An Extended Clustering Algorithm for Statistical Language Models

Joerg P. Ueberla
Forum Technology - DRA Malvern
St. Andrews Road, Malvern
Worcestershire, WR14 3PS, UK
email: ueberla@signal.dra.hmg.gb
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Abstract

Statistical language models frequently suffer from a lack of training data. This problem can be alleviated by clustering, because it reduces the number of free parameters that need to be trained. However, clustered models have the following drawback: if there is “enough” data to train an unclustered model, then the clustered variant may perform worse. On currently used language modeling corpora, e.g. the Wall Street Journal corpus, how do the performances of a clustered and an unclustered model compare? While trying to address this question, we develop the following two ideas. First, to get a clustering algorithm with potentially high performance, an existing algorithm is extended to deal with higher order N-grams. Second, to make it possible to cluster large amounts of training data more efficiently, a heuristic to speed up the algorithm is presented. The resulting clustering algorithm can be used to cluster trigrams on the Wall Street Journal corpus and the language models it produces can compete with existing back-off models. Especially when there is only little training data available, the clustered models clearly outperform the back-off models.
1 Introduction

It is well known that statistical language models often suffer from a lack of training data. This is true for standard tasks and even more so when one tries to build a language model for a new domain, because a large corpus of texts from that domain is usually not available. One frequently used approach to alleviate this problem is to construct a clustered language model. Because it has fewer parameters, it needs less training data. The main advantage of a clustered model are its robustness, even in the face of little or sparse training data, and its compactness. Particularly when a language model is used during the acoustic search in a speech recogniser, having a more compact, e.g. less complex model, can be of considerable importance. The main drawback of clustered models is that they may perform worse than an unclustered model, if there is “enough” data to train the latter. Do corpora currently used for language modeling, e.g. the Wall Street Journal corpus, contain enough data in that sense? Or, in other words, how does the performance of a clustered model compare with that of an unclustered model? In this paper, we will attempt to partly answer this question and, along the way, an extended, more efficient clustering algorithm will be developed.

In the next section (section 2), a brief review of existing clustering algorithms will be given. For the work presented here, we use the clustering algorithm proposed in [8], because, in the spirit of decision directed learning, it uses an optimisation function that is very closely related to the final performance measure we wish to maximise. Since the algorithm forms the basis of our work, its optimisation criterion is derived in detail.

In order to achieve a clustered model with potentially high performance, the algorithm is then extended (section 3) so that it can cluster higher order N-grams. We present three possible approaches for this extension and then develop the one chosen for this work in more detail.

When such a clustering algorithm is applied to a large training corpus, e.g. the Wall Street Journal corpus, with tens of millions of words, the computational effort required can easily become prohibitive. Therefore, a simple heuristic to speed up the algorithm is developed in section 4. Its main idea is as follows. Rather than trying to move each word \( w \) to all possible clusters, as the algorithm requires initially, one only tries moving \( w \) to a fixed number of clusters that have been selected from all possible clusters by a simple heuristic. This reduces the order of the complexity of
the algorithm. Of course, it may lead to a decrease in performance. However, in practice, the decrease in performance is minor (less than 5%), whereas the obtained speedup is large (up to a factor of 32).

Because of the increase in the speed of the algorithm, it can be applied more easily to the Wall Street Journal corpus and the obtained results will be presented in section 5.

2 Background and Related Work

In speech recognition, one is given a sequence of acoustic observations $A$ and one tries to find the word sequence $W^*$ that is most likely to correspond to $A$. In order to minimise the average probability of error, one should, according to Bayes’ decision rule ([3, p.17]), choose

$$W^* = \arg\max_W p(W|A).$$  \hspace{1cm} (1)

Based on Bayes’ formula (see for example [4, p.150]), one can rewrite the probability from the right hand side of equation 1 according to the following equation:

$$p(W|A) = \frac{p(W) \cdot p(A|W)}{p(A)}. \hspace{1cm} (2)$$

$p(W)$ is the probability that the word sequence $W$ is spoken, $p(A|W)$ is the conditional probability that the acoustic signal $A$ is observed when $W$ is spoken and $p(A)$ is the probability of observing the acoustic signal $A$. Based on this formula, one can rewrite the maximization of equation 1 as

$$W^* = \arg\max_W \frac{p(W) \cdot p(A|W)}{p(A)}. \hspace{1cm} (3)$$

Since $p(A)$ is the same for all $W$, the factor $p(A)$ does not influence the choice of $W$ and maximising equation 3 is equivalent to maximising

$$W^* = \arg\max_W p(W) \cdot p(A|W). \hspace{1cm} (4)$$

The component of the speech recogniser that calculates $p(A|W)$ is called the acoustic model, the component calculating $p(W)$ the language model. With
$W = w_1, ..., w_n$, one can further decompose $p(W)$ using the definition of conditional probabilities as

$$p(W) = \prod_{i=1}^{i=n} p(w_i|w_1, ..., w_{i-1}). \quad (5)$$

In practice, because of the large number of parameters in equation (5), the probability of $w_i$ usually only depends on the immediately preceding $M$ words:

$$p(w_i|w_1, ..., w_{i-1}) \approx p(w_i|w_{i-M}, ..., w_{i-1}). \quad (6)$$

These models are called $(M+1)$-gram models and in practice, mostly bigram ($M = 1$) and trigram ($M = 2$) models are used. Even in these cases, the number of parameters that need to be estimated from training data can be quite large. For a speech recogniser with a vocabulary of 20,000 words, the bigram needs to estimate roughly $20,000^2 = 4*10^8$ parameters and a trigram $20,000^3 = 8 * 10^{12}$.

One way to alleviate this problem is to use class based models. Let $G : w \rightarrow G(w) = g_w$ be a function that maps each word $w$ to its class $G(w) = g_w$ and let $|G|$ denote the number of classes. We can then model the probability of $w_i$ as

$$p(w_i|w_1, ..., w_{i-1}) \approx p_G(w_i|w_{i-M}, ..., w_{i-1}) \quad (7)$$

$$= p_G(G(w_i)|G(w_{i-M}), ..., G(w_{i-1})) * p_G(w_i|G(w_i)). \quad (8)$$

Thus, if $|G|=1000$ classes are being used, the class-based bigram model has $1,000^2 + 20,000 = 1.02 * 10^6$ parameters and the class-based trigram model $1,000^3 + 20,000 = 1.00002 * 10^9$. This constitutes a significant reduction in both cases.

