Electric dipole spin resonance in finite-size multilevel quantum dots: Floquet dynamics with tunneling

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We study electric dipole spin resonance in a multilevel finite-size quantum dot formed in a nanowire concentrating on the range of driving electric fields where a strong interplay between the Rabi spin oscillations and tunneling can occur. In the tunneling process mixed spin states are formed and determine the spin evolution. We demonstrate a strong effect of tunneling on the spin flip process and backaction of the spin dynamics on the tunneling and position of the electron. We analyze the efficiency of the spin-flip and time-dependent electron displacement and demonstrate that tunneling decreases this efficiency. Fourier analysis of the time-dependent expectation value of the electron position shows a strong effect of the spin-orbit coupling on its low-frequency components.

I. INTRODUCTION

Electric dipole spin resonance (EDSR) [1–4], being a characteristic feature of systems with spin-orbit coupling (SOC), provides an efficient tool of manipulation of electron spins in solids. This ability to manipulate spin by electric field is, in general, due to the fact that spin precession is related to the electron displacement caused by application of external electric field. There are three basic kinds of systems where the electric dipole spin resonance can be observed. First realization is given by itinerant electrons in bulk crystals [1–4], surfaces [5], and in artificial semiconductor structures [6]. Second realization, theoretically proposed in Refs. [7–15], has been experimentally realized for electrons [16] and holes [17] in single and double semiconductor quantum dots. A similar approach can be applied to the carriers in carbon nanotubes [18]. Due to a strong confinement by external potential, here the spectrum of carriers is discrete, and the carrier always remains inside the dot, either two- [16] or one-dimensional [19] even when a strong electric field is applied. Third realization occurs for electrons in potentials vanishing at large distances with the examples being: electrons on donors or impurities [20–22], in finite-range potential produced by charged gates or tips [23, 24], and in surface-based self-assembled quantum dots. This third realization is of our interest here since low-frequency electric fields can ionize the electron states by tunneling and, thus, cause additional nontrivial spin dynamics resulting from the spatial spread of the electron wavefunction. For example, even for the interwell tunneling in the double quantum dot on relatively short distances the coupled spin and spatial dynamics may cause strong nonlinearities in the Rabi frequency dependence on the driving field strength [25].

Thus, we are interested in Floquet dynamics in a tunneling system, where a combination of two long time scales becomes important. The first slow time scale is the time of spin flip driven by electric field, inversely proportional to the field amplitude. The second one is the tunneling time due to driving by electric field producing coupling of localized stats to the continuum, being strongly, approximately exponentially, dependent on the inverse field amplitude. If both these times are of the same order of magnitude, the coupled spin and charge density dynamics, studied in this paper, becomes highly nontrivial. In a multilevel quantum dot the tunneling is a well-defined semiclassical process, in contrast to that in shallow quantum dots [24]. As a result of a stronger confinement, the amplitude of the electric field required to operate a spin flip is considerably larger and, therefore, the involvement of the continuum states is qualitatively different.

An interesting aspect of this dynamics is that usually the EDSR assumes a pure spin state, defined on the Bloch sphere, where the spin rotation occurs with a minor change in the electron orbital state. However, in the presence of spin-orbit coupling the spin states become mixed.
This circumstance strongly modifies the entire spin evolution. For applications, the efficiency of the spin-flip process, that is, the ratio of the Zeeman energy to the total energy acquired by electron as a result of the electric field action, is of interest. We will demonstrate how the efficiency depends on the system parameters. Moreover, the spin-orbit coupling causes a feedback of the spin dynamics on the tunneling efficiency and resulting electron position. Even at a relatively weak spin-orbit coupling this effect can be strong, as we will demonstrate below.

This paper is organized as follows. In Sec. II we introduce the Hamiltonian of our system and discuss time-independent and periodic contributions to the electron energy. In Sec. III we describe the computational model of electron states and their dynamics and provide the specific system parameters. In Sec. IV the main results are presented: spin dynamics, structure of electron wavefunction, time dependencies of mean energy and efficiency of spin flip, localization probabilities, coordinate evolution, and its Fourier power spectra. Finally, in Sec. V we give our conclusions.

