Twist-4 effects and $Q^2$ dependence of diffractive DIS

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Abstract

In this letter we report the direct perturbative QCD evaluation of twist-4 effects in diffractive DIS. They are large and have a strong impact on the $Q^2$ dependence of diffractive structure functions at large $\beta$. Based on the AGK rules, we comment on the possible contribution from diffractive higher twists to $\propto \frac{1}{Q^2}$ corrections to proton structure function at small $x$. These corrections to the longitudinal structure function $F_L$ may be particularly large.

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The direct calculation of higher twist corrections to the proton structure function (SF) remains one of challenges of perturbative QCD (pQCD) description of inclusive deep inelastic scattering (DIS) ([1, 2], for phenomenological estimates based on departures from the DGLAP evolution see [3]). In a striking contrast to inclusive proton SF, twist-4 (T4) corrections to diffractive SF, \( F_i^{\text{D}(3)}(x_{\text{IP}}, \beta, Q^2), (i = L, T) \), are genuinely short distance dominated [4, 5] and thus calculable from the first principles of pQCD. Furthermore, they are strongly enhanced by the gluon SF of the proton squared [3], \( F_2^{\text{D}(3)}(T4; x_{\text{IP}}, \beta, Q^2) \propto \frac{1}{Q^2} G(x_{\text{IP}}, Q^2) \). Hereafter \( x, Q^2, \beta \) and \( x_{\text{IP}} = \frac{x}{\beta} \) are standard diffractive DIS variables and \( G(x, Q^2) = x g(x, Q^2) \). For instance, for \( \beta \sim 0.9 \), the twist-4 longitudinal diffractive SF was found to be comparable to, and even exceeding, the leading twist-2 (T2) transverse SF over a broad range of \( Q^2 \), making questionable [6, 7, 8, 9] applications of the standard DGLAP evolution to diffractive SF.

In this letter we extend our early analysis of twist-4 longitudinal diffractive SF [3] to similarly enhanced twist-4 contribution to the large-\( \beta \) transverse diffractive SF [6]. It enters \( F_T^{\text{D}(3)}(x_{\text{IP}}, \beta, Q^2) \) with negative sign and, as the higher twist dies out at large \( Q^2 \), the transverse SF \( F_T^{\text{D}(3)}(x_{\text{IP}}, \beta, Q^2) \) will rise with increasing \( Q^2 \). We present for the first time an analysis of the \( \beta \) and \( Q^2 \) dependence of compensations of the longitudinal and transverse twist-4 SF’s in \( F_2^{\text{D}(3)} = F_T^{\text{D}(3)} + F_L^{\text{D}(3)} \), which completes the evaluation of driving twist-4 terms in \( F_2^{\text{D}(3)} \). We compare our results with the H1 experimental data on \( F_2^{\text{D}(3)} \) [11]. Based on a connection between diffractive DIS and unitarity corrections to the proton SF suggested by the Abramovski-Gribov-Kancheli (AGK) rules [12, 13, 14], we evaluate the unitarity-driven diffractive twist-4 corrections to the small-\( x \) proton SF, which is potentially very large in the proton longitudinal SF \( F_L \).

The underlying pQCD subprocess for diffractive DIS at large \( \beta \) is the excitation of the \( q\bar{q} \) Fock states of the photon \( \gamma^* p \to X p' \), where \( X = q\bar{q} \) [3]. The relevant pQCD diagrams are shown in Fig. 1. In what follows, \( z \) and \( (1 - z) \) are the fractions of the (light–cone) momentum of the photon carried by the quark and antiquark, respectively, \( \vec{k} \) is the relative transverse momentum in the \( q\bar{q} \) pair, \( M^2 = (m_f^2 + k^2)/(1 - z) \) is the invariant mass of the diffractive system, \( \vec{p}_\perp \) is the \((p, p')\) momentum transfer and \( t = -\vec{p}_\perp^\perp \). The contribution of

\[2\] The preliminary results from this study have been reported at DIS’97 [4]. Related results were presented at DIS’97 by J.Bartels and M.Wüsthoff [3].
excitation of one open flavour of electric charge $e_f$ (in units of the electron charge) and mass $m_f$ to transverse (T) and longitudinal (L) diffractive SF’s for forward diffraction dissociation is calculable in terms of the helicity amplitudes $\bar{\Phi}_1$ and $\Phi_2$ introduced in [3]:

$$F_{T}^{D(4)}(t = 0, x_{IP}, \beta, Q^2) = \frac{4\pi e_f^2 \beta}{3\sigma_{tot}(pp)} \int \frac{dk^2(k^2 + m_f^2)}{(1 - \beta)^2 J} \alpha_s^2(Q^2) \left\{ \left[ 1 - 2\frac{k^2 + m_f^2}{M^2} \right] \bar{\Phi}_1 + m_f^2 \Phi_2 \right\}, \quad (1)$$

$$F_{L}^{D(4)}(t = 0, x_{IP}, \beta, Q^2) = \frac{4\pi e_f^2 \beta}{3\sigma_{tot}(pp)} \int \frac{dk^2(k^2 + m_f^2)}{(1 - \beta)^2 J} \alpha_s^2(Q^2) 4\beta^2(1 - z)^2Q^2\Phi_2,$$

where $J = \sqrt{1 - 4\frac{m_f^2 + k^2}{M^2}}$ is the Jacobian peak factor [3]. To the Leading Log$^{\frac{1}{2}}$ approximation [16],

$$\bar{\Phi}_1 = \frac{\bar{k}}{k^4} \int f(x_{IP}, \kappa^2) \left[ \frac{1}{k^2 + \varepsilon^2} - \frac{1}{\sqrt{a^2 - b^2}} + \frac{2\kappa^2}{a^2 - b^2 + a\sqrt{a^2 - b^2}} \right]$$

$$\approx 2\bar{k}(1 - \beta)^2G(x_{IP}, \bar{Q}^2) \left[ \frac{\beta}{(k^2 + m_f^2)^2} + \frac{(1 - \beta)m_f^2}{(k^2 + m_f^2)^3} \right],$$

$$\Phi_2 = \int \frac{dk^2}{k^4} f(x_{IP}, \kappa^2) \left[ \frac{1}{\sqrt{a^2 - b^2}} - \frac{1}{k^2 + \varepsilon^2} \right]$$

$$\approx -(1 - \beta)^2G(x_{IP}, \bar{Q}^2) \left[ \frac{2\beta - 1}{(k^2 + m_f^2)^2} + \frac{2(1 - \beta)m_f^2}{(k^2 + m_f^2)^3} \right]. \quad (3)$$

Here $f(x, \kappa^2) = \partial G(x, k^2)/\partial \log(\kappa^2)$ is the unintegrated gluon SF of the proton, $\varepsilon^2 = z(1 - z)Q^2 + m_f^2$, $a = \varepsilon^2 + k^2 + \kappa^2$, $b = 2k\kappa$,

$$Q^2 = \varepsilon^2 + k^2 = \frac{k^2 + m_f^2}{1 - \beta} \quad \text{(4)}$$

is the pQCD scale of the Leading Log$Q^2$ approximation ([4], [13]), and we also have shown the Leading Log$Q^2$ approximations for $\bar{\Phi}_1, \Phi_2$. $\bar{Q}^2$ has already been used as such in ([1], [2]) as the argument of the strong coupling $\alpha_s$. The above outlined formalism has become a standard description of DIS at large $\beta$ [3, 4, 7, 17, 18, 19]. Extra contributions, $\propto f(x_{IP}, \bar{Q}^2)$, which take over in $\bar{\Phi}_1, \Phi_2$ at $\beta \ll 1$, were found in [14], but they can be neglected at $\beta \sim 1$.

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\(^3\)We follow the convention in which diffractive structure functions $F_i^{D(4)}$ are dimensionless [3], hence $\sigma_{tot}(pp) = 40mb$ in the denominator of the r.h.s of Eqs. [4], [2]. Our definition [3] of $F_i^{D(3)}$ differs from [11, 15] by the factor $x_{IP}$, so that our $F_i^{D(3)}$ does not blow up $\sim \frac{1}{x_{IP}}$ when $x_{IP} \to 0$. 

