Article
Evaluate Radar Data Assimilation in Two Momentum Control Variables and the Effect on the Forecast of Southwest China Vortex Precipitation

Dongmei Xu 1, Gangjie Yang 1, Zheng Wu 2, Feifei Shen 1,3,4,* , Hong Li 3 and Danhua Zhai 5

1 The Key Laboratory of Meteorological Disaster, Ministry of Education (KLME)/Joint International Research Laboratory of Climate and Environment Change (ILCEC)/Collaborative Innovation Center on Forecast and Evaluation of Meteorological Disasters (CIC-FEMD), Nanjing University of Information Science & Technology, Nanjing 210044, China; 002727@nuist.edu.cn (D.X.); 20211201051@nuist.edu.cn (G.Y.)
2 Chongqing Institute of Meteorological Sciences, Chongqing 401147, China; wukgdqgh@163.com
3 Shanghai Typhoon Institute, China Meteorological Administration, Shanghai 200030, China; lih@typhoon.org.cn
4 Heavy Rain and Drought-Flood Disasters in Plateau and Basin Key Laboratory of Sichuan Province, Chengdu 610225, China
5 Chongqing Meteorological Observatory, Chongqing 401147, China; i_danhua@163.com
* Correspondence: 002746@nuist.edu.cn; Tel.: +86-13913313210

Abstract: Based on the Weather Research and Forecasting (WRF) Model and the three-dimensional variational (3DVAR) data assimilation system, this study investigates the effects of assimilation of radar reflectivity and radial velocity under different momentum control variables on the forecast of Southwest China Vortex precipitation. It is shown that the U−V control variable strengthens the wind speed and vorticity to be better matching the observation, while using ψ−χ as the control variable will produce too large increments which are unphysical. The root mean square errors (RMSE) of radar radial velocity are around 2.4 m/s in the experiment using ψ−χ control variables, while the RMSE are below 2 m/s in the experiment with U−V control variables. The composite reflectivity from the analysis of the U−V control variables matches better with the observation than that from the analysis of the ψ−χ control variables, i.e., the forecast rain band location under U−V control variables is more accurate. ψ−χ control variable enhances the cold high-pressure system in near surface, while the U−V control variable is not significant. The water vapor flux convergence in the lower layers of the ψ−χ control variable is overestimated leading excessive precipitation in the forecast. The Equitable Threat Score (ETS) of the U−V control variable is about 0.1 higher than ψ−χ control variable. In summary, the U−V control variable is superior to the ψ−χ control variable in terms of analysis and forecasting about Southwest China Vortex precipitation.

Keywords: data assimilation; momentum control variables; doppler radar data; southwest China vortex

1. Introduction
Southwest China Vortex is an α mesoscale cyclone low pressure system formed at 700 hPa or 850 hPa isolars in southwest China under the interaction of the special topography of the Qinghai–Tibet Plateau and the atmospheric circulation. The low vortex rainstorm is also a relatively complex and characteristic phenomenon. It has been commented that the intensity, frequency, and scope of influence are only second to typhoons in China. Flooding from heavy rainfall has seriously affected people’s lives and safety. Numerical weather prediction model is the main approach to predict the low vortex related rainstorm. There have been many studies on the numerical simulation of the Southwest China Vortex in recent years [1].

Atmospheric data assimilation is a relatively effective method to improve the accuracy of model initial fields by integrating different observation together. As a special type of
mesoscale convective system (MCS), Southwest China Vortex’s rainstorm forecast can be improved by radar data assimilation. Much effort has been applied during the last 10 years in research on MCS with radar data assimilation in terms of reflectivity and radial velocity from the Doppler radar [2–12]. It is found that reflectivity can effectively adjust the water vapor rate in the cloud. An indirect radar reflectivity assimilation scheme is developed by Wang et al. (2013) to assimilate retrieved rainwater and estimated in-cloud water vapor. As a result, the assimilation of reflectivity increases humidity, rainwater, and convective available potential energy in the convective region [13]. Most data assimilation studies have been conducted based on the three-dimensional variational (3DVAR) method due to its lower computational cost and its effectiveness in improving the initial field of the model. The variational assimilation is established based on Bayesian theory, which allows the stream function and velocity potential ($\psi - \chi$) to be used as control variables in the variational analysis, as well as the latitudinal and longitudinal winds ($U - V$). It was pointed out that the advantage of $\psi - \chi$ in estimating the statistical regression between the stream function and the mass field is under the Quasi-geostrophic theory [14]. However, Xie and MacDonald (2012) pointed out that 3DVAR under $\psi - \chi$ control variables requires the transformation of velocity into stream function and velocity potential by solving Poisson’s equation, which is prone to computational errors near the boundary, theoretically illustrating the disadvantage [15]. The option of $U - V$ as a control variable was updated in the Weather Research and Forecasting (WRF) Model and variational data assimilation system (WRFDA) V3.7 in 2015, which facilitates users to conduct studies of assimilation schemes based on different momentum control variables. These two control variable options were applied on seven historical convection cases occurring in the Rocky Mountain Front region and significantly improvement is found in terms of precipitation forecasts under the $U - V$ control variable [16]. The effects of different control variables were compared on a squall line individual case [17]. It was also found that radial velocity assimilation based on the $U - V$ control variable scheme significantly improved the meso-scale dynamical field under initial conditions. The enhancement of low-layer rapids, water vapor convergence, and low-layer wind shear facilitates squall line forecasting. However, the $\psi - \chi$ control variable format produces discontinuous wind fields and unrealistic convergence/divergence, which leads to less reliable precipitation forecasts. Afterwards, the effects of different control variables on individual typhoon cases were study by Shen et al. and it was found that the $U - V$ control variables simulated better typhoon vortices and warm core structures [18]. Meanwhile, Lu et al. and Thiruvengadam et al. also studied 3DVAR under different momentum control variables and obtained the same conclusion that the $U - V$ control variables capture more small-scale and meso-scale information than the $\psi - \chi$ control variables, resulting in better simulations [19,20].

