Quasinormal modes of a black hole surrounded by quintessence

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Abstract

Using the third-order WKB approximation, we evaluate the quasinormal frequencies of massless scalar field perturbation around a black hole which is surrounded by static and spherically symmetric quintessence. Our result shows that due to the presence of quintessence, the scalar field damps more rapidly. Moreover, we also note that the quintessential state parameter $\epsilon$ (the ratio of pressure $p_q$ to the energy density $\rho_q$) plays an important role for the quasinormal frequencies. As the state parameter $\epsilon$ increases, so does the real part and the absolute value of the imaginary part decreases. This means that the scalar field decays more slowly in the larger $\epsilon$ quintessence case.

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1. Introduction

Accelerated expansion is one of the most important aspects of the present universe. It seems to be strongly supported by recent cosmological observations, such as supernovae of type Ia [1], the anisotropy of the cosmic microwave background radiation [2], the large scale structure [3] and so on. However, what leads to the accelerated cosmic expansion is still uncertain. In terms of Einstein’s gravity theory, this accelerated expansion can be explained by the conjecture that at late times a considerable part of the total density of the universe is dominated by dark energy with negative pressure [4]. There are several candidates for the dark energy: the vacuum energy (the cosmological constant) [5] and dynamical components (such as quintessence [6], k-essence [7] and phantom models [8, 9]). Models of dark energy differ with respect to the size of the parameter $\epsilon$, namely, the relation between the pressure and energy density of the dark energy. For example, in the cosmological constant model, $\epsilon$ maintains the constant $-1$. For quintessence, $\epsilon$ stays in the range $-1 \leq \epsilon < 0$. Obviously, the cosmological constant can be treated as a special type of quintessence. Recently, these theoretical models have attracted much attention because they possess many advantages in the explanation of...
accelerated cosmological expansion. However, whether the dark energy is the real and unique reason behind this accelerated expansion needs to be further verified in the future.

On the other hand, the black hole is another fascinating object in modern physics. Since Hawking radiation \[10\] was discovered, a great deal of attention has been focused on many fields in black hole physics including black hole entropy \[11–14\], the loss of information paradox \[15–18\], the holographic principle \[19–21\] and so on. It is widely believed that the study of black holes may lead to a deeper understanding of the relationship between general relative theory, quantum mechanics, thermodynamics and statistics. This means that black hole physics will play an important role in fundamental physics. However, at present whether black holes exist in our universe or not is still unclear. A recent investigation shows that quasinormal modes can provide a direct way to identify black hole existence in the universe because they carry the characteristic information about black holes \[22, 23\]. Moreover, it is also found that the quasinormal modes have a close connection with AdS/CFT correspondence \[24–26\] and loop quantum gravity \[27, 28\]. Thus, the study of quasinormal modes in black hole spacetimes has become appealing in recent years \[29–40\].

From the above discussion, we know that the dark energy and the quasinormal modes are two hot topics in physics at present. Then it is natural to raise the question of whether the dark energy affects the quasinormal modes. From the literature \[36, 37\], we know that due to the presence of the cosmological constant, quasinormal frequencies of a black hole in de Sitter spacetime are clearly different from that in the asymptotically flat metrics. It seems to imply that there exist some certain connections between dark energy and quasinormal modes because the cosmological constant can be regarded as a model of dark energy in the cosmology. However, how the dark energy affects the quasinormal modes remains unclear to the best of our knowledge. Recently, Kiselev \[41\] considered Einstein’s field equations for a black hole surrounded by static and spherically symmetric quintessence whose energy–momentum tensor satisfied the additive and linear conditions, and obtained a new static solution which depends on the state parameter $\epsilon$ of the quintessence. It is very similar to the black hole solution in the de Sitter/anti de Sitter spacetime. In this paper, as a concrete case, we try to probe the relationship between the quasinormal modes and dark energy through the evaluation of the quasinormal modes of massless scalar field perturbations around the black hole which is surrounded by quintessence.

