PECULIAR HUBBLE FLOWS IN OUR LOCAL UNIVERSE

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ABSTRACT
A formalism that simultaneously searches for monopolar and dipolar peculiar velocities is presented. The formalism is applied to (1) the Mark III Catalog of Galaxy Peculiar Velocities, (2) Lauer & Postman’s Abell cluster catalog, and (3) Riess et al.’s Type Ia supernova sample. The emphasis is drawn to the monopolar peculiar velocities, i.e., peculiar Hubble flows, within these samples. The samples show inconsistent peculiar Hubble flows within a depth of ~60 h⁻¹ Mpc. Beyond a depth of ~80 h⁻¹ Mpc, the Hubble flows of all samples converge to the global Hubble flow to better than 10% at the 2σ level. The results are compared with theoretical predictions. They at face value disfavor models that predict smaller peculiar velocities, such as the tilted cold dark matter model. Limitations of the catalogs are discussed, and some are ways to improve the catalogs so that an accurate map of Hubble flows in our local universe can be drawn and be compared with theoretical predictions.

Subject headings: cosmology: observations — cosmology: theory — galaxies: distances and redshifts — large-scale structure of universe

1. INTRODUCTION
Peculiar Hubble flows (monopolar deviations from a global Hubble flow) are as important as bulk motions (dipolar deviations from a global Hubble flow) in reflecting the underlying density fluctuation of the universe. But they have not received the same attention as bulk motions have, because the uncertainty in our knowledge about the true Hubble constant $H_0$ is often greater than the expected peculiar Hubble flows at large scales. One can, however, investigate the variation of Hubble flows within a sample without knowing the value of $H_0$. Moreover, the inferred Hubble flows from samples that extend significantly beyond 100 h⁻¹ Mpc should be so close to the global Hubble flow that it is meaningful to investigate the peculiar Hubble flows within the samples. In doing so, the ultimate goal is to map out the variation of the Hubble expansion as a function of depth, which is directly related to the underlying density fluctuation.

Here I first present a formalism that simultaneously calculates the peculiar Hubble flows and the bulk motions, along with their errors, within a sample. The formalism is applied to (1) the Mark III Catalog of Galaxy Peculiar Velocities, based on the Tully-Fisher relation as a distance indicator (Willlick et al. 1997), (2) the Abell cluster catalog of Lauer & Postman (1994, hereafter LP), with brightest cluster galaxies (BCGs) as distance indicators, and (3) the sample of 20 Type Ia supernovae of Riess, Press, & Kirshner (1996). All three samples are deep or fairly deep: the Mark III Catalog extends to 11,000 km s⁻¹; the LP sample is complete up to 15,000 km s⁻¹; the supernova sample probes the deepest, 28,000 km s⁻¹. In other words, Hubble flows defined by these samples should approximate the global Hubble flow to a fairly high degree (see Table 1 below and the text).

The results from the three samples are presented and analyzed in §§ 3–5. They are compared with theoretical predictions from four representative models (Table 1): the standard cold dark matter (SCDM) model, a tilted cold dark matter (TCDM) model, a vacuum energy Λ-dominated cold dark matter (ΛCDM) model, and a cold plus hot dark matter (CHDM) model. Finally, I conclude in § 6 and comment briefly on improving the measurements of peculiar Hubble flows and their potential to test theoretical models.

2. FORMALISM
The peculiar Hubble flow and the bulk motion of a sample can be obtained by maximizing the likelihood

$$L(U_i, \delta H) = \prod_q \frac{1}{\sigma_q^2} \exp \left[ -\frac{(S_q - \delta H r_q^i - U_i r_q^i)^2}{2\sigma_q^2} \right]$$

