What Does Free Space ΛΛ Interaction Predict for ΛΛ Hypernuclei?

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Data on ΛΛ hypernuclei provide a unique method to learn details on the strangeness $S = -2$ sector of the baryon-baryon interaction. From the free space Bonn–Jülich potentials, determined from data on baryon-baryon scattering in the $S = 0, -1$ channels, we construct an interaction in the $S = -2$ sector to describe the experimentally known ΛΛ hypernuclei. After including short–range (Jastrow) and RPA correlations, we find masses for these ΛΛ hypernuclei in a reasonable agreement with data, taking into account theoretical and experimental uncertainties. Thus, we provide a natural extension, at low energies, of the Bonn–Jülich OBE potentials to the $S = -2$ channel.

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I. INTRODUCTION

In the past years a considerable amount of work has been done both in the experimental and the theoretical aspects of the physics of single and double Λ hypernuclei. Because of the lack of targets, the data on ΛΛ hypernuclei provide a unique method to learn details on the strangeness $S = -2$ sector of the baryon-baryon interaction. Ground state energies of three (the production of $^6_\Lambda H$ has been recently reported) ΛΛ hypernuclei, $^6_\Lambda$He, $^{10}_\Lambda$Be and $^{13}_\Lambda$B, have been measured. The experimental binding energies, $B_{\Lambda\Lambda} = -[M(^{A+2}_\Lambda Z) - M(^A Z) - 2m_{\Lambda}]$, are reported in Table I. Note that the $^6_\Lambda$He energy has been updated very recently, in contradiction to the old one, $B_{^6_\Lambda\Lambda} = 10.9\pm0.8$ MeV. The scarce hyperon-nucleon ($YN$) scattering data have been used by the Nijmegen (NJG), Bonn–Jülich (BJ) and Tübingen groups to determine realistic $YN$ and thus also some pieces of the $YY$ interactions. In Ref. 1 an effective ΛΛ interaction, with a form inspired in the One Boson Exchange (OBE) BJ potentials, was fitted to data, and the first attempts to compare it to the free space one were carried out. Similar studies using OBE NJG potentials have been also performed in Ref. 1 and the weak decays of double Λ hypernuclei have been studied in Ref. 2. Short Range Correlations (SRC) play an important role in these systems, but despite of their inclusion the effective ΛΛ interaction, fitted to the ΛΛ–hypernuclei data, significantly differs from the free space one deduced in Ref. 3.

II. MODEL FOR ΛΛ HYPERNUCLEI

A. Variational Scheme: Jastrow type correlations

Following the work of Ref. 3, we model the ΛΛ hypernuclei by an interacting three-body ΛΛ+nuclear core system. Thus, we determine the intrinsic wave–function, $\Phi_{\Lambda\Lambda}(\vec{r}_1, \vec{r}_2)$, and the binding energy $B_{\Lambda\Lambda}$, where $\vec{r}_{1,2}$ are the relative coordinates of the hyperons respect to the nucleus, from the intrinsic Hamiltonian.

$$H = h_{\text{sp}}(1) + h_{\text{sp}}(2) + V_{\Lambda\Lambda}(1, 2) - \vec{\nabla}_1 \cdot \vec{\nabla}_2 / M_{\Lambda}$$

