Gravitational Approach to Tachyon Matter

Kazutoshi Ohta\textsuperscript{1} and Takashi Yokono\textsuperscript{2}\textsuperscript{†}

\textsuperscript{1} Center for Theoretical Physics
Laboratory for Nuclear Science and Department of Physics
Massachusetts Institute of Technology
Cambridge, MA 02139, USA

and

\textsuperscript{1,2} Department of Physics
Kyoto University
Kyoto 606-8502, Japan

Abstract

We found a gravity solution of \(p+1\) dimensional extended object with \(SO(p) \times SO(9-p)\) symmetry which has zero pressure and zero dilaton charge. We expect that this object is a residual tachyon dust after tachyon condensation of brane and anti-brane system recently discussed by Sen. We also discuss the Hawking temperature and some properties of this object.

\textsuperscript{*}e-mail: kohta@yukawa.kyoto-u.ac.jp

\textsuperscript{†}e-mail: yokono@gauge.scphys.kyoto-u.ac.jp
1 Introduction

The studies of dynamics of the tachyon on $D_p \overline{D}_p$ brane system or unstable D-branes attract great interests for a last few years since they give the important non-perturbative and off-shell knowledge of string theory. The analysis using string field theory (SFT) and effective field theory on the brane is very useful and successful to understand the tachyon dynamics. We have been naively expecting that the end of an annihilate or decay of the brane anti-brane or unstable branes goes to nothing by emitting the graviton while the tachyon is oscillating at the bottom of the potential. However, more recently, Sen suggested the possibility of the residual tachyon matter after the tachyon condensation where the tachyon is asymptotically rolling with a constant velocity on the minimum of the runaway potential [1, 2, 3]. This tachyon matter possess two properties of the absence of plane-wave solutions around the minimum and the exponential fall off of the pressure towards zero while the energy is preserved. This kind of tachyon matter system is discussed from the point of view of boundary string field theory (BSFT) [4, 5] and also could contribute to the inflation or dark matter problems in cosmology [6]-[13].

Another approaches to these problems are not so clear and insufficient. Especially, if we want to know an annihilation or decay process of the unstable D-branes as a bulk gravitational dynamics we need a time-dependent decaying gravity solutions where the tachyon rolls down to the minimum, but we have not obtained completely except for few special examples like S-brane solutions [14, 15]. Alternative view point of the problem is given by [16, 17, 18]. They use the general non-BPS $p$-brane solution [19, 20] with three parameters and argue that these parameters correspond to the charges of brane and anti-brane and the vev of the tachyon. We can see the decay process as one-parameter deformation, which corresponds to tachyon vev, of the gravity solution. Of course, for the general value of the tachyon vev open string theory is off-shell, but we expect that off-shell open string theory also could be probed by scattering of on-shell gravitons and other massless closed string through the modified Dirac-Born-Infeld (DBI) action. Unfortunately, the three-parameter gravity solution has the $ISO(1, p) \times SO(9 - p)$ symmetry, which means there is no asymmetry between the energy and pressure, and contains no information about vev of the tachyon velocity. So this three-parameter family solution can not describe the tachyon matter conjectured by Sen.

In this paper, we use more extended solution with four parameters and the $SO(p) \times SO(9 - p)$ symmetry in order to describe the tachyon matter in a sense of the gravity of the bulk. We find the energy-momentum tensor on the brane or boundary. This additional new parameter gives an asymmetry with respect to the energy and pressure and we expect this parameter corresponds to the vev of the tachyon velocity. The asymmetry of the time and space directions in the brane metric allows the pressure to be zero by tuning the parameters even though the energy is fixed at a constant value. In addition to the
condition that the pressure of the tachyon matter is zero, we need to set the dilaton charge zero since the charge is proportional to the world-volume effective DBI action itself and it vanishes at the minimum of the tachyon potential. So we solve these conditions and find the gravity solution of the tachyon matter extended object with the zero pressure and zero dilaton charge. If we start from the situation of the same number of Dp-Dp branes, that is, the vanishing total RR charge, the solution has only one parameter which determines the total energy of the system.

This paper is organized as follows. In the next section, we give a brief review of the rolling tachyon or tachyon matter from the point of view of the world-volume effective field theory, which was discussed by Sen. And also we define the energy-momentum tensor and dilaton charge from the modified DBI action which are important in the following discussion, and discuss the relationship between the string frame and Einstein frame quantities. In Section 3, we calculate the energy-momentum tensor on the brane and dilaton charge from the bulk gravity side by using the general four-parameter gravity solution with the $SO(p) \times SO(9-p)$ symmetry. We use these results in order to determine the tachyon matter solution of the zero pressure and dilaton charge in Section 4. Finally, we derive the Hawking temperature of the zero pressure object and discuss on a stability in Section 5. Section 6 is devoted to conclusion and discussion to the future problems.

