Static Analysis of an All-Movable Rudder with Rotational Clearance based on Contact Algorithm

Yiting Wu\textsuperscript{1,*}, Yaobin Niu\textsuperscript{1,\textdegree}, Zhongwei Wang\textsuperscript{1,\textdegree\textbullet}, and Xuan Zou\textsuperscript{2,\textdagger}

\textsuperscript{1}College of Aerospace Science and Engineering, National University of Defense Technology, Changsha, China

\textsuperscript{2}College of Liberal Arts and Sciences, National University of Defense Technology, Changsha, China

\textsuperscript{\textdegree}email: niuyaobin1234@126.com, \textdegree\textbullet}email: wang_zhwei2001@163.com

\textsuperscript{\textdagger}email: zouxuan14@nudt.edu.cn

\textsuperscript{*}Yiting Wu; \textsuperscript{\textdagger}email: wuyiting@nudt.edu.cn

Abstract: The air rudder system is essential to the performance and safety of the aircraft, and the basis of the research about it is to establish a suitable mechanical model. To accurately simulate the contact behavior with gaps in the actual process, the contact algorithm is used to construct the connection between the support and the rudder’s contact surface based on the finite element model, and the mechanical relationship between the two is analyzed. The results show that under different gaps and different friction coefficients, the contact algorithm has high accuracy and credibility for describing the mechanical behavior between the support and the rudder and the presentation of the contact state. This article fully demonstrates the advantages of the contact algorithm in solving the problem of modeling with gaps, and can be a reference for the research of other similar issues.

1. Introduction

The rudder system of an aircraft is composed of an actuator, transmission mechanisms, and rudders [1]. It is mainly used to control the aircraft's attitude and maintain the stability of the flight, and is an essential part of the aircraft [2]. In the design of the rudder’s structure, not only the static strength and stiffness but also the dynamic characteristics should be considered. The dynamic characteristics of the structure are not only related to its mass, stiffness, and other linear links, but also closely related to the gaps and frictions of the connecting parts [1].

In 2007, Korean scholars Lee and Shin [3] built a rudder system model with a rotating clearance, obtained the equivalent torsional stiffness of the rudder, and analyzed the flutter characteristics of the model. Although the research constructs each link of the rudder system wholly and systematically, the article regards each structure as a rigid body in the modeling. The connection between different components is simplified to a simple ideal formula, and the influence of the contact behavior of the connection surfaces of different components in the actual process is ignored. On the basis of this research, some scholars have conducted a series of analyses on the rudder system model [4][5][6], but most of the core methods follow this article.

In 2019, Wang Ben [7] used a nonlinear spring element to equivalently simulate the nonlinear torsional stiffness of an all-movable rudder with clearance, and studied the dynamic response
characteristics of the rudder under different kinds of loads. In 2021, Gu Mingfeng [1] established a dynamic model of the air rudder system with clearance and friction, studied the influence of clearance and swing amplitude on the dynamic characteristics of the rudder system, and proposed an equivalent clearance identification method. The test results were compared to verify the correctness of the theoretical model and the equivalent calculation method.

The shortcomings of the above literature are that the focus of the research on the rudder with clearance is mainly limited to the influence of the clearance on the dynamic characteristics of the rudder (such as natural frequency, flutter speed), and there has been little modeling research on the clearance. The modeling method adopted is relatively rough, usually simplified to a single-point connection, and the mechanical relationship is an ideal formula ignoring the actual shape parameters, which cannot accurately reflect the true connection state of the structure in detail. Moreover, the mechanical model of the gap joint is mainly obtained by experiment, and it is impossible to give an accurate description of mechanical behavior in the design stage.

This paper studies the all-movable rudder of a specific type of aircraft. The gap between the connecting shaft and the rudder shaft was considered. The contact algorithm is used to statically analyze the rudder with the clearance to describe the real mechanical behavior of the connection. The influence of different clearances and different frictions on the mechanical characteristics of the joint is studied.

2. Introduction of all-movable rudder

The rudder system of an aircraft is mainly used to control the aircraft's flight attitude and maintain the stability of the flight. It is usually composed of an actuator, transmission mechanisms, and rudders. The mechanical characteristics of the rudder are essential to the design of the aircraft. There must be some clearance in the rotating mechanism of the rudder system due to the need for fit tolerance and structural wear, such as the clearance between gear and shaft, the clearance between bearing inner ring and rudder shaft, the internal clearance between ball screw-fork pair, and the clearance within bearing itself [8]. The clearance will affect the dynamic characteristics of the rudder, such as natural frequency and flutter speed.

With the continuous improvement of flight speed, the aerodynamic heat of the aircraft is becoming more and more serious. In order to isolate the high-temperature airflow from the outside, the rudder shaft is covered with thermal insulation material, and the connecting shaft is installed between the rudder shaft and the transmission mechanisms to isolate the effect of high temperature on the interior structure [9]. As shown in Figure 1, a simplified connection structure is displayed. It can be seen that there is an assembly gap between the rudder shaft and the square groove of the connecting shaft.

