Viscosity and dilepton production of a chemically equilibrating quark-gluon plasma at finite baryon density

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By considering the effect of shear viscosity we have investigated the evolution of a chemically equilibrating quark-gluon plasma at finite baryon density. Based on the evolution of the system we have performed a complete calculation for the dilepton production from the following processes: $q\bar{q} \rightarrow ll$, $q\bar{q} \rightarrow qll$, Compton-like scattering ($gq \rightarrow ql\bar{l}$, $g\bar{q} \rightarrow q\bar{l}l$), gluon fusion $gq \rightarrow c\bar{c}$, annihilation $q\bar{q} \rightarrow c\bar{c}$ as well as the multiple scattering of quarks. We have found that quark-antiquark annihilation, Compton-like scattering, gluon fusion, and multiple scattering of quarks give important contributions. Moreover, we have also found that the dilepton yield is an increasing function of the initial quark chemical potential, and the increase of the quark phase lifetime because of the viscosity also obviously raises the dilepton yield.

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I. INTRODUCTION

The Relativistic Heavy-Ion Collider (RHIC) at the Brookhaven National Laboratory and the Large Hadron Collider (LHC) being built at CERN will provide the best opportunity to study the formation and evolution of quark-gluon plasma (QGP). Dileptons have large mean free path due to the small cross section for electromagnetic interaction in the plasma, therefore can provide an ideal probe for the detection and study of the plasma.

Many authors [1-3], considering that the created QGP in collisions is a thermodynamic equilibrium system, have studied the dilepton production. Recently, the photon and dilepton productions were studied based on the evolution model of chemically equilibrating QGP, established by Shuryak, Biró and co-workers [4-6], especially, the studies of these productions in the plasma at finite baryon density were also performed [7,8]. However, most previous works were done by assuming the partonic plasma to be ideal, i.e., without any viscous effect. In principle, the viscous effects in fluid hydrodynamics should not be neglected in a realistic scenario since the dimension of the plasma is comparable to the mean free path of the partons. The viscous coefficient in the framework of hydrodynamics is composed of bulk and shear viscosity, while the bulk viscosity vanishes for a quark-gluon plasma [9]. In this work, we mainly discuss the effect of the shear viscosity, which have attracted many authors to investigate its influences on the formation and evolution of the QGP system. Authors of [9] have studied the viscous corrections to the hydrodynamic equations describing the evolution of the QGP at finite baryon density, and investigated the effect of viscosity on chemical equilibration of the system. They have found that to the viscosity the lifetime of the plasma increases, the temperature evolution of the system becomes slow, and the chemical equilibration of the system becomes fast, therefore, the reaction rate will be heightened. However, we should point out that in previous work many authors have regarded the viscous coefficients as adjustable parameters [9-12]. Indeed, they should be directly obtained from the thermodynamic quantities of the system. On the other hand, the viscous coefficients derived by Danielewicz and Gyulassy [13] based on QCD phenomenology for a baryon free-plasma, especially by Hou and Li [14] considering the Debye screening and damping rate of gluons for a baryon-rich plasma using finite-temperature QCD, are so large that the temperature of the plasma would be abnormally heightened.

In early calculations, one mainly considered the dilepton production from the process $q\bar{q} \rightarrow ll$. In recent years, possible sources of dileptons, such as $q\bar{q} \rightarrow ll$ annihilation, $q\bar{q} \rightarrow ql\bar{l}$ Compton-like scattering and $gq \rightarrow qll$ fusion, were investigated [15,16]. In addition, the contributions of gluon fusion $gq \rightarrow c\bar{c}$, quark-antiquark annihilation $q\bar{q} \rightarrow c\bar{c}$ and multiple scattering of quarks to dileptons have also been studied [17].

