A Precise Determination of (Anti)neutrino Fluxes with (Anti)neutrino-Hydrogen Interactions

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Abstract

We present a novel method to perform precision measurements of neutrino and antineutrino fluxes, which are one of the dominant systematic uncertainties affecting present and future long-baseline neutrino experiments. The limited knowledge of the (anti)neutrino fluxes is also a major bottleneck for any accelerator based neutrino scattering experiment. Using exclusive $\mu^\pm p\pi^\pm$ and $\mu^\pm n$ processes on hydrogen we obtain an overall accuracy on the relative fluxes better than 1% in the energy range covering most of the available flux. The method is based upon the approach we recently proposed to collect high statistics samples of $\nu(\bar{\nu})$-hydrogen interactions in a high resolution detector, which could serve as part of the near-detector complex in a long-baseline neutrino experiment, as well as a dedicated beam monitoring detector.

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I. INTRODUCTION

Modern wide-band (anti)neutrino beams can deliver high intensity fluxes over a broad energy range. The possibility to collect unprecedented exposures alleviates one of the primary limitations of past neutrino experiments and allows the use of high resolution detectors with a relatively small fiducial mass of a few tons to obtain a more accurate reconstruction of (anti)neutrino interactions. However, compared to electron scattering experiments, the use of the (anti)neutrino probe has to face an intrinsic limitation arising from the fact that the energy of the incoming (anti)neutrino is unknown on an event-by-event basis. Cross-sections and fluxes are thus folded into the observed event distributions and have to be both determined from the same data. Since typically (anti)neutrino experiments need to use nuclear targets to collect a sizable statistics, nuclear effects introduce a substantial smearing on the measured distributions, resulting in additional systematic uncertainties. For these reasons all (anti)neutrino scattering experiments have been limited by a poor knowledge of the incoming (anti)neutrino fluxes. Solving this outstanding problem requires to identify exclusive processes with a well known cross-section and/or a simple reconstructed topology to be used for the flux determination. An accurate knowledge of the (anti)neutrino fluxes is a necessary condition to exploit the unique features of the (anti)neutrino probe for precision measurements of fundamental interactions and to study the structure of nucleons and nuclei. The flux uncertainties are also a key element for the success of the next-generation long-baseline neutrino oscillation experiments.

In Ref. [1] we proposed a novel approach to achieve precision measurements of $\nu(\bar{\nu})$-hydrogen interactions via subtraction between dedicated CH$_2$ plastic and graphite (pure C) targets, embedded within a low mass tracker allowing a control of the targets similar to electron scattering experiments. We used a kinematic selection with efficiencies exceeding 90% to reduce backgrounds from the carbon nucleus in the CH$_2$ measurements to the 8-20% level, depending upon the specific event topology considered. The measurement of this background is entirely data-driven by subtracting it using the corresponding graphite target measurements. This concept – both simple and safe to implement – appears to be the only realistic opportunity to obtain a clean selection of high statistics samples of (anti)neutrino interactions on hydrogen, since safety and practical arguments make other techniques unfeasible.

The availability of $\nu(\bar{\nu})$-H interactions offers a solution to the longstanding problem of the determination of (anti)neutrino fluxes in both neutrino scattering and long-baseline oscillation experiments. In this paper we propose methods to achieve accurate measurements of (anti)neutrino fluxes using exclusive $\mu^\pm p\pi^\pm$ and $\mu^+ n$ quasi-elastic (QE) topologies on hydrogen. We will perform a detailed analysis using realistic assumptions for the detector smearing and physics modeling in order to evaluate the relevant uncertainties and the overall precisions achievable by the proposed techniques.

This paper is organized as follows In Sec. II we briefly describe the detection technique and the selection of the exclusive event samples for the flux determination. In Sec. III we discuss our results and in Sec. IV we summarize our findings.

