A Globally Exponentially Stable Position Observer for Interior Permanent Magnet Synchronous Motors

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Abstract

The design of a position observer for the interior permanent magnet synchronous motor is a challenging problem that, in spite of many research efforts, remained open for a long time. In this paper we present the first globally exponentially convergent solution to it. As expected in all observer tasks, a persistency of excitation condition is imposed. Conditions on the operation of the motor, under which it is verified, are given. In particular, it is shown that at rotor standstill—when the system is not observable—it is possible to inject a probing signal to enforce the persistent excitation condition.

Key words: Observers Design; Nonlinear Systems; PMSM

Nomenclature

|$\alpha\beta$| Stationary axis reference frame quantities
|v, i $\in \mathbb{R}^2$| Stator voltage and current [V, A]
|$\lambda$ $\in \mathbb{R}^2$| Stator flux [Wb]
|x $\in \mathbb{R}^2$| Active flux [Wb]
|$\theta$ $\in \mathbb{S}$| Rotor flux angle [rad]
|R| Stator winding resistance [\Ohm]
|$\psi_m$| PM flux linkage constant
|L d, L q| d and q-axis inductances [H]
|L 0| Inductance difference $L_0 := L_d - L_q$ [H]
|$L_s$| Averaged inductance $L_s := \frac{1}{2}(L_d + L_q)$ [H]
|\cdot| Euclidean norm of a vector
|p| Differential operator $p := \frac{d}{dt}$
|G(p)[w]| Action of $G(p) \in \mathbb{R}(p)$ on a signal $w(t)$
|$\nabla_x$| Gradient transpose $\nabla_x := (\frac{\partial}{\partial x})^T$
|(\dot{\cdot})| Estimation error
|\epsilon t| Exponentially decaying term

1 Introduction

Electrical motors is a benchmark system that has been intensively studied by control researchers, who have produced more than six research monographs on the topic [6,11,13,16,18,19] and hundreds of publications in our leading research journals. One of the most challenging problems that appears in this field is the so-called sensorless control, that is, the control of the motor measuring only the electrical coordinates. The problem has an enormous practical and economical significance and, theoretically, it requires the design of an observer for a highly nonlinear system. Many well-known control theorists have contributed to the mathematical solution of this problem, mainly for induction [11,17] and surface-mount [also called non-salient] permanent magnet synchronous motors (SPMSMs) [3,7,20,23,24]. For a review of the literature, the reader is referred to the aforementioned research monographs and papers.

Because of the reluctance torque and higher power density, as well as their cheaper production cost, it has been recognized in recent years that interior [also called salient] PMSMs, are more suitable for industrial and home appliances than SPMSMs or induction motors—becoming the de facto standard in these applications [18, Subsection 6.1.4]. The dynamic equations that describe the behavior of IPMSMs are far more complicated than
those of SPMSMs. Indeed, because of the rotor saliency, the model most incorporate the effect of self and mutual inductances, which vary with an electrical angle between phases and rotor axis—see the discussion in [18, Subchapter 6.2] and [20, Section VI]. In spite of an intensive research activity in the industrial electronics and the control theory communities the problem of designing a globally stable observer for IPMSMs has remained open for many years. This fact is openly recognized by one of the leading authorities in observer design in [2, Section 6] where it is stated:

“Extension to no-salient models. We are unaware of any observer for this case”.

The main purpose of this paper is to present the first globally convergent observer for IPMSMs.

The observer proposed in this paper is a gradient-descent search—an approach first proposed in observer theory in [22] and applied for the first time to PMSMs in [20]. Unfortunately, the simple construction proposed in [20] is not applicable for IPMSM. Indeed, in [20] it is shown that flux and current in SPMSMs verify an algebraic relation, which is independent of the mechanical coordinates. From this algebraic relation a quadratic criterion to be minimized—whose gradient is computable from the electrical coordinates—can be easily constructed. Unfortunately, this algebraic relation in IPMSMs depends on the rotor position—a fact that is discussed in Section 2.