In this work, starting from the shear viscous coefficient given by relativistic kinetic theory for a massless QGP under relaxation time approximation, we first estimate the mean free paths of partons in a chemically equilibrating QGP at finite baryon density, then combining with the parton energy densities, calculate the shear viscous coefficient of the QGP. Subsequently, based on our evolution model including the viscosity, we perform a complete calculation for the dilepton production from processes: $q\bar{q} \rightarrow ll$, $q\bar{q} \rightarrow qll$, Compton-like ($gq \rightarrow ql\bar{l}$, $gg \rightarrow qll$), gluon fusion $gq \rightarrow c\bar{c}$, annihilation $q\bar{q} \rightarrow c\bar{c}$ as well as multiple scattering of quarks to predict the contributions of these reaction processes, and reveal the effect of the finite baryon density and viscous phenomena on dilepton production.

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The rest of the paper is organized as follows: Sec. II describes the evolution of the dissipative QGP system. In Sec. III, we discuss the yields of dileptons of the system. We give the results and discussions in section IV. Finally, in Sec. V, we give brief summary and conclusion.

II. EVOLUTION OF THE DISSIPATIVE QGP SYSTEM

In this work, we describe the distribution functions of partons with Jüttner distributions $f_{q(i)} = \lambda_{q(i)}/(e^{p/T} + \lambda_{q(i)})$ for quarks (antiquarks) and $f_g = \lambda_g/(\epsilon^g/T - \lambda_g)$ for gluons, where fugacity $\lambda_i$ ($\leq 1$) of the parton of type $i$ is used to characterize the non-equilibrium of the system. Based on these distribution functions, we first derive the thermodynamic relations of the chemically equilibrating QGP system at finite baryon density. Expanding densities of quarks (antiquarks)

$$n_{q(i)} = \frac{g_{q(i)}}{2\pi^2} \lambda_{q(i)} \int \frac{p^2 dp}{\lambda_{q(i)} + c \epsilon + \mu_q}/T$$

(1)

over quark chemical potential $\mu_q$, we get the baryon density of the system [18]

$$n_{b,q} = \frac{g_q}{6\pi^2} [T^3(Q_1^2 \lambda_q - \bar{Q}_1^2 \lambda_{\bar{q}}) + 2\mu_q T^2(Q_1^2 \lambda_q + \bar{Q}_1^2 \lambda_{\bar{q}})]$$

$$+ T\mu_q^2(Q_1^2 \lambda_q - \bar{Q}_1^2 \lambda_{\bar{q}}) + \frac{1}{3} \mu_q^3(\lambda_q + \frac{\lambda_{\bar{q}}}{\lambda_q + 1})]$$

(2)

and the corresponding energy density

$$\epsilon_{QGP} = \frac{g_q}{2\pi^2} [T^4(Q_1^2 \lambda_q + \bar{Q}_1^2 \lambda_{\bar{q}}) + 3\mu_q T^3(Q_1^2 \lambda_q - \bar{Q}_1^2 \lambda_{\bar{q}})]$$

$$+ 3\mu_q^2 T^2(Q_1^2 \lambda_q + \bar{Q}_1^2 \lambda_{\bar{q}}) + T\mu_q^3(Q_1^2 \lambda_q - \bar{Q}_1^2 \lambda_{\bar{q}})$$

$$+ \frac{1}{4} \mu_q^4(\lambda_q + \frac{\lambda_{\bar{q}}}{\lambda_q + 1}) + \frac{g_q}{\mu_g} T^4 G_1^2 \lambda_g + \frac{2\pi^2 B_0}{g_q}$$

(3)

where $g_{q(i)}$ and $g_q$ are degeneracy factors of quarks (antiquarks) and gluons respectively. Since the convergence of the following integral factors appearing in the expansion above

$$G^n_m = \int \frac{Z^n dZ}{(\epsilon^g - \lambda_g)^m}, \quad Q^n_m = \int \frac{Z^n dZ}{(\epsilon^g + \lambda_g)^m}$$

is very rapid, it is easy to calculate these integral numerically [18].