The plan of the paper is as follows. In section 2, we review briefly Kiselev’s work and present the metric for a black hole surrounded by the static and spherically symmetric quintessence. In section 3, we evaluate the quasinormal modes of the massless scalar field in this spacetime by using the third-order WKB approximation \[42–44\]. Finally, we present a summary and conclusion.

2. The metric for a black hole surrounded by quintessence

In this section, let us briefly review Kiselev’s work \[41\]. The general metric for the static and spherically symmetric spacetime is

\[ ds^2 = e^\nu dt^2 - e^\lambda dr^2 - r^2(d\theta^2 + \sin \theta d\phi^2), \tag{1} \]

where $\nu$ and $\lambda$ are the functions of $r$. The Einstein field equations can be written as

\[ 2T^t_t = -e^{-\lambda} \left( \frac{1}{r^2} - \frac{\lambda'}{r} \right) + \frac{1}{r^2}, \tag{2} \]

\[ 2T^r_r = -e^{-\lambda} \left( \frac{1}{r^2} - \frac{\nu'}{r} \right) + \frac{1}{r^2}. \tag{3} \]
Quasinormal modes of a black hole surrounded by quintessence

\[ 2T_\theta^\theta = 2T_\phi^\phi = -\frac{1}{2} e^{-\lambda} \left( \nu'' + \frac{\nu'}{r} - \frac{\nu' - \lambda'}{2r} - \frac{\nu' \lambda'}{2} \right), \]

where the prime denotes the derivative with respect to \( r \). The energy–momentum tensor of the quintessence which satisfies the condition of the additivity and linearity can be expressed as

\[ T^r_t = T^r_r = \rho_q, \quad T^\theta_\theta = T^\phi_\phi = -\frac{1}{2} \rho_q (3\epsilon + 1), \]

where the parameter \( \epsilon \) is the state parameter of quintessence. Setting \( \lambda = -\ln(1 + f) \), we find that the variable \( f \) satisfies the equation

\[ r^2 f'' + 3(\epsilon + 1)rf' + (3\epsilon + 1)f = 0. \]

This second-order differential equation can be solved exactly and its general solution is given by

\[ f = 1 - \frac{r_g}{r} - \frac{c}{r^{3\epsilon+1}}, \]

where \( c \) and \( r_g \) are the normalization factors. Thus, the general forms of exact spherically symmetric solutions for the Einstein equations describing black holes surrounded by the quintessential matter with the energy–momentum tensor, which satisfies the condition of additivity and linearity, can be expressed by

\[ ds^2 = \left( 1 - \frac{2M}{r} - \frac{c}{r^{3\epsilon+1}} \right) dt^2 - \left( 1 - \frac{2M}{r} - \frac{c}{r^{3\epsilon+1}} \right)^{-1} dr^2 - r^2 (d\theta^2 + \sin^2 \theta d\phi^2), \]

where \( M \) is the black hole mass. Obviously, it is very similar to the de Sitter/anti-de Sitter metric. Moreover, we also note that the well-known Schwarzschild and Reissner–Nordström metrics are its special cases. It is not surprising because the Schwarzschild and Reissner–Nordström metrics describe the black holes surrounded by vacuum and electric fields, respectively.

3. Quasinormal mode of a scalar field in the black hole surrounded by quintessence

The general perturbation equation for the massless scalar field in the curved spacetime is given by

\[ \frac{1}{\sqrt{-g}} g^{\mu\nu} \partial_\mu (\sqrt{-g} g^{\nu\rho} \partial_\rho) \psi = 0, \]

where \( \psi \) is the scalar field.

