Achievable DoF and Its User Scaling Law for Opportunistic User Selection in a $K$-transmitter SIMO Interference Channel

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Abstract

In this paper, we consider a $K$-transmitter single-input multiple-output (SIMO) interference channel (IC) where each transmitter has its own user group and serves one of the users in its group. When the number of receive antennas at each user is less than $K$ (i.e., $N_R < K$), we prove a degree of freedom (DoF) of one, which is an optimal DoF in interference-free SIMO, is achieved via user selection when the number of users per transmitter, $N$, goes to infinity. Using a geometric interpretation of the interfering channels, we propose opportunistic interference alignment (OIA) as a practical implementation of interference alignment. We find that a DoF of one per transmitter is achieved using the OIA scheme when the number of users is scaled as $N \propto P^{\alpha(K-N_R)}$ where $P$ is transmit power and $\alpha$ is the relative path loss of the interfering channel in decibels. This result on the scaling law is extended and shown to be still valid for other user selection schemes such as the minimum interference-to-noise ratio (INR) user selection scheme and the maximum signal-to-interference-plus-noise ratio (SINR) user selection scheme.

Index Terms

Interference alignment, opportunistic interference alignment, user selection, single-input multiple-output (SIMO).

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I. Introduction

Inter-network interference is a major performance-limiting factor in modern wireless communication systems. Many interference mitigation strategies have been proposed to improve network spectral efficiency. By allowing partial or full cooperation among interfering base stations, the system performance can be enhanced. In a cellular network, for example, joint beamforming [1] and joint transmission [2] among base stations have been proposed to improve system performance. When no cooperation is allowed among transmitters (i.e., meaning that transmitter cooperation is not allowed when using an interference channel model), traditional strategies mainly focused on using orthogonal multiple access. In this case, all interfering transmitters exclusively share the wireless resources. In a $K$-transmitter single-input single-output (SISO) IC, for example, each transmitter can achieve $1/K$ degrees of freedom (DoF) by time division multiplexing.

In recent years, interference alignment (IA) techniques have received much attention [3]–[6], and a total of $\frac{K^2}{2}$ DoF has been shown to be achievable using IA in a $K$-transmitter SISO IC [3]. Although IA is able to obtain a substantial asymptotic capacity gain in interference channels, there are many practical challenges to implement IA techniques. IA requires global channel state information at the transmitter (CSIT), and imperfect channel knowledge severely degrades the gain of IA. In some channel configurations, symbols should be extended in the time/frequency domain for interference alignment. The large computational complexity necessitated for IA is also a major challenge.

There have been many efforts to ameliorate these practical challenges [5]–[9]. To reduce the global CSIT burden, an iterative IA scheme using time division duplexing (TDD) reciprocity between the uplink and downlink channels is proposed in [5]. To reduce computational complexity, a subspace interference alignment technique for an uplink cellular network system is proposed in [6]. In [6], there are $K$ cells each of which has $N$ users, so $K$ uplink channels compose a $K$ interfering multiple access channel. Using the subspace interference alignment technique, the achievable DoF per each cell is shown to be $N/(K-1)^{K-1}$. IA with imperfect CSIT is studied in [7]–[9]. In [7], it is shown that the maximum number of DoF, $\frac{K^2}{2}$, is achievable with limited feedback in a $K$-transmitter frequency-selective SISO IC if the feedback size is scaled as $K(L-1)\log P$ bits where $L$ is the number of taps and $P$ is transmit power. In a $K$-transmitter MIMO IC, the maximum number of DoF is also shown to be achievable if the
feedback size is scaled with the signal-to-noise ratio (SNR) \[8\]. IA using analog feedback is studied in \[9\] and shown to achieve the maximum number of DoF when the feedback power grows with the transmit power.

Recently, opportunistic interference alignment (OIA) has been proposed for a 3-transmitter \(2 \times 2\) MIMO IC \[10\] and a 3-transmitter \(N_T \times N_R\) MIMO IC \[11\], \[12\], respectively, by the authors. In the OIA scheme, each transmitter opportunistically selects a user to serve whose interfering signals are most aligned among the users associated with the transmitter. Contrary to conventional IA, global channel knowledge at each transmitter is not required in OIA, but each user only needs to feed back one scalar value. It was shown that the OIA scheme does not sacrifice the spatial dimensions in aligning interference signals and secures the full spatial DoF by exploiting the multiuser DoF for interference alignment. In a 3-transmitter IC, each user has two interfering channels and the correlation between two interfering channels is easily found and can be used for OIA scheme. In a \(K\)-transmitter IC (\(K > 3\)), however, each user has \(K - 1\) interfering channels and to measure a degree of alignment among the \(K - 1\) interfering channels is far difficult than a 3-transmitter IC case.

In this paper, we show that a form of opportunistic user selection enables each transmitter to achieve a DoF of one in a \(K\)-transmitter single-input multiple-output (SIMO) IC which is the optimal number of DoF in interference-free SIMO system. To show that, we firstly extend our OIA scheme to a \(K\)-transmitter single-input multiple-output (SIMO) IC. In the \(K\)-transmitter SIMO IC, each user has \(K - 1\) interfering channels, and we geometrically interpret the concept of interference alignment by treating the \(K - 1\) interfering channels as points on a complex \(N_R\)-dimensional hypersphere. Using the geometric concept of interfering channels, we show that the interfering channels can be aligned to a limited surface area on the hypersphere, so that each transmitter achieves a DoF of one as the number of users associated with each transmitter increases. The degree of alignment among the \(K - 1\) interfering channels at each user is quantified by the proposed interference alignment measures and used for user selection at each transmitter. User selection and postprocessing schemes are proposed and proved to achieve a DoF of one per transmitter when the number of users in each user group is scaled as \(N \propto P^{\alpha(K-N_R)}\), where \(P\) is transmit power and \(\alpha(0 \leq \alpha \leq 1)\) is the relative path loss of interfering channel in decibels. Also, a DoF per transmitter of \(1 - \alpha'(0 \leq \alpha' \leq \alpha \leq 1)\) is proved to be achieved when the number of users is scaled
as \( N \propto P^{(\alpha-\alpha')(K-N_R)} \).

The scaling law result obtained with the proposed OIA scheme is extended to practical user selection schemes. To the best knowledge of the authors, there is no work identifying the scaling laws of INR minimization and SINR maximization for a given DoF. We for the first time prove that the INR minimization scheme and SINR maximization scheme also achieve a DoF per transmitter of \( 1-\alpha' \) \((0 \leq \alpha' \leq \alpha \leq 1)\) when the number of users in each group scales as \( N \propto P^{(\alpha-\alpha')(K-N_R)} \).

