The handbag contribution to $\gamma\gamma \rightarrow \pi\pi$ and $KK$

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Abstract

We investigate the soft handbag contribution to two-photon annihilation into pion or kaon pairs at large energy and momentum transfer. The amplitude is expressed as a hard $\gamma\gamma \rightarrow q\bar{q}$ subprocess times a form factor describing the soft transition from $q\bar{q}$ to the meson pair. We find the calculated angular dependence of the cross section in good agreement with data, and extract annihilation form factors of plausible size. A key prediction of the handbag mechanism is that the differential cross section is the same for charged and neutral pion pairs, in striking contrast with what is found in the hard scattering approach.

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Introduction. The production of pion or other hadron pairs in two-photon collisions at high energies has long been a subject of great interest. Recently it has been shown \cite{1, 2, 3} that in kinematics where one of the photons has a virtuality much larger than the squared invariant mass $s$ of the hadron pair the transition amplitude factorizes into a perturbatively calculable subprocess, $\gamma^* \gamma \rightarrow q\bar{q}$, and a soft $q\bar{q} \rightarrow \pi\pi$ transition matrix element. The latter was termed the two-pion distribution amplitude in order to emphasize its close connection to the single-pion distribution amplitude introduced in the standard hard scattering approach \cite{4}. The two-pion distribution amplitude is the timelike version of a generalized parton distribution, which encodes the soft physics information in processes such as deeply virtual \cite{5} or wide-angle \cite{6, 7} Compton scattering.

Here we are interested in the complementary kinematical region of large $s$, large momentum transfer from the photons to the pions, and vanishing photon virtuality. It has long been known \cite{8, 9} that for asymptotically large $s$ the process is amenable to a leading-twist perturbative treatment, where the transition amplitude factorizes into a hard scattering amplitude for $\gamma\gamma \rightarrow q\bar{q} q\bar{q}$ and a single-pion distribution amplitude for each pion. This distribution amplitude is constrained by the photon-pion transition form factor \cite{10, 11, 12}, and it has recently become clear \cite{13} that the perturbative contribution evaluated with such a distribution amplitude is well below experimental data.

In this letter we propose an approach which is complementary to the perturbative one for large but not extremely large energies and momentum transfers. The mechanism we investigate is similar to the one in two-photon annihilation at large $Q^2$ but small $s$. The corresponding diagrams have the handbag topology shown in Fig. 1a, and we will express them as a hard scattering $\gamma\gamma \rightarrow q\bar{q}$ times a form factor describing the soft transition $q\bar{q} \rightarrow \pi\pi$ and given by a moment of the two-pion distribution amplitude in the kinematical region of interest. The handbag contribution to $\gamma\gamma \rightarrow \pi\pi$ formally represents a power correction to the leading-twist perturbative one, which will dominate at asymptotically large scales. The approach advocated here is analogous to the handbag contribution to wide-angle Compton scattering \cite{6, 7}.

Figure 1: (a) Handbag factorization of the process $\gamma\gamma \rightarrow \pi\pi$ for large $s$ and $t$. The hard scattering subprocess is shown at leading order in $\alpha_s$, and the blob represents the two-pion distribution amplitude. The second contributing graph is obtained by interchanging the photon vertices. (b) The handbag resolved into two pion-parton vertices connected by soft partons. There is another diagram with the $\pi^+$ and $\pi^-$ interchanged.
The handbag amplitude. We are interested in $\gamma\gamma$ annihilation into a meson pair at large Mandelstam variables $s \sim -t \sim -u$. For definiteness we consider a $\pi^+\pi^-$ pair, the generalization to other mesons is straightforward. As far as possible we will proceed in analogy to the calculation of the handbag contribution to wide-angle Compton scattering \[3\]. We wish to calculate the handbag diagrams in the region of phase space where the $q\bar{q} \rightarrow \pi\pi$ transition is soft. Since the $\pi\pi$ system has large invariant mass this requires the additional $q\bar{q}$ pair and possibly other partons created in the hadronization process to have soft momenta. The momenta of the initial quark and antiquark must thus approximately equal the respective momenta of the final state pions. We see from Fig. 1b that, contrary to the crossed channel process $\gamma\pi \rightarrow \gamma\pi$, the soft $q\bar{q} \rightarrow \pi\pi$ transition cannot be described in terms of individual pion light-cone wave functions: the partons connecting the two pions cannot be incoming for both of them. We can however still understand the diagram of Fig. 1b as a covariant Feynman diagram, with each blob representing a pion-parton vertex function that is purely soft in our kinematics.

The handbag diagrams also admit kinematical configurations where the blob in Fig. 1b contains hard interactions. Explicitly writing these as hard gluon exchange one obtains a subset of the graphs calculated in the leading-twist perturbative approach. Note that there are other graphs, where the two photons do not couple to the same quark line. At large $s, t, u$ they always require hard gluon exchange, and thus appear in the leading-twist calculation but not in the soft mechanism we are concerned with here.

