Ultrasound generated by laser in a coated cylinder

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Abstract. An alternative theoretical solution is proposed to predict ultrasound excited by a laser line source in a homogeneous, isotropic cylinder coated with a homogeneous and isotropic material. The spectrum and transient displacement are in agreement to author’s previous model for the cylinder without coating. Numerical results are obtained for copper on aluminium and vice versa. It is found for the surface waves that the high frequency component travels faster for hard coatings with velocities faster than that of the substrate, and vice versa for soft coatings.

1. Introduction

The inspection of coatings is important for surface engineering, where it is widely used to improve the performance of materials. To access information about a coating by surface waves, ultrasound microscope is often used to obtain the reflection coefficient through the measurement of V(z) curve [1]. It requires the sample to be immersed in water, and thus limits its application on site. Moreover it is difficult to measure a curved sample such as a cylinder. Fortunately, the laser ultrasonic technique, in which ultrasounds are generated and detected by lasers, can measure the acoustic displacement of a curved surface at a distance [2]. Recently, coatings on a plate structure have been studied [3, 4]. However, little work is available on the coated cylinder. The objective here is to develop a model to predict laser-excited surface waves on a coated cylinder.

2. Governing equations

Consider acoustic wave propagation in a coated cylinder of diameter 2a with infinite length. The coating is homogeneous with a thickness h, density ρ and Lamé coefficients λ and μ. The rod substrate is homogeneous with diameter 2b (b=a-h), density ρs and Lamé coefficients λs and μs. A system of cylindrical coordinates (r, θ, z) is chosen. The displacement U and the stress vector σ of the coating are sought in the form:

\[ U(r, θ, t) = u(r) \exp[jω(t - paθ)], \]
\[ \Sigma(r, θ, t) = σ(r) \exp[jω(t - paθ)]. \]

ω being the angular frequency, and p the circumferential slowness. Instead of searching the explicit solution of the authors’ recent publication on bi-layered cylinder [5], here the state vector approach [6] is applied to obtain the matrix differential equation as:

\[ \partial \mathbf{B}(r)/\partial r = iω\mathbf{A}(r)\mathbf{B}(r) \]

where the state vector \( \mathbf{B} = (u, \sigma_θ)^T \) is the displacement –traction and \( \mathbf{A} \) is the system matrix depending on the elastic properties of the layered medium, and on p and ω. For an isotropic and
layered cylinder, the state vector and the system matrix for P-SV coupled waves are [6]:

\[
B(r) = \begin{pmatrix}
    i \omega \mu & i \omega \sigma & \sigma & \sigma
\end{pmatrix}^T
\]

(3)

and

\[
A(r) = \begin{pmatrix}
    -r^{-1} \lambda (\lambda + 2 \mu)^{-1} & par^{-1} \lambda (\lambda + 2 \mu)^{-1} & \left(\lambda + 2 \mu\right)^{-1} & 0 \\
    pa/r & r^{-1} & 0 & \mu^{-1} \\
    \rho & -pa \xi/r^2 & -r^{-1} \lambda \left(\lambda + 2 \mu\right)^{-1} & pa/r \\
    pa \xi/r^2 & \rho - p^2 a^2 \xi/r^2 & \left(\lambda + 2 \mu\right)^{-1} & -2/r
\end{pmatrix}
\]

(4)

with

\[
\xi = 4 \mu \frac{\lambda + \mu}{\lambda + 2 \mu}
\]

(5)

3. Boundary conditions and solution

The matrix \( M(r, r_0) \) is the fundamental solution of Eq. (2) with the form:

\[
B(r) = M(r, r_0) B(r_0)
\]

(6)

where \( r_0 \) is a reference point on the \( r \)-axis. The matrix can be calculated by the Peano expansion with the dimensionless quantity \((r-r_0)/a\) to be small for the truncation to be acceptable:

\[
M(r, r_0) = I + \int_{r_0}^{r} A(\xi_1) d\xi_1 + \int_{r_0}^{r} A(\xi_1) \int_{r_0}^{\xi_1} A(\xi_2) d\xi_2 d\xi_1 + ...
\]

(7)

