Inelastic Kondo-Andreev tunnelings in a vibrating quantum dot

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Phonon-assisted electronic tunnelings through a vibrating quantum dot embedded between normal and superconducting leads are studied in the Kondo regime. In such a hybrid device, with the bias applied to the normal lead, we find a series of Kondo sidebands separated by half a phonon energy in the differential conductance, which are distinct from the phonon-assisted sidebands previously observed in the conventional Andreev tunnelings and in systems with only normal leads. These Kondo sidebands originate from the Kondo-Andreev cooperative cotunneling mediated by phonons, which exhibit a novel Kondo transport behavior due to the interplay of the Kondo effect, the Andreev tunnelings, and the mechanical vibrations. Our result could be observed in a recent experiment setup [J. Gramich et al., PRL 115, 216801 (2015)], provided that their carbon nanotube device reaches the Kondo regime at low temperatures.

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Introduction.— The hybrid quantum systems have a potential to exhibit new emergent phenomena through merging the strength of different media [1]. A quantum dot (QD) embedded between normal (N) and s-wave superconducting (S) leads (N-QD-S) is one of such devices, which has received considerable attentions from both the theoretical [2] and experimental [12–14] communities in the past two decades. In such a hybrid system, two important phenomena may arise: one is the Andreev tunneling (AT) [15] and the other is the screening of the localized spin in the QD by conduction electrons in the leads. While the former induces the Andreev bound states (ABSs) located in the superconducting gap, the latter is the famous Kondo effect [16]. The competition between these two processes results in a profound influence on the ground state properties [5] as well as the transport behaviors of the devices [2, 8, 10, 11].

For a molecular QD, it was found that vibrational degrees of freedom are easily excited when electronic tunneling takes place [17–19], which has a dramatic influence on the transport of the system due to the presence of inelastic tunneling processes mediated by emission or absorption of phonons [20–22]. In recent years, phonon-assisted inelastic AT in an N-QD-S system also leads to interesting physics on, for example, the electronic transport [24, 26], the heat generation [27], the ground-state cooling [28], the steady-state shot noise [29], as well as the transient dynamics under a step bias [30]. More interestingly, the phonon-assisted AT can lead to resonant peaks every time the bias voltage changes by one phonon energy or the gate voltage changes by half a phonon energy [31, 32], which has been unambiguously observed in a recent experiment [33]. This is somewhat reminiscent of normal systems where phonon sidebands of Kondo cotunnelings [34–36] and single-electron tunnelings [37–39] are also separated by one phonon energy in the bias voltage. Since the N-QD-S setup fabricated in the experiment [33] is indeed an ideal platform to explore the Kondo physics, it is our aim in this paper to provide a theoretical study of the Kondo transport in such a device.

Our investigation reveals that the interplay of the Kondo correlations, the superconductivity, and the mechanical vibrations of the QD gives rise to distinct transport characteristics, as compared with those arising from the conventional phonon-assisted ATs [31–39]. The main physical scenario is illustrated in Fig. 1, where elastic and inelastic AT with and without the Kondo effect are schematically shown. We set the chemical potentials of the N (µN) and S (µS) leads as µN = V, µS = 0, and the superconducting gap ∆ is taken as the largest energy scale in the problem. We consider the parameter regime where the QD-S tunnel coupling is much larger than the N-QD coupling and both are several times smaller than the onsite Coulomb repulsion, such that the Kondo effect and the onsite pairing coexist [11]. In this case, two Andreev bound states (ABSs) separated roughly by the Coulomb energy form at ±EA and their widths are determined by the N-QD coupling [2]. At zero bias V = 0, a spin↑ localized electron and a spin↓ lead electron at µN can convert to a Cooper pair in S, while another spin↓ lead electron at µN transits into the QD simultaneously [Fig. 1(a)]. This spin-flip cotunneling process, to which we refer as the Kondo-Andreev tunneling, is elastic and accounts for the zero-bias conductance peak previously observed in this system [12–13]. When the bias increases to V = εph/2 (εph the phonon energy), besides the elastic Kondo-Andreev tunneling process, additional inelastic Kondo-Andreev tunneling emitting one phonon can also take place [Fig. 1(b)]. Here, the emission of a phonon fulfills the energy conservation of the transition.
FIG. 1. (Color online) Schematics of elastic [(a), (d)] and inelastic [(b), (c), (e), (f)] electronic tunnelings in an N-QD-S system. (a)-(c) represent the Kondo-Andreev tunnelings through an interacting QD with the numbers 1, 2, 3 denoting the tunneling sequences. (d)-(f) show the conventional AT through a noninteracting QD. The wavy arrows represent the emission of phonons during the inelastic tunnelings.

that two N-lead electrons each with energy $\varepsilon_{ph}/2$ in the initial state are annihilated and a Cooper pair with zero energy is created in the final state, while the QD energy under the spin flipping remains the same. A Cooper pair with opposite spins can also transfer to the S lead through the other ABS.

