Natural Flavour-Unifying GUTs: SU(8) *

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Abstract

Some implications of flavour unification in supersymmetric Grand Unified Theories are briefly reviewed. We argue that the gauge hierarchy phenomenon properly interpreted in terms of the natural missing VEV solution for the adjoint scalar in general $SU(N)$ SUSY GUT could give insight into both of the basic features of low-energy physics accommodated in MSSM just as some strict colour-flavour (or colour-family) interrelation, so the $SU(2)$ symmetry structure of weak interactions. There, apart from the ordinary triple matter of MSSM, just three families of the pseudo-Goldstone states appear in the minimal $SU(8)$ theory to be of necessity detected at a TeV scale.

*Talk given at the Trieste Conference on Quarks and Leptons: Masses and Mixings, Trieste, October 7-11, 1996
1 Introduction

There are still left several hard problems in Supersymmetric Grand Unified Theories (SUSY GUTs) suggesting a desired stabilization of the Higgs masses against the radiative corrections [1].

The main question sounding somewhat notorious addresses, of course, the soft SUSY breaking itself – why its scale is called upon to correlate generally just with the electroweak scale $M_{EW}$ among the other ones, say, grand unifying scale $M_{GUT}$, the left-right symmetry scale $M_{LR}$, the family symmetry scale $M_F$ etc might being in particle physics? Like any purely geometric symmetry SUSY could be "blind" to the internal symmetries making no difference between them. However, looked just as an opposite case this might mean some clue towards the gauge hierarchy phenomenon in GUTs, which if properly interpreted in terms of some stable direction-dependent "grand" vacuum configuration splitting the ordinary Higgs multiplets in a particular way, could single out just the electroweak $SU(2) \otimes U(1)$ symmetry structure surviving down to low energies where SUSY softly breaks.

Another question of vital importance is that the internal symmetry breaking caused by scalar supermultiplets of GUT should guarantee together with a rather peculiar masses of quarks and leptons a nearly uniform mass spectra for their superpartners with a high degree of flavour conservation. Presently, among the other possibilities to alleviate the flavour-changing problem in SUSY GUTs some flavour symmetry $G_F$ between quark-lepton (squark-slepton) families seems to be the most natural framework [2].

Thus contemplating the above SUSY GUT issues (gauge hierarchy and flavour "democracy") one might expect that there could appear in the GUT symmetry-broken phase some interrelation between both of phenomena principally remaining down to low energies in a framework of supersymmetric unification. It seems even likely that a true solution to a doublet-triplet splitting problem in SUSY GUTs (if in contrast to a few proposed ones [3-5] was looked for in general group context) might choose itself the total starting symmetry of GUT including some family symmetry $G_F$ provided this were the case.

Following such a motivation we consider below general SU(N) SUSY GUTs. We find a stable missing VEV solution which naturally stems from general reflection-invariant adjoint scalar superpotential. Remarkably enough, the solution requires a strict equality of numbers of fundamental colours and flavours in $SU(N)$ and for the even-order groups ($N = 2n + 2$, n=1,2,...) gives in a finite parameter area the predominant breaking channel

$$SU(N) \rightarrow SU(n)_C \otimes SU(n)_F \otimes SU(2)_W \otimes U(1)_1 \otimes U(1)_2$$

which definitely favors just ab initio three-colour $SU(8)$ case among the other $SU(N)$ GUTs. The flavour subsymmetry breaking owing to some generic string inspired extra symmetries of the total Higgs superpotential appears not to affect markedly the basic adjoint vacuum
configuration in the model. Thus the both salient features of SM just as an interplay between colours and flavours (or families for due assignment of quarks and leptons), so the doublet structure of the weak interactions could be properly understood and well accommodated in the framework of the minimal SU(8) model.

