Including precision clocks into space-based net as gravitational antennas

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Abstract. Here we propose to use a precision clock in a space-based ultra-precise clock network to register sources of low-frequency gravitational waves of cosmic origin in the range of 10^-3 ÷ 0.1 Hz. We also show that the method of comparing clocks at inland and intercontinental distances (very long baseline interferometry), originally developed for radio astronomy and geodesy, can be used as a prototype method for recording gravitational waves. Estimates of the measurement accuracy are given. An analyse of precise clocks possibilities for experimental estimates for rotation parameter of Gödel universe and GW recordings is offered, which in particular opens up the prospect of registering circularly polarized gravitational waves. Some new problems of small time intervals registration from general relativity, thermodynamics and quantum mechanics points of view are discussed.

1. Introduction
Registration of gravitational waves of cosmic nature is not only one of the priority fundamental directions of experimental physics and astrophysics, namely the search for a stochastic gravitational background of gravitational waves and dark matter. Such type of registration also has practical implications, for example, in deep space navigation. Instrumentally, the problem of long-term detection of low-frequency signals requires increased reliability of measuring devices, the use of new methods of data collection and algorithms for their processing based on general relativity and quantum mechanics, improvement of operating systems for processing large databases, algorithms for identifying weak signals against the background of various measuring positions of reference sources, operating coordinates of the spacecraft and time intervals.

Unlike SRT spacetime, which can be conventionally represented as a rigid 3D frame structure with preservation of the Euclidean geometry, and with synchronized clocks in the frame nodes [1], the GRT spacetime is considered as a 4D elastic “fabric”. This fabric with an unsynchronized clock can warp and the warp is depending on the condition of the massive body producing the fabric. The degree of deformation of tension (compression), twisting or warping is determined by the movement of the test mass in the vicinity of the central massive body. The stiffness of the "fabric" is large enough, which follows from the Einstein equation [2], which connects the measure of the local curvature of spacetime with the measure of the energy-momentum density \( T^{\mu \nu} \). The indices \( \mu \) and \( \nu \) run through the values of all coordinates in 4D spacetime. In the linear approximation, the equation for the deviation of spacetime from flat looks like
\[ \left(-\frac{1}{c^2}\frac{\partial}{\partial t} + \nabla^2 \right) \hat{h}^{\mu\nu} = k T^{\mu\nu} , \]

where the coefficient \( k \approx \frac{G}{c^4} \times 10^{-42} \) can be interpreted as the rigidity of the vacuum [3]. The huge value of "stiffness" excludes the propagation of longitudinal gravitational waves, leaving transverse or circular waves. For a stationary or slowly moving source with mass density \( \rho \) (taking into account the relation \( E=mc^2 \)), the time derivatives disappear, and we can write the equation

\[ \nabla^2 \hat{h}^{\mu\nu} = \frac{G}{c^2} \rho , \]

repeating Poisson's equation in outward form. Far from the source of gravity (in empty space), equation (1) transforms into the equation

\[ \left(-\frac{1}{c^2}\frac{\partial}{\partial t} + \nabla^2 \right) \hat{h}^{\mu\nu} = 0 . \]

The expression is associated with the equation for the electromagnetic waves in Maxwell’s theory. Here the equation describes gravitational shear compression/expansion waves of space-time with deformation amplitude \( \hat{h}^{\mu\nu} \).

Accelerating masses distort spacetime, generating gravitational waves (GW). When registering GW of a cosmic nature, the experimenters faced a difficult task of registering spatial deformations due to the small deviation of spacetime from the plainness. Suffice to say that the vibration amplitude of the test body position under action of GW from distant sources was less than \( 10^{-4} \) sizes of the atomic nucleus. But in 2015, the problem was solved by the LIGO and VIRGO interferometers using [4, 5].

