Static Wind Nonlinear Analysis of Iced Transmission Lines

Yi You¹,*, Long Zhang¹, Zhitao Yan², Xiaochun Nie², Feng Wang³

¹State Grid Xinjiang Company Limited Electric Power Research Institute, Urumqi 830011, China
²School of Civil Engineering and Architecture, Chongqing University of Science and Technology, Chongqing 401331, China
³China Electric Power Research Institute, Beijing 100192, China

*Corresponding author email: yiyou@sgcc.com.cn

Abstract. The torsion of iced transmission line under the action of static wind will affect the configuration of its equilibrium state. This state is very critical for occur of the galloping. The beam element simulation method of iced transmission line is proposed. The stiffness matrix and mass matrix are derived, and the double nonlinear analysis method considering geometric nonlinearity and material nonlinearity is proposed. The static wind stability analysis of iced transmission line show that the aerodynamic nonlinearity caused by the shape is obvious. With the increase of wind loads, the torsion angle of conductor increases and the three component force coefficient changes, which leads to the nonlinear change of displacement and internal force response. At low wind speed, the iced conductor is more sensitive to the initial attack angle of wind, but at high wind speed, the conductor deformation is not sensitive to the initial attack angle of wind, and both will converge to the positive and negative values respectively.

Keywords: iced transmission lines; static; wind; nonlinear analysis.

1. Introduction

The galloping phenomenon of iced transmission line under wind loads [1] is very complex, especially the initial position of iced transmission line is very important for galloping excitation [2]. In the numerical analysis, it is generally assumed that the static wind stability analysis under the action of average wind is carried out first, and then the galloping time history analysis is carried out based on this position. The configuration of conductor will directly affect the accuracy of galloping nonlinear analysis, and the static wind nonlinear analysis of iced conductor is very important for galloping analysis of transmission line.

In the process of static wind loading on iced conductor, it is found that under certain conditions, when the wind speed is iterated in stages, the deformation of iced conductor, especially the twist angle, will change the three components force coefficient, thus changing the static wind loads. The changing wind loads will lead to the deformation of the conductor, resulting in the change of the wind attack angle. In extreme conditions, the static wind instability may occur, which is more common researched in the bridge structure [3-7], but the deformation of the bridge and the deformation and mechanical characteristics of the transmission line are essentially different. The deformation of transmission line is
larger, and the geometric nonlinearity and material nonlinearity need to be considered more. Based on
beam element, the mechanical analysis model of iced transmission line is established, and the static
wind stability finite element analysis of iced transmission line is realized by using double iteration
method.

2. Mechanical model of Iced conductor
The beam element is used to establish the analysis model of iced conductor. For iced conductors, if the
mesh is fine enough, the two-node 3D iced straight beam element can be used. For a given 3D iced
beam element, its mechanical characteristics are derived as follows.

2.1. Stiffness matrix
The stiffness matrix of beam element can be divided into two parts, one is elastic stiffness matrix, the
other is geometric stiffness matrix considering the effect of initial stress. The elastic stiffness is
deduced according to the conventional finite element interpolation function.

\[
K^e = \int \int dA \int \begin{bmatrix} B^T & DB \end{bmatrix} dx
\]

Where, \( D = \text{diag}(E, E, E, G) \), \( B \) is the strain-displacement matrix. The explicit expression
of the conventional beam element stiffness can be referred to in many references, which is not detailed
here.

The geometric stiffness matrix of beam element is the geometric stiffness matrix proposed by Yang
[8]. According to the rigid body criterion, the geometric stiffness can be derived from the rigid body
motion from \( C_1 \) state to \( C_2 \) state, as shown in Fig. 1.

