Radio-Wave Based Accurate Localization for Space Rover on Small Planetary Body without Motion Information of Mother Spacecraft

Sayaka Kanata *, Hiroaki Nakanishi **, and Tetsuo Sawaragi **

Abstract: Investigations of small planetary bodies (SPBs) such as asteroids and comets have attracted increasing attention recently. In our previous research, a method to localize a space rover was proposed. This method can provide accurate localization even if the rover is located on a small planetary body (SPB). It uses a mother spacecraft as a source of radio-waves. The localization was formulated as an optimization problem to minimize a loss function defined according to estimation error, and gradient-based optimization was applied; however, this required the equation of motion of the mother spacecraft. The purpose of this paper is to introduce a method to localize a rover on an SPB that does not require motion information for the mother spacecraft. In this paper, we propose a solution based on Powell’s conjugate direction method. The proposed method requires a large amount of computation. For computation reduction, a method to select the measurement data is proposed. Numerical simulations that assumed that a rover was located on an SPB were conducted along with indoor experiments to assess the proposed method of localization.

Key Words: optimization, localization, rovers on a small planetary body, Powell’s conjugate direction method.

1. Introduction

Recent exploration of small planetary bodies (SPBs) has included not only acquisition of surface images but also collection of rock samples to be returned to Earth for more detailed analysis. The Japanese Hayabusa probe began surveying the asteroid Itokawa in 2005 and succeeded in landing on the asteroid before returning to Earth with a sampling chamber in 2010 [1]. The observations of the Hayabusa probe have shown that even an asteroid of approximately 500 m in diameter can possess a variety of terrain [2]. From the survey of ESA’s Rosetta probe, it was found that water ice is exposed on the comet Churyumov-Gerasimenko [3]. In 2014, the probe Hayabusa-2 was launched toward the asteroid Ryugu. Three rovers are planned to land on the surface of the asteroid and then return samples to Earth [4].

To study the surface of such SPBs in detail and over a broad area, direct investigation using a rover offers one of the most promising approaches. To guide a rover to various destination points spread over a broad area, the method to localize the rover must provide absolute positions in the planetary-body-centered frame, and cover the entire surface of the SPB. In addition, the size and mass of all additional sensors on the rover must be minimized because there are strict limitations on on-board sensors owing to the launch capabilities of the rocket carrying the rover.

Various localization methods based on matching global and local information have been examined [5]–[9]. Celestial navigation [10], [11] is also a popular method, and simultaneous localization and mapping [12] has been recognized as an effective approach for self-localization. However, for rovers on SPB with diameters of less than 1,000 m, it is difficult to apply these conventional methods of localization because they have low accuracy or require excessive resources to be mounted on the rover. Edmond et al. proposed a method for determining the position and attitude of a hopping rover using optical flows of terrain obtained during hopping motions [13]. This method is attractive because it provides the position and attitude of the rover in real time while using only a single camera, but the estimated position and attitude both depend on a local frame, i.e., the planetary-body-fixed surface frame. It is therefore necessary to transform these estimations into a planetary-body-centered inertial frame.

We have proposed a method for the localization of a rover using radio waves [14]. This method provides the position of the rover based on information about the position of a mother spacecraft and the rotational motion of the planetary body. Similar to the Global Positioning System (GPS), measurement in this approach is based on the propagation delay, but our method uses the round-trip propagation delay between the rover and the radio source so that no clock synchronization is required. Only a transponder is required on the rover for this method to work. The method can 1) provide the absolute position of the rover in a planet-centered inertial frame, 2) cover the entire surface of the target planetary body, and 3) be applied to a planetary body of any size, while conventional methods cannot be applied to SPBs with diameters of less than 1000 m.

In our previous paper [15], a method to estimate the position of the rover together with the rotational parameters of the SPB was proposed. This estimation method was formulated as an optimization problem minimizing a loss function defined according to an estimation error obtained through comparison with actual measurements. The gradient-based method was applied to solve the optimization problem. This method requires the equation of motion of the mother spacecraft to calculate gradient of the loss function. This means that the gradient-based solution requires an accurate and differentiable system model,
where numerical or empirical formulas for the motion of the spacecraft are not applicable.

