$\Lambda_c \to \Lambda \ell^+ \nu_\ell$ form factors and decay rates from lattice QCD with physical quark masses

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The first lattice QCD calculation of the form factors governing $\Lambda_c \to \Lambda \ell^+ \nu_\ell$ decays is reported. The calculation was performed with two different lattice spacings and includes one ensemble with a pion mass of 139(2) MeV. The resulting predictions for the $\Lambda_c \to \Lambda \ell^+ \nu_\ell$ and $\Lambda_c \to \Lambda \mu^+ \nu_\mu$ decay rates divided by $|V_{cs}|^2$ are 0.2007(71)(74) ps$^{-1}$ and 0.1945(69)(72) ps$^{-1}$, respectively, where the two uncertainties are statistical and systematic. Taking the Cabibbo-Kobayashi-Maskawa matrix element $|V_{cs}|$ from a global fit and the $\Lambda_c$ lifetime from experiments, this translates to branching fractions of $\mathcal{B}(\Lambda_c \to \Lambda e^+ \nu_e) = 0.0380(19)_{\text{LQCD}}(11)_{\tau_{\Lambda_c}}$ and $\mathcal{B}(\Lambda_c \to \Lambda \mu^+ \nu_\mu) = 0.0369(19)_{\text{LQCD}}(12)_{\tau_{\Lambda_c}}$. These results are consistent with, and two times more precise than, the measurements performed recently by the BESIII Collaboration. Using instead the measured branching fractions together with the lattice QCD determination of the $\Lambda_c$ lifetime, this translates to branching fractions of $\mathcal{B}(\Lambda_c \to \Lambda e^+ \nu_e) = 0.0363(38)(20)$, $\ell = e$, $\mathcal{B}(\Lambda_c \to \Lambda \mu^+ \nu_\mu) = 0.0349(46)(27)$, $\ell = \mu$. In the Standard Model, the decay rates depend on six form factors that parametrize the matrix elements $\langle \Lambda(p')|\bar{s}\gamma^\mu c|\Lambda_c(p)\rangle$ and $\langle \Lambda(p')|\bar{s}\gamma^\mu\gamma_5 c|\Lambda_c(p)\rangle$ as functions of $q^2 = (p-p')^2$. These form factors have previously been estimated using quark models and sum rules $[13–29]$, giving branching fractions that vary substantially depending on the model assumptions. In the following, the first lattice QCD determination of the $\Lambda_c \to \Lambda$ form factors is reported. The calculation uses state-of-the-art methods and gives predictions for the $\Lambda_c \to \ Lambda \ell^+ \nu_\ell$ decay rates with total uncertainties that are smaller than the experimental uncertainties in Eq. (2) by a factor of two.

This work is based on gauge field configurations generated by the RBC and UKQCD collaborations with $2 + 1$ flavors of dynamical domain-wall fermions $[30, 31]$. The data sets used here are listed in Table I, and match those in Refs. $[3]$ and $[4]$, except for the addition of a new ensemble (denoted as C4) with $m_\pi = 139(2)$ MeV, and the removal of the previous “partially quenched” C14, C24, F23 data sets which had $am_{u,d} < am_{u,d}^{(\text{sea})}$. Adding the C4 ensemble significantly aids in the extrapolation of the form factors to the physical point, and removing the partially quenched data sets reduces finite-volume effects.

