Comment on Inflation and Alternative Cosmology

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Abstract

We respond to, and comment upon, a number of points raised in a recent paper by Kofman, Linde, and Mukhanov.

1 Introduction

In a recent paper [1], we argued that inflation does not provide a satisfactory explanation of the “special state” (i.e., homogeneity, isotropy, and flatness) of our universe. We then noted that the fundamental mechanism by which inflationary models produce cosmologically significant departures from homogeneity and isotropy involves the evolutionary behavior of modes whose proper wavelength is larger than the Hubble radius. Since this evolutionary behavior will occur whether or not inflation took place, we raised the possibility that one could account for the departures from homogeneity and isotropy in our universe via the same basic mechanism as in inflationary models, but without postulating that the universe actually underwent an era of inflation. We then identified a set of assumptions concerning initial conditions that would be sufficient to produce a perturbation spectrum of the same nature as that of inflationary models. Finally, we provided an explicit model in which these assumptions were made, and we carried out the calculations to show that a perturbation spectrum similar to that of inflationary models does, indeed, result.

Our arguments and conclusions have recently been criticized on a variety of grounds by Kofman, Linde, and Mukhanov [2]. The main purpose of the present paper is to respond to these criticisms and to add some general comments on irreversibility and on the use of probabilistic arguments.

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2 Pre-Planckian Initial Conditions

Inflationary models have the feature that, at sufficiently early times, the modes of cosmological interest had proper wavelength much smaller than the Hubble radius. Therefore, it seems natural to postulate that these modes were “born” in their ground state. In addition, on account of adiabatic evolution during the era when their wavelength is much smaller than the Hubble radius, the precise time at which they were “born” should not be important. In fact, upon closer inspection, these assumptions are seen to be far from obvious: In inflationary models, the modes of cosmological interest actually had wavelength very much smaller than the Planck scale at the onset of inflation. Therefore, if sub-Planckian physics deviates substantially from extrapolations of physics at scales larger than the Planck scale, the validity of inflationary models would be in doubt1. Nevertheless, inflationary models provide a simple “default assumption” concerning the initial state of modes of cosmological interest.

By contrast, in non-inflationary models, the modes of cosmological interest have proper wavelength much larger than the Hubble radius throughout the entire early evolutionary history of the universe. Therefore, one must directly confront the issue of the initial state of such modes. There is no theory presently capable of “predicting” the initial state of these modes, so one can only make hypotheses and assumptions. In [1], we put forward the hypothesis that a semiclassical description of physics may be possible at arbitrarily early times on length scales larger than some fundamental scale $l_0$. As we explained in footnote 7 of [1], we do not suggest that an accurate semiclassical description would be obtained by a naive extrapolation of a semiclassical solution to Einstein’s equation to times earlier than the Planck time, $t_P$. Rather, we proposed that some suitable “coarse graining” of the degrees of freedom of quantum gravity over length scales smaller than $l_0$ could yield an accurate (but, as yet, unknown to us) semiclassical description of nature. We further proposed that when modes of the quantum field “emerge from the spacetime foam” at lengthscale $l_0$, they do so in their ground state.

Our above assumptions concerning the initial state of the cosmologically relevant modes was criticized in [2] on the grounds that they do not follow from quantum field theory in curved spacetime. This criticism is, of course, entirely valid, since as already noted above, at the present time there does not exist any theory that is capable of predicting the initial state of these modes. Our assumptions were also criticized in [2] on the grounds that our assumed initial state depends only on $l_0$ whereas there are other scales present—in particular, the Hubble radius—that are dynamically more important in the evolution of these modes. This criticism is a more significant one. The only scale that directly enters the Lagrangian of a massless mode (see eq. (4) of [1]) is the proper wavelength of that mode, which by our assumptions is equal to $l_0$ at the semiclassical “birth” of the mode. Thus, we feel that our assumption that the modes are “born” in their ground state is not entirely unreasonable or unnatural—at least as an initial hypothesis. Nevertheless, we have no argument that other

1However, investigations of some simple modifications of sub-Planckian physics indicates that inflationary models are not very sensitive to such modifications [3].
dynamically relevant scales could not enter their initial state. Clearly, the general issue of when modes can be treated semiclassically and in what state they are “born” is a deep issue, about which very little is known at present.

