Current Issues in Perturbative QCD

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Abstract

This review talk discusses some issues of active research in perturbative QCD. Among the topics discussed are, heavy flavor and prompt photon production in hadron-hadron collisions, “small x” phenomena and the current status of $\alpha_s$. 

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1 The current value of $\alpha_s$

I will present a brief update of the value of $\alpha_s(M_Z)$. For a review of the results prior to this meeting see the article in QCD in the 1994 edition of the Review of Particle Properties [1]. The methodology adopted there will be followed here. Results from experiments using similar methods that have common systematic errors are first combined. These results are then extrapolated up to the $Z$ mass using the renormalization group. An average of these values is then made to give the final result which is quoted as a value for $\alpha_s(M_Z)$. The new results will now be discussed.

1.1 Lattice Gauge Theory.

Lattice gauge theory calculations can be used to calculated the energy levels of a $Q\bar{Q}$ system and then extract $\alpha_s$. The FNAL group [2] uses the splitting between the 1S and 1P in the charmonium system $(m_{c\bar{c}} - (3m_\psi + m_\eta_c)/4 = 456.6 \pm 0.4$ MeV) to determine $\alpha_s$. The result quoted is $\alpha_s(M_Z) = 0.108 \pm 0.006$. The splitting is almost independent of the charm quark mass and is therefore dependent only on $\alpha_s$. The calculation does not rely on perturbation theory or on non-relativistic approximation. The main errors are systematic associated with the finite lattice spacing ($a$), the matching to the perturbatively defined $\alpha_s$, and quenched approximation used in the calculation. The extrapolation to zero lattice spacing produces a shift in $\Lambda$ of order 5% and is therefore quite small. The quenched approximation is more serious. No light quarks are allowed to propagate and hence the extracted value of $\Lambda$ corresponds to the case of zero flavors. $\alpha_s(M)$ is evolved down from the scale ($\sim 2.3$ GeV) of the lattice used to the scale of momentum transfers appropriate to the charmonium system ($\sim 700$ MeV). The resulting coupling is then evolved back up with the correct number of quark flavors. Perturbative running of $\alpha_s(M)$ has to be used at small $M$.

A recent calculation [3] using using the strength of the force between two
The values of $\alpha_s(M_Z)$ determined by various methods. The symbol $\ast$ denotes a result that has been updated from that in Ref [1].

Calculations based on the $\Upsilon$ spectrum using non-relativistic lattice theory give $\alpha_s(M_Z) = 0.115 \pm 0.003$ [4]. This result includes relativistic corrections up to order $m_b(v/c)^4$. This recent result does not rely on the quenched approximation. Calculations are performed with two massless flavors. Combining this with the result in quenched approximation enables the result to be extracted for the physical case of three ($u,d$ and $s$) light quarks. It is gratifying that this result is within the error quoted on the quenched calculations [5]. Averaging the lattice results then yields $\alpha_s(M_Z) = 0.113 \pm 0.003$.
1.2 Jet counting

A recent result from CLEO [6] measuring jet multiplicities at in $e^+e^-$ annihilation at $\sqrt{s} = 10$ GeV, i.e. below the $b\bar{b}$ threshold gives a result of $\alpha_s(M_Z) = 0.113 \pm 0.006$. As with all measurements of this type, the dominant errors are systematic and arise from ambiguities in the scale at which $\alpha_s$ is evaluated and from the algorithms used to define a jet. This result is consistent with that from higher energies, in particular those from LEP [7] and SLC [8].

Data at $\sqrt{s} = 29, 58$ and 91 GeV have been fit with the same set of Monte-Carlo (fragmentation) parameters. A consistent fit is obtained providing direct evidence for the running of $\alpha_s(Q)$. [9]

The H1 collaboration working at HERA [10] has determined $\alpha_s$ from a fit to the $2 + 1$ jet rate [11]. At lowest order in QCD the final state in deep inelastic scattering contains 1+1 jets, one from the proton beam fragment and one from the quark that is struck by the electron. At next order, the struck quark can radiate another gluons giving rise to the $2 + 1$ jet final state. The determination involves data over a large $Q^2$ range and hence there is some correlation between the value of $\alpha_s$ and the structure functions that enter the computation of the event rate. The value quoted is $\alpha_s(M_Z) = 0.121 \pm 0.015$.

1.3 Upsilon decay.

The Cleo group [12] has determined $\alpha_s$ from a measurement of the ratio of Upsilon decay rates $\Upsilon \rightarrow \gamma + \text{hadrons} / \Upsilon \rightarrow \text{hadrons}$. In lowest order QCD this is given by $\Upsilon \rightarrow \gamma gg$. They quote $\alpha_s(M_Z) = 0.111 \pm 0.006$. There is non-perturbative contribution to this final state from the fragmentation of a gluon jet into a photon; this will introduce additional systematic errors into the result.
1.4 Scaling violations in Fragmentation functions.

