Quasiparticle states and quantum interference induced by magnetic impurities on a two-dimensional topological superconductor

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Abstract

We theoretically study the effect of localized magnetic impurities on two-dimensional topological superconductor (TSC). We show that the local density of states (LDOS) can be tuned by the effective exchange field $m$, the chemical potential $\mu$ of TSC, and the distance $\Delta r$ as well as the relative spin angle $\alpha$ between two impurities. The changes in $\Delta r$ between two impurities alter the interference and result in significant modifications to the bonding and antibonding states. Furthermore, the bound-state spin LDOS induced by single and double magnetic impurity scattering, the quantum corals and the quantum mirages are also discussed. Finally, we briefly compare the impurities in TSC with those in topological insulators.

(Some figures may appear in colour only in the online journal)

1. Introduction

The study of topological insulators (TIs) has attracted considerable theoretical [1–6] and experimental [7–11] interest over the past few years. These studies have opened a door for exploring the promising potential applications for spintronics with TI materials by their intrinsic strong spin–orbit coupling (SOC) nature. Very recently, a series of superconductor (SC)/TI hybrid structures, called topological superconductors (TSCs) [12, 13], were proposed to realize Majorana fermions by using the proximity effect of SC and strong SOC of TI. Various transport proposals have been suggested to detect and manipulate the Majorana fermions [14–20]. The experimental challenge of this line of research remains to be overcome. Recently, proximity induced superconductivity by Bi in topological Bi\textsubscript{2}Te\textsubscript{2}Se and Bi\textsubscript{2}Se\textsubscript{3} films has been revealed in point contact conductance measurements below 2–3 K [21]. In another way Shindou et al [22] suggested that the surface-adsorbed magnetic impurities can be effectively used to explore the Majorana fermions. The impurity interference effects on the surfaces of ordinary s-wave and d-wave superconductors [23–27] have been studied in the past decades. Unfortunately, at present a detailed and revealing study of the magnetic impurity effects on the surface spectrum of TSC is still lacking, which is unexpected since a time reversal breaking perturbation is the most natural way to reveal both the topological and the superconducting properties of TSCs.

Motivated by the above-mentioned fact, in this paper we study the impurity states and impurity interference on the two-dimensional (2D) TSC, which are different from the ordinary SC systems because of the chirality and special energy spectrum of the itinerant electron in TSC. We show that there exist critical impurity–TSC spin coupling $m_c$ (following the appellation in [25]) and critical chemical potential $\mu_c$ for a single impurity, which correspond to unpaired zero-energy states. By analyzing the spin local density of states (LDOS), a quantum transition from the spin-unpolarized
to the spin-polarized state is shown when the exchange coupling parameter exceeds $m_c$. A clear understanding of quantum interference is fundamental to our analysis of complex impurity structures. Therefore, we illustrate that the distance and the relative spin angle between two magnetic impurities can alter the quasiparticle interference effects, resulting in significant modifications to the bonding and antibonding states. For multiple impurities, we construct elliptical quantum corals to study the spectral properties of quantum mirages and the influence of quantum corals on the quantum interference effect between two magnetic impurities. We reveal that the nonmagnetic (antiferromagnetic) coral has strong (weak) influence. Finally, we briefly compare the impurity–TSC system with the impurity–TI system, and we find that for the latter, the intragap quasiparticle states can arise only when the scalar potential scatterings are taken into account. These findings, which can be detected by the scanning tunneling microscopy (STM) and scanning tunneling spectroscopy (STS) techniques, may be useful for determining the quasiparticle spectrum in the TSC surface, and possess potential applications in quantum computation.

