Further Study of $D_s^+$ Decays into $\pi^-\pi^+\pi^+$

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Received: July 23, 2007 / Revised version: February 29, 2008

Abstract. A Dalitz plot analysis of the OZI rule violating decay $D_s^+ \to \pi^-\pi^+\pi^+$ is presented using different partial wave approaches. Scalar and vector waves are described by $K$-matrices; their production is parameterized in a $P$-vector approach. Alternatively, Breit-Wigner amplitudes and Flatté parametrization are used. Special emphasis is devoted to scalar mesons. The $f_0(980)$ resonance provides the most significant contribution. Adding $f_0(1500)$ to the scalar wave leads to an acceptable fit while introduction of $f_0(1370)$ and/or $f_0(1710)$ does not lead to significant improvements. A scan of the scalar wave optimizes for $M = 1452 \pm 22$ MeV/c$^2$. When $f_0(1710)$ is added, the mass uncertainty increases, and the fit yields $M = 1470 \pm 60$ MeV/c$^2$ which is fully compatible with the nominal $f_0(1500)$ mass. The scalar wave seems to exhibit a phase motion of 270° units in the mass range from 1200 to 1650 MeV/c$^2$.

PACS: 11.80.Et Partial-wave analysis - 13.20.Fc Decays of charmed mesons - 14.40.Cs Other mesons with $S=C=0$, mass $< 2.5$ GeV

1 Introduction

Decays of charmed mesons provide an efficient tool to study meson-meson interactions at low energies. Outstanding examples are the analyses of the $D^+ \to \pi^-\pi^+\pi^+$ Dalitz plot which helped to establish the $\sigma(500)$ [1], and the two reactions $D^0 \to K^0_S\pi^+\pi^-$ [2] and $D^+ \to K^-\pi^+\pi^+$ [3,4] which revealed the existence of the $\kappa(700)$ meson. The reaction $D_s^+ \to \pi^-\pi^+\pi^+$ was proven to provide access to mesons in which the primarily formed $s\bar{s}$ state converts into a $\pi^-\pi^+$ pair [5,6,7,8] in an OZI rule violating transition. In $D$ and $D_s$ decays into three pseudoscalar mesons, a large fraction of the cross section is assigned to a pseudoscalar meson recoiling against a scalar meson; this fact makes $D$ and $D_s$ decays very well suited for investigations of the spectrum of scalar mesons and their flavor wave function. A survey of data sets and of Dalitz plot analyses of $D$ and $D_s$ decays [9] can be found in the Review of Particle Properties [10].

In spite of the large potential for illuminating contributions to our understanding of scalar mesons up to a mass of $\sim 1700$ MeV/c$^2$, the situation concerning the states $f_0(1370)$ and $f_0(1500)$ and their contributions to the reaction $D_s^+ \to \pi^-\pi^+\pi^+$ is still controversial. The evidence for the existence of $f_0(1370)$ has often been questioned [11,12,13,14]: if it exists, the flavor decomposition of the scalar states remains unclear as evidenced by the numerous mixing schemes in which a scalar glueball is supposed to intrude into the spectrum of scalar $q\bar{q}$ meson, to mix with them thus creating the observed pattern of scalar resonances. Instead of a ‘narrow’ $f_0(1370)$, a wide scalar background has been proposed which was called $f_0(1000)$ by Au, Morgan and Pennington [12], and ‘red dragon’ by Minkowski and Ochs [13]. A survey of different mixing schemes and a critical discussion of their foundations are reported in a recent review [14].

All analyses of $D_s^+$ decays into $\pi^-\pi^+\pi^+$ agree on basic features of the data even though different partial wave analysis techniques were applied which led – in important details – to rather different conclusions [5,6,7,8]. In all analyses the Dalitz plot is shown to be dominated by $f_0(980)$. There is possibly a small contribution from $\pi^+\rho^0$; significant contributions stem from $\pi^+\rho(1450)$ and $\pi^+f_2(1270)$. In the scalar isoscalar partial wave, there is a sizable contribution which leads to a peak at a mass of about $1450$ MeV/c$^2$. The origin of this enhancement is however controversial. E791 fitted the $D_s^+ \to \pi^+\pi^+\pi^+$ data with a scalar resonance for which $m_0 = 1434 \pm 18 \pm 9$ MeV/c$^2$ and $I_0 = 173\pm32\pm6$ MeV/c$^2$ were found. They had observed $f_0(1370)$ production in an analysis of the $D \to 3\pi$ Dalitz plot and identified the scalar intensity with $f_0(1370)$. The $f_0(1370)$ is not supposed to have a large $s\bar{s}$ component, but its production could be assigned to the annihilation diagram (see [13]). The Focus collaboration reported $m_0 = 1475 \pm 10$ MeV/c$^2$ and $I_0 = 112\pm24$ MeV/c$^2$. These parameters are nearly compatible with $f_0(1500)$. Thus, the important issue to which extend the two states $f_0(1370)$ and $f_0(1500)$ contribute remained unsettled.

In this paper we report on a further study of the $D_s^+ \to \pi^-\pi^+\pi^+$ Dalitz plot. The main point of the analysis is an
are observed to decay into \( \pi^+ \) with the \( \bar{u}d \) diagram. The two quarks of the decay modes \( D_s^+ \rightarrow \pi^+ \pi^+ \pi^- \) and \( D_s^+ \rightarrow \pi^+ \pi^- \pi^0 \). The former reaction is likely due to the annihilation of \( \phi \) and \( \omega \). The suppression is not as large as expected from the mixing (for a deviation \( \delta_V \sim 3.5^\circ \) from the ideal mixing angle, \( \bar{u}d \) mixing, a suppression by \( \arctg^{-2} \bar{u}d \sim 1/250 \)), suggesting other mechanisms to contribute to \( D_s^+ \rightarrow \pi^+ \pi^0 \). These branching fractions and a few further ones are collected in Table 1.

Fig. 1 depicts Feynman diagrams for the decay of charmed mesons. In Cabibbo favored decays of \( D_s^0 \) mesons (Fig. 1a) the primary \( c \) quark converts into an \( s \) quark under emission of a \( W^+ \) while the \( \bar{s} \) quark acts as a spectator particle. The most likely decay of the \( W^+ \) produces two 

\( \bar{d}d \), Conversion of a \( c \) quark into a \( d \) quark or \( W^+ \) decay into \( K^+ \) are Cabibbo suppressed. Hence most likely, a meson with hidden strangeness emerges, recoiling against a \( \pi^+ \). Evidence for this diagram can be found in a comparison of \( D_s^0 \rightarrow \pi^+ \phi \) and \( D_s^0 \rightarrow \pi^+ \omega \). The former reaction occurs with \( (4.4 \pm 0.6) \cdot 10^{-2} \) frequency, the latter reaction with \( (3.4 \pm 1.2) \cdot 10^{-3} \); the reaction \( D_s^0 \rightarrow \pi^+ \omega \) is suppressed. The suppression is not as large as expected from the mixing (for a deviation \( \delta_V \sim 3.5^\circ \) from the ideal mixing angle, \( \arctg^{-2} \bar{u}d \sim 1/250 \)), suggesting other mechanisms to contribute to \( D_s^0 \rightarrow \pi^+ \pi^0 \). These branching fractions and a few further ones are collected in Table 1.

