EFFECT OF MAGNETIC FIELD AND CONSTRICTION ON PULSATILE FLOW OF A DUSTY FLUID

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Abstract

Pulsatile flow of a Saffman’s dusty fluid through a two dimensional constricted conduit in the existence of magnetic field is investigated. Perturbation solutions have been obtained under long wave length approximation and closed form expressions have been derived for stream function, velocities of solid and fluid particles, pressure distribution and shear stress. It is found that the streamlines get altered as magnetic parameter rises. The shear stress of the fluid acting on the wall increases with magnetic parameter but the pressure decreases.

Key words : Pulsatile Flow, Dusty Fluid, Constricted channel

I. Introduction

The Pulsatile flow of blood has received lot of attention in the last few years due to its significance in several physiological systems particularly in cardiovascular system. The better description of the blood as a binary system by considering a dusty fluid for blood is given by Saffman [IX]. Nayfeh [I] considered the Pulsatile flow of a fluid through an extremely long and narrow pipe of circular cross section which consists of small rigid particles. Gupta and Gupta [X] considered the stream of a dusty gas through a canal with timely changing pressure gradient. Radhakrishnamacharya [V] deliberated the pulsatile flow of a fluid consisting of tiny solid particles through a narrow channel. Wagh and Tapi [II] investigated the pulsatile flow of a suspension in the region of mild stenosis. Afifi and Gad [VIII] considered the interface of MHD Pulsatile flow and peristaltic flow through a porous medium. Ravi Kiran et al. [III] and Kumar et al. [VII] considered pulsating flows with reference to different situations.

However, the influence of magnetic field on pulsatile flow of dusty fluid has not received much consideration. It is identified that blood demonstrates magnetic properties. Hence following saffaman’s model, in this paper, pulsatile flow of a dusty fluid through a constricted conduit under the influence of magnetic field has been
considered. Stream functions have been derived for both solid and fluid particles under long wavelength approximation and explicit solutions for pressure gradient and shear stress at the wall have been obtained. The effects of magnetic parameter and mass-concentration of solid particles on stream line pattern, velocity profiles of both the particles, pressure distribution and shear stress have been studied.

II. Formulation of The Problem

Following Radhakrishnamacharya [V], we have considered the incompressible fluid flow of a Newtonian, electrically conducting dusty-fluid through a symmetric, thin channel with longitudinal axis \( x \). A uniform magnetic field \( B_0 \) is applied to the fluid in the transverse direction. The boundary of the geometry is expressed by

\[
\eta = d + a \sin \frac{2\pi}{\lambda} x
\]

where \( a \) is height of the wall constriction.

Following Radhakrishnamacharya [V], the governing equations of the fluid containing solid particles, are

\[
\frac{\partial V}{\partial t} + (V \cdot \nabla)V = -\frac{1}{\rho} \nabla p + \nu \nabla^2 V + \frac{kN_a}{\rho} (V_s - V) + \frac{J \times B}{\rho}
\]

(2)

\[
\frac{\partial V_s}{\partial t} + (V_s \cdot \nabla)V_s = \frac{k}{m} (V - V_s)
\]

(3)

\[
\nabla \cdot V = 0
\]

(4)

\[
\nabla \cdot V_s = 0
\]

(5)

where \( V = V(u, v) \) and \( V_s = V_s(u_s, v_s) \) represents the velocities of fluid - solid particles respectively, \( \rho \) is the density of the fluid, \( m \) is the mass of the solid particle, \( p \) is the fluid pressure, \( \nu \) is the kinematic coefficient of viscosity of the fluid, \( k \) is a resistance coefficient of dust particles, \( J \) is the current density, \( \mathbf{B} = \mathbf{B}_0 + \mathbf{B}_1 \) is the overall magnetic field, \( \mathbf{B}_1 \) is the induced magnetic field and

\[
\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}
\]

The boundary conditions are

\[
\begin{align*}
  u &= v = 0 \\
  u_s &= v_s = 0
\end{align*}
\]

at \( y = \pm \eta \) (6)

in connection with the conditions of invariable volume flux. The Maxwell equations and the generalized Ohm’s law are (after neglecting displacement currents) (Srinivasacharya [IV] and Ravi Kiran [VI]),
\[ \nabla \cdot B = 0 \]
\[ \nabla \times B = \mu_0 J \]
\[ \nabla \times E = -\frac{\partial B}{\partial t} \]
\[ \mathbf{J} = \alpha (\mathbf{E} + \mathbf{V} \times \mathbf{B}) \]

where \( \mu_0 \) is the ‘magnetic permeability’ which is assumed to be constant throughout the flow field, \( \mathbf{E} \) is the ‘electric field’ and \( \alpha \) is the ‘electrical conductivity of the fluid’.

