Breakdown of ‘phase rigidity’ and variations of the Fano effect in closed Aharonov-Bohm interferometers

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Although the conductance of a closed Aharonov-Bohm interferometer, with a quantum dot on one branch, obeys the Onsager symmetry under magnetic field reversal, it needs not be a periodic function of this field: the conductance maxima move with both the field and the gate voltage on the dot, in an apparent breakdown of ‘phase rigidity’. These experimental findings are explained theoretically as resulting from multiple electronic paths around the interferometer ring. Data containing several Coulomb blockade peaks, whose shapes change with the magnetic flux, are fitted to a simple model, in which each resonant level on the dot couples to a different path around the ring.

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I. INTRODUCTION

The mesoscopic Aharonov-Bohm (ABI) interferometer (ABI) has been used widely in attempts to measure both the magnitude and the phase of the quantum transmission amplitude for an electron traversing a quantum dot (QD). Many of these experiments have been done on the closed ABI, where the QD is placed on one of the two paths which surround an area which is penetrated by a magnetic flux $\Phi$, and the two paths are connected to two reservoirs via only two terminals.1,2 In some experiments, the states on the whole path replace the QD. Unlike for the multi-terminal open ABI, for small fluxes the conductance of the closed interferometer turned out to be an even and periodic function of the Aharonov-Bohm (AB) phase $\phi = \Phi/\Phi_0$, with $\Phi_0 = \hbar c/e$ and with the period $\Delta\phi = 2\pi$. Away from resonances of the transmission, and for relatively small magnetic fields, the conductance of the ‘ideal’ closed ABI could be fitted to the simple (two-slit-like) formula $G = A + B \cos \phi$, and therefore its maxima (and minima) remained fixed at integer multiples of $\pi$, independent of the gate voltage on the QD (which only affected the values of $A$ and $B$). This phenomenon, called “phase rigidity”, has been accepted as a landmark of the closed ABI.3 Closer to a resonance, $G$ becomes a more complicated function of $\phi$, which contains higher harmonics in the flux, but continues to depend only on powers of $\cos \phi$ (and not of $\sin \phi$). Indeed, the symmetry $G(\phi) = G(-\phi)$ is now well understood, due to the Onsager relations.4 The periodicity of $G$ with $\phi$, and the resulting phase rigidity, have also been reproduced theoretically, in models which describe both the paths around the ABI ring and the leads to the reservoirs as being one-dimensional (1D).5,6 However, although phase rigidity obeys the Onsager symmetry, this rigidity does not really follow from this symmetry. In fact, many of the measurements at higher fluxes break phase rigidity.

The breakdown of phase rigidity in experiments on closed ABI’s shows up as deviations from the simple pure oscillation $G = A + B \cos \phi$, even far away from resonances. Such deviations already appeared in the pioneering work of Webb et al.8 which demonstrated the AB oscillations in normal metal rings. In these experiments (and in practically all the other experiments mentioned above), the AB oscillations appear on top of a background, whose slow variation with the magnetic flux has been described as an aperiodic fluctuation, due to the penetration of the magnetic field into conducting parts of the ABI ring. Indeed, numerical simulations have shown that such fluctuations do result from the fluxes which penetrate small areas within the finite width of the ring, whose properties fluctuate randomly.2,7 As noted by Imry,8 such fluctuations can be observed only at high fluxes, $\Phi \gg \Phi_0/x$, where $x$ is the ratio of the area of the conducting ring to the area of the hole inside it. As Stone and Imry8 note, this background may also contain beats. However, we are not aware of a detailed theoretical analysis of such beats, or of any other periodic aspect of the deviations from a simple Aharonov-Bohm oscillation.

