C}$ | $\Lambda(nlj)$ | $e_{s.p.}$ (MeV) | $B_{\Lambda}$ (MeV) | $r_{\text{rms}}^{\Lambda}$ (fm) | $B(E1)$ ($e^2\text{fm}^2$) | $B_{\Lambda}$ (exp) (MeV) |
|-------------------|----------------|------------------|-------------------|------------------|-----------------|-------------------|
| $^{9}_{\Lambda}$C | $1s_{1/2}$    | -9.478           | 7.600             | 2.160             |                 |                   |
| $^{10}_{\Lambda}$C | $1s_{1/2}$   | -10.662          | 8.821             | 2.141             |                 |                   |
| $^{11}_{\Lambda}$C | $1s_{1/2}$   | -11.615          | 9.864             | 2.136             |                 |                   |
| $^{12}_{\Lambda}$C | $1s_{1/2}$   | -12.433          | 10.787            | 2.139             | 10.75 ± 0.10[46] |                   |
|                  | $1p_{1/2}$   | -1.228           | -0.379            | 3.679             | 1.176 × 10^{-1} |                   |
|                  | $1p_{3/2}$   | -1.367           | -0.239            | 3.604             | 1.186 × 10^{-1} |                   |
| $^{13}_{\Lambda}$C | $1s_{1/2}$   | -13.156          | 11.618            | 2.144             | 11.69±0.12[47]  |                   |
|                  | $1p_{1/2}$   | -1.782           | 0.273             | 3.464             | 1.030 × 10^{-1} |                   |
|                  | $1p_{3/2}$   | -1.936           | 0.428             | 3.410             | 1.036 × 10^{-1} |                   |
| $^{14}_{\Lambda}$C | $1s_{1/2}$   | -13.563          | 12.181            | 2.172             | 12.19±0.33[47]  |                   |
|                  | $1p_{1/2}$   | -2.357           | 1.010             | 3.355             | 9.264 × 10^{-2} |                   |
|                  | $1p_{3/2}$   | -2.506           | 1.160             | 3.317             | 9.297 × 10^{-2} |                   |
| $^{15}_{\Lambda}$C | $1s_{1/2}$   | -13.941          | 12.688            | 2.199             |                 |                   |
|                  | $1p_{1/2}$   | -2.911           | 1.697             | 3.287             | 8.367 × 10^{-2} |                   |
|                  | $1p_{3/2}$   | -3.055           | 1.842             | 3.259             | 8.385 × 10^{-2} |                   |
| $^{16}_{\Lambda}$C | $1s_{1/2}$   | -14.292          | 13.083            | 2.218             |                 |                   |
|                  | $1p_{1/2}$   | -3.357           | 2.187             | 3.252             | 7.524 × 10^{-2} |                   |
|                  | $1p_{3/2}$   | -3.500           | 2.331             | 3.228             | 7.537 × 10^{-2} |                   |
| $^{17}_{\Lambda}$C | $1s_{1/2}$   | -14.633          | 13.467            | 2.236             |                 |                   |
|                  | $1p_{1/2}$   | -3.792           | 2.666             | 3.226             | 6.806 × 10^{-2} |                   |
|                  | $1p_{3/2}$   | -3.935           | 2.809             | 3.206             | 6.814 × 10^{-2} |                   |
| $^{18}_{\Lambda}$C | $1s_{1/2}$   | -14.962          | 13.839            | 2.254             |                 |                   |
|                  | $1p_{1/2}$   | -4.216           | 3.133             | 3.207             | 6.188 × 10^{-2} |                   |
|                  | $1p_{3/2}$   | -4.357           | 3.274             | 3.189             | 6.194 × 10^{-2} |                   |
| $^{19}_{\Lambda}$C | $1s_{1/2}$   | -15.281          | 14.200            | 2.270             |                 |                   |
|                  | $1p_{1/2}$   | -4.629           | 3.587             | 3.192             | 5.653 × 10^{-2} |                   |
|                  | $1p_{3/2}$   | -4.769           | 3.727             | 3.177             | 5.657 × 10^{-2} |                   |
| $^{20}_{\Lambda}$C | $1s_{1/2}$   | -15.590          | 14.549            | 2.286             |                 |                   |
|                  | $1p_{1/2}$   | -5.031           | 4.028             | 3.182             | 5.188 × 10^{-2} |                   |
