Periodically-dressed Bose-Einstein condensates: a superfluid with an anisotropic and variable critical velocity

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Two intersecting laser beams can produce a spatially-periodic coupling between two components of an atomic gas and thereby modify the dispersion relation of the gas according to a dressed-state formalism. Properties of a Bose-Einstein condensate of such a gas are strongly affected by this modification. A Bogoliubov transformation is presented which accounts for interparticle interactions to obtain the quasiparticle excitation spectrum in such a condensate. The Landau critical velocity is found to be anisotropic and can be widely tuned by varying properties of the dressing laser beams.

In numerous physical systems, particles which are confined to a medium can be treated as free particles whose properties are modified by the medium. For example, the dispersion relation of electrons in a periodic solid is changed from the free-particle relation to a band structure which is periodic in quasi-momentum space. Such electrons are then treated as particles with properties which can be purposely engineered by modifying the surrounding periodic structure. In this Letter, we consider similarly the task of engineering novel behaviour in a quantum-degenerate atomic gas by placing the gas in a properly constructed periodic medium.

A periodic potential for an atomic gas can be produced by intersecting two or more laser beams. A polarizable atomic gas illuminated by two intersecting off-resonant laser beams with identical polarization and frequency but different wavevectors experiences a spatially-periodic potential proportional to the light intensity, i.e. the atoms reside in a crystalline medium made of light. Atoms in such media have been studied in the non-degenerate and quantum degenerate regimes. A small frequency difference \( \omega = \omega_1 - \omega_2 \) can be introduced between the laser beams to induce resonant Bragg transitions between identical internal but different momentum states. Such Bragg transitions have been used to probe Bose-Einstein condensates.

In this Letter, we present a different scheme for engineering properties of an ultra-cold gas. Rather than considering a spatially periodic potential, we consider a spatially-periodic coupling between two internal states of an atomic gas, i.e. we consider a periodic medium constructed by laser beams which effect Raman rather than Bragg transitions. An atomic gas in this medium is characterized by an easily tunable dressed-state dispersion relation. In creating a Bose-Einstein condensate of such a periodically-dressed atomic gas, one may explore how the nature of free single-particle excitations can modify macroscopic properties of a quantum fluid. In particular, we develop a Bogoliubov transformation which defines the quasi-particle excitation spectrum, and show that this novel quantum fluid has a tunable and anisotropic superfluid critical velocity.

Let us consider a uniform dilute gas composed of atoms of mass \( m \) with two internal ground states, \( |a\rangle \) and \( |b\rangle \), separated by an energy \( \hbar \omega_0 \), and an excited state \( |c\rangle \) (see Fig. 1). This gas is exposed to two laser beams (labeled 1 and 2), with wavevectors \( \mathbf{k}_1 \) and \( \mathbf{k}_2 \) and frequencies \( \omega_1 \) and \( \omega_2 \), respectively. The beams are polarized so that beam 1 connects states \( |a\rangle \) and \( |c\rangle \) and beam 2 connects states \( |b\rangle \) and \( |c\rangle \), while both beams have a large detuning \( \Delta \) from these allowed dipole transitions. These laser beams can induce Raman transitions between the two internal ground states. A transition from state \( a \) to \( b \) : results in a momentum kick of \( \hbar \mathbf{k} = \hbar (\mathbf{k}_1 - \mathbf{k}_2) \). Such coupling can be represented in position space as a spatially-periodic coupling between the two internal states, proportional to \( e^{i\mathbf{k} \cdot \mathbf{r}} |b\rangle \langle a| + e^{-i\mathbf{k} \cdot \mathbf{r}} |a\rangle \langle b| \).

Using second quantized notation, a gas which is continuously illuminated by these beams can be described by the dressed-state Hamiltonian:

\[
\mathcal{H} = \sum_q \left( \frac{\hbar^2 q^2}{2m} - \frac{\hbar \omega_0}{2} \right) a_q^\dagger a_q + \left( \frac{\hbar^2 q^2}{2m} + \frac{\hbar \omega_0}{2} \right) b_q^\dagger b_q \\
+ \hbar \omega_1 \left( c_1^\dagger c_1 - N_1 - \frac{1}{2} \right) + \hbar \omega_2 \left( c_2^\dagger c_2 - N_2 - \frac{1}{2} \right) \\
+ \sum_q \left( \epsilon c_2^\dagger b_q^\dagger a_{q+k/2} a_{q-k/2} + \epsilon^* c_1^\dagger a_q^\dagger b_q b_{q+k/2} \right) \tag{1}
\]
Here $a_q$ and $b_q$ ($a^+_{q1}$ and $b^+_{q2}$) are the annihilation (creation) operators for atoms with wavevector $q$ in internal states $|a\rangle$ or $|b\rangle$, respectively. The operators $c_1$ and $c_2$ ($c^+_1$ and $c^+_2$), which are photon annihilation (creation) operators for photons in beams 1 and 2, can be replaced with $\alpha$-numbers $\sqrt{N_1}$ and $\sqrt{2N_2}$, respectively, in the limit that the photon number is large. We then define $\Omega = 2\epsilon\sqrt{(N_1 + 1)(N_2 + 1)/\hbar}$ as the (real) two-photon Rabi frequency.

Now define single-atom states $|a_q\rangle \equiv |N_1 + 1; N_2; a; q - k/2\rangle$ and $|b_q\rangle \equiv |N_1; N_2 + 1; b; q + k/2\rangle$ which are connected by a Raman transition. The notation indicates that for state $|a_q\rangle$ there are $N_1 + 1$ photons in beam 1 and $N_2$ photons in beam 2, and the atom is in state $|a\rangle$ with momentum $\hbar q + k/2$, and similar for state $|b_q\rangle$. These two states have the same total momentum $\vec{q}$, which we define as $\hbar q$ by a proper choice of reference frame. The unperturbed energies ($\epsilon \to 0$) of these states are

$$\hbar \omega^a_q = \frac{\hbar^2}{2m} \left( q - \frac{k}{2} \right)^2 + \hbar \frac{\delta}{2}$$ \hspace{1cm} (2)

$$\hbar \omega^b_q = \frac{\hbar^2}{2m} \left( q + \frac{k}{2} \right)^2 - \hbar \frac{\delta}{2}$$ \hspace{1cm} (3)

where $\delta = (\omega_1 - \omega_2) - \omega_0$ is the detuning from the Raman resonance.

Diagonalizing the Hamiltonian in the subspace of states $|a_q\rangle$ and $|b_q\rangle$ yields the dressed-state eigenstates $|\pm_q\rangle$ with energies

$$\hbar \omega^\pm_q = \frac{\hbar^2}{2m} \left( q^2 + \frac{k^2}{4} \right) \pm \frac{\hbar}{2} \sqrt{\left( \delta - \frac{\hbar q \cdot \frac{k}{m}}{m} \right)^2 + \Omega^2}$$ \hspace{1cm} (4)

and creation operators $\pi_q$ (for state $|+q\rangle$) and $\mu_q$ (for state $|-q\rangle$) defined as

$$\begin{pmatrix} \pi_q \\ \mu_q \end{pmatrix} = \begin{pmatrix} \cos \theta_q/2 & \sin \theta_q/2 \\ -\sin \theta_q/2 & \cos \theta_q/2 \end{pmatrix} \begin{pmatrix} a_q \\ b_q \end{pmatrix}$$ \hspace{1cm} (5)

The dressed states are derived from the bare states $|a_q\rangle$ and $|b_q\rangle$ by a rotation in the two-dimensional subspace by the angle $-\theta_q/2$ where $\tan \theta_q = \Omega/\left( \delta + \hbar q \cdot \frac{k}{m} \right)$.

The dressed-state dispersion relation (Eq. 4) is shown in Fig. 2. The motional ground state occurs in the lower dressed state at a wavevector $\mathbf{Q}$, which is near either $\pm k/2$ depending on the sign of $\delta$.

FIG. 2. Dressed-state dispersion relation: single particle energies $\hbar \omega^\pm_q$ vs. (normalized) wavevector $q/k$. The excitation momentum $\hbar q$ and the momentum transfer $\hbar k = \hbar (k_1 - k_2)$ are chosen to be colinear. Energies are scaled by $E_{k/2} = \hbar^2 (k/2)^2/2m$. For the traces shown, $\hbar\Omega/E_{k/2} = 2$, and $\hbar \delta/E_{k/2} = 3$ (solid lines), $\hbar \delta/E_{k/2} = 0$ (dashed lines), or $\hbar \delta/E_{k/2} = -1$ (dotted lines). For small $|\delta|$ (e.g. $\hbar \delta/E_{k/2} = -1$), the dispersion relation acquires a secondary minimum. As $\delta$ changes sign, the momentum of the lowest energy state changes discontinuously. For $\delta = 0$, the motional ground state is degenerate.

