Is the exotic $0^{-}$ glueball a pure gluon state?

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We present a new calculation of the mass and width of the exotic $0^{-}$ glueball in the framework of the QCD sum rules. We next construct a new current which couples to a pure $0^{-}$ gluon state and derive consistent and stable sum rules. A previously used current in this approach was shown to be inconsistent. We obtain for this state a mass $M_G = 6.3^{+0.4}_{-1.1}$ GeV and an upper limit for the total width $\Gamma_G \leq 235$ MeV. These values can be used as an important guide for the experimental search of this exotic state. We argue that the mixing of this glueball state with $0^{-}$ tetraquark is very small. Therefore, the exotic $0^{-}$ glueball can be considered as a pure gluon state.

The glueballs carry very important information on the gluonic sector of QCD and their study is one of the fundamental tasks in strong interaction physics. While glueballs are predicted by QCD, there has been no clear experimental evidence of their existence and so they remain as of yet a subject to theoretical and experimental research (see reviews [1, 2]). For this reason the study of glueball candidates is included in many programs of presently running and future experiments.

One of the main problems of glueball spectroscopy is the mixing of the glueballs with ordinary meson states, which leads to difficulties in disentangling the glueball components in experiments. In this connection, the discovery of the exotic $0^{-}$ glueball would be extremely useful, because it does not mix with any $q\bar{q}$ states. It is therefore very important to investigate the properties of this glueball within a QCD based approach. One of the most successful approaches to study strong interaction spectroscopy is the QCD Sum Rules (SRs) method [3, 4].

In this Letter, for the first time, a consistent SR for the exotic $0^{-}$ glueball is obtained. We calculate the Operator Product Expansion (OPE) of the correlator up to dimension-8 with a new interpolating current which couples to this pure gluon state, and show that there is good stability for the SR. From this stable SR a prediction for the mass and an upper limit of the total width of this state are found.

The QCD SR approach [3] for a bound state consists of two parts. One is the calculation of the OPE of the correlator defined by

$$\Pi(Q^2) = i \int d^4 x \epsilon^{i\rho\sigma} \langle J(0) J^\dagger(x) \rangle$$

where the current couples to the gluonic bound state $|G\rangle$ in our case as

$$\langle 0 | J | G \rangle = F_G M_G^{N-2}.$$ 

Here $Q^2 = -q^2$, $N$ is the dimension of the current $J$, $F_G$ is the decay constant and $M_G$ is the mass of the state. To construct the SR we follow for the second part, usually called phenomenological part, the pioneering work of ref. [3] and the recent study of the scalar and pseudoscalar glueballs by Forkel [4]. Putting these pieces together, the corresponding SR for a zero width resonance model of the spectral density, ($\rho \sim \delta (s - M_G^2) + \text{continuum}$), has the following form:

$$\frac{1}{\pi} \int_0^{s_0} \frac{\text{Im} \Pi_{\text{OPE}}(-s)}{s + Q^2} \text{ds} = \frac{F_G^2 M_G^{2(N-2)}}{M_G^2 + Q^2},$$

where $\Pi_{\text{OPE}}(-s)$ is the OPE of the correlator, Eq. (1), and $s_0$ is the continuum threshold. It is known that $0^{-}$ state can not couple to a three-gluon interpolating current without derivatives [5]. In the paper [6] a very specific current with derivatives has been constructed to obtain the mass of three-gluon exotic glueball. However, in [6] it has been demonstrated that this current leads to the inconsistency of QCD SR. Here we propose a new gauge invariant current with derivatives which couples to the $0^{-}$ state. It has the general form:

$$J(x) = \frac{2}{3} g^3 \epsilon^{ijk} \text{Tr} \left( (O_i G_{\mu\nu}(x)) (O_j G_{\alpha\beta}(x)) (O_k G_{\rho\sigma}(x)) \right),$$

where $G_{\mu\nu}$ is the field strength tensor, $\tilde{G}_{\mu\nu}^a \equiv G_{\mu\alpha\beta}^a \epsilon_{\alpha\beta\delta}/2$, the operators $O_i$ are the products of covariant derivatives $O_i = D_{\alpha_1} \cdots D_{\alpha_n}$. The lowest dimensional current in this form, that has nonzero LO perturbative contribution to the SR corresponds to:

$$O_1 G_{\mu\nu}(x) = D_{\alpha_1} D_{\alpha_2} D_{\alpha_3} \tilde{G}_{\mu\nu}(x),$$
$$O_2 G_{\mu\nu}(x) = D_{\alpha_1} D_{\alpha_2} G_{\mu\nu}(x),$$
$$O_3 G_{\mu\nu}(x) = D_{\alpha_3} G_{\mu\nu}(x).$$

