Production of \(\pi^0\) and \(\eta\) mesons at large transverse momenta in \(p^- p\) and \(p^- Be\) interactions at 515 GeV/c

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We present results on the production of high transverse momentum \(\pi^0\) and \(\eta\) mesons in \(p^- p\) and \(p^- Be\) interactions at 515 GeV/c. The data span the kinematic ranges \(1 < p_T < 11\) GeV/c in transverse momentum and \(-0.75 \leq y_{cm} \leq 0.75\) in rapidity. The inclusive \(\pi^0\) cross sections are compared with next-to-leading order QCD calculations and to expectations based on a phenomenological parton-\(k_T\) model.

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I. INTRODUCTION

The study of inclusive single-hadron production at large transverse momentum \(p_T\) has been a useful probe in the development of perturbative QCD (PQCD) [1, 2]. Early in the evolution of the parton model, a departure from an exponential dependence of particle production at low \(p_T\) was interpreted in terms of the onset of interactions between pointlike constituents (partons) contained in hadrons. Large \(p_T\) is a regime where perturbative methods have been applied to QCD to provide quantitative comparisons with data. Such comparisons yield information on the validity of the PQCD description, and on the parton distribution functions of hadrons and the fragmentation functions of partons.

This paper reports high-precision measurements of the production of \(\pi^0\) and \(\eta\) mesons with large \(p_T\) by 515 GeV/c \(p^-\) beams. The data were accumulated during the 1990 and 1991-92 fixed-target runs at Fermilab. The \(\pi^0\) production cross sections are compared with next-to-leading order (NLO) PQCD calculations [3]. As illustrated in a previous publication [4], our data for both inclusive \(\pi^0\) and direct-photon production are not described satisfactorily by the available NLO PQCD calculations, using standard choices of parameters. Similar discrepancies have been observed [5] between conventional PQCD calculations and other measurements of high-\(p_T\) \(\pi^0\) and direct-photon cross sections (see also [6–8]). The origin of these discrepancies can be attributed to the effects of initial-state soft-gluon radiation. Such radiation generates transverse components of initial-state parton momenta, referred to below as \(k_T\) [9]. Evidence of significant \(k_T\) in various processes, and a phenomenological model for incorporating its effect on the calculated high-\(p_T\) cross sections, have been extensively discussed in Refs. [4, 5]. Recent studies of the photoproduction of direct photons at HERA may provide additional insights [10–14]. In this paper, we follow the phenomenological prescription of Ref. [5] when comparing calculations with our \(\pi^0\) data.

II. EXPERIMENTAL APPARATUS

A. Spectrometer

Fermilab E706 was a fixed-target experiment designed to measure the production of direct photons, neutral mesons, and associated particles at high-\(p_T\) [4, 15–17]. The spectrometer, designed and built for this experiment, was located in the Meson West experimental hall and included a precision charged particle tracking system and a
large acceptance liquid argon calorimeter. Figure 1 shows the key elements of the Meson West spectrometer [18]. The tracking system consisted of silicon microstrip detectors (SSDs) in the target region, and multiwire proportional chambers (PWCs) and straw tube drift chambers (STDCs) downstream of a large aperture analysis magnet [17]. The SSD system contained sixteen planes of silicon wafers, arranged in eight modules. Each module contained two SSD planes, one providing $X$-view information and the other providing $Y$-view information. Six $3 \times 3 \text{ cm}^2$ SSD planes were located upstream of the target and used to reconstruct beam tracks. Two hybrid $5 \times 5 \text{ cm}^2$ SSD planes (25 $\mu$m pitch strips in the central 1 cm; 50 $\mu$m beyond) were located downstream of the target. These were followed by eight $5 \times 5 \text{ cm}^2$ SSD planes of 50 $\mu$m pitch. The analysis dipole imparted a transverse momentum impulse in the horizontal plane of $\approx 450 \text{ MeV}/c$ to charged particles. Downstream track segments were measured by means of four stations of four views ($XYUV$) of 2.54 mm pitch PWCs and two stations of eight ($4X4Y$) layers of STDCs with tube diameters 1.04 cm (upstream station) and 1.63 cm (downstream station).

In the 1990 run, the target consisted of two 0.8 mm thick copper foils followed by two pieces of beryllium. The upstream Be piece was 3.7 cm long, while the downstream Be piece was 1.1 cm long. For the 1991-92 run, the target was reconfigured to include a liquid hydrogen target [19] contained in a 15.3 cm long mylar flask and supported in an evacuated volume with beryllium windows at each end (2.5 mm thickness upstream and 2.8 mm thickness downstream). The liquid hydrogen target was flanked by two 0.8 mm thick copper disks upstream, and a 2.54 cm long beryllium cylinder downstream. Figure 2 shows the reconstructed vertex position along $Z$ as determined by the tracking system for representative samples from the 1990 (top) and 1991-92 (bottom) runs. The individual target elements are clearly resolved, as are several of the SSDs.

Photons were detected in a large, lead and liquid-argon sampling electromagnetic calorimeter (EMLAC), located 9 m downstream of the target [20]. The EMLAC had a cylindrical geometry with an inner radius of 20 cm and an outer radius of 160 cm. It was divided into four mechanically independent quadrants, which were further subdivided electronically to create octants. The calorimeter had 33 longitudinal cells, read out in two sections: an 11 cell front section (\$\approx 8.5$ radiation lengths) and a 22 cell back section (\$\approx 18$ radiation lengths). The longitudinal cells consisted of 2 mm thick lead cathodes (the first cathode was constructed of aluminum), double-sided copper-clad G-10 radial ($R$) anode boards followed by 2 mm thick lead cathodes, and double-sided copper-clad G-10 azimuthal ($\Phi$) anode boards. There were 2.5 mm argon gaps between each of these layers in a cell. The copper-cladding on the anode boards was cut to form strips. Signals from corresponding strips on all $R$ (or $\Phi$) anode boards in the front (or back) section were jumpered together. The copper-cladding on the radial anode boards was cut into concentric strips centered on the nominal beam axis. The width of the strips on the first $R$ board was 5.5 mm. The width of the $R$ strips on the following $R$ boards increased slightly so that the radial geometry was projective relative to the target. The azimuthal read-
out was subdivided at a radius of 40 cm into inner and outer segments, with each inner Φ strip subtending an azimuthal angle of π/192 radians, and outer Φ strips covering π/384 radians. Subdivision of the azimuthal strips in the outer portion of the detector improved both the position and energy resolution for showers reconstructed in this region. It also reduced $R$–$\Phi$ correlation ambiguities from multiple showers in the same octant of the calorimeter.

