Warehouse Design Model for Shuttle Based Storage and Retrieve System

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Abstract. Recently, SBS / RS system has been widely used and studied. This paper presents a mathematical model of travel time and cost for shuttle based storage and retrieve systems (SBS/RS), through which we can decide the optimized configuration and picking efficiency of the warehouse. By considering the operating characteristics, such as acceleration and deceleration and assuming that the storage units are uniform distributed and using probability theory, we obtain a time and cost function for a dual command storage and retrieve procedure. Finally, we validate our analysis by numerical examples and sensitivity analysis. The results show that the optimal configuration and efficiency of the warehouse are mainly affected by several key indicators, such as the width of each storage unit, the height of each shelf, etc.

1. Introduction  
Automated warehouse technologies are evolving rapidly, from AS / RS systems to AVS / RS to the recently introduced SBS / RS, and many companies are transforming their traditional warehouses with automation equipment. Automation technology undoubtedly improves the storage and retrieve efficiency and shortens picking time, resulting in more benefits.

Based on the relationship of the shuttle and the storage tier, there are usually two main situations: tier-captive and tier-to-tier. In a tier-captive configuration each floor has a shuttle and the shuttle can only move in this tier. However, in a tier-tier configuration, a shuttle may move from one tier to another tier through the lifts. According to the picking process, it can be divided into single command and dual command. In a single command, it only needs to storage or retrieve a tote at once. In a dual command, it needs to storage first and then retrieve the totes. In a typical AVS / RS warehouse, after receiving the picking order, the lift first arrives at the floor where the goods are located, and then the shuttle moves to the position where the goods are stored, moves the goods to the buffer position, and then the conveyer conveys the goods to the lift. Finally, the lift will deliver the goods to the I / O point. At this point, a picking process is completed. Inventory is the opposite process. What we usually care about is the efficiency of storage and retrieve of warehouses and the configuration of warehouses, such as the characteristics of shelves, and the cost.

In this paper, we first propose a time function for a lift and a shuttle to complete a dual command storage and retrieve procedure based on the knowledge of probability and motion. The shuttle is tier-captive. Next, we set up a cash cost function for the warehouse. Finally, by combining the above two, we get the optimal decision of the warehouse configuration and verify it with the data. Different from the past, we do not analyse our model through simulation, but through numerical analysis and theoretical derivation, and finally via programming.
The paper is organized as the following manner: Section 2 gives the literature review. In section 3, the mathematical travel time model and cost model of a dual command cycle for a SBS/RS system are presented, which consider a discrete model closer to the actual situation. In the Section 4, we validate our model through numerical analysis and sensitivity analysis and get some useful revelations. Finally, we concluded the paper in Section 5.

Fig. 1 the SBS/RS storage rack

2. Literature review

Malmborg[1] first proposed Analytical conceptualizing tools based on the features of modeling expected performance for an AVS/RS system. Since then, AVS / RS system has been widely used and studied. There are two main research areas of AVS/RS. Some of them mainly focus on the travel time model of the AVS/RS system. Malmborg[2] considered the travel time of a dual command cycle as well as the system utilization and throughput capacity. Fukunari M, Malmborg CJ[3]analyzed the cycle time based on the iterative computational scheme exploiting random storage assumptions. Some approximation strategies for transaction waiting times are proposed by Zhang et al[4], in which the approximations based on the variance of the transaction inter-arrival times can be adjusted dynamically. Kuo et al[5] provided a practical means of predicting key aspects of system performance based on five design variables. Different from others they think over about the cost. Recently, Tone, Leher et al[6] built the travel time model of both single and dual command cycles for SBS/RS system. They assumed the storage locations are uniformly distributed and proved their model through simulation. However, Tone Lerher[7] established a travel time model where the single-deep racks are replaced by the double-deep racks. So there are more places to storage goods.