2 Missing VEV solutions in SU(N): colour-flavour interrelation

We start reminding that a missing VEV conjecture [6] for doublet-triplet (or $2/(N-2)$ in general) splitting is that a heavy adjoint scalar $\Phi_{ij}^i$ ($i, j = 1...N$) of $SU(N)$ might not develop a VEV in the weak $SU(2)$ direction and through its direct coupling with fundamental chiral pairs $H_i$ and $\bar{H}^i$ (containing the ordinary Higgs doublets) could hierarchically split their masses in the desired $2/(N-2)$ way. While there was found some special realization of the missing VEV ansatz in $SO(10)$ model [4] the situation in $SU(N)$ theories looks much hopeless. The main obstacle is happened to be a presence of a cubic term $\Phi^3$ in general renormalizable Higgs superpotential $W$ leading to the impracticable trace condition $Tr\Phi^2 = 0$ for a missing VEV vacuum configuration unless there occurs the special fine-tuned cancellation between $Tr\Phi^2$ and driving terms stemming from other parts of $W$ [6].

So, the only way to a natural missing VEV solution in $SU(N)$ theories seems to exclude the cubic term $\Phi^3$ from the superpotential $W$ imposing some extra reflection symmetry on the adjoint supermultiplet $\Phi$. While an elimination of the $\Phi^3$ term itself leads usually to the trivial unbroken symmetry case an inclusion in $W$ another adjoint scalar can, as we show below, drastically change a situation.

The general renormalizable two-adjoint Higgs superpotential of $\Phi_s$ ($s = 1, 2$) satisfying the reflection symmetry $\Phi_s \rightarrow (-1)^s \Phi_s$ can be envisioned as

$$W_\Phi = \frac{M_1}{2} \Phi_1^2 + \frac{M_2}{2} \Phi_2^2 + \frac{h}{2} \Phi_1^2 \Phi_2 + \frac{\lambda}{3} \Phi_2^3$$

(2)

The total superpotential apart from the adjoints $\Phi_s$ and the ordinary Higgs supermultiplets $H$ and $\bar{H}$ includes also $N-5$ fundamental chiral pairs $(\varphi, \bar{\varphi})^p$ ($p = 1...N-5$) which break $SU(N)$ to $SU(5)$ at some high mass scale (possibly even at Planck mass $M_P$)

$$W = W_\Phi + W_H + W_\varphi$$

(3)

We do not consider for a moment the $W_\varphi$ part of the superpotential assuming that some extra symmetry (see below) makes it possible to ignore its influence on the formation of the basic adjoint vacuum configurations in the model.

The possible patterns of the adjoints $\Phi_s$ minimizing their own potential $V_\Phi$ can be sought generally in the following independent diagonal matrix forms ($s = 1, 2$)
with all decompositions of $N$, $N = m_s + q_s + r_s$. A self-consistent prerequisite to the supersymmetric minimum following from the superpotential $W_\Phi$ (4) through the vanishing $F$-terms $F_{\Phi_s} = 0 (s = 1, 2)$ for adjoint components in (4) implies that the spectrum of all basic vacuum configurations in $SU(N)$ is bound to include the following four cases only:

(i) The trivial symmetry-unbroken case ($r_1 = N$, $r_2 = N$),

(ii) The single-adjoint solutions ($r_1 = N$, $r_2 = 0$; $m_2 \equiv m$)

$$\sigma_1 = 0, \quad \sigma_2 = -\frac{M_1}{h} a x \quad (a = \frac{N \cdot m}{N - 2m}, \quad x = \frac{M_2 h}{M_1 \lambda}), \quad (5a)$$

(iii) The ”parallel” configurations, $\Phi_1 \propto \Phi_2$ ($r_1 = 0$, $r_2 = 0$; $m_1 = m_2 \equiv m$)

$$\sigma_1 = \left(\frac{2 \lambda}{h}\right)^{1/2} \frac{M_1}{h} a (x - 1)^{1/2}, \quad \sigma_2 = -\frac{M_1}{h} a, \quad (5b)$$

(iv) The ”orthogonal” configurations, $Tr(\Phi_1 \Phi_2) = 0$ ($r_1 = N - 2m_1$, $r_2 = 0$; $m_1 = m_2/2 \equiv m/2$)

$$\sigma_1 = \left(\frac{2 \lambda}{h}\right)^{1/2} \frac{M_1}{h} \left[\frac{N}{N-m}(x - 1/a)^{1/2}, \quad \sigma_2 = -\frac{M_1}{h} \right. \quad (5c)$$

where the group decomposition number $m$ is generally different in all the above cases.

