A Possible Mechanism of Biological Memories in terms of Quantum Fluids

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Abstract

A mechanism of memories, especially biological memories, is studied in terms of quantum fluids. Two-dimensional flows in central potentials $V_a(\rho) = -a^2 g_a \rho^{2(a-1)}$ ($a \neq 0$ and $\rho = \sqrt{x^2 + y^2}$) have zero-energy eigenstates that degenerate infinitely for all $a$. It is shown that stable standing waves constructed from the zero-energy flows are confined in various types of polygons which can be the minimum units of memory systems. Vortex patterns awoken in the units by stimuli correspond to the memories of the stimuli. This memory system is not a system for preserving memories as usual but that for awaking memories. The system has interesting properties; (i) the absolute economy as for the energy consumption, (ii) the infinite variety for a huge number of memories, (iii) the perfect recovery of the system from any disturbances by stimuli, and (iv) the large flexibility in the construction of the system. A process for thinking is also proposed in terms of this memory system.

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1. Introduction

Researches for the mechanism of brains are extensively going on from many different experimental and theoretical standpoints, and many interesting results have been reported. At present, however, these approaches are still not enough to understand the mechanism of brains from the viewpoint of a fundamental theory of physics. The first step of the understanding seems to be the elucidation of the mechanism of memories in brains as a physical process. The insides of our bodies are mainly occupied by many kinds of fluids, and the total unification of many organs are also preserved mainly by such fluids. It is not a peculiar idea that the mechanism in brains is governed by some kind of flows of such fluids. In this sense it is quite natural that the dynamics of flows (hydrodynamics) is one of the fundamental dynamics in our bodies. On the other hand the dynamics in brains does not seem to be a completely deterministic dynamics like classical dynamics, but some kind of probability seems to play an important role in the mechanism of thinking. Considering the coexistence of the probability and the fundamental principle of superposition in wave dynamics, we may invoke some kind of quantum process related to hydrodynamics to describe the dynamics in brains. Such a hydrodynamical approach of quantum mechanics was vigorously examined in the early stage of the development of quantum mechanics [1-8], and then some fundamental properties of the quantum flows like vortices were investigated [9-13]. Recently interesting solutions that have exactly zero-energy eigenvalue and degenerate infinitely have been found in two-dimensional quantum flows [14-16]. From the viewpoint of the energy consumption in the construction and the preservation of the mechanism of brains such zero-energy flows can be very useful tools. Furthermore the infinite degeneracy can provide the infinite variety of vortex patterns [15-17] corresponding to a huge number of memories. Note that the vortices have recently been observed in many phenomena such as condensed matters [18-20], non-neutral plasma [21-24] and Bose-Einstein gases [25-29]. Here we show a possibility of the mechanism of memories in terms of the zero-energy flows. It is shown that the memory system proposed here is not a system for preserving memories as usual but that for awaking memories, and it has quite suitable properties to represent biological memory systems.

2. A short review on zero-energy flows in two-dimensional Schrödinger equations

In this section we briefly study two-dimensional (2D) quantum flows and vortices. It has been shown that 2D Schrödinger equations with central potentials $V_a (\rho) = -a^2 g_a \rho^{2(a-1)} (a \neq 0$ and $\rho = \sqrt{x^2 + y^2}$), of which eigenvalue problems with the energy eigenvalue $\mathcal{E}$ are written as

$$\left[-\frac{\hbar^2}{2m} \Delta + V_a (\rho)\right] \psi(x, y) = \mathcal{E} \psi(x, y) \quad (1)$$

where $\Delta = \partial^2/\partial x^2 + \partial^2/\partial y^2$, have zero-energy ($\mathcal{E} = 0$) eigenstates [14-16]. Note here that in this equation the mass $m$ and the coordinate vector $(x, y)$ can represent not only those of the single particle but those of the centre of mass for a many particle system as well [17]. It has also been shown that, as far as the zero-energy eigenstates ($\psi_0$) are concerned, the Schrödinger equations for all $a$ can be reduced to the following equation in terms of the conformal transformations $\zeta_a = z^a$ with $z = x + iy$:

$$\left[-\frac{\hbar^2}{2m} \Delta_a - g_a\right] \psi_0(u_a, v_a) = 0, \quad (2)$$

