CP violation for neutral charmed meson decays to CP eigenstates

Dong-sheng Du

Institute of High Energy Physics, Chinese Academy of Sciences, P.O. Box 918 (4), Beijing 100049, China

CP asymmetries for neutral charmed meson decays to CP eigenstates are carefully studied. The formulas and numerical results are presented. The impact on experiments is briefly discussed.

I. INTRODUCTION

Up to now, we still do not have any experimental evidence for CP violation in the charm sector. Theoretically, the prediction for charm mixing is very small. This leads to small CP violating effects in charm decays. However, searching for large mixing and CP violation in charm decays is still very interesting not only for testing the standard model but also for finding new physics (for a recent review, see Ref. [1]). Because CP eigenstates are very special, if $D^0 - \bar{D}^0$ decay into the same CP eigenstates, then the CP violating asymmetry could be enhanced by interference. So we try to investigate this possibility in detail. Another advantage of CP eigenstates is that the amplitude ratio $A(D^0 \to f)/A(D^0 \to \bar{f})$ can be estimated without computing the amplitudes directly. This makes the computation of the CP asymmetries easier. In this paper, we shall concentrate on the case of CP eigenstates into which charm decays.

II. TIME-DEPENDENT CP ASYMMETRY

Define

$$CP \, |D^0\rangle = |\bar{D}^0\rangle,$$
$$|D_S\rangle = p \, |D^0\rangle + q \, |\bar{D}^0\rangle,$$
$$|D_L\rangle = p \, |D^0\rangle - q \, |\bar{D}^0\rangle,$$

$$|p|^2 + |q|^2 = 1.$$  \hspace{1cm} (1)

The corresponding eigenvalues of $|D_S\rangle, |D_L\rangle$ are

$$\lambda_S = m_S - i \frac{\gamma_S}{2}, \quad \lambda_L = m_L - i \frac{\gamma_L}{2}.$$  \hspace{1cm} (2)

Assuming CPT invariance, the time-evolved states are

$$|D^0_{p}(t)\rangle = g_+(t) |D^0\rangle + \frac{q}{p} g_-(t) |\bar{D}^0\rangle,$$
$$|\bar{D}^0_{p}(t)\rangle = \frac{p}{q} g_-(t) |D^0\rangle + g_+(t) |\bar{D}^0\rangle.$$  \hspace{1cm} (3)

where

$$g_\pm = \frac{1}{2} \left( e^{-i \lambda_p\, t} \pm e^{-i \lambda_l\, t} \right)$$

$$= \frac{1}{2} e^{-imt} \left\{ e^{i \Delta \gamma t} - e^{-i \Delta \gamma t} \pm e^{-i \Delta m t} + e^{i \Delta m t} \right\},$$

$$\Delta m = m_L - m_S, \quad m = (m_L + m_S)/2,$$

$$\Delta \gamma = \gamma_S - \gamma_L, \quad \gamma = (\gamma_L + \gamma_S)/2.$$
Define the mixing parameter

\[ x = \frac{\Delta m}{\gamma}, \quad y = \frac{\Delta \gamma}{2 \gamma}, \]  

(4)

then the decay amplitudes for final state \( f \) are

\[
A(D^0_p(t) \to f) = \langle f | H_{\text{eff}} | D^0_p(t) \rangle = g_+(t)A(f) + \frac{q}{p} g_-(t)\overline{A}(f) = A(f) \left\{ g_+(t) + \lambda_f g_-(t) \right\},
\]

(5)

where

\[
A(f) = \langle f | H_{\text{eff}} | D^0 \rangle, \quad \overline{A}(f) = \langle f | H_{\text{eff}} | D^0^\ast \rangle, \quad \lambda_f = \frac{q}{p} \frac{\overline{A}(f)}{A(f)}.
\]

(6)

Similarly, we put

\[
A(\overline{f}) = \langle \overline{f} | H_{\text{eff}} | D^0 \rangle, \quad \overline{A}(\overline{f}) = \langle \overline{f} | H_{\text{eff}} | D^0^\ast \rangle, \quad \overline{\lambda_f} = \frac{p}{q} \frac{A(\overline{f})}{\overline{A}(\overline{f})}.
\]