The spectrometer also included two other calorimeters: a hadronic calorimeter located downstream of the EMLAC within the same cryostat, and a steel and scintillator calorimeter positioned further downstream to provide instrumentation in the very forward region. The E672 muon spectrometer, consisting of a toroidal magnet, scintillators, and proportional wire chambers, was deployed immediately downstream of the calorimeters [21–24].

The beamline was instrumented with a differential Cherenkov counter [25, 26] to identify incident pions, kaons, and protons in secondary beams. This helium-filled counter was 43.4 m long and was located ≈ 100 m upstream of the experiment’s target. At the end of the beamline was a 4.7 m long stack of steel surrounding the beam pipe and shadowing the EMLAC to absorb off-axis hadrons. A water tank was placed at the downstream end of this hadron shield to absorb low-energy neutrons. Surrounding the hadron shield and neutron absorber were walls of scintillation counters (VW) to identify penetrating muons. During the 1990 run, there was one wall at the upstream end and two walls at the downstream end of the hadron absorber. During the 1991-92 run, an additional wall was added to the upstream end.

### B. Trigger

The E706 trigger selected events yielding high transverse momentum showers in the EMLAC. The selection process involved several stages: beam and interaction definitions and the pretrigger and high-$p_T$ trigger requirements. Beam particles were detected using a hodoscope consisting of three planes (arranged in $X$, $Y$, and $U$ views) of scintillator strips, located ≈ 2 m upstream of the target region. A $BEAM$ signal was generated if the beam hodoscope registered hits in time coincidence from at least two of the three planes. In addition, a $BEAM$ signal was generated if the hits in at least two of the hodoscope planes were considered to be isolated. Scintillation counters centered on the target with a 0.95 cm diameter central hole were used to reject interactions initiated by particles in the beam halo.

Interactions were detected using two pairs of scintillation counters mounted on the dipole analysis magnet, one pair on the upstream side and one pair downstream. Each pair had a central hole that allowed non-interacting beam particles to pass through undetected. An interaction was defined as a coincidence between signals from at least two of these four interaction counters. A filter was used to reject interactions that occurred within 60 ns of one another to minimize potential confusion in the EMLAC due to out-of-time interactions.

For those interactions that satisfied both the $BEAM$ and interaction definitions, the $p_T$ deposited in various regions of the EMLAC was evaluated by weighting the energy signals from the fast outputs of the EMLAC $R$ channel amplifiers by $\approx \sin \theta_i$, where $\theta_i$ is the polar angle that the $i^{th}$ strip subtended with respect to the nominal beam axis. The PRETRIGGER $HI$ signal required that the $p_T$ detected in either the inner or the outer $R$ channels of any octant was greater than the pretrigger threshold value. This signal was issued only when there was no evidence in that octant of substantial noise or significant $p_T$ attributable to an earlier interaction, and that there was no incident beam halo muon detected by the VW.

Localized trigger groups were formed for each octant by clustering the inputs from the $R$ channels into 32 groups of 8 channels. Each of the adjacent pairs of 8 channel groups formed a group of 16 strips. If the $p_T$ detected in any of these groups of 16 was above a specified high (or low) threshold, then a LOCAL $HI$ (or LOCAL $LO$) signal was generated for that octant. If a LOCAL $HI$ (or LOCAL $LO$) signal was generated in coincidence with the PRETRIGGER $HI$ in the same octant, then a SINGLE LOCAL $HI$ (or SINGLE LOCAL $LO$) trigger was generated for that octant.

Trigger decisions were also made based upon global energy depositions in an octant. A GLOBAL $LO$ signal was generated if the total $p_T$ in an octant exceeded a low global threshold value. The LOCAL $\otimes$ GLOBAL $LO$ trigger required a coincidence of the GLOBAL $LO$ signal with PRETRIGGER $HI$ and LOCAL $LO$ signals from the same octant. The LOCAL $LO$ requirement was included to suppress spurious global triggers due to coherent noise in the EMLAC.

The SINGLE LOCAL $LO$ and LOCAL $\otimes$ GLOBAL $LO$ triggers were prescaled to keep them from dominating the data sample. Prescaled samples of beam, interaction, and pretrigger events were also recorded. Further details concerning the E706 trigger can be found elsewhere [15, 17, 27, 28].

### III. ANALYSIS METHODS

This paper presents results for $\pi^0$ and $\eta$ production by 515 GeV/$c$ $\pi^-$ beam on beryllium and liquid hydrogen targets. The data sample used in this analysis corresponds to an integrated luminosity of 7.5 pb$^{-1}$ of $\pi^-\text{Be}$ data (6.1 pb$^{-1}$ from the 1990 run plus 1.4 pb$^{-1}$ from the 1991-92 run) and 0.23 pb$^{-1}$ of $\pi^-p$ data. The following subsections describe the data analysis procedures and the methods used to correct the data for losses due to inefficiencies and selection biases. Additional details may be found in several of our previous papers [15, 17, 20].
A. Reconstruction

Two major aspects of the event reconstruction procedure were particle track and calorimeter shower reconstruction. The charged-track reconstruction algorithm produced track segments upstream of the magnet using information from the SSDs, and downstream of the magnet using information from the PWCs and STDCs. These track segments were projected to the center of the magnet and linked to form the final reconstructed tracks, whose calculated charges and momenta were used in physics analyses and to determine the location of the primary interaction vertex. The charged track reconstruction is described in detail in Ref. [17].

The readout in each EMLAC quadrant consisted of four regions: left $R$ and right $R$, (radial strips of each octant in that quadrant), and inner $\Phi$ and outer $\Phi$. Strip energies from clusters in each region were fit to the shape of an electromagnetic shower as determined from Monte Carlo simulations and isolated-shower data. These fits were used to evaluate the positions and energies of the peaks in each region. Shower positions and energies were obtained by correlating peaks of approximately the same energy in the $R$ and $\Phi$ regions within the same half octant (more complex algorithms were used to handle configurations with overlapping showers in either the $R$ or $\Phi$ regions). The EMLAC readout was also subdivided longitudinally into front and back sections. This longitudinal segmentation provided discrimination between showers generated by electromagnetically or hadronically interacting particles. An expanded discussion of the EMLAC reconstruction procedures and performance can be found in Ref. [20].

