Anomalous Josephson current through a ferromagnetic trilayer junction

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We studied the anomalous Josephson current appearing at zero phase difference in junctions coupled with a ferromagnetic trilayer which has noncoplanar magnetizations. A $\pi/2$ junction with an equilibrium phase difference $\pi/2$ is obtained under suitable conditions. The equilibrium phase difference and the amplitude of the supercurrent are all tunable by the structure parameters. In addition to calculating the anomalous current using the Bogoliubov-de Gennes equation, we also developed a clear physical picture explaining the anomalous Josephson effect in the structure. We show that the triplet proximity correlation and the phase shift in the anomalous current-phase relation all stem from the spin precession in the first and third ferromagnet layers.

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I. INTRODUCTION

Usually the supercurrent in a Josephson junction vanishes, when the phase difference between the two superconductors is zero, and in the tunneling limit the current-phase relation (CPR) is sinusoidal $I(\phi) = I_c \sin(\phi)$. Recently some studies\textsuperscript{2–9} found an anomalous Josephson current flow $I_a$ exists even at zero phase difference ($\phi = 0$). The anomalous supercurrent is equivalent to the presence of an additional phase shift $\phi_0$ in the conventional CPR. i.e., $I(\phi) = I_c \sin(\phi + \phi_0)$. In fact, such CPRs have been predicted for Josephson junctions of unconventional superconductors\textsuperscript{10–14} but the experimental verification is still lacking. Recent studies have shown that the anomalous supercurrent can also exist in junctions with conventional s-wave BCS superconductors if both spin-orbit interaction (SOI) and a suitably oriented Zeeman field are present in the coupling layer.\textsuperscript{2–9} These studies revealed that the anomalous effect in conventional junctions has some intricate physics. More interesting, an anomalous Josephson current can also appear in superconductor (S)-ferromagnet(F) hybrid structure without SOI.\textsuperscript{25} In Grein’s study,\textsuperscript{25} a SFS hybrid structure with two spin-active interfaces was considered. The two spin-active interfaces are critical to the triplet proximity effect and the anomalous supercurrent in the structure, but the physics is still unclear.

In this study, we generalize the two spin-active interfaces to two ferromagnetic layers with finite thicknesses and clarify the physical mechanisms responsible for the anomalous supercurrent. In such SFFFS structures, we find that the triplet proximity correlation and the phase shift in the anomalous CPR all stem from the spin precession in the first and third F layers. According to the symmetry analysis,\textsuperscript{25} an anomalous supercurrent is possible when the symmetries of the time-reversal operator $T$ and its combination with a spin rotation operator with respect to an arbitrary spin quantum axis $\sigma_n T$ are broken at the same time. As a result, the simplest superconductor (S)-ferromagnet (F)-superconductor (S) junction for achieving an anomalous Josephson current requires the F layer to be a ferromagnetic trilayer with noncoplanar magnetizations for breaking the symmetry of the operator $\sigma_n T$. SFFFS junctions where the magnetizations of the three ferromagnetic layers need not be noncoplanar\textsuperscript{15} and SFS junctions with inhomogeneous magnetization\textsuperscript{17,18}, have been studied in order to understand the effects of triplet correlation induced in the F layers. Controllable 0-$\pi$ transition and spin-triplet supercurrents have been realized experimentally recently\textsuperscript{21,22}. In our study, we found that triplet correlation is also an important condition for the anomalous supercurrent\textsuperscript{23–27}.

We consider a junction consisting of two conventional s-wave superconductors coupled by a ferromagnetic trilayer with noncoplanar magnetizations. For convenience, hereafter we denote the three F layers sequentially by $F_1$, $F_2$, $F_3$. We start with the typical situation where the magnetizations are along the $x$, $y$, $z$ axes respectively (i.e. an $SF_x F_y F_z S$ junction), as shown in the upper panel of Fig. 1. This junction is a $\pi/2$ junction with an equilibrium phase difference $\pi/2$ under suitable conditions. The equilibrium phase difference can be tuned by the lengths, the exchange energies, and the magnetization orientations of the $F_1$ and $F_3$ layers. And the amplitude of the supercurrent can be tuned by the barriers between the F layers or by the length and the exchange energy of the middle $F_2$ layer. In this regime the Josephson junction can also act as a supercurrent rectifier\textsuperscript{28,29}.

