pQCD physics of multiparton interactions

B. Blok,1 Yu. Dokshitzer,2 L. Frankfurt,3 and M. Strikman4

1Department of Physics, Technion—Israel Institute of Technology, 32000 Haifa, Israel
2Laboratory of High Energy Theoretical Physics (LPTHE), University Paris 6, Paris, France
3School of Physics and Astronomy, Raymond and Beverly Sackler Faculty of Exact Sciences, Tel Aviv University, 69978 Tel Aviv, Israel
4Physics Department, Penn State University, University Park, PA, USA

We study production of two pairs of jets in hadron–hadron collisions in view of extracting contribution of double hard interactions of three and four partons (3 → 4, 4 → 4). Such interactions, in spite of being power suppressed at the level of the total cross section, become comparable with the standard hard collisions of two partons, 2 → 4, in the back-to-back kinematics when the transverse momentum imbalances of two pairing jets are relatively small.

We express differential and total cross sections for two-dijet production in double parton collisions through the generalized two-parton distributions, 2GPDs [1], that contain large-distance two-parton correlations of non-perturbative origin as well as small-distance correlations due to parton evolution.

A special emphasis is given to 3 → 4 double hard interaction processes that occur as an interplay between large- and short-distance parton correlations and were not taken into consideration by approaches inspired by the parton model picture. We demonstrate that the 3 → 4 mechanism, being of the same order in αs as the 4 → 4 process, turns out to be geometrically enhanced compared to the latter and should contribute significantly to 4-jet production.

The framework developed here takes into systematic consideration perturbative Q2 evolution of 2GPDs. It can be used as a basis for future analysis of NLO corrections to multi-parton interactions (MPI) at LHC and Tevatron colliders, in particular for improving evaluation of QCD backgrounds to new physics searches.

PACS numbers:

Keywords:

I. INTRODUCTION

Understanding the rates and the structure of multi-jet production in hadron–hadron collisions is of primary importance for new physics searches.

Production of high transverse momentum jets is a hard process which implies a head-on collision of QCD partons — quarks and/or gluons — from the small-distance wave functions of initial hadrons. Cross section of a hard collision is small compared with the size of hadron, wave functions of initial hadrons. Cross section of a process which implies a head-on collision of QCD particles is of primary importance for new physics searches.

We find that these large- and small-distance correlations participate in different manner in 4-jet production, and treat them in the leading logarithmic approximation of pQCD that resums collinear logarithms in all orders.

A special emphasis is given to 3 → 4 double hard interaction processes that occur as an interplay between large- and short-distance parton correlations and were not taken into consideration by approaches inspired by the parton model picture. We demonstrate that the 3 → 4 mechanism, being of the same order in αs as the 4 → 4 process, turns out to be geometrically enhanced compared to the latter and should contribute significantly to 4-jet production.

The framework developed here takes into systematic consideration perturbative Q2 evolution of 2GPDs. It can be used as a basis for future analysis of NLO corrections to multi-parton interactions (MPI) at LHC and Tevatron colliders, in particular for improving evaluation of QCD backgrounds to new physics searches.

A possibility of a double hard collision becomes more important with increase of the energy of the collision where scattering off small x partons which have much higher densities becomes possible.

In recent years multiparton collisions have attracted close attention. Following the pioneering work of Refs. [2, 3], a large number of related theoretical papers appeared [4–18], based on the parton model and geometrical picture in the impact parameter space. More recently, this topic has been intensively discussed in view of the LHC program [9, 10]. Monte Carlo event generators that produce multiple parton collisions are being developed [11, 12]; theoretical papers exploring properties of double parton distributions and discussing their QCD evolution have appeared [1, 14, 15].

In our view, however, important elements of QCD that are necessary for theoretical understanding of the multiple hard interactions issue have not yet been properly taken into account by above-mentioned intuitive approaches.

The problem is, sort of, educational: both the probabilistic picture, the MC generator technology is based upon, and the familiar Feynman diagram technique, when used in the momentum space, prove to be inadequate for careful analysis and understanding of the physics of multiple collisions.

From experience gained by treating standard (single)
hard processes, one became used to a motto that a large momentum transfer scale \( Q^2 \) ensures the dominance of small distances, \( r^2 \sim Q^{-2} \), in a process under consideration. With the multiple collisions under focus, however, one has to distinguish two space-time scales: that of localization of the parton participating in a hard interaction, \( \Delta r^2 \sim Q^{-2} \), and that of transverse separation, \( \Delta \rho \), between the two hard collision vertices. The latter can be large, of the order of the hadron size, even for large \( Q^2 \).

In order to be able to trace the relative distance between the partons, one has to use the mixed longitudinal momentum–impact parameter representation which, in the momentum language, reduces to introduction of a mismatch between the transverse momentum of the parton in the amplitude and that of the same parton in the amplitude conjugated.

Another unusual feature of the multiple collision analysis that may look confusing at the first sight is the fact that — even at the tree level — the amplitude describing the double hard interaction process contains additional integrations over longitudinal momentum components; more precisely — over the difference of the (large) light-cone momentum components of the two partons originating from the same incident hadron (see Section IV A).

In the previous short publication \( ^1 \) we have considered production of two pairs of nearly back-to-back jets resulting from simultaneous hard collisions of two partons from the wave function of one incident hadron with two partons from the other hadron (“four-to-four” processes). As we have shown, this necessitates introduction of a new object — a generalized double parton distribution, \( \Delta \)GPDF, that depends on a new transverse momentum parameter \( \Delta \), conjugate to the relative distance between the two partons in the hadron wave function. Generalized double parton distributions provide a natural framework for incorporating longitudinal and transverse correlations between partons in the hadron wave function at the \( Q^2 \) scale, and for tracing the perturbative \( Q^2 \)-evolution of the correlations.

The corresponding 4-jet cross section can be expressed in terms of \( \Delta \)GPD’s as follows

\[
\frac{d\sigma(x_1, x_2, x_3, x_4)}{dt_1 dt_2} = \frac{d\sigma^{13}}{dt_1} \frac{d\sigma^{24}}{dt_2} \times \int \frac{d^2 \Delta}{(2\pi)^2} D_a(x_1, x_2; \Delta) D_b(x_3, x_4; -\Delta). \tag{1}
\]

The factor on the second line has dimension of inverse area:

\[
\frac{1}{\pi R_{\text{int}}^2} = \int \frac{d^2 \Delta}{(2\pi)^2} D(x_1, x_2; \Delta) D(x_3, x_4; -\Delta), \tag{2}
\]

where \( D(x_i) \) are the corresponding one-parton distributions. The ratio of the product of two single-inclusive cross sections and the double-inclusive cross section (\( \pi R_{\text{int}}^2 \)) is often referred to in the literature as “an effective cross section” \( \sigma_{\text{eff}} \). We prefer, however, not to look at this quantity as a cross section, since it reflects transversal area of parton overlap as well as longitudinal correlations of the partons. At the same time, it has little to do with the measure of the strength of the interaction, which is what “cross section” represents.

In a two-parton collision, scattered partons form two nearly back-to-back jets, while additional jets (should there be any) tend to be softer and to align with the directions of initial and final partons, because of collinear enhancements due to radiative nature of secondary partons. Such will be typical characteristics of a 4-jet event, in particular. On the other hand, four jets produced as a result of a double hard collision of two parton pairs would, on the contrary, form two pairs of nearly back-to-back jets. This kinematical preference is in stark contrast with “hedgehog-like” configurations of four jets stemming from a single collision and can be used in order to single out double hard collisions experimentally.

Such experimental studies were recently carried out by the CDF and D0 collaborations who have studied production of three jets + photon \( ^3 \) \( ^4 \) \( ^5 \) \( ^6 \) \( ^7 \) using information about generalized parton distribution (GPDs) obtained from the study of hard exclusive processes at HERA has found that observed 3 jet + \( \gamma \) rates were a factor \( \geq 2 \) higher than the expected rates based on a naive model that neglected correlations between partons in the transverse plane.

The use of \( \Delta \)GPD allows one to incorporate such correlations and predict their \( Q^2 \) evolution.

On the theory side, the back-to-back enhancement has been discussed, at tree level, in a number of studies of various channels (see, for example, discussion of the 2 jets+\( \bar{b} \)b in \( ^8 \) and references therein).

In the present paper we study perturbative radiative effects in the differential 4-jet distribution in the back-to-back kinematics and derive the expression for the corresponding cross section in the leading logarithmic collinear approximation. It takes into account QCD evolution of the generalized double parton distributions as well as effects due to multiple soft gluon radiation, and turns out to be a direct generalization of the known “DDT formula” for back-to-back production of two large transverse momentum particles in hadron collisions \( ^9 \).

