Cascade Birth of Universes in Multidimensional Spaces

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Abstract—The formation mechanism of universes with distinctly different properties is considered within the framework of pure gravity in a space of \( D > 4 \) dimensions. The emergence of the Planck scale and its relationship to the inflaton mass are discussed.

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1. INTRODUCTION

The dynamics of our Universe is well described by a modern theory containing 30–40 parameters. The number of these parameters, whose values are determined experimentally, is too large for the theory to be considered final. In addition, it is well known that the range of admissible parameters must be extremely narrow (fine tuning of parameters) for the birth and existence of such complex structures as our Universe, which is difficult to explain. Extensive literature is devoted to a discussion of this problem [1]. One way of solving the problem is based on the assumption about the multiplicity of universes with different properties [2–4]. Rich possibilities for justifying this assumption are contained in the idea of multidimensionality of our space itself. The number of extra dimensions has long been a subject for debate. For example, the Kaluza–Klein model originally contained one extra dimension. At present, infinite-dimensional spaces [5] and even variable-dimensional spaces [6] are being discussed. In this paper, the concept of superspace is extended to a set of superspaces with different, unlimited (above) numbers of dimensions. Based on the introduced extended superspace, we suggest the formation mechanism of universes with distinctly different properties and the emergence mechanism of the Planck scale. The probability of the quantum transitions that produce lower-dimensional subspaces is discussed.

Let us define the superspace \( M_D = (M_D^p, g^p_{ij}) \) as a set of metrics \( g_{ij} \) in space \( M_D^p \) to within diffeomorphisms. On a space-like section \( \Sigma \), let us introduce a metric \( h_{ij} \) (for details, see the book [7] and the review [8]) and define the space of all Riemannian \((D - 1)\) metrics:

\[
\text{Riem}(\Sigma) = \{ h_{ij}(x) \mid x \in \Sigma \}.
\]

The amplitude of the transition from one arbitrarily chosen section \( \Sigma_{in} \) with the corresponding metric \( h_{in} \) to another section \( \Sigma_f \) with a metric \( h_f \) is

\[
A_{f, in} = \langle h_f, \Sigma_f | h_{in}, \Sigma_{in} \rangle = \int Dg \exp [iS(g)]. \quad (1)
\]

In what follows, we use the units \( \hbar = c = 1 \). The topologies of the sections \( \Sigma_{in} \) and \( \Sigma_f \) can be different. We will be concerned with the quantum transitions in which the topology of the hypersurface \( \Sigma_f \) is a direct product of the subspaces, \( M_{D - 1 - d} \otimes M_d \). The space \( M_f \) is assumed to be compact. Below, we will explore the question of what class of geometries on the hypersurface \( \Sigma_f \) can initiate classical dynamics.

The entire analysis is performed within the framework of nonlinear gravity in a space of \( D > 4 \) dimensions without including any matter fields. We discuss the emergence of the Planck scale and its relationship to the inflaton mass. The reduction in a lower-dimensional space is made in several steps to produce a cascade. Different cascades give rise to four-dimensional spaces with different effective theories and different numbers of extra dimensions.

The parameters of the low-energy theory turn out to depend on the topology of the extra spaces and vary over a wide range (see also [9, 10]), although the parameters of the original theory are fixed. This also applies to such fundamental concepts as, for example, the Planck mass and the topology of the extra space.

The absence of matter fields postulated here from the outset is a fundamental point. It is suggested that the metric tensor components for the extra (super)space at low energies will be interpreted as the matter fields in the spirit of theories like the Kaluza–Klein theory.
2. THE FORMATION OF SPACETIME AND PARAMETERS OF THE THEORY

Originally, the concept of superspace meant the set of various geometries [11]; subsequently, the set of all possible topologies was included in it [7]. Let us take the next step and extend the superspace to include spaces of various dimensionalities. To be more precise, let us define the extended superspace $E$ as a direct product of superspaces $M$ of various dimensionalities:

$$ E = \mathcal{M}_1 \otimes \mathcal{M}_2 \otimes \mathcal{M}_3 \otimes \cdots \otimes \mathcal{M}_D \cdots $$

(2)

Here, $\mathcal{M}_d$ is a superspace of dimensionality $n = 1, 2, \ldots$, which is the set of all possible geometries (to within diffeomorphisms) and topologies.