In [1], we also presented an explicit model—involving quantized sound waves of a perfect fluid—in which a perturbation spectrum was obtained that is of the same nature as occurs in inflationary models. As we stated very explicitly in the paper, we do not expect this model to be a realistic description of nature in the very early universe (particularly at times earlier than the Planck time). Rather, the purpose of presenting this model was to simply to illustrate how the “overdamped” evolution of modes with wavelength larger than the Hubble radius could produce cosmologically relevant perturbations in a context that is very different from inflationary models—thereby providing an “existence proof” of alternative models to inflationary ones. This model was sharply criticized in [2] for its unrealistic properties. We, of course, agree that the model we presented is not realistic, but this does not mean that there could not exist more realistic models that share its basic features.

The criticisms of [2] very nicely emphasize the point (which we also made in our paper) that in non-inflationary models, the time at which the cosmologically relevant modes have proper wavelength equal to $l_0$ occurs at times much earlier than the Planck time (or, more precisely, in a region of spacetime corresponding to times much earlier than the Planck time in naive extrapolations of classical or semiclassical models). Consequently, it is not likely that any reliable calculations can be done for non-inflationary models until our understanding of quantum gravitational physics improves significantly. However, we strongly disagree with the assertion of [2] that this fact provides an additional argument in favor of inflation. More precisely, it provides an argument in favor of inflation only in the sense that the (true) statement “If you are going to find your keys tonight, then you must have dropped them under the lamppost” provides an argument that you actually dropped your keys under the lamppost. We believe that the keys to understanding the origin of structure in our universe are likely to lie in the pre-Planckian era.

3 Irreversible Processes and the Second Law of Thermodynamics

It is clear that the present state of our universe is very “special” in the sense that its entropy is very much less than that of corresponding universes that are similar on large (i.e., Hubble) scales but are much more gravitationally clumped on small (e.g., galactic) scales. It is this “specialness” of our present universe—i.e., the fact that its entropy is very far from its maximum possible value—that gives rise to the second law of thermodynamics. In [1], we argued—following arguments previously given in [4]—that rather than seeking to use the second law of thermodynamics or other dynamical arguments to explain how the universe arrived at its current state starting from arbitrary initial conditions, we should be seeking to use the (as yet to be developed) theory of initial conditions of the universe to explain how the second law of thermodynamics came into being. We argued further that inflationary
models do not avoid the necessity of assuming “special” initial conditions for our universe. Indeed, we argued that if an expanding universe generically undergoes an era of inflation, then—unless one introduces assumptions that break time reversal invariance—a collapsing universe must generically undergo an era of “deflation”. We view this result as a reductio ad absurdum for the claim that special initial conditions are not needed for inflation to occur.

Our arguments and conclusions were criticized in [2] on a number of grounds. In particular, the simple model of a homogeneous scalar field in a flat Robertson-Walker universe was analyzed, and it was claimed that—in contradiction with our conclusions—in this model, inflation is an “attractor” whereas deflation is not. The analysis of this model given in [2] relies heavily on assumptions concerning the probability measure on initial conditions, so we will defer our discussion of this model to the next section. In the present section, we will comment on some of the other arguments concerning irreversibility given in [2].

We agree with the claims of [2] that many “irreversible processes” have occurred in the evolutionary history of our universe; indeed, the fact that irreversible processes have occurred is the main basis of our claim that the initial entropy of our universe was very low, i.e., that the universe began in a very “special” initial state. We also agree with their assertion that if one time-reversed [a very slight modification of] the initial conditions representing the present state of our universe, the subsequent evolution would not produce anything similar to the (time reverse of) the initial conditions that our universe started with. Indeed, the initial state of our universe appears to have been quite “smooth”, whereas the generic final state of a collapsing universe would be expected to be extremely “messy”, with numerous black holes forming and merging, etc. In particular, we certainly would not expect an era of deflation to occur just prior to the “big crunch”. But this is just our point: Why should not the initial state of the universe have been correspondingly messy? Why should there not have been many regions of “delayed big bang singularities” (i.e., white holes) filling the early universe, and, indeed, filling the present universe as well? We believe that the answer to these questions is that the initial state of the universe was very “special”.

We strongly disagree with the claim in [2] that particle production changes the number of degrees of freedom of a system\(^2\). Fundamentally, in quantum field theory the degrees of freedom reside in the field and are present whether or not the modes of the field are in their ground state or in excited states. The process of particle creation in the early universe is irreversible only in the same sense as the breaking of an egg, but not in any more fundamental sense. Contrary to the claim in [2], such “irreversible” dynamics in classical statistical physics is associated with a measure preserving flow on phase space.

