Comparative Study of Valency-Based Topological Descriptor for Hexagon Star Network

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Abstract: A class of graph invariants referred to today as topological indices are inefficient progressively acknowledged by scientific experts and others to be integral assets in the depiction of structural phenomena. The structure of an interconnection network can be represented by a graph. In the network, vertices represent the processor nodes and edges represent the links between the processor nodes. Graph invariants play a vital feature in graph theory and distinguish the structural properties of graphs and networks. A topological descriptor is a numerical total related to a structure that portray the topology of structure and is invariant under structure automorphism. There are various uses of graph theory in the field of basic science. The main notable utilization of a topological descriptor in science was by Wiener in the investigation of paraffin breaking points. In this paper we study the topological descriptor of a newly design hexagon star network. More precisely, we have computed variation of the Randić R, fourth Zagreb M₄, fifth Zagreb M₅, geometric-arithmetic GA, atom-bond connectivity ABC, harmonic H, symmetric division degree SDD, first redefined Zagreb, second redefined Zagreb, third redefined Zagreb, augmented Zagreb AZI, Albertson A. Irregularity measures, Reformulated Zagreb, and forgotten topological descriptors for hexagon star network. In the analysis of the quantitative structure property relationships (QSPRs) and the quantitative structure activity relationships (QSARs), graph invariants are important tools to approximate and predicate the properties of the biological and chemical compounds. We also gave the numerical and graphical representations comparisons of our different results.

Keywords: Topological indices; degree-based index; hexagon star network

1 Introduction

Cheminformatics is another field of modern sciences that connects chemistry, math, and other fields of science. Quantitative structure-activity relationship (QSAR) and Quantitative structure-activity relationship (QSPR) are the principle parts of cheminformatics which are useful to contemplate the physico-chemical properties of networks. A topological descriptor (TD) is a numerical total related to a structure that...
Topological descriptors (TD) are commonly partitioned into three sorts: degree, distance and spectrum based. The structures of networks can be scientifically demonstrated by a figure. The vertex represents the processor hub and an edge describes the link among processors. The topology of the figure of a network chooses the way by which any two vertices are linked by an edge. The topology of a network system can be used to obtain specific properties without a lot of stretches. The width is resolved as the most extreme separation between any two hubs in the system. The quantity of connections associated with a hub decides the level of that hub. If this number is the equivalent for all hubs in the system, the system is called regular.

TD can be effortlessly processed by utilizing the ideas of atomic topology (AT), an order dependent on the graph theory. Actually, AT has demonstrated to be a fantastic apparatus for quick and exact estimation of numerous physicochemical as well as biological properties [2, 3]. So as to compute topological indices, basics of AT are utilized where chemical compound is changed over into a graph, considering the atoms and bonds associated with vertices and edges of a graph. The basic definitions and notations are taken from the book [4]. The number of vertices adjacent to the vertex \( e \) is the degree of \( e \), denoted as \( d_e \).

TD which are obtained through the connectivity of two-dimensional structures, deliver significant connections to various properties of these structures via QSAR/QSPR resulting from the topological networks of these structures [5]. Since last few years, numerous researchers conducted for the expansion of TD because of their significance [6–9]. For detail study of TD see [10–21].

2 Degree-Based Indices

In this section, we define some degree based topological indices \( T(\mathbb{H}) \).

\[
T(\mathbb{H}) = \sum_{uv \in E(\mathbb{H})} \lambda(d_u, d_v),
\]

- \( \lambda(d_u, d_v) = (d_u d_v)^x \), represents \( T(\mathbb{H}) \) as the general, second, and second modified Randic’ indices if \( x \neq 0 \in \mathbb{R} \), \( x = 1 \), and \( x = -1 \) respectively.
- \( \lambda(d_u, d_v) = (d_u + d_v)^x \), represents \( T(\mathbb{H}) \) as the general sum connectivity, sum connectivity, Zagreb and hyper Zagreb indices, if \( x \neq 0 \in \mathbb{R} \), \( x = -\frac{1}{2} \), \( x = 1 \) and \( x = 2 \) respectively.
- If \( \lambda(d_u, d_v) = d_u^x d_v^y + d_u^y d_v^x \), then \( T(\mathbb{H}) \) represents generalized Zagreb index.