To be able to construct a gradient descent-based observer for IPMSMs it was proposed in [8] to follow the approach pursued in [4] for SPMSM. Namely, to derive via filtering, a linear regression equation for the flux. In [8] it was shown that, when applied to the active flux of the IPMSM [5], the procedure of [4] yields an additively perturbed linear regression. Proceeding from this regression, and neglecting the disturbance term, a gradient-descent search observer was proposed in [8]. Although experimental evidence proved the good performance of this observer, its theoretical analysis was hampered by the presence of the neglected perturbation term. In [9] another observer that takes into account the presence of the disturbance was proposed. Extensive experimental evidence proved the high performance of this observer—incorporating at low speeds the signal injection feature commonly used in sensorless control [18]. However, because of the complexity of the observer dynamics, it was not possible to carry-out the stability analysis. See Section 7 for a discussion on this matter.

In this paper we propose a modification to the observer proposed in [9] for which a complete theoretical analysis allow us to establish its global exponential stability (GES). As usual in all observer tasks, a persistency of excitation (PE) condition is imposed. Conditions on the operation of the motor, under which it is verified, are given. Interestingly, it is shown that at standstill—when the flux is not observable [14]—PE is enforced injecting a probing signal as done in [9,18,25].

The remainder of the paper is organized as follows. In Section 2 we present the model of the PMSMs and explain why the approach adopted in [20] is not applicable to IPMSMs. Section 3 presents the linear regression representation from [9], that is the basis for our observer design, which is given in Section 4. In Section 5 we discuss the PE assumption required by our main result. In Section 6 we present some simulation results, which illustrate the performance—and limitations—of the proposed observer, as well as its operation at standstill injecting a probing signal. The paper is wrapped-up in Section 7 with some concluding remarks and future research, including a discussion on the observer given in [9].

2 Difference Between SPMSM and IPMSM

In this section we present the mathematical models of the SPMSM and IPMSM and explain why the simple approach, proposed in [20] for observer design of the former, is not applicable for the IPMSM.

For both motors the magnetic energy stored in the inductors is given as

\[ H_E(\lambda, \theta) + \frac{1}{2}[\lambda - \psi_m c(\theta)]^T L^{-1}(\theta)[\lambda - \psi_m c(\theta)], \]

where \( L(\theta) \in \mathbb{R}^{2 \times 2} \) is the generalized inductance matrix, defined as

\[
L(\theta) = \begin{cases}
L_s & \text{for the SPMSM (} L_d = L_q) \\
\left[ L_s I_2 + \frac{L_0}{2} Q(2\theta) \right] & \text{for the IPMSM (} L_d \neq L_q),
\end{cases}
\]

where

\[
Q(2\theta) := \begin{bmatrix}
\cos(2\theta) & \sin(2\theta) \\
\sin(2\theta) & -\cos(2\theta)
\end{bmatrix},
\]

and we defined \( c(\theta) := \text{col}(\cos \theta, \sin \theta) \). The electrical dynamics (in the stationary \( \alpha\beta \) frame) is given by Faraday’s Law

\[
\dot{\lambda} = -Ri + v \tag{1}
\]

with the constitutive relation

\[
i = \nabla_{\lambda} H_E(\lambda, \theta) = \begin{cases}
\frac{1}{L_s}[\lambda - \psi_m c(\theta)] & \text{for the SPMSM} \\
L^{-1}(\theta)[\lambda - \psi_m c(\theta)] & \text{for the IPMSM}.
\end{cases}
\]

We underscore the fact that \( L_0 = 0 \) for SPMSM considerably simplifying the equations.
Noting that the SPMSM verifies the algebraic constraint
\[ |\lambda - L_s i|^2 - \psi_m^2 = 0, \]  
the flux observer for SPMSM proposed in [20] is a gradient-descent search for the minimization of the quadratic criterion
\[ J(\hat{\lambda}) := \frac{1}{4} \left( |\hat{\lambda} - L_s i|^2 - \psi_m^2 \right)^2. \]

Leading to
\[ \dot{\hat{\lambda}} = \hat{\lambda} - \gamma \nabla J(\hat{\lambda}) = \mathbf{v} - Ri - \gamma (|\hat{\lambda} - L_s i|^2 - \psi_m^2)(\hat{\lambda} - L_s i), \]  
As shown in [20] the flux observer (3) has some remarkable stability properties, and its excellent performance has been validated experimentally [18]. See also [15] where it is shown that the following slight variation of (3)
\[ \dot{\hat{\lambda}} = \mathbf{v} - Ri - \gamma \max\{0, |\hat{\lambda} - L_s i|^2 - \psi_m^2\}(\hat{\lambda} - L_s i), \]
has even stronger stability properties, namely, global convergence.