II. HAMILTONIAN AND OBSERVABLES

We consider a narrow nanowire, elongated along the $x-$axis located on the top of a doped substrate, as schematically presented in Fig. 1. Assuming that the transversal modes are not excited, we characterize the electron motion by a two-component wave function $\psi(x,t) = [\psi_1(x,t), \psi_2(x,t)]^T$, where $T$ stands for the transposition, and use the effective mass approximation Hamiltonian with:

$$H(t) = \frac{\hbar^2 k^2}{2m} + V(x,t) + \frac{\Delta}{2}\sigma_z + \alpha \sigma_y k.$$  

Here $m$ is the electronic effective mass, and $k = -i\partial/\partial x$ is the wave vector operator. The time-dependent potential is the sum of the static confinement potential $U(x)$, as can be produced by electrostatic gate shown schematically by a rectangle above the nanowire in Fig. 1 and external driving giving

$$V(x,t) = U(x) + Fx \sin \omega_d t,$$

where $F \equiv e\mathcal{E}_0$, $e$ is the fundamental charge and $\mathcal{E}_0$ is the field amplitude. The Rashba SOC $H_R = \alpha \sigma_y k$ while, more complicated forms of spin-orbit coupling can be found in Refs. [24–29]. The constant magnetic field $\mathbf{B} = (0,0,B)$ produces the Zeeman term in the Hamiltonian. Here the Zeeman splitting $|\Delta| = \mu_B |gB|$, where $\mu_B$ is the Bohr magneton, and $g$ is the Landé-factor. The spin-split Zeeman partners participate in the spin resonance driven by the external periodic electric field in (2). The reason for this participation is that due to the presence of SOC the eigenstates of (1) at $F = 0$ always contain both spin components. Therefore, spin-flip transitions still be caused by the electric field described with the position operator, producing the EDSR. In this case, the well-defined spin Rabi oscillations occur if one neglects the tunneling effects, that is, coupling to the other orbital and continuum states. The dynamical interaction between discrete and continuum states during the evolution driven by periodic electric field causes nontrivial effects to be discussed in the following Sections. One of the effects is illustrated schematically in Fig. 2 in the presence of SOC the direction of spin precession is different, depending on the direction of the tunneling escape, making a controlled spin flip rather challenging task especially if the tunneling is triggered by the alternating field responsible for the spin evolution.

To describe the evolution we solve the nonstationary Schrödinger equation

$$i\hbar \partial_t \psi(x,t) = H(t)\psi(x,t),$$

where $\partial_t \equiv \partial/\partial t$, and calculate expectation values for experimentally measurable observables $\mathcal{O}$ as:

$$\langle \mathcal{O}(t) \rangle = \int_{-\infty}^{\infty} \psi^\dagger(x,t)\mathcal{O}\psi(x,t)dx.$$  

The observables will be specified later in the text.

III. MODEL AND NUMERICAL APPROACH

A. Basis states and model of the dynamics

We choose the static potential $U(x) = -U_0/\cosh^2(x/d)$, assuming that it is deep such that the parameter $\xi \equiv U_0 md^2/\hbar^2 \gg 1$. Although the exact bound states for this potential are well-known analytically in terms of the Legendre functions, it is practical to consider simple approximations based on
the expansion at \( x \ll d \) as \( U(x) = -U_0 \left(1 - x^2/d^2\right) \). As a result, the lowest bound states can be described in terms of a harmonic oscillator with the energy interval \( \hbar \omega_{\text{ho}} = U_0/\sqrt{\xi/2} \ll U_0 \) and the Gaussian width \( d/(2\xi)^{1/4} \ll d \). The numerically accurate eigenenergies \( E_n^{(0)} \) and basis states of the Hamiltonian \( \hbar^2 k^2/2m + U(x) \) are found by expansion in the basis of periodic functions \( \sin(\kappa_n x + \delta_n) - \) satisfying zero boundary conditions at \( x = \pm L \), where \( 2L \) is a large wire length.