(7)

where \( \overline{f} \) is the CP conjugate state of the final state \( f \), and

\[ |\overline{f}\rangle = CP | f \rangle = \eta_{\text{CP}}(f)| f \rangle, \]

with \( \eta_{\text{CP}}(f) = \pm 1 \) is the CP eigenvalue (or CP parity). For the amplitude of \( \overline{D^0_p}(t) \to \overline{f} \), we have from Eq. (2)

\[
A(\overline{D^0_p}(t) \to \overline{f}) = \langle \overline{f} | H_{\text{eff}} | \overline{D^0_p}(t) \rangle = \frac{q}{p} g_-(t)A(\overline{f}) + g_+(t)\overline{A}(\overline{f}) = \overline{A}(\overline{f}) \left\{ g_+(t) + \overline{\lambda_f} g_-(t) \right\}.
\]

(8)

Now, it is easy to calculate the time-dependent width \( \Gamma(D^0_p(t) \to f) \) and \( \Gamma(\overline{D^0_p}(t) \to \overline{f}) \). Using Eqs. (3), (5) and (8), we have

\[
\Gamma(D^0_p(t) \to f) = |A(D^0_p(t) \to f)|^2 = |A(f)|^2 \left\{ |g_+(t)|^2 + 2Re \left[ \lambda_f g_+(t) g_-(t) \right] + |\lambda_f|^2 |g_-(t)|^2 \right\},
\]

(9)

\[
\Gamma(\overline{D^0_p}(t) \to \overline{f}) = |\overline{A}(\overline{f})|^2 \left\{ |g_+(t)|^2 + 2Re \left[ \overline{\lambda_f} g_+(t) g_-(t) \right] + |\overline{\lambda_f}|^2 |g_-(t)|^2 \right\}.
\]

(10)

In order to compute \( \lambda_f \) and \( \overline{\lambda_f} \), we need first to compute the amplitude ratios \( \overline{A}(f)/A(f) \) and \( A(\overline{f})/\overline{A}(\overline{f}) \). As an example, we consider \( D^0, \overline{D^0} \to K^+K^- \). Draw the decay diagrams (Fig. 1), we see that if we neglect the penguin diagram contribution, the \( D^0 \) and \( \overline{D^0} \) decay diagrams involve only one CKM factor \( V_{us} V_{cs}^* \) and \( V_{us} V_{cs}^* \), respectively. The only difference of \( D^0 \) and \( \overline{D^0} \) decay diagrams is that the initial and final particles change into their CP counterparts. So

\[
\frac{\overline{A}(f)}{A(f)} = \frac{\overline{A}(\overline{D^0})}{A(D^0)} \quad \text{to} \quad \frac{K^+K^-}{K^+K^-} \quad \text{is} \quad \eta_{\text{CP}}(K^+K^-) \frac{V_{us} V_{cs}^*}{V_{us} V_{cs}^*} = \eta_{\text{CP}}(K^+K^-) = +1.
\]

(11)
In Eq. (11), $V_{cs}$ and $V_{us}$ are both real in Wolfenstein parametrization for CKM matrix and $\eta_{CP}(f)$ is the CP parity of the final state $f$. Usually $\eta_{CP}(f) = \pm 1$ for different $f$. Actually, we can prove that (see the appendix in Ref. [2]) if the decays of $D^0$ and $\bar{D}^0$ only involve one CKM factor respectively, then the ratio

$$\frac{\bar{A}(f)}{A(f)} = \eta_{CP}(f) \frac{e^{-i\varphi_{wk}}}{e^{i\varphi_{wk}}} = \eta_{CP}(f).$$

(12)

The last equality holds only for charm decay because all the CKM matrix elements involved are real, if we neglect the penguin contribution.

Define

$$\rho_f = \frac{\bar{A}(f)}{A(f)}, \quad \overline{\tau}_f = \frac{A(\bar{f})}{A(f)}.$$  

(13)

From Eqs. (6) and (7), we have

$$\lambda_f = \frac{q}{p} \rho_f = \eta_{CP}(f) \left| \frac{q}{p} \right| e^{-i\varphi},$$

$$\overline{\tau}_f = \frac{p}{q} \overline{\tau}_f = \eta_{CP}(f) \left| \frac{p}{q} \right| e^{i\varphi}.$$  

(14)

After a straightforward calculation we arrive at

$$\Gamma(D^0_{p}(t) \rightarrow f) = \frac{1}{4} e^{-\gamma t} |A(f)|^2 \left\{ (1 + \left| \frac{q}{p} \right|^2)(e^{-\frac{1}{2} \Delta m t} + e^{\frac{1}{2} \Delta m t}) + 2\left(1 - \left| \frac{q}{p} \right|^2 \right) \cos \Delta m t \right\}$$

FIG. 1: Decay diagrams for $D^0, \bar{D}^0 \rightarrow K^+ K^-$. 
This is guaranteed by our approximation of neglecting the penguin, because in that case only one CKM factor appears [3].

