Nonstandard Origin of the Standard Electroweak Currents

I. T. Dyatlov

Petersburg Nuclear Physics Institute, Gatchina 188350, Russia

Abstract

Implications are considered of the hypothesis that the symplectic group $Sp(n/2)$ is the spontaneously violated gauge group of $n$ lepton flavors. Invariant Majorana masses are impossible in $Sp(n/2)$. For the local gauge symmetry $Sp(n/2)$ the dynamical spontaneous violation is only achievable for the number of flavors $n = 6$ with simultaneous parity ($R, L$–symmetry) violation. The see-saw mechanism produces here three light and three heavy Dirac neutrinos. Majorana states are unavailable here. Neglecting heavy particles in the $R, L$–symmetric system of weak and electromagnetic interactions ($R, L$–independent values of isospins $Tw$ and hypercharges $Y$ for leptons or quarks) leads to a theory with parity nonconservation and axial anomalies. Only weak left ($L$) and full ($R + L$) electromagnetic currents do not have anomalies and remain independent of the physics of heavy masses. These currents are the ones of the Standard Model. The absence of anomalies merely in the combination of currents forming the electromagnetic one presents essential difference with the SM case, where the both, left $T_W$ and $Y$, currents are deprived of anomalies independently.

PACS numbers: 12.60.-i, 12.60.NZ, 11.15.-q
1 Introduction

Neutrinos are unique particles. Firstly, they participate only in weak interactions (of those known to us). Although neutrinos have a mass, as any other particles, they are the only ones whose dynamics are characterized by chirality. One can easily assume that weak interactions themselves are determined by the properties of neutrinos (ν), whereas charged leptons and quarks adjust themselves to the structure of ν states.

This view is discussed in the present paper and is different from the conventional approach to the electroweak part of the Standard Model (SM) [1] and the theory of mass spectrum and neutrino mixing (see [2] and also reviews [3, 4] with numerous references). The differences of our approach are discussed later in this section.

Secondly, only neutrinos can have the Majorana form. This form is equivalent to chiral states: for massless particles, two states of the Majorana particle spin precisely correspond to the particle-antiparticle pair in the chiral representation (for right (R) and left (L)). It is therefore the Majorana form that can play a major role in the formation of properties of both neutrinos and weak interactions.

Thirdly, the ν mass spectrum [2] is apparently based on other principles than masses of charged and therefore compulsorily Dirac particles. The exceptional smallness of masses, the absence of any visible hierarchy of generations, and a very high value of the ratio between mass-squared differences suggest essentially different dynamics of neutrino mass formation. Using standard conventions [2], we have:

\[ 23 \lesssim \Delta m_{23}^2 / \Delta m_{12}^2 \lesssim 43 \]  

(1)

The second and third points listed above were discussed and proved to be correct for the mechanism of spontaneous ν mass formation based on the gauge coupling of n flavors in the symplectic group Sp(n/2) [5].

The Sp(n/2) group is distinguished because the invariant Majorana mass, both for right R (chiral) and left L states, is identically equal here to zero. This property is demonstrated only by the fundamental spinor representation of n in the Sp(n/2) group. Any other representations and groups are devoid of it. Therefore, the generation of Majorana masses in Sp(n/2) is only possible upon total destruction of the group: the n mass matrix may appear at once and individual diagonal and nondiagonal elements may be different from zero. This results in a whole spectrum of masses different in flavor, rather than one mass similar for all flavors. At the same time, the Dirac part of the general n mass matrix may be Sp(n/2)-invariant and Dirac masses may be the same for all flavors.

Conventionally, symmetry violation: standard electroweak symmetry, grand unification symmetry, or generation flavor symmetry ( [3,4] and report [6]), is realized through vacuum averages of the various Higgs scalar fields. At that, parity nonconservation and the number of generations are phenomenologically postulated.

The Sp(n/2) group properties become of interest if there is a possibility of non-Higgs, dynamical symmetry violation. Assuming the violation of this kind is possible, one may gain an insight into those SM aspects (parity violation dynamics and the number of generations, among others) that cannot be explained by conventional approaches. However, the nonperturbative
hypothesis and the absence of quantitative solutions certainly reduce the significance of the results.

In this connection, let us note two restrictions that are imposed on this problem:

1. The dynamics of real gauge theories are too complicated. Therefore, we consider only possible symmetry consequences of the hypothesis on the violated $Sp(n/2)$ symmetry of lepton flavors.

2. We assume that symmetry consequences and properties of mass equations (major elements in the process of mass generation) can be observed if a significant part of the gauge theory dynamics is neglected: for example, by using Nambu-Jona-Lasinio fermion propagators [7]: $S_F^{-1}(p) = U^+(\hat{p} - M_{\text{diag}})U$, where $U$ is a diagonalizing matrix. At the same time, the conclusion on the number of neutrino flavors (see below) seems to be more general.

Tuning conditions merely to sustain the system of equations ("gap equations" [7]) for the unambiguous determination of parameters of this highly complex, spontaneously appearing mass matrix ($2n \times 2n$, $L + R$ neutrinos) is difficult to achieve and imposes strict constraints on the system and the matrix itself. Tuning is only possible when considering the $n$-system as a quasi-Majorana form, where $Sp(n/2)-$covariant superpositions of chiral neutrinos-antineutrinos (analogs of Bogolyubov "particle-hole" states [8]) are assumed to define the development of process dynamics.

The physically interesting scenario for the local gauge $Sp(n/2)$ theory requires the following compulsory conditions [5]:

a) $n = 6$ for the number of $n$ flavors,

b) the Majorana mass matrix ($M_{\text{LL}}$) of $\nu$ left flavors ($L \equiv R$ is certainly possible here) vanishes.

Item (b) means the breaking of the $R, L-$symmetry, or parity. Simultaneously, this condition (b) is a requirement of implementation of one of the two necessary elements that engage the see-saw mechanism (see review [4]).

Then, the suggestion that the Majorana scale $M$ is much greater than the Dirac scale $\mu$ (the second condition of the see-saw), brings us ($Sp(3)$ to a system of three light ($\sim \mu^2/M$) and three heavy ($\sim M$) neutrinos in $Sp(n/2)$ [7]. All $\nu$ appear to be Dirac, as the characteristic equation for the Majorana spectrum resolves into mutually similar (in absolute value) eigenvalues [5].

