Correction

Correction to: Coupled Self-Organized Hydrodynamics and Stokes Models for Suspensions of Active Particles

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Correction to: J. Math. Fluid Mech. (2019) 21:6
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This note provides a list of errata and their correction for Reference [1].

• There is a typo in Eq. (5.12): the term in ($\tilde{\lambda} - 1$) should come with a negative sign. This expression becomes:

$$
\bar{\Omega} = \left[ \frac{1}{2} \rho_0 \left( 2\tilde{\lambda}k_0 \bar{p} + \tilde{b}(\tilde{\lambda} + 1)(\rho_0 \bar{k} + \bar{\rho}k_0) \right) - \frac{i}{\kappa} \hat{\rho} \right] k^\perp.
$$

(0.1)

• In the statement of Th. 5.1 (Linear Stability Analysis), we have that Eq. (5.5) is not true. As a consequence equation (5.6) is slightly modified into

$$
D(\alpha, k) = \left\{ \frac{\tilde{b}\rho_0}{2|k|^2} \left[ \left( -4\tilde{\lambda} + \tilde{\lambda} + 1 \right) (-\alpha + U_0 \cdot k) 
\right. 
- c_1 k_0 \left( -2\tilde{\lambda} k_0^2 |k|^2 + \tilde{\lambda} + 1 \right) 
\left. + \frac{i}{\kappa} c_1 \right] (|k|^2 - k_0^2) 
\right. 
- (-\alpha + U_0 \cdot k) \left[ i(-\alpha + V_0 \cdot k) - \frac{(\tilde{\lambda} - 1)\tilde{b}\rho_0 k_0^2}{2|k|^2} + \frac{\gamma|k|^2}{2} \right],
$$

(0.2)

when $\bar{k} \neq 0$. And Eq. (5.7) is valid when $\bar{k} \neq 0$. When $\bar{k} = 0$ we have a different scenario where $\bar{\rho} = 0$ and $\bar{\Omega} \neq 0$ arbitrary and

$$
\alpha = V_0 \cdot k + i \left( \frac{(\tilde{\lambda} - 1)\tilde{b}\rho_0 k_0^2}{2|k|^2} - \frac{\gamma|k|^2}{2} \right),
$$

(0.3)

which gives only stable modes since $\tilde{\lambda} \in [-1, 1]$ and so $\text{Im}(\alpha) \leq 0$. (Finally, notice that, the parameter $\eta$ in the original article does not play a role any more in this corrected version.)

The original article can be found online at https://doi.org/10.1007/s00021-019-0406-9.
To obtain these results we modify the proof of Th. 5.1 Case B, which we rewrite fully next (notice that the number of the equations that do not start by 0 refer to the number of the equations as they appear in the original article).

**Proof. Case (B)** Suppose that $k^\perp \neq 0$. Doing the inner product of Eq. (0.1) with $k$ and using that $k^\perp \cdot k = |k|^2 - k_0^2$, the dispersion relation is given by (using Eq. (5.11)):

\[
\begin{align*}
\tilde{b} \frac{\rho_0}{2|k|^2} & \left( \rho_0 \bar{k} \left( -4\bar{\lambda} \frac{k_0^2}{|k|^2} + \bar{\lambda} + 1 \right) + \rho_0 k_0 \left( -\frac{2\lambda k_0^2}{|k|^2} + \bar{\lambda} + 1 \right) \right) - \frac{i}{\kappa} \bar{\rho} (|k|^2 - k_0^2) \\
= & \left[ i\rho_0 (-\alpha + V_0 \cdot k) - (\bar{\lambda} - 1) \frac{\tilde{b} \rho_0}{2} \frac{k_0^2}{|k|^2} + \gamma |k|^2 \rho_0 \right] \bar{k},
\end{align*}
\]

and from Eq. (5.10b) we have the relation

\[
(-\alpha + U_0 \cdot k) \bar{\rho} + \rho_0 c_1 \bar{k} = 0.
\]

Next we distinguish between the cases $\bar{k} \neq 0$ and $\bar{k} = 0$. Suppose that $\bar{k} \neq 0$, then from Eq. (0.5) we have that $-\alpha + U_0 \cdot k \neq 0$ and multiplying Eq. (0.4) by $-\alpha + U_0 \cdot k \neq 0$ and using Eq. (0.5), we get the dispersion relation in Eq. (0.2), after simplifying $\bar{k}$.

If, on the contrary, $\bar{k} = 0$, then one can check that $k^\perp \cdot \bar{\Omega} = 0$ (thanks to Eq. (5.10a)) and, therefore, they are normal. Since by assumption $k^\perp \neq 0$ from Eq. (0.1) we conclude that the coefficient in front of $k^\perp$ on the right hand side must be zero. Rewriting this term using that $\bar{k} = 0$ and Eq. (5.11) we have that

\[
\tilde{b} I = 0,
\]

with

\[
I := \frac{\tilde{b} \rho_0}{2|k|^2} k_0 \left( -\frac{2\lambda k_0^2}{|k|^2} + \bar{\lambda} + 1 \right) - \frac{i}{\kappa}.
\]

From here we deduce that $\bar{\rho} = 0$ because otherwise we should have that $I = 0$ but the imaginary part of $I$ is non-zero, so $\bar{\rho} = 0$. In particular this implies that Eq. (0.5) is fulfilled and that $\bar{\Omega} \neq 0$ (otherwise we would have null perturbation). Since $\bar{\Omega} \neq 0$, from Eq. (0.1) again (remembering that $k^\perp \perp \bar{\Omega}$) we must have that the coefficient in front of $\bar{\Omega}$ is equal to zero. This gives the dispersion relation (0.3).

Now, we go back to the case $\bar{k} \neq 0$. To simplify the analysis we will restrict ourselves to the case where $k^\perp = k$, i.e. $k_0 = k \cdot \bar{\Omega}_0 = 0$ and $\bar{k} \neq 0$. This implies, in particular, that $U_0 \cdot k = V_0 \cdot k = v_0 \cdot k$. With these considerations one can simplify the dispersion relation (0.2) into

\[
\tilde{D}(\alpha, k) = 0,
\]

where $\tilde{D}(\alpha, k)$ is given in Eq. (5.7).

- There are two typos at the end of page 21: in the third line of Case (A) part b) should read $\text{Im}(\alpha)$ instead of $\text{Im}(\omega)$; and in the fifth line should read “$\bar{\Omega}$ is arbitrary with $\bar{\Omega} \cdot \bar{\Omega}_0 = 0$” rather than “$\bar{\Omega}$ is arbitrary with $\bar{\Omega}, \bar{\Omega}_0 \neq 0$”.

**Compliance with ethical standards**

**Conflict of interest** The authors declare they have no conflict of interest.

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Reference

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