2 Rolling Tachyon

We first consider the effective field theory on the Dp-Dp system or unstable D-branes, which is a modification of the ordinary DBI action [21, 22, 23]

$$S_{BI} = -\tau_p \int d\xi^{p+1} e^{-\phi} V(T) \sqrt{-\det \hat{A}_{ij}},$$

(2.1)

where

$$\hat{A}_{ij} \equiv \hat{g}_{ij} + \partial_i T \partial_j T,$$

(2.2)

and $T$ is a tachyon field and $V(T)$ is a tachyon potential. We note that all quantities in the above action like induced metric $\hat{g}_{ij}$ are defined by the string frame metric of the bulk. We apply the hatted notation to the string frame quantities in order to avoid confusing with the Einstein frame ones. If we calculate the energy-momentum tensor from the modified DBI action, we obtain

$$\hat{T}_{ij} = -\frac{\tau_p}{4} \frac{e^{-\phi} V(T)}{\sqrt{-\det \hat{g}_{ij}}} \sqrt{-\det \hat{A}_{ij}(\hat{A}^{-1})_{ij}}.$$

(2.3)

If we assume that the tachyon field depends only on time, roughly we have

$$\hat{T}_{00} \approx V(T)(\hat{g}_{00} - (\partial_0 T)^2)^{-\frac{1}{2}},$$

(2.4)

$$\hat{T}_{ii} \approx -V(T)(\hat{g}_{00} - (\partial_0 T)^2)^{\frac{1}{2}}.$$

(2.5)
Therefore if \( \hat{g}_{00} - (\partial_0 T)^2 \) approaches to zero as the same behaviour as the tachyon potential, \( \hat{T}_{00} \) remains constant in time, but the pressure becomes zero. Actually, if we take the tachyon potential as \( V(T) \propto e^{-\alpha T/2} \) for large \( T \), we find a classical solution of the modified DBI action has the following form for the large time \( (x^0) \) \[24, 3\]

\[
T = x^0 + Ce^{-\alpha x^0} + O(e^{-2\alpha x^0}),
\]
and \( \hat{T}_{00} \) and \( \hat{T}_{ii} \) behave as
\[
\hat{T}_{00} \approx \frac{1}{\sqrt{2\alpha C}}, \quad \hat{T}_{ii} \approx -\sqrt{2\alpha C}e^{-\alpha x^0}.
\]

So the pressure exponentially dumps to zero with the constant energy as expected.

In the following discussion we will treat the above energy-momentum tensor in the Einstein frame since it is more useful in the gravity calculation. Rewrite the modified DBI action in the Einstein frame and calculating the energy-momentum tensor again, we find the same expression as in the string frame up to a dilaton factor

\[
T_{ij} = -\frac{\tau_p}{4} e^{-\frac{\phi}{2}} V(T) \sqrt{- \det A_{ij}} \sqrt{\det g_{ij}} (A^{-1})_{ij},
\]
where \( A_{ij} \) is now defined by using the Einstein frame induced metric

\[
A_{ij} \equiv e^{\phi/2} g_{ij} + \partial_i T \partial_j T.
\]

So the energy-momentum tensor in the string and Einstein frame is proportional to each other

\[
\hat{T}_{ij} = e^{-\frac{\phi}{4}} \hat{A}_{ij}.
\]

Therefore a choice of the frames does not affect the discussion of the tachyon matter. If we can find the pressureless object in the Einstein frame, we expect zero pressure in the string frame too.

