Research Article

Action Strategy Analysis in Probabilistic Preference Movement-Based Three-Way Decision

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The trisecting-acting-outcome model is a methodology of “thinking in threes,” which is the main idea of the three-way decision (3WD). It consists of three components: trisecting, acting, and outcome evaluation. A strategy selection method in a movement-based three-way decision (M-3WD) has been proposed in previous work. However, conflicting information widely existing in the information system has not yet been given sufficient consideration. The conflicting information brings massive noisy strategies when mining action strategies in three regions. This paper proposed a novel three-way decision model for action strategy set, which can analyze and classify strategies by introducing credibility and coverage. The model can remove noisy strategies and choose strategies more suitable for the need of decision makers. To evaluate and select an optimal action strategy, we analyze the probabilistic preference in a movement-based three-way decision. The approach determines the probability of movement by using the evidence theory (D-S) theory. The optimal action strategy is selected by analyzing the difference between the ideal movement and the actual movement, the lower the difference, the better the strategy. We give an example of medical decision-making to illustrate the effectiveness of the proposed method.

1. Introduction

Three-way decision [1] is an effective way of solving complex problems, whose core idea can be formulated as a three-step process within trisecting-acting-outcome (TAO) [2]. The trisecting is to divide a whole into three regions that are disjoint or weakly joint. The acting is to devise action strategies to process objects in the three regions. The outcome evaluation is to evaluate the effectiveness of the action strategy. The movement-based three-way decision [3] aims to mine actionable rules in three regions and transfer objects from the unfavorable region to the favorable region. We use an example of medical-making to illustrate the main ideas of the TAO model. In medical decision-making, one typically divides a set of suspected patients into three groups: suspected patients who have the disease, suspected patients who do not have the disease, and suspected patients who still need a further examination. According to the diagnosis result, the doctor may take some actions to turn the suspected patients who have the disease into disease-free, check the suspected patients who still need a further examination whether having the disease, and retain the suspected patients who do not have the disease. At last, the doctor develops new treatment protocols by evaluating the effects of treatment and medical examination.

Since the three-way decision was putting forward, it has expanded from a special three-way decision to a general three-way decision [4]. There are many studies on theories, such as three-way classification [5, 6], three-way clustering [7–12], three-way concept analysis [13, 14], three-way conflict analysis [15, 16], three-way game theory [17, 18], three-way support systems [19], three-way granular computing [20, 21], and three-way multiple attribute decision [22, 23]. Yao [2, 24] systematically studied the main ideas of three-way decision and the granular computing model based on three-way decision. Qi and Wei [25] combined formal
concept analysis with three-way decision and put forward three-way concept analysis theory. Yu [26] introduced three-way decision into clustering analysis and proposed three-way clustering approach. Wang and Yao [27] applied contraction and expansion in mathematical morphology to the three-way decision and proposed a three-way clustering algorithm based on mathematical morphology. In terms of application, three-way decision shows its unique superiority in image recognition [28, 29], mail filtering [30], medical decision-making [19], stream computing [31], recommendation system [32], and cloud computing [33].

Researchers have long focused on constructing a tri-section from a whole [5, 6, 34]. Research on acting and outcome evaluation is still in its infancy. Gao and Yao [3] proposed a movement-based three-way decision model by mining actionable rules in three regions. Jiang and Yao [35, 36] proposed three-way decision models for quantity movement and probability movement. The corresponding outcome evaluation methods are also proposed.

In real life, the database usually stores a lot of inconsistent information [37], which is caused by missing values, duplicate, and errors. It is difficult to avoid even after many rounds of data processing. Inconsistent information refers to the data in an information system, whose condition attributes are the same but derive different decision results. In a movement-based three-way decision, the decision maker usually has a particular preference for different regions. The preference relationship will prompt the decision maker to construct action strategies in three regions. Objects will be moved from the unfavorable region to the favorable region according to the preference of decision-makers. The existence of conflicting information makes it inevitable to mine many inconsistent rules. The action strategy generated by inconsistent rules may induce multiple different decision results so that the movement-based three-way decision usually exhibits the characteristics of probability movement [35]. Determining the probability of movement and selecting an optimal action strategy are urgent problems that need to be solved.

This paper constructs a three-way decision model for strategy set by introducing the concepts of credibility and coverage [38]. The model can remove noisy strategies in the strategy set. To evaluate and select an optimal action strategy, we analyze the probabilistic preference in a movement-based three-way decision. The D-S theory determines the probability of movement. The optimal action strategy can be selected by analyzing the difference between the ideal movement and the actual movement. The actual movement. Section 5 illustrates the effectiveness of the proposed method with an example of medical decision-making. Section 6 gives a summary and planning for future work.

2. Preliminaries

In this section, we review the TAO model of three-way decision and the movement-based three-way decision.

2.1. TAO of Three-Way Decision. “Thinking in threes” [2, 39] is a kind of granular computing thought consistent with human cognition. After the summary and refinement by Yao [1, 24, 40], the three-way decision theory is formed. The TAO model of the three-way decision is given in Figure 1. The trisecting function, denoted by solid lines with arrows, is to divide a whole into three pairwise disjoint or weakly joint regions $P_1$, $P_2$, and $P_3$. The function, denoted by dashed lines, is mining actionable rules in three regions. The function of acting, denoted by dashed lines with arrows, is to devise action strategies for three regions. The three regions before acting are represented by $P_1$, $P_2$, and $P_3$. The new three regions after acting are represented by $P_1'$, $P_2'$, and $P_3'$. The outcome evaluation is to measure the effect of the trisection and action strategy.

