Is The Heavy Fermion Formalism Applicable to Meson Production Processes?

E. Gedalin*, A. Moalem† and L. Razdolskaya‡

Department of Physics, Ben Gurion University, 84105 Beer Sheva, Israel

Abstract

We consider tree diagram contributions to neutral pion production in $pp \rightarrow pp\pi^0$ using chiral perturbation theory in the relativistic and extremely non-relativistic limits. In marked difference with the results from heavy fermion formalism, the impulse and s-wave rescattering terms in the relativistic limit have equal signs and therefore add constructively, giving rise to a substantial contribution to the cross section. We argue that the power series expansion of the nucleon propagator is on the border of its convergence circle. Consequently, a finite order heavy fermion formalism does not predict nucleon pole terms correctly and therefore can not be applied to meson production.

Key Words : Chiral Perturbation, One loop, $\pi^0$ Production.

13.75.Cs, 14.40.Aq, 25.40.Ep

*gedal@bgumail.bgu.ac.il
†moalem@bgumail.bgu.ac.il
‡ljuba@bgumail.bgu.ac.il
In recent years, intensive theoretical efforts have been devoted to investigating how nuclear and hadron interactions can be understood within Chiral Perturbation Theory ($\chi$PT), an approach which is generally believed to be an effective theory of Quantum Chromodynamics (QCD) at low energies. This is an important clue toward understanding nuclear dynamics within the context of QCD, the accepted fundamental theory of strong interactions.

An outstanding, and still unsolved, problem in this area arises in the understanding of neutral pion production in $pp \rightarrow pp\pi^0$. Park et al. [1], Cohen et al. [2] and Sato et al. [3] have considered this process in $\chi$PT by applying rather similar calculation schemes based on extremely non-relativistic heavy fermion formalism (HFF), where the leading order impulse (graphs 1a-1b) and rescattering (graph 1c) contributions are found to have opposite signs, and hence leading to a cross section substantially smaller than experiment [1–3]. In this regard, it is to be noted that meson-exchange models predict equal signs for the two terms, achieving quite impressive descriptions of data near threshold [5–7,9,10]. Particularly, in covariant exchange models [5,8] the amplitudes from the impulse and rescattering terms interfere constructively. It has been argued by Hanhart et al. [9] that this sign difference between predictions of meson exchange models and HFF $\chi$PT is a genuine feature.

More recently, Tamura et al. [10] have concluded that the shape of energy spectra for the $d(p, (pp)_s)\pi^0n$ and $d(p, (pp)_s)\pi^-p$ reactions, can be explained only if the interference between these terms is constructive.

By extending calculations to chiral order $\Delta = 2$ Gedalin et al. [8] have shown that in addition to not predicting correctly the relative phase of the leading order impulse and rescattering terms, the HFF yields one loop contributions substantially larger than lower order terms. This is a serious drawback which indicates that the HFF expansion converges (if at all) slowly and therefore may not be suitable to apply to production processes. It is the purpose of the present note to call attention to the fact that in a relativistic $\chi$PT the sign and relative importance of various contributions are different from those found using HFF. First, the contribution from rescattering is substantially larger and having an equal
sign as the impulse term. Secondly, it is to be shown that these differences are mostly due to the fact that in the reduction procedure of the pion-nucleon Lagrangian, the nucleon kinetic term is reduced also, what limits the validity of the HFF Lagrangian to sufficiently small nucleon momenta. Consequently, a finite order HFF can not be applied to meson production processes which necessarily involve large momentum transfers.

To begin with we consider the tree diagram contributions (Fig. 1) to neutral pion production in $pp \rightarrow pp\pi^0$. Let $\pi$ and $N$ represent the pion and nucleon fields, $m$ and $M$ the pion and nucleon masses, then the fully relativistic pion-nucleon sector Lagrangian \cite{12,14} is,

$$L_{\pi N} = \bar{N}(\mathcal{D}^{(1)} - M)N + \frac{c'_1 m^2}{2M} \bar{N}(U^\dagger + U)N + \frac{c'_2}{4M^3} \bar{N}D^\mu D^\nu N(\Delta \mu \Delta \nu) - \frac{c'_3}{M} \bar{N}N(\Delta \cdot \Delta) + \frac{c'_4}{2M^2} \bar{N}i\gamma^\mu \bar{D}^\nu N(\Delta \mu \Delta \nu).$$