So, we can conclude that, while an ”ordinary” adjoint $\Phi_2$ having a cubic term in $W_\Phi$ (4) develops in all the non-trivial cases (ii - iv) only a ”standard” VEV ($r_2 = 0$) which breaks the starting symmetry $SU(N)$ to $SU(m) \otimes SU(N - m) \otimes U(I)$, the first adjoint $\Phi_1$ can have a new orthogonal solution of type (5c) as well with $SU(N)$ broken along the channel $SU(m/2) \otimes SU(m/2) \otimes SU(N - m) \otimes U(I)_1 \otimes U(I)_2$. This case corresponds to the missing VEV solutions just we are looking for if one identifies the $SU(N - m)$ subgroup of $SU(N)$ with the weak symmetry group while two other $SU(m/2)$ groups must be identified therewith the groups of fundamental colours and flavours, respectively,

$$N = n_C + n_F + n_W, \quad (6)$$

$$n_C = m/2, \quad n_F = m/2, \quad n_W = N - m$$

If so, we are driving at general conclusion that the missing VEV solutions in $SU(N)$ theories appear only when the numbers of colours and flavours (or families if one takes a proper assignment for quarks and leptons under the flavour group $SU(m/2)_F$) are happened to be equal. As this takes place, the realistic case with $n_W = 2$ corresponding to symmetry breaking channel (4) also occurs for the even-order $SU(N)$ groups (see Eq.(7)).
Meanwhile the supersymmetric vacuum degeneracy makes no difference between the different missing VEV solutions (5c) as well as between them in general and the ordinary adjoint ones (5a,b). One might expect that supergravity-induced lifting [1] the vacuum degeneracy modifying the potential $V_\Phi$ at the minimum to (to the lowest order in $k = M_P^{-1}$)

$$V_\Phi \simeq -3k^2|W_\Phi|^2$$  \hspace{1cm} (7)

would single out a true vacuum configuration in some finite area for parameters in the superpotential $W_\Phi$. Actually it easily can be shown by direct comparison of all vacua contributions to $V_\Phi$ (7) in $O(1/N)$ approximation that the key missing VEV configuration (1) globally dominates over all the other possible ones in (5a,b,c) within the following (and in fact somewhat favorable) parameter area

$$-2 \left( \frac{N - 2}{N + 2} \right)^{1/3} < x < 2 \left( \frac{N - 2}{N + 2} \right)^{1/3}, \quad x = \frac{M_2}{M_1} \lambda$$  \hspace{1cm} (8)

Remarkably, this dominating vacuum configuration besides the above general colour-flavour (or colour-family) interrelation provides somewhat attractive explanation just for a weak $SU(2)$ symmetry structure coming safely from a grand unifying scale down to low energies in general $SU(N)$ GUTs. In essence, there is only one parameter left for a final specialization of theory – the number of colours. It stands to reason that $n_C = 3$ and we come to the unique $SU(8)$ symmetry case with the missing VEV breaking channel (1).

3 Decoupling of flavour vacuum configurations: pseudo-goldstones

Let us consider now the other parts of the total Higgs superpotential $W$ (3). There $W_H$ is in fact the only reflection-invariant coupling of the adjoint $\Phi_1$ with a pair of the ordinary fundamental Higgs supermultiplets $H$ and $\bar{H}$

$$W_H = f \bar{H} \Phi_1 H, \quad (\Phi_1 \to -\Phi_1, \quad \bar{H} H \to -\bar{H} H)$$  \hspace{1cm} (9)

