CDM models with a BSI steplike primordial spectrum and a cosmological constant

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ABSTRACT
A class of spatially-flat models with cold dark matter (CDM), a cosmological constant and a broken-scale-invariant (BSI) steplike primordial (initial) spectrum of adiabatic perturbations, generated in an exactly solvable inflationary model where the inflaton potential has a rapid change of its first derivative at some point, is confronted with existing observational data on angular fluctuations of the CMB temperature, galaxy clustering and peculiar velocities of galaxies. If we locate the step in the initial spectrum at $k \simeq 0.05 \, h \, \text{Mpc}^{-1}$, where some feature in the spectrum of Abell clusters of galaxies was found that could reflect a property of the initial spectrum, and if the large scales flat plateau of the spectrum is normalized according to the COBE data, the only remaining parameter of the spectrum is $p$ - the ratio of amplitudes of the metric perturbations between the small scales and large scales flat plateaux. Allowed regions in the plane of parameters $(\Omega = 1 - \Omega_\Lambda, H_0)$ satisfying all data have been found for $p$ lying in the region $(0.8-1.7)$. Especially good agreement of the form of the present power spectrum in this model with the form of the cluster power spectrum is obtained for the inverted step ($p < 1$, $p = 0.7 - 0.8$), when the initial spectrum has slightly more power on small scales.

Key words: cosmology - initial spectrum of perturbations - large-scale structure of the Universe - cosmological constant.

1 INTRODUCTION

The inflationary paradigm (see the review in Linde 1990; Kolb & Turner 1990) offers an elegant solution to some of the outstanding problems of standard Big Bang cosmology. In these models, primordial quantum fluctuations (Hawking 1982, Starobinsky 1982, Guth & Pi 1982) of some scalar field(s) (inflaton(s)) are produced, which eventually form galaxies, clusters of galaxies and the large-scale structure of the Universe through gravitational instability. Though there is a variety of possible models, parametrized by a few number of constants, the increasing amount of data, for example from redshift surveys and cosmic microwave background (CMB) anisotropies measurements, severely constrain the proposed models. Hence some of them can be definitely excluded at this stage while the remaining ones are found to be viable only in some well defined region of their free parameter(s) space. Additional sharp constraints are expected from the planned satellite missions MAP and PLANCK SURVEYOR for the measurement of the CMB anisotropies up to small angular scales.

Since it is known that the simplest CDM model with a flat ($n=1$) initial spectrum of adiabatic perturbations does not agree with observational data (if normalized to the COBE data at large scales, it has too much power at small scales), a number of approaches to increase the ratio of large to small scale power were proposed. One possibility is to change the initial spectrum of perturbations. Since tilted scale-free spectra ($n < 1$) did not appear successful, the next step was to consider broken-scale-invariant (BSI) spectra arising in inflationary models with two effective scalar fields (Kofman, Linde & Starobinsky 1985; Kofman & Linde 1987; Silk & Turner 1987; Kofman & Pogosyan 1988; Gottlöber, Müller & Starobinsky 1991; Polarski & Starobinsky 1992). Recently, the CMB anisotropies for a model of double inflation (Lesgourgues & Polarski 1997) was investigated and it was found that for values of the parameters which yield a power spectrum $P(k)$ in fair agreement with observations,
the Doppler peak turns out to be low. This is related to the
effective tilt of the spectrum on very large scales.

Another possibility is to add a positive cosmological
constant leaving the initial \( n \simeq 1 \) spectrum unchanged. It
was long known that the cosmological constant is viable only
if it is accompanied by cold dark matter and, vice versa, its
inclusion improves the CDM model a great deal (as empha-
sized e.g. in Kofman & Starobinsky (1985)). This model
remains viable after the detection of the CMB anisotropies
on large angular scales by COBE (Kofman, Gnedin & Bah-
call 1993), and it is now perhaps the most promising CDM
variant (Bagla, Padmanabhan & Narlikar 1995; Ostriker &
Steinhardt 1995). As well known, one important motivation
for a positive cosmological constant \( \Lambda \) is that it provides
the possibility to accomodate both a high Hubble “constant”,
h > 0.6, and a sufficiently old universe, \( t_0 \gg 11 \) Gyr. Also,
the baryon fraction in clusters seems to imply \( \Omega < 0.55 \)
(\( \Omega = 1 - \Omega_{\Lambda} \) stands for the total matter density including
CDM and baryons). The most recent strong argument in
favour of \( \Omega < 1 \) (and \( \Omega \simeq (0.2 - 0.4) \)) follows from the evolu-
tion of rich galaxy clusters (Bahcall, Fan & Cen 1997, see
also Fan, Bahcall & Cen 1997).