In this paper, we propose a solution using Powell’s conjugate direction method, which does not require the motion information of the mother spacecraft. The proposed solution requires substantial computation, so that efficient computation reduction is necessary. For computation reduction, a method of selecting the measurement data is proposed. Numerical simulations that assumed that a rover was located on an SPB were conducted along with indoor experiments to evaluate the proposed method of localization.

Mathematical definitions, formulation of the problem as an optimization problem, and the selection method for measurement data used to conserve the sensitivity direction of the data are described in Section 2. Results from numerical simulations that assumed the rover was located on an SPB are reported in Section 3. Experimental results are discussed in Section 4, and we offer our conclusions in Section 5.

2. Accurate Localization of Rovers by Round-Trip Propagation Delay

2.1 Coordinate Definitions and Assumptions

The inertial-fixed coordinate frame used in our approach is shown in Fig. 1. The origin coincides with the center of mass of the SPB. The z-axis is parallel to the nominal direction of the rotational axis of the SPB while the x-axis is perpendicular to the z-axis and is parallel to the longitude of the ascending node of the spacecraft’s orbit. The y-axis is perpendicular to these axes and is defined by the right-hand rule. $x = [x, y, z]^T$ and $X = [X, Y, Z]^T$ denote the positions of the rover and the mother spacecraft, respectively. $\sigma = [\sigma_x, \sigma_y, \sigma_z]^T$ is a unit vector and denotes the rotational axis direction. $\omega$ denotes the angular velocity of the SPB.

Four assumptions are made in this formulation.

- Assumption 1: The rover is stationary with respect to the SPB during the localization process. The motion of the rover depends only on the rotational motion of the SPB.
- Assumption 2: The direction of the rotational axis, $\sigma$, and the angular velocity, $\omega$, are both time-invariant.
- Assumption 3: The average and variance of the processing delay of the transponder on the rover are known.
- Assumption 4: The position of the mother spacecraft relative to the SPB is known.

The motion caused by the rover’s locomotion and that caused by the rotation of the SPB cannot be decomposed. Therefore, Assumption 1 is necessary for estimation of the rotational parameters of the SPB. Even if an SPB includes precessional motion, Assumption 2 is satisfied when the period of precession is sufficiently long compared with the operational period of the rover. The paper [16] describes the estimation method in the case where precessional motion cannot be ignored. The average and variance values of the processing delay of the transponder can be measured before the launch of the rocket that carries the mother spacecraft and the rover. The relative positions of the Hayabusa probe have been estimated by using a shape model of the asteroid Itokawa combined with images from the optical navigation camera and LIDAR ranging data [17]. It is feasible to measure the position of the mother spacecraft in the planet-centered inertial frame.

2.2 Formulation as an Optimization Problem [15]

To localize the rover accurately, the rotational motion of the SPB must be estimated. We define the state vector $s$ as

$$ s = [x, y, z, \sigma_x, \sigma_y, \sigma_z, \omega]^T. \quad (1) $$

While $\sigma$ is a three-dimensional vector, it does have the constraint $||\sigma|| = 1$. Therefore, two of the components in $\sigma$ are free. In this paper, $\sigma_x$ and $\sigma_y$ are chosen as state variables and $\sigma_z$ is defined as

$$ \sigma_z = \sqrt{1 - \sigma_x^2 - \sigma_y^2}. \quad (2) $$

Note that $\sigma_z$ can take positive or negative values. A positive value is taken here because the nominal direction of the rotational axis $\sigma$ is $[0, 0, 1]^T$.

Based on Assumptions 1 and 2, the state prediction can be described as

$$ s_{t+1} = \begin{bmatrix} x_{i+1} \\ \sigma_{x,i+1} \\ \sigma_{y,i+1} \\ \omega_{i+1} \end{bmatrix} = \begin{bmatrix} R(\sigma_i, \omega_i(t\epsilon t - t))x_i \\ \sigma_{x,i} \\ \sigma_{y,i} \\ \omega_{i} \end{bmatrix} = f(s_i), \quad (3) $$

where the suffix $i$ denotes the value at the $i$-th measurement. $R(\sigma, \beta)$ is a rotation matrix and represents the rotational motion of the SPB. $R(\sigma, \beta)$ is given as

$$ R(\sigma, \beta) = \cos \beta I - \sin \beta [\sigma x] + (1 - \cos \beta)\sigma \sigma^T, \quad (4) $$

where $\beta$ is the rotation angle, $I$ is an identity matrix. $[\sigma x]$ is a skew-symmetric matrix that represents the cross product of $\sigma$, which is defined as

$$ [\sigma x] = \begin{bmatrix} 0 & -\sigma_z & \sigma_y \\ \sigma_z & 0 & -\sigma_x \\ -\sigma_y & \sigma_x & 0 \end{bmatrix}. \quad (5) $$

