Performance Analysis of Cumulative Sum Control Charts Based on Parameter Estimation

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Abstract. As an important means of quality monitoring, Cumulative Sum (CUSUM) control charts can quickly detect whether the production process is out of control. However, in the actual production process, the parameters required for the control chart—the overall mean and standard deviation are usually unknown, so it needs to be accurately estimated. This paper uses multiple estimators to estimate the mean and variance of the population by using samples in phase I. When the testing process in phase II is out of control, this paper analyzes the influence of each estimator combination on the cumulative sum control chart based on running length distribution of control charts. This paper comprehensively analyzes the average, standard deviation and percentile of the control chart running length in four different environments in order to find the parameter combination that optimizes the control chart performance. The research results show that the CUSUM control chart based on $\bar{X} - \hat{\delta}_{ pooled}$ and $WH - \hat{\delta}_{Hozo}$ estimators performs best in a non-polluted environment, while the control chart based on $\bar{X} - \hat{\delta}_{IQR}$ and $WH - \hat{\delta}_{IQR}$ estimators performs best in a polluted environment.

1. Introduction

The control chart is one of the important tools for statistical process control technology. The earliest control chart was proposed by Dr. Shewhart based on the 3σ principle in statistical theory. However, the Shewhart control chart cannot achieve an excellent monitoring effect for small and medium drifts in the production process. In 1954, Page[1] introduced the well-known Cumulative Sum (CUSUM) control chart based on the sequential probability ratio test, which can effectively monitor small and medium drifts. The CUSUM control chart has stronger monitoring capabilities for data that matches the assumed drift size. Therefore, when using the CUSUM control chart to monitor the actual production process, the parameter needs to be considered according to the actual situation in order to achieve better monitoring performance.

Traditional quality control charts are based on hypothetical process parameters, which are assumed to be known, but in many practical industrial situations, process parameters are unknown, so this assumption is invalid. Generally speaking, monitoring the production process includes two phases: Phase I and Phase II. Phase I checks the stability of the production process and estimates the actual process parameters using sample data under controlled conditions. Phase II uses the parameter estimators obtained in phase I to establish process control limits and then check whether the production status of the subsequent process has changed, in order to find the process out of control as soon as possible.
possible. Therefore, the accurate estimator in phase I has a significant impact on the performance of the control chart.

In order to study the performance of control charts based on parameter estimation, Jesen[2] studied the effect of parameter estimation on the performance of Phase I and Phase II control charts, Knoth[3] studied bilateral EWMA variance control charts based on multiple variance estimators, Schoonhoven[4] studied the effect of parameter estimation on Shewhart control charts, Nazir[5] used multiple robust position parameter estimates in the CUSUM control chart, and Zwetsloot et al.[6] proposed a new robust EWMA variance control chart for phase I for estimating parameters, Shahriari et al.[7] proposed a robust R control chart with parameter estimation, Nazir[8] et al. estimated the mean in the three control charts and compared the ability of the three control charts to detect mean drift, Francisco Aparisi[9] et al. studied the performance of S2 control charts based on variance estimation under controlled and uncontrolled conditions, Anan Tang[10] et al. proposed an adaptive EWMA median control chart based on parameter estimation.

2. CUSUM control chart

The Cumulative Sum chart can achieve excellent effect in monitoring small drifts. Assuming that the quality characteristic parameter Y follows a normal distribution, that is, \( Y \sim N(\mu, \sigma^2) \), take the cumulative sum of the upper and lower sides of the CUSUM control chart as \( C_n^+ \) and \( C_n^- \), then

\[
C_n^+ = \max \{0, C_{n-1}^+ + (Y_n - \mu) - K\} \\
C_n^- = \max \{0, C_{n-1}^- - (Y_n - \mu) - K\}
\]

For the initial values, \( C_0^+ = C_0^- = 0 \), K are allowable deviations, and H is a judgment value. Assume that the mean value of a certain quality characteristic in the production process drifts forward, when \( C_n^+ > H \), it can be judged that the process is out of control. Similarly, assuming that the mean value of a certain quality feature has a negative drift during the production process, when \( C_n^- > H \), it can be judged that the process is out of control.

