The Top-Antitop Threshold at Muon Colliders

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Abstract. Muon colliders are expected to naturally have a small spread in beam energy making them an ideal place to study the excitation curve. We present the parameter determinations that are possible from measuring the total cross section near threshold at a $\mu^+\mu^-$ collider.

INTRODUCTION

Accurate measurements of particle masses, couplings and widths are possible by measuring production cross sections near threshold. The naturally small beam energy spread of a muon collider would provide an excellent opportunity to make these measurements. Pair production of $W$-bosons, $t\bar{t}$ production and the Bjorken process $\mu^+\mu^- \rightarrow ZH$ have been considered as possible places to study thresholds at a muon collider [1–3]. There is very rich physics associated with the $t\bar{t}$ threshold, including the determination of $m_t$, $\Gamma_t$ ($|V_{tb}|$), $\alpha_s$, and possibly $m_h$ [4]. A precise value of the top-quark mass $m_t$ could prove to be very valuable in theoretical studies.

TOP-QUARK MASS MEASUREMENT AT THE $\mu^+\mu^- \rightarrow t\bar{t}$ THRESHOLD

Fadin and Khoze first demonstrated that the top-quark threshold cross section is calculable since the large top-quark mass puts one in the perturbative regime of QCD, and the large top-quark width effectively screens nonperturbative effects in the final state [5]. Such studies have since been performed by several groups [6–13]. The phenomenological potential is given at small distance $r$ by two-loop

1) Presented at the Workshop on Physics at the First Muon Collider and at the Front End of a Muon Collider, November 6-9, 1997, Fermi National Accelerator Laboratory.
perturbative QCD and for large $r$ by a fit to quarkonia spectra. In our analysis we make use of the Wisconsin potential [14] that interpolates these regimes.

The beam energy spread at a $\mu^+\mu^-$ collider is expected to naturally be small. The rms deviation $\sigma$ in $\sqrt{s}$ is given by [15,16]

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\sigma = (250 \text{ MeV}) \left( \frac{R}{0.1\%} \right) \left( \frac{\sqrt{s}}{350 \text{ GeV}} \right),
$$

where $R$ is the rms deviation of the Gaussian beam profile. With $R \lesssim 0.1\%$ the resolution $\sigma$ is of the same order as the measurement one hopes to make in the top mass. For $t\bar{t}$ studies the exact shape of the beam is not important if $R \lesssim 0.1\%$. We take $R = 0.1\%$ here; the results are not improved significantly with better resolution.\(^2\)

Changing the value of the strong coupling constant $\alpha_s(M_Z)$ influences the threshold region. Large values lead to tighter binding and the peak shifts to lower values of $\sqrt{s}$. Weaker coupling also smooths out the threshold peak. These effects are illustrated in Fig. 1.

![Figure 1](image.png)

**FIGURE 1.** The cross section for $\mu^+\mu^- \rightarrow t\bar{t}$ production in the threshold region, for $m_t = 175$ GeV and $\alpha_s(M_Z) = 0.12$ (solid) and 0.115, 0.125 (dashes). Effects of ISR and beam smearing are included.

To assess the precision of parameter determinations from cross section measurements, we generate hypothetical sample data, shown in Fig. 2, assuming that 10 fb$^{-1}$ integrated luminosity is used to measure the cross section at each energy in 1 GeV intervals. Since the top threshold curve depends on other quantities like $\alpha_s(M_Z)$, one must do a full scan to determine the shape of the curve and its overall normalization. To generate the ten data points in Fig. 2 we use nominal values of

\(^2\) The most recent TESLA design envisions a beam energy spread of $R = 0.2\%$ [17], and a high energy $e^+e^-$ collider in the large VLHC tunnel would have a beam spread of $\sigma_E = 0.26$ GeV [18]
$m_t = 175$ GeV and $\alpha_s(M_Z) = 0.12$. Following Ref. [13] we assume a 29% detection efficiency for $W \rightarrow q\bar{q}$, including the decay branching fraction. The data points can then be fit to theoretical predictions for different values of $m_t$ and $\alpha_s(M_Z)$; the likelihood fit that is obtained is shown as the $\Delta \chi^2$ contour plot in Fig. 3. The inner and outer curves are the $\Delta \chi^2 = 1.0$ (68.3%) and 4.0 (95.4%) confidence levels respectively for the full 100 fb$^{-1}$ integrated luminosity. Projecting the $\Delta \chi^2 = 1.0$ ellipse on the $m_t$ axis, the top-quark mass can be determined to within $\Delta m_t \sim 70$ MeV, provided systematics are under control. (Systematic error issues will be discussed later.) A top-quark mass of 175 GeV can be measured to about 200 MeV at 90% confidence level with 10 fb$^{-1}$ luminosity.

