B-physics with Nf=2 Wilson fermions

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Outline

**Motivation** well known:
- matrix elements for $B$ decays (cfr $|V_{ub}|$)
- $m_b(m_b)$ for perturbative computations
- reproduce $B$ mesons mass spectrum

**Method**
- CLS configurations
- HQET renormalization, matching, improvement
- large volume computations

**Results**
- predictions $f_B, f_{B_s}, \frac{f_{B_s}}{f_B}$
- postdictions $\overline{m}_{b}(m_b), m_{B_s} - m_B, m_{B^*} - m_B, m_{B^*_s} - m_{B_s}$
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  predictions $f_B$, $f_{B_s}$, $\frac{f_{B_s}}{f_B}$

  postdictions $\overline{m}_b^{MS}(m_b)$, $m_{B_s} - m_B$, $m_{B^*} - m_B$, $m_{B_s^*} - m_{B_s}$
Treatment of light quarks

\( N_f = 2 \) sea Wilson quarks

- Volume effects exponentially suppressed:
  \[ Lm_\pi \geq 4.0 \]

- Light quark mass chiral extrapolation:
  \( 190 \lesssim m_\pi \lesssim 450 \) MeV

- Discretization effects:
  - 3 lattice spacings \( a \)
    - 0.048, 0.065, 0.075 fm

- NP renormalization
- NP \( O(a) \) improvement

| id  | \( L/a \) | \( a \) [fm] | \( m_\pi \) [MeV] | \( m_\pi L \) |
|-----|-----------|-------------|-----------------|-------------|
| A4  | 32        | 0.0755      | 380             | 4.7         |
| A5  |           |             | 330             | 4.0         |
| B6  | 48        |             | 270             | 5.2         |
| E5  | 32        | 0.0658      | 440             | 4.7         |
| F6  | 48        |             | 310             | 5.0         |
| F7  |           |             | 270             | 4.3         |
| G8  | 64        |             | 190             | 4.1         |
| N5  | 48        | 0.0486      | 440             | 5.2         |
| N6  |           |             | 340             | 4.0         |
| O7  | 64        |             | 270             | 4.2         |
Treatment of light quarks

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  - NP renormalization
  - NP \( O(a) \) improvement
Treatment of $b$ quark

$m_b \gg \Lambda_{QCD}$: $b$ treated in HQET

- expansion in $1/m_b$
  \[
  \mathcal{L}_{HQET}(x) = \mathcal{L}_h^{stat} - \omega_{\text{kin}} O_{\text{kin}}(x) - \omega_{\text{spin}} O_{\text{spin}}(x)
  = \bar{\psi}_h(x) D_0 \psi_h(x) - \omega_{\text{kin}} \bar{\psi}_h(x) D^2 \psi_h(x) - \omega_{\text{spin}} \bar{\psi}_h(x) \sigma \cdot B \psi_h(x)
  \]

- restrict to processes such that $p \ll m_b$
- power divergences in $a^{-1}$ ⇒ need NP renormalization
  [Maiani, Martinelli, Sachrajda 92]

- renormalizable at every order in $1/m_b$
  ⇒ safe estimate of discretization effects

\[
\langle O \rangle = \langle O \rangle_{stat} + \omega_{\text{kin}} \sum_x \langle O O_{\text{kin}}(x) \rangle_{stat} + \omega_{\text{spin}} \sum_x \langle O O_{\text{spin}}(x) \rangle_{stat}
\]

\[
\langle O \rangle_{stat} = \frac{1}{Z} \int_{\text{fields}} \mathcal{O} \exp \left( -a^4 \sum_x [\mathcal{L}_{\text{light}} + \mathcal{L}_h^{stat}] \right)
\]
ALPHA strategy for NP renormalization & matching

- match QCD and HQET at $a^{-1} \gg M_b$
  - small volume, however $z = L M_b \gg 1$
- determine NP: $\tilde{\omega}(z) = m_{\text{bare}}(z), Z_{A}^{\text{HQET}}(z), c_{A}^{(1)}(z), \omega_{\text{kin}}(z), \omega_{\text{spin}}(z)$
- step scaling to $a$ used in large volumes
  - determine $M_b$ dependence of large volume observables

[Blossier et al. 12]
The b-quark’s mass

1. (left) Extrapolate to physical point ($m_{PS} \rightarrow m_\pi$)

$$m_B(z, m_{PS}, a, \text{HYPn}) + \frac{3\hat{g}^2}{32\pi} \left( \frac{m_{PS}^3}{f_{PS}^2} - \frac{m_\pi^3}{f_\pi^2} \right) = B(z) + Cm_{PS}^2 + D_{\text{HYPn}}a^2$$

$$\hat{g} = 0.51(2) \ [\text{Bulava et al. 10}]$$

2. (right) Interpolate $m_B(z)$ to get $M_b$:

$$m_B(z, m_\pi, a) |_{z = z_b} \equiv m_B^{\exp} = 5279.5 \text{MeV}$$

3. We get $z_b = 13.17(23)(13) \ z$ or equivalently $m_b^{\text{MS}}(m_b) = 4.23(11)(3) \ z \ \text{GeV}$. 

![Graphs showing $m_B^{\text{sub}}(z, m_\pi, 0)$ in MeV vs. $m_{PS}^2$ and $z$]
Observables in HQET

