Analysis of Student Errors on Division of Fractions

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Abstract. This study aims to describe the type of student errors that typically occurs at the completion of the division arithmetic operations on fractions, and to describe the causes of students’ mistakes. This research used a descriptive qualitative method, and involved 22 fifth grade students at one particular elementary school in Kuningan, Indonesia. The results of this study showed that students’ error answers caused by students changing their way of thinking to solve multiplication and division operations on the same procedures, the changing of mix fractions to common fraction have made students confused, and students are careless in doing calculation. From student written work, in solving the fraction problems, we found that there is influence between the uses of learning methods and student response, and some of student responses beyond researchers’ prediction. We conclude that the teaching method is not only the important thing that must be prepared, but the teacher should also prepare about predictions of students’ answers to the problems that will be given in the learning process. This could be a reflection for teachers to be better and to achieve the expected learning goals.

1. Introduction

Fraction is one of the core mathematics topics that should be learned by students in elementary school. This topic includes arithmetical operations, addition, subtraction, multiplication, and division, on proper fractions, and decimals. By studying this topic, students are expected to understand the material form of algebraic fractions operations in the next level of education.

In a current situation, students are more likely to make errors in operations because the learning material of fractions focused only on memorizing formulas and procedures, without sufficient attention to the meaning of processes. In the division of fractions for example, this is always accompanied by a divisions algorithm used by multiplying the number of students divided by the form of the inverse of the divisor. The students’ errors in solving division of fractions often caused by the error use of this algorithm.

The errors appear because the algorithm is only seen as a set of procedure that is not meaningful for students. Students do not know why they should reverse the form of fractions when they do division of fractions.

There are some different students’ mistakes in using the division algorithm. For example, students simply divide the numerator of the fraction by the fraction divisor, and also split both the denominator, and multiplying fractions divided by the fractional divisor without changing the fractional divisor in the form of its inverse\cite{1}.

Some misconceptions, one of the errors found is using the concept of multiplication in the division of fractions. For example, in the division of fractions by integers, students directly multiply the
integer with the fractions. This error occurs from an accidental students and error based on formal knowledge of the student, such as the inability of student to interpret \( \frac{1}{2} \) as “how many \( \frac{1}{2} \) in 2” ([2], [3]).

From the above description, this study focuses to investigate types of student errors that typically occur when doing division of fractions, and to explain the causes that students make. Classification errors made by participants when dividing fractions is the following [1]:

1.1. The algorithm-based error
Different ways to calculate falls into this category. This procedure includes reverse common shares in lieu of divisor or reversal before multiplying the numerator and denominator. This typically describes the results of memorization algorithms. When the algorithm displays a measure that does not mean, allow students to forget that step or change the way that it could be a mistake.

1.2. Errors are not intentional
Research on how to distribute the operation showed that students in balancing operations with fractions and integers on to explain the distribution of primer using a long way in its entire distribution model. In this distribution model of an object divided into separate figures or the smallest group

1.3. An error based on formal knowledge
Errors in the limited thinking of alleged breakdowns and lack of knowledge in connecting operations are included in this category. Lack of knowledge is probably the source of the bad response to a variety of tasks including the division of fractions.

The errors of students in solving mathematical problems can be used byteachers to know the difficulties being faced by students and to improve the next learning and teaching process. One way to reduce errors is to analyze students’ mistakes. The error analysis is an attempt to observe, discover and clarify mistakes with certain rules. Classification according to the specific rules in question are classifies errors by type of concept error, algorithm error, operation error, and random errors [4].

Cognitive state about students with intellectual upon ability to digest the subject matter at hand. Nature, weights, media and others in pursuit transfer students are all factors students' mistakes in answering problems[5].

Fractions is a part of the rational numbers that can be written in the form \( \frac{a}{b} \) where, bare integers, \( b \) is not equal to zero, and \( b \) is not a factor of \( a \). In this case “a” is called the numenator and “b” is called the denominator.

2. Experimental Method
This research used a qualitative descriptive method[6], and was conducted in the second semester of the 2015/2016 teachings of knowledge. Subjects in this study were all students of class V at one particular elementary school in Kuningan. The subjectsof this study are 22 fifth grade students of the subject who will have some students representing the error on each items to be used as a research subject respondents or interviews that match the criteria. A criterion for selection of interview subjects is determined based on the variation of the error and the number of mistakes made by students in solving each item.

The instrument of this study included ten questions and an interview guideline. Problem test was used to locate the fault of students in solving problems. Interviews were carried out, to students in order to determine causes of error factor in solving problems, after the result of the students’ written works have been analyzed.

