Higher derivative effects on $\eta/s$ at finite chemical potential

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Abstract

We examine the effects of higher derivative corrections on $\eta/s$, the ratio of shear viscosity to entropy density, in the case of a finite $R$-charge chemical potential. In particular, we work in the framework of five-dimensional $\mathcal{N} = 2$ gauged supergravity, and include terms up to four derivatives, representing the supersymmetric completion of the Chern-Simons term $A \wedge \text{Tr} (R \wedge R)$. The addition of the four-derivative terms yields a correction which is a $1/N$ effect, and in general gives rise to a violation of the $\eta/s$ bound. Furthermore, we find that, once the bound is violated, turning on the chemical potential only leads to an even larger violation of the bound.

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I. INTRODUCTION

Over the past decade the development of the AdS/CFT correspondence [1, 2, 3] has led to a new way of thinking about strongly coupled gauge theories. Although the original and best studied example of the AdS/CFT duality connects $\mathcal{N} = 4$ supersymmetric Yang-Mills to type IIB string theory on $\text{AdS}_5 \times S^5$, the duality has been extended to a variety of cases, and can describe confining gauge theories with features that are qualitatively similar to QCD. In recent years the AdS/CFT correspondence has proven to be a valuable tool for better understanding thermal and hydrodynamic properties of field theories at strong coupling. In particular, it has been applied to the realm of heavy ion collisions, with the aim of providing a more realistic description of the strongly coupled quark-gluon plasma (QGP).

In the context of RHIC physics, a quantity that has played a special role is the ratio of shear viscosity to entropy density, $\eta/s$ (see e.g. [4] and references therein). Weak coupling calculations in thermal field theory predict $\eta/s \gg 1$, while elliptic flow measurements at RHIC seem to indicate a very small ratio $0 \lesssim \eta/s \lesssim 0.3$, showing that the QGP behaves like a nearly ideal fluid, and is in the strong coupling regime. Motivated by such observations, there has been a large effort to apply AdS/CFT methods to the calculation of various transport coefficients. The AdS/CFT “program” is particularly valuable given that lattice methods (which work well for equilibrium, or thermodynamic, quantities) fail for non-equilibrium processes.

Furthermore, developments resulting from the AdS/CFT correspondence prompted Kovtun, Son and Starinets (KSS) to postulate a bound [5] for $\eta/s$, according to which all fluids would obey

$$\frac{\eta}{s} \geq \frac{1}{4\pi}. \quad (1)$$

The bound, which seems to be satisfied by all substances in nature, was later shown [6] to be saturated in all gauge theories with a dual supergravity description in the large $N$ and $\lambda = g^2_{YM}N$ limit. Moreover, the universal value $\eta/s = 1/4\pi \sim 0.08$ falls into the
experimental range observed at RHIC. Finite $\lambda$ corrections to the leading supergravity result were explored in [7], which considered curvature terms of the form $\sim \alpha'^3 R^4$ in Type IIB supergravity on $AdS_5 \times S^5$. The result was that the leading finite $\lambda$ corrections increase the ratio in the direction consistent with the bound:

$$\frac{\eta}{s} = \frac{1}{4\pi} \left[ 1 + 15 \zeta(3) \lambda^{-3/2} \right]. \quad (2)$$

However, $\eta/s$ bound violations were subsequently observed in the presence of curvature squared terms [8, 9, 10, 11]. In the context of the AdS/CFT correspondence, such terms correspond to finite $N$ corrections and lead to [12]

$$\frac{\eta}{s} = \frac{1}{4\pi} \left( 1 - \frac{c - a}{a} \right), \quad (3)$$

where $a$ and $c$ are the central charges of the dual CFT. Thus, violation will occur provided $c - a > 0$. The central charges are known to be equal in the large $N$ limit [13], with $a = c = O(N^2)$, but differ for finite $N$. For the supergravity examples studied so far, the leading $1/N$ corrections on the CFT side lead to $c - a \geq 0$, implying violation of the bound by finite $N$ effects [12]. (It is an interesting question on its own to ask whether one can have string theory constructions whose dual description allows for $c - a < 0$.)

