Universalities of weak localization

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Abstract. We present here a novel approach to evaluation of the weak localization correction (WLC) to transport properties of a mesoscopic conductor. It is based on an extension of Keldysh technique and allows one to evaluate the full counting statistics of the current in the conductor. In our opinion, it provides a fresh look on the theory of weak localization.

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1 Introduction and method

Since the discovery of weak localization twenty years ago [1] much attention has been given to its scaling properties in low dimensions in infinite homogenous media. There is also weak localization correction to transport properties of a mesoscopic conductor that is shorter than decoherence length. Such a conductor is neither scalable nor homogeneous, so that one generally expects the WLC to depend on details of sample structure. Still, the WLC exhibits some universal features.

We concentrate in this work on evaluation of WLC to transport properties of two-terminal mesoscopic diffusive conductor. We employ here a new technique that allows to calculate not only the current in the conductor but also all noise characteristics of the current, so-called full counting statistics [2]. Beside the fact that this method provides more information about transport properties of the conductor than the traditional Green's function [1] and RMT [3] methods, it gives compact and clear general expressions that are valid for inhomogeneous conductors.

Due to the lack of space, it is impossible to review all the results obtained. We concentrate on the most important ones that concern the universal features of WLC: its form for quasi-one-dimensional conductors, general formula for the current correction and its possible dependence on momentum relaxation time, universal Cooperon mode. We disregard influence of magnetic field and spin-orbit interaction of the WLC.

The method in use presents an extension of Keldysh Green function (KGF) method (see [4] for review) and is based on the following trick: let us define a one-electron Green function by means of

\[ (i \frac{\partial}{\partial t} - \hat{H} - \tau_3 \chi(t) \hat{I}) \otimes \hat{G} = \delta(1 - 1') \]  

(1)
notations of Ref. [4]), where $\hat{I}$ is for the time being an one-electron arbitrary operator, $\chi(t)$ is a time-dependent parameter, $\tau_3$ is $2 \times 2$ matrix in Keldysh index. It is easy to show by traditional diagrammatic methods that the expansion of $\hat{G}$ in $\chi(t)$ generates diagrams for higher order correlators of $\hat{I}$.

The trick can be readily applied to the problem of full counting statistics of the current.[2] To this end, we set $\hat{I}$ to the operator of the current through a cross section of the conductor, $\chi(t)$ to constant value. It is convenient to introduce the following $\chi$-dependent action to express the probability for $N$ electrons to be transferred through the conductor during time interval $t$

$$\frac{\partial S}{\partial \chi} = it \int \frac{d\varepsilon}{2\pi} \text{Tr} (\tau_3 \hat{G}(\varepsilon)); \quad \langle \langle N \rangle \rangle = \int_{-\pi}^{\pi} \frac{d\chi}{2\pi} e^{-S(\chi)-iN\chi}.$$  

Derivatives of $S$ give moments of $\langle \langle N \rangle \rangle$, first and second derivative corresponding to average current and current noise respectively.

1.1 Semiclassical limit

To calculate $S$ in semiclassical limit for diffusive conductor, one derives a diffusion equation for the new KGF in coinciding points, $\hat{G}(x, x) \equiv \pi \nu \hat{G}(x)$, $\nu$ being density of states. It turns out that $\hat{G}^2 = 1$ and the equation takes traditional form [5, 6]

$$\frac{\partial \tilde{j}_\alpha}{\partial x} = 0; \quad \tilde{j}_\alpha = -D(x) \hat{G} \frac{\partial}{\partial x}(\ln(\hat{G}_1 \hat{G}_2)).$$  

at any energy, $D$ being diffusive coefficient. The boundary conditions to Eq. 3 are set by KGF in terminals, terms with $\chi$ can be incorporated into these conditions, so that $\hat{G}$ equals

$$\hat{G}_1 = \left( \begin{array}{cc} 1 - 2f_1 & 2f_1 \\ 2(1 - f_1) & 2f_1 - 1 \end{array} \right); \quad \hat{G}_2 = \left( \begin{array}{cc} 1 - 2f_2 & 2f_2e^{ix} \\ 2(1 - f_2)e^{-ix} & 2f_2 - 1 \end{array} \right).$$  

in the right and left reservoir respectively, $f_{1,2}(\varepsilon)$ being electron distribution functions in corresponding reservoirs. The equation 3 can be solved in a rather general form yielding

$$\tilde{j}_\alpha = -D(x) \ln(\hat{G}_1 \hat{G}_2) \frac{\partial u(x)}{\partial x_\alpha}.$$  

where $u(x)$ satisfies diffusion equation with boundary conditions $u_1 = 0, u_2 = 1$. Physically, $u(x)$ is a normalized voltage distribution in the conductor. This solution leads to the following quasiclassical action:

$$-S_{class} = \frac{tG}{4e^2} \int d\varepsilon M(\varepsilon);$$  

$$M \equiv \ln^2 \left[ (2f_1 - 1)(2f_2 - 1) + 2(f_1(1 - f_2)e^{-ix} + f_2(1 - f_1)e^{ix}) \right]$$  

$$+ \frac{1}{2} \sqrt{2} \sin \frac{\chi}{2} \left[ (f_1e^{ix} + 1 - f_1)(f_2e^{-ix} + 1 - f_2)(f_2(1 - f_1)e^{-ix/2} - f_1(1 - f_2)e^{ix/2}) \right]$$