3
The advantage of forward diffraction is that the SF $F_{D_i}^{(4)}(t = 0, x_{IP}, \beta, Q^2)$ is calculable directly in terms of the conventional gluon SF. The first measurements of the $t$-dependent $F_{D_i}^{(4)}(t, x_{IP}, \beta, Q^2)$ have been reported recently by the ZEUS collaboration [15], but for large $\beta$ of our interest in this study only the $t$-integrated diffractive SF’s are available [11]. Assuming the usual $d\sigma/dt \propto \exp(-B_d p_T^2)$, one finds

$$F_{D_i}^{(3)}(x_{IP}, \beta, Q^2) = \int dt \frac{\sigma_{tot}(pp)}{16\pi} F_{D_i}^{(4)}(t, x_{IP}, \beta, Q^2) \approx \frac{\sigma_{tot}(pp)}{16\pi B_d(\beta)} F_{D_i}^{(4)}(t = 0, x_{IP}, \beta, Q^2).$$  (5)

Using Eqs. (2), (3), one readily finds that the longitudinal SF $F_{D_i}^{(3)}$ is dominated by the contribution from large $k^2 \sim \frac{1}{4}M^2$, which is further enhanced because of scaling violations in the gluon structure function of the proton. Consequently, it is perturbatively calculable [4, 5, 6] and at large $Q^2$ one finds the higher twist 4.

We emphasize that in this paper we focus on $\beta \sim 1$. As it has been shown in [17], at $\beta \ll 1$, dominated by the $q\bar{q}g$ excitation, the longitudinal diffractive SF is twist-2 and $R = \sigma_L/\sigma_T \approx 0.2$, in close similarity to inclusive DIS [20].

4
Notice, that the large-$k^2$ behaviour of the integrand of the the twist-4 transverse SF $F_T^{D(3)}(T4)$ is identical to that of $F_L^{D(3)}$, it is pQCD calculable, and for large $Q^2$, well above the charm threshold, we find

$$F_T^{D(3)}(T4; t = 0, x_{\mathbf{p}'}, \beta, Q^2) = -\frac{8\beta^4(1 - \beta)}{Q^2B_3\mathbf{p}'} \cdot \frac{e_f^2}{12}F_G(x_{\mathbf{p}'}, \beta, Q^2). \quad (11)$$

In contrast to that, the twist-2 transverse SF is dominated by $k^2 \sim m_f^2$:

$$F_T^{D(4)}(T2; t = 0, x_{\mathbf{p}'}, \beta, Q^2) \propto \int \frac{dk^2k^2}{(k^2 + m_f^2)^3} \cdot \left[ \alpha_S(\bar{Q}^2)G(x_{\mathbf{p}'}, Q^2 = \frac{m_f^2 + k^2}{(1 - \beta)}) \right]^2, \quad (12)$$

(we suppress in (12) terms containing extra factors $[m_f^2/(k^2 + m_f^2)]^n$). Still, for heavy flavours and/or sufficiently small $(1 - \beta)$, the pQCD scale $\bar{Q}^2$ is large, one is in the legitimate pQCD domain and to a logarithmic accuracy,

$$F_T^{D(3)}(T2; x_{\mathbf{p}'}, \beta, Q^2) \approx \beta(1 - \beta)^2(3 + 4\beta + 8\beta^2) \cdot \frac{e_f^2}{12} \cdot \left[ \alpha_S(\bar{Q}^2)G^2(x_{\mathbf{p}'}, Q^2) \approx \frac{m_f^2}{(1 - \beta)} \right]^2. \quad (13)$$

Both terms, $\propto \Phi_1^2$ and $\propto \Phi_2^2$ in Eq. (9), give comparable contributions to the twist-2 $F_T^{D(4)}$. Because of the finite phase space of the $k^2$ integration, $k^2 \leq \frac{1}{4}M^2 - m_f^2$, there emerge higher-twist looking small corrections $\propto \left( \frac{m_f^2}{M^2} \right)^n$, $n \geq 1$, to the above defined twist-2 SF. These corrections are logarithmically weaker than the genuine twist-4 transverse SF (11) and for this reason, we do not separate them.

