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Quark and gluon orbital angular momentum: Where are we?

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Abstract The orbital angular momentum of quarks and gluons contributes significantly to the proton spin budget and attracted a lot of attention in the recent years, both theoretically and experimentally. We summarize the various definitions of parton orbital angular momentum together with their relations with parton distributions functions. In particular, we highlight current theoretical puzzles and give some prospects.

Keywords Quark and gluon angular momentum · Parton distributions · Phase-space densities

1 Introduction

One of the major challenges in hadron physics is to unravel how the spin of the proton arises from the spin and orbital motion of its constituents. It appears that the quark and gluon spin contributions account for about 60% of the proton spin budget [1] [2] [3], implying that 40% should be accounted by the quark and gluon orbital angular momentum (OAM). This is a large fraction which reflects the relativistic nature of the quark-gluon bound state. The quark and gluon OAM, being a correlation between position and momentum, is more difficult to access experimentally than the spin. It depends also on how the total angular momentum (AM) is divided into separate quark and gluon contributions, which is intrinsically ambiguous due to quark-gluon couplings.

It has long been thought that only the kinetic (or mechanical) decomposition of the proton spin makes sense because its quark and gluon contributions can be extracted from experimental data and computed on the lattice. Recent theoretical and experimental progress have, however, shown that the canonical decomposition can also be accessed experimentally and computed on the lattice, though in a more complicated way. Kinetic and canonical decompositions appear to be complementary with their own advantages and disadvantages. For more detailed discussions, see the recent reviews [4] [5].

We present here a short summary of the theoretical status of quark and gluon OAM. First we discuss the most general gauge-invariant decomposition of the proton spin and show how it is related to the other ones proposed in the literature. We also comment on its physical significance and stress the importance of the role played by the experimental configuration and the theoretical framework in deciding which explicit form to use. Then we summarize the relations between measurable parton distributions and the different contributions to the proton spin. Having established relations at the integrated level, we finally discuss the status of those proposed at the density level. More details and discussions including recent important lattice developments can be found in Ref. [6]

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2 Gauge-invariant decomposition of angular momentum

The total AM operator in QCD can be decomposed as

\[ J = S^q + S^G + L^{q}\text{kin} + L^{G}\text{can} + L^{\text{pot}}, \tag{1} \]

where the quark spin, gluon spin, quark kinetic OAM, gluon canonical OAM and potential OAM contributions are respectively given by

\[ S^q = \int d^3r \psi^\dagger \frac{1}{2} \Sigma \psi, \quad S^G = \int d^3r E^a \times A^{a}_{\text{phys}}, \quad L^{q}\text{kin} = \int d^3r \psi^\dagger (r \times iD) \psi, \quad L^{G}\text{can} = -\int d^3r E^a (r \times D^{ab}_{\text{pure}}) A^b_{\text{phys}}, \quad L^{\text{pot}} = -\int d^3r \rho^a r \times A^a_{\text{phys}} \tag{2} \]

We followed Chen et al. \cite{7, 8} and decomposed the gauge potential into a pure-gauge field and a “physical” field \cite{9, 10}

\[ A^\mu = A^\mu_{\text{pure}} + A^\mu_{\text{phys}} \tag{3} \]

such that \( F^{\mu\nu}_{\text{pure}} = \partial^\mu A^\nu_{\text{pure}} - \partial^\nu A^\mu_{\text{pure}} - ig[A^\mu_{\text{pure}}, A^\nu_{\text{pure}}] = 0 \). Under a gauge transformation, \( A^\mu_{\text{pure}} \) and \( A^\mu_{\text{phys}} \) transform as

\[ A^\mu_{\text{pure}} \mapsto U [A^\mu_{\text{pure}} + \frac{i}{g} \partial^\mu] U^{-1}, \quad A^\mu_{\text{phys}} \mapsto U A^\mu_{\text{phys}} U^{-1}. \tag{4} \]

The pure-gauge covariant derivatives are defined like the ordinary covariant derivatives with the gauge potential \( A^\mu \) replaced by the pure-gauge field \( A^\mu_{\text{pure}} \). This ensures explicit gauge invariance of each contribution in Eq. (2).