The paper is organized as follows. In Sec. II we present the model and solve the scattering problem for quasiparticles based on the Bogoliubov-de Gennes equation. The Josephson current and Andreev bound states can be obtained from the scattering matrices. In Sec. III we show the numerical results for the anomalous supercurrent and corresponding Andreev bound states and reveal the physics. A conclusion and remarks will be given in Sec. IV.

II. MODEL AND FORMALISM

In the numerical calculation, we consider $SF_1 F_2 F_3 S$ junctions with various lengths and exchange energies for
and $h$ represents the two barriers between the F layers, and $\Delta \theta + \phi$ describes the pair potential of the system can be given by the combination of all these scattering matrices of interfaces. From the total scattering matrix, we can obtain the Andreev reflection amplitudes $a_{1\sigma}$ and $a_{2\sigma}$ of the junction where $a_{1\sigma}$ is for the reflection from an electron-like to a hole-like quasiparticle and $a_{2\sigma}$ is for the reverse process with $\sigma$ representing the spin. The stationary Josephson current can be expressed in terms of the Andreev reflection amplitudes by using the temperature Green function formalism\textsuperscript{33}

$$I_c(\varphi) = \frac{e\Delta}{4h} \sum_{\omega_n, \sigma} \frac{k_B T}{\Omega_n} \left( k^+ - k^- \right) \left( \frac{a_{1\sigma n} - a_{2\sigma n}}{k^+ - k^-} \right),$$

where $k^+ = k^+_{n, \sigma}$, $k^-_n$, and $a_{2\sigma n}$ are obtained from $k^+_{n, \sigma}$, $k_-^{\ast}$, $a_{1\sigma}$, and $a_{2\sigma}$ by analytic continuation $E \rightarrow i\omega_n$. $k^\pm$ is the wave vector for electron or hole in the superconductors and the Matsubara frequencies are $\omega_n = \pi k_B T (2n + 1)$, $n = 0, \pm 1, \pm 2, \ldots$, and $\Omega_n = \sqrt{\omega_n^2 + \Delta^2}$.

The discrete spectrum of the Andreev bound states can be determined by using the condition\textsuperscript{34}

$$\text{det}[1 - R_2 P R_1 P] = 0$$

where $R_1, R_2, P$ are 4 × 4 matrices, $P$ is the propagation matrix of modes in the $F_2$ layer, and $R_1 (R_2)$ is the reflection matrix of the right-going (left-going) incident waves.

In order to study the spin properties of the Andreev bound states formed at $F_2$ layer, we can also work out the Green’s function $G(x, x', E)$ in $F_2$ layer which is a 4 × 4 matrix\textsuperscript{35}. Now it is convenient to take the eigen spinors of $F_2$ layer, i.e., spin-parallel and spin-antiparallel with respect to the exchange field $h_2$ as the unit vectors of the spin space. Then the spin current in $F_2$ layer can be evaluated by\textsuperscript{20}

$$I_s(\varphi) = \frac{\hbar^2 k_B T}{4mi} \lim_{x' \rightarrow x} \left( \frac{\partial}{\partial x'} - \frac{\partial}{\partial x} \right) \sum_{\omega_n} \left\{ \left( \begin{array}{cc} \sigma & 0 \\ 0 & \sigma \end{array} \right) G_{\omega_n}(x, x') \right\}$$

$$= \frac{\hbar}{2e}(I_+ - I_-),$$

where $I_+$ ($I_-$) is the charge currents of electrons with parallel spin (antiparallel spin) and obviously satisfies $I_e = I_+ + I_-$. 