Evaluating the twist-2 transverse SF at not so large $\beta$, one needs a model for the small-$Q^2$ behaviour of the unintegrated gluon structure function $f(x, Q^2)$. Because of cancelation of soft gluon radiation from different quarks in the colour singlet nucleon, $f(x, Q^2)$ vanishes at $Q^2 \to 0$. The Born approximation suggests the soft $Q^2$ behaviour \cite{16, 21}

$$\frac{\partial G_B(Q^2)}{\partial \log Q^2} = \frac{4C_B\alpha_S(Q^2)}{\pi} \frac{Q^4}{(Q^2 + \mu^2)^2} \left[ 1 - G_2(Q, -Q) \right], \quad (14)$$

where $G_2(Q, -Q)$ is the two-quark form factor of the nucleus, $G(0, 0) = 1$. For harder gluons one must interpolate between the form (14) and any convenient parameterization for $G(x, Q^2)$, for instance, the GRV NLO parameterization \cite{22}:

$$G(x, Q^2) = G_B(Q^2) \left( \frac{Q_0^2}{Q_0^2 + Q^2} \right)^N + G_{GRV}(x, Q^2) \left[ 1 - \left( \frac{Q_0^2}{Q_0^2 + Q^2} \right)^N \right]. \quad (15)$$
For a similar soft $Q^2$ parameterizations see Ref. [18]. The GRV formulas hold only for $Q^2 \geq Q_c^2 = 0.4 \text{ GeV}^2$, we take $G_{GRV}(x, Q^2 < Q_c^2) = G_{GRV}(x, Q_c^2)$. We choose the parameters $C_B = 1.5$, $\mu_G = 0.15 \text{ GeV}$, $Q_0^2 = 3 \text{ GeV}^2$, $N = 1$ so as to reproduce the colour dipole cross section [23]. This way, our results for diffractive DIS are very close to those found in the colour dipole model [17, 7]. For the quark masses we take $m_c = 1.5 \text{ GeV}$, $m_s = 0.3 \text{ GeV}$ and $m_{u,d} = 150 \text{ MeV}$ as in [17]. For light flavours, Eq. (13) must not be taken literally, in this case the pQCD scale $\bar{Q}^2$ is set not by $m^2_f$ but rather by a range of rapid variation of the gluon structure function, $\bar{Q}^2 \sim 0.5 \text{ GeV}^2$.

In order to calculate $F_D^{T(3)}$ we need to know the diffraction slope $B_d$. It has been anticipated some time ago [4, 5, 14, 17] that the typical scale for the diffraction slope $B_d(\beta)$ is set by $B_{3\text{IP}} \approx 6 \text{ GeV}^{-2}$, as seen in hadronic diffraction in the so-called triple pomeron region, which in DIS corresponds to $\beta \ll 1$. This prediction has been confirmed by the first data from the ZEUS LPS: $B_d = 7.1 \pm 1.1^{+07}_{-10} \text{ GeV}^{-2}$ in diffractive DIS for $5 \text{ GeV}^{-2} < Q^2 < 20 \text{ GeV}^{-2}$ [24] and $B_d = 7.7 \pm 0.9 \pm 1.0 \text{ GeV}^{-2}$ in real photoproduction [25]. For the numerical calculation of the twist-2 transverse SF we will use the results of Ref. [9, 26], where it has been calculated that $B_d(T2) \approx B_{3\text{IP}}$ for $\beta \rightarrow 0$, it rises by $\approx 50\%$ reaching a maximum at $\beta \sim 0.5$, then drops back to $B_{3\text{IP}}$ at $\beta \sim 0.9$, has a sharp minimum at $\beta \rightarrow 1$ and ends up at $B_d(\beta = 1) \approx B_{3\text{IP}}$ in the exclusive limit of vector meson production. For the short-distance dominated twist-4 SF’s only the proton size contributes to the diffraction slope $B_d$ and, consequently, we will use $B_d(T4; \beta) \approx B_{3\text{IP}}$ [3, 24].