2. Theoretical formalism

The 2D TSC by hybridizing an ordinary s-wave SC film with a TI film (say, for example, Bi$_2$Se$_3$ film) is described by the Bogoliubov–de Gennes Hamiltonian [12]

$$H_{BdG}(k) = \begin{pmatrix} h(k) - \mu & i\Delta \sigma_y \\ -i\Delta^\ast \sigma_y & h^\ast(-k) + \mu \end{pmatrix}, \tag{1}$$

the basis for which is $(c_k^\dagger, c_{-k}^\dagger, c_{-k}^\dagger, c_k^\dagger)^T$. Here, for simplicity we just consider the strong SOC term

$$h(k) = v_F(\sigma_x k_x + \sigma_y k_y). \tag{2}$$

In equation (1) $\mu$ is the chemical potential, $\sigma_{x,y}$ are Pauli matrices of electron spin, and the s-wave gap function is simply chosen to be a constant, $\Delta = \Delta_0$. The impurity potential can be expressed as

$$V_i = \frac{1}{2}(U_i \sigma_0 + J_i S_i \cdot \mathbf{o})\tau_z, \tag{3}$$

where $S_i = S_i^\dagger$ is the classical spin (with its orientation vector $n_i$) of the $i$th magnetic impurity, $U_i$ and $J_i$ are the scalar potential and magnetic scattering strengths, and the Pauli matrix $\tau_z$ acts on the particle–hole space. We treat the impurity spin as a classical local effective exchange field $m_i = J_i S_i/2$ under the mean-field approximation. By employing the non-self-consistent $T$-matrix method, we study the quantum states and interference induced by the magnetic impurities on the TSC. The total Green’s function in the presence of $N$ impurities is written as

$$G(r, r', \omega) = G_0(r, r', \omega) + \sum_{i,j=1}^{N} G_0(r, r_i, \omega) T(r_i, r_j, \omega) G_0(r_j, r', \omega), \tag{4}$$

where the $T$-matrix is given by the Bethe–Salpeter equation

$$T(r_i, r_j, \omega) = V_{\delta i,j} + V_i \sum_{l=1}^{N} G_0(r_i, r_l, \omega) T(r_l, r_j, \omega) \tag{5}$$

with the $4 \times 4$ matrix $G_0(r, r', \omega)$ obtained from the Fourier transformation of the unperturbed Green’s function

$$G_0(k, \omega) = [i\omega - H_{BdG}]^{-1} \tag{6}$$

for the free TSC surface. At half-filling $\mu = 0$, one can find an analytical expression of $G_0(r = 0, \omega)$ for $|\omega| < \Delta_0$, written as

$$G_0(r = 0, \omega) = \frac{1}{4\pi v_F^2} \ln \left(\frac{\Delta_0^2 + \omega^2 + \sqrt{\Delta_0^2 + \omega^2}}{\Delta_0^2 + \omega^2}\right) \times (i\omega \tau_0 + \Delta_0 \tau_2 \sigma_2), \tag{7}$$

where $\mu = 1/\Sigma$ is the planar density with $\Sigma$ the area of the STC surface, and $k_F$ is the high momentum cutoff. Finally, we could get

$$G_0(r = 0, \omega) = -A(\omega)(i\omega \tau_0 + \Delta_0 \tau_2 \sigma_2)/D, \tag{8}$$

where $A(\omega) = \frac{2}{\pi} \ln(D/\sqrt{\Delta_0^2 - \omega^2})$ for $\Delta_0 \ll D$ with $D = v_F k_F$. Here, we have used $4\pi v_F^2 = D^2$ with $\mu = k_F^2/4\pi$.

The total Green’s function $G(r, r', \omega)$ in equation (4) can be rewritten as

$$G(r, r', \omega) = G_0(r, r', \omega) + G_0 T' G_0, \tag{9}$$

where

$$G_0 = \begin{pmatrix} G_0(r, r_1, \omega) & \cdots & G_0(r, r_N, \omega) \end{pmatrix}, \tag{10}$$

$$T' = \begin{pmatrix} \mathcal{H}_{imp} + \mathcal{H}_{imp} G_0 T \end{pmatrix}, \tag{12}$$

with the impurity Hamiltonian

$$\mathcal{H}_{imp} = \begin{pmatrix} V_1 & 0 & \cdots & 0 \\ 0 & V_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & V_N \end{pmatrix}, \tag{13}$$

containing all of the impurity scattering potentials $V_i$ ($i = 1, \ldots, N$). Information about the propagation between the impurities is included in $G_0$, which can be expressed as