The most fundamental issue of the three-way decision is how to construct a trisection from a whole with reasonable interpretations. Researchers have constructed three regions based on rough sets [40–43], fuzzy sets [34], shadowed sets [44, 45], intuitionistic fuzzy sets [46, 47], vague sets [48] and soft sets [49]. A three-way decision model with an ordered relationship is defined as follows.

**Definition 1.** Suppose that OB is a finite nonempty set of objects. $E: OB \rightarrow (L, <)$ is an evaluation function on set OB. For $x \in OB$, $E(x)$ is an evaluation function value of $x$. Given a pair of thresholds $(\alpha, \beta) \in V \times V$ with $\beta \leq \alpha$, we trisect OB into three pairwise disjoint regions:

$$P_1 = \{ x \in OB | E(x) \geq \alpha \},$$
$$P_2 = \{ x \in OB | \beta < E(x) < \alpha \},$$
$$P_3 = \{ x \in OB | E(x) \leq \beta \}.$$  

The three regions satisfy the following two conditions:

1. $P_1 \cup P_2 \cup P_3 = OB$
2. $P_1 \cap P_2 = \emptyset, P_1 \cap P_3 = \emptyset, P_2 \cap P_3 = \emptyset$

The $P_1$ region consists of objects with evaluation function value $E(x)$ greater than or equal to $\alpha$. The $P_3$ region consists of objects with evaluation function value $E(x)$ less than or equal to $\beta$. The $P_2$ region consists of objects with evaluation function value $E(x)$ between the two thresholds.

The acting aims to design an appropriate action strategy to process three regions, according to trisection. An efficient action strategy can make decision maker increase benefits or reduce costs. The decision maker hopes to adopt an appropriate action strategy to move the object from unfavorable to the favorable region.
The outcome evaluation is to evaluate the effect of trisecting and acting. We can construct an evaluation function to evaluate the quality of trisecting:

$$Q(\pi_{(a,b)}) = w_{p_1}Q(P_1) + w_{p_2}Q(P_2) + w_{p_3}Q(P_3),$$

where $Q(\pi_{(a,b)})$ denotes the quality of trisection by a pair of threshold $(\alpha,\beta)$, $Q(P_i)$, $i = 1, 2, 3$, denotes the qualities or utilities of each region, and $w_{p_i}$, $i = 1, 2, 3$, denotes the weights of each regions. For acting, we can construct an evaluation function to evaluate the effect of action strategy:

$$Q(\pi' | \pi) = Q(\pi') - Q(\pi),$$

where $Q(\pi)$ and $Q(\pi')$ denote the qualities or utilities of original three regions and new three regions.

### 2.2. Movement-Based Three-Way Decision

Gao and Yao [3] proposed a movement-based three-way decision by introducing actionable rules into the three-way decision. The actionable rules first proposed by Ras [50, 51]. It means that a user can mine actionable rules and moving objects to generate benefits. The movement-based three-way decision aims to mine action strategy in three regions and move objects from unfavorable regions to the favorable region.

**Definition 2** (see [3]). Suppose that $[x]$ and $[y]$ are equivalence classes in different regions. We can get two decision rules:

$$r_{[x]}: \left[ \land_{s \in A_x} s = f_s(x) \right] \land \left[ \land_{f \in A_f} f = f_f(x) \right] \Rightarrow d = f_d(x),$$

$$r_{[y]}: \left[ \land_{s \in A_y} s = f_s(y) \right] \land \left[ \land_{f \in A_f} f = f_f(y) \right] \Rightarrow d = f_d(y),$$

where $r_{[\cdot]} \in \{x, y\}$, is decision rule, $A_x$ is a set of stable attributes, $f_s(\cdot)$ is the value of attribute $s$, $A_f$ is a set of flexible attributes, $f_f(\cdot)$ is the value of attribute $f$, and $f_d(\cdot)$ is the value of decision attribute $d$.

### 3. Three-Way Decision for Action Strategy Set

This section illustrates conflicting information through an example of medical decision-making and constructs the corresponding three-way decision model.

#### 3.1. A Motivational Example

In real life, an information system often has much inconsistent information. The inconsistent information leads to a large number of inconsistent rules when mining actionable rules. It is not helpful to the decision maker and may even mislead them into choosing the wrong action strategy. The following example of medical decision-making illustrates conflicting information and the impact on the movement-based three-way decision.

Table 1 is a medical-decision information table. There are 20 suspected patients and six symptoms or attributes. All condition attributes had been discretized. The values of some attributes are grouped and reassigned as follows. Age is categorized into three groups, i.e., 0–20, 20–60, and 60+; they are reassigned to value 1 to 3, respectively. Sex is categorized as female and male; they are reassigned to value 1 and 0. Cholesterol is categorized into three groups, i.e., 0–199, 200–239, and 240+; they are reassigned to value 1 to 3, respectively. Blood sugar is categorized into three groups, i.e., 0–89, 90–139, and 140+; they are reassigned to value 1 to 3, respectively. Blood pressure is categorized into three groups, i.e., 0–3.9, 4.0–7.8, and 7.9+; they are reassigned to value 1 to 3, respectively. Blood pressure, blood pressure, and blood sugar are abbreviated as chol, ph, and bs. Age and sex are stable attributes, and others are flexible attributes. The symbol “+” stands for suspected patients who have the disease. The symbol “-” stands for suspected patients who have not the disease. The symbol “?” stands for suspected patients who need a further examination.

