The effects due to a uniformly distributed load in a non-local thermoelastic solid in a frequency domain

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Abstract. In the present study the effects on a non-local thermoelastic solid under the two-dimensional deformation effects due to a uniformly distributed load in a frequency domain have been discussed. The nonlocal thermoelastic solid is homogeneous with the effect of two temperature parameters subjected to uniformly distributed source. Fourier transforms have been used to solve the problem. The analytical expressions of displacement and stress components have been obtained in the transformed domain. For obtaining the results in the physical domain, the numerical inversion technique has been applied. The results so obtained are depicted graphically to show the effect of non-local parameter on the components of displacements and stresses and conductive temperature.

1. Introduction
The nonlocal theory of thermoelasticity was given by Eringen et al. [1-2]. The nonlocal theory considers that the physical quantities defined at any point are dependent upon a function of the values of independent constitutive variables over the whole body. For thermoelastic materials it was proved that the stress field at a particular point is dependent upon the strain at all the point of the body. Artan [3] proved that the nonlocal theory of elasticity is superior to classical elasticity theories by comparing the results of local and nonlocal elasticity theories. Eringen [4] developed nonlocal continuum field theories and formulated the basic field equations for nonlocal continuum field theories, memory-dependent elastic materials, electromagnetic and viscous fluids. Othman and Marin [5] compared different types of thermoelastic theories and studied wave propagation. Lata [6-7] depicted the effects of nonlocality during the investigation of study of reflection of plane waves. Lata and Singh [8] studied the variations due to angle of inclination and nonlocal parameter on the thermoelastic solid under the effects of an inclined load. Lata and Singh [9] studied the deformations in a nonlocal magneto-thermoelastic medium due to normal force and studied the combined effects of nonlocal parameter and hall current on various components.

Chen and Gurtin [10] gave the theory of thermoelasticity with two temperature, which considers the thermal effects in comparison to other classical theories. They suggested that there is a
temperature dependence on two distinct temperatures, namely the thermodynamic temperature and the conductive temperature. Youssef [11] and Youssef and Al-Lehaibi [12] obtained the uniqueness theorem for equations of two temperature generalized thermoelasticity and proved that the theory is able to describe the state of an elastic body more realistically as compared to one temperature. Sharma et al. [13] studied the time harmonic deformations of a transversely isotropic thermoelastic solid with two temperatures. Kumar et al. [14-15] investigated the disturbances in a homogeneous transversely isotropic thermoelastic rotating medium with two temperatures due to thermomechanical sources and proved that the thermodynamic and conductive temperature both have a significant effect. Lata and Singh [16] investigated the time harmonic interactions in a nonlocal thermoelastic solid with two temperature and studied the variations of various components due to frequency and nonlocal parameter under the effect of concentrated sources as bounding surface.

In the present study, the effects in a two-dimensional homogeneous isotropic nonlocal thermoelastic solid due to a uniformly distributed load in a frequency domain have been discussed. Fourier transforms have been used for solving the problem. The analytical expressions of displacement and stress components have been obtained in the transformed domain. The results so obtained are depicted graphically to show the effect of nonlocal parameter and frequencies on various components.

2. Basic Equations

Following Youseff [11] and Eringen [4], the equations of motion and the constitutive relations in a homogeneous non local thermoelastic solid with two temperatures are given by

\[(\lambda + 2\mu)\nabla(\nabla \cdot \mathbf{u}) - \mu (\nabla \times \nabla \times \mathbf{u}) - \beta \nabla \theta = (1 - \epsilon^2 \nabla^2) \rho \frac{\partial^2 \mathbf{u}}{\partial t^2},\]  
(1)

\[K^* \nabla^2 \varphi = \rho C^* \frac{\partial \theta}{\partial t} + \beta \theta_0 \frac{\partial}{\partial t} (\nabla \cdot \mathbf{u}).\]  
(2)

where, \(\theta = (1 - \alpha \nabla^2) \varphi\). 
(3)

\[t_{ij} = \lambda u_{k,k} \delta_{ij} + \mu (u_{ij} + u_{ji}) - \beta \theta \delta_{ij}.\]  
(4)

