Zero-Modes, Covariant Anomaly Counterparts and Reducible Connections in Topological Yang-Mills Theory

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ABSTRACT

We introduce the covariant forms for the non-Abelian anomaly counterparts in topological Yang-Mills theory, which satisfies the topological descent equation modulo terms that vanish at the space of BRST fixed points. We use the covariant anomalies as a new set of observables, which can absorb both $\delta_w$ and $\delta_{\text{BSG}}$ ghost number violations of zero-modes. Then, we study some problems due to the zero-modes originated from the reducible connections.

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One of the most important conceptual revolutions of topological field theories (TFT’s) is the unbroken phase of the general covariance or that of string theory is realized. It has been shown that the cohomological TFT’s in two dimensions are equivalent to the physical ones[1][2]. Though it is tempting to believe that TFT’s have opened new horizon of unified theory of everything, almost nothing is known how to induce the broken phases in the realistic dimensions.

Topological Yang-Mills theory (TYM)[3], which is the first example of TFT’s in the cohomological nature, has another open problem that there is no known consistent mathematical or physical method to define theory in the presence of reducible connections[4][5]. Due to a BRST-like fermionic symmetry, the entire path integral of a TFT localized to the locus of the BRST fixed points (exact semi-classical limit), which is a certain moduli space in general[3]. In TYM, the fermionic symmetry is generated by the Witten’s $\delta_W$ operator, and the basic $\delta_W$ algebra is given by

$$\delta_W A = \Psi, \quad \delta_W \Psi = -dA \Phi, \quad \delta_W \Phi = 0.$$  

(1)

Using the $\delta_W$ symmetry one restrict the configuration space of the theory to the space of instanton, which leads to another $\delta_W$ algebra

$$\delta_W \chi = \mathcal{F}^+,$$

(2)

where the superscript $+$ denotes the projection to the self-dual parts of the curvature $\mathcal{F}$. The resulting $\delta_W$ fixed action has the usual gauge symmetry, which can be fixed following the conventional BRS gauge fixing method[3][7]. Then we should consider $\delta_{BRS}$ algebra

$$\delta_{BRS} A = -dA v, \quad \delta_{BRS} \bar{v} = \text{gauge fixing condition}$$  

(3)

where $v, \bar{v}$ is the Faddev-Popov ghost and the anti-ghost respectively.

Then, the space of the $\delta_W$ and $\delta_{BRS}$ fixed points $\delta_W \chi = 0, \delta_W \Psi = 0, \delta_{BRS} A = 0, \delta_{BRS} \bar{v} = 0$ is precisely the moduli space of instanton $\mathcal{M}$ with the space of $\Phi$ zero modes and that of $v$ zero-modes. Thus, if there are no $\Phi$ and $v$ zero-modes, the path integral exactly reduce to the moduli space of instanton. In this letter, we discuss the problem of the $v, \Phi$ zero-modes originated from the reducible connections.

Another important property of TYM is that the cohomological structures of the instanton moduli space $\mathcal{M}$ - which can be summarized by the instanton complex[8]

$$0 \to A^0(ad P) \xrightarrow{d_A} A^1(ad P) \xrightarrow{d_A} A^2_+(ad P) \to 0,$$

(4)
where \( \text{ad} P = P \times_{\text{ad}} g \) denotes the adjoint bundle associated with a principle G-bundle \( P \) over a compact oriented Riemann 4-manifold \( M \), \( A^p(F) \) denotes the space of \( F \)-valued \( p \)-forms and + denotes projection to self-dual part, are realized by the zero-modes of fields \(((\Phi, \bar{\Phi}, \eta), \Psi, \chi)\) with the \( \delta_W \) ghost number \((2, -2, -1), 1, -1\), whose zero-modes are the solutions to the following equations

\[
d_A \Phi = d_A \bar{\Phi} = d_A \eta = 0, \quad d^*_A \Psi = d^*_A \bar{\Psi} = 0, \quad d_A^{\perp} \chi = 0.
\] (5)

The number of non-trivial solutions for \(((\Phi, \bar{\Phi}, \eta), \Psi, \chi)\) precisely correspond to \((h^0, h^1, h^2)\), where \( h^i = \dim H^i \) denotes the dimension \((i\text{-th Betti number})\) of the cohomology groups of the instanton complex \((\mathbb{H})\). It follows that the net \( \delta_W \) ghost number violation of zero modes equals to the formal dimension of \( M \) given by the index \( s = h^1 - h^0 - h^2 \) of the instanton complex.

