Gauge invariant $Z(2)$ vortex vacuum textures and the $SU(2)$ gluon condensate

Kurt Langfeld$^a$, Ernst–Michael Ilgenfritz$^b$, Hugo Reinhardt$^a$

$^a$ Institut für Theoretische Physik, Universität Tübingen
D–72076 Tübingen, Germany

and

$^b$ Research Center for Nuclear Physics, Osaka University
Osaka 567-0047, Japan

Abstract

For $SU(2)$ lattice gauge theory, a new $SO(3)$ cooling procedure is proposed which removes the $SU(2)/Z(2)$ coset fields from the lattice configurations and reveals a $Z(2)$ vortex vacuum texture different from the $P$–vortex content obtained in the maximal center gauge. Cooling can be restricted in a renormalization group invariant way by a parameter controlling the remaining $SO(3)$ action density. A gauge invariant $Z(2)$ vortex vacuum emerges asymptotically if cooling is not restricted. This “vortex texture” does not support the string tension $\sigma$ or a finite part of it. The $SU(2)$ action density associated with the new $Z(2)$ vortex texture has a smooth extrapolation to the continuum limit. We propose an interpretation as a mass dimension four condensate related to the gluon condensate featuring in the operator product expansion.

PACS: 11.15.Ha 14.70.Dj

keywords: center vortex, gauge invariance, confinement, gluon condensate, operator product expansion.
1 Introduction

Yang–Mills theory has become the candidate theory of strong interactions in the seventies. This confidence is based on asymptotic freedom and the validity of perturbation theory at short distances. The long distance/strong coupling regime of this theory, however, defies a sufficient understanding until now. More precisely, there are now several competing models well established on the lattice to incorporate the low energy properties of this theory. In particular, the origin of such an distinguishing feature of strong interactions like quark confinement is still under debate. One of the mechanisms, the center vortex picture will play a prominent rô le in this paper. Our main concern, however, is the intermediate region where the operator product expansion (OPE) is a reliable scheme to describe the onset of non–perturbative physics. We will point out the existence, in $SU(2)$ lattice gluon dynamics, of an alternative center vortex structure which has a good chance to explain the nature of the gluon condensate, the most important parameter of the OPE.

Relatively early in the history of QCD, ’t Hooft [1] has pointed out that choosing a particular gauge might be useful to identify the agents of confinement. In certain Abelian gauges [1], which allow for a residual $U(1)$ gauge degree of freedom, the Yang–Mills ground state appears as a dual superconductor [1, 2] where color–magnetic monopoles of quantized charge play a rô le analogous to Cooper pairs in a superconductor. After the search for monopole excitations in the lattice vacua of non–Abelian gauge theory (via Abelian projection) has started in the late eighties [3, 4], the demonstration of monopole condensation, necessary to generate confinement through a dual Meissner effect, followed in the nineties [5, 6, 7, 8] (for a caveat see [9]). This was demonstrated by evaluating the “disorder parameter” of confinement. Monopole condensation was observed for different Abelian gauges [7].

The idea that vortex free energies might serve as an order parameter for confinement was born at the end of the seventies. It dates back to another pioneering work by ’t Hooft [10] and simultaneous work by Aharonov et al. [11] and was recently confirmed by a numerical investigation [12]. Yoneya [13] and Mack at. [14] were the first to construct $Z(N)$ topological degrees of freedom from gauge invariant variables and pointed out that the so–called center vortices play an important role for the confinement of quarks. Basically, these vortices are defined by the property that they contribute a non–trivial center element $z_\alpha \neq 1$ (among the $N_{\text{color}}$–th roots of unity) to the Wilson loop if they are non–trivially linked to the latter. Random fluctuations of the vortices provide the area law for the Wilson loop, the signature of confinement [14, 15].
A revival of the vortex picture arose with the construction of the \( P \)-vortices on the lattice which could be defined after choosing the so-called maximal center gauge (MCG) \([16]\) and separating out the center elements from the other lattice degrees of freedom, in this case by center projection. For the acceptance of the \( P \)-vortices as physical objects, it was essential that evidence could be presented that they are sensible degrees of freedom in the continuum limit \([17]\): the area density of the \( P \)-vortices as well as their binary interactions properly extrapolate to the continuum \([17]\). Moreover, the \( P \)-vortex picture of the Yang–Mills ground state also gives an appealing explanation for the finite temperature deconfinement phase transition of Yang–Mills theory \([18, 19]\) as the breakdown of vortex percolation.

Subsequently, it turned out that center projection even \textit{without previous gauge fixing} reproduces the \( Q\bar{Q} \) potential, also at short distances, a finding which has somewhat obscured the relevance of the \( P \)-vortices for forming the string tension \([20]\). Moreover, in this case it was observed that other properties of the corresponding vortices (rather than the purely topological features) strongly depend on the bare lattice coupling constant (i.e. the lattice spacing), in a way that made it cumbersome to give a continuum interpretation of the apparent vortex degrees of freedom \([20]\). Thus, center projection without appropriate gauge fixing seems not to be the right way to avoid the apparent deficiency of the need to find an appropriate gauge, to fight with the corresponding technical Gribov problem etc.

Above, vortices and the center degrees of freedom were discussed exclusively under the aspect of confinement. There is a general opinion that the true confiners are random magnetic fluxes which have some transversal extendedness (thick vortices) \([21]\). The maximal center projection (after MCG fixing) however, ends up with a type of \( Z(2) \) gauge field configuration whose vortices (thin or \( P \)-vortices) live on the lattice scale \( a \) and are meant to localize the thick vortices.

In the present paper we are going to identify another vortex structure related to the underlying \( Z(2) \) degrees of freedom which is connected with the non-perturbative dynamics at short distances. In a loose sense, the \( SO(3) \) cooling method we are proposing serves to separate, among the \( P \)-vortices exhibited by center projection, those \( P \)-vortices which in fact represent extended vortices of small non–Abelian action (supported by the \( SO(3) \) part of the gauge field) referred to as \( SO(3) \) vortices, from the real scale \( a \) vortices residing in the \( Z(2) \) part of the gauge group. Only the first ones are in the position to condense and are relevant for confinement. Restricting cooling to a finite ratio of \( SO(3) \) action density to the string tension \( \sigma^2 \), a residual con-
fining force can be defined in a lattice–scale independent way such that the string tension is conserved only outside a certain cooling radius and $SO(3)$ vortices with a thickness smaller than this radius are wiped out. Infinite cooling leaves us with the scale $a$ (thin) $Z(2)$ vortex component alone.

