The minimum rotation period of millisecond pulsars

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ABSTRACT
A simple and natural explanation for the minimum period of millisecond pulsars follows from a correlation between the accretion rate and the frozen surface dipole magnetic field resulting from Ohmic diffusion through the neutron star crust in initial stages of accretion in low mass X-ray binaries.

Key words: stars: neutron – pulsars: general – pulsars: millisecond - binaries: low mass X-ray.

1 INTRODUCTION
The first millisecond pulsar, PSR J1937+214, was discovered by Backer et al. (1982). Immediately after the discovery, two groups independently proposed the idea that spin-up by accretion in low mass X-ray binaries leads to millisecond equilibrium periods if the surface dipole magnetic fields of the neutron star is in the $10^8 - 10^9 \, G$ range (Alpar, Cheng, Ruderman & Shaham 1982; Radhakrishnan & Srinivasan 1982). Millisecond pulsars would emerge from the epoch of spin-up by accretion on an initial locus in the $P - \dot{P}$ diagram, the “spin-up line” and proceed to spin-down as a radio pulsar at a rate $\dot{P} = 10^{-19} \, s^{-1}$. Subsequently discovered millisecond pulsars all have period derivatives in this range, indicating magnetic fields indeed in the $10^8 - 10^9 \, G$ range. The direct confirmation of the spin-up by accretion hypothesis came with the discovery of the first X-ray millisecond pulsar SAX 1808.4-3658 (Wijnands & van der Klis 1998). More recently sources exhibiting transitions between X-ray and radio pulsar phases have been discovered (Archibald et al. 2009; Papitto et al. 2013; Bassa et al. 2014; Jaodand et al. 2016; Papitto & de Martino 2019). The shortest millisecond pulsar period observed so far from among ~ 400 radio (Manchester et al. 1993) and 20 X-ray millisecond pulsars (see Di Salvo & Sanna 2020, for a review) is $P = 1.4 \, ms$ from PSR J1748−2446ad (Hessels et al. 2006). This is longer than the critical break-up period of neutron stars by about a factor 3 for reasonable equations of state (Haskell et al. 2018). Research on this question has focused on limitations to the fastest rotation rates reached by the growth and saturation of neutron star modes emitting gravitational radiation in the final stages of spin-up. We show that a correlation between the mass accretion rate and the surface dipole magnetic field frozen in the initial stages of spin-up by accretion explains the minimum equilibrium period of millisecond pulsars.

2 MAGNETIC FIELD EXPULSION FROM THE NEUTRON STAR

2.1 Magnetic Field Expulsion from the Superfluid-Superconducting Core
The fluid cores of neutron stars contain neutrons in the superfluid phase and protons in the Type II superconducting phase (Migdal 1959; Ginzburg & Kirzhnits 1964; Baym et al. 1969). The rotation of the core super-fluid is enabled by an array of quantized Onsager-Feynman vortex lines oriented parallel to the rotation axis while the magnetic flux is carried by an array of quantized Abrikosov flux lines parallel to the magnetic axis. External torques acting on the neutron star crust are communicated to the core by the interaction of the crust and normal (non-superfluid) matter, primarily electrons, with the vortex lines. Under an external spin-down torque the vortex lines move away from the rotation axis, thereby reducing the vortex density and achieving spin-down of the core superfluid. The coupling between electrons and vortex lines is actually very tight, so the core spin-down lags behind the crust’s spin-down by minutes or seconds (Alpar et al. 1984).

The magnetic field in the core proton superconductor can relax by motion of the flux lines whose area density determines the mean field. The mechanism for magnetic flux expulsion from the core superconductor involves the coupling of flux lines and vortex lines. The flux lines and vortex lines inevitably have junctions because of their different orientations, and they will get pinned to each other because of energy gains at the junctions where they overlap. As Sauls (1989) and Srinivasan (1989) first realized. Spin-down of the neutron star will lead to reduction of the magnetic field in the core in proportion to the decrease in the rotation rate, as the vortex lines carry pinned flux lines outward (Srinivasan 1989; Srinivasan et al. 1990). Flux expulsion may be limited at a region of toroidal flux lines at the outer core near the boundary with the crust (Sidery & Alpar 2009; Güreginolu & Alpar 2014). While the spin-down in the rotation powered pulsar epoch does not achieve significant reduction of the magnetic field in the core, the subsequent phase of spin-down by accretion from the wind of the detached binary companion does reduce the core field by a factor of 100 - 1000 in proportion to the decrease in the rotation rate (Jahan-Miri &
2.2 Ohmic Diffusion of the Magnetic Field through the Neutron Star Crust

