Successful type I Leptogenesis
with $SO(10)$-inspired mass relations

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Abstract

It is well-known that thermal leptogenesis through the decays of the lightest right-handed neutrinos encounters serious difficulties when $SO(10)$-inspired mass conditions are imposed on the Dirac neutrino mass matrix and light neutrino masses are generated through the type I see-saw mechanism. We show that these can be circumvented when the production from the next-to-lightest right-handed neutrinos and flavor effects are properly taken into account. Some conditions on the low energy parameters have to be satisfied in order for inverse processes involving the lightest right-handed neutrino not to wash-out the asymmetry. In particular we find $m_1 \gtrsim 10^{-3}$ eV, where $m_1$ is the mass of the lightest left-handed neutrino and that non-vanishing values of the mixing angle $\theta_{13}$ are preferred in the case of a normal fully hierarchical spectrum of light neutrinos.
1 Introduction

Thermal Leptogenesis \[1,2\] is an elegant model to explain the observed baryon asymmetry of the Universe and a direct consequence of the see-saw mechanism \[3\] for the explanation of the neutrino masses. Current data on neutrino masses are not only compatible with the minimal version of leptogenesis, but even exhibit interesting correlations that support the picture. The strongest one is perhaps that the solar and the atmospheric neutrino mass scales are one-two order of magnitude larger than the so called equilibrium neutrino mass scale setting the effectiveness of the wash-out processes. In this way successful leptogenesis is possible and at the same time its predictions become particularly simple and, more importantly, do not depend on the initial conditions.

On the other hand there are some well known drawbacks in the minimal version of thermal leptogenesis that relies on the simplest version of the see-saw mechanism, the type I. The most serious one is a potential conflict between leptogenesis and \(SO(10)\) grand-unified theories \[4,5\], commonly regarded as the most attractive way to embed the see-saw mechanism. Indeed, in a traditional version of leptogenesis, where the spectrum of right-handed (RH) neutrinos is hierarchical and the asymmetry is produced from the decays of the lightest ones, there is a stringent lower bound on their mass \[6\], \(M_1 > \mathcal{O}(10^9)\) GeV, for a sufficiently large baryon asymmetry to be produced. On the other hand, \(SO(10)\) grand-unified theories typically yield, in their simplest version and for the measured values of the neutrino mixing parameters, a hierarchical spectrum with the RH neutrino masses proportional to the squared of the up-quark masses, leading to \(M_1 = \mathcal{O}(10^5)\) GeV and to a final asymmetry that falls a few orders of magnitude below the observed one.

Of course, for very fine tuned choices of the parameters, it is possible to get a degenerate RH neutrino spectrum that produce an enhancement of the \(CP\) asymmetries and consequently of the final asymmetry, see Ref. \[7\] for a detailed analysis. Furthermore, non-minimal versions of leptogenesis based on a mixed type I plus type II see-saw mechanism can be adopted \[8,9,10\]. It is nevertheless fair to say that a traditional type I picture of leptogenesis, where the spectrum is hierarchical and the asymmetry is generated from the decays of the lightest RH neutrinos, encounters serious difficulties within grand-unified \(SO(10)\) theories.

Two crucial aspects have been usually neglected in the previous studies though. First, the contribution to the final baryon asymmetry from the decays of the next-to-lightest RH neutrinos \(N_2\) \[11\] and, secondly, the effect of flavor in thermal leptogenesis \[12,13\]. In the case in which the baryon asymmetry is generated by the \(N_2\)’s, the above-mentioned lower bound on \(M_1\) disappears and is replaced by a lower bound on \(M_2\), the mass of the
next-to-lightest RH neutrino. In the one-flavor approximation, that is when flavor effects are not taken into account, one can show that the baryon asymmetry generated by the $N_2$ decays is generically maximized for models characterized by having $m_1 \propto M_1^{-1}$, where $m_1$ is the lightest left-handed (LH) neutrino mass eigenvalue. Unfortunately, in their simplest version, $SO(10)$ models yield different neutrino mass relations, where $m_1 \propto M_3^{-1}$, with $M_3$ the mass of the heaviest RH neutrino. In such a case, the baryon asymmetry produced by the $N_2$’s is subsequently washed-out by the interactions involving the lightest RH neutrinos. This leads to the conclusion that $N_2$ thermal leptogenesis in the one-flavor approximation and with $SO(10)$ conditions is not able to explain the observed asymmetry. Flavor effects may help in this regard. Indeed, one can imagine the situation where the next-to-lightest RH neutrino decays predominantly in one particular flavor and that the wash-out mediated by the lightest RH neutrinos is inefficient along that flavor. This point was first made in Ref. [14] for generic models with a strong hierarchy in the RH Majorana masses arising when the up-quark–neutrino Yukawa unification is imposed. The importance of $N_2$-induced leptogenesis was also stressed in [15].

