PRODUCTION INVENTORY MODEL WITH DETERIORATING ITEMS, TWO RATES OF PRODUCTION COST AND TAKING ACCOUNT OF TIME VALUE OF MONEY

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Abstract. This paper presents production-inventory model for deteriorating items with constant demand under the effect of inflation and time-value of money. Models are developed without shortages while using two production cost functions. In the first case, production cost is divided into two parts: an initial cost which occurs at the beginning of each cycle and is applied to the entire quantity produced during the cycle and a running cost that is incurred as production progresses and is applied to the initial units produced. In the second case, the production cost is incurred at the beginning of the cycle. Numerical examples are given to illustrate the theoretical results and made the sensitivity analysis of parameters on the optimal solutions. The validation of this model's result was coded in Microsoft Visual Basic 6.0

1. Introduction. Traditional inventory models do not take into account the time-value of money and item during storage are assumed to be non-perishable. However, in reality, most products will deteriorate during storage and there is the time-value of money due to opportunity cost. With this in view, a deteriorating inventory model is developed to take into account the time-value of money. Most researches in inventory do not consider the time-value of money. This is unrealistic since the resource of an enterprise depends very much on when it is used and this is highly correlated to the return of investment. Therefore, taking into account the time value of money should be critical especially when investment and forecasting are considered. Inventory problems considering deterioration of items and constant demand were first studied by P.M.Ghare and G.F.Schrader(1963)[4].S.Eilon

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and R.V. Mallaya (1966)[3] extended the model to consider price-dependent demand. The concept of the time-value of money and inflation are commonly applied to investment and forecasting where time is an important factor. J.A. Buzacott (1979)[1] is perhaps the first author to include the concept of inflation in inventory modeling. He developed a minimum cost model for a single item inventory with inflation. According to the study of H.M. Wee (1993)[13], deteriorating items refers to the items that become decayed, damaged, evaporative, expires, invalid, devaluation and so on through time. According to the definition, deteriorating items can be classified into two categories. The first category refers to the items that become decayed, damaged, evaporative, or expires through time, like meat, vegetables, fruit, medicine, flowers, film and so on; the other category refers to the items that lose part or total value through time because of new technology or the introduction of alternatives, like computer chips, mobile phones, fashion and seasonal goods and so on. Both categories have the characteristic of short life cycle. In the first category, the items have a short natural life cycle. After a specific period (such as durability), the natural attributes of the items change and then lose usable value and economic value; In the second category, the items have short market life cycle. After a period of popularity in the market, the items lose the original economic value due to the changes in consumer preference, product upgrading and other reasons. Wee et al. (1999)[14] applied the discounted cash-flow approach to the deterministic inventory model for an item that deteriorates over time at a varying rate and derived optimal production and pricing policies to maximize the net present value of profits over a finite planning horizon. I Moon and S Lee (2000)[6] considered extensions to the EOQ model particularly recognition of the time-value of money and inflation and developed simulation model that can be used for any distribution case, setting out the associated algorithm. N. H. Shah (2006)[10] derived an inventory model by assuming constant rate of deterioration of units in an inventory, time value of money under the conditions of permissible delay in payments. The optimal replenishments and fraction of cycle time are decision variables to minimize the present value of inventory cost over a finite planning horizon. Chung and Chang (2007)[2] developed a deterministic inventory model incorporating a temporary price discount, deteriorating items and time-value of money. S.R. Singh (2009)[11] proposed to derive a deterministic inventory model for a stock with time-varying deterioration rate with a linear trend in demand over a finite planning horizon in the study and assumed that the supplier offers credit limit to the retailers during which there is no interest charged. Widyadana et al. (2010) extended I Moon and S Lee’s model (2000)[6] to examine a production system with a random life cycle, two conditions are discussed: the first is when the product life cycle ends in the production stage and the second is when the product life cycle ends in the non-production stage and developed an algorithm to derive the optimal period time and expected total cost. Barzoki et al. (2011)[7] presented the effect of inflation and time value of money on the EPQ model with rework and the total cost relation involves a exponential function. S. Singh et al. (2011)[12] developed an inventory model for decaying items with selling price dependent demand in which inflationary environment and deterioration rate is taken as two parameter Weibull distribution. Sarkar and Moon (2011)[9] considered a production inventory model for stochastic demand, with the effect of inflation and profit function is derived by using both general distribution of demand and the uniform rectangular distribution demand. A. Roy and G. P. Samanta (2011)[8] reflected the real life problem by allowing unit selling price and purchasing price to be
unequal and continuous production control inventory model for deteriorating items in which two different rates of production are available. Lia et al. (2012) developed an inventory model for exponential deteriorating items under conditions of permissible delay in payments. The objective is to determine the optimal replenishment policies, in order to maximize the systems average profit per unit of time. Factors such as demand, deteriorating rate and so on should be taken into consideration in the deteriorating inventory study. Other factors like price discount, allow shortage or not, inflation and the time-value of money are also important in the study of deteriorating items inventory. By making different combinations of these factors stated above, we can get different inventory models. Time value of money was one of the first special concepts considered as the basic EOQ model. The effect of time value of money is very important and it should reflect the development of inventory models. Since, money tied up in inventories can change its actual value over time; the effect of inflation rate can affect the optimal policies. In this paper, models were developed for an infinite planning horizon which considering time value of money. Closed formulas are obtained in models, where shortage is not allowed for the optimal policies and the corresponding cost. This paper is organized as follows. Section 2 is concerned with assumptions and notations, Section 3 presents the mathematical model for finding the optimal solutions and numerical example. Finally, the paper summary and conclusion in section 4.

2. Assumptions and notations.

2.1. Assumptions. The following assumptions are used to formulate the problem.
1. Initial inventory level is zero and planning horizon is infinite.
2. The demand rate is known to be constant and continuous.
3. Shortages are not allowed.
4. Holding cost per unit per year is known.
5. The lead time is known and constant.
6. Items are produced/purchased and added to the inventory.
7. The item is a single product; it does not interact with any other inventory items.
8. The production rate is always greater than or equal to the sum of the demand rate.
9. The deteriorating items exist in lot size Q.

