Z$_2$ orbifold compactification of heterotic string and 6D

SO(16) and $E_7 \times SU(2)$ flavor unification models

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Abstract

A Z$_2$ orbifold compactification of the heterotic string is considered. The resulting 6D GUT groups can be SO(16) or $E_7 \times SU(2)$ plus some hidden sector groups. The $N = 4$ supersymmetry is reduced to $N = 2$. In particular, the SO(16) 6D model with one spinor representation 128 can reduce to the previous 5D SO(16) or SO(14) family unification models after compactifying the sixth dimension. To obtain one spinor, we have to take into account the left-over center of SO(16). We also comment on the $E_7 \times SU(2)$ model.

[Key words: Z$_2$ orbifold, superstring, 6D SO(16) GUF, SU(2) holonomy]

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I. INTRODUCTION

With the discovery of the top quark, the three family standard model has been filled and seems to be the theory below 100 GeV energy scale. Presumably, the observed weak CP violation is the most compelling reason for the three family standard model (SM), but it lacks the theoretical reasoning of why the same fermion representation repeats three times. In this regard, the grand unification of families or flavors (GUF) attempts to unify the SM gauge couplings without the repetition of fermionic representations of the grand unification gauge group \[1\], which seems to be the most promising rationale toward interpreting multi generations. In early days of GUF, indeed there appeared models without repeated fermionic representations \[1\,2\].

But with the advent of superstring models, the GUF idea seems to be automatically implemented in superstring models. In particular, the \(Z_3\) orbifold models usually give 4D \(N=1\) supersymmetric (SUSY) three family models and \(Z_3\) seemed to be one of the fundamental principles of compactification of extra internal space \[3\,4\]. Some orbifold models are very close to the standard model \[4\], but in most cases, there appear charged SM singlet fermions, which may not be consistent with the observed weak mixing angle \(\sin^2 \theta_W \mid_{100 \,\text{GeV}} \simeq 0.232\). Therefore, one of the key points to require in the compactification is not allowing unfamiliar fermions (UF). For example, vectorlike charged fermions are UF’s since they are not appearing in the SM. For this reason, the flipped \(SU(5)\) \[4\] attracted a great deal of attention in the fermionic construction of 4D string models \[4\]. However, the orbifold compactification of \(SU(5) \times U(1)\) seemed to be not easy. Recently, a 5D \(SO(14)\) model without UF’s has been considered to give a 4D three family flipped \(SU(5)\) model which does not violate major low energy observations \[8\,9\].

From a 10D superstring theory, one has to compactify six extra internal spaces to obtain a 4D SUSY theory \[10\]. Most widely considered superstring theory is the heterotic \(E_8 \times E_8\). But the field theoretic orbifold compactification \[11\] need the string compactification of the internal space smaller than six, so that we can consider SUSY field theory in \(D \geq 5\). For the
possibility of complexifying the internal space, we consider compactifying even dimensions, and hence our consideration of the internal space is four dimensions, leading to a 6D SUSY field theory model. The field theoretic compactification need the string compactification of the internal space smaller than six, so that we can consider SUSY field theory in $D \geq 5$. For the possibility of complexifying the internal space, we consider compactifying even dimensions, and hence our consideration of the internal space is four dimensions, leading to a 6D SUSY field theory model.