In particular, the 0th cohomology \( H^0 \) is non-trivial (\( \Phi \) zero-modes) for the reducible connection, and it is a source of singularities in the moduli space \( M \). And, the elements of 1st cohomology group (\( \Psi \) zero-modes) can be identified to the tangent vectors of \( M \). Thus, the formal dimension is the actual dimension when \( h^0 = h^2 = 0 \). Then the moduli space is a smooth manifold, and the Donaldson’s invariants are well-defined.

Thus, the non-zero dimension of the instanton moduli space implies the \( \delta_W \) ghost number anomaly and appropriate set of observables should be inserted to compensate it. Such set of observables has been introduced by Witten\(^2\) and interpreted geometrically based on the universal bundle\(^3\) by Kanno\(^10\). From an obvious candidate \( \tilde{W}_0^{0,4} = \frac{1}{2} \text{Tr} (\Phi^2) \), we can find the topological descent equation after some iterations

\[
0 = \delta_w \tilde{W}_0^{0,4}, \\
\delta w \tilde{W}_0^{0,4} = \tilde{W}_1^{0,3}, \\
\delta w \tilde{W}_1^{0,3} = \tilde{W}_2^{0,2}, \\
\delta w \tilde{W}_2^{0,2} = \tilde{W}_3^{0,1}, \\
\delta w \tilde{W}_3^{0,1} = \tilde{W}_4^{0,0}, \\
\delta w \tilde{W}_4^{0,0} = 0,
\] (6)

where \( \tilde{W}_k^{0,4-k} \) denote \( k \) form with \( \delta_{\text{gss}} \) and \( \delta_w \) ghost number 0 and \( 4-k \) respectively. Integrating \( i \)-th relation over a \( i-1 \) dimensional cohomology cycle \( \gamma_{i-1} \), we can see that

\[
\delta_w \int_{\gamma_k} \tilde{W}_k^{0,4-k} \equiv \delta_w \tilde{W}^{0,4-k} = 0.
\] (7)
$\tilde{W}^{0.4-k}$ is $\delta_w$ closed. One can also easily see that the Witten’s observable is non-trivial and its $\delta_w$ cohomology class depends only on the homology class of $\gamma_k$. It is also well known that $\tilde{W}_k^{0.4-k}$ is a component of the characteristic class $\frac{1}{2} \text{Tr} \hat{F}^2 = \frac{1}{2} \text{Tr} (F + \Psi + \Phi)^2$. We can also start from a higher characteristic class $\tilde{W}_0^{0.2n} = c_n \text{Tr} \Phi^n$ and obtain $\tilde{W}_k^{0.2n-k}$ after repeating the iteration (6). And, after integrating over a $k_i$ dimensional homology cycle $\gamma_k$, we get $\tilde{W}^{0.2n-k}$, which is an element of $2n - k$ cohomology class on the orbit space $H^{2n-k}(U/G)$.

Note that the zero-modes of the Faddev-Popov ghost $v$, which are the non-trivial solutions of $d_A v = 0$, as well as those of $\Phi$ can be originated from the reducible connections. Because, in the Horne’s approach[6][7][11], both $\delta_w$ and $\delta_{\text{BRS}}$ ghost numbers should be preserved separately and the completely fixed action of TYM has both $\delta_w$ and $\delta_{\text{BRS}}$ ghost number zero, we should also insert appropriate set of observables to absorb the net violation of the $\delta_{\text{BRS}}$ ghost number. Such an observable should be $\delta_w$ as well as $\delta_{\text{BRS}}$ closed and non-trivial in the sense of global topology. If there is no such an observable, not only the topological interpretation become impossible but also the correlation function itself can not be well defined.

A way out of this problem was proposed by the author using the non-Abelian (consistent) anomaly counterparts in TYM, which can be obtained from one higher rank characteristic class $\text{Tr} \hat{F}^3$ using the extended descent equation[11]. If one insert the consistent anomaly, which is only $\delta_{\text{BRS}}$ closed but has $\delta_{\text{BRS}}$ ghost number 1, in addition to the Witten observables to absorb the $v$ zero modes, one can easily see that their contributions factorized from the correlation function at the locus of $\delta_w$ and $\delta_{\text{BRS}}$ fixed points. Thus, the original topological interpretation can be maintained. However, the use of consistent anomaly seems to be restricted to the zero modes of $v$ due to the Gribov ambiguity[12], and they have many other unpleasant aspects[11]. This motivate us to investigate the covariant forms for the consistent anomalies.

