COMPARISION OF THE PION EMISSION FUNCTION IN
HADRON-HADRON AND HEAVY-ION COLLISIONS

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Combining results on single-particle distributions with those of the Bose-Einstein
correlation analysis in the same experiment, the space-time emission function is
extracted as a function of time and longitudinal coordinate, as well as a function of
transverse coordinates. While the former function resembles a boomerang for both
types of collision (only shifted in the time direction due to the larger freeze-out time
in heavy-ion collisions), the latter function is Gaussian for heavy-ion collisions, but
ring-shaped for hadron-hadron collisions.

1 Introduction

One of the focal points of current research in high energy heavy-ion and
particle physics is the extraction of the space-time dependence of the pion
emission function of the particle production points.

In this paper we will consider the space-time distribution of correlated
pion production points in hadron-hadron and heavy-ion collisions at the CERN
SPS. It is interesting to note, that the hydrodynamical models not only hold
for heavy-ion collisions,[1–4] but also can be used for meson-proton collisions
[5–8] for which the formation of hadronic matter with macroscopic properties
is less evident. It is important to emphasize, that correlation measurements
do not contain the complete information on the geometrical and dynamical
parameters characterizing the evolution of hadronic matter. More profound
information can be provided by a combined analysis of data on two-particle
correlations and single-particle inclusive spectra.[7–16]

Here, the results of an analysis are presented on \((\pi^+ / K^+)p[5–8]\) and Pb+Pb
interactions\([2–4,17–19,25–28]\) at the CERN SPS in the framework of the hydro-
dynamical model for three-dimensionally expanding, cylindrically symmetric,
finite systems (BL-H model).[11] For both types of collision, the space-time
emission function is extracted as a function of time and longitudinal coordinate,
as well as a function of transverse coordinates.
2 Combination of Single-Particle Spectra and Two-Particle Correlations

The Bertsch-Pratt (BP) parametrization [21–24] has been used to describe the two-particle correlation function. At mid-rapidity, \( y = y_0 \), and in the Longitudinal Center of Mass System (LCMS), where the longitudinal momentum sum is zero, the effective BP radii can be approximately expressed from the BL-H parametrization as [11,14,20,21]

\[
R_L^2 = \tau_f^2 \Delta \eta^2; \quad R_0^2 = R_s^2 + \beta_t^2 \Delta \tau_s^2; \quad R_s^2 = R_s^2
\]

with

\[
\frac{1}{\Delta \eta^2} = \frac{1}{\Delta \eta^2} + \frac{M_t}{T_0}; \quad R_s^2 = \frac{R_G^2}{1 + \frac{M_t}{T_0}(\langle u_t \rangle^2 + \langle \frac{\Delta T}{T} \rangle)}.
\]

where \( M_t = 0.5(m_{T,1} + m_{T,2}) \) and the parameters \( \Delta \eta^2, T_0, \langle u_t \rangle \) and \( \langle \frac{\Delta T}{T} \rangle \) are extracted from the invariant spectra; [11] \( R_G \) is related to the transverse geometrical r.m.s. radius of the source as \( R_G \text{ (r.m.s.)} = \sqrt{2} R_G \); \( \tau_f \) is the mean freeze-out (hadronization) time; \( \Delta \tau_s \) is related to the duration time \( \Delta \tau \) of hadron emission and to the temporal inhomogeneity of local temperature, as the relation \( \Delta \tau_s \geq \Delta \tau \) holds; the variable \( \beta_t \) is the transverse velocity of the pion pair.

| Param. | NA49 | NA44 | WA98 | Averaged | NA22 |
|--------|------|------|------|----------|------|
| \( T_0 \) [MeV] | 134±3 | 145±3 | 139±5 | 139±6 | 140±3 |
| \( \langle u_t \rangle \) | 0.61±0.05 | 0.57±0.12 | 0.50±0.09 | 0.55±0.06 | 0.20±0.07 |
| \( R_G \) [fm] | 7.3±0.3 | 6.9±1.1 | 6.9±0.4 | 7.1±0.2 | 1.2±0.2 |
| \( \tau_0 \) [fm/c] | 6.1±0.2 | 6.1±0.9 | 5.2±0.3 | 5.9±0.6 | 1.4±0.1 |
| \( \Delta \tau \) [fm/c] | 2.8±0.4 | 0.01±2.2 | 2.0±1.9 | 1.6±1.5 | 1.3±0.3 |
| \( \Delta \eta \) | 2.1±0.2 | 2.4±1.6 | 1.7±0.1 | 2.1±0.4 | 1.36±0.02 |
| \( \langle \frac{\Delta T}{T} \rangle \) | 0.07±0.02 | 0.08±0.08 | 0.01±0.02 | 0.06±0.05 | 0.71±0.14 |
| \( y_0 \) | 0 (fixed) | 0 (fixed) | 0 (fixed) | 0 (fixed) | 0.082±0.006 |
| \( \chi^2/\text{NDF} \) | 163/98 | 63/71 | 115/108 | 642/683 |

TABLE 1. Fit parameters of the Buda-Lund hydro (BL-H) model in a combined (simultaneous for heavy-ion collisions) analysis of NA22, NA49, NA44, WA98 spectra and correlation data (the 5th column corresponds to the weighted average of the previous three columns and the error is composed by the statistical and systematic ones, while in the 6th column only statistical errors are presented).

