$0^{++}$ scalar glueball in finite-width Gaussian sum rules

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Abstract

Based on a semiclassical expansion for quantum chromodynamics in the instanton liquid background, the correlation function of the $0^{++}$ scalar glueball current is given, and the properties of the $0^{++}$ scalar glueball are studied in the framework of Gaussian sum rules. Besides the pure classical and quantum contributions, the contributions arising from the interactions between the classical instanton fields and quantum gluons are come into play. Instead of the usual zero-width approximation for the resonance, the Breit-Wigner form for the spectral function of the finite-width resonance is adopted. The family of the Gaussian sum rules for the scalar glueball in quantum chromodynamics with and without light quarks is studied. A consistency between the subtracted and unsubtracted sum rules is very well justified, and the values of the decay width and the coupling to the corresponding current for the $0^{++}$ resonance, in which the scalar glueball fraction is dominant, are obtained.

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I. INTRODUCTION

Glueballs, being composed of pure gluons in the limit of no quark fields, have attracted much attention since the theory of the strong interactions, quantum chromodynamics (QCD), was founded in the late of 1970s [1–4]. Estimates of glueball properties were obtained in a variety of approaches [5], ranging from model analyses [6–13], quenched lattice QCD (QLQCD) [14–18] and unquenched lattice QCD (UQLQCD) simulations [19, 20] to QCD sum rule calculations [21–33].

The lowest $0^{++}$ scalar glueball state is, in fact, the most intricate hadron state which is difficult to figure out. In lattice QCD (LQCD) the mass scale of the scalar glueball is predicted to be in the range of 1.3-1.7 GeV [14–18]. The $0^{++}$ scalar resonances closest to this energy range are $f_0(1500)$ and $f_0(1710)$ in the present data [34], and some authors favor the former as the lightest scalar glueball [35, 36], while some others prefer the latter [10, 18]. Furthermore, both $f_0(1500)$ and $f_0(1710)$ may not be pure glueballs; to the contrary, these resonances can be considered to be the mixture of glueball and mesons [7–9, 37]. In the QCD sum rule approach, the results of the mass of the scalar glueball are also different from each other. In the early days of QCD, some found a light $0^{++}$ scalar glueball in the region of 300-700 MeV [3, 30–33] using the subtracted sum rule (SSR), while the others obtained a much heavier one in the 1-2 GeV region [38, 39] by using the unsubtracted sum rule (USSR). The inconsistency between both subtracted and unsubtracted sum rules has bothered scientists for many years.

It should be noticed that, in the early QCD sum rule approach, it was already recognized that there is an onset of the nonperturbative physics (a departure from the asymptotic freedom) in the scalar glueball correlator at unusually short distances $|x| \ll \Lambda_{\text{QCD}}^{-1}$ [4]. Such hard nonperturbative effects are usually considered to be coming from direct instantons, i.e. the tunneling processes which rearrange the QCD vacuum topology in localized regions [40, 41]. This physics was ignored in the early glueball sum rules except in Ref. [42].

Recently, a sizeable instanton contribution to the QCD sum rules of the $0^{++}$ scalar glueball has been found on the basis of the instanton liquid model of the QCD vacuum [40, 41], and supported by the lattice simulations [42–46]. However, the instanton-induced continuum contributions are neglected in the early QCD sum rule approach, and thus a reliable estimate of the glueball properties cannot be obtained. On the basis of the instanton-improved operator-product expansion (OPE), the authors in Refs. [23–25] included the instanton-induced contributions to the continuum spectrum, made a great improvement to the consistency between different types of $0^{++}$ scalar glueball sum rules, and gave new predictions for the mass and decay constant confirmed later by the Gaussian sum rule (GSR) calculation [27]. Moreover, we have clarified that the stability and the consistency for the SSR and USSRs for the $0^{++}$ scalar glueball can be obtained by using a systematical semiclassical expansion of the background instanton fields in the instanton liquid model of the QCD vacuum [47].

Instantons, which make a great difference in the scalar and pseudoscalar channels [4, 48, 49], play a major role in the gluonic interactions, especially in the nonperturbative region [40]. Direct instanton contributions are included in the QCD sum rules [23, 27]. The results are found to be in good agreement with different USSRs. The compatibility between the SSR and USSRs become much better but are not satisfying yet, because only the leading classic effects are included in most of the calculations with the perturbative contribution and condensate contributions. It should be noted that the direct-instanton approximation is
criticized due to the problem of double counting \cite{24}, because both condensate and instanton contributions are included in the correlator, but most of the gluon condensates could be produced from instantons.

Up to now, most of the theoretical calculations for the $0^{++}$ scalar glueball were based on QCD Laplace sum rules (LSR), which emphasize the contributions of the lowest resonance, and have shown the power in the investigation of the nonperturbative properties of the hadron ground states. On the other hand, it should be noticed that the GSR developed later emphasizes only the contribution of the state considered, and has a cleaner background in comparison with the LSR. As a cross-check, the same problem should be investigated using GSR, because both Laplace and Gaussian sum rules are derived from the same underlying dynamical theory, and should give almost the same results.

Motivated by the above considerations, our main objective in this paper is to investigate the $0^{++}$ scalar glueball in the framework of Gaussian sum rules. For the correlation function, we include the contributions from the interactions between the quantum gluons and the classical instanton background besides the ones coming separately from these two different field configurations. For the spectral function, instead of using the usual zero-width approximation, we adopt the Breit-Wigner form for the considered resonance with correct threshold behavior, in order to get the information of not only the mass scale but also the full decay width. Moreover, without using the scheme of the mixture of the traditional condensates and the so-called direct-instanton contribution, we are working in the framework of the semiclassical expansion of QCD in the instanton liquid vacuum, a well-defined self-consistent procedure for the quantum theory justified by the path-integral quantization formalism. The paper is organized as follows: In Sec. \ref{sec:correlation} we give the expression for the correlation function of the $0^{++}$ scalar gluon current. The spectral function corresponding to this current is constructed in Sec. \ref{sec:specfunc}. Then, a family of the finite-width Gaussian sum rules are derived in Sec. \ref{sec:GSR}. In Sec. \ref{sec:numeric} the numerical simulation is carried out. Finally, a summary of our conclusions and a discussion are given in Sec. \ref{sec:summary}.

