Correlation induced phonon softening in low density coupled bilayer systems

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We predict a possible phonon softening instability in strongly correlated coupled semiconductor bilayer systems. By studying the plasmon-phonon coupling in coupled bilayer structures, we find that the renormalized acoustic phonon frequency may be softened at a finite wave vector due to many-body local field corrections, particularly in low density systems where correlation effects are strong.

We discuss experimental possibilities to search for this predicted phonon softening phenomenon.

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Much interest has recently focused on bilayer semiconductor structures, where two quantum wells are put in close proximity with a high insulating barrier in-between to suppress interlayer tunneling. Such systems are of intrinsic interest because competition between intra- and inter-layer Coulomb interaction (i.e., correlation effects), particularly at low carrier densities when Coulomb interaction dominates (or in high magnetic fields where the electronic kinetic energy is quenched), may lead to novel and exotic ground states, collective properties, and quantum phase transitions. Although much of the recent work has concentrated on bilayer quantum Hall systems, the basic issue of the competition among electron kinetic energy and intra-/inter-layer Coulomb correlations is quite generic, and interesting novel ground states and exotic collective properties are, in principle, possible even in the zero field situation provided the $r_s$ parameter defining the dimensionless ratio of Coulomb interaction strength to Fermi energy is high enough. In this Letter we predict a particularly novel possibility, namely a finite wave vector acoustic phonon softening, driven by correlation effects in low density bilayer systems. We believe that our predicted phonon softening phenomenon may be realizable in the existing (electron or hole) bilayer systems provided the carrier density (and temperature) are low enough.

Advances in nanofabrication and growth of semiconductor layered structures have recently made it possible to fabricate very high quality (and very low density) GaAs-based high-mobility double well (bilayer) two dimensional electron systems (2DES). The carrier density in these systems could be very low (large $r_s$), making them suitable for studying electronic correlation in low dimensions. As the density decreases, unusual behavior is expected due to the increasing importance of electron-electron interaction as the system goes from a weak-coupling (small $r_s$) to a strong-coupling (large $r_s$) regime. It is possible to have electron systems with $r_s$ as high as 15–20 and hole systems with $r_s$ as high as 25–30. One particularly interesting possibility is that an electronic charge density wave (CDW) instability may occur in these low density bilayer systems before the onset of Wigner crystallization at still lower density.

In bilayer structures there are two collective charge density excitations (plasmon modes) corresponding to the in-phase density fluctuation (optic plasmon, OP) and out-of-phase density fluctuation (acoustic plasmon, AP) between the layers. In the absence of any interlayer tunneling the AP mode has a linear dispersion in the 2D wave vector because the long-range Coulomb interaction is suppressed by the screening from adjacent layers. Within random phase approximation (RPA) the acoustic plasmon is shown to exist for small wave vector (long wavelength) at all densities. However, the linear dispersion relation of the acoustic plasmon is sensitive to many-electron correlation effects even in the long wavelength regime. It has been shown that electron correlations may strongly renormalize the AP mode by pushing it completely inside the single-particle electron-hole excitation (SPE) spectra, resulting in Landau damping of the mode at much smaller wave vectors than predicted by RPA. Thus, the AP mode may become overdamped due to correlation effects. Strong correlations can thus destroy the AP mode without smearing out of the boundary of the particle-hole continuum by defect or impurity scattering or thermal effects. When this happens, there is a strong shift of single particle spectral weight toward zero energy at some finite wave vector due to the disappearance of the AP mode. The appearance of these spectral features with zero energy spectral weight at finite wave vector indicates that a state with a periodic modulation of density costs relatively little energy to excite, and the system may therefore become unstable against the formation of a periodic bilayer CDW ground state due to this AP plasmon overdamping mechanism. This CDW is different from the one associated with the 2$k_F$ ($k_F$ being the Fermi wave vector) Peierls instability in one dimensional (1D) systems, where the CDW instability occurs due to the electron-acoustic phonon interaction leading to a strongly divergent 2$k_F$ response function. The bilayer correlation induced CDW instability does not happen at 2$k_F$ or any other Fermi surface related wave vector.

