A TAXONOMY FOR THE CONFIGURATIONS OF QUADRUPLY LENSED QUASARS

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ABSTRACT

A simple, novice-friendly scheme for classifying the image configurations of quadruply lensed quasars is proposed. With only six classes, it is intentionally coarse-grained. It focuses on the kitelikeness and circularity of these configurations, or the absence thereof. Other features are deliberately ignored, their importance to professional astronomers notwithstanding. Readers are invited to test drive the scheme on a sample of 12 quadruply lensed quasar systems. The theoretical underpinnings of the scheme are described in a technical appendix.

Keywords: galaxies: quasars — gravitational lensing: strong

1. INTRODUCTION

Astronomers who study quadruply lensed quasars (henceforth “experts”), can, by visual inspection, reliably discriminate between random quartets of stars and the four images of a single lensed source. The four celestial positions of the images lie in an 8-dimensional space. Random quartets fill that space far more uniformly than do lensed quasar images, which lie in a limited region of an approximately 7-dimensional subspace defined by the “configuration invariant” discovered by Kassiola & Kovner (1995).

Four of the seven dimensions are uninteresting, representing translations, rotations and scalings of otherwise identical configurations. The experts ignore these four, leaving three unitless quantities (angles and ratios of distances) that describe the salient features of the quadruple configurations.

The premise of this paper is that the three dimensional description of quadruple lensed image configurations is sufficiently simple that even novices can quickly master it.

While citizen scientists will doubtless want to develop an understanding of the theory behind the phenomenon, it can be distilled to basic ideas: a) that quadruple image configurations often resemble kites and b) the configurations are sometimes non-circular and other times nearly circular.

In distancing our scheme from theory, we have followed the example of the stellar spectral classification system. Not only is an understanding of effective temperature and surface gravity unneeded, one might argue that the scheme is
useful to professional astronomers precisely because it steps back from theory.

The classification scheme is described in Section 2. In Section 3 we discuss observational features of quadruply lensed systems that we choose to ignore. In Section 4 we explain why non-circularity is more difficult to distinguish than kitelikeness. In Section 5 we invite readers to classify a sample of a dozen systems. Appendix A contains additional material for experts. In Appendix B we give the results of theoretical fits of a singular isothermal elliptical potential to each of the sample systems.

2. THE SCHEME

We ask three questions about the four images, all qualified with the words “more or less”:

Do the four images trace the outline of a kite?

If so, are the four images symmetric around both diagonals?

If so, the system is a “super-kite.”

If not, the system is a “kite.”

If not, the system is an “un-kite.”

Do the four images lie on a circle?

If so, the system is “near-circular.”

If not, the system is “non-circular.”

In Figure 1 we show images obtained with the Hubble Space Telescope (henceforth HST) of...
representatives of each of our six classes. Both conceptually and graphically, “kites” are at the center of our scheme. These are quadruple image configurations that are approximately symmetric around one of their two diagonals.

Kites vary in the degree of asymmetry around the other diagonal, but we ignore this and call all such systems kites. The exception is that we identify as “super-kites” systems that are very nearly symmetric around both diagonals. Systems that are asymmetric around both diagonals are classified as “un-kites.”

Each of these three classes is further split by whether or not the four quasar images lie on a circle, giving “near-circular” and “non-circular” systems.

Prospective users of this scheme are discouraged from thinking too carefully about the distinctions, and are encouraged, instead, to choose whichever class seems “more nearly” to describe a particular quadruply lensed quasar configuration.

Users of the scheme are not expected to measure positions and fit theoretical models to the positions of the four images. We nonetheless show ellipses in Figure 1 that result from fitting our preferred theoretical model, the singular isothermal elliptical potential (henceforth SIEP; described further in Appendix A) to show that non-circularity can, for this model, be precisely quantified.

3. FEATURES TO IGNORE

Our three dimensional scheme ignores observable features of gravitationally lensed quasars that are nonetheless of considerable astronomical interest. The physics that governs these does not affect image positions and can be quite complicated. Expert users will want to take note of these, but for the purposes of our scheme, they are distractions.

3.1. Ignore the fluxes

We deliberately ignore the relative fluxes of the quasar images in our taxonomy as they can undergo microlensing fluctuations of two magnitudes or more (Weisenbach et al. 2021). But they can and should be taken into account in deciding whether a quartet of images is a lensed quasar. Close pairs of lensed quasar images tend to be brighter than those that are more widely separated, more so in near-circular systems (Falor & Schechter 2022), as in the upper right panel of Figure 1.

![Figure 2. An HST image of the lensed quasar system PS J0147+46 with a circle drawn through the three closest images, as described in Section 4.2. The fourth image lies well inside the circle, demonstrating the strong non-circularity of the system. In their Figure 1a, Luhtaru et al. (2021) show an ellipse passing through all four quasar images.](image)

3.2. Ignore the position of the lensing galaxy

Our classification scheme does not take account of the position of the lensing galaxy. It is often not seen in discovery images, both because it tends to be fainter than the quasar images, and because it is often crowded by them. A second reason for ignoring the lensing galaxy is that it has little effect on the image positions that drive our classification scheme.
Figure 3. HST exposures of a dozen arbitrarily selected quadruply lensed quasars. The reader is invited to classify them using the precepts of Section 1. The lensing galaxy is always visible at the center of the field. The faint background galaxies in panels A and H should be disregarded. Likewise ignore the two faint starlike images in panel I and a faint charged particle detection in panel D. The four brightest images are those of the quadruply lensed quasar. In panels D, E and G, the galaxy that hosts the quasar can be seen forming something of a ring.

In Appendix A we take the SIEP as our preferred model, but there is a second model, the singular isothermal sphere with external shear (henceforth SIS+XS) that predicts exactly the same image positions. The two models predict different positions for the galaxy (Luhtaru et al. 2021), but our classification scheme is well suited to both.

