Electric Dipole Moments as A Test of Supersymmetric Unification

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Abstract

In a class of supersymmetric grand unified theories, including those based on the gauge group $SO(10)$, there are new contributions to the electric dipole moments of the neutron and electron, which arise as a heavy top quark effect. These contributions arise from CKM-like phases, not from phases of the supersymmetry breaking operators, and can be reliably computed in terms of the parameters of the weak scale supersymmetric theory. For the expected ranges of these parameters, the electric dipole moments of the neutron and the electron are predicted to be close to present experimental limits.

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1.

Grand unification is based on the idea that, at a fundamental level, quarks and leptons are identical and cannot be distinguished [1]. This simple idea of a symmetry which relates quarks to leptons has several attractive features. For example, in $SO(10)$ theories [2], the quarks and leptons of a single generation are the components of a single spinor representation. The particle physics version of the periodic table, the $SU(3) \times SU(2) \times U(1)$ gauge quantum numbers of a generation, are immediately understood in terms of the group properties of this spinor representation. An indication that grand unification may occur at very high energies is provided by the unification of the gauge coupling constants [3], which gives agreement with the experimental value of the weak mixing angle at the present level of accuracy, providing supersymmetry is present at the weak scale [4]. Low energy supersymmetry is itself an aid in attacking the hierarchy problem, and can lead to a dynamical breaking of $SU(2) \times U(1)$ [5], thus linking the $Z$ boson mass to the scale of supersymmetry breaking.

While this picture of supersymmetric grand unification appears plausible, the mass scale of the new unified interactions, $M_G$, is enormous: $M_G \approx 10^{16}$ GeV. How can such a theory be tested?

If the weak scale theory is the standard model, then effects from the grand unified theory can appear in only two ways. Once the particle content of the standard model is fixed, the renormalizable interactions of the standard model are the most general which are consistent with the $SU(3) \times SU(2) \times U(1)$ gauge symmetry. Hence the grand unified theory can either provide relationships between the free couplings of the standard model, or it can induce additional non-renormalizable interactions. There are no other possibilities. The weak mixing angle prediction provides a well known example of the former effect. The prediction for $m_b/m_\tau$ [6] provides a second example, in fact up to seven predictions are possible in the flavor sector of $SO(10)$ theories [7]. However, the weak mixing angle prediction is unique in its conceptual simplicity, it requires only that the unified group contains $SU(5)$, whereas the flavor predictions require further assumptions.

Since the non-renormalizable interactions give effects suppressed by powers of $1/M_G$, the signals must be ones which are absent or highly suppressed in the
standard model. There are only two such signals: those which violate baryon numbers ($B$), such as proton decay, and those which violate lepton member ($L$), such as neutrino masses.

In supersymmetric grand unified theories it is not easy to reliably calculate the $B$ and $L$ violating effects from the non-renormalizable interactions. The simplest estimates give neutrino masses too small to observe in terrestrial experiments, and a proton decay rate which is already excluded. More refined calculations can lead to interesting rates, but are subject to other difficulties. For example, the rates depend on superheavy particle masses and interactions which are frequently unknown, and are certainly model dependent. Furthermore, there is no compelling minimal supersymmetric unified model.

However, in supersymmetric theories there is a third way in which the high energy interactions can be manifest at low energies [8]. As well as relating renormalizable parameters of the low energy supersymmetric theory and inducing non-renormalizable operators, the unified interactions can change the form of the soft supersymmetry breaking interactions. This window to the high energy domain is only open if the original form of the soft supersymmetry breaking interactions is not the most general allowed by the low energy symmetries, and if the supersymmetry breaking effects are present at or above the scale $M_G$. We now discuss these two points.

