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Hydrodynamic helicity and strange hyperon polarization in heavy-ion collisions

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Abstract.
We study the P-odd effects related to the vorticity of the medium formed in noncentral heavy ion collisions. Using the kinetic Quark-Gluon String Model, we perform numerical simulations of the vorticity and hydrodynamical helicity for various atomic numbers, energies and centralities. We observe vortical structures typically occupying a relatively small fraction of the fireball volume. In the course of numerical simulations a noticeable hydrodynamic helicity was observed manifesting a specific mirror behaviour with respect to the reaction plane. The effect is maximal at the NICA and FAIR energy range.

1. Introduction
The local violation [1] of discrete symmetries in strongly interacting QCD matter is now under intensive theoretical and experimental investigation. The Chiral Magnetic Effect (CME) uses the (C)P-violating (electro)magnetic field emerging in heavy-ion collisions in order to probe the (C)P-odd effects in QCD matter.

There is an even more interesting counterpart of this effect, the Chiral Vortical Effect (CVE)[2] due to coupling to P-odd medium vorticity leading to the induced electromagnetic and [3] all conserved-charge currents, in particular the baryonic ones.

The zeroth component of axial current and the corresponding axial charge are related to the hydrodynamic helicity

\[ H \equiv \int dV (\vec{v} \cdot \vec{w}), \]

being the projection of velocity \( \vec{v} \) to vorticity \( \vec{w} = \text{curl} \ \vec{v} \). This quantity is manifesting the recently discovered [4] and confirmed [5] phenomenon of separation, i.e. mirror behavior (same magnitude but different sign) in the half-spaces separated by the reaction plane.

Another important P-odd observable is the baryon polarization. The mechanisms analogous to the CVE (known as axial vortical effect, see [6] and references therein) lead to an induced axial current of strange quarks which may be converted to polarization of Λ-hyperons [3, 7, 4]. Another mechanism of this polarization is provided [8] by so-called thermal vorticity in the hydrodynamic approach.
Noncentral heavy-ion collisions could naturally generate a rotation (global or local, both related to vorticity) with an angular velocity normal to the reaction plane, which is their generic qualitative feature.

We paid special attention to the pseudoscalar characteristics of the vorticity, that is the hydrodynamic helicity 

\[ H \equiv \int dV (\vec{v} \cdot \vec{w}) \]

which is related to a number of interesting phenomena in hydrodynamics and plasma physics, such as the turbulent dynamo (providing possibly an additional mechanism of magnetic field generation in the later stages of heavy-ion collisions) and Lagrangian chaos. It might be compared to the analog of topological charge 

\[ Q = \int d^3x J^0(x) \]

where the current 

\[ J^\mu = \epsilon^{\mu
u\rho\gamma} u_\nu \partial_\rho u_\gamma \]

(as usual, the four-velocity \( u_\nu \equiv \gamma (1, \vec{v}) \)) contributes to the hydrodynamic anomaly \cite{9} and the polarization of hyperons \cite{3, 7}. The calculation of the topological charge which is the correct relativistic generalization of the hydrodynamic helicity leads to the extra factor \( \gamma^2 \) in the integrand. Still as the helicity itself is a more traditional quantity, we use it for the numerical calculations.

2. Angular momentum conservation in the kinetic model

The natural source of the P-odd observables in heavy-ion collisions is the pseudovector of angular momentum. The question immediately emerges whether it is conserved in the course of evolution governed by Quark-Gluon String Model (QGSM)\cite{10, 11}. To check this we calculated the angular momentum at various moments of collision taking into account both the contributions of participants and spectators.

We found that the participants carry about 20\% of angular momentum and that the total angular momentum of participants and spectators is conserved with a rather good accuracy. We also observe clear correlation between the fireball angular momentum and hydrodynamic helicity, pointing to the global rotation as a source of vorticity.

3. Large-scale structures of vorticity fields

We study the qualitative structure of velocity and vorticity fields. Our observation is that while velocity fields represent the "small bang" picture, vorticity fields form relatively thin toroidal tire-like structures.

4. Hyperon polarization

We compare two rather distinct methods of determining the hyperon polarization. The first one corresponds to the earlier suggested \cite{3} and explored \cite{4} relation to the induced axial current while the second one follows the procedure \cite{8} based on the thermal vorticity.