Relation (1) is easily reproduced by the appearing matrices. The great value of (1) does not mean, though, any degeneration of states (1,2), as it does in the usual Higgs system where $\Delta m^2_{12}$ is associated with the exclusively small difference of Dirac mass $\mu_{ik}$ eigenvalues. In the $Sp(3)$ system, values (1) are determined by the difference of the Majorana parameters $M_i$ that, being considerably different from each other ($m_{\nu_i} \sim \mu^2/M_i$), can reproduce (1).

If we employ several types of scalar fields with various $Sp(n/2)$ characteristics (including the $Sp(n/2)$ adjoint representation, satisfying eq. (8) ) and weak isospins, it would be possible to construct lepton mass matrices at any $n$. But such a mechanism (similar to the usual Higgs one) hardly compatible with the weak properties of leptons : Majorana and Dirac masses of neutrinos, necessarily Dirac- type masses for charged leptons, various weak properties of $R$ and $L$ fermion states. At the same time, the dynamical spontaneous violation by the use of quasi-

---

1Note that due to the smallness of neutrino masses, the parameter $M$ is very large: for $\mu \sim \mu_\tau \sim 1$ GeV \(-M \approx 10^{11}$ GeV, for $\mu \sim \mu_W \sim 10^{2}$ GeV - $M \approx 10^{15}$ GeV
Majorana lepton states helps simultaneously to determine the up and down weak components, to note a difference of its $R, L$–properties and is able to produce a proper neutrino spectrum. These were guides for a choice of the model and hypotheses in [5]. The only possible (if the phenomenon exists) number of flavours here is $n = 6$. But the weak neutrality of $Sp(n/2)$ and immediate distinguishing between up and down leptons takes away a picture for charged lepton spectrum.

The present paper is the continuation of paper [5]. We will demonstrate how the participation of $\nu$ in weak processes practically completely determines the structure of electroweak interactions with all leptons and quarks. Charged particles are present as observers, assuming the properties of $\nu$ and their spectrum.

This is related with axial anomalies generated by the spectrum of six Dirac neutrinos.

Let us explain this. Since the $\nu$ mass formation mechanism leads on its own to parity violation, it appears redundant to introduce $R, L$–symmetry violation by direct selection of specific (chiral) electroweak currents (weak isospin $T_W$ and hypercharge $Y$). Let us assume that the currents are vector and conserve parity: for all $R$ and $L$ components of leptons and quarks, $-T_W = 1/2$; for $R$ and $L$ leptons, $-Y = -1$; and for all quarks, $Y = 1/3$.

Expressions for neutrino contributions to these currents upon their transformation into quasi-Majorana and then into massive Dirac particles primarily result in tiny violations of $\nu$ interaction universality ($\sim \mu^2/M^2$) and nonconservation of lepton numbers ($\sim \mu/M$). These phenomena disappear in the low-energy region, upon elimination of heavy $\nu$. After that, however, contributions of light neutrinos become axial, i.e., they lose parity and the axial anomaly appears to be inside the dynamic system under consideration.

Only the electromagnetic current (for all light particles) and the left current of the weak isospin (compulsorily for both light leptons and quarks, neutral and charged components) do not have axial anomalies. For the neutral weak current itself, we obtain the known $Z$ boson current with the Weinberg angle $[1]$. Only these currents do not depend on large masses and remain acceptable for the theoretically consistent low-energy system of leptons and quarks.

Chiral anomalies of low-energy currents also restrict the number of quark doublets and the number of light charged leptons: they cannot exceed the number of light neutrinos, which is three. The anomalies also prohibit mixing of light and heavy neutrinos.

Currents without anomalies are the SM currents.

Although it reproduces and explains many of the well-known properties, this approach does not provide a thorough insight into the structure of the SM electroweak part. First, it does not penetrate into the Higgs part of the system, i.e., does not allow understanding of the mechanism for formation of the masses $M_W$, $M_Z$, or masses of charged fermions. The Higgs mechanism, though, offers no ultimate solution to this problem: it cannot explain the observed, clear hierarchy of charged fermion masses or the hierarchy of quark mixing angles $[2]$.

The gauge group $Sp(3)$ demonstrates, however, that Higgs scalar states cannot arise from fundamental (present in the initial Lagrangian) fields. At the same time, the spontaneous generation of Dirac masses can result in the appearance of scalars in the channel $\bar{\psi}_R \psi_L$, as it happens in the Nambu - Jona-Lasinio model $[7]$.

Secondly, the nonperturbative problem remains unsolved: what happens if the large scale $M$ increases in a system where both ”good” (not leading to an anomaly at $M \to \infty$) and ”bad” (anomalous at $M \to \infty$) currents are present? What influence and how much involvement
would heavy particles have in these ”bad” currents? In the case of neutrino weak currents, the ”anomalous parameter” that is usually considered \[9\] is exceptionally great: \(> 10^{15}\).

Using the terminology of the SM where any number of generations is possible, we can formulate a clear analog to this problem: what happens to three ”good” generations if the mass \(t'\) of the fourth generation quark (or the respective Yukawa constant) increases and if at \(m_{t'} \rightarrow \infty\) only this generation produces a chiral anomaly?

There is no solution to this problem. Indirect arguments lead one to believe that anomalous currents will drop out, whereas the three ”good” generations will reproduce the SM.

In section 2, we briefly explain the logic of paper [5] and present the formulas necessary for the purposes of this paper. In section 3, neutrino contributions to neutral vector currents are expressed through mass states \(\nu\). In section 4, all properties of SM electroweak currents are derived using the gauge \(Sp(n/2)\) approach. Section 5 presents the main results and implications of the proposed scheme.

2 Gauge mechanism for neutrino mass generation

Let us retrace the logic of paper [5] and reproduce the formulas necessary for the purposes of the present paper.

