A Novel Wind Turbine Wake Steering Model Employing the Ainslie Velocity Deficit

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Abstract. Wind turbine wakes are an important source of production losses in wind parks, and strategies to mitigate these effects are being explored. One such strategy is yaw steering, where a wind turbine is purposefully misaligned with the wind to deflect its wake away from a downstream turbine, increasing overall farm power production. In order to optimize yaw steering control, fast and accurate models for wake deflection are necessary. Several models have been proposed in literature, often based on approximations of the wake shape. In the present work, a new model is proposed that computes wake deflection based on the result of the Ainslie wake deficit model used in the Dynamic Wake Meandering (DWM) approach that has recently entered the IEC61400 standard for wind turbines. The proposed formulation makes few additional assumptions and introduces no additional fitting parameters beyond those of the DWM wake deficit model. Results obtained using the new methodology are compared to two models from literature and to LES-ALM simulations in a limited number of cases, showing satisfactory results. More extensive comparisons in a variety of cases should be performed in order to better evaluate the proposed methodology. Possible future improvements to the presented formulation are put forward.

1. Introduction
In the context of efficient wind farm operation, wake steering through yaw control is sliding into the focus of academic and industrial research alike [1]. First studies on the phenomenon have been performed almost 40 years ago [2], however deriving and validating numerical models has proven difficult, especially due to the complexity of performing validation experiments in the field [1]. Jiménez, Crespo and Migoya [3] studied the phenomenon using Large Eddy Simulations (LES) and proposed the first analytic model for wake deflection of yawed turbines. The Jiménez deflection model is based on a top-hat shaped wake velocity profile as proposed by Jensen [4], combined with a lateral momentum balance. Though this method has been used extensively, it is known to over-predict wake deflection [5, 6]. Bastankhah and Porté-Agel [5] proposed a different model based on a Gaussian wake velocity profile in the far wake, using a potential core model for the near wake. The Gaussian model has been used and elaborated in a series of recent publications: Qian and Ishihara [6] simplified the model assuming a top-hat shaped, rather than Gaussian, skew angle distribution and proposed new model parameters. Shapiro et al. [7] used Prandtl’s lifting line theory to compute the near wake, avoiding the use of the potential core model. Recently, Martinez-Tossas et al. [8] proposed a new model based on discrete, non-decaying vortexes shed from the rotor.
In this work, a novel formulation for yaw deflection is introduced. The formulation is based on a momentum balance similar to the model of Jiménez. Rather than using a prescribed wake shape, the velocity deficit is taken from the wake deficit model used in the dynamic wake meandering model (DWM) by Larsen et al. [9]. With the recent addition of DWM to the international standard for wind turbines IEC61400 [10], the DWM methodology is expected to find wide-ranging application in the industry. The wake deflection model presented in the following sections shows a formulation that is conceptually compatible with DWM and can be implemented quickly along side it and run at a minimal additional computational expense. Details and derivation of the model are laid out and results obtained from it are compared to other deflection modeling approaches from the literature.

2. Model Assumptions

The proposed wake deflection model is based on the wake deficit model of Ainslie [11]. This formulation relies on solving a set of parabolic, axi-symmetric Navier-Stokes equations to propagate an initial wake deficit downstream. For the present work, the implementation of Ainslie’s model proposed by Larsen et al. in their DWM model is used. The deflection model therefore inherits the assumptions taken in this model, including the assumption of axi-symmetry of the wake around the center line. It is known from both field measurements and LES studies that this is not true in yawed cases, where the wake assumes a characteristic kidney shape [5, 8]. This behavior was confirmed again in the present work by performing LES using an actuator line model (ALM) for a wind turbine in yaw. The turbine used for this study is a commercial offshore machine, operating at a sub-rated wind speed with a fixed yaw misalignment. The calculated wake velocity is shown in Fig. 1, and the characteristic kidney shape of the wake velocity field is observed. The assumption of a radially symmetric wake is nonetheless considered acceptable in literature as long as yaw angles remain small [3], and will be used in the present formulation as well. The wake deflection model is used to obtain a horizontal offset by which the wake center

