Destruction of integer quantum Hall effect at strong disorder: A numerical study

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A breakdown of integer quantum Hall effect (IQHE) at strong disorder is studied numerically in a lattice model. We find a generic sequence by which the integer quantum Hall plateaus disappear: higher IQHE plateaus always vanish earlier than lower ones. We show that extended levels between these plateaus do not float up in energy but keep merging together after the disappearance of plateaus, which eventually leads to a localization in the whole system. We also study this phenomenon in terms of topological properties, which provides a simple physical explanation.

It is an interesting question to ask how the integer quantum Hall effect vanishes in the strong disorder or weak magnetic field limit. Many years ago, Khmel’nitzkii and Laughlin both argued that extended levels at the centers of Landau levels would not disappear discontinuously nor merge together at strong disorder, but rather float up continuously to pass the Fermi level. This argument is a key scenario in the global phase diagram proposed by Kivelson, Lee and Zhang for the quantum Hall effect. Recently a considerable number of experimental measurements have been devoted to this issue, in which a limited floating-up feature is indeed observed for lower extended levels. However, such a floating up seems not indefinite and experiment suggests that higher extended levels eventually merge with the lowest one at weak magnetic fields. A direct transition from a higher IQHE state to an insulator state has been also found. These are in contrary to a simple floating up picture and imply that an overall destruction of IQHE may be more delicate. Recent numerical study of finite-size localization length by Liu, Xie, and Niu has also suggested no global floating up for extended levels, although no merge of extended levels was shown and actually how each of extended levels is destroyed by disorder remains unclear. Thus the problem of disappearance of the IQHE needs to be reexamined in a more careful and global way.

In this Letter, we present a numerical study of these issues based on a tight-binding model. A systematic destruction of the integer quantum Hall plateaus is unambiguously revealed, which exhibits the following sequence: higher plateaus are found to be destroyed first by strong disorder in a one-by-one order, and the lowest plateau is the last to be gone before the whole system becomes an insulator. In particular, the lowest extended level located below the last plateau is not floating up in energy, and even the critical exponent at such a delocalization point remains essentially the same as in the full IQHE case. A topological reason for such a destruction of the IQHE is given based on the Chern number which characterizes extended states: nonzero Chern numbers with opposite sign are moving down from the band center to annihilate those in the lower-energy extended levels, and eventually a total annihilation of Chern numbers leads to a global insulating phase at strong disorder. Furthermore, it is indeed found that a limited floating-up picture for the lower extended levels shows up when the electron filling number, instead of the Fermi energy itself, is fixed with the increase of disorders. At higher extended levels, we see a direct transition from the IQHE states to insulator states.

We consider a tight-binding lattice model of noninteracting electrons under a uniform magnetic field and disorders. The Hamiltonian is defined as follows:

\[ H = - \sum_{<ij>} e^{ia_{ij}} c_i^+ c_j + H.c. + \sum_i w_i c_i^+ c_i. \]

Here \( c_i^+ \) is a fermionic creation operator, with \( <ij> \) referring to two nearest neighboring sites. A uniform magnetic flux per plaquette is given as \( \phi = \sum a_{ij} = 2\pi/M \), where the summation runs over four links around a plaquette. We study the case in which the integer \( M \) is commensurate with the system width \( L \). And \( w_i \) is a random potential with strength \( |w_i| \leq W/2 \), and the white noise limit is considered with no correlations among different sites for \( w_i \).

