BEAM-BREAKUP STUDIES FOR THE 4-PASS CORNELL-BROOKHAVEN ENERGY-RECOVERY LINAC TEST ACCELERATOR

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Abstract. The Cornell Brookhaven Energy Recovery Linac (ERL) Test Accelerator (CBETA) [1] is currently under construction at Cornell University’s Wilson Laboratory. The primary structures in CBETA for beam recirculation include the Main Linac Cryomodule (MLC) and the Fixed Field Alternating Gradient (FFAG) beamline. As the electron bunches pass through the MLC cavities, higher order modes (HOMs) are excited. The recirculating bunches in turn excite HOMs further, and this feedback loop can give rise to beam breakup instability (BBU). We will first explain how BBU occurs and how we simulate the effect. Then we present the simulation results on how BBU limits the maximum achievable current of CBETA, and the potential ways to improve the threshold current.

1. Introduction
BBU occurs in recirculating accelerators when a recirculated beam interacts with HOMs of the accelerating cavities. The most dominant HOMs are the dipole HOMs which give transverse kick to the beam bunches. The off-orbit bunches return to the same cavity and excite more dipole HOMs which, if in phase with the existing dipole HOMs, can kick the bunches further in the same direction. The effect can build up and eventually result in beam loss. With a larger beam current the effect becomes stronger, so BBU is a limiting factor on the maximum achievable current, the threshold current $I_{th}$. With more recirculation passes, bunches interact with cavities for more times, and $I_{th}$ can significantly decrease [4]. The low and high target currents of CBETA are 1 mA and 40 mA respectively, for both the 1-pass mode and 4-pass mode. Simulations are required to check whether $I_{th}$ is above this limit.

2. BBU Simulation Overview
Cornell Wilson Laboratory has developed a simulation software called Bmad to model relativistic beam dynamics in customized accelerator lattices [3]. Subroutines have been established to simulate BBU and find the $I_{th}$ for a specific design. The lattice provided to the program must include at least one multipass (recirculated) cavity with HOM(s) assigned to it. The following section will explain in detail how HOMs of the MLC cavities are obtained.

The goal of BBU simulation is to find the $I_{th}$. The program starts with a test current and records the voltage of all assigned HOMs over time. As the beam pass through the cavities, the momentum exchange between the bunches and wake fields are calculated, and HOM voltages
are updated. If all HOM voltages are stable over time, the test current is considered stable, and a greater current will be tested. In contrast, if at least one HOM voltage is unstable, the test current is regarded unstable, and a smaller current will be tested. Usually $I_{th}$ can be pinned down within numerical accuracy under 30 test currents.

3. HOM SIMULATION and ASSIGNMENT

To run BBU simulation we must first obtain the HOM characteristics. Each HOM is characterized by its frequency $f$, shunt impedance $(R/Q)$, quality factor $Q$, order $m$, and polarization angle $\theta$. Since the MLC has been built and commissioned, one would expect direct measurement of HOM spectra from the cavities. Unfortunately, the measured spectra contain hundreds of HOMs, and it is difficult to isolate each individual HOM and compute their characteristics. Instead, since the cavity shapes are modelled before constructed, we can simulate the HOM profiles using the known cavity structures. This has been done using the Horizontal Test Cryomodule (HTC) program, and the HOM characteristics can be directly computed.

In reality each cavity is manufactured with unknown precision errors. The errors in the ellipse parameters of the cavity shape are typically within $\pm 125 \mu m$ (in short, $\epsilon = 125 \mu m$). Accounting for such random errors in the HTC simulation results in different HOM spectra for a single cavity. Since MLC has 6 cavities with different manufacture errors in general, for each BBU simulation we assign each cavity a different set of HOM spectrum from HTC. With multiple BBU simulations we can therefore obtain a statistical distribution of $I_{th}$ of CBETA. The results are shown in the next section.

In general we could assign each cavity many HOMs of different HOM orders $m$, but this can be computationally heavy. To save simulation time we include only the 10 most dominant dipole-HOMs ($m = 1$) from a spectrum. A dipole-HOM is more dominant if it has a greater figure-of-merit $\xi = (R/Q)\sqrt{Q/f}$ [4]. For the rest of this paper, HOM refers to dipole-HOM unless further specified.

