Late-Time Evolution of Charged Gravitational Collapse and Decay of Charged Scalar Hair - I

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Abstract

We study analytically the asymptotic evolution of charged fields around a Reissner-Nordström black-hole. Following the no-hair theorem we focus attention on the dynamical mechanism by which the charged hair is radiated away. We find an inverse power-law relaxation of the charged fields at future timelike infinity, along future null infinity and an oscillatory inverse power-law relaxation along the future outer horizon. We show that a charged hair is shedd slower than a neutral one. Our results are also of importance to the study of mass inflation and the stability of Cauchy horizons during a dynamical gravitational collapse of charged matter to form a charged black-hole.

I. INTRODUCTION

The statement that black-holes have no-hair was introduced by Wheeler in the early 1970s. The various no hair theorems state that the external field of a black-hole relaxes to a Kerr-Newman field described solely by three parameters: the black-hole’s mass, charge and angular-momentum. The mechanism responsible for the relaxation of neutral external perturbations was first studied by Price [1]. He has found that the late-time behaviour of these neutral perturbations for a fixed position, \( r \), is dominated by \( t^{-(2l+3)} \) tails (if there is no initial static field), where \( l \) is the multipole moment of the mode and \( t \) is the standard
Schwarzschild time coordinate. The behaviour of neutral perturbations along null infinity and along the future event horizon was further studied by Gundlach, Price and Pullin [2]. They have found that the neutral perturbations along null infinity decay according to an inverse power-law $u^{-(l+2)}$, where $u$ is the outgoing Eddington-Finkelstein null coordinate. Along the event-horizon the perturbations decay according to $v^{-(2l+3)}$, where $v$ is the ingoing Eddington-Finkelstein null coordinate.

In this work we study the gravitational collapse of a charged matter to form a charged black hole. In such a collapse one should expect that charged perturbations will develop outside the collapsing star. In particular we focus attention on the late-time behaviour of such charged scalar perturbations along these three asymptotic regions.

Our results are of importance for two major areas of black-hole physics:

1. The no-hair theorem of Mayo & Bekenstein [3] states that black-holes cannot have a charged scalar-hair. However, it was never before studied how a charged black-hole, which is formed during a gravitational collapse of a charged matter, dynamically sheds its charged scalar hair during the collapse. We study, here, the mechanism by which the charged hair is radiated away.

2. The mass-inflation scenario and the stability of Cauchy horizons were studied under the assumption of the existence of inverse power-law (neutral) perturbations along the outer horizon of a Reissner-Nordström black-hole. However, these models did not take into account the existence of charged perturbations which are expected to appear in the dynamical collapse of a charged star. Here, we study the asymptotic behaviour of such perturbations.

The plan of the paper is as follows. In Sec. II we describe our physical system and formulate the evolution equation. In Sec. III we study the late-time evolution of charged scalar perturbations for a collapse that leads to the formation of a (charged) black-hole. Here we generalize the formalism of Ref. [1,2] to the charged situation. We study the case
$|cQ| \ll 1$, which simplifies things enough to allow us analytical derivations of our results. In paper II in this series we will examine the problem for a general value of $|cQ|$. We find an inverse power-law behaviour of the charged perturbations along the three asymptotic regions. However, the exponents differ for those of neutral perturbations. Additionally along the outer horizon there are periodic oscillations on top of this power law decay (which do not exist for neutral perturbations).

In Sec. IV we study the behaviour of charged perturbations in the non-collapsing case (imploding and exploding shells). Qualitatively, we find the same late-time behaviour as in the collapsing situation. In Sec. V we compare the late-time behaviour of charged perturbations with the late-time behaviour of neutral perturbations. We find that the dynamical process of shedding hair is different for neutral hair and charged one, both quantitatively and qualitatively. We show that a black-hole which is formed from the gravitational collapse of a charged matter becomes “bald” slower than a neutral one due to the existence of charged perturbations. Furthermore, while the late-time behaviour of neutral perturbations is determined by the space-time curvature, the late-time behaviour of charged fields is dominated by flat space-time effects (scattering due to the electromagnetic interaction in flat space-time). We conclude in Sec. VI with a brief summary of our results.

