The gravitational and electroweak interactions unified as a gauge theory of the de Sitter group

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Abstract
The complexified gauging of the de Sitter group gives a unified theory for the electroweak and gravitational interactions. The standard spectrum for the electroweak gauge bosons is recovered with the correct mass assignments, following a spontaneous breaking of the gauge symmetry imposed by the geometry. There is no conventional Higgs sector. New physics is predicted with gravity-induced electroweak processes (at the electroweak and at an intermediate scale of about $10^{10}$ Gev) as well as with novel-type of effects (such as gravitational Aharonov-Bohm and violations of the Principle of equivalence to 1 part in $10^{17}$). The new theoretical perspectives emerging from this geometric unification are briefly discussed.

PACS numbers: 03.50z, 04.50.+h, 04.80.Cc, 04.90.+e, 11.15.-q, 11.15.Ex, 11.30.-j, 11.30.Qc, 12.10.-g, 12.60.-i, 12.60.Cn, 12.90.+b.

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1 Introduction

Following the early [1] up to the contemporary [3] efforts to unify gravity with other fundamental
interactions, one is tempted to isolate as perhaps the single most important finding the fact that
Einstein’s 4D theory can also be formulated as a gauge theory of the Poincaré group \( P_0 \) [3]. We
recall that under the conventional (that is to say, real) gauging of \( P_0 \), Einstein’s gravity emerges
as expected, namely associated with the subgroup \( \mathcal{L} \) of Lorentz rotations. Curiously, however,
there is no gauge-field output corresponding to the translational generators, for which we only
seem to have a dubious association with spin [4]. Updating earlier attempts and refining recent
preliminary results [5], we will expand here on the premise that the translational generators
of space-time are intimately associated with the electroweak interaction. We will see that
the complexified gauging of \( \mathcal{P} \), a uniquely defined 10-parameter Lie group which turns out
to be isomorphic to the de Sitter group, gives rise to a spontaneously broken \( SU(2) \times U(1) \)
gauge theory unified with gravity. In the following section we will introduce \( \mathcal{P} \) and examine
its conventional gauge theory. In section 3 we will proceed with the complexified gauging of
\( \mathcal{P} \) in the context of Riemann-Cartan geometry, but without commitment on any particular
spin-torsion interrelation. In section 4 we will trace the intricate passage from the space-time
geometry to an internal gauge symmetry. In section 5 we will uncover and examine an already
present mechanism for the spontaneous breaking of that symmetry: it is elegantly imposed by
the geometry without any \textit{ad hoc} assumptions, so the need of a Higgs sector is thus superseded.
In section 6 we will examine the relevance of our findings to the electroweak interaction [6], to
realize that we actually recover the standard mass spectrum and charges for the electroweak
gauge bosons. Our construction seems to offer fundamental upgrading to each one of the
two interactions it unifies, together with new (and apparently testable) predictions, as we will
discuss in the last section.

To establish notation [3], we briefly recall that a contemporary formulation of Einstein’s
theory in a differentiable 4D manifold \( M^4 \) involves the Einstein-Hilbert action as

\[
I_{EH} = \frac{1}{32\pi G} \int_{M^4} R_{ab} \wedge \ast (e^a \wedge e^b),
\]

wherefrom the vacuum field equations follow upon variation of the frame \( e^a \) under the require-
ment of \( SO(1,3) \) (recast from the original \( GL_4 \) co-ordinate) invariance, and the constraints of
vanishing torsion and metricity. The latter is by definition expressed as \( Dg_{ab} = 0 \) which, with
\( g_{ab} = (-1,1,1,1) \), amounts to antisymmetry of the connection \( \omega^{ab} \).

We also recall the alternative formulation of Einstein’s gravity as a gauge theory of the
The Poincaré group with generators $Q_A \equiv \{P_a, M_{aa'}\}$ assigned, respectively, to the translations and the $L$ rotations in $M^4$. According to the standard procedure, we gauge $P_0$ (and, eventually, $P$) with the introduction of the generalized potential

$$H \equiv H^A Q_A = e^a P_a + \frac{1}{2} \omega^{ab} M_{ab},$$

with covariant derivative $D$ (cf. below), and field strength

$$\Omega \equiv \Omega^A Q_A = dH + H \wedge H = (dH^A + \frac{1}{2} f_{BC}^A H^B \wedge H^C) Q_A = T^a P_a + \frac{1}{2} R^{ab} M_{ab}.$$  

To explicitly write down the torsion $T^a$ and curvature $R^{ab}$ 2-forms (namely Cartan’s structure equations), we need the structure constants $f_{BC}^A$. Utilizing the algebra of $P_0$, we find

$$T^a = D e^a \equiv d e^a + \omega^a_{\ b} \wedge e^b, \quad R^{ab} = d \omega^{ab} + \omega^a_{\ c} \wedge \omega^{cb} - \beta e^a \wedge e^b.$$

For the gauge theory of the Poincaré group. Then, variation of the Einstein-Hilbert action (in the absence of any external spinorial sources) with respect to $\omega^{ab}$ (24 independent components) sets to zero the 24 independent components of the torsion, while variation with respect to $e^a$ furnishes Einstein’s equations in vacuum.

