Mitigating Query-Flooding Parameter Duplication Attack on Regression Models with High-Dimensional Gaussian Mechanism

Xiaoguang LI†‡, Hui LI*, Haonian YAN*, Zelei Cheng†, Wenhai SUN† and Hui ZHU *

* State Key Laboratory of Integrated Service Network, School of Cyber Engineering, Xidian University, Xi’an, China
xg_li@outlook.com, lihui@mail.xidian.edu.cn, yanhaonan.sec@gmail.com, zhuhui@xidian.edu.cn
† Department of Computer and Information Technology, Purdue University, West Lafayette, USA
li3660@purdue.edu, cheng473@purdue.edu, sun841@purdue.edu

Abstract—Public intelligent services enabled by machine learning algorithms are vulnerable to model extraction attacks that can steal confidential information of the learning models through public queries [36]. Differential privacy (DP) has been considered a promising technique to mitigate this attack. However, we find that the vulnerability persists when regression models are being protected by current DP solutions. We show that the adversary can launch a query-flooding parameter duplication (QPD) attack to infer the model information by repeated queries.

To defend against the QPD attack on logistic and linear regression models, we propose a novel High-Dimensional Gaussian (HDG) mechanism for regression models. By adjusting the privacy budget to reflect the users’ privacy level on each dimension, DP can obfuscate the query results accordingly. The smaller (larger) the privacy budget is set for higher (lower) privacy, the lower (higher) accuracy of the results. However, we find that the existing DP-based mechanism for regression models cannot withstand the QPD attack while satisfying the privacy requirements. We also prepare to open-source the relevant codes to the community for further research.

Index Terms—Differential privacy, machine learning, query-flooding parameter duplication attack, high-dimensional Gaussian mechanism, optimal privacy budget allocation

I. INTRODUCTION

Service providers (SPs) train the machine learning models with massive data and use them to offer various intelligent services, such as Azure Cognitive Service [37], Amazon TextTract [1], Google Cloud Vision [3] and Speech [2]. The trained models are considered critical proprietary assets of SPs because: 1) the learning model may be trained by a private dataset. So it is unwise to publish the model without protection; 2) Training with a large dataset could be costly. SPs may prefer to keep their models confidential and charge clients to amortize the cost. Recent work shows that the adversary can exploit model extraction attacks to infer the internal states of machine learning models through the public APIs of intelligent applications [19], [31], [36], [40].

Fig. 1. Illustration of QPD Attack

Differential privacy (DP) [4], [9], [12], [14], [35], [41] has been widely recognized as an effective privacy-preserving technique to defeat the adversarial model extraction attempts. By adjusting the privacy budget to reflect the users’ privacy level on each dimension, DP can obfuscate the query results accordingly. The smaller (larger) the privacy budget is set for higher (lower) privacy, the lower (higher) accuracy of the results. However, we find that the existing DP-based mechanism for regression models cannot withstand the query-flooding parameter duplication (QPD) attack. Specifically, as shown in Fig. 1 for an n-dimensional regression model \( f(x) \), the adversary first constructs a query group \( Q \) that contains at least \( n + 1 \) linearly independent queries. The group \( Q \) is then duplicated \( r \) times before being sent to the DP-protected APIs of the services. On receiving the result vectors \( \{ y_1, \ldots, y_r \} \), the adversary can recover the real result vector \( z \) of \( Q \) by reducing the DP noise based on the law of large numbers [23]. In the end, an inferred model \( f(x) \) can be derived by analyzing the query \( Q \) and result \( z \).

The root cause of the QPD attack is that a constant privacy budget is used to generate the identically-distributed noise to protect the system. So the adversary may recover the true result by statistic analysis and further infer the model information (see Sect. [IV] for attack details). Simply blocking the duplicate queries is not a practical option. To design an effective DP mitigation of this devastating attack, we need to answer the following two challenging questions:

1. How to dynamically set privacy budgets that produce...
uncorrelated noise while satisfying user-desired privacy level on each dimension.

2. How to automatically set the privacy budgets in an optimal manner that minimum noise is introduced to meet the privacy requirements of users? – This has been an open problem concerning the DP implementation in practice [36].

In this work, we focus on the design of \((\epsilon, \delta)\)-differentially private linear and logistic regression models and propose a novel High-Dimensional Gaussian (HDG) mechanism to defend against the powerful QPD attack. To our best knowledge, HDG is the first DP scheme that can address both of the mentioned challenges. Under the hood, HDG randomly assigns the received queries into different query groups. Then it automatically sets unpredictable privacy budgets for each query group to produce the noise for QPD mitigation. The HDG mechanism requires two additional initialization parameters: Sum of privacy budgets (SPB) \(\epsilon_s\) of all dimensions of the model and the upper bound \(\rho\) of the desired distortion degree \(\Gamma\). \(\rho\) and \(\Gamma\) respectively control the upper and lower bound of the privacy level on each dimension provided by the HDG mechanism. We carefully design the privacy budget allocation algorithm to make it optimal, i.e. generating minimum noise to satisfy the privacy requirement and increase the model utility, and resilient to a mistakenly-configured large privacy budget sum \(\epsilon_s\) without significant loss of the desired privacy level. We experiment to demonstrate the practicality of the QPD attack and the effectiveness of the proposed countermeasure. The result shows that HDG outperforms other DP-based mechanisms in model protection and utility. Our contribution can be summarized as follows:

**New attack vector.** We develop a new query-flooding parameter duplication attack that can be exploited to effectively infer DP-protected models. We demonstrate its viability for regression models and discuss the potential harmful impact on other types of learning models.

**Countermeasure against QPD attack.** We propose a High-Dimensional Gaussian (HDG) mechanism to protect the linear and logistic regression models against the QPD attack. To this end, HDG can generate the uncorrelated noise while satisfying user-desired privacy level on each dimension.

**Optimal and resilient privacy budget allocation.** For the first time, we design an automatic privacy budget allocation algorithm to keep the minimum impact on model utility while meeting the expectation of privacy preservation. The algorithm is also insensitive to the misconfiguration of a large privacy budget to maintain DP functionalities.

**Comprehensive evaluation and open source for reproducibility.** We develop a prototype system for evaluation. The result shows the validity of the QPD attack and satisfactory performance of the proposed countermeasure. To help the community better understand the new attack and our defense mechanism, we will publish the relevant source code for reproducibility and further research explorations.

We introduce the related work in this section.

### II. Related Work

#### A. Differential Privacy

Differential privacy is an effective technique to protect the privacy of valuable information. DP was first introduced by Dwork et al. [9]. Since then, extensive studies have been performed to meet different privacy requirements for various settings [7], [9], [13], [15], [28], [34]. In differential privacy, how to set the key parameter – privacy budget has been attracting widely attention. To explore the best way to set the privacy budget, Kohli et al. [21] designed a chooser mechanism according to users’ preferences. Naldi et al. [29] proposed an estimated theory based method to choose privacy budget for the Laplace mechanism [11]. In [22], Lee et al. defined a new attack model and analyzed the posterior probability of privacy leakage to determine the privacy budget. In [18], the privacy budget is set by using game theory, i.e. whether data subjects opt-in to a privacy study determines the privacy budget. Unfortunately, there is no consensus on how to set the privacy budget as of now [10]. Privacy budget allocation is a complicated problem, ill-designed methods may result in flawed DP mechanisms. Li et al. discovered a multi-time query attack in [23]. If an adversary sends the same queries many times, he can infer the true results with a certain degree of confidence. They also proved that many shared DP-based mechanisms are insecure from the perspective of information theory when the number of queries is larger than 3. It is the problem that we solve in this paper.

#### B. Model Extraction Attack

Tramer et al. proposed model extraction attack for many machine learning models (e.g. logistic regression, decision tree, SVM and neural network) in literature [36]. Since that, many researchers focus on how to make model extraction attack more effective and efficient. Papernot et al. proposed a Jacobin based synthetic data augmentation technology [31] to train a synthetic DNN model and they proved this attack is practical for adversaries with no knowledge about the model. Juuti et al. proposed a new model extraction attacks [19] using novel approaches for generating synthetic queries and optimizing training hyperparameters. Shi et al. proposed an exploratory model extraction attack [40] by deep learning that can steal functionalities of Naive Bayes models and SVM classifiers with high accuracy. Duddu et al. [8] infer the depth of neural network using timing side channels and used reinforcement learning based optimization to accelerate the extraction process. Wang et al. proposed hyperparameter stealing attacks [33] for both non-kernel models and kernel models with the help of zero gradient technology.