![Fig. 1 Assembly diagram of rudder shaft and connecting shaft](image)

In order to study the influence of the gap between the rudder shaft and the connecting shaft on the mechanical behavior of the structure, the research object designed in this paper is shown in Figure 2.
The materials used for the support and the rudder are all Q235 steel, and the material parameters are recorded in Table 1.

| Material | Density (kg/m³) | E (GPa) | ν |
|----------|-----------------|---------|---|
| Q235     | 7800            | 200     | 0.3 |

The support is fixed on the platform by bolt connection, and the rudder shaft is inserted into the square groove of the support to form a connection, which is used to simulate the clearance joint between the rudder shaft and the connecting shaft. The square grooves of different sizes on the support are relatively connected with the rudder shaft to form keyway connects with different clearance. The size of the rudder shaft is 30mm*30mm*30mm, as shown in Figure 3.
The mechanical behavior between the rudder shaft and the grooves on support (such as contact, collision, friction, etc.) mainly exists in the transverse direction of the rudder shaft. The cross-sectional schematic diagram shown in Figure 4 is used to express the relative positional relationship between the rudder shaft and the grooves.

![Cross-sectional schematic diagram](image)

**Fig. 4** Schematic diagram of the cross section of the connection between rudder shaft and the grooves on the support

### 3. The modeling methods

The finite element model of the structure is established by ANSYS, as shown in fig. 5. The standard contact pairs are established between the rudder shaft and the square groove of support, and between the upper surface of support and the sides of the rudder shaft (when there is no contact, the normal pressure is 0; When the connection is closed, there is normal pressure between the two contact surfaces), and the mechanical behavior between the surfaces is simulated by using the principle of contact algorithm.

![Finite element model diagram](image)

**Fig. 5** Schematic diagram of the finite element model of rudder and support

The basic principle of the contact algorithm: several contact springs are established between the contact pairs. When the two surfaces are separated, the spring does not work. When the contact surfaces
invade each other, the spring generates a force that separates the two. The greater the penetration distance, the greater the spring force. For the same mechanical problem, the greater the contact stiffness, the higher the calculation accuracy, the more iterations the calculation requires to achieve convergence, and the longer the calculation time. In theory, the contact stiffness is infinite, and the intrusion distance between the two is always 0, but numerical simulation can't do the calculation in this case, so the finite contact stiffness is generally selected in the analysis. The value of the contact stiffness has a significant influence on the calculation results, and there are also significant differences in the contact stiffness applicable to different mechanical problems. To reduce the number of iterations, many studies have obtained appropriate contact stiffness through trial calculations. When the contact stiffness is greater than a certain value, the calculation result gradually converges, and the convergent value can be regarded as the accurate result. The appropriate contact stiffness selected is a relatively small value that can obtain accurate results (with an error of less than 5%) [10].

4. Static analysis of keyway fit

4.1. Model’s construction and boundary condition’s setting
The finite element model of the support and the rudder is established by the above method. Gravity is applied to the model, fixed constraints are applied to the inner wall of holes on the support to equal the bolt connection, and the contact between the rudder and the support is established as described above.

![Figure 6](image.png)

Fig. 6 Schematic diagram of rigid connection between rudder shaft and equivalent point

In order to obtain the relationship between the interaction torque and the relative rotation angle between the rudder and the support, the MPC multi-point constraint is used to establish a rigid connection between the rudder shaft and a point (called the "equivalent point") in the rudder shaft’s centerline (Figure 6). The degree of freedom in the UX and UY directions of this point is fixed, and an angular displacement (ROTZ) around the Z-axis (the centerline of the rudder shaft) is applied to this equivalent point. By statically calculating the overall structure and outputting the reaction force (MZ) in the equivalent point, the relationship between the interaction torque (MZ) and the relative angular displacement (ROTZ) of the two can be obtained.

In order to intuitively compare the effectiveness of the contact algorithm for constructing the mechanical relationship between the support and the rudder, the centerline of the rudder shaft coincides with the geometric centerline of the square groove to simulate an unbiased central gap connection. The two-dimensional schematic diagram of the positional relationship between the rudder shaft and the square groove is shown in Figure 7.
4.2. Selection of the contact stiffness

It can be seen from Section 3 that the selection of the contact stiffness has a significant influence on the calculation accuracy and calculation cost. If the stiffness value is too low, the calculation accuracy cannot be guaranteed; if the value is too high, the calculation time cost is too large, or even convergence cannot be achieved. Therefore, a trial calculation is needed to select a suitable contact stiffness.