The measurement of the round-trip propagation delay is expressed as

$$ \tau_{obs} = \frac{1}{c} \left( \left\| X(t_{ei}) - X(t_{ref,i}) \right\| + \left\| X(t_{ei}) - X(t_{ref,i} + \zeta_i) \right\| + \zeta_i + \eta_i \right) + g(s_i) + \zeta_i + \eta_i, \quad (6) $$

The measurement of the round-trip propagation delay is expressed as

$$ \tau_{obs} = \frac{1}{c} \left( \left\| X(t_{ei}) - X(t_{ref,i}) \right\| + \left\| X(t_{ei}) - X(t_{ref,i} + \zeta_i) \right\| + \zeta_i + \eta_i \right) + g(s_i) + \zeta_i + \eta_i, \quad (7) $$
where \( c \) is the speed of light, \( \zeta \) is the processing delay of the transponder, and \( \eta \) is the noise caused by except for the transponder with an average of zero and a known variance. \( t_{ref,i} \), \( t_{ref, j} \), \( t_{j,i} \), and \( t_{j,j} \) are the time at emission, time at reflection, and time at reception of the radio waves, respectively. \( t_{ref,j} \) and \( t_{j,j} \) are the solutions of

\[
\begin{align*}
 t_{ref,j} - t_{j,j} &= \frac{1}{c} \left| X(t_{ref,j}) - X(t_{j,j}) \right| \quad \text{and} \quad (8) \\
 t_{j,j} - t_{ref,j} &= \frac{1}{c} \left| X(t_{j,j}) - X(t_{ref,j}) \right|, \\
\end{align*}
\]

respectively.

Using the average of the processing delay, \( \bar{\zeta} \), which is a given value from Assumption 3, and using the following approximation,

\[
x(t_{ref,j} + \bar{\zeta}) \sim x(t_{ref,j} + \bar{\zeta}),
\]

(10) becomes

\[
\tau_{ob,i} - \bar{\zeta} = \frac{1}{c} \left( \left\| X(t_{ob,i}) - X(t_{ref,j}) \right\| + \left\| X(t_{ref,j}) - X(t_{ref, j} + \bar{\zeta}) \right\| \right) + (\bar{\zeta} - \xi) + \eta_i.
\]

(11)

The term \( (\bar{\zeta} - \xi) \) is a random component with an average 0 and known variance according to Assumption 3. The term \( (\bar{\zeta} - \xi) + \eta_i \) can be regarded as measurement noise of known variance with an average of \( \bar{\zeta} \). In this paper, \( \bar{\zeta} \) equals zero for simplicity.

Let us define the squared error function \( J_i \) between the actual measurement, \( \tau_{ob,i} \), and the estimated measurement, \( \hat{\tau}_i \), which is calculated from the estimated state, \( \delta \), as

\[
J_i = \sum_{i=1}^{N-1} \frac{1}{\gamma^2_i} (\tau_{ob,i} - \hat{\tau}_i)^2
\]

(12)

where \( \gamma^2_i \) denotes the variance of the measurement noise. \( \hat{\tau}_i \) denotes the estimated value and \( N \) is the number of measurements. From Eq. (7),

\[
\hat{\tau}_i = g(\delta_i).
\]

(13)

Because the state vectors \( \delta_i \) \( (i = 1, \ldots, N) \) are constrained by Eq. (3), the loss function, \( J_i \), is expressed as a function of the initial state, \( \delta_0 \). By solving the optimization problem

\[
\hat{\delta}_0^m = \arg \min_{\delta_0} J_i(\delta_0),
\]

subject to

\[
\delta_{i+1} = f(\delta_i) \quad (i=0,1,\ldots,N-1),
\]

(14)

the minimum mean square error estimation of the initial state \( \hat{\delta}_0^m \) can be derived.

