Lattice Design of a Compact Hadron Driver for Cancer Therapy

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Abstract. A next generation cancer therapy driver by using a fast-cycling induction synchrotron is given. The design of the lattice parameters with zero momentum-dispersion and high-flat momentum-dispersion region as essential parameters must be fulfilled and the optimization process with results are discussed here. Finally, the optimization process and method are developed and the lattice parameters are well defined.

1. Introduction
A design of a compact hadron driver for cancer therapy based on the induction synchrotron concept is given in Refs. 1 and 2 with parameters as in Table 1 and schematics in Fig. 1. This lattice has two-fold symmetry with a circumference of 52.8 m, a 2 m-long dispersion-free straight section, and a 3 m-long large flat dispersion straight section. By assuming a 1.5 T bending magnet, the ring can deliver heavy ions of 200 MeV/au at 10 Hz.

| Parameters          | Specification                                      |
|---------------------|----------------------------------------------------|
| Energy              | 200 MeV/nucleon for A/Q = 2 ion                   |
| Circumference, $C_0$| 52.8 m                                             |
| Ion Species         | Gaseous/Metal ions                                 |
| Ion Source (IS)     | Laser ablation IS, Electron Cyclotron Resonance IS|
| Injector            | Fast cycling (10 Hz)                               |
| Ring                | $B_{\text{max}} = 1.5$ Tesla                       |
|                    | $\rho = 2.8662$ m                                  |
|                    | FODOF cell with edge focus of B                    |
|                    | Mirror symmetry                                    |
|                    | $v_x/v_y = 1.3143/1.4635$                          |
|                    | 2m long dispersion-free region                     |
|                    | 3m long flat large dispersion region               |
|                    | $a_p=0.273088$                                     |
|                    | $\gamma_f=1.92$                                    |
| Acceleration        | Induction cells driven by SPS employing SiC-MOSFET |
| Vacuum              | $V_{\text{acc}} = \rho C_0 dB/dt$ (max 7 kV)       |
|                    | 10\textsuperscript{8} Pascal                       |
In order to realize the slow extraction technique and fast extraction in a fast-cycling synchrotron, which allows the energy sweep beam scanning, the essential features \[1,2\] required are:

i. Dispersion-free region for induction acceleration devices and injection device

ii. Localized large flat dispersion region for the slow extraction with the length of 3 m

iii. Local betatron phase advance of \(\pi/2\) for the fast extraction

![Figure 1. Hadron Driver System Schematics](image)

Details of the lattice parameters and optimization method are discussed in the next section.

2. Lattice: Basics and Configuration

The lattice of compact hadron driver is designed based on the Proton Ion Medical Machine Study (PIMMS) \[3\]. A lattice is an array of bending magnets and focusing magnets. Whereby the coefficient of restoring force, \(K(s)\), in the betatron equation is uniquely determined by this lattice. Hence, the individual common components can be treated with 3 X 3 matrices with the following equations \[4,5,6\].

Drift space transfer matrix,

\[
M_D = \begin{bmatrix}
1 & \ell & 0 \\
0 & 1 & 0 \\
0 & 0 & 1
\end{bmatrix}
\]

(1)

magnet transfer matrix,
\[ M_f = \begin{bmatrix} \cosh \sqrt{K_F} \ell_{Q_F} & \sinh \sqrt{K_F} \ell_{Q_F} & 0 \\ \sqrt{K_F} \sinh \sqrt{K_F} \ell_{Q_F} & \cosh \sqrt{K_F} \ell_{Q_F} & 0 \\ 0 & 0 & 1 \end{bmatrix} \]  
(2)

Defocusing magnet transfer matrix,
\[ M_d = \begin{bmatrix} \cos \sqrt{K_D} \ell_{Q_D} & \sin \sqrt{K_D} \ell_{Q_D} & 0 \\ -\sqrt{K_D} \sin \sqrt{K_D} \ell_{Q_D} & \cos \sqrt{K_D} \ell_{Q_D} & 0 \\ 0 & 0 & 1 \end{bmatrix} \]  
(3)

Bending magnet transfer matrix,
\[ M_B = \begin{pmatrix} \cos \theta & \rho \sin \theta & \rho(1 - \cos \theta) \\ -\sin \theta / \rho & \cos \theta & \sin \theta \\ 0 & 0 & 1 \end{pmatrix} \]  
(4)

Horizontal edge effect transfer matrix of bending magnet,
\[ M_{HE} = \begin{pmatrix} 1 & 0 & 0 \\ \tan \varepsilon / \rho & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \]  
(5)

Vertical edge effect transfer matrix of bending magnet,
\[ M_{VE} = \begin{pmatrix} 1 / \rho \left( b / \rho \cos \varepsilon - \tan \varepsilon \right) & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix} \]  
(6)