All objects are divided into following 8 equivalence classes based on their condition attributes:
Doctors divide suspected patients into three regions $P_1$, $P_2$, and $P_3$ based on the diagnosis result:

$$P_1 = \{x_1, x_5, x_6, x_7, x_9, x_{11}, x_{12}, x_{13}, x_{17}, x_{18}, x_{19}\},$$

$$P_2 = \{x_{11}, x_{12}, x_8, x_{14}, x_{15}, x_{16}, x_{20}\},$$

$$P_3 = \{x_{14}, x_{10}\}.\quad (7)$$

Taking $[x_3]$ as an example, according to equation (4), we can construct the decision rules, that is,

$$r_{[x_3]}: \text{age} = 3 \land \text{sex} = 1 \land \text{cholesterol} = 2 \land \text{blood pressure} = 3 \land \text{blood sugar} = 2 \Rightarrow \text{result} = +,$$

$$r_{[x_3]}: \text{age} = 3 \land \text{sex} = 1 \land \text{cholesterol} = 2 \land \text{blood pressure} = 3 \land \text{blood sugar} = 2 \Rightarrow \text{result} = -,$$

$$r_{[x_3]}: \text{age} = 3 \land \text{sex} = 1 \land \text{cholesterol} = 2 \land \text{blood pressure} = 3 \land \text{blood sugar} = 2 \Rightarrow \text{result} = ?.\quad (8)$$

According to equation (5), we can mine some action strategies, that is,

$$r_{[x_8]} \leadsto r_{[x_3]},$$

$$r_{[x_8]} \leadsto r_{[x_3]},$$

$$r_{[x_8]} \leadsto r_{[x_3]},$$

$$r_{[x_8]} \leadsto r_{[x_3]}\quad (9)$$

The strategy $r_{[x_8]} \leadsto r_{[x_3]}$ means moving objects from $[x_8]$ to $[x_3]$ according to equation (5). However, there are diseased, disease-free, and uncertain objects in $r_{[x_3]}$. For the

| Age | Sex | Cholesterol | Blood pressure | Blood sugar | Result |
|-----|-----|--------------|----------------|-------------|--------|
| 3   | 1   | 2            | 1              | 1           | +      |
| 3   | 1   | 2            | 1              | 1           | +      |
| 2   | 1   | 3            | 2              | 2           | -      |
| 3   | 0   | 2            | 3              | 2           | -      |
| 2   | 1   | 2            | 2              | 2           | -      |
| 3   | 0   | 1            | 2              | 3           | +      |
| 3   | 0   | 2            | 3              | 2           | -      |
| 2   | 0   | 3            | 3              | 3           | ?      |
| 3   | 0   | 2            | 2              | 2           | -      |
| 3   | 0   | 1            | 2              | 2           | -      |
| 3   | 0   | 1            | 1              | 1           | +      |
| 3   | 0   | 2            | 3              | 2           | +      |
| 3   | 0   | 2            | 3              | 2           | -      |
| 2   | 1   | 3            | 2              | 2           | -      |
| 3   | 0   | 1            | 2              | 3           | -      |
| 2   | 0   | 3            | 3              | 3           | +      |

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decision maker, the outcome of movement is difficult to determine. There are different probabilities of movement:

\[
\begin{align*}
    r_{[x]_1} \Rightarrow r_{[x]_2} : & \text{ chol: } 1 \Rightarrow 2 \land \text{bp: } 1 \Rightarrow 3 \Rightarrow \text{bs: } 1 \Rightarrow 3 \\
    \begin{cases}
        +, & = p_1, \\
        -, & = p_2, \\
        ?, & = p_3,
    \end{cases}
\end{align*}
\]

subject to: \( \text{age} = 3 \land \text{sex} = 0. \)

where \( p_1, p_2, \) and \( p_3 \) represent the proportion or probability of moving disease-free, diseased, and uncertain regions, respectively.

The existence of inconsistent information makes a lot of noisy strategies when mining action strategies, such as \( r_{[x]_1} \Rightarrow r_{[x]_2} \), and \( r_{[x]_1} \Rightarrow r_{[x]_2} \). Therefore, it is necessary to consider conflicting information and analyze strategy when mining actionable rules in three regions.

3.2. Construct Three-Way Decision Model. The actionable rules affected by inconsistent information can generate a lot of noisy strategies. These noisy strategies are not helpful to the decision maker and may cause them to make wrong judgments. Therefore, we introduce credibility and coverage concepts to construct a three-way decision model for the action strategy set.

**Definition 4.** (see [38]). Suppose that \( \text{IS} = \{U, C \cup D, V, f\} \) is an information system, and \( S \) is an action strategy mined in \( \text{IS} \) according to the target equivalence class \([x]\). The action strategy has a certain credibility. The definition is as follows:

\[
\text{Credibility}(S) = \frac{|[x] \cap P_j|}{|x|},
\]

where \([x]\) is the equivalence class divided by \( U / \text{ind}(C), i = 1, 2, \ldots, m \), and \( P_j, j = 1, 2, 3 \), is the three regions divided by evaluation function.

The strategy is certain when \( \text{Credibility}(S) = 1 \). The strategy is uncertain when \( 0 < \text{Credibility}(S) < 1 \). The credibility denotes the conditional probability of target region \( P_j \) to the target equivalence class \([x]\). When the credit is less than 1, the same condition attributes are divided into different regions. So, credibility reflects the uncertainty of strategies.

