Signatures of accretion shocks in emitted radiation from a two temperature advective flow around black holes

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Abstract. Centrifugal barrier supported boundary layer (CENBOL) of a black hole affects the spectrum exactly in the same way the boundary layer of a neutron star does. The CENBOL is produced due to standing or oscillating shock waves and these shocks accelerate electrons very efficiently and produce a power-law distribution. The accelerated particles in turn emit synchrotron radiation in presence of the magnetic field. We study the spectral properties of an accretion disk as a function of the shock strength, compression ratio, flow accretion rate and flow geometry. In the absence of a satisfactory description of magnetic fields inside the advective disk, we consider the presence of only stochastic fields and use the ratio of the field energy density to the gravitational energy density to be a parameter. Not surprisingly, stronger fields produce stronger humps due to synchrotron radiation. We not only include ‘conventional’ synchrotron emission and Comptonization due to Maxwell-Bolzmann electrons in the gas, we also compute these effects due to power-law electrons. For strong shocks, a bump is produced at a frequency just above the synchrotron self-absorption frequency at $v_{\text{pump}} \sim v_{\text{inj}}[1 + \frac{4}{3} \frac{R-1}{R_{1/2}}]^{1/2}$, where, $v_{\text{inj}}$ is the frequency of the dominant photons from the pre-shock flow, $R$ is the compression ratio of the shock located at $x_{s}$. For strong shocks, a bump at a higher frequency appears predominantly due to the power-law electrons formed at the shock front.
1. Introduction

Attempts to explain the spectra of black holes were made in the early seventies (Shapiro, 1973ab) by using spherically symmetric Bondi flows (Bondi, 1952). However, the flow was found to be very radiatively inefficient due to high advection and in the same year Shakura-Sunyaev (1973) proposed a disk model in which the angular momentum distribution is kept as the Keplerian distribution around a black hole. However, the emitted multi-colour black body radiation was not capable of explaining the spectra of Cyg X-1 and Eardley, Lightman, & Shapiro, (1975); Shapiro, Lightman & Eardley (1976) showed that it is possible to have a ‘hotter’ branch of the solution, still retaining Keplerian distribution, which can produce hard X-rays from the inner region. On the other hand, Eardley & Lightman (1975) found that the Keplerian disk with a constant \( \alpha \) was unstable due to thermal and viscous effects (1975) which was later corroborated by Eggum, Coroniti & Katz (1985) by detailed numerical simulation. They showed that the Keplerian disk collapsed in presence of constant viscosity parameter. The focus was then to introduce a mixture of sub-Keplerian and super-Keplerian (non-accreting) flows, such as thick accretion disk (Paczyński & Wiita, 1980) in the optically thick limit, or accretion torus (Rees et al. 1982) in the optically thin limit and their spectral properties were studied (e.g., Madau, 1988). Other models included ‘non-Keplerian’ components such as, magnetic corona (Galeev, Rosner & Vaiana 1979). At the same time, the sub-Keplerian transonic flow models were introduced (Muchotrzeb & Paczyński, 1982) and they were improved upon to include standing, oscillating or propagating shock waves (e.g. Chakrabarti, 1989; Chakrabarti & Molteni, 1995; Molteni, Sponholz & Chakrabarti, 1996). Subsequently, Chakrabarti & Titarchuk (1995, hereafter CT95) incorporated this flow solution in computing spectra of black holes. In recent years, it has become customary to include the sub-Keplerian flows in accretion flow models because of the compelling evidences that the Keplerian disk alone cannot explain most of the observational facts and two components were essential (Smith, Heindl & Swank, 2002; Smith, Heindl, Markwardt and Swank, 2001; see Chakrabarti, 1996 for a review). There are several other models in the literature which also include a secondary component such as Compton cloud or magnetic corona (Poutanen & Coppi, 1998; Gierliński et al. 1999; Coppi, 1999; Zdziarski et al. 2001) which also satisfactorily fit spectra till high energies. Esin et al. (1998) fitted spectra with ADAF model in which the Keplerian flow becomes ion pressure dominated in the inner region. In the present paper, we shall concentrate on spectral properties of the dynamic ‘Compton cloud’, namely, the sub-Keplerian transonic flow component and not deal with other models present in the literature.

In CT95, a two component advective flow (TCAF) solution was introduced in which the Keplerian disk was on the equatorial plane as usual, but a dynamic, sub-Keplerian halo component (basically the transonic solutions worked out earlier, see, Chakrabarti...
flanks this Keplerian disk. While the Keplerian disk was supplying soft-photons, the sub-Keplerian flow was supplying hot electrons and the relative importance of these two accretion rates determined whether the spectrum was going to be soft or hard. A pivotal role is played in this context by the CENtrifugal barrier supported BOundary Layer or CENBOL. This is the hot and the puffed-up, post-shock region which intercepts soft-photons from the Keplerian disk and reprocesses them through inverse Comptonization. It is to be remembered that the CENBOL is a direct consequence of the dominance of the centrifugal force near a black hole and it is not to be confused with puffed up corona or magnetized corona that is used by other models. We find it comfortable to use CENBOL as the Comptonizing cloud as its existence and stability have been shown beyond doubt in a large number of numerical simulations (Molteni, Lanzafame & Chakrabarti, 1994; Chakrabarti & Molteni, 1995; Molteni, Sponholz, & Chakrabarti, 1996). (Non-axisymmetric perturbations could not remove them also, see, Molteni, Tóth & Kuznetsov, 1999). We can produce the observed power-law component without taking resort to any hypothetical electron cloud. In essence, the CENBOL region in between the horizon and the shock behaves like a boundary layer where the flow dissipates its gravitational energy. This boundary layer can oscillate when the cooling is introduced and this is proposed (Molteni, Sponholz & Chakrabarti, 1996; Chakrabarti & Manickam, 2000; Chakrabarti, Acharyya & Molteni, 2004) as the cause of the Quasi-Periodic Oscillation.

