Theoretical Aspects of Heavy Flavour Physics

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Abstract

I review the status of theoretical aspects of $B$-decays. The principal difficulty in interpreting the wealth of experimental data is the control of non-perturbative QCD effects, and the talk is focused on attempts to control these effects. Lattice results for the decay constants, $B$-$\bar{B}$ mixing and semileptonic form-factors are summarized. The discrepancy of the theoretical predictions and experimental measurements for the ratio of lifetimes $\tau(\Lambda_b)/\tau(B_0)$ is discussed, as well as the status of the semileptonic branching ratio of the $B$-meson. The difficulties in making quantitative predictions for exclusive nonleptonic decays are stressed, and some recent approaches to this problem are outlined.

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1 Introduction

Weak decays of heavy quarks are a particularly fertile field of research for detailed tests of the Standard Model of Particle Physics, for measurements of its parameters (the Cabibbo-Kobayashi-Maskawa (CKM) matrix elements in particular) and for potential signatures of new physics. During the talk of P. Drell we have seen many exciting new experimental results on $B$-decays, and the flow of new data will continue for many years from existing and new facilities. The main theoretical difficulty in interpreting the experimental data is the control of the non-perturbative strong interaction effects, and this problem is the main focus of my lecture.

For some of the physical quantities discussed below I will summarize status of lattice results; these results have been taken from the recent review written together with J. Flynn, where more details can be found. The lattice formulation of QCD, together with large scale numerical simulations, enables one to calculate non-perturbative QCD effects, from first principles, with no model parameters or assumptions. The precision of the calculations is limited, however, by the available computing resources, and much effort is being devoted to reducing the systematic errors. These uncertainties, and the theoretical and computational efforts to control and reduce them, have been discussed at this conference in some detail by M. Luscher. I will therefore limit my comments to the specific computations I am discussing.

The plan for the talk is as follows. In the next section I will review the status of lattice calculations of the leptonic decay constants of heavy mesons and of the amplitudes for the important process of $B$-$\bar{B}$ mixing. Sec. 3 contains a review of exclusive semileptonic decays of $B$-mesons which are being used to extract the $V_{ub}$ and $V_{cb}$ CKM matrix elements. Inclusive semileptonic decays are discussed very briefly in sec. 4, where the recent suggestions to determine $V'_{ub}$ by measuring the hadronic invariant mass spectrum are outlined. I then digress from the mainstream of the presentation, to discuss the problem of power corrections to hard scattering and decay processes in general, and in the heavy quark effective theory (hqet) in particular (sec. 5). The next two sections contain discussions of inclusive (sec. 6) and exclusive (sec. 7) nonleptonic decays. In particular in sec. 6 I will discuss two topics which have received much attention lately, viz. the lifetimes of beauty hadrons and the semileptonic branching ratio of $B$-mesons. Finally sec. 8 contains some conclusions. $CP$-violation in $B$-decays, which is perhaps the principal motivation for the coming generation of $b$-factories, is the subject of the talk of Y. Nir, and will not be discussed below.
2 Leptonic Decays and $B^0 - \bar{B}^0$ Mixing

I start with a discussion of the calculations of the leptonic decay constants and of the $B$-parameter of $B$-$\bar{B}$ mixing.

2.1 Leptonic Decay Constants

The decay constant of a meson is a single number which contains all the information about the non-perturbative QCD effects in leptonic decays of the meson (see e.g. Fig. 1). Parity symmetry implies that only the axial component of the $V - A$ weak current contributes to the decay, and Lorentz invariance that the matrix element of the axial current is proportional to the momentum of the meson (with the constant of proportionality defined to be the decay constant):

$$\langle 0|A_\mu(0)|B(p)\rangle \equiv if_B p_\mu,$$

and similarly for the $D$-meson. Knowledge of $f_B$ would allow us not only to predict the rates for leptonic decays, but would also be very useful for describing $B$-$\bar{B}$ mixing as explained below, as well as for our understanding of other processes in $B$-physics, particularly those for which “factorization” turns out to be a useful approximation.

![Diagram](image)

Figure 1: Diagram representing the leptonic decay of the $B$-meson.

Many lattice groups have evaluated, or are evaluating, $f_D$ and $f_B$. Our view of the current status of the calculations can be summarized by the following values for the decay constants and their ratios:

$$f_D = 200 \pm 30 \text{ MeV}, \quad f_D^s = 220 \pm 30 \text{ MeV},$$

$$f_B = 170 \pm 35 \text{ MeV}, \quad f_B^s = 195 \pm 35 \text{ MeV},$$

$$f_{D_s}/f_D = 1.10 \pm 0.06, \quad f_{B_s}/f_B = 1.14 \pm 0.08.$$ (2)

All the results are presented using a normalization for which $f_{\pi^+} \simeq 131$ MeV.

In the absence of experimental results on the decay constants of the $B$-meson, the results above represent our best estimates of this quantity. During the past three years or so, experimental measurements of $f_{D_s}$ have been made by several groups, and at this years Heavy Flavours conference, the rapporteur summarized the results by:

$$f_{D_s} = (251 \pm 30) \text{ MeV}.$$ (3)

The agreement of the lattice predictions with the experimental result is very satisfying, and gives us confidence in the results for the other decay constants.

The results for the decay constants have generally remained stable for many years, but the quoted errors have not decreased significantly in this time. This is because the errors are dominated by systematic effects, and it is difficult to decrease the uncertainties due to the quenched approximation (i.e. the neglect of vacuum polarization effects) without performing reliable calculations with dynamical fermions, which will still take several more years. For example, the value of the lattice spacing typically varies by 10% or so depending on which physical quantity is used to calibrate the lattice simulation. This variation is largely due to the use of the quenched approximation. We therefore consider that $\pm 10\%$ is an irreducible minimum error in decay constants computed in quenched
At RHIC, the feasibility of obtaining (statistically) accurate results for a series of lattice spacings and actions is a new development, largely made possible by the increase in computing power. The figure also shows extrapolations to the continuum \((a = 0)\) limit. In this case the continuum values of \(f_D\) and \(f_B\) obtained using the two formulations agree remarkably well; the agreement is still acceptable (although not so remarkable) when quantities other than the string tension are used to determine the lattice spacing. We also note that in this case the dependence on the lattice spacing is milder for the improved action as expected (although further studies are needed to confirm whether this is a general feature).

The MILC collaboration has begun to study the effects of quenching, and their very early findings are that the decay constants may be a little larger (perhaps by about 10%) when the effects of quark loops are included. Further work will be done in the next few years to study this further.

### 2.2 \(B-B\)-Parameter

In order to obtain information about the unitarity triangle from experimental studies of \(B-\bar{B}\) mixing, one needs to control the non-perturbative QCD effects which are contained in the matrix element of the \(\Delta B=2\) operator:

\[
M(\mu) = \langle B^0 | \bar{b} \gamma_\mu (1-\gamma_5) q \bar{b} \gamma^\mu (1-\gamma_5) q | B^0 \rangle, 
\]  

\[
(4)
\]
where the light quark \( q = d \) or \( s \). The argument \( \mu \) implies that the operator has been renormalized at the scale \( \mu \). It is conventional to introduce the \( B_B \)-parameter through the definition

\[
M(\mu) = \frac{8}{3} f_B^2 m_B^2 B_B(\mu). \tag{5}
\]

The dimensionless quantity \( B_B \) is better-determined than \( f_B \) in lattice calculations, so that the theoretical uncertainties in the value of the matrix element \( M \), needed for phenomenology, are dominated by our ignorance of \( f_B \).

\( B_B(\mu) \) is a scale dependent quantity for which lattice results are most often quoted after translation to the \( \overline{\text{MS}} \) scheme. The next-to-leading order (NLO) renormalization group invariant \( B \)-parameter (\( \hat{B}_B^{\text{nlo}} \)) is defined by

\[
\hat{B}_B^{\text{nlo}} = \alpha_s(\mu)^{-2/\beta_0} \left( 1 + \frac{\alpha_s(\mu)}{4\pi} J_{n_f} \right) B_B(\mu), \tag{6}
\]

where \( \beta_0 = 11 - 2n_f/3 \), and \( J_{n_f} \) is obtained from the one- and two-loop anomalous dimensions of the \( \Delta B = 2 \) operator by

\[
J_{n_f} = \frac{1}{2\beta_0} \left( \gamma_0 \beta_1 - \gamma_1 \right), \tag{7}
\]

with \( \beta_1 = 102 - 38n_f/3 \), \( \gamma_0 = 4 \) and \( \gamma_1 = -7 + 4n_f/9 \). In the discussion below we choose \( \mu = m_b \).