II. DETECTION TECHNIQUE AND EVENT SELECTION

We consider the detection technique proposed in Ref. [1] to obtain $\nu(\bar{\nu})$-H interactions from the subtraction of events in dedicated CH$_2$ plastic and graphite (pure C) targets. The
key detector element is a low-density ($\rho \sim 0.16 \text{ g/cm}^3$) straw tube tracker, in which thin layers of various target materials (100% chemical purity) are alternated with straw layers so that they represent more than 95% of the total detector mass (5% being the mass of the straws). As discussed in Ref. [1] this design allows a control of the configuration, chemical composition, size, and mass of the (anti)neutrino targets in a way similar to what is typically done in electron scattering experiments. Our analysis is based upon a fiducial mass of 5 tons of CH$_2$ – corresponding to 714 kg of hydrogen – and 600 kg of graphite [1].

We simulate (anti)neutrino interactions on CH$_2$, H, and C targets with three different event generators: NuWro [2], GiBUU [3], and GENIE [4] to check the sensitivity of our analysis to the details of the input modeling. We generate inclusive Charged Current (CC) interactions including all processes available in the event generators with input (anti)neutrino spectra similar to the ones expected in the DUNE experiment [5, 6]. We then use the GEANT4 [7] program to evaluate detector effects and apply to all final state particles a parameterized reconstruction smearing consistent with the NOMAD data [8].

We assume the same event selection described in Ref. [1] and use as input for our analysis the corresponding $\nu(\bar{\nu})$-H samples obtained after the kinematic analysis and the subtraction of the small residual C background. In particular, for the various flux measurements we focus on two exclusive topologies: (a) $\nu p \rightarrow \mu^- p \pi^+$ and $\bar{\nu} p \rightarrow \mu^+ p \pi^-$, mainly from resonance (RES) production; (b) $\nu \rightarrow \mu^+ n$ quasi-elastic interactions. The former samples are selected with an efficiency of 90% and a purity of 92% (88%) for $\nu(\bar{\nu})$-H, while the latter sample is selected with 95% efficiency and 80% purity [1].

III. RESULTS AND DISCUSSION

A. Relative $\nu_\mu$ flux

Relative fluxes as a function of the (anti)neutrino energy $E_\nu$ have been determined by many modern neutrino experiments by using the measured inclusive CC interactions with small visible hadronic energy $\nu$ [9, 10]. This technique (low-$\nu$) is based on the observation that introducing a fixed $\nu_0$ cut reduces the available phase space and the corresponding energy dependence of the cross-section. This latter can be expanded in series of the ratio $\nu_0/E_\nu$, so that the number of observed events with hadronic energy $\nu < \nu_0$ can be written as $N(E_\nu, \nu < \nu_0) = k \Phi(E_\nu) f_c(\nu_0/E_\nu)$, where $\Phi$ is the (anti)neutrino flux, $k$ is an arbitrary normalization constant, and $f_c$ is a correction factor which can be calculated as a power series in $\nu_0/E_\nu$ with coefficients given by combinations of integrals of the structure functions. In practice the factor $f_c$ can be evaluated using the MC as the ratio of the cross-section with $\nu < \nu_0$ with respect to its asymptotic value at the highest energy of interest for the measurement. The correction factor $f_c$ becomes smaller by lowering the value of the cut $\nu_0$ and typically gives reliable flux predictions for $E_\nu \gtrsim 2\nu_0$. The use of low energy (anti)neutrino beams for long-baseline oscillation experiments requires to use $\nu_0$ cuts in the range 0.25–0.50 GeV [10] and the corresponding flux samples are almost entirely composed of quasi-elastic and resonant interactions.

Past and current neutrino experiments have used the low-$\nu$ approach with relatively heavy nuclear targets ranging from C to Pb. The use of such nuclear targets intrinsically limit the accuracy achievable in the determination of relative fluxes, due to the systematic uncertainties associated to the nuclear smearing including Fermi motion and binding, off-shell corrections, meson exchange currents, nuclear shadowing [11–13], neutron production, and final state
interactions [14]. The nuclear smearing directly affects the hadronic energy reconstruction and the acceptance of the cut $\nu < \nu_0$.

