External current as a coupler between the spin-vortex-induced loop current qubits

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Abstract
The spin-vortex-induced loop current (SVILC) is a nano-sized loop current predicted to exist in the CuO$_2$ plane in the bulk of hole-doped cuprate superconductors. It is a persistent loop current protected by the topological winding number associated with the wave function. It exists around a spin-vortex created by the itinerant electrons with a doped hole at its center. The direction of each SVILC can be either clockwise (winding number $-1$) or counterclockwise (winding number $+1$); and the currentless winding number zero is forbidden by the single-valued requirement of the wave function with respect to the electron coordinates. Recently, it has been demonstrated, theoretically, that this degree-of-freedom can be used for qubits. The gate-operation time by the Rabi oscillation using an electromagnetic field with electric field intensity $10^5$ V m$^{-1}$ is estimated to be several nanoseconds. This is comparable or shorter than the existing superconducting qubits. Since SVILCs do not require Cooper pairs, the SVILC qubit is free from the relaxation caused by broken Cooper pair electrons that is speculated to limit the qubit coherence time in the existing superconducting qubits. Thus, the coherence time for the SVILC qubit may exceed the value achieved by the superconducting qubit (about 100 $\mu$s). Further, since the stabilization temperature of SVILCs is expected to be the superconducting transition temperature of the cuprates, a much higher operation temperature may be possible for the SVILC qubits compared to the existing superconducting qubits, which requires the operation temperature of about 10 mK. In the present work, we show, theoretically, that coupling of two SVILC qubits can be achieved by external current feeding. This enables the construction of nano-sized qubit-couplers, and may provide a scalability that is needed in the construction of a fully fault-tolerant quantum computer.

1. Introduction
The number of qubits in a quantum computer has been increasing, steadily. It is expected that the number will reach 50, soon, where the memory space of the quantum computer eclipses that of a classical supercomputer. Since the feasibility of quantum error correction is demonstrated, experimentally [1], the next step is to equip with fault tolerance. This will eventually lead to a quantum computer with a large number (say, 100) logical qubits [2].

In order to realize such a fault tolerant quantum computer, even the most promising qubits today, superconducting qubits using Josephson junctions of the BCS superconductors, may not be good enough [3]. The reasons are following; (1) the size of a qubit with including coupler may be too large to accommodate a large number of qubits. The typical size of a superconducting qubit with including coupler is in the order of millimeters; thus, the size of a 100 logical qubit system become in the order of meters. (2) The operation temperature may be too low. The operation temperature for the superconducting qubit is in the order of mK [4]. Maintaining a large number of qubits intact with keeping such low temperature may be too difficult and too expensive [5]. (3) The coherence time is too short. The longest coherent time achieved for the superconducting...
system is about 100 μs. This is enough to accommodate error corrections for a small number of qubits; however, for a large number of qubit system, a much longer coherence time may be required.

Recently, a new qubit has been proposed that may overcome the above mentioned difficulties. It utilizes nano-sized persistent loop currents predicted to exist in the cuprate superconductor (Wakaura et al [6]). Superconductivity in the cuprate (the cuprate superconductivity) is believed to be different from the conventional one explained by the BCS theory (the BCS superconductors). It shows marked differences in many properties compared to those found in the BCS superconductors. For example, the normal state from which the superconducting state emerges is not a band metal. A theory for the cuprate superconductivity has been proposed where it is explained from the view point of a new current generation mechanism that occurs when holes are doped in the Mott insulator. In the new theory, the persistent loop current plays the role of the current element, and a macroscopic current is generated as a collection of them [7–9].

The loop current is called, the spin-vortex induced loop current (SVILC), and it is predicted to exist in the CuO$_2$ plane of the bulk of hole-doped cuprate superconductors. It is induced by the spin-vortex created by the itinerant electrons with a doped hole at its center. It is protected by the topological winding number associated with the wave function. The direction of each SVILC can be either clockwise (winding number $-1$) or counterclockwise (winding number $+1$); and the currentless winding number zero is forbidden by the single-valued requirement of the wave function with respect to the electron coordinates.

Although the presence of the SVILC is not verified, yet, there is evidence that such loop currents exist: (1) the existence of loop currents is inferred by neutron scattering measurement [10]; (2) the magnetic excitation spectrum calculated by assuming the existence of spin-vortices agrees with that obtained by the neutrons scattering experiment [11], indicating the presence of spin-vortices is very plausible; (3) the polar Kerr effect measurement [12] and the enhanced Nernst effect measurement [13] suggest the presence of loop currents.

Assuming the existence of the SVILC, we have demonstrated, theoretically, that SVILCs can be used as qubits [6]. We list expected properties of the SVILC qubits, below.

(i) All qubits can be differentiated in a controlled manner by modifying the environment of them. The controlled modification may be achieved by applying a magnetic field, applying an electric field, or feeding external currents.

(ii) The qubit operation can be achieved by irradiating an electromagnetic field with the frequency that corresponds to the energy difference between the two qubit states.

