Impact response of a nonlinear viscoelastic auxetic plate

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Abstract. Impact induced geometrically non-linear vibrations of a viscoelastic plate are investigated for the case when the shear operator is governed by the fractional derivative Kelvin-Voigt model in conjunction with the time-independent coefficient of volume extension-compression. Such a model could describe the behavior of so-called auxetic materials with negative Poisson’s ratios. The modified method of multiple time scales involving the expansion of the fractional derivative in terms of a small parameter has been utilized for solving nonlinear governing equations of motion. The asymptotic behavior of the roots of the characteristic equation for determining the frequencies of the system as a function of the retardation time is studied and three modes of vibration in the process of impact interaction are described.

1. Introduction
Nonlinear vibrations of plates caused by different dynamic loads are an important area of applied mechanics, since they can lead to dangerous non-linear phenomena. In the majority of articles on this topic, studies were carried out for a conventional isotropic plate, the material of which is described by the Hooke's law [1-3]. But recently widely spread a new class of materials called auxetics which exhibit negative Poisson’s ratio (NPR). In contrast to conventional materials (like rubber, glass, metals, etc.) they expand transversely when pulled longitudinally and contract transversely when pushed longitudinally [4].

The work, wherein materials with $\nu < 0$ were mentioned for the first time is dated by 1944 [5]. It was reported that crystals of FeS$_2$ have $\nu \approx -0.14$, but more recent studies have not confirmed this information. In 1987, Wojciechowski [6] proposed the first model of a two-dimensional isotropic thermodynamically stable systems with $\nu < 0$, composed of non-spherical particles. Potential interest in these materials has increased after the publication by Lakes the results of tests on synthesized polyester foam with $\nu = -0.7$ [7]. These new types of materials were named "auxetics" by Evans [8,9].

Auxetics cause great interest among scientists due to their special physical and mechanical properties such as resistance to indentation, fracture toughness, shear resistance, viscoelastic properties, acoustic absorption, shape memory, variable permeability, etc. [10-15]. Today auxetic properties have been identified in various classes of materials: composites, porous and granular materials, including a variety of polymers and foams, crystals of different compounds, natural and biological objects [16-18]. Much attention is paid to the synthesis of man-made auxetic materials, manufactured by using the modern technologies. A few studies have examined the modeling of metamaterials with negative Poisson's ratio by creating various geometrical configurations [8,9,19-21].

Auxetics have potentially important applications in the nearest future and can be advantageously used in the development of hydrophones and other sensors, for fiber reinforcement in composites,
shock and sound absorbers, fasteners and rivets, air filters, for thermal protection in aerospace, in manufacturing defense protective clothing, etc. [16,22].

During the past 20 years many researchers have studied the behavior of materials with negative Poisson's ratio in various dynamic problems. Argatov et al. [23] studied implications of the negative Poisson's ratio with regard to the quasi-static and dynamic indentation compliances for isotropic homogeneous materials. Aw et al. [23] considered the deformation of thin auxetic membrane structures under indentation. It is shown that negative Poisson’s ratios have direct influence on the membrane deformation domain, including the force–displacement curve, the deflection profile and the contact area. Alderson and Coenen [25] studied the problem of the low velocity impact response of auxetic carbon fibre laminates. Allen et al. [26] considered an analogous problem for auxetic and conventional open-cell polyurethane foams. The auxetic samples displayed a six times reduction in peak acceleration, showing potential in impact protector devices such as shin or thigh protectors in sports equipment applications. Strek et al. [27,28] studied the dynamic response of sandwich panels with auxetic cores. Jin et al. [29] extended this problem for sandwich structures with graded auxetic honeycomb cores under blast loading. Results show that both the graded honeycomb cores and cross-arranged honeycomb cores can significantly improve the resistance ability of the sandwich structures under blast loading.

Lim [30,31] considered thin auxetic plates and shells and found that the bending moment under uniform loading of circular plates suggests that the optimal Poisson’s ratio is −1/3 if the plate is simply-supported at the edge. Auxetic materials are not suitable for uniformly loaded and simply supported square plates, as moment minimization study requests an optimal Poisson’s ratio of 0.115, but are highly suitable for central point loaded and simply supported square plates, as moment minimization study suggests an optimal Poisson’s ratio of −1.

However, as far as the authors know, the majority of papers in the field delivered the results obtained by experimental methods for studying structures [20, 23-29, 32], while the theory of mathematical modeling of auxetic behavior is only under development [30,31,33].

