Abstract—Parallel kinematic manipulators (PKMs) are increasingly used in a wide range of industrial applications due to the characteristics of high accuracy and compact structure. However, most of the existing PKMs are structured with heavy actuators and high stiffness. In this respect, this article proposes a simple, yet effective, parallel manipulator that distinguishes itself through the following basis. First, underactuation: it employs only a single motor and a driving cable to actuate its three legs. Second, novel foot location: it uses a smart shape memory alloy clutch-based driving system (SCBDS), which catches/releases the driving cable, thus, making possible the robot underactuation. Finally, adjustable compliance: its double compliant joints on each limb with a stiffness-adjustable section, which renders a safe human–robotic interaction. To support and predict the performance of this underactuated compliant manipulator, a novel kinetostatic model was developed by considering the generalized internal loads (i.e., force and moment) in three compliant limbs and the external loads on the upper platform. Finally, based on the physical prototype, a set of experiments were conducted to validate the model proposed in this article. It was found that the proposed kinetostatic model can be validated with the average deviations of 1.8% in position and 2.8% in orientation, respectively. Furthermore, the workspace of the system (e.g., discrete and continuous workspace) was studied when different actuating strategies were employed, thus, emphasizing the advantages and the limitations of this novel system.

Index Terms—Compliant parallel kinematic manipulator (PKM), kinetostatic modeling, underactuated mechanism.

I. INTRODUCTION

In recent years, parallel kinematic manipulators (PKMs) have been increasingly used for rapid positioning and high-precision manufacturing, due to their lightweight, compact structure, high stiffness, and accuracy [1], [2]. However, most of the PKMs are structured with rigid links and joints, requiring high manufacturing tolerance and complex control algorithms [3], [4]. Also, the stiffness of the parallel kinematic system, composed of rigid parts, can be regarded as infinite, leading to the potential risk to the operators during the human–robot interaction [5]–[7].

Up to now, significant efforts have been made for developing novel compliant mechanisms (i.e., continuum robots [8]–[10] and compliant PKMs [11], [12]) to satisfy the requirements of industrial and healthcare applications [5], [13]. For example, a 6-DoF PKM was developed [14], which is composed of three pairs of notched flexure-based limbs. However, the workspace of the system was restricted by the small strokes of the 2-DoF compliant joints. In order to increase the workspace, a new concept of compliant PKM was proposed to use a fully compliant rod in the PKM [15], [16], which can provide larger stroke and better scalability. Furthermore, pneumatic actuators were utilized to construct a fully compliant PKM [17], which could achieve a 6-DoF motion by controlling the length of six limbs. Unlike the PKMs with rigid joints, these compliant robots are fully compliant resulting in a reduction of their stiffness, so significant deviations could be generated if a load is applied at the end-effector. Therefore, an approach for building PKM with an appropriate stiffness and workspace needs to be developed for safe human–robot interaction operations.

Also relevant for PKMs, different actuation methods have been studied over the past decades: conventional (e.g., electrical motors [10], hydraulics [18], and pneumatics actuators [3]) and unconventional (e.g., electromagnetic actuators [19] and piezoelectric materials [20]) approaches. Generally speaking, integrating a single actuator for each limb can make the PKM system precise and all the limb lengths vary simultaneously. However, in some applications, this could be less a priority in comparison with an underactuated system, which would rely on the successive actuation of each limb. One example is the solar panel of satellites, where the panel orientation is repeatedly adjusted in a fixed interval to point it to the light/sun. By using our new design, one motor can control the multi-DoF of end-effector with the successive actuation of each limb to reduce the overall weight of the system. For most of these systems, clutch mechanisms (e.g., electromagnetic) are adopted to switch the states of limbs (i.e., between active and passive), enabling the reduction of the number of actuators for a conventional fully actuated...
system [21]. However, as a new material with a high power/weight ratio, shape memory alloy (SMA) has a great potential to miniaturize the size and weight of the clutch systems. For example, SMA wires were used in a bat-like flapping robots [22], [23], where the wing shapes were controlled by the length of the SMA wires by adjusting the current; also, SMA-wire-based rotational joints [24]–[26] were developed to demonstrate the possibility for adopting this smart material as actuators, where the rotation of joints was directly controlled by adjusting the strain of the SMA wires. To the best knowledge of authors, no research has demonstrated using SMA actuated mechanism for minimizing the number of actuators and the weight of PKM.

Furthermore, in order to precisely control PKMs, many studies have been focused on the kinematic modeling of conventional rigid-link structures. However, if compliant rods are integrated as limbs or joints, the kinematic behavior of the overall system needs to be further studied. Up to now, different compliant rod-based mechanisms have been developed and the corresponding kinematic models were established with the consideration of the flexible rod behaviors. For example, a continuum robot was studied [27], in which the flexible rods were regarded as a compliant joint that bends as a pure circular arc for establishing the static equation of the system. A concentric-tube continuum robot [28] was developed and studied for minimally invasive surgery, where the model was established by considering the geometrically exact behavior of each tube (i.e., pure bending) under the given external loads. Furthermore, the model of a concentric-tube robot that considers the bending and rotation of each tube were derived [29] to improve the kinematic accuracy. Besides, long compliant rods were utilized as limbs to connect the base and upper platform and the Cosserat rod theory was adopted to build the overall model of the system [16], [30]. Up to now, to the best knowledge of authors, most of the research works adopted the rigid-body-model method to predict the kinematics behavior of compliant joints (i.e., joints were regarded as pure bending with constant curvature), while the Cosserat theory has not been utilized (i.e., joints were regarded as rods with varying curvature, including bending and torsion) in compliant parallel mechanism to improve the modeling accuracy.

To address the aforementioned challenges on modeling the conventional PKMs (e.g., excessive actuators and insecure human–robot interaction), a novel underactuated 6-DoF PKM, which employs SMA-based clutches to change the state of the limbs (i.e., between the actively actuated and passively locked) to enable the reduction of the number of actuators in the system (i.e., one motor was utilized to actuate the manipulator) is proposed in this article. Then, the conventional rigid-based joints (i.e., special and universal) of the PKM were replaced by length-adjustable compliant rods to alter the compliance of the system, thus, enabling improved human–robot interaction. After that, a new kinetostatic model of the proposed underactuated parallel system was established, which considers the overall loads in the system to obtain the position and orientation of the upper platform. At last, after experimentally validating the proposed models, the work volume of the system was calculated and plotted for evaluating the overall performances of the underactuated PKM.

