Spin-dependent Polarizability of Nucleon with Dispersion Relation in the Skyrme Model

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Abstract

We calculate the spin-dependent polarizability of the nucleon in the Skyrme model. The result is compared with that of a heavy baryon chiral perturbation theory (HBChPT), and is shown to be the same as that of HBChPT up to the \( \Delta \)-pole terms in the narrow width limit of the \( \Delta \) state and with the experimental physical constants. The effect of the \( \Delta + \pi \) channel is rather small and is numerically quite similar to that of the \( \Delta \) loop in the HBChPT. The electric and magnetic polarizabilities are recalculated using the transverse photon and a consistent inclusion of the \( \Delta \) width.

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The electromagnetic polarizabilities are important quantities which show the response of internal structure of nucleons to the external electromagnetic fields. These polarizabilities are extracted from the forward Compton scattering at threshold, and recently attracted a great deal of attention of both experimental and theoretical interest.

The Skyrme-soliton model is a QCD motivated model based on the idea of large $N_c$ and of the spontaneous breaking of chiral symmetry. The electromagnetic polarizability is considered to be sensitive to the pion cloud around the nucleon, so that the Skyrme model may be well-suited to the study of the polarizability. In a previous paper [1] we have calculated the electric and magnetic polarizabilities in the model using the dispersion formula with the photo-absorption cross sections of the longitudinal and transversal photons for the electric and magnetic ones, respectively. Calculations of the electric polarizability from the seagull term were also shown to be not compatible with the gauge invariance. It was shown that the chiral leading order terms of the electric and magnetic polarizabilities are exactly the same as those in the chiral perturbation theory. [2, 1] Further, we have stressed that the $\Delta$ states play important roles, and that the $N$ and the $\Delta$ states are treated as the same rotational levels of the Skyrme soliton.

In this paper we apply the method to the study of the spin-dependent polarizability $\gamma$ of the nucleon. The multipole analysis shows that $\gamma$ is small but negative: $-1.3 (-0.4) \times 10^{-4}\text{fm}^4$ for the proton (neutron). [3] On the other hand, the chiral leading order contribution is largely positive and is about $4.6 \times 10^{-4}\text{fm}^4$. There are some studies of this quantity in terms of the heavy baryon chiral perturbation theory (HBChPT). Bernard et al. [4] obtained $\gamma$ with the one-loop result and with including the effect of the $\Delta$ state. Hemmert et al. [5] calculated $\gamma$ up to $O(\epsilon^3)$ with the HBChPT and the explicit degree of freedom of the $\Delta$ state. In this approach the small parameter $\epsilon$ is taken to be either of the soft momentum, the pion mass, or the mass splitting $\Delta M = M_\Delta - M_N$ between the $N$ and $\Delta$ states. These calculations show that the contribution of the $\Delta$ state is very large and negative. In this meaning it is very interesting to study the spin polarizability in the Skyrme model, since it treats the $\Delta$ state as the equal partner as the $N$ state.

The forward Compton scattering amplitude of the nucleon is represented as

$$f_1(\omega) \, \mathbf{e} \cdot \mathbf{e}' + i f_2(\omega) \, \omega \, \mathbf{\sigma} \cdot \mathbf{e}' \times \mathbf{e},$$

(1)

where $f_1(\omega)$ and $f_2(\omega)$ are expanded at low energies as

$$f_1(\omega) = -\frac{e^2}{4\pi M} + (\bar{\alpha} + \bar{\beta}) \omega^2 + \cdots,$$

(2)
and
\[ f_2(\omega) = -\frac{e^2\kappa^2}{8\pi M^2} + \gamma \omega^2 + \cdots. \] (3)

Here, \( \bar{\alpha} \) and \( \bar{\beta} \) are the electric and magnetic polarizabilities, respectively, \( \kappa \) the anomalous magnetic moment of the nucleon, and \( \gamma \) the spin-dependent polarizability. The once-subtracted dispersion relation gives for the spin-dependent polarizability
\[ \gamma = \frac{1}{4\pi^2} \int_{-\omega_0}^{\omega_0} \frac{\sigma_{1/2} - \sigma_{3/2}}{\omega^3} \, d\omega, \] (4)
where \( \sigma_\lambda \) denotes the photo-absorption cross section with the helicity \( \lambda \).

