Sports scheduling affects incentives:
The case of the UEFA Champions League

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“Teams and leagues want to optimize their investments by playing a good schedule which seeks to meet various criteria. Good fixtures are important in order to maximize revenues, ensure the attractiveness of the games, and to keep the interest of both the media and the fans.”

Abstract
A team might lose a powerful incentive to win in a round-robin contest if its final rank does not depend on the outcome of the matches still to be played. The current paper introduces a classification scheme to identify these weakly (where one team is indifferent) or strongly (where both teams are indifferent) stakeless games. The probability of such matches can serve as a novel fairness criterion to compare and evaluate timetabling alternatives. An optimal sequence of games with respect to the proposed metric increases the utility of all stakeholders at almost no price if the scheduling constraints are appropriately defined. Its application is illustrated through the 2021/22 season of the UEFA Champions League. According to our simulation model, the same schedule is optimal across all groups and the option followed in four of the eight groups is the best under a wide set of parameters. Avoiding strongly stakeless matches is verified to be a likely goal in the computer draw of the fixture that remains hidden from the public.

Keywords: OR in sports; simulation; sports scheduling; tournament design; UEFA Champions League

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1 Source: Kendall et al. (2010, p. 1).

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1 Introduction

There is powerful support for the proposition that the level and structure of incentives influence the performance of the agents in sports (Ehrenberg and Bognanno, 1990). Consequently, one of the most important responsibilities of sports governing bodies around the world is to set the right incentives for the contestants (Szymanski, 2003).

Knockout tournaments are guaranteed to be exciting until the end since the only way to win is not to lose. On the other hand, round-robin tournaments are sometimes decided before the final round, resulting in uninteresting games that should be avoided as much as possible because:

- A team still in contention to win the championship positively affects attendance (Pawlowski and Nalbantis, 2015);
- If at least one team is completely indifferent between winning, drawing, or even losing, then it may field second-team players and take other factors such as resting before the next match into account, which is unfair to the teams that have already played against this indifferent team (Chater et al., 2021).

Therefore, the matches played in a round-robin contest can be classified into three categories: competitive (neither team is indifferent), weakly stakeless (one team is indifferent), and strongly stakeless (both teams are indifferent).

To highlight the importance of this division, let us see an illustration from the most prestigious European club competition in association football (henceforth football).

Table 1: Ranking in Group G of the 2018/19 UEFA Champions League after Matchday 5

| Pos | Team                  | W | D | L | GF | GA | GD | Pts |
|-----|-----------------------|---|---|---|----|----|----|-----|
| 1   | Real Madrid CF        | 4 | 0 | 0 | 12 | 2  | +10| 12  |
| 2   | AS Roma               | 3 | 0 | 2 | 10 | 6  | +4 | 9   |
| 3   | FC Viktoria Plzeň     | 1 | 1 | 3 | 5  | 15 | −10| 4   |
| 4   | PFC CSKA Moskva       | 1 | 1 | 3 | 5  | 9  | −4 | 4   |

Pos = Position; W = Won; D = Drawn; L = Lost; GF = Goals for; GA = Goals against; GD = Goal difference; Pts = Points. All teams have played four matches.

Example 1. (Gieling, 2022) Table 1 presents the standing of Group G in the 2021/22 season of the UEFA Champions League—a home-away round-robin tournament with four teams—before the last round. Real Madrid and Roma were guaranteed to be the group winner and the runner-up, respectively.

With the first place in the group already secured, Real Madrid had the luxury of fielding a fully rotated squad against CSKA Moskva. Perhaps not coincidentally, empty seats were everywhere when the final whistle came since Real Madrid lost a European home tie by more than two goals first in its history (Bell, 2018). However, even the remarkable performance of CSKA Moskva was insufficient to finish third in the group because Roma also suffered a shocking defeat in Plzeň.

Inspired by similar examples, our paper aims to investigate the influence of the timetable on the competitiveness of the matches played in the last round(s) of a round-robin tournament. The group stage of the UEFA Champions League is considered as a
case study to compare the schedules used in its 2021/22 season. This analysis is also essential for understanding the likely goals of the organiser—the Union of European Football Associations (UEFA)—since the official regulation provides surprisingly little information on how the group matches are scheduled (UEFA, 2021b, Article 16.02).

The main contributions of the study can be summarised as follows:

- We introduce a novel fairness measure to compare and evaluate round-robin tournament schedules, namely, the probability of (weakly and strongly) stakeless games played towards the end of the contest;
- We present that the UEFA Champions League group stage can be reliably simulated by identifying the teams with the pots from which they are drawn, guaranteeing the existence of a universal optimal schedule across all groups, independently of the outcome of the group draw;
- The timetabling option followed in four of the eight groups in the 2021/22 Champions League is found to be the best under a wide set of reasonable parameters;
- Minimising the number of strongly stakeless matches is shown to be a likely goal in the computer draw of the Champions League fixture that remains hidden from the public.

We also convey a clear message for tournament organisers: choosing an optimal sequence of games with respect to the proposed metric can increase the utility of all stakeholders at almost no price.

The remainder of the paper is organised as follows. Section 2 gives a concise overview of the literature. The game classification scheme is presented and applied to the current format of the UEFA Champions League in Section 3. The simulation model and the scheduling options are detailed in Section 4. Section 5 provides the main findings, while Section 6 contains conclusions and reflections.

2 Related literature

Although the topic can be connected to many research fields, we focus on three areas in the following: fairness, match importance, and scheduling.

Fairness

Suspicion of collusion can badly harm a tournament and even the reputation of the sports industry, independently of whether the match is actually fixed or not. Nonetheless, the history of football is full of examples of tacit coordination. The most famous match is probably the “disgrace of Gijón”, where both West Germany and Austria were satisfied by the result of 1-0 that let the opposing teams advance to the second phase of the 1982 FIFA World Cup (Kendall and Lenten, 2017, Section 3.9.1). In order to prevent similar scandals, since then almost all tournaments—including the UEFA Champions League—are designed such that the games of the last round are played simultaneously. However, this rule has not fully guaranteed fairness as we have seen.

Some recent papers have attempted to determine the best schedule for the FIFA World Cup groups. Stronka (2020) investigates the temptation to lose, resulting from the desire to play against a weaker opponent in the first round of the knockout stage. This danger is
found to be the lowest if the strongest and the weakest competitors meet in the last (third) round. Inspired by the format of the 2026 FIFA World Cup, Guyon (2020) quantifies the risk of collusion in groups of three teams, where the two teams playing the last game know exactly what results let them advance. The author identifies the match sequence that minimises the risk of collusion. Chater et al. (2021) develop a general method to evaluate the probability of any situation in which the two opposing teams might not play competitively, and apply it to the current format of the FIFA World Cup (a single round-robin contest with four teams). The scheduling of matches, in particular, the choice of teams playing each other in the last round, turns out to be a crucial factor for obtaining exciting and fair games.