The probability for a quark produced at scale $Q$ (for example in $e^+e^-$ annihilation at $\sqrt{s} = Q$) and energy $E$ to decay into a hadron of energy $zE$ is parameterized by a fragmentation function $d(z, Q)$. Just as in the case of the structure functions, the $Q$ dependence of this fragmentation function is given by perturbative QCD and depends only on $\alpha_s$. The QCD evolution of this fragmentation function also involves the fragmentation function of a gluon ($g(z, Q)$). Hence in order to determine $\alpha_s$ both $d(z, Q_0)$ and $g(z, Q_0)$ must be determined at some reference point $Q_0$. The ALEPH collaboration uses three jet events from the decay of a $Z$. Two of the jets are tagged to be from $b$–quarks using the vertex detector and the finite $b$–quark lifetime. The third is then known to be due to a gluon. This method also determines the fragmentation functions for charm and bottom quarks which do not have the same form at $Q_0$ as the light quarks. It is worth recalling that whereas higher twist corrections in deep inelastic scattering are of order $1/Q^2$, here they can be order $1/Q$. These are parameterized in the ALEPH fit by replacing $z$ by $z + c(z)/Q$. ALEPH quotes a value of $\alpha_s(M_Z) = 0.127\pm0.011$. The DELPHI collaboration, using a different method quotes $\alpha_s(M_Z) = 0.118\pm0.005$. This result does not use the independent measurements of heavy quark and gluon fragmentation functions but rather fits to a Monte-Carlo. Its error could be underestimated.

1.5 Hadronic Width of the Tau Lepton

The hadronic width (or branching ratio) of the tau lepton can be used to determine $\alpha_s$. In the decay $\tau \rightarrow \nu_\tau + \text{hadrons}$, the decay rate, $R(M)$, can be measured as a function of the invariant mass $M$ of the hadronic system. The inclusive hadronic width is then obtained by integrating over $M$, viz. $\Gamma = \int dMR(M)$. There are non-perturbative (higher twist) contributions that can be calculated using QCD sum rules. Alternatively the data can
be used to determine these quantities, which have different $M$ dependence, from the data by using $\gamma(n) = \int dM M^n R(M)$. The values obtained in this are consistent with the estimates from the sum rules. There is a new result from CLEO that gives $\alpha_s(M_Z) = 0.114 \pm 0.003$, a value somewhat below the old world average. A new result from ALEPH of $\alpha_s(M_Z) = 0.124 \pm 0.003$ is larger than the old world average. The difference between these results is due to different values of the branching ratios $R_e$ and $R_\mu$ measured for $\tau \to e\nu\nu\bar{\nu}$ and $\tau \to \mu\nu\nu\bar{\nu}$. The hadronic width is then inferred from these via $R_h = 1 - R_e - R_\mu$ as this results in a smaller error than that gotten by using $\Gamma$ directly.

1.6 Average value of $\alpha_s(M_Z)$

After taking into account this new data, the average of $\alpha_s(M_Z) = 0.117$ quoted in RPP 1994 is left unchanged. If we assume that the systematic errors associated with the different methods are uncorrelated, then we obtain an error of $\pm 0.002$. In view of the fact that most of the dominant errors are theoretical, involving such things as estimates of non-perturbative corrections and the choice of scale $\mu$ where $\alpha_s(\mu)$ is evaluated for the process in question, it is more reasonable to quote $\alpha_s(M_Z) = 0.117 \pm 0.005$ as the “world average”.

2 Heavy Quark Production in Hadron Collisions

New results are available from the CDF and D0 collaborations on the production rate of bottom quarks in $p\bar{p}$ collisions. Several methods are used. The least subject to ambiguities involves the use of fully reconstructed decays of a $B$ meson. In this case in order to get from the observed $B$ meson rate to that of $b$–quarks, the fragmentation function of a $b$–quark needs to be known. This is well constrained from data at LEP so this method of
measuring the $b$-quark production rate should be quite reliable. However there are rather few fully reconstructed events and hence this method is limited and does not permit measurement over a large range of transverse momenta.

The next method involves the use of inclusive $\Psi$ production. This method can only be used if a vertex system is available to disentangle the $\Psi$'s that come from $b$-decay from those produced directly. Here the systematic errors are a larger since one needs a model of the $b$-quark fragmentation and of the subsequent $b$-meson and $b$-baryon decays to $\Psi X$.

Finally, there is the method with the largest statistical sample and hence the greatest range in transverse momentum. Here one searches for jets which have muons or electrons associated with them. The leptons are required to have some transverse momentum of order 1 GeV or greater with respect to the jet direction ($p_\text{lab}$). Most of these leptons then arise from bottom decay: there is a small contribution from charm decay, the relative fraction being a function of $p_\text{lab}$. A model of the lepton spectrum from charm and bottom quarks is needed before the $b$-quark cross section can be extracted.

