“Measurement” by neuronal tunneling: Implications of Born’s rule

L. Polley

Institut für Physik, Universität Oldenburg, 26111 Oldenburg, FRG

Abstract
A non-collapse scenario for “conscious” selection of a term from a superposition was proposed in quant-ph/0309166: thermally assisted tunneling of neuronal pore molecules. But “observers” consisting of only two neurons appear to be at odds with Born’s rule. In the present paper, an observer is assumed to possess a large number of auxiliary properties irrelevant for the result of the measurement. Born’s rule then reduces to postulating that, prior to the result becoming conscious, irrelevant properties are in an entangled state with maximum likelihood, in the sense that phase-equivalent entanglements cover a maximal fraction of the unit sphere (leading to equal-amplitude superpositions).

1 Introduction

A persisting question of quantum theory is whether “measurement” is a unitary process, essentially determined by a Schrödinger equation with Coulomb interactions of electrons and nuclei, or whether completely different physical or even non-physical structure is involved. Any unitary scenario of measurement will have to provide a mechanism for the stochasticity inherent in a quantum measurement, and will have to explain what happens to the discarded components of a wavefunction after collapse.

In [1] a scenario was proposed in which a wavefunction does not collapse but only part of it enters the consciousness of observers. It was assumed that consciousness requires the firing of a neuron, and hence [2] the transition of a channel-pore 1 molecule from a closed to an open state. Such a transition as the basis of quantum measurement was extensively discussed already by Donald [3].

1In view of the signal-amplifying role of chemical synapses in the brain, it is probably more appropriate to envision instead the opening of fusion pore molecules (chapters 10 and 14 of [2]) which induce the release of neurotransmitters from synaptic vesicles. However, this does not affect the physical aspects of the scenario considered here.
In [3], however, the switching of pore molecules was assumed to be an irreducible stochastic process, whereas in [1] it was hypothesized that the transition involves molecular tunneling, assisted by a thermal environment. In this latter scenario, the microstates of neuronal heat baths, of all observers involved, determine the collective perception of the “measured” result.

Some potential problems of the scenario were listed in [1]. One of them, which is the subject of the present paper, is apparently due to an oversimplified model of an “observer”—assumed in [1] to consist of only two neurons plus heat baths. In such a model, the probability for obtaining the result L or R from a superposition

\[ a|L\rangle + b|R\rangle \]

is not proportional to \( |a|^2 \) or \( |b|^2 \), respectively, but rather to something like \( |a|^{1/20} \) or \( |b|^{1/20} \). This is in violation of Born’s rule, except in case of equal amplitudes \( |a| = |b| \). The latter exception suggested an approach analogous to a derivation of Born’s rule [1,2] from equal-amplitude superpositions of an extended system with many auxiliary states. Obviously, in the nervous system of a realistic observer there are many candidates for auxiliary states. What remains to be provided for a complete picture of the measurement process is an argument as to why auxiliary states should systematically be superposed with equal amplitudes.

The reason proposed here (section 2) is the following. A sufficiently complex observer, when engaged in an act of measurement, has a large number of nerval states available (ready to fire) which are irrelevant for the result of the measurement but are nevertheless affected by the process. Unitary evolution, restricted to those available states, will be determined by some effective Hamiltonian. Because the states are “irrelevant”, there is no reason for the measurement process to prefer any of them, so the process is likely to end up with a superposition belonging to a class with a large number of representatives. Whether for a given class that number is large or small, relatively, is uniquely determined since an invariant (basis-independent) measure on the unit sphere is unique up to a proportionality factor.  

An essential part of the scenario is that the response of an observer’s nervous system is not only determined by the object observed, but also by thermal agitation of neuronal channel pore molecules. Thus, when the observer’s system has interacted with the object, some neurons must be thermally induced to “actually” fire to produce a “conscious” result. This part of the measurement process has been explicated in [1]. It was tested there numerically for an “observer” consisting of two neurons plus heat baths, so it remains to be generalized to an observer with many more neurons in ready-to-fire states. This is discussed in section 3.

