THE FORMULATION AND VISUALIZATION OF 3D FRACTALS AS REAL-TIME MODELS

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Abstract

The area of fractal modeling is a present-day applicative growth. Fractals contain unlimited amount of information in contradiction to conventional geometric shapes. A well-established method of creating fractals is by means of Iterated Function Systems, with extra-ordinary work done on 2D IFS, where the rendering of the same acquired in an easy and effective manner. Though the 3D IFS transpires/takes shape as a natural world derived add-on, more research has to be carried on in it in real-world fractal science and engineering. Here 3D IFS is used to get enchanting fractals by applying algorithms. The methods used here have a widespread use in fractal science very, an example being, recursive fractals elucidated through algebraic transformations. Also presented is a suitable algorithm for processing of arrays. Finally, the outputs obtained are passed through shading and exposure to get a viewing picture. The processes used above result in producing modified versions of objects in a variety of shape and texture.

Keywords: Fractals, IFS, Self-similarity, Time Complexity, Time Image

I. Introduction

Tracing back to two decades in fractal theory [II], the domain of rise to several kinds of shapes depicting all the characteristics of fractals in nature. This research is the credit of its founder Benoit Mandelbrot, whose contribution is used by the present-day fractal world.

There are two methods for generation of fractals- the former being fractals produced recursively by iteration of algebraic equations, and the latter one is by using IFS affine transformations. This theory has a great background for significant research of image texture. This is achieved by means of a set of simple equations, which give rise to tools for generating recursive images. As is well known, we have extensive research on 2D IFS rendered fractals [III-I], using programs like Fract int, Fdesign etc., within easy reach of every one.

The mathematical exploration of these shapes leads us to ideate their 3D and 4D corresponding images in 3 and 4-dimensional space. The thrust behind this work is to
find a class of corresponding geometric shapes in 3D and 4D which are produced by applying recursive iterations in quaternions. These shapes are perceived by image construction in 3-D visual background.

This paper presents a method for creation of above shapes in 3D. The techniques presented are synonymous with generation of connected surfaces, which in turn generate boundaries in between regions recounted with the help of different criteria. These types of approaches can be used to find out and show any such surfaces.

II. 3D IFS Fractal Representation

The basic property of fractals is that they are self-similar. An IFS fractal is made up of a replica of itself, where each copy produced by affine transforms. Let us think about an IFS having transforms t₁, t₂, t₃,…….,tn. The equivalent fractals obtained are a set of n similar replicas by the appropriate transformations of n. These n replicas are appropriately identified as “t₁”, “t₂”, “t₃”, ……., “tn”. We can divide each of these n replicas further as outlined below:

Take into account the copy identified with “ti”. It’s n smaller components can have the identifiable strings as “tit₁”, “tit₂”, “tit₃”, ……., “titn”.

The results got making use of the above process is shown in Fig1. To Fig3. Fig1. Shows Ferns contained in Sierpinski Gasket, Fig2. Depicts Pyramid with trees all around it, and Fig3. Gives a picture of Clouds depicted on a carpet, from which we can draw a conclusion that IFS fractals generate fine and beautiful 3D scenes. These 3D scenes can also be colored, lighted and given mist effects, further enhancing their aestheticity.

![Fig 1: Ferns on Sierpinski Gasket](image_url)
III. Surface Determination

Fractals having invariant surfaces are obtained by recursive iteration of function, after which, track the points which meet a particular criteria after multiple iterations. The result of this is tested to trace out whether starting point lies in or out a particular invariant set. From this we conclude that if this point is inside, the particular corresponding to the point is inside. But this gives us only a sampling of the surface. The question is to find out a distinction between the grid points and their correlated volumes. The surfaces are traced out by taking a 3D grid-like surface and assessing every point on this grid along with boundary point selection, such that it correlates with the conventions presented [V, VI] irrespective of the hindrances occurring.

We also state a second method wherein, points which are farther from the surface remain uncalkulated, after which it is traced all over the grid. The interior points beside the exterior points are preserved as proven boundary points and their adjacent points serve as a candidate for the upcoming cycle. The advantage of this is reduction of function evaluation to a greater extant. The total function evaluation varies with

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the fractal dimension of the surface under consideration. This allows us to know the connectivity of the surface. From this we have a set of boundary points on the grid with the following properties:

- Take into account the boundary points lying inside the shape.
- No less than one of the neighbors of the boundary points lie out of the shape in consideration.
- The boundary points lie in connection with a set of starting points over the grid which in turn consisting only boundary points.
- The surface is outlined making use of an initial list of boundary points.

We can make interesting comparisons based on quality between “interior” and “exterior of a given mathematical boundary, and not always considering bounded and unbounded sets. This is made possible by this algorithm, as illustrated in Fig. 4. through Fig. 9.

IV. Methodology of Visualization

We have the visualization process in two steps as follows:

- Assigning illumination intensity to each vertex which conforms to vertices of an imagining light source.
- Image development corresponding to the viewers’ direction.

Step-1 has two passes through the data points. Step-2 comprises of one pass through the data in sequence. The recursive repetition of this step gives rise to a series of views of the object with different angles. The z-buffer stands as a foundation for the above two steps, wherein sorting is not necessary. Initial sorting of data is done at the outset. The surface cubes are assumed to be translucent.

Finally the points are projected in the viewer’s direction wherein intensities are used to know the brightness of a given point. The result is a direct z-buffer using an orthogonal projection. The image z-buffer contains the single isolated points which when projected to several nearby positions, given the fact where the visible and hidden surfaces enshroud each other. This results in a half-tone picture.

V. Illustrations 4 through 9

The figures used in this paper are examples shapes incorporated by making use of the above methods. Fig4. and Fig5. show 1-parameter ménage of fractals in the complex plane. A few shapes produced by recursive iteration of quadratic equations in 4D are illustrated in Fig6. through Fig 9.
Fig. 4. Two 3-D objects outlined as 1-parameter families of fractal curves.

Fig. 5. A fractal surface invariant under 7-fold iteration of a polynomial in the Quaternions.

Fig. 6. Different components of the set attracted to a cycle of length 4.

Fig. 7. Several components of the domain attracted to a cycle of length 4.

Fig. 8. One of the components of the shape defined by $\frac{1}{2}(2^n + 1)$.

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VI. Conclusion

This paper brings forth the usage of 3D IFS in generating fractals produced in real-time, along with IFS rendering of the same, along with a description of display of such shapes in three dimensions. The paper also presents the process for formulation and visualization of connected surfaces with boundary creation of surfaces entrenched by mathematical parameters. Finally, it also elucidates the method of generation of the above, along with display of a half-tone image.

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