Let us assume that \(n\) lepton flavors are related with each other by means of gauge transformations. This means that interactions which involve leptons (all \(\nu\) and \(e\) flavors) are invariant (locally or globally - is of no significance at this point) to transformations of some group. The symplectic group \(Sp(n/2)\) is distinguished because its invariant Majorana masses are identically equal to zero under the fundamental, spinor representation. Their expressions, through the commonly used chiral operators \(\psi_{R,L}(x) = 1/2(1 \pm \gamma_5)\psi(x)\), are (\(a = 1, 2, \ldots, n\) operators for massless particles): \[
\bar{\psi}^a_R h_{ab} C \psi^T_R = \psi^T_R a h_{ab} \psi_R b \equiv 0 ,
\]

In eq. (2) we omitted the argument \(x\) and will omit it further on. The skew-symmetric matrix \(h(n \times n)\):

\[
h^T = -h , \quad h_{ab} = -h_{ab} , \quad hh^+ = 1 ,
\]

– relates the equivalent conjugate representations \(\psi_a\) and \(\psi^{+T}a\). It is a matrix of alternate numbers \(\pm 1\) on the right diagonal [10]. The matrix of the charge conjugation \(C\) has common properties: \(C = -C^T, C C^+ = 1\).

In \(Sp(n/2)\), covariant analogs of Majorana states can be written in the form:

\[
\Psi_{(R,L)a} = \psi_{(R,L)a} + \gamma_5 h_{ab} C \psi^T_{(R,L)b} ,
\]

\(\gamma_5\) are present in (4) due to the antisymmetry of \(h\), eq.(3), and are absent in representations and groups with \(h = h^T\). One can easily check the fulfillment of ”Majorana conditions”:

\[
\Psi_R = \gamma_5 h C \Psi^T_R , \quad \Psi_L = -\gamma_5 h C \Psi^T_L .
\]

\(\gamma_5\) are present in (4) due to the antisymmetry of \(h\), eq.(3), and are absent in representations and groups with \(h = h^T\). One can easily check the fulfillment of ”Majorana conditions”:

\[
\Psi_R = \gamma_5 h C \Psi^T_R , \quad \Psi_L = -\gamma_5 h C \Psi^T_L .
\]

\(\gamma_5\) are present in (4) due to the antisymmetry of \(h\), eq.(3), and are absent in representations and groups with \(h = h^T\). One can easily check the fulfillment of ”Majorana conditions”:

\[
\Psi_R = \gamma_5 h C \Psi^T_R , \quad \Psi_L = -\gamma_5 h C \Psi^T_L .
\]
The phase for $\Psi_L$ in eq. (5) is selected such ($-\gamma_5$ is introduced for $L$ in (4)) that the Dirac part of the mass matrix ($\bar{\psi}_R\psi_L + \bar{\psi}_L\psi_R$ members) becomes real (see [5]).

It is rather quasi-Majorana combinations (4) than chiral states that play a major role in neutrino mass formation dynamics. Apparently, $\Psi_R$ and $\Psi_L$ are not entirely Majorana states (particle $a +$ antiparticle $n + 1 - a$): they are four-component ones, but there are common, similar states in flavor-different $\Psi_{(R,L)a}$ so that the entire number of independent states in both $\psi_{(R,L)a}$ and $\Psi_{(R,L)a}$, $a = 1, 2, \ldots, n$, ends up being the same. Such a mechanism, obviously, sets simultaneously apart ”neutrino” states from equivalent components of the weak isospin in the original Lagrangian, which is invariant to both $Sp(n/2)$ and weak interactions.

In terms of (4), operator members corresponding to the Majorana and Dirac parts of the mass matrix $\nu$ can be written in the following form:

$$\bar{\Psi}_a^R\Psi_{Rb}, \quad \bar{\Psi}_a^L\Psi_{Lb}; \quad \bar{\Psi}_a^R\psi_{Lb}, \quad \bar{\Psi}_a^L\psi_{Rb}. \quad (6)$$

In [5], it is shown how chiral representations of $Sp(n/2)$ currents (and others ones) are related with Majorana representations through operators (4). Up to this moment, using $\Psi_{(R,L)}$ instead of $\psi_{(R,L)}$ has simply been an equivalent change of variables.

This situation changes when neutrino masses are assumed to result from the spontaneous violation of the $Sp(n/2)$ symmetry, i.e., the vacuum averages of operator combinations (6) (at least part of them) become numbers other than zero. Using these vacuum averages, the complete mass matrix for neutrinos has the form ($2n \times 2n$):

$$M = \begin{bmatrix} M_{RR} & \mu_{RL} \\ \mu_{LR} & M_{LL} \end{bmatrix}. \quad (7)$$

Let us consider $M$ as a symmetric, real matrix ($CP$ is preserved). Owing to Majorana conditions (5), the elements of these matrices are related with one another:

$$M^{+T} = -h^+Mh, \quad \mu^{+T} = +h^+\mu h. \quad (8)$$

Relations (8) result in the characteristic equation for $M$ roots being only dependent on eigenvalue squares, i.e., the $M$ spectrum is $\pm \lambda_i$.

The existence of relations (8) greatly hinders the occurrence of symmetry breaking. One requires such a system of equations for the $M$ parameters that would set these parameters unambiguously. Owing to (8), the number of ”gap” equations [7], defining the parameters, exceeds the number of independent variables (elements of the matrix diagonalizing $M$ plus the number of eigenvalues).

As shown in [5], it is the transition to the Majorana form (4) that leads (in any $Sp(n/2)$-invariant model) to an additional global symmetry of ”gap” equations which turns part of these equations into identities. There is no other choice but to select a form of $M$, (7), that will meet matched conditions: the number of equations less the number of independent parameters is equal to the number of identities among the equations.

To solve the problem unambiguously requires that (physically interesting variant):

a) the number of flavors $n = 6$,

b) the matrix $M_{LL} = 0$, i.e., all its elements be equal to zero ($L \equiv R$ is certainly possible),

5
c) \( \mu_{LR} \) be a diagonal matrix with similar eigenvalues.

Condition (c) should, in fact, be realized automatically, as the invariant, Dirac part of the matrix \( M \) does not vanish in \( Sp(n/2) \). One would believe that the spontaneous violation should follow here exactly this way of the least symmetry breaking.

The first two conditions, their purpose and meaning, are discussed in detail under Introduction. The possible action of the see-saw mechanism is confirmed by condition (b). The assumption of a great difference between the Majorana scale \( M \) and the Dirac \( \mu \) resulting in the remarkable number of flavors \( \mu \) three light and three heavy - as well as the specific character of matrices \( M_{RR} \) (8) facilitate reproduction of the known properties of the \( \nu \)-spectrum (1) and lead to the coincidence with the observed number of light neutrinos.

