Unscreening modified gravity in the matter power spectrum

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Viable modifications of gravity that may produce cosmic acceleration need to be screened in high-density regions such as the Solar System, where general relativity is well tested. Screening mechanisms also prevent strange anomalies in the large-scale structure and limit the constraints that can be inferred on these gravity models from cosmology. We find that by suppressing the contribution of the screened high-density regions in the matter power spectrum, allowing a greater contribution of unscreened low densities, modified gravity models can be more readily discriminated from the concordance cosmology. Moreover, by variation of density thresholds, degeneracies with other effects may be dealt with more adequately. Specializing to chameleon gravity as a worked example for screening in modified gravity, employing N-body simulations of $f(R)$ models and the halo model of chameleon theories, we demonstrate the effectiveness of this method. We find that a percent-level measurement of the clipped power at $k < 0.3\, \text{h/Mpc}$ can yield constraints on chameleon models that are more stringent than what is inferred from Solar System tests or distance indicators in unscreened dwarf galaxies.

Introduction.— Determining the nature of the accelerated expansion of our Universe is a prime endeavor to cosmologists. In the conventional picture, the flat Λ cold dark matter (ΛCDM) concordance model based on general relativity (GR), a cosmological constant Λ contributes the bulk of the present energy density in the cosmos and drives the late-time acceleration. While alternatively, a modification of gravity may be responsible for cosmic acceleration, stringent limitations from experiments within our Solar System must be satisfied. A number of screening mechanisms have been identified that can suppress modifications of gravity in high-density regions to recover GR, whilst still generating significant modifications within lower densities on larger, cosmological scales. However, this suppression effect, along with other nonlinear effects, also prevents strong anomalies to manifest in the averaged large-scale structure of our Universe and limits the constraints that can be inferred on these gravity models from cosmology.

Given the density dependence of the screening effect, in this Letter, we propose the downweighting of high-density regions in statistical observables such as the matter power spectrum $P(k)$ to enhance, or unscreen, the signatures of modified gravity and improve observational constraints. Such a weighting is conducted in the clipping method of Ref. with the original motivation of facilitating the modeling of $P(k)$ by reducing contributions of high densities, where the assumptions of perturbation theory break down. As a worked example, we first focus on Hu-Sawicki $f(R)$ gravity, which employs the chameleon screening mechanism. We analyze effects on the power spectrum from clipping density fields in numerical simulations of the model. Using the halo model of chameleon theories, we then generalize our findings to chameleon models with arbitrary gravitational coupling and exponents of the chameleon field potential.

Modified gravity.— We first specialize to the Hu-Sawicki $f(R)\,(n = 1)$ model, where the nonlinear function $f(R) \sim -2\Lambda - f_{R0} R_0^2 / R$ of the Ricci scalar $R$ is added to the Einstein-Hilbert action. Here, bars denote quantities evaluated at the cosmological background, zeros refer to present time, $f_R \equiv df / dR$, and $f_{R0} \equiv f_R(z = 0)$. In the quasistatic approximation and for $|f_{R0}| \ll 1$, the modified Poisson equation becomes

$$\nabla^2 \Psi = \frac{16\pi G}{3} \delta\rho_m - \frac{1}{6} \delta R(f_R),$$

where $\delta$ denotes perturbations with respect to the cosmological background and $\Psi \equiv \delta g_{00} / (2g_{00})$. The scalar field equation is given by

$$\nabla^2 f_R = - \frac{8\pi G}{3} \delta\rho_m + \frac{1}{3} \delta R(f_R).$$

The chameleon mechanism works such that in a high-density region $\delta R \simeq 8\pi G \delta\rho_m$ and hence Eq. reduces to the Poisson equation of Newtonian gravity. In contrast, at low densities $\delta R \simeq \partial R / \partial f_R |_{R = \hat{R}} = 3m^2 \delta f_R$, which when applied to Eqs. (1) and (2) in Fourier space yields the unscreened modified Poisson equation

$$k^2 \Psi = -4\pi G \left\{ \frac{4}{3} - \frac{1}{3} \left[ \left( \frac{k}{ma} \right)^2 + 1 \right]^{-1} \right\} \delta\hat{\rho}_m.$$  