2.2. Notations.
1. $P$ - Production rate in units per unit time
2. $D$ - Demand rate in units per unit time
3. $Q^*$ - Optimal size of production run
4. $C_p$ - Production cost per unit
5. $\theta$ - Rate of deteriorative.
6. $C_1$ - Production cost incurred at beginning of each cycle and $C_2$ - production cost incurred at production process takes place.
7. $C_h$ - Holding cost per unit/year
8. $C_0$ - Setup cost / ordering cost
9. $T$ - Cycle time
10. $T_1$ - The time during which the stock is building up at a constant rate of $P - D$ units per unit time that is Production time.
11. $R$ - Rate of interest
3. Mathematical model. The objective of the inventory models is to determine the optimal cycle time or the corresponding optimal production quantity in order to minimize the total relevant cost. Consequently, the production time and the maximum inventory level can easily be calculated. Figure 1 represents the EPQ model with constant demand. The inventory on-hand increases with the rate $P - D$, which is the production rate minus consumption rate, until time $T_1$ when the production process stops and the inventory on hand reaches its maximum level $Q_1$. After that point, the inventory level decreases with the consumption rate $D$, until it becomes zero at the end of the cycle $T$, when the production process is resumed again.

![Figure 1. Production Inventory Cycle](image)

During the production stage, the inventory of good items increases due to production but decreases due to demand and deterioration items. Thus, the inventory differential equation is

$$\frac{dI(t)}{dt} + \theta I(t) = P - D ; \ 0 \leq t \leq T_1$$

(1)

The inventory differential equation during the consumption period with no production and subsequently reduction in the inventory level due to deterioration items is given by

$$\frac{dI(t)}{dt} + \theta I(t) = -D ; \ T_1 \leq t \leq T$$

(2)

With the boundary conditions: $I(0) = 0, I(T_1) = Q_1, I(T) = 0$ During the first cycle, the inventory level $I(t)$, at time $t$ is equal to From (1),

$$I(t) = \frac{P - D}{\theta} \left[ 1 - e^{\theta t} \right] ; \ 0 \leq t \leq T_1$$

(3)

From (1),

$$I(t) = \frac{D}{\theta} \left[ 1 - e^{\theta (T-t)} - 1 \right] ; \ T_1 \leq t \leq T$$

(4)

We know that, $I_1(T_1) = I_2(T_1)$ from the equations (3) and (4),

$$I(t) = \frac{P - D}{\theta} \left[ 1 - e^{\theta T_1} \right] = \frac{D}{\theta} \left[ e^{\theta (T-T_1)} - 1 \right]$$

(5)
In order to facilitate analysis, we do an asymptotic analysis for $I_1(t)$. Expanding the exponential functions and neglecting second and higher power of $\theta$ for small value of $\theta$. Therefore,

\[
(P - D) \left[ T_1 - \frac{1}{2} \theta T_1^2 \right] = D \left[ (T - T_1) + \frac{1}{2} \theta (T - T_1)^2 \right]
\]

From Yong He and Ju He [10], $T_1$ was considered as follows,

\[
(P - D) \left[ T_1 - \frac{1}{2} \theta T_1^2 \right] = D(T - T_1) \left[ 1 + \frac{1}{2} \theta (T - T_1) \right]
\]

From Misra [11], $T_1$ was considered as follows,

\[
T_1 = \frac{D}{P - D} (T - T_1) \left[ 1 + \frac{1}{2} \theta (T - T_1) \right]
\]

But in this model, we have considered $T_1$ as follows,

\[
(P - D)T_1 = D(T - T_1), \quad \text{Therefore,} \quad T_1 = \frac{D}{P} T
\]

The maximum inventory is as follows:

\[
I(T_1) = Q_1 \Rightarrow \frac{P - D}{\theta} (1 - e^{-\theta T_1}) = Q_1
\]

Therefore,

\[
Q_1 = (P - D)T_1
\]

3.1. Model - 1 Production inventory model with deteriorating items. The total cost comprise of the sum of the production cost, ordering cost, holding cost, deteriorating cost. They are grouped together after evaluating the above cost individually.

(i) Production Cost / unit time = $P(t)C_P \frac{T_p}{T} = DC_P$ \hspace{1cm} (7)

(ii) Ordering Cost / unit time = $\frac{C_0}{T} = \frac{D}{Q} C_0$ \hspace{1cm} (8)

(iii) Holding Cost / unit time : Holding cost is applicable to both stages of the production cycle, as described by
\[ HC = \frac{C_h}{T} \left[ \int_0^{T_1} I(t) dt + \int_{T_1}^T I(t) dt \right] \]

\[ = \frac{C_h}{T} \left[ \int_0^{T_1} \frac{P - D}{\theta} (1 - e^{-\theta t}) dt + \int_{T_1}^T \frac{D}{\theta} \left( e^{\theta(T-t)} - 1 \right) dt \right] \]

\[ = \frac{C_h}{T} \left[ \frac{P - D}{\theta^2} \left( \theta t + e^{-\theta t} \right)_{0}^{T_1} - \frac{D}{\theta^2} \left( e^{\theta(T-t)} + \theta t \right)_{T_1}^T \right] \]

\[ = \frac{C_h}{T} \left[ \frac{P - D}{\theta^2} \left( \theta T_1 + e^{-\theta T_1} - 1 \right) - \frac{D}{\theta^2} \left( 1 - e^{\theta(T-t)} + \theta(T - T_1) \right) \right] \]

\[ = \frac{C_h}{T} \left[ \frac{P - D}{\theta^2} \left( \frac{\theta^2 T_1^2}{2} \right) + \frac{D}{\theta^2} \left( \frac{\theta^2 (T - T_1)^2}{2} \right) \right] \]

\[ = \frac{C_h}{T} \left[ \frac{P T_1^2}{2T} + DT - 2DT_1 \right] \]

\[ = \frac{TC_h D (P - D)}{2P} \quad \text{from equation (5)} \tag{9} \]

(iv) Deteriorating Cost/unit time: Deteriorating cost, which is applicable to both stages of the production cycle. Therefore,

\[ DC = \frac{C_d}{T} \left[ \int_0^{T_1} \theta I_1(t) dt + \int_{T_1}^T \theta I_2(t) dt \right] \]

\[ = \frac{C_d}{T} \left[ \int_0^{T_1} \theta \frac{P - D}{\theta} (1 - e^{-\theta t}) dt + \int_{T_1}^T \frac{D}{\theta} \left( e^{\theta(T-t)} - 1 \right) dt \right] \]

Expanding the exponential functions and neglecting second and higher power of \( \theta \) for small value of \( \theta \).

\[ DC = \frac{TD\theta C_d (P - D)}{2P} \tag{10} \]

