The phenomenological implications of the M-theory limit in which supersymmetry is broken by the F-terms of five-brane moduli is investigated. In particular we calculate the supersymmetric spectrum subject to constraints of correct electroweak symmetry breaking. We find interesting differences especially in the squark sector compared with M-theory scenarios with standard embedding and weakly-coupled Calabi-Yau compactifications in the large $\mathcal{T}$-limit.
One of the most interesting developments in M-theory [1, 2, 3, 4, 5, 6, 7, 8, 9, 10] model building is that new non-perturbative tools have been developed which allow the construction of realistic three generation models [11]. In particular the inclusion of five-brane moduli $Z_n$, (which do not have a weakly coupled string theory counterpart) besides the metric moduli $T, S$ in the effective action leads to new types of $E_8 \times E_8$ symmetry breaking patterns as well as to novel gauge and Kähler threshold corrections. As a result the soft-supersymmetry breaking terms differ substantially from the weakly coupled string.

Phenomenological implications of the effective action of M-theory with standard embedding of the spin connection into the gauge fields have been investigated in [12, 13, 14, 15, 16]. Some phenomenological implications of non-standard embeddings in M-theory with and without five-branes have been studied in [17, 18]. However, a full phenomenological analysis of the interesting case when the $F$-terms associated with the five-branes dominate those associated with metric moduli ($FZ_n \gg FS, FT$), including the constraint of radiative electroweak symmetry-breaking [19] has not been performed. It is the purpose of this paper to carry out such an analysis.

The soft supersymmetry-breaking terms are determined by the following functions of the effective supergravity theory [11, 18]:

\[ K = -\ln(S + \bar{S}) - 3\ln(T + \bar{T}) + K_5 + \frac{3}{T + \bar{T}}(1 + \frac{1}{3}e_O)H_{pq}C^p_OC^q_O, \]

\[ f_O = S + B_O T, \quad f_H = S + B_H T, \]

\[ W_O = d_{pqr}C^p_OC^q_OC^r_O \]

where $K$ is the Kähler potential, $W$ the perturbative superpotential, and $f_O, f_H$ are the gauge kinetic functions for the observable and hidden sector gauge groups respectively. $K_5$ is the Kähler potential for the five-brane moduli $Z_n$ and $H_{pq}$ is some $T$-independent metric. Also

\[ e_O = b_O \frac{T + \bar{T}}{S + \bar{S}}, \quad e_H = b_H \frac{T + \bar{T}}{S + \bar{S}} \]

(2)

The various instanton numbers are given by the following expressions

\[ b_O = \beta_O + \sum_{n=1}^N (1 - z_n)^2 \beta_n \]

\[ B_O = \beta_O + \sum_{n=1}^N (1 - Z_n)^2 \beta_n \]

\[ B_H = \beta_H + \sum_{n=1}^N (Z_n)^2 \beta_n \]

\[ b_H = \beta_H + \sum_{n=1}^N z_n^2 \beta_n \]

(3)
Since a Calabi-Yau manifold is compact, the net magnetic charge due to orbifold planes and 5-branes is zero. Consequently the following cohomology condition is satisfied

$$\beta_O + \sum_{n=1}^N \beta_n + \beta_H = 0$$  \hspace{1cm} (4)$$

$S, T$ are the dilaton and Calabi-Yau moduli fields and $C^p$ charged matter fields. The five-brane moduli are denoted by $Z_n$, and $\text{Re} Z_n = z_n \in (0,1)$. The superpotential and the gauge kinetic functions are exact up to non-perturbative effects.

Given eqs(1) one can determine [11, 18] the soft supersymmetry breaking terms for the observable sector gaugino masses $M_{1/2}$, scalar masses $m_0$ and trilinear scalar masses $A$ as functions of the auxiliary fields $F^S, F^T, F^n$ of the moduli $S, T$ fields and five-brane moduli $Z_n$ respectively.

