Possible triply heavy tetraquark states

Xuejie Liu,* Yue Tan,** Dianyong Chen¹,³,‡ Hongxia Huang¹,¶ and Jialun Ping⁴,∗∗
¹School of Physics, Southeast University, Nanjing 210094, P. R. China
²School of Mathematics and Physics, Yancheng Institute of Technology, Yancheng, 224051, P. R. China
³Lanzhou Center for Theoretical Physics, Lanzhou University, Lanzhou 730000, P. R. China and
⁴Department of Physics, Nanjing Normal University, Nanjing 210023, P. R. China

In the present work, the triply heavy tetraquarks state \((QQQq, Q = c, b, q = u, d, s)\) with all possible quantum numbers are systematically investigated in the framework of the chiral quark model with the resonance ground method. Two structures, including the meson-meson configuration (the color-single channels and the hidden-color channels) and the diquark-antidiquark configuration (the color sextet-antisextet and the color triplet-antitriplet), are considered. With the studied mass region, several bound states are obtained for the \(ccq̅q̅\), \(bbq̅q̅\) and \(bcq̅q̅\) tetraquarks, respectively. From the results, an important feature can be found that the channel coupling calculation is of great significance for whether some tetraquarks will form bound states. For those predicted bound states, we expect to provide effective assistance in the experimental search for exotic states.

PACS numbers: 13.75.Cs, 12.39.Pn, 12.39.Jh

I. INTRODUCTION

The search for multiquark states has always been the most important thing in the particle world, which shall provide important hints to the understanding of the nonperturbative QCD [1–7]. Since the discovery of the quark model [8] and QCD, the progress on the experimental tools has brought to us a lot of novel phenomena in hadron physics. Looking back in history, one could be seen that there have been a sizeable number of candidates for QCD exotic. The observed exotic states immediately attracted the interest of a large number of theorists. A lot of effort has been made to understand the nature of these states, but there is still controversy about their nature.

In 2013, the Belle collaboration observed the first exotic candidate \(X(3872)\), which has hidden-charm. For the \(X(3872)\), it can not be assigned to be any radial or orbital excited state of charmonium, and should have more complicated inner quark structures than a mere \(c\bar{c}\) pair [9]. Since then, lots of charmoniumlike and botononiumlike states are found. They are called XYZ states. Besides, the exotic charmed mesons \(D_{s0}^{*}(2317)\) [10] and \(D_{s1}(2460)\) [11] were studied as the singly charmed-strange tetraquark states in Refs. [12–14].

Very recently, the LHCb Collaboration reported the discovery of two new exotic states in the \(D^-K^+\) invariant mass distributions, \(X_0(2900)\) and \(X_1(2900)\), according to the decay model, the lowest quark content of which should be \(ud\bar{c}\bar{s}\), which means that these two states could be open charm tetrquarks. In 2016, the D0 Collaboration reported a new exotic state \(X(5568)\) with different flavors in the \(B^0\pi^+\) invariant mass distribution [15]. Later, the LHCb Collaboration, the CMS Collaboration, the CDF Collaboration and the ATLAS Collaboration also investigated the \(X(5568)\) state [16–19], as a result, unfortunately no such state was found. The result led some theorists to doubt whether the \(X(5568)\) is a genuine resonance [20–22]. In 2020, the BESIII Collaboration had observed a new signal, the \(Z_{cs}(3985)\), in the \(K^+\) recoil-mass spectrum in the process \(e^+e^-\rightarrow K^+(D_s^-D_s^{*0} + D_{s1}^-D_{s0}^0)\) at the center of mass energy \(\sqrt{s}=6.81\text{ GeV} \) [23], so the \(Z_{cs}\) state, which lies near the \(D_s^-D_s^{*0}\) and \(D_{s1}^-D_{s0}^0\) thresholds, had been predicted in several theoretical models [24–30]. But the nature of \(Z_{cs}(3985)\) remains elusive. In addition, recently, the LHCb Collaboration observed an narrow state in the \(D^0\bar{D}^0\pi^+\) mass spectrum just below the \(D^+D^-\pi^+\) threshold [31, 32]. This is the first doubly charmed tetraquark \(T_{cc}\) observed in experiment.

In addition to the exotic states observed in the above experiments, for the fully heavy tetraquark, actually, in 2017, the CMS Collaboration reported a benchmark measurement of \(\Upsilon(1S)\) pair production in pp collisions at \(\sqrt{s} = 8\text{ TeV} \) [33]. Besides, a new resonance at 18.2 GeV was found in Cu + Au collisions at RHIC [34], but the LHCb and CMS collaborations [35, 36] were not observed this state in the \(\Upsilon(1S)\mu^+\mu^-\) invariant mass spectrum. However, an important turning point came, recently, a new peak at 6.9 GeV and a broad structure around 6.2 GeV to 6.8 GeV were reported by the LHCb Collaboration in the invariant mass spectrum of \(J/\psi\) pairs. The global significance for these two structures is larger than 5\(\sigma\) [37]. This important signal from the LHCb Collaboration provides us with a new ground to understand the nonperturbative behavior of QCD.

Actually, the fully heavy state had been studied in many theories before it was observed experimentally. A \(b\bar{b}b\bar{b}\) bound state was supported by various models [38–41]. QCD sum rules studies [42, 43], and a naive diffusion Monte Carlo calculation [44]. On the contrary, there were counter examples on the existence of \(QQQq\) tetraquark states. For instance, in Ref. [45], the result showed that the lowest \(cc\bar{c}\bar{c}\) state is not bound, and there are other discussions with similar conclusion [43, 46–50].

The debates on the fully heavy tetraquark states are quite intense after the LHCb findings in the \(d\bar{t} - J/\psi\) invariant mass
spectrum. In Ref. [51], the fully heavy tetraquarks system are investigated in a nonrelativistic framework, assumed to be compact and to consist of diquark-antidiquark pairs, the model predicts the mass of the fully charged tetraquark to be 5960 MeV and the mass of the full bottom tetraquark to be 18720 MeV. In the framework of nonrelativistic potential quark model, the narrow structure around 6.9 GeV can be naturally explained by the P-wave $cc\bar{c}\bar{c}$ states [52], however, in Ref. [53], the interpretation is assigning $X(6900)$ to the $2S$ multiplet. In constituent quark models [54], the reported state $X(6900)$ was explained as a compact resonance with $J^P = 0^+$. On the other hand, the production of the fully heavy states are investigated in the $\gamma\gamma$ interactions, and the result shows that the experimental study of this process can be used to investigate the existence and properties of the $T_{4Q}$ states [55]. The $X(6900)$ may be explained as a radially excited state with quark content $cc\bar{c}\bar{c}$ and spin-parity $0^+ (3S)$ or $2^+ (3S)$ or an orbitally excited $2P$ state within Bethe-Salpeter equation and Regge trajectory relation [56]. Moreover, together with the QCD sum rule study in all diquark antidiquark constructs, the results suggest that the broad structure around 6.2 GeV to 6.8 GeV can be interpreted as an S wave $cc\bar{c}\bar{c}$ tetraquark state with $J^{PC} = 0^+ \text{ or } 2^+$, and the narrow structure around 6.9 GeV can be interpreted as a P wave state with $J^{PC} = 0^+ \text{ or } 1^+$ [57]. Nevertheless, in Ref. [58], by constructing the $[8,1]_{Q\bar{Q}} \otimes [8,1]_{Q\bar{Q}}$ currents in the framework of QCD sum rules, the narrow structure around 6.9 GeV can be interpreted as the $0^+$ octet-octet tetraquark states and the broad structure around 6.2 to 6.8 GeV can be interpreted as the $0^+$ octet-octet tetraquark states. In Ref. [59–61], the predicted masses support assigning the broad structure and the narrow structure to be the radial excited state within the QCD sum rules, the string-junction picture, and the extended relativized quark model.

The discoveries of fully heavy tetraquark states make one speculate that the tetraquark state with three heavy quarks, $QQ\bar{Q}\bar{Q}$ ($q = u, d, s$), may also exist. However, as far as we know, the experimental search for this kind of tetraquark state is missing. So, in this work, we move on to the four quark systems with three heavy quarks to search for triply heavy tetraquark states. The triply heavy tetraquark states are different from the discovered quarkonium-like states, in this respect, it might offer a new platform to study the internal structure of the exotic states, in addition, it can also be an ideal source to understand the hadronic dynamic and QCD factorization approach. However, the theoretical study of this system is relatively scarce. In Ref [62], the result showed that no bound triply-heavy tetraquarks were formed in the nonrelativistic quark model. In the framework of color-magnetic interaction, the triply heavy tetraquark states with the $QQ\bar{Q}\bar{Q}$ configuration was systematically investigated by calculating the mass splitting of the $QQ\bar{Q}\bar{Q}$ tetraquark states and their rough masses, some exotic tetraquark states were found [63]. However, a recent study by Weng et al. [64] predicted that there are not subsist as stable states in the framework of an extended chromomagnetic model. Within the framework of QCD sum rules, through the construction of the interpolating currents for the possible triply heavy tetraquarks states with quantum numbers $J^P = 0^+$ and $J^P = 1^+$, the $cc\bar{c}\bar{c}$, $cc\bar{b}\bar{b}$, $bc\bar{b}\bar{q}$ may decay with a narrow width due to their mass spectra. The $bb\bar{q}$ tetraquarks were expected to be very narrow resonances [65]. The masses, lifetimes, and weak decays of the triply heavy tetraquarks $bc\bar{b}\bar{q}$ were studied, the calculations showed a possibility of existence for the stable triply heavy tetraquark $bc\bar{b}\bar{q}$ with $J^P = 1^+$ which should be verified in experiment [66]. The triply heavy tetraquark as compact topological molecules was unbound in D8–D8 [67]. Recently, in Ref [68], the author adopt an extended relativized quark model to investigate the triply heavy tetraquarks. The result indicated that the whole mass spectra for the triply heavy tetraquarks showed similar patterns, and the corresponding states were about several hundred MeV higher.

