Multiaccess Coded Caching with Private Demands

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Abstract—Hachem et al. formulated a multiaccess coded caching model which consists of a central server connected to $K$ users via an error-free shared link, and $K$ cache-nodes. Each cache-node is equipped with a local cache and each user can access $L$ neighbouring cache-nodes in a cyclic wrap-around fashion. In this paper, we take the privacy of the users’ demands into consideration, i.e., each user, while retrieving its own demanded file, cannot obtain any information on the demands of the other users. By storing some private keys at the cache-nodes, we develop a novel transformation approach to turn any non-private coded caching scheme (satisfying some constraints) into a private one.

Index Terms—Coded caching, multiaccess networks, private demands.

I. INTRODUCTION

Recently, as the wireless data traffic is increasing at an incredible rate dominated by the video streaming, wireless networks are subject to a tremendous capacity stress. Consequently, communication systems are likely to be congested during the peak traffic times. Reducing this congestion is very desirable and represents a hot topic in industrial applications and academic research. Coded caching, originally proposed in [1], is an efficient solution to turn storage into bandwidth resource, in order to reduce such congestion for “cachable” content, as for example in the case of video on demand. In the shared-link coded caching model [1] (also called MN model), a server with access to a library of $N$ files, is connected to $K$ users via an error-free shared link. Each user is equipped with a local cache of $M$ files. A coded caching scheme includes two phases: placement phase and delivery phase. In the placement phase, without knowledge of future demands, the server fills part of the popular files into the user’s cache. In the delivery phase, each user demands a file from server. After receiving the users’ demands, the server sends multicast messages such that each user’s demand can be satisfied. The goal is to minimize the amount of data transmission (referred to as load) in the delivery phase. Inspired by the seminal work in [1], the caching problem for networks with various topologies has been studied, e.g., Device-to-Device networks in [2], combination networks in [3, 4] and Hierarchical networks in [5, etc.

In the MN coded caching scheme, each user’s demand is globally known, so the privacy of the users’ demands can not be guaranteed [6]. The information theoretic coded caching model with private demands was formulated in [7] for the shared-link model. The authors in [7] introduced virtual users on the shared-link model and used the delivery strategy of MN scheme to satisfy the demands of all users including real users and virtual users, such that each real user can not distinguish the demands of other real users. The virtual-user scheme was proven to be order optimal within a constant factor except for the regime where $N > K$ and $M < N/K$. Following the new formulation in [7], variants of the private caching scheme were proposed in [8, 9] to improve the virtual user scheme in [7], in terms of either the transmission load or the sub-packetization level. Very recently, a private coded caching scheme was proposed in [10] which is based on the cache-aided linear function retrieval scheme in [11]. The key idea of the scheme in [10] is that, in addition to cached contents by the MN placement strategy, users privately cache some linear combinations of uncached subfiles in the MN placement which are regarded as keys. In such way, the effective demand of each user in the delivery phase become the sum of these linear combinations and the subfiles of its desired file, such that the real demand is concealed. The scheme in [10] has a similar load as the virtual-user scheme in [7], but with a much lower sub-packetization level.

Multiaccess caching system: The multiaccess caching model (illustrated in Fig. 1), originally proposed in [12], is motivated by the concept of “edge caching”, where caching nodes (separated from the users) can be accessed simultaneously by multiple users. Different from the MN caching model, in the multiaccess caching model, there are $K$ cache-nodes each of which can store $M$ files, and $K$ cache-less users each of which can access $L$ cache-nodes in a cyclic wrap-around fashion, while also receiving the broadcast transmission directly from the server. As assumed in [12], the cache-nodes in this model are accessed at no load cost; that is, we aim to minimize the broadcasted load from the server for any given $M$. The authors in [12] proposed a scheme, which achieves the load $K(1-LM/N)$ when $L$ divides $K$. In [13] the authors proposed a transformation approach to extend some schemes for the MN model to the multiaccess caching system. As an application, two new multiaccess coded caching schemes were obtained based on the MN scheme and partition scheme in [15]. It is worth noting that the first scheme in [14] achieves the transmission load $K(1-LM/N)$ when $L$ divides $K$. In [16]–[19], other constructions of multiaccess coded caching schemes were proposed based on uncoded cache placement (i.e., each cache-node only cache subset of library bits) and linear delivery coding. Recently, the authors in [13] studied secure multiaccess caching problem which aims to protect the security of the library content from some wiretapper.