II. CORRELATION FUNCTION

The correlation function for the scalar glueball in the Euclidean space-time with a virtuality $q^2$ is defined by

$$\Pi(q^2) = \int d^4 x e^{iq \cdot x} \langle \Omega | T O_s(x) O_s(0) | \Omega \rangle,$$  \hspace{1cm} (1)

where $|\Omega\rangle$ is the physical vacuum, and $O_s$ the scalar glueball current with the quantum numbers $J^{PC} = 0^{++}$

$$O_s = \alpha_s G_{\mu\nu}^a(B) G^{a,\mu\nu}(B),$$  \hspace{1cm} (2)

in which, $\alpha_s$ denotes the strong coupling constant. The scalar glueball current $O_s$ is gauge-invariant, and renormalization-invariant at one-loop level. In the spirit of the semiclassical expansion, and in order to maintain the $O(4)$ covariance, the gluon field strength tensor $G_{\mu\nu}^a(B)$ is considered as a functional of the full gluon potential, $B_{\mu a} = A_{\mu a} + a_{\mu a}$, with $A_{\mu a}$ and $a_{\mu a}$ being the instanton fields and the corresponding quantum fluctuations.

The theoretical expression, $\Pi^{\text{QCD}}$, for the correlation function $\Pi$ may be divided into the following three parts

$$\Pi^{\text{QCD}}(Q^2) = \Pi^{\text{pert}}(Q^2) + \Pi^{\text{inst}}(Q^2) + \Pi^{\text{int}}(Q^2),$$  \hspace{1cm} (3)
where \( Q^2 = q^2 \), and \( \Pi^{\text{pert}}(Q^2) \), \( \Pi^{\text{inst}}(Q^2) \), and \( \Pi^{\text{int}}(Q^2) \) are the contributions from the only perturbative QCD, the pure instanton dynamics, and the interactions between the instantons and the quantum gluon fields, respectively.

The perturbative contribution \( \Pi^{\text{pert}}(Q^2) \) is already known to be

\[
\Pi^{\text{pert}}(Q^2) = Q^4 \ln \left( \frac{Q^2}{\mu^2} \right) \left[ a_0 + a_1 \ln \left( \frac{Q^2}{\mu^2} \right) + a_2 \ln^2 \left( \frac{Q^2}{\mu^2} \right) \right],
\]

where \( \mu^2 \) is the renormalization scale in the \( \overline{\text{MS}} \)-dimensional regularization scheme, and the coefficients with the inclusion of the threshold effects are

\[
a_0 = -2 \left( \frac{\alpha_s}{\pi} \right)^2 \left[ 1 + \frac{659}{36} \left( \frac{\alpha_s}{\pi} \right) + 247.48 \left( \frac{\alpha_s}{\pi} \right)^2 \right],
\]

\[
a_1 = 2 \left( \frac{\alpha_s}{\pi} \right)^3 \left[ \frac{9}{4} + 65.781 \left( \frac{\alpha_s}{\pi} \right) \right],
\]

\[
a_2 = -10.1252 \left( \frac{\alpha_s}{\pi} \right)^4.
\]

for QCD with three quark flavors up to three-loop level in the chiral limit \([27, 28, 50]\), and

\[
a_0 = -2 \left( \frac{\alpha_s}{\pi} \right)^2 \left[ 1 + \frac{51}{4} \left( \frac{\alpha_s}{\pi} \right) \right],
\]

\[
a_1 = \frac{11}{2} \left( \frac{\alpha_s}{\pi} \right)^3, \quad a_2 = 0.
\]

for quarkless QCD up to two-loop level \([51]\). Both expressions for \( \Pi^{\text{pert}}(Q^2) \) with and without quark loop corrections are used in our calculation for comparison. With the assumption that the dominant contribution to \( \Pi^{\text{inst}}(Q^2) \) comes from a Belavin-Polyakov-Schwartz-Tyupkin single instanton and anti-instanton solutions \([52, 54]\) and the multi-instanton effects are negligible (see a QCD spectral sum rule (QSSR) approach \([25]\)), and in view of the gauge-invariance of the correlation function, one may choose to work in the regular gauge of the classical single instanton potential

\[
A^a_\mu = \frac{2}{g_s} \eta^a_{\mu \nu} \frac{(x - x_0)_\nu}{(x - x_0)^2 + \rho^2},
\]

where \( \eta^a_{\mu \nu} \) is the 't Hooft symbol, and \( x_0 \) and \( \rho \) denote the position and size of the instanton, respectively. The pure instanton contribution is obtained to be \([3, 23, 24, 42, 55]\)

\[
\Pi^{\text{inst}}(Q^2) = 2^5 \pi^2 \bar{n} \bar{\rho}^4 Q^4 K_2^2(\sqrt{Q^2 \bar{\rho}}),
\]

where \( K_2(x) \) is the McDonald function, \( \bar{n} = \int_0^\infty d\rho \rho n(\rho) \) and \( \bar{\rho} \) are the overall instanton density and the average instanton size in the random instanton background, respectively.

It is noticed that the contribution to \( \Pi^{\text{QCD}} \) from the interactions between instantons and the quantum gluon fields is of the order of the product of \( \alpha_s \) and the overall instanton density \( \bar{n} \). There is no reason to get rid of this contribution in comparison with the perturbative contributions of the higher order \( \alpha_s^4 \) considered in \( \Pi^{\text{pert}} \). To calculate such contribution, our key observation is that the instanton potential \( A^a_\mu \) obeys also the fixed-point gauge condition

\[
(x - x_0)_\mu A^a_\mu (x - x_0) = 0
\]
due to the antisymmetry of the ’t Hooft symbols. As a consequence, the instanton potential can be expressed in terms of the corresponding field strength tensor as follows:

$$A^a_{\mu}(x - x_0) = \int_0^1 d\nu F^a_{\mu\nu}[u(x - x_0)](x - x_0)_{\nu}$$  \hspace{1cm} (10)$$

and the gauge link with respect to the instanton fields is just the unit operator, and thus the trace of any product of the gauge-covariant instanton field strengths at different points is gauge-invariant. This allows us to conclude that the remainder quantum corrections to the gauge-invariant correlation function, arising from the interactions between the instantons and the quantum gluons, is gauge-invariant as well. Therefore, one may choose any specific gauge in evaluating the quantum correction to \(\Pi^{\text{int}}\).