We consider the reactions leading to chemical equilibrium: $gg = ggg$ and $gq = qg$. Assuming that elastic parton scatterings are sufficiently rapid to maintain local thermal equilibrium, the evolutions of gluon and quark densities can be given by the master equations, respectively. We first extend the master equations to include the viscosity as done in [9]. Similarly, the evolution of baryon density can be described by a corrected conservation equation of baryon number including a viscous term. In addition, due to viscosity, a viscous term would be contained in the conservation equation of the energy-momentum, too. Combining the master equations together with the equation of baryon number conservation and equation of energy-momentum conservation including viscous corrections, for longitudinal scaling expansion of the system, one can get a set of coupled relaxation equations (CRE) describing evolutions of the temperature $T$, quark chemical potential $\mu_q$, and fugacities $\lambda_q$ for quarks and $\lambda_g$ for gluons on the basis of the thermodynamic relations of the chemically equilibrating QGP system at finite baryon density [9,18]

$$\left(\frac{1}{\lambda_g} + \frac{G_2^2}{G_1^2}\right)\lambda_g + \frac{3 T}{T} + \frac{1}{\tau} = R_3[1 - G_1^2(\xi(3)\lambda_g)$$

$$- 2R_2[1 - \frac{2\xi(3)}{G_1^2} \frac{\eta}{\lambda_g} n_q] + \frac{\eta}{e\tau^2} (3)$$

$$+ \hat{T}\lambda_q T^3(Q_1^2 - \lambda_q Q_2^2) + 2\mu_q T^2(Q_1^2 - \lambda_q Q_2^2)$$

$$+ T\mu_q^2(Q_1^2 - \lambda_q Q_2^2) + \frac{1}{3} \mu_q^3(\lambda_q + 1)^2]$$

$$+ \hat{T}[3\lambda_q T^2 Q_1^2 + 4\lambda_q T Q_1^2 + \lambda_q Q_2^2 Q_1^2) + T[3\lambda_q T^2 Q_1^2 + 4\lambda_q T Q_1^2 + \lambda_q Q_2^2 Q_1^2)$$

$$+ \frac{1}{\tau} [2T^2 \mu_q Q_1^2 \lambda_q + \frac{1}{3} \eta \lambda_q + 1] + 6\pi^2 \eta m_b$$

$$+ \frac{g_q}{\mu_g} T^4 G_1^2 \lambda_g + \frac{2\pi^2 B_0}{g_q}$$

$$+ \hat{T}\lambda_g g_q T^4(G_1^2 + \lambda_g G_2^2) + \frac{1}{3} \mu_q^3(\lambda_q + 1)^2]$$

$$+ \hat{T}[8\lambda_q T^3 Q_1^2 + 12\lambda_q T^2 Q_1^2 + \frac{g_q}{\mu_g} T Q_1^2]$$

$$+ \mu_q[12\mu_q \lambda_q T Q_1^2 + 2\mu_q^3 \lambda_q]$$

$$+ \frac{4}{3\tau} [2T^4 Q_1^2 \lambda_q + 6T^2 \mu_q^2 \lambda_q Q_1^2 + \frac{\mu_q^4}{2} \frac{\lambda_q}{\lambda_q + 1}]$$

$$+ \frac{g_q}{\mu_g} T^4 G_1^2] + \frac{4}{3} \eta m_b$$

$$+ \frac{\eta}{e\tau^2} (5)$$

where $\bar{n}_{q(i)}$ is the value of $n_{q(i)}$ at $\lambda_{q(i)} = 1$, $n_0 = n_q/(g_q/2\pi^2)$, $n_0' = n_g/(g_g/2\pi^2)$, $\xi(3)=1.20026$, and $\eta$ the
shear viscous coefficient. The gluon and quark production rates $R_3/T$ and $R_2/T$ are respectively given by [6,18-20]

$$R_3/T = \frac{32}{3\alpha_s} \frac{\alpha_s}{\lambda_g} \left( \frac{M_2^2}{g^2 T^3/2} \right)^2 I(\lambda_g, \lambda_q, T, \mu_g),$$

$$R_2/T = \frac{g_q}{24\pi} \frac{G_1^{12}}{G_2^2} N_f \alpha_s^2 \lambda_g \ln(\frac{\lambda_g}{\alpha_s \lambda_g}),$$

$$M_2^2 = \frac{3g_2^2 T^2}{\pi^2} [2G_1 \lambda_g + 2N_f Q_1^4 \lambda_g + N_f (\frac{\mu_g}{T})^2 (\frac{\lambda_g}{\alpha_s \lambda_g} + 1)],$$

where $M_2^2$ is the Debye screening mass, $g_2^2 = 4\pi \alpha_s$, and $I(\lambda_g, \lambda_q, T, \mu_g)$ is the function of $\lambda_g, \lambda_q, T, \mu_g$, as used in [5-6]. We here take the quark flavor $\mu_q$ and fugacities $\lambda_q$ for quarks and $\lambda_g$ for gluons.