We also would like the another quantity which is a variation with respect to the dilaton field

\[
\frac{\delta S_{BI}}{\delta \phi} = \tau_p e^{-\phi} V(T) \sqrt{\det A_{ij}}.
\]

This is the modified DBI Lagrangian itself. So this quantity has to vanish at the minimum of the tachyon potential since there is no world-volume effective theory after the condensation. We note that this variation depends on the frame. If we use the Einstein frame, we can not conclude that the variation vanish at minimum. So we have to use the string frame on this quantity. We refer this quantity as a dilaton charge in the following gravity discussions.
3 Four-Parameter Brane Solution and Energy-momentum Tensor

3.1 Gravity solution

The complete solution for brane like extended objects was found by [19, 20]. They solve only the equation of motion for a coupled system of gravity, dilation and RR antisymmetric tensor fields in any dimensions. The ansatz for a \( p + 1 \) dimensional extend object has \( SO(p) \times SO(9 - p) \) symmetry, where \( SO(p) \) and \( SO(9 - p) \) are the symmetry of the space directions on the brane-like object and the \( 9 - p \) dimensional transverse directions. This solution includes the well-known BPS \( D_p \)-brane, where the symmetry on the brane enhances to \( ISO(1, p) \) Poincaré invariance. For a general non-extremal or non-BPS branes, the Poincaré symmetry breaks to \( SO(p) \).

The \( SO(p) \times SO(9 - p) \) symmetric solution includes four parameters. If we now use the notations of these parameters in [19], the metric of four-parameter solution is expressed in the Einstein frame as

\[
ds^2 = e^{2A(r)} \left(-f(r)dt^2 + dx_md^m\right) + e^{2B(r)}(dr^2 + r^2d\Omega_{8-p}^2),
\]

where

\[
f(r) = e^{-c_h(r)},
\]

\[
A(r) = \frac{7 - p}{32} \left(\frac{3 - p}{2} c_1 + \left(1 + \frac{(3 - p)^2}{8(7 - p)}\right)c_3\right) h(r)
- \frac{7 - p}{16} \ln[cosh(kh(r)) - c_2 sinh(kh(r))],
\]

\[
B(r) = \frac{1}{7 - p} \ln[f_-(r)f_+(r)] + \frac{p - 3}{64} \left((p + 1)c_1 - \frac{3 - p}{4}c_3\right) h(r)
+ \frac{p + 1}{16} \ln[cosh(kh(r)) - c_2 sinh(kh(r))]
\]

\[
\phi(r) = \frac{7 - p}{16} \left((p + 1)c_1 - \frac{3 - p}{4}c_3\right) h(r)
+ \frac{3 - p}{4} \ln[cosh(kh(r)) - c_2 sinh(kh(r))],
\]

\[
e^{\Lambda(r)} = -\sqrt{c_2^2 - 1 \sinh(kh(r)) \cosh(kh(r)) - c_2 \sinh(kh(r))},
\]

and

\[
f_{\pm}(r) = 1 \pm \left(\frac{r_0}{r}\right)^{7-p},
\]
\[ h(r) = \ln \left[ \frac{f_-(r)}{f_+(r)} \right], \tag{3.8} \]
\[ k^2 = \frac{2(8-p)}{7-p} - c_1^2 + \frac{1}{4} \left( \frac{3-p}{2} c_1 + \frac{7-p}{8} c_3 \right)^2 - \frac{7}{16} c_3^2. \tag{3.9} \]

As we can see in the above solution, the asymmetry between the time and space directions on the brane is brought by the parameter \( c_3 \) in \( f(r) \) of eq. (3.2) only. So if we set \( c_3 = 0 \), the ISO\( (1,p) \) symmetry of the world-volume is restored.

The detailed analysis for the three-parameter solution with \( c_3 = 0 \) is discussed in [16]. The solution is invariant under the \( Z_2 \) transformations of these three parameters \((r_0, c_1, c_2)\) and we can fix a physical range of parameters as \( r_0, c_1, k \in \mathbb{R}_+ \) and \( c_2 \in (-\infty, -1) \cup (1, \infty) \).

The total RR charge vanishes if \( |c_2| = 1 \), so we expect \( |c_2| = 1 \) corresponds to the situation that the number of \( Dp \) \((N)\) and \( \overline{Dp} \) \((\overline{N})\) branes is the same \((N = \overline{N})\). Moreover, in this neutral charged case, the physically relevant choice is \( c_2 = 1 \) for \( p > 3 \) and \( c_2 = -1 \) for \( p < 3 \) (for \( p = 3 \) the choices of sign are physically equivalent).

