Polarization in $\bar{B}_c \to J/\psi \mu \bar{\nu}_\mu$

Myoung-Taek Choi
Department of Physics, Hanyang University, Seoul 133-791,

Ji-Ho Jang and Jae-Kwan Kim
Department of Physics, Korea Advanced Institute of Science and Technology
373-1 Kusung-dong, Yusung-ku, Taejon 305-701, Korea.

We study the polarization of the $J/\psi$ meson of the $\bar{B}_c \to J/\psi \mu \bar{\nu}_\mu$ decay process, followed by $J/\psi \to \mu^+\mu^-$, with the help of the heavy-quark spin symmetry formalism (HQSS) of Jenkins et al. We adapt the ISGW wave function. Due to the clean signature of the decay mode, measurements of the polarization of the $J/\psi$ meson can play a special role in extracting $|V_{cb}|$, the quark mixing-matrix element. We compare the results with the predictions of other quark models.

(PACS number: 13.25.Hw)
I. INTRODUCTION

Semileptonic decays of pseudoscalar heavy mesons to vector mesons ($P \rightarrow V$) provide richer physics than those of pseudoscalar heavy mesons to pseudoscalar mesons ($P \rightarrow P$). First, the branching ratio of $P \rightarrow V$ is larger than that of $P \rightarrow P$ due to the spin structure. Second, $B \rightarrow D^* l \bar{\nu}$ decay offers a good chance to extract an accurate value of $|V_{cb}|$ because it is less affected by $1/m_Q$ corrections [2]. Finally, they allow a measurement of the polarization of the vector meson. The study of polarization of vector meson is important for the following reasons: the degree of polarization of the daughter particle, in general, strongly influences the momentum spectrum of the final particles as well as the decay rates; hence the ratio of the longitudinal to the transverse decay width is quite sensitive to form factors. Also the extraction of individual form factors proves fruitful in determining $|V_{cb}|$ [3]. Finally one can test quark models to get a better understanding of the underlying structure and the dynamics of hadrons and currents.

Several experiments have been performed to measure the $|V_{cb}|$ [4] for the decay of $B$ mesons. Also, measurements of the form factors and the polarizations in $D \rightarrow K^* l \bar{\nu}$ and $B \rightarrow D^* l \bar{\nu}$ have been done [5]. However, they suffer from large experimental uncertainties because of reconstruction due to cascade decays: $D^*$’s are reconstructed from the decay chain $D \pi$, followed by $D \rightarrow K \pi$ or $K \pi \pi$.

Recently, much interest has been paid to the study of $B_c$ meson, which is a source of interesting physics; it provides a unique probe of both strong and weak interactions. Unlike other heavy quarkonium systems which decay strongly and electromagnetically, the $B_c$ meson can decay only weakly because of the fact that it carries flavor explicitly. The decays of the $B_c$ meson can potentially provide a rich source for measuring the Cabibbo-Kobayashi-Maskawa (CKM) matrix elements. One also expects rich spectroscopy for the $(\bar{b}c)$ bound states probing the inter-quark potential at distances intermediate to the charm and the beauty quarkonium systems. Candidates for $B_c \rightarrow J/\psi l \bar{\nu}$ were recently reported for the ALEPH detector at LEP [6]. A quite large number of $B_c$ mesons are expected to be produced by the future Large Hadron Collider (LHC) experiment.

In this paper, we study the production of the transverse and the longitudinal vector meson $J/\psi$ in semileptonic decays $B_c \rightarrow J/\psi \mu \bar{\nu}_\mu$ by using the heavy-quark spin symmetry formalism (HQSS). The exclusive decays of $B_c$ mesons, which include the $J/\psi$ meson as a final state, are essential to study $B_c$ mesons because the branching ratio is large and the $J/\psi$ meson decaying to dilepton pairs is easy to identify. Among the exclusive decays, the $B_c \rightarrow J/\psi \mu \bar{\nu}_\mu$ process, followed by the decay of $J/\psi$ into a $\mu^+\mu^-$ pair, will show clean
experimental signatures\footnote{The electron can also be used instead of the muon. However, the signature of the muon is expected to be clearer than that of an electron in huge backgrounds of hadronic collisions.}: three energetic leptons coming from the secondary vertex, two of them reconstructing a $J/\psi$, and some missing transverse momentum due to neutrino. Since the measurements of the polarizations of $J/\psi$ mesons in $\bar{B}_c \to J/\psi \mu \bar{\nu}_\mu$ decays can be used to constrain the form factor behavior with respect to momentum transfer, these measurements will play a special role in extracting the $|V_{cb}|$ matrix element.

