Skewness of probability density functions of fluid particle acceleration in developed turbulence

A.K. Aringazin

Department of Theoretical Physics, Institute for Basic Research,
Eurasian National University, Astana 473021 Kazakhstan

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Within the framework of one-dimensional Laval-Dubrulle-Nazarenko type model for the Lagrangian acceleration in developed turbulence studied in the work [A.K. Aringazin and M.I. Mazhitov, cond-mat/0305186], we focus on the effect of correlation between the multiplicative noise and the additive one which models the relationship between the stretching and vorticity, and can be seen as a skewness of the probability density function of some acceleration component. The skewness of the acceleration distribution in the laboratory frame of reference should be zero in the ideal case of statistically homogeneous and isotropic developed turbulent flows but when considering acceleration component aligned to fluid particle trajectories it is of much importance in understanding of the cascade picture in the three-dimensional turbulence related to Kolmogorov four-fifths law. We illustrate the effect of nonzero cross correlation parameter \( \lambda \). With \( \lambda = -0.005 \) the transverse \( \langle x \rangle \) acceleration probability density function turns out to be in good agreement with the recent experimental data by Mordant, Crawford, and Bodenschatz. In the Random Intensity of Noise (RIN) approach, we study the conditional probability density function and conditional mean acceleration assuming the additive noise intensity to be dependent on velocity fluctuations.

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I. INTRODUCTION

Tsallis nonextensive statistics [1] inspired approach [2] was recently used [3, 4] to describe Lagrangian acceleration of fluid particle in developed turbulence; see also [5, 6]. In Ref. [7] we reviewed some refinements of this approach [8, 9].

Review and critical analysis of the applications of various recent nonextensive statistics based models to the turbulence have been made by Gotoh and Kraichnan [10]. An emphasis was made that some models lack justification of a fit from turbulence dynamics although being able to reproduce experimental data to more or less accuracy. A deductive support from the three-dimensional Navier-Stokes equation was stressed to be essential for the fitting procedure to be considered meaningful.

Recently Laval, Dubrulle, and Nazarenko [11] have developed a stochastic kind of Batchelor-Proudman rapid distortion theory approach to the three-dimensional Navier-Stokes equation using separation of large-scale and small-scale velocities and Gabor transformation (localized wave-packets) to derive one-dimensional Langevin toy model for small-scale velocity increments both in the equally interesting Eulerian and Lagrangian frames; see also recent paper [12]. The large-scale terms entering the resulting approximate small-scale equation are related to large-scale strain and inter-scale coupling and are treated as noises with a given statistics. The small scales are stochastically distorted in certain way as a combined effect of the large scales and the inter-scale coupling. Short-time correlated character of the distortions follows from the numerical study of decaying turbulence; long-time correlations and dependence on velocities which may be present here as well have not been modeled in the first step. This approach allows one to account for nonlocal interaction effects in small-scale turbulence via a simple random multiplicative process driven by coupled Gaussian white-in-time multiplicative and additive noises, while local small-scale interactions are modeled by a turbulent viscosity.

In a comparative analysis of some recent one-dimensional Langevin toy models of fluid particle acceleration in developed three-dimensional turbulence [13] we have demonstrated that the one-dimensional Laval-Dubrulle-Nazarenko (LDN) type model [14, 15], with the model turbulent viscosity \( \nu \) and delta-correlated Gaussian white multiplicative and additive noises, formulated for the Lagrangian acceleration meets the experimental data on acceleration statistics [16, 17] to a good accuracy. Particularly, it was shown that the resulting contribution to fourth order moment, \( a^4P(a) \), does peak at the same values as the experimental curve, in contrast to predictions of the most of other stochastic models [5, 6, 12].

Also, within the framework of Random Intensity of Noise (RIN) approach [18] to the LDN type model the assumption that the additive noise intensity \( \alpha \) depends on absolute value of velocity fluctuations \( u \) was found to imply the conditional probability density function \( P(u|u) \) and the conditional acceleration variance \( \langle a^2|u \rangle \) which are in a good qualitative agreement with the recent experimental data on the conditional acceleration statistics reported by Mordant, Crawford, and Bodenschatz [19]. These results have been obtained in the particular case

*Electronic address: aringazin@mail.kz. Also at Department of Mechanics and Mathematics, Kazakhstan Division, Moscow State University, Moscow 119899, Russia.
when the correlation between the multiplicative and additive noises is taken to be zero.

