Gauged NJL model at strong curvature

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Abstract

We investigate the gauged NJL–model in curved spacetime using the RG formulation and the equivalency with the gauge Higgs–Yukawa model in a modified 1/Nc–expansion. The strong curvature induced chiral symmetry breaking is found in the non-perturbative RG approach (presumably equivalent to the ladder Schwinger–Dyson equations). Dynamically generated fermion mass is explicitly calculated and inducing of Einstein gravity is briefly discussed. This approach shows the way to the non-perturbative study of the dynamical symmetry breaking at external fields.

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1 Introduction

The dynamical symmetry breaking in different quantum field theories (like QCD or Standard Model) maybe realized even in the absence of Higgs scalars with the help of the Nambu–Jona-Lasinio mechanism [1]. Moreover, NJL–like models admit the analytic treatment of composite bound states. Such models can be used as effective theories for the Standard Model and GUT’s.

It would be of interest to study the cosmological applications of the NJL–like models. In particular, in the Early Universe the phase structure of the NJL–model will be qualitatively changed due to non–zero curvature effects. The study of the non–gauged NJL– model in curved spacetime [2] has shown a quite rich phase structure and the possibility of curvature–induced transitions between chiral symmetric and chiral non–symmetric phases. Moreover, the NJL–model (in $1/N_c$ – expansion) maybe useful for the solution of the cosmological constant problem via an effective theory for the conformal factor [3].

The investigation of the more realistic gauged NJL–model is much more complicated. Recently, such a model in flat spacetime has been studied in full detail in Ref. [4] using the ladder Schwinger–Dyson (SD) equations where the renormalizability of the gauged NJL–model also has been discussed. Unfortunately, it is not clear at all how to formulate SD equations in curved spacetime [5].

In Ref. [6] the flat spacetime gauged NJL–model has been discussed using the renormalization group (RG) formalism with appropriate compositeness conditions for the running couplings. In Ref. [4] such formulation has been used to show that some class of gauge Higgs–Yukawa models in leading order with respect to a modified $1/N_c$–expansion leads to a well defined non–trivial theory which is equivalent to the gauged NJL–model. In this sense, the gauged NJL–model maybe considered as a renormalizable theory and the RG–improved effective potential [4] maybe found [4] in exact agreement with the ladder–SD effective potential [4] (for earlier discussion of the ladder SD potential see [9]).

In the present letter, using the approach of Refs. [6], [7] we study the gauged NJL–model in curved spacetime. The RG–improved effective potential is calculated at strong curvature and at weak curvature. The possible chiral symmetry breaking in curved spacetime is shown, and the induced Einstein gravity within the gauged NJL–model is described.
Let us start from the $SU(N_c)$ gauge theory with scalars and spinors in curved spacetime:

$$L_m = -\frac{1}{4} G_{\mu\nu}^a G^{a\mu\nu} + \frac{1}{2} g^{\mu\nu} \partial_\mu \sigma \partial_\nu \sigma - \frac{1}{2} m^2 \sigma^2 - \frac{\lambda}{4} \sigma^4 - \frac{1}{2} \xi R \sigma^2 + \sum_{i=1}^{N_f} \bar{\psi}_i i\hat{D} \psi_i - \sum_{i=1}^{n_f} y \sigma \bar{\psi}_i \psi_i .$$

(1)

where $\sigma$ is a single scalar, $N_f$ fermions $\psi_i$ belong to the representation $R$ of $SU(N_c)$, and $y$ is the Yukawa coupling constant. Note that in order for the theory to be multiplicatively renormalizable in curved spacetime one has to add to the Lagrangian (1) the Lagrangian of the external gravitational field $[5]$. However, this Lagrangian will not be relevant in our discussion.

Let us describe now the modified $1/N$–approximation of Ref. $[7]$ for studying the theory (1) at high energies:

a) The gauge coupling constant is assumed to be small:

$$\frac{g^2 N_c}{4\pi} << 1 ,$$

(2)

and one has to work only in the first non–trivial order on $g^2$.

b) The number of fermions should be large enough:

$$N_f \sim N_c ;$$

(3)

however, only $n_f$ fermions ($n_f \ll N_f$) have large Yukawa couplings.

c) The leading order of $1/N_c$–expansion is also considered; this means, in particular, that scalar loop contributions should be negligible

$$|\frac{\lambda}{y^2}| \leq N_c$$

(4)

(for more details, see $[3]$).