1. Interpolate $\bar{\omega}(z)$ to get $\bar{\omega}(z_b)$

2. Compute matrix elements in HQET

\[
p_{\text{stat}} = \lim_{x_0 \to \infty} \{2e^{E_{\text{stat}}x_0} \sum_{x} \langle A_0(x)A_0(0) \rangle \}^{1/2}, \quad E_{\text{stat}} = -\lim_{x_0 \to \infty} \partial_0\ln \sum_{x} \langle A_0(x)A_0(0) \rangle
\]

\[\cdots \cdots \cdots\]

3. Combine matrix elements and $\bar{\omega}(z_b)$

\[
\ln(f_B \sqrt{m_B/2}) = \ln(Z_A^{\text{HQET}}) + \ln(p_{\text{stat}}) + b_A^{\text{stat}}a m_{\text{PCAC}}^l
\]

\[+ \omega_{\text{kin}} p_{\text{kin}} + \omega_{\text{spin}} p_{\text{spin}} + c_A^{(1)} p A^{(1)}\]

\[
A_{0,R}^{\text{HQET}} = Z_A^{\text{HQET}} [A_{0}^{\text{stat}} + c_A^{(1)} A_0^{(1)}], \quad A_0^{\text{stat}} = \bar{\psi}_1 \gamma_0 \gamma_5 \psi_h, \quad A_0^{(1)} = \bar{\psi}_1 \gamma_5 \gamma_i \frac{1}{2} (\nabla_i - \overleftarrow{\nabla}_i) \psi_h
\]

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Treatment of the excited states

- GEVP with 3 light quark wavefunctions (levels of smearing)
- plateau average only where excited states contribution negligible

\[ \sigma_{\text{sys}}(t_{\text{min}}) \ll \sigma_{\text{stat}}(t_{\text{min}}) \]
Statistical analysis

Use methods described in: [Schaefer et al. 10]

- all correlations taken into account (eg with $a, \vec{\omega}(z), m_\pi, f_\pi$)
- for an observable $\mathcal{O}$: 
  \[ \delta \mathcal{O} \propto \tau_{\text{int}} = \frac{1}{2} + \sum_{1}^{\infty} \rho_{\mathcal{O}}(t) \]
- in practice restrict sum up to $W$, but 
  \[ \rho_{\mathcal{O}}(t) \xrightarrow{t \to \infty} A_{\mathcal{O}} e^{-t/\tau_{\text{exp}}} \]

E5g, $M_\pi = 440$ MeV, $a = 0.0658$ fm, $\tau_{\text{exp}} = 134$ MDU, $\tau_{\text{int}} = 36.40$ MDU

- typically $W < \tau_{\text{exp}}$
- attach a tail to $\rho_{\mathcal{O}}$

\[ \tau_{\text{int}}^{u}(\mathcal{O}) = \tau_{\text{int}}(\mathcal{O}, W_u) + \tau_{\text{exp}} \rho_{\mathcal{O}}(W_u) \]
Results: $f_B$

$$f_B \sqrt{m_B} = A \left(1 + \frac{3 + 9g^2}{8} (\tilde{y}_1 \ln \tilde{y}_1 (PS) - \tilde{y}_1 \ln \tilde{y}_1 (\pi))\right) + B (\tilde{y}_1 (PS) - \tilde{y}_1 (\pi)) + C_{HYPn} a^2$$

- continuum and chiral extrapolation (NLO HMChPT):
  - error from $a \rightarrow 0$ in stat
- $\tilde{y}_1 (PS) = \frac{m_{PS}^2}{16\pi^2 f_{PS}^2}$
- ChPT: NLO vs. linear extrapolation
- tiny cutoff effects

$$f_B = 187(12)_{stat}^{2} ChPT \text{ MeV}$$

HYP1 open symbols / dashed lines  
HYP2 filled symbols / dash-dotted lines
$f_{B_s} \sqrt{m_{B_s}} = A + B (\tilde{y}_1 (PS) - \tilde{y}_1 (\pi)) + C_{HYPn} a^2$

NLO HMChPT has no log for $f_{B_s}$ in PQ

$\kappa_s$ from scale setting by $f_K$

[Fritzsch et al. 12]

less statistics wrt $f_B$

small cutoff effects

$f_{B_s} = 224 (13)_{stat} \text{ MeV}$
Results: $f_{B_s}/f_B$

\[
\frac{f_{B_s} \sqrt{m_{B_s}}}{f_B \sqrt{m_B}} = A \left(1 - \frac{3 + 9\hat{g}^2}{8} (\tilde{\gamma}_1 \ln \tilde{\gamma}_1 (PS) - \tilde{\gamma}_1 \ln \tilde{\gamma}_1 (\pi))\right) + B (\tilde{\gamma}_1 (PS) - \tilde{\gamma}_1 (\pi)) + C_{HYPn}a^2
\]

