Halo structure of $^{17}\text{C}$

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(Dated: June 5, 2018)

Abstract

$^{17}\text{C}$ has three states below the $^{16}\text{C} + n$ threshold with quantum numbers $J^P = 3/2^+, 1/2^+, 5/2^+$. These states have relatively small neutron separation energies compared to the neutron separation and excitation energies of $^{16}\text{C}$. This separation of scales motivates our investigation of $^{17}\text{C}$ in a Halo effective field theory (Halo EFT) with a $^{16}\text{C}$ core and a valence neutron as degrees of freedom. We discuss various properties of the three states such as electric radii, magnetic moments, electromagnetic transition rates and capture cross sections. In particular, we give predictions for the charge radius and the magnetic moment of the $1/2^+$ state and for neutron capture on $^{16}\text{C}$ into this state. Furthermore, we discuss the predictive power of the Halo EFT approach for the $3/2^+$ and $5/2^+$ states which are described by a neutron in a $D$-wave relative to the core.

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I. INTRODUCTION

Halo nuclei are weakly-bound states of a few valence nucleons and a tightly-bound core nucleus [1–5]. They exemplify the emergence of new degrees of freedom close to the neutron and proton drip lines which are difficult to describe in ab initio approaches. Cluster models of halo nuclei are formulated directly in the new degrees of freedom and thus take the emergence phenomenon into account by construction, typically using a phenomenological interaction [6, 7]. These models have improved our understanding of halo nuclei significantly. However, they cannot be improved systematically and lack a reliable way to estimate theoretical uncertainties.

Halo effective field theory (Halo EFT) is a systematic approach to these systems that exploits the apparent separation of scales between the small nucleon separation energy of the halo nucleus and the large nucleon separation energy and excitation energy of the core nucleus [8, 9]. This scale separation defines (at least) two momentum scales: a small scale $M_{\text{lo}}$ and a large scale $M_{\text{hi}}$. Halo EFT provides a systematic expansion of low-energy observables in powers of $M_{\text{lo}}/M_{\text{hi}}$. Predictions made in Halo EFT can be improved systematically through the calculation of additional orders in the low-energy expansion. It uses contact interactions to parameterize the interaction between the core and the valence nucleons in terms of a few measurable observables. The absence of exchange particles in this approach indicates that the involved momentum scale is smaller than the inverse range of the interaction. Similar EFT approaches can be used for systems of atoms and nucleons at low energies [10, 11].

$^{11}\text{Be}$ represents the prototype of a one-nucleon halo nucleus and thus has been considered as a test case for Halo EFT. It has a $J^P = 1/2^+$ ground state that can be described as a neutron in an $S$-wave relative to the $^{10}\text{Be}$ core. $^{11}\text{Be}$ also has a $J^P = 1/2^-$ excited state which can be considered as a neutron in a $P$-wave relative to the core. The electric properties of the two bound states in $^{11}\text{Be}$ were studied in detail in Ref. [12] using Halo EFT. $^{11}\text{Be}$ also has a magnetic moment due to its halo neutron [13] but there are no magnetic transitions between the two states because of their opposite parity. For a recent review of Halo EFT and applications to other halo nuclei see Ref. [14].

Here, we will focus on the electromagnetic properties of $^{17}\text{C}$. This nucleus is an interesting halo candidate but has not yet been investigated using Halo EFT. Its continuum properties cannot yet be addressed using standard ab initio methods. It is too heavy for an approach that employs a combination of the no-core shell model (NCSM) and the resonating group model (RGM)[15] but it is too light to neglect center-of-mass motion effects as is done in a coupled cluster approach [16]. Recent calculations in the NCSM also seem to suggest that this nucleus is too large to obtain converged results for its spectrum [17] with the available computational resources. $^{17}\text{C}$ has a $J^P = 3/2^+$ ground state, and two excited states with $J^P = 1/2^+$ and $5/2^+$ [18]. The neutron separation energy of the ground state of about 0.7 MeV [19] is significantly smaller than the excitation energy of the $J^P = 0^+$ $^{16}\text{C}$ core, which is about 1.8 MeV [20], while the neutron separation energies of the excited states are only of order 0.4-0.5 MeV [17] (see the level scheme in Fig. 1). This suggests that $^{17}\text{C}$ may be amenable to a description using Halo EFT with $S$- and $D$-wave neutron-core interactions [21].

Recently, M1 transition rates from both excited states into the ground state were measured [17, 18]. Below, we will discuss these transition rates in the framework of Halo EFT to leading order (LO) in the Halo EFT counting. Besides these electromagnetic transitions, we will also consider static electric and magnetic properties as well as neutron capture on $^{16}\text{C}$ into $^{17}\text{C}$. We will show that future experiments and/or ab initio calculations of these quantities can provide insight in the interaction of neutrons with $^{16}\text{C}$.

This manuscript is organized as follows: In Sec. II, we introduce the theoretical foundations required to calculate the properties of halo nuclei with effective field theory. After reviewing
results for the charge radius and quadrupole moment for the $S$- and $D$-wave states in Sec. III, we calculate magnetic moments for both states. In Sec. IV we discuss E2 and M1 transitions between the different states in $^{17}$C and calculate E1 and M1 capture reactions to the $S$- and $D$-wave states. We end with a summary and an outlook.

II. HALO EFT FORMALISM FOR $^{17}$C

Our goal is to investigate the electromagnetic properties of the halo nucleus $^{17}$C using Halo EFT. As discussed above, $^{17}$C can be described as a weakly-bound state of a $^{16}$C core and a neutron. First, we need to account for the free propagation of the core and neutron degrees of freedom. The corresponding Lagrangian is

$$\mathcal{L}_0 = \bar{c} \left( i \partial_t + \frac{\nabla^2}{2M} \right) c + \bar{n} \left( i \partial_t + \frac{\nabla^2}{2m} \right) n ,$$

where $n$ denotes the spin-1/2 neutron field, $c$ the spin-0 core field, $m$ is the nucleon mass, and $M$ is the mass of the $^{16}$C core.

The first excitation of the $^{16}$C core has an energy of $E^*_{16C} = 1.766(10)$ MeV [20], while the neutron separation energy of $^{16}$C is $S_n(^{16}C) = 4.250(4)$ MeV [19]. Moreover, the neutron separation energy of $^{17}$C is $S_n(^{17}C) = 0.734(18)$ MeV [19]. This suggests that the $J^P = 3/2^+$ ground state of $^{17}$C can be described as a neutron in a $D$-wave relative to the $^{16}$C core, although the halo nature of the ground state is not commonly accepted [17, 18]. As illustrated in Fig. 1, $^{17}$C also has two excited states with $J^P = 1/2^+$ and $5/2^+$ with energies $E^*_{1/2^+} = 0.218(1)$ MeV and $E^*_{5/2^+} = 0.332(1)$ MeV [17], respectively. In Halo EFT, these two states are described by a neutron in an $S$-wave and $D$-wave relative to the core, respectively. To account for these states, we define the interaction part of the effective Lagrangian as [21]

$$\mathcal{L} = \mathcal{L}_0 + d_{J,M}^I \left[ c_2^I \left( i \partial_t + \frac{\nabla^2}{2M_{nc}} \right)^2 + \eta_2^I \left( i \partial_t + \frac{\nabla^2}{2M_{nc}} \right) + \Delta_2^I \right] d_{J,M} + g_2^I \left[ d_{J,M}^I \left[ n^{++2}_c \right]_{J,M} + \left[ n^{++2}_c \right]_{J,M}^I \right] d_{J,M} + \sigma_s^I \left[ \eta_0 \left( i \partial_t + \frac{\nabla^2}{2M_{nc}} \right) + \Delta_0 \right] \sigma_s - g_0 \left[ c^I n^I \sigma_s + \sigma_s^I n^I c \right] + \ldots ,$$

