Parameters of an avalanche of runaway electrons in air under atmospheric pressure

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Abstract. The features of runaway-electron avalanches developing in air under atmospheric pressures are investigated in the framework of a three-dimensional numerical simulation. The simulation results indicate that an avalanche of this type can be characterized, besides the time and length of its exponential growth, by the propagation velocity and by the average kinetic energy of the runaway electrons. It is shown that these parameters obey the similarity laws applied to gas discharges.

1. Introduction
In 1992 [1], the effect, which is called runaway electron breakdown (REB), was predicted. For this type of breakdown, the main role is played by runaway electrons (REs) with an energy of several hundred keV and more. It is assumed that the REB is realized in lightning discharges. Breakdown by runaway electrons is an avalanche-like multiplication in a substance of fast electrons with an energy of 0.1–10 MeV [2]. The proposed existence of REBs provoked an active interest in runaway electrons, which have been extensively studied both theoretically [1–4] and experimentally [5–8].

In this paper, using numerical calculations based on the Monte Carlo method in which the process of electron acceleration under the influence of a constant electrostatic force was modeled. The basic equation of the model is an equation describing the variation of the momentum of an electron in a constant electric field $E$:

$$\frac{dp}{dt} = -eE - F(\varepsilon_k) - (\Delta p)_{el},$$

where $p = mv/\sqrt{1 - v^2/c^2}$ is the electron momentum, $(\Delta p)_{el}$ is the variation of the electron momentum on elastic scattering and $F(\varepsilon_k)$ is a braking force. The equation for the change of momentum was solved for each of the electrons. The total number of electrons is increased by law [2] $N_{es} = N_0 \exp(l/l_a) = N_0 \exp(t/\tau_a)$, where $N_0$ is the initial number of particles (in our case $N_0 = 1$), $l$ is the distance traveled by the avalanche, $t$ the avalanche propagation time, $l_a$ and $\tau_a$ the length and the exponential rise time of the RE avalanche. The technique for modeling avalanches of RE is described in detail in [9–12].

2. Calculation of the parameters of a runaway electron avalanche
The numerical simulation was performed for air with varying gas pressure and electrostatic field. The air was assumed to consist of nitrogen (78 wt.%), oxygen (21 wt.%), and argon (1 wt.%).
The calculations were carried out, supposing that at the time zero \((t = 0)\) there was one electron, up to the occurrence of \(10^6\)–\(10^8\) runaway electrons \((N_{es})\). An avalanche having passed through a gas leaves an ion “cloud”. The parameters of the ion cloud were calculated assuming that the ions were immobile. It should be noted that not all secondary electrons become permanently accelerated. The number of REs in the calculations made 40–80% of the number of secondary electrons, \(N_{sec}\), depending on electric field and gas pressure.

The sought-for quantities were obtained as particle ensemble averages. First, the spatial characteristics of the avalanche were calculated. The radius vector of the avalanche center point, \(R_{av} \equiv \{X_{av}, Y_{av}, Z_{av}\}\), was found as

\[
R_{av} = \frac{1}{N_{es}} \sum_{i=1}^{N_{es}} R^i, \tag{1}
\]

where \(R^i \equiv \{x^i, y^i, z^i\}\) is the radius vector of the \(i\)-th runaway electron. Throughout the calculations, the \(x\) and \(y\) components of the avalanche center radius vector (normal to the electrostatic field) were negligible compared with the \(z\) component \((Z_{av})\) parallel to the field.

The radius vector of the ion cloud center point \(R_{cl} \equiv \{X_{cl}, Y_{cl}, Z_{cl}\}\), was calculated as

\[
R_{cl} = \frac{1}{N_{ion}} \sum_{i=1}^{N_{ion}} R^i_{ion}, \tag{2}
\]

where \(R^i_{ion}\) is the radius vector of the \(i\)-th ion and \(N_{ion} = N_{es} - 1\) is the number of ions in the cloud. As with the radius vector of the avalanche center point, the \(x\) and \(y\) components of the radius vector of the ion cloud center point were negligible compared with the \(z\) component.

In addition, the rms deviations from the avalanche center point were calculated as

\[
\Delta = \sqrt{\frac{1}{N_{es}} \sum_{i=1}^{N_{es}} (z^i - Z_{av})^2}, \quad \Delta_\perp = \sqrt{\frac{1}{N_{es}} \sum_{i=1}^{N_{es}} [(x^i - X_{av})^2 + (y^i - Y_{av})^2]}. \tag{3}
\]

The quantities \(\Delta_\parallel\) and \(\Delta_\perp\) characterize the avalanche dimensions in the directions parallel and normal to the electrostatic field, respectively.

Second, the avalanche characteristics depending on the runaway electron distribution in the velocity domain were determined. The average velocities in the directions parallel and normal to the electrostatic field \((V_\parallel\) and \(V_\perp\), respectively) were found as

\[
V_\parallel = \frac{1}{N_{es}} \sum_{i=1}^{N_{es}} v^i_z, \quad V_\perp = \frac{1}{N_{es}} \sum_{i=1}^{N_{es}} \frac{v^i_x (x^i - X_{av}) + v^i_y (y^i - Y_{av})}{\sqrt{(x^i - X_{av})^2 + (y^i - Y_{av})^2}} = \frac{1}{N_{es}} \sum_{i=1}^{N_{es}} v^i_\perp, \tag{4}
\]

where \(v^i \equiv \{v^i_x, v^i_y, v^i_z\}\) is the velocity vector of the \(i\)-th runaway electron.