We calculate the photo-absorption cross section in terms of the \( \gamma + N \rightarrow \pi + N \) and \( \gamma + N \rightarrow \pi + \Delta \) amplitudes in the Skyrme model. We have shown \( \square \) that the electric and magnetic Born amplitudes satisfies the low-energy theorem of the pion photo-production amplitudes at threshold except for the order \( (m_\pi/M)^2 \) term which was recently introduced by the effect of chiral loops. \( \square \) The electric Born amplitude for the \( \gamma + N \rightarrow \pi + N \) is given by
\[ \left( T_E^{(-)} \right)^N = \left( \frac{eG_{NN\pi}}{8\pi M} \right) \left\{ i\sigma \cdot \epsilon + 2\frac{i\sigma \cdot (k - q)(\epsilon \cdot q)}{m_\pi^2 + (k - q)^2} \right\}, \] (5)
where \( k \) and \( q \) are the incident photon and the outgoing pion momenta, respectively, and \( \epsilon \) is the polarization vector of the incident photon. Here, we expanded the production amplitude as
\[ T^a = i\epsilon_{a3b}^b T^{(-)} + \tau^a T^{(0)} + \delta_{a3} T^{(+)} + \delta_{a3} T^{(+)} \] (6)
We see that the \( \left( T_E^{(-)} \right)^N \) is of \( O(N^{1/2}) \), while the \( \left( T_E^{(+0)} \right)^N \) are of \( O(N^{-1/2}) \) and behave as \( O(\omega_k) \). Therefore, the latter amplitudes do not lead to finite results without unitarization of them, and are neglected in the following. The absorption cross section is calculated to be
\[ \Delta\sigma^N_E(\omega_k) = \left( \frac{e^2G_{NN\pi}^2}{8\pi M^2} \right) (1 - v^2) \ln \frac{1 + v}{1 - v}, \] (7)
where \( \Delta\sigma = \sigma_{1/2} - \sigma_{3/2} \), and \( v = q/\omega_q \) is the pion velocity. Inserting this into eq. \( \square \) we obtain
\[ \gamma^N_E = \left( \frac{e^2}{4\pi} \right) \frac{G_{NN\pi}^2}{24\pi^2 M^2 m_\pi^2}. \] (8)
In terms of the Goldberger-Treiman relation, this can be seen to be exactly the same as that of the N-loop in ChPT, as already shown by L’vov. \( \square \) The \( 1/m_\pi^2 \) dependence shows that this is the contribution from the pion cloud, and is of the leading order in the ChPT. Because the proton-neutron difference depends on the amplitude \( \left( T_E^{(0)} \right)^N \), we
predict only the average between them. It is known that the prediction of the HBChPT up to chiral order $\epsilon^3$ includes no isospin dependence, and there is no contribution of $\left( T_E^{(-)} \right)^N$, $\left( T_E^{(+0)} \right)^N$. The multipole analysis \textsuperscript{[3]} shows that $\gamma$ is possibly negative, while the above prediction is positive. The contribution of the $\Delta$ resonance is considered to reverse the sign.

We now examine the contributions from the magnetic Born terms for $\gamma + N \rightarrow \pi + N$:

\[
\left( T_M^{(-)} \right)^N = \left( \frac{eG_{NN\pi}}{8\pi M} \right) \left( \frac{\mu_V}{2M} \right) \left\{ -\frac{(\sigma \cdot q)(\sigma \cdot s)}{\omega_k} - \frac{(\sigma \cdot s)(\sigma \cdot q)}{\omega_k} + \frac{1}{2} \left[ 3s \cdot q - (\sigma \cdot q)(\sigma \cdot s) \right] \right\}
\]

\[
\left( T_M^{(+)} \right)^N = \left( \frac{eG_{NN\pi}}{8\pi M} \right) \left( \frac{\mu_V}{2M} \right) \left\{ -\frac{(\sigma \cdot q)(\sigma \cdot s)}{\omega_k} + \frac{(\sigma \cdot s)(\sigma \cdot q)}{\omega_k} - \frac{1}{2} \left[ 3s \cdot q - (\sigma \cdot q)(\sigma \cdot s) \right] \right\}
\]

