Quantitative monitoring method for analyzing the erosion of a landslide dam discharge channel using three-dimensional terrestrial laser scanning

Nan Jiang, Hai-bo Li, Qing-jian Kou and Jia-wen Zhou

ABSTRACT
While excavated discharge channels are often used for the emergency treatment of landslide-dammed lakes, the erosion of the discharge channel of a landslide dam is complex and rarely monitored on site. This work presents a quantitative monitoring method based on three-dimensional terrestrial laser scanning technology (TLS) for analyzing the erosion of a landslide dam discharge channel and then applies this method to evaluate the emergency engineering treatment of the 2018 Baige landslide-dammed lake in 2019. The proposed method uses TLS technology to accurately track the dynamic evolution of the discharge channel and to quantify the lateral erosion. Moreover, qualitatively analysis for vertical erosion is conducted based on the difference between actual and hypothetic river elevation. The results indicate that erosion processes of landslide deposition fluctuate as the results of flow discharge variations. Damage to the covering layer consisting of large particles is considered to be the main cause of this phenomenon. According to the relationship between erosion intensity ($E_I$) and flow velocity, an empirical formula is deduced for erosion analysis. The key parameter in this formula is determined by the variation rate of discharge, the flow velocity, the composition of riverbank and the flow field distribution.

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1. Introduction
A landslide-dammed lake is typically a lake formed as a result of a landslide blocking the main river channel, which is often triggered by strong earthquakes or by heavy rainfall (Li et al. 2002; Hsu and Hsu 2009; Romeo et al. 2017). A landslide dam can be formed in a wide range of geological and geomorphological settings, such as debris flows, landslides, rock/debris slides and rockfalls (Fan et al. 2017). Korup (2002)
studied 232 landslide dams in New Zealand, and found that 39% of them were triggered by earthquakes. The landslide dams usually consist of a mixture of rock and soil with uneven particle grading, low compactness and poor erosion resistance (Andrews 1983; Kundzewicz et al. 2019; Liao et al. 2020). Therefore, seepage and/or overtopping can occur on a landslide dam due to the loose composition, resulting in catastrophic floods around the downstream area and lead to considerable losses of property and life (Pickert et al. 2011; Liao et al. 2018; Chen et al. 2020). For example, the breakage of an earthquake-triggered landslide dammed-lake in 1786 on Dadu River, Southwest China, caused more than 100,000 casualties according to a historical recording (Dai et al. 2005). Therefore, it is necessary to carry out artificial interventions on landslide-dammed lakes to reduce their potential harm and risks (Kundzewicz et al. 2019). A widely used approach involves excavating the discharge channel at the top of a landslide dam. For loose landslide dams, drainage channels (artificially excavated or naturally formed) are usually eroded and expanded during the drainage process, leading to enhanced water flow in a short period of time that eventually form huge floods (Tuan et al. 2008; Xue et al. 2015a). Compared to the natural failure of a landslide dam, a discharge channel allows the dammed-lake to be released earlier, which significantly reduces the flood peak of the outburst flood during the dam failure and mitigates property damage downstream (Kundzewicz et al. 2019). The main factor controlling the peak flow of the outbreak flood is the rate of discharge channel expansion (Cao et al. 2004). The slowly expanding discharge channel can smooth the flow curves in hydrographs, delays the arrival of the flood and reduce its peak, thus providing sufficient time for flood control measures in downstream areas (Xu et al. 2015b; Chen et al. 2020). Therefore, it is of significant importance to study the erosion process of the discharge channel, which can provide a sufficient basis for regional disaster prevention and mitigation plans.

Many laboratory tests and numerical simulations have analyzed the dynamics of the discharge channels during drainage (Darby et al. 2002; Cao et al. 2004; Kim et al. 2013; Zhou et al. 2013; Zhong et al. 2017; Liu and He 2018). However, few reports have focussed on on-site quantitative monitoring. Therefore, this work proposed a quantitative monitoring method based on three-dimensional terrestrial laser scanning (TLS) technology for monitoring the dynamic erosion of discharge channels in the field. The TLS technology is a form of remote laser technology that can remotely acquire spatial information on objects (Bitelli et al. 2004; Travelletti et al. 2012). It is fast and highly accurate (Lim et al. 2005; Monserrat and Crosetto 2008) and can accurately monitor the erosion of a discharge channel. The main tasks involved in this on-site monitoring method include spatial data acquisition, data preprocessing, characteristic extraction and the quantitative calculation of erosion processes. This approach enables one to determine how a discharge channel evolves under water flow erosion, what controlling factors are involved, and how these factors affect the erosion process.

This study involved the on-site evaluation of the emergency engineering treatment of the 2018 Baige landslide-dammed lake in 2019. The 2018 Baige landslide-dammed lake formed as a result of two major rock avalanches that blocked the Jinsha River twice at the same position (Hu et al. 2020). After the second rock avalanche, risks to the barrier lake became severe, and emergency engineering treatments including excavating a discharge channel and removing residual deposits were carried out by the
Sichuan and Tibetan governments (Li et al. 2019). On-site monitoring of the discharge channel was carried out during the latter treatment. In this study, the discharge channel was divided into four zones according to geological conditions, deposition structures and morphology of riverbank. The evolution of the discharge channel was analyzed by extracting and comparing river and deposition boundaries in time series. Two indexes, the average erosion distance (AED) and erosion intensity (EI), were introduced to quantitatively analyze the lateral erosion process, and an empirical formula for calculating EI was deduced. The vertical evolution of the riverbed was analyzed in terms of the river elevation dynamics. Moreover, the initiation conditions of particles, the randomness of erosion process, the fluctuation of EI and the influence of riverbank morphology were discussed.