2. Influence of variable gravitational potential on the rate of time passage

It is known that far from the GW source, the field approach can be used to determine the effect of gravitational disturbances on measuring instruments. We will assume that the propagation of the gravitational potential \( \phi = \frac{GD}{r^4} \) (\( D \) is the quadrupole moment of a mass system) leads not only to a change in the positions of small masses, but also to a change in their velocities and the rate of time flow [6]. The quadrupole approximation is substantiated by astrophysical observations, such as the change in the orbital period of the binary pulsar PSR 1913 + 16 (accuracy 0.16%) [7], and the analysis of the results of X-ray observations during the cataclysm of binary stars [8].

The experiments of Pound - Rebka and Pound - Snyders [9, 10] confirmed a variation in the frequency of the electromagnetic signal produced by a change in the gravitational potential \( \phi \). The experimental dependence can be interpreted as a change in recording of the time pace due to \( \phi \) altering.

\[ \frac{\Delta y}{y} \approx \frac{g h}{c^2} , \]

or that \( T = f(\phi) \). Recently, researchers using NASA's Chandra X-ray Observatory have confirmed this effect outside the solar system. A gravitational redshift was observed in a two-star system (4U 1916-053) at a distance of 29 kly. (3.218688·10^{17} m) [11].

The gravitational redshift is significant in the technology for measuring the time delays of the space navigational systems. Because of the effect, clocks on the Earth's surface run slower than clocks on a satellite, as the potential of the gravitational field in whose orbit is lower, requiring the introduction of a correction factor when an object is locked to coordinates on the surface.

It was shown that the GW carries the variable potential of the gravitational field. Therefore, by comparing the time rates recorded by the reference clock (far from the GW source) – \( T_0 \) and the clock on the spacecraft exposed to GW (mobile clock) – \( T_m \), one can determine the characteristics of the GW (amplitude, frequency, polarization, direction to the source). Comparison of information from several space-based mobile clocks and laboratory clocks, firstly, will determine the characteristics of the GW:
polarization and direction to the source and, secondly, to develop technologies to increase the effective stability of mobile clocks. The latter will increase the sensitivity of the measuring system to the amplitude of the recorded GW.

Several challenges arise. This is the choice of stable time sources (natural and artificial), the development of a method for measuring the variation of the parameter associated with the time part of the signal, under the action of an alternating gravitational field, and algorithms for data collection and processing. Consequently, with an increase in the precision of mobile clocks, it will become possible to detect gravitational effects by measuring the change in the duty cycle of the pulse train of the mobile clocks. Experimental work on improving atomic clocks for their use in deep space exploration programs is being carried out on a wide scale [12 - 19]. The best error of modern laboratory clocks based on electronic transitions now is about $3.7 \cdot 10^{-18}$ (~ 1 s in 13 billion years) [13]. The current error of the clock on the spacecraft is ~ $2.3 \cdot 10^{-14} - 3.2 \cdot 10^{-15}$ [12 - 14, 16 - 18].

A promising way is the practical implementation of nuclear-optical clocks, the stability of which is ensured by the stability of nuclear transitions [19]. It is believed that the accuracy of clocks as a tool increases by an order of magnitude over a decade.

3. Breathing methods for time delays recording in astrophysical experiments

As an effect of the GW action, distortions of spatial intervals arise, as well as compression/expansion of the transit time intervals of light from sources to recorders. This effect served as the basis for the development of methods for registering GWs based on the time delays of electromagnetic (EM) signals.

One of the methods for determining changes in the parameters of EM signals due to the action of gravitational waves on them is the technique of recording microhertz GWs (sources are massive objects with masses of the order of a million solar masses during a collapse that lasts for many years) using a series of pulses received from an initially regular a sequence of EM pulses from a pulsar is known as the Pulsar Timing method.

The method is based on the assumption that GW contributes to a change in the phase of the signal and makes the sequence irregular. Although the shapes of individual pulses from a given pulsar vary considerably, the shape of the mean profile is fairly stable. Therefore, the time characterizing the moment of arrival of the pulsar pulse is stable, and the change in the time of arrival of the signal can be recorded with an estimate of the error of the pulsars as ~ $10^{-19}$ per hour [20]. The idea and development of the method belongs to Russian and American astrophysicists [21-24].

