Temperature-dependent quantum electron transport in 2D point contacts

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Abstract
We consider the transmission of electrons through a two-dimensional ballistic point contact in the low-conductance regime near the pinch-off region. The scattering of electrons by Friedel oscillations of charge density results in a contribution to the conductance proportional to the temperature. The sign of this linear term depends on the range of the electron–electron interaction and appears to be negative for the relevant experimental parameters.

( Some figures may appear in colour only in the online journal)

1. Introduction

The effects of electron–electron interaction play an essential role in the transport properties of two-dimensional systems. One of the mechanisms of these effects is scattering of electrons by Friedel oscillations of charge density arising due to inhomogeneities in them. Such oscillations are actually observed in experiments [1]. Recently, it was shown that in a two-dimensional (2D) conductor with impurities, this scattering results in a strong temperature dependence of the conductivity [2, 3]. It was also predicted that it may give rise to a zero-bias anomaly of tunneling into the edge of a 2D electron gas (2DEG) [4]. In a quantum wire the scattering by Friedel oscillations in the regions between the wire and 2D reservoirs results in the formation of quasi-bound electron states [5].

In this paper we consider the contribution to the conductance of a quantum contact of small width $a \ll \lambda_F$ from electron scattering by the Friedel oscillations arising from its walls. Unlike the paper [5] that dealt with interaction effects in the narrowest part of quantum point contacts (figure 1), we address the contribution from the regions outside the constriction and far from the contact. As the Friedel oscillations in a 2DEG rather slowly fall down with distance $x$ from a planar barrier according to the law $x^{-3/2}$, these regions should give the dominant contribution to the scattering.

Recently, we considered contacts much wider than the Fermi wavelength and obtained the temperature-dependent contribution to the conductance [6, 7] due to electron–electron interaction in the semiclassical approximation. The theoretically predicted dependences of conductance on temperature and magnetic field [6, 7] are in good agreement with the experiments [8, 9]. However, the linear in temperature contribution to the conductance persists even for contacts of width approaching the Fermi wavelength [9], where the scattering by quantum oscillations of electron density should be considerable. Therefore it is of interest to compare the contribution from the quantum effect of scattering by Friedel oscillations with the semiclassical one.

The paper is organized as follows. In section 2, we present the model and describe our general formalism. Section 3 addresses the case of noninteracting electrons, and section 4...
describes the perturbation theory. Sections 5 and 6 present the results for a point-like interaction and a generalization for an interaction of a finite range, and section 7 contains a discussion of the results.

2. General approach

We consider the effects of electron–electron interaction on the conductance of a narrow short contact at nonzero temperature. We assume that electron–electron interaction is weak so that it can be treated perturbatively. Throughout the paper we use \( k_B = \hbar = 1 \).

As the Friedel oscillations die out at a large distance from an obstacle determined by the ratio of the Fermi velocity to the temperature \( v_F/T \), we are mainly interested in scattering processes that occur in the leads outside the contact in the Fermi plane and do not focus on the exact dynamics of an electron in the narrowest part of the constriction. Therefore we consider an extremely short contact, namely, we use a typical single-slit diffraction model—a gap of width \( 2a \) in a one-dimensional barrier separating two half-planes of a 2DEG (figure 2). The width of the gap is much smaller than the Fermi wavelength \( 2a \ll \lambda_F \). Though this model of a thin wall with a slit does not show perfect steps of conductance, its dependence on the Fermi momentum exhibits precisely the same washboard-like behavior [10] as is observed in experimental papers [8, 9]. This model geometry allows us to avoid dealing with an infinite number of discrete transverse modes and to use instead the continuous representation.

Note that Friedel oscillations far from the barrier do not depend on the exact shape of the confinement potential because they are formed by electrons near the Fermi level with almost normal incidence on the barrier. Hence a smooth barrier potential should result only in a shift of their phase, which would not essentially change the correction to the conductance (see appendix B).

We obtain the conductance by using the standard Landauer approach [11] and write the conductance as a sum of transmission coefficients

\[
G = g_s \frac{e^2}{h} \int \frac{dk}{2\pi} \left( \frac{\partial f}{\partial \epsilon} \right) \sum_{\mathbf{k}, \mathbf{q}} |t(\mathbf{k}, \mathbf{q})|^2. \tag{1}
\]

Here \( g_s \) is a spin degeneracy and \( t(\mathbf{k}, \mathbf{q}) \) is the transmission amplitude from the mode with the wavevector \( \mathbf{k} \) in the left half-plane to the mode with the wavevector \( \mathbf{q} \) to the right half-plane.

First of all we calculate \( t = t_0 \) and \( G = G_0 \) for non-interacting electrons. A weak electron–electron interaction results in a scattering of electrons by the Friedel oscillations caused by the contact boundaries. We consider the oscillations arising from the barrier as one-dimensional and neglect their distortion by the gap because this effect is of higher order in the contact size. The incident electron is scattered by the Friedel oscillations before and after passing through the contact, which results in a correction to the transmission coefficient of the contact \( t(\mathbf{k}, \mathbf{q}) = t_0(\mathbf{k}, \mathbf{q}) + \delta t(\mathbf{k}, \mathbf{q}) \). The correction to the transmission coefficient may be obtained by expanding the perturbation of the wavefunction \( \delta \psi \) in plane waves. To calculate this perturbation, we solve a Schrödinger-type equation

\[
\left[ -\frac{\hbar^2}{2m} \nabla^2 + V_{\text{eff}}(\mathbf{r}) \right] \psi(\mathbf{r}) = \epsilon \psi(\mathbf{r}), \tag{2}
\]

in an iterative way [12] and obtain \( \delta \psi(\mathbf{r}) \) in the lowest order in the interaction.