(iii) Our previous calculation indicates that the gate-operation time is in the nanosecond order when an electromagnetic field with electric field intensity $10^5$ V m$^{-1}$ is used [6].

(iv) Coupling between qubits can be turned-on and off in a controlled manner as will be demonstrated in this work. The coupling is turned-off by placing qubits at a distance; the separation distance may be shortened by using barrier atoms (substituted atoms for Cu’s) between the qubits. The coupling is turned-on by feeding external currents in the region between the two target qubits.

(v) The size of each qubit is about 10 nm$^2$. The size of the qubit-coupler using the external current feeding is also in the nanometer scale.

(vi) The stabilization temperature for the SVILCs corresponds to the superconducting transition temperature $T_c$ for the cuprates [9] which is above the liquid nitrogen temperature. Thus, the qubit operation at temperatures above the liquid nitrogen temperature might be possible.

(vii) Readout process will be performed by measuring the magnetic field produced by SVILCs after turning-off the applied magnetic field and external feeding currents. It is also possible to use the response currents to the external feeding currents [14].

(viii) The SVILC is protected by the topological winding number, thus, it is expected to be robust against external perturbations. Besides, it does not require the Cooper pair formation, thus, it is free from the relaxation caused by unpaired electrons that is believed to be the major cause of limiting coherent time for the superconducting qubits using the Josephson junctions [15–19].

The purpose of the present work is to demonstrate, theoretically, that coupling between two SVILC qubits placed at a long distance and uncoupled can be turned-on by increasing the current density in the region between them by feeding external current. We also consider the substitution effect of the Cu atoms by some other atoms (we call them, ‘barrier atoms’). The coupling method proposed in this work will enable the construction of nano-sized qubit-couplers. This will provide a scalability for the SVILC qubit quantum computer.
The organization of this work is as follows: we explain the calculation method of the quantum states with SVILCs in section 2. We consider the spin–vortex quartet (SVQ), a unit of four spin-vortices, in section 3. We construct 16 orthogonal states (SVQ qubits) and calculate the electric transition dipole moments between them. In section 4, we define the coupling energy between two SVQ qubits and examine its variation as a function of the distance between them. We also calculate the coupling energy when some Cu atoms are replaced by some other atoms (barrier atoms). In section 5, we demonstrate that external feeding currents can be used as a coupler between two uncoupled SVQ qubits. In section 6, we conclude the present work.

2. Calculation of states with SVILCs

In this section, we explain how to calculate the SVILC states. The method and underlying assumptions are already explained in our previous works [6–9, 14].

The Hamiltonian we will use is a Hartree–Fock version given by

$$H_{HF} = -t \sum_{\langle i,j \rangle; \sigma} (c^\dagger_{i\sigma} c_{j\sigma} + c^\dagger_{j\sigma} c_{i\sigma}) + U \sum_j \left( \left( \frac{n_j}{2} - S_j^z \right) c^\dagger_j c_j + \left( \frac{n_j}{2} + S_j^z \right) c^\dagger_j c_j - (S_j^x - iS_j^y)c^\dagger_j c_j - (S_j^x + iS_j^y)c^\dagger_j c_j \right),$$

where $c^\dagger_{i\sigma}$ and $c_{i\sigma}$ are the creation and annihilation operators of electron at the $i$th site with the $z$-axis projection of electron spin $\sigma$, respectively; we define $i$ and $j$ as the sites of Cu atoms, by considering a single CuO$_2$ plane in the bulk; the oxygen atoms between Cu atoms are ignored; the sum of $(i, j)$ in the first term indicates that the sum is taken over the nearest neighbor pairs;

$$n_j = \sum_{\sigma} \langle c^\dagger_{j\sigma} c_{j\sigma} \rangle$$

is the number operator of electrons at the $j$th site, and $S_j = (S_j^x, S_j^y, S_j^z)$ is the electron spin at the $j$th site given by

$$S_j^x = \frac{1}{2} (c^\dagger_{j\uparrow} c_{j\uparrow} + c^\dagger_{j\downarrow} c_{j\downarrow}) = S_j \cos \xi_j \sin \zeta_j,$$

$$S_j^y = \frac{1}{2} (c^\dagger_{j\uparrow} c_{j\downarrow} - c^\dagger_{j\downarrow} c_{j\uparrow}) = S_j \sin \xi_j \sin \zeta_j,$$

$$S_j^z = \frac{1}{2} (c^\dagger_{j\uparrow} c_{j\uparrow} - c^\dagger_{j\downarrow} c_{j\downarrow}) = S_j \cos \zeta_j,$$

where $\xi$ and $\zeta$ are the azimuth and polar angles, respectively; $\langle \hat{O} \rangle$ denotes the expectation value of operator $\hat{O}$. The parameters $n_j$, $S_j^x$, $S_j^y$ and $S_j^z$ are obtained self-consistently.

