Walking Technicolor and the $Zb\bar{b}$ Vertex

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Abstract

A slowly-running technicolor coupling will affect the size of non-oblique corrections to the $Zb\bar{b}$ vertex from extended technicolor dynamics. We show that while “walking technicolor” reduces the magnitude of the corrections, they generally remain large enough to be seen at LEP.
1 Introduction

The origin of the diverse masses and mixings of the quarks and leptons remains a mystery; most puzzling is the origin of the top quark’s large mass. In technicolor models [1], the large top mass is thought to arise from extended technicolor (ETC) dynamics at relatively low energy scales. Recent work [3] has shown that the dynamics responsible for generating the large top quark mass in extended technicolor models will produce potentially large “non-oblique” [4] effects at the $Zb\bar{b}$ vertex. In this note, we discuss what happens to these effects if the technicolor beta function is assumed to walk [5]. We show that the size of the signal is reduced but that it remains quite visible at LEP for many models.

2 ETC’s Effect on the $Zb\bar{b}$ vertex

We begin by reviewing the results of ref. [3]. Consider a model in which $m_t$ is generated by the exchange of a weak-singlet extended technicolor gauge boson of mass $M_{ETC}$ coupling with strength $g_{ETC}$ to the current

$$\xi \bar{\psi}_L^i \gamma_\mu T^i_L + \xi' \bar{t}_R \gamma_\mu U^k_R$$

where $U$ and $D$ are technifermions, $i$ and $k$ are weak and technicolor indices, and the coefficients $\xi$ and $\xi'$ are extended technicolor Clebsch's expected to be of order one. At energies below $M_{ETC}$, ETC gauge boson exchange may be approximated by local four-fermion operators. For example, $m_t$ arises from an operator coupling the left- and right-handed pieces of the current in Eq. (2.2)

$$- \xi \xi' \frac{g_{ETC}^2}{M_{ETC}^2} \left( \bar{\psi}_L^i \gamma_\mu T^i_L \right) \left( \bar{U}_R^w \gamma_\mu t_R \right) + h.c.$$  

When this is Fierzed into a product of technicolor singlet densities, it is seen to generate a mass for the top quark after the technifermions’ chiral

\[\text{mass} = \frac{g^2_{ETC} M_{ETC}}{2} \langle \ldots \rangle + \ldots \]
symmetry breaking. We can use the rules of naive dimensional analysis to estimate the size of $m_t$ generated by Eq. (2.2). Assuming, for simplicity, that there is only doublet of technifermions and that technicolor respects an $SU(2)_L \times SU(2)_R$ chiral symmetry (so that the technipion decay constant, $F$, is $v \approx 250$ GeV) we have

$$m_t = \xi \xi' \frac{g_{ETC}^2}{M_{ETC}^2} (\bar{U}U) \approx \xi \xi' \frac{g_{ETC}^2}{M_{ETC}^2} (4\pi v^3). \quad (2.3)$$

In the same language, we can also show that the extended technicolor boson responsible for producing $m_t$ affects the $Zb\bar{b}$ vertex. Consider the four-fermion operator arising purely from the left-handed part of the current (2.2) – the only part containing $b$ quarks.

$$-\xi^2 \frac{g_{ETC}^2}{M_{ETC}^2} (\bar{\psi}_L^i \gamma^\mu T^i_L) (\bar{T}_L^j w^j \gamma^\mu \psi_L^j) . \quad (2.4)$$

When Fierzed into a product of technicolor singlet currents, this includes

$$-\frac{\xi^2}{2} \frac{g_{ETC}^2}{M_{ETC}^2} (\bar{\psi}_L \gamma^\mu \tau^a \psi_L) (\bar{T}_L \gamma^a \tau^a T_L) , \quad (2.5)$$

where the $\tau^a$ are weak isospin Pauli matrices. As shown in [3] this alters the $Z$-boson’s tree-level coupling to left-handed bottom quarks $g_L = \frac{\epsilon}{s_c c_b} (-\frac{1}{2} + \frac{1}{3}s^2_b)$ by

$$\delta g_L = -\frac{\xi^2}{2} \frac{g_{ETC}^2 v^2}{M_{ETC}^2 s_c c_b} (I_3) \quad (2.6a)$$

$$= \frac{1}{4} \xi' \frac{m_t}{4\xi' 4\pi v} \frac{e}{s_c c_b} \quad (2.6b)$$

Here eq. (2.6b) follows from applying eq. (2.3) to eq. (2.6a).