Ohmic diffusion of the magnetic field through the neutron star crust solid depends on the conductivity which increases with density and decreases with temperature. Accretion increases the temperature, and therefore decreases the Ohmic decay timescales and the final value of the surface field. In a series of papers Geppert & Urip found that the dipole fields of neutron stars can decay by a few orders of magnitude under accretion, reaching final frozen fields in the $10^{-7}$ - $10^{-9}$ G range observed in millisecond pulsars, with values decreasing with increasing mass accretion rate (Geppert & Urip 1994; Urip & Geppert 1995; Urip et al. 1998). Konar and Bhattacharya have made a detailed investigation of Ohmic diffusion through the crust in the presence of accretion onto the neutron star surface (Konar 2017; Konar & Bhattacharya 1997, 1999a,b). However there is also an opposing effect: As the accretion rate increases current carrying layers are pushed deeper in the crust, to denser regions where the conductivity is much higher. This effect wins over, resulting in slower magnetic field decay and a correlation between the accretion rate and the final value of the surface dipole magnetic field. Konar and Bhattacharya found that the final frozen value of the surface dipole moment is reached within some $10^5$ years into the epoch of spin-up by accretion in the LMXB epoch. Their work provides a simple and natural explanation of why the surface field does not decay down to zero but rather saturates in the observed range of $10^8$ - $10^9$ G. An understanding of the minimum rotation period observed among millisecond pulsars follows naturally from this correlation between the frozen surface field and accretion rate as we now show.

3 SPIN-UP BY ACCRETION AND THE MINIMUM EQUILIBRIUM PERIOD

The data points shown in Fig. 1 are the results for the frozen field taken from Fig. 4 (lower curve) in Konar (2017). $B_t$ denotes the initial dipole magnetic field prevailing throughout the crust at the beginning of the spin-up by accretion epoch, $B_t$ is the final, frozen field after the field decay in the initial phases of LMXB accretion. Its value is at least of the order of the typical surface dipole fields of young neutron stars, ~ $10^5$ G, and probably higher. The $B_t/B_\xi$ fraction is an increasing function of the accretion rate $M$, as deeper, higher conductivity regions of the crust determine the final field for higher accretion rates (Konar 2017). Their numerical results can be fit piece-wise with three different power laws shown in Fig 1 with the lines A, B, and C.

The equilibrium period $P_{eq}$ reached after spin-up by accretion depends on the location of the inner radius of the accretion disk and on the torques applied on the neutron star by the inner regions of the accretion disk. At the high accretion rates observed in LMXBs the inner disk radius is conventionally estimated to be $r_{in} = r_\xi = \xi r_A$, where $r_A \approx (GM)^{-1/7} \mu^{1/7} M^{-2/7}$ is the Alfvén radius, with the value of $\xi$ in the 0.5 - 1 range in many models. In a recent comprehensive study (Ertan 2021) showed that $r_{co}$ can differ significantly from $r_\xi$ for ranges of $M$ in the strong-propeller, weak-propeller and spin-up phases. The weak-propeller phase of accretion with $r_{in} \approx r_{co}$ persists for a large range of accretion rates, while the magnitude of the spin-down torque decreases, and $\tau_\xi$ approaches $r_{co}$ with increasing $M$. After torque reversal, in the spin-up phase, the equilibrium period is reached when $r_m = r_{co}$ while $r_m$ is close to the Alfvén radius, as in the conventional models. The equilibrium period $P_{eq}$ is obtained by equating the inner disk radius $r_m = \xi r_A$ to the co-rotation period $r_c = (GM/\Omega^2)^{1/3}$:

$$P_{eq} = 2.1 \left(\frac{r_{co}}{1.4 \times 10^{-7} M_{1.4} H_{20} M^{-3/7} \Omega_{12}^{-5/3}}\right)_{\xi=7} \text{ ms} \quad (1)$$

where $\xi = (\Omega/\Omega_{crit})$, $M_{1.4} = (M_*/1.4 M_\odot)$, $H_{20} = (\mu/10^{26}$ G cm$^3$) and $M_{10} = (M/10^{-10} M_\odot$ yr$^{-1}$). The equilibrium period, $P_{eq}$, depends on the dipole moment $\mu$ (corresponding to $B_t$) and $M$. Using the results in Fig. 1, we can express $P_{eq}$ as a function of $M$ only, eliminating $\mu$ using the piece-wise relations A, B and C.