It is noticed that the interference between the pure quantum part of the scalar glueball current, \((O_s)_{\text{quantum}}\), and the pure classical instanton part, \((O_s)_{\text{instanton}}\), is vanishing due to the fact that the momentum integration of the massless gluon propagator or the product of massless quark and gluon propagators has no scale parameters, and is exactly zero in the dimensional regularization scheme. The first nonvanishing contribution comes from the contraction between the two quantum glueball currents with instanton legs. Working in Feynman gauge, our result for \(\Pi^{\text{int}}\) is

$$\Pi^{\text{int}}(Q^2) = C_0 \alpha_s \bar{n}\pi + \alpha_s^2 \bar{n}[C_1 + C_2(Q\bar{\rho})^2 \ln(Q\bar{\rho})^2 + C_3 \ln(Q\bar{\rho})^2 + C_4(Q\bar{\rho})^{-2}],$$  \hspace{1cm} (11)$$

where the coefficients are

\[C_0 = 62.62, \; C_1 = 1533.15, \; C_2 = 825.81, \; C_3 = -496.33, \; C_4 = -348.89.\]  \hspace{1cm} (12)$$

It is remarkable to note that the fixed point \(x_0\), which characterizes the gauge condition, disappears in the expression of \(\Pi^{\text{int}}\), as expected from the gauge invariance of our procedure. The detail calculation for \(\Pi^{\text{int}}\) is much involved, and will appear elsewhere.

### III. SPECTRAL FUNCTION

Now, we turn to specify the spectral function for the correlation function of the scalar glueball current. The imaginary part of the correlation function Eq. (3) is

$$\text{Im} \Pi^{\text{QCD}}(s) = -\pi s^2 \left[a_0 + 2a_1 \ln \frac{s}{\mu^2} + \left(3 \ln^2 \frac{s}{\mu^2} - \pi^2\right) a_2\right]$$

$$-16\pi^4 s^2 \bar{n}\rho^4 J_2(\bar{\rho}\sqrt{s})Y_2(\bar{\rho}\sqrt{s}) + \alpha_s^2 \bar{n}\pi(C_2\bar{\rho}^2 s - C_3).$$  \hspace{1cm} (13)$$

The usual lowest resonance plus a continuum model is used to saturate the phenomenological spectral function,

$$\text{Im} \Pi^{\text{PHE}}(s) = \rho^{\text{had}}(s) + \theta(s - s_0) \text{Im} \Pi^{\text{QCD}}(s),$$  \hspace{1cm} (14)$$

where \(s_0\) is the QCD-hadron duality threshold, and \(\rho^{\text{had}}(s)\) the spectral function for the lowest scalar glueball state. Instead of using the zero-width approximation as usual, the Breit-Wigner form for a single resonance is adopted for \(\rho^{\text{had}}(s)\) in the quarkless world

$$\rho^{\text{had}}(s) = \frac{f^6 m \Gamma}{(s - m^2 + \Gamma^2/4)^2 + m^2 \Gamma^2},$$  \hspace{1cm} (15)$$
where \( f^3 = \langle \Omega | O_s | 0^{++} \rangle \) is the coupling of the lowest resonance to the scalar glueball current Eq. (2). Recall the threshold behavior for \( \rho^{\text{had}}(s) \)

\[
f^3 \to \lambda_0 s, \quad \text{for } s \to 0,
\]

which is deduced from a low-energy theorem \([4, 38, 56]\). The early QCD sum rule approach had often used \( f^3 \to \lambda_0 s \) in the whole lowest resonance region, however the obtained mass scale is too low to be expected in comparison with the lattice QCD results. In fact, the threshold behavior (16) is only proven to be valid in the chiral limit; it may not be extrapolated far away. Therefore, instead of considering the coupling \( f \) as a constant \([25]\), we choose a model for \( f \) as

\[
f^3 = \begin{cases} \lambda_0 s, & \text{for } s < m^2, \\ \lambda_0 m^2 + \lambda^3, & \text{for } s \geq m^2, \end{cases}
\]

where \( \lambda_0 \) and \( \lambda \) are some constants determined late in numerical simulation, so that the spectral function \( \rho^{\text{had}}(s) \) has the almost complete Breit-Wigner form with correct threshold behavior, and in cooperation with the threshold behavior which is important for the convergence of the corresponding integral. In Eq. (17), the constant \( \lambda \) is invoked for the discontinuity at the chiral symmetry breaking.

Although glueballs are well defined in quarkless QCD, the mixing with mesons makes the phenomenological side more complicated in full QCD; all scalar hadron states having a glue content should be interpolated by the gluonic current, and then present in the correlator with different couplings. In Refs. \([7–9, 34–36]\), the glueball is shared between the three scalar hadrons. These three hadrons are very close in mass; the assumption of single resonance maybe be not appropriate. The form of the spectral function for three resonances we will use is taken to be

\[
\rho^{\text{had}}(s) = \sum_{i=1}^3 \frac{f_i^0 m_i \Gamma_i}{(s - m_i^2 + \Gamma_i^2/4)^2 + m_i^2 \Gamma_i^2},
\]

where all \( f_i \) have the form shown in (17) with the same \( \lambda_0 \), because \( \lambda_0 \) is fixed to be 5 GeV by the low-energy theorem of QCD, and so that its value is independent of what an individual resonance considered.