Unfortunately, the approach proposed above is not applicable to IPMSM. Indeed, although the IPMSM still verifies an algebraic constraint similar to (2), that is
\[ |\lambda - L(\theta) i|^2 - \psi_m^2 = 0, \]
it is not possible to compute its gradient without the knowledge of \( \theta \). For this reason, in this paper we proceed as done in [4,8] and look for the generation, via filtering, of a linear regression in \( \lambda \).

3 A Linear Regression Equation of the IPMSM

As indicated in the previous section, the electrical dynamics of the IPMSM is given by Faraday’s Law (1), together with the constitutive relation
\[ \lambda = [L_s I_2 + \frac{L_0}{2} Q(2\theta)] i + \psi_m c(\theta). \]  
Some simple calculations [8] show that (4) may be written as
\[ \lambda = L_q i + (L_0 i^\top c(\theta) + \psi_m) c(\theta), \]  
In [8] it is proposed to obtain the rotor angle via the estimation of the active flux of the IPMSM, defined in [5] as
\[ \mathbf{x} := \lambda - L_q i. \]

The motivation to consider this signal is twofold. First, from (5) and (6), we have that
\[ \mathbf{x} = [L_0 i^\top c(\theta) + \psi_m] c(\theta). \]

Consequently,
\[ |\mathbf{x}|^2 = |L_0 i^\top c(\theta) + \psi_m|^2. \]  
Hence,
\[ \frac{\mathbf{x}}{|\mathbf{x}|} = c(\theta), \]
and the rotor angle is easily reconstructed from \( \mathbf{x} \) via
\[ \theta = \arctan \{ \frac{\mathbf{x}_2}{\mathbf{x}_1} \}. \]

A second, and most important motivation, is contained in the following lemma, whose proof was established in [9] and, to make the paper self-contained, it is also given in Appendix A.

**Lemma 1** The electrical dynamics of the IPMSM (1), (4) satisfies the following (perturbed) linear regression equation
\[ y = \Phi^\top \mathbf{x} + d + \epsilon, \]
where \( \mathbf{x} \) is the active flux defined in (6), the (unknown) perturbing signal \( d \) is given by
\[ d := -\ell \frac{\alpha p}{p + \alpha} \left[ i^\top \mathbf{x} \right], \]
and the measurable signals \( y \) and \( \Phi \) are generated as
\[
\begin{align*}
\Phi & := \Omega_1 + \Omega_2, \\
y & := L_0 \left( \frac{\alpha}{p + \alpha} \right)^\top \Omega_1 + \frac{1}{p + \alpha} |\mathbf{v} - Ri - L_q i|^2 + \frac{2}{p + \alpha} |\mathbf{x}|^2 + \frac{1}{p + \alpha} |\Omega_2^{\top} \Omega_1|,
\end{align*}
\]
where \( \ell := \psi_m L_0, \alpha > 0 \) is a tuning parameter and
\[
\begin{align*}
\Omega_1 & := \frac{\alpha}{p + \alpha} [\mathbf{v} - Ri - L_q i], \\
\Omega_2 & := \Omega_1 - L_0 \frac{\alpha p}{p + \alpha} |i|.
\end{align*}
\]

It is important to underscore that the tuning gain \( \alpha \) determines the bandwidth of the filters used for the observer design, with a larger value corresponding to faster transient responses.

Also, notice that, replacing (6) in (8) it is possible to define a perturbed, linear regression, directly for \( \lambda \). However, simpler expressions are obtained estimating \( \mathbf{x} \).
4 Main Result

In this section we present our flux/position observer that, besides boundedness of all signals, requires the following standard PE assumption [21, Subsection 2.5].