where $\Delta_a = \partial^2/\partial u_a^2 + \partial^2/\partial v_a^2$, using the variables defined by the relation $\zeta_a = u_a + iv_a$ [15, 16]. That is to say, the zero-energy eigenstates for all the different numbers of $a$ are described by the same plane-wave solutions in the $\zeta_a$ space. Furthermore it is easily shown that the zero-energy states degenerate infinitely. Let us consider the case for $a > 0$ and $g_a > 0$. Putting the function $f_n^\pm (u_a; v_a) e^{\pm ik_a u_a}$ with $k_a = \sqrt{2m g_a}/\hbar$ into (2), where $f_n^\pm (u_a; v_a)$ are polynomials of degree $n$ ($n = 0, 1, 2, \cdots$), we obtain the equation for the polynomials

$$[\Delta_a \pm 2ik_a \frac{\partial}{\partial u_a}] f_n^\pm (u_a; v_a) = 0. \quad (3)$$
Note that from the above equations we can easily see the relation \( (f_n^-(u_a; v_a))^* = f_n^+(u_a; v_a) \) for all \( a \) and \( n \). General forms of the polynomials have been obtained by using the solutions in the \( a = 2 \) case (2D parabolic potential barrier (2D PPB)) \[15\] \[16\]. Since all the solutions have the factors \( e^{\pm ik_a u_a} \), we see that the zero-energy states describe stationary flows \[14\] \[15\] \[16\]. Taking account of the direction of incoming flows in the \( \zeta_a \) plane that is expressed by the angle \( \alpha \) to the \( u_a \) axis, the general eigenfunctions with zero-energy are written as arbitrary linear combinations of eigenfunctions included in two infinite series of \( \{ \psi_{0n}^{\pm}(u_a(\alpha); v_a(\alpha)) \} \) for \( n = 0, 1, 2, \ldots \), where

\[
\psi_{0n}^{\pm}(u_a(\alpha); v_a(\alpha)) = f_n^{\pm}(u_a(\alpha); v_a(\alpha)) e^{\pm ik_a u_a(\alpha)}, \tag{4}
\]

\( u_a(\alpha) = u_a \cos \alpha + v_a \sin \alpha \), and \( 0 < \alpha \leq 2\pi \). (For the details, see the sections II and III of Ref. 15.) We briefly comment on states having non-zero energy eigenvalues. Energy eigenvalues are in general complex values, and the states with complex energies can be understood to be resonance states which are not stable and decay in some time scale \[30\]. Exact solutions not only for zero-energy but for non-zero energies as well have been solved in 2D PPB \[14\] \[30\]. Note here that we consider the motions of the \( z \) direction perpendicular to the \( xy \) plane to be free motions represented by \( e^{\pm ik_z z} \). We should, therefore, notice that the total energies \( E_T \) of the flows are given by \( E_T = E_z \), where \( E_z \) are the energies of the free motions in the \( z \) direction. We shall, however, not take account of the motions in the \( z \) direction in the following discussions on the motions in the \( \zeta_a \) plane.

Note that the zero-energy eigenfunctions can not be normalized as same as those in scattering processes \[30\]. Actually it has been shown that all the states for zero and non-zero energies are eigenstates in the conjugate spaces of the Gel’fand triplets \[13\] \[14\] \[15\] \[16\]. This fact means that the probability density \( \langle \rho(t, x, y) = |\psi(t, x, y)|^2 \rangle \) and the probability current \( \langle j(t, x, y) = \text{Re}\{\psi(t, x, y)^* (-i\hbar \nabla)\psi(t, x, y)\} \rangle \) lose the meanings. We, however, see that the velocity defined by \( \mathbf{v} = j(t, x, y) / \rho(t, x, y) \), can have the well-defined meaning, because the ambiguity arising from the normalization is cancelled in the definition. The velocity was widely discussed in early stage of quantum mechanics \[1\] \[8\]. It is also well-known that vortices appear at the zero points of the density, that is, the nodal points of the wave functions \[9\] \[13\]. Let us here remember the fact that the zero-energy states have the infinite degeneracy, and therefore we can construct wave functions having the nodal points at almost arbitrary positions in terms of linear combinations of the zero-energy states. Actually various vortex patterns have been studied \[15\] \[16\] \[17\]. It should be emphasized that we can study the vortex problems for all \( V_a(\rho) \) as a problem with the constant potential \( V_a = -g_a \) in the \( \zeta_a \) plane, because the conformal transformations do not change fundamental properties of vortices such as the numbers of vortices and the strengths of vortices \( \Gamma \). In quantum mechanical phenomena it is shown that the circulation \( \Gamma = \oint_{\zeta_a} \mathbf{v} \cdot d\mathbf{s} \) is quantized such that \( \Gamma = 2\pi l \hbar / m \), where \( l \) is an integer \[9\] \[14\]. Vortex patterns in the \((x, y)\) plane can be obtained by the inverse transformations of the conformal mappings \[15\] \[16\].