II. DESIGN OF A NOVEL UNDERACTUATED PKM

In this section, a novel underactuated PKM is introduced (see Fig. 1), which can achieve multiple-DoF output (i.e., 6-DoF) with a reduced number of actuators for three actuation limbs. For achieving this, the design enables the three free low joints to be driven along tracks with a continuous running cable. The position of the leg along the track depends on the movement of a clutch connected to the lower joint by catching the actuating cable that runs in a closed loop around the base platform. This might resemble the concept of a ski lift: a driving cable is caught by clutches; thus, enabling the transportation of the people along a predefined track. With such a generic concept, it is possible to use one motor and a continuous cable to control the movements of multi legs on a predefined line (i.e., track/guide) of a parallel manipulator.

To materialize this idea, a novel SMA-wire-based clutch was designed to engage/disengage with a continuous (closed loop) cable, which is used to change the state (i.e., move or stop) of each leg of PKM on a placing line [see Fig. 1(a)]. In this article, we define the assembly of a linear guide and its SMA clutch as one placing line [see Fig. 1(e)]. Specifically, the continuous cable is circumferentially arranged around a triangular table/base with guides provided by four fixed pulleys and one movable pulley, which is used to drive the independent motions of three clutches with only one motor. A spring [see Fig. 1(a)] is connected to the movable pulley to maintain the tension of the cable. The motor is mounted on the movable pulley to drive the cable.

Then, a 6-DoF fully elastic rod-based parallel mechanism [see Fig. 1(b)] was developed with its feet/lower joints [i.e., A, B, and C in Fig. 1(b)] connected to the proposed SMA clutch-based driving system so that the PKM is set up. With
the movement of the feet (A, B, and C) along the placing lines, the pose of the PKM platform will change. Furthermore, in order to actively adjust the performance (i.e., stiffness and stability) of the proposed mechanism, we propose here for the leg design of a combination of infinitely stiff and compliant sections [i.e., detail structure shown in Fig. 1(f)]. Thus, a new compliant limb with length-adjustable rods [see Fig. 1(d)] was developed to construct the stiffness-adjustable parallel mechanism. The three moveable feet, which are mounted on the placing lines, respectively, are driven along with linear guides by the closed-loop cable to control the configuration of the system [see Fig. 1(e)].

Finally, the overall parallel mechanism, which is composed of the smart shape memory alloy clutch-based driving system (SCBDS) and parallel mechanism, is obtained [see Fig. 1(e)]. Specifically, the ends of the three limbs [i.e., A, B, and C in Fig. 1(d)] are mounted on the corresponding three SMA clutches, respectively [i.e., A, B, and C in Fig. 1(a)], where the positions of three placing lines can be independently controlled (i.e., engaging or disengaging with the driving cable) to change the configuration of the upper platform of the manipulator.

As a result, the configuration of the proposed parallel mechanism [i.e., position and configuration of the upper platform in Fig. 1(e)] can be planned in sets of a discrete and continuous succession of poses by utilizing the different working algorithms of three placing lines in SCBDS, aspect which will be discussed later. Some of the poses can be achieved by continuously moving all the three placing lines, while for other poses only one or two placing lines can be moved at a time. Thus, by employing the aforementioned actuation method of the placing lines, a set of poses of the upper platform can be obtained to perform the given tasks. In addition, it has to be mentioned that the workspace of the upper platform (on which an end-effector/workpiece could be set) is largely decided by the arrangement of three placing lines and characteristics of compliant joints, which is necessary to be ascertained by establishing the kinematic model of the system.

A. Design of the Compliant Underactuated PKM

The generic concept of the 6-DoF underactuated parallel mechanism (i.e., three active DoFs and three passive DoFs at upper platform), which uses one motor to drive the entire system and a novel SMA-wire-based clutch that catches/releases the limbs from the driving cable, is presented in this article. For each of the limbs, the stiffness-adjustable compliant joints were employed to connect the upper platform and the SCBDS. For this, a rigid bar with a central hole, through which the elastic rod runs, is employed to separate the full length of the leg of the manipulator into two compliant universal joints and a rigid link [see Fig. 1(f)]. These compliant joints provide the rotation motion between the adjacent parts, which can be regarded as the conventional rigid universal joints for calculating the kinematic. As the motion behavior (i.e., bending angle, rotation center, and maximum stress) of the compliant joint is determined by its length, for this 6-DoF parallel mechanism, a structure of the leg that allows the adjustment of the length of the compliant joint has been adopted. Thus, the rigid bar on the long elastic rod (i.e., two ends of elastic rod are connected with base and upper platform, respectively) is combined with several small rigid segments, which allows easy adjustment of the length of the rigid part in the limb, and further to vary the length of the compliant joints at the ends of each limb [i.e., \( l_0 \)–\( l_6 \) in Fig. 2(c)]. Since the overall length of the limb is designed as fixed in this article, the length of the compliant joint can be varied by increasing/decreasing the number of the rigid bar segments.

By combining the proposed parallel mechanism and SCBDS, the overall underactuated PKM was developed (see Fig. 2), where three compliant limbs [i.e., detailed structure and working principle shown as in Fig. 2(b)] were utilized to connect the placing table and upper platform. In order to conveniently adjust the length of compliant joints for actively changing the performance of the system (i.e., stiffness and stability), the length adjustable limb was designed [see Fig. 2(c)]. Thus, six compliant joints made of super-elastic Nitinol, which could be considered kinematically equivalent as universal joints, provide the rotation motion for the upper platform. In addition, a feedback system (i.e., linear encoder) was employed on each placing line for the closed-loop control.

B. Design of the SMA Clutch-Based Driving System

The key element to enable the SMA clutch-based driving system is a clutch that is “ON” when the limb needs to catch the driving cable to move along the guide and “OFF” when the limb needs to retain a stable position. Based on this working principle (see Fig. 1), a novel design of SMA-wire-based clutch
was proposed, enabling to move and retain each limb in an independent way with only one actuator (i.e., motor).

For the use of an underactuated PKM, the miniaturized clutch needs to fulfill the following main requirements: 1) appropriate working stability to perform reliable clamping and releasing of the placing line to the base platform; 2) be small and light for optimizing the size and weight of the driving system; 3) provide two independent working functions, i.e., engage/disengage with the moving cable to move/stop the limbs separately. Based on the above-mentioned requirements, an SMA wire, which can adjust its length once its temperature is varied, was selected as the actuator of the clutch.

