Simulation of Reynolds number influence on heat exchange in turbulent flow of medium slurry

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Abstract. The paper deals with the numerical simulation of mass and heat exchange in turbulent flow of solid-liquid mixture in the range of averaged solid particle diameter from 0.10mm to 0.80mm, named further as the medium slurry. Physical model assumes that dispersed phase is fully suspended and a turbulent flow is hydro-dynamically, and thermally developed in a straight horizontal pipeline. Taking into account the aforementioned assumptions the slurry is treated as a single-phase flow with increased density, while viscosity is equals to a carrier liquid viscosity. The mathematical model constitutes time averaged momentum equation in which the turbulent stress tensor was designated using a two-equation turbulence model, which makes use of the Boussinesq eddy-viscosity hypothesis. Turbulence damping function in the turbulence model was especially designed for the medium slurry. In addition, an energy equation has been used in which a convective term was determined from the energy balance acting on a unit pipe length, assuming linear changes of temperature in main flow direction. Finally, the mathematical model of non-isothermal medium slurry flow comprises four partial differential equations, namely momentum and energy equations, equations of kinetic energy of turbulence and its dissipation rate. Four partial differential equations were solved by a finite difference scheme using own computer code. The objective of the paper is to examine the influence of Reynolds number on temperature profiles and Nusselt number in turbulent flow of medium slurry in the range of solids concentration from 0% to 30% by volume. The effect of influential factors on heat transfer between the pipe and slurry is analysed. The paper demonstrates substantial impact of Reynolds number and solids volume fraction on the Nusselt number. The results of numerical simulation are reviewed.

1. Introduction
Hydraulic transport of solids is available in a wide range of industrial applications. Designing of slurry pipeline transportation with heat exchange requires a better understanding of flow mechanism, which includes pressure drop, velocity and solids concentration profiles, deposition velocity, and as a consequence the temperature profile. Therefore a solid-liquid transport belongs to main challenges of Computational Fluid Dynamics [1-3]. Various approaches have been investigated to predict a solid–liquid mechanism during turbulent flow, however, mixture theory models are the most promising and are based on a rigorous fluid mechanics framework [3-5].

Solid-liquid flow is recognised as stationary or moving bed, heterogeneous, and pseudo–homogeneous, or as settling or non-settling [6]. Settling slurry is formed mainly by coarse particles, however, it can exist as well if fine or medium solid particles have sufficiently low velocities.

If simulation of coarse dispersive slurry flow is considered one should mention basic researches of Bagnold [7] and recent mathematical models [8-10], which include solid-solid and/or solid-liquid interactions. In order to simulate frictional head loss in slurry flow, which contains coarse or medium
solid particles, it is reasonable to assume the Newtonian model, as no one can measure rheology in such slurries [11].

If non-settling slurry is considered averaged solid particle diameters are sufficiently fine to form stable homogeneous mixture exhibiting increased apparent viscosity. Such slurry usually exhibits yield stress and require proper rheological model built into the momentum equation. Slurry with fine solid particles demonstrates increased viscous sublayer, which means that damping of turbulence appears in a near-wall region. In this case, the apparent viscosity concept, and suitable rheological model, and properly defined wall damping function is required [12-15].

Slurry with medium solid particles of averaged diameter from $d_S=0.10 \text{ m}$ to $d_S=0.80 \text{ m}$ is usually assumed as Newtonian flow or as flow of two separated phases [16]. However, if slurry velocity is sufficiently high such slurry can be considered to be non-settling and pseudo-homogeneous. It is important to emphasize that slurry with medium solid particles exhibits enhanced damping of turbulence since frictional head loss is almost equal or below that for carrier liquid flow [17,18]. Damping of turbulence depends mainly on particle size, solids concentration and flow condition. The damping of turbulence could be a consequence of thickening of a viscous sublayer, which tends to increase throughput velocity and thus promotes a drag reduction [15]. Simulation of turbulent flow of medium slurry requires a proper approach, which includes damping of turbulence, in order to predict the pressure drop and velocity distribution. The velocity distribution and the viscous sublayer are closely related to temperature distribution, which is a special interest in mining industry, pipeline erosion, food industry, medicine, etc.

