A Computational Study on the Site and Power Assignment Problem in Wireless Networks

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Abstract. In this work, we address the wireless network design problem, i.e., the problem of configuring a set of transmitters to provide service coverage to a set of receivers. It is well-known that natural formulations of this problem are sources of numerical instabilities and make the optimal resolution challenging for state-of-the-art solvers, even in small-sized instances. We tackle this limitation from a computational perspective by suggesting two implementation procedures capable of speeding the resolution of the instances of this problem. The first one consists of the employment of an extremely effective branching rule for a compact reformulation of this problem. The second one is the use of presolve operations to manage numerical instability. The approaches are validated using LTE instances kindly provided by Fondazione Ugo Bordoni. The proposed implementation techniques have proved capable of significantly accelerating the resolution of the problem, beating the performance of a standard resolution.

Keywords: wireless network design · base station deployment · power assignment · 0-1 linear programming · reduced cost fixing · fixing heuristic

1 Introduction

We address the optimal design of a wireless network, i.e., configuring a set of transmitters to provide service coverage to a set of receivers. The term configuring refers both to the optimal identification of the transmitter locations and receiver assignments, hence to the so-called base station deployment problem, and the optimal identification of some parameters of the transmitters, such as transmission power and/or frequency. This research has been carried out in collaboration with the Fondazione Ugo Bordoni (FUB) and is framed in the perspective of creating a tool used by the Municipalities. FUB is a higher education and research institution under the Ministry of Economic Development supervision that operates in the telecommunication field, providing innovative services for government bodies [14].

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Wireless networks are becoming denser due to technological advancements and increased traffic. In this context, practitioners’ traditional design approach, based on trial-and-error supported by simulation, has exhibited many limitations. The inefficiency of this approach leads to the need for optimization approaches, which are critical for lowering costs and meeting user-demanded service quality standards. Many optimization models for Wireless Network Design (WND) have been proposed over the years. However, the natural formulation on which most models are based has severe limitations since it involves strong numerical problems in the problem-solving phase, which emerge even in small instances. The reasons behind the numerical instabilities are the constraint matrix, containing coefficients that range in a huge interval, and the presence of large big-\(M\) coefficients leading to weak bounds. When combined with big instances that match real-life scenarios, memory issues may worsen numerical issues.

Most contributors used heuristic approaches to deal with large-sized instances due to their difficulty coping with practical cases. One of the first works identifying the presence of numerical issues is \cite{18}, in which a heuristic algorithm is suggested to solve very large instances of a MILP model. The model maximizes the coverage and considers power emissions and frequency channels as decision variables. A genetic algorithm for large-scale MILP problems is provided in \cite{10}. The authors of \cite{17} cope with the power assignment problem in wireless networks. Since their formulation involves big-\(M\) constraints and its exact resolution is not viable for large problems, a constructive heuristic is introduced, followed by an improving local search. A model for the optimal allocation of power emissions, frequency channels, and transmission scheme can be found in \cite{12}, in which a matherheuristic is proposed to tackle the problem. The matherheuristic combines a genetic algorithm that exploits a suitable linear relaxation of the problem and a heuristic that improves its solutions. A meta-heuristic technique is used in \cite{15} to solve a 0-1 model that minimizes the total number of deployed transmitters. Another contribution is \cite{17}, in which the architecture of 5G networks is examined using an optimization model that assigns sites and frequencies considering the limitations on electromagnetic emissions. The objective is to maximize coverage while lowering installation costs, and a heuristic is presented.

Only a minority attempted to optimally solve huge instances, mainly developing alternative non-compact formulations and/or using nonstandard optimization processes. Locations and power emissions of the transmitters are optimized in \cite{8} to maximize territorial coverage. The main proposal of the work is a non-compact binary formulation that does not use the big-\(M\) coefficients. The resolution algorithm is a combination of row-generation and branch-and-cut repeated on different combinations of power emission levels. The choice of base station locations and power emissions under the maximization of the service provider’s profit is pursued in \cite{19}. The MILP formulation described uses the big-\(M\) constraints, and the proposed exact resolution method uses combinatorial and classical Benders decomposition and valid cuts combined in a nested way. In \cite{2,3,5,9} mixed-integer formulations that take into account a wide range of decision variables, including locations and power emissions, are used. The res-
solution processes are exact and involve standard solvers, but tests are done on randomly generated instances with tiny sizes (when declared); hence the criticalities arising from practical instances are not faced. In our previous work [4], we propose a compact reformulation for the base station deployment problem that allows for the resolution of large instances simply using commercial MIP solvers.