where $h_{\text{sp}}(i) = -\vec{\nabla}_i^2 / 2\mu_{\Lambda} + V_{\Lambda\Lambda}(|\vec{r}_i|)$, $M_{\Lambda}$ and $\mu_{\Lambda}$ are the nuclear core and the Λ-core reduced masses respectively. The Λ-nuclear core potential, $V_{\Lambda\Lambda}$, is adjusted to reproduce the binding energies, $B_{\Lambda}$ ($> 0$), of the corresponding single–Λ hypernuclei, and $V_{\Lambda\Lambda}$ stands for the ΛΛ interaction in the medium. Due to the presence of the second Λ a dynamical re-ordering effect in the nuclear core is produced. Both the ΛΛ free interaction and this re-ordering of the nuclear core, contribute to $\Delta B_{\Lambda\Lambda} = B_{\Lambda\Lambda} - 2B_{\Lambda}$. However, the latter effect is suppressed with respect to the former one by at least one power of the nuclear density, which is the natural parameter in all many body quantum theory expansions. We assume the nuclear core dynamical re-ordering effects from scattering data. In this letter we consider the new datum for He and, importantly, the effect of the long range nuclear correlations (RPA) is also incorporated. Starting from the free space BJ interactions, we find a good description of the masses of He, Be and B ΛΛ hypernuclei. This has never been achieved before despite the use of different ΛΛ free space interactions. The ΛΛ set of potentials used here and the new NJG (NSC97e,b) interactions are similar in shape, though the latter ones are shifted around 0.2 fm to larger distances as compared to the BJ potentials. Due to the difficulty of including RPA effects in NJG models and since both sets of interactions give similar energies in absence of nuclear effects, in this work we have used BJ-type potentials.
to be around 0.5 MeV, as the findings of the α-cluster models of Ref. suggest, for He, Be and B ΛΛ hypernuclei, and negligible for medium and heavy ones. This estimate for the size of the theoretical uncertainties is of the order of the experimental errors of B_{AA} reported in Table. Furthermore, the RPA model used below to determine V_{AA} accounts for particle-hole (ph) excitations of the nuclear core and thus it partially includes some nuclear core reordering effects.

In Ref. both Hartree-Fock (HF) and Variational (VAR), where SRC can be included, schemes to solve the Hamiltonian of Eq. were studied. In both cases, the nuclear medium effective ΛΛ interactions fitted to data were much more attractive than that deduced from the free space YN scattering data. Since the ΛΛ interactions obtained by σ − ω exchanges behave almost like a hard-core at short distances, the VAR energies are around 30−40% lower than the HF ones (see Fig. 4 and Table 9 of Ref. ). Hence, trying to link free space to the effective interaction, V_{AA}, requires the use of a variational approach where r_{12}−correlations (SRC) are naturally considered. We have a family of 1S_{0} ΛΛ−wave functions of the form: Φ_{AA}(r_{1}, r_{2}) = N\hat{F}(r_{12})\phi_{a}(r_{1})\phi_{b}(r_{2}) \chi^{s=0}, with χ^{S=0} the spin-singlet, r_{12} = r_{1} − r_{2} and

\[ F(r_{12}) = \left(1 + \frac{a_{1}}{1 + (\frac{r_{12}}{b_{1}})^{2}}\right)^{\frac{3}{2}} \left(1 - a_{i} e^{-b_{i}r_{12}}\right) \quad (2) \]

where a_{1,2,3}, b_{1,2,3} and R are free parameters to be determined by minimizing the energy, N is a normalization factor and ϕ_{a} is the s−wave Λ−function in the single−Λ hypernucleus \(A_{Λ}^{A+1}Z\). This VAR scheme differs appreciably to that used in Ref. There, \(Φ_{AA}(r_{1}, r_{2})\) was expanded in series of Hylleraas type terms whereas here we have adopted a Jastrow−type correlation function. Hylleraas SRC, though suited for atomic physics, are not efficient to deal with almost hard core potentials, as it is the case here. Thus, to achieve convergence in Ref. a total of 161 terms (161 unknown parameters) were considered. The ansatz of Eq. (2), which has only seven parameters and thus it leads to manageable wave functions, satisfactorily reproduces all VAR results of Ref. ().