Sundaresan et al. [4], outlined a number of scientific challenges, which represent the building blocks for the comprehensive understanding of disperse flows encountered in a variety of technologies and in nature. Researchers concluded that new experiments and/or analyses are needed to cast light on the vital phenomena that cause turbulence damping or generation. The authors suggested that the experiments should be made in simple turbulent flows, such as: fully developed pipe or channel flow, or axially symmetrical flow. Regardless of geometry, experiments must include a wide range of particle parameters in a single fixed facility. Their conclusions are still outstanding in mathematical modelling of solid-liquid flow.

Silva et al. and Cotas et al. [19-21] used commercial CFD software to simulate fully developed turbulent flow of eucalyptus and pine pulps as such flows exhibit drag reduction. They applied a pseudo-homogenous approach by considering the viscosity of pulp as a function of shear rate and solids concentration. They assumed there exists lubrication layers responsible for the drag reduction, which consists of pure water with a few fibres. They adopted into account the Chang-Hsieh-Chen turbulence model [22], in which they used damping function, proposed by Bartosik [23], to study the flow of solid particles. They validated the mathematical model by comparing measured and predicted frictional head loss obtaining satisfying agreement.

Transport of solid particles is often strongly influenced by heat exchange between the transported materials and the surrounding. Researchers are developing methods and techniques to enhance or attenuate of heat exchange. Enhancement of heat exchange contributes to energy saving and preserve the environment. Those methods are recognised as active or passive [24]. The passive ones like inserted wire coils or mechanically deformed pipes, have been studied for the last several years, and have become commercial solutions. The active techniques can produce very high increases in heat exchange and there are still main challenges of fluid mechanics of two-phase flows [25].

Influence of solid particles submerged in a carrier liquid on heat exchange in turbulent flow has been experimentally investigated be several researchers [26-30]. Rozenblit et al. [30] made an experimental study on heat transfer coefficient associated with solid-liquid mixture transport in horizontal pipe using electro-resistance sensor and infrared imaging. They considered flow of acetyl-water mixture with moving bed for solids volume fraction from 5% to 15% by volume. The experimental results included the temperature profile on the heated wall and the vertical solids volume fraction profile. They observed that the local heat transfer coefficient changes from its lowest value at the bottom of the pipe, to highest values above carrier liquid at the upper heterogeneous layer.
It clearly means that the heat transfer coefficient is strongly influenced by the cross sectional profile of the solid phase in the pipe. In order to analyse the near wall structures they used algebraic three layer model to predict pressure drop and solids volume fraction distribution across the pipe [30].

Wang and Xu [31] examined experimentally thermal conductivity of nanoparticles of Al$_2$O$_3$ and CuO mixed with water, vacuum pump liquid, engine oil, and ethylene glycol. The averaged particle diameter of the Al$_2$O$_3$ was 28 nm, and the averaged particle diameter of the CuO was 23 nm. All particles were loosely agglomerated in chosen liquids. Experiments proved that the thermal conductivity of the mixture increases with the volume fraction of the Al$_2$O$_3$ particles. Increase of the mixture thermal conductivity depends on liquid and solid phase properties. They proved that increase of the mixture thermal conductivity in ethylene glycol and engine oil are the highest, whereas that in the pump liquid is the lowest. The effective thermal conductivity of ethylene glycol increases by 40% when solids concentration of Al$_2$O$_3$ was 8% by volume [31].

Summarising the literature review one can say there are no available experiments and simulations presenting the influence of Reynolds number on heat exchange in turbulent flow of medium slurry with high solids concentration.