The measured rates from the different methods are shown in figure 2 which shows the cross-section for the production of a $b$-quark of transverse momentum greater than $p_{\text{T}}^{\text{min}}$. It can be seen from this figure that the rates measured by the D0 collaboration which uses only the last method are systematically lower than those of the CDF collaboration which uses all of the methods for $b$-quark transverse momenta of less than 15 GeV. Above this value the experiments are consistent with each other. Note that in the region of disagreement CDF is able to use what should be the most reliable method. The figure also shows the theoretical expectation for this rate [22]. It is rather uncertain since the QCD predictions are not stable with respect to the choice of the scale $\mu$ at which the parton distributions and $\alpha_s(\mu)$ is evaluated in the expression for the production rate. By lowering this to $\mu = \sqrt{m_b^2 + p_T^2}/4$ consistency with the CDF data can be achieved. The D0 data can be accommodated by using larger and \textit{a priori} more reasonable
Figure 2: The inclusive cross section for the production of $b$-quarks in $p\bar{p}$ collisions at $\sqrt{s} = 1.8$ TeV. The produced quark is required to have transverse momentum greater than $p_T^{\text{min}}$ and the rate is shown as a function of $p_T^{\text{min}}$. See text for discussion.
value of \( \mu \). The cross-section now reported by CDF is lower than the values that were obtained from observation of \( \Psi \) production and the assumption, now known to be wrong, that almost all \( \psi \)'s at large transverse momenta arise from \( b \)-quark decay.

The top cross-section quoted by CDF\(^23\) is somewhat larger than that predicted by QCD for a mass of 170 GeV\(^24\), the value given by the CDF fit. Since D0 has not yet confirmed this rate\(^2\), it is premature to claim that there is a problem with QCD or that physics beyond the standard model has been discovered.

3 Production of \( \Psi \) and \( \Upsilon \) in \( p\bar{p} \) collisions.

At low transverse momentum the production of \( \psi \)'s in \( p\bar{p} \) collisions is expected to proceed dominantly via the production of \( \chi \) states followed by their decay i.e. \( g + g \rightarrow \chi \rightarrow \psi + X \)\(^26\). The analogous process at large transverse momentum is \( gg \rightarrow g\chi \). This process generates a cross section that falls off at large transverse momentum much faster than, say, the jet rate. This observation led to an assumption that \( \psi \)'s produced at large transverse momentum came almost exclusively from \( b \)-quark decay. A measurement of the rate for \( \psi \) production could then be used to infer the \( b \)-quark rate. This assumption is now known to be false. By detecting whether or not the \( \psi \)'s come from the primary event vertex, CDF is now able to test this assumption. The fraction of \( \psi \)'s produced directly is almost independent of transverse momentum and the rate of direct \( \Psi \) production at large \( p_t \) is larger than had been expected.

The dominant production mechanism of \( \psi \)'s at large transverse momentum is now believed to be the fragmentation of light quark and gluon jets into \( \chi \)'s that then decay to \( \psi \)\(^27\). CDF now has data on \( \psi, \psi', \Upsilon, \Upsilon' \) and \( \Upsilon'' \)\(^30\). While the rate for \( \psi \) production is in agreement with expectations, given the inherent theoretical uncertainties, the rate for \( \psi' \) is approximately a factor of 20 above the theoretical expectation\(^28\). The calculation does not
include the possibility of $\psi'$ production from the decay of 2P states. These states are above the $D\bar{D}$ threshold, however a branching ratio of a few percent to $\psi'$ could be enough to explain the deficit. A recent paper investigates this possibility quantitatively [29].

The predicted rates for $\Upsilon$ production should have less uncertainties due to the larger value of the $\Upsilon$ mass. Preliminary data from CDF indicate that the agreement with theoretical expectations is poor [30].

4 Prompt Photon Production.

The production of photons in $p\bar{p}$ proceeds, at lowest order in QCD, via the parton process $qg \rightarrow \gamma q$. The process provides a direct probe of the gluon distribution and can be measured more reliably than the jet cross-section whose value depends on a jet definition and upon measurements of both hadronic and electromagnetic energy. The produced photon, provided that is is produced at large transverse momentum, is well isolated from other produced particles. At higher orders in $\alpha_s$, the situation changes. Processes such as $qq \rightarrow qq\gamma$ start to contribute. This process is largest when the photon is collinear with one of the outgoing quarks. Since experiments cannot easily measure photons within jets, they search for isolated photons defined by having less than some amount $\epsilon$ of other energy in a cone of radius $\Delta R$ in rapidity-azimuth space around the photon direction. The rate then depends on $\epsilon$ and $\Delta R$; the selection criteria discriminate against the bremsstrahlung component. The fragmentation of a quark into a photon is a non-perturbative phenomena which must be modeled by a fragmentation function into which the collinear singularity is absorbed.