We finally recall the de Sitter group $[\mathbb{D}]$, with its algebra expressed as

$$[P_a, P_b] = -\beta M_{ab}, \quad [M_{aa'}, P_b] = 2\delta^{[a}_{[a'} g^{a']b} P_c, \quad [M_{aa'}, M_{bb'}] = 4\delta^{[a}_{[a'} \delta^{a']_{b']} M_{cc'}],$$

where the $\pm |\beta|$ values of the (real) parameter $\beta$ are associated with the two possible types of non-trivial topology involved (closed and open for positive and negative curvature), while the $\beta = 0$ value practically yields the standard contraction down to the Poincaré algebra. Cartan’s structure equations for the above basis of the de Sitter algebra are

$$T^a = D e^a \equiv d e^a + \omega^a_{\ b} \wedge e^b, \quad R^{ab} = d \omega^{ab} + \omega^a_{\ c} \wedge \omega^{cb} - \beta e^a \wedge e^b.$$ 

Comparing $\mathbb{P}$ $\mathbb{P}$ $\mathbb{P}$, we conclude that, besides the global distinction (from topology), the only difference (at the classical level) between the Poincaré and the de Sitter gauge theory is a comological constant proportional to $\beta$. In particular, there is again no field-strength output corresponding to the $P_a$ generators: in vacuum, the torsion has to vanish identically.

2 Introducing $\mathcal{P}$: uniqueness, isomorphism to the de Sitter group, and gauging

The mentioned peculiarity of having no field strength associated with the translational generators can actually be traced to much deeper issues such as the nature of torsion and its
relation to spin and the quantization of gravity. The apparent impas s is enhanced by the Coleman-Mandula theorem \[8\] plus the fact that the de Sitter (together with its contraction to the Poincaré) group exhausts all three possibilities for the isometry groups of maximally symmetric 4D manifolds. The best known resolutions proposed include the twistor approach and, of course, supersymmetry \[2\]. Reflecting on the above, we have been motivated to look for a generic 10-parameter Lie group \( P \) (with generators \( \Pi_a, M_{aa'} \)), not necessarily constrained to be a maximal-isometry group, but which must contain \( \mathcal{L} \) as a subgroup. This would imply that, although \( P \) will certainly act transitively on \( \mathcal{P}/\mathcal{L} \), it might not do so on space-time and, in particular, its \( \Pi_a \) generators may loose a direct interpretation as space-time translations. The latter will, of course, exist and be well defined in any case, along with whatever symmetry they may have.

Let us then formally introduce \( P \), with its generators satisfying the algebra

\[
\begin{align*}
[\Pi_a, \Pi_b] &= C_{ab}^{cc'} \Pi_c + C_{ab}^{a'b} M_{cc'}, \\
[M_{aa'}, \Pi_b] &= 2\delta_{[a}^{[a} g_{a']b} \Pi_c + C_{aa'}^{a'b} M_{cc'}, \\
[M_{aa'}, M_{bb'}] &= 4\delta_{[aa'}^{[a} g_{a']b\}] M_{cc'}. \tag{7}
\end{align*}
\]

The acceptable choices for such a simple algebra are severely restricted. In fact, the Jacobi identity dictates that \( C_{bc}^{bc} \) are precisely the structure constants for the algebra of \( SU(2) \times U(1) \), namely they are numerically equal to \( \epsilon_{jkl} \) for \( j, \ldots = \{1, 2, 3\} \) or zero if any one of the \( a, b, c \) indices is zero, while for the remaining structure constants we find

\[
\begin{align*}
C_{ab}^{cc'} &= \frac{1}{8} \left( C^{cd}_{ab} C_{cd}^{cc'} - \delta_{a}^{[c} \delta_{b]}^{c'} + \delta_{b}^{[c} \delta_{a]}^{c'} \right), \\
C_{aa'}^{a'b} &= \delta_{[a}^{[a} C^{a'b}]}_{b]} + \frac{1}{2} C_{cc'}^{[a} g_{a']b]. \tag{8}
\end{align*}
\]

It follows that \( P \) contains, in addition to \( \mathcal{L} \), a distinct \( SU(2) \) subgroup generated by \( \Pi_j \) and a \( U(1) \) associated with \( \Pi_0 \). To explicitly see this structure, we introduce the usual \( J_j = -\frac{1}{2} \epsilon_{jkl} M_{kl}, K_j = M_{0j} \) generators (spatial rotations and Lorentz boosts), so that the commutation relations \( \mathcal{L} \) may equivalently be written as

\[
\begin{align*}
[J_j, J_k] &= \epsilon_{jkl} J_l, \quad [J_j, K_k] = \epsilon_{jkl} K_l, \quad [J_j, \Pi_k] = \epsilon_{jkl} \Pi_l, \quad [J_j, \Pi_0] = 0, \\
[K_j, K_k] &= -\epsilon_{jkl} J_l, \quad [K_j, \Pi_k] = g_{jk} \Pi_0 + \frac{1}{2} \epsilon_{jkl} K_l, \quad [K_j, \Pi_0] = \Pi_j - \frac{1}{2} J_j, \\
[\Pi_j, \Pi_k] &= \epsilon_{jkl} \Pi_l, \quad [\Pi_j, \Pi_0] = \frac{1}{4} K_j. \tag{9}
\end{align*}
\]

In this basis, the \( SU(2) \) subgroup structure generated by the \( \Pi_j \) is obvious, and we also note that the commutator of each \( \Pi_j \) with \( \Pi_0 \) does not vanish but closes to a Lorentz boost along \( j \).
Further investigation has shown that $\mathcal{P}$ is, in fact, unique, modulo isomorphisms and trivial cases such as direct products of 4-parameter Lie groups with $\mathcal{L}$ or contractions of $\mathcal{P}$ (e.g., down to the Poincaré group). The de Sitter group, in particular, is actually isomorphic to $\mathcal{P}$. This can be established if one introduces the new set of (translational) generators $P_a$ with

$$ P_a = 2\sqrt{\beta} \left( \Pi_a + \frac{1}{4} C^{cc'}_{a} M_{cc'} \right) \leftrightarrow P_0 = 2\sqrt{\beta} \Pi_0, \quad P_j = 2\sqrt{\beta} \left( \Pi_j - \frac{1}{2} J_j \right), $$

(10)

to finally show that the $P_a, M_{aa'}$ set satisfies precisely the de Sitter algebra (5).

