ON THE UNCERTAINTY IN X-RAY CLUSTER MASS ESTIMATES FROM THE EQUATION OF HYDROSTATIC EQUILIBRIUM

CHRISTOPHE BALLAND
Center for Particle Astrophysics, University of California at Berkeley, 301 Le Conte Hall, Berkeley, CA 94720

AND

ALAIN BLANCHARD
Observatoire Astronomique de Strasbourg, 11 Rue de l'Université, F-67000 Strasbourg, France

Received 1995 October 24; accepted 1997 April 24

ABSTRACT

We study the uncertainty in galaxy cluster mass estimates derived from X-ray data, assuming hydrostatic equilibrium for the intracluster gas. Using a Monte Carlo procedure, we generate a general class of mass models allowing very massive clusters. We then compute the corresponding temperature profiles from the equation of hydrostatic equilibrium and compare them to observational data for some clusters. We find several massive clusters that pass the observational constraints, with integrated masses varying over quite a wide range. The resulting accuracy of the mass estimates is rather poor, the uncertainties larger than what is generally claimed. Despite the fact that the mass profile can be exactly determined mathematically from the temperature and surface brightness profiles, we find that very accurate measurements of both quantities are required to determine the actual mass with moderate accuracy. We argue that the tight constraints on cluster masses previously obtained arise from the fact that a too restricted class of mass density profiles has been investigated so far, without serious physical justification. Applying our procedure to the Perseus and then to the Coma clusters, we find that an improvement of the observational constraints results in a quite modest improvement in the accuracy of the mass estimate. For Coma, using the best current available data, we end up with an uncertainty of a factor of 2 for the mass within the Abell radius. This uncertainty rapidly increases at further radius.

Subject headings: galaxies: clusters: general — galaxies: clusters: individual (Coma, Perseus) — intergalactic medium — X-rays: galaxies

1. INTRODUCTION

The determination of the mass of galaxy clusters is of major importance in cosmology. Dynamical estimates of cluster mass provide one of the few sources of evidence that the mean density of the universe is at least 10% of the critical density, in excess of what is detected in individual galaxies. The discovery of the X-ray-emitting gas in cluster cores has revealed the presence of a large baryonic component. Under the assumption of hydrostatic equilibrium (HE), this gas offers a way to determine the radial mass distribution of clusters. This procedure yields in principle much more accurate results than determinations based on optical data (Sarazin 1986). The method has been widely used in the past, and mass estimates that were claimed to be accurate have been derived from such analyses (e.g., Cowie, Henriksen, & Mushotzky 1987; Hughes, Gorenstein, & Fabricant 1988; Hughes 1989, hereafter H89; Eyles et al. 1991; Gerbal et al. 1992; Watt et al. 1992; David, Jones, & Forman 1995; Elbaz, Arnaud, & Böhringer 1995). H89 used a combination of available data, both optical and X-ray, to simultaneously derive the mass in the gas component and the mass in the dark matter of the Coma cluster. These quantities have been refined by Briel, Henry, & Böhringer (1992, hereafter BHB) using recent ROSAT observations. Similar analyses have been performed on other clusters (see, e.g., Elbaz et al. 1995) and in a systematic way (David et al. 1995; White & Fabian 1995). These studies have reinforced the evidence for the presence of dark matter and confirmed that the fraction of baryonic material in clusters seems to be inconsistent with the overall baryon density of the universe predicted by the standard theory of cosmic nucleosynthesis in an Einstein-de Sitter ($\Omega_0 = 1$) model (White & Fabian 1995; White et al. 1993; White et al. 1994). The case of Coma has received much attention, as baryons make up a substantial fraction of the total mass ($\approx 5 \times 10^{-3} \Omega_0$; White et al. 1993), but in some other cases the baryonic fraction appears to be similar or even higher (Eyles et al. 1991; David et al. 1995; White et al. 1994; Elbaz et al. 1995).

