Numerical study of persistence in models with absorbing states

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Extensive Monte Carlo simulations are performed in order to evaluate both the local ($\theta_l$) and global ($\theta_g$) persistence exponents in the Ziff-Gulari-Barshad (ZGB) (Phys. Rev. Lett. 56, 2553, 1986) irreversible reaction model. In two dimensions and for the second-order irreversible phase transition (IPT) we find, that both the local and the global persistence exhibit power-law behavior with a crossover between two different time regimes. On the other hand, at the first-order IPT, characteristic of the ZGB, active sites are short lived and the persistence decays more abruptly, not being clear whether it shows power law behavior or not. In order to analyze universality issues, we have also studied another model with absorbing states, the contact process, and evaluated the local persistence exponent in dimensions from 1 to 4. A striking apparent super-universality is reported: the local persistence exponent seems to coincide in both one and two dimensional systems.

I. INTRODUCTION

An avalanche of articles devoted to the study of persistence in different physical contexts has appeared recently in the literature. From a theoretical point of view, the appearance of new nontrivial critical exponents as: i) the “short-time” exponent, $\lambda$, needed to characterize the two-time correlation functions in systems relaxing in the process of quenching from infinitely high temperatures to the critical temperature $T_c$ [10] and ii) the global persistence exponent $\theta_g$, related to the probability $P_g(t) \propto t^{-\theta_g}$, that the global order parameter, (e.g. the magnetization in a ferromagnet), has not changed sign up to a certain time $t$ after a quench to $T_c$ [8], constituted an exciting surprise. A number of theoretical, computational, and even experimental analysis have been performed afterwards. From the theoretical viewpoint it has been shown that for processes where the global order parameter dynamics is Markovian, that the global persistence exponent can be related to other “traditional” critical exponents: in particular, the following scaling law holds [8]:

$$\theta_g \propto \lambda - d + 1 - \eta, \quad (1)$$

where $d$ is the dimensionality and $\eta$ is the static critical exponent of the order parameter correlation function. Nevertheless, the required hypothesis is not typically satisfied for most of the usually studied statistical models; i.e. the dynamics of the global order parameter can be argued to be generically non Markovian. In these more generic cases Eq. (1) does not hold and $\theta_g$ is a new independent non-trivial exponent [8]. In general, persistence exponents depend on the system evolution as a whole and, therefore, analytical estimations of them are scarce and difficult [8]. One important theoretical contribution is by Majumdar and Sire, who proposed a method to calculate autocorrelation functions perturbatively for non-Markovian processes (perturbing around a Gaussian and Markovian process [9]).

Recently, other similar non-trivial exponents have been uncovered and measured in different systems. Some of them are:

- (i) The local persistence probability $P_l \propto t^{-\theta_l}$ defined as the probability that the local order parameter at a given point $\vec{x}$ has never changed sign after the initial time (e.g. the probability that one spin has never been flipped in a ferromagnet) [8]. Local persistence exponents have been measured in real systems as liquid crystals or soap bubbles (see [8] and references therein).

- (ii) The block persistence probability $P_L \propto t^{-\theta_L}$ defined as the probability that the order parameter integrated over a block of linear size $L$ has never changed sign since the initial time in a phase-ordering process at finite temperature [10,11]:

- (iii) The generalized persistence probabilities and exponents introduced by Dornic and Godrèche [12,13].

- (iv) The persistence exponents for domains. Found first by Krapivsky and Ben-Naim in Ising system and subsequently in diffusion by Majumdar and Cornell [12]. These can also be generalized to other different types of pattern: each pattern decays with a different exponent, a single spin or the global spin being just two specific patterns.

- (v) The persistence with partial survival first studied by Majumdar and Bray and subsequently by other authors [14]. Recently it has generated a lot of interests in the context of inelastic collapse in granular materials.
For a review of persistence studies in reversible systems see e.g. [1][2][3][5].

