Scaling Law in Laser Cooling on Narrow-Line Optical Transitions

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(Dated: 27 November 2018)

In this paper laser cooling of atoms with a narrow-line optical transition, i.e. in regimes of quantum nature of laser-light interactions resulting in a significant recoil effect, is studied. It is demonstrated that a minimum laser cooling temperature for two-level atom in standing wave reached for red detuning close to 3 recoil frequency greatly different from the theory used for a semiclassical description of Doppler cooling. A set of dimensionless parameters uniquely characterizing the time evolution and the steady state of different atoms with narrow-line optical transitions in the laser field is introduced. The results can be used for analysis of optimal conditions for laser cooling of atoms with narrow lines such as Ca, Sr, and Mg, which are of great interest for atomic clocks.

PACS numbers: 32.80.Pj, 42.50.Vk, 37.10.Jk, 37.10.De

Keywords: Laser cooling, atom kinetics, recoil effect

Nowadays deep laser cooling of neutral atoms is routinely used for wide range of modern quantum physics investigations, including metrology, atom optics, and quantum degeneracy studies. There exist well-known techniques for laser cooling below the Doppler limit, such as sub-Doppler polarization gradient cooling, velocity selective coherent population trapping, or Raman cooling restricted to atoms with energy levels degenerated over angular momentum or hyperfine structure. However, these techniques can not be applied to atoms with a single ground state, such as Sr, Ca, Mg, 174Yb, which are of great interest for atomic clocks.

For atoms with a single nondegenerated ground state the well-known theory based on a semiclassical approach predicts so-called Doppler laser cooling temperature $k_B T_D \approx \hbar \omega / 2$, with the natural linewidth $\omega$ of optical transition. One way of reaching a deeper cooling for these atoms is to use a narrow-line optical transition (clock transition) with a smaller natural linewidth $\gamma$. However, the basic semiclassical theory is not valid, since the main requirement, called the semiclassical limit $\omega / \gamma \ll 1$, is violated, that was clearly shown for atoms in a light field formed by two counterpropagating monochromatic waves (see Fig.1) resulting to standing wave with intensity modulation. The light field is close to the resonance of an atomic optical transition linewidth. Estimated Doppler limit temperatures for laser cooling at the narrow-line optical transition $^1S_0 \rightarrow ^3P_1$ for various atoms and values of the semiclassical parameter $\omega_R / \gamma$ are presented in Table I.

Note that deep laser cooling of Ca atoms at the narrow-line optical transition was made using “quenching cooling” techniques. These techniques involve the use of an additional light field, which allows increasing the effective optical transition linewidth $\gamma_{eff} \gg \gamma$ to “return back” to the semiclassical conditions $\omega_R / \gamma_{eff} \ll 1$ for the standard laser cooling regime. However, to our knowledge, no significant progress for Mg atoms has been achieved so far.

In this letter, we present a quantum theory of laser cooling in standing wave for regimes far beyond the semiclassical limit, i.e. for $\omega_R / \gamma \gtrsim 1$. This allows to clarify the laser cooling mechanisms with narrow-line optical transitions and estimate the optimal parameters for minimum cooling temperatures and cooling times especially for strong light field intensity resulting to “power broadening” regime of laser cooling. We find the laser cooling dynamics and steady-state momentum distribution can be uniquely characterized by set of dimensionless parameters for various atoms with different $\omega_R / \gamma$ ratio that results to a “scaling law” in laser cooling.

Let us consider the typical scheme of laser cooling of atoms in a light field formed by two counterpropagating monochromatic waves (see Fig.1) resulting to standing wave with intensity modulation. The light field is close to the resonance of an atomic optical transition

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| Atom | $T_D$ | $\lambda$ | $\omega_R / \gamma$ |
|------|-------|-----------|-------------------|
| $^{208}Hg$ | 32 $\mu K$ | 254 nm | 0.01 |
| $^{174}Yb$ | 4.5 $\mu K$ | 556 nm | 0.02 |
| $^{88}Sr$ | 0.17 $\mu K$ | 689 nm | 0.635 |
| $^{40}Ca$ | 10 $nK$ | 657 nm | 32.3 |
| $^{24}Mg$ | 0.75 $nK$ | 457 nm | 1100 |

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Table I. Semiclassical estimation of temperatures of laser cooling on intercombination lines $^1S_0 \rightarrow ^3P_1$.
with the natural linewidth $\gamma$. The behavior of atoms in the laser light field can be described by density matrix master equation with taking into account the quantum recoil effects in the processes of absorption and emission of photons

$$\frac{d}{dt}\hat{\rho} = -\frac{i}{\hbar} [\hat{H}, \hat{\rho}] + \hat{\Gamma} \{ \hat{\rho} \},$$  \hspace{1cm} (1)

where $\hat{H}$ is the Hamiltonian and $\hat{\Gamma} \{ \hat{\rho} \}$ is the relaxation operator due to spontaneous emission (see, for example\cite{11}). The density matrix contains all information about the internal and external states of atoms and the correlations in the processes of atom-light interactions.

