Scintillation reduction by use of multiple Gaussian laser beams with different wavelengths

Avner Peleg and Jerome V. Moloney

Arizona Center for Mathematical Sciences,
University of Arizona, Tucson, Arizona 85721, USA

Abstract

We study the scintillation index of $N$ partially overlapping collimated lowest order Gaussian laser beams with different wavelengths in weak atmospheric turbulence. Using the Rytov approximation we calculate the initial beam separation that minimizes the longitudinal scintillation. Further reduction of the longitudinal scintillation is obtained by optimizing with respect to both beam separation and spot size. The longitudinal scintillation of the optimal $N$-beam configurations is inversely proportional to $N$, resulting in a 92% reduction for a 9-beam system compared with the single beam value. The radial scintillation values for the optimal $N$-beam configurations are significantly smaller than the corresponding single beam values.

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I. INTRODUCTION

Propagation of light through atmospheric turbulence is the subject of intensive research owing to the many applications in free space laser communications [1]. In these applications it is desirable to reduce the turbulence effects on the propagating light. One promising possibility to achieve this goal is by using temporally partially coherent optical field consisting of multiple laser beams with different wavelengths [2, 3, 4, 5]. Indeed, in a typical setup in which the detector’s response time is large compared with the inverse of the frequency difference between any pair of beams in the input field, rapidly oscillating contributions to the total intensity average out. Consequently, one can expect smaller values of high moments of the intensity compared with corresponding single beam values. This would result in smaller values for the scintillation index and for the average signal to noise ratio (SNR).

Generation of temporally partially coherent light consisting of multiple beams with different wavelengths can be efficiently realized by using an array of vertical external cavity surface lasers (VECSELs). These devices have the advantage of generating high power, spectrally narrow, wavelength tunable TEM\textsubscript{00} beams (lowest order Gaussian beams) [6].

Propagation of temporally partially coherent light in atmospheric turbulence was first studied by Fante [7, 8], who obtained approximate analytic expressions for the scintillation index of a single infinite planar wave. More recently Kiasaleh studied propagation of an infinite multi-wavelength planar wave in weak atmospheric turbulence and showed that the achievable SNR is larger in the multi-wavelength case compared with the single-wavelength case [2, 3]. These previous studies focused on infinite planar waves, whereas in practice, Gaussian laser beams with finite initial spot size are employed. Since the dynamics of the optical field can strongly depend on the initial spot size it is important to take into account the finite spatial dimension of the beams. Furthermore, the optical field of \(N\) collimated TEM\textsubscript{00} beams with different wavelengths depends on the wavelength separation and also on the spatial separation between the beam centers at the transmitter. Therefore, one can exploit these two different dependences to reduce scintillation and optimize system performance against turbulence effects. The dependence of the scintillation index on the wavelength separation of multiple overlapping TEM\textsubscript{00} beams was studied in Ref. [4]. It was found that for typical lasercom setups modest scintillation reduction of about 10% can be achieved by controlling the wavelength difference between the beams. We emphasize that
only the case where the beams are completely overlapping at the transmitter plane was
considered in Ref. [4]. In the current Letter we focus attention on scintillation reduction
by varying the spatial separation between the beams at the transmitter. We show that this
approach leads to a much stronger decrease of the scintillation compared with the approach
employed in Ref. [4]. Moreover, our approach allows us to find the initial beam separation
that minimizes scintillation, thus providing a simple solution for the important problem of
optimizing temporally partially coherent sources of light against turbulence effects.

In Ref. [5] we took the first step in this approach and established the framework for
calculating the scintillation index for multiple partially overlapping beams in weak atmo-
spheric turbulence. Using the Rytov approximation and considering a typical 2-beam sys-
tem we found the initial beam separation that minimizes the longitudinal scintillation. We
showed that the longitudinal and radial scintillation for the optimal 2-beam configuration
are smaller by about 50% and 35%-40%, respectively, compared with the corresponding
single-beam values. However, two important aspects of the problem were not addressed in
Ref. [5]: (1) the N-dependence of the longitudinal scintillation reduction compared with the
single-beam case, and (2) the possibility to optimize the system with respect to both initial
beam separation and initial spot size. In this Letter we address these two central issues in
detail.