A number of groups have evaluated \( \hat{B}_B \) in quenched lattice simulations from which we deduce the preferred value

\[
\hat{B}_B^{\text{nlo}} = 1.4(1). \tag{8}
\]

The relevant quantity for \( B \rightarrow B \) mixing is \( f_B^2 \hat{B}_B \). Taking the result in eq. (8) above for \( \hat{B}_B^{\text{nlo}} \) with \( f_B = 170(35) \text{MeV} \) from eq. (8) gives

\[
f_B \sqrt{\hat{B}_B^{\text{nlo}}} = 201(42) \text{MeV} \tag{9}
\]

as our lattice estimate. For the phenomenologically important quantity \( \xi \), which relates the amplitudes for \( B_d \rightarrow \bar{B}_d \) and \( B_s \rightarrow \bar{B}_s \) mixing, we find

\[
\xi = \frac{f_B \sqrt{B_B}}{f_{B_d} \sqrt{B_{B_d}}} = 1.14(8). \tag{10}
\]

### 3 Exclusive Semileptonic Decays of B-Mesons

Semileptonic decays of \( B \)-mesons are currently being used to determine the \( V_{cb} \) and \( V_{ub} \) CKM-matrix elements. Exclusive decays are represented by the diagram in Figure 3. It is convenient to use spacetime symmetries to express the matrix elements in terms of invariant form factors (using the helicity basis for these as defined below). When the final state is a pseudoscalar meson \( P \), parity implies that only the vector component of the \( V \rightarrow A \) weak current contributes to the decay, and there are two independent form factors, \( f^+ \) and \( f^0 \), defined by

\[
\langle P(k)|V(\mu)|B(p)\rangle = f^+(q^2) \left[ (p+k)\mu - \frac{m_B^2 - m_P^2}{q^2} q^\mu \right] + f^0(q^2) \frac{m_B^2 - m_P^2}{q^2} q^\mu, \tag{11}
\]

where \( q \) is the momentum transfer, \( q = p-k \). When the final-state hadron is a vector meson \( V \), there are four independent form factors:

\[
\langle V(k,\varepsilon)|V(\mu)|B(p)\rangle = \frac{2V(q^2)}{m_B + m_V} \varepsilon^{\mu\delta\beta} \varepsilon^*_\beta p_\delta k_\beta \tag{12}
\]

\[
\langle V(k,\varepsilon)|A(\mu)|B(p)\rangle = i(m_B + m_V) A_1(q^2) \varepsilon^{*\mu} \varepsilon^* p(p+k)\mu + i \frac{A_2(q^2)}{q^2} \frac{2m_V \varepsilon^* p q^\mu}{q^2} + i \frac{A_3(q^2)}{q^2} \frac{2m_V \varepsilon^* p q^\mu}{q^2}. \tag{13}
\]
where $\varepsilon$ is the polarization vector of the final-state meson, and $q = p - k$. Below we shall also discuss the form factor $A_0$, which is given in terms of those defined above by $A_0 = A + A_3$, with

$$A_3 = \frac{m_B + m_V}{2m_V} A_1 - \frac{m_B - m_V}{2m_V} A_2.$$  \hfill (14)

Figure 3: Diagram representing the semileptonic decay of the $B$-meson. $\bar{q}$ represents the light valence antiquark, and the black circle represents the insertion of the $V-A$ current with the appropriate flavour quantum numbers.

3.1 $B \to D^*$ and $B \to D$ Decays

Exclusive $B \to D^*$ and, more recently, $B \to D$ semileptonic decays are used to determine the $V_{cb}$ element of the CKM matrix. Theoretically they are relatively simple to consider, since heavy quark symmetry implies that the six form factors are related, and that there is only one independent form factor $\xi(\omega)$, specifically:

$$f^+(q^2) = V(q^2) = A_0(q^2) = A_2(q^2) = \left[ 1 - \frac{q^2}{(M_B + M_D)^2} \right]^{-1} A_1(q^2) = \frac{M_B + M_D}{2\sqrt{M_B M_D}} \xi(\omega), \hfill (15)$$

where $\omega = v_B \cdot v_D$. Here the label $D$ represents the $D$- or $D^*$-meson as appropriate. In this leading approximation the pseudoscalar and vector mesons are degenerate. The unique form factor $\xi(\omega)$, which contains all the non-perturbative QCD effects, is called the Isgur–Wise (IW) function. Vector current conservation implies that the IW-function is normalized at zero recoil, i.e. that $\xi(1) = 1$. This property is particularly important in the extraction of the $V_{cb}$ matrix element.

The relations in eq. (15) are valid up to perturbative and power corrections. The precision with which $V_{cb}$ can be extracted is limited by the theoretical uncertainties in the evaluation of these corrections. Nevertheless we are in the fortunate situation that it is uncertainties in corrections (which are therefore small) which control the precision.

The decay distribution for $B \to D^*$ decays can be written as:

$$\frac{d\Gamma}{d\omega} = \frac{G_F^2}{48\pi^3} (M_B - M_{D^*})^2 M_{D^*}^3 \sqrt{\omega^2 - 1} (\omega + 1)^2 \left[ 1 + \frac{4\omega}{\omega + 1} \frac{M_B^2 - 2\omega M_B M_{D^*} + M_{D^*}^2}{(M_B - M_{D^*})^2} \right] |V_{cb}|^2 F^2(\omega), \hfill (16)$$

where $F(\omega)$ is the IW-function combined with perturbative and power corrections. It is convenient theoretically to consider this distribution near $\omega = 1$. In this case $\xi(1) = 1$, and there are no $O(1/m_Q)$ corrections (where $Q = b$ or $c$) by virtue of Luke’s theorem, so that the expansion of $F(1)$ begins like:

$$F(1) = \eta_A \left( 1 + 0 \frac{\Lambda_{QCD}}{m_Q} + c_2 \frac{\Lambda_{QCD}^2}{m_Q^2} + \cdots \right), \hfill (17)$$

where $\eta_A$ represents the perturbative corrections. The one-loop contribution to $\eta_A$ has been known for some time now, whereas the two-loop contribution was evaluated last year, with the result:

$$\eta_A = 0.960 \pm 0.007, \hfill (18)$$
where we have taken the value of the two loop contribution as an estimate of the error.

The power corrections are much more difficult to estimate reliably. Neubert has recently combined the results of refs. \[24,25\] to estimate that the $O(1/m_Q^2)$ terms in the parentheses in eq. (17) are about $-0.055 \pm 0.025$ and that $\mathcal{F}(1) = 0.91(3)$. Bigi, Shifman and Uraltsev \[26\] consider the uncertainties to be bigger and obtain 0.91(6). Combining the latter, more cautious, theoretical value of $\mathcal{F}(1)$, with the experimental result \[27\] $\mathcal{F}(1)|V_{cb}| = (34.3 \pm 1.6) \times 10^{-3}$, readily gives $|V_{cb}| = (37.7 \pm 1.8_{\text{exp}} \pm 2.5_{\text{th}}) \times 10^{-3}$. In considering this result, the fundamental question that has to be asked is whether the power corrections are sufficiently under control. This will be discussed in more detail in section 5.

Having discussed the theoretical status of the normalization $\mathcal{F}(1)$, let us now consider the shape of the function $\mathcal{F}(\omega)$, near $\omega = 1$. A theoretical understanding of the shape would be useful to guide the extrapolation of the experimental data, and also as a test of our understanding of the QCD effects. We expand $\mathcal{F}$ as a power series in $\omega - 1$:

$$\mathcal{F}(\omega) = \mathcal{F}(1) \left[ 1 - \tilde{\rho}^2 (\omega - 1) + \tilde{c} (\omega - 1)^2 + \cdots \right],$$

where

$$\tilde{\rho}^2 = \rho^2 + (0.16 \pm 0.02) + \text{power corrections},$$

and $\rho^2$ is the slope of the IW-function.