To discuss the effects of shear viscous coefficient, we have quoted two different expressions of it: $\eta_1$ and $\eta_2$ are taken from [9,13], respectively.

$$\eta_1 = \frac{\eta_0^{QGP}}{T}$$

$$\eta_2 = \frac{T}{\sigma_q} \left[ \frac{n_g}{\pi n_g + \pi n_q} + \frac{n_q}{\pi n_q + \pi n_g} \right]$$

where $\eta_0$ is treated as a constant [9], and $\sigma_q$ the transport cross section [13].

Now, we discuss the calculation of the viscous coefficient $\eta$ in our work. According to [13,14,21], the shear viscous coefficient using the relativistic kinetic theory for a massless QGP in the relaxation time approximation is written as:

$$\eta_i = \frac{4}{15} \epsilon_i \lambda_i,$$

where $\lambda_i$ is the mean free path of particle of type $i$ in QGP, which in a chemically equilibrating QGP for gluon is given by

$$\lambda_g = \frac{4}{9n_g} \frac{1}{2\pi \alpha_s^2} \frac{M_2^2 (M_2^2 + 9T^2/2)}{9T^2/2},$$

and for quark by

$$\lambda_q = \frac{9}{4n_q} \frac{1}{2\pi \alpha_s^2} \frac{M_2^2 (M_2^2 + 9T^2/2)}{9T^2/2},$$

where $M_2^2$ is the Debye screening mass and given by (11). Thus, we can directly calculate the viscous coefficients $\eta_g, \eta_q$ and their total $\eta$ from the thermodynamic quantities of the QGP system.

**III. DILEPTON PRODUCTION**

Based on the evolution of the QGP system, we first consider dilepton production from quark annihilation $qq\rightarrow q\bar{q}l^+l^-$ and Compton scatterings $qq\rightarrow q\bar{q}ll$ and $qq\rightarrow q\bar{q}ll$. Their production rates can be calculated by [16]

$$E \frac{dR}{dp} = \frac{1}{2(2\pi)^3} \int \frac{d^3p_1}{2E_1} \frac{d^3p_2}{2E_2} \frac{d^3p_3}{2E_3} f_1(E_1) f_2(E_2) (1 \pm f_3(E_3)) \delta^4(P_1 + P_2 - P_3 - K) \sum |M|^2,$$

where $f(E)$ is the Jüttner distribution function of partons, $\sum |M|^2$ the square of the matrix element for reaction processes summed over spins, colors and flavors. The plus sign is for the annihilation process and the minus for the two Compton processes. According to [22], above equation can be rewritten as

$$\frac{dR}{d^2M} = \frac{100}{27} \frac{\alpha_s^2}{\pi^5 M} \int ds dt u^2 + t^2 + 2s M^2 |M| \lambda^g \lambda^g$$

$$\int \frac{dE_1}{e(E_1 - \mu_q)/T + \lambda_q} \frac{dE_2}{e(E_1 + \mu_q)/T - \lambda_q} \frac{dE_3}{e(E_1 + E_2 - E_3 - \mu_q)/T - \lambda_q} \frac{dE}{E} [1 - \frac{\lambda^g}{e(E_1 + E_2 - E_3 - \mu_q)/T + \lambda_q} \theta(P(E_1, E_2)) (P(E_1, E_2))^{1/2}]$$

for the annihilation process, and

$$\frac{dR}{d^2M} = \frac{20}{27} \frac{\alpha_s^2}{\pi^5 M} \int ds dt u^2 + s^2 + 2t M^2 |M| \lambda^g \lambda^g$$