3. Standing waves and vortex patterns

Let us start with the study of zero-energy standing waves that will play an important role in the following discussions. Considering the relation \( (f_n^-(u_a; v_a))^* = f_n^+(u_a; v_a) \), we can easily make two infinite series of standing waves in terms of the zero-energy series \( \{ \psi_{0n}^{\pm}(u) \} \) as follows;

\[
\psi_{0n}^{+u}(u) = \frac{1}{2} (\psi_{0n}^{+(u)} + \psi_{0n}^{-u}(u)) = f_n^{\text{Re}(u)} \cos k_a u_a(\alpha) - f_n^{\text{Im}(u)} \sin k_a u_a(\alpha),
\]

\[
\psi_{0n}^{-u}(u) = \frac{1}{2} (\psi_{0n}^{+(u)} - \psi_{0n}^{-u}(u)) = i [f_n^{\text{Re}(u)} \sin k_a u_a(\alpha) + f_n^{\text{Im}(u)} \cos k_a u_a(\alpha)], \tag{5}
\]

where \( f_n^{\text{Re}(u)} \) and \( f_n^{\text{Im}(u)} \), respectively, stand for the real and imaginary parts of \( f_n^+ \). In order to simplify our argument we pick up only four standing waves that are constructed in terms of the linear combinations of the flows with the lowest order, that is, \( e^{\pm ik_a u_a \pm k_v v_a} \) corresponding to the choices of \( \alpha = \theta_a, \; \pi \pm \theta_a \).
and $2\pi - \theta$, where $k_u = k_\alpha \cos \theta$ and $k_v = k_\alpha \sin \theta$ with $0 < \theta \leq 2\pi$, such that
\[
\psi_{cc} = \cos k_u u_a \cdot \cos k_v v_a, \quad \psi_{ss} = \sin k_u u_a \cdot \sin k_v v_a, \quad \psi_{sc} = i \sin k_u u_a \cdot \cos k_v v_a, \quad \psi_{cs} = i \cos k_u u_a \cdot \sin k_v v_a.
\]

These wave functions can be confined in quadrangles of the $\zeta_\alpha$ plane, which are surrounded by rigid fences described by infinite potentials, because we can take zero as boundary values on the sides of the quadrangles. For example, $\psi_{cc} = \cos k_u u_a \cdot \cos k_v v_a$ becomes zero on the lines where either $k_u u_a = (2m + 1)\pi/2$ or $k_v v_a = (2n + 1)\pi/2$ ($m$ and $n$ are integers) is satisfied. In the cases where $a$ are positive integers the quadrangles in the $\zeta_\alpha$ plane are represented by 4$a$-gons in the original $xy$ plane. Now we get the standing wave states confined in the special regions which are described by the quadrangles in the $\zeta_\alpha$ plane. It should be noticed that the quadrangles have a wide variety of the sizes which are determined by the numbers of wave lengths contained in the quadrangles. That is to say, the number $N_u$ for the $u$ direction and that $N_v$ for the $v$ direction are free parameters, where $N_u$ and $N_v$ are positive integers or positive half-integers.

