Contributions to the W-Boson Anomalous Moments in the Two-Higgs-Doublet Model at Collider Energies

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Abstract

We examine the one loop contributions arising in the Two-Higgs-Doublet Model (THDM) to the W-boson anomalous magnetic dipole and electric quadrupole form factors for both photon and Z couplings relevant at collider energies. While the model parameter and \( q^2 \)-dependencies of these form factors are found to be significant, the corresponding size of these corrections are relatively small in comparison to unity. They are, however, found to be comparable in magnitude to the usual Standard Model loop corrections. Radiative corrections to the Higgs particle masses and couplings due to heavy top-quarks are included in the analysis.
Many theories which go beyond the Standard Model (SM), such as the Peccei-Quinn Model, Supersymmetry (SUSY), and $E_6$ superstring-inspired models, require the scalar sector to be enlarged by the addition of an extra Higgs doublet $[1, 2, 3]$. Such Two-Higgs-Doublet Models (THDM) have an extremely rich phenomenology$[4]$ as the particle spectrum now contains two neutral CP-even states, $h$ and $H$, a neutral CP-odd state, $A$, and a conjugate pair of charged fields, $H^\pm$. Unfortunately, although there have been direct searches for such particles at both $e^+e^-$ and hadron$[5]$ colliders as well as searches using indirect techniques$[7]$, they have failed to be observed. Of course, the searches so far conducted have only explored a small region of the allowed parameter space in these models but the advent of hadron supercolliders$[8, 9]$ and $e^+e^-$ machines in the TeV range$[10]$ will allow essentially all of the parameter space of current interest to be probed.

As the detailed predictions of the SM begin to be examined, one way to look for new physics is via loop-effects through possible small deviations away from SM expectations. Using existing data, this has already been done by a large number of authors$[11]$ for many possible SM extensions. As has been suggested$[12]$, one place to look for deviations in the future is the trilinear gauge couplings, $\gamma WW$ and $ZWW$, which are uniquely predicted in the SM. In the $\gamma WW$ case, with all particles on-shell, loop corrections to the tree-level SM predictions were considered some time ago$[13]$ for several different SM extensions including the THDM case. In this paper, we return to a discussion of the loop corrections to the $\gamma WW$ vertex within the context of the THDM, incorporating several refinements to fully cover the parameter space and make a direct connection to the physical Higgs spectrum. We then extend the previous analysis to the $ZWW$ case and generalize to the situation where the $\gamma$ or $Z$ can be off-shell. This is, of course, what will be realized experimentally when the trilinear couplings are probed in $W$-boson pair production at $e^+e^-$ colliders. In particular, we are interested in the $q^2$-dependence of the anomalous magnetic dipole and electric quadrupole moment form factors and how they differ from the expectations of the SM. We find that for both the $\gamma WW$ and $ZWW$ cases, and all interesting values of $q^2$, both the model parameter and $q^2$-dependencies of these deviations are significant. Unfortunately, we also show that the size of these deviations are always relatively small in comparison to unity although they can be comparable in magnitude to the SM predictions themselves. It may, however, be possible to probe these anomalous couplings at the few parts per mil level at the NLC$[14]$.
given sufficient integrated luminosity.

To perform our calculation we label the momentum flow in the V(= γ or Z)WW vertex as shown in Fig.1. Although we allow V to be off-shell, we assume that terms proportional to $Q^\lambda$ can be dropped since V is assumed to couple to massless fermions. Both W's are assumed to be on-shell in the discussion below. Following the notation of Hagiwara et.al. [12], the relevant part of the coupling in momentum space is given by

$$g_{VVW}^{-1} \Gamma^\lambda_{\mu\nu} = 2 f_1^V p^\lambda g_{\mu\nu} - \frac{8 f_2^V}{M_W^2} p^\lambda Q^\mu Q^\nu + 2 f_3^V (Q^\nu g^\mu\lambda - Q^\mu g^{\nu\lambda})$$

(1)

when both the W$^+$ and W$^-$ are on-shell. At tree-level, the SM gauge symmetries demand that $f_1^V = 1$, $f_2^V = 0$, and $f_3^V = 2$ with $g_{\gamma W W} = -e$ and $g_{Z W W} = -g c_W$ where $c_W = \cos \theta_W$. At one-loop, however, these parameters are now shifted and are given instead by (here s is the square of V invariant mass, i.e., $s = q^2$, which we will assume, for the moment, is positive)

$$f_1^V = 1 + \Delta g_1^V + \frac{s}{2M_W^2} \lambda_V$$
$$f_2^V = \lambda_V$$
$$f_3^V = 2 + \Delta g_1^V + \Delta \kappa_V + \lambda_V$$