Many researchers have developed algorithms for determining the clustering function $G$ automatically (see for example [2], [5], [6], [8] and [12]). Starting from an initial clustering function, the basic principle often is to move words from their current cluster to another cluster, but only if this improves the value of an optimisation criterion. The algorithms often differ in the optimisation criterion and in general, there are many possible choices for it. However, in the spirit of decision-directed learning, it makes sense to

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\footnote{This model is also sometimes referred to as bi-pos model, where pos stands for Parts Of Speech.}
use as optimisation criterion a function that is very closely related or identical to the final performance measure we wish to maximise. This way, one can be very confident that an improvement in the optimisation criterion will actually translate to an improvement of performance. We therefore chose the algorithm proposed in [8] as the basis for our work. In the following, the optimisation criterion for a bigram based model (e.g. $M = 1$) will be derived, roughly as presented in [8].

In order to automatically find classification functions $G$, the classification problem is first converted into an optimisation problem. Suppose the function $F(G)$ indicates how good the classification $G$ is. One can then reformulate the classification problem as finding the classification $G^*$ that maximises $F$:

$$G^* = \arg\max_{G \in \mathcal{G}} F(G), \quad (9)$$

where $\mathcal{G}$ contains the set of possible classifications which are at our disposal.

What is a suitable function $F$, also called optimisation criterion? Given a classification function $G$, the probabilities $p_G(w|v)$ of equation 8 can be estimated using the maximum likelihood (ML) estimator, e.g. relative frequencies:

$$p_G(w|v) = \frac{N(G(w), G(v))}{N(G(v))} \ast \frac{N(G(w), y)}{N(G(w))}, \quad (11)$$

where $N(x)$ denotes the number of times $x$ occurs in the training data. Given these probability estimates $p_G(w|v)$, the likelihood $F_{ML}$ of the training data, e.g. the probability of the training data being generated by our probability estimates $p_G(w|v)$, measures how well the training data is represented by the estimates and can be used as optimisation criterion (I). The likelihood of the training data $F_{ML}$ is simply

$$F_{ML} = \prod_{i=1}^{N} p_G(w_i|w_{i-1}) \quad (12)$$

$$= \prod_{i=1}^{N} \frac{N(g_{w_{i-1}}, g_{w_i})}{N(g_{w_{i-1}})} \ast \frac{N(g_{w_i}, w_i)}{N(g_{w_i})}. \quad (13)$$

Assuming that the classification is unique, e.g. that $G$ is a function, $N(g_{w_i}, w_i) = N(w_i)$ always holds (because $w_i$ always occurs with the same class $g_{w_i}$). Since
one tries to optimise $F_{ML}$ with respect to $G$, any term that does not depend on $G$ can be removed, because it will not influence the optimisation. It is thus equivalent to optimise

$$F'_{ML} = \prod_{i=1}^{N} \frac{N(g_{w_{i-1}}, g_{w_{i}})}{N(g_{w_{i-1}})} * \frac{1}{N(g_{w_{i}})}$$

$$=: \prod_{i=1}^{N} f(w_{i-1}, w_{i}).$$

If, for two pairs $(w_{i-1}, w_{i})$ and $(w_{j-1}, w_{j})$, $G(w_{i-1}) = G(w_{j-1})$ and $G(w_{i}) = G(w_{j})$ holds, then $f(w_{i-1}, w_{i}) = f(w_{j-1}, w_{j})$ is also true. Identical terms can thus be regrouped to obtain

$$F'_{ML} = \prod_{g_1, g_2} \left[ \frac{N(g_1, g_2)}{N(g_1)} * \frac{1}{N(g_2)} \right]^{N(g_1, g_2)},$$

(16)

where the product is over all possible pairs $(g_1, g_2)$. Because $N(g_1)$ does not depend on $g_2$ and $N(g_2)$ does not depend on $g_1$, this can again be simplified to

$$F'_{ML} = \prod_{g_1, g_2} N(g_1, g_2)^{N(g_1, g_2)} \prod_{g_1} \frac{1}{N(g_1)} \prod_{g_2} \frac{1}{N(g_2)}.$$  

(17)

After taking the logarithm, one obtains the equivalent optimisation criterion $F''_{ML}$

$$F''_{ML} = \sum_{g_1, g_2} N(g_1, g_2) * \log(N(g_1, g_2)) - \sum_{g_1} N(g_1) * \log(N(g_1))$$

$$- \sum_{g_2} N(g_2) * \log(N(g_2)).$$

(18)

$F''_{ML}$ is the maximum likelihood optimisation criterion that can be used to find a good classification $G$. However, the problem with this maximum likelihood criterion is that one first estimates the probabilities $p_G(w|v)$ on the training data $T$ and then, given $p_G(w|v)$, one evaluates the classification $G$ on $T$. In other words, both the classification $G$ and the estimator $p_G(w|v)$ are trained on the same data. Thus, there will not be any unseen event, a fact that overestimates the power for generalisation of the class based model. In order to avoid this, a cross-validation technique will be incorporated directly into the optimisation criterion in section 2.1.
2.1 Leaving-One-Out Criterion

The basic principle of cross-validation is to split the training data $T$ into a “retained” part $T_R$ and a “held-out” part $T_H$. One can then use $T_R$ to estimate the probabilities $p_G(w|v)$ for a given classification $G$, and $T_H$ to evaluate how well the classification $G$ performs. The so-called leaving-one-out technique is a special case of cross-validation (§3, pp. 75). It divides the data into $N-1$ samples as “retained” part and only one sample as “held-out” part. This is repeated $N-1$ times, so that each sample is once in the “held-out” part. The advantage of this approach is that all samples are used in the “retained” and in the “held-out” part, thus making very efficient use of the existing data. In other words, the “held-out” part $T_H$ to evaluate a classification $G$ is the entire set of data points; but when we calculate the probability of the $i^{th}$ data point, one assumes that the probability distributions $p_G(w|v)$ were estimated on all the data except point $i$.

Let $T_i$ denote the data without the pair $(w_{i-1}, w_i)$ and $p_{G,T_i}(w|v)$ the probability estimates based on a given classification $G$ and training corpus $T_i$. Given a particular $T_i$, the probability of the “held-out” part $(w_{i-1}, w_i)$ is $p_{G,T_i}(w_i|w_{i-1})$. The probability of the complete corpus, where each pair is in turn considered the “held-out” part is the leaving-one-out likelihood $L_{LO}$

$$L_{LO} = \prod_{i=1}^{N} p_{G,T_i}(w_i|w_{i-1}).$$

(19)

In the following, an optimisation function $F_{LO}$ will be derived by specifying how $p_{G,T_i}(w_i|w_{i-1})$ is estimated from frequency counts. First $p_{G,T_i}(w_i|w_{i-1})$ is rewritten as usual (see equation 8):

$$p_{G,T_i}(w|v) = p_{G,T_i}(G(w)|G(v)) \cdot p_{G,T_i}(w|G(w))$$

(20)

$$= \frac{p_{G,T_i}(g_1, g_2)}{p_{G,T_i}(g_1)} \cdot \frac{p_{G,T_i}(g_2,w)}{p_{G,T_i}(g_2)},$$

(21)

where $g_1 = G(v)$ and $g_2 = G(w)$. Now we will specify how each term in equation 21 is estimated.

