Covariant graviton propagator in anti-de Sitter spacetime

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Abstract

We construct the graviton propagator in the \( n \)-dimensional anti-de Sitter spacetime in the most general covariant gauge. We then study the behaviour of this propagator for different values of the gauge parameters. We will show that in any gauge, apart from the Landau gauge, the graviton propagator in the AdS spacetime contains a complicated term involving the derivative of a hypergeometric function which cannot be expressed in terms of elementary functions. We do our calculations in the Euclidean approach.

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1. Introduction

Anti-de Sitter (AdS) spacetime is the maximally symmetric solution to the vacuum Einstein equations with a negative cosmological constant [1]. It has the topology \( S^1 \times \mathbb{R}^{n-1} \) and can be viewed as a hyperboloid in \( \mathbb{R}^n \). There are closed time-like curves in this space. However, if we unwrap the circle \( S^1 \), then no closed time-like curves are left. By AdS spacetime, we mean the unwrapped AdS spacetime, in this paper.

AdS spacetime has attracted much attention due to AdS/CFT correspondence [2–4]. This is a correspondence between classical gravity in the bulk of AdS and the quantum field theory living on its boundary. One important aspect of AdS/CFT correspondence is the calculation of correlation functions in the type II supergravity on AdS in order to study the large \( N \) limit of \( N = 4 \) super-conformal Yang–Mills theories [5–7]. Bulk to bulk propagators are required for this purpose [8–10]. The graviton propagator in AdS spacetime in a certain Landau gauge is already known [11]. The graviton propagator for \( f(R) \)-gravity in four-dimensional AdS spacetime in a similar Landau gauge has been recently analysed [12]. The graviton propagator in AdS spacetime has also been studied in the de Donder and the Feynman gauges [13]. General higher-spin quantum field theory in AdS spacetime has been thoroughly investigated in the de Donder gauge [14–17]. However, the graviton propagator in AdS spacetime has not been studied in any other gauge. In this paper, we will derive the graviton propagator in AdS spacetime...
spacetime in the most general covariant gauge. It may be noted that the covariant graviton propagator is already known for the de Sitter spacetime [18, 19].

We will work in the Euclidean approach of Allen and Jacobson [20]. Thus, we will compute Green’s function on the \( n \)-dimensional hyperboloid and this will become the Feynman propagator in the AdS spacetime upon analytic continuation. So we will take the Euclidean vacuum [21] as the vacuum state for doing our calculations. We take the boundary condition as the standard boundary condition in the AdS spacetime, namely the fastest possible falloff at the boundary. Our propagator consists of three sectors, namely the scalar, the vector and the tensor sectors. The tensor sector is the same as that in the work of D’Hoker, Freedman, Mathur, Matusis and Rastelli [11]. The vector sector can be obtained by a trivial modification of the calculations performed for the covariant graviton propagator in the de Sitter spacetime [19]. So we really need to generalize only the scalar sector of the graviton propagator in the AdS spacetime here.

2. The field equation and the Green function

In perturbative quantum gravity, one writes the full metric in terms of a fixed background metric and small perturbations around it. We have denoted the full metric as \( g^{(f)}_{ab} \) to distinguish it from the fixed background metric \( g_{ab} \). We also denote the small perturbation around the fixed background metric as \( h_{ab} \). So we can now write

\[
g^{(f)}_{ab} = g_{ab} + h_{ab}.
\]

(1)

This small perturbation is regarded as a field that is to be quantized. The covariant derivative along with the raising and lowering of indices is with respect to the background metric.

Thus, the action for perturbative quantum gravity in AdS spacetime can be written as

\[
S = \int d^n x \sqrt{-g} [\mathcal{L}_{\text{lin}} + \mathcal{L}_{\text{int}}],
\]

(2)

where \( \mathcal{L}_{\text{lin}} \) is the free part of the Lagrangian which is quadratic in the field variable and \( \mathcal{L}_{\text{int}} \) is the part related to interactions. Here we are interested only in the free theory so we will take \( \mathcal{L}_{\text{lin}} \) as our Lagrangian. This Lagrangian is invariant under the following gauge transformations:

\[
\delta_{\Lambda} h_{ab} = \nabla_a \Lambda_b + \nabla_b \Lambda_a.
\]

(3)

We have to break this gauge invariance of the theory before quantizing it. For this purpose, we add the following gauge-fixing term to the original classical Lagrangian:

\[
\mathcal{L}_{\text{gf}} = \frac{1}{2\alpha} \left( \nabla_a h^{cb} - \frac{1 + \beta}{\beta} \nabla^b h \right) \left( \nabla^a h_{ab} - \frac{1 + \beta}{\beta} \nabla_b h \right),
\]

(4)

where \( \beta \) is a non-zero arbitrary finite number.