Introducing the variables \( \psi = e^{-\omega \Phi(r)} Y(\theta, \phi) \) and \( r_s = \int (1 - \frac{2M}{r} - \frac{c}{r^{3\epsilon+1}})^{-1} dr \), and substituting equation (8) into equation (9), we obtain a radial perturbation equation

\[ \frac{d^2 \Phi(r)}{dr_s^2} + (\omega^2 - V) \Phi(r) = 0, \]

where

\[ V = \left( 1 - \frac{2M}{r} - \frac{c}{r^{3\epsilon+1}} \right) \left( \frac{l(l+1)}{r^2} + \frac{2M}{r^3} + \frac{c(3\epsilon + 1)}{r^{3\epsilon+3}} \right). \]

It is obvious that the effective potential \( V \) depends only on the value of \( r \), angular quantum number \( l \) and ratio \( \epsilon \). Figure 1 shows the variation of the effective potential with respect to the state parameter \( \epsilon \) of quintessence for fixed \( l = 5 \) and \( c = 0.001 \). From this figure, we can find that as the absolute value \( \epsilon \) increases, the peak value of the potential barrier gets lower and the location of the peak (\( r = r_p \)) moves along the right.
Figure 1. Variation of the effective potential for the massless scalar field in a black hole surrounded by quintessence with $r$ for fixed $l = 5$, $c = 0.001$ and $\epsilon = -1/3, -0.9, -1$.

From equation (11) and figure 1, we find that the quasinormal frequencies depend on the normalized factor $c$ and state parameter $\epsilon$ of quintessence. However, in this paper, we only want to study the relationship between the quasinormal mode and the state parameter $\epsilon$. Thereafter, we take $M = 1$ and $c = 0.001$ in our calculation. Let us now evaluate the quasinormal frequencies for the massless scalar field in the background spacetime by using the third-order WKB approximation, a numerical method devised by Schutz, Will and Iyer [42–44]. This method has been used extensively in evaluating quasinormal frequencies of various black holes because of its considerable accuracy for lower-lying modes. In this approximate method, the formula for the complex quasinormal frequencies $\omega$ is

$$\omega^2 = [V_0 + (-2V''_0)^{1/2} \Lambda] - i(n + \frac{1}{2})(-2V''_0)^{1/2}(1 + \Omega),$$

where

$$\Lambda = \frac{1}{(-2V''_0)^{1/2}} \left\{ \frac{1}{8} \left( \frac{V^{(4)}_0}{V''_0} \right) \left( \frac{1}{4} + \alpha^2 \right) - \frac{1}{288} \left( \frac{V''_0}{V_0} \right)^2 (7 + 60\alpha^2) \right\},$$

$$\Omega = \frac{1}{(-2V''_0)^{1/2}} \left\{ \frac{5}{6912} \left( \frac{V''_0}{V_0} \right)^4 (77 + 188\alpha^2) \right. \right.$$  

$$- \left. \frac{1}{384} \left( \frac{V''_0}{V_0} \right)^2 \left( V^{(4)}_0 \right) (51 + 100\alpha^2) + \frac{1}{2304} \left( \frac{V^{(4)}_0}{V_0} \right)^2 (67 + 68\alpha^2) \right.$$  

$$+ \left. \frac{1}{288} \left( \frac{V''_0}{V_0} \right) (19 + 28\alpha^2) - \frac{1}{288} \left( \frac{V^{(6)}_0}{V_0} \right) (5 + 4\alpha^2) \right\},$$

and

$$\alpha = n + \frac{1}{2}, \quad V^{(n)}_0 = \frac{d^n V}{d r^n} \bigg|_{r_{n,m_{1},(r_s)}}.$$  

Substituting the effective potential (11) into the formula above, we can obtain the quasinormal frequencies for the scalar field in the background of the black hole surrounded by quintessence. The low decaying mode frequencies for fixed $l = 5$ and $c = 0.001$ are listed in table 1. From figure 2, we also find that for fixed $n$ and $l$ the absolute value of the imaginary part decreases as the real part increases. Moreover, figures 3 and 4 tell us that as the state parameter $\epsilon$ increases, the real part increases, and the absolute value of the imaginary part decreases. It means that for a larger $\epsilon$, the scalar field decays more slowly. At last, we make a comparison between tables 1 and 2, and find that the real part in the Schwarzschild spacetime
Quasinormal modes of a black hole surrounded by quintessence

Figure 2. The relationship between the real and imaginary parts of quasinormal frequencies of the scalar field in the background of a black hole surrounded by quintessence for fixed $c = 0.001$. Left: $l = 1$ and $n = 0$, right: $l = 5$.