As shown before, $p_{G,T_i}(g_2,w) = p_{G,T_i}(w)$ (if the classification $G$ is a function) and since $p_{T_i}(w)$ is actually independent of $G$, one can drop it out of the maximization and thus need not specify an estimate for it.
As will be shown later, one can guarantee that every class \( g_1 \) and \( g_2 \) has been seen at least once in the “retained” part and one can thus use relative counts as estimates for class uni-grams:

\[
p_{G,T_i}(g_1) = \frac{N_{T_i}(g_1)}{N_{T_i}} \quad (22)
\]

\[
p_{G,T_i}(g_2) = \frac{N_{T_i}(g_2)}{N_{T_i}}. \quad (23)
\]

However, in the case of the class bi-gram, one might have to predict unseen events \[ 2 \]. We therefore use the absolute discounting method ([9]), where the counts are reduced by a constant value \( b < 1 \) and where the gained probability mass is redistributed over unseen events. Let \( n_{0,T_i} \) be the number of unseen pairs \((g_1, g_2)\) and \( n_{+,T_i} \) the number of seen pairs \((g_1, g_2)\). This leads to the following smoothed estimate

\[
p_{G,T_i}(g_1, g_2) = \begin{cases} 
\frac{N_{T_i}(g_1, g_2) - b}{N_{T_i}} & \text{if } N_{T_i}(g_1, g_2) > 0 \\
\frac{n_{+,T_i} + b}{n_{0,T_i} + n_{T_i}} & \text{if } N_{T_i}(g_1, g_2) = 0
\end{cases} \quad (24)
\]

Ideally, one would make \( b \) depend on the classification, e.g. use \( b = \frac{n_1}{n_1 + 2n_2} \), where \( n_1 \) and \( n_2 \) depend on \( G \). However, due to computational reasons, we use, as suggested in [8], the empirically determined constant value \( b = 0.75 \) during clustering. The probability distribution \( p_{G,T_i}(g_1, g_2) \) will always be evaluated on the “held-out” part \((w_{i-1}, w_i)\) and with \( g_{1,i} = g_{w_{i-1}} \) and \( g_{2,i} = g_{w_i} \), one obtains

\[
p_{G,T_i}(g_{1,i}, g_{2,i}) = \begin{cases} 
\frac{N_{T_i}(g_{1,i}, g_{2,i}) - b}{N_{T_i}} & \text{if } N_{T_i}(g_{1,i}, g_{2,i}) > 0 \\
\frac{n_{+,T_i} + b}{n_{0,T_i} + n_{T_i}} & \text{if } N_{T_i}(g_{1,i}, g_{2,i}) = 0
\end{cases} \quad (25)
\]

In order to facilitate future regrouping of terms, one can now express the counts \( N_{T_i}, N_{T_i}(g_1) \) etc. in terms of the counts of the complete corpus \( T \) as

\[ 2 \text{If } (w_{i-1}, w_i) \text{ occurs only once in the complete corpus, then } p_{G,T}(w_i|w_{i-1}) \text{ will have to be calculated based on the corpus } T_i, \text{ which does not contain any occurrences of } (w_{i-1}, w_i). \]
follows:

\[ N_{T_i} = N_T - 1 \]  
\[ N_{T_i}(g_1) = N_T(g_1) - 1 \]  
\[ N_{T_i}(g_2) = N_T(g_2) - 1 \]  
\[ N_{T_i}(g_1, g_2) = N_T(g_1, g_2) - 1 \]  
\[ N_{T_i} = N_T - 1 \]  
\[ n_{+,T_i} = \begin{cases} n_{+,T} & \text{if } N_T(g_1, g_2) > 1 \\ n_{+,T} - 1 & \text{if } N_T(g_1, g_2) = 1 \end{cases} \]  
\[ n_{0,T_i} = \begin{cases} n_{0,T} & \text{if } N_T(g_1, g_2) > 1 \\ n_{0,T} - 1 & \text{if } N_T(g_1, g_2) = 1 \end{cases} \]  

All the expressions can now be substituted back into equation 19. After dropping \( p_{G,T_i}(w) \) because it is independent of \( G \), one arrives at

\[ F'_{LO} = N \prod_{i=1}^{N} \frac{p_{G,T}(g_1, g_2)}{p_{G,T_i}(g_1, g_2)} \times \frac{1}{p_{G,T}(g_1, g_2)} \]  
\[ = \prod_{g_1, g_2} \left( \frac{p_{G,T_i}(g_1, g_2)}{p_{G,T_i}(g_1, g_2)} \right)^{N_{T}(g_1, g_2)} \times \prod_{g_1} \left( \frac{1}{p_{G,T_i}(g_1)} \right)^{N(g_1)} \]  
\[ \text{where} \quad n_{1,T} = \text{the number of pairs } (g_1, g_2) \text{ seen exactly once in } T (\text{e.g. the number of pairs that will be unseen when used as "held-out" part}). \]

Taking the logarithm, we obtain the final optimisation criterion \( F''_{LO} \)

\[ F''_{LO} = \sum_{g_1, g_2: N_T(g_1, g_2) > 1} N_T(g_1, g_2) \log(N_T(g_1, g_2) - 1 - b) \]
\[ + n_{1,T} \log \left( \frac{b \cdot (n_{1,T} - 1)}{(n_{0,T} + 1)} \right) \]

\[ - \sum_{g_1} N_T(g_1) \log (N_T(g_1) - 1) - \sum_{g_2} N_T(g_2) \log (N_T(g_2) - 1). \]

### 2.2 Clustering Algorithm

Given the maximization criterion \( F_{LO}'' \), we use the algorithm in Figure 1 to find a good clustering function \( G \). The algorithm tries to make local changes by moving words between classes, but only if it improves the value of the optimisation function. The algorithm will converge because the optimisation criterion is made up of logarithms of probabilities and thus has an upper limit and because the value of the optimisation criterion increases in each iteration. However, the solution found by this greedy algorithm is only locally optimal and it depends on the starting conditions. Furthermore, since the clustering of one word affects the future clustering of other words, the order in which words are moved is important. As suggested in [8], the words are sorted by the number of times they occur such that the most frequent words, about which one knows the most, are clustered first. Moreover, infrequent words (e.g. words with occurrence counts smaller than 5) are not considered for clustering, because the information they provide is not very reliable. Thus, if one starts out with an initial clustering in which no cluster occurs only once, and if one never moves words that occur only once, then one will never have a cluster which occurs only once. Thus, the assumption we made earlier, when it was decided to estimate cluster uni-grams by frequency counts, can be guaranteed.