In this paper we investigate what happens to the $\eta/s$ ratio in the presence of non-zero chemical potential. In particular, we focus on the chemical potential corresponding to turning on a $U(1)_R$ background of the $\mathcal{N} = 2$ system. To leading order in the supergravity approximation, the $R$-charge chemical potential does not affect the calculation of $\eta/s$, as was shown in [14, 15, 16]. However, it is interesting to examine whether this is still the case once higher derivative corrections are included. Furthermore, if $\eta/s$ is affected by $R$-charge, it would be useful to see whether the KSS bound violation gets larger or smaller as a function of chemical potential$^1$.

$^1$ Ideally, one could imagine tuning the chemical potential to match observations. However, it should be noted that the $R$-charge chemical potential we are investigating is not the same as the more physically relevant chemical potential related to non-zero baryon number density.
We work in the framework of $D = 5$, $\mathcal{N} = 2$ gauged supergravity, which is dual to $\mathcal{N} = 1$ super-Yang Mills theory. In particular, we are interested in supersymmetric higher derivative terms, which have a highly constrained structure\textsuperscript{2}. The four-derivative corrections to the leading order supergravity include a mixed gauge-gravitational Chern Simons term $A \wedge \text{Tr}(R \wedge R)$. The supersymmetric completion of this term was done in \[19\], where an off-shell action was obtained for $D = 5$, $\mathcal{N} = 2$ gauged supergravity at the four-derivative level. In \[20\] we derived the corresponding on-shell Lagrangian, found corrected $R$-charged black hole solutions, and studied their thermodynamic properties. We will use many of the results of \[20\] to compute the shear viscosity. Our main result is that turning on $R$-charge not only leads to violation of the bound, but enhances the effect, pushing $\eta/s$ further below $1/4\pi$. Furthermore, while the dependence of $\eta$ and $s$ individually on the $R$-charge is quite complicated, the ratio $\eta/s$ is remarkably simple.

The general picture that emerges from such studies is that if we are interested in describing properties of the QGP (or other strongly coupled systems), we can try tuning the parameters available to us (whether $N$, $\lambda$ or the chemical potential), as long as we remain within the regime of validity of the supergravity approximation. Moreover, it is an interesting fundamental question whether violations of the bound can be related to any constraints on the dual gravitational side or consistency requirement of the underlying string theory. For instance, one may be able to relate the violation of the $\eta/s$ bound to the weak gravity conjecture of \[21\], according to which there should be some states whose $M/Q$ ratio is below the BPS bound. While this is an interesting avenue to explore\textsuperscript{3}, the solutions that we have considered do not admit a nice extremal BPS black hole limit (since the extremal solution is the superstar geometry, with a naked singularity), and therefore do not lend themselves easily to such an analysis.

The structure of this paper is as follows. In the following section, we present the

\textsuperscript{2} Four derivative corrections in the presence of a chemical potential have been partially discussed in \[17, 18\], where $R^2$ and $F^4$ corrections were considered, respectively. The supersymmetric Lagrangian, however, has $RF^2$ and $\nabla F \nabla F$-type terms as well which were not previously considered.

\textsuperscript{3} See \[22\] for investigating the effect of higher derivatives on the weak gravity conjecture.
on-shell four-derivative action and write down the non-extremal $R$-charged background. Using this as a starting point, we then compute the shear viscosity in Section III and conclude with a brief discussion in Section IV. Some of the intermediate expressions are relegated to an appendix. While this work was being completed we became aware of [23], which overlaps with our results.

II. $\mathcal{N} = 2$ GAUGED SUPERGRAVITY AND $R$-CHARGED BLACK HOLES

Our starting point is five-dimensional $\mathcal{N} = 2$ gauged supergravity. The physical fields in this theory are the metric $g_{\mu\nu}$, graviphoton $A_\mu$ and gravitino $\psi_\mu$. The supersymmetric four-derivative corrections were obtained in [19] using the superconformal tensor calculus methods worked out in [24, 25, 26, 27]. By integrating out the auxiliary fields, the Lagrangian may be put into the form [20]

$$16\pi G_5 e^{-1} \mathcal{L} = -R - \frac{1}{4} F^2 + \frac{1}{12\sqrt{3}} \left(1 - 4\bar{c}_2\right) \epsilon^{\mu\nu\rho\lambda} A_\mu F_{\nu\rho} F_{\lambda\sigma} + 12g^2$$

