Two-Photon-Exchange Effects and $\Delta(1232)$ Deformation

Hai-Qing Zhou$^1$ and Shin Nan Yang$^2$

$^1$ Department of Physics, Southeast University, NanJing 211189, China
$^2$ Department of Physics and Center for Theoretical Sciences, National Taiwan University, Taipei 10617, Taiwan

E-mail: $^1$zhouhq@seu.edu.cn, $^2$snyang@phys.ntu.edu.tw

(Received Oct 16, 2016)

The two-photon-exchange (TPE) contribution in $ep \rightarrow ep\pi^0$ with $W = M_\Delta$ and small $Q^2$ is calculated and its corrections to the ratios of electromagnetic transition form factors $R_{EM} = E_{1+}^{(3/2)}/M_{1+}^{(3/2)}$ and $R_{SM} = S_{1+}^{(3/2)}/M_{1+}^{(3/2)}$, are analysed. A simple hadronic model is used to estimate the TPE amplitude. Two phenomenological models, MAID2007 and SAID, are used to approximate the full $ep \rightarrow ep\pi^0$ cross sections which contain both the TPE and the one-photon-exchange (OPE) contributions. The genuine OPE amplitude is then extracted from an integral equation by iteration. We find that the TPE contribution is not sensitive to whether MAID or SAID is used as input in the region with $Q^2 < 2$ GeV$^2$. It gives small correction to $R_{EM}$ while for $R_{SM}$, the correction is about -10% at small $\epsilon$ and about 1% at large $\epsilon$ for $Q^2 \approx 2.5$ GeV$^2$. The large correction from TPE at small $\epsilon$ must be included in the analysis to get a reliable extraction of $R_{SM}$.

**KEYWORDS:** two-photon-exchange, pion production, $\Delta(1232)$ deformation

The study of the structure of the hadrons is one of the most important way to understand the non-perturbative properties of QCD. As the first excited state of the nucleon, the structure of $\Delta(1232)$ plays a special role as it has been established that $\Delta$ deforms. It provides a non-trivial test for theoretical models [1]. The electromagnetic excitation of the $\Delta(1232)$ provides a way to determine the deformation by measurement of the electric E2 and Coulomb C2 transitions in the electromagnetic (EM) production of pion in the $\Delta(1232)$ region. Recently, a few precise measurements of the multipoles $M_{1+}$, $E_{1+}$, and $S_1$, related to this transition have been performed [2]. The precision of the measurements of the cross sections is close to 1% which implies that electromagnetic radiative corrections should be included in the theoretical analysis for the extraction to be reliable.

It has been established that the two-photon-exchange (TPE) effects contribute non-negligibly to the elastic $ep$ scatterings [3] such that the TPE effects must be taken into account in the extraction of the ratio of the nucleon form factors $G_E/G_M$ reliably. It is natural to ask how the TPE processes would contribute to the $\Delta(1232)$ excitation when aiming at precise determination of $M_{1+}$, $E_{1+}$, and $S_1$. The TPE corrections to the EM excitation of $\Delta(1232)$ at high $Q^2$ have been considered in [4] in partonic approach with the use of GPDs. However, TPE contributions at low $Q^2$ where the deformation of the $\Delta(1232)$ is inferred, have not been estimated. In addition, in [4], only the $\Delta(1232)$ pole contribution, but not the Born term, is considered for the one-photon-exchange (OPE) mechanism in calculating the interference between OPE and TPE amplitudes. However, it is well-known that the Born term plays important role within the OPE framework for pion EM production [1]. In this work, we estimate the TPE corrections to $M_{1+}$, $E_{1+}$, and $S_1$ in the EM excitation of $\Delta(1232)$ at low momentum transfer $Q^2$ with the inclusion of the Born term in the OPE amplitude.

Within OPE approximation, the unpolarized cross section of the $ep \rightarrow ep\pi^0$ can be expressed as

$$\frac{d^2\sigma^{1+\gamma}}{d\Omega_f dE_f d\Omega} = C|\mathcal{M}^{1+\gamma}|^2 \equiv \Gamma[\sigma_0^{1+\gamma} + \sqrt{2}\epsilon(1+\epsilon)\sigma_{LT}^{1+\gamma}\cos\phi + \epsilon\sigma_{TT}^{1+\gamma}\cos\phi]$$