K- value,
\[ K = \frac{B'}{B \rho} \]  
(7)

Where:
- \( \theta \) = Bending angle of bending magnet
- \( \rho \) = Curvature of bending magnet
- \( \varepsilon \) = Edge angle of bending magnet
- \( b \) = Distance over which the fringe field drops to zero of bending magnet
- \( K_{D,F} \) = K- value of the related magnets
- \( \ell \) or \( \ell_{Q_D,F} \) = Length of the related space or magnet

The lattice configuration of compact hadron driver is shown in Fig. 2 with the half ring configuration of Cell-1 X Cell-2 X Drift space-4 X Cell-2 X Cell-1 with length of 38.2 m and the elements parameters are shown in Table 2 and Table 3.
Table 2. Parameters of drift space and quadrupole elements (*After optimization)

| Elements                          | Length, l [m] | K-value [m²] |
|-----------------------------------|---------------|--------------|
| Drift space 1, D1                 | 1.5           | ---          |
| Drift space 2, D2                 | 0.3           | ---          |
| Drift space 3, D3                 | 0.5           | ---          |
| Drift space 4, D4                 | 1.0           | ---          |
| Focusing Quadrupole magnet 1, QF1| 0.3           | 0.75568*     |
| Defocusing Quadrupole magnet, 1 QD1| 0.3       | -0.53774*    |
| Focusing Quadrupole magnet 2, QF2| 0.3           | 0.40173*     |
| Defocusing Quadrupole magnet 2, QD1| 0.3       | -0.8957*     |

Table 3. Parameters of bending magnet

| Magnetic Flux Density, B [T] | Bending Angle, θ [°] | Edge Angle, θE [°] | Curvature, ρ [m] | Length, l [m] |
|-----------------------------|----------------------|-------------------|-----------------|---------------|
| 1.5                         | 22.5                 | 11.25             | 2.8662          | 1.1256        |

By referring to Eqs. (1) to (6) with parameters of Table 2 and Table 3, the transfer matrix of individual cell can be expressed as follows:

For horizontal transfer matrix of Cell 1,

$$M_{Hcell1} = M_{D1} \cdot M_{f1} \cdot M_{D2} \cdot M_{HE} \cdot M_{B} \cdot M_{HE} \cdot M_{D2} \cdot M_{HE} \cdot M_{B} \cdot M_{HE} \cdot M_{D2} \cdot M_{f1} \cdot M_{D1}$$  \hspace{1cm} (8)

For horizontal transfer matrix of Cell 2,

$$M_{Hcell2} = M_{D3} \cdot M_{f2} \cdot M_{D2} \cdot M_{HE} \cdot M_{B} \cdot M_{HE} \cdot M_{D2} \cdot M_{HE} \cdot M_{B} \cdot M_{HE} \cdot M_{D2} \cdot M_{f2} \cdot M_{D3}$$  \hspace{1cm} (9)

For vertical transfer matrix of Cell 1,

$$M_{Vcell1} = M_{D1} \cdot M_{d1} \cdot M_{D2} \cdot M_{VE} \cdot M_{DB} \cdot M_{VE} \cdot M_{D2} \cdot M_{f1} \cdot M_{D2} \cdot M_{VE} \cdot M_{DB} \cdot M_{VE} \cdot M_{D2} \cdot M_{d1} \cdot M_{D1}$$  \hspace{1cm} (10)

For vertical transfer matrix of Cell 2,

$$M_{Vcell2} = M_{D3} \cdot M_{d2} \cdot M_{D2} \cdot M_{VE} \cdot M_{DB} \cdot M_{VE} \cdot M_{D2} \cdot M_{f2} \cdot M_{D2} \cdot M_{VE} \cdot M_{DB} \cdot M_{VE} \cdot M_{D2} \cdot M_{d2} \cdot M_{D3}$$  \hspace{1cm} (11)

Whereby the total transfer matrix can be expressed as:

For horizontal transfer matrix of half ring,

$$M_{HTotal} = M_{Hcell1} \cdot M_{Hcell2} \cdot M_{D4} \cdot M_{Hcell2} \cdot M_{Hcell1}$$  \hspace{1cm} (12)

For vertical transfer matrix of half ring,

$$M_{VTotal} = M_{Vcell1} \cdot M_{Vcell2} \cdot M_{D4} \cdot M_{Vcell2} \cdot M_{Vcell1}$$  \hspace{1cm} (13)