We work in the c.m. frame of the reaction, with axes chosen such that the process takes place in the 1-3 plane and the outgoing hadrons fly along the positive or negative 1-direction. Introducing light-cone coordinates $v = [v^+, v^-, v_\perp]$ with $v^\pm = (v^0 \pm v^3)/\sqrt{2}$ for any four-vector $v$ we then have pion momenta

$$p = \sqrt{\frac{s}{8}} \left[ 1, 1, \sqrt{2}\beta \mathbf{e}_1 \right], \quad p' = \sqrt{\frac{s}{8}} \left[ 1, 1, -\sqrt{2}\beta \mathbf{e}_1 \right],$$

with the pion velocity $\beta = \sqrt{1 - 4m^2_\pi/s}$ and $\mathbf{e}_1 = (1, 0)$. We have chosen coordinate axes with the goal in mind to describe the hadronization process by a light-cone dominated matrix element: in our coordinate system the light-cone plus-momenta of the hadrons appear in a symmetric way (as they do in the frame where the handbag contribution to wide-angle Compton scattering is calculated). The photon momenta read

$$q = \sqrt{\frac{s}{8}} \left[ 1 + \sin \theta, 1 - \sin \theta, \sqrt{2} \cos \theta \mathbf{e}_1 \right],$$

$$q' = \sqrt{\frac{s}{8}} \left[ 1 - \sin \theta, 1 + \sin \theta, -\sqrt{2} \cos \theta \mathbf{e}_1 \right],$$

where $\theta$ is the c.m. scattering angle. Up to corrections of order $m^2_\pi/s$ we have

$$\cos \theta = \frac{t - u}{s}, \quad \sin \theta = \frac{2\sqrt{tu}}{s},$$

in terms of the usual Mandelstam variables. In our symmetrical reference frame the skewness, which is defined by

$$\zeta = \frac{p^+}{(p + p')^+},$$
has a value of 1/2. Exploiting momentum conservation and introducing the plus-momentum fraction
\[ z = \frac{k^+}{(p + p')^+}, \]
we parameterize the off-mass shell quark and antiquark momenta as
\[ k = \sqrt{\frac{s}{2}} \left[ z, \bar{z} + \delta^-, \sqrt{2z\bar{z} + \delta_\perp} e_\perp \right], \quad k' = \sqrt{\frac{s}{2}} \left[ \bar{z}, z - \delta^-, -\sqrt{2z\bar{z} + \delta_\perp} e_\perp \right], \]
and their on-shell approximations as
\[ \tilde{k} = \sqrt{\frac{s}{2}} \left[ z, \bar{z}, \sqrt{2z\bar{z}} e_\perp \right], \quad \tilde{k}' = \sqrt{\frac{s}{2}} \left[ \bar{z}, z, -\sqrt{2z\bar{z}} e_\perp \right], \]
where \( \bar{z} \equiv 1 - z \) and \( e_\perp = (\cos \varphi, \sin \varphi) \). To ensure that the subprocess \( q\bar{q} \to \pi\pi \) is soft we require

(i) that all virtualities at the parton-hadron vertices be soft, of order of a squared hadronic scale \( \Lambda^2 \),

(ii) and that for each parton or system of partons in a hadron we have \( k_{\perp_i}^2/x_i \sim \Lambda^2 \), where \( k_{\perp_i} \) and \( x_i \) respectively are the transverse momentum and plus-momentum fraction in a frame where the hadron moves in the positive 3-direction.

This enforces
\[ 2z - 1, \sin \varphi, \delta^-, \delta_\perp \sim \frac{\Lambda^2}{s}, \]
and depending on whether \( \varphi \approx 0 \) or \( \varphi \approx \pi \) means that we have \( k \approx p \) or \( k' \approx p \), up to corrections of order \( \Lambda^2/s \).

We remark that condition (i) alone would constrain \( 2z - 1 \) and \( \sin \varphi \) to be of order \( \Lambda/\sqrt{s} \) only. The stronger condition (ii) arises quite naturally for light-cone wave functions \([14]\), and we also demand it here for the upper vertex in Fig. 1b. In the framework of light-cone time ordered perturbation theory \([14]\) this condition means that the light-cone energy denominator for the intermediate state with momenta \( p, k - p \) and \( k' \) must be soft of order \( \Lambda^2 \).

We now express the handbag amplitude for our process in terms of the \( \gamma\gamma \to q\bar{q} \) amplitude \( H \) and a matrix element describing the \( q\bar{q} \to \pi\pi \) transition,
\[ A = \sum_q (ee_q)^2 \int d^4k \int \frac{d^4x}{(2\pi)^4} e^{-ikx} \langle \pi^+(p) \pi^-(p') | T \bar{q}_\alpha(x) q_\beta(0) | 0 \rangle H_{\alpha\beta}(k, k') , \]
where
\[
H_{\alpha\beta}(k, k') = \left[ \epsilon \cdot \gamma \frac{(k - q) \cdot \gamma}{(k - q)^2 + i\epsilon} \epsilon' \cdot \gamma + \epsilon' \cdot \gamma \frac{(q - k') \cdot \gamma}{(q - k')^2 + i\epsilon} \epsilon \cdot \gamma \right]_{\alpha\beta}
\]
with the photon polarization vectors \( \epsilon \) and \( \epsilon' \). The summation index \( q \) refers to the quark flavors \( u, d, s \), and we have omitted terms in \( H \) suppressed by the current quark masses.

\(^4\)Such a frame is obtained for each pion via a transverse boost from the c.m., see e.g. [1].
In order to select the dominant Dirac structure of the soft matrix element, we follow [7] and perform a transverse boost to a frame where the on-shell vector $\vec{k}'$ has a zero transverse and hence also a zero minus-component. In this frame we decompose the quark field into its good and bad components,

$$q(0) = \frac{1}{2} \gamma^- \gamma^+ q(0) + \frac{1}{2} \gamma^+ \gamma^- q(0)$$

$$= \frac{1}{2k'^+} \sum_{\lambda'} \left\{ u(\vec{k}', \lambda') \left[ \bar{v}(\vec{k}', \lambda') \gamma^+ q(0) \right] + \gamma^+ u(\vec{k}', \lambda') \left[ \bar{v}(\vec{k}', \lambda') q(0) \right] \right\},$$