### 3 Measuring the Effect at LEP

We now consider how best to experimentally measure the shift in $g_L$ caused by extended technicolor. Altering the $Zb\bar{b}$ coupling will affect the decay

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3 The Fierzed form of (2.4) also includes operators that are products of weak-singlet left-handed currents; these will not affect the $Zb\bar{b}$ coupling.
width of the $Z$ boson into $b$ quarks. In addition, there are flavor universal (oblique) corrections to the width, coming from both technicolor and extended technicolor interactions. At one loop, the decay width of the $Z$ is of the form

$$\Gamma_{b}^{\text{corr.}} \equiv \Gamma(Z \rightarrow b\bar{b}) = (1 + \Delta \Gamma)(\Gamma_{b} + \delta \Gamma_{b}) \quad (3.1)$$

where $\Gamma_{b}$ is the tree-level decay width, $\Delta \Gamma$ represents the oblique corrections and $\delta \Gamma_{b}$ represents the non-oblique (flavor-dependent) corrections. We will refer to the non-oblique effect of (2.6a) on the decay width as $\delta \Gamma_{b}^{\text{ETC}}$. Ratios of $Z$ decay widths into different final states are particularly sensitive to such effects; we suggest studying the ratio\footnote{This is simply related to the ratio $\Gamma_{b}/\Gamma_{h}$ discussed in [3]:}

$$\frac{\Gamma_{b}}{\Gamma_{h\neq b}} = \frac{\Gamma_{b}/\Gamma_{h}}{1 - \Gamma_{b}/\Gamma_{h}} \quad (3.2)$$

This is accessible to the current LEP experiments.

This particular ratio has several features to recommend it. First, since it is a ratio of hadronic widths, the leading QCD corrections cancel in the limit of small quark masses. Second, eq. (3.1) implies that the fractional change in this particular ratio is approximately the fractional shift in $\Gamma_{b}$:

$$\Delta R \equiv \frac{\delta (\Gamma_{b}/\Gamma_{h\neq b})}{(\Gamma_{b}/\Gamma_{h\neq b})} \approx \frac{\delta \Gamma_{b}}{\Gamma_{b}}. \quad (3.3)$$

This is easily related to the change in $g_{L}$ that extended technicolor effects cause:

$$\Delta_{R}^{\text{ETC}} \approx \frac{\delta \Gamma_{b}^{\text{ETC}}}{\Gamma_{b}} \approx \frac{2g_{L}\delta g_{L}}{g_{L}^{2} + g_{R}^{2}}. \quad (3.4)$$

For our benchmark ETC model with two technifermion flavors,

$$\Delta_{R}^{\text{ETC}} \approx -3.7\% \cdot \xi^{2} \cdot \left(\frac{m_{t}}{100 \text{ GeV}}\right). \quad (3.5)$$

There is also a fractional shift in $\Gamma_{b}$ arising from 1-loop diagrams involving longitudinal $W$-boson exchange and internal top quarks. This has already been calculated\footnote{This is simply related to the ratio $\Gamma_{b}/\Gamma_{h}$ discussed in [3]:} it is of order -0.7% (-2.5%) for $m_{t} = 100$ (200) GeV.
This source of corrections to $\Gamma_b$ (which we shall call $\Delta_R^W$) occurs both in the standard model (where it is the dominant non-oblique correction to $\Gamma_b$) and in extended technicolor models. Note that both $\Delta_R^W$ and $\Delta_{ETC}^R$ act to decrease $\delta \Gamma_b / \delta \Gamma_{h \neq b}$. Then in comparing the size of $\Delta_R$ in the standard model with that in ETC models, we are comparing $\Delta_R^W$ to $\Delta_R^W + \Delta_{ETC}^R$. The expected LEP precision of 2.5% for measurement of $\Delta_R$ [10] should suffice to distinguish them.

4 Walking Technicolor

The dimensional estimates employed in section 2 are self-consistent so long as the extended technicolor interactions may be treated as a small perturbation on the technicolor dynamics, i.e. so long as $g_{ETC}^2 v^2 / M_{ETC}^2 < 1$ and there is no fine-tuning [7]. Note that the rules of naive dimensional analysis do not require that $M_{ETC}$ be large, only that $g_{ETC}^2 v^2 / M_{ETC}^2$ (or equivalently $m_t / 4\pi v$) be small. However, these estimates (in particular, the relationship [2.3]) between $\left(g_{ETC}^2 v^2 / M_{ETC}^2\right)$ and $\left(m_t / 4\pi v\right)$ are typically modified in “walking technicolor” models [5] where there is an enhancement of operators of the form (2.4) due to a large anomalous dimension of the technifermion mass operator.

Let us define what is meant by a “walking” technicolor coupling. The beta function for an $SU(N)$ technicolor force has the same form as that for QCD. At leading order it is simply

$$\beta(\alpha_{TC}) = -b \alpha_{TC}^2 + {\cal O}(\alpha_{TC}^3) \quad (4.1)$$

where (for technifermions in the fundamental representation) $b$ is related to the technicolor group and the number of technifermion flavors ($n_f$) by

$$b = \frac{1}{2\pi} \left( \frac{11}{3} N - \frac{2}{3} n_f \right) \quad (4.2)$$

For our benchmark model with two technifermion flavors, setting $N = 2$ yields $b = \frac{2}{\pi}$. Adding more flavors of technifermions to the model decreases $b$ so the TC coupling falls off relatively slowly with increasing momentum scale (it “walks”).