We will now determine the complexity of the algorithm. Let \( C \) be the maximal number of clusters for \( G \), let \( E \) be the number of elements one tries to cluster (e.g. \( E = |V| \)), and let \( I \) be the number of iterations. When one moves \( w \) from \( g_w \) to \( g'_w \) in the inner loop, one needs to change the counts \( N(g_w, g_2) \) and \( N(g'_w, g_2) \) for all \( g_2 \). The amount by which the counts need to be changed is equal to the number of times \( w \) occurred with cluster \( g_2 \). Since this amount is independent of \( g'_w \), one only needs to calculate it once for each \( w \). The amount can then be looked up in constant time within the loop, thus making the inner loop of order \( C \). The inner loop is executed once for every cluster \( w \) can be moved to, thus giving a complexity of the order of \( C^2 \). For each \( w \), one needed to calculate the number of times \( w \) occurred
Algorithm 1: Clustering()

start with initial clustering function $G$
iterate until some convergence criterion is met
\{ for all $w \in V$
  \{ for all $g'_w \in G$
    \{ calculate the difference in $F(G)$ when $w$ is moved from $g_w$ to $g'_w$
    \}
    move $w$ to $g'_w$ that results in biggest improvement in $F(G)$
  \}
\}
End Clustering

Figure 1: The clustering algorithm

with all clusters $g_2$. For that one has to sum up all the bigram counts $N(w, v): G(v) = g_2$, which is on the order of $E$, thus giving a complexity of the order of $E + C^2$. The two outer loops are executed $I$ and $E$ times, thus giving a total complexity of the order of $I * E * (E + C^2)$.

3 Extending the Clustering Algorithm to $N$-grams

It is well known that a trigram model outperforms a bigram model if there is sufficient training data. If we want our clustering algorithm to compete with unclustered models on a corpus like the Wall Street Journal, where the trigram indeed outperforms the bi-gram, it therefore seems logical that the clustering algorithm should be extended to deal with trigrams (and higher order $N$-grams) as well. The original clustered bigram model, as derived from equation 8, is

$$p(w_i|w_{i-1}) = p_G(G(w_i)|G(w_{i-1})) * p_G(w_i|G(w_i)).$$  \hspace{1cm} (37)
There are at least three ways of extending the clustering to \((M + 1)\)-grams, depending on how one models the probability \(p(w_i|w_{i-M}, \ldots, w_{i-1})\):

\[
\begin{align*}
& a) & p_G(G(w_i)|G(w_{i-M}, \ldots, G(w_{i-1})) & \times p_G(w_i|G(w_i)) & (38) \\
& b) & p_G(G_{M+1}(w_i)|G_{M}(w_{i-M}, \ldots, G_{1}(w_{i-1})) & \times p_G(w_i|G_{M+1}(w_i)) & (39) \\
& c) & p_G(G_2(w_i)|G_1(w_{i-M}, \ldots, w_{i-1})) & \times p_G(w_i|G_2(w_i)) & (40)
\end{align*}
\]

The tradeoff between these models is one of accuracy versus complexity. Approach \(a\), which only uses one clustering function \(G\), could produce 

\[|G|^{[V]}\]

different clusterings (for each word in \(V\), it can choose one of the \(|G|\) clusters). Approach \(b\), which uses \(M + 1\) different clustering functions, can represent 

\[\sum_{i=0}^{i=M+1} |G_i|^{V}\]

different clusterings, including all the clusterings of approach \(a\). Approach \(c\), which uses one clustering function for the tuples \(w_{i-M}, \ldots, w_{i-1}\) and one for \(w_i\), can produce 

\[|G_1|^{[V]M} + |G_2|^{[V]}\]

possible clusterings, including all the ones represented by approach \(a\) and \(b\). Approach \(c\) therefore has the highest potential for accuracy, as long as there is sufficient training data. Since the Wall Street Journal corpus is very large, we decided to use approach \(c\). Please note that for \(M = 1\), approach \(a\) gives the traditional clustered bigram approach, but approaches \(b\) and \(c\) \((c)\) collapses to \(b\) for \(M = 1\) are more general than the traditional model. Moreover, approach \(c\) is referred to in a recent publication \([10]\) as a two-sided (non symmetric) approach.

Similar to the derivation presented in section 2, one can now derive the optimisation criterion for approach \(c\). However, since it is very similar to the derivation shown in section 2, only the final formulae will be given here. The complete derivation is given in appendix A. Let \(g_1\) and \(g_2\) denote clusters of \(G_1\) and \(G_2\) respectively. The optimisation criterion for the extended algorithm is

\[
F_{LO} = \sum_{g_1, g_2 : N_T(g_1, g_2) > 1} N_T(g_1, g_2) \times \log(N_T(g_1, g_2) - 1 - b) \quad (41)
\]
Algorithm 2: Clustering()

start with initial clustering function $G$
iterate until some convergence criterion is met
{
  for all $w \in V$ and $t \in V^M$
  {
    for all $g'_w \in G_2$ and $g'_t \in G_1$
    {
      calculate the difference in $F(G)$ when $w/t$ is moved from $g_w/g_t$ to $g'_w/g'_t$
    }
    move the $w/t$ to the $g'_w/g'_t$ that results in the biggest improvement in $F(G)$
  }
}
End Clustering

Figure 2: The extended clustering algorithm

$$+ n_{1,T} \log \left( \frac{b \times (n_{1,T} - 1)}{(n_{0,T} + 1)} \right) - \sum_{g_1} N_T(g_1) \log (N_T(g_1) - 1) - \sum_{g_2} N_T(g_2) \log (N_T(g_2) - 1).$$