As the interaction is assumed to be weak, we can treat it in the Hartree–Fock approximation and neglect the correlation energy. Hence the interaction potential induced by the Friedel oscillations may be written as a sum of a direct term and an exchange one [13]

\[
V_{\text{eff}}(\mathbf{r}) = V_H(\mathbf{r}) - V_F(\mathbf{r}),
\]

where

\[
\begin{align*}
V_H(\mathbf{r}) &= g_s \int d\mathbf{r}_1 U_{ee}(\mathbf{r} - \mathbf{r}_1) n(\mathbf{r}_1, \mathbf{r}_1), \quad (3) \\
V_F(\mathbf{r}) \psi(\mathbf{r}) &= \int d\mathbf{r}_1 U_{ee}(\mathbf{r} - \mathbf{r}_1) n(\mathbf{r}, \mathbf{r}_1) \psi(\mathbf{r}_1), \quad (4)
\end{align*}
\]

where \( n(\mathbf{r}, \mathbf{r}_1) = \langle \hat{\psi}^+(\mathbf{r}_1) \hat{\psi}(\mathbf{r}) \rangle \) is the electron density matrix, \( \hat{\psi}^+ \) and \( \hat{\psi} \) are electron creation and annihilation operators, and \( U_{ee}(\mathbf{r} - \mathbf{r}_1) \) is the potential of the electron–electron interaction. Typically, it is the Coulomb interaction screened by the two-dimensional electrons and by the gate. The coefficient of spin degeneracy \( g_s \) appears only in the direct term because it involves interactions between electrons with both spin directions while the exchange interaction is possible only for electrons with the same spin.

Equations (3) and (4) result in a correction to the wavefunction in the form \( \delta \psi = \delta \psi_H - \delta \psi_F \) and, accordingly, in a correction to the conductance

\[
\delta G = \delta G_H - \delta G_F. \tag{5}
\]

The negative sign in the exchange term is explicitly shown here.
3. Noninteracting electrons

In the absence of electron–electron interaction the conductance calculation reduces to the standard problem of diffraction by a narrow gap of width $2a \ll \lambda_F$. For the three-dimensional (3D) case it was considered many times [14] and the conductance of a small 3D ballistic contact [15] was found to be proportional to the sixth power of the contact size $G \propto (k_F a)^6$.

To the best of our knowledge, the 2D problem in the limit of $a \ll \lambda_F$ was considered only once for a specifically designed model of the contact [16]. In the limit where the product of the Fermi momentum and the gap width is much smaller than unity $k_F a \ll 1$, these authors obtained $G \propto 1/\ln^2(k_F a)$, which is unphysical. Therefore we recalculate this quantity using the solution of the problem of diffraction from a narrow slit obtained many decades ago in optics [17].

We use an approach similar to Sommerfeld [17] and reduce the solution of the Schrödinger equation (2) with $V_{\text{eff}} = 0$ to a boundary-value problem. The total wavefunction may be presented in the form

\[
\begin{align*}
\psi(r') &= \psi_0(r') + \psi_t(r'), \\
\psi(r') &= \psi_t(r'),
\end{align*}
\]

where $\psi_0$ is the wavefunction in the absence of the gap and $\psi_t$ is the lowest-order correction in the gap size. The zero-order wavefunction $\psi_0$ obeys zero boundary conditions both at the barrier and the gap, while the correction $\psi_t$ obeys the zero boundary condition at the barrier and a nonzero boundary condition at the gap

\[
\begin{cases}
(V^2 + k^2) \psi_t(x, y) = 0, \\
\psi_t(x, y)|_{x=0, y \in (-a, a)} = \chi(0, y),
\end{cases}
\]

where $k^2 = 2mE$.

We assume that the incoming plane wave $\psi_t(r) = \sqrt{m/k_F} e^{ik_r x + i k_y y}$ with $k_x^2 + k_y^2 = k^2$ falls on the contact from the left in the $x$ direction. Hence the transmitted wavefunction has a form

\[
\psi_t(r) = \int_{-\infty}^{\infty} \frac{dq_y}{2\pi} \frac{1}{\sqrt{q_y}} e^{ik_y q_y} I_0(k_F, q_y).
\]

To obtain $I_0(k, q)$ we find the boundary condition $\chi(y)$ self-consistently using the continuity of the derivative of the total wavefunction at the gap (see appendix A). Expanding the transmitted wavefunction in plane waves allows us to obtain the transmission coefficient for noninteracting electrons

\[
I_0(k, q) = -\frac{\pi}{2} a^2 \sqrt{k_F q_y}.
\]

We substitute it in equation (1) and obtain

\[
G_0 = \frac{e^2}{h} \frac{\pi}{128} k_F^2 a^4 + O \left( \frac{T^2}{k_F^2} \right).
\]

This contact-size dependence is more physically plausible than that of [16] because it corresponds to the 2D analog of the Rayleigh scattering of light by small particles [14]. Indeed, the conductance is proportional to the square of the 2D particle volume.