For comparing the large-$\beta$ behaviour of the twist-4 and twist-2 contributions to the transverse SF at large $\beta$, it is sufficient to use the logarithmic approximation, in which

$$F_T^{D(3)}(T4; t = 0, x_{\text{IP}}, \beta, Q^2) \approx \frac{48 m_f^2}{(3 + 4 \beta^2 + 8 \beta^4)Q^2} \cdot \frac{B_d(T2; \beta)}{B_{3\text{IP}}} \cdot \frac{\beta^3}{1 - \beta} \cdot \left[ \frac{\alpha_S(\frac{1}{4}Q^2)G(x_{\text{IP}}, \frac{1}{4}Q^2)}{\alpha_S(\bar{Q}^2)G(x_{\text{IP}}, \bar{Q}^2 \approx \frac{m_f^2}{(1-\beta)^2}}} \right]^2. \tag{16}$$

The rise of this ratio $\propto \frac{1}{1-\beta}$ when $\beta \rightarrow 1$ is to a large extent compensated by the growth of $G^2(x_{\text{IP}}, Q^2)$ because of the rising pQCD scale $\bar{Q}^2 \propto \frac{1}{1-\beta}$ and, as a minor effect for $\beta > 0.9$, by the decrease of $B_d(T2; \beta)$ with $\beta \rightarrow 1$ as it was found in [26].
If the longitudinal polarization of the virtual photon equals $\epsilon_L$, then the twist-4 T and L SF’s enter in the combination $F_{2L} = F_T + \epsilon_L F_L$. To the leading Log$Q^2$ approximation, one readily finds

$$F_{2L}^{D(3)}(T4; x_{IP}, \beta, Q^2) = \frac{e^2}{6Q^2 B_{3IP}}\beta^3[(1 + 2\epsilon_L)(1 - 2\beta)^2 - 1] F_G(x_{IP}, \beta, Q^2),$$

and the overall twist-4 contribution is dominated by the the positive valued $F_L^{D(3)}$ for $\beta$ beyond the crossover points, $\beta > \beta_+$ or $\beta < \beta_-$, and by the negative valued $F_T^{D(4)}(T4)$ in between, where

$$\beta_{\pm} = \frac{1}{2} \left(1 \pm \frac{1}{\sqrt{1 + 2\epsilon_L}}\right)$$

and $\beta_+ = 0.79$ for $\epsilon_L = 1$. Because the subleading contributions $F_L^{D(3)}$ become larger as $\beta$ decreases, cf. Eqs. (3) and (3), the crossover is shifted to $\beta_+ \sim 0.65$. 

In Fig. 2 we show our results for $F_2^{D(3)}$ and its decomposition into the transverse and longitudinal (assuming $\epsilon_L = 1$), and the twist-2 and twist-4 components for $x_{IP} = 1.33 \times 10^{-3}$. Because of the inequality for the excited mass, $M^2 \geq 4m_c^2$, open charm excitation contributes only at $Q^2 \geq 4m_c^2 \frac{\beta}{1-\beta}$, and the charm threshold effects are clearly visible in Fig. 2. As the suppression $\propto \frac{1}{m_c^2}$ of the open charm contribution to transverse twist-2 SF is to a large extent compensated by the large pQCD scale $\bar{Q}^2$, the charm abundance is substantial even in $F_T^{D(3)}$. It is still larger in the longitudinal SF, in which the flavour dependence only enters via the lower integration limit in Eq. (7). The fractional contribution of open charm to $F_T^{D(3)}(T4)$ is similar to that in $F_L^{D(3)}$, but the charm threshold effects are smoother because $\Phi_1^2 \propto k^2$ which suppresses the near-threshold cross section. Apart from the charm threshold effects, the twist-2 transverse SF is practically flat vs. $Q^2$. Whereas for $\beta = 0.65$ the longitudinal and transverse twist-4 contributions cancel each other, at $\beta = 0.9$ the twist-4 contribution is very large. Notice how scaling violations slow down the rate of decrease of twist-4 structure functions at large $Q^2$. We find good agreement between our results and the recent experimental data on $F_2^{D(3)}$ at $x_{IP} = 1.33 \times 10^{-3}$ from the H1 collaboration [11]. In Fig. 3 we show our results for the $x_{IP}$ dependence of $F_2^D(x_{IP}, \beta, Q^2)$ in comparison with the H1 experimental data. Our formalism describes excitation of the continuum states, and the real comparison between theory and experiment is justified only for masses above the 1S and 2S vector meson.
excitation, $M^2 \gtrsim 3 \text{GeV}^2$, which for $\beta = 0.9$ implies $Q^2 = M^2 \frac{\beta}{1-\beta} \gtrsim 20-30 \text{GeV}^2$. In the spirit of the exclusive/inclusive duality \cite{8}, our results for the continuum in the vector meson excitation region must be applicable to the experimental data in which the vector meson peaks have been smeared out. With these reservations, our calculations reproduce rather well the experimentally observed $Q^2$ and $x_{\text{IP}}$ dependence of $F_2^{D(3)}(x_{\text{IP}}, \beta, Q^2)$ over the whole range of $Q^2$ and $x_{\text{IP}}$. Notice the steeper $x_{\text{IP}}$ dependence at smaller $x_{\text{IP}}$, which is a characteristic prediction of pQCD models of diffractive DIS \cite{17, 18, 19}. The accuracy of the experimental data on diffractive DIS is not yet sufficient to test different models for gluon density in the proton. Theoretical evaluations of the short-distance dominated $F_L^{D(3)}$ and $F_T^{D(3)}(T4)$ must be good to $\sim 10-20\%$ accuracy (due to uncertainties in $G(x, k^2)$ in the perturbative region), whereas for the leading twist $F_T^{D(3)}$ (where one has a large penetration into the low-$k^2$ region) a conservative estimate for the accuracy is $\sim 20-30\%$ \cite{17}.