(1)
where \( \sigma_0^{1y} = \sigma_T^{1y} + \epsilon \sigma_L^{1y} \), \( \phi \) is the pion center of mass azimuthal angle with respect to the electron scattering plane, \( \epsilon \) is the transverse polarization of the virtual photon, \( \Gamma \) is the virtual photon flux, and \( C \) is a kinematical constant. The virtual photon differential cross sections \( (\sigma_T^{1y}, \sigma_L^{1y}, \sigma_{LT}^{1y}, \sigma_{TT}^{1y}) \) are all functions of the center of mass energy \( W \), four momentum transfer squared \( Q^2 \), and the pion center of mass polar angle \( \theta_\pi \) measured from the momentum transfer direction. When using the multipoles as inputs, the cross sections can be expressed as the functions of multipoles explicitly as \( \sigma_0^{1y} \) of \( \sigma_{LT}^{1y}(Z_1, \theta_\pi) \), where \( Z_1 \)'s denote the multipoles \( E_1, M_1 \) and \( S_1 \). The superscript "th" means that they are the genuine multipoles \( M_1^{\gamma\gamma} \) and \( \bar{Z}_1^{\gamma\gamma} \) extracted from the data with TPE effects removed. The functional dependence of \( \sigma_T^{1y} \) on multipoles \( (X_1^1, \bar{Z}_1^{1y}) \) are well known.

Furthermore, the phases of \( \sigma_0^{1y} \) and \( \sigma_1^{1y} \) are fixed such that only the three multipoles \( X_1^1 = (E_1^1, M_1^1, S_1^1) \) remained to be determined. The gradient coupling is used for \( \epsilon \sigma_T^{1y} \) as required by unitarity. This reduces the variables in Eq. (3) to only three numbers, namely, \( |E_1^1| \),...
$|M_{1^+}|$, and $|S_{1^+}|$. The nonlinear equation of Eq. (3) can be solved via iteration. The multipoles $X_{1^+}$ given by either MAID2007 (with $l \leq 8$) or SAID (with $l \leq 5$), depending on which model is used to approximate the experimental cross sections, are used for $M^{1^+}$ in Eq. (3), in the first iteration. We find that only one iteration is sufficient.

In the first iteration, we have the followings from Eq. (3),

$$\frac{d\sigma^{1,M/S}}{d\Omega_\pi} \equiv \frac{d\sigma^{ex,M/S}}{d\Omega_\pi} - 2CRe[M^{1^+}(X_{1^+}^{0,M/S}, Z_{\pi}^{0,M/S})\bar{M}^{2\gamma}] = C|M^{1^+}(X_{1^+}^{1,M/S}, Z_{\pi}^{0,M/S})|^2,$$  \hspace{1cm} (4)

where superscript $M/S$ refers to either MAID or SAID and is used to approximate the cross sections and the corresponding multipoles. Superscripts 0 and 1 denote the initial guess and the resulting multipoles obtained from first iteration, e.g., $X_{1^+}^{0,M}, Z_{\pi}^{0,M} \equiv X_{1^+}(MAID), Z_{\pi}(MAID)$.

The problem now is how to determine the genuine OPE multipoles $X_{1^+}^{1,M/S}$'s from Eq. (4) with TPE contributions removed. This can be done by first noting that the multipole dependence of $\sigma^{1,M/S}$ of Eq. (4) should be the same as $\sigma^{1\gamma}$ of Eq. (2), which we define as $\sigma_{0,LT,TT}^{j\gamma}(Z_l, \theta_l)$ earlier. We then determine the absolute values of $M_{1^+}^{1\gamma}, E_{1^+}^{1\gamma}$ and $S_{1^+}^{1\gamma}$ by minimizing the following $\chi^2$

$$\chi^2 \equiv \sum_{j=0,LT,TT} \sum_{l=1^{\gamma}}^{179^{\gamma}} \omega_j[(\sigma_j^{1,M/S}(\theta_l) - \sigma_j^{1\gamma}(M_{1^+}^{1,M/S}, E_{1^+}^{1,M/S}, S_{1^+}^{1,M/S}, \theta_l))^2 \hspace{1cm} (5)$$

where $\omega_j$'s are the weights of cross sections $\sigma_{0,LT,TT}$ used in the fitting. We choose equal weights $\omega_j = 1$ in this preliminary study.

**Fig. 2.** TPE corrections to the extracted multipoles $E_{1^+}$ and $S_{1^+}$ with $\omega_j = 1$. The solid curves refer to the results with MAID2007 as input, and the dashed curves refer to the results with SAID as input.