where \(\lambda, \mu\) are material constants, \(\epsilon\) is the nonlocal parameter, \(\rho\) is the mass density, \(\mathbf{u}=(\mathbf{u}_1, \mathbf{0}, \mathbf{u}_3)\) is the displacement vector, \(\varphi\) is the conductive temperature, \(\alpha\) is two temperature parameter, \(\theta\) is absolute temperature and \(\theta_0\) is reference temperature, \(K^*\) is the coefficient of the thermal conductivity, \(C^*\) the specific heat at constant strain, \(\beta = (3\lambda + 2\mu)\alpha\) where \(\alpha\) is coefficient of linear thermal expansion, \(\epsilon_{ij}\) are components of strain tensor, \(\epsilon_{kk}\) is the dilatation, \(\delta_{ij}\) is the Kronecker delta, \(t_{ij}\) are the components of stress tensor.

3. Formulation of the problem

We consider a homogeneous non local isotropic thermoelastic body in an initially undeformed state at temperature \(\theta_0\). A rectangular Cartesian co-ordinate system \((x_1, x_2, x_3)\) is considered with
\( x_3 \) axis pointing normally into the half space. We restrict our analysis to two-dimensional problem by taking

\[
u = (u_1, 0, u_3).
\]

Using Eq.(5) in Eqs.(1)-(2), yields

\[
(\lambda + \mu) \frac{\partial u_1}{\partial x_1} + \mu \nabla^2 u_1 - \beta \frac{\partial \theta}{\partial x_1} = \left( 1 - \epsilon^2 \nabla^2 \right) \rho \frac{\partial^2 u_1}{\partial t^2},
\]

\[
(\lambda + \mu) \frac{\partial u_3}{\partial x_3} + \mu \nabla^2 u_3 - \beta \frac{\partial \theta}{\partial x_3} = \left( 1 - \epsilon^2 \nabla^2 \right) \rho \frac{\partial^2 u_3}{\partial t^2},
\]

\[
K^* \nabla^2 \varphi = \rho \ C^* \frac{\partial \theta}{\partial t} + \beta \theta_0 \frac{\partial \epsilon}{\partial t}.
\]

where, \( \epsilon = \frac{\partial u_1}{\partial x_1} + \frac{\partial u_3}{\partial x_3}, \nabla^2 = \frac{\partial^2}{\partial x_1^2} + \frac{\partial^2}{\partial x_3^2} \)

we define the following dimensionless quantities

\[
(x'_i, x'_3) = \frac{C_s}{\epsilon} (x_i, x_3), \quad (u'_1, u'_3) = \frac{C_s}{\epsilon} (u_1, u_3), \quad t' = \frac{C_s}{c_s^2} t, \quad \lambda' = \frac{C_s}{c_s^2} a, \quad K'_n = \frac{c_s^2}{\lambda \omega}, K_n.
\]

Upon introducing the quantities defined by Eq.(9) in equations Eqs.(6)-(8), and suppressing the primes, yields

\[
\left( \frac{\lambda + \mu}{\epsilon} \right) \frac{\partial^2 u_1}{\partial x_1^2} + \left( \frac{\lambda + \mu}{\epsilon} \right) \frac{\partial^2 u_3}{\partial x_3^2} + \frac{\partial^2 u_1}{\partial x_1^2} - \beta \theta_0 \frac{\partial \theta}{\partial x_1} = \left( 1 - \epsilon^2 \nabla^2 \right) \frac{\partial^2 u_1}{\partial t^2},
\]

\[
\left( \frac{\lambda + \mu}{\epsilon} \right) \frac{\partial^2 u_3}{\partial x_3^2} + \left( \frac{\lambda + \mu}{\epsilon} \right) \frac{\partial^2 u_1}{\partial x_1^2} + \frac{\partial^2 u_3}{\partial x_3^2} - \beta \theta_0 \frac{\partial \theta}{\partial x_3} = \left( 1 - \epsilon^2 \nabla^2 \right) \frac{\partial^2 u_3}{\partial t^2},
\]

\[
\nabla^2 \varphi = \frac{\beta c_s^2 \epsilon}{K' \omega^2} \frac{\partial}{\partial t} \left( \frac{\partial u_1}{\partial x_1} + \frac{\partial u_3}{\partial x_3} \right).
\]