Even though considerable effort has been devoted to the study of persistence in various models exhibiting irreversible phase transitions (IPT’s) and, in particular, in systems presenting a critical point separating an active from an absorbing phase (some exceptions can be found in [23–25]). In this context, recently, Hinrichsen and Koduvely [23] have performed a numerical study of the local and global persistence in (1 + 1)−dimensional directed percolation (DP) [26] (see also [27]). They found that the local persistence probability at the critical point (separating the absorbing from the active phase) decays as a power law with an exponent \( \theta_1^{DP} \approx 1.5 \). Also, global persistence measurements seemed to be consistent with \( \theta_g^{DP} \approx 1.5 \), a result that has to be taken with some caution since, on the one hand, the evaluation of \( \theta_g^{DP} \) requires extensive simulations (see the criticisms to this result in [24]) and on the other hand, in all the previously known models (mainly for equilibrium critical systems) \( \theta_g < \theta_1 \) [8]. Furthermore, it was conjectured that both \( \theta_g \) and \( \theta_1 \) are indeed universal exponents characterizing the DP universality class. Another interesting point is that the persistence exponents in this case seem to be independent on the initial condition at odds with what happens in other well known cases as, for example, the two-dimensional Ising model with Glauber dynamics [1]. Finally, Hinrichsen and Koduvely succeeded in relating persistence exponents in directed percolation to certain “return” probabilities with an absorbing boundary or an active source [21]. Another important contribution to the study of this IPT was presented in the recent paper by Oerding and van Wijland, in which the global persistence exponent has been calculated analytically by combining the perturbative method developed by Majumdar and Sire [3] with standard renormalization group techniques [4]. These same authors have thrown some doubts on the validity of the numerical estimations obtained by Hinrichsen and Koduvely (as will be discussed in section III).

In order to contribute further to the understanding of persistence in systems with irreversible phase transitions and try to shed some light on the aforementioned issues, the present manuscript is devoted to the numerical study and evaluation of persistence exponents of the Ziff-Gulari-Barshad (ZGB) model for the catalytic oxidation of CO [28], as well as in the contact process [29–31] in various dimensions. The ZGB model in \( d = 2 \)−dimensions has a twofold advantage: (i) it exhibits a second-order IPT unambiguously placed at the DP universality class [30], and (ii) it also exhibits a first-order IPT [28] where, as expected, the system does not show scaling neither universal behavior [31]. Therefore, the ZGB model provides a suitable framework for the study of the persistence in both, first and second order irreversible critical points. Furthermore, the present study extends the investigation of persistence exponents to higher dimensions in IPT’s, by performing numerical studies of the contact process.

The manuscript is organized as follows: In section II the ZGB model and the simulation method are described in detail. In section III we define the local and global persistence, while section IV is devoted to the presentation and discussion of the main results for both the ZGB model and the contact process. Finally in section V the main conclusions are presented.

II. THE ZGB MODEL AND THE SIMULATION METHOD

The ZGB model mimics the catalytic oxidation of carbon monoxide on a transition metal surface [28], namely \( 2CO + O_2 \rightarrow 2CO_2 \). The lattice-gas version of the ZGB model is also known as the monomer-dimer model (MD), where \( A \equiv CO \) is the monomer because it needs a single adsorption site on the surface, and \( B_2 \equiv O_2 \) is the dimer which adsorbs dissociatively, and consequently requires two neighboring sites on the catalyst surface to be adsorbed. It is assumed that the monomer-dimer reaction proceeds according to the Langmuir-Hinshelwood mechanism:

\[
A(g) + S \rightarrow A(a),
\]

\[
B_2(g) + 2S \rightarrow 2B(a),
\]

\[
A(a) + B(a) \rightarrow AB(g) + 2S,
\]

where \( S \) is an empty site on the surface, while \( A(a) \) and \( B(a) \) refer to the adsorbed and gas phase, respectively.

The MD model uses a square lattice to represent the catalytic surface. The Monte Carlo algorithm for its simulation is as follows: i) an \( A \) or \( B_2 \) molecule is selected randomly with relative probabilities \( Y_A \) and \( Y_{B_2} \), respectively. These probabilities are the relative impingement rates of both species, which are proportional to their partial pressures. Due to the normalization, \( Y_A + Y_{B_2} = 1 \), the model is characterized by a single parameter, say \( Y_A \). If the selected species is \( A \), a surface site is selected at random, and if that site is vacant, \( A \) is adsorbed on it \([\text{Eq.}(2)]\). Otherwise, if that site is occupied, the trial ends and a new molecule is selected. If the selected species is \( B_2 \), a pair of nearest-neighbor sites is randomly chosen and the molecule is adsorbed on them only if they are both vacant \([\text{Eq.}(3)]\). ii) After each adsorption event, the nearest neighbors of the added molecule are examined in order to account for the reaction given by Eq.(4). If more than one pair \([B(a), A(a)]\) are identified, one of them is randomly selected and removed from the surface.
(for more details on the MD and the simulation technique see, e.g. [22]).