Fig. 1 shows a Cabibbo allowed but color suppressed diagram. The two quarks of the \( ud \) pair need to match in color with the \( s \) and \( \bar{s} \) quark, leading to an expected \( 1/N_c \) reduction of the probability. However, \( D_s^+ \) mesons are observed to decay into \( K^+K^{*0} \) and \( K^{*+}K^0 \) with a mean rate of \( (7.9 \pm 1.4)\% \), to be compared with the \( (4.4 \pm 0.6) \cdot 10^{-2} \) frequency with which \( D_s^+ \rightarrow \pi^+ \phi \) is produced. Thus additional processes contributing to production of open strangeness are required.

The annihilation diagram of Fig. 1 contributes to \( D_s^+ \) decays with an a priori unknown fraction. The intermediate \( W^+ \) may convert into the \( 3\pi \) final state via \( \rho \pi^+ \) or \( f_2(1270)\pi^+ \) but also into a scalar and a pseudoscalar meson.

Evidence for annihilation can be deduced from leptonic decays modes \( D_s^+ \). \( D_s^+ \) mesons have a large probability to decay into \( \tau^+ \nu_\tau \); decays into \( \mu^+ \nu_\mu \) or \( e^+ \nu_e \) are suppressed due to helicity conservation. The \( u, d \) (constituent) quark masses suggest hadronic branching ratios to contribute several \% when the number of colors is included.

\begin{table}
\begin{tabular}{|c|c|c|c|}
\hline
\( D_s^+ \rightarrow \mu^+ \nu_\mu \) & \( (6.1 \pm 1.9) \cdot 10^{-3} \) & \( D_s^+ \rightarrow \tau^+ \nu_\tau \) & \( (6.4 \pm 1.5) \cdot 10^{-3} \) \\
\hline
\( D_s^+ \rightarrow \pi^+ \phi \) & \( (4.4 \pm 0.6) \cdot 10^{-2} \) & \( D_s^+ \rightarrow \pi^+ \omega \) & \( (3.4 \pm 1.2) \cdot 10^{-3} \) \\
\hline
\( D_s^+ \rightarrow \pi^+ \eta \) & \( (2.11 \pm 0.35) \cdot 10^{-2} \) & \( D_s^+ \rightarrow \pi^+ \eta' \) & \( (4.7 \pm 0.7) \cdot 10^{-2} \) \\
\hline
\( D_s^+ \rightarrow \rho^+ \eta \) & \( (13.1 \pm 2.6) \cdot 10^{-2} \) & \( D_s^+ \rightarrow \rho^+ \eta' \) & \( (12.2 \pm 2.4) \cdot 10^{-2} \) \\
\hline
\end{tabular}
\end{table}

\section{3 Detector and data analysis}

\subsection{3.1 The E791 experiment}

The data analyzed here have been collected at Fermilab in a 500 GeV/c \( \pi^- \) beam impinging on platinum and carbon targets. The pion beam was tracked in proportional wire chambers (PWC’s) and silicon microstrip detectors (SMD’s) in front of the targets; particles emerging from a hadronic reaction were detected in a spectrometer which consisted of further PWC’s and SMD’s, two magnets, 35 drift chamber (DC) planes, two gas Čerenkov counters, an electromagnetic calorimeter, a hadronic calorimeter and a muon detector composed of an iron shield and two planes of scintillation counters. A full description of the detector, of data reconstruction and of data analysis can be found in [15].

In a first analysis stage, events were required to have a reconstructed primary production vertex whose location coincided with one of the target foils. Furthermore, events had to have a well separated secondary decay vertex (more than 4\( \sigma_{\text{vertex recoi}} \) in the longitudinal separation), and 3 reconstructed tracks with a total charge +1. Particle identification was not required, tracks were assumed to originate from charged pions. The mass distribution of events due to three-pion systems surviving
falling into the 1.7 to 2.1 GeV/c$^2$ mass range is smeared out by the finite detector resolution. The reconstruction function, an exponential form was chosen. There are 1172 events due to $D_s \rightarrow \pi^- \pi^+ \pi^+$ events and 848 events due to $D_s^+ \rightarrow \pi^- \pi^+ \pi^+$. For the background function, an exponential form was used to establish the $\sigma(500)$ and its properties. For the background function, an exponential was used to establish the $\sigma(500)$ and its properties.

After a cut in the $\pi^- \pi^+ \pi^+$ invariant mass between 1.95 and 1.99 GeV/c$^2$ as indicated in Fig. 2$, 937 events are retained for further analysis. The integrated signal to background ratio was estimated to $\sim 2$.

### 3.2 The detector acceptance

The detector covers the full solid angle and, to a good approximation, the acceptance is flat over the Dalitz plot. On the other hand, some of the cuts introduce small biases and the exact shape of the acceptance needs to be known. The acceptance was determined from the reconstruction efficiency of $D_s$ production and decay and the phase space which is smeared out by the finite detector resolution. The reconstruction efficiency was derived by the E791 collaboration using a full Monte Carlo simulation, from the $\pi^+ N$ interaction to the digitalization and selection of the events. The generated events had the nominal $D_s$ mass, the mass distribution of reconstructed events was compatible with the distribution shown in Fig. 2. For the analysis presented here, this distribution was divided into 100 slices; then events were generated, for each slice, simulating $D_s^+ \rightarrow \pi^- \pi^+ \pi^+$ decays uniformly spread over the phase space. The summation of all Dalitz plot multiplet with the reconstruction efficiency gave the acceptance $Acc(s_{12},s_{13})$ as presented in Fig. 2 (s$_{12} = m^2_{\pi^- \pi^+}$, s$_{13} = m^2_{\pi^- \pi^+}$). In this way, edge bins are properly taking into account.