4. Perturbation theory

Now we take into account a weak electron–electron interaction. This interaction leads to a scattering of electrons by the Friedel oscillations induced by the barrier and results in a correction $\delta t(k, q)$ to the transmission coefficient. We substitute it in Landauer formula (1) and obtain the correction to the conductance in the case of weak interaction

\[
\delta G = -2G_0 e^2 \frac{\pi}{h} \int \frac{dk}{2\pi} \left( \frac{\partial F}{\partial k} \right) \sum_{k, q} |t_0(k, q)|^2 \delta t(k, q).
\]

Here we take into account the fact that $t_0(9)$ is an imaginary quantity. The correction to the transmission coefficient is conveniently expressed in terms of the correction to the wavefunction by expanding it in plane waves. The wavefunction is found by solving the Schrödinger equation (2) in the lowest order in the interaction. To this end, we isolate the term with $V_{\text{eff}}$ in the right-hand side and substitute the unperturbed wavefunction into it. The solution is given by

\[
\delta \psi(r) = \int dr' g(r, r') V_{\text{eff}}(r') \psi(r').
\]

Here $V_{\text{eff}}(r')$ is the scattering potential produced by the Friedel oscillations, $g(r, r')$ and $\psi(r')$ are the single-electron Green function and the total wavefunction for noninteracting electrons. We assume that the electrons are incident on the contact from the left and we measure the total current on the right, where $x > 0$. We are interested in the entire range of values of $x' \in (−\infty, \infty)$ because we consider the scattering by the Friedel oscillations on both sides of the contact. Similarly to the wavefunction (6), the one-electron Green function may be written in the form

\[
\begin{align*}
g(r, r') &= g_t(r, r'), \\
g(r, r') &= g_0(r, r') + g_t(r, r'),
\end{align*}
\]

where $g_0$ is the Green function in the absence of the gap, and $g_t \propto a^2$ is the second-order correction in the gap size. We calculate $g_t$ similarly to $\psi_t$ (see appendix A) by solving the system

\[
\begin{cases}
\frac{\hbar^2}{2m} (V^2 + k^2) g(r, r') = \delta(r - r'), \\
g(r, r')|_{x=0, y \in (-a, a)} = \chi(0, y, x'),
\end{cases}
\]

We substitute (6) and (13) into (12) and obtain in the lowest order in the contact size

\[
\delta \psi(r) = \int_{x' < 0} dr' g_t(r, r') V_{\text{eff}}(r') \psi_0(r') + \int_{x' > 0} dr' g_0(r, r') V_{\text{eff}}(r') \psi_t(r').
\]

The first term corresponds to electron scattering by Friedel oscillations in front of the contact, and the second one—behind it.

We substitute the expressions for the interaction potential in the Hartree–Fock approximation (3) and (4) into (15) and obtain the conductance as a sum of direct and exchange terms

\[
\delta G = \delta G_H - \delta G_F.
\]

Then we substitute $\psi_0$, $\psi_t$, $g_0$, and $g_t$ into
the resulting expression and after some simplifications obtain the conductance for an arbitrary interaction potential in the form

\[ \delta G_H = -g_s^2 \frac{e^2 m}{\hbar^2} \frac{1}{16} \int dx \left( -\frac{\partial f}{\partial \varepsilon} \right) k^2 \int_{-\infty}^{\infty} dy_1 \int_{0}^{\infty} dx_1 \]

\[ \times \int_{0}^{\infty} dx' U_{ee}(x' - x_1, -y_1) n(x_1) \]

\[ \times \int_{-\infty}^{\infty} dq_y \sin(2q_y x') \]  
\[ \times \int_{-\infty}^{\infty} dx \left( -\frac{\partial f}{\partial \varepsilon} \right) k^2 \int_{-\infty}^{\infty} dy_1 \]

\[ \times \int_{0}^{\infty} dx_1 \int_{0}^{\infty} dx' U_{ee}(x' - x_1, -y_1) \]

\[ \times n(x', x_1, y_1 + y') \]

\[ \times \int_{-\infty}^{\infty} dq_y \sin(q_y x') \cos(q_y y_1) \cos(q_y x_1). \]  

We use the coordinate transform \( y_1 \to y_1 + y' \) to make the interaction potential independent of \( y' \) and then integrate over \( y' \). This transform results in the independence of the Friedel oscillations of density on \( y' \) because we obtain the corrections in the lowest approximation in the contact size and use the unperturbed wavefunctions in the absence of the gap to calculate \( n \) (see appendix B)

\[ n(x', x_1, y_1 + y') = \frac{1}{2\pi} \int_{0}^{\infty} dp \left( -\frac{\partial f}{\partial p} \right) p \]

\[ \times \left[ \frac{J_1(p\sqrt{(x' - x_1)^2 + y_1^2})}{\sqrt{(x' - x_1)^2 + y_1^2}} \right. \]

\[ \left. - \frac{J_1(p\sqrt{(x' + x_1)^2 + y_1^2})}{\sqrt{(x' + x_1)^2 + y_1^2}} \right]. \]  

By setting \( x' = x_1 \) and \( y_1 = 0 \) in this expression, it is easy to obtain the electron density \( n(x_1) = n(r_1, r_1) \), which is responsible for the direct interaction term and depends only on one coordinate

\[ n(x_1) = \frac{k_1^2}{4\pi} - \frac{1}{2\pi} \int_{0}^{\infty} dp \left( -\frac{\partial f}{\partial p} \right) p \frac{J_1(2px_1)}{2x_1}. \]  

The first term here presents a uniform charge density and the second one describes its oscillations with a period \((2k_F)^{-1}\) at large distances from the barrier that decay according to the law \( x^{-3/2} \) at zero temperature. At nonzero temperature, they exponentially decay at a characteristic length \( v_F/T \).