The current work joins this line of research by emphasising that a good schedule is able to reduce the threat of tacit collusion. However, the competition examined here—home-away round-robin with four teams—is much more complicated than a single round-robin with three (Guyon, 2020; Chater et al., 2021), four (Stronka, 2020; Chater et al., 2021), or five (Chater et al., 2021) teams. The proposed match classification scheme is also different from previous suggestions: for instance, Chater et al. (2021) do not distinguish between weakly and strongly stakeless games, which seem to be important categories in scheduling the UEFA Champions League groups according to our findings.

**Match importance**

Measuring the importance of a match can be used not only to compare tournament designs but also for selecting games to broadcast, assigning referees, or explaining attendance (Goossens et al., 2012). The first metric has probably been suggested in Jennett (1984), where the importance of a game is the inverse of the number of remaining games that need to be won in order to obtain the title. Furthermore, the measure equals zero if a team can no longer be the final winner. On the other hand, a match can be deemed important if either of the opponents can still win the league (or relegate) when all other teams will draw in the rest of their games (Audas et al., 2002).

Perhaps the most widely used quantification of match importance has been provided by Schilling (1994): the significance of an upcoming match for a particular club is determined by the difference in the probability of obtaining a prize if the fixture were won rather than lost. In other words, match importance reflects the strength of the relationship between the match result and a given season outcome. This approach is usually applied with a Monte Carlo simulation of the remaining games to estimate the final standing in the ranking (Buraimo et al., 2022; Lahvička, 2015; Scarf and Shi, 2008).

There are further concepts of match importance. Goossens et al. (2012) evaluate several formats for the Belgian football league with respect to the number of unimportant games that do not affect the final outcome (league title, relegation, qualification for European cups) at the moment they are played. Geenens (2014) identifies two drawbacks of this definition as it does not take into account the temporal position of the game in the tournament and the strength of the two playing teams. Therefore, the author suggests an entropy-related measure of decisiveness in terms of the uncertainty about the eventual winner prevailing in the tournament at the time of the game. Inspired by this idea, Corona et al. (2017) analyse how the identification of decisive matches depends on the statistical approach used to estimate the results of the matches.

Faella and Sauro (2021) call a match irrelevant if it does not influence the ultimate ranking of the teams involved and prove that a contest always contains an irrelevant
match if the schedule is predetermined and there are at least five contestants. This notion is somewhat akin to our classes of matches presented in Section 1. Finally, Goller and Heiniger (2022) propose an event importance measure: the difference between the contest reward probability distributions induced by the possible outcomes of a single event. It encompasses more complex or dynamic contest designs and reward structures, which are common in the society. The authors show the association of the quantified importance of a match to in-match behaviour and the performance of the teams in seven major European football leagues.

Our main contribution to this research area resides in thoroughly analysing the role of schedule for match importance. Previous studies have only considered this as a promising direction for future study, which can be illustrated by two quotations: “research could be done to investigate the influence of the schedule on match importance” (Goossens et al., 2012, p. 239), and “The tournament design, in particular the order the games were played, obviously influences the decisiveness of those games, so we have scrupulously followed that schedule in our study” (Geenens, 2014, p. 160). Furthermore, the above metrics of match importance are not able to account for the lack of incentives. For example, the entropy-based decisiveness measure of Geenens (2014) contains the strength of the two playing teams—but Real Madrid fielded a fully rotated squad for the match discussed in Example 1, supposedly due to the lack of incentives to win.

Scheduling

The Operational Research (OR) community devotes increasing attention to the design of sports tournaments (Csató, 2021; Kendall and Lenten, 2017; Lenten and Kendall, 2021; Wright, 2014). One of the main challenges is choosing a schedule that is fair for all contestants both before and after the matches are played (Goossens et al., 2020). The traditional issues of fairness in scheduling are the number of breaks (two consecutive home or away games), the carry-over effect (which is related to the previous game of the opponent), and the number of rest days between consecutive games. They are discussed in several survey articles (Goossens and Spieksma, 2012; Kendall et al., 2010; Rasmussen and Trick, 2008; Ribeiro, 2012; Van Bulck et al., 2020).

The referred studies usually consider the teams as nodes in graphs. However, they are strategic actors and should allocate their limited effort throughout the contest, or even across several contests. Researchers have recently begun to take similar considerations into account. Krummer et al. (2017) investigate round-robin tournaments with a single prize and either three or four symmetric players. In the subgame perfect equilibrium of the contest with three players, the probability of winning is maximised for the player who competes in the first and the last rounds. This result holds independent of whether the asymmetry is weak or strong. However, the probability of winning is the highest for the player who competes in the second and third rounds if there are two prizes (Krummer et al., 2020). In the subgame perfect equilibrium of the contest with four players, the probability of winning is maximised for the player who competes in the first game of both rounds. These theoretical findings are reinforced by an empirical analysis, which includes the FIFA World Cups and the UEFA European Championships, as well as two Olympic wrestling events (Krummer and Lechner, 2017).

Two recent works focus on an issue that is similar to our topic. Yi (2020) develops an implementor-adversary approach to construct a robust schedule that maximizes suspense, the round when the winner of a round-robin tournament is decided. Gieling (2022) aims
to find the schedule having this particular property. While this binary metric of tension is clearly important for most round-robin tournaments, it does not always reflect the stakes appropriately. For instance, teams are mainly interested in obtaining the first two positions in the FIFA World Cup group stage (Chater et al., 2021).

To summarise, we add a novel fairness criterion, the probability of stakeless games, to compare and assess potential schedules for a round-robin tournament. Since the previous section has demonstrated why league organisers, teams, and fans would not like to see stakeless matches in the majority of sports, this measure can be used to evaluate any real-world sports timetable, for instance, the recently proposed and used schedule of FIFA World Cup South American Qualifiers (Durán et al., 2017).

3 Game classification in the UEFA Champions League

This section introduces the principle behind our game classification scheme and adapts the model for the current format of the UEFA Champions League, that is, a home-away round-robin tournament played by four teams.