II. CALCULATION OF THE SCINTILLATION INDEX

Consider propagation of N collimated linearly polarized TEM$_{00}$ beams with differ-
ent wavelengths $\lambda_j$, $j = 1, \ldots, N$, in weak atmospheric turbulence. Assuming that the
beams propagate along the z axis and denoting by $d_j$ the beam-center locations at the
input plane $z = 0$, the magnitude of the total electric field $E$ at $z = 0$ is
$E(r,0,t) = \sum_{j=1}^{N} U_j(r_j,0) \exp[-i\omega_j t]$, where $U_j(r_j,0) = \exp[-r_j^2/W_{0j}^2]$, $r$ is the radius vector in the $xy$
plane, $r_j \equiv r - d_j$, $t$ is time, $k_j = 2\pi/\lambda_j$ are wavenumbers, $\omega_j = k_j c$ are angular frequencies,
and $c$ is the speed of light. In addition, $W_{0j}$ are the initial spot sizes and we assume that all
beams have the same amplitude. Assuming weak turbulence, the propagation is described
by $N$ uncoupled linear wave equations

$$\nabla^2 U_j + k_j^2 [1 + 2n_1(r,z)] U_j = 0,$$

(1)
where \( n_1(r, z) \) represents the refractive index fluctuations, \(|n_1(r, z)| \ll 1\). To solve Eq. (1) we follow Ref. [1] and employ the paraxial approximation together with the Huygens-Fresnel integral and the second order (with respect to \( n_1 \)) Rylov perturbation method. The total intensity at \( z = L \) is

\[
I(r, L, t) = \sum_{j=1}^{N} I_j(r_j, L) + \sum_{j}^{N} \sum_{m \neq j}^{N} U_j(r_j, L)U_m^*(r_m, L) \exp \left[i(\omega_m - \omega_j)t\right],
\]

(2)

where \( I_j(r_j, L) = |U_j(r_j, L)|^2 \) is the intensity of the \( j \)-th beam. The intensity measured by the detector is the time average \( I_{det}(r, L) \equiv \tau^{-1} \int_0^\tau dt I(r, L, t) \), where \( \tau \) is the response time of the detector. Assuming a slow detector and \( \lambda_j \neq \lambda_m \) for \( j \neq m \), we neglect the terms \( U_j U_m^* j \neq m \), which are rapidly oscillating with time. Therefore, the measured intensity is

\[
I_{det}(r, L) \approx \sum_{j=1}^{N} I_j(r_j, L).
\]

(3)

The total scintillation index for the \( N \)-beam system is

\[
\sigma_I^2(r, L) = \langle I_{det}^2(r, L) \rangle / \langle I_{det}(r, L) \rangle^2 - 1,
\]

(4)

where \( \langle \ldots \rangle \) stands for average over different realizations of turbulence disorder. Using Eqs. (3) and (4) we obtain

\[
\sigma_I^2(r, L) = \left( \sum_{j=1}^{N} \langle I_j(r_j, L) \rangle \right)^{-2} \left[ \sum_{j=1}^{N} \langle I_j^2(r_j, L) \rangle + 2 \sum_{j}^{N} \sum_{m \neq j}^{N} \langle I_j(r_j, L)I_m(r_m, L) \rangle \right] - 1.
\]

(5)

The total scintillation index \( \sigma_I^2 \) can be decomposed into a longitudinal component \( \sigma_I^2(L) \equiv \sigma_I^2(0, L) \) and a radial component \( \sigma_r^2(r, L) \equiv \sigma_I^2(r, L) - \sigma_I^2(L) \).