Similarly, if

\[
\lambda(d_u, d_v) = \sum_{uv \in E(G)} d_u(d_u + d_v), \quad \sum_{uv \in E(G)} d_v(d_u + d_v), \quad \frac{2\sqrt{d_u d_v}}{d_u + d_v}, \quad \sqrt{\frac{d_u + d_v - 2}{d_u d_v}}, \quad \frac{2}{d_u + d_v}, \quad \frac{d_u^2 + d_v^2}{d_u d_v}, \quad \frac{d_u + d_v}{d_u d_v}, \quad \frac{d_u d_v}{d_u + d_v}, \quad d_u d_v(d_u + d_v), \quad \left( \frac{d_u}{d_u + d_v} \right)^3, \quad \frac{1}{\max\{d_u, d_v\}}, \quad |d_u - d_v|, \quad (d_u - d_v)^2, \quad (d_u + d_v - 2)^2, \quad (d_u^2 + d_v^2),
\]

we obtained fourth Zagreb \( M_4(\mathbb{H}) \), fifth Zagreb \( M_5(\mathbb{H}) \), geometric-arithmetic \( GA(\mathbb{H}) \), atom-bond connectivity \( ABC(\mathbb{H}) \), harmonic \( H(\mathbb{H}) \), symmetric division degree \( SDD(\mathbb{H}) \), first redefined Zagreb, second redefined Zagreb, third redefined Zagreb, augmented Zagreb \( AZI(\mathbb{H}) \), variation of the Randic’ \( R(\mathbb{H}) \), Albertson \( A(\mathbb{H}) \), IRM(\mathbb{H})) irregularity measures, Reformulated Zagreb, and forgotten topological indices respectively.
3 Hexagon Star Network Sheet

Interconnection systems are significant in PC systems administration and used to change information between the PC and processor. In the most recent couple of years, numerous specialists structured the new interconnection systems. In an equal PC framework, interconnection organize is accustomed to expanding the exhibition. In diagram hypothesis, organize is spoken to as a chart. In this articulation, the processor spoke to by vertex and association between the units spoke to by edges. From the topology of a system, we can decide certain properties. The level of a hub is characterized as the all outnumber of connections associated with that hub. The system is supposed to be regular if each hub in the system has the same degree. In this paper, we define a new interconnection network hexagon star network. This network is a composition of triangles around a hexagon, as shown in Fig. 1.

![Hexagon Star Network](image)

**Figure 1**: The hexagon star network sheet for $p = 2, q = 2$

4 Main Results

In this section, we give results, which are used to obtained any degree-based topological descriptors. We obtained exact results of degree-based TD for hexagon star network sheet $H$. Vetrik [22] introduced a new method to calculate the topological indices and also in [23], we follow the same technique in this paper. Now, we presents a formula, which can be used to obtain any degree based TD.

**Lemma 4.1** Let $H$ be a hexagon star network. Then $T(H) = 12pq\lambda(4, 4) + 2p(4\lambda(2, 4) - \lambda(4, 4)) + 4q(\lambda(2, 4) - \lambda(4, 4))$.

**Proof.** The graph $H$ contains $6pq + 5p + q$ vertices and $12pq + 6p$ edges. Each vertex of $H$ has degree 2 or 4, vertices of $H$ can be partitioned according to their degrees. Let

$V_i = \{v \in V(H) : d_v = i\}$.