Contribution and paper organization: Different from the secure multiaccess caching problem in [13], in this paper we formulate the multiaccess caching model with private...
Fig. 1: The \((K, L, M, N)\) multiaccess coded caching model.

demands, which aims to preserve the privacy of each user’s demand from other users. Compared to the shared-link caching model with private demands in [7], the main challenge to preserve the demand privacy in the multiaccess caching model is that the users share the cached contents at the cache-nodes while in the shared-link model, the existing private caching schemes are designed based on the fact that the composition of the each user’s cache is unknown to the other users.

Besides the novel problem formulation, our main contribution is to propose an approach to add the privacy guarantee for any non-private multiaccess coded caching scheme satisfying three requirements, such as the non-private schemes in [14], [16]. Based on the feature that each user is connected to \(L\) neighbouring cache-nodes in a cyclic wrap-around fashion, we use second-layer keys to protect the first-layer keys (i.e., the private linear combinations as in [10]), so that even if users share some cached contents, each user can privately obtain these first-layer keys.

Notations: In this paper, we use the following notations unless otherwise stated. Bold capital letter, bold lower case letter and curlicue font will be used to denote matrix, vector and set, respectively. \([a : b] := \{a, a + 1, \ldots, b\}\) and \([n] := [1 : n]\); \([\cdot]\) represents the cardinality of a set or the length of a vector. \(\text{Mod}(b, a)\) represents the modulo operation on positive integer \(b\) with positive integer divisor \(a\). In this paper we let \(\text{Mod}(b, a) \in \{1, \ldots, a\}\) (i.e., we let \(\text{Mod}(b, a) = a\) if \(a\) divides \(b\)).

II. SYSTEM MODEL

A \((K, L, M, N)\) multiaccess caching problem with private demands has a single server containing \(N\) independent files \(W_1, W_2, \ldots, W_N\). \(K\) cache-nodes \(C_1, C_2, \ldots, C_K\), and \(K\) users \(U_1, U_2, \ldots, U_K\). The server is connected to \(K\) users via an error-free shared link. We assume that each file is composed of \(B\) i.i.d. bits.

Placement phase. The server fills the cache of each cache-node without knowledge of the users’ future demands. We denote the contents cached at the cache-node \(C_k\) by \(Z_{C_k}\). In order to preserve the privacy of each user’s demand, the server generates \(K\) random variables, \(V_1\) over \(\mathcal{V}_1\), \(V_2\) over \(\mathcal{V}_2\), \ldots, \(V_K\) over \(\mathcal{V}_K\), which are independent of the library files. Hence, \(Z_{C_k} = \phi_{C_k}(W_1, W_2, \ldots, W_N, V_k)\) where \(\phi_{C_k} : [\mathbb{F}_2]^{NB} \times \mathcal{V}_k \to [\mathbb{F}_2]^{MB}\). Each user \(U_k\) where \(k \in [K]\) can retrieve the contents of \(L \leq K\) neighbouring cache-nodes in a cyclic wrap-around fashion, i.e., the cache-nodes in \(\{C_{\text{Mod}(k, K)}, C_{\text{Mod}(k+1, K)}, \ldots, C_{\text{Mod}(K+L-1, K)}\}\).

\[Z_{U_k} := \bigcup_{i \in \{\text{Mod}(k, K), \text{Mod}(k+1, K), \ldots, \text{Mod}(K+L-1, K)\}} Z_{C_i},\]
as the retrievable contents by user \(U_k\). We assume that each user can retrieve the cached contents from its connected cache-nodes without any cost.

Delivery phase. Assume that the demand vector is \(d = (d_1, d_2, \ldots, d_K)\). User \(U_k\), \(k \in [K]\), demands the file \(W_{d_k}\) from server. Notice that the users’ demands are independent of the library contents. With the knowledge of \(d\), the server broadcasts the messages

\[X = \psi(d, W_1, \ldots, W_N, V_1, \ldots, V_K),\]

where \(\psi : [N]^K \times [\mathbb{F}_2]^{NB} \times \mathcal{V}_1 \times \cdots \times \mathcal{V}_K \to [\mathbb{F}_2]^{RB}\) and \(R\) is called as the transmitted load or load. With the help of the received messages and the retrievable cached contents, each user \(U_k\) where \(k \in [K]\) must decode its demanded file \(W_{d_k}\).