$$M_{1/2} = 1 \left( \frac{S + S}{(S + S)(1 + \frac{B_0^T + B_0 T}{S + S})} \right) (F^S + F^T B_O + TF^n \partial_n B_O)$$

$$m_0^2 = V_0 + m_{3/2}^2 - \frac{1}{(3 + e_O)^2} \left[ e_O (6 + e_O) \left( \frac{|F^S|^2}{(S + S)^2} \right) + 3(3 + 2e_O) \right]$$

$$+ \frac{6e_O}{(S + S)(T + T)} \text{Re} F^S \bar{F}^S + \frac{6e_O}{b_O} \partial_n b_O \partial_n b_O F^n F^{\bar{n}}$$

$$- \frac{6e_O b_O}{b_O} \partial_n b_O \text{Re} F^S F^n + \frac{6e_O}{b_O} \partial_n b_O \text{Re} F^S F^{\bar{n}} \text{Re} F^T F^{\bar{n}} ,$$

$$A = -\frac{1}{3 + e_O} \left\{ \frac{F^S (3 - 2e_O)}{S + S} + \frac{3e_O F^T}{T + T} \right\}$$

$$+ F^n \left( \frac{3e_O b_O}{b_O} \partial_n b_O - (3 + e_O) \partial_n K_5 \right) \right\}$$

$$+ F^S \left( \frac{3e_O b_O}{b_O} \partial_n b_O - (3 + e_O) \partial_n K_5 \right) \right\}$$

where $\partial_n \equiv \frac{\partial}{\partial Z_n}$. The bilinear $B$-parameter associated with non-perturbatively generated $\mu$ term in the superpotential is given by [18]:

$$B_\mu = \frac{F^S (e_O - 3)}{(3 + e_O)(S + S)} - \frac{3(e_O + 1)F^T}{(T + T)(3 + e_O)}$$

$$+ \frac{1}{3 + e_O} \left[ (3 + e_O) F^n \partial_n K_5 - 2 F^n \frac{e_O b_O}{b_O} \partial_n b_O \right] - m_{3/2}$$

From now on we assume that only one five-brane contributes to supersymmetry-breaking $^3$. Then the auxiliary fields are given by [15, 20]

$$F^1 = \sqrt{3} m_{3/2} C (\partial_1 \partial_1 K_5)^{-1/2} \sin \theta_1$$

$^3$We assume very small $CP$-violating phases in the soft terms.
\[ F^S = \sqrt{3}m_{3/2}C(S + \bar{S}) \sin \theta \cos \theta_1 \]
\[ F^T = m_{3/2}C(T + \bar{T}) \cos \theta \cos \theta_1 \]  

The goldstino angles are denoted by \( \theta, \theta_1 \), \( m_{3/2} \) is the gravitino mass and \( C^2 = 1 + \frac{V_0}{3m_{3/2}^2} \) with \( V_0 \) the tree level vacuum energy density. The five-brane dominated supersymmetry-breaking scenario corresponds to \( \theta_1 = \frac{\pi}{2} \), i.e \( F^T, F^S = 0 \), and we take the five brane which contributes to supersymmetry breaking to be located at \( z_1 = 1/2 \) in the orbifold interval. We also set \( C = 1 \) in the above expressions assuming zero cosmological constant.

We now consider the supersymmetric particle spectrum for a single five-brane present. Our parameters are, \( e_O, \partial_1 \partial_1 K_5, \partial_1 K_5, m_{3/2}, \text{sign} \mu \) (which is not determined by the radiative electroweak symmetry breaking constraint), where \( \mu \) is the Higgs mixing parameter in the low energy superpotential. The ratio of the two Higgs vacuum expectation values \( \tan \beta = \frac{\langle H_0^2 \rangle}{\langle H_0^1 \rangle} \) is also a free parameter if we leave \( B \) be determined by the minimization of the one-loop Higgs effective potential. If \( B \) instead is given by \( (6) \), one determines the value of \( \tan \beta \). For this purpose we take \( \mu \) independent of \( T \) and \( S \) because of our lack of knowledge of \( \mu \) in \( M \)-theory. We treat \( e_O \) as a free parameter as the problem of stabilizing the dilaton and other moduli has not yet been solved, although there has been an interesting work in this area [21].

The instanton numbers are model dependent. In this paper we choose to work with the interesting example [18] with \( \beta_O = -2 \) and \( \beta_1 = 1 \) which implies \( b_O = -7/4 \). This implies that \( b_H = 5/4 \) and allow us to study the region of parameter space with \( -1 < e_O \leq 0 \), which is not accessible in strongly coupled \( M \)-theory scenarios with standard embedding. We also choose \( \partial_1 K_5 = \partial_1 \partial_1 K_5 = 1 \). We have investigated deviations from these values in order to examine the robustness of our results. We comment on that below.