Due to the QCD equations of motion \( \rho^a = g \psi^\dagger t^a \psi = D^{ab}_{\text{phys}} E^b \), the potential OAM can be interpreted as either a quark or a gluon contribution. The quark canonical OAM appearing in the (gauge-invariant) canonical decomposition is obtained by combining quark kinetic and potential OAM

\[ L^{\text{can}} = L^{q}\text{kin} + L^{\text{pot}} = \int d^3r \psi^\dagger (r \times iD_{\text{pure}}) \psi. \tag{5} \]

The gluon kinetic OAM appearing in the (gauge-invariant) kinetic decomposition is obtained by combining gluon canonical and potential OAM

\[ L^{G}\text{kin} = L^{G}\text{can} + L^{\text{pot}} = \int d^3r r \times [(A^a_{\text{phys}} \times D^{ab}_{\text{pure}}) \times E^b], \tag{6} \]

where \( D^\mu_{\text{pure}} = \frac{1}{2} (D^\mu + D^\mu_{\text{pure}}) \tag{11} \).

The decomposition of the gauge potential \cite{12} essential for ensuring gauge invariance is not unique since

\[ A^\mu_{\text{pure}} \mapsto A^\mu_{\text{pure}} + \partial^\mu C, \quad A^\mu_{\text{phys}} \mapsto A^\mu_{\text{phys}} - \partial^\mu C, \tag{7} \]

referred to as a Stueckelberg transformation \cite{12, 13}, leaves the fundamental Lagrangian invariant. In practice, this is actually not an issue since the experimental conditions combined with the theoretical framework usually provide a natural decomposition. In experiments probing the internal structure of the proton, the off-shell probe indeed provides a natural direction \cite{14} along which one can unambiguously define spin and OAM contributions \cite{13, 12, 16}.
3 Accessing angular momentum with parton distributions

Ji [17] derived a remarkable relation between the total kinetic AM of quark and gluons, and twist-2 generalized parton distributions (GPDs)

\[ \langle J_{\text{kin}}^q \rangle = \frac{1}{2} \int dx x [H^{q,G}(x,0,0) + E^{q,G}(x,0,0)]. \]  

The kinetic OAM of quarks and gluons can then be obtained by subtracting the corresponding spin contributions,

\[ \langle L_{\text{kin}}^q \rangle = \langle J_{\text{kin}}^q \rangle - \langle S^q \rangle, \]  

which are given in the $\overline{MS}$ scheme by the first Mellin moment of the quark and gluon helicity distributions \( \langle S^q \rangle = \frac{1}{2} \int dx \Delta q(x) \), \( \langle S^G \rangle = \int dx \Delta g(x) \). Alternatively, it has been shown by Penttinen, Polyakov, Shuvaev and Strikman (PPSS) [18] that the quark kinetic OAM can also be directly expressed in terms of a two-parton twist-3 GPD [19, 20, 21, 22]

\[ \langle L_{\text{kin}}^q \rangle = - \int dx x G_2^q(x,0,0). \]  

Similarly, it is thought that transverse-momentum distributions (TMDs) could also give quantitative information about OAM. Quark model calculations motivated the following relations [23, 24, 25, 26]

\[ \langle L_{\text{can}}^q \rangle = \int dx d^2k_\perp [h_1^q(x,k_\perp^2) - g_1^q(x,k_\perp^2)], \]  

\[ = - \int dx d^2k_\perp \frac{k_1^2}{2\pi^2} h_1^{q,G}(x,k_\perp^2) \]  

but they turned out to be valid only under some restricted condition, and not in general in QCD [27, 28, 29]. Alternatively, Burkardt [30, 31] suggested that a chromodynamic lensing mechanism could relate the quark Sivers TMD \( f_1^{q,G}(x,k_\perp^2) \) and the quark GPD \( E^q(x,\xi,t) \). Although this mechanism can hardly be put on a firm theoretical ground, a variation of it by Bacchetta and Radici [32] has led to a new estimate of \( \langle J_{\text{kin}}^q \rangle \), in good agreement with most common GPD extractions.