Within the above approximation, the standard one–loop RG equations for the coupling constants are (see for example $[10]$, $[11]$ for flat spacetime and, for coupling constant $\xi$, see $[4]$):
\[
\begin{align*}
\frac{dg(t)}{dt} &= - \frac{b}{(4\pi)^2} g^3(t), \\
\frac{dy(t)}{dt} &= \frac{y(t)}{(4\pi)^2} \left[ a y^2(t) - c g^2(t) \right], \\
\frac{d\lambda(t)}{dt} &= \frac{u y^2(t)}{(4\pi)^2} \left[ \lambda(t) - y^2(t) \right], \\
\frac{d\xi(t)}{dt} &= \frac{1}{(4\pi)^2} 2ay^2(t) \left( \xi(t) - \frac{1}{6} \right), \tag{5}
\end{align*}
\]

where \( b = (11N_c - 4T(R)N_f)/3, c = 6 C_2(R), a = u/4 = 2 n_f N_c. \) Here \( t = \ln(\mu/\mu_0), \) and for the fundamental representation we have \( T(R) = 1/2, C_2(R) = (N_c^2 - 1)/(2N_c). \)

The one–loop effective potential up to linear curvature terms in the above approximation maybe found as follows \( (\sigma^2 \gg |R|; \) for flat spacetime, see [7], and for curved spacetime, see [12]):

\[
V = \frac{1}{2} m^2 \sigma^2 + \frac{\lambda}{4} \sigma^4 + \frac{1}{2} \xi R \sigma^2 - \frac{a M_F^4}{2(4\pi)^2} \left[ \ln \frac{M_F^2}{\mu^2} - \frac{3}{2} \right] - \frac{a R M_F^2}{12(4\pi)^2} \left[ \ln \frac{M_F^2}{\mu^2} - 1 \right], \tag{6}
\]

where \( M_F = y\sigma \) plays the role of the effective fermion mass for the potential (6).

Similarly, in the case when the curvature is strong \( |R| \gg \sigma^2 \) (but \( |R| > R^2, R_{\mu\nu} \)-terms), the \( \sigma \)-dependent part of the effective potential is

\[
V = \frac{1}{2} m^2 \sigma^2 + \frac{\lambda}{4} \sigma^4 + \frac{1}{2} \xi R \sigma^2 - \frac{a M_F^4}{2(4\pi)^2} \left[ \ln \frac{-R/4}{\mu^2} - \frac{3}{2} \right] - \frac{a R M_F^2}{12(4\pi)^2} \left[ \ln \frac{-R/4}{\mu^2} - 1 \right], \tag{7}
\]

where \( R \) is supposed to be negative. The expressions (6), (7) will be used below.
The solution of the RG–equation (5) for the gauge coupling is ($\alpha = g^2/(4\pi)$)

$$\eta(t) \equiv \frac{g^2(t)}{g^2_0} = \frac{\alpha(t)}{\alpha_0} = (1 + \frac{b \alpha_0}{2\pi t})^{-1}.$$  \hspace{1cm} (8)

It will not be changed in the gauged NJL–model.

The analysis of solutions for Yukawa and scalar couplings in the above approximation has been done in Ref. [7] in full detail. It has been shown that the Yukawa coupling must be asymptotically free in order for the theory to be non–trivial. The condition of non–triviality and stability leads to non–trivial solution for the scalar coupling (actually the solutions lie on the line between the Gaussian fixed point and the fixed point of Ref. [13], where the reduction of couplings takes place [14], [15]).

Now we will turn to the $SU(N_c)$ gauged NJL–model with four–fermion coupling constant $G$ in curved spacetime

$$L = -\frac{1}{4} G_{\mu\nu}^2 + \sum_{i=1}^{N_f} \bar{\psi}_i \mathcal{D}_\mu \psi_i + G \sum_{i=1}^{n_f} (\bar{\psi}_i \psi_i)^2.$$  \hspace{1cm} (9)

The standard way to study such a model is to introduce an auxiliary field $\sigma$, in order to identify the NJL–model with the Higgs–Yukawa model. As it has been shown in [6], using the RG approach on can put a set of boundary conditions for the effective couplings of the gauge Higgs–Yukawa model at $t_\Lambda = \ln(\Lambda/\mu_0)$ (where $\Lambda$ is the UV–cut-off of the gauged NJL–model) in order to prove the equivalency of the gauged NJL–model with the gauge Higgs–Yukawa model (at all $\Lambda$, and even for $\Lambda \rightarrow \infty$, showing the renormalizability of the gauged NJL–model in this sense [6], see also [4]).

