Nonequilibrium Dephasing in an Electronic Mach-Zehnder Interferometer

Seok-Chan Youn,1 Hyun-Woo Lee,2 and H.-S. Sim1,*

1Department of Physics, Korea Advanced Institute of Science and Technology, Daejeon 305-701, Korea
2PCTP and Department of Physics, Pohang University of Science and Technology, Pohang, Kyungbuk 790-784, Korea

(Received 13 December 2007; published 16 May 2008)

We study nonequilibrium dephasing in an electronic Mach-Zehnder interferometer. We demonstrate that the shot noise at the beam splitter of the interferometer generates an ensemble of nonequilibrium electron density configurations and that electron interactions induce configuration-specific phase shifts of an interfering electron. The resulting dephasing exhibits two characteristic features, a lobe pattern in the visibility and phase jumps of $\pi$, in good agreement with experimental data.

Introduction.—An electronic analog of optical Mach-Zehnder interferometry (EMZI) has been recently realized [1–4] by utilizing edge channels of integer quantum Hall liquids. As it is one of the elementary types of interferometry, it can serve as an important probe of electronic coherence [5] and entanglement [6–8].

The EMZI has a simple setup consisting of two arms and two beam splitters. Recent experiments [2,4] on it, nevertheless, revealed puzzling behavior that is hard to understand within a noninteracting-electron description [9]; the interference visibility of the differential conductance shows bias-dependent lobe patterns, accompanied by phase jumps of $\pi$ at the minima of the lobes. There may exist some unnoticed fundamental physics behind it.

Electron-electron interactions may be important for the puzzling nonequilibrium behavior. Interaction effects were studied [10,11] in the tunneling regime by using bosonization methods. In Ref. [10], interactions between an EMZI channel and an additional one outside the EMZI were considered and the resulting resonant plasmon excitations were proposed as an origin of the puzzle. On the other hand, roles of the shot noise of an additional detecting channel were addressed [12,13] to understand similar lobes found in a related experiment [12].

In this work, we propose an intrinsic mechanism for the puzzling behavior, which does not require additional channels outside the EMZI. A key observation is that the shot noise at the input beam splitter of the EMZI generates an ensemble of nonequilibrium electron density configurations in the two arms. Then the electron interaction within each arm induces configuration-specific phase shifts of an interfering electron, and the ensemble average of the phase shifts leads to nonequilibrium dephasing. The combined effect of the shot noise and the interaction results in lobe patterns and phase jumps, which agree with experimental data [2,4]. We use a wave packet picture to describe the nonequilibrium density and treat the interaction phenomenologically at zero temperature. The interarm interaction is ignored.

EMZI setup.—The EMZI consists of two sources $i = 1, 2$, two drains $i = 3, 4$, two beam splitters $j = a, b$, and two arms $l = u, d$ of length $L_j$ [Fig. 1(a)]. The source $i = 1$ is biased by $|e| V$, while $i = 2$ is unbiased. Transmission probabilities at the splitter $j$ are $T_j$ and $R_j (= 1 - T_j)$. Each arm consists of a single edge channel of the integer quantum Hall liquid with the linear energy dispersion characterized by constant group velocity $v_F$. We first consider the simple case with $L_u = L_d = L$, and later discuss the case with $L_u \neq L_d$. There are two time scales, electron flight time $t_{\|} = L/v_F$ through an arm and time separation $\tau_V = 2\pi \hbar/|e| V$ between successive injections of nonequilibrium electrons (with single-particle energy $\epsilon \in [0, |e| V]$) from source 1. Their ratio gives the average number of the nonequilibrium electrons in the two arms at a given time:

$$N = \frac{t_{\|}}{\tau_V} = \frac{L|e| V}{2\pi \hbar v_F},$$

(a) Schematic EMZI setup. It has two beam splitters $j = a, b$ with transmission probabilities $T_j$ and $R_j$, two arms $l = u, d$, two sources $i = 1, 2$, and two drains $i = 3, 4$. Only source 1 is biased. (b)–(e) Schematic nonequilibrium ensemble of electron density configurations in the two arms resulting from the shot noise at the splitter (a) when the two arms have two nonequilibrium electrons. The configurations can be described by two packets (dashed lines) propagating towards the drains as marked by solid arrows. The probability $P_m$ of each ensemble element $m$ is marked.
Nonequilibrium density configurations.—At zero temperature, the nonequilibrium state of the EMZI can have two equivalent forms in the noninteracting limit, 
\[
|\Psi(t)\rangle_{\text{init}} = \hat{U}(t) \prod_{k=0}^{l-1} c^\dagger(E_k)|0\rangle = \hat{U}(t) \int_0^\infty d^4(nW)|0\rangle,
\]
where \(\hat{U}(t)\) is the time evolution operator, \(|0\rangle\) is the Fermi sea in equilibrium, and \(c^\dagger(E)\) creates an electron with the single-particle energy \(E\) in the scattering wave \(\psi(E)\) incoming from source 1. \(d^4(X = nW)\) creates an electron in the wave packet [14,15], \(\phi(X) \propto \int_0^\infty |E| dE \exp(-iEX/hv_F)\psi(E)\), centered at position \(x = X\) with packet width \(W = v_F\tau_V\). Here, \(n\) is an integer and we use the convention that both the arms amount to the range \((- L/2, L/2)\). Thus for \(- L/2 < x < L/2\), the packet center is located in the arm \(u\) or \(d\). The equivalence between the two forms in Eq. (2) can be easily verified from the identity \(d(nW), d^4(nW) = \delta_{nn'}\) and the Pauli exclusion. While the former form is more commonly used, e.g., in the linear response regime, the latter may provide more insightful understanding of the ensemble of nonequilibrium electron density configurations and the resulting intrinsic dephasing mechanism, as shown below. Later we use a combination of the two forms.

As time \(t\) passes, \(\phi(X)\) evolves to \(\phi(X + v_F t)\). When a packet arrives at the input splitter \(a\), it propagates into the arm \(u\) or into the arm \(d\) with \(T_a\). This process describes the shot noise [14,16] and generates a nonequilibrium ensemble of electron density configurations in the arms. The ensemble depends on \(N\), i.e., on how many packets have significant weight in the arms. For example, for \(N \ll 1\), only one packet has significant weight and the ensemble has two representative elements \(m = 1, 2\), in each of which the nonequilibrium density appears in the arm \(u\) (\(d\)) with probability \(P_{m=1} = R_a\) \((P_{m=2} = T_a)\) at any given time. In this case, the nonequilibrium state has the form of \(|\Psi(t)\rangle_{\text{init}} = \sum_{m=1,2} c_m|\Psi_{m\text{init}}\rangle\) where \(\left|c_m\right|^2 = P_m\) and \(\left|\Psi_{m\text{init}}\right|\) contains the packet state with significant weight in the arm \((u\) or \(d\)) and no weight in \(d\) (\(u\)). Similarly, when two packets have considerable weight [Figs. 1(b)–1(e)], the ensemble has four representative elements \(m = 1, 2, 3, 4\), with probability \(P_m = R_a^2\), \(T_a^2\), \(T_a R_a\), and \(R_a T_a\), respectively. When the interaction is turned on, each density configuration causes different phase shift to an interfering electron, resulting in the nonequilibrium dephasing.

