A Novel Micro-motion Target Parameter Estimation Algorithm in the Situation of Low SNR

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Abstract. Aiming at the problem of extracting rotor micro-motion target parameters of narrow-band radar, a novel micro-motion target parameter estimation algorithm in the situation of low SNR is studied. In this method, firstly, the echo in the range unit where the target is located after pulse compression is extracted, and the maximum instantaneous micro-Doppler frequency of the rotor is estimated by time-frequency analysis and skeleton extraction. Then the maximum instantaneous micro-Doppler frequency is taken as prior information and used to reduce the dimension of the rotating speed search matrix, so the two-dimensional joint estimation of the blade length and rotating speed is converted into step estimation. The rotating speed value is estimated by using the principle of energy accumulation, and the blade length is further extracted by the mathematical relationship. Finally, compared with other commonly estimation methods in estimation accuracy and anti-noise performance. The simulation results verify the effectiveness of the proposed method.

1. Introduction
Radar echoes produce additional Doppler frequency modulation due to micro-motion such as rotation, vibration, precession, and nutation of the target. This phenomenon is called the Micro-Doppler Effect [1]. Since the concept of micro-Doppler was proposed by V.C.Chen in 2000, the micro-Doppler study of the micro-motion target has been rapidly developed. Among these micro-Doppler components, the rotor rotation is a typical micro-motion mode, which contains the rotation speed of the rotor, blade length and other information, the use of blade length and rotating speed can be used to identify a class of helicopter targets. Therefore, it has been widely studied by scholars[2-8]. Currently, the main rotation speed estimation methods of narrow-band radar are the time-frequency analysis method[9], orthogonal matching pursuit algorithm(OMP)[9], Hough transform method[9], higher-order moment function analysis method[10-11], function construction method[12] and autocorrelation method[13]. Orthogonal matching pursuit algorithm and higher-order moment function analysis method need to use the phase information of the echo. While the Hough transform has the problem of a large amount of calculation, and it is unable to estimate the speed quickly. The function construction method has a small amount of calculation, but when the rotor scatter points are evenly distributed, it will fail, and it is difficult to obtain the rotation speed accurately. The autocorrelation method must accurately estimate the rotation speed of the rotor when the number of blades is known. Moreover, the above method has a weak speed estimation capability under low SNR conditions, in which the autocorrelation method has a certain speed estimation capability under low SNR conditions, but when it is applied to the helicopter rotor speed estimation with a regular shape, there is an integer multiple
error with the estimated rotational speed, which is equal to the number of symmetric axes of the rotor actual rotational speed. Therefore, the speed estimation method under low SNR needs further study.

In order to solve the above problems, this paper proposes a novel micro-motion target parameter estimation algorithm in the situation of low SNR. First, a model of the scattering point of the rotor target is established, and then the maximum instantaneous micro-Doppler frequency of the rotor is estimated by time-frequency analysis and skeleton extraction. Then, it is taken as prior information to reduce the dimension of the constructed search matrix, perform rough estimation and precision estimation on the rotational speed, and then use the principle of energy accumulation to obtain the rotational speed value with the best accumulation effect. The corresponding rotation speed is the estimated optimal rotation speed, and the blade length of the rotor target is further estimated by using the estimation result. By this method, the rotation speed and the blade length of the rotor can be accurately calculated. In the case of low SNR, this method still has high estimation accuracy.

2. Signal model
Assuming translational component of the target echoes is compensated, the target's motion can be considered as hovering state[14]. To simplify the analysis, assume that the target and the radar are in the same flat. As shown in figure 1, the distance from the radar to the target rotation center is $R_c$, the rotor is assumed to be model of the scattering point. The distance from scattering point $P$ to the rotor center is $r$ ($0 \leq r \leq l$, $l$ is the length of rotor blade). The distance from the point to the radar is $R_p$, while the point $P$ rotates around the center of rotation $C$ at rotor rotation speed of $\omega$, and the initial rotation angle is $\theta$.

