Analysis of the Generation and Weakening of Cogging Torque in Interior Permanent Magnet Machine

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Abstract. The permanent magnet of interior permanent magnet motor is inside the rotor. Compared with the surface-mount permanent magnet motor, its equivalent air gap is small and the cogging torque has a great influence. In this paper, the cogging torque of interior permanent magnet synchronous motor is analyzed and deduced by the energy method. Taking the 48-slot/8-pole interior V-type permanent magnet synchronous motor as the research object, the influence of the axial segmented skew of the permanent magnets, the width of the stator slot and permanent magnets segmentation on the cogging torque was analyzed by the finite element method. Studies have shown that the reasonable selection of the above three methods can effectively weaken the cogging torque of V-type interior permanent magnet motor.

1. Introduction

With the development of permanent-magnet material, power electronics and control theory, permanent magnet (PM) motors have become more and more widely used. Cogging torque arises from the interaction between permanent magnets and slotted iron structure and manifests itself in almost all types of PM motors [1], which causes torque pulsation, vibration, noise, and affects the accuracy of the motor control system.

In recent years, extensive attention has been paid to the influencing factors and weakening methods of cogging torque. Analytical method and finite element method are applied to study the effect of slot opening on the cogging torque of interior permanent magnet motor in Ref. [2]. The influence of the size and distribution of the magnetic density of each pole on the cogging torque is analyzed through an analytical method in Ref. [3]. The optimal shape of a rotor is investigated by using the continuum sensitivity analysis combined with the finite element method in Ref. [4]. A comparative study of torque-speed characteristics and cogging torque of the fractional-slot interior permanent magnet brushless ac machines having different slot openings is presented in Ref. [5]. A novel technique is adopted to reduce cogging torque and torque ripple by using an asymmetrical V-type rotor configuration in Ref. [6]. Optimal step-skew methods for cogging torque reduction in interior permanent magnet machines is proposed, which investigates end leakage flux in Ref. [7]. The partly enlarged air-gap made by rotor unequal out diameter and stator core structure with pole shoe modification is introduced in Ref. [8]. Based on two objective function, [9] presents a topology optimization method to reduce cogging torque. Unlike above studies, this paper analyzes the cogging
torque of permanent magnet motors, reveals the influence of structural parameter changes on cogging
torque through analytic expressions, and proposes three methods to weaken cogging torque in V-type
interior permanent magnet motor.

2.  Analysis of cogging torque

The cogging torque indicates the change of the magnetic field energy in the motor with the relative
position of the stator and the rotor when the permanent magnet motor is not energized. It can be
expressed as the partial derivative of the magnetic field energy with the relative position angle of the
stator and the rotor,

\[ T_{\text{cog}} = -\frac{\partial W}{\partial \alpha} \tag{1} \]

In the expression, \( W \) is the magnetic field energy in the motor, \( \alpha \) is the relative position angle of the
stator and rotor.

To facilitate the analysis, the following assumptions are made [10]:

1. The relative permeability of the iron core is infinite, so the magnetic field energy stored in the
iron core can be ignored, and only the air gap and the magnetic energy inside the permanent magnet
are considered.

2. Do not consider end effects.

3. Each permanent magnet block is consistent in both size and remanence.

4. The permeability of permanent magnets is the same as that of vacuum.

Based on the above assumptions, when the armature winding is not energized, the magnetic field
energy in the permanent magnet motor is approximately the sum of the air gap and the magnetic field
energy in the permanent magnet. Since cogging torque is the rate of change of the magnetic field
energy with the rotor position angle in the motor, more attention is paid to the change of the magnetic
field energy. For V structure, the permanent magnet is placed inside the rotor core, and it can be
approximated that the magnetic field energy in the permanent magnet does not generate cogging
 torque. The magnetic field energy in the air gap can be expressed as

\[ W_{\text{airgap}} = \frac{1}{2\mu_0} \int_{\theta} B_r^2(\theta) \left[ \frac{h_m}{h_m + \delta(\theta, \alpha)} \right]^2 dV \tag{2} \]

In the expression, \( W_{\text{airgap}} \), \( \mu_0 \), \( B_r(\theta) \), \( \delta(\theta, \alpha) \) and \( h_m \) are the magnetic field energy, vacuum
permeability, air gap magnetic density, effective air gap length, length of permanent magnet
magnetization direction.