2. Materials and methods

The dynamic erosion of a discharge channel is complex. In general, it involves three phases: downcutting of the riverbed, lateral erosion of the river banks and collapses on bank slopes (Figure 1a). The Downcutting and lateral erosion are the results of direct contact between the discharge channel and water, resulting in a large number of earth and rock particles being lifted and transported by the flow (Liu et al. 2019). As the undercutting process is typically much faster than the lateral erosion, the discharge channel will gradually form a deep V-shaped groove (Coleman et al. 2002; Luo et al. 2014). Under the lateral erosion of the slope toe, the top of the V-shaped channel gradually become unstable, leading to the occurrence of bank collapse (Figure 1b). Thus, the bank collapse is not the direct result of water erosion and it should be excluded when studying discharge channel erosion.

2.1. Data collection

The TLS is one of the most popular techniques for terrain and topography mapping. The 3-D point cloud data acquired by TLS can reflect a variety of information about

Figure 1. Evolution of the discharge channel with landslide deposits: (a) lateral erosion, vertical erosion and bank collapse in the discharge channel and (b) image of bank collapses.
observed objects such as coordinates, reflectance, amplitude, and colours (Teza et al. 2007; Fabbri et al. 2017). The scanner emits and receives the laser beam reflected from the surface of the object, and calculates spatial coordinates of the object by calculating the attitude (rotation angle and inclination angle) of the instrument and the movement time of the laser (Teza et al. 2008; Kociuba et al. 2014). Therefore, TLS technology can obtain a large number of three-dimensional spatial information in a very short time (Lim et al. 2005; Kuhn and Prüfer 2014). The measuring range of the latest TLS equipment has reached several kilometers, which provides favourable conditions for terrain deformation monitoring in mountainous areas (Bitelli et al. 2004). However, the TLS is unable to penetrate the water surface, which poses technical challenges for the terrain acquisition of underwater riverbed. In addition, when obtaining the point clouds of the bank slopes near the river, the fluctuating flow will interfere with the TLS, resulting in the incomplete point clouds of the bank slopes, especially at the junction of the flow and the river bank.

### 2.2. Quantification analysis of erosion

Most landslide dams are composed of sand, gravel, rock and boulders. These mixtures range in particle size from a few millimeters to several meters and are randomly distributed. Essentially, the erosion of these mixtures by flow is the result of the water carrying away the particles within riverbank. According to Achers and White (1973), whether or not the particles are swept away by the current depends (a) on the pressure differential that occurs as the water flows around the particles, which depends on the instantaneous water velocity at the surface of the particles, and (b) on the gravitational force soil structure, mineral and mechanical composition, dry density, plasticity index and organic species content of the mixture, which are difficult to express mathematically (Kramer 1935). In order to simplify the complex practical situation, two assumptions were made in this study: (a) the particles of landslide dam are uniform distribution and (b) the instantaneous water velocity around the discharge channel is equal to the average velocity of cross section.

In this study, the quantitative analysis of erosion process is focussed on the lateral erosion of the discharge channel, as laser scanners couldn’t effectively penetrate water

*Figure 2. Quantitative method for measuring lateral erosion: (a) the determination of river boundary and deposition boundary and (b) quantitative erosion calculation method.*
surfaces to measure an underwater channel (Monserrat and Crosetto 2008). To avoid interference from bank collapses, the river boundary was used as a characteristic line to represent the edge of the discharge channel (Figure 2a). The river boundaries (polylines) in different periods were extracted from 3-D point cloud data and projected onto a 2-D plane (X-Y plane) (Figure 2b). Therefore, it is possible to determine how discharge channel was eroded by tracing the evolution of the river boundaries. The deviation between actual and measured erosion distance is determined by water level changes. Slight water level changes would not cause significant errors, because the slope of a discharge channel is steep (70°-90°) due to a faster down cutting speed (Rinaldi et al. 2008; Luo et al. 2014; Liu et al. 2019).

For a single particle or particles of a cross section, the instantaneous erosion process is stochastic because the driving force (hydrodynamic pressure) is random. Therefore, we decided to study discharge channel erosion processes based on longer temporal episodes and larger spatial scales. We divided the discharge channel into several parts according to their material composition and constitution, and study their erosion processes over a long period of time separately. The erosion area (EA) was used to quantitatively calculate the range of erosion. The value of EA is equal to the area enclosed by the river bank before and after erosion. However, the eroded area is not usually evenly distributed along a river (Jia et al. 2010), as it is affected by the material composition and the compactness of bank as well as the flow field distribution. Therefore, the average erosion distance (AED) is used to quantify the erosion process over a large area, which is obtained through dividing the EA by the river bank length (RL) before erosion (Figure 2b). The formula for AED can be expressed as follows:

\[
AED = \frac{EA}{RL} \quad (1)
\]

where EA denotes the erosion area and RL denotes the river bank length before erosion.