5. Point-like interaction potential

Consider now the case of a point-like interaction potential \( U_{ee}(x' - x_1, -y_1) = U_p \delta(x' - x_1) \delta(y_1) \). A comparison of equations (16) and (18) shows that \( \delta G_H = g_s \delta G_F \). Therefore \( \delta G = (g_s - 1) \delta G_F \). Upon an integration over \( x_1, y_1 \) and \( q_y \), one obtains the correction in the form

\[ \delta G = \frac{1 - g_s}{g_s} g_s \frac{e^2 m}{\hbar^2} \frac{\pi}{16} \int dx \left( -\frac{\partial f}{\partial \varepsilon} \right) k^2 \int_{-\infty}^{\infty} dx' n(x') J_1(2k_Fx'). \]  

With \( n(x') \) from (20) substituted into this expression, it is easily seen that the main contribution to it is given by values \( x' \sim v_F/T \); i.e. by the ‘tail’ of the Friedel oscillations far from the barrier. We integrate over \( x' \) and \( p \) and calculate the total transmission coefficient \( \delta T(\varepsilon) \). It is a sum of two parts \( \delta T_{\text{const}}(\varepsilon) + \delta T_{\text{osc}}(\varepsilon) \) formed by the constant and the oscillating part of the electron density (20), respectively. The term \( \delta T_{\text{const}}(\varepsilon) \) is a smooth function without singularities, whereas \( \delta T_{\text{osc}}(\varepsilon) \) has a cusp at the Fermi surface of the form

\[ \delta T_{\text{osc}}(\varepsilon) \propto \frac{x}{E_F} \left[ \frac{\varepsilon}{E_F} - \frac{T}{E_F} \ln \left( 1 + \frac{\varepsilon - E_F}{T} \right) \right]. \]  

At \( T/E_F \ll 1 \), the derivative of the last term with respect to \( \varepsilon/E_F \) tends to 2 at \( \varepsilon = E_F - 0 \) and to 1 at \( \varepsilon = E_F + 0 \). This cusp (see figure 3) results in a linear temperature dependence of conductance

\[ \delta G = \frac{1 - g_s}{g_s} g_s \frac{e^2 m}{\hbar^2} \frac{\pi}{16} \frac{k_F^2 a^4 U_p}{k_F^2 a^4 U_p T}. \]  

Alternatively, this temperature dependence may be attributed to the temperature-dependent cutoff length of the Friedel oscillations.

6. Arbitrary interaction potential

6.1. Direct interaction

An isotropic finite-range interaction potential is conveniently described by its Fourier components \( U_p(p) \), which depend only on the absolute value of \( p \). Similarly to the case of a point interaction potential, the substitution of the two terms in (20) that correspond to the constant and the oscillating parts of the charge density into (16) results in a sum of two terms \( \delta G_H = \delta G_{H,\text{const}} + \delta G_{H,\text{osc}} \). The first term is easily calculated...
and equals
\begin{equation}
\delta G_{H,\text{const}} = -g_s^2 e^2 m \frac{1}{\hbar^2} \frac{1}{128} k^4 a^4 U_p(0) + O\left(e^{-E_F/T}\right). \tag{24}
\end{equation}

Here \(U_p(0)\) is the Fourier transform of the interaction potential. After some simplifications, the second term may be brought to the form
\begin{equation}
\delta G_{H,\text{osc}} = g_s^2 e^2 m \frac{1}{\hbar^2} \frac{1}{128\pi} a^4 \int \left(-\frac{\partial f}{\partial k}\right) k^3 \times \int_0^\infty dp \left(-\frac{\partial f}{\partial p}\right) p \int dp_1 \times U_p(p_1) I_1(k, p, p_1), \tag{25}
\end{equation}

