A Semi-active Vehicle Suspension Control Strategy Based on Negative Stiffness Characteristics

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Abstract. A suspension is an indispensable part of a vehicle. It is of great significance for passengers’ comfort and vehicle stability. This paper proposes a semi-active vehicle suspension model with negative stiffness, and analyzes the effect of negative stiffness characteristics upon vehicle response. When analyzing the performance of semi-active vehicle suspension, a quarter vehicle suspension model is established, and the space state model is built for numerical simulation. After contrasting and analyzing the response of semi-active passive suspension with negative stiffness and semi-active suspension controlled by Skyhook damping algorithm, we conclude that semi-active suspension with negative stiffness decreases the vibration speed of the vehicle body and the deflection of the tire in the low-frequency stage, and improves the smoothness and stability of the suspension. Hence, the negative stiffness characteristic is beneficial to improve the comfort and stability of the vehicle.

1. Introduction

In recent years, with the rapid development of the automobile industry, automobile has become an important means of transportation for people to travel. Suspension is an important part of the vehicle, which carries most of the weight of the vehicle and functions in transmitting the force between the vehicle body and the road. Therefore, suspension is of great significance to passengers’ comfort and vehicle stability. In order to achieve better performance, it has aroused widespread concern from many scholars. At present, many different control methods concerning the control of vehicle suspension have been put forward. According to distinctive control modes, these methods can be mainly classified into three strategies: passive control, active control, and semi-active control.

The passive suspension system with the characteristics of simple structure and low costs, is chiefly composed of springs and hydraulic dampers, whose damping and stiffness are kept constant. Under the condition of low road roughness, passive suspension can achieve a certain control effect; When the roughness is high, the control effect is obviously deteriorated, which affects the comfort of passengers and the stability of vehicle operation. Therefore, the optimal control effect of passive suspension is restricted to a certain frequency range. When the road excitation frequency is too high, the control effect is less effective.

The active suspension system can obtain better control effects through the cooperation of various sensors and control mechanisms. Compared with passive suspension, it has excellent control performance in a large frequency range. With the research and development of active suspension in recent decades, many control algorithms have been proposed, such as $H^\infty$ control [2], fuzzy control [3], linear optimal control (LQR) [4]. However, with a complicated structure and high costs, the active
suspension system is equipped with many sensors and actuators requiring a large amount of maintenance in practical applications. Furthermore, the performance of active suspension is easily affected by external noise, which has a negative impact upon control performance.

In addition, the rapid development of damping technology, such as electromagnetic damping, magneto-rheological damping \[5\], has stimulated the research of the semi-active vehicle suspension system. Because of its low costs and disturbance resistance, semi-active vehicle suspension has received widespread attentions from researchers so that a number of semi-active control algorithms have been proposed, such as Skyhook damping control algorithm, sliding mode control \[2\]. Through continuous optimization of the control algorithm, the control effect of semi-active vehicle suspension is close to that of active vehicle suspension.

In previous studies, the control method of active suspension always produces a force-displacement relationship with significantly negative stiffness characteristics, which contribute to elevating the vibration control performance. This kind of negative stiffness has been widely applied in some aspects, and has achieved effective control results, verifying the feasibility of this method. Similarly, the vehicle suspension system is also a typical application of it.

Therefore, this paper proposes a vehicle suspension model based on negative stiffness, and applies negative stiffness to passive suspension and semi-active suspension based on Skyhook damping control algorithm. We build a quarter vehicle suspension model, simulate the model on different levels of road excitation via MATLAB, and analyze the influence of the negative stiffness on the control effect of vehicle suspension smoothness and stability to verify the feasibility of the vehicle suspension with negative stiffness.

2. Negative Stiffness

In the previous research regarding active suspensions, the relationship between force and displacement is correlated negatively \[7\], indicating that the direction of force and deformation is opposite. This negative correlation characteristic is called negative stiffness.