**II. DESCRIPTION OF THE SYSTEM**

The external gravitational field of a spherically symmetric collapsing star of mass $M$ and charge $Q$ is given by the Reissner-Nordström metric

$$ds^2 = - \left(1 - \frac{2M}{r} + \frac{Q^2}{r^2}\right) dt^2 + \left(1 - \frac{2M}{r} + \frac{Q^2}{r^2}\right)^{-1} dr^2 + r^2 d\Omega^2 .$$

(1)

We will also use the tortoise radial coordinate $y$, defined by $dy = dr \left(1 - \frac{2M}{r} + \frac{Q^2}{r^2}\right)^{-1}$, in terms of which the metric becomes

$$ds^2 = \left(1 - \frac{2M}{r} + \frac{Q^2}{r^2}\right) (-dt^2 + dy^2) + r^2 d\Omega^2 ,$$

(2)

where $r = r(y)$. 

3
We consider the evolution of massless charged scalar perturbations fields outside a charged collapsing star. The wave equation for the complex scalar-field is

\[ \phi_{;ab} g^{ab} - ie A_a g^{ab} (2\phi_{;b} - ie A_b \phi) - ie A_{a;b} g^{ab} \phi = 0, \]  

where \( e \) is constant.

Resolving the charged scalar-field into spherical harmonics

\[ \phi = \sum_{l,m} \eta_{lm}^i (t, r) Y_{lm}^m(\theta, \varphi)/r, \]

we obtain a wave equation for each multipole moment

\[ \eta_{tt} - 2ie A_t \eta_t - \eta_{yy} + \tilde{V} \eta = 0, \]  

where

\[ \tilde{V} = \tilde{V}_{M,Q,l,e} (r) = \left( 1 - \frac{2M}{r} + \frac{Q^2}{r^2} \right) \left[ \frac{l (l + 1)}{r^2} + \frac{2M}{r^3} - \frac{2Q^2}{r^4} \right] - e^2 A_t^2. \]

Here we have suppressed the indices \( l, m \) on \( \eta \).

The electromagnetic potential satisfies the relation

\[ A_t = \Phi - \frac{Q}{r}, \]  

where \( \Phi \) is constant.

In order to get rid of the, physically unimportant quantity \( \Phi \), we introduce the auxiliary field \( \psi = e^{-ie \Phi t} \eta \), in terms of which the equation of motion becomes

\[ \psi_{tt} + 2ie \frac{Q}{r} \psi_t - \psi_{yy} + V \psi = 0, \]

where

\[ V = V_{M,Q,l,e} (r) = \left( 1 - \frac{2M}{r} + \frac{Q^2}{r^2} \right) \left[ \frac{l (l + 1)}{r^2} + \frac{2M}{r^3} - \frac{2Q^2}{r^4} \right] - e^2 \frac{Q^2}{r^2}. \]

It is well known that a gauge transformation of the form \( \eta \to e^{-i\alpha t} \eta \) (where \( \alpha \) is a real constant) merely adds a constant to \( A_t \), i.e. \( A_t \to A_t - \frac{1}{e} \alpha \).
III. EVOLUTION OF CHARGED PERTURBATIONS IN THE COLLAPSING
CASE (BLACK-HOLE FORMATION)

The general solution to the wave-equation (7) can be written as

$$\psi = \sum_{k=0}^{l} A_k r^{-k} \left[ e^{ieQ ln r} G^{(l-k)}(u) + (-1)^k e^{ieQ ln r} F^{(l-k)}(v) \right]$$

$$+ \sum_{k=0}^{\infty} \left[ B_k(r) G^{(l-k-1)}(u) + C_k(r) F^{(l-k-1)}(v) \right],$$

(9)

where $G$ and $F$ are arbitrary functions. The coefficients $A_k = A_k(l) = (l + k)!/[2^k k!(l - k)!]$ are equal to those that arise in the neutral case [1] and $B_k(r) = B_k(r; eQ, l, M), C_k(r) = C_k(r; eQ, l, M)$. Here $u \equiv t - y$ is a retarded time coordinate and $v \equiv t + y$ is an advanced time coordinate. For any function $H$, $H^{(k)}$ is the $k$-th derivative of $H^{(0)}$; negative-order derivatives are to be interpreted as integrals. The first sum in (9) represents the zeroth-order solution, i.e. neglecting terms of order $O(eQ), O(\frac{M}{r}), O(\frac{Q}{r})$ and higher.

