Transverse coherence of photon pairs generated in spontaneous parametric down-conversion

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Coherence properties of the down-converted beams generated in spontaneous parametric down-conversion are investigated in detail using an iCCD camera. Experimental results are compared with those from a theoretical model developed for pulsed pumping with a Gaussian transverse profile. The results allow to tailor the shape of correlation area of the signal and idler photons using pump-field and crystal parameters. As an example, splitting of a correlation area caused by a two-peak pump-field spectrum is experimentally studied.

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I. INTRODUCTION

Light emitted from spontaneous parametric down-conversion in a nonlinear crystal is composed of photon pairs. Two photons comprising a photon pair are called a signal and an idler photon for historic reasons. The first theoretical investigation of this process has been done in year 1968 [1]. Already this study has revealed that frequencies and emission directions of two photons in a pair are fully determined by the laws of energy and momentum conservations. For this reason, there occurs a strong correlation (entanglement) between properties of the signal and idler photons. In an ideal case of infinitely long and wide nonlinear crystal and monochromatic plane-wave pumping, a plane-wave signal photon at frequency \( \omega_s \) belongs just to one plane-wave idler photon at frequency \( \omega_i \) that is determined by the conservation of energy. Emission angles of these photons are given by the momentum conservation that forms phase-matching conditions. Possible signal (and similarly idler) emission directions lie on a cone which axis coincides with the pump-beam direction of propagation.

However, real experimental conditions have enforced the consideration of crystals with finite dimensions [2, 3], pump beams with nonzero divergence [4, 5] as well as pulsed pumping [6, 7]. During this investigation, the approximation based on a multidimensional Gaussian spectral two-photon amplitude has been found extraordinarily useful [10, 11]. The developed models have revealed that spatial characteristics of a pump beam are transferred to certain extent to these of a photon pair generated in a nonlinear crystal, especially in case of short crystals [12, 13]. These models have also been useful in quantifying real effects in applied experimental setups utilizing photon pairs [14, 15]. They have also been recently extended to photonic [18, 19] and wave-guiding [20, 21] structures. Also effects at nonlinear boundaries have been taken into account [26, 27].

In this paper, we continue the previous investigations of spatial photon-pair properties [28, 31] by experimental study of transverse profiles of the down-converted beams as well as correlation areas of the signal and idler photons using an iCCD camera [32, 33]. Special attention is paid to the role of pump-beam parameters. Experimental results are compared with a theoretical model that considers Gaussian spectrum and elliptical pump-beam profile. We note that also sensitive CCD cameras have been found useful in investigations of spatial properties of more intense twin beams [35, 37].

The paper is organized as follows. A theoretical model is presented in Sec. II. Sec. III brings theoretical analysis of parameters of a correlation area as well as spectral properties of the down-converted fields. An experimental method based on the use of an iCCD camera is discussed in detail in Sec. IV. The experimentally observed dependence of parameters of the correlation area on pump-beam characteristics and crystal length is reported in Sec. V. Sec. VI is devoted to splitting of the correlation area and its experimental observation. Conclusions are drawn in Sec. VII.

II. THEORY

The process of spontaneous parametric down-conversion is described by the following interaction Hamiltonian \( \hat{H}_{\text{int}} \) [2, 14, 18]:

\[
\hat{H}_{\text{int}}(t) = \varepsilon_0 \int_V d\mathbf{r} \chi^{(2)}(\mathbf{r}, t) \mathbf{E}_p^+(\mathbf{r}, t) \mathbf{E}_s^-(\mathbf{r}, t) \mathbf{E}_i^-(\mathbf{r}, t) + \text{H.c.},
\]

where \( \mathbf{E}_p^+(\mathbf{r}, t) \) is the positive-frequency part of the pump electric-field amplitude, whereas \( \mathbf{E}_s^-(\mathbf{r}, t) \) (and \( \mathbf{E}_i^-(\mathbf{r}, t) \)) stands for the negative-frequency part of the signal (idler) electric-field amplitude operator. Symbol \( \chi^{(2)} \) means the second-order susceptibility tensor and \( : \) is shorthand for tensor reduction with respect to its three indices. Susceptibility of vacuum is denoted as \( \varepsilon_0 \), interaction volume as \( V \) and H.c. substitutes the Hermitian-conjugated term.

We further consider parametric down-conversion in a LiIO\(_3\) crystal with an optical axis perpendicular to the \( z \) axis of fields’ propagation direction and type-I interaction. The pump field is assumed to be polarized ver-
tically (it propagates as an extraordinary wave) whereas the signal and idler fields are polarized horizontally (they propagate as ordinary waves). In this specific configuration, scalar optical fields are sufficient for the description. The interacting optical fields can then be decomposed into monochromatic plane waves with frequencies $\omega_a$ and wave vectors $k_a$:

$$E_a^+(r, t) = \int \! dk_a E_a^+(k_a) \exp(\imath k_a r - \imath \omega_a t) + \text{H.c.}$$

The signal and idler fields at a single-photon level have to be described quantally and so their spectral amplitudes $E_a^+(k_a)$ can be expressed as $E_a^+(k_a) = \sqrt{i/\hbar \omega_a / \sqrt{2 \varepsilon_0 c} A_n(\omega_a)} \hat{a}_a(k_a)$ using annihilation operators $\hat{a}_a(k_a)$ that remove one photon from a plane-wave mode $k_a$ in field $a$. Symbol $\hbar$ stands for the reduced Planck constant, $c$ is speed of light in vacuum, $A$ transverse area of a beam, and $n_a$ means index of refraction in field $a$.