![Figure 1. Rotor target motion diagram](image)

Assume the transmit signal is narrowband LFM signal[2]

$$s_i(\hat{t}, t_m) = A \text{rect}(\frac{\hat{t}}{T_p}) \exp[i2\pi(f_c + \frac{k}{2}t^2)]$$

(1)

where $A$ donates the amplitude of the transmitted signal, $T_p$ donates the pulse width, $f_c$ donates the carrier frequency, $k$ donates the slope of the frequency modulation, $t$ donates the fast time, $t_m$ donates the slow time, $t_m = mT_r$ ($m$ donates the $m$th echo, $T_r$ donates the pulse repetition period), $t$ donates the total time, and $t = \hat{t} + t_m$.

Echo after pulse compression can be expressed as

$$s_{qj}(\hat{t}, t_m) = \sum_{j=0}^{K} \sum_{n=0}^{N} \sigma_{nj} A T_p \sin c[B(\hat{t} - \frac{2R_{nj}(t_m)}{c})] \exp[-i \frac{4\pi}{\lambda} R_{nj}(t_m)]$$

(2)

In equation (2), $N$ donates the number of blades, $n$ is expressed as the $n$th blade on the rotor, $K$ is the number of scattering points on the target single blade, $j$ is expressed as the $j$th scattering point on the blade, $\sigma_{nj}$ is the scattering coefficient at the scattering point, $B$ is the bandwidth, $\lambda$ is the wavelength, $R_{nj}(t_m)$ is the range from scattering point to radar[14].

$$R_{nj}(t_m) = R_c + r_n \cos(\omega t_m + \Theta_{nj})$$

(3)
It can be seen from equation (2), the rotor target micro-Doppler phase is
\[
\phi = -\frac{4\pi}{\lambda} r_{nj} \cos(\omega t_m + \theta_n)
\]
(4)

From equation (4), it can be seen that the micro-Doppler of scattering point shows the form of the cosine function, while the period of the cosine function is consistent with the rotation period of the rotor. The micro-Doppler curves caused by the scatter points on different blade have different initial phases, while the initial phase of the same blade scatter point is consistent, and the difference is the peak Doppler frequency. Then, the rotor target micro-Doppler frequency derived from equation (4) is
\[
f_{m-D} = \frac{1}{2\pi} \frac{d\phi}{dt_m} = \frac{2\omega r_{nj}}{\lambda} \sin(\omega t_m + \theta_n)
\]
(5)

From equation (5), we can see that if we need to estimate the blade length and rotating speed of the rotor target without considering the initial phase of the blade, we can first estimate one of the parameters, and then use the mathematical relationship of equation (5) to estimate another parameter. This paper proposes a method for blade length and rotating speed estimation based on this principle.

3. Feature extraction

The rotor micro-motion target parameter estimation algorithm proposed in this paper is divided into three steps. The first step is to estimate the maximum instantaneous micro-Doppler frequency of the target using time-frequency analysis and image skeleton extraction[5]. Based on the first step, the second step reduces the amount of computation by reducing the dimension and dividing the search process into rough estimation and precision estimation. The third step is to use the energy accumulation idea to search for the rotational speed and perform energy accumulation in the time-frequency domain, using the accumulated effects to estimate the rotational speed of the rotor target, the blade length is finally estimated by the relationship among the rotating speed, the maximum instantaneous micro-Doppler frequency and blade length.

3.1. Skeleton extraction method based on Gabor transform

From equation (2), it can be seen that the micro-Doppler of the rotor is a non-stationary time-varying signal. Therefore, it is necessary to use the local analysis method to analyze the signal by the time-frequency analysis. In narrow-band radar, the fringe amplitude of the scattering point is usually in a range resolution unit, and the one-dimensional range profile shows a straight line parallel to the azimuth direction. So, only the data of this range bin needs to be taken out for time-frequency analysis. The commonly used time-frequency analysis methods include Short Time Fourier Transform(STFT), Wigner-Ville Distribution(WVD), and Smoothed Pseudo Wigner-Ville Distribution(SM-WVD), S Method(SM)[15], etc. Among them, STFT is the most commonly used time-frequency analysis method based on matched filtering. When the window function selected by STFT is a Gaussian window, it is called Gabor transform.