B. Event selection and meson signals

Events contributing to measurements of cross sections were required to have reconstructed vertices within the fiducial volume of the Be or $H_2$ targets. Both $\pi^0$ and $\eta$ mesons were reconstructed via their $\gamma\gamma$ decay modes. Only those showers which deposited at least 20% of their energy in the front part of EMLAC were considered as $\gamma$ candidates, to reduce the background due to showers from hadronic interactions. Only those $\gamma\gamma$ combinations with energy asymmetry $A_{\gamma\gamma} \equiv |E_{\gamma_1} - E_{\gamma_2}|/(E_{\gamma_1} + E_{\gamma_2}) < 0.75$ were considered in order to reduce uncertainties due to low energy photons. Photons were required to be reconstructed within the fiducial region of the EMLAC to exclude areas with reduced sensitivity. In particular, regions of the detector near quadrant boundaries (which abutted steel support plates), the central beam hole, the outer radius of the EMLAC, and octant boundaries were excluded. In addition, $\gamma\gamma$ combinations were restricted to the same octant to simplify the trigger analysis. A simple ray-tracing Monte Carlo program was employed to determine the correction for these fiducial requirements.

Signals have been corrected for the 25% loss due to the energy asymmetry cut and the branching fractions for the $\gamma\gamma$ decay modes [29]. The correction for losses due to the conversion of one or both of the photons into $e^+e^-$ pairs was evaluated by projecting each reconstructed photon from the event vertex to the reconstructed position in the EMLAC. The radiation length of material traversed was calculated based upon detailed descriptions of the detectors encountered. The photon conversion probability was evaluated and used to account for conversion losses. The average correction for conversion losses was 1.19 for the Be target in the 1990 run (1.16 in 1991-92) and 1.24 for the $p$ target. A full event simulation (described below) was employed to correct for other effects including reconstruction smearing and losses.

The $\gamma\gamma$ invariant mass distributions in the $\pi^0$ and $\eta$ mass regions for photon-pairs that satisfied the above requirements are shown in Figs. 3 and 4 for several $p_T$ intervals. A $\pi^0$ candidate was defined as a combination of two photons with invariant mass, $M_{\gamma\gamma}$, in the range $100 \text{ MeV}/c^2 < M_{\gamma\gamma} < 180 \text{ MeV}/c^2$. An $\eta$ candidate was defined as a two-photon combination in the range $450 \text{ MeV}/c^2 < M_{\gamma\gamma} < 650 \text{ MeV}/c^2$. Combinatorial background under the $\pi^0$ and $\eta$ peak regions was evaluated as follows. Sideband regions were defined to cover a mass

![FIG. 3: $\gamma\gamma$ mass distributions in the region of the $\pi^0$ for several $p_T$ bins. Curves are overlayed for those $p_T$ bins where the background to the signal was determined using a fitting procedure rather than sideband subtraction.](image)

![FIG. 4: $\gamma\gamma$ mass distributions in the region of the $\eta$ for several $p_T$ bins.](image)
range equivalent to that in the \( \pi^0 \) and \( \eta \) peak regions. Distributions from these side bands were subtracted from the distributions within the \( \pi^0 \) and \( \eta \) mass ranges to obtain the respective signals. This technique is appropriate as long as the combinatorial background depends approximately linearly on \( M_{\gamma\gamma} \). The combinatorial background shape is not linear at low \( p_T \) (below \( \approx 2 \text{ GeV}/c \)), and a fitting procedure was used to evaluate this background. The \( \gamma\gamma \) mass distributions at low \( p_T \) were fit using Gaussians for signal, and second and third order polynomials for the background. The combinatorial background in the peak regions was determined from the resultant fit parameters, and subsequently subtracted from the total entries within the peak.

C. Trigger Corrections

Trigger corrections were evaluated on an event-by-event basis using the measured efficiencies of the trigger groups responsible for the formation of a given trigger. For example, the SINGLE LOCAL HI trigger corrections were based upon the efficiencies of the 32 groups of 16 in the triggering octant. These efficiencies were evaluated as functions of the \( p_T \) reconstructed within the trigger group, using data samples that were unbiased with respect to the trigger group. From these efficiencies, a trigger probability was defined, \( P = 1 - \prod (1 - p_i) \), where \( p_i \) is the efficiency of the \( i \)th trigger group in the octant. The inverse of this probability was applied as a trigger weight to each meson candidate. Meson candidates with trigger probabilities of \( P < 0.1 \) were excluded from further consideration to avoid excessively large trigger weights. The correction for losses from this requirement was determined from Monte Carlo, and absorbed into the reconstruction efficiency.

The cross sections presented in this paper utilize the results from a combination of triggers. Going from low to high \( p_T \), the triggers were: INTERACTION, PRETRIGGER HI, LOCAL:GLOBAL LO (1990 run), SINGLE LOCAL LO (1991-92 run), and SINGLE LOCAL HI. The transition points chosen between the lower and higher threshold triggers were determined by comparing the fully corrected results from each trigger, and were different for \( \pi^0 \) and \( \eta \) mesons, and also depended on rapidity. For \( p_T > 4.0 \text{ GeV}/c \), the SINGLE LOCAL HI trigger was used exclusively for both \( \pi^0 \) and \( \eta \) mesons.

D. Beam halo muon rejection

Spurious triggers were produced by muons in the beam halo that deposited energy in the electromagnetic calorimeter in random coincidence with an interaction in the target. Particularly in the outer regions of the EMLAC, such energy depositions can produce background at low \( \gamma\gamma \) mass values due to the occasional splitting of the muon-induced showers into two closely-separated photon candidates. The pretrigger logic used signals from the VW to reject events associated with such muons in the beam halo. The off-line analysis employed expanded requirements on the latched VW signals, and imposed requirements upon the direction of reconstructed showers, the shower shape, and the total \( p_T \) imbalance in the event to further reduce this background. The effects of these off-line rejection requirements on the \( \gamma\gamma \) invariant mass distribution are shown in Fig. 5, for \( \gamma\gamma \) pairs with \( 3 < p_T < 3.5 \text{ GeV}/c \) (left) and \( 7 < p_T < 10 \text{ GeV}/c \) (right). The rejection requirements completely eliminate the large muon-induced background in the high-\( p_T \) bin while having very little effect on the signal. A more detailed description of these requirements can be found in Ref. [15].