The most natural expression for the OAM is that of a phase-space integral [33, 34]

\[ \langle L_{\text{W}}^{q,G} \rangle = \int \frac{d^2k_\perp}{2\pi^2} \frac{d^2b_\perp}{2\pi^2} (b_\perp \times k_\perp) \rho_{q,G}^L(x,k_\perp,b_\perp; W), \]  

where the Wilson line \( W \) ensures gauge invariance. In a semi-classical interpretation, the Wigner distribution \( \rho_{q,G}^L \) gives the (quasi-)probability for finding, at the transverse position \( b_\perp \) a quark or a gluon with momentum \( (x^+, k_\perp) \) inside a longitudinally polarized \( (x^+ = \pm) \) proton. This Wigner distribution is related via Fourier transform to the generalized transverse-momentum dependent distributions (GTMDs) [27, 35, 36], leading to the simple relation [33, 37, 38]

\[ \langle L_{\text{W}}^{q,G} \rangle = - \int \frac{d^2k_\perp}{2\pi^2} \frac{d^2b_\perp}{2\pi^2} F_{14}^{q,G}(x,0,k_\perp,b_\perp; W). \]  

For a staple-like Wilson line \( W_{\perp} = 1 \), Eq. [33] gives the canonical OAM \( \langle L_{\text{can}}^{q,G} \rangle = \langle L_{\text{W}}^{q,G} \rangle \) irrespective of whether the staple is future- or past-pointing [21, 22, 37, 38, 40]. For a straight Wilson line \( W_\parallel \), it gives the kinetic version of the quark OAM \( \langle L_{\text{kin}}^q \rangle = \langle L_{\text{W}}^{q,G} \rangle \) and the Ji-Xiong-Yuan (JXY) [39] definition of the gauge-invariant gluon OAM \( \langle L_{\text{G}}^{G,JXY} \rangle = \langle L_{\text{W}}^{G,JXY} \rangle \), where \( L_{\text{W}}^{G,JXY} = - \int d^3r E^{qG}(r \times D^{qG})A_{\text{phys}}^q \). Unfortunately, it is not known so far how to extract them from actual experiments, except possibly at small \( x \) [33]. Moreover, the GTMD \( F_{14} \) does not reduce to any GPD or TMD, and so cannot be directly constrained. However, in the last few years recent developments [42, 43, 44] opened the interesting possibility of computing GTMDs and OAM directly on the lattice. Moreover, phenomenological models, constrained by experimental data, also provide indirect access to GTMDs and hence OAM.
Angular momentum at the density level

Angular momentum can also be defined at the density level and, therefore, be mapped in both position and momentum spaces. Many definitions, differing by superpotential terms of the form $\partial_\alpha X^{[\alpha \nu]}$, where square brackets stand for antisymmetrization of indices, have been proposed in the literature, creating a somewhat confusing situation. It is essential to keep track of these superpotential terms, since they affect the interpretation of the density.

Hoodbhoy, Ji and Lu \[45\] defined higher moments of the kinetic AM densities and concluded that

$$\langle L_k^{q,x} \rangle (x) = \frac{i}{2} [H^q(x,0,0) + E^q(x,0,0)] - \frac{i}{2} \Delta q(x). \quad (14)$$

However, this definition is somewhat ad hoc since a complicated tower $\Delta L^{++}$ has been added to the natural simple one

$$\langle L_k^{q,x} \rangle = \frac{1}{n+1} \sum_{j=0}^{n} \int d^3 r \bar{\psi} \gamma^+ (iD^+)^j (r_\perp \times iD_\perp)_{z} (iD^+)^{n-j} \psi$$

in order to have it evolve as a leading-twist operator. Ji, Xiong and Yuan argued that the integrand of the Ji relation \[38\] could naturally be identified with an angular momentum decomposition of the transverse component of the quark angular momentum in a transversely polarized target. However, a careful inspection revealed several caveats leading to the conclusion that this interpretation is not justified \[17\].

Hattori, Hatta and Yoshida \[21\] stressed that the density of kinetic OAM is ambiguous because it involves two longitudinal momentum fractions $x_1$ and $x_2$, contrary to canonical OAM. This is related to the fact that contrary to ordinary derivatives, covariant derivatives do not commute and, therefore, do not admit a unique non-local generalization \[10\].

It is actually not so surprising that the integrand of the Ji relation \[38\] cannot be simply interpreted as the density of parton AM. Indeed, the covariant derivative in the definition of kinetic OAM necessarily implies the contribution of higher-twist parton distributions. Häger, Mukherjee and Schäfer \[20\] proposed another density of OAM which now involves both twist-2 and twist-3 GPDs

$$\langle L_k^{q,G} \rangle (x) = x [H^q(x,0,0) + E^q(x,0,0)] - \Delta q(x) \quad (16)$$

but it has been obtained within the Wandzura-Wilczek approximation, where the distinction between canonical and kinetic OAM disappears. In a detailed discussion of twist-3 GPDs, Hatta and Yoshida \[21\] stressed that the density of kinetic OAM is ambiguous because it involves two longitudinal momentum fractions $x_1$ and $x_2$, contrary to canonical OAM. This is related to the fact that contrary to ordinary derivatives, covariant derivatives do not commute and, therefore, do not admit a unique non-local generalization \[10\].

