SOFT X-RAY EXTENDED EMISSIONS OF SHORT GAMMA-RAY BURSTS AS ELECTROMAGNETIC COUNTERPARTS OF COMPACT BINARY MERGERS: POSSIBLE ORIGIN AND DETECTABILITY

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ABSTRACT

We investigate the possible origin of extended emissions (EEs) of short gamma-ray bursts with an isotropic energy of \( \sim 10^{50-51} \) erg and a duration of a few 10 s to \( \sim 100 \) s, based on a compact binary (neutron star (NS)–NS or NS–black hole (BH)) merger scenario. We analyze the evolution of magnetized neutrino-dominated accretion disks around BHs formed after the mergers and estimate the power of relativistic outflows via the Blandford–Znajek (BZ) process. We show that a rotation energy of the BH up to \( \gtrsim 10^{52} \) erg can be extracted with an observed timescale of \( \gtrsim 30(1+z) \) s with a relatively small disk viscosity parameter of \( a < 0.01 \). Such a BZ power dissipates by clashing with non-relativistic pre-ejected matter of mass \( M \sim 10^{(2-4)} M_\odot \), and forms a mildly relativistic fireball. We show that the dissipative photospheric emissions from such fireballs are likely in the soft X-ray band (1–10 keV) for \( M \sim 10^{-2} M_\odot \), possibly in NS–NS mergers, and in the BAT band (15–150 keV) for \( M \sim 10^{-4} M_\odot \), possibly in NS–BH mergers. In the former case, such soft EEs can provide a good chance of \( \sim 6 \) yr\(^{-1} \) (\( \Delta \Omega_{\text{softEE}}/4\pi \) (\( R_{\text{GW}}/40 \) yr\(^{-1} \)) for simultaneous detections of the gravitational waves with a \( \sim 0.1 \) angular resolution by soft X-ray survey facilities like the Wide-Field MAXI. Here, \( \Delta \Omega_{\text{softEE}} \) is the beaming factor of the soft EEs and \( R_{\text{GW}} \) is the NS–NS merger rate detectable by the advanced LIGO, the advanced Virgo, and KAGRA.

Key words: accretion, accretion disks – gamma-ray burst: general – X-rays: bursts

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1. INTRODUCTION

Short gamma-ray bursts (SGRBs) are usually defined by the prompt duration, i.e., gamma-ray bursts (GRBs) with a duration of a few 10 s to \( \sim 100 \) s, based on a compact binary (neutron star (NS)–NS or NS–black hole (BH)) merger scenario. We analyze the evolution of magnetized neutrino-dominated accretion disks around BHs formed after the mergers and estimate the power of relativistic outflows via the Blandford–Znajek (BZ) process. We show that a rotation energy of the BH up to \( \gtrsim 10^{52} \) erg can be extracted with an observed timescale of \( \gtrsim 30(1+z) \) s with a relatively small disk viscosity parameter of \( a < 0.01 \). Such a BZ power dissipates by clashing with non-relativistic pre-ejected matter of mass \( M \sim 10^{(2-4)} M_\odot \), and forms a mildly relativistic fireball. We show that the dissipative photospheric emissions from such fireballs are likely in the soft X-ray band (1–10 keV) for \( M \sim 10^{-2} M_\odot \), possibly in NS–NS mergers, and in the BAT band (15–150 keV) for \( M \sim 10^{-4} M_\odot \), possibly in NS–BH mergers. In the former case, such soft EEs can provide a good chance of \( \sim 6 \) yr\(^{-1} \) (\( \Delta \Omega_{\text{softEE}}/4\pi \) (\( R_{\text{GW}}/40 \) yr\(^{-1} \)) for simultaneous detections of the gravitational waves with a \( \sim 0.1 \) angular resolution by soft X-ray survey facilities like the Wide-Field MAXI. Here, \( \Delta \Omega_{\text{softEE}} \) is the beaming factor of the soft EEs and \( R_{\text{GW}} \) is the NS–NS merger rate detectable by the advanced LIGO, the advanced Virgo, and KAGRA.

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non-rotating NS is smaller than \( \sim 2.5 M_\odot \), the final outcome of such a merger will be a Kerr BH with a mass of \( M_{\text{BH}} \sim 3 M_\odot \) and a spin parameter of \( q = a/M_{\text{BH}} < 0.8 \) with an accretion disk with a mass of \( \sim 0.1 M_\odot \) and possibly beyond, and a neutron-rich ejecta with a mass of \( M \sim 10^{(2-4)} M_\odot \) with an expanding velocity of \( v_{\exp} \sim 0.1 c \) (Shibata & Taniguchi 2006; Kiuchi et al. 2009; Rezzolla et al. 2010; Hotokezaka et al. 2011, 2013). Here, we adopt this situation. If the maximum mass of a non-rotating NS is larger than \( \sim 2.5 M_\odot \), a rapidly rotating massive NS will be the final outcome and the magnetar activities may be responsible for the prompt spike and EEs of the SGRBs as well as other electromagnetic counterparts (Usov 1992; Zhang & Meszaros 2001; Gao & Fan 2006; Metzger et al. 2008; Bucciantini et al. 2010; Gompertz et al. 2013; Zhang 2013).

In this paper, we consider the huge rotational energy of a Kerr BH to be the intrinsic energy budget of the EE (see also Fan et al. 2005; Rosswog 2007; Lee et al. 2009; Barkov & Pozanenko 2011). The mass formula of the Kerr BH with a gravitational mass of \( M_{\text{BH}} \) and an angular momentum \( J \) is written as (Misner et al. 1973)

\[
M_{\text{BH}}^2 = M_{\text{ir}}^2 + \frac{J^2}{4 M_{\text{ir}}^2},
\]

where \( M_{\text{ir}} \) is the irreducible mass of the Kerr BH. Writing \( J = a M_{\text{BH}} = q M_{\text{ir}}^2 \), we have

\[
M_{\text{BH}}^2 = M_{\text{ir}}^2 + \frac{M_{\text{ir}}^2 q^2}{4 M_{\text{ir}}^2},
\]
where \( a \) is the well known Kerr parameter and \( q = a/M_{\text{BH}} < 1 \).

Equation (2) is rewritten as

\[
M_{\text{BH}}^2 = \frac{2M_a^2}{1 + \sqrt{1 - q^2}}. \tag{3}
\]

Then, the energy available by extraction of the angular momentum of the Kerr BH is given by

\[
\Delta E = M_{\text{BH}} \left(1 - \frac{1 + \sqrt{1 - q^2}}{2}\right). \tag{4}
\]

For \( q = 0.5 \), for example, \( \Delta E \) is given by

\[
\Delta E = 1.84 \times 10^{53} \text{ erg} \left( \frac{M_{\text{BH}}}{3M_\odot} \right). \tag{5}
\]

Therefore, for an \( \sim 100\% \) radiation efficiency, only \( \sim 1\% \) of the rotational energy of the BH enables fueling of the EE even if the emission is isotropic. The problem is how to extract the rotational energy of the Kerr BH on a timescale of a few \( 10^5 \) s to \( \sim 10^6 \) s.

