Proximity-induced transport in hybrid mesoscopic normal-superconducting metal structures.

A.F. Volkov, V.V. Pavlovskii and R. Seviour

* School of Physics and Chemistry, Lancaster University, Lancaster LA1 4YB, U.K.
+ Institute of Radioengineering and Electronics of the Russian Academy of Sciences, Mokhovaya str.11, Moscow 117218, Russia.
++ Institute of Physics and Technology of the Russian Academy of Sciences, Krasikov 25a, Moscow 103907, Russia.

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Using an approach based on quasiclassical Green’s functions we present a theoretical study of transport in mesoscopic S/N structures in the diffusive limit. The subgap conductance in S/N structures with barriers (zero bias and finite bias anomalies) are discussed. We also analyse the temperature dependence of the conductance variation \( \delta S(T) \) for a Andreev interferometer. We show that besides the well know low temperature maximum a second maximum near \( T_c \) may appear. We present the results of studies on the Josephson effect in 4 terminal S/N/S contacts and on the possible sign reversal of the Josephson critical current.

1. Introduction

The theory describing non-equilibrium transport in superconductors was developed 20 years ago (see, for example \[1\]) and has been used to explain phenomena such as viscous flux flow, Josephson effects in superconducting weak links and the passage of the current over the interface between a superconductor and a normal metal (an S/N interface). Technological advances achieved in the last decade have enabled the fabrication of mesoscopic structures where the dimensions are less than the energy relaxation length and with well defined physical properties. In mesoscopic systems phase coherence is maintained and the distribution of quasiparticles may differ significantly from equilibrium. Experiments on these structures have revealed a number of new effects such as, subgap conductance in S/N structures containing normal metal or superconducting loops, the non-monotonic dependence of the N film conductance on the temperature and voltage in S/N structures; long-range, phase-coherent effects, change of sign of the Josephson critical current in 4-terminal S/N/S structures when an additional current is driven through the N film. Although it is only recently that most of these effects have been explained using a non-equilibrium theory developed at the end of the 1970’s (see \[2,3\]). One method for studying these effects (ballistic systems in particular) is based on the Bogolyubov-de Gennes equations and the Landauer formula for the conductances \[4,5\]. In this paper we use another approach based on a microscopic, quasiclassical Green’s function technique, which has been successfully applied to systems with a small mean free path (i.e. diffusive regime), to analyse these effects. The method of matrix, quasiclassical Green’s functions is a convenient and powerful method for studying transport in mesoscopic S/N structures \[6\]. The quasiclassical approximation means that all the Green’s functions are spatially averaged over distances of order of the Fermi wave length \( p_F^{-1} \). This is equivalent to the integration in momentum space over the variable \( \xi_p = (p - p_F)v_F \). To describe transport and non-equilibrium properties we need to know the retarded (advanced) Green’s functions \( G^{R(A)} \) and the Keldysh function \( G \). In the case of superconducting systems each of these functions is a 2x2 matrix elements of which describe normal excitations and a condensate. These matrices are grouped together, defining the 4x4 matrix Green’s function \( \hat{G} \) as

\[
\hat{G} = \begin{pmatrix}
\hat{G}^R & \hat{G}^A \\
0 & \hat{G}^A
\end{pmatrix}
\]

(1)

where \( \hat{G}^{R(A)} \) are the matrix retarded (advanced) Green’s functions and \( \hat{G} \) is the Keldysh function. The functions \( \hat{G}^{R(A)} \) describe thermodynamical properties of the system such as the density of states (DOS), condensate currents and so on, while the function \( \hat{G} \) describes the kinetic and nonequilibrium properties of the system. The matrix \( \hat{G} \) is related to a matrix distribution function \( \hat{f} \) by

\[
\hat{G} = \hat{G}^R \hat{f} - \hat{f} \hat{G}^A
\]

(2)

Where the elements of the matrix \( \hat{f} = f_0 \hat{1} + f \hat{\sigma}_z \) describe particle and hole-like excitations. The component \( f_0 \) is used in calculating the order parameter and supercurrent, while \( f \) describes the branch imbalance and determines the spatial distribution of the electric field (see for example \[6\]). The matrices \( \hat{G}^{R(A)} \) can be presented in the form,

\[
\hat{G}^{R(A)} = G^{R(A)} \sigma_z + \hat{F}^{R(A)}
\]