- **ChPT**: NLO vs. linear extrapolation
- Less statistics wrt $f_B$
- Small cutoff effects

\[
f_{B_s}/f_B = 1.195(61)_{stat}(20)_{ChPT}
\]
Results: $m_{B_s} - m_B$

$$m_{B_s} - m_B = \frac{9\hat{g}^2}{128\pi} \left( \frac{m^3_{PS}}{f^2_{PS}} - \frac{m^3_\pi}{f^2_\pi} \right) = A + B (\tilde{y}_1(PS) - \tilde{y}_1(\pi)) + C_{\text{HYP}n} a^2$$

- **ChPT**: NLO vs. linear extrapolation
- **$a$**: fit with or without $\beta = 5.5$

$$m_{B_s} - m_B = 83.9(6.3)_{\text{stat}}(6.9)_{ChPT}(0.8) a \text{ MeV}$$
Results: $m_{B^*} - m_B$

\[ m_{B^*} - m_B = A + B (\tilde{y}_1(PS) - \tilde{y}_1(\pi)) + C_{HYPn} a \]

- $m_{B^*} - m_B = O(1/m_b)$
- despite $O(a)$ improvement, expect $O(a/m_b)$ effects
- $a$: fit in $a^2$ vs. fit in $a$

\[ m_{B^*} - m_B = 41.7(4.7)_{stat}^{(3.4)} a \text{ MeV} \]
Results: $m_{B_s^*} - m_{B_s}$

$$m_{B_s^*} - m_{B_s} = A + B(\tilde{y}_1(PS) - \tilde{y}_1(\pi)) + C_{HYPn}a$$

$m_{B_s^*} - m_{B_s} = O(1/m_b)$

despite $O(a)$ improvement, expect $O(a/m_b)$ effects

$a$: fit in $a^2$ vs. fit in $a$

$$m_{B_s^*} - m_{B_s} = 37.9(3.7)_{\text{stat}}(5.9)_{a}\text{ MeV}$$
Comparison with experimental results: postdictions

| Observable                  | ALPHA        | Exp.     | Method                           |
|-----------------------------|--------------|----------|----------------------------------|
| $m_B$ [MeV]                 | input        | 5279.5   | $e^+ e^- \text{ scat.}$         |
| $m_b^{\overline{MS}}$ [GeV] | $4.23(11) (3)_z$ | $4.18(3)$ | smeared $\sigma (e^+ e^- \rightarrow b\bar{b}) + \text{ PT}$ |
| $m_{B_s} - m_B$ [MeV]       | $83.9(6.3) (6.9)_a$ | $87.35(0.23)$ | $pp, p\bar{p} \text{ scat.}$ |
| $m_{B^*} - m_B$ [MeV]       | $41.7(4.7) (3.4)_a$ | $45.3(0.8)$ | $e^+ e^- \text{ scat.}$         |
| $m_{B^*_s} - m_{B_s}$ [MeV] | $37.9(3.7) (5.9)_a$ | $48.7(2.3)$ | $e^+ e^- \text{ scat.}$         |

- reproducing well known experimental results is a reason to be confident in our method
- **systematics** still relevant for some quantities
Comparison with Lattice averages: predictions

| Obs.       | ALPHA             | Lat. Av.\(^1\) | Experiment                                           |
|------------|-------------------|----------------|-----------------------------------------------------|
| \(f_B\) [MeV] | \(187(12)(2)_{ChPT}\) | 190.6(4.7)    | \(BR(B \to \tau\nu)_{ALPHA} = 1.065(21) \times 10^{-4}\)  |
|            |                   |                | \(BR(B \to \tau\nu)_{exp} = 1.05(25) \times 10^{-4}\)   |
| \(f_{B_s}\) [MeV] | 224(13)           | 227.6(5.0)    | \(BR(B_s \to \mu^+\mu^-)_{ALPHA} = 3.15(27) \times 10^{-9}\) |
|            |                   |                | \(BR(B_s \to \mu^+\mu^-)_{exp} = 2.9(0.7) \times 10^{-9}\) |
| \(f_{B_s}/f_B\) | 1.195(61)(20)_{ChPT} | 1.201(17)     |                                                     |

- \(BR(B \to \tau\nu)_{ALPHA}\) uses \(|V_{ub}|\) from PDG 12 (inclusive decays, \(BR(B \to \pi l\nu)\))
- \(BR(B_s \to \mu^+\mu^-)_{ALPHA}\) uses \(|V^*_{tb}V_{ts}|\) from CKM fit (mainly \(B^0_s\) splitting)
- agreement with Lattice Averages
- agreement with Experiment

\(^1\)\url{www.latticeaverages.org}
Partial contributions to the total error for $z_b$
Partial contributions to the total error for $f_B$