Let the laser excite the coating as a beam focused along the \( z \)-axis by a cylindrical lens. To model the laser source, the same boundary conditions as that in author’s previous model [7], are chosen for the two components \( \sigma_{rr} \) and \( \sigma_{r\theta} \) of the stress tensor as

\[
\begin{align*}
\sigma_{rr} \big|_{r=a} &= -\sigma_0 \delta(t) \delta(\theta) \\
\sigma_{r\theta} \big|_{r=a} &= 0
\end{align*}
\]

(8)

for the ablation generation, where \( \sigma_0 \) stands for the normal force magnitude in N\( \mu \)s\( \cdot \)m\(^2\). The delta functions of time \( \delta(t) \) and of space \( \delta(\theta) \) are used. We have

\[
B(r=a) = \begin{pmatrix}
    i \omega \mu & i \omega \sigma & -\sigma_0 & 0
\end{pmatrix}^T.
\]

(9)

The coated cylinder is assumed to have no defects along its interface. The continuity of the two non-zero components of the displacement and stress [4] are considered at the interface between cylinder and coating. The laser excited transient displacement at \( r=a \) is found to be:

\[
\begin{pmatrix}
    i \omega \mu & i \omega \sigma
\end{pmatrix}^T = \left(M_2 - M_3 M_4^{-1} M_1 \right) \begin{pmatrix}
    -\sigma_0 & 0
\end{pmatrix}^T
\]

(10)

where \( M_1, M_2, M_3, M_4 \) correspond to the left-up, right-up, left-down, and right-down off-diagonal block of \( M(a, b) \) as four 2\( \times \)2 sub-matrices [6].

4. Numerical results

First, an aluminium cylinder coated with the same material is considered, and the calculation is compared to author’s previous result [7], as shown in figure 1. The density, and the longitudinal and shear wave velocities of aluminium, are chosen to be \( \rho = 2700 \text{ kg/m}^3 \), \( V_L = 6400 \text{ m/s} \) and \( V_S = 3110 \text{ m/s} \). The diameter of the cylinder is 10mm, and the coating is 0.1mm thick through this paper. The dashed white-lines of the dispersion curves by the previous model [7] fit well to the spectrum where energy
exists. Various wave modes are clearly observable, the first mode ($R$) is a surface wave mode, the following modes are Whispering Gallery ($W_1, W_2, \ldots$) waves [8]. The transient displacement is calculated and compared with a previous model [7]. The two waveforms are identical, the direct ($L$) and reflected longitudinal waves, the reflected shear wave ($TT$), the conversion of the longitudinal and transverse waves ($LT$), and the cylindrical Rayleigh wave ($R$) are clearly observable. The agreement illustrates the effectiveness of the proposed model and solution.

Also, an aluminium rod coated by copper and vice versa were chosen for the calculations. The density, and the longitudinal and shear wave velocities of copper, are chosen to be $\rho = 8900$ kg/m$^3$, $V_L = 4700$ m/s and $V_S = 2260$ m/s. The dimension is the same as above. The spectra are obtained for the two coated cylinders in figure 2 (a) and (b). The $R$, $W_1$, $W_2$,… waves are clearly observed. As the penetration depth of the surface wave is in the order of wavelength, the slope of the R wave is quite different for both, and it approaches the surface wave velocity of the coated material. However, the waveforms of various $W$ waves make no much difference as their penetration depths are in the order of the radius.
Finally, the transient displacements are obtained as shown in figure 3 (a) and (b). The direct ($L$) and reflected longitudinal waves, the reflected shear wave ($TT$), the conversion of the longitudinal and transverse waves ($LT$) are clearly observable. However, the waveforms of Rayleigh ($R$) waves are quite different. For copper on aluminium, the high frequency component travels relatively slower than the low frequency component, and it is vice versa for aluminium on copper. This phenomenon may be explained by the penetration dependence on the frequency. The high frequency component has relatively smaller penetration than the low frequency component. As the thickness of the coating is relatively small, the high frequency component is likely affected by the coating and the counter part by the substrate. The difference of $R$ waves further illustrates the effectiveness of the proposed model and solution.

5. Conclusion

An alternative theoretical solution is provided to predict ultrasound generated by a laser line source in a homogeneous, isotropic cylinder coated with a homogeneous and isotropic material. This model is different from previous one to potentially address functionally graded cylinders. The spectrum and transient displacement are in agreement for copper on aluminium and vice versa. It is found for the surface waves that the high frequency component travels faster for hard coatings with velocities faster than that of the substrate, and vice versa for soft coatings.

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