Further, eradicating the pressure and selecting the functions \( \psi \) and \( \phi \) such that
\[ u = \frac{\partial \psi}{\partial y}; \quad v = -\frac{\partial \psi}{\partial x} \]
\[ u_s = \frac{\partial \phi}{\partial y}; \quad v_s = -\frac{\partial \phi}{\partial x} \]

and introducing the subsequent non-dimensional quantities
\[ x' = x/\lambda, \quad y' = y/d, \quad \eta' = \eta/d, \quad t' = \nu v/\lambda d, \quad \psi' = \psi'/\nu, \]
\[ \phi' = \frac{\phi}{kd^2/m}, \]

the equations (2) and (3) are transformed to (after dropping primes)
\[ \delta \left[ \frac{\partial^3}{\partial t^3} (\nabla^2 \psi) + \frac{\partial \psi}{\partial y} \frac{\partial^2 \psi}{\partial x^2} - \frac{\partial \psi}{\partial x} \frac{\partial^2 \psi}{\partial y^2} \right] = \nabla^4 \psi + \alpha (\frac{1}{R} \nabla^2 \phi - \nabla^2 \psi) - M e \frac{\partial^2 \psi}{\partial y^2} \]
\[ \delta \left[ R \frac{\partial}{\partial t} (\nabla^2 \phi) + \frac{\partial \phi}{\partial y} \nabla^2 \phi - \frac{\partial \phi}{\partial x} \nabla^2 \phi - \frac{\partial \phi}{\partial y} \nabla^2 \phi \right] = R \nabla^2 \psi - \nabla^2 \phi \]

The modified boundary conditions are
\[ \frac{\partial \psi}{\partial y} = \frac{\partial \psi}{\partial x} = 0 \]
\[ \text{at } y = \pm \eta \]
\[ \frac{\partial \phi}{\partial y} = \frac{\partial \phi}{\partial x} = 0 \]
\[ (\psi)_{y=\eta} = \text{Const}; (\psi)_{y=0} = 0 \]
where \( \nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \), \( \varepsilon = \left( \frac{d}{a} \right) \) is the amplitude ratio, \( R = \frac{um}{kd^2} \) and \( \alpha = \frac{kN_0 d^2}{\rho u} \) are non-dimensional parameters and \( M = \frac{\alpha^2 \beta^2 d^2}{\rho u} \) is the magnetic parameter.

III. Method of Solution

Equations (10) and (11) are non-linear and it is not feasible to find exact solutions for random values of distinct parameters. Hence, assuming that the slope of the channel \( \delta \) is very small, \( \psi \) and \( \phi \) may be expressed as

\[
\begin{align*}
\psi &= \psi_0 + \delta \psi_1 + \delta^2 \psi_2 + \ldots \quad (14) \\
\phi &= \phi_0 + \delta \phi_1 + \delta^2 \phi_2 + \ldots 
\end{align*}
\]

Substituting (14) in (10) - (13) and accumulating terms of identical powers of \( \delta \), both order zero and order one, the subsequent perturbed equations are obtained.

Zeroth order:

\[
\begin{align*}
\frac{\partial^4 \psi_0}{\partial y^4} + \alpha \left( \frac{1}{R} \frac{\partial^2 \phi_0}{\partial y^2} - \frac{\partial^2 \psi_0}{\partial y^2} \right) - M^2 \frac{\partial^2 \psi_0}{\partial y^2} &= 0 \\
R \frac{\partial^2 \psi_0}{\partial y^2} - \frac{\partial^2 \phi_0}{\partial y^2} &= 0
\end{align*}
\]

\[
\begin{align*}
\frac{\partial \psi_0}{\partial y} &= \frac{\partial \psi_0}{\partial x} = 0 \\
\frac{\partial \phi_0}{\partial y} &= \frac{\partial \phi_0}{\partial x} = 0
\end{align*}
\]

at \( y = \pm \eta \) \quad (16)

(\( \psi_0 \))\(_y = \text{constant}, Q \text{ (say)}; \quad (\psi_0 )\(_{y = 0} = 0 \)