where $M_{nc} = M + m$ and $d_{J,M}$ is a $(2J + 1)$-component field. We project on the $J = 3/2$ and $5/2$ parts of the resonant $D$-wave interaction via

$$\left[ n^{++2}_c \right]_{J,M} = \sum_{m_s,m_l} \left( \frac{1}{2} m_s 2m_l \right) J M n_{m_s} \sum_{\alpha,\beta} \left( 1 \alpha 1 \beta 2m_l \right) n_{m_l} \frac{1}{2} \left( \nabla_\alpha \nabla_\beta + \nabla_\beta \nabla_\alpha \right) c ,$$
where $\alpha$ and $\beta$ denote spherical indices. The D-wave interaction introduces 4 low-energy constants in the leading order (LO) Lagrangian: $c_2^f$, $\Delta_2^f$, $g_2^f$, and $\eta_2^f = \pm 1$, but only three of them are independent at LO. This increased number of parameters compared to the S-wave arises from the appearance of power divergences up to 5th order in the D-wave self-energy. Their renormalization requires effective range parameters up to the shape parameter to enter at LO [8]. In this work, we will follow Ref. [21] and use dimensional regularization with the power divergence subtraction scheme (PDS) [22, 23] for all practical calculations.

The accuracy of this approach is set by the ratio of the low-momentum scale $M_{li}$ over the high-momentum scale $M_{hi}$ which for ground states observables can be estimated as $\sqrt{S_n(17C)/E_{^5\!C}} \approx 0.64$ in our case. The expansion parameter is relatively large, and we expect slow convergence for ground state observables. However, for the excited states, the expansion parameter is approximately 0.5 which leads to 50% errors at first order and 25% errors at second order in the EFT expansion.

The dressed propagators of the $\sigma$ and $d_{J,M}$ fields are obtained by summing the bubble diagrams for the $nc$-interactions (cf. Fig. 2 for the D-wave case) to all orders. Throughout this paper, a thick single line denotes the dressed $\sigma$-propagator and a thick double line the dressed $d$-propagator in all our figures.

$\sigma$-propagator. The $\sigma$-propagator for the S-wave state is well known (see, e.g., Ref. [12]) and we quote only the final result:

$$D_{\sigma}(\tilde{p}_0) = \frac{1}{\Delta_0 + \eta_0[\tilde{p}_0 + i\epsilon] - \Sigma_{\sigma}(\tilde{p}_0)},$$

$$\Sigma_{\sigma}(\tilde{p}_0) = -\frac{g_0^2 m_{RI}}{2\pi} \left[ i\sqrt{2m_{RI}\tilde{p}_0 + \mu} \right],$$

where $\mu$ is the PDS scale [22, 23], $m_{RI}$ the reduced mass of the neutron-core system, and $\tilde{p}_0 = p_0 - \mathbf{p}^2/(2M_{nc})$ is the Galilei invariant energy.

$d$-propagator. The dressed propagator for the $d_{J,M}$ field was computed in Ref. [21]. Since we use a Cartesian representation of the D-wave, the propagator depends on four vector indices, two in the incoming channel and two in the outgoing channel. Note that Roman indices refer to Cartesian indices and Greek ones to spherical indices. Evaluating the Feynman diagrams in Fig. 2, we obtain:

$$D_{d}(\tilde{p}_0)_{ij,op} = D_{d}(\tilde{p}_0) \frac{1}{2} \left( \delta_{io}\delta_{jp} + \delta_{ip}\delta_{jo} - \frac{2}{3}\delta_{ij}\delta_{op} \right),$$

$$D_{d}(\tilde{p}_0) = [\Delta_2 + \eta_2\tilde{p}_0 + c_2\tilde{p}_0^2 - \Sigma_{d}(\tilde{p}_0)]^{-1}.$$
with the one-loop self-energy

$$\Sigma_{d}(\tilde{p}_0) = -\frac{2}{15} \frac{m_Rg_2^2}{2\pi} (2m_R\tilde{p}_0)^2 \left[i\sqrt{2m_R\tilde{p}_0 + i\epsilon - \frac{15}{8} \mu} \right].$$ (8)

The term proportional to $c_2$ in (2) is required to absorb the $\mu$-dependence from the PDS scheme. Following the arguments in Ref. [21], the terms proportional to $\eta_2$, $\Delta_2$, and $g_2$ are also required to be consistent with the threshold expansion of the scattering amplitude. In a momentum cutoff scheme, these terms absorb the linear, cubic, and quintic power law divergences in the cutoff [8].

Power counting. The canonical power counting for the $\sigma$-propagator representing a shallow $S$-wave state was given in Refs. [22, 23, 25, 26]. It implies $\gamma_0 \sim 1/a_0 \sim M_{lo}$ and $r_0 \sim 1/M_{hi}$, where $\gamma_0 = \sqrt{2m_R(S_n^{(17)C}) - E_{1/2+}^0}$ is the binding momentum of the $S$-wave state and $r_0$ the effective range. As a result, $r_0$ enters at NLO in the expansion in $M_{lo}/M_{hi}$.

The power counting for partial waves beyond the $S$-wave is more complicated and different scenarios have been proposed [8, 9, 21]. We look for a scenario that exhibits the minimal number of fine tunings consistent with the scales of the system. Bedaque et al. [9] suggested for the $P$-wave case that $a_1 \sim 1/(M_{lo}^d M_{hi})$ and $r_1 \sim M_{hi}$, where higher ERE parameters scale with the appropriate power of $M_{hi}$, given by dimensional analysis. This power counting is adequate for the excited state of $^{11}\text{Be}$ [12]. It requires only one fine-tuned constant in $\mathcal{L}$ instead of two as proposed in Ref. [8] where both $a_1$ and $r_1$ scale with appropriate powers of $M_{lo}$. In Ref. [9], the power counting was also generalized to $l > 1$. However, we employ a different power counting with a minimal number of fine tunings for $l = 2$ as proposed in Ref. [21]. In the case of the $d$-propagator, (6), two out of three ERE parameters need to be fine-tuned because $a_2 \sim 1/(M_{lo}^d M_{hi})$ and $r_2 \sim M_{lo}^2 M_{hi}$ are both unnaturally large, while $\mathcal{P}_2 \sim M_{hi}$. Higher ERE terms are suppressed by powers of $M_{lo}/M_{hi}$. Thus, the relevant fit-parameters in our EFT at LO are $\gamma_0, \gamma_2, r_2$, and $\mathcal{P}_2$, where $\gamma_2 = \sqrt{2m_R(S_n^{(17)C})}$ is the binding momentum of the $^{17}\text{C}$ ground state, while $r_2$ and $\mathcal{P}_2$ denote the $D$-wave effective range and shape parameter, respectively. For the $5/2^+$ excited state, the binding momentum is $\gamma_2' = \sqrt{2m_R(S_n^{(17)C}) - E_{5/2+}^0}$, while $r_2', \mathcal{P}_2'$ are the corresponding effective range parameters.