One of the plausible mechanisms for extracting the rotation energy of the Kerr BH is the Blandford–Znajek (BZ) process (Blandford & Znajek 1977). From the results of numerical simulations, Penna et al. (2013) found that within factors of order unity, the BZ power is expressed in units of \( c = G = 1 \) by

\[
L_{\text{BZ}} = \frac{1}{6\pi} \Omega_H \Phi^2, \tag{6}
\]

where

\[
\Omega_H = M_{\text{BH}}^{-1/3} \frac{q}{(1 + \sqrt{1 - q^2})^2 + q^2} \tag{7}
\]

and

\[
\Phi = \pi M_{\text{BH}}^2 (1 + \sqrt{1 - q^2})^2 B \tag{8}
\]

is the magnetic flux threading the horizon with \( B \) being the strength of the magnetic field formed by the disk around the Kerr BH. Recovering \( c \) and \( G \), we have

\[
L_{\text{BZ}} = \frac{\pi}{6} \left[ \frac{q(1 + \sqrt{1 - q^2})^2}{(1 + \sqrt{1 - q^2})^2 + q^2} \right]^2 \left( \frac{GM_{\text{BH}}}{c^2} \right)^2 cB^2. \tag{9}
\]

For \( q = 0.5 \), for example, \( L_{\text{BZ}} \) is expressed as

\[
L_{\text{BZ}} = 6.6 \times 10^{50} \text{ erg s}^{-1} \left( \frac{M_{\text{BH}}}{3M_\odot} \right)^2 \left( \frac{B}{10^{15} \text{ G}} \right)^2. \tag{10}
\]

Dividing Equation (5) by Equation (10), we have the characteristic time \( \delta t_{\text{BZ}} \) as

\[
\delta t_{\text{BZ}} = 2.8 \times 10^2 \text{ s} \left( \frac{M_{\text{BH}}}{3M_\odot} \right)^{-1} \left( \frac{B}{10^{15} \text{ G}} \right)^{-2}. \tag{11}
\]

This shows that if an accretion disk with \( B \sim 10^{15} \text{ G} \) and an accretion time \( \sim 100 \) s exists, up to \( \sim 10^{55} \) erg can be extracted from the Kerr BH. This is just the timescale of the EEs and only \( \sim 1\% \) efficiency is enough to explain the emissions even if they are isotropic.

This paper is organized as follows. In Section 2, we analyze the time evolution of the neutrino-dominated accretion disk with finite mass and angular momentum around the BH, and estimate the resultant BZ power and the duration. In Section 3, we consider the interaction of the BZ jets with the pre-ejected matter \((M \sim 10^{-2-4} \text{ M}_\odot)\) with an expanding velocity of \( v_{\text{exp}} \sim 0.1c \), which produces mildly relativistic fireballs. We calculate the dissipative photospheric emissions from such fireballs. There, we also argue the detectability and the association with the observed EEs. Section 4 is devoted to the discussion. We use \( Q_\odot = Q/10^5 \) in CGS units unless otherwise noted.

2. BLANDFORD–ZNAJEK JETS FROM BLACK HOLE TORI FORMED AFTER COMPACT BINARY MERGERS

Chen & Beloborodov (2007) calculated the steady-state solutions of a neutrino-dominated accretion disk around a Kerr BH, which is also the case in our setup at the initial stage. They assumed an accretion rate of \( \dot{M}(t) = \) constant, but took into account the full neutrino process and the Kerr geometry. Their important conclusions are that (1) the pressure is dominated by baryons with \( p = (\rho/m_p)kT \), (2) the disk is neutron dominated so that the electron fraction is as small as \( Y_e \sim 0.1 \), (3) the degeneracy of the electron is at most mild because, if the degeneracy is high, the neutrino cooling is lowered to increase the temperature, (4) there is an ignition accretion rate for the neutrino cooling disk that is proportional to \( \alpha^{5/3} \) where \( \alpha \) is the parameter in the so-called \( \alpha \)-disk model (Shakura & Sunyaev 1973). Kawanaka et al. (2013) performed simpler Newtonian calculations of such disks both numerically and analytically. One of the important conclusions is that their analytical model fits well with the numerical ones by Chen & Beloborodov (2007). Therefore, here we adopt a simple Newtonian analytical model to mimic the neutrino-dominated accretion disk around the Kerr BH. One of the big differences from Kawanaka et al. (2013) is that we take into account the time variation of the accretion rate for the finite disk mass and the finite disk angular momentum while they considered a constant accretion rate.

The structure of the accretion disk can be derived from (Kawanaka et al. 2013)

\[
\dot{M} = -2\pi r \times 2\rho h v_r, \tag{12}
\]

\[
2\alpha hp = \frac{M\Omega}{2\pi}, \tag{13}
\]

\[
p = \rho \Omega^2 h^2, \tag{14}
\]

where \( \dot{M}, r, \rho, h, v_r, \alpha, p, \) and \( \Omega \) are the accretion rate, \( r \) in cylindrical coordinates, the density, the half thickness of the disk, the infalling velocity, the \( \alpha \) parameter, the pressure, and the angular frequency of the disk, respectively. We can express \( h, \rho, v_r \) by \( \rho \) as

\[
h = \left( \frac{\dot{M}}{4\pi\alpha\Omega\rho} \right)^{1/3}
= 7.02 \times 10^8 \text{ cm} a^{-1/3} \rho_0^{-1/3}
\times \left( \frac{\dot{M}}{10^{-3} \text{ M}_\odot \text{ s}^{-1}} \right)^{1/3} \left( \frac{M_{\text{BH}}}{3M_\odot} \right)^{1/3} \left( \frac{r}{r_{\text{ISCO}}} \right)^{1/2}, \tag{15}
\]
\[ p = \frac{\Omega^{1/3} \rho^{1/3}}{4\pi a} \left( \frac{M}{3M_\odot} \right)^{2/3} \]
\[ = 1.04 \times 10^{25} \text{ dyn cm}^{-2} \alpha_{-1}^{-2/3} \rho_0^{-1/3}, \]
\[ \times \left( \frac{M}{10^{-3} M_\odot \text{s}^{-1}} \right)^{2/3} \left( \frac{M_{BH}}{3 M_\odot} \right)^{-4/3} \left( \frac{r}{r_{ISCO}} \right)^{-2}, \]  
\[ (16) \]
\[ v_r = -\left( \frac{\dot{M}}{4\pi \rho} \right)^{2/3} (\alpha \Omega)^{1/3} \frac{1}{r}, \]
\[ = -8.49 \times 10^{13} \text{ cm s}^{-1} \alpha_{-1}^{-1/3} \rho_0^{-2/3}, \]
\[ \times \left( \frac{M}{10^{-3} M_\odot \text{s}^{-1}} \right)^{2/3} \left( \frac{M_{BH}}{3 M_\odot} \right)^{-4/3} \left( \frac{r}{r_{ISCO}} \right)^{-3/2}. \]  
\[ (17) \]
is the innermost stable circular orbit (ISCO)\(^6\). The density \(\rho\) can be determined by the energy equation and the equation of state. Denoting the cooling rate \(\dot{q}\), the energy balance is expressed as
\[ \dot{q} = \frac{3GM_{BH}\dot{M}}{4\pi^3 \times 2h} \]  
\[ (19) \]
\[ = 7.16 \times 10^{27} \text{ erg cm}^{-3} \text{ s}^{-1} \alpha_{-1}^{-1/3} \rho_0^{1/3}, \]
\[ \times \left( \frac{\dot{M}}{10^{-3} M_\odot \text{s}^{-1}} \right)^{2/3} \left( \frac{M_{BH}}{3 M_\odot} \right)^{-7/3} \left( \frac{r}{r_{ISCO}} \right)^{-7/2}. \]  
\[ (20) \]
As for the energy loss rate, we consider two neutrino cooling processes relevant to the accretion disk we are interested in as (Itoh et al. 1989; Popham et al. 1999)
\[ \dot{q}_{URCA} = 9 \times 10^{-43} \text{ erg cm}^{-3} \text{ s}^{-1} \rho_0 T_0^6, \]  
\[ (21) \]
\[ \dot{q}_{pair} = 5 \times 10^{-66} \text{ erg cm}^{-3} \text{ s}^{-1} T_0^9, \]  
\[ (22) \]
where \(T\) is the temperature in units of [K]. The URCA process is \(p + e^- \rightarrow n + \nu_e, n + e^+ \rightarrow p + \bar{\nu}_e\), and the pair neutrino process is \(e^- + e^+ \rightarrow \nu_e + \bar{\nu}_e\). The URCA process dominates over the pair process for \(\rho_0 > 5.5 \times 10^{-24} T_0^3\). As for the pressure of the matter, we should consider
\[ p = \frac{pkT}{m_p} \text{ (gas pressure)}, \]  
\[ (23) \]
\[ p = \frac{1}{3} a_{rad} T^4 \text{ (pressure by radiation)}, \]  
\[ (24) \]
\[ p = \frac{2\pi c h}{3} \left( \frac{3Y_e \rho}{8\pi m_p} \right)^{4/3} \text{ (degenerate electron)}, \]  
\[ (25) \]
where \(a_{rad} = 7.57 \times 10^{-15} \text{ erg cm}^{-3} \text{ K}^{-4}\) is the radiation constant. The relativistically degenerate pressure dominates over the gas and the radiation pressure for \(\rho > 2.3 \times 10^{-22} \text{ g cm}^{-3} (Y_e/0.5)^{-2} T_0^4\) while the gas pressure dominates over the radiation pressure for \(\rho > 3.06 \times 10^{-23} \text{ g cm}^{-3} T_0^{-3}\).