with a sum over helicities $\lambda'/2 = \pm 1/2$. In the second line we have used the completeness relation for massless spinors and the relation $\vec{k}' \cdot \gamma = k'^+ \gamma^-$ valid in the frame we are now working in. Since the momenta at the soft parton-hadron vertices have large plus-components, but by definition no large invariants, the term with the bad components is suppressed as $\Lambda/\sqrt{s}$ compared with the good ones. Transverse boosts leave plus-components invariant, so that the decomposition in the second line of (11) also holds in the c.m. frame. By an analogous argument for $\bar{q}(x)$, we obtain

$$\mathcal{T}_a(x) q_\beta(0) H_{a\beta}$$

$$= \frac{1}{4k^+ k'^+} \sum_{\lambda=-\lambda'} \left\{ q(x) \gamma^+ u(\vec{k}, \lambda) \left[ \bar{v}(\vec{k}', \lambda') \gamma^+ q(0) \right] \left[ \bar{u}(\vec{k}, \lambda) H v(\vec{k}', \lambda') \right] + \mathcal{O} \left( \frac{\Lambda^2}{s} \right) \right\},$$

where the restriction $\lambda = -\lambda'$ implements that the hard scattering conserves chirality to leading order in the current quark masses $m_q$. The product of two bad components in $\mathcal{T}_a(x) q_\beta(0)$ is suppressed by $\Lambda^2/s$. Terms with one good and one bad component are even smaller: since they flip quark chirality in the hard scattering they come with a factor of $m_q/\sqrt{s}$ in addition to the $\Lambda/\sqrt{s}$ suppression from the soft matrix element. With a suitable phase convention for quark spinors (see e.g. [13]) and with antiquark spinors satisfying $v(k, \lambda) = -u(k, -\lambda)$ we have

$$u(\vec{k}, \lambda) \bar{v}(\vec{k}', -\lambda) = -\frac{1}{\sqrt{4k^+ k'^+}} \frac{1 + \lambda \gamma_5}{2} (\vec{k} \cdot \gamma) \gamma^+ (\vec{k}' \cdot \gamma)$$

and get, up to corrections of order $\Lambda^2/s$,

$$\mathcal{A}_{\mu\nu'} = -\sum_q (ee_q)^2 \int d^4k \frac{1}{4k^+ k'^+} \sum_\lambda \bar{u}(\vec{k}, \lambda) H_{\mu\nu'}(k, k') u(\vec{k}', -\lambda)$$

$$\times \int \frac{d^4x}{(2\pi)^4} e^{-ik \cdot x} \langle \pi^+(p) \pi^-(p') \rangle \langle T \bar{q}(x) \gamma^+ \frac{1 + \lambda \gamma_5}{2} q(0) | 0 \rangle,$$

where we have made explicit the dependence on the photon helicities $\mu$ and $\mu'$. Let us now concentrate on the term with the vector current $\bar{q}(x) \gamma^+ q(0)$ and come back to the axial current term later. From charge conjugation invariance we know that the two pions produced in the two-photon collision are in a $C$ even state, so that we can explicitly symmetrize their state vector in the soft matrix element,

$$\mathcal{S} = \frac{1}{2} \int \frac{d^4x}{(2\pi)^4} e^{-ik \cdot x} \langle \pi^+(p) \pi^-(p') + \pi^+(p') \pi^-(p) \rangle \frac{1}{2} \langle T \bar{q}(x) \gamma^+ q(0) | 0 \rangle.$$
Abbreviating
\[ \mathcal{H}_{\mu\nu} = \sum_{\lambda} \bar{u}(\bar{k}, \lambda) H_{\mu\nu}(k, k') v(k', -\lambda) \]  
we then have
\[ \mathcal{A}_{\mu\nu} = -\sum_q (ee_q)^2 \int d^4k \frac{1}{\sqrt{4k^+ k'^+}} \mathcal{H}_{\mu\nu}(k, k') S(k, k') + \text{axial current term}, \]  
where due to charge conjugation invariance
\[ S(k, k') = -S(k', k), \quad \mathcal{H}(k, k') = -\mathcal{H}(k', k). \]  
According to our hypothesis, the soft matrix element \( S(k, k') \) should be strongly peaked when (8) is fulfilled. The two regions \( k \approx p \) and \( k \approx p' \) where this is the case are related through a rotation by \( \pi \) about the 3-axis of our coordinate system. To proceed we separate the integration over \( k \) into two regions, one with \( \varphi \in [-\pi/2, \pi/2] \) and one with \( \varphi \in [\pi/2, 3\pi/2] \). Because of (18) both give the same integral, and we can write
\[ \int \frac{d^4k}{\sqrt{4k^+ k'^+}} \mathcal{H}(k, k') S(k, k') = \int \frac{dk^+ dk^- dk'^+}{\sqrt{4k^+ k'^+}} \int_{-\pi/2}^{\pi/2} d\varphi \, \mathcal{H}(k, k') S(k, k'), \]  
where the integral is dominated by the region \( k \approx p \). Since the hard scattering \( \mathcal{H}(k, k') \) depends significantly on \( k \) and \( k' \) only over scales of order \( \sqrt{s} \), we Taylor expand it around \( z = 1/2, \varphi = 0, \delta^- = \delta_\perp = 0 \). Keeping only the leading terms of the expansion in \( \delta^- \) and \( \delta_\perp \) we obtain \( \mathcal{H}_{--} = \mathcal{H}_{++} = 0 \) and
\[ \mathcal{H}_{+-} = \mathcal{H}_{-+} = 2 \left( \sqrt{u/t} - \sqrt{t/u} \right) - (z - \bar{z}) \left( s/t + s/u \right) + \mathcal{O}((z - \bar{z})^2, \varphi^2), \]  
where according to (8) the first term is of order 1, the second of order \( \Lambda^2/s \), and the terms denoted by \( \mathcal{O} \) of order \( \Lambda^4/s^2 \). It turns out that the leading term in the expansion (20) leads to a zero integral in (19). To see this we remark that due to rotation invariance about the 3-axis we have \( S(k^+, k^-, k_{\perp}) = S(k^+, k^-, -k_{\perp}) \), so that
\[ \int \frac{dk^+ dk^- dk'^+}{\sqrt{4k^+ k'^+}} \int_{-\pi/2}^{\pi/2} d\varphi \, S(k, k') = \int \frac{d^4k}{\sqrt{4k^+ k'^+}} S(k, k'), \]  
which is zero because of (18). The vanishing of what would have been the leading term is thus due to a conspiracy of invariance under charge conjugation and rotation, and it is instructive to see why this does not happen in the crossed channel process, even if one scatters on a \( C \) eigenstate. The soft handbag contribution to wide-angle Compton scattering \( \gamma \pi^0 \rightarrow \gamma \pi^0 \) is given by a convolution analogous to (17). The two possible solutions to the condition that the hadronic matrix element is soft now correspond to the photon scattering on a quark or on an antiquark. The integration over the parton momentum \( k \) is then split into regions \( k^+ > 0 \) and \( k^+ < 0 \), and these two regions are only related by charge conjugation, but not by any rotation.