### 3.3 The background

The fit in Fig. 2 assigned 568 events to $D_s$ decays and 280 events to the background. The latter was studied using Monte Carlo simulations. The most prominent background contributions stem from

1. a general combinatorial background,
2. $D^+ \rightarrow K^- \pi^+ \pi^+$ decays where a $K^+$ is wrongly interpreted as a $\pi^+$,
3. $D_s^+ \rightarrow \eta' \pi^+$ decays followed by $\eta' \rightarrow \rho(770)\gamma$, $\rho(770) \rightarrow \eta \pi^+$ with a missing $\gamma$,
4. $D^0 \rightarrow K^- \pi^+$ decays with an extra track (mostly from the primary vertex).

Amount and shape of the background is determined using both Monte Carlo simulations and data. The contribution of the $D_s^+ \rightarrow \eta' \pi^+$ background is negligible. The sum of contributions 1, 2, and 3 above, $B_{g1}$, populates uniformly the $D_s^+ \rightarrow \pi^+ \pi^- \pi^+$ Dalitz plot region. In the observed Dalitz plot, its distribution is proportional to the acceptance. This latter background represents 88 ± 2% of the total background contribution to the Dalitz plot. The shape of the background from $D^0 \rightarrow K^- \pi^+$ decays, $B_{g2}$ representing 12 ± 2% of the total background, is taken from simulations. It can be described analytically, the function is reproduced in the second part of the r.h.s. of eq. (1).

The total background, $B_g = B_{g1} + B_{g2}$, is a function of the (symmetrized) Dalitz plot variables:

$$B_g(s_{12}, s_{13}) = 1.103 \cdot Acc(s_{12}, s_{13}) + 0.0065 \cdot (g_{D_0}(s_{12}, s_{13}) + g_{D_0}(s_{13}, s_{12})) / 2,$$

where

$$g_{D_0}(s_{12}, s_{13}) = 7.2343 e^{-0.5((s_{12} - 1.55)/0.27)^2 + ((s_{13} - 1.17)/0.15)^2}}$$

After summation over the Dalitz plot, the background $B_{g1}$ ($B_{g2}$) represents 246 (34) events. The Dalitz plot distribution of the total background is shown in Fig. 3A.

**Fig. 2.** a) The $\pi^- \pi^+ \pi^+$ invariant mass spectrum. The dotted line represents the sum of the expected contributions from $D^0 \rightarrow K^- \pi^+$ and $D_s^+ \rightarrow \eta' \pi^+$ decays; the dashed line is the total background. Events in the hatched areas at the $D_s$ mass are used in this analysis. The hatched area at the $D_s$ mass was used to establish the $\sigma(500)$ and its properties. b) Detector acceptance in the mass range from 1.97 to 1.99 GeV/c$^2$.

**Fig. 3.** a) The background Dalitz plot. b) The $D_s^+ \rightarrow \pi^- \pi^+ \pi^+$ Dalitz plot. Since there are two identical particles, the plot is symmetrized.
3.4 The $D_s^+ \to \pi^- \pi^+ \pi^+$ Dalitz plot

The symmetrized Dalitz plot for the $D_s^+$ candidates shown in Fig. 3 is reproduced from [6]. Since the two $\pi^-$ are identical, the Dalitz plot is symmetrized. To avoid problems with statistical errors, all entries receive a weight $\frac{1}{4}$.

The most striking features of the Dalitz plot are the narrow horizontal and vertical bands just below $s_{12} \simeq s_{13} \simeq 1$ GeV$^2$/c$^4$ and a complex structure at 2 GeV$^2$/c$^4$. The bands correspond to $f_0(980)\pi_1^{\pm}$ decays, with $f_0(980) \to \pi_1^{+} \pi^-$. The two $\pi^+$ are identical; two interfering amplitudes contribute to the final state. The structure at 2 GeV$^2$/c$^4$ contains contributions from $f_2(1270)\pi^+$, from $\rho^0(1460)$ $\pi^+$, and from $\pi^+$ recoiling against scalar intensity. The clarification of this structure is the prime aim of this paper.

4 Partial wave analysis

4.1 The fit function

The partial wave analysis describes the Dalitz plot of Fig. 3 by a summation over possible reaction mechanisms for $D_s^+ \to 2\pi^+\pi^-$ decays plus background contributions. The different reaction mechanisms can interfere, hence they are represented by amplitudes. We consider the following reactions:

1. A $\pi^+$ recoiling against $\pi^+\pi^-$ in S-wave
2. A $\pi^+$ recoiling against $\pi^+\pi^-$ in P-wave
3. A $\pi^+$ recoiling against $\pi^+\pi^-$ in D-wave

The D-wave is always described by a $f_2(1270)$ Breit-Wigner amplitude. The S- and P-waves are alternatively represented by a sum of two-channel Breit-Wigner amplitudes or by K-matrices. Isotensor interactions are not included.

Due to Bose symmetry, the amplitudes are symmetrized with respect to the exchange of the two $\pi^+$ mesons. We use the following notations: $P = k_1 + k_2 + k_3$ is the momentum of the $D_s^+$ meson $(P^2 = s)$; $k_1$, $k_2$ and $k_3$ are the pion momenta of the $\pi^-$ and of the two $\pi^+$. Obviously, $k_1^2 = k_2^2 = k_3^2 = m_{\pi}^2$ holds. The amplitude depends on two invariant energy variables:

$$s_{12} = (k_1 + k_2)^2 \text{ and } s_{13} = (k_1 + k_3)^2. \tag{2}$$

The total amplitude includes contributions of different partial waves ($S$, $P$- and $D$- waves); it can be written in the form

$$A_{tot}(s_{12}, s_{13}) = \sum_{\alpha} [Z(k_1, k_3) A_{\alpha}(s_{12}) + Z(k_1, k_2) A_{\alpha}(s_{13})], \tag{3}$$

where $Z$ is a function referring to the spin-orbital angular momentum structure of the two mesons emerging from the decays of an intermediate state. For scalar, vector and tensor waves, they have the following form ($i = 2, 3$):

$$Z_S(k_1, k_i) = 1, \quad Z_V(k_1, k_i) = z_{1i}, \quad Z_T(k_1, k_i) = \frac{1}{2}(3z_{1i} - 1), \quad z_{1i} = \frac{k_{1i}k_{10} - (k_1k_i)}{|k_1||k_i|}. \tag{4}$$

The amplitude contains the summation over the different isobars. Each isobar depends on free fit parameters like masses, widths and coupling constants.

The intensity observed in the Dalitz plot is proportional to the squared reaction amplitude, to the phase space density $\phi$ for $D_s$ decays and the background, and to the detector acceptance $Acc$.

$$N(s_{12}, s_{13}) = |A_{tot}(s_{12}, s_{13})|^2 \cdot Acc(s_{12}, s_{13}) \phi_{D_s} \cdot Acc(5) + \alpha \cdot (Bg_1(1 - \beta) + \beta Bg_2) \phi_{Bg} \tag{5}$$

The two background functions are given in eq. (1). Note that the background definition includes the acceptance.