In the first part of this paper we concentrate on two aspects of the breakdown of phase rigidity. First, the ‘beats’. Figure 2 of Ref. 2 already showed two main peaks in the power spectrum of the flux-dependent conductance: one at the Aharonov-Bohm period and the other at a much smaller period (higher field). Ignoring the aperiodic fluctuations, both periods are clearly visible in the flux dependence of the conductance. The ratio of the two periods is presumably related to the ratio between the areas of the ring and of the hole, $x$. Similar ‘beats’ show up in practically all the experiments on closed ABI’s.1,2 The second aspect concerns the location of the conductance maxima and the related phase shift. Figure 4 of the first reference in 2 shows a contour plot of the conductance versus the gate voltage and the magnetic field. In its restricted sense, phase rigidity implies that (except close to resonances) the maxima should be at fixed fields, namely on lines parallel to the gate voltage axis. However, in the data (taken at relatively high fields) these lines have a non-zero slope relative to that axis. Similar slopes show up in many similar experimental plots (see e.g. Fig. 4 in Ref. 13, or Fig. 4 of Ref. 14). A slow change of the location of the maxima with in-
creasing magnetic fields is visible even in the original papers by Yacoby et al. Analysis of the data within a finite narrow window of fields (away from zero) would thus be described by \( G \sim A + B \cos(\phi + \delta) \), with a non-zero phase shift \( \delta \), which depends on both the flux and the gate voltage, apparently contradicting phase rigidity. (This behavior holds only at large fields; as the field goes to zero, \( \delta \) also vanishes, in accord with the Onsager requirement.) All of these papers also exhibit a slow variation of the Aharonov-Bohm oscillation amplitude \( B \) with increasing field, which is related to the ‘beats’. Below we present some additional experimental data, and give a theoretical discussion of these observations.

The second part of this paper concerns the Fano shape of the Coulomb blockade resonances. In the Coulomb blockade regime, the QD exhibits a sequence of resonances as function of the gate voltage, whenever another electron is added to its bound states. The interference between these states and the continuum of the electrons in the leads then results in the Fano effect, which modifies the shape of these resonances. These resonances are further modified once the QD is placed in the ABI. In the experimental papers, each resonance has been fitted to the ‘standard’ Fano asymmetric Breit-Wigner form, \( G \propto |e + q|^2/(e^2 + 1) \), where \( e \) is the normalized distance of the gate voltage from its resonance value and \( q \) is the so-called ‘Fano asymmetry parameter’, which can become complex at non-zero magnetic fluxes, when time-reversal symmetry is broken. Below we show that all the asymmetric resonances follow from a single unified expression for the transmission amplitude, and present fits to the data which demonstrate the utility of this representation for many resonances.

Section II presents some new data, taken from the same mesoscopic closed ABI described in Ref. 2. Section III then proceeds to describe a simple theory, which takes account of the finite width of the ring. Finally, Sec. IV uses this theory to fit experimental data from the ABI.

II. EXPERIMENTAL DATA

For a quantitative discussion of the points listed above, we start by presenting some new data, taken from the same sample described in Refs. 2. As explained there, the closed ring-shaped ABI, shown in Fig. 1, was fabricated by wet etching the 2DEG at an AlGaAs/GaAs heterostructure. Au/Ti metallic gates define the QD and control the gate voltages on the QD (sitting on the lower branch of the ring) and on the reference (upper branch). The sample was cooled by a dilution refrigerator, the base temperature of which was 30mK though the electron temperature measured from the line shape of the Coulomb oscillation was around 100mK.

All our data are found to be symmetric under \( \phi \rightarrow -\phi \), in agreement with the Onsager relations. In order to investigate the nature of electronic paths in the ring geometry, we first connected all the gates to ground. The sample was hence a simple ring without a dot at this stage. Figure 2 shows a typical flux dependence of the conductance through the ABI, for these grounding conditions. The data clearly show the Aharonov-Bohm oscillations, with a small period as demonstrated in the insets. In addition, apart from some aperiodic fluctuations, the data exhibit oscillations on a larger scale. To quantify these oscillations, Fig. 2 shows the fast Fourier transform (FFT) of these data. The top graph shows the FFT of all the data. Interestingly, the results between \(~250/T\) and \(~280/T\) seem to contain several separable peaks. The top graph also shows arrows for the Aharonov-Bohm periods associated with the contours A, B and C in Fig. 1 indicating that all the frequencies in this range can be associated with electron paths which surround the ring between contours A and C. The inset in the top frame shows the same FFT on a semi-logarithmic scale. It is interesting to note that the data contain many higher harmonics, roughly at integer multiples of the first one. The graph in the middle shows fits to these high frequency data with four and with eight Gaussians, confirming the impression that the fast oscillations are dominated by only a few electron paths. The lower graph in Fig. 3 shows the FFT of the data with fields between 0.8T and 1T. Interestingly, these data exhibit even fewer peaks, implying that these restricted data can be described by ‘beats’ of a few neighboring frequencies.