The working principle of the clutch is shown as follows. When the SMA is heated up by applying a current, the movable platform of the clutch (green part—Fig. 3, numbered 2) engages with the clutch base numbered 4 and clamps the cable numbered 3, so the whole clutch moves with the driving cable [see Fig. 3(b)]; when the SMA wire is cooled down by switching OFF the current and it stretched back to its original length, the moveable platform numbered 2 engages with the base numbered 4 and disengages with the cable numbered 3 in Fig. 3(a). In order to lock the SMA clutch once it reaches the desired position, saw tooth tracks on the base and moveable platform were employed to ensure that a foot keeps its location after being placed along with the linear guide. As the sawtooth was adopted to increase the locating stability, the resolution of placing lines, $\Delta_l$, was decided by the pitch of sawtooth.

Based on this concept of the SMA-wire-based clutch, the entire SMA clutch-based driving system for the underactuated PKM was developed (see Fig. 4). In the design example of the SCBDS, three identical placing lines are located at three sides of a triangle base plate, while the motions are actuated by a single motor located at the center of the triangle plate. In this design, the SMA-wire-based clutch can move together with the driving cable when the clutch is engaged with the cable (clutch is ON), while it stops when the clutch is engaged with the table (clutch is OFF). For achieving better positioning performance of placing lines, the position sensors (i.e., linear encoder and magnetic scale mounted on SMA clutch and system base, respectively, Fig. 4) are included for the closed-loop control of the position of the clutch.

In this section, the novel parallel mechanism and SCBDS were combined to construct an underactuated compliant robot for safe human–robot interaction with a reduced number of actuators. Due to the introduction of compliant joints in the proposed parallel mechanism, the kinetostatic model of the compliant joints, as well as for the overall underactuated system, is required to make the system work. Thus, in Section III, the kinetostatic modeling of the system, which reflects the relationship between the pose of the upper platform and the position of the clutch, is conducted.

III. MODELING OF THE UNDERACTUATED PKM

In this section, we generalized the modeling of the proposed 6-DoF underactuated parallel mechanism (i.e., three active and passive degrees of freedom, respectively), which is composed of three compliant length-adjustable limbs and one SCBDS. Unlike the conventional approaches (e.g., Pseudobody Method and Cosserat-Theory Method [32], [33]), a new kinetostatic model of the proposed manipulator was proposed. This method considers the input loads (i.e., external loads on the upper platform) and the position of the clutch-based driving system to predict the configuration of the system. Also, the new model proposed in this article aims to describe for the first time the
The modeling of a single compliant limb is divided into two stages. First, the development of the static equation of a single compliant joint. Second, the derivation of the static equation of the entire limb that has two compliant joints (to link it with upper and lower platforms of the PKM).

Due to the identical structure design, all the limbs share the same motion characteristics. First, one limb (i.e., limb \( i \) in Fig. 6) was selected as an example for building the kinetostatic modeling. Then, the necessary coordinates were established to study the kinetostatic characteristics of the system, which are the local coordinate \( \{ M \} \), the coordinate of the upper platform \( \{ U \} \), and the lower joint coordinate of the \( i \)th limb \( \{ C_i \} \).

**Step One:** (modeling single compliant joint, Fig. 6): Based on the constitutive law of Cosserat rod, the limbs can be balanced under the internal (i.e., distributed forces and moments) and external (i.e., forces and moments from tip point) loads at every location along the compliant joint. Thus, the static equation at any arbitrary point can be established as follows:

\[
\begin{align*}
\mathbf{n}_i(s) &= \int_s^{2L+s} \mathbf{f}_i(\sigma) d\sigma + m_i \mathbf{g} \\
\mathbf{m}_i(s) &= \mathbf{r}_{i,m} \times m_i \mathbf{g} + \int_s^{2L+s} \left( \mathbf{r}_i(\sigma) \times \mathbf{f}_i(\sigma) + \mathbf{l}_i(\sigma) \right) d\sigma \\
&\quad + \mathbf{r}_i(s) \times \mathbf{n}_i(s)
\end{align*}
\]

where \( \mathbf{n}_i(s) \) and \( \mathbf{m}_i(s) \) are the internal force and moment at point \( s \), along the compliant joint; \( l \) and \( L \) are the lengths of the compliant joint and rigid shaft, respectively; \( m_i \) is the mass of the \( i \)th joint; \( \mathbf{g} \) is the gravitational acceleration; \( \mathbf{r}_{i,m} \) is the position vector of the centroid of the rigid shaft in local coordinate; \( \mathbf{f}_i(\sigma) \) and \( \mathbf{l}_i(\sigma) \) are the distributed forces and moments in the compliant joint, which were caused by the plastic deformation of the compliant joints; \( \mathbf{r}_i(s) \) is the deformed shape vector at a point \( s \) of the compliant joint.

Deriving (1) with the arc length \( s \) (the arc length from point \( s \) to \( M_i \), Fig. 6), the differential equations relating to the distributed force and moment in limb \( i \) can be obtained

\[
\begin{align*}
\dot{\mathbf{n}}_i(s) + \mathbf{f}_i(s) &= 0 \\
\mathbf{m}_i(s) + \mathbf{r}_i(s) + \mathbf{f}_i(s) \times \mathbf{n}_i(s) &= 0
\end{align*}
\]

where \( \dot{\mathbf{n}}_i(s) \) and \( \dot{\mathbf{m}}_i(s) \) are the differentials of the force and moment with the length of the limb at point \( s \); \( \mathbf{f}_i(s) \) is the differential of shape vector at point \( s \).

Furthermore, the internal loads of the compliant joint (i.e., bending and torsion moments that are generated by the material deformation) are expressed by the curvature variation relative to the initial curvature as follows:

\[
\mathbf{m}(s) = \mathbf{R}(s) \mathbf{K}(s) \Delta \mathbf{u}(s)
\]

where \( \mathbf{R}(s) \in SO(3) \) is the orientation of the local moving coordinate at point \( s \) relative to the local fixed reference \( \{ M \} \); \( \mathbf{K}(s) \) is the stiffness matrix of the compliant rod by considering the bending and torsional stiffness, respectively; \( \Delta \mathbf{u}(s) \) is the variation of the curvature of the compliant joint at point \( s \), which can be expressed as \( \Delta \mathbf{u}(s) = \mathbf{u}(s) - \mathbf{u}^*(s) \), where \( \mathbf{u}(s) \) is the curvature vector of the compliant joint after deformation and \( \mathbf{u}^*(s) \) is the initial curvature vector. In the design of the proposed manipulator, the compliant limbs are directly assembled on the
base. Thus, the initial stress-free state of the rod is expressed as \( u^*_i = [0 0 0]^T \).