2.1. Fourier decomposition of \( B_r^2(\theta) \)

The polar arc coefficient of the permanent magnet is \( \alpha_p \), and the number of pole pairs is \( p \).
According to the previous assumptions, the waveform of air gap magnetic density is shown in figure 1.
Taking \( \theta \) at the center line of the magnetic pole, the air gap magnetic density waveform is an even
function, similar to a rectangular wave, which alternates positive and negative along the
circumferential direction, with a period of \( 2\pi / p \), and the width of each half wave is \( \pi \alpha_p / p \). The
waveform of \( B_r^2(\theta) \) can be obtained from the \( B_r(\theta) \) waveform, as shown in figure 2.
Figure 1. Distribution of air gap magnetic density $B_r(\theta)$ Obtained by Fourier decomposition

$B_{r}^2(\theta) = \alpha_{p}\beta_{r}^2 + \sum_{n=1}^{\infty} \left[ \frac{2\beta_{r}^2}{\pi n} \sin(n\alpha_{p}\pi) \right] \cos(2np\theta) = B_{r0} + \sum_{n=1}^{\infty} B_{rn} \cos(2np\theta) \tag{3}$

2.2. Fourier decomposition of $\left[ \frac{h_{m}}{h_{m} + \delta(\theta, \alpha)} \right]^{2}$

Without considering the leakage magnetic flux, the arc of the slot opening is $\theta_{sla}$, the air gap length corresponding to the tooth is $\delta$ and the number of stator slots is $Z$. then the distribution of $\frac{h_{m}}{h_{m} + \delta(\theta, \alpha)}$ along the air gap is shown in Figure 3.

Figure 3. Distribution of $\frac{h_{m}}{h_{m} + \delta(\theta, \alpha)}$

By Fourier decomposition, we get,
In the expression, 

\[ G_0 = \left( \frac{h_m}{h_m + \delta} \right)^2 \left( 1 - Z_{sot} \right), \]

\[ G_n = \frac{2}{n\pi} \left( \frac{h_m}{h_m + \delta} \right)^2 \sin \left( \frac{n\pi - n\pi_{sot}}{2} \right) \]

Substituting (3) and (4) into equation (2) and combining (1), the expression of cogging torque is,

\[ T_{cog}(\alpha) = \frac{\pi Z L_a}{4\mu_0} \left( R_2^2 - R_1^2 \right) \sum_{n=1}^{\infty} nG_n B_n \frac{\sin(nZ\alpha)}{2p} \]

In the expression, \( L_a \) is the axial length of the armature core, \( \mu_0 \) is the vacuum permeability, \( R_1 \) and \( R_2 \) are the inner and outer radius of the air gap, \( n \) is an integer such that \( nZ/2p \) is an integer, \( B_r \) is residual magnetic flux density of magnet.

It can be seen from expression (5) that the methods of weakening the cogging torque can be summarized into three categories, that is, the method of changing the permanent magnet magnetic pole parameters, the method of changing the armature parameters, and the reasonable combination of the number of armature slots and poles.

3. Effects of step-skewed PM on cogging torque

3.1. Basic principles

When the motor stator is skewed or the permanent magnet is skewed, the cogging torque is expressed as,

\[ T_{cog}(\alpha, N_s) = \frac{\pi L_a}{2\mu_0 N_s \theta_{s1}} \left( R_2^2 - R_1^2 \right) \sum_{n=1}^{\infty} nG_n B_n \frac{\sin(nZ\theta_{s1})}{2p} \sin \left[ \frac{nZ(\alpha + \frac{N_s \theta_{s1}}{2})}{2p} \right] \]

In the expression, \( N_s \) is the number of skewed slots in the stator, and \( \theta_{s1} \) is the stator tooth pitch expressed in radians.