In general, one might construct another interpolating currents which couple to $0^{-}$ state and include four gluons [7], for example. However, the consideration of these states is beyond of the scope of our paper and will be
the subject of our future study. The coefficient in the current Eq. 4 was chosen to have the leading term in the following form:

\[ J(x) = \frac{e^3}{2} g^{abc} g_{\mu\nu|1\tau_2}(x) g_{\nu\rho|1\tau_2}(x) g^{\rho\mu;2}(x), \]

where \[ g_{\mu\nu|1\tau_2} = \partial_1 \partial_2 \cdots \partial_2 g_{\mu\nu}. \] Using this current, Eqs. [4, 10, 11], we have calculated the OPE of the correlator up to the dimension-8 operators and is given by

\[ \Pi_{(\text{OPE})}(Q^2) = \Pi_{(\text{pert})} + \Pi_{(G3)} + \Pi_{(G4)} + \cdots = \]

\[ \frac{-5\alpha_s^3}{114\pi} Q^{20} L + \frac{\gamma G^4}{3} Q^{14} \left( \frac{gG^3}{4} - \frac{J^2}{(5 + 2L)} \right) + \frac{205\pi^2\alpha_s^2}{8e^3} Q^{12} L (\alpha_s G^4) + \cdots, \]

where \( \alpha_s = \frac{g^2}{4\pi} \) is the coupling constant, \( L = \ln(Q^2/\mu^2) \), \( \mu^2 \) is the renormalization scale, the dimension-6 condensates are \( \langle G^3 \rangle = \langle g f^{abc} G_{\mu\nu} G_{\nu\rho} C_{\rho\mu} \rangle \) and \( \langle J^2 \rangle = \langle J_\mu J_\mu \rangle \) with the quark current \( J_\mu = \bar{q} \gamma^\mu t q \), and the dimension-8 condensate is

\[ \langle \alpha_s^2 G^4 \rangle = \langle \alpha_s g f^{abc} G_{\mu\nu} C_{\alpha\beta} G_{\rho\sigma} \rangle^2 - 2 \langle \alpha_s g f^{abc} G_{\mu\nu} C_{\rho\sigma} \rangle^2. \]

We adopt Mathematica package FEYNCALC [9] to handle the algebraic manipulation.

In contrast with the previous study [6] mentioned above, we have a positive LO imaginary part and, therefore, we expect a consistent SR. We would like to emphasize that the so-called direct instantons, which effect strongly the SRs for the \( 0^{++} \) and \( 0^{-+} \) two-gluon states [4, 11], do not contribute in this case due to the symmetric color structure of the current, Eq. [3].

Following the method developed in [3], we apply the Borel transformation \( B \)

\[ B_{Q^2 \to M^2}[\Pi(Q^2)] = \lim_{n \to \infty} \left( \frac{-Q^2}{\Gamma(n)} \right)^n \frac{d^n}{dQ^{2n}} \Pi(Q^2) \]

\[ Q^2 = n M^2 \]

to both sides of the SR, Eq. [2]. Using the Borel transformation allows to reduce the SR uncertainties by suppression of the contributions from excited resonances and higher order OPE terms. After the Borel transformation the new sum rule is

\[ \sum_{\ell} \mathcal{R}_\ell(M^2, s_0) = \mathcal{R}^{(\text{res})}_0(M^2, s_0), \]

where \( M^2 \) is the Borel parameter,

\[ \mathcal{R}^{(\text{res})}_0(M^2, s_0) = M_G^{10} F_G^2 e^{-M_G^2/M^2}, \]

and \( \Pi_\ell \) denotes the different contributions to OPE of the correlator: the perturbative term (pert), and the dimension-6 (G3) and dimension-8 (G4) nonperturbative terms. To extract the mass from the SR, we use a family of derivative SRs obtained by differentiation with respect to the Borel parameter \( M^2 \):

\[ \frac{\mathcal{R}^{(\text{res})}_0(M^2, s_0)}{\mathcal{R}^{(\text{res})}_0(M^2, s_0)} = M_G^4 \frac{d}{dM^2} \frac{\mathcal{R}^{(\text{res})}_0(M^2, s_0)}{\mathcal{R}^{(\text{res})}_0(M^2, s_0)}. \]