(Kaiser 1988, 1991), where $r_q^i = r_q^i \hat{r}_q^i$, $i = x, y, z$ is the position of an object $q$ in the sample and $S_q$ is its estimated line-of-sight peculiar velocity, with an uncertainty $\sigma_q^2$; $U_i(\hat{r}_q^i, i = x, y, z)$ is the bulk motion of the sample, and $\delta H$ is its peculiar Hubble expansion rate. Maximizing $L(U_i, \delta H)$ with respect to $U_i$ and $\delta H$ yields

$$U_i = (A - RB^{-1})_{ij} \sum_q S_q r_q^j / \sigma_q^2 - B^{-1} \sum_q S_q r_q^i r_q^j / \sigma_q^2 \sigma_q^2$$

$$\delta H = B^{-1} \sum_q S_q r_q^i r_q^j / \sigma_q^2$$,

where

$$A_{ij} = \sum_q \frac{r_q^i r_q^j}{\sigma_q^2}, \quad R_{ij} = \sum_q \frac{r_q^i r_q^j}{\sigma_q^2 \sigma_q^2}, \quad B = \sum_q \frac{r_q^2}{\sigma_q^2}$$.

(1)

(2)

(3)
In linear theory, while the second term represents the contribution from noise in the data. Its Fourier transform is

$$\langle d^3 r W'(r) v_i(r) \rangle,$$

(6)

while the second term represents the contribution from noise in the data. The window function in equation (6) is

$$W'(r) = \hat{r} B r^{-1} \left( \sum_i \frac{r_i}{\sigma_i^2} \delta(r - r_i) - (A - RB^{-1})_{ij} \right) \frac{1}{\sqrt{2}} \left( \sum_i \frac{r_i}{\sigma_i^2} \right) \delta(r - r_i) - B^{-1} \left( \sum_i \frac{r_i}{\sigma_i^2} \right) \delta(r - r_i).$$

(7)

Its Fourier transform is

$$W'(k) = \frac{B^{-1}}{(2\pi)^3/2} \left( \sum_i \frac{r_i}{\sigma_i^2} e^{ik \cdot r_i} - (A - RB^{-1})_{ij} \right) \frac{1}{\sqrt{2}} \left( \sum_i \frac{r_i}{\sigma_i^2} \right) e^{ik \cdot r_i} - B^{-1} \left( \sum_i \frac{r_i}{\sigma_i^2} \right) e^{ik \cdot r_i} \sum_i \frac{r_i}{\sigma_i^2}. $$

(8)

The expected rms peculiar Hubble flow for the sample is

$$\langle \left( \frac{\delta H}{H_0} \right)^2 \rangle = \langle \left( \frac{\delta H^o}{H_0} \right)^2 \rangle + \langle \left( \frac{\delta H^{e}}{H_0} \right)^2 \rangle.$$

(9)

In linear theory,

$$\langle \left( \frac{\delta H^o}{H_0} \right)^2 \rangle = \Omega_{o,2} \int d^3 k |W'(k)k| \frac{P(k)}{k^2}.$$

(10)

From equations (4) and (5),

$$\langle \left( \frac{\delta H^{e}}{H_0} \right)^2 \rangle = B^{-2} \left( \sum_i \frac{r_i}{\sigma_i^2} - (A - RB^{-1})_{ij} \right) \frac{1}{\sqrt{2}} \left( \sum_i \frac{r_i}{\sigma_i^2} \right) \delta(r - r_i) - B^{-1} \left( \sum_i \frac{r_i}{\sigma_i^2} \right) \delta(r - r_i).$$

(11)