This seems to be a good starting point to establish a connection between the $Z(2)$ gauge field content of $SU(2)$ lattice gluon dynamics and the gluon condensate which (at least in the real world of $SU(3)$ gauge theory) is an important parameter of particle phenomenology. A large body of knowledge on low energy properties (resonance physics) and high energy scattering is incorporated in the gluon condensate and its short range, non–local structure. In the case of the hadronic spectral function approaching intermediate distances the operator product expansion (OPE) for the current–current correlators was the first systematic framework to take into account the non–trivial structure of the vacuum. In this approach, non–perturbative properties of the Yang–Mills (or QCD) vacuum are parameterized by so–called condensates the values of which are fitted to the experimental hadron correlators.

In this paper, we will demonstrate the viability of a new, gauge independent method to separate low from high energy $SO(3)$ degrees of freedom and to suppress high energy gluons. This method exhibits a gauge independent $Z(2)$ center vortex content of the $SU(2)$ vacuum, different from the well–known $P$–vortex structure discovered by center projection. We propose that the gauge invariant $Z(2)$ vortex component accounts for the average energy density after the gluon radiation is subtracted. For this purpose, we will propose cooling with respect to the $SO(3) \equiv SU(2)/Z(2)$ action. This kind of cooling eliminates the $SO(3)$ part of the links which we will refer to in the following as gluons. We find that the $SU(2)$ configurations, asymptotically emerging from the $SO(3)$ cooling procedure can be considered as configurations of an effective $Z(2)$ gauge model embedded into $SU(2)$. These configurations possess an $SU(2)$ action density which properly extrapolates to the continuum limit. We will argue that this action density acquires immediate importance as the mass dimension four condensate figuring in the OPE as gluon condensate.

The outline of the paper is as follows. In the next section we define the new cooling procedure which is based on the $SU(2)/Z(2)$ decomposition of the links. Section 3 contains numerical results characterizing the emerging gauge invariant $Z(2)$ vortex texture. We show there that these vortices do not contribute to the string tension. However, they do account for the gluon condensate as will be shown in section 4. There we also briefly review the operator product approach to hadronic correlators with a special emphasis on
Yang–Mills condensates. The $Z(2)$ vortex texture contribution to the gluon condensate is obtained there. We consider the positive plaquette model in section 5 focusing on the gluon condensate $O_4$ and find that the latter is suppressed by one order of magnitude. Our conclusions are summarized in the final section 6.

2 The gauge invariant $Z(2)$ vortex vacuum texture

2.1 Vortices and gluons

Although the problem could be posed also for $SU(3)$ gluon dynamics, in this paper, we will concentrate on 4–dimensional pure $SU(2)$ lattice gauge theory. The partition function

$$Z = \int \mathcal{D}U \exp\left\{-\beta \sum_p s^W_p\right\}, \quad (1)$$

is a high–dimensional integral over $SU(2)$ matrices $U_l = U_{x,\mu}$ associated with the links $l = \{x, \mu\}$ of the $L^4$ lattice. The inverse bare lattice coupling constant $\beta = 4/g^2$\footnote{The bare lattice coupling $\beta$ should not be confused with the renormalization group $\beta$–function $\beta(g)$.} is considered to be running with the lattice spacing $a$. Thereby, with $\beta \to \infty$, convergence can be achieved towards the continuum limit for dimensionful physical quantities, such as masses, temperatures, string tension, condensates (all given in powers of $a^{-1}$) which are extracted from expectation values with respect to the measure (1). We use the Wilson action with a density given by

$$s^W_p = 1 - \frac{1}{2} \text{tr} U_p, \quad (2)$$

while the full action in (1) is a sum over all plaquettes $p$. $U_p$ is the usual ordered product $U_p = \mathcal{P} \prod_{l \in \partial p} U_l$.

For the following considerations it is useful to introduce, besides the link variable $U_{x,\mu} \in SU(2)$, the adjoint link variable

$$O^{ab}_{x,\mu} = \frac{1}{2} \text{tr} \left\{ U_{x,\mu}, \tau^a U_{x,\mu}^\dagger \tau^b \right\} = O^{ab}[A^b_{\mu}], \quad O^{ab}_{x,\mu} \in SO(3), \quad (3)$$
which do not feel the center degrees of freedom of the links. These adjoint links transform under gauge transformations according

\[ O_{x,\mu}^\omega = \omega_x O_{x,\mu} \omega_{x+\hat{\mu}}^T, \quad \omega_x^{ab} = \frac{1}{2} \text{tr} \left\{ \Omega_x \tau^a \Omega_x^\dagger \tau^b \right\}. \quad (4) \]

Comparing this transformation property with the one of continuum gluon fields, i.e.

\[ A^{\mu}_a(x) = \omega_x^{ab} A^{b\mu}(x) + \epsilon^{aef} \omega^{ec}(x) \partial_{\mu} \omega^{fc}(x), \quad (5) \]

we are led to the identification of the gluon fields as the algebra fields of the adjoint representation, i.e.

\[ O_{x,\mu}^{ab} =: \left[ \exp \{ \epsilon^f A^{\mu}_f(x) a \} \right]^{ab}, \quad (\epsilon^f)^{ac} := \epsilon^{aef}, \quad (6) \]

where \( a \) is the lattice spacing. Here we here propose to distinguish between the gluon fields which span the SO(3) subgroup and the residual \( Z_2 \) vortex degrees of freedom.

### 2.2 Revealing the \( Z(2) \)-vortex vacuum structure

In order to detect the inherent effective \( Z(2) \) gauge model structure, we will remove the gluonic (coset) degrees of freedom from \( SU(2) \) configurations by an appropriate cooling procedure. For this purpose, we define a gluonic action density per link by

\[ s_{x,\mu}^{gl} = \sum_{\nu \neq \pm \mu} \left\{ 1 - \frac{1}{3} \text{tr}_A O_{x,\nu} \right\} = \frac{1}{3} \sum_{\nu \neq \pm \mu} F^{a}_{\mu \nu}[A] F^a_{\mu \nu}[A] a^4 + \mathcal{O}(a^6), \quad (7) \]

where \( O_{x,\mu} \) is the plaquette calculated in terms of the \( SO(3) \) link elements \( O_{x,\mu} \) (4). The sum over \( \nu \) runs over all forward and backward directions orthogonal to \( \mu \). \( F^{a}_{\mu \nu}[A] \) is the (continuum) field strength of the (continuum) gluon fields \( A_{\mu}(x) \) and \( a \) is the lattice spacing.