We define the difference of the OPE result and the continuum contribution as

\[ \mathcal{R}^{(\text{SR})}_k(M^2, s_0) = \mathcal{R}^{(\text{res})}_0(M^2, s_0) + \mathcal{R}^{(G1)}(M^2, s_0) + \mathcal{R}^{(G2)}(M^2, s_0). \]

Then the master sum rule \( (k = 0) \) and the derivative SRs \( (k > 0) \) can be expressed by the following equations:

\[ \mathcal{R}^{(\text{SR})}_k(M^2, s_0) \approx \mathcal{R}^{(\text{res})}_0(M^2, s_0). \]

The fiducial window \( M^2 \in [M_0^2, M_1^2] \) is limited by the conditions that insure the reliability of the resonance model and the OPE, i.e.,

\[ \frac{\mathcal{R}^{(G4)}_k(M^2, s_0)}{\mathcal{R}^{(\text{SR})}_k(M^2, s_0)} > \frac{1}{10}. \]

Then the QCD SRs for the mass and the decay constant can be presented in the form:

\[ M_G^2(M^2, s_0) = \frac{\mathcal{R}^{(\text{SR})}_k(M^2, s_0)}{\mathcal{R}^{(\text{SR})}_k(M^2, s_0)}, \]

\[ F_G^2(M^2, s_0) = \frac{e^{M_G^2/M^2}}{M_G^2 \mathcal{R}^{(\text{SR})}_k(M^2, s_0)}. \]

We define the mass and decay constant by minimization of the criteria \( \delta_k(s_0^0) = \delta_k^\min \) with respect to the threshold \( s_0 \) and find the best fit value \( s_0^b \):

\[ \delta_k(s_0) = \max |M_G^2(M_0^2, s_0) - M_G^2(s_0)|, \]

\[ M_G^2(s_0) = s_0^{b} + \frac{1}{n+1} \sum_{i=0}^{n} M_G^2(M_i^2, s_0), \]

where we consider \( n = 20 \) points in the fiducial interval \( M_0^2 = M_0^2 + (M_f^2 - M_0^2) n/n \). In Fig. 1 we present the \( k = 0 \) results for the glueball mass and decay constant as a function of the Borel parameter. As one can see, we have a rather good stability plateau for both quantities.

Finally, we define the decay constant and mass as an average in the fiducial interval for the best fit value of the threshold:

\[ M_G = M_G^b(s_0^b), \quad F_G^2 = \frac{1}{n+1} \sum_{i=1}^{n} \frac{e^{M_G^2/M_i^2}}{M_G^2 \mathcal{R}^{(\text{SR})}_k(M_i^2, s_0^b)}. \]
We next follow the common practice of the renormalization group improvement of the SR: in $\text{Im}\Pi_i(-s)$ all coupling constants are replaced by $\alpha_s \to \alpha_s(M^2)$. We use the strong coupling constant

$$\alpha_s(Q^2) = \frac{4\pi}{b_0 \ln(Q^2/\Lambda_{\text{QCD}}^2)},$$

with $b_0 = 11 - 2N_f/3$ and QCD scale $\Lambda_{\text{QCD}} = 300$ MeV. Since we are working in gluodynamics, we put the number of flavors $N_f = 0$ and eliminate the quark and quark-gluon condensate contributions. The dimension-6 three-gluon condensate ($gG^3$) doesn’t contribute here due to absence of the correspondent $\ln(Q^2)$ terms in the correlator, Eq. (6). For the dimension-8 gluon condensate the hypothesis of vacuum dominance yields the relation

$$\langle \alpha_s^2 G^4 \rangle = \frac{3}{24} \langle \alpha_s G^2 \rangle^2.$$

In our case, the mass of the exotic glueball is determined by the squared value of the gluon condensate $\langle \alpha_s G^2 \rangle = \langle \alpha_s G_{\mu\nu}^G G^{\mu\nu} \rangle$. Unfortunately, this value is not well known. Following the analyses carried out in refs. [12, 13], we take

$$\langle \alpha_s G^2 \rangle = 0.012 \pm 0.006 \text{ GeV}^2.$$

Implementing the QCD SR analysis described above we obtain for the prediction of the mass and the decay constant from the $k = 0$ SR (see Eqs. [11])

$$M_G = 6.3^{+1.2}_{-1.1} \text{ GeV}, \quad F_G = 67 \pm 6 \text{ keV}. \quad (11)$$

The mass and decay constant estimates for the higher values of $k = 1, 2, 3$ are in agreement, within the error bars, with $k = 0$ case considered. The SR analysis in full QCD (number of flavors $N_f = 3$ and nonzero quark condensate $\langle J^2 \rangle$) leads to a reduction of the glueball mass by 0.2 GeV. The mass of the exotic glueball in Eq. (11) is not far away from the recent unquenched lattice result $M_G = 5.166 \pm 1.0 \text{ GeV}$ [16] obtained with a rather large pion mass $m_\pi = 360$ MeV.