Due to lack of DP-based mechanisms for model extraction attacks, existing model extraction attacks concentrate on various machine learning models. We propose a new effective QPD attack for DP-protected regression models.
C. Resistance to Model Extraction Attack

In order to defend model extraction attack, Zheng et al. proposed boundary differentially private layer (BDPL) to defend model extraction attack for binary classifier. This is the only DP-based work resisting model extraction attack. Unfortunately, they do not deliberate the privacy budget setting, and the extraction rate will be near 90% when the number of queries is large. Besides BDPL, Kesavani et al. proposed model extraction monitor that quantifies the extraction status of models by continually observing the API queries and response streams of users to defend model extraction attack. Quiring et al. mitigate model-extraction attacks with the closeness-to-the-boundary concept in digital watermarking.

We solve the problem in previous DP-based mechanism and for the first time propose a dynamic privacy budget allocation in our HDG mechanism to mitigate the QPD attack.

III. PRELIMINARIES

Let \( \mathcal{X} \) be a training dataset containing \( m \) tuples \((x^{(1)}, y^{(1)}), (x^{(2)}, y^{(2)}), \ldots, (x^{(m)}, y^{(m)})\). The \( i \)-th tuple \((x^{(i)}, y^{(i)})\) includes \( n + 1 \) explanatory attributes (or “dimensions”, “features”) \( x_1^{(i)}, x_2^{(i)}, \ldots, x_n^{(i)}, y^{(i)} \), where \( (x_1^{(i)}, x_2^{(i)}, \ldots, x_n^{(i)}) \) is the input of the regression model \( f(x) \) and \( y^{(i)} \) is the predicted value (or called “label” in logistic regression) corresponding to the \( x^{(i)} \). Based on the above notations we introduce the necessary preliminary knowledge used in this work. We use “dimension” or “feature” instead “attribute”, and use “label” instead “predicted value” for logistic regression.

A. Linear Regression

**Definition 1** (Linear Regression). An \( n \)-dimensional linear regression model trained on dataset \( \mathcal{X} \) is a prediction function which returns the predicted value

\[
f(x) = a^T x + b
\]

where \( a \in \mathbb{R}^n \) and \( b \in \mathbb{R} \) are coefficients that minimize the cost function

\[
J(a, b) = \frac{1}{2m} \sum_{i=1}^{m} (y^{(i)} - a^T x^{(i)} - b)^2.
\]

B. Logistic Regression

**Definition 2** (Logistic Regression). An \( n \)-dimensional logistic regression model trained on dataset \( \mathcal{X} \) is a prediction function which returns 1 with probability

\[
f(x) = \frac{1}{1 + e^{-(a^T x + b)}}
\]

where \( a \in \mathbb{R}^n \) and \( b \in \mathbb{R} \) are coefficients that minimize the cost function

\[
J(a, b) = -\sum_{i=1}^{m} \left[ y^{(i)} \log f(x^{(i)}) + (1 - y^{(i)}) \log (1 - f(x^{(i)})) \right]
\]

C. \((\epsilon, \delta)\)-Differential Privacy

Differential privacy provides a privacy guarantee independent of adversaries with any background knowledge. Formally,

**Definition 3** ((\(\epsilon, \delta\))-Differential Privacy). A randomized algorithm \( \mathcal{M} \) satisfies \((\epsilon, \delta)\)-differential privacy iff for a pair of neighbor datasets \( \mathcal{X} \) and \( \mathcal{X}' \) which only differ in one tuple, randomized algorithm \( \mathcal{M} \) satisfies the following equation:

\[
Pr[\mathcal{M}(\mathcal{X}) \in S] \leq e^\epsilon \times Pr[\mathcal{M}(\mathcal{X}') \in S] + \delta
\]

where \( \epsilon \) is privacy budget and \( S \) is any subset of the range of \( \mathcal{M} \).

In differential privacy, privacy budget \( \epsilon \) is often set to be a small real number such as 1 or \( \ln 2 \), and \( \delta \) is set to be the inverse of any polynomial in the size of dataset, so it could be regarded as a constant when the dataset is known. When the \( \epsilon \) reduces, all of the data in dataset \( \mathcal{X} \) become almost equally likely. Thus, the smaller the privacy budget \( \epsilon \) is, the higher privacy level the mechanisms provides.

Differential privacy mechanisms are often required to calibrate the noise to global sensitivity. For different design, global sensitivity includes but not limited to \( \ell_1 \) global sensitivity and \( \ell_2 \) global sensitivity. In this paper, we only introduce the \( \ell_2 \) global sensitivity that is used in our mechanism.

**Definition 4** (\(\ell_2\) Global Sensitivity). For a function \( f : \mathcal{X} \rightarrow \mathbb{R}^n \), \( \mathcal{X} \) and \( \mathcal{X}' \) are a pair of neighbor datasets that only differ in one element. The \( \ell_2 \) global sensitivity \( \Delta_2 f \) is defined as:

\[
\Delta_2 f = \max_{X,X'} \| f(X) - f(X') \|_2
\]

From the perspective of differential privacy theory, global sensitivity of a function \( f \) captures the maximum distance by which a single tuple in the dataset can change the function \( f \) in the worst case. In addition, global sensitivity is independent of the dataset, it only relates to query function itself. Our mechanism is based on Gaussian mechanism.

**Definition 5** (Gaussian Mechanism). Gaussian mechanism is a \((\epsilon, \delta)\)-DP-based mechanism. For a function \( f : \mathcal{X} \rightarrow \mathbb{R}^n \), Gaussian mechanism \( \mathcal{M} \) injects Gaussian noise \( N \) into \( f(X) \) to protect privacy. Formally,

\[
\mathcal{M}(\mathcal{X}) = f(\mathcal{X}) + N = \mathcal{N}(f(\mathcal{X}), \sigma^2)
\]

where \( \sigma \geq \sqrt{2 \ln \left( \frac{1.25}{\delta} \right) \times \Delta_2 f / \epsilon} \).

D. Multivariate Gaussian Distribution

**Definition 6** (Multivariate Gaussian Distribution). An \( n \)-dimensional random vector \( X \) follows multivariate Gaussian distribution \( \mathcal{N}(\mu, \Sigma) \) if it has the density function:

\[
f_X(x_1, ..., x_n) = \frac{\exp(-\frac{1}{2}(X - \mu)^T \Sigma^{-1} (X - \mu))}{\sqrt{(2\pi)^n |\Sigma|}}
\]

where \( \mu \in \mathbb{R}^n \) is the mean vector of \( X \), and \( \Sigma \in \mathbb{R}^{n \times n} \) is the covariance matrix.
Multivariate Gaussian is widely generated by transformations based on the variance-covariance matrix [16].

IV. QUERY-FLOODING PARAMETER DUPLICATION ATTACK

In this section, we introduce the query-flooding parameter duplication attack. The QPD attack can be exploited to extract the coefficients of the logistic/linear regression models that are being protected by the state-of-the-art DP mechanisms [21]. We assume that the adversary can access all public APIs provided by the data owner.

Logistic and linear regression models are essentially linear equations. It is possible for an adversary to extract an n-dimensional logistic or linear regression model by querying the public APIs n + 1 times [36]. The QPD attack can be carried out by combining the naïve equation-solving model extraction attack with the multi-time query attack [23]. In a nutshell, given a DP-protected linear or logistic regression model \( f(x) \), we create \( n + 1 \) linearly independent queries denoted by Query Matrix \( Q \). Next, we duplicate each query \( q_j \in Q \) \( r \) times and get the duplicated query matrix \( Q_d \). According to the law of large numbers, the larger the \( r \) is, the more similar the extracted models are to the original models. In practice, we often choose a large number \( r \) and the probability of obtaining the true results is \( 1 - \mathcal{O}(\frac{1}{r}) \). Then, we send the query \( Q_d \) to the model \( f(x) \) and get the results \( \mathcal{Y} \). Because each query \( q_j \in Q \) is duplicated \( r \) times, each query \( q_j \in Q \) corresponds to \( r \) different perturbed results \( \{y_{i(j)}\}_{i=1}^r \in \mathcal{Y} \). Finally, we can solve the \( n + 1 \) coefficients \( a_1, \ldots, a_n \) and \( b \) based on the law of large numbers and the Cramer’s rule. The detailed process of the QPD attack is shown as follows.