The new cooling is performed by reducing the gluonic action, i.e. by minimization of \( s_{x,\mu}^{gl} \) with respect to the fields \( O_{x,\mu} \). Inspired by Ref. [23], we include a kind of self-restriction in our method. Further cooling of the adjoint link \( O_{x,\mu} \) is rejected iff the gluonic action is smaller than some threshold value

\[ s_{x,\mu}^{gl} < 8\kappa^4 a^4. \quad (8) \]

Thereby \( \kappa \) is a gauge invariant cooling scale of mass dimension one. For \( \kappa = 0 \), the cooling procedure completely removes the gluon fields from the \( SU(2) \)
lattice configurations leaving only gauge equivalents of $O_{x,\mu} = 1$. Notice that the standard cooling method (self-restricted or not) minimizes the full $SU(2)$ action, thereby affecting center as well as coset degrees of freedom. Another crucial difference to the method in [23] is that the cooling scale there measures the distance of the lattice configurations from classical solutions (instantons) while in our case the cooling scale constrains the action of the gluon fields.

In practice, the cooling procedure works as follows in the $SU(2)$ manifold. The gluonic action density $s_{x,\mu}^g$ can be written in terms of the $SU(2)$ fundamental plaquette $U_{x,\mu}$, i.e.

$$s_{x,\mu}^g = \frac{4}{3} \sum_{\bar{\nu} \neq \pm \mu} \left( 1 - \left( \frac{1}{2} \text{tr} U_{x,\mu\bar{\nu}} \right)^2 \right).$$

(9)

A local (maximal) cooling step amounts to a replacement of the link $U_{x,\mu}$ by the cooled variable $U_{x,\mu}^c$.

$$U_{x,\mu}^c = \lambda \sum_{\bar{\nu} \neq \pm \mu} B_{x,\bar{\nu}} \left( \frac{1}{2} \text{tr} U_{x,\mu\bar{\nu}} \right).$$

(10)

$$B_{x,\bar{\nu}} := U_{x,\bar{\nu}} U_{x+\bar{\nu},\mu} U_{x+\bar{\nu},\bar{\mu}}^\dagger$$

(11)

where $\lambda$ is a Lagrange multiplier ensuring $U_{x,\mu}^c U_{x,\mu}^c = 1$. This local cooling step is disregarded iff

$$1 - \frac{1}{2} \text{tr} \left[ U_{x,\mu} \left( U_{x,\mu}^c \right)^\dagger \right] < \kappa^4 a^4.$$  

(12)

The equations (10-12) define the cooling procedure to be applied in our investigations reported below. Taking into account that the normalization $\lambda$ is given by $\lambda = 1/6 + \mathcal{O}(a^4)$, the condition (12) agrees with (8) up to order $\mathcal{O}(a^6)$. One cooling sweep consists of updating once all links of the lattice in sequential order according (11), i.e. $U_{x,\mu} \rightarrow U_{x,\mu}^c$. After a finite number of cooling sweeps, the local constraint (12) is satisfied all over the lattice implying that there is no change in the link variables $U_{x,\mu}^c$ by further cooling steps. This is how the cooling procedure stops.

2.3 Gauge invariance of the texture

This cooling procedure amounts to a minimization of the $SO(3)$ action density as far as tolerated by the parameter $\kappa$, and brings the $SU(2)$ plaquettes as close as possible to $\pm 1$. In the limit $\kappa \rightarrow 0$, this cooling eliminates the
Figure 1: Separation of the $SU(2)$ action into gluonic radiation (small $f_2$) and vortex vacuum texture (large $f_2$).

$SO(3)$ part of the link variables completely. Hence, the remaining field configuration can be viewed as if generated by an underlying effective $Z(2)$ gauge theory. The field configurations of the latter are thin center vortices. Thus, the above cooling procedure extracts, in the limit $\kappa \to 0$, a structure that we call the $Z(2)$ center vortex content ("vortex texture") of a given $SU(2)$ lattice configuration. These vortices are given by co–closed manifolds of plaquettes equal to $-1$ and carry large Wilson action. The standard Wilson action density can be used as a detector for the $Z(2)$ vortex texture, even at finite $\kappa$. Although the quantity which will be used for identifying the vortices is gauge invariant, one has to make sure that the same vortex structure is obtained by the cooling procedure of subsection 2.2 when two different but gauge equivalent link configurations $\{U_{x,\mu}\}$, $\{U_{x,\tilde{\mu}}\}$, are analyzed. In order to see this, one firstly notes that the staples $B_{x,\hat{\nu}\mu}$ transform homogeneously, i.e.

$$U_{x,\mu} \to U_{x,\mu}^\Omega; \quad B_{x,\hat{\nu}\mu} \to B_{x,\hat{\nu}\mu}^\Omega = \Omega_x B_{x,\hat{\nu}\mu} \Omega_x^\dagger_{x+\hat{\mu}}.$$  \hspace{1cm} (13)

Since the trace of the plaquette is gauge invariant, one finds that the cooled configurations obtained from $\{U_{x,\mu}\}$ and $\{U_{x,\tilde{\mu}}\}$, respectively, differ by the
same gauge transformation
\[ U_{x,\mu} \rightarrow U^c_{x,\mu}, \quad U^{\Omega}_{x,\mu} \rightarrow \Omega_x U^c_{x,\mu} \Omega^\dagger_{x+\mu}. \tag{14} \]

The cooling procedure is thus gauge covariant and the distribution of gauge invariant quantities calculated on cooled configurations is independent of which gauge copy of the initial field configuration cooling is applied to.

In summary, the above introduced \(SO(3)\) cooling facilitates a gauge invariant detection of the \(Z(2)\) center vortex content of (embedded in) a \(SU(2)\) lattice configuration. As will become clear in the following, this vortex structure does not coincide with the \(P\)-vortices extracted by center projecting links after MCG fixing. If there is coincidence with part of the latter, this part becomes insignificant in the continuum limit. We mentioned in the Introduction, that in the limit \(\beta \rightarrow \infty\) the number of (dual plaquettes forming) \(P\)-vortices scales like \(a^2\) while we will find here that the corresponding number for gauge invariant \(Z(2)\) vortices scales like \(a^4\) (for fixed \(\kappa/\sqrt{\sigma} = O(1)\) and in the limit \(\kappa \rightarrow 0\)). This is because our cooling method also removes thick vortices which have a proper support within the \(SO(3)\) subgroup. One may speculate that under cooling with respect to the \(SO(3)\) part of the action, thick vortices disappear by growing in transversal extension. Opposite to this, center projection (after MCG) converts thick vortices into thin ones which can then be detected by a large \(Z(2)\) action density at single plaquettes. This is the reason why in the following we will call thick, confining vortices also \(SO(3)\) vortices.