Let us now consider the effects of Raman coupling on a two-component Bose-Einstein condensate, which is now formed of a macroscopic population of atoms in the lowest energy state $|-Q\rangle$, i.e. the condensate is formed of atoms in a coherent superposition of atomic states with differing momenta $|a_{Q-k/2}\rangle$ and $|b_{Q+k/2}\rangle$. This condensate has a two-branch excitation spectrum, reflecting the presence of two internal states (and, correspondingly, two distinct dressed-state levels). As with a scalar Bose-Einstein condensate, weak repulsive interactions should yield phonon-like excitations at low energies and a free-particle-like dispersion relation at high energies. However, unlike for a scalar condensate, we expect properties of a periodically-dressed Bose-Einstein condensate to reflect the anisotropy of the dressed-state dispersion relation.

Let us now treat explicitly the effects of atomic collisions by writing the many-body Hamiltonian as

$$\mathcal{H} = \sum_q \left( \hbar \omega^a_q \pi^+_q \pi_q + \hbar \omega^b_q \pi^+_q \pi_q + \mathcal{H}_{\text{int}} \right)$$ \hspace{1cm} (6)

Considering only elastic binary collisions (which conserve the number of atoms in each of the internal states) characterized by identical s-wave scattering lengths $a$, we may write the interaction Hamiltonian $\mathcal{H}_{\text{int}}$ as

$$\mathcal{H}_{\text{int}} = \frac{g}{2} \sum_q (n_q n_{-q} - N)$$ \hspace{1cm} (7)

Here $g = (4\pi \hbar^2 a/m) \times V^{-1}$, $V$ is the volume occupied by the gas, $N$ is the number of atoms in the gas, and $n_q$, the spatial Fourier transform of the density operator, is

$$n_q = \sum_k \left( a^+_{k+q} b^+_{k+q} \right) \left( a_k b_k \right)$$ \hspace{1cm} (8)

$$= \sum_k \left( \pi^+_k \pi_k + \mu^+_k \mu_k \right) R \left( \theta_k - \theta_{k-q} \right)$$ \hspace{1cm} (9)

Note that, since the rotation matrix $R$ (defined in accordance with Eq. 5) is not generally diagonal, elastic collisions do not generally conserve the number of atoms in the upper and the lower dressed states, respectively.

We now make use of the Bogoliubov approximation in which we assume a macroscopic population of $N_0$ atoms in the lowest energy state of wavevector $\mathbf{Q}$, and
may now approximate the Hamiltonian as $\tilde{H} \approx H - \frac{g}{2}(N^2 - N) + \sum_{q \neq 0} \frac{w_j H_{ij} v_j}{2} + \hbar \omega_q\pi \pi_q \pi_q$.

The Bose commutation relations of the dressed-state annihilation and creation operators can be expressed as $[v_i, w_j] = \delta_{ij}$. Isolating terms of order $N^2$ and $N$, we may now approximate the Hamiltonian as

$$\mathcal{H} \simeq \hbar \omega_q N + \frac{g}{2}(N^2 - N) + \sum_{q \neq 0} \frac{w_j H_{ij} v_j}{2} + \hbar \omega_q \pi \pi_q \pi_q$$

where $H$ is a 4x4 matrix of the form $H_{ij} = E_{ij} + \mu x_i x_j$. The diagonal matrix $E_{ij}$ has entries $E_{q-q} = \tilde{H}_{q-q} = \hbar (\omega_{q-q} - \omega_q)$. The chemical potential is given by $\mu = gN$, and we define

$$x = \begin{pmatrix} \cos \Delta_q \\ -i \sin \Delta_q \\ \sin \Delta_q \\ -i \sin \Delta_q \end{pmatrix}$$

where $\Delta_q = (\theta_{q-q} - \theta_{Q-q})/2$.