![Figure 1. Radial and axial cross sections through the velocity field in the wake of a turbine in 30° yaw. Radial sections at 4, 8 and 16 diameters downstream of the turbine, from LES-ALM simulations.](image)
line is displaced from the rotor axis. This assumption is similar to that of the passive tracer assumption for wake meandering proposed by Larsen et al. [9], and is therefore considered in line with the model assumptions of DWM. Furthermore, the addition of such a yaw steering offset to an implementation of DWM is straightforward, as the computed deflection can directly be added to the existing meandering time series.

3. Model Derivation

Under the described assumption, a simple model for wake deflection can be derived. Similar to the work of Jiménez et al. [3], a momentum balance is written for a streamtube enclosing the rotor as depicted in Fig. 2. The following expression is obtained:

$$\vec{F} = \dot{\vec{p}}_3 - \dot{m}_1 \vec{u}_0 - \dot{m}_2 \vec{u}_0.$$  \hspace{1cm} (1)

In this equation $\dot{\vec{p}}_3$ denotes the momentum flux over the outlet of the streamtube, $\dot{m}_1$ is the mass flux over the domain inlet, $\dot{m}_2$ the mass flux through the lateral boundaries and $\vec{F}$ the total body force exerted by the wind turbine on the flow. This force can be directly deduced from local induction $a(r)$, which is also a required input for the DWM wake deficit model and therefore necessarily available in any DWM implementation. The total thrust force of the rotor is obtained by integrating the local thrust of an annular section (given by momentum theory) over the rotor:

$$F = \int_0^R 4\pi \rho (u_0 \cos \gamma)^2 a(r)(1 - a(r))r \, dr.$$  \hspace{1cm} (2)

Figure 2. Schematic representation of the proposed wake steering model based on an Ainslie-type wake deficit model. The velocity in the wake $u(r)$ is taken from the deficit model at a given downstream distance, while the central deflection angle $\theta_c$ is retrieved from a momentum balance over the wake streamtube.
The momentum flux in both axial and lateral direction over the outlet of the streamtube of radius $b$ can be computed considering the wake skew angle $\theta(r)$ and the wake velocity $u(r)$:

\[
\dot{p}_{3,x} = \int_{0}^{b} 4\pi \rho u(r)^2 \cos\theta(r) r \, dr, \tag{3a}
\]

\[
\dot{p}_{3,y} = \int_{0}^{b} 4\pi \rho u(r)^2 \sin\theta(r) r \, dr. \tag{3b}
\]

It is worth noticing that both the momentum flux at the inlet and the entrained momentum flux only have an axial component, as the velocity field outside the wake steam tube is assumed to be uniform and directed in mean wind direction. Decomposing the momentum balance into an axial and lateral directions $x$ and $y$ yields therefore:

\[
F \cos \gamma = \dot{p}_{3,x} - Q_{m,1} u_0 + Q_{m,2} u_0, \tag{4a}
\]

\[
F \sin \gamma = \dot{p}_{3,x}. \tag{4b}
\]

A further assumption is made in order to obtain an expression for the wake deflection directly from the lateral momentum balance Eq. 4b. The wake skew angle $\theta(r)$ is assumed to follow the same distribution as the velocity deficit according to the expression:

\[
\theta(r) = \theta_c \frac{\Delta u(r)}{\Delta u_c}, \tag{5}
\]

where $\theta_c$ and $\Delta u_c$ are the center line wake deflection angle and velocity deficit respectively, while $\Delta u(r)$ is the wake deficit at distance $r$ from the wake center. This assumption is similar to what is proposed in the Gaussian deflection model developed by Bastankhah and Porté-Agel [5], where it is assumed that the skew angle follows the same Gaussian behavior as the velocity deficit.

