Open Strings on Plane Waves and their Yang-Mills Duals

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Abstract

We study the plane wave limit of $AdS_5 \times S^5/Z_2$ which arises as the near horizon geometry of D3-branes at an orientifold 7-plane in type I' theory. We analyze string theory in the resulting plane wave background which contains open strings. We identify gauge invariant operators in the dual $Sp(N)$ gauge theory with unoriented closed and open string states.
1. Introduction

Following the ideas in [1] we consider a limit of a particular large $N$ gauge theory where we can see both closed and open strings arising in the large $N$ limit. These closed and open strings move in a ten dimensional space, though the ends of the open strings move on a D7 brane located at an orientifold sevenplane. The limit is the same as the one considered in [1]. In [1] only closed strings were obtained since the Yang Mills theory contained only fields in the adjoint representation. The theory we consider here is an $\mathcal{N} = 2$ $Sp(N)$ gauge theory with a hypermultiplet in the antisymmetric representation and four fundamental hypermultiplets. Since there are fields in the fundamental we will now have open strings in the ’t Hooft limit [2]. This theory is dual to string theory on $AdS_5 \times S^5/Z_2$, where the $Z_2$ is an orientifold action [3, 4]. Here we consider the limit of large ’t Hooft coupling, we pick a $U(1)$ generator, $J$, inside the symmetry group of the theory and we consider operators that carry large charge under $J$ but have a conformal dimension close to $J$, i.e. $\Delta - J$ is small. We consider such operators at weak ’t Hooft coupling and we identify some operators that get small corrections to their dimensions which start looking like strings propagating on an orientifold of the maximally supersymmetric plane wave background [2]. We have closed strings that arise from single trace gauge invariant operators as in $\mathcal{N} = 4$ SYM and open strings that arise from gauge invariant operators with two quarks at the ends. We extrapolate these results naively to strong coupling where they produce open and closed strings moving in an orientifold background. The orientifold projection is automatically obeyed by the gauge invariant operators. The operators corresponding to open strings are such that these open strings obey the appropriate boundary conditions.

In section 2 we consider string theory in the background of the orientifolded pp wave, in section 3 we look at the field theory corresponding to the D3-D7-O(7) system. In section 4 we identify the string theory spectrum in the plane wave limit, with the gauge invariant operators. We then analyze the calculation of the anomalous dimension of operators, derive
the string bit hamiltonian and argue that we obtain the right boundary conditions for the open string.

Other papers describing orbifolds of $\mathcal{N} = 4$ gauge theories and their corresponding plane wave limits include [6, 7, 8, 11, 12]. Other aspects of plane waves were recently explored in [13, 14, 15, 16, 20, 21].

2. The orientifold of a plane wave

We will consider the $O(7)$ orientifold projection with vanishing tadpoles of the pp-wave solution of 10d IIB supergravity found in [3]. An $O(7)$ plane carries $-4$ units of D-7-brane charge, so add 4 $D7$ branes to cancel the charge locally. This produces a gauge group $SO(8)$ on the worldvolume of the D-7-branes [22].

We want to take this orientifold of the pp-wave background. We have directions $x^\pm$, and $y_{1,..6}$ and the two extra directions $y_{7,8}$. The orientifold acts by sending $y_{7,8} \rightarrow -y_{7,8}$, and leaves all other coordinates fixed. This breaks half of the 32 supersymmetries.

The metric is

$$ds^2 = -4dx^+dx^- - \mu^2 y^2(dx^+)^2 + (dy^2) \quad (2.1)$$

$$F_{+1234} = F_{+5678} = \mu \quad (2.2)$$

The fixed point set of the orientifold is at $y_{7,8} = 0$. Notice that the orientifold is parallel to the lightcone directions, so that we can take lightcone gauge and quantize the strings in this background. The quantization of the string theory on the pp wave in light cone gauge was done in [23, 24]. Once we choose light cone gauge the orientifold projection will act as a projection on the Hilbert space of the light cone gauge theory.

Let us first consider the closed string sector. Before the projection the light cone action reduced to a set of eight massive bosonic oscillators on a circle. We denote the corresponding
creation and annihilation operators by $a_{n}^{i\dagger}$, $a_{n}^{i}$ where $n$ denotes the momentum of the oscillator along the circle. Similarly we have eight massive fermions. All bosons and fermions have the same mass. We denote the fermionic annihilation and creation operators by $b_{n}^{a}$ and $b_{n}^{a\dagger}$ respectively. The index $a$ runs over the $(+, +)$ and $(-, -)$ spinor representations of $SO(4) \times SO(4)$ where $\pm$ denotes the chirality under each $SO(4)$. The orientifold leaves the closed string ground state invariant and it acts in the following way on the oscillators

$$ a_{n}^{i} \rightarrow a_{-n}^{i}, \quad i = 1, \ldots, 6, \quad a_{n}^{7,8} \rightarrow -a_{-n}^{7,8}, \quad b_{n} \rightarrow i\Gamma^{56}b_{-n} \quad (2.3) $$

Note that on the Green Schwarz light cone fermions the orientifold acts as $S_{L} \rightarrow \Gamma^{78}S_{R}$, where $S_{L}$ and $S_{R}$ are real left and right world sheet chirality fermions in definite chirality $SO(8)$ spinor representation (Here and in the following $\Gamma^{78}$ for example denotes $\Gamma_{7}\Gamma_{8}$).