having the zero VEVs $H = \bar{H} = 0$ during the first stage of the symmetry breaking. Thereupon $W_H$ turns to the mass term of $H$ and $\bar{H}$ fields depending on the basic vacuum configurations (5a,b,c) in the model. The point is the $\Phi_1$ missing VEV configuration (5c) giving generally heavy masses (of the order $M_{GUT}$) to them leaves their weak components strictly massless. Thus there certainly is a natural doublet-triplet splitting in the model although we drive at the vanishing $\mu$-term on this stage. One can argue that some $\mu$-term always appears through the radiative corrections [1] or a non-minimal choice of Kähler potential [8] or the high-order terms induced by gravity [7] in the flavour part of the superpotential $W_\phi$ we are coming to now.
The flavour symmetry breaking which is also assumed to happen on the GUT scale $M_{GUT}$ (not to spoil the standard supersymmetric grand unification picture) looks in the above favored $SU(8)$ case as

$$SU(3)_F \otimes U(I)_1 \otimes U(I)_2 \rightarrow U(1)$$ (10)

A question arises: how the missing VEV solution can survive such a high-scale symmetry breaking (10) so as to be subjected at most to the weak scale order shift?

The simplest way [7,9] could be if there appeared some generic string-inspired discrete symmetry $Z_k$ which forbade the mixing between two sectors $W_{\Phi} + W_{H} (I)$ and $W_{\varphi} (II)$ in the Higgs superpotential. Clearly, in a general SU(N) GUT the accidental global symmetry $SU(N)_I \otimes SU(N)_{II}$ (and possibly a few $U(1)'s$) being a result of a Z-symmetry must appear and pseudo-Goldstone (PG) states are produced after the global symmetry breaks. On the other hand Z-symmetries tend to strongly constrain a form of superpotential $W_{\varphi}$ itself so that contrary to the supersymmetric adjoint breaking (1) the flavour subsymmetry of $SU(N)$ can break triggered just by the soft SUSY breaking only. Another scenario [10], suggested recently, uses the anomalous $U(1)_A$ symmetry which can naturally get untie the sectors (I) and (II) and induce the high-scale flavour symmetry breaking through the Fayet-Iliopoulos D-term [1]. It stands to reason that both of ways are fully suited for our case as well to keep the missing VEV configuration going down to low energies.

4 Minimal $SU(8)$ model: low-energy predictions

The time is right to discuss now the particle spectra in the model concentrating mainly on its favored minimal $SU(8)$ version.

Having considered the basic matter superfields (quarks and leptons and their superpartners) the question of whether the above flavour symmetry $SU(3)_F$ is still their family symmetry naturally arises. Needless to say that among many other possibilities the special assignment treating the quark-lepton families as the fundamental triplet of $SU(3)_F$ comes first. Some interesting cases can be found in Ref.[7,11] showing clearly that the presented $SU(8)$ model meets a natural conservation of flavour both in the particle and sparticle sectors, respectively.

Another block, or it would be better to say a superblock of the low-energy particle spectrum is just three families of PG bosons and their superpartners of type

$$5 + \bar{5} + SU(5) - singlets$$ (11)

appearing as a result of breaking of an accidental $SU(8)_I \otimes SU(8)_{II}$ symmetry in a course of the spontaneous breakdown of the starting local $SU(8)$ symmetry to SM (1,10) and acquiring a weak scale order masses due to the soft SUSY breaking and subsequent radiative corrections or
directly through the gravitational corrections [7]. Normally they are the proper superpositions of the \(SU(5) \otimes SU(3)_F\) fragments \((5, 3) + (\bar{5}, 3)\) in the adjoints \(\Phi_s\) of \(SU(8)\) and \((5, 1)_p + (\bar{5}, 1)_p\) \((p = 1, 2, 3)\) in its fundamental flavour scalars \(\varphi_p\) and \(\bar{\varphi}_p\) (see above).

So, at a low-energy scale one necessarily has in addition to three standard families of quarks and leptons (and squarks and sleptons) just three families of PG bosons and their superpartners \((\bar{1})\) which, while beyond the one-loop approximation in the renormalization group equations (RGEs), will modify the running of standard gauge \((\alpha_1, \alpha_2, \alpha_S)\) and Yukawa \((Y_t\) primarily, \(Y_t = h_t^2/4\pi\)) couplings from GUT scale \(M_{GUT}\) down to \(M_Z\).