In Sec. II, we review the HQSS formalism of Jenkins et al.\cite{1} and calculate the decay width of $\bar{B}_c \to J/\psi \mu \bar{\nu}_\mu$. The polarization of the $J/\psi$ meson in $\bar{B}_c$ meson decay is discussed in Sec. III. We obtain the value of the longitudinal and the transverse decay widths and find an expression for the polarization parameter which is directly related to the measurement. In Sec. IV, discussion and conclusions are given. We compare the results to those of other quark models, such as ISGW2 (the improved Isgur-Scora-Grinstein-Wise (ISGW) model)\cite{3}, the original ISGW\cite{2} and the Bauer-Stech-Wirbel (BSW) model\cite{10}. In the appendix, we briefly summarize the formalisms of the ISGW and the BSW models we used in the calculation.

\section*{II. Heavy Quark Spin Symmetry of $B_c$ Meson and Semileptonic Decay Width}

The heavy-quark symmetry\cite{1} of Quantum Chromodynamics (QCD) in the infinite quark mass limit has been successfully applied to the hadrons containing a single heavy quark. In a heavy-light quark system, such as the $B$ meson, the heavy quark act as a static color source in the infinite-mass limit: light degrees of freedom do not feel the change of a $b$-quark to a $c$-quark. As a consequence, two symmetries occur: heavy-quark flavor symmetry and heavy-quark spin symmetry\cite{1}. Due to the two symmetries, the form factors of heavy-meson decay are expressed in terms of a single Isgur-Wise form factor, and the prediction of the matrix elements is quite simplified. In the $B_c$ meson, however, both quarks should move around each other to make a stable meson. Since the kinetic-energy term should be kept in the Lagrangian even at leading order, the flavor symmetry is broken explicitly.

Since the spin-spin interaction between the quarks is proportional to $1/m_b m_c$ and is expected to be small, the spin symmetry still remains. Jenkins et al.\cite{1} investigated the consequences of the heavy quark spin symmetry of the $B_c$ meson and showed that the semileptonic decay $\bar{B}_c \to J/\psi \mu \bar{\nu}_\mu$ can be described using only one form factor near zero recoil.
Heavy-quark spin symmetry implies that the pseudoscalar $B_c$ meson is degenerate with the vector $B_c^*$ meson. The consequence of the spin symmetry of the heavy-heavy meson system is compactly derived using the well-known covariant representation formalism \[12\].

The pseudoscalar $B_c$ meson of velocity $v$ is represented by a $4 \times 4$ matrix

\[ \mathcal{H}^{(bc)} = \frac{(1 + y')}{2} [B_c^{\mu*} \gamma_\mu - B_c \gamma_5] \frac{(1 - y')}{2}, \]

where $B_c$ and $B_c^*$ annihilate the pseudoscalar and the vector meson $\bar{b}c$ bound states of velocity $v$, respectively. Analogously, the $(\eta_c, J/\psi)$ spin multiplet of velocity $v'$ is given by

\[ \mathcal{H}^{(cc)} = \frac{(1 + y')}{2} [J/\psi^{\mu*} \gamma_\mu - \eta_c \gamma_5] \frac{(1 - y')}{2}. \]

The spin multiplet for $B_a$ and $B_a^*$ is given by

\[ \mathcal{H}_a^{(b)} = [B_a^{\mu*} \gamma_\mu - B_a \gamma_5] \frac{(1 - y')}{2}, \]

where the subscript $a = 1, 2, 3$ (or u,d,s) is an SU(3)$_V$ flavor index.

The amplitudes for semileptonic $B_c$ decay to lower mass states are determined by the matrix elements of the corresponding weak hadronic current between the meson states. For example, the most general form for the matrix element of $B_c$ decay to $B_a$ and $B_a^*$ is

\[ \langle B_a, v, q|\bar{q}a\Gamma_c|B_c, v \rangle = -\sqrt{m_{B_c}m_{B_a}} \text{tr} (\mathcal{H}_a^{(b)} \Omega(v, q) \Gamma \mathcal{H}^{(cb)}), \]

where

\[ \Omega(v, q) = \Omega_1 + \Omega_2 \tilde{q} \]