In this work, we aim to give some implementation details that accelerate the optimal resolution of large instances. Since the problem is notoriously challenging from a computational point of view, heuristics are usually needed for its resolution. Heuristics implemented in MIP solvers have improved significantly in recent years and are finding growing success as they do not require an accurate preliminary tuning phase. For a discussion of how the heuristics of MIP solvers have a crucial impact on solving complex problems, see [13]. A compact formulation is necessary to exploit the heuristics to their fullest potential. In addition, compact formulations are easier for telecom practitioners to manage.

A contribution of the paper is an extended formulation involving adding some variables whose structure is more manageable by numerical solvers. We show how the use of particular branching strategies makes the resolution of the subproblems generated during the branch-and-bound faster. This effect is more marked in the proposed formulation.

A second contribution is a reduced cost fixing presolve procedure to fix some variables to zero and reduce big-

\[ M \]

coefficients, strengthening the formulation. Although reduced cost fixing is a well-known technique, its use on this problem has never been tested until now (according to our knowledge) and shows significant potential thanks to the positive effect it has on the reduction of numerical problems. To get good results with this technique, we develop a heuristic producing near-optimal solutions very quickly.

The remainder of this paper is organized as follows. Section 2 reports the statement of the problem and its natural formulation. Section 3 deepens the implementation techniques we propose to accelerate the optimal resolution process. Section 4 reports the computational results, whereas conclusions are provided in Sect. 5.

### 2 A Wireless Network Design Formulation

For our purposes, a wireless network consists of radio transmitters distributing service (i.e., wireless connection) to a target area. The target area is usually partitioned into elementary areas, called testpoints, in line with the recommendations of the telecommunications regulatory bodies. Each testpoint is considered a representative receiver of all users contained in the elementary area.

Testpoints receive the signals coming from all the transmitters. The power received is classified as serving power if it relates to the signal emitted by the transmitter serving the testpoint; otherwise is classified as interfering power (see physical layer specifications of the Long-Term Evolution (LTE) standard [21]). A testpoint is regarded as served (or covered) by a base station if the ratio of
the serving power to the sum of the interfering powers and noise power (Signal-to-Interference-plus-Noise Ratio or SINR) is above a threshold \( \delta \), whose value depends on the desired quality of service.

We assume the frequency channel and the transmission scheme of the transmitters are given and equal for all of them. Furthermore, we assume that the power emissions of the activated transmitters can be represented by a finite set of power values, which fits with the standard network planning practice of considering a small number of discrete power values. The practice of power discretization for modeling purposes has been introduced in [8].

Let \( B \) be the finite set of candidate transmitters and \( T \) be the finite set of receivers located at the testpoints. Let \( \mathcal{P} = \{ P_1, \ldots, P_{|\mathcal{P}|} \} \) be the finite set of feasible power values assumed by the activated transmitters, with \( P_1 > 0 \) and \( P_{|\mathcal{P}|} = P_{max} \). Hence \( L = \{ 1, \ldots, |\mathcal{P}| \} \) is the finite set of power value indices (or simply power levels). We introduce the variables

\[
z_{bl} = \begin{cases} 1 & \text{if transmitter } b \text{ is emitting at power } P_l \\ 0 & \text{otherwise} \end{cases} \quad b \in B, l \in L \]

and

\[
x_{tb} = \begin{cases} 1 & \text{if testpoint } t \text{ is served by transmitter } b \\ 0 & \text{otherwise} \end{cases} \quad b \in B, t \in T \]

