Investigation of the influence of narrowing annular channel and Reynolds number on formation of Tayler- Görtler vortexes

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Abstract. The article investigates the Taylor-Görtler vortices arising in a swirling flow when the annular channel narrows. Several options for the geometry of the narrowed annular channel are researched. In the first case, the outer cylinder with a constant diameter and the inner cone with a variable cone angle are considered. In the second case, on the contrary an inner cylinder with a constant diameter and an outer cone with a variable cone angle are considered. All geometries were tested at different Reynolds numbers $Re. = 8.3 \cdot 10^3 \ldots 21 \cdot 10^3$. As a result, the analysis of the propagation of secondary vortices along the length of the annular channel is presented.

1. Introduction
Hydrodynamically unstable flows in a coaxial annular channel were first experimentally investigated by Taylor in 1923 [1]. He proved the existence of toroidal vortices in which fluid moves downwash periodically and then in which fluid moves upwash. Further development of research was continued by Goertler in 1955 [2]. He studied the stationary vortices formation that arises due to the imbalance of pressure and centrifugal forces in the boundary layer along concave walls. To describe the effect of instability on the Taylor vortex mechanism, we consider a coordinate system rotating together with the flow core. Then the second Taylor number can be written as:

$$Ta_2 = \frac{w_p^2 r_{in} (r_{out} - r_{in})^3}{\nu^2_{eff}},$$

where $w_p$ – characteristic tangential velocity; $r_{in}$ – internal radius of the annular channel; $r_{out}$ – external radius of the annular channel; $\nu_{eff}$ – effective kinematic viscosity.

The critical number for the formation of secondary currents is $Ta_2 = 1700$. This value is close to the Grashof number. Since the value of the critical Grashof number for the Rayleigh-Benard instability is also close to 1700.

In most of the studies that were considered earlier, vortex flows were formed as a result of the surface rotation. An attempt to investigate secondary vortices in a swirling flow with fixed surfaces was made by Kim and Choi [6]. It was a continuation of Euteneuer’s research [7]. In the study hydrodynamic instability in a swirling flow was caused by the impulsively decelerating of a rotating cylinder. It was shown that a swirling flow is formed when there is the surface impulsively decelerating. Taylor-Görtler
vortices develop in this swirling flow. However, the characteristics of secondary flows in stationary annular channels at constant flow swirling have not been studied fully.

2. Methods
In this article, we investigate the narrowing annular channels with a swirling flow. Geometry options for narrowing annular channels are shown in Figure 1.

A swirling flow is formed in a swirl generator with diameter $D = 259$ mm and length $L = 126.5$ mm. In this study we use a swirl generator with one tangential channel $35 \times 70$ mm. The annular channel narrows due to changing of the ratio of the areas at the exit from the annular channel $f_{ch}^{out}$ at $z = l = 840$ mm, and at the entrance to it $f_{ch}^{in}$ at $z = 0$. The investigated geometric and operating characteristics of the narrowing annular channels at various ratios $f_{ch}^{out}/f_{ch}^{in}$ re are given in Table 1. The studies were carried out with changing of Reynolds number $Re$ from $8.3 \cdot 10^3$ to $21 \cdot 10^3$.

Table 1. Plan of experiment

| $d_1$ | $d_2^{out}$ | Cone angle | $f_{ch}^{out}/f_{ch}^{in}$ | Re  | $d_1^{out}$ | $d_2$ | Cone angle | $f_{ch}^{out}/f_{ch}^{in}$ | Re, $10^3$ |
|-------|-------------|------------|---------------------------|-----|-------------|-----|------------|---------------------------|------------|
| 1     | 184         | 0          | 1                         | 8300| 16          | 152 | 0          | 1                         | 8300       |
| 3     | 21700       |            |                           |     | 18          | 158.9| 0.41      | 0.80                      | 8300       |
| 4     | 178.1       | 0.47       | 0.80                      | 8300| 19          | 158.9| 0.41      | 0.80                      | 8300       |
| 6     | 21700       |            |                           |     | 21          | 158.9| 0.41      | 0.80                      | 8300       |
| 7     | 152         | 171.9      | 0.92                      | 8300| 22          | 158.9| 0.41      | 0.80                      | 8300       |
| 9     | 171.9       | 0.60       | 0.80                      | 8300| 24          | 158.9| 0.41      | 0.80                      | 8300       |
| 10    | 165.5       | 1.36       | 0.40                      | 8300| 25          | 158.9| 0.41      | 0.80                      | 8300       |
| 12    | 21700       |            |                           |     | 27          | 158.9| 0.41      | 0.80                      | 8300       |
| 13    | 158.9       | 1.78       | 0.20                      | 8300| 28          | 158.9| 0.41      | 0.80                      | 8300       |
| 15    | 21700       |            |                           |     | 30          | 158.9| 0.41      | 0.80                      | 8300       |