In the present paper, we fill the gap by studying the effect of nonzero cross correlation of the noises that models a relationship between stretching and vorticity in the three-dimensional case\footnote{1} and can be seen as a skewness of the probability density function of acceleration component.

We remind that shell models of turbulence are incapable to describe the observed skewness generation along the scale of the probability density function of the longitudinal Eulerian velocity increments\footnote{1}. This skewness is of much importance in understanding of the Richardson-Kolmogorov cascade picture of the homogeneous isotropic turbulence. The skewness is particularly related to a non-zero value of the Eulerian third-order velocity structure function, and therefore to the essence of the turbulent cascade via the Kolmogorov four-fifths law\footnote{1, 20, 21}.

Intermittency of homogeneous and isotropic turbulence is usually characterized by a nonlinear dependence of the scaling exponents on the order $n$ of moment, and is well established experimentally both in the Eulerian and Lagrangian frameworks for approximately homogeneous and isotropic flows.

The Eulerian and Lagrangian anomalous scalings trace back to a local inhomogeneity of the flow and long-time correlations in the particle accelerations respectively\footnote{22}. The probability density functions of the Eulerian and Lagrangian velocity increments both exhibit the same behavior: they are approximately Gaussian at large scales and progressively develop stretched exponential tails when the spatial and time increments decrease down to the Kolmogorov length and time, respectively. In the limit of zero time increments the probability density function of the Lagrangian velocity increments converges to that of the Lagrangian acceleration. In practice, one uses nonzero time increments $\tau$ lying within the range of strong viscous dissipation, such that velocities are smoothed and the characteristic relation $u(t+\tau) - u(t) = \tau a(t)$ holds to a good accuracy. This time scale is known to be about or less than the Kolmogorov time.

In the Eulerian framework, the relative scaling exponents of the absolute moments of the longitudinal Eulerian velocity increments, $\langle |u(x+r) - u(x)|^n \rangle$, were measured to be $\zeta_n^E = 0.36, 0.70, 1.28, 1.53$ for $n = 1, 2, 4, 5$\footnote{23}, and the third-order Eulerian velocity structure function, taken as a reference, is known to scale linearly with the time increment $\tau$ due to the dimensional analysis. One therefore expects skewness of the probability density function of the Lagrangian velocity increments in time since $\zeta_n^L$ is nonzero, and hence that of the Lagrangian acceleration.

The one-dimensional LDN toy model was formulated originally in both the Eulerian and Lagrangian frameworks for velocity increments in the frame comoving with the wavepacket\footnote{1}. We use the Lagrangian formulation and the exact result for probability density function for LDN type model obtained as a stationary solution of the Fokker-Planck equation associated to the one-dimensional Langevin equation for the acceleration component\footnote{10, 17},

$$\partial_t a = (\xi - \nu k^2) a + \sigma_\perp. \quad (1)$$

Here, the Gaussian white noises $\xi$ and $\sigma_\perp$ model stochastic forces in the Lagrangian frame and are defined by

$$\langle \xi(t) \rangle = 0, \quad \langle \xi(t) \xi(t') \rangle = 2D \delta(t-t'),$$
$$\langle \sigma_\perp(t) \rangle = 0, \quad \langle \sigma_\perp(t) \sigma_\perp(t') \rangle = 2\alpha \delta(t-t'),$$
$$\langle \xi(t) \sigma_\perp(t') \rangle = 2\lambda \delta(t-t'). \quad (2)$$

All the noises are treated along a particle trajectory. Here, the free parameters $\alpha, D, \lambda$ measure intensities of the noises and their cross correlation, respectively.

Such a choice of noises is motivated not only by simplicity of their statistics but also by the DNS of decaying turbulence\footnote{17}.