3.1 The underlying idea

Let us assume that each team strives to achieve its target or improve its ranking in a round-robin tournament. The target can be anything depending on the final ranking such as being the winner, being at least the runner-up, or avoiding relegation. However, it is also reasonable to achieve a higher rank in most leagues since, for instance, a better position results in a higher revenue (Bergantínos and Moreno-Ternero, 2020). Then three categories of matches can be distinguished:

- **Competitive game**: Neither team is indifferent because they have achieved their targets or they can improve their ranking through a better performance on the field with a positive probability.

- **Weakly stakeless game**: One of the teams is completely indifferent as it has achieved its target or it cannot improve its position in the final ranking independently of the outcomes of the matches still to be played. However, it has a positive probability that the other team fails to achieve its target or obtains a higher rank through a better performance on the field.

- **Strongly stakeless game**: Both teams are completely indifferent since their targets or positions in the final ranking are not influenced by the results of the remaining matches.

The above classification differs from the definitions given in the existing literature. For example, Chater et al. (2021) call a match stakeless if at least one team becomes indifferent between winning, drawing, or even losing by 5 goals difference with respect to qualification. But this notion does not consider the incentives of the opponent, which is an important factor for the competitiveness of the game, too.

3.2 Stakeless games are relevant for the Champions League

The UEFA Champions League is probably the most prestigious annual football club competition around the world. Since the 2003/04 season, the tournament contains a single
group stage with 32 teams, divided into eight groups of four. This phase is played in a double round-robin format, that is, each team meets the other three teams in its group once home and once away. The top two clubs from each group progress to the Round of 16, where the group winners are matched with the runners-up subject to some restrictions (Klößner and Becker, 2013). The third-placed clubs go to the UEFA Europa League, the second-tier competition of European club football, while the fourth-placed clubs are eliminated.

Consequently, it can be reasonably assumed that each participating team wants to be ranked higher in the group stage. Nonetheless, a team might be indifferent with respect to the outcome of the match(es) played on the last matchday(s) since its position in the final ranking is already secured as the following illustrations reveal.

Table 2: Ranking in Group B of the 2021/22 UEFA Champions League after Matchday 4

| Pos | Team                       | W | D | L | GF | GA | GD  | Pts |
|-----|----------------------------|---|---|---|----|----|-----|-----|
| 1   | Liverpool FC               | 4 | 0 | 0 | 13 | 5  | +8  | 12  |
| 2   | FC Porto                   | 1 | 2 | 1 | 3  | 6  | −3  | 5   |
| 3   | Club Atlético de Madrid    | 1 | 1 | 2 | 4  | 6  | −2  | 4   |
| 4   | AC Milan                   | 0 | 1 | 4 | 4  | 7  | −3  | 1   |

Pos = Position; W = Won; D = Drawn; L = Lost; GF = Goals for; GA = Goals against; GD = Goal difference; Pts = Points. All teams have played four matches.

Example 2. Table 2 presents the standing of Group B in the 2021/22 Champions League with two rounds still to be played. Liverpool leads by seven points over Porto, therefore, it will certainly win the group. Thus, Liverpool will play two at least weakly stakeless games against Porto (home) and Milan (away) on the last two matchdays.

Table 3: Ranking in Group C of the 2021/22 UEFA Champions League after Matchday 5

| Pos | Team                   | W | D | L | GF | GA | GD  | Pts |
|-----|------------------------|---|---|---|----|----|-----|-----|
| 1   | AFC Ajax                | 5 | 0 | 0 | 16 | 3  | +13 | 15  |
| 2   | Sporting Clube de Portugal | 3 | 2 | 12 | 8  | +4  | 9   |
| 3   | Borussia Dortmund       | 2 | 0 | 3  | 5  | 11  | −6  | 6   |
| 4   | Beşiktaş JK             | 0 | 0 | 5  | 3  | 14  | −11 | 0   |

Pos = Position; W = Won; D = Drawn; L = Lost; GF = Goals for; GA = Goals against; GD = Goal difference; Pts = Points. All teams have played five matches.

Example 3. Table 3 presents the standing of Group C in the 2021/22 Champions League with one round still to be played. If two or more teams are equal on points on completion of the group matches, their ranking is determined by higher number of points obtained in the matches played among the teams in question, followed by superior goal difference from the group matches played among the teams in question (UEFA, 2021b, Article 17.01). Since the result of Borussia Dortmund vs. Sporting CP (Sporting CP vs. Borussia Dortmund) has been 1-0 (3-1), and Sporting CP has an advantage of three points over Borussia Dortmund, Sporting CP is guaranteed to be the runner-up. Furthermore, Ajax is the group winner and Beşiktaş is the fourth-placed team. Consequently, the outcomes of the two strongly stakeless games played in the last round do not influence the group ranking.
3.3 Identification of stakeless games in a home-away round-robin tournament with four teams

In the group stage of the UEFA Champions League, there are six matchdays. The position of a team in the group ranking can be known first after Matchday 4. In particular, a club is guaranteed to win its group after Matchday 4 if it has at least seven points more than the runner-up or if

- it leads by six points over the runner-up; and
- it leads by at least seven points over the third-placed team; and
- it has played two matches against the runner-up.

The second- and third-placed clubs cannot be fixed after Matchday 4.

A club will certainly be fourth in the final ranking after Matchday 4 if it has at least seven points less than the third-placed team or if

- it has six points less than the third-placed team; and
- it has at least seven points less than the runner-up; and
- it has played two matches against the third-placed team.

Note that these definitions are more complicated than the ones appearing in previous studies (Chater et al., 2021; Guyon, 2020) because of two reasons: (a) there are more matches due to organising the groups in a double round-robin format; and (b) tie-breaking is based on head-to-head results instead of goal difference.

It would be difficult to determine all possible cases by similar criteria after Matchday 5. Hence, we consider only the extreme cases as follows. The results of the two games played on Matchday 6 are assumed to be: (a) M-0, M-0; (b) M-0, 0-M; (c) 0-M, M-0; and (d) 0-M, 0-M, where M is a high number. The position of a team is known if it is the same in all scenarios (a) to (d).

4 Methodology

For any sports competition, historical data represent only a single realisation of several random factors. Hence, the analysis of tournament designs usually starts by finding a simulation technique that can generate the required number of reasonable results (Scarf et al., 2009).

To that end, it is necessary to connect the teams playing in the tournament studied to the teams whose performance is already known. It is achieved by rating the teams, namely, by assigning a value to each team to measure its strength (Van Eetvelde and Ley, 2019). This approach allows the identification of the teams by their ratings instead of their names.