It is interesting to comment on the possible impact of diffractive higher twist on the proton SF at small $x$. The experimentally observed small-$x$ growth of the proton structure function can not go indefinitely without running into conflict with the unitarity. In general, one can decompose the experimentally measured proton SF into the Born, alias the DGLAP, component and the unitarity correction \cite{13, 14}:

$$F_2(x, Q^2) = F_2(DGLAP; x, Q^2) + \Delta_U F_2(x, Q^2)$$

(19)

The standard DGLAP evolution, which is linear in parton densities, is strictly applicable only to the non-unitarized SF $F_2(DGLAP; x, Q^2)$. The unitarization of the proton SF is one of the open non-perturbative problems. The AGK rules suggest (i=T,L,2) \cite{12, 13}:

$$\Delta_U F_i(x, Q^2) = - \int_{x_{\text{max}}}^{1} \frac{d\beta}{\beta} F_i^{D(3)}(x_{\text{IP}}, \beta, Q^2),$$

(20)

where $x_{\text{max}} = 0.05-0.1$ is the threshold for diffractive DIS. We are interested in the twist-4 component of the unitarity correction coming from the diffractive twist-4 component of $q\bar{q}$ excitation. Because the integral (20) is dominated by the contribution from $\beta \sim 0.5$, one can put $\beta = 0.5$ in the scaling violations factor (7) and take it out of the integral (20). Then, after little algebra, one finds
\[ \Delta_U F_i(T; x, Q^2) = \frac{e_i^2}{90Q^2 B_{3ip}} \mathcal{F}_i(x, 0.5, Q^2) \begin{cases} 
 3, & \text{for } i=T \\
 -4, & \text{for } i=L \\
 -1, & \text{for } i=2 
\end{cases} \]  \hspace{1cm} (21)

which does not depend on \( x_{\text{max}} \) as soon as \( x \ll x_{\text{max}} \). It is convenient to present the diffractive twist-4 effects as the correction factor \((1 - \frac{Q^2}{Q^2_4})\) to the leading-twist structure function. Our results for the so defined \( Q^2_{4,2} \) are shown in Fig. 4. We find that \( Q^2_{4,2} \) rises with decreasing \( x \) and increasing \( Q^2 \), but it is relatively small because of the strong cancelation of the longitudinal and transverse twist-4 contributions. It is interesting to notice that diffractive twist-4 correction to the longitudinal structure function of the proton is much larger,

\[ Q^2_{4,L} = \frac{4}{R} Q^2_{4,2} \sim 20Q^2_{4,2}, \]  \hspace{1cm} (22)

where we use \( R = \sigma_L/\sigma_T \approx 0.2 \) \footnote{\textnormal{[20]}} (for an indirect experimental estimates see \footnote{\textnormal{[27]}}).

Of course, the above estimated diffractive higher twist from diffractive DIS at large-\( \beta \) is only one of the many possible sources of twist-4 corrections to the proton structure function at small \( x \). We focused on it for the reasons that it is parameter-free calculable in pQCD, is enhanced by the gluon structure function of the proton squared and receives a substantial contribution from the heavy flavour excitation. At the moment, one can not exclude that a comparable contribution to \( Q^2_{4,1} \) will come from the small-\( \beta \) diffraction.