Now let us discuss vortex patterns made from the zero-energy flows. It is known that vortex patterns are one of topological properties of flows, and then they can be stable in many physical phenomena. Following the above consideration, we can prepare many different standing waves depending on the sizes of quadrangles. The standing waves, of course, have no vortex, because we can take zero as boundary values on the sides of the quadrangles. Following the above consideration, we can prepare many different standing waves depending on the sizes of quadrangles. For example, the standing wave with the wave number vector $\vec{q} = (q_u, q_v)$, which is described by the wave function $\psi_{sc} = i \sin q_u u_a \cdot \cos q_v v_a$, into the quadrangle prepared. Note that in the process where the new flow is added we can use the free motions of the $z$ direction. We see that the ranges of $u$ and $v$ in the quadrangle are given by $|u| < u_B \equiv \pi/2k_u$ and $|v| < v_B \equiv 3\pi/2k_v$. (Hereafter we omit the suffix $a$ from $u_a$, $v_a$, $k_a$ and etc.) The total wave function is written as
\[
\Psi = \cos k_u u \cdot \cos k_v v + iC \sin q_u u \cdot \cos q_v v,
\]
where $C$ is taken to be a real number for the simplicity of the present discussion. In order that the inserted wave $\psi_{sc}$ is stable in the quadrangle, the wave function of the inserted wave must vanish on the boundaries of the quadrangle, that is, on the lines fulfilling $|u| = u_B$ or $|v| = v_B$. This constraint gives us the relations
\[
q_u = 2(L_u + 1)k_u \quad \text{and} \quad q_v = (2L_v + 1)k_v/3,
\]
where $L_u$ and $L_v$ are zero or positive integers. Let us investigate vortices in the quadrangle. From the fact that $\psi_{cc}$ has two nodal lines fulfilling $v = \pm v_B/3$ in the $\zeta$ plane and on the other hand $\psi_{sc}$ has nodal lines fulfilling $u = n\pi/q_u$ or $v = (2m + 1)\pi/2q_v$ ($n$ and $m$ are integers), we see that vortices appear at the nodal points of the total wave function $(u_V, v_V)$, where
\[
u_V = n \pi/q_u = \pm n u_B/(L_u + 1), \quad \text{and} \quad v_V = \pm v_B/3.
\]
Considering the constraint $|u| < u_B$, we obtain the following results:
\begin{itemize}
  \item in the case of $L_u = 0$ two vortices appear at $(0, \pm v_B/3)$ for $n = 0$,
  \item in the case of $L_u = 1$ six vortices appear at $(0, \pm v_B/3)$ for $n = 0$ and $(\pm u_B/2, \pm v_B/3)$ for $n = 1$,
\end{itemize}
and generally
\[2(L_u + 2)\] vortices appear on the lines of $v = \pm v_B/3$ for $L_u > 0$.
In general cases the following results are found out:
\begin{itemize}
  \item[(I)] In this vortex-formation process we find out the selection rule for the inserted wave numbers $\vec{q}$ such that only the waves with $q_u = (L_u + 1)k_u/N_u$ and $q_v = (2L_v + 1)k_v/2N_v$ can form some stable vortex-patterns in the above example. In general the ratios of $q_u/k_u$ and $q_v/k_v$ must be rational numbers.
  \item[(II)] From the observation of the number of the vortices we can read off the number $L_u$ (or $L_v$).
\end{itemize}
Furthermore we can add one more important fact:
(III) In the processes putting standing waves into the quadrangle the number $C$ that represents the magnitude of the inserted wave play no role in the determination of the vortex patterns.

4. Mechanism of memories

Here let us consider the mechanism of memories in terms of the vortex-formation process. In the present framework we consider that every vortex pattern corresponds to a memory. In this viewpoint a quadrangle confining a standing wave is not a memory corresponding to a special item, but the system in which a vortex pattern is awoken by a flow induced by a stimulus. A quadrangle can awake different vortex patterns corresponding to different wave numbers expressed by the relations like $q_u = (L_u + 1)k_u/N_u$ in the example of the section 3. We may visualize the following process; a stimulus corresponding to an item is taken by some organs and converted to flows. The flows reach at boundaries of many quadrangles with different sizes (4$a$-gons in the real space). Every quadrangle analyses the flow in terms of the standing waves given by (6). Notice that this analysis for the inserted flow is nothing but Fourier analysis in the $\zeta_a$ plane, of which boundaries are given by the lines $u = \pm u_B$ for $u$ and $v = \pm v_B$ for $v$. If the flow does not contain the waves with the wave numbers fitting to the selection rule (I), the flow cannot make any stable vortex pattern. Only the flows containing the wave numbers fitting to the rule (I) make stable vortex patterns in the quadrangles. Thus one stimulus sent to many quadrangles awakes various vortex patterns in those quadrangles. We may consider that the set of these vortex patterns made in the different quadrangles represents the memory corresponding to the stimulus. As noted in (III), the magnitude of the stimulus does not affect the vortex formation at all.