Here $F = 93$ MeV and $g_A = 1.26$ are the pion radiative decay and axial vector coupling constants; and $c'_i (i = 1-4)$ are the low energy constants \cite{14,15,16}. The covariant derivatives $D_\mu$, $\Gamma_\mu$ and axial vector field $\Delta_\mu$ are related through the expressions:

$$\mathcal{D}^{(1)} = \mathcal{D} + ig_A \Delta \gamma^5,$$

$$D_\mu = \partial_\mu + \Gamma_\mu,$$

$$\Gamma_\mu = \frac{1}{2} \left[ \xi^\dagger, \partial_\mu \xi \right],$$

$$\Delta_\mu = \frac{1}{2} \left[ \xi^\dagger, \partial_\mu \xi \right].$$

Here $\mathcal{D} = \gamma^\mu D_\mu$; $\bar{N} \hat{D} N = \bar{N}(DN) - (D^\dagger \bar{N})N$; the symbol $\langle B \rangle$ stands for the trace of the quantity B over isospin matrices and, a nonlinear realization of the chiral symmetry $U = \xi^2 = \exp(i\tau \cdot \pi / F)$ is presumed to describe the pion field.

The pion-nucleon scattering amplitude within the framework of a relativistic $\chi$PT has been derived already by Gasser et al. \cite{13}. Using their results, the Lagrangian Eqn. 4 and standard Feynman rules, the nucleon pole and sea-gull (diagrams 1a-1c) contributions to the production process are,
\[ \mathcal{M}_I^{(1)} = N_1 N_3 \frac{i g_A}{2F(q^2 - m^2)} T^{(s+u)} \chi_3^\dagger (\Pi_1 - \Pi_3) \cdot \sigma_1 \chi_1 + [1 \leftrightarrow 2, 3 \leftrightarrow 4], \]
\[ \mathcal{M}_R^{(1)} = N_1 N_3 \frac{i g_A}{2F(q^2 - m^2)} T^{(c)} \chi_3^\dagger (\Pi_1 - \Pi_3) \cdot \sigma_1 \chi_1 + [1 \leftrightarrow 2, 3 \leftrightarrow 4], \]
\[ (6) \]

with,
\[ T^{(s+u)} = N_2 N_4 \frac{g_A^2 M^2}{F^2} \chi_4 \left[ \frac{1}{(p_4 + k)^2 - M^2} kP - \frac{1}{(p_2 - k)^2 - M^2} qP - \frac{1}{M} R \right] \chi_2, \]
\[ T^{(c)} = -N_2 N_4 \frac{m^2}{MF^2} \chi_4 \left\{ -2c_1' + c_2' \frac{q(p_4 + p_2)k(p_4 + p_2)}{4m^2 M^2} + c_3' \frac{kq}{m^2} + c_4' \frac{1}{4Mm^2} [(p_4 + p_2)qkP + (p_4 + p_2)kqP] \right\} \chi_2. \]
\[ (7) \]
\[ (8) \]

Here,
\[ N_i = \sqrt{\frac{E_i + M}{2M}}, \quad \Pi_i = \frac{P_i}{E_i + M}, \]
\[ P = (1 + \Pi_2 \cdot \Pi_4 + i\Pi_2 \times \Pi_4 \cdot \sigma_2, \Pi_2 + \Pi_4 + i(\Pi_2 - \Pi_4) \times \sigma_2), \]
\[ R = 1 - \Pi_2 \cdot \Pi_4 + i\Pi_2 \times \Pi_4 \cdot \sigma_2, \]
\[ (9) \]

where \( q = p_1 - p_3, k = (\sqrt{m^2 + k^2}, k), p_1 = (E(p), p), p_2 = (E(p'), -p), p_3 = (E(p' - k/2), p' - k/2), p_4 = (E(-p' - k/2), -p' - k/2) \) stand for the momenta of the incoming and outgoing pions and nucleons (see Fig. 1) in the overall center of mass (CM) system, respectively. The bracket \([1 \leftrightarrow 2, 3 \leftrightarrow 4]\) represents the contribution from the same diagrams with the proton momenta \( p_1, p_3 \) interchanged with \( p_2, p_4 \), respectively. Note that \( T^{(s+u)} \) and \( T^{(c)} \) are analogous impulse and sea-gull contributions (graphs 2a-2c) to the \( \pi^0 N \to \pi^0 N \) conversion process, as derived by Gasser et al. \[13\].