On the other hand, the BPS \( Dp \)-brane \((N \neq 0, \overline{N} = 0)\) is appeared in the limit of \( c_2 \to \infty \) for \( p \geq 3 \) or \( c_2 \to -\infty \) for \( p < 3 \). If we consider the following scaling relations for \( p > 3 \)

\[
\begin{align*}
    r_0^{7-p} &\to \epsilon^\frac{1}{p} r_0^{7-p}, \\
    c_1 &\to c_m = \epsilon \frac{8k^2}{(p+1)(7-p)c_m}, \\
    c_2 &\to \frac{c_2}{\epsilon},
\end{align*}
\]

where \( c_m = \left( \frac{32(8-p)}{(p+1)(7-p)} \right)^\frac{1}{2} \) and \( \mu_0 = 2c_2 kr_0^{7-p} \) is fixed, then we finally obtain the metric of the BPS \( Dp \)-brane [26]

\[
\begin{align*}
    ds^2 &= H^{\frac{p-7}{2}} dx_\mu dx^\mu + H^{\frac{p+1}{2}} \left( dr^2 + r^2 d\Omega_{8-p}^2 \right), \\
    e^\phi &= H^{\frac{3-p}{2}}, \\
    C^{(p+1)}_{01...p} &= -\frac{1}{2} (H^{-1} - 1), \\
    H &= 1 + \frac{\mu_0}{r^{7-p}}.
\end{align*}
\]

The meaning of the parameter \( c_1 \) is also discussed in [16]. The authors conclude that \( c_1 = 0 \) corresponds to \( \langle T \rangle = 0 \) where \( \langle T \rangle \) is a vev of the tachyon field and \( c_1 \) mixed with \( r_0 \) has a non-trivial relation to \( \langle T \rangle \). However, it is too difficult to find exact maps between \( c_1, r_0 \) and \( \langle T \rangle \).
Now let us turn on the parameter $c_3$. The parameter $c_3$ breaks the world-volume symmetry to $SO(p)$ and the solution (3.1) with four independent parameters $(r_0, c_1, c_2, c_3)$ includes the black $p$-brane solutions of [26] at $(c_1, c_3) = \left(\frac{3-p}{2(p-7)}, -2\right)$ and the Schwarzschild solution at $(c_1, c_2, c_3) = \left(\frac{3-p}{2(p-7)}, \pm 1, -2\right)$. Introduction of $c_3$ produce the asymmetry between time and space component of the energy-momentum tensor on the brane as we will see in the following section. And also as we have mentioned in the previous section, the velocity of tachyon $\partial_0 T$ gives the different behaviour of the energy and pressure. Therefore, although we can not determine the exact relation, we expect that there is a relation between the parameter $c_3$ and a vev of the tachyon velocity $\langle \partial_0 T \rangle$. We will show in the following that a suitable choice of the parameters $c_1$ and $c_3$ makes a tachyon matter gravity solution with a positive energy and zero pressure.

### 3.2 Brane system and energy-momentum tensor

We begin with the bosonic part of Type IIA/B supergravity action in ten dimensions coupled with a boundary brane action

$$S = S_G + S_B, \tag{3.11}$$

$$S_G = \frac{1}{16\pi G_{10}} \int d^{10}x \sqrt{-g} \left\{ R - \frac{1}{2} \partial_\mu \phi \partial^\mu \phi - \frac{1}{2(p+2)!} e^{\frac{3-p}{2} \phi} F_{\mu \nu}^2 \right\}, \tag{3.12}$$

$$S_B = \int d^{10}x \delta^{(9-p)}(r) \mathcal{L}_B, \tag{3.13}$$

where $S_G$ represents a bulk supergravity action of the graviton, dilaton and anti-symmetric RR fields, and $S_B$ is a boundary term which has a delta function distribution on the brane. The Lagrangian $\mathcal{L}_B$ in the action $S_B$ is the DBI type Lagrangian of BPS branes

$$\mathcal{L}_B = -\tau_p e^{-\phi} \sqrt{- \det \left[ e^\phi g_{ij} + 2\pi \alpha' F_{ij} \right]} \tag{3.14}$$

with tension $\tau_p$, or the modified DBI Lagrangian of non-BPS branes, which we have used in Section 2

$$\mathcal{L}_B = -\tau_p e^{-\phi} V(T) \sqrt{- \det \left[ e^\phi g_{ij} + 2\pi \alpha' (F_{ij} + \partial_i T \partial_j T) \right]}, \tag{3.15}$$

which contains the tachyon field $T$ and tachyon potential $V(T)$ and we denote these actions in the Einstein frame.