where $s = \epsilon \times k$, and $\mu_V$ the vector part of the nucleon magnetic moments defined by $(\mu_p - \mu_n)/2$ in units of the nuclear magneton. Note that we introduced the nucleon- and $\Delta$-pole terms, and that we have used the relation $\mu_V^{\Delta N} = -\frac{3}{\sqrt{2}} \mu_V$ in the Skyrme model. We also see that $\left( T_M^{(\pm)} \right)^N$ reduces to $O(N_c^{1/2})$ by the cancellation among the $N$- and $\Delta$-pole terms. The term $\left( T_M^{(0)} \right)^N$ is of $O(N_c^{-1/2})$ and is neglected in the following. We rewrite $\left( T_M^{(\pm)} \right)^N$ as

\[
\left( T_M^{(\pm)} \right)^N = \left( \frac{eG_{NN\pi}}{8\pi M} \right) \left( \frac{\mu_V}{2M} \right) \left\{ t_1^{(\pm)} P_1(q, \hat{s}) + t_3^{(\pm)} P_3(q, \hat{s}) \right\}
\]

where $P_1(q, \hat{s}) = (\sigma \cdot \hat{q})(\sigma \cdot \hat{s})$ and $P_3(q, \hat{s}) = 3(q \cdot \hat{s}) - (\sigma \cdot \hat{q})(\sigma \cdot \hat{s})$ are the $P$-wave projection operators for $J = 1/2$ and $J = 3/2$, respectively, and $\hat{q} = q/q$ and $\hat{s} = s/k$.

We obtain

\[
t_1^{(-)} = -\frac{1}{3M} \frac{\Delta M q}{\omega_k + \Delta M}
\]

\[
t_3^{(-)} = \frac{1}{2M} q\omega_k \left[ \frac{\Delta M}{\omega_k^2 - \Delta M^2 + i\Delta M\Gamma_\Delta} - \frac{2}{3} \frac{\Delta M}{\omega_k(\omega_k + \Delta M)} \right]
\]

\[
t_1^{(+)} = -\frac{2}{3M} \frac{\Delta M q}{\omega_k + \Delta M}
\]

\[
t_3^{(+)} = \frac{1}{2M} q\omega_k \left[ -\frac{2\Delta M}{\omega_k^2 - \Delta M^2 + i\Delta M\Gamma_\Delta} + \frac{2}{3} \frac{\Delta M}{\omega_k(\omega_k + \Delta M)} \right].
\]
Here, we introduced the finite width of the $\Delta$ state by

$$\Gamma_\Delta = \frac{1}{6\pi} \left( \frac{G_{\Delta N\pi}}{2M} \right)^2 q^3$$

(13)

with $G_{\Delta N\pi} = -(3/\sqrt{2})G_{NN\pi}$. This is the expression given by Kokkedee without the relativistic correction.\[9\] It gives 145MeV with the experimental value of $G_{NN\pi}$ at $q = 227$MeV. Then, the contribution from the magnetic part to the difference of the absorption cross section is given by

$$\Delta \sigma^N_M = \frac{8\pi}{\omega_k} \left( \frac{G_{NN\pi}}{4\pi} \right)^2 \left( \frac{e\mu_V}{2M} \right)^2 \left\{ 2 |t_{1}^{(-)}|^2 - |t_{3}^{(-)}|^2 \right\} + \left\{ |t_{1}^{(+)}|^2 - |t_{3}^{(+)}|^2 \right\}$$

(14)

In the narrow width limit this gives for the gamma

$$\gamma^N_M \mid_{\Gamma_\Delta = 0\text{ limit}} = -\left( \frac{e^2}{4\pi} \right) \frac{\mu_V^2}{2M^2} \frac{1}{\Delta M^2}.$$  

(15)