(2)

so that the momentum space coupling can be written as

$$g_{VVW}^{-1} \Gamma^\lambda_{\mu\nu} = (1 + \Delta g_1^V + \frac{s}{2M_W^2} \lambda_V)[2p^\lambda g_{\mu\nu} + 4(Q^\nu g^{\mu\lambda} - Q^\mu g^{\nu\lambda}) - 8 \frac{\lambda_V}{M_W^2} p^\lambda Q^\mu Q^\nu + 2[\Delta \kappa_V + \lambda_V(1 - s/M_W^2)](Q^\nu g^{\mu\lambda} - Q^\mu g^{\nu\lambda})]$$

(3)

Note that both $\lambda_V$ and $\Delta \kappa_V$ are finite and $q^2$-dependent, whereas $\Delta g_1^V$ contains ultra-violet infinities corresponding to a charge renormalization. (In the case where a massless scalar is exchanged between the W's, an infra-red divergence can also be isolated in $\Delta g_1^V$ which is then cancelled by a real emission diagram.) Instead of being constants, both $\Delta \kappa_V$ and
\(\lambda_V\) are true form-factors. As can be seen from this equation, the form of the finite terms that we are interested in here \((\Delta K_V \text{ and } \lambda_V)\) are somewhat different than those used by Couture et.al.\[13\] in the case where the initial photon, as well as the W-pair, are on-shell. This problem is easily overcome by noting that the two parameterizations are simply related via

\[
\begin{align*}
(\Delta K)_\text{Couture} &= \Delta K_V + \lambda_V (1 - s/M_W^2) \\
(\Delta Q)_\text{Couture} &= -2\lambda_V
\end{align*}
\]

(4)

Our convention is now the standard one adopted by most authors in the literature\[12\].

The two classes of diagrams contributing to \(\Delta K_V\) and \(\lambda_V\) at one loop in the THDM are shown in Fig.2. In the first class, the external V couples to either a pair of W’s, a pair of charged Goldstone bosons, or one of each and the two external W’s are connected by the exchange of either h or H. In diagrams of the second class, the external V couples to H\(\pm\) and the external W’s are connected by h, H, or A exchange. (Other classes of diagrams involving the ZhA and ZHA vertices for the V=Z case are found to cancel due to Bose statistics.)

Denoting the contributions of these two sets of diagrams by the \(q^2\)-dependent quantities \(I_1\) and \(I_2\), we can write the entire THDM contribution to either \(\Delta K_V\) or \(\lambda_V\) symbolically as

\[
[f I_1(h) + I_2(H, H^\pm)] s_{\beta-\alpha}^2 + [f I_1(H) + I_2(h, H^\pm)] c_{\beta-\alpha}^2 + I_2(A, H^\pm) - SM
\]

(5)

In this equation, the arguments of \(I_{1,2}\) represent the relevant particle masses, \(c_{\beta-\alpha} = \cos(\beta-\alpha)\) and \(s_{\beta-\alpha} = \sin(\beta-\alpha)\) which are the usual mixing angle factors\[3\], and \(f\) depends on the choice made for V:

\[
f = \begin{cases} 
1, & V = \gamma \\
(1 - 2x_W)/2(1 - x_W), & V = Z 
\end{cases}
\]

(6)

Also note that in Eq.(5) we have subtracted off the SM contribution to both \(\Delta K_V\) and \(\lambda_V\) since we are interested in how much these quantities are changed when the SM is extended to the THDM case. To be specific, we have subtracted the SM contributions assuming a SM Higgs mass of 200 GeV and taking the appropriate value of \(q^2\). (Of course, a different choice
for the SM Higgs mass will only shift our results for $\Delta \kappa_V$ and $\lambda_V$ by a pair of constants.) This subtraction will be most noticeable in the case of $\lambda_V$ which would normally vanish in the limit that all the masses in the loop become large. Instead, we simply recover the negative of the SM contribution in the present case. Note that in the limit that $H$, $A$, and $H^\pm$ masses get very large and $s_{\beta-\alpha} \to 1$, we will recover the SM limit provided that $h$ is identified with the Higgs scalar of the SM in which case the difference defined in Eq.(5) will vanish.