Hence, whereas in high-density regions gravity returns to Newtonian due to the chameleon mechanism, at low densities and scales below $m^{-1}$, gravitational interactions remain enhanced by a factor of $4/3$. In particular, Eq. applies to the linear perturbation regime. Note that $f(R)$ models correspond to a Brans-Dicke scalar-tensor theory with Brans-Dicke parameter $\omega = 0$, Jordan-frame scalar field $\varphi = 1 + f_R$, and scalar field potential.

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Hence, whereas in high-density regions gravity returns to Newtonian due to the chameleon mechanism, at low densities and scales below $m^{-1}$, gravitational interactions remain enhanced by a factor of $4/3$. In particular, Eq. applies to the linear perturbation regime. Note that $f(R)$ models correspond to a Brans-Dicke scalar-tensor theory with Brans-Dicke parameter $\omega = 0$, Jordan-frame scalar field $\varphi = 1 + f_R$, and scalar field potential.
$U = (Rf_R - f)/2$. More generally, the chameleon mechanism is realized for scalar-tensor models with scalar field potential $U(\varphi) = A (1 - \varphi^p)$ with $\alpha \equiv n/(n+1) \in (0,1)$, where the gravitational coupling is maximally enhanced by a factor of $(4 + 2\omega)/(3 + 2\omega)$ with $\omega > -3/2$. Importantly, although serving as a very useful example for screening mechanisms, chameleon models do not yield a genuine self-acceleration of the cosmic expansion due to their gravitational modifications [10].

In order to obtain accurate results in the nonlinear regime of the Hu-Sawicki $f(R)$ model, we use dark matter $N$-body simulations run in Ref. [11] with the ECOSMOG code of Ref. [12], which uses particles and adaptive meshes to solve Eqs. (1) and (2). The background expansion is taken to be equivalent to that of ΛCDM, appropriate for observationally interesting $f_{\text{R0}}$ values. We use simulations of the concordance model and $f(R)$ gravity where $|f_{\text{R0}}| = 10^{-4}$ (F4), $10^{-5}$ (F5), and $10^{-6}$ (F6), all sharing an initial seed and cosmological parameters. Each simulation contains 512$^3$ particles in a box with $L = 512 h^{-1}\text{Mpc}$; $h = 0.697$, $\Omega_m = 0.281$, $\Omega_b = 0.046$, $\Omega_{\Lambda} = 0.719$, $n_s = 0.971$, and amplitude of the matter power spectrum such that $\sigma_8 = 0.82$ in ΛCDM. Note that $\sigma_8$ is larger for $f(R)$ gravity due to the enhanced forces and growth of structure. An initial power spectrum for the simulations was generated using MPGRAPHIC [13]. The particle mass in each case is $\sim 7.80 \times 10^{10}h^{-1}\text{M}_\odot$. Each simulation has exactly the same initial power spectrum ($z_i = 49$) and differences between models are confined to different strengths of enhanced perturbation growth at late times and different strengths of screening.

For the extrapolation of nonlinear physics from $f(R)$ gravity to more general chameleon models, we employ the halo model of chameleon theories developed in Ref. [2], which accounts for the density dependence of the effective gravitational coupling in the nonlinear regime and provides matter power spectra that are in good agreement with measurements in $f(R)$ $N$-body simulations (see Fig. 4 in Ref. [2]). Note that a similar density dependence enters the effective gravitational coupling at nonlinear scales in models exhibiting a Vainshtein screening effect [14].

