Predictions for $B_s \to \bar{K}^* \ell \ell$ in non-universal $Z'$ models

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The lepton flavor universality violating (LFUV) measurements $R_K$ and $R_{K^*}$ in $B$ meson decays can be accounted for in non-universal $Z'$ models. We constrain the couplings of these $Z'$ models by performing a global fit to correlated $b \to s\ell\ell$ and $b \to d\ell\ell$ processes, and calculate their possible implications for $B_s \to \bar{K}^* \ell\ell$ observables. For real new physics (NP) couplings, the 1-$\sigma$ favored parameters allow the corresponding LFUV ratio $R^{(s)}_{K^*}$ in $B_s \to \bar{K}^* \ell\ell$ to range between 0.8 – 1.2 at low $q^2$. Complex NP couplings improve the best fit only marginally, however they allow a significant enhancement of the branching ratio, while increasing the range of $R^{(s)}_{K^*}$ at low $q^2$ to 0.8 – 1.8. We find that NP could cause zero-crossing in the forward-backward asymmetry $A_{FB}$ to shift towards lower $q^2$ values, and enhancement in the magnitude of integrated $A_{FB}$. The $CP$ asymmetry $A_{CP}$ may be suppressed and even change sign. The simultaneous measurements of integrated $R^{(s)}_{K^*}$ and $A_{CP}$ values to 0.1 and 1% respectively, would help in constraining the effective NP Wilson coefficient $C_9$ in $b \to d\mu\mu$ interactions.

I. INTRODUCTION

In recent times, the most tenacious hints of physics beyond the standard model (SM) have been seen in the decays of $B$ mesons. In particular, there are several measurements in the decays involving the quark-level transition $b \to s\ell^+\ell^-$ ($\ell = e, \mu$) that deviate from the predictions of SM. These include the LFUV observables $R_K$ and $R_{K^*}$ [11,12] whose measurements disagree with the SM predictions at the level of $\sim 2.5\sigma$ [3,4]. This disagreement can be attributed to NP in $b \to se^+e^-$ and/or $b \to s\mu^+\mu^-$ [10,12]. There are also deviations from the SM expectations at the level of $\sim 4\sigma$ in other measurements involving only $b \to s\mu^+\mu^-$ transition, such as the branching ratio of

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$B_s \rightarrow \phi \mu^+ \mu^-$ [5] and angular observable $P_5'$ in $B \rightarrow K^* \mu^+ \mu^-$ decay [6–8]. Hence it is natural to try accounting for the discrepancies in all the above measurements by assuming new physics only in the muon sector.

These anomalies may be addressed in a model-independent way using the framework of effective field theory, where the effects of beyond-SM physics are incorporated by adding new operators to the SM effective Hamiltonian. Various groups have performed global fits to all available data in the $b \rightarrow s \ell^+ \ell^-$ sector in order to identify the Lorentz structure of possible new physics operators [9, 27–33]. Some of these new physics operators can be generated in $Z'$ [13–16] or leptoquark models [21–26]. It has been shown that several models with $Z'$, either light or heavy, can help account for the anomalies in $b \rightarrow s \mu^+ \mu^-$ sector [17–20].

Since the $Z'$ boson would in general couple to all generations, the imprints of such a $Z'$ would be seen in other flavor sectors as well. Therefore, it is worth extending this model to include other related decays. This will provide further insights into the NP flavor structure. In this work, we consider possible observable effects of $Z'$ models on decays induced by the quark-level transition $b \rightarrow d \mu^+ \mu^-$. 

The $b \rightarrow d \mu^+ \mu^-$ transition gives rise to inclusive semi-leptonic decays $\bar{B} \rightarrow X_d \mu^+ \mu^-$ as well as exclusive semi-leptonic decays such as $\bar{B} \rightarrow (\pi^0, \rho) \mu^+ \mu^-$, $B^+ \rightarrow \pi^+ \mu^+ \mu^-$, and $B_s \rightarrow \bar{K}^* \mu^+ \mu^-$. Till recently, the only observed decay mode among these was $B^+ \rightarrow \pi^+ \mu^+ \mu^-$ [34–35], however now LHCb has reported an evidence for the decay $B_s \rightarrow \bar{K}^* \mu^+ \mu^-$ with a measured branching ratio of $(2.9 \pm 1.1) \times 10^{-8}$ [36]. For other decays, we only have an upper bound on their branching ratios [37, 38].

A large number of $b \rightarrow d \mu^+ \mu^-$ decays, at the level of thousands or tens of thousands, would be observed after the LHC upgrade. For example, about 17000 $B^+ \rightarrow \pi^+ \mu^+ \mu^-$ events are expected to be observed after collection of the full 300 fb$^{-1}$ dataset. For $B_s \rightarrow \bar{K}^* \mu^+ \mu^-$ decays, the full angular analysis is expected to be possible after the LHCb Upgrade-II dataset, where around 4300 events could be observed [39]. This would enable the measurements of angular observables in $B_s \rightarrow \bar{K}^* \mu^+ \mu^-$ decays with a precision even better than the existing measurements of angular distributions in $B_d \rightarrow K^* \mu^+ \mu^-$ decay.

Currently, as there are not many measurements in the $b \rightarrow d$ sector, a model-independent analysis would not be very useful in constraining new physics. However, in the context of specific models (like $Z'$), some of the couplings can be constrained from the $b \rightarrow s$ sector and neutrino trident production. Therefore we choose this approach to constrain the effective couplings in the $b \rightarrow d \mu^+ \mu^-$ sector, and identify potential observables in the $B_s \rightarrow \bar{K}^* \mu^+ \mu^-$ decay where large
new physics effects are possible.

The paper is organized as follows. In section II we introduce the $Z'$ model considered in this work and indicate how it can be constrained by available measurements. We then describe our fit methodology in Section III. The fit results along with predictions of various $B_s \to \bar{K}^* \mu^+ \mu^-$ observables are presented in Section IV. We summarize in Section V.

II. THE $Z'$ MODEL AND SOURCES OF CONSTRAINTS

In the non-universal $Z'$ model that we consider, the $Z'$ boson is associated with a new $U(1)'$ symmetry. It couples to both left-handed and right-handed muons but not to leptons of other generations. It couples to both left-handed and right-handed quarks, however we assume its couplings to right-handed quarks to be flavor-diagonal, thereby avoiding contribution of new chirality flipped operators to flavor changing neutral current (FCNC) decays [66, 67]. The change in the Lagrangian density due to the addition of such a heavy $Z'$ boson is

$$\Delta L_{Z'} = J^\alpha Z'_\alpha,$$

where

$$J^\alpha \supset g^{\mu\mu}_L \bar{L} \gamma^\alpha P_L L + g^{\mu\mu}_R \bar{L} \gamma^\alpha P_R L + g^{bd}_L \bar{Q}_1 \gamma^\alpha P_L Q_3 + g^{bs}_L \bar{Q}_2 \gamma^\alpha P_L Q_3 + h.c..$$

The right-hand side in eq. (2) includes only the terms contributing to FCNC processes. Here $P_{L(R)} = (1 \mp \gamma_5)/2$, $Q_i$ is the $i^{th}$ generation of quark doublet, and $L = (\nu_\mu, \mu)^T$ is the second generation doublet. Further, $g^{\mu\mu}_{L(R)}$ are the left-handed (right-handed) couplings of the $Z'$ boson to muons, and $g^{bd}_L$ to quarks. One can integrate out the heavy $Z'$ and get the relevant terms in the effective four-fermion Hamiltonian as,

$$H_{eff}^{Z'} = \frac{1}{2M_{Z'}} J_\alpha J^\alpha \supset \frac{g^{bs}_L}{M_{Z'}} (\bar{s} \gamma^\alpha P_L b) \left[ \bar{\mu} \gamma^\alpha (g^{\mu\mu}_L P_L + g^{\mu\mu}_R P_R) \mu \right] + \frac{(g^{bs}_L)^2}{2M_{Z'}} (\bar{s} \gamma^\alpha P_L b) (\bar{s} \gamma^\alpha P_L b)$$

$$+ \frac{g^{bd}_L}{M_{Z'}} (\bar{d} \gamma^\alpha P_L b) \left[ \bar{\mu} \gamma^\alpha (g^{\mu\mu}_L P_L + g^{\mu\mu}_R P_R) \mu \right] + \frac{(g^{bd}_L)^2}{2M_{Z'}} (\bar{d} \gamma^\alpha P_L b) (\bar{d} \gamma^\alpha P_L b)$$

$$+ \frac{g^{\mu\mu}_L}{M_{Z'}} (\bar{\nu}_\mu \gamma^\alpha P_L \nu_\mu) \left[ \bar{\mu} \gamma^\alpha (g^{\mu\mu}_L P_L + g^{\mu\mu}_R P_R) \mu \right].$$

In eq. (3), the first (third) term corresponds to $b \to s(d) \mu^+ \mu^-$ transitions, the second (fourth) terms give rise to $B_s - \bar{B}_s$ ($B_d - \bar{B}_d$) mixing, whereas the fifth term contributes to the neutrino trident production $\nu_\mu N \to \nu_\mu N \mu^+ \mu^-$ ($N$ = nucleus). Consequently, the products $g^{bs}_L g^{\mu\mu}_{L,R}$ ($g^{bd}_L g^{\mu\mu}_{L,R}$) are constrained by the $b \to s(d) \mu^+ \mu^-$ data, and individual magnitudes $|g_{bs}|$ ($|g_{bd}|$) from the $B_s - \bar{B}_s$
(B_d – B̅_d) mixing. The neutrino trident production puts limits on the individual muon couplings \( g_{L,R}^{\mu} \). We now discuss constraints on the Z’ couplings arising from each of the above measurements.

