The industrial buildings reinforced concrete floor slabs with rational choice of the column pitch

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Abstract. The paper discusses the problem of determining the rational dimensions of the spans of the industrial building floor slab with a rectangular grid of columns in order to minimize the deflection of the slab, the potential strain energy and the consumption of reinforcement. The problem is solved using nonlinear optimization methods in combination with the finite element method. The optimal ratio between the marginal and middle spans of the slab is established. An analogy between the task of optimizing a slab and a multi-span continuous beam is revealed.

1. Introduction

The optimal design of reinforced concrete structures, including reinforced concrete slabs of buildings for various purposes, has been the subject of a large number of works by domestic and foreign scientists, including [1–14]. The inverse method is widely used in solving optimization problems, which consists in finding such a law of changing the characteristics of a material within a structure in which its stress-strain state is given [3-6]. Effective optimization methods also include the genetic algorithm [7-8] and the random search method [9], combining the deterministic and stochastic component.

As a method of optimizing reinforced concrete slabs, the most common is the selection of rational reinforcement [7-14]. In some publications, for example [15], models of equal strength plates of variable thickness are constructed, however, the practical implementation of such a model is associated with great difficulties, as well as the creation of artificial heterogeneity of the structure. The problems of determining the rational arrangement of the supports are mainly solved for beams [1,2]; therefore, the problem posed in this paper to determine the rational pitch of the columns under the floor slab has a scientific novelty.

2. Methods

We will consider the optimization methodology on example of a square in plane industrial building floor slab (Fig. 1). The size of the building is \( a = 24 \) m, the number of spans along the \( x \) and \( y \) axis is 4. The load is uniformly distributed over the area. The pitch of the columns of the marginal and middle rows are denoted by \( a_1 \) and \( a_2 \), respectively. The optimization problem is posed as follows: find \( a_1 \) such that the objective function takes a minimum value.

As the objective function, three quantities will be taken:
1. The maximum deflection of the plate \( w_{\text{max}} \), mm;
2. Potential strain energy (PSE) \( W \), kJ;
3. Consumption of reinforcement \( m_s \), t.
We take $a_1 = 6$ m as the initial value. The possible range of $a_1$ is assumed from 3 m to 8 m. The optimization problem is solved in the MATLAB software using the Optimization Toolbox package (fmincon function). As an optimization method, the internal point method is chosen.

**Figure 1.** Optimized floor slab

An objective function was created whose input parameters are $a_1$, the total size $a$ of the building, the thickness of the slab, the value of the uniformly distributed load, and the characteristics of concrete and reinforcement. During optimization, all values except $a_1$ are considered given. The objective function prepared by the authors, based on the input parameters, generates a finite element model, performs a calculation, and provides the value of the maximum deflection of the plate, or the potential strain energy or the reinforcement consumption. By virtue of symmetry, a quarter of the structure is considered. The selection of reinforcement is carried out in accordance with Russian design standards SR 63.13330.2018 from the conditions:

$$
\left( M_{x, \text{ult}} - M_x \right) \left( M_{y, \text{ult}} - M_y \right) - M_{xy}^2 \geq 0;
$$

$$
M_x \leq M_{x, \text{ult}};
$$

$$
M_y \leq M_{y, \text{ult}};
$$

$$
M_{xy} \leq M_{xy, \text{ult}},
$$

where $M_x, M_y$ – bending moments acting on a flat selected element, $M_{xy}$ – torque; $M_{x, \text{ult}}, M_{y, \text{ult}}, M_{xy, \text{ult}}$ – ultimate bending and torque moments perceived by a flat selected element.

The potential strain energy of the slab can be calculated by the formula:

$$
W = \frac{1}{2} \{U\}^T [K] \{U\},
$$

where $\{U\}$ is the vector of nodal displacements, $[K]$ is the stiffness matrix of the structure.

### 3. Results and Discussion

As the initial data we took concrete elastic modulus $E_b = 3 \cdot 10^4$ MPa, concrete design compressive strength $R_{cb} = 14.5$ MPa, reinforcement design compressive and tensile strength $R_{sc} = R_s = 350$ MPa, load $q = 50$ kPa, thickness $h = 20$ cm, distances from the reinforcing bars center of gravity to the lower (upper) surface of the slab: $a_x = a_x' = a_y = a_y' = 3$ cm.