### 2.3 Solution without Calculation of Differentials

In the previous paper [15], gradient-based methods were applied to solve the optimization problem (14) with the constraint (15). The method of Lagrange multipliers was applied to relax the constraint. The calculation of differentials of Eqs. (12) and (15) is required for the first derivatives of the relaxed loss function. This calculation includes a time derivative term for the position of the spacecraft, \( dX_{i,j}/dt \), i.e., it requires the motion equation of the mother spacecraft.

Powell proposed a method for finding the state \( p \) that brings out the minimum of a function \( F(p) \) while remaining independent of derivatives [18]; this method is called Powell’s conjugate direction method. The search steps are as follows:

**[Powell’s Conjugate Direction Method]**

(i) Determine linearly independent directions \( \{\xi_k\} \) \( (k = 1,2,\ldots,n) \) and set the best-known approximation to the minimum, \( p_0 \), as the starting point.

(ii) For \( k = 1,2,\ldots,n \), calculate \( \lambda_k \) such that \( F(p_{k-1} + \lambda_k \xi_k) \) is a minimum and define \( p_k = p_{k-1} + \lambda_k \xi_k \).

(iii) Find the integer \( m \) \((1 \leq m \leq n)\), such that \( F(p_{m-1}) - F(p_m) \) is a maximum and define \( \Delta = (F(p_{m-1}) - F(p_m)) \).

(iv) Calculate \( F_3 = F(2p_m - p_0) \), and define \( F_1 = F(p_0) \) and \( F_2 = F(p_1) \).

(v) If either \( F_3 \geq F_1 \) or \( 2(F_3 - 2F_2 + F_1) \cdot (F_3 - F_2 - \Delta)^2 \geq \Delta (F_1 - F_2)^2 \), then use the old directions \( \xi_1, \xi_2,\ldots,\xi_m \) for the next iteration and use \( p_m \) as the next \( p_0 \).

Otherwise define \( \xi_{m+1} = p_m - p_0 \). Calculate \( \lambda_{m+1} \) such that \( F(p_m + \lambda_{m+1} \xi_{m+1}) \) is a minimum. Use \( \xi_1, \xi_2,\ldots,\xi_m, \xi_{m+1}, \xi_{m+1}, \ldots,\xi_n, \xi_{n+1} \) as the new directions and use \( p_m + \lambda_{m+1} \xi_{m+1} \) as the starting point for the next iteration.

(vi) Repeat from (ii).

where \( \xi \) has the same dimensions as \( p \). In our problem, the function to be minimized is the loss function, \( J_i \), and the state vector is in Eq. (1). The initial search directions must span the entire state space. In our simulations and experiments, the initial search directions were the unit basis vectors.

### 2.4 Measurement Data Selection Method to Reduce Required Computations

Powell’s conjugate direction method requires many number of line search to modify the searching directions. Consequently, it requires more computation than the gradient-based method. Reduction of total amount of computation is necessary to make the proposed method practical. Reduction of the number of measurement data, \( N \), is the most effective to reduce the amount of computations. Measurements of short periods do not include enough information for accurate estimation. Therefore, a method based on selective use of the measurement data is necessary not only for reduction of the computational burden but also to maintain estimation accuracy.

In general, range measurements are sensitive to the line-of-sight direction. The proposed method uses a round-trip propagation delay such that there are two sensitivity directions for each measurement. Because the propagation delay, \( \tau \), is much smaller than the measurement interval, the sensitivity direction of the \( i \)-th measurement can be represented by one. A pseudo line-of-sight direction, \( d_i \), is given by

\[
d_i = \left( \frac{\partial f_x}{\partial \delta_0} \right)^{-1} \begin{bmatrix} X(t_{r,i}) - X(t_{s,i}) \end{bmatrix} \begin{bmatrix} X(t_{s,i}) - X(t_{r,i}) \end{bmatrix}^T
\]

(16)

where \( f_x \) is the part of the function \( f(\delta_i) \) related to the position of the rover, \( x_i \); that is,

\[
f_x = R(\sigma_i, \omega(t_{s,i} - t_i)x_i,
\]

(17)
The dimensionality of the matrix \( \frac{\partial f}{\partial x} \) is \( 3 \times 3 \). The vector \( \vec{d}_i \) is a transformed direction based on the line-of-sight direction from the rover to the mother spacecraft in the state space at time \( i = 0 \). The \( N \) directions of the pseudo line-of-sight, \( \vec{d}_i \) \((i = 1, 2, \ldots, N)\), are not biased, so an accurate estimation will be obtained.