Taking into account the explicit expressions for the compositeness conditions [6] one can show that the running Yukawa and scalar couplings in the gauged NJL–model are [6] ($g(t)$ is not changing)

$$y^2(t) = \frac{c - b}{a} g^2(t) \left[ 1 - \left( \frac{\alpha(t)}{\alpha(t_\Lambda)} \right)^{1-c/b} \right]^{-1} \equiv y_A^2(t),$$

$$\frac{\lambda(t)}{y^4(t)} = \frac{2a}{2c - b} \frac{1}{g^2(t)} \left[ 1 - \left( \frac{\alpha(t)}{\alpha(t_\Lambda)} \right)^{1-2c/b} \right] \equiv \frac{\lambda_A(t)}{y_A^4(t)}.$$  \hspace{1cm} (10)
where $t < t_A, c < b$. For the case of fixed gauge coupling, $b \to +0$, eqs. (10) are significantly simplified [7].

In addition, one should have also a compositeness condition for the mass which is explicitly written in Ref. [7]. For the case $b \to +0$, it will be

$$m^2(t) = \frac{2a}{(4\pi)^2} \left( \frac{\Lambda^2}{\mu^2} \right)^w y_A^2(t) \mu^2 \left[ \frac{1}{g_4(\Lambda)} - \frac{1}{w} \right],$$

(11)

where $g_4(\Lambda)$ is a dimensionless constant defined by $G \equiv ((4\pi)^2/a)g_4(\Lambda)/\Lambda^2$, and $w \equiv 1 - \alpha/(2\alpha_c)$, and where $y_A^2(t)$ is given by (10) at $b \to +0$, and $\alpha_c^{-1} = 3C_2(R)/\pi$. The compositeness condition for $\xi(t)$ is given as (see [12]; for non-gauged NJL-model, see also [10])

$$\xi(t) = \frac{1}{6}.$$  

(12)

Thus, we got the description of the gauged NJL–model via running coupling constants (using the equivalency with the gauged Higgs–Yukawa model).

3 RG improved potential and dynamical symmetry breaking

Let us turn now to the question of dynamical symmetry breaking in the model under consideration. For this purpose one has to calculate the effective potential, using, for example, the ladder SD equation [4], [9]. Unfortunately, as it was already mentioned it is not clear at all, how to formulate SD equations in curved spacetime. However, one may use the RG technique because we have the RG formulation of the gauged NJL–model via equivalent gauge Higgs–Yukawa model. Surprisingly, it has been shown [7] that the RG improved effective potential [8] gives the same results as the ladder SD effective potential [4].

The technique to study the RG improved effective potential is quite well-known in flat space [8] as well as in curved space [17]. Hence, we will not discuss it in detail here. Using the fact that the effective potential satisfies the RG equation, one can explicitly solve this equation by the method of characteristics as follows:

$$V(g, y, \lambda, m^2, \xi, \sigma, ..., \mu) = V(\bar{g}(t), \bar{y}(t), \bar{\lambda}(t), \bar{m}^2(t), \bar{\xi}(t), \bar{\sigma}(t), ..., \mu e^t),$$  

(13)
where the effective coupling constants $\bar{g}(t), \bar{y}(t), \ldots, \bar{\xi}(t)$ are defined by the RG equations (10), (11) and (12) at scale $\mu e^t$, and $\bar{\sigma}(t)$ is written in [7] ($t$ is left unspecified for the moment). Note that in (13) the gravitational coupling constants connected with the Lagrangian of the external gravitational field are not written explicitly. These coupling constants are not relevant in our discussion because we work in linear curvature approximation. Notice also that as boundary condition in (13) it is convenient to use the one–loop effective potential (6),(7).