The corresponding clustering algorithm, which is shown in figure 2, is a straightforward extension of the one given in section 2. Its complexity can be derived as follows. Let $C_{G_1}$ and $C_{G_2}$ be the maximal number of clusters for $G_1$ and $G_2$, let $E_1$ and $E_2$ be the number of elements $G_1$ and $G_2$ try to cluster, let $C = \max(C_{G_1}, C_{G_2})$, $E = \max(E_1, E_2)$ and let $I$ be the number of iterations. When one moves $w$ from $g_w$ to $g'_w$ in the inner loop (the situation is symmetrical for $t$), one needs to change the counts $N(g_w, g_2)$ and $N(g'_w, g_2)$ for all $g_2 \in G_2$. The amount by which the counts need to be changed is equal to the number of times $w$ occurs with cluster $g_2$. Since this amount is independent of $g'_w$, one only needs to calculate it once for each $w$. The amount can then be looked up in constant time within the loop, thus making the inner loop of order $C$. The inner loop is executed once for every cluster.
$w$ can be moved to, thus giving a complexity of the order of $C^2$. For each $w$, one needed to calculate the number of times $w$ occurred with all clusters $g_2$. For that, one has to sum up all the bigram counts $N(w, t) : G_2(t) = g_2$, which is on the order of $E$, thus giving a complexity of the order of $E + C^2$. The two outer loops are executed $I$ and $E$ times thus giving a total complexity of the order of $I \times E \times (E + C^2)$. This is almost identical to the complexity of the bigram clustering algorithm given in section 2, except that $E$ is now the number of $(M + 1)$-grams one wishes to cluster, rather than the number of unigrams (e.g. words of the vocabulary).

4 Speeding up the Algorithm

If one wants to use the clustering algorithm on large corpora, the complexity of the algorithm becomes a crucial issue. As shown in the last two sections, the complexity of the algorithm is $O(I \times E \times (E + C^2))$, where $C$ is the maximally allowed number of clusters, $I$ is the number of iterations and $E$ is the number of elements to cluster ($|V|$ in case of bigrams, $|V|M + 1$ in case of the extended algorithm). $C$ crucially determines the quality of the resulting language model and one would therefore like to chose it as big as possible. Unfortunately, because the algorithm is quadratic in $C$, this may be very costly. We therefore developed the following heuristic to speed up the algorithm.

The factor $C^2$ comes from the fact that one tries to move a word $w$ to each of the $C$ possible clusters ($O(C)$), and for each of these one has to calculate the difference in the optimisation function ($O(C)$ again). If, based on some heuristic, one could select a fixed number $t$ of target clusters, then one could only try moving $w$ to these $t$ clusters, rather than to all possible clusters $C$. This may of course lead to the situation where one does not move a word to the best possible cluster (because it was not selected by the heuristic), and thus potentially to a decrease in performance. But this decrease in performance depends of course on the heuristic function used and we will come back to this issue when we look at the practical results.

The heuristic used in this work is as follows. For each cluster $g_1$, one keeps track of the $h$ clusters that most frequently co-occur with $g_1$ in the tables $N(g_1, g_2)$. For example, if $g_1$ is a cluster of $G_1$ (the situation is symmetric for $G_2$), then the $h$ biggest entries in $N(g_1, g)$ are the $h$ clusters being stored.
When one tries to move a word \( w \), one also constructs a list of the \( h \) most frequent clusters that follow \( w \) (one can get this for free as part of the factor \( E \) in \( E + C^2 \)). One then simply calculates the number of clusters that are in both lists and takes this as the heuristic score \( H(g_1) \). The bigger \( H(g_1) \), the more similar are the distributions of \( w \) and \( g_1 \), and the more likely it is that \( g_1 \) is a good target cluster to which \( w \) should be moved. Because the length of the lists is a constant \( h \) calculating the heuristic score is also independent of \( C \). One can thus calculate the heuristic score of all \( C \) clusters in \( O(C) \). However, once one has decided to move \( w \) to a given cluster, one would have to update the lists containing the \( h \) most frequent clusters following each cluster \( g_1 \) (the lists might have changed due to the last moving of a word). Since the update is \( O(C) \) for a given \( g_1 \), the update would again be \( O(C^2) \) for all clusters. In order to avoid this, one can make another approximation at this point. One can only update the list for the original and the new cluster of \( w \). The full update of all the lists is only performed after a certain number \( u \) of words have been moved.

To sum up, we can say that one can select \( t \) target clusters using the heuristic in \( O(C) \). Following that, one tries moving \( w \) to each of these \( t \) clusters, which is again \( O(C) \). Moreover, several times per iteration (depending on \( u \)), one updates the list of most frequent clusters which is \( O(C^2) \). Thus, the complexity of the heuristic version of the algorithm is \( O(I*(E*(E+C)+C^2)) \). The complexity still contains the factor \( C^2 \), but this time not within the inner parenthesis. The factor \( C^2 \) will thus be smaller than \( E*(E+C) \), and is only given for completeness.

We will now present a practical evaluation of the heuristic algorithm. The heuristic itself is parameterised by \( h \), the number of most frequent clusters one uses to calculate the heuristic score, \( t \), the number of best ranked target clusters one tries to move word \( w \) to and \( u \), the number indicating after how many words a full update of the list of most frequent clusters is performed. In order to evaluate the heuristic for a given set of parameters, one can simply compare the final value of the approximation function and the resulting perplexity of the heuristic algorithm with that of the full algorithm.

Table 1 contains a comparison of the results using approximately one million words of training data (from the Wall Street Journal corpus) and values \( t = 10 \), \( h = 10 \) and \( u = 1000 \). The CPU Time given was measured on a DEC alpha workstation (DEC 3000, model 600), which was used in all the experiments reported in this paper. One can see that the execution
Table 1: Comparison of the algorithm with its heuristic version

| Clusters | Standard Algorithm | Heuristic Version |
|----------|--------------------|-------------------|
|          | PP     | CPU Time | PP (Δ %) | CPU Time (Δ %) |
| 10       | 746    | 1:02     | 746 (0.0) | 1:05 (4.8)     |
| 20       | 630    | 2:28     | 653 (3.6) | 1:35 (-36)     |
| 40       | 548    | 7:46     | 558 (1.8) | 2:36 (-67)     |
| 80       | 477    | 26:41    | 490 (2.7) | 4:30 (-83)     |
| 160      | 421    | 1:33:29  | 437 (3.8) | 7:59 (-91)     |
| 320      | 394    | 5:20:18  | 402 (2.0) | 14:17 (-96)    |

time of the standard algorithm seems indeed quadratic in the number of clusters, whereas that of the heuristic version seems to be linear. Moreover, the perplexity of the heuristic version is always within 4% of that of the standard algorithm, a number which does not seem to increase systematically with the number of clusters. Furthermore, the speed up of the algorithm seems to be closely related to the number of clusters divided by $t$. For example, in the case of 320 clusters, this ration is $320/10 = 32$ and the heuristic version is indeed almost 32 times as fast (the speed up is almost $1 - \frac{1}{32} = 0.97$). Judging from the time behaviour of the standard algorithm, one would expect it to take around 32 hours to run with 1000 clusters, whereas the heuristic algorithm, as will be shown later, only takes about half an hour (for $t = 10$).