Another one is the method of comparing the course of distant atomic clocks, which underlies international timekeeping, global positioning and verification of fundamental physics. This trend is the development of work on the application of very long baseline radio interferometry (VLBI) in astronomy and geodesy [25 - 28].

Fiber optic links allow comparisons of the most accurate optical clocks without degradation over inland distances of up to thousands of kilometers, but intercontinental comparisons remain limited in satellite transmission methods. However, VLBI can overcome this limit and compare distant clocks by observing extragalactic radio sources. Special mobile VLBI stations have been developed that use broadband detection and demonstrate a comparison of two optical clocks located in Italy and Japan, separated by a distance of 9000 km. This system demonstrates performance superior to satellite methods and may pave the way for future long-term stable international clock comparisons [28].

The ensemble of the Event Horizon Telescope network (EHT) of an Earth-sized interferometer was used to measure the size of the emission regions of two supermassive black holes with the largest visible event horizons: SgrA * at the center of the Milky Way and M87 at the center of Virgo A. In both cases, the dimensions correspond to the predicted silhouette caused by the strong lensing of light by the black hole. The addition of key devices in the millimeter and submillimeter wavelength range at high altitudes has opened up the possibility of visualizing such features and determining the dynamic evolution of black hole accretion. The EHT project includes theoretical and simulation research that poses questions rooted in effects at the edge of a black hole that can soon be answered through observation. By integrating existing telescopes with new systems, EHT is leveraging significant global databases to
create a ground breaking instrument with the highest possible angular resolution from the Earth's surface [29, 30].

A comparison of the fundamental features of the Pulsar Time Method and VLBI is given in [31]. A few words about the sensitivity of the Doppler method for detecting GW [32, 33]. Since the propagation of the gravitational potential $\varphi = GD\dot{h}^3$ leads not only to a change in the positions of small masses, but also to a change in their velocities, the method of Doppler tracking of a spacecraft or a spacecraft complex is used to register GW. With spacetime variations $h$ at a level of $10^{-17} \div 10^{-15}$ and a frequency of $\nu \sim 10^{-4}$ Hz, changes in the spacecraft velocity will be $\Delta \nu \sim 10^{-7} \div 10^{-5}$ m/s at a spacecraft-Earth distance of 1 AU.

4. High-precision space-based clock network

A space-based spacecraft network with precision sensors of displacements or clocks located on them should provide more reliable and accurate characteristics due to the absence of technogenic and geophysical noise compared to ground-based ones [34 - 37]. The implemented projects GRACE [38], GRAIL [39], CHAMP [40], the long-prepared and widely discussed ambitious project LISA [41], the recently considered domestic space project of the orbital gradiometer [42] are analogs of the proposed network.

Consider a network of precision clocks (for example, atomic), which are located at the nodes of a huge space system (see [35, 36] Fig. 1). The diagram shows the location of a clock in a network located in low-earth orbit around the sun. The space operating system of the clocks, covering a large spatial area in Earth's orbit, will form the chronometric space network (CSN). Below we present clock accuracy estimates for event logging and describe possible measurement results and their significance.

For the spacecraft-Earth distance $\Delta L \sim 10^6$ m, the GW amplitude $h \sim 10^{-19} \div 10^{-17}$, the GW frequency $\omega_{\mathrm{gw}} \sim 0.63 \div 63$ rad/s ($\nu \sim 10^{-1} \div 10^1$ Hz), we have $\lambda_\varphi = 3 \cdot (10^7 \div 10^9)$ m; therefore $\Delta \nu = h\nu L \sim 3 \cdot (10^{-11} \div 10^{-7})$ m/s; which gives $\Delta \omega \nu = 10^{-19} \div 10^{-15}$. With a laser clock error of $10^{-19}$, the required registration time is about 1 year.