E. Detector simulation

The Meson West spectrometer was modeled via a detailed GEANT simulation. A preprocessor was used to convert GEANT information into the hits and strip energies measured by the various detectors. The preprocessor simulated hardware effects, such as channel noise and gain variations. Monte Carlo generated events were then processed through the same reconstruction software used to analyze the data. This accounted for inefficiencies and biases in the reconstruction algorithms. Reconstruction inefficiencies for \( \pi^0 \) and \( \eta \) mesons were relatively small over most of the kinematic range. More information on
the detailed simulation of the Meson West spectrometer can be found in Ref. [15]. We employed single particle distributions, reconstructed data, and the HERWIG [31] physics generator as inputs to the GEANT simulations. HERWIG calculations of \( \pi^0 \) and \( \eta \) spectra were weighted in \( p_T \) and rapidity using our measured results in an iterative fashion so that the final corrections were based on the data distributions rather than on the output of the physics generator.

Spectral effects were particularly important to the calibration of the EMLAC’s energy response [20]. The calibration of the energy response was based on the reconstructed masses of \( \pi^0 \) mesons in the \( \gamma \gamma \) decay mode. The steeply falling \( \pi^0 \) \( p_T \) spectrum, combined with the calorimeter’s resolution, results in a small offset (\( \approx 1\% \)) in the mean reconstructed photon energies. We accounted for this offset, and for potential biases in the calibration procedure, by calibrating the simulated EMLAC in the same manner as the real detector. We also employed the simulation to evaluate the mean correction (as a function of photon energy) for energy deposited in the material upstream of the EMLAC. The impacts of detector resolution on the energy scale calibration and on the \( \pi^0 \) and \( \eta \) production spectra were incorporated in the overall reconstruction efficiency corrections.

Figure 6 compares the \( \gamma \gamma \) mass spectra in the \( \pi^0 \) and \( \eta \) mass regions to the simulation for two different minimum \( p_T \) cutoffs. In addition to providing evidence that the Monte Carlo simulated the EMLAC resolution well, the agreement between the levels of combinatorial background also indicates that the Monte Carlo provided a reasonable simulation of the underlying event structure. Figure 7 shows a comparison between the Monte Carlo simulation and the data for the background subtracted \( \gamma \gamma \) energy asymmetry in the \( \pi^0 \) signal region, for two \( p_T \) intervals. This figure illustrates that the simulation accurately describes the losses of very low-energy photons.

**F. Normalization**

Electronic scalers that counted signals from the beam hodoscope, interaction counters, and beam hole counters were used to evaluate the number of beam particles incident on the target. Other scalers logged the state of the trigger and of components of the data acquisition system. Information from these scalers was used to determine the number of beam particles that traversed the spectrometer when it was ready to record data. This number was corrected for multiple occupancy in the beam (\( \approx 3\% \)), the absorption of beam in the target material (\( \approx 6\% \) for the Be target and \( \approx 3\% \) for \( p \)), and for the \( \mu^- \) content of the beam which was measured to be \( \approx 0.5\% \) [28].

The normalization of the low \( p_T \) \( \pi^0 \) cross section was independently verified using events from the prescaled beam and interaction trigger samples. In these samples, the absolute normalization is obtained simply by counting the recorded events selected by these triggers. For these low \( p_T \) events, the normalization as determined via the scalers and via event counting techniques agreed to
An analysis of negative secondary beam production by 800 GeV/c primary protons indicates a small (≈1%) K\(^-\) component in the incident 515 GeV/c beam. The meson production cross sections were corrected for this K\(^-\) component under the assumption that meson production by K\(^-\) beam is half that of π\(^-\) beam [25, 32].

### G. Summary of systematic uncertainties

The principal contributions to the systematic uncertainty arose from the following sources: calibration of photon energy response, π\(^0\) and η reconstruction efficiency and detector-resolution unsmearing, and overall normalization. The relative systematic uncertainty for π\(^0\) production is shown as a function of \(p_T\) in Fig. 8. Included in the figure are curves showing the contributions from the major sources of systematic uncertainty. Other sources of uncertainty which contribute at the 1–2% level include: background subtraction, incident beam contamination, beam halo muon rejection, geometric acceptance, photon conversions, trigger response, and vertex finding. The total systematic uncertainty is calculated by combining in quadrature all the individual uncertainties. Other sources of uncertainty which contribute at the 1–2% level include: background subtraction, incident beam contamination, beam halo muon rejection, geometric acceptance, photon conversions, trigger response, and vertex finding.

### IV. RESULTS AND DISCUSSION

#### A. π\(^0\) production

The inclusive π\(^0\) cross sections per nucleon versus \(p_T\) are shown in Fig. 10 for 515 GeV/c π\(^-\) beams incident upon beryllium and liquid hydrogen targets. Note that the cross section is in units of pb/(GeV/c)^2 per nucleon for the Be target, and in nb/(GeV/c)^2 for the p target. The measurements on beryllium were obtained using the combined statistics from the 1990 and 1991-92 runs. Because of the steeply falling spectra, the data are plotted at abscissa values that correspond to the average values.
of the cross section in each $p_T$ bin, assuming local exponential $p_T$ dependence [33]. The cross sections are also tabulated in Tables I through IV.

In Fig. 11, the measured inclusive $\pi^0$ cross sections are compared to NLO PQCD results [3] using GRV [34] parton distributions, KKP [35] fragmentation functions, and factorization scales of $\mu = p_T/2, p_T$, and $2p_T$ (the renormalization and fragmentation scales have been set equal to the value of the factorization scale). The PQCD calculations for the Be target were adjusted to account for nuclear effects using the Hijing Monte Carlo calculation [36]. The large scale dependence in the calculations is insufficient to raise the predicted cross sections up to the values indicated by the data.