When multiple-phonon processes are involved, a series of sidebands separated by half a phonon energy are thus expected at $V = n\varepsilon_{ph}/2$ with $n = 0, \pm 1, \pm 2, \cdots$.

For comparison, we also give a general scenario of the conventional phonon-assisted ATs for a noninteracting N-QD-S system, where the QD without the onsite Coulomb interaction favors even electron occupation. In this system, there are two interleaved sets of phonon sidebands, each separated by $\varepsilon_{ph}$, in the differential conductivity, since additional phonon-emitted inelastic AT can be triggered at $V = \pm E_N + n\varepsilon_{ph}$. For $n = 0$, the AT is elastic [see Fig. 1(d) for $V = E_A$]. For $n > 0$, an N-lead electron at $\mu_N$ can transfer to the S lead through the lower [Fig. 1(e), $V = -E_A + n\varepsilon_{ph}$] or upper [Fig. 1(f), $V = E_A + n\varepsilon_{ph}$] ABS by emitting $n$ phonons, while another electron passes directly through the other ABS. Similar inelastic ATs take place from the S lead to the N lead for $n < 0$. When the two ABSs are indistinguishable (e.g., their widths being larger than their interval) or separated by multiples of $\varepsilon_{ph}$, the two sets of phonon sidebands merge into a single set of sidebands separated by one phonon energy. This is exactly the special case discussed in Ref. [31]. In the following, we perform a model calculation to demonstrate these transport scenarios.

Model and Formalism.—Our N-QD-S system is modeled by the Hamiltonian $H = H_{leads} + H_{ph} + H_{QD} + H_{tunnel}$. The first term represents the normal ($\beta = N$) and superconducting ($\beta = S$) leads, $H_{leads} = \sum_{k,\sigma} \varepsilon_k c_{k\sigma}^\dagger c_{k\sigma} - \Delta \sum_k (c_{k\uparrow}^\dagger c_{-k\downarrow} + c_{-k\uparrow} c_{k\downarrow})$, $H_{ph} = \varepsilon_{ph} a^\dagger a$ models the local phonon mode, $H_{QD} = \sum_{\sigma} \varepsilon_d n_d^\dagger n_d + U n_d^\dagger n_d + (a(a^\dagger - 1)) \sum_{\sigma} n_d n_{\sigma}$ describes an interacting single-level QD, with Coulomb repulsion energy $U$, coupled with the local phonon by $\lambda$ the Holstein-type electron-phonon interaction ($\lambda$). The last term $H_{tunnel} = \sum_{k,\sigma,\beta} (\lambda V_{c_{k\sigma}\alpha}^\dagger c_{k\alpha} + H.c.)$ describes the electronic tunneling between the dot and the leads. From the tunneling matrix elements $V_{\beta}$, the dot level $\varepsilon_d$ acquires an intrinsic broadening $\Delta_{\beta} \equiv 2\pi p_0 |V_{\beta}|^2$ with $p_0$, the density of state of lead N and lead S in normal state. By the standard Keldysh nonequilibrium Green’s function (GF) theory [40], the electronic current flowing from the N lead into the QD can be expressed as

$$I = \frac{2ie}{\hbar} \int d\omega \Gamma_N [(1 - f_N) G_{11}^{<}(\omega) + f_N G_{11}^{>}(\omega)],$$

(1)

where $f_N(\omega)$ is the Fermi distribution function of lead N. The boldfaced GF matrices are defined in the well-known 2 × 2 Nambu representation [31], from which the local density of states (LDOS) per spin can be calculated by $\rho(\omega) = -(1/\pi) \text{Im} G_{11}^{<}(\omega)$.