Similarly,

$$\langle U_l^o U_l^{e} \rangle = (A - RB^{-1})_{ij}^{-1}.$$

(12)
the whole sample (with a rate $H_1$). Thus
\[
\left\langle \left( \frac{H_1 - H_2}{H_1} \right)^2 \right\rangle = \left\langle \left( \frac{\delta H_1 + H_0 - \delta H_2 - H_0}{\delta H_1 + H_0} \right)^2 \right\rangle \approx \left\langle \left( \frac{\delta H_1}{H_0} \right)^2 + \left( \frac{\delta H_2}{H_0} \right)^2 - 2 \left( \frac{\delta H_1}{H_0} \right) \left( \frac{\delta H_2}{H_0} \right) \right\rangle 
\]
\[
= \left\langle \left( \frac{\delta H_1^{(o)}}{H_0} \right)^2 + \left( \frac{\delta H_2^{(o)}}{H_0} \right)^2 - 2 \left( \frac{\delta H_1^{(o)}}{H_0} \right) \left( \frac{\delta H_2^{(o)}}{H_0} \right) \right\rangle + \left\langle \left( \frac{\delta H_1^{(o)}}{H_0} \right)^2 + \left( \frac{\delta H_2^{(o)}}{H_0} \right)^2 - 2 \left( \frac{\delta H_1^{(o)}}{H_0} \right) \left( \frac{\delta H_2^{(o)}}{H_0} \right) \right\rangle, \quad (13)
\]
in which
\[
\left\langle \left( \frac{\delta H_1^{(o)}}{H_0} \right) \left( \frac{\delta H_2^{(o)}}{H_0} \right) \right\rangle = \Omega_0^{1/2} \int d^3 \mathbf{k} W_1^*(\mathbf{k}) \tilde{W}_2(\mathbf{k}) \tilde{k}_j \overline{P(k)} \frac{P(k)}{k^2}, \quad \left\langle \left( \frac{\delta H_1^{(o)}}{H_0} \right) \left( \frac{\delta H_2^{(o)}}{H_0} \right) \right\rangle = \left\langle \left( \frac{\delta H_1^{(o)}}{H_0} \right)^2 \right\rangle. \quad (14)
\]

Here subscript "1" refers to quantities of the entire sample, and subscript "2" refers to those of the subsample.

### 3. Peculiar Hubble Flows in the Mark III Catalog

The Mark III Catalog (Willick et al. 1997) compiles the distances and redshifts to about 3000 galaxies, based on a template Tully-Fisher relation from the 36 clusters of Han and Mould (hereafter the "HM sample"; Mould et al. 1991; Han & Mould 1992; Mould et al. 1993). Therefore the underlying Hubble expansion of the Mark III Catalog is defined by the HM sample. Table 1 lists the expected 1σ deviation of this underlying Hubble expansion rate $H_1$ from a global Hubble constant $H_0$ in various models, which is typically 2%–3%. (In this and all the following calculations, clusters are assumed to have a Gaussian random motion with a one-dimensional dispersion of 300 km s$^{-1}$, and all bulk motions are in reference to the cosmic microwave background.) Figure 1a shows the variation of Hubble flows in the HM sample with 2σ error bars calculated from equations (2)–(14) versus the depth $R$ of subsamples. Each subsample is defined to be the $N$ clusters closest to us, and $R$ is the
distance to the farthest cluster in the subsample. The errors of the variation are estimated from the second term of equation (13). In particular, they include the contribution from uncertainties in determining bulk motions.

The figure shows significant positive variation within $40 - 60 \, h^{-1} \, \text{Mpc}$, translating into significant peculiar Hubble flows $H_2 - H_0$ given the smallness of $H_1 - H_0$. In Figure 1b, the variation is compared with $2 \sigma$ expectations (with noise included) from theoretical models. Clearly, the positive values of $(H_2 - H_1)/H_1$ found exceed the $2 \sigma$ expectation from SCDM within $\sim 50 \, h^{-1} \, \text{Mpc}$, and they exceed the $2 \sigma$ expectations from CHDM, TCDM, and ΛCDM within a wider range of subsamples. The variation at scales beyond $\sim 60 \, h^{-1} \, \text{Mpc}$, however, is perfectly consistent with all models at $2 \sigma$. Not only is the variation of Hubble flows in the HM sample extremely large, its bulk flow is also extremely large, at $(740 \pm 150 \, \text{km s}^{-1}, -267 \pm 134 \, \text{km s}^{-1}, -360 \pm 141 \, \text{km s}^{-1})$, compared with the noise-free $1 \sigma$ SCDM expectation of $450 \, \text{km s}^{-1}$, or the noise-free $1 \sigma$ TCDM expectation of $240 \, \text{km s}^{-1}$.