These discrepancies have been interpreted [4, 5] as arising from additional soft-gluon emission in the initial state that is not included in the NLO calculation, and which results in sizeable parton $k_T$ before the hard collision (for a different perspective, see the discussion in Ref. [8]). Soft-gluon (or $k_T$) effects are expected in all hard-scattering processes, such as the inclusive production of jets, high-$p_T$ mesons, and direct photons [37–40]. The Collins-Soper-Sterman resummation formalism [41] provides a rigorous basis for understanding these radiative effects, and there have been several recent efforts to derive resummation descriptions for the inclusive direct-photon [42–46], jet [47], and dijet cross sections [48–50]. The calculation of Ref. [42] for inclusive direct-photon production, which includes the effects of soft-gluon resummation near the kinematic threshold limit ($x_T = 2p_T/\sqrt{s} \longrightarrow 1$), has a far smaller sensitivity to scale, compared to NLO calculations, and provides cross sections close to those of NLO calculations with a scale of $\mu = p_T/2$. Also, for our energies, the calculations of Ref. [45, 46], which simultaneously treat threshold and recoil effects in direct-photon production, yield a substantially larger cross section than the NLO result. However, no such calculations are available for inclusive meson production. In their absence, we use a PQCD-based model that incorporates transverse kinematics of initial-state partons to study the consequences of additional $k_T$ for high-$p_T$ production processes.

Because the unmodified PQCD cross sections fall rapidly with increasing $p_T$, the net effect of the “$k_T$ smearing” is to increase the expected yield. Modified parton kinematics have been implemented in a Monte Carlo calculation of the leading-order (LO) cross sections for high-$p_T$ particle production [51], with the $k_T$ distribution for each of the incoming partons represented by a Gaussian with one adjustable parameter ($\langle k_T \rangle$). Unfortunately, no such program is available for NLO calculations, and so we approximate the effect of $k_T$ smearing by multiplying the NLO cross sections by the corresponding LO $k_T$-enhancement factors. Admittedly, this procedure involves a risk of double-counting since some of the $k_T$-enhancement may already be contained in the NLO calculation. However, we expect such double-counting effects to be small.

In the calculation of the LO $k_T$-enhancement factors we employ $\langle k_T \rangle$ values representative of those found from comparisons of kinematic distributions in data involv-
ing production of high-mass $\gamma \gamma$, $\gamma \pi^0$, and $\pi^0 \pi^0$ systems with calculations relying on the same LO program (see Refs. [4, 5] for further details). For these comparisons, we used the LO versions of the GRV parton distributions and (where appropriate) KKP fragmentation functions, and an average transverse momentum of 0.6 GeV/c for the $\pi^0$ mesons relative to the fragmenting parton direction (varying this parameter in the range 0.3–0.7 GeV/c does not affect our conclusions) [52, 53].

Comparisons of the $k_T$-enhanced calculations with data at 515 GeV/c are displayed in Fig. 12, indicating good agreement for the chosen $\langle k_T \rangle$ value. Figure 13 shows the $\pi^- Be$ cross sections at 515 GeV/c versus rapidity, for several intervals in $p_T$. The shapes and normalizations of the data are in good agreement with the $k_T$-enhanced calculations.

In a previous publication [15], this experiment reported results for $\pi^0$ production in $pp$ and $pBe$ interactions at 800 and 530 GeV/c. This offers the opportunity to compare production cross sections by incident $\pi^-$ and $p$ beams. In Fig. 14, the ratio of invariant cross sections for $\pi^0$ production by 515 GeV/c $\pi^-$ and 800 GeV/c $p$ beam is shown as a function of $x_T$ (comparing results at approximately the same incident momentum per valence quark). Both theoretical and experimental uncertainties are reduced in the ratio allowing, in principle, a more sensitive test of the calculations. In the figure, the data are compared to conventional ($\langle k_T \rangle=0$) and $k_T$-enhanced NLO results using KKP fragmentation functions.
$k_T$-enhanced theory accommodates the data better than the conventional theory.

B. \( \eta \) production

Cross sections for inclusive \( \eta \) production are tabulated in Tables V through VIII. Theoretical descriptions of \( \eta \)-meson production differ from the \( \pi^0 \) case primarily because of differences in the fragmentation of partons into the particles of interest. To investigate this effect, we present \( \eta/\pi^0 \) relative production rates as functions of \( p_T \) and \( y_{cm} \) (for two \( p_T \) ranges) in Fig. 15. We see no significant dependence in this ratio — the data average to a value of \( 0.48 \pm 0.01 \) (statistical).

V. SUMMARY

The invariant cross sections for \( \pi^0 \) and \( \eta \) production have been measured for \( \pi^-p \) and \( \pi^-Be \) collisions at 515 GeV/c as functions of \( p_T \) and \( y_{cm} \), over the kinematic range \( 1 < p_T < 11 \text{ GeV/c} \), and \( -0.75 \leq y_{cm} \leq 0.75 \). The \( \pi^0 \) cross sections agree with \( k_T \)-enhanced NLO QCD calculations. The measured \( \eta/\pi^0 \) production ratio, which provides information about the relative fragmentation of partons into these mesons, is \( 0.48 \pm 0.01 \) with little dependence on \( p_T \) or rapidity.

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### APPENDIX: TABULATED CROSS SECTIONS

#### TABLE I: Invariant differential cross section \((Edσ/d^3p)\) per nucleon for the inclusive reaction \(\pi^- Be \rightarrow \pi^0 X\) at 515 GeV/c, averaged over the rapidity interval \(-0.75 \leq y_{cm} < 0.75\).