### III. RESULTS AND DISCUSSION

We start with the typical noncoplanar magnetization configuration, i.e., the $SF_x F_y F_z S$ junction. For simplicity, we introduce the dimensionless units: the energy $E \rightarrow E_E_F$, the wave vector $k \rightarrow k k_F$, the coordinate $x \rightarrow x/k_F$, and the strength of exchange field $h \rightarrow h E_F$. All physical quantities are expressed in the dimensionless units in the rest of the paper. The superconductors considered are characterized with $\Delta = 10^{-3}$ which corresponds to the BCS coherence length at zero temperature $\xi_0 = 2/\pi \Delta \approx 636.6$. 

![FIG. 1: (Color online) Upper panel: Schematic diagram of the $SF_x F_y F_z S$ junction where two barriers are present between the F layers. Lower panel: Schematic illustration of the formation of Andreev bound states with triplet correlation in the $F_y$ layer due to spin precession of electrons and holes in the $F_x$ and $F_z$ layers.](image-url)
Fig. 2 shows the charge and spin currents $I_+, I_-, I_c, I_s$ as functions of the phase difference $\varphi$ for the $SF_xF_yF_zS$ junction. The corresponding Andreev bound states are shown in Fig. 3. It is interesting to note that when there is a barrier between the F layers, ($U_0 = 2$), there is a significant anomalous Josephson current. When there is no barrier $U_0 = 0$, the anomalous Josephson current is nearly zero. This interesting dependence on the barrier strength $U_0$ will be explained below in terms of the spin characteristics of the Andreev bound states in the $F_y$ layer.

Firstly, it is useful to point out a large spin current exists in the $F_y$ layer, implying that the superconductivity correlation is mainly triplet in the $F_y$ layer. This is easily understood by considering the formation of an Andreev bound state in the $F_y$ layer. A right-going electron with spin parallel to the $y$ axis $(1, i)^T$ from the $F_y$ layer will have its spin precessing about the $z$ axis in the $F_z$ layer before it reaches the right superconductor. After the Andreev reflection from the right superconductor, a hole with reverse spin goes left and its spin continues to precess. The one-way angle of precession is approximately $(k_+ - k_-)L_3 \approx h_3L_3$ where $k_+$ ($k_-$) is the wave-vector of up-spin (down-spin) quasi-particle. Thus if the condition $h_3L_3 = n\pi + \pi/2$ ($n$ is an integer) is satisfied, the reflected hole from the right superconductor will have its spin parallel to the incident electron's spin in the $F_y$ layer. An Andreev bound state is formed, after this reflected hole travels through the $F_y$ and $F_z$ layers and Andreev reflected from the left superconductor and changes to an electron to move right to finish a cycle. If the spin rotation angle in the $F_y$ layer satisfies the same condition $h_3L_1 = n\pi + \pi/2$. The electrons and holes have identical spins (parallel to the $y$ axis) in the $F_y$ layer and the Andreev bound state formed has complete triplet correlation in the $F_y$ layer, as schematically shown in the lower panel of Fig. 1. Triplet correlation can exist in other different type of magnetic inhomogeneity too. Bergeret et al. have studied S/F/S junctions with spiral magnetization in the F layer and found spin triplet correlation there. In the present model, we found two Andreev bound states below the Fermi level with complete triplet correlation; one is "spin-up" (with respect to the $y$ axis), which carries the current $I_+$, and one is "spin-down", which carries the current $I_-$, as shown in Fig. 2. In the short junction limit, the Josephson current is totally carried by the Andreev bound states.