At low scattering energy and near threshold of the \( pp \to pp\pi^0 \) reaction, \( P = (1, \Pi_2 + i\Pi_2 \times \sigma_2), R = 1 \) so that,
\[ T^{(s+u)} \approx \frac{g_A^2 M^2}{F^2} N_4 N_2 \frac{T}{M(2Mk^0 + k^2)(2Mq^0 - q^2)}, \]
\[ (10) \]

with,
\[ T \approx k^4 - q \cdot qk^0 \left( k^0 + \frac{k^2}{M} \right), \]
\[ (11) \]
The quantity $T$ determines the magnitude as well as the sign of $T^{(s+u)}$. When both of the incoming and outgoing pions are on the mass shell (i.e. $k^2 = q^2 = m^2$ and $k^2, q^2 \ll m^2$) it is positive, $T = m^4 > 0$. For the production process however, only the outgoing pion is on the mass shell. The exchanged pion is far off mass shell ($q^2 \leq -Mm, q^2 \geq Mm$) and $T$ is negative, i.e.

$$T^{(s+u)}_{off} = -0.9/m .$$  \hfill (12)

We recall that in variance with this value the overall $s$ and $u$ nucleon pole contribution to lowest order in HFF is always positive [2],

$$T^{(s+u)}_{HFF} = \frac{g_A^2}{4MF^2} (q^2 + k^2) .$$  \hfill (13)

Thus in the transition from a fully relativistic theory to an extreme non-relativistic limit, the contribution from nucleon pole terms to off mass shell $\pi N$ scattering amplitude reverses sign. As we shall demonstrate below, this is mostly because the HFF power expansion of the nucleon propagator can not be approximated by any finite sum.

Next consider the sea-gull contribution. For low energy scattering and when both pion legs are on the mass shell, Eqn. 8 reduces to exactly the expression from HFF, i.e.

$$T^{(c)} \approx -\frac{m^2}{MF^2} N_2 N_4 \left[ -2c' + c_3 ' \left( 1 + \frac{k^2 - q \cdot k}{m^2} \right) + (c'_2 + c'_4) \left( 1 + \frac{k^2}{m^2} \right) \right] .$$  \hfill (14)

However, off the mass shell and at threshold of the production process, Eqn. 8 becomes,

$$T^{(c)}_{off} = -\frac{m^2}{MF^2} \sqrt{1 + \frac{m}{4M}} \left[ -2c'_1 + c'_2 \left( 1 + \frac{m}{4M} \right)^2 + c'_3 \frac{1}{2} + c'_4 \left( 1 + \frac{m}{4M} \right) \right] .$$  \hfill (15)

This expression departs strongly from the HFF result \cite{1,2}

$$T^{(c)}_{HFF} = -\frac{m^2}{MF^2} \left[ -2c'_1 + \frac{c'_2 + c'_3 + c'_4}{2} \right] ,$$  \hfill (16)

where a term of the order of $(c'_2 + c'_4)1/2$ is missing. Clearly, off the mass shell, the relativistic expression for the sea-gull contribution (Eqn. 8) decreases far more rapidly with the momentum transferred $q$ as compared with the respective HFF expression (Eqn. 15).
Here as well, this difference between the two approaches is the result of extending the application of the HFF Lagrangian, which is limited to low momentum transfer, outside its domain of validity. By doing so, a \( q \) momentum dependent part of the sea-gull term which is actually of the same order of magnitude, becomes part of higher chiral order terms and thus introducing sizeable discrepancy.

In table I we list values of \( T^{(c)} \) as obtained with different LEC parameter sets. The parameter Set 1 is obtained from a tree level fit of \( \pi N \) scattering data; the Set 2,3 are proposed in Ref. [3] by extracting the value of \( c'_2 + c'_3 + c'_4 \) from the effective range parameter \( b^+ \) of the low-energy pion-nucleon scattering amplitude; The Set 4 is determined by fitting pion-nucleon scattering to one loop order [16]; The Sets 5-7 are determined in Ref. [11] from fitting data to chiral order \( \Delta = 2 \). The magnitude of the sea-gull term is small on the mass shell but its sign depends on the values of the LEC used; \( T^{(c)} \) is negative with parameter Sets 2, 6 and 7 but positive with the others. Off the mass shell the sea-gull term is rather sensitive to different choices of the LEC parameter sets, though its sign is negative always. A fully relativistic approach predicts a factor 2-3 larger a contribution as compared to that from HFF. Consequently, in a fully relativistic approach the nucleon pole terms and the sea-gull term add constructively. Their overall contribution, which appears as \( T_{\text{off}}^{\text{full}} \) in Table 1, is strongly enhanced in comparison with the respective quantity \( T_{\text{offHFF}}^{\text{full}} \) from HFF. This has a dramatic impact on the calculated cross section of the \( pp \rightarrow pp\pi^0 \) reaction.