$$+ \frac{\bar{c}_2}{g^2} \left[ \frac{1}{16\sqrt{3}} \epsilon_{\mu\nu\rho\lambda\sigma} A^\mu R^{\nu\rho\delta\gamma} R^{\lambda\sigma}_{\delta\gamma} + \frac{1}{8} C_{\mu\nu\rho\sigma} + \frac{1}{16} C_{\mu\nu\rho\lambda} F^{\mu\nu} F^{\rho\lambda} - \frac{1}{3} F^\mu F_{\mu\nu} R^\nu + \frac{1}{48} RF^2 + \frac{1}{2} F_{\mu\nu} \nabla^\mu \nabla_\rho F^{\rho\nu} + \frac{1}{4} \nabla^\mu F^{\nu\rho} \nabla_\mu F_{\nu\rho} + \frac{1}{4} \nabla^\mu F^{\nu\rho} \nabla_\nu F_{\rho\mu} $$

$$+ \frac{1}{32\sqrt{3}} \epsilon_{\mu\nu\rho\lambda\sigma} F^{\mu\nu} \left(3F^{\rho\lambda} \nabla_\delta F^{\sigma\delta} + 4F^{\rho\delta} \nabla_\delta F^{\lambda\sigma} + 6F^\rho \nabla^\lambda F^{\sigma\delta} \right)$$

$$+ \frac{5}{64} F_{\mu\nu} F^{\nu\rho} F_{\rho\lambda} F^{\lambda\mu} - \frac{41}{2304} (F^2)^2 \right].$$ (4)

The four-derivative corrections are determined in terms of a single new dimensionless parameter $\bar{c}_2$ (corresponding to $c_2 g^2/24$ in the notation of [20]). Holographic computation of the Weyl anomaly [13, 28, 29, 30] allows $G_5$ and $\bar{c}_2$ to be expressed in terms of the anomaly coefficients $a$ and $c$ of the dual $\mathcal{N} = 1$ gauge theory. This was worked out in [12, 20], with the result

$$g^3 G_5 = \frac{\pi}{8a}, \quad \bar{c}_2 = \frac{c - a}{a}.$$ (5)

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4 We follow the conventions of [19] and take $[\nabla_\mu, \nabla_\nu] \nu^\sigma = R_{\mu\nu\rho} \sigma \nu^\rho$ and $R_{ab} = R_{ac} c_b$. 5
Nonextremal $R$-charged black hole solutions to the lowest order $\mathcal{N} = 2$ gauged supergravity were found in [31], and the corrections linear in $\bar{c}_2$ were worked out in [20]. Using a parameterization convenient for the shear viscosity calculation, the flat-horizon black holes are given by the metric

$$ds^2 = \frac{g^2 r_0^2}{u} \left[ \frac{f(u)}{H(u)^2} dt^2 - H(u) d\vec{x}^2 \right] - \frac{H(u)}{4 g^2 u^2 f(u)} du^2,$$

and the gauge field

$$A_t = g r_0 \sqrt{\frac{3(1 + q)^3}{q}} \left[ 1 - \frac{1}{1 + qu} - \frac{\bar{c}_2}{2} \frac{q(1 + q)^3 u^3 (1 - qu)}{(1 + qu)^4} \right].$$

The metric functions $f(u)$ and $H(u)$ are given by

$$f = (1 + qu)^3 - (1 + q)^3 u^2 + \bar{c}_2 \left[ -\frac{8}{3} q(1 + q)^3 u^3 + \frac{1}{4} (1 + q)^6 \frac{u^4}{1 + qu} \right],$$

$$H = 1 + qu - \frac{\bar{c}_2}{3} q(1 + q)^3 \frac{u^3}{(1 + qu)^2}.$$

The above solution is fixed in terms of two parameters, $r_0$ (related to non-extremality) and dimensionless $q$ (related to the $R$-charge). At the two-derivative level, the horizon is located at $u = 1$, while the boundary of AdS$_5$ is at $u = 0$. At linear order in $\bar{c}_2$, however, the horizon location gets shifted to