The question we would like to answer in this paper is whether thermal leptogenesis mediated by the next-to-lightest RH neutrinos with flavor effects can effectively work within $SO(10)$ grand-unified models where light LH neutrino masses are generated via the type I seesaw mechanism. In Ref. [14] there were no specific quantitative and full calculations performed within $SO(10)$. It was only shown that in general the wash-out from the lightest RH neutrinos may be small in some flavor even though the total wash-out is strong, while the $CP$ asymmetry generated by the decays of the next-to-lightest RH neutrino was assumed to be maximal. The problem, however, requires more care. Indeed, successful leptogenesis requires three conditions: first, the wash-out from the lightest RH neutrinos needs to be tiny in some flavor for some choice of the parameters; secondly, a large enough $CP$ asymmetry in that flavor must be produced from the decays of the next-to-lightest RH neutrinos and, third, the flavor asymmetry should not be washed out by the interactions mediated by the next-to-lightest RH neutrinos themselves. It is the interplay of all these three requirements which determines whether successful leptogenesis may be achieved.

In this Letter we show that there are regions in the parameter space where type I thermal leptogenesis with hierarchical RH neutrino masses is successful imposing simple $SO(10)$-inspired conditions on the neutrino Dirac mass matrix. Furthermore, we will show that requiring successful leptogenesis leads to interesting predictions on the low energy neutrino parameters that will be tested in future experiments. A more quantitative detailed analysis involving the many low-energy parameters is in progress [16].
The paper is organized as follows. In section 2 we describe the see-saw mechanism when $SO(10)$-like conditions are imposed on the Dirac mass matrix. In section 3 we describe the generic features of $N_2$-leptogenesis with such $SO(10)$-inspired mass relations. Finally, in section 4 we present our conclusions.

2 See-saw mechanism with hierarchical Dirac mass spectrum

The see-saw mechanism is based on a simple extension of the Standard Model where three RH neutrinos $N_i$, $i = 1, 2, 3$ (as nicely predicted within $SO(10)$), with a Majorana mass matrix $M$ and Yukawa couplings $h$ to leptons and Higgs are added. After spontaneous symmetry breaking, a Dirac mass term $m_D = h v$, is generated by the vacuum expectation value (VEV) $v = 174$ GeV of the Higgs boson. In the see-saw limit, $M \gg m_D$, the spectrum of neutrino mass eigenstates splits in two sets: three very heavy neutrinos, $N_1, N_2$ and $N_3$ respectively with masses $M_1 \leq M_2 \leq M_3$ almost coinciding with the eigenvalues of $M$, and three light neutrinos with masses $m_1 \leq m_2 \leq m_3$, the eigenvalues of the light neutrino mass matrix given by the see-saw formula \[1\],

$$m_\nu = -m_D \frac{1}{M} m_D^T.$$  

A parametrization of the Dirac mass matrix is obtained in terms of the orthogonal matrix $\Omega$ \[17\]

$$m_D = U D_m^{1/2} \Omega D_M^{1/2},$$  

where we defined $D_m \equiv \text{diag}(m_1, m_2, m_3)$ and $D_M \equiv \text{diag}(M_1, M_2, M_3)$. Neutrino oscillation experiments measure two neutrino mass-squared differences. For normal schemes one has $m_3^2 - m_2^2 = \Delta m_{\text{atm}}^2$ and $m_2^2 - m_1^2 = \Delta m_{\text{sol}}^2$, whereas for inverted schemes one has $m_3^2 - m_2^2 = \Delta m_{\text{sol}}^2$ and $m_2^2 - m_1^2 = \Delta m_{\text{atm}}^2$. For $m_1 \gg m_{\text{atm}} \equiv \sqrt{\Delta m_{\text{atm}}^2 + \Delta m_{\text{sol}}^2} = (0.050 \pm 0.001) \text{ eV}$ \[18\] the spectrum is quasi-degenerate, while for $m_1 \ll m_{\text{sol}} \equiv \sqrt{\Delta m_{\text{sol}}^2} = (0.00875 \pm 0.00012) \text{ eV}$ \[18\] it is fully hierarchical (normal or inverted). Here we will restrict ourselves to the case of normal schemes. The most stringent upper bound on the absolute neutrino mass scale comes from cosmological observations. Recently, a conservative upper bound on the sum of neutrino masses, $\sum_i m_i \leq 0.61 \text{ eV (95\% CL)}$, has been obtained by the WMAP collaboration combining CMB, baryon acoustic oscillations and supernovae type Ia observations \[19\]. Considering that it falls in the quasi-degenerate regime, it straightforwardly translates into