Identifying $b_1 = \mu_{V,\Delta N}/2$ we find that this is just the $\Delta$-pole contribution in the HBChPT.\[5\] $b_1$ is the constant of $O(\epsilon^2)$ counter term in the HBChPT, and numerically about $-2.5 \pm 0.35$, while $\mu_{V,\Delta N}/2$ is $-2.5$ with experimental values for the constants. The calculated $\gamma$ in the narrow width limit is $-4.0 \times 10^{-4}$fm$^4$, while we obtain $-2.5 \times 10^{-4}$fm$^4$ with the finite width. In the previous paper \[1\] the width of the $\Delta$ state in the direct $\Delta$ pole was proportional to $v^3$ with $v$ the pion velocity, and was chosen so as to reproduce the experimental one. However, this is not consistent with the width which appears naturally in the numerator as shown in the second line of eq. (14), so that the narrow width limit does not lead to the $\Delta$-pole term in the HBChPT. When we use the previous expression for the amplitudes we obtain $\gamma^N_M = -6.1 \times 10^{-4}$fm$^4$, which is too large compared with the case of the narrow width limit.

The interference term between the electric and magnetic terms is given by

$$\Delta \sigma^N_{EM} = \frac{q}{\omega_k} \frac{e^2\mu_V C_{NN\pi}^2}{4\pi M^2} \left\{ 2v \text{Re}t_3^{(-)} ight\}$$

$$+ \left[ \frac{1}{v} + \frac{1}{2} \left( 1 - \frac{1}{v^2} \right) \ln \frac{1 + v}{1 - v} \right] \left( \text{Re}t_1^{(-)} - \text{Re}t_3^{(-)} \right) \right\}.$$ 

(16)
The Born terms for the process $\gamma + N \rightarrow \pi + \Delta$ have been also calculated in the previous paper [1]: The amplitude is expanded as

$$T^a = i\epsilon_{abc} T^b T^{(-)} + T^a T^{(0)} + T^{+}_a T^{(+)}; \quad (17)$$

where $T^a$ is the transition isospin matrix from $N$ to $\Delta$, and $T^{+}_a = T^a \frac{1}{2} \gamma^3 + \frac{1}{2} T_{3\Delta} T^a$. The electric part is obtained by replacing $\sigma$ and $G_{NN\pi}$ in eq. (5) by the transition spin operator $S_{\Delta N}$ and $G_{\Delta N\pi}$, respectively. The magnetic part is given by

$$\left( T^{(-)}_M \right)^\Delta = \left( \frac{eG_{\Delta N\pi}}{8\pi M} \right) \left( \frac{\mu_v}{2M} \right) \left\{ -\frac{(S_{\Delta N} \cdot q)(\sigma \cdot s)}{\omega_k} - \frac{4}{5} \frac{(S_{\Delta \Delta} \cdot q)(S_{\Delta N} \cdot s)}{\omega_q} \right. \right.$$  

$$+ 2 \frac{(S_{\Delta N} \cdot q)(\sigma \cdot s)}{\omega_q} - \frac{1}{5} \frac{(S_{\Delta \Delta} \cdot q)(S_{\Delta N} \cdot q)}{\omega_q} \right\}, \quad (18)$$

$$\left( T^{(+)}_M \right)^\Delta = \left( \frac{eG_{\Delta N\pi}}{8\pi M} \right) \left( \frac{\mu_v}{2M} \right) \left\{ -\frac{(S_{\Delta N} \cdot q)(\sigma \cdot s)}{\omega_k} - \frac{1}{5} \frac{(S_{\Delta \Delta} \cdot q)(S_{\Delta N} \cdot s)}{\omega_q} \right. \right.$$  

$$+ \frac{(S_{\Delta N} \cdot q)(\sigma \cdot s)}{\omega_q} + \frac{1}{5} \frac{(S_{\Delta \Delta} \cdot q)(S_{\Delta N} \cdot q)}{\omega_q} \right\}. \quad (19)$$