However due to the fact that the mass term is of the form $S_{L}\Gamma^{5678}S_{R}$ we find that on the fermion creation operators $b_{n}^{a\dagger}$ the orientifold action is indeed as indicated in (2.3). So we see the usual statement that the orientifold reverses the direction of the string and it multiplies the $y_{7,8}$ oscillators by $-1$. If we only consider zero momentum oscillators ($n = 0$) then the orientifold projection requires that we have an even number of $a_{0}^{7,8\dagger}$ oscillators.

We also have to remember to impose the $L_{0} - \bar{L}_{0}$ condition on the states. The orientifold condition (2.3) acting on the zero momentum fermions ($n = 0$) multiplies four of them by $-1$ and leaves the other four invariant. The four that are left invariant are associated to supersymmetries that act nonlinearly on the light cone gauge Hilbert space.

Now let us consider the open strings. For these strings, the oscillators in the $y_{1,6}$ directions have Neumann boundary conditions, while the oscillators transverse to the orientifold, $y^{7,8}$, have Dirichlet boundary conditions. The fermions obey a boundary condition of the form $S_{L} = \Gamma^{78}S_{R}$. This boundary condition leaves only 8 (real components) of the massive fermion zero momentum modes $n = 0$. These split into 4 creation and 4 annihilation operators (here by creation operators we mean operators that increase the light cone energy). Again due to the specific form of the mass term we find that:
i) all the creation operators carry charge $-1/2$ with respect to $SO(2)_{56}$ acting on 56 directions, while all the annihilation operators carry charge $+1/2$.

ii) as a result, the ground state (i.e. lowest energy state) carries $SO(2)_{56}$ charge $+1$.

iii) due to the unbalanced number of zero modes, four fermions and six bosons, the open string ground state energy is $-p^- = \mu$ where $\mu$ is the mass of the oscillators. All non-zero momentum oscillators ($n \neq 0$) come in equal numbers of bosons and fermions so that their contribution to the ground state energy cancels.

These facts will be important for us in the following when we identify these states with the gauge invariant operators.

The action of these four creation operators generates the vector supermultiplet of theories of 16 supercharges. As usual, the open string ground state is odd under the projection which implies that the Chan-Paton indices are anti-symmetric and therefore are in the adjoint of $SO(8)$. We can find all open string states by keeping states invariant under the orientifold projection. One way to think about it is to say that we can form any state with the oscillators and then we arrange the $SO(8)$ indices into a symmetric or anti-symmetric representation to obey the orientifold condition.

In appendix A, we give a more detailed proof of the above statements.

II-A. The orientifolded plane wave as a limit of $AdS_5 \times S^5/Z_2$.

We write the metric of $AdS_5 \times S^5$ metric as

$$ds^2 = R^2(-\cosh^2 \rho dt^2 + d\rho^2 + \sinh^2 \rho d\Omega_3^2 + \cos^2 \theta d\psi^2 + d\theta^2 + \sin^2 \theta d\Omega_3^2) \quad (2.4)$$

where the second $d\Omega_3^2$ on $S^3$ which is part of $S^5$ is explicitly given as

$$d\Omega_3^2 = \cos^2 \theta' d\psi'^2 + d\theta'^2 + \sin^2 \theta' d\phi^2 \quad (2.5)$$
The $Z_2$ action which combines with the orientifold projection is $\psi' \rightarrow \psi' + \pi$. In these coordinates the D7 branes sit at $\theta' = \pi/2$.

We can now take the suitable limits to go to the pp wave solutions exactly as in [1]. This is a Penrose limit. The Penrose limit consists in looking at the neighborhood of a lightlike geodesic. If the geodesic does not intersect the orientifold plane we get the maximally supersymmetric type IIB plane wave [3]. It is more interesting to consider a geodesic that lies on the orientifold plane, in this way we will retain the orientifold plane as we take the limit. For this purpose we consider the case where we boost along $\psi$.

We define

$$\rho = r/R, \quad \theta = y/R, \quad x^+ = \frac{(t + \psi)}{2}, \quad x^- = \frac{R^2 t - \psi}{2}$$

(2.6)

and we take the limit $R \rightarrow \infty$. The metric then becomes (2.1) with $\mu = 1$. In these units we find that

$$-p_+ = \frac{\Delta + J}{R^2}, \quad -p_- = \Delta - J$$

(2.7)

where $\Delta$ is the energy conjugate to $t$ translations and $J$ is conjugate to translations in $\psi$. $-p_+$ is equal to the light cone hamiltonian.