In the present paper, which continues the research started in [34], the problem of the impact interaction of an elastic sphere and a viscoelastic plate is considered.

2. Problem formulation
To describe the damping features of the plate, the shear operator is preassigned in terms of the fractional derivative Kelvin-Voigt model

\[ \tilde{\mu} = \mu_0 [1 + (\tau_0^{\nu})^\gamma \tilde{D}_0^{\nu}], \]  

(1)

where \( \mu_0 \) is the relaxed shear modulus, \( \tau_0^{\nu} \) is the retardation time, \( \gamma(0 < \gamma \leq 1) \) is the fractional parameter, \( \tilde{D}_0^{\nu} \) is the Riemann-Liouville fractional derivative

\[ D_0^{\nu} x(t) = \frac{d}{dt} \int_0^t \frac{x(t')dt'}{\Gamma(1 - \gamma)(t-t')^{\gamma}}, \]  

(2)

\( \Gamma(1 - \gamma) \) is the Gamma function, and \( x(t) \) is an arbitrary function.

The bulk operator is assumed to be time-independent \( \tilde{K} = K_0 = \text{const} \), i.e.

\[ \tilde{E} = \frac{9K_0 \tilde{\mu}}{3K_0 + \tilde{\mu}}. \]  

(3)

It has been shown that such a model could describe the behaviour of so-called auxetic materials with negative Poisson's ratios [35].

Let us write Poisson’s operator \( \tilde{\nu} \) in the form [35]
\[ \ddot v = -1 + \frac{E_0}{2\mu_0} \dot \gamma (t_\nu^\nu), \]  

(4)

where \( \dot \gamma (t_\nu^\nu) \) is the dimensionless Rabotnov’s fractional operator defined in [36] as follows

\[ \dot \gamma (t_\nu^\nu) = \frac{1}{1 + t_\nu^\nu D_\nu}. \]  

(5)

Assume that an elastic or rigid sphere of mass \( M \) moves along the \( z \)-axis towards a simply supported thin rectangular nonlinear plate with thickness \( h \) and dimensions \( a \) and \( b \) along the \( x \)- and \( y \)-axes, as shown in Figure 1. Impact occurs at the moment \( t = 0 \) with the low velocity \( 0 V \epsilon \) at the point \( N \) with Cartesian coordinates \( x_0, y_0 \), where \( \epsilon \) is a small dimensionless parameter. According to the Donnell-Mushtari nonlinear shallow shell theory, the equations of motion could be obtained in terms of lateral deflection \( w \) and Airy’s stress function \( \phi \)

\[ \frac{D}{h} \left( \frac{\partial^4 w}{\partial x^4} + 2 \frac{\partial^3 w}{\partial x^2 \partial y^2} + \frac{\partial^4 w}{\partial y^4} \right) = \frac{D}{h} \left( \frac{\partial^4 \phi}{\partial x^4} + \frac{\partial^4 \phi}{\partial y^4} \right) - 2 \frac{\partial^2 w}{\partial x \partial y} \frac{\partial^2 \phi}{\partial x \partial y} + \sum_{r,s,i,j} B_{ijkl} \frac{\partial w}{\partial x} \frac{\partial \phi}{\partial x} + \frac{F}{h} - \rho \ddot w, \]  

(6)

\[ \frac{1}{E} \left( \frac{\partial^4 \phi}{\partial x^4} + 2 \frac{\partial^4 \phi}{\partial x^2 \partial y^2} + \frac{\partial^4 \phi}{\partial y^4} \right) = \frac{\partial^2 w}{\partial x^2} \frac{\partial^2 \phi}{\partial x^2} + \left( \frac{\partial^2 w}{\partial x^2} \right)^2, \]  

(7)

where \( D = \frac{E h^3}{12(1 - \nu^2)} \) is the cylindrical rigidity operator, \( \rho \) is the density, \( \ddot E \) and \( \ddot v \) are the Young’s operator and Poisson’s operator, respectively, \( F = P(t) \delta(x - x_0) \delta(y - y_0) \) is the contact force, \( P(t) \) is yet unknown function, \( \delta \) is the Dirac delta function, and overdots denote time-derivatives.