The fit parameters of the combined analysis of the single-particle spectra and the two-particle Bose-Einstein correlation functions in \((\pi^+/K^+)p\) interac-
tions,[5–8] as well as the fit parameters of the simultaneous analysis of the BL-H model to particle correlations and spectra in Pb+Pb collisions[2–4,17–19,25–28,30] are presented in Table 1.

3 Emission Function Expressed as a Function of Time and Longitudinal Coordinates

The momentum-integrated emission function along the $z$-axis, i.e., at $\mathbf{r}_t = (r_x, r_y) = (0, 0)$ is given by

$$S(t, z) \propto \exp \left( -\frac{(\tau - \tau_0)^2}{2\Delta\tau^2} \right) \exp \left( -\frac{(\eta - \eta_0)^2}{2\Delta\eta^2} \right),$$

where the parameters are taken from Table 1.

The reconstruction of the space-time distribution of hadron emission points is shown in Fig. 1, where the upper-left and lower-left diagrams correspond to the reconstructed $S(t, z)$ emission function in arbitrary units, as a function of the cms time variable $t$ and the cms longitudinal coordinate $z \equiv r_z$, for $(\pi^+/K^+)p$ and Pb+Pb interactions, respectively. Note, the coordinates $(t, z)$, are expressed with the help of the longitudinal proper-time $\tau$ and space-time rapidity $\eta$ as $(\tau \cosh \eta, \tau \sinh \eta)$.

In both types of reactions, the space-time structure resembles a boomerang, i.e., particle production takes place close to $|z| \approx t$, with gradually decreasing probability for ever larger values of space-time rapidity. We see a characteristic long tail of particle emission on both sides of the light cone, giving at least 40 fm longitudinal extension in $z$ and 20 fm/$c$ duration of particle production in $t$ for h-h collisions, while for A-A collisions they are longer than 150 fm and 80 fm/$c$, respectively. We see a sharper freeze-out hypersurface in heavy-ion than in hadron-proton collisions, indicating that in hadron-proton interactions the emission process occurs during almost all the hydrodynamical evolution (the duration time of pion emission , $\Delta\tau = 1.3 \pm 0.3$, is close to the mean freeze-out time, $\tau_f = 1.4 \pm 0.1$ fm/$c$), while in A-A collisions a large mean freeze-out time, $\tau_f = 5.9 \pm 0.6$ fm/$c$, is found with a relatively short duration of emission, $\Delta\tau = 1.6 \pm 1.5$ fm/$c$. Note, that the temporal cooling in Pb+Pb collisions seems to be stronger than in h+p collisions, which can be explained by a faster three-dimensional expansion in the former case, as compared to the essentially one-dimensional expansion in the case of h+p interactions.
Figure 1: The reconstructed $S(t, z)$ emission function in arbitrary vertical units, as a function of time $t$ and longitudinal coordinate $z$ (left diagrams), as well as the reconstructed $S(x, y)$ emission function in arbitrary vertical units, as a function of the transverse coordinates $x \equiv r_x$ and $y \equiv r_y$ (right), for h-h (upper) and A-A (lower) collisions, respectively.

4 Emission Function Expressed as a Function of Transverse Coordinates

In the transverse direction, only the rms width of the source can be directly inferred from the BP radii. However, the additional information on the values of $\langle u_t \rangle$ and on the values of $\langle \Delta T \rangle$ from the analysis of the transverse momentum distribution can be used to reconstruct the details of the transverse density profile. An exact, non-relativistic hydro solution was found in ref. [29], in terms of the parameters $\langle u_t \rangle$ and $\langle \Delta T \rangle$ using an ideal gas equation of state. In this hydro solution

$$\langle \Delta T / T \rangle \propto \frac{m\langle u_t \rangle^2}{T_0},$$

(4)

where the $<$ sign corresponds to a self-similar expanding fire-ball, while the opposite $>$ sign corresponds to a self-similar expanding ring of fire. Assuming
the validity of this non-relativistic solution in the transverse direction, one can reconstruct the detailed shape of the transverse density profile. The result looks like a ring of fire in the \((r_x, r_y)\) plane in h+p interactions (see upper-right figure of Fig. 1), while in A+A collisions it has a Gaussian distribution (see lower-right figure of Fig. 1).

So, we conclude that the pion emission function \(S(r_x, r_y)\) in h+p collisions corresponds to the formation of a ring of fire in the transverse plane. This is due to the rather small transverse flow and the sudden drop of the temperature in the transverse direction, which leads to large pressure gradients in the center and small pressure gradients and a density augmentation at the expanding radius of the fire-ring. This transverse distribution, together with the scaling longitudinal expansion, creates an elongated, tube-like source in three dimensions, with the density of particle production being maximal on the surface of the tube.

The pion emission function \(S(r_x, r_y)\) in A+A collisions corresponds to the radial expansion, which is a well established phenomenon in heavy-ion collisions from low-energy to high-energy reactions. This transverse distribution, together with the scaling longitudinal expansion, creates a cylindrically symmetric, large and transversally homogeneous fireball, expanding three-dimensionally with a large mean radial component \(<u_t>\) of hydrodynamical four-velocity.

Because of this large difference observed for those two types of collision, analysis of the emission function in \(e^+e^-\) collisions is important and has been started.

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