The Einstein equations of motion derived from (3.11) is

$$R_{\mu \nu} - \frac{1}{2} g_{\mu \nu} R = 8\pi G_{10} T^\text{total}_{\mu \nu} = 8\pi G_{10} \left( T^\text{bulk}_{\mu \nu} + \delta^{(9-p)}(r) T_{\mu \nu} \right), \tag{3.16}$$
where \( T_{\mu\nu}^{\text{bulk}} \) is an energy-momentum tensor contributing from the dilaton field and RR (p+2)-form and \( T_{\mu\nu} \) is an energy-momentum tensor localized on the branes. The solution (3.1), of course, simply satisfies the equation motion (3.16) on the all region except for at \( r = 0 \), but our present interested part of the energy-momentum tensor is the localized one on the brane which is the coefficient of the delta-function of \( r \).

If we want to extract the delta functional distribution part of the energy-momentum tensor from the general brane metric (3.1), we have to pay some attention to obtain it. To see this, we first expand the metric around the asymptotically flat region as like as

\[
g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}, \tag{3.17}
\]

where \( \eta_{\mu\nu} = \text{diag}(-1, +1, +1 \cdots) \). If we now define

\[
\varphi_{\mu\nu} \equiv h_{\mu\nu} - \frac{1}{2} \eta_{\mu\nu} h,
\]

where \( h = h^{\alpha}_{\alpha} \) and use the fact that the Einstein frame metric satisfies the following harmonic gauge

\[
\partial_{\mu} \varphi^{\mu\nu} = 0, \tag{3.19}
\]

then we obtain the following linear approximation of the Einstein equation up to the order one of \( h_{\mu\nu} \)

\[
\frac{1}{2} \nabla^2 \varphi_{\mu\nu} = -8\pi G_{10} T_{\mu\nu}^{\text{total}}. \tag{3.20}
\]

Following eq. (3.20), the delta function of \( r \) in the energy-momentum tensor comes from the singular part of the order \( 1/r^{7-p} \) in \( \varphi_{\mu\nu} \) by action of the Laplacian in the transverse space.

Expanding the metric of the solution (3.1) as

\[
g_{00} = -1 + \alpha \left( \frac{r_0}{r} \right)^{7-p} + \cdots, \tag{3.21}
\]

\[
g_{ii} = 1 + \beta \left( \frac{r_0}{r} \right)^{7-p} + \cdots, \tag{3.22}
\]

\[
g_{MM} = 1 + \gamma \left( \frac{r_0}{r} \right)^{7-p} + \cdots, \tag{3.23}
\]

where \( i = 1, \cdots, p \) and \( M = p + 1, \cdots, 9 \) represent the indices of the longitudinal and transverse space, respectively, and here we define

\[
\alpha = \frac{1}{64} \left( 16(7-p)kc_2 - 4(p-3)(7-p)c_1 + (p^2-14p-63)c_3 \right), \tag{3.24}
\]

\[
\beta = \frac{1}{64} \left( -16(7-p)kc_2 + 4(p-3)(7-p)c_1 - (p^2-14p+65)c_3 \right), \tag{3.25}
\]

\[
\gamma = \frac{1}{64} \left( 16(p+1)kc_2 - 4(p+1)(p-3)c_1 - (p-3)^2c_3 \right). \tag{3.26}
\]

7
Finally we have an expression for the energy momentum tensor on the brane in the Einstein frame.

\[
T_{00} = \frac{1}{2} (\alpha + p\beta + (9-p)\gamma) \frac{(7-p)V_p\omega_{8-p}r_0^{7-p}}{16\pi G_{10}},
\]

\[
= (16c_2k - 4(p-3)c_1 - (9+p)c_3) \frac{(7-p)V_p\omega_{8-p}r_0^{7-p}}{16\pi G_{10}},
\]

\[
T_{ii} = \frac{1}{2} (\alpha - (p-2)\beta - (9-p)\gamma) \frac{(7-p)V_p\omega_{8-p}r_0^{7-p}}{16\pi G_{10}},
\]

\[
= -(16c_2k - 4(p-3)c_1 + (7-p)c_3) \frac{(7-p)V_p\omega_{8-p}r_0^{7-p}}{16\pi G_{10}},
\]

\[
T_{MM} = \frac{1}{2} (\alpha - p\beta - (7-p)\gamma) \frac{(7-p)V_p\omega_{8-p}r_0^{7-p}}{16\pi G_{10}},
\]

\[
= 0,
\]

where we use the relation

\[
\nabla^2 \left( \frac{1}{r^{7-p}} \right) = -(7-p)V_p\omega_{8-p}\delta^{(9-p)}(r),
\]

where \(V_p\) is a normalized volume of the brane and \(\omega_{8-p}\) is a volume of \(8-p\) dimensional unit sphere.