Conclusions are given in section 4. An alternative, more intuitive way of counting representatives of classes of superpositions is given in the appendix.

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2An equivalent argument could presumably be formulated in terms of random matrices (evolution operators) drawn from the unitary circular ensemble [4].
2 Statistical dominance of equal amplitudes

We wish to assign a statistical weight to certain classes of pure quantum states. This requires a measure on the unit sphere in Hilbert space which is independent of the basis in which the states are represented. Such a measure is unique up to a multiplicative constant under very general conditions [7].

Let $n$ be the dimension of the Hilbert space, and consider a general state vector represented as

$$|\psi\rangle = \sum_{k=1}^{n} c_k |k\rangle \quad \langle k|k'\rangle = \delta_{kk'}$$

(1)

The invariant measure is proportional to

$$\delta \left( 1 - \sum_{k=1}^{n} |c_k|^2 \right) \prod_{k=1}^{n} d^2 c_k$$

Using polar coordinates so that $c_k = r_k e^{i\phi_k}$, the measure takes the form

$$\delta \left( 1 - \sum_{k=1}^{n} r_k^2 \right) \prod_{k=1}^{n} r_k dr_k \prod_{k=1}^{n} d\phi_k$$

(2)

2.1 Volume of phase-equivalent superpositions

Let us define $|\psi'\rangle = \sum_{k=1}^{n} c'_k |k\rangle$ to be phase-equivalent to $|\psi\rangle$ if there exist phase rotations of its coefficients such that $|||\psi'\rangle_{\text{rotated}} - |\psi\rangle|| < \epsilon$. Calculationally, this reduces to a condition on the absolute values of the coefficients:

$$\sum_{k=1}^{n} (r'_k - r_k)^2 < \epsilon^2$$

(3)

The reason for not requiring exact coincidence of absolute values here is that phase-equivalent sets would then all be of measure zero, so their sizes could not be compared. An alternative argument, based on discretization instead of a normalization margin, is given in appendix A.

The calculation below will simplify if we, consistently, replace the delta function of equation (2) by the step function $(4\epsilon)^{-1} \chi(1 - 2\epsilon \leq \sum r_k^2 \leq 1 + 2\epsilon)$. The ensuing constraint is automatically satisfied for the primed radii if $\langle \psi|\psi\rangle = 1$ (because of triangle inequalities).

We now calculate $V_{\text{equiv}}$, the volume on the unit sphere covered by superpositions phase-equivalent to $|\psi\rangle$. Using the integral measure (2), we first obtain a factor of $(2\pi)^n$ by integrating over the phase angles. In the radial integrations, we substitute $r'_k$ by $s_k = r'_k - r_k$ and, using $\epsilon \ll 1$, replace the factors $r'_k$ by $r_k$. The remaining integral equals the volume of an $n$-dimensional ball of radius $\epsilon$. Thus we obtain

$$V_{\text{equiv}} = \frac{1}{4\epsilon} (2\pi)^n \left( \prod_{k=1}^{n} r_k \right) V_{\text{ball}}(n, \epsilon) = \text{const} \times \prod_{k=1}^{n} |c_k|$$

(4)
2.2 Maximum likelihood for equal amplitudes

The maximum of $V_{\text{equiv}}$ given in (4), under the constraint of unit normalization, is determined by the equation

$$\prod_{k=1}^{n} |c_k| - \lambda \sum_{k=1}^{n} |c_k|^2 = \text{extremum}$$

Since all $c_k$ enter the same way, the obvious result is

$$|c_k| = \sqrt{\frac{1}{n}} \text{ for all } k$$

2.3 Fluctuations

Let us anticipate that root-mean-square deviations from a uniform amplitude configuration are small so that we need to take into account only first and second orders. Define relative deviations $\delta_k$ by

$$|c_k| = \sqrt{\frac{1}{n}} (1 + \delta_k) \quad \mathcal{O}(\delta_k^3) \text{ negligible} \quad (5)$$