Due to parity invariance the axial current term in (17) vanishes to leading order in the off-shell parameters \( \delta^- \) and \( \delta_\perp \). The first nonzero contribution to our process is then the one going with \( z - \bar{z} \) in (20), which according to (8) and (12) is parametrically of the same order.
as the parton off-shellness effects and contributions from the bad components of the fermion fields. A treatment of those is beyond the scope of this work and will among other things have to address issues of gauge invariance. Rather we will remain with the good components and the on-shell approximation, where the hard-scattering $\gamma\gamma \rightarrow q\bar{q}$ is manifestly gauge invariant. We must then at this stage consider our result as a model, or a partial calculation of the soft handbag contribution.

To proceed we thus keep the $z-\bar{z}$ term in $\mathcal{H}$. Since it is $\varphi$ independent, the integral over $k_\perp$ can be extended to the full region as in (21). For a given $k^+$ we then perform the integrals over $k_\perp$ and $k^-$. They only concern the soft matrix element $S(k, k')$, and we obtain the two-pion distribution amplitude in light-cone gauge [2],

$$\Phi^q_{2\pi}(z, \zeta = 1/2, s) = \int \frac{dx}{2\pi} e^{-i z (p+p')^+ x^-} \langle \pi^+(p) \pi^-(p') | T(x) \gamma^+ q(0) | 0 \rangle_{x=(0,x^-)}.$$  

Here we have used that for $\zeta = 1/2$ explicit symmetrization in the pion momenta is not needed, and that the time-ordering of the quark fields can be dropped after the $k^-$ integration [16].

Up to still higher orders we have $\sqrt{4k^+ k'^+} \approx 2p^+$ in (17) and obtain our final result

$$A_{+-} = A_{-+} = -4\pi \alpha_{\text{elm}} \frac{s^2}{tu} R_{2\pi}(s),$$

where we have defined the annihilation form factor by

$$R_{2\pi}(s) = \sum_q e^2_q R^q_{2\pi}(s), \quad R^q_{2\pi}(s) = \frac{1}{2} \int_0^1 dz (2z - 1) \Phi^q_{2\pi}(z, 1/2, s).$$

The operator corresponding to this form factor is the quark part of the energy-momentum tensor. This has positive $C$ parity, as needed for a pion pair produced in two-photon annihilation. Note that integrating $\Phi^q_{2\pi}(z, \zeta, s)$ over $z$ without the weight $(2z - 1)$ leads to the form factor of the quark vector current, which is $C$ odd.

The differential cross section of our process is given by

$$\frac{d\sigma}{dt}(\gamma\gamma \rightarrow \pi^+\pi^-) = \frac{8\pi \alpha_{\text{elm}}^2}{s^2} \frac{1}{\sin^2 \theta} \left| R_{2\pi}(s) \right|^2,$$

and the cross section integrated over $\cos \theta$ from $-\cos \theta_0$ to $\cos \theta_0$ reads

$$\sigma(\gamma\gamma \rightarrow \pi^+\pi^-) = \frac{4\pi \alpha_{\text{elm}}^2}{s} \left[ \frac{\cos \theta_0}{\sin^2 \theta_0} + \frac{1}{2} \ln \left( 1 + \frac{\cos \theta_0}{1 - \cos \theta_0} \right) \right] \left| R_{2\pi}(s) \right|^2.$$  

When comparing with experiment we will quote the integrated cross section for $\cos \theta_0 = 0.6$,

$$\sigma(\gamma\gamma \rightarrow \pi^+\pi^-) = 425 \text{ nb GeV}^2 \left| R_{2\pi}(s) \right|^2 / s.$$  

**Flavor symmetry.** Our results (23) to (25) easily generalize to the production of other pairs of pseudoscalar mesons. New form factors then appear, which are related by flavor symmetry. A characteristic feature of the handbag approach is the intermediate $q\bar{q}$ state, which allows
only for isospin \( I = 1 \) and \( I = 0 \). Since a \( \pi^+\pi^- \) pair in an \( I = 1 \) state is \( C \)-odd, \( \pi^+\pi^- \) as well as \( \pi^0\pi^0 \) pairs are only produced in isospin zero states. This leads to \[ R_{2\pi}^u(s) = R_{2\pi}^d(s) , \] (28)
and to the same form factors for both charge combinations, resulting in \[ \mathcal{A}_{\mu\nu}(\gamma\gamma \to \pi^+\pi^-) = \mathcal{A}_{\mu\nu}(\gamma\gamma \to \pi^0\pi^0) . \] (29)

