Calculation of the pentaquark width by QCD sum rule

A.G.Oganesian
Institute of Theoretical and Experimental Physics,
B.Cheremushkinskaya 25, 117218 Moscow, Russia

Abstract

The pentaquark width is calculated in QCD sum rules. Result for $\Gamma_\Theta$ show, that $\Gamma_\Theta$ can vary in the region less than $1\text{MeV}$. The main conclusion is, that if pentaquark is genuine states then sum rules really predict the narrow width of pentaquark $\Theta^+$, and the suppression of the width is both parametrical and numerical.

PACS: 12.39 Dc, 12.39-x, 12.38

In this paper we will discuss the the narrow exotic baryon resonance $\Theta^+$ with quark content $\Theta^+ = uudd\bar{s}$ and mass 1.54 GeV. This resonance had been discovered by two groups \[1, 2\]. Later, the existence of this resonance was confirmed by many other groups, although some searches for it were unsuccessful. Moreover, last year some groups, which have seen pentaquark, in the new experiments with higher statistics, reported null result for pentaquark signal (CLAS experiments on hydrogen and deuterium \[3\], BELLE \[4\]) but at the same time DIANA \[5\], and also LEPS, SVD-2 confirm their results with higher statistic(see \[6\] for the review).

So the status of $\Theta^+$, predicted in 1997 by D.Diakonov, V.Petrov and M.Polyakov \[7\] in the Chiral Soliton Model, till now is doubtful. (Some theoretical reviews are given in \[8, 9\]).

One of the most interesting features of $\Theta^+$ is its very narrow width. Experimentally, only an upper limit was found, the stringer bound was presented in \[2\]: $\Gamma < 9\text{MeV}$. The phase analysis of $KN$ scattering results in the even stronger limit on $\Gamma$ \[10\], $\Gamma < 1\text{MeV}$. A close to the latter limitation was found in \[11\] from the analysis of $Kd \rightarrow ppK$ reaction and in \[12\] from $K + Xe$ collisions data \[2\]. Also \[4\] from the negative result of the experiment give the upper limit for pentaquark width less than $640\text{KeV}$.

Such extremely narrow width of $\Theta^+$ (less than $1\text{MeV}$) seems to be very interesting theoretical problem. In the paper \[13, 14\] it was shown, that if pentaquark is genuine
state it width should be strongly parametrically suppressed. It was shown, that the conclusion does not depend of the choice of the pentaquark current (without derivatives). Recently in [15] the numerical estimation of pentaquark width (about 0.75 MeV for pentaquark state with positive parity) was obtained also by use QCD sum rules. The main goal of this paper is to find numerical estimation of pentaquark width by use the method offered in [13], [14].

Part 1. In the papers [13], [14] it was shown, that pentaquark width should be suppressed as \( \Gamma_0 \sim a^2_0 \langle 0 | \bar{q} q | 0 \rangle^2 \), (for any current without derivatives). Let us shortly remind the main points of the method. We start from 3-point correlator

\[
\Pi_\mu = \int e^{i(p_1 x - q y)} \langle 0 | \eta_\theta(x) j_\mu^5(y) \eta_n(0) | 0 \rangle \tag{1}
\]

where \( \eta_n(x) \) is the neutron quark current [16], \( \langle \eta_n | n \rangle = \lambda_n v_n, (v_n \) is the nucleon spinor), \( \eta_\theta \) is an arbitrary pentaquark current \( \langle 0 | \eta_\theta | \theta^+ \rangle = \lambda_\theta v_\theta \) and \( j_\mu^5 = \bar{s} \gamma_\mu \gamma_5 u \) is the strange axial current.

As an example of \( \eta_\theta \) one can use the following one (see [17], where it was first offered, and also [18], where the sum 2-point rule analysis for this current was discussed):

\[
J_A = \varepsilon^{abc} \varepsilon^{def} \varepsilon^{gcf} (u^T C d^\phi) (u^T C \gamma^\mu \gamma_5 d^\phi) \gamma^\nu c \bar{s} \bar{g} \tag{2}
\]

and we will use it farther to obtain numerical results.

As usual in QCD sum rule the physical representation of correlator (1) can be saturated by lower resonance states plus continuum (both in \( \eta_\theta \) and nucleon channel)

\[
\Pi_\mu^{phys} = \langle 0 | \eta_\theta | \theta^+ \rangle \langle \theta^+ | j_\mu | n \rangle \langle n | \eta_n | 0 \rangle \frac{1}{p_1^2 - m_\theta^2} \frac{1}{p_2^2 - m^2} + \text{cont.} \tag{3}
\]

where \( p_2 = p_1 - q \) is nucleon momentum, \( m \) and \( m_\theta \) are nucleon and pentaquark masses.