Similar problem within the Eulerian framework has been recently investigated, with the result that the noise entering Langevin type equation can be safely taken delta-correlated in scale. Namely, the Eulerian experimental study of Langevin modeling of velocity increments by Renner, Friedrich and Peinke\footnote{24} and Marcq and Naert\footnote{20} reveals that the longitudinal velocity is correlated over distances much larger than the correlation length of its spatial derivative, so that the Markovian approximation is accurate in the inertial range. Approximation of a short-correlated noise by the delta-correlated one is usually made due to the scale hierarchy validating the use of Langevin type equations. It is natural to map this result to the Lagrangian domain.

In the Eulerian LDN framework, the cross correlation parameter was found to control third-order longitudinal velocity structure function due to a kind of generalized Karman-Hovarth relationship derived in Ref.\footnote{17}. Since the noise distributions are taken to be not skewed, this suggests that the parameter $\lambda$ defined in Eq.\footnote{17} should be nonzero.

Experiments\footnote{21} show that the distribution of the Eulerian longitudinal velocity increments is slightly asymmetric in the inertial range of scales, with the skewness factor being about $S = -0.25$ for the studied $R_\lambda = 430$. 

\[\zeta_n^L = 0.56+0.01, \quad 1.34+0.02, \quad 1.56+0.06, \quad 1.8+0.2 \text{ for } n = 1, 3, 4, 5.\]
flow; the odd-order moments \((2n + 1)\) are known to be small as compared with even-order ones \((2n + 2)\). The Langevin model of Ref. 20 assumes the use of only one noise driving Eulerian velocity increments across scales, and this noise is characterized by a slightly skewed distribution (the skewness factor \(S = 0.55\)) and long tails (the flatness factor \(F = 8.5\) as compared to \(F = 3\) for a Gaussian). Gaussian approximation for this noise was used to derive the Fokker-Planck equation. It was found that this approximation and accounting for corrections coming from non-Gaussianity of the noise imply some persistent deviations from the observed scaling of third-order and fifth-order velocity structure functions. This may indicate that some different type of Langevin or Fokker-Planck equation should be used as ansatz.

The probability density function as a stationary solution of the Fokker-Planck equation associated to Eqs. (1) and (2) was calculated exactly 11,

\[
P(a) = \frac{C \exp[-\nu_k k^2]/D + F(c) + F(-c)]}{(Da^2 - 2\alpha + \alpha)^{1/2}(2Bka + \nu_k k^2)^2Dk/D^2},
\]

where we have denoted

\[
F(c) = \frac{c_1 k^2}{2c_2 D^2 c} \ln\left[\frac{2D^3}{c_1 c_2 (c - Da + \lambda)}\right]
\times (B^2(\lambda^2 + c - Da - Dao)a + c(Da^2 k^2 + c_2))],
\]

\[
c = -i\sqrt{D\lambda - \lambda^2}, \quad \nu_k = \sqrt{\nu_0^2 + B^2 a^2/k^2},
\]

\[
c_1 = B^2(4\lambda^3 + 4\nu_k^2 a^2 - 3\nu_k^2 a c - c Da + D^2(c + \lambda)\nu_k^2 k^2),
\]

\[
c_2 = \sqrt{B^2(2\lambda^2 + 2c\lambda - Da)k^2 + D^2 c^2 k^2},
\]

and \(C\) is normalization constant.

Without loss of generality one can put \(k = 1\) and \(\alpha = 1\) by appropriate rescaling of the parameters \(D, B, \nu_0,\) and \(\lambda\) 11 to make a fit of the above \(P(a)\) to the experimental data. As one can see from Eq. (3) the parameter \(\lambda\) introduced in Eq. (2) is responsible for an asymmetry of the distribution with respect to \(a \rightarrow -a\).

The fit for the particular (symmetric) case, \(\lambda = 0\), has been made in Ref. 11. The flatness factor of the studied distribution which characterizes the widening of its tails (when compared with a Gaussian) is found to be \(F = 42.5\) (for \(k = 1,\) \(\alpha = 1,\) \(D = 1.130,\) \(B = 0.163,\) \(\nu_0 = 2.631,\) \(C = 1.805\)) that deviates from the flatness of the experimental curve, \(F = 55 \pm 8\) 11. In numerical calculations we used a cutoff by restricting the integration range by \(|a|/(a^2)^{1/2} \leq 1000\).