$$m_1 < 0.2 \text{ eV (95\% CL)}.$$

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The matrix $U$ diagonalizes the light neutrino mass matrix $m_\nu$, so that
\[ U^\dagger m_\nu U^* = -D_m \] (4)
and it can be identified with the lepton mixing matrix in a basis where the charged lepton mass matrix is diagonal. We will adopt the following parametrization for the matrix $U$ in terms of the mixing angles, the Dirac phase $\delta$ and the Majorana phases $\rho$ and $\sigma$ [20]
\[
U = \begin{pmatrix}
c_{12} c_{13} & s_{12} c_{13} & s_{13} e^{-i \delta} \\
-s_{12} c_{23} - c_{12} s_{23} s_{13} e^{i \delta} & c_{12} c_{23} - s_{12} s_{23} s_{13} e^{i \delta} & s_{23} c_{13} \\
s_{12} s_{23} - c_{12} c_{23} s_{13} e^{i \delta} & -c_{12} s_{23} - s_{12} c_{23} s_{13} e^{i \delta} & c_{23} c_{13}
\end{pmatrix} \cdot \text{diag} \left(e^{i \rho}, 1, e^{i \sigma}\right),
\] (5)
where $s_{ij} \equiv \sin \theta_{ij}$, $c_{ij} \equiv \cos \theta_{ij}$ and we will adopt the following 2-σ ranges compatible with the results from neutrino oscillation experiments [18]:
\[
\theta_{12} = (31.3^\circ - 36.3^\circ), \quad \theta_{23} = (38.5^\circ - 52.5^\circ), \quad \theta_{13} = (0^\circ - 11.5^\circ).
\] (6)
With the adopted convention for the light neutrino masses, $m_1 < m_2 < m_3$, this parametrization is valid only for normal hierarchy, while for inverted hierarchy one has to perform a column cyclic permutation.

Within leptogenesis, the see-saw mechanism is also able to explain the observed baryon asymmetry of the Universe [19]
\[
\eta_{B}^{\text{CMB}} = (6.2 \pm 0.15) \times 10^{-10}.
\] (7)
The predicted baryon-to-photon ratio $\eta_B$ is related to the final value of the final $(B - L)$ asymmetry $N_{B-L}^f$ by the relation [2]
\[
\eta_B \simeq 0.96 \times 10^{-2} N_{B-L}^f
\] (8)
and we will impose a 2-σ bound $\eta_B > 5.9 \times 10^{-10}$ GeV for successful leptogenesis. Notice that the small experimental error on $m_{\text{atm}}$ can be neglected in our case and we use everywhere $m_{\text{atm}} = 0.05$ eV.

In general the final asymmetry in leptogenesis depends on all parameters in the right-hand side of (2), in particular on the ‘high-energy’ parameters in the orthogonal matrix $\Omega$ and on the three RH neutrino masses $M_i$. In this way imposing successful leptogenesis does not yield definite predictions on the low energy parameters.

However some theoretical input in $m_D$ can produce relations able to express the nine high energy parameters, six in $\Omega$ plus the three $M_i$, through the low energy parameters
and when successful leptogenesis is required, interesting predictions on the low energy parameters can follow.

This is exactly what happens within $SO(10)$ models, where the theoretical input at the grand-unified scale is conveniently plugged into $m_D$. In this sense, it proves useful to adopt the bi-unitary parametrization

$$m_D = V_L^\dagger D_{m_D} U_R,$$

(9)

where $V_L$ and $U_R$ are two unitary matrices that diagonalize $m_D$ and $D_{m_D}$ is the diagonal matrix whose elements are the eigenvalues of $m_D$: $D_{m_D} \equiv \text{diag}(\lambda_{D1}, \lambda_{D2}, \lambda_{D3})$.

The matrix $V_L$ is analogous to the CKM matrix in the quark sector. The important point is that, once the LH neutrino mass matrix $m_\nu$ is chosen, and for a given $V_L$ and $D_{m_D}$, the masses of the heavy RH neutrinos are fixed together with the matrix $U_R$. Indeed, the see-saw relation can be always written in a basis where $M$ is diagonal. Moreover, in the basis where the charged lepton mass matrix is diagonal, the neutrino mass matrix $m_\nu$ is made diagonal by the lepton mixing matrix $U$, $m_\nu = -U D_{m} U^T$. In this way it is easy to see that the matrix

$$M^{-1} \equiv D_{m_D}^{-1} V_L U D_{m} U^T V_L^T D_{m_D}^{-1},$$

(10)

is diagonalized by $U_R$ and the eigenvalues are the $M^{-1}_i$,

$$M^{-1} = U_R D_{M}^{-1} U_R^T.$$ 

(11)

Notice that we are defining the $M_i$’s to be real and positive while we incorporate the corresponding phases into the matrix $U_R$.

Given the theoretical inputs from $SO(10)$, indicating $V_L$ and $D_{m_D}$, and upon diagonalizing $M^{-1}$, one obtains the matrix $U_R$ and the RH neutrino masses $M_i$ as a function of the low energy parameters, the light neutrino masses $m_i$ and the mixing parameters in $U$. From the relation (2), one can then also calculate the orthogonal matrix $\Omega$ as a function of the low energy parameters,

$$\Omega = D_{m_D}^{-\frac{1}{2}} U^\dagger V_L^\dagger D_{m_D} U_R D_{M}^{-\frac{1}{2}}.$$ 

(12)