The initial and regularity conditions are given by

\[
u_1(x_1, x_3, 0) = 0 = \nu_1(x_1, x_3, 0),
\]

\[
u_2(x_1, x_3, 0) = 0 = \nu_2(x_1, x_3, 0),
\]

\[
\varphi(x_1, x_3, 0) = 0 = \varphi(x_1, x_3, 0) \text{ for } x_3 \geq 0, -\infty < x_1 < \infty,
\]

\[
u_1(x_1, x_3, t) = \nu_3(x_1, x_3, t) = \varphi(x_1, x_3, t) = 0 \text{ for } t > 0 \text{ when } x_3 \to \infty.
\]
Assuming the harmonic behavior as

\[
(u_1, u_3, \varphi)(x_1, x_3, t) = (u_1, u_3, \varphi)(x_1, x_3)e^{i\omega t}.
\]  

(13)

where, \( \omega \) is the angular frequency.

Using Eq.(13) in Eqs.(10)-(12) and applying Fourier transform defined by

\[
\hat{f}(x_3, \omega) = \int_{-\infty}^{\infty} f(x_1, x_3, \omega) e^{i\xi x_1} dx_1.
\]  

(14)

on the resulting quantities yields,

\[
[(1 - e^2 \omega^2) \frac{d^2}{dx_3^2} - (1 + a_1)\xi^2 + (1 + e^2 \xi^2)\omega^2] \hat{u}_1 + i\alpha_1 \xi \frac{d}{dx_3} \hat{u}_3 - i\alpha_2 \hat{\varphi} = 0, \tag{15}
\]

\[
i\alpha_1 \xi \frac{d}{dx_3} \hat{u}_1 + [(1 + a_1 - e^2 \omega^2) \frac{d^2}{dx_3^2} - \xi^2 + (1 + e^2 \xi^2)\omega^2] \hat{u}_3 - a_2 \frac{d}{dx_3} \hat{\varphi} = 0, \tag{16}
\]

\[
\beta a_3 \omega \xi \hat{u}_1 - i\beta a_3 \omega \frac{d}{dx_3} \hat{u}_3 - \left[(1 + i\omega) \frac{d^2}{dx_3^2} - \xi^2 - i\omega(1 + a\xi^2)\right] \hat{\varphi} = 0. \tag{17}
\]

This system of equations (15)-(17) possesses a nontrivial solution if determinant of coefficient \((\hat{u}_1, \hat{u}_3, \hat{\varphi})\) vanishes so as to give a characteristic equation

\[
\left[ \frac{d^6}{dx_3^6} + Q \frac{d^4}{dx_3^4} + R \frac{d^2}{dx_3^2} + S \right] (\hat{u}_1, \hat{u}_3, \hat{\varphi}) = 0, \tag{18}
\]

where,

\[Q = \frac{1}{\rho}[(1 + a_1 - \omega^2 e^2)(1 + i\omega)\{(1 + e^2 \xi^2)\omega^2 - (1 + a_1)\xi^2\} - (1 - \omega^2 e^2)(1 + a_1 - \omega^2 e^2)\xi^2 + i(1 + a_1 - \omega^2 e^2)(1 + i\omega)\{(1 + e^2 \xi^2)\omega^2 - \xi^2\} - i\beta a_2 a_3 \omega \xi(1 + a_1 - \omega^2 e^2) - \omega^2(1 + e^2 \xi^2) - \omega^2(1 + e^2 \xi^2)\xi^2 + \omega(1 + a\xi^2)\}],
\]

\[R = \frac{1}{\rho}[(1 - \omega^2 e^2)\xi^2 - (1 + e^2 \xi^2)\omega^2]\xi^2 + i(1 + a_1 - \omega^2 e^2)\xi^2 + \omega(1 + a\xi^2)\} + (1 + a_1 - \omega^2 e^2)\xi^2 + i(1 + a_1 - \omega^2 e^2)\xi^2 + \omega(1 + a\xi^2)\} + (1 + e^2 \xi^2)\omega^2 + (1 - a_1)\xi^2 + (1 + i\omega)\{(1 + e^2 \xi^2)\omega^2 - (2 + a_1)\xi^2\} - a_2^2 \xi^2(\xi^2 + i\omega(1 + a\xi^2))\}],
\]