The objective of the paper is to examine influence of the Reynolds number on the Nusselt number in turbulent flow of medium slurry in the range of solids concentration from 0% to 30% by volume.

2. The physical and the mathematical approach

Physical model assumes that slurry comprises medium solid particles with averaged particle diameter from 0.10 mm to 0.80 mm and water as a carrier liquid. All solid particles are rounded and narrowly sized with density equals to $\rho_S=2440$ kg/m$^3$ and $\rho_S=2650$ kg/m$^3$. The solids concentration (C) varies from 0% to 30% by volume. The slurry flow takes place in a horizontal pipe with sufficiently high bulk velocity, so the flow can be considered as pseudo-homogeneous. The slurry flow is stationary, turbulent, fully developed and axially symmetrical. It is assumed that slurry viscosity is equal to the carrier liquid viscosity as no one can measure rheology for such particle sizes [7]. The slurry density ($\rho_m$), specific heat ($c_p$), and thermal conductivity ($\lambda$) are dependent on temperature and volume fraction of solids (C) and were calculated, as follows:

$$\Phi_m = \Phi_L (1-C) + \Phi_S C$$  \hspace{1cm} (1)

where $\Phi_m$ is general variable for slurry (mixture of solid and liquid phase), $\Phi = (\rho, c_p, \lambda)$, and subscript L - means liquid phase, while S - means solid phase.

The final form of the mathematical model is, as follows:

- Momentum equation:

$$\frac{1}{r} \frac{\partial}{\partial r} \left[ r \left( \mu \frac{\partial U}{\partial r} - \bar{\rho}_m u'v' \right) \right] = \frac{\partial p}{\partial x}$$ \hspace{1cm} (2)

where $U$ - velocity component in main flow direction (m/s); $\rho_m$ - slurry density (kg/m$^3$); $\mu$ - molecular viscosity of carrier liquid phase (Pa s).

The component of the turbulent stress tensor, which appears in equation (2), was designated by the Boussinesq hypothesis, as follows:

$$\bar{\rho}_m u'v' = \mu_t \frac{\partial U}{\partial r}$$ \hspace{1cm} (3)

where $\rho_m$ is slurry density and subscript t - means turbulent.

Launder and Sharma developed turbulent viscosity ($\mu_t$), stated in equation (3), on the basis of dimensionless analysis, as follows [32]:

It was proved that the Launder and Sharma turbulence model has great potential to predict pseudo-homogeneous solid-liquid flows [33]. Therefore the Launder and Sharma turbulence model was chosen into the mathematical model in order to predict pressure drop, velocity and temperature profiles, and Nusselt number for medium slurry flow. The kinetic energy of turbulence (k) and its dissipation rate (ε), which exists in equation (4), were delivered from the Navier-Stokes equations [35].

For the aforementioned assumptions the final form of k and ε equations, proposed by Launder and Sharma [32], are following:

- Equation of kinetic energy of turbulence:

$$
\frac{1}{r} \left[ r \left( \mu + \frac{\mu_t}{\sigma_t} \right) \frac{\partial k}{\partial r} \right] + \mu_t \left( \frac{\partial U}{\partial r} \right)^2 = \tilde{p}_0 \epsilon + 2 \mu \left( \frac{\partial k^{1/2}}{\partial r} \right)^2
$$

- Equation of dissipation rate of the kinetic energy of turbulence:

$$
\frac{1}{r} \left[ r \left( \mu + \frac{\mu_t}{\sigma_t} \right) \frac{\partial \epsilon}{\partial r} \right] + C_1 \frac{\epsilon}{\mu_t} \left( \frac{\partial U}{\partial r} \right)^2 = C_2 \left[ 1 - 0.3 \exp \left( - \frac{Re_t}{k} \right) \right] \frac{\tilde{p}_0}{k} - 2 \frac{\mu_t}{\rho} \mu_t \left( \frac{\partial U}{\partial r} \right)^2
$$