Initially, we estimate the true result \( z^{(j)} \) for each query \( q_j \in Q \) according to the law of large numbers,

\[
z^{(j)} = \frac{1}{r} \sum_{i=1}^{r} y_{i(j)}.
\]

Second, given the true result \( z^{(j)} \) for each \( q_j \), we have \( n + 1 \) equations denoted by the augmented matrix \( Q_A \).

\[
Q_A = (Q|z) = \left(\begin{array}{cccccc}
x_1^{(1)} & \cdots & x_k^{(1)} & \cdots & x_n^{(1)} & 1 \\
\vdots & \ddots & \vdots & \ddots & \vdots & \vdots \\
x_1^{(n+1)} & \cdots & x_k^{(n+1)} & \cdots & x_n^{(n+1)} & 1 \\
\end{array}\right)
\]

where \( x_k^{(j)} \) is the value in the \( k \)-th dimension of the \( j \)-th query \( q_j \). Then we replace the \( k \)-th column of query matrix \( Q \) with the column \( z \) to construct the k-Result Matrix \( Q_k \).

\[
Q_k = \left(\begin{array}{cccccc}
x_1^{(1)} & \cdots & z^{(1)} & \cdots & x_n^{(1)} & 1 \\
\vdots & \ddots & \vdots & \ddots & \vdots & \vdots \\
x_1^{(n+1)} & \cdots & z^{(n+1)} & \cdots & x_n^{(n+1)} & 1 \\
\end{array}\right).
\]

Finally the coefficient can be solved by the Cramer’s rule

\[
a_k = \frac{\text{det}(Q_k)}{\text{det}(Q)}, \quad b = \frac{\text{det}(Q_{n+1})}{\text{det}(Q)} = \frac{Q_{n+1}}{Q}, \quad k \leq n.
\]

The QPD attack is shown in the Algorithm 1.

Algorithm 1 QPD Attack

Input: DP-protected \( n \)-dimensional regression model \( f(x) \)
Output: Extracted \( n \)-dimensional regression model \( \tilde{f}(x) \)

1: Construct a query matrix \( Q \) containing \( n + 1 \) linearly independent queries \( q_j \)
2: for each \( q_j \in Q \) do
3: Duplicate \( q_j \) \( r \) times
4: end for
5: Send the duplicated query matrix \( Q_d \) to \( f(x) \) and get the results \( \mathcal{Y} \)
6: for each \( \{y_{i(j)}\}_{i=1}^r \in \mathcal{Y} \) corresponding to \( q_j \in Q \) do
7: \( z^{(j)} = \frac{1}{r} \sum_{i=1}^{r} y_{i(j)} \)
8: end for
9: Construct column vector \( z = [z_1, z_2, \ldots, z_{n+1}]^T \)
10: for \( k \leq n \) do
11: Construct \( k \)-result matrix \( Q_k \) by replacing the \( k \)-th column in \( Q \) by \( z \)
12: \( a_k = \frac{Q_k}{Q} \)
13: end for
14: \( b = \frac{Q_{n+1}}{Q} \), \( a = [a_1, a_2, \ldots, a_n] \)
15: return \( \tilde{f}(x) = a^T x + b \)

Attack efficacy. We highlight the main evaluation results of the QPD attack and defer the detailed discussion to Section VI. We launch the QPD attack on unprotected linear regression models and BDPL-protected logistic regression models because BDPL is the only DP-based mechanism against model extraction attack in this context. We use real-world datasets [6] (see Section VI) to evaluate the attack. The main results are shown in Tab. I. In particular, we measure the utility of the extracted models to assess the performance of the
QPD attack. High utility of the extracted models represents
the high effectiveness of the QPD attack. We use Extraction
Mean Squared Error (EMSE) and Extraction Rate (ER) to
measure the similarity between the extracted models and
the original models. Smaller RMSE and higher RE indicate
higher similarity of the extracted models. Table I shows that the
efficacy of the QPD attack. On all of the datasets, the similarity
increases with the growth of \( r \), which means the QPD attack
is more effective as the \( r \) grows.

Albeit the QPD attack is shown to be effective for linear
and logistic regression models, it could be applied to other
machine learning models, such as multiclass logistic regression
model and multilayer perceptrons. Because these models are
constructed by some equations. The equations are non-linear
however, query-flooding can still impair the protection of
differential privacy and allows adversaries to infer the true
results corresponding to his queries. Thus adversaries can steal
the models by constructing the equations about the model
coefficients to solve or adapting the model implementations
for equation-solving model extraction attack.

V. HIGH-DIMENSIONAL GAUSSIAN MECHANISM

In this section, we first overview the HDG mechanism and
then describe the details of the proposal. We summarize the
notations used in HDG in Table II.

A. Overview

QPD attack uses the law of large numbers to reduce the
obfuscation produced by probability based differential privacy
technology. The key to mitigate the QPD attack is making the
data follow correlated and nonidentical distributions. Considering this, we propose a High-dimensional Gaussian (HDG)
mechanism to mitigate the powerful QPD attack. In order to
solve the first challenge that how to dynamically set privacy
budgets that produce uncorrelated noise while satisfying user-
desired privacy levels for each dimension, we use grouping
strategy and generate the optimal privacy budget for different
queries under user-desired privacy levels for each dimension.
As for the second challenge, referring to differential entropy,
we construct and solve the optimization problem about the
privacy budget such that the generated noise is minimum. The
Algorithm 1 describes the HDG mechanism.

Concretely, HDG mechanism executes the following steps.
We address the first challenge in the Step 2, 3, 4, and the
second challenge is addressed in the Step 5.

1. Initialization. Before the execution of the HDG mechanism,
\( \ell_2 \) global sensitivity is hard-coded into the HDG
mechanism. Then data administrators set the initialization
parameters SPB \( \epsilon_s \) and the upper bound \( \rho \) of the distortion
degree for the HDG mechanism (Section V-B).

2. Generate Query Groups \( Q_i \). After receiving the queries
\( Q \) from all users, HDG mechanism normalizes them and
randomly arrange all queries into different groups \( Q_i \) with
size \( n + 1 \). If the number of queries is not enough to form
a group, HDG mechanism generates some random padding
queries to pad users’ queries as an entire query group.

3. Calculate Distortion Degree set \( \Gamma \) for the Model \( f(x) \).
In this step, HDG mechanism uses \( \rho \), relative error and
feature selection algorithms (e.g. RFECV) to calculate the
distortion degree \( \Gamma_i \in \Gamma \) for the \( i \)-th dimension of the model
\( f(x) \). This allows the HDG mechanism to construct the

![Fig. 2. Workflow of HDG mechanism](image)

### Table II

**Table of Notations**

| Notation | Description |
|----------|-------------|
| \( \epsilon_s \) | Sum of privacy budgets (SPB) on all dimensions |
| \( \epsilon_e \) | Privacy budget for the \( i \)-th dimension |
| \( n \) | The number of dimensions of the model |
| \( Q \in \mathbb{R}^{m \times n} \) | All received queries, where \( m \) is the number of queries and \( n \) is the number of features |
| \( Q_i \in \mathbb{R}^{(n+1) \times n} \) | The \( i \)-th query group (matrix) after random arrangement, which contains \( n + 1 \) \( n \)-dimensional queries |
| \( Q^h_i \) | the \( h \)-Result matrix for \( Q_i \) |
| \( q^{(i)} \in Q_i \) | The \( j \)-th query vector in \( Q_i \) |
| \( f(x) \) | Original linear or logistic regression model |
| \( \Sigma \) | Covariance matrix in HDG mechanism |
| \( \Gamma \) | Set of all distortion degree for model \( f(x) \) |
| \( \Gamma_i \in \Gamma \) | Distortion degree on the \( i \)-th dimension of \( f(x) \) |
| \( \rho \) | Upper bound of the distortion degree |
| \( \Delta_f \) | \( \ell_2 \) global sensitivity of \( f(x) \) |
| \( z^{(i)} \in \mathbb{R} \) | the result corresponding to the \( i \)-th query \( q^{(i)} \) |
Algorithm 2 High-Dimensional Gaussian Mechanism

Input: \( n \)-dimensional queries \( Q \); \( n \)-dimensional linear or logistic regression model \( f(x) \); SPB \( \epsilon, \delta \); upper bound \( \rho \) of distortion degree

