Research Article

Controlled Wrinkling Analysis of Buckled Thin Films on Gradient Elastic Substrate System: A Numerical Study

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Continuous change in wrinkle patterns process of thin films to a gradient substrate is the most challenging problem regarding applying reliable and robust numerical methods for postbuckling analysis of the film/substrate system. For example, in the finite element method, the postbuckling simulation suffers from the convergence issue, while, in spectral methods, it is very difficult to capture the localized behavior in soft matters when the boundary conditions are complex. When a thin film is compressed, it can form a wrinkle of a certain amount of wavelength when the compression exceeds a critical value. The compressed compliant substrate system translates to sinusoidal wrinkles and then to period-doubling wrinkling after further compression. In this work, we investigate the mathematical model arising from the changing nature of wrinkle patterns of postbuckled thin films using a robust and efficient numerical algorithm based on the spectral method to evolve the wrinkle patterns. We consider the gradient substrates of three typical variations in the modulus, namely, the symmetry, exponential, and power-law model. It has been observed that the stable equilibrium path has two bifurcation points. At the first bifurcation point, the buckling instability of wrinkling occurs, while the period-doubling buckling instability occurs at the second bifurcation point. For the substrates of material gradients of various types, the amplitude and wavelength are obtained. This study may help in better understanding of wrinkle patterns formation which could be very useful for the designing of stretchable and flexible electronic devices of most substrate systems and to avoid resonance in the noise environment.

1. Introduction

Thin films play an important role in our daily life, and their use is dramatically increased in recent years. This is the fact that a lot of companies these days are producing a different type of coating for a variety of fields like aircraft and automation industries and even in medical device and high energy fields. The soft substrate containing stiff thin films is ancient and commonly exists in nature. Nearly every person on earth knows about the wrinkles that would be found on the skin. The animal skin composes of a stiff epidermis that is attached to the soft dermis. When the person ages, the dermis contracts and so the systems would be placed under the compressive stress. Skin will wrinkle in the response to this strain. These wrinkles have the wavelength determined by calculating the stretching of the dermis and the bending of the stiff epidermis [1]. This is not restricted to animal skin only. Fruits like plums and apples also experience wrinkles which consist of thin skin surrounding the soft interior like water and minerals. With the loss of water, the fruit dries and so the volume of the inner part of the fruit decreases resulting in the shrinkness of the skin and the rise of the wrinkles [2, 3]. Thin film that is bonded to the gradient substrate forms a buckle known as the local buckle. Hence, by changing the properties of the gradient substrate, surface patterns of buckling can be tuned. The study of wrinkles is very useful in a wide range of applications like bioengineering, micro-nanofabrication, and stretchable electronics [4–6]. In many situations, dynamic wrinkling morphologies are capable of regulating the physiological, biochemical, and physical properties of biological surfaces. For example, the
wrinkled cell membrane enables a large surface area and enhanced deformability [7, 8].

Wrinkling of the thin film is commonly observed as a result of mechanical instability. This issue was not addressed in the way as it deserved in early studies. With the growing understanding of how commonly these phenomena are found in nature, scientists have now developed an understanding of the importance of the wrinkles in nature [9]. Thin films form a number of wrinkle patterns globally such as chessboards, herringbone, stripes, and labyrinths [6]. Among these patterns, herringbone proved to have the minimum potential energy [10–12]. Scientists have shown that stripes relieve film compression in only one direction, whereas herringbones relieve compression in both directions [13]. Significantly, the patterns of wrinkle are uniform considering wavelength, amplitude, and the critical membrane force as constant. By simply considering the gradient films, the polydimethylsiloxane (PDMS) gradient embedded into a PDMS-hard matrix results in a changing gradient films, that are deposited on the soft elastic substrate [14], and the thickness-wrinkle wavelength continuously [14], and the thickness-gradient films that are deposited on the soft elastic substrate could provide a guideline for the fundamental research on several wrinkling morphologies. The change in the thickness of the film shows the nonuniform patterns of wrinkle [15].

Localization of wrinkles is the result of a change in the thickness of film [16] or by the use of functionally graded material [17] whereas wavelength and amplitude are considered to be nonuniform. In the year 2016, during the curing process, scientists tuned the in-plane temperature gradient to acquire the sample of elasticity (PDMS) [18]. To simulate the behavior of skin, several mathematical models have been developed under various mathematical loadings [19–21]. Most of these models replicate many of the complex characteristics of skin like viscoelasticity and orthotropy, while they ignore the individual contributions of the dermis, epidermis, and hypodermis, considering the skin is homogeneous material [22]. Some existing multi-layer models ignore the underlying hypodermis or the stiff stratum corneum [23, 24]. Also, prestress or natural tension in the skin is not explicitly considered for most models [25, 26].

In this paper, we apply a higher-order numerical scheme based on the spectral method for the efficient evaluations of controlled wrinkling analysis of thin films on gradient substrates. The rest of the paper is organized as follows. Section 2 consists of a mathematical model description, followed by the spectral collocation method in Section 3. Results of numerical simulation and their discussions are given in Section 4, while Section 5 is used for the conclusion.