3 Numerical results

3.1 Clustering of \(SU(2)\) action

Generally, the results of \(SO(3)\) cooling depend on the cooling scale \(\kappa\), given in units of the lattice spacing. In order to express the cooling scale in physical units, we will relate it to the string tension, given also in lattice units. To get this dimensionless ratio, we will adopt the asymptotic scaling law in the 1–loop form
\[ \sigma a^2(\beta) = 0.12 \exp \left\{ -\frac{6\pi^2}{11} (\beta - 2.3) \right\}, \quad \beta \geq 2.1, \tag{15} \]
which fixes the lattice spacing \(a\) in units of the string tension \(\sigma\) for large enough \(\beta\).
In the following, we will employ the trace of the energy momentum tensor $\theta^\mu_\mu$ to exhibit the vortex structure of the vacuum. In the present case of a $SU(2)$ Yang–Mills theory, this tensor is proportional to the Wilson action density $s^W_P$ (see e.g. [24]). We denote the Wilson action carried by an arbitrary plaquette by $f_2 = 1 - \frac{1}{2} \text{tr} U_{x,\mu\nu}$. Let $P(f_2)$ be the corresponding 1–plaquette probability distribution. This distribution is shown in figure 1 where the data come from a simulation on a $12^4$ lattice at $\beta = 2.3$. The peak at small values near $f_2 \sim 0$ can be attributed to gluon radiation, which would be eliminated by cooling completely only in the limit $\kappa/\sqrt{\sigma} \to 0$. This shows how the constraint (12) allows to control the action contained in the form of gluon radiation. As expected, it decreases with stronger cooling (decreasing energy scale $\kappa$). On the other hand, the peak at $f_2 \sim 2$ also grows with increasing cooling. This shows that the contribution of the emerging gauge invariant $Z(2)$ vortex texture to the total action density becomes more and more important with stronger and stronger cooling. As expected, the $SU(2)$ action density concentrates on single plaquettes in the limit $\kappa \to 0$ which are forming the singular vortex vacuum skeleton. Figure 2 shows the space–time distribution of the $SU(2)$ action density on a 2–dimensional hypersurface of a generic configuration generated on a $20^4$ lattice at $\beta = 2.3$. In this plot, the black spots correspond to the maximum value of the action density found on this 2–dimensional hypersurface. One clearly observes how $SO(3)$ cooling leads to the clustering of action density at points where the $Z(2)$ vortices pierce the considered hypersurface.

3.2 Does the gauge invariant $Z(2)$ vortex texture contribute to the string tension?

Let us now investigate the relation between the center vortices of the MCG projection ($P$–vortices) [16] and the gauge invariant $Z(2)$ vortex texture
which is defined by the $Z(2)$ gauge fields remaining after the $SO(3)$ cooling described above. The contribution of these $Z(2)$ vortices to the string tension is useful to look at. It was established that configurations the links of which were projected onto center elements after MCG fixing – the $P$–vortex configurations – reproduce the full string tension $\sigma$ to good accuracy (for most recent results, which also cover a discussion of the practical Gribov problem see [24, 25, 26]).

For comparison, we investigate the static $Q\overline{Q}$ attractive force and contrast the result from cooled configurations (at different cooling scales representing the remaining $SO(3)$ action) to the result obtained in full $SU(2)$ Yang–Mills theory. In each case, we used $\beta$–values ranging from 2.1 to 2.6 to check for scaling of the force as a function of the distance. Our results for a $12^4$ lattice are shown in figure 3. We find that the data points (for no cooling and a given cooling scale) obtained at different $\beta$ fall on top of the same curve, re-
spectively, thus establishing proper scaling also for the potential gained from cooled configurations. This shows the advantage to have a renormalization group invariant formulation for restricted cooling. We find that the cooling procedure strongly affects the force in a range of distances growing with decreasing cooling scale $\kappa/\sqrt{\sigma}$. At short distance this was expected since the behavior at small $r$ is dominated by the exchange of gluons, which is already partially (cf. figure 1) eliminated by cooling. Moreover, for stronger cooling (smaller cooling scale $\kappa$) the value of the full string tension is approached only asymptotically, such that lowering the cooling scale $\kappa/\sqrt{\sigma}$ shifts the asymptotic region to larger distances $r$. The explanation is that $SO(3)$ cooling washes out the $SO(3)$ vortices. We conclude that the gauge invariant $Z(2)$ vortex texture (which survives in the limit $\kappa \to 0$ of unrestricted cooling) is not related to the confinement property of the $SU(2)$ vacuum. In particular, it is not identical with the $P$-vortex ensembles constructed by means of center projection.

After the major part of the confining vortices is removed by cooling, it remains to be clarified which specific physical significance could be assigned to the exposed $Z(2)$ vortex ensembles. Following the original version of this paper, it was observed in [27] that the masses of the low-lying glueballs, $O^+$ and $2^+$, are rather insensitive against the here proposed cooling. Below, we will give further arguments supporting the idea that these configurations are not just lattice artefacts but play an important rôle in the non-perturbative physics at intermediate distances dealt with in the OPE.

4 Hadronic correlation functions and OPE

4.1 The operator product expansion

In this section, we will briefly review the operator product expansion (OPE), which is the framework to discuss the properties of hadronic resonances [22], and the inherent ambiguities. To be specific, consider the hadronic current correlation function in the vector meson channel,

$$M^{\mu\nu}(q) = i \int d^4x e^{iqx} \langle \Omega | T j^\mu(x) j^\nu(0) | \Omega \rangle ,$$

where $|\Omega\rangle$ denotes the true ground state of Yang–Mills theory. Current conservation implies

$$M^{\mu\nu}(q) = (q^\mu q^\nu - q^2 g^{\mu\nu}) M(q^2) .$$
According to Wilson [28], the bilocal, time–ordered product of two operators at short distance can be written in terms of local operators

\[
T j^\mu(x) j^\nu(0) = \sum_{n=0}^{\infty} C_n(x) \hat{O}_n(0), \quad \hat{O}_0 = 1,
\]

where \( n \) labels the canonical mass dimension of the local operators \( \hat{O}_n \). The Wilson coefficients \( C_n(x) \) contain the singular behavior occurring when the point splitting is removed as \( x \to 0 \). The vacuum expectation values of the operators \( \hat{O}_n \) correspond to physical observables, which are called condensates. Hence, sandwiching (18) with an arbitrary trial state \( |\psi\rangle \) and resorting to a dimensional analysis, one finds

\[
M(q^2) = C_0^\psi(q^2) 1^\psi + \frac{O_2^\psi}{q^2} + \frac{O_4^\psi}{q^4} + \mathcal{O}(1/q^6), \quad O_n^\psi := \langle \psi|\hat{O}_n|\psi\rangle.
\]