Hence for any user \(U_k\) it must hold that

\[H(W_{d_k}, X, d_k, Z_{U_k}) = 0.\] (1)

In addition, we impose a privacy constraint on the multiaccess caching model, i.e., each user can not get any information about other users’ demands from \(X, Z_{U_k}\) and \(d_k\); that is,

\[I(d_{-k}; X, Z_{U_k} | d_k) = 0, \quad \forall k \in [K],\] (2)

where \(d_{-k} = (d_1, \ldots, d_{k-1}, d_{k+1}, \ldots, d_K)\) denotes the demands of all the users except the demand of the user \(U_k\).

By the constraint of privacy, we can see that the loads for different demand vectors should be the same; otherwise, the transmitted load which can be counted by each user will reveal information about the other users’ demands. The objective of this paper is to minimize the load \(R\) by designing a placement and delivery strategy that satisfies each user’s demand in (1) and the privacy constraint in (2).

III. MAIN RESULT

We will propose a general approach in Section IV to add the privacy guarantee for any non-private multiaccess coded caching scheme satisfying three requirements.

Theorem 1. From any \((K, L, M', N)\) non-private multiaccess coded caching scheme with \(L < K/2 + 1\) satisfying the following requirements,

- Requirement 1: Any \(L\) neighbouring cache-nodes do not cache any common bits;
- Requirement 2: The placement phase is identically uncoded, i.e., if some cache-node stores the \(i\)-th bit of some file, then it caches the \(i\)-th bit of all files.
- Requirement 3: In the delivery phase, it treats the demand of each user as an independent file.

we obtain a \((K, L, M, N)\) private multiaccess coded caching scheme where \(M = M' + 2(1 - \frac{L^2}{K^2})\) with the same load and sub-packetization level as the original non-private scheme. \(\square\)

The proof of Theorem 1 and some detailed examples are provided in Section IV. Using the same placement and delivery strategies in Section IV and Shamir’s secret sharing in [20].
we can prove a demand-private scheme also in the regime $L \geq K/2 + 1$. In particular, for any non-private scheme $(K, L, M', N)$ satisfying the requirements in Theorem 1 we can construct a demand-private scheme with the same load and memory $M = M' + \omega(1 - LM'/N)$ where $\omega$ is an integer $\geq 2$ which will be clarified in Section V.

From Theorem 1 it follows that introducing the additional feature of demands’ privacy to a non-private multiaccess coding scheme satisfying Requirements 1, 2, and 3 incurs a cost of $M - M' = 2(1 - \frac{LM'}{N}) \leq 2$ in cache size. In other words, the additional memory size is upper bounded by an additive constant factor of 2.

By comparing the requirements in Theorem 1 and the existing non-private multiaccess coded caching schemes in [12], [14], [16]–[19], it can be seen that all of those schemes satisfy Requirements 1, 2, 3 in Theorem 1. For simplicity, here we only take the following scheme in [14] as an example.

**Lemma 1** ([14]). For the $(K, L, M', N)$ non-private multiaccess coded caching problem, the lower convex envelope of the following memory-load tradeoff points are achievable,

$$(M', R_1) = \left(\frac{Nt}{K}, \frac{K - tL}{t + 1}\right), \, \forall t \in \left[0 : \left[\frac{K}{L}\right]\right],$$

and $(M', R_1) = \left(\frac{Nt}{K}, 0\right)$.

Based on the scheme in Lemma 1 and by Theorem 1 we can obtain the following theorem.

**Theorem 2.** For the $(K, L, M, N)$ multiaccess coded caching problem with private demands, the lower convex envelope of the following memory-load tradeoff points are achievable,

$$(M, R_1) = \left(\frac{(N - 2)t}{K} + 2, \frac{K - tL}{t + 1}\right), \, \forall t \in \left[\left[\frac{K}{L}\right]\right],$$

if $L < K/2 + 1$, and $(M, R_1) = \left(\frac{Nt}{K}, 0\right)$. \hfill \Box

IV. PROOF OF THEOREM 1

Assuming that there exists a $(K, L, M', N)$ non-private multiaccess coded caching scheme, say Scheme 1, which satisfies Requirement 1 and Requirement 2. Let us then introduce the corresponding $(K, L, M = M' + 2(1 - \frac{LM'}{N}), N)$ private multiaccess coded caching scheme.