Under these conditions, the interaction Hamiltonian $\hat{H}_{\text{int}}$ in Eq. (1) takes the form:

$$\hat{H}_{\text{int}}(t) = A_n(\omega_s^0, \omega_i^0) \int \! dk_s \int \! dk_i \int \! dk_p E_p^+(k_p) \times \exp \{ \imath [\omega(k_p) - \omega(k_s) - \omega(k_i)] t \} \times \int \! d\mathbf{r} \exp [\imath (k_p - k_s - k_i) \mathbf{r}] \times \hat{a}_s^\dagger(k_s) \hat{a}_i^\dagger(k_i) + \text{H.c.} \tag{3}$$

We have assumed in deriving Eq. (3) that the function $A_n(\omega_s, \omega_i) = -\hbar \sqrt{\omega_s \omega_i} \chi^{(2)} / (2c \Lambda n_s(\omega_s) n_i(\omega_i))$ is a slowly varying function of frequencies $\omega_s$ and $\omega_i$ and can be approximated by its value taken at the central frequencies $\omega_s^0$ and $\omega_i^0$.

A quantum state $|\Psi\rangle$ of a generated photon pair can be obtained after solving the Schrödinger equation up to the first power of the interaction constant that results in the formula:

$$|\Psi\rangle = \frac{-i}{\hbar} \int_{-\infty}^{\infty} d\tau \hat{H}_{\text{int}}(t) |\text{vac}\rangle; \tag{4}$$

$|\text{vac}\rangle$ means the vacuum state. Substitution of the interaction Hamiltonian $\hat{H}_{\text{int}}$ from Eq. (3) into Eq. (1) provides the following form for the quantum state $|\Psi\rangle$:

$$|\Psi\rangle = \int \! dk_s \int \! dk_i S(k_s, k_i) \hat{a}_s^\dagger(k_s) \hat{a}_i^\dagger(k_i) |\text{vac}\rangle, \tag{5}$$

where the newly introduced two-photon amplitude $S$ takes the form:

$$S(k_s, k_i) = A_n(\omega_s^0, \omega_i^0) \int \! dk_p E_p^+(k_p) \delta(\omega_p - \omega_s - \omega_i) \times \int \! d\mathbf{r} \exp [\imath (k_p - k_s - k_i) \mathbf{r}]; \tag{6}$$

and $A_n' = -2\pi i / \hbar A_n(\omega_s^0, \omega_i^0)$. We note that squared modulus $|S(k_s, k_i)|^2$ of the two-photon amplitude gives us the probability density of simultaneous generation of a signal photon with wave vector $k_s$ and its twin with wave vector $k_i$.

Spectral resolution is usually not found in experiments with photon pairs and then the photon-pair coincidence-count rate is linearly proportional to the fourth-order correlation function $G_{s,i}$ defined as:

$$G_{s,i}(\xi_s, \delta_s, \xi_i, \delta_i) = \frac{\sin(\xi_s) \sin(\xi_i)}{c^6} \int \! d\omega_s \omega_s^2 \times \int \! d\omega_i \omega_i^2 |h(\omega_s)|^2 |S(\xi_s, \delta_s, \omega_s, \xi_i, \delta_i, \omega_i)|^2; \tag{7}$$

$S(\xi_s, \delta_s, \omega_s, \xi_i, \delta_i, \omega_i) \equiv S(k_s, k_i)$. The propagation direction of a photon is parameterized by radial emission angles $\xi_s$ (determining declination from the $z$ axis) and azimuthal emission angles $\delta_s$ (describing rotation around the $z$ axis) whereas $\xi_i$ and $\delta_i$ are not resolved, an integrated signal-field emission spectrum $S^\text{int}_{s,i}$ is observed:

$$S^\text{int}_{s,i}(\omega_s; \xi_i, \delta_i) = \int_{-\pi/2}^{\pi/2} d\xi_s \int_{-\pi}^{\pi} d\delta_s S_s(\omega_s; \xi_s, \delta_s, \xi_i, \delta_i). \tag{8}$$

Similar formulas as given in Eqs. (8) and (9) can be derived also for the idler field.