(3)
In a bulk superconductor $G^{R(A)} = \epsilon/k_{c}^{R(A)}$, $\hat{F}^{R(A)} = F^{R(A)}(i\sigma_{x} \sin \chi + i\sigma_{y} \cos \chi)$, $F^{R(A)} = \Delta /k_{c}^{R(A)}$, $\varepsilon^{R(A)} = \sqrt{(\epsilon + \Gamma)^{2} - \Delta^{2}}$, $\Gamma$ is the damping rate in the excitation spectrum of the superconductor and $\chi$ is the phase of the order parameter. The current in the system is expressed in terms of the function $\hat{G}$:

$$I = (\sigma d/8)\text{Tr}\sigma_{z} \int d\epsilon (\hat{G}^{R}\partial_{\epsilon}\hat{G} + \hat{G}\partial_{\epsilon}\hat{G}^{A})$$

(4)

This expression is written for the one-dimensional case, where $d$ is the thickness of the system. In the diffusive limit the supermatrix $\hat{G}$ in the N film of a S/N mesoscopic system obeys the equation

$$D\partial_{x}(\hat{G}\partial_{x}\hat{G}) + i\epsilon [\hat{\sigma}_{z}, \hat{G}] = 0$$

(5)

where $D$ is the diffusion coefficient and $\hat{\sigma}_{z}$ is the Pauli supermatrix. We neglect the inelastic collision integral by assuming that the length of the N film $L$ is short enough: $\epsilon_{TH} > > [\hbar\tau^{-1}_{e-e}, \hbar\tau^{-1}_{e-\text{ph}}]$, $(\epsilon_{TH} = hD/L^{2} \text{ is the Thouless energy and } \tau_{e-e}, \tau_{e-\text{ph}} \text{ are the electron-electron and electron-phonon collision times})$. To solve Eq. (5) we must have the complementary boundary condition,

$$D\left(\hat{G}\partial_{x}\hat{G}\right) = (\epsilon_{0}d_{N}) \left[\hat{G}, \hat{G}_{S}\right]$$

(6)

the $z$-axis is normal to the plane of the S/N interface. The boundary conditions for the quasiclassical Green’s functions $\hat{G}$ were derived for the general case by Zaitsev [3] and reduced to the simple form [3] by Kupriyanov and Lukichev [7] in the dirty case. The applicability of Eq. (6) is discussed in Ref. [8]. The distribution functions in the reservoirs $f_{0}$ and $f$ are assumed to be in equilibrium and for example, for the structure shown in Fig.1(b) they have the form,

$$f_{s}(\pm L) = F_{sN}(\epsilon) \equiv [\tanh((\epsilon + eV)\beta) + \tanh((\epsilon - eV)\beta)]/2,$$

(7)

$$f(\pm L) = F_{N}(\epsilon) \equiv [\tanh((\epsilon + eV)\beta) - \tanh((\epsilon - eV)\beta)]/2,$$

(8)

where $\beta = (2T)^{-1}$ and $\pm V$ are the electric potentials in the reservoirs at $x = \pm L$ (we have assumed the system is symmetrical). In order to find the distribution functions $f(\epsilon, x)$ in the N film, we consider element (12) of Eq. (6) (i.e. we consider the equation for the Keldysh matrix $\hat{G}$). Consider for simplicity a structure of the type in Fig.1(b) with one superconducting strip in the centre of the N film. Multiplying this equation by $\hat{\sigma}_{z}$ and taking the trace, we obtain,

$$\partial_{x}(M(x)\partial_{x}f) = 0$$

(9)

where $M(x) = (1/2)(1 - \hat{G}^{R}\hat{G}^{A} - \hat{F}^{R}\hat{F}^{A})$ and we have assumed that the width of the S strip ($w$) is small compared to $L$. In this limit the distribution functions $f(\epsilon, 0)$ and $f_{s}(\epsilon, 0)$ are equal to zero ($V(0) = 0$). Integrating Eq. (9) and taking into account boundary condition (3), we obtain,

$$J = (F_{N}/L) < M^{-1} >$$

(10)

here $< M^{-1} > = L^{-1} \int_{0}^{L} dx M^{-1}$ (see [3]) and $J(\epsilon)$ is the integration constant which determines the current,

$$I = (\sigma d/2) \int_{0}^{\infty} d\epsilon J(\epsilon).$$

(11)