Up to now, these two possibilities were considered as
mutually exclusive. Now we want to unite them and com-
pare the BSI CDM model including a cosmological constant with
the observational data. The reasons for this are the follow-
ing. First, it can enlarge the allowed cosmological pa-
rameters window. Second, the possibility exists that the ini-
tial power spectrum of scalar (density) perturbations in the
Universe is not scale free but has instead some non-trivial
structure near \( k = 0.05 \) \( h \) Mpc\(^{-1} \). In fact observations may
point to such a feature: the analysis of the three-dimensional
distribution of rich Abell galaxy clusters located in super-
clusters, performed in Einasto et al. (1997a), pleads for an
unexpected spatial quasi-periodicity of the data (see also
Einasto et al. 1997b, 1997c). Also, the spatial distribution of
all Abell clusters of galaxies has a well-marked peak in the
power spectrum at \( k \simeq 0.05 \) \( h \) Mpc\(^{-1} \) (Einasto et al. 1997a).
The Fourier power spectrum of the spatial distribu-
tion of APM galaxies also has a feature on the same scale,
though of a slightly different form (Caztanaga & Baugh,
1997). Note that the natural attempt to explain this feature
by Sakharov oscillations produced by the baryon admixture
to CDM does not work (Atrio-Barandela et al., 1997, Eisen-
stein et al., 1997). So this feature, if confirmed by future
improved large scale structure observations, should be as-
cribed to the initial perturbation spectrum itself.

Therefore, we need an initial spectrum which has a
non-trivial structure around some scale (preferably, with a
bump) and has essentially no tilt at larger and smaller
scales. The latter condition is necessary in order to have
sufficiently early galaxy and quasar formation. On the other
hand, this spectrum should be derivable from some first prin-
ciples (e.g., it could be generated in a concrete inflationary
model). Such a spectrum naturally arises in a well-defined
and rather generic (though idealized, of course) inflationary
model where the inflaton potential \( V(\varphi) \) has a local steplike
feature in the first derivative. An exact analytical expres-
sion for the scalar (density) perturbations generated in this
model was found in Starobinsky (1992). It has a universal
shape depending on only one parameter \( p \). Actually, it seems
to be the only example of a perturbation spectrum with the
desired properties, for which a closed analytical form exists.

Thus, we suppose that the inflaton potential \( V(\varphi) \) has a
rapid change of slope in a neighborhood \( \Delta \varphi \) of \( \varphi_0 \):

\[
V(\varphi) = V_0 + v(\varphi),
\]

\[
v(\varphi) \simeq A_+ \varphi, \quad \varphi > \varphi_0, \quad |\varphi - \varphi_0| \gg \Delta \varphi,
\]

\[
v(\varphi) \simeq A_- \varphi, \quad \varphi < \varphi_0, \quad |\varphi - \varphi_0| \gg \Delta \varphi,
\]

\[
v(\varphi_0) = 0, \quad A_+ > 0, \quad A_- > 0.
\]

The resulting adiabatic perturbation spectrum is non-flat
around the point \( k_0 = a(t_0)/H(t_0) \), \( t_0 \) being the time at
which \( \varphi = \varphi_0 \) while \( H \equiv \dot{a}/a \) is the Hubble parameter.
One can show (Starobinsky 1992) that if the width \( \Delta \varphi \)
of the singularity is small enough, namely, \( \Delta \varphi H(t_0)^2 \ll
\min(A_+, |A_+ - A_-|) \), then the adiabatic perturbation spec-
trum has maximal deviation from flatness, and acquires a
universall form that can be derived analytically:

\[
k^3 \Phi^2(k) \propto 1 - 3(p - 1) \left( \frac{1}{y} - \frac{1}{y^2} \right) \sin 2y + \frac{2}{y} \cos 2y \cdot
\]

\[
+ \frac{9}{2} (p - 1)^2 \frac{1}{y^2} \left( 1 + \frac{1}{y^2} \right) \times \frac{1}{y^2} \cos 2y - \frac{2}{y} \sin 2y \cdot
\]

\[
y = \frac{k k_0}{p}, \quad p = A_- / A_+.
\]

where \( \Phi \) is the (peculiar) gravitational potential. This
expression, plotted in Fig. 1, depends (besides the overall nor-
malization) on two parameters \( p \) and \( k_0 \). The shape of the
spectrum does not depend on \( k_0 \), \( k_0 \) only determines the
location of the step. For \( p > 1 \), the spectrum has a flat
upper plateau on larger scales, even with a small bump,
and a sharp decrease on smaller scales, with large oscilla-
tions though. For \( p < 1 \) this picture is inverted. The ratio
of power between the plateaux equals \( p^2 \), and for \( p = 1 \)
we just recover the (flat) scale-invariant Harrison-Zel’dovich
spectrum. Note that this spectrum cannot be obtained in
the slow-roll approximation (even with any finite number of
adiabatic corrections to it). In this model it is still possible
to fix freely the amount of primordial gravitational waves
(GW’s) for given \( p \) and normalization and we consider here
the model with no GW’s at all. Without the inclusion of a
cosmological constant, we would be forced to consider the
case \( p > 1 \) only, in order to increase power on large scales.
Since \( \Lambda > 0 \) already produces a desired excess of large-scale
power, we are now free to consider both cases \( p > 1 \) and
\( p < 1 \).

We use for this study observational constraints both on
the matter power spectrum \( P(k) \) on one hand, and on the
CMB anisotropies on large, intermediate and small angular
scales on the other hand, as done in Lesgourguies & Polarski
(1997), and we refer the interested reader to this article for
more details.