Besides complete triplet correlation in the $F_y$ layer, another interesting feature noted in Fig. 2 is that $I_+$ has a phase shift of $\pi/2$ while $I_-$ has a phase shift of $-\pi/2$ compared with the conventional CPR. So, these two currents move in opposite directions. Now we follow the Andreev reflection processes occurring in the formation of the bound states to find out the phase shift. For simplicity, we assume $h_1 = h_2 = h_3 = h$; thus, the wave-vectors of "spin-up" ($+$) and "spin-down" ($-$) electrons (holes) with energy $E$ at each F layer are $k^{(h)}_\pm = \sqrt{k_F^2 + \rho \epsilon(h)} \pm h$ with $\rho \epsilon(h) = \pm(-1)$. In the short junction limit and the limit $E \ll h \ll E_F$, we have $k^h_\pm \approx k^h_\pm \approx \pm k_F \pm \frac{h}{\rho}$. We start with a right-going "spin-up" electron at the position $x = L_1 + 0$, the wave function can be written as $(1, i, 0, 0)^T$. The electron moves right and acquires a phase $e^{ik_L x}$ when it arrives...
at the interface \( x = L_1 + L_2 \). To simplify the discussion we focus on the Andreev reflections at the F/S interfaces and ignore the normal reflections at the barriers which affect only the amplitude of the supercurrent but not the phase shift. When the electron travels through the \( F_z \) layer, its spin precesses. The state becomes \( (e^{ik_x L_1}e^{ik_z L_2}i e^{ik_y L_3}, 0, 0)^T e^{ik_x L_1} \) when the electron arrives at the interface \( x = L_1 + L_2 + L_3 \). Then, the electron is reflected as a hole with reverse spin and the hole wave function is \( (0, 0, -ie^{ik_y L_3}e^{ik_z L_3})^T e^{ik_x L_2} \frac{e^{i\varphi}}{\sqrt{2}} \) where \( u = \sqrt{(1 + \Omega(E))/2}, v = \sqrt{(1 - \Omega(E))/2} \) with \( \Omega = \sqrt{E^2 - \Delta^2} \). The algebraic derivation is not shown here for space limitation and the approximation \( k^z_+ \approx k^z_h \approx k^z_F \) has been used in the derivation where \( k^z_+ \) (\( k^z_h \)) is the wave-vector of electronlike (holelike) quasiparticle in the superconductors. The Andreev-reflected hole moves left and has its spin rotated in the \( F_z \) layer again and then goes back to the \( F_y \) layer \( x = L_1 + L_2 - 0 \). Now the wave function becomes \( (0, 0, -ie^{ih L_3}e^{-ih L_3})^T e^{ik_x L_2} \frac{e^{i\varphi}}{\sqrt{2}} = (0, 0, 1, -i)^T e^{ik_x L_2} \frac{e^{i\varphi}}{\sqrt{2}} \) where the condition \( hL_3 = \pi/2 \) has been used. The wave function describes a "spin-up" hole with respect to the \( y \) direction. Then the hole goes left through the \( F_y \) layer and acquires a phase \( e^{-ik_x L_2} \). So the wave function becomes \( (0, 0, 1, -i)^T e^{ik_x L_2} \frac{e^{i\varphi}}{\sqrt{2}} \) when the hole arrives at the interface \( x = L_1 \). Consequently, the hole has its spin precessed in the \( F_z \) layer and moves left to the interface \( x = 0 \) with the wave function \( \frac{1}{\sqrt{2}} [(1 - i)(0, 0, 1, 1)^T e^{ik_x L_1} + (1 + i)(0, 0, 1, -1)^T e^{-ik_x L_1}] \frac{e^{i\varphi}}{\sqrt{2}} \). The hole is Andreev-reflected as an electron with reverse spin described by \( \frac{1}{\sqrt{2}} [(1 - i)(1, -1, 1, 0)^T e^{-ik_x L_1} - (1 + i)(1, 1, 0, 0)^T e^{-ik_x L_1}] \frac{e^{i\varphi}}{\sqrt{2}} \). Then the electron goes through the \( F_x \) layer again and back to the starting position \( x = L_1 + 0 \) to finish a cycle. The final wave function is \((1, 0, 0, 0)^T e^{i\varphi} \frac{e^{i\varphi}}{\sqrt{2}} \) where \( hL_1 = \pi/2 \) is used. Comparing with the initial wave function \((1, 1, 0, 0)^T \), we can see the phase shift of the "spin-up" Andreev bound state is indeed \( \pi/2 \) when considering a conventional CPR. In the same way, we can find out the phase shift of the "spin-down" Andreev bound state is \( -\pi/2 \). In this round-trip cycle of the quasi-particle, we can clearly see that the phase shifts in Andreev bound states come from the spin precession of electron and hole in the \( F_x \) and \( F_z \) layers.