The functions $B_k(r)$ satisfy the recursion relation

$$2 \lambda^2 B_k' + 2ieQB_k r^{-1} - \lambda^2 \left( B_{k-1} \lambda^2 \right)' - \lambda^2 \left[ A_k (-k - ieQ) r^{-(k-1)-ieQ} \chi^2 \right]' -$$

$$2A_{k+1} r^{-k-2-ieQ} [ieQ (\lambda^2 - 1) + \lambda^2 (k + 1)] + V(r) \left[ A_k r^{-(k-2)-ieQ} + B_{k-1} \right] = 0 ,$$

(10)

where $\lambda^2 \equiv 1 - \frac{2M}{r} + \frac{Q^2}{r^2}$ and $B' \equiv dB/dr$.

The functions $C_k(r)$ satisfy an analogous recursion relation; however, we will not need them as the late-time behaviour of the charged scalar-field does not depend on $C_k$. We can now expand $B_k(r)$ in the form

$$B_k(r) = a_k r^{-(k+1)-ieQ} + b_k r^{-(k+2)-ieQ} + \ldots ,$$

(11)

where $a_k \equiv a_k(l, eQ), b_k \equiv b_k(l, eQ) \ldots$.

Substituting (11) in (10), one finds for the lowest order coefficients

$$a_l = -ieQ A_l \frac{2l + 1}{2(l + 1)} [1 + O(eQ)] .$$

(12)

The star begins to collapse at retarded time $u = u_0$. The world line of the stellar surface is asymptotic to an ingoing null line $v = v_0$, while the variation of the field on the stellar
surface is asymptotically infinitely redshifted \[1,3\]. This effect is caused by the time dilation between static frames and infalling frames. A static external observer sees all processes on the stellar surface become “frozen” as the star approaches the horizon. Thus, he sees all physical quantities approach a constant. Using the above effect, we make the explicit assumption that after some retarded time \( u = u_1 \), \( D_u \phi = 0 \) on \( v = v_0 \), where \( D_\mu = \partial_\mu - ieA_\mu \) is the gauge covariant derivative. This assumption has been proven to be very successful for the neutral scalar-field \[1,2\].

We start with the first stage of the evolution, i.e. the scattering of the charged field in the region \( u_0 \leq u \leq u_1 \):

The first sum in (9) represents the primary waves in the wave front, while the second sum represents backscattered waves. The interpretation of these integral terms as backscatter comes from the fact that they depend on data spread out over a section of the past light cone, while outgoing waves depend only on data at a fixed \( u \) \[1\]. It should be noted that this physical distinction between the primary waves and the backscattered waves is valid in the region \( r \ll |Q| e^{\frac{1}{Q}} \).

After the passage of the primary waves there is no outgoing radiation for \( u > u_1 \), aside from backscattered waves. This means that \( G(u_1) = 0 \). Thus, for a large \( r \) at \( u = u_1 \), the dominant term in (13) is

\[
\psi (u = u_1, r) = a_n r^{-(l+1)} G^{(-1)} (u_1) .
\]

This is the dominant backscatter of the primary waves.

After we had determined the dominant backscatter of the charged scalar field, we shall next consider the asymptotic evolution of the field. We confine our attention to the region \( y \gg M, |Q|; u > u_1 \). In this region (and for \( |eQ| \ll 1 \)) the evolution of the field is dominated by neutral flat space-time terms, i.e.

\[
\psi_{,\mu} - \psi_{,rr} + \frac{l(l+1)}{r^2} \psi = 0 .
\]

[It should be noted that for \( r \gg M, |Q| \), we have \( \psi_{,yy} \simeq \psi_{,rr} + \frac{2M}{r^2} \psi_{,r} \). However, in this region \( O(\psi_{,rr}) = O(r^{-2} \psi) \gg O(Mr^{-2} \psi_{,r}) \). So, in this region, we may replace \( r \) by \( y \).]
Thus, the solution for $\psi$ can be written as

$$
\psi = \sum_{k=0}^{l} A_k y^{-k} g^{(l-k)}(u) + (-1)^k f^{(l-k)}(v) .
$$

(15)

Comparing (15) with the initial data (13) on $u = u_1$, one finds

$$
f(v) = F_0 v^{-1} ,
$$

(16)

where

$$
F_0 = ieQG(-1)(u_1)(-1)^{l+1}(2l+1)!/(l+1)! + O(eQ^2) .
$$

(17)

For late times $t \gg y$ we can expand $g(u) = \sum_{n=0}^{\infty} (-1)^n n! g^{(n)}(t) y^n$ and $f(v) = \sum_{n=0}^{\infty} 1/n! f^{(n)}(t) y^n$. Using these expansions we can rewrite (15) as

$$
\psi = \sum_{n=-l}^{\infty} K_n y^n [f^{(l+n)}(t) + (-1)^n g^{(l+n)}(t)] ,
$$

(18)

where the coefficients $K_n$ are those given in the neutral case [1].