If the measurement data are selected to maintain the variety of the set of sensitivity directions, then any estimation performed using the selected measurement data is expected to maintain accuracy. The algorithm of the proposed method is as follows:

**[Selection Method for Measurement Data]**

(i) Calculate the directions of the pseudo line-of-sight, \( \vec{d}_i \) \((i = 1, 2, \ldots, N)\) using the nominal state, \( \vec{s}_0 \).

(ii) Derive the base vectors \( \vec{u}_k \) \((k = 1, 2, \ldots, n)\) of the set for directions, \( \{\vec{d}_i\} \).

(iii) Rank the measurement data, \( \tau_i \), in descending order of their inner products, \( \vec{u}_k \vec{d}_i \), for each base vector, \( \vec{u}_k \).

(iv) Select the same number of measurement data from each ranking of \( \vec{u}_k \) to act as representative data.

At step (iii), the measurement data are ranked in order of distance between the base vector \( \vec{u}_k \) and the sensitivity direction derived from the nominal state, \( \vec{d}_i \). By selecting the same number of data from the base vector \( \vec{u}_k \), the bias of the sensitivity directions is reduced. While estimation with less representative data would require a reduced amount of computation for optimization, the minimum search can then be trapped more easily in local minima. It is therefore necessary to determine the appropriate number of representative data in order to provide estimations with sufficiently high accuracy.

### 3. Numerical Simulations to Evaluate Estimation Accuracy

#### 3.1 Simulation Parameters

Numerical simulations that assumed that a rover was located on an SPB were conducted to evaluate the estimation accuracy of the proposed method of localization. Simulation parameters were the same as in the previous paper [15]. The planetary body assumed was a spheroid with a diameter of 300, 300, and 600 m and a uniform density of 2,500 kg/m³. The positions of the mother spacecraft were defined using Kepler’s orbit on average, with a variance of 0.2 m². The orbital elements were an inclination of 1 rad, a semi-major axis of \( 3 \times 10^3 \) m, and an eccentricity of 0.2. The longitude of the ascending node, the argument of the perigee, and the mean anomaly at the beginning of the simulation were all set to 0 rad.

The nominal position of the rover at the beginning of the simulation, \( \vec{x}_0 \), was set at \([0, 300, -100]^T\). The nominal direction of the rotational axis, \( \vec{a} \), and the angular velocity of the SPB, \( \dot{\omega}_a \), were \([0, 0, 1]^T\) and \(1.4 \times 10^{-4} \) rad/s, respectively. The uncertainty in the rover position was assumed to be 30 m in each component direction, which represented the supposed distance between the target and the actual landing point. The uncertainty in the direction of the rotational axis, \( \theta_0 \), was 0.17 rad, and the uncertainty in the angular velocity, \( \dot{\omega}_a \), was 4.2 \times 10^{-6} \) rad/s. These values reflect information about the uncertainty contained in the rotational parameters of the asteroid Itokawa [2],[19].

The prior estimation of the initial state, \( \vec{s}_0 \), was set to coincide with the nominal state. The measurement noise was assumed to be Gaussian with an average of 0 s and a variance of \(10^{-16} \) s², which reflected the performance of a recent transponder that was intended for space use. The measurement interval was 600 s. The period of observation was \(2.16 \times 10^3\) s. Measurement data was collected when the rover had the mother spacecraft in sight. The rover’s range of view was assumed to be 1.4 rad from the zenith. The iteration was set to stop when the improvement of the loss function is under a given threshold, \(10^{-2}\).