Now, in minimal $SO(10)$ scenarios [21], it is expected that a small misalignment between the charged lepton mass matrix and the Dirac neutrino mass matrix $m_D$, similar to that in the quark sector. Under the assumption of small misalignment, $V_L$ should be close to the unity matrix and we will adopt this assumption in the following. We have explicitly checked that small departures from this assumption, e.g. $V_L = V_{\text{CKM}}$, do not alter our conclusions [16]. As for the mass eigenvalues, since fermion families are 16-dimensional
spinors of $SO(10)$, from $16_F \times 16_F = 10_H + 120_H + 126_H$, one deduces that the Yukawa coupling matrices may get several contributions. If the dominant one comes from the 10-dimensional Higgs multiplet, then the relation between the up-quark mass matrix $m_u$ and the Dirac mass matrix $m_D$ reads $m_u = m_D$. On the other hand, if fermion masses are generated by the VEV of a 126-dimensional Higgs, then the $SU(4)$ symmetry relation holds $3m_u = -m_D$. A sort of mixture between these two possibilities is achieved if the 120-dimensional Higgs dominates the contribution to $m_D$. Of course, the total dominance of one Higgs representation is excluded because this would mean that all the fermion mass matrices would be simultaneously diagonalized. Besides bad mass relations, this would imply no quark and no lepton mixings in the weak currents. The fact that the minimal theory must have (at least) two Higgs representations and the fact that one can also envisage the contribution to $m_D$ from non-renormalizable interactions, lead us to assume that the Yukawa couplings are only approximately the same for the up-type quarks and neutrinos. We will therefore just assume a hierarchical pattern with $\lambda_{D3} \gg \lambda_{D2} \gg \lambda_{D1}$. This holds using a parametrization

$$\lambda_{D1} = \alpha_1 m_u, \; \lambda_{D2} = \alpha_2 m_c, \; \lambda_{D3} = \alpha_3 m_t,$$

(13)

where for the up-quark masses we use the reference values $m_u = 1 \text{ MeV}$, $m_c = 400 \text{ MeV}$ and $m_t = 100 \text{GeV}$, approximately coinciding with the up-type quark masses at $T \sim 10^9 \text{GeV}$ [22], and where the coefficients $\alpha_i = O(1)$. With this assumption the RH neutrino mass spectrum is hierarchical and of the form (for generic expressions in terms of the low energy parameters, see Ref. [7])

$$M_1 : M_2 : M_3 = (\alpha_1 m_u)^2 : (\alpha_2 m_c)^2 : (\alpha_3 m_t)^2.$$

(14)

In Fig. 1 we show the dependence of the three $M_i/\alpha_i^2$ on $m_1$ for two different sets of values of $\theta_{23}, \theta_{12}, \theta_{13}, \delta, \rho, \sigma$, as indicated in the figure caption. One can see how non-vanishing values of the phases can enhance the values of the RH neutrino masses for particular values of $m_1$.

We stress that for our considerations, the values of $\alpha_1$ and $\alpha_3$ are irrelevant (unless $\alpha_1$ is unrealistically large to push $M_1$ from $\sim 10^5 \text{GeV}$ above the lower bound $\sim 10^9 \text{GeV}$ to achieve successful $N_1$ leptogenesis). On the other hand, the value of $\alpha_2$ is only relevant to set the scale of the mass $M_2 \simeq 2(\alpha_2 m_c)^2/m_3$ (valid for $\theta_{13} \simeq 0$) of the next-to-lightest RH neutrino mass, but it does not alter other quantities crucial for thermal leptogenesis, such as the amount of wash-out from the lightest RH neutrinos. Furthermore, as we will show, realistic values of $\alpha_2 \gtrsim 3$ are already enough to provide a sufficiently large baryon asymmetry. We now have all the ingredients to calculate the final asymmetry within flavored $N_2$-leptogenesis.
Figure 1: Dependence of the three $M_i/\alpha_i^2$ on $m_1$ for $\theta_{13} = 5^\circ$, $\theta_{23} = 40^\circ$ and $\theta_{12} = 33.5^\circ$ and for three different choices of the phases: $\delta = \sigma = 0^\circ$, $\rho = 1.5^\circ$ (left); $\delta = 5.86^\circ$, $\rho = \sigma = 3^\circ$ (center); $\delta = \pi/3$, $\rho = 0.02^\circ$, $\sigma = \pi/2$ (right).

3 Thermal leptogenesis from next-to-lightest RH neutrinos

Our working assumption is that the final asymmetry is dominantly produced from $N_2$-decays. Having imposed the $SO(10)$ relations (13), the RH neutrino mass spectrum satisfies, except for special cases [7], the condition $M_1 \ll 10^9 \text{GeV} \lesssim M_2 \lesssim 10^{12} \text{GeV}$ and therefore, as explained above, the asymmetry produced from $N_1$-decays cannot reproduce the observed one.

As we outlined in the introduction, type I thermal leptogenesis in $SO(10)$ may be successful through the following chain of processes. The out-of-equilibrium decays of the $N_2$’s produce at temperatures $T \sim M_2$ asymmetries in the two flavor regime, therefore in the tauon flavor and into an asymmetry stored in leptons that are a coherent over-position of electron and muon components. Below $T \sim 10^9 \text{GeV}$ the three flavor regime holds and one has distinct asymmetries in all three flavors. Subsequently, the interactions mediated by the $N_1$’s wash-out part of these asymmetries. Successful leptogenesis may be achieved if a sufficiently large asymmetry has been generated in a given flavor from $N_2$ decays which is not washed-out by the $N_1$-mediated processes.

