Dynamic structural and topological phase transitions on the Warsaw Stock Exchange: A phenomenological approach

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We study the crash dynamics of the Warsaw Stock Exchange (WSE) by using the Minimal Spanning Tree (MST) networks. We find the transition of the complex network during its evolution from a (hierarchical) power law MST network, representing the stable state of WSE before the recent worldwide financial crash, to a superstar-like (or superhub) MST network of the market decorated by a hierarchy of trees (being, perhaps, an unstable, intermediate market state). Subsequently, we observed a transition from this complex tree to the topology of the (hierarchical) power law MST network decorated by several star-like trees or hubs. This structure and topology represent, perhaps, the WSE after the worldwide financial crash, and could be considered to be an aftershock. Our results can serve

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as an empirical foundation for a future theory of dynamic structural and topological phase transitions on financial markets.

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1. Introduction

It is only in the past two decades that physicists have intensively studied the structural and/or topological properties of complex networks \cite{1} (and refs. therein). They have discovered that in most real graphs, small and finite loops are rare and insignificant. Hence, it was possible to assume their architectures to be locally dominated by trees. These properties have been extensively exploited. For instance, it is surprising how well this assumption works in the case of numerous loopy and clustered networks\cite{1}.

Therefore, we decided on the Minimal Spanning Tree (MST) technique as a particularly useful, canonical tool in graph theory \cite{2}, being a correlation based connected network without any loop \cite{3, 10} (and refs. therein). In the graph, the vertices (nodes) are the companies and the distances between them are obtained from the corresponding correlation coefficients. The required transformation of the correlation coefficients into distances was made according to the simple recipe \cite{3, 5}.

We consider the dynamics of an empirical complex network of companies, which were listed on the Warsaw Stock Exchange (WSE) for the entire duration of each period of time in question. In general, both the number of companies (vertices) and distances between them can vary in time. That is, in a given period of time these quantities are fixed but in other peri-

\footnote{1 Nevertheless, the stability problem of the networks versus their structure and topology should be studied.}
ods can be varied. Obviously, during the network evolution some of its edges may disappear, while others may emerge. Hence, neither the number of companies nor edges are conserved quantities. As a result, their characteristics, such as for instance, their mean length and mean occupation layer \[3, 6, 11–15\], are continuously varying over time as discussed below.

We applied the MST technique to find the transition of a complex network during its evolution from a hierarchical (power law) tree representing the stock market structure before the recent worldwide financial crash \[16\] to a superstar-like tree (superhub) decorated by the hierarchy of trees (hubs), representing the market structure during the period of the crash. Subsequently, we found the transition from this complex tree to the power law tree decorated by the hierarchy of local star-like trees or hubs (where the richest from these hubs could be a candidate for another superhub) representing the market structure and topology after the worldwide financial crash.

We foresee that our results, being complementary to others obtained earlier \[17–20\], can serve as a phenomenological foundation for the modeling of dynamic structural and topological phase transitions and critical phenomena on financial markets \[21\].

2. Results and discussion

The initial state (graph or complex network) of the WSE is shown in Figure 1 in the form of a hierarchical MST. This graph was calculated for \( N = 142 \) companies present on WSE for the period from 2005-01-03 to

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2 For the construction of the MST network, we here used Prim’s algorithm \[22\], which is quicker than Kruskal’s algorithm \[22\,23\], in particular for the number of companies \( N \gg 1 \). Both algorithms are often used.
Fig. 1. The hierarchical MST associated with the WSE (and consisting of $N = 142$ companies) for the period from 2005-01-03 to 2006-03-09, before the worldwide financial crash. The companies are indicated by the coloured circles (see the legend for an additional description). We focus on the financial company CAPITAL Partners (large red circle), as later it plays a central role in the MST, shown in Figure 2. When the link between two companies is in dark grey, the cross-correlation between them is greater, while the distance between them is shorter (cf. the corresponding scale incorporated there). However, the geometric distances between companies, shown in the Figure by the lengths of straight lines, are arbitrary, otherwise the tree would be much less readable.

2006-03-09, i.e. before the worldwide financial crash occurred [16].
We focus on the financial company CAPITAL Partners\textsuperscript{3}. It is a suburban company for the most of the period in question. However, it becomes a central company for the MST presented in Figure 2 for the period from 2007-06-01 to 2008-08-12, which covers the worldwide financial crash.