In Eqs. (19) and (20), there are several parameters we need to know: the phase $\varphi$, $x = \Delta m/\gamma$, $y = \Delta \gamma/2\gamma$, $|q|/|p|$, etc. But we do know that $|x| \sim |y| \lesssim 10^{-2}$ (Ref. [1]), and $|q|/|p|$ is very close to unity. Some people assume [4] that $|q|/|p| - |p|/|q| \lesssim \pm 1\%$. As for the phase $\varphi$,

$$\frac{q}{p} = \left[ \frac{M_{12} - i\frac{1}{2} \Gamma_{12}}{M_{12} + i\frac{1}{2} \Gamma_{12}} \right]^{1/2} \frac{q}{p} e^{-i\varphi}. \quad (21)$$

In $B^0 - \bar{B}^0$ system, the box diagram dominance leads to

$$\left( \frac{q}{p} \right)_B \approx \sqrt{\frac{M_{12}}{M_{12}}} \approx e^{-2i\beta}, \quad \varphi_B = 2\beta. \quad (22)$$

In charm case, if we can use the same approximation and assume $b$-quark plays the dominant role in the corresponding box diagram, then

$$\left( \frac{q}{p} \right)_D \approx e^{-2i\gamma}, \quad \varphi_D = 2\gamma. \quad (23)$$

But it is not the case for charm. Firstly, the box diagram does not dominate. Secondly, even in the box diagram because $|V_{ub}|$ is very small compared with $|V_{ud}|$ and $|V_{us}|$, the internal $b$-quark contribution may be not important. Furthermore, $V_{ud}$ and $V_{us}$ do not carry the weak phase, $\varphi$ can also get the contribution from the $i\Gamma_{12}/2$ term. Anyway, we do not know the value of $\varphi$, so just keep it as a free parameter. For $e^{\pm 1/2 \Delta \gamma t}$, using $\Delta \gamma/(2\gamma) = y$, we have $e^{\pm 1/2 \Delta \gamma t} = e^{\pm y\gamma t} = e^{\pm yt/\tau_B^0}$, because $y \lesssim 10^{-2}$, $e^{\pm yt/\tau_B^0}$ is around unity.

$$N_f(t) \approx -4\eta_{CP}(f)\left( \frac{q}{p} - \frac{p}{q} \right)(y\gamma t) \cos \varphi + 8\eta_{CP}(f) \sin \varphi \sin \Delta mt, \quad (24)$$

Assume

$$|A(f)| = |\mathcal{A}(f)|. \quad (17)$$
\[ D_f(t) \approx 8 , \]
\[ C_f(t) \approx \eta_{\text{CP}}(f) \left\{ \frac{y\gamma t}{2} \left( \frac{p}{q} - \frac{q}{p} \right) \cos \varphi + \sin \varphi \sin \Delta mt \right\} \]
\[ = \eta_{\text{CP}}(f) \left\{ \frac{1}{2} y\gamma t \left( \frac{p}{q} - \frac{q}{p} \right) \cos \varphi + \sin \varphi \sin(x\gamma t) \right\} . \tag{26} \]