Therefore, Total Cost (TC)

\[ = \text{PurchaseCost} + \text{OrderingCost} + \text{HoldingCost} + \text{DeterioratingCost} + \text{PriceDiscount} \]

\[ = DC_P + \frac{C_0}{T} + \frac{T C_h D (P - D)}{2P} + \frac{TD\theta C_d (P - D)}{2P} \tag{11} \]
Differentiating the Total Cost w.r.t. $T$,\[\frac{\partial}{\partial T}(TC) = -\frac{C_0}{T^2} + \frac{(C_h + \theta C_d)D(P-D)}{2P} = 0\text{ and }\frac{\partial^2}{\partial T^2} = \frac{2C_0}{T^3} > 0\]
Therefore, \[T = \sqrt{\frac{2PC_0}{D(P-D)(C_h + \theta C_d)}}\text{ and }Q = \sqrt{\frac{2DPC_0}{(P-D)(C_h + \theta C_d)}}\] (12)

**Numerical example.** Let us consider the cost parameters $P = 5000$ units, $D = 4500$ units, $C_h = 10$, $C_p = 100$, $C_0 = 100$, $\theta = 0.01$ to 0.10, $C_d = 100$.

**Optimum solution.** Optimum Quantity $Q^* = 904.53$; $T_p = 0.1809$, $T = 0.2010$ , Production cost = 450,000 , Setup cost = 497.49 , Holding cost = 452.27 , Deteriorating cost = 45.22 , Price Discount = 2250 and Total cost = 453244.99

**Table 1.** Variation of Rate of Deteriorating Items with inventory and total Cost.

| $\theta$ | $Q$  | $T$   | $T_1$ | Setup Cost | Holding Cost | Deteriorative Cost | Total Cost  |
|-----------|------|-------|-------|------------|---------------|-------------------|-------------|
| 0.01      | 904.53 | 0.201 | 0.1809 | 497.49     | 452.27         | 45.22             | 450994.99   |
| 0.02      | 866.02 | 0.1924| 0.1732 | 519.61     | 433.01         | 86.60             | 451039.23   |
| 0.03      | 832.05 | 0.1849| 0.1664 | 540.83     | 416.02         | 124.81            | 451081.66   |
| 0.04      | 801.78 | 0.1782| 0.1604 | 561.25     | 400.89         | 160.36            | 451122.5    |
| 0.05      | 774.60 | 0.1721| 0.1549 | 580.95     | 387.30         | 193.65            | 451161.89   |
| 0.06      | 750.00 | 0.1667| 0.1500 | 600.00     | 375.00         | 225.00            | 451200.00   |
| 0.07      | 727.61 | 0.1617| 0.1455 | 618.47     | 363.80         | 254.66            | 451236.93   |
| 0.08      | 707.11 | 0.1571| 0.1414 | 636.4      | 353.55         | 282.84            | 451272.79   |
| 0.09      | 688.25 | 0.1529| 0.1376 | 653.84     | 344.12         | 309.71            | 451307.67   |
| 0.10      | 670.82 | 0.1491| 0.1342 | 670.82     | 335.41         | 335.41            | 451341.64   |

Note: Production cost constant=450,000

From the table 1, a study in the rate of deteriorative items with optimum quantity, cycle time, production time ($T_1$), set up cost, holding cost, deteriorating cost and total cost is observed. It is also seen that when the rate of deteriorative items increase then set up cost, deteriorative cost increases, as there is positive relationship between them and optimum quantity, cycle time, production time, holding cost decreases, there is negative relationship between them.

**Sensitivity analysis.** The total cost functions are the real solution in which the model parameters are assumed to be static values. It is reasonable to study the sensitivity i.e. the effect of making changes in the model parameters over a given optimum solution. It is important to find the effects on different system performance measures, such as cost function, inventory system, etc. For this purpose, sensitivity analysis of various system parameters for models of this research are required to be observed whether the current solutions remain unchanged or infeasible.
Table 2. Effect of Demand and cost parameters on optimal values

| Parameters | Optimum values | Total Cost |
|------------|----------------|------------|
|            | Q   | T₁   | T   |               |
| θ          | 0.01| 904.53| 0.1809| 0.201 | 450994.99 |
|            | 0.02| 866.02| 0.1732| 0.1925| 451039.23 |
|            | 0.03| 832.05| 0.1664| 0.1849| 451081.66 |
|            | 0.04| 801.78| 0.1604| 0.1782| 451122.5  |
|            | 0.05| 774.6 | 0.1549| 0.1721| 451161.89 |
| C₀         | 80  | 809.04| 0.1618| 0.1798| 450889.94 |
|            | 90  | 858.12| 0.1716| 0.1907| 450943.93 |
|            | 100 | 904.53| 0.1809| 0.201  | 450994.99 |
|            | 110 | 948.68| 0.1897| 0.2108| 451043.55 |
|            | 120 | 990.87| 0.1981| 0.2202| 451089.95 |
| Cₕ         | 8   | 1000.00| 0.2000| 0.2222| 450900.00 |
|            | 9   | 948.68| 0.1897| 0.2108| 450948.68 |
|            | 10  | 904.53| 0.1809| 0.201  | 450994.99 |
|            | 11  | 866.02| 0.1732| 0.1925| 451039.23 |
|            | 12  | 832.05| 0.1664| 0.1849| 451081.66 |
| Cₚ         | 80  | 912.87| 0.1826| 0.2029| 360985.9  |
|            | 90  | 908.67| 0.1817| 0.2019| 405990.45 |
|            | 100 | 904.53| 0.1809| 0.201  | 450994.99 |
|            | 110 | 900.45| 0.1801| 0.2001| 495999.5  |
|            | 120 | 896.42| 0.1793| 0.1992| 541003.99 |

Observations.
1. With the increase in rate of deteriorating items, optimum quantity ($Q^*$), production time ($T_1$), cycle time ($T$) decreases but total cost increases.
2. With the increase in setup cost per unit ($C_0$), optimum quantity ($Q^*$), production time ($T_1$), cycle time ($T$) and total cost increases.
3. With the increase in holding cost per unit ($C_h$), optimum quantity ($Q^*$), production time ($T_1$) and cycle time ($T$) decreases but total cost increases.
4. With the increase in production cost, optimum quantity, cycle time and production time decreases but total cost increases.