In this work, we explore the possibility of having bound states of triply heavy tetraquark systems with the quantum number $J^P = 0^+, 1^+, 2^+$. In addition, we employ a nonrelativistic quark model for two-body interaction between heavy quarks according to the Lattice QCD study of Ref. [69]. Because the interaction strengths between quarks or antiquarks may be different, it usually assumed that the diquark substructure exists in multiquark states, which is reflected in the mass spectra. So, in this work, two configurations, meson- and diquark-antidiquark, and their mixing, are considered for the triply heavy tetraquark ($QQ\bar{Q}\bar{Q}$). All the possible color and spin configurations are also taken into account. Moreover, we perform a systematic coupled-channels calculation of the triply heavy tetraquark using the resonance group method, which is a powerful method for treating the few-body problem.

The paper is organized as follows. In Sec. II, the theoretical framework is presented, including the chiral quark model and the general structure of the four-body wave function. Section III is devoted to the analysis and discussion of the obtained results. Finally, we give a summary in the last section.

II. THEORETICAL FRAMEWORK

A. Chiral quark model

The ChQM is based on the fact that nearly massless current light quark acquires a dynamical, momentum-dependent mass, namely, the constituent quark mass due to its interaction with the gluon medium. So the chiral quark model contains Goldstone-boson exchange potentials, the perturbative one-gluon interaction, and a linear-screened confining potential. The model details can be found in Ref. [70]. Here only the Hamiltonian is given:

$$H = \sum_{i=1}^{4} \left( m_i + \frac{P_i^2}{2m_i} \right) - T_{CM} + \sum_{j=i}^{4} V(r_{ij}),$$

where the center-of-mass kinetic energy, $T_{CM}$ is subtracted without losing generality since we mainly focus on the internal relative motions of the multiquark system. Besides the two-body potential in a chiral quark model contains the color
confinement, one gluon exchange, and Goldstone-Boson exchange interactions. Furthermore, only the central parts of potential are considered due to consider the S wave, the spin-orbit and tensor contributions are ignored at present.

\[ V(r_{ij}) = V_{\text{CON}}(r_{ij}) + V_{\text{OGE}}(r_{ij}) + V_k(r_{ij}) \]  

(2)

For the potential \( V_{\text{OGE}}(r_{ij}) \), which can be written as

\[ V_{\text{OGE}}(r_{ij}) = \frac{1}{4}
\begin{pmatrix}
\phi^i, j \cdot \chi_f^c \\
\frac{1}{r_{ij}} - \frac{\pi}{2} \delta(r_{ij}) (\frac{1}{m_i^2} + \frac{1}{m_j^2} + \frac{4\sigma_i \cdot \sigma_j}{3m_im_j})
\end{pmatrix} \]  

(3)

\[ \text{where} \ m_{ij} \text{and} \ \sigma \text{are the mass quark and the Pauli matrices, respectively. The} \ \chi_f^c \text{is SU(3) color matrix. The QCD-inspired effective scale-dependent strong coupling constant,} \ \alpha_{ij}^c, \text{which offers a consistent description of mesons from light to heavy-quark sector, is the quark-gluon coupling constant and it can be determined by the mass splits between different mesons. Similarly, the confining interaction} \ V_{\text{CON}}(r_{ij}) \text{can be expressed as} \]

\[ V_{\text{CON}}(r_{ij}) = -a_c \chi_f^c \cdot \chi_f^c \left( r_{ij}^2 + V_0 \right), \]  

(6)

where \( a_c \) represents the strength of the confinement potential and \( V_0 \) is the zero-point energies which can be determined by the mass shift between different mesons.

The Goldstone-boson exchange interactions between light quarks appear because of the dynamical breaking of chiral symmetry. For the \( QQ\bar{q} \) (\( Q=c, b, q=u, d, s \)) system, according to the quark components, the \( \pi, K \) and \( \eta \) exchange interactions do not work, so in this paper, the Goldstone-boson exchange interactions are not considered. The other symbols in the above expressions have their usual meanings.

The values of those parameters are listed in Table I. The calculated mesons masses in comparison with experimental values are shown in Table II.

### B. The wave function of \( QQ\bar{q} \) system

In this work, the \( QQ\bar{q} \) system is considered by using the resonance group method [73]. For the \( QQ\bar{q} \) system, there are two typical kinds of configurations of this system, which are the meson-structure and the diquark-antidiquark structure, which are listed in Fig. 1. In this paper, for the purpose of solving a manageable 4-body problem, currently, the system calculation is used to combine these two configurations to see the effect of the multi-channel coupling. Four fundamental degrees of freedom, which are color, spin, flavor, and orbit, are generally accepted by the QCD theory at the quark level. So the wave function of each structure all consists of four parts: orbit, spin, flavor, and color wave functions. In addition, the wave function of each part is constructed by coupling two sub-clusters wave functions. Thus, the multiquark system’s wave function for each channel is an internal product of color (\( \chi_f^c \)), spin (\( S_f^j \)), flavor (\( F^j \)), and orbit (\( \psi^j \)) terms.

1. The color wave function

Plenty of color structures in multiquark systems will be available with respect to those of conventional hadrons such as \( q\bar{q} \) mesons and \( qqq \) baryons. In this section, the goal is to construct the colorless wave function of a 4-quark system.

For the meson-meson configurations, the color wave functions of a \( q\bar{q} \) cluster are listed:

\[ C^1_{[111]} = \sqrt{\frac{1}{3}} (r\bar{g} + g\bar{b} + b\bar{r}), \]
\[ C^2_{[21]} = rb, C^2_{[21]} = -rb, \]
\[ C^4_{[21]} = gb, C^4_{[21]} = -gb, \]
\[ C^6_{[21]} = g\bar{r}, C^6_{[21]} = br, \]
\[ C^8_{[21]} = \sqrt{\frac{1}{2}} (-r\bar{g} - g\bar{b}), \]
\[ C^9_{[21]} = \sqrt{\frac{1}{6}} (-2r\bar{g} + 2b\bar{b}), \]  

(7)

where the subscript [111] and [21] stand for color-singlet (1) and color-octet (8), respectively. So, the SU(3) color wave functions of color-singlet (two color-singlet clusters, 1c ⊗ 1c) and hidden-color (two color-octet clusters, 8c ⊗ 8c) channels are given, respectively.

\[ \chi_1^c = C^1_{[111]} C^1_{[111]}, \]
\[ \chi_2^c = \sqrt{\frac{1}{8}} \left( C^2_{[21]} C^7_{[21]} - C^4_{[21]} C^5_{[21]} - C^3_{[21]} C^6_{[21]} + C^8_{[21]} C^9_{[21]} - C^3_{[21]} C^4_{[21]} + C^7_{[21]} C^9_{[21]} \right) \]  

(9)

For the diquark-antidiquark structure, the color wave functions of the diquark clusters are given:

\[ C^1_{[2]} = rr, C^2_{[2]} = \sqrt{\frac{1}{2}} (rg + gr), \]
\[ C^3_{[2]} = gg, C^4_{[2]} = \sqrt{\frac{1}{2}} (rb + br), \]
\[ C^5_{[2]} = \sqrt{\frac{1}{2}} (gb + bg), C^6_{[2]} = bb, \]
\[ C^7_{[11]} = \sqrt{\frac{1}{2}} (rg - gr), C^8_{[11]} = \sqrt{\frac{1}{2}} (rb - br), \]
\[ C^9_{[11]} = \sqrt{\frac{1}{2}} (gb - bg). \]  

(10)

While the color wave functions of the antidiquark clusters can
The masses of mesons take their experimental values. $m_q = 2.77$ fm$^{-1}$, $m_u = 313$ MeV, $m_s = 536$ MeV, $m_b = 5112$ MeV, $b = 0.2$ fm, $a_c = 101$ MeV fm$^{-2}$.