Then, we numerically produce basis of two-component eigenstates \( \phi_n(x) \) of the spinful Hamiltonian

\[
H_0 = \frac{\hbar^2 k^2}{2m} + U(x) + \Delta \sigma_z + \alpha \sigma_y k
\]

with the spin-split eigenenergies \( E_n \) such as \( H_0 \phi_n(x) = E_n \phi_n(x) \). As a result, the evolution of wave function is presented as

\[
\psi(x,t) = \sum_n c_n(t) e^{-iE_n t/\hbar} \phi_n(x).
\]

Thus, the problem is reduced to obtaining full set of coefficients \( c_n(t) \) by numerical solution of a large system of linear differential equations, as will be presented below with some computations details given in Ref. [24]. Here we choose the interval \( 2L = 320d \) and take 4000 basis states \( \phi_n(x) \). The initial state is chosen as \( \psi(x,0) = \phi_1(x) \), that is the ground state of \( H_0 \), where for the realistic parameters one has \( \langle \sigma_z(0) \rangle = 1 \).

B. System parameters

In the following analysis we consider a gate-formed quantum dot based on InSb quantum wire, see Fig. 1. For this material \( m = 0.0136 \) of a free electron mass \( m_0 \) and \( g = -50.6 \). The magnitude of SOC \( \alpha \) in InSb can be tuned by the gate voltage up to \( 100 \mu eV m \) [31, 32]. For the quantum dot parameters we accept \( U_0 = 27 \) meV and \( d = 50 \) nm with five discrete quantum dot levels \( E_1^{(0)}, \ldots, E_5^{(0)} \) being formed [33] with \( \omega_0 = (E_2^{(0)} - E_1^{(0)})/\hbar = 13.43 \) ps\(^{-1} \), and the associated period \( T_0 = 2\pi/\omega_0 = 0.468 \) ps gives the natural time scale for the evolution. We consider the magnetic field \( B = 0.447 \) T, providing the Zeeman splitting of the ground state corresponding to the resonant driving frequency (including the contribution of spin-orbit coupling) \( \omega_d = 2 \) ps\(^{-1} \) (0.32 THz), giving the ratio to the fundamental frequency \( \omega_d/\omega_0 = 0.149 \). We consider the effect of a relatively weak SOC \( \alpha = 5 \) meVnm and a relatively strong \( \alpha = 25 \) meVnm for various driving amplitudes. Taking into account the material parameters for spin-orbit coupling produced by external electric field in InSb [34], we obtain that the effective two-dimensional donor dopant layer concentration, as shown in Fig. 1, corresponding to this spin-orbit coupling, is in the range of \( 10^{11} - 10^{12} \) cm\(^{-2} \).

IV. SPIN DYNAMICS AND FEEDBACK ON THE TUNNELING AND POSITION

A. Time dependence of spin: the role of the tunneling

In Fig. 2 we show the time dependence of \( \langle \sigma_z(t) \rangle \) corresponding to Eq. (4) with \( O = \sigma_z \) giving

\[
\langle \sigma_z(t) \rangle = \sum_{n_1,n_2} c_{n_1}^*(t) c_{n_2}(t) e^{-i(E_{n_2} - E_{n_1})t/\hbar} \langle \phi_{n_1} | \sigma_z | \phi_{n_2} \rangle.
\]

Here and below the time is measured in the units of \( T_0 \), and we track the evolution on calculation time scales of about \( 650T_0 \) for small SOC and \( 200T_0 \) for large SOC. These ranges are determined by the spin flip processes which needs to be well-developed. As a result, one can obtain from the spin dynamics the characteristic spin evolution time. One can see that a sizable decrease in this time can be observed for moderate increase of \( F \) from 0.16 to 0.2 meV/nm which is in the framework of usual Rabi frequency dependence on the driving field amplitude. When the field grows further, the electron escapes the quantum dot quickly, before well-established spin oscillations are developed. This effect is stronger at lower values of \( \alpha = 5 \) meVnm. One can see also that at the fields \( F \geq 0.2 \) meV/nm the Rabi oscillations are accompanied by visible damping. We attribute this effect to the interactions between localized and continuum states which is enhanced at higher driving fields. The continuum states form a dense set of levels with alternating spin projections formed by Zeeman splitting [24]. Transitions to these states driven by external electric field lead to the wavefunction consisting of spin-up and spin-down states, and their relative contributions to the total norm have a tendency to equalize when the electron is pushed into the continuum.