The rest of this paper is organized as follows. In Section II, we describe the system model. In Section III, a geometric interpretation of interfering channels is provided and two interference alignment measures are proposed. In Section IV, the achievable DoF of OIA in a \( K \)-transmitter SIMO IC is analyzed. The achievable DoF of the other user selection schemes are analyzed in Section V. We provide numerical results in Section VI and conclude our paper in Section VII.

– Notations

Throughout the paper, we use boldface to denote vectors and matrices. \( A^\dagger, \lambda_i(A) \) and \( V_i(A) \) denote the conjugate transpose, the \( i \)th largest eigenvalue and the eigenvector of matrix \( A \) corresponding to the \( i \)th largest eigenvalue. For convenience, the smallest eigenvalue, the largest eigenvalue and the eigenvectors corresponding eigenvectors of \( A \) are denoted as \( \lambda_{\text{min}}(A), \lambda_{\text{max}}(A), V_{\text{min}}(A) \) and \( V_{\text{max}}(A) \), respectively. Also, \( I_n, \mathbb{C}^n \) and \( \mathbb{C}^{m \times n} \) indicate the \( n \times n \) identity matrix, the \( n \)-dimensional complex space, and the set of \( m \times n \) complex matrices, respectively.

II. System Model

Our system model is depicted in Fig. 1. There are \( K \) transmitters and each transmitter has its own user group consisting of \( N \) users. The numbers of antennas at each transmitter and each user are one and \( N_R \), respectively. Each transmitter selects a single user in its user group so that a \( K \)-transmitter SIMO IC is opportunistically constructed after user selection. In this paper, we only consider the \( N_R < K \) case because a DoF of one per transmitter can be always achieved when \( K \leq N_R \) by using zero-forcing like schemes at a receiver.

The received signal at the \( n \)th user in the \( i \)th user group becomes

\[
y_{i,n} = h_{i,n}^{(i)} x_i + \sqrt{\gamma} \sum_{k=1,k \neq i}^{K} h_{i,n}^{(k)} x_k + w_{i,n}
\]
where $h^{(k)}_{i,n} \in \mathbb{C}^{N_R \times 1}$ is the vector channel from the $k$th transmitter to the $n$th user in the $i$th user group, $x_i \in \mathbb{C}^{1 \times 1}$ is the transmitted signal from the $i$th transmitter, and $w_{i,n} \in \mathbb{C}^{N_R \times 1}$ is a circularly symmetric complex Gaussian noise with zero mean and an identity covariance matrix such that $w_{i,n} \sim \mathcal{CN}(0, I_{N_R})$.

The vector channel $h^{(k)}_{i,n} \in \mathbb{C}^{N_R \times 1}$, $\forall k$, is assumed to be perfectly estimated at a receiver.

Each user feeds one scalar value back to its own transmitter for user selection. The feedback information can be constructed in various ways, but we propose two interference alignment measures to use as feedback information in Section III.B. SNR, INR, and SINR values are also considered as feedback information in Section V. Because no information is shared among the transmitter, each transmitter independently selects the user to be served based on the collected feedback information.

Using the multiple antennas at each user, the received signal is equalized by a postprocessing vector. We denote the postprocessing vector of the $n$th user in the $i$th user group as $v_{i,n} \in \mathbb{C}^{N_R \times 1}$ such that $\|v_{i,n}\|^2 = 1$. Thus, the post-processed received signal at the $n$th user in the $i$th user group becomes

$$v_{i,n}^\dagger y_{i,n} = v_{i,n}^\dagger h_{i,n}^{(i)} x_i + \sqrt{\gamma} \sum_{k \neq i} v_{i,n}^\dagger h_{i,n}^{(k)} x_k + v_{i,n}^\dagger w_{i,n}.$$ 

If we denote the index of the selected user at the $i$th transmitter as $n_i$, the average achievable rate at
We assume that there is no collaboration among the transmitters, and the channel model is symmetric among the transmitter-selected user pairs. Also, we assume that all transmitters use the same user selection scheme, and all users process the received signal in the same way. Because the user selection at each transmitter is done independently of the other transmitters, the average achievable rate at each transmitter is the same such that $C_1 = \ldots = C_K$, and the average sum rate of the system, $C_{\text{sum}}$, is given by

$$C_{\text{sum}} = \sum_{i=1}^K C_i = K C_1.$$ 

Thus, the total achievable DoF of the system becomes

$$\lim_{P \to \infty} \frac{C_{\text{sum}} \log_2 P}{P} = K \lim_{P \to \infty} \frac{C_1 \log_2 P}{P}.$$ 

### III. Geometric Interpretation of Interfering Channels

#### A. Preliminaries

Consider an $N_R$-dimensional unit hypersphere centered at the origin. The surface of the hypersphere, $S_0$, can be defined as

$$S_0 = \{ x \in \mathbb{C}^{N_R} \mid \| x \|^2 = 1 \}$$
For an arbitrary unit vector \( \mathbf{c} \in \mathbb{C}^{N_R} \) and an arbitrary non-negative real number \( 0 \leq \lambda \leq 1 \), \( S_0 \) can be divided into two parts, \( S_1(\mathbf{c}, \lambda) \) and \( S_2(\mathbf{c}, \lambda) \), such that
\[
S_1(\mathbf{c}, \lambda) = \{ \mathbf{x} \in \mathbb{C}^{N_R} | |\mathbf{c}^\dagger \mathbf{x}|^2 \geq \lambda, \| \mathbf{x} \|^2 = 1 \}
\]
\[
S_2(\mathbf{c}, \lambda) = \{ \mathbf{x} \in \mathbb{C}^{N_R} | |\mathbf{c}^\dagger \mathbf{x}|^2 \leq \lambda, \| \mathbf{x} \|^2 = 1 \}.
\]

For graphical interpretation, \( S_1(\mathbf{c}, \lambda) \) and \( S_2(\mathbf{c}, \lambda) \) when \( \mathbf{x}, \mathbf{c} \in \mathbb{R}^3 \) are represented in Fig. 2, respectively.