where we have introduced the notation
\begin{equation}
I_1(p > k) = \frac{\theta(p_1 - 2k)}{4kp} \left[\frac{\sqrt{4p^2 - p_1^2} + p_1^2}{\sqrt{4k^2 - p_1^2}} \right] - \frac{\theta(p_1 - 2p) \theta(p_1 - 2k)}{p_1 + \sqrt{p_1^2 - 4k^2}} k \left[\frac{4p^2 - p_1^2}{p_1^2 - 4k^2}\right]. \tag{26}
\end{equation}
\begin{equation}
I_1(p < k) = \frac{\theta(p_1 - 2p) \theta(p_1 - 2k)}{4kp} \left[\frac{\sqrt{4p^2 - p_1^2} + p_1^2}{\sqrt{4k^2 - p_1^2}} \right] + \frac{\theta(p_1 - 2p) \theta(p_1 - 2k)}{k \sqrt{4p^2 - p_1^2}} \left[\frac{p_1^2}{p_1 + \sqrt{p_1^2 - 4p^2}} \right]. \tag{27}
\end{equation}
The quantity \(I_1(k, p, p_1)\) has singularities at \(p_1 = 2k\) and \(p_1 = 2p\), and the derivatives of the distribution function in (25) cut out narrow intervals of \(k\) and \(p\) of width \(\sim k_F T/E_F\) near \(k_F\). As we assume the potential \(U_p\) to be a smooth function of \(p\) at the scale \(T/E_F\), we can isolate the singular part of the integrand and substitute \(U_p(p_1) = U_p(2k_F)\) in it, while setting \(I_1(k, p, p_1) = I_1(k = k_F, p = k_F, p_1)\) in its regular part
\begin{equation}
U_p(p_1) I_1(k, p, p_1) \approx U_p(2k_F) I_1(k, p, p_1) + \left[U_p(p_1) - U_p(2k_F)\right] \times I_1(k = k_F, p = k_F, p_1). \tag{28}
\end{equation}

We calculate both terms and obtain the correction due to the direct interaction in the form
\begin{equation}
\delta G_H = -g_s^2 e^2 m \frac{1}{\hbar^2} \frac{1}{128} k^4 a^4 U_p(2k_F) \frac{T}{E_F} + g_s^2 e^2 m \frac{1}{\hbar^2} \frac{1}{64\pi} k^4 a^4 \int_0^{2k_F} dp_1 U_p(p_1) - U_p(0) \times \frac{\sqrt{4k_F^2 - p_1^2}}{\sqrt{4k_F^2 - p_1^2}}. \tag{29}
\end{equation}
The first term here presents the contribution linear in temperature and is proportional to the Fourier component of the interaction potential at \(2k_F\) while the second one presents the temperature-independent contribution and vanishes if \(U_p(p)\) is a constant.

6.2. Exchange interaction
The substitution of the two terms of (19) and (18) gives the exchange contribution to the conductance in a form \(\delta G_F = \delta G_{F,\text{const}} + \delta G_{F,\text{osc}}\) in analogy with \(\delta G_H\). The first term is easily calculated and equals
\begin{equation}
\delta G_{F,\text{const}} = -g_s^2 e^2 m \frac{1}{\hbar^2} \frac{1}{64\pi} k^4 a^4 \int_0^{2k_F} dp_1 U_p(p_1) \times p_1 \arccos\left(\frac{p_1}{2k_F}\right) + O\left(e^{-E_F/T}\right). \tag{30}
\end{equation}

After a simple rearrangement, the second term may be brought to the form
\begin{equation}
\delta G_{F,\text{osc}} = g_s^2 e^2 m \frac{1}{\hbar^2} \frac{1}{32\pi} a^4 \int \left(-\frac{\partial f}{\partial k}\right) k^2 \times \int_0^\infty dp \left(-\frac{\partial f}{\partial p}\right) \int_0^p dp_1 \int_0^k dq \times U_p(p_1 - q) \ln\left|\frac{q_3 + p_3}{q_3 - p_3}\right|. \tag{31}
\end{equation}

This term has a singularity at \(p_3 = q_3\), because the backscattering of electrons is most efficient if the \(x\) component of the electron momentum \(q_3\) coincides with the wavevector \(p_3\) of the Friedel oscillations. We write the integrand as a sum of two terms, one of which has a singularity at \(p_3 = q_3\), and the second one is a regular function, so that one may set \(k = p = k_F\) in it to obtain
\begin{equation}
U_p(p_3 - q_3) \ln\left|\frac{q_3 + p_3}{q_3 - p_3}\right| \approx U_p(0) \ln\left|\frac{q_3 + p_3}{q_3 - p_3}\right| + [U_p(p_3 - q_3) - U_p(0)] \ln\left|\frac{q_3 + p_3}{q_3 - p_3}\right| \bigg|_{p_3 = k_F}. \tag{32}
\end{equation}

We perform the integration in (31), sum the result with (30) and obtain the correction due to an exchange interaction in the form
\begin{equation}
\delta G_F = -g_s^2 e^2 m \frac{1}{\hbar^2} \frac{1}{128} k^4 a^4 U_p(0) \frac{T}{E_F} + \delta G_{F,T=0}. \tag{33}
\end{equation}

where \(\delta G_{F,T=0}\) is a temperature-independent quantity given by an integral
\begin{equation}
\delta G_{F,T=0} = g_s^2 e^2 m \frac{1}{\hbar^2} \frac{1}{32\pi^2} k^4 a^4 \int_0^{2k_F} dp_1 U_{ee}(p_1) \times \left[K\left(1 - \frac{p_1^2}{4k_F^2}\right) - E\left(1 - \frac{p_1^2}{4k_F^2}\right) - \pi \frac{p_1}{2k_F} \arccos\left(\frac{p_1}{2k_F}\right)\right] \tag{34}
\end{equation}
where \(K\) and \(E\) are full elliptic integrals of the first and second kind. The temperature-dependent correction to the conductance in (33) is determined by the long-wavelength component of the interaction potential, which is typical for the exchange interaction [18, 2]. However \(\delta G_{F,T=0}\) is determined.
by all components of $U_p$ from 0 to $2k_F$. Long-wavelength components contribute to (34) with a positive sign and short-wavelength components contribute to it with a negative sign, so that the integral is zero if $U_p$ is constant.