We also discuss and treat new specific correlations between transverse momenta of jets due to 3-parton interactions producing 4 jets, “three-to-four”. Such processes are induced by perturbative splitting of a parton from one of the hadrons, the offspring of which enter double hard collision with two partons from the wave function of the second hadron. The hard scale of this parton splitting is determined by transverse momentum imbalances of pairs of jets, \( \delta_{13}, \delta_{24}, \) and exhibits specific collinear enhancement in the kinematical region where two jet imbalances practically compensate one another, \( \delta^2 = (\delta_{13}^2 + \delta_{24}^2) \ll \delta_{13}^2 \lesssim \delta_{24}^2 \).
Consistently taking into account three-to-four parton process solves a longstanding problem of double counting in treating multi-parton interactions.

Discussion of the 2-parton distribution has a long history. It is commonly defined in the momentum space as a 2-particle inclusive quantity depending on two parton momenta, see [21] [22]. Being related to (the imaginary part of) a certain forward scattering amplitude, it therefore disregards impact parameter space geometry of the interaction. Exploring properties of 2-parton distributions so defined, an approach to the study of the multiple jet production has been recently suggested in Ref. [23]. The reason why this approach has faced difficulties, [24], and did not solve, in our view, the problem of systematics — is formulated in Section V, and the total cross section of 4-jet production in the back-to-back kinematics — is presented in Section VII.

The 4 partons are produced in pairs and have nearly opposite transverse momenta, setting the hard scale ΛQCD. For production of four jets in the back-to-back kinematics, this gives four different hard scales. As we shall see below, in the 3-partons collisions producing four jets yet another scale enters the game: δ′ = δ13 + δ24 — the total transverse momentum of the 4-jet ensemble.

In what follows we consider transverse momenta of all four jets to be of the same order, Q2, down to smaller ones, δ and δ', — is not compromised by uncertainties in determination of the transverse momenta of the jets.

The 2-GPD’s were recently used in Ref. [25] for intuitive description of the total 4-jet production cross section. However, the differential distributions were not discussed in that paper, and not all relevant pQCD contributions were included, so that our results are different from the ones obtained in [25].

The paper is organized as follows.

In Section II we recall the main ingredients of the perturbative analysis based on selection of maximally collinear enhanced contributions in all orders. In Section III we present the evolution equation for generalized two-parton distributions. Section IV is devoted to the perturbative analysis of small-distance correlations between partons. The main result of the paper — the differential distribution of 4-jet production in the back-to-back kinematics — is formulated in Section V and the total cross section of two-dijet production is described in Section VI. Conclusions and outlook are presented in Section VII.

II. PERTURBATIVE ANALYSIS

A. Hard scales

The perturbative approach implies that all hardness (transverse momentum) scales that characterize the problem are comfortably larger than the intrinsic QCD scale ΛQCD: Q2 ≫ Λ2QCD. The process under consideration may have up to five hard scales involved.

Indeed, in the leading order in αs, large transverse momentum partons are produced in pairs and have nearly opposite transverse momenta, setting the hard scale Q2 ≃ j2⊥ ≃ j2⊥. Within the parton model framework (neglecting finite smearing due to intrinsic transverse momenta of incident partons), one has dσ ∝ δ(j1⊥ + j3⊥). Secondary QCD processes — evolution of initial parton distributions and accompanying soft gluon radiation — introduce transverse momentum imbalance: j1⊥ + j3⊥ = j13 which constitutes another hard scale: Q2 ≫ δ13 ≫ Λ2QCD. For production of four jets in the back-to-back kinematics, this gives four different hard scales. As we shall see below, in the 3-partons collisions producing four jets yet another scale enters the game: δ′ = δ13 + δ24 — the total transverse momentum of the 4-jet ensemble.

In what follows we consider transverse momenta of all four jets to be of the same order, Q2, down to smaller ones, δ and δ', — is not compromised by uncertainties in determination of the transverse momenta of the jets.

B. Back-to-back kinematics

The basic 2-jet production cross section scales, asymptotically, as

$$\frac{dσ^{(2-2)}}{dt} \propto \frac{α^2}{Q^4}. \quad (3)$$

According to (1), production of four jets in simultaneous hard collisions of four partons yields

$$\frac{dσ^{(4-4)}}{dt_1 dt_2} \propto R^{-2} \left( \frac{Q^2}{R^2} \right)^2 \propto \frac{α^4}{R^2 Q^8}, \quad (4a)$$

with $R^2 = 1/\langle Δ^2 \rangle$ the characteristic distance between the two partons in the hadron wave function. At large $Q^2$ this cross section is parametrically smaller than that for production of four well separated jets with transverse momenta $j^2_{2⊥} \sim Q^2$ in a 2-parton collision:

$$\frac{dσ^{(2-4)}}{dt_1 dt_2} \propto \frac{α^4}{Q^8}, \quad (4b)$$

(with transverse momenta of two out of four jets being integrated over).

Qualitatively, the production mechanism (4a) can be labelled a “higher twist effect”. Nevertheless, it may turn out to be essential comparable with the “leading twist” 2 → 4, Eq. (4b) — if one looks at specific kinematics of the 4-jet ensemble.

Let $z$ be the direction of colliding hadron momenta. Imagine that we are triggering on two jets moving along the $x$ and $y$ axes in the transverse plane, and look for two accompanying jets inside some solid angles $ΔΩ ∼ 4π$ around the $−x$ and $−y$ directions. The production mechanism (4b) does not populate this region: the higher order 2 → 4 QCD matrix element is enhanced when two final state partons become quasi-collinear but is perfectly
smooth in the back-to-back kinematics. Therefore, its contribution will be suppressed,

\[ \left( \frac{\Delta \Omega}{4\pi} \right)^2 \propto \frac{1}{R^2 Q^2}, \]

contrary to the 4 → 4 production mechanism which is concentrated in this very kinematical region.

C. Collinear approximation

The differential 4-jet production cross section possesses two collinear enhancements. Depending on the kinematics of the jets, they are, symbolically,

\[ d\sigma^{(4→4)} \propto \frac{\alpha_s^2}{\delta_{13}^2 \delta_{24}^2} d^2 j_{3\perp} d^2 j_{4\perp} \cdot d\Sigma, \]

\[ d\sigma^{(3→4)} \propto \frac{\alpha_s^2}{\delta_{13}^2} d^2 j_{3\perp} d^2 j_{4\perp} \cdot d\Sigma, \]

\[ \delta_{13}^2 \ll Q^2, \quad \delta_{24}^2 \ll Q^2; \quad (5a) \]

\[ \delta_{13}^2 \ll \delta^2 \ll Q^2, \quad \delta_{13} \approx \delta_{24}; \quad (5b) \]

Here \( d\Sigma = d\Sigma(t_1, t_2) \) is the cross section integrated over the transverse momenta of the “backward” jets 3 and 4. The integrated cross section \( d\Sigma \) contains the squared matrix element of the four-parton production and is of the order of \( \alpha_s^2 \), cf. Eq. \( [4a] \). At the Born level, the jets in pairs are exactly back-to-back, so that \( d\sigma^{(4→4)} \propto \delta^2(\delta_{13}) \delta^2(\delta_{24}) \) in \( [5a] \). To have a non-zero value of the transverse momentum imbalance, one has to have additional large transverse momentum parton(s) produced.

In the second important contribution to the cross section, Eq. \( [5b] \), one power of the coupling emerges from the splitting of a parton from one of the incident hadrons into two, and the second power is due to production of an additional final state parton with \( \vec{k}_\perp = -\vec{\delta} \).

In both cases the smallness due to additional powers of the coupling is compensated by two broad (logarithmic) integrations over transverse momentum imbalances as indicated in \( [5] \).

D. Double Logarithmic parton form factors

In the leading order in \( \alpha_s \), it suffices to have just one parton present with \( \vec{k}_\perp = -\vec{\delta}_{13} \) in order to assure \( \delta_{13} \neq 0 \). At the same time, inclusive production of accompanying partons with transverse momenta \( \vec{k}_\perp \) turns out to be suppressed in a broad interval \( \delta_{13}^2 \ll k_{\perp}^2 \ll Q^2 \), as long as one wants to preserve the collinear enhancement factor \( \delta_{13}^2 \) in the jet correlation \( [5a] \).

This dynamical “veto” has two consequences.