Neither the large variation of Hubble flows within $\sim 50 \, h^{-1} \, \text{Mpc}$ nor the large bulk flows may be surprising. Figure 2a projects the positions of the inner 20 clusters (with a depth of $56 \, h^{-1} \, \text{Mpc}$) and the direction of their bulk motion on the sky. The four clusters in the lower right quadrant of the map, Telescopium at $24 \, h^{-1} \, \text{Mpc}$, OC 3627 at $29 \, h^{-1} \, \text{Mpc}$, Pavo II at $37 \, h^{-1} \, \text{Mpc}$, and OC 3742 at $40 \, h^{-1} \, \text{Mpc}$, all show positive radial peculiar velocities. Since they lie roughly between us and the Great Attractor (GA, at $\sim 4200 \, \text{km s}^{-1}$ and in the direction of Hydra-Centaurus; Lynden-Bell et al. 1988), their positive radial peculiar velocities may be the result of the gravitational pull of the GA. The clusters not in the general direction of the GA are farther away from it and thus experience a smaller infall into the GA. As a result, the subsample shows a large dipole in the direction of the four clusters, plus a large positive monopolar flow. Even the entire HM sample with 36 clusters going up to $11,000 \, \text{km s}^{-1}$ may show the influence of the GA, since, from Figure 2b, a projection of the outer 16 clusters, no new cluster beyond the distance of the GA is added in the same direction as the four clusters, and almost all the outer 16 clusters are in the opposite hemisphere. To further show the effect of the four clusters, Figure 3 plots $(H_2 - H_1)/H_1$ versus the depth $R$ of

![Figure 2](image-url)

**Fig. 2.** (a) Projection of the inner 20 clusters of the HM sample and their bulk motion in Galactic coordinates. Solid circles denote clusters with negative radial peculiar velocities; open circles denote clusters with positive radial peculiar velocities. The cross is the direction of the bulk motion. (b) Projection of the outer 16 clusters of the HM sample in Galactic coordinates. The dotted line is the equator of the sky when the bulk motion of the inner 20 clusters is chosen as a pole.
Fig. 3.—Variation of Hubble flows in the HM sample less the four clusters in the lower right quadrant of Fig. 2a, with 2σ error bars.

Fig. 4.—Projection of (a) 277 groups of the Mathewson et al. (1992) sample and (b) 11 clusters of the Willick (1991) sample, in Galactic coordinates. Symbols are as in Fig. 2.
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Fig. 5.—Similar to Fig. 1, but for the M + W sample. Errors and theoretical expectations are calculated only for one subsample and the entire sample.

Subsamples without the four clusters. The positive variation of Hubble flows at 40–60 \( h^{-1} \) Mpc becomes much less significant and no longer exceeds the 2 \( \sigma \) expectations of the four chosen models. The bulk motion of the remaining 32 clusters, however, is still large, at \((718 \pm 205 \text{ km s}^{-1}, -216 \pm 140 \text{ km s}^{-1}, -344 \pm 191 \text{ km s}^{-1})\). Clearly, the HM sample is so sparse and inhomogeneous that spurious bulk motions and peculiar Hubble flows can result from structures such as the GA. The window functions \( W_i(k) \) of the sample, on the other hand, may not fully reflect the density waves that give rise to these structures. Therefore, the theoretical expectations based on these window functions can be too small. This also implies that the assumption that the HM sample well represents the true Hubble flow may be rather poor.