The Gabor transform of equation (2) can be used to obtain the time-frequency diagram of the micro-motion target signal. The expression[9] is
\[
STFT(t_m, \omega) = \int_{-\infty}^{\infty} s_{w}(t, t_m) \frac{1}{\pi ^{\frac{1}{2}} \sigma ^{\frac{1}{2}}} \exp(-\frac{(t_m-t)^2}{2\sigma ^2}) \exp(-i\omega t_m) dt
\]
(6)

From equation (5), we can see that in the time-frequency diagram, the micro-Doppler curve of the rotor target is a three-parameter sinusoidal curve, and the signal energy is concentrated on the curve. In order to reduce the number of subsequent processing operations and improve the accuracy of parameter extraction, skeleton extraction is performed on the results of time-frequency analysis. From the perspective of the image domain, the time-frequency image is denoised, smoothed, binarized, and then extracted. Then extract the skeleton of the micro-motion target sinusoidal curve. At this time, the image matrix contains only two values of “0” and “1”. The position where “1” appears is the skeleton of the characteristic curve.
3.2. Parameter estimation method based on dimensionality reduction

In addition to traditional rotating speed estimation methods, many micro-motion parameter extraction methods use micro-Doppler sinusoidal modulation curves as prior knowledge, such as complex empirical mode decomposition (CEMD), matched Fourier transform, extended Hough transform, etc. But the above methods are computationally complex and computationally intensive. Therefore, this paper proposes a parameter estimation method with dimension reduction at time-frequency domain. This method will change the two-dimensional joint estimation of the blade length parameter and the rotation speed parameter to stepwise estimation method by using the maximum instantaneous micro-Doppler frequency of the extracted f micro-motion target as prior information. To estimate the rotation speed. Firstly, the search function constructed.

\[ h(r, \omega, \theta, t_m) = \exp[-i\frac{4\pi}{\lambda} r_m \cos(\omega t_m + \theta)] \]  

(7)

In order to facilitate the analysis, it is assumed that the initial phase \( \theta_m \) is known. From the analysis, the micro-Doppler curve of the rotor target is a sine curve. Its expression can be written as

\[ f_{m-D} = f_{m-D_{\max}} \sin(\omega t_m + \theta_m) \]  

(8)

From equation (5) and (8), it can be seen that there must be a certain value to make the instantaneous micro-Doppler frequency reach the maximum value. At this time,

\[ r_m = \frac{2f_{m-D_{\max}}}{2\omega} \]  

(9)

At this point, the search function is rewritten as

\[ h(\omega, t_m) = \exp[-i2\pi \frac{f_{m-D_{\max}}}{\omega} \cos(\omega t_m + \theta_m)] \]  

(10)

Then the search matrix is represented as

\[
H = \begin{bmatrix}
  h(\omega_1, t_1) & h(\omega_1, t_2) & \ldots & h(\omega_1, t_m) \\
  h(\omega_2, t_1) & h(\omega_2, t_2) & \ldots & h(\omega_2, t_m) \\
  \vdots & \vdots & \ddots & \vdots \\
  h(\omega_q, t_1) & h(\omega_q, t_2) & \ldots & h(\omega_q, t_m)
\end{bmatrix}_{q \times m}
\]

(11)

Perform Gabor transform on each row of \( H_{q \times M} \) can obtain \( q \) time-frequency data matrices

\[ V = [V_1, V_2, \ldots, V_{q-1}, V_q] \]  

(12)

Since the time-frequency analysis method uses time variables and frequency variables to characterize the energy distribution density of the signal, the energy accumulation can be performed by performing a dot product between the data matrices \( V \) and the echo signal matrix \( J \) after time-frequency analysis. The effect of energy accumulation is only related to the \( \omega \) in the search function.

\[ F = J \cdot V \]  

(13)

When the accumulated energy reaches the maximum value, the rotating speed value \( \omega \) in the search function is the estimated value \( \hat{\omega} \) of the rotating speed.

\[ \hat{\omega} = \max(E(F)) \]  

(14)

According to the analysis, when the search interval \( \Delta \omega \) is smaller in the search function, the search result is more accurate, but because the algorithm needs to perform time-frequency analysis on each possible value of the rotational speed, the calculation amount will increase dramatically. Therefore, in this paper, the rough estimation and the precise estimation are performed in the search process. The approximate range of the actual rotating speed value is found by using the rough estimation, then, using precise estimation to search for the rotating speed value in this range, finally, the rotating speed estimation value is obtained.
After estimating the rotating speed of the rotor, the blade length of the target can be estimated by equations (14).