First order:

\[
\begin{align*}
\frac{\partial^4 \psi_1}{\partial y^4} + \alpha \left( \frac{1}{R} \frac{\partial^2 \phi_1}{\partial y^2} - \frac{\partial^2 \psi_1}{\partial y^2} \right) - M^2 \frac{\partial^2 \psi_1}{\partial y^2} &= \frac{\partial^3 \psi_0}{\partial \psi_0 \partial y} + \frac{\partial \psi_0}{\partial \psi_0} \frac{\partial^2 \psi_0}{\partial \psi_0 \partial y} + \frac{\partial \psi_0}{\partial \psi_0} \frac{\partial^2 \psi_0}{\partial \psi_0 \partial y} \\
R \frac{\partial^2 \psi_1}{\partial y^2} - \frac{\partial^2 \phi_1}{\partial y^2} &= R \left( \frac{\partial^2 \phi_0}{\partial \psi_0 \partial y} + \frac{\partial \phi_0}{\partial \psi_0} \frac{\partial^2 \phi_0}{\partial \psi_0 \partial y} + \frac{\partial \phi_0}{\partial \psi_0} \frac{\partial^2 \phi_0}{\partial \psi_0 \partial y} \right)
\end{align*}
\]
Further, we assume
\[
\begin{align*}
\psi_0 &= \psi_{00}(x, y) e^{i\omega t} \\
\phi_0 &= \phi_{00}(x, y) e^{i\omega t} \\
\psi_1 &= \psi_{10}(x, y) + \psi_{11}(x, y) e^{i\omega t} + \psi_{12}(x, y) e^{2i\omega t} \\
\phi_1 &= \phi_{00}(x, y) + \phi_{11}(x, y) e^{i\omega t} + \phi_{12}(x, y) e^{2i\omega t}
\end{align*}
\]

(21)

Substituting eq.(21) in (15) to (20), and following the solution procedure as in Radhakrishnamacharya [V], we get the solutions

\[
\psi = QA_3 \left( \sinh(My) - My \cosh(My) \right) e^{i\omega t} + \delta \left[ E_1 A_1 \left( \frac{y \sinh^2(My)}{\eta} \right) - \frac{\sinh(My) \sinh(My)}{8M^3} - \frac{y}{\eta} A_2 + \frac{E_1 \eta y \cosh^2(My)}{2M^3} - \frac{E_1 \eta y \cosh(My) \cosh(My)}{2M^3} \right] \left[ 1 + \frac{e^{2i\omega t}}{2} \right] + \right.
\]

\[
A_3 \left[ A_2 A_3 \left( -\eta y \cosh(My) \sinh(My) \right) - \frac{\eta \cosh(My) \sinh(My)}{2M^2 \sinh(My)} + \frac{y}{2M^2} \cosh(My) \right] \right]
\]

(22)

Further, we assume
\[
\begin{align*}
\eta_0 &= \eta_{00}(x, y) e^{i\omega t} \\
\zeta_0 &= \zeta_{00}(x, y) e^{i\omega t} \\
\eta_1 &= \eta_{10}(x, y) + \eta_{11}(x, y) e^{i\omega t} + \eta_{12}(x, y) e^{2i\omega t} \\
\zeta_1 &= \zeta_{00}(x, y) + \zeta_{11}(x, y) e^{i\omega t} + \zeta_{12}(x, y) e^{2i\omega t}
\end{align*}
\]

(21)

Substituting eq.(21) in (15) to (20), and following the solution procedure as in Radhakrishnamacharya [V], we get the solutions

\[
\phi = R \psi - R Q e^{i\omega t} + R^2 \delta \left[ 1 + \frac{e^{2i\omega t}}{2} \right] D_1 \left[ \frac{\eta^2}{M} + \frac{\eta \cosh(My) \sinh(My)}{M^2} \right] - \left( \frac{y \cosh(My) \sinh(My)}{M^2} - \frac{2 \sinh(My) \sinh(My)}{M^3} - \frac{\eta \cosh(My) \sinh(My)}{M^2} \right) - \left( \frac{\eta y + y \cosh(My) \sinh(My)}{M^2} - \frac{2 \sinh^2(My)}{M^3} \right) + \right.
\]