We may now proceed with the (real) gauging of $\mathcal{P}$, utilizing the basis in (7) with components of the generalized potential $(e^a, \frac{1}{2} \omega^a_{aa'})$, to find Cartan’s equations as

$$ \hat{T}^a = T^a = de^a + \frac{1}{2} C^{a}_{bc} e^b \wedge e^c + \frac{1}{2} \omega^a_{b} \wedge e^b, $$

$$ \hat{R}^{ab} = R^{ab} = d \omega^{ab} + \frac{1}{2} \omega^a_{b} \wedge \omega^b_c + \frac{1}{2} \left( C^{a}_{cd} \omega^c_{cb} + C^{b}_{cd} \omega^c_{ac} + C^{ab}_{cd} \omega^c_{cd} \right) \wedge e^d + C^{ab}_{cd} \omega^d_c \wedge e^d, $$

with $C^{ab}_{cd}$ given by (8). Due to the established isomorphism between $\mathcal{P}$ and the de Sitter group, one might expect that the content of the sets (6) and (11) is identical. In fact, this is not true because of the different gauging involved in each case. To better compare the two sets, we may re-cast (11) as

$$ \hat{T}^a = D e^a \equiv de^a + \omega^a_{b} \wedge e^b, \quad \hat{R}^{ab} = d \omega^{ab} + \omega^a_{c} \wedge \omega^c_b - \frac{1}{4} e^a \wedge e^b \frac{1}{2} C^{ab}_{cd} \omega^c_d, $$

(12)

expressed in terms of the new connection

$$ \omega^{ab} = \hat{\omega}^{ab} - \frac{1}{2} C^{ab}_{cd} \omega^c_d, $$

(13)

and its respective covariant derivative $D$. As compared to the gauge theory of the de Sitter group (namely (8) etc.), the above equations supply a formally identical expression for the torsion, but there are differences in the curvature. Besides the modification in the contribution for the cosmological constant, the presence of the additional term proportional to $D e^a$ (namely to the torsion) is non-redundant and cannot be transformed or gauged away. The reason is that, unlike the case for the de Sitter group, variation of the Einstein-Hilbert action (written in terms of either of the expressions (II,12)) with respect to $\hat{\omega}^{aa'}$ shows that the torsion cannot vanish in the gauge theory of $\mathcal{P}$, in fact it turns out to be

$$ T^a = \frac{1}{6} C^{a}_{bc} e^b \wedge e^c. $$

(14)

We observe that we now have a non-vanishing field strength associated with the $\Pi_a$ generators. To understand the difference in view of the identical expressions for the torsion in (8,12), we
observe that these expressions are definitions of $T^a$, while (14) is its field equation (whose counterpart in the de Sitter case is $T^a = 0$). Obviously, this field equation is an algebraic one, in accord with the general result that torsion does not propagate \[3\]. In fact, we will eventually also get a propagating field (the electroweak) associated with the $\Pi_a$, following the complexified gauging of $\mathcal{P}$.

3 Complexified gauging of $\mathcal{P}$

By complexified gauging of $\mathcal{P}$ we simply mean that we now let $H$ in (2-3) become complex, namely we re-write (11) or (12) with the substitutions

\[ e^a \rightarrow \tilde{e}^a = e^a + ih^a, \quad \omega^{ab} \rightarrow \tilde{\omega}^{ab} = \omega^{ab} + i\left(K^{ab} + S^{ab}\right), \tag{15} \]

or, equivalently,

\[ e^a \rightarrow \tilde{e}^a = e^a + ih^a, \quad \tilde{\omega}^{ab} \rightarrow \tilde{\omega}^{ab} + i\left(\frac{1}{2}C^{abc}h_c + K^{ab} + S^{ab}\right), \tag{16} \]

with $\omega^{ab}$ still as defined by (13). Before we proceed to discuss the physical interpretation of this complexification, we note that what is indicated as the imaginary part of the connection in (15) has been split into two pieces for later convenience. In $K^{ab} = -K^{ba}$ we have chosen to segregate all contributions coming from (and thus been determined by) $h^a$, while $S^{ab} = -S^{ba}$ carries the needed 24 of the total 48 (real+imaginary) independent components of $\tilde{\omega}^{ab}$. The antisymmetry has been imposed so that metricity is maintained.

At least formally, we could exploit the use of the complex covariant derivative defined with $\tilde{\omega}^{ab}$ as a complex connection. The latter notion is already familiar from the treatment of non-abelian gauge theory and connections are not observables anyway, so there is really no problem involved here. On the other hand, this is not quite the case with $\tilde{e}^a$ (frames are geometrical objects), so we will briefly examine the physical interpretation associated with the present employment of a complex frame.

Let us recall our earlier re-definition of a connection in (13), which facilitated our comparative examination of two gauge theories, whereby a contribution from the $e^a$ frame (the $\frac{1}{2}C_{abc}e^c$ term) was effectively absorbed in the connection. In the present context, the contribution of what is indicated in (15) as the imaginary part of $\tilde{e}^a$, namely $h^a$, can be likewise absorbed in the already existing complex connection. As a result, in addition to its gauging aspect, the above complexification also admits the following geometrical interpretation. Let us start with
the conventional (general relativistic) description of $M^4$ given in terms of the real basis $e^a$ and the also real $\hat{\omega}^{ab}$ as its Christoffel connection. Following the complexification (13), the quantities $e^a$ and $\hat{\omega}^{ab}$ will retain both, their reality as well as their previous identification (even though the may change in value as a result of additional sources in whatever will turn up as Einstein’s equations). However, the connection of the resulting space-time will no longer be simply Christoffel (just like the $\omega^{ab}$ in (13) is not). In other words, $\hat{\omega}^{ab}$ will be enlarged with a tensorial contribution, namely with what is defined as contorsion in the context of Riemann-Cartan geometry (given by $-\frac{1}{2}C^a_{bc}e^c$ in (13)). The only difference from that context is that in our case the contorsion will be complex: it will receive mixed (real+imaginary) contributions from $h^a$, $K^{ab}$, $S^{ab}$. The real part of this contorsion will be observable through its conventional geometric interpretation. The imaginary part will also be observable, but in a gauge-theoretic (notably for the electroweak interaction) and topological (e.g., gravitational Aharonov-Bhm effects) context.