$$\int \frac{dE_1}{e(E_1 - \mu_q)/T + \lambda_q} \frac{dE_2}{e(E_1 + \mu_q)/T - \lambda_q} \frac{dE_3}{e(E_1 + E_2 - E_3 - \mu_q)/T - \lambda_q} \frac{dE}{E} [1 - \frac{\lambda^g}{e(E_1 + E_2 - E_3 - \mu_q)/T + \lambda_q} \theta(P(E_1, E_2)) (P(E_1, E_2))^{1/2}]$$

for Compton scatterings, where $P(E_1, E_2) = -(tE_1 + (s + t)E_2) / 2 + 2E_3(s + t / E_2 - t / E_2) - s^2E^2 + s^2t + st^2, \theta$ is the step function, $\alpha$ the fine-structure constant, and $\alpha_s$ the running coupling constant. The minus sign is for the Compton process $qq\rightarrow q\bar{q}ll$ and the plus for $qq\rightarrow q\bar{q}ll$. The letters $s, t$ and $u$ are the Mandelstam variables. And the integration are performed over $-(s - M^2) + k_t^2 \leq t \leq -k_c^2$ and $M^2 + 2k_c^2 \leq s \leq \infty$ [16]. The cutoff $k_c^2$ is replaced by the thermal quark mass $2m_q^2$ [22]. For a chemically equilibrating QGP system at finite baryon density $m_q^2$ is given by

$$m_q^2 = \frac{4\alpha_s T^2}{3\pi} \left[ 2(G_1^2 \lambda_g + Q_1^4 \lambda_q) + (\frac{\mu_q}{T} \frac{\lambda_q}{\alpha_s \lambda_q}) \right]$$

where integral factors $G_1^2$ and $Q_1^4$ have been given above.

Obviously, above calculations give up the infrared contribution because of introducing the infrared cutoff $k_c^2$. Authors of [22] have discussed the infrared contribution to photon production. Following their calculation, in this
where we have given an assessment of the contribution from the infrared part to the dilepton production. The dilepton production rate with total energy \( E \) and total momentum \( p \) can be calculated by [22,23]

\[
dR \over dE dp = \frac{\alpha}{12\pi^4} \frac{1}{P^2} \times \frac{1}{e^{E/T} - 1} \Im \Pi_{\mu\nu},
\]

where \( \Pi_{\mu\nu} \) is the retarded photon self-energy. The infrared divergence mentioned above is caused by propagation of soft momenta. To cure the problem it is necessary to dress one of the quark propagators (as done in Fig. 3 of [22]). Then the retarded photon self-energy can be written as [22]

\[
\Pi_{\mu\nu}(p) = \frac{5}{3} \frac{e^2}{2\sqrt{2}} \sum_{k_0} \int \frac{d^4k}{(2\pi)^4} \text{Tr} \left[ S^*(k) \gamma_{\mu} S(p-k) \gamma_{\nu} \right],
\]

where \( S^*(k) \) is the dressed propagator for a quark with four-momentum \( k \), and \( S(q) \) is the bare propagator for a quark with four-momentum \( q = p - k \). Following [22], after performing some calculations and applying the elegant method developed by Braaten, Pisarski and Yuan [23], one can finally obtain a simplified expression of the imaginary part of the retarded photon self-energy

\[
\Im \Pi_{\mu\nu} \approx \frac{5e^2}{6\pi} \left( e^{E/T} - 1 \right) \int_{0}^{k_c} dk \int_{-k}^{k} d\omega f(\omega) f(E-\omega)(k-\omega) \beta_+(\omega, k),
\]

where \( f \) is the Jüttner distribution function of partons, and

\[
\beta_+(\omega, k) = \frac{1}{k} m_2^2 (k-\omega) \frac{[\omega/k - m_2^2 (Q_0(z) - Q_1(z))]^2 + [1/2\pi m_2^2 (1-z)]^2}{(k-\omega - m_2^2 (Q_0(z) - Q_1(z)))^2},
\]

where \( z = \omega/k \) [24], \( Q_0 \) and \( Q_1 \) are the Legendre functions of the second kind. After performing integral over the dilepton energy \( (E \geq M) \) one can obtain the dilepton production rate over square invariant mass of dileptons \( dR/dM^2 \).