In other words, for this period of time, CAPITAL Partners is represented by a vertex which has a much larger number of edges (or it is of a much larger degree) than any other vertex (or company). This means that it becomes a dominant hub (superhub) or a giant component.

In the way described above, the transition between two structurally (or topologically) different states of the stock exchange is realized. We observed the transition from hierarchical (power law) MST (consisting of a hierarchy of local stars or hubs) to the superstar-like (or superhub) MST decorated by the hierarchy of trees (hubs).

In Figure 3 we compare discrete distributions of vertex degrees\textsuperscript{4}.

Although the distributions obtained are power laws, we cannot say that we are here dealing with a Barabási–Albert (BA) type of complex network with their rule of preferential linking of new vertices\textsuperscript{24}. This is because for both our trees, the power law exponents are distinctly smaller than 3 (indeed, the exponent equal to 3 characterizes the BA network), which is a typical observation for many real complex networks\textsuperscript{11}.

Remarkably, the rhs plot in Figure 3 makes it, perhaps, possible to consider the tree presented in Figure 2 as a power law MST decorated

\textsuperscript{3} The full name of this company is CAPITAL Partners. It was listed on the WSE from 20 October 2004. The main activities of the company are capital investment in various assets and investment advice.

\textsuperscript{4} The discrete distribution of the vertex degree is normalized by a factor equal to the total number of vertices $N$ fixed for a given period of time.
Fig. 2. The superstar-like graph (or superhub) of the MST (also consisting of \( N = 142 \) companies of the WSE) observed for the period from 2007-06-01 to 2008-08-12, which covers the worldwide financial crash. Now CAPITAL Partners becomes a dominant hub (or superhub). It is a temporal giant component, i.e. the central company of the WSE.

by a temporal dragon king\(^5\). This is because the single vertex (representing

\footnote{The equivalent terms ‘superextreme event’ and ‘dragon king’ stress that [25]: (i) we are dealing with an exceptional event which is completely different in comparison with the usual events; (ii) this event is significant, being distinctly outside the power law. For instance, in paper [26] the sustained and impetuous dragon kings were defined and discussed.}
Fig. 3. The comparison of power law discrete distributions $f(k)$ vs. $k$ (where $k$ is the vertex degree) for the hierarchical MST shown in Figure 1 and the superstar-like MST decorated by the hierarchy of trees shown in Figure 2. One can observe that for the latter MST there is a single vertex (rhs plot), which has a degree much larger (equalling 53) than any other vertex. Indeed, this vertex represents the company CAPITAL Partners, which seems to be a superextreme event or a dragon king [25–28], being a giant component of the MST network [1].

CAPITAL Partners) is located far from the straight line (in the log-log plot) and can be considered as a temporally outstanding, superextreme event or a temporal dragon king [25–28], which condenses the most of the edges (or links). Hence, the probability $f(k_{\text{max}}) = 0.007 = 1/142$, where $k_{\text{max}} = 53$ is the degree of the dragon king (which is the maximal degree here). We suggest that the appearance of such a dragon king could be a signature of a crash.\footnote{Obviously, this is a promising hypothesis which requires, however, a more systematic study.}

For completeness, the MST was constructed for $N = 274$ companies of
Fig. 4. The hierarchical graph of the MST decorated by several local star-like trees for the WSE for the period from 2008-07-01 to 2011-02-28, that is after the worldwide financial crash (cf. Figure 1). Apparently, CAPITAL company is no longer the central hub, but has again become a marginal company (vertex). When the link between two companies is in dark gray, the cross-correlation between them is greater, while the distance between them is shorter. However, the geometric distances between companies, shown in the figure by the length of the straight lines, are arbitrary, otherwise the tree would be much less readable.

the WSE for a third period of time, from 2008-07-01 to 2011-02-28, i.e. after the worldwide financial crash (cf. Figure 1). It is interesting that several new (even quite rich) hubs appeared while the single superhub (superstar)
disappeared (as it became a marginal vertex). This means that the structure and topology of the network strongly varies during its evolution over the market crash. This is also well confirmed by the plot in Figure 5, where several points (representing large hubs) are located above the power law. Apparently, this power law is defined by the slope equal to $-2.62 \pm 0.18$ and cannot be considered as a BA complex network. Rather, it is analogous to the internet, which is characterized by almost the same slope [29, 30].

It would be an interesting project to identify the actual local dynamics (perhaps nonlinear) of our network, which subsequently creates and then annihilates the temporal singularity (i.e. the temporal dragon king).