UEFA widely uses such a measure, the UEFA club coefficient, to determine seeding in its club competitions. First, we provide some information on this statistic and the draw of the UEFA Champions League group stage in Section 4.1. After that, Section 4.2 evaluates some simulation models. Finally, Section 4.3 outlines some valid schedules of the Champions League groups.
4.1 Seeding in the UEFA Champions League

The UEFA club coefficient depends on the results achieved in the previous five seasons of the UEFA Champions League, the UEFA Europa League, and the UEFA Europa Conference League, including their qualifying (UEFA, 2018). In order to support emerging clubs, the coefficient equals the association coefficient over the same period if it is higher than the sum of all points won in the previous five years.

For the draw of the Champions League group stage, a seeding procedure is followed to ensure homogeneity across groups. The 32 clubs are divided into four pots and one team is assigned from each pot to a group, subject to some restrictions: two teams from the same national association cannot play against each other, certain clashes are prohibited due to political reasons, and some clubs from the same country play on separate days where possible (UEFA, 2021b).

Seeding is based primarily on the UEFA club coefficients prior to the tournament. Before the 2015/16 season, Pot 1 consisted of the eight strongest teams according to the coefficients, Pot 2 contained the next eight, and so on. The only exception was the titleholder, guaranteed to be in Pot 1. In the three seasons between 2015/16 and 2017/18, the reigning champion and the champions of the top seven associations were in Pot 1. The effects of this reform have been extensively analysed in the literature (Corona et al., 2019; Dagaev and Rudyak, 2019). Since the 2018/19 season, Pot 1 contains the titleholders of both the Champions League and the Europa League, together with the champions of the top six associations. The other three pots are composed in accordance with the club coefficient ranking. The seeding rules are discussed in Csató (2020) and Csató (2021, Chapter 2.3). Engist et al. (2021) estimate the effect of seeding on tournament outcomes in European club football. In our simulation models, all teams are classified according to the seeding system used in the given season for the sake of simplicity.
Figure 1 plots the club coefficients for the 32 teams that participated in the Champions League group stage in four different seasons. As it has already been mentioned, Pot 1 does not contain the teams with the highest ratings in the 2015/16 and 2021/22 seasons.

4.2 The simulation of match outcomes

In football, the number of goals scored is usually described by a Poisson distribution (Maher, 1982; Van Eetvelde and Ley, 2019). Dagaev and Rudyak (2019) propose such a model to evaluate the effects of the seeding system reform in the Champions League, introduced in 2015. Consider a single match between two clubs, and denote by $\lambda_{H,A}$ and $\lambda_{A,H}$ the expected number of goals scored by the home team $H$ and the away team $A$, respectively. The probability of team $H$ scoring $n_{H,A}$ goals against team $A$ is given by

$$P_H(n_{H,A}) = \frac{\lambda_{H,A}^{n_{H,A}} \exp(-\lambda_{H,A})}{n_{H,A}!},$$

whereas the probability of team $A$ scoring $n_{A,H}$ goals against team $H$ is

$$P_H(n_{A,H}) = \frac{\lambda_{A,H}^{n_{A,H}} \exp(-\lambda_{A,H})}{n_{A,H}!}.$$  

In order to determine the outcome of the match, parameters $\lambda_{H,A}$ and $\lambda_{A,H}$ need to be estimated. Dagaev and Rudyak (2019) use the following specification:

$$\log(\lambda_{H,A}) = \alpha_H + \beta_H \cdot (R_H - \gamma_H R_A),$$

$$\log(\lambda_{A,H}) = \alpha_A + \beta_A \cdot (R_A - \gamma_A R_H),$$

with $R_H$ and $R_A$ being the UEFA club coefficients of the corresponding teams, and $\alpha_i, \beta_i, \gamma_i$ ($i \in \{A, H\}$) being parameters to be optimised on a historical sample. A simpler version containing four parameters can be derived by setting $\gamma_H = \gamma_A = 1$.

We have studied two options for quantifying the strength of a club: the UEFA club coefficient and the seeding pot from which the team is drawn in the Champions League group stage. The latter can take only four different values. Furthermore, each group is guaranteed to consist of one team from each pot, consequently, the dataset contains the same number of teams for each possible value, as well as the same number of matches for any pair of ratings.

Assuming the scores to be independent may be too restrictive because the two opposing teams compete against each other. Thus, if one team scores, then the other will exert more effort into scoring (Karlis and Ntzoufras, 2003). This correlation between the number of goals scored can be accounted for by bivariate Poisson distribution, which introduces an additional covariance parameter $c$ that reflects the connection between the scores of teams $H$ and $A$ (Van Eetvelde and Ley, 2019).

To sum up, five model variants are considered:

- 6-parameter Poisson model based on UEFA club coefficients (6p coeff);
- 4-parameter Poisson model based on UEFA club coefficients (4p coeff);
- 6-parameter Poisson model based on pot allocation (6p pot);
- 4-parameter Poisson model based on pot allocation (4p pot);
Table 4: Model parameters estimated by the maximum likelihood method on the basis of Champions League seasons between 2003/04 and 2019/20

| Model | \(\alpha_H\) | \(\alpha_A\) | \(\beta_H\) | \(\beta_A\) | \(\gamma_H\) | \(\gamma_A\) | \(c\) |
|-------|--------|--------|--------|--------|--------|--------|------|
| 6p coeff | 0.335  | 0.087  | 0.006  | 0.006  | 0.833  | 0.963  | —    |
| 4p coeff | 0.409  | 0.102  | 0.006  | 0.006  | —      | —      | —    |
| 6p pot | 0.464  | 0.143  | -0.177 | -0.182 | 0.91   | 0.922  | —    |
| 4p pot | 0.424  | 0.108  | -0.169 | -0.175 | —      | —      | —    |
| Bivariate | 0.335  | 0.087  | 0.006  | 0.006  | 0.833  | 0.963  | \(\exp(-12.458)\) |

- 7-parameter bivariate Poisson model based on UEFA club coefficients (Bivariate).

All parameters have been estimated by the maximum likelihood approach on the set of \(8 \times 12 \times 17 = 1632\) matches played in the 17 seasons from 2003/04 to 2019/20. They are presented in Table 4. The optimal value of \(c\), the correlation parameter of the bivariate model is positive but almost zero, hence, the bivariate Poisson model does not improve accuracy. This is in accordance with the finding of Chater et al. (2021) for the group stage of the FIFA World Cup. The reason is that the bivariate Poisson model is not able to grab a negative correlation between its components, however, the goals scored by home and away teams are slightly negatively correlated in our dataset.