1. \( b \rightarrow s (d)^{\pm} \mu^{-} \) decays

The effective Hamiltonian for \( b \rightarrow q \mu^{\pm} \mu^{-} \) transition in the SM is

\[
H_{\text{eff}}^{\text{SM}} = -\frac{4G_{F}}{\sqrt{2}\pi} V_{tb}^{\ast} V_{tb} \sum_{i=1}^{6} C_{i} O_{i} + C_{7b}^{bq} \frac{e}{16\pi^{2}} [\bar{q} \sigma_{\mu\nu} (m_{q} P_{L} + m_{b} P_{R}) b] F_{\mu\nu} + C_{8}^{bq} O_{8} + C_{9}^{bq,\text{SM}} \frac{\alpha_{\text{em}}}{4\pi} (\bar{q} \gamma_{\mu} P_{L} b) (\bar{\nu}_{\gamma_{\mu} \mu}) + C_{10}^{bq,\text{SM}} \frac{\alpha_{\text{em}}}{4\pi} (\bar{q} \gamma_{\mu} P_{L} b) (\bar{\nu}_{\gamma_{\mu} \gamma_{5} \mu}) \]

where \( G_{F} \) is the Fermi constant and \( V_{ij} \) are the Cabibbo-Kobayashi-Maskawa (CKM) matrix elements. The Wilson coefficients (WC) \( C_{i} \) of the four-fermi operators \( O_{i} \) encode the short-distance contributions to the Hamiltonian in the SM, where the scale-dependence is implicit, i.e. \( C_{i} \equiv C_{i}(\mu) \) and \( O_{i} \equiv O_{i}(\mu) \). The operators \( O_{i} \) (\( i = 1, \ldots, 6, 8 \)) contribute to these processes through the modifications \( C_{7,9}^{bq}(\mu, q^{2}) \), where \( q^{2} \) is the invariant mass-squared of the final state muon pair. We drop the superscript “eff” from here on for the sake of brevity. Addition of the new Z’ boson to the SM particle spectrum modifies the WCs as \( C_{9,10}^{bq} \rightarrow C_{9,10}^{bq,\text{SM}} + C_{9,10}^{bq,\text{NP}} \), where

\[
C_{9,10}^{bq,\text{NP}} = -\frac{\pi}{\sqrt{2}G_{\alpha} V_{tb} V_{tb}^{\ast}} \frac{g_{bq}^{L} (g_{L}^{\mu\mu} + g_{R}^{\mu\mu})}{M_{Z'}^{2}},
\]

\[
C_{10}^{bq,\text{NP}} = -\frac{\pi}{\sqrt{2}G_{\alpha} V_{tb} V_{tb}^{\ast}} \frac{g_{bq}^{L} (g_{L}^{\mu\mu} - g_{R}^{\mu\mu})}{M_{Z'}^{2}}.
\]

In the Z’ models, \( C_{9,10}^{bq,\text{NP}} \) are in general independent. Two of the one-parameter scenarios, \( C_{10}^{bq,\text{NP}} = 0 \) (popularly known as \( C_{9,10}^{bq,\text{NP}} < 0 \)) and \( C_{9}^{bq,\text{NP}} = -C_{10}^{bq,\text{NP}} \), can be realized by substituting \( g_{L}^{\mu\mu} = g_{R}^{\mu\mu} \) and \( g_{L}^{\mu\mu} = 0 \), respectively.

2. \( B_{s(d)} - \bar{B}_{s(d)} \) mixing

The dominant contribution to \( B_{q} - \bar{B}_{q} \) mixing within the SM comes from the virtual top quark in the box diagram. The Z’ boson contributes to \( B_{q} - \bar{B}_{q} \) mixing at the tree-level. The combined contribution to \( M_{12}^{B} \), the dispersive part of the box diagrams responsible for the mixing, is

\[
M_{12}^{B} = \frac{1}{3} M_{B_{q}} f_{B_{q}}^{2} \tilde{B}_{B_{q}} \left[ N C_{V,L}^{\text{SM}} + \left( \frac{g_{L}^{bq}}{2 M_{Z'}^{2}} \right)^{2} \right],
\]

where
where

\[ N = \frac{G_F^2 M_W^2}{4\pi^2} (V_{tb} V_{tq}^*)^2, \]

\[ C_{VLL}^{SM} = \eta_B \left[ 1 + \frac{9}{1-x_t} - \frac{6}{(1-x_t)^2} - \frac{6x_t^2 \ln x_t}{(1-x_t)^3} \right], \quad (7) \]

with \( x_t \equiv m_t^2/M_W^2 \). Here \( \eta_B = 0.84 \) is the short-distance QCD correction calculated at NNLO [40], \( f_B \) is the decay constant, and \( \tilde{B}_B \) is the bag factor. The mass difference \( \Delta M_q = 2|\Delta M_{12}^q| \) is

\[ \Delta M_q = \Delta M_q^{SM} \left[ 1 + \frac{(g_{bL}^{bs})^2}{2N C_{VLL}^{SM} M_{Z'}^2} \right], \quad (8) \]

while the relevant weak phase \( \phi_q \) is

\[ \phi_q = -2\beta_q = \arg(M_{12}^q). \quad (9) \]

3. Neutrino trident production

Within the \( Z' \) models, the modification of the cross section \( \sigma \) for neutrino trident production, \( \nu_\mu N \rightarrow \nu_\mu N \mu^+ \mu^- \) may be parameterized as [41]

\[ R_\nu = \frac{\sigma}{\sigma_{SM}} = \frac{1}{1 + (1 + 4s_W^2)} \left[ \left( 1 + \frac{v^2 g_{L}^{\mu\mu}(g_{L}^{\mu\mu} - g_{R}^{\mu\mu})}{M_{Z'}^2} \right)^2 + \left( 1 + 4s_W^2 + \frac{v^2 g_{L}^{\mu\mu}(g_{L}^{\mu\mu} + g_{R}^{\mu\mu})}{M_{Z'}^2} \right)^2 \right], \quad (10) \]

where \( v = 246 \text{ GeV} \) and \( s_W = \sin \theta_W \).

III. FIT METHODOLOGY

We now determine favored values of the new physics couplings \( g_{bL}^{bs}, g_{bL}^{bl}, g_{L}^{\mu\mu} \) and \( g_{R}^{\mu\mu} \). We nominally take the mass of the \( Z' \) boson to be \( M_{Z'} = 1 \text{ TeV} \). Note that since \( M_{Z'} \) only appears through the combination \( g^2/M_{Z'}^2 \), the constraints on couplings can be appropriately scaled with the actual value of \( M_{Z'} \).

In \( b \rightarrow s\mu^+\mu^- \) decays, we consider the following observables: (i) \( B_s \rightarrow \mu^+\mu^- \) branching ratio [42-44], (ii) the updated value of \( R_K \) by the LHCb collaboration [4], (iii) \( R_{K^*} \) measured by LHCb [8] and its new Belle measurements, reported at Moriond’19 [45] (for Belle results, we use measurements in the bins 0.045 GeV^2 < q^2 < 1.1 GeV^2, 1.1 GeV^2 < q^2 < 6.0 GeV^2, and 15.0 GeV^2 < q^2 < 19.0 GeV^2, for \( B^0 \) as well as \( B^+ \) decays), (iv) the differential branching ratios of \( B_d \rightarrow K^*\mu^+\mu^- \) [46,49], \( B^+ \rightarrow K^{*+}\mu^+\mu^- \), \( B_d \rightarrow K\mu^+\mu^- \), \( B^+ \rightarrow K^+\mu^+\mu^- \) [47,50], and
\(B \rightarrow X_s \mu^+ \mu^- [51]\) in several \(q^2\) bins, (v) various \(CP\)-conserving and \(CP\)-violating angular observables in \(B_d \rightarrow K^* \mu^+ \mu^- \) [7, 47, 49, 52, 53], (vi) the measurements of differential branching ratio and angular observables of \(B_s \rightarrow \phi \mu^+ \mu^- [5]\) in several \(q^2\) bins.

All the observables in the \(b \rightarrow s \mu^+ \mu^-\) sector put constraints on the combinations \(g_{bsL}^{\mu\mu}\) and \(g_{bsR}^{\mu\mu}\). For the fit related to \(b \rightarrow s \mu^+ \mu^-\), we closely follow the methodology of Ref. [27]. The \(\chi^2\) function for all the \(b \rightarrow s \mu^+ \mu^-\) observables listed above is calculated as

\[
\chi^2_{b \rightarrow s \mu^+ \mu^-}(C_i) = (O_{\text{th}}(C_i) - O_{\exp})^T C^{-1} (O_{\text{th}}(C_i) - O_{\exp}),
\]

where \(C_i = C_{bs, NP}^{9, 10}\). Here \(O_{\text{th}}(C_i)\) are the theoretical predictions of \(b \rightarrow s \mu^+ \mu^-\) observables calculated using \textit{flavio} [54], and \(O_{\exp}\) are the corresponding experimental measurements. The total covariance matrix \(C\) is obtained by adding the individual theoretical and experimental covariance matrices.