Taking recourse to \( U \)-spin symmetry, i.e., the symmetry under the exchange \( d \leftrightarrow s \), we can relate the form factor for the production of a \( K^+K^- \) pair to that for pion pairs. Since the photon behaves as a \( U \)-spin singlet while \( (K^+ , \pi^+) \) and \( (K^-, \pi^-) \) are doublets, \( U \)-spin conservation leads to
\[ \mathcal{A}_{\mu\nu}(\gamma\gamma \to K^+K^-) \simeq \mathcal{A}_{\mu\nu}(\gamma\gamma \to \pi^+\pi^-) \] (30)
and corresponding relations among the two sets of form factors. In contrast to (29), which is characteristic of the handbag approach, (30) holds in any dynamical approach respecting \( SU(3) \) flavor symmetry. Finally, isospin links the \( K^+K^- \) form factors \( R_{2K}^d(s) \) to those for \( K^0\bar{K}^0 \) production. Putting everything together we have the following set of relations:
\[ R_{2\pi}^u(s) = R_{2\pi}^d(s) \simeq R_{2K}^u(s) = R_{K^0\bar{K}^0}^d(s) \]
\[ = R_{2\pi}^s(s) \simeq R_{2K}^s(s) = R_{K^0\bar{K}^0}^d(s) , \] (31)
\[ R_{2\pi}^s(s) = R_{2\pi}^a(s) \simeq R_{2K}^a(s) = R_{K^0\bar{K}^0}^d(s) . \] (32)

We neglect isospin breaking while, in general, flavor symmetry violations cannot numerically be ignored. This is indicated in (30) to (32) by the approximate symbol.

To the extent that flavor symmetry holds there are only two independent form factors, a valence quark one, \( R_{2\pi}^u \), and a non-valence one, \( R_{2\pi}^s \). Following our discussion of the soft handbag amplitude one may expect that \( |R_{2\pi}^u| \ll |R_{2\pi}^s| \). In order to be soft these form factors require the parton entering the meson to take most of its momentum, and it is plausible to assume that this parton is most likely a valence quark. This is in accordance with experience from parton densities in the limit \( x \to 1 \). Except for \( K^0\bar{K}^0 \) production, the contribution from the non-valence form factor to the amplitudes is further suppressed by the charge factor \( e_d^2 / (e_u^2 + e_d^2) = 1/5 \). For pions it thus seems to be quite safe to neglect the non-valence form factor and we arrive at
\[ R_{2\pi}(s) = (e_u^2 + e_d^2) R_{2\pi}^u(s) . \] (33)

For the process \( \gamma\gamma \to K^0\bar{K}^0 \), on the other hand, we have
\[ R_{K^0\bar{K}^0}(s) \simeq e_u^2 R_{2K}^d(s) + (e_d^2 + e_u^2) R_{2K}^u(s) , \] (34)
and see that this process is more sensitive to the non-valence form factor. Neglecting the non-valence contribution nevertheless, we obtain
\[ R_{K^0\bar{K}^0}(s) \simeq e_d^2 + e_u^2 e_u^2 + e_d^2 R_{2K}(s) . \] (35)

\(^5\)The sign in this and the following relations depends on the phase convention for the different meson states, cf. Appendix A of \[17\].
and a corresponding relation between $\mathcal{A}_{\mu\nu}(\gamma\gamma \rightarrow K^+K^-)$ and $\mathcal{A}_{\mu\nu}(\gamma\gamma \rightarrow K^0\bar{K}^0)$.

Notice that all we have used in our discussion of flavor symmetry is the general structure of the handbag amplitude (6) with its $q\bar{q}$ intermediate state, plus valence quark dominance in the case of (33) and (35). Our predictions relating $\pi^+\pi^-$ with $\pi^0\pi^0$ and $K^+K^-$ with $K^0\bar{K}^0$ production do therefore not require the technical approximations we needed to arrive at (23) and (24), like the neglect of off-shell corrections or of the bad components of the quark fields. As already mentioned, the relation (30) is yet more general.

Comparison with experiment. The new measurements of $\gamma\gamma \rightarrow \pi^+\pi^-$, $K^+K^-$ performed by ALEPH [18] and DELPHI [19] allow for an experimental determination of the annihilation form factors quite analogous to the measurements of electromagnetic form factors. One thus extracts moments of the two-pion distribution amplitude. This amplitude is related by crossing to the ordinary parton distributions in the pion, which have been extracted from Drell-Yan data in pion-nucleon scattering. The annihilation form factors and the two-pion distribution amplitude can as yet not be calculated within QCD. As follows from our earlier remark they do not admit a direct representation as overlaps of light-cone wave functions [20] either. A recent investigation [21] has sought to circumvent this restriction using a Bethe-Salpeter approach. To our knowledge, no model calculation is presently available for the annihilation form factors in the $s$ range where we need them.

![Figure 2: The scaled annihilation form factors $s|R_{2\pi}(s)|$ (left) and $s|R_{2K}(s)|$ (right) versus $s$. The preliminary ALEPH and DELPHI data is taken from [18, 19] and plotted according to (27). Dashed lines represent our fitted values (36) and (40).](image)

In order to avoid the resonance region, we restrict ourselves to data with $\sqrt{s} \gtrsim 2.5$ GeV here and in the following. We have used (27) to extract the form factor $R_{2\pi}(s)$ from the preliminary data on $\gamma\gamma \rightarrow \pi^+\pi^-$ [18, 19]. As Fig. 2 reveals, the form factor scaled by $s$ is compatible with a constant over a large range of $s$, within the still large experimental errors. A fit provides

$$s|R_{2\pi}(s)| = 0.75 \pm 0.07 \text{ GeV}^2.$$

The annihilation form factor is comparable in magnitude with the timelike electromagnetic
form factor of the pion, which is related to the first moment of the two-pion distribution amplitude \([2, 22]\). We have performed a combined fit to the admittedly poor \(e^+e^- \rightarrow \pi^+\pi^-\) data \([23]\) in the range \(4 \text{ GeV}^2 < s < 9 \text{ GeV}^2\), and to the branching ratio of \(J/\Psi \rightarrow \pi^+\pi^-\) \([24]\), which to a good approximation provides the form factor at \(s = M^2_{J/\Psi}\) \([23]\). This yields

\[
s|F_\pi(s)| = 0.93 \pm 0.12 \text{ GeV}^2.
\]  