On the basis of dimensionless analysis the turbulent Reynolds number in equation (6) can be expressed, as follows [35]:

$$
Re_t = \frac{\tilde{p}_0 k^2}{\mu \epsilon}
$$

The crucial point in the above turbulence model is proper expression of turbulence damping function (f_μ), stated in equation (4). The function is known as the wall damping or turbulence damping function. This is an empirical function, which causes decrease of turbulent viscosity in the vicinity of solid wall and in consequence decrease of the turbulent stress tensor component described by equation (3). As it is an empirical function it is possible to redesign the function (f_μ) for certain application, like medium slurry flows for instance. Recently, Ruffin and Lee [34] successfully used the standard k-ε model of Launder and Spalding [35] and a new wall damping function for the unstructured Cartesian grid solver.

Taking into account medium slurry flows, which exhibits enhanced damping of turbulence; author developed a new turbulence damping function [23]. The new turbulence damping function includes averaged particles diameter (d), and solids concentration by volume (C). The new turbulence damping function proposed to medium slurry flow is described, as follows:

$$
f_\mu = 0.09 \exp \left[ - \frac{3.4 \left( 1 + A d^2 \right) (8 - 88 A d) C^{0.5}}{(1 + \frac{Re_t}{50})^2} \right]
$$

where A is an empirical constant, and Re_t is turbulent Reynolds number described by equation (7).

If particle diameter and/or solids concentration goes to zero the new turbulence damping function, described by equation (8), approaches the standard turbulence damping function of a single-phase flows proposed by Launder and Sharma [32]. This is in accordance with our expectation as the mathematical model should be suitable for single-phase flow if d_s=0 and/or C=0. The new turbulence damping function works well if slurries with averaged solid particle diameters from d_s=0.10 mm to d_s=0.80 mm are considered. The new turbulence damping function proposed for the medium slurry
flows, described by equation (8), demonstrates that averaged solid particle diameter plays primarily role, while the solids concentration plays secondary role in damping of turbulence near a pipe wall. This is in accordance with previous observations made by several researchers [15, 17-18]. The new turbulence damping function, described by (8), was developed on the basis of comparison between numerical predictions and measured global parameters in comprehensive range of averaged solid particle diameter equals to \( d_S = (0.125; 0.240; 0.471; 0.780) \) mm and solids concentration \( C = (10\%; 20\%; 30\%; 40\%) \).

In order to simulate heat exchange in turbulent flow of medium slurry the energy equation, written in form of the temperature equation, is needed. Taking into account aforementioned assumptions one can write final form of the temperature equation, as follows:

- Temperature equation:

\[
\bar{p}_m \bar{U} \frac{\partial T}{\partial x} = \frac{1}{r} \frac{\partial}{\partial r} \left[ r \left( \frac{\mu}{Pr} + \frac{\mu_t}{Pr_t} \right) \right] \frac{\partial T}{\partial r} \tag{9}
\]

where \( Pr \) is the Prandtl number.

In reference to the turbulent Prandtl number, researches of Blom [36] confirmed that in the flow on a plate the turbulent Prandtl number is about \( Pr_t = 0.5 \), while for the boundary layer is about \( Pr_t = 0.9 \).

The convective term, which appears in the equation (9), was determined from the energy balance. Taking into account the steady heat flux acting on the unit pipe length and assuming that the temperature in main flow direction \((ox)\) varies linearly, the final form of the convective term can be described, as follows:

- Convective equation:

\[
\frac{\partial T}{\partial x} = \frac{2q}{\rho_b u_b (c_p)_b R^2} \tag{10}
\]

where \( q \) is an input power of heat per unit pipe length \((W/m)\), \( c_p \) is specific heat at constant pressure \((J/(kg \, K))\) and \( R \) is the pipe radius \((m)\), while subscript b - means cross sectional averaged value.