The registered change in the sequence of pulses received from the reference clock will signal the amplitude and frequency of the GW. The delay of signals from different network nodes can be associated with the direction of the gravitational wave front and its polarization. Obtaining other data (for example, the position of the apparatus with the clock and its relative movement) can be used to check some conclusions of general relativity (for example, Shapiro's delays). A functioning CSN will also play the role of a gravitational antenna. As an initial step towards the creation of a CSN, one can limit us to the presence of central precision clocks and ground-based repeaters on satellites. In this scheme (Doppler tracking), the ground station first sends an electromagnetic signal $\nu (t)$, which is compared with a high-quality frequency standard $\nu_0$, then records the mirror response from the spacecraft reproduced by the repeater, $\nu (t - T)$, where $T$ is time travel from a ground station to a spacecraft. Thereafter, the frequency offsets of the transmitted and reference signals are compared. The relative frequency difference $\delta \nu (t) = [\nu(t - T) - \nu (t)]/\nu_0$, ($\nu_0$ is the central frequency of the microwave signal) will be zero if there is no noise, systematic drift, and the GW signal with a wavelength $\lambda$ is comparable with the dimensions of the detector. Otherwise, the dependence $\delta \nu (t)$ has a three-pulse character (which is a distinctive feature of the method and serves as the basis for signal filtering). According to already known algorithms for calculating the cross-correlation of time residuals from several spacecraft, the result is compared with possible gravitational signals to confirm their registration [33].

We have to point out the difficulties of project implementation:
1. Obtaining long-term and stable readings of the clock on board the spacecraft under observance of cryogenic conditions of measurements
2. The presence of new types of noise (for example, plasma noise in the signal path)
3. The cost of creating and maintaining a CSN can exceed the cost of creating and operating ground-based GW observatories or an array of radio telescopes by tens of times.
5. The principle of registering slow rotations with a clock and some new problems

The spin of a particle can change its behavior after the passage of the GW. Let we suspend a particle in a chamber in a laboratory and measure the speed and direction of its rotation when a gravitational wave packet is sweeping; then measure it again, but after the wave has passed. The difference in the behavior of the spin of the particle would reveal a different kind of memory about the wave, which can be done with the help of post-temporal rotation sensors [34, 42, 43].

The possibility of registering the transient torsion caused by the rotation of a massive body of mass M with angular velocity, which creates a gravimagnetic field with the Lense - Thirring metric, has already been discussed. The metric has the form

\[ ds^2 = \left( 1 - \frac{2GM}{c^2r} \right) c^2dt^2 - \left( 1 + \frac{2GM}{c^2r} \right) \left( dx^2 + dy^2 + dz^2 \right) - 2\frac{2GS}{c^3} \frac{y}{r^3} cdt dx + 2\frac{2GS}{c^3} \frac{x}{r^3} cdt dy, \ldots \]

(5)

where the \( \Omega_z \) axis is directed along the angular velocity, \( x, y, z \) are the current coordinates, \( r \) is the distance to the observer, \( S = \boldsymbol{\Omega} \) is the angular momentum of the body. To solve an experimental problem: determining the angular velocity of rotation, some schemes were proposed in which centrifugal forces were created. In turn, these forces caused longitudinal stresses in a cylindrical solid sample perpendicular to the \( \Omega \) axis. The stresses can be measured using a composite magnetostrictor-SQUID sensor [44].

However, there is another possibility here. Since the rotation of the clock relative to the center of the Earth under the action of a gravitational wave is relatively slow, it is possible to use the formula of the Lorentzian time dilation when the clock moves on a satellite with a speed of \( \Delta v = \frac{GGM}{c^2L} \). An estimation is \( \Delta t \approx \frac{1}{h_{GW}} (\Delta v/c)^{\gamma-10^{-36}s} \), with a measurement time of 10 s.