In the following, for simplicity, we restrict ourselves to the fixed gauge coupling case $b \to +0$. (One can also discuss the general case without any problems; however, then the expressions are getting very complicated and also non–explicit ones). Chosing the condition of vanishing of logarithmic terms in the effective potential (7) for finding $t$ we will get:

$$e^t = \left(\frac{-R}{4\mu^2}\right)^{1/2}.$$  \hspace{1cm} (14)

Using (10) – (12), (14) in the RG–improved potential (13) with the effective potential (7) as boundary condition, one can evaluate the RG improved effective potential at strong curvature as follows (the convenient RG invariants are given in [7], [12]):

$$\frac{(4\pi)^2}{2a} \frac{V}{\mu^4} = \frac{x^2}{2} \left(\frac{A^2}{\mu^2}\right)^w \left[\frac{1}{g_4(\Lambda)} - \frac{1}{w}\right]$$
$$+ \frac{x^4}{4} \left(\frac{-R}{4\mu^2}\right)^{\frac{2}{\alpha c}} \left[\frac{3}{2} + \frac{\alpha c}{\alpha} - \frac{\alpha c}{\alpha} \left(\frac{\sqrt{-R}}{2\Lambda}\right)^{\frac{2}{\alpha c}}\right]$$
$$+ \frac{Rx^2}{12\mu^2} \left(\frac{-R}{4\mu^2}\right)^{\frac{2}{\alpha c}} \left[\frac{1}{2} + \frac{\alpha c}{\alpha} - \frac{\alpha c}{\alpha} \left(\frac{\sqrt{-R}}{2\Lambda}\right)^{\frac{2}{\alpha c}}\right],$$  \hspace{1cm} (15)

where $x = y_\Lambda(\mu)\sigma_\Lambda(\mu)/\mu$. The expression (15) gives the RG improved effective potential for the gauged NJL–model (with finite cut-off) at strong curvature.

Taking the limit $\Lambda \to \infty$ we obtain for the renormalized effective potential (see [4], [7] for a discussion of renormalization of the four–fermion coupling constant):
\[
\frac{(4\pi)^2}{2a} \frac{V}{\mu^4} = \frac{x_*^2}{2} \left[ \frac{1}{g_{4R}(\mu)} - \frac{1}{g_{4R}^*} \right] + \frac{x_*^4}{4} \left( -\frac{R}{4\mu^2} \right)^{\frac{\alpha_c}{\alpha}} \left[ \frac{3}{2} + \frac{\alpha_c}{\alpha} \right] + \frac{R}{12\mu^2} \left( \frac{-R}{4\mu^2} \right)^{\frac{\alpha_c}{\alpha}} \left[ \frac{1}{2} + \frac{\alpha_c}{\alpha} \right] ,
\]

(16)

where \( x_* = y_* \sigma(\mu)/\mu \) and \( R < 0 \).

Taking into account that \( \alpha_c/\alpha \approx (2\pi/3)(4\pi/N_c g^2) \gg 1 \) one can study the possible chiral symmetry breaking in the model under discussion. From Eqs. (16), taking the first derivative with respect to \( x_* \), we will find:

\[
x_*^2 = -\frac{\alpha}{\alpha_c} \left[ \frac{1}{g_{4R}(\mu)} - \frac{1}{g_{4R}^*} \right] \left( -\frac{R}{4\mu^2} \right)^{\frac{\alpha_c}{\alpha}} - \frac{R}{6\mu^2} \left( -\frac{R}{4\mu^2} \right)^{\frac{\alpha_c}{\alpha}} .
\]

(17)

Hence, even in the case when there is no chiral symmetry breaking in flat space (i.e. \( g_{4R}^{-1}(\mu) - (g_{4R}^* (\mu))^{-1} \geq 0 \)) one may expect that chiral symmetry breaking occurs at strong curvature (if the second term in (17) is the dominant one). One can also find the critical curvature between symmetric and non-symmetric phases. Hence, we found the condition defining the chiral symmetry breaking at strong curvature (the right–hand side of eq. (17) should be positive). Taking into account the fact that in flat spacetime the dynamical chiral symmetry breaking in ladder SD approach [4] maybe re–obtained using the RG–improved effective potential [7] we are led to the following conjecture: Eqs. (16), (17) on dynamical symmetry breaking in the NJL–model in curved spacetime should be the same as in the ladder SD formulation (which is not developed in curved spacetime yet). It is quite remarkable that non–perturbative results on dynamical symmetry breaking maybe extended to curved spacetime. Presumably our approach maybe useful also for quantum gravity in frames of \( 1/N \)–expansion.