In this Letter we study the electron-acoustic phonon interaction in low density bilayer systems. We investigate the plasmon (both OP and AP) mode coupling to the acoustic-phonons of the system and the consequent possibility of a softening of acoustic phonon modes caused by the electron-phonon interaction in 2D bilayer systems.
The plasmon-phonon mode coupling phenomenon, which hybridizes the collective plasmon modes of the electron gas with the acoustic-phonon modes, gives rise to coupled plasmon-phonon modes. The mode coupling is particularly strong at wave vectors around the plasmon-phonon resonance condition \( \omega_{pl} \approx \omega_{ph} \). In 2D GaAs systems the resonant mode-coupling phenomenon normally occurs at an extremely low density \( n < 1.2 \times 10^8 \text{ cm}^{-2} \) in the absence of correlation effects because the acoustic phonon energy is typically very low and one has to go to extremely low densities for the plasma frequency to be low enough to satisfy the \( \omega_{pl} \approx \omega_{ph} \) condition. Our important finding is that in the strongly correlated bilayer systems the plasmon-phonon mode coupling mechanism is sufficiently enhanced to give rise to strong (resonant) mode coupling effect at rather high densities (noninteracting electron and acoustic phonon kinetic energy is typically very low and one particularly strong at wave vectors around the plasmon-phonon modes. The mode coupling is parametrized by the deformation-potential coupling, which becomes soft at the wave vector around the resonance. We find that bilayer correlation effects in currently calculated bilayer systems the plasmon-phonon mode coupling constant via the deformation-potential interaction of an electron in the \( l \)-th layer with an electron in the \( l' \)-th layer with matrix elements are given by \( V_{ll'}(q) = \langle q | f_l(z) | q' \rangle \), where \( f_l(z) = 2e^z/\left( \epsilon(k) - \epsilon(k') \right) \). We note that the Coulomb interaction between the \( l \)-th and \( l' \)-th layer \( \epsilon(k) = k^2/2m \) the kinetic energy of the electron (we take \( h = 1 \) throughout this paper), and \( \omega_{Q} \) the bulk phonon frequency. \( V_{ll'}(q) \) is the Coulomb interaction between \( l \)-th and \( l' \)-th layer whose matrix elements are given by \( V_{ll'}(q) = \langle q | f_l(z) | q' \rangle \), where \( v_{q} = 2\pi e^2/\left( \kappa a \right) \) with \( \kappa \) as the background lattice dielectric constant and \( f_l(z) \) are the form factors describing the finite well width effects. We consider only the lowest subband (for an infinite square well potential) in each layer for simplicity. For two identical layers we have \( V_{11} = V_{22} \) and \( V_{12} = V_{21} \). In Eq. (2), \( f_l(z) = \int dz \phi_l(z)^* \phi_l(z) e^{iqz} \), where \( \phi_l(z) \) is the subband wave function in \( l \)-th layer and \( M_l(Q) \) is the coupling constant between the \( l \)-th layer electron and bulk GaAs phonons. In the jellium model \( M_l(Q) = (2\pi e^2 \omega_{Q} / Q^2)^{1/2} \). GaAs/Al\(_x\)Ga\(_{1-x}\)As systems, electrons couple to acoustic phonons via deformation potential and piezoelectric couplings. We consider only the deformation potential coupling, which dominates in the present context. The electron-acoustic phonon coupling constant via the deformation-potential interaction is given by \( |M_l(Q)|^2 = D^2 Q^2 / (2\rho \omega_{Q} Q) \) where \( \rho \) is the mass density of the lattice, \( D \) the known deformation-potential coupling constant, and \( \omega_{Q} = \omega_{Q} Q \). The Coulomb interaction between the \( l \)-th layer and an electron in the \( l' \)-th layer is obtained in RPA by summing all the bubble diagrams

\[
W_{ll'}(q, \omega) = \psi_{ll'}(q, \omega) + \sum_m \psi_{lm} \Pi(q, \omega) W_{mm'}(q, \omega),
\]

where \( \psi_{ll'}(q, \omega) = V_{ll'}(q) + U_{ll'}(q, \omega) \), and the \( U_{ll'}(q, \omega) \) is the phonon-mediated electron-electron interaction which is given by summing over the phonon wave vector in the \( z \)-direction (the 2D layers are in the \( x-y \) plane):

\[
U_{ll'}(q, \omega) = \sum_q M_l(q, q_z) M_{l'}(q, q_z) |f_l(q_z)|^2 D^0(Q, \omega),
\]

where \( D^0(Q, \omega) = 2\omega_{Q} Q / (\omega_{Q}^2 + i\gamma) \) is the free-phonon propagator. We note that except for the form factor \( |f_l(q_z)|^2 \) the integrand in Eq. (4) does not vary rapidly with \( q_z \). We therefore crudely approximate it with that factor multiplied by the integrand at \( q_z = 0 \). (Explicit calculations show that this approximation is excellent.) Then we get