We have then formed a quantum dot in the lower branch, by applying the gate voltages \(-0.255\, V, -0.215\, V\) for \( V_L, V_R \) respectively and connecting the other gates to ground. The conductance then shows a series of Fano resonances, the most prominent of which is indicated by the arrows. The insets are blowups in two different regions of magnetic field.
FIG. 3: Fourier power spectrum of the data in Fig. 2 (see text). (a) FFT result for the entire field region in Fig. 2. A, B, C are the frequencies, which correspond to the areas indicated by the contours A, B, C in Fig. 1. The inset is a log-plot of the same data, enhancing the higher harmonics. (b) Results of the fitting to the main peak in (a) by 4 Gaussians (solid line) and by 8 Gaussians (broken line). (c) Result of the same analysis as in (a) for the field region from 0.8T to 1T.

FIG. 4: False color image plots of the conductance, versus gate voltage $V_g$ and magnetic field. See the text for other voltages. The linear baseline from 2.6 to 2.8 in $(e^2/h)$ was subtracted.

the ideal 1D model, between resonances the maxima remain fixed at integer multiples of the flux unit, as follows from phase rigidity. In addition, maxima and minima can suddenly interchange between resonances, in a ‘phase lapse’ which is attributed to a Fano vanishing of the conductance. In contrast, the maxima in Fig. 4 never stay on lines parallel to the gate voltage axis. Instead, they move continuously towards larger fields, indicating a non-zero phase shift $\delta$. The variation of the maxima can be characterized by three typical forms, indicated by white arrows: in form A, the maximum moves almost linearly with the gate voltage, so that $\phi$ changes by $\pi$, returning to the value it had before the previous resonance. In form B, one observes a fast change in the location of the maximum, about half way between the resonances. Although reminiscent of the Fano jump by $\pi$, this change has a finite width and seems continuous. Finally, in form C the maximum moves by $\pi$ over the resonance, but very soon it moves quickly back to its location before the resonance, so that the line of this maximum remains almost parallel to the gate voltage axis for a range of gate voltages.

More experimental data are presented below, in Sec. IV.

III. THEORY

As stated, most of the existing theoretical descriptions of the ABI use models in which all the links are 1D. This means that the ABI “ring” surrounds a well defined unique area, which is penetrated by a single valued magnetic flux $\Phi$. It is this uniqueness that then results in the periodicity of $G$ with $\phi$, resulting with phase rigidity. To explain the deviation of our (and practically everyone else’s) data from this periodic-
We shall demonstrate that the ABI in the Coulomb blockade regime is a simplified version of that discussed in Refs. 10 and 11: in 1D (see Fig. 1), where the phase \( \phi \) is controlled by the gate voltage. (Here, \( \phi \) is given in units of \( \pi /2 \), and then \( J \) is a real hopping energy (a is the lattice constant on the leads), we write the wave functions on the right and right leads as \( u(e^{ikna} + e^{-ikna}) \) and \( u'e^{ikna} \)).

Indeed, away from a resonance one has \( |S_{xy}| \ll 1 \), the field dependence of \( G \) in Eq. (3) is dominated by the numerator \( 4 \sin^2 ka |S_{xy}|^2 \), and then Eq. (5) has the form

\[
|S_{xy}|^2 \approx A + B_1 \cos (\phi(1) + B_2 \cos (2)) + C \cos [\phi(1) - \phi(2)].
\]

Since both \( \phi(1) \) and \( \phi(2) \) correspond to paths through the 'upper' branch of the ABI, it is reasonable to expect that these two fluxes are quite close to each other. Assuming a ratio \( (1 + x) \) between the areas surrounded by the two paths, we denote \( \phi(1) = \phi \) and \( \phi(2) = (1 + x)\phi \), and then Eq. (5) has the form

\[
|S_{xy}|^2 \approx A + [B_1 + B_2 \cos x \phi(1)] \cos \phi - B_2 \sin \phi(1) \sin \phi + C \cos \phi(1) \\
\equiv \bar{A} + \bar{B} \cos (\phi + \delta),
\]

with

\[
\tan \delta = \frac{B_2 \sin \phi(1)}{[B_1 + B_2 \cos \phi(1)]},
\]

\[
\bar{B} = \sqrt{B_1^2 + B_2^2 + 2B_1B_2 \cos \phi(1)},
\]

\[
\bar{A} = A + C \cos \phi(1).
\]
For $|x| \ll 1$, the parameters $\tilde{A}$, $\tilde{B}$ and $\delta$ vary slowly with $\phi$, and therefore within a limited window of magnetic fields the data look like in the two-slit open ABI, with a phase shift $\delta$ which varies with $\phi$ and with the gate voltage, represented by $\epsilon_d$. On larger field ranges, Eq. (6) exhibits beats, similar to those observed experimentally. Note that the parameters in Eq. (7) may change quite significantly as $\phi$ changes from zero to $\pi/x$. Needless to say, the expression in Eq. (4), and therefore also $G$, is symmetric under field reversal [$\phi(n) \rightarrow -\phi(n)$ for all $n$], as expected from the Onsager relations.