On the other hand excluding resolution in emission directions, spectral correlations between the signal and idler fields are characterized by a two-photon spectral amplitude $\Phi_{s,i}$ which squared modulus is defined as:

$$|\Phi_{s,i}(\omega_s, \omega_i)|^2 = \frac{\omega_s^2 \omega_i^2}{c^6} \int \! d\xi_s \int \! d\delta_s \int \! d\xi_i \sin(\xi_s) \sin(\xi_i) |h(\omega_s) h(\omega_i)|^2 \times |S(\xi_s, \delta_s, \omega_s, \xi_i, \delta_i, \omega_i)|^2. \tag{10}$$

We further consider a Gaussian pump beam with the electric-field amplitude $E_p^+(r, t)$ in the form:

$$E_p^+(r, t) = \int \! d\omega_p A_p(\omega_p) \exp(\imath k_{px} z - \imath \omega_p t) \times \frac{1}{W_{px}(z)} \exp \left(-\frac{x^2}{W_{px}^2(z)}\right) \exp \left[-\imath k_p \frac{x^2}{2R_{px}(z)}\right]$$
In Eq. \([18]\), \(\tau_p\) denotes pump-pulse duration, \(a_p\) stands for a chirp parameter, and \(\xi_p\) is the pump-field amplitude. We note that the amplitude width \(\Delta \omega_p\) (given as full width at 1/e of the maximum) of the pulse written in Eq. \([18]\) equals \(4\sqrt{1 + a_p^2}/\tau_p\).

Considering the pump-field amplitude \(E_p^{(+)}\) as given in Eq. \([15]\) the two-photon amplitude \(S\) defined in Eq. \([9]\) can be recast into the form:

\[
S(\xi_s, \delta_s, \omega_s, \xi_i, \delta_i, \omega_i) = c A_p(\omega_s + \omega_i) \times F_p(k_{sx}, k_{sy}) \exp\left\{ -\frac{i}{2} [k_{pz}(\omega_s + \omega_i) - k_{sz}(\omega_s) - k_{iz}(\omega_i)] L_z \right\} \\
\times \exp\left\{ -i \frac{[k_{pz}(\omega_s + \omega_i) - k_{sz}(\omega_s) - k_{iz}(\omega_i)] L_z}{2} \right\};
\]

\[
sinc(x) = \sin(x)/x. \quad \text{In deriving Eq. \([19]\), we have assumed that the crystal extents from } z = -L_z \text{ to } z = 0, L_z \text{ being the crystal length. The transverse profile of crystal is also assumed to be sufficiently wide.}
\]

### III. CORRELATION AREA, SPECTRAL PROPERTIES

Correlation area is defined by the profile of probability density of detecting a signal photon in the direction described by angles \((\xi_s, \delta_s)\) provided that its idler twin has been detected in a fixed direction given by angles \((\xi_i, \delta_i)\). In coherence theory, this probability is given by the fourth-order correlation function \(G_{s,i}\) defined in Eq. \([7]\). Because the correlation function \(G_{s,i}\) is usually a smooth function of its arguments, it can be conveniently parameterized using angular widths (given as full-widths at 1/e of maximum) in the radial \((\Delta \xi_s)\) and azimuthal \((\Delta \delta_s)\) directions. In general, parameters of the correlation area depend on properties of crystal material as well as crystal length, pump-field spectral bandwidth, and transverse pump-beam profile. The last two parameters allow to tailor characteristics of the correlation area in wide ranges.

In the theoretical analysis of Sec. III, we use radial \((\xi)\) and azimuthal \((\delta)\) angles inside a nonlinear crystal. The reason is that we want to exclude the effect of mixing in spatial and frequency domains at the output plane of the crystal in the discussion. However starting from Sec. IV radial \((\xi)\) and azimuthal \((\delta)\) angles outside the nonlinear crystal are naturally used in the presentation of experimental results.

In radial direction, crystal length and pump-field spectral bandwidth as well as transverse pump-beam profile play a role. The dependence of radial width of the correlation area on the crystal length \(L_z\) emerges through the phase matching condition in the \(z\) direction. This condition is mathematically described by the expression sinc\((\Delta k_z L_z/2)\) in Eq. \([19]\); \(\Delta k_z = k_{pz} - k_{sz} - k_{iz}\). Actual
radial width is determined by this condition and conservation of energy ($\omega_p = \omega_s + \omega_i$). According to the formula in Eq. (19), the longer the crystal, the smaller the radial width. Analytical theory also predicts narrowing of the signal- and idler-field spectra with an increasing crystal length. If pulsed pumping is considered, the wider the pump-field spectrum, the greater the radial width and also the greater the signal-field spectral width (compare Figs. 1c, d with Figs. 1a, b). This can be understood as follows: more pump-field frequencies are present in a wider pump-field spectrum and so more signal- and idler-field frequencies are allowed to obey the phase-matching conditions in the $z$ direction and conservation of energy. In more detail and following the graphs in Figs. 1c, d, signal-field photons with different wavelengths are emitted into different radial emission angles $\xi_s$. Superposition of photon fields emitted into different radial emission angles $\xi_s$ then broadens the overall signal-field spectrum. It is important to note that all idler-field photons have nearly the same wavelengths which means that signal-field photons emitted into different radial emission angles $\xi_s$ use different wavelengths of the pulsed-pump spectrum. The transverse pump-beam profile affects the radial width through the phase-matching condition in the radial plane. This radial phase matching condition is an additional requirement that must be fulfilled by a generated photon pair. Qualitatively, the more the pump beam is focused, the wider its spatial spectrum in radial direction and so the weaker the radial phase-matching condition. However, this dependence is quite small in radial angles, as follows from the comparison of graphs in Figs. 1a, b and Figs. 1c, f. On the other hand, focusing of the pump beam leads to considerable broadening of the signal- and idler-field spectra in all radial emission directions. Finally, if a focused pulsed pump beam is assumed (see Figs. 1e, h), broadening of the correlation area in radial direction as well as broadening of the overall signal- and idler-field spectra is observed due to a final pump-field spectral width. On the top, broadening of the signal- and idler-field spectra corresponding to any radial emission angle $\xi_s$ occurs as a consequence of pump-beam focusing. This behavior is related to the fact that indexes of refraction of the interacting fields are nearly constant inside the correlation area. We can say in general, that spectral widths of the signal and idler fields behave qualitatively in the same way as the radial width of correlation area.