To find the Feynman propagator, we have to first find the Euclidean Green’s function for \( \mathcal{L} = \mathcal{L}_{\text{lin}} + \mathcal{L}_{\text{gf}} \) and then analytically continue it back to AdS. The Euler–Lagrange field equation for this total Lagrangian is given by

\[
\frac{\partial \mathcal{L}}{\partial h_{cd}} - \nabla_a \left( \nabla^a \frac{\partial \mathcal{L}}{\partial \nabla_a h_{cd}} \right) = 0.
\]

(5)
This equation can be written as

\[
L_{ab\cd} h_{cd} = -\frac{1}{2} \nabla_c \nabla^c h_{ab} + \left( \frac{1}{2} - \frac{1}{2\alpha} \right) \left( \nabla_c \nabla_c h_{ab} + \nabla_h \nabla_h h_{ab} \right)
- \left( \frac{1}{2} - \frac{1}{2\alpha} \right) \nabla_a \nabla_b h - \left( \frac{(\beta + 1)^2}{\alpha \beta^2} - \frac{1}{2} \right) g_{ab} \nabla_c \nabla_c h
- \frac{1}{2} g_{ab} \left[ 1 - \frac{2(1 + \beta)}{\alpha \beta} \right] \nabla_c \nabla_d h^c d - R^{-2} h_{ab} - \frac{n - 3}{2} R^{-2} g_{ab} h
= 0.
\] (6)

Here, \( R \) is the radius of the hyperbolic space from which the AdS spacetime is obtained.

Now we split the field \( h_{ab} \) as follows:

\[
h_{bc} = A_{bc} + B_{bc} + C_{bc},
\] (7)

where \( C_{bc}, A_{ab} \) and \( B_{ab} \) are the scalar, vector and tensor sectors, respectively. We decompose the scalar part into a complete set of modes \( C^{\lambda \sigma}_{ab} \) constructed from \( \phi^{\lambda \sigma} \), where

\[
\nabla^2 \phi^{\lambda \sigma} = -\lambda \phi^{\lambda \sigma},
\] (8)

and \( \sigma \) represents all the other labels. The modes \( C^{\lambda \sigma}_{ab} \) can be written as a sum of a traceless part \( W^{\lambda \sigma}_{ab} \) and the trace \( X^{\lambda \sigma}_{ab} \):

\[
C^{\lambda \sigma}_{ab} = X^{\lambda \sigma}_{ab} + W^{\lambda \sigma}_{ab},
\] (9)

where

\[
X^{\lambda \sigma}_{ab} = \frac{1}{\sqrt{n}} g_{ab} \phi^{\lambda \sigma},
\] (10)

\[
W^{\lambda \sigma}_{ab} = \sqrt{\frac{n}{n - 1}} \frac{1}{\sqrt{\lambda (\lambda + nR^{-2})}} \left( \nabla_c \nabla_b + \frac{\lambda}{n} g_{ab} \right) \phi^{\lambda \sigma}.
\] (11)

Here, the factors \( 1/\sqrt{n} \) and \( \sqrt{n}/\sqrt{(n - 1)\lambda + nR^{-2}} \) are the normalization factors. The trace of \( W^{\lambda \sigma}_{ab} \) obviously vanishes. The vector part \( A_{ab} \) is defined by

\[
A_{ab} = \nabla_a A_b + \nabla_b A_a,
\] (12)

where the divergence of \( A_a \) vanishes, \( \nabla^a A_a = 0 \), and this in turn implies that the trace of \( A_{ab} \) also vanishes, \( g^{ab} A_{ab} = 0 \). The vector part can also be decomposed using a complete set of modes \( A^{\lambda \sigma}_{ab} \) constructed from vector modes, \( A^{\lambda \sigma}_{ab} = \nabla_a A^{\lambda \sigma}_b + \nabla_b A^{\lambda \sigma}_a \), where [22]

\[
\nabla^a A^{\lambda \sigma}_a = 0,
\]

\[
\nabla^2 A^{\lambda \sigma}_a = (-\lambda + R^{-2}) A^{\lambda \sigma}_a.
\] (13)

