GALACTIC MASERS AND THE MILKY WAY CIRCULAR VELOCITY

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ABSTRACT

Masers found in massive star-forming regions can be located precisely in six-dimensional phase space and therefore serve as a tool for studying Milky Way dynamics. The non-random orbital phases at which the masers are found and the sparseness of current samples require modeling. Here, we model the phase-space distribution function of 18 precisely measured Galactic masers, permitting a mean velocity offset and a general velocity dispersion tensor relative to their local standards of rest, and accounting for different pieces of prior information. With priors only on the Sun’s distance from the Galactic Center and on its motion with respect to the local standard of rest, the maser data provide a weak constraint on the circular velocity at the Sun of $V_c = 246 \pm 30 \text{ km s}^{-1}$. Including prior information on the proper motion of Sgr A* leads to $V_c = 244 \pm 13 \text{ km s}^{-1}$. We do not confirm the value of $V_c \approx 254 \text{ km s}^{-1}$ found in more restrictive models. This analysis shows that there is no conflict between recent determinations of $V_c$, from Galactic Center analyses, orbital fitting of the GD-1 stellar stream, and the kinematics of Galactic masers; a combined estimate is $V_c = 236 \pm 11 \text{ km s}^{-1}$. Apart from the dynamical parameters, we find that masers tend to occur at post-apocenter, circular-velocity-lagging phases of their orbits.

Key words: [insert key words here]

1. INTRODUCTION

The value of the circular orbital velocity at the Sun’s radius in the Milky Way is of considerable interest in Galactic and extragalactic astrophysics. It is necessary to correct observed velocities of stars and galaxies for the motion of the Sun around the Galactic Center. The circular velocity also plays a large role in characterizing the mass of the Milky Way in comparison with other spiral galaxies, placing it in a cosmological context, e.g., when asking whether the Milky Way matches the Tully–Fisher relation (e.g., Klypin et al. 2002; Flynn et al. 2006) or what is its total star formation efficiency (e.g., Smith et al. 2007; Xue et al. 2008).

The circular velocity at the Sun’s radius has typically been established by measuring the Sun’s motion with respect to an object assumed to be at rest with respect to the Galaxy (Sgr A*: Reid & Brunthaler 2004; the stellar halo: Sirko et al. 2004), or by using a tracer population assumed to be angle-mixed, i.e., having a uniform distribution of orbital phases, in a steady-state Galaxy (e.g., Feast & Whitelock 1997). Recently, a competitive estimate has been obtained by a different approach using a narrow stellar stream that is assumed to be tracing out an orbit (Koposov et al. 2009).

In this paper, we re-analyze a new population of tracers of Milky Way dynamics: masers associated with star-forming regions (Reid et al. 2009, R09). Using the Very Long Baseline Array (VLBA) and the Japanese VLBI Exploration of Radio Astronomy (VERA), precise measurements of the parallaxes, proper motions, and line-of-sight velocities of masers have been made (see R09, and references therein). These give accurate full six-dimensional phase-space information in the disk of the Galaxy. Since these massive star-forming regions are associated with spiral arms and their shocks, the dense molecular gas regions that produce masers do not lie on exactly circular orbits, nor are they detected at random points on their orbits. Therefore, modeling approaches that assume a uniform distribution of the orbital phases of the tracer population cannot give accurate determinations of the dynamics of the Galaxy. For the existing maser data, the problem of non-random orbital phases is exacerbated by the sparseness of the sample—only 18 masers with accurate six-dimensional phase-space information have been measured at present—and by the spatially non-uniform selection of the current sample of masers.

In this paper, we perform an analysis of the R09 maser data that deals simultaneously with the sparseness of the data, the spatial non-uniformity of the sampling, the non-random orbital phase distribution of masers, and prior information. Assuming a flat rotation curve, $V_c(R) = \text{constant}$, we use a simple model for the distribution of the maser velocities with respect to their local standards of rest: a mean offset from circular rotation $V_c(R)$ and a general velocity dispersion tensor fixed in Galactocentric cylindrical coordinates. In the probabilistic inference framework that we use—described in Section 2—we can marginalize over the uncertainty in the inferred distribution function of masers, take prior information on the dynamics of the Galaxy into account, use the sparse data set as efficiently as possible, and then ask what information on $V_c$ the maser data provide. Our results presented in Section 3 show that allowing for a finite velocity dispersion tensor in the model for the maser peculiar-velocity distribution function leads to lower values of $V_c$ than the large value reported in R09, in whose analysis the maser velocity dispersion was (implicitly) assumed to vanish. Adding in informative prior information about $R_0$, inferred from monitoring stellar orbits around the black hole at the center of the Galaxy (Ghez et al. 2008; Gillessen et al. 2009) and from the measurement of the proper motion of Sgr A* (Reid & Brunthaler 2004), we find that the best circular velocity estimate is $V_c = 244 \pm 13 \text{ km s}^{-1}$, but that the current maser data set adds little information. We discuss this measurement and its limitations in the light of other recent determinations in Section 4.