Using the equations given in Fig. 1 together with Eq. (1), we find that $P_{eq}$ is proportional to $M^{-0.32}$, $M^{0.129}$, and $M^{0.33}$ for the lines A, B and C respectively. $P_{eq}$ decreases with increasing $M$ along A, while it is an increasing function of $M$ along the segments B and C, going through a shallow minimum. The equilibrium period for each millisecond pulsar is the minimum period it achieves, depending on the accretion rate and the correlated surface magnetic field, through spin-up by accretion in the LMXB epoch. After the millisecond pulsar emerges as a radio pulsar on the 'birth line' in the $P - P$ diagram it will proceed to spin-down. Having thus noted that $P_{eq}$ is the minimum period in the history of any millisecond pulsar, we now proceed to search for the minimum equilibrium period $P_{min}$ of all millisecond pulsars. This $P_{min}$ is obtained with the accretion rate corresponding to the intersection point of the lines A and B. Denoting this rate by $M_{0}$, we find $M_0 = 10^{-5.5} M_\odot$ yr$^{-1}$ $\approx 10^{15}$ g s$^{-1}$, which corresponds to log($B_t/B_\xi$) = 5.02. For a given solution, $B_\xi$ should be consistent with the field strengths of young neutron stars, while the $B_t$ values should be in agreement with the observed range of the millisecond pulsars from a few $10^7$ G to above $10^9$ G. Taking $B_t = 5 \times 10^{12}$ G, and $\xi = 0.7$, we find that the $M$ values in the range ~12 < log $M$ < ~9 give a dipole moment range 0.17 < $\mu_{26} < 10$. For this normalization, $P_{min}$ is produced with $B_t \approx 5 \times 10^7$ G. Table 1 shows the $P_{min}$ values obtained with this calculation, for each of the numerical data points seen in Fig. 1.

These results are not sensitive to different $\xi$ and $B_t$ values and the density layer down to which the current loops are pushed. Solutions obtained with different densities can be seen in Fig. 4 of Konar.
and $B_i = 5 \times 10^{12} \text{ G}$. See the text for details.

| $\mu_{26}$ | $\log M$ | $P_{\text{min}}$ (ms) |
|-----------|----------|---------------------|
| 0.17      | -12.0    | 2.00                |
| 0.24      | -11.6    | 1.75                |
| 0.30      | -11.3    | 1.59                |
| 0.40      | -11.0    | 1.51                |
| 0.63      | -10.6    | 1.51                |
| 0.93      | -10.3    | 1.57                |
| 1.48      | -10.0    | 1.74                |
| 2.81      | -9.6     | 2.03                |
| 5.24      | -9.3     | 2.57                |
| 9.98      | -9.0     | 3.33                |

(2017). Only one of these model curves is a close representation of the accretion rate - frozen field relation. Similar ranges of $P_{\text{min}}$ values are obtained for a range of $B_i$ and $\xi$ values with each choice of density. The correlation between the frozen field strength and the long-term accretion rate establishes a robust barrier to $P_{\text{min}}$, with no millisecond pulsars reaching critical rotation rates.

4 DISCUSSION AND CONCLUSIONS

We have shown that the minimum equilibrium period of recycled millisecond pulsars can be understood naturally in terms of the initial conditions at the beginning of the spin-up by accretion in low mass X-ray binaries rather than by final conditions when the equilibrium period is defined by a steady state of gravitational wave emitting modes of the neutron star. The final “frozen” value of the surface magnetic field and the accretion rate are correlated, because higher accretion rates settle currents to higher density and thereby higher conductivity layers in the crust, as established by Konar and Bhattacharya (Konar 2017; Konar& Bhattacharya 1997, 1999a,b). As these authors show, the frozen field value is reached at times much shorter than the duration of the spin-up by accretion LMXB epoch. Thus the surface field value can be taken as an initial condition correlated with average mass accretion rate throughout the LMXB epoch. The equilibrium period reached when the Alfvén radius and the corotation are equal can be expressed as a function of the accretion rate $M$ alone. Adopting the results of Konar & Bhattacharya we have found that the equilibrium period indeed has a minimum at $M_0 = 10^{-10.8} M_\odot \text{ yr}^{-1} \approx 10^{15} \text{ g s}^{-1}$, which corresponds to $B_i = 5 \times 10^{12} \text{ G}$. This result is not sensitive to parameters of the crustal ohmic diffusion. The minimum equilibrium period $P_{\text{min}}$ for the millisecond pulsar population is about a factor 3 above the critical neutron star rotation period simply because of the frozen field - accretion rate correlation. The shallowness of the minimum obtained using the results of Konar & Bhattacharya also qualitatively accounts for the clustering of millisecond pulsar periods near the shorter values.

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