|                  | $1p_{3/2}$   | -5.170           | 4.167             | 3.168             | 5.190 × 10^{-2} |                   |
| $^{21}_{\Lambda}$C | $1s_{1/2}$   | -15.890          | 14.887            | 2.302             |                 |                   |
|                  | $1p_{1/2}$   | -5.422           | 4.457             | 3.174             | 4.780 × 10^{-2} |                   |
|                  | $1p_{3/2}$   | -5.559           | 4.595             | 3.162             | 4.782 × 10^{-2} |                   |
| $^{22}_{\Lambda}$C | $1s_{1/2}$   | -16.038          | 15.097            | 2.315             |                 |                   |
|                  | $1p_{1/2}$   | -5.648           | 4.742             | 3.191             | 4.405 × 10^{-2} |                   |
|                  | $1p_{3/2}$   | -5.787           | 4.881             | 3.178             | 4.407 × 10^{-2} |                   |
| $^{23}_{\Lambda}$C | $1s_{1/2}$   | -16.176          | 15.29114          | 2.326             |                 |                   |
|                  | $1p_{1/2}$   | -5.853           | 5.000             | 3.208             | 4.070 × 10^{-2} |                   |
|                  | $1p_{3/2}$   | -5.992           | 5.140             | 3.195             | 4.072 × 10^{-2} |                   |
TABLE II: Properties of isotopes $^A$C: the calculated total binding energy $B_{\text{cal}}$ (MeV), the mass rms radius $r_{\text{rms}}^A$ (fm), and the mass rms radius of the core $r_{\text{rms}}^{\text{coreA}}$ (fm) in hypernucleus $^{A+1}$C. The experimental data of $B_{\text{exp}}$ are taken from Ref. [48]. The experimental uncertainty in the parentheses is given in unit of keV. The experimental data of $r_{\text{rms}}^A$ are taken from Refs. [38, 39].

| Nucleus | $B_{\text{cal}}$ (MeV) | $B_{\text{exp}}$ (MeV) | $r_{\text{rms}}^A$ (fm) | $r_{\text{rms}}^{\text{coreA}}$ (fm) | $\Lambda(nl)$ | $r_{\text{rms}}^{\text{exp}}$ (fm) |
|---------|------------------------|-------------------------|--------------------------|---------------------------------|-------------|-----------------------------|
| $^8$C   | 31.078                 | 24.812(18)              | 2.5573                   |                                 | 1s$_{1/2}$  | 2.5020                      |
| $^9$C   | 46.633                 | 39.037(2)               | 2.4408                   |                                 | 1s$_{1/2}$  | 2.4120                      |
| $^{10}$C| 62.278                 | 60.320(0)               | 2.4098                   |                                 | 1s$_{1/2}$  | 2.3902                      |
| $^{11}$C| 77.855                 | 73.441(0)               | 2.4094                   |                                 | 1s$_{1/2}$  | 2.3942                      |
|         |                        |                         |                          |                                 | 1p$_{1/2}$  | 2.4169                      |
|         |                        |                         |                          |                                 | 1p$_{3/2}$  | 2.4158                      |
| $^{12}$C| 93.330                 | 92.162(0)               | 2.4228                   | 2.35±0.02                       | 1s$_{1/2}$  | 2.4103                      |
|         |                        |                         |                          |                                 | 1p$_{1/2}$  | 2.4290                      |
|         |                        |                         |                          |                                 | 1p$_{3/2}$  | 2.4280                      |
| $^{13}$C| 100.716                | 97.108(0)               | 2.5095                   | 2.28±0.04                       | 1s$_{1/2}$  | 2.4943                      |
|         |                        |                         |                          |                                 | 1p$_{1/2}$  | 2.5125                      |
|         |                        |                         |                          |                                 | 1p$_{3/2}$  | 2.5116                      |
| $^{14}$C| 108.421                | 105.284(0)              | 2.5860                   | 2.30±0.07                       | 1s$_{1/2}$  | 2.5690                      |
