Oblique Confinement and Phase Transitions in Chern-Simons Gauge Theories

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We investigate non-perturbative features of a planar Chern-Simons gauge theory modeling the long-distance physics of quantum Hall systems, including a finite gap $M$ for excitations. By formulating the model on a lattice we identify the relevant topological configurations and their interactions. For $M > M_{cr}$, the model exhibits an oblique confinement phase, which we identify with Laughlin’s incompressible quantum fluid. For $M < M_{cr}$, we obtain a phase transition to a Coulomb phase or a confinement phase, depending on the value of the electromagnetic coupling.

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Planar gauge fields play an important role in effective field theories describing the low-energy degrees of freedom of two-dimensional condensed matter systems [1]. When the discrete $P$ and $T$ symmetries are either explicitly or spontaneously broken, the dynamics of the gauge fields is usually governed by the topological Chern-Simons term. In particular, the theory with Lagrangian (we use units $c = 1$, $\hbar = 1$)

$$
\mathcal{L} = \frac{\kappa}{\pi} A_\mu \epsilon^{\mu \alpha \nu} \partial_\alpha B_\nu + \frac{\eta}{\pi} B_\mu \epsilon^{\mu \alpha \nu} \partial_\alpha B_\nu
$$

(1)

has been proposed [2] as the effective field theory describing the long distance physics of chiral incompressible fluids. Here, the current $j^\mu \equiv \frac{\kappa}{\pi} \epsilon^{\mu \alpha \nu} \partial_\alpha B_\nu$ describes matter fluctuations of charge $\kappa$ above the ground state. The first term in (1) is the standard electromagnetic coupling, while the second term describes the kinetic term for matter fluctuations. This can be written as a non-local Hopf interaction; the introduction of the effective pseudovector gauge field $B_\mu$ allows however to avoid non-local terms in the effective field theory [3]. For $\eta = \text{even integer}$, the effective field theory (1) describes the long-distance physics of chiral spin liquids [4]; in this case, the $P$ and $T$ symmetries are spontaneously broken. For $\eta = \text{odd integer}$, the same theory describes the long-distance physics of Laughlin’s incompressible quantum fluids, which are the matter ground states at the plateaus of the quantum Hall effect [5]. In this case the $P$ and $T$ symmetries are explicitly broken by the external magnetic field and $1/\eta$ plays the role of the filling fraction [3] as can be easily recognized by integrating out the matter degrees of freedom and computing the current induced by a constant, uniform electric field.

Up to now, only perturbative analyses of the model (1) are available. Vortices have been coupled to the system only as external sources, in order to classify their quantum numbers [4]. In this paper we do two things. First we enlarge the model (1) by adding to it the three possible terms of dimension $[\text{mass}^4]$ coupling the dual field strengths $F^\mu \equiv \epsilon^{\mu \alpha \nu} \partial_\alpha A_\nu$ and $f^\mu \equiv \epsilon^{\mu \alpha \nu} \partial_\alpha B_\nu$:

$$
\mathcal{L} = -\frac{1}{2e^2} F_\mu F^\mu + \frac{\kappa}{\pi} A_\mu \epsilon^{\mu \alpha \nu} \partial_\alpha B_\nu - \lambda F_\mu f^\mu - \frac{1}{2g^2} f_\mu f^\mu + \frac{\eta}{\pi} B_\mu \epsilon^{\mu \alpha \nu} \partial_\alpha B_\nu
$$

(2)

These can be interpreted as the next-to-leading terms appearing in a derivative expansion of a relativistic, gauge invariant effective action for the incompressible quantum fluids. They provide dynamics for the gauge fields $A_\mu$ and $B_\mu$, which become propagating degrees of freedom. We stress that this dynamics is not meant to reproduce exactly the dynamics of matter fluctuations about incompressible quantum fluids; rather it is meant to incorporate
one essential feature of this dynamics which is not described by (1), namely the existence of a finite gap $M$ for the excitations.

Secondly, we probe non-perturbative features of our model, by formulating the corresponding Euclidean action on a lattice. Indeed, the two Abelian gauge symmetries of (2) have to be considered as compact $U(1)$ symmetries. The compactness of the gauge groups leads to the existence of topological excitations [4], which can drive phase transitions [8].