Obviously, in the limit of massless kaon

\[
\langle \theta^+ | j_\mu | n \rangle = g^A_{\theta n} \bar{v}_n \left( g^{\mu \nu} - \frac{q^\mu q^\nu}{q^2} \right) \gamma^\nu \gamma_5 v_\theta \tag{4}
\]

where axial transition constant \( g^A_{\theta n} \) is just we are interesting in (the width is proportional to the square of this value). Such a method for calculation the width in QCD sum rules is not new, see, e.g. [19]. In the case of massive kaon the only change is in denominator of second term in r.h.s of the eq. (4), i.e.

\[
\langle \theta^+ | j_\mu | n \rangle = g^A_{\theta n} \bar{v}_n \left( g^{\mu \nu} - \frac{q^\mu q^\nu}{q^2 - m_k^2} \right) \gamma^\nu \gamma_5 v_\theta \tag{5}
\]

It is clear that the second term vanishes at small \( q^2 \).

Substituting \( \langle 0 | \eta_n | n \rangle = \lambda_n v_n \), and \( \langle 0 | \eta_\theta | \theta^+ \rangle = \lambda_\theta v_\theta \) in eq (3) and take the sum on polarization one can easily see, that (in the limit of small \( q^2 \)) correlator (1) is proportional to \( g^A_{\theta n} \).

\[
\Pi_\mu^{phys} = \lambda_\theta g^A_{\theta n} \frac{1}{p_1^2 - m_\theta^2} \frac{1}{p_2^2 - m^2} (-2 \hat{p}_1 \gamma_5 p_1^\mu + \ldots) \tag{6}
\]
where dots in r.h.s mean other kinematic structures (proportional to $q$ e.t.c). For our sum rules we will use invariant amplitude just at the kinematical structure $\hat{p}_1 p_1^\mu$, because, as it was discussed in [20], [21], [22], [23] the choice of the kinematic structures with maximal number of momentum lead to better sum rules.

By use of the equation of motions the eq.(4) close to the mass shell can be rewritten

$$\langle \theta^+ | j_\mu | n \rangle = g_{\theta n} \bar{v}_n \left( \gamma^\mu + \frac{m_\theta + m_n}{q^2} q^\mu \right) \gamma_5 v_\theta$$  \hspace{1cm} (7)

At the same time, the second term in (4,7) correspond to the kaon contribution to $\theta - n$ transition with lagrangian density $L = ig_{\theta nk} v_n \gamma^5 v_\theta$ so one can write

$$\langle \theta^+ | j_5^\mu | n \rangle = g_{\theta nk} \frac{q^\mu f_k}{q^2 - m_k^2} \bar{v}_n \gamma^5 v_\theta$$  \hspace{1cm} (8)

Comparing (7) and (8) one can found (if we for a moment neglect the kaon mass)

$$g_{\theta nk} f_k = (m_n + m_\theta) g_{\theta n}$$  \hspace{1cm} (9)

This is the analog of the Golderberger-Trieman relation. Of course the accuracy of this relation is about the scale of SU(3) violation but as estimation of the value of $g_{\theta nk}$ it is enough good. Before we discuss numerical sum rules for $g_{\theta n}$, let us remind the common properties of correlator (1). In [13], [14] it was shown, that correlator(1) vanishes in the chiral limit for any pentaquark current without derivatives, so axial constant $g_{\theta n}$ should be proportional to the quark condensate. In this papers also the another reason of small value of $g_{\theta n}$ was discussed. Let us again consider correlator (1). One can easily note, that unit operator (as well as $d = 3$ operator e.t.c.) contributions to the correlator are expressed in the terms of the following integrals

$$\int e^{i(p_1 \cdot x - q \cdot y)} \frac{d^4x d^4y}{((x - y)^2)^n (x^2)^m} \equiv \int e^{i p_1 x} e^{-i q t} \frac{d^4x d^4t}{(x^2)^m (t^2)^n}$$  \hspace{1cm} (10)

It is clear that such integrals have imaginary part on $p_1^2$ and $q^2$ - the momentum of nucleon and axial current - but there is no imaginary part on $p_1^2$ - the momentum of pentaquark. So we come to the conclusion that such diagrams correspond to the case, when there is no $\Theta^+$ resonance in the pentaquark current channel (this correspond to background of this decay). (Note, that this conclusion don’t depend on the fact that one of the quark propagators should be replaced by condensate, as we discuss before).

The imaginary part on $p_1^2$ (i.e. $\Theta^+$ resonance)appears only if one take into account hard gluon exchange. So we come to conclusion, that if $\Theta^+$ is a genuine 5-quark state (not, say, the $NK$ bound state), then the hard gluon exchange is necessary, what leads to additional factor of $\alpha_s$. We see, that pentaquark width $\Gamma_{\Theta} \sim \alpha_s^2 \langle 0 | \bar{q} q | 0 \rangle^2$, i.e., $\Gamma_{\Theta}$ has strong parametrical suppression.