However, the effect of data assimilation under different control variables on the simulation and forecast of the Southwest China Vortex and its precipitation has been relatively little studied. Because of its complex terrain and strong effect of large-scale weather system, whether $U - V$ control variables can still show its strengths is unknown. With high temporal and spatial resolution, radar observations are useful for the analysis and study of the Southwest China Vortex. In previous studies on the effect of different momentum control variables on radar data assimilation, only radial velocity is assimilated without radar reflectivity. In real practice, both radial velocity and radar reflectivity are essential to improve multi-variables in terms of dynamical and microphysics conditions. Therefore, this study will discuss the impact of radar radial velocity and reflectivity assimilation on the Southwest Chinese Vortex precipitation forecast based on different momentum control variables.

The structure of this paper is as follows. In Section 2, the cost function of WRFDA and the radar observation operator will be introduced. Experimental configurations and case presentations will be provided in Section 3. Section 4 examines the experiment result, including the performance of single observation test, the impact of radar data assimilation and the forecast results. Conclusions and discussion are presented in Section 5.
2. Materials and Methods

2.1. Cost Function in the WRFDA

The WRF-3DVAR system is used in the study and the cost function in the data assimilation system is listed as

\[
f(x) = \frac{1}{2}(x - x_b)^T B^{-1} (x - x_b) + \frac{1}{2} [y_o - H(x)]^T R^{-1} [y_o - H(x)]
\]  (1)

The left side of the Equation (1) is the objective function, also known as the cost function. The first term on the right is the distance from the background field \( x_b \) weighted by the inverse matrix of the background error covariance \( x \), where \( x \) is the atmospheric state variable, \( x_b \) denotes the background field of the atmospheric state, and \( B \) is the background error covariance matrix. The second term on the right is the distance to the observation \( y_o \), weighted by the inverse matrix of the observation error covariance \( x \), where \( y_o \) stands for the observation. In addition, \( H(x) \) denotes the observation operator and \( R \) is the covariance matrix of the observation error. The analytical field \( x^{ana} \) produced by the 3D-Var assimilation is the solution with the smallest objective function \( f(x) \). Because the background error covariance matrix \( B \) is too large to store or compute, \( B \) is often simplified by using a new set of completely uncorrelated analysis variables called control variables. Using \( x' = x - x_b = UV \) with \( H(Uv + x_b) = H(Uv) + H(x_b) \), Equation (1) is modified as

\[
f(v) = \frac{1}{2} v^T v + \frac{1}{2} [y_o - H(Uv)]^T R^{-1} [y_o - H(Uv)]
\]  (2)

where \( v \) is control variable, \( y_o = H(x_b) \) is the innovation vector, \( H \) is the linearization operator of observation operator, and \( U \) is the transformation operator of the control variables. As Equation (2), after transforming the control variables, \( B = UU^T \) is implied in \( H \). As \( U \) includes horizontal transformation \( (u_h) \), vertical transformation \( (u_v) \), and physical transformation \( (u_p) \) [21], \( B \) can be described as \( B = U_0 U_1 U_2 U_3 U_4 U_5 \). In the WRF-3DVAR, there are two different sets of momentum control variables which are \( \psi - \chi \) and \( U - V \) related to Control Variable Option 5 (CV5) and Control Variable Option 7 (CV7) respectively in WRFDA. To be specific, CV5 includes stream function \( (\psi) \), unbalanced velocity potential \( (\chi) \), and pseudo-relative humidity \( (RH_h) \). The transform \( U_p \) is defined as

\[
\nabla^2 \psi = \frac{\partial \chi}{\partial y} \quad \text{and} \quad \nabla^2 \chi = \frac{\partial \psi}{\partial x} - \frac{\partial \chi}{\partial y}
\]  (3)

On the other hand, CV7 applies latitude wind \( (u) \), longitude wind \( (v) \), surface pressure \( (P_s) \), temperature \( (T) \), and relative humidity \( (RH_r) \) as the control variables. In the experiments, the background error covariance \( B \) is derived from a forecast difference ensemble generated by the WRF forecast during June 2019 using the “NMC method” [22], \( \psi - \chi \) control variable by setting the option CV5 while \( U - V \) control variable by setting the option CV7.