Given that a significant fraction of the accretion flow could pass through a standing, oscillating or a propagating shock, it is likely that the hot electrons may be accelerated by them just as the high energy cosmic rays (see, Kazanas & Ellison, 1986, Molteni & Sponholz, 1994, Chakrabarti, 1996) are produced by the transient super-novae shocks (Bell 1978ab, Longair 1981). In the present analysis, we shall incorporate the power-law component of the electrons to compute the synchrotron radiation and the Comptonization of these photons by these electrons. The inclusion of the synchrotron emission generated by non-thermal electrons is not new. For example, Lin & Liang (1999) considered a hybrid model which included a few percent of non-thermal electrons along with the usual thermal electron obeying Maxwell-Boltzmann distribution. The emission included thermal and magnetic bremsstrahlung but no Comptonization. In a parametric study, with the percentage of non-thermal electrons as a free parameter, Wardzinski & Zdziarski (2001) showed that the soft photons generated could significantly change the spectra. These works do not address the origin of the non-thermal electrons quantitatively but propose that they could be produced in magnetized corona. In contrast, we assume that all the non-thermal electrons are produced in the accretion shocks, though, some electrons from magnetized corona could not be ruled out and these should be included for a completely consistent solution. Some of the examples of changes of spectral states of the black holes are presented in Mandal & Chakrabarti (2004).
In the next Section, we present the basic equations and discuss the relevant parameters for the problem. We assume the flow to be around a Schwarzschild black hole. For the sake of simplicity, we use the well-known Paczyński & Wiita (1980) pseudo-Newtonian potential \( \phi_{PN} = -\frac{GM}{r_g} \), where \( r_g = 2GM/c^2 \) is the Schwarzschild radius of the black hole, \( M \) is the mass of the black hole, \( G \) and \( c \) are the universal gravitational constant and the velocity of light respectively.) to describe the black hole geometry. In Section 3, we discuss the nature of heating and cooling processes in an advective flow. In Section 4, we present our procedure to compute the Comptonized spectra using power-law electrons. In Section 5, we explain the solution procedure. In Section 6, we present the emergent spectra for various parameters and discuss their significance. Finally, in Section 7, we present concluding remarks.

2. Basic hydrodynamic equations

We consider a wedge shaped, axi-symmetric, low angular momentum flow (Chakrabarti 1990) having the number density distribution:

\[
n(x) = \frac{\dot{M}}{\Omega m_p x^2 v(x)}.
\]  

Here, \( \Omega \) is the solid angle subtended by the flow:

\[
\Omega = 4\pi \cos(\Theta),
\]  

\( \Theta \) is the angle made by the surface of the flow with the vertical axis, \( m_p \) is the mass of the proton, \( x \) is the radial distance \( r \) in units of \( r_g \) and \( \dot{M} \) is the accretion rate. The radial velocity,

\[
v(x) \sim (x - 1)^{-1/2}
\]  

increases rapidly in this sub-Keplerian flow. In presence of pure hydrogen, Thomson scattering will be the most dominating scattering process and the corresponding optical depth \( \tau(x) \) can be computed using Eqs. (1) and (3). In the absence of any satisfactory description of the magnetic field in accretion disks, we assume only a random or stochastic field of strength \( B(r) \). This may or may not be in equipartition with the ionized flow. We define a parameter \( \beta \) which is the ratio of the magnetic energy density and the gravitational energy density:

\[
\beta = \frac{B^2(r - 2GM/c^2)}{8\pi GM \rho}.
\]  

Normally, \( \beta \lesssim 1 \) and for \( \beta > 1 \), the magnetic field may buoyantly leave the disk.

The ions are expected to supply Coulomb energy to the electrons which in turn become cooler using thermal and magnetic bremsstrahlung processes. In the presence of soft photons, they can loose energy via inverse Compton processes.

The energy balance equation for the protons and electrons in our model could be written as:
where, $\gamma_{pol}$ is $5/3$ for non-relativistic electron temperatures ($T_e \leq m_e c^2/k$) and $4/3$ for relativistic electron temperatures ($T_e > m_e c^2/k$). $k$ is the Boltzmann constant. Since protons are much heavier than the electrons, $T_p$ always remains in the non-relativistic domain. $\Gamma$ and $\Lambda$ contain contributions from all the heating and cooling processes respectively. We shall describe various terms shortly below.

One can integrate these equations simultaneously using a fourth order Runge-Kutta method. We impose a condition that a steady accretion shock occurs at $x = x_s$ through which the energy flux and the mass flux are conserved. In Chakrabarti (1989) it was shown that the shocks in a Schwarzschild geometry typically occur at around $x_s \sim 10^{-80} r_g$ depending on specific angular momentum $\lambda$. Instead of using $\lambda$ as a free parameter we use $x_s$ to be the free parameter. We use $R$, the compression ratio, to be a free parameter as well. Normally, in a transonic solution the shock location and the compression ratio are obtained simultaneously (Chakrabarti 1989).

3. Nature of the heating and cooling Processes

Protons lose energy through Coulomb interaction $\Lambda_{ep}$ and inverse bremsstrahlung $\Lambda_{ib}$:

$$\Lambda_p = \Lambda_{ep} + \Lambda_{ib}.$$  \hfill (6)

Here, the subscript $p$ stands for protons. Electron-proton coupling supplies energy to the electrons (from protons) through $\Lambda_{ep}$ which it lost due to various cooling processes. This behaves as a heating term as far as the electrons are concerned and is given by:

$$\Lambda_{ep} = 1.6 \times 10^{-13} \frac{k \sqrt{m_e} \ln \Lambda_0}{m_p} n^2 (T_p - T_e) T_e^{-3/2},$$  \hfill (7)

where, $\ln \Lambda_0$ is the Coulomb logarithm, $m_p$ and $m_e$ are the rest masses of protons and electrons respectively. Electrons are heated through this Coulomb coupling.

$$\Gamma_e = \Lambda_{ep}.$$  \hfill (8)

Subscript $e$ stands for electrons.