Interest in the MD model arises due to its rich and complex phenomenology. In fact, in two dimensions and for the asymptotic regime ($t \to \infty$), the system reaches a stationary state whose nature depends solely on the parameter $Y_A$. In fact, for $Y_A \leq Y_{1A} \approx 0.387368$ ($Y_A \geq Y_{2A} \approx 0.52554$) the surface becomes irreversibly poisoned by $B$ ($A$) particles. On the other hand, for $Y_{1A} < Y_A < Y_{2A}$ a steady state with sustained production of $AB$ is generated. Figure 1 shows plots of the rate of $AB$ production ($R_{AB}$) and the surface coverages of $A$ ($\theta_A$) and $B$ ($\theta_B$) versus $Y_A$, respectively. Just at $Y_{1A} = Y_{2A}$ the MD model exhibits irreversible phase transitions (IPT’s) between the reactive regime and poisoned states, which are of second and first order respectively. The second-order IPT belongs to the universality class of directed percolation [29] and is rather well understood [4]. Furthermore, as it is shown in Figure 1, when $Y_A$ increases towards $Y_{2A}$ the catalytic activity increases, but when $Y_{2A}$ is reached large $A$ clusters suddenly emerge and cover the whole lattice. The transition occurs abruptly, with discontinuities of the coverages and activity, unveiling its first-order nature.

III. LOCAL AND GLOBAL PERSISTENCE

In a system, such as directed percolation, possessing absorbing states, the local persistence probability can be measured by starting the system with a homogeneous random initial conditions, and evaluating the probability that a given, originally inactive, site has not become active up to time $t$ [29]. It should be noticed that this quantity depends upon the whole system evolution up to time $t$, and therefore it can be envisaged as an infinite-point correlation function in the time direction. Consequently $P_l(t)$ is a rather non-trivial quantity, difficult to study analytically as discussed before. Numerical simulations in a $(1+1)$-dimensional DP model show that $P_l(t)$ decays algebraically at criticality:

$$P_l(t) \propto t^{-\theta_l},$$

with $\theta_l \approx 1.5$ [4]. In the case of the ZGB model, and for the continuous phase transition, the absorbing state corresponds to the surface of the catalyst fully covered by $B$-species. Such species plays the role of inactive sites (for the other transition, i.e. the first-order one, the roles of the $A$ and $B$ particles are exchanged). We define $P^1_Z(t)$ as the probability that a given inactive site does not become active up to time $t$. Consequently, for the second-order (first order) transition simulations are started with lattices partially covered at random by $A$-species ($B$-species) ($\theta_{A(B)}(t = 0)$). The only mechanism capable to activating a site is the reaction $A + B \rightarrow AB$ as specified by equation 4.

We have numerically evaluated the persistence distribution function $D^Z_{ZGB}(t)$, that is, the probability for a given site to become active in the time interval between $t$ and $t + dt$. Accordingly, one has

$$D^Z_{ZGB}(t) \propto t^{-(\theta^Z_{ZGB} + 1)},$$

where $\theta^Z_{ZGB}$ is the persistence exponent ($D^Z_{ZGB}(t)$ is simply the time derivative of $P^Z_{ZGB}$).

In systems exhibiting reversible phase transitions, the global persistence is usually defined as the probability that the global order parameter (e.g. the total magnetization in the Ising model) does not change sign up to time $t$. In contrast, for systems with absorbing states, such as DP, the global order parameter given by the density of active sites $\rho_{AS}(t)$, is a strictly positive quantity and therefore the standard definition is not applicable. Hinrichsen and Koduvely [20] have proposed that, instead, one can evaluate the probability that the deviation of the order parameter from its mean value, $\Delta \rho_{AS}(t) = \rho_{AS}(t) - \langle \rho_{AS}(t) \rangle$, does not change sign up to time $t$. In the present study, and close to the second-order IPT, we have considered