The considerations given above are confirmed in the plots shown in Figures 6 and 7. There well-defined absolute minima of the normalized length and mean occupation layer vs. time at the beginning of 2008 are clearly shown, respectively.

As usual [13, 15], the normalized length of the MST network simply means the average length of the edge directly connecting two vertices.

Apparently, this normalized length vs. time has an absolute minimum close to 1 at the beginning of 2008 (cf. Figure 6), while at other times much shallower (local) minimums are observed. This result indicates the existence of a more compact structure at the beginning of 2008 than at other times.

Furthermore, by applying the mean occupation layer defined, as usual [13, 15], by the mean number of subsequent edges connecting a given vertex of a tree with the central vertex (here CAPITAL Partners), we obtained quite similar results (cf. the solid curve in Figure 7). For comparison, the result based on the other central temporal hubs (having currently the largest

\footnote{Obviously, the edge between two vertices is taken into account only if any connection between them exists.}
Fig. 5. The power law discrete distribution $f(k)$ vs. $k$ (where $k$ is the vertex degree) for the MST shown in Figure 4. Six points (associated with several different companies) appeared above the power law. This means that several large hubs appeared instead of a single superhub. Apparently, the richest vertex has here the degree $k_{\text{max}} = 30$ and the corresponding probability $f(k_{\text{max}}) = 0.0036 = 1/274$. However, this vertex cannot be considered as a superextreme event (or dragon king) because it is not separated far enough from other vertices.

degrees) was also obtained (cf. the dotted curve in Figure 7). This approach is called the dynamic one. Fortunately, all the approaches used above give fully consistent results which, however, require some explanation.

In particular, both curves in Fig. 7 coincide in the period from 2007-06-01 to 2008-08-12 having common absolute minimum located at the beginning of 2008. To plot the dotted curve the company, which has the largest de-
Fig. 6. Normalized length of the MST vs. time (counted in trading days (td)). Apparently, the well-defined absolute minimum of the curve is located at the beginning of 2008. This localization (in the period from 2007-06-01 to 2008-08-12, covering the crash), together with the corresponding length so close to 1.0, confirm the existence of a network, which is significantly more compact than others.

grees, was chosen at each time as a temporal central hub. In general, such a company can be replaced from time to time by other company. However, for the period given above indeed the CAPITAL Partners has largest degrees (while other companies have smaller ones, of course). This significant observation is clearly confirmed by the behavior of the solid curve constructed at a fixed company assumed as a central hub, which herein it is the CAPITAL Partners. Just outside this period, the CAPITAL Partners is no more a central hub (becoming again the peripheral one) as other companies play then
Fig. 7. Mean occupation layer for the MST vs. time (counted in trading days, (td)), where CAPITAL Partners was assumed to be the central hub (the solid curve). For comparison, the result based on the central temporal hubs (having currently the largest degrees, the dotted curve) was obtained. Apparently, the well-defined absolute minimum, common for both curves, is located at the beginning of 2008 (in the period from 2007-06-01 to 2008-08-12).

his role, although not so spectacular. This results from the observation that the dotted curve in Fig. 7 is placed below the solid one outside the second period (i.e. from 2007-06-01 to 2008-08-12). Hence, we were forced to restrict the period on August 12, 2008 and do not consider other period such as between September 2008 and March 2009, where the most serious draw-
down during the worldwide financial crash 2007-2009 occurred\(^\text{8}\). Perhaps, some precursor of the crash is demonstrated herein by the unstable state of the WSE. Anyway, the subsequent work should contain a more detailed analysis of the third period.

The existence of the absolute minimum (shown in Figure 7) for CAPITAL Partners, and simultaneously the existence of the absolute minimum shown in Figures 6 in the first quarter of 2008 (to a satisfactory approximation) confirms the existence of the star-like structure (or a superhub), as a giant component of the MST, centered around CAPITAL Partners. We may suppose that the evolution from a marginal to the central company of the stock exchange and again to a marginal company, is stimulated perhaps by the most attractive financial products offered by this company to the market only in the second period of time (i.e. in the period from 2007-06-01 to 2008-08-12).

3. Concluding remarks

In this work, we have studied the empirical evolving connected correlated network associated with a small size stock exchange, the WSE. Our result seem somewhat embarrassing that such a marginal capitalization company as CAPITAL (less than one permil of a typical WIG20 company, like for instance KGHM) becomes a dominant hub (superhub) in the second period considered (see Fig. 2 for details).

