On the Super-NLS Equation and its Relation with $N=2$ Super-KdV within Coset Approach.

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Abstract

A manifestly $N = 2$ supersymmetric coset formalism is introduced to describe integrable hierarchies. It is applied to analyze the super-NLS equation. It possesses an $N = 2$ symmetry since it can be obtained from a manifest $N = 2$ coset algebra construction. A remarkable result is here discussed: the existence of a Bäcklund transformation which connects the super-NLS equation to the equations belonging to the integrable hierarchy of one particular (the $a = 4$) $N = 2$ super-KdV equation. $N = 2$ scalar Lax pair operators are introduced for both super-KdV and super-NLS.

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Introduction.

Studying the hierarchies of integrable differential equations is an interesting subject not only on a purely mathematical basis but also, on physical grounds, due to their connection with the discretized versions of the two-dimensional gravity. Looking for supersymmetric extensions of such hierarchies is therefore quite important because it is commonly believed that any consistent 2-dimensional theory which includes gravity is necessarily supersymmetric to avoid problems arising from tachyonic states in the purely bosonic sector. Even if no discretized supergravity formulation is available at present there has been some attempts to bypass this step and to formulate such theories directly in terms of supersymmetric integrable hierarchies.

The understanding of bosonic hierarchies is rather good at present (see [1]), while the supersymmetric case is much less understood.

In this letter we use an \( N = 2 \) manifestly supersymmetric formalism to treat with \( N = 2 \) hierarchies and we apply it to analyze the simplest \( N = 2 \) model, namely the super-NonLinear Schrödinger Equation [2-5].

It has been shown in [5] that what is commonly denoted as \( N = 1 \) super-NLS equation [2-4] possesses a hidden \( N = 2 \) supersymmetry structure. In this letter we will show that such hidden \( N = 2 \) supersymmetry structure finds a natural explanation due to the fact that the hierarchy can be obtained as an \( N = 2 \) coset of an \( N = 2 \) superalgebra.

A remarkable connection [5] between the super-NLS equation and the special \( N = 2 \) supersymmetric version of KdV obtained by setting the parameter \( a = 4 \) (see [6]) is discussed in the framework of coset approach. There exists indeed a Bäcklund transformation which maps the super-NLS equation into the second flow of the hierarchy associated to the \( N = 2, a = 4 \), KdV equation, the super-KdV itself being associated to the third flow.

The crucial point which allows recognizing the existence of such Bäcklund transformation is the existence of a disguised version [7] of the bosonic NLS equation which admits a Virasoro \( \times \hat{U}(1) \) Kac-Moody algebra structure; such equation admits an immediate and natural \( N = 2 \) generalization by accommodating these fields into a single \( N = 2 \) super stress-energy tensor \( J \) (see [5]).

The Bäcklund transformation is non-polynomial in the original fields (and their derivatives) which give rise to the super-NLS equation. This explains why at a bosonic level (this statement is true also in the supersymmetric case) a rational \( \mathcal{W} \) algebra structure associated to the coset model [8] can be mapped into a polynomial \( \mathcal{W} \) algebra of the transformed equation.

We are able to construct for both the super-NLS equation and the \( N = 2, a = 4 \) super-KdV equation their corresponding manifestly \( N = 2 \) supersymmetric scalar Lax operators which generate the infinite tower of hamiltonians in involution.

Particularly interesting is the Lax operator for the super-NLS equation: even if non-polynomial in the superfields and their derivatives, nevertheless it generates the infinite tower of local (polynomial) hamiltonians in involution. This fact can be understood due to the existence of a similarity transformation which maps the Lax operator into a purely polynomial one.
2 The $N=2$ formalism and the $N=2 \hat{U}(2)$ algebra.

In this section we will introduce our conventions concerning the $N=2$ superfield formalism and we will define the $N=2$ algebras which allow introducing the super-NLS hierarchy.

Our superfields $\Phi(x, \theta, \bar{\theta})$ depend on two conjugate Grassman variables $\theta, \bar{\theta}$.

