A super-resolution face enhancement algorithm based on eigen transformation

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Abstract: A tensor feature transformation and enhancement algorithm are proposed in this paper based on PCA algorithm to address the challenges in face detection from low-resolution images. A 2D-PCA face super-resolution enhancement algorithm of a second-order tensor replaces the traditional 1D-PCA method for feature transformation to improve the accuracy of feature tensor solution and reduce the computational complexity. Furthermore, a coupling PCA method is applied for resolution enhancement of the combination of global information and local information of face image. Experimental results verify the stable and superior performance of the proposed algorithm.

1. Introduction
Low-resolution face recognition refers to the recognition of low-resolution or low-quality face images. With the wide use of remote surveillance cameras in recent years, low-resolution face recognition has gradually attracted attention and research. Among all the methods, the super-resolution method is the most widely used and most concerned.

The research of super-resolution has its practical significance. Image super-resolution (SR) refers to the process of fusing low resolution (LR)\textsuperscript{[1]} images or image sequences to obtain an ideal higher resolution image (HR).

In the process of long-distance face detection, a high-resolution face image can be obtained, and its performance directly affects the effect of face image recognition. At present, there is limited research on long-distance low-resolution face detection. For a long time, super-resolution image reconstruction (SR) has been used as a preprocessing step to obtain a high-resolution face. Then the high-resolution image is input into the face recognition system for recognition. In this paper, we mainly focus on the low-resolution image in the process of long-distance face detection, using the resolution enhancement algorithm based on feature transformation.

2. Feature conversion resolution enhancement algorithm

2.1 PCA feature conversion
Wang et al.\textsuperscript{[2]} proposed a face super-resolution enhancement algorithm based on PCA feature transformation in 2005. The algorithm makes full use of the powerful feature expression ability of PCA to find the correlation between high and low-resolution face samples and realize the super-resolution enhancement of the face. The algorithm starts with the observation model of image
super resolution SR, projects the observation model to the feature face space, converts the super-resolution reconstruction from the spatial domain to the feature face space, and finally reconstructs the projection coefficient high-resolution image.

Let X be the high-resolution image to be converted with a size of \( m \times n \). \( Y_i \) is the \( i \)th known low-resolution observation image with a size of \( s \times s \) \( (1 \leq i \leq k, s \) is a positive integer). Convert X and \( Y_i \) to \( mn \times 1 \) and \( \frac{mn}{s^2} \times 1 \) by column, according to the observed image imaging model, it results in:

\[
Y_i = H_iX + N_i, \quad 1 \leq i \leq k
\]  

There, \( H_i \) is the point spread function and the size is \( \frac{mn}{s^2} \times mn \), \( N_i \) is \( \frac{mn}{s^2} \times 1 \) noise vector.

When using PCA to reduce the dimension, converting the two-dimensional image matrix into a one-dimensional vector is necessary. Because the dimension of the original image matrix is usually very high, the sample dimension obtained after matrix column quantization is very high, which significantly improves the difficulty of covariance calculation. Moreover, the process of matrix vectorization is considered to destroy the internal structure of the matrix [3]. Based on this, many improved PCA methods have emerged in recent years, among which the two-dimensional principal component analysis (2D-PCA) is the most representative[4].

2.2. Improved tensor based feature transformation resolution enhancement algorithm

The most significant difference between 2D-PCA and PCA is that its processing object is a two-dimensional matrix. 2D-PCA does not need to convert the image matrix into a vector but directly uses the original image matrix for projection. As a result, the covariance matrix calculated by 2DPCA is more direct than that calculated by PCA, with fewer data and less time-consuming feature extraction.

Let X represent a \( w \)-dimensional normalized column vector, and image L is regarded as a random matrix of \( h \times W \). The basic idea of 2D-PCA is to project matrix L onto vector X through \( y = LX \) linear transformation to obtain an \( h \)-dimensional normalized column vector \( y \), which is called the feature vector of image L. X is called the projection vector. The 2D-PCA feature extraction is to obtain the projection vector X.

Let \( L_1, L_2, ..., L_n \) be the \( n \) amplitude of centralized training, and the size is \( m \times n \) low-resolution face image. Where \( L_K = [(L_K^{(1)})^T(L_K^{(2)})^T... (L_K^{(n)})^T]^T \) \( (k = 1,2, ..., n) \), \( L_K^{(i)} \) represents the \( i \)th row vector of matrix \( L_k \), \( L \) is the mean image of \( n \) images, then \( \bar{L} \) can be expressed as \( \bar{L} = [(L^{(1)})^T(L^{(2)})^T... (L^{(m)})^T]^T \).