$$G_0 = \begin{pmatrix} G_0(r_1, r_1) & G_0(r_1, r_2) & \cdots & G_0(r_1, r_N) \\ G_0(r_2, r_1) & G_0(r_2, r_2) & \cdots & G_0(r_2, r_N) \\ \vdots & \vdots & \ddots & \vdots \\ G_0(r_N, r_1) & G_0(r_N, r_2) & \cdots & G_0(r_N, r_N) \end{pmatrix}. \tag{14}$$
As a result, one can obtain

$$T = \frac{\mathcal{H}_{\text{imp}}}{I - \mathcal{H}_{\text{imp}} G_0},$$

where $I$ is a $4N \times 4N$ unity matrix. Finally, by using these equations, one can easily finish the numerical calculations with $N$ impurities, and then get the LDOS $\rho(r, E)$ of TSC with impurities, which is given by

$$\rho(r, \omega) = -\frac{1}{\pi} \text{Im} \sum_{\omega_{\text{res}}} \frac{G_{ii}(r, r, \omega)}{\omega - \omega_{\text{res}}}$$

$$\equiv -\frac{1}{\pi} \text{Im} \text{Tr} G^{(p)}(r, r, \omega),$$

where $G^{(p)}(r, r', \omega)$ is the particle Green function. Furthermore, the spin LDOS is written as

$$s(r) = -\frac{1}{2\pi} \text{Im} \text{Tr} G^{(p)}(r, r, \omega) a.$$ (17)

For definiteness we set $k_F = 0.3\ldots0.4 \beta^{-1}$, $v_F = 2.55$ eV $\AA^{-1}$, and $\Delta_0 = 1$ meV, which give out a coherence length $\xi = v_F/\Delta_0 = 255$ nm. The lattice constant is chosen as $a_0 = 4.25$ A, and $0^+ = 0.02$ meV in the following calculations. Here the chosen value of $\Delta_0$ is consistent with recent theoretical prediction [12] and experimental implication [29] on Cu$_4$Bi$_2$Se$_3$.

3. Results and discussions

3.1. Single magnetic impurity

Similar to the ordinary s-wave SC systems, the purely scalar potential scattering cannot form an impurity bound state in the gap of TSC. Therefore, we focus on the magnetic scattering effects in the following calculations. Firstly, we consider a single magnetic impurity in STC.

We know that the $T$-matrix possesses poles at resonance frequencies $\omega_{\text{res}}^{(1,2)}$, which can reflect the presence of bound states. The spectroscopic evidence for bound states is a pair of peaks in the LDOS, which changes with $m$ as well as $\mu$ of the impurity–TSC system. As $m$ increases, $\omega_{\text{res}}^{(1,2)}$ approaches the chemical potential, and at the critical coupling $m_c$, $\omega_{\text{res}}^{(1,2)}$ become degenerate at the zero energy. If $|\omega|$ is not close to $\Delta_0$, one has a reasonable approximation $A(\omega) \approx A(0)$, and then from the Green’s function $G_0(r = 0, \omega)$ in equation (7), we can easily obtain the $T$-matrix at half-filling $\mu = 0$, written as

$$T(\omega) = [V^{-1} - G_0(r = 0, \omega)]^{-1}$$

(18)

for a single impurity case, and the critical coupling is given by

$$m_c(U) \simeq \sqrt{4D^2/(\Delta_0 A(0))^2 + U^2},$$

(19)

which is dependent on the ratio of parameters $D = v_F k_F$ and $\Delta_0$. This means that the appearance of a zero-energy resonance peak can be attributed to the interplay of strong spin–orbit coupling and pairing-condensation energy. At this critical point, the ground state of the topological superconductor becomes unstable against the spontaneous creation of a quasiparticle excitation. The bound-state energies

$$\omega_{\text{res}}^{(1,2)} = -2c_m \pm \sqrt{c_m^2 + \Delta_0^2}$$

(20)

for $m < m_c(U)$, where $c_m = 2Dm/[A(0)(4m^2 - U^2)]$ and $c_u = 2DU/[A(0)(4m^2 - U^2)]$. It is clear from equation (19) that taking into account the scalar potential $U$ in the impurity potential can effectively increase the critical coupling $m_c(U)$.