We now turn to \(B_q - \bar{B}_q\) mixing. Here we consider constraints from \(\Delta M_d, \Delta M_s\), and the two \(CP\)-violating phases. Using \(f_{B_d} \sqrt{\hat{B}_{B_d}} = (225 \pm 9)\) MeV [58], along with other input parameters from ref. [59], eq. [8] gives \(\Delta M_d^{\text{SM}} = (0.547 \pm 0.046)\) ps\(^{-1}\). With \(\Delta M_d^{\exp} = (0.5065 \pm 0.0019)\) ps\(^{-1}\) [57], the contribution of \(\Delta M_d\) to \(\chi^2\) is

\[
\chi^2_{\Delta M_d} = \left( \frac{\Delta M_d - \Delta M_d^{\exp,m}}{\sigma_{\Delta M_d}} \right)^2,
\]

where we denote the experimental mean value of an observable \(X\) by \(X^{\exp,m}\), and the uncertainty in the observable by \(\sigma_X\). In order to obtain \(\sigma_X\), we add the experimental and theoretical uncertainties in quadrature. Here, \(\sigma_{\Delta M_d}\) is dominated by the theoretical uncertainty.

In order to minimize the impact of theoretical uncertainties, we use \(\Delta M_s\) constraints through the ratio \(M_R = \Delta M_d/\Delta M_s\). In the SM,

\[
M_R^{\text{SM}} = \left| \frac{V_{td}}{V_{ts}} \right|^2 \frac{1}{\xi^2} \frac{M_{B_d}}{M_{B_s}},
\]

where \(\xi = \frac{f_{B_d}^2 \hat{B}_{B_d}}{f_{B_s}^2 \hat{B}_{B_s}}\). Using \(\xi = 1.2014^{+0.0065}_{-0.0072} [56]\) and \(\left| V_{td}/V_{ts} \right| = 0.2088^{+0.0016}_{-0.0030} [55]\), we obtain \(M_R^{\text{SM}} = 0.0297 \pm 0.0009\), where we have added the errors in quadrature. Wherever there are asymmetric errors, we take a conservative approach and use the larger of the errors on two sides. The value of \(M_R^{\exp} = 0.0285 \pm 0.0001 [57]\), so the contribution to \(\chi^2\) due to this ratio is

\[
\chi^2_{M_R} = \left( \frac{M_R - M_R^{\exp,m}}{\sigma_{M_R}} \right)^2.
\]

The observables \(\Delta M_d\) and \(M_R\) constrain \(|g_{bs}^{\mu\mu}|\) and \(|g_{bd}^{\mu\mu}|\).
The $CP$-violating constraints from $J/\psi\phi$ and $J/\psi K_S$ decays contribute to the $\chi^2$ as

$$\chi^2_{J/\psi\phi} = \left( \frac{S_{J/\psi\phi} - S_{J/\psi\phi}^{\exp,m}}{\sigma_{S_{J/\psi\phi}}} \right)^2, \quad \chi^2_{J/\psi K_S} = \left( \frac{S_{J/\psi K_S} - S_{J/\psi K_S}^{\exp,m}}{\sigma_{S_{J/\psi K_S}}} \right)^2,$$  \hspace{1cm} (15)

where $S_{J/\psi\phi} = -\text{Im}[M_{12}^d]/|M_{12}^d|$ and $S_{J/\psi K_S} = \text{Im}[M_{12}^d]/|M_{12}^d|$. Here we have taken the measurements to be $S_{J/\psi\phi}^{\exp} = 0.02 \pm 0.03$ and $S_{J/\psi K_S}^{\exp} = 0.69 \pm 0.02$ [59].

For the constraints from neutrino trident production, we use the quantity $R_\nu = \sigma/\sigma_{SM}$, whose theoretical expression is given in eq. (10). We have taken $R_\nu^{\exp} \equiv 0.82 \pm 0.28$ [60 75]. The contribution to the total $\chi^2$ is

$$\chi^2_{\text{trident}} = \left( \frac{R_\nu - R_\nu^{\exp,m}}{\sigma_{R_\nu}} \right)^2.$$  \hspace{1cm} (16)

This observable constraints $g_L^{\mu\mu}$ and $g_R^{\mu\mu}$.

The $b \to d\mu^+\mu^-$ decays are CKM-suppressed as compared to $b \to s\mu^+\mu^-$. In our analysis, we include constraints from the branching ratios of $B^+ \to \pi^+\mu^+\mu^-$ and $B_d \to \mu^+\mu^-$ decays. We do not include the measurements of observables in $B_s \to \bar{K}^*\mu^+\mu^-$ decay in our fit, since we are interested in obtaining predictions for these.

The theoretical expression for $\mathcal{B}(B^+ \to \pi^+\mu^+\mu^-)$ in the $Z'$ model can be obtained from Ref. [61], by adding the NP contribution as given in eq. (5). The contribution to $\chi^2$ from this decay is

$$\chi^2_{B^+\to\pi\mu\mu} = \left( \frac{\mathcal{B}(B^+ \to \pi\mu\mu) - \mathcal{B}(B^+ \to \pi\mu\mu)^{\exp,m}}{\sigma_{\mathcal{B}(B^+ \to \pi\mu\mu)}} \right)^2,$$  \hspace{1cm} (17)

where $\mathcal{B}(B^+ \to \pi\mu\mu)^{\exp} = (1.83 \pm 0.24) \times 10^{-8}$ [35]. Following Ref. [61], a theoretical error of 15% is included due to uncertainties in the $B \to \pi$ form factors [62].

The branching ratio of $B_d \to \mu^+\mu^-$ in our model is given by

$$\mathcal{B}(B_d \to \mu^+\mu^-) = \frac{G_F^2 c^2 |M_{B_d}|^2 f_{B_d}^2}{16\pi^3} |V_{ud}V_{tb}|^2 \sqrt{1 - 4(m_\mu^2/M_{B_d}^2)} \left| C_{10}^{bd,SM} + C_{10}^{bd,NP} \right|^2,$$  \hspace{1cm} (18)

and the contribution to $\chi^2$ is

$$\chi^2_{B_d\to\mu\mu} = \left( \frac{\mathcal{B}(B_d \to \mu\mu) - \mathcal{B}(B_d \to \mu\mu)^{\exp,m}}{\sigma_{\mathcal{B}(B_d \to \mu\mu)}} \right)^2.$$  \hspace{1cm} (19)

We have used $\mathcal{B}(B_d \to \mu^+\mu^-)^{\exp} = (3.9 \pm 1.6) \times 10^{-10}$ [63], $f_{B_d} = (190 \pm 1.3)\text{MeV}$ [50], and other inputs from [59].

Finally, combining all the above constraints, we obtain

$$\chi^2_{\text{total}} = \chi^2_{8\to\mu\mu} + \chi^2_{\Delta M_d} + \chi^2_{M_H} + \chi^2_{J/\psi\phi} + \chi^2_{J/\psi K_S} + \chi^2_{\text{trident}} + \chi^2_{B^+\to\pi\mu\mu} + \chi^2_{B_d\to\mu\mu}.$$  \hspace{1cm} (20)

In the next section, we present our fit results, along with predictions of several observables in $B_s \to \bar{K}^*\mu^+\mu^-$ decay.
FIG. 1: The \((g_{bd}^{L}, g_{L}^{\mu\mu})\) parameter space corresponding to a \(Z'\) model with \(g_{L}^{\mu\mu} = g_{R}^{\mu\mu}\) (i.e. \(C_{10}^{bd, NP} = 0\)), for \(M_{Z'} = 1\) TeV. The blue curve is the boundary of the 1\(\sigma\)-favored region due to constraints from measurements in \(b \to d\) sector and neutrino trident production. The pink shaded region represents the 1\(\sigma\)-favored parameter space after including additional constraints from \(b \to s\mu^+\mu^-\) data and \(B_s - \bar{B}_s\) mixing.

IV. FIT RESULTS AND PREDICTIONS

In a model-independent analysis, it is hard to obtain useful constraints on the new physics couplings for \(b \to d\ell\ell\) decays, since there are only a few measurements in this sector. However, within the context of a \(Z'\) model, one can obtain useful constraints using correlated \(b \to s\) and \(b \to d\) processes. This can be seen from Fig. 1, which depicts the allowed \((g_{bd}^{L}, g_{L}^{\mu\mu} = g_{R}^{\mu\mu})\) parameter space corresponding to a \(Z'\) model which generates the 1D scenario \(C_{10}^{bd, NP} = 0\). The elliptical region represents the 1\(\sigma\)-favored parameter space with constraints only from \(b \to d\) sector, i.e., branching ratios of \(B^{+} \to \pi^{+}\mu^{+}\mu^{-}\) and \(B_{d} \to \mu^{+}\mu^{-}\) decays, \(B_d - \bar{B}_d\) mixing, and neutrino trident production. The two shaded regions represent the 1\(\sigma\)-favored parameter space obtained by including additional constraints from all relevant measurements related to \(b \to s\mu^+\mu^-\) decays and \(B_s - \bar{B}_s\) mixing. It can be seen that the allowed range of NP couplings, in particular \(g_{L,R}^{\mu\mu}\), reduces considerably after including constraints from the \(b \to s\) sector. Therefore, it is worth studying implications of several measurements in the \(b \to s\) sector on the observables in \(b \to d\mu^+\mu^-\) decays.