### III. TIME-INTEGRATED CP ASYMMETRY

In order to have more statistics, we integrate the time-dependent observables with time. We first list some useful quantities:

\[ G_+ = \int_0^\infty dt \ |g_+(t)|^2 = \frac{2 + x^2 - y^2}{2\gamma(1 + x^2)(1 - y^2)} \approx \frac{1}{\gamma} , \tag{27} \]
\[ G_- = \int_0^\infty dt \ |g_-(t)|^2 = \frac{x^2 + y^2}{2\gamma(1 + x^2)(1 - y^2)} \approx \frac{x^2 + y^2}{2\gamma} , \tag{28} \]
\[ G_{+-} = \int_0^\infty dt \ g_+(t)g_-(t) = \frac{-y(1 + x^2) + ix(1 - y^2)}{2\gamma(1 + x^2)(1 - y^2)} \approx \frac{-y + ix}{2\gamma} , \tag{29} \]

for \( x^2, y^2 \ll 1 \). It is straightforward to get the integrated decay width. From Eqs. (9) and (10) we have

\[ \Gamma(D^0_\mu \to f) = \int_0^\infty dt \ \Gamma(D^0_\mu(t) \to f) = |A(f)|^2 \left\{ G_+ + 2\text{Re}(\lambda_f G_-) + |\lambda_f|^2 G_- \right\} , \tag{30} \]
\[ \Gamma(D^0_\mu \to \overline{f}) = \int_0^\infty dt \ \Gamma(D^0_\mu(t) \to \overline{f}) = |\overline{A(f)}|^2 \left\{ G_+ + 2\text{Re}(\overline{\lambda_f} G_-) + |\overline{\lambda_f}|^2 G_- \right\} . \tag{31} \]

Again we assume \(|A(f)| = |\overline{A(f)}|\), then the time-integrated CP asymmetry is

\[ C_f = \frac{\Gamma(D^0_\mu \to f) - \Gamma(D^0_\mu \to \overline{f})}{\Gamma(D^0_\mu \to f) + \Gamma(D^0_\mu \to \overline{f})} = \frac{N_f}{D_f} , \tag{32} \]
\[ N_f = -2\eta_{\text{CP}}(f) \left[ y\left( \frac{p}{q} - \frac{q}{p} \right) \cos \varphi - x\left( \frac{q}{p} + \frac{p}{q} \right) \sin \varphi \right] + (x^2 + y^2)(\frac{q}{p}^2 - \frac{p}{q}^2) , \tag{33} \]
\[ D_f = 4 - 2\eta_{\text{CP}}(f) \left[ y\left( \frac{p}{q} + \frac{q}{p} \right) \cos \varphi + x\left( \frac{q}{p} - \frac{p}{q} \right) \sin \varphi \right] + (x^2 + y^2)(\frac{q}{p}^2 + \frac{p}{q}^2) \]
\[ \approx 4 . \tag{34} \]

Finally, neglecting the \((x^2 + y^2)\) term in Eq. (33), one can obtain

\[ C_f \approx \eta_{\text{CP}}(f) \left\{ -\frac{y}{2} \left( \frac{p}{q} - \frac{q}{p} \right) \cos \varphi + x \sin \varphi \right\} . \tag{35} \]

In Ref.\[3\], the first term in Eq. (35) is omitted and the CP parity factor \(\eta_{\text{CP}}(f)\) is missing.

Up to now, we have only discussed incoherent \(D^0 - \overline{D^0}\) decays. Sometimes \(D^0 - \overline{D^0}\) pairs are produced coherently, such as in \(e^+e^-\) colliding machines (BES and CLEO-c).

The time-evolved coherent state of \(D^0 - \overline{D^0}\) pair can be written as \[3\]

\[ |i\rangle = |D^0(k_1, t_1)\overline{D^0}(k_2, t_2)\rangle + \eta|\overline{D^0}(k_1, t_1)D^0(k_2, t_2)\rangle , \tag{36} \]

where \(\eta\) is the charge conjugation parity or orbital angular momentum parity of the \(D^0 - \overline{D^0}\) pair.

Because \(D^0 \to l^+X\) and \(\overline{D^0} \to l^-X\) only, we can use the semileptonic decay to tag one of the two time-evolved states \(D^0(t)\) and \(\overline{D^0}(t)\). We define the leptonic-tagging CP asymmetry \(C_{fi}\) as

\[ C_{fi} = \frac{N(l^-, f) - N(l^+, \overline{f})}{N(l^-, f) + N(l^+, \overline{f})} , \tag{37} \]
Comparing Eq. (45) with Eq. (35), we find that

\[ N(l^-, f) = \int_0^\infty dt_1 dt_2 |\langle l^-, f | H_{\text{eff}} | i \rangle|^2 \]  

is proportional to the number of events in which \( D^0_\ell(k, t) \rightarrow l^- X \) as tagging in one side and the other side is the decay \( \bar{D}^0_\ell(k, t) \rightarrow f \) or vice versa. Similarly,

\[ N(l^+, \bar{f}) = \int_0^\infty dt_1 dt_2 |\langle l^+, \bar{f} | H_{\text{eff}} | i \rangle|^2 . \]