Therefore, the problem of finding the $I(V)$ characteristics is reduced to finding the retarded (advanced) Green’s functions which determine the function $M(\epsilon, x)$. Using the well known substitution, $G^{R(A)} = \cosh u^{R(A)}$ and $F^{R(A)} = \sinh u^{R(A)}$, the equation for the retarded (advanced) Green’s functions $\hat{G}^{R(A)}$ may be presented in the form,

$$\partial_{x} u^{R(A)} - \left(k^{R(A)}\right)^{2} \sinh u^{R(A)} = -k_{s}^{2} w(\delta(x + L_{1}) + \delta(x - L_{1})) \left[\hat{F}^{R(A)} \cosh u - G^{R(A)} \sinh u\right]$$

(12)

We have used Eqs. (3) and (1) and assumed that $w << L$, where $k_{s}^{2} = \epsilon_{b}/D$, $(k^{R(A)})^{2} = \gamma \pm 2i\epsilon$. The term on the right hand side of Eq. (12) is the source of the condensate (the proximity effect). Solutions to Eq. (12) can be written in an explicit form in some limiting cases. For example in the case of a weak proximity effect ($|F^{R(A)}| << 1$) Eq. (12) can be linearised (i.e. $\sinh u^{R} \approx u^{R}$), then a solution of Eq. (12) is easily found. This case occurs if the ratio of the resistance of the N film $R_{N}$ and the S/N interface resistance $R_{S/N}$ is small, $r \equiv R_{N}/R_{S/N} << 1$. The comparison of an exact numerical solution and the solution of the linearised Eq. (12) shows that even when $r \approx 1$ the difference
between these solutions is less than 10%. Another limiting case corresponds to a short N film ($\epsilon_{Th} >> (T, V)$), now Eq. (12) can be averaged over the length [10][11]. We use this procedure to analyse the subgap conductance of the structure shown in Fig.1(a).

2. Subgap conductance of S/N/N' structures

Considering the S/N/N' system shown in Fig.1(a) and assuming the middle N film is short, i.e. $\epsilon_{Th} \equiv D/L^2 >> [T, eV]$ (L is the length of the N film and V the applied voltage), we can average Eq. (12) over the length using the boundary condition (6) and obtain for $F^{R(A)}$ and $G^{R(A)}$,

$$F^{R(A)} = \frac{\epsilon_g}{\sqrt{\left(\epsilon + i\gamma\right)^2 - \epsilon_g^2}}, \quad G^{R(A)} = \frac{\epsilon \pm i\gamma}{\sqrt{\left(\epsilon + i\gamma\right)^2 - \epsilon^2}}$$

(13)

Here $\epsilon_g = \rho D/(2R_{bSNC}L)$, $\gamma = \rho D/(2R_{bNCG}L)$ and we have assumed that $\Delta >> (\epsilon, \epsilon_g)$. $R_{bSNC}, R_{bNCG}$ are the interface resistance per unit area. The energy $\epsilon_g$ is a minigap in the excitation spectrum of the N film induced by the proximity effect [2][3]. The damping constant $\gamma$ is due to the N/N' contact, which increases with decreasing the N/N' interface resistance. In Fig.2(a) we show the energy dependence of the DOS in the normal film for different $\gamma$. (14)

We see that if $\gamma >> \epsilon_g$ the DOS in the N film for $T < T_c$ is close to the DOS when $T > T_c$, and a subgap arises when $\gamma << \epsilon_g$.

In order to calculate the conductance of the structure, we need to find the distribution function $f$ from Eq.(4), taking into account boundary condition (6). Carrying out some simple calculations, we obtain the dimensionless conductance $F^{s}$,

$$S = R_N \frac{dI}{dV} = \int_0^\infty d\epsilon \beta F^s_N[M^{-1} + \frac{1}{\nu_d} \frac{\epsilon_{Th}}{\epsilon_g} (\frac{\epsilon_g}{\gamma}) + \frac{\epsilon_{Th}}{\nu_d} A_{\epsilon_g}^{-1}]$$

(15)

Here $F^s_N = \partial F_N/\partial(eV)$, M is defined in Eq.(11) and,

$$A = \frac{1}{4}[(G^R - G^A)(G^R - G^A) - (F^R + F^A)(F^R + F^A)]$$

(16)