2 CONFRONTATION WITH OBSERVATION

In this work we restrict ourselves to the case of a spatially
flat universe, containing cold dark matter, baryonic matter
with \( \Omega_B h^2 = 0.015 \), and a cosmological constant \( \Lambda \). Hence,
the two cosmological parameters \( h = H_0/100 \) and \( \Omega_\Lambda \) are free. The present power spectrum \( P(k) \) reads:

\[
P(k) = \frac{4}{9} \frac{k^4}{H_0^4} T^2(k) \frac{\Omega_\Lambda}{\Omega} \left( 1 - \frac{H}{a} \int_0^t a \, dt \right)^2 .
\]

where \( \Phi(k) \), given by Eq. (3), is the gravitational potential at the matter dominated stage for large redshifts \( z \gg 1 \) when \( \Omega \gg 1 \), and we put the light velocity \( c = 1 \). The scale factor \( a(t) \) is given by the expression \( a(t) = a_1 \sinh(\sqrt{2} H_{\infty} t) \), where \( H_{\infty} = \sqrt{\Lambda/3} = H_0 \sqrt{1 - \Omega_{\Lambda}, a_1 = \text{const}} \). The transfer function \( T(k) \) is computed with the fast Boltzmann code CMBFAST by Seljak & Zaldarriaga (1996), for each value of the cosmological parameters.

The power spectrum is normalized to the four years COBE DMR data (Bennett et al., 1996), using \( Q_{\text{rms}} = 0.015 \). In the relevant cases, COBE scales will always correspond to the small-\( k \) flat plateau of the initial spectrum. Afterwards, we use the following tests to discriminate between each set of values of the cosmological, resp. inflationary, parameters \( h, \Omega_\Lambda, \text{resp.} \rho, k_0 \):

(i) The “optical” \( \sigma_8 \). White, Efstathiou and Frenk (1993) give \( \sigma_8 = 0.57 \pm 0.06 \), with conservative errorbars. This is a sharp constrain at wavenumbers \( k \sim 0.2 \) h Mpc\(^{-1}\). More recent determinations of this quantity have a tendency to decrease it to \( \sigma_8 \sim 0.5 \), and even a bit lower (Ebe, Cole & Frenk 1996; Viana & Liddle 1996; Ying, Mo & Börner 1997). Still, we shall use the former value (the exponent of \( \Omega \) corresponds to the case of a flat Friedmann-Robertson-Walker model with a \( \Lambda \)-term, see below).

(ii) Peculiar velocities, deduced from the Mark III catalog, and POTENT reconstruction of the density field. In our case, the power spectrum has got strong oscillations, so we cannot simply use direct estimates of \( P(k) \) at given wavenumbers (Kolatt & Dekel 1997), which would give precise constraints in the case of a smooth spectrum. We will rather use the rms bulk velocity in a sphere of radius \( R \):

\[
\left( \langle V_{\text{rms}}^2 \rangle_R \right) = \frac{f^2(\Omega) H_0^2}{2\pi^2} \int_0^\infty \frac{dk P(k) W_R^2(k)}{k},
\]

where \( W_R(k) \) is the Fourier transform of the top-hat window function of radius \( R \), and \( f(\Omega) \equiv H^{-1} D/D(\Omega) \) is the linear growth factor for inhomogeneities. Expressing \( f \) as a power law, \( f(\Omega) = \Omega^{\alpha} \), one can easily compute the index for a given \( \Omega \) and \( \Omega_\Lambda \). In the interesting range \( 0.2 \leq \Omega \leq 1 \), \( r = 0.57 \pm 0.60 \) for an open universe with \( \Omega_\Lambda = 0 \), but \( r = 0.55 \pm 0.56 \) for a flat universe with \( \Omega + \Omega_\Lambda = 1 \). In the following we will take \( f(\Omega) \) to be \( \Omega^{0.56} \).

The Mark III POTENT result at \( R = 50h^{-1}\) Mpc (with a gaussian smoothing at \( R_s = 12h^{-1}\) Mpc) is \( V_{\text{rms}} = 375 \pm 85 \) km s\(^{-1}\) (Kolatt & Dekel 1997). The cosmic variance (the possible dissimilarity between the rms value of \( \langle V_{\text{rms}}^2 \rangle_R \) and the particular realization in our local neighborhood) is quite large for this quantity (~100 km s\(^{-1}\)), and can be added in quadrature with the previous errorbar, leading to a global uncertainty \( \sigma \sim 130 \) km s\(^{-1}\). This test is mainly sensitive to wavenumbers \( 0.01 \leq k \leq 0.06 \) h Mpc\(^{-1}\).

(iii) Redshift surveys. Since they are strongly bias-dependent, redshift surveys give indications about the shape of \( P(k) \). Here again, due to the oscillations, instead of using some sets of estimates at given wavenumbers, one has to convolve the spectrum with the window functions of a given experiment and compare with the raw data. We use the count-in-cells analysis of large-scale clustering of the Stromlo-APM redshift survey. Taking other experiments into account would slightly improve the precision, but not change the results, since Stromlo-APM is in very good agreement with other redshift surveys, as can be seen in Peacock & Dodds (1994). After normalizing the spectrum to \( \sigma_8 = 1 \), we compute the variance \( \sigma_8^2 \) in cells of size \( l h^{-1}\) Mpc, and compare it with the data (Loveday et al., 1992), consisting of nine points (assumed to be independent, with error bars treated as \( 2\sigma \) ones), through a \( \chi^2 \) analysis. Since we can vary four parameters (plus the overall normalisation, which is irrelevant for this test), \( \chi^2 \leq 5 \) is excellent, whereas \( \chi^2 \geq 15 \) is bad.