Performing a fit to the relevant observables in \(b \to s\) and \(b \to d\) sectors, we determine the 1\(\sigma\)-favored parameter space of the couplings \(g_{bd}^{L}, g_{bs}^{L}, g_{L}^{\mu\mu}\) and \(g_{R}^{\mu\mu}\), considering \(g_{bs}^{L}\) and \(g_{bd}^{L}\) to be (i) real, (ii) complex. These can be used to find constraints on the NP Wilson coefficients \((C_{9}^{bd, NP}, C_{10}^{bd, NP})\), and to put limits on the allowed NP in the following observables in \(B_s \to K^{*}\mu^+\mu^-\) decay: differential branching ratio, the LFUV ratio \(R_{K}^{(s)}\), muon forward-backward asymmetry \(A_{FB}\), longitudinal polarization fraction \(F_{L}\), and direct \(CP\) asymmetry \(A_{CP}\).
The matrix element for the decay amplitude of $B_s \to \bar{K}^* \mu^+ \mu^-$ can be written as

\[ \mathcal{M} = \frac{G_F \alpha}{\sqrt{2}} V_{tb} V^*_{td} \left\{ C_9^{bd} \langle \bar{K}^* | \bar{q} \gamma^\mu P_L b | B_s \rangle - \frac{2m_b}{q^2} C_7^{bd} \langle \bar{K}^* | \bar{q} i \sigma_{\mu\nu} q_V P_R b | B_s \rangle \right\} (\bar{\mu} \gamma_\mu \mu) 
+ C_{10}^{bd} \langle \bar{K}^* | \bar{q} \gamma^\mu P_L b | B_s \rangle (\bar{\mu} \gamma_\mu \gamma_5 \mu) \right\}, \]  

(21)

where $C_9^{bd,SM}$ and $C_{10}^{bd,SM}$ are taken from Ref. [70] and Ref. [69] respectively. The matrix elements appearing in eq. (21) have been calculated using form factors obtained by a combined fit to lattice calculations and QCD sum rules on the light cone (LCSR) [68]. We also include the non-factorizable corrections due to soft gluon emission and charmonium resonance, which have been computed for $B_d \to K^{*} \ell \ell$ [71, 72], and parameterized as corrections to $C_9^{SM}$. These effects are assumed to be roughly the same for $B_s \to \bar{K}^* \ell \ell$ due to flavor symmetry [65].

The decay $B_s \to \bar{K}^* \mu^+ \mu^-$ may be described in terms of the four-fold distribution as [65]

\[ \frac{d\Gamma}{dq^2} = \int_{-1}^{1} d \cos \theta_t d \cos \theta_V \int_{0}^{\pi} d\phi \frac{d^4\Gamma}{dq^2 d \cos \theta_V d \cos \theta_t d\phi} = \frac{1}{4} (3I_1^s + 6I_1^s - I_2^s - 2I_2^s), \]  

(22)

where $q^2$ is the lepton invariant mass, $\theta_V$ and $\theta_t$ are the polar angles, and $\phi$ is the angle between the dimuon plane and $K^*$ decay plane. The relevant observables can be obtained from the four-fold distribution as

\[ \frac{dB}{dq^2} = \tau_{B_s} \frac{d\Gamma}{dq^2}, \quad R_{K^*}^{(s)}(q^2) = \frac{d\Gamma(B_s \to \bar{K}^* \mu^+ \mu^-)/dq^2}{d\Gamma(B_s \to K^{*+} e^+ e^-)/dq^2}, \]

\[ A_{FB}(q^2) = \frac{1}{d\Gamma/dq^2} \left[ \int_{-1}^{1} - \int_{0}^{1} \right] d \cos \theta_t \frac{d^4\Gamma}{dq^2 d \cos \theta_t} = \frac{-3I_6^s}{3I_1^s + 6I_1^s - I_2^s - 2I_2^s}, \]

\[ F_L(q^2) = \frac{3I_1^s - I_2^s}{3I_1^s + 6I_1^s - I_2^s - 2I_2^s}, \quad A_{CP}(q^2) = \frac{dB/dq^2 - d\bar{B}/dq^2}{dB/dq^2 + d\bar{B}/dq^2}, \]  

(23)

where the functions $I_i$ can be expressed in terms of the transversity amplitudes [64]. Here $\bar{B}$ corresponds to the decay mode $B_s \to K^{*+} e^+ e^-$.  

We present our results for the above observables at four benchmark NP scenarios as given in Table 1

| Scenario | NP1     | NP2     | NP3     | NP4     |
|----------|---------|---------|---------|---------|
| $C_9^{bd,NP}$ | +0.98   | -0.80   | (-1.4 + 4.9 i) | (-0.6 + 0.8 i) |
| $C_{10}^{bd,NP}$ | -0.17   | +0.19   | (+0.7 - 2.3 i) | (+0.2 - 0.2 i) |

TABLE I: Values of NP Wilson coefficients for benchmark scenarios NP1, NP2 corresponding to a $Z'$ model with real couplings, and scenarios NP3, NP4 with complex couplings.
A. Real couplings

In order to quantify how well the $Z'$ model is able to account for all data in the $b \to s$ and $b \to d$ sectors, we define $\Delta \chi^2 = \chi^2_{\text{SM}} - \chi^2_{\text{NP}}$, where the minimum $\chi^2$ in the SM, and in the presence of NP $Z'$ couplings, is denoted by $\chi^2_{\text{SM}}$ and $\chi^2_{\text{NP}}$, respectively. For the case of real couplings, we find the best fit values to be $g_{bd}^{L} = \pm 0.3 \times 10^{-3}$, $g_{\mu\mu}^{L} = \mp 0.4$, and $g_{\mu\mu}^{R} = \mp 0.2$. The value of $\chi^2_{\text{SM}} \approx 221$ and $\Delta \chi^2 \approx 41$, so that the SM point corresponding to $g_{\mu\mu}^{L} = g_{\mu\mu}^{R} = g_{bd}^{L} = 0$ is highly disfavoured.

The $1\sigma$-favored parameter space of the couplings $(g_{bd}^{L}, g_{\mu\mu}^{L}, g_{\mu\mu}^{R})$ is shown in Fig. 2. It can be seen from $(g_{bd}^{L}, g_{\mu\mu}^{L})$ and $(g_{bd}^{L}, g_{\mu\mu}^{R})$ planes that, while $g_{\mu\mu}^{L} = 0$ is barely disfavored within $1\sigma$, a rather wide strip $|g_{L}^{\mu\mu}| \leq 0.25$ lies beyond the $1\sigma$-favored region. This is because the anomalies in
**FIG. 3**: The predictions for observables in $B_s \to \bar{K}^* \mu^+\mu^-$ decay, with real $Z'$ couplings, for SM and the benchmark scenarios NP1 and NP2 as in Table I.

$b \to s\mu\mu$ decays need a non-zero value of $C_{bs}^{9,\text{NP}}$, which in turn require a non-zero value of $g_L^{\mu\mu}$ or $g_R^{\mu\mu}$. Furthermore, the scenario $C_{9}^{\text{obs, NP}} = -C_{10}^{\text{obs, NP}}$ [27], which provides a good fit, favors $g_R^{\mu\mu} = 0$, thus requiring $g_L^{\mu\mu}$ to be away from zero. Note that the results in the $(g_L^{\mu\mu}, g_R^{\mu\mu})$ plane indicate the class of favored solutions that lie along $g_L^{\mu\mu} = g_R^{\mu\mu}$, corresponding to $C_{10}^{\text{obs, NP}} \approx 0 \approx C_{10}^{\text{obs, NP}}$.

1. **Predictions for $dB/dq^2$, $R_{K^*}(s)(q^2)$, $A_{FB}(q^2)$ and $F_L(q^2)$**

The top left panel of Fig. 3 shows predictions for the differential branching ratio corresponding to real $Z'$ couplings, for the SM as well as two benchmark scenarios NP1 and NP2 from Table I. These scenarios roughly correspond to the maximum deviation on either side from the SM predictions in the 1σ favored NP parameter space. The maximum enhancement (suppression) in the differential branching ratio corresponds roughly to a maximum positive (negative) value of $C_{9,10}^{\text{obs, NP}}$. It can be seen from the figure that only a marginal enhancement or suppression over the SM value is possible in the differential branching ratio. A clean distinction among the predictions of different scenarios is difficult owing to the large uncertainties (about 20%) arising from the form-factors.
A measurement of the LFUV ratio $R^{(s)}_{K^*}(q^2)$ in a few $q^2$ bins would be possible with the LHCb upgrade-II data set \[39\]. The predictions for this quantity in the benchmark scenarios NP1 and NP2 are shown in the top right panel of Fig. 3. In the SM, $R^{(s)}_{K^*}(q^2)$ is unity in the entire low-$q^2$ region, while an enhancement up to 1.3 and a suppression up to 0.8 is allowed. The maximum enhancement (suppression) roughly corresponds to the maximum positive (negative) value of $C^{bd,NP}_{9}$.