To understand the interplay of tunneling and spin flip we first evaluate the upper limit for the semiclassical tunneling rate \( w_{\text{tun}} \) in the model triangular potential \( Fx \) neglecting tunneling from the excited states as:

\[
\ln \frac{\omega_0}{w_{\text{tun}}} = \frac{4\sqrt{2}}{3} \xi^{1/2} \frac{U_0}{F d}
\]

Equation 5 gives the upper estimate of the quantity since the real tunneling barrier is lower and more narrow than the triangular one. Although this equation shows that the tunneling probability rapidly increases at \( Fd \) being a sizable fraction of \( U_0 \), the actual tunneling rate, very strongly \( F \) - dependent, can only be obtained numerically. Since spin-flip Rabi frequency \( \Omega_R \) is a linear function of \( F \) (being strictly linear when the two-level approximation is applicable) with \( \Omega_R T_0 \ll 1 \), the effect of tunneling on the spin-flip processes occurs in a relatively narrow range of the driving fields when two rates are comparable.

For a stronger Rashba coupling the spin dynamics becomes faster and well-defined Rabi oscillations have
enough time to develop before the tunneling occurs, as it can be seen in Fig.3(b). However, when the field exceeds 0.2 meV/nm, the electron tunnels from the dot so quickly that the spin flip has no time to be established, regardless of the spin-orbit coupling strength, and the continuum states play the dominant role. In other words, high enough driving field amplitudes produce the tunneling rate $F \geq 0.2$ meV/nm in the spin dynamics demonstrates visible damping of the Rabi oscillations, as caused by the interaction with continuum states increasing with the driving field amplitude.

At $t = 0$, we have $|\psi_2(x,0)| \ll |\psi_1(x,0)|$ with $\langle \sigma_z(0) \rangle \approx 1$, such that the initial spin state has a high purity with $\text{tr} \rho^2(0) \approx 1$. Note that a state is pure if and only if $\psi_1(x,t) = c \psi_2(x,t)$, where $c$ is a complex constant. With the course of time, two components of $\psi(x,t)$ begin to develop different shapes, producing spin density matrix in Eq. (9) with $\text{tr} \rho^2(t) < 1$ characterizing a mixed spin state.

An example of the structure of mixed spin states in the driven spin dynamics is shown in Fig. 4 at the end of the computational evolution $t = 655T_0$ for $\alpha = 5$ meVnm and $F = 0.2$ meV/nm. Although at this time and driving field, the electron density still has a well-defined peak inside the confining potential, a broad contribution of the continuum states with a relatively low total probability density has already been formed at $d < |x| < L$, possessing more than half of the total norm. This Figure clearly demonstrates that $\psi_1(x,t)$ and $\psi_2(x,t)$ are strongly different, both in the real and imaginary parts. Analysis of the density of spin components $|\psi_1(x,t)|^2$ and $|\psi_2(x,t)|^2$ (not shown in the Figure) demonstrates that they are shifted with respect to each other, corresponding to the effects shown in Fig. 2 where direction of the spin precession depends on the direction of the tunneling since the momentum operator coupled to the spin, is spatially odd.

By evaluating the spin flip times in Fig. 3 one may notice that they are of the order of $(30 \ldots 40)T_d$ for $\alpha = 5$ meVnm and $(5 \ldots 8)T_d$ for $\alpha = 25$ meVnm where $T_d = 6.71T_0$ is the driving field period. A difference to Ref. [24]...
is that the electric field amplitude required to operate a spin flip in a multilevel dot is three orders of magnitude larger than that for the shallow dot. We attribute these differences to the considerably stronger localization and energy scales of a multilevel dot, where one needs larger electric fields to induce the spin flip in a desired time. These larger scales, however, have side effects such as a broader energy interval of the continuum states that actively participate and modify both the position and spin evolution.