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2. DATA AND METHODOLOGY

Throughout the analysis that follows we use the standard cylindrical Galactocentric coordinate frame \((R, \phi, z)\), with associated unit vectors \((\mathbf{e}_R, \mathbf{e}_\phi, \mathbf{e}_z)\) pointing toward the Galactic center, in the direction of Galactic rotation, and toward the North Galactic Pole, respectively.

2.1. Data from Reid et al. (2009)

The data we analyze here consist of the Galactic coordinates, parallaxes, proper motions, and line-of-sight velocities of 18 Galactic masers, as well as their associated uncertainties, presented in Table 1 of Reid et al. (2009). Following R09, we add a 7 km s\(^{-1}\) uncertainty in quadrature to the uncertainties in the velocity components of each maser to describe the random, virial motion in the massive star-forming region of the individual massive star associated with each maser.

The line-of-sight velocities have been “corrected” by the radio observatories’ pipelines for the motion of the Sun with respect to the Local Standard of Rest (LSR). This correction assumed a value of 20 km s\(^{-1}\) toward \(\alpha(B1900.0) = 18^h, \delta(B1900.0) = +30^\circ\) for the Solar motion \(v_\odot\), although it is unclear whether all observatories used this standard value (M. Reid 2009, private communication). We undo this correction, after which the currently accepted correction for \(v_\odot\) can be applied; however, as we will describe below, this correction will become part of our model and, therefore, the correction for \(v_\odot\) does not occur during the preprocessing of the data.

Beyond these two corrections, no processing of the Reid et al. (2009) data has been done.

2.2. Probabilistic Framework

Parameter estimation in a probabilistic framework by necessity uses Bayes’s theorem to connect the probability of the model parameters given the data \([x_i^{\text{obs}}, y_i^{\text{obs}}]\) to the probability of the observed data given the model parameters (e.g., Jaynes 2003). This requires us (1) to identify all the parameters that need to be included in the model, (2) to write down the likelihood of parameters given the data \(\mu\), (3) to specify suitable priors for the model parameters. Although the model space needs to be exhaustive, the probabilistic framework allows integration over uninteresting parameters.

Here, we put forward a model for the maser kinematics in which the maser velocities are most easily modeled in Galactocentric cylindrical coordinates. In order to go from the raw data described in Section 2.1 to the velocity of each maser in Galactocentric coordinates, we need to (1) correct the measured velocity for \(v_\odot\), (2) add to this velocity the circular velocity around the Galactic center at the Sun’s radius, and (3) project this velocity onto the Galactocentric coordinate frame (the details of this transformation are described in the Appendix of R09). Since the latter procedure includes geometrical projection factors depending on the distance \(R_0\) of the Sun from the Galactic Center, the model parameters need to include the three components of \(v_\odot, R_0, V_c\). However, it is more practical to assume that Sgr A* is at rest with respect to the Galaxy, and to use the proper motion \(\mu_{\text{Sgr A*}}\) of Sgr A* (Reid & Brunthaler 2004) as a model parameter instead of the circular velocity, as \(\mu_{\text{Sgr A*}}\) is very tightly constrained independently of \(R_0\). These two parameters are related simply by multiplying the proper motion of Sgr A* by \(R_0\) and correcting this for \(v_\odot\). The circular velocity then becomes a parameter derived from the actual model parameters, which is no problem in the probabilistic framework, where it is easy to propagate uncertainties correctly. As we will assume that the rotation curve is flat, no extra parameters to model the shape of the rotation curve need to be included in the model.