4. Bmad Simulation Result

Hundreds of simulations with different HOM assignments were run to obtain a statistical distribution of $I_{th}$ for each specific CBETA design. Three distributions are presented as histograms in this section:

Case 1: CBETA 1-pass with $\epsilon = 125 \mu m$
Case 2: CBETA 4-pass with $\epsilon = 125 \mu m$
Case 3: CBETA 4-pass with $\epsilon = 250 \mu m$

The third case with manufacture precision errors within $\pm 250 \mu m$ is investigated for academic interest.

4.1. Case 1) CBETA 1-Pass with $\epsilon = 125 \mu m$

The design current of CBETA 1-pass is 1 mA (the lower goal) and 40 mA (the higher goal). Figure 1 shows that all 500 simulations results exceed the lower goal of 1 mA, and 499 of them are above 40 mA. The result is quite promising.

4.2. Case 2) CBETA 4-Pass with $\epsilon = 125 \mu m$

The design current of CBETA 4-pass is the higher goal of 40 mA. Figure 2 shows that out of 500 simulations, 494 of them exceed the 40 mA goal. This implies that with certain undesirable combinations of HOMs in the cavities, the beam current can not reach the 40 mA goal due to BBU.
4.3. Case 3) CBETA 4-Pass with $\epsilon = 250 \, \mu m$
It is interesting to see how $I_{th}$ distribution changes with a different manufacture error ($\epsilon$) for the 4-pass lattice (see Fig. 3). For $\epsilon = 250 \, \mu m$, all 500 simulations are above 40 mA, which is better than the $\epsilon = 125 \, \mu m$ case. Some might thus wonder if a greater $\epsilon$ could statistically improve the threshold current. In fact, greater deviation in cavity shapes results in greater spread in the HOM frequencies. This allows the HOMs across cavities to act less coherently when kicking the beam, thus potentially increases the $I_{th}$. However, a greater deviation also tends to undesirably increase the $Q$ (and possibly $R/Q$) of the HOMs, which usually lowers $I_{th}$. Compensation between the frequency spread and HOM damping implies that a greater manufacture error in cavity shapes can not reliably improve $I_{th}$.

5. Aim for higher $I_{th}$
To achieve a higher $I_{th}$, three ways have been proposed, and their effects can be simulated. The first way is to change the bunch frequency $f_b$ (from injector) by an integer multiple. Simulations on a CBETA 1-pass and 4-pass lattice show a change of $I_{th}$ fewer than 5% over several choices of $f_b$, implying that varying $f_b$ is not effective in improving CBETA $I_{th}$. Rigorous calculation [4] has shown that $I_{th}$ depends on $f_b$ in a non-linear way for a multi-pass ERL, and it will be interesting to experiment this effect on the realistic CBETA. The other two ways involve either varying the phase advances or introducing x-y coupling between the cavities. The simulation results with these two methods are presented in the following sections.

6. Effect on $I_{th}$ by varying phase advance
$I_{th}$ can potentially be improved by changing the phase advances (in both x and y) between the multi-pass cavities. This method equivalently changes the $T_{12}$ (and $T_{34}$) element of the transfer
matrices, and smaller $T_{12}$ values physically correspond to a greater $I_{th}$ in 1-pass ERLs [4]. To vary the phase advances in Bmad simulations, a zero-length matrix element is introduced right after the first pass of the MLC LINAC. In reality the phase advances are changed by adjusting the quad strengths around the accelerator structure. In simulation the introduction of the matrix may seem arbitrary, but this gives us insight on how high $I_{th}$ can reach as phase advances vary.

For each simulation, each cavity is assigned with three “$\epsilon = 125 \mu m$” HOMs in x, and three identical HOMs in y (polarization angle = $\pi/2$). The $I_{th}$ is obtained for a choice of $(\phi_{x}, \phi_{y})$, each from 0 to $2\pi$. Several simulations were run for both the 1-pass and 4-pass CBETA lattice, and mainly 4-pass results are presented below.

