Fast high-fidelity composite gates in superconducting qubits: Beating the Fourier leakage limit

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We present a method for quantum control in superconducting qubits, which overcomes the Fourier limit for the gate duration imposed by leakage to upper states. The method uses composite pulses, which allow for the correction of various types of errors, which naturally arise in the system, by destructive interference of these errors. We use our approach to produce complete and partial population transfer between the qubit states, as well as the three basic single-qubit quantum gates. Our simulations show a substantial reduction of the typical errors and a gate speed-up by an order of magnitude.

I. INTRODUCTION

Quantum computing (QC) promises to revolutionize computer science by introducing an astonishing paradigm shift in how we think about information. It has been shown that, for certain problems, quantum algorithms significantly outperform the best known classical analogues \cite{1}. Until recently, quantum computing was, to a large extent, concentrated in academic institutions and was considered mostly an interesting scientific exercise with unclear potential for “real-world” applications. This was mainly due to the difficulties in scaling-up quantum computers to a large number of qubits. These difficulties themselves lead to the development of new types of algorithms (e.g. VQE \cite{2, 3}, QAOA \cite{4}) which aimed at achieving useful results by utilizing the quantum computers, which are currently available. Quantum computing entered the so-called NISQ (Noisy Intermediate-Scale Quantum) era \cite{5}.

With the expansion of quantum computing from academia to industry came a (maybe temporary) shift in the focus on the physical systems, used to represent the qubits. While in academic institutions the interest is spread among a variety of qubit systems, including trapped ions, atoms, photons, quantum dots, etc., in the commercially available quantum computers the clear front runner is the superconducting qubit, mostly due to fabrication facilitation. The most popular type of superconducting qubit is the so-called transmon, which is essentially an LC circuit, where the linear inductance is replaced by a Josephson junction \cite{6}. For its lowest states, such a system can be described as an anharmonic oscillator, where the lowest two energies represent the qubit. The anharmonicity prevents from exciting the higher states, referred to as leakage errors, if the driving fields are long enough, such that their Fourier spectrum does not cover the leakage transition.

The control over transmon qubits \{0, 1\}, used to produce quantum gates and algorithms, is performed by external microwave fields. Through this manipulation, several types of errors may arise \cite{7}, such as decoherence \cite{8}, population leakage outside the computational subspace \{9, 10\}, coherent errors \{11\}, and measurement errors \{12, 13\}. Among these, the most crucial limitation derives from the leakage of population upwards the anharmonic ladder \{0\} \rightarrow \{1\} \rightarrow \{2\} \rightarrow \cdots, which limits the gate duration. The limitation stems from the Fourier bandwidth of a pulse: if the pulse is too short then its Fourier bandwidth is too large and contains frequency components close to resonance with the unwanted upper transition \{1\} \rightarrow \{2\}. The probability of such a leakage is reduced by increasing the gate duration (and hence squeezing the Fourier bandwidth), by pulse shaping or, furthermore, by using the so-called DRAG pulses \cite{14}, which can be seen as an extension of the popular “short-cuts to adiabaticity” method \cite{15}. In either cases, the idea is to eliminate the frequency components near resonance with the leakage transition \{1\} \rightarrow \{2\}. In two-qubit gates, leakage can be reduced by using a suitable synchronization of control parameters \cite{16}.

In this work, we introduce a drastically different approach to eliminate the probability for leakage \{1\} \rightarrow \{2\} even with (previously prohibitively short) pulses which frequency spectrum covering the \{1\} \rightarrow \{2\} transition. We use the technique of composite pulses, which allows to enhance or reduce a certain probability generally at will. The composite pulses are sequences of pulses with well-defined relative phases, which are used as control parameters to distort the excitation profile (and even the entire propagator) as desired. In the present context the leakage transition \{1\} \rightarrow \{2\} is strongly suppressed by the destructive interference enabled by the composite sequence, even though every single pulse in the sequence produces non-negligible excitation of state \{2\}.