### IV. FINITE-WIDTH GAUSSIAN SUM RULES

A family of Gaussian sum rules can also be constructed from the Borel transformation of the correlation function in Eq. (3) \([57]\)

\[
G^{\text{had}}_k(s_0; \hat{s}, \tau) = G^{\text{QCD}}_k(s_0; \hat{s}, \tau) + \frac{1}{\sqrt{4\pi \tau}} \exp \left[ -\frac{\hat{s}^2}{4\tau} \right] \Pi(0) \delta_{k,-1},
\]

where \( \Pi(0) \) comes from the subtraction to the corresponding dispersion relation due to the degree of divergence of the correlation function of the scalar glueball, and

\[
G^{\text{QCD}}_k(s_0; \hat{s}, \tau) = G^{\text{QCD}}_k(\hat{s}, \tau) - G^{\text{cont}}_k(s_0; \hat{s}, \tau),
\]

\[
G^{\text{had}}_k(s_0; \hat{s}, \tau) = \frac{1}{\sqrt{4\pi \tau}} \int_0^{s_0} ds s^k \exp \left[ -\frac{(s - \hat{s})^2}{4\tau} \right] \frac{1}{\pi} \rho^{\text{had}}(s),
\]
where $G^\text{cont}_k(s_0; \hat{s}, \tau)$ is the contribution of continuum,

$$G^\text{cont}_k(s_0; \hat{s}, \tau) \equiv \frac{1}{\sqrt{4\pi\tau}} \int_{s_0}^\infty ds \exp \left[ -\frac{(s - \hat{s})^2}{4\tau} \right] \frac{1}{\pi} \text{Im}\Pi^\text{QCD}(s)$$

and $G^\text{QCD}_k(\hat{s}, \tau)$ is defined as

$$G^\text{QCD}_k(\hat{s}, \tau) \equiv \frac{2\tau}{\sqrt{4\pi\tau}} \mathcal{B} \left[ \text{Im}\left(\hat{s} + iQ^2\right)^k \Pi^\text{QCD}(-\hat{s} - iQ^2) \right],$$

with the Borel transformation $\mathcal{B}$ being defined by

$$\mathcal{B} \equiv \lim_{N \to \infty, Q^4 \to \infty} \frac{(-1)^N}{(N - 1)!} \frac{Q^4}{N} \left( \frac{d}{dQ^4} \right)^N.$$ (24)

The Gaussian sum rule emphasizes only the contribution of the hadron state considered, and suppresses the background exponentially (according to the Gaussian distribution). Recall that the Laplace sum rule stress only the contribution from the lowest state, and suppresses the other contributions exponentially (according to the exponential distribution).

For $k = -1, 0, \text{ and } 1$, a straightforward but tedious manipulation leads to

$$G^\text{QCD}_{-1}(\hat{s}, \tau) = -\frac{1}{\sqrt{4\pi\tau}} \int_0^\infty ds \exp \left[ -\frac{(s - \hat{s})^2}{4\tau} \right]
\times \left[ (a_0 - \pi^2 a_2) + 2a_1 \ln \left( \frac{s}{\sqrt{\tau}} \right) + 3a_2 \ln^2 \left( \frac{s}{\sqrt{\tau}} \right) \right]
- \frac{1}{\sqrt{4\pi\tau}} \int_0^\infty ds \exp \left[ -\frac{(s - \hat{s})^2}{4\tau} \right] 16\pi \frac{3}{2} \hat{n}^2 \rho J_2(\hat{s}) Y_2(\rho \sqrt{s})
+ \frac{1}{\sqrt{4\pi\tau}} \int_0^\infty ds \exp \left[ -\frac{(s - \hat{s})^2}{4\tau} \right] \hat{n}^2 \rho^2 (C_2 \rho)^2
+ \frac{1}{\sqrt{4\pi\tau}} \exp \left[ -\frac{s^2}{4\tau} \right] \left[ C_4 \hat{n}^2 \rho^2 \left( \frac{\hat{s}}{4\tau} \right)^2 - (C_0 \alpha_s \pi \rho + C_1 \alpha_s \bar{n} + 2^7 \pi^2 \bar{n}) \right]
+ \frac{1}{\sqrt{4\pi\tau}} \exp \left[ -\frac{s^2}{4\tau} \right] \hat{n} \alpha_s C_3 \left\{ \frac{3\pi}{12} \left[ \text{erf} \left( \frac{\hat{s}}{2\sqrt{\tau}} \right) \right]^2 + \frac{\pi}{2} \text{erf} \left( \frac{\hat{s}}{2\sqrt{\tau}} \right) \right\] - \ln \left( \sqrt{\tau} \rho^2 \right) + \frac{\gamma}{2} - \ln 2 \right\},$$

(25)
\[ G_{0}^{QCD} (\hat{s}, \tau) = -\frac{1}{\sqrt{4\pi\tau}} \int_{0}^{\infty} ds s^2 \exp \left[ -\frac{(s - \hat{s})^2}{4\tau} \right] \]
\times \left[ (a_0 - \pi^2 a_2) + 2a_1 \ln \left( \frac{s}{\sqrt{\tau}} \right) + 3a_2 \ln^2 \left( \frac{s}{\sqrt{\tau}} \right) \right] \\
- \frac{1}{\sqrt{4\pi\tau}} \int_{0}^{\infty} ds s^2 \exp \left[ -\frac{(s - \hat{s})^2}{4\tau} \right] 16\pi^3 \bar{n}\rho J_2 (\bar{\rho}\sqrt{s}) Y_2 (\bar{\rho}\sqrt{s}) \\
+ \frac{1}{\sqrt{4\pi\tau}} \int_{0}^{\infty} ds \exp \left[ -\frac{(s - \hat{s})^2}{4\tau} \right] \bar{n}\alpha_s^2 (C_2\rho^2 s - C_3) \\
+ \frac{1}{\sqrt{4\pi\tau}} \exp \left[ -\frac{s^2}{4\tau} \right] C_4\alpha_s^2 \bar{n}\rho^2, \quad (26) \]