Similar calculations were performed for the 277 galaxy groups of Mathewson, Ford, & Buchhorn (1992) plus the 11 clusters of Willick (1991) (hereafter the “M + W sample”) in the Mark III Catalog. The Aaronson et al. (1982) sample is too shallow to be included. The Mathewson et al. (1992) sample is much denser than the HM sample. It also goes up to \( \sim 10,000 \text{ km s}^{-1} \), but it only covers the southern sky. The Willick (1991) sample is too sparse, but it is not overly influenced by the GA (thanks to the large distances of its members from the GA). Figure 4 shows the projections of the two samples on the sky. Figure 5a shows the variation \((H_2 - H_1)/H_1\) versus the depth \( R \) of the subsamples of the M + W sample. Figure 5b shows the comparison between this variation and 2 \( \sigma \) theoretical expectations (with noise included). The errors and noise-included theoretical expectations are calculated only for two representative points, because both quantities are monotonic, slowly varying functions of \( R \). It should also be pointed out that \( H_1 \) is the Hubble expansion rate of the HM sample because it provides the template Tully-Fisher relation for the Mark III Catalog. Therefore, instead of equation (13), we have

\[
\left\langle \left( \frac{H_2 - H_1}{H_1} \right)^2 \right\rangle \approx \left\langle \left( \frac{\delta H^{(0)}}{H_0} \right)^2 + \left( \frac{\delta H^{(5)}}{H_0} \right)^2 \right\rangle + \left\langle \left( \frac{\delta H^{(0)}}{H_0} \right)^2 \right\rangle .
\]

The bulk motion of the M + W sample is relatively small, \((256 \pm 57 \text{ km s}^{-1}, -343 \pm 61 \text{ km s}^{-1}, 170 \pm 47 \text{ km s}^{-1})\), while the noise-free expectation of SCDM is \(670 \text{ km s}^{-1} \) and that of TCDM is \(361 \text{ km s}^{-1} \).

There are two conclusions that can be drawn from Figure 5: (1) the M + W sample shows Hubble flows that are 6%–8% faster than the Hubble flow of the HM sample, with \( \sim 4 \sigma \) significance; (2) the differences between the HM sample of 36
clusters and the combined M + W sample are consistent with theoretical expectations within 2 σ, although the consistency is marginal for models that predict smaller peculiar velocities, such as TCDM and ΛCDM. But because of the poor sampling of the HM sample, whether the first conclusion implies a significant positive peculiar Hubble flow in the M + W sample is murky.

4. PECULIAR HUBBLE FLOWS IN THE LP CATALOG

Lauer & Postman’s Abell cluster catalog is a volume-limited sample that includes 119 Abell clusters with redshifts of ≲ 0.05 and a Galactic latitude above 13° on both hemispheres. Its advantages over the HM sample are that it is volume limited, more homogeneous, and deeper. As shows, its Hubble flow is expected to deviate from the global Hubble flow by only ~1%. The standard candle of the catalog is taken to be the luminosity L of the brightest cluster galaxies as a function of the second parameter, α, the power index of L as a function of the aperture. The distance to a BCG is taken to be its cosmological redshift, i.e.,

\[
\rho_q = (cz_q - U_i r_i^q - \delta H_1 r_q) / H_0 = (cz_q - U_i r_i^q) / (H_0 + \delta H_1) = (cz_q - U_i r_i^q) / H_1,
\]

where \( H_1 \) is the Hubble expansion rate defined by the entire sample. The estimated line-of-sight peculiar velocity is

\[
S_q = cz 10^{0.4 M_g(\alpha_q) - M_g(2 - \alpha_q)},
\]

where \( M_g \) is the absolute magnitude of BCG and \( M_g(\alpha_q) \) is the magnitude of the standard candle at \( \alpha = \alpha_q \).