The Hamiltonian in a resonant light field in the rotating wave approximation (RWA) has the following form:

$$\hat{H} = \frac{\vec{p}^2}{2M} - \hbar \delta \hat{P}_e + \hat{V}_{ed},$$  \hspace{1cm} (2)

The first term describes the kinetic energy of atoms, $\hat{P}_e = \langle e \rangle \langle e \rangle$ is the projection operator to the excited state $\langle e \rangle$, $\hat{V}_{ed} = \hbar \Omega / 2 \langle e \rangle \langle g \rangle + h.c.$ is the atom-light coupling of the ground (g) and excited (e) states in the electric dipole approximation, and $\Omega$ is the Rabi frequency. In standing light field the Rabi frequency has a spatial modulation $\Omega(z) = 4 \Omega_0 \cos^2(kz)$, where $\Omega_0$ is the Rabi frequency per one wave. Here $\delta = \omega - \omega_0$ is detuning of the laser light frequency $\omega$ from the atom optical transition frequency $\omega_0$.

First of all, let us emphasize the light field and atomic parameters that define the time evolution and the steady state of atom kinetics in the light field. The master equation (1) is determined by the following parameters of frequency dimension: $\gamma$, $\delta$, $\Omega$, and $\omega_R$.

The semiclassical regime is characterized by small recoil frequency

$$\omega_R / \gamma \ll 1,$$  \hspace{1cm} (3)

that is valid for major atoms with strong dipole optical transitions where laser cooling was realized (see for example\cite{21}).

Let us mention, within the framework of a two-level model (an atom with nondegenerate ground state) the atoms steady state distribution of laser cooling in the limit \cite{3} can be equally described by only two dimensionless parameters:

$$\delta / \gamma, \quad \Omega / \gamma.$$  \hspace{1cm} (4)

The cold atoms steady-state momentum distribution function for low saturation $S = |\Omega|^2 / (\delta^2 + \gamma^2 / 4) \leq 1$ and red detuning scales in these dimensionless parameters as

$$F_p = F \left( \frac{p}{\bar{p}}, \frac{\Omega}{\gamma}, \frac{\delta}{\gamma} \right), \quad \bar{p} = \hbar k \sqrt{\gamma / \omega_R}$$  \hspace{1cm} (5)

that represents the scaling law in semiclassical limit \cite{3}. The momentum distribution for all atoms in the semiclassical limit \cite{3} get equivalent from for momentum expressed in Doppler momentum units $\bar{p}$ \cite{3}. In particular, the steady-state average kinetic energy in the semiclassical limit scales in $\gamma$ units and it is a function of light field dimensionless parameters \cite{1}:

$$E_{\text{kin}} = \langle p^2 / 2M \rangle = \hbar \gamma F_S(\delta / \gamma, \Omega / \gamma)$$  \hspace{1cm} (6)

The function $F_S(\delta / \gamma, \Omega / \gamma)$ (see for example\cite{11}) for two-level atom has the minimum $\simeq 1 / 6$ at $\delta / \gamma = -1 / 2$ for $\Omega / \gamma \to 0$, so called Doppler cooling limit.

The momentum distribution on Fig.2 is obtained by numerical solution of density matrix equation (1) with the use of the method suggested by us in\cite{23,24}. The momentum distribution function is well scaled for introduced set of parameters, and in the limit of small ratio $\omega_R / \gamma$ the difference between the curves becomes less noticeable in low intensity (see Fig.2(a) and (b)). Here for the Fig2(c) and (d) we choose larger field intensity to get difference between the curves more visible.

The evolution of the momentum distribution of an atomic cloud and the laser cooling rate are determined by the slowest processes of atom momentum distribution function modification due to interaction with light field photons through the exchange of photon momentum. The cooling rate of slow atoms with $p < \hbar k \sqrt{\gamma / \omega_R}$ in semiclassical limit \cite{3}, can also be represented in equivalent form for various atoms by introducing a dimension-
less time $t/t_S$

$$t_S = \omega_R^{-1} \cdot \tau_S (\Omega/\gamma, \delta/\gamma),$$

that scales in $\omega_R^{-1}$ units and is a function of dimensionless parameters $\Omega/\gamma$ and $\delta/\gamma$ only.