The quasi-particle energies and their creation and annihilation operators are found by diagonalizing the matrix $H$. Given the invertible matrix $M$ for which $M H M^{-1} = H$ is diagonal, allowing $v$ to transform as a column vector $(\tilde{v}_i = M_{ij} v_j)$ and $w$ as a row vector $(\tilde{w}_j = w_i M_{ij}^{-1})$, we find the Bose commutation relations $[\tilde{v}_i, \tilde{w}_j] = \delta_{ij}$ to be preserved for the quasi-particle creation and annihilation operators $(\tilde{\mu}_q, \tilde{\pi}_q, \tilde{\pi}_q, \tilde{\pi}_q)$ defined by the elements of $\tilde{v}$ and $\tilde{w}$. One finds the diagonal elements of $\hbar^{-1} H$ to be $\tilde{H}_{q-q} = \hbar (\omega_{q-q} - \omega_q)$, and $\tilde{H}_{q-q} = \hbar (\omega_{q-q} - \omega_q)$ which define the lower and upper quasi-particle excitation energies at wavevector $Q \pm q$.

Figure 3 shows the quasiparticle spectrum calculated for excitations parallel to the Raman transition momentum transfer $k$. In choosing parameters for this calculation, we have in mind an experimentally convenient realization with a Bose-Einstein condensate of $^{87}$Rb. One may choose the internal ground hyperfine states $|a\rangle = |F = 1, m_F = -1\rangle$ and $|b\rangle = |F = 2, m_F = 1\rangle$ which can be connected by a two-photon Raman transition. Choosing these states has the benefit that the Raman transition frequency is insensitive to magnetic field fluctuations, both hyperfine states are magnetically trapped, and, specifically in $^{87}$Rb, inelastic collisions are scarce. Furthermore, the scattering lengths for all elastic collisions are nearly the same, justifying the assumption made in Eq. 2.

We consider the case of counter-propagating Raman laser beams which are nearly resonant with the $D_2$ transition ($k \simeq 4\pi/\lambda$ where $\lambda = 780$ nm). The chemical potential $\mu = \hbar \times 2.5$ kHz corresponds to a condensate density of $3 \times 10^{14}$ cm$^{-3}$.

As shown in the figure, the quasi-particle energies are higher than the free dressed-state energies due to the repulsive interactions between atoms. Comparing the quasi-particle spectrum to that for the two-component condensate in the absence of Raman coupling ($\Omega \rightarrow 0$), one sees that the effect of the dressing lasers is to introduce avoided level-crossings to the spectrum. The spectrum for quasi-particle excitations near the condensate momentum $Q$ (i.e. wavevectors $Q + q$ for small $q$) is linear, describing phonon-like excitations. For excitations parallel to the direction of momentum transfer and making small $q$ approximations to the Hamiltonian, we find the lower excitation spectrum to have the limiting value $\hbar \omega_{Q+q} = \epsilon^* \hbar q$ where $\epsilon^* = \sqrt{\mu/m^*}$ is the Bogoliubov speed of sound corresponding to an effective mass $m^*$ determined by the curvature of the dressed-state dispersion relation at its minimum. We note further that the complex matrix $H$ which appears in the Hamiltonian has two positive and two negative real eigenvalues $\epsilon^*$, as is required for the stability of the condensate.

Finally, we consider the implications of the dressed-state dispersion relation for the superfluidity of the periodically-dressed Bose-Einstein condensate. An explanation for the dissipation-less flow of a superfluid below a critical velocity $v_c$ was provided by Landau, who used kinematic arguments to define $\epsilon_L = \min E(q)/\hbar q$ where $E(q)$ is the quasi-particle excitation energy at wavevector $q$. For weakly interacting scalar Bose-Einstein condensates, the Bogoliubov quasi-particle dispersion relation gives a Landau critical velocity $v_c = c$ which is equal to the speed of sound $c = \sqrt{\mu/m}$, and the onset of scattering of microscopic impurities thus occurs by the scattering of phonons. This contrasts with...
the behaviour of superfluid $^4\text{He}$ in which excitations are phonon-like at low momenta, while at higher momenta there exists a secondary minimum in the excitation spectrum corresponding to rotons [12]. The Landau critical velocity in that case is set by the roton minimum [13].