In Fig. 3 we draw cross sections calculated, taking into account contributions from the impulse and rescattering terms only. Corrections due to initial state interactions (ISI) and final state interactions (FSI) are introduced using the approximation II of Ref. [8]. The cross sections calculated with the different LEC sets vary within a factor of 2 and, except for results obtained with the Set 2, all others underestimate data [18,19] by a factor 1.5-2. We would like to stress though that the curves presented in Fig. 3 do not account for contact terms and loop contributions, and therefore may serve for illustrative purposes only, not for explaining data. The inclusion of the contact terms (Fig. 1d-1f) must awaits a reliable (and consistent with NN interactions) determination of many LEC’s [13,20]. Unfortunately,
it is not straightforward to extend the procedure previously applied in the context of the HFF [14,21,8] to determine the parameters for the contact terms (Fig. 1d-1f) [20,22].

It is to be noted also, that we have not included lower order loop contributions. Restricting our discussion to one loop only, there are more than 40 possible diagrams which may contribute to the production process. In the HFF $\chi$PT calculations [8], many of these are proportional to three momentum of the meson produced and therefore their contribution to the cross section at threshold is negligibly small. In the relativistic $\chi$PT these contributions become proportional to the pion mass and can no longer be neglected. Because of fine mutual cancellations amongst them, all these diagrams must be included in a systematic way as in Ref. [8]. However, it should be noted that an inspection of the expressions derived by Gasser et al. [13] for analogous contributions to $\pi^0 N \rightarrow \pi^0 N$, reveals that such one loop contributions to the production process are smaller than the overall contribution from the impulse and rescattering terms.

Clearly, the relative importance of various contributions and their signs in a fully relativistic $\chi$PT approach differ significantly from those obtained in the non-relativistic HFF approach. To point out the origin of these differences we recall that in the transition from a relativistic $L_{\pi N}$ to non-relativistic HFF Lagrangian one reduces the nucleon kinetic term which affects the nucleon propagator strongly. By doing so, the validity of the HFF Lagrangian becomes limited to sufficiently small momenta and therefore is not applicable to meson production. Both of the s and u pole diagrams, Fig. 1a, 1b, involve a covariant nucleon propagator,

$$S_N = i \frac{\not{p} + M}{p^2 - M^2},$$

which we may separate into a positive and negative energy parts. To do this let us write the numerator in Eqn. [17] as

$$\not{p} + M = M(1 + \not{v}) + (\not{p} - M\not{v}) = 2MP_+ + l(\not{P}_+ + \not{P}_-),$$

where $v^\mu$ denotes the four velocity of the nucleon with $v^2 = 1$, $l^\mu = p^\mu - Mv^\mu$ is its residual momentum, and $P_{\pm} = (1/2)(1 \pm \not{v})$ are operators which project the nucleon Dirac
field into large and small components. Following the usual reduction procedure, we take 
\( p_\perp = p^\mu - (pv)v^\mu \) to be the transversal component of the nucleon momentum and write the 
nucleon propagator in the form,

\[
S_N = i \left[ 2MP_+ + \mathcal{I}(P_+ + P_-) \right] \frac{1}{2(M + T(p_\perp))} \left[ \frac{1}{vl - T(p_\perp)} - \frac{1}{2M + vl + T(p_\perp)} \right].
\]  

(19)

with \( T(p_\perp) = \sqrt{M^2 + p_\perp^2} - M \). In the limit of low kinetic energy the negative energy part 
reduces to \( S_N \approx -iP_+/2M \). It is easy to show now that, the respective negative energy 
contributions from s and u channels sum up to be,

\[
T_Z \approx \frac{g_4^2}{4MF^2} (vq)(vk).
\]  