2.2. Radar Observation and Its Observation Operator

The radar data in the experiments is obtained from the Wanzhou S-band radar (Chinese Next Generation Weather Surveillance Radar 1998 Doppler, 108.683°E, 30.802°N, altitude 1031.1 m, type CINRAD/SB) for the time 2100UTC04 to 0000UTC05, June 2019. In order to improve the reliability of the radar data, quality control such as unfolding aliased Doppler velocity and removing noise is performed before the assimilation [23,24].

In WRF-3DVAR, the following formulas are used to assimilating Doppler radar data including radial velocity and reflectivity. The relation between the radial velocity and the model variables is listed as:

\[
V_r = u \frac{x - x_i}{r_i} + v \frac{y - y_i}{r_i} + (w - v_e) \frac{z - z_i}{r_i}
\]  (4)

where \((u, v, w)\) are the wind components, \((x, y, z)\) are the radar location, \((x_i, y_i, z_i)\) are the location of the radar observation, \(r_i\) is the distance between the radar and the observation, and \(v_e\) is the terminal velocity. The \(v_e\) is related to rainwater mixing ratio \( (q_r) \). For the radar
reflectivity, an indirect radar reflectivity assimilation scheme is applied [13], using retrieved rainwater and water vapor derived by adding two elements to cost function:

$$J_{qr} = \frac{1}{2} (q_r - q_r^o)^{T} B_{qr}^{-1} (q_r - q_r^o) + \frac{1}{2} (q_r - q_r^o)^{T} R_{qr}^{-1} (q_r - q_r^o)$$

(5)

$$J_{qv} = \frac{1}{2} (q_v - q_v^o)^{T} R_{qv}^{-1} (q_v - q_v^o)$$

(6)

where $q_r$ and $q_v$ represent the rainwater and water vapor of the atmospheric state, respectively; $q_r^o$ and $q_v^o$ are the retrieved rainwater and water vapor from radar reflectivity observations; and $R_{qr}$ and $R_{qv}$ are the observation error variance of rainwater and water vapor, respectively.

3. Case Overview and Experimental Setup

3.1. Synoptic Overview

On 5 June 2019, a strong convective system occurred in Chongqing, China from the west to the east, under the combined effects of a high-layer trough, low-layer vortex and surface cyclone. The process had a maximum cumulative rainfall of 99.7 mm and a maximum hourly rain intensity of 61.7 mm/h, accompanied by widely distributed short-term intense rainfall and lightning activity, and significant ground-based wind bands. Short-term heavy rainfall and ground flash activity in the western part of Chongqing is the most obvious, thunderstorm and strong wind at about 20:00 on 4 June from the western part of Chongqing, gradually moving east, the maximum wind speed appeared in Tongliang District, reaching 30.3 m/s. Heavy rainfall and thunderstorm wind caused tens of thousands of people and thousands of hectares of crops affected, causing a serious impact on people’s production and life.

Figure 1 shows the circulation situation from 1200 UTC to 1800 UTC 4 June 2019. The potential height of 1200UTC 04 (Figure 1a) shows a low vortex near the location $(28^\circ N, 104^\circ E)$ reaching below 140 gpm along with a large temperature gradient within the center. From 1200 to 1800 UTC, the isotherms extending northward from the center of the Southwest China Vortex are dense and merge with those to the north, forming a narrow northeast–southwest trending band. Narrow and dense bands of isotherms indicate areas of large temperature gradients, which generally occur near fronts. The distribution characteristics of the temperature field show that a cold front exists to the west of Chongqing at 1800 UTC, and the warm center of the warm air mass in front of the cold front is above $22^\circ$. It can be seen that there is a strong south wind before the cold front, leading the southern water vapor constantly transported to the low-pressure center from the Pacific Ocean. With the large amount of water vapor gathered in the west of Chongqing, the precipitable water reached more than 60 g/kg. At the same time, the cold front behind the dry cold air and cold front continues to strengthen in front of the wet warm air convergence, which provides favorable conditions for the subsequent precipitation.

3.2. Model and Experimental Setup

Experiments are conducted with the version 4.3 of the WRF model. The model domain applied for the experiments has a $651 \times 601$ horizontal grid and 50 vertical layers and the horizontal grid spacing is 5 km (Figure 2). The simulations are conducted with the following physical schemes: WRF Single–moment 6–class Scheme [25]; the shortwave radiation scheme of the fifth-generation [26]; the Rapid Radiative Transfer Model (RRTM) longwave radiation scheme [27]; the Yonsei University scheme (YSU) boundary layer scheme [28]; the Noah Land Surface Model for land surface processes scheme, and no cumulus parameterization scheme [29]. The NCAR/NCEP 0.25$^\circ \times 0.25^\circ$ GDAS/FNL data are used as initial and boundary conditions.