The heterotic string theory has $N = 4$ in the 4D viewpoint. To obtain chiral fermions, we have to reduce $N = 4$ down to $N = 1$. The most famous internal space for this purpose is the Calabi-Yau space with $SU(3)$ holonomy. The $Z_3$ orbifold also reduces $N = 4$ down to $N = 1$. It can be understood by observing that blowing up the singularities of the orbifold fixed points leads to the Calabi-Yau space with the $SU(3)$ holonomy, because $Z_3$ is the center of $SU(3)$. To obtain a 6D model by compactifying four internal space, we cannot have an $SU(3)$ holonomy since 6D SUSY theory must be $N = 2$. But we find that $Z_2$ orbifold works in reducing $N = 4$ down to $N = 2$. It is because an $SO(4)$ vector $4$ is $(2, 2)$ under $SU(2) \times SU(2)$ and giving a vacuum expectation value to $4$ can break one $SU(2)$ but leaves the other $SU(2)$ unbroken, which leads to an $SU(2)$ holonomy. Then SUSY is reduced by half. Since $Z_2$ is the center of $SU(2)$, we obtain 6D $N = 2$(in 4D viewpoint) theory. To obtain an $N = 1$ theory, one has to introduce another $Z'_2$ to break the remaining $SU(2)$, which we do not consider in this paper. With the $Z_2$ orbifold, we obtain the simplest 6D orbifold family unification model $SO(16)$, which can be compared to the simplest 4D orbifold model $E_6 \times SU(3)$. It can lead to 5D flavor unification models considered in Refs. [8,9]. Also, it is possible to consider a 6D $E_7 \times SU(2)$ model as a flavor unification model.
II. $\mathbb{Z}_2$ ORBIFOLD

In this spirit, let us proceed to consider the orbifold compactification of $T^m/\mathbb{Z}_n$ with $m = 4$ and $n = 2$ in mind. The twisting of the $m$-dimensional internal space and gauge groups are,

- **internal space**: $\exp(2\pi i [\frac{1}{2} J_{67} + \frac{1}{2} J_{89}])$,
- **gauge groups**: 
  \begin{align*}
  v_{1I} &= \frac{1}{n}(w_I), \quad (E_8 : I = 1, \ldots, 8) \\
  v_{2I} &= \frac{1}{n}(w'_I), \quad (E'_8 : I = 9, \ldots, 16).
  \end{align*}

We define $\alpha$ is the eigenvalue of the internal space rotation,

$$
\exp \left( 2\pi i \sum_{a=1}^{m/2} \frac{r_a}{n} J_{2a+4,2a+5} \right),
$$

satisfying $\alpha^n = 1$.

There are conditions to be satisfied \[3\].

\begin{align*}
\sum r_a &= \sum w_I = \sum w'_I = 0 \mod 2. \\
\sum r_a^2 &= \sum w_I^2 + \sum w'_I^2 \mod \epsilon_n n,
\end{align*}

where $\epsilon_N = 2(1)$ for even(odd) $n$. The first equation in Eq. (2) is from the definition of twist of order $n$ up to fermionic degree of freedom, and the second one is the modular invariance condition.

There are only few possibilities for the lattice vector $v_1$ and $v_2$ of $E_8 \times E'_8$ satisfying these conditions. For $2v_i$ should be again lattice vector in $E_8$, $v_i^2$ is $\frac{1}{2}$ times an integer. Also it is known that any lattice vector in $E_8$ is within the distance 1 from some lattice point, so $v_i^2 \leq 1$. There are only three allowed combinations which are shown in Table I.

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1Even though we are interested in $m = 4$ and $n = 2$, some formulae include $m$ and $n$ explicitly to compare with the well-known case $m = 6$ and $n = 3$ \[3\].
Table I. $Z_2$ orbifolds of the heterotic string.

| Case | $E_8$ shift | $E'_8$ shift | 6D gauge group |
|------|-------------|--------------|---------------|
| (i)  | ($\frac{1}{2}$ $\frac{1}{2}$ $0$ $0$ $0$ $0$ $0$ $0$) | (0 $0$ $0$ $0$ $0$ $0$ $0$ $0$) | $E_7 \times SU(2) \times E'_8$ |
| (ii) | $1$ $0$ $0$ $0$ $0$ $0$ $0$ | ($\frac{1}{2}$ $\frac{1}{2}$ $0$ $0$ $0$ $0$ $0$ $0$) | $SO(16) \times SU(2)' \times E'_7$ |
| (iii)| ($\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $0$ $0$ $0$ $0$) | ($\frac{1}{2}$ $\frac{1}{2}$ $0$ $0$ $0$ $0$ $0$ $0$) | $SO(16) \times SU(2)' \times E'_7$ |