In the above analysis it makes no difference if some or all of the $h^a$, $K^{ab}$, $S^{ab}$ become complex (e.g., as a result of the employment of a particular representation or gauge). Although no degrees of freedom will be gained or lost, it will be useful to have a clear perspective on that as well as on potentials and their field strengths. By the generic definitions in (2-3) applied for the algebra (7), let $\tilde{H}$ be the new (complex) generalized potential with field strength $\tilde{\Omega}$. The components of $\tilde{H}$ are specified as

$$\tilde{H}^a = e^a + ih^a, \quad \tilde{H}^{aa'} = \frac{1}{2} \left( \hat{\omega}^{ab} + i \left( \frac{1}{2}C^a_{bc}h^c + K^{ab} + S^{ab} \right) \right),$$

while those of $\tilde{\Omega}$, namely $\tilde{T}^a$, $\frac{1}{2}\tilde{R}^{ab}$, will be calculated shortly.

The independent variables, namely those to be varied in the classical action, are 16+24 for the real $e^a$ and $\hat{\omega}^{aa'}$ plus an additional real count 16+24 coming from $h^a$ (actually $A^I$ in terms of which $h^a$ will be defined - cf. below), and $S^{aa'}$. Due to certain calculational subtleties, increased care is required in the choice of the connection and its variation, otherwise one may end up with virtually unmanageable complexity. In particular we note that our choice of the basis in (8) and the connection (13), essentially a choice of gauge, may not be optimal. Practically, the variation of $\omega^{aa'}$ seems preferable and equivalent to a variation of $\hat{\omega}^{aa'}$. However, in the former case, under the independent variation $\delta e^a$ of the frame, one should simultaneously vary the connection as $\delta \omega^{ab} = -\frac{1}{2}C^a_{bc}\delta e^c$. The rest of the independent variables, namely $S^{aa'}$, are expected to be associated with the fermionic content of space-time but they will not be really considered any further in the present treatment.
Re-tracing the steps leading to (11) or (12) (practically, just substituting (15) in (12), having set $S_{ab} = 0$) we find Cartan’s equations as

$$\tilde{T}^a = de^a + \tilde{\omega}_b^a \wedge e^b, \quad \tilde{R}^{ab} = d\tilde{\omega}^{ab} + \tilde{\omega}_c^a \wedge \tilde{\omega}^{cb} - \frac{1}{4} e^a \wedge e^b + \frac{1}{2} C_{abc} \tilde{T}^c,$$

(18)

These quantities specify directly observable field strengths and the Lagrangian, so their imaginary parts should vanish identically or behave as needed. In particular they should not give unacceptable imaginary contributions, starting with the classical action. They essentially carry the entire content of the present theory (except for its symmetry breaking aspect, introduced in section 5). Although one can recognize the desired sectors (notably the gravitational) already forming in (15), a rather delicate handling is required for uncovering what actually is an internal gauge symmetry from space-time: the endeavoured electroweak interaction from the $SU(2)$ and $U(1)$ subgroups (generated by $\Pi_j$ and $\Pi_0$, as we have seen).

4 Internal symmetry extracted from space-time

To proceed, we need explicit expressions for $h^a$ and $K^{ab}$ in (13). To uncover them, let us recall that the action of $P$ on space-time may not be isometric, as mentioned, but $M^4$ may well be chosen to be diffeomorphic to $P/L$. We will make this assumption so that there exist in $M^4$ realizations of the algebra of $P$. This algebra contains, as we have seen, the two sub-algebras for the $SU(2)$ and $U(1)$ subgroups generated by $\Pi_a$. Although it is customary (in GUTs etc.) to denote the presence of these subgroups as a direct product, we actually have a semi-direct one in view of the $[\Pi_j, \Pi_0]$ commutator as given by (9). However, we have already seen from (7) that the structure constants $C^a_{bc}$ are precisely those of the direct-product $SU(2) \times U(1)$ algebra, which will be refered to as $u_2$. Next, we will introduce a vierbein $\theta^a_j$ which will relate vectors in the two 4D vector spaces, namely $u_2$ and the tangent space at each point in $M^4$.

Let us then introduce in $M^4$ a set (frame) of four real vector fields $\theta_j^a = \theta^a_j e_a$, enumerated by the index $J = \{0, j\} = \{0, 1, 2, 3\}$ and with $e_a$ the dual of $e^a$. The $\theta_J$ are chosen so that they satisfy the commutation relations

$$[\theta_J, \theta_K] = C^{I}_{JK} \theta_I, \quad C^{I}_{JK} \equiv \theta^I_a \theta^b_j \theta^c_k C^a_{bc},$$

(19)

where the commutators are defined as usual by the respective Lie derivatives and the $C^I_{JK}$ are defined in terms of the $C^a_{bc}$ as indicated, with $\theta^I_a$ the matrix inverse of $\theta^a_I$.

