Mathematical and physical modeling shape memory polymer composites

B D Annin¹, E V Karpov¹, E V Moskvichev² and A Yu Larichkin¹

¹Lavrentiev Institute of Hydrodynamics Siberian Branch Russian Academy of Sciences, Novosibirsk, Russia
²Institute of Computational Technologies Siberian Branch Russian Academy of Sciences, Krasnoyarsk Branch Office, Krasnoyarsk, Russia

E-mail: larichking@gmail.com

Abstract. In the article the influence of fiber orientation on the deformation character of specimens from composites with shape memory (EMC) under transverse and longitudinal bending is studied. Also, restoration of the initial configuration of the sample upon activation of the shape memory effect (SME) is considered. The effect of fiber orientation on the formation of systems of irreversible structural defects during molding is revealed. The degree of shape restoration upon SME implementation and the reproducibility of the original form upon repetition of the shape fixing-recovering cycles depend on these defects. Variants of a material model with linear hardening and a Voigt model for describing the shape memory effect in modeling the stages of molding and restoration are considered.

1. Introduction

The use of smart materials is an actual task for the aerospace industry. The stability and accuracy of signals transmitted from satellites depend in particular on the stiffness of the antenna reflectors, which can be increased by installing frames with shape memory [1]. At the stage of bringing the antenna to the transport state, it is heated, wrapped and cooled in a fixed state. When the satellite is delivered into orbit, the antenna heats up and the antenna and the frame are deployed to the operating state due to the implementation of the shape memory effect. In this article carbon-fiber reinforced plastic with shape memory is considered, from which a similar frame can be made. The behavior of the material at different stages of the shape fixing-recovering cycles with various types of deformation is investigated. The material consists of several layers of plain weave fabric made of carbon fibers ST 12073 impregnated with Diaplex MP5510 recoplast, which is a shape memory polymer (SMP). An external stimulus activating the shape memory effect of this polymer is heating.

2. Methods and materials

In the experiments samples of composite material with two types of orientation of orthogonal fiber families ([0,0,0] and [0,45,-45]) were used. The samples consisted of three layers of fabric, had a length $l_0 = 100$ mm, a width $a = 10$ mm, and a thickness $h = 0.8$ mm. The mechanical parameters are obtained in the case of uniaxial tension for these samples (elastic modulus, Poisson's ratio, temporary strength, and strain at break). A series of tensile tests [2] and transverse (three-point) bending were carried out. Tests for longitudinal bending were also carried out by converging the opposite ends of the specimen fixed in articulated supports. The tests were carried out on a Zwick Roell Z100 universal...
testing machine using a climate chamber. The temperature of the samples was measured using a CEM BE-8833 pyrometer with a chromel-copel thermocouple.

![Figure 1](image1.png)

**Figure 1.** Type of samples: *a, c, e, f* – after shape fixing (*a, c* – transverse bending, *e, f* – longitudinal bending), *b, d* – after shape recovery. Orientation of the fibers *[0,0,0]* – *a, b, e*, *[0,45,-45]* – *c, d, e*.

![Figure 2](image2.png)

**Figure 2.** The loading diagram of flat EMC specimens with fiber orientation *[0,0,0]* and *[0,45,-45]* under tension to failure.

### Table 1. EMC Tensile Test Results at T = 20 °C.

| #  | Orientation | $E$, GPa | $\sigma_{max}$, MPa | $\varepsilon_{max}$, % | $\nu_1$ |
|----|-------------|---------|-------------------|---------------------|--------|
| 1  | *[0,0,0]*   | 48.507  | 632.0             | 1.43                | 0.153  |
| 2  | *[0,45,-45]*| 8.201   | 158.0             | 15.10               | 0.855  |

### 3. Material Models

To describe the behavior of the material during deformation, the approach of constructing a mathematical model of the material based on the rheological models of the bodies of Hooke, Newton, Saint-Venant is adopted. Combining their serial and parallel connections, we can obtain a model that adequately describes the experimental observations. In this paper, as applied to the stages of operation of structures made of EMC, Voigt models and linear elastic-plastic material with hardening were considered. The sequential stages of uniaxial loading of a strip of composite were considered:

1. heating to the temperature of the highly elastic state of the composite matrix $T = 70$ °C;
2. loading by moving the loose end of the strip;
3. shape fixation (stress relaxation) $u(t) = \text{const}$;
4. sample cooling to $T = 20$ °C with a fixed shape;
5. sample unloading;
6. heating to $T = 70$ °C to realize the shape memory effect;
7. cooling to $T = 20$ °C.
Figure 3 shows the history of loading and changes in the parameters of the Voigt model. The model does not take into account stress relaxation, but it allows us to describe the shape memory effect. Curve 1 corresponds to the displacement history \( u(t,T) \), and curve 2 corresponds to the temperature history \( T(t) \). When moving the loose end at a speed \( u_0 \) at \( T = 70 \, ^\circ\text{C} \), the stress is specified by the following equation:

\[
\sigma(t,T) = \sigma_1(t,T) + \sigma_2(t,T) = E(T)\varepsilon(t,T) + \eta(T)\dot{\varepsilon}(t,T),
\]

where \( \varepsilon(t,T) = \ln(1 + t \dot{u}_0 / L_0) \).