The $S_3$ (at $\theta' = \pi/2$) in $S^5$ where the orientifold is lying has a symmetry $SO(4) = SU(2)_R \times SU(2)_L$, we can define

$$J = J^3_{SU(2)_R} + J^3_{SU(2)_L}, \quad J' = J^3_{SU(2)_R} - J^3_{SU(2)_L}$$

(2.8)

In terms of the coordinates in the metric (2.1) we see that $J'$ performs rotations in the $y^5, 6$ plane, while $J$ is the one appering in (2.6)

3. The gauge theory

Consider a theory where we have $N$ D3 branes parallel to the $O(7)$ plane. The D3 branes have a gauge group $Sp(N)$. $N$ denotes the number of D3 branes after we do the
orientifold (i.e. in the covering space we have $2N$ D3 branes). We consider the low energy limit where we decouple the gauge theory living on the D3 branes from the rest. The $SO(8)$ gauge symmetry of the D7 branes becomes a global symmetry of the theory on the D3 branes.

The theory on the $D3$ branes is an $\mathcal{N} = 2$ theory, with a hypermultiplet in the antisymmetric representation of $Sp(N)$. These antisymmetric representations describe the motion of the D3 brane along the directions of the D7 brane. We will split them into two chiral fields $Z,Z'$. The scalar superpartner of the gauge field will be called $W$, it describes motions of the branes in the direction transverse to the orientifold. It can be shown that the theory of the D3-branes at the origin in moduli space is conformal [25, 26, 27]. We also have the $D3 - D7$ strings, which are in the fundamental representation of $Sp(N)$. These are four hypermultiplets in the fundamental, which we label in chiral language by $q_i, \tilde{q}_i$. Since this representation is real the global symmetry group is $SO(8)$, as expected also from the fact that this is the gauge group of the D7 theory. This theory has a $U(1) \times SU(2)_R$ R-symmetry and also a $SU(2)_L \times SO(8)$ global symmetry. The chiral fields $Z,Z'$ are doublets of $SU(2)_L$, they have zero charge under $U(1)$, and they, together with their complex conjugates, form a doublet of $SU(2)_R$. The field $W$ is a singlet of both $SU(2)$ symmetries and carries charge 1 under $U(1)$. Finally the fields $q, \tilde{q}$ carry zero $U(1)$ charge, are singlets under $SU(2)_L$ and together with their complex conjugates they form a doublet of $SU(2)_R$. Remembering the definition of the charge $J$ \eqref{eq:charge} we find that $Z$ has $J = 1$ and $Z'$ has $J = 0$. We also find that $q^i, \tilde{q}^\dagger$ have $J = 1/2$.

We also have the relations $-p_+ = \Delta - J$ and $-p_- \sim (\Delta + J)/R^2$, where $R^2$ is the curvature radius of $AdS$ in string units. In summary, we consider the limit of large ’t Hooft coupling and we consider operators with large charge $J$ (with $J/R^2 \sim J/(g_{YM}^2 N)^{1/2}$ fixed), and small values of $\Delta - J$.

The superpotential of the theory is given by $\mathcal{N} = 2$ SUSY. In $\mathcal{N} = 1$ language it
reads (up to normalizations of the fields)

\[ W \sim (W_{ab}q^a_{\bar{q}}q^b_{\bar{q}} + W_{ab}\Omega^{bc}Z_{cd}\Omega^{de}Z'_{ef}\Omega^{fa}) \]  

(3.1)

The F-terms are given by

\[ F_{\bar{q}} = Wq, \quad F_q = W\bar{q} \]  

(3.2)

\[ F_{Z' ad} = W_{ab}\Omega^{bc}Z_{cd} - (a \leftrightarrow d) \]  

(3.3)

\[ F_{Z ad} = W_{ab}\Omega^{bc}Z'_{cd} - (a \leftrightarrow d) \]  

(3.4)

\[ F_{W cb} = Z_{cd}\Omega^{da}Z'_{ab} + (b \leftrightarrow c) + (q\bar{q})(cb) \]  

(3.5)

The action written in terms of components has a potential which is given by the square of the \( F \) terms plus the square of the \( D \) terms.

A chiral operator with angular momentum \( J \) and \( \Delta - J = 0 \) is given by

\[ (Z_{ab}\Omega^{bc})^J = \text{tr}[(Z\Omega)^J] \]  

(3.6)

we need the invariant tensor \( \Omega \) of \( Sp(N) \) to raise one of the indices so that we can use matrix multiplication. The indices range from 1 to \( 2N \). This will be identified with the closed string ground state with \(-p_- = 2J/R^2\).