A theoretical prediction of the prompt photon rate then depends on, the gluon and quark distributions, the fragmentation functions, and the scales $\mu$ and $Q$ at which these functions and $\alpha_s(Q)$ are evaluated (these scales need not be the same). The dependence on these scales is an indication of the
uncertainties in the theoretical predictions; if the process were calculated to all orders in perturbation theory, the dependence on $\mu$ and $Q$ would, at least in principle, disappear. $\mu$ and $Q$ should be of order $p_t$, beyond that theory provides no guidance. There is a longstanding problem in that at small values of $x_\perp = 2p_t/\sqrt{s}$, the data tend to be larger than the theoretical predictions. At values of $x_\perp$ that are probed at the Fermilab collider a smaller value of $\mu$ results in a larger predicted cross-section.

There have been several new developments in this field. Measurements of structure functions at small $x$ at HERA\cite{37} have indicated that the gluon distribution is larger at small values of $x$ than used to be assumed. This change increases the predicted rate at small $x_\perp$. There has been a reassessment of the importance of fragmentation \cite{31} and finally new data are available.

Figure 3 shows that data from the CDF collaboration \cite{34}. The data fall very rapidly with increasing $p_t$ so, to facilitate comparison with theory, the data are shown relative to the calculation of ref \cite{32}. The tendency for the data to have a steeper dependence on $p_t$ than the theory can be seen in this figure. A reduction in the scale to $\mu = p_t/2$ brings the data into better agreement with the theory. Figure 4 shows the same data compared to the calculation of ref \cite{31}. Here the scale $\mu = p_t/2$ has been used along with the GRV structure functions \cite{33}. The predictions of MRSD\textsuperscript{−′} structure functions\cite{39} are almost identical. This theoretical result has a slightly larger fragmentation component. The tendency for the data to have a steeper $p_t$ slope than the theory is still apparent in this plot, although the agreement is quite good. A reduction in the scale $\mu$ to the, possibly unreasonable, value of $p_t/3$ improves the agreement further.

Preliminary results presented at this meeting from D0\cite{35} lie somewhat below those of CDF and are therefore in better agreement with theoretical estimates. Figure 5 shows a comparison with theory. Note that that theory is the same as that used in figure 3. The tendency for the data to have a steeper $p_t$ slope than the theory is not evident in the D0 results although the systematic errors are such that there is no significant disagreement with
Figure 3: A comparison of the CDF data on prompt photon production. Data refers to the quantity $\frac{d\sigma}{dp_{d\eta}}$ at $\eta = 0$. Theory refers to the calculation of Ref [32] using the CTEQ2M structure functions [36] and $\mu = p_T$. 
Figure 4: As Figure 3 except that the theory refers to the calculation of Ref [31] using the GRV structure functions [38] and $\mu = p_t/2$.

CDF.

There is a preliminary measurement of the di-photon rate from CDF [41]. This measurement is important since, at the LHC, one of the decay mechanisms proposed to search for the Higgs boson is its decay to two photons [42]. The very large background is expected to occur from $q\overline{q} \rightarrow \gamma\gamma$ and $gg \rightarrow \gamma\gamma$. The rate observed by CDF is consistent with the expectation from calculations using these mechanisms [43]. We can now have more confidence in the ability of LHC to see the Higgs signal.

5 Small-$x$ and related phenomena

QCD perturbation theory is an expansion powers of $\alpha_s(Q)$. Two conditions must be satisfied to have a reliable prediction. First, the scale $Q$ must be large and second the perturbation series must contain no large coefficients.
The value of some measured dimensionless quantity $P$ and be expressed as a power series.

$$P = A\alpha_s^n(Q)(1 + b\alpha_s(Q) + \cdots)$$

If $b\alpha_s(Q) \sim 1$, then the perturbation series useless. A physical prediction can be recovered if the large terms can be isolated order by order in perturbation theory and the terms summed up. Once these pieces are absorbed into $A$, the resulting series may be well behaved and a prediction possible.

The simplest example of a resummation of this type is that of the Altarelli-Parisi (DGLAP) equation which sums terms of the type $(\alpha_s \ln(Q^2))^n$ that arise in Deep Inelastic Scattering [44]. At very small values of Bjorken-$x$, $b$ can contain terms of the type $\log(1/x)$. These terms can be resummed using the BFKL equation [45]. The result of this resummation is a structure function that rises very rapidly at small-$x$. While this behaviour is seen at HERA [37], it cannot be used to distinguish between evolution expected from BFKL and DGLAP [46].

Figure 5: As Figure 3 except that the experiment refers to the results of the D0 collaboration [40].
The behaviour of the structure functions at very small $x$ is connected with attempts to calculate the total cross-section in perturbative QCD. The same resummation that leads to BFKL is also responsible for the appearance in perturbative QCD of the pomeron\[47\]. This connection has recently been clarified\[48\]. I will now discuss some phenomena related to the pomeron.