The corresponding wave function renormalization constants for the $1/2^+, 3/2^+$, and $5/2^+$ states at LO are:

$$Z_\sigma = \frac{2\pi}{m_R^2 g_0^2} \gamma_0, \quad Z_d^{3/2} = -\frac{15\pi}{m_R g_2^2} \frac{1}{r_2 + \mathcal{P}_2 \gamma_2}, \quad Z_d^{5/2} = -\frac{15\pi}{m_R^2 g_2^2} \frac{1}{r_2' + \mathcal{P}_2' \gamma_2'},$$ (9)

respectively. At NLO, $Z_\sigma$ is modified by a factor $(1 + \gamma_0 r_0)$. The constants $Z_d^{3/2}$ and $Z_d^{5/2}$ will not be required beyond LO.

III. STATIC ELECTROMAGNETIC PROPERTIES OF $^{17}$C

We first consider the static electromagnetic properties of $^{17}$C. These are usually easier to measure experimentally than dynamical properties. They can also be calculated in ab initio approaches that provide the wave functions of the involved states. In particular, we will consider the charge radii and magnetic moments of the $^{17}$C states. It is convenient to calculate all form factors in the Breit frame where the photon transfers no energy, $q = (0, \mathbf{q})$, and to choose the photon to be moving in the $\hat{z}$ direction $\mathbf{q} = |\mathbf{q}| \hat{z}$.

5
A. Charge radii

The form factor of a general $S$-wave one-neutron halo nucleus was calculated in Ref. [12]. The electric charge radius of the $S$-wave state at NLO is given by:

$$\langle r_{E}^{2} \rangle_{\pi}^{(\sigma)} = \frac{f^{2}}{2\gamma_{0}^{2}}(1 + r_{0}\gamma_{0}) ,$$  \hspace{1cm} (10)

where $f = m_{R}/M$ is a mass factor. The LO result can be obtained by setting $r_{0} = 0$ in Eq. (10). At next-to-next-to-leading order (NNLO) a counterterm related to the radius of the core contributes. In the standard power counting, the factors of $f$ are counted as $O(1)$, although they can become rather small for large core masses. As a consequence, the counterterm contribution is enhanced numerically. Up to NLO, one can interpret the Halo EFT result as a prediction for the radius relative to the core [12].

Using the measured one-neutron separation energy of the $1/2^+$ state, we obtain for the charge radius of the excited $S$-wave state of $^{17}$C relative to the charge radius of $^{16}$C at LO:

$$\langle r_{E}^{2 \, 1/2^+} \rangle_{^{17}C} - \langle r_{E}^{2} \rangle_{^{16}C} = 0.0743 \text{ fm}^{2} ,$$  \hspace{1cm} (11)

where the error from NLO corrections is about 50%. To make a numerical prediction for the full charge radius of $^{17}$C, we have to add the charge radius of $^{16}$C, $\langle r_{E}^{2} \rangle_{^{16}C}$, to our result. We use the point-proton radius $R_{p}$ from Ref. [27] and the proton radius $r_{p}$ to obtain $\sqrt{\langle r_{E}^{2} \rangle_{^{16}C}} = \sqrt{R_{p}^{2} + r_{p}^{2}} = 2.55(4) \text{ fm}$. This yields $\sqrt{\langle r_{E}^{2 \, 1/2^+} \rangle_{^{17}C}} = 2.56(4) \text{ fm}$.

To date, there is no experimental data for the charge radius of the $1/2^+$ excited state to compare with. As a consistency check, we compare with the experimental value for the $3/2^+$ ground state of $^{17}$C extracted in Ref. [27], $\sqrt{\langle r_{E}^{2 \, 3/2^+} \rangle_{^{17}C}} = 2.57(4) \text{ fm}$, which is very close to our result for the $1/2^+$ excited state.

The charge radius of a $D$-wave state has recently been calculated in Ref. [21] at LO and yields:

$$\langle r_{E}^{2 \, (d)} \rangle_{^{17}C} = -\frac{6L_{C0E}^{(d) \text{ LO}}}{r_{2} + P_{2}\gamma_{2}^{2}} .$$  \hspace{1cm} (12)

Here, the counterterm $L_{C0E}^{(d) \text{ LO}}$ already contributes at LO while the loop contribution is suppressed.

For the $D$-wave state, we also find a quadrupole moment which yields at LO:

$$\mu_{Q}^{(d)} = \frac{40L_{C02}^{(d) \text{ LO}}}{3(r_{2} + P_{2}\gamma_{2}^{2})} ,$$  \hspace{1cm} (13)

where another counterterm enters at LO. Both $D$-wave observables have the same denominator of effective range parameters $(r_{2} + P_{2}\gamma_{2}^{2})$ which is related to the Asymptotic Normalization Coefficient (ANC) of the $D$-wave state, $A_{2} = \sqrt{2\gamma_{2}^{4}/(-r_{2} - P_{2}\gamma_{2}^{2})}$. Similar to the correlation between $\mu_{Q}^{(d)}$ and B(E2) in Ref. [21], we find a smooth correlation between $\langle r_{E}^{2 \, (d)} \rangle$ and $\mu_{Q}^{(d)}$:

$$\mu_{Q}^{(d)} = -\frac{20}{9} \frac{L_{C02}^{(d) \text{ LO}}}{L_{C0E}^{(d) \text{ LO}}} \langle r_{E}^{2 \, (d)} \rangle ,$$  \hspace{1cm} (14)

which implies that ab initio calculations with different phaseshift-equivalent interactions should show a linear correlation between the quadrupole moment and the charge radius.
Figure 3. Diagrams contributing to the magnetic moment. The first diagram is the coupling of a vector photon to the charge of the core arising from minimal substitution in the Lagrangian. The second diagram displays a vector photon coupling to the magnetic moment of the neutron. The last diagrams shows a two-body current. The thick solid line denotes the dressed $\sigma$-propagator.

B. Magnetic moments

The magnetic properties of shallow bound states are predominantly determined by the magnetic moments of its degrees of freedom. The magnetic moment of a single particle is introduced into the Lagrangian through an additional magnetic one-body operator [13, 28]. An additional counterterm enters via a two-body current. Assuming a spin-0 core, the effective Lagrangian is

$$\mathcal{L}_M = \kappa_n \mu_N n^\dagger \sigma \cdot B n + 2\mu_N L_M^J S_J \cdot B \Phi ,$$

where $\Phi$ is a place holder for the relevant auxiliary field ($\sigma_s, \pi_s, d_{J,M}, ...$), $S_J$ is the corresponding spin matrix for spin $J$, $\mu_N$ denotes the nuclear magneton, and $L_M^J$ the coupling constant for the magnetic two-body current. For the neutron anomalous magnetic moment we use $\kappa_n = -1.91304$.