Case (i) breaks down to $E_7 \times SU(2) \times E'_8$. Cases (ii) and (iii) break down to $SO(16) \times SU(2)' \times E'_7$. The cases ($v_1^2 = 1$, $v_2^2 = 0$), ($v_1^2 = v_2^2 = 1$) with entries $\frac{1}{2}$, and ($v_1^2 = v_2^2 = \frac{1}{2}$) do not satisfy the two conditions given above. We will mainly discuss $SO(16)$ and comment on $E_7 \times SU(2)$ flavor unification group at the end.

A. $SO(16)$

Let us consider the $SO(16)$ first which can give 5D flavor unification models \[ \text{[8,9]} \]. Even though Cases (ii) and (iii) give the same theory, we will treat them separately, since the introduction of Wilson lines can be studied more easily with two different shift vectors. It is easy to understand Case (ii). It is simply separating $248$ of $E_8$ into vector and spinor parts, where the vector forming the adjoint representation of $SO(16)$ and the spinor forming the matter fields $128$ of $SO(16)$ \[ \text{[3]} \]. Thus, we discuss Case (iii) only in detail since it has not been discussed in the literature and can be a potential GUF(grand unification of families) $SO(16)$ \[ \text{[8,4]} \]. The gauge group from $SU(2)' \times E'_7$ can be considered as the "hidden" sector needed for SUSY breaking.

Now, from the mass shell condition

$$\frac{m^2}{4} = \frac{p^2}{2} - 1$$

we have massless states as follows:

On the left-moving side there are

$$\alpha^0 : \alpha^\mu_{-1}|0\rangle \quad \text{and} \quad 112$$

$$\alpha^1 : \alpha^i_{-1}|0\rangle, \alpha^\bar{i}_{-1}|0\rangle \quad \text{and} \quad 128$$

$$5$$
where we have not displayed the hidden sector. The corresponding lattice vectors are shown in the following tables.

Table II. Root vectors $p_I$ in untwisted sector transforming like $\alpha^0$. The underlined entries allow permutations and those in the $[ ]$ bracket allow even numbers of sign flips.

| vector                  | number of states |
|-------------------------|------------------|
| $\begin{pmatrix} 0 & 0 & 0 & \pm 1 & 1 & 0 & 0 \end{pmatrix}$ | 24               |
| $\begin{pmatrix} \pm 1 & 1 & 0 & 0 & 0 & 0 \end{pmatrix}$ | 24               |
| $\begin{pmatrix} [\frac{1}{2} & \frac{1}{2} & \frac{1}{2}] [\frac{1}{2} & \frac{1}{2} & \frac{1}{2}] \end{pmatrix}$ | 64               |

In the Tables, we have the convention that the underlined entries allow permutations and those in the square bracket $[ ]$ allow even numbers of sign flips. We have 112 winding states and from the oscillators we have 8 $U(1)$ generators. They will form the adjoint representation $\mathbf{120}$ of $SO(16)$.

Table III. Root vectors $p_I$ in untwisted sector transforming like $\alpha^1$.

| vector                  | number of states |
|-------------------------|------------------|
| $\begin{pmatrix} \pm 1 & 1 & 0 & 0 & 0 \end{pmatrix}$ | 64               |
| $\begin{pmatrix} [\frac{1}{2} & \frac{1}{2} & \frac{1}{2}] [\frac{1}{2} & \frac{1}{2} & \frac{1}{2}] \end{pmatrix}$ | 64               |