Output: HDG-protected result \( f'(Q) \)

1. Normalize and randomly arrange the received queries \( Q \) into different query groups \( Q_1, Q_2, \ldots \) with size \( n + 1 \).
2. if The size of \( Q_i \) is less than \( n + 1 \) then
3. Generate random padding queries for \( Q_i \) such that the size is \( n + 1 \).
4. end if
5. Calculate distortion degree \( \Gamma \) for all dimensions in the model based on \( \rho \)
6. Construct two types of privacy budget constraints for all query groups \( Q_i \) based on the distortion degree \( \Gamma \) and SPB \( \epsilon, \delta \)
7. for each query group \( Q_i \) do
8. Solve the optimal privacy budget \( \epsilon \) under the privacy budget constraints
9. for each query \( q^{(j)} \in Q_i \) do
10. Based on \( \epsilon \), generate \( (n + 1) \)-dimensional Gaussian noise vector \( N_j \)
11. Split \( N_j = [N_j^{input}, N_j^{output}] \).
12. \( q^{(j)*} = q^{(j)} + N_j^{input} \)
13. \( f'(q^{(j)*}) = f(q^{(j)*}) + N_j^{output} \)
14. end for
15. \( f'(Q_i) = \bigcup_j f'(q^{(j)}) \)
16. end for
17. return \( f'(Q) = \bigcup_i f'(Q_i) \)

We prove that the HDG mechanism satisfies the differential privacy in the Theorem 1.

Theorem 1. HDG mechanism satisfies \((\epsilon, \delta)\)-differential privacy, where \( \epsilon = \sum_{i=1}^{n} \epsilon_i \), \( \delta = \sum_{i=1}^{n} \delta_i \), \( \epsilon_i \) and \( \delta_i \) are parameters allocated in the \( i \)-th dimension of the model.

Proof. The projection of the multivariate Gaussian noise in each dimension is one-dimensional Gaussian distribution. So the HDG mechanism satisfies \((\epsilon_i, \delta_i)\)-differential privacy in the \( i \)-th dimension. According to sequential composition [26], HDG satisfies \((\epsilon, \delta)\)-differential privacy where \( \epsilon = \sum_{i=1}^{n} \epsilon_i \) and \( \delta = \sum_{i=1}^{n} \delta_i \).

Security. The HDG mechanism guarantees the following security properties.

- HDG mechanism can defend against the QPD attack while keeping the utility of the model. This is because the two types of privacy budget constraints allow the HDG mechanism to solve the optimal privacy budgets while satisfying the desired privacy level on each dimension. Besides, the privacy budgets and noise are dynamically generated every time, it is difficult for adversaries to predict and reduce the noise.
- HDG is resilient to misconfigured SPB. In differential privacy, larger privacy budget provides lower privacy level. However, the HDG mechanism mitigates this problem by ensuring the solved privacy budgets are under the two types of privacy budget constraints even if a misconfigured SPB is set initially.

B. Global Sensitivity

In general, calculating global sensitivity is an NP-hard problem [30]. However, we prove that the \( \ell_2 \) global sensitivity \( \Delta_2 f \) for both linear and logistic regression models is a constant. Then we can hard-code \( \Delta_2 f \) into the HDG mechanism so that it is not necessary to calculate it again. We solve the \( \Delta_2 f \) for linear and logistic regression simultaneously, because these models are essentially linear functions with an identical form. Linear regression models can be represented as: \( f(x) = a^T x + b \); logistic regression models can be represented as: \( -ln(\frac{1}{x} - 1) = a^T x + b \). Hence, both models can be written as \( h(x) = a^T x + b \). So we only need to solve the \( \Delta_2 f \) on this linear function. The proof is simple. Specifically, we first create a pair of neighbor training datasets \( \mathcal{X} \) and \( \mathcal{X}' \) such that the \( \Delta_2 f = \max_{\mathcal{X}, \mathcal{X}'} \|f(\mathcal{X}) - f(\mathcal{X}')\| \), where \( f(\mathcal{X}) \) and \( f(\mathcal{X}') \) are the models trained by the datasets \( \mathcal{X} \) and \( \mathcal{X}' \). Then we can get the analysis formulas of the two linear models, given which we can derive the \( \ell_2 \) global sensitivity \( \Delta_2 f \). Theorem 2 shows that \( \Delta_2 f \) for both linear and logistic regression models is equal to a constant \( \sqrt{3} \).

Theorem 2. Let \( \mathcal{X} \) and \( \mathcal{X}' \) be two normalized \( n \)-dimensional neighbor training datasets. Let \( f_\mathcal{X}(x) \) and \( f_\mathcal{X}'(x) \) be linear or logistic regression models trained on the dataset \( \mathcal{X} \) and \( \mathcal{X}' \). Then the \( \ell_2 \) global sensitivity \( \Delta_2 f \) is \( \sqrt{3} \).

Proof. See Appendix A
C. Distortion Degree and Privacy Level

In our scheme, we regard the injected noise and the original model \( f(x) \) as the HDG-protected model \( f'(x) \). By this way, we use relative error between the coefficients in the \( f(x) \) and \( f'(x) \) to define the privacy level and distortion degree. The privacy level is defined as follows.

**Definition 7 (Privacy Level).** Given the original linear or logistic regression model \( f(x) \), the corresponding HDG-protected model \( f'(x) \), the privacy level on the \( i \)-th dimension is

\[
\Gamma_i = \frac{a_i - a'_i}{a_i} = 1 - \frac{a'_i}{a_i}
\]

where \( a_i \) and \( a'_i \) are the coefficients of the \( i \)-the dimension of the function \( f(x) \) and \( f'(x) \) respectively.

Larger relative error leads to larger obfuscation, and the privacy level thus is higher. Hence the above definition is reasonable. Given this definition, we find that privacy level is determined by \( \frac{a'_i}{a_i} \). Thus, we define this item as distortion degree \( \Gamma_i \). Formally,

**Definition 8 (Distortion Degree).** Given the privacy level \( 1 - \frac{a'_i}{a_i} \) for the \( i \)-th dimension, the distortion degree \( \Gamma_i \) is defined by:

\[
\Gamma_i = \frac{a'_i}{a_i} \quad (9)
\]

By Definition [8] we find that the closer the \( \Gamma_i \) is to 1, the lower the privacy level is. This is reasonable in that when \( \Gamma_i \) is approaching 1, \( a'_i \) gets closer to \( a_i \). When \( \Gamma_i = 1 \), \( a'_i \) equals to \( a_i \), thus no privacy being provided at all.

The distortion degree \( \Gamma_i \) in our design is adopted as a measurement of the lower bound of the magnitude of the noise. It indicates the minimum noise magnitude required to achieve the privacy level. As the privacy level is defined as an absolute value, the domain of distortion degree \( \Gamma_i \) can be split into two equivalent parts: \( [\infty, 1] \) and \( [1, \infty] \), from the perspective of privacy level. Because for any \( \Gamma_i \in [\infty, 1] \) that makes privacy level \( 1 - \Gamma_i = m \), there always exists another \( \Gamma'_i \in [1, \infty] \) that also makes \( 1 - \Gamma'_i = m \). As a result, we only discuss \( \Gamma_i \in [1, \infty] \) in this paper for ease of demonstration.

Initialization for Distortion Degree. Next, we describe a conservative approach to set \( \Gamma_i \) for each dimension. First, we use feature selection algorithms (e.g. RFECV [24]) to score each dimension of the model according to their importance. Thus we algebraically transform the score to domain \([1, \infty]\) as the distortion degree. In practice, an infinite distortion degree is unreasonable, so we need to set an upper bound \( \rho \) for \( \Gamma_i \). Given a group of scores, we generate a function to map the scores to the range \([1, \rho]\). In the HDG mechanism, a larger \( \rho \) represents a higher distortion degree and makes our scheme provide a higher privacy level. The mapping function is as follows

\[
\Gamma_i = \rho - \frac{\rho - 1}{s_{\max} - s_{\min}} \times (s_i - s_{\min}) \quad (10)
\]

where \( s_i \) is the score for the \( i \)-th dimension, and \( s_{\max} \) and \( s_{\min} \) denote the maximum and the minimum respectively in the group of scores. For example, if \( \rho = 3 \) and a group of scores is \([0.1, 0.7, 0.9] \), then the distortion degree \( \Gamma = [1, 2.5, 3] \).