Using the boundary conditions for small $r$ (regularity as $y \to -\infty$, at the horizon of a black-hole, or at $r = 0$, as in the next section), one finds that at late times the terms $h(t) \equiv f(t) + (-1)^l g(t)$ and $f^{(2l+1)}(t)$ must be of the same order (see [2] for additional details). Thus, we conclude that

$$
f(t) \simeq F_0 t^{-1} ,
$$

(19)

$$
g(t) \simeq (-1)^{l+1} F_0 t^{-1}
$$

(20)

and

$$
h(t) = O\left[t^{-2(l+1)}\right] .
$$

(21)

Hence, we find that the late-time behaviour of the charged scalar-field for $|Q|e^{\frac{1}{2r^2}} \gg t \gg y \gg M, |Q|$ is

$$
\psi \simeq 2K_{l+1} y^{l+1} f^{(2l+1)}(t) = -2K_{l+1} F_0 (2l+1)! t^{-2(l+1)} y^{l+1} + O\left(eQ^2\right) .
$$

(22)
This is the late-time behaviour of the charged scalar field at timelike infinity $i_+$. 

From equations (15), (16) and (20) one finds that the behaviour of the charged scalar field at future null infinity $\mathcal{I}^+$ (i.e., at $u \ll v \ll |Q|/e^{1/|Q|}$) is

$$\psi(v \gg u, u) \simeq A_0 g^{(l)}(u) \simeq -F_0 l! u^{-(l+1)} .$$

Finally, we go on to consider the behaviour of the charged scalar-field at the black-hole outer horizon $r_+$:

As $y \to -\infty$ the wave-equation (7) can be approximated by the equation

$$\psi_{tt} + 2ieQ r_+ \psi_t - \psi_{yy} - e^2 Q^2 r_+^2 \psi = 0 ,$$

whose general solution is

$$\psi = e^{-ieQ r_+} [\alpha(u) + \gamma(v)] .$$

In order to match the $y \ll -M$ solution (26) with the $y \gg M$ solution (22), we make the ansatz $\psi \simeq \psi_{stat}(r) t^{-2(l+1)}$ for the solution in the region $y \ll -M$ and $t \gg |y|$. In other words, we assume that the solution in the $y \ll -M$ region has the same late-time $t$-dependence as the $y \gg M$ solution. Using this assumption, one finds $\psi_{stat} = \Gamma_0 e^{ieQ r_+ y}$ and $\gamma(t) = \Gamma_0 e^{ieQ r_+ t^{-2(l+1)}}$ (where $\Gamma_0$ is a constant). Thus, the late-time behaviour of the charged scalar-field at the horizon is (for $v \ll |Q|e^{1/|Q|}$)

$$\psi(u \to \infty, v) = \Gamma_0 e^{ieQ r_+ y} v^{-(l+1)} .$$

### IV. EVOLUTION OF CHARGED PERTURBATIONS IN THE NON-COLLAPSING CASE (IMPLODING AND EXPLODING SHELLS)

We now consider the case of imploding and exploding shells of charged scalar field. In this situation, the charge of the space-time falls to zero as the evolution proceeds (the charged
scalar field, which is the source of that charge, escapes to infinity). One might think that this dissipation of the charge of the spacetime would lead to a late-time evolution which is identical with the neutral case. However, this is not the case; as we have seen, the late-time behaviour of the charged scalar field at constant $r$ and at $scri_+$ depends on the backscattering of the initially outgoing waves. This backscattering is different for the charged scalar field compared with the neutral one, and hence would lead to a different late-time behaviour. It should be noted that the backscattering is taking place in an early stage ($u < u_1$), when the spacetime still contains a considerable amount of charge. Thus, the initial data on $u = u_1$ is still given by (13). Furthermore, the late-time behaviour of the charged scalar field at constant $r$ and at $scri_+$ are independent of the small-$r$ nature of the background (this is the situation in the neutral case as well [1,2]). Thus, we expect that the non-collapsing charged field would produce a similar late-time behaviour compared with the collapsing one.

Finally, it should be noted that in this situation the space-time has no internal “infinity” (no event horizon) and thus $r_+ = \infty$.