Diagonalization of the mass matrix meeting conditions (a, b, c) and (8) results in the eigenfunctions of matrix (7) (see [5], eq.(36)):

\[
\Psi_{\pm D} = U_{\pm D}^a (\cos \Theta_{\pm D} \Psi_{Ra} + \sin \Theta_{\pm D} \Psi_{La}),
\]

\[
\psi_{\pm D} = U_{\pm D}^a (-\sin \Theta_{\pm D} \Psi_{Ra} + \cos \Theta_{\pm D} \Psi_{La}),
\]

\( D = 1, 2, 3 \). Here, \( U \) is the orthogonal matrix diagonalizing \( M_{RR}(6 \times 6) \). \( \Psi_D \) corresponds to the heavy masses \( \nu(M_D \sim M) \) and \( \psi_D \) represents the light particles \( (m_D = \mu^2/M_D) \). Values of \( \cos \Theta_D \) and \( \sin \Theta_D \) are:

\[
\cos \Theta_D = \frac{1}{\sqrt{1 + (\mu/M_D)^2}}, \quad \sin \Theta_D = \frac{\mu/M_D}{\sqrt{1 + (\mu/M_D)^2}},
\]

\[
\cos \Theta_{-D} = \cos \Theta_D, \quad \sin \Theta_{-D} = -\sin \Theta_D.
\]

\( \Psi_{Ra} \) and \( \Psi_{La} \) are massless "Majorana" states (4), in all 24. Massive eigenstates of matrix (7) \( \Psi_D \) and \( \psi_D \), also 24 in number, have properties similar to (5):

\[
\gamma_5 h C \Psi_D^{TD} = \Psi_{-D}, \quad \gamma_5 h C \psi_D^{TD} = -\psi_{-D},
\]

\( h_{D',D} \equiv h_{-DD} = -h_{-DD} \). Equations (11) relate states with opposite mass signs. Equations (9)-(11) will help solve the principal problem in the next section: expressing massless Majorana states \( \Psi_{(R,L)a} \) (or chiral \( \psi_{(R,L)a} \)) through functions of massive Dirac neutrinos. Further, in section 4, we will express electroweak currents in terms of physical, massive (Dirac) \( \nu \).

### 3 Representation of chiral vector currents through Dirac massive neutrinos

To solve this problem, let us determine Dirac states by combining two quasi-Majorana spinors (9) with masses equal in absolute value into one entity. This can be done in a standard way. First, let us build true Majorana massive states (all masses are positive) for heavy neutrinos:

\[
\Psi_D^{(1)} = \frac{\Psi_D + C\Psi_D^{DT}}{\sqrt{2}} = \frac{\Psi_D + \gamma_5 h \Psi_{-D}}{\sqrt{2}},
\]

\( 6 \)
\[
\Psi_D^{(2)} = -\gamma_5 h \frac{\Psi_D - C\Psi_{DT}}{\sqrt{2}i} = \Psi_D - \gamma_5 h \Psi_D,
\]
and for light neutrinos:
\[
\chi_D^{(1)} = \frac{\psi_D + C\psi_{DT}}{\sqrt{2}} = \frac{\psi_D - \gamma_5 h \psi_D}{\sqrt{2}},
\]
\[
\chi_D^{(2)} = \gamma_5 h \frac{\psi_D - C\psi_{DT}}{\sqrt{2}i} = \frac{\psi_D + \gamma_5 h \psi_D}{\sqrt{2}i},
\]
(13)

The signs are chosen for convenience. The second equations in (12) and (13) are obtained by using relations (11). Equations (12) and (13) are the first \(Sp(n/2)\)-noncovariant formulas, in which \(h \equiv h^{D-D} = \pm 1\).

Dirac states can be chosen by different, but physically equivalent methods. The difference lies in the definition of what is considered a particle and what an antiparticle in systems of heavy and light states (or, which is the same, in the definition of \(\psi\) and \(\bar{\psi}\) for Dirac particles).

The obvious definitions of Dirac states (for heavy and light \(\nu\), respectively):
\[
\Psi_{MD} = \frac{\Psi_D^{(1)} + i\Psi_D^{(2)}}{\sqrt{2}} \equiv \Psi_D, \quad \psi_{\mu D} = \frac{\chi_D^{(1)} + i\chi_D^{(2)}}{\sqrt{2}} \equiv \psi_D,
\]
(14)
after inverting expressions (9), result in the following formulas:
\[
\Psi_{La} = U_a^{+D}(-\gamma_5 h^+ \Psi_{MD}^C \sin \Theta_D + \cos \Theta_D \psi_{\mu D}) + U_a^{+D}(\Psi_{MD} \sin \Theta_D - \cos \Theta_D \gamma_5 h^+ \psi_{\mu D}^C),
\]
\[
\Psi_{Ra} = U_a^{+D}(\gamma_5 h^+ \Psi_{MD}^C \cos \Theta_D + \sin \Theta_D \psi_{\mu D}) + U_a^{+D}(\Psi_{MD} \cos \Theta_D + \sin \Theta_D \gamma_5 h^+ \psi_{\mu D}^C).
\]
(15)

To make eq.(15) covariant, the symbol \(h^+\) is introduced which corresponds to \(h\) in (12)-(13). One should mention, however, that all \(h\) and \(h^+\) are \((\pm 1)\) and selecting either value makes no essential difference. \(\Psi^C\) and \(\psi^C\) are antiparticle operators: \(\psi^C = C\bar{\psi}^T\).

The same symbol \(D\) stands for both large and small masses. In the see-saw model, it is reasonable to use the same symbol for masses with different sign; this is how pairs of large-small masses are made up here [4]. Then, the corresponding Dirac particles should be determined by the formulas:
\[
\Psi_{MD} = \frac{\Psi_D^{(1)} - i\Psi_D^{(2)}}{\sqrt{2}} \equiv C\Psi_D^T, \quad \psi_{\mu D} = \frac{\chi_D^{(1)} - i\chi_D^{(2)}}{\sqrt{2}} \equiv \psi_D.
\]
(16)

In eq.(16) we retained the small mass as a "particle". Relation (11) is used again in the first expression of (16).