As for materials parameter values, we use the lattice constant $a = 0.4$ nm, the transfer integral $t = 130$ meV, and the on-site Coulomb repulsion $U = 8t$ [20]. We also consider only the case where $\zeta_j = \pi/2$ at all sites. In this case the spin is polarized in the CuO$_2$ plane as is observed in the parent compound of the cuprate [21]. There must be a mechanism that stabilizes such spin polarization; however, we take it as a phenomenological condition without identifying the responsible interaction in this work.

From an energy minimization requirement, we obtain single–particle wave functions,

$$|\tilde{\gamma}_j\rangle = \sum_j [e^{-\xi_j/2}D^\dagger_{j\uparrow}c^\dagger_{j\uparrow} + e^{\xi_j/2}D^\dagger_{j\downarrow}c^\dagger_{j\downarrow}]|\text{vac}\rangle,$$

where $D^\dagger_{j\uparrow}$, $D^\dagger_{j\downarrow}$ and $\xi_j$ are obtained numerical parameters. The many-electron wave function is usually constructed as a Slater determinant of them. However, the above single–particle wave functions are multi-valued with respect to electron coordinates due to the presence of spin–vortices; thus, the obtained many-electron wave function is also multi-valued, which is against the requirement that the total wave function should be a single-valued wave function of the electron coordinates.

Let us explain how the multi-valuedness of the wave function $|\tilde{\gamma}_j\rangle$ arises, and a way to remedy it. A spin-vortex in the CuO$_2$ plane is described by a non-zero winding number of $\xi$ for some loop $C_{\xi}$, where the winding number of $\xi$ for $C_{\xi}$ is defined by

$$w_{\xi}[\xi] = \frac{1}{2\pi} \sum_{i=1}^{N_{\xi}} (\xi_{C_{\xi}(i+1)} - \xi_{C_{\xi}(i)}),$$

where $N_{\xi}$ is the number of sites for $C_{\xi}$; $C_{\xi}(i)$ denotes the $i$th site in $C_{\xi}$ with the boundary condition $C_{\xi}(N_{\xi} + 1) = C_{\xi}(1)$.

When the phase factor $e^{\pm \xi_{C_{\xi}(1)}}$ is evaluated starting from $C_{\xi}(1)$ by integrating $\nabla \xi \approx (\xi_{C_{\xi}(i+1)} - \xi_{C_{\xi}(i)})$ along closed path $C_{\xi}$, it becomes.


\[ e^{\pm i(\ell_i x + 2\pi \psi_i)} = (-1)^{w_i} e^{\pm i\ell_i x} \]  \tag{6}

Then, if \( w_i [\xi] \) is odd, sign-change occurs. This means that the wave function \( |\gamma\rangle \) in equation (4) is multi-valued.

We construct a single-valued single-particle wave function \( |\gamma\rangle \) as

\[ |\gamma\rangle = \sum_j e^{i\psi_j} e^{i\frac{\xi_j}{\pi} T} |c_j^+ \rangle + e^{i\frac{\xi_j}{\pi} T} |c_j^- \rangle |\text{vac}\rangle, \]  \tag{7}

by including the phase factor \( e^{-i\psi_j} \). The way to obtain \( \psi_j \) will be explained below. This single-valued wave function is used as a basis function for constructing the Slater determinant.

The angular variable \( \chi \) is determined in the following manner; first, note that the single-valued requirement is satisfied if the following condition is satisfied by \( \chi \):

\[ w_i [\chi] + w_i [\xi] = \text{even for all loops} \quad G_r. \]  \tag{8}

The angular variable \( \chi \) is determined by minimizing the total energy with taking into account the above condition. This is achieved by minimizing the following functional,

\[ F[\nabla \chi] = E[\nabla \chi] + \sum_{r=1}^{N_{\text{loop}}} \lambda_r \left( \oint_{C_r} \nabla \chi \cdot dr - 2\pi \bar{w}_r \right), \]  \tag{9}

where \( E[\nabla \chi] \) is the total energy depends on \( \nabla \chi \); the second term is the one arising from the constraint with \( \lambda_r \) being the Lagrange multiplier; \( \bar{w}_r \) is the winding number of \( \chi \) along a loop \( C_r \) that satisfies the constraint in equation \( (8) \); \( N_{\text{loop}} \) is the number of independent loops, where any loop in the system can be constructed by combining the \( N_{\text{loop}} \) independent loops.

A set of values for \( \bar{w}_r \) in equation \( (9) \) specifies a particular current distribution; in other words, by changing the values of \( \bar{w}_r \) for the independent loops, solutions with different current distributions are obtained. In the following, we obtain states with different current patterns by changing only \( \nabla \chi \), and fixing values of \( D_j^a \) and \( \xi_j \).