In the following calculations of $\mu \neq 0$, for briefness we ignore the influence of the scalar potential term ($U = 0$). From numerical calculations, we find that the critical magnetic moment $m_c = 3.0, 1.15, 0.98$, and 0.71 eV for $U = 0$ corresponding to the chemical potential $\mu = 0.05, 0.2, 0.25$, and 0.37 eV respectively. On the other hand, in figure 1 we present the LDOS at the impurity site $r = (0, 0)$ as a function of $\mu$ for a fixed exchange field $m = 0.8$ eV. One can see that the impurity resonance phenomenon nearly vanishes when tuning the chemical potential to half-filling ($\mu = 0$). The observable intragap resonant states with energies of $\omega_{\text{res}}^{(1,2)}$ evolve from non-zero $\mu$, and their energy difference $(|\omega_{\text{res}}^{(2)} - \omega_{\text{res}}^{(1)}|)$ decreases with increasing $\mu$. Particularly, at a critical value $\mu_c$ of the chemical potential, the particle- and hole-like bound states are degenerate at zero energy with equal spectral weight. In figure 1 we determine this critical chemical potential to be $\mu_c = \pm 320 \Delta_0$ for the occurrence of zero-energy bound states (see the red curves). Therefore, we can conclude that by tuning the spin coupling or the host’s chemical potential, the zero-energy bound states could be detected on 2D TSC by employing spectral techniques such as STS.

The occurrence of zero-energy bound states is a result of competition among the pairing-condensation energy, the magnetic interaction, and strong SOC. At these critical points, the ground state of STC may become thermodynamically unstable, causing the quantum transformation of the system from the spin-unpolarized to the spin-polarized state. As regarding the spin polarization, one should consider the spin LDOS around the impurity, from which the spatially resolved

Figure 1. The LDOS $\rho(r = 0, \omega)$ of a single impurity located at the origin with $m = 0.8$ eV, $S \parallel \hat{z}$, and $U = 0$, as a function of frequency $\omega$ for a series of chemical potentials $\mu$ in a TSC. The unpaired zero-energy states are illustrated by red curves as $\mu$ increases to $\mu_c \approx 0.12$ eV.
spin polarization can be obtained by performing integration over energy,

\[ \langle s(r) \rangle = -\frac{1}{\pi} \int_{-\infty}^{0} d\omega \langle s(r, \omega) \rangle, \tag{21} \]

as well as the total spin polarization \( \langle s \rangle = \sum s(r) \). The typical spatial distribution of spin LDOS is plotted in figure 2 at resonance frequency \( \omega_{\text{res}}^{(1)} = -0.38 \Delta_0 \) for two choices of spin orientations of the single magnetic impurity. Figure 2(a) is the case where the local moment \( \mathbf{S} \parallel \hat{z} \), while figure 2(b) corresponds to \( \mathbf{S} \parallel \hat{x} \). It is clear that the impurity induces not only a \( z \)-direction spin polarization which decays with oscillations, but also an \( xy \)-plane spin polarization in the present TSC system. The spin LDOS possesses rotation symmetry about the \( z \)-axis for a \( z \)-direction impurity spin, shown as figure 2(a). However, the rotation symmetry is broken when the impurity spin lies in-plane.

We find from figure 2(b), for example, that the in-plane component \( s_x(r, \omega) \) rotates anticlockwise (clockwise) in the upper (lower) half-plane. These results can be understood by the effective magnetic field \( \mathbf{B}_{\text{eff}} = v_F \mathbf{k} \) induced by the strong SOC in the 2D TSC, which is similar to the case of a magnetic impurity on the TI surface \([28]\). Because it is locked to the momentum of the electron, with the movement of the electron, the spin undergoes a gyroscopic precession in the plane perpendicular to the propagation orientation, which arises from the corresponding slant of spin LDOS \( s(r, \omega) \).

Taking a further step, corresponding to figure 2(a), we find that the total spin polarization \( \langle s_x \rangle \approx 0.04 < 1/2 \), which indicates a negligibly small spin polarization. This is consistent with our choice of \( m \), which is lower than \( m_c \); when \( m > m_c \), an obvious spin polarization is observed. As a result, the resonance peaks at the negative (positive) energy region possess particle-like and hole-like spin-chiral states \( |p, \rangle \) and \( |h, \rangle \), respectively, where \( \rangle \) and \( \langle \) represent two different spin chiralities. This is different from that on the ordinary \( s \)-wave SC surface with classical magnetic impurity, where the particle/hole spin is parallel or antiparallel to the impurity spin \( \mathbf{S} \) \([25\text{-}27]\). Notice that we have not found a quantum transformation from spin-unpolarized to the spin-polarized state by increasing \( \mu \).