Our work provides an empirical evidence that there is a dynamic structural and topological the first order phase transition in the time range dom-

\(^8\) However, our complementary calculations for the Frankfurt Stock Exchange support our approach. The role of central hub (or superhub) plays there the GITTARSALZ AG Stahl und Technologie company.
inated by a crash. Namely, before and after this range the superhub (or the unstable state of the WSE) disappears and we observe the power law MST and power law MST decorated by several hubs, respectively. Therefore, our results consistently confirm the existence of the dynamic structural and topological phase transitions, which can be roughly summarized as follows:

- phase of power law MST - a stable state
  ⇒ phase of the superstar-like or superhub MST decorated by hierarchy of trees or hubs - perhaps an unstable state
  ⇒ phase of power law MST decorated by several star-like trees or hubs
    (where the richest hub could be a candidate for another superhub)
    - perhaps a stable state.

We assume the hypothesis that the first transition can be considered as a signature of a stock exchange crash, while the second one can be understood to be an aftershock. Nevertheless, the second transition related to the third period requires a more detailed analysis. Indeed, in this period the PKOBP to much resemble a superhub (see Figs. 4 and 5 for details), which could play a role of other stable state of the WSE. In other words, our work indicates that we deal perhaps with indirect transition (the first order one) between two stable components, where the unstable component is surprisingly well seen among them.

One of the most significant observations contained in this work comes from plots in Figures 3 and 5. Namely, the exponents of all degree distributions are smaller than 3, which means that all variances of vertex degrees

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9 The best candidate for the superhub within the third period (from 2008-07-01 to 2011-02-28) could be, perhaps, PKOBP (see Figs. 4 and 5 for details), which is a richest vertex having degree equals 30.
diverge. This indicates that we are here dealing with criticality as the range
of fluctuations is compared with the size of the graph. This means that
the network evolution from 2005-01-03 to 2011-020-028 takes place within
the scaling region\textsuperscript{10} containing a critical point. Apparently, we are
here dealing with scale-free networks, which are ultrasmall worlds
\textsuperscript{11}.

It should be stressed that similar results we also obtained for Frankfurt Stock
Exchange\textsuperscript{12}.

We suppose that our results are complementary to those obtained earlier
by Drożdż and Kwapien\textsuperscript{13}. Their results focused on the slow (stable)
component (state). Namely, they constructed the MST network of 1000
highly capitalized American companies. The topology of this MST show
its centralization around the most important quite stable node being the
General Electric. This was found both in the frame of binary and weighted
MSTs.

Noteworthily, the fact should be stressed in this context that the discon-
tinuous phase transition (i.e. the first order phase one) evolves continuously
before the continuous phase transition (i.e. before the second order one).
This discontinuous phase transition goes over the unstable state involving,
perhaps, a superheating state such as the superhub in our case. This cannot
be considered as a noise\textsuperscript{13} in the system but rather should be considered
as a result of the natural evolution of the system until the critical point is

\textsuperscript{10} Note that the scaling region it is a region where both the first order and signatures
of the second order phase transitions are present together. We suppose herein that
this is, indeed, our case.

\textsuperscript{11} For the ultrasmall world the mean length between two vertices of a graph is propor-
tional to $\ln \ln N$ instead of that for the small world proportional only to $\ln N$.

\textsuperscript{12} In fact, we obtained results analogous to those presented in all our Figs. 1–7.

\textsuperscript{13} Indeed, the case of the noise was discussed in this context in paper\textsuperscript{37}. 
reached (cf. [20] and refs. therein, where the role of stable states (or slow components) on NYSE or NASDAQ was considered by using binary and weighted MSTs).

We suppose that the phenomenological theory of cooperative phenomena in networks proposed by Goltsev et al. [38] (based on the concepts of the Landau theory of continuous phase transitions) could be a promising first attempt.

An alternative view of our results could consider the superhub phase as a temporal condensate [1] (and refs. therein). Hence, we can reformulate the phase transitions mentioned above as representing the dynamic transition from the disordered phase into a temporal condensate, and then the transition from the condensate again to some disordered phase.

We hope that our work is a good starting point to find similar topological transitions at other markets. For instance, we also studied a medium size stock exchange, the Frankfurt Stock Exchange. Because the results obtained resemble very much those found for the Warsaw Stock Exchange, we omitted them here. Furthermore, the analytical treatment of the dynamics of such a network remains a challenge.

We can summarize this work with the conclusion that it could be promising to study in details the phase transitions considered above, which can define the empirical basis for understanding of stock market evolution as a whole.

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