In order to eliminate the moment variation of (2) in compliant joint \( i \), the derivative was taken on (3) by the arc length \( s \)

\[
\dot{u}_i = R_i (K_i (\dot{u}_i - \dot{u}^*_i) + u^*_i K_i (u_i - u^*_i))
\]  

Combining (2) and (4) to eliminate the differential of the moment \( \dot{u}_i \) (s), the variation of curvature \( \ddot{u} \) related to distributed loads and rod parameters can be established. For simplicity, the detailed process in obtaining the derivative of the moment was omitted here [28]. Here, for the simplification, the notation \( s \) was inputted in the equation as follows:

\[
\ddot{u}_i = \ddot{u}^*_i - K_i^{-1} \left( (\dot{u}_i K_i + \dot{K}_i) (u_i - u^*_i) + \dot{\hat{e}}_3 R_i^T \left( \int^l_s f_i (\sigma) d\sigma + m_i \dot{g} \right) + R_i^T \dot{L}_i \right).
\]  

The boundary conditions for the compliant limb can be expressed as follows:

\[
g_i (0) = \begin{bmatrix} R_{C,i} & P_{C,i}^T \\ 0_{1 \times 3} & 1 \end{bmatrix}
\]

\[
m_i \left( 2l + L \right) = M_{U,i}
\]  

where \( R_{C,i} \) and \( P_{C,i}^T \) are the orientation matrix and position vector, respectively, at the initial point \( M_i \) of the rod related to the assembling characteristic (see Fig. 6). Thus, (5) allows finding the curvature variation along the compliant limb with the given boundary conditions, (6).

**Step Two:** For establishing the static equation of the rigid shaft in the \( i \)th limb, the following coordinates were defined, which represent the orientation and position of the compliant joints and rigid shaft: the coordinates \( \{ O_{U,i} \} \) and \( \{ O_{C,i} \} \) are located at the tip and end of the compliant limb to define their local moving coordinates [i.e., \( g (l + L) \) and \( g (0) \) in (16)], respectively; the coordinates \( \{ O_{M,i} \} \) and \( \{ O_{N,i} \} \) are located at the two ends of rigid shaft (i.e., distal and proximal) to define their positions and orientations, respectively; the coordinate \( \{ O_{R,i} \} \) is located at the geometrical centroid of the rigid shaft, which is parallel with \( \{ O_{M,i} \} \) and \( \{ O_{N,i} \} \).

Furthermore, considering the mass of the rigid shaft, the variation of the distributed force and moment at two ends of

\[
\text{the rigid part of the shaft keep its balance (see Fig. 7)}
\]

\[
\begin{cases}
\dot{f}_i (M_i) = -f_i (N_i) - m_i g \\
I_i (M_i) = -I_i (N_i) + r_i (M_i) \times (f_i (N_i) + m_i g) - r_i (N_i) \times f_i (N_i) - r_i,m \times m_i g
\end{cases}
\]  

where \( f_i (M_i) \) and \( I_i (M_i) \) are the distributed force and moment at point \( M_i \); \( f_i (N_i) \) and \( I_i (N_i) \) are the distributed force and moment at point \( N_i \); \( r_i (M_i) \) and \( r_i (N_i) \) are the position vectors of points \( M_i \) and \( N_i \) in local coordinate \( \{ M \} \).

After establishing the load-deformation differential equations of a general Cosserat rod (i.e., single compliant joint), the kinetostatic model of a single compliant limb (two compliant rods and one rigid shaft) was developed, (7). With the established model, the configuration of the limb can be calculated under any given boundary conditions (i.e., external loads \( F_{U,i} \) and \( M_{U,i} \) in Fig. 6). Furthermore, the kinetostatic model of the overall system (constructed by one upper platform and three compliant limbs) can be progressed (see Section IV).

**B. Kinetostatic Modeling of the Whole System**

As the proposed compliant PKM is structured with an upper platform and three compliant limbs, the position and orientation of the upper platform is determined by the configurations of the limbs. In this section, with the previously established model of a single compliant limb, the forward kinetostatic model of the entire system was developed to obtain the pose of the upper platform (i.e., position and orientation) under any given inputs of the SMA clutch-based driving system (i.e., positions of three clutches along the placing lines).

1) **Static Constraint on the Upper Platform:** For each configuration of the manipulator (i.e., different positions of the SMA clutches along the placing lines), the kinetostatic equations of the upper platform can be obtained (see Fig. 8). According to the previous research [34], [35], the static equation of the compliant parallel manipulators can be established in the form of the force and moment constraints of the upper platform (i.e., from external constraints).
and internal loads of the compliant joints) (see Fig. 8)

\[
\begin{align*}
\sum_{i=1}^{3} F_{U,i} &= F_e = 0 \\
\sum_{i=1}^{3} F_{U,i} \times (R_{U} U_i + p_e) + \sum_{i=1}^{3} M_{U,i} + F_e &= \sum_{i=1}^{3} \varepsilon_{i} = 0
\end{align*}
\]

(8)

where \( F_e \) and \( M_e \) are the force and moment on the upper platform, respectively; \( F_{U,i} \) is the internal force in the \( i \)-th compliant joint; \( M_{U,i} \) is the internal moment of the \( i \)-th compliant joint in local coordinate \( \{M\} \); \( F_e \) is the external force applied on the upper platform in local coordinate \( \{M\} \); \( M_e \) is the external moment of the upper platform in local coordinate \( \{M\} \); \( R_{U} \) is the rotation matrix of the upper platform in local coordinate \( \{M\} \); \( U_i \) is the position vector of the \( i \)-th joint in upper platform coordinate \( \{U\} \); \( p_e \) is the position vector of the upper platform in local coordinate \( \{M\} \).

