The physics of pulses in gamma-ray bursts: emission processes, temporal profiles and time lags.

Frédéric Daigne and Robert Mochkovitch
Institut d’Astrophysique de Paris, 98 bis bd. Arago, 75014 Paris, France

5 December 2018

1 INTRODUCTION

Cosmic gamma-ray burst (hereafter GRBs) exhibit a great diversity of duration and profiles. The distribution of durations is clearly bimodal with two peaks at about 0.2 and 20 seconds. GRB light curves are highly variable but can often be interpreted in terms of a succession of elementary pulses which possibly overlap (Norris et al. 1994). These pulses appear as the building blocks of the profiles and understanding their physical origin would certainly represent a clue for a better description of the whole GRB phenomenon. The pulse temporal evolution has often been described by a fast rise followed by an exponential decay (the so-called FRED shape; see Fishman et al. 1994) but other mathematical behaviors such as stretched exponentials, gaussian (Norris et al. 1996) or power-law decays (Ryde & Svensson 2000) have been also proposed. Spectral hardness decreases during pulse decay and two relations between the temporal and spectral properties, the HIC (hardness-intensity correlation; Golenetskii et al. 1983) and the HFC (hardness-fluence correlation; Liang & Kargatis 1996) appear to be satisfied by a substantial fraction of GRB pulses during the decay phase. Pulse profiles peak earlier in higher energy bands and the corresponding time lags between different energy channels correlate to pulse hardness and peak luminosity (Norris et al. 2000). These observational results must be reproduced by the models and may help to discriminate among different possibilities.

Two distinct mechanisms have been proposed to explain the origin of pulses in GRBs. In the external shock model they are formed when a relativistic shell ejected by the central engine is decelerated by the circumstellar material (Meszaros & Rees 1993). An homogeneous medium leads to a single pulse but an irregular, clumpy environment can produce a complex profile if a large number of small clouds are present (Dermer & Mitman 1999). In the internal shock model (Rees & Meszaros 1994) the central engine generates a relativistic flow with a highly non uniform distribution of the Lorentz factor and the pulses are made by collisions between rapid and slower parts of the flow. In the two scenarios the variability of the profiles has a very different interpretation. In one case it provides a “tomography” of the burst environment while in the second it reveals the activity of the central engine.

In this paper we consider in some details the mechanism of pulse formation by internal shocks. Three characteristic time scales may be relevant during pulse evolution: the time $t_{\text{rad}}$ required to radiate the energy dissipated in shocks; the dynamical time $t_{\text{dyn}}$, i.e. the time taken by internal shocks to travel throughout the flow and the angular spreading...
time \( t_{\text{ang}} \) corresponding to the delay in arrival time of photons emitted from a spherical shell. A short radiative time \( t_{\text{rad}} \ll t_{\text{dyn}}, t_{\text{ang}} \) appears to be mandatory to avoid adiabatic losses and maintain a sufficient efficiency. This condition is satisfied by the synchrotron process which is the most commonly invoked radiation mechanism in GRBs. If the thickness of colliding shells is small compared to their initial separation, \( t_{\text{dyn}} \ll t_{\text{ang}} \) and the pulse temporal evolution is fixed by geometry; conversely if the source produces a continuous wind rather than a series of discrete, well separated shells, \( t_{\text{dyn}} \gtrsim t_{\text{ang}} \) and hydrodynamical effects control the pulse shape.

Pulse evolution has been studied extensively when it is dominated by geometry (see e.g. Fenimore et al. (1996); Kobayashi et al. (1997)) but discrepancies between model predictions and the observations (Soderberg & Fenimore 2001) have cast some doubt about the validity of the internal shock model. Our purpose is to see if the situation can be improved when the hydrodynamical point of view is adopted. We first summarize in Sect. 2 some basic informations regarding pulse temporal and spectral evolution. We then develop in Sect. 3 a simple model where pulses are formed when a fast moving wind is decelerated by a comparatively slower shell. Spectral evolution is considered in Sect. 4 where constraints are obtained on the GRB radiation condition is satisfied by the synchrotron process which is well separated shells, \( t_{\text{ang}} \gtrsim t_{\text{rad}} \) and hydrodynamical effects control the pulse shape.