If we neglect the second-harmonic term in the CPR, the charge current carried by the two Andreev bound states can be written as \( I_\pm \approx I_0^+ \sin(\varphi + \pi/2), I_- \approx I_0^- \sin(\varphi - \pi/2) \),

where \( I_0^+ \) (\( I_0^- \)) is the amplitude of the "spin-up" ("spin-down") charge current. When the barriers are absent, the normal scattering at the two F/F interfaces can be ignored and we can have \( I_0^+ \approx I_0^- \). As a result, the total charge current \( I_c = I_+ + I_- \) is very small and only the second-harmonic term remains, as shown in Fig. 2(a).

At zero phase difference, the charge current is very small and the spin current in the \( F_y \) layer is almost a pure spin current.

When the barriers are present, the normal scattering at the barriers reduces the amplitudes of the two charge currents \( I_0^+ \) and \( I_0^- \). The transmission probability through the double delta function barriers of electrons or holes depends on the wave-vector of the particle in the \( F_y \) layer and reaches the maximum when resonance transmission occurs. Here in the \( F_y \) layer, the "spin-up" Andreev bound state couples a "spin-up" electron with a "spin-up" hole which have the same wave-vector \( k_+ \approx k_F + \frac{h_2}{h_1} \) while the "spin-down" Andreev bound state couples a "spin-down" electron with a "spin-down" hole which have the same wave-vector \( k_- \approx k_F + \frac{h_2}{h_1} \). The difference in the wave vector between the two Andreev bound states leads to the difference in the transmissions through the \( F_z \) layer. Consequently, we can make a large difference between \( I_0^+ \) and \( I_0^- \) as shown in Fig. 2(b) by using two barriers as well as suitable exchange field strength and length of the \( F_y \) layer. In this way, an anomalous Josephson current appears at zero phase difference. The CPR of the junction has a phase shift of \( \pm\pi/2 \) in comparison with the conventional CPR where the sign of the phase shift depends on the relative magnitude of \( I_0^+ \) and \( I_0^- \).

Since the phase shift of the anomalous CPR stems from the spin precession of electrons and holes in the \( F_x \) and \( F_z \) layers, we can modulate the phase shift by tuning the parameters of these two layers. If the conditions \( \phi_1 = \phi_3 = 0 \) and \( h_1L_1 = h_3L_3 = (n + 1/2)\pi \) are satisfied, the complete equal-spin triplet correlation in the \( F_y \) layer is maintained. Now the phase shifts of the "spin-up" and "spin-down" Andreev bound states are \( \pm(\pi + \theta_3 - \theta_1) \) according to the above discussion. Fig. 4 shows the tuning of the equilibrium phase difference by varying \( \theta_3 \). Bergeret et al. \( \text{[12]} \) have found that the relative orientation