From the stationary condition of \( F[\nabla \chi] \), \( \nabla \chi \) is obtained as the solution of

\[ 0 = \frac{\delta F[\nabla \chi]}{\delta \nabla \chi} = \frac{\delta E[\nabla \chi]}{\delta \nabla \chi} + \sum_{r=1}^{N_{\text{loop}}} \lambda_r \frac{\delta}{\delta \nabla \chi} \oint_{C_r} \nabla \chi \cdot dr \]  \tag{10}

with the constraint,

\[ \oint_{C_r} \nabla \chi \cdot dr - 2\pi \bar{w}_r = 0, \quad r = 1, \ldots, N_{\text{loop}}. \]  \tag{11}

From equation \( (10) \), the current density is given by

\[ j = -\frac{2e}{h} \sum_{r=1}^{N_{\text{loop}}} \lambda_r \frac{\delta}{\delta \nabla \chi} \oint_{C_r} \nabla \chi \cdot dr. \]  \tag{12}

This clearly indicates that the current is generated by the phase factor \( e^{-i\chi} \) in a non-perturbative way. It is given as a sum of loop currents called, ‘SVILCs’.

The external feeding current contribution is included by adding extra loops in equation \( (9) \). For the detail of the method, consult \[8, 14] \).

3. Transition dipole moments of SVQ states in quadratic inhomogeneous magnetic field

The SVQ we consider is shown in figure 1. It is composed of four SVILCs. The sum of the winding numbers for the spin-vortices is zero, and the nearby spin-vortices have the opposite winding numbers. This arrangement of the spin-vortices is the minimal energy one.

The winding number of \( \chi \) for each spin-vortex is either +1 or −1, thus, totally 16 different SVILC states are possible from a single SVQ. These 16 current patterns are depicted in figure 2. We apply a quadratic inhomogeneous magnetic field in the direction perpendicular to the CuO2 plane to remove all degeneracies of the SVILC states, and construct orthogonal states that can be used as qubit states. We call them SVQ states or SVQ qubit states.

The inhomogeneous magnetic field we apply is

\[ B = 0.178x^2 + 6.0x + 0.07y^2 + 6.0y \quad \text{(T)}, \]  \tag{13}

where \( x \) and \( y \) are given in the units of lattice constant \( a \).

The interaction Hamiltonian between the electric current and the applied magnetic field \( \nabla \times \mathbf{A}^m = (0, 0, B) \) is given by
Figure 1. Spin structure of SVQ. An arrow on each lattice point indicates the electron spin whose direction is specified by $\zeta_j$, where $j$ is the site index. 'M' and 'A' denote spin-vortex with $w_\ell(\zeta) = 1$ and $w_\ell(\zeta) = -1$, respectively, where $\ell$ denotes the loop of 8 sites around each center.

Figure 2. Sixteen current patterns for SVQ states. These 16 states are derived by applying a magnetic field in the $z$ direction given by $B = 0.178x^2 + 6.0x + 0.07y^2 + 6.0y$ T (the unit of $x$ and $y$ is $a$, where $a$ is the lattice constant of the CuO$_2$ plane). The arrows indicate current directions. 'L' and 'R' denote the centers of SVILCs with $w_{\ell}(\chi) = 1$ and $w_{\ell}(\chi) = -1$, respectively, where $w_{\ell}(\chi)$ is the winding number of $\chi$ around each center.
In this section, we calculate the coupling energy between a pair of SVQ qubits chosen from $\{\gamma\}$ by diagonalizing the matrix for the operator $H_b$ whose $(a, b)$ element is given by $\langle \Phi_a | H_b | \Phi_b \rangle$. The energy of the $|\Phi_a\rangle$ state is calculated as $(\Phi_a| H_b |\Phi_a)^{HF} + H_b|\Phi_a\rangle$ [6]. We use the resulting orthogonal basis $\{|\Phi_a\rangle\}$ as qubit states.

In figure 2, all current patterns for the qubit states are displayed in the energy-increasing order. In figure 3, energy levels for all SVQ states are depicted. It is clear that energies of states with non-zero sum of the four winding numbers for $\chi$ ($|(g)\rangle - |(p)\rangle$) are significantly larger than those with zero-sum states ($|(a)\rangle - |(l)\rangle$). Therefore, the zero-sum states are more favorable for the qubit states. Some of transition dipole moments between two states in figure 2 are tabulated in table 1. Current patterns (c)–(f) have very large dipole moments, thus, they will be favorable as qubits. Note that the values for dipole moments in table 1 are slightly different from those in our previous work because the magnetic field we applied is different [6]; in the present work, we have used the quadratic inhomogeneous magnetic field to remove the degeneracies of all 16 states, while in the previous work, we used the linear-gradient magnetic field, thus, some of the degeneracies remained.

4. Coupling energy between two SVQ qubits with changing their distances

In this section, we calculate the coupling energy between a pair of SVQ qubits chosen from $(a)$–$(p)$ in figure 2 by changing the distance between them. First, we examine the uncoupling condition of two SVQ qubits for the system denoted by $|\alpha\rangle$, where $\alpha$ and $\bar{\alpha}$ are the SVQ states in the left-hand and right-hand sides, respectively, as shown in figure 4; its energy is denoted as $E_{|\alpha\rangle}$ [6]. Note that the two SVQs have a $C_2$ symmetry with respect to the center of the CuO$_2$ plane.