In order to compare the effect of two control variable schemes, we set up three contrast experiments as the Table 1 showing: a control experiment (CTRL) with no data assimilation, $\psi – \chi$ control variable experiment and $U – V$ control variable experiment with radial velocity and reflectivity assimilation. Figure 3 shows the experimental process: CTRL simulation
starts at 1500 UTC 04 and ends at 0600 UTC 5 June 2019 while assimilation experiment starts at 1500 UTC 04 to 2100 UTC 04 as “spin up” and then do once data assimilate every hour until 0000 UTC 5 June 2019. Finally, the analysis field after the last assimilation as the initial field for the 6 h deterministic forecast.

Figure 1. Geopotential height (black solid lines; units: gpm), temperature (red dotted line, units: K) and wind fields (wind barbs), and precipitable water (shaded; units: mm) at (a) 1200 UTC and (b) 1800 UTC 4 June 2019.

Figure 2. The WRF model domain with the altitude (shade, unit: m) also indicated is the WanZhou radar locations (blue asterisks) and its maximum range coverage circles (blue dotted line).
Table 1. Experiment scheme.

| Number | Experiment | Scheme |
|--------|------------|--------|
| 1      | CTRL       | No data assimilation |
| 2      | ψ−χ        | radial velocity and reflectivity assimilation with ψ−χ control variable |
| 3      | U−V        | radial velocity and reflectivity assimilation with U−V control variable |

Figure 3. The flow chart for the data assimilation experiments.

4. Results of Analysis and Forecasting

4.1. Single Observation Test

In order to better show the effect of radar data assimilation of different control variables on the mode variables, a single observation test is conducted. The test observation location is (30.802°N, 108.683°E), with an altitude of 3356.2 m, close to the 700 hPa layer. The radial wind speed is 10.5 m/s, the radial wind observation error is 2 m/s, and the reflectivity factor is 28 dBZ. The initial field is obtained by interpolating the NCEP/GFS reanalysis data of 2100 UTC 5 June 2019. In order to objectively compare the assimilation effects under the two control variables, it is necessary to balance their background error covariances. Therefore, we reduce the background error covariances of CV5 and CV7 to the default 0.3 and 0.4, respectively.

Since the assimilated radial winds are away from the radar, a wind increment away from the radar direction is expected as the southeast wind increment at the observation location (Figure 4). Both assimilation schemes of the control variables show a southeast wind increment at the observation point location. Figure 4a shows that there is a negative increment of x-component wind near the observation point, while a positive increment of symmetric x-component wind existed around it. The increment shows a circulation pattern when ψ−χ is applied as the control variable with mass field balance. Figure 4c shows that there are also negative increments of the x-component near the observation location and the increments of x-component and y-component wind both show a standard Gaussian distribution. Figure 4b,d show the wind increments of the y-component, which exhibit the same form as the x-component wind increments. It can also be found that although the background error covariance of CV5 is reduced to a larger extent than that of CV7, the influence range of CV5 is still significantly larger. This suggests that the real winds are better observed when using the U−V control variables, and that the background error characteristic scales are better suited for the assimilation of small-scale and meso-scale observations.
Figure 4. The increments of (a,c) U and (b,d) V wind (shaded; units: m s\(^{-1}\)) and wind vector at 700 hPa from single observation experiments using (a,b) the \(\psi - \chi\) control variable, and (c,d) U-V control variable.

4.2. The Impact on the Analysis

Single observation experiments show how radar data assimilation affects the wind. In this section, the effect of radar data assimilation has on the wind field by verifying the wind analysis field and the wind increment field are investigated.

Figure 5 shows the background field and analysis field of the wind at 21:00 on 4 June 2019. From the wind field of the 500 hPa background field (Figure 5a), it can be found that there is a strong southwest wind at the west of Chongqing, and the west side of the southwest wind band is westerly, and there is strong and obvious horizontal wind shear in the region. After assimilating the radar data (Figure 5d,g), the southwesterly winds in some areas have changed to southerly or even southeasterly winds, and the change in the \(\psi - \chi\) control variable is more obvious. It indicates that the assimilation makes the horizontal wind shear in the region enhanced, which is favorable to the enhancement of convective activities. The background field (Figure 5b) at 700 hPa, on the other hand,
exhibits a clear north–south wind shear line at 105°E with northwest winds to its west and southwest winds to its east. The eastern part of the wind shear line changes to southeasterly winds after assimilating the radar data, which is especially evident in the analyzed field of the $\psi-\chi$ control variable (Figure 5e). At 850 hPa (Figure 5c), a cyclone appears in the west of Chongqing, which is the previously mentioned southwest vortex, and it has moved further northwestward compared to 1800 UTC. After the assimilation (Figure 5f,i), the front bottom of this low vortex has an enhancement of southwest winds and the wind of the $\psi-\chi$ control variable is changed more pronounced. Generally, the radar data assimilation makes the wind shear enhanced and convective activity enhanced, while the assimilation under the $\psi-\chi$ control variable changes the wind field more significantly.