To reflect the dynamic of erosion over time, the erosion intensity (EI) is calculated through dividing the AED by time interval (T). The formula for EI is outlined as follows:

\[
EI = \frac{AED}{T} \quad (2)
\]

where T is the time interval between two scanning operations.

2.3. Calculation of flow velocity

It is of great importance to determine the flow velocity of a river, which pose significant importance for erosion process of the discharge channel (Aleixo et al. 2011). However, with seasonal changes, rainfall, and lateral and vertical erosion, the flow discharge and cross-sectional area change continuously, resulting in constant changes in flow velocity. Moreover, the actual flow velocity of a river is complex and is
measured as a gradient distribution from the river surface to the riverbed. Therefore, for a uniform flow and trapezoidal river channel, a simplified calculation method using Manning’s formula is employed to calculate the average river flow velocity (Attari and Hosseini 2019; Tuozzolo et al. 2019). Manning’s formula is expressed as follows:

$$V = \frac{1}{n} \cdot R^{\frac{2}{3}} \cdot i^{\frac{1}{2}}$$  \hspace{1cm} (3)

where $n$ is the roughness coefficient of the riverbed; $R$ is the hydraulic radius and $i$ is the riverbed slope. The flow chart of the calculations is shown in Figure 3.

3. The 2018 Baige landslide-dammed lake

3.1. Overview

The present work evaluated the emergency engineering treatment of the 2018 Baige landslide-dammed lake in 2019. At the end of 2018, two major rock avalanches occurred on the right bank of the Jinsha River (E 98°42'17.98", N 31°04'56.41") (Li et al. 2019), bordering the Tibet Autonomous Region and Sichuan Province and roughly
54 kilometers away from the downstream Yebatan Hydropower Station (under construction) (Figure 4).

The first rock avalanche occurred at 22:06 on October 10. Approximately 15 million cubic meters of soil and rock slid into the river channel along the slope (Figure 5a), blocking the Jinsha River and forming a large landslide-dammed lake. The landslide dam reached approximately 60-115 m in height, and two days later the lake water overflowed the dam, and a natural discharge channel formed (Figure 5c). After 7 hours, the incoming upstream water and outflow reached a balance, and the dam gradually disintegrated under water erosion. At 17:21 on December 3, the second rock avalanche occurred. Approximately $6.0 \times 10^6$ m$^3$ of soil and rock fell from the top of the first rock avalanche (Figure 5a) and directly blocked the natural discharge channel (Figure 5c). The new landslide dam was roughly 30 meters taller than the former one. Five days later, an emergency engineering treatment team arrived at the site, and three days later, an artificial discharge channel was excavated at the top of the landslide dam (Figure 5b). The water level of the barrier lake reached the bottom of the discharge channel at 04:54 on December 12. As the discharge channel began to drain, the discharge channel then began to down cut and formed a V-shaped breach. Bank collapses occurred on the top of the breach, expanding the cross-section rapidly (Figure 5c). A balance between inflows and outflows was reached the next morning. After assessing potential risks of the remnant rock slope, another round of emergency treatment, excavation and residual deposition removal was carried out to prevent the river from becoming blocked again. The project started in June 2019 and was
completed on 11 July 2019 (Figure 6). The majority data for this study were collected during this project while early data acquisition taking place on 26 April.

### 3.2. Hydrometeorological conditions

Water erosion is one of the most important factors that shapes the evolution of a discharge channel. The flow of the Jinsha River is mainly affected by rainfall with pronounced flood and dry seasons. The flood season occurs from June to October every year. Figure 7 shows monthly rainfall in this region from 2016 to 2019 (China...
Meteorological Data Service Center) and the discharge of Jinsha River for 2019. The flow discharge of Jinsha River is based on the diversion flow of the Yebatan Hydropower Station (collected every 12 hours).

3.3. On-site work

For this work, we mainly analyzed the discharge channel on the right bank of the Jinsha River. This part of the river bank had not been affected by human activity.

![Figure 6](image1.jpg)

**Figure 6.** The evolution of the discharge channel of Baige landslide dam: (a) the discharge channel after the second emergency engineering treatment; (b\(_1\)) - (b\(_2\)) the landslide dam before and after the first emergency engineering treatment; (c\(_1\)) - (c\(_3\)) satellite images of the original river channel taken on 2011/03/04, 2015/02/22 and 2017/07/18.

![Figure 7](image2.jpg)

**Figure 7.** Regional monthly rainfall statistics for 2016, 2017, 2018 and 2019 and Jinsha River discharge in 2019.

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The Rigel VZ-2000i was used to collect 3-D point cloud data. Both the horizontal and vertical angular resolution were set to 0.01 degrees. A total of 14 period of point cloud data were collected between 2019/04/26 and 2019/07/11. With the exception of the first and last sets of data, all scans were operated at three fixed scanning positions to minimize registration deviation (Figure 8).

According to the structure and material compositions, the discharge channel was divided into four zones: I, II, III and IV (Figure 8a-b). The characters of the four zones were shown in Table 1. Zone I located at the upstream of the landslide dam consisting of landslide deposits. Zone II and III were the main body of the landslide dam, which were eroded into V-shaped by flood during the drainage. Zone IV located at the downstream of the landslide dam. It consisted of flood sediment and original landslide deposits including fine sands, gravels, rocks and boulders.