The chiral and antichiral fermionic derivatives $D, \bar{D}$ are introduced respectively through

$$D = \frac{\partial}{\partial \theta} - \frac{1}{2} \bar{\theta} \partial_x ,$$

$$\bar{D} = \frac{\partial}{\partial \bar{\theta}} - \frac{1}{2} \theta \partial_x .$$

They satisfy the following relations

$$D^2 = \bar{D}^2 = 0 ,$$

$$\{D, \bar{D}\} = -\partial_x .$$

(2.1)

An $N=2$ supersymmetric (bosonic) delta function $\delta(X, Y)$ is introduced by assuming

$$\int dX_1 \delta(X_1, X_2) A(X_1) = A(X_2)$$

(2.3)

for any $N=2$ superfield $A(X)$.

Here $X \equiv x, \theta, \bar{\theta}$ denotes the $N=2$ superspace; the integration measure is $dX \equiv dx d\theta d\bar{\theta}$; the bosonic integration is understood over the whole real line (or over the circle $S^1$), while the standard convention for the Berezin integration over fermionic variables is assumed.

Explicitly we have

$$\delta(X_1, X_2) = \delta(x_1 - x_2)(\theta_1 - \theta_2)(\bar{\theta}_1 - \bar{\theta}_2) .$$

(2.4)

The $N=2$ extension of the $\hat{U}(1)$ Kac-Moody algebra is introduced through the spin $\frac{1}{2}$ fermionic superfields $H, \bar{H}$, satisfying the following Poisson brackets relations

$$\{H(1), H(2)\} = \{\bar{H}(1), \bar{H}(2)\} = 0 ,$$

$$\{H(1), \bar{H}(2)\} = -D_1 \bar{D}_2 \delta(1, 2) ,$$

(2.5)

where, for simplicity, we have just denoted with numbers the superspace coordinates.

Notice that it is consistent with the above relations to assume that $H, \bar{H}$ are chirally (and respectively antichirally) constrained superfields, that is

$$DH = \bar{D} \bar{H} = 0$$

(2.6)

The notion of chirally covariant fermionic derivatives can now be introduced in full generality. Let us just remember here that covariant derivatives have been introduced in \cite{9} as an elegant way to simplify the analysis of $W$-algebras containing Kac-Moody subalgebras; further it has been shown in \cite{8, 4} that they could be useful for constructing integrable hierarchies.
At first we have to precise the notion of charged \( N = 2 \) superfields: they will be denoted as \( \Phi_{q, \bar{q}} \) \((q, \bar{q} \) are respectively the chiral and antichiral charges) and assumed to satisfy the following Poisson brackets relations with \( H, \overline{H} \):

\[
\begin{align*}
\{ H(1), \Phi_{q, \bar{q}}(2) \} &= qD_1 \delta(1, 2) \Phi_{q, \bar{q}}(2) , \\
\{ \overline{H}(1), \Phi_{q, \bar{q}}(2) \} &= \overline{q}D_1 \delta(1, 2) \Phi_{q, \bar{q}}(2) .
\end{align*}
\]

(2.7)

The fermionic covariant derivatives \( \mathcal{D}, \overline{\mathcal{D}} \) can be introduced through the following definitions:

\[
\begin{align*}
\mathcal{D}\Phi_{q, \bar{q}} &= D\Phi_{q, \bar{q}} - \overline{q}H\Phi_{q, \bar{q}} , \\
\overline{\mathcal{D}}\Phi_{q, \bar{q}} &= \overline{\mathcal{D}}\Phi_{q, \bar{q}} + q\overline{H}\Phi_{q, \bar{q}} , \\
\mathcal{D}^2 &= \overline{\mathcal{D}}^2 = 0 .
\end{align*}
\]

(2.8)

They map \( \Phi_{q, \bar{q}} \) into new superfields \( \mathcal{D}\Phi_{q, \bar{q}}, \overline{\mathcal{D}}\Phi_{q, \bar{q}} \) which still have \( q, \bar{q} \) charges, i.e. they satisfy (2.7).

At this point we can introduce what can be called the \( N = 2 \) extension of the affine \( \hat{U}(2) \) algebra \([10, 11]\), which is obtained by adding to \( H, \overline{H} \) the spin \( \frac{1}{2} \) fermionic superfields \( F, \overline{F} \). They are assumed to be charged, with charges \((q = 1, \bar{q} = -1), (q = -1, \bar{q} = 1)\) respectively.