Figure 5. (a–c) background and ($\psi-\chi$: d–f; $U-V$: g–i) analysis wind speed (shaded; units: m s$^{-1}$) and wind vector at 500, 700, 850 hPa at 2100 UTC 4 June.
To better show wind field change under two control variables, Figure 6 shows the increment of wind field obtained by assimilating the radar data with two control variables at 2100 UTC 4 June 2019. After assimilating the radar data, both sets of experiments exhibit significant wind increments in Chongqing West. ψ−χ control variables exhibit strong easterly wind increments in the middle and upper layers (Figure 6a,b), while there are cyclonic wind field increments generated, which can also suggest that ψ−χ control variables can cause the u/v variables to not satisfy the uncorrelated hypothesis condition during variational assimilation. In contrast, the wind increments of the U−V control variables (Figure 6d,e) are smaller and more concentrated. For the two data assimilation experiments, the areas of the large wind increments are closed. At 850 hPa, there is an incremental northeasterly wind for both (Figure 6c,f), which essentially weakens the top-front wind field of the low vortex, and the ψ−χ control variable set has a larger range.

In conclusion, the U/V control variables assimilate the radar data while changing the atmospheric state outside the radar data coverage less, which is consistent and convincing with the expected effect of assimilation.

Figure 6. The increments of wind vector at 500 hPa, 700 hPa and 850 hPa after the assimilation of WZRD radial velocity and reflectivity using (a–c) the ψ−χ control variable and (d–f) U−V control variable. (Shaded; units: m s\(^{-1}\)).

The effect of radar data assimilation on the wind field is discussed in the previous analysis, which inevitably also leaves some traces in the change of radial velocity, since radial velocity is derived from the wind field. To further examine the effect on the effect of assimilating radar information under different control variables, the variation of the root mean square errors (RMSE) of radar radial velocity before and after assimilation for the two sets of experiments is shown in Figure 7. It can be found that the RMSE decreases after each assimilation within the assimilation window, and the RMSE decreases the most after the first assimilation of radial velocity and reflectivity of the radar, by about 2.8 m/s for the ψ−χ control variable test and by about 3.2 m/s for the U−V control variable experiment. This is caused by the weak wind simulation on the background field of the model during the first assimilation, so the increment of the first assimilation is also the largest.
the later assimilation sessions, the RMSE of the $\psi - \chi$ control variable experiment stabilized around 2.4 m/s, while the error of the $U-V$ control variable experiment was much smaller, below 2 m/s. Overall, although the RMSE decreased after assimilating radial wind and reflectivity for different control variables compared with those before assimilation, the errors under the $U-V$ control variable were significantly lower than those under the $\psi - \chi$ control variable throughout the assimilation time. It indicates that $U-V$ as a control variable is more advantageous in the variational assimilation.

![Figure 7](image_url)

**Figure 7.** The forecast and analysis (sawtooth pattern during DA cycling) of RMSE of radial velocity (m s$^{-1}$) for $\psi - \chi$ and $U-V$ control variable from 2100 UTC 04 June to 0000 UTC 5 June 2019 for the 4 data assimilation cycles indicated in x axis.

The effect of the data assimilation is further examined on the rain band simulations by examining the composite radar reflectivity of the analysis and forecast. Figure 8 shows the comparison of the echoes at the four analysis moments added to the radar data for assimilation. Since the information from a single radar such as Wanzhou Radar is selected for the real case, the comparison is made only within the observation range of this radar. As can be seen from Figure 8e–h, the convective system of CTRL shows a southwest–northeast zonal echo, while the observation data is changing from zonal (2100 UTC) to block echo. The echo range of CTRL is obviously narrow, and the structure and movement process of convective system is not obvious. The assimilation experiments then roughly simulate the location and extent of the convective system. The two sets of assimilation experiments at 2300 UTC simulate the main body of the convective system and are closer in location and range to the real situation than CTRL. Where the echo range of the $\psi - \chi$ control variable is slightly larger than that of the observation, while the echo range of the $U-V$ control variable converges more toward the observation range, i.e., the echo location of the $U-V$ control variable is more accurate than that of the $\psi - \chi$ control variable. At 0000 UTC, the echo map also shows that the echo range of the $U-V$ control variable is more accurate and convergent than that of the $\psi - \chi$ control variable. The strong echoes in the $\psi - \chi$ control variable are loosely distributed in a point-like pattern and do not match the observation. The $U-V$ control variable shows a south–north band-like strong echo, and it can be seen in the observation that there is indeed a south–north band-like strong echo on the west side of the Wanzhou radar. It is indicated that the $U-V$ control variable is more consistent with
the real situation. It is concluded that the composite reflectivity simulated by CTRL is quite imprecise, while the accuracy of the assimilation experiment with radar data assimilation is much improved. For the radar data assimilation experiments, the performance of the $U-V$ control variable is better than that of the $\psi-\chi$ control variable in terms of simulation of the radar reflectivity.

Figure 8. The composite reflectivity (shaded; units: dBZ) from observation (a–d), CTRL (e–h), $\psi-\chi$ (i–l), $U-V$ (m–p) at 2100 UTC 04 (the first column), 2200 UTC 04 (the second column), 2300 UTC 04 (the third column) and 0000 UTC 05 (the last column).