#### 3.2 Validation of Convergence

To validate the convergence of the proposed method, the values of the loss function and the estimation errors were analyzed in accordance with the iteration number. Each initial error-sample assumed a maximum difference from the nominal state; differences of +30 or −30 in each rover position component and differences of +ω_a or −ω_a in the angular velocity were assumed. The direction of the rotational axis was assumed to be inclined at \( \theta_0 \) from the nominal direction, and eight directions were considered for simplicity:

\[
\sigma_s^* = \begin{bmatrix}
\sin \delta_1 \sin \theta_0 \\
\cos \delta_1 \sin \theta_0 \\
\cos \theta_0 
\end{bmatrix},
\]

where \( \delta_1 = \pi \ell/4 \) (rad), \( (\ell = 0, 1, \ldots, 7) \), and \( ^* \) denotes the actual value. 128 samples of initial errors were used in this analysis.

The values of the loss function and the estimation error are plotted in Figs. 2 and 3, respectively. In these figures, the average, maximum, and minimum values of the loss function over 128 error-samples are plotted. Figure 2 shows that the value of the loss function decreased in accordance with iterations. This indicates that the proposed method works for any initial error sample. Figure 3 shows that the estimation error also decreased in accordance with the iteration number. It is confirmed that the minimum search of \( J_s \) in (12) leads to the appropriate estimation. The maximum estimation error among the 128 samples was \(|\vec{s}_0 - \vec{s}_s^*| = 10.8\) m, which consists of a position error of 10.8 m, a directional error of 3.90 \times 10^{-2} rad, and an angular velocity error of \(8.34 \times 10^{-8}\) rad/s. The error sample that gave the maximum estimation error was

\[
\vec{s}_0 - \vec{s}_s^* = \begin{bmatrix}
\hat{x}_0 - x_s^* \\
\hat{y}_0 - y_s^* \\
\hat{z}_0 - z_s^* \\
\hat{\delta}_{x,0} - \sigma_{x,0}^* \\
\hat{\delta}_{y,0} - \sigma_{y,0}^* \\
\hat{\delta}_{z,0} - \sigma_{z,0}^* \\
\hat{\dot{\omega}}_a - \omega_a^*
\end{bmatrix} = \begin{bmatrix}
-30 \\
-30 \\
-30 \\
-\sin \theta_a \\
0 \\
-\omega_a
\end{bmatrix}.
\]

From these results, we can conclude that the proposed method provides sufficiently accurate estimation of both the rover’s position and the rotational parameters of the SPB.

#### 3.3 Validation of Estimation Accuracy Using Selected Measurements

The actual state was set to be the worst case of Eq. (19), and 100 measurement noise samples were simulated to validate
the proposed selection method of measurement data. The easiest way to reduce the number of measurement data is to make the measurement period longer. Measurement selection with equal time intervals was compared with the proposed selection method. The following results were provided by a computer with an Intel Core i7 running at 2.93 GHz using Matlab 2009a software.

The estimation error and computation time when using the selected data are plotted in Figs. 4 and 5, respectively. In these figures, the solid lines denote the results of the proposed selection method and the dashed lines denote the results of measurement selection with equal time intervals. The maximum and minimum errors over 100 measurement noise samples are shown with error bars. The proposed selection method was superior to measurement selection with equal time intervals from the perspectives of estimation error and computation time. While both methods of measurement selection required the same computation time for each iteration, the proposed selection method was faster overall. This is because the proposed selection method considers the sensitivity direction of the measurement data. We can conclude that the proposed selection method can effectively reduce the computation time while maintaining estimation accuracy.

4. Indoor Experimental Results

4.1 Overview and Purpose of Experiments

To evaluate the feasibility of the proposed method, simple experiments using a range measurement tool were conducted. The experimental setup is shown in Fig. 6. One end of a straight stick was attached to a pan-tilt unit (PTU). The PTU rotated the stick, which simulated the rotational motion of the SPB. At the other end of the stick, a cylindrical reflector with a diameter of $1.1 \times 10^{-2}$ m and a height of $1.0 \times 10^{-2}$ m was fixed, representing the reflection of the radio waves at the transponder on a rover. A Total Station (TS: Topcon GTS-810A) [20] was used to collect the measurement data, which simulated a mother spacecraft. The TS output the position of a target in a local frame as well as the distance between TS and the target. The distance measured by the TS, $\rho_i$, was transformed into the round-trip propagation delay, $\tau$, as

$$\tau = \frac{2\rho}{c_a},$$

(20)

where $c_a$ is the speed of light in air. It is assumed that the rover can receive a signal from its upper hemisphere. The measurement data $\tau_i$, when the position of the reflector, $x_i$, and the position of the TS, $X_i$, satisfy

$$x_i^T (x_i - X_i) \geq 0,$$

(21)

was used for estimation.