We define $\bar{\alpha}$ state as the SVQ state with all $w_j |\nabla \chi\rangle$'s reversed with respect to those of $\alpha$ state. Then, the coupling energy, $V_{\bar{\alpha} \alpha}$, is defined as the energy difference given by

$$ V_{\bar{\alpha} \alpha} = E_{|\bar{\alpha}\rangle} - E_{|\alpha\rangle}. $$

For example, when $\alpha$ is $(a)$ and $\bar{\alpha}$ is $(b)$ in figure 2, the coupling energy is given by $V_{ab} = E_{|ab\rangle} - E_{|aa\rangle}$.

We calculate the coupling energy in the $4a \times 6a$ sized lattice shown in figure 4. To save the computational time, we use the non-orthogonal basis $\{|\Phi_a\rangle\}$ states to calculate $V_{\bar{\alpha} \alpha}$. We checked that the calculation using the orthogonalized basis gives almost the same results.

If an SVQ qubit has a $C_2$ symmetry with respect to its center ($(a)$, $(b)$, $(o)$, and $(p)$), the separation distance ($r_s$) dependence of the coupling energy behaves like a screened Coulomb potential. We consider that when the coupling energy is less than $10^{-3}$, the two qubits are uncoupled. When $\alpha$ is $(o)$ and $\bar{\alpha}$ is $(e)$ (then, $\bar{\alpha}$ is $(f)$), the two qubits are considered to be uncoupled at $r_s = 32$ $(a)$ as shown in figure 5; instead, if $\bar{\alpha}$ is $(p)$, the two qubits are uncoupled at $r_s = 36$ $(a)$.
There are cases in which the coupling is not negligible even if the magnitude of the coupling energies is very small. They occur when the current direction around the site located a\(^2\) to the right of the center of \(\alpha\) qubit, and that around the site located a\(^2\) to the left of the center of \(\Upsilon\) qubit are neither parallel nor anti-parallel. In this case, the coupling energy does not monotonically decrease with the increase of \(r_x\). Concretely, when \(\alpha\) is (\(e\)) or (\(f\)) and \(\Upsilon\) is (\(b\)), (\(a\)), (\(c\)) or (\(d\)), the coupling energy is small even at small separations but the two qubits are coupled. For example, the energy difference between \(\left| \psi_{ec} \right\rangle\) and \(\left| \psi_{ed} \right\rangle\) is only \(-1.0\) at \(r_x = 6\) (a), however, the coupling exists and the coupling energy does not decrease monotonically with the increase of \(r_x\). This behavior occurs because the coupling of SVQs are governed not only by the separation distance but also by the direction and intensity of the current at the qubit sites.

Next, we calculate the coupling energy when some of the Cu atoms are replaced by other atoms (barrier atoms). The barrier atoms may be used to reduce the coupling between the two SVQ qubits. The effect of the barrier atom substitution is taken into account by the change of the transfer integral for the bonds that include the barrier atom sites to \(t' = 0.5t\) and the Coulomb repulsion on the barrier atom sites to \(U' = 10r\). In figure 6,

Table 1. Transition dipole moments between states with different current patterns for the spin-vortex quartet (SVQ) depicted in figure 2. The value of the \(y\) component is tabulated in the lower-left (upper-right) triangle positions of the table. The unit of the moment is \(10^{-30}\) Cm. Underlined ones have extremely large values; they are the \(y\) component of dipole transition moment between (e) in figure 2 (denoted by \(RL\) in the table) and (f) (denoted by \(LR\) in the table), and the \(x\) component between (c) (\(LL\)') and (d) (\(RR\')).

| \(\mu_{y}^{\alpha}\) (10\(^{-30}\)Cm) | \(I\) | RL | LL | RL | LR | RR | LR |
|---|---|---|---|---|---|---|---|
| \(k\) | RL | 0.334 | 2.67 | 4.259 | 6.82 | 1.998 |
| RL | LL | 0.0501 | 8.882 | 8.595 | 22.721 | 6.82 |
| RR | LR | 8.642 | 2.871 | 0.662 | 8.595 | 2.769 |
| LL | RR | 7.47 | 2.457 | 2.656 | 2.871 | 0.334 |
| LR | 22.2 | 7.47 | 8.642 | 8.837 | 0.501 |

Figure 4. Two SVQs on the 44\(\times\) 6\(\times\) sized lattice of the CuO\(_2\) plane. \(r_x\) is the distance between the centers of the two SVQs. (i) The spin structure when \(r_x = 8\) (a). (ii) The spin structure when \(r_x = 36\) (a).
the case in which Cu atoms on \( \mathcal{V} \) are replaced with barrier atoms is depicted. The magnitude of coupling energies between any SVQ qubits is reduced by \( 1/10 \) as shown in figure 7. This may be used to reduce the size of the SVQ qubit systems since the qubits can be uncoupled with shorter distances.