The RIN approach extends the LDN type model 11 by assuming certain relationship of noise intensities and in general other model parameters to velocity fluctuations \(u\), and enables one to study acceleration statistics conditional on velocity fluctuations 11. A dependence of acceleration distribution on velocity fluctuations is known to violate Kolmogorov 1941 local homogeneity of the flow 11.

The paper is organized as follows.

II. THE SKEWNESS

A. The unconditional probability density function

In the present paper we generalize the consideration made in Ref. 10 by letting the cross correlation parameter to be nonzero, \(\lambda \neq 0\). For this case, the exact probability density function 15 will be used.
A sample fit of the distribution [3] is presented in Fig. 1 and the contribution to the fourth order moment is plotted in Fig. 2 in which the signature of a skewness of the experimental distribution (dots) of the acceleration component can be seen as a small difference of about 5% in heights of the two peaks. Geometrically, this component of acceleration corresponds to the direction transverse to the axial symmetry (z) axis of the large-scale forcing of studied flow [17].

We remind that the turbulence was generated in a flow between counter-rotating disks in a cylindrical container, and the flow significantly deviates from the ideal of homogeneity and isotropy. By the large-scale flow symmetry the two transverse components (x and y) are taken statistically equivalent, and distinct from the axial component (z), in the Cartesian laboratory frame of reference. Particle accelerations were measured in a small volume in the center of the flow chamber within which the flow is approximately homogeneous and isotropic. Only one transverse (x) and the axial (z) components were actually measured. Statistical properties of the unmeasured y component are expected to be identical to those of the x component [17,18]. Throughout the paper we consider the data for the x component of acceleration.

One observes a better agreement of the skewed curve (solid line in Fig. 2) for the fitted value

$$\lambda = -0.005$$

\((k = 1, \alpha = 1, D = 1.100, B = 0.155, \nu_0 = 2.910, C = 3.230)\) with the data points in the intermediate range of positive and negative accelerations, \(|a_0|/(\langle a^2 \rangle)^{1/2} \approx 10\), as compared with (symmetric) curves implied by the stretched exponential fit (dashed line in Fig. 2) [18], the RIN chi-square Gaussian model fit [12], the RIN log-normal model [4], and the Reynolds model [9].

We note that better result was recently obtained [18] by processing the original (unfitted) curve of Ref. [9] in exactly the same way as that for the data but some departure from the experimental data still persists. Particularly, the associated contribution to fourth order moment, \(a^4 P(a)\), does not peak at the same value as that for the experimental curve, being however very close to it. Since in statistically homogeneous and isotropic turbulent flows for the x, y, or z component of acceleration one should have zero skewness, we could attribute the observed small skewness to anisotropy of the studied \(R_\lambda = 690\) flow.

We remind that the relative root-mean-square (rms) uncertainty of the experimental \(P(a)\) is about 3% for \(|a_0|/(\langle a^2 \rangle)^{1/2} \leq 10\) (the most accurate part of the distribution), and is less than 40% for \(|a_0|/(\langle a^2 \rangle)^{1/2} \leq 40\) [19]. Clearly, less uncertainties are required to obtain good quantitative description of the skewness as it appears to be a quite tiny effect. Nevertheless, in the present section we use the available experimental data to demonstrate the effect of nonzero \(\lambda\).

Recently reported high precision data [19] show that the mean acceleration conditional on velocity fluctuations is nonzero and increases with the increase of velocity. This was claimed to be related to the anisotropy of the studied flow, although it was pointed out that DNS of homogeneous isotropic turbulence also shows slightly nonzero mean acceleration. We will consider this issue below and in Sec. [11].