The terms in the square brackets in Eq.(16) determine the normalised resistances of the N film, N/N' and N/S interfaces respectively. The first term in Eq.(16) stems from the usual quasiparticle current, which is non-zero when $eV > \Delta$. The second term in Eq.(16) represents the subgap conductance due to Andreev reflection processes and is non zero when $\epsilon < \Delta$ (if the damping $\Gamma$ in the superconductors is negligible). This term also leads to the so called interference current in Josephson tunnel junctions. In our case this term arises due to the proximity effect inducing a condensate in the normal film. In Fig.2(b) we plot the dependence of the zero-temperature conductance S versus normalised voltage, $S = G_n(dI/dV)$, where $G_n$ is the normal state conductance. The parameter $\epsilon_{Th}/\epsilon_g$, the ratio of the S/N interface resistance to the N film resistance is chosen large so that the resistance of the system is determined by the interface resistance. If the damping rate $\gamma$ is large in comparison with $\epsilon_g$, then the DOS in the normal film is only weakly disturbed by the proximity effect and the peak arising in the subgap conductance is located at zero voltage. The S/N interface resistance dominates in this case, and the subgap conductance is caused by the interference current (the second term in Eq.(16)), or by Andreev reflection processes. In the opposite case of a small damping ($\gamma << \epsilon_g$) or a large N/N' interface resistance a peak in the DOS of the N film appears in the voltage dependence of the subgap conductance (curves for $\gamma = 0.1$ and $\gamma = 0.5$). The peak in $S(V)$ is caused by a peculiarity in the quasiparticle current at $eV = \epsilon_g$ (the second term in Eq.(16)).

It is interesting to note that the minigap in the N film $\epsilon_g$ may be modulated by an external magnetic field if two superconductors (instead of one) are used in the structure (see for example [14]), then $\epsilon_g$ depends on the phase difference between the superconductors $\phi, \epsilon_g = \epsilon_g(\phi) \cos \phi$. We see from Fig.2(b) that increasing $\gamma$ with respect to $\epsilon_g$ may lead to an increase in the conductance for some voltages. This means that an applied magnetic field $H$ ($\phi \sim H$) may cause an increase in the conductance. This effect was predicted for a two-barrier structure in refs [14][15] and observed experimentally [23][24]. Note that the subgap conductance was first observed in a SISm system (Sm is a heavily doped semiconductor) by Kastalsky and Kleinsasser [31] and later by others [32][33], and theoretically studied in many works [1][36][37].

3. Nonmonotonic temperature and voltage dependence of the conductance.

In this section we analyse the voltage and temperature dependence of the conductance for the structure shown in Fig.1(b) (Andreev interferometer). Since the late 1970’s it has been known that the conductance ($G$) of S/N
mesoscopic structures depends on temperature \( T \) (and voltage \( V \)) in a non-monotonic way (see reviews \cite{2,3}). This behaviour was first predicted in Ref. \cite{1} where a simple point S/N contact was analysed. The authors of Ref. \cite{1}, using a microscopic theory and assuming that the energy gap in the superconductor \( \Delta \) is much less than the Thouless energy \( \epsilon_{Th} \equiv \hbar D/L^2 \), showed that the zero-bias conductance \( G \) coincides at zero temperature with its normal state value \( (G_n) \). With increasing \( T \), \( G \) exhibits a non-monotonic behaviour, increasing to a maximum of \( G_{max} \approx \epsilon_{Th} T_m \approx \Delta(T_m) \) and then decreasing to \( G_n \) for \( T > T_m \).

Recently mesoscopic S/N structures have been fabricated in which the limit \( \Delta \gg \epsilon_{Th} \) is realised. In this case Nazarov and Stoof \cite{17} also (see \cite{15, 21}) argued that the temperature dependence of the conductance \( G \) has a similar non-monotonic behaviour with a maximum at a temperature comparable with the Thouless energy, while simultaneously Volkov, Allsopp and Lambert \cite{21} predicted that the voltage dependence of the conductance in an S/N mesoscopic structure (Andreev interferometer) has a similar form with a maximum at \( \epsilon V_m \approx \epsilon_{Th} \). This non-monotonic behaviour has been observed both in very short S/N contacts \cite{22} and in longer mesoscopic S/N structures \cite{23, 25, 27}.