The final form of the mathematical model of non-isothermal medium slurry flow comprises four partial differential equations, namely momentum equation (2), temperature equation (9), equation for kinetic energy of turbulence (5), and its dissipation rate (6), together with the wall turbulence damping function (8), and complementary equations (3), (4), (7) and (10).

All constants in the turbulence model are the same as those proposed by Launder and Sharma [35] and are equal to: \( C_1 = 1.44; \ C_2 = 1.92; \ \alpha_1 = 1.0; \ \alpha_2 = 1.3, \ Pr_t = 0.9 \). The constant in the turbulence damping function was found as equal to \( A = 100 \).

The mathematical model assumes non-slip velocity at the pipe wall, i.e. \( U = 0 \), and \( k = 0, \ v = 0 \). Axially symmetrical conditions were applied at the pipe centre, therefore: \( dU/dr = 0, \ dT/dr = 0, \ dk/dr = 0 \) and \( d\varepsilon/dr = 0 \). Taking into account boundary conditions the equations set have been solved by finite difference scheme using own computer code. After determining boundary and convergence conditions the mathematical model was solved for 80 nodal points distributed on the radius of the pipe. The majority of the nodal points were localized in a close vicinity of the pipe wall to ensure the convergence process. The number of nodal points was set up experimentally to ensure nodally independent computations. Numerical simulations were performed for a priori set of \( dp/dx \).

The mathematical model is able to predict frictional head loss, velocity and temperature profiles, and Nusselt number in turbulent flow of slurries with averaged solid particle diameters from \( d_S = 0.10 \) mm to \( d_S = 0.80 \) mm, known as medium slurries.
3. Simulation results
The mathematical model of slurry flow with solid particles of averaged diameter from \( d_S = 0.10 \) to \( d_S = 0.80 \) mm, presented in chapter 2, was validated for isothermal turbulent flow only [23]. The predictions and measurements of slurry frictional head loss were matched for two different types of solid particles, named as Canasphere and Sand. Solid particle density of Canasphere and Sand were \( \rho_s = 2440 \) kg/m\(^3\) and \( \rho_s = 2650 \) kg/m\(^3\), respectively. The measurements and simulations were performed for solids concentration from 10\% to 40\% by volume. Measurements and simulations demonstrate that frictional head loss significantly depends on particle size and solids concentration [23]. Experiments and simulations confirmed that in slurry flow with averaged solid particles diameter about \( d_S = 0.50 \) mm the frictional head loss is equal or below that for carrier liquid flow even for solids concentration equal up to 30\% by volume. It is worth to emphasise that for averaged solid particle diameters, which are below or above \( d_S = 0.50 \) mm, the frictional head loss was higher than that for the carrier liquid flow, however, was still below values we expected.

The mathematical model, presented in chapter 2, was not validated for non-isothermal slurry flow, as there are not any available experimental data in literature. Limited validation has been made for carrier liquid flow only. Figure 1 presents comparison of simulated and empirically calculated Nusselt number expressed by equation (11) for the carrier liquid flow (water) in the range of Reynolds number from \( Re = 10,000 \) to \( Re = 140,000 \). Simulations confirmed that the relative error of the predicted Nusselt number vary from +2.5\% to -3.3\%.

\[
Nu = 0.023 \, Re^{0.8} \, Pr^{1/3}
\]  

(11)

**Figure 1.** Comparison of simulations with empirically predicted Nusselt number for carrier liquid flow (water): \( q = 200 \) W/m, \( T_w = 303.15 \) K, \( D = 0.0127 \) m.