On the right moving side, there are vectors transforming like $\alpha^0 = 1$ and $\alpha^1 = e^{2\pi i (1/2)} = -1$,

$$\alpha^0 : \quad \tilde{b}^\mu_{-\frac{1}{2}} |0\rangle_{NS}, ~ |+; 1\rangle_R$$

$$\alpha^1 : \quad \tilde{b}^i_{-\frac{1}{2}} |0\rangle_{NS}, ~ \tilde{b}^i_{-\frac{1}{2}} |0\rangle_{NS}, ~ |-; 2\rangle_R$$

where $\tilde{b}^M_{-\frac{1}{2}}$s are NS creation operators. Note that, the first entry of spinorial(Ramond) representation is chirality in six dimension. The $|+; 1\rangle_R$ (leading to gaugino after combining with the left movers) and $|--; 2\rangle_R$ (leading to the chiral matter after combining with the left movers) have opposite chiralities. For $D = 2n$, a spinor of $SO(2n)$ can be represented
by $n$-tuples of spin $\frac{1}{2}$ eigenstates as, $|s_1 \ s_2 \ldots \ s_n\rangle$ where $s_i$ can be either $\pm \frac{1}{2}$. Then, the chirality is defined by the eigenvalue of

$$\Gamma = 2^n s_1 s_2 \ldots s_n.$$  \hfill (8)

Compactification of some internal space of even dimensions (e.g., from 10D to 6D) kicks out some factors of $s_i$ in Eq. (8) and the product of the remaining $s_i$’s in Eq. (8) determines the chirality in the uncompactified space. For example, the chirality in 6D in our compactification of 10D down to 6D is $\Gamma^7 = 4s_1 s_2$. The ten dimensional 8 of right-moving ground state is decomposed into two 2’s and four 1’s with opposite chirality in six dimension.

Combining the right movers of the preceding paragraph with the left movers into $Z_2$ invariant states, we have an adjoint representation 120 for the gauge multiplet and a spinor representation 128 for the hyper-multiplet. Defining the chirality of the gauge multiplet as left-handed, the chirality of the hyper-multiplet becomes right-handed. In addition, there appear the 6D supergravity multiplets, etc.

Notice, however, that combining the right-handed $|2\rangle$ with the left-handed 128 seems to give 2 spinors.\footnote{In $Z_3$ orbifold, this is the reason that we obtain three copies of chiral fermions from the untwisted sector.} But note that the center of $SO(2n)$ is $Z_2 \times Z'_2$ for an even $n$ and $Z_4$ for an odd $n$. Our orbifolding under $Z_2$ and assigning at the center of $SO(16)$ still allow $Z'_2$ freedom. Thus, the two spinors of $SO(16)$ are connected by $SO(16)$ gauge transformation, and we have to divide the number of spinors by the left-over center $Z'_2$. Thus, the untwisted sector allows only one spinor 128 of $SO(16)$. It is similar to the mechanism of calculating the domain wall number in axionic models [13].

**B. Twisted sector**

In the twisted sector, the twisted mode expansion gives a different zero point energy. The zero point energy of a bosonic string is given by $\sum_{n=1}^{\infty} (n + \eta) = -\frac{1}{24} + \frac{1}{4} \eta (1 - \eta)$, where
\( \eta = \frac{1}{n} \) is a shift. In our \( Z_2 \) model, the zero point energy is \( E_0 = -1 + 4 \frac{1}{4} \frac{1}{2} \frac{1}{2} = -\frac{3}{4} \) Then the mass shell condition becomes

\[
\frac{m_R^2}{8} = \frac{m_L^2}{8} = \frac{(p^I + v^I)^2}{2} + \tilde{N} - \frac{3}{4} = 0. \tag{9}
\]

where \( \tilde{N} \) is the oscillator number. For \( \tilde{N} = \frac{1}{2} \), there is no vector in the \( E_8 \) lattice, satisfying Eq. (8). For \( \tilde{N} = 0 \), we have some lattice vectors satisfying the above condition. We can find two sixteen states satisfying Eq. (9) given in Table IV.

