Suppression, persistence and reentrance of superfluidity in overflowing nuclear systems

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Based on a microscopic description of superfluidity in overflowing nuclear systems, it is shown that continuum coupling plays an important role in the suppression, the persistence and the reentrance of pairing. In such systems, the structure of the drip-line nucleus determines the suppression and the persistence of superfluidity. The reentrance of pairing with increasing temperature leads to additional critical temperatures between the normal and superfluid phases.

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Overflling many-body fermionic systems exist in various situations going from the crust of neutron stars to ultra-cold atoms. Interestingly, these systems offer the possibility to study the coupling between two fluids with very different pairing properties. In such systems, one fluid is localized inside an initial container, such as for instance a nuclear potential, and a second fluid is overflowing towards a larger container. Being in different environments, these two fluids can acquire different pairing gaps. In this Letter, we address the question of the coupling between the two superfluids and their finite temperature properties.

At the transition between its outer and inner crust, neutron stars provide an example of such microscopic overflowing systems, commonly called neutron driping. In the outer crust, nuclei form a Coulomb lattice which gets more and more neutron rich as the density increases. When the maximum number of neutrons that a nucleus can sustain is reached, the overproduced neutrons drip out of nuclei. These neutrons populate the continuum states and shall be described within the band theory. It should be noted that nuclei surrounded by an infinite neutron gas could exist as a stable configuration in neutron stars where they are bounded by gravitation, while isolated nuclei that exist for instance on earth are limited to the drip lines.

The first prediction of the suppression of pairing in overflowing $Z = 50$ nuclear systems was performed in Ref. [1]. It was proposed to attribute this suppression to the large coherence length of the weakly-superfluid neutrons gas: the neutron gas can penetrate the dense nuclear system and could impose its weak pairing field. It was however also noted in the same work that neutrons could impose its weak pairing field. It was proposed to attribute this suppression to the large coherence length of the weakly-superfluid neutrons gas: the neutron gas can penetrate the dense nuclear system and could impose its weak pairing field. It was however also noted in the same work that neutrons gas: the neutron gas can penetrate the dense nuclear system and could impose its weak pairing field.

In this Letter a systematic analysis based on several overflowing isotopic chains is performed and it is shown that pairing quenching and continuum coupling are strongly related. A pairing reentrance phenomenon with increasing temperature is predicted for $Z = 50$ overflowing systems.

In the present work, an HFB approach in coordinate representation is employed. This model has already been applied to describe nuclei and Wigner-Seitz cells in a fully self-consistent framework (see Refs. [15] and references therein). The Skyrme SLy4 interaction is used in the mean-field channel, and is completed by the ISS pairing force which is adjusted to the BCS pairing gap predicted by bare nucleon-nucleon potentials. All the bound states are considered, the angular momentum projection and reentrance of pairing remains to be studied. Pairing correlations in the ground state of weakly-bound nuclei are commonly described by the Hartree-Fock-Bogoliubov (HFB) theory [12, 13]. In most of the HFB calculations the continuum is discretized by solving the HFB equations with box boundary conditions [14].

For these selected isotopes, the neutron-drip number $N_{drip}$, defined as the neutron number of the last nucleus with very different pairing properties [4, 5]. In such systems, one fluid is localized inside an initial container, such as for instance a nuclear potential, and a second fluid is overflowing towards a larger container. Being in different environments, these two fluids can acquire different pairing gaps. In this Letter, we address the question of the coupling between the two superfluids and their finite temperature properties.

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The total occupation number of resonance states for the drip-line nuclei $N_{\text{res}}$ is shown in the last column (see the text for more details).

TABLE I: Isotope acronym, number of protons $Z$, number of neutrons of the last nucleus before the drip line $N_{\text{drip}}$, for the selected set. Isotopes for which the drip-line nucleus is non-magic (magic) belongs to the group $A_1$ ($A_2$, respectively). The total occupation number of resonance states for the drip-line nuclei $N_{\text{res}}$ is shown in the last column (see the text for more details).

| Isotope | $Z$ | $N_{\text{drip}}$ | group | $N_{\text{res}}$ |
|---------|-----|-----------------|-------|-----------------|
| Ni      | 28  | 60              | $A_1$ | 3.0             |
| Kr      | 36  | 82              | $A_2$ | 0.0             |
| Sr      | 38  | 82              | $A_2$ | 0.0             |
| Zr      | 40  | 84              | $A_1$ | 2.2             |
| Mo      | 42  | 90              | $A_1$ | 8.0             |
| Ru      | 44  | 92              | $A_1$ | 3.0             |
| Sn      | 50  | 126             | $A_2$ | 0.0             |
| Te      | 52  | 126             | $A_2$ | 0.0             |

FIG. 1: (color online) Neutron pairing gaps versus neutron density $N$ (top panel) and versus $N - N_{\text{drip}}$ (bottom panel) for the isotopes in the group $A_1$ (see the text for more details).
instance.