This means that the set $V_i$ contains the vertices of degree $i$. The set of vertices with respect to their degrees are as follows:

$V_2 = \{v \in V(H) : d_v = 2\}$

$V_4 = \{v \in V(H) : d_v = 4\}$

Since, $|V_2| = 4p + 2q$ and $|V_4| = 6pq + p - q$. We partite the edges of $H$ into sets based on degrees of its end vertices. Let

$E_{2,4} = \{uv \in E(H) : d_u = 2, d_v = 4\}$

$E_{4,4} = \{uv \in E(H) : d_u = 4, d_v = 4\}$. 
Note that \( E(\mathbb{H}) = \Xi_{2,4} \cup \Xi_{4,4} \). The number of edges incident to one vertex of degree 2 and other vertex of degree 4 is \( 8p + 4q \), so \( |\Xi_{2,4}| = 8p + 4q \). Now, the remaining number of edges are those edges which are incident to two vertices of degree 4, i.e., \( |\Xi_{4,4}| - |\Xi_{2,4}| = 12pq - 2p - 4q \).

Hence,
\[
I(\mathbb{H}) = \sum_{u \in E(\mathbb{H})} \lambda(d_u, d_v) = \sum_{u \in \Xi_{2,4}} \lambda(2, 4) + \sum_{u \in \Xi_{4,4}} \lambda(4, 4) = (8p + 4q)\lambda(2, 4) + (12pq - 2p - 4q) \lambda(2, 4).
\]

After simplification, we get
\[
I(\mathbb{H}) = 12pq\lambda(4, 4) + 2p(4\lambda(2, 4) - \lambda(4, 4)) + 4q(\lambda(2, 4) - \lambda(4, 4)).
\]

Now we obtained the well-known degree based TD of hexagon star network in the following theorem.

**Theorem 4.2** For the hexagon star network \( \mathbb{H} \), we have

the general Randic’ index of \( \mathbb{H} \) is,
\[
R_\alpha(\mathbb{H}) = 12pq(16)^\alpha + 2p(4(8)^\alpha - (16)^\alpha) + 4q((8)^\alpha - (16)^\alpha),
\]

the Randic’ index of \( \mathbb{H} \) is
\[
R_\frac{1}{2}(\mathbb{H}) = 3pq + \frac{p}{2} \left( 4\sqrt{2} - 1 \right) + q \left( \sqrt{2} - 1 \right),
\]

the second Zagreb index of \( \mathbb{H} \) is
\[
R_1(\mathbb{H}) = 192pq + 32p - 32q
\]

the second Zagreb index of \( \mathbb{H} \) is
\[
R_{-1}(\mathbb{H}) = \frac{3}{4}pq + \frac{7}{8}p + \frac{1}{4}q.
\]

**Proof.** For \( R_\alpha(\mathbb{H}) \) which is the general Randic’ indices of \( \mathbb{H} \), we have \( \lambda(d_u, d_v) = (d_u d_v)^\alpha \), therefore \( \lambda(2, 4) = (8)^\alpha \) and \( \lambda(4, 4) = (16)^\alpha \). Thus by Lemma 4.1,
\[
R_\alpha(\mathbb{H}) = 12pq(16)^\alpha + 2p(4(8)^\alpha - (16)^\alpha) + 4q((8)^\alpha - (16)^\alpha).
\]

For \( \alpha = \frac{1}{2} \), the Randic’ index
\[
R_{\frac{1}{2}}(\mathbb{H}) = 12pq(16)^{\frac{1}{2}} + 2p(4(8)^{\frac{1}{2}} - (16)^{\frac{1}{2}}) + 4q((8)^{\frac{1}{2}} - (16)^{\frac{1}{2}}).
\]

After simplification, we get
\[
R_{\frac{1}{2}}(\mathbb{H}) = 3pq + \frac{p}{2} \left( 4\sqrt{2} - 1 \right) + q \left( \sqrt{2} - 1 \right).
\]

For \( \alpha = 1 \), the second Zagreb index is
\[
R_1(\mathbb{H}) = 12pq(16) + 2p(4(8) - 16) + 4q(8 - 16) = 192pq + 32p - 32q.
\]

