Atomic Quantum Corrals for Bose-Einstein Condensates

Hongwei Xiong\(^1,2,\)\(^\ast\) and Biao Wu\(^3,\)\(^†\)

\(^1\)State Key Laboratory of Magnetic Resonance and Atomic and Molecular Physics, Wuhan Institute of Physics and Mathematics, Chinese Academy of Sciences, Wuhan 430071, China
\(^2\)Center for Cold Atom Physics, Chinese Academy of Sciences, Wuhan 430071, P. R. China
\(^3\)Institute of Physics, Chinese Academy of Sciences, Beijing 100190, China
(Dated: July 23, 2009)

We consider the dynamics of Bose-Einstein condensates in a corral-like potential. Compared to the electronic quantum corrals, the atomic quantum corrals have the advantage of allowing direct and convenient observation of the wave dynamics. Our numerical study shows that this advantage not only allows people to explore the rich dynamical structures in the density distribution but also makes the corrals useful in many other aspects. In particular, the corrals for atoms can be arranged into a stadium shape for the experimental visualization of quantum chaos, which has been elusive with the electronic quantum corrals.

I. INTRODUCTION

Quantum corrals were first demonstrated by arranging iron adatoms into a ring on a copper surface \(^1,2\). Several years later, an optical analogy to the electronic quantum corrals was proposed theoretically and realized in experiment by a skillful arrangement of nanoscale pillars \(^3,4\). However, in both of the quantum corrals only static wave properties were studied experimentally. Since the density image was obtained by sequential scanning over a period of time, the experimental study of wave dynamics in these corrals is difficult if not impossible.

In this paper, we study the dynamics of atomic quantum corrals for Bose-Einstein condensates (BECs). Since the density distribution of a BEC can be imaged snapshot by snapshot with a charge coupled device (CCD), the atomic quantum corrals offer a great advantage over their electronic and optical counterparts, namely, the possible experimental study of the wave dynamics inside the corrals. Our numerical simulation shows that rich dynamical structures of a condensate can arise in the quantum corrals due to reflection and interference. Moreover, with the great deal of experimental control over the condensates \(^5,6\), one is possible to study novel quantum behaviors, which are unimaginable for electronic or optical quantum corrals. For example, the dynamic evolution of a quantized vortex confined inside quantum corrals can be studied.

Of particular importance, the atomic quantum corrals proposed here can be used as a laboratory to study quantum chaos. Even since the first creation of quantum corrals, it has been pursued to build stadium-shaped quantum corrals to visualize experimentally the “scar” states, a signature of quantum chaos \(^7\). The stadium-shaped quantum corrals were built; however, the visualization of quantum chaos has remained elusive because the quantum corrals are too “leaky” for electrons \(^7\). Our atomic quantum corrals are very flexible and can be made to minimize the “leakage” so that be used to explore quantum chaos experimentally. As an example, we demonstrate with numerical simulation that with atomic quantum corrals of stadium shape, one should be able to visualize experimentally the quantum chaotic behavior predicted in Ref. \(^8\). We also show how the interference between two BECs can be destroyed by chaos.

The paper is organized as follows. In Sec. II, we consider the wave packet dynamics for a BEC in a corral-like potential. In particular, the vortex evolution in a corral-like potential is studied. In Sec. III, we study the dynamic quantum chaos for a BEC in stadium-shaped quantum corrals. A brief summary and discussion is given in the last section.

II. WAVE PACKET DYNAMICS IN CORRAL-LIKE POTENTIAL

We consider here a scheme to create quantum corrals for a BEC illustrated in Fig. 1. An opaque sheet with a
ring of holes is placed between a lens and a blue-detuned laser beam to create a corral-like repulsive potential for the condensate. The lens is to focus and adjust the sizes of the “corrals”. The holes in the opaque sheet are identical with a radius around or larger than 50 µm, which is large enough so that the diffraction of the laser beam can be ignored. This setup is similar to the one used to create a rotating quasi-2D optical lattice by a mask with a set of holes [9] and a random potential [10] for BECs. One may also create this kind of quantum corrals by spatial light modulator or microlens array [11].