We can introduce the natural formulation of the WND problem, containing the so-called SINR inequalities used to assess service coverage conditions. To formulate the SINR inequalities, we refer to the discrete big-M formulation reported, e.g., in [8], which considers a discretization of the power range. Let \( a_{tb} > 0 \) be the fading coefficient applied to the signal received in \( t \in T \) and emitted by \( b \in B \). Then a receiver \( t \) is served by a base station \( \beta \in B \) if the SINR of the serving power to the sum of the interfering powers and noise \( \mu > 0 \) is above a given SINR threshold \( \delta > 0 \), namely

\[
a_{t\beta} \sum_{l \in L} P_l z_{\beta l} \geq \delta \quad \mu + \sum_{b \in B \setminus \{\beta\}} a_{tb} \sum_{l \in L} P_l z_{bl} \geq \delta \quad t \in T, \beta \in B : x_{t\beta} = 1. \]  

(1)

Following [8], we can rewrite the SINR condition (1) using the following big-M constraints

\[
a_{t\beta} \sum_{l \in L} P_l z_{\beta l} - \delta \sum_{b \in B \setminus \{\beta\}} a_{tb} \sum_{l \in L} P_l z_{bl} \geq \delta \mu - M_{t\beta}(1 - x_{t\beta}) \quad t \in T, \beta \in B \]  

(2)

where \( M_{t\beta} \) is a large (strictly) positive constant. When \( x_{t\beta} = 1 \), (2) reduces to (1); when \( x_{t\beta} = 0 \) and \( M_{t\beta} \) is sufficiently large, (2) becomes redundant. We can set, e.g.,

\[
M_{t\beta} = \delta \mu + \delta P_{max} \sum_{b \in B \setminus \{\beta\}} a_{tb} \]  

(3)
like in [8].

To enforce a minimum territorial coverage, namely a minimum integral number \( \alpha \in [0, |T|] \) of served testpoints, we need

\[
\sum_{b \in B} \sum_{t \in T} x_{tb} \geq \alpha. \tag{4}
\]

Each testpoint must be covered by at most one serving base station, namely

\[
\sum_{b \in B} x_{tb} \leq 1 \quad t \in T. \tag{5}
\]

To enforce the choice of only one (strictly positive) power level for each activated transmitter we use

\[
\sum_{l \in L} z_{bl} \leq 1 \quad b \in B. \tag{6}
\]

Variable upper bound constraints

\[
x_{tb} \leq \sum_{l \in L} z_{bl} \quad t \in T, b \in B, \tag{7}
\]

can be included in the formulation – though redundant – to improve the lower bounds and speed up the resolution, as stressed in [4] for the case with fixed power emissions. They enforce that a testpoint \( t \) can be assigned to a transmitter \( b \) only if \( b \) is activated.

A considerable goal to pursue is the citizens’ welfare; therefore, we propose a model that aims at identifying solutions with low environmental impact in terms of electromagnetic pollution and/or power consumption. Reducing electromagnetic pollution indeed involves reducing the power emitted by the transmitters [6]. Hence, we aim to minimize the total number of activated base stations with a penalization on the use of stronger power levels; the cost associated with the use of a power level equal to \( l \in L \), namely \( c_l \), is greater the greater is \( P_l \).

To summarize, a natural formulation for the WND problem is the following 0-1 Linear Programming:

\[
\min_{x,z} \sum_{b \in B} \sum_{l \in L} c_l z_{bl} \quad \forall (x, z) \in S \tag{8}
\]

where \( S \) is the feasible region defined as

\[
S = \{(x, z) \in \{0, 1\}^{n+m} : \text{satisfying (2), (4), (5), (6), (7)}\}
\]

with \( x = (x_{tb})_{t \in T, b \in B} \), \( z = (z_{bl})_{b \in B, l \in L} \) and \( n = |T| \times |B|, m = |B| \times |L| \).
3 Implementation Details

In principle, model (8) can be solved by MIP solvers. However, it is well-known (see, e.g., [4, 8, 11]) that the following issues arise when trying to solve real-life instances:

- the power received in each testpoint lies in a large interval, from very small values to huge, which makes the range of the coefficients in the constraint matrix very large and the solution process numerically unstable and possibly affected by error;
- the big-M coefficients lead to poor quality bounds that impact the effectiveness of standard solution procedures;
- real-world problems lead to models with a huge number of variables and constraints.

