Neutrino masses from Planck-scale lepton number breaking

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We consider an extension of the Standard Model by right-handed neutrinos and we argue that, under plausible assumptions, a neutrino mass of $\mathcal{O}(0.1)\text{eV}$ is naturally generated by the breaking of the lepton number at the Planck scale, possibly by gravitational effects, without the necessity of introducing new mass scales in the model. Some implications of this framework are also briefly discussed.

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INTRODUCTION

The masses of the third generation electrically charged fermions are known to a fairly high precision: the top quark mass is $m_t = 173.1 \pm 0.6\text{ GeV}$, the bottom quark mass is $4.18^{+0.04}_{-0.03}\text{ GeV}$ and the tau lepton mass is $1776.86 \pm 0.12\text{ MeV}$ [1]. In the framework of the Standard Model, these masses are generated by postulating an $\mathcal{O}(1)$ top-Yukawa coupling and $\mathcal{O}(0.01)$ bottom- and tau-Yukawa couplings to the Higgs field, which lead after the spontaneous electroweak symmetry breaking to the measured masses. The mass of the third generation neutrino, on the other hand, is not positively known. However, oscillation experiments indicate that it should be in the sub-eV range [2, 3].

The huge hierarchy between the third generation charged fermion and neutrino masses, at least nine orders of magnitude, suggests the existence of a different mechanism of mass generation for the neutral fermions, other than just a Yukawa coupling to the Higgs field. Arguably, the simplest and most economical framework to explain the differences between the electrically charged fermion masses and the neutrino masses is the seesaw mechanism [4–8]. The gauge symmetry of the model allows a (lepton number breaking) Majorana mass term for the right-handed neutrinos, possibly much larger than the electroweak scale, as well as a Yukawa coupling of the lepton doublets and the right-handed neutrinos to the Standard Model Higgs doublet, which generates after electroweak symmetry breaking a (lepton number conserving) Dirac mass term. The interplay between the heavy Majorana mass and the Dirac mass leads to a neutrino mass eigenstate which can be naturally much lighter than the Dirac mass. In this way, the smallness of the neutrino mass can be related to the breaking of the lepton number at very high energies.

The seesaw model, on the other hand, cannot predict the concrete value of the neutrino masses, as the Yukawa couplings and the right-handed Majorana masses are a priori undetermined. Furthermore, in contrast to the Standard Model, the parameters of the Lagrangian cannot be determined univocally from experiments, as right-handed neutrinos are not present in the low energy particle spectrum. On the other hand, they are constrained by the requirement of reproducing the measured neutrino oscillation parameters. For instance, assuming a Yukawa coupling of $\mathcal{O}(1)$, reproducing a neutrino mass scale $\sim 0.1\text{eV}$ requires to postulate a right-handed neutrino mass $\sim 10^{14}\text{GeV}$. Whereas this is a phenomenologically acceptable explanation, it is theoretically unsatisfactory, since in order to explain the origin of the 0.1 eV mass scale it has been necessary to introduce ad hoc another mass scale in the theory.

The Standard Model in curved spacetime contains two mass scales: the Higgs mass parameter (or alternatively the electroweak symmetry breaking scale) and the (reduced) Planck mass. It has been argued that gravity effects do not preserve global symmetries, such as lepton number [9, 11]. Therefore, a natural value for the right-handed Majorana neutrino mass is the reduced Planck scale, $M_P \simeq 2 \times 10^{18}\text{ GeV}$. Under this reasonable assumption, and assuming also an $\mathcal{O}(1)$ neutrino Yukawa coupling, the predicted neutrino mass is $m_\nu \sim 10^{-5}\text{eV}$ [12], far too small to explain neutrino oscillation experiments.