(19)

Since an infinite number of degrees of freedom is encoded in the wave function \( |\psi\rangle \), the matrix element of the unit operator, \( 1^\psi = \langle \psi|\hat{1}|\psi\rangle \) is generically depending on the state under discussion. Usually this is included in the coefficient function. Therefore the function \( C_0^\psi(q^2) \) in (19) does depend on the state (see below and [29] for an illustration). This fact is just a reflection of the scale anomaly [30]. The other Wilson coefficient functions are defined in (18) without reference to a particular state. Note also that a logarithmic dependence of \( C_0^\psi(q^2) \) on \( q^2 \) is not excluded by the dimensional analysis. In particular, choosing \( |\psi\rangle = |0\rangle \), the perturbative vacuum satisfying \( \hat{O}_n^0 = 0 \) for \( n > 0 \), yields

\[
C_0^0(q^2) = M_{\text{pert}}(q^2),
\]

(20)
where \( M_{\text{pert}} \) is the correlator (17) calculated by summing perturbative diagrams up to a finite order using tree–level propagators and vertices. In this case, one finds up to second order in the gauge coupling constant \( g \)

\[
C_0^0(q^2) = -\frac{1}{8\pi^2} \left( 1 + \frac{\alpha_s}{\pi} \right) \ln \frac{q^2}{\nu^2} + \mathcal{O}(\alpha_s^2) ,
\]

(21)

where \( \nu \) is the renormalization point and \( \alpha_s = \frac{g^2}{4\pi} \).

It has been known for some time that the condensates are ambiguous since a splitting like (19) into a so–called perturbative part and contributions from the OPE corrections is not well–defined (see e.g. [31]). This conclusion is based on the observation that a partial re–summation of perturbative diagrams to all orders can generate, after renormalization, contributions \( \exp(-\frac{1}{g^2}) \) to correlators. Containing essential singularities in the coupling constant, these contributions are imitating OPE condensate corrections (see e.g. [31]). These results do not invalidate the considerations above since the re–summation of perturbative diagrams to all orders bears the potential to change the properties of the reference state \( |\psi\rangle \) under discussion. This has been illustrated for the two–dimensional Gross–Neveu model in Ref. [29]. The model is solvable in the large \( N_f \) limit where \( N_f \) is the number of fermion flavors. In leading order of the \( 1/N_f \)–expansion, the ladder re–summation of perturbative diagrams results in an exact, non–perturbative gap equation which admits a non–trivial solution corresponding to the true ground state for \( N_f \to \infty \), which we call \( |BCS\rangle \). Doing this re–summation of all perturbative diagrams which contribute in the large \( N_f \) limit yields

\[
M(q^2) = C_0^{BCS}(q^2) + \frac{O_1^{BCS}}{q^2} + \frac{O_4^{BCS}}{q^4} + \mathcal{O}(1/q^6) .
\]

(22)

In this example, the numbers \( O_n^{BCS} \), which vanish in the infinite \( N_f \) limit, account for correlations which are not present in the wave function \( |BCS\rangle \) and which become non–negligible at finite \( N_f \). Moreover, \( C_0^{BCS}(q^2) \) contains already the power–like OPE corrections. If one does not perform the summation over an infinite number of diagrams, one obtains from the OPE

\[
M(q^2) = C_0^0(q^2) + \frac{O_2^0}{q^2} + \frac{O_4^0}{q^4} + \mathcal{O}(1/q^6) ,
\]

(23)

where \( C_0^0(q^2) \) is calculated from a large but finite number of perturbative diagrams (see (20)). The superscript 0 signals that the condensates \( O_n^0 \) carry all the information on the true vacuum state, since contributions which could imitate a condensate are lacking in \( C_0^0(q^2) \). In particular, the parameter \( O_2^0 \)
reflects the dynamical generated fermion mass which is present in the true vacuum of the infinite $N_f$ limit but goes beyond perturbation theory.

Both (23) and (22), respectively, represent the unique answer for $M(q^2)$ at large values of $q^2$. The example of the Gross–Neveu model demonstrates that choosing for $|\psi\rangle$ the true ground state $|\Omega\rangle$ does not necessarily imply a unique definition of the condensates $O^\Omega_n$. In addition, usually one demands that, with $M_{\text{per}}$ given in (21),

$$C^\Omega_0(q^2) = M_{\text{per}}(q^2),$$

which is indeed satisfied in the case of (23). The summation of a finite (even large) number of perturbative diagrams cannot imitate the renormalization group invariant dependence of the condensates on the coupling $g$. Therefore, the choice (24) is specific in as far it shifts the maximum content of information on the non–trivial properties of the true vacuum $|\Omega\rangle$ to the condensates $O^\Omega_n$.

### 4.2 The Yang–Mills condensate on the lattice

The local condensates $O_n$ entering the OPE (18) are required to be physical observables. Hence, in the case of Yang–Mills theory, they consist of gauge invariant and renormalization group invariant quantities. In the present paper, we will concentrate onto the condensate $O_4$ of canonical mass dimension four.

Using the trace of the energy momentum tensor $\theta^\mu_\mu$ for the operator $\hat{O}_4$ meets the gauge invariance condition. However, it has been known for a long time that the contribution from gluon radiation yields divergent results for the expectation value $\langle \theta^\mu_\mu \rangle$, hence, violating the second criterion of renormalization group invariance. For later convenience, we illustrate this observation for the case of lattice regularization. Calculating $\theta^\mu_\mu$ by studying scale variations, one finds

$$\langle \theta^\mu_\mu \rangle = \frac{1}{a^4} \frac{d\beta(a)}{d\ln a} \langle 1 - \frac{1}{2} \tr U_p \rangle ,$$

where $\beta(a)$ is the running bare lattice coupling. Using (lattice) perturbation theory one finds that

$$\langle 1 - \frac{1}{2} \tr U_p \rangle = \frac{3}{4\beta} + \mathcal{O}(1/\beta^2).$$

Given (15), this implies that $\langle \theta^\mu_\mu \rangle$ strongly diverges in the continuum limit $a \to 0$. 

