Entanglement charge of thermal states

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Entanglement charge is an operational measure to quantify nonlocalities in ensembles consisting of bipartite quantum states. Here we generalize this nonlocality measure to single bipartite quantum states. As an example, we analyze the entanglement charges of some thermal states of two-qubit systems and show how they depend on the temperature and the system parameters in an analytical way.

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It has been indicated that ensembles consisting of bipartite or multipartite quantum states can exhibit a kind of nonlocality that is different from quantum entanglement [1]. So far, most efforts have been devoted to addressing conditions that can be used to determine whether a given ensemble can exhibit this nonlocality [2–13]. Compared to the numerous studies about quantum entanglement, profound understandings on this intriguing ensemble nonlocality and its implications are still awaited. In a recent paper, we have introduced “entanglement charge” as an operational measure to quantify nonlocalities in ensembles consisting of bipartite quantum states [14]. The quantification of nonlocalities in ensembles has also been discussed in ref. [15].

In this paper we generalize the idea of entanglement charge to single bipartite quantum states. This generalization can be done is due to the fact that quantum states have ensemble decompositions [16]. Based on the entanglement charge, two kinds of nonlocalities on bipartite states are introduced, which are different from quantum entanglement. Therefore, this work can enrich our knowledge about quantum states. The paper is organized as follows. We first give a brief introduction to entanglement charge defined for ensembles consisting of bipartite quantum states. Then we generalize the concept of entanglement charge to single bipartite quantum states and apply it to some thermal states. Finally, a summary is given.

1 Entanglement charge of ensembles

Suppose $\varepsilon = \{p_X, \rho_X^{AB}\}$ is an ensemble consisting of bipartite states. The ensemble can be used to describe a bipartite quantum system $AB$ which is in state $\rho_X^{AB}$ with probability $p_X$. A measurement can be implemented on $AB$ to get information about which state the system really is in, and the obtained information can be measured by the mutual information between the random variable representing the measurement result and the random variable $X$ that determines which state the system is in. The maximal achievable information is denoted by $I_{\text{Global}}(\varepsilon)$ when there is no restriction on the measurement. If the measurement can only be implemented through local operations and classical communication (LOCC), the maximal achievable information will be denoted by $I_{\text{LOCC}}(\varepsilon)$. When $I_{\text{LOCC}}(\varepsilon) < I_{\text{Global}}(\varepsilon)$, some en-
tanglement can be consumed in addition to LOCC to get the same amount of information as $F_{\text{Global}}(\varepsilon)$. When $P_{\text{LOCC}}(\varepsilon) = F_{\text{Global}}(\varepsilon)$, some entanglement may be distilled in the process of getting the same amount of information as $F_{\text{Global}}(\varepsilon)$ through LOCC. The entanglement charge is defined to quantify the amount of entanglement consumed or distilled in the above two cases when we consider many copies of the same ensemble [14].

More precisely, the entanglement charge of the ensemble $\varepsilon = \{p_X, \rho_{AB}^X\}$ is defined through considering its tensor power $\varepsilon^{\otimes n} = \{p_X, \rho_{AB}^{X^n}\}$, where $p_X = p_{X_1}p_{X_2} \cdots p_{X_n}$, $\rho_{X_1A_1}^{X^n} = \rho_{X_1}^{A_1} \otimes \rho_{X_2}^{A_2} \cdots \otimes \rho_{X_n}^{A_n}$ and $X_n$ are independent and identically distributed classical variables as $X$. Suppose Alice holds $A^n$ and Bob holds $B^n$. To get the information about the value of $X^n$, they can make a measurement that satisfies the conditions: (1) the mutual information between $X^n$ and the measurement result $Y$ satisfies $I(X^n; Y) \geq F_{\text{Global}}(\varepsilon^{\otimes n}) - \delta_n$ with $\lim_{n \to \infty} \delta_n = 0$; (2) it is implemented through LOCC plus $n \times \alpha_n$ bits of entanglement; (3) when the measurement result $Y$ with probability $p_Y$ is obtained, $n \times \beta_n$ bits of entanglement are distilled. The entanglement charge of the ensemble $\varepsilon = \{p_X, \rho_{AB}^X\}$ is defined as:

$$N(\varepsilon) = \inf_{n \to \infty} \left( \alpha_n - \sum_{p_Y} p_Y \times \beta_n \right),$$

(1)

where the infimum operation is taken over all measurements satisfying the above conditions [14].

The entanglement charge $N(\varepsilon)$ of the ensemble $\varepsilon$ may be positive, negative or zero [14]. The ensembles with positive $N(\varepsilon)$ are defined to have information nonlocality and those with negative $N(\varepsilon)$ are defined to have entanglement nonlocality [14]. In both cases the entanglement charge $N(\varepsilon)$ or its absolute value $|N(\varepsilon)|$ can be used as a measure to quantify the corresponding nonlocality.