Explicitly we have

\[
\begin{align*}
\{ H(1), F(2) \} &= D_1 \delta(1, 2) F(2) , \\
\{ H(1), \overline{F}(2) \} &= -D_1 \delta(1, 2) \overline{F}(2) , \\
\{ \overline{H}(1), F(2) \} &= -\overline{D}_1 \delta(1, 2) F(2) , \\
\{ \overline{H}(1), \overline{F}(2) \} &= \overline{D}_1 \delta(1, 2) \overline{F}(2) .
\end{align*}
\]

(2.9)

The algebra is completed with the following relations among \( F, \overline{F} \):

\[
\begin{align*}
\{ F(1), F(2) \} &= \{ \overline{F}(1), \overline{F}(2) \} = 0 , \\
\{ F(1), \overline{F}(2) \} &= -2D_1\overline{D}_2 \delta(1, 2) + \delta(1, 2) F(2) \overline{F}(2) ,
\end{align*}
\]

(2.10)

where in the above formulas the compact notation which makes use of the covariant derivatives has been introduced, being understood that \( \delta(1, 2) \) has the required charge properties.

The Jacobi identities are satisfied for the above algebra only when covariantly chiral–anti-chiral constraints for \( F, \overline{F} \) are taken into account:

\[
\begin{align*}
\mathcal{D}F &\equiv (D + H) F = 0 , \\
\overline{\mathcal{D}}F &\equiv (\overline{D} - \overline{H}) \overline{F} = 0 .
\end{align*}
\]

(2.11)

The above algebra has the structure of a non-linear \( W \) algebra when expressed in terms of \( N = 2 \) superfields due to the presence of non-linear terms in the right hand side. However, in terms of component fields, the algebra turns out to be a standard linear algebra. The non-linearity on the right hand side is the price we have to pay for disposing of a manifestly \( N = 2 \) supersymmetric formalism\([10, 11]\).
It is interesting to notice that in [4] the super-NLS equation was derived by coseting the $N = 1 \tilde{sl}(2)$ Kac-Moody algebra. Such algebra is not $N = 2$ supersymmetric, while its coset has a hidden $N = 2$ structure [5]. The algebra here described coincides with an $N = 1 \tilde{sl}(2) \times \hat{U}(1)$ algebra which possesses a manifest $N = 2$ supersymmetry. The coset itself, which is taken by quotienting the subalgebra generated by $H, \overline{H}$ is manifestly $N = 2$ supersymmetric. It is associated to a whole super-NLS hierarchy of integrable equations. Stated more precisely, there exists an infinite tower of super-hamiltonian densities which belong to the enveloping algebra of the $N = 2 \hat{U}(2)$ algebra and have vanishing Poisson brackets with respect to $H, \overline{H}$. The corresponding hamiltonians are all in involution with respect to the Poisson brackets structure provided by the $N = 2 \hat{U}(2)$ algebra.

Due to their coset structure the whole set of hamiltonian densities are obtained as chargeless homogeneous differential polynomials in $F, \overline{F}$ and the covariant derivatives $\mathcal{D}, \mathcal{D}$. The first two hamiltonians are completely specified by their dimensionality and symmetry properties, while for higher order hamiltonians, considerations based on the symmetry properties allows to drastically reduce their possible form, up to free parameters which must be fixed by requiring compatibility conditions for the different flows. Just to give an idea of the simplifications introduced with this formalism, the third hamiltonian of the hierarchy which will be produced below is given by the sum of two terms and it is sufficient (up to an overall normalization factor) to specify one single free parameter. When the covariant derivatives are expanded in terms of their $H, \overline{H}$ component superfields, the same hamiltonian turns out to be expressed as the sum of 23 different terms.

3 The super-NLS hierarchy as $N = 2$ coset.

As discussed in the previous section, there exists an infinite tower of hamiltonians, all in involution, which belong to the $N = 2 \hat{U}(2)$ coset algebra. This statement will be proved later, with the introduction of the generating Lax operator.