$G$ being full conductance. This generalizes results of [3], those were obtained by RMT methods. One can find in [3] why both methods give the same results.
1.2 Weak localization correction

The advantage of the method is that one obtains the WLC directly to $S$. It is given by a usual Cooperon ladder \[1\] (Fig. 1) made of new KGF. Due to its ladder nature, the correction can be express in terms of eigenvalues $1 + \lambda_n$ of the ladder section operator

\[
K^{ab,cd}(1, 2; 1', 2') = G^{ac}(1, 1')U(1' - 2')G^{bd}(2, 2')
\]

where $U(1' - 2')$ presents impurity potential. It reads

\[
-S_{wlc} = t \int \frac{d\epsilon}{\pi} \sum_n \ln \lambda_n
\]

Here dimensionless $\lambda_n$ characterize Cooperon propagation. We derive corresponding diffusion equation for these eigenvalues and matrix eigenfunction $\hat{f}$,

\[
-\frac{\partial}{\partial x_\alpha}D(x)(\hat{G} \frac{\partial}{\partial x_\alpha} \hat{f} + \hat{f} \frac{\partial}{\partial x_\alpha} \hat{G}^T) = \lambda(\hat{G} \hat{f} - \hat{f} \hat{G}^T)\tau(x)
\]

It is essential that the eigenvalues do depend on $\tau(x)$, the momentum relaxation time. This is not the same as transport scattering time that enters diffusion coefficient and in principle shall be considered as an independent parameter. The relevant branch of eigenvalues corresponds to $\hat{f} \propto \tau^2$. That simplifies the equation considerably. Finally we arrive at the following relation for eigenvalues $\lambda_n$

\[
-\tau^{1/2}(x)\frac{\partial}{\partial x_\alpha}D(x)\frac{\partial}{\partial x_\alpha} \tau^{1/2}(x)f - D(x)\tau(x)M(\epsilon)(\nabla_\alpha u)^2f = \lambda f
\]

that implicitly presents the result for the WLC in the most general form.

2 Results and discussion

2.1 Universality in a general conductor

We note an interesting feature of the results: although each eigenvalue $\lambda$ does depend on $\tau(x)$, the WLC does not since the corrections to different $\lambda$ cancel each other. So
that we can replace $\lambda$ by effective $\tilde{\lambda}$ those are given by equation that does not contain $\tau(x)$

$$-\frac{\partial}{\partial x_\alpha} D(x) \frac{\partial}{\partial x_\alpha} f - D(x) M(x) (\nabla_\alpha u)^2 f = \tilde{\lambda} f \quad (13)$$

Since $M \approx i4\chi(f_1 - f_2)$ at $\chi \to 0$, and the average current $\propto \partial S/\partial \chi(0)$, the WLC to conductivity is obtained by perturbation expansion in $M$ yielding

$$G_{wlc} = \frac{-4e^2}{\pi} \sum_n \frac{<D(x) (\nabla_\alpha u)^2>_n}{\lambda_n(M = 0)} \quad (14)$$

where brackets denote average with respect to $n$-th Cooperon state at $M = 0$.

2.2 Universality in a quasi-one-dimensional conductor

There is even more universality in the case of quasi-one-dimensional conductor where $D(x)$ depends only on single coordinate and only one Cooperon branch has to be taken into account. The eigenvalues reduce to $\pi^2n + M$, $n \geq 1$ and the spatial dependence of diffusion coefficient is of no importance. The sum in (10) can be taken explicitly yielding

$$S_{wcl} = t \int \frac{d\varepsilon}{\pi} \ln \left( \frac{\sin(\sqrt{-M})}{\sqrt{-M}} \right) \quad (15)$$

2.3 Universal gapless mode

At $\chi = 0$ Cooperon eigenvalues never approach zero. This is a kind of finite-size effect specific for mesoscopic conductor. We note that there is a universal Cooperon mode with eigenfunction $\sin(\pi u(x))$ that becomes gapless in the absence of magnetic field provided $M \to -\pi^2$. This indicates a strong divergence of WLC in this regime that can be lifted by magnetic field. $M$ approaches $-\pi^2$ in the limit of $\chi \to i\infty$, $f_1 = 0$, $f_2 = 1$ that corresponds to a current fluctuation which is much smaller than the average current. These fluctuations are exponentially suppressed by $S_{class}$, and diverging $S_{wlc}$ provides anomalous magnetic dependence of their probability.

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