1) Placement Phase: Each cache-node caches two parts of cached contents. The first part is the same as the contents cached at cache-nodes in the placement phase of Scheme 1. As the placement of Scheme 1 is identically encoded, we can divide each file $W_n$ where $n \in [N]$ into subfiles, $W_n = \{W_{n, Q} | Q \subseteq [K]\}$, where $W_{n, Q}$ represents the set of bits in $W_n$ which are uniquely cached by cache-nodes in $\{C_i | i \in Q\}$. Since the placement phase is identically encoded, we define $x_Q = |W_{1, Q}| = \cdots = |W_{N, Q}|$.

For each $k \in [K]$, we define $T_k = \{Q \subseteq [K] | Q \cap \{\text{Mod}(k, K), \cdots, \text{Mod}(k + L - 1, K)\} = \emptyset \}$. Hence, each subfile $W_{n, Q}$ where $Q \in T_k$ cannot be retrieved by user $U_k$ from its connected cache-node.

From Requirement 1 and Requirement 2, we have for each user $k \in [K]$, the number of bits of each file which user $k$ cannot retrieve from its connected cache-nodes is

$$\sum_{Q \in T_k} x_Q = (1 - LM'/N)B.$$

In the second part, by the following two steps, each cache-node additionally stores some linear combinations of subfiles as the ‘keys’ which can be retrieved by the connected users.

- **Step 1.** The server randomly generates $K$ vectors where $p_k = (p_{k,1}, p_{k,2}, \ldots, p_{k,N}) \in \mathbb{F}_2^N$, $k \in [K]$, which are independent of the library files. Then for any $Q \in T_k$, the server generates a linear combination

$$S_{k, Q} = \sum_{n \in [N]} p_{k,n} W_{n, Q}. \quad (3)$$

- **Step 2.** For each $k \in [K]$ and each $Q \in T_k$, the server generates a random variable $A_{k, Q}$, which is uniformly i.i.d. over $\mathbb{F}_2^Q$ and is independent of the library files. Then for each $k \in [K]$, the server generates the key set

$$P_k = \{S_{k, Q} \oplus A_{k, Q} | \forall Q \in T_k\} \cup \{A_{\text{mod}(k + L - 1 + K, K)} | \forall Q \in T_{\text{mod}(k + L - 1, K)}\}$$

and places it into the cache of cache-node $C_k$.

Each cache-node totally caches $M'B + 2(1 - LM'/N)B = MB$ bits, satisfying the memory size constraint.

For each $k \in [K]$, user $U_k$ can retrieve $S_{k, Q} \oplus A_{k, Q}$ where $Q \in T_k$ from $C_k$. Similarly, user $U_k$ can also obtain $A_{k, Q}$ where $Q \in T_k$ from cache-node $C_{\text{mod}(k + L - 1, K)}$. Hence it can successfully recover $S_{k, Q}$ where $Q \in T_k$. Let us define that $S_k := \{S_{k, Q} | \forall Q \in T_k\}$. For each $j \in [K] \setminus \{k\}$ and each $Q \in T_j$, $S_{j, Q} \oplus A_{j, Q}$ is only cached by cache-node $C_j$, while $A_{j, Q}$ is only cached by cache-nodes $C_{\text{mod}(j + L - 1)}$. So except user $U_j$, none of the other users is simultaneously connected to cache-nodes $C_j$ and $C_{\text{mod}(j + L - 1)}$ if $L < K/2 + 1$. Hence, we have the following key point for our scheme:

**Key Point:** Each user $U_k$ can retrieve $S_k$ from its connected cache-nodes and can not get any information about $S_j$ where $j \in [K] \setminus \{k\}$.

Since $S_j, Q$ is locked by a key with the same length, it can be seen from [21] that user $U_k$ cannot get any information about $S_j, Q$. As a result, we also have

$$I(p_1, \ldots, p_K; Z_{U_k} | p_k, W_1, \ldots, W_N) = 0, \forall k \in [K]. \quad (4)$$

We then illustrate the placement strategy by an example.

**Example 1.** Consider the $(K, L, M, N) = (3, 2, 5/3)$ private multiaccess coded caching problem. We design the following placement phase based on the $(K, L, M', N) = (3, 2, 1, 3)$ non-private multiaccess coded caching scheme in [16].