Figure 3. Variation of the real (left) and imaginary (right) parts of quasinormal frequencies with the state parameter $\epsilon$ for $l = 1$ and $n = 0$.

Table 1. The low overtone quasinormal frequencies of the massless scalar field in a black hole surrounded by quintessence for fixed $l = 5$ and $c = 0.001$.

| $3\epsilon + 1$ | $\omega (n = 0)$ | $\omega (n = 1)$ | $\omega (n = 2)$ | $\omega (n = 3)$ | $\omega (n = 4)$ |
|----------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| 0              | 1.057 97–0.096 41i | 1.048 39–0.290 37i | 1.030 48–0.487 29i | 1.006 01–0.687 89i | 0.976 51–0.891 88i |
| −0.2           | 1.057 57–0.096 43i | 1.048 00–0.290 43i | 1.030 10–0.487 38i | 1.005 65–0.688 01i | 0.976 17–0.892 04i |
| −0.4           | 1.057 07–0.096 45i | 1.047 51–0.290 50i | 1.029 62–0.487 51i | 1.005 20–0.688 18i | 0.975 75–0.892 24i |
| −0.6           | 1.056 45–0.096 49i | 1.046 90–0.290 61i | 1.029 04–0.487 68i | 1.004 65–0.688 42i | 0.975 24–0.892 53i |
| −0.8           | 1.055 67–0.096 54i | 1.046 14–0.290 75i | 1.028 32–0.487 92i | 1.003 98–0.688 70i | 0.974 63–0.892 91i |
| −1.0           | 1.054 70–0.096 61i | 1.045 20–0.290 96i | 1.027 43–0.488 24i | 1.003 18–0.689 16i | 0.973 92–0.893 42i |
| −1.2           | 1.053 49–0.096 70i | 1.044 04–0.291 24i | 1.026 36–0.488 69i | 1.002 22–0.689 73i | 0.973 10–0.894 09i |
| −1.4           | 1.051 98–0.096 84i | 1.042 60–0.291 63i | 1.025 07–0.489 30i | 1.001 11–0.690 51i | 0.972 19–0.894 97i |
| −1.6           | 1.050 09–0.097 03i | 1.040 84–0.292 18i | 1.023 52–0.490 13i | 0.999 83–0.691 53i | 0.971 19–0.896 09i |
| −1.8           | 1.047 74–0.097 30i | 1.038 68–0.292 94i | 1.021 70–0.491 26i | 0.998 41–0.692 87i | 0.970 14–0.897 48i |
| −2.0           | 1.044 81–0.097 68i | 1.036 04–0.294 00i | 1.019 56–0.492 79i | 0.996 85–0.694 59i | 0.969 12–0.899 17i |

Table 2. The low overtone quasinormal frequencies of the massless scalar field in a Schwarzschild black hole for fixed $l = 5$.

| $\omega (n = 0)$ | $\omega (n = 1)$ | $\omega (n = 2)$ | $\omega (n = 3)$ | $\omega (n = 4)$ |
|-----------------|-----------------|-----------------|-----------------|-----------------|
| 1.059 57–0.096 34i | 1.049 97–0.290 17i | 1.032 01–0.486 96i | 1.007 48–0.687 44i | 0.977 89–0.891 34i |

is larger and the magnitude of imaginary parts is smaller. In other words, due to the presence of quintessence, the oscillations damp more rapidly.
Figure 4. Variation of the real (left) and imaginary (right) parts of quasinormal frequencies of the scalar field with the state parameter $\epsilon$ for $l = 5$.

4. Summary

Using the third-order WKB approximation, we evaluated the quasinormal modes of the scalar field in a black hole spacetime surrounded by quintessence. Our results show that due to the presence of quintessence, the scalar field damps more rapidly. Moreover, we also note that the state parameter $\epsilon$ plays an important role for the quasinormal frequencies. As the state parameter $\epsilon$ increases, so does the real part and the absolute value of the imaginary part decreases. This means that the scalar field decays more slowly in the larger $\epsilon$ quintessence.

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