The stiffness matrix expression of reference [8] is directly quoted as Eq.(2).
\[
K_g = \begin{bmatrix}
 p & h_a & -p & h_b \\
 h_a^T & i_a & -h_a^T & 0 \\
 -p & -h_a & p & -h_b \\
 h_b^T & 0 & -h_b^T & i_b
\end{bmatrix}
\] (2)

Where, \(0\) is the zero matrix of 3x3.

\[
p = \begin{bmatrix}
0 & -\frac{1}{2}F_{yb} / L & -\frac{1}{2}F_{zb} / L \\
-\frac{1}{2}F_{yb} / L & \frac{1}{2}F_{ab} / L & 0 \\
-\frac{1}{2}F_{zb} / L & 0 & \frac{1}{2}F_{ab} / L
\end{bmatrix}, h_p = \begin{bmatrix}
0 & 0 & 0 \\
\frac{1}{2}M_{yb} / L & -\frac{1}{2}M_{zb} / 2L & 0 \\
\frac{1}{2}M_{zb} / L & 0 & -\frac{1}{2}M_{ab} / 2L
\end{bmatrix}
\] (3)

For iced conductors, the stiffness caused by icing eccentricity should be considered as Eq.(4).

\[
K_{icc} = -g \int \int dA \int N^T S_c N \, dx
\] (4)

where, \(S_c = \text{diag}\{0 \quad 0 \quad S_z\}^T\) (5)

2.2. Mass matrix
Considering the icing of conductor, the mass matrix of conductor is determined according to Eq.(6).

\[
m = \int N^T \mu N \, dx
\] (6)

Where, \(N\) is the conventional shape function of the 3D beam element.

\[
\mu = \rho \begin{bmatrix}
A & 0 & 0 & 0 \\
0 & A & 0 & -S_y / r \\
0 & 0 & A & S_z / r \\
0 & -S_y / r & S_z / r & J_k / r^2
\end{bmatrix}
\] (7)

Where, \(S_y\), \(S_z\) denotes the static moment of the section to the y and z axes respectively, \(r\) is the radius of the section, \(A\) represents the section area and \(\rho\) is the density of the wire.

3. Wind loads
The iced transmission line is affected by lift, resistance and moment under wind load, and its coordinate system is shown in Fig. 3. The lift, drag and moment are shown in Eq.(8).
4. Stability analysis under static wind

The mechanical analysis of transmission line under wind load is a typical problem of large deformation and small strain. In general, the method of linearization is used to solve the nonlinear equation, that is to transform the nonlinear problem into a series of linear problems. Generally, Newton Raphson method is used to deal with nonlinear problems by linear iteration. Based on the traditional Newton-Raphson iterative method, the nonlinear analysis of wind load under static wind is carried out considering the change of three component forces under wind attack angle. It can be divided into the following steps:

(1) The initial wind attack angle is determined according to the icing shape, and the initial static three component force coefficient is determined according to the wind attack angle.

(2) Considering geometric nonlinearity, static analysis of transmission lines is carried out step by step.

(3) The displacement increment is obtained, and the stiffness matrix is modified based on the new deformation to check whether the joint forces are balanced.

(4) Determine the increment of force and displacement, and judge whether the Euclidean norm of increment of force and displacement converges to a small quantity.

(5) According to the obtained displacement, the static three component force coefficient and the static three component force are determined. If the deviation from the force load obtained in Step (3) is too large, the average or weighted forces of the current step and previous iteration step is taken. Determine the new loading step and proceed to the Step (2).

(6) If it converges, the displacement and force of each step are output.