Visibility was compromised when scanning zone I at the fixed scanning positions, resulting in a loss of data for this area. Therefore, the follow-up studies mainly focus on zones II, III and IV.

4. Results

4.1. Discharge channel evolution

Figure 9a shows the three-dimensional evolution of the discharge channel from 2019/04/26 to 2019/7/11. Deposits on the left bank were manually excavated and transferred to nearby low-lying areas (yellow area in Figure 9a). Areas of zones I and IV reduced by approximately $3.2 \times 10^4$ m$^2$ in total, and the maximum erosion distance for zone IV reached roughly 36 m (Figure 9b-c). Zone II located at the foot of the “11.03” rock avalanche, locating at the bend of the Jinsha River (turning angle of approximately 60°). Totally, approximately $5.0 \times 10^5$ m$^3$ of deposits on the right bank

Figure 8. Scanning scheme and data collection: (a) 3-D point cloud data for the study area and study area zoning and (b) 3-D model of the study area.
of the discharge channel collapsed from 2019/04/26 to 2019/7/11. It’s worth noting that compared to Zone II, the discharge channel in zone III was rarely damaged during the monitoring period, as bedrock at the bottom of zone III protected deposits from being eroded by water flows. The bedrock was roughly 112 m long, and deposits on the bedrock reached roughly 100 meters high with a slope angle of 40°. As

| Zone | Material composition | Cause of formation | Structural features |
|------|----------------------|--------------------|--------------------|
| I    | Landslide deposits   | Rock avalanche     | Relatively flat, natural deposited, saturated |
| II   | Landslide deposits   | Rock avalanche     | V-shaped, natural deposited and consolidated |
| III  | Bedrock              | Scraping and weathering | (a) V-shaped deposition on the top and (b) scraped and weathered bedrock on the bottom |
| IV   | Landslide deposits and flood sediment (sand, gravel and rock) | Rock avalanche and flood | Flat, consolidated |

Figure 9. Discharge channel evolution from 2019/04/06 to 2019/07/11: (a) 3-D deviation of the discharge channel from 2019/04/06 to 2019/07/11; (b) - (c) images of zone IV taken on 2019/04/26 and 2019/07/11.
another noteworthy phenomenon, the river water level declined with the start of the flood season, showing that the riverbed continued to undercut and that the evolution of the discharge channel was not yet complete.

4.2. Lateral erosion: bank erosion and collapse

Figure 10 illustrates the evolution of the right bank under water erosion from 2019/04/26 to 2019/07/11 in a 2-D view. Four typical cross-sections (A-A’, B-B’, C-C’, and D-D’) of the discharge channel are shown in detail in Figure 11.

Section A-A’ belongs to zone I (Figure 11b). The erosion distance reached roughly 31 m and the water level declined from EL. 2904.3 m to EL. 2899.9 m. Due to sight obstructions, data for this cross-section are only available from the first (2019/4/26) and last (2019/7/11) rounds of scanning. Section B-B’ is located in the center of zone II and the bend of the Jinsha River (Figure 11c). The river boundary advanced roughly 46 m towards the bank, causing 60 meter-high deposits to collapse from 2019/4/26 to 2019/7/11. Compared to the others, cross-section C-C’ was unique due
to the presence of bedrock on the bottom (Figure 11d). The bedrock protected the discharge channel from direct erosion by water flow. Before the rock avalanche, this part of the bedrock was well protected by the original outer covering (boulders and pebbles) (Figure 6c1-c3). However, after the rock avalanche, the original outer covering was destroyed and the inner weathered bedrock was exposed. Under the erosion of water flow, overhanging structures on the bedrock were formed close to the water. With the development of the erosion process, the overhanging structures gradually collapsed. As is shown in Figure 11d, the protruding length of the bedrock reached approximately 31 m on 2019/04/26 and was later reduced to 23 m on 2019/07/11. Cross-section D-D’ was located in zone IV (Figure 11e). This area was made up with landslide deposits (rocks) and flood sediment (sands, gravels, boulders) ranging from a few millimeters to several meters. From the first to last scanning periods, the erosion distance of this cross-section spanned roughly 56 m, and the water level was reduced by approximately 2.7 m.

4.3. Quantification of the erosion process

Based on Figure 10, the cumulative AED is calculated and shown in Figure 12a. It is clear that there is a positive relationship between the AED and the discharge of the Jinsha River, especially during the flood season. As the discharge rapidly increases, the values of AED in different zones rise significantly. There is a high degree of consistency between the red (deposition in zone II) and yellow curves (deposition in zone IV) due to similar material compositions and structures. Meanwhile, there are clear differences between the green (bedrock in zone III) and other two curves, showing that the weathered bedrock can be eroded by water flows even when the process is slow.
EI is used in this study as a quantitative index representing the erosion process. We first calculated the flow velocity based on Manning’s formula (Figure 3), and the results had been shown in Table 2. We then calculated the EI in a continuous time series where the value of EI is obtained by dividing the area enclosed by two adjacent river boundaries by their time interval. As the result, the value of EI varies with three pronounced fluctuations with increasing flow discharge (Figure 12b). Four stages of discharge changes have been noted and there is reason to believe that the increase of flow discharge is responsible for the abrupt changes in EI. The illustration of the fluctuation of EI is as follow.