\( ARL_0 \) is the average run length from the start to the first false alarm when the process is in control, and when the process is out of control, \( ARL_1 \) is the run length from the state change to when a drift is detected. Therefore, when designing a control chart, make sure that the larger \( ARL_0 \) when the process is controlled, the smaller \( ARL_1 \) when the process is out of control, and the higher the performance of the control chart. Define the variable \( K = k\sigma \), \( H = h\sigma \). When designing the control chart, the values of k and h should be considered. Usually, the values of k and h should be set as the constant when the predetermined value of \( ARL_0 \) is satisfied.

3. Parameter estimation

In actual production, the data in phase I may contain abnormal observations, which will affect the estimation of parameters, resulting in the weakening ability of the control chart to detect process characteristic changes in phase II. Therefore, the key issue is how to obtain a robust estimate so as not to affect phase II. In the CUSUM control chart, the process mean \( \mu \) and standard deviation \( \sigma \) are unknown and it needs to be estimated by controlled historical data in phase I. In phase I, m groups of samples with a capacity of n are drawn, expressed by \( X_{ij} \), where \( i = 1,2, ..., m; j = 1,2, ..., n \). Take a set of samples \( X_{i1}, X_{i2}, ..., X_{im}, \ell = 1,2, ..., m \) and sort the n observations in ascending order to get an ordered order statistic \( X_{i(1)}, X_{i(2)}, ..., X_{i(n)} \). The following estimators are used to estimate the population mean:

(1) Sample mean estimator

The process mean is estimated by the following estimator

\[
\bar{X} = \frac{\sum_{i=1}^{m} X_i}{m}
\]

(1)

Where \( \bar{X}_i \) is the sample mean of the i-th sample, defined as \( \bar{X}_i = \frac{\sum_{j=1}^{n} X_{ij}}{n} \).

The research of Rousseeuw[11] shows that the estimated value is sensitive to outliers in the sample, and if there is only one inconsistent observation value, it can change its value.
(2) Median estimator
When the sample size \( n \) is odd, the median of the \( i \)-th group is expressed as \( \tilde{X}_i = X_{\frac{i(n+1)}{2}} \).

When the sample size \( n \) is even, the median of the \( i \)-th group is expressed as \( \tilde{X}_i = \frac{1}{2}(X_{\frac{i(n)}{2}} + X_{\frac{i(n)+1}{2}}) \).

Then define the estimator \( \bar{X} \) of the population mean as the average of the median of the \( m \) groups of samples, that is,
\[
\bar{X} = \frac{1}{m} \sum_{i=1}^{m} \tilde{X}_i
\]
(2)

The study of Dixon [12] shows that with the increase of sample size, the efficiency of the median of the sample is reduced relative to the mean of the sample.

(3) Midrange estimator
The midrange of the \( i \)-th sample is shown as \( MR_i = \frac{1}{2}(X_{i(1)} + X_{i(n)}) \).

Where \( X_{i(1)} \) is the smallest order statistic in the sample, and corresponding \( X_{i(n)} \) is the largest order statistic in the sample. The estimate \( \bar{MR} \) of the population mean is the average of the mid-range estimates of the \( m \) sample groups, that is,
\[
\bar{MR} = \frac{1}{m} \sum_{i=1}^{m} MR_i
\]
(3)

(4) Walsh estimator
A group of samples \( X_{11}, X_{12}, \ldots, X_{1m}, l = 1, 2, \ldots, m \) comes from the same normal distribution, and the average of any two numbers is calculated. The median of the new data is used as the Walsh average.

In this paper, the mean of the Walsh values \( WH \) of \( m \) samples is taken as the overall estimator, which is defined as
\[
WH = \frac{1}{m} \sum_{i=1}^{m} WH_i
\]
(4)

Where \( WH_i \) is the Walsh average of the \( i \)-th group?

The estimators for estimating the population standard deviation \( \sigma \) from samples are as follows:

(1) Sample average standard deviation \( \bar{S} = \frac{1}{m} \sum_{i=1}^{m} S_i \).

Where \( S_i \) is the unbiased standard deviation of sample \( i \), which is defined as \( S_i = \sqrt{\frac{1}{n-1} \sum_{j=1}^{n} (X_{ij} - \bar{X}_i)^2} \).