$$\rho_{AS}(t) = 1 - \theta_B = \theta_A + \theta_V$$

where $\theta_V$, $\theta_A$ and $\theta_B$ are the density of empty and occupied by $A$ and $B$-species sites, respectively. However, in finite systems as directed percolation the deviation probability depends on the sign of $\Delta \rho_{AS}(t)$. This asymmetry should vanish in the thermodynamic limit where $\Delta \rho_{AS}(t)$ becomes a Gaussian process. In finite systems, however, the renormalized variance of the fluctuations is no longer constant but increases with the value of $\Delta \rho_{AS}(t)$ causing the effective time scales for positive and negative fluctuations to be different [20,24]. Therefore, $P^g_{-}(t)$ ($P^g_{+}(t)$) is defined as the probability of $\Delta \rho_{AS}(t)$ to remain negative (positive) from an initial time $t_i$ up to time $t$. We have also evaluated the distribution function $D^g_{-}(t)$, i.e., the probability that $\Delta \rho_{AS}(t)$ changes sign in the time interval between $t$ and $t + dt$. In order to evaluate $\Delta \rho_{AS}(t)$ we have first to calculate a “calibration curve” given by $\rho_{AS} \propto t^{-\theta}$ where $\theta \approx 0.4505$ is the corresponding DP exponent in $(2+1)$-dimensions [22,24], describing the time decay of a homogeneous initial condition at criticality: every-time $\rho_{AS}(t)$ intersects this curve, $\Delta \rho_{AS}(t)$ changes sign.

A second important issue related to the global persistence exponent, worth to discuss before proceeding, is the time regime in which it should be measured. Following [24], the global exponent is well defined in the regime in which the dependence on the initial state has been erased and, on the other hand, there is an upper bound to the validity of measurements, given by the upper cutoff induced by the finite system size. Putting together these two constraints

$$L^z \gg t \gg \theta_B^{2/(d-n)}$$


IV. RESULTS AND DISCUSSION.

In this section we report our main numerical findings. The value of the second order critical point of the ZGB model reported previously, $Y_A = 0.387368$ [30], corresponds to the infinite size limit; while for the system sizes we will consider, namely $128 \times 128$ and $256 \times 256$, finite size corrections shift the critical value to slightly larger values: in particular, the simulations at the critical point reported in what follows are performed at $Y_A = 0.3907$ [34].

Figure 2 shows a log-log plot of $D_l(t)$ versus $t$ at the second-order IPT. Two regimes can clearly be observed: i) The short time one, for $t < 200$, where $D_l(t)$ exhibits a power law behavior $D_l(t) \propto t^{-(\theta_l + 1)}$, with $\theta_l \approx 1.00 \pm 0.05$, and (ii) the asymptotic regime, for $t > 200$, with the exponent $\theta^{ZGB} \approx 1.50 \pm 0.10$. Since the phase transition belongs in the DP universality class [36], it is quite surprising that the obtained exponent, within error bars, is almost the same as the one reported by Hinrichsen et al. [21], for a DP process in $(1 + 1)$-dimensions, namely $\theta^{DP} \approx 1.50 \pm 0.03$. This finding suggests that for DP, the difference between the persistence exponents in $(1+1)$- and $(2+1)$-dimensions is very small. In addition, we have also verified the lack of dependence of these exponents upon modifications of the initial condition, as shown in figure 3. In fact, runs performed using $0.05 \leq \theta_D(t = 0) \leq 0.20$ show clearly that neither the short-time regime nor the persistent regime depend on the initial density of inactive sites. This finding is in accordance with Hinrichsen et al. observation [24].

In order to gain some insight on the origin of the crossover observed in figures 2 and 3, we have evaluated the time dependence of the fraction of persistent sites with $NN = 0, 1, 2, 3$ and 4 empty nearest-neighbor respectively, as a function of time, as it is shown in figure 4. It is clear that different $NN$ values vanish at distinct characteristic times, e.g. $t \approx 10, (NN = 0); t \approx 15, (NN = 1); t \approx 30, (NN = 2); t \approx 120, (NN = 3); \text{and} t \approx 300, (NN = 4)$, respectively. Therefore comparing the results of figures 2-3 and 4 we conclude that the crossover in figures 2-3 takes place when almost all persistent sites are blocked by inactive (absorbing) ones. The physical interpretation of the observed crossover is therefore the following: There is an initial regime in which the persistence of a given site is dominated by its open neighboring sites: particles can land directly in the neighboring sites causing desorption, and consequently the death of persistent sites. However, as the number of such open sites is reduced on average to a value close to zero, there is a crossover to the true asymptotic regime in which one has to wait for local rearrangements in order to reach persistent sites.