7. Discussion

The presence of a barrier in a 2DEG results in Friedel oscillations of electron density with a period of half of the Fermi wavelength that slowly decay with distance from the barrier. The cutoff length $v_F/T$ for them is set by the temperature. These oscillations result in an efficient backscattering of electrons incident on the Fermi level near pinch-off. The estimate for realistic values of distance $d$ leads to a large cutoff length due to a scattering by impurities or a finite size of the sample, so that $\delta G(G_0)_{\text{semi}}$ is roughly proportional to $G_0$. Should this correction be extrapolated to narrow contacts, it would be proportional to $G_0^2$ because both the number of injected and incident electrons is proportional to $G_0$. Hence the correction from Friedel oscillations must dominate at small contact widths.

The quantum correction from the Friedel oscillations is more sensitive to the shape of the interaction potential than the semiclassical one. In particular, its sign is determined by the factor $|U_p(0) - 2U_p(2k_F)|$ for the interaction potential between the electrons in the gas is given by (see appendix C)

$$U(q) = \frac{4\pi e^2}{\varepsilon_d (\coth |qd| + 1) |q| + 4\pi e^2 v_2},$$

which leads to the correction of the form

$$\frac{\delta G_T}{G_0} = T \frac{2\kappa_2 d}{E_F} \left[ 1 + \frac{2\kappa_2}{2k_F \coth(2k_F d) + 1} + \kappa_2 \right].$$

where $\kappa_2$ is the inverse screening length. Figure 4 shows the regions in the $(k_F d, \kappa_2 d)$ plane where the correction is positive or negative.

Unfortunately, we are unaware of detailed measurements of the temperature dependence of the contact conductance near pinch-off. The estimate for realistic values of distance $d = 100$ nm, the inverse screening length $\kappa_2 = 2\pi e^2 v_2/\varepsilon_d = 1.93 \times 10^6$ cm$^{-1}$ and electron density $n_s = \ldots$
10^{-10} \text{ cm}^{-2} \text{ for temperature 1 K results in } \delta G_T/G_0 = -15\% . Measurements of conductance in the region of the 0.7 anomaly, indeed, reveal a negative slope of G(T) dependence [19]. However, a quantitative comparison with our predictions is not possible.

In the multichannel regime, this negative correction should be suppressed by the positive one from the scattering of oppositely moving electrons [6], and one should observe a change in the sign of the slope of its temperature dependence. However, if the actual \( k_2 \) is smaller due to the low electron concentration, the sign of the correction from the electron–electron interaction may remain positive for all contact sizes. Therefore one may estimate the actual screening length from the sign and slope of the temperature dependence of conductance for narrow contacts.

Though our results were obtained for a sharp potential in the form of an infinitely narrow and very high barrier, they should also survive for reasonably smooth potentials. It is easily seen that the smoothness of potential results only in a phase shift of the Friedel oscillations and preserves the \( x^{-3/2} \) dependence of their amplitude far from the barrier (see appendix B). That is why the linear temperature dependence and change of sign of slope of G(T) should be robust with respect to the exact shape of the barrier.

In summary, we calculated the conductance of a narrow and short quantum point contact at nonzero temperature taking into account the electron–electron interaction. The conductance linearly depends on temperature and is proportional to the fourth power of the contact size, and the relative correction does not depend on the contact size. The sign of the linear temperature-dependent term depends on the competition between direct and exchange interactions. Measurements of the slope of its temperature dependence allow one to determine the parameters of electron–electron interaction.

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**Appendix A. Calculation of transmitted wavefunction**

We use the approach described by Sommerfeld [17] for the problem of 2D diffraction and solve the system (7) assuming a continuity of the derivative of the wavefunction \( \psi = \psi_0 + \psi_t \) at \( x = 0 \), where \( \psi_0 \) is the wavefunction in the absence of the gap and \( \psi_t \) is the correction in the lowest order in a contact size. We expand the unknown function into a Fourier integral

\[
\psi_t(x, y) = \int_{-\infty}^{\infty} \frac{dk_y}{2\pi} e^{-ik_y y} \psi_t(x, k_y) \tag{A.1}
\]

and then search a solution in the form of outgoing waves \( \psi_t(x, k_y) = c_1 \times e^{-ik_y x} \). The constant \( c_1 \) is easily expressed from the boundary condition

\[
c_1 = \int_{-\infty}^{\infty} dy' e^{ik_y y'} \chi(0, y'). \tag{A.2}
\]

We sequentially substitute these formulas into (A.1) and obtain

\[
\psi_t(x, y) = \int_{-\infty}^{\infty} dy' \chi(0, y') K(x, y, y'), \tag{A.3}
\]

where the kernel is given by

\[
K(x, y, y') = -\frac{i}{2} k_x \frac{H_1^{(1)}(k\sqrt{x^2 + (y-y')^2})}{\sqrt{x^2 + (y-y')^2}}. \tag{A.4}
\]