Special case. If the production system is considered to be ideal that is no deterioration is produced, it means the value of $\theta$ is set to zero. In that case, equations (11) and (12) reduce to the classical economic production quantity model as follows:

\[
\text{TotalCost}(TC) = \text{PurchaseCost} + \text{OrderingCost} + \text{HoldingCost} + \text{ShortageCost} = DC_P + \frac{C_0 P (P - D) T_1^2}{2 TD} + \frac{C_S (P - D)}{2 TP D} (DT - PT_1)^2
\]

\[
T = \sqrt{\frac{2PC_0(C_h + C_S)}{(P - D)DC_hC_S}}, \quad \text{Therefore } Q = \sqrt{\frac{2PC_0(C_h + C_S)}{(P - D)C_hC_S}}
\]

3.2. Model - 2 Production inventory model with present value money. Case(i). Usually, in the analysis of an inventory system, normally three types of costs are considered. These are production cost, inventory carrying cost and
holding cost. But purchasing cost is constant. This is not so if we considered the value of money, hence this cost will be included in the analysis. In the first case, the production cost \( C_p \), has two components. The cost \( C_1 \), that is incurred at the beginning of each cycle and is applied to the total quantity produced during the cycle and a second cost \( C_2 \) which is incurred as the production process takes place. To incorporate the effect of time-value of money into the equations, the difference between the interest rate and inflation rate is calculated as \( r \), the inflation free or real interest rate representing the time value of money. That is, \( r = R - f \).

(i) Ordering Cost \( = C_0 \) 

(ii) Production Cost \( = QC_1 + C_2 P \int_0^{T_1} e^{-rT} dt \) 

\[
DTC_1 + \frac{PC_2}{r} \left(1 - e^{-rT} \right) \text{ where } C_P = C_1 + C_2
\] 

(iii) Holding Cost: Holding cost is applicable to both stages of the production cycle, as described by

\[
HC = C_h \left[ \int_0^{T_1} I(t)e^{-rt} dt + \int_{T_1}^{T} I(t)e^{-rt} dt \right]
\]

\[
= C_h \left[ \int_0^{T_1} \frac{P-D}{\theta} \left(1 - e^{-\theta t}\right) e^{-rt} dt + \int_{T_1}^{T} \frac{D}{\theta} \left(e^{\theta(T-t)} - 1\right) e^{-rt} dt \right]
\]

\[
= C_h \left[ \int_0^{T_1} \frac{P-D}{\theta} \left(e^{-rt} - e^{-(r+\theta)t}\right) dt + \int_{T_1}^{T} \frac{D}{\theta} \left(e^{\theta(T-(r+\theta)t)} - e^{-rt}\right) dt \right]
\]

\[
= C_h \left[ \frac{P-D}{\theta} \left\{ \frac{e^{-(r+\theta)t} - e^{-rt}}{r+\theta} \right\} \left[ \begin{array}{c} _0^T_1 \frac{e^{\theta T-(r+\theta)t}}{r+\theta} \\ \frac{D}{\theta} \left( \frac{e^{-rt}}{r} - \frac{e^{\theta T-(r+\theta)t}}{r+\theta} \right) \end{array} \right] \right]
\]

\[
= \frac{C_h}{r\theta(r+\theta)} \left[ \begin{array}{c} (P-D) \left\{ re^{-(r+\theta)T_1} - (r+\theta)e^{-rT_1} + \theta \right\} \\ +D \left\{ (r+\theta)e^{-rT} - re^{rT} \right\} \\ -(r+\theta)e^{-rT_1} + re^{\theta T-(r+\theta)T_1} \end{array} \right]
\]

\[
= \frac{C_h}{r\theta(r+\theta)} \left[ \begin{array}{c} \Pr \left\{ e^{-(r+\theta)T_1} - e^{-rT_1} \right\} + P\theta \left(1 - e^{-rT_1} \right) \\ -D\theta \left(1 - e^{-rT} \right) + Dr \left(e^{\theta T} - 1\right) e^{-(r+\theta)T_1} \end{array} \right]
\]

Substitute the value of and simplify.

\[
HC = \frac{C_h}{r\theta(r+\theta)} \left[ \begin{array}{c} \Pr \left( e^{-(r+\theta)DT} - e^{-rDT} \right) + P\theta \left(1 - e^{-rDT} \right) \\ -D\theta \left(1 - e^{-rT} \right) + Dr \left(e^{\theta T} - 1\right) e^{-(r+\theta)DT} \end{array} \right]
\]
(iv) Deteriorating Cost/unit time: Deteriorating cost, which is applicable to both stages of the production cycle. Therefore,

\[
DC = \theta C_d \left[ \int_0^{T_1} I_1(t)e^{-rt}dt + \int_{T_1}^T I_2(t)e^{-rt}dt \right]
\]

\[
= \frac{\theta C_d}{r \theta (r + \theta)} \left[ Pr \left( \frac{e^{-(r+\theta)DT}}{1-e^{-rT}} - \frac{e^{-rDT}}{1-e^{-rT}} \right) + P \theta \left( 1 - \frac{e^{-(r+\theta)DT}}{1-e^{-rT}} \right) - D \theta \left( 1 - \frac{e^{-rT}}{1-e^{-rT}} \right) + Dr \left( e^{\theta T} - 1 \right) e^{-r(T+\theta)} DT \right]
\]

Total Cost TC (T) = Purchase Cost + Ordering Cost + Holding Cost + Deteriorating Cost.

Assuming continuous compounding and a constant setup cost \(C_0\), the present value of the total cost of the inventory system for the first cycle, TC (\(T\)), can be expressed as,

\[
TC(T) = C_0 + QC_1 + \frac{PC_2}{r} \left( 1 - \frac{e^{-rDT}}{1-e^{-rT}} \right)
+ \frac{(C_h + \theta C_d)}{r \theta (r + \theta)} \left[ Pr \left( \frac{e^{-(r+\theta)DT}}{1-e^{-rT}} - \frac{e^{-rDT}}{1-e^{-rT}} \right) + P \theta \left( 1 - \frac{e^{-(r+\theta)DT}}{1-e^{-rT}} \right) - D \theta \left( 1 - \frac{e^{-rT}}{1-e^{-rT}} \right) + Dr \left( e^{\theta T} - 1 \right) e^{-r(T+\theta)} DT \right]
\]

According to Figure 2, the present value TC (\(T\)), for the first period is repeated at the start of each of the subsequent cycles. The present value of the total cost for \(N\) cycles, TC (\(T\)) can be calculated as follows:

\[
TC(T) = P \left( 1 + e^{-rT} + e^{-2rT} + ... + e^{-(N-1)rT} \right)
\]

\[\text{Figure 2. Cash Flow Diagram}\]