D. Privacy Budget Constraints for Each Dimension

In what follows, we discuss how to construct the privacy budget constraints for each query group \( Q_k \) in terms of utility and security. We propose the Type I privacy budget constraints for utility by analyzing the impact of privacy budgets on the utility of the model and the Type II privacy budget constraints for security by analyzing the impact of privacy budgets on adversaries.

1) Type I Privacy Budget Constraints: we construct the Type I privacy budget constraints by quantifying the similarity between the HDG-protected model \( f'(x) \) and the original model \( f(x) \). The more similar \( f'(x) \) is to \( f(x) \), the more utility \( f'(x) \) has. The main idea is that we estimate the coefficient \( a'_i \) in the \( f'(x) \) and \( a_i \) in \( f(x) \), then we make the ratio of \( a'_i \) to \( a_i \) larger than \( \Gamma_i \). We let \( N_i \) denote the noise matrix injected into the query matrix \( Q_i \), and \( N_i^{kR} \) be the noise matrix injected into the \( k \)-Result matrix \( Q_i^{kR} \). According to the Cramer’s rule, the \( k \)-th coefficient in the HDG-protected model \( f'(x) \) is \( |Q_i^{kR}| / |Q_i| \). Then we can get the privacy budget constraints by solving the bound of this item. However, because noise matrix \( N_i \) and \( N_i^{kR} \) contain unknown items \( \sigma_i \), directly solving the analysis formula of the determinant of the sum of the two matrices costs exponential time [32]. Therefore, we propose a conservative approach to estimate the bound.

Initially, we find that for any two matrices \( A \) and \( B \) whose entries are the same order of magnitude, the determinant \( |A| \) is closer to the determinant \( |A + B| \) with the increased order of magnitude of matrix \( A \). Thus, we can estimate \( |Q_i^{kR}| + N_i^{kR} |Q_i^{kR}| \approx |Q_i| + N_i |Q_i| \approx |Q_i| \) by intentionally enlarging the matrices \( Q_i \) and \( Q_i^{kR} \). On the other hand, we cannot enlarge the order of the magnitude infinitely because the noise is designed to match the normalized queries in our scheme. If the query matrices are enlarged too much, the noise will mismatch the queries and the mismatching degree will increase as the order of magnitude grows. Therefore, we need to find the minimum enlarging order of magnitude when the estimated error is negligible. In practice, we enlarge the matrix by 3 orders of magnitude, because the relative error of the estimated determinant is only a few tenths and independent of the dimensionality of the matrix. Fig. [3] shows an example indicating that our estimation is practical. It illustrates that the relative error of the estimated matrices is less than 0.13 for all matrices under 100-dimensional. The Theorem [3] shows the Type I privacy budget constraints.
Theorem 3. In the HDG mechanism, given a distortion degree \( \Gamma_i \), a query matrix \( Q_k \), and the original linear or logistic regression model \( f(x) \). The Type I privacy budget constraints on the \( i \)-th dimension in \( Q_k \) are as follows:

\[
a_i \times \Gamma_i \leq \sqrt{\frac{n}{2\ln n} \times \frac{\Delta f}{\epsilon_i} + 3\sigma_p^2 \times \frac{n}{2\ln n} \times \frac{\Delta f}{\epsilon_i}}\]

(11)

where \( \hat{Q}_k \) is obtained by enlarging the entries in \( Q_k \) by 3 orders of magnitude, and \( \hat{x} \) and \( \hat{z} \) are entries in matrix \( \hat{Q}_k \).

Proof. See Appendix B.

Since each projection in HDG is a Gaussian mechanism, we can substitute \( \sigma_i = \sqrt{2\ln n \times \frac{\Delta f}{\epsilon_i} + 3\sigma_p^2} \) into Equation (11) and get the Type I privacy budget constraints. Note that although we analyze the privacy budget constraints on the enlarged query matrices, the noise can still protect the original queries against the QPD attack. This is because the noise generated by Type I privacy budget constraints is negligible for the enlarged queries.

2) Type II Privacy Budget Constraints: We construct the Type II privacy budget constraints by quantifying the similarity between the coefficients of the original model \( f(x) \) and those of the extracted model \( \hat{f}(x) \). The lower the similarity is, the better protection the HDG mechanism provides. The idea is that we estimate the coefficient \( \hat{a}_i \) in \( \hat{f}(x) \) and make the ratio of \( \hat{a}_i \) to \( a_i \) is larger than \( \Gamma_i \). There is a difference between coefficient \( a_i \) and \( \hat{a}_i \): adversaries can only obtain the perturbed results and his queries when extracting models, and the extracted \( i \)-th coefficient is thus \( \hat{a}_i = \frac{\hat{Q}_k^F \cdot N_i^F}{Q_k^F} \), where \( N_i^F \) is a noise column vector on the \( k \)-th dimension of \( Q_k^F \).

Theorem 4. In the HDG mechanism, given a distortion degree \( \Gamma_i \), a query matrix \( Q_k \), and the original linear or logistic regression model \( f(x) \). The Type II privacy budget constraints on the \( i \)-th dimension in \( Q_k \) are as follows:

\[
a_i \times \Gamma_i \leq \sum_{j=1}^{n} \left( s^{(j)} + 3\sigma_a \right) \times M_{ji} \]

(12)

where \( M_{ji} \) is the \((j, i)\) minor of the original query matrix \( Q_k \).

Proof. See Appendix C.

Similarly, we can substitute \( \sqrt{2\ln n \times \frac{\Delta f}{\epsilon_i} + 3\sigma_p^2} \) into Equation (12) and get the Type II privacy budget constraints. Thus, given a sum of all privacy budgets \( \epsilon_s \), we constrain the privacy budgets as Equations (13) and (14):

\[
\epsilon_s = \epsilon_z + \sum_{i=1}^{n} \epsilon_i \]

(13)

Next, we propose an algorithm to optimize the privacy budgets to introduce minimum noise while satisfying the two types of constraints.

E. Optimizing Privacy Budget

There exist many solutions of \( \epsilon \) that meet the constraints (13) and (14) and not all of the solutions lead to minimum noise. To solve this problem, we measure the magnitude of the multivariate Gaussian noise via differential entropy (27). We find that the variance of all \( \epsilon \)'s determines the magnitude of the noise. Theorem 5 proves that the noise is minimum when the variance of all \( \epsilon \)'s is minimum.

Theorem 5. Given the SPB, the noise is minimum when the variance of all \( \epsilon \)'s is minimum.

Proof. The proof is shown in the Appendix D.

Consequently, finding the minimum multivariate Gaussian noise is equal to finding a solution of \( \epsilon \)'s whose variance is minimum under the constraints (13) and (14). So we convert this problem to an optimization problem, and the objective function is \( Var(\epsilon_1, ..., \epsilon_n) \). We formulate the optimization problem in Equation (15).

Algorithm 3 Privacy budget optimization

Input: Objective function \( f \), constraint functions \( h \) and \( g \)

Output: Optimal solution of \( \epsilon \)’s

1: 25 groups of \( \epsilon \)'s are chosen randomly from the search space as the initial population
2: for round \( \leq 150 \) do
3: Generate fitness function \( F = F(f, h, g) \) by penalty factor \( M_i \)
4: Use \( F \) to fitness each group of \( \epsilon \)
5: Based on roulette selection, produce the next generation of groups of \( \epsilon \)'s
6: end for
7: return The solution of the optimal \( \epsilon \)'s

To solve this constrained optimization problem, we adopt global optimization algorithms (e.g., genetic algorithm) instead of local optimization algorithms. The reason is that the non-linear extent of the constraints is too high, so the solutions are subject to converging to the local optimal points in local optimization algorithms, which causes large noise and poor
utility. In the genetic algorithm, we use the following common initialization settings:
- Set encoding mode of the solutions to be binary coding.
- Set fitness function to be $\text{Var}(\epsilon_1, ..., \epsilon_n)$ with penalty function. The fitness function will be discussed later.
- Set selection mode to be roulette wheel.
- Set genetic operator to be uniform crossover.
- Set mutation mode to be uniform mutation.