In calculating intensity moments we assume that the perturbation field in the Rylov approximation is a Gaussian random variable and that the turbulence is statistically homogeneous and isotropic. Consequently, the average intensity of the \( j \)-th beam is given by

\[
\langle I_j(r_j, L) \rangle = \frac{W_{0j}^2}{W_j^2} \exp \left[-\frac{2r_j^2}{W_j^2} + H_{ij}(r_j, L)\right],
\]

(6)
where \( W_j \) is the spot size at distance \( L \), and \( H_{1j} \) is expressed in terms of a double integral of the spectral density of the refractive index fluctuations \( \Phi_n(\kappa) \) over wavenumber \( \kappa \) and propagation distance \( z \). [See Ref. [3], Eq. (18)]. The average of the second moment is

\[
\langle I_j^2(r_j, L) \rangle = \langle I_j(r_j, L) \rangle^2 \exp [H_{2j}(r_j, L)],
\]

(7)

where \( H_{2j} \) is given by another double integral of \( \Phi_n(\kappa) \) over \( \kappa \) and \( z \). [See Ref. [3], Eq. (20)]. The cross-intensity term \( \langle I_j(r_j, L)I_m(r_m, L) \rangle \) is given by [5]

\[
\langle I_j(r_j, L)I_m(r_m, L) \rangle = \langle I_j(r_j, L) \rangle \langle I_m(r_m, L) \rangle \times
\]

\[
\exp \{ E_{2jm}(r_j, r_m; k_j, k_m) + E_{2mj}(r_m, r_j; k_m, k_j) +
2\text{Re} [E_{3jm}(r_j, r_m; k_j, k_m)] \},
\]

(8)

where \( E_{2jm}, E_{2mj} \) and \( E_{3jm} \) are three different integrals of \( \Phi_n(\kappa) \) over \( \kappa \) and \( z \). [See Ref. [3], Eqs. (22-24)].

We consider two typical free space laser communication setups, in which the central wavelength is \( \lambda_c = 10^{-6}\text{m} \), the wavelength spacing is \( \Delta\lambda = 10^{-8}\text{m} \), all beams are collimated and have the same initial spot size and on-axis amplitude. In setup A \( L = 1\text{km} \) and the refractive index structure parameter is \( C_n^2 = 3.0 \times 10^{-15}\text{m}^{-2/3} \), and in setup B \( L = 10\text{km} \) and \( C_n^2 = 10^{-16}\text{m}^{-2/3} \). Both setups correspond to weak atmospheric turbulence conditions, where the Rytov variance \( \sigma_R^2 = 1.23C_n^2k^{7/6}\lambda^{11/6} \) is about 0.1 and 0.23, respectively, for all beams. We use the Von Kármán spectrum to describe the refractive index fluctuations. Thus, \( \Phi_n(\kappa) = 0.033C_n^2(\kappa^2 + \kappa_{\text{out}}^2)^{-11/6} \exp (-\kappa^2/\kappa_{\text{in}}^2) \), where \( \kappa_{\text{in}} = 5.92/l_0, \kappa_{\text{out}} = 1/L_0, l_0 \) and \( L_0 \) are the turbulence inner and outer scales, respectively, and \( l_0 = 1.0\text{mm}, L_0=1.0\text{m} \) are used.

For even \( N \) we consider initial configurations in which the beam centers are located on a circle with diameter \( d \) centered about the \( z \)-axis with equal angles between \( \mathbf{d}_{j-1} \) and \( \mathbf{d}_j \). Thus, for \( N = 4 \), for example, the centers are at \( \mathbf{d}_1 = d\hat{x}/2, \mathbf{d}_1 = d\hat{y}/2, \mathbf{d}_1 = -d\hat{x}/2, \) and \( \mathbf{d}_1 = -d\hat{y}/2 \). For odd \( N \) we consider the same geometry as in the \( N - 1 \) case, with an additional beam on the \( z \)-axis.