We found that the MSSM predictions for \(\alpha_S(M_Z)\) and \(M_{GUT}\) changed as the PG supermultiplets were included in the RGEs for the starting large values of top-Yukawa coupling \(Y_t\) on the GUT scale \(Y_t(M_{GUT}) \geq 0.1\) evolving rapidly towards its infrared fixed point.

Our results are summarized in Table 1. Performing the running of the gauge and top-Yukawa couplings from \(M_{GUT}\) down to \(M_Z\) (by considering two-loop \(\beta\)-functions [12] up-dated for the \(SU(8)\) case with PG supermultiplets) the threshold corrections related with PG states due to one-loop RGE evolution as well as the overall one-loop supersymmetric threshold corrections associated with the decoupling of supersymmetric particles at some effective (lumped) scale \(T_{SUSY}\) had also been included. We took for \(T_{SUSY}\) the relatively low values \(T_{SUSY} = M_Z\) to keep the sparticle masses typically in a few hundred GeV region [13].

### Table 1: The \(SU(8)\) vs MSSM (in square brackets) predictions for \(\alpha_S(M_Z), \tan\beta, \alpha_{GUT}\) and \(M_{GUT}\).

| \(\alpha_S(M_Z)\) | \(\tan\beta\) | \(\alpha_{GUT}\) | \(M_{GUT}/10^{16}\) GeV |
|-----------------|---------------|----------------|-----------------------------|
| 0.121 \([0.124]\) | 1.12 \([1.44]\) | 0.137 \([0.042]\) | 5.08 \([2.91]\) |

As one can see from Table 1 the \(\alpha_S(M_Z)\) value in \(SU(8)\) model (contrary to MSSM even for the most beneficial, while still perturbative, value of \(Y_t(M_{GUT}) = 0.3\)) is brought into essentially stable World average value [14] \(\alpha_S(M_Z) = 0.118 \pm 0.003\).

Yet another significant outcome of the \(SU(8)\) model turns out to be very low values of \(\tan\beta\) if one takes the \(Y_t\) fixed-point solution. As a result the very likely reduction of the theoretically allowed upper bound on the MSSM lightest Higgs mass \(m_h\) down to \(M_Z\) is expected [15]. If so, the Higgs boson \(h\) might be accessible at LEPII.

Meanwhile \(b-\tau\) unification, while not generally evolved from the effective Yukawa couplings of the \(SU(8)\) model [7,11], was found very sensitive to new PG states and, if even were taken on a GUT scale, actually broke down for any reasonable b-quark mass value \(m_b(m_b)\).
5 Final remarks

To conclude any extended $SU(N)$ SUSY GUT containing MSSM appears in general not to be observationally distinguishable from the standard supersymmetric $SU(5)$ model [1] provided that its flavour subsymmetry $SU(N-5) \otimes U(1)$ breaks at some superhigh scale $M_{N-5} \geq M_5$. However, if its basic adjoint and flavour vacuum configurations are proved to be essentially untied there evolve pseudo-Goldstone states (bosons and fermions) at a scale where supersymmetry softly breaks. So, one could judge of the starting symmetry $SU(N)$ and its breaking process to SM by a particular PG spectrum at low energies. Depending on the details of their mixing pattern with the ordinary Higgs sector of MSSM the PG states could influence appreciably on the particle phenomenology expected at a TeV scale [16]. Three full-multiplet families of them of type $[\mathbf{3} \oplus \mathbf{1}]$, if observed, could say just about $SU(8)$ GUT naturally projected down to low energies. Otherwise, ”between the idea and the reality ... falls the shadow” [17].

Acknowledgments

It is a pleasure to thank Goran Senjanović, Alexei Smirnov and their colleagues at ICTP for organizing such a stimulating Trieste Meeting on flavour physics. I am indebted to Riccardo Barbieri, Zurab Berezhiani, Barbara Mele, S.Ranjbar-Daemi, Alexei Smirnov and Gena Volkov for interesting discussions and useful comments and S.Ranjbar-Daemi also for a warm hospitality at ICTP High Energy Department where part of this talk was prepared. Many helpful discussions with my collaborators Ilia Gogoladze and Archil Kobakhidze are also gratefully acknowledged.

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