Let us first describe the additional terms in (2). The $F_\mu F^\mu$ term is the standard Maxwell term of (2+1)-dimensional electrodynamics. The corresponding coupling constant $e^2$ has dimension [mass]: in modeling quantum Hall systems it can be viewed as the fundamental length scale set by the external magnetic field, namely $\alpha/\ell$, with $\alpha$ the fine structure constant and $\ell$ the magnetic length. The $f_\mu f^\mu$ term represents a current-current interaction term for the matter: it produces a mass $2\eta g^2/\pi$ for free matter fluctuations, the so called magnetophonons [9]. Finally, the $F_\mu f^\mu$ term can be viewed (upon an integration by parts) as an electromagnetic coupling of the matter vorticity $e^{\mu\nu\alpha} \partial_\alpha j_\nu$. In our model, the coupling constant $\lambda$ is fixed as follows. By integrating out the electromagnetic gauge field $A_\mu$, we obtain an effective theory for the matter degrees of freedom:

\[
\mathcal{L}_{\text{eff}}^B = -\frac{1}{2} \left( \frac{1}{g^2} - e^2 \lambda^2 \right) f_\mu f^\mu + \frac{e^2 \kappa^2}{2\pi^2} B_\mu \left( \delta^{\mu\nu} - \frac{\partial_\mu \partial_\nu}{\Delta} \right) B_\nu + \frac{\eta - \kappa \lambda e^2}{\pi} B_\mu e^{\mu\nu\alpha} \partial_\alpha B_\nu . \tag{3}
\]

For generic $\lambda$, this theory contains both a Higgs mass and a topological Chern-Simons mass. We shall fix $\lambda$ by the requirement that the induced Chern-Simons term cancels exactly the bare one, so that only the Higgs mass survives:

\[
\lambda = \frac{\eta}{\kappa e^2} . \tag{4}
\]

As we shall see below, it is exactly this requirement which allows a consistent formulation of the model (2) on the lattice.

When (4) is satisfied, (3) describes a parity doublet of excitations with spin $\pm 1$ and mass

\[
M = \frac{e g \kappa}{\sqrt{1 - \frac{\eta^2 g^2}{\kappa^2 e^2}}} . \tag{5}
\]

This mass represents the effective gap for matter excitations, when electromagnetic fluctuations are taken into account. The mass scale $g^2$ can be expressed in terms of the fundamental mass scale $e^2$ by introducing a dimensionless parameter. By choosing
\[ g^2 = \frac{\pi^2 x^2}{(1 + \pi^2 x^2)} |\kappa^2 e^2/\eta^2 | \] the gap becomes \( M = x\kappa^2 e^2/\eta \) and has the correct scaling with charge and filling fraction \( \rho \) to model the quantum Hall gap. Here \( x \) can be thought of as a phenomenological parameter encoding all microscopic effects which have an influence on the gap, like the range and strength of the inter-particle interaction and the amount of impurities. With this choice of \( g^2 \), the original model \( \Pi \) is recovered in the limit \( e^2 \to \infty \).

In the following we shall study the phase structure of our model. To this end we study the topological excitations due to the compactness of the two Abelian gauge symmetries of \( \Pi \). We introduce a cubic lattice with lattice spacing \( l \) and lattice sites denoted by \( x \), on which we define the following forward and backward lattice derivatives and shift operators:

\[
\begin{align*}
    d_\mu f(x) &\equiv \frac{f(x + \hat{\mu}l) - f(x)}{l}, & S_\mu f(x) &\equiv f(x + \hat{\mu}l), \\
    \hat{d}_\mu f(x) &\equiv \frac{f(x) - f(x - \hat{\mu}l)}{l}, & \hat{S}_\mu f(x) &\equiv f(x - \hat{\mu}l),
\end{align*}
\]

where \( \hat{\mu} \) denotes a unit vector in direction \( \mu \). To each link \( \{x, \mu\} \) of the lattice we assign two real gauge fields denoted by \( A_\mu(x) \) and \( B_\mu(x) \). These are compact variables defined on a circle of radius \( 2\pi/l \): \( -(\pi/l) < A_\mu, B_\mu < (\pi/l) \). This definition fixes the normalization of the gauge fields; consequently, no coupling constant in \( \Pi \) can be reabsorbed in a redefinition of \( A_\mu \) and \( B_\mu \).