According to Kramer (1935) and Costa (2016), the initiation of a sediment particle is determined by the driving force ($F_D$), lifting force ($F_L$) and the underwater gravity ($W'$):
Table 2. The River elevation, River width, River depth and Flow velocity are the average value for each zone. They are obtained by calculating the average of five cross sections in each zone. The River elevation and River width are measured through 3D point cloud data, and the River depth and Flow velocity are calculated by Manning’s formula (Fig. 3).

| Date  | Discharge (m³/s) | Zone II | Zone III | Zone IV |
|-------|-----------------|---------|----------|---------|
|       | River elevation (m) | River width (m) | River depth (m) | Flow velocity (m/s) |
|       | River elevation (m) | River width (m) | River depth (m) | Flow velocity (m/s) |
| 4/26  | 956             | 2900.50  | 78.14    | 3.27    | 3.74  |
|       |                 | 2894.50  | 75.45    | 2.32    | 5.45  |
| 6/11  | 948             | 2899.40  | 85.77    | 3.07    | 3.61  |
|       |                 | 2897.25  | 80.64    | 2.25    | 5.84  |
| 6/12  | 968             | 2899.30  | 86.09    | 2.76    | 4.07  |
|       |                 | 2897.17  | 81.75    | 1.94    | 7.14  |
| 6/13  | 1030            | 2899.20  | 86.40    | 2.93    | 4.07  |
|       |                 | 2897.08  | 87.67    | 2.29    | 5.11  |
| 6/14  | 1040            | 2899.10  | 86.72    | 2.86    | 4.20  |
|       |                 | 2897.00  | 87.91    | 2.45    | 4.83  |
| 6/15  | 1120            | 2899.18  | 87.03    | 2.97    | 4.33  |
|       |                 | 2896.96  | 87.98    | 2.46    | 5.17  |
| 6/16  | 1150            | 2899.27  | 87.35    | 3.01    | 4.38  |
|       |                 | 2896.92  | 88.06    | 2.49    | 5.25  |
| 6/17  | 1170            | 2899.35  | 87.66    | 3.04    | 4.38  |
|       |                 | 2896.83  | 88.30    | 2.56    | 5.30  |
| 6/18  | 1260            | 2899.43  | 88.22    | 3.11    | 4.45  |
|       |                 | 2896.79  | 88.40    | 2.48    | 5.61  |
| 6/19  | 1230            | 2899.52  | 88.77    | 3.12    | 4.34  |
|       |                 | 2896.71  | 88.63    | 2.42    | 5.70  |
| 6/20  | 1220            | 2899.60  | 89.13    | 3.11    | 4.45  |
|       |                 | 2896.63  | 89.30    | 2.83    | 6.72  |
| 6/21  | 1480            | 2899.90  | 90.52    | 3.50    | 4.61  |
|       |                 | 2896.58  | 89.92    | 2.74    | 7.05  |
| 6/22  | 1700            | 2900.20  | 91.97    | 3.67    | 4.72  |
|       |                 | 2896.54  | 90.03    | 2.72    | 7.24  |
| 6/23  | 1780            | 2900.50  | 93.43    | 3.88    | 4.73  |
|       |                 | 2896.50  | 90.15    | 2.80    | 7.93  |
| 6/24  | 1780            | 2900.40  | 93.52    | 3.91    | 4.86  |
|       |                 | 2896.50  | 90.26    | 2.80    | 7.32  |
| 6/25  | 1700            | 2900.30  | 94.00    | 3.94    | 4.86  |
|       |                 | 2896.50  | 90.36    | 2.84    | 7.67  |
| 6/26  | 1830            | 2900.20  | 94.50    | 3.97    | 4.92  |
|       |                 | 2896.50  | 90.30    | 2.84    | 7.67  |
| 6/27  | 1990            | 2900.10  | 95.00    | 4.00    | 5.14  |
|       |                 | 2896.50  | 90.40    | 2.84    | 7.67  |
| 6/28  | 1970            | 2900.00  | 95.50    | 4.03    | 5.14  |
|       |                 | 2896.50  | 90.50    | 2.84    | 7.67  |
| 6/29  | 2150            | 2899.90  | 96.00    | 4.06    | 5.29  |
|       |                 | 2896.50  | 90.60    | 2.84    | 7.67  |
| 6/30  | 2570            | 2899.80  | 96.50    | 4.16    | 5.80  |
|       |                 | 2896.50  | 90.70    | 2.84    | 7.67  |
| 7/1   | 2510            | 2899.70  | 97.00    | 4.20    | 5.80  |
|       |                 | 2896.50  | 90.80    | 2.84    | 7.67  |
| 7/2   | 2320            | 2899.60  | 97.50    | 4.24    | 5.80  |
|       |                 | 2896.50  | 90.90    | 2.84    | 7.67  |
| 7/3   | 2160            | 2899.50  | 98.00    | 4.28    | 5.80  |
|       |                 | 2896.50  | 91.00    | 2.84    | 7.67  |
| 7/4   | 2290            | 2899.40  | 98.50    | 4.32    | 5.80  |
|       |                 | 2896.50  | 91.10    | 2.84    | 7.67  |
| 7/5   | 2290            | 2899.30  | 99.00    | 4.36    | 5.80  |
|       |                 | 2896.50  | 91.20    | 2.84    | 7.67  |
| 7/6   | 2440            | 2899.20  | 99.50    | 4.40    | 5.80  |
|       |                 | 2896.50  | 91.30    | 2.84    | 7.67  |
| 7/7   | 2720            | 2899.10  | 100.00   | 4.44    | 5.80  |
|       |                 | 2896.50  | 91.40    | 2.84    | 7.67  |
| 7/8   | 2770            | 2899.00  | 100.50   | 4.48    | 5.80  |
|       |                 | 2896.50  | 91.50    | 2.84    | 7.67  |
where $d$ is the particle size; $\gamma$ is the bulk density of water; $\gamma_s$ is the bulk density of the particle; $g$ is the gravitational acceleration, $C_D$ and $C_L$ are the driving and lifting