When $x = 0$, one has $\delta = 0$ and $\tilde{A}$, $\tilde{B}$ remain constant, as for the simple 1D model. In that limit, the maxima between resonances remain fixed, with possible jumps by $\pi$ when $(B_1 + B_2)$ changes sign as function of the gate voltage $\epsilon_d$.

The above simple results change close to a resonance. To demonstrate the full behavior of the conductance, we present an example with $N = 2$, with the parameters $ka = \pi/2$, $j_\ell = j_\ell = J_{\ell}(n) = J_{\ell}(n) = 0.5 J$, $E_{res} = J$, $U = 3 J$ and $x = 0.1$. Figure 6 shows the calculated transmission, Eq. (3), for several values of the gate voltage $\epsilon_d$. In addition to seeing the beats away from resonances, we note the asymmetric beats closer to resonances. We also note the gradual shifts in the maxima. These shifts are highlighted in Fig. 7 (top), which shows only the locations of the maxima. This figure contrasts the behavior of the maxima between the ideal 1D case, with $x = 0$ (bottom) and the case described above, $x = 0.1$ (top). Note particularly the qualitative change with increasing flux between the two resonances: for $x = 0$ one observes a sharp ‘phase lapse’, where the maxima jump from even to odd multiples of $\pi$, due to an exact vanishing of the conductance which results from the Fano interference between the two resonances. These ‘lapses’ are no longer sharp when $x \neq 0$: in the range $1 < \phi/\pi < 2$ there appears a relatively fast change of the maximum from around $\phi = 2\pi$ down to $\phi \approx \pi$, similar to form B in Fig. 4. However, as $\phi$ increases this ‘lapse’ becomes smoother, and near $\phi = 7\pi$ one no longer sees such a ‘lapse’ at all, as in form A in Fig. 4. At fluxes of order $\pi/x$, the interplay between the two phases $\phi(1)$ and $\phi(2)$ destroys the exact vanishing of the transmission between resonances, and thus also destroys the ‘phase lapses’.

The interplay between the two fluxes also affects the dependence of the transmission on the gate voltage at fixed magnetic flux. Figure 5 shows this dependence for $\phi$ equal to integer multiples of $\pi$. Although qualitatively similar, the curves are not periodic in $\phi$, and one can see variations of the Fano asymmetric shapes of the resonances with increasing flux.

Similar graphs also arise when one considers larger numbers of resonances. As long as one has $x(n) = 1 + x(n) |\phi|$, with $|x(n)| \ll 1$, the overall shapes of the graphs are found to be similar to those presented above. Apparently, in the vicinity of a specific resonance the results are mainly affected by neighboring resonances, so that only a small number of different fluxes participate in the ‘beats’.

FIG. 6: Transmission through the closed ABI of Fig. 5 versus $\phi$. Graphs are shifted up by 1 as $\epsilon_d$ increases by 1.

FIG. 7: The location of the maxima of $T$ in the $\phi/\pi - \epsilon_d$ plane. Top: $x = 0.1$. Bottom: $x = 0$.

FIG. 8: Transmission through the ABI of Fig. 5 versus gate voltage $\epsilon_d$, at $\phi = m\pi$. The graphs shift up by 1 as $\phi$ increases.
IV. FITS TO EXPERIMENTAL DATA

Equation (3) should represent a good approximation for any sequence of Coulomb blockade peaks, with and without a magnetic flux. Indeed, in Ref. 17 this equation has been applied to produce a reasonable imitation of the data found by Gores et al. for a mesoscopic single electron transistor. The Fano asymmetry parameter for each resonance is in fact determined by the influence of all the other resonances, and therefore there is no need for individual fits of $q$ for each resonance and each set of parameters.