Similar to the preceding treatment, the production rate of quark-antiquark annihilation \( q\bar{q} \rightarrow l\bar{l} \) can be given by [25]

\[
dR \over dM^2 = \frac{5}{24\pi^4} M^2 \sigma_{hl}(M^2) \times \int_{0}^{\infty} dp_1 f_q(p_1) \int_{M^2/4p_1}^{\infty} dp_2 f_{\bar{q}}(p_2),
\]

where \( \sigma_{hl}(M^2) = \frac{20}{9} \pi e^2 / 3M^2 \) is the quark annihilation cross section.

Aurenche, Gelis, and their co-workers have studied the dilepton production from bremsstrahlung and off-shell annihilation where the quark undergoes multiple scattering in the medium as shown in Fig.1 of [26]. Their calculation includes the Landau-Pomeranchuk-Migdal effect and is concluded that this contribution is important. According to these authors’ approach, the imaginary part of the retarded current-current correlator \( \Pi_{\mu\nu}(P) \) in Eq.(21) can be computed by

\[
\Im \Pi_{\mu\nu} \approx \frac{5}{6\pi} \int_{-\infty}^{+\infty} dq_0 [f(k_0) - f(q_0)] \times \Re \int \frac{d^2q_{\perp}}{(2\pi)^2} \left[ \frac{q^2_{\perp} + k^2_0}{2(q_0 k_0)} \cdot q_{\perp} \cdot f(q_{\perp}) + \frac{1}{\sqrt{|q_0 k_0|}} \frac{P^2}{p^2} g(q_{\perp}) \right],
\]

with \( k_0 \equiv q_0 + E \), \( f \) is the Jüttner distribution function of partons again, and the dimensionless functions \( f(q_{\perp}) \) and \( g(q_{\perp}) \) respectively obey the integral equations [26]

\[
2q_{\perp} = i\delta E f(q_{\perp}) + \frac{4}{3} g^2_\perp T \int \frac{d^2l_{\perp}}{(2\pi)^2} C(l_{\perp}) \times [f(l_{\perp}) - f(q_{\perp} + l_{\perp})]
\]

and

\[
2\sqrt{|q_0 k_0|} = i\delta E g(q_{\perp}) + g^2_\perp C(l_{\perp}) \int \frac{d^2l_{\perp}}{(2\pi)^2} C(l_{\perp}) \times [g(l_{\perp}) - g(q_{\perp} + l_{\perp})].
\]

Using the method described in [26], we recast (27) and (28) as differential equations and solve them using a simple algorithm, finally, can get \( \Re [f d^2q_{\perp}/(2\pi)^2 q_{\perp} \cdot f(q_{\perp}) \) and \( \Re [f^2 d^2q_{\perp}/(2\pi)^2 g(q_{\perp}) \) and the corresponding dilepton production rate of multiple scattering process.

For the QGP system, produced in collisions at RHIC energies, with very high initial temperature \((\approx 57 \text{ GeV}) [6,18]\), thermal charmed quark production and its contribution to lepton pairs should be contained, especially, those from the gluon fusion \( gg \rightarrow c\bar{c} \) and quark-antiquark annihilation \( q\bar{q} \rightarrow c\bar{c} \). Similar to the calculation for \( q\bar{q} \rightarrow l\bar{l} \), replacing the cross section \( \sigma_{hl}(M^2) \) appears in the expression (25) with those of the reactions \( q\bar{q} \rightarrow c\bar{c} \) and \( gg \rightarrow c\bar{c} \) in leading order QCD, we can compute the yields of charm pairs in the QGP. Almost all of the produced thermal charmed quarks would eventually hadronize to D-mesons [17]. Considering that the D-meson decays to leptons with a 17% branching ratio for charged D-mesons [17,27,28], finally one can obtain the contribution of charmed quarks from reactions \( gg \rightarrow c\bar{c} \) and \( q\bar{q} \rightarrow c\bar{c} \) to lepton pairs.