From equation (6), to eliminate the cogging torque harmonics of the nth order in cogging torque, then

\[ \sin \left( \frac{nZ\theta_{s1}}{2} \right) = 0 \]

Then,

\[ N_s = \frac{360}{\theta_{s1} N_p Z} = \frac{1}{N_p} \]

Where \( N_p \) is the number of fundamental cogging torque cycles for a pitch, and its value is

\[ N_p = \frac{2p}{\text{HCF}(Z, 2p)} \]
In the expression, \( HCF(Z, 2p) \) is expressed as the greatest common divisor of the number of slots \( Z \) and the number of poles \( 2p \).

It can be seen from the above equations that if \( N_s = \frac{1}{N_p} \), then all cogging torque harmonics are eliminated.

When each pole permanent magnet is divided into \( k \) segments of equal length in the axial direction, the total cogging torque is to be zero, then

\[
N_s = \frac{1}{N_p k}
\]  

(10)

3.2. Finite element simulation verification

Taking a 48-slot/8-pole V-type interior permanent magnet synchronous motor as an example, the structure and specific parameters of the motor are shown in Figure 4 and Table 1. Finite element simulation is performed on the weakening of cogging torque when the permanent magnet is segmented skew. The simulation results are shown in Figure 5.

As can be seen from Fig. 5, the cogging torque can be weakened by using the rotor segmented skew. When not segmented, the peak cogging torque is 1.4184Nm. However, when the number of segments is \( k = 5 \), the peak cogging torque is 0.1402Nm. The cogging torque was reduced by 90.1%. On the other hand, the cogging torque amplitude decreases as the number of segments increases. It should be noted that as long as the number of segments is not infinite, some specific cogging torque harmonics cannot be eliminated, which also means that cogging torque cannot be completely eliminated.

![Figure 4 The configuration of V-Type IPMSM](image)

| Table 1 The specific parameters of V-Type IPMSM |
|-----------------------------------------------|
| Rated power | 50kW | PM thickness | 6.48mm | Rotor outer diameter | 160.4mm |
| Rated speed  | 3000rpm | Stator outer diameter | 269.24mm | Rotor inner diameter | 110.64mm |
| PM width     | 32mm | Stator inner diameter | 161.9mm | Shaft length        | 83.82mm  |
There are certain disadvantages to the methods of reducing the cogging torque by step-skewed PM. It also reduces the back-EMF and output electromagnetic torque. Skewed slots of the stator also make winding difficult. Moreover, the stator skewed slot, and rotor step-skewed PM generate additional axial forces.

**4. Effect of stator slot width on cogging torque**

From equation (5), we can know that the cogging torque can be weakened by changing the number of Fourier decomposition $G_n$ of the relative air gap permeance. On the other hand, the change in the slot width results in a change in the relative air-gap permeability, which in turn affects the magnitude of the cogging torque. The finite element simulation of the relationship between the slot width $b_0$ and the cogging torque is performed on the above-mentioned V-type permanent magnet synchronous motor. The simulation result is shown in figure 6.

![Figure 5. Cogging torque for different step-skewed PM](image)

**Fig.6** The comparison of cogging torque with different slot-opening

It can be seen from figure 6 that when the slot width gradually changes from 1.93mm to 0.13mm, the cogging torque decreases monotonically from a high value, the peak value decreases from 1.4184Nm to 0.0875Nm, and the cogging torque decreases by 93.8%. Of course, the lower the slot width, the more difficult the armature winding wire. On the other hand, the armature slot width affects the phase winding back-EMF and electromagnetic torque. Therefore, in terms of the overall design performance of the motor, comprehensive considerations are needed, and the main performance of the motor cannot be reduced solely to improve a certain performance.
5. Effect of permanent magnet block on cogging torque

Each permanent magnet is divided into $N_b$ blocks of the same width and uniformly distributed. The width of each block is $\theta$, and the interval between two adjacent blocks is $\gamma$, as shown in Figure 7. Regardless of the end effect and saturation, the total cogging torque of the permanent magnet motor can be regarded as the superposition of the cogging torque produced by each permanent magnet block.