The cross section for the process is given by the electric and magnetic terms and their interference term:

$$\Delta \sigma^\Delta = \Delta \sigma^\Delta_E + \Delta \sigma^\Delta_M + \Delta \sigma^\Delta_{EM} \quad (20)$$

with

$$\Delta \sigma^\Delta_E = \frac{e^2 G_{NN\pi}^2 v}{4\pi M^2} b \left[ \frac{a - b^2}{b^2} - \frac{a^2 - b^2 v^2}{2b^2 v} \ln \frac{a + bv}{a - bv} \right],$$

$$\Delta \sigma^\Delta_M = \frac{e^2 G_{NN\pi}^2 \mu^2 \Delta M^2 v^3}{24\pi M^4} b,$$

$$\Delta \sigma^\Delta_{EM} = -\frac{e^2 G_{NN\pi}^2 \mu \nu \Delta M^3 v^2}{8\pi M^3 \frac{b^3}{3}} \times \left[ \frac{2bv}{3} - \frac{a^2}{bv} + \frac{a(a^2 - b^2 v^2)}{2b^2 v^2} \ln \frac{a + bv}{a - bv} \right]. \quad (21)$$

where $b = 1 + d\sqrt{1 - v^2}$ and $a = (1 + b^2)/2$ with $d = \Delta M/m_\pi$.

In Table 1, we give numerical results of the spin-dependent polarizability $\gamma$ for parameter sets I, II and III. Set I is that of Adkins. In Set II $f_\pi$ is the experimental one and the Skyrme parameter $e = 4.0$ is chosen for $g_A$ to be reproduced. In Set III all the constants such as $G_{\pi NN}$, $\mu \nu$ in the above are taken to be the empirical values. Here,
Table 1: Spin-dependent polarizability. Those with the suffices $E$, $M$ and $EM$ are from the electric, magnetic and interference terms between the electric and magnetic ones, respectively. The superscripts $N$ and $\Delta$ denote the contributions from the $N + \pi$ and $\Delta + \pi$ channels, respectively. $\gamma^N = \gamma^N_E + \gamma^N_M + \gamma^N_{EM}$, $\gamma^\Delta = \gamma^\Delta_E + \gamma^\Delta_M + \gamma^\Delta_{EM}$, and $\gamma = \gamma^N + \gamma^\Delta$. All values are in units of $10^{-4}$ fm$^4$. See text for the parameter sets.

| Set | $\gamma^N_E$ | $\gamma^N_M$ | $\gamma^N_{EM}$ | $\gamma^N$ | $\gamma^\Delta_E$ | $\gamma^\Delta_M$ | $\gamma^\Delta_{EM}$ | $\gamma^\Delta$ | $\gamma$ |
|-----|--------------|--------------|-----------------|-----------|-------------------|-------------------|-------------------|-----------|-------|
| I   | 3.9          | -1.2         | -1.3            | 1.4       | -0.3              | 0.0               | -0.3              | 1.1       |       |
| II  | 2.9          | -2.8         | -0.8            | -0.7      | 0.0               | 0.0               | -0.3              | -0.9      |       |
| III | 5.0          | -2.5         | -2.4            | 0.2       | -0.4              | 0.1               | 0.0               | -0.3      | -0.1  |

$\gamma = \gamma^N + \gamma^\Delta$ with $\gamma^{N,\Delta} = \gamma^{N,\Delta}_E + \gamma^{N,\Delta}_M + \gamma^{N,\Delta}_{EM}$. The suffices $E$, $M$ and $EM$ denote the contributions from the electric, magnetic terms and their interference terms, respectively.

The result of Set III can be compared with that of the HBChPT. Hemmert et al. showed in ref. [5] that $\gamma$ is given by $[4.5 (N \text{ loop}) - 4.0 (\Delta \text{ pole}) - 0.4 (\Delta \text{ loop})] \times 10^{-4}$ fm$^4$. As already shown, $\gamma^N_E$ is the same as the $N$-loop term in the HBChPT. Note that we did not use the Goldberger-Treiman relation here. $\gamma^N_M$ reduces to the $\Delta$-pole term at the narrow width limit. For the finite width case $\gamma^N$ is close to that sum of the $N$-loop and $\Delta$-pole terms in the HBChPT. The effect of the $\Delta + \pi$ channel is small and is also close to that of the $\Delta$ loop in the HBChPT. In the latter the contribution of magnetic terms is not included, so that the electric term can be compared with that of $\Delta$ loop. We see that the total result is very similar to that of the HBChPT. Numerical results show that the magnetic terms for the $\Delta + \pi$ channel are small but have an opposite sign.