Coupled vibrations of the impactor (sphere) and the target (nonlinear viscoelastic plate) are described by an infinite set of coupled nonlinear ordinary differential equations of the second order with respect to time for defining the generalized coordinates which has been obtained in [34]:

\[ \ddot \xi_{mn}(t) + \ddot \Omega_{mn} \xi_{mn}(t) + \frac{32\pi^4 \dot \nu}{a b^4 \rho \sum \sum \sum \sum \sum \sum \sum B_{ijkl} \xi_{ij}(t) \xi_{kl}(t) + \frac{4M}{ab \rho h} \sin \left( \frac{m \pi x_0}{a} \right) \sin \left( \frac{n \pi y_0}{b} \right) \sum \sum \ddot \xi_{ij}(t) \sin \left( \frac{p \pi x_0}{a} \right) \sin \left( \frac{q \pi y_0}{b} \right) = 0, \]  

(8)

where coefficients \( B_{ijkl} \) are represented in [37], \( \ddot \Omega_{mn} \) is the viscoelastic operator corresponding to the natural frequency of the \( mn \)th mode of the plate vibration defined as

\[ \ddot \Omega_{mn} = \Omega_{mn}^2 \ddot d; \quad \ddot d = [1 + \eta D' - \zeta \dot \gamma (A \dot \alpha')], \]  

(9)

with

\[ \Omega_{mn}^2 = \frac{\pi^4 h^2}{\rho} \left( \frac{m^2}{a^2} + \frac{n^2}{b^2} \right)^2 \left( 9 \lambda_0 + 10 \mu_0 \right), \quad \eta = \frac{12(\lambda_0 + \mu_0)}{(9 \lambda_0 + 10 \mu_0)}, \quad \zeta = \frac{(3 \lambda_0 + 2 \mu_0)^2}{(9 \lambda_0 + 10 \mu_0)}, \quad A = \frac{4(\lambda_0 + \mu_0)}{(\lambda_0 + 2 \mu_0)}. \]

The last term in each equation from (8) describes the influence of the coupled impact interaction of the target with the impactor of the mass \( M \) applied at the point with the coordinates \( x_0, y_0 \).
It is known [38] that during nonstationary excitation of thin bodies not all possible modes of vibration would be excited. Moreover, the modes which are strongly coupled by any of the so-called internal resonance conditions are initiated and dominate in the process of vibration, in so doing the types of modes to be excited are dependent on the character of the external excitation.

Thus, in order to study the additional nonlinear phenomenon induced by the coupled impact interaction due to equation (8), we suppose that only two natural modes of vibrations are excited during the process of impact, namely, $\Omega_{\alpha\beta}$ and $\Omega_{\gamma\delta}$, in so doing each type of impact subjection should be considered separately.

3. Method of solution

The set of equations (8) is reduced to the following two nonlinear differential equations:

$$2^2 3^2 1^1 1^2 1^6 1^7 = 0,$$

$$\dot{\Omega}_{\alpha\beta} + \dot{\Omega}_{\gamma\delta} + \dot{\Omega}_{\alpha\beta} + \dot{\Omega}_{\gamma\delta} = 0,$$ (10)

$$2^2 2^1 2^2 2^6 2^7 = 0,$$

$$\dot{\Omega}_{\alpha\beta} + \dot{\Omega}_{\gamma\delta} + \dot{\Omega}_{\gamma\delta} + \dot{\Omega}_{\gamma\delta} = 0,$$ (11)

where coefficients $p_{ij}$ ($i=1,2$, $j=3,4,5,7$) are represented in [10] and $p_{ij}$ ($i=1,2$, $j=3,4,5$) = 0.

In order to solve a set of two nonlinear equations (10) and (11), let us apply the method of multiple time scales [39] via the following expansions:

$$\xi(t) = \varepsilon X_0(T_0),$$

where $ij = \alpha\beta$ or $\gamma\delta$, $T_0 = \varepsilon^0 t$ are new independent variables, among them: $T_0 = t$ is a fast scale characterizing motions with the natural frequencies, and $T_1 = \varepsilon t$ and $T_2 = \varepsilon^2 t$ are slow scales characterizing the modulation of the amplitudes and phases of the modes with nonlinearity.