4. PRECISE ESTIMATES OF ELLIPTICITY VERSUS CRUDE GUESSTIMATES OF NON-CIRCULARITY

4.1. The problem

The ellipses shown in Figure 1 were derived by fitting our preferred theoretical model, the SIEP to the positions of the four images. They are characterized by a precise “ellipticity” obtained from the ratio, $q$, of the short symmetry axis to the long symmetry axis.

Unfortunately, four image positions do not yield a unique ellipse without additionally specifying the direction of the gravitational potential’s symmetry axis. This is illustrated in Figure 2 of Wucknitz (2002), where the ellipses drawn through each configuration have a range of orientations and a range of ellipticities.

How then is the classifier expected to determine non-circularity? Crudely!

Our scheme does not require precise quantification of non-circularity, but only the binary choice between near-circular and non-circular. In the author’s experience, a quartet of lensed
images for which the SIEP model has axis ratio $0.9 < q < 1$ looks quite circular, while a quartet for which $q < 0.85$ looks very non-circular.

Using the letters $C$ and $N$ to designate near-circular and non-circular, one might use $C?$ and $N?$ for systems about which one has doubts. The classifier is urged to choose between the two, the lack of certainty notwithstanding.

4.2. Relief for the ambiguity intolerant

Though we stipulate that the classifier choose between circular and non-circular, we recognize some will want to eliminate all ambiguity. One can do this using high school geometry.

It takes only three points to draw a circle (with the center where the bisectors of the chords intersect). If one draws a circle through three images of a quadruple configuration, the fourth image will lie close to that circle if the system is near-circular. This works best if one chooses the three closest images.

Figure 2 shows this construction for the system PS J0147+46, one of the more non-circular lenses known. A circle is drawn through the three closest images. The fourth image lies well inside that circle.

5. TEST DRIVE

We invite readers to decide for themselves whether the present scheme might be useful. In Figure 3 we show an arbitrary selection of 12 HST images of quadruply lensed quasars. We believe that even novices will find it easy to decide upon the kitelikeness of these systems.

As discussed in the previous section, distinguishing between near-circular and non-circular systems is more challenging. Non-circularity is most easily recognized in the doubly symmetric super-kites, and also fairly obvious in at least some kites, again by virtue of their symmetry. It is least obvious in the doubly asymmetric unkites.

We urge readers to make a best effort to classify all 12 systems. Additional material presented in Appendix B may help in rectifying first timers’ mistakes.

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APPENDIX

A. NOTES FOR EXPERTS

The taxonomy presented here was inspired by the work of Falor & Schechter (2022) who studied the quadruple image configurations of what they call the “asymptotically circular lens.” The configurations are determined by the position of the center of the circle within a true astroid, which marks the transition from four images to two. The image positions perfectly satisfy the configuration invariant discovered by Kassiola & Kovner (1995). They showed that the invariant is very nearly satisfied by most known quadruply lensed quasars.

Our three kite classes correspond to having the center of the circle be a) close to the center of the astroid, b) close to one of its two axes or c) close to neither axis.
Figure 4. For each of the systems in our test sample, orange circles have been drawn through the three closest images. The fourth image in systems D and E, both un-kites, lie very close to the circle. By contrast, in system H, also an un-kite, the fourth image lies very far from the circle. By coincidence, a charged particle detection lies on the circle of system D.

The singular isothermal elliptical potential (SIEP), the singular isothermal sphere with external shear (SIEP+XS), and the singular isothermal elliptical potential with parallel shear (SIEP+XS\parallel) all produce elliptical configurations that are “scronched” (Ellenberg 2021) versions of the asymptotically circular configurations, adding a third dimension and giving us the distinction between nearly-circular and non-circular systems.

The simplest scheme for determining the orientation of the lensing potential on the sky involves solving for Witt’s hyperbola (Witt 1996), which in turn requires solving a system of 3 linear equations in 3 unknowns. As this may be out of reach for many citizen scientists, we describe in Section 4.2 a simpler, ad-hoc construction to quantify the deviation from non-circularity.

We do not refer to “crosses”, “folds” and “cusps” for two reasons. First, we have avoided jargon where possible. Second, these are limiting cases of our three, more broadly defined kite classes.

We likewise do not refer to the classification scheme developed by (Saha & Williams 2006). Their scheme does not explicitly allow for the elongation of image configurations. But it does make a distinction between what they call long-axis quads and short-axis quads, indicating the axis of the potential on which the source lies. We would call these wide and narrow kites, respectively, as source position is a theoretical concept that we have avoided mentioning. The distinction between the two
is very small except for the most non-circular systems. We have therefore chosen not to add this distinction to our scheme.

Our taxonomy is built on two commonly used and widely understood words – the noun kite and the adjective circular. A system is then described as a prefix-adjective prefix-noun. While the word “rhombus” might be mathematically more rigorous, we use super-kite to emphasize that it is the degree of kitelikeness that distinguishes our three classes.

### B. DEVIATIONS FROM CIRCULARITY

As discussed in Section 4, ellipses with a range of orientations and axis ratios can be drawn through four images that are not perfectly symmetric. We described a construction that permits a quantitative estimate of non-circularity.

While our hope is that novices will quickly develop confidence in their ability to make a crude distinction between nearly circular and non-circular systems, this construction may prove useful to first time users who do not yet trust their own judgement. We have carried this out for each of the 12 systems in our test sample, with the resulting circles shown in Figure 4.

Table 1 reports ellipticities, defined as $\epsilon \equiv 1 - q$, where $q$ is the axis ratio. They were obtained from fits of theoretical SIEP models to the test sample.

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