Considering the system shown in Fig.1(b) and assuming a weak proximity effect (an anomalous term which determines the voltage dependence of the normal state value \( < M > \) can be presented in the form, \( < M > = 1 - < m > \)). In order to calculate the variation of the normalised conductance \( < M > \), one has to solve Eqs. (10-11) and to find the functions \( F^{RA}(A) \). The function \( < M > \) which determines the voltage dependence of the zero-temperature conductance is equal to,

\[
< M > = \frac{1}{8} \frac{T}{\bar{R}} \left( 2 F^R F^A - 2 F^R F^A \right)
\]

where \( r \) is the ratio of the N film resistance and the S/N interface resistance in the normal state, \( \phi \) is the phase difference between the superconductors, for simplicity we assume that \( L_1 << L \). Knowing \( F^{RA}(A) \) we can easily calculate \( < m > \) and \( \delta S \) using Eqs. (10-11). The function \( < m > \) which determines the voltage dependence of the zero-temperature conductance is equal to,

\[
< m > = \frac{T^2}{8} \left[ \frac{\text{Re} \left( \sin 2\theta / 2\theta - 1 \right) / (\theta \cosh \theta)^2 - \left( \sin 2\theta_1 / 2\theta_1 - \sin 2\theta_2 / 2\theta_2 \right) / (\theta \cosh \theta)^2}{1 + \cos \phi} \right]
\]

where \( \theta = \theta_1 + i \theta_2 \). The first term in Eq. (20) is related to a change in the DOS and the second term is due to the anomalous term \( < F^{RA} > \). In Fig.3 we present the total conductance variation \( \delta S \) vs the normalised voltage \( V \) (in units of the Thouless energy) at zero temperature, the contribution \( \delta S_{DOS} \) due to a change of the DOS, and the anomalous term \( \delta S_{an} \) for \( \phi = 0 \). The variation \( \delta S \) has a maximum at \( \epsilon V \approx \epsilon_{Th} \). The temperature dependence \( \delta S(T) \) found with the help of Eqs. (10-11) has a similar form provided \( T << \Delta, \epsilon_{Th} \ll \Delta \) \cite{17, 21}. It is interesting to note that as \( T \) increases \( \delta S_{an} \) decays slowly (non-exponentially) as \( \epsilon_{Th} T \) at \( T >> \epsilon_{Th} \). This law leads to long-range, phase-coherent effects in mesoscopic structures \cite{24, 25, 26}. One can see from Eq. (19) that the functions \( |F^{RA}| \) have a peak at \( \epsilon \approx \Delta \) because the functions \( F_{S}^{RA} \) have a singularity at the energy \( |\epsilon| = \Delta \) (see Eq. (3)). With increasing temperature \( T \) the order parameter \( \Delta(T) \) starts decreasing near \( T_c \) and may be comparable with \( \epsilon_{Th} \), hence the function \( < m > \) and \( \delta S \) may have one more maximum. In Fig.4 the temperature dependence of \( \delta S \) is presented over a wide temperature range \( (0 < T < T_c) \) for a structure similar to that shown in Fig.1b (where \( L_1 = 0 \), i.e. only one superconductor is in contact with the normal film). We see that besides the main, low-temperature maximum in \( \delta S(T) \) there is an additional maximum near \( T_c \). The magnitude of both maxima depends strongly on the depairing rate \( \gamma \) in the N film (in order to account for \( \gamma \), the energy \( \epsilon \) must be replaced with \( \epsilon \pm i \gamma \) \cite{35}.

4. Negative Josephson current in a 4 terminal S/N/S structure.

In the preceding section we calculated the conductance \( S \) of the Andreev interferometer (see Fig.1b), determined by the nonequilibrium distribution function \( f \) (see Eq. (6)) which varies in space. In the limit of a weak proximity effect we obtain from Eqs. (6)
Here \( J(\epsilon) \) is a function even in energy \( \epsilon \) and defined in Eq. (10). Therefore \( f(\epsilon, x) \) increases almost linearly from zero (at \( x = 0 \)) to \( F_N \) (at \( x = L \)). The function \(< m >\) determined by the condensate functions depends on the phase difference \( \phi \) (see Eq. (20)), increasing \( \phi \) for example, in an applied magnetic field) causes the function \(< m >\) and the conductance \( S \) to oscillate. This oscillatory behaviour of the conductance has been observed in many experiments 23, 24, 26, 32, 33, 41 starting from the pioneering work 40 and later explained theoretically 14, 17, 42.