Of course, there are many studies which pay more attention to the performance and design of the warehouse. Miki Fukunari, Malmborg[8]studied a queuing approach to estimate the performance of the AVS/RS systems by using opportunistic interleaving. Ekren, B Y[9] et al focused on the rack configuration of AVS/RS. They observe the change of picking time by changing the number of tiers, aisles, bays and setting up a regression model. Then, Ekren, B Y[10] et al compared the performance of the two systems ,AVS/RS and CBAS/RS, five performance measures are considered, such as average flow time, device utilization, waiting time in queue, average number of jobs waiting in queue and cost. Finally, they found the AVS/RS system is more efficiency than the traditional one. After the regression model, Ekren, B Y[11] et al applied the DOE in the analytical model and measure the performance of the system by simulation. Gino Marchet[12] et al established an analytical model to evaluate the performance of the warehouse, however, they just considered the cycle time and waiting time. Both the performance and design decision were analyzed by Debjit Roy[13] et al, the model also contained the allocation of resources to zones, and the vehicle assignment rules.
Different from the past, we considered both the travel time and cost, and established a mathematical model without simulation, programmatically solve the optimal configuration of the warehouse. And when the problem was complex enough, we used heuristics to find the possible best solution. Finally, the effect of each parameter on the optimal decision is obtained.

3. Analytical model for SBS/RS

In this section we mainly propose a travel time model of dual command based on the probability and physical movement knowledge. Later on, the total cost of the SBS/RS will be presented. Considering the above two cost model, both time and cash, we will get an optimal configuration of the SBS/RS warehouse.

3.1. Notations and assumptions

The notations used in this paper are presented below:

- \( k \): number of the columns
- \( m \): number of the tiers
- \( n \): number of the single-deep racks
- \( d \): length of each storage unit
- \( d_{\text{max}} \): the maximum height of the racks
- \( h_{\text{max}} \): the maximum length of the racks
- \( w_i \): width of each storage unit
- \( w_a \): width of the aisle
- \( a_c \): accelerated/retarded velocity of the lift
- \( v_c \): the maximum velocity of the lift
- \( a_s \): accelerated/retarded velocity of the shuttle
- \( v_s \): the maximum velocity of the shuttle
- \( P \): unit time cost
- \( c_{\text{g}} \): cost of each storage unit
- \( N_1 \): number of columns the shuttle required to accelerate to the maximum speed
- \( N_2 \): number of tiers the lift required to accelerate to the maximum speed
- \( c_{\text{g}} \): cost of each shuttle
- \( c_{\text{i}} \): cost of each lift
- \( c_{\text{a}} \): cost of unit area

All the hypotheses in this article:

1. The number of layers and columns is enough to allow the lift and shuttle to accelerate to maximum speed. That is: \( m > N_2 + 2 \), \( k > N_1 + 2 \).
2. The distance the lift and shuttle required to accelerate to their maximum speed is greater than the height of the each tier or the length of each storage unit. That is: \( \frac{v_c^2}{2a_c} > h \), \( \frac{v_s^2}{2a_s} > d \)
3. The warehouse requires at least 10000 storage units. That is: \( 2mkn > 10000 \)

3.2. Dual command travel time model

In the dual command travel, after receiving the storage command the lift reaches the ith floor of the rack with a tote first from the I/O point which usually is in the ground and middle of the single-deep rack, the lift unloads the tote and the shuttle receives it, takes it to the xth column of the ith tier, which of course is an empty storage position. Then a retrieve command is sent to the lift, it moves to the next jth tier. At the same time the shuttle starts to pick the tote at the yth column of the jth tier then unload it on the lift. Finally, the lift carries the tote to the I/O point.

3.2.1. Lift travelling from I/O point to the ith tier

The time that the lift needs from the I/O point to the ith tier lies on the distance of them. In this model we assume the distance required for the lift to accelerate to the highest speed exceeds the height of the single tier, Namely, \( \frac{v_c^2}{2a_c} > h \). So the lift needs at least \( \lfloor \frac{v_c^2}{2ha_c} \rfloor = N_2 \) tiers. So we could get the relationship between the vertical velocity and the time (see figure 2). The time-tier function also can be gained as follow:
Then the expected time the lift needs from I/O point to the $i$th tier is as follow:

$$E(t_i) = \left[ \sum_{i=1}^{N_2} \sqrt{4ih / a_y} + \sum_{N_2+1}^{m} (ih / v_y + v_y / a_y) \right] \frac{1}{m}$$  \hspace{1cm} (2)

There are $m^2$ cases in total. The expected time of it can be calculated by the matrix method. The matrix of the time from the $i$th to $j$th tier is:

$$T_{2ij} = \begin{pmatrix}
0 & t_{12} & \ldots & \ldots & t_{1m} \\
t_{12} & 0 & \ldots & \ldots & \ldots \\
\ldots & \ldots & \ldots & \ldots & \ldots \\
\ldots & \ldots & \ldots & \ldots & \ldots \\
t_{m-1,1} & \ldots & \ldots & t_{m-1,m} & 0
\end{pmatrix}$$