Recently, using analyticity and unitarity properties of the amplitudes, as well as the heavy quark symmetry, Caprini and Neubert have derived an intriguing result for the curvature parameter $\tilde{c}$ \[28\]. This result would have effectively removed one of the parameters from the extrapolation of the experimental data to $\omega = 1$. The derivation of this relation has been criticized in ref. \[29\] primarily for discarding sub-threshold $(B_c)$ contributions, and the expectation is that the corrected relations give somewhat weaker information. Caprini et al. are preparing a revised slope-curvature relation \[30\]. Preliminary indications were presented in ref. \[31\].

Finally in this section I consider $B \to D$ semileptonic decays, which are beginning to be measured experimentally with good precision (see refs. \[32\] and references therein). The decay distribution can be written as:

$$\frac{d\Gamma}{d\omega} = \frac{G_F^2}{48\pi^3} (M_B + M_D)^2 M_B^2 (\omega^2 - 1)\frac{\hat{G}(\omega)}{2} V_{cb}^2 G^2(\omega),$$

where again $G(\omega)$ is the IW-function combined with perturbative and power corrections. Theoretically the first complication is that the $1/m_Q$ corrections to $G(1)$ do not vanish. However, as pointed out by Shifman and Voloshin \[24\], these corrections would vanish in the limit in which the $b$ and $c$-quarks are degenerate, and hence are suppressed. Ligeti, Nir, and Neubert estimate the $1/m_Q$ corrections to be between approximately $-1.5\%$ to $+7.5\%$. The $1/m_Q^2$ corrections for this decay have not yet been studied systematically. A recent theoretical estimate for $G(1)$ gives $0.98 \pm 0.07$ \[33\].

### 3.2 $B \to \rho$ and $B \to \pi$ Decays

In this subsection we consider the heavy-to-light semileptonic decays $B \to \rho$ and $B \to \pi$ which are now being used experimentally to determine the $V_{ub}$ matrix element \[34\]. Heavy quark symmetry is less predictive for heavy-to-light decays than for heavy-to-heavy ones. In particular, there is no normalization condition at zero recoil corresponding to the relation $\xi(1) = 1$, which is so useful in the extraction of $V_{cb}$. The lack of such a condition puts a premium on the results from nonperturbative calculational techniques, such as lattice QCD or light-cone sum rules. Heavy quark symmetry does, however, give useful scaling laws for the behaviour of the form factors with the mass of the heavy quark ($m_Q$) at fixed $\omega$:

$$f^+, A_0, A_2, V \sim \sqrt{m_Q}; \ A_1, f^0 \sim \frac{1}{\sqrt{m_Q}}; \ A_3 \sim m_Q^{\frac{3}{2}}.$$
These scaling relations are particularly useful in lattice simulations, where the masses of the quarks are varied. Moreover, the heavy quark spin symmetry relates the $B \rightarrow V$ matrix elements (where $V$ is a light vector particle) of the weak current and magnetic moment operators, thereby relating the amplitudes for the two physical processes $\bar{B}^0 \rightarrow \rho^+ l^- \bar{\nu}_l$ and $\bar{B} \rightarrow K^* \gamma$, up to $SU(3)$ flavour symmetry breaking effects. These relations also provide important checks on theoretical, and in particular on lattice, calculations.

Recent work includes detailed lattice studies by several groups, but in particular by the UKQCD collaboration, who try to exploit all possible symmetries in order to optimize the available information about the form factors and calculations using light-cone sum rules.

3.3 Lattice Calculations of $B \rightarrow \rho, \pi$ Semileptonic Decays and of the Rare Decay $\bar{B} \rightarrow K^* \gamma$

The techniques required to extract the form-factors for $\bar{B}^0 \rightarrow \pi^+ l^- \bar{\nu}_l$ and $\bar{B}^0 \rightarrow \rho^+ l^- \bar{\nu}_l$ semileptonic decays are very similar to those used to compute the short distance contribution to the rare radiative decay $\bar{B} \rightarrow K^* \gamma$, so I consider them together. For completeness, I define here form factors for the matrix element of the magnetic moment operator responsible for this decay:

$$\langle K^*(k, \varepsilon) \mid 8\sigma_{\mu\nu} q^\nu b_R \mid B(p) \rangle = \sum_{i=1}^{3} C_i^i T_i(q^2),$$

where $q = p - k$, $\varepsilon$ is the polarization vector of the $K^*$ and

$$C_{\mu}^1 = 2 \epsilon_{\mu\nu\lambda\rho} \varepsilon^* \nu^{*} \rho^{*} k^{\lambda},$$

$$C_{\mu}^2 = \varepsilon^*_{\mu} (m_B^2 - m_{K^*}^2) - \varepsilon \cdot q (p + k)_\mu,$$

$$C_{\mu}^3 = \varepsilon^* \cdot q \left( q_{\mu} - \frac{q^2}{m_B^2 - m_{K^*}^2} (p + k)_\mu \right).$$

$T_3$ does not contribute to the physical $\bar{B} \rightarrow K^* \gamma$ amplitude for which $q^2 = 0$, and $T_1(0)$ and $T_2(0)$ are related by,

$$T_1(q^2=0) = i T_2(q^2=0).$$

Hence, for the process $\bar{B} \rightarrow K^* \gamma$, we need to determine $T_1$ and/or $T_2$ at the on-shell point $q^2 = 0$.

From lattice simulations we can only obtain the form factors for part of the physical phase space for all the decays. In order to control discretization errors we require that the three-momenta of the $B$, $\pi$ and $\rho$ mesons be small in lattice units. This implies that we determine the form factors at large values of momentum transfer $q^2 = (p_B - p_{\pi, \rho, K^*})^2$. Experiments can already reconstruct exclusive semileptonic $b \rightarrow u$ decays (see, for example, the review in [3]), and as techniques improve and new facilities begin operation, we can expect to be able to compare the lattice form factor calculations directly with experimental data at large $q^2$. A proposal in this direction was made by UKQCD [11] for $\bar{B}^0 \rightarrow \rho^+ l^- \bar{\nu}_l$ decays. To get some idea of the precision that might be reached, they parametrize the differential decay rate distribution near $q_{\text{max}}^2$ by:

$$\frac{d\Gamma(\bar{B}^0 \rightarrow \rho^+ l^- \bar{\nu}_l)}{dq^2} = 10^{-12} \frac{G_F^2 |V_{ub}|^2}{192\pi^3 m_B^6} q^2 \lambda^2 (q^2) a^2 (1 + b(q^2 - q_{\text{max}}^2)),$$

where $a$ and $b$ are parameters, and the phase-space factor $\lambda$ is given by $\lambda(q^2) = (m_B^2 + m_\rho^2 - q^2)^2 - 4m_B^2 m_\rho^2$. The constant $a$ plays the role of the IW function evaluated at $\omega = 1$ for heavy-to-heavy transitions, but in this case there is no symmetry to determine its value at leading order in the heavy quark effective theory. UKQCD obtain [14]

$$a = 4.6^{+0.3}_{-0.3} \pm 0.6 \text{ GeV} \quad \text{and} \quad b = (-8^{+4}_{-3}) \times 10^{-2} \text{ GeV}^2.$$
The fits are less sensitive to $b$, so it is less well-determined. The result for $a$ incorporates a systematic error dominated by the uncertainty ascribed to discretization errors and would lead to an extraction of $|V_{ub}|$ with less than 10% statistical error and about 12% systematic error from the theoretical input. The prediction for the $d\Gamma/dq^2$ distribution based on these numbers is presented in Fig. 4. With sufficient experimental data an accurate lattice result at a single value of $q^2$ would be sufficient to fix $|V_{ub}|$.

In principle, a similar analysis could be applied to the decay $\bar{B}^0 \to \pi^+ l^- \bar{\nu}_l$. However, UKQCD find that the difficulty of performing the chiral extrapolation to a realistically light pion from the unphysical pions used in the simulations makes the results less certain. The $B \to \pi$ decay also has a smaller fraction of events at high $q^2$, so it will be more difficult experimentally to extract sufficient data in this region for a detailed comparison.