3.2. Two magnetic impurities

When the two impurities are close to each other, electrons will be scattered by both impurities, resulting in quantum interference of electronic waves. The interference effect is related to the scattering strength, the distance as well as the spin angle between two impurities. There are two important consequences that occur due to the interference effect: (i) one is that the interference can change the formation of the so-called bonding and antibonding states \( |p(h)_{b,a}\rangle = (|p(h),1\rangle \pm |p(h),2\rangle)/\sqrt{2} \) for the particle-like (hole-like) states, with \( |p, i = 1, 2 \rangle \) and \( |h, i = 1, 2 \rangle \) being the bound states of each impurity; (ii) the other one is that the spin polarization will be altered too, which can be observed from the spin LDOS patterns.

Figure 2. The spatial distribution of spin LDOS at resonance frequency \( \omega_{\text{res}} = -0.38 \Delta_0 \) for a single magnetic impurity located at the origin with \( z \)-direction (a) and \( x \)-direction polarization. The background color denotes the \( z \)-component of spin LDOS \( s_z(r, \omega) \) while the arrow indicates the \( xy \)-plane component \( s_x(r, \omega) \). The parameters are chosen as \( m = 0.8 \text{ eV} \) and \( \mu = 0.2 \text{ eV} \).

Here, we focus on two identical magnetic impurities located at \( r_1 = (0, 0) \) and \( r_2 = (\Delta r, 0) \) with the same exchange field \( m = 0.8 \text{ eV} \). Firstly, the LDOS for two impurities located at \( r_1 = (0, 0) \) and \( r_2 = (8a_0, 0) \) is shown by the red curve in figure 3(a), which exhibits four resonance peaks with peaks \( \Omega_{1,2} \) corresponding to the particle-like states \( |p\rangle_{b,a} \) and peaks \( \Omega_{3,4} \) to the hole-like states \( |h\rangle_{b,a} \). If the interference of \( |p, \rangle |p, \rangle \) and \( |p, \rangle |p, \rangle \) is constructive (destructive) between two impurities, the bonding state \( |p(h)\rangle \) \( (|p(h)\rangle \) is formed, which can be observed from the spatial LDOS patterns. For this purpose, we plot in figures 3(b) and (c) the spatial LDOS corresponding to the particle-like state peaks \( \Omega_1 = -0.74 \Delta_0 \) and \( \Omega_2 = -0.27 \Delta_0 \) respectively (the positions of two impurities are indicated by black dots). It is obvious from figure 3(b) that the LDOS are located in the middle region between two impurities and reach a maximum at \( \Delta r = 4a_0 \), therefore, peak \( \Omega_1 \) corresponds to a bounding state \( |p\rangle \). On the other hand, peak \( \Omega_2 \) should
correspond to an antibonding state $|p⟩$ since the LDOS vanishes at the middle point between two impurities, see figure 3(c).

However, the interference pattern between the bonding and antibonding states changes by varying the distance between the two impurities, i.e. the parity and resonance energy of the two-impurity states oscillate with increasing impurity separation. This can be observed from figure 3(d), which shows the two-impurity distance $Δr$ dependency of LDOS. For instance, $|p⟩$ corresponds to peak $Ω_2$ while $|p⟩$ corresponds to peak $Ω_1$ when $Δr = 5a_0$ or $20a_0$. The curves in figure 3(d) are moved in order to illustrate the finer and clearer resonance frequencies. The frequencies of the resonance oscillations and at the same time the amplitude of LDOS as well as their energy width changes. Moreover, we find that there exists a critical distance $Δr_c ≈ 2a_0$ similar to the critical exchange parameter $μ_c$ and critical chemical potential $μ_c$ for the single impurity case. At this point, the bound-state energy of particle-like $|p⟩$ (hole-like $|h⟩$) crosses zero, and the state transforms into hole-like $|h⟩$ (particle-like $|p⟩$). A similar result has been theoretically observed in the ordinary s-wave SC materials [26]. Whether or not a quantum spin polarization transition occurs with increasing $Δr$ to exceed $Δr_c$ is not clear, and further work is under way to study this interesting issue. Besides, it is obvious that when the distance of two impurities is as small as $Δr ≲ 4a_0$, only two intragap bound states are found, while there appear four non-degenerate bound states with increasing $Δr$ larger than $4a_0$. With further increasing $Δr$, the interference effect during the electron scattering processes by both impurities gets so weak that the bound states become degenerate.