We use the following experimental bounds from unsuccessful searches at LEP and Tevatron for supersymmetric particles [23]. We require the lightest chargino \( M_{\chi_1^+} \geq 95 \) GeV, and the lightest Higgs, \( m_{h_0} > 79.6 \) GeV. A lower limit on the mass of the lightest stop \( m_{\tilde{t}_2} > 86 \) GeV, from \( \tilde{t}_2 \rightarrow c\chi_1^0 \) decay in D0 is imposed. The stau mass eigenstate (\( \tilde{\tau} \)) should be heavier than 53 GeV from LEP2 results.

The soft masses start running from a mass \( R_1^{-1} \sim 7.5 \times 10^{15} \) GeV with \( R_1 \) the extra \( M \)-theory dimension. This is perhaps the most natural choice, although values as low as \( 10^{13} \) GeV are possible and have been advocated by some authors [4]. However, the analysis of [4] disfavours such scenarios. For the most part of our analysis we shall therefore consider the former value of \( R_1 \), but we shall also comment on the consequences of the latter. Then using \( (5),(6) \) as boundary conditions for the soft terms, one evolves the renormalization group equations down to the weak scale and determines the sparticle spectrum compatible with the constraints of correct electroweak symmetry breaking and the above experimental constraints on the sparticle spectrum.
Electroweak symmetry breaking is characterized by the extrema equations

\[ \frac{1}{2} M_Z^2 = \frac{\bar{m}_{H_1}^2 - \bar{m}_{H_2}^2 \tan^2 \beta}{\tan^2 \beta - 1} - \mu^2 \]

\[ - B\mu = \frac{1}{2}(\bar{m}_{H_1}^2 + \bar{m}_{H_2}^2 + 2\mu^2) \sin 2\beta \]  
(8)

where

\[ \bar{m}_{H_1, H_2}^2 \equiv m_{H_1, H_2}^2 + \frac{\partial \Delta V}{\partial v_{1,2}} \]  
(9)

and \( \Delta V = (64\pi^2)^{-1} \text{Str} M^4 \left[ \ln \left( \frac{M^2}{Q^2} \right) - \frac{3}{2} \right] \) is the one loop contribution to the Higgs effective potential. We include contributions only from the third generation of particles and sparticles.

Since \( \mu^2 \gg M_Z^2 \) for most of the allowed region of the parameter space [22], the following approximate relationships hold at the electroweak scale for the masses of neutralinos and charginos, which of course depend on the details of electroweak symmetry breaking.

\[ m_{\chi^\pm_1,2} \sim m_{\chi_0}^2 \sim 2m_{\chi_0} \]
\[ m_{\chi_3,4} \sim m_{\chi^\pm_2} \sim |\mu| \]  
(10)

In (10) \( m_{\chi^\pm_{1,2}} \) are the chargino mass eigenstates and \( m_{\chi_i}, i = 1 \ldots 4 \) are the four neutralino mass eigenstates with \( i = 1 \) denoting the lightest neutralino. The former arise after diagonalization of the mass matrix.

\[ M_{ch} = \begin{pmatrix} M_2 & \sqrt{2}m_W \sin \beta \\ m_W \cos \beta & -\mu \end{pmatrix} \]  
(11)

The stau mass matrix is given by the expression

\[ M_\tau^2 = \begin{pmatrix} \mathcal{M}_{11}^2 & m_\tau(A_\tau + \mu \tan \beta) \\ m_\tau(A_\tau + \mu \tan \beta) & \mathcal{M}_{22}^2 \end{pmatrix} \]  
(12)

where \( \mathcal{M}_{11} = m_L^2 + m_E^2 - \frac{1}{2}(2m_W^2 - M_Z^2) \cos 2\beta \) and \( \mathcal{M}_{22} = m_E^2 + m_L^2 + (M_W^2 - M_Z^2) \cos 2\beta \). where \( m_L^2, m_E^2 \) refer to scalar soft masses for lepton doublet, singlet respectively.