Then, \(\Psi_{La}\) and \(\Psi_{Ra}\) are equal to:
\[
\Psi_{La} = U_a^{+D}(-\gamma_5 h^+ \Psi_{MD} \sin \Theta_D + \cos \Theta_D \psi_{\mu D}) + U_a^{+D}(\Psi_{MD}^C \sin \Theta_D - \cos \Theta_D \gamma_5 h^+ \psi_{\mu D}^C),
\]
\[
\Psi_{Ra} = U_a^{+D}(\gamma_5 h^+ \Psi_{MD} \cos \Theta_D + \sin \Theta_D \psi_{\mu D}) + U_a^{+D}(\Psi_{MD}^C \cos \Theta_D + \sin \Theta_D \gamma_5 h^+ \psi_{\mu D}^C).
\]
(17)

Formulas (15) and (17) allow expressing \(Sp(n/2)\)-invariant chiral vector currents through massive states. At first, by means of direct substitution we obtain:
\[
\tilde{\bar{\psi}}_L \gamma_\rho \psi_{La} = -\frac{1}{2} \tilde{\bar{\psi}}_L \gamma_\rho \gamma_5 \Psi_{La}, \quad \tilde{\bar{\psi}}_R \gamma_\rho \psi_{Ra} = +\frac{1}{2} \tilde{\bar{\psi}}_R \gamma_\rho \gamma_5 \Psi_{Ra}.
\]
(18)
Note that the Majorana vector currents:

\[ \bar{\Psi}_L^a \gamma_\rho \Psi_{La} = \bar{\Psi}_R^a \gamma_\rho \Psi_{Ra} \equiv 0 \]

– are absolute neutrality of Majorana multiplets.

Transforming eq. (18) by using (15) and (17), we obtain:

\[ \bar{\psi}_a^L \gamma_\rho \psi_{La} = -\sin^2 \Theta_D \bar{\Psi}_M^D \gamma_\rho \gamma_5 \Psi_{MD} - \cos^2 \Theta_D \bar{\psi}_{\mu D}^C \gamma_\rho \gamma_5 \psi_{\mu D} - R_\rho (\Psi_{MD}, \psi_{\mu D}) \frac{1}{2} \sin 2\Theta_D , \]

\[ \bar{\psi}_a^R \gamma_\rho \psi_{Ra} = \cos^2 \Theta_D \bar{\Psi}_M^D \gamma_\rho \gamma_5 \Psi_{MD} + \sin^2 \Theta_D \bar{\psi}_{\mu D}^C \gamma_\rho \gamma_5 \psi_{\mu D} - R_\rho (\Psi_{MD}, \psi_{\mu D}) \frac{1}{2} \sin 2\Theta_D , \]

where, for (15) and (17), respectively, the vector \( R_\rho \) is equal to:

\[ R_\rho (\Psi_{MD}, \psi_{\mu D}) = \begin{cases} \bar{\Psi}_M^D \gamma_\rho \gamma_5 \Psi_{MD} + \bar{\psi}_{\mu D}^C \gamma_\rho \gamma_5 \psi_{\mu D} \\ \bar{\Psi}_M^D \gamma_\rho \gamma_5 \Psi_{MD} + \psi_{\mu D} \gamma_\rho \gamma_5 \Psi_{MD} \end{cases} . \]

Certainly, in formulas (20) and (21), summation over \( D = 1, 2, 3 \) is implied. To obtain these expressions, we used the orthogonality of the matrices \( U \).

The axial current has a simple formula, which is similar for both cases (15) and (17):

\[ \bar{\psi}^a \gamma_\rho \gamma_5 \psi_a = \bar{\Psi}_M^D \gamma_\rho \gamma_5 \Psi_{MD} + \bar{\psi}_{\mu D}^C \gamma_\rho \gamma_5 \psi_{\mu D} . \]

The vector current is:

\[ \bar{\psi} \gamma_\rho \psi = \cos 2\Theta_D (\bar{\Psi}_M^D \gamma_\rho \gamma_5 \Psi_{MD} - \bar{\psi}_{\mu D}^C \gamma_\rho \gamma_5 \psi_{\mu D}) - \sin 2\Theta_D R_\rho (\Psi_{MD}, \psi_{\mu D}) . \]

Let us note the two properties of vector currents:

1. With regard to heavy particles (\( \sim M \)), transitions between light and heavy neutrinos are possible, with both conservation and nonconservation of the lepton number. This effect is negligible at \( \mu << M \) (\( \sim \mu/M \)).

2. The presence in (23) of \( \cos 2\Theta_D \) attests to a small (at \( \mu << M \)) nonuniversality of neutrino vector interactions (\( \sim \mu^2/M^2 \)).

Both formulas (22) and (23) demonstrate that the transition to massive states in currents (i.e., neutral currents, the subject of the next section) does not produce transitions between the types of light neutrinos (between flavors).

In conclusion, let us point out that the sign of the \( \sim \sin 2\Theta_D \) contribution in eq. (23) depends on selection of \( h \Rightarrow \pm 1 \).

### 4 Electroweak currents

During the spontaneous transition to massive neutrinos, \( R, L \)–symmetry is not preserved. Let us show that this is a good enough reason for distinguishing the observed currents of
electroweak interaction, namely, currents of $W$-bosons and the photon, from common vector (nonchiral) currents which lack parity violation.

In the SM, the interaction of $W$ only with left chiral currents is a postulate: all left fermion components are weak isospin doublets and all right ones are singlets. In addition to weak isospin $T_W$ currents in the SM, there is the $R,L-$asymmetric current of the hypercharge $Y$: $Y = 1/3$ for $L$ quarks, $Y = -1$ for $L$ leptons; for $R$ quarks $Y = 4/3$ for $u$ and $Y = -2/3$ for $d$ $R$ quarks; $Y = -2$ for charged $R$ leptons, and $Y = 0$ for $R$ neutrinos [1]. As a result, we obtain the electromagnetic and left weak currents, which explain the observed phenomena.

Let us take, as the basis for the electroweak theory in this paper, $R,L-$symmetrical (i.e., preserving parity) full vector currents of all initially massless leptons and quarks. In such currents, the characteristics of both right and left particles should be similar. Therefore, not only $L$ but also $R$ components are doublets of the weak isospin; all $L$ and $R$ quarks have $Y = 1/3$; and all $L$ and $R$ leptons have $Y = -1$:

$$J_{W,\rho} = g \sum_f (\bar{\ell}_\rho T_W \ell + \bar{q}_\rho T_W q), \quad J_{Y,\rho} = g' \sum_f \left( \bar{\ell}_\rho \frac{Y}{2} \ell + \bar{q}_\rho \frac{Y}{2} q \right).$$

(24)

All fermion operators have a four-component Dirac form, and the sum is carried out over all types of particles (including color for quarks). The lepton part of currents is also invariant to $Sp(3)$, whereas quarks can be neutral towards this group.