For defining the rotation matrix, \( R_{U} \), the \( XYZ \) Euler angle was used to define the orientation of the upper platform. The process can be stated as follows: the upper platform coordinate was originally transformed from point \( O_M \) in local coordinate \( \{M\} \) to point \( O_U \); then, the upper platform coordinate was rotated around the axes \( Z, Y, \) and \( X \) with angles \( \theta_z, \theta_y, \) and \( \theta_x \), respectively. Thus, the rotation matrix can be expressed as follows:

\[
R_{U} = \begin{bmatrix}
\cos \theta_z \cos \theta_y & \cos \theta_z \sin \theta_y & -\sin \theta_z \\
\sin \theta_z \cos \theta_y & \sin \theta_z \sin \theta_y & \cos \theta_z \\
-\sin \theta_y & \cos \theta_y & 0
\end{bmatrix}
\]

(9)

where \( \theta = \sin \theta \) and \( \theta = \cos \theta \).

2) Geometrical Constraints on Compliant Limbs: By considering the geometrical constraints, the tip position and orientation of the limb [the upper compliant joint (\( O_{U,i}, \) Fig. 7)] can be obtained. Thus, the geometrical constraints can be established for ascertaining the configuration of the system.

As the stiffness of the rigid shaft is much higher than the compliant rods, the elastic deformations of the limb were considered to occur at the compliant rods. Thus, the following equations can be established regarding the position and orientations at points \( M_i \) and \( N_i \) based on the geometrical constraints of the rigid shaft:

\[
\begin{align*}
R(M_i) &= R(N_i) \\
r(M_i) &= r(N_i) + R(N_i) \begin{bmatrix} 0 & 0 & L \end{bmatrix}^T
\end{align*}
\]

(10)

where \( R(M_i) \) and \( R(N_i) \) are the attached local moving coordinate of points \( M_i \) and \( N_i \); \( r(M_i) \) and \( r(N_i) \) are the position vectors of attached frames in local coordinate; \( L \) is the length of the rigid shaft (see Fig. 6).

Based on the differential equations [i.e., (5) and (6)] and geometrical constraints [see (10)] of the compliant rods and rigid shaft, the position and orientation of the limb tip can be calculated to obtain the geometrical constraints of the PKM

\[
\begin{bmatrix} \log(\mathbf{R}((2L + L_i)^T \mathbf{R}(U_i) \mathbf{R}_{ei})) \end{bmatrix}^\vee = 0, \quad i = 1, 2, 3
\]

(11)

where \( \log \) is a natural logarithm of the matrix, which maps \( SO(3) \) to \( so(3) \); \( \vee \) denotes the conversion of a matrix from \( so(3) \) to its corresponding \( \mathbb{R}^3 \); \( \mathbf{R}(U_i) \) is the rotation matrix of the upper platform; \( \mathbf{R}_{ei} \) is the angular displacement vector, which represents the predefined offset orientation between the upper platform and assembling vector of the compliant limb. Note: \( \mathbb{R}^3 \) is the real matrix group; \( SO(3) \) and \( so(3) \) are the special orthogonal and Euclidean groups, respectively.

Furthermore, the position vector loop constraint of three compliant limbs (see Fig. 8) can be expressed as follows:

\[
\mathbf{R}(U_i) U_i = \mathbf{r}(2L + p_e), \quad i = 1, \ldots, 3
\]

(12)

As the three placing lines have fixed orientations on the base platform, the position vector of SMA clutch, \( C_i \), in the \( i \)-th placing line can be expressed as follows:

\[
C_i = A_i + d_i \mathbf{e}_i, \quad i = 1, \ldots, 3
\]

(13)

where \( A_i \) is the position vector of point \( A_i \) in the global reference; \( d_i \) is the distance between SMA clutch and point \( A_i \); \( \mathbf{e}_i \) is the unit vector of the \( i \)-th placing line.

Hence, based on the static constraints [see (1)–(7)] and the geometric boundary conditions [see (10) and (11)] of the compliant limbs, an iteration approach was utilized to find the solutions that satisfy (8), which is the configuration of the upper platform.

Since a novel underactuated compliant system is presented, a new kinetostatic method for analyzing the configuration of the system was introduced, which is different from the conventional approaches (e.g., Pseudobody and Cosserat-Theory Methods). This method considers the generated loads (i.e., internal force and moment) of the compliant joints and the underactuated structure, which can predict the noncircular deflection of the joints. Based on this analysis of the joints, the kinetostatic behavior of the under-actuated system is described, which can also be utilized for other underactuated compliant parallel systems.

IV. EXPERIMENTAL VALIDATION AND PERFORMANCE STUDY

In Section III, the forward kinetostatic model of the compliant rod-based parallel system was introduced [see (8)–(11)]. In order to validate the proposed kinetostatic model, a set of experiments was conducted by moving the upper platform by controlling the positions of three clutches along the track of the base platform. In the experiments, the configurations of the upper platform were measured by a vision-based tracking system.

A. Model Validation for a Single Compliant Limb

First, the kinematic performances of a single compliant limb were studied under different static load conditions. To validate the proposed model of a limb, one of its ends was mounted while measuring the deflection of the limb when loading the other end (see Fig. 9). A grid article was utilized as the background for scaling the limb trajectory during the experiments, which were then compared with the results from the theoretical calculations. Visual measurements were employed for validating the model of a single compliant joint, but the more accurate measurements (i.e., VICON) were adopted for the experiments of the overall system.

As superelastic NiTi rods were employed to replace the conventional rigid universal joints, the kinetostatic characteristics
of the compliant limb were carefully studied based on the Cosserat rod theory. The parameters of the NiTi rods utilized in experiments are listed in Table I.

With one limb configured as a cantilever beam (see Fig. 9), the weights with different masses (i.e., 0.29 N and 0.62 N, respectively) were hanged at the end of the limb. Then, the kinetostatic modeling of a single compliant limb proposed in Section III was implemented with the same boundary conditions to calculate the limb trajectories.

From Fig. 10, it can be seen that the simulation results match those from the experiments (average error: 4.7%) well at the measured points. Under two different loads (i.e., 0.29 N and 0.62 N), the maximum error between the simulation and experimental results is 0.75 and 0.87 mm, respectively. The values of the deformations of the compliant joint presented in these tests (see Fig. 10) represent that happens during the operation of the PKM. Hence, it can be concluded that the proposed model of a single compliant limb is accurate for predicting the kinetostatic performance of a single leg. Based on the structure design of the PKM, each limb is deformed within 20°; thus, the current experiment under 0.29 N and 0.62 N (deformation angles are 12° and 21°, respectively) end loads can validate the modeling of a single limb. Then, based on the validated model of a single limb, the model of the entire parallel kinematic system was tested and presented in the following part.