On a base of laser cooling theory in semiclassical limit from the general relations (4) - (7) one may conclude that the narrow-line optical transitions with small natural linewidth might be perspective for deep and fast laser cooling, been considering same optimal field parameters as semiclassical theory predicts. This naive picture is not correct. For the laser cooling regimes far beyond the semiclassical limit, the recoil effects becomes essential as it was demonstrated in (12) for $\sigma_+ - \sigma_-$ field configuration. Here we may also expect significant modification of the scaling law.

In the quantum regime being considered

$$\omega_R \geq \gamma$$

the simple scheme of two-level atom interaction with light standing wave (Fig.1(b)) is significantly modified. The internal states can be written in a momentum representation as a set of families with a different momentum $p$ with a difference $\Delta p = 2\hbar k$ in the ground and excited states:

$$|\psi(t)\rangle = \sum_n \alpha_n(t)|g, p_0 + 2n\hbar k\rangle + \beta_n(t)|e, p_0 + (2n + 1)\hbar k\rangle.$$  

with quasi-momentum $p_0$ in the range $-\hbar k \leq p_0 \leq \hbar k$. Coupling schemes for families for $p_0 = \hbar k$ and $p_0 = 0$

are shown in Fig. 3. In the regime (8) the processes of induced absorption and emission of light field photons are beyond the resonance contour $\gamma$, and have different detuning:

$$\delta_p = \delta - \omega_R (1 \mp 2p/\hbar k),$$

depending on the atom momentum on the ground state, $p$. Here the sign “-” is related to induced emission of the photon to copropagating wave or absorption from counterpropagating wave, and the sign “+” related to reverse processes.

Let us define the scaling law in the regime (8). Here the equivalent parameters universally describing laser cooling of various atoms (4) and (7), are no longer valid. In this regime $\gamma$ is the smallest parameter and the steady-state solution of the master equation is governed by different set of dimensionless parameters:

$$\delta/\omega_R, \Omega/\omega_R,$$

defining the equivalence of laser cooling of various atoms with the use of the narrow-line transitions, that is, in the case of an essential quantum recoil effect in the atom-light interaction. The momentum distribution of atoms is a function of parameters scales not in $\gamma$, but in $\omega_R$ units, contrary to semiclassical limit:

$$F_p = F \left( \frac{p}{\hbar k}, \frac{\Omega}{\omega_R}, \frac{\delta/\omega_R}{\omega_R} \right)$$

and the average kinetic energy is determined by

$$E_{kin} = \langle p^2/2M \rangle = \hbar \omega_R F_Q(\delta/\omega_R, \Omega/\omega_R)$$

with $F_Q(\delta/\omega_R, \Omega/\omega_R)$ is dimensionless function of introduced parameters.

To confirm this, we perform a numerical simulation of the master equation (11) for various atoms (Sr, Mg, Ca) with narrow-line optical transitions for a set of different parameters (11).

The steady-state momentum distribution of atoms was obtained by solving the master equation (11) numerically by a method developed by us in (12). The method proposed allows one to obtain a direct solution for a density matrix containing all information on internal and external states without any restrictions and limitations.

Fig. 3 shows the results of steady state momentum distribution of various atoms ($^{88}$Sr, $^{24}$Mg, $^{40}$Ca) for the set of field parameters (11) in $\omega_R$ units. We see strong equivalence for the steady-state distribution of laser cooled $^{24}$Mg ($\omega_R/\gamma \approx 1100$), $^{40}$Ca ($\omega_R/\gamma \approx 32.3$), and $^{88}$Sr ($\omega_R/\gamma \approx 0.635$) atoms at the narrow-line $^1S_0 \rightarrow ^3P_1$ optical transitions that confirm the scalability principle (11-13) of laser cooling on narrow-line optical transitions.

First of all, notice sharp peaks in the momentum distribution at $p = \pm \hbar k$. These peaks represent the effects of velocity-selective coherent population trapping (12) for the $\Lambda$ scheme of the families (9) with $p_0 = \hbar k$ (Fig.3(a)) and was first demonstrated in a two-level system for $^{2}S_1 \rightarrow ^3P_2$ optical transition in metastable helium with $\omega_R/\gamma \approx 0.22$. 
that is far beyond optimal condition of laser cooling in semiclassical limit and close to one was shown in\textsuperscript{13} for $\sigma_+ - \sigma_-$ light field configuration. The momentum distribution is significantly nongaussian function for all considered range of detuning Fig\textsuperscript{5(b)}.