Where \( \bar{X}_i \) is the mean of the \( i \)-th sample, then the unbiased estimator of the population is expressed as
\[
\hat{\sigma}_{\text{mean}} = \frac{\bar{S}}{c_4(n)}
\]
(5)

\( c_4(n) \) is the correction factor, which is defined as \( c_4(n) = \sqrt{\frac{2}{n-1} \Gamma(n/2)} \).

(2) Sample pooled standard deviation
\[
S_{\text{pooled}} = \sqrt{\frac{1}{m} \sum_{i=1}^{m} S_i^2}
\]

Where \( S_i \) is the unbiased standard deviation of sample \( i \), then the overall estimator \( \hat{\sigma}_{\text{pooled}} \) is as follows
\[
\hat{\sigma}_{\text{pooled}} = \frac{S_{\text{pooled}}}{c_4(m(n-1)+1)}
\]
(6)

Research by Mahmond[13] show that this estimator is the most effective estimator under normal distribution.

(3) Hozo standard deviation estimation method
Hozo[14] et al. proposed a method for estimating the population standard deviation based on the five-number generalization method commonly used in statistics. When the sample size \( n \) is less than 15, the population variance is defined as \( S_i^2 = \frac{n+1}{4bn(n-1)^2} \left[ 4n^2(b - a)^2 + (n^2 + 3)(a - 2c + b)^2 \right] \).
Where a and b are the minimum and maximum values of sample i, and c is the median of the sample. In this paper, sample size m is considered, and the estimator $\hat{\sigma}_{Hozo}$ of the overall standard deviation is expressed as

$$\hat{\sigma}_{Hozo} = \frac{1}{m} \sum_{i=1}^{m} \sqrt{s_i^2}$$  \hspace{1cm} (7)

(4) Interquartile range estimation

The interquartile range estimator defined by Douglas [15] is as $\hat{\sigma}_{IQR} = \frac{1}{m} \sum_{i=1}^{m} IQR_i$.

In the formula of $IQR_i = X_{i(b)} - X_{i(a)}$, $X_{i(a)}$ represents the a-th order statistics of sample i in ascending order, $a = \lceil n/4 \rceil + 1$, $b = n - a + 1$. When n is 5, the unbiased estimator of the population standard deviation $\hat{\sigma}_{IQR}$ is defined as

$$\hat{\sigma}_{IQR} = \frac{\overline{IQR}}{0.99}$$  \hspace{1cm} (8)

4. Performance of parameter estimation CUSUM control chart

In actual production, the sample data in phase I comes from a polluted environment, so it is not appropriate to consider only whether the samples follow the normal distribution. Therefore, this paper considers four environments to analyze the performance of the estimator in multiple environments.

Normal distribution environment: all the samples in stage I are derived from the normal distribution with mean $\mu$ and variance $\sigma^2$. Take $\mu = 0$, $\sigma^2 = 1$ in this paper.

Scattered symmetrical variance polluted environment: each observation in the sample comes from the standard normal distribution $N(0,1)$ at a 95% probability, and each observation in the sample comes from the normal distribution $N(0,4)$ at a 5% probability.

Dispersed asymmetric variance polluted environment: each observation in the sample comes from the standard normal distribution $N(0,1)$ at a 95% probability, and it comes from the standard normal distribution and 1.5 times the chi-square distribution with a freedom of 1 at a 5% probability, that is, $N(0,1) + 1.5 \chi_1^2$.

Local variance infected environment: the entire sample group comes from the standard normal distribution $N(0,1)$ at a 95% probability, and comes from the normal distribution $N(0,4)$ at 5%.

This section builds the CUSUM control chart in phase II based on the estimates of the mean and standard deviation proposed by formulas (1) to (8) in the previous section, and analyzes their performance in different environments. To design the CUSUM control chart, it needs to determine the parameters k and h. The previous research shows that the CUSUM control chart has the best detection performance under the condition of $k = 0.5$, however, h is difficult to calculate mathematically, so it is generally calculated by Monte Carlo method. Let the sample size $m = 50$ and the sample size $n = 5$. Assume that the data in phase I comes from a normal distribution environment. The $\mu$ and $\sigma$ of the population are estimated by different estimators to ensure that the running length of the CUSUM control chart is 370 when the process is in control. Then the value of h was obtained by Monte Carlo simulation 100,000 times. The specific results are shown in Table 1.