Both the trace and divergence of the tensor part \( B_{ab} \) vanish, \( \nabla^a B_{ab} = g^{ab} B_{ab} = 0 \). Thus, the tensor part can be decomposed using a complete set of tensor modes \( B^{\lambda \sigma}_{ab} \), where [22]

\[
\nabla^a B^{\lambda \sigma}_{ab} = 0,
\]

\[
g^{ab} B^{\lambda \sigma}_{ab} = 0,
\]

\[
\nabla^2 B^{\lambda \sigma}_{ab} = (-\lambda + 2R^{-2}) B^{\lambda \sigma}_{ab}.
\] (14)

Furthermore, Green’s function is given by

\[
L_{cd} G_{abcd'} (x, x') = \delta_{cd} \delta_{ab'} (x, x'),
\] (15)

where for any smooth function \( f^{cd} (x) \), we have

\[
\int d^4 x \sqrt{g} \delta_{cd} \delta_{ab'} (x, x') f^{cd} (x) = f_{ab'} (x').
\] (16)
Now we can decompose this delta function and Green’s function into scalar, vector and tensor modes as follows:
\[
\delta_{cdab}(x, x') = \delta^{(C)}_{cdab}(x, x') + \delta^{(A)}_{cdab}(x, x') + \delta^{(B)}_{cdab}(x, x'),
\]
(17)
\[
G_{cdab}(x, x') = G^{(C)}_{cdab}(x, x') + G^{(A)}_{cdab}(x, x') + G^{(B)}_{cdab}(x, x'),
\]
(18)
where
\[
\delta^{(C)}_{cdab}(x, x') = \sum_{\lambda} \sum_{\sigma} C_{cd}^{\lambda\sigma}(x) C^{\lambda\sigma}_{ab}(x'),
\]
(19)
\[
\delta^{(A)}_{cdab}(x, x') = \sum_{\lambda} \sum_{\sigma} A_{cd}^{\lambda\sigma}(x) A^{\lambda\sigma}_{ab}(x'),
\]
(20)
\[
\delta^{(B)}_{cdab}(x, x') = \sum_{\lambda} \sum_{\sigma} B_{cd}^{\lambda\sigma}(x) B^{\lambda\sigma}_{ab}(x'),
\]
(21)
and
\[
G^{(C)}_{cdab}(x, x') = \sum_{\lambda} \sum_{\sigma} c_1^{\lambda\sigma} C_{cd}^{\lambda\sigma}(x) C^{\lambda\sigma}_{ab}(x'),
\]
(22)
\[
G^{(A)}_{cdab}(x, x') = \sum_{\lambda} \sum_{\sigma} c_2^{\lambda\sigma} A_{cd}^{\lambda\sigma}(x) A^{\lambda\sigma}_{ab}(x'),
\]
(23)
\[
G^{(B)}_{cdab}(x, x') = \sum_{\lambda} \sum_{\sigma} c_3^{\lambda\sigma} B_{cd}^{\lambda\sigma}(x) B^{\lambda\sigma}_{ab}(x');
\]
(24)
here, the sum is a shorthand notation and includes integrals as the AdS spacetime is non-compact. The constants $c_1^{\lambda\sigma}, c_2^{\lambda\sigma}, c_3^{\lambda\sigma}$ are determined by
\[
L_{cd}^{ab} C^{(C)}_{ab}(x, x') = \delta^{(C)}_{cdab}(x, x'),
\]
(25)
\[
L_{cd}^{ab} C^{(A)}_{ab}(x, x') = \delta^{(A)}_{cdab}(x, x'),
\]
(26)
\[
L_{cd}^{ab} C^{(B)}_{ab}(x, x') = \delta^{(B)}_{cdab}(x, x').
\]
(27)
We discuss the solutions of these equations in the following sections.