Within the SM, the forward-backward asymmetry $A_{FB}(q^2)$ is predicted to vanish around $q^2 \approx 3.5$ GeV$^2$, and the zero-crossing is from negative to positive, as can be seen from the bottom left panel in Fig. 3. The maximum value of $A_{FB}(q^2)$ in the SM is $\approx 10 \%$. The positive (negative) value of $C^{bd,NP}_{9}$ also shifts the zero-crossing towards lower (higher) $q^2$ value. The integrated value of $A_{FB}$ over $q^2 = (1 - 6)$ GeV$^2$ bin is $(-0.6 \pm 1) \%$ within the SM. The predictions for integrated $A_{FB}$ for the benchmark scenarios NP1 and NP2 are $(3.1 \pm 1.4)\%$ and $(-5 \pm 1.7)\%$, respectively.

The predictions for longitudinal polarization fraction $F_L(q^2)$ are shown in the bottom right panel of Fig. 3. Within the SM, the peak value of $F_L(q^2)$ is $\approx 0.9$ around $q^2 \approx 1.8$ GeV$^2$. The shape of $F_L(q^2)$ does not change with NP and only a marginal deviation from SM is allowed for the benchmark NP scenarios considered here.

Thus, in the case of real couplings, $R^{(s)}_{K^*}(q^2)$ is useful to distinguish the predictions of the two benchmark NP scenarios from the SM expectation, while the predictions for the differential branching ratio, $A_{FB}(q^2)$, and $F_L(q^2)$ may not have distinct NP signatures, owing to the large form factor uncertainties.

2. Integrated branching ratio and $R^{(s)}_{K^*}$ in the low-$q^2$ region

The results obtained for integrated $dB/dq^2$ and $R^{(s)}_{K^*}(q^2)$ over the $q^2 = (1 - 6)$ GeV$^2$ bin are presented in Fig. 4. These results are depicted in the $(C^{bd,NP}_9, C^{bd,NP}_{10})$ plane, with different colors and symbols indicating the values of integrated branching ratio (left panel) and integrated $R^{(s)}_{K^*}$ (right panel). At each 1$\sigma$-favored value of $(C^{bd,NP}_9, C^{bd,NP}_{10})$, we vary the values of form factor parameters within their 1$\sigma$ range \[68\] with a gaussian distribution of uncertainties.

In the case of integrated branching ratio, the errors due to form factors are about 20%. Due to such large errors, even by considering branching ratio values as different as $(5 - 6) \times 10^{-9}$ and $(12 - 14) \times 10^{-9}$, we find a significant overlap in the $(C^{bd,NP}_9, C^{bd,NP}_{10})$ plane. Hence, a measurement of integrated branching ratio may not be very helpful to put limits on the allowed values of the NP couplings.

In the case of $R^{(s)}_{K^*}$, the uncertainties due to form factors cancel in the ratio. The lack of
overlap between the regions of integrated \( R_{K^*}^{(s)} \) values in the range (0.7–1.4) indicates that a future measurement of integrated \( R_{K^*}^{(s)} \) with an accuracy of \( \sim 10\% \) in this decay mode would make it possible to identify the ranges of \( (C_{9}^{bd,NP}, C_{10}^{bd,NP}) \) more precisely. Even with a preliminary measurement, an enhancement in the value of \( R_{K^*}^{(s)} \) above unity would indicate a positive value of \( C_{9}^{bd,NP} \) and a negative value of \( C_{10}^{bd,NP} \), while a suppression would imply a negative \( C_{9}^{bd,NP} \) and positive \( C_{10}^{bd,NP} \). This feature may be understood from the approximate analytic form of the LFUV ratio, \( R_{K^*}^{(s)} \propto (\text{Re}[C_{9}^{bd,NP}] - \text{Re}[C_{10}^{bd,NP}]) \) [73].

\[
\begin{align*}
\text{B. Complex couplings} \\
\text{We would now like to see how the predictions for the above observables in the } B_s \rightarrow K^* \mu^+ \mu^- \text{ decay would change if the couplings } g_{L}^{bd} \text{ and } g_{R}^{bd} \text{ are allowed to be complex. Note that since the leptonic current in eq. (3) is self-conjugate, } g_{L}^{\mu\mu} \text{ and } g_{R}^{\mu\mu} \text{ must be real. We also study the impact of these complex couplings on the direct } CP \text{ asymmetry in this decay.}
\end{align*}
\]

Fig. 5 shows the 1\( \sigma \)-favored regions of the couplings \( \text{Re}[g_{L}^{bd}], g_{L}^{\mu\mu} \), and \( g_{R}^{\mu\mu} \). The minimum \( \chi^2 \) in the presence of the complex NP couplings is \( \chi^2_{\text{NP}} \approx 178 \), so that \( \Delta \chi^2 \approx 43 \), thereby providing a slightly better fit as compared to the case of real couplings (\( \Delta \chi^2 = 41 \)). The corresponding best fit values are \( \text{Re}[g_{L}^{bd}] = \pm 2.7 \times 10^{-3}, \text{Im}[g_{L}^{bd}] = \mp 3.8 \times 10^{-3}, g_{L}^{\mu\mu} = \mp 1 \), and \( g_{R}^{\mu\mu} = \mp 0.255 \). As \( \chi^2_{\text{NP}} \) for complex couplings is lower compared to that for real couplings, the 1\( \sigma \)-favored parameter space shifts further away from the SM point, which has \( \text{Re}[g_{L}^{bd}] = 0 \). A larger parameter space is allowed for the muon couplings compared to the real case, \( |g_{L}^{\mu\mu}| \leq 2.5 \text{ and } |g_{R}^{\mu\mu}| \leq 1.4 \). The 1\( \sigma \) favored region encompasses \( g_{L}^{\mu\mu} = 0 \), whereas a rather large region around \( g_{L}^{\mu\mu} = 0 \) (i.e. \( |g_{L}^{\mu\mu}| \leq 0.4 \)) is disfavoured within 1\( \sigma \). The allowed range of \( \text{Im}[g_{L}^{bd}] \) is qualitatively similar to that of \( \text{Re}[g_{L}^{bd}] \). Note
FIG. 5: The 1σ-favored (Re$[g^L_{b\mu}]$, $g^\mu\mu_L$, $g^\mu\mu_R$) parameter space for a $Z'$ model with complex couplings, for $M_{Z'} = 1$ TeV. The colors red to blue in the bottom left 3D parameter space correspond to decreasing values of $g^\mu\mu_R$.

that the complex nature of $g^L_{b\mu}$ is constrained only from $B_d - \bar{B}_d$ mixing measurements, since no $CP$-violating measurements are currently available in the $b \to d\mu\mu$ sector.

1. Predictions for $dB/dq^2$, $R^{(s)}_{K^+}(q^2)$, $A_{FB}(q^2)$ and $F_L(q^2)$

The predictions for differential branching ratio and $R^{(s)}_{K^+}(q^2)$ for the $Z'$ model with complex couplings are shown in the top panel of Fig. 6, for SM as well as the benchmark scenarios NP3 and NP4 in Table I. These scenarios are the 1σ-favored ones with a maximum value of Im[$C^L_{b\mu,NP}$] and a minimum value of Re[$C^L_{b\mu,NP}$], respectively, and are observed to provide close to maximal allowed deviation from the SM predictions. A significant enhancement in the branching ratio is possible
in NP3, which could be useful in identifying deviations from the SM. A large enhancement is also possible in the LFUV ratio $R_{K^*}^{(s)}(q^2)$ in the NP3 scenario, with the maximum value of $R_{K^*}^{(s)} = 1.8$ at $q^2 = 6 \text{ GeV}^2$. While the scenario NP4 cannot be distinguished from the SM using only the branching ratio, the value of $R_{K^*}^{(s)}(q^2)$ in this scenario can be as low as 0.85. Therefore, $R_{K^*}^{(s)}(q^2)$ would be useful to identify deviations from the SM.

A marginal enhancement in $A_{FB}(q^2)$ is possible for the scenario NP3, which would also display zero-crossing at much lower $q^2$ values ($q^2 \approx 2.5 \text{ GeV}^2$) compared to that in the SM ($q^2 \approx 3.5 \text{ GeV}^2$). A marginal suppression in $F_L(q^2)$ is also possible in NP3. The scenario NP4, on the other hand, does not show significant deviations from the SM for these two observables.

2. Direct CP asymmetry $A_{CP}(q^2)$

The direct CP asymmetry in the $b \to d \mu^+ \mu^-$ sector is expected to be about an order of magnitude larger than $b \to s \mu^+ \mu^-$. As direct CP violation in $b \to s \mu^+ \mu^-$ sector is expected to be $\sim 0.1\%$, its experimental observation would be possible only if some new physics provides...
an order of magnitude enhancement to bring it up to the level of a few percent. In \( b \to d \mu^+ \mu^- \) decays, the \( A_{CP} \) in SM itself is at the level of a few per cent, and can be within experimental reach.