The persistence of superfluidity upon overflow of bound neutrons might not be the only consequence of continuum coupling: in the case where pairing is suppressed, the increase of temperature may generate the reentrance of superfluidity. The finite-temperature HFB model [24], is employed to study the reentrance of pairing in the thermal state of overflowing even-even nuclear systems. The temperature-averaged pairing gap for \(160, 176, 180, 200\) Sn is shown in Fig. 3. In \(160\) Sn and \(200\) Sn it behaves as expected from HFB theory: the pairing gap vanishes at the critical temperature \(T_c = 0.57\Delta(T \approx 0)\) (see Ref. [28] and references therein). In the case of \(176\) Sn and \(180\) Sn, the reentrance of superfluidity is observed with increasing temperature. This reentrance is induced by the presence of resonances states in the spectrum of these nuclear systems: Being slightly too high in energy, these states are not occupied at zero temperature (see Table I), while at finite temperature, they can be partially occupied from the Fermi-Dirac distribution. At low temperature, the pairing correlations can therefore be switched on allowing the reappearance of the superfluid state. The reentrance critical temperature depends on the step in energy between the last occupied bound state and the first resonance one, which changes from one system to another, as observed in Fig. 3 for \(176\) Sn and \(180\) Sn. In the case of \(200\) Sn, this energy step is too large to give rise to the reentrance of superfluidity before the highest critical temperature is reached. The \(180\) Sn overflowing system has an interesting phase diagram including three critical temperatures: with increasing temperature, two of them correspond to the vanishing of superfluidity \((T_{c1} \sim 11\) keV and \(T_{c3} \sim 1\) MeV) and one to its reappearance \((T_{c2} \sim 300\) keV). The lowest critical temperature \(T_{c1}\) is associated to the transition from the superfluid to the normal state in the overflowing neutron gas. The highest critical temperature \(T_{c3}\) is similar for \(160, 176, 180, 200\) Sn, indicating that superfluidity has been restored in the seed nucleus of \(176, 180\) Sn between \(T_{c2}\) and \(T_{c3}\). This superfluidity is mainly built on resonances populated at finite temperature. More generally, pairing reentrance in hot systems is observed for isotopes belonging to the group \(A_2\) where resonances are too high in energy to participate to pairing at zero temperature but close enough to the last occupied state to be reached at finite temperature.

Reentrance of superfluidity at finite temperature have been predicted in nuclear systems such as in odd-nuclei [19], rotational motion of nuclei [20], and the deuteron pairing channel in asymmetric infinite matter [21]. It was aslo predicted in polarised \(^3\)He [22–24] and in spin asymmetric cold atom gas [25, 26]. In all these systems, pairing at zero temperature is generated by an attraction among Fermions of different spin or isospin. Superfluidity is therefore maximum in spin or isospin symmetric systems for which there is a matching of the Fermi levels of the constituent Cooper-pairs. Breaking the spin or isospin symmetry disfavor pairing while temperature in asymmetric systems acts in favor of restoring the broken symmetry and can eventually induce a reentrance of pairing. At variance with this mechanism the pairing reentrance phenomenon discussed in this Letter is based on a novel mechanism in finite systems where resonance states play a major role.

In summary, we have investigated the pairing properties of nuclear systems upon overflowing superfluid neutrons. Suppression, persistence and reentrance of superfluidity can occur in these finite systems. From a systematic HFB calculations on 8 isotopic chains, the pairing properties is shown to be strongly correlated to the continuum coupling, both at zero and finite temperature. At zero temperature, the coupling between the seed nucleus and the gas is weak, and a formal separation of their
properties into a nucleus plus a gas provides a qualitative understanding of the suppression and the persistence of superfluidity. With increasing temperature in the normal state, the Fermi-Dirac distribution can populate the resonance states giving rise to the reentrance of superfluidity. The pairing correlations in the nuclear system are switched on again and consecutive critical temperatures are predicted.