Here, we discuss about the Drell-Hearn-Gerasimov(DHG) sum rule[10] in the Skyrme model. The low-energy limit of the Compton scattering amplitude is given by the nucleon Born terms with the spatial component of the electromagnetic current, and we obtain

$$T_{N\text{-pole}} = \frac{1}{4\pi} \left\{ \frac{\langle N(p)|\epsilon' \cdot J_{em}|N(p+k)\rangle \langle N(p+k)|\epsilon \cdot J_{em}|N(p)\rangle}{-\omega_k} \right. $$

$$+ \left. \frac{\langle N(p)|\epsilon \cdot J_{em}|N(p-k)\rangle \langle N(p-k)|\epsilon' \cdot J_{em}|N(p)\rangle}{\omega_k} \right\}. \quad (22)$$

The spin-dependent part of the $N$-pole terms is then given by

$$T_{N\text{-pole}} = -\frac{e^2 \mu_N^2}{8\pi M_N^2} \omega_k i\sigma \cdot \epsilon' \times \epsilon. \quad (23)$$
where $\mu_N$ is the nucleon magnetic moment in units of nuclear Bohr magneton. Consequently, the unsubtracted dispersion relation is

$$\frac{e^2 \mu_N^2}{8\pi M^2} = \frac{1}{4\pi^2} \int_{\omega_0}^\infty \frac{\sigma_{1/2} - \sigma_{3/2}}{\omega} d\omega. \quad (24)$$

This is different from the DHG sum rule, in which the left-hand side is given by the anomalous magnetic moment instead of the total magnetic one. How to resolve this has been shown by Low [7]: The Born terms in terms of the time-component of the relativistic electromagnetic current removes this discrepancy; however, the effect is highly relativistic and cannot be obtained in a nonrelativistic approach such as in the Skyrme model. The contribution from the electric Born terms is already calculated by L’vov [2] and is given by

$$\frac{e^2 G_{NN\pi}^2}{32\pi^3 M^2}, \quad (25)$$

while those from the magnetic terms are calculated to be in the narrow width approximation for the $\Delta$ state

$$\frac{e^2 \mu_N^2}{8\pi M^2}. \quad (26)$$

Noting that $\mu_N = \mu_S + \tau_3 \mu_V$ with $\mu_S/2M$ and $\mu_V/2M$ to be $O(N_c^{-1})$ and $O(N_c)$, respectively, we see that the left-hand side of eq. (24) is of $O(N_c^2)$, but the right-hand side is of $O(N_c)$, so that there seems to be an inconsistency in the $N_c$ dependence of the DHG sum rule. However, the term of $O(N_c^2)$ in the left-hand side is completely canceled by the contribution from the $\Delta$ resonance as shown in eq. (26), and it turns out that the left-hand side of the sum rule becomes of $O(1)$, but the right-hand side is of $O(N_c)$. It is not clear if the sum rule is down to that of $O(1)$, because of the nonrelativistic approach. A similar situation was also shown in the Adler-Weisberger (AW) relation [11]: The square of the axial-vector constant is of $O(N_c^2)$, but other terms is at most of $O(N_c)$. Therefore, there appears an inconsistency in the $N_c$ dependence of the AW relation, but the contribution of the $\Delta$ states again cancels the term of $O(N_c^2)$ at the narrow width limit. In this case the remaining terms of $O(N_c)$ reduces to be $O(1)$, due to further cancellations among the isospin odd forward scattering amplitudes.

Finally, we calculate the spin-independent part; namely, the electric and magnetic polarizabilities of the nucleon. In ref. [1] the electric polarizability $\bar{\alpha}$ is derived from the absorption cross section by the longitudinal photon, thereby the magnetic one can be obtained by means of the Baldin sum rule. [12] However, it is possible to calculate
Table 2: Electric and magnetic polarizabilities, $\bar{\alpha}$ and $\bar{\beta}$. Notations are the same as those of the spin-dependent polarizability $\gamma$. All values are in units of $10^{-4} \text{fm}^3$. See text for the parameter sets.