Now we want to check, does this parametrical suppression really lead to numerical smallness of the pentaquark width. It is necessary to note, that in the sense of discussion before (see [14]) it is quite necessary to keep only those part of the correlator, which has really imaginary part both on $p_1^2$ and $p_2^2$ (i.e pentaquark and nucleon 4-momenta square). This mean, at least, double Borel transformation (for $p_1^2$ and $p_2^2$...
independently). From (5) it is clear that for invariant amplitude at the kinematic structure $p_1 p_\mu$ we have the following sum rules

$$\lambda_n \lambda_0 g_{0n}^A e^{-(m_n^2/M_n^2+m_0^2/M_0^2)} = (-1/2) B_\theta B_n \Pi^{QCD}$$

(11)

where $B_\theta$, $B_n$ mean Borel transformation on pentaquark and nucleon momenta correspondingly, and continuum extraction is supposed.

For $\Pi^{QCD}$ (QCD representation of correlator (1)) corresponding diagrams are shown fig.1a,b. But as was discussed before, diagrams at Fig.1a has no imaginary part on pentaquark 4-momentum. (They are proportional to terms as in eq. (10)). So the only contribution come from diagrams on the Fig.1b The calculation of this diagrams is technically enough complicated. One should pay special attention to extract the terms, which have no imaginary part on pentaquark 4-momenta correctly. We perform calculation in the x-representation, using standard exponential representation of propagators and the relation $Be^{-br^2} = \delta(b-1/M^2)$. In this short paper we do not stop on the discussion of calculation features. Unfortunately we can not write down pure analytical answer, because the final answer is expressed in terms of a very large number of different double (and ordinary) integrals and it total size is very large (about some hundred terms). That’s why we prefer at last stage make numerical analysis of results. Of course we check, that all integrals converge if $Q^2(= -q^2)$ is not equal to zero. At the same time we use in calculation the fact, that the ratio $A1 = M_n^2/M_\theta^2$, (where $M_n^2$, $M_\theta^2$ are nucleon and pentaquark Borel masses) should be of order of ratio of the corresponding mass square $m_n^2/m_\theta^2$, so we can threats $A1$ as small parameter.

Really, from the sense of sum rule, it is clear that we can found axial constant $g_{0n}^A$ only at $Q^2$ not close to zero (about 1 GeV$^2$ or higher). Of course, as was discussed before, the result of QCD part (we will denote it as $R^{QCD} = B_n B_\theta \Pi^{QCD}$) is proportional to quark condensate and strong coupling constant $\alpha_s a$, where $a = -(2\pi)^2 (0|\bar{q}q|0)$. As a characteristic virtuality we chose the Borel mass for nucleon, but one should note, that because $\alpha_s a^2$ do not depend on normalization point, this choice is rather unessential. We use the value of $\alpha_s a^2 = 0.23$ GeV$^6$, $\lambda_n^2*32\pi^4 = 3.2$ GeV$^6$, $\lambda_\theta^2*(4\pi)^8 = 12$ GeV$^{12}$, obtained [18]. The continuum dependence is weak, we use standard value $s_0 = 1.5$ GeV$^2$ for nucleon and $s_0 = 4 - 4.5$ GeV$^2$ for pentaquark current [18].

On Fig.2 the value of axial constant $g_{0n}^A$, obtained from sum rules, are shown at two values of $Q^2$: $Q^2 = 1$ GeV$^2$, $Q^2 = 2$ GeV$^2$ as a function of Borel mass of nucleon (the Borel mass of pentaquark is supposed to be $M_\theta^2 = 3M_n^2$). One can see that the Borel mass behavior of $g_{0n}^A$ is reasonable. I should note, that this is only contribution of the first non vanishing operator (dimension 3), and the calculation the contribution of higher dimension operators are in progress, but first estimations show, that they expected not to be extremely large (we expect contribution no more than 40%). On the Fig.3 the $Q^2$ dependence of $g_{0n}^A$ is shown (at $M_n^2 = 1$ GeV$^2$ and $M_\theta^2 = 3M_n^2$ - thin line, and the same for $M_\theta^2 = 1.2$ GeV$^2$ - thick line ). Really we are interest the value $g_{0n}^A$ in the limit $Q^2 \to 0$, which can’t be calculated directly from S.R., obviously. But one can see from Fig.3, that $Q^2$ behavior is found to be almost linear (which is physically expected at small $Q^2$) so one can extrapolate it to zero. Of course such an extrapolation is lead to large inaccuracy, especially before we have no calculate the higher order corrections and that’s why don’t know the region of $Q^2$ where sum rules are reliable. We suppose them to be reliable starting $Q^2 = 1$ GeV$^2$, then we estimate averaged (on Borel mass)
\( g_{\theta n}^A = 0.02 \) at \( Q^2 = 0 \) with inaccuracy about a factor two. The other sources of inaccuracy are:

a) the method itself (the accuracy is of the order of the SU(3) violation)

b) possible contribution of higher dimension operators (according the estimations, most likely can change the value about 30-50%)

c) in the value of \( \lambda_\theta \) (accuracy about 20-30%)

d) inaccuracy of sum rule approach, especially for pentaquark case (see, for example, discussion in [25]), and also the possible effects of the size of pentaquark.