\(^6\) In fact, the location of the ISCO depends on \(a/a_*/M\). However, we are using Newtonian gravity so the exact treatment of ISCO is not possible. One can take into the account the change of ISCO by putting a different value of \(r/6GM_{BH}/c^2\) into all the equations. In this case, various quantities are modified by powers of the above factor, but our results do not change qualitatively.

Let us consider the case for \(3.06 \times 10^{-23} T_0^{-3} < \rho_0 < 1.4 \times 10^{-19} T_0^3\), where the pressure is determined by gas pressure and the cooling process is dominated by the URCA process so that Equations (20) and (21) give
\[ 7.18 \times 10^{27} \alpha_{-1}^{-1/3} \rho_0^{1/3}, \]
\[ \times \left( \frac{M}{10^{-3} M_\odot \text{s}^{-1}} \right)^{2/3} \left( \frac{M_{BH}}{3 M_\odot} \right)^{-7/3} \left( \frac{r}{r_{ISCO}} \right)^{-7/2}, \]  
\[ = 9 \times 10^{-43} \rho_0 T_0^6, \]  
\[ (26) \]
with \(p = \rho kT/m_p\).

From Equations (16), (23), and (26), \(\rho\) is expressed as
\[ \rho = 6.47 \times 10^9 \text{ g cm}^{-3} \alpha_{-1}^{-13/10}, \]
\[ \times \left( \frac{M}{10^{-3} M_\odot \text{s}^{-1}} \right)^{-17/10} \left( \frac{r}{r_{ISCO}} \right)^{-51/20}. \]  
\[ (27) \]
We then have
\[ h = 3.77 \times 10^5 \text{ cm}^{-1} \alpha_{-1}^{1/10}, \]
\[ \times \left( \frac{M_{BH}}{3 M_\odot} \right)^{9/10} \left( \frac{r}{r_{ISCO}} \right)^{27/20}, \]  
\[ (28) \]
\[ p = 1.94 \times 10^{28} \text{ dyn cm}^{-2} \alpha_{-1}^{-11/10}, \]
\[ \times \left( \frac{M}{10^{-3} M_\odot \text{s}^{-1}} \right)^{9/10} \left( \frac{M_{BH}}{3 M_\odot} \right)^{-19/10} \left( \frac{r}{r_{ISCO}} \right)^{-57/20}, \]  
\[ (29) \]
\[ T = 3.63 \times 10^{10} K \alpha_{-1}^{1/5} \left( \frac{M_{BH}}{3 M_\odot} \right)^{-1/5} \left( \frac{r}{r_{ISCO}} \right)^{-3/10}, \]  
\[ (30) \]
\[ v_r/v_{cep} = -2.00 \times 10^{-3} \alpha_{-1}^{6/5}, \]
\[ \times \left( \frac{M_{BH}}{3 M_\odot} \right)^{-1/5} \left( \frac{r}{r_{ISCO}} \right)^{7/10}, \]  
\[ (31) \]
\[ v_{cep} = 1.22 \times 10^{10} \text{ cm s}^{-1} \alpha_{-1}^{-1/2} \left( \frac{r}{r_{ISCO}} \right)^{-1/2}, \]  
\[ (32) \]
Then, the surface density of the disk (\(\Sigma\)) is given by
\[ \Sigma = 2.9 \rho h = 4.88 \times 10^{15} \text{ g cm}^{-2} \alpha_{-1}^{-6/5}, \]
\[ \times \left( \frac{\dot{M}}{10^{-3} M_\odot \text{s}^{-1}} \right) \left( \frac{M_{BH}}{3 M_\odot} \right)^{-4/5} \left( \frac{r}{r_{ISCO}} \right)^{-6/5}. \]  
\[ (34) \]
Let us introduce the coordinate \(x\) by \(r = x \times r_{ISCO}\). We assume here that the disk has a minimum and a maximum \(x\) as \(x_{min} = 1\) and \(x_{max}\), respectively. Then for a given total mass \(m_d\) and the total angular momentum \(J_\ell\), we have
\[ m_d = 1.36 \times 10^{-4} M_\odot \alpha_{-1}^{-6/5} (x_{max}^{4/5} - 1) \]
\[ \times \left( \frac{\dot{M}}{10^{-3} M_\odot \text{s}^{-1}} \right) \left( \frac{M_{BH}}{3 M_\odot} \right)^{6/5}. \]  
\[ (35) \]
\[ J_t = 5.48 \times 10^{45} \text{ g cm}^2 \text{ s}^{-1} \alpha_{-1}^{-6/5} \left( x_m^{13/10} - 1 \right) \]
\[ \times \left( \frac{M}{10^{-3} M_\odot \text{ s}^{-1}} \right) \left( \frac{M_{\text{BH}}}{3 M_\odot} \right)^{11/5} \].

For a given value of \( m_d, J_t, \alpha, \) and \( M_{\text{BH}} \), from Equations (35) and (36), we can determine \( M \) and \( x_{\text{max}} \) in general. Let us define a new variable \( \beta \) by \( J_t = m_d \beta \sqrt{GM_{\text{BH}}r_{\text{ISCO}}} \), that is, the mean value of the angular momentum is \( \beta(>1) \) times the minimum value of the specific angular momentum at the ISCO. Then, \( x_{\text{max}} \) is determined by
\[ \frac{x_{\text{max}}^{13/10} - 1}{x_{\text{max}}^{4/5} - 1} = 1.625 \beta. \] (37)

It is easily shown that the left-hand side of Equation (37) is a monotonically increasing function for \( x_{\text{max}} > 1 \) and has a minimum value of 1.625 at \( x_{\text{max}} = 1 \) so that for an arbitrary value of \( \beta > 1 \), there is an unique solution \( x_{\text{max}} > 1 \). In the accretion process, the total angular momentum of the system will be conserved in our case, since the Kerr metric has a rotational Killing vector, i.e., a stationary axisymmetric system. Some of the angular momentum is absorbed by the BH from the ISCO so that \( \beta \) increases as a function of time. Note that the spin up of the BH due to accretion is negligible in our case. If we denote the mass and the angular momentum of the accreted blob into the BH as \( \Delta m_d(\leq 0) \) and \( \Delta J = \Delta m_d \sqrt{GM_{\text{BH}}r_{\text{ISCO}}} \), we have
\[ \Delta \beta = (1 - \beta) \frac{\Delta m_d}{m_d} > 0. \]