Since the THDM contains many unknown quantities, it is very difficult to explore all of the parameter space. The above results, while being valid in any THDM whose scalar sector conserves CP, are difficult to analyse numerically. In order to simplify our results and to reduce the number of free parameters we have assumed that the THDM Higgs masses and couplings are as given by SUSY-version of the Model with the one-loop heavy top-quark mass corrections included\[15\]. Specifically, we have chosen the same input parameters as used in the effective potential analysis of Barger et.al.\[8\] except that we have taken the top-quark mass to be 130 GeV. Our results are not qualitatively sensitive to this particular choice of parameters. With this great simplification, the only remaining free parameters are the ratio of the two Higgs doublet vacuum expectation values, $\tan \beta = v_t/v_b$, and the mass of the CP-odd field $A$, $m_A$. All other masses and mixing angle parameters are then fixed. For numerical purposes, will we assume $M_W = 80.14$ GeV \[16\] and $M_Z = 91.175$ GeV \[17\] with $c_W = M_W/M_Z$ in the analysis that follows. The functions $I_{1,2}$ can be expressed generically in terms of a small set of integrals which reduce to those of Couture et al.\[13\] in the limit that $q^2 \to 0$. For example, in the $\lambda_V$ case, we define the integral

$$J(r_q, r_p, r_0) = 6a \int_0^1 dt \ t^3(1-t) \int_0^1 du \ u(1-u)[A - Bu(1-u)]^{-1}$$

with $B \equiv r_q t^2$ and $A \equiv t^2 - t(1 + r_0 - r_p) + r_0$. The $r'$s are given by

$$r_q = q^2/M_W^2$$
$$r_p = m_{H^+}^2/M_W^2$$
$$r_0 = m_0^2/M_W^2$$

with $m_0$ being a generic mass for a neutral Higgs field. Note that we have defined the
dimensionless parameter, \( a \), as

\[
a = \frac{g^2}{96\pi^2}
\]  

(9)

which sets the typically small scale for the size of the loop-induced anomalous couplings. We now find that \( I_{1,2} \) in Eq.(5) can now be simply expressed as

\[
I_1^\lambda(h, H) = J(r_1, 1, r_{h,H}) \\
I_2^\lambda(h, H, A; H^\pm) = J(r_q, r_p, r_{h,H,A})
\]  

(10)

in obvious notation. A similar set of relations can be developed for the case of \( \Delta \kappa_V \); defining the integrals

\[
K(r_q, r_p, r_0) = -6 a \int_0^1 dt \int_0^1 du t(3ut - 1) \log|A - Bu(1 - u)| \\
L(r_q, r_p, r_0) = 3 a \int_0^1 dt \int_0^1 du t^2(4 + 2tr_0 - 2r_0)[A - Bu(1 - u)]^{-1}
\]  

(11)

with \( A \) and \( B \) as given above, we find that

\[
I_1^{\Delta \kappa}(h, H) = K(r_q, 1, r_{h,H}) - (1 - r_q)J(r_q, 1, r_{h,H}) \\
I_2^{\Delta \kappa}(h, H, A; H^\pm) = K(r_q, r_p, r_{h,H,A}) - (1 - r_q)J(r_q, r_p, r_{h,H,A}) + L(r_q, r_p, r_{h,H,A})
\]  

(12)

The above integrals are relatively easy to perform numerically once care is given to potential imaginary parts. Since \( \Delta \kappa_V \) and \( \lambda_V \) are themselves generated only at the one-loop level, they contain no imaginary parts at this order; such imaginary parts will, however, arise at the two-loop level. (The imaginary part of the one-loop vertex function is in fact buried in the quantity \( \Delta g_1^V \) introduced above.) These results are seen to simply reduce to those obtained earlier assuming \( V = \gamma \) and taking \( q^2 \) to zero.

In order to get a feeling for the relative size of the THDM contributions we note that the SM with \( m_t = 130 \text{ GeV} \) and \( M_H = 200 \text{ GeV} \) predicts \( \Delta \kappa_\gamma = 13.5a \) and \( \lambda_\gamma = -0.6a \) for \( q^2 = 0 \). (Note that the predicted size of \( \lambda_\gamma \) is more than an order of magnitude smaller than that of \( \kappa_\gamma \).)
than that of $\Delta \kappa_\gamma$.) As we will see, the THDM contributions are quite comparable to these values in size. Unfortunately, since $a$ is a small quantity, $\Delta \kappa_{\gamma, Z}$ will almost always be less than $\simeq 0.01$ in magnitude (with $\lambda_{\gamma, Z}$ even smaller) in the THDM case. It is important to remember that such SM contributions are subtracted out in what follows as purely SM radiative corrections are already well understood in processes such as $e^+e^- \rightarrow W^+W^-$ in which our anomalous couplings might be measured.