3. Scalar sector

To find the total propagator, we first deal with the scalar sector. First recall that the scalar sector was decomposed using $c_{ab}^{\lambda\sigma}$, and this was expressed in terms of $X_{ab}^{\lambda\sigma}$ and $W_{ab}^{\lambda\sigma}$, where
\[
X_{ab}^{\lambda\sigma} = \frac{1}{\sqrt{n}} g_{ab} \phi^{\lambda\sigma},
\]
(28)
\[
W_{ab}^{\lambda\sigma} = \sqrt{\frac{n}{n-1}} \frac{1}{\sqrt{\lambda (\lambda + nR^{-2})}} \left( \nabla_a \nabla_b + \frac{\lambda}{n} g_{ab} \right) \phi^{\lambda\sigma}.
\]
(29)
Furthermore, here $\lambda$ satisfies [22]
\[
\lambda \geq \frac{(n-1)^2}{4R^2}.
\]
(30)
Now we define $K_{ij}$, $i, j = 1, 2$, by
\[
L_{cd}^{ab} X_{ab}^{\lambda\sigma} = K_{11}^{\lambda\sigma} X_{ab}^{\lambda\sigma} + K_{12}^{\lambda\sigma} W_{ab}^{\lambda\sigma},
\]
(31)
We then find $c_{ij}$, $i, j = 1, 2$, by
\[
\begin{pmatrix}
  c_{11} & c_{12} \\
  c_{21} & c_{22}
\end{pmatrix}
= \begin{pmatrix}
  K_{11} & K_{12} \\
  K_{21} & K_{22}
\end{pmatrix}^{-1}.
\]
We then find $c_{12}^2 = c_{21}^2$ and
\[
c_{11} = \beta^2 \frac{(n-2)\alpha - 2(n-1)}{n-2} \lambda - (n-1)\beta R^{-2}
\]
\[
= \frac{2(n-1)\beta - \frac{n-2}{2}\alpha\beta}{n-2} R^{-2},
\]
\[
c_{22} = -\frac{2}{(n-1)(n-2)R^2} \left( \lambda + nR^{-2} \right) + \frac{\alpha}{(n-1)^2 R^2} \lambda
\]
\[
+ \frac{2(n-1)\beta - \frac{n-2}{2}\alpha\beta}{n-2} R^{-2}.
\]
where
\[
c_{12}^2 = \frac{c_{12}^2}{\sqrt{(n-1)\lambda(\lambda + nR^{-2})}},
\]
\[
c_{22}^2 = \frac{nc_{22}^2}{(n-1)\lambda(\lambda + nR^{-2})}.
\]
Define $\Delta_k(x, x')$ to be the scalar propagator with mass $kR^{-2}$ and let $\mu(x, x')$ be the geodesic distance between spacelike separated points $x$ and $x'$ in AdS spacetime and $z = \cosh^2(\mu/(2R))$; then, the scalar propagator is given by [20]
\[
\Delta_k(z) = q_0 e^{-a_0} F[a_0, a_0 - c_0 + 1; a_0 - b_0 + 1; z^{-1}],
\]
with
\[
q_0 = \frac{\Gamma(a_0)\Gamma(a_0 - c_0 + 1)}{\Gamma(a_0 - b_0 + 1)\pi^{n/2-2}} |R|^{2-n},
\]
where
\[
a_0 = \frac{1}{2} \left[ (n-1) + \sqrt{(n-1)^2 + 4k^2} \right],
\]
\[
b_0 = \frac{1}{2} \left[ (n-1) - \sqrt{(n-1)^2 + 4k^2} \right],
\]
\[
c_0 = \frac{1}{2} n.
\]
Further, let
\[
\Delta_k^{(1)}(x, x') = \frac{1}{R^{-2}} \frac{\partial}{\partial k} \Delta_k(x, x').
\]
Then, the scalar sector of the graviton propagator in AdS, in the Euclidean approach, is given by

\[ G^{(C)}_{\alpha\beta}(x,x') = g_{\alpha\beta}(x)g_{\alpha'b}(x')X(x,x') + g_{\alpha b}(x) \left( \nabla_\alpha \nabla_{\beta'} - \frac{1}{n} g_{\alpha b}(x) \nabla \nabla c \right) Y(x,x') \\
+ g_{\alpha b}(x) \left( \nabla_\alpha \nabla_b - \frac{1}{n} g_{\alpha b}(x) \nabla \nabla c \right) Y(x,x') \\
+ \left( \nabla_\alpha \nabla_b - \frac{1}{n} g_{\alpha b}(x) \nabla \nabla c \right) \left( \nabla_\alpha' \nabla_{b'} - \frac{1}{n} g_{\alpha' b'} \nabla \nabla c' \right) Z(x,x'). \tag{43} \]

where

\[ X(x,x') = \frac{\beta^2 n^2}{n - 2} \left( \frac{n - 2 - 2(n - 1)}{n - 2} \Delta^{(1)}_{-(n-1)\beta}(x,x') \right) \tag{44} \]

