s\bar{s} dominance of the $f_0(980)$ meson

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Abstract
We prove that recent data demonstrates unequivocally that the scalar meson $f_0(980)$ is mostly composed of $s\bar{s}$ quarks and that the coupling of $f_0$ to photons and mesons is in agreement with expectations from the linear sigma model.

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The observation[1] of the decay mode $\phi(1020) \rightarrow \pi^0\pi^0\gamma$ was reported very recently for the first time. The experiment clearly shows that the process is dominated by the $f_0(980)\gamma$ channel. This may be contrasted with the decay process $J/\psi \rightarrow \omega\pi\pi$, which was measured a decade ago[2] and where it was found that the $\omega f_0(980)$ channel was highly suppressed but that the $\omega\sigma(500)$ and $\omega f_2(1270)$ channels predominated. In this letter we wish to give an interpretation of these results and show that the $f_0(980)$ scalar meson is mostly composed of strange quarks.

First we recall that, according to quark models, the vector meson $\omega(782)$ is 97% nonstrange (viz. $\text{NS}=(u\bar{u}+d\bar{d})/\sqrt{2}$) and 3% strange ($\text{S}=s\bar{s}$). Conversely, the other vector meson $\phi(1020)$ has the opposite composition: 3%

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NS and 97% S. We will show that the above two experiments strongly suggest that the \( f_0(980) \) is largely S. Independently and for different reasons the phenomenological study of low-energy \( \pi\pi \) scattering and the phase shift analysis of \( \pi K \) scattering have led to the same conclusion about the composition of \( f_0(980) \).

The effective coupling constants, extracted from ref. [1], are based upon an inferred \( f_0 \) total decay width of \( 188^{+48}_{-33} \) MeV:

\[
g_{f_0\pi^+\pi^-}^{\text{eff}}/4\pi = 0.51^{+0.13}_{-0.09} \text{ GeV}^2 \quad \text{or} \quad |g_{f_0\pi^+\pi^-}^{\text{eff}}| \approx 2.5 \pm 0.3 \text{ GeV}, \quad (1)
\]

\[
g_{f_0K^+K^-}^{\text{eff}}/4\pi = 2.10^{+0.88}_{-0.56} \text{ GeV}^2 \quad \text{or} \quad |g_{f_0K^+K^-}^{\text{eff}}| \approx 5.1 \pm 0.9 \text{ GeV}. \quad (2)
\]

These values roughly follow from a dynamically generated theory of the SU(3) linear sigma model (L\( \sigma \)M) Lagrangian where a scalar meson nonet pattern is demanded: \( \sigma_{NS}(670) \), \( \kappa(810) \), \( \sigma_S(940) \), \( a_0(984) \). A related description is that the scalar mesons are composed of 4 quarks (\( qq\bar{q}\bar{q} \)), which is the description adopted by ref. [1].

In order to bring the observed isoscalar mesons \( \sigma(600) \) and \( f_0(980) \) into the L\( \sigma \)M picture, we must first consider the NS-S mixing basis,

\[
|\sigma\rangle = \cos \phi_s |\sigma_{NS}\rangle - \sin \phi_s |\sigma_S\rangle, \quad |f_0\rangle = \sin \phi_s |\sigma_{NS}\rangle + \cos \phi_s |\sigma_S\rangle \quad (3)
\]

in a manner similar to \( \eta - \eta' \) mixing. For the orthogonal mixed states \( \langle \sigma|f_0\rangle = 0 \), we find from (3),

\[
m_{\sigma_{NS}}^2 = m_\sigma^2 \cos^2 \phi_s + m_{f_0}^2 \sin^2 \phi_s, \quad m_{\sigma_S}^2 = m_\sigma^2 \sin^2 \phi_s + m_{f_0}^2 \cos^2 \phi_s. \quad (4)
\]

Inserting the dynamically generated NJL-type masses,

\[
m_{\sigma_{NS}} = 2\hat{m} \approx 670 \text{ MeV}, \quad m_{\sigma_S} = 2m_s \approx 940 \text{ MeV}, \quad (5)
\]

along with \( m_{f_0} \approx 980 \) MeV, one gets

\[
m_\sigma = [m_{\sigma_{NS}}^2 + m_{\sigma_S}^2 - m_{f_0}^2]^{1/2} \approx 610 \text{ MeV}, \quad \phi_s = \arcsin \left[ \frac{m_{f_0}^2 - m_{\sigma_{NS}}^2}{m_{f_0}^2 - m_\sigma^2} \right]^{1/2} \approx 20^\circ. \quad (6)
\]

Such a value for the scalar mixing angle was proposed several years ago and it is worth noting that the proximity in mass between \( \sigma_S(940) \) and \( f_0(980) \)
has its counterpart in the vector mesons with \( \phi_S(985) \) and \( \phi(1020) \). Thus one should not be surprised that the \( f_0(980) \) meson (with \( \cos 20^\circ \approx 0.94 \)) is principally an \( s\bar{s} \) state.