For \( \alpha = -1 \), the second modified Zagreb index is
\[
R_{-1}(\mathbb{H}) = 12pq \left( \frac{1}{16} \right) + 2p \left( \frac{4}{8} - \frac{1}{16} \right) + 4q \left( \frac{1}{8} - \frac{1}{16} \right) = \frac{3}{4}pq + \frac{7}{8}p + \frac{1}{4}q.
\]

We gave graphical comparison of Theorem 4.2 in Fig. 2 and numerical values Tab. 1.
In the next theorem, we determined general sum-connectivity index, first Zagreb index and hyper-Zagreb index of the hexagon star network $H$.

**Theorem 4.3** For the hexagon star network $H$, we have

The general sum-connectivity index of $H$ is

$$\chi_s(H) = 12pq(8)^x + 2p(6)^x - (8)^x + 4q((6)^x - (8)^x),$$
the sum-connectivity index of $H$ is

$$\chi_{-1/2}(H) = 3\sqrt{2}pq + p\left(\frac{8 - \sqrt{3}}{\sqrt{6}}\right) + q\left(\frac{4 - 2\sqrt{3}}{\sqrt{6}}\right),$$

the first Zagreb index of $H$ is

$$\chi_1(H) = 96pq + 32p - 8q,$$

the hyper-Zagreb index of $H$ is

$$\chi_2(H) = 768pq + 160p - 112q.$$

**Proof.** For $\chi_x(H)$ which is the general sum-connectivity index of $H$, we have $\lambda(d_v, d_v) = (d_v + d_v)^x$, therefore $\lambda(2, 4) = (6)^x$ and $\lambda(4, 4) = (8)^x$. Thus by Lemma 4.1,

$$\chi_x(H) = 12pq(8)^x + 2p(4(6)^x - (8)^x) + 4q((6)^x - (8)^x).$$

For $x = -\frac{1}{2}$, sum-connectivity index of $H$

$$\chi_{-1/2}(H) = 12pq(8)^{-\frac{1}{2}} + 2p(4(6)^{-\frac{1}{2}} - (8)^{-\frac{1}{2}}) + 4q((6)^{-\frac{1}{2}} - (8)^{-\frac{1}{2}}).$$

After simplification, we get

$$\chi_{-1/2}(H) = 3\sqrt{2}pq + p\left(\frac{8 - \sqrt{3}}{\sqrt{6}}\right) + q\left(\frac{4 - 2\sqrt{3}}{\sqrt{6}}\right).$$

For $x = 1$, the first Zagreb index is

$$\chi_1(H) = 12pq(8)^1 + 2p(4(6)^1 - (8)^1) + 4q((6)^1 - (8)^1) = 96pq + 32p - 8q.$$  

For $x = 2$, the hyper-Zagreb index is

$$\chi_2(H) = 12pq(8)^2 + 2p(4(6)^2 - (8)^2) + 4q((6)^2 - (8)^2) = 768pq + 160p - 112q.$$

We gave graphical comparison of Theorem 4.3 in Fig. 3 and numerical values Tab. 2.

**Theorem 4.4** For the hexagon star network $H$, we have

the geometric-arithmetic index of $H$,

$$GA(H) = 12pq + 2p\left(\frac{8\sqrt{2}}{3} - 1\right) + 4q\left(\frac{2\sqrt{2}}{3} - 1\right)$$

the atom-bond connectivity index of $H$,

$$ABC(H) = 3\sqrt{6}pq + p\left(\frac{8\sqrt{2} - \sqrt{6}}{2}\right) + q\left(2\sqrt{2} - \sqrt{6}\right)$$

the augmented Zagreb index of $H$,

$$AZI(H) = \frac{2048}{9}pq + \frac{704}{27}p - \frac{1184}{27}q.$$
Proof. For $GA(H)$ which is the geometric-arithmetic index of $H$, we have
$$\lambda(\frac{d_v + d_e}{2}, d_v) = \sqrt{\frac{d_v + d_e}{2}}$$
therefore $\lambda(\frac{1}{2}, \frac{1}{2}) = \sqrt{\frac{1}{4}}$ and $\lambda(\frac{1}{2}, \frac{1}{2}) = \frac{\sqrt{6}}{4}$. Thus by Lemma 4.1,

$$GA(H) = 12pq + 2p \left( \frac{8\sqrt{2}}{3} - 1 \right) + 4q \left( \frac{2\sqrt{2}}{3} - 1 \right)$$