For example, the horizontal transfer matrix of Cell 1 can be expressed as:
\[
M_{Hcell1} = \begin{pmatrix} m_{11} & m_{12} & m_{13} \\ m_{21} & m_{22} & m_{23} \\ 0 & 0 & 1 \end{pmatrix}_{Hcell1}
\]

\[
M_{Hcell1} = \begin{pmatrix} 1 & \ell_1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \left( \begin{array}{ccc}
\cos \sqrt{K_F} \ell_{Q_F} & \sin \sqrt{K_F} \ell_{Q_F} & 0 \\
-\sqrt{K_F} \sin \sqrt{K_F} \ell_{Q_F} & \cos \sqrt{K_F} \ell_{Q_F} & 0 \\
0 & 0 & 1
\end{array} \right) \begin{pmatrix} 1 & \ell_2 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}
\]

\times \left( \begin{array}{ccc}
\cos \theta & \rho \sin \theta & \rho (1 - \cos \theta) \\
-\frac{\sin \theta}{\rho} & \cos \theta & \sin \theta \\
0 & 0 & 1
\end{array} \right) \left( \begin{array}{ccc}
\tan \varepsilon & 1 & 0 \\
\frac{\rho}{\tan \varepsilon} & 1 & 0 \\
0 & 0 & 1
\end{array} \right) \begin{pmatrix} 1 & \ell_2 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}
\times \left( \begin{array}{ccc}
\frac{\sinh \sqrt{K_D} \ell_{Q_D}}{\sqrt{K_D}} & \frac{\cosh \sqrt{K_D} \ell_{Q_D}}{\sqrt{K_D}} & 0 \\
\sqrt{K_D} \sinh \sqrt{K_D} \ell_{Q_D} & \sqrt{K_D} \cosh \sqrt{K_D} \ell_{Q_D} & 0 \\
0 & 0 & 1
\end{array} \right) \begin{pmatrix} 1 & \ell_2 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}
\times \left( \begin{array}{ccc}
\frac{\sin \sqrt{K_F} \ell_{Q_F}}{\sqrt{K_F}} & \frac{\cos \sqrt{K_F} \ell_{Q_F}}{\sqrt{K_F}} & 0 \\
-\sqrt{K_F} \sin \sqrt{K_F} \ell_{Q_F} & \sqrt{K_F} \cos \sqrt{K_F} \ell_{Q_F} & 0 \\
0 & 0 & 1
\end{array} \right) \begin{pmatrix} 1 & \ell_1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}
\]

Therefore, from Eqs. (1) to (15) the essential lattice parameters which are betatron tune, phase advance and dispersion function that can be expressed as follows:

For horizontal or vertical phase advance of half ring,

\[
\mu_{HTotal, Vtotal} = \left\{ \cos^{-1} \left( \frac{m_{11} + m_{22}}{2} \right) \right\}_{HTotal, VTotal}
\]

(16)

Fractional part of horizontal or vertical tune with unknown integer part,

\[
\Delta \nu_{HTotal, Vtotal} = \left\{ \frac{\mu}{2 \pi} \right\}_{HTotal, VTotal}
\]

(17)

For dispersion function at particular transfer location,

\[
\left( \frac{D_i}{D} \right) = \left\{ \frac{1}{2 - (m_{11} + m_{22})} \left( m_{13} + m_{12}m_{23} - m_{22}m_{13} \right) \right\}_{i}
\]

(18)

Whereby, Eqs. (16), (17) and (18) are essential to optimize the K-value of individual quadrupole elements as described at next section.
3. Lattice: Optimization Process and Result

For such evaluation, momentum deviation is assumed as 0.01 and three cases have been evaluated as follows. The lattice of hadron driver can be simplified with four FODO cells and treated as a super-period. They are grouped into two types of cells, CELL-1 and CELL-2, and aligned as CELL-1, CELL-2, Drift4, CELL-2 and CELL-1 as shown in Fig. 3.

The crucial requirements of this lattice are noted in Fig. 3 with large flat dispersion and dispersion-free region. Therefore, the aim is to optimize the K-value of quadrupole to achieve the (D, D')~(arbitrary number, 0) at location 1 and (D, D')~(0, 0) at location 2 (Fig. 3) by scanning the vertical tune νv, while νh is fixed (D may be slightly modified). The detail process and successive iteration are shown in Fig. 4 and Fig. 5.