2. Model Description

Consider a hard thin film on a soft substrate having a thickness $h$ as shown in Figure 1(a). The deformation of the substrate is modeled as an isotropic elastic half-space. Therefore, substrates have represented a set of parallel springs and the spring that is individual is in the uniaxial strain state with thickness $\kappa$. Assume that before compression, the system is initially flat, and after compression, it remains uniform in the $x$-axis, while it deforms in $x - z$ direction as shown in Figures 1(b) and 1(c). The reaction at any point of the compliant substrate is proportional to deflation due to the elastic nature of the wrinkle. The buckle wavelength is much smaller than the length in the $x$-direction. Since the shear traction has a very small effect on the solution of the thin film, the wavelength provided is much larger than the deflection; therefore, its effect is neglected, and the film equation of equilibrium for uniaxial deformation is the following ordinary differential equation [9, 11, 27, 28]:

$$
\begin{aligned}
&M \frac{d^4 \zeta}{dx^4} + N \frac{d^2 \zeta}{dx^2} + \kappa(X) \zeta = 0, \\
&\zeta(0) = 0, \zeta(1) = 0, \\
&\zeta'(0) = 0, \zeta'(1) = 0,
\end{aligned}
$$

where $\zeta$ is the deflection, $\kappa$ is the stiffness of the film, $M$ represents the bending stiffness, and $N$ refers to the membrane force of the film. The dimensionless form of the equilibrium equation can be obtained by introducing the dimensionless variables as $x = X/L$ and $\phi = \zeta/h$. The dimensionless wrinkle stiffness is introduced as:

$$
x = \frac{X}{L}, \phi = \frac{\zeta}{h},
$$

where $L$ and $h$ are the length and thickness of the film, respectively. The dimensionless form of the equilibrium equation is then given by [29]

$$
\begin{aligned}
&\frac{d^4 \phi}{dx^4} + \frac{N}{M} \frac{d^2 \phi}{dx^2} + \kappa(X) \phi = 0, \\
&\phi(0) = 0, \phi(1) = 0, \\
&\phi'(0) = 0, \phi'(1) = 0.
\end{aligned}
$$

3. Chebyshev–Galerkin Method

The governing equation is the differential equation of the fourth order. Hence, even for the simple function of wrinkle stiffness, the analytical solution is very difficult to obtain. The Chebyshev–Galerkin method is used to discretize the film-substrate system. To get the solution of this equation using the spectral method, replace the spatial derivative by a suitable basis function. As we have both Dirichlet and Neumann boundary conditions, therefore, the ideal choice of the basis function has the form.
Figure 1: Schematic illustration of wrinkling patterns of the thin film. (a) Undeformed thin film (before compression). (b) The sinusoidal wrinkles form (compresses by $\Delta L$). (c) Doubling periodic configuration can be seen when compresses by $L$, with wavelength $2\lambda_0$.

Figure 2: Effects of material gradient on wrinkle stiffness and normalized amplitude based on the power-law model.
Using the definition of inner product,
\[
\begin{bmatrix} \sin \left( \frac{m\pi}{L} x \right), \sin \left( \frac{n\pi}{L} x \right) \end{bmatrix} = \begin{cases} \frac{L}{2} & m = n, \\ 0 & m \neq n, \end{cases}
\]
so equation (8) reduces to
\[
\left( \frac{m\pi}{L} \right)^4 a_n = K(x) a_n - N \left( \frac{m\pi}{L} \right)^2 a_n.
\] (11)

By taking \( L/2 \) is a common factor [30]. Hence, the problem is now reduced to a system of linear algebraic equation with \( a_n \) unknown coefficients, which is then solved by using a standard numerical technique for the algebraic equation.

4. Results and Discussion

The wrinkle patterns of film bonded to gradient substrate are investigated by considering the reference wrinkle stiffness to be constant as \( k_0 = 10^7 \). Material gradient effects on wrinkle patterns are illustrated by plotting the normalized amplitude and wrinkle stiffness with several material gradients. Wrinkle patterns changing from global to local are observed for a material gradient substrate, while they are highly depending upon wrinkle stiffness. We investigate the wrinkle patterns of the power-law model, the exponential model, and the symmetric model. Power-law function defines the wrinkle stiffness as
\[
K = K_0 x^\alpha,
\] (12)
where \( \alpha \) is the material gradient. As shown in Figure 2, the wrinkle stiffness increases with the increasing distance. As \( \alpha \)
increases, the wrinkle stiffness also increases with the increasing distance, and the area of the wrinkle region is smaller.

Exponential function defines the wrinkle stiffness as
\[ K = K_0 e^{-\beta x}, \]  
(13)
where \( \beta \) is material gradient. From Figures 2–4, it can be observed that the wrinkle stiffness decreases steadily with the increase of the distance. With the increase of \( \beta \), the stiffness of the wrinkle decays keeping the area of the wrinkle region small. Here, the transition from global to local wrinkle patterns can be observed. Figures 5–7 show the effects of the material gradient using different parameter values on wrinkle stiffness based on the power-law model, exponential model, and symmetric model, respectively.

Hence, the conclusion can be made that the material gradient strongly affects the localization of wrinkle patterns. Symmetric function defines the wrinkle stiffness as
\[ K = K_0 (x - \gamma)^2, \]  
(14)
From Figure 4, it can be observed that the symmetric wrinkle patterns are obtained.
5. Conclusion

In this paper, Chebyshev spectral method is used to simulate the wrinkling behavior of the film-gradient substrate system. Three types of gradient substrates are considered, the power-law, the exponential, and the symmetric model. With the increasing material gradient, the evolution of wrinkle patterns from global to local can be observed. The normalized amplitude and the wavelength of the wrinkles for various material gradient substrates are obtained. The wrinkling pattern is not uniform and so it is concluded that the material gradient strongly affects the localization of wrinkle patterns.

Data Availability

No data were used to support this study.

Conflicts of Interest

The author declares no conflicts of interest.

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![Figure 7: Effects of material gradient on wrinkle stiffness based on the symmetric model.](image-url)
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