Since the $q^2 = 0$ situation has already been discussed previously for the case $V = \gamma$ we will mention it here only to illustrate the $q^2$ dependence of the two form factors. It will, however, show many features in common with cases where $q^2 \neq 0$ and where $V = Z$. Figs.3a and 3b show $\Delta \kappa_\gamma$ and $\lambda_\gamma$ as functions of $m_A$ for different choices of $\tan \beta$ assuming $q^2 = 0$. The first thing we observe is the apparent lack of sensitivity to the choice of $\tan \beta$, a property that will persist in all other cases as we will see below. Both $\Delta \kappa_\gamma$ and $\lambda_\gamma$ have their greatest magnitudes when $m_A$ is small. The reason for this is clear; as $m_A$ increases the corresponding masses for $H$ and $H^\pm$ also become large leaving only $h$ light. The $h$ contribution is then largely cancelled by the usual SM subtraction; this is particularly true in the $\lambda_\gamma$ case where decoupling in the heavy mass limit is most effective. We also note again the general feature that the relative sizes of $\Delta \kappa_\gamma$ and $\lambda_\gamma$ generally differ by more than an order of magnitude; this effect will occur in all the cases we will examine below. As advertised, the THDM contributions to both of these parameters are generally comparable to their SM values in magnitude, particularly for small values of $m_A$. These results were found to be only moderately sensitive to the existence of heavy top-quark loop corrections to the Higgs’ masses and couplings, particularly, at smaller values of $m_A$, and only mildly sensitive to the particular SUSY parameter values used to calculate these corrections. These properties continue to valid in all of the results we present below.

To verify the lack of $\tan \beta$ sensitivity, we show in Figs.3c and 3d the values of $\Delta \kappa_\gamma$ and $\lambda_\gamma$ as functions of $\tan \beta$ for different values of $m_A$ assuming $q^2 = 0$. Quite generally, we see that the values of these parameters are approximately constant for $\tan \beta$ larger than unity which is just the region in which we expect the SUSY THDM mass and coupling realtions from the effective potential approach that we have used in these calculations to be most valid.
What happens when we increase $q^2$ and we are no longer probing the static moments? Figs. 4a-d and 5a-d show the results akin to Figs. 3a-b for different values of $q^2$ and we see a significant change in the shape of the curves when the W-pair threshold is exceeded. The general lack of sensitivity to $\tan\beta$ is still evident except for some very small ranges of $m_A$; this we have examined explicitly for several fixed values of $q^2$. In the case of $\lambda_\gamma$, we note that the magnitude decreases significantly as $q^2$ increases. This behaviour might have been anticipated by the results in Eq.(10). $\Delta\kappa_\gamma$ also decreases in magnitude once the value of $q^2$ exceeds the W-pair threshold but does not decrease significantly beyond $\sqrt{s}=200$ GeV. To explore this behaviour further, we display the explicit $q^2$-dependencies of both $\Delta\kappa_\gamma$ and $\lambda_\gamma$ in Figs. 6a-b assuming $\tan\beta = 2$ for different values of $m_A$. (Due to the general overall lack of sensitivity to $\tan\beta$, these results are largely independent of the value of this parameter.) Except for the region near the W-pair threshold, these functions are generally smooth and $\lambda_\gamma$ is seen to tend to zero in the large $q^2$ limit as expected. The asymptotic behavior of $\Delta\kappa_\gamma$ is somewhat more complex in this limit. We do, however, see the general result that the $q^2$-dependence of both $\Delta\kappa_\gamma$ and $\lambda_\gamma$ in the THDM cannot be ignored in that the deviations expected for the static moments in the THDM are quite different from those one would measure at a high energy collider producing W-pairs. Unfortunately, although the size of these corrections we have found are comparable to the SM results themselves, they are numerically quite small.