B. ΛΛ Interaction in the Nuclear Medium

The potential V_{AA} represents an effective interaction which accounts for the dynamics of the ΛΛ pair in the nuclear medium, but which does not describe their dynamics in the vacuum. This effective interaction is usually approximated by an induced interaction \(\hat{V}_{AA}^{ind}\) (V_{AA}^{ind}) which is constructed in terms of the ΛΛ → ΛΛ (G_{AA}), ΛΛ → ΛN (G_{AN}) and NN → NN (G_{NN}) G−matrices, as depicted in Fig. The induced interaction, V_{AA}^{ind} combines the dynamics at short distances (accounted by the effective interaction G_{AA}) and the dynamics at long distances which is taken care of by means of the iteration of ph excitations (RPA series) through the effective interactions G_{AN} and G_{NN}. Near threshold (2m_{Λ}), the S = −2 baryon−baryon interaction might be described in terms of only two coupled channels ΛΛ and ΞN. For two hyperons bound in a nuclear medium and because of Pauli-blocking, it is reasonable to think that the ratio of strengths of the ΛΛ → ΞN → ΛΛ and the diagonal ΛΛ → ΛΛ (with no ΞN intermediate states) transitions is suppressed respect to the free space case. This is explicitly shown for \(\frac{G_{AA}}{G_{NN}}\) in Ref. (11), though a recent work (12), using a NJG model, finds increases of the order of 0.4 MeV in the calculated B_{AA} values, for He, Be and B ΛΛ hypernuclei, due to ΞN components. In any case 0.4 MeV is of the order of the experimental and other theoretical uncertainties discussed above, and we will assume that the data of ΛΛ hypernuclei would mainly probe the free space, V_{AA}^{free}, diagonal ΛΛ element of the ΛΛ − ΞN potential. Hence G_{AA} might be roughly approximated by V_{AA}^{free}, and the interaction V_{AA} can be split into two terms V_{AA} = V_{AA}^{free} + δV_{AA}^{RPA}, where the first one accounts for the first diagram of the rhs of Fig. and δV_{AA}^{RPA} does it for the remaining RPA series depicted in this figure. Let us examine in detail each of the terms.

1. Free space ΛΛ interaction

We use the BJ models for vacuum NN (15) and YN interactions (16) to construct the free space diagonal ΛΛ potential. We consider the exchange between the two Λ hyperons of (I = 0, J^{P} = 0^{+}), ω and φ (I = 0, J^{P} = 1^{−}) mesons. The free space ΛΛ potential, V_{AA}^{free}, in coordinate space (non-local) and for the 1S_{0} channel, can be found in Eqs. (24) and (25) of Ref. (17) for σ− and ω−exchanges respectively. The φ−exchange potential can be obtained from that of the ω−exchange by the obvious substitutions of masses and couplings. Besides, monopolar form−factors are used (18), which leads to extended expressions for the potentials (see Eq. (19) of Ref. (17)). In the spirit of the BJ models, SU(6) symmetry is used to relate the couplings of the ω− and φ−mesons to the Λ hyperon to those of these mesons to the nucleons. We adopt the so-called “ideal” mixing angle \(\tan\theta_{\omega} = 1/\sqrt{2}\) for which the ω meson comes out as a pure ss state and hence one gets a vanishing φNN coupling (15). This also determines the φΛ couplings in terms of the ΛΛ ones. Couplings (g_{σΛΛ}, g_{ωΛΛ}, f_{φΛΛ}) and momentum cutoffs (A_{σΛΛ}, A_{ωΛΛ}) appearing in the expression of the σ− and ω−exchange ΛΛ potentials can be found in Table 2 of Ref. (17) which is a recompilation of model Α of Ref. (17), determined from the study of YN scattering. The φ meson couplings are given in Eq. (65) of Ref. (17). Because the φ meson does not couple to nucleons, there exist much more uncertainties on the value of A_{ωΛΛ}. Assuming that this cutoff should be similar to A_{ωΛΛ} and bigger than the φ meson mass, we have studied three values, 1.5, 2 and 2.5 GeV.
2. RPA contribution to the ΛΛ interaction

Here, we perform the RPA resummation shown in the rhs (from the second diagram on) of Fig. 1. We will do first in nuclear matter and later in finite nuclei.

a. Nuclear Matter: Let us consider two Λ hyperons inside of a non-interacting Fermi gas of nucleons, characterized by a constant density ρ. The series of diagrams we want to sum up correspond to the diagrammatic representation of a Dyson type equation, which modifies the propagation in nuclear matter of the carriers (σ, ω and φ mesons) of the strong interaction between the two Λ’s. This modification is due to the interaction of the carriers with the nucleons. Because in our model the φ meson does not couple to nucleons, its propagation is not modified in the medium and will be omitted in what follows.