First of all, it results in reduction of the hardness scale of the parton distributions from the natural scale \( Q^2 \) (scale of the parton distributions in the integrated cross section) down to the observation-induced scale \( \delta_{13}^2 \ll Q^2 \).

Then, it introduces double logarithmic (DL) form factors of participating initial state partons, since the transverse momentum of the jet pair can be compensated not only by a hard (energetic) parton from inside initial parton distributions but also by a soft gluon whose radiation did not affect inclusive parton distributions due to real–virtual cancellation.

The presence of the DL form factors depending on the logarithm of a large ratio of scales, \( \ln(Q^2/\delta_{13}^2) \), is typical for the so-called “semi-inclusive” processes \( [20, 26] \).

Production of massive lepton pairs in hadron collisions (the Drell–Yan process) is a classical example of a two-scale problem. Here enter form factors of colliding quarks that depend on the ratio of the invariant mass \( q^2 \) to the transverse momentum of the lepton pair, \( \alpha_s \ln^2(q^2/q_{\perp}^2) \), in the dominant kinematical region \( q_{\perp}^2 \ll q^2 \):

\[
\frac{d\sigma}{dq^2 dq_{\perp}^2} = \frac{d\sigma_{\text{tot}}}{dq^2} \times \frac{\partial}{\partial q_{\perp}^2} \left\{ D_q^\alpha(x_1, q_{\perp}^2) D_{\bar{q}}^\alpha(x_2, q_{\perp}^2) S_q^2(q^2, q_{\perp}^2) \right\}. \quad (6)
\]

\( S_q \) is the double logarithmic QCD quark form factor.

Sudakov quark and gluon form factors can be expressed via the exponent of the total probability of the parton decay in the range of virtualities (transverse momenta) between the two hard scales:

\[
S_q(Q^2, \kappa^2) = \exp \left\{ - \int_{\kappa^2}^{Q^2} \frac{dk^2 \alpha_s(k^2)}{2\pi} \int_0^{1-k/Q} dz P_q^\alpha(z) \right\}, \quad (7a)
\]

\[
S_{\bar{q}}(Q^2, \kappa^2) = \exp \left\{ - \int_{\kappa^2}^{Q^2} \frac{dk^2 \alpha_s(k^2)}{2\pi} \int_0^{1-k/Q} dz \left[ z P_{\bar{q}}^\alpha(z) + n_f P_{\bar{g}}^\alpha(z) \right] \right\}. \quad (7b)
\]

Here \( P_i^k(z) \) are the non-regularized one-loop DGLAP splitting functions (without the “+” prescription):
the upper limit of $z$-integrals properly regularizes the soft
gluon singularity, $z \to 1$ (in physical terms, it can be
looked upon as a condition that the energy of a gluon
should be larger than its transverse momentum, [20]).

The case of hadron interactions producing large trans-
verse momentum partons (instead of colorless objects like
a Drell–Yan pair or an intermediate boson) is more in-
volved since here the transverse momentum imbalance
may be compensated by QCD radiation from the final
state partons too.

The azimuthal correlation between two nearly back-
to-back large transverse momentum particles was consid-
ered in [20]. An analog of the “DDT formula” has been
derived in the collinear approximation, which expression
contained the product of four form factors, two initial
parton distributions and two fragmentation functions.

E. Single Logarithmic soft gluon effects

The case when jets are being reconstructed in the final
state is more complicated to analyze as it yields an an-
swer depending on the jet finding algorithm. The prob-
lem has been addressed by Banfi and Dasgupta in [27]
where a smart way of defining the final state jets was
formulated that permitted to write down a resummed QCD formula for soft gluon effects in “2 partons \to 2
jets” cross sections.

Collinear logarithms due to hard splittings of the fi-
nal state partons do not pose a problem: such secondary
partons populate the jets. Partons that appear as sep-
arate out-of-jet radiation — and are relevant for trans-
verse momentum imbalance compensation — have to be
produced at sufficiently large angles with respect to the
jet axis. This is the domain of large-angle gluon radia-
tion. Production of soft gluons in-between jets is also
logarithmically enhanced and induces single logarithmic (SL) corrections, $\left[\alpha_s \ln(Q^2/\delta^2)\right]''$, that may also be sig-
nificant and should be resummed in all orders.

Contrary to collinear enhanced effects (that drive evolu-
tion of parton distributions and fragmentation func-
tions and determine the Sudakov form factors), the large-
angle gluon radiation cannot be attributed to one or an-
other of the partons participating in the hard scattering.
It is coherent and depends on the kinematics and color
topology of the hard parton ensemble as a whole. As a
result, resummation of these SL corrections becomes a
matrix problem that involves tracing various color states
of the parton system, see [27] and references therein.

In the present paper we concentrate on resummation of
collinear enhanced DL and SL terms and avoid com-
plications due to soft SL corrections. This means ig-
noring color transfer effects in hard interactions. Thus,
production of four jets with large transverse momenta
$j_\perp \sim Q$ and pair imbalances $\delta_{ij}$ will be equivalent, in our
treatment, to production of two colorless Drell–Yan pairs
with invariant masses $O(Q)$ and transverse momenta $\delta_{13}$
and $\delta_{24}$. Generalization of the results of [27] to the case
do double parton scattering seems straightforward and
should be considered separately.

III. GENERALIZED DOUBLE PARTON DISTRIBUTION

A. Geometry of $z$GPD

The $z$GPD in the expression for the multiparton pro-
duction cross section has a meaning of a two body form
factor when partons 1 and 2 receive transverse momenta
$\Delta$ and $-\Delta$ leaving the hadron intact. Nonrelativistic
analogue of this form factor is familiar from the double
scattering amplitude in the momentum space representa-
tion of the Glauber model, see e.g. [28]. Recall that [1] the
scale $\Delta$ in $z$GPD is conjugate to the relative transverse
distance between the two partons in the $z$GPD in the
impact parameter representation considered in [2, 3, 14].

Two partons may originate from soft low-scale fluc-
tuations inside the hadron; they can also emerge from a
perturbative splitting of a common parent parton at
relatively large momentum scales. It is clear that these
two contributions to $z$GPD will have essentially different
dependence on the parameter $\Delta$.

The first contribution we will denote

$$[2]D_{abc}^\alpha(x_1, x_2; q_1^2, q_2^2; \Delta),$$

with the subscript [2] stressing that here the partons $b$
and $c$ emerge from the non-perturbative wave function of
the hadron $a$. It should decrease rapidly at scales larger
than a natural scale of short-range parton correlation in
a hadron (this scale may be slightly different for quarks
and gluons and could in principle be significantly larger
than the $1/r_N$ as there exists another non-perturbative
scale of the chiral symmetry breaking which maybe as
large as 700 MeV).
Fast decrease of the product of two squared form factors. The parameter $m_F$ gluon form factor $G_g(x, q_1^2)$ to the geometrical factor Eq. (12) is given by

$$D_{ab}^c(x_1, x_2; q_1^2, q_2^2; \Delta) = F_g(\Delta^2; x_1, q_1^2) F_g(\Delta^2; x_2, q_2^2) \times G_g^c(x_1, q_1^2) G_g^c(x_2, q_2^2), \quad (10a)$$

where $G$ are the single-parton distributions and the two-gluon form factor $F_g$ can be parametrized as

$$F_g(\Delta^2) = \left(1 + \frac{\Delta^2}{m_g^2(x)}\right)^{-2}. \quad (10b)$$

The parameter $m_g^2$ is of the order of 1 GeV$^2$ for $x \sim 10^{-2}$ and gradually drops with decrease of $x$ and with increase of virtuality $Q^2$.

The second contribution we will denote

$$D_{ab}^{bc}(x_1, x_2; q_1^2, q_2^2; \Delta), \quad (11)$$

where the subscript $[1]$ stands as a reminder of the fact that $a$ and $c$ originate from perturbative splitting of a single parton from the hadron wave function. This contribution is practically $\Delta$-independent and should decrease with $\Delta$ much more slowly, due to logarithmic pQCD effects. (A steep power falloff starts only when $\Delta^2$ exceeds the relevant hard scale, $q^2$.)

By total cross section in the present context we mean the back-to-back 4-jet cross section integrated over pair jet imbalances in the dominant logarithmic region $\delta_{ik} \ll j_{i\perp} \simeq j_{k\perp}$.