Do these general features persist when we look at the case $V=Z$ as we might naturally expect? Fig. 7 answers this question in the affirmative; here we see repeated lack of dependence of our results on $\tan\beta$ and the drastic reduction in magnitude of $\lambda_Z$ as $m_A$ increases. Generally, the magnitudes we find for $\lambda_Z$ and $\Delta\kappa_Z$ are comparable to what was seen in the case $V=\gamma$. In fact, the numerical results for the two quantities in the $V=Z$ and $V=\gamma$ cases are found to be very similar for most values of the parameters. Fig. 8 shows that the $q^2$-dependence of the form factors in both cases is also extremely similar as one might expect. Thus in the case of W-pair production at $\sqrt{s}=200$ GeV, the anticipated size of $\lambda_\gamma$ and $\lambda_Z$ (as well as $\Delta\kappa_\gamma$ and $\Delta\kappa_Z$) are quite comparable in both the SM and the THDM.

We have examined the contributions to the anomalous WWZ and WW$\gamma$ trilinear couplings within the context of the THDM over a wide range of model parameters and
values of $q^2$. We have incorporated the one-loop radiative corrections due to a heavy top-quark into the mass and coupling relationships amongst the various physical Higgs fields. The major results that we have obtained in our analysis are as follows:

(i) Both $\Delta \kappa_V$ and $\lambda_V$ were found to be relatively insensitive to the choice of $\tan\beta$ once this quantity was greater than unity. The sensitivity to the value of $m_A$ was particularly strong especially for $m_A$ values below about 200 GeV. The usual SM results were obtained in the limit of large $m_A$. Although the THDM contributions did not lead to order of magnitude changes in the anomalous moment for factors, we did see that the additional contributions could be quite comparable to the SM results.

(ii) The anomalous couplings were seen to be of comparable magnitude in both the $V=Z$ and $V=\gamma$ cases. Thus it would be impossible to separate these two sources of anomalous couplings using $e^+e^-$ data alone. Although the values we have obtained for the anomalous moment form factors in the THDM remain reasonably small, measurements taken at the NLC may help to constrain the THDM parameter space.

(iii) The two form factors we examined were observed to exhibit a strong $q^2$ dependence. Their values, as would be measured via W-pair production at an $e^+e^-$ collider, are substantially different than their static values obtained at $q^2=0$. $\lambda_V$ was found to decrease in magnitude quite rapidly with increasing $q^2$.

(iv) The influence of incorporating the heavy top-quark loop corrections to the Higgs’ masses and couplings was found to be significant particularly in the range of small $m_A$ values. The specific results were, however, quite insensitive to the particular choice of the SUSY parameters that were employed.

Precision measurements of the anomalous moment couplings of the W at $e^+e^-$ colliders and elsewhere may lead to the first indications of the existence of new physics beyond the Standard Model.
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Figure Captions

Figure 1. Feynman diagram for the VWW vertex with the momenta labeled as used in the text.

Figure 2. The two classes of triangle diagrams responsible for generating the anomalous moments of the W-boson in the THDM.

Figure 3. The $\gamma$WW anomalous moments for the case $q^2=0$. (a) $\Delta\kappa_{\gamma}$ as a function of $m_A$ for $\tan\beta = 1, 2, 5, 10, 30$; (b) same as (a) but for $\lambda_{\gamma}$; (c) same as (a) but as a function of $\tan\beta$ for $m_A = 50$ (top solid), 100(dashed-dot), 250(dashes), 500(dots), or 1000(bottom solid) GeV; (d) same as (c) but for $\lambda_{\gamma}$ with the top solid curve now corresponding to $m_A = 1000$ GeV and the order of the subsequent curves in (c) now reversed.

Figure 4. $m_A$ dependence of $\Delta\kappa_{\gamma}$ for $\sqrt{s} = (a) 100; (b) 200; (c) 500; or (d) 1000$ GeV with $\tan\beta$ as given in Fig.3.

Figure 5. Same as Fig.4 but for $\lambda_{\gamma}$.

Figure 6. $\sqrt{s}$ dependence of (a) $\Delta\kappa_{\gamma}$ and (b) $\lambda_{\gamma}$ assuming $\tan\beta = 2$. In (a), from top to bottom on the left hand side, the curves correspond to $m_A = 50, 100, 200, 500$, or 1000 GeV while in (b) the same order is reversed.

Figure 7. $m_A$ dependence of $\Delta\kappa_Z$ and $\lambda_Z$ for the same values of $\tan\beta$ as in Fig.3 with either $\sqrt{s} = 200, 500$, or 1000 GeV.

Figure 8. Same as Fig.6 but for $\Delta\kappa_Z$ and $\lambda_Z$. 

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