The solution of Equation (38) is given by
\[ \beta = 1 + \left( \beta_0 - 1 \right) \frac{m_d^0}{m_d}, \] (39)
where \( \beta_0 \) and \( m_d^0 \) are the initial values of \( \beta \) and \( m_d \), respectively. Therefore, \( \beta \gg 1 \) in the later phase of the accretion so that \( x_{\text{max}} \gg 1 \) and the \( x_{\text{max}} \approx (1.6 \beta)^2 \approx [1.6(\beta_0 - 1) \times m_d^0/m_d]^2 \) will be a good approximation. Inserting this expression into Equation (35), we have
\[ \frac{m_d}{m_d^{13/5}} = -3.47 \text{ s}^{-1} \left[ m_d^0(\beta_0 - 1) \right]^{-8/5} \alpha_{-1}^{-6/5} \left( \frac{M_{\text{BH}}}{3 M_\odot} \right)^{-6/5}. \] (40)

Integration of Equation (40) yields
\[ m_d = \frac{m_d^0}{(t/A + 1)^{5/8}}, \] (41)
\[ A = 0.18 s \left( \beta_0 - 1 \right)^{8/5} \alpha_{-1}^{-6/5} \left( \frac{M_{\text{BH}}}{3 M_\odot} \right)^{6/5}, \] (42)
\[ \dot{m}_d = -\frac{5m_d^0}{8A(t/A + 1)^{13/8}}. \] (43)

The method we adopted here to solve the evolution of the accretion disk is similar to the quasi-static evolution of the star where the nuclear timescale is much longer than the free fall time so that at each time, the star can be regarded in gravitational equilibrium (see, e.g., Kippenhahn & Weigert 1994). In our case, from Equation (32), the accretion velocity is much smaller than the Kepler velocity which determines the dynamical timescale so that we can regard the disk as stationary at each time. The decrease in total mass and angular momentum can be regarded as a decrease in total nuclear energy and a change of the composition in the stellar evolution case, which are very slowly changing in a dynamical timescale.

Let us assume that \( m_0 = 0.1 M_\odot, \beta_0 = 2, M_{\text{BH}} = 3 M_\odot, \) and \( q = 0.5 \) as suggested by numerical relativity calculations (Shibata & Taniguchi 2006; Kiuchi et al. 2009; Rezzolla et al. 2010; Hotokezaka et al. 2011, 2013). Solid lines in Figure 1 show the accretion rates as a function of \( \alpha \) for the representative time such as 1 s, 3 s, 10 s, 30 s, 100 s, and 300 s. We are interested in the late time behavior (\( t > 30 \text{ s} \)) where the accretion rate decreases as a function of \( \alpha \) for a fixed time. This is because \( A \) in Equation (42) is smaller for larger \( \alpha \) so that the accretion rate is smaller. Physically, the accreting velocity is larger for larger viscosity as is clear from Equation (31), where the consumption of the disk mass is faster. The dashed lines are the neutrino cooling ignition accretion rate obtained by Ch\'en & Beloborodov (2007) for \( q = 0 \) and \( q = 0.95 \) (Equation (42) of their paper). Neutrino cooling is effective only above these dashed lines.

(A color version of this figure is available in the online journal.)

The dashed lines are the neutrino cooling ignition accretion rate obtained by Chen & Beloborodov (2007) for \( q = 0 \) and \( q = 0.95 \) (Equation (42) of their paper). Neutrino cooling is effective only above these dashed lines.
To check this, we show in Figure 3 the time evolution of the luminosity for \( m_d^0 = 0.1 \, M_\odot, \beta_0 = 2, M_{\mathrm{BH}} = 3 \, M_\odot, \alpha = 0.01, \) and \( \alpha = 0.01. \) The red and blue solid lines show the numerical solution, which is obtained by integrating Equations (35)–(37) directly over time, the analytic approximation (Equation (43)). The dashed horizontal lines show the neutrino cooling ignition accretion rates for \( q = 0 \) (upper) and \( q = 0.95 \) (lower), respectively. We see that the analytic approximation only slightly overestimates the luminosity. We can justify the use of the analytic approximate solution to Equation (37).

It is suggested that the typical \( \alpha \sim 0.01\text{--}0.02 \) from various numerical simulations (Davis et al. 2010; Guan & Gammie 2011; Blaes et al. 2011; Parkin & Bicknell 2013; Jiang et al. 2013). To have possible isotropic EEs, the typical luminosity of \( \sim 10^{50} \, \text{erg s}^{-1} \) is needed at \( t \sim 10 \, \text{s} \). From Figure 1, if \( \alpha \lesssim 0.01 \), which is relatively smaller than that suggested by various numerical simulations, the accretion rate is above the ignition rate for neutrino cooling up to the observed time of \( t \sim 30(1+z) \, \text{s} \), and the luminosity can be above \( \sim 10^{50} \, \text{erg s}^{-1} \) so that enough luminosity for EEs seems to be obtained via the BZ mechanism.

In Newtonian gravity, a numerical calculation of the accretion disk exists for the present problem, i.e., the time evolution of an accretion disk of mass \( 0.1 \, M_\odot \) after the merger of an NS–NS binary (Metzger et al. 2009). Our treatment also uses Newtonian gravity so that we can compare our analytic results with their numerical calculations to confirm the quantitative agreement. In their calculations, the mass of the central BH is \( 3 \, M_\odot \) and they solved the time evolution of a \( z \)-direction integrated quantity such as the surface density \( \Sigma \) with neutrino and advection cooling. The initial surface density is given as

\[
\Sigma \propto (r/r_{d,0})^5 \exp(-7r/r_{d,0}), \tag{45}
\]

with \( r_{d,0} = 3 \times 10^6 \, \text{cm} \sim 6GM_{\mathrm{BH}}/c^2 \). Since \( \Sigma^2 \) peaks at \( r_{d,0} \), the initial specific angular momentum is \( \sim \sqrt{GM_{\mathrm{BH}} \times 6GM_{\mathrm{BH}}/c^2} \). In our crude model, we assumed the disk boundary is at \( r_{\mathrm{ISCO}} = 6GM_{\mathrm{BH}}/c^2 \) while in their calculation inner edge (=disk boundary) is \( 10^6 \, \text{cm} \sim 2GM_{\mathrm{BH}}/c^2 \) so that we define the coordinate \( x \) by \( r = x \times 2GM_{\mathrm{BH}}/c^2 \) in this paragraph. In the Newtonian calculation, there is no ISCO so that the minimum specific angular momentum of their calculation is \( \sim \sqrt{GM_{\mathrm{BH}} \times 2GM_{\mathrm{BH}}/c^2} \) which is different from our model of \( \sqrt{GM_{\mathrm{BH}} \times 6GM_{\mathrm{BH}}/c^2} \). As for \( \alpha \), they adopt 0.3 so that we need to rewrite Equations (35) and (36) as

\[
\dot{m}_d = 1.53 \times 10^{-5} \, M_\odot \left( \frac{J_{\max}^{1/5}}{5} - 1 \right) \times \left( \frac{\dot{M}}{10^{-3} \, M_\odot \, \text{s}^{-1}} \right) \left( \frac{M_{\mathrm{BH}}}{3 \, M_\odot} \right)^{6/5}, \tag{46}
\]

\[
J_{\max} = 3.52 \times 10^{44} \, \text{g cm}^2 \text{s}^{-1} \left( \frac{J_{\max}^{1/10}}{1} - 1 \right) \times \left( \frac{\dot{M}}{10^{-3} \, M_\odot \, \text{s}^{-1}} \right) \left( \frac{M_{\mathrm{BH}}}{3 \, M_\odot} \right)^{11/5}. \tag{47}
\]