The constraint of unit normalization implies

$$\sum_{k=1}^{n} \delta_k = -\frac{1}{2} \sum_{k=1}^{n} \delta_k^2$$

Hence, taking the logarithm of the number of phase-equivalent superpositions and expanding in $\delta_k$, we obtain

$$\sum_{k=1}^{n} \log (1 + \delta_k) = \sum_{k=1}^{n} \delta_k - \frac{1}{2} \sum_{k=1}^{n} \delta_k^2 + \mathcal{O}(\delta_k^3) = -\frac{3}{2} \sum_{k=1}^{n} \delta_k^2$$

Thus, the probability of relative RMS deviation $\delta = \sqrt{\frac{4}{n} \sum_{k=1}^{n} \delta_k^2}$ is a Gaussian of width $(3n)^{-1/2}$,

$$p(\delta) \propto \exp \left( -\frac{3}{2} n \delta^2 \right) \quad (6)$$

2.4 Further constraints: Conservation of observables

Let us assume that the observer’s nervous system, when engaged in an act of measurement, has $n$ basis states available for reaction. Unitary evolution, restricted to those states, will be determined by some effective Hamiltonian. Let us assume the properties of those states to be “irrelevant” in the sense that the result of the measurement is solely contained in a tensorial factor $|L\rangle$ or $|R\rangle$ associated with
each of the $n$ nerval states. Thus, prior to thermal agitation of the channel pore molecules, the entangled state is of the form

$$\sum_{k=1}^{m} c_k |k\rangle |L\rangle + \sum_{k=m+1}^{n} c_k |k\rangle |R\rangle$$

where nerval states associated with $|L\rangle$ have been relabeled so as to appear as the first $m$ summands.

Measurement should not change the value of the measured quantity. Hence, let us assume that the unitary operator of the process commutes with the observable. If the original state of the quantum system is

$$a|L\rangle + b|R\rangle$$

we must impose on the coefficients of (7) the constraints

$$\sum_{k=1}^{m} |c_k|^2 = |a|^2 \quad \sum_{k=m+1}^{n} |c_k|^2 = |b|^2$$

We already know from sections 2.2 and 2.3 that amplitude configurations with maximum likelihood under the constraints (9) are characterized by

$$|c_k| = \frac{|a|}{\sqrt{m}} \quad k = 1, \ldots, m \quad |c_k| = \frac{|b|}{\sqrt{n-m}} \quad k = m+1, \ldots, n$$

Thus, by equation (4), the number of phase-equivalent configurations is proportional to

$$\frac{|a|^m}{(\sqrt{m})^m} \frac{|b|^{n-m}}{(\sqrt{n-m})^{n-m}}$$

The logarithmic derivative of this expression with respect to $m$ is

$$-\frac{1}{2} \log m + \frac{1}{2} \log(n-m) + \log |a| - \log |b|$$

which is zero for

$$\frac{|a|}{\sqrt{m}} = \frac{|b|}{\sqrt{n-m}}$$

Hence, by (10), all $|c_k|$ are equal. We have thus shown that the equal-amplitude superpositions considered by Deutsch [4] and Zurek [5] are distinguished by their statistical dominance in a sufficiently complex unitary scenario.

Deviations from equal amplitudes have the probability determined in section 2.3. The probability tends to zero with $n \to \infty$.  

5
3 Selectivity

The analytical and numerical study of [1] showed that thermally assisted neural tunneling (modeled with a hypothetical choice of parameters) is capable of selecting a single term from a superposition of two terms—the firing amplitude of one neuron highly exceeds the firing amplitude of the other neuron in the majority of cases.

In the present, more complex scenario a single term would have to be selected from a superposition of \( n \) quantum states of the observer’s nervous system. Peculiar to extreme-value statistics (Gumbel distribution), this does not imply a dramatic decline in selectivity even with \( n \to \infty \).