Recall that the first and the second time derivatives are defined, respectively, as follows [39]

$$\frac{d}{dt} = D_0 + \varepsilon D_0 + \varepsilon^2 D_2 + \ldots,$$

$$\frac{d^2}{dt^2} = D_0^2 + 2\varepsilon D_0 D_1 + \varepsilon^2 \left(D_0^2 + 2D_0 D_2\right),$$ (14)

where $D_n^i = \partial^n / \partial T^n_i$ ($n=1,2$, $i=0,1$).
Following Rossikhin and Shitikova [40], the fractional derivative is interpreted as the fractional power of the differential operator

\[ D_\gamma^\gamma = \left( \frac{d}{dt} \right)^\gamma = D_\gamma^\gamma + \varepsilon_\gamma D_0^{\gamma-1} D_1 + \ldots \]  

(15)

Now let us expand the dimensionless Rabotnov's fractional operator (5) in a Taylor series. As a result we have

\[ \mathcal{A}_\gamma^\gamma (\tau^\gamma) = \frac{1}{1 + \tau^\gamma D_0^{\gamma-1}} = \left[ 1 + \tau^\gamma (D_0^{\gamma-1} D_1) \right]^{-1} \] 

\[ = (1 + \tau^\gamma D_0^{\gamma-1})^{-1} - \varepsilon(1 + \tau^\gamma D_0^{\gamma-1})^2 \tau^\gamma D_0^{\gamma-1} D_1 + \ldots \]  

(16)

Substituting relationships (12)-(16) in (10) and (11), after equating the coefficients at like powers of \( \varepsilon \) to zero, we are led to a set of recurrence equations at various orders:

- to order \( \varepsilon \)
  \[ p_1 D_0^\gamma X_1 + p_{12} D_0^\gamma X_2 + \tilde{\Omega}_2^\gamma X_1^2 = 0, \]  
  \[ p_2 D_0^\gamma X_1 + p_{22} D_0^\gamma X_2 + \tilde{\Omega}_2^\gamma X_2^2 = 0; \]  

- to order \( \varepsilon^2 \)
  \[ p_1 D_0^\gamma X_1^2 + p_{12} D_0^\gamma X_2^2 + \tilde{\Omega}_2^\gamma X_1^2 = -2p_1 D_0 D_1 X_1^2 - 2p_{12} D_0 D_1 X_2^2 - \tilde{I}_{11} \gamma D_0^{\gamma-1} D_1 X_1^2, \]  
  \[ p_2 D_0^\gamma X_1^2 + p_{22} D_0^\gamma X_2^2 + \tilde{\Omega}_2^\gamma X_2^2 = -2p_2 D_0 D_1 X_1^2 - 2p_{22} D_0 D_1 X_2^2 - \tilde{I}_{22} \gamma D_0^{\gamma-1} D_1 X_2^2; \]  

- to order \( \varepsilon^3 \)
  \[ p_1 D_0^\gamma X_1^3 + p_{12} D_0^\gamma X_2^3 + \tilde{\Omega}_2^\gamma X_1^3 = -2p_1 D_0 D_1 X_1^3 - 2p_{12} D_0 D_1 X_2^3 - p_1 \left( D_1^\gamma + 2D_0 D_2 \right) X_1^3 \]  
  \[ - p_{12} \left( D_1^\gamma + 2D_0 D_2 \right) X_2^3 - \tilde{I}_{11} \gamma D_0^{\gamma-1} D_1 X_1^3 - p_{16} \left( X_1^3 \right)^3 E - p_{17} X_1^2 \left( X_1^3 \right)^2 E, \]  
  \[ p_2 D_0^\gamma X_1^3 + p_{22} D_0^\gamma X_2^3 + \tilde{\Omega}_2^\gamma X_2^3 = -2p_2 D_0 D_1 X_1^3 - 2p_{22} D_0 D_1 X_2^3 - p_{21} \left( D_1^\gamma + 2D_0 D_2 \right) X_1^3 \]  
  \[ - p_{22} \left( D_1^\gamma + 2D_0 D_2 \right) X_2^3 - \tilde{I}_{22} \gamma D_0^{\gamma-1} D_1 X_2^3 - p_{26} \left( X_1^3 \right)^3 E - p_{27} X_1^2 \left( X_1^3 \right)^2 E, \]  

where for simplicity it is denoted \( X_1^k = X_{4\lambda}^k, \ X_2^k = X_{5\lambda}^k, \ X_1^2 = X_{4\sigma}^2, \ X_2^2 = X_{5\sigma}^2, \ \tilde{\Omega}_1 = \tilde{\Omega}_{4\gamma}^\sigma, \ \tilde{\Omega}_2 = \tilde{\Omega}_{5\gamma}^\sigma, \) and \( \tilde{I}_{1(2)} = \Omega_{1(2)}^\sigma \left[ \eta_{\sigma}^\gamma - \zeta(1 + A_{0\sigma}^\gamma D_0^{\gamma})^{-2} A_{0\sigma}^\gamma \right]. \)

The solution of the set of equations (17) and (18) we will seek in the form

\[ X_1 = A_1(T_1, T_2) e^{\lambda_1 T_0} + A_2(T_1, T_2) e^{\lambda_2 T_0} + cc, \]  
\[ X_2 = \alpha_1 A_1(T_1, T_2) e^{\lambda_1 T_0} + \alpha_2 A_2(T_1, T_2) e^{\lambda_2 T_0} + cc, \]  

where \( \lambda_1, \lambda_2, \alpha_1, \alpha_2 \) are yet unknown constants, \( cc \) is the complex conjugate part to the preceding terms.