Comparison of the signal- and idler-field spectra in Figs. 1c, d valid for pulsed pumping with those in Figs. 1a, b for cw pumping leads to a remarkable observation. Photon pairs generated into different signal- and idler-field radial emission angles $\xi_s$ use different pump-field frequencies. There occurs spectral asymmetry between the signal and idler fields that originates in different detection angles considered; whereas the idler-field detection angle is fixed, the angle of a signal-photon detection varies. This asymmetry determines the preferred direction of the signal- and idler-field frequency correlations as they are visible in the shape of squared modulus $|\Phi_{s,i}|^2$ of two-photon amplitude introduced in Eq. (11). A large signal-field detector is assumed. Contour plot of the squared modulus $|\Phi_{s,i}|^2$ of two-photon amplitude has a typical cigar shape. In cw case, the main axis of this cigar is rotated by 45 degrees counter-clockwise with respect to the $\lambda_i$ axis (see Fig. 2a) in order to describe perfect frequency anti-correlation. If pulsed pumping is taken into account, the cigar axis tends to rotates clockwise; the broader the pump-field spectrum, the greater the rotation angle. Even states with positively correlated signal- and idler-field frequencies can be observed for sufficiently broad pump-field spectra (see Fig. 2a). We note that different dispersion properties at different propagation angles have been fully exploited in the method of achromatic phase matching that allows to generate photon pairs with an arbitrary orientation of the two-photon spectral amplitude [15, 12, 43].

The azimuthal width of correlation area is determined
are rotationally symmetric with respect to the azimuthal direction. As material dispersion characteristics of the crystal the phase-matching conditions in the azimuthal direction are practically influence spectral properties of the signal and idler fields.

We illustrate the dependence of correlation area on pump-beam focusing using a 5 mm long crystal and both cw and pulsed pumping in Fig. 3. We can see in Fig. 3a that the signal-field azimuthal width $\Delta \xi_s$ is inversely proportional to the width $W_p^{0,f}$ (full-width at 1/e of the maximum; $W_p^{0,f} \equiv W_{px}^{0,f} = W_{py}^{0,f}$) of the pump-beam waist whereas the radial width $\Delta \xi_r$ does not practically depend on the width $W_p^{0,f}$ of the pump-beam waist. This is caused by the fact that the phase-matching condition in the $z$ direction is much stronger than that in radial direction for a 5-mm long crystal and so the radial width $\Delta \xi_r$ is sensitive only to the pump-field spectral width in this case. Pulsed pumping gives a broader correlation area in radial direction as well as broader signal-field spectrum compared to cw case (see Fig. 3b). Increasing pump-beam focusing releases phase-matching conditions and naturally leads to a broader signal-field spectrum.

Contrary to the azimuthal width, the radial width $\Delta \xi_r$ depends on the pump-field spectral width $\Delta \lambda_p$. The larger the pump-field spectral width $\Delta \lambda_p$ the greater the radial width $\Delta \xi_r$ and also the greater the signal-field spectral width $\Delta \omega_s$, as documented in Fig. 3 for a focused pump beam. We can also see in Fig. 3 that the radial width $\Delta \xi_r$ reaches a constant value for sufficiently narrow pump-field spectra. This value is determined by the phase-matching condition in the $z$ direction for the central pump-field frequency $\omega_p^0$ and so depends on the crystal length $L_z$ (together with material dispersion properties of the crystal). The longer the crystal the smaller the radial width $\Delta \xi_r$.

The above described dependencies allow to generate photon pairs with highly elliptic profiles of the correlation area provided that the pump-beam profile in the transverse plane is highly elliptic. As an example, we consider a pump beam having $W_{py}^{0,f}/W_{px}^{0,f} = 10$. The dependence of the radial ($\Delta \xi_r$) and azimuthal ($\Delta \xi_s$) widths and signal-field spectral width $\Delta \omega_s$ on the central azimuthal signal-photon emission angle $\delta_s$ is shown in Fig. 3 in this case. Whereas the radial and azimuthal widths are comparable for the azimuthal signal-field emission angle $\delta_s = \pi/2$, their ratio $\Delta \xi_s/\Delta \xi_r$ equals approx. 20 for $\delta_s = 0$. Focusing the pump beam from 200 $\mu$m to 20 $\mu$m in radial direction results in doubling the signal-field spectral width $\Delta \omega_s$ as documented in Fig. 3b (see also Figs. 1b, and e).