**Summary and Conclusions.**

We have presented the calculation of the Born approximation for the higher twist-4 corrections to the transverse diffractive structure function of the proton. It is similar to the longitudinal diffractive structure function which to the Born approximation is entirely twist-4. Both twist-4 diffractive structure functions are short-distance dominated and are perturbatively calculable. Their contribution to the diffractive structure function is large and dominates \( F_2^{D(3)} \) at \( \beta \gtrsim 0.9 \). Evidently, this twist-4 component of \( F_2^{D(3)} \) must be removed before venturing the DGLAP evolution analysis of the experimentally measured diffractive structure function. Because the pQCD calculation of the twist-4 corrections to \( F_2^{D(3)} \) is parameter free, such a correction of the diffractive structure function must not cause any ambiguities.
Taking for guidance the AGK rules, we also evaluated the impact of the diffractive twist-4 to the twist-4 terms in the proton structure function. The estimate (22) serves as a warning that diffractive twist-4 corrections to small-\(x\) longitudinal structure functions of the proton can be surprisingly large.
References

[1] E.V. Shuryak and A.I. Vainshtein, *Nucl. Phys.* **B199**, 451 (1982).

[2] R.L. Jaffe and M. Soldate, *Phys. Rev.* **D26**, 49 (1982).

[3] M. Virchaux and A. Milztajn, *Phys. Lett.* **B274**, 221 (1992).

[4] N. N. Nikolaev and B. G. Zakharov, *Z. Phys.* **C49**, 607 (1991).

[5] N. N. Nikolaev and B. G. Zakharov, *Z. Phys.* **C53**, 331 (1992).

[6] M. Genovese, N. N. Nikolaev and B. G. Zakharov, *Phys. Lett.* **B380**, 213 (1996).

[7] M. Genovese, N. N. Nikolaev and B. G. Zakharov, *Phys. Lett.* **B378**, 347 (1996).

[8] N. N. Nikolaev and B. G. Zakharov, DIS’96: Deep Inelastic Scattering and Related Phenomena, Editors G. D’Agostini and A. Nigro, World Scientific, Singapore, pp. 347-353.

[9] N. N. Nikolaev and B. G. Zakharov, Phenomenology of Diffractive DIS, [hep-ph/9706343](http://arxiv.org/abs/hep-ph/9706343). To be published in Proceedings of 5th International Workshop on Deep Inelastic Scattering and QCD. Chicago, Illinois, USA, April 14-18, 1997. American Institute of Physics Proceedings series, eds. M. Krakauer and J. Repond, 1997, in press.

[10] J. Bartels and M. Wüsthoff, Higher twist in diffractive dissociation. *ANL-HEP-CP-97-51* (1997). To be published in Proceedings of 5th International Workshop on Deep Inelastic Scattering and QCD. Chicago, Illinois, USA, April 14-18, 1997. American Institute of Physics Proceedings series, eds. M. Krakauer and J. Repond, 1997, in press.

[11] H1 Collab., P. Newman, DIS’96: Deep Inelastic Scattering and Related Phenomena, Editors G. D’Agostini and A. Nigro, World Scientific, Singapore, pp. 331-339; C. Adloff et al. *DESY-97-158* (1997), [hep-ex/9708016](http://arxiv.org/abs/hep-ex/9708016).

[12] V. Abramovski, V. N. Gribov and O. V. Kancheli, *Sov. J. Nucl. Phys.* **18**, 308 (1974); J. Bartels and M. G. Ryskin, *Z. Phys.* **C76**, 241 (1997).
[13] V.Barone, M.Genovese, N.N.Nikolaev. E.Predazzi and B.G.Zakharov, Phys. Lett. B326, 161 (1994).

[14] N. N. Nikolaev and B.G.Zakharov, JETP 78, 598 (1994); Z. Phys. C64 (1994) 631.

[15] ZEUS Collaboration; J.Breitweg et al. DESY 97-184 (1997), hep-ex/9709021.