Defining \( J_{\max} \) by \( J_{\max} = m_d \dot{\beta} \sqrt{GM_{\mathrm{BH}} \times 2GM_{\mathrm{BH}}/c^2} \), we have the same equation as Equation (37). The argument for deriving equations corresponding to Equations (38) and (39) is also the same by changing \( \Delta J = \Delta m_d \sqrt{GM_{\mathrm{BH}} \times 2GM_{\mathrm{BH}}/c^2} \) and \( \beta_0 \sim \sqrt{3} \), which gives

\[
\frac{m_d}{m_{\max}^{13/5}} = -51.0 \, \text{s}^{-1} \left( \frac{m_d}{m_{\max}} \right)^{-8/5} \left( \frac{M_{\mathrm{BH}}}{3 \, M_\odot} \right)^{-6/5}. \tag{48}
\]

Integration of Equation (48) yields

\[
m_d = 0.1 \, M_\odot \left( \frac{t}{1.22 \times 10^{-2} \, \text{s}} + 1 \right)^{-5/8}, \tag{49}
\]

\[
\dot{m}_d = -5.12 \, M_\odot \, \text{s}^{-1} \left( \frac{t}{1.22 \times 10^{-2} \, \text{s}} + 1 \right)^{-13/8}. \tag{50}
\]

Now let us compare Equation (50) with those of the numerical calculations by Metzger et al. (2009). Their Figure 3 shows an
accretion rate at \( r = 10^4 \) cm for \( t = 0.01 \) s, 0.1 s and 1 s are \( 1 M_\odot \) s\(^{-1}\), 0.1 \( M_\odot \) s\(^{-1}\) and 4.5 \( \times 10^3 \) \( M_\odot \) s\(^{-1}\), respectively. While Equation (50) yields 1.93 \( M_\odot \) s\(^{-1}\), 0.139 \( M_\odot \) s\(^{-1}\), and 3.9 \( \times 10^3 \) \( M_\odot \) s\(^{-1}\). We can say that our analytic model agrees rather well with the numerical calculations at ISCO especially for later times, which is indispensable for the use of our analytic model to study EEs.

It has been shown that, in the late phase of the accretion of dense debris such as we consider here, the disk wind driven by energy injection via viscous heating and the recombination of nucleons into alpha-particles becomes relevant (Metzger et al. 2008; Beloborodov 2008; Metzger et al. 2009; Lee et al. 2009; Fernández & Metzger 2013). Although our calculation does not include this effect, since such outflows are predominantly triggered after the viscous timescale of the disk, our results can be still viable up to this timescale, e.g., \( t \sim 30(1 + z) \) s for \( \alpha \lesssim 0.01 \) (see Figure 1).

3. EXTENDED X-RAY EMISSION AS AN ELECTROMAGNETIC COUNTERPART OF A COMPACT BINARY MERGER

In the previous section, we showed that the rotational energy of the Kerr BH up to \( E_{\text{BZ}} \sim 10^{52} \) erg can be extracted as the Poynting outflow via the BZ process with a timescale of \( \delta t_{\text{BZ}} \gtrsim 30 \) s if the accretion of the debris \( \sim 0.1 M_\odot \) occurs with \( \alpha \lesssim 0.01 \). Hereafter, we argue the resultant emissions from such outflows and their detectability.

In the course of compact binary mergers, a fraction of baryons of mass \( M \sim 10^{-2} - 10^{-3} M_\odot \) can be ejected with an expansion velocity of \( v_\exp \sim 0.1c \) (Hotokezaka et al. 2013). A certain duration after the merger, say \( \delta t \sim 0.1 \) s, the hypermassive NS collapses into a BH due to the loss of rotational support by emitting gravitational waves (GWs) and/or Poynting fluxes. The Poynting outflow by the BZ process, which is relativistic, clashes with the pre-ejecta. The BZ outflow or jet will be more or less beamed and drill through the pre-ejecta, forming a hot plasma cocoon surrounding the jet. Recently, such a situation has been investigated numerically (Nagakura et al. 2014; Murguia-Berthier et al. 2014), although the jet injection timescale is set to be \(< 1\) s, considering jets responsible for prompt emissions of SGRBs. These studies show that the jet dynamics are significantly affected by the pre-ejecta, especially for a jet luminosity of \( \lesssim 10^{53} \) erg s\(^{-1}\) and a pre-ejecta mass ejection rate of \( \lesssim 10^{-3} M_\odot \) s\(^{-1}\), which we are interested in here. In particular, if the jet launch is delayed more than \( \delta t \gtrsim 0.1 \) s from the pre-ejecta launch, such a jet will be choked in the ejecta and a significant fraction of its energy will be dissipated inside the ejecta, forming a cocoon fireball. In our case, the assumed duration of the outflow injection is \( \gtrsim 10 \) s, thus the BZ outflow would more easily penetrate the pre-ejecta if the onset time of outflow injection is not delayed significantly. Nevertheless, even after penetration, a fraction of the BZ-outflow energy can dissipate at the interaction surface with the pre-ejecta or the cocoon, which typically occurs at

\[
r_o \sim (0.1 - 1) \times c \delta t_{\text{BZ}} \sim 3.0 \times 10^{11} - 12 \text{ cm} \delta t_{\text{BZ}, 2},
\]

where \( \delta t_{\text{BZ}} \) corresponds to the time \( t \) in the previous section. Note that the BZ outflow is most likely magnetically dominated at the launching radius. In the case of magnetically dominated jets, the dynamics including the cocoon formation are different from those of hydrodynamical jets (e.g., Bromberg et al. 2014), and the energy dissipation process is rather uncertain. Hereafter, we simply parameterize such a dissipation process by the fraction of dissipated energy, \( \xi \ll 1 \) and the beaming factor of the dissipation region, \( \Delta \Omega \).

After dissipation, the heated pre-ejecta can be regarded as a fireball. The temperature can be estimated as \( T' \approx (3 \xi E_{\text{BZ}}/4 \pi \delta t_{\text{BZ}}) \Delta \Omega^{-1/4} \delta t_{\text{BZ}, 2}^{-1/4} v_{\text{ph}}^{-1/4} r_o^{-1/2} \). (52)

We note that the optical depth at this radius is large, \( \tau_y \sim Y_T \kappa_T \rho \nu \gtrsim 4.2 \times 10^7 (Y_e/0.2) M_\odot r_r^{-1/2} \delta t_{\text{BZ}, 2}^{-1/2} \). Here, \( \kappa_T \sim 0.2 \) g cm\(^{-2}\) is the opacity of the Thomson scattering, \( \rho \sim M/4 \pi r_r^2 c \delta t \sim 5.3 \times 10^{-4} M_\odot r_r^{-2} \delta t_{\text{BZ}, 2} \) g cm\(^{-3}\) is the mean density of the ejecta, and \( M_{\odot} = M/10^{-2} M_\odot \) is the isotropic mass ejection. Though the mass ejection, in general, is anisotropic (e.g., Hotokezaka et al. 2013), we can take into account this effect by changing \( \Delta \Omega \) and \( M \) appropriately. The fireball is accelerated due to the large internal energy, and the Lorentz factor saturates at \( \Gamma \sim 4 \pi \xi E_{\text{BZ}}/\Delta \Omega M c^2 \), or

\[
\Gamma \sim 7.0 \xi E_{\text{BZ}, 2} M_{\odot}^{-2} \Delta \Omega^{-1},
\]

which occurs at

\[
r_r \sim 12 \times 10^{13} cm \xi E_{\text{BZ}, 2} M_{\odot}^{-2} \Delta \Omega^{-1} r_o, 12. (54)
\]