IV. EXPERIMENTAL SETUP

We have used a negative uniaxial crystal made of LiIO$_3$ cut for non-critical phase matching, i.e. the optical axis was perpendicular to the pump-beam propagation direction. We have considered crystals of two different lengths ($L_z=2$ mm and 5 mm) pumped both by cw and pulsed lasers. As for cw pumping, a semiconductor laser Cube 405 (Coherent) delivered 31.6 mW at 405 nm and with spectral bandwidth $\Delta \lambda_p = 1.7$ nm. The second-harmonic field of an amplified femtosecond
FIG. 4: a) Radial width $\Delta \xi_s$ of correlation area and b) signal-field spectral width $\Delta \omega_s$ as functions of pump-field spectral width $\Delta \lambda_p$ for a 5-mm (circles) and 10-cm (triangles) long crystal assuming a focused pump beam; $W_{0,f}^{p} = 200 \ \mu m$. Logarithmic scales on the $x$ and $y$ axes are used.

FIG. 5: a) Radial ($\Delta \xi_s$, solid curve) and azimuthal ($\Delta \delta_s$, dashed curve) widths of correlation area and b) signal-field spectral width $\Delta \omega_s$ as they depend on central azimuthal signal-field emission angle $\delta_{s0}$ for a highly elliptic pulsed pump beam ($W_{0,f}^{p} = 200 \ \mu m$, $W_{0,f}^{p} = 20 \ \mu m$, $\Delta \lambda_p = 5 \ \text{nm}$); $L_z = 5 \ \text{mm}$.

Tisapphire system (Mira+RegA, Coherent) providing pulses at 800 nm and $\sim 250$ fs long was used in the pulsed regime. The mean SHG power was 2.5 mW at the crystal input for a repetition rate of 11 kHz. Spectral bandwidth was adjusted between 4.8 and 7.4 nm by fine tuning of the SHG process. A dispersion prism was used to separate the fundamental and SHG beams (for details, see Fig. [D]).

Transverse profile of the pump beam and its divergence were controlled by changing the focus length of converging lens $L_1$ or using a beam expander (BE2X, Thorlabs). The used focal lengths $f_{L1}$ of lens $L1$ laid in the interval from 30 to 75 cm. As we wanted the pump beam to be as homogeneous as possible along the $z$ axis, we chose the distance $z_{L1}$ between the lens $L1$ and the nonlinear crystal such that the beam waist was placed far behind the crystal, i.e. $z_{L1} < f_{L1}$. Spatial spectrum of the pump beam in the transverse plane as a very important parameter in our experiment was measured by a CCD camera (Lu085M, Lumenera) placed at the focal plane of a converging lens $L3$. Spatial spectra in horizontal and vertical directions were determined as marginal spectra and parameters $\tilde{W}_{px}$ and $\tilde{W}_{py}$ characterizing their widths were found after fitting the experimental data. A fiber-optic spectrometer (HR4000CG-UV-NIR, Ocean Optics) was used to obtain the pump-beam temporal spectrum after propagation through the nonlinear crystal.

The experiment was done with photon pairs degenerate in frequencies ($\lambda_{s0} = \lambda_{i0} = 800 \ \text{nm}$) and emitted in opposite parts of a cone layer (the central radial emission angle was 33.4 deg behind the crystal). As shown in Fig. [F], the signal beam was captured directly by a detector whereas the idler beam propagated to the detector after being reflected on a high-reflectivity mirror. Both beams were detected on a photocathode of an iCCD camera with image intensifier (PI-MAX:512-HQ, Princeton Instruments). Before detection, both beams were transformed using a converging lens $L2$, one narrowbandwidth and two high-pass edge filters. The geometry of the setup was chosen such that the lens $L2$ mapped the signal and idler photon emission angles to positions at the photocathode; the photocathode was placed in the focal plane of lens $L2$. For convenience, lenses $L2$ with different focal lengths ($f_{L2} = 12.5, 15, \text{ and } 25 \ \text{cm}$) were used. The applied bandwidth filter was 11 nm wide and centered at 800 nm. Edge filters (Andover, ANDV7862) had high transmittances at 800 nm (98%) and blocked wavelengths below 666 nm.

An active area of the photocathode in the form of a rectangular 12.36 mm wide (see Fig. [I]) was divided into 512 $\times$ 512 pixels. Spatial resolution of the camera was 38 $\mu m$ (FWHM) and its main limitation came from imperfect contrast transfer in the image intensifier. In order to make data acquisition faster the resolution was further decreased by grouping $4 \times 4$ or $8 \times 8$ pixels into one superpixel in the hardware of the camera. Consequently, several tens of camera frames were captured in one second. The overall quantum detection efficiency including com-
components between the crystal and photocathode was 7%, as derived from covariance of the signal and idler photon numbers. Widths of the signal and idler strips are given by the bandwidth filter and lens L2 focal length. As for timing, a 10 ns long gate of the camera was used synchronously with laser pulses. In cw case, a 2 µs long gate was applied together with internal triggering. This timing together with appropriate pump-field intensities assured that the probability of detecting two photons in a single super-pixel was negligible. In other words, the number of detection events divided by quantum detection efficiency had to be much lower than the number of detection events divided by quantum detection efficiency.