| \(p_T\) (GeV/c) | \(\pi^- Be \rightarrow 515 GeV/c\) (nb/(GeV/c)^2) |
|----------------|-----------------------------------------------|
| 1.00 – 1.20    | 255 ± 35                                      |
| 1.20 – 1.40    | 97 ± 14                                       |
| 1.40 – 1.60    | 39.5 ± 5.9                                    |
| 1.60 – 1.80    | 19.8 ± 2.5                                    |
| 1.80 – 2.00    | 9.5 ± 1.2                                     |
| 2.00 – 2.20    | 3.64 ± 0.36 ± 0.43                            |
| 2.20 – 2.30    | 2.11 ± 0.053 ± 0.25                           |
| 2.30 – 2.40    | 1.46 ± 0.038 ± 0.17                           |
| 2.40 – 2.50    | 955 ± 27 ± 111                                |
| 2.50 – 2.60    | 673 ± 17 ± 78                                 |
| 2.60 – 2.70    | 496 ± 14 ± 57                                 |
| 2.70 – 2.80    | 315.4 ± 9.6 ± 36.1                            |
| 2.80 – 2.90    | 243.8 ± 7.2 ± 27.8                            |
| 2.90 – 3.00    | 170.2 ± 4.3 ± 19.3                            |
| 3.00 – 3.10    | 127.5 ± 3.3 ± 14.4                            |
| 3.10 – 3.20    | 90.7 ± 2.6 ± 10.2                             |
| 3.20 – 3.30    | 72.1 ± 1.9 ± 8.1                              |
| 3.30 – 3.40    | 48.3 ± 1.3 ± 5.4                              |
| 3.40 – 3.50    | 35.38 ± 1.00 ± 3.9                            |
| 3.50 – 3.60    | 26.53 ± 0.65 ± 2.9                            |
| 3.60 – 3.70    | 19.73 ± 0.50 ± 2.2                            |
| 3.70 – 3.80    | 14.88 ± 0.36 ± 1.6                            |
| 3.80 – 3.90    | 11.15 ± 0.30 ± 1.2                            |
| 3.90 – 4.00    | 8.28 ± 0.11 ± 0.91                            |
| 4.00 – 4.10    | 6.220 ± 0.039 ± 0.69                          |
| 4.10 – 4.20    | 4.770 ± 0.034 ± 0.53                          |
| 4.20 – 4.30    | 3.604 ± 0.027 ± 0.40                          |
| 4.30 – 4.40    | 2.825 ± 0.022 ± 0.31                          |
| 4.40 – 4.50    | 2.166 ± 0.019 ± 0.24                          |
| 4.50 – 4.60    | 1.654 ± 0.015 ± 0.18                          |
| 4.60 – 4.70    | 1.277 ± 0.013 ± 0.14                          |
| 4.70 – 4.80    | 990 ± 11 ± 108                                |
| 4.80 – 4.90    | 756.4 ± 9.5 ± 83.0                            |
| 4.90 – 5.00    | 605.8 ± 8.5 ± 66.6                            |
| 5.00 – 5.10    | 475.9 ± 7.2 ± 52.4                            |
| 5.10 – 5.20    | 366.0 ± 6.0 ± 40.3                            |
| 5.20 – 5.30    | 298.1 ± 5.4 ± 32.9                            |
| 5.30 – 5.40    | 237.8 ± 4.8 ± 26.3                            |
| 5.40 – 5.50    | 191.4 ± 4.3 ± 21.2                            |
| 5.50 – 5.60    | 151.3 ± 3.7 ± 16.8                            |
| 5.60 – 5.70    | 116.5 ± 3.2 ± 13.0                            |
| 5.70 – 5.80    | 91.0 ± 2.8 ± 10.2                             |
| 5.80 – 5.90    | 77.2 ± 2.6 ± 8.6                              |
| 5.90 – 6.00    | 57.3 ± 2.2 ± 6.4                              |
| 6.00 – 6.25    | 41.3 ± 1.2 ± 4.7                              |
| 6.25 – 6.50    | 25.37 ± 0.90 ± 2.9                            |
| 6.50 – 6.75    | 15.27 ± 0.67 ± 1.7                            |
| 6.75 – 7.00    | 8.71 ± 0.50 ± 1.0                             |
| 7.00 – 7.50    | 4.48 ± 0.25 ± 0.52                            |
| 7.50 – 8.00    | 1.30 ± 0.13 ± 0.16                            |
| 8.00 – 9.00    | 0.366 ± 0.049 ± 0.045                         |
| 9.00 – 10.00   | 0.044 ± 0.022 ± 0.006                         |
| 10.00 – 12.00  | 0.0031 ± 0.0031 ± 0.0004                      |

#### TABLE II: Invariant differential cross section \((Edσ/d^3p)\) for the inclusive reaction \(\pi^- p \rightarrow \pi^0 X\) at 515 GeV/c, averaged over the rapidity interval \(-0.75 \leq y_{cm} < 0.75\).

| \(p_T\) (GeV/c) | \(\pi^- p \rightarrow 515 GeV/c\) (nb/(GeV/c)^2) |
|----------------|-----------------------------------------------|
| 1.00 – 1.50    | 161 ± 57                                      |
| 1.50 – 2.50    | 5.2 ± 4.1                                     |
| 2.50 – 2.75    | 455 ± 34 ± 52                                 |
| 2.75 – 3.00    | 170 ± 14 ± 19                                 |
| 3.00 – 3.25    | 96.3 ± 8.3 ± 10.9                             |
| 3.25 – 3.50    | 34.3 ± 3.7 ± 3.8                              |
| 3.50 – 3.75    | 18.5 ± 2.6 ± 2.1                              |
| 3.75 – 4.00    | 9.8 ± 1.4 ± 1.1                               |
| 4.00 – 4.25    | 4.67 ± 0.12 ± 0.51                            |
| 4.25 – 4.50    | 2.586 ± 0.085 ± 0.28                         |
| 4.50 – 4.75    | 1.303 ± 0.048 ± 0.14                         |
TABLE III: The averaged invariant differential cross section \((E\sigma/d^2p)\) per nucleon as a function of rapidity and \(p_T\) for the inclusive reaction \(\pi^-\text{Be} \rightarrow \pi^0\text{X}\) at 515 GeV/c.

| \(y_{cm}\) | \(1.00 - 1.50\) | \(1.50 - 2.00\) | \(2.00 - 2.50\) | \(2.50 - 3.00\) |
|-----------|----------------|----------------|----------------|----------------|
|           | \((\mu b/(GeV/c))^2\) | \((\mu b/(GeV/c))^2\) | \((\mu b/(GeV/c))^2\) | \((\mu b/(GeV/c))^2\) |
| \(\pm 0.075\) | \(\pm 0.0625\) | \(\pm 0.050\) | \(\pm 0.0375\) | \(\pm 0.025\) |
| 0.000 - 0.125 | 60.8 ± 1.7 | 2.198 ± 0.071 | 0.467 ± 0.018 | 1.262 ± 0.018 |
| 0.0125 - 0.250 | 86.6 ± 3.9 | 18.56 ± 0.53 | 4.562 ± 0.40 | 1.229 ± 0.18 |
| 0.0250 - 0.500 | 84.6 ± 6.0 | 18.58 ± 0.60 | 4.526 ± 0.50 | 1.218 ± 0.17 |
| 0.0500 - 0.125 | 71.1 ± 2.9 | 17.31 ± 0.65 | 4.067 ± 0.38 | 1.177 ± 0.17 |
| 0.000 - 0.125 | 61.0 ± 5.8 | 11.59 ± 0.95 | 2.766 ± 0.57 | 0.770 ± 0.16 |
| 0.0125 - 0.250 | 73.3 ± 8.2 | 14.27 ± 0.46 | 3.309 ± 0.44 | 0.863 ± 0.16 |
| 0.0250 - 0.500 | 68.6 ± 3.3 | 14.96 ± 0.57 | 3.668 ± 0.41 | 0.955 ± 0.17 |
| 0.0500 - 0.125 | 77.1 ± 2.8 | 17.31 ± 0.65 | 4.067 ± 0.38 | 1.177 ± 0.17 |
| 0.000 - 0.125 | 60.8 ± 1.7 | 2.198 ± 0.071 | 0.467 ± 0.018 | 1.262 ± 0.018 |
| 0.0125 - 0.250 | 86.6 ± 3.9 | 18.56 ± 0.53 | 4.562 ± 0.40 | 1.229 ± 0.18 |
| 0.0250 - 0.500 | 84.6 ± 6.0 | 18.58 ± 0.60 | 4.526 ± 0.50 | 1.218 ± 0.17 |
| 0.0500 - 0.125 | 71.1 ± 2.9 | 17.31 ± 0.65 | 4.067 ± 0.38 | 1.177 ± 0.17 |
TABLE IV: The averaged invarient differential cross section \((Ed\sigma/d^3p)\) as a function of rapidity and \(p_T\) for the inclusive reaction \(\pi^- p \rightarrow \pi^0X\) at 515 GeV/c.