In order to formulate an Euclidean version of \( \Pi \) on the lattice, we have to face the problem of defining a lattice version of the (Euclidean) Chern-Simons operator \( \epsilon_{\mu\alpha\nu} \partial_\alpha \), which can be viewed as the square root of the familiar Maxwell operator. It turns out that on a cubic lattice there is no gauge invariant local operator whose square reproduces the Maxwell operator. Rather, one can define the following two lattice operators \( \Pi \):

\[
\begin{align*}
    K_{\mu\nu} &\equiv S_\mu \epsilon_{\mu\alpha\nu} d_\alpha, & \hat{K}_{\mu\nu} &\equiv \epsilon_{\mu\alpha\nu} \hat{d}_\alpha \hat{S}_\nu,
\end{align*}
\]

where there is no summation over equal indices. These operators are gauge invariant,

\[
\begin{align*}
    K_{\mu\nu} d_\nu &\equiv \hat{d}_\mu K_{\mu\nu} = 0, & \hat{K}_{\mu\nu} d_\nu &\equiv \hat{\mu} K_{\mu\nu} = 0,
\end{align*}
\]

and their product reproduces the Maxwell operator,

\[
K_{\mu\alpha} \hat{K}_{\alpha\nu} = \hat{K}_{\mu\alpha} K_{\alpha\nu} = -\delta_{\mu\nu} \nabla^2 + d_\mu \hat{d}_\nu,
\]
where $\nabla^2 \equiv \hat{d}_\mu d_\mu$ is the three-dimensional, Euclidean Laplace operator on the lattice. Note also that the two operators $K_{\mu\nu}$ and $\hat{K}_{\mu\nu}$ are interchanged upon a summation by parts on the lattice.

In order to take into account the periodicity of the link variables $A_\mu$ and $B_\mu$ we introduce four sets of integer link variables and we posit the following Euclidean model of the Villain type \cite{8}:

$$Z = \sum_{\{n_\mu\},\{l_\mu\},\{m_\mu\},\{k_\mu\}} \int_0^{2\pi} DA_\mu DB_\mu \exp(-S),$$

$$S = \frac{l^3}{2e^2} \left( F_\mu + \frac{2\pi}{l^2} n_\mu \right)^2 - \frac{i l^3 \kappa}{2\pi} \left( A_\mu + \frac{2\pi}{l} l_\mu \right) K_{\mu\nu} \left( B_\nu + \frac{2\pi}{l} m_\nu \right) - \frac{i l^3 \eta}{\kappa e^2} \left( B_\mu + \frac{2\pi}{l} m_\mu \right) \left( K_{\mu\nu} + \hat{K}_{\mu\nu} \right) \left( B_\nu + \frac{2\pi}{l} m_\nu \right),$$

where we have introduced the notation $DA_\mu \equiv \prod_x dA_\mu(x)$ and we define the lattice dual field strengths as $F_\mu \equiv K_{\mu\nu} A_\nu$ and $f_\mu \equiv K_{\mu\nu} B_\nu$. Due to the property (9), the terms $\sum_{x,\mu} F_\mu^2$ and $\sum_{x,\mu} f_\mu^2$ reproduce the familiar lattice Maxwell action. The partition function (10) is clearly invariant under shifts $A_\mu \to A_\mu + 2\pi i_\mu/l$ and $B_\mu \to B_\mu + 2\pi j_\mu/l$ with integer $i_\mu$ and $j_\mu$, since these can be reabsorbed by a redefinition of the integer link variables $n_\mu$, $m_\mu$, $l_\mu$ and $k_\mu$.

Gauge invariance, instead requires the quantization of the parameters $\kappa$ and $\eta$. The boundary conditions are such that the dual field strengths $F_\mu$ and $f_\mu$ vanish modulo $2\pi/l^2$ at infinity. Consider now a gauge transformation $A_\mu \to A_\mu + d_\mu A$ which wraps non-trivially around one of the three directions, say $A(x) = 0$, $\Lambda(+\infty, x^2, x^3) = 2\pi n$ with $n$ an integer. Under such a gauge transformation the lattice action (10) changes by the surface term obtained by summing by parts the second term in (10):

$$\Delta S = \sum_{x^2,x^3} -i \kappa n \ t(+\infty, x^2, x^3),$$

where $t \equiv 2\pi l K_{1\nu} m_\nu$ is an integer multiple of $2\pi$. Gauge invariance requires that $\Delta S$ vanishes modulo $i2\pi$. This is realized when the coupling constant $\kappa$ satisfies an integer
quantization condition. Correspondingly, gauge invariance under topologically non-trivial
gauge transformations $B_\mu \rightarrow B_\mu + d_\mu \Lambda$ requires the integer quantization of the coupling
constant $\eta$: $k = p \in \mathbb{Z}$, $\eta = q \in \mathbb{Z}$. Thus, the quantization of the inverse filling fraction
$1/\eta$ is a consequence of the compactness of the effective gauge field $B_\mu$.