The hamiltonian densities $H_k$, for integers $k = 1, 2, \ldots$ are bosonic and $k$ is the spin-dimension. Explicitly we have for the first four hamiltonian densities the following solutions up to total derivatives:

\[
\begin{align*}
H_1 &= F\overline{F}, \\
H_2 &= F'\overline{F}, \\
H_3 &= F''\overline{F} - \frac{1}{2} \mathcal{D}F \cdot \mathcal{D}\overline{F} \cdot F\overline{F}, \\
H_4 &= F'''\overline{F} + \frac{3}{2} \mathcal{D}F \cdot \mathcal{D}\overline{F} \cdot F\overline{F} + F\overline{F}'F\overline{F},
\end{align*}
\]

where in the above relations we have denoted with a prime the operator $-\{\mathcal{D}, \overline{\mathcal{D}}\}$ (i.e. the ”covariant” space derivative).

The different flows are defined through the position

\[
\frac{\partial}{\partial t_k} \Phi(X) = \{\Phi(X), \int dY H_k(Y)\}
\]

for any given $\Phi$ superfield. The Poisson brackets are given by the $N = 2 \hat{U}(2)$ algebra relations (2.3, 2.9-2.11).
The first flow just gives the covariantly chiral equations of motion for $F, \overline{F}$

$$\frac{\partial}{\partial t_1} F = F', \quad \frac{\partial}{\partial t_1} \overline{F} = \overline{F'},$$  \hspace{1cm} (3.14)

while the second Hamiltonian is the one which gives the $N = 2$ super-NLS equations:

$$\frac{\partial}{\partial t_2} F = F'' - \overline{F} \overline{D} (\overline{F} \overline{D} F), \quad \frac{\partial}{\partial t_2} \overline{F} = -\overline{F}'' + \overline{F} \overline{D} (\overline{F} \overline{D} F).$$  \hspace{1cm} (3.15)

As for the third flow we have explicitly

$$\frac{\partial}{\partial t_3} F = F''' - \frac{3}{2} \left[ (\overline{D} F') \overline{D} F F + F' (\overline{D} F) \overline{D} F + F'' \overline{F} \overline{F} \right],$$

$$\frac{\partial}{\partial t_3} \overline{F} = \overline{F}''' - \frac{3}{2} \left[ \overline{D} (\overline{F}') \overline{D} F F + F' (\overline{D} F) \overline{D} F + F'' \overline{F} \overline{F} \right].$$  \hspace{1cm} (3.16)

As we will show in the next section these equations are related with $N = 2$ super-KdV equation via Bäcklund transformation.

Since all the Hamiltonians $H_k$ have vanishing Poisson brackets with the superfields $H, \overline{H}$, then

$$\frac{\partial}{\partial \xi_k} H = \frac{\partial}{\partial \xi_k} \overline{H} = 0$$  \hspace{1cm} (3.17)

for any $k$.

It is therefore consistent to set, at the level of the equations of motion, $H = \overline{H} = 0$, which implies in particular that the covariant derivatives in the expressions for $F, \overline{F}$ can be replaced by the ordinary fermionic derivatives.

The formalism of coset here used allows a simpler analysis than the procedure based on Dirac’s constraints: $H = \overline{H} = 0$ is obtained as a consequence of the equations of motion and not imposed as an external constraint; it is therefore not necessary to use Dirac’s brackets. This simplification is very much evident when using $N = 2$ superfield formalism since the algebra between $H, \overline{H}$ involves a non-invertible operator $(\overline{D} D)$ acting on the delta-function, which puts extra-complications in the Dirac procedure in this case.

### 4 The Bäcklund transformation between super-NLS and super-KdV.

At the bosonic level it is very well known that there exists a mapping between the fields $J_{\pm}$ entering the standard NLS equation and the fields $R, S$ of ref. which satisfy an equation equivalent to the NLS one. The $N = 2$ generalization of such mapping is provided by the following Bäcklund transformation, local but not polynomial in the fields $F, \overline{F}$ and their covariant derivatives, mapping them into a single spin 1 $N = 2$ superfield $J$:

$$J = \frac{1}{4} \overline{F} F - \frac{1}{2} \frac{D \overline{F}'}{D \overline{F}}.$$  \hspace{1cm} (4.18)
Notice that this transformation is not invertible.