If we denote the spherical area of \( S_i(\mathbf{c}, \lambda) \) as \( A(S_i(\mathbf{c}, \lambda)) \) for \( i \in \{0, 1, 2\} \), it is satisfied that
\[
A(S_0) = A(S_1(\mathbf{c}, \lambda)) + A(S_2(\mathbf{c}, \lambda)). \tag{1}
\]

The surface area of an \( N_R \)-dimensional complex unit hypersphere is known as
\[
A(S_0) = \frac{2\pi^{N_R}}{(N_R - 1)!},
\]
and \( A(S_1(\mathbf{c}, \lambda)) \) is invariant with \( \mathbf{c} \) and given by [13]
\[
A(S_1(\mathbf{c}, \lambda)) = \frac{2\pi^{N_R}(1 - \lambda)^{N_R-1}}{(N_R - 1)!}.
\]

Therefore, we can find \( A(S_2(\mathbf{c}, \lambda)) \) from (1) such as
\[
A(S_2(\mathbf{c}, \lambda)) = \frac{2\pi^{N_R}(1 - (1 - \lambda)^{N_R-1})}{(N_R - 1)!}.
\]

Lemma 1 ([14]): For arbitrary unit vector \( \mathbf{c} \in \mathbb{C}^{N_R} \) and a constant \( \lambda \in [0, 1] \), the probability that \( n \) isotropic and i.i.d. \( N_R \)-dimensional random unit vectors are contained in \( S_2(\mathbf{c}, \lambda) \) denoted as \( p_{[n]}(\lambda) \) becomes
\[
p_{[n]}(\lambda) = \left(1 - (1 - \lambda)^{N_R-1}\right)^n \tag{2}
\]
which is invariant with \( \mathbf{c} \).

Proof: The probability that a single isotropic random \( N_R \)-dimensional unit vector is in \( S_2(\mathbf{c}, \lambda) \) becomes the ratio of \( A(S_2(\mathbf{c}, \lambda)) \) and \( A(S_0) \) given by \( p_{[1]}(\lambda) = 1 - (1 - \lambda)^{N_R-1} \). Thus, the probability that \( n \) isotropic and i.i.d. random unit vectors are contained in \( S_2(\mathbf{c}, \lambda) \), which is \( p_{[n]}(\lambda) \), becomes \( [p_{[1]}(\lambda)]^n \) given in (2).
B. \(K-1\) interfering channels aligned in \(N_R - 1\)-dimensional subspace in \(\mathbb{C}^{N_R}\)

In our system model, the interfering channels are \(N_R\)-dimensional vectors whose elements are i.i.d. complex Gaussian random variables so that each interfering channel is isotropic in \(\mathbb{C}^{N_R}\). To secure a DoF of one at a user having \(N_R\) receive antennas, \(K-1\) (\(\geq N_R\)) interfering channels should be aligned in \(N_R - 1\)-dimensional subspace in \(\mathbb{C}^{N_R}\). Thus, we start with the basic question, “How should the \(K-1\) interfering vectors be aligned in a \((N_R - 1)\)-dimensional subspace, and how can we measure the degree of interference alignment of the \(K-1\) interfering channels?”

Consider a user suffering from \(K-1\) (\(\geq N_R\)) interferers. If we denote the normalized \(K-1\) interfering channels as \(g_1, \ldots, g_{K-1}\), then \(g_i\) becomes an \(N_R\)-dimensional unit vector isotropic in \(\mathbb{C}^{N_R}\) which is independent of \(g_j\) for \(i \neq j\). Because \(g_1, \ldots, g_{K-1}\) are i.i.d. and isotropic in \(\mathbb{C}^{N_R}\), the space spanned by the interfering channels becomes \(N_R\)-dimensional space with probability 1. Although the interfering channels span \(N_R\)-dimensional space, the interfering channels can be closely aligned in a smaller dimensional subspace. To quantify the degree of alignment among \(K-1\) interfering channels in an \((N_R - 1)\)-dimensional subspace, we propose two interference alignment measures \(q_1(\cdot)\) and \(q_2(\cdot)\) each of which is a mapping from \(K-1\) unit vectors to a non-negative number such that

\[
q_1(g_1, \ldots, g_{K-1}) \in [0, 1] \\
q_2(g_1, \ldots, g_{K-1}) \in [0, (K-1)/N_R].
\]

Firstly, we consider the following optimization problem

\[
\min_{c, \lambda} \quad A(S_2(c, \lambda)) \tag{3}
\]

subject to

\[
S_2(c, \lambda) \supset \{g_1, \ldots, g_{K-1}\}, \\
\|c\|^2 = 1, \quad \lambda \in [0, 1].
\]

If we denote the solution of the problem as \((c^*, \lambda^*)\), \(S_2(c^*, \lambda^*)\) has the smallest area among all \(S_2(c, \lambda)\) containing \(g_1, \ldots, g_{K-1}\). In (3), \(c^*\) and \(\lambda^*\) can be represented as

\[
c^* = \arg \min_{\|c\|=1} \max_{1 \leq k \leq K-1} |c^\dagger g_k|^2
\]

\[
\lambda^* = \min_{\|c\|=1} \max_{1 \leq k \leq K-1} |c^\dagger g_k|^2. \tag{4}
\]
Thus, (3) is equivalent to the following optimization problem

\[
\begin{align*}
\text{minimize} & \quad \lambda \\
\text{subject to} & \quad |c^\dagger g_k|^2 \leq \lambda \quad \text{for } 1 \leq k \leq K - 1, \\
& \quad \|c\|^2 = 1, \quad 0 \leq \lambda \leq 1,
\end{align*}
\]

which can be solved by linear programming [15], [16].

Using \( c^* \), we can divide an \( N_R \)-dimensional space into two subspaces which are the one-dimensional subspace spanned by \( c^* \) and the residual \( N_R - 1 \)-dimensional subspace orthogonal to \( c^* \) denoted by \( \mathcal{U} \). If there exists \( c^* \) such that \( c^* \perp \{g_1, \ldots, g_{K-1}\} \), it is satisfied that \( S_2(c^*, 0) \supset \{g_1, \ldots, g_{K-1}\} \) and thus \( \lambda^* = 0 \). Accordingly, \( \text{span}(g_1, \ldots, g_{K-1}) \subset \mathcal{U} \) and we can say \( \{g_1, \ldots, g_{K-1}\} \) are perfectly aligned in \( N_R - 1 \)-dimensional subspace in \( \mathbb{C}^{N_R} \). Note that \( S_2(c^*, 0) \) is an \( N_R - 1 \)-dimensional subspace orthogonal to \( c^* \) and \( S_2(c^*, 1) \) is the \( N_R \)-dimensional complex hypersphere, \( S_0 \). It can be seen that the smaller \( \lambda^* \) is the more closely the vectors are aligned in the \( N_R - 1 \)-dimensional subspace, \( \mathcal{U} \). Thus, \( \lambda^* \) can be used as an interference alignment measure to quantify how much \( K - 1 \) \( N_R \)-dimensional interfering channels are aligned in an \( N_R - 1 \)-dimensional subspace. We use \( \lambda^* \) as the first interference alignment measure such that

\[
q_1(g_1, \ldots, g_{K-1}) = \min_{\|c\|=1} \max_{1 \leq k \leq K-1} |c^\dagger g_k|^2
\]

\[
= \lambda^* \quad (\lambda^* \in [0, 1]).
\]

Note that \( \lambda^* = 0 \) when \( K < N_R \). When \( g_1, \ldots, g_{K-1} \in \mathbb{R}^3 \), the concept of the alignment measure \( \lambda^* \) can be easily explained using the graphical interpretation in Fig. 2.