**TABLE I:** Model parameters of the $qar{q}$ and $qq$ pairs. The masses of mesons take their experimental values. $m_q = 2.77$ fm$^{-1}$, $m_u = 313$ MeV, $m_s = 536$ MeV, $m_b = 5112$ MeV, $b = 0.2$ fm, $a_c = 101$ MeV fm$^{-2}$.

| Parameter | $V_{0u}(MeV)$ | $V_{0d}(MeV)$ | $V_{0s}(MeV)$ | $V_{0c}(MeV)$ | $V_{0b}(MeV)$ | $V_{0u}(MeV)$ | $V_{0s}(MeV)$ |
|-----------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|
| Value     | -3.7467        | -2.8684        | -2.6750        | -1.9211        | -1.7566        | -0.7367        | -1.0557        |

**TABLE II:** The Masses (in units of MeV) of the ground mesons. Experimental values are taken from the Particle Data Group (PDG) [72].

| System | $M_1$ | $M_2$ | $M_3$ | $M_4$ | $M_5$ | $M_6$ | $M_7$ | $M_8$ |
|--------|-------|-------|-------|-------|-------|-------|-------|-------|
| $\eta$ | 313 MeV | 272 MeV | 546 MeV | 630 MeV | 101 MeV | 67 MeV | 143 MeV | 77 MeV |

**FIG. 1:** Two types of configurations in $QQ\bar{q}$, $QQ\bar{q}$, and $QQ\bar{q}$ tetraquarks. For the $QQ\bar{q}$ system, there are two structures: the meson-meson configuration (diagram(a)) and the diquark-antidiquark configuration (diagram(b)). For the the $QQ\bar{q}$ system, two types of configurations are considered. diagram(c) is the meson-meson configuration and diagram(d) is the diquark-antidiquark configuration. For the $QQ\bar{q}$ system, it has two types of the meson-meson structure: diagram(e) and diagram(f), respectively. diagram(g) is the diquark-antidiquark structure.

be written as:

\[
C_{[22]}^1 = \frac{1}{2}(\bar{r}\bar{g} + \bar{g}\bar{r}), \\
C_{[22]}^3 = \frac{1}{2}(\bar{g}\bar{g}), \\
C_{[22]}^5 = \sqrt{\frac{1}{2}(\bar{g}\bar{g} + \bar{b}\bar{b})}, \\
C_{[22]}^7 = \frac{1}{2}(\bar{r}\bar{g} - \bar{g}\bar{r}), \\
C_{[22]}^9 = \frac{1}{2}(\bar{g}\bar{b} - \bar{b}\bar{g}).
\]

(11)

The color-singlet wave functions of the diquark-antidiquark configuration can be the product of color sextet and antisextet clusters ($6_s \otimes \bar{6}_s$) or the product of color-triplet and antitriplet
body clusters are 0 to 2. All of them are considered. The wave functions of two
clusters and the relative motion wave function between two
problems, to solve the Schrödinger-like 4-body bound state
extensive tools to solve eigenvalue problems and scattering

\begin{align}
\chi_3^c &= \sqrt{\frac{1}{6}} (C^1_{[2]} C^1_{[22]} - C^2_{[2]} C^2_{[22]} + C^3_{[2]} C^3_{[22]}) \\
&\quad + C^4_{[2]} C^4_{[22]} - C^5_{[2]} C^5_{[22]} + C^6_{[2]} C^6_{[22]}), \\
\chi_4^c &= \sqrt{\frac{1}{3}} (C^7_{[11]} C^7_{[211]} - C^8_{[11]} C^8_{[211]} + C^9_{[11]} C^9_{[211]}).
\end{align}

(12) (13)

2. The flavor wave function

For the flavor degree of freedom, since the quark content
of the investigated 4-quark system is \(QQ\bar{Q}\bar{q}\) (Q=c, b, q=u, d, s), the isospin \(I = 1/2\) and \(I = 0\) will be discussed. Here, we adopt \(F_m\) and \(F_d\) to denote the flavor wave functions of the tetraquark system with meson-meson and diquark-antidiquark
configurations, respectively. In this paper, the flavor wave function of the \(QQ\bar{Q}\bar{q}\) system can be categorized into three types: \(QQ\bar{Q}\bar{q}\), \(QQ\bar{Q}\bar{q}\) and \(QQ\bar{Q}\bar{q}\), respectively.

For the \(QQ\bar{Q}\bar{q}\) system, the flavor wave functions can be written as

\[ F_m^1 = (QQ)(\bar{Q}\bar{q}), \quad F_d^2 = (QQ)(\bar{Q}\bar{q}). \]

(14)

For the \(QQ\bar{Q}\bar{q}\) system, the flavor wave functions can be read as

\[ F_m^3 = (QQ')(\bar{Q}\bar{q}), \quad F_d^4 = (QQ')(\bar{Q}\bar{q}). \]

(15)

For the \(QQ'\bar{Q}\bar{q}\) system, the flavor wave functions can be obtained as below

\[ F_m^5 = (QQ)(\bar{Q}' \bar{q}), \quad F_m^6 = (QQ')(\bar{Q} \bar{q}), \quad F_d^7 = (QQ')(\bar{Q}\bar{q}). \]

(16)

3. The spin wave function

For the spin, the total spin \(S\) of tetraquark states range from
0 to 2. All of them are considered. The wave functions of two
body clusters are

\[ \chi_{11} = \alpha \alpha, \]
\[ \chi_{10} = \sqrt{\frac{1}{2}} (\alpha \beta + \beta \alpha), \]
\[ \chi_{1-1} = \beta \beta, \]
\[ \chi_{00} = \sqrt{\frac{1}{2}} (\alpha \beta - \beta \alpha). \]

(17)

Then, the total spin wave functions \(S_i\) are obtained by considering the coupling of two subcluster spin wave functions
with SU(2) algebra, and the total spin wave functions of four-quark states can be read as

\[ S_0^1 = \chi_{00}\chi_{00}, \]
\[ S_0^2 = \sqrt{\frac{1}{3}} (\chi_{10}\chi_{1-1} - \chi_{10}\chi_{10} + \chi_{1-1}\chi_{11}), \]
\[ S_1^3 = \chi_{10}\chi_{11}, \]
\[ S_1^4 = \chi_{11}\chi_{00}, \]
\[ S_1^5 = \sqrt{\frac{1}{2}} (\chi_{11}\chi_{10} + \chi_{10}\chi_{11}), \]
\[ S_2^6 = \chi_{11}\chi_{11}. \]

(18)

4. The orbital wave function

In the spatial space, we define different Jacobi coordinates for different diagrams in Fig. 1. For the \(QQ\bar{Q}\bar{q}\) system, the Jacobi coordinates for the meson-meson configuration in Fig. 1(a) and Fig. 1(c) can be written as

\[ R_1 = r_1 - r_2, R_2 = r_3 - r_4, \]
\[ R = \frac{r_1 + r_2 - r_3 + r_4}{2}; \]

(19)

whereas, for the \(QQ'\bar{Q}\bar{q}\) system, the Jacobi coordinates of two meson-meson types from the Fig. 1(c)-(f) can be obtained, where the Jacobi coordinates of Fig. 1(e) can be described as

\[ R_1 = r_1 - r_2, R_2 = r_3 - r_4, \]
\[ R = \frac{r_1 + r_2 - r_3 + r_4}{2}; \]

(20)

and by interchanging \(r_2\) with \(r_4\) one can obtained the Jacobi coordinates for Fig. 1(f), so the Jacobi coordinates can be defined as,

\[ R_1 = r_1 - r_2, R_2 = r_3 - r_4, \]
\[ R = \frac{r_1 + r_4 - r_3 + r_2}{2}. \]

(21)

As for the diquark-antidiquark structure as shown in Fig. 1(b),(d),(g), the Jacobi coordinates can change to

\[ R_1 = r_1 - r_2, R_2 = r_2 - r_4, \]
\[ R = \frac{r_1 + r_3 - r_2 + r_4}{2}. \]

(22)

With Jacobi coordinates defined above, we use the resonating
group method (RGM) [73], which is one of the most
effective tools to solve eigenvalue problems and scattering
problems, to solve the Schrödinger-like 4-body bound state
equation. The total orbital wave functions can be constructed by the product of the orbital wave function of two internal clusters and the relative motion wave function between two
clusters, which is

\[ \psi_{4q}^{(L)} = \psi_1(R_1)\psi_2(R_2)\chi_{1L}(R), \]

(23)

where \(R_1\) and \(R_2\) are the internal coordinates for the cluster 1 and cluster 2. \(R = R_1 - R_2\) is the relative coordinate between
the two clusters 1 and 2. The \( \psi_1(R_1) \) and \( \psi_2(R_2) \) are the internal cluster orbital functions of the clusters 1 and clusters 2, and \( \chi_L(R) \) is the relative motion wave function between two clusters, which is expanded by the gaussian bases

\[
\chi_L(R) = \sqrt{\frac{1}{4\pi}} \frac{3}{2n!^2} \sum_{i=1}^{n} C_i \times \int \exp[-\frac{3}{4b^2} (R - s_i)^2] Y_{LM}(\hat{s}_i) d\hat{s}_i
\]  

(24)

where \( n \) is the number of gaussian bases, which is determined by the stability of the results. By including the center of mass motion,

\[
\phi_c(R_c) = \left( \frac{4}{\pi b^2} \right)^{3/4} e^{-\frac{R_c^2}{2b^2}},
\]

the ansatz, Eq. (23), can be rewritten as

\[
\psi_{q_1q_2} = \sum_{i=1}^{n} C_{iL} \int \frac{d\hat{s}_i}{\sqrt{4\pi}} \left[ \prod_{a=1}^{4} \phi_a(S_i) \right] \prod_{a=3}^{4} \phi_b(-S_i),
\]  

(26)

where \( \phi_a(S_i) \) and \( \phi_b(-S_i) \) are the single-particle orbital wave functions, their specific form are shown below,

\[
\phi_a(S_i) = \left( \frac{1}{\pi b^2} \right)^{3/4} e^{-\frac{(r_s - \frac{1}{2} S_i)^2}{2b^2}};
\]

(27)

Finally, to fulfill the Pauli principle, the complete wave function is written as

\[
\psi = A[[\psi^L S^L_{1L_M}, F^L_{1L}]]
\]

(28)

where \( A \) is the antisymmetry operator of triply heavy tetraquarks. For \( QQ\bar{Q}q \) system, we have

\[
A = 1 - (13).
\]

(29)

For \( QQ\bar{Q}q \) system, the antisymmetry operator becomes

\[
A = 1 - (13).
\]

(30)

For \( QQ\bar{Q}q \) system, due to the absence of any homogeneous quarks the antisymmetry operator is defined as

\[
A = 1.
\]

(31)

In this work, after applying the antisymmetry operator, some wave functions will vanish, which means that some states are forbidden. For example, for the \( ccb\bar{q} \) system with \( J^P = 0^+ \), when considering the diquark-antidiquark structure with the spin wave function forced to choose \( S^L_1 \), the color wave function \( \chi^L_1 \) would be excluded due to the constrain that the total wave function must be antisymmetric.