Then, the fundamental theory for massless particles, invariant to $Sp(3)$, $SU_W(2)$, $SU(3)$ (color) and $Y$, has only vector currents, lacks anomalies, and is entirely defined and renormalizable. Of course, it should be supplemented with the mass formation mechanism for $W$, $Z$, and charged leptons and quarks. Let us assume that this mechanism involves violation only of the weak $T_W$ group and that no new $R,L-$symmetry violations occur.

Compulsory participation of Majorana states in the $Sp(3)$-group spontaneous violation model under discussion requires some additional explanations in connection with the appearance of the quantum numbers $T_W$ and $Y$. As is known, $\psi$ and $\bar{\psi}$ have different values of $T_3$ and $Y$ and it appears impossible to combine them into a Majorana object. We have earlier explained, however, that the dynamically active states (4) being used are quasi-Majorana ones, and their $\psi$ and $\bar{\psi}$ have different characteristics ($Sp(n/2)$ representation components, $a$ and $n + 1 - a$). The states of this kind resemble Bogolyubov [8] systems: a particle with one spin projection + "hole" with the opposite spin projection. The above complexity, however, is of purely technical significance: what should be the form of state (4) (for example, a doublet). As such, it has no implications for our discussion of vector currents; the results of this paper and paper [5] are not affected.

Let us express the neutrino contribution in eq.(24) through Dirac mass states, i.e., substitute in (24) expression (23) and neglect contributions from heavy masses. For neutral currents, we obtain:

$$- \bar{\psi}_{\mu D} \gamma_\rho \gamma_5 \left( \frac{T_3}{Y/2} \right) \psi_{\mu D} + \{\text{vector part } T_3 \text{ or } Y/2, \}$$

(25)

$L + R$ components of charged leptons and $u$ and $d$ quarks.
So, the low-energy part appears to be axial and contains anomalies induced by contributions from interactions between vector bosons of the theory presented by the well-known triangle diagrams \([1]\).

Only those anomalies are important that are caused by weak and hypercharge vector bosons: \(W^\pm, W_0\) and \(B\). Color currents of quarks apparently do not introduce anomalies as they have no relation to neutrinos. The gauge bosons \(Sp(3), F_\mu\) in terms of \([5]\), should acquire heavier masses if such a complex spontaneous violation of the group (involving all its operators) does take place. At the energy of the order of these masses, no anomalies are present due to the full vectorness of the whole interaction system.

There are six possible combinations of weak \(W\) and hypercharge bosons \(B\) in which an anomaly can occur. These combinations are calculated and sorted by \(R\) and \(L\), for leptons and quarks. We do not cancel quark and charged lepton contributions due to their vectorness, i.e., in the sum of \(L + R\) contributions: as the left world is now different from the right, it is important to know the state of things in each of them separately. The following table shows the results of the calculations.

**Low-energy anomalies by \(R, L\) sectors; leptons \((\ell)\) and quarks \((q)\)**

|            | \(R\) | \(L\) |
|------------|-------|-------|
| \(\ell\)   | \(+2/9\) | \(+2\) |
| \(q\)      | \(+2\) | \(+2/9\) |
| \(BBB\)    | \(-2\) | \(0\) |
| \(W_3BB\)  | \(-2\) | \(0\) |
| \(BW^+W^-\) | \(-2\Theta_{qR}^2\) | \(+2\Theta_{\ell}^2\) |
| \(W_3W^+W^-\) | \(-2\Theta_{qR}^2\) | \(+2\Theta_{\ell}^2\) |
| \(BW_3^2\) | \(+2\) | \(+2\) |
| \(W_3W_3^2\) | \(+2\) | \(+2\) |

The calculated numbers are coefficients at anomalous divergencies \([11]\):

\[
\partial_\mu j_\mu^{(S)} = \frac{g_1 g_2 g_3}{(4\pi)^2} \text{Tr}FF^\dagger. \tag{26}
\]

They take into account signs and values of the \(Y(Y_\ell = -1, Y_q = 1/3)\) and \(2T_3\) components \((\pm 1)\) as well as the difference in sign of chiral \(R(+1)\) and \(L(-1)\) contributions.

When calculating table contributions from neutral currents, we used the obvious formulas of the low-energy expression (25) for one generation (for light masses \(\mu_D\) for \(\nu\) and \(m_d\) for \(\ell\)):

\[
J^{(T_3)} = \bar{\nu}_L \gamma \nu_L - \bar{\ell}_L \gamma \ell_L - \bar{\nu}_R \gamma \nu_R - \bar{\ell}_R \gamma \ell_R + \bar{u}_i \gamma u_i - \bar{d}_i \gamma d_i, \tag{27}
\]

\[
J^{(Y)} = -\bar{\nu}_L \gamma \nu_L - \bar{\ell}_L \gamma \ell_L + \bar{\nu}_R \gamma \nu_R - \bar{\ell}_R \gamma \ell_R + \frac{1}{3} \bar{u}_i \gamma u_i + \frac{1}{3} \bar{d}_i \gamma d_i,
\]

summed over the color \(i\) In eq.(27), the charges \(g/2\) and \(g^1/2\), respectively, are omitted. Sign difference of the \(R\) and \(L\) parts results from axiality of the neutrino contribution in eq.(25).