The hydrodynamical helicity should give rise to the polarization of \( \Lambda^- \)–hyperons with the sign differing for the particles with "up" and "down" \( y \)–components of their momenta, so that the hyperons acquire the helicity in the course of their motion transverse to the reaction plane. As we already suggested earlier \cite{3}, the effect is pronounced at moderate (NICA) energies due to a large (strange) chemical potential. The current investigation shows that, luckily, the helicity at these energies is also noticeable.

For a semi-quantitative estimate of this effect one may use the average strange chiral charge produced by the zeroth component of the respective current

\[ Q_s^5 = \frac{N_c}{2\pi^2} \int d^3x \mu^2(x) \epsilon^{ijk} u_i \partial_j u_k = \frac{\mu^2 > N_c H}{2\pi^2}, \]

where we use the mean value theorem to extract the value of the square of strange chemical potential at some point inside the integration region and get the helicity from the remaining integral. Assuming that the strange chirality is carried by the \( \Lambda^- \)– hyperons whose average
number in each event is $< N_\Lambda >$ one gets the estimate for its average polarization as

$$< P_\Lambda > \sim \frac{< \mu^2 >}{2\pi^2} \frac{N_c H}{< N_\Lambda >}.$$ 

For numerical estimates at NICA energies, we take $H = 30 \text{ fm}^2$ ($c = 1$) and, as typical values, $< \mu^2 > = 900 \text{ MeV}^2$ and $< N_\Lambda > = 15$ to get $< P_\Lambda > \sim 0.8\%$. This value is not large, but does not exclude the opportunity to measure the effect. Note that it is indirectly supporting the actual calculations of helicity as the obtained expression respects the density matrix positivity [15] limit $P_\Lambda \leq 1$. Should the helicity be much larger, a much larger number of hyperon production (and/or $K^*-\text{mesons}$) would be required to preserve the positivity of the density matrix. This is an example of the situation when the spin-dependent effects may be used [15] to bound the spin-averaged cross-sections from below.

Figure 1. Upper: Tire-like structure of vorticity field for fixed orientation of reaction plane. Lower: Tire-like structure of vorticity field for orientations of reactions plane averaged in the whole $2\pi$ region of azimuthal angles.
Of course, more detailed calculations of polarization taking into account the spatial distribution of the chemical potential and the kinematics of produced hyperons will be required.

5. Conclusions and Outlook
We have investigated vorticity and hydrodynamic helicity in noncentral heavy-ion collisions in the framework of the kinetic quark-gluon string model. We have observed that the vorticity is predominantly localized in a relatively thin layer ($2 \div 3 \, fm$) on the boundary between the participant and spectator nucleons. This might be qualitatively understood in the spirit of the core-corona type models [13, 14].

Thus, the gradients of the velocities in the region occupied by the participants are small due to the compensation of momenta between the target and projectile particles in the c.m. frame. As a result the vorticity is substantial only in the thin transition layer between the participant (i.e., core) and the spectator (i.e., corona) regions. We found the novel effect of the helicity separation in heavy-ion collisions when it has the different signs below and above of the reaction plane. We have investigated its dependence on the type of nuclei and collision energy and observed that it is maximal in the NICA energy range. We have also calculated the degree of alignment of the velocity and vorticity which is maximal for the Beltrami flows whose relativistic generalization is currently under investigation.

This pattern may be compared with the distribution of P-odd correlation of particle momenta, so-called handedness [5] which manifests a similar separation phenomenon. The handedness averages to zero when averaged over the whole momentum space and shows mirror pattern in its separate octants.

We used the obtained values of helicity for estimates of $\Lambda$ hyperons polarization in heavy-ion collisions at NICA energy range due to the earlier suggested [3] mechanism. The resulting polarization is about 1% and may be studied experimentally. Of course, more detailed theoretical investigations are required.

In particular, the discovery of an extra $T^2$ term [16] raises again the question why polarization was not observed at RHIC. Here one may refer to the (exponential) dilution of polarization by temperature effects (similar to what happens at much larger scale in polarized targets) although this problem certainly requires further investigation.

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