The charm quark is implemented using an anisotropic clover action, with parameters tuned to produce the correct $J/\psi$ relativistic dispersion relation as quantified by the “speed of light”, $c$, and the correct spin-averaged mass $\overline{m} = \frac{2}{3} m_{J/\psi} + \frac{1}{3} m_{\eta_c}$. On the new C4 ensemble,
TABLE II. Hadron masses in lattice units obtained from exponential fits to two-point functions.

| Set | \(am_{\Lambda_c}\) | \(am_\Lambda\) | \(am_{D_s}\) | \(am_{D}\) |
|-----|-----------------|----------------|---------------|----------------|
| CP  | 1.3194(36)      | 0.6483(33)    | 1.12902(39)  | 1.0720(13)     |
| C54 | 1.3706(40)      | 0.7348(30)    | 1.13156(49)  | 1.0763(13)     |
| C53 | 1.3647(60)      | 0.7096(47)    | 1.11550(59)  | 1.0763(13)     |
| F43 | 1.0185(67)      | 0.5354(29)    | 0.85447(47)  | 0.81185(91)    |
| F63 | 1.0314(40)      | 0.5514(23)    | 0.85663(39)  | 0.81722(56)    |

TABLE III. Residual matching and improvement coefficients for the \(c \rightarrow s\) vector and axial vector currents, computed using automated lattice perturbation theory [40, 41]. The notation is the same as in Eqs. (18)-(21) of Ref. [3].

The currents was performed for all source-sink separations (instead of just a subset as in Refs. [3, 4]). Examples for the ratios and \(t \rightarrow \infty\) extrapolations are shown in Fig. 1.

The ground-state form factors obtained in this way for the different data sets and different discrete momenta are shown as the data points in Fig. 2. To obtain parametrizations of the form factors in the physical limit \((a = 0, m_\pi = m_{\pi,\text{phys}}, m_\eta = m_{\eta,\text{phys}})\), fits were then performed using \(z\)-expansions [42] modified with additional terms to describe the dependence on \(a, m_\pi,\) and \(m_\eta\). In the physical limit, the fit functions reduce to the form

\[
f(q^2) = \frac{1}{1 - q^2/(m_\text{pole}^2)} \sum_{n=0}^{n_{\text{max}}} a_n[z(q^2)]^n,\]

where \(z(q^2) = \sqrt{t_+ - q^2} - \sqrt{t_+ - t_0}\) with \(t_0 = q^2 = (m_\Lambda_c - m_\Lambda)^2\) and \(t_+ = (m_D + m_K)^2\). The \(D_s\) meson pole masses are \(m_\text{pole} = 2.112\text{ GeV}, m_\text{pole} = 2.318\text{ GeV}, m_{D_s,\text{pole}} = 2.460\text{ GeV}, m_{D_s,\text{pole}} = 1.968\text{ GeV} [37],\) and to evaluate \(t_+\), the masses \(m_D = 1.870\text{ GeV}\) and \(m_K = 494\text{ MeV}\) are used. Following Refs. [3, 4], two separate fits were performed: a “nominal fit”, giving the central values and statistical uncertainties of the form factors, and a “higher-order fit”, used to compute system-
The higher-order fit had the same form as Eq. (36) of Ref. [4], but with $n_{\text{max}} = 3$. In addition to the $z^3$ terms, this fit also includes terms of higher order in $a, m_a, m_{a_3}$, and was performed after modifying the data correlation matrix to include the uncertainties from the renormalization and $O(a)$-improvement coefficients, from finite-volume effects (1.0\%, rescaled from Ref. [4] according to $e^{-\min(m_a - L)}$, and from the missing isospin breaking/QED corrections (0.5\%, 0.7\%). The priors for the higher-order parameters were chosen as in Ref. [4], except that the coefficients $a_{f 2}^T$ were left unconstrained and the priors for $a_{f 3}^T$ were set to $0 \pm 30$. The fit results for the parameters $a_{n}^T$ that describe the form factors in the physical limit are given in Table IV, and the form factors are plotted in Fig. 2. The lattice results do not significantly constrain the $z^3$ terms (note that $z_{\text{max}} \approx 0.08$), so that their uncertainties are governed by the priors.