Figure 4 shows a typical way $I_{th}$ varies with the two phase advances. Depending on the HOM assignment, the $I_{th}$ can reach up to 200 mA with an optimal choice of $(\phi_{x}, \phi_{y})$. This implies that changing phase advances does give us advantages in improving $I_{th}$ for the 1-pass CBETA lattice (the improvement can range from +200 mA to +400 mA depending on the HOMs assigned). Note that $\phi_{x}$ and $\phi_{y}$ affect $I_{th}$ rather independently. That is, at certain $\phi_{x}$ which results in a low $I_{th}$ (the “valley”), any choice of $\phi_{y}$ does not help increase $I_{th}$, and vice versa. It is also observed that $I_{th}$ is more sensitive to $\phi_{x}$, and the effect of $\phi_{y}$ becomes obvious mostly at the “crest” in $\phi_{x}$. Physically this is expected since many lattice elements have a unit transfer matrix in the vertical direction, and the effect of varying $T_{12}$ is more significant than $T_{34}$. In other words, HOMs with horizontal polarization are more often excited. As we will see this is no longer true when x-y coupling is introduced.

It is also observed that the location of the “valley” remains almost fixed when HOM assignments are similar. Physically the valley occurs when the combination of phase-advances results in a great $T_{12}$ which excites the most dominant HOM. Therefore, the valley location depends on which cavity is assigned with the most dominant HOM, and is consistent with the simulation results.

Figure 4. A scan of BBU $I_{th}$ over the two phase advances for the CBETA 4-pass lattice. Each cavity is assigned with a random set of 3 dipole HOMs in both x and y polarization. ($\epsilon = 125 \mu m$). For this particular HOM assignment, $I_{th}$ ranges from 61 mA to 193 mA.

Figure 5. A scan of BBU $I_{th}$ over the two free phases for the CBETA 4-pass lattice with x-y coupling. Each cavity is assigned with a random set of 3 dipole HOMs in both x and y polarization. ($\epsilon = 125 \mu m$). For this particular HOM assignment, $I_{th}$ ranges from 89 mA to 131 mA.

7. Effect on $I_{th}$ with x-y coupling
Another method potentially improves $I_{th}$ by introducing x-y coupling in the transverse optics, so that horizontal HOMs excite vertical oscillations and vise versa. This method has been shown very effective for 1-pass ERLs [5]. To simulate the coupling effect in Bmad simulation, a different
non-zero length is again introduced right after the first pass of the LINAC. The matrix couples the transverse optics with two free phases ($\phi_1, \phi_2$) to be chosen. These two phases are not the conventional phase advances, but can also range from 0 to $2\pi$.

Figure 5 shows a typical way $I_{th}$ varies with the two free phases for the 4-pass lattice. Depending on the HOM assignment, the $I_{th}$ can reach up to 131 mA with an optimal choice of ($\phi_1, \phi_2$). Because the transverse optics are coupled, the two phases no longer affect $I_{th}$ in an independent manner. That is, there is no specific $\phi_1$ which would always result in a relatively high or low $I_{th}$. Both phases need to be varied to reach a relatively high $I_{th}$. Therefore introducing x-y coupling can still improve $I_{th}$ for the 4-pass lattice (about +60 mA), but not as significantly as varying phase advances.

8. Summary

Bmad BBU simulation has shown that for the current CBETA design lattice, both the 1-pass and 4-pass machine can always reach the low design current (1 mA), and can surpass the high goal of 40 mA over 98% of the time depending on the HOMs assigned.

To potentially increase the $I_{th}$, we can either adjust the injector bunch frequency, or vary the lattice optics (by introducing additional phase advances or x-y coupling). While the former is shown ineffective by simulation, the later provides room for improvement. For the 1-pass lattice, both optic-varying methods allow great improvement in $I_{th}$ (about +200 mA to +400 mA). For the 4-pass lattice, the method of varying phase advances allow more improvement (about +150 mA) than x-y coupling (about +60 mA).

In reality, introducing x-y coupling requires installation of skew quadrupole magnets, so varying phase advances remains the most effective method to improve the $I_{th}$ of CBETA.

References

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