**Clipping the density fields.**—The spatial distribution of matter on cosmological scales may be quantified by the fractional overdensity field $\delta(x) \equiv \rho_m/\bar{\rho}_m - 1$. We construct the density fields from the simulations using a cloud-in-cell interpolation on a 256$^3$ Cartesian mesh in each cell. Clipping is a local density transformation characterized by enforcing a maximum fractional overdensity $\delta_0$ such that

$$\delta_c(x) = \begin{cases} \delta_0, & \delta(x) > \delta_0, \\ \delta(x), & \delta(x) \leq \delta_0. \end{cases}$$

We shall also make use of applying a minimum instead of a maximum threshold, which is equivalent to clipping the negative field $-\delta(x)$. While this may prove more challenging to apply to real data, due to the lower signal to noise associated with cosmic voids, it will serve as a useful validation test in our simulations and support for the concept of weighting to unscreen or screen gravitational modifications. We quantify the clipping strength in terms of the fractional loss of power in the lowest $k$-bin, applying a simple iterative procedure to determine the threshold $\delta_0$ required to establish the desired fraction. Defining clipping strength directly in terms of $\delta_0$ is another possibility, but complicates the comparison of results from fields with different choice of smoothing length.

Given the nature of the chameleon mechanism, the clipping transformation promises the extraction of more information on the gravitational physics, as it allows to focus on the less dense, unscreened regions of the Universe, where there exists a greater difference to GR. In the left panel of Fig. [11] we compare the deviation in power between the F6 and ACDM simulations as a function of clipping strength. With increased clipping strength, the difference in $P(k)$ is enhanced. At $k = 0.5 h/\text{Mpc}$ this difference more than doubles, greatly facilitating the discrimination between the two models. In practice, the amount of clipping that can realistically be applied will depend on the noise levels, limited by the number density of galaxies. To see the reverse of this effect, we also apply the clipping transformation to the negative field $-\delta(x)$. Since clipping voids, this reduces the contributions from unscreened regions to $P(k)$. Therefore, as expected, stronger clipping leads to convergence between the modified gravity and ΛCDM density fields. Note that we do not expect a full unscreening of modified dynamics by clipping high-density peaks since the chameleon mechanism not only self-screens large local densities but also introduces a shielding based on environment that is not nullified. In the opposite case of clipping low-density troughs, however, only self-screened regions with GR dynamics remain.

In the middle panel of Fig. [11] we show the ratios of the power spectra of F4-6 to that of ACDM simulations and the corresponding ratios after clipping has been applied such that the large-scale power is reduced by 50%. We also show the results of instead applying a logarithmic density transform [15]. Both local transformations suppress contributions from the densest regions of the field. Note that while the shape recovered from the logarithmic transform is closer to linear theory, for sparsely sampled fields the logarithmic transform becomes unstable whereas clipping is insensitive to the number density of sources [16]. Regarding the proximity to linear theory, Ref. [16] demonstrated that the clipped ΛCDM matter power spectrum is well described by a linear combination of the linear and one-loop contributions, where higher-order terms are strongly suppressed. Thereby, increased clipping downweights the one-loop relative to the linear contribution. We verify the recovery of linear theory for $f(R)$ gravity in the right-hand panel of Fig. [11] using F4 simulations, where chameleon screening affects
In particular the growth of structure can be strongly altered by variations of $\Omega_m$ or $\Omega_b$. Using HALOFIT [20], we estimate effects of varying these parameters on the clipped power by comparing signatures in the linear and nonlinear $P(k)$. While enhancing the amplitude of $P(k)$ with increasing $\sigma_8$, the shape of the nonlinear enhancement resembles that of a chameleon model. Hence, with absent information on the absolute amplitude of $P(k)$ due to galaxy bias, there clearly is a degeneracy between variations in $\sigma_8$ and $f_{R0}$. However, the $\Lambda$CDM case with a larger value of $\sigma_8$ will experience a strong reduction of the enhancement in power at large $k$ after clipping.