![Diagram showing the relationship between contact stiffness and interaction torque]

**Fig. 8** The curves of interaction torque between the rudder shaft and the support and the iteration number

When the side length of the square groove is 30.06mm, and the friction coefficient $\mu$ is 0, a rotation angle of 0.003 rad is applied to the rudder shaft, and static calculations are conducted under different contact stiffnesses. The curves of interaction torque between the rudder shaft and the support and the iteration number are shown in Figure 11. The relative contact stiffness is the ratio of the actual contact stiffness to the default contact stiffness.

It can be seen from Figure 8 that when the relative contact stiffness is greater than 100, the number of iterations required for the convergence continues to increase, but the calculation result’s change is very little. When the relative contact stiffness is 100 and 500, the relative difference of the interaction torque between the support and the rudder is only 1.31%. Therefore, the relative contact stiffness of all examples in this paper is taken as 100.
4.3. Research on the mechanical behavior of keyway fits under different clearance

When the friction coefficient is 0, the mechanical relationship between the support and the rudder under different square groove side lengths (30.00mm/30.02mm/30.06mm/30.12mm) is calculated, and the relationship between the interaction torque and the relative rotation angle is obtained as shown in Figure 9.

![Fig. 9 Schematic diagram of mechanical relationship between support and rudder under different clearance](image)

Taking the side length of the square groove as 30.06mm for example, according to the geometric relationship shown in Figure 10, the critical angle when the two structures touch is calculated by formula 1:

$$\theta_0 = \frac{\pi}{4} - \arctan\left(\frac{\sqrt{15^2 \times 2 - 15.03^2}}{15.03}\right) = 0.002 \text{ rad} \quad (1)$$

When $\theta \leq \theta_0$, as the friction coefficient is 0, the interaction torque between the rudder shaft and the support should be 0; When $\theta > \theta_0$, the interaction torque should gradually increase, which is very consistent with the trend shown in Figure 12. When the side length of the square groove takes other values, the same conclusion can be verified in the same way.

When $a=30.06mm$ and $\theta=0.003$ rad, the displacement cloud picture of the rudder and the support is shown in Figure 10. The left picture is the displacement cloud picture of the combined structure, and the right picture is the comparison picture of the rudder before and after the deformation.

![Fig. 10 Displacement cloud picture of rudder and support](image)
The contact stress of the contact surface between the rudder shaft and the square groove of the support is shown in Figure 1. It can be seen from Figure 14 that the contact stress mainly exists near the four edges of the rudder shaft. Among the two adjacent surfaces close to a certain edge, only the side that is compressed during rotation has contact stress, which is very consistent with the actual physical process.

Fig. 11 Contact stress cloud diagram of rudder shaft and support

In summary, it can be concluded that the contact algorithm has high accuracy and credibility for the description of the mechanical behavior and the contact state between the support and the rudder under different gaps and the restoration of the contact state.

4.4. Research on the mechanical behavior of keyway fits under different friction coefficients

When a=30.06mm and $\mu=0/0.05/0.15/0.3$ respectively, with the repeated loading and unloading of the torque, the relationship between interaction torque and relative rotation angle between rudder and support is shown in Figure 12.

Fig. 12 Relationship between interaction torque and relative rotation angle between rudder and support when material friction coefficient $\mu$ is different
It can be seen that the torque-angle curve in this figure is a typical hysteretic curve, and with the increase of $\mu$, the torque needed to overcome by the rudder’s rotation at the same angle increases gradually. When $\theta<\theta_0$, if $\mu=0$, the torque to be overcome by rotation will always be 0 with the increase of $\theta$. If $\mu>0$, the torque that the rudder needs to overcome is always a constant with the increase of $\theta$, and the larger the friction coefficient, the greater the constant. When $\theta>\theta_0$, the interaction torque between the support and the rudder increases with the increase of the rotation angle, and the growth rate is the same. The calculated results have the same trend as the rudder’s torque-rotation angle curves obtained by ground experiments in literature [11], which shows that the results are highly reliable.

Through the above research, we can see the advantages of contact algorithm in solving contact problems with gaps, which can be extended to other types of structure modeling with gaps. Therefore, the contact algorithm used in this paper can be used as a universal method to obtain the mechanical relationship of parts.

5. Conclusion

(1) The static characteristics of the rudder with rotating clearance are studied, which lays a foundation for further analysis of the rudder’s performance and safety.

(2) The connection relationship between the support and the rudder with clearance is constructed through the contact algorithm based on the finite element model, which describes the mechanical behavior between the two well.

(3) The relationship between the interaction torque and the relative rotation angle between the rudder and the support under different clearance and different clearances is studied, consistent with the theoretical calculation and the existing literature, which shows that the result is reasonable.

(4) This research fully demonstrates the advantages of the contact algorithm in mechanics analysis with gaps. It can be a reference for the research of other similar issues.

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