Compared with general positive stiffness, negative stiffness manifests that the instantaneous direction of external force is opposite to the instantaneous direction of deformation. Studies have shown that negative stiffness is beneficial to the effectiveness of vibration control and obtain better shock absorption effects \[6\]; using elastic elements connected in parallel with positive and negative stiffness can effectively reduce the natural frequency of the vehicle suspension system resulting in the decrease of the vibration of the system. The force-displacement relationships of the positive and negative stiffness devices are shown in Figure 1:

![Force-displacement relationship](image)

Figure 1. Force-displacement relationship

The deflection of the suspension spring can be categorized into two parts: static deflection and dynamic deflection. The static deflection is actually caused by the weight of the car body and the load weight, which can realize the function of supporting the car body and the load; the dynamic deflection is caused by vehicle vibration, and it transmits the mass under the spring to the car body. In order to achieve larger carrying capacity, we hope that the greater the stiffness of the suspension for static
deflection, the better; however, in order to reduce the vibration of the vehicle body, the stiffness for
dynamic deflection must be restricted to improve the riding comfort[9].

In order to achieve the above two goals, we designed a vehicle suspension device with negative
stiffness, which is in parallel with a spring of negative stiffness characteristics at the working balance
point of the spring elasticity of the ordinary elastic suspension, so that it has a small stiffness near the
working balance point [7].

![Figure 2. Parallel characteristics of positive and negative stiffness](image)

Figure 2 shows the stiffness characteristic of parallel connection. The red dotted line represents
the original stiffness characteristic, and the blue curve represents the stiffness characteristic of parallel
connection. The use of the positive and negative stiffness parallel device can not only enhance
the bearing capacity of the vehicle suspension, but also decrease vehicle vibration. It can realize the
reduction of vehicle vibration as well as the increase of vehicle stability and comfort while the
load-bearing capacity remains the same.

3. Quarter-car Suspension Model
Because the vertical vibration of the front and rear suspension systems is almost independent, the car
can be simplified to a quarter semi-active suspension model. The quarter suspension model is
presented for studying the suspension system simulation, which is widely used in vehicle suspension.
In this study, a quarter car model is utilized to evaluate the performance of the suspension system.
Figure 3 shows a quarter vehicle suspension model, including a wheel, a quarter of the body mass and
suspension components.

![Figure 3. Quarter-car semi-active suspension model](image)

Where $z_s$ is the suspension mass (upper-spring mass, including the car body, etc.); $m_u$ is the
non-suspension mass (the mass of the lower part of the spring, including the wheels, axles, etc.);
$k_s, k_u$ respectively, the suspension and tire stiffness, $C_{semi}$ is the damping coefficient of the
suspension system.
The damping of semi-active vehicle suspension \( C_{\text{semi}}(t) \) has a certain range of variation:

\[
0 \leq C_{\text{semi}}(t) \leq C_{\text{max}} \tag{3.1}
\]

In the case of adjustable damping, we carry out dynamic analysis on the quarter vehicle model, and the differential equation of the vehicle model is as follows:

\[
\begin{aligned}
    m_s z_s + C(\dot{z}_s - \dot{z}_u) + k_s(z_s - z_u) &= -C_{\text{semi}}(\dot{z}_s - \dot{z}_u) \\
    m_u z_u + k_u(z_u - q) - C(\dot{z}_s - \dot{z}_u) - k_s(z_s - z_u) &= C_{\text{semi}}(\dot{z}_s - \dot{z}_u)
\end{aligned} \tag{3.2}
\]

In order to analyze the state variables of the vehicle model in depth, the state space expression (3.3) is presented according to the differential equation of the vehicle model.

\[
\dot{x} = Ax + Bu + B_w q \tag{3.3}
\]

Where \( x \) is the state space variable, \( A \) is the system matrix, \( B_c \) is the control matrix, \( B_w \) is the road noise input matrix, \( u \) is the control force input vector, \( q \) is the road roughness input vector.

Comfort and stability are the main performance indicators of automobile suspensions. Comfort depends on the acceleration of the spring-loaded mass, and smoothness depends on the deflection of the tire. The state space variables \( x(t) = [x_1(t), x_2(t), x_3(t), x_4(t)]^T \) are built as follows:

\[
\begin{aligned}
    x_1(t) &= z_s(t) - z_u(t) \\
    x_2(t) &= \dot{z}_u(t) \\
    x_3(t) &= z_u(t) - q(t) \\
    x_4(t) &= \dot{z}_s(t)
\end{aligned} \tag{3.4}
\]

Where \( x_1(t) \) is the dynamic deflection of suspension, \( x_2(t) \) is the absolute speed of spring mass, \( x_3(t) \) is the deformation of tire, \( x_4(t) \) is the absolute speed of spring mass.