| \(y_{cm}\)       | \(2.50 - 3.00\)          | \(3.00 - 3.50\)          | \(3.50 - 4.00\)          | \(4.00 - 4.50\)          |
|-----------------|--------------------------|--------------------------|--------------------------|--------------------------|
|                 | (nb/(GeV/c)^2)           | (nb/(GeV/c)^2)           | (nb/(GeV/c)^2)           | (nb/(GeV/c)^2)           |
| 9.00 – 10.00    | 1.95 ± 0.29 ± 0.21       | 2.92 ± 0.34 ± 0.32       | 3.55 ± 0.34 ± 0.39       | 3.79 ± 0.28 ± 0.42       |
| 7.00 – 8.00     | 1.21 ± 0.20 ± 0.22       | 2.44 ± 0.29 ± 0.39       | 3.56 ± 0.29 ± 0.39       | 4.03 ± 0.20 ± 0.44       |
| 5.00 – 5.25     | 4.42 ± 0.20 ± 0.22       | 6.39 ± 0.34 ± 0.39       | 7.39 ± 0.28 ± 0.42       | 4.19 ± 0.20 ± 0.46       |
| 4.20 – 4.40     | 2.44 ± 0.099 ± 0.099     | 3.90 ± 0.13 ± 0.13       | 4.74 ± 0.25 ± 0.30       | 3.96 ± 0.20 ± 0.44       |
| 3.80 – 4.00     | 8.10 ± 0.11 ± 0.11       | 12.4 ± 0.16 ± 0.16       | 16.6 ± 0.45 ± 0.51       | 3.86 ± 0.25 ± 0.42       |
| 4.00 – 4.20     | 2.44 ± 0.099 ± 0.099     | 3.90 ± 0.13 ± 0.13       | 4.74 ± 0.25 ± 0.30       | 3.96 ± 0.20 ± 0.44       |
| 3.50 – 3.75     | 2.44 ± 0.099 ± 0.099     | 3.90 ± 0.13 ± 0.13       | 4.74 ± 0.25 ± 0.30       | 3.96 ± 0.20 ± 0.44       |
| 3.00 – 3.25     | 2.44 ± 0.099 ± 0.099     | 3.90 ± 0.13 ± 0.13       | 4.74 ± 0.25 ± 0.30       | 3.96 ± 0.20 ± 0.44       |
| 2.50 – 2.75     | 2.44 ± 0.099 ± 0.099     | 3.90 ± 0.13 ± 0.13       | 4.74 ± 0.25 ± 0.30       | 3.96 ± 0.20 ± 0.44       |
| 2.00 – 2.25     | 2.44 ± 0.099 ± 0.099     | 3.90 ± 0.13 ± 0.13       | 4.74 ± 0.25 ± 0.30       | 3.96 ± 0.20 ± 0.44       |
| 1.50 – 1.75     | 2.44 ± 0.099 ± 0.099     | 3.90 ± 0.13 ± 0.13       | 4.74 ± 0.25 ± 0.30       | 3.96 ± 0.20 ± 0.44       |
| 1.00 – 1.25     | 2.44 ± 0.099 ± 0.099     | 3.90 ± 0.13 ± 0.13       | 4.74 ± 0.25 ± 0.30       | 3.96 ± 0.20 ± 0.44       |
| 0.50 – 0.75     | 2.44 ± 0.099 ± 0.099     | 3.90 ± 0.13 ± 0.13       | 4.74 ± 0.25 ± 0.30       | 3.96 ± 0.20 ± 0.44       |

TABLE V: Invariant differential cross section \((Ed\sigma/d^3p)\) per nucleon for the inclusive reaction \(\pi^- Be \rightarrow \eta X\) at 515 GeV/c, averaged over the rapidity interval \(-0.75 \leq y_{cm} \leq 0.75\).

| \(p_T\) (GeV/c) | \(\pi^- Be \rightarrow \eta X\) (nb/(GeV/c)^2) |
|-----------------|---------------------------------------------|
| 3.00 – 3.20     | 9.46 ± 0.73 ± 0.69                        |
| 3.20 – 3.40     | 23.8 ± 3.9 ± 3.0                         |
| 3.40 – 3.60     | 16.9 ± 2.1 ± 2.1                         |
| 3.60 – 3.80     | 7.91 ± 0.97 ± 0.94                       |
| 3.80 – 4.00     | 4.42 ± 0.20 ± 0.52                       |
| 4.00 – 4.20     | 2.440 ± 0.099 ± 0.28                     |
| 4.20 – 4.40     | 1.506 ± 0.062 ± 0.17                     |
| 4.40 – 4.60     | 922 ± 41 ± 107                           |
| 4.60 – 4.80     | 571 ± 25 ± 66                            |
| 4.80 – 5.00     | 353 ± 16 ± 41                            |
| 5.00 – 5.25     | 192.9 ± 9.5 ± 22.4                       |
| 5.25 – 5.50     | 110.4 ± 6.3 ± 12.9                       |
| 5.50 – 5.75     | 69.3 ± 4.7 ± 8.1                         |
| 5.75 – 6.00     | 33.4 ± 3.0 ± 3.9                        |
| 6.00 – 6.50     | 20.1 ± 1.5 ± 2.4                        |
| 6.50 – 7.00     | 6.23 ± 0.73 ± 0.75                      |
| 7.00 – 8.00     | 1.21 ± 0.22 ± 0.15                       |
| 8.00 – 9.00     | 0.224 ± 0.083 ± 0.029                    |
| 9.00 – 10.00    | 0.035 ± 0.039 ± 0.005                    |