In particular, we demonstrate how three of the most common sources of errors in a superconducting qubit — due to decoherence, leakage, and control inaccuracies — can be mitigated by a single unified approach. Decoherence errors are addressed by shortening the gate duration. This is usually associated with a higher population leakage, due to the increased Fourier bandwidth. Furthermore, the CPs produce robust excitation profiles, therefore lower errors due to pulse imperfections. Below, we explain our approach in more details and provide sim-
ulations of the performance of the method.

II. DESCRIPTION OF THE METHOD

The transmon qubit can be described as a quantum anharmonic oscillator with energy levels

\[ \omega_n = \left( \omega + \frac{\delta}{2} \right) n - \frac{\delta}{2} n^2, \quad (1) \]

where \( \omega \) is the frequency of the qubit and \( \delta \) is its anharmonicity. Normally, due to this anharmonicity, one restricts only to states \(|0\rangle\) and \(|1\rangle\), which compose the qubit, and neglects all higher-energy states. This approximation, however, sets a limit on the gate speed. The faster the gate is, the broader its Fourier spectrum and hence the larger the off-resonant excitation. To deal with this problem, we include the off-resonant state \(|2\rangle\) in our description. When driven by an external microwave field, moving into the interaction frame and making the rotating wave approximation, the system can be described by the Hamiltonian

\[
H = \frac{\hbar}{2} \begin{bmatrix} 0 & \Omega & 0 & 0 \\ \Omega^* & 0 & \sqrt{2}\Omega & 0 \\ 0 & \sqrt{2}\Omega^* & -2\delta \\ 0 & 0 & 0 & \delta \end{bmatrix}, \quad (2)
\]

where \( \Omega \) is the Rabi frequency, which measures the strength of the coupling between the field and the qubit. The evolution of the system is described by the propagator \( U = e^{-iHt} \). To address the problem of off-resonant excitation to state \(|2\rangle\), we are going to use the tool of composite pulses. Instead of a single pulse, described by the propagator \( U \), we are going to apply a sequence of \( N \) pulses, each with a particular relative phase \( \phi_k \) and Rabi frequency \( \Omega_k \). The total propagator is given by the product

\[ U^{(N)} = U(\Omega_N, \phi_N) \cdots U(\Omega_2, \phi_2)U(\Omega_1, \phi_1), \quad (3) \]

where \( U(\Omega_k, \phi_k) \) is the single-pulse propagator, corresponding to Hamiltonian \( \{\Omega_k\} \), with the complex Rabi frequency \( \Omega_k e^{i\phi_k} \). We are going to use the relative phases and Rabi frequencies as control parameters to minimize the leakage of population to state \(|2\rangle\) and shape the excitation profile in a desired fashion. We achieve this goal by maximizing the fidelity between a target gate and the actual propagator over a certain range of deviations in the Rabi frequencies, while trying to keep the overall pulse area as small as possible. For a more detailed discussion on composite pulses, we refer the reader to the vast literature on the topic.

Before proceeding with the next section, we make a brief note on some realistic values for the parameters of our system, which will help us setting up more realistic simulations. These values can be easily found at the IBM Quantum Experience documentation. A typical transition frequency for the transmon qubit is \( \omega \sim 2\pi \times 5 \) GHZ, and the anharmonicity is \( \delta \sim 2\pi \times 0.3 \) GHZ. Furthermore, the duration of a single-qubit gate is \( T \sim 50 \) ns, which leads to a Rabi frequency of \( \Omega \sim \pi/T \approx 2\pi \times 10 \) MHz. Therefore, we note that in dimensionless units we have \( \delta T \sim 100 \), and numerical simulations have shown that the leakage starts to become substantial at \( \delta T \gtrsim 20 \). In our simulation we are going to assume that \( \delta T \sim 0.5 \), which corresponds to a \( 40 \) speed-up of the single interaction, and a pulse duration close to \( 1 \) ns. Further decrease of \( T \) is expected to break the rotating-wave-approximation and requires additional investigation, which is outside the scope of the current work.