5. Calculation examples

The cross section of iced conductor is shown in Fig. 4. The diameter of bare conductor is 27.6 mm and the ice thickness are assumed to be 20 mm. Other key parameters at wind attack angle of 0° are shown in Table 1. The axial stiffness of the conductor is $EA = 31.1 \times 106$N, and the torsional stiffness is $GJ = 159 \text{mm}^2\text{rad}^{-1}$. The bending stiffness is $1965 \text{Nm}^2$ and the elastic modulus E is $69000 \text{N/mm}^2$. 

$$\begin{align*}
F_L &= 0.5\rho_a U^2 D LC_L(\alpha) \\
F_D &= 0.5\rho_a U^2 D LC_D(\alpha) \\
F_M &= 0.5\rho_a U^2 D^2 LC_M(\alpha)
\end{align*} \tag{8}$$

Where, $U$ is wind speed, $\alpha$ is wind attack angle, $\rho_a$ is air density, $D$ is diameter of conductor, $L$ is length of element, $C_L$ is lift coefficient, $C_D$ is drag coefficient and $C_M$ is torsional coefficient.
Figure 4. Cross section of crescent shaped iced conductor

Table 1. Parameters of iced conductor

| Parameters                                | Ice Shape |
|-------------------------------------------|-----------|
| Ice thickness (mm)                        | 20.00     |
| Cross-sectional area of bare conductor (mm²) | 598.28    |
| Cross-sectional area of ice (mm²)         | 343.04    |
| Mass of bare conductor (kg/m)             | 1.51      |
| Mass of ice (kg/m)                       | 0.308     |
| Section eccentricity (mm)                 | 19.17     |
| Eccentricity of cross section $e_{10}$ (mm) | 3.25      |
| Eccentricity of cross section $e_{20}$ (mm) | 0.00      |

For this icing shape, the relationship between aerodynamic load and wind attack angle is shown in Fig. 5, which show that the aerodynamic load increases with the increase of icing amount. In this ideal ice shape, the lift and moment coefficients are antisymmetric at the angle of attack of 180 ° and the drag coefficients are symmetric at the angle of attack of 180 °. When the angle of attack is 0 °, the lift and moment coefficients are zero.

Figure 5. Aerodynamic coefficients on crescent shaped iced conductors

Assuming the initial angle of attack is 10 ° and the wind speed of 20m / s, the wind loads are evenly divided into 50 load steps, and the influence of double nonlinearity is considered to solve the problem. Fig. 6 shows that if the iteration of aerodynamic nonlinearity is not considered, with the increase of load, the mean displacement and moment of the conductor will increase linearly, showing obvious linear characteristics. It is shown that although the transmission line is a typical large deformation structure, it still shows obvious linear characteristics under the initial tension, and the geometric nonlinearity is not obvious. However, the aerodynamic nonlinearity caused by deformation is obvious. Figs. 6 (a) show that with the increase of load, the torsion angle of conductor increases, the drag coefficient increases, and the displacement and drag in y direction increase nonlinearly.
Fig. 6 (b) show that with the increase of load, the torsion angle of conductor increases, the lift coefficient increases first and then decreases, and the displacement and resistance in z direction show the same nonlinear trend. Fig. 6 (c) and (d) show that with the increase of load, the torsion angle of conductor increases, the torque coefficient increases first and then decreases, and the torsion angle increases steadily, but the torque increases first and then decreases.

6. Conclusions

The beam element simulation method of iced transmission line is proposed, the stiffness matrix and mass matrix are derived, and the double nonlinear analysis method considering geometric nonlinearity and material nonlinearity is proposed. The following conclusions are obtained:

1) If the iteration of aerodynamic nonlinearity is not considered, the displacement and torque uniformity of the conductor will increase with the increase of load, showing obvious linear characteristics.

2) The aerodynamic nonlinear phenomenon caused by deformation is obvious. With the increase of load, the torsion angle of conductor increases and the three component force coefficient changes, which leads to the nonlinear change of displacement and internal force response.

3) Under low wind speed, iced conductor is more sensitive to the initial angle of attack. Different initial angle of attack will produce different initial configuration. With the increase of load, the initial shape of conductor changes, which may cause the transmission line galloping.

4) At high wind speed, initial configuration is not sensitive to the initial angle of attack because the twist angle will converge to positive and negative values respectively. If the aerodynamic force
galloping is easy to occur under these two angles, the galloping of transmission line under strong wind is inevitable.

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