One can formulate the inverse problem, how the non-equilibrium distribution function affects, for example, the condensate current, which is determined by the expression

\[
f(\epsilon, x) = J(\epsilon)[x + \int_0^x dx_1 m(x_1)]
\]

(21)

where \( m \) is a function of energy \( \epsilon \) and determined in Eq. (14). By taking the trace of this equation in the limit of a weak proximity effect we obtain,

\[
\partial_x [(1 - m_o) \partial_x f_o] = 0
\]

(23)

where \( m_o = (1/8) Tr [\hat{F}^R R - \hat{F}^A A] f_o \). The current \( I_s \) is of the order of the small parameter \( r^2 \), therefore in finding \( f_o \) we can neglect \( m_o \) and obtain that \( f_o \) is constant in space and equal to \( f_o(\epsilon) = F_N \) (see Eq. (14)). The function \( F_N(\epsilon) \) is an equilibrium distribution function (odd in \( \epsilon \)) in the \( N \) reservoirs at a given potential \( V \), and is shifted with respect to the function \( F_{OS} = \tanh(\epsilon / \beta) \) by the value \( V \) (the potential in the superconductors is set equal to zero). The function \( F_N \) is an even function of \( V \) and therefore does not depend on whether the additional (control) current flows from the \( N \) reservoirs to the superconductors (the potential \( V \) has the same sign in both reservoirs) or from one \( N \) reservoir to another one (\( V \) has opposite signs in the reservoirs). A relative shift of the distribution functions in \( N \) and \( S \) controlled by the applied voltage \( V \) leads to a dependence of the Josephson critical current \( I_c \) on \( V \). In order to determine this dependence, we need to find the condensate functions \( \hat{F}^R(A) \), obeying the linearised Usadel equation which in this case has the form,

\[
\partial_{xx} \hat{F}^R(A) - (k^{R(A)})^2 \hat{F}^R(A) = -k^o w \hat{F}^R(A) [\delta(x + L_1) + \delta(x - L_1)]
\]

(24)

the functions \( \hat{F}^R(A) \) are given by Eq. (14). Solving Eq. (24) and substituting the solutions into Eq. (23), we find

\[
I_s = I_c \sin \phi,
\]

where

\[
I_c(V)R_L = (\pi T)r^2 \Re \sum_{n=0}^{\infty} \frac{\Delta^2}{\Delta^2 + (\omega_n + i e V)^2} \frac{\sin^2(\theta_n(L - L_1)/L)}{\theta_n \sin(2\theta_n)}
\]

(25)

here \( \theta_n = L[(2\omega_n + i e V + \gamma)/D]^{1/2}, \omega_n = \pi T(2n + 1) \) is the Matsubara frequency. In Fig. 5 we show the dependence of the Josephson critical current \( I_c \) on \( V \). We see that \( I_c \) changes sign when \( e V \approx c_T h \). The \( I_c \) sign reversal effect (\( \pi \) - contact) was first predicted in Ref. 13 where a \( S/M/S \) contact was considered (\( M \) is an insulating layer with magnetic impurities). The same effect was analysed later in Ref. 44 where a \( S/F/S \) contact was considered. The possibility to change of the \( I_c \) sign by a control voltage (or current) in a 4-terminal \( S/N/S \) diffusive structure was first studied in Ref. 45 for the 1-dimensional case of a short spacing between superconductors \((L_1 \ll L_s)\) and in Refs. 20, 29, 30, 44 for the case of a longer spacing. The physical reason for the sign reversal of \( I_c \) in 4 terminal \( S/N/S \) structures is a difference in the distribution functions \( f_o \) in the \( N \) film and in the superconductors. These functions have an equilibrium form corresponding to the electric potential \( V \) and zero respectively. The 1-dimensional case was analysed in Refs. 20, 29, 45, 44. In Ref. 47 the 2-d case was analysed, however in finding the distribution function \( f_o \) and the condensate functions \( F^R(A) \) the authors considered opposite limits, \( d_s << d < d_s >> d \), where \( d_s, d \) are the width of the \( S \) and \( N \) films respectively. Therefore the region of applicability of the results obtained in Ref. 47 remains unclear. One can show that qualitatively Eq. (27) in limit \( d_s << d \) remains valid for the 2-d case, i.e. \( I_c R_N \) is proportional to the small parameter \( r^2 \) \((r \) is the ratio of the \( N \) film resistance to the resistance of the \( S/N \) contact). Similar effects in ballistic systems were studied in Refs. 19, 51.