\(T_{MM} = 0\) means that the object is not extended in transverse space and all components of the energy-momentum tensor are localized on the brane at \(r = 0\). \(T_{00}\) is well known as the Arnowitt-Deser-Misner (ADM) mass \(M_{ADM}\), which agrees with other derivations of \([27, 28]\). As we have noted before, if we set \(c_3 = 0\) the world-volume symmetry is restored to ISO\((1,p)\) and we have a restricted relation \(T_{00} = -T_{ii}\). So we can not reach at the situation of the tachyon matter without the parameter \(c_3\).

### 3.3 Dilaton charge

Following the discussion in Section 2, the variation of the unstable Dp-brane action with respect to the dilation field must vanish at the minimum of the tachyon potential. This means that the source term of the bulk dilation field also vanishes. Since this argument is frame dependent, we derive the string frame equation of motion of the dilaton from the action \([3.11]\)

\[
\sqrt{-\hat{g}}e^{-2\phi} \left( \hat{R} + 4\partial_\mu \phi \partial^\mu \phi \right) + 4\partial_\mu \left( \sqrt{-\hat{g}}e^{-2\phi} \hat{g}^\mu\nu \partial_\nu \phi \right) = -8\pi G_{10}Q_D\delta^{(9-p)}(r).
\]

The contribution to the delta function comes from the total derivative term in \(\hat{R}\) and Laplacian of \(\phi\). If we again use the weak field approximation around the asymptotically flat region, we have the following linearized Laplace equation

\[
\nabla^2 (\hat{h}_{MM} - \hat{h} + 4\phi) = -8\pi G_{10}Q_D\delta^{(9-p)}(r).
\]
Picking up the delta function pole from the order of \(1/r^{7-p}\), we obtain
\[
Q_D = (16c_2 k - 4(p+1)c_1 - (p-3)c_3) \left( \frac{(7-p)V_p \omega_{8-p} r_0^{7-p}}{128 \pi G_{10}} \right). \tag{3.33}
\]

\(Q_D = 0\) is a necessary condition at the minimum of the tachyon potential.

From the world-volume effective theory analysis, we can see that the space components of the energy-momentum tensor \(\hat{T}_{ii}\) and the dilation charge are proportional to each other up to some overall factor, but (3.28) and (3.33) in our analysis from the gravity side are obviously not proportional. However, if we use the effective field theory from BSFT \([4, \ 5]\), the pressure and dilation charge are simply not proportional to each other. From our standpoint, we can not determine the explicit coupling of the bulk fields to world-volume theory. So we do not straightaway think this fact is contradiction.

### 4 Tachyon Matter

Now we solve the condition for the tachyon matter, namely \(T_{ii} = 0\) and \(Q_D = 0\) while \(T_{00} \neq 0\). In the following arguments, we assume \(3 < p < 7\) and set \(c_2 = 1\) since we would like to fix a physical region of the parameters and consider the case of the vanishing total RR charge \((N = \overline{N})\). From the eqs. (3.28) and (3.33), we find
\[
(c_1, c_3) = \left( \frac{1}{2} \sqrt{\frac{8-p}{7-p}}, -2 \sqrt{\frac{8-p}{7-p}} \right), \tag{4.1}
\]
\[
k = \frac{1}{2} \sqrt{\frac{8-p}{7-p}}, \tag{4.2}
\]
where there is an alternative choice of sign in \(c_1, c_3\) and \(k\) as a solution, but we have fixed by using the physical condition of \(T_{00} \geq 0\). Therefore, in the above parameter choice, the energy or ADM mass is positive definite
\[
M_{ADM} = \frac{2\sqrt{(8-p)(7-p)V_p \omega_{8-p} r_0^{7-p}}}{\pi G_{10}}, \tag{4.3}
\]
which includes only one parameter of \(r_0 \geq 0\). We also note that all parameters are in the physical region if \(p < 7\).

Using eqs. (4.1) and (4.2), we find the relation of \((c_1, c_3) = (k, -4k)\) and substituting into the metric (3.1), we have the metric of the tachyon matter in the Einstein frame
\[
ds^2 = e^{-k h(r)} \left[ -e^{4k h(r)} dt^2 + dx_m dx_m + (f_- (r) f_+(r)) \frac{2}{r^2} \left( dr^2 + r^2 d\Omega_{8-p}^2 \right) \right], \tag{4.4}
\]
\[
\phi(r) = k h(r), \tag{4.5}
\]
where \(h(r)\) and \(f_\pm(r)\) are also given by (3.8) and (3.7). This metric has the \(SO(p) \times SO(9-p)\) symmetry and is static. This static gravity solution could describe the tachyon
matter object which reaches a constant tachyon vev $\langle T \rangle$ and velocity $\langle \partial_0 T \rangle$ after the rolling tachyon condensation.