Although supersymmetry can break in a great many ways, for our proposes the only crucial question is whether the superpartners first feel the effects of supersymmetry breaking above or below the scale $M_G$. For example, if supersymmetry is broken dynamically by some new force at scale $\Lambda \ll M_G$, and if this breaking is communicated to the superpartners by gauge interactions at this scale, then the grand unified interactions can only modify the form of the supersymmetry breaking by effects which are suppressed by powers of $\Lambda/M_G$. The third window to the unified interactions is closed. On the other hand, if the soft supersymmetry breaking interactions are present at the scale $M_G$, they can be directly modified by the unified interactions and the window is open. An example of the latter case is when the supersymmetry breaking occurs at the intermediate scale, and is communicated to the superpartners by supergravitational interactions [9]. In this case the soft operators for the superpartners are present up to the Planck scale, the situation assumed in this letter.
What form do the soft operators have at the Planck scale? If the form is the most general allowed by the gauge symmetry, then flavor changing processes [10] and the neutron electric dipole moment [11] force the scale of supersymmetry breaking to be unnaturally far above the weak scale. Hence in this letter we follow the common practice of assuming a boundary condition for the soft operators which does not distinguish generations and which conserves CP. The generation and CP dependence of the soft operators at low energies then reflects the interactions of the grand unified theory.

The crucial imprint which the high energy interactions leave on the soft operators [8] can be seen from the following simple example. Consider an $SO(10)$ unified model in which the 16-plet of the ith generation, $16_i$, has a coupling $\lambda_i 16_i AB$ where $A$ and $B$ are any fields, subject only to the condition that at least one of them is superheavy with a mass $M_G$. This interactions provides a 1-loop radiative correction to the soft mass-squared matrix for the superpartners in the $16_i$

$$\frac{\Delta m^2_{ij}}{m^2_0} \approx \frac{\lambda_i \lambda^*_j}{\pi^2} \ln \frac{M_P}{M_G}$$

where $m_0$ is the common scalar mass at the Planck mass $M_P$. The choice of normalization, $1/\pi^2$, is justified by the precise equations which follow. Thus we see that $\lambda_i = 1$ leads to $\approx 50\%$ corrections in the eigenvalues of the soft masses. Furthermore, $\lambda_i \neq \lambda_j$ leads to flavor and CP violation. Thus, not only do the effects of the interactions of superheavy particles not decouple with powers of $1/M_G$, but they lead to enormous effects at the weak scale if the coupling constants have strength unity. As a further advantage, there is only a logarithmic sensitivity to the mass of the superheavy particle, which will not be precisely known.

A recent analysis shows that the top quark Yukawa coupling of any grand unified theory leaves an imprint in the soft operators which leads to a violation of individual lepton number violation, $L_i$ [12]. The argument can be summarized as follows. All grand unified models must have a large superpotential interaction

\textsuperscript{*} It may be objected that if we see such flavor changing and CP violating effects we cannot be sure that they originate from the grand unified interactions rather than from small effects at the Planck scale in the boundary conditions. While this is clearly true, the crucial difference is that we know how to reliably compute the grand unified effects, whereas it is not known how to compute small generation dependent or CP violating terms at the Planck scale.
which generates the top quark mass: $\lambda_t$. Quark-lepton unification implies that this interaction breaks $\tau$ number. Flavor mixing of the quarks implies flavor mixing of the leptons, so this interaction also breaks $\mu$ and $e$ number. Hence the large top Yukawa coupling imprints the soft scalar operators with $e, \mu$ and $\tau$ violation, leading to 1-loop weak scale contributions to the processes $\mu \to e\gamma$, $\tau \to \mu\gamma$, $\mu \to 3e$ and $\mu \to e$ conversion. Over much of the space of the soft parameters, the rates are within range of future experiments which aim to push two orders of magnitude beneath present experimental bounds [13].

In this letter we discuss how CP violating phenomena can probe supersymmetric unification via the soft operators, and compute the size of the effects induced by the top quark Yukawa coupling. The most sensitive probes of CP violation beyond the standard model are the electric dipole moments of the neutron ($d_n$) and electron ($d_e$). This is because in the standard model the other CP observables, such as $e, e'$, sin $2\alpha$ and sin $2\beta$ are not particularly suppressed, other than by small intergenerational mixing angles. On the other hand, a naive guess of $d_n \approx 10^{-25}$ e cm from the weak scale is incorrect, because the chiral nature of the weak interaction leads to vanishing electric dipole moments at both one and two loop order. The actual prediction from the standard electroweak interaction is much smaller, of order $10^{-30}$ e cm [14].

2.