From the density of the covariant anomalies in Yang-Mills theory[13], one can readily obtain the density of covariant anomaly counterpart in TYM as

\[(n + 1)c_{n+1} \text{Tr} (v F^n) \rightarrow (n + 1)c_{n+1} \text{Tr} (v \hat{F}^n) \equiv \tilde{W}_{2n}^{-1}, \quad (8)\]
where the superscript denote $\delta_{\text{BRS}}$ ghost number. For $n = 2$, the covariant anomaly counterpart $\mathcal{W}_4^1$ which can be written in the components
\begin{align*}
\mathcal{W}_4^{1,0} &= \frac{1}{2} \text{Tr} \left( v \mathcal{F}^2 \right), \\
\mathcal{W}_3^{1,1} &= \frac{1}{2} \text{Tr} \left( v(\mathcal{F} \bar{\Psi} + \Psi \mathcal{F}) \right), \\
\mathcal{W}_2^{1,2} &= \frac{1}{2} \text{Tr} \left( v(\mathcal{F} \Phi + \Phi \mathcal{F} + \Psi^2) \right), \\
\mathcal{W}_1^{1,3} &= \frac{1}{2} \text{Tr} \left( v(\Psi \Phi + \Phi \Psi) \right), \\
\mathcal{W}_0^{1,4} &= \frac{1}{2} \text{Tr} \left( v \Phi^2 \right).
\end{align*}
(9)

Then a covariant anomaly is given by
\begin{equation}
W^{1,4-k} \equiv \int_{\gamma_k} \mathcal{W}_k^{1,4-k},
\end{equation}
where $\gamma_k$ is a $k$ dimensional homology cycle of $M$.

Note that a covariant anomaly, which is gauge invariant, is not closed under the action of $\delta_{\text{BRS}}$ operator ($\delta_{\text{BRS}} v = -v^2$)
\begin{equation}
\delta_{\text{BRS}} W^{1,2n-k}(v, \cdot) = -W^{2,2n-k}(v^2, \cdot).
\end{equation}
(11)

The above transformation rule motivate us to redefine the $\delta_{\text{BRS}}$ algebra slightly such that
\begin{align*}
\tilde{\delta}_{\text{BRS}} A &= -dv - \{A, v\}, \\
\tilde{\delta}_{\text{BRS}} v &= 0, \\
\tilde{\delta}_{\text{BRS}} \bar{v} &= \pi, \\
\tilde{\delta}_{\text{BRS}} \pi &= 0, \\
\tilde{\delta}_{\text{BRS}} \hat{\mathcal{F}} &= -[v, \hat{\mathcal{F}}].
\end{align*}
(12)

Note that the replacement $\delta_{\text{BRS}} \rightarrow \tilde{\delta}_{\text{BRS}}$ does not change the fixed point of BRS symmetry $d_A v = 0$ and $\tilde{\delta}_{\text{BRS}}$ is nilpotent at the fixed point $\tilde{\delta}_{\text{BRS}}^2 A = d_A v^2$. The $\tilde{\delta}_{\text{BRS}}$ operator can be regarded a covariant derivative for $\delta_{\text{BRS}}$ acting only on the basic $\delta_{\text{BRS}}$ triplet $(v, \bar{v}, \pi)$. That is, if we introduce the anti-ghost $\bar{v}$ and auxiliary field $\pi$ and use a particular $\delta_{\text{BRS}}$ algebra as
\begin{align*}
\delta_{\text{BRS}} v &= -v^2, \quad \delta_{\text{BRS}} \bar{v} = \pi - [v, \bar{v}], \quad \delta_{\text{BRS}} \pi = -[v, \pi],
\end{align*}
(13)
we can read off corresponding $\tilde{\delta}_{\text{BRS}}$ algebra as $[12]$. Thus, deformation of $\delta_{\text{BRS}}$ to $\tilde{\delta}_{\text{BRS}}$ change essentially nothing. Now the covariant anomaly is $\tilde{\delta}_{\text{BRS}}$-closed.
Though $W^{1,4-k}$ are not $\delta_w$ closed in general, one can easily see that $W^{1,4} = 3c_3 \text{Tr} (v \Phi^2)$ is $\delta_w$ closed and non-trivial ($\delta_w v = 0$). The situation is quite similar to the Witten’s basic observable $\tilde{W}^{0,4} = c_2 \text{Tr} \Phi^2$ which is gauge invariant, $\tilde{\delta}_{\text{BRS}}$ and $\delta_w$ closed and non-trivial. This motivate us to find an analogue of the topological descent equation (6). One can easily find that