Fig. 7 shows \( A_{CP}(q^2) \) in the low-\( q^2 \) region for the decay \( B_s \to K^* \mu^+ \mu^- \), considering the benchmark scenarios NP1 and NP2 (real couplings), as well as NP3 and NP4 (complex couplings). It can be seen from the left panel of the figure that for real couplings, \( A_{CP}(q^2) \) is either marginally below the SM prediction or almost consistent with it. For complex couplings, however the suppression in \( A_{CP}(q^2) \) can be quite large. It can even lead to \( A_{CP}(q^2) \) falling below a per cent level, hence making its measurement extremely difficult. In some scenarios (e.g. NP3), it is even possible for \( A_{CP}(q^2) \) to be negative for very low \( q^2 \) values. After scanning over the 1\( \sigma \)-favored parameter space, we find no significant enhancement in \( A_{CP}(q^2) \). So an NP signal can be established if the measurements put an upper bound which is firmly below the SM prediction of \( A_{CP}(q^2) \).

3. Integrated \( R^{(s)}_{K^*} \) and \( A_{CP} \)

As observed in the case of real couplings, the integrated branching ratio does not help much in narrowing down the range of effective NP Wilson coefficients. Hence, in this section, we focus on the integrated values of \( R^{(s)}_{K^*} \) and \( A_{CP} \) over \( q^2 = (1-6) \) GeV\(^2 \) bin. Fig. 8 depicts these results in the (\( \text{Re}[C_9^{bd, NP}], \text{Im}[C_9^{bd, NP}] \)) plane, with different colors and symbols indicating the values of integrated \( R^{(s)}_{K^*} \) (left panel) and \( A_{CP} \) (right panel). At each 1\( \sigma \)-favored complex value of \( (C_9^{bd, NP}, C_{10}^{bd, NP}) \), we vary the values of form factor parameters within their 1\( \sigma \) range \(^{[68]} \) with a gaussian distribution of uncertainties.
FIG. 8: The integrated values of the LFUV ratio $R_K^{(s)}$ (left panel) and the direct $CP$ asymmetry $A_{CP}$ (right panel) in the 1σ-favored parameter space of ($Re[C_g^{bd,NP}]$, $Im[C_g^{bd,NP}]$) for the $Z'$ model with complex couplings.

As in the case of real NP couplings, integrated $R_K^{(s)}$, below the SM prediction of unity could indicate a negative value of $Re[C_g^{bd,NP}]$. An enhancement in integrated $R_K^{(s)}$ upto (1.2 – 1.6) is possible for large positive or negative values of $Im[C_g^{bd,NP}]$. These features may be understood from the observation that in the case of complex couplings, $R_K^{(s)}$ has contributions both from $Re[C_g^{bd,NP}]$ and $|C_g^{bd,NP}|^2$.

The right panel of Fig. 8 shows that a large positive value of $Im[C_g^{bd,NP}]$ can decrease the integrated $A_{CP}$ to less than a per cent. The negative values of $Im[C_g^{bd,NP}]$ do not seem to affect $A_{CP}$ much, keeping it close to the SM prediction of 2.5%. Therefore, a simultaneous measurement of integrated $R_K^{(s)}$ and $A_{CP}$, with a precision of 0.1 and 1%, respectively, may help identify the sign of $Im[C_g^{bd,NP}]$. We find that the measurements of integrated $R_K^{(s)}$ and $A_{CP}$ values are not very useful in identifying the allowed ranges of $Re[C_{10}^{bd,NP}]$ and $Im[C_{10}^{bd,NP}]$.

V. SUMMARY AND CONCLUSIONS

In non-universal $Z'$ models, instrumental in accounting for the flavor anomalies, the observables in $b \to s\ell\ell$ and $b \to d\ell\ell$ processes would be correlated. In this paper, we study the constraints on the couplings of a non-universal $Z'$ model from the measurements in $b \to q\mu\mu$ ($q = s,d$) decays, $B_q - \bar{B}_q$ mixing, and neutrino trident production. These couplings give rise to new additional contributions to the Wilson coefficients $C_g^{bd}$ and $C_{10}^{bd}$. Using the above constraints, we perform a global fit to determine 1σ-favored regions in the parameter space of the $Z'$ couplings $g_L^{bd}$, $g_L^{bs}$, $g_L^{\mu\mu}$, and $g_R^{\mu\mu}$. We analyze the cases when quark-$Z'$ couplings $g_L^{bd}$ and $g_L^{bs}$ are (i) real, and (ii) complex. We also present our predictions for some important observables in $B_s \to \bar{K}^* \mu\mu$ decays — the
differential branching ratio $d\mathcal{B}/dq^2$, the LFUV ratio $R_{K^*}^{(s)}$, the angular observables $A_{FB}$ and $F_L$, and the $CP$ asymmetry $A_{CP}$ — for some benchmark scenarios.

It is observed from our analyses that the $Z'$ model improves the global fit over the SM by $\Delta \chi^2 \approx 41$ (real couplings) and $\Delta \chi^2 \approx 43$ (complex couplings). The favored regions in the parameter space lie along $g_{L}^{\mu\mu} \approx g_{R}^{\mu\mu}$, corresponding to $C_{9}^{bd,NP} \approx 0$, while the region around $g_{L}^{\mu\mu} = 0$ is disfavored. These are mainly dictated by the $R_K$ and $R_{K^*}$ anomalies in $b \to s$ sector.

For the observables in $B_s \to \bar{K}^* \mu^+\mu^-$ decays, when the couplings are real, we find that the enhancement and suppression in $d\mathcal{B}/dq^2$ cannot be cleanly identified due to the large uncertainties in the SM prediction. However, the value of $R_{K}^{(s)}(q^2)$ can substantially deviate from the SM prediction of unity — it can range from 0.8 to 1.3. The enhancement (suppression) corresponds to positive (negative) values of $C_{9}^{bd,NP}$. A marginal enhancement and suppression in $A_{FB}(q^2)$ is possible compared to the SM predictions, with the zero-crossing shifting towards lower (higher) $q^2$ values for positive (negative) values of $C_{9}^{bd,NP}$. There is no significant deviation from SM in the predictions of $F_L(q^2)$, and the predictions of $A_{CP}(q^2)$ also stay close to the SM expectation for all the favored values of NP Wilson coefficients. Further, we find that a measurement of integrated $R_{K}^{(s)}$ in the low-$q^2$ bin with a precision of $\sim 0.1$ can help narrow down the ranges of $(C_{9}^{bd,NP}, C_{10}^{bd,NP})$.

In the case of complex couplings, a larger NP parameter space is allowed, leading to larger possible deviations in the $B_s \to \bar{K}^* \mu^+\mu^-$ observables. In particular, a $\sim 50\%$ enhancement in $d\mathcal{B}/dq^2$ is allowed. Moreover, the LFUV ratio $R_{K}^{(s)}(q^2)$ can be enhanced up to 1.8 in scenarios with large positive and negative $\text{Im}[C_{9}^{bd,NP}]$. There can also be a significant enhancement in $A_{FB}(q^2)$ for positive values of $\text{Re}[C_{9}^{bd,NP}]$ and large $\text{Im}[C_{9}^{bd,NP}]$, with the zero-crossing shifting towards lower $q^2$. A significant suppression in $A_{CP}(q^2)$ compared to the SM prediction of 2.5$\%$ is possible for large positive values of $\text{Im}[C_{9}^{bd,NP}]$, which may lead to $A_{CP}(q^2)$ falling below a per cent level. We find that a measurement of integrated $R_{K}^{(s)}$ and $A_{CP}$, with a precision 0.1 and 1$\%$, respectively, would be needed to narrow down the allowed ranges of $(\text{Re}[C_{9}^{bd,NP}], \text{Im}[C_{9}^{bd,NP}])$.

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[1] G. Hiller and F. Kruger, “More model-independent analysis of $b \to s$ processes”, Phys. Rev. D 69, 074020 (2004) [hep-ph/0310219].

[2] M. Bordone, G. Isidori and A. Pattori, “On the Standard Model predictions for $R_K$ and $R_{K^*}$,” Eur. Phys. J. C 76, no. 8, 440 (2016) [arXiv:1605.07633 [hep-ph]].

[3] R. Aaij et al. [LHCb Collaboration], “Test of lepton universality with $B^0 \to K^{*0}\ell^+\ell^-$ decays”, JHEP 1708, 055 (2017) [arXiv:1705.05802 [hep-ex]].

[4] R. Aaij et al. [LHCb Collaboration], “Search for lepton-universality violation in $B^+ \to K^+\ell^+\ell^-$ decays”, Phys. Rev. Lett. 122, no. 19, 191801 (2019) [arXiv:1903.09252 [hep-ex]].

[5] R. Aaij et al. [LHCb Collaboration], “Angular analysis and differential branching fraction of the decay $B^0_s \to \phi\mu^+\mu^-$”, JHEP 1509, 179 (2015) [arXiv:1506.08777 [hep-ex]].