1. Magnetic moment of the 1/2$^+$ state

We reproduce the results obtained by Fernando et al. [13], who calculated electromagnetic form factors for $S$-wave states of one-neutron halo nuclei. Up to NLO, only the two last diagrams in Fig. 3 contribute to the magnetic form factor in the Breit frame:

$$\frac{eQ_c}{2M_{nc}} G_M(q^2) = Z_\sigma \mu_N \left( g_0^2 \kappa_n \frac{m m_R}{\pi q} \arctan \left[ \frac{q m_R}{2 m \gamma_0} \right] + \tilde{L}_M^\sigma \right),$$

with

$$Z_\sigma = \frac{2 \pi \gamma_0}{m^2_R g_0^2} (1 + \eta_0 \gamma_0), \quad \text{and we define} \quad \tilde{L}_M^\sigma = \frac{2 \pi \tilde{L}_M^\sigma}{m^2_R g_0^2} .$$

The magnetic moment $\kappa_\sigma$ is obtained by evaluating the form factor at $q^2 = 0$:

$$\kappa_\sigma = \frac{eQ_c}{2M_{nc}} G_M(0) = (\kappa_n + \tilde{L}_M^\sigma \gamma_0) (1 + \eta_0 \gamma_0),$$

where $\kappa_\sigma$ is given in units of $\mu_N$. Naive dimensional analysis with rescaled fields $[\tilde{\sigma}] = 2$ [12] determines the scaling of the counterterm $\tilde{L}_M^\sigma \sim M_{hi}^{-1}$. As a consequence, $\tilde{L}_M^\sigma$ contributes at NLO. At LO, the magnetic moment of the 1/2$^+$ state is thus given by the magnetic moment of the neutron, $\kappa_n$. 

2. Magnetic moments of the $3/2^+$ and $5/2^+$ states

In the case of the $D$-wave, the only contribution to the magnetic moment at LO is the two-body current in Eq. (15), which corresponds to the last diagram in Fig. 3, and we obtain:

$$\frac{eQ_c}{2M_{nc}} G_M(q^2) = Z_d \mu_N L^d_M = -\frac{\mu_N \tilde{L}^d_M}{r_2 + P_2 \gamma_2^2}, \quad \text{(19)}$$

with

$$Z_d = -\frac{15\pi}{m_R g_2^2} \frac{1}{r_2 + P_2 \gamma_2^2}, \quad \text{and} \quad \tilde{L}^d_M = \frac{15\pi L^d_M}{m_R g_2^2}. \quad \text{(20)}$$

This yields for the magnetic form factor at LO:

$$\kappa_d = -\frac{\tilde{L}^d_M}{r_2 + P_2 \gamma_2^2}, \quad \text{(21)}$$

where $\kappa_d$ is again given in units of $\mu_N$. Beyond LO we also need to consider the two loop diagrams in Fig. 3. Therefore, we require additional counterterms to renormalize the corresponding divergences. This makes predictions even harder, and for that reason, we do not calculate the NLO contribution to the magnetic form factors for the $D$-wave state explicitly.

In general, the magnetic moment of the $D$-wave states will thus differ significantly from the magnetic moment of the neutron since $\kappa_n$ is a NLO contribution.

IV. ELECTROMAGNETIC TRANSITIONS AND CAPTURE REACTIONS OF $^{17}$C

A. E2 transitions

The ground state and the two excited states of $^{17}$C have positive parity and differ at most by 2 units in total angular momentum. All states can therefore be connected by E2 transitions.

The transition strength for $S \rightarrow D$ has been calculated at LO in Ref. [21] for the transition:

$$B(\text{E2}: 1/2^+ \rightarrow 5/2^+) = -\frac{4}{5\pi} \frac{Z_{\text{eff}}^2 e^2}{r_2' + P_2' \gamma_2'^2} \gamma_0 \left[ \frac{3\gamma_0^2 + 9\gamma_0 \gamma_2' + 8\gamma_2'^2}{(\gamma_0 + \gamma_2')^3} \right]^2, \quad \text{(22)}$$

where the effective charge for $^{17}$C, $Z_{\text{eff}} = (m/M_{nc})^2 Q_c \approx 0.021$ [29], comes out of the calculation automatically. At NLO, there is an unknown short-range contribution that enters via a counterterm.

For the transition strength $B(\text{E2}: 1/2^+ \rightarrow 3/2^+)$, we get the same result for the amplitude but with different Clebsch Gordan coefficients (leading to a relative factor of 3/2) and the appropriate binding momentum and renormalization constant for the $3/2^+$ ground state:

$$B(\text{E2}: 1/2^+ \rightarrow 3/2^+) = -\frac{8}{15\pi} \frac{Z_{\text{eff}}^2 e^2}{r_2' + P_2' \gamma_2'^2} \gamma_0 \left[ \frac{3\gamma_0^2 + 9\gamma_0 \gamma_2' + 8\gamma_2'^2}{(\gamma_0 + \gamma_2')^3} \right]^2. \quad \text{(23)}$$

Following the approach in Ref. [21], we can also calculate the E2 transition for $D \rightarrow D$. However, we do not display the result here since the relevant diagram diverges cubically and, therefore, additional counterterms are required for this observable already at LO.
B. M1 transitions

1. $S \rightarrow D$

We will first consider the M1 transition strength from the $3/2^+$ ground state ($D$-wave) to the first excited $1/2^+$ state ($S$-wave) in $^{17}$C since it was measured in Refs. [17, 18]. The experimental result is small compared with typical M1 transitions strengths in nuclei, i.e. $B$(M1: $1/2^+ \rightarrow 3/2^+) = 1.04^{+0.03}_{-0.12} \times 10^{-2} \mu_N^2$ [17] or $0.58 \times 10^{-2}$ W.U. expressed in Weisskopf units.

In the neutron-core picture of Halo EFT, the M1 transition from a $D$-wave to an $S$-wave state is forbidden for one-body currents which is in agreement with the experimental suppression of the transition. The non-zero transition strength can only be accounted for by a two-body current which takes short-ranged (core) physics into account. We therefore add the gauge-invariant counterterm

$$\mathcal{L}_M = -\mu_N \bar{L}_{M1}^d \sigma_{m} d_{m'} \left( \frac{1}{2} m \mu \right) \left( \frac{3}{2} m' \mu \right) B_1.$$  \hspace{1cm} (24)

By rescaling the fields to absorb unnaturally large coupling constants, leading to $[\hat{\sigma}] = 2$, $[\hat{d}] = 0$, and using naive dimensional analysis for the rescaled fields [30], we find $L_{M1}^d \approx M_{hi} l_{M1}^d g_0 \mu \pi^2$ with $l_{M1}^d$ of order one. To obtain the magnetic transition amplitude we calculate the vertex function

$$\Gamma_{m m' i} = \left( \frac{1}{2} m \mu \right) \left( \frac{3}{2} m' \mu \right) \mu_N \bar{L}_{M1}^d \epsilon_{i j k} k_j,$$  \hspace{1cm} (25)

with $\bar{L}_{M1}^d = \sqrt{m_{hi}^2 g_0^2} L_{M1}^d$. If we consider the case $m = -m' = \pm 1/2$ and choose the photon to be traveling in $\hat{z}$ direction, we find

$$\bar{\Gamma}_{\pm \mp, \mp 1} = \mp \mu_N L_{M1}^d \pi^2.$$  \hspace{1cm} (26)

This yields for the M1 transition strength:

$$B$(M1: $1/2^+ \rightarrow 3/2^+) = \frac{3}{4\pi} \left( \frac{\bar{\Gamma}_{\mp \mp, \mp 1}}{\omega} \right)^2 = -\frac{1}{4\pi r_2 + \mathcal{P}_2 \gamma_2} \left( \bar{L}_{M1}^d \right)^2 \mu_N^2.$$  \hspace{1cm} (27)

Moreover, combining Eqs. (27) and (23), we find a correlation between $B$(E2) and $B$(M1):

$$B$(E2: $1/2^+ \rightarrow 3/2^+) = \frac{32}{15} \frac{Z_{eff}^2 e^2}{\left( \bar{L}_{M1}^d \right)^2 \mu_N^2} \left[ \frac{370^2 + 9 \gamma_0 \gamma_2 + 8 \gamma_2^2}{(\gamma_0 + \gamma_2)^3} \right] B$(M1: $1/2^+ \rightarrow 3/2^+).$$  \hspace{1cm} (28)