$$a_i \times \Gamma_i \leq \min \left\{ \sqrt{\sum_{q=2}^{n} [z(q) - z(1) + 6\sigma z]_i^2} \prod_{p=1, p \neq i}^{n} \sqrt{\sum_{q=2}^{n} [x_p(q) - x_p(1) + 6\sigma_p]_i^2}, \sum_{j=1}^{n} (\epsilon_{i,j}^{(j)} + 3\sigma z) \times M_{ji} \right\}$$

$$\min f(\epsilon_1, ..., \epsilon_n, \epsilon_z) = \text{Var}(\epsilon_1, ..., \epsilon_n, \epsilon_z)$$

$$s.t. \quad g_0(\epsilon_1, ..., \epsilon_n, \epsilon_z) = \epsilon_s - \epsilon_z - \sum_{i=1}^{n} \epsilon_i = 0$$

$$g_i(\epsilon_1, ..., \epsilon_n, \epsilon_z) =$$

$$a_i \Gamma_i - \min \left\{ \sqrt{\sum_{q=2}^{n} [z(q) - z(1) + 6\sigma z]_i^2} \prod_{p=1, p \neq i}^{n} \sqrt{\sum_{q=2}^{n} [x_p(q) - x_p(1) + 6\sigma_p]_i^2}, \sum_{j=1}^{n} (\epsilon_{i,j}^{(j)} + 3\sigma z) \times M_{ji} \right\} \leq 0$$

Now we explain our fitness function. The fitness function controls the probability that a solution is selected to generate the next solution. It is easier to choose the solution if its fitness function is higher. In order to remove the solutions that do not satisfy the constraints, we resort to the penalty function \(39\). The penalty function is used to convert the constrained optimization problems to unconstrained versions, whose solutions, in turn, can converge to the solutions of the original constrained optimization problems. In particular, we multiply each constraint function \(h\) and \(g_i\) by a penalty factor \(M_i\) to reduce the fitness of the solutions that do not satisfy the constraints. Then, these solutions will be eliminated in the next generation. The penalty factor \(M\) is defined as follows:

$$M_i = \begin{cases} 0, \quad \epsilon_i's \text{ satisfies the } i\text{-th constraints } g_i \\ -\infty, \quad \text{otherwise} \end{cases}$$

In theory, \(M\) should be infinite, but in reality, \(M_i\) is often set to be a large number, such as \(10^6\) or \(10^7\). So the fitness function can be written as:

$$f + \sum_{i=0}^{n} M_i \times g_i$$

We set the size of the initial population to be \(25\) and set the number of iteration to be \(150\) because in this setting Algorithm \(3\) reaches the trade-off between the performance and efficiency in our experiment.

VI. Evaluation

We evaluate the effectiveness of HDG-protected linear and logistic regression models against the QPD attack. Then we compare the utility and the security of ours with other existing work under the same attack. We also study the impact of the SPB \(\epsilon_s\) and upper bound \(\rho\) on the proposed HDG mechanism.

A. Setup

| Dataset | #Instances | #Number of Dimension | Type       |
|---------|------------|----------------------|------------|
| Iris    | 100        | 4                    | Classification |
| Mushroom| 8124       | 22                   | Regression |
| Bank    | 45210      | 16                   |            |
| Forestfires | 517     | 13                   |            |
| GeoOriginal | 1059   | 68                   |            |
| UJIIndoorLoc | 21048  | 529                  |            |

Datasets and Machine Learning Models. Six datasets from UCI machine learning repository \(6\) are used in our experiment – Iris, Mushroom and Bank for the logistic regression model and Forestfires, GeoOriginal and UJIIndoorLoc for the linear regression model. All the datasets are split into 70% for training and 30% for test. All categorical items are encoded by one-hot-encoding \(17\). Missing values are replaced by the mean of this attribute, which is a common and effective method in this situation. One-hot-encoding does not need to assume that the machine learning models understand the
order among the dimensions of the models, which improves the performance of the model by eliminating the redundant order information. We also normalize the datasets before model training because the proposed HDG is restricted for normalized data. The information about the datasets are listed in Table III.

QPD Attack. In order to evaluate our protection scheme, we launch the QPD attack to HDG-protected linear and logistic regression models. In the attack, the fine-tuned queries are constructed using the methods described in [25], [36]. The queries are linearly independent and the distribution of queries is correlated to training datasets. By this way, and we can thus use as few as possible queries to effectively extract models.

Evaluation Metrics. We use the following evaluation metrics in our experiment.

- **Accuracy** measures the proportion of the correct classification results for the logistic regression model. It indicates the utility of the model from users’ perspective. Formally, given the logistic regression model \( f(x) \), the number of tuples \( m \), the \( i \)-th tuple \( x^{(i)} \) and the corresponding label \( y^{(i)} \),

\[
\text{Accuracy} = \frac{1}{m} \sum_{i=1}^{m} I\left(f(x^{(i)}) = y^{(i)}\right) \tag{17}
\]

where \( I \) is an indicator function that equals 1 if \( f(x^{(i)}) = y^{(i)} \), otherwise 0.

- **Mean square error** (MSE) indicates the utility of the linear regression model. Specifically, it measures the average squared error between the results returned by the HDG-protected model \( f'(x) \) and the true results returned by the original model \( f(x) \). In general, a lower MSE represents higher model utility. Formally, given the linear regression model \( f(x) \), the number of tuples \( m \), the \( i \)-th tuple \( x^{(i)} \) and the corresponding predicted value \( y^{(i)} \),

\[
\text{MSE} = \frac{1}{m} \sum_{i=1}^{m} \left(f(x^{(i)}) - y^{(i)}\right)^2 \tag{18}
\]

- **Extraction Rate** (ER) measures the similarity between the extracted logistic regression model and the original model. Larger ER indicates the similarity is higher, and the extraction attack is thus more effective. Formally, given the extracted logistic regression model \( \tilde{f}(x) \), the number of tuples \( m \) and the \( i \)-th tuple \( x_i \),

\[
\text{ER} = \frac{1}{m} \sum_{i=1}^{m} I\left(f(x^{(i)}) = \tilde{f}(x^{(i)})\right) \tag{19}
\]

where \( I \) is an indicator function that equals 1 if \( f(x^{(i)}) = \tilde{f}(x^{(i)}) \), otherwise 0.

- **Extraction MSE** (EMSE) measures the similarity between the extracted linear regression model and the original model. Lower EMSE indicates the similarity is higher, and the extraction attack is thus more effective. Formally, given the linear regression model \( \tilde{f}(x) \), the number of tuples \( m \) and the \( i \)-th tuple \( x_i \),

\[
\text{EMSE} = \frac{1}{m} \sum_{i=1}^{m} \left(f(x^{(i)}) - \tilde{f}(x^{(i)})\right)^2 \tag{20}
\]

B. Overall Evaluation and Comparison Result

We evaluate the HDG-protected linear and logistic regression models against using other differential privacy mechanisms, i.e., MVG [4], DPBA [14] and BDPL [41], in terms of utility and security. Among the three, BDPL is the only one that can defend against the general model extraction attack for binary classification models. So we only compare our scheme with BDPL in the setting of logistic regression models. For linear regression models, MVG serves as a comparison base for it only fits this scenario. DPBA is suitable and used for both models because it injects the noise into the cost function during the training process. However, it not able to resist QPD attack since the linear property of models still persists. We set the default \( \epsilon = 1 \) for all mechanisms and the zone parameter \( \Delta = \frac{1}{2} \) for the BDPL. To unify the evaluation indicator, we set SPB \( \epsilon_s = \epsilon \times n \) for the HDG mechanism, where \( n \) is the number of model dimensions.

**Utility.** We send fine-tuned queries to the DP-protected models and plot MSE and Accuracy as the functions of the number of queries \( r \). In Fig. 4 the results show that MSE of all mechanisms on the datasets Forestfires, GeoOriginal and UJIIndoor fluctuates slightly as the number of queries grows. It is worth noting that the MSE of the DPBA-protected model is lower than the rest. This is because the DPBA only injects the noise into the cost function and does not break the linear relationship between the new queries and their corresponding results. Except for DPBA, our HDG scheme exhibits better model utility (lower MSE) than MVG, as it is impossible for MVG to generate minimum noise by allocating the constant privacy budget for each dimension by their experience.