To simulate the dynamics of the cigar-shaped BEC in the corrals, we can integrate out the axial degree of freedom. In this case, the BEC dynamics is described by a two dimensional Gross-Pitaevskii (GP) equation [6]:

$$i \frac{\partial \Phi}{\partial t} = - \left( \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right) \Phi + V_{oc} \Phi + V_{ht} \Phi + g_{2D} |\Phi|^2 \Phi,$$

where $\Phi$ is a normalized wave function. The above equation has been made dimensionless with the length unit $L_0$ and the energy unit $E_0 = \hbar^2 / 2 m L_0^2$. The time unit is then $T_0 = \hbar / E_0$. $V_{oc}$ is the corral-like potential while $V_{ht} = m \omega_z^2 L_0^2 (x^2 + y^2) / 2 E_0$ with $\omega_z$ being the harmonic frequency in $x$ and $y$ directions. The dimensionless coupling constant $g_{2D} = 2 \sqrt{\pi N} \hbar^2 a / m \omega_z L_0^2$, where $a$ is the $s$-wave scattering length and $L_0 = \sqrt{\hbar / m \omega_z}$ with $\omega_z$ being the trapping frequency in the $z$ direction. We numerically solve this GP equation to illustrate the wave dynamics inside the quantum corrals. In the numerical calculations, we consider $2 \times 10^4$ Na atoms in the condensate and use $L_0 = 5$ µm, $T_0 = 1.8$ ms, and $\omega_z = 4 \times 2 \pi$ Hz.

The $i$th Gaussian potential $V_i (x, y, z)$ created by a focused laser beam propagating along $z$ direction has the following form

$$V_i (x, y, z) \sim \frac{P}{\pi \sigma^2(z)} e^{-\left[\left(x-x_i\right)^2 + \left(y-y_i\right)^2\right] / \sigma^2},$$

where $P$ is the total power of a laser beam. $\sigma(z) = \sigma_0 \sqrt{1 + z^2 / z_R^2}$ with the Rayleigh length $z_R = 2 \pi \sigma_0^2 / \lambda$. In the following calculations, $\sigma_0 = 5$ µm. For a focused laser beam with wave length $\lambda = 500$ nm, we have $z_R = 314$ µm, which is much larger than the length (about 10 µm for the parameters in this paper) of the cigar-shaped condensate. In this situation, along the $z$ direction, the condensate feels almost the same corral-like potential in the $x-y$ plane. Thus, the two dimensional GP equation can be used very well to study the dynamics of a condensate in the corral-like potential.

To show clearly the fundamental properties of the atomic quantum corrals, we calculate numerically the evolution of the condensate confined in circle-shaped quantum corrals, which can be described as a series of Gaussian potentials $V_{oc} = \sum_{j=1}^{M} \gamma e^{-\left[\left(x-x_j\right)^2 + \left(y-y_j\right)^2\right] / \sigma^2}$.

![FIG. 2: Density distribution of a condensate in a corral-like potential at different times: (a) $t = 0$, (b) $t = 1$, (c) $t = 2$, (d) $t = 3$, (e) $t = 4$, (f) $t = 5$. The coordinates are in unit of $L_0$. In Fig. a, the corral-like potential is also shown.](image)

with $\{x_j, y_j\}$ distributed uniformly along a circle of radius $R$. With the parameters $\gamma = 20$, $M = 20$, $R = 10$, $\sigma = 1$, and $\omega_\perp = 70 \times 2 \pi$, we first get the ground-state wave function by using the widely used ITP (imaginary time propagation) method. Then the evolution of the condensate is solved numerically from the GP equation [11], after suddenly switching off the harmonic trap. In this situation, the condensate will evolve under the confinement of the corral-like potential. In Fig. a, both the initial density distribution $|\Phi|^2$ and the corral-like potential are shown. In Figs. b-f, the evolution of $|\Phi|^2$ is given, after suddenly switching off the harmonic trap. The rich and colorful structures in $|\Phi|^2$ originate from the following two physical mechanisms:

(i) After switching off the harmonic trap, and with the expansion of the condensate, the condensate will be reflected by the corral-like potential. The expanding and reflected condensates will overlap, and lead to clear interference patterns. In Figs. 2c-2f, a series of ring-shaped interference fringes are clearly shown.