is the most general Dirac matrix that can be written in terms of the vectors $q$ and $v$. Explicit evaluation of Eq. \[4\] gives

\[ \langle B_a, v, q|V_{\mu}|B_c, v \rangle = 2\sqrt{m_{B_c}m_{B_a}} [\Omega_1 v_\mu + \Omega_2 q_\mu], \]

\[ \langle B_a^*, v, q|V_{\mu}|B_c, v \rangle = -2i\sqrt{m_{B_c}m_{B_a}} \epsilon_{\mu\nu\alpha\beta} \epsilon^{*\nu} q^\alpha v^\beta, \]

\[ \langle B_a, v, q|A_{\mu}|B_c, v \rangle = 2\sqrt{m_{B_c}m_{B_a}} [\Omega_1 v_\mu + \Omega_2 \epsilon^* \cdot q v_\mu], \]

where $V_\mu$ and $A_\mu$ refer to the vector and the axial vector currents, respectively, and $\epsilon_\mu$ is the polarization vector of $B_a^*$. Here, six form factors are expressed in terms of 2 independent form factors.

One very different point with respect to single heavy-quark systems is that the form factors are not normalized at the zero-recoil point. The value depends on how exactly one calculates the bound-state wave function. Even in order to obtain an estimate of the corresponding decay width, we need to extrapolate to the larger recoil region, which requires
a reasonable model. The form factor $\Omega_2$ is irrelevant for $\mu$- or $e$-lepton semileptonic decay because the contribution of $\Omega_2$ to the decay amplitude will be proportional to the lepton mass. In addition, $\Omega_2$ does not contribute to decay amplitudes at zero recoil, $q^2 = 0$.

In the case of $\bar{B}_c \to J/\psi$ decay, there is an additional spin symmetry of the produced antiquark ($\bar{c}$), which forbids a form factor proportion to $q$. The spin symmetry requires only one single form factor at the zero-recoil point for $\bar{B}_c \to J/\psi \mu \bar{\nu}_\mu$. The matrix element for the decay of $\bar{B}_c$ to $J/\psi$ is expressed by a single Isgur-Wise-like function $\Delta(v \cdot v')$, which will be denoted as $\Delta$ hereafter:

$$\langle J/\psi, v'|\bar{c}\Gamma|B_c, v\rangle = -\sqrt{m_{B_c}m_{J/\psi}} \Delta(v \cdot v') \text{tr} (H^{(bc)}_\Gamma H^{(\bar{c}\bar{c})})$$

where $\epsilon_\mu$ is the polarization vector of the $J/\psi$. From the above formula, one finds

$$\langle J/\psi, v'|V_\mu|B_c, v\rangle = 0,$$  \hspace{1cm} (9)

$$\langle J/\psi, v'|A_\mu|B_c, v\rangle = 2\sqrt{m_{B_c}m_\eta} \Delta(v \cdot v') \epsilon_\mu^*,$$  \hspace{1cm} (10)

where $V_\mu$ and $A_\mu$ refer to the vector and the axial vector currents respectively.

In the limit of vanishing lepton mass, we find the differential decay width:

$$\frac{d\Gamma}{d\omega} = \frac{G_F^2}{48\pi^3} |V_{cb}|^2 m_{B_c}^2 m_{J/\psi}^3 (\omega^2 - 1)^{1/2} F(r, \omega)$$  \hspace{1cm} (11)

with

$$F(r, \omega) = 8(1 - 2\omega r + r^2) \Delta^2, \hspace{1cm} B_c \to J/\psi_T \mu \bar{\nu}_\mu,$$  \hspace{1cm} (12)

$$F(r, \omega) = 4(\omega - r)^2 \Delta^2, \hspace{1cm} B_c \to J/\psi_L \mu \bar{\nu}_\mu.$$  \hspace{1cm} (13)

In the above equation,

$$w = v \cdot v' = \frac{m_{B_c}^2 + m_{J/\psi}^2 - q^2}{m_{B_c} + m_{J/\psi}}$$  \hspace{1cm} (14)

and $r = m_{J/\psi}/m_{B_c}$, where $q^2$ is the momentum transfer to the lepton pair.