(iv) CMB anisotropies. We compute the curves \( (l+1)C_l \) using CMBFAST, and compare it with some preliminary measurements. At the moment, there are still many uncertainties, and we only have global indications on the \( C_l \)’s curve. As far as the first peak is concerned, a sixth order polynomial fit to the full available data set gives \( A_{\text{peak}} \equiv (l+1)C_l/2\pi^{1/2} = 28 \times 10^{-6} \) with \( l = 260 \) (Lineweaver & Barbosa, 1997), but it is very difficult to calculate an errorbar for this quantity in the general case. CAT and OVRO give an indication on the amount of power on small scales, but do not constraint the position and height of the secondary peaks.

Since the precision of these measurements is increasing very quickly, we do not intend in this work to perform a full \( \chi^2 \) analysis, using each result and the corresponding window function (to find which parameters yield the best agreement). We prefer to calculate the \( C_l \)’s and comment our results in such way that in a few years one could easily update the analysis, restrict the allowed parameters window and eventually rule out the model. This is why we concentrate on the position and height of the peaks. We will use CMB data to eliminate parameters only if there is an obvious discrepancy between the predicted curve and the observations.

### 3 RESULTS

Starting with a flat spectrum, we explore the range \( h = 0.5, 0.6, 0.7 \) and \( 0 \leq \Omega_\Lambda \leq 1 \). For each value of \( h \), there are some \( \Omega_\Lambda \)’s in agreement with the \( \sigma_8 \) constraint: \( (h = 0.5, 0.45 \leq \Omega_\Lambda \leq 0.50), (h = 0.6, 0.55 \leq \Omega_\Lambda \leq 0.60) \) and \( (h = 0.7, 0.65 \leq \Omega_\Lambda \leq 0.70) \). These windows are inside the limits \( \Omega h \simeq 0.25 \pm 0.30 \) found in Kofman, Gnedin & Bahcall (1993), and are in good agreement with other tests: bulk velocity, with \( V_{\text{rms}} \simeq 300 \) km s\(^{-1}\), and count-in-cells, with \( 3 < \chi^2 < 6 \). There is no obvious contradiction with CMB measurements, and the first Doppler peak is fairly high: \( A_{\text{peak}} \simeq (26 - 29) \times 10^{-6} \). Therefore, it is not necessary to depart from a flat spectrum in order to explain all observations (apart, of course, from a possible spike in the spectrum at \( k \approx 0.05 h \) Mpc\(^{-1}\) if \( h \) and \( \Omega_\Lambda \) turn out to be close to these values.

However, as we shall see, the steplike spectrum is compatible with a larger subset \((h, \Omega_\Lambda)\). It also predicts some specific features in \( P(k) \) and \( C_l \) curves that could easily be observed or ruled out by future experiments, so we are not
just adding some extra degeneracy. More precisely, one can think of a spectrum with:

A. $p > 1$, in order to get less power on small scales in the primordial spectrum. To compensate the loss of power in $P(k)$, we will allow smaller values of the cosmological constant. This will lower the CMB anisotropies. Then, to avoid a problematic collapse of the first acoustic peak, like in double inflation (Lesgourgues & Polarski, 1997), $k_0$ must be chosen so that multipoles up to $l \sim 200$ (at least) are given by the upper plateau of the primordial spectrum. This means that we can forget any $k_0 < 0.03\, h\, \text{Mpc}^{-1}$. For $k_0 \simeq 0.03\, h\, \text{Mpc}^{-1}$, the first peak is even enhanced by a few percents by the global maximum of the primordial spectrum (the “bump” at the extremity of the upper plateau).

B. $p < 1$, in order to get more power on small scales in the primordial spectrum, and therefore allow some higher values of the cosmological constant which would be excluded by small scale constraints (for instance, $\sigma_8$ in case of a flat spectrum ($p = 1$). Since CMB peaks grow with $\Omega_{\Lambda}$, multipoles $l > 2k_0/\sigma_8^2 H_0$ will be unusually large. A priori, in this case, the step is anywhere between the COBE and $\sigma_8$ scales: $0.003 \lesssim k_0 \lesssim 0.1\, h\, \text{Mpc}^{-1}$. However, in this case the spectrum (3) has a well-pronounced sharp maximum at $y \simeq 3.5$ for the values $p \simeq 0.8$ which are the most interesting ones as will be seen below (for $p < 1$ the maximum is located at $y \simeq 3.14$). So, if we want to use this bump to explain the feature in the cluster spectrum at $k \simeq 0.05\, h\, \text{Mpc}^{-1}$, $k_0$ should be taken $\approx 0.015\, h\, \text{Mpc}^{-1}$. Note that this possibility was not expected and discussed before.

3.1 The case $p > 1$

We consider first the case $h = 0.5$. Assumption A turns out to be successful with respect to the first three tests in many cases: for any $0 \leq \Omega_{\Lambda} \leq 0.5$, one can find a large allowed window in the $(p, k_0)$ plane. Of course, a smaller $\Omega_{\Lambda}$ will lead to a higher range for $p$. For instance, when $\Omega_{\Lambda} = 0.3$, we find $1.3 \leq p \leq 1.7$ and $0.03 \leq k_0 \leq 0.06\, h\, \text{Mpc}^{-1}$. We must stress that satisfying the three tests is a success, since most other models satisfy two of them at most. For instance, if a tilted spectrum leads to a correct $\sigma_8$ and $\chi^2$, the bulk velocity will be generally too low, unless a very large cosmic variance is invoked. Double inflation will predict a higher $V_{50}$, but still under the $-1\sigma$ errorbar. In the present model, $V_{50}$ is much higher, in very good agreement with observations, as can be seen on table I (second line) for one example, because there is at least as much power on scales $0.01 \leq k \leq 0.06\, h\, \text{Mpc}^{-1}$ as for a scale invariant spectrum.