5. External current as a coupler between two SVQ qubits

We consider the situation that two SVQ qubits are separated enough and uncoupled. Let us denote the SVQ states \((e)\) and \((i)\) in figure 2 as \( |D\rangle \) and \( |U\rangle \), respectively, and treat them as \( |0\rangle \) and \( |1\rangle \) states of a single qubit,
is larger than in addition, the two qubits are coupled, and the controlled gate-operation becomes possible.

and an applied electromagnetic is shown. The change of the kinetic energy for the two DCQ system is a parabolic function of regions of the two qubits do not overlap, thus, their interaction is negligible.

respectively. Because, they have the significant y component of dipole moment $22.2 \times 10^{-30}$ Cm as shown in table 1. We call it the dipole-current qubit (DCQ), and use the gradient-magnetic field perpendicular to the CuO$_2$ plane

$$B = 0.4\alpha \ (T)$$

(17)

to remove the degeneracy of $|D\rangle$ and $|U\rangle$ of the both side of DCQs and to split the energy levels of them differently (the unit of $x$ is $a$, where $a$ is the lattice constant of the CuO$_2$ plane). The intensity of applied magnetic field at the right edge of the system is 18 T to emphasize the effect of feeding external currents.

We may use the right-directed and left-directed current states ($|c\rangle$ and $|d\rangle$ in figure 2) as a single qubit since they have also significant dipole moments; however, their splitting by the linear-gradient magnetic field is very small. Hence, we use the DCQ as a single qubit. The two-qubit system we consider is the one with a 44$a_i \times 6a_i$ sized lattice shown in figure 3 with the distance between the two DCQs is 36 $a_i$. The coupling energy between the two DCQ, $V_{XY}$, is less than $10^{-4}\gamma$, thus, the two qubits are considered to be uncoupled. The current flowing regions of the two qubits do not overlap, thus, their interaction is negligible.

Next, we feed external currents at the site $(x, y) = (10, 1)$ and $(23, 7)$ and drain them at $(x, y) = (23, 1)$ and $(35, 7)$. Then, the coupling between the two DCQs arises if the magnitude of the currents is sufficient as shown in figure 9. In the region between the two DCQs, an extra current density appears by the external feeding currents, causing the overlap between the current regions around the two qubits.

In figure 10, the energy levels of two DCQ system are shown. Without the external magnetic field and feeding currents, all four states are degenerate. By turning on the external magnetic field, two states of a single qubit split, thus, the single qubit gate-operation is possible; however two qubits are still uncoupled. By turning on the external current $J_{ex}$ in addition, the two qubits are coupled, and the controlled gate-operation becomes possible. This indicates that external currents can be used as a coupler.

The electric transition dipole moments between the four states of the two DCQ system are tabulated in table 2. We note that the $x$ components of the dipole moments are essentially zero. However, $y$ components of the dipole moments are over $1.0 \times 10^{-30}$ Cm, thus, the Rabi oscillations can be achieved using the lasers with the $y$ polarization component.

In figure 11, the change of the sum of the single particle energies with varying the feeding current magnitude $J_{ex}$ is shown. The change of the kinetic energy for the two DCQ system is a parabolic function of $J_{ex}$. The coupling energy can be modified by changing the magnitude of $J_{ex}$, and also the energy difference between the states. Thus, using a suitable $J_{ex}$ and an applied electromagnetic field frequency, we can manipulate the Rabi oscillations for the quantum gate-operation. If $J_{ex}$ is larger than $0.05(2\pi/t\ h)$ the crossing of the energy levels occurs; the order of $|UU\rangle$ and $|DD\rangle$ is reversed. This reversing of the order may cause the flipping of the states, and lead to undesirable transitions. Thus, we should use the external current less than $0.05(2\pi/t\ h)$. Note also
that feeding external current larger than $0.08(2\pi/\hbar)$ reverse the order of $|DU\rangle$ and $|DD\rangle$, and the order of $|UU\rangle$ and $|UD\rangle$.

Let us examine the effect of using barrier atoms on the performance of the current feeding coupler. We calculate the energy levels and dipole moments between the qubit states of the two DCQ system with barrier atoms. The barrier atoms are located at $(x, y) = (22, 1), (22, 2), \cdots, (22, 7)$ and $(x, y) = (24, 1), (24, 2), \cdots, (24, 7)$ as shown in figure 6. The transfer integral of all the bonds including barrier atoms is taken to be $t' = 0.5t$ and the Coulomb parameter on the sites of barrier atom is $U' = 10t$.

We feed and drain one of the external currents through the sites between the two arrays of the barrier atoms. Remarkably, we can minimize the effect of the barrier atoms on the current feeding couplers in this way; as you Figure 8. The current distribution of four states of the two DCQ qubit system. One lattice distance corresponds to the current of the magnitude $1/3$ in the units of $2\pi/\hbar$. The four states are indicated as $|DU\rangle, |UU\rangle, |DD\rangle$ and $|UD\rangle$, respectively. The centers of the two DCQs are $(x, y) = (5, 4)$ and $(41, 4)$.