The heavy-quark symmetry itself does not predict the exact structure of the form factor $\Delta(\omega)$, which includes all the non-perturbative dynamics of the system. Ref. 1 showed that by using the operator product expansion, $\Delta(q)$ could be expressed by wave function overlaps of the initial and the final mesons

$$\Delta(q) = \int d^3x \ e^{-i\vec{q} \cdot \vec{x}} \Psi_{J/\psi}(x) \Psi_{B_c}(x).$$  \hspace{1cm} (15)

We adopt the non-relativistic wave function of the ISGW model [9] to estimate the polarization of the $J/\psi$. $\Delta(\omega)$ is expressed as,
\[ \Delta(\omega) = \left( \frac{2\beta_{B_c}\beta_{J/\psi}}{\beta_{B_c}^2 + \beta_{J/\psi}^2} \right)^{3/2} \exp \left( -\frac{m_{sp}^2}{\kappa^2(\beta_{B_c}^2 + \beta_{J/\psi}^2)}(\omega - 1) \right), \]  

where \( \beta_{B_c} \) and \( \beta_{J/\psi} \) are parameters of the model, \( m_{sp} \) is the mass of the spectator quark, and \( \kappa \) is introduced to account for the relativistic recoil effect. Since \( \beta_{B_c} \) is not the same as \( \beta_{J/\psi} \), the form factor \( \Delta(\omega) \) is not normalized to unity at the zero-recoil point, which reflects breaking of the heavy flavor symmetry. The resulting decay width is shown in Table 1 for \( \kappa = 0.7 \) (the value used in the Ref. 9).

### III. POLARIZATION OF J/ψ MESON

In \( B_c \to J/\psi l\bar{\nu} \) decay, followed by \( J/\psi \) decaying into two lepton pairs, the decay amplitude with helicity \( \lambda \) for \( J/\psi \) is \[ M \propto \frac{G_F}{\sqrt{2}} V_{cb} \sum_\lambda L_\lambda H_\lambda d_{\lambda l+}^{\lambda l-} \]  

where \( \lambda_{l+} - \lambda_{l-} = \pm 1 \). Here, \( L_\lambda \) describe the \( W^{*-} \to l\bar{\nu} \) decay, \( H_\lambda \) are the three \( \bar{B} \to D^*_\lambda W^{*-}_\lambda \) decay amplitudes, and the Wigner \( d \)-function describes the vector meson decay into two fermion pairs, \( J/\psi \to l^+ l^- \).

The angle distribution of \( l^- \) in the rest frame of \( J/\psi \) is given by \[ \frac{d\Gamma}{d\cos \theta_{l-}} \propto 1 + \alpha' \cos^2 \theta_{l-}, \]  

where we find the polarization parameter \( \alpha' \) is expressed \[ \alpha' = \frac{\Gamma_T - 2\Gamma_L}{\Gamma_T + 2\Gamma_L}. \]  

In the case of a vector meson decaying into two pseudoscalar mesons, like \( D^*_\lambda \to D\pi \) decay, the Wigner function should be replaced by \( J = 1 \) spherical harmonics \( Y^1_\lambda \). Then, the polarization parameter is of the familiar form \[ \alpha = 2\frac{\Gamma_T}{\Gamma_L} - 1. \]

We note that the parameter \( \alpha' \) is expressed as the ratio of the longitudinal and the transverse decay widths, as shown in the Eq. \( \text{[13]} \). As we saw in the previous section, \( \Delta(1) \) is not normalized to 1 due to the breaking of the heavy-flavor symmetry, and a model is required to predict the value. However, \( \Delta(1) \) is canceled in Eq. \( \text{[19]} \); therefore, it is irrelevant to \( \alpha' \). As a result, the right-hand side of Eq. \( \text{[19]} \) is only a function of the slope parameter. One cannot predict the slope of the form factor at the non-zero recoil point,
which is essentially non-perturbative, without referring to the quark-model, even in a heavy-
light meson system like $B$. The experimental measurement of $\alpha'$ can be used to constrain
the value of the slope parameter in the semileptonic decay of $B_c$ meson, but we will have to
wait for this until sufficient data are gathered in the future.