Lobe patterns in transmission probability.—We focus on a weak interaction regime where the energy relaxation rate is sufficiently smaller than \(1/\tau_{\text{rel}}\) [5]. In this regime, a single-particle energy \(E_0\) \((\in [0, |eV|])\) is a well-defined quantity, and thus one can consider the interference of the scattering plane wave \(|\psi(E_0)\rangle\). Within the arms, \(|\psi(E_0)\rangle\) shows the superposition, \(|\psi(E_0)\rangle = \hat{U}(t) |\psi(E_0)\rangle + t|d(E_0)\rangle\), of the plane waves \(||E_0\rangle\rangle\) in the arms \(l = u, d\), where \(|l|^2 = R_a\) and \(|d|^2 = T_a\). The phase accumulation of \(||E_0\rangle\rangle\) is affected by the interaction between \(|\psi(E_0)\rangle\rangle \) and the rest of the nonequilibrium electrons. We call the rest as the environment of the electron in \(|\psi(E_0)\rangle\rangle \).

To see how the interaction affects the phase accumulation, we write the total nonequilibrium state as \(|\Psi\rangle = |\Psi(t)\rangle_{\text{init}} = |\psi(E_0)\rangle\otimes |\Phi_{E_0}\rangle\). We will later turn on a weak interaction between \(|\psi(E_0)\rangle\rangle \) and \(|\Phi_{E_0}\rangle\), and ignore the interaction among the environment electrons, as a weak interaction may only slightly modify the nonequilibrium densities from the noninteracting case. Both \(|\psi(E_0)\rangle\rangle \) and the environment electrons in \(|\Phi_{E_0}\rangle\) are injected from source 1 and end in either drain 3 or 4. As \(|\Phi_{E_0}\rangle\), will be traced out to obtain the transmission probability \(T(E_0)\) of \(|\psi(E_0)\rangle\rangle\) to a drain, one has freedom of choosing either the wave packet or the plane wave description for \(|\Phi_{E_0}\rangle\). We choose the former, since it is more insightful and convenient for obtaining the density configurations of \(|\Phi_{E_0}\rangle\) in the arms, which is essential to describe the interaction. In the former, \(|\Phi_{E_0}\rangle\) is described by the moving train of wave packets constructed by single-particle states with energy \((\in [0, |eV|])\) except \(E_0\); the exclusion of \(E_0\) negligibly modifies the packets from \(\phi(X)\). As discussed before, due to the shot noise at the splitter \(j = a\), \(|\Phi_{E_0}\rangle\) has the superposition, \(|\Phi_{E_0}\rangle = \sum_m c_m|\Phi_{E_0,m}\rangle\) of the multiple-particle states, each corresponding to an element \(m\) (\(with P_m = |c_m|^2\)) of the nonequilibrium density ensemble. As the spatial configurations of the moving packets are repeated in time with periodicity \(\tau_V\), time-ensemble average over \([0, \tau_V]\), in addition to the average over the nonequilibrium density ensemble, is necessary to obtain \(T(E_0)\). We remark that the ensemble averages cannot be captured by a mean-field approach.

We turn on a weak interaction between \(|\psi(E_0)\rangle\rangle\) and \(|\Phi_{E_0}\rangle\). It causes the phase shift of \(\Psi\) described by an operator, 
\[
\hat{U}_{\text{ph}}(t_0) = \sum_{i=u,d} e^{-i\langle\langle U_{0,i}/b\rangle\rangle} \int_0^\infty d\hat{\Phi}_{E_0}(i)|l(E_0)\rangle\langle\langle l(E_0)|.\]
Here, \(U_0\) is the interaction strength (assumed [17] to be independent of \(|eV|\), \(t_0 \in [0, \tau_V]\) is the time-ensemble index, and \(\hat{\Phi}_{E_0}(i) = \int_1^2 \hat{\Phi}_{E_0}(x, t) dx\) acts on \(|\Phi_{E_0}\rangle\) to measure the number of the environment electrons in arm \(l\) at time \(t\). The time ordering operator is dropped in \(\hat{U}_{\text{ph}}\) as the packet dynamics \(\phi(X + v_F t)\) makes \(\hat{\Phi}_{E_0}(t^*')\) a c number [13].