| Set | $\bar{\alpha}^N$ | $\bar{\alpha}^\Delta$ | $\bar{\alpha}$ | $\bar{\beta}_E^N$ | $\bar{\beta}_M^N$ | $\bar{\beta}_{EM}^N$ | $\bar{\beta}^N$ | $\bar{\beta}_E^\Delta$ | $\bar{\beta}_M^\Delta$ | $\bar{\beta}_{EM}^\Delta$ | $\bar{\beta}^\Delta$ | $\bar{\beta}$ |
|-----|------------------|-------------------|----------------|-------------------|-------------------|-------------------|----------------|-------------------|-------------------|-------------------|----------------|-----|
| I   | 10.8             | 6.4               | 17.2           | 1.1               | 4.3               | 0.1               | 5.5            | 0.1               | 0.5               | -1.3              | -0.7             | 4.8           |
|     | (5.6)            | (16.4)            | (16.4)         | (16.4)            | (16.4)            | (16.4)            | (16.4)         | (16.4)            | (16.4)            | (16.4)            | (16.4)          | (16.4)        |
| II  | 8.0              | 5.3               | 13.3           | 0.8               | 7.7               | -0.3              | 8.2            | 0.1               | 0.4               | -1.1              | -0.5             | 7.7            |
|     | (4.7)            | (12.7)            | (12.7)         | (12.7)            | (12.7)            | (12.7)            | (12.7)         | (12.7)            | (12.7)            | (12.7)            | (12.7)          | (12.7)        |
| III | 13.9             | 8.2               | 22.1           | 1.4               | 9.0               | 0.3               | 10.7           | 0.1               | 1.4               | -2.4              | -0.9             | 9.8            |
|     | (7.2)            | (21.1)            | (21.1)         | (21.1)            | (21.1)            | (21.1)            | (21.1)         | (21.1)            | (21.1)            | (21.1)            | (21.1)          | (21.1)        |

The difference between two approaches appears only in the case of the $\Delta + \pi$ channel for the final states. Here we give the result directly them using the transverse photon. The difference between two approaches appears only in the case of the $\Delta + \pi$ channel for the final states. Here we give the result with use of the transverse photon. In this calculation we also used the amplitudes for the direct $\Delta$-pole terms to the $N$ + $\pi$ channel given in eq. (12). The magnetic Born term to the $\pi + N$ channel at the narrow width limit is again the same as that of the $\Delta$-pole term in the HBChPT:

$$\beta_M^N |_{r_{\Delta}=0\text{limit}} = \left( \frac{e^2}{4\pi} \right) \frac{\mu_V^2}{M^2} \frac{1}{\Delta M}.$$  \hspace{1cm} (27)

The numerical results for the electric and magnetic polarizabilities are shown in Table 2. The numbers in parentheses are values calculated using the longitudinal photon. Empties show no change for this case. We can see that the results for Set III are very close to those of the HBChPT: Hemmert et al. showed in the calculation up to $O(\epsilon^3)$ that $\bar{\beta}$ is given by $[1.2(N \text{ loop}) + 12(\Delta \text{ pole}) + 1.5(\Delta \text{ loop})] \times 10^{-4} \text{fm}^3$. The effect of the finite width is seen to reduce the value of the $\Delta$-pole term. $\beta_E^\Delta$ is, however, small, but the interference term $\beta_{EM}^\Delta$ is rather large and negative. The large interference term is due to the high-energy behavior of the amplitudes of the magnetic part. This may need further consideration. In the previous paper the calculated results for $\bar{\beta}$ are 7.8 and 21.3 in units of $10^{-4} \text{fm}^3$ for Set I and III, respectively. Therefore, we see that the inconsistent inclusion of the $\Delta$ width leads to too large values for the magnetic polarizability. The effect of the finite width makes the magnetic polarizabilities rather small in the consistent

\[\text{footnote}{5\text{There was an error in the expression and the numerical results in the contribution from the }\Delta + \pi\text{ channel for the magnetic polarizabilities.}^{13}}\]
Inclusion of this paper.

In summary we have calculated the spin polarizability of the nucleon in the Skyrme model, where the pion photo-production Born amplitudes are employed for obtaining the absorption cross section in the dispersion relation. The electric and magnetic Born terms agree with the $N$-loop and the $\Delta$-pole terms of the HBChPT, at the limit of the narrow width of the $\Delta$ state. The electric and magnetic polarizabilities were also calculated using the transverse photon and by the consistent treatment of the $\Delta$ width.
References

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