Tables 2 to 4 contain a more detailed analysis of the influence of the parameters $t$, $u$, and $h$ on the heuristic version of the algorithm, this time with a maximal number of allowed clusters of 1000. The first point to note is that in all tables, a change in the value of the optimisation function is very closely related to a change in perplexity. This is a very reassuring finding, because it indicates that the clustering algorithm actually tries to optimise the correct criterion.

From table 2, one can see that an increase in $t$ leads to an increase in execution time, but also to an increase in performance. This is because as $t$ increases, the chances of the heuristic missing the overall best target cluster for a given word $w$ decreases.

In table 3, one can see that the effect of $u$ on the algorithm is very minor. This could be explained by the fact that even though the full lists of most frequent clusters are not updated at every move, the update in clusters $g_w$
and $g'_w$, which is performed at every move, contains the most important
changes.

Finally, in table 4, one can see that the performance of the algorithm
decreases with an increase in $h$. This in a way counter intuitive result could
be explained by the following hypothesis. If the suitability of a target cluster
is determined by a small number of very frequently co-occurring clusters, then
increasing $h$ could make the heuristic perform worse, because the effect of
the most important clusters is perturbed by a large number of less important
clusters (the heuristic only counts the number of clusters in both lists and
does not weigh them).

Based on the results of these experiments, we chose $h = 5$, $t = 10$ and
$u = 1000$ for future experiments with the heuristic version of the algorithm.
5 Results

In the following, we will present results of clustered language models on the Wall Street Journal (WSJ) corpus. The work reported here was performed on the WSJ0 corpus, using the verbalised pronunciation (VP) and non verbalised pronunciation (NVP) versions of the corpus with the 20K open vocabulary. As mentioned on the CDROM (and as discussed in [14]), the results for open vocabularies are usually not meaningful, if the unknown words are taken into account when calculating the perplexity. One way to solve this problem ([13]) is to simply skip the unknown words, when calculating the perplexity. Since all our experiments were performed with the open vocabulary, this is the approach taken here, except when indicated otherwise. In order to investigate the influence of the amount of training data on the results, we used seven different sets of training data, T1 to T7, with about 2K, 12K, 60K, 350K, 1.7M, 8.5M and 40M words respectively. All perplexity results were calculated on approximately 2.3 million words of text that were not part of the training material. The clustered models were produced with the extended heuristic version of the algorithm. To run to completion, it took less than 12 hours real time for the bigram case, and several days for the trigram case.

As a yardstick for the performance of the clustered models, we implemented the commonly used compact back-off model ([7], [11]). Because the bigram counts were not smoothed, the probability mass, that could be redistributed to unseen events, was only gained through events that fell below the cut-off threshold. If a given cut-off threshold did did not lead to any gained probability mass for a particular distribution (because no event was below the threshold and thus no probability mass could be redistributed), the cut-off threshold of this distribution was set to the lowest value, that would lead to some gain in probability mass. Table 5 and 6 give the perplexity of back-off models with various cut-off thresholds $C$ for verbalised and non-verbalised pronunciation respectively. First, one can see that a bigger value of $C$ leads to a higher perplexity. This is because as $C$ increases, more and more bigram counts are discarded and replaced by unigram, rather than bigram, probability estimates. However, a good reason why higher values of $C$ might still be of interest is that they lead to substantially smaller models and this can be of crucial importance for the time performance of a recogniser. Second, and more importantly for our purposes, the results seem comparable
| Training Set | $C = 250$ | $C = 50$ | $C = 10$ | $C = 2$ |
|--------------|-----------|-----------|-----------|-----------|
| T1           | 2180      | 2180      | 2150      | 2240      |
| T2           | 1350      | 1320      | 1210      | 1130      |
| T3           | 1030      | 936       | 750       | 621       |
| T4           | 812       | 645       | 480       | 373       |
| T5           | 620       | 462       | 330       | 254       |
| T6           | 456       | 323       | 236       | 190       |
| T7           | 324       | 233       | 182       | 159       |

Table 5: Back-off perplexity results on VP data

| Training Set | $C = 250$ | $C = 50$ | $C = 10$ | $C = 2$ |
|--------------|-----------|-----------|-----------|-----------|
| T1           | 3170      | 3170      | 3150      | 3220      |
| T2           | 1980      | 1950      | 1790      | 1610      |
| T3           | 1440      | 1310      | 1060      | 878       |
| T4           | 1170      | 936       | 696       | 537       |
| T5           | 928       | 693       | 488       | 369       |
| T6           | 666       | 466       | 332       | 265       |
| T7           | 464       | 327       | 250       | 216       |

Table 6: Back-off perplexity results on NVP data
### Table 7: Perplexity results for (2000,2000) bigram clusters (VP)

| Training | back-off | clustered | improvement (%) |
|----------|----------|-----------|-----------------|
| T1       | 2240     | 1750      | 22              |
| T2       | 1130     | 831       | 27              |
| T3       | 621      | 515       | 17              |
| T4       | 373      | 324       | 13              |
| T5       | 254      | 231       | 9.1             |
| T6       | 190      | 188       | 1.1             |
| T7       | 159      | 172       | -8.2            |

### Table 8: Perplexity results for (2000,2000) bigram clusters (NVP)

| Training | back-off | clustered | improvement (%) |
|----------|----------|-----------|-----------------|
| T1       | 3220     | 2420      | 25              |
| T2       | 1610     | 1230      | 24              |
| T3       | 878      | 762       | 13              |
| T4       | 537      | 491       | 8.6             |
| T5       | 369      | 345       | 6.5             |
| T6       | 265      | 267       | -0.76           |
| T7       | 216      | 240       | -11             |

to other results reported in the literature. In [1] for example, the perplexity results for the non-verbalised data with open vocabulary is 205, quite close to our 216 (for $C = 2$). However, it is quite likely that the probabilities of unknown words were taken into account for the calculation of the 205 value and our model also gives a perplexity of 205 in that case. The back-off results of tables 5 and 6 therefore constitute a reasonable yardstick to evaluate the performance of the clustered language models.