\[ \hat{f} = \frac{\lambda f_{w,t,\omega_{\text{max}}}}{2\hat{\omega}} \]  

(15)

4. Simulation analysis

In order to obtain the method of this article on the micro-motion target rotation frequency \( f_{\text{rot}} = (2\pi)^{-1}\omega \) and blade length extraction more clearly, this simulation selected 3 rotor blades with 10 strong scattering points. The model is shown in figure 2. The length of the blades is \( l = 6m \), and the SNR defined in this paper is the SNR after pulse compression. Specific simulation parameters are presented in Table 1.

| Simulation parameters |
|------------------------|
| PRF  | \( T_p \) | B  | \( f_s \) | \( T_a \) |
| 4000Hz | 100μs | 1MHz | 2MHz | 0.512s |

Figure 3 is the time-frequency diagram processed of the target echo at SNR = 0dB and \( f_{\text{rot}} = 3.14Hz \). Figure 4 is the result of skeleton extraction of time-frequency diagram. Figure 5 is the rough estimation result of the rotation frequency when the search range of the rotation frequency is [1Hz, 5Hz] and the search interval is 0.5Hz. Figure 6 is the first precision estimation result after changing the search interval to 0.05Hz. Figure 7 shows the second precision estimation result after changing the search interval to 0.005Hz.
In figure 3, it can be seen that the fretting frequency of the target is sinusoidally modulated. The time-frequency diagram can be used to extract the time-frequency diagram skeleton as shown in figure 4, and the skeleton can be used to estimate the maximum micro-Doppler instantaneous frequency. In figure 5, when \( f_{\text{rot}} = 3.14 \text{Hz} \), there is a mismatch occurred between the rotation frequency and the search interval, this method can only estimate the approximate range of the target's rotation frequency, and the estimation error is unacceptable. Therefore, it is necessary to make a precise estimation. In figure 6 and figure 7, the rotation frequency is estimated by changing the search interval, and the estimation result is gradually approached to the actual rotation frequency. After the second precision estimation, the method can accurately estimate the rotation frequency of the rotor target.

Figure 8 is a comparison of the estimation accuracy and anti-noise performance of the rotation frequency estimation with the presented method, the higher-order moment function analysis method (method 1), the orthogonal matching pursuit method (method 2), and the Hough transform method (method 3). Figure 9 shows the error values estimated by the method of this paper for rotor target’s blade length under different SNR.

From figure 8, we can see that the accuracy of the rotational frequency estimation is obviously better than the other three algorithms. Therefore, this method can perform relatively fast rotational frequency estimation under the condition of lower SNR. Compared with other methods, the proposed method can also extract the feature of blade length. As can be seen from figure 9, the estimation error of the method for the blade length is very small and almost negligible at \( \text{SNR} > -5 \text{dB} \). At \(-18 \text{dB} < \text{SNR} < -5 \text{dB}\), the estimation error is still within the acceptable range.
5. Conclusions
In this paper, a novel micro-motion target parameter estimation algorithm in the situation of low SNR is proposed. By using the prior information of the maximum instantaneous micro-Doppler frequency of the rotor, the dimension of search matrix constructed is reduced, and then energy is accumulated from the time-frequency domain. From the point of view, rough estimation and two precise estimations are performed on the rotational speed, and the best accumulated rotational speed value is the estimated optimal rotational speed, as well as the blade length is estimated through the rotational speed estimation value and the maximum instantaneous micro-Doppler frequency estimate. It can be seen from the simulation results that the rotational speed can be estimated quickly by this method. In the case of low SNR, this method still has high estimation accuracy. However, this method is relying on skeleton extraction results when the estimation is performed. Under extremely low SNR conditions, the estimation accuracy of the method will drop sharply or even fail to be estimated. The next step needs to be analyzed from the perspective of time-frequency analysis and image processing to further improve the estimation accuracy of the proposed method under extremely low SNR.

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