**Table IV. Root vectors \( p_I \) in the twisted sector**

| vector | number of states |
|--------|-----------------|
| \((-\frac{1}{2} - \frac{1}{2} - \frac{1}{2} - \frac{1}{2} [\frac{1}{2} \frac{1}{2} \frac{1}{2} \frac{1}{2}])\) \((0 0 0 0 0 0 0 0)\) | 8 |
| \((-1 - 1 0 0 0 0 0 0)\) \((0 0 0 0 0 0 0 0)\) | 6 |
| \((-1 - 1 - 1 - 1 0 0 0 0)\) \((0 0 0 0 0 0 0 0)\) | 1 |
| \((0 0 0 0 0 0 0 0)\) \((0 0 0 0 0 0 0)\) | 1 |
| \((-\frac{1}{2} - \frac{1}{2} - \frac{1}{2} - \frac{1}{2} [\frac{1}{2} \frac{1}{2} \frac{1}{2} \frac{1}{2}])\) \((-1 - 1 0 0 0 0 0)\) | 8 |
| \((-1 - 1 0 0 0 0 0 0)\) \((-1 - 1 0 0 0 0 0 0)\) | 6 |
| \((-1 - 1 - 1 - 1 0 0 0 0)\) \((-1 - 1 0 0 0 0 0)\) | 1 |
| \((0 0 0 0 0 0 0 0)\) \((-1 - 1 0 0 0 0)\) | 1 |

Combining with the right-moving states, we have two 16’s in the twisted sector.

In the \( Z_3 \) orbifold model, we encounter only fixed points. However in a general \( Z_n \) orbifold model with \( n \) even, there are also fixed tori which have a different topology and can give a different number for the twisted sector states.

For the case of the \( Z_2 \) orbifold, there are 16 fixed points. However, the effective multiplicity is 8 rather than 16. ³

³This can be more transparent by reading off the partition function. In terms of the notation in Ref. [3], projector operator is

\[
P_\theta = \frac{1}{2}(\chi(\theta,1)\Delta^0_\theta + \chi(\theta,\bar{\theta})\Delta^1_\theta)
\]
Therefore, there appear sixteen ($8 \times 2$) copies of $16$ from the twisted sector. These have the same chirality as the matter representation from the untwisted sector.

C. Anomaly

In 6D, there can exist a square anomaly. The anomaly of $SO(2N)$ for $N \geq 5$ is \cite{14},

$$+ 2(N - 4) \text{ for the adjoint representation},$$

$$- 2^{N-5} \text{ for the spinor representation},$$

with the normalization in units of the anomaly of the left-handed vector representation. It is important that in this model we have gaugino and matter fermion with opposite chirality. In six dimension, Weyl spinor is self-dual, or charge conjugate of one has the same chirality of itself.

We can check the anomaly cancellation with the fermion spectrum obtained in the preceding section,

$$A = A(L \text{- handed Adjoint}) + A(R \text{- handed spinor})$$

$$+ 16A(R \text{- handed vector}) = 8 + 8 + 16(-1) = 0.$$  

This again shows the consistency of having only one spinor representation due to the nontrivial center of $SO(16)$ after orbifolding with $Z_2$.

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with $\chi(\theta^m, \theta^n)$ is the Euler number of compact manifold twisted by $\theta^m, \theta^n$ in the each worldsheet direction. Here, $\Delta_\theta = \exp 2\pi i [(p + \tilde{v})\tilde{v} - (r + w + v)v]$ where $p$ and $(r + w)$ are the lattice of the left and right movers, respectively. In the $Z_2$ case, $\chi(\theta, \theta^n) = 16$ where $n = 0, 1$ and $\Delta_\theta$ can be either 1 or -1.