V. CHARGED SCALAR-HAIR VS. NEUTRAL SCALAR HAIR

The no-hair theorems for black-holes state that black-hole can have neither neutral scalar hair [6,7] nor a charged scalar-field hair [3]. Price [1] has investigated the mechanism which leads to the relaxation of such external neutral scalar hair. However, it was never before investigated how a charged black-hole, which is formed during a gravitational collapse of a charged matter, dynamically sheds its charged scalar hair during the process of the collapse.

Here, we have shown that the two mechanisms are quite different, both quantitatively and qualitatively. While the late-time behaviour of a neutral scalar field outside a black-hole is dominated by $\sim t^{-(3+2l)}$ tails, the relaxation of the charged scalar hair outside a charged black-hole is dominated by a $\sim t^{-(2+2l)}$ behaviour. Therefore, we conclude that charged perturbations die slower than neutral ones, i.e. a charged black-hole, which is formed during a gravitational collapse of a charged matter, is expected to loose its charged
scalar-hair and relax to its final state slower than a neutral one. In a more pictorial way, a black-hole, which is formed from the gravitational collapse of a charged matter, becomes “bald” slower than a neutral one.

Mathematically, it is the relation of \( r \) to \( y \) which determines the dominant initial backscattering (and therefore the behaviour of the late-time tails) for neutral perturbations \([1]\). This means that to a leading order in \( M \) the evolution of neutral perturbations depends on the space-time curvature (on \( M \)) in the first step \((u_0 < u < u_1)\), but not in the second step.

On the other hand, the flat space-time charged terms in the evolution equation for the charged scalar-field are of critical importance in determining the dominant initial backscattering for charged perturbations. Indeed, the flat space-time evolution equation is

\[
\psi_{,tt} + 2ieQ \frac{Q}{r} \psi_{,t} - \psi_{,rr} + \frac{l(l+1) - e^2Q^2}{r^2} \psi = 0, \tag{28}
\]

whose general solution can be written as

\[
\psi = \sum_{k=0}^{\infty} r^{-k} \left[ e^{-ieQlnr} C_k p_{(-k)} (u) + e^{ieQlnr} D_k q_{(-k)} (v) \right], \tag{29}
\]

where

\[
C_k = C_k (l, eQ) = \frac{1}{2k!} \prod_{n=0}^{k-1} \left[ l(l+1) - n(n+1) - ieQ (2n+1) \right],
\]

\[
D_k = D_k (l, eQ) = (-1)^k C_k^*, \tag{30}
\]

for \( k \geq 1 \) and \( C_0 = D_0 \equiv 1 \).

For \( eQ = 0 \) this infinite series is cut-off at \( k = l + 1 \), i.e.

\[
\frac{C_{l+1}}{C_l} = \frac{D_{l+1}}{D_l} = -ieQ \frac{2l+1}{2(l+1)}. \tag{31}
\]

In other words, for \( eQ = 0 \) there is no backscatter of the waves.

For \( |eQ| \ll 1 \), we may rewrite (29) as

\[
\sum_{k=0}^{l} A_k r^{-k} \left[ e^{-ieQlnr} G^{(l-k)} (u) + (-1)^k e^{ieQlnr} F^{(l-k)} (v) \right] +
\sum_{k=l+1}^{\infty} r^{-k} \left[ e^{-ieQlnr} C_k G^{(l-k)} (u) + e^{ieQlnr} D_k F^{(l-k)} (v) \right], \tag{32}
\]
where $A_k$ are the coefficients given in sec. III and $C_k = A_k + O(eQ)$ for $0 \leq k \leq l$. Thus, we conclude that the dominant backscatter of the primary waves, for $|Q| \ll r \ll |Q|e^{|r|}$, is given by $C_{l+1}r^{-(l+1)}G^{(-1)}(u_1)$ where $C_{l+1} = -ieQ A_l \frac{2l+1}{2(l+1)} [1 + O(eQ)]$. This is just the result obtained earlier, see (12).

The physical significance of this result is the conclusion that unlike neutral perturbations the late-time behaviour of charged scalar-field is entirely determined by flat space-time effects. In other words, the scattering is caused by the electromagnetic interaction in flat space-time.