For all this reason we clearly understand, that really we can estimate only the order of magnitude of the value of axial constant \( g_{\theta n}^A(0) \) at \( Q^2 = 0 \), which can to be varied from 0 to 0.06 with central point (formally) \( g_{\theta n}^A = 0.02 \). So we can more or less reliable estimate only upper limit for \( g_{\theta n}^A < 0.06 \). By use of eq. (9) one can easily express the pentaquark width in terms of \( (g_{\theta n}^A)^2 \), and we come to conclusion, that our result give upper limit for pentaquark width \( \Gamma_\theta < 1 MeV \). More precise prediction can be done only after higher dimension operator corrections will be estimated. Note, that our result for width of the pentaquark (with positive parity) does not contradict to those (0.75 MeV), obtained in [15] also in sum rules, but by quite different method.

Our estimation of the pentaquark width also doesn’t contradict to the result ([4] (0.36 MeV with accuracy about 30%), obtained from the ratio between numbers of resonant and non-resonant charge exchange events.

The main conclusion is, that if the \( \theta^+ \) is genuine states then sum rules really predict the narrow width of pentaquark, and the suppression of the width is both parametrical and numerical.

Author is thankful to B.L.Ioffe for useful discussions and advises. This work was supported in part by CRDF grant RUP2-2621-MO-04 and RFFI grant 06-02-16905.

References

[1] T.Nakano et al., Phys.Rev.Lett. 91, 012002 (2003).

[2] V.V.Barmin, A.G.Dolgolenko et al., Yad.Fiz. 66, 1763 (2003). (Phys.At.Nucl. 66, 1715 (2003)).

[3] M. Battaglieri et all (CLAS Coll.) hep-ex/0510061

[4] R. Mizuk et al (BELLE Coll.) Phys Lett. B362, 173, (2006)

[5] Barmin, V.V, A.G.Dolgolenko et al, hep-ex/0603017

[6] Volker D.Burkert, hep-ph/0510309

[7] D.Diakonov, V.Petrov and M.Polaykov, Z.Phys. A359, 305 (1997).

[8] D.Diakonov, hep-ph/0406043

[9] V.B.Kopeliovich, Uspekhi Fiz.Nauk, 174, 323 (2004)
[10] R.A. Arndt, I.I. Strakovsky and R.L. Workman, nucl-th/0311030
[11] A. Sibirtsev, J. Heidenbauer, S. Krewald and Ulf-G. Meissner, hep-ph/0405099
[12] A. Sibirtsev, J. Heidenbauer, S. Krewald and Ulf-G. Meissner, nucl-th/0407011.
[13] B.L. Ioffe and A.G. Oganesian, JETP Lett. 80, (2004) 386
[14] A.G. Oganesian, hep-ph/0410335
[15] F.S. Navarra, M. Nielsen and R. Rodrigues da Silva, hep-ph/0510202
[16] B.L. Ioffe, Nucl. Phys. B188, 317 (1981).
[17] Shoichi Sasaki, Phys. Rev. Lett. 93, 152001 (2004)
[18] Oganesian A.G., hep-ph/0510327
[19] V.L. Eletsky, B.L. Ioffe and Ja.I. Kogan Phys. Lett. B122, 423 (1983)
[20] V.M. Belyaev and B.L. Ioffe Sov. Phys. JETP 56, (1982) 493
[21] V.M. Belyaev and B.L. Ioffe Sov. Phys. JETP 57, (1983) 716
[22] V.M. Belyaev and B.L. Ioffe Nucl. Phys. B310 (1988) 548
[23] B.L. Ioffe and A.V. Smilga Nucl. Phys. B232 (1984), 109
[24] B.L. Ioffe Prog. Part. Nucl. Phys. 56 (2006) 232
[25] Matheus, R.D and Narison, S. hep-ph/0412063
Figure 1:

Figure 2: \( g_{\theta n}^A \) dependence on Borel mass for \( Q^2 = 1 GeV^2 \) - upper line and for \( Q^2 = 2 GeV^2 \) - lower line
Figure 3: $Q^2$ dependence of $g_{\theta n}^A$ for $M_n^2 = 1.2 GeV^2$ - upper line and for $M_n^2 = 1 GeV^2$ - lower line