![FIG. 4: (Color online) \( I_c \) versus \( \varphi \) for the SF\(_F\), SF\(_F\), SF\(_S\) junction with different \( \theta_3 \): varying \( \theta_3 \) from 0 to \( \pi \) with a step \( \pi/4 \). \( \phi_3 = 0 \). The other parameters are chosen as: \( U_0 = 2, h_1 = h_2 = h_3 = 0.1, L_1 = L_2 = L_3 = 5\pi, T/T_c = 0.5 \).](image-url)
of the two magnetizations in S/F/I/F/S junctions can change the critical current. For the particular structure considered in Fig. 4, which is different from theirs, the orientation of the magnetization in the third layer has no strong effect on the critical current. However, for other values of $h_2$, the orientation can also modify the critical current. On the other hand, the amplitude or even the sign of the supercurrent can be changed by the barrier strength, the exchange field or the length of the $F_y$ layer, as shown in Fig. 5. The dependence of the supercurrent on the barrier strength is because of the condition of resonance transmission through double delta barriers $\sin(2kL_2) = -4U/(U^2+4)$ with $k$ the wave vector of particles. \Fig. 6 shows the anomalous supercurrent at zero phase difference $I_s(\varphi = 0)$ for the $SF_xF_yF_zS$ junction as functions of $h_2$ with $L_2 = 10\pi$, and (b) $L_2$ with $h_2 = 0.1$. The other parameters are chosen as: $h_1 = h_3 = 0.1$, $L_1 = L_3 = 5\pi$, $U_0 = 2$, $T/T_c = 0.5$.

LIBRARY PHASE DIFFERENCE IS TUNED BY THE LENGTH OF THE $F_x$ AND $F_z$ LAYERS. TO STUDY THE CHARACTERISTICS OF COOPER PAIRS IN THE $F_y$ LAYER IN DETAIL, THE PAIR FUNCTION CAN BE DEFINED BY THE ANOMALOUS GREEN FUNCTION AND BE DECOMPOSED INTO FOUR COMPONENTS.

$$\sum_{\omega_n>0} G_{\omega_n}^{eh}(x,x) = i \sum_{\nu=0}^{3} f_\nu(x)\sigma_\nu\sigma_2,$$

where $G_{\omega_n}^{eh}$ is the anomalous electron-hole correlation function, $\sigma_0$ is the unit matrix and $\sigma_\nu(\nu = 1,2,3)$ are three Pauli matrices. In Eq. (6), the frequency summation is only made over positive frequencies because the triplet pair functions are odd functions of frequency. $f_0$ $(f_3)$ is the pairing function of spin-singlet (spin-triplet) pairs with spin structure of $[\uparrow\downarrow] - (+) [\downarrow\uparrow]$ / $\sqrt{2}$. The pairing functions of $[\uparrow\uparrow]$ and $[\downarrow\downarrow]$ pairs are given by $f_{\uparrow\uparrow} = if_2 - f_1$ and $f_{\downarrow\downarrow} = if_2 + f_1$, respectively. Fig. 8 (a) shows the absolute values of pairing functions at

\FIG. 5: (Color online) $I_s$ versus $\varphi$ for the $SF_xF_yF_zS$ junction with different $U_0$ and $h_2$: (a) varying $U_0$ from $-2$ to $2$ with a step $1$, $h_2 = 0.1$, $L_2 = 5\pi$; (b) varying $h_2$ from $0.05$ to $0.15$ with a step $0.02$, $L_2 = 10\pi$, $U_0 = 2$. The other parameters are chosen as: $h_1 = h_3 = 0.1$, $L_1 = L_3 = 5\pi$, $T/T_c = 0.5$.

\FIG. 6: The anomalous supercurrent at zero phase difference $I_s(\varphi = 0)$ for the $SF_xF_yF_zS$ junction as functions of: (a) $h_2$ with $L_2 = 10\pi$, and (b) $L_2$ with $h_2 = 0.1$. The other parameters are chosen as: $h_1 = h_3 = 0.1$, $L_1 = L_3 = 5\pi$, $U_0 = 2$, $T/T_c = 0.5$.