III. RESULTS AND DISCUSSIONS

In the present calculation, the \( QQ\bar{Q}q(Q = c, b, q = u, d, s) \) system is evaluated by taking into account the meson-meson and diquark-antidiquark configurations in the ChQM, which have been shown in Fig 1. To exhaust all possible configurations of the \( QQ\bar{Q}q(Q = c, b, q = u, d, s) \) system, this system can be divided into three classes: 1) the \( QQ\bar{Q}q \) (Q stand for the same patical): \( (cc\bar{c}q, b\bar{b}q, cc\bar{c}q, bb\bar{s}) \); 2) the \( QQ\bar{Q}q \): \( (ccb\bar{q}, ccb\bar{q}, cc\bar{b}q, bb\bar{c}q); \) and 3) the \( QQ\bar{Q}q \): \( (cc\bar{b}q, cc\bar{b}q, cc\bar{b}q, cc\bar{b}q) \). Besides, in present work, the \( QQ\bar{Q}q \) tetraquarks only consider the calculation of the S-wave state, so this means that the total orbital angular momenta \( L \) is equal to zero. According to the relationship between the spin, the total orbital angular momenta and the total angular momentum, the total angular momentum, \( J \), coincides with the total spin, \( S \), and can take values of 0, 1 and 2. For the \( QQ\bar{Q}q(Q = c, b, q = u, d, s) \) system, all possible channels would be considered through the symmetry of the wave function. All of allowed channels are listed in Table III. From the Table III, for the meson-meson structure, the color singlet-singlet (1\( \times \) 1\( _c \)) and the color octet-octet (8\( _c \) \( \times \) 8\( _c \)) are taken into account in ChQM. Moreover, for the diquark-antidiquark structure, two color configurations, antitriplet-triplet (3\( _c \) \( \times \) 3\( _c \)) and sextet-antisextet (6\( _c \) \( \times \) 6\( _c \)) are also considered.

A. The \( QQ\bar{Q}q \) system

For the \( QQ\bar{Q}q \) system, which include the \( cc\bar{c}q, b\bar{b}q, cc\bar{c}q \) and \( bb\bar{s} \) system \( (q = u, d) \), so in order to reveal the nature of these states that may exist under these systems, we perform rigorous dynamical calculations on the systems. Now let us proceed to describe in detail our theoretical findings for each sector of the \( QQ\bar{Q}q \) tetraquarks.

In Table IV and Table V, we list our estimations of the lowest-lying triply heavy tetraquark states with quantum numbers \( J^P = 0^+, 1^+ \) and \( 2^+ \). In these tables, all the allowed meson-meson and diquark-antidiquark configurations are listed, which are shown as \( P - \) channel. The experimental values of the thresholds for the physical channels \( E_{th} \) are also presented. In the table, \( E_{cc}, E_{cc\bar{c}}, E_{cc\bar{c}} \) and \( E_{cc} \) are the eigen of every single channel, the eigen of the couple channels for the meson-meson configurations, the eigen of the couple channels for the diquark-antidiquark configurations, and the estimated eigen by simultaneously considering the meson-meson and diquark-antidiquark configurations, respectively. \( P\% \) is the percentages of each channel for the lowest-lying eigenenergies \( E_{CC} \).

For the \( cc\bar{c}q, b\bar{b}q, cc\bar{c}q \) and \( bb\bar{s} \) tetraquarks, the single-channel and channel-coupling calculations are individually performed in the present work. The results of the \( cc\bar{c}q, b\bar{b}q, cc\bar{c}q \) and \( bb\bar{s} \) tetraquarks are shown in the Table IV and Table V, respectively. From the Table IV and Table V, we shall proceed to discuss below the \( J^P = 0^+, 1^+ \) and \( 2^+ \) channels individually.

For the \( cc\bar{c}q, b\bar{b}q, cc\bar{c}q \) and \( bb\bar{s} \) tetraquarks in the \( J^P = 0^+ \), there are four channels in the meson-meson struc-
tured while two diquark-antidiquark channels are found. According to the method of the present work, the single-channel and channel-coupling calculation are all considered. From the Table IV and Table V, for the single-channel calculation, the results of every single channel for meson-meson structure are higher than the corresponding theoretical threshold, which means that the formation of a bound state is impossible. For the diquark-antidiquark structure, the lowest eigenenergies of every single channel are still above the theoretical threshold of the physical channel \(\eta D^*\), \(\eta B\), \(\eta D_s^*\) and \(\eta B_s\), their behavior similar to that of the meson-meson structure. Besides, we have performed a coupled-channels calculation on certain configurations. From the Table IV and Table V, \(E_{\text{CC1}}\) represents the results of all channel coupling in the meson-meson configuration while \(E_{\text{CC2}}\) is defined as a consequence of all channel coupling in the diquark-antidiquark. Compared to the lowest energy of the physical channel, the value of \(E_{\text{CC1}}\) and \(E_{\text{CC2}}\) still higher. These results confirm the nature of scattering states \(\eta D^*, \eta B, \eta D_s^*\) and \(\eta B_s\). Moreover, in a complete coupled-channels study, the lowest energy of 4851 MeV for \(\eta D^*\), 14681 MeV for \(\eta B\), 4954 MeV for \(\eta D_s^*\) and 14767 MeV for \(\eta B_s\) states are remained, so no bound states are found.

We have investigated the composition of this state in Table IV and Table V. The \(P\%\) gives the percentages of each channel for the lowest eigenstate which energy is given in the row marked by \(E_{\text{CC}}\) in the channel coupling calculation. From Table IV and Table V, the dominant structure is of the meson-meson contributing 99.9% to the total wave function, but the effect of the diquark-antidiquark structure almost can be ignored.

For the quantum number \(J^p = 1^+\) in the \(cc\bar{c}\bar{q}\), \(bb\bar{b}q\), \(cc\bar{c}\bar{s}\) and \(bb\bar{b}s\) tetraquarks, the behavior is similar to that of the quantum number \(J^p = 0^+\). The meson-meson and diquark-antidiquark configurations are taken into account. There are nine channels in this case which include three-color singlet channels and three hidden color channels in the meson-meson structure and three channels in the diquark-antidiquark arrangement. The formation of a bound state is not successful when combining single-channel and channel coupling calculations are considered. In addition to this, the percentages for each channel are calculated by the eigenvector of the total wave function, these results are consistent with those of the bound state calculations. From the results, the \(J/\psi D^*(96.24\%)\), the \(\eta B(99.98\%)\), the \(\eta D_s^*(99.91\%)\) and the \(\eta B_s(99.97\%)\) for the color single-channel account for a large effect in the bound state calculations, while the effect of the hidden color channel in the meson-meson structure and the diquark-antidiquark channel is negligible, and on the other hand the effect of all channel coupling has little influence on the bound state dynamics calculations for the quantum number \(J^p = 1^+\).

For the quantum number \(J^p = 2^+\) in the \(cc\bar{c}\bar{q}\), \(bb\bar{b}q\), \(cc\bar{c}\bar{s}\) and \(bb\bar{b}s\) tetraquarks, The only physical channel, which is \(J/\psi D^{**}\) for the \(cc\bar{c}\bar{q}\) tetraquark, \(\gamma B^*\) for the \(bb\bar{b}q\) tetraquark, \(J/\psi D^{**}\) for the \(cc\bar{c}\bar{s}\) tetraquark and \(\gamma B^*\) for the \(bb\bar{b}s\) tetraquark, respectively, exists. One hidden color channel and one diquark-antidiquark channel are also allowed due to a much richer combination of color, spin and flavor wave functions which fulfills the Pauli Principle. Firstly, for the single channel calculation, no bound state is found in the \(cc\bar{c}\bar{q}\), \(bb\bar{b}q\), \(cc\bar{c}\bar{s}\) and \(bb\bar{b}s\) tetraquarks. Secondly, the channel couplings in one structure are extremely weak. However, the coupled-channels computation considering the color single configuration, along with hidden-color and diquark-antidiquark ones, results in a mass around 5095 MeV, 14787 MeV, 5211 MeV and 14876 MeV for the different systems, respectively. So for the \(cc\bar{c}\bar{q}\) tetraquark, a bound state \(J/\psi D^{**}\)