In order to link up to the recently extracted interactions (1) and (2), we first state the predicted \( L\sigma_M \) Lagrangian couplings\(^{[5]} \) (for brevity we refer to either the \( \pi^+\pi^- \) or \( K^+K^- \) final states as \( \pi\pi \) or \( KK \)),

\[
\begin{align*}
g_{\pi\sigma NS\pi} & = (m_{\sigma NS}^2 - m_{\pi}^2) / 2f_\pi \simeq 2.3 \text{ GeV}, \\
g_{K\sigma NSK} & = (m_{\sigma NS}^2 - m_{K}^2) / 4f_K \simeq 0.45 \text{ GeV}, \\
g_{K\sigma SK} & = (m_{\sigma S}^2 - m_{K}^2) / 2\sqrt{2}f_K \simeq 2.1 \text{ GeV},
\end{align*}
\]

(7)

where we have substituted the experimental values \( f_\pi \simeq 93 \text{ MeV}, f_K / f_\pi \simeq 1.22 \). From these and the mixing angle \( \phi_s \simeq 20^\circ \) we may compute the effective\(^{[9]} \) \( L\sigma_M \) couplings,

\[
\begin{align*}
geff_{f_0\pi\pi} & = 2\sin \phi_sg_{\pi\sigma NS\pi} \simeq 1.6 \text{ GeV}, \\
geff_{f_0KK} & = 2\sin \phi_sg_{K\sigma NSK} + 2\cos \phi.sg_{K\sigma SK} \simeq 4.3 \text{ GeV}.
\end{align*}
\]

(8) (9)

Although these predictions (8) and (9) seem shy of (1) and (2) by 64% and 84% respectively, we have not yet considered other data.

Specifically, the detailed Particle Data Group (PDG) Tables\(^{[7]} \) quote the average decay rate \( f_0 \rightarrow \gamma\gamma \) to be 0.56±0.11 keV. In combination with the \( f_0 \rightarrow \gamma\gamma \) branching ratio\(^{[7]} \) of \( (1.19 \pm 0.33) \times 10^{-5} \), one may derive a total \( f_0 \) width of 47±16 MeV. Given the \( f_0 \rightarrow \pi\pi \) branching ratio, one may deduce the PDG effective coupling\(^{[9]} \),

\[
|g_{f_0\pi\pi}^{\text{eff}}|_{PDG} = 1.1 \pm 0.4 \text{ GeV}.
\]

(10)

\(|g_{f_0KK}^{\text{eff}}|_{PDG} \) is unknown, because of the negligible phase space.)

This said, we note that our theoretical values (8) and (9) lie between the Novosibirsk and PDG values. See Table 1. Regardless of the final phenomenological resolution of the discrepancy, we must emphasize that the important result is the contrast between the measured \( \pi\pi \) invariant mass spectra of refs. \([1]\) and \([2]\).
### Table 1: Comparison of effective couplings (in GeV) of $f_0$ to the pseudoscalar mesons.

| Coupling | New Data  | LoSM Theory | Particle Data Group |
|----------|-----------|-------------|---------------------|
| $g^{eff}_{f_0\pi\pi}$ | 2.5 ± 0.3 | 1.6         | 1.1 ± 0.4           |
| $g^{eff}_{f_0KK}$   | 5.1 ± 0.9 | 4.3         | -                   |

For the reader’s convenience, we display the observed invariant $\pi^0\pi^0$ mass spectrum in $\phi \to \pi^0\pi^0\gamma$ in Figure 1; our quark model interpretation is given alongside, in Figure 2. The observed $\pi\pi$ mass spectra for $J/\psi \to \omega\pi\pi$ are displayed in Figure 3 to show the stark difference from Figure 1; again we provide the quark model interpretation alongside, in Figure 4. The resonance bumps $f_0(980)$ in $\phi \to \pi^0\pi^0\gamma$ and $\sigma(500)$ in $J/\psi \to \omega\pi\pi$ along with the near absence of an $f_0(980)$ bump in $J/\psi \to \omega\pi\pi$ and of $\sigma(500)$ in $\phi \to \pi^0\pi^0\gamma$ is telling us that $\sigma(500)$ is mostly NS while $f_0$ is mainly S. This is in consonance with the linear sigma model and a scalar mixing angle of $\phi_s \sim 20^\circ$.