\[ Y(x,x') = \frac{2\beta}{n(n - 2)} \left[ n + (n - 1)\beta - \frac{n - 2}{2}\alpha\beta \right] \Delta^{(1)}_{-(n-1)\beta}(x,x'), \tag{45} \]

\[ Z(x,x') = -\frac{2}{(n - 1)(n - 2)R^2} \Delta_0(x,x') + \frac{\alpha}{(n - 1)^2R^2} \Delta_0(x,x') \\
+ \frac{2(n - 1) - (n - 2)\alpha}{(n - 1)(n - 2)R^2} \Delta^{(1)}_{-(n-1)\beta}(x,x') \\
- \frac{2n + (n - 1)\beta - \frac{n - 2}{2}\alpha\beta}{(n - 1)(n - 2)R^2} \Delta^{(1)}_{-(n-1)\beta}(x,x'). \tag{46} \]

Now after finding the graviton propagator in the most general gauge, we will discuss certain limits of this propagator. In the gauge, where \( \alpha = 0 \) and \( \beta = n/(1 - n) \), we have

\[ X(x,x') = \frac{2}{(n - 1)(n - 2)} \Delta_n(x,x') \tag{47} \]

and

\[ Y(x,x') = Z(x,x') = 0. \tag{48} \]

So we can see that this part of the graviton propagator can be written in a simple form in the gauge when \( \alpha = 0 \) and \( \beta = n/(1 - n) \). This is the gauge chosen in the work of D'Hoker, Freedman, Mathur, Matusis and Rastelli \cite{11}. Furthermore, for \( \beta = 0 \), we have

\[ X(x,x') = Y(x,x') = 0 \tag{49} \]

and

\[ Z(x,x') = -\frac{2}{(n - 1)(n - 2)R^2} \Delta_n(x,x') \\
+ \left[ \frac{\alpha}{(n - 1)^2R^2} + \frac{2(n - 1) - (n - 2)\alpha}{(n - 2)(n - 1)^2R^2} \right] \Delta_0(x,x') \\
- \frac{2n}{(n - 1)(n - 2)R^2} \Delta^{(1)}_0(x,x'). \tag{50} \]

Finally we take \( \alpha = 0 \) and \( \beta = n(n - 2)/4(n - 1) \). For this value of \( \beta \), the scalar propagator \( \Delta^{(1)}_{-(n-1)\beta}(x,x') \) becomes the propagator for the conformally coupled case. Thus, for \( \alpha = 0 \)
and $\beta = n(n - 2)/4(n - 1)$, we have
\[
X(x, x') = \frac{(n - 2)}{2(n - 1)} \Delta_m(x, x') - \frac{n(n - 1)(n + 2)}{2(n - 2)} R^{-2} \Delta_m^{(1)}(x, x'),
\]
\[
Y(x, x') = \frac{n(n + 2)}{2(n - 1)} \Delta_m^{(1)}(x, x'),
\]
\[
Z(x, x') = -\frac{2}{(n - 1)(n - 2)R^{-4}} \Delta_m(x, x') + \frac{2}{(n - 2)(n - 1)R^{-4}} \Delta_m(x, x')
- \frac{2}{n(n + 2)} \frac{1}{(n - 1)(n - 2)R^{-4}} \Delta_m^{(1)}(x, x'),
\]
where $m = n(2 - n)/4$.

### 4. Total propagator

To calculate the total graviton propagator we first find Green’s function for the vector part. Green’s function for the vector part of the graviton propagator in the AdS spacetime is obtained by a trivial modification of the calculations done to calculate Green’s function for the vector part of the graviton propagator in de Sitter spacetime [19]. However, it will turn out that unlike Green’s function for the vector part in de Sitter spacetime, Green’s function for the vector part in AdS spacetime contains a very complicated term which cannot simply be expressed in terms of elementary functions.