(37)

Omitting the \(J/\Psi\) data would increase the errors but not significantly alter the central value of the fit. It is amusing to note that the data giving access to the annihilation form factor is more precise than the one for the well-known and extensively discussed electromagnetic one, where improvement would be highly welcome. The similarity between (36) and (37) is reminiscent of the spacelike region, where the form factors for wide-angle Compton scattering off protons and the Dirac form factor also have similar \(s\) behavior and are of comparable magnitude \([4]\).

The \(s\) dependence of both the annihilation and the electromagnetic form factor is in agreement with the dimensional counting rule behavior. At this point we must realize that, since we have calculated the soft part of the handbag diagrams, the form factor appearing in our result \([23]\) is only the soft part \(R^\text{soft}_{2\pi}(s)\) of the matrix element defined by \([23]\) and \([24]\). At very large \(s\) this is power suppressed compared to the hard perturbative part \(R^\text{pert}_{2\pi}(s)\), which scales like \(1/s\). Asymptotically the cross section \([25]\) for \(\gamma\gamma \rightarrow \pi\pi\) therefore decreases faster than \(1/s^4\) at fixed angle \(\theta\) and thus is indeed a power correction to the leading twist contribution. Our fit \([30]\) of the form factor to the available data does not display a falloff faster than \(1/s\). In the absence of a dynamical model for \(R^\text{soft}_{2\pi}(s)\) we cannot say at which \(s\) this falloff will start and how rapid it will be. In the case of wide-angle Compton scattering on the proton, an explicit model in terms of light-cone wave functions has shown how the soft overlap part of the Compton form factor can mimic dimensional counting behavior over a finite range of \(s\) \([26]\).

As to the hard part \(R^\text{pert}_{2\pi}\) of the annihilation form factor, it is readily obtained from the leading-twist expression of the two-pion distribution amplitude at large \(s\) \([27]\). Taking the asymptotic form of the single-pion distribution amplitude we get \(s|R^\text{pert}_{2\pi}| \sim \alpha_s \times 0.1 \text{ GeV}^2\). This is indeed negligible compared to \([30]\), and for simplicity we write \(R_{2\pi}(s)\) instead of \(R^\text{soft}_{2\pi}(s)\) throughout this work.

Clearly, the handbag diagrams are not the only ones to provide a soft physics contribution to \(\gamma\gamma \rightarrow \pi\pi\). A different contribution coming to mind is due to the hadronic components of the photons, which one may model using vector meson dominance. Unfortunately, no data is available for elastic or quasielastic meson-meson scattering at large c.m. energy and angle. We can only observe that experimentally many other hadronic processes at large angle show an \(s\) behavior compatible with dimensional scaling. If this were also true for \(\rho\rho \rightarrow \pi\pi\) then the vector dominance contribution to two-photon annihilation would decrease faster than the data in Fig. 2 by a power of \(1/s\) at the amplitude level. We also remark that for wide-angle Compton scattering off the proton one can estimate the vector dominance part if, following quark model ideas, one relates \(\rho\rho \rightarrow \rho\rho\) to \(\pi\pi \rightarrow \pi\pi\). Using the data for the latter, one finds that for \(\theta \approx 90^\circ\) and \(s\) between 8 and 10 GeV\(^2\) the corresponding contribution to \(\gamma p \rightarrow \gamma p\) is about an order of magnitude below the measured cross section \([23]\). One also observes that its suppression scales like \(1/s\) in the amplitude. We finally remark that the flavor symmetry relations we elaborated for the soft handbag are not generically satisfied by the vector dominance mechanism, since a \(\rho^0\rho^0\) pair can couple to isospin \(I = 2\).
In Ref. [22] a simultaneous expansion of the two-pion distribution amplitude in eigenfunctions of the corresponding evolution kernel and in partial waves of the $\pi\pi$ system has been given. The moment (24) of the two-pion distribution amplitude involves two of the coefficients in that expansion:

$$R_{2\pi}(s) = \frac{5}{18} \int_0^1 dz (2z - 1) \Phi_{2\pi}^u(z, 1/2, s) = \frac{1}{6} \left[ B_{10}^u(s) - \frac{1}{2} B_{12}^u(s) \right], \quad (38)$$

where we have neglected the non-valence contribution $R_{2\pi}^s$. Here $B_{nl}^u$ is the expansion coefficient of the two-pion distribution amplitude for $u$ quarks, with $n$ giving the order of the Gegenbauer polynomial, and $l$ the partial wave of the $\pi\pi$ system. We remark that for $s = 0$ the coefficient $B_{12}$ can be expressed in terms of the ratio $M_Q$ of momentum carried by quarks in a single pion [17, 22]. Furthermore, a soft pion theorem [22] provides the relation $B_{10}(0) = -B_{12}(0)$. With these two inputs we obtain

$$\left|R_{2\pi}(s = 0)\right| = \frac{5}{36} M_Q. \quad (39)$$

Taking the LO GRS parameterization of parton distributions in the pion [29], one finds $M_Q$ between 0.7 and 0.5 at renormalization scales $\mu^2$ from 0.26 GeV$^2$ to 36 GeV$^2$. The size of $R_{2\pi}(s)$ at $s = 0$ is thus comparable to the one which our fit (36) gives for $s$ around 6 GeV$^2$, just above the resonance region.