The credibility of strategy \( S \) only considers the proportion of the equivalence class \([x]\) in the target region. However, it is not enough to consider credibility in an inconsistent information system. It is necessary to consider the coverage of the strategy in the target region: how many objects induce the strategy.

**Definition 5.** (see [38]). Suppose that \( \text{IS} = \{U, C \cup D, V, f\} \) is an information system, and \( S \) is an action strategy mined in \( \text{IS} \) according to the target equivalence class \([x]\). The action strategy has a certain coverage. The definition is as follows:

\[
\text{Coverage}(S) = \frac{|[x] \cap P_j|}{|P_j|},
\]

where \([x]\) is the equivalence class divided by \( U / \text{ind}(C), i = 1, 2, \ldots, m \), and \( P_j, j = 1, 2, 3 \), is the three regions divided by evaluation function.

The coverage describes the proportion of objects meeting the strategy constraints in the target region. If the strategy’s coverage is small, the objects that derive the strategy occupy a small part of the region \( P_j \). Therefore, the causality of this strategy lacks sufficient data to support it.

Therefore, it is necessary to consider both the strategy’s credibility and coverage when selecting a strategy. Based on the above description, combined with the trilevel thinking proposed by Yao [52], we constructed a novel three-way decision model for the strategy set. The model is given in Figure 2.

Given a strategy set \( AS = \{S_1, S_2, \ldots, S_n\} \). The strategy set is divided into a top-down trilevel granularity structure according to the credibility threshold \( \text{cre} \) and the coverage threshold \( \text{cov} \). The strategy set is gradual granulation from top to down.

**The first level:**

\[
\text{AS} = \{S_1, S_2, \ldots, S_n\}.
\]

**The second level:**

\[
\text{AS/cre} = \{\{S_1, S_2, \ldots, S_m\}, \{S_{m+1}, S_{m+2}, \ldots, S_n\}\}.
\]

**The third level:**

\[
\text{AS/cre/\text{cov}} = \left\{\{S_1, \ldots, S_m\}, \{S_{m+1}, \ldots, S_m\}\right\} \cup \left\{\{S_{m+1}, \ldots, S_n\}, \{S_{n+1}, \ldots, S_n\}\right\}.
\]
The model of three-way decision for strategy set is as follows:

\[
\begin{align*}
    \text{POS}(S) &= \{s_i \in S \mid \text{credibility}(s_i) \geq \text{cre} \land \text{coverage}(s_i) \geq \text{cov} \}, \\
    \text{NEG}(S) &= \{s_i \in S \mid \text{credibility}(s_i) \leq \text{cre} \land \text{coverage}(s_i) \leq \text{cov} \}, \\
    \text{BND}(S) &= (\text{POS}(S) \cup \text{NEG}(S))^C \\
    &= \{s_i \in S \mid \text{credibility}(s_i) > \text{cre} \land \text{coverage}(s_i) < \text{cov} \} \cup \\
    &\quad \{s_i \in S \mid \text{credibility}(s_i) < \text{cre} \land \text{coverage}(s_i) > \text{cov} \}.
\end{align*}
\]

(16)

The model introduces the concepts of credibility and coverage. In the first level, the strategy set does not introduce any concept. In the second level, the strategy set is divided into two subsets \( AS_1 \) and \( AS_2 \) through the credibility threshold \( \text{cre} \). In the third level, the subsets \( AS_1 \) and \( AS_2 \) are classified again by the coverage threshold \( \text{cov} \), which are above the threshold \( \text{cov} \) and below the threshold \( \text{cov} \). Finally, the strategies whose credibility and coverage are above the threshold \( \text{cre} \) and \( \text{cov} \) are divided into POS region; the strategies whose accuracy and coverage are lower than the threshold \( \text{cre} \) and \( \text{cov} \) are divided into NEG region, and the remaining strategies are divided into BND region. The decision maker will give priority to the strategy in the POS region when selecting a strategy. The model enables the decision maker to choose a strategy with high credibility and coverage as much as possible, which can effectively avoid the influence of conflicting information.

At last, we summarize the key steps to construct the three-way decision for the strategy set. The approach is outlined in Algorithm 1.

4. Optimal Action Strategy Selection in M-3WD

This section analyzed the probabilistic preference in three regions and proposed an evidence theory-based approach to determine the probability of movement. The determination of the movement’s probability is the basis for selecting the optimal action strategy. The optimal strategy is selected by comparing the difference between the ideal movement and the actual movement.

4.1. Probabilistic Preference in M-3WD. The model of the three-way decision for the strategy set can effectively avoid the influence of conflicting information by mining action strategy with high credibility and coverage. However, there may be many action strategies with high credibility and coverage. Therefore, we need to determine the probability of moving to three regions under different action strategies, which will help us select an optimal action strategy.

In a movement-based three-way decision, the decision maker has different preferences for different regions. For example, doctors want patients to move from a diseased region to a disease-free region in medical decision-making. According to the preference of decision makers, there are two types of movement. If the object’s movement matches the preference of decision makers, we call it a favorable movement. If the object’s movement does not match decision makers’ preference, we call it an unfavorable movement. In a movement-based three-way decision, we first divide a set of objects into three different regions according to decision makers’ needs. Then, we can mine action strategies between different regions.