Fig. 1 shows an overall picture for the Hall conductance calculated by the Kubo formula, with a flux strength \( \phi = 2\pi/8 \) at a 16 \( \times \) 16 lattice size (only \( E < 0 \) part is shown in the figure due to the antisymmetry of the Hall conductance on the two sides of the band center \( E = 0 \)). In the weak disorder case (\( W = 1 \)), there are three well-defined IQHE plateaus at \( \sigma_H = \nu e^2/h \) (\( \nu = 1, 2, 3 \)), corresponding to four Landau levels which are centered at the jumps of the Hall conductance at \( E < 0 \). With disorder strength \( W \) varying from 1 to 6, we see a systematic destruction of these IQHE plateaus. At \( W = 2 \), the third plateau (closest to the band center) begins to disappear. An increase of \( W \) to 3 and 4 will further result in the disappearance of the second IQHE plateau, while the lowest plateau still remains well defined. In the inset of Fig. 1, such the lowest IQHE plateau at \( W = 4 \) is shown with sample sizes varying as \( 8 \times 8, 16 \times 16 \) and \( 24 \times 24 \). While the quantization of this
\( \nu = 1 \) IQHE plateau remains at different lattice sizes, the region of the transition towards zero is continuously narrowed with all the curves crossing at a fixed-point corresponding to the Hall conductance \( \frac{1}{2} e^2 / h \), which should be extrapolated to a sharp step of jump in the thermodynamic limit. This resembles a typical scaling behavior found for the IQHE [1] in the weak disorder case and the lattice-size-independent fixed-point represents the mobility edge of the delocalization region. (Later it will be shown that even the critical exponent at this delocalization energy point remains essentially the same as in the full IQHE case.) On the other hand, the Hall conductance in the higher energy region at \( W = 4 \) has no more IQHE structure and is seen to continuously decrease with the increase of the lattice size. A breakdown of the lowest IQHE plateau is eventually found at larger \( W \)'s (\( W = 6 \) case is shown in Fig. 1). Here the Hall conductance in the whole energy region is much less than \( e^2 / h \), which continuously decreases at larger sample sizes (from 8 \times 8, 16 \times 16 to 24 \times 24), indicating that it will scale to zero in the thermodynamic limit with all states being localized.

To further inspect the critical behaviors shown in Fig. 1, we use a different finite-size scaling method in terms of the Thouless number. [12] This method has been previously used by Ando [13] to study a similar problem, but much larger sample sizes (from 16 \times 16 to 96 \times 96) with more random configurations (200 – 30,000) can be achieved here by using Lanczos diagonalization method, in order to accurately decide the critical points and exponents. Fig. 2 shows localization lengths versus energy obtained by a finite-size scaling calculation. [14] Only the data for \( W = 4 \) and 5 are presented here where only the lowest IQHE plateau still remains. At \( W = 4 \), Fig. 2 shows that the localization length follows a scaling behavior \( \xi \sim |E - E_{c1}|^{-x} \) with the exponent \( x = 2.4 \pm 0.1 \) (which is the same as in the case when full IQHE plateaus are present at weak disorders [13]) on the two sides of the divergent point \( E_{c1} \approx -3.40 \). Such a scaling law verifies the existence of a delocalization fixed-point shown in the inset of Fig. 1 and also confirms that the lowest IQHE plateau at \( \nu = 1 \) is robust as delocalization point \( E_{c1} \) is well separated by a localization region from the higher-energy part. The localization length \( \xi \) exhibits an another dramatic increase which can be extrapolated as \( |E - E_{c2}|^{-4.0} \) when one approaches to \( E_{c2} \approx -2.46 \) from below. Such a second delocalization point \( E_{c2} \) is more clearly shown at \( W = 5 \) case in Fig. 2. Above \( E_{c2} \) one finds a large but finite localization length which is rather flat up to the band center and whose magnitude is reduced at \( W = 5 \). A recent numerical calculation by using one-parameter finite-size scaling analysis also suggests such a region to be localized. [15] Thus our results show that after losing the IQHE plateaus at higher energies, original extended levels within this region merge and move down to \( E_{c2} \) at \( W = 4 \) and 5. Fig. 2 also shows that \( E_{c1} \) is only slightly reduced (from \(-3.40 \) to \(-3.48 \)) with essentially the same critical exponent \( x \) when \( W \) is increased from 4 to 5, whereas \( E_{c2} \) is much more quickly moving down (from \(-2.46 \) to \(-2.96 \)). Correspondingly the \( \nu = 1 \) IQHE plateau between \( E_{c1} \) and \( E_{c2} \) becomes narrower, and eventually these last two extended levels merge and disappear at a critical \( W_c \). We find that the localization length becomes finite at \( W = 6 \) in the whole regime, which is consistent with the conclusion drawn from the Hall conductance calculations, and places \( W_{c} \) at a range: \( 5 < W_{c} < 6 \). At smaller disorder strengths, like \( W = 3 \), we also see that more divergence points emerge in \( \xi \), in correspondence with more extended levels and additional IQHE plateaus shown in Fig. 1.