As a result of these issues, real-life instances of this problem are difficult to solve. Therefore, this paper aims to report some implementation details that can make the solution of this problem more efficient and fast.

3.1 Reformulation

We can rewrite (2) as

\[ M_{t \beta} (1 - x_{t \beta}) \geq \delta \sum_{b \in B \setminus \{\beta\}} a_{tb} \sum_{l \in \mathcal{L}} P_l z_{bl} - a_{t \beta} \sum_{l \in \mathcal{L}} P_l z_{\beta l} + \delta \mu \quad t \in T, \beta \in B. \]  

(9)

We introduce the auxiliary binary variables \( w_{tb} \), defined as

\[ w_{tb} \leq 1 - x_{tb} \quad t \in T, b \in B \]

\[ w_{tb} \in \{0, 1\}. \]  

(10)

Hence, \( w_{tb} \) is forced to zero whenever a testpoint \( t \) is served by a base station \( b \).

We can rewrite (9) using \( w_{tb} \)

\[ M_{t \beta} w_{t \beta} \geq \delta \sum_{b \in B \setminus \{\beta\}} a_{tb} \sum_{l \in \mathcal{L}} P_l z_{bl} - a_{t \beta} \sum_{l \in \mathcal{L}} P_l z_{\beta l} + \delta \mu \quad t \in T, \beta \in B. \]  

(11)

Theorem 1. Inequalities (10) and (11) are valid for \( S \).

Proof. By (10) we get that when \( x_{t \beta} = 1 \) we must have \( w_{t \beta} = 0 \) and (2) and (11) enforce the same condition on the SINR. When instead \( x_{t \beta} = 0 \), the big-M activates in constraint (2) so that no condition is enforced on the corresponding SINR; in turn from (10) we have that \( w_{tb} \in \{0, 1\} \) so that (11) does not enforce any condition on SINR too. \( \square \)

We will denote by reformulated WND the following 0-1 Linear Programming:

\[ \min_{x, w, z} \sum_{b \in B} \sum_{l \in \mathcal{L}} c_l z_{bl} \quad (x, w, z) \in S' \]  

(12)
where the feasible region is given by

\[ S' = \{(x, w, z) \in \{0, 1\}^{2n+m} : \text{satisfying } (1), (5), (6), (7), (10), (11) \}\]

with \( w = (w_{tb})_{t \in T, b \in B} \).

Reformulation (12) opens the possibility of testing new branching rules. According to computational testing (see Sect. 4), branching with a priority on the \( w \) variables typically yields faster results than alternative branching rules.

### 3.2 Coefficient Tightening

Reducing the model size is crucial because real-life instances typically involve many variables and constraints. In this direction, we propose using a Reduced Cost Fixing method, in short RCF (see [1] for a survey on presolve techniques). Although this procedure is well-known and widespread, no one has ever tried to see its effects on this type of problem (based on our knowledge).

We get the lower bound \( lb \) solving the linear relaxation of the problem and the corresponding reduced costs \( \tilde{c}_{bl} \) associated with the sole variables \( z_{bl} \) in the optimal solution of the linear relaxation. Then, given an upper bound \( ub > lb \), if

\[ \tilde{c}_{bl} \geq ub - lb \text{ for some } b \in B, l \in L \]

the corresponding \( z_{bl} \) must be at its lower bound in every optimal solution; hence we can fix \( z_{bl} = 0 \).

Whenever the fixing of a variable \( z_{bl} \) occurs at a given \( l \in L \) such that \( P_l = P_{max} \), we can recompute and reduce the big-\( M \), resulting in a tightening of the formulation. Indeed, in the big-\( M \) (see (3)) appears a sum over \( b \in B \) of the coefficients \( a_{tb} \) weighted according to the maximum power that \( b \) can emit, that is \( P_{max} \). After the RCF, we may have some cases in which a certain transmitter \( b \) cannot emit at the maximum power level \( l \) such that \( P_l = P_{max} \), since the corresponding \( z_{bl} \) variable has been fixed to zero. In such cases, the value by which \( a_{tb} \) is weighted in the big-\( M \) can be decremented to the highest power value that \( b \) can assume.