Including CMB anisotropies in the tests provides two independent constraints:

- on one hand, the position and height of the first peak are essentially related to cosmological parameters, not inflationary parameters. Indeed, as we said previously, in the relevant cases ($k_0 \geq 0.03\, h\, \text{Mpc}^{-1}$), the first peak is deduced from an essentially scale invariant spectrum (with only a little enhancement proportional to $p$, but $\leq 10\%$, on $A_{\text{peak}}$ in viable cases), and depends only on $h$ and $\Omega_{\Lambda}$.

  The position and height of the first peak are not precisely constrained by observations at the moment, neither by Saskatoon (whose calibration is under progress: Leicht, in preparation), nor by MSAM. Inside the allowed $(p, k_0)$ window found previously, we find $24 \times 10^{-6} < A_{\text{peak}} < 30 \times 10^{-6}$, in good agreement with current limits, so we cannot exclude any set of parameters.

- on the other hand, experiments on the secondary peaks scales constrain $p$ and $k_0$. The global height of the multipoles at $400 < l < 1500$ depends on $p$, whereas $k_0$ gives the detailed shape at these scales (for instance, the ratio between the peaks), by shifting the maxima and minima of the primordial spectrum in $k$-space.

At the moment, observations do not indicate a detailed shape, but from CAT and OVRO we know that multipoles on such scales should range basically between $A_i = [l(l + 1)C_l/2\pi^{1/2}] = 10 \times 10^{-6}$ and $A_i = 25 \times 10^{-6}$. Since we are not dealing with the detailed window function of each measurement, we must be extremely conservative. Using the second point of CAT, one can state that a $C_l$ curve that would not reach $A_i = 12 \times 10^{-6}$ (the $-1\sigma$ value) in the range $550 < l < 720$ (for which the window function is above half of its peak value) can be confidently excluded. The reason for which we use this particular point is that the associated window function does not interfere with the first acoustic peak: it is probing power only on the scales of the secondary ones. This restriction provides, for each value of the cosmological constant, an upper limit on $p$, and we find that for $0 \leq \Omega_{\Lambda} < 0.2$, all sets of parameters are ruled out. This is an indirect constraint on $\Omega_{\Lambda}$. For $0.2 \leq \Omega_{\Lambda} \leq 0.5$, the previously found windows still hold.

Finally, for $0.2 \leq \Omega_{\Lambda} \leq 0.5$, all the constraints can be satisfied by some values of $p$ and $k_0$ in the ranges $1 \leq p \leq 1.7$ and $0.03 \leq k_0 \leq 0.07\, h\, \text{Mpc}^{-1}$. To illustrate this case, we show in table I (second line) a particular example: $h = 0.5$, $\Omega_{\Lambda} = 0.3$, $p = 1.3$, $k_0 = 0.03\, h\, \text{Mpc}^{-1}$ ($\Omega_{\Lambda}$ is chosen to obtain the preferred value $A_{\text{peak}} = 28 \times 10^{-6}$, and $p$ is as low as possible, in order to maximize small scales anisotropies). We also give an example of the case $\Lambda = 0$, $p = 2.1$, though it is excluded by CAT. The corresponding power spectra are plotted in Fig. 3, the CMB anisotropies in Fig. 4.

This type of model could be easily discriminated by the forthcoming improvements of redshift surveys and CMB observations. The former might state about the little well predicted in the $P(k)$ around $k \simeq (0.1 - 0.2)\, h\, \text{Mpc}^{-1}$. The latter will soon indicate:

- first, the position and amplitude of the first peak, i.e., $h$ and $\Omega_{\Lambda}$ (in the framework of this model).
- second, power on small scales, i.e. $p$.
- finally, the shape of secondary peaks, i.e. $k_0$.

This model is very unlikely to be degenerate with some other one (for instance, other cosmological parameters plus tilted spectrum) from the point of view of CMB anisotropies, because it predicts a tremendously high ratio between multipoles at scales $l \sim 200$ and $l \sim 600$ (recall that, in contrast with tilted, $n < 1$, or with double inflationary models, small scales are lowered however intermediate scales are preserved).

A similar analysis can be performed for higher $h$ values. Since increasing $h$ lowers the CMB multipoles, $p$ is more restricted now by the constraints on small scales anisotropies. At $h=0.6$, possible models have $0.4 \leq \Omega_{\Lambda} \leq 0.6$, $1 \leq p \leq 1.5$. 

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and $0.03 \leq k_0 \leq 0.07 \, \text{h Mpc}^{-1}$. The first peak reaches lower values as well: $26 \times 10^{-6} \leq A_{\text{peak}} \leq 27 \times 10^{-6}$. When $h = 0.7$, we find $0.5 \leq \Omega_\Lambda \leq 0.7$, $1 \leq p \leq 1.4$, $0.03 \leq k_0 \leq 0.07 \, \text{h Mpc}^{-1}$ and furthermore $24.5 \times 10^{-6} \leq A_{\text{peak}} \leq 26 \times 10^{-6}$. The resulting allowed region in the $(h, \Omega_\Lambda)$ plane is plotted in Fig. 2. For a few successfull examples, we give the results of the tests in Table I.