We start by noting that the product of two small-distance parton fluctuations, $[1]D \times [1]D$, does not contribute to the process we are interested in. Indeed, in this case the integral over $\Delta$ in Eq. (1) formally diverges and yields a hard scale (instead of $Q^2$) in the numerator. This means a significant contribution to the cross section but not the one we are looking for. Below in Section IVB we will explicitly verify that a double hard collision of two parton pairs each of which originates from perturbative splitting, lacks the back-to-back enhancement. In fact, the product $[1]D \times [1]D$ corresponds to a one-loop correction to the “leading twist” perturbative production of four jets in a hard collision of two partons (“two-to-four”) whose distribution is smooth in the back-to-back region and as such gets subtracted as background.

Keeping this in mind, the back-to-back 4-jet production cross section is proportional to the inverse “interactions area” $S$ described by the expression

$$1 \over S = \int \frac{d^2 \Delta}{(2\pi)^2} \left( [2]D_a(\Delta) [2]D_b(\Delta) + [2]D_a(\Delta) [1]D_b(\Delta) + [1]D_a(\Delta) [2]D_b(\Delta) \right), \quad (12)$$

where indices $a$, $b$ mark two interacting nucleons. This expression is somewhat symbolic; a careful analysis of the “interaction area” will be carried out below in Sec. IV (see Eq. (32)).

The first term in Eq. (12) we will refer to as a “four-to-four” process: two partons from the wave functions of the hadron $a$ interact with two partons from the hadron $b$ producing four jets. The second and the third terms in Eq. (12) describe hard collisions of one parton from one hadron with two partons from the second hadron. Until recently, these “three-to-four” processes were commonly ignored in the literature (see, however, [25]). At the same time, they turn out to be somewhat enhanced.

Indeed, the contribution due to four-to-four processes to the geometrical factor Eq. (12) is given by

$$\int \frac{d^2 \Delta}{(2\pi)^2} F_g^2(\Delta^2) \times F_g^2(\Delta^2) = \frac{m_g^2}{\pi}, \quad (13a)$$

Fast decrease of the product of two squared form factors leads to fast convergence of the integral whose median is positioned at a value as low as $\Delta^2 \approx 0.1 m_g^2$. The case of the three-to-four process is different. This process corresponds, as we explained above, to interaction of the offspring of the perturbative splitting of a parton from the wave function of one hadron, with two partons from the non-perturbative wave function of the second colliding hadron. On the side of $[1]D$, the parameter $\Delta$ enters “perturbative loop” due to parton splitting and as a result the dependence of $[1]D$ on $\Delta$ turns out to be only logarithmic, that is, parametrically much slower than that of the non-perturbative form factor $F_g(\Delta^2)$. Thus, the three-to-four contribution to the double interaction cross section reduces to

$$\int \frac{d^2 \Delta}{(2\pi)^2} [1]D(x_1, x_2; \Delta) F_g^2(\Delta^2) = [1]D_{\Delta=0} \int \frac{d^2 \Delta}{(2\pi)^2} F_g^2(\Delta^2), \quad (13b)$$

(where we have neglected the logarithmic $\Delta$-dependence
of \([1] D\). This corresponds to the fact that in the impact parameter space, the distance between partons coming from a perturbative splitting is much smaller than the hadron size, so that the answer is proportional to the density of non-perturbative two-parton correlation at small distances — “in the origin”:

\[
\int \frac{d^2 \Delta}{(2\pi)^2} F_g(\Delta^2) = \frac{m_g^2}{3\pi}
\]  

(13c)

Comparison with the estimate (13a) shows that the contribution to the cross section of the “interference term” \(1 + 2 \rightarrow 4\) is enhanced, relative to the \(2 + 2 \rightarrow 4\) process, by the factor

\[
\frac{7}{3} \times 2 \sim 5.
\]  

(14)

(For the case of the Gaussian form factors this enhancement is 15% smaller — a factor of 4.) This estimate was obtained for the case when all four partons participating in the hard collisions are gluons. A detailed numerical study of the \(x_1\)-dependence of the effective interaction area \(S\) will be presented in the paper under preparation.

So we conclude that the three-to-four processes may provide a sizable contribution to the cross section even if they constitute a small correction to 2GPD.

### C. Perturbative QCD effects in 2GPD

Thus, we represent the generalized double parton distribution 2GPD as a sum of two terms:

\[
D_a^{b,c}(x_1, x_2; q_1^2, q_2^2; \Delta) = [2]D_a^{b,c}(x_1, x_2; q_1^2, q_2^2; \Delta) + [1]D_a^{b,c}(x_1, x_2; q_1^2, q_2^2; \Delta). 
\]  

(15)

The term \([2] D\) describes the distribution of two partons from the non-perturbative wave function of the hadron \(a\) that are independently evolved to large perturbative scales \(q_1^2\) and \(q_2^2\) according to the standard one-parton evolution equation. The perturbative evolution involves momentum scales much larger than the hadron wave function correlation scale \((\Delta^2) \sim Q_0^2\). Therefore, the evolution practically does not affect the \(\Delta\)-dependence of the two-parton spectrum. This piece of the 2GPD acquires but a mild additional logarithmic dependence at the tail of the \(\Delta\)-distribution in addition to a non-perturbative power falloff \([10]\).

The integral QCD evolution equation for \([2] D_a^{b,c}\) reads

\[
[2]D_a^{b,c}(x_1, x_2; q_1^2, q_2^2; \Delta) = S_b(q, Q^2) S_c(q, Q^2) [2]D_a^{b,c}(x_1, x_2; Q_0^2, Q_0^2; \Delta)
\]

\[
+ \sum_{b'} \int_{Q_{\text{min}}^2} q_{b'}^2 \frac{dk^2}{k^2} \alpha_s(k^2) \frac{1}{2\pi} S_b(q_{b'}^2, k^2) \int \frac{dz}{z} P_{b'}^{k}(z)[2]D_a^{b',c}(x_1, x_2; k^2; \Delta)
\]

\[
+ \sum_{c'} \int_{Q_{\text{min}}^2} q_{c'}^2 \frac{dk^2}{k^2} \alpha_s(k^2) \frac{1}{2\pi} S_c(q_{c'}^2, k^2) \int \frac{dz}{z} P_{c'}^{k}(z)[2]D_a^{b,c'}(x_1, x_2; q_{c'}, k^2; \Delta).
\]  

(16)

Here \(P_k^k(z)\) are the non-regularized one-loop DGLAP splitting functions \([5]\) and \(S_i\) — the double logarithmic Sudakov parton form factors defined in Eq. \([7]\).

The lower limit of the perturbative evolution in Eq. \([16]\),

\[
Q_{\text{min}}^2 = \max(Q_0^2, \Delta^2) \simeq Q_0^2 + \Delta^2,
\]  

(17)

is the only source of additional (logarithmic) \(\Delta\)-dependence. It emerges when \(\Delta^2\) exceeds — and substi-
The $\Delta$-dependence of $[1]D$ is very mild as it emerges solely from the lower limit of the logarithmic transverse momentum integration $Q_\min^2$.

### IV. ANALYSIS OF PERTURBATIVE TWO-PARTON CORRELATIONS

#### A. $3 \to 4$

Let us analyze the lowest order interaction amplitude shown in Fig. 1 that produces a double hard collision and involves parton splitting.

![Figure 1: $3 \to 4$](image)

We express parton momenta $k_i$ in terms of the Sudakov decomposition using the light-like vectors $p_a, p_b$ along the incident hadron momenta:

$$k_1 = x_1p_a + \beta p_b + k_\perp, \quad k_3 \simeq (x_3 - \beta)p_b;$$
$$k_2 = x_2p_a - \beta p_b - k_\perp, \quad k_4 \simeq (x_4 + \beta)p_b;$$
$$\vec{k}_\perp = \vec{\delta}_{12} = -\delta_{34} (\beta' \equiv 0); \quad k_0 \simeq (x_1 + x_2)p_a.$$

Here $k_0, k_3$ and $k_4$ are momenta of incoming (real) partons, and $k_1$ and $k_2$ — virtual ones. Light-cone fractions $x_i, i=1,\ldots,4,$ are determined by jet kinematics (invariant masses and rapidities of jet pair). The fraction $\beta$ that measures the difference in longitudinal momenta of the two partons coming from the hadron $b$, is arbitrary. Fixed values of the parton momenta $x_3 - \beta$ and $x_4 + \beta$ correspond to the plane wave description of the scattering process in which the longitudinal distance between the two scatterings is arbitrary. This description does not correspond to the physical picture of the process we are interested in. In order to ensure that the partons 3 and 4 originate from the same hadron of finite size, we have to introduce an integration over $\beta$ in the amplitude, in the region $\beta = \mathcal{O}(1)$.

