Student difficulties in solving geometry problem based on Van Hiele thinking level

D L Sulistiowati¹*, T Herman² and A Jupri²

¹Departemen Pendidikan Matematika, Sekolah Pascasarjana, Universitas Pendidikan Indonesia, Jl. Dr. Setiabudi No. 229, Bandung 40154, Indonesia
²Departemen Pendidikan Matematika, Universitas Pendidikan Indonesia, Jl. Dr. Setiabudi No. 229, Bandung 40154, Indonesia

*dwilaila.sulistiowati@student.upi.edu

Abstract. The background of this study is the lack of problem-solving ability of students. The evidence in the school shows that there are difficulties experienced by students in solving geometry problems. This fact gave rise to this study which aims to analyze students’ difficulties in solving geometry problems based on Van Hiele thinking level. The descriptive qualitative research was used in this study. Van Hiele geometry test and problem-solving test were administered, followed by interviews. The subjects of the study were 38 students grade VIII in one of the Secondary school in Bandung and 6 of them were interviewed afterward. The results showed that the main difficulty of students who at level 1 (visualization) is interpreting problems into a mathematical model. While the main difficulty of students who at level 2 (analysis) and level 3 (deduction informal) is in the solution processes. We conclude that problem-solving ability on geometry is important to be taught to all students even though they are at different Van Hiele levels.

1. Introduction

One of the important objectives that students need to have in mathematics is problem-solving ability. These problem-solving abilities include the ability to understand the problem, to devise a plan, to carry out the plan, and to look back [1]. These steps also apply to problem-solving in geometry. Furthermore, geometry is one of the mathematics branches that related to problem-solving. Through geometric learning, students can train their problem-solving skills and facilitate in studying various mathematical topics as well as other sciences [2].

Geometry is a mathematics branch that considered difficult and feared by students [3]. The difficulty students often encounter in geometry is solving geometry problems [4-6]. The difficulties students experience in solving geometry problems include the difficulty in understanding the given problem, determining appropriate problem-solving strategies, making mathematical models, and performing correct mathematical procedures [7]. In addition to the awareness of these difficulties in solving geometry problems, it is also crucial to consider the level of geometric thinking achieved by students. In this study, we use the Van Hiele theory to assess the level of geometric thinking achieved by students. The theory divides the geometric thinking level into 5 levels, namely level 1 (Visualization), level 2 (Analysis), level 3 (informal deduction), level 4 (deduction), and level 5 (Rigor) [7-9]. These levels are characterized by certain characteristics that appropriate to the students' geometric thinking process.
These levels describe the way of thinking and the types of geometric ideas thought with rather than indicating how much information is known [10].

The geometric thinking level of students influences their mathematical proficiency generally and geometric thinking skill specifically [11]. Students who possess high proficiency in mathematics and geometry, are likely to reach the high Van Hiele level of geometric thinking as well. Thus, each student on Van Hiele level possesses different skills in geometry. These differences allow different difficulties experienced by students in solving geometry problems at each Van Hiele level. As a result, it seems important to analyze the difficulties experienced by students in solving geometry problems viewed from Van Hiele level. Therefore, the question in this study is: What are the students’ difficulties in solving geometry problems based on the Van Hiele thinking level?

2. Method
To answer the research question, we conducted qualitative research through individual written tests and interviews. The subjects were 38 students of grade VIII in one of the Secondary school in Bandung, and 6 of them were interviewed afterward. The instruments consisted of Van Hiele geometry test, problem-solving test, and interview guidelines. The Van Hiele geometry test used was developed by the Cognitive Development and Achievement in Secondary School Geometry projects (CDASSG) in the form of multiple choices containing 25 items with 5 questions in each Van Hiele thinking level. Van Hiele geometry test is used to know the Van Hiele level that students achieved. Meanwhile, the problem-solving test consisted of four questions. The problem-solving test is required to collect data about students’ difficulties in solving geometry problems. The instrument was designed based on Polya's problem-solving techniques. Furthermore, interviews were conducted to strengthen the analysis of students’ difficulty in solving geometry problems. Here are geometry problems which were given as in Figure 1, 2, 3, and 4.