III. SIMULATIONS

To demonstrate the performance of our method, we produce two types of simulations. The first type is only concerned with the population of states \(|0\rangle\) and \(|1\rangle\) after the interaction, neglecting the phases in the probability amplitudes. The second type simulates the production of quantum gates, and therefore phases become important; obviously, this second type is far more demanding. Therefore, we will analyze these separately.

The simulations are performed as follows. We calculate the propagator \( \{\Omega_k\} \) for a \( N \)-pulse sequence, where we assume that the Rabi frequencies of the constituent pulses may have a common systematic deviation \( \epsilon \) and are therefore given by \( \Omega_k(1+\epsilon)e^{i\phi_k} \). Our goal is now to choose the parameters \( \Omega_k \) and \( \phi_k \) so that we obtain the desired excitation, as a function of \( \epsilon \).

Before proceeding with the two separate types of composite sequences, we also note that in all of our simulations we have used the value of \( \delta T = 20 \) for the single-pulse case and \( \delta T = 0.5 \) for the composite-pulse case. These, as discussed in the preceding section, correspond to realistic experimental values.

A. Population transfer

For the population transfer simulations, we are going to consider that our system is initially in state \(|0\rangle\). Our goal is to produce a predefined transition probability in a robust manner. In particular, we consider the case of complete population transfer, with transition probability \( P_1 = 1 \), and half population transfer with \( P_0 = P_1 = \frac{1}{2} \). We find the optimal values for the set of parameters \( \{\Omega_k\} \) and \( \{\phi_k\} \) by minimizing the cost function

\[ f(\{\Omega_k\}, \{\phi_k\}) = \sum_{i=0}^{1} \left| P_i - |U^{(N)}_{i1}|^2 \right|, \quad (4) \]

over the widest possible region of values for \( \epsilon \), where \( P_i \) is the target population for state \(|i\rangle\) (\( i = 0, 1 \)). In addition, we favour solutions with minimal total pulse area \( \sum \Omega_k T \), which implies maximal speed.
we plot the excitation profiles generated
method to three common gates, namely the
posite sequences is
as seen from the table, the total pulse area of the com-
be produced, in a robust and accurate way. Furthermore,
single-pulse profiles. As seen from the figure both com-
by these composite pulses, and compare them with the
solutions, with different number of constituent
pulses, have been found too. We have selected to present
the ones which seem to offer the “sweet spot” in terms
of accuracy, robustness, and total pulse area.

Two of the best solutions we have derived are pre-
presented in Table I (top part). We provide a single solution
for each of our target transition probabilities, although
multiple solutions, with different number of constituent
pulses, have been found too. We have selected to present
the ones which seem to offer the “sweet spot” in terms
of accuracy, robustness, and total pulse area.

In Fig. 1 we plot the excitation profiles generated
by these composite pulses, and compare them with the
single-pulse profiles. As seen from the figure both com-
plete population transfer and an equal superposition can
be produced, in a robust and accurate way. Furthermore,
as seen from the table, the total pulse area of the com-
posite sequences is \(\sim 4\pi\). As discussed in the previous
section, the single pulse speed-up is a factor of \(\sim 40\).
Therefore, we achieve a factor of \(40/4 = 10\) effective
speed-up, using our composite sequences.