The diagrams depicted in Fig. 2 plus the corresponding momentum transferred between the two Λ’s, Table 3 of Ref. [8].

Let us consider two Λ hyperons and ΛΛ mesons (of the strong interaction between the two Λ’s). Of the interaction feels different densities when it is inside of a non-interacting Fermi gas of nucleons, characterized by a constant density ρ.

In the medium and will be omitted in what follows. The σ − ω propagator in the medium, D(Q), has been already studied in the context of Fermi-liquids in Ref. [19] and it is determined by the Dyson equation

\[ D(Q) = D^0(Q) + D^0(Q) \Pi(Q) D(Q) \]

where \( D^0(Q) \) is a 5 × 5 matrix composed of the free σ and ω propagators, and the \( \Pi \) matrix is the medium irreducible σ − ω selfenergy

\[ \Pi(Q) = \begin{bmatrix} \Pi^\sigma(Q) & \Pi^\omega(Q) \\ \Pi^\omega(Q) & \Pi_\sigma(Q) \end{bmatrix} \]

where \( \Pi^\sigma \) and \( \Pi_\sigma \) account for excitations over the Fermi sea driven by the ω and σ mesons respectively and \( \Pi_\sigma \) generates mixings of scalar and vector meson propagations in the medium. Obviously, this latter term vanishes in the vacuum. Having in mind the findings of Ref. [8], \( -V_{\Lambda\Lambda}^{\text{free}} \) should give us the bulk of \( V_{\Lambda\Lambda} \) and thus we have performed some approximations to evaluate \( \Pi(Q) \): i) We approximate \( G_{\Lambda N} \) and \( G_{NN} \) in Fig. 2 by the free space diagonal \( \Lambda N \) and \( NN \) potentials, which are well described by σ and ω exchanges in the isoscalar \( ^1S_0 \) channel. The ΛΛ and ΛΛω vertices were discussed in the previous subsection while the \( NN \) and \( NN \omega \) Lagrangians can be found in Ref. [8]. The corresponding coupling constants and form-factors can be found in Ref. [8] and in Table 3 of Ref. [8]. ii) We have only considered \( \text{ph} \) excitations over the Fermi sea. This corresponds to evaluate the diagrams depicted in Fig. 2 plus the corresponding crossed terms which are not explicitly shown there. iii) We work in a non-relativistic Fermi sea and we evaluate the \( \text{ph} \) excitations in the static limit.

With all these approximations and taking the four-momentum transferred between the two Λ’s, \( Q' = (q^0 = 0, 0, 0, q) \), the elements of the \( \Pi(Q) \) matrix read

\[ \Pi_{ij}(0, q) = U(0, q; \rho) C_i^N(q) C_j^N(q) ; \ i, j = 1, \ldots, 5 \] (6)

where \( C^B(q) \equiv (g_{\alpha BB}(q), 0, 0, g_{\sigma BB}(q)) \) with

\[ g_{\alpha BB}(q) = g_{\alpha BB} \left( \frac{\Lambda^2_{\alpha BB} - m_{\alpha}^2}{\Lambda^2_{\alpha BB} + q^2} \right) ; \ \alpha = \sigma, \omega; \ \Lambda = N \]

(7)

\[ \delta V_{\Lambda\Lambda}^{\text{RPA}}(q, \rho) = \sum_{ij=1}^5 C_i^N(q) \left[ D(Q) - D^0(Q) \right]_{ij} C_j^N(q) \]

(8)

In the non-relativistic limit adopted to evaluate \( \delta V_{\Lambda\Lambda}^{\text{RPA}} \), and for consistency, we have neglected the spatial and tensor \( f_{\omega\Lambda\Lambda} \) couplings of the \( \omega \) meson to the Λ.