In models with weak scale supersymmetry, $d_n$ is generated by the 1-loop diagrams of Figure 1. We take the particle content of the low energy theory to be that of the minimal supersymmetric standard model (MSSM), but allow for general soft supersymmetry breaking scalar interactions:

$$V_{soft} = Q_i^U U^c H_2 + Q_i^D D^c H_1 + L_i^E E^c H_1 + h.c.$$  
$$+ Q_i^\dagger m^2_Q Q + U_i^c m^2_U U^c + D_i^c m^2_D D^c + L_i^L m^2_L L + E_i^c m^2_E E^c$$  

(2)

where $Q$ and $L$ are squark and slepton doublets, and $U^c$, $D^c$ and $E^c$ are squark and slepton singlets. The parameters $i_U$, etc, are $3 \times 3$ matrices. Working in a basis in which the squark mass matrices $m^2_Q, m^2_U$ and $m^2_D$ are diagonal, the diagram of Figure 1 is proportional to the quantity $X$, where

$$X = Im \left\{ V_{L_{1i}}^T (\zeta_D + \lambda_D \mu \tan \beta)_{ij} V_{R_{1j}} \right\} I_{ij} (m^2_Q, m^2_D)$$  

(3)
where $\mathcal{U}, D, E$ are the quark and lepton Yukawa coupling matrices. In general there is an additional similar contribution involving internal up squarks. The parameter $\mu$ is the coefficient of the Higgs superpotential interaction, $\mu H_1 H_2$, while $\tan \beta = v_2/v_1$ is the ratio of vacuum expectation values. The matrix $\mathbf{V}_L (\mathbf{V}_R)$ is the relative rotation between left-handed (right-handed) quarks and squarks to reach the mass eigenstate basis. The function $I_{ij}$ results from the loop integral and depends on the squark mass eigenvalues. Here and later we ignore the electroweak $D^2$ and supersymmetric contributions to the squark and slepton masses.

Before studying unified theories we consider the MSSM. The universal boundary condition allows us to choose $\mathbf{V}_R$ to be the unit matrix at $M_P$. Furthermore, a non-trivial $\mathbf{V}_R$ does not get generated by renormalization group (RG) scaling. Hence the quantity $X$, of equation (3), becomes

$$X_{MSSM} = \text{Im} \left( \mathbf{V}_L^T (\zeta_D + \lambda_D \mu \tan \beta)_{i1} \right) I_{i1}. \quad (4)$$

Keeping only the RG scalings induced by the top Yukawa coupling, it is possible to choose a basis in which $\mathcal{U}, U$ and the squark masses are all real and diagonal. In this basis it is clear that there is no contribution to the electric dipole moment from diagrams with internal up squarks. The Yukawa couplings have the form

$$W_{MSSM} = Q^\mathcal{U} U^c H_2 + Q^\mathcal{D} D^c H_1 \quad (5)$$

where

$$\mathcal{D} = \mathbf{V}^* \mathcal{D}, \quad (6)$$

$\mathbf{V}$ is the Kobayashi-Maskawa (KM) matrix, and $\mathcal{U}$ and $\mathcal{D}$ are real and diagonal. For the MSSM, $\mathbf{V}_L = \mathbf{V}$. It may appear from (4) that a non-zero contribution to $X$ results when $i = 3$, with a size of order $\text{Im}(V_{td}^2)m \lambda_d I_{31}$, where $m$ is the scale of supersymmetry breaking. However, this is incorrect. A study of the RG equations show that at low energies $\zeta_{D31} \propto \lambda_{D31} \propto V_{31}^* \lambda_{D11}$. Hence the quantity in (4) is proportion to $\text{Im}(V_{td}V_{td}^*) = 0$.

There is a deeper reason for this null result, which applies even in the case of large $\tan \beta$, when RG scalings induced by $\lambda_b$ must also be kept. The structure of CP violation in the MSSM with universal boundary conditions is the same as for the standard model. At any scale there is a single CP violating phase and
it appears in $V$. Furthermore if any two eigenvalues of $\hat{\gamma}_U$ or of $\hat{\gamma}_D$ are equal then this phase can be removed. The one loop diagram we have considered has a contribution which is proportional to $m_d$. Since it vanishes when $m_s = m_d$ and it is independent of $m_s$, it is forced to vanish.