\[S = \frac{1}{\rho}[(2 + a_1)(1 + e^2 \xi^2)\omega^2 \xi^2 + \omega(1 + a\xi^2)] - (1 + a_1)\xi^2 - i\omega(1 + a\xi^2) + (1 + e^2 \xi^2)\omega^2 + (1 + a_1)\xi^2 + (1 + e^2 \xi^2)\omega^2 + (1 + a_1)\xi^2 + i(1 + a_1)(1 + e^2 \xi^2)\omega^2 + \xi^2 + i\beta a_2 a_3 \omega \xi^2][(1 + e^2 \xi^2)\omega^2 - \xi^2\}],
\]

\[P = (1 - \omega^2 e^2)(1 + a_1 - \omega^2 e^2)(1 + i\omega).\]

The roots of Eq. (18) are \( \pm \lambda_i \) (\( i = 1,2,3 \)) which satisfy the radiation condition \( \hat{u}_1, \hat{u}_3, \hat{\varphi} \rightarrow 0 \) when \( x_3 \rightarrow \infty \), the solutions of equation can be written as,

\[
\hat{u}_1 = A_1 e^{-\lambda_i x_3} + A_2 e^{-\lambda_2 x_3} + A_3 e^{-\lambda_3 x_3},
\]  

(19)
\[ \ddot{u}_3 = d_1 A_1 e^{-\lambda_1 x_3} + d_2 A_2 e^{-\lambda_2 x_3} + d_3 A_3 e^{-\lambda_3 x_3}, \]  
\[ \ddot{\phi} = l_1 A_1 e^{-\lambda_1 x_3} + l_2 A_2 e^{-\lambda_2 x_3} + l_3 A_3 e^{-\lambda_3 x_3}. \]  
where,
\[ d_i = \frac{P' \lambda_i^3 + Q' \lambda_i}{s' \lambda_i^4 + T' \lambda_i^2 + U'}, \quad i = 1, 2, 3. \]
\[ l_i = \frac{P'' \lambda_i^2 + Q''}{s' \lambda_i^4 + T' \lambda_i^2 + U'}, \quad i = 1, 2, 3. \]

where,
\[ P' = \alpha_1 \xi (1 + i \omega \xi), \quad Q' = -\alpha_1 \xi [\xi^2 + \omega (1 + a \xi^2)] + \beta a_3 a \omega s' = (1 + a_1 - \omega^2 \xi^2)(1 + i \omega), \quad T' = (1 + e^2 \xi^2)(1 + i \omega \xi^2) \omega^2 - (1 + a_1 - \omega^2 \xi^2)[\xi^2 + i \omega(1 + a \xi^2)] - i \beta a_3 a \omega \omega^2, \quad P'' = -\beta a_3 a \omega (1 - \omega^2 \xi^2), \quad Q'' = \beta a_3 a \omega [\xi^2 - (1 + e^2 \xi^2) \omega^2]. \]

4. Applications
The boundary conditions are
\[ \frac{\partial}{\partial x_3} \varphi(x_1, x_3, t) + h_1 \varphi = 0 \quad \text{at} \ x_3 = 0. \]

where, \( F_1 \) is the magnitude of the force applied, \( \psi_1 (x) \) specify the source distribution function along \( x_1 \) axis, \( h_1 \) is the heat transfer coefficient where \( h_1 \to 0 \) for insulated boundary and \( h_1 \to \infty \) for isothermal boundary.