To formalize it, let us define the set \( B^R \subseteq B \) of base stations affected by RCF, i.e., such that \( b \in B^R \) if the variable \( z_{bl} \) has been fixed to zero for at least one \( l \in L \). Then, for a given \( b \in B^R \), we can define the set \( L_b \subseteq L \) of power levels that \( b \) can assume after the RCF. We denote by \( P_{b, max} \) the power value corresponding to the maximum power level that \( b \in B^R \) can assume. Since \( L_b \subseteq L \), we have that \( P_{b, max} \leq P_{max} \). Using this notation, we can write down the new value of the big-\( M \)

\[ M'_{l_0} = \delta \mu + \delta \left( P_{max} \sum_{b \in B \setminus \{b, B^R\}} a_{tb} + \sum_{b \in B^R} P_{b, max} a_{tb} \right) = \delta \mu + \delta \sum_{b \in L \setminus \{\beta\}} \tilde{P}_b a_{tb} \]

which satisfies \( M'_{l_0} \leq M_{l_0} \) since

\[ \tilde{P}_b = \begin{cases} P_{max} & \text{if } b \in B \setminus B^R \\ P_{b, max} & \text{if } b \in B^R \leq P_{max} \end{cases} \]
The smaller the optimality gap given by the estimated lower and upper bounds, the greater the number of \( z_{bl} \) variables that can be fixed to zero, and the smaller the big-\( M \) coefficients. Hence applying a standard algorithm, as implemented in MIP commercial solvers, to the tightened formulation—got after the RCF—produces stronger bounds and a faster resolution.

We can apply another reduction (already proposed in [4]). Since we have information on the maximum number of transmitters that can be installed from the \( ub \), we can further reduce the value of the big-\( M \) by replacing the sum of all the interfering contributions in the testpoint \( t \) with the sum of the major interfering contributions in \( t \). In particular, only the strongest \( \gamma \) interferers are considered, where \( \gamma \) is the maximum number of transmitters that can be activated. Since no more than \( \gamma \) transmitters can be activated simultaneously, there could be no more than \( \gamma \) interferers. Note that in this way, all the big-\( M \) coefficients are affected by this operation, i.e., are reduced. Hence, a reduced big-\( M \) is

\[
M_{t,\beta}' = \delta \mu + \delta \sum_{b \in A_t \setminus \{\beta\}} \hat{P}_b a_{tb} \leq M_{t,\beta}' \leq M_{t,\beta}
\]

where \( A_t \subset B \) is the set of the \( \gamma \) base stations emitting the strongest signals received in \( t \), i.e, \( |A_t| = \gamma \). The smaller \( \gamma \), the smaller the big-\( M \), and the better the formulation. Hence, the estimate of \( \gamma \) must be as accurate as possible.

We observe that the presence of constraints (7) in \( S' \), that naturally lead to a good bound, makes obtaining a suitable lower bound trivial. Getting a good upper bound is more hard instead. Commercial MIP solvers can be employed to identify a good feasible solution, but it may take a long time. Hence, we develop a fixing heuristic based on the observation that often, the fractional values of the variables in the LP relaxation are good predictors of zero/non-zero variables in an optimal ILP solution. This happens especially when the LP relaxation is very tight because it could mean that the fractional solution is very close to the optimal. The scheme of our fixing heuristic is:

1. solve the LP relaxation of the problem and take the fractional solution;
2. identify the set of fractional variables that are likely to be zero (i.e., whose value is less than a very low threshold in the fractional solution);
3. fix the selected variables to zero and perform bound strengthening to propagate implications;
4. solve the resulting ILP problem to get a near-optimal feasible solution.

The selected variables rounded to zero correspond to the power levels not needed by the transmitters to cover the target area.

4 Computational Experiments

The computational experiments are done over a set of eleven realistic instances provided by FUB concerning the LTE signal in the Municipality of Bologna. Each instance differs in the quality of service required, i.e., it is characterized
by a threshold \( \delta \) chosen in the typical range of LTE services \([0.100 \text{ W}, 0.316 \text{ W}]\), and the minimum number of receivers covered, i.e., \( \alpha \) ranges in \([0.95|T|, |T|]\).