For an infinite planning horizon, \(N \rightarrow \infty\) and TC (\(T\)) can be expressed as

\[
TC(T) = P \left( \frac{1}{1-e^{-rT}} \right)
= \frac{C_0 + DTC_1}{1 - e^{-rT}} + \frac{PC_2}{r} \left( 1 - \frac{e^{-rDT}}{1-e^{-rT}} \right)
+ \frac{(C_h + \theta C_d)}{r \theta (r + \theta)} \left[ Pr \left( \frac{e^{-(r+\theta)DT}}{1-e^{-rT}} - \frac{e^{-rDT}}{1-e^{-rT}} \right) + P \theta \left( 1 - \frac{e^{-(r+\theta)DT}}{1-e^{-rT}} \right) - D \theta \left( 1 - \frac{e^{-rT}}{1-e^{-rT}} \right) + Dr \left( e^{\theta T} - 1 \right) e^{-r(T+\theta)} DT \right]
\]

The present value of the total cost function TC (\(T\)) is a function of the length of the cycle \(T\). Taking the first derivative of TC (\(T\)) w.r.t. \(T\) and equating to zero, and using a common denominator and after some simplifications, the equation can
be written as follows:

\[
\frac{dT}{dt}(TC) = \left(1 - e^{-rT}\right) DC_1 - re^{-rT} (C_0 + DTC_1) + \frac{PC_2}{r} \left\{ \frac{(1 - e^{-rT}) \left( \frac{P}{D} e^{-rDT} \right)}{1 - e^{-rT}} \right\} + e^{-rT} \left( \frac{P}{D} e^{-rDT} \right) + \frac{C_h + \theta C_d}{\frac{r}{P}(r + \theta)} \left\{ \frac{(e^{rT} - 1) \left( \frac{D}{P} e^{-rDT} \right)}{r(1 - e^{-rDT})} \right\} = 0
\]

Multiply both sides by \(e^{rT}\) and simplify

\[
\left( e^{rT} - 1 \right) DC_1 - r(C_0 + DTC_1) + \frac{PC_2}{r} \left\{ \frac{(e^{rT} - 1) \left( \frac{D}{P} e^{-rDT} \right)}{r(1 - e^{-rDT})} \right\} + \frac{C_h + \theta C_d}{\frac{r}{P}(r + \theta)} \left\{ \frac{(e^{rT} - 1) \left( \frac{D}{P} e^{-rDT} \right)}{r(1 - e^{-rDT})} \right\} = rC_0
\]
The optimal cycle time can be defined as

\[
T^* = \sqrt{\frac{2PC_0}{PD_rC_1 + DC_2r(P - D)(rC_2 + C_h + \theta C_d)}}
\]  (18)

Applying this expression and ignoring the cubit and higher terms, which are very close to zero, the above equation can be reduced to,

\[
\frac{DC_1}{2}r^2T^2 + \frac{PC_2}{r^2} \left( \frac{PDr^3T^2 - r^3D^2T^2}{2P^2} \right) + \frac{(C_h + \theta C_d)}{r(r + \theta)} \left[ \begin{array}{c}
\Pr \left\{ \frac{r(r+\theta)^2D^2T^2}{2P^2} - \frac{r^3D^2T^2}{2P^2} \right. \\
+ \frac{r^2(r+\theta)DT^2}{2P^2} + \frac{r^3DT^2}{2P^2} \\
\left. + Dr \left\{ \frac{-r\theta(r+\theta)DT^2}{2P} + \frac{r^2T^2}{2} + \frac{r^2\theta^2}{2} \right. \right. \
\left. \left. - D^2\theta(r+\theta) \frac{2P}{2P} + \frac{Dr\theta}{2} \right) \right) \right] = rC_0
\]

\[
r^2D \left[ \frac{PC_1 + C_2(P - D)}{2P} \right] T^2 + \frac{C_h + \theta C_d}{\theta} \left[ \frac{r\theta D^2}{2P} - \frac{Dr\theta}{2} \right] T^2 = rC_0
\]

\[
r^2D \left[ \frac{PC_1 + C_2(P - D)}{2P} \right] T^2 + (C_h + \theta C_d) \left[ \frac{rD^2}{2P} - \frac{D^2r}{P} + \frac{Dr}{2} \right] T^2 = rC_0
\]

\[
rD \left( \frac{PC_1 + C_2(P - D)}{2P} \right) T^2 + (C_h + \theta C_d) \left[ \frac{-D^2}{2P} + \frac{D}{2} \right] T^2 = C_0
\]

\[
rD(\frac{PC_1 + C_2(P - D)}{2P}) T^2 + (C_h + \theta C_d)D(P - D) T^2 = C_0
\]

\[
[Dr(\frac{PC_1 + C_2(P - D)}{2P}) + (C_h + \theta C_d)D(P - D)] T^2 = 2PC_0
\]

\[
T^2 [PD_rC_1 + DC_2r(P - D) + D(P - D)(C_h + \theta C_d)] = 2PC_0
\]

Therefore, \( T^2 = \frac{2PC_0}{PD_rC_1 + DC_2r(P - D) + (C_h + \theta C_d)D(P - D)} \)
and $\frac{d^2}{dT^2} (TC) > 0$. The corresponding production quantity is

$$Q^* = \sqrt{\frac{2PDC_0}{PrC_1 + (P - D)(rC_2 + C_h + \theta C_p)}}$$

(19)

**Numerical example.** Let us consider the cost parameters $P = 5000$ units, $D = 4500$ units, $C_h = 10$, $C_1 = 10$, $C_2 = 90$, $C_0 = 100$, $\theta = 0.01$ to $0.10$, $r = 0.01$ to $0.10$.

**Optimum solution.** Optimum Quantity $Q^* = 835.27$; $T_1 = 0.1671$, $T = 0.1856$, $Q_1 = 83.53$ Production cost = 449661.72, Setup cost = 538.75, Holding cost = 417.14, Deteriorating cost = 41.71, and Total cost = 450659.32.