In Fig. 5 we observe that the Accuracy of all mechanisms exceeds 84% for the logistic regression model, which indicates that the HDG-protected logistic regression models keep a superb utility. Regardless of the number of queries, DPBA and our HDG have almost unchanged Accuracy measurements, about 90% and 86% respectively. However, the Accuracy of BDPL reduces as the number of queries increases, especially on datasets Mushroom and Bank. The reason behind it is: The error produced by BDPL will gradually converge to a constant if the privacy budget is constant. However, dataset Iris is too small (only 100 instances). Consequently, the model trained on this dataset might be underfitting and the Accuracy is about 84% at the beginning.

In summary, the model utility with our HDG scheme is independent of the query numbers and shows performance comparable to other state-of-the-art mechanisms for both linear and logistic regression models.

**Protection against QPD Attack.** We launch the QPD attack to all DP mechanisms and plot the EMSE and the ER as the functions of the number of queries \( r \). We only compare
the EMSE of our design to the unprotected linear regression model because of no known mitigation of model extraction using differential privacy in this setting. Fig. 6 and 7 show the protection provided by the HDG mechanism for the linear and logistic regression models. The large gap between the EMSE on NoPrivacy and HDG-protected linear regression models in Fig. 6 shows that the protections of HDG on linear regression models are outstanding and stable. The EMSE of the extracted linear regression model with our scheme is greater than 15 on all three tested datasets, much larger than that of the unprotected case. Fig. 7 shows that HDG provides excellent protection for logistic regression models, since the ER of the extracted logistic regression model is significantly lower than that of the BDPL and NoPrivacy cases, i.e. only about 50% with all datasets. Besides, the ER of our scheme remains almost the same as the number of queries increases. On the
other hand, the protection by BDPL dramatically decreases with more queries (i.e. much lower ER than HDG). This is due to the constant privacy budget used in BDPL while we adopt group strategy, and compute different privacy budgets and noise for different query groups.

C. Impact of $\epsilon$ and $\rho$

We evaluate the utility and protection performance of the proposed HDG mechanism with respect to different $\epsilon$ and $\rho$.

In this experiment, we vary $\epsilon$ from 0.2 to 1 and vary $\rho$ from 1 to 2. Besides, we set the number of queries $r$ to be the maximum number in our experiments – 20$k$ in order to reduce the influence due to the lack of queries.

**Utility Evaluation.** To assess the utility performance with different $\epsilon$ and $\rho$, we send 20$k$ queries to models and plot MSE and Accuracy as the functions of $\epsilon$ and $\rho$. Fig. 8 and 9 show the utility evaluation of HDG-protected linear and logistic regression models.
regression models with varied $\epsilon$ and $\rho$. Fig. 8 exhibits that the maximum MSE of our mechanism is about 0.45 (in dataset UIIIndoor), and the MSE decreases with the growth of $\epsilon$ and reduction of $\rho$, reaching the bottom when $\epsilon = 1$ and $\rho = 1$. In Fig. 9, the Accuracy of the logistic regression model is larger than 0.8 on all datasets. The Accuracy increases with a growth of $\epsilon$ and a decrease of $\rho$, reaching the summit when $\epsilon = 1$ and $\rho = 1$. In Fig. 8 and 9, the utility of the HDG-protected models will reduce as the privacy budget decreases, which is coincident with the definition of differential privacy. On the other hand, the growth of $\rho$ leads to the increment of distortion degree. Hence, the utility will reduce as the $\rho$ increases. In conclusion, the utility of both models is maximized when $\epsilon$ is maximum but $\rho$ is minimum.

Protection Evaluation. To see the influence of varied $\epsilon$ and $\rho$ on protection, we plot EMSE and ER as the functions of $\epsilon$ and $\rho$. In Fig. 10, the EMSE reduces from over 17 to 15 as $\epsilon$ increases and $\rho$ decreases, which means the similarity between the extracted linear regression model and the HDG-protected model is increasing. In other words, the protection provided by HDG will reduce as $\epsilon$ increases and $\rho$ decreases. Fig. 11 depicts that the ER of the extracted logistic regression model on three datasets is around 0.5, which means that the result is almost random. From the perspective of information theory, the disorder degree of the extracted logistic regression model is maximum. As a result, the HDG mechanism provides better protection than BDPL does.

D. Resilience Verification

In this section, we evaluate the resilience of our HDG mechanism. Resilience is a crucial property of HDG. It guarantees that our HDG can provide a strong protection under the misconfigured excessive large SPB $\epsilon_s$. To unify the evaluation indicator as above, we still set the number of queries $r$ to be $20k$, $\rho = 2$ and the SPB $\epsilon_s = \epsilon \times n$ in this evaluation. In Table [IV], we compare the EMSE and the ER among different $\epsilon$ for HDG-protected linear and logistic regression models. The results demonstrate that with the reduction of $\epsilon$, the EMSE decreases to at least 14 for all datasets; the ER increases to at least 0.63 for all datasets. In general, $\epsilon$ is set to be a small real number such as 1 or $\ln 2$, but the resilience evaluation results show that even the SPB is large, our scheme still provides good protection for linear and logistic regression models.

VII. DISCUSSION

The QPD attack is very powerful and can cause potential damage to other DP-protected machine learning models. For example, similar vulnerabilities may exist in regression models, multiclass logistic regression models and multilayer perceptrons. This is because the QPD attack can allow adversaries to construct the true results corresponding to their queries that could help them extract the models. Therefore, it is worth continuing to study similar attack techniques, which allows us to better understand the related vulnerabilities and provides insightful guidance for corresponding defense strategy development.

Although the proposed HDG mechanism is effective to withstand the QPD attack, it also has some limitations. For instance, the cost of the optimization process of HDG will be expensive for high-dimensional models, which may not be friendly to some time-sensitive applications. A potential solution is to find the correlation among all dimensions of the noise and reduce the optimization complexity. Another limitation is that the presented countermeasure cannot be easily extended to protecting other machine learning models. This gives rise to an urgent call for investigations on the development of QPD-resistant differential privacy techniques and presents potential opportunities to the research community.

VIII. CONCLUSION

In this work, we develop a new powerful query-flooding parameter duplication attack. By only accessing the public APIs, the attack can effectively infer the private machine learning models that are being protected by the state-of-the-art differential privacy mechanisms. We analyze the cause of the attack and propose a High-dimentional Gaussian mechanism as the countermeasure for regression models. HDG can produce the uncorrelated noise to disable the QPD attack and automatically optimize the required noise to the minimum. The scheme is also resilient to misconfigured privacy budgets. The proposed attack and defense have been comprehensively verified by experiments and will be made open-source for further research.

REFERENCES

[1] Amazon web services. Amazon textract [online]. Available: https://aws.amazon.com/textract. Accessed on: Aug 27, 2019.

[2] Google. Google cloud speech to text. [online]. Available: https://cloud.google.com/speech-to-text. Accessed on: Aug 27, 2019.

[3] Google. Google cloud vision OCR. [online]. Available: https://cloud.google.com/vision/docs/ocr. Accessed on: Aug 27, 2019.

[4] Thee Chanyawad, Alex Dytso, H Vincent Poor, and Prateek Mittal. Mvg mechanism: Differential privacy under matrix-valued query. Computer and communications security, pages 230–246, 2018.

[5] SS Dragomir, Josip Pecaric, and L-Erik Persson. Some inequalities of hadamard type. Soochow J. Math, 21(3):335–341, 1995.

[6] Dheeru Dua and Casey Graff. UCI machine learning repository, 2017.

[7] J. C. Duchi, M. I. Jordan, and M. J. Wainwright. Local privacy and statistical minimax rates. In 2013 IEEE 54th Annual Symposium on Foundations of Computer Science, pages 429–438, Oct 2013.

[8] Vasiht Duddu, Debasis Samanta, D Vijay Rao, and Valentina E Balas. Stealing neural networks via timing side channels. arXiv preprint arXiv:1812.11720, 2018.

[9] Cynthia Dwork. Differential privacy: A survey of results. In International Conference on Theory and Applications of Models of Computation, pages 1–19. Springer, 2008.

[10] Cynthia Dwork, Nitin Kohli, and Deirdre Mulligan. Differential privacy in practice: Expose your epsilon! Journal of Privacy and Confidentiality, 9(2), 2019.

[11] Cynthia Dwork, Frank McSherry, Kobbi Nissim, and Adam Smith. Calibrating noise to sensitivity in private data analysis. In Theory of Cryptography Conference, pages 265–284. Springer, 2006.