There are other protected BPS operators, or elements of the chiral ring, (see [3]) whose quantum numbers are those of

\[ \text{tr} ((Z\Omega)^J(Z'O)^{l_1}(W\Omega)^{2l_2}) \]  

(3.7)

More precisely, we have to symmetrize over the positions of \( Z'O \) and of the \( W\Omega \) insertions. The reason is that, as usual, any operator proportional to \( \partial W = 0 \) (the \( F \) terms in [1,2]) is not a primary operator. The \( F_Z = 0 \) and \( F_{Z'} = 0 \) equations tell us that \( W\Omega \) can be commuted freely past \( Z\Omega \) and \( Z'O \), and the \( F_W = 0 \) equation that \( Z'O \) can be commuted
past $Z\Omega$. So the operator is actually

$$\sum_{i_1,...,i_l,j_1,...,j_{l_2}=1}^{j} \text{tr}((Z\Omega)^{i_1}(Z'\Omega)(Z\Omega)^{i_2}...(W\Omega)^{j_{l_2}}(Z\Omega)^{J-j_{l_2}})$$  \hspace{1cm} (3.8)$$

Now $l_2$ should be even since $Z$ and $Z'$ are antisymmetric matrices, whereas $W$ is a symmetric matrix, thus the symmetrized sum $\sum ((Z\Omega)^{i}(Z'\Omega)^{i_1}(W\Omega)^{s})$ is a symmetric matrix if $s$ is even, and therefore gives 0 when it multiplies another $W\Omega$. This implies that we have an even number of $W$.

We can also have elements of the chiral ring with the same quantum numbers as the operator

$$Q_i\Omega(Z\Omega)^{i}(Z'\Omega)^{j}Q_j$$ \hspace{1cm} (3.9)$$

where $Q_i$, $i = 1, \cdots, 8$ is any of $q^l, \tilde{q}^l$. The F-terms imply that we should again symmetrize (sum) over all possible positions of the $Z'\Omega$ insertions and then the operator

$$\sum_{k_1,...,k_l=1}^{J} Q_i\Omega(Z\Omega)^{k_1}(Z'\Omega)...(Z'\Omega)^{k_i}(Z\Omega)^{J-k_i}Q_j$$ \hspace{1cm} (3.10)$$

is antisymmetric in $i,j$. These represent the massless open strings at the orientifold fixed point.

We cannot have elements of the chiral ring where we insert $W$ and $Q$ simultaneously, because using the F-terms we can commute the $W$ past anything else so that it lies adjacent to $Q$, and then the operator vanishes when we impose that the $F_q, F_{\tilde{q}}$ are zero. In other words, $F_Z = 0, F_{Z'} = 0$ tell us that we should sum over the positions of the $W\Omega$ insertions with equal weight, and $F_q = 0, F_{\tilde{q}} = 0$ tell us that the weight at the endpoints should be zero.

\footnote{the commutator is a $q\tilde{q}$ which splits the single trace into 2 traces, this is subleading in $1/N$ but it seems to be important for interactions involving open strings.}
4. The string bit hamiltonian

We now discuss the operators corresponding to the string states that we found above.

The closed and open string ground states are identified as the operators

\[ \text{tr}((Z\Omega)^J) \]  \hspace{1cm} (4.1)

\[ Q_i\Omega(Z\Omega)^J Q_j \]  \hspace{1cm} (4.2)

Note that \( \Delta - J = 0 \) for (4.1), while \( \Delta - J = 1 \) for (4.2) since \( \Delta - J = 1/2 \) for \( Q_i \). This is in precise agreement with the non-vanishing ground state energy that we found for open strings above. Furthermore, since \( Q \) is \( SU(2)_R \) doublet, it follows that the operator (4.2) carries \( J' = +1 \), which agrees with what we found for the open string vacuum. Replacing the operators \( Q_j \leftrightarrow Q_i \) does not produce a new state, but just a minus sign. This follows because \( \Omega(Z\Omega)^J \) is antisymmetric, and so the \( i, j \) indices are in the adjoint of \( SO(8) \). Of course this had to work since it is a rephrasing of the comparison done in [4].

In analogy with the discussion in [1] we associate each oscillator on the string with the insertion of an operator with \( \Delta - J = 1 \) along the “string of \( \Omega Z \)s” that we have in (4.1, 4.2). The first four \( y^i \) oscillators are associated to the insertion of a derivative, as in [1]. \( y^{5,6} \) are related to the insertion of \( Z' \) or \( \bar{Z}' \). The \( y^{7,8} \) oscillators are related to the insertion of \( W \) and \( \bar{W} \) in the operator. We can also insert the fermion operators with \( \Delta - J = 1/2 \). We have the fermions in the vector multiplet and also the fermions in the antisymmetric representation which are in a hypermultiplet. Once we have chosen the components with \( J = 1/2 \) these two types of fermions are distinguished by their \( J' \) eigenvalue. On the string Hilbert space these two correspond to the two possible eigenvalues of \( i\Gamma^{56} \) on \( b_{n\dagger}^{a} \). Then the string states are obtained by summing over all possible positions inside the trace of the insertions of \( \partial_i Z, Z', \bar{Z}', W, \bar{W} \) and fermions with \( J = 1/2 \). We can act with these on either the closed string ground state, or the open string ground state. We have seen that if we only
act with zero momentum modes (corresponding to elements of the chiral ring), we can only act with an even number of $a_0^{7,8\dagger}$'s (corresponding to $W, \bar{W}$ insertions) on the closed string ground state, and with no $a_0^{7,8\dagger}$'s on the open string ground state. Similarly the fermion zero mode creation operators in open string come with $J' = -1/2$ which translates into the gauge theory statement that the chiral ring involves only the insertion of anti-symmetric fermion and not the adjoint ones. That is as it should be, from the orientifold projection. Once we include nonzero momentum oscillators, we can act with any number of $a_n^{7,8\dagger}$'s.