3.2 The case $p < 1$

Again, we first consider the case $h = 0.5$. When $0.45 \leq \Omega_\Lambda \leq 0.7$, one can find some $(p, k_0)$ in good agreement with $\sigma_8$, $V_50$ and $\chi^2$. For instance, when $\Omega_\Lambda = 0.6$, the allowed region is $0.75 \leq p \leq 1$ and $0.003 \leq k_0 \leq 0.04 \, \text{h Mpc}^{-1}$. In the last subsection, it was found that for $p > 1$, the most compelling constraint was on $\sigma_8$. Now the three tests play an important part in the definition of the allowed region. Indeed, the above mentioned sharp maximum appears in $P(k)$, preceded at larger scales by a depression at $y \approx 1.2$ (the inverted bump of the case $p > 1$). As a result, the power spectrum $P(k)$ has no pronounced second maximum at the place where it exists for $p = 1$, namely $k \approx 0.05 \, \Omega^{-1} \text{h Mpc}^{-1}$. Note that this depression would become very pronounced in the case $p \ll 1$, its position in this limit being given by $y = \sqrt{2 \pi p}$ (Starobinsky, 1992). When $k_0 > 0.015 \, \text{h Mpc}^{-1}$, the bulk velocity is sometimes too small due to this little depression. On the contrary, when $k_0 < 0.015 \, \text{h Mpc}^{-1}$, the maximum often generates excessive bulk velocities.

As expected, the CMB anisotropies are amplified by both the cosmological constant and the primordial spectrum step. The basic picture is that the $C_l$’s are enhanced by a factor $p^2$ for $l > 2k_0/a_0 H_0 \approx 120000$. When $k_0 = 0.01 - 0.02 \, \text{h Mpc}^{-1}$, the first peak is enhanced by the maximum of the primordial spectrum, so its location and maximum value are highly dependent on all parameters, including $k_0$ and $p$ (in contrast with the case $p < 1$). The secondary peaks are given by an almost flat region of the primordial spectrum, so they depend on all parameters, $k_0$ excepted.

As in the previous subsection, we can use the last CAT point to reduce the allowed window, confidently excluding any $C_l$ curve that would not pass $A_1 = 21 \times 10^{-6}$ (the $-1\sigma$ value) in the range $550 < l < 720$. This rules out many low $p$ values for a given $\Omega_\Lambda$. In fact models with $\Omega_\Lambda > 0.5$ do not survive. At $\Omega_\Lambda = 0.5$ we find the allowed window: $0.85 \leq p \leq 1$, $0.003 < k_0 < 0.04 \, \text{h Mpc}^{-1}$. Similarly, when $h = 0.6$, successful models can be found for $0.55 \leq \Omega_\Lambda \leq 0.65$, extending the validity range of the scale invariant model. At $\Omega_\Lambda = 0.65$ the allowed window is $0.80 < p < 0.85$, $0.003 \leq k_0 \leq 0.04 \, \text{h Mpc}^{-1}$. Finally, when $h = 0.7$, $0.65 \leq \Omega_\Lambda \leq 0.75$ is allowed. At $\Omega_\Lambda = 0.72$ we find $0.80 < p < 0.85$, $0.003 \leq k_0 \leq 0.04 \, \text{h Mpc}^{-1}$. These results are also summarized in Fig. 1. Table I contains a few examples, and for one of them the power spectrum and CMB anisotropies are illustrated in Fig. 2 and Fig. 3.

At first sight, the case $p < 1$ is not interesting since it does not extend very much the allowed region for $(h, \Omega_\Lambda)$; good results are obtained for cosmological parameters that are not in conflict with the scale invariant model. The interest of the $p < 1$ steplike spectrum lies in the prediction of specific features, namely:

- a sharp maximum in $P(k)$. The steplike model with $k_0 \approx 0.015 \, \text{h Mpc}^{-1}$ and $p \approx 0.8$ could perfectly explain the form of the cluster spectrum with a peak at $k_0 \approx 0.05 \, \text{h Mpc}^{-1}$ (see Fig. 3).
- large CMB anisotropies. The Saskatoon experiment (Netterfield et al. 1997) indicates a too high first peak that cannot be explained by current flat CDM models (unless $h = 0.2 - 0.3$ is allowed). These measurements might be contaminated by some systematic effects, as indicated by MSAM third flight result (Cheng 1997). However, if the Saskatoon points are confirmed, the flat $\Lambda$+CDM steplike model with $p < 1$ would be a good candidate, since it predicts high anisotropies, without getting in conflict with constraints on $P(k)$, and without requiring $h < 0.5$. For instance, when $p = 0.85$, the $C_l$ values increase by 40% at the scales of secondary peaks, and even of the first peak if $k_0 \leq 0.01 \, \text{h Mpc}^{-1}$. When $h = 0.6$, $\Omega_\Lambda = 0.65$, $p = 0.85$, and $k_0 = 0.01 \, \text{h Mpc}^{-1}$, we find $A_{\text{peak}} = 35 \times 10^{-6}$.