The interference signal of the interfering electron with energy \(E_0\) can be obtained from the reduced density matrix \(T_{\text{env}}[\hat{U}_{\text{ph}}(t_0)|\Psi\rangle\langle\Psi|\hat{U}_{\text{ph}}(t_0)\rangle\), which is found to be 
\[
R_a|u(E_0)\rangle\langle u(E_0)| + T_a|d(E_0)\rangle\langle d(E_0)| + \sum_m P_m (|t|^2|u(E_0)\rangle\langle u(E_0)| + H.c.),
\]

where \(e^{i\delta} = e^{-i\langle\langle U_{0,i}/b\rangle\rangle} \int_0^\infty d\hat{\Phi}_{E_0}(i)|\Phi_{E_0}\rangle\langle\langle \Phi_{E_0}|\cdot \cdot \cdot |\Phi_{E_0,m}\rangle\cdot \cdot \cdot |\Phi_{E_0,m'+m}\rangle\cdot \cdot \cdot\). The trace \(T_{\text{env}}\) over all the orthogonal multiparticle states \(|\Phi_{E_0,m}\rangle\rangle\) of the environment electrons is evaluated using the fact that \(|\Phi_{E_0,m}\rangle\rangle\) and \(|\Phi_{E_0,m'+m}\rangle\rangle\)
are “severely” orthogonal to each other, i.e., \( \langle \Phi_{\text{env}, m} | \sum_{k_{\text{max}}} n_k(x_k) \rangle_{\Phi_{\text{En}, n}} \langle \Phi_{\text{env}, m} | \sum_{k_{\text{max}}} n_k(x_k) \rangle_{\Phi_{\text{En}, n}} \) is nonzero only for \( m = m' \) where \( k_{\text{max}} \) and \( k_{\text{max}}' \) are finite positive integers; any finite number of local density operators can not transform a packet of finite width into another orthogonal packet. The off-diagonal part of the reduced density matrix in Eq. (3) describes the dephasing of \( |\psi(E_0)\rangle \) due to the interaction.

The factor \( (e^{i\delta})_m \) can be further evaluated as

\[
(e^{i\delta})_m = e^{i(\delta(t_0))_m} \approx e^{-iU_0(t_0)/h} \int_{0}^{t_0} dt (\Delta \tilde{N}(t))_m,
\]

where \( \Delta \tilde{N}(t) = \tilde{N}_a(t) - \tilde{N}_d(t) \). In Eq. (4), we ignore the number fluctuation, \( \langle \Delta \tilde{N}(t) \Delta \tilde{N}(t') \rangle_m - \langle \Delta \tilde{N}(t) \rangle_m \langle \Delta \tilde{N}(t') \rangle_m \), or \( \langle \delta(t_0) \delta(t_0) \rangle_m - \langle \delta(t_0) \rangle_m^2 \), based on the observation that it is not a crucial factor for the nonequilibrium dephasing [18].

From Eq. (3), the transmission probability of the electron in \( \psi(E_0) \) to the drain \( i = 3 \) is obtained as

\[
T = T(E_0) = T_0 + T_1 D \cos(\Phi_B + \eta_D),
\]

where \( T_0 = T_a T_b + R_a R_b, \quad T_1 = 2 \sqrt{T_a T_b R_a R_b}, \quad \Phi_B \) is the Aharonov-Bohm phase, \( D \) is the nonequilibrium dephasing factor, and \( \eta_D \) is the phase shift of \( T \). Notice that \( T \) is independent of \( E_0 \) when \( L_u = L_d \) and that Eq. (5) reproduces the noninteracting result when \( U_0 = 0 \).