If we had uniformly sampled the phase space of masers and full prior knowledge of the phase-space distribution function of massive star-forming regions, this would uniquely specify the likelihood of the model, as the probability of the measured position and velocity of each maser would simply be given by the distribution function of the masers convolved with the observational uncertainty. However, we have neither a uniform sample of masers nor much prior information about the distribution of masers throughout the Galaxy. To account for the spatial non-uniformity of the sample we will focus on the distribution of velocities at the actually observed position of the maser, instead of using the full six-dimensional phase space distribution function to evaluate the likelihood. For this distribution, we will assume that it only depends on the peculiar velocity \(v_{\text{pec}}\) of the maser in Galactocentric cylindrical coordinates. We will assume that this distribution of peculiar velocities is given by a Gaussian distribution characterized by a mean, a 3-vector \(\langle \mathbf{v} \rangle\), the offset from circular motion, and a general velocity dispersion tensor, a symmetric 3 \(\times\) 3 tensor \(\sigma\) with six free parameters. Since there have been no measurements of either the mean offset from circular motion of the masers or their velocity dispersion, we will use flat priors on these quantities. This model is essentially a generalization of the model used in Reid et al. (2009) where the velocity dispersion tensor was assumed to vanish; this was a poor assumption as we will show below.

The probability of a single maser is thus given by

\[
p(\mathbf{x}_i^{\text{obs}}, \mathbf{v}_i^{\text{obs}} | \mu, \mathbf{R}_0, v_\odot, \mathbf{\sigma}) = N(\mathbf{v}_{\text{pec}}(\mathbf{x}, \mathbf{v}) | \mathbf{\bar{v}}, \mathbf{\sigma}) \otimes p(\mathbf{x}, \mathbf{v} | \mathbf{x}_i^{\text{obs}}, \mathbf{v}_i^{\text{obs}}),
\]

where we have suppressed the dependence of \(v_{\text{pec}}\) on the dynamical parameters, and where the convolution with the observational uncertainty distribution \(p(\mathbf{x}, \mathbf{v} | \mathbf{x}_i^{\text{obs}}, \mathbf{v}_i^{\text{obs}})\) has been included. The posterior distribution for the 14 model parameters is then given by

\[
p(\mu, \mathbf{R}_0, v_\odot, \mathbf{\sigma} | \mathbf{x}_i^{\text{obs}}, \mathbf{v}_i^{\text{obs}}) \propto p(\mu, \mathbf{R}_0, v_\odot) \times \prod_i p(\mathbf{x}_i^{\text{obs}}, \mathbf{v}_i^{\text{obs}} | \mu, \mathbf{R}_0, v_\odot, \mathbf{\sigma}),
\]

where the first factor on the right-hand side is the prior probability distribution for these parameters and the product is the likelihood. We have used flat priors for \(\mathbf{v}\) and \(\sigma\), which is why they do not appear explicitly.

For \(\mu\) we use a Gaussian prior with a mean of 30.24 km s\(^{-1}\) kpc\(^{-1}\) and a standard deviation of 0.12 km s\(^{-1}\) kpc\(^{-1}\) (Reid & Brunthaler 2004). For \(R_0\) we combine current state-of-the-art determinations of \(R_0\) from Galactic Center orbits with equal weights: 8.0 \pm 0.6 kpc found by Ghez et al. (2008) and 8.33 \pm 0.35 kpc found by Gillessen et al. (2009). This prior is shown as the gray curve in Figure 2. For \(v_\odot\) we use the value and uncertainties obtained from Hipparcos data (Hogg et al. 2005), although the clumpiness of the velocity distribution of nearby stars (Dehnen 1998; Bovy et al. 2009) implies an uncertainty more on the order of a few km s\(^{-1}\) in the value of \(v_\odot\) (J. Bovy & D. W. Hogg 2010, in preparation). The implied prior for the circular velocity is shown as the thick gray curve in Figure 1. To investigate how informative the maser
from dropping the informative prior on $\mu_{Vc}$.

The prior probability distribution is shown as the thick gray curve; its mean is $243 \pm 16$ km s$^{-1}$. The posterior and its mean (bottom label) obtained from dropping the informative prior on $\mu_{Vc}$ is shown as the thin gray curve, illustrating that the maser data themselves constrain $Vc$ relatively weakly. The quoted uncertainty in mean value is the standard deviation $\sqrt{(Vc^2) - (Vc)^2}$.

measurements are about $Vc$ and $R_0$, we will consider the effect of dropping (some combination of) these priors below.

The framework described here can easily be generalized to more general descriptions of the distribution of the peculiar velocities of the masers. In what follows, we will use a distribution function that is the sum of two Gaussian distributions, the second having half of the weight and twice the dispersion of the first Gaussian, to determine the possible effect of outliers.