Using the dimensionless quantities defined by Eq. (9), then Eqs. (3), (4), (13), (14) in Eqs. (22)-(24) and substituting values of \( \ddot{u}_1, \ddot{u}_3 \) and \( \ddot{\phi} \) from Eqs. (19)-(21), we obtain the components of displacement, normal stress, tangential stress and conductive temperature as
\[ \ddot{u}_1 = -F_1 \dot{\psi}_1 (x) e^{i\omega t}, \]
\[ \ddot{u}_3 = -F_1 \dot{\psi}_3 (x) e^{i\omega t}, \]
\[ \ddot{\phi} = -F_1 \dot{\psi}_3 (x) e^{i\omega t}, \]
\[ \ddot{\varepsilon}_3 = -F_1 \dot{\psi}_3 (x) e^{i\omega t}, \]
\[ \ddot{\varepsilon}_3 = -F_1 \dot{\psi}_3 (x) e^{i\omega t}, \]
\[ \ddot{\varepsilon}_{11} = -\mu F_1 \dot{\psi}_1 (x) e^{i\omega t}, \]

where,
\[ \Delta = \Delta_{11} M_{21} - \Delta_{12} M_{22} + \Delta_{13} M_{23}, \]
\[ M_{11} = \Delta_{22} \Delta_{33} - \Delta_{32} \Delta_{23}, \quad M_{12} = \Delta_{31} \Delta_{23} - \Delta_{33} \Delta_{21}, \quad M_{13} = \Delta_{32} \Delta_{21} - \Delta_{31} \Delta_{22}. \]
\[ \Delta_{1j} = \lambda_j d_j (\lambda + 2 \mu) + \beta j, \quad \Delta_{2j} = i \xi d_j - \lambda_j, \quad \Delta_{3j} = l_j a_j + h_1; \quad j = 1, 2, 3. \]
\[ M_{21} = \Delta_{22} \Delta_{33} + \Delta_{32} \Delta_{23}, \quad M_{22} = \Delta_{31} \Delta_{23} + \Delta_{33} \Delta_{21}, \quad M_{23} = \Delta_{32} \Delta_{21} + \Delta_{31} \Delta_{22}. \]
\[ N_{1j} = -\lambda_j d_j (\lambda + 2\mu) - \beta \theta_0 (1 + a\xi^2) l_j + \beta \theta_0 a\lambda_j^2 l_j ; \ j = 1,2,3. \]
\[ N_{2j} = \iota_\xi (\lambda + 2\mu) - \beta \theta_0 (1 + a\xi^2) l_j + \beta \theta_0 a\lambda_j^2 l_j ; \ j = 1,2,3. \]

Influence function

The method to obtain the solution due to distributed load applied on the half space is obtained by setting
\[
\psi_1(x) = \begin{cases} 1 & \text{if } |x| \leq m \\ 0 & \text{if } |x| > m \end{cases}
\]
(31)
The Fourier transform of \( \psi_1(x) \) with respect to the pair \((x, \xi)\) for the case of a uniform strip load of non-dimensional width 2m applied at origin of co-ordinate system \( x_1 = x_3 = 0 \) in the dimensionless form after suppressing the primes is given by
\[
\hat{\psi}_1(\xi) = \left[ 2 \sin (\xi m) / \xi \right] \xi \neq 0.
\]
(32)
The expressions for displacement, stresses and conductive temperature can be obtained for uniformly distributed normal force by replacing \( \hat{\psi}_1(\xi) \) from Eq.(32) in Eqs. (25)-(30).

5. Particular cases
- If \( \epsilon = 0 \), then from Eqs.(25)–(30), the corresponding expressions for displacement components, stress components and conductive temperature for local homogeneous isotropic solid are obtained.
- If \( \epsilon = \alpha = 0 \), then from Eqs.(25)–(30), the corresponding expressions for displacement components, stress components and conductive temperature for local homogeneous isotropic solid without two temperature are obtained.

6. Inversion of the transformation

For finding the solution in physical domain, we are required to invert the transforms in Eqs. (25)–(30). Here the displacement components, normal and tangential stresses and conductive temperature are functions of \( x_3 \) and the parameters of Fourier transform \( \xi \) and thus of the form \( f(\xi, x_3) \). For obtaining the function \( f(x_1, x_3) \) in the physical domain, we first invert the Fourier transform as used by Sharma et al. [17], using
\[
f(x_1, x_3) = \frac{1}{2\pi} \int_{-\infty}^{\infty} e^{-i\xi x_1} \hat{f}(\xi, x_3) d\xi = \frac{1}{2\pi} \int_{-\infty}^{\infty} \left[ \cos(\xi x_1) f_e - i \sin(\xi x_1) f_0 \right] d\xi.
\]
(33)
where, \( f_e \) and \( f_0 \) are respectively the even and odd parts of \( \hat{f}(\xi, x_3) \). The method for evaluating this integral is described in Press et al. [18], which involves the use of Romberg’s integration with adequate step size. The results from successive refinements of the extended trapezoidal rule followed by extrapolation of the results to the limit when the step size tends to zero are also used.