The expected effect of a walking technicolor coupling on $\Delta_{ETC}^R$ can be outlined fairly briefly. When the technicolor coupling becomes strong and
the technifermion condensate $\langle \bar{T}T \rangle$ forms, a dynamical mass $\Sigma(p)$ is also generated for the technifermions. As discussed in ref. [5], having $\alpha(p)$ fall off slowly with increasing $p$ causes $\Sigma(p)$ to decrease more slowly with rising $p$ than it would in a ‘running’ TC theory. Since the technifermion condensate

$$\langle \bar{T}T \rangle \sim \int_0^{M_{ETC}^2} dk^2 \Sigma(k),$$

(4.3)

enhancing $\Sigma$ increases $\langle \bar{T}T \rangle$. According to eq. (2.3) this means that a walking TC coupling increases $m_t$ for a given ETC scale $M_{ETC}$. The factor $\left(\frac{\alpha_T^2}{M_{ETC}^2}\right)$ appearing in our expression (2.6a) for $\delta g_L$ is therefore smaller than $\left(\frac{m_t}{4\pi v}\right)$ in an ETC model with walking TC. Thus, the expected size of $\Delta_{ETC}^{ETC}$ is reduced.

5 Numerical Results

To illustrate the effect of walking technicolor on the size of $\Delta_{ETC}^{ETC}$, we have studied coupled ladder-approximation Dyson-Schwinger equations [5] for the dynamical technifermion and top quark masses, $\Sigma(p)$ and $m_t(p)$. The gap equations always possess a chiral symmetry preserving solution with $m_t$ and $\Sigma$ both equal to zero. Our interest is in finding chiral symmetry violating solutions with both $m_t$ and $\Sigma$ non-zero. We have focused on $SU(N+1)_{ETC} \rightarrow SU(N)_{TC}$ models with a full family of technifermions.

In Landau gauge and after the angular integrations have been performed, we approximate the gap equations by \[11\]

$$\Sigma(p) = C_{ETC}^2 \int_0^\infty \frac{3\alpha_{TC}(M[p,k])}{\pi M[p^2,k^2]} \frac{\Sigma(k)}{k^2 + \Sigma^2(k)} k^2 dk^2$$

$$+ c_1 \int_0^\infty \frac{3\alpha_{TC}(M[p,k,M_{ETC}])}{\pi M[p^2,k^2,M_{ETC}^2]} \frac{\Sigma(k)}{k^2 + \Sigma^2(k)} k^2 dk^2$$

$$+ c_2 \int_0^\infty \frac{3\alpha_{TC}(M[p,k,M_{ETC}])}{\pi M[p^2,k^2,M_{ETC}^2]} \frac{m_t}{k^2 + m_t^2} k^2 dk^2$$

$$+ C_{QCD}^2 \int_0^\infty \frac{3\alpha_{QCD}(M[p,k,M_{ETC}])}{\pi M[p^2,k^2,M_{ETC}^2]} \frac{\Sigma(k)}{k^2 + \Sigma^2(k)} k^2 dk^2$$

(5.1)
\[ m_t = c_3 \int_0^\infty \frac{3\alpha_{TC}(M[p, k, M_{ETC}])}{\pi M[p^2, k^2, M_{ETC}^2]} \frac{m_t}{k^2 + m_t^2} k^2 dk^2 \]

\[ + c_4 \int_0^\infty \frac{3\alpha_{TC}(M[p, k, M_{ETC}])}{\pi M[p^2, k^2, M_{ETC}^2]} \frac{\Sigma(k)}{k^2 + \Sigma(k)^2} k^2 dk^2 \]

\[ + C_2^{QCD} \int_0^\infty \frac{3\alpha_{QCD}(M[p, k, M_{ETC}])}{\pi M[p^2, k^2, M_{ETC}^2]} \frac{\Sigma(k)}{k^2 + \Sigma^2(k)} k^2 dk^2 \] (5.2)

where \( M[x, y] \) signifies the greater of \( x \) and \( y \), \( C_2^{TC} = \frac{N^2 - 1}{2N} \), \( C_2^{QCD} = \frac{4}{3} \) and the coefficients \( c_i \) are

\[ c_1 = \frac{1}{2N(N + 1)} \quad c_2 = \frac{1}{2} \quad c_3 = \frac{N}{2(N + 1)} \quad c_4 = \frac{N}{2} \] (5.3)

We have ignored the mass splittings between the extended technicolor gauge bosons and used \( M_{ETC} \) to stand for the masses of all the heavy extended technicolor gauge bosons in the gap equations.