**Lemma 2:** When \( K > N_R \), the probability that the alignment measure \( q_1(g_1, \ldots, g_{K-1}) \) is smaller than \( \lambda \in [0, 1] \), i.e., \( Pr[\lambda^* \leq \lambda] \), satisfies that

\[
p_{[K-N_R]}(\lambda) \leq Pr[\lambda^* \leq \lambda],
\]

where \( \lambda^* = q_1(g_1, \ldots, g_{K-1}) \) in (7).

\(^1\)Actually, \( \lambda^* = 0 \) is with probability zero in our configuration. Because \( g_1, \ldots, g_{K-1} \) are i.i.d. and isotropic in \( \mathbb{C}^{N_R} \), the case of \( c^* \perp \{g_1, \ldots, g_{K-1}\} \) cannot be happened.
Proof: Because the random variable $\lambda^*$ is generated from the optimization problem in (4), the probability $Pr[\lambda^* \leq \lambda]$ is hard to obtain directly. Instead, we find the bounds of $Pr[\lambda^* \leq \lambda]$. If we define a unit vector $\bar{c} \in \mathbb{C}^{N_R}$ such that $\bar{c} \perp \{g_1, \ldots, g_{N_R-1}\}$, it is satisfied that $S_2(\bar{c}, 0) \supset \{g_1, \ldots, g_{N_R-1}\}$.

Consider the following three events.

$E_1$: $\{g_1, \ldots, g_{K-1}\} \subset S_2(\bar{c}, \lambda)$

$E_2$: $\{g_1, \ldots, g_{K-1}\} \subset S_2(c^*, \lambda)$ (or, equivalently, $\lambda^* \leq \lambda$)

When $\{g_1, \ldots, g_{K-1}\} \subset S_2(\bar{c}, \lambda)$, it is satisfied that $\lambda^* \leq \lambda$ from the definition of $\lambda^*$ in (4), so

$$\{g_1, \ldots, g_{K-1}\} \subset S_2(c^*, \lambda^*) \subset S_2(c^*, \lambda).$$

Thus, when $E_1$ is true, $E_2$ is true, i.e., $E_1 \Rightarrow E_2$, and $Pr[E_1] \leq Pr[E_2]$, where

$$Pr[E_1] = Pr[\{g_{N_R}, \ldots, g_{K-1}\} \subset S_2(\bar{c}, \lambda)] = p_{[K-N_R]}(\lambda)$$

obtained in Lemma [1].

As an alternative interference alignment measure, the minimum of sum projected power of $K-1$ normalized interfering channels on a direction can be used and is given by

$$q_2(g_1, \ldots, g_{K-1}) = \min_{\|c\|^2 = 1} \sum_{i=1}^{K-1} |c^\dagger g_i|^2$$

$$= \Lambda_{\min} \left( \sum_{i=1}^{K-1} g_i g_i^\dagger \right),$$

since $\|c\|^2 = 1$. This value is upper bounded on

$$\Lambda_{\min} \left( \sum_{i=1}^{K-1} g_i g_i^\dagger \right) \leq \frac{1}{N_R} \sum_{j=1}^{N_R} \Lambda_j \left( \sum_{i=1}^{K-1} g_i g_i^\dagger \right)$$

$$\overset{(a)}{=} \frac{1}{N_R} tr \left( \sum_{i=1}^{K-1} g_i g_i^\dagger \right)$$

$$\overset{(b)}{=} \frac{(K-1)}{N_R},$$

where equality $(a)$ holds from the fact that the sum of the eigenvalues of a matrix is the trace of a matrix, and equality $(b)$ is satisfied using $tr(g_i g_i^\dagger) = g_i^\dagger g_i = 1$. Thus, we can find that $q_2(g_1, \ldots, g_{K-1}) \in [0, (K-1)/N_R]$. 

Lemma 3: The relationship between two interference measures defined in (6) and (9) becomes

\[ q_1 \leq q_2 \leq (K - 1)q_1, \]  

where \( q_i = q_i(g_1, \ldots, g_{K-1}) \).

Proof: For arbitrary unit vector \( c \in \mathbb{C}^{NR} \), it is satisfied that

\[
\max_{1 \leq k \leq K-1} |c^\dagger g_k|^2 \leq \sum_{k=1}^{K-1} |c^\dagger g_k|^2 \leq (K - 1) \max_{1 \leq k \leq K-1} |c^\dagger g_k|^2.
\]

Therefore, we can find that

\[
\min_{\|c\|=1} \max_{1 \leq k \leq K-1} |c^\dagger g_k|^2 \leq \min_{\|c\|=1} \sum_{k=1}^{K-1} |c^\dagger g_k|^2 \leq (K - 1) \min_{\|c\|=1} \max_{1 \leq k \leq K-1} |c^\dagger g_k|^2,
\]

which is equivalent to (10) from the definitions of \( q_1 \) and \( q_2 \) given in (6) and (9), respectively.