In order to further explore the dependence of the persistence exponents on the dimensionality we have also performed extensive simulations [31], of another system with absorbing states belonging to the DP universality class, namely the contact process (CP) [22,29]. In this model, proceeding analogously, we observe no crossover analogous to the one described for the ZGB, asymptotic regimes can be obtained with less computational effort, and the exponent values are far more accurate. We simulate the CP in dimensions from 1 to 4. The largest system sizes considered are $10^4$ in $d = 1, 256 \times 256$ in $d = 2, 50^3$ in $d = 3$, and $32^4$ in $d = 4$. Our results are $\theta_l \approx 1.50 \pm 0.01$ for $(1 + 1)$-, and $\theta_l \approx 1.50 \pm 0.01$ for $(2 + 1)$-dimensions [3] (see Fig. 5). In fact, the two curves in Figure 5 are strikingly parallel. This fact confirms our rather strange finding, that the local persistence exponent is (almost?) the same in one and two-dimensional systems with absorbing states. It could be the case, that there is some small difference between the exponent values in $d = 1$ and $d = 2$, but within our numerical accuracy they are absolutely indistinguishable. The apparent coincidence between the results in one and two dimensions is quite intriguing, and so far, we do not have any satisfactory explanation for this surprising result.

For the sake of completeness, and in order to test whether this apparent “super-universality” extends also to higher dimensions, we have also measured the local persistence exponents in $d = 3$ and $d = 4$ as said before. The results for $\theta_l$ are $1.33 \pm 0.03$ in $d = 3$, and $1.15 \pm 0.05$ in $d = 4$, indicate a monotonous diminution of the local persistence as a function of dimensionality [3].

It is well known that for equilibrium models the persistence exhibits generically a power-law behavior below the critical temperature [1,12]. In order to establish analogies with irreversible systems having absorbing states, we have also measured the local persistence away from criticality, i.e. within the reactive regime of the ZGB model, as it is shown in figure 6. For $Y_A = 0.40$ (i.e. very close to the critical threshold) the power-law characteristic of the short-time regime is still observed, but the persistent regime shows a clear departure from the behavior observed at criticality. Going deeper into the reactive
regime, e.g. for $Y_A = 0.45$, the plot shows a marked curvature and the existence of a power-law behavior can be ruled out. On the other hand, in the absorbing phase persistence curves tend to a constant for large values of $t$, i.e. there is a non-vanishing asymptotic probability to persist indefinitely. This is a direct consequence of the fact that eventually the system reaches an absorbing configuration and the dynamics is arrested. Therefore, we conclude that unlike reversible systems where a power-law behavior holds for a wide range below the critical temperature, in systems with absorbing states the power-law behavior of the persistence is restricted to criticality.

In what respect the global persistence, Figure 7 shows a double logarithmic plot of $D_{ZGB}^2(t)$ versus $t$ obtained at the second-order IPT for two different lattice sizes. As in the case of the local persistence, two time regimes observed: i) a short time regime ($t \lesssim 300$) where a power law behavior with a slope $\sim 1.25 \pm 0.05$, and ii) an asymptotic regime ($t > 300$) with a power law behavior with exponent $\theta_{ZGB} = 2.5 \pm 0.5$. The onset of the long time regime depends on $L$ as it shows in figure 6 for $L = 256$ and $L = 512$, however $\theta_{ZGB}^2$ appears to be size independent. As it was already discussed for the case of the local persistence, the onset of the crossover between the different time regimes can be linked with the results shown in figure 4. Consequently the asymptotic regime in the ZGB model starts when persistent sites are almost surrounded by inactive sites.

Our result for the persistence exponents, namely $\theta_l < \theta_g$, is quite surprising at first sight, since from all previously accumulated experience, mainly for equilibrium critical systems, it is known that $\theta_l > \theta_g$. It seems that irreversible critical systems may depart from this behavior.