The limitations discussed above can be overcome by considering a single exclusive process on an elementary target like hydrogen (free proton). The use of a single process rather than an inclusive sample offers the advantage of a well defined cross-section, while the availability of a hydrogen target eliminates the bottleneck arising from nuclear effects. As a result, hadronic uncertainties in the determination of relative fluxes can be dramatically reduced.

The simplest topology available in $\nu$-H interactions is the process $\nu_\mu p \rightarrow \mu^- p\pi^+$, dominated by resonance production. Since all final state particles can be accurately reconstructed in the low-density tracker described in Sec. II, the unfolding of the detector response is controlled by the momentum resolution $\delta p/p \sim 3\%$. These features make the $\nu_\mu p \rightarrow \mu^- p\pi^+$ topology an excellent tool for the determination of the relative $\nu_\mu$ fluxes as a function of $E_\nu$. The absolute flux normalization is rather provided by the purely leptonic process $\nu e^- \rightarrow \nu e^-$, which can be measured accurately in the low-density tracker of Sec. II, up to the available statistics. The relevant model uncertainties are the ones affecting the energy dependence of the RES cross-section on hydrogen, which is controlled by the proton form factors. These uncertainties are substantially smaller than in any nuclear target, due to the absence of nuclear effects. In order to estimate their effect on the determination of the relative fluxes we vary the axial and vector form factors in the event generators and repeat our analysis. The results shown in Fig. 1 (left plot) indicate flux shape uncertainties of the order of 2–5% depending upon the neutrino energy considered. We can further reduce such uncertainties by restricting our analysis to events with low hadronic energy $\nu$. Given the typical invariant mass of resonant processes and the absence of QE processes in $\nu$-H interactions, cuts down to $\nu < 0.5$ GeV are feasible. Figure 1 (right plot) demonstrates that the use of this cut with $\mu^- p\pi^+$ events on H can reduce the hadronic uncertainties on the flux determination to the sub-percent level. This effect arises from the flattening of the energy dependence of the RES cross-section at

$$1\text{ Elastic scattering off electrons (mainly NC) can also provide complementary information on relative flux, free from nuclear effects. However, the limited statistics and the additional smearing associated to the outgoing neutrino and the beam divergence result in larger uncertainties compared to the ones achievable with } \nu(\bar{\nu})\text{-H interactions.}$$
FIG. 2. Left plot: reconstructed $Q^2$ distribution of $\mu^-p\pi^+$ events for the complete statistics of selected events on H. The solid circles (mock-data) with statistical uncertainties correspond to the nominal GENIE cross-section. The solid upper and lower curves show the sensitivity to a modification of the axial form factor by $M_A \pm 20\%$. Right plot: reconstructed $Q^2$ distribution of QE $\mu^+n$ events for the complete statistics of selected events on H. The same notations as in the previous plot are used.

We have shown that large variations of the proton form factors result in small uncertainties on the relative flux determination from $\mu^-p\pi^+$ interactions on H at $\nu < 0.5$ GeV. To this end, in Fig. 1 we consider variations of the vector mass by $\pm 10\%$ and of the axial mass by $\pm 20\%$. However, this estimate may not cover more general variations of the form factors – which cannot be simply described in terms of axial and vector masses [15] – nor larger unexpected discrepancies. We can address such effects by directly extracting the relevant proton form factors from the reconstructed $Q^2$ distribution in the complete $\mu^-p\pi^+$ sample on H without the $\nu$ cut. Using the exposures from Ref. [1], the total statistics expected is about $2.24 \times 10^6$ reconstructed $\mu^-p\pi^+$ events. This statistics provides a stringent test against arbitrary model variations and a good $Q^2$ coverage to fit the relevant form factors from data. The sensitivity to model variation is illustrated in Fig. 2 (left plot) for the $\mu^-p\pi^+$ sample on H: the same variations of form factors resulting in sub-percent uncertainties on the relative fluxes (Fig. 1) produce large changes in the measured $Q^2$ distribution, easily detectable. It is worth noting that the fraction of overlap events between the flux sample with $\nu < 0.5$ GeV and the total sample $\mu^-p\pi^+$ considered here is only about 25%, allowing a robust in-situ measurement of the proton form factors and a further reduction of the corresponding flux uncertainties.
\[ \mu - p\pi^- + \mu + p\pi^- - \mu + n \] 