As has been noted in [18], in the case when only the five-branes contribute to supersymmetry-breaking the ratio of scalar masses to gaugino mass, \( m_0/|M_{1/2}| > 1 \) for \( e_O > -0.65 \). This is quite interesting since scalar masses larger than gaugino masses are not easy to obtain in the weakly-coupled heterotic string or M-theory compactification with standard embedding. In the case of non-standard embeddings without five-branes there is small region of the parameter space where it is possible to have \( m_0/|M_{1/2}| > 1 \).
In fig.1 we present the sparticle spectrum plotted against \( \tan \beta \) for \( \epsilon_O = -0.6 \), \( m_{3/2} = 165 \text{ GeV} \) and \( \mu < 0, \partial_1 K_5 = \partial_1 \bar{K}_5 = 1 \). For this choice of \( \epsilon_O \), \( m_0/|M_{1/2}| \sim 1.2 \) at the unification scale. The scalar masses are bigger than the gaugino masses, in contrast to weakly coupled string scenarios emerging from the large \( T \)-limit of Calabi-Yau compactifications where gaugino masses are generically bigger than scalar masses. One can see that the lightest supersymmetric particle is the lightest neutralino. It is a linear combination of the superpartners of the photon, \( Z^0 \) and neutral-Higgs bosons,

\[
\chi_1^0 = c_1 \tilde{B} + c_2 \tilde{W}^3 + c_3 \tilde{H}_1^0 + c_4 \tilde{H}_2^0
\]  

(13)

The neutralino \( 4 \times 4 \) mass matrix can be written as

\[
\begin{pmatrix}
M_1 & 0 & -M_Z A_{11} & M_Z A_{21} \\
0 & M_2 & M_Z A_{12} & -M_Z A_{22} \\
-M_Z A_{11} & M_Z A_{12} & 0 & \mu \\
M_Z A_{21} & -M_Z A_{22} & \mu & 0
\end{pmatrix}
\]

with

\[
\begin{pmatrix}
A_{11} & A_{12} \\
A_{21} & A_{22}
\end{pmatrix}
= \begin{pmatrix}
\sin \theta_W \cos \beta & \cos \theta_W \cos \beta \\
\sin \theta_W \sin \beta & \cos \theta_W \sin \beta
\end{pmatrix}
\]

The next to lightest supersymmetric particle (NLSP) in this scenario is the lightest chargino. In this figure we plot chargino masses heavier than the experimental bound from LEP, \( m_{\chi^+} \geq 95 \text{ GeV} \). In fact this experimental bound imposes \( m_{3/2} > 160 \text{ GeV} \). The lightest Higgs eigenstate (\( h \)) satisfies \( m_h > 85 \text{ GeV} \) for \( \tan \beta = 2.5 \) and \( m_h \geq 109 \text{ GeV} \) for \( \tan \beta = 10 \). Also \( m_{\tilde{t}_2} \geq 183 \text{ GeV} \) for all values of \( \tan \beta \) and the lightest stau satisfies \( m_{\tilde{\tau}_2} \geq 127 \text{ GeV} \). The lightest stau mass eigenstate is the next to the NLSP (NNLSP) mass eigenstate. This is to be contrasted with the extreme M-theory limit with standard embedding for a goldstino angle chosen to get vanishing scalar masses at the unification scale. In the latter scenario the lightest stau is the NLSP and for values of \( \tan \beta \geq 12 \) becomes the LSP. It differs from the M-theory scenario with goldstino angle \( \frac{\pi}{8} \) and \( \tan \beta \geq 10 \) since in this case again the lightest stau is the NLSP. It also differs from these M-theory scenarios in the squark sector. In particular, the lightest stop is always much lighter than the CP odd Higgs mass eigenstate \( A^0 \). This is not the generic case with M-theory scenarios with standard embeddings [12] and weakly coupled CY compactifications in the large \( T \)-limit [20]. Similarly the lightest sbottom \( \tilde{b}_2 \) (sb2 in the figure) is almost degenerate with the CP odd Higgs mass (is lighter for small \( \tan \beta \) and heavier for large \( \tan \beta \)) in contrast to M-theory scenarios with standard embedding and various goldstino angles. In the latter case the lightest sbottom is much heavier than the CP-odd Higgs mass [12, 13] in most region of the parameter space. We also observe from figure 1 that the lightest stop is lighter than the lightest sbottom mass eigenstate.
For $\tan \beta \leq 2$ and for this particular gravitino mass the lightest Higgs becomes lighter than the current experimental bound of 79.6 GeV. In figure 3 we present the sparticle spectrum plotted against $\tan \beta$ for $\mu > 0$. Through the region of parameter space the lightest neutralino is the LSP and the lightest chargino the NLSP. The lightest stau is the NNLSP.