Third, the energy characteristics of the electron avalanche were calculated. The average kinetic energy of the runaway electrons was found as

\[
\bar{\varepsilon}_k = \frac{1}{N_{es}} \sum_{i=1}^{N_{es}} \varepsilon^i_k, \tag{5}
\]

where \(\varepsilon^i_k\) is the kinetic energy of the \(i\)-th runaway electron.

The avalanche “temperature” profiles along and transverse to the electrostatic field \((T_\parallel\) and \(T_\perp\), respectively) were calculated using the formulas

\[
kT_\parallel = \frac{m}{N_{es}} \sum_{i=1}^{N_{es}} (v^i_z - V_\parallel)^2, \quad kT_\perp = \frac{m}{2N_{es}} \sum_{i=1}^{N_{es}} (v^i_\perp - V_\perp)^2, \tag{6}
\]
where \( k \) is Boltzmann’s constant. The factors in (6) were determined proceeding from the fact, known from thermodynamics, that \( \frac{1}{2}kT \) is accounted for by each degree of freedom. Certainly, the avalanche “temperatures” bear no relation to actual avalanches; they only characterize the rms deviations from average avalanche velocities.

3. Calculated parameters of a runaway electron avalanche

Figures 1–3 present the results of the simulation of a runaway electron avalanche propagating in air at atmospheric pressure at an electric field strength of 200 kV/cm. The calculations were carried out up to occurring \( 10^8 \) runaway electrons in the avalanche. From figures 1–3 it can be seen that at the early stage of avalanche propagation (at a small number of REs), the calculated quantities vary irregularly due to transient phenomena. Once the number of REs in the avalanche reaches \( 10^3 \)–\( 10^4 \), the transients cease and the variations in avalanche parameters become gradual.

In figure 1, the spatial characteristics of the avalanche calculated using formulas (1)–(3) are plotted versus the number of electrons in the avalanche. It can be seen that the characteristic avalanche dimensions along and transverse to the field increase with number of REs approximately in proportion with \( \ln N_{es} \). The distance between the avalanche and ion cloud center points, \( Z_{av} - Z_{cl} \) (curve 1); the dimension along the field, \( \Delta_\parallel \) (curve 2); and the dimension transverse to the field, \( \Delta_\perp \) (curve 3).

![Figure 1](image_url)

**Figure 1.** Dimensions of an RE avalanche in air at a pressure of 1 atm and an electrostatic field of 200 kV/cm versus number of REs: the distance between the avalanche and ion cloud center points, \( Z_{av} - Z_{cl} \) (curve 1); the dimension along the field, \( \Delta_\parallel \) (curve 2); and the dimension transverse to the field, \( \Delta_\perp \) (curve 3).
Figure 2. Velocities of an RE avalanche in air at a pressure of 1 atm and an electrostatic field of 200 kV/cm versus number of REs: the avalanche velocity along the field, $V_\parallel$ (curve 1); and transverse to the field, $V_\perp$ (curve 2).

$\sim 2.3 \times 10^{10}$ cm/s. The characteristic exponential growth length and time of the avalanche are related with its propagation velocity as $\tau_a \approx l_a/V_\parallel$.

The existence of the transverse velocity of an avalanche is due to two factors. First, when a fast electron ionizes a gas atom, the velocity vector of the secondary electron is directed almost normal to that of the primary electron (if the energy of the primary electron is substantially greater than that of the secondary one, $\varepsilon_2' \ll \varepsilon_k$). As the fast electrons travel along the field, the velocity vector of the secondary electrons is originally directed mainly transverse to the field. Second, the elastic scattering of electrons by gas atoms also gives rise to the electron velocity transverse to the field. For fast electrons, the cross section for elastic scattering is highly nonisotropic [13], and so, when participating in single collisions, they scatter within small angles. Therefore, the effect of elastic collisions is most pronounced in low electrostatic fields for which the exponential growth length of an RE avalanche is large. As can be seen from figure 2, the transverse velocity $V_\perp$ also tends to a stationary value, but, because of the elastic scattering effects, it reaches a stationary value within a longer time than $V_\parallel$ does.

Figure 3 presents the energy characteristics of an avalanche calculated using formulas (5), (6) and plotted versus the number of electrons in the avalanche. It can be seen that when $N_{es} > 10^4$, all avalanche characteristics take stationary values, and we have $\bar{\varepsilon}_k \gg kT_\parallel \gg kT_\perp$. This situation is typical for avalanches in strong electric fields; for the low electric fields close to $E_c$ (where $E_c$ is the electric field corresponding to the minimum braking force), we have $\bar{\varepsilon}_k \gg kT_\parallel \approx kT_\perp$ due to pronounced elastic scattering. The fact that the energy characteristics become stationary indicates that the RE velocity distribution function becomes invariable in form, and only the number of electrons in the avalanche increases.
4. Conclusion

The features of RE avalanches developing in air have been investigated using a numerical simulation. The numerical model took into account the electron braking due to inelastic energy loss, the elastic scattering of electrons by the gas atoms, and the generation of secondary electrons. The simulation results have shown that the avalanche propagation velocities, $V_\parallel$ and $V_\perp$, and “temperatures”, $T_\parallel$ and $T_\perp$, and the average kinetic energy of the electrons, $\bar{\varepsilon}_k$, tend to stationary values with increasing number of runaway electrons. Thus, these quantities, along with the parameters $l_a$ and $\tau_a$, can be considered characteristics of exponentially growing RE avalanches.

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