\begin{table}[h]
\centering
\begin{tabular}{|c|c|c|c|c|}
\hline
\( \nu \) cut (GeV) & \( \nu < 0.50 \) & \( \nu < 0.75 \) & \( \nu < 0.50 \) & \( \nu < 0.75 \) & \( \nu < 0.10 \) & \( \nu < 0.25 \) \\
\hline
Low energy beam & 25.2\% & 14.9\% & 46.8\% & 76.0\% \\
High energy beam & 45.6\% & 20.7\% & 39.3\% & 68.0\% \\
\hline
\end{tabular}
\caption{Average efficiencies of the various \( \nu \) cuts for the \( \mu p\pi \) and \( \mu + n \) QE samples on hydrogen used in our analysis for the two beam configurations considered.}
\end{table}

B. Relative \( \bar{\nu}_\mu \) flux

The process \( \bar{\nu}_\mu p \rightarrow \mu^+ p\pi^- \) on hydrogen has the same experimental signature and features as the corresponding \( \nu_\mu p \) discussed in Sec III A. We can therefore perform a similar analysis and use the sample with \( \nu < 0.5 \) GeV to determine the relative \( \bar{\nu}_\mu \) flux as a function of \( E_\nu \). Considering the exposures from Ref. [1], the overall efficiency of the cut \( \nu < 0.5 \) GeV on the reconstructed hadronic energy is about 15\% for the \( \mu^+ p\pi^- \) topologies on H (Tab. I). The efficiency of the \( \nu \) cut is lower for the antineutrino samples due to the larger contribution from higher resonances and non-resonant events to the inclusive \( \mu^+ p\pi^- \) topologies. Model systematics on the flux determination are similar to the ones discussed in Sec III A.

In addition to the \( \mu^+ p\pi^- \) topologies, we also have the exclusive \( \bar{\nu}_\mu p \rightarrow \mu^+ n \) QE process in \( \bar{\nu}_\mu \) CC interactions on hydrogen. These QE events can be reconstructed with good efficiency [1] in the detector described in Sec. II and can also be used to determine the relative \( \bar{\nu}_\mu \) flux in a way similar to the \( \mu^+ p\pi^- \) events. The QE sample allows a lower cut on the reconstructed hadronic energy down to \( \nu < 0.25 \) GeV, which has an overall efficiency of 76\% with the beam spectrum of Ref. [1] (Tab. I). With the event selection and exposures of Ref. [1] we expect a total of about 812,000 reconstructed QE events on H, out which 617,000 have \( \nu < 0.25 \) GeV. The model uncertainties on the relative flux can be directly constrained by extracting the relevant form factors from the measured \( Q^2 \) distribution (Sec III A), as illustrated in Fig. 2. The overlap with the flux sample can be reduced below 50\% with a lower cut \( \nu < 0.1 \) GeV (Tab. I). The large statistics of the complete reconstructed QE sample without the \( \nu \) cut provides a good sensitivity to constrain arbitrary model variations.