Investigating charged current participation in \(BW^+W^-\) and \(W_3W^+W^-\) is more complicated, although more informative. At energies \(E << M\), there is no interaction of light...
neutrinos from eqs. (15), (17) with right charged leptons ("electrons"), because large contributions in (15), (17) for \( R \) neutrinos, \( \nu_R = (1/2)(1 + \gamma_5)\Psi_R \), include only heavy particles \( (M_D) \). Therefore, at \( E << M \) charged currents contain only \( \ell_L \) components:

\[
\bar{\ell}^c_L \gamma \nu_{La} + \bar{\ell}^c_R \gamma \nu_{Ra} + \text{c.c.} \simeq \bar{\ell}^c_L \gamma \frac{1}{2}(1 - \gamma_5) \left( U^+_a - D \nu_{\mu D} - U^+_a D \gamma_5 \gamma^T \mu_D \right) + \text{c.c.} \tag{28}
\]

The four-component Dirac spinor of the massless electron \( \ell \) should be expressed through massive states of all electron flavors (at that, the origin of \( \ell \) masses is of no importance). Generally, we have:

\[
\ell_a = A^d_a \ell_{md} + \tilde{A}^d_a \ell_{Md}, \tag{29}
\]

\( A^d_a, \tilde{A}^d_a (6 \times 3) \). Where \( m_D \) and \( M_D \) are masses of light and heavy electrons and \( A^d_a \) and \( \tilde{A}^d_a \) are some matrices \( (6 \times 3) \). We have assumed that the number of light electrons is equal to the number of light neutrinos. An inequality would lead to the lack of non-anomalous currents in a low-energy system, i.e., to the absence of a low-energy limit independent of heavy masses (see the discussion of eq.(37)).

The superpositions

\[
\ell^{(1)}_{LD} = U_{-D} a A^d_a \ell_{mdL} \quad \text{and} \quad \ell^{(2)}_{LD} = U_D a A^d_a \ell_{mdL} \tag{30}
\]

appear in eq.(28) as representatives of \( L \)-electron states. The interaction of \( \ell_{LD} \) with \( R \) and \( L \) neutrinos and antineutrinos does not conserve the lepton number

\[
\bar{\ell} \gamma + \text{c.c.} = \bar{\ell}^{(1)}_{LD} \gamma \nu_{\mu D L} + \bar{\ell}^{(2)}_{LD} \gamma C\nu^T_{\mu D R} + \text{c.c.} \tag{31}
\]

The interaction \( W^\pm \) with \( \ell^{(2)}_{LD} \) results in four additional anomalous contributions\(^3\), which are not included in the Table. These contributions are proportional to quantities that can be termed as generalized sums of squares of lepton mixing angle cosines. In the presence of both members (31), we have two sums over all low-energy lepton flavors (refer to a single generation):

\[
\Theta^2_{\ell L} = \frac{1}{3} \sum_{D,d} A^a_D U^+_a - D \nu_{-D a} \ell^d_m, \quad \Theta^2_{\ell R} = \frac{1}{3} \sum_{D,d} A^a_D U^+_a D \nu_{D a} \ell^d_m. \tag{32}
\]

At \( \Theta^2_{\ell R} \neq \Theta^2_{\ell L} \) (using Table designation \( \Theta_{\ell L} \equiv \Theta^2_{\ell L} \)) but at \( \Theta^2_{\ell R} \neq 0 \), currents without anomalies are not present at all and there is no low-energy limit independent of large masses. At \( \Theta^2_{\ell R} = \Theta^2_{\ell L} \) (for which there is no apparent reason), there is one non-anomalous electromagnetic current (see eq. (35)), whereas \( R \) and \( L \) weak \( T^\pm, T^3 \) currents are anomalous.

The only scenario that results in a complete spectrum of independent low-energy currents is the exclusion of the interaction that violates the lepton number. In this scenario, the matrix \( A \) should have the form:

\[
A^d_a = U^+_a - D V^d_D \rightarrow \Theta^2_{\ell R} = 0, \tag{33}
\]

---

\(^3\)The propagator of the antiparticle \( \nu^- \) assings a sign opposite to normal \( R, L \) cases to the liner divergence of the diagrams \([II]\) and consequently to the anomaly: \(<C \bar{\nu}_R, \nu_R^T C >= -C <\bar{\nu}_R, \nu_R^T C > C \rightarrow -C \hat{\rho}^T C = -\hat{p} \).
since for the unitary matrix $U$ we have $U_D^a U_a^{-D} = 0$. Then

$$\Theta_l^2 = \Theta_{\ell L}^2 = \frac{1}{3} \sum_{d,D} V^+_{dD} V^D_d = \frac{1}{3} \sum_{D,d} \cos^2 \Theta_{dD}, \quad (34)$$

$V^d_D$ is a matrix and $\Theta_{dD}$ are mixing angles of light leptons. The Table shows this very pattern of anomaly distribution. The quantity $\Theta_{qL}^2 = \Theta_{qR}^2 \equiv \Theta_q^2$ for quark generations is similar to (34) for leptons.

Then, based on the Table, the left current $T_{3L}$ (the interaction with $W_3$) is apparently non-anomalous. At the same time, the electromagnetic current interacting with the photon $[1]$

$$j_{em} = e Q = e \left( T_3 + \frac{Y}{2} \right), \quad e = \frac{g g'}{\bar{g}}, \quad \bar{g} = \sqrt{g^2 + g'^2} \quad (35)$$

is also non-anomalous provided that ($T_3$ and $Y/2$ refer to both $L$ and $R$ components):

a) the quantity $\Theta^2_{qR}$ can be excluded from consideration

b) $\Theta^2_q = \Theta^2_{\ell L}$.

Right lepton contributions to charged current $T_{\pm}^\pm$ anomalies are absent owing to the low-energy interaction structure (31) resulting from (33):

$$\bar{\ell} \gamma \nu + c.c. = \bar{\ell}_{mL} V^+_{dD} \gamma \nu_{dL} + c.c. \quad (36)$$

The contribution $\Theta^2_{qR}$ from right quark currents can be neglected in the system of only left weak currents. Condition (a) means selecting only left weak currents. The equality (b) of lepton and quark expressions (34) independent of each other makes sense only in one instance, namely, when systems of light leptons and quarks are complete:

$$\Theta^2_{\ell L} = \Theta^2_q = 1. \quad (37)$$

This means that there should be three flavors of light electrons, there should be no mixing of light and heavy $L$ leptons, and there should be only three generations of quarks. At $d = D = 3$, the representation of eq. (33) for $A^D_a$ is already equivalent to the absence of mixing.

Also note that eq. (33) indicates a close relationship between neutrino and electron spectra: $U$ is the matrix diagonalizing the Majorana mass matrix. This relationship is in conflict with the Higgs mechanism of mass formation where spectra of up and down components are independent, being determined by different $SU_L(2)$ invariants of Yukawa couplings.