The resulting Standard-Model predictions for the $\Lambda_c \to \Lambda \ell^+ \nu_\ell$ differential decay rates, without the factor of $|V_{cs}|^2$, are shown in Fig. 3. The $q^2$-integrated rates are

$$\Gamma(\Lambda_c \to \Lambda \ell^+ \nu_\ell) / |V_{cs}|^2 \propto \begin{cases} 0.2007(71)(74) \text{ ps}^{-1}, & \ell = e, \\ 0.1945(69)(72) \text{ ps}^{-1}, & \ell = \mu, \end{cases}$$

where the two uncertainties are from the statistical and total systematic uncertainties in the form factors. Using the world average of $\Lambda_c$ lifetime measurements, $\tau_{\Lambda_c} = 0.200(6) \mu$s [37], and $|V_{cs}| = 0.97344(15)$ from a CKM unitarity global fit [8] then yields the branching fractions

$$B(\Lambda_c \to \Lambda \ell^+ \nu_\ell) = \begin{cases} 0.0380(19)_{\text{LQCD}}(11)_{\tau_{\Lambda_c}}, & \ell = e, \\ 0.0369(19)_{\text{LQCD}}(11)_{\tau_{\Lambda_c}}, & \ell = \mu, \end{cases}$$

where the uncertainties marked “LQCD” are the total form factor uncertainties from the lattice calculation. These results are consistent with, and two times more precise than, the BESIII measurements shown in Eq. (2). This is a valuable check of the lattice methods which were also used in Refs. [1–4].

Combining instead the BESIII measurements (2) and $\tau_{\Lambda_c} = 0.200(6) \mu$s with the results in Eq. (4) to determine $|V_{cs}|$ from $\Lambda_c \to \Lambda \ell^+ \nu_\ell$ gives

$$|V_{cs}| = \begin{cases} 0.951(24)_{\text{LQCD}}(14)_{\tau_{\Lambda_c}}(56)_{\beta}, & \ell = e, \\ 0.947(24)_{\text{LQCD}}(14)_{\tau_{\Lambda_c}}(72)_{\beta}, & \ell = \mu, \\ 0.949(24)_{\text{LQCD}}(14)_{\tau_{\Lambda_c}}(49)_{\beta}, & \ell = e, \mu, \end{cases}$$

where the last line is the correlated average over $\ell = e, \mu$. This is the first determination of $|V_{cs}|$ from baryonic decays. The result is consistent with CKM unitarity, and the uncertainty can be reduced further with more precise measurements of the $\Lambda_c \to \Lambda \ell^+ \nu_\ell$ branching fractions.

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FIG. 2. Lattice QCD results for the $\Lambda_c \to \Lambda$ form factors, along with the modified $z$-expansion fits evaluated at the lattice parameters (dashed and dotted lines) and in the physical limit (solid lines, with statistical and total uncertainties indicated by the inner and outer bands).

FIG. 3. Predictions for the $\Lambda_c \to \Lambda \ell^+ \nu_{\ell}$ differential decay rates (divided by $|V_{cs}|^2$) in the Standard Model. For clarity, the uncertainties are shown only for $\ell = e$; the inner and outer bands correspond to the statistical and total uncertainties.
the parameter values with more digits and the full covariance.

TABLE IV. Results for the $a_{0}^{f}$.

| Parameter | Nominal fit | Higher-order fit |
|-----------|-------------|------------------|
| $a_{0}^{f}$ | 1.30 ± 0.06 | 1.28 ± 0.07 |
| $a_{1}^{f}$ | −3.27 ± 1.18 | −2.85 ± 1.34 |
| $a_{2}^{f}$ | 7.16 ± 11.6 | 7.14 ± 12.2 |
| $a_{3}^{f}$ | −1.08 ± 30.0 | |
| $a_{0}^{g}$ | 0.81 ± 0.03 | 0.79 ± 0.04 |
| $a_{1}^{g}$ | −2.89 ± 0.52 | −2.38 ± 0.61 |
| $a_{2}^{g}$ | 7.82 ± 4.53 | 6.64 ± 6.07 |
| $a_{3}^{g}$ | −1.08 ± 29.8 | |

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