Summary: In this paper we focus on $r$-neighbor bootstrap percolation on finite graphs, which is a process on a graph evolving in discrete rounds where, initially, a set $A_0$ of vertices gets infected. Now, subsequently, an uninfected vertex becomes infected if it is adjacent to at least $r$ infected vertices. Call $A_f$ the set of vertices that are infected after the process stops. More formally, we let $A_t := A_{t-1} \cup \{v \in V : |N(v) \cap A_{t-1}| \geq r\}$, where $N(v)$ is the neighborhood of some vertex $v$. Then $A_f = \bigcup_{t \geq 0} A_t$. We are mainly interested in the size of the final set $A_f$. We present a theorem which bounds the size of the final infected set for degenerate graphs in terms of $|A_0|, d$ and $r$. To be more precise, for a $d$-degenerate graph, if $r \geq d+1$, the size of $A_f$ is bounded from above by $(1 + \frac{d}{r-d})|A_0|$.

For the entire collection see [Zbl 1398.90008].

MSC:
60K35 Interacting random processes; statistical mechanics type models; percolation theory
05C80 Random graphs (graph-theoretic aspects)
60K37 Processes in random environments

Keywords:
bootstrap percolation; degenerate graphs

Full Text: DOI, arXiv

References:
[1] Chalupa, J., Leath, P. L., & Reich, G. R. (1979). Bootstrap percolation on a Bethe lattice. $Journal of Physics C: Solid State Physics$, $12$(1). · doi:10.1088/0022-3719/12/1/008
[2] Levin, A. D., Łuczak, M. J., & Peres, Y. (2010). Glauber dynamics for the mean-field ising model: cut-off, critical power law, and metastability. $Probability Theory and Related Fields$, $146$(1-2), 223-265. · Zbl 1187.82076 · doi:10.1007/s00440-008-0189-z
[3] Holroyd, A. E. (2003). Sharp metastability threshold for two-dimensional bootstrap percolation. $Probability Theory and Related Fields$, $125$(2), 195-224. · Zbl 1042.60065 · doi:10.1007/s00440-002-0239-x
[4] Balogh, J., Bollabás, B., Duminic-Copin, H., & Morris, R. (2012). The sharp threshold for bootstrap percolation in all dimensions. $Transactions of the American Mathematical Society$, $364$(5), 2667-2701. · Zbl 1238.60108 · doi:10.1090/S0002-9947-2011-05552-2
[5] Bradonjić, M., & Saniee, I. (2014). Bootstrap percolation on periodic trees. In $2015 Proceedings of the Twelfth Workshop on Analytic Algorithmics and Combinatorics (ANALCO)$, SIAM$\$, (pp. 89-96).
[6] Bollobás, B., Gunderson, K., Holmgren, C., Janson, S., & Przykucki, M. (2014). Bootstrap percolation on Galton-Watson trees. $Electronic Journal of Probability$, $19$.$\$ · Zbl 1290.05035
[7] Balogh, J., Peres, Y., & Pete, G. (2006). Bootstrap percolation on infinite trees and non-amenable groups. $Combinatorics, Probability and Computing$, $15$(5), 715-730. · Zbl 1102.60086 · doi:10.1017/S0963548305007610
[8] Riedl, E. (2012). Largest and smallest minimal percolating sets in trees. $The Electronic Journal of Combinatorics$, $19$(1), P64. · Zbl 1254.05196
[9] Riedl, E. (2010). Largest minimal percolating sets in hypercubes under 2-bootstrap percolation. $The Electronic Journal of Combinatorics$, $17$(1), P1. · Zbl 1214.05067
[10] Benevides, F., & Przykucki, M. (2015). Maximum percolation time in two-dimensional bootstrap percolation. $SIAM Journal on Discrete Mathematics$, $29$(1), 224-251. · Zbl 1371.60169 · doi:10.1137/130941584
[11] Benevides, F., & Przykucki, M. (2012). Maximum percolation time in hypercubes under 2-bootstrap percolation. $The Electronic Journal of Combinatorics$, $19$(1), 1-13. · Zbl 1254.82017
[13] Bollobás, B., Holmgren, C., Smith, P., & Uzzell, A. J. (2014). The time of bootstrap percolation with dense initial sets. *The Annals of Probability*, 42(4), 1337-1373. - Zbl 1311.60113 - doi:10.1214/12-AOP818

This reference list is based on information provided by the publisher or from digital mathematics libraries. Its items are heuristically matched to zbMATH identifiers and may contain data conversion errors. It attempts to reflect the references listed in the original paper as accurately as possible without claiming the completeness or perfect precision of the matching.