Gravity effects in inclined air showers induced by cosmic neutrinos

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Abstract

The Randall-Sundrum model with a small curvature is considered in which five-dimensional Planck scale lies in the TeV region, and a spectrum of Kaluza-Klein gravitons reminds that in one flat extra dimension. The cross sections for interactions of ultra-high energy cosmic neutrinos with nucleons are calculated. It is shown that effects related with massive graviton excitations can be detected in deeply penetrating inclined air showers induced by these neutrinos. The expected number of air showers at the Auger Observatory is estimated as a function of two parameters of the model.

1 Warped extra dimension with the small curvature

One of the most important problem of the modern particle physics is a hierarchy problem, i.e. unnaturally large ratio of the gravity scale ($10^{19}$ GeV) to the electroweak scale ($10^2$ GeV). To solve this problem, theories with large extra dimensions have been proposed [1]. However, they could only explain a huge value of the Planck mass by introducing another large scale, namely, a size of extra flat dimensions. Thus, the hierarchy problem was not really solved, but reformulated in terms of this new scale.

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The model which does solve the problem most economically is the Randall-Sundrum (RS) model \cite{2} with a single extra dimension and warped background metric \cite{3}:

\[ ds^2 = e^{2\kappa r_c |y|} \eta_{\mu\nu} dx^\mu dx^\nu + dy^2. \]  

(1)

Here \( y = r_c \theta ( -\pi \leq \theta \leq \pi ) \), \( r_c \) being the “radius” of the extra dimension, while \( \{ x^\mu \}, \mu = 0, 1, 2, 3 \), are the coordinates in four-dimensional space-time. The parameter \( \kappa \) defines the scalar curvature in five dimensions. Note that the points \( (x^\mu, y) \) and \( (x^\mu, -y) \) are identified, and the periodicity condition, \( (x^\mu, y) = (x^\mu, y + 2\pi r_c) \), is imposed. The tensor \( \eta_{\mu\nu} \) is the Minkowski metric.

It is assumed that there are two 3-dimensional branes with equal and opposite tensions located at the point \( y = 0 \) (called the Plank brane) and point \( y = \pi r_c \) (referred to as the TeV brane). All SM fields are confined to the TeV brane, while the gravity propagates in five dimensions. The following relation between the 4-dimensional (reduced) Planck mass, \( \bar{M}_{Pl} \), and (reduced) gravity scale in five dimensions, \( \bar{M}_5 \), can be derived:

\[ \bar{M}_{Pl}^2 = \bar{M}_5^3 \kappa \left( e^{2\pi \kappa r_c} - 1 \right). \]  

(2)

The masses of the Kaluza-Klein (KK) graviton excitations are proportional to the curvature parameter \( \kappa \):

\[ m_n = x_n \kappa, \quad n = 1, 2, \ldots, \]  

(3)

where \( x_n \) are zeros of the Bessel function \( J_1(x) \). On the TeV brane, the zero graviton mode, \( h^{(0)}_{\mu\nu} \), and massive graviton modes, \( h^{(n)}_{\mu\nu} \), are coupled to the energy-momentum tensor of the matter, \( T^{\mu\nu} \), as follows:

\[ \mathcal{L}_{\text{int}} = -\frac{1}{\bar{M}_{Pl}} T^{\mu\nu} h^{(0)}_{\mu\nu} - \frac{1}{\Lambda_\pi} T^{\mu\nu} \sum_{n=1}^{\infty} h^{(n)}_{\mu\nu}, \]  

(4)

with

\[ \Lambda_\pi = \left( \frac{\bar{M}_5^3}{\kappa} \right)^{1/2} \]  

(5)

being a physical scale on this brane.

Let us note that the metric (1) differs from that presented in the original paper \cite{2} in which both \( \bar{M}_5 \) and \( \kappa \) have to be taken as large as the Planck
mass ($\bar{M}_5 \sim \kappa \sim \bar{M}_{Pl}$). Moreover, the size of the warped extra dimension should be extremely small ($r_c \simeq 60 l_{Pl}$). Thus, in order to explain the huge value of $\bar{M}_{Pl}$ in such a scheme, one has to introduce new mass scales of the same order, namely, $\bar{M}_5$, $\kappa$, and $r_c^{-1}$.

However, the hierarchy problem can be successfully solved in the RS scenario, but with the metric (1). The equation (2) allows us to consider the small curvature option of the RS model [3]-[5].

$$\kappa \ll \bar{M}_5 \sim 1 \text{ TeV}.$$  \hspace{1cm} (6)

In such a case, we get an almost continuous spectrum of low-mass graviton excitations with the small mass splitting $\Delta m \simeq \pi \kappa$. Note that in the standard scenario of the RS model [2] one has a series of KK graviton resonances with the lightest one having a mass around 1 TeV.