(ii) In Figs. 2c and 2d, the density distribution shows sunflower-like structure. This sunflower-like structure is due to the discrete characteristic of the corral-like potential. The expanding and reflected condensates will overlap, and lead to clear interference patterns. In Figs. 2c and 2d, the number of small peaks distributed along an interference fringe is found to be exactly 20. By varying $M$, we have verified numerically that the number of small peaks distributed along a ring-shaped interference fringe is always equal to $M$, the number of corrals.

The coherent interaction between atoms and external field (such as laser and magnetic field) can provide us im-
FIG. 3: Evolution of the density distribution in the presence of both a corral-like potential and a harmonic potential. The dynamic evolution is due to the sudden decreasing of the harmonic frequency. (a) $t = 0$, (b) $t = \pi/5$, (c) $t = 2\pi/5$, (d) $t = \pi/2$, (e) $t = 3\pi/5$, (f) $t = 7\pi/10$, (g) $4\pi/5$, (h) $\pi$. Here the coordinates are in unit of $L_0$. Fig. i gives the time evolution of the standard deviation $\delta$ of the density distribution. In Fig. a, the corral-like potential and harmonic potential are also shown. The height of these two potentials has been adjusted in Fig. a.

FIG. 4: Density distributions of a quantized vortex in a corral-like potential at different times. The left column is for a vortex rotating counterclockwise at (a) $t = 2.5$, (b) $t = 3.8$, (c) $t = 6.3$. The right column is for a vortex rotating clockwise at (d) $t = 2.5$, (e) $t = 3.8$, (f) $t = 6.3$. The coordinates are in units of $L_0$. The dashed circles mark the position of the circular corrals.
of the vortex has larger velocity. In Figs. 4c and 4f, a series of flyer-like peaks distributed along the circumference of the outmost circle distinguish obviously two vortices with different rotational directions. This feature may be applied to detect the rotational direction of a vortex.

It is still an open problem to experimentally investigate the behavior of a quantized vortex in electronic quantum corrals. For an atomic condensate, all the rich dynamics of a vortex shown above can be readily observed experimentally as a macroscopically quantized vortex can be generated experimentally in a BEC with mature technologies\cite{12}.

III. QUANTUM CHAOS FOR A BEC IN CORRAL-LIKE POTENTIAL

We now turn to consider the dynamics of a BEC confined in stadium-shaped quantum corrals as shown in Fig. 5a. Stadium billiard is a typical system to study both classical and quantum chaos\cite{8}. The energy-level distribution was proposed in Ref. \cite{13} to reveal the quantum chaos for a BEC. Most recently, the scars in the steady-state density profiles of parametrically driven condensates was proposed to study the quantum chaos \cite{14}. Our work here focuses on the dynamic manifestation of quantum chaos with a BEC.

The stadium potential can be described by \( V_{oc} \) with \( \{x_1, y_1\} \) uniformly distributing along the circumference of the stadium as shown in Fig. 5a. In our numerical computation, the parameters that we choose for the stadium-shaped quantum corrals are \( M = 50, \sigma = 1, \gamma = 500, R = 10, \) and \( d = 30\pi/19 \). The groundstate wave function is calculated for the condensate confined in the harmonic trap with \( \omega_\perp = 87.5\pi \) and the stadium-shaped quantum corrals. This condensate is then given an initial velocity of \( v \). After switching off the harmonic trap, we study numerically the evolution of this moving condensate in the stadium-shaped quantum corrals. For \( v = 2.7 \text{ mm/s} \), Figs. 5a-5f show a series of snapshots for the condensate evolution. The regular density distribution is destroyed after a small number of reflections. This is in stark contrast with the dynamics of a moving BEC in circle-shaped quantum corrals, which stays regular even after long-time evolution as shown in Fig. 6.

This quantum chaotic behavior was firstly revealed in Ref. \cite{8} for a free particle in a continuous stadium billiard. In this work, Tomsovic and Heller showed that the chaotic wave dynamics shown in Fig. 5 can be regarded as the supposition of millions of classical trajectories. The random looking in Fig. 5 is just a reflection of the chaotic classical trajectories. In contrast to a free particle, there is interatomic interaction in a BEC and the corral potential is discrete. As a result, one might expect with slight hesitation that a BEC in stadium-shaped quantum corrals exhibits typical quantum chaotic wave dynamics. Our numerical calculations above show that the hesitation can be cast aside. In fact, no obvious difference is found, compared to the situation of noninteracting BEC. We have also considered the situation with increasing coupling constant. It is found that increasing the coupling constant has the effect of enhancing the dynamical quantum chaotic behavior.