If we use the experimental result for $B$(M1: $1/2^+ \rightarrow 3/2^+) = 1.04^{+0.03}_{-0.12} \times 10^{-2} \mu_N^2$ and employ naive dimensional analysis for the counterterm $\bar{L}_{M1}^d \sim M_{hi} \approx 0.28$ fm$^{-1}$, we can make a rough prediction for $B$(E2),

$$B$(E2: $1/2^+ \rightarrow 3/2^+) \approx 3 \times 10^{-2} e^2 \text{fm}^4.$$  \hspace{1cm} (29)

Moreover, we can compare the M1 and E2 transition strengths for $^{17}$C if we look at the transition rates [31],

$$T(R\lambda) = \frac{8\pi(\lambda + 1)}{\lambda[(2\lambda + 1)!!]^2} k^{2\lambda+1} B(R\lambda),$$  \hspace{1cm} (30)
that have, in contrast to B(M1) and B(E2), the same units. Here \( R \) stands for E or M and \( \lambda \) denotes the order of the transition. Using the naive dimensional analysis result for \( \tilde{L}_{\sigma d}^{dd} \) from above we find:

\[
\frac{T(E2)}{T(M1)} = \frac{32k^2}{125} \frac{Z^2 e_{eff}^2}{(\tilde{L}_{\sigma d}^{M1})^2} \left[ \frac{3\gamma_0^2 + 9\gamma_0\gamma_2 + 8\gamma_2^2}{(\gamma_0 + \gamma_2)^3} \right]^2 \approx 1 \times 10^{-5},
\]

which implies that the M1 transition strongly dominates over E2 for 17C.

2. D \( \rightarrow \) D

The M1 transition strength from the 3/2\(^+\) ground state (D-wave) to the second excited 5/2\(^+\) state (D-wave) in 17C was also measured in Ref. [18]: B(M1: 5/2\(^+\) \( \rightarrow \) 3/2\(^+\)) = 7.12\(^{+1.97}_{-0.96}\) \times 10\(^{-2}\) \( \mu^2 \). Compared to the D \( \rightarrow \) S-state M1 transition strength, it is around one order of magnitude larger. This is in agreement with the fact that M1 transitions are allowed for neutron-core systems with one-body currents by the usual selection rules. We calculate both loop diagrams in Fig. 4 and find that we need additional counterterms to absorb all divergences. Moreover, we obtain results for the M3 and M5 transition. We find that two different counterterms are needed for the M1 transition and also two for the M3 transition.

In the following, we concentrate the discussion on the M1 transition. In this case, the two counterterms are given by:

\[
L_M = -L_{M1a}^{dd} \mu_N d_{ij}^d d_{ij}^d \sigma_k^{m_s m_s'} B_k - L_{M1b}^{dd} \mu_N d_{ij}^d \nabla \cdot A d_{ij}^d.
\]

The first counterterm is needed to renormalize the scale dependence from diagram (a) with the magnetic photon coupling to the neutron and the second one renormalizes the scale for the vector photon coupling in diagram (b), respectively. For the calculation it is convenient to define:

\[
\tilde{L}_{M1a}^{dd} = \frac{15\pi}{m^2_{R g^2 g_2}} L_{M1a}^{dd} + \frac{15}{4} (\gamma_2^2 + \gamma_2^2) \kappa_n \mu, \tag{33}
\]

\[
\tilde{L}_{M1b}^{dd} = \frac{15\pi}{m^2_{R g^2 g_2}} L_{M1b}^{dd} + \frac{15}{4} (\gamma_2^2 + \gamma_2^2) \frac{m R Q_c}{M} \mu, \tag{34}
\]

where \( \mu \) is the PDS scale.
Again, the photon has four-momentum \( k = (\omega, \mathbf{k}) \), and its polarization index is denoted by \( \mu \). The computation of both diagrams yields a vertex function \( \Gamma_{mm'm} \), where \( m \) is the total angular momentum projection of the \( 3/2^+ \) state and \( m' \) denotes the spin projection of the \( 5/2^+ \) state. We compute the vertex function with respect to the specific components of the \( D \)-wave interaction:

\[
\Gamma_{mm'm} = \sum_{\alpha\beta\delta\mu} \frac{1}{m_s 2m_l} \left( \frac{1}{2} m_s 2m_l \right) (1\alpha1\beta|2m_l) \left( \frac{1}{2} m_{s'} 2m_{l'} \right) (1\delta1\eta|2m_{l'}) \tilde{\Gamma}_{\alpha\beta\mu}. \tag{35}
\]

We calculate the irreducible vertex in Coulomb Gauge so that we have \( k \cdot p = 0 \) for real photons. Additionally, we choose \( k \cdot \epsilon = 0 \), where \( \epsilon \) denotes the incoming momentum of the \( D \)-wave state.

As a result, the space-space components of the vertex function in Cartesian coordinates for the left diagram can be written as:

\[
\tilde{\Gamma}_{ijop} = \Gamma_{ijop}^{(a)} \epsilon_{abk} \sigma_a m_{s} m_{s'} k_b \left( \frac{\delta_{io}\delta_{jp} + \delta_{ip}\delta_{jo}}{2} - \frac{1}{3} \delta_{ij}\delta_{op} \right), \tag{36}
\]

and for the right one:

\[
\tilde{\Gamma}_{ijop} = \Gamma_{ijop}^{(b)} \left( \frac{\delta_{io}\delta_{jp} + \delta_{ip}\delta_{jo}}{2} - \frac{1}{3} \delta_{ij}\delta_{op} \right) + \Gamma_{E2} \left[ k_i \left( \frac{\delta_{jp}\delta_{ko} + \delta_{jo}\delta_{kp}}{2} - \frac{1}{3} \delta_{jk}\delta_{op} \right) + \cdots \right]. \tag{37}
\]

In the left diagram, the photon couples to the spin of the neutron and we get a spin flip \( m_s \neq m'_s \). In the case of the right diagram there is no spin flip so that \( m_s = m'_s \). By choosing the photon to be traveling in \( \hat{z} \) direction it follows from the tensor structure of \( \tilde{\Gamma}_{ijop} \) that \( m_l = m'_l \) and \( \mu \neq 0 \).

For the case that \( m = \pm 1/2 = -m' \) we get:

\[
\tilde{\Gamma}_{+-,1} = \Gamma_{+-,-1} = \frac{\sqrt{6}}{5} \Gamma_{M}^{(a)} \sqrt{2} k, \tag{38}
\]

and for \( m = m' \) we get 0 for all possible values. This yields for the \( \text{B(M1: } 3/2^+ \rightarrow 5/2^+ \text{)} \) transition:

\[
\text{B(M1: } 3/2^+ \rightarrow 5/2^+ \text{)} = \frac{3}{4 \pi} \left( \frac{\Gamma_{+-,-1}}{\omega} \right)^2 = \frac{9}{25 \pi} \left( \frac{\Gamma_{M}^{(a)} \omega}{\omega} \right)^2
\]

\[
= \frac{9\mu^2}{25\pi} \frac{\gamma_2^2}{r_2^2 + P_2}\left[ \frac{\tilde{L}_{M1a}^{dd'} + 2\gamma_2^2\kappa_n}{(\gamma_2^2 + \gamma_2^2) + 2\kappa_n (\gamma_2^2 + \gamma_2^2)} \right]^2,
\]

with the renormalized, irreducible vertex \( \tilde{\Gamma}_{M} = \sqrt{Z_d Z_{d'}} \Gamma_{M} \). By rescaling the fields, \([\hat{d}] = [\hat{d}]' = 0\), and using dimensional analysis we find that the counterterm scales as \( L_{M1a}^{dd'} \sim M_{h_i}^3 g_{dd'g2 ineffective} g_{2'} m_{R}^2 \), with \( L_{M1a}^{dd'} \) of order one. In contrast, the contribution from the loop scales as \( M_{lo}^2 \) which means that in LO only the counterterm contributes to the M1 transition and the loop diagram is suppressed by \( (M_{lo}/M_{hi})^3 \). Thus the M1 transition is strongly dominated by short-range physics.