Assuming \(|A(f)| = |\overline{A(f)}|\) and \(|A(l^+)| = |\overline{A(l^-)}|\), after a tedious calculation, we have

\[ N(l^-, f) = |\overline{A(l^-)}A(f)|^2 \left\{ G_+^2 + G_-^2 + 2|\lambda_f|^2[G_+ G_- + \eta |G_+|^2] 
+ 2(1 + \eta) G_+ \text{Re}(\lambda_f G_+^*) + (1 + \eta) G_+ \text{Re}(\lambda_f G_-^*) + 2 \eta \text{Re}(G_+^2) \right\} , \]

\[ N(l^+, \bar{f}) = |A(l^+)\overline{A(\bar{f})}|^2 \left\{ \overline{G_+^2} + \overline{G_-^2} + 2|\overline{\lambda_f}|^2[\overline{G_+ G_-} + \eta |\overline{G_+}|^2] 
+ 2(1 + \eta) \overline{G_+} \text{Re}(\overline{\lambda_f G_+^*}) + (1 + \eta) \overline{G_+} \text{Re}(\overline{\lambda_f G_-^*}) + 2 \eta \text{Re}(\overline{G_+^2}) \right\} , \]

\[ C_{fl} = \frac{N(l^-, f) - N(l^+, \bar{f})}{N(l^-, f) + N(l^+, \bar{f})} = \frac{N_f}{D_f} , \]

\[ N_f = 2(1 + \eta) \frac{x^2 + y^2}{2\gamma^2} \left\{ \left( \frac{y}{p} \right)^2 + (1 + \eta) \eta_{CP}(f) \frac{x^2 + y^2}{2\gamma^2} \left[ \frac{p}{q} \right] \cos \varphi 
- x \left( \frac{q}{p} \right) \sin \varphi \right\} 
+ x \left( \frac{q}{p} \right) \sin \varphi \right\} \approx \frac{2(1 + \eta) \eta_{CP}(f)}{\gamma^2} \left\{ \frac{y}{2} \left( \frac{q}{p} \right) \cos \varphi + x \sin \varphi \right\} , \]

\[ D_f = \frac{2}{\gamma^2} + \frac{(x^2 + y^2)^2}{2\gamma^2} + \frac{1}{2\gamma^2} \left( \frac{p}{q} \right) \left( \frac{q}{p} \right) (2 + \eta)(x^2 + y^2) 
+ (1 + \eta) \eta_{CP}(f) \frac{x^2 + y^2}{2\gamma^2} \left[ -y \left( \frac{p}{q} \right) + \left( \frac{q}{p} \right) \right] \cos \varphi 
- x \left( \frac{q}{p} \right) \sin \varphi \right\} 
+ x \left( \frac{q}{p} \right) \sin \varphi \right\} 
\approx \frac{2}{\gamma^2} . \]

Finally

\[ C_{fl} = \frac{N_f}{D_f} = (1 + \eta) \eta_{CP}(f) \left\{ -\frac{y}{2} \left( \frac{q}{p} \right) \cos \varphi + x \sin \varphi \right\} . \]

Comparing Eq. (45) with Eq. (35), we find that \( C_{fl} \) is just twice as large as \( C_f \) when the charge conjugation parity or the orbital angular momentum \( l \) is even. This is surprising. From Eq. (35), the order of magnitude of \( C_f \) is \( \lesssim 10^{-3} \), because \( x \sim y \lesssim 10^{-2} \). Now we present \( C_f \) (theory), \( C_f \) (exp.), branching fractions for \( D^0, \bar{D}^0 \) decay into CP eigenstates and the number of \( D - \bar{D} \) pairs needed for testing CP asymmetry for 1σ signal lower bound in TABLE I, where for the branching ratios we take most of them from the 2006 particle data book [1]. For the measured CP asymmetries listed in Table I are also taken from the 2006 particle data book [1]. We use the formula for \( N_{\overline{D}} \)

\[ N_{\overline{D}} = \frac{1}{BC^2_f} \quad \text{for 1σ signature} ; \]