The analysis of the preliminary data for the production of charged kaon pairs, see Fig. 2, gives for the kaon annihilation form factor

$$s |R_{2K}(s)| = 0.64 \pm 0.04 \text{ GeV}^2, \quad (40)$$

which is close to the value (36) for charged pions. Taking the central values of our fits we find that flavor symmetry violation lead to a suppression of the kaon form factor by about 15%, which according to phenomenological experience is a rather typical value. For the cross sections our fits (36) and (40) give a ratio of $K^+K^-$ to $\pi^+\pi^-$ production between 0.54 and 1 within one standard deviation, with a central value of 0.73. We regard this as compatible with the $U$-spin relation

$$\frac{d\sigma}{dt}(\gamma\gamma \rightarrow K^+K^-) \simeq \frac{d\sigma}{dt}(\gamma\gamma \rightarrow \pi^+\pi^-). \quad (41)$$

In Fig. 3 we compare our results with the CLEO data for the integrated cross section [30], where pions and kaons have not been separated, and find rather good agreement.

The $1/\sin^4 \theta$ behavior of the differential cross section, which represents a characteristic result of our handbag calculation (25), is confronted with experiment in Fig. 4. Good agreement with the preliminary ALEPH data [18] for pions and kaons can be observed. The large $s$ data from the other experiments [19, 30] are comparable with a $1/\sin^4 \theta$ behavior, too.

We recall that, besides the angular dependence, there is another parameter-free prediction of the handbag approach:

$$\frac{d\sigma}{dt}(\gamma\gamma \rightarrow \pi^0\pi^0) = \frac{d\sigma}{dt}(\gamma\gamma \rightarrow \pi^+\pi^-). \quad (42)$$

In the integrated cross section the statistical factor $1/2$ for identical particles in the final state must be taken into account. Unfortunately the existing data for the $\pi^0\pi^0$ channel is either at
Figure 3: The CLEO data \cite{30} for the cross section $\sigma(\gamma\gamma \rightarrow \pi^+\pi^-)+\sigma(\gamma\gamma \rightarrow K^+K^-)$ integrated with $|\cos \theta| < 0.6$. The solid line is the result of the handbag approach with our fitted annihilation form factors \cite{30} and \cite{10}. The dashed line is the estimate of the leading-twist perturbative contribution described below.

too small energies or has too large errors. Likewise, no data is available on $K^0\bar{K}^0$ production at $\sqrt{s} \gtrsim 2.5$ GeV. To the extent that the non-valence form factor can be neglected we find

$$
\frac{d\sigma}{dt}(\gamma\gamma \rightarrow K^0\bar{K}^0) \simeq \frac{4}{25} \frac{d\sigma}{dt}(\gamma\gamma \rightarrow K^+K^-).
$$

(43)

It would be interesting to examine this relation experimentally. Deviations from \cite{13} may provide information on the non-valence form factor $R_{2K}$ and on the pattern of flavor SU(3) breaking. The generalization to other pseudoscalar pairs like $\eta\eta$ and $\eta'\eta'$ is also possible. Based on flavor symmetry we expect production rates of similar size as for $K^0\bar{K}^0$.

**Comparison with the leading-twist perturbative approach.** Let us now discuss a few characteristic differences between our handbag approach and the hard scattering picture of Brodsky and Lepage. In the perturbative approach there are two $q\bar{q}$ pairs in the intermediate state, which allows for a non-zero isospin $I = 2$ amplitude. Hence, our relation (29) does not hold in this approach. In fact, if one uses a pion distribution amplitude which is compatible with the photon-pion transition form factor \cite{11,11,12}, the differential cross section for $\pi^0\pi^0$ production is found about an order of magnitude smaller than that for $\pi^+\pi^-$ pairs \cite{8}. This implies $I = 0$ and $I = 2$ transitions of nearly the same magnitude, in sharp contrast to the situation in the handbag approach. Flavor SU(3) violations also manifest themselves differently in the two mechanisms. Since the single-meson distribution amplitudes are normalized to the respective decay constants, a factor of $(f_K/f_\pi)^4 \approx 2.2$ appears in favor of the $\gamma\gamma \rightarrow K^+K^-$ cross section in the perturbative picture. In order to obtain a $K^+K^-$ cross section comparable to or smaller than the one for $\pi^+\pi^-$, one needs a narrower shape for the kaon than for the pion distribution amplitude.

As already mentioned in the introduction, the perturbative result is way below the experimental data if single-pion distribution amplitudes consistent with other data are employed \cite{13}. 

Studies of the spacelike pion form factor $F_\pi(Q^2)$ suggest that for processes of the type we are considering higher order corrections in $\alpha_s$ can be substantial. One may hope to keep their size moderate by using the BLM prescription for setting the scale of the running coupling \[31\]. This cannot be done for our process as long as the next-to-leading order corrections in $\alpha_s$ have not been calculated, but one may take the spacelike pion form factor as a guideline, where $\mu_{BLM}^2 \approx 0.05 Q^2$ \[32\]. For most of the $s$ range we are dealing with the corresponding scale is then too low to use the perturbative expression of the running coupling, and one has to make an ansatz for its behavior in the infrared region. This is a highly nontrivial problem, and a wide choice of options is discussed in the literature. For simplicity we evaluate the leading-twist expression with a fixed coupling $\alpha_s = 0.5$, a size suggested by different lines of investigation \[33\]. Following \[13\] we take the asymptotic form for both pion and kaon distribution amplitudes, which in light of our above remark should rather over- than underestimate the $K^+K^-$ cross section. The leading-twist prediction thus obtained amounts to about 15% of our fitted handbag result as shown in Fig. 3. In view of this we consider that we make an acceptably small error in our analysis by altogether neglecting the hard perturbative part compared with the soft handbag mechanism. Note that taking it into account would require us to fit both the magnitude and the phase of $R_{2\pi}(s)$ since the two contributions must be added at amplitude level. Also, a careful study would be necessary to avoid double counting because the leading-twist expression evaluated with an infrared saturated coupling contains soft physics effects, including the diagrams with handbag topology.