$$F_D = C_D a_1 d^2 \gamma \frac{u_b^2}{2g}$$  \hspace{1cm} (4)$$

$$F_L = C_L a_2 d^2 \gamma \frac{u_b^2}{2g}$$  \hspace{1cm} (5)$$

$$W' = a_3 (\gamma_s - \gamma) d^3$$  \hspace{1cm} (6)$$

Figure 13. Quantitative analyses of the erosion intensity (EI): (a) statistics of the relationship between $EI$ and flow velocity and (b) fitted curve of $EI$ and flow velocity based on the least squares.
coefficient; \( a_1 \) and \( a_2 \) the area coefficients of the headwaters and substrates of the particle; \( a_3 \) is the volume coefficient of the particle; \( u_b \) is the effective instantaneous velocity acting on the particle surface;

With the increase of river flow, the small particle will start to mobilize first when the combined force of \( F_D \) and \( F_L \) is greater than \( W' \). As fine sediments are eroded, the remaining large particles act as a protective layer to prevent the continued erosion of small particles within the riverbank. If the river flow stops increasing (remains stable or decreases), the erosion process will stop and the bank morphology will thus be temporarily stabilized due to the existence of the protective layer. However, this temporary equilibrium will no longer exist when the flow increases again. The increase of discharge will break the force balance of particles (driving force greater than gravitational force), and lead to the initiation of larger particles within the riverbed. This is the reason why a sudden fluctuation of erosion intensity appeared at the turning point of discharge in Figure 12b.

The direct factor influencing the erosion process is the instantaneous velocity of the water flow on the particle surface, so we calculated and statistically correlated the \( EI \) with the flow velocity (Figure 13a). The statistics of \( EI \)-flow velocity for each zone were divided into two separate groups according to the increment of discharge. The mathematical expression can be summarized as:

\[
EI = f(k, v, \Delta_d)
\]  

(7)

where the \( k \) is the erosion coefficient which is determined by the distribution of flow field, the value of discharge, the increment of discharge and the composition of riverbank; \( v \) is flow velocity and \( \Delta_d \) is the increment of flow discharge.

Further processing of Eq. (7) using the least squares method yields the empirical formula as follow:

\[
EI = \begin{cases} 
  k_1 * v & \Delta_d \in (a_1, a_2) \\
  k_2 * v & \Delta_d \in (b_1, b_2) \\
  \vdots & 
\end{cases}
\]  

(8)

where \( a_1 \), \( a_2 \), \( b_1 \) and \( b_2 \) are the different value ranges of discharge increment.

The value of parameter \( k \) in this study is shown in Figure 13b and Table 3. Its value is related to the value of discharge, the increment of discharge, the composition of riverbank as well as the distribution of flow field. For other discharge channel or riverbank, the value of \( k \) should be calculated according to the measured data.

| Zone | \( \Delta d \) | II | III | IV |
|------|---------------|----|-----|----|
| 0-8% | 5.1E-6        | 6.1E-7 | 7.2E-6 |
| 9%-11% | 2.4E-5   | 1.7E-6 | 1.9E-5 |

Table 3. The value of parameter \( k \) of Baige discharge channel.
4.4. Vertical erosion: down cutting of the discharge channel

The evolution of a riverbed involves not only erosion by water flow but also sedimentation, including incoming sediment from upstream and bank collapse (Lajczak 2003). Compared to lateral erosion, the vertical erosion of the underwater part of discharge channel cannot be observed by naked eyes, photograph or TLS. It is thus difficult to depict the erosion as well as the sediment process of riverbed in a running river. Therefore, we tried to qualitatively analyze the vertical evolution process of riverbed using the variation of river surface elevation and the river depth ($D_r$) calculated by Manning’s formula. The change of water elevation is considered as two independent processes, one is caused by the change of discharge, the other is caused by the change of riverbed elevation. The undercutting of riverbed can be calculated through the following equation:

$$H_u = EL_a - EL_m$$  \hspace{1cm} (9)

where $H_u$ is the undercutting depth of riverbed; $EL_m$ is the actual elevation of river surface measured by TLS, its value is determined by the change of flow discharge as well as the riverbed elevation; $EL_a$ is the assumed elevation of river surface, assuming that the riverbed is stable since 2019/4/26 (without undercutting or sediment), and the elevation of river surface is only determined by the variation of flow discharge. As the results, the difference between the assumed and actual river surface is equal to the elevation change of the riverbed.