B. Efficiency of spin flip

The studies of coupled spin and coordinate dynamics can be facilitated by tracking the time dependence of the energy-related variables such as \( \langle E(t) \rangle \) defined as

\[
\langle E(t) \rangle = \sum_n |c_n(t)|^2 E_n \tag{10}
\]

where the sum is taken over all basis states with coefficients \( c_n(t) \) in the wavefunction \( \psi(t) \) for \( E_n \) defined with \( \psi(0) \) for the unperturbed Hamiltonian \( H_0 \).

With the external driving, the energy is pumped into the system, and the period-averaged expectation value of \( \langle E(t) \rangle \) grows with time. When \( \langle E(t) \rangle \) passes the threshold \( \langle E(t) \rangle = 0 \) between the localized and continuum states, the electron has effectively tunneled from the quantum dot into the continuum.

In Fig. 5(a) we plot \( \langle E(t) \rangle \) for \( \alpha = 5 \) meVnm and the same electric fields as in Fig. 3. One can see that its magnitude increases with increasing field amplitude, and for strong fields greater than 0.2 meV nm the electron escapes into continuum during \( \sim 10 \) periods of driving. The influence of SOC on the energy evolution can be illustrated by Fig. 5(b): at the given time interval and at zero SOC the energy grows faster than for finite SOC presented in the Figure. This effect can be attributed to stronger coupling of all spin-resolved states via the SOC, which slows to some extent the motion to higher energy states. This picture, however, cannot be used for a direct comparison of tunneling in the coordinate space when considering the transitions from localized states into continuum, and looking onto the curves with various \( \alpha \). The reason is that for different \( \alpha \) one has different continuum states with different properties, and their direct comparison in terms of tunneling into continuum is difficult when looking only onto the energy dependence. We will return to this question by considering the spatial dependence for the electron position in the next subsection.

In addition to the time dependence \( \langle E(t) \rangle \), it is of interest to study the spin-flip efficiency \( \eta(t) \) defined as the ratio of the energy pumped into the spin degree of freedom to the total change of the mean energy:

\[
\eta(t) = \frac{\Delta \langle \sigma_z(t) \rangle - \langle \sigma_z(0) \rangle}{2 \langle E(t) \rangle - \langle E(0) \rangle}, \tag{11}
\]

where \( \langle E(0) \rangle \equiv E_1 \). In Fig. 6 we show this \( \eta(t) \) for some of the electric fields from Fig. 3. One can see that its time dependence is qualitatively independent of the SOC strength. It reaches the maximum when the first full or almost full spin flip is achieved. After this, the magnitude of \( \eta(t) \) decreases with time. The explanation is straightforward: the spin dynamics and in particular the Zeeman energy have an oscillating character (at low driving strength) or oscillating plus decaying character (at high driving strength). So, the numerator in (11) does not grow with time, while the denominator in general increases, as it can be seen from the energy dependence in Fig. 5. As to the magnitude of (11), it is greater for bigger SOC strength since the stronger SOC coupling provides a quicker spin flip with lower corresponding change of the mean energy, giving the smaller numerator in (11) at the moment when the maximum numerator is reached.

C. Feedback on the tunneling

To visualize the electron escape from the quantum dot, we use the stay probability defining the weight of the lo-
FIG. 6. Time dependent spin-flip efficiency (11): (a) $\alpha = 5$ meVnm, (b) $\alpha = 25$ meVnm, for some of the driving fields (marking the plots) from Fig. 3.

FIG. 7. Time dependence of localization probability (12) for (a) short evolution times for $\alpha = 5$ (dashed lines) and $\alpha = 25$ (solid lines) meVnm and (b) long evolution times for $\alpha = 0$ (solid lines) and $\alpha = 5$ (dashed lines) meVnm. The plots are marked with the values of the driving field amplitudes.

localized eigenfunctions in the discrete part of the spectrum as:

$$ P(t) = \sum_{i(\text{loc})} |c_i(t)|^2, $$

(12)

where summation is taken over the contribution of localized $\phi_i(x)$-states.