3. Results and Discussion
The research was conducted in grade five, in one of elementary schools inKuningan, Indonesia, in the second semester of the 2015/2016 academic year. Based on the research results, the language is
relaxed, clear and understandable, the teacher explained the purpose of the learning to be achieved at the time of learning. This is in accordance with aspects of learning in Piaget's theory, namely, using language that is appropriate to the child's way of thinking. Because of the language and the way children think differently from adults. Elementary school children are in the concrete operational stage according to Piaget's theory. Where children are invited to think logically start with the help of concrete objects. The teacher started the lesson by asking students to recall simple fraction multiplication, so that children do not feel confused [5]. The results of analysis of student errors are shown in Table 1.

| Categories                                   | Pattern of Error                                                                 | The students answer |
|----------------------------------------------|----------------------------------------------------------------------------------|---------------------|
| The algorithm-based error                    | Students assume that the completion of the operation of division of fractions is  |
|                                              | equal to solve addition operations on fractions, i.e., by equating the          |
|                                              | denominator.                                                                     | $6 : \frac{2}{3} = \frac{6}{1} \times \frac{2}{3} = \frac{18}{3} \times \frac{2}{3} = \frac{36}{3} = 12$. |
| An error based on formal knowledge           | Students complete the operation of division of fractions and integers by        |
|                                              | dividing those numbers directly.                                                 | $\frac{5}{9} : 3 \frac{1}{3} = \frac{5}{9} \frac{1}{3} = \frac{5}{3}$ |
|                                              | Students assume that the division of fractions and integers, wherever the       |
|                                              | location of the breakdown of the fractions that have to be reversed.            | $5 \frac{1}{2} : 3 = \frac{6}{2} \times \frac{2}{3} = \frac{6}{6} = 1$ |
|                                              | Students think that the changing of fractions into equivalent fractions, can be   |
|                                              | solved by multiplying the numerator and the denominator.                         | $5 \frac{1}{2} : 3 = \frac{6}{2} : \frac{3}{3} = \frac{6}{2} \times \frac{3}{5} = \frac{18}{10}$ |

Type of errors of the fifth grade students to solve problems in the division of fractions. On the basis of an understanding of division of fractions algorithm, we found the following patterns of errors:

- Students assume that the division of fractions and integers, wherever the location of the breakdown, the fractions have to be reversed. This error occurs because of a lack of understanding of the basic concepts of fractions and division of integers.
- Students assume that the completion of the division of fractions is equal to solve addition operations on fractions, by equating the denominator. This error occurs because of the application of the law and relevant strategies that students who do not use the concept of fractional sum in solving division operations on fractions.
- Students complete the operations division of integer fractions by dividing direct those numbers. This error occurs because of a lack of understanding of the basic concepts of
fractions and division of integers or vice versa, as well as a lack of understanding of the basic concepts of integer division.

- Student think the alternation fractions into equivalent fractions can be solved by simplifying the numerator and the denominator. Students made a mistake in understanding the breakdown of the fractions, including the numerator and the denominator; do not adjust the numerator of the fraction, and cannot change a whole number into a fraction [8].

Based on the above analysis, one of the causes of these errors is to be short of mastery of prerequisite skills, such as students do not know that integers can be expressed as a fraction. In addition, other factors such as a lack of understanding of the concept of multiplying fractions with integers.

4. Conclusion

Based on the results and discussion as described in the previous section, we conclude that the mistakes made by students in solving fractions, can be classified into four types of errors are: (1) Students assume that the division of fractions and integers, wherever the location of the breakdown of the fractions that have to be reversed; (2) Students assume that the completion of the operation division of fractions equal to solve addition operations on fractions, by equating the denominator; (3) Students completing the operations division of the integer fractions by dividing direct those numbers; and (4) Students think that the change fractions into equivalent fractions, can be solved by simplifying the numerator and the denominator remains.

The causes of student mistakes are the following: (1) Implementation of laws and strategies that are not relevant, one example of the application of the law and the evidence irrelevant strategy that students use the concept of the sum fractions in solving division operations on fractions; (2) Be short of understanding of the basic concepts of multiplication and division of whole numbers with fractions; (3) Be short of mastery of prerequisite skills on fractions. For example, students do not know that integers can be expressed as a fraction.

Based on this results, we suggest that (1) Teachers should give emphasis to the material preconditions on the material fractions so that students are not making a mistake in applying the arithmetic operation algorithm shards;(2) Teachers should often give a lot of exercises fractional arithmetic operation so that students are more skilled in solving them;(3) For researchers who want to do research on a material matter fractions should be used when the test is more focused research materials (e.g.specialized on addition and subtraction of fractions or multiplication and division fractions only) in order to be in a study in order to obtain more accurate results.

6. References

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