We now rewrite (10) in a fashion which exposes explicitly the topological configurations and their interactions. To this end we decompose $n_\mu$ and $k_\mu$ as
\[ n_\mu \equiv lK_\mu l_\nu + a_\nu, \quad k_\mu \equiv lK_\mu m_\nu + b_\nu, \]
with $a_\mu$ and $b_\mu$ integers. Correspondingly, the sum over all configurations \{\$n_\mu\$\} and \{\$k_\mu\$\} in (10) can be replaced by a sum over all configurations \{\$a_\mu\$\} and \{\$b_\mu\$\}. By
changing variables $A_\mu \rightarrow A_\mu + (2\pi/l)a_\mu$ and $B_\mu \rightarrow B_\mu + (2\pi/l)b_\mu$ in the integration and
performing the sum over all configurations \{\$l_\mu\$\} and \{\$m_\mu\$\} we obtain
\[
Z = \sum_{\{a_\mu\}} \sum_{\{b_\mu\}} \int_{-\infty}^{+\infty} \mathcal{D}A_\mu \mathcal{D}B_\mu \exp(-S),
\]
\[
S = \sum_{x,\mu} \frac{l^3}{2e^2} F^2_\mu - \frac{i^3l}{\pi} A_\mu K_{\mu\nu} B_\nu + \frac{l^3q}{e^2} F_\mu f_\mu + \frac{l^3}{2g^2} f^2_\mu - \frac{i^3q}{\pi} B_\mu K_{\mu\nu} B_\nu
\]
\[
+ \frac{2\pi^2}{le^2} a_\mu^2 + \frac{2\pi^2}{lg^2} b_\mu^2 + \frac{4\pi^2q}{lpe^2} a_\mu b_\mu
\]
\[
+ \frac{2\pi l}{e^2} A_\mu \hat{K}_{\mu\nu} \left( a_\nu + \frac{q}{p} b_\nu \right) + \frac{2\pi l}{e^2} B_\mu \hat{K}_{\mu\nu} \left( e^2 \frac{g^2}{b_\nu} + \frac{q}{p} a_\nu \right).
\]
Finally, we perform the Gaussian integrations over $A_\mu$ and $B_\mu$. Here the condition (4) is
 crucial: indeed, for other values of $\lambda$ the quadratic kernels in the Gaussian integrals would
not be invertible. The final result takes the form $Z = Z_0 \ Z_{Top}$, where $Z_0$ is the lattice
partition function for the non-compact, Euclidean version of (2) and $Z_{Top}$ is given by
\[
Z_{Top} = \sum_{\{a_\mu\}} \sum_{\{b_\mu\}} \exp \left[ -\frac{2\pi^2}{le^2} \left( a_\mu + \frac{q}{p} b_\mu \right) \frac{M^2 \delta_{\mu\nu} - d_\mu \hat{d}_{\nu}}{M^2 - \nabla^2} \left( a_\nu + \frac{q}{p} b_\nu \right) \right]
\]
\[
- \frac{2e^2p^2}{lM^2} b_\mu \frac{M^2 \delta_{\mu\nu} - d_\mu \hat{d}_{\nu} b_\nu - i^4\pi p}{l} b_\mu \frac{\hat{K}_{\mu\nu}}{M^2 - \nabla^2} \left( a_\nu + \frac{q}{p} b_\nu \right) \).
\]
The integer link variables $a_\mu$ and $b_\mu$ form the topological configurations of the model. They arise as the integer parts of the dual field strengths $F_\mu$ and $f_\mu$, respectively. Therefore
the corresponding topological excitations can be interpreted as magnetic flux strings and quasi-particle current strings. The quasi-particles represent localized, collective matter excitations and have to be distinguished from the original matter particles of the underlying
microscopic model. The strings can be closed (rings), in which case \( \hat{d}_\mu a_\mu = 0 \) and \( \hat{d}_\mu b_\mu = 0 \), or open, in which case the integers \( l\hat{d}_\mu a_\mu \) and \( l\hat{d}_\mu b_\mu \) represent the monopoles corresponding to the two compact Abelian gauge symmetries of (10). In our Euclidean formalism, these monopoles describe tunneling events corresponding to the formation (or destruction) of magnetic fluxes and quasi-particles.