To construct nonzero modes we need to add momentum on the worldsheet, and the momentum $n$ is related to a phase proportional to the $J$ charge to the left of the operator, as in (1). Let us see how the orientifold projection arises for them. Any state that does not obey the orientifold projection is automatically zero once we take into account the symmetry or anti-symmetry of the various indices of the operators. The action of the orientifold is essentially taking the transpose of the operator. For example, we can consider the operator

$$\sum_l e^{i2\pi l J} Tr [W\Omega(Z\Omega)^l Z'\Omega(Z\Omega)^{-l}]$$

(4.3)

We can think of this state as $a_{-n}^{7+8\dagger} a_n^{5+6\dagger} |0, p_-\rangle_{l.c.}$. Under the orientifold action this state is mapped to $-a_{-n}^{7+8\dagger} a_n^{5+6\dagger} |0, p_-\rangle_{l.c.}$. So the surviving state is the combination $(a_{-n}^{7+8\dagger} a_n^{5+6\dagger} - a_{-n}^{7+8\dagger} a_n^{5+6\dagger}) |0, p_-\rangle_{l.c.}$. Now we would like to prove that the operator (4.3) can be interpreted as containing both terms in precisely this combination. To see this we can take the transpose of (4.3). We get a minus sign from the transpose of $W$. And we easily see that we effectively change the momentum from $n \to -n$. So we see that we indeed have the precise combination we have in the lightcone after performing the orientifold. The two possible $J = 1/2$ fermions, the one coming from the vector multiplet and the one from the antisymmetric hypermultiplet, get a relative minus sign when we take the transpose. As we said above these two fermions are distinguished by their eigenvalue of $J'$. This is related to the fact that under the orientifold projection the fermions on the string worldsheet get
a sign proportional to their $J'$ eigenvalue.

The $L_0 - \bar{L}_0$ condition is imposed by the cyclicity of the trace as in [1].

The large $N$ Feynman diagrams of this theory are similar to the SU($N$) theory except that the double lines that represent particles do not have oriented edges. They have un-oriented edges. This implies that that we can build non-orientable surfaces, etc. When we twist a field in the symmetric or anti-symmetric representations we get different minus signs [28].

In our case we will be interested only in planar diagrams. So the only difference with SU($N$) will be in the labeling of states, as we discussed above. The gauge invariant states automatically implement the orientifold projection.

The derivation of the string hamiltonian is completely parallel to what we discussed in [1]. Here we just note that there is a simple way of doing the computation if we keep track of what terms in the bosonic potential come from the $F$ terms and which from the $D$ terms.

Then it becomes rather simple to compute the conformal dimension of an operator of the form

$$\sum_l e^{i2\pi nl} Tr[Z\Omega(Z'\Omega)(Z\Omega)^{l-1}]$$

which contains only chiral fields but with an extra phase for the field $Z'$. These are holomorphic operators. Then we know that when the phases are zero, all diagrams that contribute to the anomalous dimension vanish. In this case diagrams coming from the $F$ terms cancel by themselves, while the ones coming from photons, $D$-terms and self energy corrections of each individual propagator, cancel each other (see figure 1). These latter diagrams do not change the position of the fields, so they will also cancel when we include phases.

So the total anomalous dimension comes from the diagrams involving $F$ terms (see figure 2). These diagrams exchange the position of two fields and give a result that depends on the phases. Since the $F$ terms contain commutators we see that the diagrams in figure 2
correctly contain the relative \((-1)\) signs between the terms that exchange position relative to the ones that do not exchange position. This is why the contributions to the anomalous dimensions gives terms that involve \((Z'_i - Z'_{i+1})^2\) in the effective action of the string.

Figure 1: The corrections involving D terms, photons and self-energies cancel each other if all fields are holomorphic. They do not exchange the position of the fields. We denoted the propagator of the auxiliary \(D\) field by a dotted line, this is just a delta function in position space.

Figure 2: The diagrams involving \(F\) terms give non vanishing contributions which involve terms exchanging the position of the fields. We denoted the propagator of the auxiliary \(F\) field by a dotted line, this is just a delta function in position space.