| \(QQ\bar{q}\) Channel | \(QQ\bar{q}\) Channel | \(QQ\bar{q}\) Channel | \(QQ\bar{q}\) Channel |
|----------------------|----------------------|----------------------|----------------------|
| \(1/2^+\) \[2,1,1\] | \(1/2^+\) \[3,1,1\] | \(1/2^+\) \[5,1,1\] | \(1/2^+\) \[6,1,1\] |
| \(1/2^+\) \[5,1,2\] | \(1/2^+\) \[6,1,2\] | \(1/2^+\) \[3,1,2\] | \(1/2^+\) \[6,1,2\] |
| \(1/2^+\) \[2,2,2\] | \(1/2^+\) \[3,1,2\] | \(1/2^+\) \[5,1,2\] | \(1/2^+\) \[6,1,2\] |
| \(1/2^+\) \[2,2,1\] | \(1/2^+\) \[3,1,2\] | \(1/2^+\) \[5,1,2\] | \(1/2^+\) \[6,1,2\] |
| \(1/2^+\) \[1,5,1\] | \(1/2^+\) \[3,1,2\] | \(1/2^+\) \[5,1,2\] | \(1/2^+\) \[6,1,2\] |
| \(1/2^+\) \[1,5,2\] | \(1/2^+\) \[3,1,2\] | \(1/2^+\) \[5,1,2\] | \(1/2^+\) \[6,1,2\] |
| \(1/2^+\) \[2,3,3\] | \(1/2^+\) \[3,3,3\] | \(1/2^+\) \[5,3,3\] | \(1/2^+\) \[6,3,3\] |
| \(1/2^+\) \[2,4,4\] | \(1/2^+\) \[3,3,3\] | \(1/2^+\) \[5,3,3\] | \(1/2^+\) \[6,3,3\] |
| \(1/2^+\) \[2,5,4\] | \(1/2^+\) \[3,3,3\] | \(1/2^+\) \[5,3,3\] | \(1/2^+\) \[6,3,3\] |
| \(1/2^+\) \[1,6,1\] | \(1/2^+\) \[3,6,1\] | \(1/2^+\) \[5,6,1\] | \(1/2^+\) \[6,6,1\] |
| \(1/2^+\) \[1,6,2\] | \(1/2^+\) \[3,6,1\] | \(1/2^+\) \[5,6,1\] | \(1/2^+\) \[6,6,1\] |
| \(1/2^+\) \[2,6,4\] | \(1/2^+\) \[4,6,4\] | \(1/2^+\) \[5,6,4\] | \(1/2^+\) \[6,6,4\] |
is obtained with the binding energy $-9$ MeV. Besides, the composition of each channel is also performed, which is listed in Table IV. The overwhelming dominant channel is $J/ψD^{∗∗}$, contributing approximately 75% to the total wave function while the share of the hidden color channel ($J/ψD^{∗∗})^8$ and the diquark-antidiquark channel (cc(with$cq$)) are about 11% and 13%, respectively. From the analysis of the above results, it can be seen that the channel coupling effect plays an important role in the existence of bound states for the $J^{P} = 2^{+}$ system.

To find which interaction leads to the form of the bound states and to see more intuitively the influence of the channel coupling effect on the (cc(with$cq$)) tetraquark with the quantum number $J^{P} = \frac{1}{2} 2^{+}$, the contribution of each term in the system hamiltonian to the total energy of the system is shown in Table VI. Since the special characteristics of the quark components of the system, the $V^{π}, V^{k}$ and $V^{η}$ do not work for the whole system, which is not listed in the table. According to the Hamiltonian, the mass and the kinetic $V_{T}$ are shown in Table VI, besides, the confining interaction ($V_{CON}$) and the one-gluon-exchange interaction ($V_{OGE}$) are considered. From Table VI, as the number of the channel coupling structures increases, the interaction properties of the kinetic energy $V_{T}$ change from repulsion to attraction, however, the major contribution to repulsion has always been the $V_{CON}$ and $V_{OGE}$. So, according to the results of the table, the strong repulsive interaction from the $V_{CON}$ and $V_{OGE}$ can not completely neutralize the attraction of the kinetic energy $V_{T}$ when considering the channel coupling calculations, a phenomenon that more intuitively indicates that the effect of channel coupling is important for the formation of bound states in the system.

B. The $QQQ \bar{q}$ system

In this sector, we proceed here to analyze the $QQQ \bar{q}$ ($Q$ and $\bar{q}$ represent the different particle) system with quantum number $J^{P} = 0^{−}$, $1^{+}$ and $2^{+}$. At addition, due to the special nature of the quark components, quarks and antiquarks can not annihilate each other, so if there is an exotic state in the $QQQ \bar{q}$ system, it must be there. the $QQQ \bar{q}$ system contains $ccbq, b\bar{c}q, cc\bar{b}s$ and $b\bar{c}\bar{s}$ tetraquarks. To determine the presence of exotic states in this regime, we perform the dynamical bound state calculations for these systems. The results of the calculations are presented on Table VII and Table VIII. The details are as follows.

For the quantum number $J^{P} = 0^{−}$ in the $ccbq, b\bar{c}q, cc\bar{b}s$ and $b\bar{c}\bar{s}$ tetraquarks, six channels can be obtained by constructing the wave function, which have two structure including four the meson-meson and two diquark-antidiquark configurations. From Table VII and Table VIII, the fact can be found that no bound is found in each single channel calculation. When taking into account the coupling channel calculation of one certain structure or the whole configuration, the lowest mass of the $ccbq, cc\bar{b}s$ and $b\bar{c}\bar{s}$ tetraquarks still are above the corresponding threshold of the physics channel and from the calculation results, a certain fact is noticed that the energy variation interval is very small, which means the effect of the channel coupling in the $ccbq, cc\bar{b}s$, and $b\bar{c}\bar{s}$ tetraquarks are very weak. For the $b\bar{c}\bar{q}$ tetraquarks, the single-channel calculations indicate that the eigenenergies of every single channel are higher than the corresponding thresholds, which demonstrates that the formation of a bound state is impossible. Then, we consider the coupling of the meson-meson structure, and coupled channel estimations indicate that the lowest energies are still above the corresponding lowest meson-meson threshold. As for the diquark-antidiquark configuration, our estimations indicate that the energy obtained is almost constant. Finally, we perform a completed coupled channels estimations by involving the mixing between the meson-meson and diquark-antidiquark configurations, and we find the estimated eigenenergy is about 11553 MeV, which is below the corresponding threshold $B_{s}^{+}\bar{B}_{s}$, with the binding energy of $-2$ MeV. Additionally, from the table, we find that the meson structure under channel coupling plays a large role in the formation of bound states, which is $B_{s}^{+}\bar{B}_{s}(94.25%)$.

For the quantum number $J^{P} = 1^{+}$ in the $ccbq, b\bar{c}q, cc\bar{b}s$ and $b\bar{c}\bar{s}$ tetraquarks, three color-single channels and three hidden color channels in the meson-meson structure are listed in table while three diquark-antidiquark channels are also presented in table. From the Table VII and Table VIII, the result is similar to the quantum number $J^{P} = 0^{−}$. For the $ccbq, cc\bar{b}s$ and $b\bar{c}\bar{s}$ tetraquarks, one can conclude that no bound state is obtained when the three kinds of the bound calculation (single channel analysis, coupled-channels computation in each configuration and complete coupled-channels investigation) are performed. The main component of the lowest eigen-states are $B_{s}^{+}D_{s}^{∗} (98.00%), B_{s}^{+}D_{s}^{∗} (99.91%)$ and $B_{s}^{+}\bar{B}_{s}$, (98.59%), respectively, which reflect the lack of the coupling channels effect for the $ccbq, cc\bar{b}s$ and $b\bar{c}\bar{s}$ tetraquarks. Then, for the $b\bar{c}\bar{q}$ tetraquarks, the single-channel calculation shows that no bound state is formed while considering the coupling between channels in the same configurations, the eigenenergies are estimated to be 11581 and 11759 MeV for the meson-meson and diquark-antidiquark configurations, respectively. Further, complete coupled-channels calculations predict a bound state with a mass 11565 MeV, which is below the threshold of $B_{s}^{+}B_{s}^{∗}$. This phenomena reflect the importance of channel coupling calculations. Moreover, a similar conclusion can be drawn from the percentages of each channel, the coupling channel effect for the meson-meson structure accounts for about 80% and that of diquark-antidiquark structure accounts for about 20%, separately.

For the quantum number $J^{P} = 2^{+}$ in the $ccbq, b\bar{c}q, cc\bar{b}s$ and $b\bar{c}\bar{s}$ tetraquarks, three channels (the color-single channel, the hidden color channel and the diquark-antidiquark channels) can be obtained in Table VII and Table VIII. For the $ccbq, cc\bar{b}s$ and $b\bar{c}\bar{s}$ tetraquarks, one can see that three contributing channels are all above the corresponding theoretical thresholds, and the lowest mass are 8309 MeV ($B_{s}^{−}D_{s}^{∗} + D_{s}^{−}D_{s}^{∗}$), 8413 MeV ($B_{s}^{−}D_{s}^{∗} + D_{s}^{−}D_{s}^{∗}$) and 11716 MeV $B_{s}^{−}B_{s}^{∗}$, respectively. Moreover, this fact is not changed when a compete coupled-channels calculation is performed. For the $b\bar{c}\bar{q}$ tetraquark, a bound states, which is $B_{s}^{−}\bar{B}_{s}$ (11613) with the binding energy $-12$ MeV, exist when considering the coupling channels. Form the Table VII, the meson-meson structure is the primary,
TABLE IV: The lowest-lying eigenenergies of the $cc\bar{q}q$, and $bb\bar{q}q$ tetraquarks in the ChQM.