We may now express $h^a$ in (13) in terms of some $u_2$-valued 1-form $A^J = A^J_a e^a$ (where $A^J_a$ are differentiable functions in $M^4$) dotted to the $\theta^a_J$. To do that, we obviously need an inner
product between vectors in $u_2$. We are thus prompted to the definition

$$h^a \equiv \gamma^I_j \theta^a_I A^J = (\cos \vartheta_W \theta^0_i A^0_k + \sin \vartheta_W \theta^i_j A^j_k) e^b,$$

(20)

where the particular value chosen for the constant matrix $\gamma^I_j$ has been generically expressed (in view of the nature of the $u_2$ algebra) as a mixing by a single constant angle parameter $\vartheta_W$.

To re-express (18), we substitute for $h^a$ its definition in terms of $A^I$, which obviously does not change the real count of 24 independent variables. The result of this calculation is

$$\tilde{T}^a = T^a + \gamma_j^I \theta^b_I A^J \wedge K^a_b + i \left( \gamma_j^I (D\theta^0_I) \wedge A^J + \gamma_j^I \theta^a_I F^J + K^a_b \wedge e^b \right),$$

(21)

$$\tilde{R}^{ab} = R^{ab} - K^a_c \wedge K^c_b + \frac{1}{2} C^c_{ab} \bar{T}^c + \frac{1}{4} \gamma_j^I \gamma^L \theta^a_I \theta^b_K A^J \wedge A^K + i \left( D K^{ab} - \frac{1}{4} \gamma_j^I (\theta^a_I e^b + \theta^b_I e^a) \wedge A^J \right).$$

(22)

In the above expressions, we have made use of the definitions

$$D \theta^a_I \equiv d\theta^a_I + \omega^a_{ib} \theta^b_I + \omega^I_j \theta^a_J,$$

(23)

$$F^I \equiv dA^I + \frac{1}{2} C^I_{JK} A^J \wedge A^K,$$ (with $\theta^a_I \theta^b_J = \delta^I_a$).

(24)

It should be noted that the definitions of $T^a$ and $R^{ab}$ as given by (12) are clearly retained, although these quantities will acquire different values through the field equations. The result (14), in particular, is not expected to hold in the present context, because it will be replaced by the new field equation resulting from the variation of $\omega^{ab}$. We also observe that the field strength $F^I$ is defined as usual for the gauge potential $A^I$, while the covariant derivative $D$ remains the same, as long as it does not meet a $u_2$ index, otherwise (as in (23)), there is an extra term from the A-connection in $u_2$, implemented by

$$\omega^I_j \equiv \frac{1}{2} C^I_{jK} A^K.$$

(25)

For reasons which will be discussed shortly, we will assume that the $\theta^a_I$ is covariantly constant, namely we will set $D\theta^a_I = 0$ in (23) (this may be viewed as analogous to the $De^a = 0$ in conventional general relativity, imposed there as the zero-torsion constraint. Then, the indicated as imaginary part of the torsion in (21) vanishes identically if

$$K^a_b \wedge e^b = -\theta^a \cdot F.$$

(26)

Reflecting upon the structure of (22), we see that a Yang-Mills sector associated with $F$ can be expected to emerge automatically as part of the Einstein-Hilbert action written for $\tilde{R}^{ab}$. In realizing that, we will see the space-time metric $g_{ab}$ transformed by the vierbein $\theta^a_I$ into $g_{IJ}$ as

$$\theta^a_I \theta^b_J g_{ab} \equiv \frac{32\pi G}{g^2} g_{IJ},$$

(27)
up to an overall factor specified by the parameter $g$. The latter will, in fact, turn out to be the
gauge coupling, which will obviously deviate from a constant to the extent that $D\theta^a_j = 0$ is
violated. Such deviations would, in any case, be negligible at energies below those associated
with the epoch of homogenization of the universe. Moreover, to secure correct relative signs
in the Yang-Mills sector, the signature of $g_{IJ}$ must be $\delta_{IJ}$ (all pluses), attainable with a ‘Wick
rotation’ of $\theta_0$ to $i\theta_0$, hereafter re-defined as $\theta_0$, by which all our results remain formally intact.

We are now ready to write down the Einstein-Hilbert action in $M^4$ with the curvature tensor
(22). Dropping the imaginary surface term and the gauge-fixing terms (which, however, would
be of importance for quantization as well as for certain topological effects discussed in the last
section), we end up with three contributions. The first one is precisely the Einstein-Hilbert
action (1) written for $R^{ab}$ (which includes a comological constant, as mentioned), obviously
associated with the gravitational sector. The second one gives the Yang-Mills action (with the
correct relative sign when considered as a source for gravity) of the unbroken
$SU(2) \times U(1)$ gauge
theory. The third contribution describes a set of novel gravity-induced electroweak processes.
We will omit the details of this calculation because they can be fully recovered from the case
with spontaneous breaking of the gauge symmetry, to which we now turn.

5 Spontaneous breaking of the gauge symmetry

Let us recall a fundamental symmetry of the Einstein-Hilbert action (1), by which the latter
remains invariant if the connection $\omega_{ab}$ is changed by any 1-form $\lambda$ to

$$\omega^{ab} \rightarrow \omega^{ab} + \lambda g^{ab}. \quad (28)$$

If $\lambda$ is real, as we will assume, this special kind of a projective transformation is known as
Einstein’s $\lambda$ transformation [1]. Obviously, the new connection violates metricity as

$$Dg_{ab} = -2\lambda g_{ab}, \quad (29)$$

which apparently is the reason why $\lambda$-transformations have rarely been used in recent times
(see, however, [2]). Thus uncovered, a degeneracy of the vacuum exists in the sense that the
same Einstein-Hilbert action (1) describes not only the ‘symmetric vacuum’ $M^4$ corresponding
to $\lambda = 0$, but equally well any other vacuum $M^4_\lambda$ with any $\lambda \neq 0$. Remarkably enough,
if we repeat the construction of the previous section not in $M^4$ but rather in $M^4_\lambda$, then the
gauge symmetry of the Yang-Mills sector is broken. Conforming to standard terminology, we
may characterize the breaking as *spontaneous*. The magnitude of this breaking is directly proportional to the scale of $\lambda$ which, at the outset, has nothing to do with the Planck scale.