$$u_+ = 1 + \frac{\bar{c}_2}{12} \frac{(1 + q)(3 - 26q + 3q^2)}{2 - q}.\tag{9}$$

The temperature and entropy density were obtained in [20]

$$T = \frac{g^2 r_0 (2 - q)(1 + q)^{1/2}}{2 \pi} \left[ 1 - \frac{\bar{c}_2}{8} \frac{10 - 59q - 4q^2 - 3q^3}{(2 - q)^2} \right],$$

$$s = \frac{(gr_0)^3 (1 + q)^{3/2}}{4 G_5} \left[ 1 + \frac{\bar{c}_2}{8} \frac{21 + 14q - 3q^2}{2 - q} \right].\tag{10}$$

Note that, for $q = 0$, we may write the entropy density in terms of the temperature as

$$s = 2 \pi^2 a \left[ 1 + \frac{\frac{9}{4} \frac{c - a}{a} }{T^3} \right] T^3,$$

where we used the holographic relations [5]. This reduces to the familiar $s = \pi^2 N^2 T^3/2$ for $\mathcal{N} = 4$ SYM, where $a = c = N^2/4$. 

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III. COMPUTATION OF THE SHEAR VISCOSITY

We compute the shear viscosity using the Kubo formula, following the methods developed in [7, 8]. In particular, we introduce a scalar channel perturbation to the metric

$$g_{xy} \rightarrow g_{xy} + h_{xy},$$

(12)

where, for convenience, we define $h_{xy} = \phi(t, u, \vec{x})$. Expanding the Lagrangian (11) to second order in the perturbation yields

$$S = \frac{1}{16\pi G_5} \int \frac{d^4k}{(2\pi)^4} \int_0^1 du \left[ A\phi'_{-k}\phi_{-k} + B\phi'_{k}\phi'_k + C\phi'_{-k} + D\phi_{-k} - k\phi_{-k} \\
+ E\phi''_{-k} + F\phi''_{k}\phi'_k \right],$$

(13)

where the fourier components of $\phi$ are defined by

$$\phi(t, u, \vec{x}) = \int d^3x dt \phi_k(u)e^{i(k \vec{x} - \omega t)}.$$  

(14)

We note that this parameterization of the action with coefficients $A, \ldots, F$ was originally used in [7] to handle the $R^4$ correction of IIB supergravity. However, it is general enough to accommodate the present case. The coefficients are even functions of the momentum, and are given explicitly in the appendix.

Varying this action with respect to $\phi$ yields a fourth order differential equation. However, since the higher derivative terms are multiplied by $\bar{c}_2$, we may reduce the order of the equation by working perturbatively in $\bar{c}_2$. To see this, we first consider the lowest order equation of motion

$$\phi'' + \left( \frac{f_0'}{f_0} - \frac{1}{u} \right) \phi' + \frac{\omega^2 H_0^3}{u f_0^2} \phi = 0,$$

(15)

where we have defined the dimensionless frequency

$$\omega^2 = \frac{\omega^2}{4 g^4 r_0^2}.$$  

(16)

The lowest order metric functions

$$f_0 = (1 + qu)^3 - (1 + q)^3 u^2, \quad H_0 = 1 + qu,$$

(17)
are obtained by setting $\bar{c}_2 = 0$ in (8). Taking additional derivatives of (15) allows us to eliminate $\phi'''$ and $\phi''''$ terms in the full equation of motion. The result is rather simple:

$$\phi'' + \left( \frac{f'}{f} - \frac{1}{u} - \bar{c}_2 \frac{(1+q)^3u}{(1+qu)^3} \right) \phi' + \frac{\bar{\omega}^2 H^3}{uf^2} \phi = 0.$$  \hspace{1cm} (18)

Notice that the form of this equation is almost identical to that of (15), the lowest order equation of motion, modified only by the presence of the corrected metric functions $f$ and $H$ as well as one new term, which is explicitly $O(\bar{c}_2)$.