**Table 1.** h values under different combinations of mean and standard deviation

| $\hat{\mu}$ | $\hat{\sigma}_{mean}$ | $\hat{\sigma}_{pooled}$ | $\hat{\sigma}_{Hozo}$ | $\hat{\sigma}_{IQR}$ |
|---|---|---|---|---|
| $\bar{X}$ | 4.765 | 4.776 | 5.329 | 4.598 |
| $\bar{X}$ | 4.79 | 4.802 | 5.38 | 4.63 |
| $MR$ | 4.782 | 4.798 | 5.345 | 4.612 |
| $WH$ | 4.758 | 4.768 | 5.318 | 4.588 |
This paper adds offset to the mean in phase II to shift the process mean from \( \mu \) to \( \mu + \delta \). \( \delta = 0.25, 0.5, 0.75, 1 \). At \( \delta = 0 \), the process is in control. When the process shifts, the performance of the control chart is evaluated by its ability to detect deviations under different parameter combinations. Because the ARL value of a control chart with parameter estimates does not follow a geometric distribution, this paper uses the running length standard deviation (SDRL) and the size of the 10th, 50th, and 90th percentiles to evaluate the performance of the control chart. See Table 2 for specific data in a normal distribution environment.

When the production process is in control, the smaller the SDRL, the smaller the fluctuation in the running length and the better the performance; when the 10th percentile of the run length is small, it means that 10% of the values are lower than this value, which means that the false alarm rate is high and the performance is poor in a controlled environment. According to the results in the table, in a normal distribution environment, when the process is in control, \( ARL_0 \) is about 370, and the SDRL and percentile of each estimator combination are not much different. The SDRL value of combination \( \bar{X} - \delta_{\text{pooled}} \) is 444.5, which is the smallest among all combinations, so its performance is the best. When using \( \bar{X} \) to estimate the mean, the value of SDRL is larger than other mean estimators and the CUSUM control chart performance is poor; when \( \delta_{\text{IQR}} \) is used to estimate the standard deviation, the SDRL value is larger than the SDRL value of other standard deviation estimators, and the percentile value is the smallest. The control chart performance is poor. When the process is out of control, a small value of \( ARL_1 \) indicates that the CUSUM control chart can quickly find that the process is out of control, and the performance of the control chart is good. In addition, the smaller the 10th percentile, the higher the alarm rate of the small running step, the higher the control chart performance. The \( WH - \delta_{\text{HoGo}} \) combination has the smallest \( ARL_1 \), SDRL, and the 10th percentile, so it has the best performance in detecting shift.

This paper uses the value of \( h \) determined in the non-polluted normal distribution environment as the control limit of the CUSUM control chart to detect the ARL, SDRL, and percentiles of the running length of the control chart in a polluted environment. The specific values are shown in Table 3-5. It can be seen from the table that the ARL value of control chart in the polluted environment is greatly improved, and the performance of the control chart is decreased. For example, in a non-polluted normal distribution environment, The \( ARL_0 \) value of the control chart based on the combination of \( \bar{X} - \delta_{\text{mean}} \) estimator is 370, but it becomes 1804.1 in scattered symmetrical variance polluted environment. When the process is in a controlled state, the ARL value of the CUSUM control chart based on \( \bar{X} - \delta_{\text{IQR}}, WH - \delta_{\text{IQR}}, \) and \( M\bar{R} - \delta_{\text{IQR}} \) is less affected, and the control chart performance is better than other combinations. When using \( \delta_{\text{pooled}} \) to estimate the standard deviation, the SDRL values of the control charts are large. In addition, when \( M\bar{R} \) estimator is used in dispersed asymmetric variance polluted environment, their control chart performance is poor, so \( M\bar{R} \) and \( \delta_{\text{pooled}} \) estimators are not recommended.
Table 2. ARL value of the parameter estimation control chart in normal distribution environment (SDRL; 10th, 50th, 90th)

| μ   | σ   | δ = 0 | δ = 0.25 | δ = 0.5 | δ = 0.75 | δ = 1 |
|-----|-----|-------|-----------|---------|----------|-------|
| 252 | 100 | 252.1 | 15.57     | 18.14   | 19.6     | 21.4  |
| 252 | 102 | 253.1 | 15.57     | 18.14   | 19.6     | 21.4  |
| 252 | 103 | 254.1 | 15.57     | 18.14   | 19.6     | 21.4  |
| 252 | 104 | 255.1 | 15.57     | 18.14   | 19.6     | 21.4  |
| 252 | 105 | 256.1 | 15.57     | 18.14   | 19.6     | 21.4  |
| 252 | 106 | 257.1 | 15.57     | 18.14   | 19.6     | 21.4  |
| 252 | 107 | 258.1 | 15.57     | 18.14   | 19.6     | 21.4  |