[12] Cynthia Dwork, Aaron Roth, et al. The algorithmic foundations of differential privacy. Foundations and Trends® in Theoretical Computer Science, 9(3–4):211–407, 2014.

[13] Ulfar Erlingsson, Vasyl Pihur, and Aleksandra Korolova. Rappor: Randomized aggregatable privacy-preserving ordinal response. In Proceedings of the 2014 ACM SIGSAC conference on computer and communications security, pages 1054–1067. ACM, 2014.

[14] X. Fang, F. Yu, G. Yang, and Y. Qu. Regression analysis with differential privacy preserving. IEEE Access, 7:129353–129361, 2019.
| Datasets for Logistic Regression (ER) | Datasets for Linear Regression (EMSE) |
|--------------------------------------|-------------------------------------|
| | Forestfires | GeoOriginal | UJIIndoor | Mushrooms | Iris | Bank |
| | | | | | | |
| $\epsilon = 5$ | 15.3 | 15.2 | 15.9 | 0.51 | 0.53 | 0.49 |
| $\epsilon = 10$ | 14.9 | 15.7 | 15.4 | 0.55 | 0.57 | 0.49 |
| $\epsilon = 15$ | 14.5 | 15.1 | 14.9 | 0.56 | 0.62 | 0.53 |
| $\epsilon = 20$ | 14.3 | 14.9 | 14.1 | 0.58 | 0.65 | 0.53 |

[15] Q. Geng, P. Kairouz, S. Oh, and P. Viswanath. The staircase mechanism in differential privacy. *IEEE Journal of Selected Topics in Signal Processing*, 9(7):1176–1184, Oct 2015.

[16] James E. Gentle. *Computational Statistics*. Springer Publishing Company, Incorporated, 2009.

[17] Sarah Harris and David Harris. *Digital design and computer architecture: arm edition*. Morgan Kaufmann, 2015.

[18] J. Hsu, M. Gaboardi, A. Haeberlen, S. Khanna, A. Narayan, B. C. Pierce, and A. Roth. Differential privacy: An economic method for choosing $\epsilon$. In *Computer Security Foundations Symposium*, pages 398–410, July 2014.

[19] M. Juuti, S. Szyller, S. Marchal, and N. Asokan. Prada: Protecting against dnn model stealing attacks. In *2019 IEEE European Symposium on Security and Privacy (EuroS P)*, pages 512–527, June 2019.

[20] Manish Kesarwani, Bhaskar Mukhoty, Vijay Arya, and Sameep Mehta. Model extraction warning in mlas paradigm. In *Proceedings of the 34th Annual Computer Security Applications Conference*, pages 371–380. ACM, 2018.

[21] N. Kohli and P. Laskowski. Epsilon voting: Mechanism design for parameter selection in differential privacy. In *2018 IEEE Symposium on Privacy-Aware Computing (PAC)*, pages 19–30, Sep. 2018.

[22] Jaewoo Lee and Chris Clifton. How much is enough? choosing $\epsilon$. In *International Conference on Information Security*, pages 325–340. Springer, 2011.

[23] Xiaoguang Li, Hui Li, Hui Zhu, and Muyang Huang. The optimal upper bound of the number of queries for laplace mechanism under differential privacy. *Information Sciences*, 503:219–237, 2019.

[24] B. Liu, X. Li, J. Li, Y. Li, J. Lang, R. Gu, and F. Wang. Comparison of machine learning classifiers for breast cancer diagnosis based on feature selection. In *2018 IEEE International Conference on Systems, Man, and Cybernetics (SMC)*, pages 4399–4404, Oct 2018.

[25] Daniel Lowd and Christopher Meek. Adversarial learning. In *Proceedings of the eleventh ACM SIGKDD international conference on Knowledge discovery in data mining*, pages 641–647. ACM, 2005.

[26] Frank D McSherry. Privacy integrated queries: an extensible platform for privacy-preserving data analysis. In *Proceedings of the 2009 ACM SIGMOD International Conference on Management of data*, pages 19–30. ACM, 2009.

[27] Joseph Victor Michalowicz, Jonathan M Nichols, and Frank Bucholtz. *Handbook of differential entropy*. Chapman and Hall/CRC, 2013.

[28] Takao Murakami and Yusuke Kawamoto. Utility-optimized local differential privacy mechanisms for distribution estimation. In *28th USENIX Security Symposium (USENIX Security 19)*, pages 1877–1894, Santa Clara, CA, August 2019. USENIX Association.

[29] Maurizio Naldi and Giuseppe D’Acquisto. Individual differential privacy: A utility-preserving formulation of differential privacy guarantees. *IEEE Transactions on Information Forensics and Security*, 12(6):1418–1429, June 2017.

[30] E. Quiring, D. Arp, and K. Rieck. Forgotten siblings: Unifying attacks on machine learning and digital watermarking. In *2018 IEEE European Symposium on Security and Privacy (EuroS P)*, pages 488–502, April 2018.

[31] J. Soria-Comas, J. Domingo-Ferrer, D. Schneez, and D. Megias. Individual differential privacy: A utility-preserving formulation of differential privacy guarantees. *IEEE Transactions on Information Forensics and Security*, 12(6):1418–1429, June 2017.

[32] E. Yeniay. Penalty function methods for constrained optimization with genetic algorithms. *Mathematical and Computational Applications*, 10(1):45–56, 2005.

[33] Yi Shi, Y. Sagduyu, and A. Grushin. How to steal a machine learning classifier with deep learning. In *2017 IEEE International Symposium on Technologies for Homeland Security (HST)*, pages 1–5, April 2017.

[34] Huadi Zheng, Qingqing Ye, Haibo Hu, Chengfang Fang, and Jie Shi. Bdpl: A boundary differentially private layer against machine learning model extraction attacks. In *European Symposium on Research in Computer Security*, pages 66–83. Springer, 2019.

**APPENDIX A**

**PROOF OF THEOREM**

**Proof.** Assume that the training dataset contains $n$ tuples and each tuple is denoted by $X_i = [x_1, x_2, \ldots, x_d, y_i]$. In order to find a pair of neighbor datasets such that the sensitivity of the linear model $-\ln \frac{1}{y} = a_1 x_1 + \ldots + a_d x_d + b$ trained on them is maximum, we firstly algebraically transform each attribute $y$ to $z = -\ln \frac{1}{y}$. After normalization, the domain of the training dataset is a $d$-dimension hypercube with side length 1. Let the coordinate axis be $(x_1, \ldots, x_d, z)$, and the coordinate axis of the output attribute is $z$. Considering the most extreme case, all data points are located at hyperplane $a_1 x_1 + \ldots + a_d x_d$, and we propose a method to generate a pair of neighbor datasets on this hyperplane as follows.

First we choose the vertex $(1, 1, \ldots, 0, 1)$ as $v_1$ on the hyperplane $a_1 x_1 + \ldots + a_d x_d$ and put $(n - d + 1)$ data points on $v_1$, then we choose the neighbour vertex $v_2$ of $v_1$ and put one data point on $v_2$, next we choose the neighbor vertex $v_3$ of $v_2$ and put one data point on $v_3$, etc. Until we put the last point.
Then we create the neighbour dataset \( DB' \). We change a data point on \( v_d \), and put it on another vertex that satisfies following two properties:

1) \( z \) is equal to 1.
2) this vertex is the neighbour of a vertex which has at least one data point.

At this time, the linear model \( f_{DB'} \) is:

\[
0 \times x_1 + ... + 1 \times x_i + ... + 0 \times x_d + 0 \times z = 1
\]

Then the \( \Delta_2f \) between \( f_{DB} \) and \( f_{DB'} \) is:

\[
\sqrt{(0-1)^2 + (1-0)^2 + (0-1)^2} = \sqrt{3}
\]  

\[(21)\]

\[\]
Now let \( x = S_k \), because \( k \) is a constant, we consider the function

\[
g(x) = x^k(S - x),
\]

because the maximum is at the point where the derivative is 0, then we have

\[
g'(x) = kx^{k-1}S - kx^k - x^k = 0
\]

\[\iff kS - kx - x = 0\]

\[\iff x = S_k = \frac{k}{k+1} S\]

Thus

\[
\epsilon_{k+1} = S - x = S \frac{S}{k+1} = \epsilon_1 = \ldots = \epsilon_k,
\]

and \( Var(\epsilon_1, \epsilon_2, \ldots, \epsilon_{k+1}) \) is the smallest at this time. \( \square \)