We would also like to know the full $q^2$ dependence of the form factors, which involves a large extrapolation in $q^2$ from the high values where lattice calculations produce results, down to $q^2 = 0$. In particular the radiative decay $B \to K^* \gamma$ occurs at $q^2 = 0$, so that existing lattice simulations cannot make a direct calculation of the necessary form factors. In order to determine the form factors at lower values of $q^2$ from their measurements the lattice collaborations, and the UKQCD collaboration in particular, exploit a number of important constraints. I now briefly outline these in turn:

- An interesting contribution to the problem of extrapolation to low $q^2$ has been suggested by Lellouch for $\bar{B}^0 \to \pi^+ l^- \bar{\nu}_l$ decays. Using dispersion relations constrained by UKQCD lattice results at large values of $q^2$ and kinematical constraints at $q^2 = 0$, one can tighten the bounds on form factors at all values of $q^2$. This technique relies on perturbative QCD in evaluating one side of the dispersion relations, together with general properties of field theory, such as unitarity, analyticity and crossing. It provides model-independent results which are illustrated in Fig. 5. The results (at 50% CL – see Ref. 32 for details) are

$$f^+(0) = 0.10 - 0.57 \quad \text{and} \quad \Gamma(\bar{B}^0 \to \pi^+ l^- \bar{\nu}_l) = 4.4 - 13 |V_{ub}^2| \text{ps}^{-1}. \quad (30)$$

Unfortunately these bounds are not very restrictive when constrained by existing lattice data. In principle, this method can be applied to $B \to \rho$ decays also, but is more complicated there, and the calculations have yet to be performed. Recently, Becirevic has applied the method...
for the process $\bar{B} \to K^*\gamma$, using lattice results from the APE collaboration as constraints. However, he has not applied the kinematic constraint at $q^2 = 0$ and the resulting bounds are not informative: they become so, however, once he uses a light-cone sum rule evaluation of the form-factors for the process $\bar{B} \to K^*\gamma$ as an additional constraint. These dispersive methods can be used with other approaches in addition to lattice results and sum rules, such as quark models, or even in direct comparisons with experimental data, to check for compatibility with QCD and to extend the range of results.

- UKQCD make use of the kinematic constraints on the form factors at $q^2 = 0$:
  \[ f^+(0) = f^0(0), \quad T_1(0) = iT_2(0), \quad A_0(0) = A_3(0). \]  
  (31)

- In spite of all the constraints, model input is required to guide $q^2$ extrapolations. We can ensure that any assumed $q^2$-dependence of the form factors is consistent with the requirements imposed by heavy quark symmetry, as shown in (31), together with the kinematical relations of eq. (31). UKQCD also verify that the expected relations between form factors of the different processes (at fixed $\omega$)
  \[ A_1(q^2(\omega)) = 2iT_2(q^2(\omega)), \quad V(q^2(\omega)) = 2T_1(q^2(\omega)), \]  
  (32)
are indeed well satisfied in the infinite mass limit. Even with all these constraints, however, current lattice data do not by themselves distinguish a preferred $q^2$-dependence. Fortunately, more guidance is available from light-cone sum rule analyses which lead to scaling laws for the form factors at fixed (low) $q^2$ rather than at fixed $\omega$ as in eq. (22). In particular all form factors scale like $M^{-3/2}$ at $q^2 = 0$:
  \[ f(0)\Theta = M^{-3/2}gf \left( 1 + \frac{\delta_f}{M} + \frac{\epsilon_f}{M^2} + \cdots \right), \]  
  (33)
where $f$ labels the form factor, $M$ is the mass of the heavy-light meson and $\Theta$ is a calculable leading logarithmic correction.
A combined fit to the lattice data for the form factors (obtained at large values of $q^2$) satisfying all these constraints is shown in Fig. 6. The quark masses have been chosen to correspond to the $K^*$ vector-meson. The figure demonstrates the large extrapolation needed to reach $q^2 = 0$.

Our preferred results for $\bar{B}_0 \to \pi^+ l^- \bar{\nu}_l$ and $\bar{B}_0 \to \rho^+ l^- \bar{\nu}_l$ come from the UKQCD constrained fits\textsuperscript{39}. Their best estimates of the total rates are:

$$\Gamma(\bar{B}_0 \to \pi^+ e^- \bar{\nu}_e) = 8.5^{+3.3}_{-1.4} |V_{ub}|^2 \text{ps}^{-1} \quad \text{and} \quad \Gamma(\bar{B}_0 \to \rho^+ e^- \bar{\nu}_e) = 16.5^{+3.5}_{-2.3} |V_{ub}|^2 \text{ps}^{-1}. \quad (34)$$

There are also preliminary results for heavy-to-light form factors from FNAL, JLQCD and a Hiroshima-KEK group (see the reviews in\textsuperscript{40, 41}) and the different lattice calculations are in agreement for the form factors at large $q^2$ where they are measured.

For the form factor $T_1(0)$ of $\bar{B} \to K^* \gamma$ decays, the combined fits give

$$T_1(0) = 0.16(3). \quad (35)$$

Using this value to evaluate the ratio (given at leading order in QCD and up to $O(1/m_B^2)$ corrections\textsuperscript{42})

$$R_{K^*} = \frac{\Gamma(\bar{B} \to K^* \gamma)}{\Gamma(b \to s \gamma)} = 4 \left(\frac{m_B}{m_b}\right)^3 \left(1 - \frac{m_{K^*}^2}{m_B^2}\right)^3 |T(0)|^2 \quad (36)$$

results in

$$R_{K^*} = 16(3)^\% . \quad (37)$$

which is consistent with the experimental result $18(7)^\%$ from CLEO\textsuperscript{43}. Discrepancies between $R_{K^*}$ calculated using $T(0)$ and the experimental ratio $\Gamma(\bar{B} \to K^* \gamma)/\Gamma(b \to s \gamma)$ could reveal the existence of long-distance effects in the exclusive decay. It has been proposed that these effects may be significant for the process $\bar{B} \to K^* \gamma$\textsuperscript{44-46}, but within the precision of the experimental and lattice results, there is no evidence for them.
3.4 Light Cone Sum Rule Studies of the Form Factors for $B^0 \to \pi^+ \nu_l$ and $B^0 \to \rho^+ \nu_l$ decays

The second technique which has been used to study heavy $\to$ light decays is QCD sum rules. Ball and Braun [33] have recently clarified the origin of the discrepancy in results obtained using different types of sum rules for $B \to \rho$ semileptonic decays at large recoil [37, 38]. These authors explain why the standard sum rules, in which one performs an Operator Product Expansion (OPE) in terms of operators of increasing dimension, fail in this region of phase space. They stress the necessity of using light cone sum rules, in which the contributions are classified by the components of different “twist” of the distribution amplitude of the $\rho$-meson (strictly speaking of its moments). This work exploits and extends earlier studies [30-32]. A careful investigation of the dominant contributions to the form factors at large recoil yields the scaling law in eq. (33) for the behaviour of the form factors with the mass of the heavy quark at small fixed $q^2$ (rather than fixed $\omega$).

The form factors derived in ref. [30] are shown in Fig. 7 together with the results from the UKQCD collaboration at large values of $q^2$. In view of the uncertainties in both sets of calculations, the agreement between the sum rule and lattice results is remarkable. For the total rate, Ball and Braun find

$$\Gamma(B^0 \to \rho^+ e^{-}\nu_e) = (13.5 \pm 1.0 \pm 1.3 \pm 0.6 \pm 3.6)|V_{ub}|^2 ps^{-1},$$

(38)

to be compared to the lattice result in eq. (34). The second error in eq. (38) is due to the uncertainty in the mass of the $b$-quark, the remaining errors are estimates of various uncertainties in the light-cone sum rule calculation and in the distribution amplitude of the $\rho$-meson.

4 Inclusive Semileptonic Decays - $V_{ub}$

The energy of the electron ($E_e$) in semileptonic $B$-decays is limited kinematically to a very narrow window:

$$\frac{m_B^2 - m_D^2}{2m_B} \approx 2.3 \text{ GeV} \leq E_e \leq \frac{m_B}{2} \approx 2.6 \text{ GeV}. $$

(39)

This window contains only a small fraction (about 10%) of all the $b \to u$ decays, and it is difficult to make theoretical predictions for the spectrum.