In contrast, the $f(R)$ power spectrum increases the enhancement at large $k$ after clipping, becoming more linear and unscreened. Therefore variations in $f_{R0}$ and $\sigma_8$ respond to clipping in a qualitatively different manner, breaking the degeneracy. Modifications in the shape of $P(k)$ due to changes in $\Omega_m$ are qualitatively different from $f(R)$ modifications, e.g., changing baryon acoustic oscillation features. Although a partial suppression in the change of power attributed to $\Omega_m$ variations is seen in nonlinear compared to linear theory, it does not reproduce the strong screening effect of chameleon models. Finally, note that since focusing on differences in the shape of $P(k)$, effects of linear galaxy bias can be neglected. However, through redshift-space distortions, in chameleon models the ratio between galaxy and dark matter density becomes scale dependent. Since adding to the deviations between the shape of modified and $\Lambda$CDM galaxy power spectra [21], we conservatively assume this ratio to be constant when estimating potential observational bounds on chameleon models.

**Outperforming Solar System constraints.—** The requirement that the Milky Way dark matter halo screens the Solar System sets a constraint on the chameleon field amplitude of $|\varphi_0 - 1| \lesssim 5 \times 10^{-6}/(6 + 4\omega)$ [9]. Similarly strong constraints can be obtained from the absence of deviations in luminosity distances from different types of distance indicators in unscreened dwarf galaxies [22]. Current cosmological constraints are about two orders of magnitude weaker than Solar System bounds [23, 24]. We refer to Ref. [25] for a review of constraints on chameleon gravity. Having analyzed the effect of clipping on the matter power spectrum, we estimate the constraints that can be inferred from applying this method to observa-
tions of galaxy clustering. While fractional errors in the measurement can be kept approximately constant, clipping reduces systematic errors from modeling uncertainties of the nonlinear structure contributing at large $k$ such that constraints on cosmological parameters can be improved (see Ref. [20]). We assume that a future measurement of the clipped $P(k)$ can discriminate 1% deviations from the fiducial ΛCDM shape at $k \leq 0.3 \, h/Mpc$. Note that this is a conservative estimate with surveys already yielding sub-percent-level measurements of the acoustic features [27]. We compute the modified nonlinear power employing the halo model of chameleon theories [9]. As clipping chameleon densities mainly removes regions where GR is recovered, we approximate the clipped chameleon power spectrum by removing the difference of unclipped to clipped power obtained from the ΛCDM simulation. For $k \leq 0.3 \, h/Mpc$, this simple approach reproduces the absolute clipped power for F4 and F6 at the few percent and permille level, respectively, and recovers the measured fractional difference to ΛCDM within 20%, increasingly underestimating it from F4 to F6. Employing this method and varying the background field value $\phi_0$, the coupling strength set by $\omega$, and the exponent of the scalar field potential $\alpha$, we set constraints where the deviation between the clipped chameleon and ΛCDM $P(k)$ at $k = 0.3 \, h/Mpc$ exceeds 1%.

We present our results in Fig. 2 comparing them to Solar System, astrophysical distance indicator, and current cosmological bounds. We find that for the simulated Hu-Sawicki model ($\alpha = 0.5$), clipping constraints on $f_{R0}$ can improve upon existing cosmological constraints, using clusters [23] or the matter power spectrum [24], by 2-3 orders of magnitude and outperform the Solar System and astrophysical bounds. For smaller values of $\omega$, corresponding to larger force modifications, the improvement of constraints from clipping over Solar System bounds is even greater. This is not surprising since larger force modifications also imply a more efficient screening of the Solar System region whereas gravitational dynamics in unscreened low densities is modified even more strongly. Constraints in this region of parameter space may also confirm or rule out chameleon models as explanation of the observed cored density profiles of dwarf spheroidal galaxies in the Milky Way [28]. In the opposite limit of increasing $\omega$, i.e., weakening gravitational coupling, Solar System bounds strengthen and surpass the decreasing clipping constraints. Importantly, these power spectrum constraints clearly depend on the exponent of the scalar field potential $\alpha$.