2.3. Exploration of the Posterior Probability Distribution

In order to explore the posterior distribution for all of the model parameters in light of the maser data we use a simple Markov Chain Monte Carlo (MCMC) method (Mackay 2003). This procedure is described in some detail in the Appendix.

The practical complication in evaluating the likelihood given in Equations (1) and (2) for each of the masers comes from the fact that the observational uncertainties are Gaussian in the space of observed quantities—more specifically, for the parallax—but are non-Gaussian in the space of the peculiar velocities. However, if the relative parallax uncertainty is small ($\leq 10\%$) we can confidently propagate the uncertainties to the space of peculiar velocities, where the convolution of the Gaussian velocity distribution model for the peculiar velocities with the observational Gaussian uncertainty distribution is simple. A few of the masers have relative parallax uncertainties larger than $10\%$, but we have nonetheless propagated the uncertainties in the Gaussian approximation. To check that this does not bias our results we have also run our analysis using a full numerical convolution with the actual observational uncertainties and we find results that are barely distinguishable from the results presented below.

### Figure 1

Marginalized posterior probability distribution for the circular velocity $Vc$, shown as the black curve, and its mean (top label) from $10^6$ MCMC samples. The prior probability distribution is shown as the thick gray curve; its mean is $Vc = 243 \pm 16$ km s$^{-1}$. The posterior and its mean (bottom label) obtained from dropping the informative prior on $\mu_{Vc}$ is shown as the thin gray curve, illustrating that the maser data themselves constrain $Vc$ relatively weakly. The quoted uncertainty in mean value is the standard deviation $\sqrt{(Vc^2) - (Vc)^2}$.

### Figure 2

Marginalized posterior probability distribution for the distance $R_0$ to the Galactic center, shown as the black curve, from $10^6$ MCMC samples. The prior probability distribution is shown as the thick gray curve; its mean is $R_0 = 8.2 \pm 0.5$ kpc.

3. RESULTS

The main scientific goal of this paper is to understand what the maser measurements tell us about $Vc$. The posterior probability distribution for $Vc$, fully marginalized over all of the parameters of the maser distribution function, the Solar motion with respect to the LSR, the distance to the Galactic Center, and the proper motion of Sgr A*, is shown in Figure 1. The analogously marginalized posterior distribution for $R_0$ is shown in Figure 2. Also shown in Figure 1 is the posterior we obtained when we drop the informative prior on $\mu_{Vc}$. The posterior distributions for the proper motion of Sgr A* and for the components of $v(\odot)$ are not shown here. They are all basically identical to their prior distributions, implying that the masers—not surprisingly—cannot inform us about these quantities.

While the prior on $Vc$ in Figure 1 peaks at $244$ km s$^{-1}$ with a $1\sigma$ uncertainty of $16$ km s$^{-1}$, the posterior for $Vc$ is peaked at a value of $244$ km s$^{-1}$ with a $1\sigma$ uncertainty of about $13$ km s$^{-1}$. This equal value for $Vc$ after analyzing the masers is in qualitative contrast to the initial analysis of R09, who found that it raised the peak to $254$ km s$^{-1}$. This difference arises mainly from our more general model for the distribution function of the masers. If we insist within our analysis that the velocity dispersion of the masers is zero, we find a posterior distribution for the circular velocity that is peaked at $255$ km s$^{-1}$, in rough agreement with the R09 results. The light gray line in Figure 1 shows what happens when we drop the informative prior on $\mu_{Vc}$, while keeping the $R_0$ prior: $Vc = 246 \pm 30$ km s$^{-1}$. This and the fact that the posterior probability is barely narrower than the prior tells us that the current maser measurements have not much power to constrain $Vc$. The posterior estimate for the distance to the Galactic Center is $R_0 = 8.2 \pm 0.4$ kpc; this shows that the masers lead to a small improvement to our knowledge of the Sun’s distance to the Galactic Center. Without the informative prior on $\mu_{Vc}$
the posterior estimate for $R_0$ is the same as the prior estimate: $R_0 = 8.2 \pm 0.5$ kpc.