**Assumption 1.** \( \Phi \) is PE. That is, there exist \( \delta > 0 \) and \( T > 0 \) such that

\[
\int_t^{t+T} \Phi(s) \Phi^T(s) ds \geq \delta I_2, \quad \forall t \geq 0.
\]

**Proposition 1.** Consider the electrical dynamics of the IPMSM (1), (4), with the signal \( \Phi \), defined in (9), verifying the PE Assumption 1. Define the active flux/position observer

\[
\dot{\lambda} = v - R i + \gamma \Phi \left( y - \Phi^T \tilde{x} + \frac{\alpha p}{p+\alpha} \left[ i^T \tilde{x} \right] \right),
\]

\[
\dot{x} = \dot{\lambda} - L q i,
\]

\[
\dot{\theta} = \arctan \left( \frac{\tilde{x}_2}{\tilde{x}_1} \right),
\]

with \( y \) defined in (9), and \( \gamma > 0 \) a tuning parameter. There exists \( \alpha_{\text{max}} > 0 \) and \( \gamma_{\text{max}} > 0 \) such that for all \( \alpha \leq \alpha_{\text{max}} \) and \( \gamma \leq \gamma_{\text{max}} \) we have

\[
|\dot{\lambda}(t)| \leq me^{-\rho t}|\dot{\lambda}(0)|, \quad \forall t \geq 0,
\]

for some \( m > 0, \rho > 0 \). Moreover, \( \lim_{t \to \infty} |\dot{\theta}(t)| = 0 \) (exp.).

**Proof 1.** The observation error dynamics is

\[
\begin{align*}
\dot{x} &= \dot{x} - x \\
&= \gamma \Phi (y - \Phi^T \tilde{x} - \tilde{d}) \\
&= -\gamma \Phi (\Phi^T \tilde{x} + \tilde{d}),
\end{align*}
\]

where, to simplify the notation, we defined

\[
\tilde{d} := -\ell \frac{\alpha p}{p+\alpha} \left[ i^T \tilde{x} \right],
\]

Notice that

\[
\begin{align*}
\dot{d} &= -\ell \frac{\alpha p}{p+\alpha} \left[ i^T (\tilde{x} - x) \right] \\
&= -\frac{\alpha p}{p+\alpha} \tilde{w}^T (i, x, \tilde{x}),
\end{align*}
\]

where we have moved the constant \( \ell \) inside the filter and defined the continuous mapping \( \tilde{w} : \mathbb{R}^2 \times \mathbb{R}^2 \times \mathbb{R}^2 \rightarrow \mathbb{R}^2 \). The existence of this factorization is ensured invoking the Lagrange remainder representation of the Taylor series expansion and noticing that the term in parenthesis in the first equation of (12) is zero at \( \tilde{x} = 0 \).

The equation (12) admits a state-space realization

\[
\begin{align*}
\dot{z} &= -\alpha z + \alpha^2 \tilde{w}^T \tilde{x} \\
\dot{d} &= z - \alpha \tilde{w}^T \tilde{x},
\end{align*}
\]

with \( z \in \mathbb{R} \). Replacing (13) in (11) yields the state-space model of the error equation

\[
\begin{align*}
\dot{x} &= -\gamma \Phi (\Phi^T \tilde{x} + z - \alpha \tilde{w}^T \tilde{x}) \\
\dot{z} &= -\alpha z + \alpha^2 \tilde{w}^T \tilde{x}
\end{align*}
\]

To express the system (14) as a perturbed GES system, we rewrite the dynamics (14) as

\[
\begin{align*}
\dot{\chi} &= \left[ -\gamma \Phi \Phi^T (t) - \gamma \Phi(t) \right] \chi + \alpha \left[ -\gamma \Phi \tilde{w}^T \right] \tilde{x} \\
\gamma \alpha \Phi^T (t) - \alpha
\end{align*}
\]