To demonstrate the effectiveness of Eq. (3), we have fitted it to data from Ref. 2 and to similar new data, which exhibit a sequence of Coulomb blockade resonances of the ABI, at various values of the magnetic flux. Here we chose a sequence of measurements, done on the device shown in Fig. 1 for gate voltages between $-0.142$ V and $-0.014$ V and magnetic fields in the range $0.9100 - 0.9132$ T. Figures 9 and 10 show fits to our data, using Eq. (3). As explained in Ref. 2, our device allows the pinch off of the reference path. Fitting Eq. (3) with $j_r = j_r = 0$ to the pinched off data, we have determined $E_D(n)$, $J^0_r(n)$ and $J_r(n)$, with $\phi(n) = 0$. The results of this fit are shown in Fig. 9. We have then added the reference path, and used the data from the ABI to determine $j/r/\sqrt{E_{ref}^{1/2}}$, $j_r/\sqrt{E_{ref}^{1/2}}$ (we use $\epsilon = 0$, to represent electrons at the Fermi level, deep inside the band) and the phases $\phi(n) = [1 + x(n)] \phi_1$ for one particular value of the magnetic field, $B_1 = 0.9116$ T. The results of this fit appear as the fifth graph from the bottom in Fig. 10. Repeating this fit for $B_2 = 0.9100$ T (the curve at the bottom in Fig. 10), using the same parameters except for replacing $\phi_1$ by a fitted $\phi_2$, we then found the coefficient $C$ in the relation $\phi = \phi_1 + C(B - B_1)$, $C = 976.9T^{-1/2}$. In addition, our fit allows a background conductance which contains a component linear in the gate voltage, in addition to a constant. The resulting parameters are listed in Table I. Using these parameters, we have then produced the theoretical curves for all the other values of $B$, with no further adjustments (Fig. 10). The fits clearly capture all the qualitative changes in the shapes of the resonances at different magnetic fluxes. We find the results quite satisfactory, confirming the assumptions of our theoretical model. We note that the fitted parameters have practically equidistant resonances, with $U \approx 0.02$ V. This Coulomb energy is consistent with the capacitance and area of the quantum dot.

Having obtained the parameters in Table I we have then plotted the theoretical contour plot of the conductance, see Fig. 11. Qualitatively, this figure is similar to Fig. 4 (taken with different parameters): the maxima move continuously with the magnetic field and with the gate voltage, imitating typical experimental data.
TABLE I: The parameters of our fits to Eq. (3). The conductance which penetrates the wave function in the ring which couples quantum dot can be associated with a single magnetic flux, having a closed ABI with a ring which has a finite width. For ABI is shifted by $\phi_1$.

| $n$ | $J_0(n) / \sqrt{\epsilon J}$ | $J_r(n) / \sqrt{\epsilon J}$ | $E_D(n) / e$ | $x(n)$ | $n$ | $j_0 / \sqrt{\epsilon E_{ref}}$ | $j_r / \sqrt{\epsilon E_{ref}}$ |
|-----|-------------------------------|-------------------------------|-------------|--------|-----|---------------------------------|-----------------------------|
| 1   | 0.5886                        | 3.860                         | 0.0008      | 0      | 1   | 0.4224                          | 0.1003                     |
| 2   | -3.761                        | 0.3806                        | -0.0159     | -5.913 | 2   | 1639.13                         | 0.5392                     |
| 3   | 3.708                         | 0.4211                        | -0.0350     | -0.069 | 3   | 0.7391                          | -0.01768                   |
| 4   | 3.876                         | 0.2988                        | -0.0543     | -0.085 | 4   | 5                               |                            |
| 5   | 3.716                         | 0.3073                        | -0.0752     | 5.669  | 5   | 1639.13                         | 0.5392                     |
| 6   | 3.161                         | 0.4030                        | -0.0956     | -0.058 | 6   | 0.7391                          | -0.01768                   |
| 7   | 3.420                         | 0.4155                        | -0.1168     | -0.006 | 7   | 5                               |                            |
| 8   | 4.170                         | 0.2934                        | -0.1353     | 0.069  | 8   | 1639.13                         | 0.5392                     |

V. CONCLUSION

In this paper we have concentrated on the periodic effects of having a closed ABI with a ring which has a finite width. For a semiconductor ring, we argue that each resonance on the quantum dot can be associated with a single magnetic flux, which penetrates the wave function in the ring which couples to that resonance. A simple theoretical formula then captures all the qualitative features observed in many experiments.

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