\[ G_{1}^{QCD} (\hat{s}, \tau) = -\frac{1}{\sqrt{4\pi\tau}} \int_{0}^{\infty} ds s^3 \exp \left[ -\frac{(s - \hat{s})^2}{4\tau} \right] \\
\times \left[ (a_0 - \pi^2 a_2) + 2a_1 \ln \left( \frac{s}{\sqrt{\tau}} \right) + 3a_2 \ln^2 \left( \frac{s}{\sqrt{\tau}} \right) \right] \\
- \frac{1}{\sqrt{4\pi\tau}} \int_{0}^{\infty} ds s^3 \exp \left[ -\frac{(s - \hat{s})^2}{4\tau} \right] 16\pi^3 \bar{n}\rho J_2 (\bar{\rho}\sqrt{s}) Y_2 (\bar{\rho}\sqrt{s}) \\
+ \frac{1}{\sqrt{4\pi\tau}} \int_{0}^{\infty} ds s^2 \exp \left[ -\frac{(s - \hat{s})^2}{4\tau} \right] \bar{n}\alpha_s^2 (C_2\rho^2 s - C_3). \quad (27) \]

V. NUMERICAL ANALYSIS

Now, we specify the input parameters in the numerical calculation. The color and flavor numbers are taken to be \( N_c = 3 \) and \( N_f = 3 \), respectively. The expressions for two-loop quarkless (\( N_f = 0 \)) running coupling constant \( \alpha_s (Q^2) \) at renormalization scale \( \mu \) \[58, 59\]

\[ \frac{\alpha_s^{(2)} (\mu^2)}{\pi} = \frac{1}{\beta_0 L} - \frac{\beta_1}{\beta_0} \ln L, \quad (28) \]

and for the three-loop running coupling constant with three flavors (\( N_f = 3 \))

\[ \frac{\alpha_s (\mu^2)}{\pi} = \frac{\alpha_s^{(2)} (\mu^2)}{\pi} + \frac{1}{(\beta_0 L)^3} \left[ L_1 \left( \frac{\beta_1}{\beta_0} \right)^2 + \frac{\beta_2}{\beta_0} \right], \quad (29) \]

are used, where the central value of the \( \overline{\text{MS}} \) QCD scale \( \Lambda \) is taken to be 120 MeV, and

\[ L = \ln \left( \frac{\mu^2}{\Lambda^2} \right), \quad L_1 = \ln^2 L - \ln L - 1, \]
\[ \beta_0 = \frac{1}{4} \left[ 11 - \frac{2}{3} N_f \right], \quad \beta_1 = \frac{1}{4\pi} \left[ 102 - \frac{38}{3} N_f \right], \]
\[ \beta_2 = \frac{1}{4\pi} \left[ \frac{2857}{2} - \frac{3033}{5} N_f + \frac{325}{34} N_f \right]. \quad (30) \]
Recall here that research on the renormalization group improvement for Gaussian sum rules indicates that $\mu^2 = \sqrt{\tau}$ [60]. The subtraction constant $\Pi(0)$ is fixed by QCD low-energy theorem [3]

$$\Pi(0) = \frac{32\pi}{9}\langle\alpha_s G^2 \rangle \simeq 0.6 \text{ GeV}^4.$$  \hfill (31)

The values of the average instanton size and the overall instanton density are adopted from the instanton liquid model [42]

$$\bar{n} = 1 \text{ fm}^{-4} = 0.0016 \text{ GeV}^4, \quad \bar{\rho} = \frac{1}{3} \text{ fm} = 1.689 \text{ GeV}^{-1}.$$  \hfill (32)

Finally, the mass of the neutral pion is taken from the experimental data, i.e. $m_\pi = 135$ MeV.

In order to measure the compatibility between both sides of the sum rules (19) realized in our numerical simulation, we introduce a variation, $\delta$, defined by

$$\delta = \frac{1}{N} \sum_{i=1}^{N} \frac{[L(\tau_i) - R(\tau_i)]^2}{|L(\tau_i)R(\tau_i)|},$$  \hfill (33)

where the interval $[\tau_{\text{min}}, \tau_{\text{max}}]$ is divided into 100 equal small intervals, $N = 101$, and $L(\tau_i)$ and $R(\tau_i)$ are left-hand side and right-hand side of Eq. (19) evaluated at $\tau_i$.

To determine the values of the resonance parameters in Eq. (15), we match both sides of sum rules Eq. (19) optimally in the fiducial domain. In doing so, the value of $\hat{s}$ should approximately be set to be the corresponding mass squared of the resonance $m^2$. To suppress the continuum contribution, we require $\hat{s} \leq m^2$. The conditions for determining the value of $s_0$ are: first, it should be greater than $m^2$; and second, it should guarantee that there exists a sum rule window for our Gaussian sum rules. We note that the upper limit $\tau_{\text{max}}$ of the sum rule window is determined by requiring that the contribution from the continuum should be less than that of the resonance

$$G_k^{\text{cont}}(s_0; \hat{s}, \tau_{\text{max}}) \leq G_k^{\text{QCD}}(s_0; \hat{s}, \tau_{\text{max}}),$$  \hfill (34)

while the lower limit $\tau_{\text{min}}$ of the sum rule window is obtained by requiring the contribution of pure instantons to be greater than 50% of $G_k^{\text{QCD}}(s_0; \hat{s}, \tau)$, because such classical contributions should be dominant in the low-energy region. Moreover, to require that the multi-instanton corrections remain negligible, we simply adopt a rough estimate

$$\tau_{\text{min}}^{-1} \leq (2\hat{\rho})^4 \sim \left(\frac{2}{0.6 \text{ GeV}}\right)^4.$$  \hfill (35)

With these requirements, the figures and numerical results are given below.