An additional result of quantum interference between two impurities is the change with the position of STM tip $r$ in the number of observable low-energy resonance peaks. For illustration we present in figure 4(a) the LDOS as a function of $r = (r, 0)$ for two impurities with parallel spins located at $r_1 = (0, 0)$ and $r_2 = (8a_0, 0)$. From four main low-energy intragap resonances $Ω_{i=1,...,4}$, one can find that the resonance amplitude of particle-like (hole-like) peaks $Ω_1$ and $Ω_2$ ($Ω_3$ and $Ω_4$) varies obviously (not obviously) with $r$. For example, for the uppermost curve $r = (3a_0, 0)$, the peak $Ω_1$ ($Ω_2$) is sharp (smooth), while it is weaker (stronger) when the tip moves to $r = (9a_0, 0)$, see the bottom curve in figure 4(a). This is consistent with figure 3(b) because $Ω_1$ corresponds to a bonding state $|p⟩$. As mentioned above, the quantum interference effect could also be tuned through varying the angle $α_2$ between two impurity spins, as shown in figure 4(b) for two impurities $8a_0$ apart. We find that the resonance frequencies $Ω_i$ change with $α_2$.

We also investigate the spin LDOS interference patterns of two impurities $8a_0$ apart with parallel spins in the $z$-direction, and the results are presented in figure 5. It is found from figures 5(a) to (b) that on one hand, at the midpoint $Δr/2$ between the two impurities, the $z$-component $s_z(r, ω)$ reaches its maximal (minimal) value, while the in-plane component $s_∥(r, ω)$ vanishes when the bonding (antibonding) state is formed. Thus the spin in the bonding (antibonding) particle-like state $|p⟩$ (hole-like $|h⟩$) is completely parallel (antiparallel) at $Δr/2$ to the impurity spins. On the other hand, the in-plane component $s_∥(r, ω)$ rotates anticlockwise (clockwise) between the two impurities.

Figure 3. (a) LDOS at the impurity site for the clean surface (black curve), a single impurity (blue curve), and two impurities (red dotted curve) located at $r_1 = (0, 0)$ and $r_2 = (8a_0, 0)$ with parallel spins along the $z$-direction, respectively. (b), (c) Spatial dependence of LDOS corresponding to peaks $Ω_1$ and $Ω_2$ in (a), respectively. The locations of impurities are denoted by black dots. (d) LDOS at $r = 0$ as a function of $Δr$ for two identical impurities with parallel spins. The red dotted curve is for $Ω_2$.
when the bonding (antibonding) state is formed. These findings may contribute to potential applications in the spin selection and also may be useful for analyzing the bound-state electron mediated Ruderman–Kittel–Kasuya–Yosida (RKKY) spin–spin interaction between magnetic impurities in TSC.

3.3. Multiple impurities—quantum corral

The nanostructures, such as quantum corrals constructed by impurities on STC, are also an issue of importance since different impurity structures usually lead to different quantum interference behaviors. Here, as an illustrative example, we briefly consider the spectral property of elliptical quantum corrals with semi-axes $a = 10a_0$, $b = 8.66a_0$, and foci $f_\pm = (\pm 5a_0, 0)$, which are constructed by 30 nonmagnetic or magnetic impurities on STC. In the following, we fix $\mu = 0.2$ eV and define $U_i = 0.8$ eV for scalar impurities ($J_i = 0$), while $m_i = 0.8$ eV ($U_i = 0$) for magnetic ones.