However, to stress that this small mixing angle of $\phi_s$ is not the central issue, we close this letter by considering $f_0 \to 2\gamma$ decays which proceed via a quark loop. We may compare this radiative channel with $\pi^0 \to \gamma\gamma$, which is accurately estimated by a nonstrange quark triangle; the latter provides the effective amplitude $\alpha_{N_c}/3\pi f_\pi \simeq 0.025$ GeV$^{-1}$ and agrees with the data to within 2% for $N_c = 3$, $f_\pi \simeq 93$ MeV.

If $f_0$ were purely nonstrange too, the (isoscalar) $f_0\gamma\gamma$ effective amplitude would be given by

$$|M(f_{0NS} \to 2\gamma)| = 5\alpha_{N_c}/9\pi f_\pi \simeq 0.042 \text{ GeV}^{-1},$$

predicting a decay rate

$$\Gamma(f_{0NS} \to 2\gamma) = m_{f_0}^3|M_{NS}|^2/64\pi \sim 8 \text{ keV}. \quad (12)$$

On the other hand, if $f_0$ were a pure $s\bar{s}$ scalar, its effective $\gamma\gamma$ amplitude would be

$$|M(f_{0S} \to 2\gamma)| = \alpha_{N_c}g_{f_0ss}/9\pi m_s \sim 0.0081 \text{ GeV}^{-1},$$

for a constituent mass $m_s \simeq 490$ MeV and a strange $f_0ss$ coupling$^5$ of $\sqrt{2}g_{\pi^0qq} = \sqrt{2}2\pi/\sqrt{3}$. The decay rate would then be

$$\Gamma(f_{0S} \to 2\gamma) = m_{f_0}^3|M_S|^2/64\pi \sim 0.3 \text{ keV}. \quad (14)$$
From (12) we see that the pure $f_{0NS} \to \gamma\gamma$ decay rate is about 15 times larger than the PDG\cite{7} average rate of 0.56 keV, while the pure $f_{0S} \to \gamma\gamma$ rate (14) is within striking range of the experimental value. The comparison can be improved by including the small mixing angle $\phi_s$. But in any case the quark triangle description of radiative meson decays reinforces the fact that $f_0(980)$ is mostly $s\bar{s}$ rather than a $(u\bar{u} + d\bar{d})$ scalar meson.

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Effective couplings are defined via $\Gamma_{f_0 \pi\pi} = 3|g_{f_0 \pi\pi}^{\text{eff}}|^2/16\pi m_{f_0}^2$ and they yield $g_{f_0 \pi\pi}^{\text{eff}} \simeq 1.12\text{ GeV}$ for $\Gamma_{f_0 \pi\pi} = 0.781 \times 47\text{ MeV}$. We need to double the Lagrangian couplings by a factor of two to compare them with $g^{\text{eff}}$.

**Figure Captions**

Fig. 1. The measured $\pi^0\pi^0$ invariant spectrum in $\phi \to \pi^0\pi^0\gamma$. Reprinted from hep-ex/9807016 by kind permission from Budker Institute.

Fig. 2. Theoretical interpretation of $\phi \to \gamma f_0 \to \gamma\pi^0\pi^0$, as due to three gluon exchange.

Fig. 3. Fit of the $\pi\pi$ distribution in $J/\psi \to \omega\pi\pi$, as obtained by the DM2 group, by kind permission from Dr A Calcaterra. Reprinted from Nucl. Phys. B320, 1, Copyright (1989), with permission from Elsevier Science.

Fig. 4. Theoretical interpretation of $J/\psi \to \omega\sigma \to \omega\pi\pi$. 
Figure 1: Delbourgo
Figure 2: Delbourgo

$\phi(1020)$

$\gamma$

$\pi$

$\pi$

$f_0(980)$