The parameters of the optimization are: \(|\mathcal{B}| = 135\), \(|\mathcal{T}| = 4693\), \(|\mathcal{L}| = 3\), \(\mathcal{P} = \{20 \text{ W}, 40 \text{ W}, 80 \text{ W}\}\), \(c_l = (1, 2, 4)\), \(\mu = 7.998 \times 10^{-14} \text{ W}\). The values of the received power \(a_{tb}P_l\), and of the noise \(\mu\) by consequence, are scaled by \(10^{-10}\) to get better accuracy on the optimal solutions, as suggested in [11]. Hence, \(a_{tb}P_l\) ranges in \((10^{-4}, 10^{5})\) after this scaling.

The code has been implemented in Python, and the experiments have been carried out on a Ubuntu server with an Intel(R) Xeon(R) Gold 5218 CPU with 96 GB RAM. Gurobi Optimizer 9 [16] has been used to get all the results, including the feasible solutions for the RCF. The setting used on Gurobi to solve the problems using formulations (8) and (12) consists of a standard B&B with more time spent on identifying feasible solutions, which is critical in closing the problem resolution.

The purpose of this section is to validate the two proposed approaches. As regards the first, in Table 1 there is a comparison between the natural formulation (8) under standard resolution — what is called Basic framework (indicated with B) — and its reformulation (12) under standard resolution (indicated with R).

The metrics reported in Table 1 are the number of branching nodes and the computational times on each tested instance (identified by the ID code) and for each framework; the lower times are in bold and the instances reported are ordered according to a complexity that grows going downwards. The performance obtained on these two frameworks shows little difference: R reaches optimum faster in 6/11 instances (almost 55%). The quality of subproblems produced by branching on the \(w\) variables may be the source of this sporadic speeding since the reformulation cannot yield better bounds. We provided priority to the \(w\) variables during branching for this reason. The performance is displayed under the column R + PB\(w\) of Table 1 where R indicates the reformulation and PB\(w\) denotes the priority branching followed by the variable on which the priority is given (i.e., \(w\)). By prioritizing the \(w\) variables when branching, we can close the MIP gap more quickly than in the B/R framework in 10/11 instances (more than 90%).

To show that such good results cannot be obtained by prioritizing the branching of the \(x\) variables directly on the natural formulation, we provide the results obtained in this case — indicated with B+PB\(x\) — in Table 1. We also tested a framework that employs the reformulation but gives priority to \(x\) variables while branching, which we refer to as R+PB\(x\). For what concerns B+PB\(x\), even though it is faster than B in most of the cases, we have that R+PB\(w\) is faster than it in 8/11 instances (more than 70%). Compared to B/R, we get better results when branching with priority to the \(x\) or the \(w\) in the reformulation. This means that the computational difference in using a natural formulation and its reformulation is visible in adopting particular branching strategies. As for the choice between giving priority to \(x\) or \(w\) in the reformulation, we recommend R+PB\(w\) as the computational times are, on average, slightly lower; however, the number of explored nodes is identical in R+PB\(w\) and R+PB\(x\).
Table 1. Comparison between the natural formulation under standard resolution (B), its reformulation under standard resolution (R), and both the formulations with a branching rule giving priority to some variables (B+PBx, R+PBx, R+PBw)