Here, we evaluate the value of $\alpha'$ using Eq. (16). Integrating Eq. (11) over the possible
kinetic-energy range, the following result is obtained (the parameters we used are shown in
the Table 1):

\[
\begin{align*}
\Gamma_L &= 9.6 \times 10^{-15} \text{ GeV}, \\
\Gamma_T &= 7.0 \times 10^{-15} \text{ GeV}, \\
\frac{\Gamma_L}{\Gamma_T} &= 1.37, \\
\alpha &= -0.47.
\end{align*}
\]

Fig. 1 shows the plot of the transverse and the longitudinal decay widths with respect to
$\omega$ for $\kappa = 0.7$. $\Gamma_L$ dominates at low $q^2$ (high $\omega$): at $q^2 = 0$, $J/\psi$ has its maximum possible
momentum, and the helicities are aligned to give $S_z = 0$. $\Gamma_T$ dominates near $q^2 = q^2_{\text{max}}$ (low
$\omega$): at small $J/\psi$ velocity the probabilities are uncorrelated with the spin of $J/\psi$; hence,
$H_+ = H_- = H_L$. This gives $\Gamma_T/\Gamma_L = 2$.

The authors of Ref. 9 obtained the value of the correction factor $\kappa$ from the pion form
factor by comparison with experiment. In our case, however, there are no available data to
determine the correct value of $\kappa$. Therefore, we varied $\kappa$ from 0.6 to its maximum of 1 and
Fig. 2 shows $\alpha'$ as a function of the relativistic correction factor $\kappa$. Within the assumption
of the validity of Eqs. (9) and (10) measuring $\alpha'$ determines $\kappa$.

IV. DISCUSSION AND CONCLUSIONS

Polarizations of vector meson have been studied by several authors [3,15–18]. We com-
pare the above predictions with those of other quark models which have been generally
regarded as giving a successful description of the semileptonic decays of heavy mesons like
the $B$ meson. The result is given in the Table 1. We find that $\Gamma_L/\Gamma_T$ is rather large in the
heavy-quark spin symmetry formalism when the ISGW model is adopted.

We consider some uncertainties of our method. First, strictly speaking, the expressions
of Eq. (14) are valid near the zero-recoil point. Additional form factors might contribute at
the large recoil point. The exact expression of the matrix elements at large recoil within
the heavy-spin symmetry formalism is beyond the scope of this paper. However, since the
recoil momentum of $J/\psi$ is small ($\omega - 1 \simeq 0.26$) due to its heavy mass, we expect that the
assumption of Eq. (14) being applicable to other kinematic points is not too wrong. Second,
in order to obtain an estimate of the decay width, we need to extrapolate the form factor to
the large-recoil region, which requires a reasonable model. Since we used the ISGW model wave function in the calculation of the form factor, model dependences cannot be avoided. The value of the polarization is also sensitive to the parameter $\kappa$. Unfortunately, one cannot say which value of $\kappa$ should be used for the form factor; experiments should provide that information. We checked that $\Gamma_L/\Gamma_T$ is insensitive to the parameter set, such as quark mass. We note that even in $B \to D^{*} l \nu$, there are large discrepancies in the $\Gamma_L$ and $\Gamma_T$ between the heavy-quark effective theory (HQET) and other models, although they all are consistent within the experimental error.

Finally, we remark that the measurement of the polarization of the $J/\psi$ meson in the $\bar{B}_c \to J/\psi \mu \bar{\nu}_\mu$ decay may provide an alternate way to extract $|V_{cb}|$. Knowing both the form factor behavior on the right-hand side of Eq. (11) and the measured value of the total decay width on the left-hand side allows us to determine the value of $|V_{cb}|$. As we saw in Eq. (15), $\Delta(1)$ is a product of two meson wave functions at rest. Although the evaluation of the wave function of the meson at rest depends on the model, the uncertainty is expected to be small compared to that of a non-zero recoil meson. Even the value of $\Delta(1)$ can be obtained from lattice QCD with small uncertainty. In the case of $B \to D^{*} l \nu$ decay, $\Delta(1)$ is normalized to one as a consequence of HQET. Hence, the measurement of the slope parameter through the polarization of $D^*$ (Eq. (20)) and the measurement of the decay width determines the $|V_{cb}|$ model independently. Further discussions will be published elsewhere [7].

ACKNOWLEDGMENTS

We would like to thank K. Y. Lee for helpful discussions and for reading the manuscript. This work was supported by the Korea Research Foundation and by the Korea Science and Engineering Foundation.