Two groups have recently proposed to use the hadronic invariant mass ($M_h$) spectrum instead of the electron energy spectrum [35, 36]. About 90% of $b \to u$ decays satisfy $M_h < M_D$. The spectrum takes the form

$$\frac{d\Gamma}{dM_h^2} = G_F^2 \frac{m_b^5}{192 \pi^3} |V_{ub}|^2 S(y) \text{ where } y = \frac{M_h^2}{\Lambda m_b} \text{ and } \Lambda = m_B - m_b.$$  

(40)

The use of the inclusive hadronic energy spectrum for the determination of $V_{ub}$ had been proposed in ref. [37].
The non-perturbative QCD effects are contained in $S(y)$, and specifically in the parameters $a_i$ which can be expressed as matrix elements of composite operators:

$$\langle H(v)|\bar{h}_\nu (iD_{\mu_1}) (iD_{\mu_2}) \cdots (iD_{\mu_n}) h_\nu |H(v)\rangle = 2m_B a_n v_{\mu_1} v_{\mu_2} \cdots v_{\mu_n}, \quad (41)$$

where $a_0 = 1$, $a_1 = 0$ and $a_2$ is given in terms of the kinetic energy of the heavy quark in the meson.

It is proposed to determine $V_{ub}$ from the integral of the spectrum up to some maximum hadronic mass; the precision will depend on the value of the cut-off which can be attained experimentally (the precision on $V_{ub}$ is estimated to be 10-20% for values of the cut-off from $M_D$ down to 1.5 GeV or so). Much theoretical and experimental work is needed to extract the optimal results from this method.

### 5 Power Corrections

I now digress from the main discussion of $B$-decays to consider the evaluation of power corrections to hard scattering and decay processes. Since there are many confusing statements in the literature, and because the evaluation of higher order terms in the heavy quark expansion is very important in $B$-physics, it may be useful to clarify some of the key points. Although the discussion is presented in the context of $B$-decays, it can readily be applied to other processes for which an OPE is useful and where the power corrections are given by the matrix elements of higher-twist or higher-dimension operators.

The approach presented here was developed together with G. Martinelli, where references to the key papers can be found.

Consider some physical quantity $\mathcal{P}$ for which the OPE allows us to write the theoretical prediction in terms of an expansion in inverse powers of $m_b$:

$$\mathcal{P} = C_0(m_b, \mu) \langle f | O_0(\mu) | i \rangle + \frac{1}{m_b^2} C_1(m_b, \mu) \langle f | O_1(\mu) | i \rangle + \cdots \quad (42)$$

where the ellipsis represents higher order terms in the OPE which we will not consider further. For simplicity I assume here that there is only a single operator in each of the first two orders of the expansion; if this is not the case then there will be an additional mixing of operators which only requires a minor modification of the discussion below. For clarity of notation I suppress the dependence of the Wilson coefficient functions ($C_i$) on the strong coupling constant.

I assume that we are interested in evaluating the $O(1/m_b^n)$ corrections in $\mathcal{P}$. The theory of power corrections is very delicate, since they are exponentially small compared with the terms of the perturbation series for $C_0$ ($1/m_b \sim \exp(-c/\alpha_s(m_b))$, where $c$ is a constant). Thus in order to evaluate the power corrections, we need to control the exponentially small tail of the perturbation series for $C_0$. This raises the problems of the Borel summability and uniqueness of the sum of this perturbation series, and the question of renormalon ambiguities which I will not discuss further here.

Frequently, renormalization schemes based on the dimensional regularization of ultraviolet divergences are used to define renormalized operators (e.g. the $\overline{MS}$ scheme). One consequence of the problems mentioned above is that higher dimensional operators (such as $O_1$) are not uniquely defined in the $\overline{MS}$ scheme; the remaining ambiguity in the matrix elements of $O_1$ is of $O(\Lambda_{QCD}^n)$, i.e. of the same order as the matrix elements themselves). The exceptions to this are operators whose matrix elements give the leading contribution to some physical quantity, e.g. the dimension 5 chromomagnetic operator

$$\lambda_2 = \frac{1}{3} \frac{\langle B | \bar{h}_2 G^{ij} h | B \rangle}{2M_B} \quad (43)$$
and $h$ is the field of the (static) heavy quark.

In order to avoid the renormalon ambiguity it is necessary to introduce a hard ultraviolet cut-off (or subtraction scale), $\mu$, so as to provide a well-defined boundary between long- and short-distance contributions. In lattice calculations this occurs naturally; the lattice spacing $a$ is such a cut-off ($\mu = a^{-1}$). In continuum calculations, a hard cut-off has been used in refs.\textsuperscript{56,57,58}.

With a hard cut-off it is clear that the matrix elements of higher dimensional operators diverge as powers of $\mu$ and in themselves do not have a natural physical interpretation, e.g.

$$\langle f|O_1(\mu)|i \rangle \sim \mu^n.$$  \hfill (44)

Since physical quantities cannot depend on the cut-off, the term proportional to $\mu^n$ present in the matrix element in eq. (44) must be cancelled in the full prediction of $\mathcal{P}$. Specifically, this term is cancelled by corresponding ones in the coefficient function $C_0$, which now takes the form

$$C_0 = \sum_{i=0}^{\infty} c_i \alpha_s + \frac{\mu^n}{m_b^n} \sum_{i=1}^{\infty} d_i \alpha_s.$$  \hfill (45)

The second term on the right hand side of eq. (45) arises from the matching of full QCD onto the hQET, when the operator $O_1$ is included in the OPE. The important point is that the coefficient function $C_0$ is evaluated perturbatively and hence, in practice, this cancellation will only be achieved partially. In order to evaluate (or even to estimate) the $O(\Lambda_{QCD}^2/m_b^n)$ corrections to $\mathcal{P}$, we must evaluate $C_0$ to a sufficiently high order of perturbation theory to reach the corresponding precision. We believe that this is not the case in present calculations, and certainly has not been demonstrated to be so\textsuperscript{6}.

For calculations in heavy quark physics the above discussion implies that there is no “natural” definition of parameters such as the binding energy $\bar{\Lambda}$ (see eq. (40)) or the kinetic energy $\lambda_1$

$$\lambda_1(B) = \frac{1}{2m_B} \langle B| \bar{h}(D)^2 b | B \rangle .$$  \hfill (46)

Different definitions of these parameters, due to different renormalization prescriptions for the operators, differ by terms of $O(\Lambda_{QCD})$ and $O(\Lambda_{QCD}^2)$ respectively. It is therefore of little use to compare values of these parameters, obtained using different definitions (often the definitions are implicit and must be inferred from the details of the calculations); nevertheless, this is frequently done.

6 Non-Leptonic Inclusive Decays

In this section I discuss two very interesting problems in the phenomenology of $B$-decays, that of lifetimes and the semileptonic branching ratio. The discussion will use the formalism of Bigi et al.(see ref.\textsuperscript{60} and references therein), developed and used by them and many other groups, in which inclusive quantities are expanded in inverse powers of the mass of the heavy quark, e.g.

$$\Gamma(H_b) = \frac{G_F^2 m_b^5 |V_{cb}|^2}{192\pi^3} \left\{ c_3 \left( 1 + \frac{\lambda_1 + 3\lambda_2}{2m_b^2} \right) + c_5 \frac{\lambda_2}{m_b^4} + O \left( \frac{1}{m_b^6} \right) \right\} ,$$  \hfill (47)

where $\Gamma$ is the full or partial width of a beauty hadron $H_b$, $c_{3,5}$ are coefficients which can be computed in perturbation theory and $\lambda_{1,2}$ are the parameters introduced in section 5 above. Here I will not rediscuss the cancellation of renormalon ambiguities present in $c_3$, $\lambda_1$ and the quark mass; below we will consider ratios of physical quantities for which the cancellation is more transparent. An important feature of the general expression in eq. (47) is the absence of terms of $O(1/m_b)$, which is a consequence of the absence of any operators of dimension 4 which can appear in the corresponding OPE\textsuperscript{6}.