It is well-known that, under the boundedness and PE Assumption 1, the unperturbed part of (15), namely, the linear time-varying system

\[
\begin{align*}
\dot{\chi} &= \left[ -\gamma \Phi \Phi^T (t) - \gamma \Phi(t) \right] \chi =: f(\chi, t)
\end{align*}
\]

with \( \chi := \text{col}(\tilde{x}, z) \), is GES, see e.g., [1, Theorem 2.3]. On the other hand, from [12, Theorem 4.14] we know that the unperturbed GES system, (16) admits a Lyapunov function \( V(\chi) \), verifying

\[
\begin{align*}
c_1 |\chi|^2 \leq V(\chi) \leq c_2 |\chi|^2 \\
(\nabla V)^T f(\chi, t) \leq -c_3 |\chi|^2 \\
\|\nabla V\| \leq c_4 |\chi|,
\end{align*}
\]

for some positive constant \( c_i, i = 1, \ldots, 4 \). Evaluating the time-derivative of the Lyapunov function, along the trajectories of (15) and using the bounds above, yields

\[
\begin{align*}
\dot{V} &\leq -c_3 |\chi|^2 + \alpha (\nabla V)^T \left[ -\gamma \Phi \tilde{w}^T \right] \tilde{x} \\
&\leq -c_3 |\chi|^2 + \alpha c_4 \left\| -\gamma \Phi \tilde{w}^T \right\| |\chi|^2.
\end{align*}
\]
with $\| \cdot \|$ the induced matrix norm.

Now, it is shown in [1, Corollary 2.1] that the rate of exponential stability is proportional to $\gamma$ for sufficiently small $\gamma$. More precisely, it is shown that—for small $\gamma$—there exists a positive constant $k$ such that

$$c_3 = k\gamma + O(\gamma^2).$$

Given the boundedness assumption, we can always find a small $\alpha > 0$ such that

$$c_3 > \alpha c_4 \left\| \begin{bmatrix} -\gamma \Phi w^T \\ -\Phi^T + \alpha w^T \end{bmatrix} \right\|_\infty,$$

with $\| \cdot \|_\infty$ the $L_\infty$ norm, ensuring the GES of (15) and completing the proof. \hfill $\Box$

5 The Persistence of Excitation Condition

In this section, we give some verifiable sufficient conditions that ensure the PE Assumption 1 of $\Phi$.

Proposition 2 Consider the dynamics (1), (4). The vector $\Phi$ is PE if any of the following conditions are satisfied:

$\mathbf{C1}$ the signal $(v - Ri - L_s p_i)$ is PE;

$\mathbf{C2}$ the position-parameterized port signal

$$p[-(L_0/2)i + (L_0i^T c(\theta) + \psi_m)c(\theta)]$$

is PE;

$\mathbf{C3}$ at standstill ($\theta = \text{const}$), the time derivative of $i$ is PE.

Proof 2 Some calculations show

$$\frac{1}{2} \Phi = \frac{\alpha}{p + \alpha} [v - Ri - L_s \alpha p + \alpha i]$$

$$= \frac{\alpha}{p + \alpha} [v - Ri - L_s p_i].$$

It is well-known that a PE signal filtered by an asymptotically stable transfer function is still PE [21, Lemma 2.6.7], thus

$$(v - Ri - L_s p_i) \in \text{PE} \implies \Phi \in \text{PE},$$

verifying the claim $\mathbf{C1}$. Now, from the motor dynamics we have

$$(v - Ri - L_s p_i) = p[\lambda - L_s i] \in \text{PE}$$

$$= p[(L_0 - L_s)i + (L_0i^T c(\theta) + \psi_m)c(\theta)]$$

$$= p[-(L_0/2)i + (L_0i^T c(\theta) + \psi_m)c(\theta)],$$

which proves the second claim. To prove the third claim we carry-out the following computations with $\theta$ constant:

$$p[-(L_0/2)i + (L_0i^T c(\theta) + \psi_m)c(\theta)]$$

$$= p[-(L_0/2)i + (L_0i^T c(\theta))c(\theta)]$$

$$= L_0[\frac{1}{2}i + i^T (c(\theta)c(\theta))]$$

$$= L_0[\frac{1}{2}i + (c^2_1 c_1 c_2 + c^2_2)]$$

$$= L_0[-\frac{1}{2}i + (c^2_1 c_1 c_2 + c^2_2)]$$

$$= L_0[-\frac{1}{2}i + (c(\theta)c^T(\theta)i)]$$

$$= L_0[\frac{-I_2}{2} + c(\theta)c^T(\theta)]p[i].$$

Furthermore, we have

$$\det \left( \frac{-I_2}{2} + c(\theta)c^T(\theta) \right) = -\frac{1}{4},$$

concluding the proof. \hfill $\Box$

We bring to the readers attention the practical relevance of Condition $\mathbf{C3}$. It is well-known that, because of a lack of observability at zero speed [14], for slow-speed operation of the motor it is necessary to use an active, probing-signal injection method to estimate the flux [18]. Condition $\mathbf{C3}$ shows that injecting a high-frequency signal to the the stator currents guarantees $\Phi \in \text{PE}$ at standstill. Simulation results, presented in the next section, corroborate this fact. See [9,25] for further discussion on this matter.

6 Simulations

Simulations were conducted to evaluate the performance of the proposed position observer with the motor parameters listed in Table 6, which were taken from the experimental rig used in [25]. The IPMSM is controlled by a classical decoupling field-oriented speed regulation scheme, and the observer is not connected to the closed-loop.
Table 1
Parameters of the IPMSM

| Parameter                                | Value |
|------------------------------------------|-------|
| Number of pole pairs                     | 6     |
| PM flux linkage constant ($\psi_m$) [Wb] | 0.11  |
| d-axis inductance ($L_d$) [mH]           | 5.74  |
| q-axis inductance ($L_q$) [mH]           | 8.68  |
| Stator resistance ($R$) [Ω]              | 0.43  |
| Drive inertia [kg·m²]                    | 0.01  |

Fig. 1. Reference for the rotor speed $\omega$.

6.1 Non-zero speed behaviour

We first consider the motor speeding up from 10 rad/s to 100 rad/s with a constant torque equal to 0.5 N·m, see Fig. 1. The parameters and initial conditions of the observer are selected as $\alpha = 20$, $\gamma = 10$ and $\lambda(0) = [0.5, 2]$. The performance of the position estimator is shown in Fig. 2, where perfect tracking is observed. Fig. 3 gives the trajectory of the vector $\Phi$, clearly satisfying the PE condition. A load shifting is considered in Fig. 4 at $t = 0.5$ s, illustrating how the observer is invariant to the state changes.

To evaluate the conservativeness of our main proposition we tested the case with a large $\alpha = 200$, whose simulation results are given in Fig. 5. As expected, a small steady-state position estimation error is observed. This stems from the fact that the the perturbation term in (15) cannot be dominated by the GES part for large $\alpha$. To show that the same phenomenon appears increasing $\gamma$, in Fig. 6 we took $\gamma = 100$, where a notable performance degradation is observed.

6.2 Signal injection at zero speed

In this subsection we simulate the motor operating at zero velocity and verify the PE condition injecting a probing signal. As indicated in condition C3 of Proposition 2, $\Phi$ is PE at standstill if $p[i]$ is PE. Fig. 7 shows the behaviour of the proposed observer with or without high-frequency injection when the motor is at a standstill.

Fig. 2. Angle $\theta$ and its estimate $\hat{\theta}$. ($\gamma = 10$, $\alpha = 20$)

Fig. 3. Trajectory of the vector $\Phi$.

Fig. 4. Angle $\theta$ and its estimate $\hat{\theta}$ with a load torque shifting.

Fig. 5. Angle $\theta$ and its estimate $\hat{\theta}$ with a large $\alpha$. ($\gamma = 10$, $\alpha = 200$)
This observer is motivated by the fact that, for small $\alpha > 0$, the conditions are imposed in the stability proof to be able to "dominate" a disturbance term that appears in the conditions are not far from being necessary. The latter used in the observer are "not too fast" and the gain under a reasonable PE assumption, provided the filters for IPMSMs in [8,9]. GES of the observer is established, proposed for PMSMs in [20], and successfully pursued "linear regression plus gradient search" approach first was presented. The observer is designed following the classically important and theoretically challenging IPMSM problem.