It should be emphasized that for the studied \(R_\lambda = 690\) flow the ratio between the variances of the x and z components of acceleration was measured to be slightly different from unity due to \(a_{0x}/a_{0z} \approx 1.06\) [17]. It was pointed out that experimental biases do not affect acceleration measurements (they depend mostly on the velocity) except to the extent that the acceleration and velocity are correlated. The shapes of the probability distributions for the x and z components of acceleration were measured to be approximately the same at high Reynolds numbers. This level of x-to-z anisotropy was found to persist for higher Reynolds numbers (data presented for \(R_\lambda = 970\)) that may indicate a fundamental character of this phenomenon. Namely, while the K41 theory postulates complete universality it is still an open question to what extent statistical properties of the 3D turbulence in the inertial range do not depend on the details of large-scale forcing. As the presence of finite injection scale is felt through the entire inertial range via the anomalies of the Eulerian and Lagrangian scaling exponents (K62) it is natural to expect that the statistics of fluid particle acceleration, which is generally associated to small scales of the flow, should reflect the anisotropy of the large scale forcing. From this point of view, the observation supports the view that the induced anisotropy in the acceleration statistics is a rule rather than exception in the context of developed turbulence dynamics: Forced anisotropy at large scales is not washed out in the inertial range of a high-Reynolds-number flow and seems to be felt at small scales, smaller than the Kolmogorov length. This kind of anisotropy in acceleration statistics may not be directly related to the parameter \(\lambda\).

The cross correlation parameter \(\lambda\) is nonzero for longitudinal and zero for transverse velocity increments by construction [12]. For the Lagrangian acceleration, this reads that for the component of Lagrangian acceleration, \(a_x\), pointed along the corresponding Lagrangian velocity at some point of the particle trajectory the parameter \(\lambda\) is nonzero while for the transverse component \(a_y\) it is zero. The experimental data on \(a_x\) and \(a_z\) time series do not allow one to extract separately \(a_x\) and \(a_z\), in order to verify that the \(a_x\) acceleration PDF is skewed while the \(a_z\) acceleration PDF is symmetric relative to the change of sign of acceleration. This requires obtaining the data on the whole set of the acceleration and corresponding velocity components, to have an access to geometry of individual trajectories of the tracer particle. Also, one should suppress the effect of above mentioned induced anisotropy. The influence of the large-scale anisotropy on \(a_x\) and \(a_z\) acceleration PDFs may occur to be differ-
ent. In contrast to the experiment where it seems to be difficult to provide sufficiently high level of isotropy and extract statistical data on $a_\tau$ and $a_n$, the DNS could be used to test skewness of the $a_\tau$ and $a_n$ acceleration PDFs. The isotropy is well satisfied in DNS and at least it is free from strong large-scale anisotropy of the experimentally studied von Karman flow.

The obtained small value $\lambda$ of the fitted cross correlation parameter $\lambda$, as compared with the used values of the noise intensities $D$ and $\alpha$, is in a good agreement with the results of numerical Rapid Distortion Theory (RDT) analysis of the noise cross correlators in decaying turbulence. Particularly, $\lambda$ turned out to be about two orders of magnitude smaller than $D$ and $\alpha$. The time scale of the cross correlation as well as of the autocorrelation of additive noise.

Below we use the RIN extension of the LDN model to obtain and study the conditional probability density function.

B. The conditional probability density function

In the RIN approach the result is treated in general as a probability density function conditional on the parameters involved in the model. It was found that for $\lambda = 0$ only a variation of the additive noise intensity $\alpha$, with $\alpha = e^{\nu/\nu_0}$, qualitatively meets (i) the observed variation of the shape of experimental probability density function of the transverse component of acceleration conditional on the transverse component of velocity fluctuations $u$ with variation of $u$, and (ii) the polynomial type increase of the normalized conditional acceleration variance with an increase of $|u|$. In the present paper we study the effect of nonzero $\lambda$. In general one may expect that the variation of $\lambda$ causes not only essential variation of the skewness but at the same time a considerable variation of the acceleration variance.