The RS model with the large extra dimension has been checked by the DELPHI Collaboration [9]. The gravity effects were searched for by studying photon energy spectrum in the process $e^+e^- \rightarrow \gamma + E \perp$. The limit on $\bar{M}_5$ obtained [9] is

$$\bar{M}_5 > 0.92 \text{ TeV}.$$ \hspace{1cm} (7)

The search for large extra dimensions in the diphoton channel using data collected by the CDF and DØ Collaborations at $\sqrt{s} = 1.96 \text{ TeV}$ are presented in Refs. [10]. The measured $p_\perp$-distributions are in a good agreement with the SM background, that allowed us to obtain the bound [8]:

$$\bar{M}_5 > 0.81 \text{ TeV}.$$ \hspace{1cm} (8)

The discovery limit of the LHC in the two-photon production (requiring a 5σ effect) has also been derived for two values of the integrated luminosity $\mathcal{L}$ [8]:

$$\bar{M}_5 = \begin{cases} 6.3 \text{ TeV}, & \mathcal{L} = 100 \text{ fb}^{-1} \\ 5.1 \text{ TeV}, & \mathcal{L} = 30 \text{ fb}^{-1} \end{cases}$$  \hspace{1cm} (9)

Previously, the gravity effects in the RS model with the small curvature were already looked for in a number of different processes (see Refs. [5]-[8]). In the present paper we will estimate an upper bound on $\bar{M}_5$ which can be reached by the Auger ground array in detecting quasi-horizontal air showers induced by ultra-high energy (UHE) cosmic neutrinos.

1For numerical estimates, the region $0.5 \text{ GeV} \leq \kappa \leq 1.5 \text{ GeV}$ will be used (see Fig. [3]).
2 Gravity effects in interactions of cosmic neutrinos with atmospheric nucleons

A promising possibility to detect effects induced by the low-mass KK gravitons is to look for their contributions to the scattering of the SM fields in the trans-Planckian kinematical region:

\[ \sqrt{s} \gtrsim M_5 \gg |t| , \]  

with \( \sqrt{s} \) being the colliding energy and \( t = -q^2 \) four-dimensional momentum transfer. It is also assumed that inequality (10) is satisfied. As we will see below, in the trans-Planckian region the gravity contribution to the scattering of UHE cosmic neutrinos off the atmospheric nucleons can dominate the SM contribution.

In the eikonal approximation which is valid in the kinematical region (10), elastic scattering amplitude is given by the sum of gravi-Reggeons, i.e. reggeized gravitons in the \( t \)-channel. Because of a presence of extra dimension, the Regge trajectory of the graviton is splitting into an infinite sequence of trajectories enumerated by the KK number \( n \):

\[ \alpha_n(t) = 2 + \alpha'_g t - \alpha'_g m_n^2, \quad n = 0, 1, \ldots \]  

(11)

In string theories, the slope of the gravi-Reggeons is universal, and \( \alpha'_g = M_s^{-2} \), where \( M_s \) is the string scale. For more details, see Refs. [11].

Correspondingly, the gravity Born amplitude for the neutrino scattering off a point-like particle looks like:

\[ A_{\text{grav}}^B(s, t) = \frac{\pi \alpha'_g s^2}{2\Lambda_s^2} \sum_{n \neq 0} \left[ i - \cot \frac{\pi \alpha_n(t)}{2} \right] \left( \frac{s}{M_5} \right)^{\alpha_n(t)-2} . \]  

(12)

The differential neutrino-proton cross section is of the form:

\[ \frac{d\sigma}{dy} = \frac{1}{16\pi s} |A_{\nu p}(s, t)|^2 . \]  

(13)

The inelasticity \( y = -t/s \) defines a fraction of the neutrino energy transferred to the nucleon. \( A_{\nu p} \) is the neutrino-proton amplitude which is related to the

\[ \text{Remember that the KK gravitons interact universally with the SM fields.} \]
The hadronic Born amplitude in (15) is defined by the gravity amplitude (12) and skewed ($t$-dependent) parton distributions $F_i(x, t)$:

$$A_{\nu p}^B(s, t) = \sum_{i=q, \bar{q}, g} \int_0^1 dx A_{\nu p}^B(xs, t) F_i(x, t).$$

(16)

The $t$-dependent distributions have the Regge-like form [3]:

$$F_i(x, t) = f_i(x) \exp \left[ t(r_0^2 - \alpha'_P \ln x) \right],$$

(17)

where $\alpha'_P$ is the Pomeron slope, while $f_i(x)$ is the distribution of the parton of the type $i$ inside the proton. The values of the parameters are [12]:

$$r_0^2 = 0.62 \text{ GeV}^{-2}, \quad \alpha'_P = 0.094 \text{ GeV}^{-2}.$$  

(18)

We will use a set of parton distribution functions $f_i(x)$ from Ref. [13].

In Fig. 1 and Fig. 2 we present total neutrino-nucleon cross sections calculated by using Eqs. (12)-(16) for two values of the curvature $\kappa$ and different values of the reduced fundamental gravity scale $\bar{M}_5$. 