All predictions presented in the paper have been made for slurries with averaged particle diameter equals to 0.240 mm. Simulations of heat exchange in turbulent flow of medium slurry indicate that Reynolds number and solids concentration have significant influence on the process. Figure 2 presents temperature profiles across the pipe radius in the carrier liquid and in the medium slurry flows at solids concentration from 10\% to 30\% by volume, where \( y \) - is the distance from the pipe wall. If solids concentration arises from 10\% to 30\% the temperature difference between the pipe wall and the centre of the pipe increases. If temperature difference increases the coefficient of heat transfer decreases because the heat flux per unit pipe length \( q \) and area perpendicular to the heat flux are constant. If solids concentration increases, the Nusselt number decreases as the coefficient of heat transfer decreases.
Figure 2. Temperature profiles in carrier liquid and slurry flows: $d_S=0.24$ mm; $\rho_S=2440$ kg/m$^3$; $C=0\%$, 10\%, 20\%, 30\%; $T_w=303.15$ K, $q=200$ W/m, $D=0.0127$ m.

Figure 3 presents influences of Reynolds number on the Nusselt number in the flow without solid particles ($C=0\%$) and in flow with solids concentration equals to 10\%, 20\% and 30\% by volume.

Figure 3. Influence of Reynolds number on the Nusselt number for different solids concentration $d_S=0.24$ mm; $\rho_S=2650$ kg/m$^3$; $C=0\%$, 10\%, 20\%, 30\%; $T_w=303.15$ K, $q=200$ W/m, $D=0.0127$ m.

Simulations made for turbulent flow of medium slurry emphasized substantial influence of Reynolds number and of solids concentration on the temperature profile and as a consequence on the Nusselt number.
4. Discussion

Simulations of heat exchange in turbulent flow of medium slurry requires mathematical model, which is suitable to predict increased damping of turbulence. The mathematical model presented in the paper has been successfully validated for isothermal slurry flow in a comprehensive range of solid particle diameters, solids concentration and Reynolds number [23]. The crucial point of the mathematical model is a properly designed turbulence damping function incorporated into the turbulence model, which has been described by equation (8).

The mathematical model of heat exchange in turbulent flow of medium slurry was not validated, as there are no such experimental data available in literature. Such experiments are extremely tough for performance. Measurements at high solids concentration of the dispersed phase are prevented because of the risk of damage or contamination of intrusive hot-film or hot wire probes. With optical methods, attenuation of the beams by particles occurs. As a result of these difficulties, most of the measurements in the literature concern gas-liquid or gas-solid flows.

There are evidences in literature that some slurries exhibit increased damping of turbulence. Slurries with averaged solid particle diameters in the range from $d_s=0.10$ mm to $d_s=0.80$ mm exhibit increased damping of turbulence; however, the highest damping of turbulence takes place if averaged solid particle diameter is about $d_s=0.50$ mm [17,18]. Schreck and Kleis [37] proved that swirls with sizes lower than solid particle diameter, drastically reduce particles shade causing that the level of turbulence decreases. Furthermore, there are evidences in literature that solids concentration decreases towards a pipe wall, which depends on diameter of solid particles [11,17-18, 33]. This could be mainly caused by lift forces, which act from the pipe wall towards symmetry axis. Such forces causes that the contact of solid particles with a pipe wall is less intensive. This could be responsible for thickening of viscous sublayer as larger solid particles are pushed away from the pipe wall and are replaced by very fine particles. Very fine particles are responsible for enhancement of viscous forces in vicinity of a pipe wall. If the viscous forces are increasing at the pipe wall the laminarisation of the flow takes place [38]. However, it should be noted that the complexity of turbulent slurry flow causes that many parameters influence the flow phenomena and some of them could act oppositely to each other. Therefore, slurry flow still belongs to main challenges of Computation Fluid Dynamics. In literature, there is still lack of a simple parameter, which can resolve whether there is an enhancement or attenuation of turbulence in slurry flow.

Numerical simulations presented in the paper, which have been made for slurry flow with averaged solid particle diameter equals to $d_s=0.240$ mm, demonstrate the influence of Reynolds number and volume fraction of solids on the heat exchange process. Both mentioned factors affect the velocity profile across the pipe and as a result the temperature profile too. Changes of temperature profile are due to changes in heat transfer coefficient, which affects the Nusselt number.

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