Fig. 2 shows the isofield of deflections for a quarter of the slab at $a_1 = 6$ m. From this graph it can be seen that with a constant pitch of the columns, maximum displacements occur in marginal spans. The same goes for internal forces. The values of $\{w_{\text{max}}, W, m_s\}$ with a uniform column pitch were $\{43.6$ mm, 72.7 kJ, 4.98 t\}. The values of $W$ and $m_s$ are calculated hereafter for a quarter of the slab.
Figure 2. Deflections isofield (m) for a quarter of a slab 24x24 m with $a_1 = 6$ m.

Figure 3 shows the change in the values of the objective functions depending on the parameter $a_1$ with respect to the values calculated at a regular column pitch ($w_0 = 43.6$ mm, $W_0 = 72.7$ kJ, $m_0 = 4.98$ t).

As a result of optimization, the value $a_1 = 5.39$ m was obtained from the condition of minimum deflection, $a_1 = 5.42$ m from the condition of minimum PSE and $a_1 = 5.35$ m from the condition of minimum reinforcement consumption. The ratio $a_1 / a_2$ for each of the variants was 0.815, 0.824, and 0.805, respectively. When optimizing for deflection, alignment of displacements in the marginal and middle spans occurs, as can be seen from Fig. 4. The maximum deflection when choosing $w_{max}$ as the optimized parameter was 27 mm, which differs from the initial value with a uniform column pitch by 38%.
We also considered a square slab 36x36 m (Fig. 5). The total number of spans was 6. The steps $a_1$ and $a_2$ were used as variable parameters. The values of $\{w_0, W_0, m_0\}$ with $a_1 = a_2 = a_3 = 6$ m were $\{43.8 \text{ mm}, 142.13 \text{ kJ}, 10.14 \text{ t}\}$. Surfaces showing the change in objective functions depending on parameters $a_1$ and $a_2$ are shown in Fig. 6 – Fig.8.

The values $a_1$, $a_2$ and $a_3 = a/2 - a_1 - a_2$ obtained as a result of optimization by three criteria, as well as the corresponding values of the objective functions with respect to the values calculated at a regular column pitch are presented in Table 1. From this table it can be seen that the results determined based on three criteria are quite close. Also note that the steps $a_2$ and $a_3$ were almost equal to each other. As for the 24x24 plate, during optimization, the maximum displacements and internal forces are aligned in the marginal and middle spans.
Figure 7. The dependence of the potential strain energy on the parameters $a_1$ and $a_2$

Figure 8. The dependence of the reinforcement consumption on the parameters $a_1$ and $a_2$

Table 1. Optimization results for the slab 36x36 m

| From a minimum of a deflection | $a_1$, m | $a_2$, m | $a_3$, m | $w_{\text{max}}/w_0$ | $W/W_0$ | $m_0/m_0$ |
|--------------------------------|----------|----------|----------|----------------------|---------|-----------|
| 5.                             | 6.       | 6.       | 0.5      | 0.                   | 0.      | 0.        |
| 2                              | 39       | 41       | 39       | 899                  | 937     |
| From a minimum of PSE          | 5.       | 6.       | 6.       | 0.5                  | 0.      | 0.        |
| 24                             | 38       | 38       | 43       | 899                  | 938     |
| From a minimum of reinforcement consumption | 5.       | 6.       | 6.       | 0.5                  | 0.      | 0.        |
| 15                             | 42       | 43       | 55       | 900                  | 937     |

The problem was also solved for a square slab 48x48 m with variable parameters $a_1$, $a_2$, $a_3$ and a rectangular slab 48x36 m with variable parameters $a_1$, $a_2$, $a_3$, $b_1$, $b_2$ (Fig. 9). In the first case, with regular column pitch, the values of $\{w_0, W_0, m_0\}$ were $\{43.8$ mm, $233.36$ kJ, $17.01$ t$\}$, and $\{43.8$ mm, $182.27$ kJ, $13.12$ t$\}$ in the second. The optimization results for slabs 48x48 m and 48x36 m are presented in the Tables 2 and 3 respectively.