\FIG. 7: (Color online) $I_s$ versus $\varphi$ for the $SF_xF_yF_zS$ junction with different $L_1$: varying $L_1$ from $0$ to $5\pi$ with a step $\pi$, $L_3 = L_1$. The other parameters are chosen as: $U_0 = 2$, $h_1 = h_3 = 0.1$, $L_2 = 5\pi$, $T/T_c = 0.5$. 

$$\sum_{\omega_n>0} G_{\omega_n}^{eh}(x,x) = i \sum_{\nu=0}^{3} f_\nu(x)\sigma_\nu\sigma_2,$$
lous supercurrent is nearly proportional to the equal-spin current shown in Fig. 8 (b), we can see that the anomalous current-phase relation is also a result of the spin precession of electrons and holes in the $F_1$ and $F_3$ layers. If the condition $h_1 L_1 = h_3 L_3 = (n + 1/2)\pi$ is satisfied, an electron incident to the left (right) superconductor will precess its spin by $\pm \pi$ in the $F_1$ ($F_3$) layer before it arrives at the superconductor and the Andreev-reflected hole proceeds to precess the spin by $\mp \pi$ when it goes back to the $F_2$ layer. Thus the Andreev-reflected hole will have the same spin with the incident electron and the complete triplet correlation arises in the $F_2$ layer. The two spin-resolved Andreev bound states carry two spin-polarized supercurrents which have opposite phase shifts and different amplitude thus leading to an anomalous Josephson current. And the phase shift in the anomalous current-phase relation is also a result of the spin precession of electron and hole in the $F_1$ and $F_3$ layers. The equilibrium phase difference of the anomalous supercurrent can be tuned by the lengths, the exchange energies, and the magnetization orientations of the $F_1$ and $F_3$ layers. And the amplitude of the supercurrent can be tuned by the barriers between the $F$ layers or by the length and the exchange energy of the $F_2$ layer.

FIG. 8: (Color online) The absolute values of pair functions in the $F_y$ layer (a) and the anomalous supercurrent at zero phase difference (b) as functions of the length of the $F_x$ and $F_z$ layers for the $SF_xF_yF_zS$ junction. $L_3 = L_1$. The other parameters are the same as in Fig. 7.

In summary, we predict a tunable anomalous Josephson effect in $SF_1F_2F_3S$ junction where the three $F$ layers have noncoplanar magnetizations. The superconducting correlation can be completely triplet in the $F_2$ layer due to the spin precession of electrons and holes in the $F_1$ and $F_3$ layers. Compared with the anomalous supercurrent shown in Fig. 5(b), we can see that the anomalous supercurrent is nearly proportional to the equal-spin triplet correlations.

IV. CONCLUSION

In summary, we predict a tunable anomalous Josephson effect in $SF_1F_2F_3S$ junction where the three $F$ layers have noncoplanar magnetizations. The superconducting correlation can be completely triplet in the $F_2$ layer due to the spin precession of electrons and holes in the $F_1$ and $F_3$ layers. If the condition $h_1 L_1 = h_3 L_3 = (n + 1/2)\pi$ is satisfied, an electron incident to the left (right) superconductor will precess its spin by $\pm \pi$ in the $F_1$ ($F_3$) layer before it arrives at the superconductor and the Andreev-reflected hole proceeds to precess the spin by $\mp \pi$ when it goes back to the $F_2$ layer. Thus the Andreev-reflected hole will have the same spin with the incident electron and the complete triplet correlation arises in the $F_2$ layer. The two spin-resolved Andreev bound states carry two spin-polarized supercurrents which have opposite phase shifts and different amplitude thus leading to an anomalous Josephson current. And the phase shift in the anomalous current-phase relation is also a result of the spin precession of electron and hole in the $F_1$ and $F_3$ layers. The equilibrium phase difference of the anomalous supercurrent can be tuned by the lengths, the exchange energies, and the magnetization orientations of the $F_1$ and $F_3$ layers. And the amplitude of the supercurrent can be tuned by the barriers between the $F$ layers or by the length and the exchange energy of the $F_2$ layer.

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