![Fig. 6. Angle $\theta$ and its estimate $\hat{\theta}$ with a large $\gamma$. ($\gamma = 100$, $\alpha = 20$)](image)

The initial angle error was 90°, that is, we used as an initial value

$$\hat{\lambda}(0) = \psi_m \cos(\theta + \pi/2), \sin(\theta + \pi/2).$$

For $0 \leq t \leq 0.05$ s, the estimation flux $\hat{x}$ stays in the initial value because $\Phi$ is the zero vector. For $t \geq 0.05$ s, a rotating high-frequency voltage in the stationary frame is injected, as suggested in [10]. Then, the trajectory of $i$ and $\Phi$ form an ellipse as shown in Fig. 7 (e) and (f). Consequently, $\Phi$ satisfies the PE condition, even at standstill.

7 Concluding Remarks and Future Research

The first GES flux/position observer for the practically important and theoretically challenging IPMSM was presented. The observer is designed following the “linear regression plus gradient search” approach first proposed for PMSMs in [20], and successfully pursued for IPMSMs in [8,9]. GES of the observer is established, under a reasonable PE assumption, provided the filters used in the observer are “not too fast” and the gain of the gradient search is sufficiently small. The latter conditions are imposed in the stability proof to be able to “dominate” a disturbance term that appears in the GES part of the error system. Simulation results show that these conditions are not far from being necessary.

In [9] the following observer is suggested

$$\begin{align*}
\dot{\lambda} &= \psi_m \cos(\theta + \pi/2), \sin(\theta + \pi/2) \\
\dot{x} &= \lambda - L_q i
\end{align*}$$

where

$$\Lambda := -\psi_m L_0 \frac{\alpha p}{p + \alpha} \left( [\hat{x}]^2 I_2 - \hat{x} \hat{x}^T \right) \frac{1}{[\hat{x}]^2}.$$

This observer is motivated by the fact that, for small $\alpha$,

$$\frac{\partial}{\partial \hat{x}} (y - \Phi^T \hat{x} - \hat{d})^2 = (\Phi + \Lambda)(y - \Phi^T \hat{x} - \hat{d}).$$

Hence, (17) is a bona fide gradient search. Notice the absence of the matrix $\Lambda$ in the observer (10) proposed in this paper. Unfortunately, the analysis of (17) is a daunting task.

Current research is under way in the following directions.

- To carry-out a simulation and experimental comparison of the observer proposed in this paper and (17).
- As the conditions of “small” $\alpha$ and $\gamma$ of Proposition 1 seem necessary, and this restrictions limit the transient performance of the observer, we are looking for some modifications to the observer to relax these conditions.
- Although simulation results have shown that the observer can still be used—adding a probing signal—at standstill, further theoretical analysis is required to provide a solid foundation to this modification and, in particular, the transition from an active to a passive approach—see [9] for a detailed discussion on this matter.

Appendix

A Proof of Lemma 1

From (7), and after some complicated but straightforward calculations, it is possible to prove that

$$L_0 i^T x = [x]^2 - \psi_m^2 - \psi_m L_0 i_d.$$

Applying the filter $\frac{\alpha p}{p + \alpha}$, we get

$$L_0 \frac{\alpha p}{p + \alpha} [i^T x] = \frac{\alpha p}{p + \alpha} [x]^2 + d + \epsilon_t. \quad (A.1)$$

Now, in [8, Lemma 2] the following variation of the well-known Swapping Lemma [21, Lemma 3.6.5] is established. Given two smooth functions $u, v$ and a constant $\alpha > 0$, we have that

$$\frac{\alpha p}{p + \alpha} [uv] = \frac{\alpha p}{p + \alpha} [u] v + \frac{\alpha}{p + \alpha} [u] \frac{\alpha p}{p + \alpha} [v] - \frac{1}{p + \alpha} [u] \frac{\alpha p}{p + \alpha} [v].$$

The proof of Lemma 1 is completed applying the identity above to (A.1) and rearranging terms. □□□

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