| [i,j,k] | $cc\bar{q}q$ | $bb\bar{q}q$ |
|---------|----------------|----------------|
|         | $P$-channel | $E_{ib}$ | $E_{iw}$ | $P_{ib}$ | $E_{ib}$ | $E_{iw}$ | $P_{ib}$ |
| $I^{F} = \frac{1}{2}^{+}$ | | | | | | | |
| [1,1,1] | $(\eta_{c}D_{s}^+)^{1}$ | 4849 | 4851 | 99.91% | $(\eta_{b}B_{s}^+)^{1}$ | 14679 | 14681 | 99.99% |
| [1,2,1] | $(J/\psi D_{s}^+)^{1}$ | 5104 | 5106 | 0.01% | $(\Upsilon B_{s}^+)^{1}$ | 14785 | 14787 | 0.01% |
| [1,1,2] | $(\eta_{c}D_{s}^+)_{s}^{2}$ | 5550 | 0.01% | $(\eta_{b}B_{s}^{*})^{8}$ | 15302 | ~ 0% |
| [1,2,2] | $(J/\psi D_{s}^{**})_{s}^{8}$ | 5563 | 0.03% | $(\Upsilon B_{s}^{*})^{8}$ | 15342 | ~ 0% |
| [2,1,3] | $(cc)(\bar{c}\bar{q})^{5}$ | 5624 | 0.01% | $(bb)(\bar{b}\bar{q})^{8}$ | 15359 | ~ 0% |
| [2,2,4] | $(cc)(\bar{c}\bar{q})^{5}$ | 5421 | 0.03% | | | | |
| $E_{CC}$ | 4851 | | | 4851 | | | 4851 |
| $I^{F} = \frac{1}{2}^{0}$ | | | | | | | |
| [1,3,1] | $(\eta_{c}D_{s}^+)^{1}$ | 4991 | 4993 | 1.68% | $(\eta_{b}B_{s}^+)^{1}$ | 14724 | 14726 | 99.98% |
| [1,4,1] | $(J/\psi D_{s}^+)^{1}$ | 4962 | 4964 | 96.24% | $(\Upsilon B_{s}^+)^{1}$ | 14740 | 14742 | 0.01% |
| [1,5,1] | $(J/\psi D_{s}^{**})_{s}^{8}$ | 5104 | 5106 | 0.19% | $(\Upsilon B_{s}^{*})^{1}$ | 14785 | 14787 | 0.01% |
| [1,3,2] | $(\eta_{c}D_{s}^+)_{s}^{2}$ | 5522 | 0.05% | $(\eta_{b}B_{s}^{*})^{8}$ | 15292 | ~ 0% |
| [1,4,2] | $(J/\psi D_{s}^{**})_{s}^{8}$ | 5526 | 0.09% | $(\Upsilon B_{s}^{*})^{8}$ | 15294 | ~ 0% |
| [1,5,2] | $(J/\psi D_{s}^{**})_{s}^{8}$ | 5518 | 0.65% | $(\Upsilon B_{s}^{*})^{8}$ | 15306 | ~ 0% |
| [2,3,3] | $(cc)(\bar{c}\bar{q})^{5}$ | 5588 | 0.37% | $(bb)(\bar{b}\bar{q})^{8}$ | 15348 | ~ 0% |
| [2,4,4] | $(cc)(\bar{c}\bar{q})^{5}$ | 5364 | 0.06% | $(bb)(\bar{b}\bar{q})^{8}$ | 15127 | ~ 0% |
| [2,5,3] | $(cc)(\bar{c}\bar{q})^{5}$ | 5428 | 0.17% | $(bb)(\bar{b}\bar{q})^{8}$ | 15147 | ~ 0% |
| $E_{CC}$ | 4964 | | | 4963 | | | 4963 |
| $I^{F} = \frac{1}{2}^{0}$ | | | | | | | |
| [1,6,1] | $(J/\psi D_{s}^{**})_{s}^{1}$ | 5104 | 5106 | 75.23% | $(\Upsilon B_{s}^{*})^{1}$ | 14785 | 14787 | 99.99% |
| [1,6,2] | $(J/\psi D_{s}^{**})_{s}^{8}$ | 5494 | 11.70% | $(\Upsilon B_{s}^{*})^{8}$ | 15273 | ~ 0% |
| [2,6,4] | $(cc)(\bar{c}\bar{q})^{5}$ | 5442 | 13.06% | $(bb)(\bar{b}\bar{q})^{8}$ | 15153 | 0.01% |
| $E_{CC}$ | 5106 | | | 5106 | | | 5106 |
| $E_{CC}$ | 5442 | | | 5442 | | | 5442 |
| $E_{CC}$ | 5095 | | | 5095 | | | 5095 |

almost accounting for 77.79% due to the channel coupling effect. However, the diquark-antidiquark structure in the all-channel coupling calculation is also not be overlooked, which has almost 22.21%.

Besides, to investigate the contribution of each interaction term to the binding state of the $bb\bar{q}q$ tetraquark, the results are listed in Table IX. From Table IX, one fact that can be seen is that the major contribution to the attraction of the $bb\bar{q}q$ tetraquark is from the one-gluon-exchange interaction $(V_{OGE})$, right next to the kinetic energy $(V_{T})$. In contrast, the confinement potential $(V_{CON})$ appears to be very strong repulsive interactions. So, there is a delicate competition between the confinement potential and the sum of the kinetic energy and the one-gluon-exchange interaction, according to Table IX. The total interaction is expressed as attractions, which is necessary for forming a bound state. In addition, from the table, the nature of the equivalent potential energy would change from repulsion to attraction as the number of coupling channels increases. Therefore, one important conclusion from the table is obtained, which is that the channel coupling effect is not negligible.

C. The $QQ\bar{q}\bar{q}$ system

The $QQ\bar{q}\bar{q}$ system, is investigated in this sector. According to the possible quark components of the $QQ\bar{q}\bar{q}$ system, the possible tetraquark systems are $cb\bar{c}q$, $bc\bar{b}q$, $cb\bar{c}q$ and $bc\bar{b}q$. The sophisticated dynamical calculations have been performed in order to determine the existence of exotic states in these tetraquark systems, and the corresponding results have been presented in Table X and Table XI. Then, we analyze these results in detail.

For the quantum number $J^{P} = 0^{+}$ in the $bc\bar{c}q$, $bc\bar{b}q$, $cb\bar{c}q$ and $bc\bar{b}q$ tetraquarks, they have twelve channels including eight meson-meson structures (the color-singlet and hidden-color configurations) and four diquark-antidiquark structures, respectively. The calculated masses of each single channel are above the corresponding theoretical threshold of the physics channel, and no bound states are observed. Additionally, after coupling between the same kind of configurations, one can conclude that the coupling is weak in di-meson case and it is quite comparable in diquark-antidiquark structure. The nature of scattering for the lowest state $D_{s}^{*}B_{s}^{*}$, $\eta_{b}D_{s}^{*}$, $D_{s}^{*}B_{s}^{*}$ and $\eta_{b}D_{s}^{*}$ remains in the complete coupled-channels calculations for the $QQ\bar{q}\bar{q}$ tetraquark. In addition, the percentage con-
distribution of each channel in the lowest energy under the complete channel coupling calculation, is listed in Table XII. From the Table XII, the $D_s^* B_c^-$, $\eta_c D_s^*$, $D_s^* D_s^*$ and $\eta_c D_s^*$ have a great contribution, up to 99%, which mean that the coupling channel bound calculation does not work for the $bc\bar{c}\bar{s}$, $bcb\bar{q}$, $bc\bar{c}s$ and $bcb\bar{s}$ tetraquarks.

For the quantum number $J^P = 1^+$ in the $bc\bar{c}\bar{s}$, $bcb\bar{q}$, $bc\bar{c}s$ and $bcb\bar{s}$ tetraquarks, there are twelve meson-meson channels (the color-singlet and hidden-color configurations) and six diquark-antidiquark channels. The single channel calculation produces masses which are above the corresponding threshold of the physics channel and all states are scattering ones. The coupled-channels results for each kind of structure reveals weak in di-meson configuration and stronger ones for the diquark-antidiquark structures. Now, when considering the fully coupled-channels calculation, the only one bound

| TABLE V: The lowest-lying eigenenergies of the $cc\bar{c}s$ and $bb\bar{b}s$ tetraquarks in the ChQM. |
|---|---|---|---|---|---|---|---|
| P-channel | $E_{th}$ | $E_{sc}$ | $P_{th}$ | $I_{P} = 00^+$ |
| $[1,1,1]$ | $(\eta_c D_s^*)^3$ | 4952 | 4954 | 99.98% |
| $[1,2,1]$ | $(J/\psi D_s^*)^3$ | $5209$ | $5210 \sim 0\%$ |
| $[1,1,2]$ | $(\eta_c D_s^*)^3$ | $5640 \sim 0\%$ |
| $[1,2,2]$ | $(J/\psi D_s^*)^3$ | $5614 \sim 0\%$ |
| $[2,1,3]$ | $(cc)(\bar{c}\bar{s})$ | $5697 \sim 0\%$ |
| $[2,2,4]$ | $(cc)(\bar{c}\bar{s})$ | $5498 \sim 0\%$ |
| $E_{cc1}$ | $4954$ |
| $E_{cc2}$ | $5487$ |
| $E_{cc}$ | $4954$ |

| TABLE VI: Contributions of each terms in Hamiltonian to the energy of the $cc\bar{c}\bar{s}$ tetraquark for the bound state in ChQM. $E_{M^*\text{-channel}}$ stands for the sum of the theoretical thresholds of lowest physical channel. (unit: MeV). |
|---|---|---|---|
| $I_{P} = 01^+$ |
| $[1,3,1]$ | $(\eta_c D_s^*)^3$ | $5096$ | $5098 \sim 0\%$ |
| $[1,4,1]$ | $(J/\psi D_s^*)^3$ | $5065$ | $5067 \sim 99.91\%$ |
| $[1,5,1]$ | $(J/\psi D_s^*)^3$ | $5209$ | $5211 \sim 0\%$ |
| $[1,3,2]$ | $(\eta_c D_s^*)^3$ | $5610 \sim 0\%$ |
| $[1,4,2]$ | $(J/\psi D_s^*)^3$ | $5614 \sim 0\%$ |
| $[1,5,2]$ | $(J/\psi D_s^*)^3$ | $5585 \sim 0\%$ |
| $[2,3,3]$ | $(cc)(\bar{c}\bar{s})$ | $5661 \sim 0\%$ |
| $[2,4,4]$ | $(cc)(\bar{c}\bar{s})$ | $5445 \sim 0\%$ |
| $[2,5,3]$ | $(cc)(\bar{c}\bar{s})$ | $5508 \sim 0\%$ |
| $E_{cc1}$ | $5067$ |
| $E_{cc2}$ | $5442$ |
| $E_{cc}$ | $5066$ |