APPENDIX A: ISGW MODEL

The meson state vector is expressed by the following nonrelativistic expression:

$$|X(p_X; s_x)\rangle = \sqrt{2m_x} \int d^3 p \sum C_{m_L m_S} s_x \phi_X(p) L m_L \chi_{s \bar{s}}^{m_S}$$

$$|q(\frac{m_q}{m_x} p_X + p, s) \bar{q}(\frac{m_s}{m_x} p_X - p, \bar{s})\rangle,$$

where $\chi_{s \bar{s}}^{m_S}$ is the spin wave function of the quark-antiquark pair in the state with total spin $S$ and spin projection $m_S$; $C_{m_L m_S} s_x$ is the coupling between the orbital momentum $L$ and the total spin $S$ of a system with the total momentum $s_x$; $\phi_X(p) L m_L$ is the corresponding nonrelativistic wave function; $p_X$ is the meson momentum; and $p$ is the relative momentum of quarks. In the model, the meson mass is equal to the sum of quark masses. As the probe wave functions, the nonrelativistic oscillator wave functions are used.
In the ISGW model, the hadronic matrix elements are defined as

\[ <m(k) \mid V_\mu(0) \mid M(P) > = f_+(q^2)(P+k)_\mu + f_-(q^2)(P-k)_\mu, \]  
\[ <m(k, e^*) \mid V_\mu(0) \mid M(P) > = ig(q^2)\epsilon_{\mu\nu\rho\sigma}e^*(P+k)^\rho(P-k)^\sigma, \]  
\[ <m(k, e^*) \mid A_\mu(0) \mid M(P) > = f(q^2)e^* + a_+(q^2)(e^* \cdot P)(P+k)_\mu 
+ a_-(q^2)(e^* \cdot P)(P-k)_\mu, \]  

Expressions of the form factors were also obtained (see Refs. 9 and 15). We define

\[ \bar{H}_\pm = [f(q^2) \mp 2MKg(q^2)], \]  
\[ \bar{H}_0 = \frac{M}{2m\sqrt{y}}[(1 - \frac{m^2}{M^2} - y)f(q^2) + 4K^2a_+(q^2)]. \]

For the transition into a pseudoscalar meson final state, one obtains

\[ \bar{H}_\pm = 0, \ \bar{H}_0 = -2\frac{K}{\sqrt{y}}f_+(q^2). \]  

In this case,

\[ \frac{d\Gamma}{d\omega} = \frac{G_F^2 \mid V_{Qq} \mid^2 K^3M^2}{24\pi^3} \mid f_+(q^2) \mid^2 . \]

For the vector-meson final states, in the \( l\nu \) frame

\[ H = 2i\sqrt{\omega}Mg(q^2)e^* \times k - f(q^2)e^* - 2(e^* \cdot P)a_+(q^2)k, \]  
\[ a_- \] does not contribute. Then

\[ \frac{d\Gamma}{d\omega} = \frac{G_F^2 \mid V_{Qq} \mid^2 KM^2\omega}{96\pi^3}[\mid \bar{H}_+ \mid^2 + \mid \bar{H}_- \mid^2 + \mid \bar{H}_0 \mid^2]. \]

**APPENDIX B: BSW MODEL**

The meson is considered as a relativistic bound state of a quark \( q_1 \) and an antiquark \( \bar{q}_2 \) in a system of infinitely large momentum:

\[ |P, m, j, j_z > = \sqrt{2}(2\pi)^{3/2} \sum_{s_1, s_2} \int d^3p_1 \ d^3p_2 \delta^3(p - p_1 - p_2) \]
\[ L^{j, j_z}_{m}(p_{1\perp}, x, s_1, s_2) a_1^{s_1\dagger}(p_1) b_2^{s_2\dagger}(p_2) |0 >, \]

where \( P_\mu = (P, 0, 0, P) \); as \( P \to \infty \), \( x = p_{1z}/p \) corresponds to the momentum fraction carried out by the nonspectator quark; and \( p_{1\perp} \) is the transverse momentum. For the orbital part of the wave functions, the solution of the relativistic oscillator is used
\[ L_m(p_t, x) = N_m \sqrt{x(1-x)} \exp(-\frac{p_t^2}{2\omega^2}) \exp[-\frac{m^2}{2\omega^2}(x - \frac{1}{2} - \frac{m^2 q_1 - m^2 q_2}{2m^2})^2]. \]  

The set of (dimensionless) form factors of the BSW model is related to that of the ISGW model:

\[ F_1(q^2) = f_+(q^2), \]
\[ V(q^2) = (M + m)g(q^2), \]
\[ A_1(q^2) = (M + m)^{-1}f(q^2), \]
\[ A_2(q^2) = -(M + m)a_+(q^2). \]