Definition 6. Suppose that \( \pi = \{P_1, P_2, P_3\} \) is a tripartition regions. We can mine action strategies and construct corresponding strategy sets:

\[
    \text{AS} = (S_1, S_2, \ldots, S_n),
\]

where AS denotes the set of all strategies for objects in \( P_i \) region to move to \( P_j \) region. Objects in three regions may have nine movements:

\[
\begin{align*}
    &P_1 \rightarrow P_1', P_1 \rightarrow P_2', P_1 \rightarrow P_3', \\
    &P_2 \rightarrow P_1', P_2 \rightarrow P_2', P_2 \rightarrow P_3', \\
    &P_3 \rightarrow P_1', P_3 \rightarrow P_2', P_3 \rightarrow P_3',
\end{align*}
\]

(18)

where \( P_i \rightarrow P_j', i, j = 1, 2, 3 \) denotes the object in \( P_i \) region move to \( P_j \) region and \( P_i' \) and \( P_j' \) denote the region before and after the movement. In movement-based three-way decision, each kind of movement will generate the corresponding strategy set AS.

Due to the inconsistent information in the information system, the object moves from the unfavorable region to the favorable region with probability. According to the preference of decision maker, the probabilistic preference [53] movement matrix can be constructed as follows:

\[
\begin{align*}
    &P_{11}, P_{12}, P_{13}, \\
    &P_{21}, P_{22}, P_{23}, \\
    &P_{31}, P_{32}, P_{33},
\end{align*}
\]

(19)

where \( p_{ij} \) denotes the probability of object’s movement from \( P_i \) region to \( P_j \) region.

How to determine the probability of movement is an urgent problem to be solved. Considering the advantages of evidence theory in information fusion, we use the credibility of the equivalence class divided under a particular flexible attribute as the mass function of evidence theory. The
Input: IDIS = (U, C ∪ D, V, f), evaluation function E(x), a pair of thresholds (α, β), cre, and cov.
Output: The three-way decision for strategy set.

(1) Divide equivalence classes according to condition attributes C.
(2) for x ∈ U do
(3) Calculate the evaluation function E(x).
(4) if E(x) ≥ α then
(5) Divide x into P₁ region.
(6) end
(7) else if E(x) ≤ β then
(8) Divide x into P₂ region.
(9) end
(10) else
(11) Divide x into P₃ region.
(12) end
(13) Choose equivalence classes [x] that need to be moved.
(14) Mining action strategies in P₁, P₂, and P₃ to generate corresponding strategy set AS.
(15) end
(16) for S ∈ AS do
(17) if credibility(S) ≥ cre ∧ coverage(S) ≥ cov then
(18) Divide S into POS region.
(19) end
(20) else if credibility(S) < cre ∧ coverage(S) < cov then
(21) Divide S into NEG region.
(22) end
(23) else
(24) Divide S into BND region.
(25) end
(26) end

Algorithm 1: Three-way decision for the action strategy set.

The probability of movement can be obtained by fusing multiple mass function under flexible attributes.

Evidence theory, also known as D-S evidence theory, is an uncertainty reasoning theory first proposed by Dempster. Shafer further researched and developed it. Evidence theory has been widely used in many fields such as information fusion [54], medical diagnosis [55], and uncertainty decision-making [56]. Suppose that m: 2ᵦ → [0, 1] is a function on the recognition framework Θ. The function satisfies the following two conditions:

\[
\begin{align}
(1) \quad m(\emptyset) &= 0, \\
(2) \quad \sum_{A \subseteq \Theta} m(A) &= 1,
\end{align}
\]

where the subset A denotes the focal elements, which meet the \( m(A) > 0 \). The value of \( m(A) \) denotes the belief that supports the occurrence of A. The function is called a mass function or a basic credibility distribution function.

The synthesis rule [57] is the core of evidence theory. If there is multiple evidence under the recognition framework, different mass functions will be derived. The evidence synthesis formula synthesizes multiple mass functions to obtain a fused mass function. The fused mass function is used for decision-making, which can improve accuracy. Given the recognition framework Θ, for \( \forall A \subseteq \Theta \), two mass functions under the same recognition frame Θ obtained from different evidences are \( m₁ \) and \( m₂ \). The synthesis formula of evidence \( m₁ \) and \( m₂ \) is as follows:

\[
(m₁ \oplus m₂)(A) = \frac{1}{1 - K} \sum_{B \cap C = \emptyset} m₁(B)m₂(C), K = \sum_{B \cap C = \emptyset} m₁(B)m₂(C),
\]

where \( K \) is the confidence assigned to the null value. In reality, the null value has no confidence, that is, \( m(\emptyset) = 0 \). When the two pieces of evidence are combined, the constraint condition \( K < 1 \) must be satisfied. If \( K = 1 \), the evidence is completely contradictory, and there is no \( m₁ \oplus m₂ \) at this time.

We use the credibility \( cre_a \) of the equivalence class \([x]\)
under a certain flexible attribute \( a \in C \) as the mass function, that is, \( m(P_j/[x]_a) \). The meaning of credibility is the proportion of equivalence classes divided by the condition attribute \( a \in C \) in the target region \( P_j \). The larger the ratio, the higher the
reliability. The mass function needs to satisfy two constraints in equation (20). The corresponding proof is as follows.

**Proof.** The mass function needs to satisfy two constraints, namely, \( m(\phi) = 0 \) and \( \sum_{A \in \mathcal{A}} m(A) = 1 \). According to the definition of credibility,

\[
m(\phi/[x]_a) = \frac{[x]_a \cap \phi}{[x]_a} = 0,
\]

\[
\sum_{j=1}^{3} m \left( \frac{P_j}{[x]_a} \right) = \sum_{j=1}^{3} \frac{[x]_a \cap P_j}{[x]_a} = \frac{[x]_a \cap (P_1 \cup P_2 \cup P_3)}{[x]_a} = 1.
\]

(22)

The credibility satisfies the above two constraints.