2 TEMPORAL AND SPECTRAL EVOLUTION DURING PULSE DECAY

We consider a pulse characterized by a photon flux \( N(t) \) in the energy range \((E_1, E_2)\), a peak energy \( E_{\text{p}}(t) \) of the \( E^2 N(E,t) \) spectrum and a photon fluence defined by

\[
\frac{d\Phi(t)}{dt} = N(t) = \int_{E_1}^{E_2} \Phi(E,t)dE .
\]

(1)

The HIC and the HFC are then given by

\[
E_{\text{p}}(t) \propto N(t)^{\alpha} \]

(2)

and

\[
E_{\text{p}}(t) \propto e^{-\alpha N(t)} ,
\]

(3)

where \( \alpha \) is an exponential decay constant. For pulses satisfying both the HIC and the HFC, Ryde & Svensson (2004) have shown that the photon flux and the peak energy follow simple power laws during the decay phase

\[
N(t) = \frac{N_0}{1 + t/\tau}
\]

(4)

and

\[
E_{\text{p}}(t) = \frac{E_{\text{p},0}}{(1 + t/\tau)^{\beta}} ,
\]

(5)

where \( t = 0 \) corresponds to the maximum of \( N(t) \). Ryde & Svensson (2004) performed a detailed analysis of the decay behavior of a sample of 25 long and bright pulses to check whether it was indeed described by eqs 4 and 5. They found that to account for the temporal and spectral evolution of all the pulses, eq. 4 had to be replaced by the more general expression

\[
N(t) = \frac{N_0}{(1 + t/\tau)^{\alpha}} .
\]

(6)

If \( n \neq 1 \), the \( \delta \) indices appearing in eqs 2 and 5 are different and following Ryde & Svensson (2002) we then write

\[
E_{\text{p}}(t) = E_{\text{p},0}/(1 + t/\tau)^{\delta} ,
\]

(7)

with \( \delta = n\alpha \). Ryde & Svensson (2002) found that the distribution of \( n \) in their sample was sharply peaked at \( n = 1 \) with however a secondary bump at \( n \lesssim 3 \). The values of \( \delta \) were all smaller than 1.5 for the \( n = 1 \) pulses but could reach 3.5 when \( n \approx 3 \). The distribution of \( \delta \) was narrower with 0.5 \( \lesssim \delta \lesssim 1 \) in most of the sample.

Once the decay behavior of \( N(t) \) and \( E_{\text{p}}(t) \) has been specified, it becomes possible to obtain the evolution of the bolometric energy flux \( F_E(t) \) since

\[
F_E(t) = \int_0^\infty N(E,t)EdE = E_p^2 \int_0^\infty N(x,t)xdx ,
\]

(8)

where \( x = E/E_p \). We suppose that the temporal and spectral behavior can be separated in \( N(x,t) \) :

\[
N(x,t) = A(t)B(x) ,
\]

(9)

\( B(x) \) representing the spectrum shape. The photon flux in the energy range \((E_1, E_2)\) is then given by

\[
N(t) = \int_{E_1}^{E_2} \Phi(E,t)dE = \frac{F_E(t)}{E_p(t)} \Phi(E_p) ,
\]

(10)

so that

\[
F_E(t) = N(t)E_p(t) \Phi(E_p) \Phi(E_p) \frac{\phi_0}{\phi_0} = \frac{N_0 E_{p,0}}{(1 + t/\tau)^{1+\beta}} \phi_0 \phi_0
\]

(11)

and

Figure 1. Derivative of \( \Phi(E_p) \) for three Band functions with \( \beta = -2.5 \) and \( \alpha = -2/3 \) (dotted line) \( \alpha = -1 \) (full line) and \( \alpha = -1.5 \) (dashed line). The two vertical lines limit the BATSE spectral range. The average slope during pulse decay typically lies between 0 and 1.
The physics of pulses in GRBs: emission processes, temporal profiles and time lags

Figure 2. Solution for $\gamma(\tau)$ corresponding to eq. 21 with $\gamma_0 = 0.25$. The dashed line is the approximation given by eq. 23.