Table 3. ARL value of the parameter estimation control chart in scattered symmetrical variance polluted environment (SDRL; 10th, 50th, 90th)

| μ   | σ   | δ = 0 | δ = 0.25 | δ = 0.5 | δ = 0.75 | δ = 1 |
|-----|-----|-------|-----------|---------|----------|-------|
| 252 | 100 | 252.1 | 15.57     | 18.14   | 19.6     | 21.4  |
| 252 | 102 | 253.1 | 15.57     | 18.14   | 19.6     | 21.4  |
| 252 | 103 | 254.1 | 15.57     | 18.14   | 19.6     | 21.4  |
| 252 | 104 | 255.1 | 15.57     | 18.14   | 19.6     | 21.4  |
| 252 | 105 | 256.1 | 15.57     | 18.14   | 19.6     | 21.4  |
| 252 | 106 | 257.1 | 15.57     | 18.14   | 19.6     | 21.4  |
| 252 | 107 | 258.1 | 15.57     | 18.14   | 19.6     | 21.4  |
Table 4. ARL value of the parameter estimation control chart in dispersed asymmetric variance polluted environment (SDRL; 10th, 50th, 90th)

| \( \mu \) | \( \sigma \) | \( \delta = 0 \) | \( \delta = 0.25 \) | \( \delta = 0.5 \) | \( \delta = 0.75 \) | \( \delta = 1 \) |
|---|---|---|---|---|---|---|
| 552.1 | 770.6 | 1097.6 | 106.6 | 19.94 | 751.2 | (1065/62,405,1863) |
| 788.3 | (962/32,233,1325) | 222.1 | 53.3 | 17.9 | (1598/36,494,2911) |
| 550.3 | (1430/20,253,2106) | 103.2 | 28.4 | 14.1 | (1089/62,416,1846) |
| 425.6 | (965/33,227,1367) | 63.7 | 22.3 | 12.1 | (674/8.33,209,996) |
| 414.3 | (507/3.2,133,669) | 76.4 | 24.8 | 13.1 | (1267/58,454,2249) |
| 682.3 | (744/29,186,944) | 154.8 | 36.9 | 15.6 | (1171/2) |
| 682.3 | (123/6,12,42,164) | 229.7 | 45.1 | 16.8 | (1718/20,499,3232) |
| 436.6 | (123/6,4,18,210) | 214.4 | 45.1 | 16.8 | (1310/62,444,2317) |
| 232.2 | (132/9,12,1,155) | 53.5 | 20.3 | 11.1 | (746/9) |
| 733.2 | (404/19,115,530) | 196.6 | 51.8 | 16.8 | (761/36,229,1163) |
| 971.3 | (557/9,16,64,374) | 349.2 | 95.5 | 27.8 | (721/48,303,1269) |
| 753.2 | (1689/14,29,2788) | 214.4 | 45.1 | 16.8 | (1388/53,446,2341) |
| 214.4 | (651/2,62,402) | 170.1 | 22.3,78 | 19.4 | (766/4,297,1321) |
| 29.3 | (1241/2,6,19,738) | 29.3 | 12.1 | 3.6 | (351/4) |
| 99.8 | (621/2,17,830) | 237.7 | 45.8 | 15.9 | (585/6,26,16,818) |
| 81.6 | (237/7,11,66,212) | 81.6 | 25.2 | 13.1 | (449/9) |
| 170.1 | (142/9,12,15,70) | 170.1 | 25.2 | 13.1 | (1187/60,433,2052) |
| 359. | (803/3,19,104) | 170.1 | 25.2 | 13.1 | (1138/1) |
| 170.1 | (142/9,12,15,70) | 170.1 | 25.2 | 13.1 | (1667/25,493,3154) |
| 83.6 | (511/4,14,57,303) | 90.8 | 25.6 | 13.2 | (956/9) |
| 24.2 | (136/6,13,47,179) | 23.6 | 19.50 | 8.3 | (1355/32,271,499) |
| 95.8 | (485/3,220,1100) | 95.8 | 20.2 | 11.6 | (722/34,221,1096) |