Let us recall that in [1] the distribution function for extremal values \( \Phi \) of elongations of the pore molecule was approximated by the Gumbel form [8]

\[
F(\Phi) = \exp \left( - \exp \left( - \frac{\Phi - \mu}{\sigma} \right) \right)
\]

(11)

with \( \sigma \) parameterizing the thermal fluctuations induced by the neuronal heat bath\(^3\). The maximum of the tunnel amplitude was proportional to \( \exp(\kappa \Phi) \) with \( \kappa \) deriving from the parameters of the internal molecular tunnel barrier.

In the approximation (11) we can easily obtain the probability for the largest elongation in an ensemble of \( n \) draws of \( \Phi \) to differ from the second-largest elongation by an amount greater than \( a \) (some chosen margin of selectivity). We have to sum the probabilities for \( \Phi_k (k=1,\ldots,n) \) taking some value while all other elongations are smaller than \( \Phi_k - a \). This gives

\[
p(n,a) = n \int dF(\Phi) (F(\Phi - a))^{n-1} = \frac{1}{(1 - \frac{1}{n}) e^{a/\sigma} + \frac{1}{n}}
\]

(12)

using \( F(\Phi - a) = (F(\Phi))^{e^{a/\sigma}} \) and integrating over the variable \( F \) from 0 to 1. In particular, we recover the result \( p(2,a) = 1 - \tanh(a/2\sigma) \) of section 3.2 of [1].

Aiming at the selection by an outstanding tunnel amplitude of a term in a superposition, we are interested in cases where the probability (12) is close to unity, so that \( a/\sigma \ll 1 \). To first order in \( a/\sigma \) we have

\[
p(n,a) \approx 1 - \left( 1 - \frac{1}{n} \right) \frac{a}{\sigma}
\]

Hence, for arbitrary \( n \) the probability for insufficient selectivity, \( 1 - p(n,a) \), cannot be more than twice that fault probability with \( n = 2 \) (which case was illustrated in Figure 3 of [1]).

\(^3\)To the extent that local elongations \( \phi(\vec{s},t) \) of a harmonically oscillating phononic heat bath can be regarded as statistically independent at sampling times \( t_1,\ldots,t_N \), the Gumbel form is the exact limit distribution of the maxima for \( N \to \infty \). We are assuming \( n \ll N \) here.
4 Conclusion

In continuation of [1], the present approach aims to recover the dynamical pattern of quantum measurement in the “conscious” parts of some unitary, Schrödinger-type dynamics of a suitably constituted “observer”. One of the potential problems listed in [1] has been eliminated—Born’s rule is satisfied if an “observer” consists of many more than two neurons. The solution of the problem, utilizing the large number of “auxilliary” states of such a nervous system, was suggested by an elementary derivation of Born’s rule in [4] and [5]. Two points were to be demonstrated in the present paper:

• The most likely superpositions of auxilliary states have equal amplitudes.

• Thermally assisted tunneling of neuronal pore molecules leads to dominance of one summand in a superposition of many neuronal states with the same efficiency as with only two states.

This was shown on the basis of fairly standard results: Uniqueness of the unitarily invariant measure on the unit sphere, and the Gumbel distribution for extreme values of Gaussian fluctuations.

The unitary structure of Hilbert space was taken for granted here. In particular, state vectors were assumed to be normalized in the usual way. It should be noted that this does not imply a problem of circularity in the present context. We did not undertake to derive Born’s rule but to show that it is consistent with unitary quantum-mechanical time evolution of a system which has some relevant physical structure in common with a conscious observer engaged in a measurement.

A Statistics of discretized amplitudes

Let us divide the plane of complex probability amplitudes into squares of length $d$ in the real and imaginary direction. The number of squares traced by a circle of radius $r_k$ is, in average over a small range of radii, $2\pi r_k/d$. Hence the total number of $2n$-dimensional cells representing phase-equivalent superpositions,

$$N_{\text{equiv}} = \left[\frac{2\pi}{d}\right]^{n} \prod_{k=1}^{n} |c_k|$$

Equation (4) is thus recovered (with a very large proportionality factor).
References

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