As the scheme in [16], we divide each file $W_n$ where $n \in [3]$ into 3 equal-length subfiles, $W_n = \{W_{n, 1}, W_{n, 2}, W_{n, 3}\}$, where each subfile has $\frac{2}{3}$ bits.

\footnote{Since $L \leq K$, we have $L - 1 < K$. Hence, cache-nodes $C_k$ and $C_{\text{mod}(k + L - 1, K)}$ are two different cache-nodes.}
For the first part of cached contents, for each $n \in [3]$, the server places the subfiles $W_{n,1}$ into the cache-node $C_1$; places the subfiles $W_{n,2}$ into the cache-node $C_2$; places the subfiles $W_{n,3}$ into the cache-node $C_3$. It can be checked that the memory size used for the first part of cached contents is equal to $M_l = 3 \times \frac{1}{3} = 1$. Recall that in this example, user $U_1$ is connected to cache-nodes $C_1$ and $C_2$; user $U_2$ is connected to cache-nodes $C_2$ and $C_3$; user $U_3$ is connected to cache-nodes $C_3$ and $C_1$. Hence, we have $T_1 = \{\{3\}\}$, $T_2 = \{\{1\}\}$, $T_3 = \{\{2\}\}$. For the second part of cached contents, the server generates $K = 3$ vectors where for any $k \in [3]$, $p_k = (p_{k,1}, p_{k,2}, p_{k,3}) \in \mathbb{F}_2^3$. By \ref{eq:3}, we have
\[
S_1 = p_{1,1}W_{1,3} + p_{1,2}W_{2,3} + p_{1,3}W_{3,3}, \\
S_2 = p_{2,1}W_{1,3} + p_{2,2}W_{2,3} + p_{2,3}W_{3,3}, \\
S_3 = p_{3,1}W_{1,2} + p_{3,2}W_{2,2} + p_{3,3}W_{3,2}.
\]

\textbf{Step 2.}  Let $A_1, A_2, A_3$ be random variables, each of which is uniformly i.i.d. over $\mathbb{F}_2^{1/3}$. Then cache-nodes $C_k, k \in [K]$ caches the following key sets respectively,
\[
P_1 = \{S_1, A_1, A_3\}, \\
P_2 = \{S_2, A_2, A_1\}, \\
P_3 = \{S_3, A_3, A_2\}.
\]
In total, each cache-node $C_k$ where $k \in [3]$ caches $Z_{C_k}$, where $Z_{C_1} = \{W_{n,1}|\forall n \in [3]\} \cup P_1$, $Z_{C_2} = \{W_{n,2}|\forall n \in [3]\} \cup P_2$, and $Z_{C_3} = \{W_{n,3}|\forall n \in [3]\} \cup P_3$. Hence, the size of all the cached contents for each cache-node $C_k$ equals the capacity $\frac{5}{3} = MB$. Then the users can retrieve the contents as follows,
\[
Z_{U_1} = \{W_{n,1}, W_{n,2}|\forall n \in [3]\} \cup P_1 \cup P_2, \\
Z_{U_2} = \{W_{n,2}, W_{n,3}|\forall n \in [3]\} \cup P_2 \cup P_3, \\
Z_{U_3} = \{W_{n,3}, W_{n,1}|\forall n \in [3]\} \cup P_3 \cup P_1.
\]
Among $S_1, S_2, S_3$, user $U_1$ can only recover $S_1$; user $U_2$ can only recover $S_2$; user $U_3$ can only recover $S_3$.

\textbf{2) Delivery Phase:} User $U_k$ requests the file $W_{d_k}$ where $d_k \in [N]$. Now for any $k \in [K]$, we treat the following linear combination
\[
W_{d_k} = \left( \begin{array}{c}
p_{k,1}W_1 \\
p_{k,2}W_2 \\
p_{k,3}W_3
\end{array} \right)
\]
where $q_{k,j} = p_{k,j}$ for $j \in [N] \setminus \{d_k\}$ and $q_{k,d_k} \neq p_{k,d_k}$. We also let $q_k = (q_{k,1}, \ldots, q_{k,N})$. To preserve the privacy of the demand of each user $U_k$ where $k \in [K]$, we assume that the actual request of user $U_k$ becomes the virtual file $W_{d_k}$. Hence, user $U_k$ should recover
\[
W_{d_k,Q} := q_{k,1}W_{1,Q} \oplus \cdots \oplus q_{k,N}W_{N,Q}, \forall Q \in T_k.
\]
In the delivery phase, for the successful decoding, the server first broadcasts $q_1, \ldots, q_K$ to the users; thus, from the viewpoint of the other users the demand of user $U_k$ is $W_{d_k}$. We then use the delivery strategy of Scheme 1 to broadcast messages, such that each user can recover $W_{d_k,Q}$.