Assuming the dynamics of the order parameter to be Markovian, and using the known scaling relations, and values of the directed percolation universality class, one obtains that Eq. (1) can be written as $[2,3]$

$$\theta_g = 1 + d/(2z) \approx 1.565 \quad (d = 2).$$

Our numerical value is far from this result, implying that the process is strongly non-Markovian. Oerding and van Wijland have shown [24], using field theoretical tools combined with the perturbative expansion method by Majumdar and Sire [3], that up to first order in epsilon expansion, $\theta_g = 2 + 0.056 + O(\epsilon^2)$. This implies $\theta > 2$ (at least up to first order) as, in fact, is the case in our measurements.

The physical interpretation for the fact that $\theta_l < \theta_g$ is qualitatively the following: at the critical point, in the activity density decay process it is more likely to fluctuate around the global average density (which corresponds to annihilating more (or less) activity than the average at that time), than invading absorbing regions in which persistent sites exist. This is very reasonable since in general, at the critical point and for large enough times, activity is restricted to some patches, which become smaller and smaller as time runs. It is therefore, more likely to have fluctuations in the density of these patches (controlled by the global persistence exponent) that to invade persistent sites (controlled by the local exponent) typically surrounded by absorbing regions at large times.

The insert of figure 7 shows a log-log of $D_{ZGB}^2(t)$ and $D_{\eta}^2(t)$ versus $t$ as obtained for lattices of size $L = 512$. Since the results are almost independent of $L$, finite size effects may be almost negligible. Figure 8 also shows that at coexistence the persistent sites are short-lived and consequently the persistence drops off abruptly. The data can be fitted by a power law with an exponent $\theta_l \approx 3.0 \pm 0.3$. However, since the range of the fit is narrow and there seems to be a systematic curvature, we cannot disregard an exponential (or an stretched exponential) decay for longer times. Further clarification of this issue will have to wait for computationally expensive large-scale numerical simulations. It should be noticed that the operation of power-law (scale-invariance) in the dynamical critical properties of the first-order IPT of the ZGB model has recently been ruled out [31]. But given the lack of a general theoretical framework, we do not know whether or not algebraic decay of persistence functions should be expected at non-equilibrium first order transitions.

### V. CONCLUSIONS

A numerical study of persistence in the ZGB dimer-monomer model for the catalyzed reaction $A + \frac{1}{2} B_2 \rightarrow A B_2$ is presented. It is found that for the second-order IPT the time dependence of both the local and the global persistence exhibit a crossover between a short- and a long-time regimes. A physical explanation for such a crossover has been provided. Persistence exponents are evaluated within the long-time regime giving $\theta_{ZGB}^2 \approx 1.50 \pm 0.01$ for the local one, and $\theta_{ZGB}^2 \approx 2.5 \pm 0.5$ for the global one. The former, which should correspond to the DP universality class in $(2+1)$—dimensions is very close to the value reported by Hinrichsen et al. [23] for DP in $(1+1)$—dimensions, namely $\theta_{DP} \approx 1.50 \pm 0.03$. Simulations in the contact process (another model belonging in the same universality class) confirm this indistinguishability between the one and two-dimensional local persistence exponents with higher accuracy (see the striking parallelism of the curves in figure 5). This finding implies a theoretical and numerical puzzle: Is $\theta_{DP}$ independent of the dimensionality for $d \leq 2$, or is it just that
the one and two-dimensional exponents incidentally take very similar numerical values? The likely possibility of them being equal is quite intriguing since the return probabilities in systems with absorbing states are expected to be rather different in different spatial dimensions. For the contact process, in dimensions larger than $d = 2$ we find, as expected, dimension dependent exponents.

The fact that $\theta_3^{ZGB} > \theta_3^{A}$ (in agreement with field theoretical predictions) is in marked contrast to well established findings in the field of reversible transitions, where quite generically $\theta_3 < \theta_1$. A physical explanation for such a discrepancy has been provided.

We have also observed that power laws for persistence are obtained only at criticality, and the associated exponents do not depend upon the considered initial conditions, ad hoc with what is known for other well known reversible systems.

Persistence probabilities have also been evaluated at the first-order IPT of the ZGB model. Due to the short-lived behavior of the persistent sites we obtained an exponent $\theta_3^{ZGB} \approx 3.0 \pm 0.3$. However, further effort will be required in order to clarify if this apparent power-law behavior merely reflects a short-time regime followed by a cut-off or it is a real asymptotic effect.