Usually it is hard to compute $N(\varepsilon)$. However, when the states $\rho_{AB}^X$ in the ensemble $\varepsilon = \{p_X, \rho_{AB}^X\}$ are pure states, the entanglement charge $N(\varepsilon)$ satisfies the following bounds [14]:

$$N(\varepsilon) \leq S(\rho_{AB}^X) - S(\rho^B),$$

(2)

$$N(\varepsilon) \leq S(\rho_{AB}^X) - S(\rho^A),$$

(3)

$$N(\varepsilon) \geq \sum_{p_X} p_X S(\rho_{AB}^X) - I_{\text{log}}(A; B) - \Delta(\varepsilon),$$

(4)

where $\rho_{AB}^X = \text{Tr}_{B^n} \rho_{AB}^{X^n}$, $\rho_{AB}^X = \sum_X p_X \rho_{AB}^{X^n}$, $\rho^B = \text{Tr}_{A} \rho_{AB}^{X^n}$, $\rho^A = \text{Tr}_{B} \rho_{AB}^{X^n}$, $S(c)$ is the quantum entropy, $I_{\text{log}}(A; B) = S(\rho^A) + S(\rho^B) - S(\rho_{AB}^X)$ is the quantum mutual information [16], and $\Delta(\varepsilon) = S(\rho_{AB}^X) - F_{\text{Global}}(\varepsilon)$. We note that there is $F_{\text{Global}}(\varepsilon) = S(\rho_{AB}^X)$ when the pure states in $\varepsilon$ are mutually orthogonal [14].

Especially, when $\rho_{AB}^X$ in $\varepsilon = \{p_X, \rho_{AB}^X\}$ are $d \times d$ mutually orthogonal maximally entangled pure states, the upper bounds (2) and (3) and the lower bound (4) of $N(\varepsilon)$ become the same value and the entanglement charge will be given by an analytical expression [14]:

$$N(\varepsilon) = S(\rho_{AB}^X) - S(\rho^B) = S(\rho_{AB}^X) - \log d.$$

(5)

Eq. (5) will be used when we address the entanglement charge of some thermal states.

2 Entanglement charge of bipartite states

Consider the bipartite quantum state $\rho_{AB}^{XY}$. If it is a mixed state, it has many pure state ensemble decompositions [16]. For example, the two-qubit state $\rho_{AB}^{XY} = \frac{1}{2} I_{XY} + \frac{1}{2} \rho_{AB}$ can be decomposed as an ensemble consisting of the four computational basis states with equal probabilities or an ensemble consisting of the four Bell states with equal probabilities. Among all the pure state ensemble decompositions of $\rho_{AB}$, we can select a specific one and define the entanglement charge of $\rho_{AB}$ as the entanglement charge of this selected ensemble. The question is which ensemble should be selected. To define the entanglement charge of $\rho_{AB}$, we select the pure state ensemble that has the maximal entanglement charge. More rigorously, the entanglement charge $N(\rho_{AB}^{XY})$ of the state $\rho_{AB}^{XY}$ is defined as:

$$N(\rho_{AB}^{XY}) = \max_{\rho^{AB} \in \mathcal{D}} \left[ N\left( \left\{ p_i, |\Psi_i\rangle \langle \Psi_i| \right\} \right) \right],$$

(6)

where $N(\left\{ p_i, |\Psi_i\rangle \langle \Psi_i| \right\})$ denotes the entanglement charge of the ensemble $\left\{ p_i, |\Psi_i\rangle \langle \Psi_i| \right\}$.

The entanglement charge $N(\rho_{AB}^{XY})$ of the state $\rho_{AB}^{XY}$ defined above can be understood as follows. Usually there are two cases where we use a density operator $\rho_{AB}^{XY}$ to describe the state of the system $AB$: the system $AB$ together with a reference system $C$ is in a pure state or the state of the system $AB$ is described by a pure state ensemble. Let us consider the latter case. In this case we only know the system $AB$ is in some known pure state with some probability and we can ask what minimal amount of entanglement is needed in addition to LOCC to get the maximal obtainable information about the true state of the system. If we are only concerned with measurements on $AB$, the density operator $\rho_{AB}^{XY}$ can be introduced as a convenient tool to calculate the probabilities of measurement outcomes. However when we use the density operator $\rho_{AB}^{XY}$ instead of a pure state ensemble to describe the system $AB$, we lose information about the possible pure states of the system. So when we use the density operator $\rho_{AB}^{XY}$ to describe the system $AB$, it represents many situations where each has a pure state ensemble description. Among all these situations, the entanglement charge $N(\rho_{AB}^{XY})$ defined above just quantifies the minimal amount of entanglement needed in addition to LOCC in the worst situation to get the maximal obtainable information about the true state of the system in the asymptotic limit.

It is usually hard to compute $N(\rho_{AB}^{XY})$, however it is obvious from eq. (2) that $N(\rho_{AB}^{XY}) \leq S(\rho_{AB}^{XY}) - S(\rho^B)$ and when $\rho_{AB}^{XY}$ has an ensemble decomposition that consists of $d \times d$ mutually orthogonal maximally entangled pure states there is $N(\rho_{AB}^{XY}) = S(\rho_{AB}^{XY}) - S(\rho^B)$.

It was mentioned in the previous section that the value of the entanglement charge of ensembles may be positive, zero or negative, so it is with the entanglement charge of a state.