Research on Motion Generation for Machining Involute Tooth Profile Surface Based on Analysis Method

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Abstract. Starting from the configuration characteristics of gear, the pose relationship between grinding wheel and involute tooth profile surface is clarified. Pose matrix of grinding wheel relative to involute tooth profile surface is established, and the necessary motion modules are obtained. A method for designing motion module combination for machining involute tooth profile surface is created, and module combinations are screened. An effective module combination and a reasonable schematic diagram of motion module combination are given. The method adopted in this paper is beneficial to motion module design for machining involute tooth profile surface.

Keywords: Generative motion, Involute tooth profile surface, Matrix of position and orientation, Motion module.

1. Introduction

Gear is a kind of precision part which is widely used in mechanical field. Its machining methods mainly include hobbing and grinding [1]. With the continuous progress of science and technology, aviation, automobile and other fields have higher and higher requirements for gear accuracy, and the demand for gear processing is increasing day by day, and the motion design of involute tooth surface machining machine tool is particularly important.

For a long time, people have designed the structure of machine tools according to existing experience, and rarely considered how to innovate the structure of machine tools, which largely limits the diversification of machine tool configurations. Ref [2] studied how to use slice milling cutter to process shaft parts. Ref [3] proposed a machine tool for machining parts with non-circular and equal width cross-section. The problem of machining axisymmetric aspheric surfaces is discussed in Ref [4]. Based on the characteristics of involute tooth profile surface configuration, this paper puts forward the motion module combination method of involute tooth profile surface machining machine tool.

2. Basic theory of involute tooth profile

As shown in Fig. 1, unfold angle of the AK segment of the involute is expressed by $\theta_k$, radial diameter is represented by $r_k$, pressure angle is represented by $\alpha_k$, $\angle BOK=\alpha_k$, $u=\theta_k+\alpha_k$, and $r_b$ is base circle radius.
3. Design of the motion module combination
This paper chooses flake grinding wheel to process involute tooth profile surface, as shown in Fig.2.

3.1. Mathematical Description of Grinding Wheel Surface
The transformation vector (TV) is shown in Fig.3. \( O_p \) is the grinding wheel coordinate system, \( O_c \) is the contact point \( C \) coordinate system, and \( r_c \) is grinding wheel cutting part radius. According to Ref [5], the transformation process (TP) is shown in Fig. 4. \([^pT_c]\) is the homogeneous transformation matrix (HTM) of \( O_c \) relative to \( O_p \).
3.2. Mathematical Description of Involute Tooth Profile Surface

The TV is shown in Fig. 5. \( O_w \) is the involute tooth profile coordinate system. \( O_s \) is the cutting point \( S_0 \) coordinate system. TP from \( O_w \) to \( O_s \) is shown in Fig. 6 and HTM is expressed as:

\[
\begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
\end{bmatrix} = \begin{bmatrix}
r_c & 0 & 0 & 0 \\
l_c & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
\end{bmatrix}
\]

(1)

3.3. Relationship between Grinding Wheel and Involute Tooth Profile

Point \( C \) coincides with point \( S_0 \) when grinding wheel cut involute tooth profile surface. TM between grinding wheel and involute tooth profile \([^wT_p]\) can be obtained:

\[
[^wT_p] = [^wT_s][^wT_c]^{-1}
\]

(3)

So, we can obtain:

\[
[^wT_p] = \begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & \cos \theta & -\sin \theta & y' \cos \theta - z' \sin \theta \\
0 & \sin \theta & \cos \theta & y' \sin \theta + z' \cos \theta \\
0 & 0 & 0 & 1 \\
\end{bmatrix} = \begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & \cos \delta & \sin \delta & r_c \sin \delta \\
0 & -\sin \delta & \cos \delta & R + r_c \cos \delta \\
0 & 0 & 0 & 1 \\
\end{bmatrix}
\]

(4)

3.4. Machine Tool Motion Module

According to Eq. (5), the machine tool motion module combination matrix for machining involute tooth profile surface is determined as follows:
According to Ref [6], 48 kinds of reasonable motion module combination can be obtained by removing unreasonable modules. Fig. 7 gives the diagram of machine tool motion module combination.

Here $a_1, a_2, a_3, a_p, b_2, b_3, b_p, c_1, c_3, c_p$ is the horizontal transformation, which are the constant values. $\alpha_i$ is the rotational motion about $X$ axis. $Y$ and $Z$ represent the linear motion.

According to $[^yT_p]=[^yT_w]$, the following equations can be obtained:

$$
\begin{align*}
\theta - \delta &= \alpha_i \\
y' \cos \theta - r_z \sin(\theta - \delta) - (R + z') \sin \theta &= b_2 + b_3 + Y + b_p \cos \alpha_i - c_p \sin \alpha_i \\
y' \sin \theta + r_z \cos(\theta - \delta) + (R + z') \cos \theta &= c_1 + c_3 + Z + b_p \sin \alpha_i + c_p \cos \alpha_i
\end{align*}
$$

(6)

The motion modules can be expressed as: $G/Z\alpha_i/\beta I$, where “$G$” is grinding wheel and “$I$” is involute tooth profile.

Since the lead is a straight line, it is not difficult to obtain that the lead needs only $X$ linear motion. That is, $W/X/Y$.

### 3.5 Comprehensive Analysis

After comprehensive analysis, the motion module required for processing involute tooth profile surface is $G/X/Y\alpha_i/\beta I$. So twenty-four Motion module combinations can be obtained as $G/X/Y\alpha_i/\beta I$, $G/X/Y\alpha_i/\beta I$, $G/X/Y\alpha_i/\beta I$, $G/X/Y\alpha_i/\beta I$, $G/X/Y\alpha_i/\beta I$, $G/X/Y\alpha_i/\beta I$, $G/X/Y\alpha_i/\beta I$, $G/X/Y\alpha_i/\beta I$, $G/X/Y\alpha_i/\beta I$, $G/X/Y\alpha_i/\beta I$, $G/X/Y\alpha_i/\beta I$, $G/X/Y\alpha_i/\beta I$, $G/X/Y\alpha_i/\beta I$, $G/X/Y\alpha_i/\beta I$, $G/X/Y\alpha_i/\beta I$, $G/X/Y\alpha_i/\beta I$, $G/X/Y\alpha_i/\beta I$, $G/X/Y\alpha_i/\beta I$. Here use “$-$” to represent the bed and connect the motion module. Each motion module can get five combinations, so 120 combinations can be obtained.

### 4. Optimization of Motion Schemes

According to Ref [6], 48 kinds of reasonable motion module combination can be obtained by removing unreasonable modules. Fig. 7 gives the diagram of $G/X/Y\alpha_i/\beta I$ machine tool motion module combination.
5. Conclusions
This paper presents a method for designing motion module combination for machining involute tooth profile surface. Starting from the configuration characteristics of gear, the pose relationship between grinding wheel and involute tooth profile surface is clarified. Pose matrix of grinding wheel relative to involute tooth profile surface is established, and the necessary motion modules are obtained. Module combinations are screened. An effective module combination and a reasonable schematic diagram of motion module combination are given. The method adopted in this paper is beneficial to the motion module design for machining involute tooth profile surface.

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