Since $T_{2ij}$ is a symmetric matrix, so the mathematical expectation of it can be shown as follow:

$$E(t_2) = \sum_{i=1}^{m-N_2} \left[ \sum_{j=1}^{i+N_2} \sqrt{4(j-i)h / a_y} + \sum_{j=i+N_2+1}^{m} ((j-i)h / v_y + v_y / a_y) \right] \frac{2}{m^2} + \sum_{j=m-N_2}^{m} \left( \sum_{i=1}^{m} \sqrt{4(j-i)h / a_y} \right) \frac{2}{m^2}$$

3.2.3. Lift travelling from the $j$th tier to the I/O point

Apparently, this period is exactly the reverse of the first case, namely lift traveling from I/O point to the $i$th tier. So the expected time it takes can be deduced directly.
3.2.4. Shuttle travelling from buffer position to the ith column

Similar to the lift, the time that the shuttle used from the buffer position to the ith column relies on the distance between them. We assume the distance needed for the shuttle to accelerate to the highest speed exceeds the height of the single column, that is: \[ v_s^2 / 2a_s > d \]. Hence, the lift needs at least \( [v_s^2 / 2da_s] = N_1 \) columns. So we could get the horizontal velocity-time relationship The time-column function also can be gained as follow:

\[
t_4 = \begin{cases} 
\sqrt{4id / a_s} & i \leq N_1 \\
 id / v_s + v_s / a_s & N_1 < i \leq k 
\end{cases} 
\]  

(5)

Then the expected time the lift needs from buffer position to the ith column is as follow:

\[
E(t_4) = \left[ \sum_{i=1}^{N_1} \sqrt{4id / a_s} + \sum_{N_1+1}^{m} (id / v_s + v_s / a_s) \right] \frac{1}{m} 
\]  

(4)

3.2.5. Shuttle travelling from the ith column to the jth column

First, the time the shuttle takes from the ith to jth column is similar to the lift, which can be seen as follow:

\[
t_{s_{ij}} = \begin{cases} 
\sqrt{4|i-j|d / a_s} & |i-j| \leq N_1 \\
 |i-j|d / v_s + v_s / a_s & N_1 < |i-j| \leq k 
\end{cases} 
\]  

(7)

There are \( k^2 \) cases in total. The time(from the ith column to the jth column) matrix is:

\[
T_{s_{ij}} = \begin{pmatrix} 
0 & t_{s_{12}} & \cdots & \cdots & t_{s_{1k}} \\
 t_{s_{21}} & 0 & \cdots & \cdots & \cdots \\
 \vdots & \vdots & \ddots & \cdots & \vdots \\
 t_{s_{k-1,1}} & \cdots & \cdots & 0 & \cdots \\
 t_{s_{k1}} & \cdots & \cdots & t_{s_{kk}} & 0 
\end{pmatrix} 
\]

So the mathematical expectation of \( t_{s_{ij}} \) is:

\[
E(t_s) = \sum_{i=1}^{k} \left[ \sum_{j=1}^{i} \sqrt{4(j-i)d / a_s} + \sum_{j=i+N_1+1}^{k} ((j-i)d / v_s + v_s / a_s) \right] \frac{2}{k} + \sum_{i=k-N_1}^{k} \left[ \sum_{j=i}^{k} \sqrt{4(j-i)d / a_s} \right] \frac{2}{k^2} 
\]

3.2.6. Shuttle travelling from the jth column to the buffer position

It also can be treated as a reverse of the travelling from the buffer position to the ith column. Hence, the expected time it takes is:

\[
E(t_5) = \left[ \sum_{i=1}^{N_1} \sqrt{4id / a_s} + \sum_{N_1+1}^{k} (id / v_s + v_s / a_s) \right] \frac{1}{k} 
\]  

(8)

At this point, we have completed all the discussion of the travel time in a dual command storage and retrieve process for a SBS/RS system. So the total travel time is:

\[
E(t) = E(t_1) + E(t_2) + E(t_3) + E(t_4) + E(t_5) + E(t_6) \]  

,namely:
In this part, we mainly consider the cash cost of racks, shuttles, lifts, area of storage. It is obvious that there are $2mkn$ storage position, $m*n$ shuttles, $n$ lifts and the total area of storage is:

$$T_{C_1} = c_s * 2mkn + c_a * mn + c_r * n + c_m (w_a + 2w_j) * k * d * n$$  \hspace{1cm} (9)$$

In fact, the cost of the warehouse also includes many hidden parts, which may be difficult to measure, such as operating costs, labor costs, other facilities costs, etc. However, this has little effect on our decision of warehouse configuration or the two are not relevant. Consider the above two situations, the total cost (both time and cash) of a SBS/RS warehouse is: $E(t) * p + T_{C_1}$. We consider the time cost together with the cash cost, which is quite different from the past, and decide the optimal configuration of the warehouse by minimizing the total cost of it.