**FIGURE 2.** Sample data for $\mu^+\mu^- \rightarrow t\bar{t}$ obtained assuming a scan over the threshold region devoting 10 fb$^{-1}$ luminosity to each data point. A detection efficiency of 29% has been assumed [13] in obtaining the error bars. The threshold curves correspond to shifts in $m_t$ of 200 MeV increments. Effects of ISR and beam smearing have been included, and the strong coupling $\alpha_s(M_Z)$ is taken to be 0.12.

Since the exchange of a light Higgs boson can affect the threshold shape, a scan of the threshold cross section can in principle yield some information about the Higgs mass and its Yukawa coupling to the top quark. Figure 4 shows the dependence of the threshold curve on the Higgs mass, $m_h$. However, it may be difficult to disentangle such a Higgs effect from two-loop QCD effects, which are not yet fully calculated [19].

QCD measurements at future colliders and lattice calculations will presumably determine $\alpha_s(M_Z)$ to 1% accuracy (e.g. ±0.001) [20] by the time muon colliders are constructed so the uncertainty in $\alpha_s$ will likely be similar to the precision obtainable at a $\mu^+\mu^-$ and/or $e^+e^-$ collider with 100 fb$^{-1}$ integrated luminosity. If the luminosity available for the threshold measurement is significantly less than 100 fb$^{-1}$, one can regard the value of $\alpha_s(M_Z)$ coming from other sources as an input, and thereby improve the top-quark mass determination.
FIGURE 3. The $\Delta \chi^2 = 1.0$ and $\Delta \chi^2 = 4.0$ confidence limits for the sample data shown in Fig. 2. The “+” marks the input values from which the data were generated.

There is some theoretical ambiguity in the mass definition of the top quark. The theoretical uncertainty on the quark pole mass due to QCD confinement effects is of order $\Lambda_{QCD}$, i.e., a few hundred MeV [21]. In the $\overline{\text{MS}}$ scheme of quark mass definition, the theoretical uncertainty is better controlled.

Systematic errors in experimental efficiencies are not a significant problem for the $t\bar{t}$ threshold determination of $m_t$. This can be seen from Fig. 2, which shows that a 200 MeV shift in $m_t$ corresponds to nearly a 10% shift in the cross section on the steeply rising part of the threshold scan, whereas it results in almost no change in $\sigma$ once $\sqrt{s}$ is above the peak by a few GeV. Not only will efficiencies be known to much better than 10%, but also systematic uncertainties will cancel to a high level of accuracy in the ratio of the cross section measured above the peak to measurements on the steeply-rising part of the threshold curve.

As Fig. 4 shows, it will be important to know the Higgs mass and the $ht\bar{t}$ coupling strength in order to eliminate this source of systematic uncertainty when extracting other quantities.

The measurements described in this section can be performed at either an $e^+e^-$ or a $\mu^+\mu^-$ collider. The errors for $m_t$ that we have found for the muon collider are smaller than those previously obtained in studies at the NLC electron collider primarily because the smearing of the threshold region by the energy spread of the beam is much less, and secondarily due to the fact that the reduced amount of initial state radiation makes the cross section somewhat larger.

CONCLUSION

With an integrated luminosity of 10 (100) fb$^{-1}$, the top-quark mass can be measured to 200 (70) MeV, using a 10-point scan over the threshold region, in 1 GeV
FIGURE 4. The dependence of the threshold region on the Higgs mass, for $m_h = 50, 100, 150$ GeV. Effects of ISR and beam smearing have been included, and we have assumed $m_t = 175$ GeV and $\alpha_s(M_Z) = 0.12$.

intervals, to measure the shape predicted by the QCD potential. In the $t\bar{t}$ threshold study, differences of cross sections at energies below, at, and above the resonance peak, along with the location of the resonance peak, have different dependencies on the parameters $m_t$, $\alpha_s$, $m_h$ and $|V_{tb}|^2$ and should allow their determination. To utilize the highest precision measurements achievable at the statistical level, theoretical uncertainties and other systematics need to be under control. We are confident that uncertainty in $\alpha_s$ will not be a factor and we have noted that ratios of above-peak measurements to measurements on the steeply rising part of the threshold cross section will eliminate many experimental systematics related to uncertainties in efficiencies.

ACKNOWLEDGMENTS

I thank V. Barger, J. F. Gunion and T. Han for a pleasant collaboration on the issues reported here. This work was supported in part by the U.S. Department of Energy under Grant No. DE-FG02-91ER40661.

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