Since the function $f(u)$ vanishes linearly at the horizon $u_+$, the point $u = u_+$ is a regular singular point of the equation of motion (18). This suggests that we write

$$\phi(u) = f(u)^\nu F(u),$$  \hspace{1cm} (19)

where $F(u)$ is assumed to be regular at the horizon. The exponent $\nu$ is then obtained by solving the indicial equation. In the hydrodynamic limit, the lowest order solution is known [14, 15] and is given by:

$$\phi_0 = f_0(u)^{\nu_0} \left\{ 1 - \frac{\nu_0}{2} \left[ \Delta \ln \left( \frac{\Xi - \alpha_1 - 1 + 2\alpha_3 u}{(\Xi + \alpha_1 + 1)(\Xi - \alpha_1 - 1)} \right) + 3 \ln \left( 1 + (\alpha_1 + 1)u - \alpha_3 u^2 \right) \right] \right\},$$  \hspace{1cm} (20)

where

$$\alpha_1 \equiv 3q, \quad \alpha_2 \equiv 3q^2, \quad \alpha_3 \equiv q^3, \quad \Xi \equiv (1 + q)(1 + 4q)^{1/2}, \quad \Delta \equiv -3 \frac{q + 1}{\Xi}.$$  \hspace{1cm} (21)

The exponent $\nu_0$ is given by

$$\nu_0 = -\frac{i\bar{\omega}}{(2 - q)(1 + q)^{1/2}},$$  \hspace{1cm} (22)

and may be re-expressed as $\nu_0 = -i\omega/4\pi T_0$, where $T_0$ is the lowest order temperature given in (10). Note that we have chosen incoming wave boundary conditions at the horizon as appropriate to the shear viscosity calculation.

Adding higher derivative terms will have two effects on this solution, one being a correction to the function $F(u)$ and the other a modification of the exponent $\nu$ defined
above. For the exponent, solving the indicial equation gives

\[
\nu = -\frac{i\omega}{(2-q)(1+q)^{1/2}} \left(1 + \frac{\bar{c}_2 10 - 59q - 4q^2 - 3q^3}{(q - 2)^2}\right) = -\frac{i\omega}{4\pi T},
\]

(23)

where the relation to the temperature (10) is valid to linear order in \(\bar{c}_2\). We may now substitute \(\phi(u) = f(u)^\nu F(u)\) into the equation of motion (18) and linearize in \(\bar{c}_2\) to obtain an equation for \(F(u)\). While this is difficult to solve exactly, since we only need a solution in the hydrodynamic regime, it is sufficient to work to first order in \(\omega\) (or equivalently \(\nu\)).

The solution for \(F(u)\) is quite complicated and can be found in the appendix.

Given this solution, it remains to evaluate the on-shell value of the action. As explained in [7], the bulk action (13) must be paired with an appropriate generalization of the Gibbons-Hawking term. In general, the fourth order equation of motion yields a boundary value problem for the two-point function where additional data must be specified (e.g. fields and their first derivatives at the endpoints). However, when working perturbatively in \(\bar{c}_2\), the equation of motion reduces to a second order one, given by (18). This allows us to use a generalized Gibbons-Hawking term of the form

\[
\mathcal{K} = -A\phi_k\phi'_{-k} - \frac{F}{2}\phi'_k\phi'_{-k} + E(p_1\phi'_k + 2p_0\phi_k)\phi'_{-k},
\]

(24)

where

\[
p_1 = \frac{f'_0}{f_0} - \frac{1}{u}, \quad p_2 = \frac{\bar{\omega}^2 H^3_0}{u f'^2_0}
\]

(25)

are the coefficients in the lowest order equation of motion (15).

Evaluating the on-shell action then amounts to evaluating a boundary term

\[
S = \int \frac{d^4k}{(2\pi)^4} \mathcal{F}_k \bigg|_0^{1},
\]

(26)

where

\[
\mathcal{F}_k = \left[ (B - A - \frac{F'}{2})\phi'_k\phi'_{-k} + \frac{1}{2}(C - A')\phi_k\phi_{-k} - E\phi''_k\phi_{-k} + \frac{f'_0}{f_0} - \frac{1}{u}\right] \phi'_k\phi'_{-k} + 2E\bar{\omega}^2 H^3_0 f'^2_0\phi'_k\phi'_{-k}.
\]