Inserting Eq. 2 and 3b into Eq. 4b, assuming small angles, substituting Eq. 5 and solving for the center line wake deflection $\theta_c$ finally yields the following, simplified expression:

\[
\theta_c \approx \frac{\cos^2 \gamma \sin \gamma \int_{0}^{R} a(r)(1 - a(r)) r \, dr}{\int_{0}^{b} \left(1 - \frac{\Delta u(r)}{u_0}\right)^2 \frac{\Delta u(r)}{\Delta u_c} r \, dr}. \tag{6}
\]

The integrals in the denominator can only be solved numerically, as the center line and local wake deficit, $\Delta u_c$ and $\Delta u(r)$, are only known on a discrete radial grid from the Ainslie model. The integration is performed using piece-wise constant numerical quadrature, which was found to be sufficient due to the regular behavior of the integrand. Integrating the tangent of $\theta_c$ numerically along $x$ finally gives the deflection of the wake center $\delta$:

\[
\delta = \int_{0}^{X} \theta_c \, dx. \tag{7}
\]

4. Comparisons with other deflection models

Results of the newly developed approach are compared with two models commonly employed in wind turbine research and industry, the model of Jiménez et al. [3] and the Gaussian model of Bastankhah and Porté-Agel [5]. Comparisons were run for a three-bladed commercial offshore wind turbine with a rotor diameter of approximately 150 meters and nominal power of 6 MW.
The wind conditions correspond to a case of standard operation at below-rated wind speed. The Jiménez model was parametrized according to standard literature [12], while the Gaussian model was parametrized according to recent work of Altun [13]. The parameters used in both models are summarized in Tab. 1. Furthermore, simulations with yaw error were run using the ALM-LES set-up described by Bénard et al. [14].

Fig. 3 shows a horizontal cross section at hub height through the time-averaged LES result. Results for center line deflection as given by the Gaussian and Jiménez models as well as the new formulation are plotted over the LES results. The center line of the LES wake is, defined by taking the midpoint between the two positions with wind speed equal to the 95% of the free-stream velocity [3] is also displayed.

It is remarked that none of the presented results can be considered a perfect benchmark. In particular, the LES-ALM model for the turbine in question has not been validated with field data in yawed conditions and is lacking realistic turbulence, which plays a large role in wake propagation. It can therefore only be regarded as an indication of wake behavior compatible with the physics of fluid flow, rather than as a reference. The Jiménez model is known to overestimate the wake deflection in the far wake [5], where it predicts the largest deflection among the approaches. The Gaussian model, in its here-presented form, has been optimized for the turbine set-up in question and is therefore expected to give satisfactory results.

Results show, that the Gaussian model tends to predict a slightly lower mean center line deflection than the LES-ALM approach, while the Jiménez model predict a significantly higher deflection. Results of the novel formulation are more in line with the Jiménez model for the presented test case. However, unlike Jiménez, the predicted deflection does not appear to diverge in the far wake, a behavior which is not expected to occur in actual HAWT wakes, and which is not predicted by the Bastankhah/Porté-Agel and LES-ALM results.

5. Discussion
Predictions for wake center line deflection of the newly proposed model are overall satisfactory compared to prevailing literature. For small downstream distances, in the typical range of inter-turbine spacing, results of the novel wake deflection formulation closely match with the model proposed by Jiménez et al. [3]. For larger downstream distances, predictions of the novel formulation are lower than those of the Jiménez model, but still higher than the prediction of the model of Bastankhah and Porté-Agel. The new formulation therefore brings an improvement compared to the Jiménez model, which is known to overestimate deflection at large downstream distances. LES-ALM simulations at two yaw configurations gave results for wake center line deflection that match closely with the model of Bastankhah and Porté-Agel. Given the current validation status of the LES-ALM computation the results cannot be used as a reference [14].