When the load is removed and cooling occurs, we have

\[
\varepsilon(t,T) = \varepsilon(t_1,T) \exp\left(-(t - t_1)E(T)/\eta(T)\right),
\]

\[
\varepsilon(t_1,T) = \sigma(t_1,T)/E(T)(1 - \exp\left[-t_1E(T)/\eta(T)\right]).
\]

Here \( L_0 \) is initial sample length, \( t_1 \) is moment of unloading, \( E(T) \) is Young's modulus of Hooke's body, \( \eta(T) \) is damper fluid viscosity. Both parameters of the Voigt model depend on the history of temperature changes:

\[
E(T) = E_0 \cdot \exp\left[-(T - T_0)/a\right], \quad \eta(T) = \eta_0 \cdot \exp\left[-(T - T_0)/b\right],
\]

where \( a \) and \( b \) are parameters that need to be selected based on experimental data. The temperature dependence of the elastic modulus of EMC is presented in [1].

Figure 4 shows the history of loading and changes in the parameters of the linearly hardening body. In this model, the bodies of Hooke and Saint-Venant, connected in parallel, are connected in series with another body of Hooke. Curves 1 – 4 correspond to the histories of displacement, temperature, stresses and stress values, upon reaching which hardening of the material occurs.

The deformation at the stage of active loading is as follows:

\[
\varepsilon(t,T) = \varepsilon_1(t,T) + \left(\varepsilon_2(t,T)\right) = \sigma(t,T)/E_1(T) + \left[\left(\sigma(t,T) - \sigma_\alpha(T)\right)/E_2(T)\right].
\]

Saint-Venant's body begins to work when stress \( \sigma_\alpha(T) \) is reached. Then the stresses are

\[
\sigma(t,T) = \sigma_\alpha(T) + \lambda(T)E_1(T)E_2(T)/(E_1(T) + E_2(T))(t - t_\alpha).
\]

When the shape is fixed, stress relaxation does not occur in the body, and when the load is removed, the total deformation decreases by the deformation of the first Hooke body \( E_1(T) \). Thus, after cooling and unloading the sample, energy is stored in the model, which, when the temperature is raised again, is released and spent on restoring the shape of the sample. The model parameters are selected so that at high temperature the body of Saint-Venant does not resist the second body of Hooke.
$E_x(T)$. An original model of additional stiffness with decreasing temperature as applied to fibrous composites is presented in [5].

Thus, the model stores energy during unloading of the Hook body $E_x(T)$ after cooling the sample. This energy is released upon repeated heating and is expended in restoring the shape of the body.

The considered models have their advantages and disadvantages. It is necessary to determine the values of the parameters on the basis of experimental data for their application and to evaluate the limits of the use of models.

Models have their advantages and disadvantages. To apply them, it is necessary to determine the values of parameters based on experimental data and assess the model limits. Next, it is necessary to take into account the material anisotropy properties to evaluate the effect of fiber orientation on samples shape recovery. An original model of additional stiffness with decreasing temperature as applied to fibrous composites is presented in [3].

To solve the problems, variational formulations of continuum mechanics are required, both for the case of small and large deformations. In [5], solutions are given to the problems of bending beams and plates with various boundary conditions. It should be noted that the solution of transverse bending problem satisfactorily describes the implemented test for transverse bending only with small values of deflection. This can be explained by touching a sample of a bending tool along an area, but not along a line as it describe in mathematical model.

For the case of longitudinal bending, a solution was given in [5]. This solution allows to relate the critical buckling force with the linear moment and axial force in the beam for the case of large strains $\varepsilon_{xx}(x) = u'(x) + 1/2 w'(x)^2 - zw''(x)$, where $x$ is the coordinate along the length of the beam, $z$ is the coordinate along the thickness of the beam, $(\cdot)'$ is the derivative with respect to $x$, $u(x)$ is the displacement along $x$, $w(x)$ is the displacement along $z$. Is the component of the strain tensor along $x$.