We integrate these production rates over the space-time volume of the reaction. According to Bjorken’s model, the volume element is \( d^4x = d^2x d\tau dy d\sigma \), where \( \tau \) is the evolution time of the system and \( y \) the rapidity of the fluid element. We consider Au\textsuperscript{197} + Au\textsuperscript{197} central collisions, so the integration over transverse coordinates just yields a factor of \( d^2x = \pi R_A^2 \), where \( R_A \) is the nuclear radius. Finally we obtain the dilepton spectra of the system

\[
\frac{dN}{dy dM^2} = \pi R_A^2 \int \tau d\tau \frac{dR}{dM^2}.
\]
also note that the temperature is unreasonably height-
ened for viscous coefficient \( \eta_2 \). For \( \eta_1 \), the evolution of temperature seems to be reasonable, however, the viscosity \( \eta_2 \) is only obtained through adjusting the parameter \( \eta_0 \). In this work, we have calculated the viscous coefficients \( \eta \) by the thermodynamic quantities of the QGP system using (14)–(16). From Fig.1, we note that the calculated temperature distribution is reasonable. In addition, the temperature is also a increasing function of the initial quark chemical potential. Fig.2 shows the value of \( \eta \) as a function of the initial temperature, where the black, red, green and blue curves denote, in turn, the calculated \( \eta \) for initial quark chemical potentials \( \mu_\eta=0.000, 0.284, 0.568, \) and \( 0.852 \) GeV at the same initial conditions as given in Fig.1.

The estimated evolution paths of the system in the phase diagram have been shown in Fig.3, where black, red, green and blue curves are, in turn, the calculated evolution paths for initial quark chemical potentials \( \mu_\eta=0.000, 0.284, 0.568, \) and \( 0.852 \) GeV. The solid line is the phase boundary between the quark phase and hadronic phase. The lines with open circles denote the evolution of system without viscous effect, while the lines with solid circles are from the system with viscosity. The time interval between the two circles is 0.3 fm. The corresponding equilibration rates of gluons and quarks, \( \lambda_q \) and \( \lambda_g \), are shown in Fig. 4. The solid lines are for the cases of viscosity, and the short dash lines denote ideal cases, where the black, red, green and blue curves are, in turn, the calculated values for initial quark chemical potentials \( \mu_\eta=0.000, 0.284, 0.568, \) and \( 0.852 \) GeV. From
The time interval between the two circles is 0.3 fm.

The lines with open circles denote the evolution of system without viscous effect while the lines with solid circles are the evolution of the system with viscosity. The initial quark chemical potentials \( \mu_q = 0.000, 0.284, 0.568, \) and \( 0.852 \) GeV. One can see that the calculated spectrum goes up with increasing the initial quark chemical potential. The law is valid for processes \( qg \rightarrow q\bar{q} \) and multiple scattering as shown in panels (a) and (c) in Fig.6.

Based on the evolution of the system described in Figs.3 and 4, we have calculated dilepton spectra from quark annihilation processes \( q\bar{q} \rightarrow l\bar{l} \) and \( q\bar{q} \rightarrow gl\bar{l} \). The signs of the curves are the same as those in Fig.4.

Figs.3 and 4 we see that the evolution of the system becomes slower due to viscosity, whereas the equilibration rate of the plasma becomes faster compared to the one in the ideal case. And also the effect of the initial quark chemical potential on the evolution is in accordance with previous conclusion [18].
will lead to a suppression of the production from process \( \bar{q}g \rightarrow \bar{q}l \bar{l} \), as shown in panel (b) of Fig. 6. And also we can see from Figs. 5 and 6 that the dilepton production is a increasing function of viscosity, which would be mainly attribute to the increase of the quark phase life-time due to the viscosity.