\[ N_{D} = \frac{9}{BC^2_f} \quad \text{for 3σ signature} . \]
TABLE I: The number of $D\overline{D}$ pairs needed for testing CP asymmetry

| $D^0 \rightarrow f^+$ | $C_f$ (theory) | $C_f$ (exp.) | BR | $N_{D\overline{D}}$ (1σ lower bound) |
|----------------------|----------------|--------------|----|-----------------------------------|
| $K^+ K^-$            | 0.014 ± 0.010  | (3.84 ± 0.10) $\times 10^{-3}$ | 2.60 $\times 10^7$ |
| $K_s K_s$            | −0.23 ± 0.19   | (3.7 ± 0.7) $\times 10^{-4}$ | 2.70 $\times 10^8$ |
| $K^+ K^-$ $\sim 10^{-3}$ | 1.0 $\times 10^{-2}$ (BSW) | 1 $\times 10^7$ |
| $\pi^+ \pi^-$       | 0.013 ± 0.012  | (1.364 ± 0.032) $\times 10^{-3}$ | 7.4 $\times 10^7$ |
| $\pi^0 \pi^0$       | 0.00 ± 0.05    | (7.9 ± 0.8) $\times 10^{-4}$ | 1.26 $\times 10^8$ |
| $\rho^+ \rho^-$     | (3.2 ± 0.4) $\times 10^{-3}$ | 3.13 $\times 10^7$ |
| $\rho^+ \rho^0$     | 1.3 $\times 10^{-3}$ (BSW) | 7.69 $\times 10^6$ |
| $\phi \pi^0$        | (7.4 ± 0.5) $\times 10^{-4}$ | 1.35 $\times 10^8$ |
| $\phi \eta$         | (1.4 ± 0.4) $\times 10^{-4}$ | 7.14 $\times 10^8$ |
| $K^{*+} K^{*-}$     | (7 ± 5) $\times 10^{-5}$ | 1.43 $\times 10^9$ |

IV. SUMMARY AND CONCLUSION

We have computed the time-dependent and time-integrated CP asymmetry for neutral charmed meson decays into CP eigenstates. We present CP asymmetry not only for incoherent $D^0 - \overline{D^0}$, but also coherent $D^0 \overline{D^0}$ pairs. We find that the time-integrated CP asymmetries are very small (order of $\lesssim 10^{-3}$). We also give the lower bound for the number of $D\overline{D}$ pairs needed for testing the CP asymmetries. At present, the integrated luminosities for $e^+e^-$ colliders are:

- BES II: 27 pb$^{-1}$
- BES III: 20 fb$^{-1}$ for 4 years data taking.
- CLEO $-c$: 281 pb$^{-1}$

The corresponding $D\overline{D}$ pairs are:

- BES II: $10^5$
- BES III: $10^7$
- CLEO $-c$: $10^6$

In Table I, for $D \rightarrow VV$ decays, only when both vector mesons are longitudinally polarized the $VV$ final states are CP eigenstates. For the corresponding branching ratios in Table I, we assume that the $VV$ final states for which both $V$ are longitudinally polarized dominate. From Table I we see that the only hope is relying on BES III and $B$ factories. At $B$-factories, because the large data sample of charmed meson, both time-dependent asymmetry and time-integrated asymmetry can be measured. While for BES III, only time-integrated CP asymmetry can be tested. Of course, if there is new physics, some surprise may happen. We can also see from Table I that all the measured CP asymmetries are consistent with zero.

Acknowledgments

I thank Hai-bo Li, Cai-dian Lü, Mao-zhi Yang and Zhi-zhong Xing for discussions. This work is supported in part by the National Natural Science Foundation of China under grants NSFC 90103011, 10375073 and 90403024.

[1] I. Shipsey, hep-ex/0607070 (2006).
[2] D.S. Du, I. Dunietz and D.D. Wu, Phys. Rev. D 34, 3414 (1986).
[3] D.S. Du, Phys. Rev. D 34, 3428 (1986), Eqs. (11), (12) and (13).
[4] Z.Z. Xing, Phys. Rev. D 55, 196 (1997).
[5] http://pdg.lbl.gov, page 34-44.
[6] M. Bauer, B. Stech and M. Wirbel, Z. Phys. C 34, (1989).