\(^2\)We note however that, while particle production does not change the number of degrees of freedom, the expansion of the universe does produce an effective change in the number of semiclassical degrees of freedom in the universe, since new semiclassical modes presumably “emerge from the spacetime foam” as the universe expands. In both inflationary models and in our proposal, these modes are assumed to emerge in their ground state and, hence, their emergence is not associated with particle creation. However, it is possible that this phenomenon may play a key role in accounting for the thermodynamic arrow of time.
4 The Probability of Inflation

We begin this section with two completely general remarks concerning the use of probabilistic arguments. First, probabilistic arguments can be used reliably when one completely understands both the nature of the underlying dynamics of the system and the source of its "randomness". Thus, for example, probabilistic arguments are very successful in predicting the (likely) outcomes of a series of coin tosses. Conversely, probabilistic arguments are notoriously unreliable when one does not understand the underlying nature of the system and/or the source of its randomness. For example, if asked to estimate the conditional probability that if cows were to be discovered on a distant planet, then their color would be green, a physicist might proceed by taking the frequency bandwidth of the green part of the visible spectrum and dividing this by the bandwidth of the entire visible spectrum. By contrast, a chemist might take the number of common chemical compounds that are green and then divide this quantity by the total number of common chemical compounds. On the other hand, a biologist might first estimate the likelihood of the color of the planet itself by methods similar to that of the chemist, and then take camouflage and other survival factors into account to estimate the probability of the cow being green. Although each of these methods of estimation is arguably reasonable, it is not likely that any of them is reliable, particularly if the planet and the cows living on it are very different from anything we have experienced.

The second comment concerns the general situation where one has a manifold $M$ representing the possible states of a system, on which there is defined a measure $\mu$ such that $\mu(M) = \infty$. Consider a property, $Q$, of the system that corresponds to a (measureable) subset, $S$, of $M$. Suppose that we wish to know the probability, $p(Q)$, that property $Q$ holds. Then there are precisely three possibilities:

- $\mu(S) < \infty$. In this case, $p(Q) = 0$.
- $\mu(M - S) < \infty$. In this case $p(Q) = 1$.
- $\mu(S) = \infty$ and $\mu(M - S) = \infty$. In this case $p(Q)$ is undefined.

In the last case, one might be tempted to define $p(Q)$ by considering a nested family of sets, $K_n$, of finite measure whose union is $M$, and considering the limit as $n \to \infty$ of $\mu(S \cap K_n) / \mu(K_n)$. In practice, this might be done by introducing coordinates on $M$ and taking $K_n$ to be the coordinate ball of radius $n$. However, it is easy to see that one can get any answer for $p(Q)$ that one wishes by making a suitable choice of the sets $K_n$ or, equivalently, by a suitable choice of coordinates on $M$.

In [2], the model of a homogeneous scalar field in a spatially flat ($k = 0$) Robertson-Walker universe was considered, and it was claimed that this model provides a counterexample to our claim that the probability of inflation should equal the probability of deflation. In fact, this model—including its generalization to the $k = \pm 1$ cases—was previously analyzed in detail by Hawking and Page [5]. The $k = +1$ case is particularly relevant, since in this case all (or essentially all) solutions should start from a "big bang" singularity and end in a "big crunch" singularity, so the probabilities of inflation and deflation can be compared.
The model of [2] and [5] is a constrained Hamiltonian system on a 4-dimensional phase space, $M$. We choose as coordinates on $M$ the Robertson-Walker scale factor, $a$, the value of the scalar field, $\phi$, and the canonically conjugate momenta, $p_a = -a \dot{a}$ and $p_{\phi} = a^3 \dot{\phi}$, respectively, where the overdot denotes the derivative with respect to proper time. The Hamiltonian of this system is given by

$$
H = \frac{1}{2}(-a^{-1} p_a^2 + a^{-3} p_{\phi}^2 - ka + m^2 a^3 \phi^2)
$$

(1)

and the constraint hypersurface, $C$, is given by $H = 0$.

As is well known, the Liouville measure, defined by the volume element

$$
\epsilon^{(4)} = dp_a \wedge da \wedge dp_{\phi} \wedge d\phi,
$$

(2)

is invariant under dynamical evolution. It seems to be much less widely known that this measure induces a natural invariant measure, $\mu$, on the constraint hypersurface, $C$. This measure is given by the volume element $(3) \epsilon$ on $C$ that is determined by the condition that on $C$ we have

$$
dH \wedge (3) \epsilon = \epsilon^{(4)}
$$

(3)

(see, e.g., section 7 of [3]). Consequently, one way of defining the “probability of inflation”, $p(I)$, that appears to be very natural at least from a mathematical point of view would be to set $p(I) = \mu(I)/\mu(C)$, where $I$ denotes the region of $C$ that is occupied by dynamical orbits that undergo an era of inflation. It is obvious that with this definition, we have $p(I) = p(D)$, where $p(D)$ denotes the similarly defined probability of deflation. However, by an analysis similar to that given in [3], it is not difficult to show that $\mu(C) = \infty$, $\mu(I) = \infty$, and $\mu(C - I) = \infty$, so, by our general remarks above, the probabilities of inflation and deflation are not defined.