**Table 3.** Variation of Rate of Deteriorating Items with inventory and total Cost

| $\theta$ | $T$   | $Q$   | Setup cost | Production cost | Holding cost | Deteriorating cost | Total cost |
|---------|-------|-------|------------|-----------------|--------------|--------------------|------------|
| 0.01    | 0.1856| 835.27| 538.75     | 449661.72       | 417.14       | 41.71              | 450659.32 |
| 0.02    | 0.1788| 804.66| 559.24     | 449674.11       | 401.87       | 80.37              | 450715.6  |
| 0.03    | 0.1727| 777.19| 579.01     | 449685.24       | 388.17       | 116.45             | 450768.87 |
| 0.04    | 0.1672| 752.35| 598.12     | 449695.3        | 375.78       | 150.31             | 450819.51 |
| 0.05    | 0.1622| 729.76| 616.64     | 449704.45       | 364.5        | 182.25             | 450867.85 |
| 0.06    | 0.1576| 709.08| 634.63     | 449712.82       | 354.19       | 212.51             | 450914.15 |
| 0.07    | 0.1534| 690.06| 652.11     | 449720.52       | 344.7        | 241.29             | 450958.62 |
| 0.08    | 0.1494| 672.5 | 669.14     | 449727.64       | 335.93       | 268.75             | 451001.46 |
| 0.09    | 0.1458| 656.22| 685.75     | 449734.23       | 327.81       | 295.02             | 451042.81 |
| 0.1     | 0.1424| 641.06| 701.96     | 449740.37       | 320.24       | 320.24             | 451082.81 |

From the table 3, it is observed that a study of rate of deteriorative items with cycle time, optimum quantity, setup cost, production cost, holding cost, deteriorating cost and total cost. When the rate of deteriorative items increases then setup cost, production cost, deteriorating cost and total cost also increases, then there is positive relation between them. When the rate of deteriorative items increases then the cycle time, optimum quantity, holding cost decreases then there is negative relationship between them.

**Table 4.** Variation of rate of interest with inventory and total Cost

| $r$ | $T$   | $Q$   | Setup cost | Production cost | Holding cost | Deteriorating cost | Total cost |
|-----|-------|-------|------------|-----------------|--------------|--------------------|------------|
| 0.01| 0.1856| 835.27| 538.75     | 449661.72       | 417.14       | 41.71              | 450659.32 |
| 0.02| 0.1733| 779.81| 577.06     | 449368.35       | 389.05       | 38.91              | 450373.39 |
| 0.03| 0.1631| 734.11| 612.98     | 449108.05       | 365.92       | 36.59              | 450123.55 |
| 0.04| 0.1546| 695.61| 646.92     | 448873.11       | 346.44       | 34.64              | 449901.12 |
| 0.05| 0.1472| 662.59| 679.15     | 448658.26       | 329.75       | 32.97              | 449700.14 |
| 0.06| 0.1409| 633.87| 709.93     | 448459.71       | 315.24       | 31.52              | 449516.4  |
| 0.07| 0.1352| 608.58| 739.42     | 448274.67       | 302.47       | 30.25              | 449346.81 |
| 0.08| 0.1302| 586.1 | 767.79     | 448101.04       | 291.11       | 29.11              | 449189.06 |
| 0.09| 0.1258| 565.94| 795.14     | 447937.16       | 280.94       | 28.09              | 449041.33 |
| 0.1  | 0.1217| 547.72| 821.58     | 447781.72       | 271.75       | 27.17              | 448902.23 |
From the table 4, a study of rate of interest with cycle time, optimum quantity, setup cost, production cost, holding cost, deteriorating cost and total cost. When the rate of interest increases then setup cost increases, then there is positive relation between them. When the rate of interest items increases then the cycle time, optimum quantity, production cost, holding cost, deteriorating cost and total cost decreases then there is negative relationship between them.

**Sensitivity analysis.** The total cost functions are the real solution in which the model parameters are assumed to be static values. It is reasonable to study the sensitivity i.e. the effect of making changes in the model parameters over a given optimum solution. It is important to find the effects on different system performance measures, such as cost function, inventory system, etc. For this purpose, sensitivity analysis of various system parameters for the models of this research are required to observe whether the current solutions remain unchanged, the current solutions become infeasible, etc.

**Table 5.** Effect of Demand and cost parameters on optimal values

| Parameters | T   | T₁  | Q   | Q₁  | Total Cost    |
|------------|-----|-----|-----|-----|--------------|
| θ          | 0.01| 0.1856 | 0.167 | 835.27 | 83.53 | 450659.32 |
|            | 0.02| 0.1788 | 0.1609 | 804.66 | 80.47 | 450715.6  |
|            | 0.03| 0.1727 | 0.1554 | 777.19 | 77.72 | 450768.87 |
|            | 0.04| 0.1672 | 0.1504 | 752.35 | 75.23 | 450819.51 |
|            | 0.05| 0.1622 | 0.146  | 729.75 | 72.98 | 450867.85 |

**Observations.**
1. With the increase in the rate of deteriorating items (θ), cycle time, production time and optimum quantity, the maximum inventory decreases but total cost increases.
2. With the increase in the rate of interest (r), cycle time, production time and optimum quantity, the maximum production and total cost decreases.
3. With the increase in setup cost per unit (C₀), cycle time, production time, optimum quantity, the maximum time and total cost increases.
4. With the increase in holding cost per unit (Cₙ), cycle time, production time, optimum quantity, the maximum inventory and total cost increases.
5. Similarly, other parameters C₁, C₂ can also be observed from the table 5.

**Note.** The above formulae can be reduced to the standard equation of the optimal production quantity and cycle time when the value of C₁, C₂, r, and θ are equal to zero, then

\[ T^* = \sqrt{\frac{2PC_0}{C_nD(P-D)}} \] and \[ Q^* = \sqrt{\frac{2DPC_0}{(P-D)C_n}} \]

**Case (ii).** In the second case, the setup cost and the production cost are assumed to be incurred at the beginning of each cycle and the inventory holding cost is incurred continuously during the period T. To incorporate the effect of time value of money into the equations, the difference between the interest rate and inflation
rate is calculated as \( r \), the inflation free or real interest rate representing the time value of money. That is, \( r = R - f \).