$\varphi_0 = \int_{0}^{\infty} x B(x) dx$. (12)

The derivative of $\varphi(E_p)$ has been represented in Fig. 1 for the BATSE spectral range (20, 1000 keV), using a standard Band function (Band et al. 1993) with low and high energy indices $\alpha = -2/3$, $-1$ or $-1.5$ and $\beta = -2.5$. At low (resp. high) $E_p$, $\varphi(E_p)$ is given by a simple power-law $E_p^{-\beta+1}$ (resp. $E_p^{-\alpha+1}$) but for intermediate values ($E_1 < E_p < E_2$) which are representative of the decay phase in the Ryde & Svensson (2002) sample, $\varphi(E_p)$ does not have a simple analytical form. Assuming that it can still be approximated by a power-law, $\varphi(E_p) \propto E_p^{-\zeta+1}$ where $\zeta$ is a weighted average of the low and high energy spectral indices ($-2 < \zeta < -1$) the bolometric energy flux also follows a power-law

$F_b(t) \propto \frac{1}{t^{\epsilon}}$, (13)

with $\epsilon = n + (2 + \zeta) \delta_*$. (14)

The slope of the HIC is then given by

$\delta = \frac{\delta_*}{n} = \frac{1}{\epsilon/\delta_* - (2 + \zeta)}$. (15)

If the temporal and spectral evolution during pulse decay is due to geometrical effects alone $\epsilon = 3$ and $\delta_* = 1$ (Granot et al. 1999) which leads to $\delta = \frac{1}{\zeta}$. With $-2 < \zeta < -1$ the resulting value $0.3 < \delta < 0.5$ lies below what is found in most observed pulses (Soderberg & Fenimore 2001).

Geometrical effects govern pulse evolution if the shell thickness is small compared to their initial separation. But if a continuous outflow emerges from the central engine the hydrodynamical time scale can play a dominant role during pulse decay. We have then developed a simple model to check whether a better agreement can be found with the observations when the hydrodynamical aspect of the flow is taken into account.

3 A SIMPLE PULSE MODEL

We consider a relativistic wind where a slow shell of mass $M_0$ and Lorentz factor $\Gamma_0$ decelerates a more rapid part of the flow characterized by a constant mass flux $\dot{M}$ (in the source frame) and Lorentz factor $\Gamma_1 > \Gamma_0$. We do not solve the true hydrodynamical problem but rather approximate the flow evolution by considering that fast material is "accreted" by the slow shell. The accretion rate is given by

$\frac{dM}{d\tau} = \dot{M} (1 - \gamma^2)$, (16)

where $t$ is the observer time and $\gamma = \Gamma / \Gamma_1$ ($\Gamma$ and $M$ being the current Lorentz factor and mass of the slow shell). Due to the accretion of fast moving material, the Lorentz factor of the slow shell increases. When a mass element $dM$ is accreted the Lorentz factor becomes

$\Gamma + d\Gamma = \left( \frac{\Gamma_1 \Gamma_1 \dot{M} \tau + \Gamma_0 M}{\Gamma_0 \dot{M} \tau + \Gamma_1 M} \right)^{1/2}$, (17)

so that

$\frac{d\gamma}{dM} = \frac{1 - \gamma^2}{2M}$, (18)

which can be integrated to give

$\mu = \frac{1 + \gamma}{1 - \gamma} / \left( \frac{1 + \gamma_0}{1 - \gamma_0} \right)$, (19)

where $\mu = M/M_0$ and $\gamma_0 = \Gamma_0 / \Gamma_1$. Introducing $t_0 = M_0 / \dot{M}$ and $\tau = t/t_0$, eqs 16–19 yield

$\frac{d\gamma}{d\tau} = Q(1 - \gamma^2)(1 - \gamma)^2$ (20)

with $Q = \frac{1}{4} \left( \frac{1 + \gamma}{1 - \gamma} \right)$. Equation 20 has the analytical solution