4 CONCLUSIONS AND DISCUSSION

We have compared the CDM+$\Lambda$ cosmological model with a BSI initial spectrum of adiabatic perturbations given by Eq. (3) with recent observational data. The model is determined by four fundamental parameters $\Omega_\Lambda$, $A_+$, $A_- \equiv pA_+$, and $k_0$, (in addition to the Hubble constant $H_0$) out of which one ($A_-$) is fixed by the normalization to the COBE data. The number of observational tests we use is sufficient to rule out many primordial spectra well-motivated by inflationary theories. For instance, to enlarge the allowed $(h, \Omega_\Lambda)$ window, one could think of introducing a tilted or double inflationary spectrum to reconcile observations on large scales (COBE) and small scales ($\sigma_8$). However, there will be a generic lack of power on intermediate scales (bulk velocity, first CMB peak). Moreover, exactly at these scales, there may be an unexpected excess of power. The initial spectrum that we study here allows a significant enlargement of the allowed $(h, \Omega_\Lambda)$ region, especially smaller $\Omega_\Lambda$’s, without supressing power at intermediate scales.

We have found allowed regions in the $(h, \Omega_\Lambda)$ parameter plane for $p$ lying in the region $(0.8 - 1.7)$. These allowed regions are larger than in the case of a flat initial spectrum ($p = 1$). The most interesting, and altogether unexpected, successfull model appears to be that with an inverted step $p < 1$, where the power at intermediate scales is even more enhanced. It appears that this latter case is suitable for the description of the feature in the cluster spectrum found in Einasto et al. (1997a, 1997b, 1997c). The most distinctive feature of the class of models in question is the suppression of the second and higher acoustic (Doppler) peaks in the case $p > 1$, and their enhancement in the opposite case. That is why the CAT CMB experiment appears the most restrictive for the model. So, the exact measurement of $C_l$ for $l \approx 500$, i.e. around the second acoustic (Doppler) peak, will be the crucial test for this model. The forthcoming improvements of CMB anisotropies measurements, especially balloon and satellite experiments, should be able either to rule out this model or to detect its signature in the next ten years.

On the other hand, the increase of the allowed region in the $(h, \Omega_\Lambda)$ plane and the allowed range for $p$ itself are not large. This shows the remarkable robustness of the CDM+$\Lambda$ cosmological model with the simplest inflationary
Table 1. Results of the tests for the different models. For each value of $h$, we show the best model with a flat spectrum ($p = 1$), a step towards large scales ($p > 1$), and a step towards small scales ($p < 1$). For $h = 0.5$ we also give the best model with $\Omega_\Lambda = 0$.

| $h$ | $\Omega_\Lambda$ | $p$ | $k_0$ (h Mpc$^{-1}$) | $\Omega H^2_0 \sigma^8$ | $V_{50}$ (km s$^{-1}$) | $\chi^2$ | $l_{\text{peak}}, A_{\text{peak}}$ | $l_{\text{peak}}, A_{\text{peak}}$ |
|-----|-----------------|-----|---------------------|----------------------|-------------------|---------|------------------|------------------|
| 0.5 | 0.21           | 0.030 | 0.63               | 390                  | 7.6               | 215, 26 × 10$^{-6}$ | 475, 10 × 10$^{-6}$ |
| 0.3 | 1.3            | 0.030 | 0.63               | 345                  | 6.2               | 225, 28 × 10$^{-6}$ | 515, 16 × 10$^{-6}$ |
| 0.5 | 1              | (flat) | 0.54               | 300                  | 3.7               | 235, 29 × 10$^{-6}$ | 555, 22 × 10$^{-6}$ |
| 0.5 | 0.85           | 0.015 | 0.63               | 310                  | 3.1               | 260, 32 × 10$^{-6}$ | 555, 25 × 10$^{-6}$ |
| 0.6 | 0.55           | 1.2 | 0.030 | 0.53               | 330                  | 4.6       | 220, 27 × 10$^{-6}$ | 510, 18 × 10$^{-6}$ |
| 0.6 | 1              | (flat) | 0.54               | 305                  | 3.5               | 225, 27 × 10$^{-6}$ | 530, 21 × 10$^{-6}$ |
| 0.6 | 0.8            | 0.015 | 0.57               | 300                  | 2.6               | 255, 31 × 10$^{-6}$ | 530, 27 × 10$^{-6}$ |
| 0.7 | 0.60           | 1.2 | 0.030 | 0.57               | 340                  | 4.7       | 210, 25 × 10$^{-6}$ | 485, 18 × 10$^{-6}$ |
| 0.7 | 1              | (flat) | 0.57               | 315                  | 4.1               | 215, 26 × 10$^{-6}$ | 500, 20 × 10$^{-6}$ |
| 0.7 | 0.8            | 0.015 | 0.59               | 310                  | 2.7               | 240, 28 × 10$^{-6}$ | 505, 26 × 10$^{-6}$ |