In this letter, we argue that the seesaw model with Planck scale lepton number breaking can naturally generate, under simple and plausible assumptions, an $\mathcal{O}(0.1)\text{eV}$-neutrino mass scale. To this end, we extend the Standard Model with two right-handed neutrinos, and we assume that at the cut-off scale of the model (which we identify with the Planck scale), one of the right-handed neutrinos has a Planck scale mass, while the other right-handed neutrino is massless. In this way, the only fundamental parameter of the model that breaks lepton number has a Planck scale size. We show that, in general, quantum effects induce a mass for the lighter right-handed neutrino. Furthermore, if the heaviest right-handed neutrino interacts with the left-handed leptons with an $\mathcal{O}(1)$ Yukawa coupling, then the seesaw mechanism generates an effective neutrino mass with size $\mathcal{O}(0.1)\text{eV}$, in qualitative agreement with experiments, fairly independently of the value of the Yukawa couplings of the lighter right-handed neutrino.
QUANTUM EFFECTS ON RIGHT-HANDED NEUTRINO MASSES

We consider for simplicity a model with one generation of lepton doublets, $L$, and two right-handed neutrinos, $N_1$ and $N_2$. The part of the Lagrangian involving the right-handed neutrinos reads:

$$\mathcal{L}_N = \frac{1}{2} \bar{N}_i \gamma^\mu \partial_\mu N_i - \bar{Y}_i L \bar{H} N_i - \frac{1}{2} M_i \bar{N}_i^c N_i + \text{h.c.}, \quad (1)$$

where $\bar{H} = i \tau_2 H^*$, with $H$ the Standard Model Higgs doublet. We take as cut-off of the theory the Planck scale and we assume that the parameters of the model at that scale are $M_2 = O(M_P)$, $M_1 = 0$, such that the lepton number breaking occurs at the Planck scale, possibly by gravity effects, and we assume $Y_2 = O(1)$. We leave $Y_1$ unspecified.

If quantum effects were neglected, the model would predict the existence at low energies of a pseudo-Dirac neutrino pair, with masses $m_\nu \simeq Y_1 \langle H^0 \rangle \pm \frac{1}{2} Y_2^2 \langle H^0 \rangle^2 / M_2$, where $\langle H^0 \rangle \simeq 174$ GeV is the Higgs vacuum expectation value. This conclusion is, however, completely altered when properly including quantum effects on the right-handed masses.

We note that when $Y_1, Y_2 = 0$ and $M_1, M_2 = 0$ the Lagrangian Eq. (1) is invariant under the global $U(1)_L$ transformation $L \rightarrow e^{i \alpha} L$ and $U(2)_N$ transformation $N \rightarrow V N$, with $N = (N_1, N_2)$ and $V$ a $2 \times 2$ unitary matrix. However, when setting $Y_1, Y_2 \neq 0$ and $M_2 \neq 0$, the global $U(1)_L \times U(2)_N$ symmetry is completely broken, even if $M_1 = 0$. Therefore, and since there is no symmetry protecting the lightest right-handed neutrino mass against radiative effects, it will be generated via loops, and will be proportional to the order parameter of the lepton number breaking, $M_2$. We also note that if any of the parameters $Y_1, Y_2$ or $M_2$ is equal to zero, the symmetry of the Lagrangian is enhanced and this symmetry will protect $M_1$ against quantum effects.

An explicit calculation confirms this expectation. At the one-loop level, one finds corrections to the right-handed masses which are proportional to themselves $[13, 14]$, such that $M_1$ remains massless. However, at the two-loop level one finds non-vanishing contributions to $M_1$, through the diagram depicted in Fig. 1. Concretely, for the toy Lagrangian Eq. (1), we find that the right-handed neutrino masses, evaluated at the scale $\mu = M_2$, approximately read:

$$M_1 \bigg|_{\mu = M_2} \simeq \frac{4 Y_2^2 Y_2^2}{(16 \pi^2)^2} M_2 \log \left( \frac{M_2}{M_P} \right), \quad (2)$$

$$M_2 \bigg|_{\mu = M_2} \simeq M_2, \quad (3)$$

where all parameters in the right-hand side of these equations are evaluated at the cut-off scale. As anticipated, the lightest right-handed neutrino mass is proportional to $Y_1, Y_2$ and $M_2$, such that it vanishes when any of these is equal to zero. Finally, below the scale $\mu = M_2$, quantum corrections induced by the Yukawa coupling $Y_1$ will modify the value of the lightest right-handed neutrino mass at the scale $\mu = M_1$. These corrections are, however, typically small and will not affect our main conclusions.