The credibility uses the value of each attribute and the decision result, that is, it makes full use of all the information in the information system. Therefore, the movement probability determined by evidence theory is more accurate.

The importance of different attributes to the decision maker is different. Considering the need for accuracy, the decision maker needs to consider the importance of different attributes when synthesizing the approximate movement probability. The attribute importance is taken as the weight factor, the greater the attribute’s importance, the greater the weight of the mass function. The weight of the mass function under different attributes is represented by \( w_a \).

Suppose that \( IS = (U, C \cup D, V, f) \) is an information system, action strategies \( S \) derived in IS, the weight \( w_a \) of attribute \( a \), and the mass function \( m(P_j/[x]_a) \). The model of approximate movement probability assignment in three regions is as follows:

\[
\begin{cases}
p_j \neq \phi, & \frac{P_j}{[x]} = \phi, \\
\sum_{1 \leq i \leq m} \prod_{a \in C} w_a m(P_j/[x]_a) \prod_{a \in C} w_a & \prod_{a \in C} w_a m(P_j/[x]_a),
\end{cases}
\]

(23)

The preferences of movement widely exist in the movement-based three-way decision. The preferences are often expressed in the form of probability. We analyze the probabilistic preference in the movement-based three-way decision. The evidence theory-based method is proposed to determine the probability of movement. The model uses all the information in the system and considers the importance of different attributes to improve the fusion results’ reliability and objectivity.

### 4.2. Optimal Action Strategy Selection

The movement-based three-way decision model aims to move objects from the unfavorable region to the favorable region. The movement of objects brings benefits and costs. We can select the optimal action strategy by analyzing the benefits and costs. However, the benefits and costs are difficult to determine. To select an optimal action strategy, we propose an action strategy selection method by analyzing the difference between the ideal movement and actual movement, the smaller the difference, the better the strategy. The ideal movement is determined by the decision maker, which can bring the highest utility.

Given the equivalence class \([x] = \{x_1, x_2, \ldots, x_n\}\) in the unfavorable region, the decision maker wants to move \([x]\) to a favorable region. We describe the equivalence class’s probabilistic preference \([x]\) moving to three regions under the action strategy through a probability mass function. The probability mass function is the probability of a discrete random variable at each particular value. All values of the probability mass functions are nonnegative, and the sum of the probability is equal to 1.

For the equivalence class \([x]\), the probability mass function of moving to three regions under the action strategy is represented by

\[
P_e = \{p(x_1), p(x_2), \ldots, p(x_n)\}.
\]

(24)

For example, objects in the equivalence class \([x] = \{x_1, x_2, x_3, x_4, x_5\}\) are in \(P_3\) region. After the action strategy, the distribution of objects in three regions is as follows:

\[
P_1 = \{x_1, x_2, x_4\},
\]

\[
P_2 = \{x_3\},
\]

\[
P_3 = \{x_5\}.
\]

(25)

The probability mass function of \([x]\) under the action strategy is \(p_e = \{0.6, 0.2, 0.6, 0.6, 0.2\}\). It can be seen from the probability mass function that \(p(P_1) + p(P_2) + p(P_3) = 0.6 + 0.2 + 0.2 = 1\). The probability mass function of ideal movement for decision maker is as follows:

\[
P_e = \{p(x_1), p(x_2), \ldots, p(x_n)\}.
\]

(26)

We use \(u_j(x), j = 1, 2, 3\), to represent the quality or utility of object \(x\) in different regions. So, the quality or utility of \([x]\) in three regions is represented as follows:
where the first-order moment is the expectation and the second-order moment is the variance.

The proposed method selects an optimal action strategy by comparing the ideal movement and the actual movement, the smaller the difference, the better the strategy. The first moment compares the expectations between the ideal movement and the actual movement. The second-order moment compares the variance between the ideal movement and the actual movement. The framework can further consider the variance based on the expectations and help the decision maker select an optimal action strategy.

Finally, we summarize the key steps of approximate movement probability assignment and effectiveness measures framework for the trisecting-acting-outcome model. The specific procedure is given in Algorithm 2.

5. An Illustrative Example

In this section, we use an example of medical decision in Table 2 to illustrate the proposed method. There are 468 suspected patients and six symptoms or attributes. Chol, bp, and bs stand for cholesterol level, blood pressure, and blood sugar, respectively. The condition attributes are age, sex, chol, bp, and bs. The result is a decision attribute. Symbols “+,” “−” and “?” stand for a suspected patient who has the disease, does not have the disease, and uncertain, respectively. For the convenience of representation, we divide objects into ten equivalence classes based on condition attributes. The right side of the decision attribute value indicates the number of objects in the equivalence class whose decision result is this type. For example, there are 50 objects in [x]10, of which there are five objects with a decision result of “+,” 30 objects with a decision result of “−,” and 15 objects with a decision result of “?.” We divide all objects into P1, P2, and P3 according to the value “−,” “+,” and “?” of the decision attribute.