Figure 9. The current distribution of four states of 2-qubit system of DCQ with feeding currents $J_{ex}$. The external currents enter the system at $(x, y) = (10, 1)$ and $(23, 7)$, and exit at $(x, y) = (23, 1)$ and $(35, 7)$. The magnitude of the external currents is $0.049(2\pi/\hbar)$ for each source and drain. The blue arrow indicates the source or drain of the external currents.
can see in figure 12, the energy diagram in this case is almost the same as that of the system without the barrier atoms. In table 3, the transition dipole moments for the system with the barrier atoms are tabulated. They differ only slightly from those of the system without the barrier atoms. If $J_{ex}$ is larger than $0.05 \text{ (2eV)}$, the order of $\ket{UD}$ and $\ket{UU}$ is reversed in a similar manner as the system without the barrier atoms as shown in figure 13. By feeding external currents larger than $0.1 \text{ (2eV)}$, the order of energy alters as $\ket{DD}, \ket{UD}, \ket{DU}$ and $\ket{UU}$. We note that this crossing point is slightly shifted from the value for the system without the barrier atoms. The changing of the current distributions due to the feeding currents are shown in figure 14. Overall, the presence of the barrier atoms enhances the change of the energies by $J_{ex}$ slightly. However, it does not cause any qualitative changes.

Table 2. The $x$ and $y$ components of transition dipole moments of the two DCQ qubit system.

| $k$  | $l$  | $\mu_{x}^{k}(10^{-30}\text{ Cm})$ | $\mu_{y}^{k}(10^{-30}\text{ Cm})$ |
|------|------|---------------------------------|---------------------------------|
| $\ket{DD}$ | $\ket{UU}$ | $1.326 \times 10^{-5}$ | $0$ |
| $\ket{DD}$ | $\ket{DU}$ | $9.475$ | $7.097 \times 10^{-6}$ |
| $\ket{DD}$ | $\ket{UD}$ | $9.481$ | $4.043 \times 10^{-5}$ |
| $\ket{UU}$ | $\ket{DU}$ | $9.481$ | $4.043 \times 10^{-5}$ |
| $\ket{UU}$ | $\ket{UD}$ | $9.475$ | $7.097 \times 10^{-6}$ |
| $\ket{DU}$ | $\ket{UD}$ | $3.084 \times 10^{-7}$ | $0$ |

Figure 10. Energy levels of the two DCQ qubit system. Left: $B = 0$, $J_{ex} = 0$. Middle: $B = (0, \delta Bx)$ with $\delta B = 0.4 \text{ (T a}^{-1})$, $J_{ex} = 0$. Right: $B = (0, \delta Bx)$, $J_{ex} = 0.049 \text{ (2eV)}$. $B$ removes the degeneracy of the single qubit state. $J_{ex}$ couples the two qubit states.

Figure 11. The plot of the sum of single particle energies of the two DCQ system versus $J_{ex}$ for the four qubit states $\ket{DU}$, $\ket{UU}$, $\ket{DD}$, and $\ket{UD}$. The zero of energy is that with $B = 0$ and $J_{ex} = 0$. 

Figure 12. The energy levels of the two DCQ qubit system. In table 3, the transition dipole moments for the system with the barrier atoms are tabulated. They differ only slightly from those of the system without the barrier atoms.
6. Concluding remarks

In this work, we have examined a possible qubit coupler for SVILC qubits. We have used an SVQ qubit composed of four SVILCs as a qubit since the SVQ is a stable unit of SVILCs. We have demonstrated that two SVQ qubits placed at a distance (and uncoupled) can be coupled by feeding external currents. This coupling is achieved by increasing the current density in the region between the two SVQs and causing an overlap of the current flowing regions centered at the two SVQs. A notable point of this coupling method is that it is very...
compact; thus, it enables a construction of a nano-sized coupler. This nano-sized couple will be an advantage of the SVILC quantum computer from the scalability viewpoint.

We have also examined the effect of replacing some Cu atoms with some other atoms (barrier atoms) existing between the two qubits. It is shown that the introduction of the barrier atoms does not significantly change the effect of external currents as the coupler. However, it enables the shortening of the distance for the uncoupling, thus, the size of the SVQ qubit system is reduced. This will further increase the scalability of the SVQ qubit quantum computer.

In figure 15, an architecture that might be used to implement SVQ qubits is depicted. It uses a strip of CuO₂ plane, where SVQ qubits are arranged in one-dimension. It composed of three layers; they are readout layer, qubit controlling layer, and qubit fixing layer. In the top layer, the qubit states are determined by measuring the magnetic field produced by the SVILCs; this may be achieved by placing atoms or molecules with unpaired electrons and measuring the energy level spacing by ESR devices [22]. The middle layer is the qubit layer; qubits

Figure 14. The current distribution of four states of the two DCQ qubit system with barrier atoms at \((x, y) = (22, 1), (22, 2), \ldots, (22, 7)\) and \((24, 1), (24, 2), \ldots, (24, 7)\). The two states \(|DU\rangle\) and \(|UU\rangle\) out of four are shown, respectively. The centers of the two DCQs are \((x, y) = (5, 4)\) and \((41, 4)\). The current distribution with feeding currents are depicted in bottom two figures, where the external currents enter the system at \((x, y) = (10, 1)\) and \((23, 7)\), and exit at \((x, y) = (23, 1)\) and \((35, 7)\). The magnitude of the external currents is 0.049\((2et/h)\) for each source and drain. The blue arrow indicates the source or drain of the external currents.