Numerical simulation is carried out in three-dimensional formulation with using of the software package ANSYS Fluent 15.0. The solution is made on a structured hex mesh. A mesh-independent solution was found. As a result, a grid with 9 million cells is used, where the minimum size is $2.5 \cdot 10^{-6}$ m (at the surfaces $y^+ \approx 1$) and the maximum dimension is $3.5 \cdot 10^{-3}$ m (in the swirling generator). The numerical simulation was verified on the basis of the physical experiment. The relative errors of the experimentally determined values did not exceed 2.7% for the air flow rate and 0.4% for the velocity. Three turbulence models were compared: standard $k$-$\varepsilon$ with curvature corrections; SST $k$-$\omega$ with curvature corrections and Reynolds stress model. Figure 2 shows a comparison of the experimental and
numerical distributions of dimensionless tangential $w_\phi = w_\phi/V_{in}$ and axial $\bar{w}_z = w_z / V_{in}$ ($V_{in}$ - is the average velocity in the input channel) components of the velocity vector in the cross section. The measurements were carried out in the sections of the annular channel at $z = 290$ mm, $z = 538$ mm, and $z = 808$ mm ($z$ is the longitudinal coordinate measured from the swirl generator along the annular channel axis).

Figure 2. Full velocity components in the cross section of the swirl generator.

The velocity distributions most accurately describe the results of calculations obtained using Menter's two-parameter model of shear stresses SST $k$-$\omega$ with curvature corrections, and the solution with the standard $k$-$\varepsilon$ with curve correction model gives the lowest accuracy.

3. Results
The study was carried out for dry air at an inlet temperature of 20 °C. when entering the Reynolds number $Re_{in} = 8.3\cdot10^3...21\cdot10^3$ and Prandtl number $Pr_{in} = 0.7$. One-sided local input into the swirling generator determines azimuthal non-uniformity of total velocity distribution and its components. It leads to asymmetric airflow into the annular channel [1]. Figure 3 shows the distribution of the total velocity components (tangential $w_\phi$, axial $w_z$ and radial $w_r$) in the cross section of the flow swirl generator.

Figure 3. Full velocity components in the cross section of the swirl generator

Based on the figure, it can be confirmed that the tangential component of the velocity is prevalent among all the components of the total velocity. The axial and radial components of the velocity have
lesser significant influence. It should be noted that the swirl generator was smooth in all geometry modifications. Consequently, the aerodynamics in each of the considered cases was identical.

It should also be noted that periodic large-scale unsteady vortices are formed from the leading edge of the outer cylinder of the swirl generator. These vortex structures are characterized by Strouhal numbers equal to 0.22. In turn it leads to the formation of structures like Taylor-Görtler vortices near the outer wall. These secondary streams are distributed over the entire space of the annular gap. In this case the axes of the secondary vortex structures coincide with the direction of motion of the main swirling flow.

In Figure 4 (a), where a physical experiment is visualizing, confirmation of the presence of secondary flows in annular channels during flow swirling was obtained. This secondary flow has a complex, three-dimensional and unsteady separation flow. The manifestation of instability is explained by the action of centrifugal forces in the boundary layer when flowing around concave surfaces. Figure 4 (b) shows a numerical calculation which qualitatively repeats the experimentally obtained data.

\[ \frac{f_{ch, out}}{f_{ch, in}} \]

(a) \hspace{2cm} (b)

**Figure 4.** Secondary flows similar to Taylor-Görtler vortices

First, we consider the case when the diameter of the outer surface \( d_2 \) doesn't change. In this case the diameter of the inner surface at the outlet \( d_{1, out} \) changes in accordance with the data in Table 1. Figure 5 shows the distribution of the radial velocity along the length of the annular channel for various ratios \( f_{ch, out}/f_{ch, in} \) and at \( Re = 21 \cdot 10^3 \).