4. Numerical examples and sensitivity analysis

4.1. Numerical examples

(1). Assuming $d=0.8m$, $h=0.5m$, $d_{max} = 40m$, $h_{max} = 10m$, $a_y = 2m/s^2$, $v_y = 3m/s$, $a_z = 3m/s^2$, $v_z = 3m/s$, $w_a = 0.5m$, $c_s = 30S$, $c_a = 7000S$, $c_m = 50S$, $c_i = 30000S$. Thus, the best configuration of the warehouse is: 18tiers, 47 columns, 6 single-deep racks and the total cost is 1243630$.

(2). Assuming all parameters keep constant except $d=1.2m$. Thus, the best configuration of the warehouse is: 19tiers, 33 columns, 8 single-deep racks and the total cost is 1608190$.

(2). Assuming all parameters keep constant except $d_{max} = 60m$. Thus, the best configuration of the warehouse is: 17tiers, 74 columns, 4 single-deep racks and the total cost is 902681$.

Based on the above examples, we can see the total cost drops sharply when the maximum length of the warehouse increases, which mainly because it can contain more storage unit in one tier. However, when the length of unit storage increases the total cost rises dramatically. The reason is that each layer can accommodate fewer storage units. So it is vital for decision maker.

4.2. Sensitivity analysis

In this paper, we first propose a travel time model and cash cost model of dual command storage and retrieve process for a SBS/RS system warehouse to analyses the best decision of the warehouse configuration. Next, by minimize $E(t) * p + T_{C_1}$ we calculated and get the numerical examples through C++ program. (See Fig 3). We observe the optimal decision and the total cost by changing the value of each parameter and get some management implications.

From these figures we can observe that the number of tiers augments first then keeps later and the number of single-deep racks rises when the length of unit column increases, but the number of columns decreases. Since we assume that the total number of storage units is not less than 10000, as the length of unit column increases each tier can accommodate fewer storage units. Therefore, the number of columns reduces and the number of tiers increases. However, when the length of a storage unit exceeds a threshold, the increase of the number of tiers will reach a limit, so the storage requirement can be satisfied by adding a single-deep rack.
For the height and length of the warehouse, their changes lead to an increase in the number of tiers and columns. Similar to the total cost, while the number of single-deep racks will decrease. The changes in the accelerated velocity and the maximum speed of the lift and shuttle seem to have no effect on the optimal decision, while the total cost will be reduced. This is because changes in speed and acceleration only affect the total expected travel time. Since the size of the warehouse we are considering is not large enough, the cost of such a short time period to total cost is insignificant. However, we mainly often also care about the picking efficiency of the warehouse, or the time it takes to complete a dual command process.

The impact of shuttle or lift maximum speed and acceleration on storage and retrieve time is easy to understand. Time reduces when the speed increase, the faster it accelerates the little time it needs. But the fact is not always the case, when the speed is high, the car will move to farther place to access the totes, the total time actually is increased.

Fig. 3 The optimal decision sensitivity to all parameters

5. Conclusion
This study focused on the design model of a warehouse based on the SBS/RS system, in which the shuttle was tier-captive and executed a dual command at once. By considering the cost and travel time, we obtain the total cost function and get the optimal warehouse configuration by solving the function.

First, we assume that each storage location is equally uniform distributed and based on the kinematics of the shuttle and the lift we propose a travel time model. Then, considering the expectations of travel time, use its expectation to represent the average time of storage and retrieve. Next, we also consider the cash cost of the warehouse, including storage costs, area costs, shuttle costs and lift costs. Finally, we programmatically calculate the optimal configuration of the warehouse for a given situation and the efficiency of storage and retrieve.

Of course, our model can be more complex in future. For example, we can change the single-deep rack to double-deep. The shuttles can be tier-to-tier. Each aisle has multiple lifts. The lift executes single and dual commands alternately, or considers the time the shuttle and the goods waits in queue.
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