For the amplitude $A_{tot}(s_{12}, s_{13})$ we use different approaches. The scalar amplitude contains a choice of a series of poles for $\sigma(500)$, $f_0(980)$, $f_0(1370)$, $f_0(1500)$, and $f_0(1710)$. The vector amplitude comprises up to three poles at 770 MeV/c$^2$, 1460 MeV/c$^2$ and 1740 MeV/c$^2$. The tensor wave is described by a Breit-Wigner amplitude for the $f_2(1270)$ meson. In most cases, we use fixed masses and widths.

As dynamical function describing meson resonances, we use K-matrices or relativistic Breit-Wigner amplitudes.

1. In a first approach, we use K-matrices to describe scalar resonances. For vector mesons, either a K-matrix or Breit-Wigner functions are introduced.

2. In the second method, the $f_0(980)$ is represented by the Flatté formula. The total amplitude is formed as a sum of Flatté and Breit-Wigner amplitudes.

4.2 Breit-Wigner formalism

4.2.1 Breit-Wigner amplitudes

The use of Breit-Wigner amplitudes offers large flexibility and good control of the fit ingredients. The price is the violation of unitarity when two Breit-Wigner amplitudes overlap.

In the Breit-Wigner approach, the dynamical amplitude $A_{tot}(s_{12}, s_{13})$ is given by a sum of relativistic Breit-Wigner amplitudes which can be written in the form

$$A_R = BW_R(s) = \frac{\lambda_R^\pi}{|M_R^2 - s - i\Gamma M_R^2|}, \quad \Gamma_R = \Gamma_R \left(\frac{p}{p_R}\right)^{2L} \left(\frac{p}{p_R}\right) B_L(p, p_R) \tag{6}$$

where $M_R$ is the mass and $\Gamma_R$ the partial width of the resonance $R$. The complex number $\lambda_R^\pi = A_{D^+\pi} g_{\pi\pi}$ is given by the product of the production amplitude $A_{D^+\pi}$ of the resonance $R$ in $D_s$ decays, and the coupling constant $g_{\pi\pi}$ for its decay into $\pi\pi$. The (running) decay momentum is $p = \frac{1}{2}\sqrt{(s - 4m_{\pi}^2)}$ while the decay momentum at the nominal mass of the resonance is given by $p_R = \frac{1}{2}\sqrt{(M_R^2 - 4m_{\pi}^2)}$. The phase space is written as $\rho(s) = \sqrt{(s - 4m_{\pi}^2)}/s$ and $\rho_R(M_R^2) = \sqrt{(M_R^2 - 4m_{\pi}^2)/M_R^2}$, respectively.
For the three lowest orbital angular momenta, the Blatt-Weisskopf factors $B'_L$ have the form

$$B'_0(p, p_R) = 1 \quad B'_1(p, p_R) = \sqrt{1 + \frac{z}{1 + z}}$$
$$B'_2(p, p_R) = \sqrt{\frac{(z^2 - 3) + 9z}{(z^2 - 3) + 9z}}$$

where $z = (p|d|p)^2$ is the meson radius. In the fits, the scale parameter $d$ is restricted to $d \leq 0.8$ fm.

4.2.2 The Flatté parametrisation

The $f_0(980)$ is taken into account using the Flatté parametrisation

$$BW_{f_0(980)}(s) = \frac{\lambda_{f_0(980)}^{(980)} + i\rho_K\lambda_{f_0(980)}^{(980)}}{M_{f_0(980)}^2 - s - i\eta^2\rho_{\pi\pi} - i\eta^2\rho_{KK}}$$

where $M_{f_0(980)}$ and $\Gamma_{f_0(980)}$ are the $f_0(980)$ mass and width, $\lambda_{f_0(980)}^{(980)}$ and $\alpha_{f_0(980)}^{(980)}$ are complex numbers. The first number ($\lambda^{\pi\pi}$) is the same as the one used in the Breit-Wigner parametrisation. The second expression ($\lambda^{KK}$) refers to non-resonant two-kaon production and rescattering into $\pi\pi$. The coupling constants for $f_0(980)$ decay into $\pi\pi$ and $KK$ are denoted as $g_{\pi\pi}$ and $g_{KK}$; the two-particle phase space is written as $\rho_{\pi\pi} = \sqrt{(s - 4m_K^2)/s}$ and $\rho_{KK} = \sqrt{(s - 4m_K^2)/s}$.

4.2.3 Parameters of resonances

The $T$-matrix poles of the most important resonances are listed in Tables 2 and 3. In most of the fits described below, masses, widths and some coupling constants are frozen.

**Table 2.** The $\rho_0(1450)$ and $f_2(1270)$ mass and width ($M + i\frac{\Gamma}{2}$)

| Mass/Width | $\rho(1450)$ | $f_2(1270)$ |
|------------|--------------|--------------|
| $M(\rho)$  | 1460 + i 150 | 1275 + i 92.5 |

**Table 3.** Masses and widths ($M + i\frac{\Gamma}{2}$ in GeV/c²) of scalar resonances used in fits.

| Mass | $f_0(1470)$ | $f_0(1500)$ | $f_0(1510)$ |
|------|-------------|-------------|-------------|
| $\sigma(500)$ | 0.55 + i 200 | 1.32 + i 155 | 1.490 + i 58.0 |
| $f_0(1510)$ | 1.769 + i 170 |  |

The $f_0(980)$ is described by the Flatté formula; the following parameters are used [16]:

$$M_{f_0(980)} = 1.023 \text{ GeV};$$
$$g_{\pi\pi}^2 = 0.15 - 0.20 \text{ GeV}^2; \quad \frac{g_{\pi\pi}^2}{g_{KK}^2} = 2.4$$

In all cases, we keep the names of Particle Data Group even though we assign different masses to the particles.