Alternatively, following [3], one could choose an initial data surface, $\Sigma$ on $C$—i.e., a two-dimensional surface on $C$ that is intersected transversely by all (or almost all) of the dynamical orbits on $C$—and try to define a measure directly on $\Sigma$. As noted in [3], the pullback of the symplectic form, $\Omega = dp_a \wedge da + dp_{\phi} \wedge d\phi$, to $\Sigma$ yields a mathematically natural volume element on $\Sigma$ that is invariant under dynamical evolution. One could then set $p'(I) = \mu'(I)/\mu'(\Sigma)$ where $\mu'$ denotes the measure on $\Sigma$ associated with the pullback of $\Omega$ to $\Sigma$, and $I$ now denotes the region of $\Sigma$ corresponding to initial data for inflationary solutions. In contrast to the probability $p(I)$ defined in the previous paragraph, the probability $p'(I)$ will depend upon the choice of initial data surface $\Sigma$—although it will be invariant under dynamical evolution of $\Sigma$. For a choice of initial data surface of the form $f(\phi) =$ const., it was shown in [3] that $\mu'(\Sigma) = \infty$, $\mu'(I) = \infty$, and $\mu'(\Sigma - I) = \infty$. Thus, for the measure $\mu'$, as with the measure $\mu$ above, the probability of inflation is not defined. Indeed, section 5 of [3] provides a nice illustration of the above general point that if one attempts to define $p'(I)$ by a limiting procedure, the answer one obtains will depend upon how the limit is taken.

Alternatively, following [2], if one restricts attention to the case $k = 0$, then—after taking the constraint $\mathcal{H} = 0$ into account—the dynamical evolution equations for $\phi$ can be expressed
in terms of $\phi$ and $\dot{\phi}$ alone, i.e., they do not depend upon $a$. One may then (arbitrarily) ignore the coordinate $a$ on the constraint hypersurface $C$ and work in a new two dimensional space $\tilde{C}$ parametrized by $\phi$ and $\dot{\phi}$. (This cannot be done for the cases $k = \pm 1$ since the dynamics of $\phi$ in these cases depends on $a$, so this procedure is very special to the case $k = 0$.) One may then (arbitrarily) define a Euclidean metric, $g$, on $\tilde{C}$ by treating $\phi$ and $\dot{\phi}$ as though they were Cartesian coordinates. If one then chooses an initial data surface $\tilde{\Sigma}$ on $\tilde{C}$, this Euclidean metric $g$ will induce a Riemannian metric $h$ on $\tilde{\Sigma}$ and, thereby, a measure, $\tilde{\mu}$, on $\tilde{\Sigma}$. This measure will not be preserved under dynamical evolution. One could then set $\tilde{p}(I) = \tilde{\mu}(I)/\tilde{\mu}(\tilde{\Sigma})$ where $I$ now denotes the region of $\tilde{\Sigma}$ corresponding to initial data for inflationary solutions. This is precisely what was done in [2], with the initial data surface chosen to be $H = l_p^{-1}$, where $H = \dot{a}/a$ denotes the Hubble constant (which is related to $\dot{\phi}$ and $\phi$ by the constraint). In this case, $\tilde{\mu}(\tilde{\Sigma}) < \infty$, so a well defined result for $\tilde{p}(I)$ is obtained. The numerical investigations of [2] establish that $\tilde{p}(I)$ is nearly 1 if $m \ll m_p$ (with most of the initial data giving rise to inflation satisfying $\phi \gg m_p$). However, since the measure used in [2] is not preserved under dynamical evolution, a different answer will be obtained if one uses the same procedure to calculate probabilities on a time evolved initial data surface. It was shown in [2] that a very small value would be obtained for the probability, $\tilde{p}(I)$, that inflation occurred in the early universe if one were to do the corresponding calculations using a late time initial data surface. This was interpreted in [2] as showing that the probability for deflation is small.

Clearly, many other possible choices of measure could be made and, correspondingly, many other conclusions of all varieties could be drawn. Indeed, the situation becomes even more murky when one considers more general, inhomogeneous models. In our view, the probability of inflation cannot be reliably estimated until one has some understanding of the physical processes that determine the initial state of our universe.

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