Tables 7 and 8 give the results of a clustered bigram with 2000 clusters for both $G_1$ and $G_2$, for verbalised and non-verbalised pronunciation respectively. For better comparison, the matching results of the back-off models are repeated and the difference is given in percent. Even though the clustered model performs worse than the back-off model on the largest set of data, it outperforms the back-off model in almost all other cases. This clearly shows the superior robustness of the clustered models.
Table 9: Results for bigram and trigram clusters (VP)

| Training | clustered bigram (3000,3000) | clustered trigram (7000,1000) | improvement(%) |
|----------|------------------------------|-------------------------------|----------------|
| 5.2M     | 191                          | 208                           | -8.9           |
| 41M      | 167                          | 151                           | 9.6            |

Table 9 shows the results for a clustered tri-gram \(^3\) with 7000 and 1000 clusters for \(G_1\) and \(G_2\) on VP data. Because these results were obtained on slightly different training and testing texts, the table also contains the results of the clustered bi-gram on the same data. One can see that the clustered trigram outperforms the clustered bigram, at least with sufficient training data. But even with only five million words of training data, the clustered trigram is only slightly worse than the clustered bigram, showing again the robustness of the clustered language models.

From all the results given here, one can see that the clustered language models can still compete with unclustered models, even when a large corpus, such as the Wall Street Journal corpus, is being used.

6 Conclusions

In this paper, an existing clustering algorithm is extended to deal with higher order \(N\)-grams. Moreover, a heuristic version of the algorithm is introduced, which leads to a very significant speed up (up to a factor of 32), with only a slight loss in performance (5\%). This makes it possible to apply the resulting algorithm to the clustering of bigrams and trigrams on the Wall Street Journal corpus. The results are shown to be comparable to standard back-off bigram models. Moreover, in the absence of many million words of training data, the clustered model is more robust and clearly outperforms the non-clustered models. This is an important point, because for many real world speech recognition applications, the amount of training data available for a certain task or domain is in general unlikely to exceed several million words. In those cases, the clustered models seem like a good alternative to back-off models and certainly one that deserves close investigation.

\(^3\)Only the 500,000 most frequent bigrams were clustered using \(G_1\).
The main advantage of the clustering models, its robustness in the face of little training data, can also be seen from the results and in these situations, the clustered algorithm is preferable to the standard back-off models.

Appendix A: Deriving the Optimisation Function

In this appendix, we will present the derivation of the optimisation function for the extended clustering algorithm in detail. It is a generalisation of [15], where the derivation was given for \( M = 1 \).

Let \( G \) be a short hand to denote both classification functions \( G_1 \) and \( G_2 \). Following the same approach as in section 3, one can estimate the probabilities in equation 40 using the maximum likelihood estimator

\[
p_G(w|v_M, ..., v_1) = p(G_2(w)|G_1(v_M, ..., v_1)) \cdot p(w|G_2(w))
\]

\[
= \frac{N(g_1, g_2)}{N(g_1)} \cdot \frac{N(g_2, y)}{N(g_2)},
\]

where \( g_1 = G_1(v_M, ..., v_1) \), \( g_2 = G_2(w) \) and \( N(x) \) denotes the number of times \( x \) occurs in the data. Given these probability estimates \( p_G(w|v_M, ..., v_1) \), the likelihood \( F_{ML} \) of the training data, e.g. the probability of the training data being generated by our probability estimates \( p_G(w|v_M, ..., v_1) \), measures how well the training data is represented by the estimates and can be used as optimisation criterion (39). The likelihood of the training data \( F_{ML} \) is simply

\[
F_{ML} = \prod_{i=1}^{N} p_G(w_i|w_{i-M}, ..., w_{i-1})
\]

\[
= \prod_{i=1}^{N} \frac{N(G_1(w_{i-M}, ..., w_{i-1}), G_2(w_i)) \cdot N(G_2(w_i), w_i)}{N(G_1(w_{i-M}, ..., w_{i-1})) \cdot N(G_2(w_i))}.
\]

Assuming that the classification is unique, e.g. that \( G_1 \) and \( G_2 \) are functions, \( N(G_2(w_i), w_i) = N(w_i) \) always holds (because \( w_i \) always occurs with the same class \( G_2(w_i) \)). Since one is trying to optimise \( F_{ML} \) with respect to \( G \), one can remove any term that does not depend on \( G \), because it will not influence
the optimisation. It is thus equivalent to optimise
\[
F_{ML}' = \prod_{i=1}^{N} \frac{N(G_1(w_{i-M}, ..., w_{i-1}), G_2(w_i))}{N(G_1(w_{i-M}, ..., w_{i-1}))} \cdot \frac{1}{N(G_2(w_i))}
\]
\[
=: \prod_{i=1}^{N} f(w_{i-M}, ..., w_{i-1}, w_i).
\]
If, for two tuples \((w_{i-M}, ..., w_{i-1}, w_i)\) and \((w_{j-M}, ..., w_{j-1}, w_j)\), \(G_1(w_{i-M}, ..., w_{i-1}) = G_1(w_{j-M}, ..., w_{j-1})\) and \(G_2(w_i) = G_2(w_j)\) is true, then \(f(w_{i-M}, ..., w_{i-1}, w_i) = f(w_{j-M}, ..., w_{j-1}, w_j)\) also holds. One can thus regroup identical terms to obtain
\[
F_{ML}' = \prod_{g_1, g_2} \left[ \frac{N(g_1, g_2)}{N(g_1)} \cdot \frac{1}{N(g_2)} \right]^{N(g_1, g_2)}
\]
where the product is over all possible pairs \((g_1, g_2)\). Because \(N(g_1)\) does not depend on \(g_2\) and \(N(g_2)\) does not depend on \(g_1\), one can simplify this again to
\[
F_{ML}' = \prod_{g_1, g_2} N(g_1, g_2)^{N(g_1, g_2)} \prod_{g_1} \frac{1}{N(g_1)} \prod_{g_2} \frac{1}{N(g_2)}^{N(g_2)}
\]
Taking the logarithm, one obtains the equivalent optimisation criterion
\[
F_{ML}'' = \sum_{g_1, g_2} N(g_1, g_2) \log(N(g_1, g_2)) - \sum_{g_1} N(g_1) \log(N(g_1)) - \sum_{g_2} N(g_2) \log(N(g_2)).
\]
\(F_{ML}''\) is the maximum likelihood optimisation criterion which could be used to find a good classifications \(G\). However, the problem with this maximum likelihood criterion is the same as in section 2. In the following, a leaving-one-out criterion is therefore developed.