| $I_{P} = 02^+$ |
| $[1,6,1]$ | $(J/\psi D_s^*)^3$ | $5209$ | $5211 \sim 99.79\%$ |
| $[1,6,2]$ | $(J/\psi D_s^*)^3$ | $5598 \sim 0\%$ |
| $[2,6,4]$ | $(cc)(\bar{c}\bar{s})$ | $5526 \sim 0\%$ |
| $E_{cc1}$ | $5211$ |
| $E_{cc2}$ | $5526$ |
| $E_{cc}$ | $5211$ |
state with the binding energy -6 MeV, is observed in the tetraquark, which is the \(D^*\bar{B}_c\) state with the mass 8159 MeV. For the \(bc\bar{c}\bar{q}\), \(bc\bar{c}\bar{s}\) and \(bc\bar{b}\bar{s}\) tetraquarks, there is no bound state. Furthermore, the percentage of each channel is also presented. From the Table X, one can find that the color singlet channel of the \(D^*\bar{B}_c\) is about 92%, while the remaining channels make up 8% for the \(bc\bar{c}\bar{q}\) tetraquark, the result indicate that the coupling channels calculation must be considered to study the nature of the tetraquark system. For the \(bc\bar{c}\bar{q}\), \(bc\bar{c}\bar{s}\) and \(bc\bar{b}\bar{s}\) tetraquarks, the percentage of color single channels is about 99%, which mean that the role of the remnant channel is trivial.

For the quantum number \(J^P = 2^+\) in the \(bc\bar{c}\bar{q}\), \(bc\bar{b}\bar{q}\), \(bc\bar{c}\bar{s}\) and \(bc\bar{b}\bar{s}\) tetraquarks, four meson-meson structure and two diquark-antidiquark structure are studied. Regardless of the single channel or the channel coupling calculation of the same structure, the \(bc\bar{c}\bar{q}\), \(bc\bar{b}\bar{q}\), \(bc\bar{c}\bar{s}\) and \(bc\bar{b}\bar{s}\) tetraquarks do not reveal bound states. However, two only bound states \(D^*\bar{B}_c\) and \(D_{s}^*\bar{B}_c\) can be turned up in the \(bc\bar{c}\bar{q}\) and \(bc\bar{c}\bar{s}\) tetraquarks by considering the complete channel coupling calculation. From the Table X and Table XI, the masses of two bound states with the binding energy -35 MeV and -3 MeV, are 8272 MeV and 8409 MeV, respectively. For the \(bc\bar{c}\bar{q}\) and \(bc\bar{c}\bar{s}\) tetraquark, the situation is similar to the quantum number \(J^P = 0^+, 1^+\), no bound states can be formed, the details would not be elaborated. Moreover, from the percentage of the result, one important fact can be seen that the channel coupling calculation is very important for looking for some exotic states.

We have investigated the contribution of each term of three bound states \(D^*\bar{B}_c\), \(D^{**}\bar{B}_c\) and \(D_{s}^*\bar{B}_c\) in Table XII-XIII. For the \(bc\bar{c}\bar{q}\) and \(bc\bar{c}\bar{s}\) tetraquarks, whether considering coupled-channels calculation of meson-meson or diquark-antidiquark structure, the kinetic energy, the one-gluon-exchange interaction (\(V_{OGE}\)) and the confinement potential (\(V_{CON}\)) are repulsive. However, under the complete coupled-channels calculation, the primary attraction contribution are from the kinetic energy while the repulsive potential are the one-gluon-exchange interaction (\(V_{OGE}\)) and the confinement potential (\(V_{CON}\)). From the results, the strong repulsion from \(V_{OGE}\) and \(V_{CON}\) would not be neutralized by the attractive effect of the kinetic energy, which contributes the sizable attraction to bind the \(bc\bar{c}\bar{q}\) and \(bc\bar{c}\bar{s}\) tetraquarks.
IV. SUMMARY

In this work, we have systematically investigated the masses of triply heavy tetraquarks $QQQq$ ($Q=c, b, q = u, d, s$) in the chiral quark model. For the triply heavy tetraquarks, the meson-meson structure including the color-singlet and hidden-color channel, and the diquark-antidiquark structure (color sextet-antisextet, color triplet-antitriplet) are considered herein. The total Hamiltonian of the total possible triply heavy tetraquark, including the kinetic potential, the confinement, and one-gluon exchange interaction, are solved within the resonance group method.

From the results, two important conclusions can be drawn that, 1): some bound states can be obtained in the particular tetraquarks. For the $ccar{c}q$ tetraquark in the quantum number $I^J = \frac{4}{2}^+$, a bound state $J/ψ D^{∗*}$ is observed with the binding energy $-9$ MeV, the mass of which is $5095$ MeV. For the $bbar{c}q$ tetraquark, three bound state are found, which are $B_c \bar{B}$ with $I^J = \frac{1}{2}^+$, $B^{∗*}B$ with $I^J = \frac{1}{2}^+$ and $B^{∗*}B^{∗*}$ with $I^J = \frac{3}{2}^+$, respectively. Their mass and the binding energy are $(11553$ MeV, $-2$ MeV), $(11565$ MeV, $-15$ MeV), and $(11613$ MeV, $-12$ MeV), respectively. Meanwhile, three bound states with the mass and binding energy also appear in the $bcar{c}q$ and $bcar{c}q$ tetraquarks, which are $D^+ B^{∗*}$ (8159 MeV, $-6$ MeV) with $I^J = \frac{3}{2}^+$, $D^{∗*}B$ (8272 MeV, $-35$ MeV) with $I^J = \frac{3}{2}^+$ and $D^{∗*}B^{∗*}$ (8409 MeV, $-3$ MeV) with $I^J = 0^+$, respectively. 2): the effect of the channel coupling plays a very important role in the study of multiquark systems. From the results, some single states are not bound at first, but they become bound by the channel-coupling calculation. The main reason is that the triply heavy tetraquarks we investigate here is all in S wave, and the channel coupling between all channels is through the central force, the role of which has been verified to be much more important than the tensor force in our quark level calculation [74]. Therefore, to explore the multiquark states, the effect of channel coupling can not be neglected. We hope our calculations and predictions of the triply heavy tetraquarks may provide valuable information for future experimental searches.

Acknowledgments

This work is supported partly by the National Natural Science Foundation of China under Contract Nos. 11775050, 11775118 and 11535005, and is also supported by the Fun-
TABLE IX: Contributions of each terms in Hamiltonian to the energy of the $bbc\bar{q}$ tetraquark for the bound state in ChQM. $E_{\text{Mi}(\text{channel})}$ stands for the sum of the theoretical thresholds of lowest physical channel. (unit: MeV).

| $I(J')$ | $I(J') = \frac{1}{2}(0^+)$ | $I(J') = \frac{1}{2}(1^+)$ | $I(J') = \frac{3}{2}(2^+)$ |
|---------|-----------------|-----------------|-----------------|
| $E_{\text{CC}^1}$ | $12265.0$ | $12265.0$ | $12265.0$ |
| $E_{\text{CC}^2}$ | $12265.0$ | $12265.0$ | $12265.0$ |
| $E_{\text{CC}}$ | $12265.0$ | $12265.0$ | $12265.0$ |
| $E_{\text{M}^1;\text{CC}^1}$ | $12265.0$ | $12265.0$ | $12265.0$ |
| $\Delta E_{\text{M}^1;\text{CC}^1}$ | $0.0$ | $0.0$ | $0.0$ |
| $\Delta E_{\text{CC}^1}$ | $0.0$ | $0.0$ | $0.0$ |
| $\Delta E_{\text{CC}^2}$ | $0.0$ | $0.0$ | $0.0$ |
| $\Delta E_{\text{CC}}$ | $0.0$ | $0.0$ | $0.0$ |

Theoretical values of mass $m$ for $|\psi(0^+)|$ and $|\psi(1^+)|$ states of the $bbc\bar{q}$ tetraquark for the bound state in ChQM. $E_{\text{Mi}(\text{channel})}$ stands for the sum of the theoretical thresholds of lowest physical channel.