The experimental conditional distributions $P(a|u)$ for $u$ ranging from 0 to 3 rms velocity were found to be almost of the same shape as the experimental unconditional distribution $P(a)$ by Crawford, Mordant, and Bodoenschatz. Dashed line: stretched exponential fit, $\beta = 0.513$, $\sigma = 0.563$, $\gamma = 1.600$, $C = 0.733$. Solid lines: the model $\lambda$ at $\lambda = -0.005$, $-0.025$, $-0.05$ ($k = 1$, $\alpha = 1$, $D = 1.100$, $B = 0.155$, $v_0 = 2.910$). $x = a/(a^2)^{1/2}$.

We conclude that within the framework of RIN approach the cross correlation parameter $\lambda$ (as well as the parameters $D$, $\nu_0$, and $B$ as shown in Ref.), with $\lambda = \lambda(u)$, could not be responsible for the specific visible change of the shape of experimental conditional acceleration probability density function with an increase of velocity fluctuations.

We are thus left with the only possibility: to assign the observed essential dependence of $P(a|u)$ on velocity fluctuations $u$ to the additive noise intensity $\alpha$. In general, this is in an agreement with the RDT approach by Laval, Dubrulle, and Nazarenko, particularly with the approximate LDN model, in which only the additive noise intensity is characterized by a dependence on small scale velocity fluctuations coupled to large scale velocity fluctuations.

It should be emphasized here that in the original LDN
model the other parameters of the model, $B$, $\nu_0$, and $\lambda$, were not assumed to depend on small scale velocity fluctuations, and the multiplicative noise intensity $D$ was found to depend only on large scale velocity fluctuations. Nevertheless, in Ref. [10] we have evaluated the effect caused by each of these parameters, $D(u)$, $B(u)$, $\nu_0(u)$, and $\lambda(u)$, in order to verify independently whether this may qualitatively correspond to the set of experimental conditional distributions $P(a|u)$, $u/\langle u^2 \rangle^{1/2}$ at $u = 0$, $\langle a|u\rangle/\langle a^2\rangle^{1/2}$. Despite we obtained a negative result, presence of some weak dependencies of these parameters on $u$ can not be ruled out on the basis of made qualitative comparison.

Adopting the point of view that $\alpha = \alpha(u)$ as a first approximation, in the next section we turn to a consideration of the mean acceleration conditional on velocity fluctuations $u$.

III. THE CONDITIONAL MEAN ACCELERATION

The conditional mean acceleration should be zero in homogeneous and isotropic turbulence, and departures from zero reflect the anisotropy of the studied flow although DNS of homogeneous isotropic turbulence has also shown slight departures from zero [10].

As the first step, the conditional mean acceleration can be calculated under the assumption that only $\alpha$ depends on velocity fluctuations $u$. We take an exponential dependence, $\alpha = e^{u_0/u}$, $u_0 = 3$, which was found to be relevant from both the theoretical and experimental points of view [10].

Assuming that the unconditional distribution $P(a)$ is approximately of the same shape as the conditional distribution $P(a|u)$ at $u = 0$, and using the same set of fitted parameters as in Sec. [10] ($\lambda = -0.005$) we obtain that the conditional mean acceleration only slightly deviates from zero,

$$\frac{\langle a|u\rangle}{\langle a^2|u\rangle^{1/2}} \simeq 0.002,$$

for all values of $|u|$ ranging from zero to 2.5 $\text{rms}$ velocity, with the tendency to decrease down to zero with the increase of $|u|$, as shown in Fig. 5 (triangles).

In general, this is in agreement with almost symmetric shapes of the experimental probability density functions of the component of acceleration conditional on the same component of velocity fluctuations [10]. Alas, an illustrative character of the presented experimental plots and the increasing experimental uncertainty of $P(a|u)$ at big $|u|/\langle u^2 \rangle^{1/2}$ does not allow us to make a definite conclusion since the skewness effect is very small to be readily seen from the experimental $P(a|u)$. Also, we note that the predicted value $\langle a|0\rangle/\langle a^2|0\rangle^{1/2} \simeq 0.0024$ (see Fig. 5) does not contradict to that shown in Fig. 6b of Ref. [10], $\langle a|0\rangle/\langle a^2 \rangle^{1/2} \leq 0.05$ (uncertainty of the presented data points does not allow us to give the upper bound more precisely).