[16] N. N.Nikolaev and B.G.Zakharov, Phys. Lett. B332, 177 (1994).

[17] M. Genovese, N.N.Nikolaev and B.Zakharov, JETP 81 625 (1995).

[18] J. Bartels, H. Lotter and M. Wüsthoff, Phys. Lett. B379 (1996) 239; H. Lotter, hep-ph/9612415.

[19] E.M.Levin, A.D.Martin, M.G.Ryskin and T.Teubner, Z. Phys. C74 (1997) 671.

E.Gotsman, E.Levin and U.Maor, Nucl. Phys. B493 (1997) 354.

[20] N. N. Nikolaev and B.G.Zakharov, Phys. Lett. B327, 149 (1994); B327, 157 (1994).

[21] V. Barone, M. Genovese, N.N. Nikolaev, E. Predazzi and B.G. Zakharov, Int. J. Mod. Phys. A8 (1993) 2779.

[22] M. Glück, E. Reya and A. Vogt, Z. Phys. C67, 433 (1995).

[23] J. Nemchik, N.N. Nikolaev and B.G. Zakharov, Phys. Lett. B341, 228 (1994).

[24] ZEUS Collab., M.Grothe, to be published in Proceedings of 5th International Workshop on Deep Inelastic Scattering and QCD. Chicago, Illinois, USA, April 14-18, 1997. American Institute of Physics Proceedings series, eds. M.Krakauer and J.Repond, 1997, in press.

[25] ZEUS Collab., G.Briskin, to be published in Proceedings of 5th International Workshop on Deep Inelastic Scattering and QCD. Chicago, Illinois, USA, April 14-18, 1997. American Institute of Physics Proceedings series, eds. M.Krakauer and J.Repond, 1997, in press.

[26] N.N.Nikolaev, A.V.Pronyaev and B.G.Zakharov, paper in preparation.

[27] H1 Collab., T.Ahmed et al., Phys. Lett. B393, 452 (1997).
Figure captions

Figure 1: One of the 16 Feynman diagrams for diffractive cross section.

Figure 2: The $Q^2$ dependence of the diffractive structure function and its decomposition into the transverse (T) and longitudinal (L) components and into the twist-4 (T4) and twist-2 (T2) components. The open charm threshold in $F_2^{D(3)}(x_F, \beta, Q^2)$ is seen from the comparison of solid curves for excitation of 4 flavours ($u+d+s+c$) and dotted curves for excitation of light flavours ($u+d+s$). The predictions are compared with the H1 experimental data on $F_2^{D(3)}(x_F, \beta, Q^2)$ at $x_F = 1.33 \times 10^{-3}$ [11] for $\beta = 0.65$ (box a) and $\beta = 0.9$ (box b).

Figure 3: Our predictions for the $x_F$ dependence of diffractive structure function for $\beta = 0.65$ (the two top raws) and $\beta = 0.9$ (the two bottom raws) in comparison with the H1 experimental data [11] for several values of $Q^2$.

Figure 4: The scale $Q_{4,2}^2$ for the AGK evaluation of the diffractive higher twist contribution to the proton structure function at small $x$. 
Figure 1: One of the 16 Feynman diagrams for diffractive cross section.
Figure 2: The $Q^2$ dependence of the diffractive structure function and its decomposition into the transverse (T) and longitudinal (L) components and into the twist-4 (T4) and twist-2 (T2) components. The open charm threshold in $F_2^{D(3)}(x_I, \beta, Q^2)$ is seen from the comparison of solid curves for excitation of 4 flavors $(u + d + s + c)$ and dotted curves for excitation of light flavors $(u + d + s)$. The predictions are compared with the H1 experimental data on $F_2^{D(3)}(x_I, \beta, Q^2)$ at $x_I = 1.33 \times 10^{-3}$ [1] for $\beta = 0.65$ (box a) and $\beta = 0.9$ (box b).
Figure 3: Our predictions for the $x_P$ dependence of diffractive structure function for $\beta = 0.65$ (the two top rows) and $\beta = 0.9$ (the two bottom rows) in comparison with the H1 experimental data [11] for several values of $Q^2$. 
Figure 4: The scale $Q^2_{4,2}$ for the AGK evaluation of the diffractive higher twist contribution to the proton structure function at small $x$. 