Substituting equations (23) and (24) in (17) and (18) yields

\[ \begin{align*} 
\{ p_{11}^{\lambda_1^2} + \Omega_{10}^{\sigma} + [1 + \eta_{\sigma}^{\gamma} \lambda_1^{\gamma} - \zeta(1 + A_{0\sigma}^{\gamma} \lambda_1^{\gamma})^{-2}] + \alpha_1 p_{12} \lambda_2^{\gamma} \} A_1 e^{\lambda_1 T_0} & + \{ p_{12}^{\lambda_1^2} + \Omega_{12}^{\sigma} + [1 + \eta_{\sigma}^{\gamma} \lambda_1^{\gamma} - \zeta(1 + A_{0\sigma}^{\gamma} \lambda_1^{\gamma})^{-2}] + \alpha_2 p_{12} \lambda_2^{\gamma} \} A_2 e^{\lambda_2 T_0} = 0, \\
\{ p_{21}^{\lambda_1^2} + \Omega_{20}^{\sigma} \alpha_1 + [1 + \eta_{\sigma}^{\gamma} \lambda_2^{\gamma} - \zeta(1 + A_{0\sigma}^{\gamma} \lambda_2^{\gamma})^{-2}] + \alpha_1 p_{22} \lambda_2^{\gamma} \} A_1 e^{\lambda_1 T_0} & + \{ p_{22}^{\lambda_1^2} + \Omega_{22}^{\sigma} \alpha_2 + [1 + \eta_{\sigma}^{\gamma} \lambda_2^{\gamma} - \zeta(1 + A_{0\sigma}^{\gamma} \lambda_2^{\gamma})^{-2}] + \alpha_2 p_{22} \lambda_2^{\gamma} \} A_2 e^{\lambda_2 T_0} = 0. 
\end{align*} \]  

(25) (26)
In order to satisfy equations (25) and (26), it is a need to vanish to zero each bracket in these equations. As a result, from four different brackets we have

$$\alpha_i = -\frac{p_{11} \lambda_i^2 + \Omega_{01}^2 [1 + \eta_{\sigma} \lambda_i^2 - \zeta (1 + At_{\sigma} \lambda_i^2)^{-1}]}{p_{12} \lambda_i^2}$$  \hspace{1cm} (27)

$$\alpha_i = -\frac{p_{22} \lambda_i^2 + \Omega_{02}^2 [1 + \eta_{\sigma} \lambda_i^2 - \zeta (1 + At_{\sigma} \lambda_i^2)^{-1}]}{p_{22} \lambda_i^2}$$  \hspace{1cm} (28)

Since the left-hand side parts of relationships (27) and (28) are equal, then their right-hand side parts should be equal as well. Now equating the corresponding right-hand side parts, we are led to one and the same characteristic equation for determining the values $\lambda_1$ and $\lambda_2$

$$(p_{11} p_{22} - p_{12}^2) \lambda^4 + \Omega_{01}^2 [1 + \eta_{\sigma} \lambda^2 - \zeta (1 + At_{\sigma} \lambda^2)^{-1}] \lambda^2 + \Omega_{01}^2 = 0.$$  \hspace{1cm} (29)

Let us study the asymptotic behavior of the roots of equation (29), supposing for this purpose that $t_{\sigma}$ is a small value. Then equation (29) at any $\gamma (0 \leq \gamma \leq 1)$ could be written in the form

$$(p_{11} p_{22} - p_{12}^2) \lambda^4 + (p_{11} \Omega_{01}^2 + p_{22} \Omega_{01}^2 - \Omega_{01}^2 (1 - \zeta) \lambda^2 + \Omega_{01}^2 \Omega_{02}^2 (1 - \zeta)^2 = 0,$$  \hspace{1cm} (30)

where $p_{11} p_{22} - p_{12}^2 > 0$, and $1 - \zeta = \frac{16 \mu_0 (\mu_0 + \lambda_0)}{(9 \lambda_0 + 10 \mu_0)(\lambda_0 + 2 \mu_0)} > 0$.