To integrate the gap equations, we use the following 1-loop form for the running of the technicolor coupling:

\[ \alpha_{TC}(p) = \begin{cases} 2\alpha_{TC}^c & p < \Lambda_c \\ \frac{2\alpha_{TC}^c}{1 + b\alpha_{TC}^c \ln \left( \frac{p^2}{\Lambda_c^2} \right)} & p \geq \Lambda_c \end{cases} \] (5.4)

where \( \alpha_{TC}^c \equiv \pi/3C_{2}^{TC} \) is the 'critical' value for chiral symmetry breaking, and \( \Lambda_c \) is to be determined by requiring that the model reproduce the correct electroweak gauge boson masses. At energies below the extended technicolor scale \( M_{ETC} \), the beta-function parameter \( b \) is given by eq. (4.2); above \( M_{ETC} \), it is

\[ b_{ETC} = \frac{1}{2\pi} \left( \frac{11}{3}(N + 1) - \frac{2}{3}n_f \right) \]

\[ = b + \frac{1}{6\pi} \] (5.5)

We set the scale of the chiral symmetry breaking and the dynamical masses by using the calculated \( \Sigma(p) \) to compute the technipion decay con-
In one-family technicolor models, \( f \approx 125 \text{ GeV} \).

For a given value of \( b \), we vary the ETC scale, \( M_{ETC} \), until we obtain a chiral symmetry violating solution to the gap equations with a particular value of \( m_t \). Knowing \( M_{ETC} \) allows us to use equations (2.6a) and (3.4) to find the value of \( \Delta^{ETC}_R \) associated with our initial values of \( b \) and \( m_t \). In applying (2.6a) we recall that \( g_{ETC}^2 \equiv 4\pi\alpha_{TC}(M_{ETC}) \) and we set \( \xi = \xi' = \frac{1}{\sqrt{2}} \) as is appropriate for our ETC models.

Our numerical results for an \( SU(3)_{ETC} \rightarrow SU(2)_{TC} \) model are shown in fig. 1. Here, \( \Delta^{ETC}_R \) is plotted as a function of \( A \equiv (bo_{ETC}^{-1})^{-1} \) for several values of the top quark mass. Similar results for an \( SU(5)_{ETC} \rightarrow SU(4)_{TC} \) model are plotted in fig. 2. In the small-A ("running") regime of the plots, \( \Delta^{ETC}_R \) is of order a few percent, in good agreement with the estimates from naive dimensional analysis. As one moves towards the large-A ("walking") regime, the size of the effect decreases as we expected. Note that the decrease is very gradual. For the large top quark masses shown, \( \Delta^{ETC}_R \) generally remains big enough to be visible at LEP even if the TC coupling runs very slowly.

Fig. 3 compares the variation of \( \Delta^{ETC}_R \) with \( A \) found for several \( SU(N + 1)_{ETC} \rightarrow SU(N)_{TC} \) models with \( m_t \) set to 140 GeV. Note that the size of \( \Delta^{ETC}_R \) grows with \( N \) and that \( \Delta^{ETC}_R \) depends much less strongly on \( A \) as \( N \) increases.

6 Conclusions

In this note, we have discussed the degree to which extended technicolor effects reduce the \( Zb\bar{b} \) coupling in models with a walking technicolor beta-function. We chose the variable \( \Delta_R \) (fractional shift in the ratio of \( Z \) hadronic widths \( \Gamma_b/\Gamma_{b\neq b} \)) as most suitable for measurement of the shift in the coupling. We indicated why one expects models with a slowly running technicolor beta function to have a smaller \( \Delta^{ETC}_R \) than models with a running TC beta function. Then we presented a numerical analysis of dynamical chiral symmetry breaking to illustrate how strongly the technicolor beta function affects the size of \( \Delta^{ETC}_R \). Our results show that while \( \Delta^{ETC}_R \) is reduced in
walking technicolor models, it generally remains large enough to be visible at LEP.

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Figure 1: Plot of $\Delta_{ETC}^{R}$ as a function of walking parameter $A$ in an $SU(3)_{ETC} \to SU(2)_{TC}$ model. The dotted (solid, dashed) curve is for a top quark mass of 100 (140, 180) GeV.

Figure 2: Plot of $\Delta_{ETC}^{R}$ as a function of walking parameter $A$ in an $SU(5)_{ETC} \to SU(4)_{TC}$ model. The dotted (solid, dashed) curve is for a top quark mass of 100 (140, 180) GeV.

Figure 3: Plot of $\Delta_{ETC}^{R}$ as a function of walking parameter $A$ for $m_t = 140$ GeV in several $SU(N + 1)_{ETC} \to SU(N)_{TC}$ models.