The possible qualitative explanation of optimal value of detuning (14) can be given as follows. For a low field intensity the population of the excited state is negligibly small. In this case, the atom distribution in the momentum space can be represented as a set of families (11) with nonzero amplitudes $\alpha_e(p_0)$ near $p_0 = 0$ (i.e. $|g, p = 0\rangle$ and $|g, p = \pm 2hk\rangle$ states), that results in a main peak in the momentum distribution with $p = 0$ and two side peaks at $p = \pm 2hk$ (Fig\textsuperscript{4(b)}). To obtain a minimum kinetic energy, it is necessary to suppress the amplitudes of these side peaks. The depopulation of $|g, p = \pm 2hk\rangle$ states is reached for detuning $\delta_\rho$ providing resonance for transitions $|g, p = \pm 2hk\rangle \rightarrow |e, p = \pm h\rangle$, i.e. for $\delta_\rho = \delta + 3\omega_R = 0$ according to (10), see Fig\textsuperscript{5(b)}. Thus the detuning $\delta = -3\omega_R$ results effective suppression of side peaks of momentum distribution at $p = \pm 2hk$ Fig\textsuperscript{5(b)} and effective population of $|g, p\rangle$ state around $p = 0$.

Finally, an important question is the time evolution of atomic distribution and cooling time. In the quantum regime being considered (8), the atom cooling rate can be represented in equivalent form that is different form the semiclassical case (3). The scaling low for cooling rate on narrow lines can be written for dimensionless time $t/t_Q$ in the following form:

$$t_Q = \gamma^{-1} \cdot \tau_Q (\Omega/\omega_R, \delta/\omega_R)$$

where time $t_Q$ scales in inverse $\gamma$ units apart from the semiclassical definition and a function of set of dimensionless parameters $\Omega/\omega_R$ and $\delta/\omega_R$ that define scaling law in quantum regime of laser cooling (12).

Fig\textsuperscript{6} shows the time evolution of the momentum distribution obtained by the quantum Monte-Carlo method\textsuperscript{26} with averaging over 3000 trajectories. Here we consider the laser cooling from an initial momentum distribution $p_\sigma = 20hk$ corresponding the initial temperature of Mg atoms, $T \approx 1.5mK$, that is, the laser cooling temperature in a MOT with use of the strong dipole optical transition $^1S_0 \rightarrow ^1P_1$, which can be treated as an initial stage of laser cooling for further subcooling cooling temperatures with the use of the narrow-line optical transition.

As was discussed above, the slow cooling rate with use of narrow-line optical transition is the main obstacle for realization of effective laser cooling in monochromatic light. The different modification like cooling with broadband light and “quenching cooling” techniques we considered\textsuperscript{17,20}. However in “scaling law” for considered quantum the regimes of laser cooling significantly different from the semiclassical one. In particular, the field dependence in semiclassical regime $\Omega/\gamma$ changes to $\Omega/\omega_R$ that gives possibility to use significantly stronger laser field intensity without increasing much the steady-state temperature of cold atoms Fig\textsuperscript{7}(a). Thus the “power broadening” allows to significantly increase the cooling rate without much lose in the temperature. The cool-
larger steady-state kinetic energy of the atoms (Fig. 7(b)).

\[ \text{stronger light field results in faster evolution but in a} \]

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semiclassical description of the standard Doppler theory

atom fraction with the momentum \( |p| < 3\hbar k \) time evolution

\[ \text{in} \gamma^{-1} \text{units (b)).} \]

\[ \text{In attempts to reach deep laser cooling of atoms with} \]

nondegenerate ground states, such as \( ^{24}\text{Mg}, \ ^{40}\text{Ca}, \) and \( ^{88}\text{Sr} \) the researchers tried to use the narrow-line optical

transitions. However, due to the limitation of the

semiclassical description of the standard Doppler theory of

laser cooling, the optimal light field parameters differ

significantly. This may not lead to the expected subrecoil laser cooling temperatures in experiments with light field detuning and intensity extrapolated from standard the semiclassical theory. The laser cooling at the narrow-line optical transition requires special study we perform here. We have shown the equivalence of laser cooling of various atoms with \( \omega_R \geq \gamma \) for a set of dimensionless light field parameters \( (\Omega/\omega_R \text{ and } \delta/\omega_R) \) scaled in recoil frequency units that represent a scaling low in laser cooling on narrow-line optical transition. In introduced dimensionless units the momentum distribution of various atoms are equivalent for momentum scaled in \( \hbar k \) units. We have demonstrated that in standing wave the kinetic energy of cold atoms reached it minimum for detuning close to \( \delta \sim -3\omega_R \), i.e. is far from the Doppler theory of laser cooling.

The cooling time with the use of narrow-line optical transition scales in the \( \gamma^{-1} \) units differ from the semiclassical limit (cooling time scales in the \( \omega_R^{-1} \) units) and efficient cooling rate can be achieved with “power broadening” for the Rabi of few recoil frequency.

The research was supported by RSF (project No. 16-12-00054).

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