Since the asymmetry production from $N_2$ decays at $T \sim M_2$ occurs in a two-flavor regime, when only the interactions mediated by the $\tau$-Yukawa couplings are in equilibrium, the $(B - L)$ asymmetry from $N_2$-decays is the sum of two contributions

$$N_{B-L}^{T \sim M_2} \simeq \varepsilon_{2\tau} \kappa(K_{2\tau}) + \varepsilon_{2e+\mu} \kappa(K_{2e+\mu}),$$

(15)
where the flavored $CP$ asymmetries in the flavor $\alpha$ are defined as
\[ \varepsilon_{2\alpha} \equiv -\frac{\Gamma_{2\alpha} - \Gamma_{2\alpha}^*}{\Gamma_2 + \Gamma_2^*}. \]  
(16)

Here $\Gamma_{2\alpha}$ ($\Gamma_{2\alpha}^*$) is the partial decay rate of the $N_2$’s into the $\alpha$-flavored lepton (anti-lepton) and $(\Gamma_2 + \Gamma_2^*)$ is the total decay rate into lepton and anti-leptons. They can be calculated using the expression ($x_2 = M_2^2/M_1^2$) [23]
\[ \varepsilon_{2\alpha} \simeq \frac{3}{16\pi (h^\dagger h)_{22}} \left\{ \text{Im} [h_{\alpha 2}^* h_{\alpha 3} (h^\dagger h)_{23}] \frac{\xi(x_3/x_2)}{\sqrt{x_3/x_2}} + \frac{2}{3(x_3/x_2 - 1)} \text{Im} [h_{\alpha 2}^* h_{\alpha 3} (h^\dagger h)_{32}] \right\}, \]
\[ \xi(x) = \frac{2}{3} x \left[ (1 + x) \ln \left( \frac{1 + x}{x} \right) - \frac{2 - x}{1 - x} \right]. \]  
(17)

We are neglecting the term, negligible for $M_2 \gg M_1$, arising from the interference between the tree level contribution with one loop graphs containing the lightest RH neutrino in the propagator. Furthermore, in the two flavour regime, the asymmetry $\varepsilon_{2e+\mu}$ stands for $\varepsilon_{2e+\mu} = \varepsilon_{2e} + \varepsilon_{2\mu}$.

In Fig. 2 we plotted the dependence of $\varepsilon_2$, $\varepsilon_{2\tau}$ and $\varepsilon_{2e+\mu}$ on $m_1$ for the same values of the mixing angles as in Fig. 1. In the left (right) panel the values of the phases are the same as in the center (right) panel of Fig. 1. The efficiency factor is given by
\[ \kappa(x) = \frac{2}{x z_B(x)} \left[ 1 - \exp \left( -\frac{1}{2} x z_B(x) \right) \right], \]
\[ z_B(x) \simeq 2 + 4 x^{0.13} \exp \left( -\frac{2.5}{x} \right). \]  
(18)
and the flavored decay parameters are

$$K_{i\alpha} = \frac{\Gamma_{i\alpha}}{H} \bigg|_{T=M_2} = \frac{|(m_D)_{\alpha i}|^2}{m_* M_i}.$$ (19)

Here $H$ is the Hubble rate and

$$m_* = \frac{16 \pi^{5/2} \sqrt{g_*}}{3\sqrt{5}} \frac{v^2}{M_{Pl}} \approx 1.08 \times 10^{-3} \text{eV},$$ (20)

where $g_*$ is the effective relativistic degrees of freedom and $M_{Pl}$ is the Planck mass. Again, in the two-flavor regime, $K_{2e+\mu}$ stands for $K_{2e+\mu} = K_{2e} + K_{2\mu}$. In Fig. 3 we plotted $K_2$, $K_{2\tau}$ and $K_{2e+\mu}$ versus $m_1$ for the same values of the mixing angles and of the phases as in Fig. 2. It can be noticed how the wash-out from the same $N_2$ inverse processes is quite strong in the $\tau$ flavor in the first (left panel) and in the third example (right panel). In the second example (center panel) the wash-out is almost absent but there is some $CP$ asymmetry suppression compared to the maximum value. A suppression $\sim 1/30$ of $N_{B-L}^{T\sim M_2}$ is therefore actually unavoidable compared to a maximum potential value. For this reason, and because $\varepsilon_2 \propto M_2$, the typical order of magnitude of $M_2$ for the mechanism to work is $M_2 \sim 10^{11} \text{GeV}$, two orders of magnitude higher than the usual lower bound in $N_1$-leptogenesis [6] and one order of magnitude higher than the lower bound in $N_2$ leptogenesis without any condition on $m_D$ [11]. In Fig. 4 we finally plotted $N_{B-L}^{T\sim M_2}$ (dashed line) for the same values of the parameters as in Fig. 2 and in Fig. 3. We would like to stress that in the third example (right panel) for a very fine tuned valued of $m_1$ and for the chosen values of $\alpha_i$, the values of $M_2$ and $M_3$ are equal and therefore there is a resonant $CP$ asymmetry enhancement ($\xi(M_2^2/M_3^2) \gg 1$) just at the center of the
peak in the right panel of Fig. 3. However, far from the peak one has $M_3 \gtrsim M_2$ implying no resonant $CP$ asymmetry enhancement. Therefore it must be clearly stressed that the success of the mechanism does not rely at all on a $CP$ asymmetry enhancement due to $M_2 \sim M_3$. This will be even more clear when we will perform a general scan in the space of parameters imposing a very restrictive condition $M_3/M_2 > 10$ and we will find points with large enough asymmetry.