The Feynman amplitude contains the product of two virtual propagators. The virtualities $k_1^2$ and the $k_2^2$ that enter the denominator of the amplitude in terms of the Sudakov variables read

$$k_1^2 = x_1\beta s - k_\perp^2, \quad k_2^2 = -x_2\beta s - k_\perp^2,$$

with $s = 2p_ap_b$ and $k_\perp^2 \equiv (\vec{k}_\perp)^2 > 0$ the square of the two-dimensional transverse momentum vector.

A singular contribution we are looking for originates from the region $\beta \ll 1$, so that precise form of the longitudinal smearing does not matter and the integral yields

$$N \int \frac{d\beta}{(x_1 + x_2) k_\perp^2} = \frac{2\pi i N}{(x_1 + x_2) k_\perp^2}.$$

The numerator of the amplitude is proportional to the first power of the transverse momentum $k_\perp$. As a result, the squared amplitude (and thus the differential cross section) acquires the necessary factor $1/\delta^2$ that enhances the back-to-back jet production.

#### B. $2 \to 4$

Now we should verify that the diagram of Fig. 2 where both incident partons split, and their offspring engage into double hard scattering, does not favor back-to-back jet kinematics. In other words, it does not lead to a small imbalance factor $1/\delta^2$ in the differential cross section.

![Figure 2: $2 \to 4$](image)
To get an enhanced contribution we have to have parton virtualities that enter the denominator of the Feynman amplitude to be relatively small, of the order of $\delta^2 \ll Q^2$. This implies $|\alpha|, |\beta| \ll 1$ in the essential integration region. Adopting this approximation, we can simplify parton propagators and reduce the longitudinal momentum integrations to the product of two independent integrals:

$$\frac{i}{s} \int \frac{d\beta}{2\pi i} \frac{1}{\beta x_1 s - k_\perp^2 + i\epsilon} \frac{1}{\beta x_2 s + k_\perp^2 - i\epsilon}$$

$$\times \int \frac{d\alpha}{2\pi i} \frac{1}{\alpha x_3 s - k_\perp'^2 + i\epsilon} \frac{1}{\alpha x_4 s + k_\perp'^2 - i\epsilon}$$

The remaining transverse momentum integration takes the form

$$\int \frac{d^2 k_\perp}{(2\pi)^2} \frac{V}{k_\perp^2 (k_\perp + \delta)^2}$$

Due to gauge invariance the numerator of the diagram — the “vertex factor” $V$ — is linear in transverse momenta of the loop partons: $V \propto k_\perp^\mu k_\perp'^\nu$. Therefore, the integral \[20\] produces no more than a logarithmic enhancement factor, $\ln(Q^2/\delta^2)$, instead of the power back-to-back singularity $Q^2/\delta^2$ we were looking for.

So, the diagram Fig. 2 with double parton splitting constitutes but a negligible loop correction to the usual "hedgehog" 4-jet kinematics typical for $2 \rightarrow 4$ QCD processes. The fact that this loop diagram does not produce a pole singularity in $\delta^2$ could have been extracted, e.g., from numerical studies of double $Z$-boson production in two-parton collisions \[30\] and, more generally, of multileg parton amplitudes \[31\].

The logarithmic character of this correction has been recently confirmed by the systematic study of “box integrals” in \[24\].

The presence of the double parton splitting contribution of Fig. 2 is being treated in the literature as a source of potential problem of double counting (see, e.g., \[15, 31, 32\]). The present paper solves this problem.

C. $3 \rightarrow 4$ with additional parton emission

We have to return now to the $3 \rightarrow 4$ process and examine the possibility of producing an additional parton, in collinear enhanced manner, in order to lift off the Born level kinematical constraint $\delta' = 0$.

Consider the diagram of Fig. 3a. The momenta of quasi-real colliding partons are

$$k_0 \simeq (x_1 + x_2 + \alpha)p_a; k_3 \simeq (x_3 - \beta)p_b; k_4 \simeq (x_4 + \beta + \frac{\delta'^2}{\alpha s})p_b;$$

and the radiated on-mass-shell parton carries momentum

$$\ell - k_2 = \alpha p_a + \frac{\delta'^2}{\alpha s} p_b - \delta', \quad \delta' = \delta_{13} + \delta_{24}.$$  

We have three virtual propagators subject to integration over $\beta$:

$$k_1 = x_1 p_a + \beta p_b + \delta_{13},$$  \hspace{1cm} (21a)

$$k_2 = x_2 p_a - \left(\beta + \frac{\delta'^2}{\alpha s}\right)p_b + \delta_{24},$$  \hspace{1cm} (21b)

$$\ell = (x_2 + \alpha)p_a - \beta p_b - \delta_{13}.$$  \hspace{1cm} (21c)
Closing the contour around the pole $k_1^2 + i\varepsilon = 0$, we obtain $\beta s = \delta_{13}^2/x_1$ and

$$-\ell^2 = (x_2 + \alpha)\beta s + \delta_{13}^2 = x_2 + \frac{x_2 + \alpha}{x_1}, \quad (22a)$$

$$-k_2^2 = x_2\left(\beta + \frac{\delta_{21}^2}{\alpha s}\right) s + \delta_{24}^2 = \frac{x_1\delta_{24}^2 + x_2\delta_{13}^2 + x_2 \delta_{13}^2}{x_1}, \quad (22b)$$

Taken together with the residue of the $\beta$-integration, $1/x_1$, Eq. (22a) produces the universal factor present in all the amplitudes considered, (including the diagrams with parton emission off the external lines, see Fig. 4 below):

$$P^{-1} = x_1\delta_{24}^2 + x_2\delta_{13}^2 + \frac{x_1 x_2 \delta_{21}^2}{\alpha}. \quad (23)$$

We observe that both propagators (22) are enhanced in the back-to-back kinematics. The amplitude of Fig. 3a gives a double collinear enhanced contribution to the cross section in the region

$$\delta_{13}^2 \ll \delta_{24}^2 \simeq \delta_{13}^2. \quad (24a)$$

The inequality (24a) corresponds to the following physical picture. An incident parton $k_0$ splits into $k_1$ and $\ell$ early, at time $O(\sqrt{s}/\delta_{13}^2)$ corresponding to some comparatively low perturbative scale $\delta_{13}$. At this time scale

the parton 1 collides with 3, while the parton $\ell$ keeps evolving and scatters off 4 with a much larger momentum transfer $\delta_{24}$. Evolution of the parton $\ell$ in between these two scales is the origin of probable (logarithmically enhanced) production of additional parton(s).

Analogously, the diagram Fig. 3b with a parton produced off the virtual line 1 contributes in the complementary kinematical region

$$\delta_{24}^2 \ll \delta_{13}^2 \simeq \delta_{13}^2. \quad (24b)$$

Full perturbative analysis of the production of a parton from inside the “splitting fork”, Fig. 3 together with emission off the incoming line “0” (to be treated below in Sec. V B) is sketched in the Appendix.

V. DIFFERENTIAL DISTRIBUTION

A. $2 + 2 \to 4$

Now that we know the structure of the perturbative corrections to $2GPD$, Eqs. (15)–(18), we are in a position to write down the generalization of the DDT formula (6) for (the first contribution to) the differential cross section of 4-jet production in nearly back-to-back kinematics. It reads

$$\pi^2 \frac{d\sigma^{(4\to4)}}{d^2\delta_{13}d^2\delta_{24}} = \frac{d\sigma_{\text{part}}}{dt_1 dt_2} \frac{\partial}{\partial \delta_{13}^2} \frac{\partial}{\partial \delta_{24}^2} \left\{ [2] D_{a}^{1,2}(x_1, x_2; \delta_{13}^2, \delta_{24}^2) \times [2] D_{b}^{3,4}(x_3, x_4; \delta_{13}^2, \delta_{24}^2) \right\}.$$  \quad (25)

Here $d\sigma_{\text{part}}$ is the cross section of double hard parton scattering, and $S_i$ stand for Sudakov form factors of four participating partons. Sum over parton species and convolution over $\Delta$ as in Eq. (1) is implied.

Taking derivative over the scale $\delta^2$ of the function depending on $\alpha_s \log \delta^2$, produces the factor $\alpha_s/\delta^2$. Differentiating the Sudakov form factor of a given parton describes the situation when the jet imbalance is compensated by radiation of a soft gluon off this parton. Differentiation of the parton distribution corresponds to the situation when a hard parton takes the recoil.