At the same time, the delay increases as the radius of rotation increases. Therefore, we will consider an experiment involving cosmological distances. Since the time of I. Newton, the cosmological principle has been known, according to which in modern wording an observer located at any point in the Universe and at any moment of time "sees" the Universe as homogeneous and isotropic. In other style, the average values of mass density and pressure obtained after observing large, including galaxies and their clusters, spatial regions (about 250 million light years) and long time intervals, but small in comparison with the size and age of the Universe, are the same in all directions and are the same over time. The statement implies that at any point in space all physical laws are fulfilled in the same way. However, the nature of the principle is more philosophical than physical and has no logical foundation. The reasons for its application were the absence of any significant observed deviations from homogeneity and isotropy in the indicated cosmological limits. After analyzing part of the data of the Planck project (mission), ESA (Planck Mission) determined that the observed deviations of anisotropy are statistically significant and can no longer be ignored [45]. In addition, various other astronomical observations, including the discovery in June 2021 of the structure, the Giant Arc, extending approximately 9.2 billion light years and consists of galaxies, galactic clusters, gas, and dust, question the validity of the cosmological principle [46].

Along with the well-known cosmological models that satisfy the requirements of relativistic invariance and have their own privileged time axis, there are other models in which there is no isotropy. These include K. Gödel model. It is believed that at every point in the universe there is some semblance of rotation. This rotation is described by the Gödel metric, which is an exact solution of the Einstein field equations, in which the energy-momentum tensor contains two terms, the first of which represents the density of matter of a uniform distribution of rotating dust particles (which can represent galaxies), and the second one is associated with a nonzero cosmological constant

\[ ds^2 = \frac{1}{2\omega^2} \left[ -\left( cdt + e^s dy \right)^2 + dx^2 + \frac{1}{2} e^{2s} dy^2 + dz^2 \right], \ldots \]

(6)

where \( \omega \) is a nonzero real constant, which turns out to be the angular velocity of the surrounding dust grains around the \( y \)-axis, measured by a "non-rotating" observer located on one of the dust grains. Rotation is reflected in the pace of time, and therefore can be measured with a watch. "Non-rotation" means that the observer does not feel centrifugal forces, but in this coordinate system he will actually
rotate around an axis parallel to the y-axis. As can be seen, the dust grains remain at constant values of x, y, and z. Their density in this coordinate system increases with increasing x, but their density in their own reference frames is the same everywhere.

If measurements are carried out in the Earth's orbit around the Sun $L \approx 1.5 \times 10^{11}$ m, then with an experiment duration of one year, the time delay will be $\Delta \tau \approx 0.8 \times 10^{15} \mu s$, or the limiting value of the constant $\omega$ in Gödel metric, which can be experimentally recorded $\omega \approx 1.2 \times 10^{-16}$ m/s. The measurement error can be reduced by the duration (three to five years) of the mission.

For further reasoning, we will use the following definition of time using a clock “[Time] is considered to be measured by a clock (an ideal periodic process) - [an object] of insignificant spatial extent. The time of the event at the point is defined as the time shown on the clock simultaneously with the event” [47], i.e. we have to deal with the readings of the clock, and not with an abstract definition of time that is not observable. In other words, the clock only measures the time interval between two events, not the absolute time of each of them. This instrumental definition becomes necessary, for example, when we discuss the time it takes for an electron to tunnel through a potential barrier. Another example from classical mechanics - how to establish that two particles began to fall from a certain height (Galileo's experiment) at the same time?

In recent publications [48, 49], a question was discussed that can be formulated in a simplified form as follows. What remains of a black hole after it evaporates?

Theoretical analysis pointed out possible residual phenomena manifested in "homeopathic" deviations of the state of test masses, from the ideal three-dimensional crystal with synchronized clocks assumed in flat space-time conditions, three spatial displacements, three residual angular rotations and three Lorentz transformations (Poincaré group). Those symmetries of the special theory of relativity (SRT) as asymptotes in the absence of a gravitational field can manifest themselves as one of the sets (groups) of boundary conditions of the equations of the general theory of relativity (GR).