The key advantage of the newly proposed formulation compared to other models for wake deflection found in literature is that no additional calibration parameters beyond those of the underlying Ainslie deficit model are required. This eliminates a source of uncertainty and results in a consistent wake modeling framework. The direct dependency requires a deficit

| Parameter | Value |
|-----------|-------|
| $k_e$ ($k_\sigma$) | Jiménez [3] 0.04 |
| | Gaussian [13] 0.039$I_{amb}$ +0.006 |
| $\alpha$ | 3.0782 |
| $\beta$ | 0.0213 |

Table 1. Table caption.
Figure 3. Comparison the new deflection model to the formulations of Jiménez [3] ($k_e=0.039$) and Bastankhah [5] ($k_e=I_{amb} +0.006$, $\alpha=3.08$, $\beta=0.0213$ [13]) at (a) 15 degree and (b) 30 degree yaw misalignment. Center line deflection predicted by the models is plotted on top of the average wake velocity field computed using LES-ALM [14].

model that gives reliable results for wakes generated by a yawed turbine. In the here-presented computations, the yaw angle was considered in the Ainslie formulation by assuming the local induction at each radial position to be a direct function of the rotor-normal incoming wind speed. No further yaw corrections were applied, but this possibility could be be explored in further work. The proposed deflection model is tailored to be implemented alongside the dynamic wake meandering (DWM) model as proposed in the IEC61400 standard [10], where the computed wake deflection can be added directly to the meandering offset already present in the model. Lastly, it has been shown that in multi-turbine configurations, added wake turbulence has a large impact on wake deflection [13]. While such effects cannot be implemented in a Jiménez approach, and have to be implemented explicitly in the case of a Gaussian model, the here-presented model takes these effects into account intrinsically since added turbulence is already part of the wake modeling tool. No validation steps on multiple-turbine configurations have been performed, however fair performance of this model is expected.

Due to its comparatively simple assumptions, the newly introduced model suffers from some inherent weaknesses who’s importance remains to be assessed. Firstly, the underlying Ainslie model assumes a radially symmetrical wake, even in the case of yaw. This is known to be a questionable approximation that is accepted by Jiménez et al. [3], while Bastankhah and Porté-Agel [5] choose an elliptic deficit shape in their formulation. Taking into account the true shape of the deflected wake requires a more detailed analysis of the vortex structure behind a
yawed turbine. Such a model is proposed by Martínez-Tossas et al. [8], however the impact on such a detailed treatment on the center line deflection is not well understood, especially in the context of wake meandering and turbulence. Secondly, the Ainslie wake deficit, as proposed in the IEC61400 standard [10], relies on the assumption of immediate wake expansion (see Madsen et al. [15]), meaning that the wake deficit computed in the immediate near wake is inherently incorrect. This in not taken into consideration in the here presented model, meaning that the deflection predicted in the immediate near wake is likely to be exaggerated. This leads to an offset on the total deflection farther downstream that should be corrected for. Similarly, neglecting the pressure term, as is done in the Ainslie model, might no longer be adequate when trying to model a yawed turbine, as the pressure distribution in the near wake could impact the wake deflection.

Further comparisons to other models and LES studies on different turbines and wind conditions should be undertaken to assess performance of the novel formulation, and ideally a validation with LIDAR data from field measurements on yawed turbines should be performed.

6. Conclusions

A promising new approach for wake deflection modeling is derived, implemented and tested. The proposed model is designed to be easily added onto any implementation of the DWM wake model proposed in the IEC61400 standard [10]. First comparisons to state-of-the-art deflection models show results similar to the formulation originally proposed by Jiménez et al. [3] for intermediate downstream distances. Unlike other engineering models, the newly proposed formulation does not rely on additional hypothesis and parameters beyond those of the underlying wake deficit model, reducing uncertainty and the risk of over-fitting. Possible improvements of the methodology include the use of a dedicated model for induction in yaw for the Ainslie deficit model [16], a correction for the assumption of radial symmetry in the wake, as well as a correction for the assumption of instantaneous wake expansion. Lastly, it is noted that improvements to the DWM wake deficit model are inherited directly by the newly proposed deflection model.

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