In this paper we investigate whether accurate mass estimates can be obtained from the HE equation (note that our aim is not to discuss the relevance of the assumption of HE in cluster central regions, though evidence for recent mergers of subcluster clumps challenges such an hypothesis; e.g., Burns et al. 1994; Roettiger, Burns, & Pinkney 1995). The possible uncertainty in mass estimates is generally believed to be small once the temperature is reasonably well known (Loewenstein 1994; Schindler 1996; Evrard, Metzler, & Navarro 1996). In order to investigate this in detail, we consider a new class of mass profiles and use a Monte Carlo procedure to investigate a large range of acceptable models. We discuss the X-ray mass estimate method in § 2 and our specific procedure in § 3. Our main results are presented in § 4. For this purpose, we take the Perseus and Coma clusters as examples. Our conclusions are given in § 5.

2. MASS ESTIMATES

Under the assumption of hydrostatic equilibrium, the temperature of the X-ray gas traces the gravitational poten-
tial of the whole cluster. This may be written (assuming spherical symmetry) as

\[ T(r) = -\frac{1}{\gamma_g + \gamma_T} \frac{GM(r)}{r} . \]  

(2.1)

In principle, the slope of the radial gas distribution \( \gamma_g \) and the temperature gradient \( \gamma_T \) are accessible through observations of \( T(r) \) and the luminosity profile \( s(r) \) arising from the thermal bremsstrahlung emission of the gas, provided that the observed quantities are deprojected. In the absence of cooling flows, the emissivity profile of clusters is well fitted by a standard \( \beta \) profile (Cavaliere & Fusco-Femiano 1976);

\[ s(r) \propto \left[ 1 + (r/r_c)^2 \right]^{-3\beta + 1/2} , \]  

(2.2)

where \( r_c \) denotes the gas core radius and \( \beta \) is a fitting parameter (physically, the ratio of galaxy kinetic energy to gas energy). Once deprojected, \( s(r) \) gives the radial distribution of the gas with satisfactory accuracy. One might therefore expect from equation (2.1) that the mass at a given radius is uncertain to a precise degree comparable to the uncertainty in the temperature. One should, however, be extremely cautious when trying to use equation (2.1) to infer \( M(r) \) from the observed temperature profile \( T(r) \), as equation (2.1) is actually a differential equation. In fact, the solutions obtained through the HE equation are very sensitive to the boundary condition (as noted by some authors; see, e.g., Loewenstein 1994). Several rules have been used in the past to limit the solutions that are actually investigated. Some authors have argued that solutions for which the temperature diverges or reaches zero at some finite radius must be ruled out, i.e., that only critical solutions for which the pressure goes smoothly down to zero at infinity should be kept (e.g., Loewenstein 1994). These assumptions severely limit the class of solutions under investigation, without being well motivated on a physical basis; the hot gas in clusters is probably heated by shocks during the phase of nonlinear collapse and a shock front is to be expected. The assumption of hydrostatic equilibrium is certainly wrong for radii larger than the shock radius, which may be of the order of the virial radius or less. Clearly, a reliable boundary condition would include assumptions concerning the discontinuity at the front shock.

In most analyses to date, only one physical scale is introduced in the mass profile; viz., the core radius of the gas component. Actually, some authors have noted that the mass may be more concentrated in the cores of clusters than in their outer regions (Gerbal et al. 1992; Loewenstein 1994). This tendency has been confirmed by determinations of the mass of cluster centers through gravitational lensing effects (Hammer 1991; Mellier, Fort, & Kneib 1993). Therefore, the mass profile might differ considerably from the profile of the gas component and might possess several physical scales. For instance, significant temperature variation in the intracluster gas has been observed in some clusters (e.g., the Coma Cluster; Honda et al. 1996). This might partly reflect a complex mass density structure. Even where the temperature is rather constant up to a few core radii, it is natural to expect that the gas temperature will drop to zero far away from the center in an isolated bounded cluster. If the temperature does not present a strong discontinuity but rather decreases smoothly beyond some radius, this radius might correspond to an additional scale length in the mass distribution. It is thus important to allow for more freedom in investigating mass estimates of X-ray clusters.