In figure 2 we present the sparticle spectrum with respect to the parameter $e_O$ for $\tan \beta = 2.5$, and $m_{3/2} = 250$ GeV. We note the following very interesting features. As the parameter $e_O$ increases through $-0.65$ the scalar masses become heavier than the gaugino masses. The imprint in the low energy spectra is that the lightest chargino is the NLSP, and for $e_O > -0.5$ the lightest stop becomes the NNLSP. In the limit $e_O \to 0$ the scalar masses are much heavier than gaugino masses, $m_0 \gg M_{1/2}$. For smaller (more negative) values of $e_O$ the lightest stau becomes lighter as the gaugino masses become heavier and eventually for $e_O \leq -0.7$ becomes the NLSP. For instance for $e_O = -0.7, |M_{1/2}|/m_O \sim 1.32$ while for $e_O = -0.8, |M_{1/2}|/m_O \sim 2.33$. In the region $-1 < e_O < -0.7$, the spectrum resembles strongly coupled $M$-theory scenarios without five-branes for some values of the goldstino angle, e.g. $\theta = \frac{\pi}{20}$. In contrast for this particular $\tan \beta$ in $M$-theory scenarios without five branes and for $\theta = \frac{\pi}{8}$ the lightest chargino is the NLSP.

When the $B_\mu$ term is given by (4) correct electroweak symmetry breaking occurs for $18 \leq \tan \beta \leq 20.5$ in the region $-0.7 \leq e_O \leq -0.2$. In fig.4 we present the supersymmetric particle spectrum with respect to $e_O$ for $\tan \beta = 20$ and $m_{3/2} = 230$ GeV. The same qualitative features hold as in the case of low $\tan \beta$ discussed earlier. The neutralino is the LSP in most of the parameter space ($-0.85 < e_O < 0$). For $e_O > -0.65$ the lightest chargino is the NLSP while for $-0.85 < e_O < -0.7$ the lightest stau is the NLSP. For $e_O \geq -0.5$ the lightest stop is the NNLSP.

We also investigated the supersymmetric particle spectrum for $\partial_1 \partial_2 K_5 = 2, 0.5$. The same qualitative features discussed earlier hold also for this choice of the five-brane Kahler potential metric.

In conclusion we have calculated the supersymmetric particle spectrum in the five-brane dominated limit of $M$-theory subject to the constraint of correct electroweak symmetry breaking and experimental bounds from accelerator physics. For most of the parameter space the lightest neutralino is the LSP. Depending on the value of $e_O$ and $\tan \beta$ the NLSP is either the lightest chargino or the the lightest stau. In the region of parameter space for which $m_0/|M_{1/2}| > 1$ the lightest chargino is the NLSP. For $\tan \beta = 2.5$, $m_{3/2} = 250$ GeV, and for $e_O > -0.5$ the lightest stop becomes the NNLSP. For $e_O < -0.5$ the lightest stau is the NNLSP and eventually as $e_O$ decreases becomes the NLSP. In the region of parameter space in which $m_0 > |M_{1/2}|$ the five-brane dominated scenario leads to qualitative differences from $M$-theory scenarios with standard embeddings and Calabi-Yau compactifications in the large $T$-limit, especially in the squark sector. The resulting particle spectra should be subject of experimental investigation. They also motivate the study of how well the dark matter constraints are satisfied. We expect that for large values of the $e_O$ close to 0 the dark matter constraints can become quite important in setting upper limits on the gluino mass. Also in this region the higgsino component of the lightest neutralino increases.
and this can be quite interesting for direct dark matter detection. These matters will
be a subject of a future publication [24].

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Figure 1: Sparticle spectrum vs $\tan\beta$ for $e_O = -0.6, \partial_1 K_5 = \partial_1 \bar{\partial}_1 K_5 = 1, m_{3/2} = 165 GeV \mu < 0$. 
Figure 2: Sparticle spectrum vs $e_O$ for fixed $\tan \beta = 2.5, m_{3/2} = 250 GeV$. 
Figure 3: Sparticle spectrum vs tan $\beta$ for $e_O = -0.6, \partial_1 K_5 = \partial_1 \tilde{\partial}_1 K_5 = 1, m_{3/2} = 165 GeV, \mu > 0.$
Figure 4: Sparticle spectrum vs $\frac{r_{\phi}}{12}$ for $\tan \beta = 20, m_{3/2} = 230 GeV$