### 4.3 K-matrix approach

4.3.1 The formalism

As reference fit, we use a K-matrix in the $P$-vector approach of Anisovich and Sarantsev [16], applied to the analysis of $D$ and $D_s$ decays in [2]. The S-wave amplitude has the form:

$$A_i(s_{12}) = \frac{P_j(s_{12})}{I - iP_{ij}K_{ij}(s_{12})}$$

where

$$P_j(s_{12}) = \sum_{\alpha} \frac{A_{\alpha}g_{ij}^\alpha}{M_{\alpha}^2 - s_{12}} + d_j$$

$$K_{ij}(s_{12}) = \left\{ \sum_{\alpha} \frac{g_{\alpha}^j g_{ij}^\alpha}{M_{\alpha}^2 - s_{12}} + f_{ij}^{\text{scatt}} 1 \text{ GeV}^2 - s_{12}^{\text{scatt}} \right\} \times \left( \frac{(s_{12} - s_A m_\pi^2/2)(1 - s_{A0})}{(s_{12} - s_{A0})} + c_{ij} \right)$$

The production of a two-meson intermediate state $j$ is enhanced due to formation of resonances. The summation in (10) extends over all considered scalar resonances $\alpha$ with mass $M_{\alpha}$. The enhancement is proportional to their production strengths $A_{\alpha}$ (for which we use $S_\alpha$, $V_\alpha$, and $T_\alpha$ to denote the strength to produce scalar, vector or tensor resonances), and their couplings $g_{ij}^\alpha$ to the decay channel $j$. The constants $d_j$ allow for nonresonant production of that channel; however, this flexibility is not exploited in this paper. The rescattering series is summed up in the $K$-matrix. Transitions from unobserved channels $i$ into the observed channel $j$ by resonant rescattering are included, nonresonant meson-meson scattering processes are taken into account by matrix elements $c_{ij}$. The $\rho_{ij}$ form the phase space matrix elements:

$$\rho_{ij} = \frac{1}{s} \sqrt{(s - (m_i + m_j)^2)(s - (m_i - m_j)^2)}$$

The rescattering process $KK \rightarrow \pi\pi$ is allowed below the $KK$ threshold; in this kinematical region, $\rho_{ij}$ becomes imaginary. The parameters $f_{ij}^{\text{scatt}}$ and $g_{ij}^{\text{scatt}}$ describe a slowly varying part of the $K$-matrix elements. The masses $M_{\alpha}$ and the couplings $g_{ij}^\alpha$, $f_{ij}^{\text{scatt}}$ are process-independent properties; they were determined in [16] from a large number of data sets and enter as fixed parameters into the analysis presented here. The constants $A_{\alpha}$ and $d_j$ determine the dynamics of a production process; they are free parameters to be determined from the fits.

In a scattering situation, Chiral Perturbation Theory forces the $\pi\pi$ amplitude to vanish at small energies. This is a kinematical effect which is not enforced in production. The suppression of the $\pi\pi$ scattering amplitude at small energies is taken into account by the Adler-Weinberg zero,

$$(s - s_A m_\pi^2/2)(1 - s_{A0})/(s - s_{A0}).$$

We choose $s_A = 1$, $s_{A0} = -0.15$. 


For higher partial waves, only the resonant terms are required to describe data. The $K$-matrix now contains Blatt-Weisskopf barrier factors $B_L$

$$K_{ij}(s_{12}) = \sum_{\alpha} \frac{1}{B_{L(ij)}} \frac{g_{ij}^{\alpha} g_{ij}^{\bar{\alpha}}}{M_{ij}^2 - s_{12} B_{L(ij)}}$$

which depend on the orbital angular momentum $L$ between the two mesons, the c.m.s. momentum of the two mesons in the initial and final state and one scale parameter. The Blatt-Weisskopf factors are given in eq. (7). The production vector for channel $P_j$ is multiplied by one Blatt-Weisskopf factor taking into account the angular momentum barrier for channel $j$.

In this paper, the $K$-matrix from [10] is used for the $S$-wave which included five resonances. These five resonances are observed when the $T$-matrix poles of the scattering amplitude are inspected. The $T$-matrix is defined by

$$T = (I - iK \cdot \rho)^{-1} K .$$

The $T$-matrix poles depend on the $K$-matrix masses and the couplings $g_{ij}^{\alpha}$ and are thus independent of the production process. The poles were identified with $f_0(980)$, $f_0(1370)$, $f_0(1500)$, $f_0(1710)$ of the Particle Data Listings[10] plus a broad resonance $f_0(1470)$ which was interpreted as scalar glueball.

$K$-matrix masses and coupling constants $g_{ij}^{\alpha}$ and $f_{ij}^{\text{scatt}}$ are taken from [10]; the values are reproduced in Table 4 and 5 for two cases. In Table 4 a solution is given which includes a pole for $f_0(1370)$; the parameters in Table 5 are optimized for the case where $f_0(1370)$ is omitted. The $K$-matrix parameters reproduce the $T$-matrix poles given in Tables 2 and 3

For the vector mesons, a $K$-matrix is used which contains $\rho(770)$, $\rho(1450)$, and $\rho(1770)$. The $K$-matrix masses and coupling constants are reproduced in Table 6

### 4.4 Fit results

#### 4.4.1 Study of vector and tensor contributions

Our fit strategy is as follows: we first describe the $S$-wave and $P$-waves using the $K$-matrices with fixed pole structure as given in Tables 4 and 5. The $f_2(1270)$ is included as Breit-Wigner resonance. This fit has 13 parameters to describe the 77 independent cells. Hence there are 64 degrees freedom. The $\chi^2$ of the fit is 63.49; the fit quality is very satisfactory. Figs. 4 and 5 show mass square distributions and, respectively, the $\chi^2$ per Dalitz plot cell. A negative sign is attached to the $\chi^2$ whenever the fit result exceeds data.

Results of this fit and the $\chi^2$ achieved is given in Table 7 as solution 1 (Sol. 1). The (complex) production amplitudes are given without errors; the statistical errors are small in comparison to the spread of results when the fit hypothesis is varied.

### Table 4. $K$-matrix parameters for the scalar isoscalar wave using 5 $K$-matrix poles. Masses and coupling constants are in GeV. Only the $i = 1 f_{ij}^{\text{scatt}}$ terms are listed since they are the only values relevant to the three-pion decay.

|  | $m_\alpha$ | $g_{\pi\pi}^{\text{scatt}}$ | $g_{KK}^{\text{scatt}}$ | $g_{\pi\pi}$ | $g_{\eta\eta}$ | $g_{\eta\eta}^{\prime}$ |
|---|---|---|---|---|---|---|
| 1 | 0.65100 | 0.22889 | 0.55377 | 0.00000 | 0.039899 | 0.34639 |
| 2 | 1.20306 | 0.94128 | 0.55905 | 0.00000 | 0.39065 | 0.31503 |
| 3 | 1.55817 | 0.36856 | 0.23388 | 0.55639 | 0.18340 | 0.18681 |
| 4 | 1.21000 | 0.33650 | 0.40907 | 0.85679 | 0.19906 | 0.00098 |
| 5 | 1.82206 | 0.18171 | 0.17558 | 0.83651 | 0.00035 | 0.22358 |