Let us see the characteristic features of this memory system. From the mechanical standpoint the fundamental properties of a quadrangle are determined by the parameters of the potential inside the quadrangle, that is, $a$ and $g_a$, where $a$ determines the form ($4a$-gon) of the $xy$ plane and $g_a$ does the fundamental wave number $k_a$. It must be noticed that further steps are needed for making a quadrangle confining a standing wave. One is the division of the fundamental wave number between two directions $u_a$ and $v_a$, following the relation $k_a^2 = k_u^2 + k_v^2$, that is, $\vec{k}_a = (k_u, k_v)$ is the wave number vector in the $\zeta_a$ plane. In order to determine the actual lengths of the sides of the quadrangle, we need one more step for the decision of two numbers $N_u$ and $N_v$ which determine the lengths of the sides of the quadrangle. Summarizing the above argument, the following three steps are needed to set up a quadrangle:

1. the choice of $a$ and $g_a$ (the determination of the potential inside the quadrangle),
2. the division of $k_a$ between $u_a$ and $v_a$ (the determination of the wave vector $\vec{k}_a$ confined in the quadrangle), and
3. the choice of the numbers of the wave lengths contained in the quadrangle, $N_u$ and $N_v$ (the determination of the lengths of the sides of the quadrangle).

Through these processes a quadrangle confining a definite standing wave is determined. We see that in these processes we have five parameters, that is, $a$, $g_a$, $N_u$, $N_v$ and one more parameter $\theta_a$ to determine $k_a$ such that $k_u = k_a \cos \theta_a$ and $k_v = k_a \sin \theta_a$. The angle parameter $0 < \theta_a \leq 2\pi$ is a continuous free-parameter, and therefore the two wave numbers $k_u$ and $k_v$ can be also continuous numbers limited by $k_a$. This fact means that from one fundamental wave number $k_a$ we can make quadrangles having arbitrary values as for the ratio of the lengths of two perpendicular sides, which is given by $\tan \theta_a$. Taking also account of the free parameters $N_u$ and $N_v$, we have large freedom to the determination of the sizes and forms of quadrangles. We can, thus, provide quadrangles which can correspond to almost arbitrary inserted wave number vectors $\vec{q}$ even if $\vec{q}$ must be satisfied by the selection rule given in (I) to make stable vortex patterns in the quadrangles.

Let us continue to discuss about the mechanism of memories. As already noted, the quadrangles are not the mechanism for preserving memories but that for awaking memories. One quadrangle characterized by a set of the four parameters $k_a$, $\theta_a$, $N_u$ and $N_v$ can fundamentally produce infinite numbers of different vortex patterns corresponding to the differences of the wave numbers of the inserted waves expressed by the relations $q_u = L_u k_a/N_u$ and $q_v = L_v k_a/N_v$, where $L_u$ and $L_v$ are positive integers or positive half-integers. It should once more be emphasized that one quadrangle awakes only one pattern
for a stimulus but the pattern corresponding to the stimulus is generally different from others. Considering the limit of the discrimination of the vortex patterns, each quadrangle may have a certain maximum values \( L_{\text{max}} \) for \( L_u \) and \( L_v \). In such a case, for the system where \( N_q \) number of quadrangles cooperate we can roughly estimate the number \( (N_{VP}) \) of the different vortex patterns that can be distinguished by the system as

\[
N_{VP} = \prod_{i=1}^{N_q} (L_{u,max} \times L_{v,max}).
\]

Provided that the mean number of \( (L_{u,max} \times L_{v,max}) \) is 10 and \( N_q \) is a large number, we obtain a huge number of the distinguishable vortex patterns counted to be \( N_{VP} \sim 10^{N_q} \). Even if the mean number is 2 and \( N_q \) is not a very large number, e.g. 100, we still have a huge number \( 2^{100} \sim 10^{30} \). From this fact we may consider that a total system consisting of a huge number of quadrangles is divided into many small blocks, and each block plays a fairly independent role in the total system.

We would like here briefly to note on the role of the higher order standing waves given in (5). Taking account of the freedom introduced in linear combinations of those standing waves, we will find out that the boundary conditions are expressed by some complicated relations for \( u_a \) and \( v_a \). On the other hand, from (5) we see that all the terms of the standing waves have one of the four factors \( \cos k_u u_a \), \( \sin k_u u_a \), \( \cos k_v v_a \) and \( \sin k_v v_a \). Therefore, as far as the discussions of vortex patterns are concerned, we can perform arguments similar to those done in the lowest cases. In actual situations, however, the fences of quadrangles (4\( a \)-gons in real spaces) will be made of some membranes in living beings. They are, of course, not perfectly rigid. In such cases the wave functions do not generally vanish at the boundaries of the quadrangles. Boundaries are, therefore, not given by strict quadrangles but by some deformed forms, and the values of wave functions on the boundaries are not strictly 0 but some values determined by the continuity conditions of wave functions. The infinite degeneracy of the standing waves can possibly play a role to solve the problem. That is to say, we will be able to provide various forms corresponding to various boundary conditions in terms of the infinite degeneracy. The vortex positions must also be changed. By gathering the terms with the same factor (cos or sin) discussions similar to those done in the lowest order cases will be also performed here. This infinite degeneracy and the potential parameters \( a \) and \( g_a \) will bring a large flexibility in the construction of the minimum units of the memory system.