Where \( \bar{L}^{(i)} \) represents the \( i \)th row vector of the matrix \( \bar{L} \). Then the covariance \( C \) of the image matrix is:

\[
C = \frac{1}{N} \sum_{i=1}^{n} (L_k^{(i)} - \bar{L}^{(i)})(L_k^{(i)} - \bar{L}^{(i)})^T \quad (2)
\]

It can be seen from (2) that \( C \) is a positive definite square matrix, and its dimension is much lower than the dimension of covariance obtained by PCA.

In order to determine the optimal projection vector X, the overall divergence of the post projection training sample can be used to measure the discrimination ability of the projection vector X, and the overall divergence of the post projection training sample can be expressed by the trace \( tr(S_c) \) of the covariance matrix of the projection eigenvector. There is the following relationship between this trace and the image covariance matrix column C:

\[
tr(S_c) = X^T E[(L-EL)^T(L-EL)]X = X^TCX \quad (3)
\]

Therefore, the following criterion functions can be defined:

\[
J(X) = X^TCX \quad (4)
\]

Here, the standard orthogonal condition constraint of the projection vector is introduced to find the optimal projection vector group \( x_1, ..., x_p \) corresponding to the first \( p \) maximum eigenvalues according to Fig. (4), to maximize the class dispersion of the two characteristic quantities Y as shown below:

\[
\{X_1,...,X_p\} = \arg \max J(X) \quad (5)
\]

For a low-resolution face image \( G_L \), the optimal projection vector X obtained from the above
solution is used for projection to obtain the projection vector $Y_k$, as shown below:

$$Y_k = G_k X_k, \quad k = 1, 2, \ldots, p$$  \hspace{1cm} (6)

Similarly, $N$ face training sample images are projected using the optimal projection vector to obtain the projection vector corresponding to each image, as shown below:

$$Y_i = L_i X, \quad i = 1, 2, \ldots, N$$

The basic principle of feature transformation is to find the corresponding relationship between high-resolution and low-resolution samples. The combined weight distribution is obtained for a low-resolution face image in the low-resolution face training samples through the “feature face” expression. Using this weight distribution, the high-resolution face training samples are weighted and combined to enhance the face resolution. According to equation (6), an input low-resolution face image $G_{1L}$ can be expressed as follows:

$$G_{1L} = Y_{1L} X^T \Rightarrow X^T = Y_{1L}^T G_{1L}$$  \hspace{1cm} (8)

From equations (7) and (8):

$$Y_{1L}^T G_{1L} = \frac{1}{N} \sum_i Y_i^T L_i \Rightarrow G_{1L} = \frac{1}{N} \sum_i Y_{1L} Y_i^T L_i = \bar{R} L$$

Where $\bar{R}$ is the reconstructed weight coefficient matrix, i.e. $\bar{R} = Y_{1L} Y^T = \begin{bmatrix} r_1, r_2, \ldots, r_N \end{bmatrix}$. As seen from equation (9), an input low-resolution face image can be represented by the weighted sum of low-resolution face training samples based on the reconstructed weight system matrix. Using the high-resolution face training sample $H_i$ instead of the corresponding low-resolution face training sample $A_i$ in equation (9), we can get:

$$G_{H}^{ET} = \frac{1}{N} \sum_i Y_{1L} Y_i^T H_i = \bar{R} H$$

Where $G_{H}^{ET}$ is the approximate image of the desired high-resolution face. However, there are still some differences between the down-sampled image of the approximate face image and the input low-resolution face image. In a sense, the face enhancement image is only the global information expression of the original high-resolution face image. It is necessary to further enhance the quality of the global face image by using the residual information of the low-resolution face input image through residual compensation technology. Here, we use the coupled PCA statistical model [5] proposed by Liu et al. to learn the relationship between high and low-resolution residual spaces. The objective function of the coupled PCA is as follows:

$$\arg \min \left\{ A_l, A_h \right\} \sum_i \left\| H_i - A_h A_l^T L_i \right\|$$

Here, the high and low-resolution residual spaces $L_i \in \mathbb{R}^m$ and $H_i \in \mathbb{R}^n$ are no longer 2D matrices but transformed into 1D vectors, i.e., $m = h \times w$ and $M = H \times W$. Therefore, $L_i \in \mathbb{R}^{m \times N}$ and $H_i \in \mathbb{R}^{M \times N}$. In addition, $A_l \in \mathbb{R}^{m \times d}$, $A_h \in \mathbb{R}^{M \times d}$ are the two orthogonal coupling mapping matrices [6], and $d$ is the dimension of the unified feature space.