\[
U_{ll'}(q, \omega) = \frac{U_0(q, \omega)}{\pi(q, \omega)} [\theta(\omega - \omega_{Q}) I_{ll'}(qs) - \theta(\omega_{Q} - \omega) I_{ll'}(qs)],
\]
where $s(q, \omega) = \sqrt{|1 - \omega^2/\omega_D^2|}$, and $U_0(q) = D^2q^2/(2\rho c_D^2)$ for the deformation potential and $U_0(q) = v(q)$ for the jellium model, respectively, and $I_{\mu\nu}(q) = \int dz dz' |\phi|^2|\phi|e^{-i\mathbf{q}z'}a_{\mu\nu}(z')$, $J_{\mu\nu}(q) = \int dz dz' |\phi|^2|\phi|e^{-i\mathbf{q}z'}a_{\mu\nu}(z')$. 

The effective electron-electron interaction matrix, Eq (3), can be diagonalized through a canonical transformation by introducing symmetric-antisymmetric (SAS) operators $(\alpha_{\pm}, \pm [\alpha_{+, 1} \pm \alpha_{-, 2}]/\sqrt{2})$ in the Hamiltonian. In the SAS representation the effective interaction then becomes decoupled by virtue of the symmetric nature of the bilayer system. The diagonalized elements of the effective interaction are given by

$$W_{\pm}(q, \omega) = \frac{\psi_{\pm}(q, \omega)}{1 + \psi_{\pm}(q, \omega)\Pi(q, \omega)} = \frac{V_{\pm}(q)}{\epsilon_{\pm}(q, \omega)},$$

(6)

where the plus (minus) label corresponds to the in-phase (out-of-phase) charge density modulation of the two layers. $\psi_{\pm}(q, \omega) = V_{\pm} + U_{\pm}$, where $V_{\pm}(q) = V_{11}(q)[1 - G_{11}(q)] \pm V_{12}(q)[1 - G_{12}(q)]$ is the Coulomb interaction matrix elements with the proper intralayer and interlayer local field corrections $G_{11, 12}(q)$, and $U_{\pm}(q, \omega) = U_{11}(q, \omega) \pm U_{12}(q, \omega)$ is the phonon mediated interaction matrix element; and $\Pi(q, \omega)$ the noninteracting 2D polarizability function. $\epsilon_{\pm}(q, \omega)$ is the total dielectric function of the coupled electron-phonon system and is given by

$$\epsilon_{\pm}(q, \omega) = 1 + V_{\pm}(q)\Pi(q, \omega) - \frac{U_{\pm}(q, \omega)}{V_{\pm}(q) + U_{\pm}(q, \omega)}.$$ 

(7)

The dispersion relation of the coupled plasmon-acoustic phonon mode is given by the zeros of the dielectric function $\epsilon_{\pm}(q, \omega)$. We can rewrite Eq. (6) in terms of the screened Coulomb interaction and renormalized phonon interaction

$$W_{\pm}(q, \omega) = \frac{V_{\pm}(q)}{\epsilon_{\pm}(q, \omega)} + \frac{M_{\pm}^2(q, \omega)}{\epsilon_{\pm}(q, \omega)} D_{\pm}(q, \omega).$$

(8)

The first term is the screened Coulomb interaction due to the collective motion of the electrons, and the second term represents the renormalized electron-phonon interaction with the renormalized vertex $M_{\pm}/\epsilon_{\pm}$ and phonon propagator $D_{\pm}$. $\epsilon_{\pm}(q, \omega)$ is the symmetric (antisymmetric) dielectric function of the electron system and is given by $\epsilon_{\pm}(q, \omega) = 1 - V_{\pm}(q)\Pi(q, \omega)$. The renormalized phonon propagator $D_{\pm}(q, \omega)$ is

$$D_{\pm}(q, \omega) = \frac{2\omega_q}{\omega^2 - \omega_q^2 + 2\omega_q M_{\pm}^2(q, \omega)\chi_{\pm}(q, \omega)},$$