(27)
In order to compute the shear viscosity we need only the limit of the above action as \( u \) approaches the AdS boundary (i.e. \( u \to 0 \)). It turns out that only the first and third terms contribute. This yields a value for the shear viscosity via the Kubo relation

\[
\eta = \lim_{\omega \to 0} \lim_{u \to 0} \frac{1}{\omega} \text{Im} F_k = \frac{(gr_0)^3}{16\pi G_5} (q + 1)^{3/2} \left( 1 + \frac{\bar{c}_2}{8} \frac{5 + 6q + 5q^2}{2 - q} \right). \tag{28}
\]

Finally, dividing this by the entropy density \( s \) gives a value for the shear viscosity to entropy density ratio of

\[
\frac{\eta}{s} = \frac{1}{4\pi} \left[ 1 - \bar{c}_2 (1 + q) \right] = \frac{1}{4\pi} \left[ 1 - \frac{c - a}{a} (1 + q) \right], \tag{29}
\]

where we have rewritten \( \bar{c}_2 \) in terms of the anomaly coefficients \( c \) and \( a \) using (5).

**IV. DISCUSSION**

The expression for \( \eta/s \), given in (29), is surprisingly simple, given that both \( \eta \) and \( s \) are individually rather more complicated functions of the parameter \( q \). This is presumably related to some form of universality, which holds even in an \( R \)-charged background\(^5\). It is instructive to examine the contribution of the various terms in the four-derivative action to the result (29). We find that only four terms in (4) are important. Writing

\[
16\pi G_5 e^{-1} \mathcal{L} = -R - \frac{1}{4} F^2 + \cdots + \frac{\bar{c}_2}{g^2} \left[ \alpha_1 C_{\mu \nu \rho \sigma}^2 + \alpha_2 C_{\mu \nu \rho \sigma} F_{\mu \nu}^\rho F_{\rho \sigma} + \alpha_3 \nabla^\mu F_{\mu \nu}^\rho \nabla_\rho F_{\nu \sigma} + \alpha_4 \nabla^\mu F_{\mu \nu}^\rho \nabla_\rho F_{\nu \sigma} + \cdots \right], \tag{30}
\]

we may arrive at the result

\[
\frac{\eta}{s} = \frac{1}{4\pi} \left[ 1 - 4\bar{c}_2 \left( 2\alpha_1 - q(\alpha_1 + 6\alpha_2 - 6\alpha_3 + 3\alpha_4) \right) \right]. \tag{31}
\]

Note that setting \( \alpha_i \) to their actual values in (4) reproduces (29).

\(^5\) Of course, the simplest result possible would have been to obtain \( \eta/s \) independent of \( q \). But this is clearly not the case here.
The shear viscosity to entropy density ratio was independently derived in [23], where it was found to depend only on terms explicitly involving the Riemann tensor \( \alpha_1 \) and \( \alpha_2 \) terms in (30). This appears to differ from the result found above. However, by the use of Bianchi identities and integration by parts we can cast the gradient terms into the form

\[
\alpha_3 \nabla^\mu F^{\nu\rho} \nabla_\mu F_{\nu\rho} + \alpha_4 \nabla^\mu F^{\nu\rho} \nabla_\nu F_{\rho\mu} =
(2\alpha_3 - \alpha_4) \left[ - F_{\mu\nu} \nabla^\nu \nabla_\rho F^{\mu\rho} + F^{\mu\rho} F_{\rho\nu} R_\mu^\nu - \frac{1}{2} R_{\mu\nu\rho\sigma} F^{\mu\nu} F^{\rho\sigma} \right]. \tag{32}
\]

The first two terms do not contribute to the \( \eta/s \) ratio, while the last term will add to the original \( \alpha_2 \) term to give an effective \( \tilde{\alpha}_2 = \alpha_2 - \alpha_3 + \alpha_4/2 \), so that (31) may be rewritten as

\[
\eta \over s = \frac{1}{4\pi} \left[ 1 - 4\bar{c}_2 (2\alpha_1 - q(\alpha_1 + 6\tilde{\alpha}_2)) \right]. \tag{33}
\]

This agrees with the result of [23] provided the difference in signature conventions is taken into account.