|         |                        |                         |                          |                                 | 1p$_{1/2}$  | 2.5865                      |
|         |                        |                         |                          |                                 | 1p$_{3/2}$  | 2.5856                      |
| $^{15}$C| 110.946                | 106.503(1)              | 2.6570                   | 2.50±0.08                       | 1s$_{1/2}$  | 2.6385                      |
|         |                        |                         |                          |                                 | 1p$_{1/2}$  | 2.6554                      |
|         |                        |                         |                          |                                 | 1p$_{3/2}$  | 2.6545                      |
| $^{16}$C| 113.752                | 110.753(4)              | 2.7193                   | 2.70±0.03                       | 1s$_{1/2}$  | 2.6999                      |
|         |                        |                         |                          |                                 | 1p$_{1/2}$  | 2.7160                      |
|         |                        |                         |                          |                                 | 1p$_{3/2}$  | 2.7152                      |
| $^{17}$C| 116.835                | 111.486(17)             | 2.7747                   | 2.72±0.03                       | 1s$_{1/2}$  | 2.7545                      |
|         |                        |                         |                          |                                 | 1p$_{1/2}$  | 2.7700                      |
|         |                        |                         |                          |                                 | 1p$_{3/2}$  | 2.7692                      |
| $^{18}$C| 120.193                | 115.670(31)             | 2.8243                   | 2.82±0.04                       | 1s$_{1/2}$  | 2.8037                      |
|         |                        |                         |                          |                                 | 1p$_{1/2}$  | 2.8186                      |
|         |                        |                         |                          |                                 | 1p$_{3/2}$  | 2.8179                      |
| $^{19}$C| 123.819                | 116.242(95)             | 2.8692                   | 3.13±0.07                       | 1s$_{1/2}$  | 2.8484                      |
|         |                        |                         |                          |                                 | 1p$_{1/2}$  | 2.8628                      |
|         |                        |                         |                          |                                 | 1p$_{3/2}$  | 2.8620                      |
| $^{20}$C| 127.707                | 119.220(240)            | 2.9102                   | 2.98±0.05                       | 1s$_{1/2}$  | 2.8894                      |
|         |                        |                         |                          |                                 | 1p$_{1/2}$  | 2.9032                      |
|         |                        |                         |                          |                                 | 1p$_{3/2}$  | 2.9025                      |
| $^{21}$C| 130.470                | 119.154(588)            | 3.0054                   |                                 | 1s$_{1/2}$  | 2.9833                      |
|         |                        |                         |                          |                                 | 1p$_{1/2}$  | 2.9952                      |
|         |                        |                         |                          |                                 | 1p$_{3/2}$  | 2.9944                      |
| $^{22}$C| 133.175                | 119.262(242)            | 3.0095                   | 3.44±0.08                       | 1s$_{1/2}$  | 3.0762                      |
TABLE III: The same as Fig. 1 but for B isotopes. The p-states are unbound in $^8_{\Lambda}$B. Experimental data of $B_{\Lambda}$ are taken from refs. [40, 49].