Figure 15. Implementation of SVQ qubits with a three layer architecture. Top layer: qubit state readout layer. SVQ states are determined using magnetic field detectors. Middle: qubit layer. SVQ quits are generated in a strip of the CuO₂ plane of the cuprate. Leads are connected to the strip and currents are fed. They control each qubit state and also coupling between qubits. Bottom layer: qubit fixing layer. Negatively biased electrodes are used to fix the positions of qubits.
composed of SVQs are located there. The separation of the nearby qubits is long enough to decouple each qubit; this will require the distance longer than 14 nm. Leads are connected to a strip of the CuO$_2$ plane; they are used to control the energy levels of each qubit, and also used as couplers of the qubits. By controlling the qubit energy levels, each qubit becomes differentiated and can be manipulated, separately. The qubit coupling is achieved by feeding currents in the region between two qubits. The leads considered in the present work is in the order of the lattice distance, however, much thicker lead (say 10 nm) may be used. It has been theoretically demonstrated that such a thick lead also brings about a similar effect [14]. Then, a reasonable parameter for the qubit distance will be about 30 nm if the current feeding leads with thickness 10 nm is used.

The third layer contains electrodes that fix the locations of the SVQ’s in the middle layer. Experimental data suggest that holes in the CuO$_2$ plane are more stable in the place close to the doping ion (for example, Sr$^{2+}$ for La$_{2-x}$Sr$_x$CuO$_4$) in the reservoir layers of the cuprate. If this is the case, the location of the SVQ’s may be fixed using the negatively biased electrodes. Also the number of holes in the CuO$_2$ plane will be controlled by them.

There are three major sources of the relaxation of the qubit state. The first one is the flipping of the loop current directions. Second one is the destruction of spin-vortices by flipping of spin directions. Third one is the hopping of the hole. At present, we do not know exact magnitude of each contribution; however, theoretical estimates for the energy scales are, $t^2/U^2$ (the values used in this work are $t = 130$ meV and $U = 8t$) for the loop current fluctuation [23], $t^2/U$ for the spin-vortex fluctuation, and 0.2 eV for the hole hopping [24]. Thus, they will be negligible in the liquid nitrogen temperature. The charge fluctuation for the SVQ qubit will be negligible since the on-site Coulomb repulsion parameter $U$ for the SVQ qubit system is very large (in the order of eV). The loop currents and spin-vortices are protected by topology, and the hole hopping requires the destruction of loop currents and spin-vortices, thus, all major qubit relaxation processes are topologically protected collective one. Therefore, the SVQ qubit should be a robust one. We also speculate that the coherence time of the SVQ qubits is rather long by the same reason.

The applicability of the present coupling method is only between nearby qubits. However, it is possible to construct a fault tolerant quantum computer using the surface code [25–28]. In figure 16, a schematic layout for an implementation of two-dimensional surface code using the SVILC qubits is depicted. This is a two-dimensional extension of the one shown in figure 15. There are sections that are not the CuO$_2$ plane. From these regions, two types of external currents are fed, one for the coupling between nearby SVQ qubits, and the other for the controlling the single qubit states of SVQs. The qubit controlling external current may be used to initialize the qubit since the energy of states with different current patterns depend on the external current pattern [14]. It may be possible to change the energy levels of (e) and (f) in figure 2 so that they become single and first excited states by controlling the environment. The above each qubit, a molecule that measures the magnetic field from the qubit is placed. This two-dimensional version is ideal one since it is compact and equipped with the error correction. However, the construction of it may be highly demanding at present.

Figure 16. Implementation of two-dimensional surface code using the SVILC qubits. Each qubit is an SVQ qubit composed of four SVILCs. White square regions are non-CuO$_2$ plane regions used for feeding external currents. There are two types of external currents: one with filled arrow is for the coupling between nearby SVQ qubits, and one with open arrow is for the controlling the single qubit states of SVQs.
As listed in the introduction, the SVILC qubit is expected to have a number of advantages over the currently available qubits. However, even the experimental confirmation for the presence of the SVILC is lacking. Thanks to the development of the nanotechnology, it will be possible to verify their existence and usability as qubits. A possible method to detect SVILCs is given in [14]. The architecture similar to the one in figure 15 will be used to demonstrate the qubit usability. The atomic scale manipulation of the CuO2 plane using the molecular beam epitaxy method is currently possible [29]. Thus, the SVILC qubit quantum computer will be something to be realized using the currently available technology if SVILCs really exist.

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