C. Absolute \( \bar{\nu}_\mu \) flux

The availability of large samples of \( \bar{\nu}_\mu p \rightarrow \mu^+ n \) QE events on hydrogen also allows a determination of the absolute \( \bar{\nu}_\mu \) flux, in addition to the relative one as a function of \( E_\nu \) discussed in Sec III B. The cross-section for the \( \bar{\nu}_\mu \) QE process on hydrogen in the limit of \( Q^2 \rightarrow 0 \) can be written as:

\[
\frac{d\sigma}{dQ^2} \bigg|_{Q^2=0} = \frac{G_F^2 \cos^2 \theta_c}{2\pi} \left[ F_V^2(0) + F_A^2(0) \right]
\]

where \( F_V \) and \( F_A \) are the vector and axial form factors, \( \theta_c \) is the Cabibbo angle, \( G_F \) the Fermi constant, and we have neglected terms in \( (m_\mu/M)^2 \). The cross-section in Eq.(1) at \( Q^2 = 0 \) is determined by the neutron \( \beta \) decay to a precision \( \ll 1\% \). Experimentally, we can select low \( Q^2 \) QE events and determine the asymptotic value by fitting the measured \( Q^2 \) distributions (Fig. 2). Considering the exposures from Ref. [1], we expect about 135,000
FIG. 3. Summary of the expected statistical and systematic uncertainties in the $\nu_\mu$ relative flux determination using $\mu^- p\pi^+$ exclusive processes on hydrogen. Two different input spectra similar to the ones used in the DUNE long-baseline oscillation experiment are considered: (a) a low-energy beam optimized to search for CP violation (left plot) with a cut $\nu < 0.5$ GeV; (b) a high-energy beam optimized to detect the $\nu_\tau$ appearance (right plot) with a cut $\nu < 0.75$ GeV. See text for details.

reconstructed $\mu^+ n$ QE events with $Q^2 < 0.05$ GeV$^2$. We note that in a detector like the one discussed in Sec. II neutrons can be detected down to a much lower threshold than protons, thus enhancing the reconstruction efficiency of $\bar{\nu}_\mu$ QE on H at very small $Q^2$ values. The measurement of the absolute $\bar{\nu}_\mu$ flux using QE interactions on H requires a calibration of the absolute neutron detection efficiency, which can be performed using dedicated test-beam exposures of the relevant detector elements.

D. Flux uncertainties

In order to study the the statistical and systematic uncertainties on the $\nu_\mu$ and $\bar{\nu}_\mu$ fluxes achievable with the method we propose, we consider the realistic case study of the fluxes and exposures from Ref. [1]. The same beam and detector assumptions were the basis of a proposal to enhance the sensitivity to long-baseline oscillations in LBNF/DUNE and to define an extensive program of precision tests of fundamental interactions [16, 17]. As an illustration of the flexibility of the method we consider two different beam spectra with the exposures of Ref. [17]: (a) a low energy beam similar to the default one optimized for the CP violation in DUNE [5, 6]; (b) a high energy beam option optimized for the $\nu_\tau$ appearance from long-baseline oscillations.

The expected statistical uncertainties are the ones related to the selection of the exclusive $\mu^\pm p\pi^\pm$ and $\mu^\pm n$ QE topologies on H described in Sec. III, for the assumed exposures. We study the effect of three different sources of systematic uncertainties on the fluxes determined from $\nu(\bar{\nu})$-H interactions: (i) muon energy scale; (ii) hadronic energy reconstruction and $\nu$ cut; (iii) modeling of form factors and cross-sections.

Since the flux samples (Sec. III) include events with small hadronic energy $\nu$, the dominant
FIG. 4. Summary of the expected statistical and systematic uncertainties in the $\bar{\nu}_\mu$ relative flux determination using $\mu^+ n$ QE exclusive processes on hydrogen. Two different input spectra similar to the ones used in the DUNE long-baseline oscillation experiment are considered: (a) a low-energy beam optimized to search for CP violation (left plot) with a cut $\nu < 0.25$ GeV; (b) a high-energy beam optimized to detect the $\nu_\tau$ appearance (right plot) with a cut $\nu < 0.25$ GeV. See text for details.

contribution to the visible energy comes from the muon. The accuracy in determining the muon energy scale is therefore crucial for all the flux measurements, requiring a low density tracking detector, as well as a precise calibration of the measured momenta for the charged particles. The density of the detector described in Sec. II $\rho \sim 0.16$ g/cm$^3$ and its track sampling are well suited for these measurements. Following the technique used by the NOMAD experiment [8] – based upon a similar detector concept – we can calibrate the momentum scale of charged particles with the mass peak of the large samples of reconstructed $K_0$ decays. In our study we assume the same muon energy scale uncertainty of 0.2% achieved by the NOMAD experiment.