After the flavor asymmetries are generated at $T \sim M_2$, they are subsequently washed out by the interactions mediated by the $N_1$’s. This takes place at $T \sim M_1 \ll 10^9$ GeV, in the three flavor regime, where the interactions mediated by all the three charged lepton Yukawa interactions are in equilibrium. Therefore, we calculate the final asymmetry by projecting $N_{B-L}^{T \sim M_2}$ onto the three flavors using the expression

$$N_{B-L}^{f} \simeq \varepsilon_{2e} \kappa(K_{2e+\mu}) e^{-\frac{m_{\nu}}{M_2}} + \varepsilon_{2\mu} \kappa(K_{2e+\mu}) e^{-\frac{m_{\nu}}{M_2}} K_{1\mu} + \varepsilon_{2\tau} \kappa(K_{2\tau}) e^{-\frac{m_{\nu}}{M_2}} K_{1\tau}.$$  (21)

Notice that we are neglecting the matrix relating the asymmetries stored in the lepton doublets to the $(B/3 - L_\alpha)$ asymmetries. These would further decrease the wash-out from the lightest RH neutrino. However, it has been noticed in [24] that when the Higgs asymmetry is taken into account as well, the diagonal terms sum up approximately to unity and it is a good approximation to neglect their effect.

The first important thing to notice is that adopting the $SO(10)$ relation (13) in the expression (17) one can see that

$$\varepsilon_{2\tau} : \varepsilon_{2\mu} : \varepsilon_{2e} = (\alpha_3 m_t)^2 : (\alpha_2 m_c)^2 : (\alpha_1 m_u)^2,$$  (22)

This hierarchy is lost if the matrix $V_L$ differs significantly from the unity matrix. To our understanding this is the case in Ref. [10] after translating their parametrization into ours.
from which we deduce that $\varepsilon_{2\tau} \simeq \varepsilon_2$ and that most of the asymmetry generated by the $N_2$ decays is produced predominantly along the tau flavor. This can be clearly seen in the two examples in Fig. 2. Furthermore, in the limit of normal hierarchy for the light LH neutrinos, adopting tri-bimaximal mixing for $U$, ($\theta_{13} = 0$, $\theta_{12} \simeq 35.3^\circ$, $\theta_{23} = 45^\circ$) and setting, as we wrote earlier, the matrix $V_L$ equals to the unity matrix, we find

$$\varepsilon_{2\tau} \simeq \frac{9}{4\pi} \left(\frac{\alpha_2 m_c}{v}\right)^2 \frac{m_1 \text{Im} \left[ (U_R^{*})_{32}^2 \right]}{m_3 (U_R)_{32}}.$$  \hspace{1cm} (23)

Indeed, since $\varepsilon_{2\tau} \propto (M_2/M_3)$ and $M_3 \propto m_1^{-1}$, we immediately deduce that a large lepton asymmetry in the tau flavor may be produced only for sufficiently large values of $m_1$. This is rather easy to understand. If $m_1$ tends to zero, we go into the so-called decoupling limit, $M_2/M_3 \simeq 0$. As the CP asymmetry needs (at least) two heavy states to be generated at the one-loop level, and disregarding the contribution from the $N_1$, $\varepsilon_{2\tau}$ must vanish.

The wash-out of such an asymmetry by the next-to-lightest neutrinos themselves is governed by the parameter

$$K_{2\tau} = \frac{(\alpha_2 m_c)^2 |(U_R)_{32}|^2}{m_* M_2},$$

$$| (U_R)_{32} | \simeq \frac{(m_{\nu})_{11} (m_{\nu})_{23} - (m_{\nu})_{12} (m_{\nu})_{13} \alpha_2 m_c}{(m_{\nu})_{11} (m_{\nu})_{22} - ((m_{\nu})_{12})^2 \alpha_3 m_t},$$

$$M_2 \simeq \frac{\alpha_2 m_c^2 |(m_{\nu})_{11}|}{|((m_{\nu})_{12})^2 - (m_{\nu})_{11} (m_{\nu})_{22}|}.$$  \hspace{1cm} (24)

For instance, using the same limits adopted for the computation of the CP asymmetry, we find

$$K_{2\tau} \simeq \frac{m_3}{2m_* \left[1 - 2s_{13} \cos(2\delta - \sigma)\right]}.$$  \hspace{1cm} (25)