3. METHOD

We have investigated the possibility that the mass contains several scales, keeping in mind that the mass density profile may become shallower in the outer parts of clusters. We therefore assume a simple mass distribution for which the density profile in the central region of clusters corresponds to the isothermal case (the gas distribution following, in this case, an isothermal \( \beta \) profile):

\[ \rho(r) \propto \frac{3 + (r/r_c)^2}{[1 + (r/r_c)^2]^{3/2}} , \]  

(3.1)

where the central dark matter density \( \rho_0 \) is left as a free parameter. In the outer parts of the cluster, beyond some transition radius \( r_t \), we assume that the mass density is a power law with index \( x \),

\[ \rho(r > r_t) \propto (r/r_t)^x , \]  

(3.2)

where \( x \) is a free parameter allowed to vary between \(-3 \) and \( 0 \) (the asymptotic slope of the isothermal solution being \(-2 \)). We do not attempt to argue that such slopes are realistic from a physical point of view; although the slope of the dark matter distribution is known to depend on the power spectrum as well as on the density of the universe (Crone, Evrard, & Richstone 1994), some of the profiles we use are rather extreme and may appear unrealistic. We fully appreciate this point, but as we investigate possible uncertainties from a purely phenomenological point of view, we do not want to add any theoretical prejudice to our analysis that cannot be directly tested by the observations. We also investigate profiles that are peaked in the central region (referred to as “centrally peaked profiles”). For these, we add a term \( 1/[1 + (r/r_c)^2]^{n/2} \) to equation (3.1), where \( n \) is taken between 2.25 and 3. We generate mass models according to equations (3.1)-(3.2) by a Monte Carlo procedure, computing the corresponding temperature profiles through equation (2.1) and comparing them to various sets of available data. We keep any solutions that satisfy the observational constraints without imposing any further restrictions based on (uncertain) physical assumptions. The observational data used are the emissivity profile and the distance up to which the emission has been confidently detected, as well as the information on the temperature profile. This procedure is efficient as long as the number of constraints is neither very high nor of extremely good quality; otherwise, the required number of models to investigate is prohibitive. The following quantities are drawn at random from a uniform distribution over the specified range: (1) the total mass \( M_A \) within the Abell radius, which is allowed to vary between half and 6 times the mass corresponding to the isothermal solution within the same radius, (2) the central dark matter density \( \rho_0 \), varying within at most 10\% of the isothermal value, and (3) the slope \( x \). The trial is kept only if a radius \( r_t \) can be chosen consistently with equations (3.1)-(3.2).

4. RESULTS

Integrating equation (2.1) with fixed central temperature as a boundary condition, we find that an isothermal solution is obtained for a very precise value of the central
binding density $\rho_0$. Any departure from this particular fine-
tuned value leads to a dramatic behavior of the solution. A
difference as small as 0.5% yields temperature profiles that
fall to zero within a few core radii or that diverge rapidly.
This extreme sensitivity of the hydrostatic equation to the
boundary conditions has been noticed but not emphasized,
and its consequences have not been fully appreciated. In
addition, the temperature solutions exhibit a surprising
result: the temperature decreases in the outer region of the
most massive pro-
files, rather than increasing as one would
expect from a naive examination of the hydrostatic equation (2.1). We present the results of our simulations in
the particular cases of two well-known clusters: A426
(Perseus) and A1656 (Coma).