The performance of the models has been evaluated on two disjoint test sets, the seasons of 2020/21 and 2021/22. They are treated separately because most games in the 2020/21 edition were played behind closed doors owing to the COVID-19 pandemic, which might significantly affect home advantage (Benz and Lopez, 2021; Bryson et al., 2021; Fischer and Hauccap, 2021).

Two metrics have been calculated to compare the statistical models. Average hit probability measures how accurately a model can determine the exact score of a match: we pick up the probability of the actual outcome, sum up these probabilities across all matches in the investigated dataset, and normalise this value by the number of matches and seasons. For instance, assume that two games have been played in a season such that the predicted probability for their known outcome is 0.1 and 0.06, respectively. The average hit probability will be \((0.1 + 0.06)/2 = 0.08\). A simple baseline model serves as a benchmark, where the chances are determined by relative frequencies in the seasons from 2003/04 to 2018/19.

The results are provided in Table 5. The baseline model shows the worst performance, which is a basic criterion for the validity of the proposed methods. The bivariate Poisson variant does not outperform the 6-parameter Poisson based on UEFA club coefficients. Even though the club coefficient provides a finer measure of strength than the pot allocation, it does not result in a substantial improvement with respect to average hit probability.

The average hit probability does not count whether the prediction fails by a small margin (the forecast is 2-2 and the actual result is 1-1) or it is completely wrong (the forecast is 4-0 and the actual result is 1-3). However, there exists no straightforward “distance” among the possible outcomes. If the differences in the predicted and actual goals scored by the home and away teams are simply added, then the result of 2-2 will be farther from 1-1 than 2-1. But 1-1 and 2-2 are more similar than 1-1 and 2-1 from a sporting perspective since both 1-1 and 2-2 represent a draw. To resolve this issue, we have devised a distance metric for the outcome of the matches generated by the scalar product with a specific matrix, which has been inspired by the concept of Mahalanobis
Table 5: Average hit probability for the statistical models (in %)

| Model      | 2003/04-2019/20 | Season(s)   |
|------------|-----------------|-------------|
|            | 2020/21         | 2021/22     |
| 6p coeff   | 7.016 (1)       | 6.682 (2)   | 6.148 (3)  |
| 6p pot     | 6.869 (4)       | 6.443 (5)   | 6.297 (1)  |
| 4p coeff   | 7.012 (3)       | 6.683 (1)   | 6.146 (5)  |
| 4p pot     | 6.868 (5)       | 6.456 (4)   | 6.297 (1)  |
| Bivariate  | 7.016 (1)       | 6.682 (2)   | 6.148 (3)  |
| Baseline   | 6.123 (6)       | 5.482 (6)   | 5.485 (6)  |

Baseline model: The probability of any match outcome is determined by the relative frequency of this result in the training set (all seasons between 2003/04 and 2019/20).

The ranks of the models are indicated in bracket.

Table 6: Selected values for the suggested distance of match scores

| Result | 0-0 | 0-1 | 1-0 | 1-1 | 2-0 | 2-1 | 2-2 | 3-0 | 3-1 | 3-2 | 3-3 |
|--------|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|
|        | 0.447 | 0.447 | 0.894 | 1.342 | 1.183 | 1.483 | 2    | 2.145 | 1.183 | 1.183 | 1.183 |
|        | 0.447 | 0.447 | 0.894 | 1.342 | 1.183 | 1.483 | 2    | 2.145 | 1.183 | 1.183 | 1.183 |
|        | 0.447 | 0.447 | 0.894 | 1.342 | 1.183 | 1.483 | 2    | 2.145 | 1.183 | 1.183 | 1.183 |
|        | 0.447 | 0.447 | 0.894 | 1.342 | 1.183 | 1.483 | 2    | 2.145 | 1.183 | 1.183 | 1.183 |
|        | 0.447 | 0.447 | 0.894 | 1.342 | 1.183 | 1.483 | 2    | 2.145 | 1.183 | 1.183 | 1.183 |
|        | 0.447 | 0.447 | 0.894 | 1.342 | 1.183 | 1.483 | 2    | 2.145 | 1.183 | 1.183 | 1.183 |
|        | 0.447 | 0.447 | 0.894 | 1.342 | 1.183 | 1.483 | 2    | 2.145 | 1.183 | 1.183 | 1.183 |
|        | 0.447 | 0.447 | 0.894 | 1.342 | 1.183 | 1.483 | 2    | 2.145 | 1.183 | 1.183 | 1.183 |
|        | 0.447 | 0.447 | 0.894 | 1.342 | 1.183 | 1.483 | 2    | 2.145 | 1.183 | 1.183 | 1.183 |

Darker colour indicates a higher value.

distance (De Maesschalck et al., 2000).

Let the final score of the game be \( R_1 = (h_1, a_1) \), where \( h_1 \) is the number of goals for the home team, and \( a_1 \) is the number of goals for the away team. Analogously, denote by \( R_2 = (h_2, a_2) \) the predicted result of this game. The distance between the two outcomes equals

\[
\Delta(R_1, R_2) = \sqrt{\left[ h_1 - h_2 \right]^2 + \left[ a_1 - a_2 \right]^2} \left[ \frac{1}{-9/10} \right] \left[ \frac{-9/10}{1} \right].
\]  

For instance, with the final score of 2-0 and the forecast of 1-2, \( h_1 - h_2 = 1 \) and \( a_1 - a_2 = -2 \), which leads to

\[
\Delta(R_1, R_2) = \sqrt{\left[ 1 - 2 \right]^2 + \left[ 1 + 2 \times 0.9 \right] \left[ -0.9 - 2 \times 1 \right]} = \sqrt{2.8 - 2 \times (-2.9)} = \sqrt{8.6} \approx 2.933. \tag{6}
\]

The distances between the outcomes defined by this metric can be seen in Table 6. For instance, the error derived in equation (6) can be found at the intersections of 2-0 and 1-2 (eighth row, twelfth column; twelfth row, eighth column) since formula (5) is symmetric. The measure is called distance of match scores in the following.

\(^2\) The value of \(-9/10\) controls the relative cost of adding one goal for both teams. If it would be equal
Table 7: Average distance of match scores for the statistical models

| Model    | Season(s)          | 2003/04–2019/20 | 2020/21 | 2021/22 |
|----------|--------------------|-----------------|---------|---------|
| 6p coeff |                    | 2.041 (3)       | 2.199 (3) | 2.163 (3) |
| 6p pot   |                    | 1.958 (2)       | 2.068 (2) | 2.013 (2) |
| 4p coeff |                    | 2.054 (5)       | 2.218 (5) | 2.175 (5) |
| 4p pot   |                    | 1.957 (1)       | 2.066 (1) | 2.012 (1) |
| Bivariate|                    | 2.041 (3)       | 2.199 (3) | 2.163 (3) |
| Baseline |                    | 2.095 (6)       | 2.247 (6) | 2.210 (6) |

*Baseline model:* The probability of any match outcome is determined by the relative frequency of this result in the training set (all seasons between 2003/04 and 2019/20).