5.1 Jets with Large Rapidity Separation

Events are selected in $p\bar{p}$ collisions having a pair of jets with transverse momenta $p_1$ and $p_2$ and rapidities $\eta_1$ and $\eta_2$ with azimuthal angle $\phi$ between them. At lowest order in perturbative QCD, $p_1 = p_2$, $\cos(\pi - \phi) = 1$ and the rate is given by

$$\frac{d\sigma}{dp_1 d\eta_1 d\eta_2} = \frac{1}{16\pi s} g(x_1, Q^2) g(x_2, Q^2) \frac{d\sigma(gg \rightarrow gg)}{dt}$$

Here I have assumed that only gluons contribute. If $\eta_1 = -\eta_2 = y$, then $x_1 = x_2 = \frac{2p_1}{\sqrt{s}} \cosh y / 2$. If $y$ is very large, the center of mass energy of the parton system ($= x_1 x_2 s$) becomes large and the partonic cross section can be approximated by

$$\frac{d\sigma}{dt} = \frac{9\pi \alpha_s^2}{2p_1^2 p_2^2}$$

If we now integrate over $p_1$ and $p_2$ greater than some scale $M$

$$\frac{d\sigma}{d\eta_1 \eta_2} \sim \frac{\alpha_s^2}{M^2} x_1 g(x_1) x_2 g(x_2)$$

At order $\alpha_s^3$ in perturbation theory, several phenomena occur. A third jet is emitted and the correlation in azimuth and equality of $p_1$ and $p_2$ is lost. More important is that this order $\alpha_s^3$ process modifies the result for $\frac{d\sigma}{dp_1 d\eta_1 d\eta_2}$ by a factor of $(1 + 3\alpha_s |\eta_1 - \eta_2| / \pi)$ (for large values of $|\eta_1 - \eta_2|$). If $|\eta_1 - \eta_2|$ is large enough this factor can be so large that perturbation theory is not reliable. In this case the leading terms at all orders in perturbation theory can be resummed to give a factor of $exp(3\alpha_s |\eta_1 - \eta_2| / \pi)$ \[49\]. This
growth is not observable at the Tevatron since it is more than compensated by the drop off caused by the falling structure functions. (Note that \(x_1\) and \(x_2\) increase as \(y\) increases.) It may be observable at LHC \cite{50}. However the other effects should be observable. The rapidity region between the two jets is filled with many mini-jets since there is no penalty of \(\alpha_s\) to pay for each emission. The correlation in \(\phi\) between the two trigger jets should show a rapid fall off as \(y\) is increased. The D0 collaboration\cite{51} has searched for this effect by selecting events with two jets one of which has \(p_t > 20\) GeV and the other has \(p_t > 50\) GeV. The \(\phi\) correlation is then plotted as a function of the rapidity separation. The data show a decorrelation. However it is a much slower fall off than predicted and is consistent with that expected from a fixed order \(\alpha_s^3\) calculation or from showering Monte-Carlos such as HERWIG\cite{52}. It is possible that the rather asymmetric trigger could be masking the effect in this case.

5.2 Rapidity Gaps.

Consider the production of two jets at large rapidity separation in a \(p\bar{p}\) collision. At lowest order in QCD perturbation theory one contribution to this arises from quark-quark scattering via gluon exchange. Before the scattering each quark forms a color singlet state with the rest of the quarks and gluons from its parent (anti-)proton. After scattering, this is no longer the case since the gluon exchange causes color charge to be transferred between the quarks. As the parton system hadronizes into jets, color must be exchanged between the outgoing jets. This color exchange manifests itself as soft (low transverse) momentum particles that fill the rapidity interval between the jets. Contrast this with the situation if a colorless object (such as a photon) were exchanged. Now the struck quark and the remnant of its parent are still in a color singlet and can hadronize without communication with the other quark. There is no necessity for color exchange and hence no need for particle production in the rapidity interval between the jets. Both the CDF\cite{53} and
D0 collaborations have searched for events with rapidity gaps. CDF uses the charged particle multiplicity while D0 uses the energy flow as measured by the calorimeter.

In the D0 case, events with two jets each of transverse energy of at least 30 GeV are selected. The jets are separated by rapidity $\eta$. Events are determined to have a gap if there are no calorimeter towers in the region between the two jets with an electromagnetic energy deposit of more than 200 MeV. Figure 6 shows the fraction ($f$) of events that have such gaps as a function of the rapidity separation of the jets. If all of the events are due to jet production involving color exchange, one expects that $f$ will fall rapidly with increasing $\eta$. While this behaviour is observed at small $\eta$ there is clear evidence for a plateau in $f = f_0$ at large values of $\eta$ indicating the presence of color singlet exchange.
CDF tags two jets of rapidity $\eta_1$ and $\eta_2$ ($\eta_2$ is assumed to be greater than $\eta_1$). They then look at the the multiplicity of charged tracks in region G defined by $\eta_1 < \eta < \eta_2$ and region N defined by the remainder of the rapidity range. N then covers the rapidity range between each jet and its parent (anti)proton. If color singlet exchange is contributing to the jet production, then one should expect events with zero multiplicity in region G. Region N always has color flow across it and can therefore be used as a control region. A KNO type multiplicity plot is made for the G and N regions, see figure 7. The shapes of the distributions in the G and N regions are the same except for an excess in the zero multiplicity bin in the G region. This provides clear evidence for events with a rapidity gap at a rate $f = (0.86 \pm 0.12)\%$. There is no evidence for any dependence of $f$ on either the transverse momentum of the jets ($E_t$) or the width of the gap ($\eta_2 - \eta_1$). The rate is too large to be due to photon exchange and must represent the exchange of another color singlet object. The obvious candidate is the pomeron.