Now let us discuss the open strings in more detail. The bulk of the open string is the same as the bulk of the closed string. New aspects arise when an excitation approaches the boundary. We need to understand the origin of the Dirichlet and Neuman boundary conditions. In principle, when we insert excitations with some phases we can put phases that mimic any boundary condition. So we need to understand the first order correction to the Hamiltonian. If we insert a field \(Z'\) at position \(k\) along the “open string of \(Z\)” in
we find an effective Hamiltonian of the form
\[ \sum_k g_{YM}^2 N (b_k + b_k^\dagger - (b_{k-1} - b_{k-1}^\dagger))^2 + b_k b_k^\dagger \]  

(4.5)

We are interested in the term proportional to \( g^2 N \). This is a quadratic form which is proportional to the following matrix

\[
M = \begin{pmatrix}
1 & -1 & 0 & 0 & \ldots \\
-1 & 2 & -1 & 0 & \ldots \\
0 & -1 & 2 & -1 & \ldots \\
\vdots & \ddots & \ddots & \ddots & \vdots \\
0 & \ldots & 0 & -1 & 1
\end{pmatrix}
\]  

(4.6)

and it is a discretized version of the second derivative. Here the only thing to notice are the terms at the upper left and bottom right corner of the matrices which come from examining the potential terms coming from \( F \) terms in (3.2). We want to write the theory in terms of normal modes, so we want to write it in terms of eigenvectors of the above matrix. It is a standard exercise in coupled harmonic oscillators to show that the eigenvectors of the above matrix have the form \( \cos \pi nk/J \). So that we effectively have Neumann boundary conditions.

One can also see this explicitly by looking at the first component of the eigenvalue problem for \( M \), \( (M - \lambda)v = 0 \). This takes the form \( v^1 - v^2 = \lambda v^1 \), since the eigenvalue will be proportional to \( 1/J^2 \) in the limit of large \( J \) we see that this reduces to \( \partial_{x^1} v \sim v(0)/J \sim 0 \) in the large \( J \) limit.

Similarly we can consider an insertion of a \( W \) field along the “open string of Zs” (4.2). Now the matrix to be diagonalized has the form

\[
W = \begin{pmatrix}
2 & -1 & 0 & 0 & \ldots \\
-1 & 2 & -1 & 0 & \ldots \\
0 & -1 & 2 & -1 & \ldots \\
\vdots & \ddots & \ddots & \ddots & \vdots \\
0 & \ldots & 0 & -1 & 2
\end{pmatrix}
\]  

(4.7)
where the only difference with the previous one is the factor of 2 in the lower right and upper left corners of $W$. This arises because there is an extra potential term coming from the $|F_q|^2$ terms in the bosonic potential. Again it can be seen that we recover the Dirichlet boundary condition. One can see this by looking at the first equation for an eigenvector $v^k$ of eigenvalue $\lambda$. This becomes $v(0) - \partial_\sigma v/J \sim \lambda v(0)$ which becomes $v(0) = 0$ in the $J \to \infty$ limit.

In summary, we see that open strings obey the appropriate boundary conditions. These boundary conditions depend on the form of the interactions in the gauge theory.

5. Conclusions

In this paper we have considered the dual pair of $\mathcal{N} = 2$ superconformal $Sp(N)$ gauge theory and the string theory in the near horizon geometry $AdS_5 \times S^5/Z_2$ in the Penrose limit. In particular for the Penrose limit which includes the D7 branes, string theory has open string sectors. Following the proposal of [1] we compare a class of gauge invariant operators in the gauge theory with the oscillator states of the string theory, both in the unoriented closed and open string sectors. In particular, we find that the operators which are bilinear in the fundamental hypermultiplets are dual to open string states. By studying the eigenvectors of the string bit hamiltonian we find the Dirichlet and Neumann boundary conditions at the level of gauge invariant operators.

A natural extension of this paper is to consider open strings ending on baryons. The BPS sector of those open strings was considered in [29]. If we consider a baryon or a giant graviton with $J = N$ which are represented by D3 branes on $S^5$ and we take the Penrose limit along a geodesic on the D3 then we find a plane wave with a D3 along $x^\pm$ and two of the transverse coordinates. In this case we expect that the open string spectrum is given again by putting oscillators with phases on the open strings discussed in [29]. We plan to study this further.
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Appendix A: String spectrum on the orientifold of a plane wave

In this appendix we discuss in more detail some aspects of the string spectrum on the orientifolded plane wave.