For $ABC(H)$ which is the atom-bond connectivity index of $H$, we have
$$\lambda(d_v, d_e) = \frac{\sqrt{d_v + d_e - 2}}{d_v d_e}$$
therefore $\lambda(2, 4) = \frac{1}{\sqrt{2}}$ and $\lambda(4, 4) = \frac{\sqrt{6}}{4}$. Thus by Lemma 4.1,
\[ ABC(\mathbb{H}) = 12pq \left( \frac{\sqrt{6}}{4} \right) + 2p \left( 4 \left( \frac{1}{\sqrt{2}} - \frac{\sqrt{6}}{4} \right) \right) + 4q \left( \frac{1}{\sqrt{2}} - \frac{\sqrt{6}}{4} \right) \]

After simplification, we get
\[ ABC(\mathbb{H}) = 3\sqrt{6}pq + p\left( \frac{8\sqrt{2} - \sqrt{6}}{2} \right) + q\left( 2\sqrt{2} - \sqrt{6} \right). \]

For \( AZI(\mathbb{H}) \) which is the augmented Zagreb index of \( \mathbb{H} \), we have \( \lambda(d_v, d_v) = \left( \frac{d_v}{d_v + d_v - 2} \right)^3 \), therefore \( \lambda(2, 4) = 8 \) and \( \lambda(4, 4) = \frac{512}{27} \). Thus by Lemma 4.1,
\[ AZI(\mathbb{H}) = 12pq \left( \frac{512}{27} \right) + 2p \left( 4(8) - \frac{512}{27} \right) + 4q \left( 8 - \frac{512}{27} \right) = \frac{2048}{9}pq + \frac{704}{27}p - \frac{1184}{27}q. \]

We gave graphical comparison of Theorem 4.4 in Fig. 4 and numerical values Tab. 3.

\[ \begin{array}{cccc}
| p, q | & GA(\mathbb{H}) & ABC(\mathbb{H}) & AZI(\mathbb{H}) \\
\hline
[1, 1] & 17.314 & 12.159 & 209.78 \\
[2, 2] & 58.627 & 39.016 & 874.67 \\
[3, 3] & 123.94 & 80.570 & 1994.7 \\
[4, 4] & 213.25 & 136.82 & 3569.8 \\
[5, 5] & 326.57 & 207.77 & 5600.0 \\
\end{array} \]

**Figure 4:** Graphical comparison of Theorem 4.4

**Table 3:** Numerical representation of Theorem 4.4.
Theorem 4.5 For the hexagon star network $\mathbb{H}$, we have
the symmetric division degree index of $\mathbb{H}$,
$SDD(\mathbb{H}) = 24pq + 16p + 2q$.

the Albertson index of $\mathbb{H}$,
$A(\mathbb{H}) = 16p + 8q$

the harmonic index of $\mathbb{H}$
$H(\mathbb{H}) = 3pq + \frac{13}{6}p + \frac{1}{3}q$.

Proof. For $SDD(\mathbb{H})$ which is the symmetric division degree index of $\mathbb{H}$, we have $\lambda(d_v, d_v) = \frac{d_v^2 + d^2_v}{d_v d_v}$, therefore $\lambda(2, 4) = \frac{5}{2}$ and $\lambda(4, 4) = 2$. Thus by Lemma 4.1, $SDD(\mathbb{H}) = 12pq(2) + 2p \left(4 \cdot \frac{5}{2} - 2\right) + 4q \left(\frac{5}{2} - 2\right) = 24pq + 16p + 2q$.