| Type | Riverbed composition and characteristics | Channel morphology and flow pattern | River bank characteristics | $n$ |
|------|-----------------------------------------|------------------------------------|---------------------------|-----|
| I    | Flat riverbed of sand                   | Straight channel, regular profile, smooth flow | Regular river bank of soil/gravel | 0.020 ~ 0.024 |
| II   | Flat riverbed of gravel/pebble/rock    | Straight channel, regular profile, smooth flow | Regular river bank of soil/gravel/rock | 0.022 ~ 0.026 |
| III  | Uneven riverbed of sand                 | Slightly curved channel, blocked flow with local backflow | River bank with soil and weeds | 0.025 ~ 0.029 |
| 1    | Uneven riverbed of gravel/pebble       | Straight channel, regular profile, smooth flow | Regular river bank of soil/gravel/rock with weeds or trees | 0.025 ~ 0.029 |
| IV   | River bed of fine sand with aquatic plant | Straight channel bended at both ends, blocked flow | River bank of soil with sparse or dense vegetation, bank collapse | 0.030 ~ 0.034 |
| 2    | Uneven riverbed of gravel/pebble       | Straight channel bended at upstream side, regular profile, slight backflow | Regular river bank of gravel/rock with vegetation | 0.030 ~ 0.034 |
| V    | Uneven riverbed of gravel/rock/boulder | Straight channel is sandwiched between two curves, regular profile, block flow with oblique flow/backflow/stagnant water | Regular river bank of gravel/rock with vegetation | 0.035 ~ 0.040 |
The $E_{La}$ can be calculated according to the following equation:

$$E_{La} = E_{Lor} + D_r$$

where $E_{Lor}$ is equal to the original riverbed elevation in 2019/4/26 (EL. 2897.23 m, EL. 2895.60 m and EL. 2892.18 m in zones II, III and IV, respectively); the $D_r$ refers to the depth of water. The $E_{Lor}$ and $D_r$ can be obtained from Table 2.

In this method, the $D_r$ obtained by Manning’s formula is essential to the accuracy of the results, and the roughness coefficient $n$ has a great impact of the $D_r$. Table 4 represents recommended values of natural channel roughness coefficient provided by Hydraulic Calculation Manual (2nd Edition) published by Wuhan University, China (in Chinese, page 363). The value of $n$ is determined by the composition, morphology and characteristics of the discharge channel as well as the flow pattern. In order to determine the value of $n$, a sensitivity analysis has been conducted for evaluating the choice of $n$ on $E_{La}$ (Figure 14). Six of roughness coefficients from 0.020 to 0.033 were selected according to Table 4, and the maximum deviation of the $E_{La}$ is approximately 1 to 2 meters. It is obvious that the roughness coefficient $n$ has a certain influence on the accuracy of vertical erosion analysis. According to the material composition, morphology, characteristics and flow pattern, the description in Table 4 that matches the discharge channel is highlighted, and the value of $n$ is set to 0.025 in this study.

The results of vertical evolution of riverbed are shown in Figure 15. In general, the difference between $E_{Lm}$ and $E_{La}$ increased with the discharge, indicating that the discharge channel experienced continuous vertical erosion. The $E_{Lm}$ in each zone had a short rise in a certain period of time and a rapid decline when it reached the peak. For example, the $E_{Lm}$ in zone II raised about 1.3 meters from June 14 to June 24,
and then continue to decline afterwards. Moreover, the $EL_m$ both in zones III and IV experienced decrease in a stepwise manner since June 25. To illustrated this phenomenon, two factors, $D_r$ and $H_u$ are considered responsible:

$$
\Delta EL_m = \Delta D_r - H_u
$$

where $\Delta EL_m$ is the variation of $EL_m$, $\Delta D_r$ is the variation of river depth which is determined by the (a) flow discharge, (b) roughness and slope of riverbed, and (c) the shape change of cross section; and the $H_u$ is determined by the (a) initiation velocity, diameter and density of riverbed particles, (b) the flow velocity and its distribution, (c) the river depth $D_r$ and (d) sediment and bank collapse.

From early June to July 9, the river flow raised rapidly to its annual peak, resulting in a quick growth in $EL_m$ and flow velocity. There would not exhibit massive undercutting in the riverbed if the flow velocity is less than the initiation velocity of most particles. Instead, the sediment deposited on the discharge channel from upstream and bank collapse might raise the elevation of riverbed to a certain extent, leading to the decrease of $H_u$ and increase of $EL_m$. With the further increase of river flow, the undercutting began to occur after the flow velocity exceeds the initiation velocity of most particles in the riverbed. Then, the $EL_m$ would exhibit a significant decrease when the undercutting of riverbed exceeded the increment of water level caused by the increase of river flow.
5. Discussion

5.1. Randomness of erosion process

The erosion process of riverbank may be random in a small area or in a short period of time. The velocity of river is one of the most important factors affecting the erosion process. This is because the instantaneous velocity of water over a particle’s surface is the direct factor in determining whether the particle will start moving or not. However, due to the complexity of real channel morphology and the instantaneous distribution of the flow field, studying the movement of a single particle or erosion process of a particular cross section is difficult and pointless. Besides, the shape and position of the particles are random variables, and even for the uniform particles, the instantaneous driving force or lifting force are also random variables.