1. H. X. Chen, W. Chen, X. Liu and S. L. Zhu, Phys. Rept. 639 (2016), 1-121 doi:10.1016/j.physrep.2016.05.004 [arXiv:1601.02092 [hep-ph]].
2. E. S. Swanson, Phys. Rept. 429 (2006), 243-305 doi:10.1016/j.physrep.2006.04.003 [arXiv:hep-ph/0601110 [hep-ph]].
3. M. B. Voloshin, Prog. Part. Nucl. Phys. 61 (2008), 455-511 doi:10.1016/j.ppnp.2008.02.001 [arXiv:0711.4556 [hep-ph]].
4. R. Chen, X. Liu and S. L. Zhu, Nucl. Phys. A 954 (2016), 406-421 doi:10.1016/j.nuclphysa.2016.04.012 [arXiv:1601.03233 [hep-ph]].
5. A. Esposito, A. Pilloni and A. D. Polosa, Phys. Rept. 668 (2017), 1-97 doi:10.1016/j.physrep.2016.11.002 [arXiv:1611.07920 [hep-ph]].
6. R. F. Lebed, R. E. Mitchell and E. S. Swanson, Prog. Part. Nucl. Phys. 93 (2017), 143-194 doi:10.1016/j.ppnp.2016.11.003 [arXiv:1610.04528 [hep-ph]].
7. F. K. Guo, C. Hanhart, U. G. Meißner, Q. Wang, Q. Zhao and B. S. Zou, Rev. Mod. Phys. 90 (2018) no.1, 015004 doi:10.1103/RevModPhys.90.015004 [arXiv:1705.00141 [hep-ph]].
8. M. Gell-Mann, Phys. Lett. 8 (1964), 214-215 doi:10.1016/S0031-9163(64)92001-3
9. S. K. Choi et al. [Belle], Phys. Rev. Lett. 91 (2003), 262001 doi:10.1103/PhysRevLett.91.262001 [arXiv:hep-ex/0309032 [hep-ex]].
10. B. Aubert et al. [BaBar], Phys. Rev. Lett. 90 (2003), 242001 doi:10.1103/PhysRevLett.90.242001 [arXiv:hep-ex/0304021 [hep-ex]].
11. D. Besson et al. [CLEO], Phys. Rev. D 68 (2003), 032002 [erratum: Phys. Rev. D 75 (2007), 119908] doi:10.1103/PhysRevD.68.032002 [arXiv:hep-ex/0305100 [hep-ex]].
12. H. Y. Cheng and W. S. Hou, Phys. Lett. B 566 (2003), 193-200 doi:10.1016/S0370-2693(03)00834-7 [arXiv:hep-ph/0305038 [hep-ph]].
TABLE X: The lowest-lying eigenenergies of the $b c \bar{c} \bar{q}$ and $b c \bar{b} \bar{q}$ tetraquarks in the ChQM.

| [i,j,k] | $P$-channel | $E_{th}$ | $E_w$ | $P_{th}$ |
|---------|-------------|---------|-------|----------|
| $\sqrt{1}^+$ | $\sqrt{1}^+$ |
| [5,1.5] | $\eta_b B^+_1$ | 8266 | 8266 | 0% |
| [5,2.1] | $J/\psi B^+_1$ | 8422 | 8424 | 0% |
| [5,2.2] | $\eta_b B^+_1$ | 8717 | 0% |
| [5,3.1] | $J/\psi B^+_1$ | 8663 | 0% |
| [5,3.2] | $\eta_b B^+_1$ | 8307 | 8309 | 0% |
| [5,4.1] | $J/\psi B^+_1$ | 8870 | 0% |
| [5,4.2] | $\eta_b B^+_1$ | 8815 | 0% |
| [5,5.1] | $J/\psi B^+_1$ | 8864 | 0% |
| [5,5.2] | $\eta_b B^+_1$ | 8643 | 0% |
| $\sqrt{2}^+$ | $\sqrt{2}^+$ |
| [5,6.1] | $J/\psi B^+_1$ | 8422 | 8424 | 0% |
| [5,6.2] | $J/\psi B^+_1$ | 8713 | 11.04% |
| [5,7.1] | $\eta_b B^+_1$ | 8307 | 8309 | 0% |
| [5,7.2] | $J/\psi B^+_1$ | 8866 | 1.38% |
| [5,7.3] | $\eta_b B^+_1$ | 8841 | 4.53% |
| [5,7.4] | $\eta_b B^+_1$ | 8657 | 10.77% |
| $\sqrt{3}^+$ | $\sqrt{3}^+$ |
| [5,8.1] | $J/\psi B^+_1$ | 8422 | 8424 | 0% |
| [5,8.2] | $J/\psi B^+_1$ | 8713 | 11.04% |
| [5,9.1] | $\eta_b B^+_1$ | 8307 | 8309 | 0% |
| [5,9.2] | $J/\psi B^+_1$ | 8866 | 1.38% |
| [5,9.3] | $\eta_b B^+_1$ | 8841 | 4.53% |
| [5,9.4] | $\eta_b B^+_1$ | 8657 | 10.77% |

[13] V. Dmitrasinovic, Phys. Rev. D 70 (2004), 096011 doi:10.1103/PhysRevD.70.096011
[14] L. Maiani, F. Piccinini, A. D. Polosa and V. Riquer, Phys. Rev. D 71 (2005), 014028 doi:10.1103/PhysRevD.71.014028 [arXiv:hep-ph/0412098 [hep-ph]].
[15] V. M. Abazov et al. [D0], Phys. Rev. Lett. 117 (2016), no.2, 022003 doi:10.1103/PhysRevLett.117.022003 [arXiv:1602.07588 [hep-ex]].
[16] R. Aaij et al. [LHCb], Phys. Rev. Lett. 117 (2016), no.15, 152003 doi:10.1103/PhysRevLett.117.152003 [arXiv:1608.00435 [hep-ex]].
[17] A. M. Sirunyan et al. [CMS], Phys. Rev. Lett. 120 (2018), no.20, 202005 doi:10.1103/PhysRevLett.120.202005 [arXiv:1712.06144 [hep-ex]].
| [i,j,k] | P-channel | $bc\bar{c}$ | $bc\bar{b}$ |
|--------|-----------|-------------|-------------|
| $J^P = \frac{1}{2}^+$ | $\frac{1}{2}^+$ | $\frac{1}{2}^+$ | $\frac{1}{2}^+$ |
| $[5,1,1]$ | $(\eta_1) \bar{B}^+_1$ | $E_{bc}$ | $E_{bc}$ | $P_{bc}$ |
| $[5,2,1]$ | $(J/\psi \bar{B}^+_1)$ | $8351$ | $8352$ | $0\%$ | $11367$ | $11368$ | $99.99\%$ |
| $[5,2,2]$ | $(J/\psi \bar{B}^+_1)^8$ | $8796$ | $0\%$ | | $12135$ | $0\%$ |
| $[6,1,1]$ | $(D_s^* \bar{B}^+_1)$ | $8243$ | $8244$ | $99.92\%$ | $11642$ | $11643$ | $0\%$ |
| $[6,2,1]$ | $(D_s^* \bar{B}^+_1)^8$ | $8412$ | $8413$ | $0\%$ | $11715$ | $11716$ | $0\%$ |
| $[7,1,1]$ | $(bc)\bar{c}\bar{c}$ | $8924$ | $0\%$ | | $12035$ | $0\%$ |
| $[7,1,4]$ | $(bc)\bar{c}\bar{c}$ | $8922$ | $0\%$ | | $12018$ | $0\%$ |
| $[7,2,3]$ | $(bc)\bar{c}\bar{c}$ | $8924$ | $0\%$ | | $12077$ | $0\%$ |
| $[7,2,4]$ | $(bc)\bar{c}\bar{c}$ | $8715$ | $0\%$ | | $11976$ | $0\%$ |
| $I_{bc}$ | $E_{bc}$ | $8244$ | $11368$ |
| $I_{bc}$ | $E_{bc}$ | $8244$ | $11368$ |

**Table XI:** The lowest-lying eigenenergies of the $bc\bar{c}$ and $bc\bar{b}$ tetraquarks in the ChQM.
TABLE XII: Contributions of each terms in Hamiltonian to the energy of the $bc\bar{c}q$ tetraquark for the bound state in ChQM. $E_{M^c_{\text{channel}}}$ stands for the sum of the theoretical thresholds of lowest physical channel. (unit: MeV).

| term       | mass     | $V_T$    | $V_{CON}$ | $V_{OGE}$ |
|------------|----------|----------|-----------|-----------|
| $I(J^P)=\frac{1}{2}^-(1^1)$ | (1) | 8881.0   | 1662.7    | -1984.6   | -391.6   |
| $E_{CC1}$  | 8881.0   | 1661.9   | -1985.3   | -391.0    |
| $E_{CC2}$  | 8881.0   | 1856.1   | -1870.1   | -312.9    |
| $E_{CC}$   | 8881.0   | 1625.7   | -1975.2   | -372.4    |
| $E_{M_{D^*B_c^+}}$ | 8881.0   | 1660.2   | -1984.6   | -391.6    |
| $\Delta E_{D^*B_c^+}$ | 0.0      | 2.5      | 0.0       | 0.0       |
| $\Delta E_{CC1}$ | 0.0      | 1.7      | -0.7      | 0.6       |
| $\Delta E_{CC2}$ | 0.0      | 195.9    | 114.5     | 78.7      |
| $\Delta E_{CC}$ | 0.0      | -34.5    | 9.4       | 19.2      |

$T A B L E X X I I I$: Contributions of each terms in Hamiltonian to the energy of the $bc\bar{c}\bar{q}$ tetraquark for the bound state in ChQM. $E_{M_{\text{channel}}}$ stands for the sum of the theoretical thresholds of lowest physical channel. (unit: MeV).

| term       | mass     | $V_T$    | $V_{CON}$ | $V_{OGE}$ |
|------------|----------|----------|-----------|-----------|
| $I(J^P)=\frac{1}{2}^-(2^1)$ | (1) | 9104.0   | 1176.7    | -1474.3   | -392.6   |
| $E_{CC1}$  | 9104.0   | 1176.3   | -1475.0   | -392.3    |
| $E_{CC2}$  | 9104.0   | 1347.7   | -1396.2   | -347.2    |
| $E_{CC}$   | 9104.0   | 1161.5   | -1471.2   | -384.4    |
| $E_{M_{D^*B_c^+}}$ | 9104.0   | 1175.9   | -1474.3   | -392.7    |
| $\Delta E_{D^*B_c^+}$ | 0.0      | 0.8      | 0.0       | 0.1       |
| $\Delta E_{CC1}$ | 0.0      | 0.2      | -0.7      | 0.4       |
| $\Delta E_{CC2}$ | 0.0      | 171.8    | 78.1      | 45.5      |
| $\Delta E_{CC}$ | 0.0      | -14.4    | 3.1       | 8.3       |