Electrons are cooled by bremsstrahlung at the rate of $\Lambda_b$, by cyclo-synchrotron at the rate of $\Lambda_{cs}$ and by Comptonization at the rate of $\Lambda_{mc}$ of the soft photons due to cyclo-synchrotron radiation. The net cooling of the electrons is:

$$\Lambda_e = \Lambda_b + \Lambda_{cs} + \Lambda_{mc}.$$  \hfill (9)

Explicit expressions for these cooling terms for electrons satisfying Maxwell-Bolzmann (MB) distribution are:

$$\Lambda_{ib} = 1.4 \times 10^{-27} n^2 \left( \frac{m_e}{m_p} T_p \right)^{1/2},$$  \hfill (10a)
$$\Lambda_b = 1.4 \times 10^{-27} n^2 T_e^{3/2} (1 + 4.4 \times 10^{-10} T_e),$$

$$\Lambda_{cs} = \frac{2\pi}{3c^2} kT_e(x) \frac{\nu_a^3}{x},$$

where $\nu_a$ is the critical frequency at which the self-absorbed synchrotron radiation spectrum is peaked and it can be determined from the relation,

$$\nu_a = \frac{3}{2} \nu_0 \theta_e x_m,$$

where,

$$\nu_0 = 2.8 \times 10^6 B,$$

$$\theta_e = \frac{kT_e}{m_e c^2}.$$

Procedure to determine $x_m$ will be discussed below.

So far, we discussed the situation when the electrons obey MB velocity distribution law. However, in our present situation, a section of the flow passes through the shock and a fraction $\zeta$ of the electrons will have a power-law distribution. For instance, the cooling term due to cyclo-synchrotron photons would be given by (Longair 1981),

$$\Lambda_{cs} = A \xi B^{(p+1)/2} (\nu_{max}^{(3-p)/2} - \nu_{min}^{(3-p)/2})$$

where,

$$A = \frac{(3\pi)^{1/2} K e^3}{m_e c^2 (1 + p)(3 - p)} \frac{2\pi m_e^2 c^5}{3c} (1-p)^{1/2},$$

$$G = \frac{\Gamma(p/4 + 19/12)\Gamma(p/4 - 1/12)\Gamma(p/4 + 5/4)}{\Gamma(p/4 + 7/4)}.$$

and $K$ is the normalization constant of power-law electron distribution,

$$n(E)|_{x_s} = K E^{-p},$$

which is obtained using the constraint that the electron number is conserved during the shock acceleration. Here $\nu_{min}$ is the minimum frequency considered for integration. When self-absorption is included, this may be treated as $\nu_a$ defined above. The Comptonization is computed by using this cooling term augmented by the enhancement factor $F$:

$$\Lambda_{mc} = \Lambda_{cs} F,$$

For MB distribution, $F$ is given by,

$$F = \eta \left\{ 1 - \left( \frac{x_a}{3 \theta_e} \right)^{n_2} \right\},$$

where,

$$\eta = \frac{P(A - 1)}{(1 - PA)},$$

$$P = 1 - \exp(-\tau_{cs}),$$

is the probability that an escaping photon is scattered.

$$A = 1 + 4\theta_e + 16\theta_e^2,$$
is the mean amplification factor in the energy of a scattered photon when the scattering
electrons have a Maxwellian velocity distribution of temperature $\theta_e$, $\eta_2 = 1 - \frac{\ln p}{m}$ and
$x_a = h\nu_a/m_e c^2$. The seed photons generated by the electrons obeying power-law distri-
bution will also be comptonized by the thermal electrons and the amplification factor
(Dermer, Liang & Canfield, 1991) can be written as,

$$F = \eta_1[1 - \left(\frac{s}{\phi_s}\right)\frac{x_{s max}^2 - x_s^2}{3\theta_c(\phi - 1)(x_{max}^2 - x_a^2)}],$$

(20)

where,

$$x_{max} = \frac{h\nu_{max}}{m_e c^2},$$

$$s = \frac{5 - p}{2},$$

$$\phi_s = \phi + \frac{3 - p}{2},$$

and

$$\phi = \eta_2 - 1.$$

Similarly, the amplification factor (Rybicki & Lightman 1979) for the seed photons
Comptonized by the electrons obeying power-law distribution can be written as,

$$F = \frac{4}{3} \sigma_T R_c K \frac{\mathcal{E}_{max}^{3-p} - \mathcal{E}_{min}^{3-p}}{(3-p)},$$

(21)

where $R_c$ is the size of the Comptonized region. $\mathcal{E}_{max}$ and $\mathcal{E}_{min}$ are maximum and mini-
mum energy of the power-law electrons respectively.