### Table 5. $K$-matrix parameters for the scalar isoscalar wave using 4 $K$-matrix poles. The pole representing $f_0(1370)$ was omitted resulting, in [10], in an increased $\chi^2$. Masses and coupling constants are in GeV.

|  | $m_\alpha$ | $g_{\pi\pi}^{\text{scatt}}$ | $g_{KK}^{\text{scatt}}$ | $g_{\pi\pi}$ | $g_{\eta\eta}$ | $g_{\eta\eta}^{\prime}$ |
|---|---|---|---|---|---|---|
| 1 | 0.69566 | 0.66859 | 0.71636 | 0.00000 | 0.10133 | 0.17721 |
| 2 | 1.24458 | 0.87180 | 0.59038 | 0.00000 | 0.38115 | 0.39085 |
| 3 | 1.54280 | 0.36228 | 0.24167 | 0.50412 | 0.15035 | 0.45126 |
| 4 | 1.84274 | 0.07290 | 0.01118 | 0.30778 | 0.03018 | 0.17852 |

### Table 6. $K$-matrix parameters for the vector isoscalar wave. Masses and coupling constants are in GeV, $d$ is given in fm. Only the $i = 1 c_{ij}$ terms are listed since they are the only values relevant to the three-pion decay.

|  | $m_\alpha$ | $d$ | $g_{\pi\pi}$ | $g_{\pi\pi}$ | $g_{\pi\pi}$ |
|---|---|---|---|---|---|
| 1 | 0.77873 | 0.34469 | 0.69461 | -0.50000 | 0.00000 |
| 2 | 1.50000 | 0.80000 | -0.20000 | 0.54000 | -0.65181 |
| 3 | 1.90000 | 0.80000 | 0.18042 | -0.71729 | -0.30664 |

In a next step we explore the role of vector and tensor mesons maintaining the full flexibility of the $S$-wave description. First, we replace the $K$-matrix by three Breit-Wigner amplitudes. A slightly improved $\chi^2$ is obtained, $\delta \chi^2 = -0.85$ (Sol. 2). The results do not change significantly when $\rho(770)$ is removed from the fit. Without $\rho(770)$, $\chi^2$ increases by 0.21 compared to solution 2, the number of parameters by 2 (Sol. 3). Removing $\rho(1770)$ changes $\chi^2$ by 1.75 (Sol. 4), and removing both, $\rho(770)$ and $\rho(1770)$, leads to $\chi^2 = 64.41$ (Sol. 5), an increase in $\chi^2$ by about 2 units while four parameters are spared. We conclude that $\rho(1460)$ is sufficient to describe the contribution of vector mesons to the data. However, if the latter reso-
nance or the $f_2(1270)$ is taken out of the fit, $\chi^2$ increases by 20 or more. Both these resonances are required to get a good fit. A further small improvement is achieved when mass and width of the $\rho$ and $f_2$ resonances are allowed to vary freely. The resulting values remain compatible with PDG values. Both are included with fixed values for mass and width in all subsequent fits (see Table 8). We note that some fits prefer to split $\rho(1450)$ into a $\rho(1250)$ and a $\rho(1500)$ even though the statistical evidence for the split $\rho(1500)$ is weak ($\delta \chi^2 = 4$ for two more parameters).

### 4.4.2 Study of the scalar partial wave

Table 8 presents a series of fits in which amplitudes contributing to the $\pi\pi$ $S$-wave are explored. For convenience of the reader we reproduce solution 5 of Table 8 as solution 6 in Table 8. We then add a constant $N_1 = d_{\pi\pi \pi}$ describing direct $\pi\pi$ production. This gives an improvement of 2.67 in $\chi^2$ (Sol. 7). Direct production of a $KK$ pair ($N_2$) and subsequent rescattering into $\pi\pi$ is unimportant: $\chi^2$ changes by 0.32 units only (Sol. 8).

We now take out $f_0(1370)$. This is an alternative solution presented in [10]. In a fit to a large number of reactions, the omission of $f_0(1370)$ resulted in a worse $\chi^2$. The $K$-matrix parameters obtained in that fit are listed in Table 8. In the fit to the $D_s^+ \rightarrow \pi^+\pi^+\pi^-$ data, $\chi^2$ changes by 6.44 units when solutions 6 (with parameter $N_1 = 0$) is compared to solution 9, and by 4.38 when solution 7 with $N_1 \neq 0$ is compared to solution 10, see Table 8. Including $N_2$ in the fit with no $f_0(1370)$ gives again a marginal improvement.

In a next set of fits, we used the Flatté parametrization to describe the $f_0(980)$, and Breit-Wigner amplitudes for all other resonances. The three states $f_0(980)$, $\rho(1450)$, and $f_2(1270)$ are always included. For the $S$-wave, the maximal set of resonances includes, apart from $f_0(980)$, the $\sigma(500)$, $f_0(1370)$, $f_0(1500)$, and $f_0(1710)$. All masses

### Table 7. Fit parameters for five different solutions exploring the role of vector and tensor resonances. The full $\pi\pi$ $S$-wave is always included. $S_1$ and $S_2$ interfere to make the $f_0(980)$, $S_3 = f_0(1370)$, $S_4 = f_0(1500)$, $S_5 = f_0(1710)$. There are up to three vector resonances, $V_1 = \rho(770)$, $V_2 = \rho(1450)$, and $V_3 = \rho(1770)$, and the tensor $T_1 = f_2(1270)$. The experimental mass distributions are compared to the PDG values. Both are included with fixed values for mass and width of the resonances. The fit parameters for five different solutions exploring the role of vector and tensor resonances are shown in Table 8. We then add a constant $N_1 = d_{\pi\pi \pi}$ describing direct $\pi\pi$ production. This gives an improvement of 2.67 in $\chi^2$ (Sol. 7). Direct production of a $KK$ pair ($N_2$) and subsequent rescattering into $\pi\pi$ is unimportant: $\chi^2$ changes by 0.32 units only (Sol. 8). We now take out $f_0(1370)$. This is an alternative solution presented in [10]. In a fit to a large number of reactions, the omission of $f_0(1370)$ resulted in a worse $\chi^2$. The $K$-matrix parameters obtained in that fit are listed in Table 8. In the fit to the $D_s^+ \rightarrow \pi^+\pi^+\pi^-$ data, $\chi^2$ changes by 6.44 units when solutions 6 (with parameter $N_1 = 0$) is compared to solution 9, and by 4.38 when solution 7 with $N_1 \neq 0$ is compared to solution 10, see Table 8. Including $N_2$ in the fit with no $f_0(1370)$ gives again a marginal improvement.