### B. Validation of the Underactuated Manipulator Model

As commented, the underactuated PKM system proposed here can perform a 6-DoF manipulation, which is actuated by positioning the SMA-based clutches along three placing lines using a single driving cable. Thus, the forward kinetostatic model is of key importance to predict the output of the system (i.e., workspace of the upper platform) for given inputs (i.e., positions of three placing lines).

The validation process of the proposed kinetostatic modeling was performed based on the prototyped PKM (see Fig. 11). After connecting the SMA clutch-based driving system and upper platform by the three same structured complaint limbs (i.e., the ends of limbs are fixed firmly on the clutch and upper platform, respectively, to avoid the relative rotation during the working of the system), the PKM is configured.

After planning the motion trajectories of three placing lines, the control system was employed to actuate the three placing lines to the desired positions, respectively. Then, the developed kinetostatic model and camera system were used to get the outputs of the upper platform. At last, the results from the two methods were compared and plotted for the model evaluations.

The experimental setup (see Fig. 11) includes the underactuated system (5), a host computer, control system, power supply, and a motion capture system [(1)–(4)], VICON (UI-5480VP), that was deployed to measure the position and orientation of
the upper platform for the model validation. In the test, two coordinate systems (i.e., the upper platform and base coordinate systems, respectively, constructed by the corresponding markers) were set up in the camera system. Then, the kinematic models [(8)–(12)] were employed to obtain the configuration of the upper platform in the local coordinate system (see Fig. 6).

In the manipulator, one motor (Maxon DCX32L) integrated with a gearbox (Maxon GPX32, reduction ratio - 138:1) and encoder (Maxon ENC16 with 1024 pulses) was used to drive a polyvinyl chloride coated cable (i.e., outer diameter: 2.3 mm, inner steel wire diameter: 1.5 mm). Three identical placing lines, which are composed of SMA clutches, guiding system, and linear encoder, are located at the three sides of the triangle base. Then, three linear encoders (receiver: RLC2IC, resolution: 0.244 μm) have been used to measure the positions of three placing lines. Based on the properties of SMA wire, the time for each set of movement of SMA clutch (from one end to another end of stroke) is around 6.2 s (i.e., 1 s for contracting the SMA wire to clamp the cable, 3.2 s for driving the clutch to move, and 2 s for cooling down the clutch to release the cable).

The control strategy for implementing the clutch-based driving system is shown in Fig. 12.

![Fig. 12. Control strategy for the clutch-based driving system (4th placing line was selected).](image)

### TABLE II
**Prototype Parameters for the Model Validation (see Fig. 8)**

| Variables | Variable description | Value (mm) |
|-----------|----------------------|------------|
| $A_1$ | The vector of point $A_1$ in (O) | [193.7, -64.3, 0] |
| $A_2$ | The vector of point $A_2$ in (O) | [-75.4, 188.5, 0] |
| $A_3$ | The vector of point $A_3$ in (O) | [-140, -148.4, 0] |
| $e_1$ | The unit vector of the moving direction of placing line-1 | [0.42, 0.91, 0] |
| $e_2$ | The unit vector of the moving direction of placing line-2 | [-0.42, -0.91, 0] |
| $e_3$ | The unit vector of the moving direction of placing line-3 | [1, 0, 0] |
| $U_1$ | The coordinate of the upper compliant joint $U_1$ in (u) | [61.5, 35.5, 0] |
| $U_2$ | The coordinate of the upper compliant joint $U_2$ in (u) | [-61.5, 35.5, 0] |
| $U_3$ | The coordinate of the upper compliant joint $U_3$ in (u) | [0, -71, 0] |
| $d_1, d_2, d_3$ | Motion stroke of the placing line | [0, 210] |

Unit: mm.

### TABLE III
**Positions of the Placing Lines (Inputs) for the Validation of the Model Determining the Position and Orientation of the Upper Platform (Outputs)**

| Test number | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
|-------------|---|---|---|---|---|---|---|---|---|----|----|----|
| Placing Line 1 (mm) | 0 | 0 | 0 | 40 | 40 | 80 | 80 | 80 | 80 | 120 | 120 | 120 |
| Placing Line 2 (mm) | 40 | 40 | 80 | 0 | 0 | 80 | 0 | 0 | 0 | 0 | 40 | 0 |
| Placing Line 3 (mm) | 80 | 120 | 120 | 80 | 120 | 120 | 40 | 120 | 120 | 40 | 80 | 80 |

Two key parameters were varied for validating the kinetostatic model of the entire underactuated PKM: the positions of the lower joints (given by the position of the clutches) and the compliance of the PKM.

First, the inputs of the three placing lines (i.e., the position of the clutches) were planned (see Table III) based on which, by using the forward kinematics, the corresponding configurations of the upper platform were evaluated, while the experimental data were captured by the camera system. This enabled the comparison of the simulated and experimental results.

For better understanding of the behavior of the system when varying the length of compliant joints (see Table III), the configurations of the upper platform were measured under different lengths of the compliant joints (i.e., 20, 30, and 40 mm, respectively). Here, as the base for discussions, three groups of the experiments (i.e., numbers 3, 8, and 10 in Table III) have been selected to briefly describe the configuration variations of the PKM with different inputs (see Fig. 13).

It can be seen from Fig. 13 that the length of the compliant joint has a significant effect on the performance of the system. Taking test 8 as an example, with the same input (i.e.,}
and orientation, respectively, for the workspace (see Fig. 15). Fluctuations between 0.2% and 3.8%, respectively. However, for lengths 30 and 40 mm closing with each other, and they fluctuate between 0.2% and 3.8%, respectively. Nevertheless, this “advantage” in this mechanism will increase the motion variety of the upper platform, which increases the complexity of the workspace analysis.

As the saw teeth have been adopted to increase the locking ability of the SMA clutch along each placing line (see Fig. 3), the locating position of each SMA clutch is not continuous anymore, which results in the discretization of the workspace of the system. Based on this observation, in the following, the workspaces have been evaluated by the designed characteristics of the placing lines (i.e., the pitch of saw teeth of the moveable foot and base platform).

1) Discrete Workspace of the System: This refers to the evaluation of workspace, which is dependent on the location of the feet on the track (given “resolution” of the saw tooth indexing features) as well as on their movement direction along the placing lines. This was followed by the use of the kinetostatic model to obtain the poses of the upper platform of the manipulator.