7. Numerical results and discussion

Magnesium material is chosen for the purpose of numerical calculation and physical data for which is given by Dhaliwal and Singh [19] as follows:
\[ \lambda = 9.4 \times 10^{10} Nm^{-2}, \quad \mu = 3.278 \times 10^{10} Nm^{-2}, \quad K^* = 1.7 \times 10^{2} Wm^{-1} K^{-1}, \quad \rho = 1.74 \times 10^{3} Kg m^{-3}, \quad \theta_0 = 298 K, \quad C^* = 10.4 \times 10^{2} J Kg^{-1} deg^{-1}, \quad \omega_1 = 3.58, \quad a = 0.05 \]

A comparison of values of normal displacements \( u_1 \) and \( u_3 \), normal stresses \( t_{11} \) and \( t_{33} \), tangential stress \( t_{31} \) and conductive temperature \( \varphi \) for a homogeneous isotropic thermoelastic solid with distance \( x \) has been made for local and nonlocal parameter \( (\varepsilon = 2.0) \) and is presented graphically for the non-dimensional frequencies \( \omega = 0.25, \omega = 0.5 \) and \( \omega = 0.75 \).

1) The solid dark black colored line with squares as centre symbol, blue colored line with upward triangles and green colored line with downward triangles, respectively corresponds to nonlocal parameter \( (\varepsilon = 2) \) with frequencies \( \omega = 0.25, \omega = 0.5 \) and \( \omega = 0.75 \).

2) The solid red colored line with centre symbol circles, purple colored line with centre symbol downward triangles and orange colored line with centre symbol downward triangles, respectively represents local parameter \( (\varepsilon = 0) \) with frequencies \( \omega = 0.25, \omega = 0.5 \) and \( \omega = 0.75 \).

**Fig. 1.** Variation of displacement component \( u_1 \) with displacement \( x \)

**Fig. 2.** Variation of displacement component \( u_3 \) with displacement \( x \)
Fig. 1 and Fig. 2, shows the variations in values of normal displacements $u_1$ and $u_3$ respectively. The behavior followed is oscillatory in both figures. Non-locality is clearly playing a major part while the variations are more for $\omega = .25$ as compared to $\omega = .5$ and $\omega = .75$. Fig. 3 depicts the variations of values of normal stress $t_{11}$. The behavior followed is oscillatory with maximum deflections for $\omega = .25$. The effects of non-local parameter can be clearly noticed. Fig. 4 describes the variations of normal stress $t_{33}$. Nonlocality is visibly causing the differences. The behavior is less oscillatory for $\omega = .75$ as compared to $\omega = .5$ and most oscillatory for $\omega = .25$. Fig. 5 illustrates the variation of tangential stress $t_{31}$. The behavior for $\omega = .25$ is more oscillatory with higher magnitude of values as compared to other frequency values. Nonlocality is clearly causing more differences at $\omega = .75$ relative to $\omega = .25$ and $\omega = .5$. Fig. 6 shows the variation of conductive temperature $\varphi$. The oscillatory behavior is more for $\omega = .25$ and $\omega = .5$. But as the trend goes there is difference for local and non-local parameter values.

8. Conclusion
From above study it is clear that nonlocal parameter and two temperature parameter together have a highly significant impact on displacement components, stress components and conductive temperature. The variation of the components is graphically observed for the nonlocal parameter as well as the variations in the frequencies. The results of this investigation can be very helpful for the researchers working in the field of material science, geophysics, acoustics etc. The research also motivates to study the nonlocal parameter further for time harmonic interactions.

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