Table 5. ARL value of the parameter estimation control chart in local variance infected environment (SDRL; 10th, 50th, 90th)

| \( \mu \) | \( \sigma \) | \( \delta = 0 \) | \( \delta = 0.25 \) | \( \delta = 0.5 \) | \( \delta = 0.75 \) | \( \delta = 1 \) |
|---|---|---|---|---|---|---|
| 322.4 | 551.2 | 94.8 | 28.1 | 13.7 | (1820/36,597,3654) |
| 551.2 | 209.3 | 12,45,197 | 78.5 | 19,52 | (1053/27,200,1317) |
| 1058.5 | 391.8 | 78.4 | 23.2 | (2241/0,303,4863) |
| 1293.3 | 1022/14,81,864 | 276.9 | 21,128 | (1849/0,265,3289) |
| 538.7 | 87.8 | 25.6 | 13.6 | (1293/3) |
| 543.2 | 104.8 | 27.2 | 13.2 | (1146/4) |
| 1278.2 | (323/5,11,42,201) | 42.7 | 14,853 | (1736/22,173,132) |
| 706.7 | 114.4 | 28.8 | 13.8 | (1278/2) |
| 607.6 | (351/3,12,45,217) | 43.3 | 18,955 | (1795/41,570,3498) |
| 1026.3 | 408.6 | 87.3 | 25.1 | (1357/6) |
| 1026.3 | (1055/14,80,931) | 359.1 | 19,27,126 | (1820/6,238,3257) |
| 563.8 | 106.8 | 26.9 | 13.8 | (1263/4) |
| 1156 | 30.4 | 13.7 | (1138/9) |
| 584.6 | 115.6 | 30.4 | 13.7 | (1757/2,434,2328) |
| 1082 | 108.2 | 13.7 | (1285/6) |
| 1337 | 13.7 | (1778/39,587,3554) |
| 1023.6 | 387.5 | 83.6 | 23.1 | (1354/2) |
| 1023.6 | (359/19,27,126) | 359.1 | 19,27,126 | (1263/307,4566) |
| 551.3 | 95.3 | 26.4 | 13.5 | (1793/44,562,3453) |
| 1137 | 27.5 | 13.5 | (1130/3) |
| 564.9 | 98.5 | 27.5 | 13.5 | (1726/26,430,3274) |
| 213/2,12,46,208 | 33.6 | 18,54 | (1147/2,171,1379) |
| 1314.8 | 98.5 | 27.5 | 13.5 | (1726/26,430,3274) |
| 1064 | 400.9 | 82.9 | 24.2 | (1314.8) |
| 1064 | 400.9 | 82.9 | 24.2 | (1314.8) |
| 517.6 | 109.1 | 28.3 | 13.4 | (1144/2) |
| 517.6 | (327/9,11,42,203) | 58.1 | 17,58,5 | (1763/23,431,3314) |

\( \delta = 0 \) for 10th, 50th, 90th
5. Conclusion
The accuracy of the estimator has a large impact on the performance of the control chart. The research results show that each estimator combination has an excellent performance under the non-polluted normal distribution environment, and the control chart based on the $\bar{X} - \tilde{\sigma}_{\text{pooled}}$ and $\overline{WH} - \delta_{\text{Hozo}}$ estimators has the best performance. In the polluted environment, the performance of the control charts using parameter estimation is greatly reduced, and $\bar{X} - \tilde{\sigma}_{\text{IQR}}$ and $\overline{WH} - \delta_{\text{IQR}}$ are the least affected, which can be used as the best estimator combination in the polluted environment. In actual production, the sample data may hardly obey the normal distribution, so it is important to choose the parameter combination. The size of the samples and number of samples in this paper are relatively in small size. Increasing the size of the samples and number of samples can effectively increase the accuracy of the estimate. The author can further study the influence of different estimators, sample sizes and capacities on the performance of control charts in the future.

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