Regularities in electromagnetic decay widths

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October 11, 2018

Abstract

We revisit Sakurai’s remark on the regularities of lepton-pair widths for mesons, extending the panorama to radiative $X \rightarrow \gamma \gamma$ decays. The regularities persist, and somehow surprisingly some of them seem to relate with Fermi’s constant $-or Z^0$.

Back in 1978, Sakurai took the opportunity of a festscrift [11] to remark how the study of vector meson decays $V \rightarrow \gamma \rightarrow e^+ e^-$, when extended to $J/\Psi$ and $\Upsilon$, seemed to have confirmed an empirical rule of Yennie [14] about the universality of $\Gamma(V \rightarrow e^+ e^-)$. On other hand a universality in the effective coupling $g_{X \gamma \gamma}$ for pseudoscalars $X = \pi, \eta$ is well known popularly and we have recently noticed that it seems to extend to vector mesons, particularly to $J/\Psi$, when interpreted as a virtual process for total decay width. And also radiative transitions as $\Sigma^0 \rightarrow \Lambda^0 \gamma$ happen to be in the adequate range.

Our purpose here is to review all the radiative decays well measured in the modern data tables in order to establish how significant these regularities are.

1 All total decay widths

We can try to get some perspective by plotting all the decay rates of subnuclear particles. This can be done straightly from the tables provided by the particle data group [15] in its website [http://pdg.lbl.gov/]. One can see three layers corresponding to decays strong, electromagnetic or weak, with the neutron being apart from the rest of weak decaying particles due to its quark composition. The area of strong decay is actively pushed forward by experimentalists, and one can see how the actual advances hit on one hand the need of having a noticeable width in the resonance plot, say $\Gamma < E_0$, and on the other hand enough area. We have plotted the lines $E_0 \Gamma = (1 GeV)^2$, $\Gamma = kE_0$ ($k = 1, 0.5, 0.25$) as reference, but we excuse ourselves about plotting the mass against spin dependence famously reported in Chew-Frautschi plots.

When particles can not decay strongly -either because there are no other hadronic mode available, or because any available mode should change quark flavours or because asymptotic freedom activates OZI rule- they give way to electromagnetic decay or to weak decay.

In the weak decay area spectator models and other approaches (even plain dimensional analysis) point to a $\Gamma \sim m^5$ scaling, and we have drawn one such line from the muon. Note that such line in a spectator model should be used to meet the mass value of the decaying quark, not of the whole particle.

The intermediate area, electromagnetic or radiative decays (we are using both terms as exchangeable ones in this context), is the one we are interested. We have

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marked it in both figures with a scaling $\Gamma \propto m^3$, partly because of the usual $\pi^0$ decay calculation, partly because of dimensional comparison with weak decays: the weak decays have gained a dimensionful coupling constant (Fermi constant) from the breaking of electroweak symmetry, and then an additional $\propto m^2$.

It is interesting to stop a little to think how should the points in the plot move if electroweak symmetry were restored, playing with the parameters in the Higgs sector, thus altering the value of Fermi constant until a phase transition is reached and it disappears. It is even possible to use mass formula of mesons and baryons to determine when a given strong decay becomes available/unavailable and some particle leaves/enters the electroweak zone.

Also it is a funny try to look for alternative visualizations of the non strong sector (hint: in modern notation, the strong sector particles are the ones that carry a mass value between parenthesis as part of its name). It has been suggested to use base 137 for the logs in the decay width, and also to center the masses via an $\arctan(x/M_0)$ map, adjusting $M_0$ to the value of the proton.

2 Non weak, non strong decays

The list of known particles whose main decay mode is neither strong nor weak is very short. In order of increasing mass we meet:

- The neutral pion
- $\eta$, although it has already a mix with pions that makes use of the strong force.
- $\Sigma^0$, the only baryon in the party.
- Lower charmonium, this is $J/\Psi$, $\Psi(2S)$, $\eta_c$ (mostly strong mixed), and some $1P$ wave states: $\chi_{c0}, \chi_{c1}, \chi_{c2}$.
- Lower bottomonium, this is $\Upsilon(1S), \Upsilon(2S), \Upsilon(3S)$
- $Z^0$ and $W^\pm$. Well, why not? They do not decay strongly, and we can not say strictly that they decay weakly, no more than we can claim that Death is dead.

If we calculate the reduced decay widths $\beta \equiv \Gamma/M^3$ for the above group, only the $\Upsilon$ are noticeably apart.

As I have noted in the introduction, and we will see now in the next section, the coincidence between $\pi^0$ and $\eta$ increases if we only consider $\gamma\gamma$ decay.