A key thing to understand the above evolution of IQHE plateaus is the localization-delocalization transition, and it is a well-known fact that an extended state can be characterized in terms of a topological quantity, namely, nonzero Chern number [10]. And the boundary-condition-averaged Hall conductance is a summation of all the Chern numbers carried by states below Fermi surface: \( \rho_{E_{ ext}}(H) = \frac{2\pi}{eh} \sum_{E < E_{f}} C(m) \). Here the Chern number \( C(m) \) is always an integer. Since its distribution in the thermodynamic limit decides delocalization regions, one may define states with nonzero Chern integers as extended states. [16] Plots of the density of extended states \( \rho_{ext} \) are presented in Fig. 3a with the same flux strength as in Fig. 1. In the weak-disorder case (\( W = 1 \)), well-defined peaks of \( \rho_{ext} \) are at centers of Landau-level bands, separated by localized regions represented by plateaus in Fig. 1. Widths of these peaks will approach to zero in the thermodynamic limit. [17] Total Chern number for each of three lower-energy peaks is found to be exactly +1, which is the reason leading to three quantized Hall plateaus at +1, +2, and +3 in unit of \( e^2 / h \) shown in Fig. 1, when the Fermi surface is located between these peaks. The last peak closest to the band center in Fig. 3 carries a total Chern number –3, which guarantees that the Hall conductance in Fig. 1 falls back to zero beyond the third plateau when the Fermi energy approaches \( E = 0 \). This is a peculiar feature of a lattice model since when the whole band is half-filled, the Hall conductance has to be zero. With the decrease of the flux strength and increase of the number of Landau levels or peaks of \( \rho_{ext} \) at the center of the Landau levels, one always sees that the last peak near the band center carries a large Chern number with opposite sign and a magnitude equal to the total sum of those at lower-energy peaks.

In Fig. 3a, upper two peaks at \( W = 2 \) start to merge due to the disorder scattering. At \( W = 3 \) they are pretty much mixed together. Because these two peaks carry total Chern numbers of +1 and –3, respectively, an annihilation of these opposite-sign Chern numbers occurs with the disappearance of the third plateau in Fig. 1, which leads to a substantial reduction of the magnitude
of the Hall conductance in that region. In fact, the overall magnitude of the Hall conductance beyond the second plateau is less than the value of the second plateau at $W = 3$, suggesting that the negative Chern numbers have already moved down from the original $-3$ peak and dominated this region. Notice that the lowest peak near $-3.4$ in Fig. 3a still remains separated from the rest spectrum at $W = 4$. The inset in Fig. 3a shows such a peak becomes narrower and sharper with the increase of lattice sizes. This is consistent with the finite-size scaling result in Fig. 2 that a well-defined mobility edge still exists at $E_{c1} = -3.4$. The inset of Fig. 3a also shows a second peak emerging near $-2.5$, which coincides with $E_{c2}$ in Fig. 2 and implies that negative Chern numbers would be located in this neighborhood at a large lattice size, representing a delocalization region beyond which the Hall conductance jumps back to zero from the $\nu = 1$ plateau. Thus when the Fermi energy is well above $E_{c2}$, the system becomes an insulator with zero Hall conductance. In other words, a direct transition from original $\nu = 2$ IQHE state to an insulator state is realized here, which is consistent with the experimental finding. Finally, at $W = 6$, the negative Chern numbers reaches to $E_{c1}$ and annihilates the last $+1$ Chern number peak in Fig 3a, where we find a monotonic decrease of $\rho_{\text{ext}}$ with larger lattice sizes which corresponds to a localization in the whole region as shown by previous analyses.