![Figure 7 Schematic diagram of the evenly block permanent magnet pole.](image)

The cogging torque generated by each permanent magnet block can be expressed as

$$T_{\text{cog}}(\alpha) = \sum_{n=1}^{\infty} T_n \sin(nN_p Z \alpha)$$

In the expression, $T_n$ is the amplitude of the cogging torque harmonics, $\alpha$ is the stator and the relative position of the rotor.

Since each permanent magnet is divided into $N_b$ blocks of the same width and uniformly distributed, the total cogging torque can be regarded as the superposition of the cogging torque of $N_b$ permanent magnet motors.

$$T_{\text{cog}}(\alpha) = \sum_{m=1}^{N_b} \sum_{n=1}^{\infty} T_n \sin\left\{nN_p Z \left[\alpha + (m-1)\Delta \beta\right]\right\}$$

In the expression, $\Delta \beta$ is the offset angle of two adjacent permanent magnet blocks, which can be expressed as the permanent magnet block width and the permanent magnet block interval in figure 7 as.

$$\Delta \beta = \theta + \gamma$$

To eliminate the $n$th harmonic of cogging torque, the offset angle $\Delta \beta$ is

$$\sum_{m=1}^{N_b} \sin\left\{nN_p Z \left[\alpha + (m-1)\Delta \beta\right]\right\} = 0$$

Due to the relative position of the permanent magnet and the armature slot cogging torque distribution influential, so $\Delta \beta$ offset angle is an integer multiple of the general form of a cogging torque cycle.

$$\Delta \beta = \frac{2\pi}{2N_p} k + \Delta \beta_s$$
In the expression, \( k \) is a positive integer and \( \Delta \beta_s \) is the relative position change angle of adjacent permanent magnet blocks, as shown in figure 8.

Equation (14) can be expressed as

\[
\sum_{m=1}^{N_b} \sin \left[ nN_p Z \left( \alpha + (m-1)\Delta \beta_s \right) \right] = \frac{\sin \left( \frac{1}{2} nN_b N_p Z \Delta \beta_s \right)}{\sin \left( \frac{1}{2} nN_p Z \Delta \beta_s \right)} \sin \left[ nN_p Z \left( \alpha + \frac{N_b - 1}{2} \Delta \beta_s \right) \right]
\]  

(16)

By properly selecting \( \Delta \beta_s \) so that (16) is zero, the nth harmonic of the cogging torque can be eliminated. According to the conclusion in Ref. [11], the value of \( \Delta \beta_s \) for eliminating the cogging torque fundamental wave is

\[
\Delta \beta_s = \frac{2k\pi}{N_b N_p Z}
\]  

(17)

Taking the above-mentioned V-type interior permanent magnet synchronous motor as an example, the permanent magnet is divided into 3 blocks, and the finite element simulation is performed under the conditions of \( \gamma = 1\text{mm} \), \( \gamma = 2\text{mm} \) and \( \gamma = 4\text{mm} \) respectively. The simulation model in the case of evenly block is shown in figure 9, and the simulation results see figure 10.
Fig. 10 The cogging torque with permanent magnets evenly divided

It can be known from the simulation results that as the distance between the permanent magnet blocks increases, the cogging torque decreases monotonously. When the block separation distance $\gamma = 4\text{mm}$, the peak cogging torque is $0.1097\text{Nm}$, and the cogging torque is weakened by $91.6\%$. It should be noted that if the distance between the permanent magnet blocks is too large, it will cause a series of problems such as back-EMF, which need to be considered in many aspects.

6. Conclusion
This paper aims at the cogging torque weakening method of V-type interior permanent magnet synchronous motor. The analytic expression of cogging torque is studied. It can be seen from the analysis that the methods of reducing cogging torque can be summarized into three major methods class. Three methods of skewed PM, optimized slot width and permanent magnet block are applied to V-type interior permanent magnet synchronous motor. The verification shows that the above three methods have obvious effects on reducing the cogging torque of V-type interior permanent magnet synchronous motor.

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