| ID | B Nodes | B Time[s] | B+PBx Nodes | B+PBx Time[s] | R Nodes | R Time[s] | R+PBx Nodes | R+PBx Time[s] | B+RCF Nodes | B+RCF Time[s] | R+RCF Nodes | R+RCF Time[s] | R+PBw+RCF Nodes | R+PBw+RCF Time[s] |
|----|---------|-----------|-------------|--------------|---------|---------|-------------|--------------|-------------|--------------|-------------|--------------|----------------|------------------|
| I-10 | 81 | 652.33 | 587 | 941.11 | 189 | 629.35 | 528.38 | 189 | 537.11 |
| I-9.5 | 80 | 629.35 | 244 | 727.43 | 308 | 987.71 | 82 | 544.11 | 82 | 540.74 |
| I-9 | 88 | 751.76 | 227 | 495.73 | 341 | 2052.64 | 88 | 435.03 | 88 | 433.70 |
| I-8.5 | 83 | 652.87 | 390 | 643.51 | 87 | 686.63 | 87 | 686.86 | 87 | 662.00 |
| I-8 | 2097 | 13939.16 | 169 | 535.40 | 328 | 1076.34 | 525 | 609.97 | 525 | 727.12 |
| I-7.5 | 2102 | 1587.88 | 282 | 621.91 | 251 | 800.53 | 39 | 348.67 | 39 | 385.21 |
| I-7 | 350 | 1093.30 | 80 | 365.61 | 356 | 1214.33 | 165 | 393.10 | 165 | 395.94 |
| I-6.5 | 1528 | 1331.78 | 1528 | 1371.79 | 521 | 1102.29 | 1141 | 883.88 | 1141 | 902.55 |
| I-6 | 1973 | 1265.77 | 2471 | 1432.41 | 1480 | 1096.09 | 997 | 841.16 | 997 | 852.44 |
| I-5.5 | 76 | 2474.90 | 76 | 2470.98 | 76 | 2538.30 | 76 | 2348.91 | 76 | 2486.41 |
| I-5 | 75 | 2934.47 | 72 | 2460.33 | 75 | 2355.57 | 76 | 2403.94 | 76 | 2428.77 |

Notice that we only present the results from the most promising branching rules. However, we also tested branching with priority on z variables, Pseudo Reduced Cost Branching, Pseudo Shadow Price Branching, Maximum Infeasibility Branching, and Strong Branching, all producing unsuccessful results.

The other technique to speed up the resolution of the problem is a presolve based on reduced cost fixing. We denote this framework by B+RCF when applied to the natural formulation and R+RCF when applied to its reformulation. In Table 1, we compare the computational results obtained with this technique on the two formulations to the results obtained with the basic framework B. In addition, we tried to combine the two proposals by experimenting with the RCF applied to the reformulation and using priority branching on w when solving the MIP after the RCF – we denote this framework by R+PBw+RCF. From a computational perspective, applying the RCF-based presolve is preferable in most cases, regardless of the formulation or branching rules. In fact, RCF produces smaller problems that are typically faster to solve. For what concerns the best approach among the three reported, the R+PBw+RCF case seems to be the worst, though still better than B on average. Instead, both B+RCF and R+RCF produce good results. R+RCF has a greater number of times the lowest time, but it is less stable than B+RCF; thus, we conclude that applying the RCF to the natural formulation is the best strategy.

In Table 3, we analyze the effect of the RCF presolve on the problem sparsity and the big-M range and assess the importance of having a fast heuristic for achieving such good results. The evaluation metrics of Table 3 are the number of non-zeros for what concerns the sparsity, the maximum value of the big-M, and some metrics about the time, including the total resolution time (denoted by TotTime) needed to evaluate the proposed method. The total resolution time is given by the sum of the time to get the lower bound (not reported), the time to get the upper bound using our heuristic (denoted by HTime), and the time to solve the problem after the RCF-presolve (denoted by Time).
Table 2. Comparison between the basic framework and the reduced cost fixing presolve applied to the natural formulation (B+RCF) and the reformulation (R+RCF, R+Pbw+RCF)