b. Finite Nuclei: The Fourier transform of Eq. 8 gives the RPA ΛΛ nuclear matter interaction, \( \delta V_{\Lambda\Lambda}^{\text{RPA}}(r_{12}, \rho) \), in coordinate space. It depends on the constant density \( \rho \). In a finite nucleus, the carrier of the interaction feels different densities when it is traveling from one hyperon to the other. To take this into account, we average the RPA interaction over all different densities felt by the carriers along their way from the first hyperon to the second one. Assuming meson straight-line trajectories and using the local density approximation, we obtain

\[ \delta V_{\Lambda\Lambda}^{\text{RPA}}(1, 2) = \int_0^1 d\lambda \ \delta V_{\Lambda\Lambda}^{\text{RPA}}(r_{12}, \rho(|r_{12} + \lambda r_{12}|)) \]

(10)
III. RESULTS AND CONCLUDING REMARKS

Using the numerical constants and the YNG [21] (He) and BOY [22] (Be, B, Ca, Zr, Pb) nuclear core potentials given in Ref. [21], we obtain the results of Table I, where all the effect of the φ−exchange ΛΛ potential. The obtained values are 3.12, 6.71, 11.37, 18.7, 22.0, and 26.5 MeV.

\[
\begin{array}{cccccc}
\Lambda & \Lambda & \Lambda & \Lambda & \Lambda & \Lambda \\
\text{He} & 7 & 2.5 & 30.3 & 38.2 & 39.1 & 40.1 \\
\text{Be} & 17 & 7.7 & 0.4 & 24.2 & 25.4 & 27.0 \\
\text{B} & 27.5 & 0.7 & 22.5 & 22.6 & 23.2 & 23.8 \\
\text{Ca} & - & 37.2 & 37.3 & 37.7 & 38.1 & 38.3 \\
\text{Zr} & - & 44.1 & 44.2 & 44.4 & 44.7 & 46.4 \\
\text{Pb} & - & 53.1 & 53.1 & 53.3 & 53.4 & 53.4 \\
\end{array}
\]

Table I: Binding energies \( B_{ΛΛ} \) (MeV). Experimental values taken from Refs. [37] (He) [38] (Be) and [39] (B). We show theoretical results with and without RPA effects and with different treatments of the φ−exchange ΛΛ potential. The used \( B_{A} \) values are 3.12, 6.71, 11.37, 18.7, 22.0, and 26.5 MeV.

| \( B_{ΛΛ}^{\text{exp}} \) | \( B_{ΛΛ} \) | \( B_{ΛΛ} \) |
|-------------------|------------------|------------------|
| He                | 7.25 ± 0.33      | 6.15 ± 0.22      |
| Be                | 17.7 ± 0.4       | 13.1 ± 1.2       |
| B                 | 27.5 ± 0.7       | 22.5 ± 2.2       |
| Ca                | -                | 37.2 ± 3.3       |
| Zr                | -                | 44.1 ± 4.4       |
| Pb                | -                | 53.1 ± 5.3       |

The RPA re-summation leads to a new nuclear density or \( A \) dependence of the ΛΛ potential in the medium which notably increases \( ΔB_{ΛΛ} \) and that provides, taking into account theoretical and experimental uncertainties, a reasonable description of the currently accepted masses of the three measured ΛΛ hypernuclei (see last column of Table I). This is achieved from a free space OBE BJ potential determined from \( S = 0, -1 \), baryon-baryon scattering data. Hence, our calculation does not confirm the conclusions of Ref. [13] about the incompatibility of the He, and Be and B data. The binding energies of \(^{16}\) He and \(^{20}\) B might change if the single hypernuclei produced in Be and B events were produced in excited states [3]. The modified Be and B masses would then favor a different set of \( \Lambda_{A} \) and φΛΛ potentials (see columns 8−10 in Table I and discussion on \( SU(6) \) violations).

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