3. Result and discussion

3.1. Analysis of Van Hiele thinking level
A total of 38 students participated in the Van Hiele geometry test. Based on the results, it was found that 21 students were at level 1, 13 students were at level 2, 4 students were at level 3 and no students could reach level 4 and level 5. Here is the recapitulation of Van Hiele thinking level achievement.

| Achievement Level         | Percentage |
|---------------------------|------------|
| Level 1 (Visualization)   | 55%        |
| Level 2 (Analysis)        | 34%        |
| Level 3 (Informal deduction) | 11%     |
| Level 4 (deduction)       | 0%         |
| Level 5 (rigor)           | 0%         |
Table 1 shows that most of the students’ achievement at the level of Van Hiele level is visualization level. The best achievement of Van Hiele level that can be achieved by students is informal deduction level. It is consistent with another research finding by Burger & Shaughnessy [12] which stated that the highest level of thinking of junior high students in learning geometry is informal deduction level and most of them are at visualization level. This statement is also supported by Walle [12] that most of the junior high school students are between visualization level to informal deduction level. Based on the interview, no students could reach level 4 and level 5 because they had not yet studied the material on the questions for these levels. It means that the Van Hiele level is related to the amount and type of geometric experiences and learning activities that they have. This is not due to age because the Van Hiele level does not depend on age in terms of Piaget’s developmental stage [11,12].

3.2. Analysis of student difficulties in problem-solving
Students were given four geometry problems. Once the students had solved these problems, they were later interviewed to clarify their answers. Based on the analysis of the problem-solving test and the interview results, the following describes students’ difficulties based on Van Hiele level that they achieved.

3.2.1. Analysis of Student’s Difficulties at the visualization level. In solving the first problem in Figure 1, all students at this level can understand the problem and have a strategy to solve the problem. Based on interviews, they had worked on the similar problem before. However, some students are hampered due to their inability to choose the appropriate strategy. As an illustration, students’ answer is presented in Figure 5 below.

![Figure 5](image_url)

Figure 5. The answer of HR for the first problem.

Figure 5 shows that the student sums up the area of the rectangle and the area of the triangle to find the length of the fence needed. The student applies the inappropriate strategy to solve the problem. There are also some students who find the area of the trapezoid to find the required length of the fence. In addition, there is an error in writing the selected formula. For example, in writing a formula for finding the area of a triangle, students write that formula is the base of the triangle multiplied by the height of the triangle. Many errors occurred in the use of the Pythagorean theorem to find one of the trapezoid sides. The students use the Pythagorean theorem like c = a + b, where c is the hypotenuse and a, b are the lengths of other two sides of the right-angled triangle. Errors in the use of these formulas are possibly due to the fact that students simply memorize the formula [13]. To clarify the students’ answers, the following is the interview with HR:

R: How do you solve this problem?
HR: I find the area of the plane and then multiplying it by the cost of the fence per meter.
R: Why do you find the area of the plane?
HR: To find the length of the fence needed.
R: Why do you sum up the area of the rectangle and the triangle?
HR: Because the plane is a combination of both.

From the conversation, we know that the student does not know that the plane is actually a trapezoid. According to the student, the trapezoid has no the right angle. They do not recognize the right trapezoid. This is in line with the characteristics of visualization level. They are only able to identify shapes because similar in shape to previously encountered [7-9].
In solving the second problem, 9 students at this level do not understand the problem. Based on the results of interviews, they found the problem confusing because too many sentences are involved in this problem. They are unable to visualize the field and the inside garden into geometric shapes and comprehend the problem at all. Students have not been able to use the geometry model in problem-solving [14]. Some students at this level who have no trouble understanding the problem, have difficulty in choosing the correct strategy to solve the problem. It is probably for the reason that they have no knowledge of non-routine ways [6]. Some students who have a strategy to solve the problem, turn out to choose an inappropriate strategy. They use an incorrect formula. These students find the area of the rhombus then divided it by the distance between the lights. It should, students find the perimeter of the rhombus.

In solving the third problem, there are students who do not understand the given problem, which consequently caused the students confusion on writing down the answers. Students who are able to understand the problem, hamper in representing the problem in the geometric shapes. To illustrate, the following is student’ answer in Figure 6.