After four cycles of assimilation, a 6-h deterministic forecast is launched. Since the live conditions are selected from a single radar in Wanzhou, the comparison is made only within the observation range of that radar. The composite reflectivity at four forecast moments is shown in Figure 9 at 0100, 0200, 0300, and 0400 UTC 5 June 2019. From the observation plots (Figure 9a–d), it can be seen that the convective system is moving eastward, and the main body moves out of Chongqing at 0400 UTC. The location, range, and intensity of strong echo are unrealistic in CTRL. It seems the convective system moved obviously too fast that the main body has moved out of Chongqing at 0300 UTC in CTRL. In the contrast, the forecast echoes obtained from the assimilation experiments are more accurate. The range of strong echoes of the $\psi-\chi$ control variable is larger and more concentrated than
CTRL range with a relative accurate location of the strong echoes. However, the radar reflectivity is over-estimated to some extent using the $\psi - \chi$ control variable. On the other hand, the hourly radar reflectivity forecasts from the experiment using the $U - V$ control variables are more concentrated in the strong echo region compared to experiments using the $\psi - \chi$ control variables. To be specific, at the 0200 UTC, observations show a comma-like echo shape, while $\psi - \chi (j)$ has a looser echo shape. It is found that CTRL (b) and $U - V (n)$ both predicted the southern rain band, while $\psi - \chi (j)$ failed. At 0400 UTC, the comma echo of observation (d) is more elongated, and similar comma echoes appear in both simulated experiments. Compared the three experiments of CTRL, $\psi - \chi$ and $U - V$, the echo shape of CTRL (h) is looser demonstrating discontinuity of the echoes on the south side. Data assimilation experiments shows the similarity of more concentrated echoes, except at the top of Chongqing. An extreme echo is found in $\psi - \chi (l)$ while an obviously weakened echo is shown in $U - V (p)$. In general, the echoes of the $U - V$ control variable experiment reflect a clear advantage, and its forecasting effect is significantly better than that of the CTRL and $\psi - \chi$ control variable experiment.

Figure 9. The same as Figure 8 but at 0100 UTC (a,e,i,m), 0200 UTC (b,f,j,n), 0300 UTC (c,g,k,o), and 0400 UTC (d,h,l,p) 5 June 2019.
To quantitatively describe the effect of the three experiments on composite reflectivity, Threat Score (TS) and Equitable Threat Score (ETS) with thresholds of 30 dBZ for the last 3 h of the experiments are shown in Figure 10. The scores for both assimilation experiments are higher than that for CTRL. At 0400 UTC and 0500 UTC, the scores of $U-V$ control variable and $\psi-\chi$ control variable experiments are similar, the scores of the $U-V$ control variable experiment are slightly higher than those of $\psi-\chi$ control variable experiment. At the last verification hour, both ETS and TS for $U-V$ control variable experiment are about 0.05 higher than that in the $\psi-\chi$ control variable experiment. The TS and ETS are consistent with the previous results from the Figure 9 and quantify that result to make it more intuitive.

![Figure 10. ETSs and TSs of composite reflectivity from 0400 UTC to 0600 UTC at the 30 dBZ thresholds for three experiments (CTRL: green line, $\psi-\chi$ control variable: blue line, $U-V$ control variable: red line).](image)

To investigate how the composite reflectivity in the previous analysis occurs, the perturbation potential and perturbation pressure are further illustrated at 925 hPa at 2100 UTC. Figure 11 shows the perturbation potential temperature field and the perturbation pressure field at 925 hPa at the time of the first assimilation (2100 UTC). From the perturbed potential temperature fields (Figure 11a–c), it can be found that there is an obvious cold pool structure at the west of Chongqing. The cold pool is generated due to the strong precipitation dragging and evaporative cooling, which causes the temperature of the lower layers to drop. The potential temperature gradient at the cold pool front can reach more than 4 K. This not only causes a lift trigger, but also can force the enhancement of vertical wind shear at the lower layer, leading to the strengthening of boundary layer updrafts and the development of convection. The lowest perturbed potential temperature in the center of the cold pool can reach $\sim$4 K, and its location corresponds to the strong convective echo region of the convective system. Comparing the three sets of experiments, it is found that the cold pool structure of the $\psi-\chi$ control variable assimilation experiment is stronger than that of other two. There is an expansion of the region reaching below $-4$ K in the cold pool range, while it is not obvious for the $U-V$ control variable. Figure 11e shows the perturbed potential temperature increments under the $U-V$ control variable, and it can be seen that the magnitudes of the changes are small, all below 0.1 K. In contrast, the perturbed potential temperature increment under the $\psi-\chi$ control variable (Figure 11d), the overall range of perturbed potential temperature is reduced and the warm area in front
of the cold pool is also cooled. The temperature gradient at the edge of the cold pool is not changed much, the convection facilitation will not be so obvious and the precipitation will not be enhanced too much. It supports the phenomenon that the radar composite reflectivity of the analyzed fields does not differ much at this moment.

Figure 11. The perturbation potential (a–e) and perturbation pressure (f–j) from CTRL (a,f), $\psi - \chi$ (b,g), $U - V$ (c,h), $\psi - \chi$ minus CTRL (d,i) and $U - V$ minus CTRL (e,j) at 925 hPa at 2100 UTC 4 June 2019.