TABLE VI: Invariant differential cross section \((Ed\sigma/d^3p)\) for the inclusive reaction \(\pi^- p \rightarrow X\) at 515 GeV/c, averaged over the rapidity interval \(-0.75 \leq y_{cm} \leq 0.75\).

| \(p_T\) (GeV/c) | \(\pi^- p \rightarrow X\) (nb/(GeV/c)^2) |
|-----------------|---------------------------------------------|
| 3.00 – 3.50     | 69 ± 62 ± 9                               |
| 3.50 – 4.00     | 9.1 ± 3.8 ± 1.1                           |
| 4.00 – 4.50     | 1.76 ± 0.31 ± 0.20                       |
| 4.50 – 5.00     | 400 ± 110 ± 60                            |
| 5.00 – 5.50     | 121 ± 30 ± 14                             |
| 5.50 – 6.00     | 19 ± 17 ± 2                              |
| 6.00 – 7.00     | 7.8 ± 4.7 ± 0.9                          |
| 7.00 – 8.00     | 0.37 ± 0.97 ± 0.05                       |
| 8.00 – 9.00     |                                            |
| 9.00 – 10.00    |                                            |
TABLE VII: The averaged invariant differential cross section \((E\sigma/d^3p)\) per nucleon as a function of rapidity and \(p_T\) for the inclusive reaction \(\pi^-\text{Be} \to \eta\Xi\) at 515 GeV/c. Units are pb/(GeV/c)^2.

| \(y_{cm}\) | \(3.00 - 4.00\) | \(4.00 - 4.50\) | \(4.50 - 5.00\) |
|----------|-----------------|-----------------|-----------------|
| -0.750 - -0.625 | 14000 ± 6200 ± 1700 | 680 ± 260 ± 79 | 287 ± 88 ± 33 |
| -0.625 - -0.500 | 880 ± 220 ± 100 | 278 ± 61 ± 32 |
| -0.500 - -0.375 | 1290 ± 180 ± 150 | 393 ± 51 ± 39 |
| -0.375 - -0.250 | 1810 ± 170 ± 210 | 456 ± 45 ± 53 |
| -0.250 - -0.125 | 1710 ± 150 ± 200 | 589 ± 53 ± 67 |
| -0.125 - 0.000 | 2210 ± 140 ± 260 | 556 ± 47 ± 64 |
| 0.000 - 0.125 | 1750 ± 130 ± 200 | 578 ± 46 ± 67 |
| 0.125 - 0.250 | 2400 ± 140 ± 280 | 650 ± 55 ± 75 |
| 0.250 - 0.375 | 2360 ± 150 ± 270 | 683 ± 51 ± 79 |
| 0.375 - 0.500 | 2280 ± 140 ± 260 | 688 ± 51 ± 80 |
| 0.500 - 0.625 | 2030 ± 120 ± 240 | 650 ± 46 ± 75 |
| 0.625 - 0.750 | 2070 ± 120 ± 240 | 595 ± 43 ± 69 |

| \(y_{cm}\) | \(5.00 - 5.50\) | \(5.50 - 6.50\) | \(6.50 - 8.00\) |
|----------|-----------------|-----------------|-----------------|
| -0.750 - -0.625 | 37 ± 15 ± 4.3 | 6.8 ± 4.6 ± 0.8 | 0.48 ± 0.48 ± 0.06 |
| -0.625 - -0.500 | 79 ± 22 ± 9.1 | 22.9 ± 4.8 ± 2.7 |
| -0.500 - -0.375 | 141 ± 22 ± 16 | 18.5 ± 4.3 ± 2.2 | 1.31 ± 0.48 ± 0.16 |
| -0.375 - -0.250 | 125 ± 17 ± 15 | 32.2 ± 5.6 ± 3.8 |
| -0.250 - -0.125 | 157 ± 17 ± 18 | 36.5 ± 4.5 ± 4.3 |
| -0.125 - 0.000 | 143 ± 19 ± 17 | 42.5 ± 5.5 ± 5.0 | 3.63 ± 0.72 ± 0.44 |
| 0.000 - 0.125 | 137 ± 19 ± 16 | 33.4 ± 5.2 ± 3.9 |
| 0.125 - 0.250 | 197 ± 22 ± 23 | 45.6 ± 5.5 ± 5.4 | 3.57 ± 0.78 ± 0.43 |
| 0.250 - 0.375 | 225 ± 21 ± 26 | 48.0 ± 5.4 ± 5.6 |
| 0.375 - 0.500 | 218 ± 22 ± 25 | 46.0 ± 6.4 ± 5.4 | 4.36 ± 0.85 ± 0.53 |
| 0.500 - 0.625 | 184 ± 20 ± 21 | 41.5 ± 5.8 ± 4.9 | 3.86 ± 0.77 ± 0.47 |
| 0.625 - 0.750 | 173 ± 18 ± 20 | 35.9 ± 5.1 ± 4.2 |

TABLE VIII: The averaged invariant differential cross section \((E\sigma/d^3p)\) as a function of rapidity and \(p_T\) for the inclusive reaction \(\pi^-p \to \eta\Xi\) at 515 GeV/c. Units are pb/(GeV/c)^2.

| \(y_{cm}\) | \(4.00 - 5.00\) | \(5.00 - 6.00\) |
|----------|-----------------|-----------------|
| -0.750 - -0.625 | 550 ± 550 ± 64 |
| -0.625 - -0.500 | 950 ± 490 ± 110 | 74 ± 42 ± 8.6 |
| -0.500 - -0.250 | 1170 ± 360 ± 140 | 39 ± 38 ± 4.5 |
| 0.000 - 0.250 | 1840 ± 430 ± 210 | 147 ± 47 ± 17 |
| 0.250 - 0.500 | 1020 ± 270 ± 120 | 153 ± 50 ± 18 |
| 0.500 - 0.750 | 960 ± 220 ± 110 | 44 ± 43 ± 5.1 |