IV. OPPORTUNISTIC INTERFERENCE ALIGNMENT IN A K-TRANSMITTER SIMO IC

A. Achievable DoF per Transmitter

Without loss of generality, we consider the first transmitter and the first user group. The average achievable rate of the first transmitter and the selected user pair becomes

\[
C_1 = \mathbb{E} \log_2 \left( 1 + \frac{P|v_{1,n_1}^\dagger h_{1,n_1}^{(1)}|^2}{1 + P^\alpha \sum_{k=2}^{K} |v_{1,n_1}^\dagger h_{1,n_1}^{(k)}|^2} \right). \tag{11}
\]

We decompose (11) into the capacity gain term and the capacity loss term denoted as \( C_{\text{gain}} \) and \( C_{\text{loss}} \) such that

\[
C_{\text{gain}} = \mathbb{E} \log_2 \left( 1 + P|v_{1,n_1}^\dagger h_{1,n_1}^{(1)}|^2 + P^\alpha \sum_{k=2}^{K} |v_{1,n_1}^\dagger h_{1,n_1}^{(k)}|^2 \right), \tag{12}
\]

\[
C_{\text{loss}} = \mathbb{E} \log_2 \left( 1 + P^\alpha \sum_{k=2}^{K} |v_{1,n_1}^\dagger h_{1,n_1}^{(k)}|^2 \right). \tag{13}
\]

Then, \( C_1 \) can be represented as \( C_1 = C_{\text{gain}} - C_{\text{loss}} \). At the first transmitter, the achievable DoF becomes

\[
\lim_{P \to \infty} \frac{C_1}{\log_2 P} = \lim_{P \to \infty} \frac{C_{\text{gain}} - C_{\text{loss}}}{\log_2 P} = 1 - \lim_{P \to \infty} \frac{C_{\text{loss}}}{\log_2 P}. \tag{14}
\]
Remark 1: In (14), it can be easily found that at least a DoF of $1 - \alpha$ is achieved at the first transmitter because $\lim_{P \to \infty} \frac{C_{\text{loss}}}{\log_2 P} \leq \alpha$. When the number of users in each user group, $N$, is finite, there is a residual interference satisfying $\lim_{P \to \infty} \frac{C_{\text{loss}}}{\log_2 P} = \alpha$ so that a DoF of $1 - \alpha$ is achieved at each transmitter.

In the following subsection, we will find the number of users to achieve a DoF more than $1 - \alpha$ at the first transmitter.

B. Opportunistic Interference Alignment Using the Geometry of Interfering Channels (OIA1)

Opportunistic interference alignment can be implemented in many ways. Firstly, we consider OIA using the interference alignment measure defined in (6) to opportunistically select a user, and we call this scheme as OIA1 for convenience. In a $K$-transmitter SIMO IC, each user finds the smallest $S_2(c, \lambda)$ packing $K - 1$ interfering channels. For example, the $n$th user in the first user group having $K - 1$ interfering channels, $h^{(2)}_{1,n}, \ldots, h^{(K)}_{1,n}$, finds $c_n$ and $\lambda_n$ such that

$$c_n = \arg \min_{\|e\|^2=1} \left[ \max_{2 \leq i \leq K} |c^\dagger \tilde{h}^{(i)}_{1,n}|^2 \right]$$

$$\lambda_n = \min_{\|e\|^2=1} \left[ \max_{2 \leq i \leq K} |c^\dagger \tilde{h}^{(i)}_{1,n}|^2 \right]$$

$$= \max_{2 \leq i \leq K} |(c_n)^\dagger \tilde{h}^{(i)}_{1,n}|^2,$$

where $\tilde{h} = h/\|h\|$. Note that $\lambda_n$ is the interference alignment measure defined in (6) such that

$$\lambda_n = q_1(\tilde{h}^{(2)}_{1,n}, \ldots, \tilde{h}^{(K)}_{1,n}).$$

Each user feeds $\lambda_n$ back to the transmitter, and the first transmitter selects the user $n^{\text{OIA1}}_1$ such that

$$n^{\text{OIA1}}_1 = \arg \min_{1 \leq n' \leq N} \lambda_{n'}.$$

The feedback information of the selected user can be represented as $\lambda_{n^{\text{OIA1}}_1}$.

**Lemma 4:** The average of $\lambda_{n^{\text{OIA1}}_1}$ is upper bounded by

$$\mathbb{E}[\lambda_{n^{\text{OIA1}}_1}] < N^{-\frac{1}{\kappa - \lambda R}}.$$
Proof: Using Lemma \([2]\) the complementary CDF of \(\lambda_{n}^{\text{QRAI}}\) is bounded as

\[
Pr[\lambda_{n}^{\text{QRAI}} \geq z] = Pr[\lambda_1 \geq z \cap \cdots \cap \lambda_N \geq z] \\
= (1 - Pr[\lambda_1 < z])^N \\
\leq [1 - p(K-N_R)(c, z)]^N \\
= [1 - (1 - (1 - z)^{N_R-1})^{K-N_R}]^N,
\]

where \(c \in \mathbb{C}^{N_R}\) in \((19)\) is an arbitrary unit vector. Using this bound, \(\mathbb{E}[\lambda_{n}^{\text{QRAI}}]\) is upper bounded by

\[
\mathbb{E}[\lambda_{n}^{\text{QRAI}}] = \int_0^1 Pr[\lambda_{n}^{\text{QRAI}} \geq z] \, dz \\
\leq \int_0^1 [1 - (1 - z)^{N_R-1})^{K-N_R}]^N \, dz \\
\leq \int_0^1 [1 - (1 - z)]^{K-N_R}]^N \, dz \\
= \frac{1}{K-N_R} \beta \left( N + 1, \frac{1}{K-N_R} \right) \\
= \frac{\Gamma \left( 1 + \frac{1}{K-N_R} \right)}{\Gamma \left( N + 1 + \frac{1}{K-N_R} \right)} \\
< N^{-\frac{1}{K-N_R}} ,
\]

where the inequality \((a)\) is due to \((1 - z)^{N_R-1} \leq (1 - z)\) since \(0 \leq z \leq 1\), and the equality \((b)\) holds from the representation of beta function \([17,\text{p.324}]\)

\[
\int_0^1 x^{p-1}(1 - x^q)^{r-1} \, dx = \frac{1}{q} \beta \left( p, q \right) .
\]

The equality \((c)\) comes from the definition of the beta function \(\beta(p, q) = \frac{\Gamma(p)\Gamma(q)}{\Gamma(p+q)}\) and the property of the Gamma function \(\Gamma(p+1) = p\Gamma(p)\). In RHS of equality \((c)\), it holds \(\Gamma(1 + \frac{1}{K-N_R}) < 1\) because \(0 < \Gamma(x) < 1\) for \(1 \leq x \leq 2\). Also, it is satisfied that

\[
\frac{\Gamma \left( N + 1 \right)}{\Gamma \left( N + 1 + \frac{1}{K-N_R} \right)} < \left( N + 1 + \frac{1}{K-N_R} \right)^{-\frac{1}{K-N_R}} \\
< N^{-\frac{1}{K-N_R}} ,
\]

from the Gautschi’s inequality \([18]\) such that

\[
\frac{\Gamma(x + s)}{\Gamma(x + 1)} < (x + 1)^{s-1}, \quad \text{for} \quad x > 0, \quad 0 < s < 1 ,
\]
with \( x = N + \frac{1}{K-NR} \) and \( s = 1 - \frac{1}{K-NR} \). Thus, the inequality (d) holds.