C. E1 neutron capture on \( ^{16}\text{C} \)

1. E1 capture into the \( 1/2^+ \) state

E1 capture proceeds dominantly through the vector coupling of the photon to the halo core. The corresponding leading order operator is generated through minimal substitution in Eq. (2). The diagram that contributes at LO to this process is shown in Fig. 5. It is the time-reversed
diagram of the photodissociation reaction considered in Ref. [12]. At LO, the amplitude is

$$\bar{\Gamma}^i = \frac{\epsilon^i \cdot p}{M} \frac{\sqrt{Z^e} e Q_c g_0 2m_R}{\gamma_0^2 + (p - \frac{m}{M_{nc}} k)^2}.$$  \hspace{1cm} (39)

where \(i\) is the photon polarization, \(p\) denotes the relative momentum of the \(nc\) pair and \(k\) the photon momentum. Throughout this section we choose the \(nc\) pair to be traveling in \(\hat{z}\) direction which means that \(p = |p|\hat{e}_z\). Since \(m/M_{nc}\) is small and it follows from power counting that \(p \sim \gamma_0 \sim M_0\) and \(k \sim M_0^2/M_{hi}\), we can neglect the recoil term \(\sim p \cdot k\) in the denominator. By averaging over the neutron spin and photon polarization and summing over the outgoing \(S\)-wave spin we obtain at LO \((m/M_{nc} k \ll p)\):

$$\frac{d\sigma_{cap}}{d\Omega} = \frac{m_R k}{4\pi^2 p} |\mathcal{M}^{(1/2)}|^2 = \frac{e^2 Z_{eff}^2 p\gamma_0 \sin^2 \theta}{\pi m_R^2 (p^2 + \gamma_0^2)},$$  \hspace{1cm} (40)

with \(k \approx (p^2 + \gamma_0^2)/2m_R\), \(\hat{k} \cdot \hat{p} = \cos \theta\), \(Z_{eff} = (m_R/M)Q_c \approx 0.353\) and

$$|\mathcal{M}^{(1/2)}|^2 = \frac{1}{2} \sum_{i,ms,M} |\bar{\Gamma}^i|^2 \delta_{ms,M},$$  \hspace{1cm} (41)

where \(m_s\) denotes the neutron spin and \(M\) the \(S\)-wave polarization. Since the neutron spin is unaffected by this reaction, \(m_s\) and \(M\) have to be the same. After integration over \(d\Omega\) we get

$$\sigma_{cap} = \frac{m_R k}{\pi p} |\mathcal{M}^{(1/2)}|^2 = \frac{8e^2 Z_{eff}^2 p\gamma_0}{3m_R^2 (p^2 + \gamma_0^2)} = \frac{32\pi \alpha Z_{eff}^2}{3m_R^2 (p^2 + \gamma_0^2)},$$  \hspace{1cm} (42)

with the fine-structure constant \(\alpha = e^2/(4\pi)\). Exploiting the detailed balance theorem, the capture cross section \(\sigma_{cap}\) can be related to the photodissociation cross section \(\sigma_{dis}\) [32],

$$\sigma_{cap} = \frac{2(2j_{\pi} + 1)}{(2j_n + 1)(2j_c + 1)} \frac{k^2}{p^2} \sigma_{dis} = \frac{2k^2}{p^2} \sigma_{dis}.$$  \hspace{1cm} (43)

Our numerical results for the E1 capture into \(^{17}\text{C}\) and photodissociation of \(^{17}\text{C}\) obtained using Eq. (43) at LO are shown in Fig. 6. At NLO, there is an additional contribution from the effective range \(r_0\). By assuming that \(r_0\) scales as \(1/M_{hi}\), we can estimate the size of the NLO contribution and add an error band to our LO results in Fig. 6.
Figure 6. Left panel: E1 capture cross section into $^{17}$C as a function of the center-of-mass energy $E_{cm}$. Right panel: E1 photodissociation cross section as a function of $E_{cm}$. The solid (blue) line denotes the LO result and the dashed (red) lines show an estimate of the NLO corrections.

Figure 7. Relevant diagrams for E1 capture to D-wave states at LO. For a more detailed description of the lines, see Fig. 2.

2. E1 capture into the $3/2^+$ and $5/2^+$ states

In this section, we calculate E1 neutron capture to the $3/2^+$ D-wave ground state and $5/2^+$ excited state of $^{17}$C. The relevant diagrams that emerge from minimal substitution in our Lagrangian (2) are shown in Fig. 7. They yield

$$\Gamma_{m,JM}^i = \sum_{m',m_l} \left( \frac{1}{2} m_{s'} 2m_l \left| JM \right) \sum_{\alpha \beta} (1 \alpha 1 \beta | 2m_l) \sqrt{Z_g g_{2e} Q_c} \frac{2m_R}{M} \times \frac{1}{2m_{nc}} k \right)_{\alpha} \left( p - \frac{m_{nc}}{M} k \right)_{\beta} \epsilon^\iota \cdot p + \epsilon^i \left( p_{\beta} - \frac{m_{nc}}{2M} k_{\beta} \right) \delta_{m,m'},$$

where the charge of the core $Q_c$, the photon momentum $k$, the relative momentum of the incoming $nc$ pair $p$, the photon polarization $\iota$ and $JM$ denoting the spin and polarization of the D-wave. Note that the neutron spin is unaffected by the E1 capture process up to this order. If we project out the $J = 3/2$ part of the amplitude $M^{(3/2)}$ and average and sum over incoming and outgoing spins, respectively, we finally find the differential cross section for the E1 capture process at LO ($\frac{m}{M_{nc}} k \ll p$):

$$\frac{d\sigma^{cap}}{d\Omega} = \frac{m_R}{4\pi^2 p} \left| M^{(3/2)} \right|^2 = \frac{15}{2\pi} \frac{(p^2 + \gamma_2^2)}{m^2_{Rb}} \frac{e^2 Z_{eff}^2}{-r_2 - P_2 \gamma_2} X(\theta) = \frac{30\alpha Z_{eff}^2}{-r_2 - P_2 \gamma_2} \frac{(p^2 + \gamma_2^2)}{m^2_{Rb}} X(\theta),$$

with $\gamma_2$ the photon energy, $r_2$ the point of closest approach, $P_2$ the photon polarization, and $X(\theta)$ the spin eigenvalue.
with the fine-structure constant $\alpha$, $Z_{\text{eff}} = (m_R/M)Q_c$ and

$$|\mathcal{M}^{(3/2)}|^2 = \frac{1}{2} \sum_{i,m,s,M} |\Gamma_{i,m,3/2M}|^2,$$  

(46)

and

$$X(\theta) = \frac{1}{15} \left[ 2p^2(13 - \cos(2\theta)) + \frac{4p^4 \sin^2(\theta)}{(\gamma_2^2 + p^2)} \left( \frac{p^2}{\gamma_2^2 + p^2} + 2 \right) \right].$$  