(20)

This in fact has the form of a contact term. In passing by, we note that by separating the 
nucleon propagator into negative and positive energy parts the s and u channels (graphs 
1a, 1b) split into direct and Z-graph contributions. In the non-relativistic limit the 
Z-graph contribution appears in exactly the form of Eqn. 20, as a local rescattering term. 
Thus in the transition to non-relativistic limit, the negative energy part of the nucleon pole 
terms ”converts” into a sea-gull contact term. An even more serious a drawback concerns 
the direct part of the nucleon pole terms. It is not always possible to calculate the direct 
part, which is a non-local term, within the frame of HFF. To see this consider the expansion 
of \( S_N \), Eqn. 19 in power series. The series expansion for the factors \( 1/(M + T(p_\perp)) \) and 
\( 1/(2M + vl + T(p_\perp)) \) converges up to high energies. However, the series

\[
\frac{1}{vl - T(p_\perp)} = \frac{1}{vl} \sum \left[ \frac{T(p_\perp)}{vl} \right]^n,
\]  

(21)

converges only for \( T(p_\perp)/vl < 1 \). In the instance of \( \pi N \to \pi N \) scattering and in the limit 
\( T(p_\perp) \ll vl \) this series converges rather well. However for a production process \( NN \to 
NNX \), the virtual nucleon in the graph 1b has a residual momentum \( l = (-m_X/2, l); 1 \cdot l = 
Mm_X \), so that \( T(p_\perp)/vl \approx -1 \). Thus the power series is on the border of its convergence 
circle and therefore it can not be approximated by any finite sum. This shows, in fact, 
that the HFF can not possibly predict the impulse term correctly, and therefore excludes
the possibility that a finite chiral order HFF based $\chi$PT calculations can explain meson production in NN collisions.

In summary, we have considered tree diagram contributions to neutral pion production in $pp \rightarrow pp\pi^0$ in a fully relativistic $\chi$PT and in the extremely non-relativistic HFF. We have found that in the relativistic approach, the relative phase of the impulse and rescattering terms is +1 and the two terms add constructively, giving rise to a substantial contribution to the cross section. This stands in marked difference with the HFF results where these two terms have opposite signs, the rescattering term being considerably smaller and their overall contribution to the cross section is small. The usefulness and success of non-renormalizable (and renormalizable) effective field theories, depend on how fast the respective perturbative expansion converges. It was demonstrated that for meson production in NN collisions, the HFF series of the nucleon propagator is on the border of its convergence circle. Consequently, meson production falls outside the HFF validity domain, making the predictions from a non-relativistic $\chi$PT for such processes impossible. Since the preparation of this note, we have noticed that the Jülich group \cite{23} have reported on similar results.

Acknowledgments This work was supported in part by the Israel Ministry Of Absorption. We are indebted to Z. Melamed for assistance in computation.
REFERENCES

[1] B. -Y. Park et al., Phys. Rev. C53, 1519 (1996).

[2] T. D. Cohen et al., Phys. Rev. C53, 2661 (1996).

[3] T. Sato et al., Phys. Rev. C56, 1246 (1997).

[4] C. J. Horowitz et al., Phys. Rev. C49, 1337 (1994).

[5] E.Hernandez and E.Oset, Phys.Lett.B 350, 158(1995).

[6] C.Hanhart et al.,Phys.Lett.B 358, 21 (1995)

[7] E. Gedalin, A. Moalem and L. Razdolskaya, [nucl-th/9611005] and E. Gedalin, A. Moalem and L. Razdolskaya, submitted to Nucl. Phys. A.

[8] E.Gedalin, A.Moalem and L.Razdolskaya, [nucl-th/9803029] and submitted to Phys. Lett.B.

[9] C.Hanhart et al., Phys.Lett. B 424, 8 (1998).

[10] K.Tamura et al., To be published and private communication with A. Moalem.

[11] N.Fettes et al.,Nucl.Phys.A 640, 199 (1998).

[12] J.Gasser and H.Leutwyler, Nucl. Phys. B 250,465 (1985).

[13] J.Gasser et al. Nucl.Phys. B 307, 779 (1988).

[14] T.S. Park, D. -P. Min and M.Rho, Phys. Rep. 233, 341 (1993).

[15] V. Bernard, N. Kaiser and Ulf-G.Meissner, Int. J. Mod. Phys. E4, 193 (1995).