We first consider the regime of \( N \ll 1 \) and \( U_0 t_0 / h \gg 1 \). Though this parameter regime is unphysical [17], it is nevertheless instructive since it allows the analytic evaluation of \( D \) and \( \eta_D \). In this regime, the packet \( \phi \) may be approximated as a square packet of extension \( W \) with constant density of \( 1/W \), and the nonequilibrium density ensemble has two representative elements, one packet partially in the \( l = u \) arm with \( P_m = R_a \) or in \( d \) with \( P_2 = T_a \), as discussed before. Then, \( \langle \delta(t_0) \rangle_m \) in Eq. (4) has the constant value of \( -1 \)^\( m \) \( \delta \),

\[
\delta = N \frac{U_0 t_0}{h} = \frac{|e| V t_0 U_0 t_0}{2\pi h},
\]

and one finds, using \( \arg[.] = \arctan(\Im \cdot / \Re \cdot) \),

\[
D = \sqrt{\cos^2 \delta + (R_a - T_a)^2 \sin^2 \delta},
\]

\[
\eta_D = \arg[\cos \delta - i(R_a - T_a) \sin \delta].
\]

Notice that \( \delta \) is proportional to the bias \( V \) and that \( D(\delta) \) (thus \( T \)) shows lobe patterns with periodic minima of value \( |R_a - T_a| \) at \( \delta = \pi/2, 3\pi/2, \ldots \). At \( T_a = 0.5, \eta_D \) shows sharp jumps of \( \pi \) at the minima, while it is zero or \( \pi \) otherwise. The jumps become smeared as \( T_a \) deviates from 0.5. All these features are related to the which-path information [19] of the nonequilibrium electrons. As \( |e|V \) increases, the phase shift \( \langle \delta \rangle \) increases (decreases) by \( \delta \) with probability \( T_a (R_a) \) as the environment electrons are in arm \( d (u) \). At \( T_a = 0.5 \), the two possible phase shifts in the opposite direction are balanced in probability, resulting in the sharp jumps in \( \eta_D \) at the lobe minima. When \( T_a \neq 0.5 \), the balance is broken, leading to the smearing of the jump.

Hereafter, we discuss numerical results for the weak interaction regime with \( U_0 t_0 / h < \pi \) [17]. A quantitative calculation needs to take account of a larger number of nonequilibrium ensemble elements than Fig. 1 might imply, since the packet \( \phi \) has a rather slowly decaying tail and, more importantly for given \( N \), about \( 2N \) packets appear in the arms during \( t_0 \). Thus the number of ensemble elements for a given \( N \) is \( 2^{2N} \); in the calculation, \( 2^6 \) elements are considered in the regime of \( N \leq 3 \).

Such variety (of \( (e^{i\delta})_m \)) modifies the lobes [Figs. 2(b) and 2(c)]. Interestingly, the lobes in \( D \) and the jumps in \( \eta_D \) of the \( N \ll 1 \) case are maintained, though the lobes now acquire a decaying envelope and the minimum positions of the lobes are shifted. This robustness can be understood as follows. Among the \( 2^{2N} \) elements of the nonequilibrium density ensemble, those with probability \( P_m = R_a^N \) or \( T_a^N \) have \( \langle \Delta \tilde{N} \rangle_m = N \) or \( -N \), generating the same phase shift of \( \pm \delta \) at all \( t_0 \)'s as in the \( N \ll 1 \) case, while in the others \( \langle \Delta \tilde{N} \rangle_m \) varies with \( t_0 \) within a range smaller than \( \delta \). After the ensemble average over \( t_0 \), the lobes themselves survive due to the former, while the latter has less contribution to \( T \), giving rise to the decaying envelope and the shift of the lobe positions. For smaller \( U_0 t_0 / h \), a larger number of packets are involved (at a given \( \delta \)), resulting in more rapidly decaying envelope and thus smaller number of visible lobes [Fig. 2(c)].

\section*{Lobe patterns in \( dI/dV \).}

---From the zero-temperature current \( I = (|e|/h) \int_0^V dE_i \langle \Phi_{\text{En}, n} | (e^2/h) V T \rangle \), one evaluates \( dI/dV = (e^2/h) T_0 [1 + F(V) \cos[\Phi_B + \eta_F(V)]] \). Here, \( F(V) \) gives the visibility of \( dI/dV \), \( (dI/dV)_{\max} - (dI/dV)_{\min} / (dI/dV)_{\max} + (dI/dV)_{\min} \), and \( \eta_F \) is the phase shift of \( dI/dV \); remember \( \delta \propto V \) and \( L_u = L_d \).