In the minimal supersymmetric $SU(5)$ grand unified model, the Yukawa interactions are

$$W_{SU(5)} = T\hat{\gamma}_U T H + TP\hat{\gamma}_D P'$$

(7)

where $T$ and $F$ are 10 and 5 representations of matter, $H$ and $\overline{H}$ are 5 and $\overline{5}$ Higgs supermultiplets, $\hat{\gamma}_U$ and $\hat{\gamma}_D$ are diagonal $3 \times 3$ Yukawa coupling matrices, $V$ is the KM matrix and $P$ is a diagonal phase matrix with two physical phases.

The reason for these two additional phases compared with the MSSM structure of equations (5) and (6) is that if $P$ is removed from the down couplings by rephasing $T$, it appears in the up couplings. In the MSSM it can be removed from the up couplings by relative notations of $Q$ and $U^c$, which is not possible in $SU(5)$ as these fields are unified into $T$.

Taking universal boundary conditions, $\hat{1}_D = A\hat{\gamma}_D, \hat{1}_U = A\hat{\gamma}_U$ and the squarks are degenerate at $M_P$. Including $RG$ scaling effects induced by the top Yukawa coupling maintains the diagonality of $\hat{\gamma}_U, \hat{1}_U$ and the squark masses. The structure of (7) is also preserved, with the parameters becoming scale dependent.

In particular, no right-handed angles are generated: $V_R$ of equation (3) is the unit matrix, so that like the MSSM the quantity which determines the 1 loop contribution to $d_n$ is that of equation (4). Although the top quark Yukawa coupling causes some elements of $1_D$ to scale in the $SU(5)$ theory, it does not change their phase. This means that once $M_G$ is reached and the heavy states decouple, the phase matrix $P$ can be removed, by rotating $Q$ and $U^c$ fields, in both the renormalizable and soft terms. Thus the phases of $P$ in the $SU(5)$ model do not lead to large contributions to $d_n$ generated by the top Yukawa coupling. The only physical phase of the low energy theory is in the KM matrix, and the same argument as for the MSSM shows that there is no contribution to the quantity $X$ at order $Im(V_{td}^2)m_\lambda d_{I31}$.

3.

In the MSSM and $SU(5)$ models discussed above, we argued that the right handed mixing angles, those appearing in $V_R$ of equation 3, are zero. This is
because the universal boundary condition on the soft scalar interactions allows these angles to be defined away at $M_P$, and they are not generated by $RG$ scaling. Another crucial feature that allows these angles to be defined away at $M_P$, is that those theories both allow independent rotations on the $Q_i$ fields and the $D_c^i$ fields. In $SO(10)$ theories, however, $Q_i$ and $D_c^i$ are unified in the spinor representation, 16, so it is not possible to perform such relative rotations.

In $SO(10)$ theories, the single Yukawa interaction $16 \cdot 16 \phi$, where $\phi$ is a 10 dimensional Higgs multiplet, does not allow for any intergeneration mixing, since the basis for the 16, can be chosen to make $\gamma$ diagonal. We introduce a minimal $SO(10)$ model by the Yukawa interactions

$$W_{SO(10)} = 16 \gamma_U 16 \phi_U + 16 \gamma_D 16 \phi_D.$$  \hspace{1cm} (8)

As always, we take the representation structure beneath $M_G$ to be that of the MSSM, and we assume that the doublet $H_2$ lies solely in $\phi_U$ and the doublet $H_1$ lies solely in $\phi_D$. Thus $\gamma_U (\gamma_D)$ is responsible for the up (down) quark masses.

At $M_P$ we can choose a basis for the 16, in which $\gamma_U$ is real and diagonal. However, no further basis redefinitions are then possible. Since $\gamma_D$ is symmetric it can be diagonalized by a single unitary matrix $U$: $\gamma_D = U^\dagger \gamma_D U$. Hence we can write

$$W_{SO(10)} = 16 \gamma_U 16 \phi_U + 16 U^\dagger \gamma_D U 16 \phi_D$$  \hspace{1cm} (9)

where $\gamma_U$ and $\gamma_D$ are diagonal. The matrix $U$ is a general $3 \times 3$ unitary matrix with 3 angles and 6 phases. It can be written as

$$U = P^* VP,$$  \hspace{1cm} (10)

where $V$ is the scale-dependent $KM$ matrix, and $P$ and $P'$ are diagonal phase matrices.