These data leave several questions unanswered. $f$ cannot be directly interpreted in terms of the strength of the coupling of the pomeron to quarks and gluons since, once two jets are produced by this mechanism, we do not know how often particles are emitted into the gap region by the rest of the event and hence what fraction of these events survive to be detected by the experiments. (Bjorken \cite{55} uses the term survival probability $S$ for this.) Hence $f = S \frac{\text{pomeron-rate}}{\text{perturbative QCD-rate}}$. More data are needed on the $E_t$ dependence of $f$. If, for example, the pomeron couples to quarks and gluons with a form factor as opposed to a hard coupling, then one would expect $f$ to fall as $E_t$ increases. A constant $f$ would indicate that it coupled in a similar way to gluons.

A similar phenomenon has been observed at HERA. In the usual picture of deep-inelastic scattering a quark is struck by the virtual photon and ejected from the target proton. This quark then hadronizes into a jet (the current jet) and since its color is compensated by the target remnant, particles are produced in the rapidity region between the current jet and the beam proton.
Figure 7: The event rate plotted against particle multiplicity. Two jets are selected, separated in rapidity by 2.8 units of rapidity. The G (N) region is defined as the interval between the jets (between each jet and the end of the physical region closest to it). There is clear evidence for an excess of events in the zero multiplicity bin in the G region over that expected from a KNO fit (solid curve). No such excess is visible in the N region. See ref [53] for more details.
$\eta_{\text{max}}$ is defined as the rapidity of the particle with the largest rapidity in a particular event. (The proton is initially moving in the positive rapidity direction.) One would expect that there are always particles produced near the initial proton and so the $\eta_{\text{max}}$ distribution would have a peak at large positive value. Figure 8 shows the distribution as measured by ZEUS [56]. The data show, in addition to the expected peak, a large number of events where $\eta_{\text{max}}$ is very small. Approximately 8% of the events have no hadrons in the direction of the initial proton; the fraction is independent of the mass $Q^2$ of the virtual photon. Similar phenomena have been observed by the H1 collaboration [57].

The rate of events in this region of $\eta_{\text{max}} \sim 0$ is much larger than expected from a Monte-Carlo based on this picture of Deep inelastic scattering. The excess of events can be explained if there is some color neutral component of the proton which itself can be disassociated by the virtual photon. This component will have some fraction of the proton’s momentum. After interaction with the virtual photon the hadronization and color neutralization need only take place among the fragments of this color neutral system. There is no necessity for particle production in the rapidity region between the object and its parent proton. A second peak at smaller values of $\eta_{\text{max}}$ will then appear. One candidate for this object is the pomeron [58], which can be thought of as an object similar to other hadrons with quark constituents. A model of this type where the object (pomeron) has a structure function of the form $f(x) \sim x(1-x)$ is compatible with the ZEUS data [56].

The simplest candidate for this object in QCD is a two-gluon object [59] as shown in figure 9. This simple picture has been extended to and builds up the BFKL pomeron [60]. This picture is also in qualitative agreement with the data. Note that, as in the case of events with rapidity gaps at hadron colliders, the relative normalization of these color singlet pieces is difficult to extract from the data.
Figure 8: The $\eta_{\text{max}}$ distribution (see text) as measured by the ZEUS collaboration [56].
Figure 9: The simplest contribution to deep inelastic scattering in QCD with the possibility to produce an event with low $\eta_{max}$. The simplest object in QCD to play the role of the pomeron is a two gluon system.
6 Particle Multiplicity in Heavy Quark Jets.

While the particle multiplicity cannot be calculated in perturbative QCD, its growth with energy can be. Consider the radiation of a gluon off a quark produced of mass $M$ say in $e^+ e^-$ annihilation. If the gluon has energy $E$ and is emitted at angle $\theta$ with respect to the quark, then the emission probability behaves as

$$d\sigma = \frac{\theta^2 d\theta^2}{\theta^2 + \delta^2} \frac{dE}{E}$$

This has two consequences, radiation at $\theta < \delta$ is suppressed resulting a what is called a “dead-cone” and a heavy quark radiates less than a light quark [61]. We should expect the particle multiplicity ($N$) from a heavy quark pair ($Q$) to be less than that from a light quark pair ($q$)

$$N(Q\bar{Q}, \sqrt{s}) = N(q\bar{q}, \sqrt{s}) - N(q\bar{q}, M)$$

Note that the difference in multiplicities is independent of $\sqrt{s}$. The naive expectation based on the available phase space

$$N(Q\bar{Q}, \sqrt{s}) = N(q\bar{q}, \sqrt{s} - 2M)$$

predicts a difference that is not independent of $\sqrt{s}$. Figure 10 shows data at various energies[62]. While the total charged particle multiplicity rises with $\sqrt{s}$, the difference between multiplicity in $b-$quark events (tagged using the finite $b$ lifetime) and average events does not.