On $J$ the reality condition $J = J^*$ can be imposed.

A feature which is absolutely new in the super-case and is not present in the bosonic case is the following: the above Bäcklund transformation connects two very well studied hierarchies, the super-NLS one with the $a = 4 \ N = 2$ super-KdV [5]. In the bosonic case the KdV equation is obtained only after performing a field reduction [7], while here we have an exact equivalence (we recall that $N = 2$ super-KdV contains one extra-field in the bosonic sector with respect to the standard KdV).

In [12, 13] it has been shown that the $N = 1$ super-KdV can be obtained after a suitable reduction of the $N = 1$ super-NLS equation, but the exact equivalence with the $N = 2$ super-KdV has not been remarked.

It can be easily realized that the hamiltonians (3.12) of the super-NLS hierarchy, and the corresponding flows for $F, \overline{F}$, can be reobtained respectively from the hamiltonians of the $N = 2$ $a = 4$ super-KdV hierarchy and the corresponding flows for $J$, after reexpressing $J$ in terms of (4.18).

We have, as first order hamiltonian densities for such a hierarchy [6]:

\[
\begin{align*}
\mathcal{H}_1 &= J , \\
\mathcal{H}_2 &= J^2 , \\
\mathcal{H}_3 &= J^3 + \frac{3}{4} [D, \overline{D}] J , \\
\mathcal{H}_4 &= J^4 - \frac{1}{2} (J')^2 + \frac{3}{2} [D, \overline{D}] J .
\end{align*}
\]

The second Poisson brackets structure is just given by the $N = 2$ superconformal algebra; explicitly we have in the $N = 2$ superfield formalism

\[
\{ J(X_1), J(X_2) \} = (\frac{1}{2} [D, \overline{D}] \partial + J(X_2) \partial + (\overline{D} J(X_2)) D + (D J(X_2)) \overline{D} + J(X_2) ') \delta(X_1, X_2)
\]

where all derivatives in the above formula are applied in $X_2$.

The following flows can be derived, with a suitable overall normalization for the hamiltonians in (4.18):

\[
\begin{align*}
\frac{\partial}{\partial t_1} J &= J' , \\
\frac{\partial}{\partial t_2} J &= [D, \overline{D}] J' + 4 J' J , \\
\frac{\partial}{\partial t_3} J &= -J''' + 6 (\overline{D} J D J)' - 6 (J[D, \overline{D}] J)' - 4 (J^3)' .
\end{align*}
\]

The second flow corresponds precisely to the equation derived from the super-NLS equation once the Bäcklund transformation (4.18) has been taken into account, while the third flow is just the $a = 4$ (in the language of [5]) $N = 2$ super-KdV equation.

The third flow (3.16) generates for the Bäcklund transformed superfield $J$ just the above super-KdV equation.

\footnote{Here we assume for simplicity the reduction $H = \overline{H} = 0$, but the general case works as well by substituting the fermionic derivatives with the covariant ones}
It should be noticed that the Bäcklund transformation (4.18) connects through field redefinitions the two hierarchies of equations of motion, but it cannot be lifted to a Poisson map relating the two Poisson structures, the one given by the $N = 2$ $U(2)$ algebra, with the one given by $N = 2$ super-Virasoro. This means in particular that (4.18) is not a Sugawara-type realization of the $N = 2$ super-Virasoro superfield $J$; this statement holds even if we replace the standard fermionic derivative with the covariant one in (4.18).

5 The Lax pairs.

In this section we construct new bosonic manifestly $N = 2$ supersymmetric Lax pairs for the super-NLS hierarchy and the super-KdV hierarchies.

At first we will compute the Lax operator $L$ in terms of the $J$ superfield. It can be constructed by assuming as an Ansatz that it has a non-standard form (see [14]), already introduced in a slightly different context and for a different Lax operator in [15]: the different flows are defined through the equation

$$\frac{\partial}{\partial t_k} L = [(L^k)_{\geq 1}, L]$$

(5.22)

for integer values of $k = 1, 2, \ldots$. The underscript $\geq 1$ means that only the purely derivative part must be considered [15].