For $A(\mathbb{H})$ which is the Albertson index of $\mathbb{H}$, we have $\lambda(d_v, d_v) = |d_v - d_v|$, therefore $\lambda(2, 4) = 2$ and $\lambda(4, 4) = 0$. Thus by Lemma 4.1,
$A(\mathbb{H}) = 16p + 8q$.

For $H(\mathbb{H})$ which is the harmonic index of $\mathbb{H}$, we have $\lambda(d_v, d_v) = \frac{2}{d_v + d_v}$, therefore $\lambda(2, 4) = \frac{1}{3}$ and $\lambda(4, 4) = \frac{1}{4}$. Thus by Lemma 4.1,
$H(\mathbb{H}) = 12pq \left(\frac{1}{4}\right) + 2p \left(4 \cdot \frac{1}{3} - \frac{1}{4}\right) + 4q \left(\frac{1}{3} - \frac{1}{4}\right) = 3pq + \frac{13}{6}p + \frac{1}{3}q$.

We gave graphical comparison of Theorem 4.5 in Fig. 5 and numerical values Tab. 4.

Theorem 4.6 For the hexagon star network $\mathbb{H}$, we have
the first redefined Zagreb index of $\mathbb{H}$,
$ReZG_1(\mathbb{H}) = 6pq + 5p + q$.

the second redefined Zagreb index of $\mathbb{H}$,
\[ \text{ReZG}_2(\mathbb{H}) = 24pq + \frac{20}{3}p - \frac{8}{3}q \]

the third redefined Zagreb index of \( \mathbb{H} \)

\[ \text{ReZG}_3(\mathbb{H}) = 1536pq + 128p - 320q. \]

**Proof.** For \( \text{ReZG}_1(\mathbb{H}) \) which is the first redefined Zagreb index of \( \mathbb{H} \), we have \( \lambda(d_v, d_v) = \frac{d_u + d_v}{d_u d_v} \), therefore \( \lambda(2, 4) = \frac{3}{4} \) and \( \lambda(4, 4) = \frac{1}{2} \). Thus by Lemma 4.1,

**Figure 5:** Graphical comparison of Theorem 4.5

**Table 4:** Numerical representation of Theorem 4.5

| \( [p, q] \) | \( \text{SDD}(\mathbb{H}) \) | \( A(\mathbb{H}) \) | \( H(\mathbb{H}) \) |
|---|---|---|---|
| [1,1] | 42 | 24 | 5.5000 |
| [2,2] | 132 | 48 | 17.0 |
| [3,3] | 270 | 72 | 34.500 |
| [4,4] | 456 | 96 | 58.0 |
| [5,5] | 690 | 120 | 87.500 |
| [6,6] | 972 | 144 | 123.0 |
| [7,7] | 1302 | 168 | 164.50 |
| [8,8] | 1680 | 192 | 212.0 |
| [9,9] | 2106 | 216 | 265.50 |
| [10,10] | 2580 | 240 | 325 |
ReZG_1(H) = 12pq\left( \frac{1}{2} \right) + 2p\left( \frac{3}{4} - \frac{1}{2} \right) + 4q\left( \frac{3}{4} - \frac{1}{2} \right) = 6pq + 5p + q.

For ReZG_2(H) which is the second redefined Zagreb index of H, we have \( \lambda(d_v, d_v) = \frac{d_v d_v}{d_v + d_v} \), therefore \( \lambda(2, 4) = \frac{4}{3} \) and \( \lambda(4, 4) = 2 \). Thus by Lemma 4.1,

\[
ReZG_2(H) = 12pq(2) + 2p\left( \frac{4}{3} - 2 \right) + 4q\left( \frac{4}{3} - 2 \right) = 24pq + \frac{20}{3}p - \frac{8}{3}q.
\]