[16] V. Bernard, N. Kaiser and Ulf-G.Meissner, Nucl. Phys. A615, 483 (1997).

[17] V. De Alfaro, S. Fibini, G. Furlan, C. Rossetti, ”Currents In Hadron Physics”, North-Holland, Amsterdam 1973, p. 171.

[18] A. Bondar et al., Phys. Lett. B356, 8 (1995).
[19] H. O. Meyer et al., Nucl. Phys. A539, 683 (1992).

[20] G. Ordóñez et al., Phys. Rev. C 53, 2086 (1996).

[21] U. van Kolck et al., Phys. Lett. B388, 679 (1996).

[22] E. Gedalin et al., to be published.

[23] V. Bernard, N. Kaiser and Ulf-G. Meissner, nucl-th/9806013, v2 Aug (1998).
TABLES

| Set No. | $c'_1$ | $c'_2 + c'_4$ | $c'_3$ | $mT_{on}^c$ | $mT_{off}^c$ | $mT_{off}^{c_{HFF}}$ | $mT_{off}^{full}$ | $mT_{off}^{full HF F}$ |
|---------|--------|---------------|--------|-------------|--------------|---------------------|-------------------|----------------------|
| 1       | -1.63  | 6.28          | -9.86  | 0.135       | -1.53        | -0.48               | -2.41            | 0.42                 |
| 2       | -1.63  | 8.46          | -9.86  | -0.63       | -2.29        | -0.87               | -3.17            | 0.03                 |
| 3       | -1.87  | 6.28          | -9.86  | 0.007       | -1.69        | -0.64               | -2.57            | 0.26                 |
| 4       | -1.74  | 6.28          | -9.94  | 0.08        | -1.59        | -0.54               | -2.47            | 0.34                 |
| 5       | -2.38  | 6.07          | -11.14 | 0.105       | -1.77        | -0.76               | -2.65            | 0.14                 |
| 6       | -2.76  | 6.03          | -11.27 | -0.1        | -2.0         | -0.98               | -2.88            | -0.08                |
| 7       | -2.87  | 6.05          | -11.63 | -0.054      | -2.0         | -1.0                | -2.88            | -0.1                 |

**TABLE I.** The sea-gull term and pion-nucleon scattering amplitude for different LEC parameter sets. $T_{on}^c$ represents the on mass shell sea-gull term contribution to the pion-nucleon scattering amplitude. The quantities $T_{off}^c$ and $T_{off}^{c_{HFF}}$ denote the off mass shell sea-gull term in the relativistic and HFF limits, respectively. The amplitudes $T_{off}^{full}$ and $T_{off}^{full HF F}$ represents the respective overall contribution of the impulse and sea-gull terms in relativistic and non-relativistic limits. The pole term is equal to $mT^{s+u} = -0.88$ in the relativistic approach and $mT^{s+u}_{HFF} = 0.88$ in HFF $\chi$PT. The different LEC parameter sets are listed in columns 2-4; the Set 1 is taken from Ref. [15], Set 2, 3 from Ref. [3], Set 4 from Ref. [16] and the Sets 5-7 from Ref. [11].

|       | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|-------|---|---|---|---|---|---|---|
| $\mathcal{M}$ (fm) | 92 | 121 | 98 | 94 | 101 | 110 | 111 |
| $\mathcal{M}_{HFF}$ (fm) | -16 | -1.1 | -10 | -13 | -5.3 | 3 | 3.8 |

**TABLE II.** The full primary production amplitudes $\mathcal{M}$ and $\mathcal{M}_{HFF}$ at threshold for different sets of $c'_i$. 

12
FIG. 1. Various tree level diagram to the $NN \rightarrow NN\pi^0$ reaction: (a) s-channel nucleon pole impulse term, (b) u-channel nucleon pole impulse term, (c) rescattering term, (d)-(f) various contact term contributions.
FIG. 2. Pole and sea-gull terms contributing to the $\pi N \rightarrow \pi N$
FIG. 3. S-wave production cross section for the $pp \rightarrow ppp\pi^0$ from relativistic $\chi$PT. Only the impulse and s-wave rescattering terms are included. All curves are corrected for ISI and FSI using the approximation II of \cite{8}. The labels of the curves denote the LEC parameter sets. See caption of table I. The data are taken from Ref. \cite{18,19}.