B. Quantum gates

We now follow the same approach to produce high-
fidelity error-protected quantum gates. We apply our
method to three common gates, namely the \(X\), the
Hadamard \(H\), and the phase \(T\) gates. The optimal phases
and Rabi frequencies are found by maximizing the fidelity
function

\[
F(\{\phi_k\}, \{\Omega_k\}) = \frac{1}{2} \left| \text{Tr} \left( U^{(N)} G \right) \right|
\]

over the widest possible range of values for \(\epsilon\), where \(G\) is
the target (goal) gate and \(U^{(N)}\) is the actual composite
gate. Again, smaller values for the total pulse area \(A\) are
favoured. The results are presented in Table I (bottom
part) and are illustrated in Fig. 2 where the solid lines
represent the composite-pulse fidelity, while the dotted
lines correspond to the fidelity achieved by a single-pulse
approach. The latter are produced by a \(\pi\) rotation on
the Bloch sphere about the \(x\) axis for the \(X\) gate, and a
\(\pi/2\) rotation about the \(y\) axis for the \(H\) gate. As seen
from the figure, our method produces a robust and high
fidelity for all the three gates. Furthermore, the compari-
on with the single-pulse fidelity shows a substantial
improvement of the robustness of the \(X\) gate and a
slight advantage of the \(H\) gate, compared to the single-pulse
approach. In terms of speed-up, however, both the \(X\) and \(H\)
sequences significantly outperform the single-pulse ana-
lougues. Namely, the speed-up factor is \(40/6.96 = 5.7\) for
the \(X\) gate and \(40/7.55 = 5.3\) for the \(H\) gate.

We note that, of course, the quantum gate sequences
can be used for population transfer too. However, the
dedicated population transfer sequences in Table I o-
fer the better performance for this purpose as they are
shorter.

IV. CONCLUSIONS AND DISCUSSION

In this work we have described a method for quantum
control in transmon qubits, which suppresses errors from
various sources. Namely, we show that our approach
is capable of reducing the errors, due to i) population

| Target \( (\phi_1, \phi_2, \ldots, \phi_N) \) \( (\Omega_1, \Omega_2, \ldots, \Omega_N) \) \( A \) |
|---|---|---|
| Population transfer \( P_1 = 1 \) \( (0, 0.5530, 1.9143, 1.9720, 0.2462, 1.3857) \) \( (0.1586, 0.6205, 0.4461, 0.4994, 1.0982, 0.8116) \) \( 3.63\pi \) |
| \( P_2 = \frac{\beta}{2} \) \( (0.5003, 1.1833, 1.1037, 1.5215, 0.6296, 0.4988) \) \( (0.2997, 0.4086, 0.4242, 0.6462, 0.6771, 0.7331, 0.8712) \) \( 4.06\pi \) |
| Quantum gates \( X \) \( (0.4318, 0.6684, 0.6746, 0.6175, 1.3619, 0.8914, 1.4236, 0.8910) \) \( 6.96\pi \) |
| \( H \) \( (0.8289, 0.9549, 1.1362, 1.2669, 0.8429, 1.4784, 1.0455) \) \( 7.55\pi \) |
| \( T \) \( (0.2647, 0.1738, 0.6147, 0.8168, 0.1897, 0.6416, 0.7425) \) \( 3.44\pi \) |

TABLE I: Relative phases (in units \(\pi\)) and Rabi frequencies (in units \(1/T\)) of the composite sequences, producing specific transition probability (top part) or specific quantum gates (bottom part). The total pulse area is denoted by \(A\).
leakage outside of the computational basis, ii) decoherence, and iii) deviations in the control parameters. This is achieved by using a sequence of pulses with suitably chosen Rabi frequencies and relative phases. In such a way, one manages to both cancel transitions to higher states, by destructive interference, and at the same time to shorten significantly the pulse duration. The former leads to a higher accuracy, while the latter allows for larger circuit depth, which is limited by the coherence time of the system. Moreover, the control parameters of the composite sequences allow to improve the robustness with respect to the so-called coherent errors, such as deviations in the Rabi frequency.

Finally, our technique can be seen as an alternative to the popular method of DRAG pulses \cite{14}, which is currently the standard approach to reduce leakage errors in weakly nonlinear qubits, and is based on exploiting the shortcuts to adiabaticity method to cancel the coupling to the non-computational subspace. In contrast to DRAG pulses, our method can be used to reduce not only the leakage, but also coherent errors, arising from various imperfections in the control parameters.

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