Figure 6 shows that the student has difficulty in representing the problem in a sketch of the geometric figure. The student draws a sketch of the original rectangular cake with the size 14 cm × 12 cm and sketches a piece of cake with the size 4 cm × 2 cm. But they do not make a sketch how to cut the cake into small pieces. This is caused inability to translate the problem into mathematical model [6]. Some students have been able to represent the problem in a sketch of the geometric figure, but they state that there is the leftover cake that cannot be cut into small pieces with size 4 cm × 2 cm. Actually, the cake can be divided into 21 pieces with a size of 4 cm × 2 cm.

The fourth problem seems to be difficult for students at visualization level. Ten students at this level do not answer the problem because they are unable to understand the given problem. Other students who have a strategy to solve the problem, are having difficulty in interpreting the problem into a mathematical model. To illustrate these findings, students’ answer presented in Figure 7. Figure 7 shows that the student cannot formulate a mathematical model of the length and the width of the frame. So that, the student chooses a size of the frame length to the specific size they desired i.e. 25 cm. According to interviews, it happened because the student is not familiar with the problem that given unknown number. Students have difficulty in interpreting and operating symbols to represent the unknown. It seems to be caused by lack of connecting geometry and algebra.

Students’ error that occurs in each given problem is computational error. From the first problem to the fourth problem, there are students who make the computational error either the strategies they used are appropriate or not. Computational errors occur due to inaccuracy in calculating [13].

3.2.2. Analysis of Student Difficulties at the analysis level. In solving the first problem, all students at analysis level can understand the problem. Some students solve this problem by finding the unknown side BC by applying a Pythagorean theorem that had been studied previously. Students use their prior knowledge to solve the problem [15,16]. Some students already use the appropriate strategy, but there are mistakes in solution processes. For example, they make calculation error when calculating the perimeter of the trapezoid and the cost of installing the fence of the garden.
In solving the second question, students at this level understand the given problem. Students have no difficulty in representing problems in the form of drawing. Students find the unknown rhombus side using the Pythagorean theorem. After that, students find the perimeter of the rhombus by multiplying the resulted side value with the number of sides on the rhombus. Here is the conversation with a student in the interview process:

R: Why do you use “side length multiplied by 4” formula to find the perimeter of the rhombus?
AK: Since formula to find the perimeter of the rhombus is sum up all sides. Because the length of the rhombus sides is the same, it can be concluded that the rhombus perimeter is 4s.
R: Are you sure that the length of the rhombus sides is the same? Why?
AK: Yes. It is based on the property of rhombus, that is the four sides are the same length.

In this case, the students are able to identify the properties of a shape in accordance with students' geometric thinking ability at level 2 [17]. There is students’ difficulty in solving the second problem. They make mistakes in solution processes. Some students have calculation errors in calculating the perimeter of rhombus and finding the number of the lights.

In solving the third problem, all of the students at this level also understand the given problem. Students try to represent the problems into a sketch drawing. The resulting problem sketches vary from one student to another. However, there are some students whose drawings do not use the appropriate scale and do not give details on the drawing. Thus, students at this level can connect the information (physical objects) provided and develop it in geometry model (without using a scale) [14].

In solving the fourth problem, some students at this level have difficulty in making solution processes. As an illustration, students’ answer presented in Figure 8 below.

$$L_{new} = P \times 4$$
$$= (p-6) \times (p-16)$$
$$= P^2 - 6P \times 16 \times 96$$
$$= P^2 - 22 + 96 \text{ cm}$$

**Figure 8.** The answer of AS for the third problem.

Figure 8 shows that students’ error in applying arithmetic operations in algebraic expressions. The student makes an error in writing mathematics symbol in the third row. This is caused by the inability to use the correct mathematics [6].

### 3.2.3. Analysis of Student Difficulties at informal deduction level.

All students at this level are able to understand all the problems given. Students also have strategies to solve problems. The main difficulty experienced at this level is in solution processes include calculation error. This could be due to students’ inaccuracy in the calculations [13]. In addition, based on interviews, some students did not re-examine the accuracy of the answers obtained. Other observed students’ difficulties at this level concern applying arithmetic operation in algebraic expressions. Here is the students’ answer as in Figure 9.

$$f = p \times 4$$
$$= (x-6) \times (x-10-6)$$
$$= x^2 + x - 16 - 24$$

**Figure 9.** The answer of HS for the fourth problem.

Figure 9 shows students’ error in applying arithmetic operations in algebraic expressions. The student raises the number x-4 as the result of the count operation x-10-6. When interviewed, the student said that -4 as the result of -10 added to 6 and she did not know where to get the final result. Observed difficulties concerned inability to use the correct mathematics [6]. Although students make mistakes in solution processes, students can identify and use meaningful strategies or reasoning to solve problems.
Students can relate the information (physical objects) provided and develop it in the geometry model (using the scale) [14]. They can make deductions simply. They also use a formula that they know beforehand [16]. In accordance with this level, students can recognize geometric shapes and understand its properties [18]. Students use their knowledge about geometric shapes and their properties to solve problems. This is visible when they solve given problems.