Clearly, canonical OAM is more amenable to a description at the density level. Following the work of Jaffe and Manohar \[54\], a natural density of quark and gluon canonical OAM in the light-front frame has been given by Harindranath and Kundu \[51\] and later improved by Bushinsky and Jaffe \[12\] who provided an explicit expression invariant under residual gauge transformations. Recently, it has been shown that the density of OAM can directly be defined in a gauge-invariant way in phase-space \[33\].

$$\langle L_k^{q,G} \rangle (x, k_\perp, b_\perp) \rho_{1+}^q (x, k_\perp, b_\perp; W). \quad (17)$$

Contrary to the integrated version, the density depends on whether the staple points toward the future $W_\perp$ or the past $W_\perp$. Using the constraints imposed by parity and time-reversal symmetries, the Wigner distribution of unpolarized quarks and gluons inside a longitudinally polarized proton can be decomposed into four contributions \[52\]

$$\rho_{1+}^{q,G} = \rho_{1+}^q (x, b_\perp \cdot k_\perp) \rho_{2+}^q (x, b_\perp \cdot k_\perp) \rho_{3+}^q (x, b_\perp \cdot k_\perp) \rho_{4+}^q (x, b_\perp \cdot k_\perp), \quad (18)$$

where $\rho_{1}^{q,G} \equiv \rho_{1}^{q,G} (x, k_\perp^2, (b_\perp \cdot k_\perp)^2, b_\perp^2; W)$. The functions $\rho_1$ and $\rho_2$ are related via Fourier transform to the real and imaginary parts of the GTMD $F_{11}$, and similarly for $\rho_3$ and $\rho_4$ with $F_{14}$. The coefficient $(b_\perp \cdot k_\perp)$ implies that $\rho_2$ and $\rho_4$ are naive T-odd, i.e. they change sign under $W_\perp \rightarrow W_\perp$. Integrating over $b_\perp$ or $k_\perp$, one is left with just $\rho_3$, i.e. the real part of $F_{14}$. On the other hand, a straight Wilson line $W_\perp$ leads to the density of JXY quark and gluon OAM \[39\] provided that $k_\perp$ is integrated over $40$.

The Ji relation \[38\] has also been discussed in position space. Since the information about the spatial distribution of quarks and gluons is encoded in the $t$ dependence of GPDs \[53\], Polyakov \[55\]
suggested that the Ji relation generalized to $t \neq 0$ should provide information about the spatial distribution of kinetic OAM

$$\langle J_{\text{kin}}^{q,G}(t) \rangle = \frac{1}{4} \int dx x [H^{q,G}(x,0,t) + E^{q,G}(x,0,t)].$$

(19)

Interestingly, Adhikari and Burkardt [57] observed within the scalar diquark model with Pauli-Villars regularization that the quark kinetic and canonical OAM spatial densities do not coincide $\langle L_{\text{kan}}^{q}(b_\perp) \rangle \neq \langle L_{\text{kan}}^{q}(b_\perp) \rangle$, contrary to their integrated counterparts $\langle J_{\text{kin}}^{q}(t) \rangle = \langle J_{\text{kan}}^{q}(t) \rangle$. It should however be noted that these spatial densities have been defined as the Fourier transform of the $t$-dependent distribution. This is however not justified in all cases. For example, the total AM, is given by the Fourier transform of the combination $\langle J_{\text{kin}}^{q}(t) \rangle + t \frac{q}{2} \langle J_{\text{kin}}^{G}(t) \rangle$, not just $\langle J_{\text{kin}}^{q}(t) \rangle$. Note also that the spatial density of quark kinetic OAM has been defined as $\langle L_{\text{kan}}^{q}(b_\perp) \rangle = \langle J_{\text{kin}}^{q}(b_\perp) \rangle - \langle S^q \rangle (b_\perp)$, but a superpotential ruins this relation at the density level [4]. Further investigations are therefore needed.

5 Summary

We briefly reviewed the recent developments about kinetic and canonical decompositions of the proton spin. We discussed in particular the issue of gauge invariance and the link with measurable parton distributions. We also critically commented the various definitions of quark and gluon angular momentum densities.

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