$$d \text{Tr} (v \Phi^2) = -\delta_w \text{Tr} (v(\Psi \Phi + \Phi \Psi)) + \text{Tr} (d_A v \Phi^2).$$

(14)

Repeating the same procedure as (6), one can derive a descent equation

$$0 = -\delta_w W^{1,4},$$

$$dW_0^{1,4} = -\delta_w W_1^{1,3} + \text{Tr} (d_A v \Phi^2),$$

$$dW_1^{1,3} = -\delta_w W_2^{1,2} + \text{Tr} (d_A v (\Psi \Phi + \Phi \Psi)), $$

$$dW_2^{1,2} = -\delta_w W_3^{1,1} + \text{Tr} (d_A v (F \Phi + \Phi F + \Psi^2)), $$

$$dW_3^{1,1} = -\delta_w W_4^{1,0} + \text{Tr} (d_A v (F \Psi + \Psi F)), $$

$$dW_4^{1,0} = \text{Tr} (d_A v F^2).$$

(15)

The above descent equation contains an extra term unlike the topological descent equation (6), which vanishes at the $\tilde{\delta}_{\text{BRS}}$ fixed point $\delta_{\text{BRS}} \mathcal{A} = \tilde{\delta}_{\text{BRS}} \mathcal{A} = 0$. After integrating the $i$th relation above over a $i-1$ dimensional cycle $\gamma$, we can see that $W^{1,4-k}$ is $\delta_w$ closed at the $\delta_{\text{BRS}}$ fixed point. If we integrate the extra term by part, we can see that $W^{1,4-k}$ is $\delta_w$ closed at the $\delta_{\text{BRS}}$ fixed points $\delta_w \Psi = \delta_w \chi = 0$. Then, we can conclude that a covariant anomaly $W^{1,4-k}$ is $\delta_w$ closed if we restrict the configuration space the locus of $\delta_w$ and $\delta_{\text{BRS}}$ fixed points. One can also easily find that $W^{1,4-k}$ is non-trivial and its $\delta_w$ cohomology class depends only on the homology class of $\gamma_k$ as the Witten’s observables. Thus, the covariant anomaly can be used as an topological observable as long as the reduction of the theory to the locus of BRST fixed points is exact. One can also start from $W^{1,2n}_0 = (n+1)c_{n+1} \text{Tr} (v \Phi^n)$ and obtain $W^{1,2n-k}_k$, which satisfy the same kind of descent equation (15). Clearly, an arbitrary $W^{1,2n-k}$ is $\delta_w$ closed at the locus of the BRST fixed points.
Let $\Delta U$ and $\Delta u$ denote the net violation of $\delta_W$ and $\delta_\nu$ ghost number, respectively, due to the zero-modes. Then the correlation function

$$\left\langle \prod_{i=1}^{r} \tilde{W}^{0,4-k_i} \prod_{j=1}^{\Delta u} W^{1,4-\ell_j} \right\rangle$$

(17)

will be non-zero for

$$\Delta U = h^1 - h^0 = \sum_{i=1}^{r} (4 - k_i) + \sum_{j=1}^{\Delta u} (4 - \ell_j),$$

(18)

where we will always assume that $h^2 = 0$ and restrict $n = 2$ for simplicity. In the semi-classical limit the correlation function (17) will be reduced to the integration of the wedge products of certain closed differential form over the moduli space of the instanton with the space of $\Phi$ and $v$ zero-modes.

Now we will discuss possible branches; (A) There is no reducible connection- thus all possible zero-modes of $v$ is originated from the Gribov ambiguity and the formal dimension is the actual dimension. (B) There are reducible connections and all the zero-modes of $v$ is originated from them - thus the number of $v$ zero-modes equals that of $\Phi$ zero-modes.