The above result on the conditional mean transverse acceleration (Fig. 5) is however in a sharp contrast with the reported experimental dependence of the conditional mean transverse acceleration on $u$ (Fig. 6b of Ref. [10]) which displays that $\langle a|u\rangle/\langle a^2 \rangle^{1/2}$ increases from about zero at $|u| = 0$ to about 0.3 at $|u|/\langle u^2 \rangle^{1/2} = 2.5$, in some nonlinear way.

In essence the experimental data show that with the increase of velocity fluctuations $|u|$ the mean of the conditional transverse acceleration distribution $P(a|u)$ normalized to unit unconditional acceleration variance increases by small but appreciable amount. One would like to know whether this holds for the $z$ component of acceleration. We expect that the mean for this component is bigger than that for the $x$ component, partially due to about 3% smaller variance of the $z$ component of acceleration [17].

More detailed study is required to explain this highly remarkable phenomenon, which may indicate stronger coupling of the multiplicative noise to the additive one for bigger velocity fluctuations $|u|$ in the Lagrangian frame.

Below, we use definitions of the noises provided by the LDN approach [15] and the results for the velocity-dependence of noise intensities [10] to give a tentative explanation of this phenomenon.

The additive noise intensity considerably increases for bigger $|u|$ due to the relationship $\alpha \sim e^{[u]}$ (incoherent noisy background responsible for the random walk behavior of acceleration is much intensified) while the multiplicative noise intensity remains at approximately the same level (very intense vortical structures responsible for the random multiplicative process are relatively frozen in time and characterized by a saturation level of $|u|$ for a given Reynolds number and vorticity). This means an increase of large scale effects produced by the interaction between small and large scales (the effect of
nonlocal interactions). Since the large scales are the only unifying agent between the noises the cross correlation of the noises becomes more pronounced for bigger \(|u|\). Whereas a direct effect of the increase of additive noise intensity tends to symmetrize the distribution (see Fig. 6), due to a higher degree of chaoticity (higher statistical isotropy), bigger \(|u|\) may imply a variation of the cross correlation, to which the skewness is highly sensitive, so that the overall effect is a small but appreciable increase of the mean of the conditional distribution. To be more precise, the net effect depends on competition between \(\lambda\) and \(\alpha\) as the velocity \(|u|\) increases.

Also, it is worthwhile to note that the effect of discrete Kolmogorov turbulent cascade may be of importance here since it is characterized by a relationship between high-amplitude harmonics of the basic ratio (nonlocal interactions).

The following remark is in order.

Note that in Sec. IV B we used the experimental data on unconditional probability density function, and estimated its value, \(\alpha = e^{\mu/3}\) and the other parameters to be constant. Hence, the additive noise tends to symmetrize acceleration distribution.

In summary, the predicted conditional mean acceleration \(\langle a|u|\rangle/\langle a^2|u|\rangle^{1/2}\) is negligibly small and decreases down to zero for larger \(|u|\) under the assumption that \(\alpha = e^{\mu/3}\) and the other parameters to be constant. An increase of \(|u|\): to assign some dependence of the parameter \(\lambda\), which measures correlation between the additive noise and the multiplicative one, on velocity fluctuations \(u\). It should be noted that this is in agreement with the LDN model, in which the additive noise depends on small-scale velocity fluctuations.

It should be stressed however that as it has been mentioned above the observed nonzero mean acceleration is associated to the flow anisotropy which may be not related to the effect described by the parameter \(\lambda\).

IV. DISCUSSION AND CONCLUSIONS

(i) We have shown that the cross correlation parameter \(\lambda\) of the LDN type model \(\delta\) for a particle acceleration could be used to explain a skewness of the acceleration probability density function, and estimated its value, \(\zeta = -0.005\), by using a fit to the recent experimental statistics data on the transverse component of acceleration.