When applying equations (1)-(4) to the LP catalog, because \( M_g \) and \( \alpha_q \) depend on \( \rho_q \) and thus on \( U_i \) and \( \delta H_1 \), one has to iterate equations (1)-(4) to obtain a self-consistent result. Assuming no peculiar Hubble flow, Lauer & Postman (1994) obtained a bulk flow for the sample of \( U_x = 477 \pm 250 \) km s\(^{-1}\), \( U_y = -142 \pm 273 \) km s\(^{-1}\), \( U_z = 635 \pm 198 \) km s\(^{-1}\) relative to the rest frame of cosmic microwave background radiation. I first repeat their calculation, log-linearly interpolating Table 3 of LP to find the dependence of \( M_g \) and \( \alpha_q \) on \( \rho_q \). I obtain \( U_x = 475 \pm 285 \) km s\(^{-1}\), \( U_y = -123 \pm 308 \) km s\(^{-1}\), \( U_z = 648 \pm 225 \) km s\(^{-1}\), in very good agreement with LP’s result. The errors are estimated from equation (12) and verified by Monte Carlo simulations similar to those of LP.

I then consider the possibility of a nonzero \( \delta H_1 = H_1 - H_0 \). Since the distance scale is established within the sample itself, \( \delta H_1 \) of the sample is really not a variable but a well-defined value. In particular, one can always redefine the true Hubble expansion rate as \( H_0 = H_0 + \delta H_1 = H_1 \) so that variation of Hubble flows in the sample can be investigated without referring to \( H_0 \), just as in the previous section.

However, if one simply follows LP’s way of calibrating \( M_g(\alpha) \), redefining \( H_1 \) to zero cannot be done. This is due to the fact that the nonlinear relation between \( S_q/cz_q \) and \( M_g(\alpha) - M_g \) always skews the distribution of \( S_q/cz_q \) to the negative direction when \( M_g(\alpha) - M_g \) is Gaussian, as calibrated by regression from the \( M-\alpha \) distribution. An unphysical and negative \( \delta H_1/H_0 \) will always result for such a calibration, regardless of the value of \( H_0 \). For the LP sample, \( \delta H_1/H_0 \approx -1.2\% \).

One way to get rid of the unphysical \( \delta H_1 \) and set \( \delta H_1 \) to zero is to calibrate \( M_g(\alpha) \) by regression from the \( S_q/cz_q \)-\( \alpha \) distribution (my calculation shows that the resulting \( S_q/cz_q \) residual is Gaussian with a standard deviation of 0.166 at the 6% confidence level). The resultant bulk motion of the sample is then \( U_x = 528 \pm 285 \) km s\(^{-1}\), \( U_y = -272 \pm 311 \) km s\(^{-1}\), \( U_z = 607 \pm 225 \) km s\(^{-1}\), \( U_y \), being the most uncertain component, shows the greatest difference from the LP result. The resultant \( \delta H_1 \) converges to zero to a high precision.

Once \( M_g(\alpha) \) is calibrated, one can calculate the variation of Hubble flows within the sample. Figure 6 shows the variation with 2 σ error bars versus the depth of the LP subsamples and its comparison with noise-included 2 σ theoretical expectations. The error bars are once again calculated using equation (13). They are checked with Monte Carlo simulations, and consistency is found. The figure shows evidence for negative \( (H_2 - H_1)/H_1 \) within a radius of ~60 h\(^{-1}\) Mpc at the 2 σ level, indicating a negative peculiar Hubble flow at this scale given the tiny deviation of \( H_1 \) from \( H_0 \) (Table 1). But before the negative peculiar Hubble flow is explained with overdensities, possible systematic effects have to be considered, as in the case of the Mark III Catalog.

Possible biases in the analysis have been discussed extensively in LP. Among them are several radially dependent biases that affect the calculation of peculiar Hubble flows. The first is the selection bias. A luminosity-limited catalog introduces an artificial Hubble outflow due to the missing low-brightness galaxies. This is apparently not the case with LP’s catalog, because it is volume limited. Second, the estimated peculiar velocity depends on the deceleration parameter \( q_0 \) assumed. But for a catalog extending only to \( z \approx 0.05 \), the estimated radial peculiar velocity is only changed by \( \lesssim 1\% \) if the true \( q_0 \) is changed by 0.5. Since the standard candle in LP’s catalog is established internally, influenced mostly by outlying clusters, the effect on the peculiar Hubble flow of inner clusters is \( \lesssim 1\% \). A third bias comes from the random peculiar velocities of BCGs due to local nonlinearities, which tend to scatter more BCGs to lower measured redshifts than to higher redshifts. But given a typical value of this random radial velocity of 300 km s\(^{-1}\), the velocity bias introduced on the 6000 km s\(^{-1}\) scale is only 0.5% (LP). Another important concern is whether the BCGs of the inner Abell clusters belong statistically to the same population of the entire BCG sample. Table 2 lists the statistical properties of the inner BCG subsamples and the entire BCG sample. They are consistent statistically.