In this work, we focus on the strong EPI regime. It is thus appropriate to make the non-perturbative Lang-Firsov transformation [42] $\tilde{H} = e^{S_h} H e^{-S_h}$ with $S = (\lambda/\varepsilon_{ph})(a^\dagger - a) \sum_{\sigma} \rho_{\sigma} d_\sigma$ to eliminate the linear EPI. This gives us $\tilde{H} = H_{leads} + H_{ph} + H_{QD} + H_{tunnel}$, where $H_{QD} = \sum_k \tilde{\varepsilon}_d n_d^\dagger n_d + \tilde{U} n_d^\dagger n_d$ and $H_{tunnel} = \sum_{k,\sigma,\beta} (\tilde{V}_{c_{k\sigma}\alpha}^\dagger c_{k\alpha} + H.c.)$, with $\tilde{\varepsilon}_d = \varepsilon_d - \varepsilon_{ph}$, $\tilde{U} = U - 2\varepsilon_{ph}$, $\tilde{V}_{\beta} = V_{\beta} X$, and $X = \exp\{-(\lambda/\varepsilon_{ph})(a^\dagger - a)\}$. Here a dimensionless measure of EPI $g \equiv X^2/\varepsilon_{ph}^2$ is introduced. As in dealing with the localized polarons, we adopt the approximation replacing the operator $X$ with its expectation value $\langle X \rangle = \exp\{-(g/N_{ph} + 1/2)\}$, where the average is taken over the independent phonon bath $H_{ph}$, and $N_{ph}$ is the Bose distribution. Hence, the renormalized $\Gamma_{\beta} = \langle X \rangle^2 \Gamma$. This approximation is valid when $V_{\beta} \ll \lambda$ and has been widely employed in both the N-QD-N [35, 37, 43] and N-QD-S systems [24, 27, 29, 32].

Applying the above decoupling scheme and the Feynman disentangling technique [44], one obtains $G_{11}^{<}(\omega) = \sum_{n=-\infty}^{\infty} L_n \tilde{G}_{11}^{<}(\omega - n\varepsilon_{ph}) + \frac{1}{2} \tilde{G}_{11}^{<}(\omega - n\varepsilon_{ph}) - \frac{1}{2} \tilde{G}_{11}^{<}(\omega + n\varepsilon_{ph})$ and $G_{11}^{>}(\omega) = \sum_{n=-\infty}^{\infty} L_n \tilde{G}_{11}^{>}(\omega + n\varepsilon_{ph})$, where $L_n = \exp\{-(\beta/2N_{ph} + 1)\} \exp\{n\beta\varepsilon_{ph}/2\} L_n(x)$, with $x = 2g\sqrt{N_{ph}(N_{ph} + 1)}$ and $I_n(x)$ being the modified Bessel function of the first kind. Note that the new GFs $\tilde{G}$ is defined according to the Hamiltonian $\tilde{H}$ in which the Bose degrees of freedom is totally decoupled.

We proceed to solve the retarded GF $\tilde{G}^{\sigma}(\omega)$ using the equation of motion method within the truncation scheme adopted by Sun et al. [3]. The solution of $\tilde{G}^{\sigma}(\omega)$, in the
large gap limit $\Delta \to \infty$, is obtained as

$$
(\tilde{g}_0^{-1} - \tilde{\Sigma}_0^\dagger - \tilde{U} \tilde{P} \tilde{Q}) \tilde{G}^\dagger = \mathbf{I} + \tilde{U} \tilde{P} \tilde{N},
$$

where $[\tilde{g}_0^{-1}(\omega)]_{11} = \omega - \varepsilon_d$, $[\tilde{g}_0^{-1}(\omega)]_{22} = \omega + \varepsilon_d$, $[\tilde{g}_0^{-1}(\omega)]_{21} = [\tilde{g}_0^{-1}(\omega)]_{12} = 0$, $P_{11}(\omega) = (\omega + \varepsilon_d + \bar{U} + 3\varepsilon_d \Gamma_N(\omega)/2)/\Omega(\omega)$, $P_{22}(\omega) = -\left[ P_{11}(\omega) \right]^*, P_{12}(\omega) = P_{21}(\omega) = -\Gamma_S/2\Omega(\omega)$, $\Omega(\omega) = (\omega - \varepsilon_d - \bar{U} + 3\varepsilon_d \Gamma_N(\omega)/2)(\omega + \varepsilon_d + \bar{U} + 3\varepsilon_d \Gamma_N(\omega)/2) - \Gamma_S^2/4$, $Q_{11}(\omega) = -\sum_k \frac{|\tilde{V}_k|^2 f_N(\omega)|\varepsilon_k - \omega - \varepsilon_d|^2}{\omega - \varepsilon_d}$, $Q_{22}(\omega) = \left[ Q_{11}(\omega) \right]^*$, $Q_{12}(\omega) = -Q_{21}(\omega) = -\Gamma_S/2$, $N_{11} = (n_{d1})$, $N_{22} = -(n_{d2})$, and $N_{12} = N_{21} = (d_1 d_\uparrow)$. The noninteracting retarded self-energy $\tilde{\Sigma}_0$ can be exactly obtained \[41\]. Note that the elements of matrix $\tilde{N}$ should be calculated by the equations $\left\langle n_{d1} \right\rangle = \int \frac{d\omega}{2\pi} \tilde{G}_{11}^\dagger(\omega)$ and $\left\langle d_1 d_\uparrow \right\rangle = \int \frac{d\omega}{2\pi} \tilde{G}_{21}^\dagger(\omega)$, where the lesser and greater GFs are related to the retarded one through the kinetic equation $\tilde{G}^{<}(\omega) = \tilde{G}^{>}(\omega) \tilde{G}^{a}(\omega)$ with $\tilde{G}^{a}(\omega) = (\tilde{G}^{\dagger}(\omega))$. We further approximate the lesser and greater self-energies $\tilde{\Sigma}^{<}(\omega)$ by their noninteracting counterparts $\tilde{\Sigma}^{<}(\omega)$ \[45\].\[46\]. The formulae are thus closed, which can be self-consistently calculated to determine the GFS $\tilde{G}^{\dagger}$, $\tilde{G}^{<}$, and $\tilde{G}^{>}$, and subsequently the current $I$, differential conductance $G \equiv dI/dV$, and LDOS $\rho(\omega)$.