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In order to arrive at a sensible (i.e. gauge and RG invariant) definition of $O_4$, one closely follows the perturbative approach augmented by OPE corrections \cite{22} and defines

$$O_{4}^{\text{sub}} := \langle : \theta_\mu^\mu : \rangle := \langle \theta_\mu^\mu \rangle^{\text{sub}} \quad (26)$$

where $\langle \theta_\mu^\mu \rangle^{\text{sub}}$ denotes the expectation value from which the contribution from gluon radiation is subtracted. A technique subtracting perturbative contributions was firstly employed in lattice calculations in the early eighties \cite{32, 33, 34}, and was pursued to high order in a recent study of the OPE corrections \cite{35}.

By subtracting perturbative gluonic parts, both, the continuum approach which distinguishes between perturbative gluon radiation and non–perturbative vacuum properties as well as the lattice approach, have the shortcoming that they do not specify the remnants of the vacuum which form the gluon condensate once gluonic contributions have been subtracted to all orders.

Here, we define the condensate through the subtraction of the contribution of the gluons (i.e. subtracting the $SO(3)$ coset fields) by means of the above cooling procedure rather than by subtracting perturbation theory, i.e.,

$$O_{4} := \lim_{\kappa \to 0} O_{4}(\kappa) ,$$

$$O_{4}(\kappa) a^4 = \frac{24}{\pi^2} \left\langle 1 - \frac{1}{2} \text{tr} U_p \right\rangle_{SO(3) \text{ cooled with scale } \kappa} .$$

This definition amounts to the choice - in the sense of eq. (24) -

$$C_0^0(q^2) = M_{SO(3)}(q^2) .$$

(28)

We point out that $M_{SO(3)}$ not necessarily contains only perturbative gluon contributions. Hence, in general, $M_{SO(3)} \neq M_{\text{per}}$.

This identification offers the appealing feature that the condensate of mass dimension four, $O_4$, could be given by the vacuum energy density of the $Z_2$ gauge system which remains after suppressing the contribution of the coset fields by the $SO(3)$ cooling method. It remains to show that the condensate $O_4$ \cite{28} properly extrapolates to the continuum limit $a \to 0$, which will be done in the next subsection.

### 4.3 Numerical results for the $Z(2)$ vortex induced gluon condensate

In section \ref{sec:results}, we have demonstrated a string like clustering of $SU(2)$ action density which corresponds to a gauge invariant $Z(2)$ vortex vacuum texture.
In this subsection, we will show that, in addition to gauge invariance, this action density, which is carried exclusively by the $Z(2)$ gauge fields, possesses the correct renormalization group dependence on the $SU(2)$ inverse coupling $\beta$ inherited from the full $SU(2)$ lattice gluon dynamics. Hence, this action density can be considered as a physical observable and can be used to define the mass dimension four condensate $O_4$ as outlined in the previous subsection.

Recalling (28), $O_4$ should be compared with what is defined in the literature as the gluon condensate

$$\left\langle \frac{1}{4\pi^2} : F_{\mu\nu}^a F_{\mu\nu}^a : \right\rangle,$$

which is obtained from fits to the hadronic spectral function \cite{22}. We emphasize, however, that there is abuse of notation in (29) since the field strength $F_{\mu\nu}^a$ is defined in terms of the coset (gluon) fields which do not contribute to the above defined condensate $O_4$ by construction. First results which employ the Wilson loop for extracting the gluon condensate were obtained in the
Figure 6: The gluon condensate $O_4$ in units of the string tension $\sigma$ squared, as function of the cooling scale $\kappa/\sqrt{\sigma}$ (see text).

The most recent value for the non–natural case of the pure $SU(2)$ gauge theory is given by

$$O_4 \approx 0.15 \text{ GeV}^4$$

and was calculated from field strength correlation functions (for a recent review of the method see e.g. [37]).

In order to assign a physical meaning to the quantity (28), it is important to check whether this yields a finite and non–vanishing value in the continuum limit $a \to 0$. We have calculated $f_2 a^4 = \pi^2 G_2 a^4 / 24$ as function of $\beta$ for cooled configurations, fixing the cooling scale $\kappa^2 = 0.5 \sigma$. The result is shown in figure 5. Our results nicely meets with the expectation from perturbative scaling (dashed line) for $\beta > 2.2$. This observation only tells us that we can safely extract a signal for the desired expectation value which remains valid in the continuum limit. Again, it turns out advantageous to have the
self-restriction of the cooling in a scale independent way.

In order to show the stripping off of the gluonic contributions from the OPE parameter $O_4$ (28), we can follow the limit $\kappa \to 0$. The result for $O_4(\kappa)$ (28) as function of $\kappa$ given in physical units is presented in figure 3. Data from various $\beta$ in the scaling region $\beta > 2.2$ fall on the same curve. For $\kappa > \sqrt{\sigma}$, a significant contribution of the gluon radiation to the "gluon condensate" is still present, and one asymptotically expects $O_4(\kappa) \propto \kappa^4$. This behavior is confirmed by our lattice data (see figure 3). Roughly at the confining scale, set by $\kappa \approx \sigma^{1/2}$, the contribution of the gluons to $O_4(\kappa)$ becomes comparable with the gluon condensate carried by the $Z(2)$ vortex vacuum texture. For clearly displaying the contribution of the texture, we compare the lattice data with the model fits

\[ O_4/\sigma^2 = a_0 + a_1 \kappa^4, \quad \text{(fit A)} \]
\[ O_4/\sigma^2 = b_1 \kappa^4, \quad \text{(fit B)} \]

In both cases, the terms proportional to $\kappa^4$ parameterize the gluonic contribution while the $a_0$ term of fit A specifies the $Z_2$ vortex content. The lattice data clearly favorites fit A. We find it convincing that the "gluon condensate" $O_4(\kappa)$ approaches a finite value in the limit $\kappa \to 0$ which is roughly consistent with the known value (30) (dash–dotted line in figure 3).