$\tau = \frac{1}{Q} \left[ F(\gamma) - F(\gamma_0) \right]$ (21)

where the function $F(\gamma)$ is given by

$F(\gamma) = \frac{1}{8} \log \left( \frac{1 + \gamma}{1 - \gamma} \right) + \frac{1}{4(1 - \gamma)} + \frac{1}{4(1 - \gamma)^2}$. (22)
The solution $\gamma(\tau)$ corresponding to eq. [21] has been represented in Fig. 4 for $\gamma_0 = 0.25$. When $\tau \gtrsim 2$, it is well approximated by

$$\gamma(\tau) \simeq 1 - \frac{1}{2\sqrt{\Gamma_1^2}}. \quad (23)$$

Once $\gamma(\tau)$ is known it is possible to calculate the dissipated power

$$\dot{E}(\tau) = \frac{M\Gamma_1 c^2}{2}(1 - \gamma^2)(1 - \gamma)^2, \quad (24)$$

which has been represented in Fig. 3. At large $\tau$, it behaves as $\tau^{-3/2}$ since

$$\dot{E}(\tau) \propto (1 - \gamma)^3(1 + \gamma) \propto \tau^{-3/2} \quad (25)$$

for $\tau \gtrsim 2$. Pulse evolution is essentially completed at $\tau \sim 10$ when $\Gamma/\Gamma_1 > 0.8$ and $\dot{E}$ has decreased by more than an order of magnitude.

The dissipated power given by eq. [24] is slightly different from what the observer will see since the energy released at time $t$ is spread over an interval $\Delta t$ corresponding to the difference in arrival time for photons emitted by a shell of radius $r$ moving at a Lorentz factor $\Gamma$

$$\Delta t = \frac{r}{2c\Gamma^2}. \quad (26)$$

The solution for $\dot{E}$ including angular spreading has been obtained numerically and is also shown in Fig. 3. It differs from the analytical expression (eq. [21]) at early times but preserves the power law decay of slope $\epsilon = 3/2$ at late times.

4 SPECTRAL EVOLUTION AND EMISSION PROCESSES

We now use the analytical model to follow the spectral evolution during pulse decay. If the dissipated energy is radiated by the synchrotron process the peak energy $E_\text{p}$ is

$$E_\text{p} = E_{\text{syn}} \propto \Gamma B\Gamma_1^2, \quad (27)$$

where $B$ is the magnetic field and $\Gamma_e$ the characteristic electron Lorentz factor behind the shock. With classical equipartition assumptions $B$ and $\Gamma_e$ can be expressed as

$$B = (8\pi\alpha_B \rho c^2)^{1/2} \quad (28)$$

and

$$\Gamma_e = \frac{\alpha_e}{\zeta m_e c \epsilon}, \quad (29)$$

where $\rho$ is the density and $\epsilon c^2$ the dissipated energy per unit mass (both in the comoving frame); $\alpha_B$ and $\alpha_e$ are the equipartition parameters and $\zeta$ is the fraction of electrons which are accelerated. Finally,

$$E_{\text{syn}} \propto \Gamma \rho^{1/2} \epsilon^{5/2}, \quad (30)$$

where the comoving density $\rho$ is proportional to $r^{-2}$ ($r$ being the shock radius $r \sim \Gamma^2 c t$) and $\epsilon$ is obtained from $\dot{E} = \frac{dM}{dt} \Gamma \epsilon c^2$ and eq. [21]

$$\epsilon = \frac{(1 - \gamma)^2}{2\gamma}. \quad (31)$$

This leads to the following expression for $E_{\text{syn}}$

$$E_{\text{syn}} \propto \frac{(1 - \gamma)^5}{\gamma^{7/2} t}, \quad (32)$$

which behaves as a power law ($E_\text{p} \propto t^{-7/2}$) when $(1 - \gamma) \sim t^{-1/2}$. This is much steeper than the observed spectral evolution of pulses which satisfy both the HIC and the HFC.

Instead of using eq. [30] we therefore parametrize the peak energy with the more general phenomenological expression

$$E_\text{p} \propto \Gamma \rho^x \epsilon^y \propto \frac{(1 - \gamma)^{2y}}{\gamma^{4x + y - 1 + 2y}}, \quad (33)$$

which becomes

$$E_\text{p} \propto \frac{1}{\epsilon^{2x+y}} \quad (34)$$

at late times. The exponents $x$ and $y$ can be different from their standard synchrotron values $1/2$ and $5/2$ if the equipartition parameters $\alpha_B$, $\alpha_e$, or $\zeta$ vary with $\rho$ or $\epsilon$. For example, Daigne & Mochkovitch (1998) adopted a fraction $\zeta$
The physics of pulses in GRBs: emission processes, temporal profiles and time lags