In our case the integrals of motion $I_n$ are obtained from the constant term of $L^n$, that is

$$H_n = I_n = (-\frac{1}{2})^n \int dX (L^n)_0 ,$$

(5.23)

where underscript 0 means the constant part of an operator. They coincide with the integrals of the hamiltonian densities given in (4.19). This statement has been explicitly checked, with computer computations, up to the fourth order hamiltonian of the series.

It should be noticed that our prescription to compute the integrals over the residue is not the canonical one. In the standard prescription for the $N = 2$ formalism the integrals are computed from the term $[D, \overline{D}] \partial^{-1}$. For our Lax operator all these integrals are vanishing. The only possibility left to compute integrals of motion which is consistent due to dimensional reasons and the bosonic character of the $N = 2$ integration measure is precisely the one here given. Our computations prove that this is indeed so. We believe that our Lax operator is the first representative of a new class of manifestly $N = 2$ supersymmetric Lax operators admitting such property.

The explicit expression for our Lax operator which provides the flows for the $N = 2$ $a = 4$ super-KdV hierarchy is given by the following formula:

$$L = \partial - 2J - 2\overline{D}\partial^{-1}(DJ) .$$

(5.24)

Here $\partial$ means the standard bosonic $x$-derivative. The brackets in the above expression means the action of $D$ on $J$, (that is $D$ must not be commuted with $J$).

Inserting the Bäcklund transformation (4.18) in the above Lax pair allows us to produce, with the same prescription as before, the integrals of motion for the super-NLS
hierarchy. We have in this case

\[ L = \partial - \frac{1}{2} \bar{F}F + \frac{DF'}{DF} - \frac{1}{2} D\partial^{-1}F(D\bar{F}). \]  \tag{5.25}

One should immediately notice that the above Lax pair is not polynomial in the superfields \( F, \bar{F} \) and their derivatives. Despite of this, it is quite remarkable that the conserved quantities which are produced are local, polynomial expressions which coincides, of course, with the hamiltonians already computed for the super-NLS hierarchy.

The arising of polynomial hamiltonians can be clearly understood once realized that there exists a similarity transformation which maps \( L \) into an operator \( \tilde{L} \) through

\[ L = \frac{1}{(DF)} \tilde{L}(DF), \] \( \tilde{L} = \partial - \frac{1}{2} \bar{F}F - \frac{1}{2} (DF)\bar{D}\partial^{-1}F. \] \tag{5.26}

\( \tilde{L} \) does not produce consistent equations of motion, but its formal adjoint operator \( \tilde{L}^* \), given by

\[ \tilde{L}^* = \partial + \frac{1}{2} \bar{F}F - \frac{1}{2} F\bar{D}\partial^{-1}(DF) \] \tag{5.27}

turns out to be a polynomial Lax operator which provides the flows and conserved integrals of motion for the super-NLS hierarchy (with the same conventions as before).

In \cite{13} analogous steps (that is at first taking a similarity transformation and then the formal adjoint of the transformed operator) were introduced to produce a polynomial Lax pair operator for the super-NLS hierarchy out of a starting non-polynomial one. The Lax operators here produced are different from those of \cite{13} which are expressed in terms of \( N = 1 \) superfields.

6 Conclusions.

In this paper we considered a remarkable and unexpected connection \cite{5} between the super-NLS equation and the \( N = 2 \) super-KdV hierarchy in the framework of coset approach. The deep reason for the existence of the Bäcklund transformation relating the two hierarchies is still mysterious to us. However it sheds light on the fact that hierarchies which are produced in completely different manners and apparently have nothing in common can be intimately related. This provides further motivation for looking at other such kinds of relations.

Some problems are more naturally studied in one framework rather than the other one. So, for instance, Lax pairs are more easily obtainable in the context of the super-KdV, while the coset derivation is very transparent in the super-NLS language.

We plan in future to further develop our \( N = 2 \) formalism to study and produce more complicated \( N = 2 \) hierarchies.
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