At low energies the heavy neutrinos can be integrated out, leading to an effective neutrino mass

$$m_\nu \simeq \left( \frac{Y_2^2}{M_1} \bigg|_{\mu = M_1} + \frac{Y_2^2}{M_2} \bigg|_{\mu = M_2} \right) \langle H^0 \rangle^2 \simeq \left( \frac{(16 \pi)^2}{4 Y_2^2 \log(M_2/M_P)} + \frac{Y_2^2}{M_2} \right) \langle H^0 \rangle^2. \quad (4)$$

Here, we have again neglected the effect of quantum corrections between the scale $\mu = M_1$ and the scale of oscillation experiments, first discussed in [15], since they will not affect our conclusions.

For perturbative values of $Y_2$, namely $Y_2 \leq \sqrt{4 \pi}$ the first term in Eq. (4) dominates. So, the neutrino mass is mostly generated by the interaction of the lepton doublet with $N_1$ and takes the value

$$m_\nu \simeq \left( \frac{(16 \pi)^2}{4 \log(M_2/M_P)} \right) \frac{\langle H^0 \rangle^2}{Y_2^2 M_2} \simeq O(0.1) \text{eV} \left( \frac{Y_2}{1} \right)^{-2} \left( \frac{M_2}{2 \times 10^{18} \text{GeV}} \right)^{-1}, \quad (5)$$

which is naturally of the correct size for $M_2 \sim M_P$ and $Y_2 \sim 1$ (here, we have approximated $\log(M_2/M_P) \approx 1$). It is notable that this result holds independently of the value of the Yukawa coupling $Y_1$ (as long as it is non-zero), and correspondingly of the value of the lightest right-handed neutrino mass, which can be either $M_1 \sim 10^{14}$ GeV when $Y_1 \sim 1$, as can be checked from eq. (2), or much lighter, depending on the value of $Y_1$.

THE THREE GENERATION CASE

This discussion can be extended to the realistic case of three generations of lepton doublets. Here we just sketch
the basic ideas and we defer a detailed discussion to a forthcoming publication [16].

The neutrino Yukawa coupling, $Y$, and the right-handed neutrino mass matrix, $M$, are in this case $3 \times 3$ complex matrices with eigenvalues $y_1 \leq y_2 \leq y_3$, and $M_1 \leq M_2 \leq M_3$. We assume $y_3 = O(1)$, $M_3 = O(M_P)$ and $M_1, M_2 = 0$. For a rank-3 Yukawa matrix, the global flavor group $U(3)_L \times U(3)_N$ is completely broken and therefore no symmetry protects $M_1$ or $M_2$ against quantum effects, and hence they will be generated through loops (we note that a rank-2 Yukawa matrix suffices to break completely the global flavor group).

Concretely, one can show that $M_2$ is generated at order $O(y_2^3/(16\pi^2)^2)$ and $M_1$ at order $O(y_2^4 y_3^2/(16\pi^4))$ [10]. Therefore, the right-handed neutrino masses can be determined as a function of the Yukawa eigenvalues and mixing angles, which are in our framework unspecified (apart from the assumption $y_3 = O(1)$). Following a similar rationale as above, one obtains for the largest neutrino mass eigenvalue:

$$m_{\nu_3} \sim \frac{y_3^2 (H^0)^2}{M_2} \sim \left(\frac{(16\pi^2)^2}{4\log(M_3/M_P)}\right) \frac{(H^0)^2}{y_3^2 M_3} \sim O(0.1)eV \left(\frac{y_3^2}{1}\right)^{-2} \left(\frac{M_3}{2 \times 10^{18} \text{GeV}}\right)^{-1},$$  \hspace{1cm} (6)

which is of the right order of magnitude.

Generating a mild neutrino mass hierarchy from the seesaw mechanism requires special choices of the right-handed parameters [17]. Nevertheless, for appropriate mixing angles in the right-handed sector the neutrino eigenvalues take the form

$$m_{\nu_2} \sim \frac{y_2^2}{M_1} (H^0)^2, \quad m_{\nu_1} \sim \frac{y_1^2}{M_3} (H^0)^2,$$  \hspace{1cm} (7)

such that by adjusting the Yukawa eigenvalues it is possible to reproduce the mild neutrino mass hierarchy inferred from oscillation experiments $m_{\nu_3}/m_{\nu_2} \sim 6$. We note that in this framework the lightest neutrino mass is proportional to the squared of the smallest Yukawa coupling and is suppressed by a Planck-scale mass, therefore it is expected to be very small (a mild neutrino mass hierarchy could also be naturally obtained, without the necessity of adjusting parameters, by introducing a second Higgs doublet to the model [18, 19]).