For ReZG_3(H) which is the third redefined Zagreb index of H, we have \( \lambda(d_v, d_v) = d_v d_v (d_v + d_v) \), therefore \( \lambda(2, 4) = 48 \) and \( \lambda(4, 4) = 128 \). Thus by Lemma 4.1,

\[
ReZG_3(H) = 12pq(128) + 2p(4(48) - 128) + 4q(48 - 128) = 1536pq + 128p - 320q.
\]

We gave graphical comparison of Theorem 4.6 in Fig. 6 and numerical values Tab. 5.

\[
\begin{align*}
\text{Table 5: Numerical representation of Theorem 4.6} \\
| p, q | ReZG_1 | ReZG_2 | ReZG_3 |
\hline
| 1, 1 | 12 | 28 | 1344 |
| 2, 2 | 36 | 104 | 5760 |
| 3, 3 | 72 | 228 | 1324 |
| 4, 4 | 120 | 400 | 23808 |
| 5, 5 | 180 | 620 | 37440 |
| 6, 6 | 252 | 888 | 54144 |
\end{align*}
\]

(Continued)
Theorem 4.7  For the hexagon star network $\mathbb{H}$, we have
the Randić index of $\mathbb{H}$,

$$R'(\mathbb{H}) = 3pq + \frac{3}{2}p.$$  

the Reformulated Zagreb index of $\mathbb{H}$,

$$RZ(\mathbb{H}) = 432pq + 56p - 80q$$

the forgotten index of $\mathbb{H}$

$$F(\mathbb{H}) = 384pq + 96p - 48q.$$  

the irregularity measures of $\mathbb{H}$

$$IRM(\mathbb{H}) = 32p + 16q.$$  

Proof. For $R'(\mathbb{H})$ which is the Randić index of $\mathbb{H}$, we have $\lambda(d_v, d_v) = \frac{1}{\max\{d_v, d_v\}}$, therefore $\lambda(2, 4) = \frac{1}{4}$ and $\lambda(4, 4) = \frac{1}{4}$. Thus by Lemma 4.1,

$$R'(\mathbb{H}) = 12pq\left(\frac{1}{4}\right) + 2p\left(4\left(\frac{1}{4}\right) - \frac{1}{4}\right) + 4q\left(\frac{1}{4} - \frac{1}{4}\right) = 3pq + \frac{3}{2}p.$$  

For $RZ(\mathbb{H})$ which is the Reformulated Zagreb index of $\mathbb{H}$, we have $\lambda(d_v, d_v) = (d_v + d_v - 2)^2$, therefore $\lambda(2, 4) = 16$ and $\lambda(4, 4) = 36$. Thus by Lemma 4.1,

$$RZ(\mathbb{H}) = 432pq + 56p - 80q.$$  

For $F(\mathbb{H})$ which is the forgotten index of $\mathbb{H}$, we have $\lambda(d_v, d_v) = (d_v^2 + d_v^2)$, therefore $\lambda(2, 4) = 20$ and $\lambda(4, 4) = 32$. Thus by Lemma 4.1,

$$F(\mathbb{H}) = 384pq + 96p - 48q.$$  

For $IRM(\mathbb{H})$ which is the irregularity measures of $\mathbb{H}$, we have $\lambda(d_v, d_v) = (d_v - d_v)^2$, therefore $\lambda(2, 4) = 4$ and $\lambda(4, 4) = 0$. Thus by Lemma 4.1,

$$IRM(\mathbb{H}) = 32p + 16q.$$  

4 Conclusion  

The study of graphs and networks through topological descriptors is important to understand their underlying topologies. Such investigations have a wide range of applications in cheminformatics, bioinformatics and biomedicine fields, where various graph invariants based assessments are used to deal
with several challenging schemes. In the analysis of the quantitative structure property relationships (QSPRs) and the quantitative structureactivity relationships (QSARs), graph invariants are important tools to approximate and predicate the properties of the biological and chemical compounds. In this paper, we study the valency-based topological descriptor for hexagon star network.

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