\[ \frac{f_{ch, out}}{f_{ch, in}} \]

\[ W_r \]

\[ 0 \quad 0.1 \quad 0.2 \quad 0.3 \quad 0.4 \quad 0.5 \quad 0.6 \quad 0.7 \quad 0.8 \]

\[ z = 0.08 \]

\[ f_{ch, out}/f_{ch, in} \]

\[ 1 \quad 0.8 \quad 0.6 \quad 0.4 \quad 0.2 \]

\[ 0 \quad 0.1 \quad 0.2 \quad 0.3 \quad 0.4 \quad 0.5 \quad 0.6 \quad 0.7 \quad 0.8 \]

**Figure 5.** Distribution of the radial velocity along the length of the annular channel when the diameter \( d_{1, out} \) changes with different ratios \( f_{ch, out}/f_{ch, in} \) and at \( Re = 21 \cdot 10^3 \).

When \( d_2 \) is unchanged, the formation of secondary Taylor-Görtler vortices for all \( f_{ch, out}/f_{ch, in} \) begins at \( z = 0.08 \). It is due to the insignificant effect of the narrowing in the initial sections of the annular channel. At ratios \( f_{ch, out}/f_{ch, in} \) from 1 to 0.8 secondary vortices propagate along the entire length of the annular channel. It should be noted that the number of vortices increases when there is moving along the annular channel. At \( f_{ch, out}/f_{ch, in} \geq 0.6 \) the secondary vortices begin to decay at \( z \geq 0.5 \). It is due to the decrease in
the intensity of the flow swirl. Along the length of the annular channel, the axial component of the velocity increases, while the tangential and radial components decrease.

Figure 6 shows the distribution of the radial velocity along the length of the annular channel at various ratios \( f_{ch}^{out}/f_{ch}^{in} \) and at \( Re = 8.3 \cdot 10^3 \).

A decrease in the value of Reynolds number leads to the later formation of secondary Taylor-Görtler vortices at \( z \geq 0.1 \). Due to the decrease in the input speed, more intense decrease in the flow swirl occurs. However, when there is decrease of Reynolds number, the decay of secondary vortex structures occurs at \( f_{ch}^{out}/f_{ch}^{in} \geq 0.6 \). The decay of secondary vortices also occurs at the higher values of the length \( z \).

Next, we consider the case when the diameter of the inner surface \( d_1 \) doesn’t change. In this case, the diameter of the outer surface at the outlet \( d_{2out} \) will change in accordance with the data in Table 1. Figure 7 shows the distribution of the radial velocity along the length of the annular channel for various ratios \( f_{ch}^{out}/f_{ch}^{in} \) and at \( Re = 21 \cdot 10^3 \).

When \( d_1 \) is unchanged, the formation of secondary Taylor-Görtler vortices for all \( f_{ch}^{out}/f_{ch}^{in} \) begins at \( z \geq 0.08 \). Disintegration occurs when the secondary flow \( f_{ch}^{out}/f_{ch}^{in} \leq 0.4 \). In this case the length of the decay of vortex structures is \( z \geq 0.6 \). Figure 8 shows the distribution of the radial velocity along the length of the annular channel at various ratios \( f_{ch}^{out}/f_{ch}^{in} \) and at \( Re = 8.3 \cdot 10^3 \).
When Reynolds number Re decreases, decay of secondary vortices occurs later. Thus, secondary vortex structures are retained at $f_{ch}^{out}/f_{ch}^{in} \geq 0.4$. At $f_{ch}^{out}/f_{ch}^{in} = 0.2$ the decay of vortices begins with a length $z \geq 0.55$.

In all the cases, that were considered, the secondary Taylor-Görtler vortices propagate along the entire length of the annular channel and disintegrate into smaller structures. These structures are destroyed downstream due to secondary instabilities and the development of small-scale turbulence. However, there is a region in which secondary vortices will dominate the flow before they will decay.

4. Conclusion
The results obtained for conical annular channels indicate a noticeable effect of the channel width on the state of secondary vortices. There is a decrease in the flow swirl with a decrease in the width of the beneficial section area $f_{ch}^{out}$. This follows from the stability criterion of the second Taylor number, in which the value of the width of the annular gap is raised to a cube. The restructuring of the flow, that is related to its acceleration and derotation leads to the disintegration and gradual degeneration of secondary flows such as Taylor-Görtler vortex flows. Secondary vortices exist for the longest time in straight annular channels with lower Reynolds numbers Re. However, when there is narrowing of the annular channel, the more uniform distribution of secondary vortices occurs in the cross section.

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5. References
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