Our results show that mass-inflation in a gravitational collapse of a charged scalar field will be stronger than in the collapse of a neutral fields. Mass inflation models have relied heavily on the existence of inverse power-law tails along the outer horizon of a Reissner-Nordström black-hole. However, these models did not take into account the existence of charged perturbations outside the collapsing star (during the gravitational collapse of a charged star to form a Reissner-Nordström black-hole we, of course, do expect to find charged perturbations outside the star). We have investigated the behaviour of these charged perturbations on the outer horizon of a RN black-hole. We do find a rather similar behaviour of the charged field along the outer horizon (see (27)), although with a different exponent and with periodic oscillations (which do not exist for neutral perturbations). The power-law fall-off (times periodic oscillations) of the charged perturbations suggests that mass-inflation should occur in the gravitational collapse of a charged matter in which a charged black-hole forms. Moreover, since charged perturbations have smaller dumping exponents compared with neutral ones, they will dominate the influx through the outer horizon and hence will be the main cause for the mass-inflation phenomena. One should remember that the mass function diverges like $m(v) \approx v^{-p}e^{k_0v}$ (for $v \to \infty$, near the Cauchy horizon), where $\frac{1}{2}p$ is the dumping exponent of the field 8.
VI. SUMMARY AND CONCLUSIONS

We have studied the gravitational collapse of a charged matter to form a charged black-hole. The main issue considered is the late-time behaviour of charged scalar perturbations outside the collapsing star. We have shown that power-law tails develop at timelike infinity (at fixed radius at late times) and along null infinity. Along the outer horizon there is an oscillatory power-law tail. The period of these oscillations is determined by the quantity $eQ/r_+$. The exponents of these inverse power-law tails are all smaller compared with neutral perturbations. Thus, we conclude that a black-hole which is formed from the gravitational collapse of a charged matter becomes “bald” slower than a neutral one due to the presence of charged perturbations.

While the late-time behaviour of neutral perturbations is determined by the space-time curvature (mathematically, the relation between $y$ and $r$), the asymptotic behaviour of charged fields is dominated by flat space-time effects.

Our work reveals the dynamical mechanism by which the charged scalar-hair is radiated away leaving behind a Reissner-Nordström black-hole. We have shown that this mechanism differs from the neutral one both quantitatively (different power-law exponents and oscillatory behaviour along the black-hole outer-horizon) and qualitatively (the initial backscattering, and thus the late-time behaviour are dominated by flat space-time terms, namely, by the electromagnetic interaction, rather than by curvature effects).

Furthermore, we have shown that the late-time behaviour of charged fields in the non-collapsing situation (imploding and exploding shells) is dominated by a similar inverse power-law behaviour both at a fixed radius (and late times) and along null infinity.

Finally, our results are of importance for the mass-inflation scenario and stability of Cauchy horizons. Here, we have shown that the asymptotic behaviour of charged perturbations along the outer-horizon of a RN black-hole is characterized by an oscillatory inverse power-law behaviour (with smaller exponents compared with neutral tails). Thus, one should expect that these inverse power-law charged perturbations will cause a mass-inflation singu-
larity during the gravitational collapse of a charged matter that forms a charged black-hole. Moreover, the slower relaxation of charged perturbations makes them the dominant cause for the divergence of the mass-function.

The most significant shortfall in our analysis is the limitation to the case $|\epsilon Q| \ll 1$. In an accompanying paper (II) we extend our analytical results to include general values of $\epsilon Q$ (using a spectral decomposition) and we confirm them numerically. On the other hand, the main advantage of this approach is the fact that it gives a clear picture of the physical mechanism responsible for the late-time behaviour of charged perturbations, namely, the tail arises because of backscattering of the charged field off the electromagnetic potential far away from the black-hole. The physical picture that arises from this paper is clear — dealing with charged (massless) perturbations, one may neglect any curvature effects.

In accompanying papers we study the fully nonlinear gravitational collapse of a charged scalar field to form a charged black-hole. In order to numerically confirm our analytical predictions we will first focus attention on the asymptotic behaviour of the charged field outside the dynamically formed charged black-hole.

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REFERENCES

[1] R.H. Price, Phys. Rev. D5, 2419 (1972).

[2] C. Gundlach, R.H. Price, and J. Pullin, Phys. Rev. D49, 883 (1994).

[3] A.E. Mayo and J.D. Bekenstein, Phys. Rev. D54, 5059 (1996).

[4] S.W. Hawking and G.F.R. Ellis, The Large Scale Structure of Space Time (Cambridge University Press, Cambridge, England, 1973).

[5] J. Bičák, Gen. Relativ. Gravitation 3, 331 (1972).

[6] J.E. Chase, Commun. Math. Phys. 19, 276 (1970).

[7] J.D. Bekenstein, Phys. Rev. Letters 28, 452 (1972); Phys. Rev. D5, 1239 and 2403 (1972).

[8] E. Poisson and W. Israel, Phys. Rev. D41, 1796 (1990).