The workspaces (Fig. 15 with views from Z and X directions) are presented in the following conditions of the manipulator: 1) all three placing lines are located in random positioning; 2) two of the placing lines (i.e., 2 and 3) are located randomly, while the remaining placing line (i.e., 1) keeps a fixed position.

It can be seen that the workspace of the proposed system is almost symmetrical, with respect to Y and Z axes (see Fig. 15). Nevertheless, this is dependent on the accuracy of the setup (e.g., the angle between the placing lines, relative positions between the saw tooth tracks, manufacturing errors, etc.). However, a very interesting point here is that the resolution of positioning the center point (O1) in Fig. 8) of the platform is dependent on the resolution [i.e., pitch, Δl in Fig. 3(b)] of the saw teeth track.

C. Workspace Analysis

Due to the introduction of the SCBDS in the overall system, it is becoming possible to use one motor to control the independent motion of three placing lines further, to control a 6-DoF PKM. However, this “advantage” in this mechanism will increase the motion variety of the upper platform, which raises the complexity of the workspace analysis.

Based on the motion characteristics of the proposed system (i.e., the three placing lines can only move to one direction at any time), the workspace of the system can be divided into two groups: one is the discrete workspace, which is generated by the discrete/random motions of the three placing lines; another is the continuous workspace that is generated by the continuous motions of one, two, or three placing lines simultaneously.

The three SMA clutches having the same positions), the balanced configuration of the upper platform varies significantly (i.e., the average difference is 10.3 mm) with different lengths of the compliant joints. The reason for the position deviation of the upper platform can be attributed as the static and kinematic performance of the limbs is sensitive/dependent with the length of the compliant joints, where the internal force distribution and stiffness is affected under different configurations of the P. Thus, the position and orientation of the upper platform satisfying (8) vary when different lengths of the compliant joints are utilized.

Fig. 14(a) illustrates the overall position deviations of the upper platform under the given sets of inputs. It can be seen that the model presented in this article can be validated with a maximum deviation of 8.7% and an average deviation of 1.9% [i.e., the absolute errors are ±5.9 mm and ±2.6° in position and orientation, respectively, for the workspace (see Fig. 15)]. In addition, the deviation of the upper platform when joint lengths are 30 and 40 mm is closing with each other, and they fluctuate between 0.2% and 3.8%, respectively. However, for the compliant joint with the length of 20 mm, the deviations fluctuate similarly with 30 and 40 mm joint length from test 1 to 8, but then it quickly increases to around 4.2% afterward, this is mainly due to the structure singularity in some specific configuration of PKM, causing the system instability.

Fig. 14(b) displays the orientation deviations of the upper platform for the given set of tests with different lengths of the compliant joints. It can be found (see Fig. 14) that the orientation deviations of the upper platform are within 6% during the tests. The average orientation deviations with three types of the compliant joint (i.e., l = 20, 30, and 40, respectively) are 3.2%, 2.7%, and 2.4%, respectively, which proves the correctness of the model proposed in this article.

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Fig. 16. Examples of the discrete workspace of the system with placing lines 2 and 3 randomly moving within their strokes, while the placing line 1 is fixed. (a) and (b) Top view and side view of the workspace.

One reason is that the kinematic characteristics of the PKM (i.e., the relationship between the locations of three clutches and the position of upper platform) are varying with different configurations of system; another reason is that the feet of the manipulator can be placed only in the positions that are defined by the conjugate profiles of the sawtooth features of the placing line and base platform. It can also be seen that the position boundaries (mm) of the proposed PKM in X, Y, and Z directions are [−34, 34], [−47, 28], and [114, 162], respectively, while for the pose boundaries (degree) in X, Y, directions are within [−19, 19], for Z direction is within [−37, 37].

By this inference, the system resolution, $\Delta_i$, can be calculated from (8) to (13) by inputting the position of the feet and the value of the resolution, $\Delta_i$, of the saw tooth tracks. This leads to the notable observation that the resolution of the center point of the manipulator, $\Delta_i$, being of lower value (average 0.35 mm) in the center region of the workspace and of higher value (average 0.68 mm) at its outer region (see notations in Fig. 15).

Fig. 16 is the workspace of the system with two placing lines (i.e., 2 and 3) moving along their guides while placing line 1 is fixed in the middle of its stroke.

As expected, the workspace captures only one section of the full workspace, while the distance between the positions of the center point of the upper platform is not uniform although the placing lines have been moved with the same increment, i.e., smaller distances toward the center of the workspace (see Fig. 16). Interesting to note that a position of the upper platform along the moving curves is dependent on the resolution of the linear encoders, but in a full 3-D (spacing between the moving curves) it depends again on the resolution of the saw teeth, $\Delta_i$.

2) Continuous Workspace of the System: In this case, one, two, or all (three) placing lines move continuously and in the same direction (imposed by the movement of the driving cable). Interesting to observe that under these restrictions, the “workspaces” become families of moving curves in a three-dimensional (3-D) space. Of course, the resolution of positioning the upper platform along the moving curves is dependent on the resolution of the linear encoders, but in a full 3-D (spacing between the moving curves) it depends again on the resolution of the saw teeth, $\Delta_i$.

Thus, based on the working principle of the SMA clutch-based driving system, the motion characteristics of the placing lines can be divided into the following three cases. Case 1: one of the placing lines moves within the stroke, while the rest two placing lines remain stationary. Case 2: two of the placing lines move within their strokes at the same speed and direction, but the third one remains stationary. Case 3: three placing lines move at the same speed and direction to actuate the PKM. The workspaces for all of the three cases are plotted in Fig. 17. The resolution of the system was studied, which is the displacement of the upper platform when the placing lines move with a minimal move one pitch (1.5 mm—saw teeth width of the placing line).