Note, a formal analogy between the two possibilities of the \( I_c \) sign reversal effect: a structure with magnetic material and a shift of the distribution functions in a 4-terminal \( S/N/S \) structure. In the first case the Green’s functions \( F^R(A) \) in the \( F \) layer differ from those in the \( N \) layer by a shift in energy \( \epsilon \rightarrow \epsilon + h \), where \( h \) is the exchange energy 44. The distribution function \( f_o \) in the \( F \) layer coincides with the distribution function in the superconductor \( F_G = \tanh(\epsilon / \beta) \). Meanwhile in the 4-terminal \( S/N/S \) contact the distribution function in the \( N \) layer is
shifted with respect to $F_{20}$, $f_0(\epsilon) = [F_{0S}(\epsilon+eV) + F_{0S}(\epsilon-eV)]/2$. Making the substitution $\epsilon \to \epsilon \pm eV$ in the integral in Eq.\(\text{(23)}\), one can reduce the case of a 4-terminal S/N/S contact to the case of a S/F/S contact with $h = eV$. The sign reverse of $I_c$ in S/F/S contacts has, presumably, not yet been observed. On the other hand in a recent publication \[\text{[28]}\] experimental evidence for the change of the sign of $I_c$ in 4-terminal S/N/S contacts was obtained.

5. Conclusions.

The quasiclassical Green’s function technique we have used complements the approach based on the scattering matrix method and the Landauer formula for the conductance. This technique allows one to describe the transport properties of S/N mesoscopic structures over a wide range of parameters. With the help of this approach the conductance of a double-barrier S/N’/N’ structure was calculated. We have shown that in the differential subgap conductance $S$ has a peak situated at zero or finite bias voltage (depending on the relation between the S/N and N/N’ interface resistances). When the temperature is lowered below $T_c$, the variation of the conductance may be both positive and negative. Therefore the conductance decreases or increases with increasing an external magnetic field.

The conductance in S/N structures of the types of the Andreev interferometer or S/N point contact depends on $T$ or $V$ in a non-monotonic way. As a function of temperature $T$ the zero-bias conductance variation is negative. Therefore the conductance decreases or increases with increasing an external magnetic field.

An interesting effect may occur in 4-terminal S/N/S mesoscopic structures (see for example Fig.1b). If an additional (control) current flows through the N film, then the distribution functions in the N and S films are different. Due to this difference the critical Josephson current may change sign and the S/N/S contact may become a $\pi$-contact. Most of the effects discussed in this paper have been observed experimentally, although some predictions such as the appearance of the second maximum near $T_1$ of the conductance have, presumably, not yet been observed experimentally.

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![Diagram](a) Barriers Normal Reservoir Normal Metal Superconducting Reservoir (b) Superconductors Normal Reservoir Normal Metal

**FIG. 1.** The structures considered.

![Diagram](a) Energy dependence of the DOS for the structure in Fig.1a, for different γ; here ∆ = 20, Γ = 0.1 and εd = 10.
(b) The zero temperature conductance S (Eq.(15)), normalised to the conductance of the normal metal in the norml state, dependence on voltage, using the same parameters as for Fig.2a. All energies are measured in units of εTh.

**FIG. 2.** (a) The energy dependence of the DOS for the structure in Fig.1a, for different γ; here ∆ = 20, Γ = 0.1 and εd = 10.
(b) The zero temperature conductance S (Eq.(15)), normalised to the conductance of the normal metal in the norml state, dependence on voltage, using the same parameters as for Fig.2a. All energies are measured in units of εTh.
FIG. 3. Normalized conductance $\delta S$ vs. normalized voltage $eV/\epsilon_L$ at $T = 0$ and vs. normalized temperature $T/\epsilon_L$ at $V = 0$ for the structure shown in Fig.1b ($L_1 = 0$).

FIG. 4. Dependence of $\tilde{\delta}S$ on temperature, for the cross geometry in the weak proximity limit, for different depairing rates $\gamma$. With $\Delta_0 = 20, \Gamma = 0.1, L_1/L = 0$. The inset is an enlargement of the second maximum around $T/T_c = 1.0$. Note the $\delta S = \delta S(2r_b)^2$.

FIG. 5. Critical current $I_c$ dependence on voltage for the 4 terminal Andreev interferometer, for several temperatures ($T$). The critical current is measured in units $\epsilon_T r_0/ (eR_L r^2)$ and $T, V$ in $\epsilon_T; \Delta = 20, \gamma = 0.2, L_1/L = 0.5$. 