In the current situation both the thermal and the non-thermal electrons are present.
In this case, $x_m$ is calculated by equating the total loss due to the thermal and the
non-thermal emission from the CENBOL with the contributions from the corresponding
source functions from the CENBOL surface at the self-absorption frequency $\nu = \nu_a$, i.e.,

$$[\epsilon(\nu)_{MB} + \epsilon(\nu)_{PL}] V = \pi[(1 - \zeta)p(\nu)_{MB} + \zeta p(\nu)_{PL}] S,$$

(22)

where,

$$\epsilon(\nu)_{MB} = 4.43 \times 10^{-30} \frac{4\pi n\nu}{K_2(1/\theta_e)} I\left(\frac{x_m}{\sin(\psi)}\right),$$

(23)

$x_m$ is calculated from eq. (11):

$$x_m = \frac{2\nu}{3\nu_0 \theta_e^2},$$

(24a)

and $\psi$ is the angle between the velocity vector of the electron and the direction of the
local magnetic field. Averaging over $\psi$ for an isotropic velocity distribution, $I\left(\frac{x_m}{\sin(\psi)}\right)$
gets replaced by $I'(x_m)$ and is given by (Narayan & Yi, 1995),

$$I'(x_m) = \frac{4.0505}{x_m^{1/6}} \left(1 + \frac{0.40}{x_m^{1/4}} + \frac{0.5316}{x_m^{1/2}}\right)^{\exp(-1.8899x_m^{1/3})}. \tag{24b}$$

This is the emissivity of the electrons obeying MB distribution. The emissivity of the
electrons obeying power-law distribution is given by,

$$\epsilon(\nu)_{PL} = \frac{3 - p}{2} AGB^{(p+1)/2}(\nu^{(1-p)/2}). \tag{25}$$
The source function for thermal emission in the low frequency regime is given by,

\[ p(\nu)_{MB} = 2m_e \nu^2 \theta_e. \]  

(26)

The source function of power-law emission is given by,

\[ p(\nu)_{PL} = \sqrt{\frac{\pi}{c^2(1 + p)}} \left( \frac{2\pi m_e^2 \nu^5}{3e} \right)^{1/2} \mathcal{H}B^{-1/2}\nu^{5/2}, \]  

(27a)

where,

\[ \mathcal{H} = \frac{\Gamma(p/4 + 19/12)\Gamma(p/4 - 1/12)\Gamma(3/4)}{\Gamma(p/4 + 11/6)\Gamma(p/4 + 1/6)\Gamma(5/4)} \]  

(27b)

\( \zeta \) is the percentage of electrons acquiring a power-law energy distribution. In Eqn. (22), \( V \) is the volume element under consideration and \( S \) is the corresponding surface area.

We ignore the effects of power-law electrons on the emission of bremsstrahlung radiation since this radiation is very weak.

4. Computation of the spectral index

In the previous Section, we mentioned how the injected photon intensity is enhanced due to the inverse Comptonization process. In this Section, we discuss how the spectral slope is computed. In CT95, the electrons in the pre-shock and the post-shock regions were assumed to be following Maxwell-Bolzmann distribution. Hence the spectral index was computed from Titarchuk & Lyubarskij (1995).

The electrons from the pre-shock flow will be accelerated at the shock and a fraction of post-shock electrons will obey the power-law distribution. So, the CENBOL has both the MB as well as the power-law electrons. The energy spectral index \( \alpha \) of the outgoing radiations due to Comptonization depends on the distribution function of electrons. We follow the prescription suggested in Titarchuk & Lyubarskij (1995) and Gieseler & Kirk (1997) to calculate \( \alpha \). A brief discussion about the procedure of calculating \( \alpha \) is given below.

It is well known that a spectrum can be characterized by the dimensionless electron temperature \( \theta_e = kT_e/m_e c^2 \) and the optical depth \( \tau \) of the medium (Sunyaev & Titarchuk 1985). The condition that has to be satisfied is:

\[ C_0(\theta_e, \alpha)\lambda(\tau_0) = 1. \]  

(28)

Here, \( \lambda(\tau_0) = exp(-\xi) \), and

\[ \xi = \frac{\pi^2}{3(\tau_0 + 2/3)^2}(1 - exp(-7\tau_0) + exp(-1.4\tau_0)) \ln \frac{4}{3\tau_0}, \]  

(29)

for spherical geometry and \( \tau_0 \) is the total optical depth of the CENBOL. The quantity \( C_0 \) itself is to be obtained from,

\[ C_0(\theta_e, \alpha) = 3\pi \int_0^1 v^2 dv \frac{\tilde{f}(v)}{\gamma^2} C_0, \]  

(30)
where $v$, $f(v)$ and $\gamma$ are the velocity of electrons in units of the velocity of light, electron
velocity distribution function and the Lorentz factor respectively and

$$\dot{C}_0 = \frac{3}{8\gamma^2} [I_{0,0} + I_{2,2} - \frac{1}{3} (I_{2,0} + I_{0,2})].$$

(31)

The tabulated integrals $I_{i,j}$ is given by,

$$I_{i,j} = \int_{-1}^{1} \frac{y^j}{(1 + v y)^{\alpha + 3}} dy \int_{-1}^{1} x^i (1 + v x)^{\alpha} dx.$$ 

(32)

In the case of MB distribution,

$$f(v) = \frac{\gamma^5 \exp(-\gamma/\theta_e)}{4\pi \theta_e K_2(1/\theta_e)},$$

(33a)

where $K_2(1/\theta_e)$ is the modified Bessel function of second kind of order two and for
power-law distribution (eq. 15),

$$f(v) = \frac{1 - p}{4\pi (\gamma_{max} - \gamma_{min})} v^{\gamma - p}.$$ 

(33b)

Here, $\gamma_{max}$ and $\gamma_{min}$ are obtained from particle energy as will be discussed in next section.
Integrating over different $f(v)$ we get different $\alpha$ for the same set of flow parameters. Using
these $\alpha$ we calculate the spectrum from CENBOL in the same way as described in CT95.

5. Solution Procedure

The geometry of the flow is chosen to be conical and the flow surface makes an angle
$\Theta$ with the z-axis. We use the Fourth-order Runge-Kutta method to integrate the en-
ergy equations 5(a-b). These two first order differential equations required two initial
conditions when the flow geometry (determined mainly by the momentum equations)
and the accretion rate (determined by the continuity equation) are provided. At the
shock, $x = x_s$, the velocity $v(x)$ is reduced by a factor $R$, the compression ratio, i.e.,
$v_+(x_s) = v_-(x_s)/R$. The number density as obtained from Eq. (1) also goes up by a
factor $R$.