B. $1 + 2 \to 4$

The differential transverse momentum imbalance distribution due to the cross-terms $[2] D \times [1] D$ contains two pieces.

1. Two compensating partons

The first one has the same structure as Eq. (25):

$$\pi^2 \frac{d\sigma^{(3\to4)}}{d^2\delta_{13}^2 d^2\delta_{24}^2} = \frac{d\sigma_{\text{part}}}{dt_1 dt_2} \frac{\partial}{\partial \delta_{13}^2} \frac{\partial}{\partial \delta_{24}^2} \left\{ [1] D_{a}^{1,2}(x_1, x_2; \delta_{13}^2, \delta_{24}^2) \times [2] D_{b}^{3,4}(x_3, x_4; \delta_{13}^2, \delta_{24}^2) \right\}.$$  \quad (26)
Sum over parton species and convolution over \( \vec{\Delta} \) is implied as above in Eq. (25).

As above, taking the derivatives in Eq. (26) corresponds to fixing transverse momenta of two final state partons that compensate jet pair imbalances: \( -\vec{\delta}_{13} \) and \( -\vec{\delta}_{24} \).

Consider now the correlation term \( [1]D \) of Eq. (26). If we apply the derivatives to the parton distributions \( D \) in the integrand of Eq. (18) for the correlation term \( [1]D \), this contribution will correspond to production of two momentum compensating quanta in the course of evolution of the system of two partons with account of small-distance perturbative correlation between them. In this case the scale of the core parton splitting stays smaller than the two external scales \( \delta_{13}, \delta_{24} \), and is being integrated over.

2. One compensating parton

The correlation term \( [26] \) contains an additional option. Namely, instead of creating intermediate state partons that keep evolving up to external scales, the perturbative splitting may produce that very parton that gets engaged in the hard scattering. This possibility is also contained in Eq. (26): it corresponds to the differentiation of the upper limit of the virtuality integral in Eq. (18) over the smaller of the two imbalances, \( \min\{\delta_{13}, \delta_{24}\} \). One of the two parton distribution functions in the integrand then collapses to \( \delta(1-x/y) \). Taking the second derivative over the larger imbalance of the second parton distribution produces the contribution described by the diagram of Fig. 3 that we have discussed above in Section IV C.

C. \( 1 + 2 \to 4 \), endpoint contribution

Finally, there is a possibility that both partons emerging from the perturbative splitting in \( [3]D \) experience hard collisions straight away, without any further evolution. Jet imbalances stemming from such an eventuality are no longer independent but, on the contrary, are strongly correlated. At the “Born level” one has

\[
\frac{d\sigma}{d\delta_{13}d\delta_{24}} \propto \frac{\alpha_s}{\delta^2} \delta(\vec{\delta}_{13} + \vec{\delta}_{24}), \quad \delta^2 \equiv \delta_{13}^2 = \delta_{24}^2.
\]

With account of additional radiation, the delta-function gets replaced by the pole enhancement of the differential cross section in the imbalance of imbalances in a specific region of jet momenta when the pair imbalances are practically equal-and-opposite:

\[
\frac{d\sigma}{d\delta_{13}d\delta_{24}} \propto \frac{\alpha_s^2}{\delta^2 \delta'^2}, \quad \delta'^2 \ll \delta^2 \equiv \delta_{13}^2 \simeq \delta_{24}^2.
\]

Importantly, this contribution is also double-collinear enhanced and therefore has to be taken into full consideration. This eventuality is not incorporated in Eq. (26) and should be taken care of separately.

In this kinematical region the parton that carries the compensating transverse momentum \( -\vec{\delta}' \) can be produced off one of the external legs, as shown in Fig. 4.

FIG. 4: \( 3 \to 4 \) with real parton emission off the external lines

The corresponding contributions to the differential 4-jet production cross section can be combined into the following
relatively compact expression:

\[
\frac{\pi^2 \, \sigma_0^{(3-4)}}{d^2 \delta_1 \, d^2 \delta_2} = \frac{d \sigma_{\text{part}}}{dt_1 \, dt_2} \cdot \alpha_s(\delta^2) \sum_c P_{c}^{1,2} \left( \frac{x_1}{x_1 + x_2} \right) S_1(Q^2, \delta^2) S_2(Q^2, \delta^2) \\
\times \frac{\partial}{\partial \delta^2} \left( S_c(\delta^2, \delta^2) \frac{G_2^c(x_1 + x_2; \delta^2, Q^2)}{x_1 + x_2} S_2(Q^2, \delta^2) S_4(Q^2, \delta^2) \times 2 \, d \sigma_{\text{part}}^{3,4} (x_3, x_4; \delta^2, \delta^2) \right) + \left\{ a \leftrightarrow b \right\}.
\]  

(27)

Once again, integral over \( \Delta \) on the r.h.s. is implicit here. In this formula the splitting of the parton “c” from the hadron “a” is written explicitly, represented by the splitting function \( P_{c}^{1,2} \). The splitting occurs at the virtuality scale \( \delta^2 \). However, accompanying production of secondary real particles is vetoed starting from a much smaller scale, \( \delta^2 \ll \delta^2 \). This explains appearance of the Sudakov form factors of five participating partons in \( 27 \).

In the impulse approximation, there is one parton produced with the transverse momentum \(-\vec{\delta}\) compensating the 4-jet imbalance, while transverse momenta of all other real partons are smaller: \( k_{2}^{\perp} \ll \delta^2 \). Production probability of this parton integrated over its energy (rapidity) is embedded into the derivative of the product of three Sudakov form factors and of the initial parton distributions depending on \( \delta^2 \) as the upper evolution scale. As before in Eq. \( 25 \), by differentiating form factors we obtain soft gluon radiation off three external lines (“0”), “3” and “4”. Differentiation of the parton distribution functions describes real parton production due to “hard” splittings.

The full answer is given by the sum of Eqs. \( 25 \) – \( 27 \):

\[
\frac{\pi^2 \, d \sigma^{(4-4)}}{d^2 \delta_1 \, d^2 \delta_2} + \frac{\pi^2 \, d \sigma_0^{(3-4)}}{d^2 \delta_1 \, d^2 \delta_2} + \frac{\pi^2 \, d \sigma_0^{(3-4)}}{d^2 \delta_1 \, d^2 \delta_2}.
\]  

(28)

D. Distribution over jet imbalances in the extreme back-to-back limit \( \delta^2, \delta^2 \rightarrow 0 \)

As we have repeatedly stressed above, double parton collisions constitute but a small correction to 4-jet production if characterized by their contribution to the total cross section. However, “higher twist” four-to-four and three-to-four processes are specific in that they populate the kinematical region of relatively small pair jet imbalances, \( \delta_1^2, \delta_2^2, \delta^2 = (\delta_1^2 + \delta_2^2) \ll Q^2 \).

Being a multi-scale hard process, back-to-back jet production cross section possesses double logarithmic parton form factors. The Sudakov suppression is the price one pays for having the differential distribution peaked, \( d \sigma \propto \delta^{-2} \), which enhancement implies vetoing the bremsstrahlung radiation of gluons with transverse momenta in a broad interval ranging from \( k_{2}^{\perp} > \delta^2 \) all the way up to \( k_{2}^{\perp} < Q^2 \).

The formulas \( 25 \) – \( 27 \) for the differential imbalance spectra were derived in the impulse approximation in which it is a single parton (soft gluon) that compensates the transverse momentum imbalance, while radiation of other partons with \( k_{2}^{\perp} > \delta^2 \) is strictly vetoed. However, in the extreme \( \delta^2 \rightarrow 0 \) limit the form factor suppression becomes so strong as to override the pole enhancement. In these conditions the impulse approximation is no longer valid, and instead of vetoing accompanying radiation one has to impose the condition that the real partons carry, in aggregate, the given transverse momentum: \( \sum k_{2}^{\perp} = -\delta \). This implies exponentiating soft gluon radiation in the impact parameter space, and evaluating the inverse Fourier transform to access the resulting \( \delta \)-spectrum \( 33, 35 \). The result is the flattening of the spectrum in the \( \delta^2 \rightarrow 0 \) limit.

The precise width of this plateau depends on parton form factors as well as on parton distributions, and should be calculated numerically. However, a rough semi-quantitative estimate of the value of the critical transverse momentum \( p_0 \) where the spectrum starts to flatten out can be obtained from the following simplified consideration.