For the case of quarkless QCD with the lowest resonance in the $0^{++}$ channel, we adopt the isolated lowest resonance model (15) for the spectral function, the optimal parameters governing the sum rules are listed in Table I. The corresponding curves for the left-hand side and right-hand side of (19) of $k = -1, 0, \text{ and } +1$ are displayed in Fig. I where the solid lines are the right-hand side (QCD) of Eq. (19), and the dashed lines are the left-hand side (HAD) of Eq. (19), and the dotted lines are for the right-hand side (QCD) excluding the contribution of interactions between the instantons and the quantum gluons (the same for hereafter). Taking the average, the values of the mass and width of the lowest $0^{++}$ scalar
FIG. 1: The curves for the left-hand side and right-hand side of the Eq. (19) for quarkless QCD with only the lowest resonance considered. The solid line denotes the right-hand side (QCD), dashed line for left-hand side (HAD), and dotted line for the right-hand side (QCD) without the interaction contribution of the Gaussian sum rules (19).
TABLE I: The fitting values of the mass \( m \) and width \( \Gamma \) of the lowest \( 0^{++} \) scalar glueball, and of the parameters \( \lambda \) and \( f \), \( s_0 \), \([\tau_{\text{min}}, \tau_{\text{max}}]\), and \( \delta \) characterizing the couplings to the lowest resonance, the continuum threshold, the sum rule window, and the compatibility measure for finite-width Gaussian sum rules \((19)\) of \( k = -1, 0, \) and \( 1 \) in quarkless QCD for a given \( \hat{s} \), the real part of the complex \( q^2 (q^2 = \hat{s} + i Q^2) \).

| \( \hat{s}(\text{GeV}^2) \) | \( k \) | \( m (\text{GeV}) \) | \( \Gamma (\text{GeV}) \) | \( \lambda (\text{GeV}) \) | \( f (\text{GeV}) \) | \( s_0 (\text{GeV}^2) \) | \([\tau_{\text{min}}, \tau_{\text{max}}]\) (GeV\(^4\)) | \( \delta \) |
|---|---|---|---|---|---|---|---|---|
| 1.49\(^2\) | −1 | 1.49 | 0.04 | 1.481 | 1.495 | 4.40 | [0.75-2.0] | 2.40 \(\times 10^{-5}\) |
| 1.50\(^2\) | 0 | 1.50 | 0.15 | 1.523 | 1.534 | 4.65 | [1.0-2.6] | 7.79 \(\times 10^{-6}\) |
| 1.50\(^2\) | 1 | 1.51 | 0.09 | 1.517 | 1.530 | 4.68 | [0.8-1.5] | 2.80 \(\times 10^{-6}\) |

The fitting values of the mass \( m \) and width \( \Gamma \) of the lowest \( 0^{++} \) scalar glueball living in quarkless QCD, and the corresponding optical fit parameters are predicted to be

\[
\begin{align*}
m &= 1.52 \pm 0.18 \text{ GeV}, \quad \Gamma = 0.2 \pm 0.15 \text{ GeV}, \\
f &= 1.47 \pm 0.13 \text{ GeV}, \quad s_0 = 3.8 \pm 0.9 \text{ GeV}^2.
\end{align*}
\tag{36}
\]

where the errors are estimated from the uncertainties of the spread between the individual sum rules, and by varying the phenomenological parameters, \( \Lambda \) and \( \langle \alpha_s G^2 \rangle \), appropriately away from their central values (the same for hereafter)

\[
\begin{align*}
\Lambda &= 120 - 200 \text{ MeV}, \\
\langle \alpha_s G^2 \rangle &= 0.6 - 0.8 \text{ GeV}^4.
\end{align*}
\tag{37, 38}
\]

For quarkless QCD, there is only one well-defined scalar bound state of gluon suggested by lattice QCD and also from our investigation just described above. Including quarks enhances the difficulty of the task since many states possessing the same quantum numbers may be present in the correlator. Even so, at the first step as comparison, we still consider only the lowest resonance in the \( 0^{++} \) channel for the case of QCD with three massless quarks as usual for the scalar glueball mass \( m \) of about 600 MeV, because that resonance \( f_0(600) \) may be considered to be well isolated. The optimal parameters governing the sum rules are listed in Table II. The corresponding curves for the left-hand side and right-hand side of \((19)\) of \( k = -1, 0 \) and, +1 are displayed in Fig.2. Taking the average, the values of the mass and width of the (probable isolated) lowest \( 0^{++} \) scalar glueball in the world of QCD with three massless quarks, and the corresponding optical fit parameters are predicted to be

\[
\begin{align*}
m &= 1.54 \pm 0.17 \text{ GeV}, \quad \Gamma = 0.23 \pm 0.13 \text{ GeV}, \\
f &= 1.64 \pm 0.14 \text{ GeV}, \quad s_0 = 3.8 \pm 0.9 \text{ GeV}^2.
\end{align*}
\tag{39}
\]

The above one isolated lowest resonance assumption is questioned from the admixture with quarkonium states, and from the experimental data that three \( 0^{++} \) scalar states are around the mass scale of 1500 MeV [namely \( f_0(1370) \), \( f_0(1500) \), and \( f_0(1710) \)]. Therefore, we adopt the three-resonance model for the phenomenological side of the sum rules for the case of QCD with three massless quarks in the \( 0^{++} \)-channel when \( m \) is above 1 GeV; the optimal parameters governing the sum rules are listed in Table III. The corresponding curves for
FIG. 2: The curves for the left-hand side and right-hand side of the Eq. (19) for three-flavor QCD in chiral limit with only the lowest resonance considered. The solid line denotes the right-hand side (QCD), dashed line for left-hand side (HAD), and dotted line for the right-hand side (QCD) without the interaction contribution of the Gaussian sum rules (19).
FIG. 3: The curves for the left-hand side and right-hand side of the Eq. (19) for three-flavor QCD in chiral limit with the lowest three resonances considered. The solid line denotes the right-hand side (QCD), dashed line for left-hand side (HAD), and dotted line for the right-hand side (QCD) without the interaction contribution of the Gaussian sum rules (19).
TABLE II: The fitting values of the mass $m$ and width $\Gamma$ of the lowest $0^{++}$ scalar glueball, and of the parameters $\lambda$ and $f$, $s_0$, $[\tau_{\text{min}}, \tau_{\text{max}}]$, and $\delta$ characterizing the couplings to the lowest single resonance, the continuum threshold, the sum rule window, and the compatibility measure for finite-width Gaussian sum rules \cite{19} of $k = -1, 0, 1$ in QCD with three massless quarks for a given $\hat{s}$, the real part of the complex $q^2 (q^2 = \hat{s} + iQ^2)$.