Results and discussions.—In the numerical results presented below, we take all the renormalized parameters to be freely tunable. $\tilde{\Gamma}_N$ is taken as the energy unit and the temperature is always set at zero. We consider first the phonon-assisted inelastic AT in the Kondo regime. To this end, we adopt the parameters $\varepsilon_d = -2.5$, $\bar{\Gamma}_S = 4$, and $\bar{U} = 10$ such that the Kondo effect and the on-dot paring coexist. In Fig. 2(a), it is shown that remarkable differential conductance peaks, in addition to the zero-bias Kondo peak, develop whenever the bias voltage varies by half a phonon energy. These Kondo sidebands are consistent with the scenarios previously discussed in Figs. 1(a)–(c), and are very different from those occurring in N-QD-N systems that are separated by one phonon energy \[34\] [35]. Note also that the Kondo sidebands at positive bias are much weaker than those at negative bias, which can be ascribed to the Kondo effect is suppressed (enhanced) at positive (negative) bias since the dot energy level gets away from (closer to) the Fermi level of lead N. Furthermore, as compared with the Kondo resonance at zero EPI [see the red dashed curve in Fig. 2(a)], the zero-bias peak at finite EPI is significantly reduced and narrowed.

The underlying physics about why the conductance peaks are separated by $\varepsilon_{ph}/2$ can be acquired by examining the LDOS, since the conductance from the Kondo-Andreev tunneling processes is roughly proportional to the convolution of electron and hole density of states \[13\] [17]. Fig. 2(b) presents the LDOS for several bias voltages decreasing in a step of $\varepsilon_{ph}/4$. In equilibrium, multiple Kondo satellites ($\omega = n \varepsilon_{ph}$) exhibit on each side of the main Kondo resonance ($\omega = 0$) due to the EPI. In the following, we will focus on the nearest two satellites around the main resonance. In nonequilibrium, the main resonance and the two Kondo satellites all split into two subpeaks, resulting in totally six Kondo peaks in the LDOS as indicated by $L_1 (\omega = V)$, $L_2 (\omega = V - \varepsilon_{ph})$, $L_3 (\omega = V + \varepsilon_{ph})$, $R_1 (\omega = V)$, $R_2 (\omega = V - \varepsilon_{ph})$, and $R_3 (\omega = V + \varepsilon_{ph})$ in Fig. 2(b). When the bias is tuned to $V = -\varepsilon_{ph}/2$, the two peaks $L_1$ and $R_2$, as well as $L_3$ and $R_1$, merge into a single pronounced resonance (marked by red circles), respectively. Clearly, the convolution of these two merged Kondo resonances is larger than the convolution of $L_1$ and $R_1$ at $V = -\varepsilon_{ph}/4$, thereby cooperatively giving rise to a conductance peak at $V = -\varepsilon_{ph}/2$. Similarly, at $V = -\varepsilon_{ph}$, the two Kondo satellites $L_3$ and $R_3$ get merged at $\omega = 0$ and thus results in a conductance peak. In short words, the Kondo sidebands always appear in the conductance at the bias voltage $V$ under which the LDOS exhibits Kondo-peak cooperative enhancement within the bias window $\omega \in [-V, V]$. The cotunneling processes associated with some Kondo peaks in the LDOS are illustrated in Fig. 2(c). It is shown that the cotunneling processes of the $L_i$ ($i = 1, 2, 3$) and $R_3$ Kondo peaks are of the second and fourth order, re-
spectively. This explains why the $L_i$ Kondo resonances are stronger than the $R_i$ resonances. Specifically, in the Kondo process of $L_1$, a localized spin-$\uparrow$ electron tunnels out to lead N, followed closely by a spin-$\downarrow$ electron at $\mu_N$ tunneling into the QD. At low temperatures, a coherent superposition of such second-order spin-flip cotunneling events yields a many-body spin singlet comprising of the localized and N-lead electrons, which manifests itself as the sharp Kondo resonance $L_1$ in the LDOS. When $\tilde{\Gamma}_S > \tilde{\Gamma}_N$ as in our case, the AT can also take part in the Kondo cotunneling process. For example, in the Kondo-Andreev process of $R_1$, the localized spin-$\downarrow$ electron first tunnels out to $\mu_N$ and a Cooper pair in the S lead splits into two electrons with opposite spins. The split spin-$\downarrow$ electron then tunnels into the QD while the other electron transfer through the QD to the empty state with energy $-V$ in lead N. The coherent superposition of such fourth-order spin-flip cotunneling events leads to the weak Kondo resonance $R_1$ in the LDOS [2][3][6]. Other Kondo peaks such as $L_2$, $L_3$, $R_2$, and $R_3$ are produced by similar Kondo and Kondo-Andreev cotunneling processes but with one phonon being emitted.