\( A \) is the system state matrix, as follows:

\[
A = \begin{bmatrix}
0 & 1 & 0 & -1 \\
-k_s/m_s & -C/m_s & 0 & C/m_s \\
0 & 0 & 0 & 0 \\
-k_s/m_u & C/m_u & -k_s/m_u & -C/m_u
\end{bmatrix}; \tag{3.5}
\]

The control matrix \( B_c \) and road noise input matrix \( B_w \) are as follows:

\[
B_c = \begin{bmatrix} 0 & 1/m_s & 0 & -1/m_s \end{bmatrix}^T; \quad B_w = \begin{bmatrix} 0 & 0 & -1 & 0 \end{bmatrix}^T \tag{3.6}
\]

In previous studies, the optimal control strategy is a principle (two-state algorithm) which switches from the minimum damping to the maximum damping. By adopting the above-mentioned quarter vehicle suspension model and skyhook control algorithm, the semi-active suspension system with negative stiffness is analyzed. As a typical semi-active control method, Crosby and Karnopp first proposed skyhook control algorithm in 1973. Skyhook control algorithm is the most widely used control strategy in semi-active suspension system. The mathematical description of skyhook control algorithm is as follows (3.7):

\[
F_{\text{sky}} = \begin{cases}
    c_{in} = c_{\text{max}}, & \dot{z}_s(\dot{z}_s - \dot{z}_u) \geq 0 \\
    c_{in} = c_{\text{min}}, & \dot{z}_s(\dot{z}_s - \dot{z}_u) < 0
\end{cases} \tag{3.7}
\]

Where \( F_{\text{sky}} \) is the damping force, \( \dot{z}_s \) is the speed of the sprung mass, \( \dot{z}_u \) is the speed of the free sprung mass. It can be seen from the mathematical expression that when the velocity direction of the damper is not consistent with the direction of the ideal damping force, the damper will be closed.
4. Numerical Simulation

The change of the height of the road surface $q$ relative to the reference plane along the length $I$ is called the road roughness function [1]. According to the “Draft Representation Method for Pavement Roughness” proposed by the International Standardization Organization in the ISO/ TC108/ SC2N67 document, the pavement power spectrum uses formula (4.1) as the fitting expression:

$$G_q(n) = G_q(n_0)(\frac{n}{n_0})^w$$  \hspace{1cm} (4.1)

Where $n$ is the spatial frequency, which represents the number of waves contained in each meter length; $n_0$ is the reference spatial frequency, $n_0 = 0.1m^{-1}$; $G_q(n_0)$ is the road surface power spectral density value at the reference frequency of the control, called the road roughness coefficient; $w$ is the frequency index, which is double The slope of the diagonal line on the logarithmic coordinate determines the frequency structure of the power spectral density.

$G_q(n_0)$, the power spectrum of the road surface, is the power spectrum of the vertical displacement road surface, and the first derivative of the unevenness density to the longitudinal length $I$, that is, the speed power spectral density can also be used to describe the unevenness of the road surface. When the spatial frequency index $w= 2$, the calculation formula of the road speed power spectrum is as follows:

$$G_q(n) = (2\pi n_0)^2 G_q(n_0)$$ \hspace{1cm} (4.2)

At this time, the amplitude of the road speed power spectral density is a constant in the entire frequency range, that is, white noise, and the amplitude is only related to the unevenness coefficient.

In the research of this paper, the spatial roughness input uses spatial frequency index $w=2$ and spatial reference frequency $n_0 = 0.1m^{-1}$. Figure 4 (a) shows the variation of road roughness with the distance traveled by the vehicle, and Figure 4 (b) shows the power spectral density of road roughness.