To see explicitly how this mechanism works, we will repeat the steps leading to (22), but now we must complexify starting with the connection (28) for the vacuum $M^{\lambda}_{\lambda}$. In other words, instead of $\tilde{\omega}^{ab}$, we must employ the connection

$$\omega^{ab} = \omega^{ab} + \lambda g^{ab} + iK_{(\lambda)}^{ab}.$$  (30)

In this expression we have anticipated that the contorsion will change (while retaining its antisymmetry), as it indeed does described by

$$K_{(\lambda);b}^a \wedge e^b = -\gamma^I_J \theta^a_I \left( F^J + \lambda \wedge A^J \right).$$  (31)

This result, which obviously replaces the previous expression (26), follows from the new set of Cartan’s equations which replace (21,22) as

$$\tilde{T}^{a}_{(\lambda)} = T^a + \lambda \wedge e^a + \gamma^I_J \theta^b_J A^J \wedge K_{(\lambda);b}^a - \gamma^I_J \theta^a_I \lambda \wedge A^J + i\mathcal{O}^a$$  (32)

$$\tilde{R}^{ab} = R^{ab} + g^{ab} D\lambda - K_{(\lambda);c}^a \wedge K_{(\lambda);b}^c + \frac{1}{2} C^{abc}_{\cdots} \tilde{T}^{\cdots}_{(\lambda)} + \frac{1}{4} \gamma^I_J \gamma^J_L \theta^i_I \theta^j_L K_{(\lambda)}^a \wedge A^J \wedge A^L + i\mathcal{O}^{ab}$$  (33)

where we have defined

$$\mathcal{O}^a \equiv \gamma^I_J \left( D\theta^a_I \wedge A^J + \theta^a_I F^J + \theta^a_I \lambda \wedge A^J \right) + K_{(\lambda);b}^a \wedge e^b,$$  (34)

$$\mathcal{O}^{ab} \equiv DK_{(\lambda)}^{ab} - \frac{1}{4} \gamma^I_J \left( \theta^a_I e^b + \theta^b_I e^a \right) \wedge A^J.$$  (35)

The above expressions obviously reduce to (21,22) at the $\lambda = 0$ limit. The most prominent difference between the two sets is the already noted presence of the term proportional to $\lambda$ in (31). It is precisely this term which will give masses to the gauge bosons, as we will see in the next section. The components of $K_{(\lambda)}^{ab}$ can be calculated explicitly from (31), if we also take into account the specific value for the $\gamma^I_J$ introduced in (20). The result of this calculation is

$$2K_{(\lambda);abc} = \cos \vartheta_W C_{abc} + \sin \vartheta_W S_{abc},$$  (36)

$$C_{abc} = \theta_{0a} \left( F^{0}_{bc} + \lambda_b A^0_c - \lambda_c A^0_b \right) + \theta_{0b} \left( F^{0}_{ca} + \lambda_c A^0_a - \lambda_a A^0_c \right) - \theta_{0c} \left( F^{0}_{ab} + \lambda_a A^0_b - \lambda_b A^0_a \right),$$  (37)

$$S_{abc} = \theta_{ja} \left( F^{j}_{bc} + \lambda_b A^j_c - \lambda_c A^j_b \right) + \theta_{jb} \left( F^{j}_{ca} + \lambda_c A^j_a - \lambda_a A^j_c \right) - \theta_{jc} \left( F^{j}_{ab} + \lambda_a A^j_b - \lambda_b A^j_a \right),$$  (38)

wherefrom we may recover, if needed, the explicit values for the components of $K^{ab}$ by setting $\lambda = 0$. We thus have at our disposal everything we need to find explicitly the classical action of the theory.
Again dropping the imaginary surface and gauge-fixing terms (essentially the $O$ terms in (32,33)), we obtain the Einstein-Hilbert action

$$I_{(\lambda)EH} \equiv \frac{1}{32\pi G} \int_{M^4_\lambda} \tilde{R}_{(\lambda)ab} \wedge \star (e^a \wedge e^b)$$

expressed as

$$I_{(\lambda) EH} = \int_{M^4_\lambda} \mathcal{L}_{EH} + \mathcal{L}_{YM} + \mathcal{L}_{gw} + \mathcal{L}_{mass}. \quad (40)$$

The first contribution in (40) describes the gravitational sector with

$$\mathcal{L}_{EH} = \frac{1}{32\pi G} R_{ab} \wedge \star (e^a \wedge e^b), \quad (41)$$

reproducing precisely the action (1). The second contribution gives the Yang-Mills action for the $SU(2) \times U(1)$ gauge field with

$$\mathcal{L}_{YM} = \left(\frac{\cos^2 \vartheta W}{2g^2} F^{0ab} F_{0b} + \frac{\sin^2 \vartheta W}{2g^2} F^{jab} F_{jb} \right) e^0 \wedge e^1 \wedge e^2 \wedge e^3, \quad (42)$$

and the indicated couplings. The third contribution is too long (to be in good taste and context), so it will be described only formally as

$$\mathcal{L}_{gw} = \frac{1}{g^2} \left( \frac{1}{2L^2} S^{ab}_{\alpha \beta} \theta^\alpha \theta^\beta A^I_a A^I_b + \frac{1}{4} T^{ab}_{\alpha \beta} \theta^\alpha \theta^\beta F^{I}_{ac} F_{b}^c \right) + \frac{1}{gL\sqrt{32\pi G}} U^{K}_{\alpha \beta \gamma} \theta^\alpha \theta^\beta \theta^\gamma A^I_a F^J_{bc}. \quad (43)$$