Also the level of noise was monitored in the third narrow strip; 1.82% of detection events came from noise. Detailed analysis has shown that 90% of noise photons were red photons originating from fluorescence inside the crystal. Scattered pump photons contributed by 8.4% and only 1.6% of noise counts were dark counts of the crystal. Only 1.6% of noise counts were dark counts of the crystal. Scattered pump photons contributed by 8.4% of noise photons originating from fluorescence inside the crystal. Detailed analysis has shown that 90% of noise photons were red photons originating from fluorescence inside the crystal. Scattered pump photons contributed by 8.4% and only 1.6% of noise counts were dark counts of the crystal.

The experimental signal-idler correlation functions $g_x$ and $g_y$ in the transverse plane described by horizontal ($x'$) and vertical ($y'$) coordinates of the reference system in this plane have been determined after processing many experimental frames. The formula for the determination of correlation function $g_x$ can be written as follows (see also Fig. 7):

$$ g_x(x'_s, x'_i) = \sum_{p=1}^{N} \sum_{m=1}^{M_p} \sum_{l=1}^{L_p} \sum_{i=1}^{\delta} \delta(x'_s - x'_s) \delta(x'_i - x'_i). $$

In Eq. (20), $p$ indexes frames ($N$ gives the number of frames) and $m$ ($l$) counts signal (idler) detection events [up to $M_p$ ($L_p$) in the $p$-th frame]. Symbol $x'_s$ ($x'_i$) denotes horizontal position of the $l$-th detection in the signal (idler) strip of the $p$-th frame. Correlations in the vertical direction given by the correlation function $g_y$ can be determined similarly. The formula in Eq. (20) takes into account all possible combinations of pairwise detection events. Only some of them correspond to detection of both photons from one pair. The remaining combinations are artificial in the sense that they do not correspond to detection of a photon pair. This poses the following restriction to the method. The number of artificial combinations that occur at random positions has to be large enough in order to create a plateau in a 2D graph of correlation function $g_x(x'_s, x'_i)$. Real detections of photon pairs are then visible on the top of this plateau (see Fig. 9 later).

Cartesian coordinates $x'_j$ and $y'_j$, $j = s, i$, in the transverse plane can be conveniently transformed into angles $\beta_j$ and $\gamma_j$ measured from the middle ($x^\text{cent}_j, y^\text{cent}_j$) of the $j$th strip and defined in Fig. 8 using the formulas:

$$ \gamma_j = \arctan \left( \frac{(x'_j - x^\text{cent}_j)}{f_{L2}} \right), $$

$$ \beta_j = \arctan \left( \frac{(y'_j - y^\text{cent}_j)}{f_{L2} \cos(\gamma_j)} \right), \quad j = s, i; $$

FIG. 7: Photocathode with registered photons after a) illumination by light coming from 20 000 consecutive pump pulses, b) one pump pulse. The signal and idler strips image small sections of the cone layer and are slightly curved. The curvatures are oriented in the same sense in both strips since the idler beam is reflected on a mirror.

FIG. 6: Experimental setup used for the determination of angular widths: a) Entire setup that includes both cw and pulsed pumping as well as pump-beam diagnostics (for more details, see the text). b) Detail of the setup showing paths of the signal and idler beams.

iCCD camera.
The photon emission direction is described by radial and azimuthal emission angles. In detector plane, Cartesian coordinates \( x' \) and \( y' \) are useful. Photons propagating in the model for the determination of expected parameters of correlation area along the radial angle \( \gamma \) as a mean value over all possible values of the signal-field azimuthal angle \( \beta \).

Moreover, consideration of different idler-photon detection positions means averaging over the angles \( \gamma_i \) and \( \beta_i \). The averaging is indicated by symbol \( \langle \rangle \). Mathematically, the radial cross-section \( \langle G_{s,i} \rangle_{\beta_s} \) expressed in the coordinate \( x'_i \) can be derived along the formula

\[
\langle G_{s,i} \rangle_{\beta_s}(x'_i) = \sum_{x'_i} g[x'_s - x'^{\text{mid}}(x'_i), x'_i],
\]

where the function \( x'^{\text{mid}}(x'_i) \) gives the central position (given as a locus) of the cut of the histogram \( g(x'_s, y'_i) \) for a fixed value of the coordinate \( x'_i \). In the theory, the radial cross-section \( \langle G_{s,i} \rangle_{\beta_s} \) is determined using the fourth-order correlation function \( G_{s,i} \) written in Eq. (7), substitution of angles \( \xi_s, \delta_s, \xi_i, \) and \( \delta_i \) by angles \( \gamma_s, \beta_s, \gamma_i, \) and \( \beta_i \) [inverse transformation to that in Eq. (22)] and finally integration over the angles \( \gamma_s, \gamma_i, \) and \( \beta_i \). Similarly, the azimuthal cross-section \( \langle G_{s,i} \rangle_{\gamma_s} \) of the correlation area along the azimuthal angle \( \beta_s \) arises after averaging over the angles \( \gamma_s, \gamma_i, \) and \( \beta_i \) and can be determined by a formula analogous to that given in Eq. (23). The radial and azimuthal cross-sections \( \langle G_{s,i} \rangle_{\beta_s} \) and \( \langle G_{s,i} \rangle_{\gamma_s} \) corresponding to the pump beam with characteristics defined in Figs. 9 and 10 are plotted in Figs. 9e and f. Solid lines in Figs. 9e and f refer to the results of numerical model and are in a good agreement with the experimental data.

\[
\langle G_{s,i} \rangle_{\gamma_s}(x'_s, y'_i) = \sum_{x'_s} g[x'_s, y'_i],
\]

where \( \beta_i \) arises after averaging over the angles \( \gamma_s, \gamma_i, \) and \( \beta_i \).