The productions of soft dileptons, which are connected with the infrared contribution, have been computed following the method represented in [23] for initial quark
chemical potentials $\mu_\text{q0}=0.000$, 0.284, 0.568, and 0.852 GeV. In Fig.7 we have shown the results from all processes and their total for the initial quark chemical potential $\mu_\text{q0}=0.284$ GeV. Curves 1–9 represent, in turn, the calculated spectra for $q\bar{q}\rightarrow c\bar{c}$, $gg\rightarrow c\bar{c}$, $q\bar{q}\rightarrow q\bar{q}l\bar{l}$, soft dileptons, multiple scattering, $qg\rightarrow q\bar{q}l\bar{l}$, $q\bar{q}\rightarrow q\bar{q}l\bar{l}$, and their total. From Fig.7, one can see that the spectra from the quark-antiquark annihilations $q\bar{q}\rightarrow l\bar{l}$ and $q\bar{q}\rightarrow q\bar{q}l\bar{l}$ dominate. The infrared contribution is as important as that of reaction $q\bar{q}\rightarrow q\bar{q}l\bar{l}$ and even higher than the later one in the range of small invariant mass. The contributions from Compton-like scattering $qg\rightarrow q\bar{q}l\bar{l}$, multiple scattering, and annihilation $gg\rightarrow cc$ can not also be neglected.

We have also given the total yields of all processes of the system for initial conditions mentioned above, as shown in Fig. 8. The black, red, green and blue curves represent the total yields for $\mu_\text{q0}=0.000$, 0.284, 0.568, and 0.852 GeV, respectively. To understand the effect of viscosity on the dileptons production, we also give the yields for ideal QGP system, which are denoted by dash lines. It shows clearly that the dilepton yield of the system goes up with increasing initial quark chemical potential. However, previous authors have found that dileptons produced in a thermodynamic equilibrium QGP system are suppressed with increasing initial quark chemical potential [1]. In this work, since that both the quark chemical potential and the temperature of the system are functions of time, compared with the baryon-free QGP it necessarily takes a long time for value $(\mu_\text{q}, T)$ of the system to reach a certain point of the phase boundary to make the phase transition. Furthermore, in the calculation we have found that with increasing the initial quark chemical potential the production rate of gluons goes up, and thus their equilibration rate goes down, leading to the little energy consumption of the system, i.e., slow cooling of the system. Since gluons are much more than quarks in the system, with increasing the initial quark chemical potential the cooling of the system further slows down. These cause the quark phase life-time to further increase, as seen in Fig.3. These effects will heighten the dilepton yield and compensate the dilepton suppression, leading the spectrum of the system to be an increasing function of the initial quark chemical potential. On the other hand, as seen in Fig.3, due to the viscosity the evolution of the system becomes even slower, so that the dilepton yield will be heightened, as seen in Fig.8. From Fig.8 one can note that the dilepton yields are remarkably heightened due to the effect of the viscosity of the QGP system.

V. SUMMARY AND CONCLUSION

In this work, taking into account reactions $gg\rightarrow ggg$ and $gg\rightarrow q\bar{q}$ leading to the chemical equilibrium of the QGP system, and conservations of energy-momentum and baryon number, as well as viscosity of the QGP system, we have derived a set of coupled CRE of the chemically equilibrating QGP system with viscosity at finite baryon density, produced from $Au^{197}+Au^{197}$ central collisions at RHIC energies, which describes the space-time evolution of the system. Then, we have solved the CRE, and directly obtained the viscous coefficients from the thermodynamic quantities of the QGP system. We note that the calculated results of the viscous coefficients are reasonable. Subsequently, based on the evolution of the QGP system we have computed the dilepton spectra of the QGP system, we have found that the spectra is dominated by the quark-antiquark annihilation $q\bar{q}\rightarrow l\bar{l}$ and $q\bar{q}\rightarrow q\bar{q}l\bar{l}$, following by the multiple scattering of quarks, compton-like scattering $qg\rightarrow q\bar{q}l\bar{l}$ and annihilation $gg\rightarrow cc$. While we have also calculated the infrared contribution and found it very important at the range of small invariant mass of dileptons. Furthermore, we note that the increase of the dilepton yield with increasing the initial quark chemical potential can compensate the dilepton suppression, thus eventually leading to the dilepton spectrum to be an increasing function of the initial quark chemical potential. Especially, we have found that the dilepton yield of the system is obviously enhanced due to the viscous effect because this effect makes the evolution of the system slow down and thus the life-time of the QGP system increases.

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