But a problem arises when one tests whether the detection is dominated by a small number of clusters. Figure 7 projects the inner 11 clusters (which show the negative peculiar Hubble flow at 2 σ) on the sky. The subsample is very inhomogeneous because of its small size. In particular, there is one (and only one) cluster (A262) that roughly aligns with the bulk motion of the subsample. Since the bulk motion and the peculiar Hubble flow are not determined independently, the cluster will

\(^1\) The cosmological distortion is small and is omitted; see the discussion at the end of this section.
certainly play a dominant role in determining both quantities. If the cluster is excluded, as shown in the inset to Figure 6a, no variation of Hubble flows is found at the 2 $\sigma$ level. Therefore, once again, the poor sampling of the subsample renders its result susceptible to systematic effects such as small-scale structures, unrepresented structures, and small number statistics.

5. PECULIAR HUBBLE FLOWS IN A TYPE Ia SUPERNOVA SAMPLE

Type Ia supernova samples probe much deeper ($\sim 10^3$ Mpc) and have very precise distance measurements ($\sim 5\%$). Therefore, even for the 20 Type Ia supernovae compiled by Riess et al. (1996), Table 1 shows that the expected 1 $\sigma$ deviation of its Hubble flow from the global value is only $\sim 1\%$. But because of its extremely sparse sampling, its usefulness to investiga-

\begin{table}[h]
\centering
\begin{tabular}{lllll}
\hline
$N^a$ & $\langle x \rangle^b$ & $\langle M_q - M_q(x) \rangle_x^c$ & $\sigma_{M_q - M_q(x)}^d$ & $P_{KS}^e$ \\
\hline
8      & 0.61   & 0.141 & 0.255 & 83 \\
9      & 0.61   & 0.113 & 0.253 & 71 \\
10     & 0.59   & 0.109 & 0.239 & 84 \\
11     & 0.59   & 0.106 & 0.227 & 93 \\
119... & 0.57   & 0     & 0.245 & 39 \\
\hline
\end{tabular}
\caption{Comparison of Statistical Properties in the LP Sample}
\end{table}

\begin{itemize}
    \item $^a$ Number of clusters in the (sub)sample.
    \item $^b$ Average $x$ in the (sub)sample.
    \item $^c$ Average $M_q - M_q(x)$ in the (sub)sample.
    \item $^d$ Standard deviation of $M_q - M_q(x)$.
    \item $^e$ Confidence level that $M_q - M_q(x)$ is Gaussian according to a Kolmogorov-Smirnov test.
\end{itemize}
tions of peculiar Hubble flows is significantly compromised. Figure 8 shows the variation of Hubble flows in the Type Ia supernova sample. No significant variation is found at any scale, because of its sparse sampling. Since the errors in the plot roughly scale as the inverse square root of the number of objects, as more Type Ia supernovae are being observed with good time coverage, they can certainly provide more accurate results at a wide range of depths in the future.