As mentioned at the beginning, it has remained elusive to visualize experimentally the quantum chaotic behavior in the electronic quantum corrals despite a great deal of effort\cite{7}. The main reason is that the quantum corrals are too “leaky” for electrons. Our above numerical studies show that the BEC system provides a very good chance to experimentally visualize the quantum chaos: The stadium-shaped corrals can be built with the scheme shown in Fig. 1. The BEC can gain a velocity with the method used for the experimental stud-
ies of quantum reflection\[15\]. Considering the length unit is 5 \(\mu m\) in Figs. 2-6, a imaging resolution below 5 \(\mu m\) is necessary to reveal the structure in these figures. Fortunately, high resolution absorption imaging below 4 \(\mu m\) has been achieved [16]. The chaotic behavior has been studied experimentally with nondegenerate ultracold atomic gases [17]; quantum chaos has been visualized with classical wave systems, such as sound and microwave [18]. An eventual implementation of our scheme will be an experimental realization of quantum chaos with coherent matter wave, marking an important step forward in the experimental studies of chaos.

Besides visualizing the known quantum chaotic behavior, we can try to explore more on quantum chaos with BECs in a stadium-shaped corral. For example, we can study the thermal effect on quantum chaos by placing non-condensed cold atoms and partially condensed cold atoms into the corrals. This is impossible for the electronic quantum corrals. One can also study how the interference between two BECs is affected by quantum chaos. We have shown in Fig. 7 the evolution of two initially coherently separated condensates [19]. The significance of this study lies in that a classical particle can not be regarded as two coherently separated particles. Figs. 7(a1)-7(a6) give the evolution of two coherently separated Gaussian wavepackets with the same condition in Fig. 5. The initial width and distance are \(\sqrt{2/5}\) and 4. For short-time evolution, the interference between two condensates is seen in Fig. 7(a2). After long-time evolution, the interference is destroyed by quantum chaos as seen in Figs. 7(a4)-7(a6). As a comparison, we have also computed the evolution of these two BECs in circle-shaped corrals. The results are shown in Fig. 7(b1-b6), where the interference between two condensates is preserved partly even for long-time evolution. In the inset of Fig. 7(b5), the fork-like structure implies the information of two initially coherently separated condensates.

### IV. SUMMARY AND DISCUSSION

In summary, we have studied an atomic analogy to electronic quantum corrals. A scheme is proposed to study the dynamic evolution of a Bose-Einstein condensate confined in a corral-like potential. In particular, with these atomic corrals, it is now promising to study experimentally the dynamic quantum chaotic behavior [8]. The atomic quantum corrals proposed here can also be applied to study other quantum gases, such as molecular Bose-Einstein condensates [20], degenerate Fermi gases [21], ultracold Fermi gases in the unitarity limit [22], and BCS (Bardeen-Cooper-Schrieffer) superfluid [23] in the corral-like potential. The different descriptions of the order parameter, quantum statistics and evolution equation [24] mean that there should be very rich phenomena for discovery.

FIG. 7: The left and right columns give the evolution of two initially coherently separated condensates in a stadium-shaped and circle-shaped potentials, respectively. Each figure in the left and right columns is 30 \(\times\) 40 and 30 \(\times\) 30 with coordinate unit \(L_0\). Here the time unit is \(T = 2(R + d)/v\) and \(T = 2R/v\) for the left and right columns.