Solving equation (30), we find

$$\lambda_{1,2} = \frac{1 - \zeta}{2(p_{11} p_{22} - p_{12}^2)} \pm \sqrt{\left(\frac{1}{2(p_{11} p_{22} - p_{12}^2)} \right)^2 + \frac{4 \Omega_{01}^2 \Omega_{02}^2 p_{12}^2}{(p_{11} p_{22} - p_{12}^2)}}$$  \hspace{1cm} (31)

From (31) it follows that $\lambda_1$ and $\lambda_2$ are purely imaginary values, i.e. $\lambda_{1,2} = i \omega_{1,2}$ and

$$e^{\lambda_{1,2}} = e^{i \omega_{1,2}}$$  \hspace{1cm} (32)

where $\omega_1$ and $\omega_2$ are real values. In this case, the plate performs undamped vibrations.

It is easy to show that at $t_{\sigma} \to 0$ operator $I_1(21) = 0$ and $\Omega_{mn} = \omega_{mn}$, where $\Omega_{mn}$ is a constant value corresponding to the natural frequency of the $mn$th mode of the elastic plate.

Now let us suppose that $t_{\sigma}$ is a large value. Then equation (29) at any $\gamma (0 \leq \gamma \leq 1)$ could be written in the form

$$(p_{11} p_{22} - p_{12}^2) \lambda^4 + (p_{11} \Omega_{01}^2 + p_{22} \Omega_{01}^2) - \Omega_{01}^2 \lambda^2 + \Omega_{01}^2 \Omega_{02}^2 \gamma = 0,$$  \hspace{1cm} (33)

where $\lambda = \lambda^2 \gamma$. Solving equation (33), we find

$$\lambda_{1,2} = \sqrt{2\left(\frac{1}{2(p_{11} p_{22} - p_{12}^2)} \right) \pm \sqrt{\left(\frac{1}{2(p_{11} p_{22} - p_{12}^2)} \right)^2 + \frac{4 \Omega_{01}^2 \Omega_{02}^2 p_{12}^2}{(p_{11} p_{22} - p_{12}^2)}}}$$  \hspace{1cm} (34)

Rewriting the solution of (34) in the form $\lambda_{1,2} = -B_{1,2}$ when $B_{1,2} > 0$, we will obtain

$$\lambda_{1,2} = \sqrt{\frac{1}{2(p_{11} p_{22} - p_{12}^2)}} \left[\cos(2 - \gamma)^{-1} \pi + i \sin(2 - \gamma)^{-1} \pi\right].$$  \hspace{1cm} (35)

Considering (35), relationships (32) are reduced to

$$e^{\lambda_{1,2}} = e^{i \omega_{1,2}}$$  \hspace{1cm} (36)

where $\omega_{1,2} = B_{1,2}^2 \gamma \cos(2 - \gamma)^{-1} \pi$, and $\omega_{1,2} = B_{1,2}^2 \gamma \sin(2 - \gamma)^{-1} \pi$. 

\[\text{ToPME} \hspace{1cm} \text{IOP Conf. Series: Materials Science and Engineering 489 (2019) 012038 doi:10.1088/1757-899X/489/1/012038} \]
It follows from equation (36) that at $\gamma \neq 0$ and $\gamma \neq 1$ damped vibrations take place. At $\gamma = 0$, values $\alpha_{1,2} = 0$ and vibrations become undamped, while at $\gamma = 1$, values $\omega_{1,2} = 0$ and the plate possesses the aperiodic mode of vibration.

4. Conclusion

Large amplitude (geometrically non-linear) vibrations of viscoelastic rectangular plate were investigated for the case when the shear operator is governed by the fractional derivative Kelvin-Voigt model in conjunction with the time-independent coefficient of volume extension-compression, what is verified by experimental data. It has been shown that such a model could describe the behaviour of so-called auxetic materials with negative Poisson's ratios.

The governing set of nonlinear differential equations has been obtained using the modified method of multiple time scales. The asymptotic behavior of the roots of the characteristic equation for determining the frequencies of the system as a function of the retardation time was studied, and three types of vibration in the process of impact interaction were described. It has been shown that the characteristic equation (30) describes only one vibrational mode – undamped, while the characteristic equation (33) describes three vibrational modes at once: undamped, damped and aperiodic.

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