7 Conclusions.

The past few years has seen a continuing development in our understanding of QCD. The strong coupling constant is now known to a precision of order 5% . Its precision can only be expected to improve slowly in the near future since most of the measurements are now limited by various theoretical uncertainties. An improved measurement of the hadronic width of the $Z$ from
Figure 10: The behaviour of the charged particle multiplicity in $e^+e^-$ events as a function of $\sqrt{s}$ [92]. The plot shows the total multiplicity $n_{\text{had}}$ as well as the difference in multiplicity between events tagged as being from or not from the production of a $b\bar{b}$ pair ($\Delta n_b$).
LEP is one of the few areas where a more precise measurement of a physical quantity will yield a more accurate value of $\alpha_s$. The recent developments involving lattice gauge theory calculations with propagating light quarks is another area where one can hope for increased precision.

There are still some experimental results that, while accommodated by perturbative QCD, are not entirely satisfactorily explained. While the long standing problem of the prompt photon rate in $p\overline{p}$ collisions may now be going away, the production rate of bottom quarks is still not fully digested. Interesting data on $\psi$ and $\Upsilon$ production from CDF are yet to be fully understood.

Much interest, both theoretical and experimental, has occurred in the area of semi-hard (or small-$x$) QCD. Diffractive phenomena, for a long time dismissed as incalculable and hence uninteresting, are finally being given the attention that they deserve. There are many “predictions” for phenomena in this region of phase space. However, a systematic procedure for calculating the subleading corrections to the BFKL equation is lacking. Such a procedure is badly needed, for, until we can determine the size of these terms, we cannot say how accurate predictions using BFKL can be expected to be.

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References

[1] Review of Particle Properties  *Phys. Rev.* **D50**, 1174 (1994).
[2] A.X. El-Khadra et al., Phys. Rev. Lett. 69, 729 (1992). A. X. El-Khadra, presented at 1993 Lattice conference, OHSTPY-HEP-T-93-020, A.X. El-Khadra et al., FNAL 94-091/T (1994).

[3] Martin Luscher, Rainer Sommer, Peter Weisz and Ulli Wolff. Nucl. Phys. B413, 481 (1994).

[4] C.T.H. Davies et al. OHSTPY-HEP-T-94-013, Aug 1994.

[5] G.P. Lepage, and J. Sloan presented at 1993 Lattice conference. [hep-lat/9312070]

[6] R. Balest, et al. CLEO-CONF-94-28, Jul 1994.

[7] For a recent review see S. Bethke PITHA-94-29, Aug 1994.

[8] K. Abe, et al. Phys. Rev. Lett. 71, 2528 (1994).

[9] D.A. Bower et al. LBL-35812, M. Aolki et al. Submitted to the XXVII International Conference on High Energy Physics (Glasgow 1994), D. Decamp et al. Phys. Lett. B284, 163 (1992).

[10] S. Soldner-Rembold, submitted to Int. Conf. on High Energy Physics, Glasgow, Scotland, Jul 20-27, 1994.

[11] G. Ingelman, J. Rathsman, TSL-ISV-94-0096, May 1994.

[12] CLEO collaboration, reported at the QCD94 Conference in Montpellier.

[13] G. Cowan, submitted to Int. Conf. on High Energy Physics, Glasgow, Scotland, Jul 20-27, 1994.

[14] P. Abreu, et al. Phys. Lett. B311, 408 (1993).

[15] S. Narison and A. Pich Phys. Lett. B211, 183 (1988), E. Braaten, S. Narison and A. Pich, Nucl. Phys. B373, 581 (1992)
[16] M.A. Shifman, A.I Vainshtein, and V.I. Zakharov Nucl. Phys. B147, 385 (1979).

[17] J. Alexander, et al. CLEO-CONF-94-26, Jul 1994, Submitted to Int. Conf. on High Energy Physics, Glasgow, Scotland, Jul 20-27, 1994.

[18] D. Buskulic, et al. Phys. Lett. B307, 209 (1993).

[19] ALEPH collaboration reported at the QCD94 Conference in Montpelier.

[20] F. Abe, et al. Phys. Rev. Lett. 71, 2396 (1993), Phys. Rev. Lett. 71, 500 (1993) and CDF collaboration these proceedings.

[21] G. Alves and T. Huen for the D0 collaboration, these proceedings.

[22] P. Nason, S. Dawson and R.K. Ellis, Nucl. Phys. B303, 607 (1988).