The condition of continuity of the derivative of total wavefunction at the gap may be written as

\[
\frac{\partial \psi_0(x, y)}{\partial x} \bigg|_{x=0^-} + \frac{\partial \psi_t(x, y)}{\partial x} \bigg|_{x=0^+} = \frac{\partial \psi_t(x, y)}{\partial x} \bigg|_{x=0}, \tag{A.5}
\]

where

\[
\psi_0(x, y) = \frac{m}{\sqrt{k_x}} e^{ik_y y} (e^{ik_y x} - e^{-ik_y x}). \tag{A.6}
\]

We substitute the kernel (A.4) into (A.5) and obtain an integral equation

\[
\int_{-a}^{a} dy' \chi(0, y') \frac{kH_1^{(1)}(k|y-y'|)}{|y-y'|} = 2 \frac{m}{\sqrt{k_x}} e^{ik_y y}. \tag{A.7}
\]

This integral equation appears in the problem of 2D diffraction by a narrow slit [17]. We are interested in the case of \( k \approx k_f \), so the limit \( k_f a \ll 1 \) is equivalent to \( ka \ll 1 \). The solution of the integral equation in the limit of \( ka \ll 1 \) is given by [20]

\[
\chi(0, y') = -i k_x \sqrt{\frac{m}{k_x}} \sqrt{a^2 - y'^2}. \tag{A.8}
\]

We substitute (A.8) and (A.4) into (A.3) and obtain

\[
\psi_t(x, y) = \frac{\pi}{4} k_x ka |x| \sqrt{\frac{m}{k_x}} \frac{H_1^{(1)}(kr)}{r}. \tag{A.9}
\]

It is conveniently presented in the form

\[
\psi_t(x, y) = -\frac{i \pi}{2} \sqrt{\frac{m}{k_x}} a^2 k_x \int_{-\infty}^{\infty} \frac{dk}{2\pi} e^{ik(x+kr)}. \tag{A.10}
\]

In a similar way we solve the system (14) for the Green’s function of the Schrödinger equation \( g_t \), and obtain

\[
g_t = \frac{\pi}{8} \frac{m}{\hbar^2} k_x a^2 |x| \sqrt{\frac{m}{k_x}} \frac{H_1^{(1)}(kr)H_1^{(1)}(kr')}{rr'}. \tag{A.11}
\]

**Appendix B. Friedel oscillations of electron density**

The electron density matrix is given by

\[
n(r, r_1) = \langle \hat{\psi}^+(r_1) \hat{\psi}(r) \rangle = \sum_{\alpha} f(\epsilon_\alpha) \psi_\alpha^*(r_1) \psi_\alpha(r), \tag{B.1}
\]
where \( \psi \) are the eigenfunctions of the noninteracting Hamiltonian. In the lowest approximation of the contact size we neglect the distortion of the Friedel oscillations of electron density due to the gap and consider them as arising from a solid barrier. Because of the translational symmetry in the direction parallel to the barrier we may use instead of \( \alpha \) momentum \( p \) and the eigenfunctions

\[
\psi_p(r) = e^{i p \cdot r} \left[ e^{i p \cdot r} + r(p_o) e^{-i p \cdot r} \right]. \tag{B.2}
\]

With the help of the distribution function \( f(p) \), equation (B.1) may be rewritten in the form

\[
n(r, r_1) = \int \frac{d^2 p}{(2\pi)^2} f(p) \psi_p^* (r_1) \psi_p(r). \tag{B.3}
\]

In the case of a sharply high barrier, the reflection coefficient \( r(p_o) = -1 \) for any \( p_o \) and the substitution of \( \psi_p(x, y) \) into (B.3) gives us, after simple rearrangements

\[
n(r, r_1) = 4 \int \frac{d^2 p}{(2\pi)^2} \cos[p_x(y - y_1)] \times \sin[p_x x] \sin[p_x x_1]. \tag{B.4}
\]

Then we go to cylindrical coordinates \( (p, \varphi) \), expand the product of trigonometric functions into a sum of four terms and perform the integration over \( \varphi \). The remaining integral over \( \varphi \) is a sum of four integrals of the form

\[
\int_{-\pi/2}^{\pi/2} \cos(a \cos \varphi + b \sin \varphi) = \pi J_0(\sqrt{a^2 + b^2}). \tag{B.5}
\]

Calculations give the electron density matrix in the form

\[
n(r, r_1) = \int_0^{\infty} \frac{dp}{2\pi} \left[ J_0(p \sqrt{(x-x_1)^2 + (y-y_1)^2}) - J_0(p \sqrt{(x+x_1)^2 + (y+y_1)^2}) \right]. \tag{B.6}
\]

Upon integrating by parts, one arrives at

\[
n(r, r_1) = \frac{1}{2\pi} \int_0^{\infty} dp \left[ \frac{\partial f}{\partial p} \right] \times p \left[ J_1(p \sqrt{(x-x_1)^2 + (y-y_1)^2}) \right.
\]

\[
- \left. J_1(p \sqrt{(x+x_1)^2 + (y+y_1)^2}) \right]. \tag{B.7}
\]

In the case of \( r_1 = r \) the Friedel oscillations of density depend only on \( x \), so

\[
n(r, r) = n(x) = \frac{1}{2\pi} \int_0^{\infty} dp \left( -\frac{\partial f}{\partial p} \right) \times \left[ \frac{p^2}{2} - \frac{p J_1(2px)}{2x} \right]. \tag{B.8}
\]