(47)

After integrating over $d\Omega$ we find for the total cross section:

$$\sigma^{\text{cap}} = \frac{\alpha Z_{\text{eff}}^2}{-r_2' - \mathcal{P}_2\gamma_2^2} \frac{32\pi p (5\gamma_2^4 + 11p^4 + 14\gamma_2^2 p^2)}{3m_R^2}.$$  

(48)

From an experimental measurement of the capture (or dissociation) cross section we can therefore extract the numerical value of the combination of $D$-wave effective range parameters $1/(r_2' - \mathcal{P}_2\gamma_2^2)$ For the $5/2^+$ state we project out the $J = 5/2$ part of the amplitude $M^{(5/2)}$ and obtain:

$$\frac{d\sigma^{\text{cap}}}{d\Omega} = \frac{m_R k}{4\pi^2 p} |\mathcal{M}^{(5/2)}|^2 = \frac{45}{4\pi} \frac{(p^2 + \gamma_2^2)}{m_R^2p} \frac{e^2Z_{\text{eff}}^2}{-r_2' - \mathcal{P}_2\gamma_2^2} X(\theta) = \frac{45\alpha Z_{\text{eff}}^2}{-r_2' - \mathcal{P}_2\gamma_2^2} \frac{(p^2 + \gamma_2^2)}{m_R^2p} X(\theta),$$  

(49)

where $X(\theta)$ is the same as for the $J = 3/2$ cross section. After integrating over $d\Omega$ we find for the total cross section:

$$\sigma^{\text{cap}} = \frac{\alpha Z_{\text{eff}}^2}{-r_2' - \mathcal{P}_2\gamma_2^2} \frac{16\pi p (5\gamma_2^4 + 11p^4 + 14\gamma_2^2 p^2)}{m_R^2} \left( \frac{\gamma_2^2}{\gamma_2^2 + p^2} \right),$$  

(50)

which is the same result as the $J = 3/2$ cross section multiplied by a factor of $3/2$ and different numerical values for $\gamma_2$, $r_2$ and $\mathcal{P}_2$.

**D. M1 neutron capture on $^{16}$C**

1. **M1 capture into the 1/2$^+$ state**

Similar to E1 capture, we can calculate the M1 capture cross section. The main difference between both processes is the parity conservation in the M1 matrix element. Therefore, the loop
Diagram (b) shown in Fig. 8 is also relevant at LO for M1 capture since initial state interactions in the S-wave channel have to be taken into account. Additionally, the photon now couples to the magnetic moment of the halo neutron. For diagram (a) in Fig. 8 we get:

$$\bar{\Gamma}_{im,s,m'}^{(a)} = -2\sqrt{Z_\sigma g_0 M_N} \frac{\sigma_j^{m,m'}(k \times e^i)_j}{\gamma_0^2 + (p - \frac{M}{M_N} k)^2},$$

(51)

with the Pauli matrices $\sigma_j$, the photon polarization index $i$, and the relative momentum of the incoming nc pair $p$.

Since the power counting stipulates $p \sim \gamma_0 \sim M_{lo}$ and $k \sim M_{lo}^2/M_{hi}$, we can neglect the recoil term $\sim p \cdot k$ in the denominator of Eq. (51).

$$\bar{\Gamma}_{im,s,m'}^{(a)} = -2\sqrt{2\pi \gamma_0 \epsilon_0 M_N} \frac{\sigma_j^{m,m'}(k \times e^i)_j}{\gamma_0^2 + p^2}. \tag{52}$$

Diagram (b) with the intermediate $S$-wave state yields

$$\bar{\Gamma}_{im,s,m'}^{(b)} = -\sqrt{Z_\sigma g_0^2 M_N} \frac{2\pi}{g_0^2 m R} \frac{\sigma_j^{m,m'}(k \times e^i)_j}{\gamma_0^2 + p^2 + ip} \int \frac{d^3l}{(2\pi)^3} \frac{2m_R}{p^2 - l^2} \frac{2m_R}{(l + \frac{m}{m} k)^2},$$

(53)

with the loop momentum $l$, which leads at LO to

$$\bar{\Gamma}_{im,s,m'}^{(b)} = 2\sqrt{2\pi \gamma_0 \epsilon_0 M_N} \frac{\sigma_j^{m,m'}(k \times e^i)_j}{\gamma_0 + ip} \frac{1}{\gamma_0 - ip} = -\bar{\Gamma}_{im,s,m'}^{(a)}. \tag{54}$$

As a consequence, both diagrams cancel each other at LO. In coordinate space, this process is given by an overlap integral between two orthogonal wave functions. At NLO, there is an additional contribution from the effective range $r_0$ as discussed for the E1 capture process before, which will give a correction of order $\gamma_0 r_0 \approx 40\%$.

a. Recoil corrections - Subleading recoil corrections are usually dropped in EFT calculations for capture reactions such as this one. Taking recoil corrections into account, the first diagram (a) will give non-zero contributions to higher multipoles through higher partial waves in the initial state. The second diagram (b) in Fig. 8 contributes only when the core and the nucleon are in a relative $S$-wave in the initial state.

The denominator in Eq. (51) for diagram (a) can be written in coordinate space as

$$\frac{1}{\gamma_0^2 + (p - \frac{M}{M_N} k)^2} = \int dr^3 \exp\left(-\gamma_0 r\right) \frac{1}{4\pi r} \exp\left[i \left(p - \frac{M}{M_N} k\right) \cdot r\right]. \tag{55}$$

The expression on the right-hand side can be viewed as the overlap integral of halo wave function, a plane wave scattering wave function and the current operator $\exp(i \frac{M}{M_N} k \cdot r)$. After spherically expanding the plane wave scattering wave function, we obtain for the different partial waves

$$\frac{1}{\gamma_0^2 + (p - \frac{M}{M_N} k)^2} = -\sum_l (2l + 1) i^{2l} P_l(\hat{p} \hat{k}) \frac{M_{nc}}{2Mk} \Re \left\{ Q_l \left( -\frac{M_{nc}}{2Mk} \left(p^2 + \frac{M^2}{M_{nc}^2} k^2 + \gamma_0^2\right)\right) \right\}, \tag{56}$$

where $Q_l(x)$ denotes the Legendre function of the second kind and we used

$$\exp(ip \cdot r) = 4\pi \sum_{l=0}^{\infty} \sum_{m=-l}^{l} i^l j_l(pr) Y_{lm}(\hat{p}) Y_{lm}^*(\hat{r}). \tag{57}$$
As an example, we consider the S-wave result for Eq. (56)

$$- \frac{1}{a} \ln \left( 1 - \frac{a}{\gamma_0^2 + \left( p + \frac{M_{nc}}{M_{nc}}k \right)^2} \right), \quad (58)$$

with $a = M_{nc}/(4M_kp)$, which is in perfect agreement with Eq. (52) if we set $k \sim 0$ and expand the logarithm.