The localization probability (12) is shown in Fig. 7 for two time intervals. One can see that for short evolution times shown in Fig.7(a) both values of SOC strength produce similar evolution $P(t)$, and the values of (12) for smaller $\alpha$ are in general slightly higher than for the bigger one. The situation is, however, somewhat different on long times to 600 $T_0$ shown in Fig.7(b) for small value of $\alpha = 5$ meVnm together with the $\alpha = 0$ realization. Here two curves for a given driving strength and for different SOC parameters exchange their places more frequently at some moments of time with respect to the localization probability amplitude. So, it is harder to predict in which case the final localization probability will be greater if the SOC parameter is small or equal to zero. We attribute these differences to the more effective coupling between the localized and continuum states for higher values of SOC parameter $\alpha$ compared to lower ones. As a result, the electric field-induced transitions to continuum are more effective for higher values of $\alpha$ (not necessarily meaning the higher instant value of mean energy, see Fig.5(b)), and the electron may tunnel into continuum faster than for the lower $\alpha$, creating the lower localization probability (12) for the stronger SOC.

An important element for understanding of the feedback effect of spin motion on the time-dependent position and tunneling is the anomalous [36] spin-dependent velocity [33]:

$$ v_{so} = \frac{i}{\hbar} [\alpha \sigma_y k, x] = \frac{\alpha}{\hbar} \sigma_y, $$

(13)

and the corresponding acceleration:

$$ \frac{d}{dt} v_{so} = \frac{i}{\hbar} \frac{\Delta \alpha}{2} [\sigma_z, \sigma_y] = \frac{\alpha \Delta}{\hbar^2} \sigma_x. $$

(14)

This term leads to variation in the velocity due to the spin precession, as was observed in experiments on high-frequency conductivity of two-dimensional electron gas [37], and the effect of time-dependent $\langle \sigma_y(t) \rangle$ on $\langle x(t) \rangle$ appears as a result. The corresponding local probability
flux satisfying equation \( \partial_t (\psi^\dagger (x, t) \psi(x, t)) + \partial_x j(x, t) = 0 \), where \( \partial_x \equiv \partial / \partial x \), is given by:

\[
j(x, t) = \frac{i\hbar}{2m} \left[ \psi(x, t) \partial_x \psi^\dagger(x, t) - \psi^\dagger(x, t) \partial_x \psi(x, t) \right] + \frac{\alpha}{\hbar} \psi^\dagger(x, t) \sigma_\alpha \psi(x, t).
\] (15)

Note that for a relatively small coupling constant \( \alpha = 5 \text{ meV} \text{nm} \), the maximum value of the \( v_{\text{so}} \) velocity \( \alpha/\hbar \approx 8 \text{ nm} / \text{ps} \) can sufficiently modify the tunneling process.

**D. Evolution of the position**

1. **Time-dependence of the expectation values**

Along with the spin evolution, the expectation value of the coordinate \( \langle x(t) \rangle \), is of interest for understanding of the dynamics. It provides information of the electron localization domain, and can help in studies of the tunneling effect. For example, the sufficient criteria for tunneling into the continuum may be formulated in terms of the \( \langle x(t) \rangle \) amplitude: if it steadily exceeds the quantum dot size \( d \), the electron is out of the dot. Also, this expectation value produces a time-dependent electric field outside of the quantum dot, which under certain conditions, can be experimentally tracked.

We calculate the dynamics of \( \langle x(t) \rangle \) in analogy with \( \langle \psi^\dagger (x, t) \psi(x, t) \rangle \) where the spin operator is replaced by \( \psi^\dagger \psi \) with some examples of coordinate dynamics presented in Fig. 4.

The upper panel shows the dynamics for the low amplitude \( F = 0.16 \text{ meV} \text{nm} \) and the bottom panel shows the dynamics for \( F = 0.22 \text{ meV} \text{nm} \). By looking in Fig. 8(a) one can conclude that for low driving amplitude \( F = 0.16 \text{ meV} \text{nm} \) the electron is still confined within the quantum dot, and its spin exhibits well-defined Rabi oscillations (see Fig. 3). When the driving amplitude grows, the delocalization trend is manifested, and for the highest amplitude \( F = 0.22 \text{ meV} \text{nm} \) the amplitude of \( \langle x(t) \rangle \) well exceeds the quantum dot size, as one can see in Fig. 8(b).