In the case of a smooth yet impenetrable barrier the wavefunction \( \psi_p \) away from it still may be presented in the form (B.2) with a momentum-dependent reflection coefficient \( r(p_o) = -\exp[i \delta(p_o)] \), where \( \delta(p_o) \) presents the phase shift of the reflected wave with respect to the case of zero boundary conditions at \( x = 0 \). We substitute (B.2) into equation (B.3) and consider the coordinate-dependent oscillating part of electron density. With the help of cylindrical coordinates \( (p, \varphi) \) we can write it in the form

\[
n_{osc}(r, r) = -\frac{1}{\pi^2} \int_0^{\infty} dp f(p) \int_{0}^{\pi/2} dp \cos(2px \cos \varphi + \delta(p \cos \varphi)). \tag{B.9}
\]

At large distances from the barrier \( 2px \gg 1 \) we can estimate \( n_{osc}(r, r) \) using the stationary phase method

\[
n_{osc}(r, r) = -\frac{1}{\pi^2} \int_0^{\infty} dp f(p) \left[ \frac{\pi}{px} \cos(2px - \pi/4 + \delta(p)) \right.
\]

\[
+ \left. \left( \frac{1}{2px} \right)^{3/2} \right]. \tag{B.10}
\]

If \( \delta \partial \delta(p)/\partial p \ll 2x \), it is possible to integrate this equation by parts and to obtain

\[
n_{osc}(r, r) = -\frac{1}{4(\pi x)^{3/2}} \int_0^{\infty} dp \left( -\frac{\partial f}{\partial p} \right) \times p^{1/2} \cos(2px + \delta(p) + \pi/4). \tag{B.11}
\]

It is easily seen that at low temperatures and far from the contact, \( n_{osc} \) exhibits the same asymptotic behavior as the oscillating part of equation (B.5) except for the phase shift \( \delta(p) \).

We elucidate now the conditions for validity of (B.11). If the screening in the system is 2D and the potential of the barrier falls off according to a power law, \( \delta \partial \delta(p)/\partial p \sim x_0(p) \), where \( x_0(p) \) is the classical turning point for the electrons at the barrier. Even if the width of the barrier \( 2x_0(p_F) \) is of the order of the Fermi wavelength, there is a large interval of distances \( p_F^{-1} \ll x \ll p_F/T \) where equation (B.11) holds, and it is precisely this interval that dominates the temperature-dependent contribution from Friedel oscillations to the conductance.

### Appendix C. Screened Coulomb potential

We calculate the Coulomb potential screened by a gate and 2D electrons. Consider the system shown in figure C.1 with a positively charged particle \( e_0 \) at point \( (0, 0, 0) \). The total potential induced by the charged particle, the 2D electrons, and the gate satisfies the Poisson equation

\[
-\nabla^2 \phi(r_1, z) = \frac{4\pi}{e} \left[ \rho^{ext} + \rho^{ind} \right]. \tag{C.1}
\]

where \( \rho^{ext} = e_0 \delta(r_1) \delta(z) \) is the density of the particle charge and \( \rho^{ind} = -e^2 v_F^2 \delta(z) \phi(r) \) is the density of the induced charge calculated in the Thomas–Fermi approximation. We take a Fourier transform of (C.1) with respect to the in-plane coordinates and integrate it with respect to \( z \) over a small
vicinity of $z = 0$ to arrive at an equation
\[ \frac{\partial \phi(q, z)}{\partial z} \bigg|_{z=0} = -2\kappa_2\phi(q, 0) = -\frac{4\pi e_0}{\varepsilon}. \] (C.2)

Here $\kappa_2 = 2/a_B$ is the inverse 2D screening length. We assume that the layer of 2DEG is thin and the potential is continuous at $z = 0$, i.e.
\[ \phi(q, z = 0) = \phi(q, z = +0). \] (C.3)

We solve equations (C.2) and (C.3) with boundary conditions
\[ \phi(q, d) = 0 \] (C.4)
\[ \phi(q, -\infty) = 0 \] (C.5)

and write the potential in the form
\[ \phi(q, 0) = \frac{4\pi e_0}{\varepsilon} \left( \frac{1}{\coth(qd) + 1} + q + 2\kappa_2 \right). \] (C.6)

The potential in the coordinate space is obtained by the inverse Fourier transform of equation (C.6) and may be conveniently expressed in terms of a dimensionless coordinate $x = qr_\parallel$
\[ \phi(r_\parallel, 0) = \frac{2e_0}{\varepsilon r_\parallel} \int dx \frac{J_0(x)}{\coth(xd/r_\parallel) + 1 + 2\kappa_2 r_\parallel / x}. \] (C.7)

In experiments [8, 9] the case of $d \gg \kappa_2^{-1}$ is realized. An evaluation of the integral (C.7) gives for different limiting cases

\[ \phi(r_\parallel, 0) \begin{cases} \frac{e_0}{2\kappa_2^2 r_\parallel^2}, & r_\parallel \gg d \gg \kappa_2^{-1} \\ \frac{e_0}{\varepsilon r_\parallel}, & d \gg r_\parallel \gg \kappa_2^{-1} \\ \frac{e_0}{\varepsilon r_\parallel}, & d \gg \kappa_2^{-1} \gg r_\parallel. \end{cases} \] (C.8)

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