At the same time, the MCMC procedure provides fully marginalized posterior distributions for the parameters of the conditional velocity distribution function of masers, which are given in Figure 3: shown are the posterior distributions for the three components of the mean offset from circular velocity of the masers, i.e., the mean peculiar velocity, in cylindrical coordinates (toward the Galactic Center, in the direction of Galactic rotation, and toward the North Galactic Pole) as well as for the trace of the velocity dispersion tensor. From this we confirm the mean lag of 15 km s$^{-1}$—we find a lag of 14 ± 5 km s$^{-1}$—of the masers with respect to their local standards of rest previously found by R09. Figure 3 shows that the masers have a mean velocity toward the Galactic Center of 7 ± 6 km s$^{-1}$. Taken together, these mean peculiar velocities imply that the masers are typically just past the apocenter of their orbits. We also find a mean velocity component of 3 ± 3 km s$^{-1}$ in the direction toward the North Galactic Pole.

From the posterior distribution for the trace of the velocity dispersion tensor we see that the masers have a relative large velocity dispersion—$\text{Trace}(\sigma) \sim [29 \text{ km s}^{-1}]^2$—larger than might be expected from a comparison with the velocity dispersion of young stars in the Solar neighborhood, whose trace is about $[14 \text{ km s}^{-1}]^2$ (Hogg et al. 2005). Since we put no restrictions on the form of $\sigma$ we also obtain posterior probability distributions for all of the components of $\sigma$: for the diagonal components we find $\sqrt{\sigma_{RR}} = 22 \pm 8$ km s$^{-1}$, $\sqrt{\sigma_{\phi\phi}} = 18 \pm 7$ km s$^{-1}$, and $\sqrt{\sigma_{zz}} = 12 \pm 5$ km s$^{-1}$. As we discuss below, the fact that we obtain these large values could be because our model for the conditional velocity distribution is too restrictive.

In order to assess the possible affect of outliers on our inference, we have performed the same analysis assuming a distribution of the peculiar velocities which consists of a mixture of two Gaussian distributions, identical in every aspect except that the second Gaussian has half of the weight and twice the dispersion of the first Gaussian (by doubling each component of the velocity dispersion tensor). We find the same posterior distributions for the dynamical parameters and the
mean offset; the inferred dispersion of the masers is, predictably, somewhat smaller: the trace of the covariance matrix peaks at \( [22 \text{ km s}^{-1}]^2 \). Two specific candidate outliers, the sources NGC 7538 and G 23.6–0.1, were identified and removed from the sample by R09, because they displayed large post-fit residuals. To assess whether these two sources affect our results significantly, the same analysis as described above of the R09 basic sample of 16 masers was performed, leaving out the sources NGC 7538 and G 23.6–0.1. We find basically the same result: \( V_c = 245 \pm 13 \text{ km s}^{-1} \). Thus, as opposed to R09, who found that these two sources significantly raise the circular velocity derived from the maser data, our result is robust with respect to their inclusion.

4. DISCUSSION

We have re-analyzed the recent maser kinematics from R09, to see what they tell us about \( V_c(R_0) \) and the maser orbits. Our analysis differs from that of R09 by allowing for a more general model for the distribution of the velocities of the masers with respect to their local standards of rest, by using a proper probabilistic framework that includes proper marginalization over uninteresting parameters, and by the explicit inclusion of suitable prior information. From this, we find an estimate of \( V_c \) of \( 244 \pm 13 \text{ km s}^{-1} \), the same value as the mode of our prior, and substantially lower than the estimate of R09. Our analysis has also shown that the current maser measurements have only limited power to constrain \( V_c \) beyond the prior; dropping the prior coming from the measured proper motion of Sgr A* we find \( V_c = 246 \pm 30 \text{ km s}^{-1} \); further dropping the prior information on \( R_0 \), the maser data provide no constraint on \( V_c \) at all.

The value for \( V_c \) that we have inferred in this paper from the kinematics of Galactic masers compares favorably with other recent measurements of the circular velocity. As is clear from Figure 1, the posterior probability distribution for the circular velocity is peaked at about the same value as the prior probability distribution obtained from combining the precise measurements of the distance to the Galactic Center, the proper motion of Sgr A*, and the Solar motion in the direction of Galactic rotation. It is also consistent with the value of \( V_c = 221 \pm 18 \text{ km s}^{-1} \) from a recent measurement based on the completely different principle of fitting an orbit to the GD-1 stellar stream (Koposov et al. 2009). Combining these estimates by inverse variance weighting we find a value for the circular velocity of \( V_c = 236 \pm 11 \text{ km s}^{-1} \).