6. DISCUSSION

After investigating the variation of Hubble flows in the Mark III Catalog (Willick et al. 1997), the Lauer & Postman (1994) sample, and the Type Ia supernova sample of Riess et al. (1996), there are three conclusions that I would like to draw:

1. Some significant peculiar Hubble flows are found in the Mark III Catalog and at 50–60 $h^{-1}$ Mpc in the Lauer & Postman sample. They at face value disfavor cosmological models that predict smaller peculiar velocities, such as TCDM. However, because of the sparse and inhomogeneous sampling of the HM cluster sample, on which the Hubble expansion of the Mark III Catalog is defined, and because of the sparse sampling of the LP sample within the 60 $h^{-1}$ Mpc scale, the
peculiar Hubble flows found are dominated by statistics of a small number of clusters and thus cannot be trusted at their face value. Further supporting this is the fact that the peculiar Hubble flows found in the different samples are inconsistent: the peculiar Hubble flows found are dominated by statistics of a small number of clusters and thus cannot be trusted at their face value. Further supporting this is the fact that the peculiar Hubble flows found in the different samples are inconsistent: the peculiar Hubble flows found are dominated by statistics of a small number of clusters and thus cannot be trusted at their face value.

2. All samples agree that the Hubble flows beyond a depth of \( \sim 80 \, h^{-1} \) Mpc are true to the global Hubble flow to better than 10% at the \( \sim 2 \sigma \) confidence level. In each sample, the variation of Hubble flows at a depth of \( \geq 80 \, h^{-1} \) Mpc, with respect to the Hubble flow of entire sample, is found to be less than 10% at \( 2 \sigma \). This, combined with the small expected deviation (1%–3% at 1 \( \sigma \)) see Table 1) of the Hubble flow defined by each sample from the global Hubble flow, ensures that the Hubble flows at \( \geq 80 \, h^{-1} \) Mpc conform to the global Hubble flow to better than 10% at the \( \sim 2 \sigma \) confidence level. At depths below \( 80 \, h^{-1} \) Mpc, the limits on peculiar Hubble flows are somewhat weaker. For example, at the depth of the Coma Cluster (70 \( h^{-1} \) Mpc), while the LP sample indicates a less than 15% deviation from the true Hubble flow at \( 2 \sigma \), all others imply a stronger limit—less than 10% deviation from the true Hubble flow at \( 2 \sigma \) confidence level.

3. Since the peculiar Hubble flows detected in the samples are untrustworthy at the moment, so is their comparison with theoretical predictions. However, this is not to say that any such comparison will be useless, because the major limiting factor to a meaningful comparison between models and observations is neither our uncertain knowledge of the Hubble constant nor the uncertainties in the distance scales, but rather the imperfect sampling of our local universe. The errors achieved by the current Mark III Catalog are comparable (\( \sim 1% \) at \( \sim 60 \, h^{-1} \) Mpc) to the differences in noise-free model predictions. Therefore, if samples were improved to yield a consistent and reliable detection of peculiar Hubble flows, measurements of peculiar Hubble flows would certainly be able to test models.

While the first and second conclusions represent an attempt to map out Hubble flows in our local universe, which are extremely interesting in their own right, the third conclusion aims at finding a potential tool to gain further information about cosmological parameters and therefore warrants further discussion. Since errors in the question are roughly proportional to the square root of the number of objects in a sample, if a deep (\( > 100 \, h^{-1} \) Mpc), homogeneous (covering all directions and depth), and dense (at least \( 10^2 \) clusters to yield a template Tully-Fisher relation and \( \sim 10^3 \) clusters and groups within \( 100 \, h^{-1} \) Mpc to be sampled) sample can be assembled with the Tully-Fisher relation, a reliable detection of peculiar Hubble flows may be made with errors at the \( \lesssim 1\% \) level, sufficient to test many models at the \( \sim 60 \, h^{-1} \) Mpc scale. Or, if more than 100 Type Ia supernovae are adequately observed as distance indicators, the errors of their measured peculiar Hubble flows can be cut by more than half from the current level, to \( \sim 1\% \). Better yet, if the Tully-Fisher relation is calibrated with Type Ia supernovae, by observations of Type Ia supernovae in clusters, not only will the overall Hubble flow of the Tully-Fisher sample represent the global Hubble flow better, the two samples can be combined to yield better statistics.

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