The presence of a secondary minimum in the dispersion relation of periodically-dressed Bose-Einstein condensates suggests an analogy to superfluid $^4\text{He}$ and a reduction of the superfluid velocity below the speed of sound. Figure 4 shows the Landau critical velocity calculated for impurity velocities parallel to the Raman momentum transfer $k$, as the Raman laser detuning $\delta$ is varied. For large values of $|\delta|$, the critical velocity is equal to the speed of sound $\sqrt{\mu/m}$ as for a single-component Bose-Einstein condensate. As $|\delta|$ is lowered, the critical velocity becomes anisotropic. In one direction, $v_L$ is dramatically lowered due to scattering at the secondary minimum (an “artificial roton”) in the dispersion-relation. We may approximate the “artificial roton” minimum as occurring at energy $\hbar|\delta|$ and quasi-particle momentum $\hbar k$, obtaining $|v_L| \approx |\delta/k|$ in this regime. Thus, the superfluid properties can be controlled by varying the detuning $\delta$ or by varying $k$ by changing the relative angle between the Raman laser beams. In the other direction, $v_L = c^* = \sqrt{\mu/m^*}$ as determined by phonon scattering.

![Image](image_url)

**FIG. 4.** Landau critical velocity in a periodically-dressed Bose-Einstein condensate. Velocities aligned with (positive $v_L$) or counter to (negative $v_L$) the Raman momentum transfer $k$ are considered, and we take $\hbar\Omega = 2E_{k/2}$. At large detunings $|\delta|$, $v_L$ has the same magnitude for flow in both directions, with a value approaching the Bogoliubov speed $c = \sqrt{\mu/m} = 3.3\text{ mm/s}$ for a $^{87}\text{Rb}$ condensate at the density of $3 \times 10^{14}\text{ cm}^{-3}$. At smaller detunings, an anisotropy in $v_L$ develops. For the regime $\{\delta > 0, v_L > 0\}$ and $\{\delta < 0, v_L < 0\}$, $v_L$ is determined by the speed of Bogoliubov sound modified by the effective mass $m^*$. In the regimes $\{\delta > 0, v_L < 0\}$ and $\{\delta < 0, v_L > 0\}$, the magnitude of $v_L$ is lowered due to the secondary minimum in the dispersion relation.

One may also vary the intensity of the Raman beams, thereby changing $\Omega$. An important impact of changing the Raman coupling strength is not only altering the Landau critical velocity (changing both the effective mass $m^*$ and the position of the secondary minimum), but also controlling the degree to which quasi-particles in the lower or upper excitation branches can be created by scattering off an obstacle. That is, the Landau criterion determines the onset of dissipation for a moving superfluid, but does not describe the strength of such dissipation. A large Raman coupling would enhance scattering into secondary minimum of the lower excitation branch, while in the limit $\Omega \to 0$, this scattering rate clearly vanishes. A detailed calculation of this dissipation rate (similar to Refs. [14][15]) will be given elsewhere.

In summary, we have described a means of engineering a novel quantum fluid composed of dressed-state atoms in a spatially-periodic Raman coupling medium. This quantum fluid should be amenable to study using current methods for probing ultracold atomic gases. The quasi-particle dispersion relation can be probed by studying collective excitations (i.e. density and magnetization modulations) at various length scales (see reviews in [17]). Aspects of superfluidity in gaseous Bose-Einstein condensates have been investigated in recent experiments [18][19], and similar methods can be used to investigate the superfluidity of a periodically-dressed condensate.

For example, the anisotropic Landau critical velocity can be probed by studying the scattering of an impurity gas which propagates through the fluid [13]. Further theoretical work should address a number of issues. For instance, the current treatment describes the quantum fluid exclusively in momentum space. Experimentally, such a fluid would be produced within a trapped volume. Excitations with wavelengths much smaller than the size of the fluid (as probed with Bragg scattering [3]) should be well described by our treatment, while the description of longer wavelength excitations will require further theoretical considerations. Another interesting consideration is the behaviour of the quantum fluid with the Raman lasers tuned precisely to the Raman resonance, $\delta = 0$, at which point the motional ground state is doubly degenerate. We suspect that in the presence of an external trapping potential, such a quantum fluid would be described as evolving in a double-well potential in momentum space. Finally, our scheme may also be extended to condensates with more than two internal states, such as spinor Bose-Einstein condensates [14], giving greater flexibility in engineering properties of gaseous quantum fluids.

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