The specific process of residual information compensation: (1) Smooth downsampling of the high-resolution face enhancement image $G_{H}$ to obtain the low-resolution face enhancement image; (2) The low-resolution residual information, $\Delta L$, is obtained based on the difference between $G_{H}$ and $G_{H}^{ET}$; (3) By coupling the mapping matrix columns $A_l$ and $A_h$, i.e. $H_i - A_h A_l^T L_i$, the high-resolution residual information $\Delta H$ is obtained; and (4) The learned high-resolution residual information is superimposed on the high-resolution face enhancement image $G_{H}^{ET}$ to obtain the final high-resolution face enhancement image, that is, $G_{H} = G_{H}^{ET} + \Delta H$.

2.3. Design of enhanced algorithm framework

Based on the PCA algorithm, this paper uses a 2D-PCA method of second-order tensor to replace the traditional 1D-PCA method for feature transformation. It proposes a tensor feature transformation enhancement algorithm to improve the accuracy of feature tensor solutions and reduce the
computational complexity. The feature transformation method obtains the “global” enhanced face with rich low-frequency information, while the high-frequency local information is missing. In order to solve this problem, this paper introduces the coupled PCA method to obtain the high-resolution local residual information between the “global” enhanced face and the low-resolution input samples. It superimposes the local residual information based on the “global” enhanced face to realize the resolution enhancement framework of the combination of global information and local information of face image, as shown in Figure 1.

Fig.1 Face resolution enhancement framework based on tensor eigen transformation.

3. Experimental analysis
In order to verify the feasibility and effectiveness of the algorithm proposed in this paper, the influence of low resolution on face resolution enhancement is mainly studied, and the factors such as illumination and posture are not considered temporarily. Therefore, good illumination conditions and frontal face samples are mainly selected for experiments. We implemented a comparative experiment on the ORL face database and used the tensor feature transformation model to enhance face resolution. The overall test framework is shown in Figure 1. Our goal is to enhance the face images with different resolutions to obtain the final high-resolution face image.

3.1. Face database
The experiment is carried out in the ORL face database. ORL face database contains 400 face images of 40 people. Among the 400 face images, we selected 160 face images without glasses for the recognition test, as shown in Figure 2. These images are first smoothly downsampled to three different resolutions: 1-10×9, 2-14×12, and 3-28×23, and then upsampled to 112×92 high resolution. One image is randomly selected for testing for each person, and the other images are used for training.

Figure 2 Some training samples selected from the ORL database

3.2. Experimental results
We use the traditional PCA, 2D-PCA and improved 2D-PCA feature extraction methods to perform recognition tests on the above three different resolution images, and the results are shown in Figure 3.
Table 1 The result of comparison of recognition performance

| Algorithm       | Resolution-1(H) | Resolution-2(M) | Resolution-3(L) |
|-----------------|-----------------|-----------------|-----------------|
| PCA             | 65.7            | 59.3            | 50.3            |
| Tradition 2D-PCA| 73.4            | 68.0            | 62.4            |
| Proposed method | 89.5            | 87.3            | 85.7            |

The experimental data show that the proposed method is better than the other two methods, especially in the lower resolution, and the performance remains relatively stable with the decline of resolution. Compared with the other two methods, the proposed method remains relatively stable and achieves better performance with the reduced resolution.

4. Conclusion
In this paper, a super-resolution enhancement algorithm of face image based on tensor feature transformation is proposed. Based on the PCA algorithm, a 2D-PCA method of a second-order tensor is used to replace the traditional 1D-PCA method for feature transformation. Furthermore, a tensor feature transformation enhancement algorithm is proposed. The coupled PCA method is introduced to enhance the resolution of the face image further. The experiments verify the feasibility and effectiveness of the proposed enhancement algorithm.

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