The $SO(10)$ RG equations can now be solved, keeping only the scalings induced by the large top Yukawa coupling. The matrices $\gamma_U, \gamma_U$ and the scalar mass matrix $m^2$ remain diagonal. It might be thought that at $M_G$, when the superheavy states are decoupled, it will be possible to rotate the $Q_i$ and $D_c^i$ states to remove the right-handed angles and extra phases, as was possible in $SU(5)$. In $SO(10)$ models this is not possible. At $M_G$ the scalar mass squared
matrix takes the diagonal form

\[
m^2 = \begin{pmatrix}
m_0^2 & 0 & 0 \\
0 & m_0^2 & 0 \\
0 & 0 & m_0^2 - I
\end{pmatrix}
\]  

(11)

where \( I \) is obtained from the RG equation for \( m_3^2 \) the third eigenvalue of \( \mathbf{m}^2 \)

\[
I = \frac{5}{8\pi^2} \int_{M_G}^{M_P} (2m_3^2 + m_\phi^2 + A_3^2)\lambda^2 dt
\]  

(12)

where \( \lambda \) is the running top Yukawa coupling. It is not possible to do a superfield rotation on \( D^c \) to remove the right-handed angles, because they reappear as a non-diagonal \( m_D^2 \). Theories which unify all quarks of a single generation will have right-handed mixing angles which are physical by virtue of large top Yukawa coupling effects. At \( M_G \), decoupling the superheavy states, we can write the Yukawa couplings in the form

\[
W'_{SO(10)} = \bar{Q}u U^c H_2 + Q(\bar{V}^* D^c P^{*2} V^d)D^c H_1
\]  

(13)

where \( \mathbf{V} \) is the running \( KM \) matrix. We could now redefine the \( D^c_{\xi} \) super fields to put the matrix \( P^{*2} V^d \) in the soft operators. This is logically preferable, since it is the presence of the soft operators which allows the non-decoupling of these extra phases and mixing angles. However, for the calculation of \( d_n \) it is preferable to remain in the present basis, which diagonalizes the squark mass matrix. The presence of right-handed mixing angles means that there is no chiral symmetry argument requiring \( d_n \propto \lambda_d \), the small down quark Yukawa coupling. Similarly there is no argument that CP violation must vanish when two quark mass eigenvalues are degenerate. As we will see below, this allows very large contributions to \( X: X \approx Im(V_{\alpha 3}^2)\lambda_b I_{33} \).

Our claim of such large effects is based on \( I \) being a substantial modification to \( m_0^2 \) for the third generation scalars. Taking \( A_3 = 0 \), and computing \( I \) of equation (12) in the one loop insertion approximation gives \( I/m_0^2 \simeq 0.2\lambda^2 \ln \frac{M_P}{M_G} \) where \( \lambda_G = \lambda_t(M_G) \). Present top quark mass data require \( \lambda_G > 0.5 \), so \( I/m_0^2 > 0.25 \). This should be considered a lower bound: in the integral of equation (12) the effects of evolving \( \lambda \) and of allowing \( A_3 \neq 0 \) both lead to larger values for \( I/m_0^2 \). Furthermore, the prediction for \( m_b/m_c \) requires \( \lambda_G \) larger than unity.
While this is relaxed if $\lambda_b$ is itself large, this requires $\tan \beta$ large (of order 50) which will enhance the dipole moment, as can be seen from equation (3).

4.

We now compute the contribution of the diagrams of Figure 1 to $d_n$, and an analogous contribution to $d_e$, in the minimal $SO(10)$ theory. In the superfield basis which diagonalizes the squark mass matrices, the Yukawa interactions are given by (13) at $M_G$. Modifications to this form will be induced by $\lambda_t$ scalings from $M_G$ to $M_W$. These effects are calculable, but they are not large and we ignore them for simplicity. Hence we take (12) to apply also at the weak scale.