5 TEMPORAL PROFILES

We obtain the temporal profile of synthetic pulses from eqs 6, 24 and 33 of our model. We have represented in Fig. 4 a pulse formed when a wind of Lorentz factor \( \Gamma_1 = 400 \) and power \( M \Gamma_1 c^2 = 10^{52} \text{ ergs s}^{-1} \) is decelerated by a slow shell with \( \Gamma_0 = 100 \). We adopt \( t_0 = 0.4 \text{ s} \) and \( x = y = 1/4 \) and \( z = 1 \). The profile is computed in the BATSE range \((20 - 1000 \text{ keV})\) and the constant of proportionality in eq. 33 is fixed to get a peak energy \( E_p = 300 \text{ keV} \) for the whole pulse spectrum. The pulse duration is close to 10 \((1 + z) t_0\) as expected from the results obtained in Sect.3.

The evolution after maximum is initially close to a \( 1/t \) decay (i.e. \( n \sim 1 \) in eq. 6) as can be seen in Fig. 4 where \( 1/N(t) \) has been also represented. This can be simply understood from eq. 14 which, for the decay slopes of the dissipated power \( \epsilon = 1 \) (eq. 25) and of the peak energy \( \delta_\ast = 2x + y = 0.75 \) gives

\[ n = -0.75 \zeta . \]  

(35)

With \(-2 \lesssim \zeta \lesssim -1\), the central value of \( n \) is indeed close to unity. Since the decay phase of our synthetic pulse can be described by eqs. 6 and 14 it should also satisfy both the HIC and the HFC. This is checked in Fig. 4 where the two relations have been plotted in the time interval delimited by the two vertical lines in Fig. 4. The HIC is not a strict power law but its average slope \( \sim 0.9 \) is in global agreement with the observations. The HFC is satisfied to a better accuracy since the relation between \( \phi \) and \( \psi \) factor in eq. 9 is in different energy channels obtained from eq. 9. The first factor in eq. 9 is \( \psi(t) = \frac{F_p(t)}{E_p(t)} \),

\[ \psi(t) = \frac{F_p(t)}{E_p(t)} , \]  

(37)

which behaves as \( t^{2x+y-1.5} \) during pulse decay. The sign of \( \Delta \) is of great importance in determining the time \( t_{\max} \) of maximum count rate and the related time lags.

6 TIME LAGS

Norris et al. \((2000)\) have shown that time lags between different energy channels correlate with spectral hardness and possibly also with the burst peak luminosity. GRBs are distributed in a triangular domain of the time lag–hardness ratio diagram (the hardest bursts having the shortest time lags; see Fig. 6 and the time lag–luminosity relation obtained from 6 bursts with known redshifts takes the form

\[ L_{31} \simeq 130 \left( \frac{\Delta t_{31}}{0.01 \text{ s}} \right)^{−1.14} , \]  

(36)

where \( \Delta t_{31} \) is the time lag between BATSE channels 3 and 1 and \( L_{31} \) is the luminosity in units of \( 10^{51} \text{ ergs s}^{-1} \). In our model, we estimate time lags by cross-correlating profiles in different energy channels obtained from eq. 9. The first factor in eq. 9 is \( \psi(t) = \frac{F_p(t)}{E_p(t)} \),

\[ \psi(t) = E_p(t) \]  

(37)

which behaves as \( t^{2x+y-1.5} \) during pulse decay. The sign of \( \Delta \) is of great importance in determining the time \( t_{\max} \) of maximum count rate and the related time lags.
temporal and spectral properties are probably governed by the hydrodynamics of the flow rather by the geometry of the emitting shells.

We finally checked if our model was able to reproduce the time lag – luminosity correlation (eq. 36). When $x = y = 1/4$, we do find that the lags decrease with increasing luminosity. This is a consequence of the HIC and the time lags – hardness ratio relation discussed above. The results are shown in Fig. 4 where the three lines correspond to the wind cases with $\Gamma_1 = 300, 400$ and 600 already considered in Fig. 6. It can be seen that there is an overall agreement between the model predictions and eq. 36. If this is confirmed by the analysis of more GRBs with known redshifts, the time lags between energy channels decrease with increasing pulse hardeness and peak luminosity. We therefore conclude that if GRB pulses are produced by internal shocks, their temporal and spectral properties are probably governed by the hydrodynamics of the flow rather than by the geometry of the emitting shells.