| ID   | B       | B + RCF | R + RCF | R + Pbw + RCF |
|------|---------|---------|---------|---------------|
|      | Nodes   | Time[s] | Nodes   | Time[s]       | Nodes   | Time[s]       |
| I-10 | 80      | 652.33  | 36      | 405.55        | 49      | 484.46        | 65      | 393.4        |
| I-9.5| 81      | 629.35  | 248     | 556.98        | 156     | 493.38        | 156     | 606.58       |
| I-9  | 88      | 751.76  | 125     | 647.30        | 186     | 460.41        | 375     | 1590.17      |
| I-8.5| 83      | 652.87  | 116     | 430.66        | 86      | 601.51        | 106     | 438.11       |
| I-8  | 2097    | 13939.16| 149     | 497.47        | 1647    | 8772.59       | 404     | 567.39       |
| I-7.5| 2102    | 1587.88 | 154     | 486.41        | 105     | 430.23        | 157     | 563.69       |
| I-7  | 350     | 1093.30 | 154     | 461.14        | 111     | 1036.27       | 111     | 1069.02      |
| I-6.5| 1528    | 1331.78 | 124     | 544.68        | 175     | 496.49        | 256     | 829.04       |
| I-6  | 1973    | 1265.77 | 258     | 593.12        | 75      | 459.39        | 75      | 454.56       |
| I-5.5| 76      | 2474.90 | 63      | 2795.88       | 37      | 2368.67       | 37      | 2546.26      |
| I-5  | 75      | 2934.47 | 340     | 2780.13       | 211     | 3768.62       | 211     | 3780.96      |

Table 3. Comparison between the basic framework (B) and the reduced cost fixing presolve applied to the natural formulation (B+RCF)

| ID   | B       | B + RCF |
|------|---------|---------|
|      | Non-zeros MaxBig-M Time[s] | Non-zeros MaxBig-M RTTime[s] | Time[s] | TotTime[s] |
| I-10 | 12573655 915958.73 652.33 | 7435231 45111.35 65.55 | 318.92 | 405.55 |
| I-9.5| 12573655 103179.39 629.35 | 7950820 51589.60 80.71 | 150.91 | 556.98 |
| I-9  | 12573655 115769.18 751.76 | 7950820 57884.48 51.04 | 562.77 | 647.30 |
| I-8.5| 12573655 129895.16 652.87 | 7950820 64947.46 49.36 | 356.77 | 430.66 |
| I-8  | 12573655 145744.77 13939.16| 7950820 72872.25 54.40 | 416.21 | 497.47 |
| I-7.5| 12573655 161284.63 1587.88| 7950820 79119.94 48.84 | 410.17 | 486.41 |
| I-7  | 12573655 183481.79 1093.30| 7950820 91740.72 56.12 | 381.49 | 461.14 |
| I-6.5| 12573655 205869.96 1331.78| 7950820 102934.78 56.84 | 462.92 | 544.68 |
| I-6  | 12573655 230989.89 1265.77| 7950820 115494.73 127.56 | 439.38 | 593.12 |
| I-5.5| 12620585 259174.92 2474.90| 7950820 3781038 62347.46 | 37.96 | 2714.18 |
| I-5  | 12620585 290799.04 2934.47| 7950820 3768647 69955.01 | 36.60 | 2687.69 |

From the results in Table 2 we can state that RCF-based presolve induces more sparsity in the model as the number of non-zeros is noticeably lower. It also intervenes on each big-M by decreasing its value (the maximum value is halved on average) and experimentally reduces computational times. It should be noted that such good total time results could only be achieved thanks to a good and rapid heuristic. While this approach is a promising and valid tool to efficiently solve large-scale problems of site and power assignment, it has one limitation. It fails in instances where a very high quality of service is required (i.e., I-5.5, I-5), that is, in the most complex scenarios.
In conclusion, by comparing the two best implementation techniques, it can be stated that B+RCF reaches the optimum with an average number of nodes equal to 160.64 and with average times equal to 927.21 seconds, while R+PBw takes on average 315 nodes and 911.27 seconds. Both proposed strategies significantly beat the basic framework, which takes, on average, 777.73 nodes and 2483.05 seconds.

5 Conclusions

This work aimed at tackling site and power assignment problems in wireless networks. Our research focused on giving implementation suggestions to make the optimal resolution of this problem faster. We proposed a reformulation leading to a more efficient branching and presolve operations for reducing the big-$M$ and the problem size. The presolve is based on a reduced cost fixing procedure. Computational tests on several realistic instances of this problem confirmed the efficiency of our proposals, which, compared to standard resolution of the natural formulation of the problem, resulted in faster resolution times. However, it makes sense to apply the reduced cost fixing procedure only when proper upper bounds can be computed quickly: this is not trivial in all cases, particularly when high-quality service is required. In this direction, our fixing heuristic was crucial to the success of this approach.

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