\textsuperscript{6}In lattice calculations it may be possible to calculate the coefficients $d_i$ to reasonably high orders by using the Langevin stochastic formulation of lattice field theory\textsuperscript{59}.\textsuperscript{13}
6.1 Beauty Lifetimes

Using the expression in eq. (47) for the widths one readily finds the following results for the ratios of lifetimes:

\[
\frac{\tau(B^-)}{\tau(B^0)} = 1 + O\left(\frac{1}{m_b^3}\right) \tag{48}
\]

\[
\frac{\tau(\Lambda_b)}{\tau(B^0)} = 1 + \frac{\mu_\pi^2(\Lambda_b) - \mu_\pi^2(B)}{2m_b^2} + c_G \frac{\mu_G^2(\Lambda_b) - \mu_G^2(B)}{m_b^2} + O\left(\frac{1}{m_b^3}\right)
\]

\[
= (0.98 \pm 0.01) + O\left(\frac{1}{m_b^3}\right), \tag{49}
\]

where \(\mu_\pi^2 = -\lambda_1\) and \(\mu_G^2 = 3\lambda_2\). In order to obtain the result in eq. (49), one needs to know the difference of the kinetic energies of the \(b\)-quark in the baryon and meson. To leading order in the heavy quark expansion we have:

\[
\mu_\pi^2(\Lambda_b) - \mu_\pi^2(B) = -\frac{M_B M_D}{2} \left( \frac{M_{\Lambda_b} - M_{\Lambda_c}}{M_B - M_D} - \frac{3 M_{B^*} - M_{D^*}}{4} \right). \tag{50}
\]

From equation (50), and using the recent measurement of \(m_{\Lambda_b}\) from CDF, one finds that the right hand side is very small (less than about 0.01 GeV\(^2\)). The matrix elements of the chromomagnetic operator are obtained from the mass difference of the \(B^*-\) and \(B\)-mesons (see eq. (43)) and from the fact that the two valence quarks in the \(\Lambda_b\) are in a spin-zero state. The theoretical predictions in eqs. (48) and (49) can be compared with the experimental measurements

\[
\frac{\tau(B^-)}{\tau(B^0)} = 1.06 \pm 0.04 \quad \text{and} \quad \frac{\tau(\Lambda_b)}{\tau(B^0)} = 0.79 \pm 0.05. \tag{51}
\]

The discrepancy between the theoretical and experimental results for the ratio \(\tau(\Lambda_b)/\tau(B^0)\) in eqs. (49) and (51) is notable. It raise the question of whether the \(O(1/m_b^3)\) contributions are surprisingly large, or whether there is a more fundamental problem. I postpone consideration of the latter possibility and start with a discussion of the \(O(1/m_b^3)\) terms.

At first sight it seems strange to consider the \(1/m_b^3\) corrections to be a potential source of large corrections, when the \(O(1/m_b^3)\) terms are only about 2%. However, it is only at this order that the
“spectator” quark contributes, and so these contributions lead directly to differences in lifetimes for hadrons with different light quark constituents (consider for example the lower diagram in Fig. 3 for which, using the short-distance expansion, one obtains operators of dimension 6). Moreover, the coefficient functions of these operators are relatively large, which may be attributed to the fact that the lower diagram in Fig. 3 is a one-loop graph, whereas the corresponding diagrams for the leading contributions are two-loop graphs (see, for example, the upper diagram of Fig. 3). The corresponding phase-space enhancement factor is $16\pi^2$ or so. We will therefore only consider the contributions from the corresponding four-quark operators, neglecting other $O(1/m_b^6)$ corrections which do not have the phase space enhancement. For each light-quark flavour $q$, there are four of these

$$
O_1 \equiv \bar{b}\gamma_\mu(1 - \gamma^5)q \bar{q}\gamma^\mu(1 - \gamma^5)b \\
O_2 \equiv \bar{b}(1 + \gamma^5)q \bar{q}(1 + \gamma^5)b \\
T_1 \equiv \bar{b}\gamma_\mu(1 - \gamma^5)T_\alpha q \bar{q}\gamma_\alpha(1 - \gamma^5)T_\alpha b \\
T_2 \equiv \bar{b}(1 + \gamma^5)T_\alpha q \bar{q}(1 + \gamma^5)T_\alpha b
$$

where $T_\alpha$ are the generators of colour $SU(3)$. Thus we need to evaluate the matrix elements of these four operators.

For mesons, following ref. 2, I introduce the parametrization

$$
\langle B|O_{1}|B\rangle_{\mu=m_b} \equiv B_1 f_B^2 M_B^2 \quad ; \quad \langle B|T_1|B\rangle_{\mu=m_b} \equiv \epsilon_1 f_B^2 M_B^2,
$$

where $\mu$ is the renormalization scale. We have chosen to use $m_b$ as the renormalization scale. Bigi et al. 9 prefer to use a typical hadronic scale, and estimate the matrix elements using a factorization hypothesis at this low scale. Operators renormalized at different scales can be related using renormalization group equations in the hqet (sometimes called hybrid renormalization 9). For example, if we assume that factorization holds at a low scale $\mu$ such that $\alpha_s(\mu^2) = 1/2$, then, using the (leading order) renormalization group equations, one finds $B_1 = B_2 = 1.01$ and $\epsilon_1 = \epsilon_2 = -0.05$.\footnote{I use the notation of ref. 9.}

In the limit of a large number of colours $N_c$, $B_1 = O(N_c^0)$ whereas $\epsilon_i = O(1/N_c)$.

For the $\Lambda_b$, heavy quark symmetry implies that

$$
\langle \Lambda_b|O_2|\Lambda_b\rangle = -\frac{1}{2} \langle \Lambda_b|O_1|\Lambda_b\rangle \quad \text{and} \quad \langle \Lambda_b|T_2|\Lambda_b\rangle = -\frac{1}{2} \langle \Lambda_b|T_1|\Lambda_b\rangle,
$$

so that there are only two parameters. It is convenient to replace the operator $T_1$, by $\tilde{O}_1$ defined by

$$
\tilde{O}_1 \equiv \bar{b}\gamma_\mu(1 - \gamma^5)q^j \bar{q}^j\gamma^\mu(1 - \gamma^5)b^i,
$$

where $i, j$ are colour labels, and to express physical quantities in terms of the two parameters $\tilde{B}$ and $r$ defined by

$$
\langle \Lambda_b|\tilde{O}_1|\Lambda_b\rangle_{\mu=m_b} \equiv -\tilde{B} \langle \Lambda_b|O_1|\Lambda_b\rangle_{\mu=m_b} \quad \text{and} \quad \frac{1}{2M_{\Lambda_b}} \langle \Lambda_b|O_1|\Lambda_b\rangle_{\mu=m_b} \equiv -\frac{f_B^2 M_B}{48} r
$$

We do not know the values of these parameters. In quark models $\tilde{B} = 1$, and $r = 0.2$–0.5. Using experimental values of the hyperfine splittings and quark models, it has been suggested that $r$ may be larger\footnote{By factorization we mean that if the $B_i$’s and $\epsilon_i$’s had been defined at this scale (instead of $m_b$) they would have been 1 and 0 respectively.}, e.g.

$$
\frac{M_{\pi^+} - M_{\pi^0}}{M_{\pi^+} - M_{\pi^-}} = 0.9 \pm 0.1 \quad \text{and} \quad r \simeq \frac{M_{\pi^+}^2 - M_{\pi^-}^2}{M_{\pi^+}^2 - M_{\pi^-}^2} = 1.8 \pm 0.5.
$$
The lifetime ratios can now be written in terms of the six parameters $B_{1,2}, \epsilon_{1,2}, \tilde{B}$ and $r$ (as well as $f_B$):

$$\frac{\tau(B^-)}{\tau(B^0)} = 1 + \left( \frac{f_B}{200 \text{ MeV}} \right)^2 \left\{ 0.02B_1 + 0.00B_2 - 0.70\epsilon_1 + 0.20\epsilon_2 \right\} \quad (60)$$

$$\frac{\Lambda_b}{\tau(B^0)} = 0.98 + \left( \frac{f_B}{200 \text{ MeV}} \right)^2 \left\{ -0.00B_1 + 0.00B_2 - 0.17\epsilon_1 + 0.20\epsilon_2 \right. \right. + \left( -0.01 - 0.02\tilde{B} \right) r \right\}, \quad (61)$$

where the effective weak Lagrangian has been renormalized at $\mu = m_b$. The central question is whether it is possible, with “reasonable” values of the parameters, to obtain agreement with the experimental numbers in eq. (51). At this stage in our knowledge, the answer depends somewhat on what is meant by reasonable. For example, Neubert, guided by the arguments outlined above, has considered these ratios by varying the parameters in the following ranges:

$$B_i, \tilde{B} \in \left[ \frac{2}{3}, \frac{4}{3} \right] ; \quad \epsilon_i \in \left[ -\frac{1}{3}, \frac{1}{3} \right] ; \quad r \in [0.25, 2.5] ; \quad \left( \frac{f_B}{200 \text{ MeV}} \right)^2 \in [0.8, 1.2]. \quad (62)$$