It predicts a novel kind of electroweak processes induced by gravity (we will call them gravitoweak) which are in fact of two generic types. The first one involves couplings of the order of $g^2$ and $L^2$ (the Planck scale cancels out in view of (27)). The second type, described by the last term in (43), is at a much higher scale, roughly given by the geometric mean of $L$ (the electroweak breaking scale) and the Planck scale, namely at about $10^{10}$ GeV. In general, the couplings (and scattering amplitudes) specified by the matrices $S, T, U$ are fully calculable, but the explicit tree-level predictions which are thus available will be meaningful only after the $A^I$’s have been rotated to physical states. This rotation will be effected by $\Delta^I_J$, the diagonalizer of the mass matrix $M_{IJ}$, the latter specifying the last contribution in (40) as

$$\mathcal{L}_{mass} = \frac{1}{g^2 L^2} M_{IJ}(\lambda, \theta, \vartheta W) A^I_a A^J_a e^0 \wedge e^1 \wedge e^2 \wedge e^3. \quad (44)$$

$M_{IJ}$ is fully expressible in terms of the parameters $\lambda, \theta, \vartheta W$ (which fully determine the symmetry-breaking pattern, as we will see in the next section. The contribution (44) can be viewed as a gravitational substitute for the Higgs sector. The latter is hereby abrogated because we have no option on the spontaneous breaking of the gauge symmetry presented in this section: it is imposed by the geometry and the only existing freedom is in the choice of the scale and orientation of $\lambda$. 

12
6 Relevance to the electroweak interaction

With the generic mixing introduced by (20), the mass matrix as defined by (44) turns out to be

\[
M_{00} = \theta_a^0 \theta_b^0 (\lambda^2 g_{ab}) - \lambda_a \lambda_b \cos^2 \vartheta_W, \\
M_{0i} = M_{i0} = -\theta_a^0 \theta_i^0 \lambda_a \lambda_b \sin \vartheta_W \cos \vartheta_W, \\
M_{ij} = \theta_i^a \theta_j^b (\lambda^2 g_{ab} - \lambda_a \lambda_b) \sin^2 \vartheta_W. 
\] (45)

More explicit values will obviously depend on the particular choice of \( \lambda \). For example, we can orient \( \lambda \) (without loss of generality) to lie in the \((\theta_0, \theta_1)\) plane as

\[
\lambda_i = \frac{g L}{\sqrt{32\pi G}} (l_0 \theta_a^0 + l_1 \theta_1^0), 
\] (46)

so, with \( L \) and the ratio \( l_1/l_0 \) (or \( l_W/l \)) as two independent real constant parameters, we have

\[
\lambda^2 = \lambda^a \lambda_a = \frac{l_0^2 + l_1^2}{L^2} \equiv \frac{l_0^2}{L^2}, \\
l_0^2 \equiv l_0^2 \sin^2 \vartheta_W + l_1^2 \cos^2 \vartheta_W. 
\] (47)

From (45, 46), we find for the mass matrix and its diagonalizer

\[
M_{IJ} = \begin{pmatrix}
l_1^2 \cos^2 \vartheta_W & -l_0 l_1 \sin \vartheta_W \cos \vartheta_W & 0 & 0 \\
-l_0 l_1 \sin \vartheta_W \cos \vartheta_W & l_0^2 \sin^2 \vartheta_W & 0 & 0 \\
0 & 0 & l_2^2 \sin^2 \vartheta_W & 0 \\
0 & 0 & 0 & l_2^2 \sin^2 \vartheta_W 
\end{pmatrix}, 
\] (48)

\[
\Delta_I^J = \begin{pmatrix}
l_0/l_W & (l_1/l_W) \cos \vartheta_W & 0 & 0 \\
-(l_1/l_W) \cos \vartheta_W & (l_0/l_W) \sin \vartheta_W & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1 
\end{pmatrix}. 
\] (49)

Taking also into account the scale from (44), we obtain the mass spectrum

\[
m_0^2 = 0, \quad m_1^2 = \frac{l_0^2}{g^2 L^2} \equiv m_Z^2, \quad m_2^2 = m_3^2 = \frac{l_2^2}{g^2 L^2} \sin^2 \vartheta_W \equiv m_W^2, 
\] (50)

obviously acquired by the physical bosons

\[
B = \frac{l_0}{l_W} \sin \vartheta_W A^0 + \frac{l_1}{l_W} \cos \vartheta_W A^1, \quad Z = -\frac{l_1}{l_W} \cos \vartheta_W A^0 + \frac{l_0}{l_W} \sin \vartheta_W A^1, \quad W^+ = A^2, \quad W^- = A^3. 
\] (51)

We can trade \( L \) for \( m_W \) (or \( m_Z \)) and \( l_W/l \) (or \( l_1/l_0 \)) for the positive parameter

\[
\rho \equiv \frac{m_Z^2 \cos^2 \vartheta_W}{m_W^2} = \frac{l_0^2 \sin^2 \vartheta_W + l_1^2 \cos^2 \vartheta_W}{(l_0^2 + l_1^2) \tan^2 \vartheta_W} = \left( \frac{l_W}{l \tan \vartheta_W} \right)^2, 
\] (52)
so that, following the $\rho = 1$ choice (see, however, following remarks), we recover precisely the mass spectrum of the standard electroweak model. One may now proceed to fully determine the gravitoweak sector given by (43), which however is beyond our present scope and will be examined elsewhere.