The relationship between stress and strain is determined by the principle of virtual work $N/A_0 = E \left[ u'(x) + 1/2 w'(x)^2 \right]$, linear moment $M = -EIw''(x)$, where $E$ is Young's modulus, $I$ is the polar moment of inertia. The author derives the condition of stationarity of the functional based on the principle of virtual work $P_{cr} = \left\{ \int_0^L EI w'(x)^2 dx \right\} / \left\{ \int_0^L w'(x)^2 dx \right\}$, in this case, $P_{cr}$ is the critical buckling load. The complete solution regarding the stability of the elastic articulated plate, taking into account large deflections, includes elliptic integrals, and the solution is calculated based on tables of elliptic integrals. The geometry of a bent elastic plate can be described parametrically:

$$
\begin{align*}
z &= 2m/\lambda \cos \phi \\
x &= \left[ 2(\tilde{E} - \tilde{E}(m,\phi) - F + F(m,\phi)) \right]/\lambda'
\end{align*}
$$

where $\tilde{E}(m,\phi) = \int_0^\phi (1 - m^2 \sin^2 \phi)^{1/2} d\phi$, $F(m,\phi) = \int_0^\phi (1 - m^2 \sin^2 \phi)^{-1/2} d\phi$, $\tilde{E} = \tilde{E}(m,\pi/2)$, $F = F(m,\pi/2)$, $m = \sin Q_0/2$, $Q_0$ — initial angle between the tangent at the point of hinged support of the plate and the $x$ axis $\lambda = \sqrt{P_{cr}/(EI)}$. In the same monograph, the solutions of Shenley and Karman for the longitudinal bending of an elastoplastic rod are presented.

Figure 5a shows axial force dependence of the displacement of the beam edge during the longitudinal bending of the elastic plate under conditions of plane deformation at a temperature $T = 70^\circ C$ for three finite element models: 1 — a model composed of three types of material: matrix, longitudinal fibers, transverse fibers (Figure 5, b); 2 — a model approximating the real geometry of the EMC layers (Figure 5, c); 3 — a variant of model 2 with initial sections. In Figure 5a, markers 4 and 5 denote the Euler forces of buckling for cases of fiber orientation [0,0,0] and [0,45,-45] respectively.
Figure 5. The dependence of the reaction force on displacement during longitudinal bending. Modeling in MSC. Marc.

Figures 5 d, e, and f show the bending shapes and equivalent stresses in the middle of the samples under longitudinal bending for models 1, 2, and 3, respectively. The simulation results showed that the two-dimensional approximation does not give the value of the maximum efforts observed in the experiment. For the case of model 3, buckling is similar to the experiment for [0,45,-45]. However, in all modeling cases there were no localization of deflections is observed in the middle of the sample, as is typical for samples with orientation [0,0,0].

4. Defects formation during shape fixation

The results of the samples shape fixation and the degree of their recovery after SME implementation depend on the orientation of the fibers and the type of bending. When samples with fiber orientation [0,0,0] are bent, one of the fiber families is oriented along the axis of the sample. Some of the fibers of this family are compressed, and some are stretched. As a result, defect systems appear, which are local buckling of longitudinal fibers. In places of buckling, a shift of tissue layers relative to each other occurs and their delamination.

When specimens with fiber orientation [0,45,-45] are bent, both fiber families are deformed equally. Local buckling in this case does not appear.

Defects that appear do not affect the degree of shape recovery in the case of lateral bending (the degree of recovery is about 93% for both types of orientation). Differences in the degree of shape fixation are observed after hot deformation and cooling. The degree of shape fixation is about 50% for samples with fiber orientation [0,0,0], part of the longitudinal fibers of which are in a tense state. The degree of shape fixation is about 100% for samples with fiber orientation [0,45,-45], i.e. springing practically does not occur after cooling and unloading.

The appearance of local defects has an unexpectedly positive effect in the case of longitudinal bending, in which the point of the sample with maximum curvature is not set by the testing tool. For
samples with orientation [0,0,0], the presence of a system of local delaminations ensures shape repeatability when repeating the cycles of shape fixing-recovering.

![Image](image_url)

**Figure 6.** Local bulging of tissue layers and their location on the sample.

The point with maximum curvature appears in a new place each time for samples with fiber orientation [0,45,-45]. The result of this is the asymmetry of the bent sample, and a large spread of the fixed shapes with the same fixation method. The parameter characterizing shape recovery for samples with fiber orientation [0,45,-45] decreases much faster than for samples with fiber orientation [0,0,0] when repeating the cycles of shape fixing-recovering. This is obviously due to the fact that in samples with fiber orientation [0,0,0], the largest SMP deformation occurs in the same place each time due to localized defects. At the same time, in samples with fiber orientation [0,45,-45], the sites with the highest strain values are distributed over a significant part of the sample length and the residual strains that persist after SME implementation are summed up.

5. Conclusion

A series of tests of samples from a composite with shape memory has been carried out, approaches to modeling the process of loading, fixing the shape and unloading samples at a given temperature change program have been determined. A significant difference in the properties of the material is shown, depending on the direction of application of forces to the orientation of the fibers.

**Acknowledgments**

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