He concludes that, within these ranges, it is just possible to obtain agreement at the two standard deviation level for large values of $r$ ($r \geq 1.2$) and negative values of $\epsilon_2$. Lattice studies of the corresponding matrix elements are underway; a recent QCD sum-rule calculation has found a small value of $r$, $r \simeq 0.1 - 0.3$.66

If the lattice calculations confirm that the parameter $r$ is small, or find that the other parameters are not in the appropriate ranges, then we have a breakdown of our understanding. If no explanation can be found within the standard formulation, then we will be forced to take seriously the possible breakdown of local duality. This is beginning to be studied in toy field theories.67,68

6.2 The “Baffling” Semileptonic Branching Ratio

This was the name given by Blok et al.69 to the observation that the experimental value of the semileptonic branching ratio

$$B_{SL} = \frac{\Gamma(B \to X\ell\bar{\nu})}{\sum_i \Gamma(B \to Xl\bar{\nu}) + \Gamma_{had} + \Gamma_{rare}} \quad (63)$$

Figure 9: Theoretical Prediction of the semileptonic branching ratio and charm counting. The data points are the experimental results from high-energy (LEP) and low energy (i.e. at the Υ(4S) from CLEO) experiments.
appeared to be lower than expected theoretically. In eq. (63) the sum is over the three species of lepton, and $\Gamma_{had}$ and $\Gamma_{rare}$ are the widths of the hadronic and rare decays respectively. Bigi et al. concluded that a branching ratio of less than 12.5% cannot be accommodated by theory. Since then Bagan et al. have completed the calculation of the $O(\alpha_s)$ corrections, and in particular of the $b \to c\bar{c}\bar{s}\bar{s}$ component (including the effects of the mass of the charm quark), these have the effect of decreasing $B_{SL}$. With M. Neubert, we used this input to reevaluate the branching ratio and charm counting ($n_c$, the average number of charmed particles per $B$-decay) finding, e.g.

$$B_{SL} = 12.0 \pm 1.0\% \ (\mu = m_b) \quad n_c = 1.20 \mp 0.06 \ (\mu = m_b)$$

$$B_{SL} = 10.9 \pm 1.0\% \ (\mu = m_b/2) \quad n_c = 1.21 \mp 0.06 \ (\mu = m_b/2) .$$  \ (64)

$\mu$ is the renormalization scale and the dependence on this scale is a reflection of our ignorance of higher order perturbative corrections. The experimental situation is somewhat confused, see Fig. 9. In his compilation at the ICHEP conference last year, Richman found that the semileptonic branching ratio obtained from $B$-mesons from the $\Upsilon(4S)$ is:

$$B_{SL}(B) = (10.23 \pm 0.39)\% ,$$  \ (65)

whereas that from LEP is:

$$B_{SL}(b) = (10.95 \pm 0.32)\% .$$  \ (66)

The label $b$ for the LEP measurement indicates that the decays from beauty hadrons other than the $B$-meson are included. Using the measured fractions of the different hadrons and their lifetimes, and assuming that the semileptonic widths of all the beauty hadrons are the same, one finds:

$$B_{SL}(b) = (10.95 \pm 0.32)\% \Rightarrow B_{SL}(B) = (11.23 \pm 0.34)\% ,$$  \ (67)

amplifying the discrepancy. It is very difficult to understand such a discrepancy theoretically, since the theoretical calculation only involves $\Gamma_{SL}$ (and not $\Gamma_{had}$ for which the uncertainties are much larger). In view of the experimental discrepancy, I consider the problem of the lifetime ratio $\tau(\Lambda_b)/\tau(B^0)$, described in subsection 6.1 above, to be the more significant one.

### 7 Exclusive Nonleptonic Decays

In this section I consider two-body nonleptonic decays of $B$-mesons, for which a large amount of data is becoming available, particularly from the CLEO collaboration. This is an exciting new field of investigation, which will undoubtedly teach us much about subtle aspects of the standard model. Unfortunately, at our present level of understanding we are not able to compute the amplitudes from first principles, and are forced to make assumptions about the non-perturbative QCD effects; frequently these assumptions concern factorization. These assumptions may well be wrong. Thus the analyses are limited to a semi-quantitative level. In this talk I will briefly describe some recent attempts to understand nonleptonic exclusive decays; at this stage it is not possible to endorse these approaches with any confidence.

#### 7.1 Generalized Factorization Hypothesis

Neubert and Stech suggest an approach based on keeping the leading order terms in the limit of a large number of colours ($N_c$). Consider $B \to D\pi$ decays, for which the effective Hamiltonian is given by:

$$\mathcal{H}_{eff} = \frac{G_F}{\sqrt{2}} V_{cb} V_{ud}^* \left\{ c_1(\mu) \ (\bar{d}u) \ (\bar{c}b) + c_2(\mu) \ (\bar{c}u) \ (\bar{d}b) \right\} ,$$  \ (68)

\(^{17}\)Note that the rapporteur at the 1997 EPS conference argued that the branching ratio had been overestimated by the LEP collaboration.
where, at the renormalization scale $\mu = m_b$, $c_1(m_b) = 1.13$ and $c_2(m_b) = -0.29$, and the $V-A$ structure of the current is implied, e.g. $(\bar{d}u)(\bar{c}b) = (d\gamma^\mu(1 - \gamma^5)u)(\bar{c}\gamma_\mu(1 - \gamma^5)b)$.

For the class-1 decay $B^0 \to D^+\pi^-$, using colour and spinor Fierz identities we can write:

$$A_{B^0 \to D^+\pi^-} = \left(c_1 + \frac{c_2}{N_c}\right)\langle D^+\pi^- \mid (\bar{d}u)(\bar{c}b)\mid B^0\rangle + 2c_2\langle D^+\pi^- \mid (\bar{d}T^a u)(\bar{c}T^a b)\mid B^0\rangle .$$

Following ref.\cite{ref} we write

$$F_{(BD)\pi} \equiv \langle \pi^- \mid (\bar{d}u)\mid 0\rangle \langle D^+ \mid (\bar{c}b)\mid B^0\rangle \tag{70}$$

and

$$A_{B^0 \to D^+\pi^-} \equiv a_1 F_{(BD)\pi}, \quad \text{with} \quad a_1 = \left(c_1 + \frac{c_2}{N_c}\right)\left[1 + \epsilon_1^{(BD)\pi}\right] + c_2\epsilon_8^{(BD)\pi}. \tag{71}$$

So far this is only a parametrization of the amplitude. The two factors on the right-hand side of eq. (70) are given in terms of the decay constant $f_\pi$ and the form-factors for semileptonic $B \to D$ decays respectively. The naive factorization hypothesis would imply that $a_1 = c_1 + c_2/N_c$. Neubert and Stech argue that the large-$N_c$ expansion may be a better guide than factorization, in which case we have $a_1 = c_1 + O(1/N_c^2)$.

For the class-2 process $B^0 \to D^0\pi^0$, the relations corresponding to eqs. (69) to (71) are:

$$A_{B^0 \to D^0\pi^0} = \left(c_2 + \frac{c_1}{N_c}\right)\langle D^0\pi^0 \mid (\bar{c}u)(\bar{d}b)\mid B^0\rangle + 2c_1\langle D^0\pi^0 \mid (\bar{c}T^a u)(\bar{d}T^a b)\mid B^0\rangle \tag{72}$$

$$F_{(B\pi)D} \equiv \langle D^0 \mid (\bar{c}u)\mid 0\rangle \langle \pi^0 \mid (\bar{d}b)\mid B^0\rangle \tag{73}$$

$$A_{B^0 \to D^+\pi^-} \equiv a_2 F_{(B\pi)D} \tag{74}$$

$$a_2 = \left(c_2 + \frac{c_1}{N_c}\right)\left[1 + \epsilon_1^{(B\pi)D}\right] + c_1\epsilon_8^{(B\pi)D} \tag{75}.$$

In this case naive factorization would imply that $a_2 = c_2 + c_1/N_c$, whereas using the large $N_C$ expansion $a_2 = c_2 + c_1(1/N_c + \epsilon_8^{(B\pi)D}) + O(1/N_c^3)$, (the leading terms in $a_2$ are all of $O(1/N_c)$). This approximation preserves the correct renormalization group behaviour (up to corrections of $O(1/N_c^2)$).