This gives $K_{2\tau} \simeq 25$, in agreement with the three numerical examples in Fig. 3. This leads to a suppression in the efficiency factor given by $\kappa(K_{2\tau}) \lesssim 10^{-2}$ that is not enough to prevent that the hierarchy in the CP asymmetries translates into a hierarchy in the flavor asymmetries. Therefore, the total asymmetry produced from $N_2$-decays at $T \sim M_2/5$ is dominantly in the tau flavor. It follows that in order for the mechanism to work we have to impose the subsequent wash-out mediated by the $N_1$'s is inefficient along the tau flavor, that is $K_{1\tau} \lesssim 1$. In Fig. 5 we plotted $K_1$ and the three $K_{1\alpha}$'s depending on $m_1$ for the usual two sets of values. One can see that in these two examples there are values for $m_1$ where $K_{1\tau} \lesssim 1$. Correspondingly one can see in Fig. 4 (solid line) that $N_{B-L}^T \simeq N_{B-L}^{T-M_2}$ meaning that the wash-out from the lightest RH neutrinos is circumvented: the mechanism proposed in [14] is therefore working in this case. The existence of regions in the space
of parameters where $K_{1\tau} \lesssim 1$, one of the key points in our discussion, can be understood analytically. Indeed an analytical expression for $K_{1\tau}$ is easily found when exploiting again the $SO(10)$ relation \[^{[13]}\]

$$K_{1\tau} = \frac{(\alpha_3 m_1^2 |(U_R)_{31}|^2}{m_*}, \quad |(U_R)_{31}| \simeq \frac{|(m_\nu)_{13}|}{|(m_\nu)_{11}|}, \quad M_1 \simeq \frac{(\alpha_1 m_u)^2}{|(m_\nu)_{11}|},$$

$$|(m_\nu)_{13}|^2 \simeq \frac{1}{9} \left[ m_1^2 + m_2^2 - 2m_1 m_2 \cos 2\rho - 2s_{13} \right]$$

$$\left( \sqrt{2}m_1^2 \cos (4\rho + \delta) - \sqrt{2}m_1 m_2 \cos (2\rho + \delta) + \frac{m_1 m_2}{\sqrt{2}} \cos (2\rho + \delta) - \frac{m_1^2}{\sqrt{2}} \cos \delta - \frac{3m_1 m_3}{\sqrt{2}} \cos (\delta - 2\sigma - 2\rho) + \frac{3m_2 m_3}{\sqrt{2}} \cos (\delta - 2\sigma) \right),$$

$$(m_\nu)_{11} \simeq \frac{2}{3} m_1 e^{-2i\rho} + \frac{1}{3} m_2,$$ (26)

where we have retained only terms linear in $s_{13}$. From these expressions, we can already deduce some important results. We see, for instance, that, for $s_{13} = 0$, $K_{1\tau}$ is minimized by $\rho = 0 \text{ (mod } 2\pi\text{) irrespectively from the values of } \delta \text{ and } \sigma$, leading to

$$K_{1\tau}^{\min} (s_{13} = 0) \simeq \frac{1}{3} \frac{(m_2 - m_1)^2}{(2m_1 + m_2) m_*},$$ (27)

suggesting that for no mixing between the first and the third LH neutrino generation, the wash-out of the baryon asymmetry produced by the $N_2$ decays via the interactions mediated by the $N_1$'s is already mild, $K_{1\tau} \sim 3$ if the LH spectrum is hierarchical, $m_1 \ll m_2 \ll m_3$. The more the spectrum of the LH neutrinos is degenerate, $m_1 \simeq m_2 \simeq m_3 \simeq \bar{m}$, the more the wash-out decreases, down to the value $K_{1\tau} \simeq (\Delta m_{sol}^2)^2/(36\bar{m}^4 m_*) \simeq 10^{-2}$.  

Figure 5: Dependence of $K_1$ (black solid line), $K_{1\tau}$ (red long-dashed line), $K_{1\mu}$ (blue dashed line) and $K_{1e}$ (dotted yellow line) on $m_1$ for the same value of the parameters as in Fig. 2.
However, for $s_{13} = 0$, but for non-vanishing values of $\rho$, $K_{1\tau}$ gets very large in the degenerate case, 

$$K_{1\tau} \simeq (m/3m_\star)(1 - \cos 2\rho)/(5 + \cos 2\rho)^{1/2}.$$ 

For non-vanishing values of $s_{13}$, one can find values of the parameters for which $K_{1\tau}$ is smaller than unity. For instance, for a hierarchical spectrum of LH neutrinos and independently of the value of $\rho$, requiring $K_{1\tau} \lesssim 1$ leads to