Taking [x]10 as an example of the equivalence class that needs to be moved, from the information table, there are 105 objects in the equivalence class. Among them, 0 object does not have the disease, 100 objects have the disease, and 5 objects are uncertain. For diseased objects in [x]10, action strategy are mined from the table according to equations (4)
and (5), where age and sex are stable attributes and chol, bp, and bs are flexible attributes. The action strategy of \([x]_{10}\) is as follows:

\[
S_1 = \text{chol}: 3 \rightarrow 2 \land \text{bp}: 3 \rightarrow 2 \land \text{bs}: 3 \rightarrow 2,
S_2 = \text{chol}: 3 \rightarrow 2 \land \text{bp}: 3 \rightarrow 3 \land \text{bs}: 3 \rightarrow 2,
S_3 = \text{chol}: 3 \rightarrow 2 \land \text{bp}: 3 \rightarrow 2 \land \text{bs}: 3 \rightarrow 2,
S_4 = \text{chol}: 3 \rightarrow 1 \land \text{bp}: 3 \rightarrow 1 \land \text{bs}: 3 \rightarrow 2,
S_5 = \text{chol}: 3 \rightarrow 1 \land \text{bp}: 3 \rightarrow 2 \land \text{bs}: 3 \rightarrow 3.
\]

Table 2: Table for medical decision-making.

| Age | Sex | chol | bp | bs | Result |
|-----|-----|------|----|----|--------|
| [x]_1 | 3 | 1 | 2 | 1 | 1 | + = 5 |
| [x]_2 | 2 | 1 | 3 | 2 | 3 | + = 30 |
| [x]_3 | 3 | 0 | 2 | 3 | 2 | + = 35 |
| [x]_4 | 3 | 0 | 1 | 2 | 1 | + = 15 |
| [x]_5 | 1 | 1 | 3 | 2 | 2 | + = 30 |
| [x]_6 | 3 | 0 | 1 | 1 | 2 | + = 2 |
| [x]_7 | 3 | 0 | 1 | 2 | 3 | + = 20 |
| [x]_8 | 2 | 0 | 1 | 2 | 2 | + = 30 |
| [x]_10 | 3 | 0 | 3 | 3 | 3 | + = 0 |

Table 3: Credibility and coverage of each strategy.

|    | $S_1$ | $S_2$ | $S_3$ | $S_4$ | $S_5$ |
|----|-------|-------|-------|-------|-------|
| cre | 40/50 | 35/50 | 15/50 | 2/2   | 20/40 |
| cov | 40/207| 35/207| 15/207| 2/207 | 20/207|

The three regions have a partial order relationship of POS > BND > NEG. When the decision maker chooses a strategy, the strategy in the POS region is preferred. The accuracy of strategy $S_4$ is 1, but the coverage is 2/207. Two hundred and seven objects have no disease in the information system, but only two objects derived strategy $S_4$. This strategy is most likely a noise strategy that is affected by inconsistent information. This strategy is not helpful for the decision maker. Therefore, the strategy $S_4$ is effectively divided into the BND region. The strategy $S_1$ has an accuracy of 40/50 and coverage of 40/207. Both credibility and coverage exceed the thresholds. Therefore, strategy $S_1$ is more effective and more reliable.

Both credibility and coverage are positive evaluation criteria, that is, the higher the value of two criteria, the better the strategy. This model can effectively avoid the influence of conflicting information for the decision maker. There are two action strategies in the POS region: $S_1$ and $S_2$. They both have high credibility and coverage. We select an optimal action strategy by the approximate movement probability assignment model and the effectiveness measures the framework.

The credibility of all flexible attributes of the strategies $S_1$ and $S_5$ in the POS region is used as a mass function. The approximate movement probability assignment is synthesized by the mass function under each flexible attributes. A total of 150 decision objects derived from strategy $S_1$ under the attribute chol, 80 of them do not have the disease, 45 of them have the disease, and 25 of them are uncertain. Strategy $S_1$ has a total of 261 objects under the attribute bp, 165 of them have not the disease, 45 of them have the disease, and 51 of them are uncertain. The strategy $S_1$ has a total of 183...
objects under the attribute bs, 137 of them do not have the disease, 26 of them have the disease, and 20 are uncertain. The basic credibility allocation of strategy under the three attributes is shown in Table 4.

Given the flexible attribute chol, bp, and bs. The attribute weights are as follows:

\[ w_{\text{chol}} = \frac{1}{3}, \]
\[ w_{\text{bp}} = \frac{1}{3}, \]
\[ w_{\text{bs}} = \frac{1}{3}, \] (33)

The approximate movement probability distribution of strategy \( S_1 \) is calculated by equation (23):

\[
m(P_1) = \frac{\sum_{1 \leq i \leq m} \prod_{a \in C} w_{a} m(P_i / [x]_a)}{1 - \sum_{1 \leq i \leq m} \prod_{a \in C} w_{a} m(P_i / [x]_a)} = 0.96,
\]
\[
\cap P_i / [x]_a = \phi
\]

\[
m(P_2) = \frac{\sum_{1 \leq i \leq m} \prod_{a \in C} w_{a} m(P_2 / [x]_a)}{1 - \sum_{1 \leq i \leq m} \prod_{a \in C} w_{a} m(P_2 / [x]_a)} = 0.03,
\]
\[
\cap P_2 / [x]_a = \phi
\]

\[
m(P_3) = \frac{\sum_{1 \leq i \leq m} \prod_{a \in C} w_{a} m(P_3 / [x]_a)}{1 - \sum_{1 \leq i \leq m} \prod_{a \in C} w_{a} m(P_3 / [x]_a)} = 0.01.
\]
\[
\cap P_3 / [x]_a = \phi
\] (34)