5. Interesting properties of the system composed of the zero-energy flows

We have used only the zero-energy states for making the memory system. We would like to point out four interesting properties of the present system composed of the zero-energy flows [17]:

(i) the absolute economy as for the energy consumption in the construction of the memory system and also the preservation of the system, where no flow in the \( z \) direction should be taken, and therefore the total energy \( E_T = 0 \).
(ii) the almost infinite variety of the vortex patterns provided for a huge number of memories, and
(iii) the perfect recovery of the memory system from shocks induced by stimuli, because the zero-energy \( (E_T = 0) \) states have no time dependence at all, and therefore the standing waves in the minimum units of the system are perfectly recovered even if any disturbances induced by flows decaying or diffusing in some time scale are taken place [17].

These properties are suitable for interpreting the mechanism of memories in brains. That is to say, stimuli taken in sense organs are analyzed in terms of the states including both of zero- and nonzero-energy states in the conjugate spaces of Gel’fand triplets, and after some time scale only the zero-energy flows selected by the rule (I) remain in the minimum units and make stable vortex patterns. The vortex patterns are sent to other places by the free motions of the inserted waves in the direction perpendicular to the \( xy \) plane, and then every unit perfectly recover the initial state. We may point out one more important property:

(iv) the large flexibility due to the infinite degeneracy and the parameters \( a \) and \( g_a \) in the potentials is provided in making the minimum units like the quadrangles.

6. A comment on thinking processes
Finally we would like to present a conjecture about thinking processes. At present we do not know how the mechanism of memories presented here does work in thinking processes (algorithm of thinking processes). We shall here point out a possibility that the memory system proposed here plays a role to distinguish accustomed stimuli, which have been repeatedly experienced in the past, from many stimuli included in an item caught by sense organs. We may consider that the same stimulus repeatedly given in our experience provokes organizations in brains to make a memory system consisting of many minimum units like quadrangles (possibly neurons in our brains) for selecting the very stimulus among many stimuli. Some systems like autonomic nervous system are made in the womb of mother. Such a memory system provided for the selection of some special stimuli will be made in a rather small block of the total system.

Now we can consider the following process for thinking: When an item is caught by some sense organs, the stimuli taken by the organs will be gathered in a special place of brains, where the stimuli are converted into signals that fit to the detection by the memory system. From the place the signals are delivered to many blocks, each of which consist of a certain number of minimum units. The signal sent to one of the blocks are analyzed by each unit in the block in terms of the standing waves, and then every unit selects the standing waves, following the selection rule (1). If there exist wave numbers fitting to the rule, the unit make a vortex pattern. Thus the signal sent to the block is converted to a set of vortex patterns. This set represents a recognition of an accustomed experience. Thus the stimuli caught by the organs are represented by a set of such recognitions. The recognitions characterized by various vortex patterns are once more delivered to a certain number of new blocks (call them higher blocks) and gathered together in each block. In this process the transportations will be carried out by using the free motions of the direction perpendicular to the standing waves. It should be noticed that the vortex patterns which are a kind of topological property can be very stable in such transportations. Then they proceed to the next step carried out in the higher blocks, where interactions between vortices take place. Some vortices will be created and/or annihilated \[17\], and a new vortex pattern appears in each block. Such new patterns produced in the higher blocks will be sent to more higher blocks, where the higher step is performed. The same processes will be repeated, until the brain gets a set of vortex patterns that is recognized to be the final answer. Such hierarchy may be the base of our thinking processes. As we know, flows of fluids are very much flexible and easily deformed, but not easy to control them perfectly. Experiences and trainings in daily life must play a very important role in the construction of this hierarchy. At present we have so many unknown problems such as the definite correspondence between the vortex patterns and the recognitions, the dynamics in the higher blocks, and so on. We are still at the entrance of the understanding. The view given here, however, is possibly an idea for the understanding of thinking in terms of real physical processes.

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