Since the cold pool itself is composed of a high density of cold air and is a high-pressure area (thunderstorm high pressure) compared to its surroundings, the formation of a strong cold pool creates a strong thunderstorm high pressure near the ground. From Figure 11f–g, thunderstorm high pressure centers exist at the Sichuan–Chongqing junction for all three sets of experiments. Among them, the $\psi - \chi$ control variable assimilation experiment has the strongest high pressure, the $U - V$ control variable assimilation experiment is the second strongest, and the control experiment is the smallest. From the difference field, it can be seen that the $\psi - \chi$ control variable assimilation experiment has a significant barometric pressure enhancement of 1 hPa near the west-central Chongqing, and the center of this increment of barometric pressure coincides with the region of strong radar echoes. In contrast, the differential field of the $U - V$ control variables does not show a clear incremental center, which is a large area of weak barometric pressure enhancement coupled with an irregular weakening of a small part of the area. Collectively, a distinct cold high-pressure system with a cold pool structure is present in western Chongqing, while the $\psi - \chi$ control variable assimilation test significantly enhances this cold high-pressure system in terms of potential temperature and barometric pressure compared to CTRL, and the $U - V$ control variable assimilation experiment is less altered.

It is well known that atmospheric precipitation is closely related to the source and sink of water vapor. The water vapor flux divergence is a diagnostic quantity which is helpful to analysis vapor. Figure 12 shows the water vapor flux divergence for the three sets of experiments at 850 hPa at 0000 UTC 5 June 2019. The negative water vapor flux divergence indicates that there is water vapor convergence, and the smaller the value, the stronger the convergence. From Figure 12, a strong low vortex is formed in west–central Chongqing, and the water vapor irradiation is located at the front of the low vortex. From the radar echo (Figure 8d,h,i,p), it can be found that the locations where the dispersion of the water vapor flux is smaller for the three sets of experiments consistent with exactly where the reflectivity is larger. The water vapor convergence area in CTRL (Figure 12a)
is matching the observed strong echo region, while the $\psi-\chi$ control variable experiment (Figure 12b) has stronger water vapor convergence than that in the CTRL. In addition, there is more than $3 \times 10^{-6}$ g (hPa cm$^2$ s$^{-1}$) water vapor convergence, which is likely to cause the excessive forecast precipitation. The $U-V$ control variable experiment is closer to CTRL and stronger at $(31^\circ N,108^\circ E)$ than CTRL, which also leads to increased precipitation and thus enhanced radar reflectivity at this location. The above analysis shows that the $\psi-\chi$ control variable experiment resulted in a negative impact of reduced water vapor flux divergence and excessive water vapor irradiation in the precipitation region.

![Figure 12. Horizontal wind vector and water vapor flux divergence (shaded; units: 10$^{-6}$ g cm$^{-2}$ hPa$^{-1}$ s$^{-1}$) at 850 hPa at 0000 UTC 5 June 2019: (a) CTRL; (b) $\psi-\chi$ control variable; (c) $U-V$ control variable.](image)

The winds obtained from the $\psi-\chi$ control variable tend to develop toward the cyclone, which can be seen from the increment of wind field. It will additionally enhance the cyclone west of Chongqing to enhance the water vapor flux divergence. In contrast, the winds of $x/y$ component of the $U-V$ control variables are independent of each other and therefore do not exhibit as pronounced convergence enhancement. Therefore, in terms of the water vapor flux divergence, the $U-V$ control variables are better simulated.

4.3. The Forecast of Precipitation

In the previous section, changes in the model variables were analyzed. In this section, the final revealed form of these changes in precipitation will be objectively compared in the differences between the three sets of experiments by 6 h cumulative rainfall and ETS.

The 6 h cumulative precipitation for the observations and the three sets of experiments are shown in Figure 13. Precipitation observations are from ground meteorological observation station. From the observation (Figure 13a), it is found that the precipitation mainly occurs in the junction area of Chongqing, Shanxi, and Hubei province, with one heavy precipitation center in the north and one in the south. The heavy precipitation center in the north is located at the north of Chongqing (center at about $31.8^\circ N$), and its northeast is the area of its larger precipitation value (>25 mm); the heavy precipitation center in the south is located at the junction of Chongqing East and Hubei (center at about $29.5^\circ N$), forming a southwest-northeast precipitation belt. For the northern rain band: the main area of this precipitation is forecasted on CTRL (Figure 13b), and the larger value area of precipitation to the north is forecast to be flatter, smaller in extent, and weaker in intensity compared to the observed precipitation. Meanwhile, the center of heavy forecast precipitation by CTRL is scattered at the top of Chongqing, which is located northward compared with the actual situation. Compared with CTRL, the $\psi-\chi$ control variable assimilation experiment forecasts the heavy precipitation center well, and its location is in
good agreement with the observation. However, its precipitation intensity is somewhat larger, especially for the extreme area with accumulated precipitation exceeding 100 mm. Compared with $\psi - \chi$ control variable, the $U-V$ control variable assimilation experiment also forecasts the intense precipitation center of the rain band, with lighter intensity of the intense precipitation center than that of $\psi - \chi$ control variable. The phenomenon of the overestimated precipitation is alleviated with the accumulated precipitation less than 100 mm for almost the whole domain.