| $\hat{s}$(GeV$^2$) | $k$ | $m$(GeV) | $\Gamma$(GeV) | $\lambda$(GeV) | $f$(GeV) | $s_0$(GeV$^2$) | $[\tau_{\text{min}}, \tau_{\text{max}}]$(GeV$^4$) | $\delta$ |
|------------------|----|---------|---------------|--------------|---------|-------------|---------------------------------|------|
| 1.45$^2$         | -1 | 1.52    | 0.10          | 1.590        | 1.602   | 4.5         | [0.5-2.0]                        | 6.87 $\times 10^{-5}$ |
| 1.50$^2$         | 0  | 1.53    | 0.10          | 1.635        | 1.646   | 4.7         | [1.5-5.0]                        | 1.34 $\times 10^{-5}$ |
| 1.52$^2$         | 1  | 1.53    | 0.10          | 1.663        | 1.674   | 4.7         | [1.5-2.7]                        | 3.04 $\times 10^{-5}$ |

TABLE III: The fitting values of the mass $m$ and width $\Gamma$ of the lowest $0^{++}$ scalar glueball, and of the parameters $\lambda$ and $f$, $s_0$, $[\tau_{\text{min}}, \tau_{\text{max}}]$, and $\delta$ characterizing the couplings to the three closely located resonances, the continuum threshold, the sum rule window, and the compatibility measure for finite-width Gaussian sum rules \cite{19} of $k = -1, 0, 1$ in QCD with three massless quarks for a given $\hat{s}$, the real part of the complex $q^2 (q^2 = \hat{s} + iQ^2)$.

| $\hat{s}$(GeV$^2$) | $k$ | $m$(GeV) | $\Gamma$(GeV) | $\lambda$(GeV) | $f$(GeV) | $s_0$(GeV$^2$) | $[\tau_{\text{min}}, \tau_{\text{max}}]$(GeV$^4$) | $\delta$ |
|------------------|----|---------|---------------|--------------|---------|-------------|---------------------------------|------|
| 1.35$^2$         | -1 | 1.50    | 0.10          | 1.510        | 1.523   | 4.2         | [2.0,8.0]                        | 2.90 $\times 10^{-4}$ |
| 1.35$^2$         | 0  | 1.50    | 0.10          | 1.583        | 1.595   | 4.0         | [3.0,8.0]                        | 6.42 $\times 10^{-6}$ |
| 1.35$^2$         | 1  | 1.50    | 0.10          | 1.600        | 1.612   | 4.0         | [3.0,8.8]                        | 5.13 $\times 10^{-6}$ |

Taking the average, the values of the widths of the three lowest $0^{++}$ scalar resonances in the world of QCD with three massless quarks, and the corresponding optical fit parameters are predicted to be

\begin{equation}
  m = 1.37 \pm 0.06 \text{ GeV}, \quad \Gamma = 0.30 \pm 0.10 \text{ GeV}, \quad f = 1.10 \pm 0.13 \text{ GeV}, \quad (40)
\end{equation}

for the resonance $f_0(1370)$, and

\begin{equation}
  m = 1.50 \pm 0.10 \text{ GeV}, \quad \Gamma = 0.10 \pm 0.06 \text{ GeV}, \quad f = 1.60 \pm 0.11 \text{ GeV}, \quad (41)
\end{equation}

for $f_0(1500)$, and

\begin{equation}
  m = 1.71 \pm 0.11 \text{ GeV}, \quad \Gamma = 0.14 \pm 0.08 \text{ GeV}, \quad f = 1.10 \pm 0.14 \text{ GeV}. \quad (42)
\end{equation}
for \( f_0(1710) \).

Figures 1-3 show the consistent match between the both sides of Eq. (19) for \( k = -1, 0, \) and 1, respectively, with the fitting parameters. The matching between both sides of the sum rules is very well over the whole fiducial region with a very little departure.

These results are in good accordance with the experimental data of \( f_0(1500), m = 1505 \pm 5 \) MeV, \( \Gamma = 109 \pm 7 \) MeV \[34\], and the sum rule calculation of Ref. \[24\], \( m = 1.53 \pm 0.2 \) GeV, \( f = 1.01 \pm 0.25 \) GeV (see Table IV). We do not calculate the higher moments sum rule, because for \( k > 1 \), the continuum contributions become very large.

| Methods     | Mass (GeV) | Width (GeV) | Coupling (GeV) | \( s_0 \) (GeV\(^2\)) | References |
|-------------|------------|-------------|----------------|------------------------|------------|
| GSR         | 0.8 - 1.6  | 0.4 - 0.6   | 2.3            | \( s_0 \)              | \[28\]     |
| QLQCD       | 1.3 - 1.7  | 0.035 - 0.873 | 2.3            | \[14, 18\]             |
| QSSR        | 1.4 \pm 0.2 | 2.56 - 2.61 | 5.0 \pm 0.1    | \[23\]                 |
| LSR         | 1.25 \pm 0.2 | 1.05 \pm 0.1 | 5.0 \pm 0.1    | \[24\]                 |
|             | 1.52 \pm 0.2 | 0.39 \pm 0.145 | 4.2 \pm 0.2    | \[29\]                 |
| model       | 1.666      |             | 5.0 \pm 0.1    | \[11\]                 |
|             | 1.633      |             | 4.2 \pm 0.2    | \[11\]                 |

VI. DISCUSSION AND CONCLUSIONS

The properties of the \( 0^{++} \) scalar glueball are examined in a family of the finite-width Gaussian sum rules. The correlation function is calculated in a semiclassical expansion, a well-defined process justified in the path-integral quantization formalism, of QCD in the instanton background, namely the instanton liquid model of the QCD vacuum. Besides the contributions from pure gluons and instantons separately, the one arising from the interactions between the classical instanton fields and the quantum gluon ones are taken into account as well. Instead of using the usual zero-width approximation for the spectral function of the considered resonances, the Breit-Wigner form for the resonances with a correct threshold behavior is adopted. With the QCD standard input parameters, three Gaussian sum rules with the \( k = -1, 0, \) and 1-th moments are carefully studied.