The basic credibility allocation of strategy \( S_2 \) under the three attributes is shown in Table 5. The approximate movement probability allocation of strategy \( S_2 \) is calculated by equation (23):

\[
m(P_1) = \frac{\sum_{1 \leq i \leq m} \prod_{a \in C} w_{a} m(P_1 / [x]_a)}{1 - \sum_{1 \leq i \leq m} \prod_{a \in C} w_{a} m(P_1 / [x]_a)} = 0.74,
\]
\[
\cap P_1 / [x]_a = \phi
\]

\[
m(P_2) = \frac{\sum_{1 \leq i \leq m} \prod_{a \in C} w_{a} m(P_2 / [x]_a)}{1 - \sum_{1 \leq i \leq m} \prod_{a \in C} w_{a} m(P_2 / [x]_a)} = 0.25,
\]
\[
\cap P_2 / [x]_a = \phi
\]

\[
m(P_3) = \frac{\sum_{1 \leq i \leq m} \prod_{a \in C} w_{a} m(P_3 / [x]_a)}{1 - \sum_{1 \leq i \leq m} \prod_{a \in C} w_{a} m(P_3 / [x]_a)} = 0.01.
\]
\[
\cap P_3 / [x]_a = \phi
\] (35)

### Table 4: Basic credibility assignment of strategy \( S_1 \).

|      | \( P_1 \)   | \( P_2 \)   | \( P_3 \)   |
|------|-------------|-------------|-------------|
| \( m(\text{chol}) \) | 80/150      | 45/150      | 25/150      |
| \( m(\text{bp}) \)   | 165/261     | 45/261      | 51/261      |
| \( m(\text{bs}) \)   | 137/183     | 26/183      | 20/183      |

The probability quality function that the decision-maker ideal movement is \( \text{pe} \) and the probability quality of strategies \( S_1 \) and \( S_2 \) is \( \text{pr}_{S_1} \) and \( \text{pr}_{S_2} \), respectively. Their utility is expressed as follows:

\[
\text{Pe}([x]_{[0]}) = \{ \{ u_{P_1} (x) \} \times 100, \{ u_{P_2} (x) \} \times 0, \{ u_{P_3} (x) \} \times 0 \},
\]
\[
\text{Pe}_{S_1}([x]_{[0]}) = \{ \{ u_{P_1} (x) \} \times 96, \{ u_{P_2} (x) \} \times 3, \{ u_{P_3} (x) \} \times 1 \},
\]
\[
\text{Pe}_{S_2}([x]_{[0]}) = \{ \{ u_{P_1} (x) \} \times 74, \{ u_{P_2} (x) \} \times 25, \{ u_{P_3} (x) \} \times 1 \}.
\] (36)

In medical decision-making, the decision maker wants all objects in the diseased region to move to the disease-free region with appropriate action strategies, which will bring the greatest benefits to the decision-maker. The decision maker wants that the object in the diseased region in \([x]_{[0]}\) to move completely to the disease-free region with appropriate strategies. Therefore, the ideal probability distribution of the decision maker is recorded as \( \text{pe} = [1, 0, 0] \). Due to the influence of random errors in the actual cases, the strategy's effect is often difficult to meet the expectations of the decision maker. The above approximate movement probability allocation is used as its real movement probability distribution.

The objects in the \( P_1, P_2, \) and \( P_3 \) regions are assigned utility of 3, 1, and 2, respectively. That is to say, the utility of the object in the not disease region is 3, the utility of the object in the diseased region is 1, and the utility of the object in the uncertainty area is 2. According to equation (30), we can calculate the difference between the ideal expectation of the decision maker and strategy \( S_1 \):

\[
D(P_i \parallel \text{pe}, S_1) = \left\{ \sum_{k=1}^{2} \frac{E(p_{k}^i) - E(p_{k}^f)}{k!} \right\}^{1/2} = 0.1128.
\] (37)

The difference between the ideal expectation of the decision maker and strategy \( S_2 \) is as follows:

\[
D(P_i \parallel \text{pe}, S_2) = \left\{ \sum_{k=1}^{2} \frac{E(p_{k}^i) - E(p_{k}^f)}{k!} \right\}^{1/2} = 0.7357.
\] (38)

The degree of difference of \( S_1 \) is smaller, that is to say, the action strategy \( S_1 \) is more effective, with less uncertainty and more in line with the needs of the decision maker.
6. Conclusion

In this paper, we analyze the existence of conflicting information in a movement-based three-way decision. We construct a three-way decision model for the action strategy set by introducing the credibility and coverage. The model can effectively avoid the influence of conflicting information. To select an optimal action strategy, we analyzed the probabilistic preference in a movement-based three-way decision. An evidence-theory-based method for determining the probability of movement has been proposed. The optimal action strategy can be selected by analyzing the difference between the ideal movement and the actual movement, the smaller the difference, the better the strategy. We illustrate the practicability of the proposed method through an example of medical decision-making. This model improves and enriches the movement-based three-way decision and widens its application range.

In future work, we will systematically study conflicting information in movement-based three-way decisions and establish a more effective evaluation framework to evaluate and select the optimal action strategy. Thresholds for credibility and coverage are given by expert experience, which is too subjective. In the future, we will study how to determine optimal thresholds. This paper only considers the case where the same region’s objects have the same quality or utility. Therefore, the application of utility theory is also a future research direction.

Data Availability

All data, models, and code generated or used during the study are included within the article.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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Table 5: Basic credibility allocation of strategy $S_2$

|    | $P_1$ | $P_2$ | $P_3$ |
|----|-------|-------|-------|
| $m$ (chol) | 80/150 | 45/150 | 25/150 |
| $m$ (bp)   | 35/155 | 110/155| 10/551 |
| $m$ (bs)   | 137/183| 26/183 | 20/183 |
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