In other words, the vacuum of GR with its symmetries makes possible the existence of a memory matrix that stores this information in the Universe, even after the disappearance of the black hole. These are the shifts of the Poincaré group, which connect the points infinitely distant from the gravitating body. The discovered abundance of symmetries, known as the BMS group, gives empty space-time tremendous latent complexity. The localization of such places in space-time can later serve as a reference point for navigation, similar to observing an overgrown ravine or abandoned mines on topological maps.

Estimates utilizing a toy model with Lense-Thirring metric give us the upper optimistic limit of the residual linear deformation at 5% of the GW already recorded value. Therefore, a multiple series of measurements of time intervals, as well as the position of the end mirrors of the LIGO (VIRGO) interferometer, should indicate the residual displacement.

On the other hand, on the assumption that SRT is not the only asymptote of GR when the gravitational field disappears, it is necessary to discuss the problem of mismatching reference IFRs and introduce the Lorentz transformation into the communication protocol, which connects their reference frames [50].

In conclusion, we would like to note some aspects of measuring time intervals, which will affect the accuracy of timekeeping, and, consequently, the accuracy of recording gravitational waves. These aspects are not directly related to measurement technology and engineering problems, but rather indicate possible connections between the theory of gravity and quantum mechanics.

The first aspect. The point of view of A. Einstein [47], stated above, indicates the clock as a dynamic system that cannot be a stationary object, and the absence of global time. That. it is necessary to introduce the true (proper) time of the observer $d\tau^2 L := -dt^2/c^2$, defined in a curved space-time along its trajectory of motion $\Gamma$ as $\tau = \int_{\Gamma} \sqrt{-g_{\mu\nu}dx^\mu dx^\nu}, \quad \{\mu,\nu\} = \{0,1,2,3\}$. The insignificance of the spatial extent of clocks (for example, atomic, and even better time crystals) can be interpreted as a quantum dot with many internal degrees of freedom, which moves in a curved spacetime. To measure the time interval, it is necessary to introduce such measures that change covariantly in the group of time transformations acting on the used clock system. Such descriptions of quantum clocks have been
developed in the context of quantum metrology. In addition, it is necessary to take into account the influence of the gravitational field on the position of energy levels for a given accuracy of measuring time intervals. Time can then be determined through measurements of quantum systems that serve as clocks. Now we can raise the question of the influence of the GW on the difference in clock readings from a quantum point of view. At the same time, covariant observables in time make it possible to rigorously formulate the time-energy uncertainty relation [51, 52].

The second aspect is how to synchronize the quantum (in terms of size) clocks to a fraction of a tick? What is the impact of the measurement process? It is shown that there are universal limits for the accuracy of measuring time intervals. According to thermodynamics, a watch is a machine that converts one type of energy into another (gravitational into mechanical, mechanical into electrical, nuclear into heat, etc.). That is, any thermodynamically irreversible process can be used as a clock, for example, simply by observing the equilibration process as an indicator of time. Since energy conversion is accompanied by losses, the accuracy of a quantum clock is directly proportional to the created entropy [53]. It was experimentally shown measuring time requires more than just a certain increase in an entropy [54]. It was realized that the marginal relationship between the accuracy of the clock ticks and the entropy produced could be a universal characteristic of timekeeping. The result can make a difference for technologies that depend on accurate timing. A conceptual diagram of a clock for thermodynamic accuracy analysis is shown in fig.1.

![Conceptual clock model](image)

**Figure 1.** Conceptual clock model from the point of view of thermodynamics in the analysis of the accuracy of measuring time intervals.