[6] R. Aaij et al. [LHCb Collaboration], “Measurement of Form-Factor-Independent Observables in the Decay $B^0 \to K^{*0}\mu^+\mu^-$”, Phys. Rev. Lett. 111, 191801 (2013) [arXiv:1308.1707 [hep-ex]].

[7] R. Aaij et al. [LHCb Collaboration], “Angular analysis of the $B^0 \to K^{*0}\mu^+\mu^-$ decay using 3 fb$^{-1}$ of integrated luminosity”, JHEP 1602, 104 (2016) [arXiv:1512.04442 [hep-ex]].

[8] S. Descotes-Genon, T. Hurth, J. Matias and J. Virto, “Optimizing the basis of $B \to K^{*}\ell\ell$ observables in the full kinematic range”, JHEP 1305, 137 (2013) [arXiv:1303.5794 [hep-ph]].

[9] M. Alguer, B. Capdevila, A. Crivellin, S. Descotes-Genon, P. Masjuan, J. Matias and J. Virto, “Emerging patterns of New Physics with and without Lepton Flavour Universal contributions”, arXiv:1903.09578 [hep-ph].

[10] D. Bhatia, S. Chakraborty and A. Dighe, “Neutrino mixing and $R_K$ anomaly in U(1)$_X$ models: a bottom-up approach”, JHEP 1703, 117 (2017) [arXiv:1701.05825 [hep-ph]].

[11] B. Capdevila, A. Crivellin, S. Descotes-Genon, J. Matias and J. Virto, “Patterns of New Physics in $b \to s\ell^+\ell^-$ transitions in the light of recent data”, JHEP 1801, 093 (2018) [arXiv:1704.05340 [hep-ph]].

[12] J. Kumar and D. London, “New physics in $b \to s\ell^+\ell^-$?” [arXiv:1901.04516 [hep-ph]].

[13] A. Crivellin, G. D’Ambrosio and J. Heeck, “Addressing the LHC flavor anomalies with horizontal gauge symmetries”, Phys. Rev. D 91, no. 7, 075006 (2015) [arXiv:1503.03477 [hep-ph]].

[14] S. M. Boucenna, A. Celis, J. Fuentes-Martin, A. Vicente and J. Virto, “Non-abelian gauge extensions for B-decay anomalies”, Phys. Lett. B 760, 214 (2016) [arXiv:1604.03088 [hep-ph]].

[15] S. M. Boucenna, A. Celis, J. Fuentes-Martin, A. Vicente and J. Virto, “Phenomenology of an SU(2) × SU(2) × U(1) model with lepton-flavour non-universality”, JHEP 1612, 059 (2016) [arXiv:1608.01349 [hep-ph]].

[16] W. Altmannshofer, S. Gori, M. Pospelov and I. Yavin, “Quark flavor transitions in $L_{\mu} - L_{\tau}$ models”, Phys. Rev. D 89, 095033 (2014) [arXiv:1403.1269 [hep-ph]].
[17] A. Datta, J. Liao and D. Marfatia, “A light Z′ for the R_K puzzle and nonstandard neutrino interactions”, Phys. Lett. B 768, 265 (2017) [arXiv:1702.01099 [hep-ph]].

[18] A. J. Buras and J. Girrbach, “Left-handed Z′ and Z FCNC quark couplings facing new b → sμ+μ− data”, JHEP 1312, 009 (2013) [arXiv:1309.2466 [hep-ph]].

[19] D. Aristizabal Sierra, F. Staub and A. Vicente, “Shedding light on the b → s anomalies with a dark sector” Phys. Rev. D 92, no. 1, 015001 (2015) [arXiv:1503.06077 [hep-ph]].

[20] B. Allanach, F. S. Queiroz, A. Strumia and S. Sun, “Z′ models for the LHCb and g−2 muon anomalies”, Phys. Rev. D 93, no. 5, 055045 (2016) [arXiv:1511.07447 [hep-ph]].

[21] B. Gripaios, M. Nardecchia and S. A. Renner, “Composite leptoquarks and anomalies in B-meson decays”, JHEP 1505, 006 (2015) [arXiv:1412.1791 [hep-ph]].

[22] S. Fajfer and N. Konik, “Vector leptoquark resolution of R_K and R_D(∗) puzzles”, Phys. Lett. B 755, 270 (2016) [arXiv:1511.06024 [hep-ph]].

[23] I. de Medeiros Varzielas and G. Hiller, “Clues for flavor from rare lepton and quark decays”, JHEP 1506, 072 (2015) [arXiv:1503.01084 [hep-ph]].

[24] R. Alonso, B. Grinstein and J. Martin Camalich, “Lepton universality violation and lepton flavor conservation in B-meson decays”, JHEP 1510, 184 (2015) [arXiv:1505.05164 [hep-ph]].

[25] L. Calibbi, A. Crivellin and T. Ota, “Effective Field Theory Approach to b → sll(′), B → K(∗)νν and B → D(∗)τν with Third Generation Couplings”, Phys. Rev. Lett. 115, 181801 (2015) [arXiv:1506.02661 [hep-ph]].

[26] R. Barbieri, G. Isidori, A. Pattori and F. Senia, “Anomalies in B-decays and U(2) flavour symmetry”, Eur. Phys. J. C 76, no. 2, 67 (2016) [arXiv:1512.01560 [hep-ph]].

[27] A. K. Alok, A. Dighe, S. Gangal and D. Kumar, “Continuing search for new physics in b → sμμ decays: two operators at a time”, JHEP 1906, 089 (2019) [arXiv:1903.09617 [hep-ph]].

[28] M. Ciuchini, A. M. Coutinho, M. Fedele, E. Franco, A. Paul, L. Silvestrini and M. Valli, “New Physics in b → sℓ+ℓ− confronts new data on Lepton Universality”, arXiv:1903.09632 [hep-ph].

[29] G. D’Amico, M. Nardecchia, P. Panci, F. Sannino, A. Strumia, R. Torre and A. Urbano, “Flavour anomalies after the R_K measurement”, JHEP 1709, 010 (2017) [arXiv:1704.05438 [hep-ph]].

[30] A. Datta, J. Kumar and D. London, “The B Anomalies and New Physics in b → se+e−”, arXiv:1903.10086 [hep-ph].

[31] J. Aebischer, W. Altmannshofer, D. Guadagnoli, M. Reboud, P. Stangl and D. M. Straub, “B-decay discrepancies after Moriond 2019”, [arXiv:1903.10434 [hep-ph]].

[32] K. Kowalska, D. Kumar and E. M. Sessolo, “Implications for new physics in b → sμμ transitions after recent measurements by Belle and LHCb”, Eur. Phys. J. C 79, no. 10, 840 (2019) [arXiv:1903.10932 [hep-ph]].

[33] A. Arbey, T. Hurth, F. Mahmoudi, D. M. Santos and S. Neshatpour, “Update on the b → s anomalies”, Phys. Rev. D 100, no. 1, 015045 (2019) [arXiv:1904.08399 [hep-ph]].

[34] R. Aaij et al. [LHCb Collaboration], “First observation of the decay B+ → π+μ+μ−”, JHEP 1212.
21

(2012) 125 [arXiv:1210.2645 [hep-ex]].

[35] R. Aaij et al. [LHCb Collaboration], “First measurement of the differential branching fraction and CP asymmetry of the $B^\pm \rightarrow \pi^\pm \mu^+\mu^-$ decay”, JHEP 1510, 034 (2015) [arXiv:1509.00414 [hep-ex]].

[36] R. Aaij et al. [LHCb Collaboration], “Evidence for the decay $B_S^0 \rightarrow K^{*0}\mu^+\mu^-$”, JHEP 1807, 020 (2018) [arXiv:1804.07167 [hep-ex]].

[37] J.-T. Wei et al. [Belle Collaboration], “Search for $B \rightarrow \pi l^+l^-$ Decays at Belle”, Phys. Rev. D 78 (2008) 011101 [arXiv:0804.3656 [hep-ex]].

[38] J. P. Lees et al. [BaBar Collaboration], “Search for the rare decays $B \rightarrow \pi l^+l^-$ and $B^0 \rightarrow \eta l^+l^-$”, Phys. Rev. D 88, no. 3, 032012 (2013) [arXiv:1303.6010 [hep-ex]].

[39] A. Cerri et al., “Opportunities in Flavour Physics at the HL-LHC and HE-LHC”, arXiv:1812.07638 [hep-ph].

[40] G. Buchalla, A. J. Buras and M. E. Lautenbacher, “Weak decays beyond leading logarithms”, Rev. Mod. Phys. 68, 1125 (1996) [hep-ph/9512380].

[41] A. K. Alok, B. Bhattacharya, D. Kumar, J. Kumar, D. London and S. U. Sankar, “New physics in $b \rightarrow s\mu^+\mu^-$: Distinguishing models through CP-violating effects”, Phys. Rev. D 96 (2017) no.1, 015034 [arXiv:1703.09247 [hep-ph]].