In this analysis, all the fixed placing lines start to be placed at the location of 30 mm from one end of their strokes and move toward the other end with an increment displacement of 1.5 mm (the pitch of the placing lines). Fig. 17 presents the workspace of the system when the fixed placing lines are positioned at the locations of 30, 50, 70, 90, and 110 mm, respectively (Case 1—lines 2 and 3; Case 2—line 3). Although the feet moved with identical increments, it can be seen that the resolution of the system ($\Delta_i$) varies in the entire workspace when the system takes different configurations; see the change of the value $\Delta_i$ in Fig. 17(a) and (b). In Case 1 [see Fig. 17(a)], the value of the resolution of the system decreases from the border to the center regions of the workspace. Specifically, resolutions $\Delta_1$–$\Delta_5$ are 0.77, 0.58, 0.48, 0.37, and 0.29 mm, when the fixed placing lines are at the locations of 30, 50, 70, 90, and 110 mm, respectively. The same tendency was also found in the discrete workspace study [see Fig. 15(a)]. In Fig. 17(b), a similar result was identified; the resolution of the family curves decreased (i.e., $\Delta_1$: 0.55, $\Delta_2$: 0.48, $\Delta_3$: 0.45, $\Delta_4$: 0.41 and $\Delta_5$: 0.38 mm, respectively) from the border to the center of the workspace. In Case 3, all the placing lines move simultaneously from one
end of their strokes to the other one. Hence, just one continuous curve of the upper platform position can be generated. In this case, the upper platform has small movements in $X$ and $Y$ directions, which is close to a circular arc shape (i.e., center: around $[0, 0]$, radius: 4 mm), while the height of the upper platform has a big variation (from 115.3 mm at the beginning to 160.8 mm at the middle, and then drops to 148 mm at the end).

From all these results, it can also be found that, by inputting a unit displacement from the actuation, the upper platform can move faster in the outer region of the workspace, while slower in the center area. This is an interesting observation that enables the operator to use different regions of the system workspace for distinct operations (e.g., slow and fine manipulations in the center region of the workspace, while fast and course maneuverings in the outer area).

In this section, the experimental validation for the proposed kinetostatic modeling was conducted on the proposed PKM. By varying the locations of all three SMA clutches, the configurations of the upper platform (i.e., position and orientation) were calculated and measured, respectively. It could be observed that the proposed kinetostatic models have been proved accurate (i.e., overall position deviation 1.8% and orientation deviation 2.8%, respectively). Then, with the validated kinetostatic model developed in this article, the workspace of the PKM was studied, which is important for understanding how the movements of the placing lines influence the movement of the upper platform of the underactuated PKM. After studying the discrete (i.e., three placing lines move independently to achieve the maximum workspace of the system) and a continuous workspace of the system, the resolution variation of the system was studied under the different working characteristics of placing lines. It was found that the resolution of the PKM is unevenly distributed among the workspace (i.e., higher resolution within the center region of the workspace, 0.35 mm, and lower resolution in the outer region of workspace, 0.68 mm).

V. CONCLUSION

In this article, a novel underactuated 6-DoF PKM, which uses a drive cable in combination with smart SMA clutches, three stiffness-adjustable limbs, and an upper platform, was proposed to perform a multiple degrees operation with a single actuator. Based on the proposed manipulator, the novelties can be summarized as follows.

First, an advanced compliant parallel mechanism, in which conventional rigid joints were replaced with stiffness-adjustable joints, has been developed to allow the safe human–robotic interaction. To achieve this, superelastic NiTi rods were selected to generate the motions (i.e., by material deformation) as conventional rigid joints. Furthermore, in order to actively control the stiffness of the system, the length of the compliant joints was designed to be adjustable, which was operated by covering the NiTi alloy rod with a set of small fixed-length rigid segments. Hence, in this article, a traditional universal/spherical joint-based PKM with complicated structure and strict assembling requirements has been transferred to a novel PKM.

Furthermore, a novel cable-driven SMA clutch-based driving system has been developed for actuating the parallel manipulators with a reduced number of actuators (i.e., using a single actuator to control a 6-DoF manipulator), which provides a new way for actuating, in a simple and efficient way, multiple-DoF parallel manipulators only with one motor. Furthermore, an SMA-wire-based clutch (characteristics: small size, powerful output, and quick response) that enables the underactuation of the manipulator has been demonstrated; it is simple but efficient design is a viable alternative to bulky conventional (e.g., electromagnetic) clutches.

In addition, for studying the kinematic characteristics (i.e., positioning and orientation accuracy) of the proposed underactuated system, a novel kinetostatic model was developed to predict the performances of an end-effector by considering the external and internal loads. Specifically, by accounting for the bending and torsion characteristics of each compliant joint and force and moment of the upper platform, the generated forces and moments in three limbs have been calculated to establish the overall static equation of the upper platform. Based on the proposed models, the outputs of the system (i.e., position and orientation of the upper platform) can be predicted with the given inputs of the system (i.e., positions of three SMA clutches). Then, the validation process was conducted based on the proposed prototype (i.e., a triple manipulator) and its control system, which yielded average deviations 1.8% in position and 2.8% in orientation (see Fig. 14). The analysis of discrete and continuous workspaces indicates that the proposed parallel mechanism (i.e., structured by three limbs) can be used as a 6-DoF manipulator with only one motor as the actuator. It was also found that the resolution of the system is unevenly distributed within the workspace, i.e., higher in the centre region (0.35 mm), lower in the outer region (0.68 mm), which mainly depends on the pitch of the saw-tooth features of the clutches (see Fig. 3). Hence, for operations with distinct movement requirements (e.g., fine positioning - assembly of small components; coarse positioning - pick and place in logistic lines), the system design could be customized for different regions of the workspace, in order to optimize the manipulation time and energy consumption.

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Nan Ma received the B.S. and M.E. degrees in mechanical engineering from Shenyang Ligong University and NCUT in 2012 and 2015, respectively. He is currently a research fellow in University of Nottingham. His current research interests are design, static and dynamic analysis of parallel mechanisms (i.e. walking and flying robots), as well as the soft material-based robotics (e.g. continuum and stiffening robots).

Xin Dong received the B.S. and M.S. degrees from the Dalian University of Technology and Beihang University in 2008 and 2011 respectively, and the Ph.D. degree from the University of Nottingham, in 2015, and the postdoctoral period from the University of Nottingham. He is currently the Assistant Professor working with the University of Nottingham. His research interests are extra slender continuum robot and reconfigurable hexapod robots with novel actuation solutions for the application in Aerospace, Nuclear, Oil&Gas, Marine and rescue.

Dragos Axinte was appointed Lecturer in Manufacturing Engineering (2005) and successively promoted to Associate Professor (2007), Reader (2010) and Professor (2011). He is currently the Fellow of the Institution of Mechanical Engineer (IMechE) and Fellow of the International Academy of Production Engineering (FCIRP). Since 2009 he is the Director of the Rolls-Royce University Technology Centre (UTC) in Manufacturing and On-Wing Technology.

Dr. Axinte is also Editor-in-Chief of the International Journal of Machine Tools and Manufacture.