Finally, we return to the \( \mathcal{N} = 1 \) SYM shear viscosity result of (29). In order to express this in terms of physical quantities, we wish to relate the parameter \( q \) to the \( R \)-charge chemical potential and temperature. Since \( q \) only enters into (29) at the next-leading order, we can use the leading order expressions in pinning down \( q \). The chemical potential for \( R \)-charge \( \Phi \) is identified as the difference of \( A_t \) between horizon and boundary [33, 34]. At lowest order, (7) yields

\[
\Phi = gr_0 \sqrt{3q(1 + q)}. \tag{34}
\]

Comparing this to the temperature

\[
T_0 = \frac{g^2 r_0}{2\pi} (2 - q)(1 + q)^{1/2}, \tag{35}
\]

allows us to write

\[
q = \frac{3}{2\Phi^2} \left( 1 + \frac{4}{3} \Phi^2 - \sqrt{1 + \frac{8}{3} \Phi^2} \right), \tag{36}
\]
where $\bar{\Phi} = g\Phi/2\pi T$ is the dimensionless chemical potential. Note that $q$ is an increasing function with respect to $\bar{\Phi}$, with $q = 0$ when $\bar{\Phi} = 0$. The possible value of $q$ ranges as

$$0 \leq q \leq 2.$$  (37)

Substituting (36) into (29) then gives

$$\frac{\eta}{s} = \frac{1}{4\pi} \left[ 1 - \frac{c - a}{a} \left( 1 + \frac{3}{2\Phi^2} \left( 1 + \frac{4}{3}\Phi^2 - \sqrt{1 + \frac{8}{3}\Phi^2} \right) \right) \right].$$  (38)

Since $q$ is non-negative, this demonstrates that turning on an $R$-charge chemical potential only increases violation of the $\eta/s$ bound, provided $c - a > 0$. Taking the range (37) into account, we see that adjusting the $R$-charge yields a range of values

$$\frac{1}{4\pi} \left( 1 - 3 \frac{c - a}{a} \right) \leq \frac{\eta}{s} \leq \frac{1}{4\pi} \left( 1 - \frac{c - a}{a} \right),$$  (39)

where we have again assumed $c - a > 0$.

In conclusion, we have explored the effect of a background $R$-charge on the shear viscosity to entropy density ratio $\eta/s$. While the leading order ratio $\eta/s = 1/4\pi$ is universal, $R$-charge corrections do turn up at the $1/N$ order. For known theories with a holographic dual, where $c - a > 0$, the conjectured $1/4\pi$ bound is generally violated for arbitrary chemical potential. We caution, however, that this is a parametrically small violation appearing at $O(1/N)$ in the large $N$ limit. In principle, it would be desirable to obtain a more robust result. However, this is hindered by difficulties in obtaining exact solutions to the full equations of motion (i.e. beyond the linearized limit). While this can be done in certain cases such as Gauss-Bonnet gravity, the natural supersymmetric organization of the higher derivative Lagrangian (4) is not of this form. It would be interesting to see if a modified universality relation for $\eta/s$ can be obtained for arbitrary forms of the higher derivative gravity theory.

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VI. APPENDIX

The quadratic action for the scalar channel perturbation $\phi$ is given in (13) in terms of six coefficients $A, \ldots, F$. Here we present their explicit forms:

$$A(u) = \frac{4}{u} f_0 + \bar{c}_2 \left[ -\frac{\omega^2}{2u^2} \frac{H_0^3}{3g^2} + \frac{2uf_0(1 + q)^3(5qu - 1)}{H_0^3} - \frac{32g^2u^2(1 + q)^3}{3H_0^3} + \frac{g^2u^3(1 + q)^6}{H_0^3} \right],$$

$$B(u) = \frac{3f_0}{u} + \bar{c}_2 \left[ -\frac{\omega^2}{2u^2} \frac{H_0^3}{3g^2} + \frac{g^2(4qu + 1)^2H_0^3}{3u} - \frac{g^2u(1 + q)^3(56q^2u^2 + 7qu + 11)}{6} 
+ \frac{g^2u^2(1 + q)^6(26q^2u^2 - 17qu + 17)}{6H_0^3} \right],$$