Finally, on the same way as above one can find the renormalized effective potential in the situation when \( \sigma^2 \gg |R| \) (i.e. using eq. (6) as boundary condition). Explicitly, for \( \Lambda \to \infty \) we have (see also [12]):

\[
\frac{(4\pi)^2 V}{2a\mu^4} = \frac{x_*^2}{2} \left[ \frac{1}{g_{4R}(\mu)} - \frac{1}{g_{4R}^*} \right] + \frac{x_*^{4/(2-w)}}{4} \left[ \frac{3}{2} + \frac{\alpha_c}{\alpha} \right] + \frac{R}{12\mu^2} \frac{x_*^{2/(2-w)}}{2} \left[ \frac{1}{2} + \frac{\alpha_c}{\alpha} \right] .
\]

(18)
This potential again leads to the dynamical symmetry breaking in weakly–curved spacetime. The dynamical fermionic mass is given in this case as follows:

\[ x^*_\alpha \simeq -\frac{\alpha}{\alpha_c} \left[ \frac{1}{g_{4R}(\mu)} - \frac{1}{g_{4R}^*} \right] - \frac{R}{6\mu^2}. \]  

(19)

Hence, we showed the possibility of dynamical symmetry breaking at strong curvature as well as at weak curvature.

4 Induced Einstein gravity

It is quite evident that the NJL–model potential under discussion supports the inflationary universe in the same way as the usual Higgs scalar potential. Indeed, we have the flat effective potential of \( \sigma \) which gives the effective cosmological constant by its minimum; but such vacuum energy serves as a source of inflation.

Here, we will discuss the gravitational effective action taking into account \( V(\sigma) \) since such an action maybe useful in the study of inflation based on induced gravity (due to NJL–model). At GUT’s epoch, the linear curvature approach is enough to study the applications of the effective potential to quantum cosmology. Hence, the gravitational effective action in gauged NJL-model (for fixed gauge coupling) maybe taken as follows

\[ S = \int d^4x \sqrt{-g} \left[ \Lambda - \frac{1}{\kappa^2} R - V(\sigma) \right] \]  

(20)

where \( V(\sigma) \) is given by eq. (18), and \( \Lambda, \kappa^2 \) denote the cosmological and gravitational constants due to other effects.

From (20) one can obtain the induced cosmological and gravitational coupling constants as follows:

\[ \Lambda_{ind} \simeq \Lambda - \frac{a\mu^4}{(4\pi)^2} \left[ \left( \frac{1}{g_{4R}(\mu)} - \frac{1}{g_{4R}^*} \right) x^2 + \frac{\alpha_c}{2\alpha} x^4 \right], \]

\[ \frac{1}{\kappa_{ind}^2} \simeq \frac{1}{\kappa^2} + \frac{a\mu^2}{6(4\pi)^2} \frac{\alpha_c}{\alpha} x^2. \]  

(21)
Hence, Einstein gravity maybe induced by purely quantum effects in the gauged NJL–model (i.e. even for $\Lambda = \kappa^{-2} = 0$). In this case, $\Lambda_{ind}, \kappa_{ind}^{-2}$ maybe easily defined by the minimum of the effective potential using (19).

Thus, induced Einstein gravity maybe succesfully obtained within the gauged NJL–model (for induced Einstein gravity in GUT’s see, for example, [3]). It would be of interest to investigate the details of inflationary scenario in such a model.

In summary, using the RG language and the equivalency with the gauge Higgs–Yukawa model in a modified $1/N_c$–expansion we studied the gauged NJL–model in curved spacetime. Applying the RG improved effective potential the curvature–induced chiral symmetry breaking is investigated and (presumably non–perturbative) dynamically generated fermionic mass is found at strong and at weak curvature. The possiblity of inducing Einstein gravity is briefly discussed.

It should be of interest to discuss the same model at some specific background (like DeSitter universe) what maybe useful for developing of a ”four–fermion model of inflation”.