If we now rewrite (4.4) to the string frame metric by using relation $\hat{g}_{\mu\nu} = e^{\phi/2} g_{\mu\nu}$, we obtain

$$d\hat{s}^2 = -\left(\frac{f_-(r)}{f_+(r)}\right)^2 \sqrt{\frac{8-p}{7-p}} dt^2 + dx^m dx^m + (f_-(r) f_+(r))^{\frac{3-p}{p}} \left(dr^2 + r^2 d\Omega^2_{8-p}\right),$$

and the dilaton field is again given by (4.5).

This solution is controlled only by the parameter $r_0$. If the total energy of the system decreases due to the emission of the graviton or evaporation, $r_0$ also must decrease. In the limit of $r_0 \to 0$, we have $f_{\pm}(r) \to 1$. So, the metric (4.4) or (4.6) become a flat 10-dimensional space in the limit of $M_{\text{ADM}} \to 0$ as expected.

5 Hawking temperature

To discuss a quantum stability of the object found in Section 4, we need the semi-classical thermodynamics of the above gravitational system, namely the Hawking temperature of the brane-like object. The Hawking temperature is determined by avoiding conical singularity on the plane of the Euclidean time and the radial coordinate. The some useful expressions of the Hawking temperature and entropy for the general black $p$-brane solution are derived in [28].

In our case of $c_2 = 1$, if we define $\epsilon^{7-p} \equiv (r^{7-p} - r_0^{7-p})/r_0^{7-p}$, the Hawking temperature is given by the surface gravity at the horizon ($r \to r_0$ or $\epsilon \to 0$)

$$T_H = \frac{1}{2\pi} \left[ \frac{d}{dr} \left( \frac{f(r)^{1/2} e^{A(r)}}{e^{B(r)}} \right) \right]_{r \to r_0},$$

$$= \frac{\xi}{2\pi r_0} e^\lambda \bigg|_{\epsilon \to 0},$$

where

$$\xi = \frac{7-p}{128} \left(16(7-p)k - 4(p-3)(7-p)c_1 + (p^2 - 14p - 63)c_3\right),$$

$$\lambda = -(8-p) + \frac{7-p}{32} \left(16k - 4(p-3)c_1 - (p+9)c_3\right).$$

This result means that if $\lambda > 0$ then $T_H = 0$ and if $\lambda = 0$ then $T_H$ is finite, but if $\lambda < 0$ then the Hawking temperature diverges as long as $\xi$ is a positive constant.

Now substituting our choice of the parameters in the pressureless object (4.1) and (4.2), we find

$$\xi = \frac{7}{4} \sqrt{(8-p)(7-p)},$$
\[ \lambda = -(8-p) + \sqrt{(8-p)(7-p)}, \quad (5.5) \]

and \( \lambda \) is always negative for \( p < 7 \). This signals that the Hawking temperature of the object we found is infinite and using the Hawking radiation argument the object is highly unstable and may evaporate and disperse.

However, this conclusion of the semi-classical analysis is presumably too naive since we do not use any information about non-interacting closed string pressureless gas localized on the tachyon matter at the horizon. We need further discussions for the stability of this object.

6 Conclusion and Discussion

In this paper, we found the metric of the \( p + 1 \) dimensional extended object without the pressure and dilation charge. However, there is no conclusive evidence that our gravity solution describes the tachyon matter discussed by Sen since we do not have sufficient knowledge of the relationship between the world-volume open string tachyon and parameters in the closed string bulk gravity solution.

We also do not know the explicit relation between the vevs in the world-volume effective theory \( \langle T \rangle, \langle \partial_0 T \rangle \) and the parameters in the gravity solution \( (c1, c3) \). Information about the correspondence gives justification on the gravity description of the tachyon matter.

To understand more deeply the nature of the tachyon matter, we have to develop closed string theory on the background \([4,3]\). String theory has a upper limit of the temperature which is known as the Hagedorn temperature. So our semi-classical analysis on the Hawking temperature may fail at the Hagedorn temperature of the closed string on the brane. We hope that a string dynamics at high temperature gives us reason for zero pressure and true properties of the tachyon matter.