After user selection, the selected user uses \( c_{\text{n1OIA1}} \) given in (15) as a postprocessing vector. We denote the achievable rate, the capacity gain term, and the capacity loss term using OIA1 at the first transmitter as \( C^\text{OIA1} \), \( C^\text{gain} \) and \( C^\text{loss} \), respectively. Thus, \( C^\text{OIA1} = C^\text{gain} + C^\text{loss} \).

**Theorem 1:** Using the OIA1 defined in (15), (17) and (18), a DoF of one per transmitter is achieved when the number of users in each group is scaled as \( N \propto P^{\alpha(K-NR)} \).

**Proof:** Using \( c_{\text{n1OIA1}} \) as a postprocessing vector at the selected user, \( C^\text{loss} \) is upper bounded on

\[
C^\text{loss} = \mathbb{E} \log_2 \left[ 1 + P^\alpha \sum_{k=2}^{K} |(c_{\text{n1OIA1}})^\dagger h_{1,n1}^{(k)}|^2 \right]
= \mathbb{E} \log_2 \left[ 1 + P^\alpha \sum_{k=2}^{K} \|h_{1,n1}^{(k)}\|^2 |(c_{\text{n1OIA1}})^\dagger \tilde{h}_{1,n1}^{(k)}|^2 \right] 
\leq \mathbb{E} \log_2 \left[ 1 + P^\alpha N_R \sum_{k=2}^{K} |(c_{\text{n1OIA1}})^\dagger \tilde{h}_{1,n1}^{(k)}|^2 \right] \tag{20}
\leq \log_2 \left( 1 + P^\alpha N_R (K-1) \mathbb{E}[\lambda_{\text{n1OIA1}}] \right). \tag{21}
\]

In the above equations, the inequality (a) is using \( \mathbb{E}\|h_{1,n1}^{(k)}\|^2 = N_R \) and Jensen’s inequality. Also, from the definition of \( \lambda_{\text{n1OIA1}} \) in (16), it is satisfied that \( \sum_{k=2}^{K} |(c_{\text{n1OIA1}})^\dagger \tilde{h}_{1,n1}^{(k)}|^2 \leq (K-1) \lambda_{\text{n1OIA1}} \), and the inequality (b) holds owing to the Jensen’s inequality. To maintain \( C^\text{OIA1} \leq \delta \), it is sufficient that

\[
\log_2 \left( 1 + P^\alpha N_R (K-1) \mathbb{E}[\lambda_{\text{n1OIA1}}] \right) \leq \delta. \tag{22}
\]

From \( \mathbb{E}[\lambda_{\text{n1OIA1}}] < N^{- \frac{1}{K-NR}} \) as shown in Lemma 4, a DoF of one per transmitter is achievable if \( N^{- \frac{1}{K-NR}} \leq \frac{\delta}{P^\alpha N_R (K-1)} \). Therefore, the required number of users, \( N \), to achieve a DoF of one per transmitter is obtained by

\[
N > \left[ \frac{P^\alpha N_R (K-1)}{2^\delta - 1} \right]^{K-NR}.
\]

This means that the number of users should be scaled as \( N \propto P^{\alpha(K-NR)} \) in the high SNR region to achieve a DoF of 1 at each transmitter. ■
Theorem 2: Using the OIA1 scheme defined in (15), (17) and (18), a DoF of \(1 - \alpha'\) \((0 \leq \alpha' \leq \alpha \leq 1)\) per transmitter is achieved when

\[ N \propto P^{(\alpha - \alpha')(K-N_R)}. \]

Proof: From (14), a DoF of \(1 - \alpha'\) at each transmitter is achieved when

\[
\lim_{P \to \infty} \frac{\log_2 \left(1 + P^\alpha N_R (K-1)E[\lambda_{n_{\text{OIA1}}}] \right)}{\log_2 P} = \alpha'.
\]

Using Lemma 4, we can find that the scaling of the number of users \(N = O(1)P^{(\alpha - \alpha')(K-N_R)}\) enables the network to obtain a DoF of \(1 - \alpha'\) at each transmitter.

C. Alternative Implementation of OIA (OIA2)

Although \(c_n\) and \(\lambda_n\) in (15) and (17) can be found by solving the equivalent problem as in (5), it requires high computational complexity at each user. In this subsection, we consider an alternative OIA scheme proposed in [10]–[12] for practical implementation, and we call this scheme as OIA2. In OIA2, each user uses the alternative interference alignment measure defined in (9).

In the first user group, for example, the \(n\)th user equalizes its received signal by using the postprocessing vector given by

\[
\mathbf{v}_{1,n}^{\text{OIA2}} = \arg\min_{\|\mathbf{v}\|^2 = 1} \sum_{k=2}^{K} |\mathbf{v}^\dagger \tilde{\mathbf{h}}_{1,n}^{(k)}|^2 = V_{\text{min}} \left( \sum_{k=2}^{K} \tilde{\mathbf{h}}_{1,n}^{(k)} \tilde{\mathbf{h}}_{1,n}^{(k)\dagger} \right)
\]

and each user feeds back the information \(\sigma_n\) determined by

\[
\sigma_n = \min_{\|\mathbf{v}\|^2 = 1} \sum_{k=2}^{K} |\mathbf{v}^\dagger \tilde{\mathbf{h}}_{1,n}^{(k)}|^2 = \Lambda_{\text{min}} \left( \sum_{k=2}^{K} \tilde{\mathbf{h}}_{1,n}^{(k)} \tilde{\mathbf{h}}_{1,n}^{(k)\dagger} \right).
\]

Note that \(\sigma_n\) is interference alignment measure at the \(n\)th user defined in (9) such that \(\sigma_n = q_2(\tilde{\mathbf{h}}_{1,n}^{(2)}, \ldots, \tilde{\mathbf{h}}_{1,n}^{(K)})\).

The transmitter selects the \(n_1^{\text{OIA2}}\)th user such that

\[
n_1^{\text{OIA2}} = \arg\min_{1 \leq n' \leq N} \sigma_{n'},
\]
and the feedback information of the selected user becomes

\[ \sigma_{n_1^{OIA2}} = \min_{1 \leq n' \leq N} \sigma_{n'} \].