¿From another side the above approach maybe useful to study the non–perturbative aspects of quantum gravity (in particular, $R^2$–gravity [4]) interacting with the gauged NJL–model. This question deserves further study.

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References

[1] Y. Nambu and G. Jona-Lasinio, Phys. Rev. 122 (1961) 345; W. A. Bardeen, C. N. Leung and S. T. Love, Phys. Rev. Lett. 56 (1986) 1230

[2] T. Inagaki, T. Muta and S. D. Odintsov, Mod. Phys. Lett. A8 (1993) 2117; E. Elizalde, S. Leseduarte and S. D. Odintsov, Phys. Rev. D49 (1994) 5551; E. Elizalde and S. D. Odintsov, Phys. Rev. D51 (1995) 5990; T. Inagaki, S. Mukaigawa and T. Muta, Phys. Rev. D52 (1995) R4267; E. Elizalde, S. Leseduarte, S. D. Odintsov and Yu. I. Shilnov, Phys. Rev. D, to be published 1996; T. Inagaki, preprint HUPD -9522, 1995.

[3] E. Elizalde and S. D. Odintsov, Mod. Phys. Lett. A10 (1995) 733; R. Floreanini and R. Percacci, Nucl. Phys. B (1995)
[4] K.-I. Kondo, M. Tanabashi and K. Yamawaki, Progr. Theor. Phys. 89 (1993) 1249; Mod. Phys. Lett. A8 (1993) 2859; K.-I. Kondo, S. Shuto and K. Yamawaki, Mod. Phys. Lett. A6 (1991) 3385.

[5] I. L. Buchbinder, S. D. Odintsov and I. L. Shapiro, Effective Action in Quantum Gravity, IOP Publishing, Bristol and Philadelphia, 1992

[6] W. Bardeen, C. Hill and M. Lindner, Phys. Rev. D41 (1990) 1647

[7] M. Harada, Y. Kikukawa, T. Kugo and H. Nakano, Progr. Theor. Phys. 92 (1994) 1161

[8] S. Coleman and E. Weinberg, Phys. Rev. D7 (1973) 1888; M. B. Einhorn and D. R. T. Jones, Nucl. Phys. B211 (1983) 29; G. B. West, Phys. Rev. D27 (1983) 1402; K. Yamagishi, Nucl. Phys. B216 (1983) 508; M. Sher, Phys. Repts. 179 (1989) 274; C. Ford, D. R. T. Jones, P.-W. Stevenson and M. B. Einhorn, Nucl. Phys. B395 (1993) 17; M. Bando, T. Kugo, N. Maekawa and H. Nakano, Progr. Theor. Phys. 90 (1993) 405; J. A. Casas, J. R. Espinosa and M. Quiros, Phys. Lett. B342 (1995) 171

[9] W. A. Bardeen and S. T. Love, Phys. Rev. D45 (1992) 4672

[10] N. P. Chang, Phys. Rev. D10 (1974) 2706; T. P. Cheng, E. Eichten and L.-F. Li, Phys. Rev. D9 (1974) 2259

[11] D. J. Gross and F. Wilczek, Phys. Rev. D8 (1973) 3633; H. D. Politzer, Phys. Rev. Lett. 30 (1973) 1346

[12] B. Geyer and S. D. Odintsov, Chiral symmetry breaking in gauged NJL model in curved spacetime, NTZ-preprint 02/96, Leipzig 1996

[13] B. Pendleton and G. Ross, Phys. Lett. B98 (1981) 291

[14] J. Kubo, K. Sibold and W. Zimmermann, Phys. Lett. B220 (1989) 191

[15] B. L. Voronov and I. V. Tyutin, Yad.Fiz. (Sov. J. Nucl. Phys.) 23 (1976) 664.

[16] C. T. Hill and D. S. Salopek, Ann. Phys. 213 (1992) 21; T. Muta and S. D. Odintsov, Mod. Phys. Lett. A6 (1991) 3641

[17] I. L. Buchbinder and S. D. Odintsov, Class. Quant. Grav. 2 (1985) 721; E. Elizalde and S. D. Odintsov, Phys. Lett. B303 (1993) 240; Z.Phys. C64 (1994) 699; G. Devaraj and M. Einhorn, preprint gr-qc/9504024