Previously, low-scale gravity effects in cosmic neutrino interactions were calculated in models with compactified extra dimensions (see [14,15] and references therein). Recently, the gravity effects on the neutrino-nucleon cross sections in the eikonal approximation were estimated for the case of infinitely thin branes embedded in five extra dimensions [16]. The black hole

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3Since $A_{\nu p}^B \sim s^2$, the integral converges rapidly at $x = 0$. 

Figure 1: The total neutrino-proton cross section as a function of the neutrino energy (in mb). The curves correspond (from above) to $\bar{M}_5 = 3, 5, 7$ TeV. The parameter $\kappa$ is equal to 100 MeV. The straight line: SM neutral current cross section.

production cross sections in cosmic neutrino interactions were also calculated (see, for instance, Ref. [17]).

By comparing Figs. 1, 2 with figures from Ref. [16], one can see that the neutrino cross sections in the small curvature scenario of the RS model and those in the ADD model have different energy dependences. The formers are significantly smaller at $E_\nu \lesssim 10^9$ GeV, but exceed the ADD cross sections at $E_\nu \gtrsim 10^{10}$ GeV (at comparable values of gravity scale $\bar{M}_5$ in both models).

The number of neutrino induced air showers is given by

$$\frac{dN_{ev}}{dt} = \int_{E_{th}}^{E_{\max}} dE_\nu \int_0^1 dy \theta(E_{sh} - E_{th}) \frac{d\sigma(E_\nu)}{dy} \Phi(E_\nu) A_{eff}(E_{sh}, E_\nu),$$

(19)

where $E_\nu$ is the energy of the cosmic neutrino, $\Phi(E_\nu)$ denotes its flux, and

$$E_{sh} = y E_\nu$$

(20)
Figure 2: The total neutrino-proton cross section as a function of the neutrino energy (in mb). The curves correspond (from above) to $M_5 = 1$, TeV, 3 TeV, and 5 TeV. The parameter $\kappa$ is equal to 1 GeV. The straight line: SM neutral current cross section.

is the energy of the air shower produced. Effective aperture for the UHE neutrinos is defined by the neutrino flux attenuation $\text{att}(E_\nu)$ and detector efficiency $P(E_{\text{sh}})$:

$$A_{\text{eff}}(E_{\text{sh}}, E_\nu) = \text{att}(E_\nu) P(E_{\text{sh}}) A_p(E_{\text{sh}}). \quad (21)$$

The attenuation $\text{att}(E_\nu)$ depends (besides neutrino-nucleon total cross section) on $X_{\text{obs}}$, the depth within which air shower is visible for the ground array detector, and $X_{\text{uno}}$, the minimum atmospheric depth a neutrino must reach in order to induce an observable shower to these detectors.

In order to isolate neutrino-induced events at the Auger Observatory, deeply penetrating quasi-horizontal air showers should be looked for [18, 19]. We impose the following bounds on the zenith angle of the incoming neutrino: $75^\circ \leq \theta_{\text{zenith}} \leq 90^\circ$. The functions $P(E_{\text{sh}})$ and $A_p(E_{\text{sh}})$ as well as values of the parameters $X_{\text{obs}}$ and $X_{\text{uno}}$ are taken from Ref. [20]. In particular, the deeply penetrating events must satisfy the condition $X_{\text{uno}} \geq 1700$ g/cm$^2$. 
Figure 3: The expected rate of the neutrino induced inclined air showers ($75^\circ \leq \theta_{\text{zenith}} \leq 90^\circ$) at the Auger Observatory for the Waxman-Bahcall flux (in yr$^{-1}$).

Since, on average, ultra-high energy air shower develops to its maximum after traversing 800 g/cm$^2$, it is set $X_{\text{obs}} = 1300$ g/cm$^2$ (see [19, 20] for more details).

The threshold energy in [19] is taken to be $E_{\text{th}} = 5 \cdot 10^7$ GeV, and maximum energy $E_{\text{max}} = 10^{12}$ GeV. The result of our calculations for the Waxman-Bahcall neutrino flux [21] is presented in Fig. 3. It shows the rate of the inclined air showers at the Auger detector as a function of two parameters of the model.

In particular, the number of the inclined air showers is equal to 1.54, 0.68,
0.37 for $\bar{M}_5 = 9$ TeV and $\kappa = 0.5$ GeV, 1 GeV, 1.5 GeV, respectively. These estimates can be compared with the SM prediction, 0.22 events per year for the same neutrino flux and $\theta_{\text{zenith}} \geq 70^\circ$ [14], that corresponds to $\simeq 0.13$ SM events for our case.

We conclude that the search limit of the Auger Observatory for the 5-dimensional Planck scale $\bar{M}_5$ can reach 9 TeV (depending on other parameter $\kappa$), i.e. be even larger than the discovery limit of the LHC [9] derived in the framework of the same scenario.

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