One finds

\[ H_\pm(q^2) = (M + m)A_1(q^2) \pm 2 \frac{MK}{M + m} V(q^2), \]  
\[ H_0(q^2) = \frac{1}{2m\sqrt{q^2}} \left[ (M^2 - m^2 - q^2)(M + m)A_1(q^2) - 4 \frac{M^2 K^2}{M + m} A_2(q^2) \right] \]  

with

\[ K = \frac{1}{2M} [(M^2 - m^2 - q^2)^2 - 4m^2 q^2]^{1/2}. \]
REFERENCES

[1] E. Jenkins, M. Luke, A. V. Manohar, and M. J. Savage, Nucl. Phys. B390, 463 (1993).
[2] M. Neubert, Phys. Lett. B264, 455 (1991).
[3] K. Hagiwara, A. Martin, and M. Wade, Nucl. Phys. B327, 596 (1989).
[4] ALEPH Collaboration, Phys. Lett. B359, 236 (1995); ARGUS Collaboration, Z. Phys. C57, 533 (1993), Phys. Lett. B324, 249 (1994); C. W. Kim, J. Korean Phys. Soc., Vol. 26, Supplementary Issue, p. S319.
[5] CLEO Collaboration, Phys. Rev. Lett. 76, 3898 (1996); ARGUS Collaboration, Phys. Lett. B219, 121 (1989).
[6] ALEPH Collaboration, Contributed paper to the Warsaw Conference, July 1996.
[7] M. T. Choi and J. K. Kim, Phy. Lett. B (will be published).
[8] D. Scora and N. Isgur, Phys. Rev. D 52, 2783 (1995).
[9] N. Isgur, D. Scora, B. Grinstein, and M. Wise, Phys. Rev. D39, 799 (1989).
[10] M. Bauer, B. Stech and M. Wirbel, Z. Phys. C29, 637 (1985).
[11] N. Isgur and M. B. Wise, Phys. Lett. B232, 113 (1989); B237, 527 (1990).
[12] A. F. Falk, H. Georgi, B. Grinstein, and M. B. Wise, Nucl. Phys. B343, 1 (1990); J. D. Bjorken, talk given at Les Rencontres de la Valle d'Aoste La Thuile, Aosta Valley, Italy, March 1990, SLAC preprint SLAC-PUB-5278 (1990).
[13] M. Galdon and M. A. Sanchis-Lozano, Z. Phys. C71, 227 (1996).
[14] J. Körner and G. Schuler, Z. Phys. C46, 93 (1990).
[15] D. Scora and N. Isgur, Phys. Rev. D40, 1491 (1991).
[16] M. Bauer and M. Wirbel, Z. Phys. C42, 671 (1989).
[17] F. Gilman and R. Singleton, Phys. Rev. D41, 142 (1990).
[18] J. Körner and G. Schuler, Z. Phys. C38, 511 (1988).
[19] M. Lusignoli and M. Masetti, Z. Phys. C51, 549 (1991).
FIGURES

1. Plot of the transverse and the longitudinal decay widths as functions of $\omega = v \cdot v'$ (in units of $10^{-15}$ GeV).

2. Plot of $\alpha'$ as a function of $\kappa$.
Table 1. The widths ($10^{-15}$ GeV) of $\bar{B}_c \rightarrow J/\psi \mu \bar{\nu}_\mu$, and $\Gamma_L/\Gamma_T$ of the $J/\psi$ meson: first row, heavy-quark spin symmetry [1]; second row, the ISGW model [9]; third row, values from Ref. 8; fourth row, the BSW model [16]. We set $V_{cb} = 0.04$, $m_{B_c} = 6.3 \text{ GeV}$, $m_b = 4.75 \text{ GeV}$, $m_c = 1.5 \text{ GeV}$, $\kappa = 0.7$. Also, $\beta_{B_c} = 0.82$, $\beta_{J/\psi} = 0.61$, $\omega_{B_c} = 0.8 \text{ GeV}$, $\omega_{J/\psi} = 0.6 \text{ GeV}$, as used in Ref. 19.

| Model  | $\Gamma$  | $\Gamma_L/\Gamma_T$ |
|--------|-----------|---------------------|
| Here   | 16.6      | 1.37                |
| ISGW   | 12.4      | 0.73                |
| ISGW2  | 15.7      | 0.87                |
| BSW    | 19.0      | 1.04                |
$\frac{d\Gamma}{d\omega} \left( 10^{-15} \text{ GeV}^{-1} \text{ sec}^{-1} \right)$

Transverse

Longitudinal