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REFERENCES

Atrio-Barrio F., Einasto J., Gottlöber S., Müller V. and Starobinsky A., 1997, JETP Lett., 66, 397; astro-ph/9708128
Bagla J. S., Padmanabhan T., Narlikar J. V., E-print Archive astro-ph/9511102
Bahcall N. A., Fan X., Cen R., 1997, ApJ, 485, L53
Baker J. et al., in preparation, presented at the Cambridge CMB Conference
Bennett C. L. et al., 1996, ApJ, 464, L1
Czatzanaga E. and Baugh C. M., accepted in MNRAS, E-print Archive astro-ph/9704240
Cheng E. S. et al., submitted, E-print Archive astro-ph/9705041
Eke V. R., Cole S., Frenk C. S., 1996, MNRAS, 282, 263
Einasto J., Einasto M., Gottlöber S., Müller V. et al., 1997a, Nature, 385, 139
Einasto J., Einasto M., Frisch P., Gottlöber S. et al., 1997b, MNRAS, 289, 801
Einasto J., Einasto M., Frisch P., Gottlöber S. et al., 1997c, MNRAS, 289, 813
Eisenstein D. J., Hu W., Silk J., Szalay A. S., E-print Archive astro-ph/9710303
Fan X., Bahcall N. A., Cen R., E-print Archive astro-ph/9709265
Gottlöber S., Müller V., Starobinsky A. A., 1991, Phys. Rev. D, 43, 2510

Guth A. H., Pi S.-Y., 1982, Phys. Rev. Lett., 49, 1110
Hawking S. W., 1982, Phys. Lett. B, 115, 295
Kofman L. A., Gnedin N. Y., Bahcall N. A., 1993, ApJ, 413, 1
Kofman L. A., Linde A. D., 1987, Nucl. Phys. B, 282, 555
Kofman L. A., Linde A. D., Starobinsky A. A., 1985, Phys. Lett. B, 157, 361
Kolff L., Pogosyan D., 1988, Phys. Lett. B, 214, 508
Kofman L., Starobinsky A. A., 1985, Sov. Astron. Lett., 11, 271
Kolatt T., Dekel A., 1997, ApJ, 479, 592
Kolb E., Turner M., 1990, The Early Universe, Addison-Wesley
Leicht E. et al., in preparation
Leicht E. et al., in preparation, presented at the Cambridge CMB Conference
Lesgourgues J., Polarski D., to be published in Phys. Rev. D; astro-ph/9710083
Linde A., 1990, Particle Physics and Inflationary Cosmology, Cambridge University Press
Liddle A. R., White S. D. M., Efstathiou G., Frenk C. S., 1993, MNRAS 262, 1023
Netterfield C. B. et al., 1997, ApJ, 474, 47
Ostriker J. P., Steinhardt P. J., 1995, Nature, 377, 600
Peacock J. A., Dodds S. J., 1994, MNRAS, 267, 1020
Polarski D., Starobinsky A. A., 1992, Nucl. Phys. B, 385, 623
Séljak U., Zaldarriaga M., 1996, ApJ, 469, 7
Silk J., Turne M. S., 1987, Phys. Rev. D, 35, 419
Starobinsky A. A., 1990, Particle Physics and Inflationary Cosmology, Cambridge University Press
Tanaka S. T. et al., 1996, ApJ, 468, L81
Viana P. A., Liddle A. R., 1996, MNRAS, 281, 323
White S. D. M., Efstathiou G., Frenk C. S., 1993, MNRAS 262, 1023
Ying Y. P., Mo H. J., Börner G., E-print Archive astro-ph/9707106

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Figure 1. Primordial power spectrum for \( (p = 0.8, k_0 = 1 \, h \, \text{Mpc}^{-1}) \) and \( (p = 1.7, k_0 = 1 \, h \, \text{Mpc}^{-1}) \). At this stage, the normalisation of each spectrum is arbitrary. It is important to note that in both cases, the maximum is located at the extremity of the upper plateau.

Figure 2. Allowed region in the cosmological parameters plane \((h, \Omega_\Lambda)\). The lower hatched region corresponds to models with \( p \geq 1 \), the upper one to models with \( p \leq 1 \). Inside the intersection, a scale-invariant spectrum \((p = 1)\) is allowed. The steplike model is seen to enlarge significantly the allowed region.

Figure 3. Power spectrum for a few models from Table 1: a model with \( \Omega_\Lambda = 0 \) (in conflict with CAT), and three viable models with \( p > 1 \), \( p = 1 \) and \( p < 1 \).

Figure 4. CMB anisotropies for the same models as in the previous figure. We also plot a few measurements, including Saskatoon (Netterfield 1997) recalibration (Leicht, in preparation) and new preliminary CAT (Baker, in preparation) and OVRO (Leicht, in preparation) results. We have in order of appearance for growing \( l \): COBE (3 points), Tenerife, South Pole, Saskatoon (5 points), MAX (2 points), MSAM, CAT (2 points) and OVRO.

Figure 5. Theoretical power spectrum for \( h = 0.7, \Omega_\Lambda = 0.72, p = 0.75 \) ans \( k_0 = 0.016 \, h \, \text{Mpc}^{-1} \) compared with the power spectrum of rich Abell galaxy clusters, taken from Einasto et al. (1997a) and divided by \( b^2 = 5 \).
\[ k^2 \phi^2(k) \]

\[ p = 0.8 \]
\[ p = 1.7 \]

\[ k_0 \]

\[ k \text{ (h Mpc}^{-1}) \]
$\frac{P(k)}{(h^{-3} \text{Mpc}^3)}$ vs $k$ (h Mpc$^{-1}$)

- $h = 0.5$, $\Omega_A = 0.0$, $p = 2.1$
- $h = 0.5$, $\Omega_A = 0.3$, $p = 1.3$
- $h = 0.5$, $\Omega_A = 0.5$, $p = 1.0$
- $h = 0.7$, $\Omega_A = 0.7$, $p = 0.8$