From the result of this study, we learn that difficulties in solving geometry problems experienced by students vary at each Van Hiele level that they achieve. Difficulties in solving geometry problems experienced by students who are at a lower level are more compared to the difficulties experienced by students who are at a higher level. In other words, students who experienced the most difficulties in solving geometry problems are students at visualization level. This indicates that the Van Hiele level achieved by students is a good indicator to demonstrate their proficiency and skills in mathematics generally and in geometry particularly. This is consistent with the results of the study by Alex and Mammen [11] that the students’ geometric thinking level influences their general mathematical understanding and geometric problem-solving ability in particular.

The differences that occur in solving geometry problems at each level of Van Hiele are also influenced by the geometry skills that students have at each level. Hoffer categorized the geometry skill into five skills: visual, drawing, verbal, logical and applied [19]. The geometry skills that students have at each level of geometry they achieve, especially the drawing skills and applied skills that students have are vary. These cause difficulties in representing problems in the form of images that students experience at each level are also vary.

4. Conclusion
Data analysis revealed that interpreting the problem into the mathematical model is the main difficulty for the students who at level 1 (visualization). Other difficulties experienced by students at visualization level were lack of understanding the problem, lack of strategy knowledge, and the inability to use the correct mathematics. Meanwhile, the main difficulty of students who at analysis and informal deduction level was in the solution processes. Difficulties experienced by students at analysis level were the inability to interpret problems into a mathematical model and the inability to use the correct mathematics and difficulties experienced by students at informal deduction level were the inability to use the correct mathematics and computational error. We suggest for mathematics teacher to assist their students to develop their problem-solving ability, it is essential that they aware of their difficulties first. Problem-solving ability in geometry is important to be taught to all students even though they are at different Van Hiele levels.

Acknowledgments
The authors thank to students and the teacher for their help and their active participation in this research.

References
[1] Polya G 1957 How to solve it (New York: Princenton University Press) pp 16-17
[2] Kaufman L 2008 Journal Education Research 50 163– 175
[3] Adolphus 2011
[4] Mahdayani R 2016 Jurnal pendas Mahakam 1 86-98
[5] Seifi M, Haghverdi M, Azismohamadi F 2012 Journal of Basic and Applied Scientific Research 2 2923-2928
[6] Yeo J 2009 International Education Journal for Mathematics Teaching and Learning 10 1-30
[7] Haviger dan Vojkuvkova 2014 Social and behavioral sciences 112 977 – 981
[8] Alex J K and Mammen K J 2012 Anthropologist 14 123-129
[9] Alex J K 2016 Eurasia Journal of Mathematics, Science, and Technology Education 12 173-188
[10] Walle J A V D 1994 Elementary school mathematics (Ney York: Longman) pp 325-326
[11] Alex J K and Mammen K J 2016 Eurasia journal of mathematics, science and technology education 12 173-188
[12] Burger W F and Shaughnessy J M 1986 *Journal for Research in Mathematics Education* **17** 31-48
[13] Ozerem A 2012 *Journal of Trends in Art, Sport and Science Education* **1** 23-35
[14] Muhassanah N, Sujadi I, Riyadi 2014 *Jurnal Elektronik Pembelajaran Matematika* **2** 54-66
[15] Ionas I G, Cemusca D, Collier H L 2012 *International Journal of Teaching and Learning in Higher Education* **24** 349-358
[16] Aydogdu M Z and Kesan C 2014 *Journal of Educational and Instructional Studies* **4** 53-62
[17] Ural A 2016 *Journal of humanities and social science* **21** 13-19
[18] Nuraeni E 2010 *Jurnal saung guru* **1** 28-34
[19] Hoffer A 1981 *NCTM Journal* **74** 11-18