Actual values of the reflector’s initial position, $x^*_0$, the direction of the rotational axis, $\sigma^*$, and the variance of the measurement noise, $\gamma^*$, are needed to evaluate the feasibility of the proposed method. In our experiments, these actual values were determined by a statistical method as
Details of the method was explained in Appendix. The angular velocity, $\omega^*$, is controllable using the PTU. In this experiment, the value used was $0.100 \text{ rad/s}$.

The prior estimation of the initial state was set as

$$x_0 = x_0^* + [0.1, \ 0.1, \ 0.1]^T \tag{25}$$

$$\sigma = [0, \ 0, \ 1]^T \tag{26}$$

$$\omega = \omega^* - 0.03, \tag{27}$$

where $x_0$ and $\omega$ were defined such that the initial error rates $(x_0 - x^*)/|x_0^*|$ and $(\omega - \omega^*)/|\omega^*|$ were almost the same as those in the numerical simulations. The iteration was set to stop when the improvement of the loss function per number of measurements, $J_f/N$ was under a given threshold, $10^{-3}$. The following results were provided by a computer with an Intel Core i7 running at 2.93 GHz with Matlab 2009a software.

As explained in Section 3, the movement of the mother spacecraft is relatively smaller than the rotational motion of the SPB. The small movement of the mother spacecraft leads to lack of variation in sensitivity direction. First, the TS was fixed to simulate the estimation under the minimal variation in sensitivity direction. To compare with this case, the TS was moved to collect measurement data with sufficiently high variation in sensitivity directions. Finally, the data-selection method to reduce computation time was also applied to data with high variation in sensitivity directions.

### 4.2 Estimation by Data with Minimal Variation in Sensitivity Directions

The TS was fixed and measured the distance to the reflector at 100 different angles of rotation. 56 data satisfied Eq. (21), which were used for estimation.

The above figure of Fig. 7 is the values of the loss function (above) and estimation error (below) in case of minimal variation in sensitivity directions.

In reality, the long measurement period provides high variation in sensitivity directions, if the orbit of the mother spacecraft is appropriate. The polar orbit with the rover in sight is appropriate while the orbit with the constant latitude leads poor variations in sensitivity directions [14].

### 4.3 Estimation by Data with High Variation in Sensitivity Directions

In order to confirm the effect of the variation of the sensitivity directions, TS measured the distance at different four fixed points. 56 measurement data satisfied Eq. (21), which were used for estimation. The same number of measurement data was used as the estimation described in Section 4.2.

The above figure of Fig. 8 plots the values of the loss function and the below figure plots the estimation error. Unlike the estimation under the minimal variation in sensitivity directions, the value of loss function as well as the estimation error decreased in accordance with iteration. From the results described in Sections 4.2 and 4.3, we can conclude that the variation of the sensitivity directions influences the estimation error. Sufficient variation in sensitivity directions is necessary for the proposed method to work properly.

### 4.4 Discussion on Number of Selected Data

The proposed selection method for the measurement data was also assessed. The TS measured the distance at different four fixed points. At each point, the TS measured the distance to the reflector at 100 different angles of rotation. 400 data were collected in total. 223 data satisfied eq. (21), which were used for estimation.

Figure 9 plots the estimation error and Fig. 10 plots the required computation time together with the number of measurement data. The horizontal axis of Fig. 9 and 10 shows the number of selected data. It is confirmed that the proposed selection method can reduce the required computation time without losing estimation accuracy. It is also shown that too much reduction of the number of measurements causes the increase of

$$x_0 = [0.784, -0.961, 0.271]^T \tag{22}$$

$$\sigma = [0.307, 0.149, 0.940]^T \tag{23}$$

$$\bar{y}_i = 4.04 \times 10^{-3}/a_i \tag{24}$$
In the loss function (12), only contains measurement noise. If the estimated state \( \hat{s}_n \) is equal to the actual state, then the term \((T_{\text{obs}} - \hat{T}_i)\) in the loss function (12), only contains measurement noise. If sufficient measurement data are obtained, the expected value of \( J_s/N \) satisfies

\[
J_s/N \sim 1.
\]

(28)

Both estimation error and computation time. It is confirmed that the proposed selection method can provide as accurate an estimation as the approach without data selection, and excessive selection of measurement data causes poor estimation.