Figure 13. 6 h cumulative precipitation (shaded; units: mm). (a) Observation, (b) CTRL, (c) $\psi - \chi$ control variable, (d) $U-V$ control variable.

Regarding of the south rain band, the shape of the rain band in the south of CTRL is not obvious, and there is excessive forecast precipitation in Hubei province, with some heavy precipitation that does not actually exist, and the location of the heavy precipitation is not concentrated, and the location of the center of heavy precipitation in the south of the rain band is miss. $\psi - \chi$ control variable assimilation experiment also misses the center of heavy precipitation, and the precipitation area is not accurate, with excessive forecast precipitation in central Chongqing and excessive forecast precipitation in Hubei, which is different from the observation. $U-V$ control variable assimilation experiment better forecasts the band structure of the southern rain band, and the precipitation area is more convergent than $\psi - \chi$ but it still does not show the location of the center of heavy precipitation. For that, it may be related to the fact that only Wanzhou radar data are assimilated, and the information on the southern rain band is not well captured. Overall, the WRF–3DVar assimilation experiments have substantially improved the forecasts compared to CTRL though also has
some deviations to the actual observed precipitation. Besides, the $U-V$ control variable assimilation experiment forecasts are better than the $\psi-\chi$.

To quantitatively describe the effect of the three experiments on precipitation forecasting, ETS with thresholds of 3 mm/h, 5 mm/h, and 10 mm/h for the last 6 h of the experiments are shown in Figure 14. From the precipitation forecasts at all layers throughout the forecast period, it can be found that the ETS under the $U-V$ control variable is almost 0.1 higher than $\psi-\chi$ control variable. At 0100 UTC with 10 mm/h threshold, $\psi-\chi$ control variable’s source is significantly higher than $U-V$ control variable. The outlier probably caused by the excessive precipitation at some points. In general, the application of the $U-V$ control variable has significantly improved the precipitation forecast skill. The $\psi-\chi$ control variable, on the other hand, is probably due to the excessive change in the flow field in the area beyond the radar coverage due to the large characteristic scale, and although the cumulative precipitation forecast near the radar is closer to the observation than CTRL as can be seen in Figure 13, the ETS for the area doing the overall precipitation scoring drops even less than CTRL.

![Figure 14. ETSs of 1-h accumulated precipitation from 0100 UTC to 0600 UTC at the 3 mm$^{-1}$ (a), 5 mm$^{-1}$ (b) and 10 mm$^{-1}$ (c) thresholds for three experiments (CTRL: green line, $\psi-\chi$ control variable: blue line, $U-V$ control variable: red line).](image-url)
5. Conclusions

Radar data assimilation is able to improve the forecast of Southwest China Vortex precipitation, and the use of different momentum control variables will have different effects on the analysis. This study investigates the impact of radar radial velocity ($V_r$) and reflectivity ($Z$) assimilation on the simulation and forecasting of Southwest China Vortex precipitation using the WRF-3DVAR system based on the comparison of different momentum control variables ($\psi - \chi$ and $U - V$). For a rainfall case on 5 June 2019, a single observation experiment is conducted before assimilating the radar data and then the analysis and forecast are explored.

As the single observation test and wind analysis shown that $U - V$ as the control variable strengthens the wind speed and vorticity to be better matching the observation, while $\psi - \chi$ as the control variable will produce too large unphysical increments. The RMSE of radar radial velocity decreases significantly after each assimilation and is lower for $U - V$ as a control variable than for $\psi - \chi$ control variable. In terms of the composite reflectivity, the analysis field of $U - V$ as a control variable is more in accordance with the observation than that of the $\psi - \chi$ control variable. This effect is far-reaching and leads to better forecasts for the $U - V$ control variable than the $\psi - \chi$ control variable as well, implying that the $U - V$ control variable forecasts more accurate rain band locations. In terms of the surface perturbation potential temperature, there is an enhancement of the cold pool structure and a deepening of the cold high pressure for the $\psi - \chi$ control variable, while the change is not very significant under the $U - V$ control variable. In the last data assimilation cycle, the water vapor fluxes are ultra to converge in the lower layers of the $\psi - \chi$ control variable, which probably caused the excessive forecast precipitation. The $U - V$ control variable gets the higher ETS with more accurate precipitation location. In summary, $U - V$ control variable is superior to $\psi - \chi$ control variable in terms of analysis and forecasting about Southwest China Vortex precipitation. Although the results from the $U - V$ control variables is more promising than $\psi - \chi$ control variable in terms of the momentum variables, other choice of the momentum control variables will also be considered such as the vorticity and divergence. It is of also of interest to explore the impact of the adding other radar reflectivity related control variables that are affecting the microphysics schemes to introduce more thermodynamic information from the reflectivity [33,34].

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