5 The positive plaquette model (PPM)

The numerical results presented above were obtained with the $SU(2)$ Wilson action, which includes a definite prescription of the interaction between center and coset fields. The question arises how the residual action density which is carried by the $Z(2)$ gauge fields after $SO(3)$ cooling does depend on the choice of the lattice action used in the simulation. In order to get some information on this dependence, we adopt an extreme point of view in this section and repeat the analysis of the previous section using the positive plaquette model (PPM). This model is defined by the partition function

\[ Z_{\text{ppm}} = \int \mathcal{D}U \prod_{x,\mu\nu} \theta(\tr U_{x,\mu\nu}) \exp \left\{ \frac{\beta}{2} \tr U_{x,\mu\nu} \right\}. \quad (31) \]

The Gibbs weight is that of the Wilson action up to the fact that link configurations which would lead to negative plaquettes are rejected. One expects that the latter constraint strongly influences the asymptotic $Z(2)$ gauge field configurations remaining after cooling.
A set of low energy quantities – the string tension, the glueball masses and the topological susceptibility – have been studied in great detail within the PPM in [38]. It was found that such observables in units of the string tension become independent of the lattice regulator $a$ when the continuum limit is approached within a certain scaling window $\beta \in [1.3, 2.1]$. Moreover, the values of the above observables quantitatively agree with the values obtained with Wilson action. However, the renormalization group scaling of the lattice spacing with $\beta$ does not match with the expectations from continuum perturbation theory in the investigated scaling window of the PPM. In fact, a scaling

$$\sqrt{\sigma} a(\beta) = 0.36 - 0.3 (\beta - 1.3), \quad \beta \in [1.3, 2.1]$$  \hspace{1cm} (32)

is consistent with the numerical data presented in [38]. In order to test our algorithm for simulations of the PPM, we have re–calculated the quark–antiquark force in physical units using $a(\beta)$ (32) as input. We could verify
that the data points obtained for several values of the lattice spacing fall on top of the same curve (see fig. 7).

The continuum limit, however, is problematic to assess. Notice that we cannot assume that (32) is valid upto \( a = 0 \), otherwise the equation suggests a strong coupling UV fix point at

\[
g_{\text{fix}} := g(a \to 0) \approx 1.26, \quad (\beta = 4/g^2) \tag{33}
\]

and a renormalization group \( \beta \)-function of

\[
\bar{\beta}(g) \approx \frac{g}{\Lambda} \frac{d\Lambda}{dg} = 1.25 - \frac{2}{g^2} \tag{34}
\]

where we have defined the UV cutoff by \( \Lambda = \pi/a \). Such a fix point contradicts asymptotic freedom. The equations (33,34) are based on extrapolation of numerical data to the continuum limit. Therefore, it cannot be excluded that the onset of perturbative physics is postponed to the region \( \beta > 2.1 \). Note, however, that at \( \beta = 2.1 \) the UV cutoff is of order \( \Lambda \approx 11 \text{ GeV} \) (if we take the reference scale \( \sqrt{\sigma} = 440 \text{ MeV} \)), where perturbative scaling should dominate.

Cooling (as well as gauge fixing) is a non–local procedure on the link configurations. Therefore, it is not excluded that we obtain negative plaquettes

Figure 8: The \( SU(2) \) action density as function of the cooling scale \( \kappa \) in the PPM.
after $SO(3)$ cooling although we started from a configuration with positive plaquettes only\footnote{The same $SO(3)$ action is used for the cooling, the positive plaquette constraint is ignored during the cooling.}. Formally repeating the analysis of the previous section for the PPM, we present our result for the “gluon condensate” as function of the cooling scale in figure 8 (left panel), compared with the corresponding result obtained with Wilson action in a double–logarithmic plot. Again the condensate value for each cooling scale does not depend on the bare inverse coupling $\beta$ of the PPM as long as the scaling (32) is applied. We find in the case of the PPM that the asymptotic value (for $\kappa \to 0$) is non–vanishing, but significantly smaller than in the case of the Wilson action. In fact, we find that our data for $O_4(\kappa)$ are well reproduced by the fit (see the semi–logarithmic plot in figure 8, right panel)

$$O_4(\kappa) = 0.45 \sigma^2 \exp\left\{2.63 \frac{\kappa}{\sqrt{\sigma}}\right\} .$$

Extrapolating (35) to $\kappa = 0$, we find that the gluon condensate of the PPM is non–vanishing, but one order of magnitude smaller than the gluon condensate if the Wilson action is used.

The fact that we have obtained a renormalization group invariant and non–vanishing gluon condensate also in the PPM, in which the plaquettes are constrained to be positive, stirs the hope that our above defined gluon condensate gets physical significance independent of the choice of the lattice action. We attribute the fact that we don’t find the same value of the condensate in the case of Wilson action and in the PPM to the deficiency of the latter to match with perturbative scaling at very short distance. This is because the gluon condensate appears as the first correction to a perturbative calculation.

6 Conclusions

In this paper, we have separated the $SU(2)$ gauge field degrees of freedom into thin $Z(2)$ center vortices and $SO(3)$ coset fields. Since the $SO(3)$ coset fields are isomorphic to algebra valued fields, these degrees of freedom have been identified with the gluonic ones.

A new self–restricted cooling algorithm which reduces the $SO(3)$ action of the coset fields facilitates the gradual removal of the gluon fields from the lattice configurations while preserving the center degrees of freedom.
SO(3) cooling procedure is gauge covariant. Hence, the remaining SU(2) Wilson action density reveals the gauge invariant Z(2) vortex texture of the SU(2) vacuum.

Extracting the string tension for several values of the gauge invariant cooling scale $\kappa/\sqrt{\sigma}$, we have found that the string tension vanishes in the limit of unlimited cooling. This shows that the Z(2) vortex ensembles remaining after SO(3) cooling cannot be identified with the confining P–vortices found in the MCG.

The operator product expansion (OPE) does not offer an unambiguous prescription for identifying the condensates. We have suggested here an appealing picture: the mass dimension four condensate is given by the action density of the effective Z(2) gauge model configurations which remain after the SO(3) cooling procedure.

This proposal gets support from the following numerical observations: first, the SU(2) action density for a given cooling scale properly scales towards the continuum limit; second, this action density approaches a renormalization group invariant constant in the limit of infinite cooling, when the SU(2) field is reduced to its Z(2) (vortex) content. This quantity gets immediate importance as the gluon condensate figuring in the OPE approach.

Acknowledgments:
We thank M. Engelhardt for critical discussions. K. L. gratefully acknowledges discussions with G. Burgio and F. Di Renzo.

References

[1] G. ’t Hooft, Nucl. Phys. B79 (1974) 276;  
G. ’t Hooft, in: High energy physics , Bologna 1976;  
G. ’t Hooft, Nucl. Phys. B190 (1981) 455.

[2] S. Mandelstam, Phys. Rep. C23 (1976) 245.