(ii) The mean acceleration is found to be very close to zero, \(\langle a|u|\rangle/\langle a^2|u|\rangle^{1/2} \leq 0.0024\), when the predicted mean acceleration \(a^4P(a)\) is fitted to the experimental data. The mean acceleration should vanish for homogeneous isotropic turbulence. The observed mean acceleration can be attributed to small anisotropy (imperfection) of the studied DNS, for which isotropy is well satisfied, indicates slight departures from zero. Whether this is of some importance for developed turbulence should be clarified. In any case, the observed noticeable nonzero mean of Lagrangian acceleration, which is usually associated to extremely small scales of the flow, in the high-Reynolds-number flow that is anisotropic at large scale, deserves a separate study.

(iii) Using the RIN approach which extends the LDN type model by assuming certain relationship of noise intensities and in general other model parameters to velocity fluctuations \(u\) we have studied acceleration statistics conditional on velocity fluctuations. We found that the assumption \(\lambda = \lambda(u)\) could not be responsible for the experimentally observed characteristic variation of the shape of conditional distribution \(P(a|u)\) with variation of \(u\). Taken together with the result of our previous work \(\delta\) this implies that only the additive noise inten-
sity $\alpha$ reveals an essential dependence on $u$ for which we
used the exponential function, $\alpha = e^{u/u_0}$, relevant from
both the phenomenological and experimental points of
view. The additive noise tends to symmetrize acceleration
distribution for larger $|u|$; the simulated conditional
mean acceleration is very small and decreases down to
zero as shown in Fig. 5. This is not in agreement with the
experimental data even qualitatively. However, the observed
mean acceleration would be attributed mainly to
anisotropy (imperfection) of the studied
study is of interest and can be made elsewhere. To be stressed however that in the proper RIN approach
it is the marginal distribution \[10\] obtained from \[4\]
by averaging over random noise intensities with some
judiciously chosen distributions assigned to them, that
should be fitted to the experimental unconditional $P(a)$.
The simplest choice is to assume that only $\alpha$ is a random
parameter, inverse of which follows chi-square or
log-normal distribution in the spirit of simple RIN mod-
els. \[11\] Particularly, for $\alpha = e^{u/u_0}$ the choice of the log-
normal distribution is equivalent to that $u$ is normally
distributed with zero mean \[13\]. It should be empha-
sized here that only absolute value of $u$ contributes to the
marginal distribution. In this case the distribution \[4\]
is treated as a conditional probability distribution function $P(a|\alpha(u))$ (see Fig. 7 in Ref. \[10\]), which could be in
principle fitted to the experimental conditional distribution
$P(a|u)$. Although we have found a good qualitative
agreement of the model with the experimental data on
conditional statistics, an illustrative quality of the rep-
resentation of experimental $P(a|u)$ in Ref. \[12\] does not
allow us to make a reliable numerical fit of the proposed
$P(a|u) = P(a|\alpha(u), \lambda(u))$. For good fit results, high ac-
curacy experimental data on $P(a|u)$ and on the contrib-
tion to fourth order conditional moment, $a^4P(a|u)$, for
$|u|$ ranging from zero to three $rms$ velocity with the step
0.5, would be required.

(b) This would give a possibility to identify the depen-
dence $\lambda(u)$, for which one can try a polynomial or
exponential function.

(c) Despite similarity one observes some differ-
ence between the experimental distributions $P(a)$ and
$P(a|u)$ \[11\] which could be described in terms of the RIN
approach to LDN type model \[4\] along the line of rea-
soning given in the present paper. Namely, with $P(a|u)$
taken to be the distribution $P(a|\alpha(u), \lambda(u))$ and $P(a)$
to be derived from it by integrating out velocity fluctua-
tions, Eq. \[10\].

(d) Reynolds-number dependence of the acceleration
statistics is not considered in the present paper and can
be studied elsewhere. Also, it is of interest to study vari-
ation of the value of flatness factor of acceleration distri-
bution measuring Lagrangian intermittency as a function
of finite time increment used for low-pass filtering. This
dependence exhibits a fine structure of the viscous dis-
sipation range of time-scales. The flatness factor of the
$R_\lambda = 690$ flow varies by about 15% when the filter width
is changed from 0.23$\tau_\eta$ to 0.31$\tau_\eta$, where $\tau_\eta$ is Kolmogorov
time \[19\].

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