To determine the effects of the hadronic energy reconstruction on the flux measurements we consider a realistic detector smearing and event selection from Ref. [1]. The acceptance for individual final state particles is folded into the analysis and the reconstruction smearing on the hadronic energy is evaluated as a function of $\nu$. In addition to the unfolding of the detector response, we vary the $\nu$ cut applied to define the flux samples according to the expected resolution around the cut values. We also study the effect of different $\nu$ cuts in the range 0.25–0.75 GeV to optimize the sensitivity of the analysis for different beam spectra and $\nu(\bar{\nu})$-H topologies.

Model uncertainties are estimated by varying the vector form factor by $\pm 10\%$ and the axial form factor by $\pm 20\%$, as described in Sec III. These variations are relatively large and provide an upper limit on the corresponding expected uncertainties, since we deliberately ignore here the in-situ constraints on the form factors extracted from the measured $Q^2$ distributions (Fig. 2).

For each variation of the relevant parameters within their systematic uncertainties we repeat our analysis and evaluate the difference in the extracted fluxes as a function of $E_\nu$. To this end, we consider both positive and negative variations and symmetrize the corresponding
results. Since the determination of relative fluxes is defined up to an arbitrary constant, we normalize these measurements to unit area by dividing them by the integral of the measured distributions. The absolute flux normalization is provided independently by the $\nu e^- \rightarrow \nu e^-$ elastic scattering for $\nu_\mu$ and by the $\bar{\nu}_\mu p \rightarrow \mu^+ n$ QE on H at $Q^2 = 0$ for $\bar{\nu}_\mu$ (Sec. III).

The statistical and systematic uncertainties expected on the relative $\nu_\mu$ flux determined from $\mu^- p \pi^+$ topologies on H are shown in Fig. 3 for both the low energy and high energy beam options considered. In the former case we use a cut $\nu < 0.5$ GeV (Sec. III), while in the latter a higher cut $\nu < 0.75$ GeV turns out to be more appropriate. In the energy ranges where we expect the bulk of the fluxes the total uncertainties – including both the statistical and systematic ones added in quadrature – are below 1%. The dominant systematic uncertainty is related to the muon energy scale, as hadronic and model uncertainties are small for interactions on hydrogen. The statistical uncertainty is dominating the results in the tails of the available spectra. Following Ref. [1], the statistical uncertainty shown in Fig. 3 has to be increased by about 30% to account for the subtraction of the residual C background using a mass of graphite of 600 kg. Uncertainties with the high energy beam option are smaller than with the low energy beam due to the higher statistics and to the broader energy spectrum. The level of accuracy on the flux determination demonstrated in Fig. 3 cannot be achieved by other known techniques using nuclear targets. As illustrated by the comparison between two different spectra, the method discussed can be easily adapted to a wide range of beam configurations, provided the exposures are large enough to offer the required statistics $\mathcal{O}(10^6)$ for the various exclusive samples considered.

Figure 4 shows the statistical and systematic uncertainties on the relative $\bar{\nu}_\mu$ flux determined from $\mu^+ n$ QE interactions on H. We can apply cuts $\nu < 0.1$ GeV and $\nu < 0.25$ GeV for both the low and the high energy beam options, given the larger efficiencies (Tab. I). Similar considerations as for the relative $\nu_\mu$ fluxes can be made.