Firstly, we consider the quantum mirage effect of a magnetic impurity that is projected from the occupied into the empty focus of elliptical corrals. The numerical spectral properties of mirage versus the energy $\omega$ are presented in figures 6(a) and (b) for nonmagnetic and antiferromagnetic quantum corrals, respectively. The resonance peaks $\Omega_{1,2}$ of the mirage at empty focus $f_-$ (blue curve) are in good agreement with those at occupied focus $f_+$ (red curve), i.e. the mirage is clear. Comparing this with the case without corral, we find that the nonmagnetic corral unsymmetrically shifts the particle-like and hole-like peaks. In figure 6(a) the particle-like peak is shifted down by $\delta \Omega_1 = 0.27\Delta_0$, while the hole-like peak is shifted up by $\delta \Omega_2 = 0.44\Delta_0$. Surprisingly, although the mirage is obvious in the antiferromagnetic corral (figure 6(b)), the resonance peaks without the corral are not shifted by the antiferromagnetic one in 2D TSCs.
Furthermore, the influence of quantum corrals on the quantum interference effects between two spin-parallel magnetic impurities $2f_z$ (red curve) and empty focus $f_z$ (blue curve) for a nonmagnetic $U = 0.8$ eV (a) and antiferromagnetic $m = 0.8$ eV (b) corral with a magnetic impurity located at $f_x$. (c), (d) Splitting of the resonance peaks in LDOS at $f_x$ with (red curves) and without (green dotted curves) a nonmagnetic corral (c) and an antiferromagnetic corral (d) respectively, for two magnetic impurities with parallel spins. The chemical potential is $\mu = 0.2$ eV.

4. Comparison with an impurity–TI system

Before ending this paper, we would like to qualitatively compare the magnetic impurities on TSC with those on a TI surface. For brevity, we just consider one- and two-impurity cases. In the presence of magnetic impurities, the TI surface is described by

$$H = h(k) + \mu \sigma_0 + \Delta_0 \sigma_z + \sum_{i=1}^{N} [U \sigma_0 + 4S/\sigma_2] \delta(r - r_i),$$

where $2\Delta_0$ in the third term is the energy gap induced by the effective mass of Dirac fermions. The eigenvalues of the free part of equation (22) are

$$\epsilon_k = \pm \sqrt{(v_F k)^2 + \Delta_0^2 + \mu},$$

which indicates that the chemical potential is not important for impurity scattering in TI since it just move the levels up and down.

The numerically calculated LDOSs at impurity site $r_1 = (0, 0)$ for the TI surface with $\mu = 0.2$ eV are shown in figure 7, in which the black curves are for clean surface, and the blue (red) curves are for TI surface with one (two) impurity. In figures 7(a) and (b), we set the scalar scattering potential $U = 0$, while in (c) $U = 2$ eV. The magnetic moment is chosen as $m = 0.8$ eV.

Figure 7. (a) LDOS at impurity site $r_1 = (0, 0)$ on gapless (a) and gapped $\Delta_0 = 25$ meV (b), and (c) topological insulator surface. The black, blue, and red curves correspond to clean surface, one single impurity, and two impurities $2\Delta_0$ apart, respectively. In (a) and (b) $U = 0$, while in (c) $U = 2$ eV. The magnetic moment is chosen as $m = 0.8$ eV.

In summary, we have studied the spectral properties of quasiparticle states induced by localized classical magnetic impurities on 2D TSC. For the single impurity case, the critical exchange field $m_e$ and critical chemical potential $\mu_e$ are determined, and moreover, the spin LDOS is discussed. Spin polarization transformation has been found in the TSC.
system when $m > m_c$. For two-impurity scattering, we have discussed the interference effects by changing the distance and relative spin angle between two impurities. We have shown that the bound states can be changed by varying the distance between the two impurities. The quantum mirages and the influence of quantum corrals on the quantum interference effect between two impurities have also been analyzed. The nonmagnetic corral exhibits stronger influence than the antiferromagnetic one. Finally, we briefly compare the impurities in TSC with those in TI. The results illustrated that the intragap resonance states could be found in TI so long as the scalar scattering potential is taken into account. These predictions, which could be observed by STM and STS techniques, may be useful for exploring the electronic structures of TSC as well as TI materials.

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