**Lemma 5:** \( \sigma_{n_1^{OIA2}} \) is bounded as

\[ \lambda_{n_1^{OIA1}} \leq \sigma_{n_1^{OIA2}} \leq (K-1)\lambda_{n_1^{OIA1}}. \] (26)

**Proof:** From Lemma 3, we can find that \( \lambda_{n_1^{OIA1}} \leq \sigma_{n_1^{OIA1}} \leq (K-1)\lambda_{n_1^{OIA1}} \) and \( \lambda_{n_1^{OIA2}} \leq \sigma_{n_1^{OIA2}} \leq (K-1)\lambda_{n_1^{OIA2}} \). Also, \( \lambda_{n_1^{OIA1}} \leq \lambda_{n_1^{OIA2}} \) and \( \sigma_{n_1^{OIA2}} \leq \sigma_{n_1^{OIA1}} \) from the definitions of \( n_1^{OIA1} \) and \( n_1^{OIA2} \) in (18) and (25). Thus, it is satisfied that

\[ \lambda_{n_1^{OIA1}} \leq \lambda_{n_1^{OIA2}} \leq \sigma_{n_1^{OIA2}} \leq \sigma_{n_1^{OIA1}} \leq (K-1)\lambda_{n_1^{OIA1}} \leq (K-1)\lambda_{n_1^{OIA2}}, \]

and (26) holds.  

**Theorem 3:** Using the OIA2 defined in (24) and (25), a DoF of one per transmitter can be achieved when

\[ N \propto P^{\alpha(K-N R)}, \]

and a DoF per transmitter of \( 1 - \alpha' \) (\( 0 \leq \alpha' \leq \alpha \leq 1 \)) is achieved when

\[ N \propto P^{(\alpha-\alpha')(K-N R)}, \]

which are the same results with Theorem 1 and Theorem 2 by OIA1.

**Proof:** Taking the same procedure with (20) in the proof of Theorem 1 we can easily show that the capacity loss term using OIA2 is upper bounded as

\[ C_{loss}^{OIA2} \leq \log_2 \left( 1 + P^\alpha N_R E[\sigma_{n_1^{OIA2}}] \right). \]

Using Lemma 5 we can find the following inequalities

\[ O(1) \leq \lim_{P \to \infty} \log_2 \left( 1 + P^\alpha N_R E[\sigma_{n_1^{OIA2}}] \right) \leq O(1) \]

are satisfied when \( N \propto P^{\alpha(K-N R)} \). Thus, \( \lim_{P \to \infty} \frac{C_{loss}^{OIA2}}{\log_2 P} = 0 \) and a DoF of one per transmitter is achieved when \( N \propto P^{\alpha(K-N R)} \). With the same procedure in the proof of Theorem 2 we can also find that a DoF of \( 1 - \alpha' \) (\( 0 \leq \alpha' \leq \alpha \)) at each transmitter is achieved when \( N \propto P^{(\alpha-\alpha')(K-N R)} \) because

\[ O(\log_2 P^{\alpha'}) \leq \lim_{P \to \infty} \log_2 \left( 1 + P^\alpha N_R E[\sigma_{n_1^{OIA2}}] \right) \leq O(\log_2 P^{\alpha'}) \]
V. USER SCALING LAWS FOR OTHER OPPORTUNISTIC USER SELECTION SCHEMES

Conventional opportunistic user selection schemes can also mitigate interference as the proposed OIA scheme. However, the user scheduling laws for conventional opportunistic user selection schemes are not fully understood. In this section, we extend the scaling law analysis to three other user selection schemes and find the user scaling laws to achieve DoF larger than $1 - \alpha$ at each transmitter.

A. SNR Maximizing User Selection (MAX-SNR)

In the maximum SNR user selection, each user adopts a postprocessing vector maximizing SNR so that the postprocessing vector for the $n$th user in the $i$th group is given by

$$v_{SNR}^{i,n} = \frac{h_{i,n}^{(i)}}{\|h_{i,n}^{(i)}\|},$$

and the corresponding SNR becomes $P\|h_{i,n}^{(i)}\|^2$. Each user feeds the SNR back to the transmitter and the user who has the largest SNR is selected at the transmitter.

B. INR Minimizing User Selection (MIN-INR)

In the minimum INR user selection, each user adopts a postprocessing vector minimizing INR, so the postprocessing vector of the $n$th user in the $i$th group is given by

$$v_{INR}^{i,n} = V_{\text{min}}(B_{i,n}),$$

where $B_{i,n} = \sum_{k \neq i} h_{i,n}^{(k)}(h_{i,n}^{(k)})^\dagger$, and the corresponding INR becomes $\Lambda_{\text{min}}(B_{i,n})$. Each user feeds the INR back to the transmitter, and the user who has the smallest INR is selected.

C. SINR Maximizing User Selection (MAX-SINR)

In the maximum SINR user selection, the transmitter selects the user who has the maximum SINR value. Each user adopts the postprocessing vector maximizing SINR and feeds the SINR value to the
transmitter. To maximize the SINR value at each transmitter, the \( n \)th user in the \( i \)th user group finds the postprocessing vector \( \mathbf{v}_{i,n}^{\text{SINR}} \) such that

\[
\mathbf{v}_{i,n}^{\text{SINR}} = \arg \max_{\|\mathbf{v}'\|_2 = 1} \frac{P\|\mathbf{v}'^\dagger \mathbf{h}_{i,n}^{(i)}\|^2}{1 + P\alpha \sum_{k=1, k\neq i}^K |\mathbf{v}'^\dagger \mathbf{h}_{i,n}^{(k)}|^2}.
\]

\( \mathbf{v}_{i,n}^{\text{SINR}} \) is obtained by solving a generalized eigenvalue problem and becomes

\[
\mathbf{v}_{i,n}^{\text{SINR}} = \mathbf{L}_{i,n}^{-1} \cdot \mathbf{V}_{\text{max}} \left( \mathbf{L}_{i,n}^{-1} \mathbf{D}_{i,n} (\mathbf{L}_{i,n}^\dagger)^{-1} \right), \tag{28}
\]

where \( \mathbf{D}_{i,n} = P\mathbf{h}_{i,n}^{(i)} (\mathbf{h}_{i,n}^{(i)})^\dagger \) and \( \mathbf{L}_{i,n} \) is chosen using the Cholesky decomposition of the positive semi-definite matrix \( \mathbf{I}_{N_R} + P\alpha \mathbf{B}_{i,n} \) such that

\[
\mathbf{L}_{i,n} \mathbf{L}_{i,n}^\dagger = \mathbf{I}_{N_R} + P\alpha \mathbf{B}_{i,n}.
\]

For \( \mathbf{v}_{i,n}^{\text{SINR}} \) in (28), the corresponding SINR becomes \( \Lambda_{\text{max}}(\mathbf{L}_{i,n}^{-1} \mathbf{D}_{i,n} (\mathbf{L}_{i,n}^\dagger)^{-1}) \).