(9)

where $\chi_{\pm}(q, \omega) = \Pi(q, \omega)/\epsilon_{\pm}(q, \omega)$. The quasiparticle energies of the screened longitudinal vibrations are the poles of $D(q, \omega)$. The poles of the $D(q, \omega)$ are complex and hence we need an analytic continuation of the response function. However, it is a good approximation to take the static limit ($\omega = 0$) in the dielectric function and to replace $\omega$ by the real frequency $\Omega(q)$ in the imaginary part of the response function. Then we have the renormalized phonon modes $\Omega_{\pm}$ and damping of the phonon $\gamma_{\pm}$: $\Omega_{\pm}^2(q) = \omega_q^2 - 2\omega_q M_{\pm}^2(q, \omega)\chi_{\pm}(q, \omega)$, $\gamma_{\pm}(q, \omega) = M_{\pm}^2(q, \omega)\chi_{\pm}(q, \omega)/[\omega_q^2 - 2M_{\pm}^2(q, \omega)]$. The width of the resonance in $D_{\pm}(q, \omega)$ is given by $2\gamma_{\pm}(q, \omega)$.

In Figs. 1 and 2 we show the results for the out-of-phase charge density modulation of the two layers in the bilayer system (the in-phase modulation, which is not shown, is similar to the single layer case). We use parameters corresponding to electrons in GaAs quantum wells: $e = 0.067 m_e$, $\kappa = 12.5$, $c_0 = 5.14 \times 10^5$ cm/s, $\rho = 5.3$ g/cm$^3$, and $D = -16.0$ eV for all our results. In Fig. 1 we show the coupled plasmon-acoustic phonon modes and the loss function, $\text{Im}[1/\epsilon_{\pm}(q, \omega)]$ which gives the mode spectral weight, as a function of energy for fixed wave vectors. Arrows in (b) are the energies of the bare phonon mode at given wave vectors.
and layer separation $d = 300\,\text{Å}$. The area bounded by the light solid lines indicates the SPE Landau damping continuum. When correlation effects are turned off (i.e., $G \equiv 0$) we have a well defined acoustic plasmon mode above the electron hole continuum and a well defined acoustic phonon mode with the bare phonon dispersion $\omega_{ph} = cq$. There is essentially no mode coupling without correlations because of the large difference in the plasmon and phonon energy scales. Local field effects give rise to a suppression of the AP mode above the SPE and its reappearance at large wave vectors beyond the single particle continuum in spite of it being overdamped (by Landau damping) at long wavelengths. This reappearance of the AP mode at large wave vectors, which is purely a local field effect, produces strong plasmon-phonon mode coupling. In Fig. 1(b) we show the spectral weight of the coupled modes. We find two peaks corresponding to the coupled modes, one below the bare acoustic phonon mode and the other above. Since the in-phase plasmon mode $\omega_s(q)$ is essentially unaffected by correlation effects and has much higher energy than the phonon mode, the coupling of the in-phase plasmon mode to the acoustic phonon mode is negligible. (This is also the reason why the phonon softening phenomenon predicted in this Letter cannot occur in single layer 2DES.)

In Fig. 2 we show the renormalized acoustic phonon mode dispersion and the damping for different layer separations by finding the complex poles of the renormalized phonon propagator, Eq. (9). The renormalized phonon mode is very different from the bare phonon mode with the bare phonon dispersion $\omega_{ph} = 300\,\text{Å}$. For $d = 690\,\text{Å}$ we find phonon softening, where the acoustic phonon mode is renormalized to zero frequency $\omega(q_c) = 0$. The critical wave vector depends on both density and layer separation, since it is a correlation effect. We emphasize that the bilayer phonon softening takes place due to the resonant coupling of GaAs acoustic phonons to the out-of-phase charge density modulation associated with the bilayer acoustic plasmons, which can become very strong at low enough carrier density. Strong acoustic phonon - acoustic plasmon coupling is the mechanism causing the bilayer phonon softening. Our predicted phonon softening thus necessarily requires both low carrier density (large $r_s$) and a bilayer system.

We conclude by briefly discussing possible experimental means to observe the predicted phonon softening. The most direct method would be to measure the bilayer phonon dispersion using ballistic phonon spectroscopy as was done for single layer 2D systems some years ago. The other possibility is to look for phonon softening indirectly through bilayer drag measurements where the magnitude of the drag should develop interesting anomalies in the soft mode regime since the drag is mediated partly by the phonon exchange mechanism. It may also be possible to see the softening in bilayer transresitivity measurements. In principle, it may be easier to see the softening phenomenon in hole bilayers where very large

$\omega(q) = q^2$.