(23)
\[ A_i = \frac{\eta^2 \sinh(M\eta)}{M^3} - \frac{1}{8M^4} \eta \cosh(M\eta) + \frac{\sinh(M\eta)}{8M^5} + \frac{\eta^2 \cosh(M\eta)}{4M^2} \]

\[ A_2 = E_i \sinh(M\eta) \left[ \frac{\eta^2 \sinh(M\eta)}{4M^3} - \frac{5}{4M^4} \eta \cosh(M\eta) + \frac{\sinh(M\eta)}{8M^5} \right] \]

\[ A_3 = \frac{1}{(\sinh(M\eta) - M\eta \cosh(M\eta))} \]

\[ A_4 = \frac{\eta}{M^2} - \frac{1}{2M^3} \sinh(2M\eta) \]

\[ A_5 = (1 + S)iM^2 QA_3 \]

\[ E_1 = \left( \frac{1 + S}{2} \right) D_1 \]

\[ D_1 = Q^2 M^2 \sinh(M\eta) \frac{d\eta}{dx} A_3^3 \]

and

\[ S \left( \alpha R = \frac{N_0 m}{\rho} \right) \text{is the 'mass concentration' of dust particles.} \]

The pressure gradient after non-dimensionalisation, obtained as

\[ P = \frac{\partial p}{\partial x} = \nabla^2 \frac{\partial \psi}{\partial y} - \delta \left( \frac{\partial^2 \psi}{\partial y^2} + \frac{\partial \psi}{\partial y} \frac{\partial^2 \psi}{\partial x \partial y} - \frac{\partial \psi}{\partial x} \frac{\partial^2 \psi}{\partial y^2} \right) + \alpha \left( \frac{1}{R} \frac{\partial \phi}{\partial y} - \frac{\partial \psi}{\partial y} \right) \quad (24) \]

In perspective of (21), we assume

\[ P = (P_{00} + \delta P_{01} + \delta^2 P_{02} + \ldots) e^{iwT} \quad (25) \]

Substituting eq.(14), (21) and (25) in (24), likening like terms and on rearrangement, we get

\[ P = \left\{ QM^3 A_3 \cosh(My) + \right. \]

\[ \delta \left\{ \frac{(1 + S)iwQA_3}{2} \left[ M^2 A_3 \cosh(My) \cosh(My) - \frac{\eta^2 M^3 A_3 \cosh(My)}{\sinh(M\eta)} + \right. \]

\[ 3MC \cosh(My) + M^2 \eta \sinh(My) - \frac{\eta M^2 \cosh(My) \cosh(My)}{\sinh(M\eta)} - \]

\[ 2M \cosh(My) + 2M \cosh(My) \left\} \right\} e^{iwT} \quad (26) \]
The shear stress performing on the wall in non-dimensional form is given by

\[ \tau_{xy} | y = \eta = \frac{\tau_{xy}}{\rho d^2} = A_3 Q M^2 \sinh(M\eta) e^{iwt} + \]

\[ \delta \left[ A_3 E\sin^2(h(M\eta)) \left( \frac{\eta \cosh(M\eta)}{8M^2} \right) - \frac{\eta \sinh(M\eta)}{8M^2} \right] \] 

\[ + E_2 \sinh(M\eta) \left( \frac{\eta \sinh(M\eta)}{4M} - \frac{15\sinh(M\eta)}{8M^2} \right) - \frac{E_2 \sinh(M\eta) \cosh(M\eta)}{M^2} \]

\[ + \frac{E_2 \eta^2 \cosh(M\eta)}{2M} \left[ 1 + \frac{\eta^2}{2} \right] + \frac{A_4}{2M} \left[ \eta \cos(k_{cr}M\eta) \sin(k_{cr}M\eta) - M\eta \right] + \frac{2\sin(k_{cr}M\eta)}{M} e^{iwt} \]

(27)

IV. Results Discussion

The streamlines for the fluid particles are shown in Figs.1-5. It can be observed that the constricted boundary has a very little influence on the stream lines closer to the center line. This result agrees with that of Radhakrishnamacharya [V]. It is interesting to note that the streamlines form closed loops for \( wt = \frac{\pi}{2} \). The effect of magnetic parameter \( M \) on streamlines is not significant for \( wt = \frac{\pi}{4} \) but for \( wt = \pi / 2 \), the closed loops get significantly altered. Further, the effect of dust concentration \( S \) on streamline pattern is not very significant.

The velocity profiles of the fluid particles are presented in Figs.6-8. We can observe that the velocity is maximum on the center line of the channel and decreases as \( y \) increases up to a certain value of \( y \) and then increases. The character of the velocity profiles is the same at various cross sections i.e. for different values of \( x \). It can be seen that the velocity becomes negative beyond a certain value of \( y \) and the point at which the velocity becomes negative varies with axial distance \( x \). Further, the velocity decreases with magnetic parameter but increases with dust concentration. However, the increase with dust concentration is very insignificant.

