Did Hypatia Know about Negative Numbers?

Hypatia lived in Alexandria in the 4th-5th century AD. She was one of the most remarkable women in history. Her father Theon, presumably the last member of the Museum, instructed her in mathematics, philosophy and classical Greek literature. With time she herself became a brilliant teacher of Neoplatonic philosophy and mathematics. Numerous young aspirants from wealthy Christian and pagan families came to Alexandria with the sole purpose of joining the sophisticated inner circle of her students.

Theon and Hypatia edited several of the major mathematical treatises available in their time: *Arithmetica* by Diophantus, *Almagest* by Ptolemy, *Conics* by Appolonius of Perga, and *Elements* by Euclid. A significant part of this work, in particular on the most difficult and demanding *Arithmetica*, she did alone. To some of these treatises she added commentaries and exemplary exercises.

A charismatic figure in the city’s intellectual life and a close friend to many important officials, Hypatia had a significant influence on the Alexandrian elite and politics. She was admired for her knowledge, wisdom, righteousness and personal charm. Her public lectures for the Alexandrian intelligentsia attracted a great deal of interest. However, her elevated social position and popularity did not prevent her from becoming the victim of a brutal murder in 415 AD by the fanatic mob who accused her of conducting forbidden magical practices. Groundless rumors about Hypatia’s dealings with witchcraft were most likely inspired by her political enemies. Bishop Cyril was certainly one of them, but the allegations concerning his direct involvement have never been proven. The tragic circumstances of her death created the romantic legend of Hypatia that persists in Western literature, art and drama.

There have been two absorbing books published recently which deal specifically with the figure of Hypatia. Each of them, in its own way, sets to demythologize her life, legacy or legend. In *Hypatia of Alexandria*¹ Maria Dzielska proves that the real Hypatia was anything but a young and liberated rebel fighting against Christianity, whereas Michael A. B. Deakin in *Hypatia of Alexandria: Mathematician and Martyr*² argues that she was not a mathematical genius, and her contributions to mathematical knowledge were slight.

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¹Maria Dzielska (tr. F. Lyra), *Hypatia of Alexandria*. Cambridge (Mass.): Harvard University Press, 1995. (*Revealing Antiquity*, 8). Pp. xi + 157. ISBN 0-674-43775-6.

²Michael A. B. Deakin, *Hypatia of Alexandria: Mathematician and Martyr*, Amherst (N.Y.): Prometheus Books, 2007; also: Michael A.B. Deakin “Hypatia and Her Mathematics”, *The American Mathematical Monthly*, March 1994, Volume 101, Number 3, pp. 234-243.
or non-existent.

In this Letter we comment on one particular aspect of Hypatia’s enigmatic biography by translating into English a short poem that appeared in a recent review of the third revised Polish edition of Maria Dzielska’s book. It poses a simple and specific question: did Hypatia know about the negative numbers?

Diophantus noticed that equations like $4 = 4X + 20$ have no solution and called them “absurd” or “false”. Did Hypatia, his careful editor and commentator, share this opinion? The Chinese and Indians had known about the negative numbers long before Hypatia, and used them in their pure mathematics as well as in practical book-keeping: a debt cut off from nothingness becomes a credit; a credit cut off from nothingness becomes a debt. Knowing certainly about credit and debt, could Hypatia realize that $X = -4$ is the solution to the “absurd” equation of Diophantus? Neither Dzielska, nor Deakin, nor any other scholar, knows the answer to this question. There are no manuscripts and we have no evidence. But the absence of evidence should not be mistaken for the evidence of absence.

**Ageometretos**

By the shore of the uninviting sea,
Far away from Greece,
I discussed with another Hypatia
The Negative Numbers.

Now her name was Paola,
Her lashes superbly long,
And her slim hands
Cradled the memory of Tergeste.

We were amused by the petty thing,
A Diophantine problem,
Perhaps known to Hypatia:
Among whole numbers
Find all the threes
Of the roots of an equation
$X + Y + Z = 3$
$X^3 + Y^3 + Z^3 = 3$

One such three,
$(X, Y, Z) = (+1, +1, +1)$

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3Marek Abramowicz, “Niezwykła uroda równań Diofantosa”, Świat Nauki, October 2010, pp. 88-92.
Is apparent to all.
Everyone understands also
That in other solutions,
If they exist,
At least one of the three
Is a Negative Number.
For example
\((X, Y, Z) = (+4, +4, -5)\)

Did divine Hypatia
Know the Negative Numbers?
Could she discover them
In the vague reference
Of Diophantus?

None of us can know it.
\((X, Y, Z) = (+1, +1, +1)\)
\((X, Y, Z) = (+4, +4, -5)\)

Are there any other
Matching threes?
How to find them All?
I know, and so does Paola,
And perhaps
Hypatia knew it.

But this cannot be deciphered
By those missing the Platonic gift
For mathematics.

Following the idea outlined in the poem, let us assume (without loss of generality) that \(Z < 0\) and write,

\[
X + Y = 3 - Z \tag{1}
\]

\[
X^3 + Y^3 = 3 - Z^3 \tag{2}
\]

After dividing Eq. (2) side by side by Eq. (1) we arrive at,

\[
X^2 + Y^2 - XY - Z^2 - 3Z - 9 = \frac{24}{Z - 3}. \tag{3}
\]

By substituting \(Y = 3 - Z - X\) into Eq. (3), we may write,

\[
X^2 + (Z - 3)X - 3Z = \frac{8}{Z - 3}. \tag{4}
\]
From the above equation we see that \( Z - 3 \) must divide 8. This condition reduces the set of possible solutions for \( Z \) to just two,

\[
Z = (-5, -1),
\]

(5)
of which \( Z = -1 \) should be rejected. Thus, all possible solutions to Hypatia’s problem are,

\[
(X, Y, Z) = (+1, +1, +1) \\
(+4, +4, -5) \\
(+4, -5, +4) \\
(-5, +4, +4).
\]

(6)
The above problem was presented to the competitors of the Polish 1963 Mathematical Olympiad, of which one of the authors of this Letter was a Finalist. We do not know the original author of the problem.

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