Neubert and Stech propose a “generalized” factorization hypothesis for processes in which a large amount of energy is released, based on the large $N_C$ expansion described above, and on the concept of colour transparency applied also to class-2 decays\cite{ref} (for more formal arguments see also \cite{ref}).

$$a_1 = c_1(m_b) \quad \text{and} \quad a_2 = c_2(m_b) + \zeta c_1(m_b), \tag{76}$$

where \(\zeta\) is a process independent parameter (for two-body decays). They study a wide variety of processes and conclude that $a_{1,2}$ are process independent within the available precision:

$$a_1 = 1.10 \pm 0.07 \pm 0.17 \quad \text{and} \quad a_2 = 0.21 \pm 0.01 \pm 0.04. \tag{77}$$

It is now important, not only to extend the theoretical and experimental studies to improve the precision and determine the range of validity of the hypothesis, but above all to try and understand the theoretical foundations (if any) for it.

\footnote{Neubert and Stech also apply factorization assumptions to processes in which the energy of the outgoing particles are not necessarily large.}
7.2 B-Decays to Two Light Hadrons

The recent data from the CLEO collaboration on B-decays into two light mesons has stimulated many theoretical papers (see e.g. refs. 76 and for decays (inclusive as well as exclusive) with an \( \eta' \) in the final state in particular see refs. 77). Many of the decays have suppressed tree level contributions, so that loop effects, which are sensitive to the presence of new physics, are important. As always, the principal difficulty in drawing quantitative conclusions from the experimental data is our inability to control the non-perturbative QCD effects. For example, control of the penguin contribution to the decay \( B \rightarrow \pi^+ \pi^- \), which may well be significant, is needed in the forthcoming studies of CP-violation and in the determination of the angle \( \alpha \) of the unitarity triangle.

Among the recent phenomenological studies of the CLEO data is a standard analysis by Ali and Greub 78 who include the next-to-leading order coefficient functions, and estimates of the hadronic matrix elements based on factorization. They find that all the \( B \rightarrow K \pi \) and \( \pi \pi \) rates or upper limits can be accommodated, within this picture. Their estimates of the effects of charm quarks in penguin diagrams are based on perturbation theory. This has been criticized by Ciuchini et al. who stress that the relevant regions of phase space are close to the charm threshold and hence intrinsically non-perturbative. Their book-keeping of penguin effects within the OPE is briefly outlined in the next subsection. Concerning the phenomenological conclusions, Ciuchini et al. question whether the consistency of theoretical predictions and experimental measurements or bounds found in ref. 77 can be achieved for all processes with a single set of non-perturbative parameters. For example, Ciuchini et al. find it difficult to satisfy simultaneously the experimental branching ratio for the process \( B \rightarrow \eta' K^0 \) and the bound on the branching ratio for \( B_d \rightarrow \pi^+ \pi^- \). Such debates in the community, on what is a very new field of study, will lead to considerable insights into subtle features of the Standard Model.

7.3 Charming Penguins

The analysis of non-leptonic b-decays generally starts with a classification of the relevant operators and a calculation of their coefficient functions in perturbation theory. For example:

\[
Q_1^u = (bd)_{V-A}(\bar{u}u)_{V-A} \quad Q_2^u = (bd)_{V-A}(\bar{c}c)_{V-A} \\
Q_1^c = (\bar{b}u)_{V-A}(\bar{u}d)_{V-A} \quad Q_2^c = (\bar{b}c)_{V-A}(\bar{c}d)_{V-A},
\]

(78)

plus QCD and electroweak penguin operators. The coefficients of \( Q_1^u, Q_1^c \) are of \( O(1) \) whereas those of the remaining operators are of \( O(\alpha_s) \) and are generally small.

Ciuchini et al. 77 have recently emphasized the point, previously made by Buras and Fleischer 80 that the matrix elements of the operators \( Q_1^u, Q_1^c \) have penguin-like contractions (when the up or charm quark fields are Wick contracted). For the case of the charm quark, the effects of these “charming” penguins are enhanced in decays where the emission diagrams are Cabibbo suppressed relative to the penguin diagram. Although the precise definition of charming penguins (i.e. the separation of penguin effects between the explicit contributions of the further operators \( Q_3, \ldots \)) and those in the matrix elements of the operators \( Q_1^u, Q_1^c \) is a matter of convention, they do contain non-perturbative QCD effects which must be included in the calculations of decay amplitudes. From their recent analysis, Ciuchini et al. conclude that large contributions are likely from charming penguins, e.g. that it is necessary to include them in order to obtain branching ratios of order \( 1 \times 10^{-5} \) for the decays \( B^+ \rightarrow K^0 \pi^+ \) and \( B_d \rightarrow K^+ \pi^- \) as recently found by the CLEO collaboration 81. By using recent results from CLEO, Ciuchini et al. constrain many of the relevant hadronic matrix elements and are able to make predictions for a large number of processes which have not yet been observed. Some of these processes, including \( B \rightarrow \rho K \), are predicted to have branching ratios close to the current experimental bounds, and will provide an excellent testing ground for our understanding of penguin effects.
7.4 Lattice Calculations

The amplitudes for non-leptonic decays of hadrons \( H \rightarrow h_1 h_2 \) are difficult to evaluate in lattice simulations in principle. Maiani and Testa pointed out that in lattice simulations one obtains the following average of matrix elements\(^{79}\):

\[
\frac{1}{2} \left( \langle h_1 h_2 | H_{\text{eff}} | H \rangle + \langle h_1 h_2 | H_{\text{eff}} | H \rangle \right),
\]

and hence no information about the phase of the final state interactions. They also show that from the large time behaviour of the correlation functions one can extract the unphysical matrix element:

\[
\langle h_1(p_{h_1} = 0) h_2(p_{h_2} = 0) | H_{\text{eff}} | H \rangle.
\]

Together with chiral perturbation theory, this may be useful to obtain good estimates of the amplitudes for kaon decays, but it is not very useful for \( B \)-decays.

The Maiani-Testa theorem\(^{79}\) implies that it is not possible to obtain the phase of the final state interactions without some assumptions about the amplitudes. The importance of developing reliable quantitative techniques for the evaluation of non-perturbative QCD effects in non-leptonic decays cannot be overstated, and so attempts to introduce “reasonable” assumptions to enable calculations to be performed (and compared with experimental data) are needed urgently. Ciuchini et al.\(^{82,83}\) have recently shown that by making a “smoothness” hypothesis about the decay amplitudes it is possible to extract information about the phase of two-body non-leptonic amplitudes. Studies to see whether their proposals are practicable and consistent are currently beginning.

8 Conclusions

During recent years, through the combined work of experimentalists and theorists, there has been enormous progress in our understanding of heavy quark physics. In this talk I have reviewed some of this work, and underlined a few of the areas in which further progress is urgently needed in developing control of the non-perturbative QCD effects. Among the main points were:

- The lattice community will continue to refine the computations of a wide variety of physical quantities (e.g. leptonic decay constants, \( B \)-parameter of \( B-\bar{B} \) mixing, form factors of semileptonic decays, the parameters of the \( hqet \)), reducing the systematic errors, in particular those due to the use of the quenched approximation.

- \( V_{ub} \) can be determined by studying the hadronic invariant mass spectrum in inclusive semileptonic decays. The authors of ref.\(^{50}\) estimate that this will lead to an error of about 10% in \( V_{ub} \) whereas those of ref.\(^{51}\) that the error will not exceed 10–20%.

- We need to understand whether the discrepancy of the theoretical prediction for the ratio \( \tau(\Lambda_b)/\tau(B) \) with the corresponding experimental measurement is due to some effect which can be controlled (such as a matrix elements which is larger than currently expected) or to violations of local duality.

- We need progress in understanding whether power corrections to hard scattering and decay processes can be controlled numerically.

- New theoretical ideas are urgently required to interpret quantitatively the wealth of data on non-leptonic exclusive decays from current and future experiments. In particular progress in this area is needed for studies of CP-violation at the forthcoming \( b \)-factories, and in the attempts to fix the angles of the unitarity triangle.
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