The sediment from upstream, bank collapse and construction of artificial river bank also pose great influence on the evolution of river bank. The sediment and bank collapse may impede the erosion process while the construction of artificial bank may enhance the lateral erosion on the opposite bank. For example, in Figure 10, the evolution process of the river bank in zone II was determined greatly by the construction progress on the opposite bank and the dynamics of bank collapses, so there is no obvious spatial or temporal regularity of the erosion process (see the part in zone II). In contrast, the evolution of river bank in zone IV is more regular in a wave-shaped manner (see the part in zone IV). The change of riverbank morphology in this area is considered to be only affected by water flow. Even so, the dynamic evolution of this wave-shaped bank varied spatially. The erosion process in the upstream (near zone III) developed along the river, while the erosion direction in the downstream is almost perpendicular to the channel. Therefore, even if the two study sections are very close, the lateral erosion may be significantly different. The
quantitative analysis of a single profile or small area might not reflect completely the overall erosion process determined by the material composition, morphology, characteristics and flow pattern of the riverbed. Therefore, integrating the regions of similar erosion characteristics and analyzing their average variation from the perspective of large-scale and long-term series (such as $EA$, $AED$ and $EI$ in this study) are helpful to deepen the understanding of erosion.

5.2. Fluctuation of $EI$

To further examine the fluctuations of $EI$, we analyzed the relationship between the variation rate of discharge and $EI$ (Figure 16). Numerous field observations, experiments and theoretical studies have confirmed that bank erosion occurs under approximately three steps (Kramer 1935): (a) Low discharge: erosion does not occur because the particles of riverbed cannot reach the initiation conditions; (b) Increased discharge: the fine sands and small gravels can reach the initiation conditions while the medium or large blocks cannot. The small particles are washed away, and the erosion occurs and develops rapidly. The retained large particles will form a covering layer which prevents the small particles within the riverbank from being washed away. If the discharge continues to increase, the retained large particles will be washed away and the erosion process continues; (c) Stable or decreased discharge: the flow velocity will decrease due to the increase of cross section area or the decrease of discharge, and the covering layer will prevent further erosion of the riverbed.

Apparently, the discharge channel in this study was constantly repeating the process from (b) to (c) in flood season (Figure 16). The increase of discharge will damage the covering layer of discharge channel, leading to the increase of $EI$. And the decrease of discharge will form new covering layer and thus slow the erosion process down.
5.3. Water flow field

The strike and morphology of the riverbed have an important influence on the distribution of the flow field, which in turn affects the development of riverbank morphology. The initiation condition of a particle includes not only the value of discharge and the characteristics of the particle, but also the distribution of flow filed. For different cross sections, even if the discharge or average velocity is the same, the different distribution of the flow field will result in different instantaneous velocities and different initiation conditions. And two important factors affecting the distribution of the flow field are the strike and morphology of the river bank. Figure 17c show the differences between the cumulated AED curves of zones II (B-B’) and IV (D-D’), representing that the erosion of zone II is greater than that of zone IV, even though their bank compositions are similar. This was observed because zone II is positioned at the bend of the Jinsha River (Figure 17b), where there is a bend in circulation flow that runs perpendicular to the river (Figure 17a). The centrifugal force produced by the curvilinear motion of the water flow in the curved section orients the surface flow towards the concave bank and the bottom flow towards the convex bank, producing closed transverse circulation in the cross-section (Núñez-González et al. 2018). This perpendicular bend circulation flow combined with the longitudinal flow form a spiral flow along the flow direction, increasing water erosion along the concave bank (Rinaldi et al. 2008).

6. Conclusions

This work presents a quantitative method for the on-site monitoring of the drainage processes of a landslide-dammed lake. Based on the 3-D point cloud data collected by TLS and the flow characteristics calculated by Manning’s formula, the lateral and vertical erosion of the discharge channel are analyzed in quantitative and qualitative methods, respectively. The following conclusions can be drawn from this study:

(1) Based on the river boundary extracted from the point cloud data captured in different temporal episodes, the lateral evolution of the discharge channel is described in detail, and the erosion intensity EI and an empirical formula for calculating EI is obtained.

(2) Based on the quantitative analysis of relationship between EI and river discharge, it is found that the fluctuations of EI is consistent with variation rate of discharge. The value of EI is related to discharge change rate, flow velocity and erosion coefficient k.

(3) The choice of roughness coefficient is explained, and the sensitivity analysis is carried out. It is found that n = 0.025 is more in line with the actual situation of the discharge channel.

(4) Qualitative analysis of riverbed undercutting is carried out according to the difference between the actual river elevation and a hypothetic river surface. The stepwise water surface elevation is affected by both the variation of discharge and the evolution of the riverbed.
(5) The erosion process has strong randomness in a short time and small scope due to the evolution of river channel morphology and the variation of flow field. Appropriate research scale and long-term data series should be selected in studying the erosion process.

Data availability statement

The data used to support the findings of this study are available from the corresponding author upon request.

Disclosure statement

No potential conflict of interest was reported by the authors.

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