[23] F. Abe, et al. Phys. Rev. Lett. 73, 225 (1994).

[24] E. Laenen J. Smith, and W.L. van Neerven, Phys. Lett. B321, 254 (1994).

[25] S. Abachi, et al. Phys. Rev. Lett. 72, 138 (1994).

[26] R. Baier and R. Ruckl, Zeit. fur Physik c19, 251 (1983), F. Halzen et al. Phys. Rev. D30, 700 (1984).

[27] E. Braaten and T.-C. Yuan Phys. Rev. Lett. 71, 1673 (1993).

[28] E. Braaten et al. Phys. Lett. B333, 548 (1994).

[29] D.P. Roy and K. Sridhar, CERN-TH-7434-94.

[30] Vaia Papadimitriou, et al. FERMILAB-CONF-94-221-E, Aug 1994.

[31] M. Gluck, et al. DO-TH-94-02-REV, Feb 1994.
[32] H. Baer, J. Ohnemus and J. Owens  *Phys. Rev.* **D42**, 61 (1990).

[33] M. Gluck, E. Reya, and A. Vogt  *Phys. Lett.* **B306**, 391 (1993).

[34] F. Abe, *et al.*  FERMILAB-PUB-94-208-E, 1994.

[35] S. Fahey for the D0 collaboration, these proceedings.

[36] James Botts, *et al.*  *Phys. Lett.* **B304**, 159 (1993).

[37] For a review see, for example, Gunter Wolf DESY-94-178, 1994.

[38] M. Gluck, E. Reya and M. Voit, *et al.*  *Zeit. fur Physik* **C53**, 127 (1992).

[39] A.D. Martin, R.G. Roberts and W.J. Stirling HEPPH-9409257, Jul 1994.

[40] D0Collaboration, these proceedings.

[41] R. Blair, *et al.*  FERMILAB-CONF-94-269-E.

[42] For a recent review see, for example, J.F. Gunion, UCD-94-24, (1994).

[43] B. Bailey, J.F. Owens and J. Ohnemus  *Phys. Rev.* **D46**, 2018 (1992).

[44] V. N. Gribov and L.N. Lipatov  *Sov. J. Nucl. Phys.* 15, 438 (1972), L.N. Lipatov,  *Sov. J. Nucl. Phys.* 20, 181 (1974), Yu. L. Dokshitzer,  *Sov. Phys. JETP* **46**, 641 (1977), G. Altarelli and G. Parisi  *Nucl. Phys. B126*, 298 (1977).

[45] E.A. Kurayev, L.N. Lipatov and V.S. Fadin,  *Sov. Phys. JETP* **45**, 119 (1977), Ya. Ya. Bailitsky and L.N. Lipatov,  *Sov. J. Nucl. Phys.* 28, 882 (1978).

[46] By R.K. Ellis, Z. Kunszt and E.M. Levin,  *Nucl. Phys. B420*, 517 (1994).

[47] L.N. Lipatov  *Sov. Phys. JETP* **63**, 904 (1986).
[48] A.H. Mueller and B. Patel  *Nucl. Phys.* **B425**, 471 (1994), A.H. Mueller  
*Nucl. Phys.* **B415**, 373 (1994), N.N. Nikolaev, B.G. Zakharov and V.R. Zoller  
*J.Exp.Theor.Phys.* **78**, 806 (1994).

[49] A.H. Mueller, H. Navelet  *Nucl. Phys.* **B282**, 87 (19).

[50] Vittorio Del Duca and Carl R. Schmidt, DESY-94-114. W.J. Stirling  
*Nucl. Phys.* **B423**, 56 (1994).

[51] D0 Collaboration, these proceedings.

[52] G. Marchesini and B.R. Webber,  *Nucl. Phys.* **B310**, 461 (1988).

[53] F. Abe, *et al.* FERMILAB-PUB-194-94E, *Phys. Rev. Lett.* (to appear).

[54] S. Abachi, *et al.*  *Phys. Rev. Lett.* **72**, 2332 (1994).

[55] For a recent review see, J.D. Bjorken, SLAC-PUB-6463, (1994),

[56] M. Derrick, *et al.*  *Phys. Lett.* **B332**, 228 (1994).

[57] T. Ahmed, *et al.*  DESY-94-133 (1994).

[58] G. Ingelmann and P.E. Shlein,  *Phys. Lett.* **152B**, 256 (1985).

[59] S. Nussinov,  *Phys. Rev.* **d41**, 246 (1976), S. Donnachie and P.V. Landshoff,  
*Nucl. Phys.* **B244**, 322 (1984).

[60] N. Nikolaev and B.G. Zakharov,  *Zeit. fur Physik C53*, 331 (1992)

[61] B. A. Schumm, L. Dokshitser and V, A. Khoze  *Phys. Rev. Lett.* **69**, 3025 (1992).

[62] K. Abe, *et al.*  *Phys. Rev. Lett.* **72**, 3145 (1994).