As for the initial conditions, we fix the outer boundary at a large distance (say,
$10^6 r_g$) and supply matter (both electrons and protons) with the same temperature (say,
$T_p = T_e = 10^6 K$). During integration, we obtained temperature and radiation emitted by
the flow through bremsstrahlung and synchrotron radiation. These low energy photons
are then inverse Comptonized by the hot electrons in the flow.

A shock of compression ratio $R$ causes the formation of power-law electrons of slope
(Bell, 1978ab),

$$p = (R + 2)/(R - 1).$$

(34)

This power-law electrons produce a power-law synchrotron emission with an index $q$
given by (Longair, 1981):

$$q = (1 - p)/2.$$ 

(35)
The power-law electrons have energy minimum at

$$E_{\text{min}} = m_e c^2 \gamma_{\text{min}}$$

(36a)

and energy maximum at

$$E_{\text{max}} = m_e c^2 \gamma_{\text{max}}.$$  

(36b)

The $E_{\text{max}}$ was obtained self-consistently by conserving the number of power-law electrons and by computing the number of scatterings that the electrons undergo inside the disk before they escape. This yields,

$$\gamma_{\text{max}} = \gamma_{\text{min}} \left[ 1 + \frac{4}{3} \frac{R - 1}{R} \frac{1}{xs} \right]^{1/2}.$$  

(37)

The minimum value of the Lorentz factor $\gamma_{\text{min}}$ is obtained from the temperature of the injected electrons. This temperature is obtained self-consistently through our integration procedure. While obtaining the net cooling, we have used this local temperature and the local optical depth. To compute the spectrum, we divided the pre-shock region into several annuli and the post-shock region has been kept as a single zone. The number of intercepted soft-photons which participated in the Comptonization process has been calculated using local optical depth.

To obtain a realistic idea of how much matter actually passes through the shock, we take resort to the numerical simulations of Molteni, Lanzafame & Chakrabarti (1994). This fraction will depend on the angular momentum: for lower angular momentum, the flow passes close to the equatorial plane and passes through the shock formed very close to the hole. Very little matter is lost as winds. For higher angular momentum, the CENBOL size increases and a profuse amount of wind/outflow is formed after being processed through the CENBOL. The fraction can also be time-dependent. In the absence of a fully time dependent solution, we assume the percentage of electrons $\zeta$ acquiring a power-law energy distribution to be a free parameter. The goal is to obtain as varied spectra as possible in order to constrain the parameters of a system.

In presence of the magnetic field, the pre-shock flow and the CENBOL would emit the synchrotron radiation. While computing this we have incorporated the synchrotron self-absorption by the emitting medium when it becomes optically thick. Eqn. (11) gives the expression for the self-absorption frequency $\nu_a$.

Though there are several approaches in the literature (see, e.g., Poutanen & Coppi, 1998; Gierliński et al. 1999; Coppi, 1999; Zdziarski et al. 2001) we followed the procedures presented in CT95 and Titarchuk & Lyubarskij (1995) while computing the Comptonized spectrum due to the scattering from MB electrons and the power-law electrons. However, unlike CT95 where a Keplerian disk was supplying soft photons, here the source is everywhere, i.e., distributed throughout. This has been taken into account. At the end, we add contributions from all the components to get the net photon emissions from the flow.
In the next Section, we present the spectral characteristics of the wedge shaped thin flow around a $10\, M_\odot$ black hole (unless otherwise stated) having $\Theta = 75^\circ$. We generally use the strong shock $R = 4$ except in Fig. 4, (where $R$ itself is varied) and saturated magnetic field $\beta = 1$ except in Fig. 7 (where $\beta$ is varied). We vary other solution parameters: $x_s$, $\zeta$ and $\dot{m}$ to obtain results for different cases. Here, $\dot{m}$ is the mass accretion rate in units of Eddington rate $\dot{M}_{\text{Edd}}$. We generally use a black hole of mass $10\, M_\odot$. However, we probed the results when the mass was chosen to be $10^6\, M_\odot$ as well.

6. Results and Interpretations

First, we show the temperature distribution of the incoming matter in presence of cooling. Figures 1(a-b) show the variation of the electron and proton temperatures ($T_e$ and $T_p$ respectively) as functions of the radial distance $x$ (measured in units of the Schwarzschild radius $r_g$). Both the axis are in logarithmic scale. Solid curves show the proton temperatures.

In Fig. 1a, results for $\dot{m} = 0.01\, M = 10\, M_\odot$ (long-dashed curve of $T_e$), and $M = 10^6\, M_\odot$ (dotted curve of $T_e$) are shown. The other parameters are: $\zeta = 0.05$ and $x_s = 35$. For the higher mass, the density is lower and electrons are less efficiently cooled. In Fig. 1b, $\dot{m} = 0.1$ (long-dashed curve of $T_e$) and $\dot{m} = 0.5$ (dotted curve). Other parameters are $M = 10\, M_\odot$, $\zeta = 0.2$, and $x_s = 25$. Higher accretion rate causes higher cooling of the electrons and thus $T_p$ and $T_e$ start to deviate from each other farther from the black hole. Second, the initial deviation from proton temperature is also higher for higher rates. It is generally observed that just after the shock, electrons of the flow with lower accretion rate cool faster. This is because the CENBOL has more synchrotron photons to cool fewer electrons. Thus the role of the CENBOL is crucial to change the nature of cooling quantitatively.