The ranks of the models are indicated in bracket.

Figure 2: The distribution of goals for the historical data and the simulation model chosen

Table 7 evaluates the six statistical models (including the baseline) according to the average distances of match scores over three sets of games. In contrast to the average hit probability, now a lower value is preferred. There is only a slight difference between the performance of variants based on UEFA club coefficients and pot allocation—and the latter provides a better estimation in both cases. Using six parameters instead of four does not improve accuracy. Since the schedule of group matches will depend on the pots of the teams and UEFA club coefficients are not able to increase the predictive power, we have decided for the 4-parameter Poisson model based on pot allocation to simulate the group matches played in the Champions League.

Finally, the chosen specification is demonstrated to describe well the unknown score-to −1, then the prediction of 1-1 (2-1) instead of 0-0 (1-0) would not be punished.
Table 8: The number of matches with a given outcome in the UEFA Champions League and the expected number of matches from the selected simulation model (4p pot)

(a) Seasons between 2003/04 and 2019/20

| Final score | 0         | 1         | 2         | 3         | 4         |
|-------------|-----------|-----------|-----------|-----------|-----------|
|             | (103.0 ± 9.8) | (124.6 ± 10.7) | (81.2 ± 8.7) | (38.1 ± 6.0) | (14.0 ± 3.7) |
| 0           | 115       | 109       | 83        | 48        | 20        |
|             | (159.4 ± 12.0) | (175.8 ± 12.5) | (106.4 ± 9.9) | (46.8 ± 6.7) | (16.6 ± 4.0) |
| 1           | 157       | 175       | 96        | 38        | 19        |
|             | (133.8 ± 11.0) | (135.2 ± 11.1) | (74.7 ± 8.4) | (30.3 ± 5.4) | (9.9 ± 3.1) |
| 2           | 138       | 139       | 76        | 25        | 5         |
|             | (80.5 ± 8.6) | (75.6 ± 8.5) | (38.5 ± 6.1) | (14.3 ± 3.8) | (4.1 ± 0.2) |
| 3           | 84        | 72        | 36        | 14        | 2         |
|             | (38.9 ± 6.1) | (34.0 ± 5.7) | (16.0 ± 4.0) | (5.4 ± 2.3) | (1.6 ± 1.3) |
| 4           | 44        | 22        | 18        | 5         | 2         |

(b) Season 2020/21

| Final score | 0         | 1         | 2         | 3         | 4         |
|-------------|-----------|-----------|-----------|-----------|-----------|
|             | (6.1 ± 2.4) | (7.3 ± 2.6) | (4.8 ± 2.1) | (2.2 ± 1.5) | (0.8 ± 0.9) |
| 0           | 5         | 3         | 8         | 4         | 4         |
|             | (9.4 ± 2.9) | (10.3 ± 3.0) | (6.3 ± 2.4) | (2.8 ± 1.6) | (1.0 ± 1.0) |
| 1           | 6         | 9         | 7         | 3         | 1         |
|             | (7.9 ± 2.7) | (8.0 ± 2.7) | (4.4 ± 2.0) | (1.8 ± 1.3) | (0.6 ± 0.8) |
| 2           | 7         | 5         | 6         | 2         | 0         |
|             | (4.7 ± 2.1) | (4.4 ± 2.1) | (2.3 ± 1.5) | (0.8 ± 0.9) | (0.2 ± 0.5) |
| 3           | 7         | 5         | 4         | 0         | 1         |
|             | (2.3 ± 1.5) | (2.0 ± 1.4) | (0.9 ± 1.0) | (0.3 ± 0.6) | (0.1 ± 0.3) |
| 4           | 2         | 1         | 0         | 0         | 0         |

(c) Season 2021/22

| Final score | 0         | 1         | 2         | 3         | 4         |
|-------------|-----------|-----------|-----------|-----------|-----------|
|             | (6.1 ± 2.4) | (7.3 ± 2.6) | (4.8 ± 2.1) | (2.2 ± 1.5) | (0.8 ± 0.9) |
| 0           | 6         | 5         | 1         | 3         | 1         |
|             | (9.4 ± 2.9) | (10.3 ± 3.0) | (6.3 ± 2.4) | (2.8 ± 1.6) | (1.0 ± 1.0) |
| 1           | 9         | 7         | 8         | 4         | 2         |
|             | (7.9 ± 2.7) | (8.0 ± 2.7) | (4.4 ± 2.0) | (1.8 ± 1.3) | (0.6 ± 0.8) |
| 2           | 10        | 7         | 3         | 2         | 0         |
|             | (4.7 ± 2.1) | (4.4 ± 2.1) | (2.3 ± 1.5) | (0.8 ± 0.9) | (0.2 ± 0.5) |
| 3           | 2         | 3         | 3         | 2         | 0         |
|             | (2.3 ± 1.5) | (2.0 ± 1.4) | (0.9 ± 1.0) | (0.3 ± 0.6) | (0.1 ± 0.3) |
| 4           | 5         | 2         | 2         | 0         | 0         |

Goals scored by the home team are in the rows, goals scored by the away team are in the columns. Games where one team scored at least five goals are not presented. The numbers in parenthesis indicate the average number of occurrences based on simulations ± standard deviations.
generating process. First, Figure 2 shows the real goal distributions and the one implied by our Poisson model that gives the same forecast for each season as the teams are identified by the pot from which they are drawn. Second, the final scores of the games are analysed: Table 8 presents the number of matches with the given outcome in the corresponding season(s) and the number of occurrences for these events according to the chosen simulation model. Again, the forecast is the same for any season since the groups cannot be distinguished by the strengths of the clubs.

To summarise, the 4-parameter Poisson model based on pot allocation provides a good approximation to the empirical data. This is essential for further analysis: since each group contains one team from each pot, the groups are identical with respect to our simulation model. Consequently, the performance of any schedule is the same for the game classification scheme. Otherwise, the predicted probability of a (weakly/strongly) stakeless match might depend on other characteristics of the clubs, for instance, their UEFA club coefficients, which are not known before the group draw.