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Table 1: [Inverse square root of] Reduced full decay widths $\beta = \Gamma/M^3$ for EM (generically, non strong non weak) decaying particles

| particle | $\beta^{-\frac{1}{2}}$ (GeV) |
|----------|-----------------------------|
| $\pi^0$  | 561                         |
| $\eta$   | 357                         |
| $\Sigma^0$ | 138                       |
| $J/\Psi$  | 571                         |
| $\Psi(2S)$ | 422                       |
| $\Upsilon(1S)$ | 3996                  |
| $W^\pm$  | 495                         |
| $Z^0$    | 551                         |

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1 Amateur suggestion of Yuri Danoyan
Figure 1: Log-log plot of $\Gamma(M)$ (units in MeV) from data in http://pdg.lbl.gov/2005/mcdata/mass_width_2004.csv
3 All the non negligible radiative widths

3.1 Decay via $\gamma$ to lepton pairs

For electron pair production, this was reported by Sakurai [11]. Today we can complete it with decays to muons and (for $\Upsilon$) tau.

| particle | mass | total $\Gamma$ | $\Gamma(l^+l^-)$ | $\Gamma(e^+e^-)$ |
|------|------|------------|----------------|----------------|
| $\rho(770)$ | 775.8 | 150.3 | .01385766 | .00701901 |
| $\omega(782)$ | 782.59 | 8.49 | .001373682 | .00609582 |
| $\phi(1020)$ | 1019.46 | 4.26 | .00248784 | .00126948 |
| $J/\Psi$ | 3097 | 0.091 | .0107471 | .0053963 |
| $\Upsilon$ (1S) | 9460 | 0.053 | .0039909 | .0012614 |

The analysis was aimed to conclude that $\Gamma$ was almost constant in this kind of decays. We are more interested in the fact that the amplitudes have values near the cubic scaling. This is even truer if following [14] we consider a proportion 1:9 between $\rho$ and $\omega$.

3.2 Decay to $\gamma\gamma$

This kind of decay – and optionally its descendant $\gamma e^+e^-$ – is useful to separate strong force effects from electromagnetic ones in decays of $\eta$ and $\eta'$; see [5] for calculations and estimates of mixing effects. It has also been used to argument against a quark composition of scalars $a_0$, $f_0$, on the basis of a failure of $m^3$ scaling [7].

We omit this pair of scalars because their decay and total width is not completely established yet.

Besides, it does exist data about this kind of decay for $\eta_c$ and $\eta_c(2S)$ and for some spin 2 particles $a_2$, $f_2$, $f_2'$ which we list for completion.

| particle | mass | $\Gamma(\gamma\gamma)$ |
|------|------|----------------|
| $\pi^0$ | 134.97 | .00000778 |
| $\eta$ | 547.75 | .0000509 |
| $\eta'$ | 957.78 | .0042824 |
| $f_2$ | 1275.4 | .00260991 |
| $a_2$ | 1318.3 | .0010058 |
| $f_2'$ | 1525 | .00008103 |
| $\eta_c'$ | 3642.9 | .0013 |
| $\eta_c$ | 2980 | .0074 |

3.3 Decay to anything plus $\gamma$

The particle data group list decays to $X\gamma$ for about a dozen of strongly decaying mesons but the corresponding widths are still scattered along various orders of magnitude and some extra organizing principle is still needed.

For the baryons, the dominant (100%) $\Sigma^0 \to \gamma A^0$ is the only decay measured directly, although via Primakoff method, and it fits very nicely in the expected range. We could try to add some decays built from the fit of $N$ and $\Delta$ resonances, for instance from the database of [1] (see [4] for a theoretical model and some experimental plots too), but we felt so blocked as in the meson case.

3.4 Whole widths

Besides the above cases of $\Sigma^0$ and $\pi^0$ whose radiative decay is practically the whole decay, we are interested on considering the Charmonium and Bottomonium areas, where OZI rule (a combination of asymptotic freedom and conservation laws) forbids strong decay and hadron production also proceeds mainly via the electroweak forces.
Figure 2: Widths of electromagnetic decaying particles, including excited states of charmonium and bottomonium, and its approximate cubic dependence of mass. Note that we have included Υ(4S), which is strongly decaying, and also two points for η (total decay and γγ only).

We find the amazing fact that J/Ψ total width scales respective to Z0 (!) total decay as \( m^3 \), while Υ could be thought to scale from it as \( m^5 \) (but the states of Υ having allowed strong decays are closer to the cubic scaling than to quintic one).

4 Discussion

The plot in figure 2 summarizes the observations –we can barely call them results. While the decay to leptons stated by Sakurai persists, it is to us be of a minor consideration when one takes into account that the whole J/Ψ decay must be electromagnetically dominated. In this case the property that becomes interesting is the alignment according a scaling \( \Gamma \propto M^3 \) already noticed in [10] and which now incorporates Sakurai’s vector meson decays for ω and φ as well as the baryon \( \Sigma^0 \). Besides, the consideration of the decay to \( \gamma\gamma \) of \( \eta \) and \( \eta' \), instead of the total width, lets one to put such particles near the approximate scaling rule, but this was perhaps already known [12, pg 10] in the decay lore. In total we have eight electromagnetically decaying strong particles (ten if we count \( W^+ \) and Ψ(2S) following a scaling rule across four orders of magnitude in mass and mysteriously fitting a purely electroweak quantity, the decay of \( Z^0 \) (that is controlled by \( \sin \theta_W \) basically)). Among spin 0 and spin 1 mesons, only Υ, \( \rho \) and some excited states do not fit straightly into this rule, and really kinematic and symmetry arguments could be invoked to fit every except Υ; note for instance already in [14] the expected 9:1 proportion between \( \rho \) and \( \omega \).

As a collateral remark, it is noticeable that the double gamma decay of \( \eta_c \) resolves near of the leptonic decay of J/Ψ; perhaps a rule can be built relating both.

I want to thank K. Illinsky from the PDG by providing a corrected version of
the file `mass.width.2004.csv`; this work was retaken as a way to double check the values of such file.

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