**Figure 3.** Configuration of half ring (1 denote as large flat dispersion region and 2 denote as dispersion-free region)

**Figure 4.** Optimization procedure of K-value

For each stage, the dispersion function at location 1 and 2 must be evaluated according to the Eq. (18). In additional, the QD is used for fine tuning to fulfill the 3 lattice requirements of the compact hadron driver. Does this mean you change the length QF1 and QD1 and calculate new v using eq 15?
According to the process as described in Fig. 4, the K-value of quadrupole elements have been optimized with the result as shown in Table 4. Meanwhile, the dispersion function at location 1 and location 2 must be evaluated as well to confirm the desirable dispersion magnitude. If the large flat and zero dispersion regions haven’t been realized by this optimization process. Then the optimization process should be restarted from the beginning.

| Parameters | Specifications | Remarks |
|------------|----------------|---------|
| K-Value of QF$_1$ [m$^{-2}$] | 0.75568 | Optimized |
| K-Value of QD$_1$ [m$^{-2}$] | -0.53774 | Optimized |
| K-Value of QF$_2$ [m$^{-2}$] | 0.40173 | Optimized |
| K-Value of QD$_2$ [m$^{-2}$] | -0.89570 | Optimized |
| D$_2$ [m] | -5.50X10$^{-8}$ | Dispersion-free region (i) |
| D$_2$ [m] | 2.22×10$^{-16}$ | Sustainable for along 2 m (i) |
| D$_1$ [m] | 5.8725 | Large flat dispersion region (ii) |
| D$_1$ [m] | 3.24×10$^{-16}$ | Sustainable along 3 m (ii) |
| Phase advance | $\pi/2$ | For fast extraction (iii) |

The result shown in Table 4, the lattice requirements of compact hadron driver: dispersion region, large flat dispersion region and betatron phase advance for fast extraction has been fulfilled with the optimized K-value of quadrupole elements. Hence, the beta function can be plotted by using the Courant-Snyder parameters, which is discussed in the next section.

### 4. Result: Courant-Snyder Parameters and Beta Function

A beam comprised of particles will move at different movements along the design orbit in a transport line, where magnets are aligned. All particles in a beam are positioned in a phase space. If such a phase space is scanned along the beam orbit, there revealed a surface enclosing all particles. This surface is called as Beam Envelope. From the standpoint of accelerator design, it is important to know the beam behavior rather than the behavior of each particle. In the phase space, the beam envelope is given by an ellipse as shown in Fig. 6, which is defined by the following formula [4,5,6]:

$$\beta x'^2 + 2\alpha xx' + \gamma x^2 = \varepsilon$$  \hspace{1cm} (19)

Where,

$\beta, \alpha$ and $\gamma$ = Courant-Snyder Parameter coefficients depending on the orbit position $s$. 

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**Figure 5.** Schematic of tune drift during successive iteration
\[ \varepsilon = \text{Emittance and defined by an ellipse area divided by } \pi. \]

![Diagram of Courant-Snyder parameters with phase space revolution along the ring.](image)

**Figure 6.** Courant-Snyder parameters with phase space revolution along the ring

Since the Courant-Snyder parameter satisfies the following relation, the number of independent parameters is two.

\[ \beta \gamma = 1 + \alpha^2 \]  

(20)

The Twiss parameter is transferred by a matrix constructed by a corresponding transfer matrix given as \( M \). Here, \( m_{ij} \) is the component of the transfer matrix.

\[
\begin{pmatrix}
\beta \\
\alpha \\
\gamma
\end{pmatrix}_{s} = M \begin{pmatrix}
\beta_0 \\
\alpha_0 \\
\gamma_0
\end{pmatrix}, \quad M \equiv \begin{pmatrix}
m_{11}^2 & -2m_{11}m_{12} & m_{12}^2 \\
-m_{11}m_{21} & m_{11}m_{22} + m_{12}m_{21} & -m_{12}m_{22} \\
m_{21}^2 & -2m_{21}m_{22} & m_{22}^2
\end{pmatrix}
\]  

(21)

Whereby, the dispersion function is constructed by a corresponding transfer matrix given by \( M \) (give another subscript). Here, \( m_{ij} \) is the component of the transfer matrix.

\[
\begin{pmatrix}
D \\
D'
\end{pmatrix}_{s} = M \begin{pmatrix}
D_0 \\
D'_0
\end{pmatrix}, \quad M \equiv \begin{pmatrix}
m_{11} & m_{12} & m_{13} \\
m_{21} & m_{22} & m_{23} \\
0 & 0 & 1
\end{pmatrix}
\]  

(22)

From Eqs. (21, 22) with the lattice parameters from Table 2., the beta and dispersion functions of the compact hadron driver is plotted as shown in Fig. 7.
Fig. 7 shows that the requirements of the dispersion free region and the large flat dispersion region with sustainability of 4 m are fulfilled.

5. Conclusion
Using the optimization method of tune scanning, the lattice parameters of the compact hadron driver have been defined. The requirements of the dispersion free region, the large flat dispersion region and the phase advance of $90^\circ$ condition for fast extraction have been fulfilled as well. For further consideration, non-linear perturbation terms must be studied and the magnetic component designs must be able to be realized with the designed lattice. Hence, the lattice should be modified to comply with the engineering aspects of the magnetic components.

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7. References
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