We have argued that at $M_G$ the third generation scalars are much lighter than those of the first two generations. This will also be true at the weak scale, although gluino contributions to the squark masses could reduce the fractional difference. In this letter will therefore only calculate the contributions to $d_n$ arising from internal $b$ squarks. The $\tilde{b}_L$ and $\tilde{b}_R$ squarks are degenerate at $M_G$. Scalings induced by $\lambda_t$ will mean that at the weak scale $\tilde{b}_L$ is lighter than $\tilde{b}_R$. We ignore this effect for simplicity and take $\tilde{b}_L$ and $\tilde{b}_R$ to be degenerate at the weak scale with mass $m_{\tilde{b}}$. We now rotate the quarks (not the squarks) to diagonalize the quark mass matrix so that we can compute the mixing matrices $V_L$ and $V_R$ of equation (3). We find $V_L = V$ and $V_R = VP^2$. Hence we find

$$X = Im(V_{td}^2 P_{11}^2 (\zeta_{D_{33}} + \lambda_{D_{33}} \mu \tan \beta)) I_{33}(m_{\tilde{b}}^2, m_{\tilde{b}}^2).$$

(14)

We write $\lambda_{D_{33}} = \lambda_b (V_{33} P_{33})^{*2}$ and $\zeta_{D_{33}} = A_b m_{\tilde{b}} \lambda_b (V_{33} P_{33})^{*2}$, where $A_b$ and $\lambda_b$ are real. To relate $A_b$ to other $A$ parameters of the theory would require solving the $RG$ equation for $i$. This will be worth doing once the parameters of the MSSM are measured, but for now we take $A_b$ to be an unknown parameter. Hence

$$X = \lambda_b A_b' m_{\tilde{b}} |V_{td}|^2 \sin \phi I_{33}(m_{\tilde{b}}^2, m_{\tilde{b}}^2)$$

(15)

where $A_b' = A_b + \frac{\mu}{m_{\tilde{b}}} \tan \beta$, and we have set $|V_{tb}| = 1$. The phase $\phi$ is a physical observable and is given by $\phi = 2(\phi_{td} - \phi_{tb}) + 2(\phi_{11} - \phi_{33})$, where $\phi_{td}$ and $\phi_{tb}$ are the phases of $V_{td}$ and $V_{tb}$, $P_{11} = e^{i\phi_{11}}$ and $P_{33} = e^{i\phi_{33}}$. Field rephasings can change $\phi_{td}$, $\phi_{tb}$, $\phi_{11}$ and $\phi_{33}$, but the phase $\phi$ is a physical observable. Calculating the diagrams of Figure 1, with the gluino mass set equal to the $b$ squark mass
\( \bar{m}_3 = m_\tilde{b}, \) we find

\[
d_n = e \frac{2 \alpha_s}{81 \pi} |V_{td}|^2 \sin \phi \frac{m_b}{\eta_b} A'_b \frac{1}{m^2_b} \tag{16}
\]

where \( \eta_b \) is the QCD correction required to run the \( b \) quark mass up to the scale of the superpartners. To obtain this formula we have used the quark model result \( d_n = 4d_d/3. \)

There is a similar contribution to the electric dipole moment of the electron, \( d_e, \) coming from a diagram with internal bino and tau sleptons. In the minimal \( SO(10) \) model, the slepton mass matrix at \( M_G \) is given by (12) and in this basis the lepton Yukawa matrix is given by \( \tilde{\gamma}_E = V^{\dagger} P^{\dagger} V \); it is identical to \( \tilde{\gamma}_D \) given in (14). Using an analogous set of assumptions, and working in the approximation that the bino is a mass eigenstate, one derives

\[
X_e = \lambda_\tau A'_\tau |V_{td}|^2 \sin \phi I_{33}(m^2_\tau, m^2_\tau)
\]

where \( A'_\tau = A_\tau + \frac{\mu}{m_\tau} \tan \beta. \) Here \( m_\tau \) is the mass of both right and left-handed tau scalars. Taking the bino degenerate with the tau slepton, \( \bar{m}_1 = m_\tau, \) we find

\[
d_e = -e \frac{\alpha}{48 \pi c^2} |V_{td}|^2 \sin \phi m_\tau A'_\tau \frac{1}{m^2_\tau} \tag{17}
\]

where \( c^2 = \cos^2 \theta_W. \) Using the unification constraint on the gaugino masses: \( \bar{m}_3/\bar{m}_1 = \tilde{\alpha}_3/\tilde{\alpha}_1 \approx 7, \) one can deduce that \( d_n/d_e \approx -0.4 \frac{A'_b}{A'_\tau} \tag{18} \) for the case that the scalars and gaugino in any loop are degenerate. The relative minus sign is due to the fact that the product of the hypercharges of left and right-handed sleptons is negative. We are unable to predict the absolute sign of \( d_n \) or \( d_e \) since we do not know the sign of \( A'_b \) or \( A'_\tau. \) Since we expect \( A'_b \) and \( A'_\tau \) to be comparable, \( d_n \) and \( d_e \) are comparable in this limit.