We expect our numerical findings to constitute a valuable step for the development of a theoretical framework in the field of irreversible phase transitions, which is certainly needed.

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FIG. 1. Average coverage of $B$–species ($\theta_B$), $A$–species, ($\theta_A$) and the $AB$ production rate ($R_{AB}$), respectively, obtained under steady state operation, as a function of $Y_A$ for the ZGB model. Irreversible transitions of second- and first-order occurs at $Y_1 \simeq 0.387368$ and $Y_2 \simeq 0.52554$, respectively.

FIG. 2. Log-log plot of $D(t)$ versus $t$, corresponding to the ZGB model at the second-order IPT and obtained for lattices of size $L = 128$ and $L = 256$, respectively. The dashed (full) line shows the short- (long-)time regime and has slope $\theta_l + 1 = 2.00$ ($\theta^{ZGB}_l + 1 = 2.5$), respectively. Results are obtained starting simulations with $\theta_B(t = 0) = 0.1$ and averaging over $10^3 (2.5 \times 10^3)$ realizations for $L = 128 (L = 256)$, respectively.

FIG. 3. Log-log plot of $D_i(t)$ versus $t$, corresponding to the ZGB model at the second-order IPT and obtained for lattices of size $L = 256$. Results obtained starting simulations with different values of $\theta_B(t = 0)$, as indicated in the figure, and averaging over $2.5 \times 10^3$ realizations. No significative modification in the slope is observed upon changing the initial condition.

FIG. 4. Linear-logarithmic plot of the average fraction of persistent sites with $NN$ empty neighbors, for $NN = 0, 1, 2, 3$, and 4 respectively, as a function of time $t$. Results are obtained using the same conditions as in figure 2. For times larger than $10^3$ the fraction of empty sites neighboring persistent ones becomes very small; i.e. typically, activity takes place away from persistent sites, and local outbursts of activity are required in order to reach persistent sites.
FIG. 5. Local persistence for one and two dimensional simulations of the contact process. In \( d = 1 \) \((d = 2)\) the system size is \( L = 10^4 \) \((256 \times 256)\) respectively. Observe that both slopes are indistinguishable. The straight line has a slope \( \theta_l = -1.5 \).

FIG. 6. Log-log plot of \( D_l(t) \) versus \( t \), corresponding to the ZGB model. Results obtained using lattices of size \( L = 256 \), with \( \theta_B(t = 0) = 0.10 \), and averaging over \( 2.5 \times 10^3 \) realizations. Different values of the parameter \( Y_A \) were used, as indicated in the figure. The dashed (full) line shows the short- (long-)time regime and has slope \( 2.00 \) \((\theta^{ZGB}_B + 1 = 2.5)\), and have been drawn to allow comparison with figure 2. See more details in the text.

FIG. 7. Log-log plot of \( D_g^+(t) \) versus \( t \) corresponding to the ZGB model at the second-order IPT and obtained for lattices of side \( \Delta, L = 256 \) and \( \bigcirc, L = 512 \), respectively. The dashed (full) line shows the short- (long-)time regime and has slope \( 1.25 \) \((\theta^{ZGB}_B + 1 = 3.5)\), respectively. Results have been obtained starting the simulations with \( \theta_A(t = 0) = 0.10 \) and averaging over \( 10^6 \) \((2.5 \times 10^7)\) realizations for \( L = 256 \) \((L = 512)\), respectively. The inset shows a log-log plot of \( D_g^-(t) \) versus \( t \), obtained for \( L = 512 \): the asymptotic slope coincides with that of the main plot.

FIG. 8. Log-log plot of \( D_l(t) \) versus \( t \) obtained at the coexistence point \( Y_2 = 0.52554 \) for lattice sizes \( L = 256 \) and \( L = 512 \). The straight line has slope \( \theta_l + 1 = 4.00 \). Results have been obtained starting simulations with \( \theta_A(t = 0) = 0.1 \) and averaging over \( 10^4 \) \((2.5 \times 10^7)\) realizations for \( L = 256 \) \((L = 512)\), respectively. The upward bending of the last two points indicate that the value of the critical point we are considering is slightly in the subcritical phase for the finite system sizes under analysis.