Closed strings

Let us recall the basic facts. In the light cone gauge \( x^+ = p^+ \tau \), the 8 transverse bosonic and Green-Schwarz fermionic coordinates become massive. The mode expansion for the light cone variables are [24]:

\[
\begin{align*}
    x^I(\sigma, \tau) &= i \sum_n \frac{1}{\sqrt{2\omega_n}} (e^{-i\omega_n \tau + ins} a_n^I - e^{i\omega_n \tau - ins} (a_n^I)^\dagger) \\
    \theta^1(\sigma, \tau) &= [e^{-i\mu p^+ \tau} b_0 - \sum_{n>0} c_n e^{-i\omega_n \tau} (e^{ins} b_n + \frac{\omega_n - n}{\mu p^+} e^{-ins} b_{-n})] + c.c. \\
    \theta^2(\sigma, \tau) &= [-e^{-i\mu p^+ \tau} i \Pi b_0 - i \Pi \sum_{n>0} c_n e^{-i\omega_n \tau} (e^{-ins} b_{-n} - \frac{\omega_n - n}{\mu p^+} e^{ins} b_n)] + c.c. \quad (A.1)
\end{align*}
\]

where

\[
\omega_n = \sqrt{n^2 + (\mu p^+)^2}, \quad (A.2)
\]

\( c_n \) are some normalization constants which we shall not need here and \( \Pi \) is the reflection operator \( \Gamma_{5678} \) along 4 directions \( x^5, ..., x^8 \). This can be compared with eqs.(2.5-2.7) of [24], by the substitution \( (a_n^I)^\dagger = \sqrt{\frac{n^2}{2}} \alpha_n^{1I} \) and \( (a_{-n}^I)^\dagger = \sqrt{\frac{n^2}{2}} \alpha_n^{2I} \) for \( n > 0 \). The creation and annihilation operators are \( a_n^\dagger \) and \( a_n \) respectively for all integers \( n \). Similarly \( b_n = \theta_n^1 \), \( b_{-n} = i \Pi \theta_n^2 \) for \( n > 0 \) and the zero mode \( b_0 = \theta_0^1 + i \Pi \theta_0^2 \). For all integers \( n \), \( b_n \) and \( b_n^\dagger \) are annihilation and creation operators respectively. Note that this choice of vacuum for the fermion zero modes corresponds to the lowest energy state and the light cone Hamiltonian comes with vanishing zero point energy. The level matching condition in this notation
becomes the vanishing of the sum of the momenta \( n \) of all the oscillator modes of the creation operators, \( \sum_i n_i = 0 \). This form of level matching condition is convenient for the comparison with the gauge theory operators.

The \( Z_2 \) projection in the present case is \( \Omega(-1)^{F_L} R \) where \( R \) reflects the two directions \( \bar{y} \). On the fermions \( R \) therefore acts as \( \theta^I \rightarrow \Gamma_{78} \theta^I \). The world sheet parity operator exchanges \( \theta^1 \) with \( \theta^2 \) while \( (-1)^{F_L} \) gives a minus sign to \( \theta^1 \). Combining all this one finds that the \( Z_2 \) action on the oscillator modes is

\[
a_I^I \rightarrow a_{-n}^I, \text{ for } I = 1, \ldots, 6; \quad a_I^I \rightarrow -a_{-n}^I, \text{ for } I = 7, 8;
\]
\[
b_n \rightarrow i \Gamma_{56} b_{-n}
\]

Note that, in particular only half of the fermion zero modes are \( Z_2 \) even, signalling that supersymmetry is now half compared to the type IIB case. In particular we note that these \( Z_2 \) even fermion zero mode annihilation (creation) operators come with definite eigenvalue w.r.t. \( i \Gamma_{56} \) implying that they have \( J' \) charge \(-1/2 (+1/2)\). This is important in the identification with the gauge theory chiral operators since this \( J' \) charge corresponds to an insertion of the anti-symmetric fermion as opposed to the adjoint fermion.

The physical states are the ones that are even under the above \( Z_2 \) and satisfy level matching condition. We take the vacuum state \( |0, p^+ > \) to be even under \( Z_2 \). The states that are obtained by applying bosonic creation operators are

\[
\psi(n_1, \ldots, n_k; I_1, \ldots, I_k) = \Pi_{r=1}^{k} (a_{n_r}^{I_r})^\dagger |0, p^+ >
\]

which under \( Z_2 \) action transform as

\[
\psi(n_1, \ldots, n_k, I_1, \ldots, I_k) \rightarrow (-1)^\ell \psi(-n_1, \ldots, -n_k; I_1, \ldots, I_k)
\]

where \( \ell \) is the number of creation operators in \( \psi \) along 7 and 8 directions.

**Open string sector**
In type I', there are also open strings that are stretched between the 8 D7 branes. In the Penrose limit under consideration these D7 branes are located at the origin of the 78 plane. This means that $x^7$ and $x^8$ must satisfy Dirichlet boundary conditions. The boundary conditions at $\sigma = 0$ and $\pi$ are therefore:

$$\partial_\sigma x^I = 0, \text{ for } I = 1, ..., 6; \quad x^I = 0 \text{ for } I = 7, 8 \quad \theta^1 = \Gamma_{78}\theta^2 \quad (A.6)$$