After averaging and summing over incoming and outgoing spins, respectively, we obtain for the differential cross section the general result:

$$\frac{d\sigma^{\text{cap}}}{d\Omega} = \frac{m_R k}{4\pi^2 p} |\mathcal{M}^{(1/2)}|^2 = \frac{m_R k^3}{m^2 p} \left[ \frac{4\alpha^2 \kappa_n^2 \gamma_0}{\gamma_0^2 + \left( p - \frac{M_{nc}}{M_{nc}}k \right)^2} \right]^2, \quad (59)$$

with the fine structure constant $\alpha$ and

$$|\mathcal{M}^{(1/2)}|^2 = \frac{1}{2} \sum_{i,m_i,m_{i'}} |\bar{\Gamma}^{(a)}_{im_i m_{i'}}|^2. \quad (60)$$

2. M1 capture into the 3/2+ and 5/2+ states

In this section, we calculate M1 neutron capture to the 3/2+ D-wave ground state or 5/2+ excited state of 17C. At LO for the D-wave, we only have to consider diagram (a) in Fig. 8 where the photon couples to the magnetic moment of the neutron. The loop diagram contributes at NLO. This yields:

$$\bar{\Gamma}^i_{m_s JM} = \sum_{m_{s'}, m_{l}} \left( \frac{1}{2} m_{s'} 2m_l \right) |J M \rangle \sum_{\alpha, \beta} (1 \alpha 1 \beta | 2m_l \rangle \sqrt{Z_d g_2} 2m_R \kappa_n \mu_N \times \sigma^m_{m_{s'}, m_{l}} (k \times e)^j (p + \frac{M_{nc}}{M_{nc}}k)^\alpha (p + \frac{M_{nc}}{M_{nc}}k)^\beta \frac{\gamma_0^2}{\gamma_0^2 + \left( p - \frac{M_{nc}}{M_{nc}}k \right)^2}, \quad (61)$$

with the magnetic moment of the neutron $\kappa_n$, the D-wave polarizations $\alpha$ and $\beta$, the photon momentum $k$, photon polarization $i$ and the relative momentum of the incoming $nc$ pair $p$. If we project out the $J = 3/2$ part of the amplitude $M^{(3/2)}$ we finally find the differential cross section for the M1 capture process at LO:

$$\frac{d\sigma^{\text{cap}}}{d\Omega} = \frac{m_R k}{4\pi^2 p} |\mathcal{M}^{(3/2)}|^2 = \frac{m_R k^3}{m^2 m_{nc}^2} \frac{k^3 p^3 (\gamma_0^2 + p^2)^2}{r_2^2 - p_2 \gamma_0^2}, \quad (62)$$

with the fine-structure constant $\alpha$ and

$$|\mathcal{M}^{(3/2)}|^2 = \frac{1}{2} \sum_{i,m_i,M} |\bar{\Gamma}^{i}_{m_i,3/2M}|^2. \quad (63)$$

After integration over $d\Omega$ we obtain for the total cross section:

$$\sigma^{\text{cap}} = \frac{4\pi \alpha \kappa_n^2 (\gamma_0^2 + p^2)^2 p^3}{m^2 m_{nc}^2 m_{nc}^2 - r_2^2 - p_2 \gamma_0^2}, \quad (64)$$
where we replaced $k$ by $(\gamma_2^2 + p^2)/(2m_R)$. From an experimental measurement of the cross section we can extract therefore the numerical value of the combination of D-wave effective range parameters $1/(-r_2 - p_2\gamma_2^2)$ which is directly related to the ANC of the D-wave, $A_2 = \sqrt{2\gamma_2^4/(-r_2 - p_2\gamma_2^2)}$. At NLO, there is an additional unknown short-range contribution that enters via a counterterm. Moreover, we have to consider final state interactions similar to the right diagram in Fig. 8 for the S-wave case. For the 5/2+ state we project out the $J = 5/2$ part and obtain the same result as for $J = 3/2$ with a relative factor of $3/2$:

$$\sigma^{cap} = \frac{m_R k}{\pi p} |M^{(5/2)}|^2 = \frac{6\pi\alpha_n^2 \gamma_2^2 (\gamma_2^2 + p^2) p^3}{m_R^2 m^2 - r_2^2 - p_2\gamma_2^2} \quad (65)$$

If we compare the cross section for E1 capture into the D-wave state with the result above, we find that M1 capture is suppressed approximately by a factor of $p^2 M^2/(m^2 m^2) \sim 10^{-2}$ for $^{17}\text{C}$.

V. SUMMARY

Halo nuclei are weakly bound systems of a tightly bound core nucleus and a small number of valence nucleons. Their structure can be probed experimentally by measuring capture reactions, dissociation cross sections, and charge radii. In this work, we have discussed these observables for S- and D-wave halo states using the framework of Halo EFT.

We have considered the nucleus $^{17}\text{C}$ as a halo nucleus consisting of a $^{16}\text{C}$ core and a neutron. $^{17}\text{C}$ is an interesting halo candidate since it has three S- and D-wave neutron-core states with small neutron separation energies in its spectrum. We have calculated the key observables relevant to this system, including radii, magnetic moments as well as electric and magnetic transition rates. Moreover, we showed that capture reactions can provide insight into the continuum properties of the neutron-$^{16}\text{C}$ system.

We found that predictions of many observables for states with angular momentum larger than zero need additional input parameters, beyond the neutron separation energy. This limits the predictive power of Halo EFT for such states. However, these counterterms can be matched to experiment or other theoretical calculations. For example, the counterterms appearing in the expressions for the S- to D-wave transitions can be determined in this way. Shell model calculations for $^{17}\text{C}$ were carried out in Ref. [27] using effective interactions derived from first principles, and this approach could be extended to calculate the transitions in our work. The results could then be used to predict capture cross sections since the counterterms in capture cross sections and transition strengths are related. This strategy would provide insights into the continuum properties of the neutron-$^{16}\text{C}$ system.

Some of the observables discussed in this work have been studied extensively in the case of the deuteron which can be considered the lightest halo nucleus, consisting of a neutron and a proton core [28, 33]. One-neutron halo nuclei can therefore have similar electromagnetic properties to the deuteron. For example, the expression for the LO charge radius of an S-wave neutron halo nucleus shown in Eq. (10) is the same as for the deuteron. However, the deuteron consists of two spin-1/2 particles and interacts resonantly in the spin-triplet and spin-singlet S-wave channels. This leads to a relatively large M1 capture cross section between the unbound spin-singlet and the spin-triplet
channel in which the deuteron resides. The absence of a second resonantly interacting channel leads a strong suppression of magnetic capture in the case of $^{17}\text{C}$.

We hope that our investigation will motivate further theoretical and experimental investigations of $^{17}\text{C}$. The expressions presented in this paper should be useful for the analysis of experimental and/or ab initio data on $^{17}\text{C}$ in order to establish the halo nature of $^{17}\text{C}$. The combination of Halo EFT and ab initio calculations as was done in Refs. [34–36] could provide insights into the continuum properties of $^{17}\text{C}$ and should facilitate a test of the power counting that was used in this work.

Future extensions of our calculation to NLO and beyond would improve this comparison quantitatively, but a growing number of counterterms may invalidate this advantage.

ACKNOWLEDGMENTS

We acknowledge useful discussions with Thomas Papenbrock and Wael Elkamhawy. JB thanks the University of Tennessee, Knoxville and the Joint Institute for Nuclear Physics and Applications for their hospitality and partial support. This work has been supported by Deutsche Forschungsgemeinschaft under grant SFB 1245, by the BMBF under grant No. 05P15RDFN1, by the Office of Nuclear Physics, U.S. Department of Energy under Contract No. DE-AC05-00OR22725 and the National Science Foundation under Grant No. PHY-1555030.

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