In the double logarithmic approximation one can neglect effects due to parton distributions and state that the plateau develops when the product of relevant Sudakov form factors just compensates the pole enhancement:

\[
\frac{d}{d \ln \kappa^2} \ln \left( \prod_i S_i(Q^2, \kappa^2) \right) = 0.
\]

From Eqs. \( 7 \) we obtain the condition

\[
\Sigma_C \, \frac{\alpha_s(p_0^2)}{\pi} \ln \frac{Q}{p_0} = 1, \tag{29}
\]

where \( \Sigma_C \) is the sum of the “squared color charges” (Casimir operators) of participating partons. Substituting the one-loop expression for the running coupling, we have

\[
\ln \frac{p_0^2}{\Lambda^2_{\text{QCD}}} = \ln \frac{Q^2}{\Lambda^2_{\text{QCD}}} \cdot (1 + \frac{\beta_2}{2 \Sigma_C})^{-1}, \tag{30}
\]

with \( \beta_2 = 11 \nu_C / 3 - 2n_f / 3 \) the leading coefficient of the QCD \( \beta \)-function. The estimate of the “flattening momentum” follows:

\[
p_0^2 \propto \Lambda^2_{\text{QCD}} \left( \frac{Q^2}{\Lambda^2_{\text{QCD}}} \right)^\gamma, \quad \gamma = \left( 1 + \frac{\beta_2}{2 \Sigma_C} \right)^{-1}. \tag{31}
\]

Using \( \beta_2 = 9 \) for \( n_f = 3 \) light quark flavors, the exponent \( \gamma \) in Eq. \( 31 \) equals 0.372 for a quark pair (\( \Sigma_C = 8 / 3 \),...
0.456 for a system of a quark and a gluon ($\Sigma_C = 13/3$), and 0.571 for two gluons ($\Sigma_C = 6$).

The value $p_0$ characterizes a typical total transverse momentum of the final state produced in a hard 2-parton collision, be it a Drell–Yan pair, $W/Z$ or a system of four hadron jets (or, say, three jets and a large transverse momentum photon).

In the case of four-(three-) parton collisions, one has double back-to-back enhancement, accompanied by the momentum photon).

hadron jets (or, say, three jets and a large transverse collision, be it a Drell–Yan pair, $W/Z$ momentum of the final state produced in a hard 2-parton collisions (2 → 4), are comparable. This makes the experimental studies of double hard collisions not an easy task. To extract MPI, one should learn to reliably subtract the $2 \to 4$ contributions. This “background” should be theoretically predicted and generated at the NLO level (for the amplitude) in order to accommodate loop correction effects discussed above in Section IV B.

We conclude the discussion of the differential distribution with two remarks.

As we have seen above, in the back-to-back kinematics the 4-jet production in double parton collisions (4 → 4, 3 → 4), and due to higher order QCD effects in standard two-parton collisions (2 → 4), are comparable. This makes the experimental studies of double hard collisions not an easy task. To extract MPI, one should learn to reliably subtract the $2 \to 4$ contributions. This “background” should be theoretically predicted and generated at the NLO level (for the amplitude) in order to accommodate loop correction effects discussed above in Section IV B.

On the experimental side, the Tevatron experiments [16–18] have set a fashion of searching for MPI by looking at angular correlations between transverse momentum imbalances in $\gamma + 3$ jets (and $\gamma + 2$ jets, [15]) events. Such a strategy can well be used to signal the presence of MPI. At the same time, it is not suited for quantitative analysis as it mixes together contributions of hard and soft physics. In order to extract and study double hard collisions, and thus to get hold of inter-parton correlations inside hadrons, one should instead base the measurements on the values (and relative geometry) of transverse momentum imbalances.

VI. TOTAL CROSS SECTION

Integrating the differential distribution [25] over imbalances $d^2\delta_{13}, d^2\delta_{24}$ we obtain the total cross section of the two dijet production in the double parton collision process in the leading collinear approximation:

$$
\frac{d\sigma(x_1, x_2, x_3, x_4)}{dt_1 dt_2} = \frac{d\sigma^{13}}{dt_1} \frac{d\sigma^{24}}{dt_2} \times \left\{ \frac{1}{S_4} + \frac{1}{S_3} \right\}
$$

(32a)

Here $S_4$ and $S_3$ are $x_i$- and $Q^2$-dependent parameters that describe effective interaction areas characterizing 4- and 3-parton collisions, correspondingly. They are given by the following expressions:

$$
S_4^{-1}(x_1, x_2, x_3, x_4; Q^2) = \int \frac{d^2\Delta}{(2\pi)^2} \left\{ [2]D_a(x_1, x_2; Q^2, Q^2; \tilde{\Delta}) [2]D_b(x_3, x_4; Q^2, Q^2; -\tilde{\Delta})
\right.
$$

$$
+ [2]D_a(x_1, x_2; Q^2, Q^2; \tilde{\Delta}) [1]D_b(x_3, x_4; Q^2, Q^2; -\tilde{\Delta})
+ [1]D_a(x_1, x_2; Q^2, Q^2; \tilde{\Delta}) [2]D_b(x_3, x_4; Q^2, Q^2; -\tilde{\Delta}) \right\},
$$

(32b)

and

$$
S_3^{-1}(x_1, x_2, x_3, x_4; Q^2) = \sum_c \int \frac{d^2\Delta}{(2\pi)^2} p_{1,2}^{1,2} \left( \frac{x_1}{x_1 + x_2} \right) \int Q^2 \frac{d\delta^2}{\delta^2} \frac{\alpha_s(\delta^2)}{2\pi} \prod_{i=1}^{4} S_i(Q^2, \delta^2)
$$

$$
\times \frac{Q_a^c(x_1 + x_2, \delta^2, Q_0^2)}{x_1 + x_2} \left\{ [3]D_b^{3,4}(x_3, x_4; \delta^2, \delta^2; \tilde{\Delta}) \right. \left.+ (a \leftrightarrow b; 1,2 \leftrightarrow 3,4) \right\}
$$

(32c)

Unlike the case of $4 \to 4$, at the level of the differential spectrum the contribution $3 \to 4$ due to an interplay of non-perturbative and perturbative two-parton correlations is not given by an expression with full derivatives over two relevant scales; therefore the final expression for $S_3^{-1}$ contains logarithmic integration over the hard scale $\delta^2$ running up to the overall hardness scale of the parton scattering $Q^2$.

VII. CONCLUSIONS AND OUTLOOK

In this work we aimed at generalization of the QCD factorization theorem to multiple parton interactions in hadron–hadron collisions. Such generalization is necessary for carrying out systematic studies — both theoretical and experimental — of the parton correlations inside hadrons, and allows one to incorporate in a model independent way higher order pQCD effects. In the small-$x$ domain, generalization of the MPI analysis presented here may be looked upon as realization of the Gribov Pomeron Calculus, with exact account of energy-
m
t
momentum conservation.

We considered four jet production and have shown that double hard parton collisions play a significant role in the back-to-back kinematics. This kinematics one selects by demanding the transverse momentum imbalances of two pairs of jets, \( \delta_1^2 \equiv (\vec{J}_\perp + \vec{J}_\perp')^2 \) and \( \delta_2^2 \equiv (\vec{J}_\perp + \vec{J}_\perp')^2 \), to be much smaller than the overall hard scale of the scattering process: \( \delta_1^2, \delta_2^2 \ll Q^2 \sim J^2 \).

We were able to analyze and calculate corresponding differential spectra and total cross sections with account of all collinear enhanced pQCD radiative effects giving rise to Sudakov form factors and to scaling violations in of all collinear enhanced pQCD radiative effects giving
differential spectra and total cross sections with account
developed here provides the framework, and
in particular, in the present study we did not hunt
loose itselfs in DIS processes, etc. In addition, the
able in the conventional DIS processes, etc. In addition,
parton correlations in the nucleon and the nuclei, for the
formalism developed in this paper will give the

3 \to 4 transitions already discussed in \[1\]:

\[
\frac{d\sigma}{d^2\delta_1^2 d^2\delta_2^2} \propto \frac{1}{\delta_1^2} \cdot \frac{1}{\delta_2^2}.
\]

The corresponding differential distribution is a direct generalization of the “DDT formula” \[20\] containing derivatives over transverse momentum imbalances, with Eq. \(25\) describing 4 \to 4 collisions and Eq. \(26\) — a crosstalk between large- and small-distance two-parton correlators.