The displacements presented in Fig. 8 can be compared with the results for oscillations amplitude \( x_0^{\text{[loc]}} \) for a particle localized in the potential \( U_0 x^2 / d^2 \), and a free electron driven by a periodic field, \( x_0^{[\text{cl}]} \), where

\[
x_0^{[\text{loc}]} = \frac{d^2}{2U_0} F, \quad x_0^{[\text{cl}]} = \frac{1}{m \omega^2} F.
\] (16)

For \( F = 0.16 \text{ meV} \text{nm} \), estimated amplitude \( x_0^{[\text{loc}]} \approx 8 \text{ nm} \), in a good agreement with the calculations at \( t < 100 \tau_0 \) (not shown in the Figure). Here the velocity amplitude \( v_0^{[\text{loc}]} = \omega d x_0^{[\text{loc}]} \approx 16 \text{ nm} / \text{ps} \), meaning that the anomalous velocity \( \alpha / \hbar \) plays essential role in the electron dynamics. For a delocalized electron, Eq. (15) yields for \( F = 0.22 \text{ meV} \text{nm} \), the amplitude \( x_0^{[\text{cl}]} \approx 720 \text{ nm} \), considerably larger than the calculated values. This difference is due to the fact that in the tunneling ionization a broadly distributed probability density rather than a well-defined wavepacket is being formed, resulting in a relatively small amplitude of \( \langle x(t) \rangle \). This can be illustrated by the example for the wavefunction component profile shown in Fig. 4 which can be compared with the localization probability in Fig. 8(b): for \( F = 0.2 \text{ meV} \text{nm} \) the low value of localization probability for the final moment of computation \( (P \approx 0.2) \) corresponds to Fig. 4 which describes a wavefunction with substantial impact of continuum states demonstrating a widely delocalized profile rather than a well-localized wavepacket. As to the velocity, taking into account that the numerically obtained amplitude, \( x_0^{[\text{tun}]} \approx 150 \text{ nm} \) we find that the corresponding velocity amplitude \( v_0^{[\text{tun}]} \approx 300 \text{ nm} / \text{ps} \), being comparable to the maximum anomalous velocity at \( \alpha = 25 \text{ meV} \text{nm} \), confirming the importance of spin-orbit coupling for the electron displacement.
corresponding wavevector, $k_{\text{tun}}^0 = v_{\text{tun}}^0 m/\hbar$ and the spin precession rate $\Omega_{\text{tun}}^0 = 2\alpha k_{\text{tun}}^0 / \hbar$. For the given set of system parameters and $\alpha = 5$ meV nm, we obtain at $F = 0.22$ meV nm, $\Omega_{\text{tun}}^0 \approx 0.53$ ps$^{-1}$. In the panel (a) of Fig. 5 on can see that at $F = 0.22$ meV nm, the characteristic time of spin evolution (time, at which the expectation value $\langle \sigma_z(t) \rangle = 0$, is close to 45 ps, considerably larger than $\pi/\Omega_{\text{tun}}^0 \approx 6$ ps. This difference means that the main effect of the tunneling on the spin evolution is due to formation of the mixed states rather than due to the driven tunneling-induced spin precession.

By analyzing the results in Fig. 5 - Fig. 8, one can see that the tunneling becomes efficient at few periods of driving field when the driving amplitude exceeds the values of about 0.18...0.2 meV nm. At lower fields the tunneling is slow enough to allow for a well-defined spin flip. For the fields higher than 0.22 meV nm the potential well opens so effectively that the electron escapes into continuum before the well-established Rabi oscillations are developed. So, one needs to choose some optimal intermediate driving fields to achieve good spin flip in a proper time. Besides, the very strong dependence of the tunneling probability on the driving field amplitude requires a well-defined window of the field amplitude in order to achieve the desired spin flip if one is interested in still keeping the electron inside the hosting quantum dot.

As to the dependence of the tunneling on the spin, and, more precisely, on the SOC strength $\alpha$, one can see that the patterns of $\langle x(t) \rangle$, the localization parameter $P(t)$ and the mean energy $\langle E(t) \rangle$ are similar but still quantitatively different depending on the presence and value of the SOC strength. Namely, for stronger SOC the localization probability is slightly lower at all times and for most driving fields, i.e. the tunneling is in general faster with increasing SOC. We attribute this effect to a more intensive EDSR development at higher SOC. Stronger SOC enlarges the matrix elements for the states with opposite spins, leading not only to a faster spin flip but also to more intense occupation of higher energy levels, speeding up to some extent the tunneling.