Hereafter, we simply assume \( \Gamma \propto r \) in the acceleration phase. For \( r > r_s \), the fireball moves as a shell with a shell width \( \sim r_s \) so that the temperature decreases as \( T' \propto (r/r_s)^{-2/3} \), while in the lateral direction \( y \), it expands as \( y \sim r T' / T \) (e.g., Mészáros & Rees 2000). Then, the fireball begins to expand almost spherically, irrespective of the initial beaming angle beyond the radius given by

\[
T' \propto (r/r_{\exp})^{-1} \Gamma \propto (r/r_{\exp})^{-1} \delta t_{\text{BZ}, 2}^{-1/2} \delta t_{\text{BZ}, 2}^{-1/2} r_{\exp}. (55)
\]

The temperature decreases as \( T' \propto (r/r_{\exp})^{-1} \) for \( r > r_{\exp} \). The photospheric emission from the fireball can be expected around the photospheric radius,

\[
r_{\text{ph}} \sim \left( \frac{Y_T \kappa_T M}{4 \pi} \right)^{1/2} \approx 2.5 \times 10^{14} \text{ cm} (Y_e/0.2)^{1/2} M_{\odot}^{1/2}. (56)
\]

The temperature of the fireball in the comoving frame evolves as \( T' \propto (r/r_s)^{-1} \) for \( r_s < r < r_s \), \( T' \propto (r/r_s)^{-2/3} \) for \( r_s < r < r_{\exp} \), and \( T' \propto (r/r_{\exp})^{-1} \) for \( r_{\exp} < r \). From Equations (55) and (56), \( r_{\text{ph}} > r_{\exp} \) is satisfied if \( M \) is larger than the critical mass given by

\[
M_{\odot} > 0.76 (Y_e/0.2)^{-1/5} E_{\text{BZ}, 2}^{4/5} \Delta \Omega^{-4/5} r_{\text{BZ}, 2}^{2/5} r_o, 12. (57)
\]

In the case of dirty fireworks with \( M_{\odot} \sim 1 \), \( r_{\text{ph}} > r_{\exp} \) is expected. The temperature at the photospheric radius becomes \( T_{\text{ph}} \sim T'_{\text{ph}} \sim T'_{\text{ph}} (r_s/r_s)^{-1}(r_{\exp}/r_s)^{-2/3}(r_{\text{ph}}/r_{\text{ph}})^{-1} \), which is expressed as

\[
k_B T_{\text{ph}}' \sim 22 eV (Y_e/0.2)^{-1/2} \times \xi^{7/12} E_{\text{BZ}, 2}^{7/12} M_{\odot}^{-1/2} \delta t_{\text{BZ}, 2}^{-1/2} r_{\text{BZ}, 2}^{-1/2} \delta t_{\text{BZ}, 2}^{-1/2} \delta t_{\text{BZ}, 2}^{-1/2} r_o, 12. (58)
\]

\footnote{If only a tiny fraction of Poynting energies is dissipated at \( r = r_s \), the fireball is still magnetically dominated, and the evolution of the Lorentz factor is generally different from the above scaling; \( \Gamma \propto r^{\mu} \) with \( 1/3 \lesssim \mu \lesssim 1 \).}
and the peak photon energy of the resultant photospheric emission can be estimated as $\epsilon_{\text{peak}} \approx 2.83 k_B T_{\text{ph}}'\Gamma/(1 + z)$, which is given by

$$\epsilon_{\text{peak}} \approx 0.40 \text{ keV} \left(1 + z\right)^{-1} \left(\epsilon_{\text{cut}}/0.2\right)^{-1/2} \times 10^{19/12} E_{\text{BZ,52}}' M_{-2}^{-11/6} \delta_{\text{BZ,2}}^{-1/4} \Delta\Omega^{-1/9} r_{\text{ph},12}^{-1/2}. \quad (59)$$

The peak intensity in the comoving frame can be approximated as $I_{\nu,\text{peak}}' \approx 2\left(\nu_{\text{peak}}'/c\right)^2 \times 2.83 k_B T_{\text{ph}}' \exp(-2.83)$, where $\nu_{\text{peak}}' \approx 2.83 k_B T_{\text{ph}}'/h$ with the Planck constant $h = 6.62 \times 10^{-27}$ erg s.\(^8\)

In the observer frame, using the fact that $L_{\nu}/\nu^3$ is Lorentz invariant (e.g., Rybicki & Lightman 1979), the corresponding energy flux is given by

$$F_{\text{peak}} \approx \frac{\left(1 + z\right)^2}{\Gamma^2} \left(\frac{r_{\text{ph}}}{d_L}\right)^2 \frac{\epsilon_{\text{peak}}^3}{h^2 c^2} \times 1.1 \times 10^{-23} \text{ erg cm}^{-2} \text{ s}^{-1} \text{ Hz}^{-1} \left(1 + z\right) \times \left(\frac{d_L}{200 \text{ Mpc}}\right)^{-2} \left(\frac{\epsilon_{\text{ph}}}{0.1}\right)^{-1/2} \times \xi^{11/4} E_{\text{BZ,52}}' M_{-2}^{-3/2} \delta_{\text{BZ,2}}^{-3/2} \Delta\Omega^{-11/4} r_{\text{ph},12}^{-1/2}. \quad (60)$$

$$\nu_{\text{peak}}' F_{\text{peak}} \sim 1.2 \times 10^{-6} \text{ erg cm}^{-2} \text{ s}^{-1} \times \left(\frac{d_L}{200 \text{ Mpc}}\right)^{-2} \left(\frac{\epsilon_{\text{ph}}}{0.1}\right)^{-1} \times \xi^{13/3} E_{\text{BZ,52}}' M_{-2}^{-13/3} \delta_{\text{BZ,2}}^{-13/3} \Delta\Omega^{-13/3} r_{\text{ph},12}^{-2}. \quad (61)$$

In general, the observed duration of the photospheric emission is given by

$$t_{\text{dur}} \sim (1 + z) \times \max\{r_{\text{ph}}/c \Gamma^2, \delta_{\text{BZ}}\}. \quad (62)$$

In the case of dirty fireballs, $r_{\text{ph}} > r_{\exp} \sim c \delta_{\text{BZ}} \Gamma^2$ (see Equations (51) and (55)), and thus the first term on the right-hand side of the above equation is always larger than the second one. Then,

$$t_{\text{dur}} \sim 170 \text{ s} \left(1 + z\right) \left(\epsilon_{\text{cut}}/0.2\right)^{1/2} \xi^{-2} E_{\text{BZ,52}}' M_{-2}^{-5/2} \Delta\Omega^2. \quad (63)$$

We remark that EE durations of $t_{\text{dur}} > 100 \text{ s}$ do not necessarily require durations of the BZ jet of $\delta_{\text{BZ}} > 100 \text{ s}$ if the fireball is dirty and the duration is determined by Equation (63). For a fiducial parameter set ($\xi = 1$, $E_{\text{BZ,52}}' = 1$, $M_{-2} = 1$, $\delta_{\text{BZ,2}} = 1$, $\Delta\Omega = 1$), the emission is characterized by $\epsilon_{\text{peak}} \sim 0.42 \text{ keV}$, $v_{\text{peak}} F_{\text{peak}} \sim 1.2 \times 10^{-6} \text{ erg cm}^{-2} \text{ s}^{-1}$, and $t_{\text{dur}} \sim 180 \text{ s}$ from $d_L = 200 \text{ Mpc}$, and $\epsilon_{\text{peak}} \sim 0.29 \text{ keV}$, $v_{\text{peak}} F_{\text{peak}} \sim 6.0 \times 10^{-9} \text{ erg cm}^{-2} \text{ s}^{-1}$, and $t_{\text{dur}} \sim 260 \text{ s}$ from $z = 0.5$.