In the experiment, spatial and temporal spectra of the pump beam have been characterized first. Typical results are shown in Figs. 9a and b and have been used in the model for the determination of expected parameters of the correlation area. The correlation area, or more specifically its radial and angular profiles, have been characterized using histograms \( g_x(x'_s, x'_i) \) and \( g_y(y'_s, y'_i) \). Histogram \( g_x(x'_s, x'_i) \) gives the number of paired detections with a signal photon detected at position \( x'_s \) together with an idler photon registered at position \( x'_i \). These histograms usually contain experimental data from several hundreds of thousands of frames. As graphs in Figs. 9c and d show detections of correlated photon pairs lead to higher values in histograms \( g_x \) and \( g_y \) around diagonals going from upper-left to lower-right corners of the plots. Finite spreads of these diagonals have their origin in non-perfect phase matching and can be characterized by their widths \( \Delta x'_s \) and \( \Delta y'_s \). Or more conveniently by uncertainties in the determination of angles \( \beta_s \) and \( \gamma_s \); \( \Delta x'_s, \Delta y'_s \) from several hundred frames. As detailed inspection of the histogram \( g_x \) in Fig. 9d has shown, cuts of this histogram along the lines with constant values of \( x'_s (y'_s) \) do not depend on the value of \( x'_i (y'_i) \). This reflects the fact that idler photons detected at different positions inside the investigated area on the photocathode have identical (signal-photon) correlation areas. This allows us to combine the data obtained for idler photons detected at different positions together and increase the measurement precision this way. This approach thus provides the radial cross-section \( \langle G_{s,i} \rangle_{\gamma_s} \) of the correlation area along the radial angle \( \gamma \) as a mean value over all possible values of the signal-field azimuthal angle \( \beta \).

Moreover, consideration of different idler-photon detection positions means averaging over the angles \( \gamma_i \) and \( \beta_i \). The averaging is indicated by symbol \( \langle \rangle \). Mathematically, the radial cross-section \( \langle G_{s,i} \rangle_{\beta_s} \) expressed in the coordinate \( x'_i \) can be derived along the formula

\[
\langle G_{s,i} \rangle_{\beta_s}(x'_i) = \sum_{x'_i} g[x'_s - x'^{\text{mid}}(x'_i), x'_i],
\]

where the function \( x'^{\text{mid}}(x'_i) \) gives the central position (given as a locus) of the cut of the histogram \( g(x'_s, y'_i) \) for a fixed value of the coordinate \( x'_i \). In the theory, the radial cross-section \( \langle G_{s,i} \rangle_{\beta_s} \) is determined using the fourth-order correlation function \( G_{s,i} \) written in Eq. (7), substitution of angles \( \xi_s, \delta_s, \xi_i, \) and \( \delta_i \) by angles \( \gamma_s, \beta_s, \gamma_i, \) and \( \beta_i \) [inverse transformation to that in Eq. (22)] and finally integration over the angles \( \beta_i, \gamma_i, \) and \( \beta_i \). Similarly, the azimuthal cross-section \( \langle G_{s,i} \rangle_{\gamma_s} \) of the correlation area along the azimuthal angle \( \beta_s \) arises after averaging over the angles \( \gamma_s, \gamma_i, \) and \( \beta_i \) and can be determined by a formula analogous to that given in Eq. (23). The radial and azimuthal cross-sections \( \langle G_{s,i} \rangle_{\beta_s} \) and \( \langle G_{s,i} \rangle_{\gamma_s} \) corresponding to the pump beam with characteristics defined in Figs. 9 and 10 are plotted in Figs. 9e and f. Solid lines in Figs. 9e and f refer to the results of numerical model and are in a good agreement with the experimental data.

The radial width \( \Delta \gamma_{\gamma_s} \) (measured as full-width at 1/e of the maximum) of radial cross-section \( \langle G_{s,i} \rangle_{\beta_s} \) depends mainly on the pump-beam spectral width \( \Delta \lambda_p \). It holds that the greater the pump-beam width \( \Delta \lambda_p \) the larger the radial width \( \Delta \gamma_{\gamma_s} \) as documented in Fig. 10 for crystals 2- and 5-mm long. In the experiment, 11-nm wide frequency filters have been applied to cut noise. However, certain amount of photons comprising a photon pair has also been blocked. According to the theoretical model, this has also resulted in a small narrowing of the radial cut of the correlation area (compare solid and dashed curves in Fig. 10). The theoretical curve in Fig. 10 has been experimentally confirmed for several values of the width \( W_{p_x} \) and \( W_{p_y} \) of pump-beam waist both for cw and pulsed pumping.