4.3 Scheduling options

The regulation of the UEFA Champions League provides surprisingly little information on how the group matches are scheduled (UEFA, 2021b, Article 16.02): “A club does not play more than two home or two away matches in a row and each club plays one home match and one away match on the first and last two matchdays.” Therefore, the eight schedules of group matches used in the 2021/22 Champions League are regarded as valid solutions and options available for the tournament organiser. Since each group consists of one team from each of the four pots, the clubs are identified by their pot in the following, that is, team $i$ represents the team drawn from Pot $i$.

Table 9 outlines these alternatives. From our perspective, five different patterns exist as the prediction of match outcome does not depend on the schedule, and game classification starts after Matchday 4:

- The schedules of Groups A and C differ only in one game played on Matchdays 3 and (consequently) 4;
- The schedules of Groups A and F coincide;

3 At first sight, one might conclude that UEFA has fixed the schedule of group matches until the 2020/21 season. For instance, the regulation of the competition for 2020/21 (UEFA, 2020, Article 16.02) says that:

“*The following match sequence applies:*

*Matchday 1: 2 v 3, 4 v 1;*
*Matchday 2: 1 v 2, 3 v 4;*
*Matchday 3: 3 v 1, 2 v 4;*
*Matchday 4: 1 v 3, 4 v 2;*
*Matchday 5: 3 v 2, 1 v 4;*
*Matchday 6: 2 v 1, 4 v 3."

However, the meaning of the numbers remains unknown. They certainly do not correspond to the pots from which the teams have been drawn since, on Matchday 1 in the 2020/21 season, FC Bayern München (Pot 1) hosted Club Atlético Madrid (Pot 2) in Group A and FC Dynamo Kyiv (Pot 3) hosted Juventus (Pot 1) in Group G. Furthermore, the match sequence cannot be determined before the identity of the clubs are known due to the other—obvious but unannounced—constraints.

4 In the Champions League seasons from 2003/04 to 2020/21, Matchday 4/5/6 was the mirror image of Matchday 3/1/2, respectively. Consequently, the same two teams played at home on the first and last matchday in the previous seasons. This arrangement has been changed in the 2021/22 season such that Matchday 4/5/6 is the mirror image of Matchday 3/2/1, see Table 10.
Table 9: Group schedules in the 2021/22 UEFA Champions League

| Gr. A | Gr. B | Gr. C | Gr. D | Gr. E | Gr. F | Gr. G | Gr. H |
|-------|-------|-------|-------|-------|-------|-------|-------|
| H     | A     | H     | A     | H     | A     | H     | A     |
| 1     | 3     | 1     | 3     | 1     | 2     | 2     | 1     |
| 4     | 2     | 4     | 2     | 4     | 3     | 4     | 2     |
| 3     | 1     | 4     | 2     | 1     | 3     | 1     | 2     |
| 2     | 1     | 3     | 1     | 2     | 3     | 2     | 1     | 1     |
| 3     | 4     | 2     | 3     | 4     | 2     | 4     | 3     |
| 1     | 4     | 1     | 2     | 4     | 1     | 1     | 4     | 1     | 1     |
| 2     | 3     | 3     | 4     | 3     | 2     | 2     | 3     | 3     | 3     | 2     |

The numbers indicate the pots from which the teams are drawn.

Table 10: Scheduling alternatives from the 2021/22 UEFA Champions League

| Matchday 5 | Matchday 6 |
|------------|------------|
| Home       | Away       | Home       | Away       |
| Schedule A | Pot 1      | Pot 2      | Pot 3      | Pot 1      |
|            | Pot 4      | Pot 3      | Pot 2      | Pot 4      |
| Schedule B | Pot 1      | Pot 4      | Pot 3      | Pot 1      |
|            | Pot 2      | Pot 3      | Pot 4      | Pot 2      |
| Schedule D | Pot 1      | Pot 3      | Pot 2      | Pot 1      |
|            | Pot 4      | Pot 2      | Pot 3      | Pot 4      |
| Schedule E | Pot 4      | Pot 1      | Pot 3      | Pot 4      |
|            | Pot 2      | Pot 3      | Pot 3      | Pot 4      |
| Schedule G | Pot 1      | Pot 3      | Pot 4      | Pot 1      |
|            | Pot 2      | Pot 4      | Pot 3      | Pot 2      |

- The schedules of Groups A and H differ in the two games played on Matchdays 3 and (consequently) 4.

On the other hand, the schedules of Groups A, B, D, E, and G are worth assessing for the frequency of weakly and strongly stakeless games.

The five scheduling options are summarised in Table 10. Note that only the last two matchdays count from our perspective.

5 Results

According to Section 4.2, the best simulation model is the 4-parameter Poisson based on pot allocation (4p pot), which will be used throughout this section. We focus on the probability of a stakeless game as a function of the group schedule. For sample size $N$, the error of a simulated probability $P$ is $\sqrt{P(1-P)/N}$. Since even the smallest $P$ exceeds
Table 11: The average distances of match scores for the statistical models

| Schedule          | A     | B     | D     | E     | G     |
|-------------------|-------|-------|-------|-------|-------|
| Weakly played on Matchday 5 | 2.61 (2) | 3.95 (5) | 2.60 (1) | 2.85 (3) | 3.92 (4) |
| Weakly played on Matchday 6  | 35.38 (3) | 37.27 (5) | 28.35 (2) | 26.87 (1) | 36.69 (4) |
| Strongly played on Matchday 6 | 8.01 (1) | 8.83 (2) | 10.23 (5) | 9.71 (4) | 8.85 (3) |

The ranks of the schedules are indicated in bracket.

2.5% and 1 million simulation runs are implemented, the error always remains below 0.016%. Therefore, confidence intervals will not be provided because the averages differ reliably between the candidate schedules.

Table 11 shows the likelihood of a stakeless game in a given round. The probability of a weakly stakeless game at the first point where it might occur, on Matchday 5, varies approximately between 2.5% and 4%. It is the lowest for schedules A and D, while schedules B and G are poor choices to avoid these matches.

With respect to the probability of a weakly stakeless game played in the last round, the potential timetables differ to a higher degree. The worst schedules (B and G) increase the danger of a weakly stakeless match by 35% (more than 10 percentage points) compared to the best schedule E. The most widely used option, schedule A—followed in four groups of the 2021/22 Champions League—becomes unfavourable from this point of view.

However, schedule A is the best alternative to minimise the chance of strongly stakeless games that are totally unimportant with respect to the group ranking. Now the timetabling alternatives vary less in absolute terms, the probability of such a situation remains between 8% and 10.5%.