Conclusions.— Modifications of gravity potentially explaining cosmic acceleration need to employ a screening mechanism that allows modifications in low densities at large scales while suppressing them in high-density regions such as the Solar System. Such screening mechanisms also suppress anomalies in the averaged cosmological large-scale structure, limiting observational constraints inferred on such models. By clipping the density fields at a maximal threshold, contributions of screened high-density regions to the matter power spectrum can be downweighted, enhancing modified gravity effects. We apply this method to chameleon models with particular emphasis on $f(R)$ gravity to demonstrate that it can improve cosmological constraints on the models to a level stronger than the currently most stringent bounds from Solar System and astrophysical tests. We also expect our method to be applicable to other screening effects like the Vainshtein mechanism [1] or k-mouflage [3], however, not to linear shielding [4].

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1. A. Vainshtein, Phys. Lett. B39, 393 (1972).
2. J. Khoury and A. Weltman, Phys. Rev. Lett. 93, 171104 (2004).
3. E. Babichev, C. Deffayet and R. Ziour, IJMPD 18, 2147 (2009).
4. L. Lombriser and A. Taylor, arXiv:1405.2896.
5. H. Oyaizu, M. Lima and W. Hu, Phys. Rev. D78, 123524 (2008).
6. F. Simpson, J. B. James, A. F. Heavens and C. Heymans, Phys. Rev. Lett. 107, A261301 (2011).
7. W. Hu and I. Sawicki, Phys. Rev. D76, 064004 (2007).
[8] H. A. Buchdahl, Mon. Not. Roy. Astron. Soc. 150, 1 (1970).
[9] L. Lombriser, K. Koyama and B. Li, JCAP 1403, 021 (2014).
[10] J. Wang, L. Hui and J. Khoury, Phys. Rev. Lett. 109, 241301 (2012).
[11] A. Mead, J. Peacock, L. Lombriser and B. Li, arXiv:1412.5195.
[12] B. Li, G.-B. Zhao, R. Teyssier and K. Koyama, JCAP 1, 51 (2012).
[13] S. Prunet and C. Pichon, MPgrafic: A parallel MPI version of Grafic-1, 2013, Astrophysics Source Code Library, ascl:1304.014.
[14] F. Schmidt, W. Hu and M. Lima, Phys. Rev. D81, 063005 (2010).
[15] M. C. Neyrinck, I. Szapudi and A. S. Szalay, Astrophys. J. 698, L90 (2009).
[16] F. Simpson, A. F. Heavens and C. Heymans, Phys. Rev. D88, 083510 (2013).
[17] H. Motohashi, A. A. Starobinsky and J. Yokoyama, Prog. Theor. Phys. 124, 541 (2010).
[18] M. Baldi et al., Mon. Not. Roy. Astron. Soc. 440, 75 (2014).
[19] E. Puchwein, M. Baldi and V. Springel, Mon. Not. Roy. Astron. Soc. 436, 348 (2013).
[20] Virgo Consortium, R. Smith et al., Mon. Not. Roy. Astron. Soc. 341, 1311 (2003).
[21] L. Lombriser, J. Yoo and K. Koyama, Phys.Rev. D87, 104019 (2013).
[22] B. Jain, V. Vikram and J. Sakstein, Astrophys. J. 779, 39 (2013).
[23] A. Terukina et al., JCAP 1404, 013 (2014).
[24] J. Dossett, B. Hu and D. Parkinson, JCAP 1403, 046 (2014).
[25] L. Lombriser, Annales Phys. 526, 259 (2014).
[26] F. Simpson et al., in prep.
[27] L. Anderson et al., Mon. Not. Roy. Astron. Soc. 441, 24 (2014).
[28] L. Lombriser and J. Peñarrubia, arXiv:1407.7892.