IV. SUMMARY

We proposed a novel method to achieve a precise determination of (a) relative $\nu_\mu$ fluxes; and (b) relative and absolute $\bar{\nu}_\mu$ fluxes using exclusive $\mu^\mp p \pi^\pm$ and $\mu^\mp n$ quasi-elastic processes on hydrogen. These event topologies can be efficiently selected with the simple and safe technique we previously proposed, based upon the subtraction between dedicated CH$_2$ plastic and graphite (pure C) targets.

We performed a detailed study of the relevant experimental and model uncertainties in the proposed method for the flux determination. To this end, we considered a realistic case study with (anti)neutrino beams similar to the ones planned for the DUNE experiment. Our results show that (anti)neutrino fluxes can be measured to an overall accuracy better than 1% in the main energy ranges – including both statistical and systematic uncertainties – with a selection of $\nu(\bar{\nu})$-H exclusive topologies. The analysis appears to be statistics limited and can be easily generalized to arbitrary (anti)neutrino input spectra. This level of accuracy cannot be achieved by other techniques using nuclear targets.
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[1] H. Duyang, B. Guo, S. R. Mishra, and R. Petti, (2018), arXiv:1809.08752 [hep-ph].
[2] C. Juszczak, J. A. Nowak, and J. T. Sobczyk, NuInt05, proceedings of the 4th International Workshop on Neutrino-Nucleus Interactions in the Few-GeV Region, Okayama, Japan, 26-29 September 2005, Nucl. Phys. Proc. Suppl. 159, 211 (2006), [,211(2005)], arXiv:hep-ph/0512365 [hep-ph].
[3] O. Buss, T. Gaitanos, K. Gallmeister, H. van Hees, M. Kaskulov, O. Lalakulich, A. B. Larionov, T. Leitner, J. Weil, and U. Mosel, Phys. Rept. 512, 1 (2012), arXiv:1106.1344 [hep-ph].
[4] C. Andreopoulos et al., Nucl. Instrum. Meth. A614, 87 (2010), arXiv:0905.2517 [hep-ph].
[5] R. Acciarri et al. (DUNE), (2015), arXiv:1512.06148 [physics.ins-det].
[6] R. Acciarri et al. (DUNE), (2016), arXiv:1601.02984 [physics.ins-det].
[7] S. Agostinelli et al. (GEANT4), Nucl. Instrum. Meth. A506, 250 (2003).
[8] J. Altegoer et al. (NOMAD), Nucl. Instrum. Meth. A404, 96 (1998).
[9] S. R. Mishra, Proceedings of the Workshop on Hadron Structure Functions and Parton Distributions (edited by D. Geesaman et al., World Scientific) , 84 (1990).
[10] A. Bodek, U. Sarica, D. Naples, and L. Ren, Eur. Phys. J. C72, 1973 (2012), arXiv:1201.3025 [hep-ex].
[11] S. A. Kulagin and R. Petti, Nucl. Phys. A765, 126 (2006), arXiv:hep-ph/0412425 [hep-ph].
[12] S. A. Kulagin and R. Petti, Phys. Rev. D76, 094023 (2007), arXiv:hep-ph/0703033 [HEP-PH].
[13] S. A. Kulagin and R. Petti, Phys. Rev. C90, 045204 (2014), arXiv:1405.2529 [hep-ph].
[14] L. Alvarez-Ruso et al., Prog. Part. Nucl. Phys. 100, 1 (2018), arXiv:1706.03621 [hep-ph].
[15] B. Bhattacharya, R. J. Hill, and G. Paz, Phys. Rev. D84, 073006 (2011), arXiv:1108.0423 [hep-ph].
[16] R. Petti, Workshop on Near Detector Physics at Neutrino Experiments, CERN, 18-22 June 2018, https://indico.cern.ch/event/721473/contributions/3034869/.
[17] P. Bernardini et al., Proposal to European Strategy Group for Particle Physics, December 2018, https://indico.cern.ch/event/765096/contributions/3295805/.