Brodsky and Lepage \[8\] have proposed a formula for meson pair production which looks similar to \[23\], except for a different charge factor and the appearance of the timelike electromagnetic meson form factor instead of the annihilation form factor $R(s)$. This formula was obtained from the leading-twist result by neglecting part of the amplitudes with opposite photon helicities. As has been pointed out in \[3\], this part is however not approximately in-
dependent of the pion distribution amplitude and not generically small. We also remark that the appearance of $F_\pi(s)$ in the $\gamma\gamma \rightarrow \pi^+\pi^-$ amplitude is no longer observed if corrections from partonic transverse momentum in the hard scattering process are taken into account, and that these corrections are not numerically small for the values of $s$ we are dealing with \cite{13}. Notice further that two-photon annihilation produces two pions in a $C$-even state, whereas the electromagnetic form factor projects on the $C$-odd state of a pion pair. In contrast, our annihilation form factor $R_{2\pi}(s)$ is $C$-even as discussed after (24). Finally, due to a particular charge factor, the Brodsky-Lepage formula leads to a vanishing cross section for $\gamma\gamma$ annihilation into pairs of neutral pseudoscalars.

On the other hand, its apparent phenomenological success for $\pi^+\pi^-$ and $K^+K^-$ production is not a surprise because of its similarity to our result (25). This success is achieved if one takes a suitable value for the timelike electromagnetic form factor, related to our annihilation form factor via $|F_\pi(s)|_{BL} = |R_{2\pi}(s)|/\sqrt{2}$. Our fit \cite{16} amounts to $s|F_\pi|_{BL} = 0.53 \pm 0.05 \text{ GeV}^2$, which is clearly smaller than the experimental value \cite{37} we extracted for $s|F_\pi|$, and at the same time larger than the leading-twist perturbative result for this form factor given in \cite{11}. In view of this we do not think that the presently available data on $F_\pi(s)$ and on $\gamma\gamma$ annihilation into $\pi^+\pi^-$ and $K^+K^-$ can be considered as a success of the Brodsky-Lepage formula or of the leading-twist perturbative approach.

**Summary.** We have discussed the soft handbag contribution to two-photon annihilation into pseudoscalar meson pairs at large energy and large momentum transfer. Our main result is to express the amplitude as a product of a parton-level subprocess, $\gamma\gamma \rightarrow q\bar{q}$, and an annihilation form factor given by a moment of the two-meson distribution amplitude at skewness $\zeta = 1/2$. The operator associated with this form factor is the quark part of the energy-momentum tensor. To obtain our result we have neglected quark off-shell effects in the hard scattering and the bad components of the corresponding field operators. A closer investigation of these corrections, which as far as we could establish are of the same parametric order as the terms we retained, is an open task. We remark that according to Radyushkin it may be possible to treat the processes under investigation in the framework of double distributions \cite{34}.

Although the handbag contribution formally represents a power correction to the asymptotically leading perturbative contribution, it seems to dominate at experimentally accessible energies. We find that the data for $\pi^+\pi^-$ and $K^+K^-$ production is compatible with annihilation form factors behaving as $1/s$ for $s$ between $6 \text{ GeV}^2$ and $36 \text{ GeV}^2$, a counting rule behavior typical of many exclusive observables. Fitting the form factors to the preliminary ALEPH \cite{18} and DELPHI \cite{19} data, we find that for pions the annihilation form factor is comparable in size to the timelike electromagnetic form factor, and that for kaons it is suppressed by an amount consistent with moderate flavor $SU(3)$ breaking. A severe test of our approach is the $1/\sin^4 \theta$ angular dependence of the cross section, which agrees well with the preliminary ALEPH and DELPHI data. We also find good agreement with the CLEO data \cite{30} on the combined cross section for pion and kaon production.

A key prediction of the handbag mechanism is that the differential cross sections for $\pi^+\pi^-$ and $\pi^0\pi^0$ production should be the same. This is in sharp contrast to the leading-twist perturbative approach, where $\pi^0\pi^0$ is found suppressed by about an order of magnitude. Measurement of the production ratio of neutral and charged pion pairs would thus be most valuable to help us understand the dynamics of such processes. Under further assumptions, the handbag
mechanism also predicts the production ratio of $K^0\bar{K}^0$ and $K^+K^-$. Amusingly, our expression for the cross section is very similar to the formula proposed by Brodsky and Lepage [8], where instead of our annihilation form factor the timelike electromagnetic one appears. We would however like to emphasize that the dynamical origins of the two expressions are completely different. We also recall that the Brodsky-Lepage formula does not represent the full leading-twist perturbative result, and we found that it has normalization problems when compared with presently available data.

The factorization of the soft handbag diagrams is analogous to the one in wide-angle Compton scattering. For the latter it has recently been shown that this factorization remains valid when taking into account next-to-leading corrections in $\alpha_s$ to the parton-level subprocess [35]. We are tempted to expect that this also holds in the annihilation process considered here.

It is straightforward to extend the results of this letter to the case where one or two of the photons is off-shell by an amount not significantly bigger than the large scale $s$ in the process. Another generalization is the production of vector meson or baryon-antibaryon pairs, where several form factors describing the spin structure of the final state will appear. Finally, the time reversed process of $p\bar{p}$ annihilation into photon pairs can be described in the same way.

Acknowledgments. We would like to thank E. Leader for correspondence, and A. Finch and K. Grzelak for providing us with the numbers of the preliminary ALEPH and DELPHI data.

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