Here we would like to note that there are three sources of uncertainties in the above analysis for the mass and decay constant: i) the variation of the gluon condensate; ii) the stability of the SR triggering the Borel parameter $M^2$ dependence in terms of the criteria $\delta_{\text{res}}^{\text{min}}$; and iii) the roughly estimated SR uncertainty coming from the OPE truncation. The latter uncertainty for the decay constant comes from the definition of the fiducial interval, Eq. (9), in the standard assumption that the contribution from the missing terms is of the order of the last included nonperturbative term squared: $(1/3)^2 \sim 10\%$. The same error for the mass can be expected to be suppressed since the related errors for $\mathcal{R}_k^{(\text{SR})}$ and $\mathcal{R}_k^{(\text{res})}$ are correlated. The presumable underestimation of uncertainties related to the OPE truncation is unlikely due to conservative choice of the gluon condensate uncertainty. The considered three sources of uncertainty can be given in percentage of the final uncertainty for the mass and the decay constant

$$M_G = 6.3^{+1.2}_{-1.1} \pm 0.5\% \pm 0\% \text{ GeV},$$

$$F_G = 67^{+2\%}_{-3\%} \pm 0.6\% \pm 5\% \text{ keV},$$

where the first uncertainty is related to gluon condensate variation, the second is representing the stability of SR, and the third is OPE truncation uncertainty.

The best fit threshold value is $s^b_0 = 52.4^{+12.6\%}_{-16.2\%} \text{ GeV}^2$ when only the uncertainty of the gluon condensate is included. Note that the fiducial interval for the central value of the gluon condensate is $M^2 \in [3.7, 7.3] \text{ GeV}^2$.

The glueball width can be estimated in the QCD SR approach also using the broad resonance distribution. The good stability of the zero width resonance based SR, Eq. (2), shows that we can extract only the upper limit of the glueball width from the QCD SR. The simplest way to introduce the width is by using unit step functions [11]:

$$\text{Im}\Pi_i^{(\text{res})}(-s) = \frac{\pi(m^2)^{N-2} f^2}{2m\Gamma} \left( \Theta(s - m^2 + m\Gamma) - \Theta(s - m^2 - m\Gamma) \right).$$

Requiring that the stability of the broad resonance based SR is better than the stability of zero width resonance based SR,

$$\max \left\{ \mathcal{R}_k^{(\text{res})}(M^2_i, s_0) \right\} \leq \max \left\{ \mathcal{R}_k^{(\text{SR})}(M^2_i, s_0) \right\},$$

we obtain an upper limit for the glueball width, $\Gamma_G \leq 235$ MeV. The used stability test was chosen for the simplicity and transparency of the width estimation keeping the level of accuracy at the level of SR accuracy for mass and decay constant. In the new SR we vary only the width value while the values for condensate, mass and decay constant remain fixed. The Borel parameter value is varied in the interval $M^2_i \in [3.7, 7.3] \text{ GeV}^2$. This result indicates that the $0^{-+}$ glueball should be rather narrow. Therefore, it can be seen in the appropriate experiments.

By quantum numbers the exotic glueball could mix with the exotic $0^{-+}$ tetraquark. However, a recent study
with QCD SR for this tetraquark has obtained a small mass, $M_{\text{tetra}} = 1.66 \pm 0.14$ GeV. The large mass difference between the two states leads us to expect a very small mixing between them. Thus, we can consider the exotic $0^{--}$ glueball as a pure gluon state.

Summarizing, we have presented a QCD SR study for the exotic three-gluon glueball state with quantum numbers $J^{PC} = 0^{--}$ using a new interpolating current. We have analyzed the QCD SR consisting of contributions of operators up to dimension-8 and have obtained an estimation of the mass, the decay constant and an upper limit for the width of the exotic glueball. These results provide a clear guide for the search of this important state in the experiments.

After the paper was completed we were informed of the negative result of the search of the low mass exotic $0^{--}$ glueball by the Belle Collaboration [18].

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