2. Fourier analysis of the dynamics

It is of interest to study the driven evolution in terms of its spectral properties, namely, the Fourier power spectrum applied to a finite sequence of data for $\langle x(t) \rangle$ collected on the evolution interval $T = NT_0$ with $N$ points spaced by $T_0$. Here it is appropriate to use the discrete version of the Fourier transform written as:

$$X_p = \sum_{n=0}^{N-1} \langle x(nT_0) \rangle e^{-i\omega_p/\omega_0}.$$  \hspace{1cm} (17)

In (17) $X_p$ is the output for the Fourier harmonic at frequency $\omega_p = \omega_0 N$, and (17) gives Fourier data for the frequency interval $0 \ldots \omega_0/2$ with frequency spacing $\omega_0/N$, so the index $p$ takes the values $1 \ldots N/2$, where it is convenient to express the frequencies in units the of $\omega_d$. For our numerical simulations we have the values $N = 200 \ldots 600$, where the discrete spectra obtained from (17) form a dense quasicontinuous set of states.

Here one may expect the local maxima at both driving frequency $\omega_d$ and in the low-frequency part associated with slow spin evolution coupled to the coordinate via SOC. In Fig. 9 we show examples of Fourier power spectra to compare the $\alpha = 0$ system with the system having moderate SOC for driving with $F = 0.16$ meV nm and $F = 0.22$ meV nm. We may see that the presence of SOC modifies mainly the low frequency part of $X_p$ in the $\sim 10^{-2}$ THz frequency domain. This is natural due to the SOC coupling of the coordinate with spin which affects mainly lower frequencies associated with the spin evolution which is lower than the driving frequency of the electric field. The peak at $\omega_p/\omega_d = 1$ is present for all SOC parameters and driving field amplitudes and corresponds to the dominant frequency of driven dynamics. Another interesting feature of Fourier spectra is the difference between the zero- and nonzero SOC cases which increases at high driving fields as it can be seen by comparing panels (a) and (b) in Fig. 9. With the increase in
the driving field, at $\alpha = 0$, the contribution of the large-displacement dynamics at frequency $\omega_d$ dominates over the low-frequency part of the displacement spectrum. However, a finite SOC due to the anomalous velocity produces a still relatively intense low-frequency motion. Thus, on one hand, moderate SOC gives enough time for the intermediate processes of tunneling between localized and extended states to develop, and, on the other hand, the coupling of spin and coordinate leads to a more complicated dynamics of \( \langle x(t) \rangle \) than for the $\alpha = 0$ realization, especially at high driving fields.

It is interesting to note that in the non-relativistic nonlinear atomic optics, frequent re-collisions of a driven electron with the singular Coulomb potential of the remaining ion \( \text{38} \) lead to a strong enhancement in the high-frequency radiation. In our paper we take into account a relativistic effect in the form of the spin-orbit coupling, albeit for a nonsingular potential. It will be of a future interest to study high-frequency harmonics produced in the presence of these relativistic effects.

V. CONCLUSIONS

We studied the electric dipole spin resonance with simultaneous tunneling and position evolution in a multilevel quantum dot formed in a nanowire. We demonstrated a strong effect of tunneling on the driven spin flip processes with disappearance of well-defined Rabi spin oscillations when the tunneling rate is of the order of the corresponding Rabi frequency. For multilevel quantum dots this matching happens in the electric field amplitudes close to 0.2 meV/nm. The strong effect of the tunneling on the spin dynamics is due to the formation of the mixed spin states, shifting the spin vector inside the Bloch sphere. In addition, we demonstrated backaction of the spin dynamics on the tunneling probability and position of the electron. This backaction is qualitatively attributed to the anomalous spin-dependent velocity, proportional to the spin-orbit coupling strength. These effects should be taken into account for the development of qubit architecture based on such structures.

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