The current and conductance as a function of the dot level $\tilde{\varepsilon}_d$ are further investigated [Fig. 2(d)]. As we can see, both quantities change monotonously with $\tilde{\varepsilon}_d$. This is different from those in conventional AT regime where characteristic peaks show up whenever the dot level $\tilde{\varepsilon}_d$ changes by $\varepsilon_{ph}/2$ [31][32]. The featureless nature of our $I$ vs $\tilde{\varepsilon}_d$ and $G$ vs $\tilde{\varepsilon}_d$ curves can be readily understood. As long as $\tilde{\varepsilon}_d$ is always restricted in the Kondo regime, the resulting Kondo resonances are robust and no additional phonon-assisted channel could be opened or closed when $\tilde{\varepsilon}_d$ is varied.

For comparison, we now turn to investigate the conventional inelastic AT in a noninteracting ($\tilde{U} = 0$) N-QD-S with the QD level $\tilde{\varepsilon}_d = 0$ fixed at the Fermi energy. In this parameter regime, the two ABSs appear at $\pm E_A$ with $E_A = \tilde{\Gamma}_S/2$. In Figs. 3(a)-3(c), the conductance is displayed for three values of $\tilde{\Gamma}_S$. Different from the conductance behaviors in the Kondo regime, there are indeed two sets of phonon sidebands at $V_{\pm,n} \equiv \pm E_A + n\varepsilon_{ph}$, each separated by one phonon energy, in agreement with our previous discussions of Figs. 1(d)-1(f). Generally, the two sets of sidebands are interleaved [Figs. 3(a) and 3(b)]. For $E_A = \varepsilon_{ph}/2$ [Fig. 3(c)], the two sets of sidebands merge with each other. This corresponds to the $I$-$V$ staircases addressed previously [31]. These conductance behaviors displayed can also be traced back to the LDOSs at different bias voltages, as shown in Figs. 3(d)-3(f). At zero bias, only hole-type (electron-type) sidebands of the upper (lower) ABS appear at $E_A + n\varepsilon_{ph}$ ($-E_A - n\varepsilon_{ph}$), with $n > 0$, which can be attributed to the fact that the upper (lower) ABS is fully empty (occupied) and the phonon absorption is unavailable at zero temperature [37]. For finite bias larger than $E_A$, the upper ABS becomes occupied, therefore phonon sidebands develop on both sides of each ABS. Upon adjusting $\tilde{\Gamma}_S$ such that $E_A = n\varepsilon_{ph}/2$ the sidebands associated with the two ABSs merge together [see Fig. 3(f)].

**Conclusions.**—We have predicted in N-QD-S systems a series of differential conductance subpeaks developed at $V = n\varepsilon_{ph}/2$ and resulting from phonon-assisted inelastic Kondo-Andreev cotunnelings. These structure are truly remarkable when compared with the transport characteristics of i) the conventional inelastic AT in N-QD-S systems [31][33] and ii) the inelastic Kondo cotunneling in the N-QD-N systems [33][36]. Our prediction might be observed in the carbon nanotube device fabricated by J. Gramich et al. [33] as long as the Kondo regime is achieved at low temperatures. Similar phenomena can also be expected when the device is driven by a microwave [48] instead of the electron-phonon coupling.

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