Acknowledgements

We would like to thank S. Kawamoto for useful discussions. KO would like to thank A. Fayyazuddin and A. Sen for very useful discussions and comments. KO would also like to thank B. Zwiebach for a lucid lecture on tachyon matter. TY would like to thank M. Fukuma, K. Hashimoto, K. Hotta for useful discussions. The work of KO and TY is supported in part by Japan Society for the Promotion of Science Research Fellowship (\#02809 and \#02845). KO is also supported in part by funds provided by the U.S. Department of Energy (D.O.E.) under cooperative research agreement #DF-FC02-94ER40818.
References

[1] A. Sen, “Rolling Tachyon”, JHEP 0204 (2002) 048, hep-th/0203211.

[2] A. Sen, “Tachyon Matter”, hep-th/0203265.

[3] A. Sen, “Field Theory of Tachyon Matter”, hep-th/0204143.

[4] S. Sugimoto, S. Terashima, “Tachyon Matter in Boundary String Field Theory”, hep-th/0205085.

[5] J. Minahan, “Rolling the tachyon in super BSFT”, hep-th/0205098.

[6] G. W. Gibbons, “Cosmological Evolution of the Rolling Tachyon”, Phys. Lett. B537 (2002) 1, hep-th/0204008.

[7] M. Fairbairn, M. H. G. Tytgat, “Inflation from a Tachyon Fluid?”, hep-th/0204070.

[8] S. Mukohyama, “Brane cosmology driven by the rolling tachyon”, hep-th/0204084.

[9] A. Feinstein, “Power-Law Inflation from the Rolling Tachyon”, hep-th/0204140.

[10] T. Padmanabhan, “Accelerated expansion of the universe driven by tachyonic matter”, hep-th/0204150.

[11] A. Frolov, L. Kofman, A. Starobinsky, “Prospects and Problems of Tachyon Matter Cosmology”, hep-th/0204187.

[12] D. Choudhury, D. Ghoshal, D. P. Jatkar, S. Panda, “On the Cosmological Relevance of the Tachyon”, hep-th/0204204.

[13] G. Shiu, I. Wasserman, “Cosmological Constraints on Tachyon Matter”, hep-th/0205003.

[14] M. Gutperle, A. Strominger, “Spacelike Branes”, JHEP 0204 (2002) 018, hep-th/0202210.

[15] K. Hashimoto, “Dynamical Decay of Brane-Antibrane and Dielectric Brane”, hep-th/0204203.

[16] P. Brax, G. Mandal, Y. Oz, “Supergravity Description of Non-BPS Branes”, Phys. Rev. D63 (2001) 064008, hep-th/0005242.

[17] R. Rabadan, J. Simon, “M-theory lift of brane-antibrane systems and localised closed string tachyons”, JHEP 0205 (2002) 045, hep-th/0203243.
[18] H. Kim, “Supergravity Approach to Tachyon Potential in Brane-Antibrane Systems”, hep-th/0204191.

[19] V. D. Ivashchuk, V. N. Melnikov, “Multidimensional Classical and Quantum Cosmology with Intersecting p-branes”, J. Math. Phys. 39 (1998) 2866-2888, hep-th/9708157.

[20] B. Zhou, C-J. Zhu, “The Complete Black Brane Solutions in D-dimensional Coupled Gravity System”, hep-th/9905146.

[21] M. R. Garousi, “Tachyon couplings on non-BPS D-branes and Dirac-Born-Infeld action”, Nucl. Phys. B584 (2000) 284-299, hep-th/0003122.

[22] E. A. Bergshoeff, M. de Roo, T. C. de Wit, E. Eyras, S. Panda, “T-duality and Actions for Non-BPS D-branes”, JHEP 0005 (2000) 009, hep-th/0003221.

[23] J. Kluson, “Proposal for non-BPS D-brane action”, Phys. Rev. D62 (2000) 126003, hep-th/0004106.

[24] G. Gibbons, K. Hori, P. Yi, “String Fluid from Unstable D-branes”, hep-th/0009061.

[25] M. J. Duff, K. S. Stelle, “Multi-membrane solutions of D=11 supergravity”, Phys. Lett. B253 (1991) 113.

[26] G. T. Horowitz, A. Strominger, “Black Strings and p-Branes”, Nucl. Phys. B360 (1991) 197.

[27] R. C. Myers, M. J. Perry, “Black Holes in Higher Dimensional Space-Time”, Ann. Phys. 172 (1986) 304.

[28] R. Argurio, “Brane Physics in M-theory”, hep-th/9807171.