Quantum fluctuations generate various geometries in each of the superspaces (spacetime foam) [7, 11]. The probability of the quantum birth of “long-lived” 3-geometries and the conditions under which this occurs are discussed below in Section 4. In this section, we consider the corollaries of the hypothesis about the existence of an extended superspace.

Let us choose a space $M_D$ of some dimensionality. Its topological structure can change under the effect of quantum fluctuations [7, 12]. In particular, topologies are possible that admit of space foliations by space-like surfaces $\Sigma$, as implied in Eq. (1) for the transition amplitude. In what follows, we consider spaces that admit of a partition in the form

$$ M_D = \mathbb{R} \otimes M_D^{(\text{space})}, $$

(3)

where $\mathbb{R}$ is the time-like direction.

Let us concretize the topology and metric on the space-like section $\Sigma_f$ of amplitude (1) by subjecting them to the following conditions:

(i) the topology of the section $\Sigma_f$ has the form of a direct product,

$$ \Sigma_f = M_D^{(\text{space})} = M_{D_1} \otimes M_{d_1}, $$

(4)

where $D_1$ and $d_1$ are the dimensionalities of the corresponding subspaces (in what follows, the compact subspace is denoted by $M_{d_n}$, $n = 1, 2, \ldots$);

(ii) the condition for the curvatures of the subspaces $M_{D_1}$ and $M_{d_1}$ is satisfied:

$$ R_{D_1}(g_{ab}) \leq R_{d_1}(\gamma_{ij}); $$

(5)

(iii) in the set of subspaces $M_{d_1}$, we will choose the maximum symmetric spaces with a constant curvature $R_{d_1}$, which is related to the curvature parameter in the standard way:

$$ R_{d_1}(\gamma_{ij}) = kd_1(d_1 - 1). $$

(6)

Otherwise, the topology and geometry of the subspace $M_{D_1}$ are arbitrary.

Let us choose the dynamical variables and the Lagrangian. We will write the metric of the space $M_D$ as [13]

$$ ds^2 = G_{AB}dx^A dx^B $$

$$ = g_{ac}(x)dx^a dx^c - b^2(x)\gamma_{ij}(y)dy^i dy^j $$

(7)

Here, $g_{ac}$ is the metric of the subspace $R \otimes M_{D_1}$, with signature (+ − − − − −), $b(x)$ is the radius of curvature of the compact subspace $M_{d_1}$, and $\gamma_{ij}(y)$ is its positively defined metric. For a given foliation of the space by space-like surfaces, we can always choose the normal Gaussian coordinates, which is used in the last equality in (7).

The Einstein–Hilbert action for a gravitational field, linear in curvature $R$, completely describes the physical phenomena at low energies where gravity is important. However, it is clear that quantum effects inevitably lead to nonlinear corrections in the expression for the action [14]. In this case, the action contains terms with higher derivatives in the form of polynomials of various degrees in Ricci scalar and other invariants. Thus, whatever the gravitational action we take as the basis, after applying the quantum corrections, it takes the form

$$ S = \int d^N x \left( R + \epsilon_1 R^2 + \epsilon_2 R^3 + \epsilon_3 R^4 + \ldots + \alpha_1 R_{AB} R^{AB} + \ldots \right) $$

with a set of unknown coefficients dependent on the topology of the space [15–17]. However, the problem is not that acute, since the nonlinear (in Ricci scalar) theories can be reduced to the linear theory by a conformal transformation [18]. Moreover, in our papers [19], we suggest a more general method for reducing arbitrary Lagrangians to the standard Einstein–Hilbert form in the low-energy limit. In this case, the problem of stabilizing the sizes of the extra dimensions [13] turns out to be quite solvable.

Thus, using theories with higher derivatives is inevitable when the quantum effects are taken into account. At low energies, such theories can provide the same predictions as general relativity, while having richer possibilities. For simplicity, we will restrict ourselves to an action quadratic in Ricci scalar:

$$ S_D = \frac{1}{2} \int d^D x \sqrt{-G} \left( R_D(G_{AB}) + CR_D(G_{AB})^2 - 2\Lambda \right) + \int_{\partial M_D} K d^{D-1} \Sigma. $$

(8)

The contribution from the boundary $\partial M_D$ is a term introduced by Hawking and Gibbons ($K$ is the spur of the second fundamental form of the boundary [20]). In the subsequent analysis, the parameters $C$ and $\Lambda$ are