One can see from the Table that all anomalies from all $R$ and $L$ components of light particles in superposition (35) are reduced. The electromagnetic current (35) includes the currents $T_3$ and $Y/2$ of both $L$ and $R$ sectors.

Consequently, the Table corresponds to a system of non-anomalous currents consisting of three left weak currents that interact with $W$ and the full electromagnetic current (35) that interacts with the photon. Only these currents are independent of the high-energy part of the scheme. Let us point out that the difference of currents other than $T_L$ and $Q$ is not that $T_R = 0$, as it is believed in the SM, but that these currents are anomalous, i.e., require involvement of heavy particles. $W$ also interacts with $R$ components; at that, participation of
heavy fermions is imperative, as in the instance of the charged current with $\ell_R$ participation (see eqs. (15) and (17)). Therefore, the current interacting with the vector $Z$-boson $[\Pi]$ orthogonal to the photon:

$$\bar{g}(T_3 - \sin^2 \Theta_W Q), \quad \sin \Theta_W = \frac{g'}{\bar{g}},$$

is non-anomalous and is consistently low-energy only if $T_3 \equiv T_{3L}$, i.e., contains only left components of $\ell$ and $q$ massive states, as is postulated in the SM.

The system of left weak and full electromagnetic currents is the very system of SM fermion currents. It appears to be the only anomaly-free system and therefore is independent of high-energy physics in our scenario. Our mechanism of the low-energy anomaly contraction for weak currents essentially differs from the same in SM. There we had independent contraction for any row of the Table. Here the anomaly is absent only for the sum of currents forming the observed electromagnetic one.

The analysis of charged currents $T^\pm$ proved to be heuristically important:

1. The anomalous character of $R$-charged current has been proved: taking it into consideration leads to the quark anomaly $\Theta^2_{qR}$ that is not compensated in $R$ components. This does not permit turning the neutral electromagnetic current into non-anomalous.

2. The $L$-quark anomaly is coupled with the $L$-lepton anomaly and they compensate each other. Therefore, although the $R, L$-symmetry was violated only in the lepton sector, non-anomalous electromagnetic current would not be possible in the absence of quarks.

3. It is charged currents that limit the number of quark generations.

The SM allows any number of generations. In the proposed scenario, attempts to change the number of quark generations and light electrons (only three kinds of light neutrinos are possible, see [5]) would lead to the disappearance of the entire system of non-anomalous low-energy currents independent of high-energy physics. These currents exist only in three generations of particles included in the SM.

The consistent and decisive consideration of the problem would include investigating the change in the properties of a well defined system (without anomalies) with the increase in the large ”anomalous” parameter ($M$). This could shed light on the ”fate” of all currents and define more specifically the influence of heavy masses on currents that remain anomalous at $M \to \infty$. We can only indicate that in the proposed scheme currents that are non-anomalous, and therefore the best-suited for independent low-energy survival, are the ones of the SM: the left weak and the total electromagnetic currents.

As discussed in Introduction, our study is not complete. We only can repeat here that the existence of doublets of both left and right components does not allow inclusion of the Higgs scalar field with required properties ($T_W = 1/2$) in the fundamental Lagrangian with Yukawa couplings. The immediate result is the violation of the weak $SU(2)$ symmetry. At the same time, scalar fields $T_W = 0$ and $T_W = 1$ may participate in the dynamic of mass formation.

### 5 Conclusion

Notwithstanding the unexplained masses $W, Z$ and charged leptons-quarks, the hypothesis of the gauge $Sp(3)$ symmetry of lepton flavors is remarkable in that it allows one to relate and
explain the observed facts and proposed properties that form the basis of the SM, as well as offer answers to a number of fundamental questions.

1. The number of light neutrinos is three (plus three very heavy ones). The same applies to charged leptons. Light leptons do not mix with heavy ones.

2. There may only be three generations of quarks (there is no reason to expect a greater number of generations).

3. The $R - L$ asymmetry in spontaneous neutrino mass formation is the sufficient cause of weak parity nonconservation.

4. The see-saw mechanism is necessary for the appearance of $\nu$ masses.

5. There can be similar hypercharge values and a common weak isotopic pattern for Dirac states of all leptons and quarks.

In weak currents, before the mass $\nu$ formation mechanism steps in, parity could conserve.

6. Electromagnetic, $Z$ bozon and weak charged currents are distinguished at low energies.

7. A simple explanation of neutrino spectrum features becomes possible.

8. The double $\beta$—decay is absent in the dynamics under consideration.

The author is grateful to Ya. I. Azimov, G. S. Danilov and V. Yu. Petrov for useful and enlightening discussions.
References

[1] L. B. Okun. *Leptons and Quarks* (Ed. M. Editorial "Nauka", Moskva, 1981)

[2] Particle Date Group, Phys.Lett. B 667, 36, 517 (2008).

[3] H. Fritzsch and Z. Z. Xing, Prog.Part.Nucl.Phys. 45, 1 (2000); G. Altarely and F. Ferguglio, New.J.Phys. 6, 106 (2004); A. Yu. Smirnov, arXiv:0810.2668 v.1 [hep-ph].

[4] R. N. Mohapatra and A. Y. Smirnov, Arxiv:0603118 v.1 [hep-ph]; Ann.Rev.Nucl.Part.Sci. 56, 569 (2006).

[5] I. T. Dyatlov, Ya.F. 72, 2121 (2009); arXiv:0910.0153 [hep-ph].

[6] J. G. Pati, Arxiv:0507307 v.1 [hep-ph].

[7] Y. Nambu and G. Jona-Lasinio, Phys.Rev. 122, 345 (1961); W. A. Bardeen, C. T. Hill, and M. Lindner, Phys.Rev. D 41, 1647 (1990); S. P. Klevansky, Rev.Mod.Phys. 64, 649 (1992).

[8] N. N. Bogolyubov, Izv.AN SSSR, ser.fiz. 11, 77 (1947); 34, 735 (1958).

[9] J. Preskill, Ann.Phys. (N.Y.) 210, 323 (1991).

[10] R. E. Berehnds, J. Dreitlein, C. Fronsdal, and B. W. Lee, Rev.Mod.Phys. 34, 1 (1962).

[11] S. Adler, Phys.Rev. 177, 2426 (1969); J. Bell and R. Jackiw, Nuovo Cimento A 60, 47 (1969).