For experimental research, physicists have created a simple optomechanical "nano-clock" consisting of a membrane whose deflection is controlled by an electric field. The membrane is driven by heat input to it and electrical work used to measure the displacement. During operation, the watch converts the received energy into a membrane deflection signal and releases excess heat, producing entropy. Thus, by measuring this entropy, we can determine the amount of energy consumed. The implication is that clocks, like other machines such as motors and computers, are subject to a fundamental performance limit determined by the laws of thermodynamics. Although the system was nanoscale, it was too large for quantum mechanical analysis, and physically completely different from the previously studied quantum clock. However, the researchers found a relationship between precision and entropy in the classical clock model, the same as in the quantum clock. The relationship between the clock accuracy and the change in entropy in the experiment is also consistent with the theoretical model of the researchers, confirming that this pattern can be preserved for both classical and quantum models.

Although the researchers have experimentally tested the relationship between entropy and accuracy for only one particular implementation of a classical clock, they argue that the similarity to a quantum result suggests this may be true for any clock. The researchers also propose to redefine the "optimal" watch as one that has the highest possible accuracy with the least entropy dissipation (consistent with the discovered relationship between accuracy and entropy), regardless of the physical details of the watch. The result was the establishment of the dependence of the accuracy of measuring the oscillation period
\[ N_{osc} = \frac{(t_{tick})^2}{(\Delta t_{tick})^2} \]

from the entropy produced during the oscillation period – \( \Delta S_{tick} \)

\[ N_{osc} = \frac{2\pi^2}{k_B} \frac{T_C}{T_N} \Delta S_{tick}, \]

where \( k_B \) is the Boltzmann constant; \( T_C \) is the effective temperature of the amplifier in the refrigerator; \( T_N \) is the effective temperature determined by the electronic noise (Johnson noise) during data recording.

Since all clocks use an increase in entropy in one form or another to quantify the passage of time, a better understanding of the measurement of time can provide a new understanding of how heat, work and the arrow of time are related, which will affect the determination of the accuracy limits of the temporal parameters of GW.

The third aspect. One can also raise questions about the minimum distance between two quantum clocks, due to the reaction of space-time to their presence and limiting the joint measurability of time along the nearest world lines [55]; on the influence of the watch masses on the limitation of the measurement time. Among them there is also the question of the quantum entanglement of quantum particles - clocks, since clocks, being, in the final analysis, quantum systems, make it possible to apply the principle of superposition to their states, and, consequently, to the superposition of proper times [56].

The authors of [55] argue that, at least in principle, the proposed scheme can also be used to detect time dilation effect caused by GW. However, at present, the detection of a gravitational wave according to the proposed scheme is still a difficult task.

For example, the precession of the spin of particles when a magnetic field is applied can be used as a quantum clock. Here we can turn on (turn off) the clock by applying a non-zero (zero) magnetic field. In this case, the accuracy of the two separate clocks will also not be limited by the fit, since we can turn off the two clocks when we measure them. However, in this case, the magnetic field must instead be precisely controlled to the level of the required circuit accuracy, which will be much more difficult than achieving the required accuracy with two naturally defined energy eigenstates, as in a conventional quantum clock circuit.

It is noted that an entangled pair of non-degenerate quantum clocks can be used as a specialized detector to accurately measure the difference in proper times experienced by each constituent quantum clock. In this case, several experiments are considered in which the acceleration effects are mutually compensated. The proposed scheme can be used in precision testing of relativistic time dilation.

In [56], two clocks are considered prepared in localized impulse wave packets moving in Minkowski space. Let clock B move with a wave packet with an average localized momentum \( \bar{p}_B \), while clock A moves in a superposition of wave packets with a localized average momentum \( \bar{p}_A \) and \( \bar{p'}_A \). It is demonstrated that, on average, quantum clocks A measure time dilation relative to clock B in accordance with STR special theory of relativity, when the state of their center of mass is localized in momentum space. However, when the state of their center of mass is in a superposition of such states with a localized momentum, then for clocks A there additionally takes place the effect of quantum time dilation relative to clock B. This quantum time dilation is due to their non-classical state of motion and depends on the parameters that determine the superposition of pulses. An estimate of the order of magnitude of this effect indicates that quantum time dilation can be observed using modern technology.

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