Finally, user $U_k$ can decode the subfile $W_{d_k,Q}$ where $Q \in T_k$ from $W_{d_k,Q}$ and $S_k,Q$, thus it can recover its requested file $W_{d_k}$. In terms of the load incurred by the coefficients $q_1, \ldots, q_K$, it is easy to argue that in the information theoretic limit of large file size $(B \rightarrow \infty)$ and fixed system parameters $K$ and $N$, the excess load of communicating $q_1, \ldots, q_K$ is vanishing, since the coefficients require $K N$ bits, and $K N / B$ can be made arbitrarily small for sufficiently large $B$. For the privacy, intuitively, from the viewpoint of the other users the demand of user $U_k$ is $W_{d_k}$. The information-theoretical proof for the privacy constraint in \ref{eq:2} is given in Appendix A.

\textbf{Example 2.} Let us return to Example 1 with $K = N = 3$ and $L = 2$. Assuming that the demand vector $d = (1, 2, 3)$. From \ref{eq:5a}, we have three virtual files as follows,
\[
W_1 = (p_{1,1} + 1)W_1 + p_{1,2}W_2 + p_{1,3}W_3, \\
W_2 = p_{2,1}W_1 + (p_{2,2} + 1)W_2 + p_{2,3}W_3, \\
W_3 = p_{3,1}W_1 + p_{3,2}W_2 + (p_{3,3} + 1)W_3.
\]
From \ref{eq:6}, user $U_1$ requires the subfile $W_{1,1}$ as follows,
\[
W_{1,1} = (p_{1,1} + 1)W_{1,1} + p_{1,2}W_{2,1} + p_{1,3}W_{3,1}.
\]
User $U_2$ requires the subfile $W_{2,2}$ as follows,
\[
W_{2,2} = p_{2,1}W_{1,2} + (p_{2,2} + 1)W_{2,2} + p_{2,3}W_{3,2}.
\]
User $U_3$ requires the subfile $W_{3,3}$ as follows,
\[
W_{3,3} = p_{3,1}W_{1,3} + p_{3,2}W_{2,3} + (p_{3,3} + 1)W_{3,3}.
\]
According to the delivery strategy of non-private multiaccess coded caching scheme in \ref{eq:16}, server sends the following signal to users,
\[
W_{1,1} \oplus W_{2,2} \oplus W_{3,3}.
\]
From \ref{eq:7}, user $U_1$ can decode $W_{1,1}$, user $U_2$ can decode $W_{2,2}$, and user $U_3$ can decode $W_{3,3}$; respectively. Hence user $U_3$ can obtain $W_{1,1}$ from $W_{1,1}$ and $S_1, S_3$, user $U_2$ can obtain $W_{2,2}$ from $W_{2,2}$ and $S_2, S_3$, user $U_3$ can decode $W_{3,2}$ from $W_{3,2}$ and $S_3, S_2$. Finally each user can recover its requested file.

\textbf{V. EXTENSION FOR $L \geq K/2 + 1$}

When $L > K/2 + 1$, after some slight modification of the proposed scheme in Section \ref{sec:5}, we can guarantee the privacy constraint of (2). Specifically, the server encrypts each key, say $S_{k,Q}, k \in [K]$ and $Q \in T_k$, into $\omega \geq 2$ shares by Shamir’s secret sharing in \ref{eq:20}, say $S_{k,Q}^{(h)}$, $h \in [\omega]$, and then places each share into some appropriate cache-node. By Shamir’s secret sharing, $S_{k,Q}$ could be re-constructed from $\omega$ shares, and we cannot get any information about $S_{k,Q}$ by any $\omega - 1$ shares nor less.