REFERENCES

Band D., Matteson J., Ford L., Schaefer B., Palmer D., Teegarden B., Cline T., Briggs M., Paciesas W., Pendleton G., Fishman G., Kouveliotou C., Meegan C., Wilson R., Lestrade P., 1993, ApJ, 413, 281

Daigne F., Mochkovitch R., 1998, MNRAS, 296, 275

Derner C. D., Mitman K. E., 1999, ApJ, 513, L5

Fenimore E. E., Madras C. D., Nayakshin S., 1996, ApJ, 473, 998

Fishman G. J., Meegan C. A., Wilson R. B., Brock M. N., Horack J. M., Kouveliotou C., Howard S., Paciesas W. S., Briggs M. S., Pendleton G. N., Koshut T. M., Mallozzi R. S., Stollberg M., Lestrade J. P., 1994, ApJS, 92, 229

Golenetskii S. V., Mazets E. P., Aptekar R. L., Ilinskii V. N., 1983, Nat, 306, 451

Granot J., Piran T., Sari R., 1999, ApJ, 513, 679

Kobayashi S., Piran T., Sari R., 1997, ApJ, 490, 92

Liang E., Kargatis V., 1996, Nat, 381, 49

Meszaros P., Rees M. J., 1993, ApJ, 405, 278

Norris J. P., Marani G. F., Bonnell J. T., 2000, ApJ, 534, 248

Norris J. P., Nemiroff R. J., Bonnell J. T., Scargle J. D., Kouveliotou C., Paciesas W. S., Meegan C. A., Fishman G. J., 1996, ApJ, 459, 393

Rees M. J., Meszaros P., 1994, ApJ, 430, L93

Ryde F., Svensson R., 2000, ApJ, 529, L13

Ryde F., Svensson R., 2002, ApJ, 566, 210

Soderberg A. M., Fenimore E. E., 2001, in Costa E., Frontera F., Hjorth J., eds, Gamma-ray Bursts in the Afterglow Era Berlin Heidelberg: Springer, p. 87

7 CONCLUSION

We have developed a simple model where GRB pulses are produced when a rapid part of a relativistic outflow is decelerated by a comparatively slower shell. We do not solve the true hydrodynamical problem but rather assume that the slow shell “accretes” the fast moving material which allows to obtain an analytical solution for the dissipated power $\dot{E}$. During pulse decay $\dot{E} \propto t^{-3/2}$ as $\dot{E} \propto t^{-3}$ when the evolution is fixed by shell geometry. To compute the spectral evolution of our synthetic pulses we parametrize the peak energy as $E_p \propto \rho^x c^y \Gamma$ where $\rho$, $cc^2$, and $\Gamma$ are respectively the post shock values of the density, dissipated energy (per unit mass) and Lorentz factor. At late times, we get $E_p \propto t^{-(2x+y)}$ which constraints $x$ and $y$ since in most observed bursts $E_p \propto t^{-5/3}$ with $\delta \lesssim 1.5$. The synchrotron process with standard equipartition assumptions corresponds to $x = 1/2$ and $y = 5/2$ (i.e. $2x + y = 3.5$) and gives a much too steep spectral evolution. One has then to suppose that the equipartition parameters $\alpha_c$, $\alpha_B$ and $\zeta$ vary with $\rho$ or and $\epsilon$ to reduce $x$ and $y$ (a possible alternative being that energy is radiated by another process – different from synchrotron – but which can still be approximated by eq. 36).

We have considered the case $x = y = 1/4$ and the resulting pulses then have temporal and spectral properties in excellent agreement with the observations. They follow both the HIC and the HFC during the decay phase and the time lags between energy channels decrease with increasing pulse hardeness and peak luminosity. We therefore conclude that if GRB pulses are produced by internal shocks, their temporal and spectral properties are probably governed by the hydrodynamics of the flow rather than by the geometry of the emitting shells.