In the branch A, a zero-mode of $v$ can not be defined globally. It will have different value in the different coordinate patch and it has no relation with the cohomology structures of the moduli space. Thus, inserting $W^{1,4-\ell_j}$ should not in principle affect the global topological meaning of the correlation function. It will be sufficient to use $\Delta u$ copies of the $W^{1,0}$ only to control the zero-modes of $v$. Then, one can immediately see that the correlation function (17) factorized

$$\left\langle \prod_{i=1}^{r} \tilde{W}^{0,4-k_i} \right\rangle_{\delta_W \chi = 0} \left\langle \prod_{j=1}^{\Delta u} W^{1,0} \right\rangle_{\delta_{BRS} A = 0},$$

(19)

where

$$\dim(\mathcal{M}) = \Delta U = \sum_{i=1}^{r} (4 - k_i),$$

and $\langle \cdots \rangle_{\delta_W \chi = 0}$ denotes the integration of the wedge product of $\delta_W$ cohomology classes over the moduli space of instanton $\mathcal{M}$. The cohomology classes can be obtained by replacing $(\mathcal{F}, \Psi, \Phi)$ in $\tilde{W}^{0,4-k}$ by their (instanton value, zero-modes $\psi_i$, $<\Phi>$), respectively, where $<\Phi>$ is given by

$$<\Phi> = -\int_M \frac{1}{d_A d_A^*} [\psi, * \psi].$$
Then $\tilde{W}^{0,4-k_i}$ reduces to a closed differential $4-k$ form on the moduli space

$$f_{4-k_i} = f_{j_1\ldots j_{4-k_i}} \psi^{j_1} \ldots \psi^{j_{4-k_i}}.$$  

The cohomology class $f_{4-k_i}$ is Poincaré dual to a codimension $4-k_i$ homology class in $\mathcal{M}$ and the correlation function reduce to intersection number of the homology classes, which is an invariant of smooth four-manifold[4].

The additional term $<\ldots>_{\delta W, A=0}$ denotes the integration of the wedge product of the local $\delta_{BRS}$ cohomology classes over the space of $v$ zero-modes, which may be related to the group cohomology - note that $\tilde{W}^{1,0}$ is a $\tilde{\delta}_{BRS}$ closed one form in the space of gauge group $\mathcal{G}$ and $\tilde{\delta}_{BRS}$ cohomology is equivalent to $\delta_{BRS}$ cohomology in the BRS fixed points. Thus, the original topological meaning of the correlation function is maintained in the branch A.

The theory in the branch B is far more subtle and we do not know the final answer. What we will try to do is just a formal analysis. Note that in the branch B both anti-commuting $v$ and commuting $\Phi$ zero-modes obey the same kind of equation $d_A\Phi = d_A v = 0$. Consequently, the numbers of the $v$ and $\Phi$ zero-modes are identical (equal to $h^0 = \Delta u$) and the spaces of the zero-modes are isomorphic. Thus, the $v$ and $\Phi$ zero-modes always appear in the pair and we can assign a certain supersymmetry between them. Another important point is that the products of the observables in the correlation function (17) obeying the selection rule (18) is not the top form in the moduli space $\mathcal{M}$, which actual dimension is given by $\Delta U + h^0$, unlike the branch A. Note that the inserted observables carry $\delta_{BRS}$ ghost number $h_0$. The above properties motivate us to examine a possibility to transform the $\delta_{BRS}$ ghost number into $\delta_W$ ghost number, such that we can obtain the desired top form in $\mathcal{M}$.

Note that $v$ and $\Phi$ have ($\delta_w, \tilde{\delta}_{BRS}$) ghost number (0,1) and (0,2) respectively. Then, we redefine a $\tilde{\delta}_{BRS}$ algebra

$$\tilde{\delta}_{BRS} v = -\Phi.$$  

Note that by the modified $\tilde{\delta}_{BRS}$ algebra the Witten’s observables are trivialized. One can easily prove in general