It may have been already noticed that our construction carries certain aspects of a Kaluza-Klein setting [2]. These could be profitably exploited in spite of obvious fundamental differences in dimensionality or the fact that $\mathcal{P}$ is not necessarily an isometry group (although it is expected to be so asymptotically, e.g., in models which attain the homogenization mentioned earlier). In any case, there seems to be no obstruction in applying existing methods to obtain results such as the quantization of the electric charge and the computability of $g, \vartheta_W, \rho$ [10]. As related to that, we may already demonstrate in the present context an elegant formulation of the generalized minimal coupling prescription, which is precisely carried by $\tilde{e}_a$ (the dual of $\tilde{e}^a$) and automatically assigns the correct charges to the electroweak gauge bosons. For an explicit expression let us assume that the imaginary part of $\tilde{e}^a$ in (15) is small so that, to lowest order in $\hbar$, we have

$$\tilde{e}_a = (\delta_b^a + i\gamma^I_J A_I^J) e_b,$$

where we should actually rotate the $A_I$ to the physical states (51). To better recognize this result, one may convert to holonomic co-ordinates and disregard the non-abelian contributions. Then, (53) further reduces to the electromagnetic minimal coupling prescription $\partial_a - ie A_a$, with the electric charge $e$ emerging through the identification of the relevant charge operator.

7 Discussion and conclusions

Shortly after its discovery as a possible space-time symmetry, the de Sitter group has been repeatedly advocated as an option superior compared to the Poincaré group [7]. Here we have seen that the complexified gauging of the de Sitter group, albeit with its algebra expressed in the particular basis (7) for the isomorphic $\mathcal{P}$, has uncovered a unified description of the gravitational and electroweak interactions (cf. also comments at the beginning of section 2). The construction is fixed by less than five adjustable parameters (at best only the two scales) among the $G, g, m_W, \vartheta_W, \rho$. We have seen that the known association of gravity with the generators of the Lorentz group $\mathcal{L}$ has been retained, while the $\Pi_a$ have been uniquely associated with the electroweak interaction. Our findings offer new theoretical perspectives and predictions which apparently could upgrade general relativity as well as the standard electroweak model. The
implied programme is obviously vast so we will only list what appear at the moment as its major aspects, also to be thought of as testing grounds on its worthiness.

To the extent that one can isolate gravity, Einstein’s theory remains unchanged except for one major issue, namely the dramatic reduction of the immense body of all possible global topologies \[ \mathcal{P}/\mathcal{L} \] to just \( \mathcal{P}/\mathcal{L} \). Einstein’s equations (possibly with the addition of appropriate external sources) will still have to be solved, and even the asymptotic-flatness boundary conditions may certainly retain most (but not all) of their practical applicability, e.g., for an isolated local source. Clearly however, solutions with topology consistent to that of \( \mathcal{P}/\mathcal{L} \) (which includes several models with Bianchi-type symmetry \[ \mathcal{P}/\mathcal{L} \]) would be of special interest within the present context. Transcending the gravitational sector, novel effects are expected such as those related to violations of metricity and to alterations of the general relativistic junction conditions on the surface of appropriately chosen sources. The former should be observable through tests for violation of the principle of equivalence, expected to be positive if their accuracy exceeds one part in \( 10^{17} \) (the ratio of the electroweak to the Planck scale). The second type uncovers a new generation of Aharonov-Bohm-like effects and a novel insight for the Blackett effect, as already discussed in a related context \[ [12] \].

Our findings also provide a unification of the electroweak sector when considered by itself. The association of the group generators with a space-time vierbein (which could offer an elegant explanation of the origin and uniqueness of the \( SU(2) \times U(1) \) choice) is in no way contradicted by any rigorous result in the Coleman-Mandula theorem, but it does supply a counter-example to some fundamental assumptions therein \[ [8] \]. The gravitoweak sector involves, as we have seen, two types of gravity-induced electroweak interactions with couplings at a low (essentially electroweak) scale and at an intermediate scale of about \( 10^{10} Gev \).

On major open issues, we note that the modifications in the commutation relations \[ (9) \], e.g., the association of the \( [\Pi_j, \Pi_0] \) with a Lorentz boost along \( j \), will be clarified once the representations of \( \mathcal{P} \) (with the related kinematics etc.) have been worked out in full detail. We also note that the predicted absence of a Higgs sector, congruous as it may be with current doubts, will expose the electroweak sector to contamination by the renormalization and quantization impasses for gravity, possibly made worse by unitarity problems. Contributions to the latter could also come from the Goldstone-boson analogue expected from the symmetry breaking: although Einstein’s \( \lambda \)-transformation is an invariance of the classical vacuum, it does contribute with surface and gauge terms, as seen from \[ (33) \].

Such contributions can also be expected from terms in \[ (32, 33) \] (e.g. the \( O \)'s) which were
dropped in the classical action \((14)\). On the other hand, the same terms seem to convey a welth of topological configurations and novel effects, including the classical ones mentioned above. Additional (and possibly exploitable) novel aspects may be itemized as follows: the simple group \(\mathcal{P}\) replacing the (essentially responsible for the mentioned impasses) \(\mathcal{P}_0\); novel insight on the CPT theorem and chiral behavior, or on spin and space-time parallelizability, through their association (together with the electroweak bosons) with the globally defined \(\theta^a\)-frame \([13]\); the uncovered gravitoweak processes and novel effects; the rigorous foundation of the minimal coupling prescription offered by \(\tilde{c}_a\); the expected predictability and protection of the \(g, \vartheta_W, \rho = 1\) values; and the hereby anticipated explanation of Dirac’s large-number conjecture. We finally note the rather unexpected (and certainly peculiar) synthesis of several major theoretical aspects: grand-unification with \(\mathcal{P}\) (albeit geometrical); superseding of the Coleman-Mandula theorem (without supersymmetry); spontaneous breaking of the gauge symmetry (without Higgs fields); complexification (without complex or twistor structures \([1]\)); and Kaluza-Klein aspects (without extra dimensions!). Amusing as it may be, this assortment may also supply options to be further pursued.
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