4.1. Perseus

In Figure 1, we give examples of emission-weighted pro-
jected temperature profiles fitted roughly to the tem-
perature data from Spacelab 2 (Eyles et al. 1991; solid bars).
Also shown are new data from ASCA (Arnaud et al. 1994;
dashed bars). As our aim is to compare our mass estimates
with those previously proposed (e.g., Loewenstein 1994), we
do not try to fit our models to this new set of data. The core
radius and the $\beta$ parameter have been taken from Eyles et
al. (1991), in good agreement with Jones & Forman (1984),
yielding $r_c \sim 9.1$ and $\beta \sim 0.57$. Note that Perseus has a
central cooling flow that we do not try to model. Estimates of
the mass within 1.3 $h_{50}^{-1}$ Mpc may vary between $\sim 4.1 
\times 10^{14} h_{50}^{-3} M_\odot$ and $\sim 1.3 \times 10^{15} h_{50}^{-3} M_\odot$, with an
uncertainty factor of 3 (throughout this work, we assume
$h_{50} = 1$). This must be compared with the range 4.4–4.8
$\times 10^{14} h_{50}^{-3} M_\odot$ derived by Loewenstein (1994), who inte-
grates the hydrostatic equation inward from boundary con-
ditions taken at infinity. At the Abell radius ($3 \ h_{50}^{-1}$ Mpc),
where no data are available, mass estimates appear to be
almost unconstrained (with uncertainty of more than a
factor of 10). If we now impose the condition that the tem-
perature be nonzero within $70'$, the radius up to which sig-
nificant X-ray emission is detected (Eyles et al. 1991), this
reduces the uncertainty of the mass estimate to a factor of
1.5 at 1.3 Mpc and to a factor 3 at the Abell radius. As we
can see, the uncertainty remains quite substantial. It might
be thought that this is due to the fact that data are of rather
poor quality and that an improvement in the data quality
will result in a substantial reduction of the uncertainty. As
we will illustrate in the case of Coma, for which the data are
significantly better, the actual improvement is rather
modest.

4.2. Coma

For Coma, we first apply the following constraints: (1)
the temperature does not drop to zero within $\sim 50$ from
the cluster center and (2) the averaged synthetic temperature
within the EXOSAT and Tenma beams (0.75° and 3°,
respectively) yields the observed values (8.5 $\pm 0.5$ keV, and
7.5 $\pm 0.2$ keV at the $2 \sigma$ level, respectively). H89 uses similar
constraints. However, in his analysis, the entire Tenma spec-
trum is fitted, and the ratios of center to off-center tem-
perature and flux measurements from EXOSAT are used.
In addition, both optical and X-ray data are used to better
constrain the mass of Coma. Therefore, strictly speaking, a
comparison between our work and the results found by
H89 is not relevant; however, his estimated upper limit on
the mass of Coma derives from the X-ray constraint alone.
Furthermore, our procedure allows us to directly test the
improvement in the mass determination due to detection of
X-ray emission up to a larger radius using ROSAT detec-
tion, as we show below. For each of our model temperature
profiles that meets criterion (1), we calculate the average
temperature weighted by the (projected) luminosity profile
in order to reproduce exactly the observational procedure.
Because the temperature is allowed to be zero after only $50'$,
we find that the uncertainty of mass estimates is large at the
Abell radius (a factor of 3) and more than 1 order of magni-
uite at 5 $h_{50}^{-1}$ Mpc. Moreover, the mass estimates we derive
within these radii are typically larger than those inferred by
H89. This is a direct consequence of using shallower pro-
files. A stringent constraint comes from the fact that
ROSAT detected gas emission of Coma up to $\sim 95'$ with an
emissivity well fitted by equation (2.2), with $\beta \sim 0.75$ and
$r_c \sim 10.5$ (BHB). From this, we impose the further condition
that the temperature does not drop to zero within $95'$, and we
apply the same procedure as before. In general
"centrally peaked profiles" are preferred. Some of the syn-
thetic profiles passing the new constraints are shown in
Figure 2. The corresponding mass profiles appear in Figure
3 (solid lines), together with some mass profiles allowed
when the constraint that the temperature is nonzero within
$95'$ is relaxed to $50'$ (dashed lines). Despite the fact that
the emission has been detected by ROSAT over twice as great a
range as before, the spread in the mass estimates at the
Abell radius is still a factor of $\sim 2$, i.e., $M(<3 \ h_{50}^{-1}$
Mpc) $\sim 1.2$–$2.5 \times 10^{15} M_\odot$, and reaches up to a factor of
$\sim 7$ at 5 Mpc, i.e., $M(<5 \ h_{50}^{-1}$ Mpc) $\sim 1.3$–$9.2 \times 10^{15} M_\odot$.
Very few mass models from our Monte Carlo samples
actually satisfy the temperature measurements from
EXOSAT and Tenma at the $2 \sigma$ level. Specifically, about
10% of our trial profiles satisfy equations (3.1)–(3.2), $\sim 2%$
of which pass the combined Tenma and EXOSAT con-
straints. Models that satisfy the average temperature in
EXOSAT and Tenma beams at the $3 \sigma$ level are much more
Fig. 2.—Synthetic temperature profiles for Coma. All the profiles pass the constraint imposed by the temperature measurements of the Tenma and EXOSAT satellites and are nonzero within 95' from the cluster center.