$$s_{13} \cos(\delta - 2\sigma) \gtrsim \frac{m_2}{3\sqrt{2}m_3} \simeq 0.04.$$ 

To get the feeling of the figures involved, we may set $\delta \simeq 2\sigma$ and find that the wash-out mediated by the $N_1$’s vanishes for an experimentally allowed value of the mixing between the first and the third generation of LH neutrinos, $\theta_{13} \simeq 2.3^\circ$. We stress again that these conclusions do not depend on the value of $\alpha_2$ defined in Eq. (14). These simple analytical insights together with the numerical results demonstrate that it is possible to find regions of the low-energy neutrino parameters where the wash-out mediated by the lightest RH neutrinos is totally negligible. Furthermore, for a hierarchical spectrum of LH neutrinos, it seems that small, but non-vanishing values of $\theta_{13}$ may be preferred. Going back to the minimal value of $m_1$ required to have enough baryon asymmetry, we may estimate

$$\eta_B \simeq 5 \times 10^{-3} \varepsilon_{2\tau} \simeq 5 \left(\frac{\alpha_2^2 m_1}{m_3}\right) \cdot 10^{-10},$$ 

which requires

$$m_1 \gtrsim \left(\frac{5}{\alpha_2}\right)^2 10^{-3} \text{eV},$$ 

for a normal hierarchical spectrum of light neutrinos. In the left panel of Fig. 6 we performed a random scan in the space of parameters, in the case of the mixing angles within the $2\sigma$ ranges and for $\log(m_1/\text{eV})$ spanning in the interval $[-3.5, -0.3]$. The yellow stars are those for which $\eta_B > 3 \times 10^{-10}$ while the green circles, found with a frequency $\sim 10^{-4}$, are those corresponding to successful leptogenesis for $\eta_B > 5.9 \times 10^{-10}$. We imposed $M_3/M_2 > 10$ and therefore, we can conclude that successful leptogenesis is possible without relying on any $CP$ asymmetry enhancement due to $M_2 \simeq M_3$. In the right panel of Fig. 6 we also show the corresponding values of the three RH neutrino masses in the $M_i - m_1$ plane. We see that successful leptogenesis may be achieved for values of $M_2$ as small as $10^{10.5}$ GeV. On the other hand one can see that there is a lower bound $m_1 \gtrsim 10^{-3}$ eV and that for a fully hierarchical spectrum ($m_1 \lesssim m_{\text{sol}} \simeq 10^{-2}$ eV) only non-vanishing values of $\theta_{13}$ in the range $\sim (3^\circ - 9^\circ)$ are allowed. A more detailed analysis of the allowed values of all parameters will be presented in [16].
Figure 6: Scan plot in the $m_1 - \theta_{13}$ plane (left panel) for $V_L = I$ and $\alpha_i = 1, 5, 1$. The hatched area is excluded by the cosmological upper bound on $m_1$ (cf. (3)). Yellow stars and green circles correspond respectively to $\eta_B > 3 \times 10^{-10}$ and $\eta_B > 5.9 \times 10^{-10}$ and corresponding values of the RH neutrino masses $M_i$ (right panel). Notice that all points have been obtained imposing $M_3/M_2 > 10$ and therefore a resonant $CP$ asymmetry enhancement is not necessary for the mechanism to work.

4 Conclusions

In this paper we have studied the thermal leptogenesis mechanism within $SO(10)$-inspired models where the LH neutrino masses are generated via the type I see-saw mechanism and $SO(10)$ inspired conditions are imposed on the neutrino Dirac mass matrix. We have shown that a large enough baryon asymmetry may be created by the decays of the next-to-lightest RH neutrinos when flavor effects are accounted for. Indeed, it is possible to find regions of the parameter space where the asymmetry in a given flavor, in our case the tau one, is not erased by the processes mediated at much lower temperatures by the lightest RH neutrinos. This result has been obtained assuming a hierarchical spectrum of RH neutrinos, therefore without resorting to any resonant enhancement factor in the $CP$ asymmetries. Let us also remark that for thermal leptogenesis to be valid, the initial temperature for the evolution of the system has to be higher than about $M_2/5$ and from the right panel of Fig. 6 one can see that values of the reheating temperature as small as $10^{10}$ GeV may be sufficient for generating a large enough baryon asymmetry. This point becomes relevant when considering the super-symmetrized version of the scenario where the gravitino bound on the reheating temperature after inflation [25] becomes an
issue. We have also found both analytically and numerically an interesting lower bound on the mass of the lightest LH neutrino, \( m_1 \gtrsim 10^{-3} \text{eV} \), and that non-vanishing values of the mixing angle \( \theta_{13} \) are preferred. A more complete analytical and numerical analysis, relaxing the assumptions made here for simplicity (e.g. \( V_L = 1 \), normal scheme for LH neutrinos), is now in preparation [16].

When this paper was being prepared, Ref. [10] appeared where flavored leptogenesis based on a mixed type I plus type II see-saw mechanism was studied. The comparison between our results and those in Ref. [10] is made difficult by the fact that there the type I see-saw case has been marginally studied as a limit of a more general left-right framework. We believe we have found regions of the parameter space where leptogenesis is successful, not fully investigated in Ref. [10]. In particular, in our study the asymmetry is dominantly produced along the tau flavor and we have given more emphasis to the impact of low energy parameters.

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