The present experimental values for these quantities are

\[
d_n = (-30 \pm 50) \times 10^{-27} e \, cm \tag{19}
\]

and

\[
d_e = (1.8 \pm 1.2 \pm 1.0) \times 10^{-27} e \, cm \tag{20}
\]
from references [15] and [16] respectively. Since the present experimental sensitivity to $d_e$ is approximately 30 times greater than to $d_n$, equation (18) implies that, in the degenerate mass case, $d_e$ provides a much stronger probe of the theory than $d_n$. If one takes $\tilde{m}_1 = m_\tau = 100$ GeV, and $\tilde{m}_3 = m_\nu = 700$ GeV, the central value predicted by (17), with $A'_\tau = 1$, is already excluded by about a factor of 4, while the central prediction of (16), with $A'_b = 1$, is about an order of magnitude smaller than the experimental limit. Consistency with data requires increasing the superpartner masses so that the colored ones are above a TeV. Thus the present data already suggest that it is unlikely that the scalars and gauginos are degenerate.

Given the large hierarchy of $\tilde{m}_3/\tilde{m}_1 \simeq 7$, it is reasonable to expect that the bino is lighter than the scalar tau. In the limit that $\tilde{m}_1 \ll m_\tau$ we find

$$d_e = -e \frac{\alpha}{8\pi c^2} |V_{td}|^2 \sin \phi m_\tau A'_\tau \frac{\tilde{m}_1}{m_\tau^3}$$

(21)

Evaluating (16) and (21) with representative masses we find:

$$d_n = 40 \times 10^{-27} e \text{ cm} \left( \frac{|V_{td}|^2}{10^{-4}} \right) \left( \frac{m_b/\eta_b}{2.7 \text{ GeV}} \right) \left( \frac{A'_b}{1} \right) \left( \frac{250 \text{ GeV}}{m_\nu} \right)^2 \left( \frac{\sin \phi}{0.5} \right)$$

(22)

and

$$d_e = -3.0 \times 10^{-27} e \text{ cm} \left( \frac{|V_{td}|^2}{10^{-4}} \right) \left( \frac{m_\tau}{1.78 \text{ GeV}} \right) \left( \frac{A'_\tau}{1} \right) \left( \frac{\tilde{m}_1}{35 \text{ GeV}} \right) \left( \frac{200 \text{ GeV}}{m_\tau} \right)^3 \left( \frac{\sin \phi}{0.5} \right)$$

(23)

We have normalized these results to $\sin \phi = 0.5$. While the phase $\phi$ is not known, it is expected to be large. It has the same origin as the physical phase in the Kobayashi-Maskawa matrix, which we know is not small. With these parameter choices both predictions lie close to present experimental limits. Some regions of parameter space of the minimal SO(10) theory are already excluded, for example those with light scalar taus and large $\tan \beta \approx m_t/m_b$. A complete numerical calculation must be done to determine precisely which regions are already excluded, and how further improvements of the experiments will constrain the theory.
To what extent will the above calculations for the minimal $SO(10)$ model apply to more general models? The main assumption of the minimal $SO(10)$ model is that the quark and lepton masses arise from the two renormalizable operators of equation 8. However, in general there could be very many operators, including non-renormalizable ones, contributing to the Yukawa interactions of the MSSM. Thus, just beneath the GUT scale the Yukawa interactions need not have the minimal $SO(10)$ from of equation (13). While a quark basis can always be found to make $\tilde{U}$ diagonal, as shown in (13), the matrix $\tilde{D}$ need not be symmetric, so a general form for $\tilde{D}$ should be taken: $\tilde{D} = V \tilde{D} V_R$. Thus in equation (15) for $X$, and for $X_e$, one should replace $|V_{td}|^2$ with $|V_{td}V_{R_{td}}|$, and similarly in equations (22) and (23) for $d_n$ and $d_e$. The definition of the phase $\phi$ will also change.