Figure 9. A quarter of the slab 48x48 and 36x36 m

From tables 2 and 3 it can be seen that the optimal values of $a_1$, $a_2$, $a_3$ for the slabs 48x48 m and 48x36 m practically coincide. Also, optimal values $b_1$, $b_2$, $b_3$ for the slab 48x36 m slightly differ from values $a_1$, $a_2$, $a_3$ for the slab 36x36 m. This suggests that the optimal column pitch in $x$ does not depend on the optimal pitch in $y$ and is determined only by the number of spans.
Also, these tables show that with the optimal arrangement of columns, the pitches of the middle rows should be equal to each other. In addition, three optimization criteria chosen by us give very close results.

### Table 2. Optimization results for the slab 48x48 m

| $a_1$, m | $a_2$, m | $a_3$, m | $a_4$, m | $w_{\text{max}}/w_0$ | $W/W_0$ | $m_x/m_0$ |
|---|---|---|---|---|---|---|
| From a minimum of a deflection | 5.13 | 6.27 | 6.32 | 6.39 | 6.39 | 0.521 | 0.905 | 0.944 |
| From a minimum of PSE | 5.16 | 6.28 | 6.28 | 6.28 | 6.28 | 0.53 | 0.905 | 0.945 |
| From a minimum of reinforcement consumption | 5.06 | 6.3 | 6.32 | 6.32 | 6.32 | 0.514 | 0.906 | 0.944 |

### Table 3. Optimization results for the slab 48x36 m

| $a_1$, m | $a_2$, m | $a_3$, m | $a_4$, m | $b_1$, m | $b_2$, m | $b_3$, m | $w_{\text{max}}/w_0$ | $W/W_0$ | $m_x/m_0$ |
|---|---|---|---|---|---|---|---|---|---|
| From a minimum of a deflection | 5.11 | 6.27 | 6.3 | 6.32 | 5.22 | 6.39 | 6.39 | 0.521 | 0.905 | 0.944 |
| From a minimum of PSE | 5.16 | 6.28 | 6.28 | 6.28 | 5.24 | 6.38 | 6.38 | 0.53 | 0.905 | 0.945 |
| From a minimum of reinforcement consumption | 5.06 | 6.3 | 6.32 | 6.32 | 5.14 | 6.42 | 6.44 | 0.537 | 0.906 | 0.944 |

Note that there is an analogy between the task of optimizing a slab and a multi-span continuous beam. Let us consider the following problem. There is a four-span continuous beam of constant stiffness $EI$ loaded with a uniformly distributed load $q$ (Fig. 10). For $a = l$, the maximum bending moments arise on the supports 1 and 3. Let us find a ratio $a/l$ of the marginal spans to the middle spans at which the bending moments on all the supports are equal to each other.

![Figure 10. Continuous four-span beam](image)

For solution, we use the equation of three moments:

$$l_i M_{i-1} + 2(l_i + l_{i+1}) M_i + l_{i+1} M_{i+1} = -6 \Delta_{ip} EI.$$  

(3)

Having compiled equation (3) for $i = 1, 2$, taking into account the fact that $M_1 = M_3$, we get:

$$2(a+l)M_1 + lM_2 = -6 \Delta_{1p} EI;$$

$$2IM_1 + 4IM_2 = -6 \Delta_{2p} EI.$$  

(4)

Coefficients $\Delta_{ip}$ are determined using the Mohr integral and have the form:

$$\Delta_{1p} = \frac{q}{24EI} \left( a^3 + l^3 \right); \Delta_{2p} = \frac{qI^3}{12EI}. $$  

(5)

Substituting (5) into (4) and assuming $M_1 = M_2$, we obtain:
This $a/l$ ratio also provides a minimum of potential strain energy. Approximately the same ratio of the marginal span size to the middle span size was obtained when optimizing the slab 24x24 m. It can be shown that for $\frac{a}{l} = \frac{\sqrt{3}}{3}$ the bending moments on supports will be equal to each other for any number of spans.

As a simplification, the value $\frac{a}{l} = \frac{\sqrt{3}}{3}$ can be used in the design of slabs.

4. Conclusions
Based on the results, the following conclusions were made:

1. The optimal columns pitch along the $x$ axis does not depend on the pitch along the $y$ axis. It is determined only by the number of spans.
2. Only the distance $a_1$ between the marginal columns can be taken as a variable parameter. The spans of the middle rows at an optimal location are equal to each other.
3. The optimal arrangement of columns using three optimization criteria (deflection, potential strain energy, reinforcement consumption) differs slightly.
4. There is an analogy between the task of optimizing a slab and a multi-span continuous beam.

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