To do so we first find the equation of motion for the vector part by substituting $h_{ab} = A_{ab}$ in equation (6). Thus, the equation of motion for the vector part is given by
\[
\frac{1}{\alpha} [-\nabla^2 + (n - 1)R^{-2}] A_a = 0.
\]
Now decomposing $A_a$ into a complete set of vector modes and using equation (13), we get
\[
\frac{1}{\alpha} [\lambda_a + (n - 2)R^{-2}] A_a = 0.
\]
Green’s function for this equation can be obtained by repeating the calculations that were done for obtaining Green’s function of the vector part in the four-dimensional de Sitter spacetime [19] for the $n$ dimensional AdS spacetime. Thus, Green’s function for equation (53) can be written as
\[
G_{ab}(z) = -P_{ab}(z) - \frac{1}{4(n - 1)^2} \nabla_a \nabla_b \Delta_0(z).
\]
If $n_a$ and $n_b$ are unit tangent vectors along the geodesics at $x$ and $x'$, respectively, then the parallel transport $g_{ab}(z)$ is defined by $g_{ab}(z) = -n_a n'(x')$. Now in the Allen and Jacobson formalism [20], the propagator $P_{ab}(z)$ is given by
\[
P_{ab}(z) = a(z)g_{ab}(z) + b(z)n_a n_b,
\]
where $a(z)$ and $b(z)$ again only depend on $z$ which in turn only depends on the geodesic distance between $x$ and $x'$. They are given by
\[
a(z) = \left[ \frac{1}{n - 1} R \sinh(\mu R^{-1}) \frac{d}{d\mu} + \cosh(\mu R^{-1}) \right] \gamma(z),
\]
\[
b(z) = \left[ \frac{1}{n - 1} R \sinh(\mu R^{-1}) \frac{d}{d\mu} + \cosh(\mu R^{-1}) - 1 \right] \gamma(z),
\]
where
\[
\gamma(z) = \left[ \frac{\partial}{\partial m^2} [q^a F(a_1, b_1; c_1; z)] \right]_{m^2 = 2R^{-2}(n - 1)},
\]
So Green’s function for the tensor part in the covariant gauge can be written as

\[ G = \frac{(1 - n)\Gamma(a_1)\Gamma(b_1)}{\Gamma(c_1)2\pi^{(n+1)/2}m^{2-n}}R^{-n}. \]  

Here, \(a_1, b_1\) and \(c_1\) are given by

\[ a_1 = \frac{1}{2}[(n + 1) + \sqrt{(n - 3)^2 + 4m^2R^2}], \]
\[ b_1 = \frac{1}{2}[(n + 1) - \sqrt{(n - 3)^2 + 4m^2R^2}], \]
\[ c_1 = \frac{1}{2}n + 1. \]

Now by repeating the argument used to calculate Green’s function of the vector part in the four-dimensional de Sitter spacetime \([19]\), for the \(n\) dimensional AdS spacetime we obtain the final expression for Green’s function for the vector part of the graviton propagator in the AdS spacetime

\[ G^{(A)}_{a\beta a'b'}(x, x') = a[\nabla_a \nabla_{a'} G_{b\beta}(x, x') + \nabla_b \nabla_{a'} G_{a\beta}(x, x') + \nabla_a \nabla_b G_{aa'}(x, x')] + \nabla_a G_{a\beta}(x, x') + \nabla_b G_{b\beta}(x, x'). \]  

As the tensor part does not depend on the gauge parameters, it will be the same in all gauges and the full graviton propagator has already been obtained in a certain Landau gauge \([11]\). Thus, Green’s function for the tensor part in the covariant gauge will be the same as that which was obtained in that Landau gauge; however, we will include it here for completeness. So Green’s function for the tensor part in the covariant gauge can be written as

\[ G^{(R)}_{a\beta a'b'}(x, x') = \sum_{i=1}^{5} G_i(\mu)O^{(i)}_{a\beta a'b'}(x, x'), \]

where \(O^{(i)}_{a\beta a'b'}\) are constructed from all possible linear combinations of the metrics and unit normals at \(x\) and \(x'\) and along with the parallel transport. So we have

\[ O^{1}_{a\beta a'b'} = g_{ab}g_{a'b'}, \]
\[ O^{2}_{a\beta a'b'} = n_a n_{a'} n_b n_{b'}, \]
\[ O^{3}_{a\beta a'b'} = g_{ab}g_{a'b'} + g_{a'b}g_{a'b'}, \]
\[ O^{4}_{a\beta a'b'} = g_{ab}n_a n_{a'} n_{b'} + n_a n_b g_{a'b'}, \]
\[ O^{5}_{a\beta a'b'} = g_{ab}n_{a'b'} n_b + g_{a'b}n_a n_{b'}. \]