For the quarkless QCD, we have, in fact, changed the value of \( \hat{s} \), and found that the value of the mass of the lowest resonance is approximately proportional to \( \hat{s} \), and the value of \( s_0 \) arrives almost at its maximum for \( \hat{s} \) lying between 1.50 GeV\(^2\) and 1.70 GeV\(^2\), where the couplings to the state are almost the same for different \( k \), and the corresponding widths become small and stable. We have only shown the situation with the optimal compatibility. We note here that the value of \( \delta \) for \( m = 650 \) MeV is one or two-orders lower than the optimal one, and the values of \( f \) are not coincident for different \( k \), so that the mass scale of the lowest \( 0^{++} \) scalar glueball may not be lower than 1 GeV. The mass and width of the lowest glueball without quark loop corrections are predicted in Eq. (36).
For QCD with three massless flavors and by considering only single scalar resonance, the same behavior with respect to the changing of $\hat{s}$ appears. Namely, the value of the mass of the resonance is approximately proportional to $\hat{s}$, and the value of $s_0$ arrives almost at its maximum for $\hat{s}$ lying between 1.35 GeV$^2$ and 1.70 GeV$^2$, where the couplings to the state are almost the same for different $k$, and the corresponding widths become small and stable. When $\hat{s} = 1.70^2$ GeV$^2$ (the situation with optimal compatibility), all physical parameters are almost the same for different $k$. We note here that the value of $\delta$ for $m = 650$ MeV is nearly one order lower than the optimal one, and the values of $f$ are not coincident for different $k$ as in the case of pure QCD, so that the mass scale of the lowest $0^{++}$ scalar glueball, even in the world of QCD with massless quarks, may not be lower than 1 GeV. The mass and width of the lowest glueball with quark loop corrections under the assumption of no mixture between glueball and $q\bar{q}$ state are predicted in Eq. (39).

For QCD with three massless flavors and by considering three closely located $0^{++}$ scalar resonances $f_0(1370)$, $f_0(1500)$, and $f_0(1710)$ (namely both $\hat{s}$ and the resonance masses are given input parameters), the behavior with respect to the changing of $\hat{s}$ is changed. Namely, the values of the widths and $s_0$ for the three resonances remain as almost invariant, and only their couplings slightly increase when increasing $\hat{s}$. The optimal compatibility is arrived at $\hat{s} = 1.35^2$ GeV$^2$, as shown in Table III and Fig.3. The widths of the lowest three $0^{++}$ resonances coupled to the glueball current $O_s$ are predicted in (40), from which we can read off the corresponding couplings $f^3$ to the three resonances $f_0(1370)$, $f_0(1500)$, and $f_0(1710)$ with masses 1.35, 1.47, and 1.70 GeV as

$$0.95 \pm 0.47 \text{ GeV}^3, \quad 3.92 \pm 0.85 \text{ GeV}^3, \quad 1.42 \pm 0.51 \text{ GeV}^3,$$

respectively.

In summary, we may conclude that, first, any $0^{++}$ scalar resonance below 1 GeV, such as $f_0(600)$, contains almost no component of the scalar glueball; second, the values of the mass and decay width of the $0^{++}$ resonance, in which the fraction of the scalar glueball state is dominant, are $m = 1.50 \pm 0.10$ GeV and $\Gamma = 0.10 \pm 0.06$ GeV, respectively, and the value of its coupling to the corresponding current is $f^3_{s \geq m^3} = 3.92 \pm 0.85$ GeV$^3$; third, the fractions of the scalar glueball contained in the other nearby $0^{++}$ scalar resonances, $f_0(1370)$ and $f_0(1710)$, are also appreciable. These are not only compatible with lattice QCD simulation [14–18] and other estimations [23–25, 29, 61], but also in good accordance with the experimental data of the low scalar resonances [34–36].

It is also remarkable that the three Gaussian sum rules lead to almost the same results, a consistency between the subtracted and unsubtracted sum rules is very well justified. We note that we have not been working within the mixed scheme, namely with including condensates, and in the same time, adopting the so-called direct-instanton approximation, but simply with a self-consistent framework, a quantum theory in a classical background, without the problem of double counting. In this aspect, our results further justified the instanton liquid model for QCD among many other justifications.

In our semiclassical expansion, the leading contribution to the sum rules comes from instantons themselves, especially in the region below the threshold $s_0$. It is the amount of this contribution that determines the low bound of the sum rule window. This means that the nonlinear configurations of gluons have a dominant role with respect to the quantum fluctuations in the low-energy region.

The contribution of the interactions between the classical instanton fields and quantum gluon ones, considered in this paper but neglected in earlier sum rule calculations [23, 24, 27–
is in fact not negligible. To the contrary, its amount is approximately double or even triple that from the pure quantum fluctuations in the whole fiducial domain, expected from a view point of the semiclassical expansion. Moreover, it is obviously seen from Figs. 1.3 that, without taking the contribution from the interactions between instantons and quantum gluons into account, the departures between $G_{\text{had}}^{(s_0; \hat{s}, \tau)}$ and $G_{\text{QCD}}^{(s_0; \hat{s}, \tau)}$ without interaction become large, and all three Gaussian sum rules become less stable, and thus less reliable.

Finally, it should be noticed that the imaginary part of the instanton contribution is an oscillating, amplifying and nonpositive defined function, and so is the imaginary part of the correlation function. This property which is a fatal problem for the QCD sum rule calculation with the instanton background, may make the contribution of continuum too large to be under control. Hilmar Forkel introduced a Gaussian distribution for the instanton to get rid off this trouble, and obtained a smaller $0^{++}$ mass scale: $1.25 \pm 0.2 \text{ GeV}$ [23] compared to the earlier result $1.53 \pm 0.2 \text{ GeV}$ [24]. We did not use this Gaussian distribution, but simply chose a smaller fitting parameter to avoid this problem.

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