In general, the spectral shape is determined by additional dissipation processes, e.g., internal shocks or magnetic reconnectons occurring in $r_{\nu} < r < r_{\text{ph}}$, which may slightly boost the peak energy and most likely produce a quasi-thermal spectrum,

$$F \approx F_{\text{peak}} \times \frac{\left(\epsilon/\epsilon_{\text{peak}}\right)^2}{\left(\epsilon/\epsilon_{\text{peak}}\right)^\beta} \quad \text{for } \epsilon < \epsilon_{\text{peak}}, \quad \text{for } \epsilon_{\text{peak}} < \epsilon < \epsilon_{\text{cut}}. \quad (64)$$

\(^8\) Note that here $h$ is not the half thickness of the disk but the Planck constant. Also we use $\beta$ as the spectral index in this section.
 (>20 keV). We take into account the sky coverage of the Wide-Field MAXI ∼20% and the anticipated duty cycle, ∼80% for the Wide-Field MAXI and ∼90% for the second generation GW network. In the case of dirty fireballs, the beaming factor can be relatively large, e.g., ΔΩ_{softEE} ∼ 1, and the above estimate gives ∼0.5 yr^{-1} for R_{GW} ∼ 40 yr^{-1} and f_{softEE} ∼ 1. With a planned detection threshold flux of the Wide-Field MAXI ∼1.0 × 10^{-9} erg s^{-1} cm^{-2}, such soft EEs can be detectable from z ∼ 0.5, which corresponds to a luminosity distance of d_L = 2.8 Gpc and a comoving volume of V_{softEE,MAXI} = 28 Gpc^3.

The anticipated total detection rate can be estimated as

\sim 430 \text{ yr}^{-1} f_{softEE} \Delta \Omega_{softEE} (R_{GW}/40 \text{ yr}^{-1}). \tag{66}

Next let us consider the observed EEs in our scenario. As we argued above, the dissipative photophoretic emissions from dirty fireballs are too soft and too dim for Swift BAT as far as M_{-2} ∼ 1 and ΔΩ ∼ 1. Importantly, the typical beaming angle of the observed EEs can be estimated to be much smaller as follows. The EEs are being detected predominantly by Swift BAT with the image trigger. Here we set the trigger threshold as 2 photon cm^{-2} in 64 s (Toma et al. 2011). In this case, the trigger threshold by BAT for a burst with β = −1 can be calculated as F_{thr,BAT} = 1.9 × 10^{-9} erg s^{-1} cm^{-2}. For an EE with a mean luminosity of L_{iso,15-150 keV} ∼ 10^{49} erg s^{-1} (like SGRB 061006), the detection horizon also becomes d_L = 6.6 Gpc, corresponding to z = 1 and V_{EE,BAT} = 150 Gpc^3. For an effective total observation time for the BAT of T_{obs,BAT} = 0.8 × 6 yr, and a sky coverage of the BAT of 15%, the number of detectable EEs becomes ∼1.4 × 10^7 (R_{GW}/40 yr^{-1}). Given that 14 EEs have been identified in this interval (Gompertz et al. 2013), the fraction of EE bursts f_{EE} and the beaming factor ΔΩ_{EE}/4π can be constrained as f_{EE}(ΔΩ_{EE}/4π) ∼ 9.7 × 10^{-5} (R_{GW}/40 yr^{-1})^{-1}, or

ΔΩ_{EE} \sim 4.9 × 10^{-3} \left(\frac{f_{EE}}{0.25}\right)^{-1} \left(\frac{R_{GW}}{40 \text{ yr}^{-1}}\right)^{-1}, \tag{67}

which is small compared to the inferred beaming factor of the prompt spikes, ΔΩ_{GPS}/4π ∼ 10^{-3} (Fong et al. 2012). We should mention that candidates of “orphan” EEs without prompt spikes, i.e., long GRBs with T_{90} ≥ 100 whose redshifts and host galaxies are not identified, have been detected by Swift BAT. The detection rate of those candidates is roughly comparable to that of SGRBs with EEs. Given this fact, the constraint on the beaming factor (Equation (67)) is a lower limit and can be larger by a factor. Nevertheless, the possible simultaneous detection rate of such EEs and GWs from NS–NS mergers would be quite small, ∼0.15 × 0.9 × R_{GW} f_{EE}(ΔΩ_{EE}/4π) ∼ 10^{-3} yr^{-1}. Note that the estimated value is independent of the relatively uncertain binary NS merger rate.

In the context of our picture, a smaller ΔΩ corresponds to a relatively narrow fireball. We note that the mass ejection associated with the BH–NS merger is orders of magnitude smaller in the polar direction than the average angled one, which has been confirmed by numerical simulations (Kiyotoku et al. 2013) where the BH spin is set to be parallel to the orbital angular momentum. In general, it is expected that they are misaligned due to the kick velocity in the formation process of NSs. Although more numerical simulations using general initial conditions are needed to know what happens in BH–NS mergers, the simulations by Kiyotoku et al. (2013) suggest that the matter along the BZ jet axis is small compared with an NS–NS binary. As a result, one can expect that a smaller fraction of energy is dissipated in a smaller region of the pre-ejecta compared with the previous NS–NS cases where the outflows via the BZ process clash with denser pre-ejecta. Therefore, we take ΔΩ ∼ 10^{-4}, M ∼ 10^{-4} M_{⊙}, ΔΩ ∼ 10^{-2} as a fiducial value.

Such fireballs are clean in that r_{ph} < r_{o}, i.e.,

M_{-4} < 2.3 (Y_e/0.2)^{-1/3} \xi^{2/3}_{-3} E_{BZ,52}^{1/3} \Delta \Omega^{-2/3}_{-2} \delta t_{o,12}^{-3/2}. \tag{68}

The Lorentz factor of such clean fireballs and the comoving temperature at the photospheric radius are T_{ph} ∼ 76 (r_{ph}/r_{o})^{-2} and Γ ∼ 10^{-4}/r_{ph}/r_{o}, respectively, and the peak energy of the photospheric emission \epsilon_{\text{peak}} ∼ 2.83 k_B T_{ph}^2 Γ/(1 + z) can be estimated as

\epsilon_{\text{peak}} \sim 4.3 \text{ keV} (1 + z)^{1/3} \xi^{-1/3}_{-3} E_{BZ,52}^{1/4} \delta t_{o,12}^{-1/2}. \tag{69}

From Equation (60), the corresponding peak flux is

v_{\text{peak}} F_{\text{peak}} \sim 1.0 \times 10^{-5} \text{ erg cm}^{-2} \text{ s}^{-1} \frac{d_L}{(200 \text{ Mpc})^2} \approx \xi^{-3}_{-3} E_{BZ,52} \delta t_{o,12}^{-1} \Delta \Omega^{-2}_{-2}. \tag{70}

In the case of clean fireballs, the emission duration is t_{\text{dur}} ∼ (1 + z) × \delta t_{BZ}.