We now present a typical spectrum of an accretion flow around a $M = 10\, M_\odot$ black hole (Fig. 2a) and around a $M = 10^6\, M_\odot$ black hole (Fig. 2b). Other parameters are exactly the same as in Fig. 1a. The curve marked 1 is due to the synchrotron emission by the thermal electrons from the pre-shock region and that marked 2 is due to the Comptonization of these photons. These curves were drawn after the self-absorption was taken into account. The curves marked 3 and 4 are due to the synchrotron emission by the thermal electrons and the Comptonization of these photons by the same in the post-shock region respectively. The curve marked 5 represents the synchrotron emission by the power-law electrons in the post-shock region. The curve marked 6 is due to Comptonization of the power-law generated soft photons. The curve marked 7 is the total spectrum emitted from the entire flow. The bumps in the spectrum at around $\log(\nu) \sim 14 - 16$ are the distinct signatures synchrotron emission at the CENBOL. The bump at the lower energy is due to the thermal electrons and the bump at the higher
Fig. 1. Variation of the proton (solid) and electron temperatures as functions of dimensionless radial distance $x$ in presence of a strong shock ($R = 4$) with saturated magnetic field ($\beta = 1$).

In (a), $\dot{m} = 0.01$ and $M = 10M_\odot$ (long-dashed) and $M = 10^6M_\odot$ (dotted). Other parameters are $x_s = 35$, $\zeta = 0.05$. For higher mass, the density becomes lower and the electrons are less efficiently cooled. In (b) $\dot{m} = 0.1$ (long-dashed) and $\dot{m} = 0.5$ (dotted). Other parameters are $M = 10M_\odot$, $\zeta = 0.2$ and $x_s = 25$. For higher accretion rates, the pre-shock flow cools faster and the temperatures deviate from each other farther away.

Energy is due to the power-law electrons. The photons around $\log(\nu) \sim 18 - 22$ are produced mainly due to Comptonization of the thermal and non-thermal electrons. When it is dominated by the emission from the non-thermal electrons, it also carries the signature of the CENBOL as well. Notice in curve 7, how the spectral slope $\alpha$ changes from that of the thermal photons $\alpha_{th}$, to that of the non-thermal photons $\alpha_{nth}$. The major difference between the two cases is that in Fig. 2a, the spectrum is softer since the electron number density is higher for the low mass black hole, while in Fig. 2b, the spectrum is harder as the electron number density is lower for the high mass black hole and the electrons are difficult to cool by the synchrotron photons emitted by lower density electrons.

The location of the shock can change depending on the specific angular momentum and specific energy of the flow. When the shock is located at a larger distance, much bigger volume of the gas becomes hotter and the spectrum is expected to become harder. When the shock moves in, it is easy to cool the small sized CENBOL and the spectrum could become softer. We see this effect in Fig. 3 where we vary the shock location. The solid, dotted and the dashed curves denote the cases when $x_s = 50$, 25 and 10 respectively. Other parameters are $\dot{m} = 0.05, \zeta = 0.2$. When the shock location is very close the number of excursion by the electrons before escaping is fewer and hence $\gamma_{max}$ is lower (see, eq. 37). This causes the higher energy bump to disappear. When the shock is farther away, the CENBOL radiation dominates the spectrum. Effectively, these three cases can be thought of having an extremely rotating Kerr black hole with contra-rotating disk, a Schwarzschild black hole, and an extremely rotating Kerr black hole with co-rotating disk respectively.
Fig. 2. Typical complete spectra from a sub-Keplerian accretion flow with all the contributions shown for the same parameters as in Fig. 1a except for the black hole mass: (a) $M = 10M_\odot$ and (b) $M = 10^6M_\odot$. See text for details of the components marked individually. In (a), the spectrum is softer and in (b), the spectrum is harder due to lower density.

We show now that the extent to which the pre-shock and the post-shock flows dominate the spectral feature depends on the compression ratio of the shock. It is well known (Landau & Lifshitz, 1956) that when the shock is strong, its compression ratio $R$ is given by,

$$R_{\text{strong}} = \frac{\gamma_{\text{pol}} + 1}{\gamma_{\text{pol}} - 1},$$

where $\gamma_{\text{pol}}$ is the polytropic index of the equation of state $P \propto \rho^{\gamma_{\text{pol}}}$. In the case of the mono-atomic gas, $\gamma_{\text{pol}} = 5/3$ and $R_{\text{strong}} = 4$. However, for weak shocks, the strength may be $R \sim 2 - 3$. For the weakest shock (i.e., no shock), $R = 1$. For an extremely relativistic flow, $\gamma_{\text{pol}} = 4/3$ and the shock may have a strength up to 7. In Fig. 4, we show the variation of the spectrum as a function of the compression ratio $R$. The accretion rate is $\dot{m} = 0.5$ and $\zeta = 0.2$. Solid curves are drawn for $R = 4$, the dotted curves are for $R = 2.6$ and the dashed curves are for $R = 1.5$. For a weak shock, the jump in density $\rho_+ = R\rho_-$, where $+$ and $-$ denote the post and pre-shock values respectively, at the shock is not so high and post-shock flow does not leave any distinct signature (long-dashed). Exactly opposite phenomenon occurs for a strong shock (solid). The accretion rate being high, it is difficult to cool the electrons, especially when the compression ratio is large, i.e., the electrons in the CENBOL are hotter. As a result, the spectrum is softer for $R = 1.5$ and harder for $R = 4$.

In Fig. 5, we show the variation of the spectrum when $\dot{m}$ is varied. Here, $\zeta = 0.2$ is used. A higher accretion rate is expected to increase the density of the flow which enhances the magnetic field and therefore the synchrotron cooling is also increased.
Fig. 3. Variation of the total emitted spectrum when the shock location $x_s$ is varied. The solid, dotted and dashed curves are drawn for $x_s = 50, 25, 10$ respectively. The spectrum becomes softer when $x_s$ is reduced.