(i) Production Cost = \( QC_p = DTC_p \) \hspace{1cm} \text{(20)}

(ii) Ordering Cost = \( C_0 \) \hspace{1cm} \text{(21)}

(iii) Holding Cost : Holding cost is applicable to both stages of the production cycle, as described by

\[
HC = C_h \left[ \int_0^{T_1} I(t)e^{-rt}dt + \int_{T_1}^T I(t)e^{-rt}dt \right]
\]

\[
= C_h \left[ \int_0^{T_1} \frac{P - D}{\theta} (1 - e^{-\theta t}) e^{-rt}dt + \int_{T_1}^T \frac{D}{\theta} \left( e^{\theta(T-t)} - 1 \right) e^{-rt}dt \right]
\]

\[
= C_h \left[ \int_0^{T_1} \frac{P - D}{\theta} \left( e^{-rt} - e^{-(r+\theta)t} \right) dt + \int_{T_1}^T \frac{D}{\theta} \left( e^{\theta(T-(r+\theta)t)} - e^{-rt} \right) dt \right]
\]

\[
= C_h \left[ \frac{P - D}{\theta} \left( \frac{e^{-(r+\theta)t}}{r + \theta} - \frac{e^{-rt}}{r} \right) \bigg|_0^{T_1} + \frac{D}{\theta} \left( \frac{e^{-rt}}{r} - \frac{e^{\theta(T-(r+\theta)t)}}{r + \theta} \right) \bigg|_0^{T} \right]
\]
some simplifications, the cycle (\( T \)) of the production cycle. Therefore,

\[
TC = C_0 + DTCP + \frac{Ch + \theta C_d}{r\theta(r + \theta)} \left[ Pr \left( e^{-(r+\theta)DT} - e^{-rDT} \right) + P\theta \left( 1 - e^{-rDT} \right) \right] (24)
\]

Assuming continuous compounding and a constant setup cost \( C_0 \), the present value of the total cost of the inventory system for the first cycle, \( TC(T) \), can be expressed as,

\[
TC(T) = P \left( \frac{1}{1 - e^{-rT}} \right)
\]

The present value of the total cost:

\[
\frac{C_0 + DTCP}{1 - e^{-rT}} + \frac{Ch + \theta C_d}{r\theta(r + \theta)} \left[ Pr \left( e^{-(r+\theta)DT} - e^{-rDT} \right) + P\theta \left( 1 - e^{-rDT} \right) - D\theta \right] (25)
\]

The present value of the total cost function \( TC(T) \) is a function of the length of the cycle \( T \). Taking the first derivative of \( TC(T) \) w.r.t. \( T \) and equating to zero, and using a common denominator and same procedure used in case (i) and after some simplifications,

\[
T^* = \sqrt{\frac{2PC_0}{PDrCP + (Ch + \theta C_d)D(P - D)}} (26)
\]
and \( \frac{\partial^2}{\partial T^2} > 0 \). The corresponding production quantity is

\[
Q^* = \sqrt{\frac{2DPC_0}{PrC_p + (P - D)(C_h + \theta C_p)}}
\]

(27)

**Numerical example.** Let us consider the cost parameters \( P = 5000 \) units, \( D = 4500 \) units, \( C_h = 10, C_p = 100, C_0 = 100, \theta = 0.01 \) to 0.10, \( r = 0.01 \) to 0.10.

**Optimum solution.** Optimum Quantity \( Q^* = 654.65; T_1 = 0.1309, T = 0.1455, Q_1 = 65.46 \), Production cost = 450,000, Setup cost = 687.38, Holding cost = 327.02, Deteriorating cost = 32.70, and Total cost = 451047.11.

**Table 6.** Variation of Rate of Deteriorating Items with inventory and total Cost

| \( \theta \) | T    | Q    | Setup cost | Production cost | Holding cost | Deteriorating cost | Total cost    |
|----------|------|------|------------|-----------------|--------------|-------------------|---------------|
| .01      | 0.1455 | 654.65 | 687.38    | 450000         | 327.02       | 32.7              | 451047.11     |
| .02      | 0.1421 | 639.6  | 703.56    | 450000         | 319.51       | 63.9              | 451086.98     |
| .03      | 0.1391 | 625.54 | 719.37    | 450000         | 312.5        | 93.75             | 451125.62     |
| .04      | 0.1361 | 612.37 | 734.85    | 450000         | 305.92       | 122.37            | 451163.38     |
| .05      | 0.1333 | 600    | 750       | 450000         | 300          | 150               | 451199.62     |
| .06      | 0.1307 | 588.35 | 764.85    | 450000         | 293.93       | 176.36            | 451235.14     |
| .07      | 0.1283 | 577.35 | 779.42    | 450000         | 288.44       | 201.91            | 451269.77     |
| .08      | 0.126  | 566.95 | 793.72    | 450000         | 283.25       | 226.6             | 451303.57     |
| .09      | 0.1238 | 557.09 | 807.77    | 450000         | 278.32       | 250.49            | 451336.59     |
| .10      | 0.1217 | 547.72 | 821.58    | 450000         | 273.65       | 273.65            | 451368.88     |

From the table 6 is a study of the rate of deteriorative items with cycle time, optimum quantity, setup cost, production cost, holding cost, deteriorating cost and total cost. When the rate of deteriorative items increases, the setup cost, deteriorating cost and total cost increases, as a result there is positive relation between them. When the rate of deteriorative items increases, the cycle time, optimum quantity and holding cost decreases that results in a negative relationship between them.

The Table 7 is a study of the rate of interest with cycle time, optimum quantity, setup cost, production cost, holding cost, deteriorating cost and total cost. When the rate of interest increases, the setup cost and total cost increases. Then there is positive relation between them. When the rate of interest items increases, the cycle time, optimum quantity, production cost, holding cost and deteriorating cost decreases. Then there is negative relationship between them.

**Sensitivity analysis.** The total cost functions are the real solutions in which the model parameters are assumed to be static values. It is reasonable to study the sensitivity i.e. the effect of making changes in the model parameters over a given optimum solution. It is important to find the effects on different system performance measures, such as cost function, inventory system, etc. For this purpose, sensitivity
Table 7. Variation of rate of interest with inventory and total Cost

| r   | T   | Q   | Setup cost | Production cost | Holding cost | Deteriorating cost | Total cost  |
|-----|-----|-----|------------|----------------|--------------|-------------------|-------------|
| .01 | 0.1455 | 654.65 | 687.38 | 450000 | 327.02 | 32.7 | 451047.1 |
| .02 | 0.1197 | 538.82 | 835.16 | 450000 | 268.99 | 26.9 | 451131.1 |
| .03 | 0.1041 | 468.52 | 960.47 | 450000 | 233.8 | 23.38 | 451217.7 |
| .04 | 0.0933 | 420.08 | 1071.21 | 450000 | 209.54 | 20.95 | 451301.7 |
| .05 | 0.0853 | 384.11 | 1171.54 | 450000 | 191.54 | 19.15 | 451382.2 |
| .06 | 0.0791 | 356.03 | 1263.92 | 450000 | 177.48 | 1.75 | 451459.2 |
| .07 | 0.0741 | 333.33 | 1350 | 450000 | 166.12 | 16.61 | 451532.7 |
| .08 | 0.0699 | 314.48 | 1430.91 | 450000 | 156.68 | 15.67 | 451603.3 |
| .09 | 0.0663 | 298.51 | 1507.48 | 450000 | 148.69 | 14.87 | 451671 |
| .10 | 0.0633 | 284.74 | 1580.35 | 450000 | 141.8 | 14.18 | 451736.3 |

Analysis of various system parameters for the models of this research are required to be observed whether the current solutions remain unchanged, the current solutions become infeasible, etc.