In a next set of fits, we used the Flatté parametrization to describe the $f_0(980)$, and Breit-Wigner amplitudes for all other resonances. The three states $f_0(980)$, $\rho(1450)$, and $f_2(1270)$ are always included. For the $S$-wave, the maximal set of resonances includes, apart from $f_0(980)$, the $\sigma(500)$, $f_0(1370)$, $f_0(1500)$, and $f_0(1710)$. All masses

### Table 8. Study of the scalar wave in the $K$-matrix approach. See caption of Table 8.

| Sol.6 | Sol.7 | Sol.8 | Sol.9 | Sol.10 |
|-------|-------|-------|-------|--------|
| $S_1$ | 6.78+16.31 | 6.50+14.39 | 6.81+14.22 | 9.05+11.07 | 8.42+12.19 |
| $S_2$ | -7.53+i3.23 | -7.78+i2.31 | -5.39+i10.13 | -8.74+i0.53 | -9.71+i0.36 |
| $S_3$ | -1.82+i9.28 | 0.88+i4.43 | 0.45+i1.48 | -1.79+i1.49 | -2.51+i0.62 |
| $S_4$ | -0.75+i11.21 | 2.68+i4.32 | -1.18+i2.64 | — | — |
| $S_5$ | 11.63-i0.70 | 2.68+i11.03 | -0.02+i3.52 | 10.97-i3.34 | 4.73-i7.80 |
| $N_1$ | — | -1.71+i14.12 | -2.41+i6.10 | — | 0.05+i4.69 |
| $N_2$ | — | — | -1.30+i12.76 | — | — |
| $V_1$ | 2.15+i0.85 | 2.86+i0.29 | 2.80+i0.37 | 1.70+i0.20 | 2.58-i0.25 |
| $T_1$ | -5.58+i10.00 | -5.07+i10.00 | -4.92+i10.00 | -5.39+i10.00 | -5.05+i10.00 |
| $\chi^2$ | 64.41 | 61.74 | 61.42 | 70.85 | 66.12 |
Table 9. Fits using Breit-Wigner amplitudes and Flatté parametrization for $f_0(980)$. $S_1 = \sigma(500)$; the two lines for $f_0(980)$, $S_2$ and $S_2'$, give the couplings $\lambda^{\eta\eta}$ and $\lambda^{\pi\pi}$. $S_3 = f_0(1370)$, $S_4 = f_0(1500)$, $S_5 = f_0(1710)$, $V_2 = \rho(1450)$, $T_1 = f_2(1270)$.

| Sol. 11  | Sol. 12  | Sol. 13  | Sol. 14  | Sol. 15 |
|---------|---------|---------|---------|---------|
| $S_1$   | 0.73-i0.14 | —       | —       | —       |
| $S_2$   | 3.57+i5.18 | 0.89+i5.35 | 1.99+i3.03 | 1.66+i4.48 | 1.45+i5.79 |
| $S_2'$  | 2.53+i4.94 | -4.62+i3.27 | -3.59+i10.45 | -2.78+i13.10 | -1.73+i5.93 |
| $S_3$   | 0.96+i0.58 | —       | -2.13-i4.57 | —       | —       |
| $S_4$   | 0.71-i0.71 | —       | —       | 0.38-i0.98 | —       |
| $S_5$   | 4.31+i1.36 | —       | —       | —       | 3.76+i2.09 |
| $V_2$   | 2.79+i1.89 | 0.11+i3.02 | 1.28+i1.96 | 1.07+i2.46 | 1.38+i2.94 |
| $T_1$   | -5.39+i4.00 | -3.77+i0.00 | -3.89+i0.00 | -3.45+i0.00 | -5.40+i0.00 |
| $\chi^2$ | 62.47   | 110.18  | 78.48   | 70.56  | 81.40  |

Table 10. Fits using Breit-Wigner amplitudes and Flatté parametrization for $f_0(980)$. See caption of Table 9.

| Sol. 16  | Sol. 17  | Sol. 18  | Sol. 19  | Sol. 20  |
|---------|---------|---------|---------|---------|
| $S_1$   | 0.73-i0.14 | —       | —       | —       |
| $S_2$   | 3.57+i5.18 | 1.39+i14.12 | 1.76+i14.10 | 1.77+i3.27 | 1.78+i3.76 |
| $S_2'$  | 2.53+i4.94 | -1.33+i12.50 | 1.17+i11.87 | -2.60+i12.26 | -2.97+i12.00 |
| $S_3$   | 0.96+i0.58 | -0.15+i0.67 | —       | -2.03-i0.38 | -0.81-i0.40 |
| $S_4$   | 0.71-i0.71 | 0.66-i0.61 | 0.56-i0.70 | —       | 0.29-i0.75 |
| $S_5$   | 4.31+i1.36 | 2.72-i0.92 | 2.15-i1.35 | 2.34+i1.82 | —       |
| $V_2$   | 2.79+i1.89 | 1.82+i1.83 | 1.85+i1.82 | 1.97+i1.89 | 1.27+i12.15 |
| $T_1$   | -5.39+i4.00 | -5.55+i4.00 | -5.55+i4.00 | -5.80+i0.00 | -5.61+i0.00 |
| $\chi^2$ | 62.47   | 65.53   | 65.97   | 73.20  | 69.30  |

and widths are fixed to the $T$-matrix pole positions obtained in [10]. Fit results are collected in Table 9 and 10.

A fit which includes all resonances (Sol. 11) yields the best $\chi^2$, 62.47 for 16 parameters. The $f_0(980)$ is not sufficient to describe the scalar intensity: when $\sigma(500)$, $f_0(1370)$, $f_0(1500)$, and $f_0(1710)$ are removed, $\chi^2$ increases dramatically to 110.18 (Sol. 12). The fit is unacceptable. We now test the $\chi^2$ changes when only one of the three high-mass resonances is included in the fit. Inclusion of $f_0(1500)$ gives the smallest yield and the largest $\chi^2$ gain (Sol. 14). The other two resonances (Sol. 13 and 15) need a large yield (to describe intensity at the wrong mass) and bring a smaller gain in $\chi^2$. We conclude that $f_0(1500)$ helps best to describe the data efficiently.

Fit 16 reproduces the results of solution 11. Removing $S_1 = \sigma(500)$ changes $\chi^2$ by 3.06 units, and saves two parameters (Sol. 17). The $\chi^2$ change is certainly not sufficient to claim that $\sigma(500)$ is present but some contribution can also not be excluded. Its integrated fractional contribution to the Dalitz plot is estimated to $\sim 3\%$. In solution 18, the $S_3 = f_0(1370)$ is removed additionally, with leads to a $\chi^2$ increase of 0.44 units while two parameters are spared; the $f_0(1370)$ does not help to improve the fit. If $f_0(1500)$ is removed (Sol. 19), $\chi^2$ increases by 7.67. In solution 20, $f_0(1710)$ is removed, and $\chi^2$ changes by 3.77 units. Again, $f_0(1500)$ has the largest impact on the fit quality. The best $\chi^2$ per degree of freedom is achieved when the scalar wave is described by $f_0(980)$, $f_0(1500)$, and $f_0(1710)$.