However, the cooling is not sufficient when the optical depth enhancement is also taken into account. Thus, the spectrum becomes harder despite of this cooling. The Comptonized spectrum becomes harder. At lower accretion rate, it is easy to cool the flow and the spectrum becomes softer. The curves are drawn for: $\dot{m} = 1.0$ (solid), 0.5 (dotted) and 0.1 (dashed).

As we discussed earlier, not all the accreting matter actually passes through the shock. Low angular momentum matter (for a given specific energy) produces lesser jet and if shock exists, most of the matter passes through the shock located closer to the hole. High angular momentum inflow produces profuse outflows because the CENBOL size becomes very high. The fraction of electrons which have power-law distribution $\zeta$ thus varies from 0 to 1 depending on the flow parameters. In Fig. 6, we show the variation of the outgoing spectrum as a function of $\zeta$. The dashed curve is for $\zeta = 0$ and the dotted and solid curves are for $\zeta = 0.5$ and $\zeta = 0.7$ respectively. We use $\dot{m} = 0.05$ and $x_s = 25$. The spectrum changes from soft to hard with the increase of $\zeta$. Note that the bump around $\nu \sim 10^{18}$Hz is also weaker for smaller $\zeta$.

In all the cases mentioned above, we have assumed that the stochastic magnetic field is amplified sufficiently to become equipartition value at each radial distance. There are several reasons in both for and against this argument. In a conical wedge shaped flow as we used here, $B \sim 1/r^2$ and the magnetic pressure $\sim 1/r^4$ increases much faster
Variation of the spectrum when the compression ratio $R$ is varied. Solid, dotted and dashed curves are for $R = 4.0, 2.6$ and $1.5$ respectively. In general, the spectrum hardens with the increase of the compression ratio.

compared with the gas pressure $P_{\text{gas}} \sim 1/r^{5/2}$. Thus, it is not unlikely that magnetic field, equipartition with gas pressure at a given outer radius, could remain in equipartition in all the radii by appropriately releasing magnetic energy. However, the time scale of amplification could be comparable to the buoyancy time-scale (Nandi et al. 2001) and it is not necessary that the low angular flow would get enough time to be amplified to the equipartition value. In the absence of any time-dependent simulation results in this direction, we put the ratio of the magnetic energy to the gravitational energy, namely, $\beta$ to be a parameter. This enables us to understand the importance of the magnetic field. Since we are not considering any Keplerian disk, the only source of soft photon is the synchrotron radiation, the intensity of which directly depends on the magnetic field. Figure 7 shows the spectral variation. The solid, dotted and the dashed curves are drawn for $\beta = 1, 0.1$, and $0.01$ respectively. As expected, at low $\beta$, the spectrum is hard (dashed) and at high $\beta$, the spectrum is soft (solid). Thus, the spectral transition may also be triggered by the degree of magnetization of the flow.

While discussing Fig. 5, we have already mentioned that, when the accretion rate is increased the cooling takes place but the spectrum is hardened. This behaviour is generally seen in Figs. 8(a-b) where we present the variation of (a) the average electron temperature and (b) the spectral indices of the thermal ($\alpha_{\text{th}}$, solid) and ($\alpha_{\text{nth}}$, dotted) with the mass accretion rate. The parameters are $x_s = 35$ and $\zeta = 0.1$. In this case, a bump is seen around $\dot{m} \sim 0.1$ where the electron temperature rises momentarily. Below this bump, the synchrotron cooling dominates and above this bump the Compton cooling dominates. Close to the bump these are of comparable magnitude. Both the thermal and
non-thermal electrons show similar behaviour, namely, hardening with the increase of the accretion rate. It is to be noted that this is markedly different from CT95 result where the flow was becoming softer with the accretion rate. This is because, in this paper, we are plotting the variation with the halo rate (sub-Keplerian flow), while in CT95, it was the Keplerian rate that was considered. There too it was found that the spectral index goes down with the increase in the halo rate. Thus the behaviour in presence of synchrotron radiation (not present in CT95) is qualitatively similar.

7. Discussion and Concluding Remarks

In this paper, we have highlighted one major issue: to take care of the effect of accretion shocks on the spectral properties. To our knowledge, no such studies have been done in the literature. An accretion shock separates a flow into two parts: the pre-shock flow is of lower density but of higher volume, while the post-shock flow (CENBOL) is of higher density but of lower volume. More importantly, the shock accelerates particles and generate non-thermal electrons. We consider this generation self-consistently. It is true that we used the shock location and the compression ratio as independent parameters, but since we did not explicitly use the angular momentum of the matter, this re-parametrization is totally consistent. In this way, we have thus incorporated the emission from the Maxwell-
Boltzmann electrons and the power-law electrons as well as Comptonization of these emissions. Since the power-law electrons are present in the CENBOL, the radiation from these electrons is also the signature of the CENBOL.

In this paper, we have shown that, generally speaking, the spectral transition from a hard-state to a soft-state can be triggered by (a) reducing shock location (Fig. 3), (b) by reducing the compression ratio (Fig. 4), (c) by reducing the accretion rate (Fig. 5), (d) by reducing power-law electron contents (Fig. 6), and (e) by increasing the magnetic field strength inside the flow (Fig. 7). Note that, so far, we did not introduce the soft-photons from a Keplerian disk as was done in CT95. There, we did see that for a given rate of the Keplerian flow, if the accretion rate of the halo (sub-Keplerian flow) is increased, the spectrum becomes harder – an effect reflected in Fig. 5 above even when synchrotron photons are present. In future, we shall include the Keplerian disks as well. We shall also compare our results with observed spectra in order to pinpoint signatures of shocks in an accretion flow. In presence of a Keplerian disk, the soft X-ray bump will be produced at around a few KeV, while for shocks the bump due to Maxwell-Boltzmann electrons is located in UV while that of the power-law electrons is located at a few hundred keV to a few MeV. Thus, it is possible that these two distinct bumps can be distinguished through observation.