$$C(u) = \frac{2g^2(4qu - 3)H_0^3}{4u^2} - \frac{2g^2(1 + q)^3(2qu + 1)}{H_0^3}$$
$$+ \bar{c}_2 \left[ -\frac{\omega^2}{6uf_0} \left((4qu + 1)H_0^3 - (1 + q)^3(-11qu^3 + 13u^2)H_0 \right) 
- \frac{g^2(1 + q)^3(4q^2u^2 + 45qu + 3)}{3H_0^3} + \frac{g^2u^2(1 + q)^6(4q^3u^3 - 7q^3u^2 - 32qu + 15)}{2H_0^3} \right],$$

$$D(u) = \frac{2g^2H_0^3 - g^2qu^3(1 + q)^3}{u^3H_0^3} + \frac{\omega^2}{4u^2f_0} \frac{H_0^3}{H_0^2}$$
$$+ \bar{c}_2 \left[ \frac{\omega^4}{g^2} \frac{H_0^3}{12uf_0^2} + \frac{\omega^2g^2(1 + q)^3}{48f_0^2} \left(2(31qu - 9)H_0^3 - 3u^2(1 + q)^3(5q^2u^2 - 4qu + 11) \right) 
- \frac{19g^2q(1 + q)^3}{3H_0^3} - \frac{3g^2u(1 + q)^6(6q^2u^2 - 17qu + 1)}{2H_0^3} \right],$$

$$E(u) = \bar{c}_2 \frac{4uf_0^2}{3g^2H_0^3},$$

$$F(u) = \bar{c}_2 f_0 \frac{2(4qu + 1)H_0^3 - u^2(1 + q)^3(7qu + 4)}{3H_0^3}. \quad (40)$$

Here we also present the $O(\bar{c}_2)$ solution for $\phi$. Writing $\phi(u) = f(u)^\nu F(u)$, we may expand $F(u)$ to first order in both $\bar{c}_2$ and $\omega$

$$F(u) = F_0(u, \omega) + \bar{c}_2 (F_{10}(u) + \omega F_{11}(u)). \quad (41)$$
Since \( F(u) \) satisfies a second order equation (after linearizing in \( \bar{c}_2 \) and using the lowest order equation of motion), it is consistent to choose the boundary conditions such that \( F(u) \) is normalized at the boundary \( (F(0) = 1) \) and is regular at the horizon.

The function \( F_0(u, \omega) \) is given by the expression in the curly brackets in (20), while the remaining functions are

\[
\begin{align*}
F_{10}(u) &= 0, \\
F_{11}(u) &= \frac{(1 + q)^{3/2}(11q^6 + 4q^4 + 179q^3 - 10q^2 - 8q - 16)}{32q^2(1 + q)^2(q - 2)^3} \left[ i \ln(q^3u^2 - 3qu - u - 1) + \pi \right] \\
&\quad + \frac{i(q + 1)^{3/2}(60q^6 + 99q^5 + 648q^4 - 69q^3 - 154q^2 - 104q - 16)}{16(4q + 1)^{3/2}(q + 1)^2(q - 2)^3} \times \\
&\quad \left[ \tanh^{-1} \frac{(1 + 3q)}{(4q + 1)^{1/2}(q + 1)} - \tanh^{-1} \frac{2q^3u - (1 + 3q)}{(4q + 1)^{1/2}(q + 1)} \right] \\
&\quad - \frac{i \ln(1 + qu)(1 + q)^{3/2}}{8q^2} - \frac{i(q + 1)^{3/2}(-4q^5 + 21q^4 + 143q^3 - 21q^2 - 39q - 6)}{8q^4(4q + 1)(q - 2)^2} \\
&\quad - \frac{i(q + 1)^{3/2}(4q^7 - 27q^6 + 64q^5 + 511q^4 + 137q^3 - 128q^2 - 57q - 6)q^u^2}{8(1 + qu)q^4(q^3u^2 - 3qu - u - 1)(4q + 1)(q - 2)^2} \\
&\quad + \frac{i(-12q^6 + 102q^5 + 605q^4 + 63q^3 - 177q^2 - 63q - 6)u}{8(1 + qu)q^4(q^3u^2 - 3qu - u - 1)(4q + 1)(q - 2)^2} \\
&\quad - \frac{i(4q^5 + 21q^4 + 143q^3 - 21q^2 - 39q - 6)}{8(1 + qu)q^4(q^3u^2 - 3qu - u - 1)(4q + 1)(q - 2)^2}.
\end{align*}
\]

(42)

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