Mathematical modeling of water supply and sewerage systems

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Abstract. The paper describes special aspects of mathematical modeling of water supply and sewerage systems. These systems are complex units consisting of extensive pipeline networks, structures, installations, and equipment supposed to work coherently. This task is challenging due to the large number of parameters and technological processes that occur in sewerage and water supply systems. Therefore, in practice, different types of modeling, including mathematical modeling, are used to cope with these tasks. Modeling is a form of reflection of reality. A developed model can only be useful if it precisely corresponds to the real system. The more perfect the model, the closer it is to the reality.

1. Introduction
Limitation is the main condition for all systems. There are no unlimited systems, since the meaning of their systematization is lost [1].

A system is a set of interactions and properties of material objects and phenomena in a given or restricted area. All systems are characterized by factors that use variables to describe them.

2. Problem specification
Table 1 summarises existing classifications of systems and their definitions as given in Paper [1].

| System name  | System essence                                               |
|--------------|--------------------------------------------------------------|
| Discrete     | variables change abruptly, reflect "external" factors, and are not time-dependent |
| Continuous   | variables change constantly, reflect "internal" changes, and depend on time |
| Regular      | a set of natural properties and interactions                 |
| Sporadic     | a set of random properties and interactions                  |
| Real         | a set of all material factors                                |
| Abstract     | a set of factor functions                                    |

Table 1. Systems classification.
A common classification of modeling types is difficult to create due to the ambiguity of the concept of "model" in science and technology. Still, some researchers suggest designing it while using different grounds [2], that is:

- according to the type of models;
- according to the properties of model objects;
- according to the spheres of modeling application.

"Modeling" as a term means to study objects of cognition using their models [3]:

- design and study of models of phenomena and objects that actually exist, including inanimate and living systems, engineering structures, various processes (physical, chemical, biological, social);
- design and study of models of objects under modeling.

Modeling is a form of reflection of reality. A developed model can only be useful if it precisely corresponds to the real system. The more perfect the model, the closer it is to the reality [1].

Thus, any classification of modeling methods will be incomplete. Besides, terminology in this area is based not so much on the “strict” rules, but on language, scientific and practical traditions. Terminology usually exists within a specific context and has no generally accepted meaning without this context.

3. Discussion

Modeling is used to study real systems. In the field of water supply and sewerage, modeling means developing models of environments, processes, engineering structures and equipment.

A model differs from the modeling object in its scale or properties. In the first case we have real models, in the second case we deal with imaginary models. In contrast to real systems, models have the ability to reproduce their state both in different conditions and over time. Thus, as a result, this opens up great opportunities for obtaining knowledge about real systems [4].

The main goal of modeling is to reduce the time and cost to get reliable information about the object of study. As for water supply and sewerage systems, the practical tasks that are solved by modeling methods include the following [5]:

- conducting expert assessments of existing water supply and sewerage systems;
- making a forecast about water supply and sewerage systems operation under new operating conditions;
- management of water supply and sewerage systems and technological processes connected with them;
- implementation of engineering and technological design of new water supply and sewerage structures;
- conducting scientific research of water supply and drainage systems and technological processes;
- search for optimal technical and economic solutions.

Models are developed to solve technological and theoretical problems. The sequence of their design is shown in Figure 1.

Researchers distinguish between the following types of modeling:

- objects modeling when the study is carried out on a model that reproduces main physical, geometric, dynamic and functional characteristics of the original object;
- physical modeling when both the model and nature have the same physics;
- sign modeling which requires the use of diagrams, graphs, drawings, graphs, formulas, words, and sentences;
- mathematical or logical-mathematical modeling, carried out by means of the language of mathematics and logic, belongs to sign modeling.

The modern form of mathematical modeling implementation is modeling by means of electronic data processing machines / computers [6].
4. Results

In mathematical modeling, there is always a question about the practical applicability of the model. Without an answer to this question, the task of modeling water supply and sewerage systems will turn into an end in itself, which will eventually call into question the pragmatic value of the entire system analysis [7].

The selection of the project solution option is related to the evaluation of its effectiveness. The same applies to selecting the operating mode of the system or unit. The impact of changes in external conditions can be evaluated depending on the response time of the system or block. A deterministic mathematical model satisfies these conditions [8, p. 36].