In actual applications, the estimation error is unknown while the value of the loss function, \( J_s \), can be calculated. An index for quality of estimation is necessary, which can be used in actual applications. An index \( J_s/N \) is useful in judging whether or not the estimated state \( \hat{s} \) is close to the actual state [15]. If the estimate \( \hat{s} \) is equal to the actual state, then the term \((T_{\text{obs}} - \hat{T}_i)\) in the loss function (12), only contains measurement noise. If sufficient measurement data are obtained, the expected value of \( J_s/N \) satisfies

\[
J_s/N \sim 1.
\]

In Fig. 11, the values of the index \( J_s/N \) are plotted, where the loss function \( J_s \) was calculated for 223 data and the estimated state, \( \hat{s}_n \), was derived from the selected data. From Fig. 11, the index \( J_s/N \) is sufficiently close to 1 when the estimation error was small; see Fig. 9. We can conclude that the index \( J_s/N \) calculated using all of the data provides an effective reference value for checking whether or not the estimated state is close to the actual state.

5. Conclusion

In this paper, we proposed a localization method for a space rover on an SPB, which does not require motion information of a mother spacecraft. The method was formulated as an optimization problem and a solution based on Powell’s conjugate direction method was proposed. The proposed method does not require the calculation of any derivatives. Instead, it requires large amounts of computation. In order to reduce the computation time, a selection method for measurement data was proposed based on sensitivity directions. Numerical simulations and indoor experimental results showed that the proposed method is able to localize a rover with sufficient accuracy. The proposed selection method was compared with the selection of equal time intervals. The comparison proved that the proposed method of selection can provide more accurate results than the estimation using data with equal time intervals, and even requires less computation time. These results suggested that insufficient measurement data led the estimated results into local minima. We confirmed that the index \( J_s/N \) in (28) is efficient for judging the estimated state is a local minimum or not. Using the estimated parameters of the rotational motion of the SPB, a method of localization based on a Kalman filter [14] can provide real-time localization for a rover.

Acknowledgments

This work was partly supported by JSPS Grant-in-Aid for Young Scientists (B)25820185.

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Appendix  Parameter Determination in Experimental Setup

The position of the TS was determined by using prisms. Prisms were fixed at three different points: $O_1$, $O_2$, and $O_3$. $O_1$ was located at the center of the rotational plane formed by the PTU. $O_2$ and $O_3$ were located such that $O_1$, $O_2$, and $O_3$ were not in a line. In this experiment, $O_1 = [0, 0, 0]^T$, $O_2 = [0.7738, -0.6328, -0.0273]^T$, and $O_3 = [0.0959, 0.9394, -0.3293]^T$.

The rotational plane formed by the PTU was determined using the reflector’s positions by principal component analysis (PCA). The first and the second PCA bases denote the base vectors of the rotational plane if there are sufficient data for the reflector’s positions. The third component lies parallel to the direction of the rotational axis. The determined direction of the rotational axis in this experiment was

$$\sigma^2 = [0.307, 0.149, 0.940]^T.$$  \hspace{0.5cm} (A. 1)

which used 400 data for the reflector’s position. The rotational plane was inclined at 0.340 rad from the horizontal plane.

The initial position of the reflector, $x_0$, was determined by the TS’s output. In this experiment, the initial reflector position was

$$x_0 = [-0.784, -0.0961, 0.271]^T.$$  \hspace{0.5cm} (A. 2)

The difference between the measurement and the identified values was $2.38 \times 10^{-4}$ m on average, and the variance was $4.04 \times 10^{-5}$ m$^2$. The variance of the measurement noise in eq. (12) was set $\tilde{\gamma}_i^2 = 4.04 \times 10^{-5} / \sigma_i^2 (i = 0, 1, \cdots, N)$.