We do not see any floating up of extended levels before they vanish. In fact, extended levels are all moving down at strong disorders except $E_{c1}$ which is essentially not changed until this last extended level is completely annihilated by negative Chern numbers moving down from the band center. Nevertheless, a limited floating-up picture may still be seen in the case if the Landau-level filling number is fixed as shown in Fig. 3b. If we look at $\rho_{\text{ext}}$ within the lowest Landau level (with the occupation number $n_L \leq 1$), the peak position does shift towards higher occupation number at a larger $W$. Since the peak of $\rho_{\text{ext}}$ will represent a mobility edge in the thermodynamic limit, this upward shifting implies that for a given filling number in the lowest Landau level, the extended states can float up to pass the Fermi surface when the disorder is strong enough, leading to an insulating transition. Such a “floating up” is due to the reason that the number of localized states below the lowest extended level at $E_{c1}$ is increased at stronger disorder, so that when the electron filling number is fixed the Fermi level is actually moving down, or relatively, the extended level is shifting up. This effect is indeed consistent with experimental observations for lower extended levels. Hall conductances at weaker magnetic fields (thus with more Landau levels as well as IQHE plateaus) have been also calculated with flux strength $\phi = 2\pi/M$ at $M = 11, 16$ and 24. All of them exhibit the same generic features as shown in $M = 8$ for the destruction of the IQHE at strong disorder. It suggests that at large $M$ case (weak-field limit), lower extended levels are always fixed at lower energies until higher extended levels move down to merge with them after higher IQHE plateaus are destroyed in a one-by-one manner. In completing the present work, we also became aware of a recent Chern number calculation by Yang and Blatt [13] at $M = 3$, by which they concluded that a “floating up” picture is correct. Such a floating up of the lowest-delocalization region seen by them is actually the same as shown in Fig. 3b. But we point out that $M = 3$ is a very special case where only one plateau ($\nu = 1$) exists at $E < 0$, and thus one cannot find the sequence of the destruction of higher plateaus as well as the merge of higher delocalization levels, which are essential in our discussion of the global IQHE evolution at strong disorder.

In conclusion, our numerical calculations unequivocally demonstrate that the integer quantum Hall effect in a lattice model is destroyed by strong disorder by the following sequence: higher integer plateaus closer to band center are first to vanish when lower plateaus still remain well defined. Once an IQHE plateau disappears, two delocalization levels separated by it will merge together and move down. Since extended states at the band center carry a total Chern number with opposite sign as compared to those in the lower delocalization levels, the sequence of the disappearance of the IQHE plateaus actually corresponds to that Chern numbers at the band center continuously move down towards the band edge and annihilate those Chern numbers with opposite sign in all the lower extended levels. No floating up in energy is found for extended levels during such a destruction procedure. However, a relative upwards shifting of lower delocalization levels is indeed seen if the electron filling number, not the Fermi energy, is fixed. Our results thus provide a consistent explanation of both “floating up” as well as direct transitions from higher IQHE to insulator states in the experimental measurements.

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Fig. 1. The Hall conductance $\sigma_H$ as a function of energy $E$ is plotted for different disorder strength $W$'s, at a lattice size $16 \times 16$. The inset shows the evolution of the $\nu = 1$ IQHE plateau at $W = 4$ with lattice size varying form $8 \times 8$ (○), $16 \times 16$ (●), to $24 \times 24$ (+).

Fig. 2. Localization length $\xi$ obtained by a finite-size scaling based on the Thouless number is shown as a function of energy $E$. Critical behaviors near divergent points are fitted (dotted curves) by scaling laws (see text).

Fig. 3 (a) The density of extended states $\rho_{ext}$ versus $E$. Disorder strength $W$'s are the same as in Fig. 1 with a lattice size $8 \times 8$ (about 400 random-potential configurations are used). The inset shows $\rho_{ext}$ at $W = 4$ with lattice sizes varying as $8 \times 8$ (●), $16 \times 16$ (×), and $24 \times 24$ (*); (b) $\rho_{ext}$ versus Landau-level occupation number $n_L$ with the same parameters as in (a).