As presented in Table 11, there are three objectives to be optimised. While schedule A dominates both schedules B and G, the remaining three alternatives can be optimal depending on the preferences of the decision-maker. In order to evaluate them, it is worth considering a weighting scheme. The cost of a weakly stakeless game played on Matchday 6 can be fixed at 1 without losing generality. It is reasonable to assume that the cost of a weakly stakeless game played on Matchday 5 is not lower than 1. Regarding strongly stakeless games, there are two contradictory arguments:

- If a team has something to play for but its opponent has no such incentives (weakly stakeless game), then the match is exposed to the risk of manipulation or—depending on the outcome—to the impression that the match has been sold. This problem does not emerge if no team can improve its ranking (strongly stakeless game).

- Weakly stakeless games can generate attendance because at least one team should exert effort. On the other hand, strongly stakeless games might be completely boring.

Hence, the relative cost of weakly and strongly stakeless games greatly rests on the preferences of the decision-maker, and can differ even by an order of magnitude.

Figure 3 calculates the price of timetabling alternatives as a function of the cost ratio between a strongly and weakly stakeless game played in the last round (be aware of the

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5 We thank an anonymous referee for this remark.
(a) A weakly stakeless game has the double cost on Matchday 5 compared to Matchday 6.

Figure 3: The prices of the Champions League group stage schedules

(b) A weakly stakeless game has the double cost on Matchday 5 compared to Matchday 6.
the relative cost of a strongly stakeless game is at most 5, however, schedule A should be chosen if this ratio is higher. If the goal is to avoid strongly stakeless games, schedules D and E are unfavourable options.

The key findings can be summed up as follows:

- Schedule A dominates schedules B and G with respect to stakeless games;
- Schedules B and G have approximately the same cost, which can be explained by their similarity on the last two matchdays, where only the positions of the teams drawn from Pots 3 and 4 are interchanged (see Table 10);
- Schedule E is preferred to schedule D except for the case when an unlikely high weight is assigned to weakly stakeless games played on Matchday 5;
- Schedule E (and, to some extent, D) should be followed if strongly stakeless games are not judged to be substantially more harmful than weakly stakeless games;
- Schedule A can decrease the probability of a strongly stakeless game by at least 10%, hence, it needs to be implemented to reduce the number of matches that do not affect the final group ranking.

In the 2021/22 Champions League, UEFA has essentially used schedule A in four groups and schedules B, D, E, and G in one group, respectively. Therefore, it is likely that minimising the number of strongly stakeless games has been a crucial goal in the computer draw of the fixture, which remains unknown to the public.

6 Discussion

The paper has proposed a novel classification method for games played in a round-robin tournament. The selection criterion is connected to the incentives of the teams, it depends on whether the position of a team in the final ranking is already known, independently of the outcomes of matches still to be played. In particular, a game is called (1) competitive if neither opposing team is indifferent; (2) weakly stakeless if exactly one of the opposing teams is indifferent; or (3) strongly stakeless if both teams are indifferent. Avoiding stakeless matches should be an imperative aim of the organiser because a team might play with little enthusiasm if the outcome of the match cannot affect its final position, which probably reduces attendance and is unfair to the teams that have already played against this particular team (see Example 1).

The group stage of the most prestigious European club football competition is currently organised as a home-away round-robin contest with four teams. We have built a simulation model to simulate the matches of the UEFA Champions League and to compare several feasible sequences for the group matches with respect to the probability of games where one or both clubs cannot achieve a higher rank. Five candidate timetables have been identified based on the 2021/22 season. The prevailing schedule—applied in half of the eight groups—is found to be optimal under a wide set of costs assigned to different stakeless matches. Reducing the number of strongly stakeless games has probably been a crucial goal in the computer draw of the fixture.

We have argued that a tournament schedule is fairer if the probability of games where at least one team has few incentives to win is lower. These situations threaten with one team or both teams playing intentionally below their full potential, which might lead to the
opponent scoring/not conceding a goal. Since the number of matches is fixed, reducing the likelihood of stakeless games maximises the number of competitive games that are more exciting to watch. Therefore, the assessment and simulation method suggested here allows for the organiser to choose an optimal timetable without altering other characteristics of the tournament (number of teams, number of matches, qualification rules, points system, etc.). Then the final ranking will better reflect the true strengths of the teams as remaining less affected by the unwanted incentives attributable to the tournament schedule. Having fewer stakeless games is also beneficial for the teams that are less likely to suffer from unfair results of matches played by other teams and for fans who can see more matches where both teams give their best. To conclude, picking up an optimal sequence of games with respect to the proposed metric increases the utility of all stakeholders at almost no price if the scheduling constraints are appropriately defined.

Our study has some limitations. The simulation model may be refined and sensitivity analysis can be carried out with various assumptions on the outcomes of the games. There are other aspects of scheduling fairness, for example, balancing the kick-off times of the matches (Krumer, 2020) or the home games played on non-frequent days between the teams (Goller and Krumer, 2020). The suggested game classification scheme does not deal with the sequence of matches played in the first four rounds. Furthermore, stakeless games are identified in a deterministic framework but a team may exert lower effort still if its position is known with a high probability. Since the difference between the value of the first two places is probably smaller in the UEFA Champions League groups than the difference between the value of the second and the third positions, and winning a match is awarded by the revenue distribution system (Csató, 2022b; UEFA, 2021a), the true incentive scheme is not binary as in our model. Finally, the number of stakeless games can also be reduced through more radical changes in the tournament design. For example, teams that have performed best during the preliminary group stage can choose their opponents during the subsequent knockout stage in order to provide a strong incentive for exerting full effort even if the position of the team in the final ranking is already known (Guyon, 2022). Analogously, additional draw constraints can also contribute to avoiding unfair situations (Csató, 2022a) that might include stakeless games.

Last but not least, it has not been taken into account that the group fixtures are not necessarily independent of each other. For instance, FC Internazionale Milano (drawn from Pot 1 to Group D) and AC Milan (drawn from Pot 4 to Group B) share the same stadium, but both of them should have played at home on Matchday 5 if their groups would have been organised according to schedule A, which is clearly impossible. Similar constraints might prevent choosing the optimal schedule in all groups, and can (partially) explain the variance of schedules used in the 2021/22 season of the UEFA Champions League. Therefore, UEFA is strongly encouraged to increase the transparency of how its competitions are scheduled by announcing these restrictions.

Despite the caveats mentioned above, our study has hopefully managed to uncover an important aspect of tournament design and can inspire further research by scheduling experts to optimise various measures of competitiveness beyond the classical criteria of fairness.

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