The coefficient \(G_i(\mu)\) also only depends on \(z\) and has been explicitly calculated in terms of a single function \(g(z)\) as \([11]\)

\[ G_1 = \frac{1}{n(n-2)} \left( \frac{4z(z-1)^2}{n+1} g''(z) + 4z(z-1)(2z-1)g'(z) + (4nz(z-1) + n-2)g(z) \right), \]
\[ G_2 = -4(z-1)^2 \left( \frac{z^2}{n(n+1)} g''(z) + \frac{2(n+2)}{n(n+1)} g'(z) + g(z) \right), \]
\[ G_3 = \frac{1}{2(2-n)} \left( \frac{4(n-1)z^2(z-1)^2}{n(n+1)} g''(z) + \frac{4(n-1)z(z-1)(2z-1)}{n} g'(z) + \frac{n}{4(n-1)z(z-1) + n-2)g(z)} \right), \]
\[ G_4 = -4z(z-1) \left( \frac{z(z-1)}{n+1} g''(z) + (2z-1)g'(z) + ng(z) \right). \]
\[ G_5 = -\frac{x - 1}{n - 2} \left( \frac{2(n-1)z^2(z-1)}{n(n+1)} g'(z) + \frac{z(4n-1)z - 3n + 4}{n} g(z) \right) + (2(n-1)z - n + 2) g(z), \]

where \( g(z) \) is given by

\[ g(z) = g_0(z) + g_1(z) + g_2(z) + g_3(z), \]

with

\[ g_0(z) = \frac{n(n+1)\Gamma((n-2)/2)}{(n-1)2^{n+2}\pi^{n/2}} \int_0^\infty \frac{d\zeta}{[\zeta(z-1)]^{(n+2)/2}}, \]

\[ g_1(z) = -\frac{n(n+1)\Gamma((n+2)/2)(4n-1)z}{2(n-1)n \pi^{n/2} \Gamma((n-1)/2)^2}, \]

\[ g_3(z) = \frac{n(n+1)\Gamma((n+2)/2)}{(n-2)(n-1)2^{n+2}\pi^{n/2}} \frac{2z - 1}{[\zeta(z-1)]^{(n+2)/2}}, \]

\[ g_4(z) = (-1)^{(n-1)/2} \frac{\Gamma((n+3)/2)n}{(n+2)(n-1)} \frac{1}{2\pi^{n/2}} F[2, n + 1; (n + 4)/2; 1 - z]. \]

Thus, the total graviton propagator in the AdS spacetime is given by the sum of the scalar, vector and tensor parts of the graviton propagator. We can now write the graviton propagator, in the AdS spacetime, in the most general covariant gauge as

\[ G_{\mu\nu}(x, x') = G_{\mu\nu}(x, x')^{(C)} + G_{\mu\nu}(x, x')^{(A)} + G_{\mu\nu}(x, x')^{(B)}, \]

where the scalar contribution to the graviton propagator \( G_{\mu\nu}(x, x')^{(C)} \) is given by equation (43), the vector contribution to the graviton propagator \( G_{\mu\nu}(x, x')^{(A)} \) is given by equation (60), and the tensor contribution to the graviton propagator \( G_{\mu\nu}(x, x')^{(B)} \) is given by equation (61).

5. Conclusion

In this paper, we have derived the graviton propagator in the most general covariant gauge. This propagator can be used to calculate the correlation functions in the bulk of AdS spacetime, which in turn can be used to study certain aspects of the AdS/CFT correspondence. It may be noted that the graviton propagator in the AdS spacetime has already been derived in a certain Landau gauge [11] and used to study certain aspects of the AdS/CFT correspondence [23–27].

We have also seen in this paper that in any gauge, apart from the Landau gauge, the graviton propagator in the AdS spacetime has a very complicated form. This is because the the vector part of the graviton propagator in the AdS spacetime contains a derivative of the hypergeometric function which cannot be simply expressed in terms of elementary functions. As the vector part of the graviton propagator is proportional to the gauge parameter \( \alpha \), it vanishes in the Landau gauge. It may be noted that in the de Sitter spacetime, this problem does not arise and it is possible to write the derivative of the hypergeometric function in terms of elementary functions, at least in three dimensions [18, 19]. As in the work of D’Hoker, Freedman, Mathur, Matusis and Rastelli [11], a certain Landau gauge was chosen, the AdS graviton propagator they derived did not contain this complicated term and thus could be easily written in terms of elementary functions.

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