| Nucleus | $\Lambda((nlj) \epsilon_{x.p.}$ | (MeV) | $B_{\Lambda}$ (MeV) | $r_{rms}^\Lambda$ (fm) | $B(E1)$ ($e^2$fm$^2$) | $B_{\Lambda}$ (exp) (MeV) |
|---------|---------------------------------|-------|----------------------|-----------------------|----------------------|----------------------|
| $^8_{\Lambda}$B | 1s$_{1/2}$ | -8.750 | 6.670 | 2.148 |
| $^9_{\Lambda}$B | 1s$_{1/2}$ | -9.917 | 7.892 | 2.132 |
| $^{10}_{\Lambda}$B | 1s$_{1/2}$ | -10.877 | 8.968 | 2.128 | 8.1±0.1 [49] |
| $^{11}_{\Lambda}$B | 1s$_{1/2}$ | -11.712 | 9.932 | 2.131 |
| $^{12}_{\Lambda}$B | 1s$_{1/2}$ | -12.457 | 10.805 | 2.137 | 11.38 [40] |
| | 1p$_{1/2}$ | -1.229 | -0.386 | 3.674 | 8.1524 × 10$^{-2}$ |
| | 1p$_{3/2}$ | -1.370 | -0.245 | 3.599 | 8.2226 × 10$^{-2}$ |
| $^{13}_{\Lambda}$B | 1s$_{1/2}$ | -12.843 | 11.375 | 2.168 |
| | 1p$_{1/2}$ | -1.787 | 0.364 | 3.503 | 7.3314 × 10$^{-2}$ |
| | 1p$_{3/2}$ | -1.925 | 0.502 | 3.454 | 7.3684 × 10$^{-2}$ |
| $^{14}_{\Lambda}$B | 1s$_{1/2}$ | -13.205 | 11.885 | 2.198 |
| | 1p$_{1/2}$ | -2.331 | 1.061 | 3.402 | 6.6020 × 10$^{-2}$ |
| | 1p$_{3/2}$ | -2.465 | 1.195 | 3.367 | 6.6216 × 10$^{-2}$ |
| $^{15}_{\Lambda}$B | 1s$_{1/2}$ | -13.544 | 12.277 | 2.218 |
| | 1p$_{1/2}$ | -2.765 | 1.549 | 3.352 | 5.9055 × 10$^{-2}$ |
| | 1p$_{3/2}$ | -2.898 | 1.682 | 3.323 | 5.9190 × 10$^{-2}$ |
| $^{16}_{\Lambda}$B | 1s$_{1/2}$ | -13.876 | 12.660 | 2.237 |
| | 1p$_{1/2}$ | -3.193 | 2.028 | 3.315 | 5.3143 × 10$^{-2}$ |
| | 1p$_{3/2}$ | -3.325 | 2.160 | 3.291 | 5.3233 × 10$^{-2}$ |
| $^{17}_{\Lambda}$B | 1s$_{1/2}$ | -14.203 | 13.034 | 2.255 |
| | 1p$_{1/2}$ | -3.613 | 2.497 | 3.287 | 4.8090 × 10$^{-2}$ |
| | 1p$_{3/2}$ | -3.744 | 2.628 | 3.266 | 4.8151 × 10$^{-2}$ |
| $^{18}_{\Lambda}$B | 1s$_{1/2}$ | -14.522 | 13.399 | 2.273 |
| | 1p$_{1/2}$ | -4.026 | 2.956 | 3.265 | 4.3743 × 10$^{-2}$ |
| | 1p$_{3/2}$ | -4.156 | 3.086 | 3.247 | 4.3784 × 10$^{-2}$ |
| $^{19}_{\Lambda}$B | 1s$_{1/2}$ | -14.834 | 13.754 | 2.290 |
| | 1p$_{1/2}$ | -4.430 | 3.403 | 3.249 | 3.9977 × 10$^{-2}$ |
| | 1p$_{3/2}$ | -4.559 | 3.532 | 3.233 | 4.0005 × 10$^{-2}$ |
| $^{20}_{\Lambda}$B | 1s$_{1/2}$ | -15.138 | 14.099 | 2.306 |
| | 1p$_{1/2}$ | -4.825 | 3.839 | 3.236 | 3.6695 × 10$^{-2}$ |
| | 1p$_{3/2}$ | -4.952 | 3.966 | 3.222 | 3.6713 × 10$^{-2}$ |
| $^{21}_{\Lambda}$B | 1s$_{1/2}$ | -15.276 | 14.306 | 2.319 |
| | 1p$_{1/2}$ | -5.034 | 4.112 | 3.254 | 3.3640 × 10$^{-2}$ |
| | 1p$_{3/2}$ | -5.163 | 4.241 | 3.239 | 3.3659 × 10$^{-2}$ |
| $^{22}_{\Lambda}$B | 1s$_{1/2}$ | -15.406 | 14.497 | 2.330 |
| | 1p$_{1/2}$ | -5.224 | 4.360 | 3.271 | 3.0932 × 10$^{-2}$ |
| | 1p$_{3/2}$ | -5.353 | 4.489 | 3.257 | 3.0952 × 10$^{-2}$ |