DISCUSSION

Our assumption of a rank-1 right-handed mass matrix at the cut-off scale is purely phenomenological, and similar conclusions would follow if the tree level mass hierarchy among the right-handed masses is very large, such that the radiative contribution dominates over the tree level values. Nevertheless, it is interesting to speculate about the possible origin of such structure. If the right-handed neutrino mass matrix is of gravitational origin, so that one can relate the lepton number breaking to physics at the Planck scale, it is natural to identify the overall right-handed neutrino mass scale with the Planck mass. Besides, and since gravitational effects do not distinguish among generations, one can speculate that in the interaction basis the flavor structure of the mass matrix at the cut-off scale is of the form

$$M(M_P) = \omega M_P \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix},$$  \hspace{1cm} (8)

with $\omega = O(1)$, which has eigenvalues $\omega M_P \{0, 0, 3\}$. (An analogous ansatz was proposed in [20] for a gravitationally induced effective neutrino mass matrix.) On the other hand, and although the origin of the Yukawa couplings remains a mystery to this day, it is feasible that the right-handed neutrino Yukawa interactions will distinguish among generations, as is the case for the known quark and charged lepton Yukawa couplings. Hence, a rank-1 right-handed neutrino mass matrix and a Yukawa coupling displaying non-trivial mixing in the right-handed sector seem to be physically plausible assumptions.

In this framework, two of the right-handed masses are determined purely from quantum effects. Therefore, this framework renders a higher predictivity. More specifically, in the most general seesaw framework with three generations of lepton doublets and right-handed neutrinos, the Lagrangian contains 18 free parameters: 9 moduli and 6 phases in the Yukawa matrix, as well as 3 right-handed masses. With our assumption of a rank-1 right-handed mass matrix, the number of free parameters at the cut-off scale reduces to 8 moduli and 3 phases in the Yukawa matrix, and 1 right-handed mass. The higher predictivity, especially in the phases, could have implications for testing leptogenesis with low energy observables [16].

In this work we have been motivated by generating dynamically the right-handed neutrino mass scale necessary to reproduce the neutrino mass inferred from oscillation experiments. However, a similar rationale can be applied in other frameworks to generate a mass scale for a fermion singlet. For instance, several works have advocated a keV mass sterile neutrino as dark matter candidate [21, 22]. This choice was purely based on various phenomenological considerations, but lacked theoretical justification. Using the mechanism presented in this work, the keV mass scale could be related to the breaking of lepton number at very high energies, which is transmitted via loops suppressed by small Yukawa couplings to a right-handed neutrino with vanishing mass at the cut-off scale. In this way, no new mass scales have to be introduced in the model.

Finally, we would like to emphasize that quantum ef-
fects on the right-handed neutrino masses can also be important in frameworks where the scale of lepton number breaking is below the Planck scale (as in some Grand Unification Models) and/or in frameworks where the right-handed mass matrix at the cut-off scale has rank $>1$, especially when the tree-level mass hierarchy is very large. In this case, quantum effects induced by the heavier mass eigenvalues can radiatively induce masses for the lighter eigenvalues which can be much larger than the tree-level values, possibly affecting the phenomenology of the model.

**CONCLUSIONS**

We have considered an extension of the Standard Model by a Planck scale mass right-handed neutrino, motivated by the fact that the lepton symmetry is likely to be broken by gravitational effects at the Planck scale, and several massless right-handed neutrinos. The model therefore does not contain any new mass scale, but just the ones already existing in the Standard Model in curved spacetime.

We have argued that, in general, the masses of the lighter right-handed neutrinos are not protected by any symmetry and therefore they should be generated by quantum effects. We have explicitly shown that the next-to-heaviest right-handed neutrino mass is generated at the two-loop level. Furthermore, we have shown that when the heaviest right-handed neutrino interacts with the lepton doublets with an $\mathcal{O}(1)$ Yukawa coupling, the seesaw mechanism generates an effective neutrino mass which is naturally of $\mathcal{O}(0.1)$ eV, as suggested by neutrino oscillation experiments. This result supports the seesaw mechanism with Planck scale lepton number breaking as the origin of neutrino masses.

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