[42] R. Aaij et al. [LHCb Collaboration], “Measurement of the $B_S^0 \rightarrow \mu^+\mu^-$ branching fraction and search for $B^0 \rightarrow \mu^+\mu^-$ decays at the LHCb experiment”, Phys. Rev. Lett. 111, 101805 (2013) [arXiv:1307.5024 [hep-ex]].

[43] V. Khachatryan et al. [CMS and LHCb Collaborations], “Observation of the rare $B^0_S \rightarrow \mu^+\mu^-$ decay from the combined analysis of CMS and LHCb data”, Nature 522, 68 (2015) [arXiv:1411.4413 [hep-ex]].

[44] M. Aaboud et al. [ATLAS Collaboration], “Study of the rare decays of $B^0$ and $B^0_s$ mesons into muon pairs using data collected during 2015 and 2016 with the ATLAS detector”, JHEP 1904, 098 (2019) [arXiv:1812.03017 [hep-ex]].

[45] A. Abdesselam et al. [Belle Collaboration], “Test of lepton flavor universality in $B \rightarrow K^*\ell^+\ell^-$ decays at Belle”, arXiv:1904.02440 [hep-ex].

[46] R. Aaij et al. [LHCb Collaboration], “Measurements of the S-wave fraction in $B^0 \rightarrow K^{*}\pi^-\mu^+\mu^-$ decays and the $B^0 \rightarrow K^{*(892)^0}\mu^+\mu^-$ differential branching fraction”, JHEP 1611, 047 (2016) [arXiv:1606.04731 [hep-ex]].

[47] CDF Collaboration, “Updated Branching Ratio Measurements of Exclusive $b \rightarrow s\mu^+\mu^-$ Decays and Angular Analysis in $B \rightarrow K^{(*)}\mu^+\mu^-$ Decays”, CDF public note 10894.

[48] S. Chatrchyan et al. [CMS Collaboration], “Angular analysis and branching fraction measurement of the decay $B^0 \rightarrow K^{*0}\mu^+\mu^-$”, Phys. Lett. B 727, 77 (2013) [arXiv:1308.3409 [hep-ex]].

[49] V. Khachatryan et al. [CMS Collaboration], “Angular analysis of the decay $B^0 \rightarrow K^{*0}\mu^+\mu^-$ from pp collisions at $\sqrt{s} = 8$ TeV”, Phys. Lett. B 753, 424 (2016) [arXiv:1507.08126 [hep-ex]].

[50] R. Aaij et al. [LHCb Collaboration], “Differential branching fractions and isospin asymmetries of $B \rightarrow K^{(*)}\mu^+\mu^-$ decays”, JHEP 1406, 133 (2014) [arXiv:1403.8044 [hep-ex]].
[51] J. P. Lees et al. [BaBar Collaboration], “Measurement of the $B \to X_s l^+ l^-$ branching fraction and search for direct CP violation from a sum of exclusive final states”, Phys. Rev. Lett. 112, 211802 (2014) [arXiv:1312.5364 [hep-ex]].

[52] M. Aaboud et al. [ATLAS Collaboration], “Angular analysis of $B_0^0 \to K^* \mu^+ \mu^-$ decays in pp collisions at $\sqrt{s} = 8$ TeV with the ATLAS detector”, JHEP 1810, 047 (2018) [arXiv:1805.04000 [hep-ex]].

[53] CMS Collaboration [CMS Collaboration], “Measurement of the $P_1$ and $P'_5$ angular parameters of the decay $B_0 \to K^* \mu^+ \mu^-$ in proton-proton collisions at $\sqrt{s} = 8$ TeV”, CMS-PAS-BPH-15-008.

[54] D. M. Straub, “flavio: a Python package for flavour and precision phenomenology in the Standard Model and beyond”, arXiv:1810.08132 [hep-ph].

[55] J. Charles et al. [CKMfitter Group], “CP violation and the CKM matrix: Assessing the impact of the asymmetric $B$ factories”, Eur. Phys. J. C 41, no. 1, 1 (2005) [hep-ph/0406184].

[56] D. King, A. Lenz and T. Rauh, “$B_s$ mixing observables and $|V_{td}/V_{ts}|$ from sum rules”, JHEP 1905, 034 (2019) [arXiv:1904.00940 [hep-ph]].

[57] Y. S. Amhis et al. [Heavy Flavor Averaging Group], “Averages of $b$-hadron, $c$-hadron, and $\tau$-lepton properties as of 2018”, arXiv:1909.12524 [hep-ex].

[58] S. Aoki et al. [Flavour Lattice Averaging Group], “FLAG Review 2019”, arXiv:1902.08191 [hep-lat].

[59] M. Tanabashi et al. [Particle Data Group], “Review of Particle Physics”, Phys. Rev. D 98, no. 3, 030001 (2018).

[60] S. R. Mishra et al. [CCFR Collaboration], “Neutrino tridents and $W Z$ interference”, Phys. Rev. Lett. 66 (1991) 3117.

[61] J. J. Wang, R. M. Wang, Y. G. Xu and Y. D. Yang, “The Rare decays $B_{u}^{+} \to \pi^{+} l^{+} l^{-}, \rho^{+} l^{+} l^{-}$ and $B_{d}^{0} \to l^{+} l^{-}$ in the $R$-parity violating supersymmetry”, Phys. Rev. D 77 (2008) 014017 [arXiv:0711.0321 [hep-ph]].

[62] P. Ball and R. Zwicky, “New results on $B \to \pi, K, \eta$ decay formfactors from light-cone sum rules”, Phys. Rev. D 71 (2005) 014015 [hep-ph/0406232].

[63] Y. Amhis et al. [HFLAV Collaboration], “Averages of $b$-hadron, $c$-hadron, and $\tau$-lepton properties as of summer 2016”, Eur. Phys. J. C 77 (2017) no.12, 895 [arXiv:1612.07233 [hep-ex]].

[64] W. Altmannshofer, P. Ball, A. Bharucha, A. J. Buras, D. M. Straub and M. Wick, “Symmetries and Asymmetries of $B \to K^{*} \mu^{+} \mu^{-}$ Decays in the Standard Model and Beyond”, JHEP 0901, 019 (2009) [arXiv:0811.1214 [hep-ph]].

[65] B. Kindra and N. Mahajan, “Predictions of angular observables for $\bar{B}_{s} \to K^{*} \ell \ell$ and $\bar{B} \to \rho \ell \ell$ in the standard model”, Phys. Rev. D 98, no. 9, 094012 (2018) [arXiv:1803.05876 [hep-ph]].

[66] V. Barger, L. Everett, J. Jiang, P. Langacker, T. Liu and C. Wagner, “Family Non-universal $U(1)$-Prime Gauge Symmetries and $b \to s$ Transitions”, Phys. Rev. D 80, 055008 (2009) [arXiv:0902.4507 [hep-ph]].

[67] V. Barger, L. L. Everett, J. Jiang, P. Langacker, T. Liu and C. E. M. Wagner, “$b \to s$ Transitions in Family-dependent $U(1)$-Prime Models”, JHEP 0912, 048 (2009) [arXiv:0906.3745 [hep-ph]].
[68] A. Bharucha, D. M. Straub and R. Zwicky, “$B \rightarrow V\ell^+\ell^-$ in the Standard Model from light-cone sum rules”, JHEP 1608, 098 (2016) [arXiv:1503.05534 [hep-ph]].

[69] H. M. Asatrian, K. Bieri, C. Greub and M. Walker, “Virtual corrections and bremsstrahlung corrections to $b \rightarrow d\ell^+\ell^-$ in the standard model”, Phys. Rev. D 69, 074007 (2004) [hep-ph/0312063].

[70] A. K. Alok, A. Dighe and S. Ray, “CP asymmetry in the decays $B \rightarrow (X_s,X_d)\mu^+\mu^-$ with four generations”, Phys. Rev. D 79, 034017 (2009) [arXiv:0811.1186 [hep-ph]].

[71] A. Khodjamirian, T. Mannel, A. A. Pivovarov and Y.-M. Wang, “Charm-loop effect in $B \rightarrow K^{(*)}\ell^+\ell^-$ and $B \rightarrow K^*\gamma$”, JHEP 1009, 089 (2010) [arXiv:1006.4945 [hep-ph]].

[72] S. Descotes-Genon, L. Hofer, J. Matias and J. Virto, “Global analysis of $b \rightarrow s\ell\ell$ anomalies”, JHEP 1606, 092 (2016) [arXiv:1510.04239 [hep-ph]].

[73] G. Hiller and M. Schmaltz, “Diagnosing lepton-nonuniversality in $b \rightarrow s\ell\ell$”, JHEP 1502, 055 (2015) [arXiv:1411.4773 [hep-ph]].

[74] LHCb collaboration, “Framework TDR for the LHCb Upgrade: Technical Design Report”, CERN-LHCC-2012-007. LHCb-TDR-012.

[75] W. Altmannshofer, S. Gori, J. Martn-Albo, A. Sousa and M. Wallbank, “Neutrino Tridents at DUNE”, arXiv:1902.06765 [hep-ph].

[76] J. Alda, J. Guasch and S. Penaranda, “Some results on Lepton Flavour Universality Violation”, Eur. Phys. J. C 79, no. 7, 588 (2019) [arXiv:1805.03636 [hep-ph]].