The lobes in \( T \) give rise to similar lobes in \( dI/dV \). At \( T_a = 0.5 \), the visibility \( F(V) \) shows a lobe pattern whose
minima reach zero and the phase $\eta_F$ jumps by $\pi$ at the minima of the lobes while staying constant in other regions. The number of visible lobes in $F(V)$ becomes smaller for smaller $U_0 f_0 / h$, similar to $D$ [Figs. 3(a) and 3(b)]. The first zero of $F(V)$ appears even for small $U_0 f_0 / h$ where the second lobe of $T$ almost vanishes [Figs. 2(c) and 3(b)]. As $T_a$ deviates from 0.5, the minimum values of $F(V)$ increase from zero, similarly to those of $D$, and the jumps of $\eta_F$ become smeared. In this case the jump around the first minimum of $F(V)$ is sharper than that around the second [Fig. 3(c)].

Discussion.—The lobes and phase jumps in Fig. 3 qualitatively agree with the experimental data [2,4] at $T_a = 0.5$, where $N \approx 0.4$ at $V = 8 \mu V$ in the EMZI with $L = 10 \mu m$ and $\nu_F = (2-5) \times 10^4 m/s$. The first minimum of $F(V)$ occurs around $\delta = 3\pi/8$ (where $N \approx 0.4$) in Fig. 3, while around $V = 8 \mu V$ in the data. From this, we estimate $U_0 \approx 1.5 - 9 \mu eV$ for the experimental EMZIs. The dependence of the number of visible lobes in $F(V)$ on $U_0 f_0 / h$ [17] indicates that the number depends on the magnetic field, disorder, equilibrium electron density, etc., as in the data. Note that the minima of $F(V)$ look periodic in $V$ in Fig. 3(a), although not in general.

The case with $\Delta L = L_u - L_d \neq 0$ can be understood from Eqs. (5), (7), and (8). $D$ and $\eta_D$ have the same forms as in Eqs. (7) and (8) except for the replacements, $\delta \rightarrow aV(L_u + L_d)/2$ and $\eta_D \rightarrow \eta_D + E_0 \Delta L / h \nu_F - aV \Delta L / 2$, where $a = |e| U_0 f_0 / (2 \pi h^2 \nu_F)$. Then, the shift of the lobe-minimum positions is governed by $(L_u + L_d)/2$, and for $U_0 f_0 / h < \pi$, $\eta_F$ becomes an undulating function that no longer shows sharp jumps. These modifications are, however, negligible for $\Delta L \ll W = 2\pi h \nu_F / |e| V$, in agreement with experimental data [2].

We suggest that the combined experimental analysis of both $I$ and $dI/dV$ may be useful since $I$ can provide the direct information of the dephasing factor $D$. And our formalism may be applicable to possible nonequilibrium dephasing in other electron interferometers [12]. Finally, careful treatment of the number fluctuation [18], ignored in Eq. (4), may give further understanding, as it may modify the decaying envelope and the lobe positions.

In summary, we have shown that the nonequilibrium density ensemble, generated by the shot noise at the input beam splitter, can cause the nonequilibrium dephasing in the EMZI. Our result suggests that the experimental data in Ref. [2] may only be the first of its kind with more nonequilibrium quantum effects waiting for their discovery.

We thank I. Neder for sending us experimental data and M. B"uttiker for useful discussions. We are supported by KRF (2006-331-C00118 and 2005-070-C00055) and by MOST through the leading basic S & T research projects.

Note added.—A recent Letter [20] addressing shot-noise effects in the EMZI was reported. It considers a regime of the interaction strength stronger than ours.