The results in this paper are unaffected by the uncertainty in the value of the Solar motion with respect to the LSR. If we use a larger uncertainty in the value of \( v_0 \) of 3 km s\(^{-1}\) in each component, as suggested by an analysis of the effect of moving groups on \( v_0 \) (J. Bovy & D. W. Hogg 2010, in preparation), we retrieve the same estimate \( V_c = 244 \pm 14 \text{ km s}^{-1} \) as before. Even when we use an uncertainty of 15 km s\(^{-1}\) in the value of each component of \( v_0 \), we find a slight increase in the uncertainty, but still the same value \( V_c = 244 \pm 20 \text{ km s}^{-1} \). Thus, the uncertainty in \( v_0 \) only affects our conclusions if it is larger than about 10 km s\(^{-1}\).

We also learned that the masers on average lag \( V_c \), and are moving toward the Galactic Center. This fact is illustrated in Figures 3 and 4, where the orbital phases of the masers are shown for a logarithmic potential \( \Phi = V_c^2 \ln r \) (e.g., Equation (3.14) in Binney & Tremaine 2008) assuming \( R_0 = 8.2 \text{ kpc} \) and \( V_c = 244 \text{ km s}^{-1} \). This will be interesting to analyze in the context of spiral shock models.

Our analysis implies that the present maser data do not lead to a substantive improvement of our knowledge of \( R_0 \) and \( V_c \), as most of the information in the data is spent on determining the properties of the conditional velocity distribution of the masers. It is also remarkable that, given all of the prior information, the masers are much more informative about \( R_0 \) than they are about the angular rotation speed at the Sun’s radius, as the posterior distribution for \( \Omega_0 \) is barely distinguishable from the prior distribution.

Despite the fact that most of the information content in the maser data is already being used to infer the distribution function, it is possible that our model for the distribution function is not general enough. For one, it is very likely that the distribution function of the masers depends on the Galactocentric radius and, in particular, that the mean velocity offset in the direction toward the Galactic center depends on radius. Indeed, there is some indication of that already in our results, as the large velocity dispersion of the masers is mostly driven by a large velocity dispersion in the direction toward the Galactic Center; this could be due to an unmodeled radial dependence of the distribution function.

The measurement of the dynamics of the Galaxy performed here uses a tracer population that is obviously non-angle mixed but has no unambiguous non-angle-mixed interpretation—such as a stellar stream tracing out an orbit. Such a measurement has the fundamental problem that structure in the distribution function of the masers is, in a sense, exchangeable with complexity of the potential. Therefore, detailed measurements of the potential of the Galaxy using larger samples of masers will very likely be fundamentally limited by our lack of knowledge about the distribution function of the masers. As more masers with precise kinematic information become available—as many as 400 are possible over the next few years (M. Reid 2009, private communication)—more detailed inferences of the distribution function will have to be made simultaneously with more precise
measurements of the potential of the Galaxy from these masers. The method described and used in this paper is flexible enough to handle these more general distribution functions and more general models for the potential of the Galaxy.

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APPENDIX

MCMC EXPLORATION OF THE POSTERIOR DISTRIBUTION

We explore the posterior probability distribution using a Metropolis–Hastings (MH) MCMC algorithm (e.g., Mackay 2003). The MH algorithm works by proposing new model parameters \( x' \) from a proposal distribution \( Q(x'; x^{(t)}) \) that can only depend on the current values \( x^{(t)} \) of the parameters. One then computes the quantity

\[
a = \frac{p(x' | \{x_i, v_i\}) Q(x^{(t)}; x')} {p(x^{(t)} | \{x_i, v_i\}) Q(x'; x^{(t)})}. \tag{A1}
\]

If \( a \geq 1 \) one accepts the new state; if \( a < 1 \), the new state is accepted with probability \( a \). If the new state is rejected, the old state is added again as a sample of the posterior. This procedure converges to give samples from the posterior.

As proposal distributions we use (1) the prior for the components of \( \nu_0 \), (2) a Gaussian for \( R_0 \) and \( \mu_{Sgr A^*} \) centered on the current values with widths of 0.5 kpc and 0.12 km s\(^{-1}\) kpc\(^{-1}\), respectively, (3) a Gaussian for the mean offset centered on the current values with a width of \( \sim 10 \) km s\(^{-1}\) for each component, and (4) a Wishart distribution for the velocity dispersion tensor with mean equal to the current tensor and shape parameter \( \sim 20 \). The widths of these last three proposal distributions were chosen so as to give an acceptable acceptance rate of about 50%. Monte Carlo chains were run with different sets of parameters of the proposal distributions and the resulting posterior probability distributions were found to be independent of the parameters of the proposal distributions.

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