$$\tilde{\delta}_{BRS} W^{1,2n-k_i} = -\tilde{W}^{0,2n+2-k_i}.$$  

It does not, however, leads that the correlation function vanishes because the action $\tilde{\delta}_{BRS}$ on $v$ change the total ghost number by one while the both ghost number should be preserved independently[7].
If we interpret a $v$ zero-mode $\hat{v}_i$ as a one-form (a Grassman variable) on the space $V$ of $v$ zero-modes, which is a space of $h^0$ Grassman variables, and a $\phi$ zero-mode $\phi_i$ as a one-form on the space $W$ of $\Phi$ zero-modes, the action of the $\tilde{\delta}_{\text{BRS}}$ transformation (20) on $\hat{v}_i$ is an exotic supersymmetry

$$\tilde{\delta}_{\text{BRS}} \hat{v}_i = -\phi_i. \quad (22)$$

Furthermore, we interpret the action of $\tilde{\delta}_{\text{BRS}}$ on $\hat{v}_i$ as a Grassmann differential;

$$\tilde{\delta}_{\text{BRS}} \hat{v}_j \equiv \left\{ \frac{\partial}{\partial \hat{v}_i}, \hat{v}_j \right\} \phi_i. \quad (23)$$

Note that a Witten’s observable $\tilde{W}^{0,4-k_i}$ corresponds to a $4 - k_i$ form in $\mathcal{M}$ and a zero-form in $V$, and $\overline{W}^{1,4-\ell_j}$ corresponds to a $4 - \ell_j$ form in $\mathcal{M}$ and a one-form in $V$. Thus we use the convention that $f_{(\ell,k)j_1...j_{i_1}...i_k} \hat{v}_{j_1} \cdot \cdot \cdot \hat{v}_{i_k} \psi_{i_1} \cdot \cdot \cdot \psi_{i_k}$ denote a $(\ell, k)$ form on $V \times \mathcal{M}$.

Now the correlation function (17) reduce to the integral

$$\int_{\mathcal{M}} \int_{W} \int_{V} f_{0,4-k_i} \wedge \cdot \cdot \cdot \wedge f_{0,4-k_i} \wedge f_{1,4-\ell_j} \cdot \cdot \cdot \wedge f_{1,4-\ell_j} \triangle \psi. \quad (24)$$

Note that the integration over $V$ is a Berezin integral and the integrand is a top form on $V$. Then, we can perform the integral which is identical to the differentiation. From the relation (21), one can infer that the integral (24) reduce to

$$\int_{\mathcal{M}} \int_{W} f_{0,4-k_i} \wedge \cdot \cdot \cdot \wedge f_{0,4-k_i} \wedge f_{1,5-\ell_j} \cdot \cdot \cdot \wedge f_{1,5-\ell_j} \triangle \psi. \quad (25)$$

where the superscript 1 denotes that $f_{1,5-\ell_j}$ is a one-form in $W$. After integrating over $W$ eq. (25) reduces to

$$\int_{\mathcal{M}} f_{0,4-k_i} \wedge \cdot \cdot \cdot \wedge f_{0,4-k_i} \wedge \tilde{f}_{0,5-\ell_j} \cdot \cdot \cdot \wedge \tilde{f}_{0,5-\ell_j} \triangle \psi. \quad (26)$$

Now the integrand of the integral (26) is a top-form in $\mathcal{M}$, however, the problem is to determine whether it is a topological invariant. Because the space $W$ of $\Phi$ zero-modes is non-compact in a serious way, the convergence of the integrals (25)(26) can not be guaranteed, and because the moduli space $\mathcal{M}$ has singularities, the semi-classical approximation becomes doubtful. Note that we have inserted a new set of observables into the correlation function because of the singularities. Then, there can be two possibilities; (a) the new set of observables $\overline{W}^{1,2n-k_i}$ in general may acts as a regulator for the singularity such
that we can maintain the topological meaning of eq. (26), (b) the semi-classical exactness of the path integral is broken by the singularity such that the new set of observables is no longer $\delta W$ invariant, then eq. (26) is no longer topological. If the second possibility is the case, we can conclude that the singularity of $\mathcal{M}$ induce a topological symmetry breaking mechanism. However, I do not know the answer.

Finally, I will just mention that we can generalize the gauge fixed action $L$ of TYM to a perturbed one

$$L \rightarrow L + \sum_{\alpha} t_{\alpha} \bar{W}^{0,\alpha} + \sum_{\beta} s_{\beta} W^{1,\beta},$$

where $t_{\alpha}, s_{\beta}$ are some deformation parameter. Then the partition function for the perturbed action will reduce to the correlation function of an appropriate set of observables of the unperturbed theory.$^1$
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