Observations.

1. With the increase in the rate of deteriorating items (θ), cycle time, production time and optimum quantity, the maximum inventory decreases but total cost increases.
2. With the increase in the rate of interest (r), cycle time, production time and optimum quantity, the maximum production decreases but total cost increases.
3. With the increase in setup cost per unit (C₀), cycle time, production time, optimum quantity, the maximum time and total cost increases.
4. With the increase in holding cost per unit (Cₕ), cycle time, production time, optimum quantity, the maximum inventory decreases but total cost increases.
5. Similarly, other parameters Cₚ, C₇, can also be observed from the table 8.

Note. The above formulae can be reduced to the standard equations of optimal production quantity and cycle time when the value of r and θ are equal to zero.

\[ T^* = \sqrt{\frac{2PC₀}{CₕD(P - D)}} \text{ and } Q^* = \sqrt{\frac{2DPC₀}{(P - D)Cₕ}} \]

4. Conclusion. The deterministic inventory control problem was considered for the determination of optimal production quantities for items with constant demand rate, while considering the effect of time value of money. Two different models are developed. In the first model, deterioration is considered and in the second model, deterioration with time value of money is considered. In the second model, two different production cost functions are considered. In the first case, the production cost is incurred at the beginning of the cycle, while in the second, the production cost consists of two parts: an initial cost, which is incurred at the beginning of the cycle and is applied to the entire quantity produced during the cycle and a running cost that is incurred as production progresses and is applied to the individual units produced. Closed formulae were obtained for the optimal cycle time and the corresponding optimal production quantity for the models without shortage. Numerical
Table 8. Effect of Demand and cost parameters on optimal values

| Parameters | Optimum values | | | | |
|------------|----------------|----------------|----------------|----------------|----------------|
|            | T | T1 | Q | Q1 | Total cost |
| \( \theta \) | 0.01 | 0.1455 | 0.1309 | 654.65 | 65.46 | 451047.1 |
|            | 0.02 | 0.1421 | 0.1279 | 639.6 | 63.96 | 451087 |
|            | 0.03 | 0.139 | 0.1251 | 625.54 | 62.55 | 451125.6 |
|            | 0.04 | 0.1361 | 0.1225 | 612.37 | 61.24 | 451163.1 |
|            | 0.05 | 0.1333 | 0.12 | 600 | 60 | 451199.6 |
| r | 0.01 | 0.1455 | 0.1309 | 654.65 | 65.46 | 451047.1 |
| | 0.02 | 0.1197 | 0.1078 | 538.81 | 53.88 | 451131.1 |
| | 0.03 | 0.1041 | 0.0987 | 468.52 | 46.85 | 451217.7 |
| | 0.04 | 0.0933 | 0.084 | 420.08 | 42.01 | 451301.7 |
| | 0.05 | 0.0854 | 0.0768 | 384.11 | 38.41 | 451382.2 |
| \( C_0 \) | 80 | 0.1301 | 0.1171 | 585.54 | 58.55 | 450938.6 |
| | 90 | 0.138 | 0.1242 | 621.06 | 62.11 | 450993.4 |
| | 100 | 0.1455 | 0.1309 | 654.65 | 65.46 | 451047.1 |
| | 110 | 0.1526 | 0.1373 | 686.61 | 68.66 | 451098.2 |
| | 120 | 0.1594 | 0.1434 | 717.14 | 71.71 | 451147 |
| \( C_h \) | 8 | 0.1529 | 0.1376 | 688.25 | 68.82 | 450963.3 |
| | 9 | 0.1491 | 0.1342 | 670.82 | 67.08 | 451005.9 |
| | 10 | 0.1455 | 0.1309 | 654.65 | 65.46 | 451047.1 |
| | 11 | 0.1421 | 0.1279 | 639.6 | 63.96 | 451087 |
| | 12 | 0.139 | 0.1251 | 625.54 | 62.55 | 451125.6 |
| \( C_p \) | 80 | 0.1529 | 0.1376 | 688.25 | 68.82 | 361032 |
| | 90 | 0.1491 | 0.1342 | 670.82 | 67.08 | 406039.4 |
| | 100 | 0.1455 | 0.1309 | 654.65 | 65.46 | 451047.1 |
| | 110 | 0.1421 | 0.1279 | 639.6 | 63.96 | 496055 |
| | 120 | 0.139 | 0.1251 | 625.54 | 62.55 | 541063.1 |
| \( C_d \) | 80 | 0.1462 | 0.1316 | 657.79 | 65.78 | 451039 |
| | 90 | 0.1458 | 0.1312 | 656.22 | 65.62 | 451043.1 |
| | 100 | 0.1455 | 0.1309 | 654.65 | 65.46 | 451047.1 |
| | 110 | 0.1451 | 0.1306 | 653.1 | 65.31 | 451051.2 |
| | 120 | 0.1448 | 0.1303 | 651.56 | 65.16 | 451055.2 |

Examples were presented and sensitivity analysis was performed. This research can be extended as follows:

1. In developing the models, only one concept was introduced at a time, along with time value of money. One may investigate models with a combination of several concepts and determine the optimal policies for these cases.
2. Another extension to this research could be an attempt to prove the convexity of the total cost function where the interest rate is included in the total cost function.
3. The models developed in this research were considered for a single item. One may relax this assumption and consider models with multiple items.
4. The production rate in all the models was constant and the demand rate was either constant or increasing linearly over time. Other extension to this research could be to consider probabilistic demand or production rate.
The proposed model can assist the manufacturer and retailer in accurately determining the optimal quantity, cycle time and inventory total cost. Moreover, the proposed inventory model can be used in inventory control of certain items such as food items, fashionable commodities, stationery stores and others.

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