The outflows produced from a CENBOL itself can have shocks. The shocks in outflows are located very close to the black holes. Thus, a pre-shock flow has a very little volume. Post-shock outflows cool off rapidly and the cooler electrons ($T < 10^6$K) can also produce
Variation of the emitted spectrum when the ratio $\beta$ between the magnetic energy density and the gravitational energy density is varied. Solid, dotted and dashed curves are for $\beta = 1$, 0.1 and 0.01 respectively. The spectrum softens when the magnetic energy is increased.

a low-energy, power-law spectrum. On the other hand, there could be several shocks in the jets. Their effects from our CENBOL would be discussed elsewhere.

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References

Bell, A.R., 1978a, MNRAS 182, 147
Bell, A.R., 1978b, MNRAS 182, 443
Bondi, H., 1952, M.N.R.A.S., 112, 195
Chakrabarti, S. K. 1989, ApJ 347, 365
Chakrabarti, S. K. 1990, Theory of Transonic Astrophysical Flows (World Scientific: Singapore)
Chakrabarti, S. K. 1996, Phys. Rep. 266, 229
Chakrabarti, S. K., Acharyya, K., Molteni, D., 2004, A & A, 421, 1
Chakrabarti, S. K., Manickam, S. G. 2000, ApJ 531, L41
Chakrabarti, S. K. & Molteni, D., 1995, 272, 80
Chakrabarti, S. K., Nandi, A., Manickam, S. G., Mandal, S. and Rao, A. R., 2002, ApJ 579, 21
Chakrabarti, S. K., Titarchuk, L.G. 1995, ApJ 455, 623 (CT95)
Coppi, P.S., 1999, in ASP Conf. Ser. v. 161, High Energy Processes in Accreting Black Holes, Eds. J. Poutanen & R. Svensson (San Francisco: ASP), 375
Fig. 8. Variation of the (a) average electron temperature and (b) thermal (solid) and non-thermal (dotted) spectral indices as functions of the mass accretion rate.

Dermer, C.D., Liang, E.P. & Canfield, E., 1991, ApJ, 369, 410
Eardley, D., Lightman, A.P., 1975 ApJ, 200, 187
Eardley, D., Lightman, A.P., & Shapiro, S.L., 1975 ApJ, 199, 153
Eggum, G. E., Coroniti, F. V. & Katz, J. I., 1985, ApJ 298, L41
Esin, A. A. & Narayan, R., Cui, W., Grove, J. E. & Zhang, S.-N., 1998, ApJ, 505, 854
Galeev, A. A., Rosner, R. & Vaiana, G. S., 1979, ApJ, 229, 318
Gierliński, M. et al., 1999, MNRAS 309, 496
Gieseler, U. D. J., Kirk, J. G. 1997, A & A 323, 259
Kazanas, D. & Elision, J., 1986, ApJ, 384, 1986
Landau, L., Lifshitz, E. M. 1959, Fluid Mechanics (Pergamon Press: New York)
Lin, D. & Liang, E.P., 1999, A&A, 341, 954
Longair, M. S. 1981, High Energy Astrophysics, (Cambridge Univ. Press: UK)
Madau, P., 1988, ApJ, 327, 116
Mandal, S. & Chakrabarti, S.K., 2004, Astrophys. & Space Sc., Proceedings of the The Multiwavelength Approach To Unidentified Gamma-Ray Sources, Eds. K.S. Chang & R. Romero (in press).
Molteni, D., Lanzafame, G., Chakrabarti, S. K. 1994, ApJ 425, 161
Molteni, D., & Sponholz, H., 1994, MNRAS, 271, 233
Molteni, D., Sponholz, H., Chakrabarti, S.K., 1996, ApJ, 457, 805
Molteni, D., Tóth, G., Kuznetsov, O. A. 1999, ApJ 516, 411
Muchotrzeb, B. & Paczynski, B., 1982, Acta Astron. 32, 1
Nandi, A., Chakrabarti, S. K., Vadawale S. V., Rao, A. R., 2001, A & A 380, 245
Narayan, R. & Yi, I., 1995, ApJ, 452, 710
Paczyński, B. & Wiita, P.J., 1980, A & A 88, 23
Poutanen, J. & Coppi, P.S. 1998, Physica Scripta, T77, 57
Rees, M.J., Begelman, M.C., Blandford, R.D., & Phinney, E.S. 1982, Nature, 295, 17
Ryu, D., Chakrabarti, S. K., Molteni, D. 1997, ApJ 474, 378
Shakura, N.I. and Sunyaev, R., 1973, A & A 24, 337
Shapiro, S.L., 1973a, ApJ, 180, 531
Shapiro, S.L., 1973b, ApJ, 185, 69
Shapiro, S.L., Lightman, A.P. & Eardley, D., 1976 ApJ, 204, 187
Smith, D. M., Heindl, W. A., Markwardt, C. B., Swank, J. H. 2001, ApJ 554, L41
Smith, D. M., Heindl, W. A., Swank, J. H. 2002, ApJ 569, 362
Sunyaev, R. A., Titarchuk, L. 1985, A & A 143, 374
Titarchuk, L.G., Lyubarskij, Y. 1995, ApJ 450, 876
Zdziarski, A.A., Grove, J.E., Poutanen, J., Rao, A.R. & Vadawale, S.V. 2001, ApJ, 554, L45
Wardzinski, G. & Zdziarski, A. A. 2001, MNRAS 325, 963