### D. Scaling Law to Achieve a DoF per Transmitter Larger Than \( 1 - \alpha \)

Using the same notations defined in Section IV-A, the achievable rates at the first transmitter using OIA2, MAX-SNR, MIN-INR and MAX-SINR can be decomposed, respectively, as

\[
C_{1}^{\text{OIA2}} = C_{\text{gain}}^{\text{OIA2}} - C_{\text{loss}}^{\text{OIA2}}, \quad C_{1}^{\text{SNR}} = C_{\text{gain}}^{\text{SNR}} - C_{\text{loss}}^{\text{SNR}},
\]

\[
C_{1}^{\text{INR}} = C_{\text{gain}}^{\text{INR}} - C_{\text{loss}}^{\text{INR}}, \quad C_{1}^{\text{SINR}} = C_{\text{gain}}^{\text{SINR}} - C_{\text{loss}}^{\text{SINR}},
\]

where \( C_{\text{gain}} \) and \( C_{\text{loss}} \) for each scheme are obtained by substituting its postprocessing vector into (12) and (13). In the case of the MAX-SNR scheme, the achievable rate of each transmitter becomes \( 1 - \alpha \) because \( \lim_{P \to \infty} C_{\text{loss}}^{\text{SNR}} = \alpha \) regardless of \( N \). Substituting (23) and (27) into (13), we can easily find that

\[
C_{\text{loss}}^{\text{INR}} \leq C_{\text{loss}}^{\text{OIA2}}, \tag{29}
\]

by noting that \( C_{\text{loss}}^{\text{INR}} \) given by

\[
C_{\text{loss}}^{\text{INR}} = \mathbb{E} \min_{n'} \log_2 \left( 1 + \min_{\mathbf{v}'} P\alpha \sum_{k=2}^K |\mathbf{v}'^\dagger \mathbf{h}_{i,n}^{(k)}|^2 \right) \tag{30}
\]
is smaller than the capacity loss terms using any other schemes.

**Theorem 4:** When the number of users in each user group is scaled as \( N \propto P^\alpha (K-N_R) \) for \( \alpha' \) (with \( 0 \leq \alpha' \leq \alpha \leq 1 \)), a DoF of \( 1 - \alpha' \) per transmitter is also achievable by both INR minimization and SINR maximization schemes.
Proof: From (29), we can find that

\[
\lim_{P \to \infty} \frac{C_{\text{INR}}^{\text{loss}}}{\log_2 P} \leq \lim_{P \to \infty} \frac{C_{\text{OIA2}}^{\text{loss}}}{\log_2 P}.
\]

From (14) and Theorem 3, the OIA2 scheme achieves a DoF per transmitter of \(1 - \alpha'\) is achievable when \(N \propto P^{(\alpha - \alpha')(K - NR)}\) for \(\alpha' (0 \leq \alpha' \leq \alpha \leq 1)\), and hence the INR minimization does. The SINR maximization scheme can also achieve a DoF of \(1 - \alpha'\) at each transmitter with the same scaled number of users in each user group because \(C_1^{\text{SINR}} \geq C_1^{\text{INR}}\).

VI. NUMERICAL RESULT

In Fig. 3, the achievable rates at each transmitter using various schemes in a 4-transmitter \(1 \times 3\) SIMO IC are plotted when \(\alpha = 1\) and \(K = 10\). In this figure, the achievable rate is saturated at high SNR region and the achievable DoF per transmitter becomes zero. As mentioned in Remark 1, a DoF of \(1 - \alpha\) (= 0) is achieved at each transmitter when each user group has finite users.

In Fig. 4, the achievable rates per transmitter using various schemes in a 4-transmitter \(1 \times 3\) SIMO IC are plotted when \(\alpha = 1\) and the number of users are scaled as \(N = O(P^{\alpha(K - NR)}) = O(P^1)\). To make user scaling \(O(P)\), we consider two cases \(N = P\) and \(N = 0.5P\). As shown in Fig. 4, a DoF of one is achieved for both cases of the OIA2 scheme as predicted in Theorem 3. It is also confirmed that a DoF...
Fig. 4. Achievable rates per transmitter using various schemes when $\alpha = 1$ and the number of users in each group scaled as $N = P$ and $N = 0.5P$, respectively.

Fig. 5. Achievable rates per transmitter using various schemes when $\alpha = 1$ and the number of users in each group is scaled as $N = P$ and $N = P^{0.5}$, respectively. According to Theorem 3 and Theorem 4, the achievable DoFs of MAX-SINR, MIN-INR, and OIA2 schemes become $1 - \alpha'$ when the number
of users is scaled as $N = O(P^{1-\alpha'})$. Fig. 5 confirms Theorem 5 and Theorem 4 that DoFs of 1 and 0.5 are achieved when $N = P^1$ and $N = P^{0.5}$, respectively, when MAX-SINR, MIN-INR and OIA2 schemes are employed.

In Fig. 6, achievable rates per transmitter are plotted according to the number of users in each user group when $\alpha = 1$ and SNR is fixed to 10dB. As the number of users in each group increases, the achievable rate of each scheme increases. Note that our main concern is on user scaling to achieve a certain DoF as a function of the interference power so that the behavior of achievable rates according to the number of users at fixed SNR is beyond the scope of this paper. It is our on-going research item.

VII. Conclusion

In this paper, we show that employing a form of user selection in an interference channel enables the network to achieve the same DoF as the interference-free DoF in a $K$-transmitter SIMO IC. We have extended the OIA scheme to a $K$-transmitter SIMO IC when $K > N_R$. From a geometrical interpretation of a properly chosen interference alignment measure, it has been shown that a DoF of one can be achieved by opportunistic user selection. From the geometrical interpretation of interference alignment, the proposed opportunistic interference alignment schemes (OIA1 and OIA2) achieve a DoF of $1 - \alpha'$ ($0 \leq \alpha' \leq \alpha \leq 1$) when the number of users in each group increases as $O(P^{(\alpha-\alpha')(N_R-K)})$. It was also proved that the other
user selection schemes such as MAX-SINR and MIN-INR scheme also achieve DoF of $1 - \alpha'$ when $N = O(P^{(\alpha - \alpha')(K - N_R)})$. 
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