Let us focus on user 1. Let $w$ be the minimum value satisfying that there exists some set $L_1 \subseteq L$ with cardinality $w$, where $L_1 \not\subseteq \Mod([k, k + 1, \ldots, k + L - 1], K)$ for each
Let \( k \in [2 : K] \). We assume that the \( h \)th element in \( L \) is \( j_h \), for each \( h \in [\omega] \). For each \( Q \in T \), we place the share \( S_{i,Q}^{(h)} \) into cache-node \( C_{j_h} \), where \( h \in [\omega] \). Hence, user \( U_1 \) can retrieve \( S_{1,Q} \) from the cache-nodes indexed by \( L_1 \), while any other user cannot get any information about \( S_{1,Q} \).

For each user \( k \in [2 : K] \), we let \( L_k = \{ \text{Mod}(j_1 + k - 1, K), \text{Mod}(j_2 + k - 1, K), \ldots, \text{Mod}(j_h + k - 1, K) \} \) and place the shares by a similar way as described above. Then it can be seen that the Key point in Section [IV] holds. By the symmetry, the memory size of each cache-node is \( M = M' + \omega(1 - \frac{d}{d+1}) \). Using the same delivery strategy in Section [IV] a \((K, L, M, N)\) private multiaccess coded caching scheme where \( M = M' + \omega(1 - \frac{d}{d+1}) \) with the same load and sub-packetization level as the original \((K, L, M', N)\) non-private scheme satisfying the three requirements in Theorem [U]. Note that when \( L \leq K/2 + 1 \), we have \( \omega = 2 \), the above scheme is equivalent to the proposed scheme in Section [IV]

VI. CONCLUSION

This paper formulated the multiaccess caching system with private demands, where each user cannot get any information about the demands of other users. We then proposed an approach to guarantee the privacy for any non-private multiaccess coded caching scheme satisfying some constraints. On-going works include the consideration on the private multiaccess coded caching problem with more general topologies.

APPENDIX

We will prove that the private multiaccess coded caching scheme satisfies the privacy constraint in [2]. Note that from [4] and that the demands are independent of the placement phase, we have

\[
I(p_1, \ldots, p_K; Z_{U_k} | d_k, p_k, W_1, \ldots, W_N) = 0; \quad (8)
\]

and

\[
I(p_1, \ldots, p_K; Z_{U_k} | d_k, p_k, W_1, \ldots, W_N) = 0. \quad (9)
\]

With [9] and the demands are independent of the placement phase, we further have

\[
I(q_1, \ldots, q_{k-1}, q_{k+1}, \ldots, q_K; Z_{U_k} | d_k, p_k, W_1, \ldots, W_N) = 0. \quad (10)
\]

Focus on user \( U_k \) where \( k \in [K] \), we have

\[
I(d_{-k} | X, Z_{U_k} | d_k) \leq I(d_{-k} | X, q_1, \ldots, q_K, Z_{U_k}, W_1, \ldots, W_N | d_k) \quad (11a)
\]

\[
= I(d_{-k} | q_1, \ldots, q_K, Z_{U_k}, W_1, \ldots, W_N | d_k) \quad (11b)
\]

\[
= I(d_{-k} | Z_{U_k} | d_k, q_1, \ldots, q_K, W_1, \ldots, W_N) \quad (11c)
\]

\[
= H(Z_{U_k} | d_k, q_1, \ldots, q_K, W_1, \ldots, W_N) - H(Z_{U_k} | d_k, q_1, \ldots, q_K, W_1, \ldots, W_N) \quad (11d)
\]

\[
= H(Z_{U_k} | d_k, p_k, q_1, \ldots, q_K, W_1, \ldots, W_N) \quad (11e)
\]

\[
= H(Z_{U_k} | p_k, d_k, W_1, \ldots, W_N) - H(Z_{U_k} | p_k, W_1, \ldots, W_N) \quad (11f)
\]

\[
= H(Z_{U_k} | p_k, W_1, \ldots, W_N) - H(Z_{U_k} | W_1, \ldots, W_N) \quad (11g)
\]

\[
= 0. \quad (11h)
\]

where (11b) follows from the fact that \( X \) is a function of \((q_1, \ldots, q_K, W_1, \ldots, W_N)\), (11c) follows from the fact that \((q_1, \ldots, q_K, W_1, \ldots, W_N)\) are independent of the users’ demands, (11d) follows from (8) and (10), (11g) follows from the fact that the placement is independent of the demands.

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