On the other hand and in our geometry, it is the width \( W_{p_x} \) of the pump-beam waist that determines the angular width \( \langle \Delta \beta_s \rangle_{\gamma_s} \) of angular cross-section \( \langle G_{s,i} \rangle_{\gamma_s} \). Predictions of the model for 2- and 5-mm long crystals are shown in Fig. 11 by a solid curve. This curve has been shown experimentally for several values of the width
FIG. 9: Typical measurement of a correlation area for pulsed pumping having 327,600 frames; $L_z = 5\, \text{mm}$. a) Spatial spectrum of the pump beam determined in the focal plane of lens L3. b) Temporal intensity spectrum of the pump beam as determined by a spectrometer (diamonds), solid line represents a multi-peak Gaussian fit. c), d) Experimental histograms $g_x(x', x'_s)$ (c) and $g_y(y', y'_s)$ (d). e) Experimental radial ($\langle G_{s,i}\rangle_{\beta_s}$) and azimuthal ($\langle G_{s,i}\rangle_{\gamma_s}$) cross-sections of the correlation area (rectangles, circles) together with theoretical predictions (solid lines).

FIG. 10: Radial width $\langle \Delta \gamma_s \rangle_{\gamma_s}$ as a function of pump-beam spectral width $\Delta \lambda_p$. Experimental points have been obtained for $L_z = 5\, \text{mm}$, $W_{py}^{0,f} > 140\, \mu\text{m}$ (triangles), $L_z = 5\, \text{mm}$, $W_{py}^{0,f} < 140\, \mu\text{m}$ (diamonds), $L_z = 2\, \text{mm}$, $W_{py}^{0,f} > 140\, \mu\text{m}$ (crosses), and $L_z = 2\, \text{mm}$, $W_{py}^{0,f} < 140\, \mu\text{m}$ (circles) both for cw and pulsed pumping. The theoretical model gives the same dependence for both crystal lengths $L_z$ in cases without (solid curve) as well as with (dashed curve, 11-nm wide) spectral filters.

VI. ENGINEERING THE SHAPE OF A CORRELATION AREA

As the above results have shown parameters of a correlation area can be efficiently controlled using pump-beam parameters, namely temporal spectrum and transverse profile. Even the shape of correlation area can be considerably modified. Splitting of the correlation area into two parts that occurs as a consequence of splitting of the pump-field temporal spectrum can serve as an example.

$W_{py}^{0,f}$ of the pump-beam waist both for cw and pulsed pumping. We note that these curves do not depend on the pump-beam spectral width $\Delta \lambda_p$. We can see in Fig. 11 that the measured points agree with the theoretical curve for smaller values of the width $W_{py}^{0,f}$, larger values of the width $W_{py}^{0,f}$ lead to small angular widths $\langle \Delta \beta_s \rangle_{\beta_s}$ that could not be correctly measured because of the limited spatial resolution of the iCCD camera.
Using our femtosecond pump system, we were able to experimentally confirm this behavior. We have generated a pump beam with the spatial spectrum given in Fig. 12. Its temporal spectrum containing two peaks as was acquired by a spectrometer is plotted in Fig. 12g. The experimental radial width $\langle \Delta \gamma_{s,i} \rangle / \beta$ of cross-section $\langle G_{s,i} \rangle / \beta$, given in Fig. 12h shows that the two-peak structure of the pump-field spectrum results in splitting of the correlation area into two parts. On the other hand and in agreement with the theory, the angular cross-section $\langle G_{s,i} \rangle / \gamma$, was not affected by the pump-field spectral splitting (see Fig. 12i). For comparison, the theoretical profile of the correlation area given by the correlation function $G_{s,i}$ and appropriate for the pump-beam parameters given in Figs. 12a and b is plotted in Fig. 12j. It indicates a good agreement of the model with experimental data. Moreover, the squared modulus $|\Phi_{s,i}|^2$ of theoretical two-photon spectral amplitude reveals that splitting of the correlation area is accompanied by splitting of the signal-field spectrum (see Fig. 12k).

VII. CONCLUSIONS

We have developed a method for the determination of profiles of a correlation area using an intensified CCD camera. Single detection events in many experimental frames are processed and provide histograms from which cross-sections of the correlation area can be recovered. This method has been used for investigations of the dependence of parameters of the correlation area on pump-beam characteristics and crystal length. The experimentally obtained curves have been successfully compared with a theoretical model giving fourth-order correlation functions. Radial profile of the correlation area depends mainly on pump-field spectrum and crystal length. On the other hand, azimuthal profile of the correlation area is sensitive only to the transverse profile of the pump beam. Splitting of the correlation area caused by a two-peak structure of the pump-field spectrum has also been experimentally observed.

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FIG. 12: Determination of a correlation area for pulsed pumping composed of two spectral peaks; $L_z = 5$ mm. a) Spatial spectrum of the pump beam. b) Temporal pump-field intensity spectrum (experimental points are indicated by diamonds, solid line represents a multi-peak Gaussian fit). c), d) Experimental radial ($\langle G_{s,i} \rangle_{\beta_s}$, c) and azimuthal ($\langle G_{s,i} \rangle_{\gamma_s}$, d) cross-sections of the correlation area (rectangles and circles) together with theoretical predictions (solid line). e) Contour plot of the theoretical correlation function $G_{s,i}$. f) Contour plot of the squared modulus $|\Phi_{s,i}|^2$ of the theoretical two-photon spectral amplitude.