The precise values for $V_{R_{td}}$ are model dependent, indeed this becomes the dominant uncertainty due to the unknown nature of the grand unified theory. Nevertheless, the unified theory before symmetry breaking is left-right symmetric, hence one expects $V_{R_{td}}$ to be comparable to $V_{td}$. The predictions of this letter for the minimal $SO(10)$ model apply equally to arbitrary $SO(10)$ or $E_6$ models providing $|V_{td}|^2$ is replaced by $|V_{td}V_{R_{td}}|$. This does not alter the central value of the prediction, but does enlarge the uncertainty of the prediction due to the model dependence of the origin of quark and lepton masses.

5.

In this letter we have shown that supersymmetric theories, which unify the quarks and leptons of a generation into a single multiplet, lead to predictions for $d_e$ and $d_n$ close to present experimental limits. These predictions can be reliably computed in terms of the parameters of the low energy supersymmetric theory. The origin of this effect is the grand unified gauge symmetry and the top quark Yukawa coupling. The unification of different particles into an irreducible representation prevents basis rotations which remove flavor mixing angles. The weak unification of $(u_L, d_L)$ into $Q$ prevents the removal of the Kobayashi-Maskawa mixing matrix in the standard model. Similarly, the unification $(Q, U^c, D^c)$ leads to a flavor mixing matrix in the right-handed down quark sector, and the unifications $(Q, U^c, E^c)$ and $(Q, U^c, L)$ lead to flavor mixing matrices in the right-handed and left-handed lepton sectors, respectively.
The super-rotations of $D^c$, $E^c$ and $L$ relative to $Q$ and $U^c$, needed to diagonalize the fermion mass matrices, can only be performed beneath $M_G$, where the grand unified symmetry is broken. However, at this point the scalars are non-degenerate, and the flavor mixing reappears in the scalar mass matrices. This non-degeneracy of the scalars, another unavoidable feature of unification, occurs because the $D^c$, $E^c$ and $L$ particles also have interactions induced by the same large parameter, $\lambda$, that induces the top quark mass.

Recently, large individual lepton number ($L_i$) violating processes, such as $\mu \to e\gamma$, have also been proposed as a signal for supersymmetric unification [12]. The origin of such signals is similar to that discussed here for the CP violating signals of $d_n$ and $d_e$, and a few brief comparisons can be made. Both $L_i$ and CP violating signals are based on the three assumptions of weak scale supersymmetry, grand unification and supersymmetry breaking operators present close to the Planck scale. The $L_i$ violating signals are more general, in that they also apply to the case of $SU(5)$ and require only that top quark and tau lepton are unified. For $SO(10)$ models the $L_i$ and CP violating processes, very broadly speaking, provide comparable signals. For example, for slepton masses of 200 GeV, both the $\mu \to e$ conversion rate and $d_e$ are very close to present experimental sensitivities. An important theoretical advantage of the CP violating signals is that as the scale of supersymmetry breaking, $m$, increases so the dipole moments scale as $1/m^2$, while the $L_i$ violating processes have rates which drop off more rapidly as $1/m^4$. Thus, improvements of the experimental limits by the same factor would mean that the CP violating signals would ultimately provide the most powerful probe of $SO(10)$ theories.

We believe that a continued experimental search for electron and neutron electric dipole moments will provide a very powerful probe of supersymmetric $SO(10)$ unification. Further numerical theoretical work is necessary to determine precisely how present and future measurements will constrain the parameter space of the low energy theory. Suppose that supersymmetry is discovered, and the weak-scale parameters associated with the superpartners are measured. It will then be possible to compute the values of $d_n$ and $d_e$, subject only to the mixing angle uncertainties. If the moments are not seen at the predicted level, the theory will be excluded. This should be contrasted with the signals of proton decay and neutrino masses. In general $SU(5)$ or $SO(10)$ models, the size of these
signals cannot be reliably computed, and we can never foresee the exclusion of superunification on these grounds.
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Figure Caption

A Feynman diagram which provides a contribution to the neutron electric dipole moment.
This figure "fig1-1.png" is available in "png" format from:

http://arxiv.org/ps/hep-ph/9411273v1