Eq. (13) describes the interactions between these topological excitations. Note that magnetic flux strings appear only in the combination \([a_\mu + (q/p)b_\mu]\). Due to the mass gap \( M \), all interactions are short-range; it is therefore a good approximation to neglect the off-diagonal terms in the interaction kernels and to assign a free energy

\[
F = \left\{ \frac{2\pi^2}{le^2} \left( a + \frac{q}{p}b \right)^2 + \frac{2e^2p^2}{LM^2} b^2 - \mu \right\} N \tag{14}
\]

to a string of length \( L = lN \) carrying magnetic and quasi-particle quantum numbers \( a \) and \( b \). The last term in (14) represents the entropy of the string: the parameter \( \mu \) is given roughly by \( \mu = \ln 5 \), since at each step the string can choose between 5 different directions. In a dilute instanton approximation, in which all values \( a_\mu, b_\mu \geq 2 \) are neglected it can be proved that the correct value of \( \mu \) is the same for open and closed strings [13]. In (14) we have neglected all subdominant functions of \( N \), like a \( \ln N \) correction to the entropy and a constant term due to the monopole contribution to the energy for open strings. Moreover, we have neglected the imaginary term in the action (13). This is justified self-consistently, since the contribution of this term vanishes in all phases of the model, as we now show.

Long strings with quantum numbers \( a \) and \( b \) condense in the ground state if the coefficient of \( N \) in (14) is negative. When two or more condensates are possible, one has to choose the one with the lowest free energy. The condensation condition describes the interior of an ellipse with semi-axes \( le^2\mu/2\pi^2 \) and \( LM^2\mu/2e^2p^2 \) on a non-rectilinear lattice of magnetic and quasi-particle charges. For generic values of \( p \) and \( q \) the phase structure may be quite complex, displaying various oblique confinement phases [14]. Here we discuss the simpler case of matter of charge \( p=1 \). In this case we find the following phase structure:

\[
le^2\mu < 2\pi^2 \rightarrow \begin{cases}
\frac{M^2}{2e^2} = \frac{x^2}{2q^2} > \frac{1}{le^2\mu} , & \text{oblique confinement} ; \\
\frac{M^2}{2e^2} = \frac{x^2}{2q^2} < \frac{1}{le^2\mu} , & \text{Coulomb} ;
\end{cases}
\]

\[
le^2\mu > 2\pi^2 \rightarrow \begin{cases}
\frac{M^2}{2e^2} = \frac{x^2}{2q^2} > \frac{1}{2\pi^2} , & \text{oblique confinement} ; \\
\frac{M^2}{2e^2} = \frac{x^2}{2q^2} < \frac{1}{2\pi^2} , & \text{confinement} .
\end{cases}
\tag{15}
In the oblique confinement phase the ground state consists of a condensate of strings carrying magnetic flux $a = \pm q$ and quasi-particle number $b = \mp 1$. The confinement phase is characterized by a ground state consisting of a condensate of magnetic flux strings. In the Coulomb phase, instead, there is no condensation of topological excitations in the ground state.

The order parameter distinguishing the various phases is the Wilson loop expectation value in the corresponding ground states. We shall present the details of this computation elsewhere \[15\]. Here we report only the results. In the oblique confinement phase, the only free (non-confined) excitations carry magnetic and quasi-particle quantum numbers in the ratio $a/b = -q$. A magnetic flux must consequently carry fractional quasi-particle number $-1/q$. This composite object is therefore an anyon \[1\] excitation with fractional statistics. The free excitations in the confinement phase are magnetic fluxes: in this phase quasi-particles are confined. In the Coulomb phase, both magnetic fluxes and quasi-particle constitute free excitations.

It is known \[16\] that the microscopic Laughlin wave functions \[17\] for the incompressible quantum fluids describe a state in which an odd number of statistical fluxes are bound to the electrons. This fact is at the basis of Jain’s theory \[18\] of composite electrons and of most field theoretic treatments \[19\] of the quantum Hall effect. We are thus led to identify the oblique confinement phase of our model (for odd $q$) with the incompressible quantum fluid phase of quantum Hall systems. Note however, that our mechanism of condensation of composite objects is different from the usual one, in which fictitious, statistical flux is attached to the physical electrons. In our model the dual mechanism takes place: physical magnetic flux is attached to collective quasi-particle excitations and the so formed composite objects condense in the ground state. Moreover, in previous field theoretic formulations of the quantum Hall effect, flux is attached to the electrons by the introduction of a Chern-Simons term for the statistical gauge field. This is a kinematical mechanism, since it follows from the Chern-Simons Gauss law constraint. In our model, instead, the condensation of composite objects is a dynamical mechanism. Correspondingly we obtain the flux-unbinding phase transition to a Coulomb or to a confinement phase, depending on the value of the electromagnetic coupling $\lambda e^2 \mu$. This transition takes place when the parameter $x/q$ diminishes under a critical value (depending on $\lambda e^2 \mu$) i.e. when either the phenomenological gap parameter $x$ or the filling fraction $1/q$ become too small.
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