**Acknowledgments**

We acknowledge useful discussions with Baolong Lü, Cheng Chin, and Jin Wang. This work was supported by NSFC under Grant Nos. 10875165, 10825417, 10634060, and NBRPC 2006CB921406.
[1] M. F. Crommie, C. P. Lutz, and D. M. Eigler, Nature (London) 363, 524 (1993); M. F. Crommie, C. P. Lutz, and D. M. Eigler, Science 262, 218 (1993).
[2] G. A. Fiete and E. J. Heller, Rev. Mod. Phys. 75, 933 (2003).
[3] G. C. des Frans et al., Phys. Rev. Lett. 86, 4950 (2001).
[4] C. Chicanne et al., Phys. Rev. Lett. 88, 097402 (2002).
[5] Nature Insight: Ultracold Matter [Nature (London) 416, 205 (2002)]; J. P. Yin, Phys. Rep. 430, 1 (2006).
[6] F. Dalfovo, S. Giorgini, L. P. Pitaevskii, and S. Stringari, Rev. Mod. Phys. 71, 463 (1999); A. J. Leggett, Rev. Mod. Phys. 73, 307 (2001); C. J. Pethick and H. Smith, Bose-Einstein Condensation in Dilute Gases (Cambridge University, Cambridge, 2002); L. P. Pitaevskii and S. Stringari, Bose-Einstein condensation (Clarendon, Oxford, 2003).
[7] E. J. Heller, M. F. Crommie, C. P. Lutz, and D. M. Eigler, Nature 369, 464 (1994); M. F. Crommie, C. P. Lutz, D. M. Eigler, and E. J. Heller, Physica D 83, 98 (1995); M. F. Crommie, C. P. Lutz, D. M. Eigler, and E. J. Heller, Surf. Sci. 361, 864 (1996).
[8] S. Tomsovic and E. J. Heller, Phys. Rev. Lett. 67, 664 (1991); Phys. Rev. E 47, 282 (1993).
[9] S. Tung, V. Schweikhard and E. A. Cornell, Phys. Rev. Lett. 97, 240402 (2006).
[10] D. Clement et al., New J. of Phys. 8, 165 (2006).
[11] K. Henderson et al., Preprint arXiv:0902.2171 (2009); A. Itah et al., Preprint arXiv:0903.3282 (2009); V. Boyer et al., Phys. Rev. A 73, 031402(R) (2006).
[12] A. L. Fetter, Preprint arXiv:0801.2952 (2008); M. R. Matthews et al., Phys. Rev. Lett. 83, 2498 (1999); K. W. Madison, F. Chevy, W. Wohlleben, and J. Dalibard, Phys. Rev. Lett. 84, 806 (2000); J. R. Abo-Shaeer, C. Raman, J. M. Vogels, and W. Ketterle, Science 292, 476 (2001).
[13] C. W. Zhang, J. Liu, M. G. Raizen, and Q. Niu, Phys. Rev. Lett. 93, 074101 (2004).
[14] N. Katz and O. Agam, Preprint arXiv: 0903.0968 (2009).
[15] T. A. Pasquini et al., Phys. Rev. Lett. 93, 233201 (2004).
[16] N. Gemelke et al., Preprint arXiv:0904.1532 (2009); private communication with Dr. Cheng Chin about the improved imaging resolution of about 1.5 μm.
[17] V. Milner, J. L. Hanssen, W. Campbell, and M. G. Raizen, Phys. Rev. Lett. 86, 1514 (2001); D. A. Steck, W. H. Oskay, and M. G. Raizen, Science. 293, 274 (2001); N. Friedman et al., Phys. Rev. Lett. 86, 1518 (2001).
[18] H. J. Stöckmann, Quantum Chaos: an Introduction (Cambridge, Cambridge, 1999).
[19] M. R. Andrews et al., Science 275, 637 (1997).
[20] S. Jochim et al., Science 302, 2101 (2003); M. Greiner, C. A. Regal, and D. S. Jin, Nature 426, 537 (2003); M. W. Zwierlein et al., Phys. Rev. Lett. 91, 250401 (2003).
[21] B. D. Marco, and D. S. Jin, Science 285, 1703 (1999).
[22] K. M. O’Hara et al., Science 298, 2179 (2002).
[23] C. A. Regal, M. Greiner, and D. S. Jin, Phys. Rev. Lett. 92, 040403 (2004); M. W. Zwierlein et al., Phys. Rev. Lett. 92, 120403 (2004); M. Bartenstein et al., Phys. Rev. Lett. 92, 120401 (2004); J. Kinast et al., Phys. Rev. Lett. 92, 150402 (2004).
[24] S. Giorgini, L. P. Pitaevskii, and S. Stringari, Rev. Mod. Phys. 80, 1215 (2008).