Let \(T_i\) denote the data without the pair \((w_{i-M}, ..., w_{i-1}, w_i)\) and \(p_{G,T_i}(w|v_M, ..., v_1)\) the probability estimates based on a given classification \(G\) and training corpus \(T_i\). Given a particular \(T_i\), the probability of the “held-out” part \((w_{i-M}, ..., w_{i-1}, w_i)\) is \(p_{G,T_i}(w_i|w_{i-M}, ..., w_{i-1})\). The probability of the complete corpus, where each pair is in turn considered the “held-out” part is the leaving-one-out likelihood \(L_{LO}\)
\[
L_{LO} = \prod_{i=1}^{N} p_{G,T_i}(w_i|w_{i-M}, ..., w_{i-1}).
\]
In the following, we will derive an optimisation function $F_{LO}$ by specifying how $p_{G,T_i}(w_i|w_{i-M},...,w_{i-1})$ is estimated from frequency counts. One first rewrites $p_{G,T_i}(w_i|w_{i-M},...,w_{i-1})$ as usual (see equation 40):

$$p_{G,T_i}(w_i|v_M,...,v_1) = \frac{p_{G,T_i}(g_1,g_2)}{p_{G,T_i}(g_1)} * \frac{p_{G,T_i}(g_2,w)}{p_{G,T_i}(g_2)},$$

(52)

where $g_1 = G_1(v_M,...,v_1)$ and $g_2 = G_2(w)$. Now we will specify how we estimate each term in equation 53.

As before, $p_{G,T_i}$ can be dropped from the optimisation criterion and relative frequencies can be used as estimators for the class unigrams:

$$p_{G,T_i}(g_1) = \frac{N_{T_i}(g_1)}{N_{T_i}}, \quad (54)$$

$$p_{G,T_i}(g_2) = \frac{N_{T_i}(g_2)}{N_{T_i}}. \quad (55)$$

In the case of the class bi-gram, one can again use the absolute discounting method for smoothing. Let $n_{0,T_i}$ be the number of unseen pairs $(g_1, g_2)$ and $n_{+,T_i}$ the number of seen pairs $(g_1, g_2)$, leading to the following smoothed estimate

$$p_{G,T_i}(g_1, g_2) = \begin{cases} \frac{N_{T_i}(g_1, g_2) - b}{N_{T_i}} & \text{if } N_{T_i}(g_1, g_2) > 0 \\ \frac{n_{+,T_i} - b}{n_{0,T_i} + n_{+,T_i}} & \text{if } N_{T_i}(g_1, g_2) = 0 \end{cases}$$

(56)

Again, the empirically determined constant value $b = 0.75$ is used during clustering. The probability distribution $p_{G,T_i}(g_1, g_2)$ will always be evaluated on the “held-out” part $(w_{i-M},...,w_{i-1}, w_i)$ and with $g_{1,i} = G_1(w_{i-M},...,w_{i-1})$ and $g_{2,i} = G_2(w_i)$ one obtains

$$p_{G,T_i}(g_{1,i}, g_{2,i}) = \begin{cases} \frac{N_{T_i}(g_{1,i}, g_{2,i}) - b}{N_{T_i}} & \text{if } N_{T_i}(g_{1,i}, g_{2,i}) > 0 \\ \frac{n_{+,T_i} - b}{n_{0,T_i} + n_{+,T_i}} & \text{if } N_{T_i}(g_{1,i}, g_{2,i}) = 0 \end{cases}$$

(57)
In order to facilitate future regrouping of terms, one again expresses the counts \( N_{Ti}, N_{Ti}(g_1) \) etc. in terms of the counts of the complete corpus \( T \) as follows:

\[
N_{Ti} = N_T - 1 \quad (58)
\]

\[
N_{Ti}(g_1) = N_T(g_1) - 1 \quad (59)
\]

\[
N_{Ti}(g_2) = N_T(g_2) - 1 \quad (60)
\]

\[
N_{Ti}(g_1,i, g_2,i) = N_T(g_1,i, g_2,i) - 1 \quad (61)
\]

\[
N_{Ti} = N_T - 1 \quad (62)
\]

\[
n_{+, Ti} = \begin{cases} 
      n_{+, Ti} & \text{if } N_T(g_1,i, g_2,i) > 1 \\
      n_{+, Ti} - 1 & \text{if } N_T(g_1,i, g_2,i) = 1
   \end{cases} \quad (63)
\]

\[
n_{0, Ti} = \begin{cases} 
      n_{0, Ti} & \text{if } N_T(g_1,i, g_2,i) > 1 \\
      n_{0, Ti} - 1 & \text{if } N_T(g_1,i, g_2,i) = 1
   \end{cases} \quad (64)
\]

After dropping \( p_{G,T,i}(w) \) and substituting the expressions back into equation \( 51 \), one obtains:

\[
F'_{LO} = \prod_{i=1}^{N} p_{G,T,i}(g_1,i, g_2,i) \frac{1}{p_{G,T,i}(g_2,i)} \quad (65)
\]

\[
= \prod_{g_1,g_2} (p_{G,T,i}(g_1, g_2))^{N(g_1, g_2)} \prod_{g_1} \left( \frac{1}{p_{G,T,i}(g_1)} \right)^{N(g_1)} \prod_{g_2} \left( \frac{1}{p_{G,T,i}(g_2)} \right)^{N(g_2)} \quad (66)
\]

One can now substitute equations \( 54, 55 \) and \( 57 \) using the counts of the whole corpus of equations \( 58 \) to \( 64 \). After having dropped terms independent of \( G \), one obtains

\[
F''_{LO} = \prod_{g_1,g_2: N(g_1, g_2) > 1} (N_T(g_1, g_2) - 1 - b)^{N_T(g_1, g_2)} \frac{(n_{+, Ti} - 1) \times b^{N_{Ti}}}{(n_{0, Ti} + 1)} \quad (67)
\]

\[
\times \prod_{g_1} \left( \frac{1}{(N_T(g_1) - 1)} \right)^{N_T(g_1)} \prod_{g_2} \left( \frac{1}{(N_T(g_2) - 1)} \right)^{N_T(g_2)}
\]

where \( n_{1,T} \) is the number of pairs \((g_1, g_2)\) seen exactly once in \( T \) (e.g. the number of pairs that will be unseen when used as “held-out” part). Taking the logarithm, one obtains the final optimisation criterion \( F'''_{LO} \):

\[
F'''_{LO} = \sum_{g_1,g_2: N_T(g_1, g_2) > 1} N_T(g_1, g_2) \times \log(N_T(g_1, g_2) - 1 - b) \quad (68)
\]
\begin{align*}
&+ n_{1,T} \log \left( \frac{b \cdot (n_{1,T} - 1)}{(n_{0,T} + 1)} \right) \\
&- \sum_{g_1} N_T(g_1) \log(N_T(g_1) - 1) - \sum_{g_2} N_T(g_2) \log(N_T(g_2) - 1).
\end{align*}

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