Finally, we use the most constraining temperature data available for Coma, i.e., Ginga data (Jones & Forman 1992). Note that these data are not fully consistent with the constraints from Tenma and EXOSAT. We consider the Ginga profile as a template and use it in an attempt to illustrate the improvement in the mass estimates due to better information on temperatures. In Figure 4 we present emission-weighted projected profiles from our sample fitted to these data (solid lines). Once the underlying mass profile is determined in our simulation, the two parameters relevant to the fit are \( \rho_0 \) and the central temperature \( T_0 \). The dotted line corresponds to a modified “isothermal” profile, i.e., an isothermal profile with a value of \( \rho_0 \) slightly different from the isothermal value. The mass associated with this profile within the Abell radius is \( 1.95 \times 10^{15} h_{50}^{-1} M_{\odot} \). The maximum mass we obtain within the Abell radius is \( 3 \times 10^{15} h_{50}^{-1} M_{\odot} \) (Fig. 4, solid lines). Also shown is the model temperature used by Makino (1993) (dashed line), who derives a mass of \( 1.6 \times 10^{15} h_{50}^{-1} M_{\odot} \) within the same radius using the same information on the luminosity profile, namely the ROSAT emissivity profile for Coma. (Note that the lowest value for the mass at the Abell radius given by our Monte Carlo procedure is \( 1.8 \times 10^{15} h_{50}^{-1} M_{\odot} \).)

These results together, we find a spread in the allowed mass estimates of about a factor 2 within the Abell radius. It is clear that even with considerably improved data, the uncertainty in estimating the mass remains large at this radius, of the order of a factor 2. It is important to keep in mind that this is a minimum value, as more complex profiles might allow a wider mass range.

Recent analyses of the baryonic fraction in clusters are based on the assumption of hydrostatic equilibrium (David et al. & Fabian 1995; White & Fabian 1995). The range of fractions obtained in such analyses is therefore optimistically narrow. Sadat (1995) also pointed out that mass estimates based on the virial equation are rather uncertain when the density profile is not constrained. Thus, in order to obtain reliable mass estimates for clusters, the use of numerical simulations is required (White et al. 1993; Evrard et al. 1996; Roettiger, Burns, & Locken 1996). Such an approach, followed by, for example, Evrard (1997), confirms the reality of a high baryonic fraction in clusters and supports the idea that some
analyses based on the assumption of hydrostatic equilibrium may underestimate the mass of galaxy clusters.

5. CONCLUSION

We have generated a large number of mass models and derived the corresponding temperature profiles via the equation of hydrostatic equilibrium. For the sake of illustration, we have applied this procedure to evaluate the possible range of mass estimates for the specific examples of the Coma and Perseus clusters. From available observations of the temperature and X-ray emission, we find that constraints on cluster mass estimates derived from the hydrostatic equation are much less severe than what is usually believed: in our low-mass range we recover the typical masses that were previously inferred, but also find more massive profiles that fit the observations identically well. This is reminiscent of the results of The & White (1988). Moreover, the mass estimates corresponding to our massive profiles are in better agreement with those inferred from numerical simulations, although the comparison is not directly meaningful. Even with the most constraining observations we use for Coma (which are much more restrictive than those currently used in mass estimates based on the HE equation), we find that the uncertainty in estimates of the total mass within the radius up to which data are available remains of the order of a factor 2, despite the use of a high signal-to-noise temperature profile. This number is a lower limit, and the uncertainty would be larger for clusters for which the available observations are of poorer quality. We conclude that cluster masses cannot be determined accurately by the sole use of the equation of hydrostatic equilibrium even when X-ray data of good quality are available.