\footnote{The $r + \omega$ vector for the right movers in the notation of Ref. \cite{3} can be $(\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}), (\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, \frac{1}{2}), (-\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2})$, and $(\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, \frac{1}{2})$. The chirality is the product of the third and the fourth entries and turns out to be the same as the hypermultiplet from the untwisted sector.}
D. $E_7 \times SU(2)$

In 6D the square anomalies of $133$ (adjoint) and $56$ of $E_7$ are absent. As for the anomaly cancellation, therefore, $E_7$ in 6D is like $SO(N)$ in 4D. As we studied the $SO(16)$ model previous subsection, we can repeat a similar analysis. The gauge group is $E_7 \times SU(2) \times E_8'$ or $E_7 \times SU(2) \times SO(16)'$. The matter in the untwisted sector is listed in Table IV. In this subsection, we will comment the $E_7 \times SU(2)$ flavor unification briefly. In the untwisted sector, matter fields of Table IV arise. The states in Table IV constitute $(56,2)$ of $E_7 \times SU(2)$ since the center of $E_7 \times SU(2)$ is $Z_2 \times Z_2$. We divided the total number by 2, as we have done in the $SO(16)$ case. The flavor group is $SU(2)$. Thus, we obtain the flavor doublet of $56$. It is a 6D model.

Table IV. Root vectors $p_I$ in untwisted sector transforming like $\alpha^1$.

| vector                       | number of states |
|------------------------------|-----------------|
| $(1 \ 0 \ \pm1 \ 0 \ 0 \ 0 \ 0 \ 0)$  | (12,2)          |
| $(-1 \ 0 \ \pm1 \ 0 \ 0 \ 0 \ 0 \ 0)$ | (12,2)          |
| $([1 \ \frac{1}{2} \ \frac{1}{2} \ \frac{1}{2} \ \frac{1}{2} \ \frac{1}{2} \ \frac{1}{2} \ \frac{1}{2}])$ | (32,2)          |

In the twisted sector, we obtain following states satisfying $(p + v)^2 = 3/2$,

\[
-1 \ 0 \ \pm1 \ 0 \ 0 \ 0 \ 0 \ 0, \quad 2(12,1)
\]

\[
(-\frac{1}{2} - \frac{1}{2} \ [1 \ \frac{1}{2} \ \frac{1}{2} \ \frac{1}{2} \ \frac{1}{2} \ \frac{1}{2} \ \frac{1}{2} \ \frac{1}{2}]), \quad (32,1)
\]

which constitute $(56,1)$. Since there are 16 fixed points, there are 16 copies of $(56,1)$. If we somehow remove the $56$'s from the twisted sector, we can have a four generation model. Indeed, we can devise such a scheme by compactifying the 6th dimension by $S_1/Z_2$ or by the Scherk-Schwarz mechanism. Since the number of components of spinors in 6D and 5D are the same, by the Scherk-Schwarz mechanism for example, we just distinguish the representation property. We assign the anti periodic boundary condition for all the wave functions in the compactification of the 6th dimension. The $2\pi$ rotation in the 6th dimension is embedded
in the $SU(2)$ group space so that the spinors of $SU(2)$ get an extra minus sign for the $2\pi$ rotation. Therefore, sixteen $(56,1)$’s are projected out. In 5D we obtain an $E_7 \times SU(2)$ model with the matter $(56,2)$.

The $E_7$ branching of 56 to $E_6$ representations is

$$56 \to 27 + \overline{27} + 1 + 1.$$  (11)

Thus, it is possible to pick up two 27’s in 4D by picking one 27 from the left-handed and one $\overline{27}$ from the right-handed. Then, we obtain a four generation model since 56 is a flavor group doublet.

III. CONCLUSION

We considered the $Z_2$ orbifold compactification of the heterotic string to obtain a 6D $SO(16)$ model with one spinor. We pointed out that the untwisted sector matter fields should be properly counted by considering the center of $SO(16)$. One $SO(16)$ spinor can split into two $SO(14)$ spinors, and a 5D $S_1/Z_2 \times Z'_2$ compactification leads to reasonable family unification models [9]. We also commented on the $E_7 \times SU(2)$ flavor unification.

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