At the same time, the parton splitting may occur just at the scale \(\delta^2\) and produce two partons that, without any further evolution, interact with two partons from the wave function of the second hadron. Such eventualty also has a necessary back-to-back enhancement but of a different structure:

\[
\frac{d\sigma}{d^2\delta_1^2 d^2\delta_2^2} \propto \frac{1}{\delta_1^2} \cdot \frac{1}{\delta_2^2} \cdot \delta^2 \equiv (\sum_{i=1}^4 \vec{J}_\perp^i)^2 \sim \delta_1^2 \sim \delta_2^2.
\]

Such contributions are absent in the parton model. They cannot be represented as the product of two GPDs and must be taken into account separately, see Eq. \[27\]. A detailed numerical investigation of these contributions will be presented elsewhere \[36\].

Formulas derived in this work can be used to address various aspects of the LHC physics. Our results are directly applicable to the case of double Drell–Yan processes at LHC. In particular they can be applied to the recently observed production of heavy quarkonia at the LHC \[37\]. The formalism developed in the present paper should help to improve the accuracy of the prediction of QCD backgrounds in searches for new physics. It also can be used for further studies of the nucleon structure, in particular of the short-range non-perturbative inter-parton correlations in the nucleon and the nuclei, for the study of the higher twist contributions that are unavailable in the conventional DIS processes, etc. In addition, it would be interesting to extend the formalism developed here to investigate QCD medium effects that manifest themselves in AA collisions in recent experiments at CERN.

On the experimental side, observation of the parton splitting processes discussed in this paper will give the

\[2\]GPDs, that have been introduced in \[1\]. Our main

\[3\]AA

\[4\]PASS

\[5\]AA

\[6\]CERN. 

\[7\]AA

\[8\]AA

\[9\]AA

\[10\]AA

\[11\]AA

\[12\]AA

\[13\]AA

\[14\]AA

\[15\]AA

\[16\]AA

\[17\]AA

\[18\]AA

\[19\]AA

\[20\]AA

\[21\]AA

\[22\]AA

\[23\]AA

\[24\]AA

\[25\]AA

\[26\]AA

\[27\]AA

\[28\]AA

\[29\]AA

\[30\]AA

\[31\]AA

\[32\]AA
first direct evidence of an interplay between large- and short-distance QCD correlations in hard processes. To achieve this goal one has to look for enhancement of 4-jet production in the kinematical region of jet transverse momenta

\[(\hat{j}_1 + \hat{j}_2 + \hat{j}_3 + \hat{j}_4)^2 \ll (\hat{j}_1 + \hat{j}_3 + \hat{j}_4)^2 \approx (\hat{j}_2 + \hat{j}_4)^2.\]

Acknowledgments

The authors are grateful to the Galileo Galilei Institute for Theoretical Physics, Florence, for an opportunity to get together and fine tune this work during the QCD after the start of LHC Workshop in September–October 2011.

Appendix

Here we present the analysis of real parton production in the course of \(1 \to 2\) parton splitting, \(0 \to 1 + 2\), determining the correlated 4-jet production. We consider three contributing diagrams: that of Fig. 4(a) for emission off the initial parton "0", and those of Fig. 3 for emission off the offspring partons "1" and "2".

1. Vertices

It is straightforward to write down exact expressions for the amplitude using the economic technique of quasi-parton states developed by Bukhvostov, Frolov, Lipatov and Kuraev (BFKL; not to be confused with BFKL) and described in [38]. In the BFKL technique the quark and (axial gauge) gluon Green functions are replaced by propagators of two physical \(\pm\) helicity states. In general, in this framework additional terms looking like contact four-vertex factors are replaced by propagators of two physical states developed by Bukhvostov, Frolov, Lipatov and Kuraev.

Effective three-parton interaction vertices are proportional to the linear combination of the transverse momenta of participating partons weighted with the longitudinal light-cone momentum fractions:

\[K_{\perp}^\mu = \alpha_i k_{i\perp}^\mu - \alpha_j k_{j\perp}^\mu.\]  

(A.1)

In the matrix element the vector index is contracted with the gluon polarization vector (for details see [38]).

The products of two vertices for our amplitudes read

\[
\begin{align*}
0 & : T_0 \cdot (x_1 + x_2 + \alpha)\delta' \times (x_2\delta_{13} - x_1\delta_{24}) \quad (A.2a) \\
1 & : T_1 \cdot (-(x_1 + x_2)\delta_{13} - x_1\delta_{24}) \times (x_1 + x_2)\delta_{24} \quad (A.2b) \\
2 & : T_2 \cdot ((x_2 + \alpha)\delta_{24} + x_2\delta_{13}) \times (x_1 + x_2)\delta_{13} \quad (A.2c)
\end{align*}
\]

Here the first vertex corresponds to the real parton (soft gluon) radiation, the second vertex — to the internal hard splitting.

2. Amplitudes

Adding the virtual propagators

\[
\begin{align*}
\alpha \frac{\delta'}{\delta^2}, \quad x_2 \frac{\delta_{13}}{\delta_{24}} \quad \text{and} \quad x_1 \frac{\delta_{13}}{\delta_{13}}
\end{align*}
\]

correspondingly, and extracting the factor \((x_1 + x_2 + \alpha)(x_1 + x_2)\) we get the amplitudes

\[
\begin{align*}
0 & = T_0 \cdot \frac{\alpha\delta'}{\delta^2} \times x_2\delta_{13} - x_1\delta_{24} \cdot x_1 + x_2 \quad (A.3a) \\
1 & = T_1 \cdot \frac{-x_1\delta' - x_2\delta_{13}}{x_1 + x_2 + \alpha} \times x_1\delta_{24} \cdot x_1 + x_2 + \alpha \quad (A.3b) \\
2 & = T_2 \cdot \frac{x_2\delta' + \alpha\delta_{24}}{x_1 + x_2 + \alpha} \times x_1\delta_{13} \cdot x_1 + x_2 + \alpha \quad (A.3c)
\end{align*}
\]

Here \(P\) is the common propagator factor defined in [23]:

\[
P^{-1} = x_1\delta_{24} + x_2\delta_{13} + (x_1 + x_2) r^{-1} \delta^2, \quad r = \frac{\alpha(x_1 + x_2)}{x_1 x_2}. \tag{A.4}
\]

3. Limits

In the case of soft gluon radiation, \(\alpha \ll x_i\), the parameter \(r\) defined in (A.4) becomes numerically small. Therefore one should examine, separately, two kinematical regions:

1. \(\delta^2 \ll r \cdot \delta^2\)

Only "0" contributes:

\[M \simeq T_0 \cdot \frac{\alpha\delta'}{\delta^2} \times \frac{\delta_{13}}{(x_1 + x_2)\delta_{24}^2} \tag{A.5a}\]

2. \(\delta^2 \gg \delta^2 \gg r \cdot \delta^2\)

Here the amplitudes (A.3b) and (A.3c) contribute, and we obtain

\[M \simeq (T_1 + T_2) \cdot \frac{x_1 x_2 \delta' \delta_{13}}{x_1 + x_2 + \alpha \delta_{13}^2} \quad (A.5b)\]

reproducing the structure of (A.5a). Emissions from 1 and 2 are coherent, and due to conservation of the color current, \(T_1 + T_2 = T_0\), the two expressions coincide. As a result, (A.5a) applies in the entire kinematical region \(\delta^2 \ll \delta^2\), irrespectively to the value of \(r\).

This contribution — real parton production off the incoming line "0" — is a part of the \(3 \to 4\) process, Eq. (27), described in Section \(\text{XIII}\).

Additional collinear enhanced contributions arise in two complementary regions:

3. \(\delta_{24}^2 \ll \delta_{13}^2 \simeq \delta^2\)

Here the amplitude 1 dominates:

\[M \simeq T_1 \cdot \frac{\alpha}{x_1 + x_2 + \alpha} \frac{\delta_{13} \delta_{24}}{\delta_{13} \delta_{24}} \tag{A.6a}\]
4. $\delta_{13}^2 \ll \delta_{24}^2 \simeq \delta^2$

Alternatively, here the dominant amplitude is that with emission off the line 2:

$$M \simeq T_2 \cdot \frac{\alpha}{x_1 + x_2 + \alpha} \frac{\delta_{13} \delta_{24}}{\delta_{13} \delta_{24}},$$

(A.6b)

The contributions (A.6) are related with internal evolution of the 2GPD of the hadron $a$ and are contained in Eq. (25). Specifically, in the kinematical region where the imbalances of two jet pairs are significantly different, these contributions correspond to the case when the differentiation over the smallest of the two imbalances applies to the 2GPD and acts upon the integration limit of the correlation $|1|D$ term in (18).