We find that the TPE corrections to $M_{1^+}$ are very small. The TPE corrections to the multipoles $E_{1^+}$ and $S_{1^+}$ with the MAID2007 and SAID as inputs are presented in the Fig. 2, where the ratio between the multipoles $X_{1^+}^{0,M/S}$ used in the input and the extracted genuine multipoles $X_{1^+}^{1\gamma,M/S}$ after TPE effects are removed. Note that the multipoles $X_{1^+}^{0,M/S}$'s which contain some TPE effects are labelled as $X_{1^+}^{1\gamma+2\gamma}$ in the figures. It is seen that the $\epsilon$ dependence of the TPE corrections are similar with different models.
as inputs, while the $Q^2$ dependence of the corrections are rather different for $Q^2 > 2$ GeV$^2$. In this higher $Q^2$ region, the SAID multipoles agree better with the experiments.

The absolute TPE corrections to the $R_{EM}$ and $R_{SM}$ at fixed $\epsilon$ are presented in the Fig. 3 where $R_{EM} \equiv \text{Im}[E_{1\gamma}]/\text{Im}[M_{1\gamma}]$ and $R_{SM} \equiv \text{Im}[S_{1\gamma}]/\text{Im}[M_{1\gamma}]$. It is interesting to note that even though the obtained TPE corrections $S_{1\gamma}/S_{1\gamma}^{1\gamma+2\gamma}$ are very different with different input models in the $Q^2 \sim 2 - 4$ GeV$^2$ region as seen in Fig. 2, the corrections $\delta R_{SM}$ are almost same. When comparing our results with those given in [4], we see that when $\epsilon = 0.2$ our $\delta R_{EM}$ are much smaller than theirs, while our $\delta R_{SM}$ are much larger than theirs in the intermediate $Q^2$. One of the main reason of this difference lies in the fact that only the $\Delta$ pole term is included for $M_{1\gamma}$ in [4]. Namely, the background contributions to $M_{1\gamma}$ are not considered there when calculating the interference term between $M_{1\gamma}$ and $M_{2\gamma}$.

In summary, we evaluate the TPE corrections in the $ep \rightarrow epn^0$ at $W = M_\Delta$ in the low $Q^2$ region in the hadronic approach. We include the background contribution in the OPE amplitude. We find that TPE corrections $\delta R_{SM} = R_{SM}^{1\gamma} - R_{SM}^{1\gamma+2\gamma}$ are not sensitive w.r.t. whether MAID2007 or SAID is used as input model. Our results differ considerably with [4] in $Q^2 \sim 2 - 4$ GeV$^2$ region.

ACKNOWLEDGEMENTS

This work is supported in part by the National Natural Science Foundations of China under Grant No. 11375044, the Fundamental Research Funds for the Central Universities under Grant No. 2242014R30012 for H.Q.Z. and the National Science Council of the Republic of China (Taiwan) for S.N.Y. under grant No. NSC101-2112-M-002-025. H.Q.Z. would also like to gratefully acknowledge the support of the National Center for Theoretical Science (North) of the National Science Council of the Republic of China for his visit in the January of 2016.

References

[1] V. Pascalutsa, M. Vanderhaeghen, and Shin Nan Yang: Phys. Repts. 437 (2007) 125.
[2] N. Sparveris et al.: Eur. Phys. J. A 49 (2013) 136; J. M. Kirkpatrick et al.: Phys. Rev. C 84 (2011) 028201; S. Stave et al.: Phys. Rev. C 78 (2008) 025209.
[3] J. Arrington, P. Blunden, W. Melnitchouk: Prog. Nucl. Part. Phys. 66, (2011) 782.
[4] V. Pascalutsa, C. E. Carlson, M. Vanderhaeghen: Phys. Rev. Lett. 96 (2006) 012301.
[5] S. Kondratyuk, P. G. Blunden: Nucl. Phys. A 778 (2006) 206.
[6] Hai-Qing Zhou, C. W. Kao, Shin Nan Yang: Phys. Rev. Lett. A 99 (2015) 262001.
[7] D. Drechsel, S. S. Kamalov, and L. Tiator: Eur. Phys. J. A 34 (2007) 69.
[8] R. A. Arndt, W. J. Briscoe, I. I. Strakovsky, and R. L. Workman: Phys. Rev. C 66 (2002) 055213.
[9] Hai-Qing Zhou, Shin Nan Yang: Eur. Phys. J. A 51 (2015) 105.