![Road roughness](image)

Figure 4. Road roughness

In this part, we mainly use MATLAB / Simulink for numerical simulation of semi-active suspension with negative stiffness and the one controlled by Skyhook algorithm. The simulation parameters of the quarter suspension model are shown in Table 1:

| Parameters | Value |
|------------|-------|
| $m_s$      | 496.5kg |
| $m_u$      | 57.3kg  |
| $k_s$      | 12700N/m |
| $k_u$      | 24700N/m |

Table 1. Quarter-car suspension model parameters

Figure 5 (a) and Figure 5 (b) show the response of the tire deflection in the time and frequency domains using Skyhook control and the one with negative stiffness. According to Fig.5 (a), the root
mean square (RMS) of tire deflection is 1.293mm and 1.122mm, respectively. It can be observed from the frequency domain response of Fig.5 (b) that in the low-frequency stage, negative stiffness can reduce the tire deflection; in the high-frequency stage, the negative stiffness does not have too many effects upon tire deflection.

Figure 5. Tyre deflection

Figure 6 (a) and Figure 6 (b) show the response of the body acceleration of the semi-active suspension with Skyhook control and negative stiffness in the time and frequency domains, respectively. According to Fig.6 (a), the root mean square (RMS) of the body acceleration is 23.6 mm/s² and 21.3 mm/s², respectively. It can be observed from the frequency domain response of Fig.6 (b) that in the low-frequency stage, negative stiffness can significantly reduce the acceleration of the vehicle body; in the high-frequency stage, the improvement of the vehicle acceleration is negatively limited.

Figure 6. Sprung mass acceleration

5. Results and discussions

In this paper, the evaluation of vehicle performance mainly focuses on comfort and stability. Comfort is defined as the vibration acceleration of the vehicle body, the lower the acceleration of the body, the higher the comfort of the vehicle; the stability is the deflection of the tire, the smaller the deflection of the tire, the higher the stability of the vehicle. The analysis of the simulation results is as follows:

(1) Body acceleration: In the low-frequency phase, compared with the semi-active vehicle suspension controlled by Skyhook algorithm, the vehicle vibration speed of the semi-active vehicle suspension with negative stiffness is significantly lower, and the vehicle comfort is improved; in the high-frequency phase, the vibration speed of the body of the semi-active vehicle suspension controlled by Skyhook is similar to that of the semi-active vehicle suspension with negative stiffness, and the vehicle comfort obeys the same variation.

(2) Tire deflection: In the low-frequency phase, the semi-active vehicle suspension with
negative-stiffness significantly lowers tire deflection, and improves the ride comfort as well; in the high-frequency phase near the wheel resonance point, the one with negative stiffness characteristics is similar in tire deflection to the one adopting Skyhook algorithm. The smoothness effect is similar.

From the above analysis, it can be concluded that in the low frequency phase, the semi-active vehicle suspension with negative stiffness characteristics can significantly suppress vehicle vibration and can also improve comfort and stability of the vehicle; in the high frequency phase, the vehicle suspension with negative stiffness has a general effect on the suppression of vehicle vibration, and has little impact on vehicle comfort and stability, which proves the effectiveness and feasibility of the negative stiffness characteristics applied in the vehicle suspension system.

6. Conclusion
This paper proposes a semi-active vehicle suspension model based on negative stiffness characteristics, and analyzes the effect of negative stiffness upon the performance of the vehicle suspension system. A quarter vehicle suspension model is established, and the differential equations of the vehicle model are listed using dynamic theory to derive the space state equation. Under certain road excitation, the simulations of the semi-active suspension with negative stiffness and the semi-active vehicle suspension controlled by Skyhook algorithm are carried out on MATLAB/ Simulink. According to the response of the vehicle model in time and frequency domain, we analyze the stability and comfort of the two vehicle suspensions.

The numerical simulation results show that: in the low-frequency vibration stage, the semi-active vehicle suspension with negative stiffness can reduce vehicle vibration speed and tire deflection, and enhance the comfort and stability of the vehicle. Finally, we conclude that a semi-active vehicle suspension with negative stiffness can improve the comfort and stability of the vehicle, and verify the effectiveness of the negative stiffness device in the control of semi-active vehicle suspension.

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