[3] A. S. Kronfeld, G. Schierholz, U.-J. Wiese, Nucl. Phys. B293 (1987) 461;  
F. Brandstater, U. J. Wiese and G. Schierholz, Phys. Lett. B272 (1991) 319.

[4] V. G. Bornyakov et al., Phys. Lett. B261 (1991) 116.
[5] T. L. Ivanenko, A. V. Pochinsky and M. I. Polikarpov, Nucl. Phys. Proc. Suppl. B 30 (1993) 565.

[6] H. Shiba and T. Suzuki, Nucl. Phys. Proc. Suppl. B 34 (1994) 182; Phys. Lett. B351 (1995) 519; Phys. Lett. B395 (1997) 275.

[7] L. Del Debbio, A. Di Giacomo, G. Paffuti and P. Pieri, in: Quark Confinement and the Hadron Spectrum, proceedings. N. Brambilla and G. M. Prosperi (Eds.), World Scientific, 1995; Nucl. Phys. Proc. Suppl. B 42 (1995) 234; Phys. Lett. B355 (1995) 255; A. Di Giacomo, B. Lucini, L. Montesi and G. Paffuti, Phys. Rev. D61 (2000) 034503,034504.

[8] K. Schilling, G.S. Bali and C. Schlichter, Nucl. Phys. Proc. Suppl. 73 (1999) 638.

[9] K. Langfeld and A. Schäfke, Phys. Rev. D61 (2000) 114506.

[10] G. ’t Hooft, Nucl. Phys. B138 (1978) 1.

[11] Y. Aharonov, A. Casher and S. Yankielowicz, Nucl. Phys. B146 (1978) 256.

[12] T. G. Kovacs and E. T. Tomboulis, Phys. Rev. Lett. 85 (2000) 704.

[13] T. Yoneya, Nucl. Phys. B144 (1978) 195.

[14] G. Mack, Phys. Rev. Lett. 45 (1980) 1378; G. Mack and V. B. Petkova, Ann. Phys. (NY) 125 (1980) 117; G. Mack, in: Recent Developments in Gauge Theories, G. ’t Hooft et al. (Eds.), Plenum, New York, 1980; G. Mack and E. Pietarinen, Nucl. Phys. B205 [FS5] (1982) 141.

[15] T. G. Kovacs and E. T. Tomboulis, Phys. Rev. D57, (1998)4054; Nucl. Phys. Proc. Suppl. 63 (1998) 534; Phys. Lett. B443 (1998) 239.

[16] L. Del Debbio, M. Faber, J. Greensite and S. Olejnik, Nucl. Phys. Proc. Suppl. 53 (1997) 141; L. Del Debbio, M. Faber, J. Giedt, J. Greensite and S. Olejnik, Phys. Rev. D58 (1998) 094501.
[17] K. Langfeld, H. Reinhardt and O. Tennert, Phys. Lett. B419 (1998) 317;
   M. Engelhardt, K. Langfeld, H. Reinhardt and O. Tennert, Phys.
   Lett. B431 (1998) 141.

[18] K. Langfeld, O. Tennert, M. Engelhardt and H. Reinhardt, Phys.
    Lett. B452 (1999) 301;
    M. Engelhardt, K. Langfeld, H. Reinhardt and O. Tennert, Phys.
    Rev. D 61 (2000) 054504.

[19] J. Gattnar, K. Langfeld, A. Schäfke and H. Reinhardt,
    Phys. Lett. B489 (2000) 251.

[20] M. Faber, J. Greensite and S. Olejnik, JHEP 9901 (1999) 008;
    M. C. Ogilvie, Phys. Rev. D59 (1999) 074505.

[21] J. Greensite, M. Faber and S. Olejnik, Nucl. Phys. Proc. Suppl. 73
   (1999) 572;
   M. Faber, J. Greensite and S. Olejnik, Phys. Rev. D57 (1998) 2603.

[22] M. A. Shifman, A. I. Vainshtein and V. I. Zakharov, Nucl. Phys.
    B147 (1979) 385;
    A. J. Buras, Rev. Mod. Phys. 52 (1980) 199.
    P. Pascual and R. Tarrach, QCD renormalization for the practitioner,
    Springer 1984;
    Stephan Narison, QCD spectral sum rules, World Scientific, 1989.

[23] M. Garcia Perez, O. Philipsen and I. Stamatescu, Nucl. Phys. B551
    (1999) 293.

[24] V. G. Bornyakov, D. A. Komarov, M. I. Polikarpov and A. I. Veselov,
    JETP Lett. 71 (2000) 231.

[25] R. Bertle, M. Faber, J. Greensite and Š. Olejník, JHEP 0010 (2000)
    007;
    Nucl. Phys. Proc. Suppl. 94 (2001) 482.

[26] J. D. Stack and W. Tucker, Nucl. Phys. Proc. Suppl. 94 (2001) 529.

[27] K. Langfeld and A. Schafke, Phys. Lett. B493 (2000) 350.

[28] K. G. Wilson, Phys. Rev. 169 (1969) 1499.

[29] K. Langfeld, L. von Smekal, H. Reinhardt, Phys. Lett. B362 (1995)
    128.

25
[30] J. C. Collins, A. Duncan and S. D. Joglekar, Phys. Rev. D16 (1977) 438.

[31] V. I. Zakharov, Nucl. Phys. B385 (1992) 452.

[32] J. Kripfganz, Phys. Lett. B101 (1981) 169.

[33] R. Kirschner, J. Kripfganz, J. Ranft and A. Schiller, Nucl. Phys. B210 (1992) 567.

[34] E. M. Ilgenfritz and M. Müller-Preussker, Phys. Lett. B119 (1982) 395.

[35] G. Burgio, F. Di Renzo, G. Marchesini and E. Onofri, Phys. Lett. B422 (1998) 219.

[36] E. M. Ilgenfritz, Field Strength Correlators and the Instanton Liquid, to appear in the proceedings of the International Symposium on "Quantum Chromodynamics and Color Confinement", Osaka, Japan, March 7-10, 2000.

E.-M. Ilgenfritz, S. Thurner, SU(2) Field Strength Correlators: a Comparison of Cooling and RG Smoothing, in preparation.

[37] A. Di Giacomo, Lectures at 17th Autumn School: QCD: Perturbative or Nonperturbative?, Lisbon, Portugal, 29 Sep - 4 Oct 1999, e-Print Archive: hep-lat/9912010.

[38] J. Fingberg, U. M. Heller, V. Mitrjushkin, Nucl. Phys. B435 (1995) 311.