We wish to thank Monique Arnaud, Joe Silk, Gary Mamon, Yoel Rephaeli, David Valls-Gabaud, and Jim Bartlett for numerous fruitful discussions regarding the results presented in this paper. C. B. acknowledges financial support from the Center for Particle Astrophysics of the University of California at Berkeley.

REFERENCES

Arnaud, K. A., et al. 1994, ApJ, 436, L67
Briel, U. G., Henry, J. P., & Böhringer, H. 1992, A&A, 259, L31 (BHB)
Burns, J. O., Roettiger, K., Ledlow, M., & Klypin, A. 1994, ApJ, 427, L87
Cavaliere, A., & Fusco-Femiano, R. 1976, A&A, 49, 137
Cowie, L. L., Henriksen, M., & Mushotzky, R. 1987, ApJ, 317, 593
Cowie, M. M., Evrard, A. E., & Richstone, D. O. 1994, ApJ, 434, 402
David, L. P., Jones, C., & Forman, W. 1995, ApJ, 445, 578
Elbaz, D., Arnaud, M., & Böhringer, H. 1995, A&A, 293, 337
Evrard, A. E. 1997, preprint astro-ph/9701148
Evrard, A. E., Metzler, C. A., & Navarro, J. F. 1996, ApJ, 469, 494
Eyles, C. J., Watt, M. P., Bertram, D., Church, M. J., Ponman, T. J., Skinner, G. K., & Willmore, A. P. 1991, ApJ, 376, 23
Gerbal, D., Durret, F., Lima-Neto, G., & Lachièze-Rey, M. 1992, A&A, 253, 77
Hammer, F. 1991, ApJ, 383, 66
Honda, H., et al. 1996, ApJ, 473, L71
Hughes, J. P. 1989, ApJ, 337, 21 (H89)
Hughes, J. P., Gorenstein, P., & Fabricant, D. 1988, ApJ, 329, 82
Jones, C., & Forman, W. 1984, ApJ, 276, 38
Jones, C., & Forman, W. 1992, in Clusters and Superclusters of Galaxies, ed. A. C. Fabian (Dordrecht: Kluwer), 49
Loewenstein, M. 1994, ApJ, 431, 91
Makino, N. 1993, PASJ, 46, 139
Mellier, Y., Fort, B., & Kneib, J.-P. 1993, ApJ, 407, 33
Roettiger, K., Burns, J. O., & Locken, C. 1996, ApJ, 473, 651
Roettiger, K., Burns, J. O., & Pinkney, J. 1995, ApJ, 453, 634
Sadat, R. 1995, Ap&SS, 234, 303
Sarazin, C. L. 1986, X-Ray Emissions from Clusters of Galaxies (Cambridge: Cambridge Univ. Press)
Schindler, S. 1996, MNRAS, 280, 309
The, L. S., & White, S. D. M. 1988, AJ, 95, 15
Watt, M. P., Ponman, T. J., Bertram, D., Eyles, C. J., Skinner, G. K., & Willmore, A. P. 1992, MNRAS, 258, 738
White, D. A., Fabian, A. C., Allen, S. W., Edge, A. C., Crawford, C. S., Johnstone, R. M., Stewart, G. C., & Voges, W. 1994, MNRAS, 269, 589
White, S. D. M., Navarro, J. F., Evrard, A. E., & Frenk, C. S. 1993, Nature, 366, 429