The velocity profiles of the solid particles are shown in Figs.9-11. It can be seen that the velocity is positive for all the values of magnetic parameter and dust concentration. The velocity first decreases down stream and then increases i.e, it initially decreases as \( x \) increases and then increases with \( x \). Further, as magnetic parameter increases, the velocity decreases up to a certain value of \( y \) and then increases. Also, the velocity increases with dust concentration but this increase is very insignificant. Moreover, the velocities of the dust particles near the axis are found to be maximum and also the dust particles demonstrate backflow in the vicinity of the boundaries.

Figs.12-14 show that the shear stress of the fluid acting on the wall increases with magnetic parameter and dust concentration. Further, it also increases with time \( wt \). The shear stress shows oscillatory nature with axial distance.

The pressure distribution for the fluid increases with time \( wt \) (Fig.15) and dust concentration (Fig.16) but decreases with magnetic parameter (Fig.17). The increase in pressure distribution with mass concentration is prominent along the
Also, the change in pressure with the change in Magnetic parameter shows a similar but an inverse behavior to that of dust concentration. Further, there is a non-uniform pressure distribution throughout the channel. This result is in good agreement with the conclusion of Radhakrishnamacharya [V].

Fig. 1 Streamlines for Fluid Particles
\((M=1,S=0.2,Q=0.3, \varepsilon =0.2, \delta =0.2, wt=\pi /4)\)

Fig. 2 Streamlines for Fluid Particles
\((M=1,S=0.2,Q=0.3, \varepsilon =0.2, \delta =0.2, wt=\pi /2)\)
Fig. 3 Streamlines for Fluid Particles
\( (M=5, S=0.2, Q=0.3, \varepsilon =0.2, \delta =0.2, wt=\pi /4) \)

Fig. 4 Streamlines for Fluid Particles
\( (M=5, S=0.2, Q=0.3, \varepsilon =0.2, \delta =0.2, wt=\pi /2) \)
Fig. 5 Streamlines for Fluid Particles
(M=5, S=0.4, Q=0.3, ε =0.2, δ =0.2, wt= π /2)

Fig. 6 Velocity Profiles for Fluid Particles
(M=1, S=0.2, Q=0.3, ε =0.2, δ =0.2, wt= π /2)
Fig. 7 Velocity Profiles for Fluid Particles
(M=5, S=0.2, Q=0.3, \( \varepsilon = 0.2 \), \( \delta = 0.2 \), wt= \( \pi /2 \))

Fig. 8 Velocity Profiles for Fluid Particles
(M=5, S=0.4, Q=0.3, \( \varepsilon = 0.2 \), \( \delta = 0.2 \), wt= \( \pi /2 \))

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Fig. 9  Velocity Profiles for Solid Particles
\((M=1,R=5,S=0.2,Q=0.3, \varepsilon =0.2, \delta =0.2, wt= \pi / 2)\)

Fig. 10  Velocity Profiles for Solid Particles
\((M=5,R=5,S=0.2,Q=0.3, \varepsilon =0.2, \delta =0.2, wt= \pi / 2)\)

Fig. 11  Velocity Profiles for Solid Particles
\((M=5,R=5,S=0.4,Q=0.3, \varepsilon =0.2, \delta =0.2, wt= \pi / 2)\)
Fig. 12  Shear Stress Distribution for the Fluid at the Wall
(M=5, S=0.2, Q=0.3, $\varepsilon = 0.2$, $\delta = 0.2$)

Fig. 13  Shear Stress Distribution for the Fluid at the Wall
(M=5, Q=0.3, $\varepsilon = 0.2$, $\delta = 0.2$, wt= $\pi$/2)
Fig. 14  Shear Stress Distribution for the Fluid at the Wall
(S=0.2,Q=0.3,ε=0.2,δ=0.2,wt=π/2)

Fig. 15  Pressure Distribution of the Fluid
(M=5,S=0.2,Q=0.3,ε=0.2,δ=0.2)
Fig. 16 Pressure Distribution for the Fluid  
\((M=5, Q=0.3, \varepsilon=0.2, \delta=0.2, \text{wt}=\pi/2)\)

Fig. 17 Pressure Distribution for the Fluid  
\((S=0.2, Q=0.3, \varepsilon=0.2, \delta=0.2, \text{wt}=\pi/2)\)

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