The understanding of the suppression, persistence and reentrance of superfluidity in nuclear systems, deeply related to the continuum coupling, opens wide perspectives for discoveries in weakly bound nuclei, as well as it sheds new light on the transition between the inner and outer crusts in neutron stars. The role of resonances around the neutron drip not only changes the microscopic understanding of the neutron drip-out mechanism, but it also modifies the thermal properties of the crust through the strength of the pairing interaction [20]. The temperatures at work during cooling are typically of the order of 10 to 500 keV [1] and coincide with the critical temperatures of the reentrance phenomenon. As a consequence, the novel pairing reentrance phenomenon analyzed in this Letter is expected to modify the thermodynamical and cooling properties in the crust of neutron stars. The links with other superfluid Fermi-systems shall be investigated in the near future. For instance in cold Fermionic atoms overflowing from an inner trap to a larger one [2, 3], it is known that stable path close to the centrifugal energy are the classical analog of the quantal resonances. It will therefore be interesting to investigate the role of these stable paths on the superfluid properties of overflowing cold Fermionic atoms.

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[1] P. Haensel, A. Y. Potekhin, and D. G. Yakovlev, Neutron Stars I, vol. 3 (2007).
[2] D. Stamper-Kurn et al., Phys. Rev. Lett. 81, 2194 (1998).
[3] L. Viverit, S. Giorgini, L. P. Pitaevskii, and S. Stringari, Phys. Rev. A 63, 033603 (2001).
[4] M. Grasso, E. Khan, J. Margueron, and N. Van Giai, Nucl. Phys. A 807, 1 (2008).
[5] P. Schuck and X. Viñas, Phys. Rev. Lett. 107, 205301 (2011).
[6] N. W. Ashcroft and N. D. Mermin, Solid State Physics (Saunders College Publishing, 1976).
[7] N. Chemel, S. Naimi, E. Khan, and J. Margueron, Phys. Rev. C 75, 055806 (2007).
[8] A. Bulgac, preprint No. FT-194-1980, Institute of Atomic Physics, Bucharest, 1980, nucl-th/9907088
[9] M. Grasso, N. Sandulescu, N. Van Giai, and R. J. Liotta, Phys. Rev. C 64, 064321 (2001).
[10] Y. Zhang, M. Matsuo, and J. Meng, Phys. Rev. C 83, 054301 (2011).
[11] K. Hagiino, and H. Sagawa, Phys. Rev. C 84, 011303(R) (2011).
[12] P. G. De Gennes, Superconductivity of metals and alloys (Addison-Wesley, 1989).
[13] P. Ring and P. Schuck, The Nuclear Many-Body Problem (Springer-Verlag, 1980).
[14] J. Dobaczewski, H. Flocard, and J. Treiner, Nucl. Phys. A 422, 103 (1984).
[15] F. Grill, J. Margueron, and N. Sandulescu, Phys. Rev. C 84, 065801 (2011).
[16] E. Chabanat et al.; Nucl. Phys. A 635, 231 (1998).
[17] S. B. Rüster, M. Hempel, and J. Schaffner-Bielich, Phys. Rev. C 73, 035804 (2006).
[18] S. Hilaire and M. Girod, E. Phys. J. A 33, 237 (2007).
[19] R. Balian, H. Flocard, M. Veneroni, Phys. Rep. 317, 251 (1999).
[20] D. J. Dean, K. Langanke, H. Nam, and W. Nazarewicz, Phys. Rev. Lett. 105, 212504 (2010).
[21] A. Sedrakian, T. Alm, and U. Lombardo, Phys. Rev. C 55, R582 (1997).
[22] G. Frossati, K. S. Bedell, S. A. J. Wiegers, and G. A. Vermeulen, Phys. Rev. Lett. 57, 1032 (1986).
[23] P. A. Crowell and J. D. Reppy, Phys. Rev. Lett. 70, 3291 (1993).
[24] G. A. Csathy, E. Kim, and M. H. W. Chan, Phys. Rev. Lett. 88, 045301 (2002).
[25] P. Castorina, M. Grasso, M. Oertel, M. Urban, D. Zappalà, Phys. Rev. A, 025601 (2005).
[26] K. Levin and Q. Chen, Ultracold Fermi Gases (Proc. Int’l School of Physics ”Enrico Fermi”), Vol. CLXIV, 751 (2008). ArXiv:0610006 (cond-mat.str-el) (2006).
[27] K. Sandulescu, Phys. Rev. C 70, 025801 (2004).
[28] E. Khan, N. Van Giai, and N. Sandulescu, Nucl. Phys. A 789, 94 (2007).
[29] M. Fortin, F. Grill, J. Margueron, D. Page, and N. Sandulescu, Phys. Rev. C, 82, 065804 (2010).