The general solutions to the equations of motion in the light-cone gauge subject to the above boundary conditions are:

$$x^I(\sigma, \tau) = \sum_{n \geq 0} \frac{1}{\sqrt{2}\omega_n} \cos n\sigma (e^{-i\omega_n \tau a^I_n} - e^{i\omega_n \tau (a^I_n)^\dagger}) \text{ for } I = 1, ..., 6$$

$$x^I(\sigma, \tau) = \sum_{n > 0} \frac{1}{\sqrt{2}\omega_n} \sin n\sigma (e^{-i\omega_n \tau a^I_n} - e^{i\omega_n \tau (a^I_n)^\dagger}) \text{ for } I = 7, 8 \quad (A.7)$$

The solution for the $\theta^1$ and $\theta^2$ are the same as in the closed string case eq.(A.1) subject to the condition

$$b_n = i\Gamma_{56}b_{-n} \quad (A.8)$$

There are a few important points to note. While the bosonic Neumann directions $x^I$ for $I = 1, ..., 6$ have zero modes, the two Dirichlet directions $x^7$ and $x^8$ do not have zero modes. For fermions there are on the other hand 8 zero modes (as opposed to 16 in the closed string case) of which 4 are creation and 4 annihilation operators. From (A.8) it follows that all the annihilation operators $b_0$ carry the $SO(2)_{56}$ charge $J' = +1/2$. Since the state $(b_0^\dagger)^4|0 >$ carries +2 units of $J'$ charge above that of the vacuum $|0 >$ we conclude that $|0 >$ must carry $J' = +1$. The vacuum energy now is $\mu$ due to the fact that the number of boson zero modes is 6 while that of fermion zero modes is 8. We will see this fact also by matching the states obtained by applying the fermion zero mode creation operators on the vacuum state to the vector multiplet living on the D7 brane world volume.
We can decompose the states in the vector multiplet of D7 brane in the light cone gauge in terms of $SO(4) \times SO(2)_{56} \times SO(2)_{78}$ where $SO(4)$ is the rotation group acting on 1234 directions, (recall that $J'$ is the $SO(2)_{56}$ charge). The vector multiplet contains a light cone vector which has the 4 components $V_i$ transforming as $(v, 0, 0)$, and a complex $V$ (and $\bar{V}$) in $(1, \pm 1, 0)$. The complex scalar $\phi$ (and $\bar{\phi}$) in the vector multiplet transforms as $(1, 0, \pm)$. The fermions are $\psi_{\pm}$ transforming as $(sp, \pm 1/2, \mp 1/2)$ and $\psi'_{\pm}$ in $(sp', \pm 1/2, \pm 1/2)$. Here $v, sp$ and $sp'$ denote the vector, spinor and spinor’ representations of $SO(4)$. Denoting by $b_{0L}$ and $b_{0R}$ the projections $(1 + \Pi)b_0/\sqrt{2}$ and $(1 - \Pi)b_0/\sqrt{2}$, we get the above table for the bosonic states. In this table $c$ is the energy of the open string vacuum. To show that $c = 1$ we need only to determine its value from the scalar field equation for say $\phi$. Recalling that D7 branes are located at the origin of the 78 plane, we can carry out the analysis of the linearized equations of motion for these fields in the pp wave background exactly as in [24]. For example the scalar field equation in the pp background $\Box \phi = 0$ gives rise to the following light cone Hamiltonian:

$$H = -p_{+} = \frac{1}{2p_{+}} \sum_{i=1}^{6} (p_i^2 - \mu^2 p_i^2 \partial_{\mu}^2)$$  \hspace{1cm} (A.9)

This is just the Schrödinger equation for a non-relativistic 6-dimensional harmonic oscillator and the zero point energy $E_0$ is simply $6/2\mu = 3\mu$. Comparing with the table above and noting that the scalar fields appear in 2nd and 3rd rows in the table, we conclude $c = 1$. A similar analysis for gauge fields can be carried out where the Chern-Simons terms play a crucial role in splitting the energies of $V, \bar{V}$ and $V_i$. 

| open string states | $E_0/\mu$ | $SO(4) \times SO(2)_{56} \times SO(2)_{78}$ | vector multiplet states |
|-------------------|-----------|---------------------------------|------------------------|
| $|0>$               | $c$       | $(1,+1,0)$                       | $V$                    |
| $(b_{0L}^\dagger)^2|0>$ | $c+2$     | $(1,0,-1)$                       | $\bar{\phi}$          |
| $(b_{0R}^\dagger)^2|0>$ | $c+2$     | $(1,0,+1)$                       | $\phi$                |
| $b_{0L}^\dagger b_{0R}^\dagger|0>$ | $c+2$     | $(v,0,0)$                        | $V_i$                  |
| $(b_0^\dagger)^4|0>$ | $c+4$     | $(1,-1,0)$                       | $\bar{V}$             |
Finally, the $Z_2$ action on these different oscillator modes are as follows (as can be seen by taking $\sigma$ to $\pi - \sigma$ together with $(-1)^{E_L}$ and the reflection in the 78 plane):

$$a_n^I \rightarrow (-1)^n a_n^I, \quad I = 1, \cdots, 8; \quad b_n \rightarrow (-1)^n b_n$$

(A.10)
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