The \( d \)-dependence of the longitudinal scintillation for 2-, 3-, 4-, 5-, 8-, and 9-beam configurations in setup A with initial spot sizes \( W_0 = 1\text{cm} \) is shown in Fig. 1 together with the corresponding value for a single TEM\(_{00} \) beam with the same total power and initial spot size. One can see that in each of the \( N \geq 2 \) cases the \( \sigma_{i,l}^2 \)-curve exhibits a minimum.
at an intermediate $d$ value, $d_0 = 2.8\text{cm}, 4.4\text{cm} 3.6\text{cm}, 4.6\text{cm}, 4.0\text{cm} \text{ and } 4.8\text{cm},$ for the $2-, 3-, 4-, 5-, 8-, \text{ and } 9$-beam configurations, respectively. These minima correspond to the optimal configurations of the $N$ beams for the given physical conditions and geometric arrangements, where optimization is with respect to longitudinal scintillation. Comparison with the single-beam value shows that the longitudinal scintillation is reduced by 53.4%, 63.2%, 82.2%, 84.4%, 88.1%, and 92.0% for the $2-, 3-, 4-, 5-, 8-, \text{ and } 9$-beam optimal configurations, respectively. Moreover, analysis of the longitudinal scintillation values for the optimal configurations shows that $\sigma_{I,l}^2$ decreases like $1/N$ with increasing $N$.

An important question concerns the possibility to further reduce the scintillation by optimizing with respect to the initial spot size. This question is addressed in Fig. 2, which shows the $d$-dependence of the longitudinal scintillation for different $W_0$ values for a 5-beam system in setup B. One can see that the minimum value of $\sigma_{I,l}^2$ first decreases with increasing $W_0$ and then increases. Hence, the optimal configuration for the 5-beam system in setup B, is the one with $W_0 \simeq 4\text{cm}$ and $d_0 = 13.2\text{cm}$. Notice that the final free space spot size of the beams for the optimal configuration in this case is only 8.8cm.

When the spot size is comparable with the radius of the receiver’s collecting lens, the radial scintillation becomes important. In this case it is essential to understand whether the radial scintillation of the optimal $N$-beam configurations, where optimization is with respect
FIG. 2: Longitudinal scintillation $\sigma_{l,l}^2$ vs initial beam separation $d$ for different $W_0$ values for a 5-beam system in setup B. The solid, dashed, dotted, dashed-dotted, short-dashed, and short-dotted lines correspond to $W_0=0.5\,\text{cm}$, 1cm, 2cm, 4cm, 5cm, and 6cm, respectively.

FIG. 3: Circularly averaged radial scintillation index $\sigma_{rr}^2$ vs radius $r$ for the optimal 4-beam (squares) and 2-beam (circles) configurations, and for a single beam with the same power and initial spot size (triangles), in setup A.

to longitudinal scintillation, is sufficiently small compared with the single-beam value. Notice that in the $N$-beam case $\sigma_{rr}^2(r, L)$ is not radially symmetric. To enable comparison with the single-beam case we define the circularly averaged radial scintillation in the $N$-beam case as $\sigma_{rr}^2(r, L) \equiv \langle \sigma_{rr}^2(r, L) \rangle_\theta$, where $\langle \ldots \rangle_\theta$ denotes averaging over the angle $\theta$. Figure 3 shows the $r$-dependence of $\sigma_{rr}^2$ for the optimal 4- and 2-beam configurations and for a single beam with the same power and initial spot size, all in setup A. One can see that the radial scintillation for
the optimal 4-beam and 2-beam configurations are smaller by about 65%-80% and 35%-40%, respectively, compared with the corresponding single beam values. Therefore, optimization of the \( N \)-beam configurations with respect to the longitudinal scintillation leads to significant reduction in the radial scintillation, and this reduction effect grows with increasing \( N \).

III. CONCLUSION

We calculated the scintillation index for an array of \( N \) partially overlapping collimated TEM\(_{00}\) beams with different wavelengths in weak atmospheric turbulence using the Rytov perturbation method. We showed that both the longitudinal and the radial scintillation can be significantly reduced compared with the corresponding single-beam values by optimizing the beam array with respect to initial beam separation and spot size. These reduction effects grow with increasing \( N \), resulting in a 92% reduction in the longitudinal scintillation for an optimal 9-beam system.

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