Working with a simulator makes it possible to choose the optimal mode of operation or the optimal design of the system. The experiment with the technological complex itself or its reduced model is not economically justified. It requires a significant amount of time to complete.

The process of the mathematical model algorithm development is shown in Figure 2.

![Figure 2. Mathematical model algorithm development [1].](image)

When developing a mathematical model algorithm, the most difficult task is to make its mathematical description [9].

No model can act as the original object fully and comprehensively. Often, it turns out that in practice it is advisable to use a less "perfect" model that reflects only certain features of the original and is not at all similar to the original from other points of view. In some cases, it is reasonable to model one and the same original object using different models that are not similar to each other.
Typically, the complexity or simplicity of a mathematical model is related to the number of parameters it includes. These parameters consider the features of the object and distinguish the object from other objects [1].

Models written by equations or systems of equations are very complex [10]. Their complexity depends on the number of equations and the type of equations applied. Differential equations are more difficult to solve than algebraic ones, and partial differential equations are more difficult than ordinary differential equations. Noticeable difficulties are observed in the transition from linear to nonlinear equations. Systems of linear algebraic or ordinary differential equations can be solved analytically in general form, in case the number of these equations is reasonable. Non-linearity complicates the solution procedure [11].

Modern ways to improve the efficiency of water supply and sewerage systems are based on the design of mathematical models of networks, which make it possible to perform calculations using static and dynamic modes. Calculations are always associated with designing a solving a system of differential and algebraic equations. It is also advisable to solve them by numerical methods [2].

If calculations of large industrial water supply systems are performed, there is a limited amount of information on the details of distribution network elements and consumers that is available for practical use. This feature is caused by the need to use special methods of macromodeling that do not require excessive detail of network characteristics, with sufficient accuracy for practical application [2].

Macromodeling of water supply systems involves the following principles described in Paper [12]:

- network areas that have a complex network connection scheme containing a large number of relatively low-power consumers are considered as a single equivalent consumer;
- consumers connected directly to the main network are considered as independent consumers;
- in case it is required to carefully consider the parameters of hydraulic modes within individual network areas, a transition to the next level of the macromodel can be performed. Such a model is supposed to reflect the state of consumers and newly allocated network areas within the network area. In this case, the object that is considered as a consumer at the top level of the macromodel acts as a source for the network area [13,14].

The approach described above makes it possible to build an identical mathematical macromodel of a complex distributed production system of an enterprise water supply system in conditions of limited initial data. Such an approach makes it possible to make up and solve hydraulic equations in the VisSim modeling and visualization environment. This software product has a number of features that can significantly simplify and speed up the calculation process [15,16].

5. Conclusions
The paper describes special aspects of mathematical modeling of water supply and sewerage systems. In the course of their research, the authors managed to:

- perform a comprehensive analysis of the process of mathematical models design;
- study different types of systems and various methods of their modeling;
- describe the process of water supply and sewerage systems mathematical modelling;
- determined such main concepts used in modeling as 'system', 'modeling', 'model', and 'mathematical model'.

The researchers also proved that when working with water supply and sewerage systems, it was possible to solve the following practical tasks by modeling methods:

- conducting expert assessments of existing water supply and sewerage systems;
- making a forecast about water supply and sewerage systems operation under new operating conditions;
- management of water supply and sewerage systems and technological processes connected with them;
- implementation of engineering and technological design of new water supply and water disposal structures;
- conducting scientific research of water supply and drainage systems and technological processes;
- search for optimal technical and economic solutions.

Besides, the paper shows that the modern form of mathematical modeling implementation is modeling by means of electronic data processing machines / computers. It stresses that the main goal of modeling is to reduce the time and cost to get reliable information about the object of study.

The article points out that the time spent on modeling depends on the task, the developed mathematical model and the number of parameters used in the model. Models that are written as equations or systems of equations are very complex. Their complexity depends on the number of equations and the type of equations applied. Differential equations are more difficult to solve than algebraic ones, and partial differential equations are more difficult than ordinary differential equations.

The paper provides examples of mathematical modeling for elements of water supply and sewerage systems.

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