For a fiducial parameter set (ξ = 10^{-3}, E_{BZ,52} = 1, M_{-4} = 1, δt_{BZ,2} = 0.7, ΔΩ_{-2} = 1), the emission is characterized by \epsilon_{\text{peak}} ∼ 3.5 keV, v_{\text{m,ph}} ∼ 1.2 × 10^{-7} erg cm^{-2} s^{-1}, and t_{\text{dur}} ∼ 100 s from z = 0.4337 (see Figure 4 where we also plot an observed flux spectrum of EE associated with SGRB 061006 at z = 0.4337). The observed EEs can be consistently interpreted as the high energy tail of the photospheric emission from such a clean fireball. To test this scenario, simultaneous detections in the soft X-ray band (< 10 keV) are crucial.

4. DISCUSSIONS

As for the EEs from fireballs, we need to model, e.g., the subphotospheric dissipation processes in detail to predict the spectra more precisely. Nevertheless, a key message here is that the typical energy of EEs is likely to be in soft X-ray bands. In our scenario, this is essentially due to the relatively large launching radii of the fireball, r_o ≥ 10^{11} cm (Equation (51)) compared with that of the conventional GRB fireball, r_o ≤ 10^{8} cm, which results in a lower initial temperature (Equation (52)). The importance of soft X-ray bands has been implied from the observed soft photon index of EEs and the increase of the fraction of SGRB with EEs in softer bands.

We strongly encourage soft X-ray survey facilities like the Wide-Field MAXI, which can provide a useful electromagnetic counterpart to GWs from compact binary mergers with an angular resolution of ∼0:1. Such EE counterparts are also important in terms of time domain astronomy since they would be observed only ∼1 s after the mergers. If a larger detection rate such as Equation (66) is realized, a statistical technique using a stacking approach might also be possible for the detection of GWs with the aid of soft EE counterparts.

In our scenario, an EE duration of a few 10 s is attributed to a relatively small disk viscosity of α ≤ 0.01 and the effect of the disk spreading during the accretion. On the other hand, such a relatively long duration may also be realized if the disk accretion

http://swift.gsfc.nasa.gov/archive/grb_table/
is suspended, but still the rotation energy of the BH is extracted via the interaction between the BH and the disk magnetosphere (van Putten & Levinson 2003).

So far, we have focused on the emission mechanism of EEs, and not discussed that of the initial spike. In our picture, the initial spike can be provided by the initial inhomogeneity in the ejecta. If there is a direction with low column density, either the BZ jet, the neutrino–antineutrino pair annihilation jet, or the magnetic tower jet would cause the initial spike. Whatever the origin of the initial spike is, our point here is that the major component of SGRBs can be the EEs in terms of the energetics. If a half opening angle of the outflow responsible for the initial spike is \( \sim 0.1 \), the total energy of the initial spike can be two to three orders of magnitude smaller than that of the EEs. Relatively soft EEs without initial spikes might already have been detected by, e.g., Swift, but misidentified as other types of events. One might think that soft EEs should have already been detected by MAXI\(^{11}\) though the rapid sky sweeping (\( \sim 4\pi/90 \) minutes) makes it difficult to identify the \( \sim 100 \) s emissions. Our scenario can be more clearly tested by future soft X-ray observations.

We note that the pressure in Equation (29) is a factor of five larger than that estimated by the general relativistic calculation by Chen & Beloborodov (2007), which is due to our Newtonian treatment. This causes a factor of five overestimate of the BZ power through Equation (44). To take into account this fact as well as the difference between our Newtonian dynamics and the general relativistic one, we add a new phenomenological parameter \( \xi_B \) in Equation (44) as

\[
\frac{B^2}{8\pi} = \xi_B P_{\text{ISCO}},
\]

Figure 5 shows the total energy of a BZ jet as a function of \( \alpha \) for \( m_0^B = 0.1 \ M_\odot \), \( \beta_0 = 2 \), \( M_{\text{BH}} = 3 \ M_\odot \), and \( q = 0.5 \) for three values of \( \xi_B \). The total energy is obtained by integrating the BZ luminosity over time with a mass accretion rate that is larger than the ignition rate of neutrino cooling, which is determined by Chen & Beloborodov (2007) for \( q = 0.95 \) (Equation 42 of their paper). The dashed lines show the rotational energy of a BH with \( q = 0.5 \) and \( q = 0.95 \), respectively. For low \( \alpha \), the figure shows that either the effect of a back reaction is needed, \( \xi_B \) is small, or the disk accretion is suspended for a while as discussed in the text.

(A color version of this figure is available in the online journal.)

\(^{11}\) http://maxi.riken.jp/top/

Figure 5. Total BZ energy as a function of \( \alpha \) for \( m_0^B = 0.1 \ M_\odot \), \( \beta_0 = 2 \), \( M_{\text{BH}} = 3 \ M_\odot \), and \( q = 0.5 \). The total energy is obtained by integrating the BZ luminosity over time with a mass accretion rate larger than the ignition rate of neutrino cooling, which is determined by Chen & Beloborodov (2007) for \( q = 0.95 \) (Equation 42 of their paper). The red lines show the results for the different efficiency parameters \( \xi_B \) in Equation (71): \( \xi_B = 1.0 \) for the solid, \( \xi_B = 0.1 \) for the dashed, and \( \xi_B = 0.01 \) for the dotted lines, respectively. The dashed black lines show the rotational energy of a BH with \( q = 0.5 \) and \( q = 0.95 \), respectively. For low \( \alpha \), the figure shows that either the effect of a back reaction is needed, \( \xi_B \) is small, or the disk accretion is suspended for a while as discussed in the text.

1. The reason is simple and clear. Let us consider the non-relativistic one-dimensional diffusion equation for some quantity \( Q(t,x) \),

\[
\frac{\partial Q}{\partial t} = D \frac{\partial^2 Q}{\partial x^2},
\]

where \( D \) is the diffusion constant. If we set the delta function source as

\[
\frac{\partial^2 Q}{\partial x^2} - D^{-1} \frac{\partial Q}{\partial t} = -C \delta(x) \delta(t),
\]

the solution for \( t > 0 \) is expressed as

\[
Q = C \sqrt{\frac{1}{4\pi Dt}} \exp \left( -\frac{x^2}{4Dt} \right).
\]

The above solution clearly shows that the initial disturbance at \( t = 0 \) and \( x = 0 \) propagates to any \( x \) even for any very small value of \( t > 0 \), which means the causality is violated, i.e., the information propagates with infinite speed. The simple rule of changing the non-relativistic equation into a general relativistic one such as changing the derivative to the covariant derivative and using the projection of the tensor with \( \gamma^{\mu\nu} \equiv g^{\mu\nu} + u^\mu u^\nu \) does not help to guarantee the causality. We need to add several new terms with undetermined parameters to the basic equations such as done by Israel & Stewart (1979). If we can start from the generalized relativistic Boltzmann equation, there is no problem as for the causality in principle. However, in practice, we should treat the distribution function that depends on three coordinates and three momenta, but it is beyond the ability of the present computer power to simulate such a problem. One may think that the general relativistic resistive MHD simulations in three dimensions are enough to solve this problem since the causality is not violated in such a system. However, the Boltzmann equation should be solved anyway since the gyration radius of the proton is typically comparable to the mean free path of \( p-n \) collision at around the ISCO.
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