Instead of using masses and widths from [10], we may chose values given in the PDG listings. The overall $\chi^2$ deteriorates by 2 units; the $\chi^2$ changes remain close to the ones shown in Tables 9 and 10. When the $f_0(1370)$ parameters are changed, mass and width can adopt nearly arbitrary values. Its parameters cannot be deduced from this data. A free fit to $f_0(1710)$ properties puts its mass to the limit of the phase space, with a marginal $\chi^2$ improvement.

The fractional contributions of the various isobars to the Dalitz plot are not well defined quantities. First, interference effects make it impossible to assign a fraction of a data to an amplitude. The second argument is more technical: the fit fraction depends on the model used. Tables 9 and 10 show the spread of results when the fit hypothesis is varied. From solution 14 we estimate that nearly 70% of the intensity comes from $f_0(980)$, 8% from $f_0(1500)$, 8% from $\rho(1450)$, and 15% from $f_2(1270)$.

4.5 Mass scans

In mass scans, the mass of one resonance is changed in steps, and the change of $\chi^2$ as a function of the imposed mass is inspected. In a well behaved situation, the $\chi^2$ distribution exhibits a local minimum at the optimal mass.

The resonance is produced with an arbitrary phase relative to the production of another meson. When the mass of the physical meson under study and the Breit-Wigner test mass coincide (and their respective widths), the Breit-Wigner amplitude and phase, and amplitude and phase of the physical resonance agree over the full mass range of the resonance. The overall phase difference of the resonances under study relative to the other mesons is determined in the fit; this phase is denoted here as $\phi_0$. When the test mass is changed, the Breit-Wigner amplitude and phase do no longer match the complex amplitude in the data. In this situation, the fitted amplitude adjusts its phase in a way that the true phase and the fitted phase match in the mass range in which the true resonance and the test Breit-Wigner amplitude have a large overlap. In a mass scan, the phase motion is thus approximately reproduced by the fitted phase. This method was developed in [12]; a first application showed that the $\eta(1440)$ region hosts one un-split resonance [18]. The analytical form of a Breit-Wigner amplitude leads to an overall sign change of the amplitude when going far below the nominal mass to far above the resonance, independent of its coupling to other channels, independent of inelasticities. The method explores directly the phase of a Breit-Wigner amplitude.

We test this idea by monitoring the phase of the $f_2(1270)$. Fig. 6 shows a scan of the $f_2$ mass region. The minimum in $\chi^2$ reached at 1280 MeV (see Fig. 6). The 'observed' phase resulting from the fit shows a strong variation as a function of the imposed $f_2$ mass, stronger than expected for a single Breit-Wigner. A phase variation is observed as expected but quantitatively, the expected phase variation is not reproduced. This may serve as a warning that the method is not without risk.

Fig. 7 shows the scan of a scalar amplitude. We start from solution 13 of Table 9, fix the mass of the scalar
meson at pre-set values, and determine the \( \chi^2 \) as a function of the pre-set mass. The scalar mass is varied from 1.2 to 1.7 GeV/\( c^2 \). A deep minimum in \( \chi^2 \) is observed at \( M = 1452 \pm 22 \) MeV/\( c^2 \) reproducing findings of previous analyses. At a mass of 1680 MeV, a weaker but still pronounced local minimum is seen which we tentatively identify with \( f_0(1710) \). The mass shift with respect to the expected value is possibly due to the phase space limitation.

The phase motion observed in the scan and shown in Fig. 7, exhibits a surprise: in the range from 1.2 to 1.7 GeV/\( c^2 \) it covers more than 2\( \pi \). The region must house more than a single resonance. This result encouraged us to introduce a \( f_0(1710) \) with fixed parameters. The phase motion expected for two resonances (plus a small contribution from \( f_0(980) \)) agrees reasonably well with the 'measured' phase, see Fig. 7.

As a next step, we include \( f_0(1710) \) in the fit with fixed parameters and study the remaining scalar wave in a scan with a second scalar resonance. The result is shown in Fig. 8, the associated phase motion in Fig. 8. There is now one \( \chi^2 \) minimum which is wider than the central minimum seen in Fig. 7. When \( f_0(1710) \) is added, the mass uncertainty increases, and the fit yields \( M = 1470 \pm 60 \) MeV/\( c^2 \), a value which is not incompatible with 1500 MeV. If instead of \( f_0(1710) \) the \( f_0(1370) \) resonance is included, the production strength of the second scalar resonance gets small, and the \( \chi^2 \) minimum is found at 1400 MeV. Obviously, the interference of two close-by resonances can mimic the observed structure but we discard this solution.

The phase motions shown in Figs. 7 and 8, are suggestive but should be discussed with some reservations. The observed phase is certainly influenced by the data but the fit does not determine the local phase. The phase is derived from a comparison of data and a Breit-Wigner amplitude of finite (110 MeV) width; thus, the phase motion is smeared out. Second, the phase is determined from fits in which the mass of the resonance under studied is intentionally detuned. It is hence not guaranteed that the fit does not explore different local minima. In order to test the method, we have made the scans with different widths of the test Breit-Wigner amplitude (with \( \Gamma = 60 \) or 160 MeV/\( c^2 \)). No significant changes were found supporting the hypothesis that the observed phase motions reflect some physics content.

Thus we go one step beyond, and try to determine the number of resonances by comparing the observed phase motions with Breit-Wigner phase motions of resonances with fixed masses and widths. In Fig. 8 the 'observed' phase motion of Fig. 6 is shown together with two fits. Both fits use \( f_0(1500) \) and \( f_0(1710) \), fit (a) adds \( f_0(1370) \), fit (b) adds the wide background amplitude \( f_0(1000) \) suggested by Au, Morgan, and Pennington [12], and by Mindskowski and Ochs [13]. Both possibilities reproduce the 'observed' phase shift reasonably well even though \( f_0(1370) \) leads to an expected phase motion exceeding slightly the observed one while introducing \( f_0(1000) \) leads to a perfect match.

We have tried to perform a mass-independent analysis by choosing complex amplitudes for the scalar partial wave in each mass bin, and offering these to the minimization process. This mass independent partial wave amplitude was partly restricted to 2, 3, or 4 mass bins. The results were unstable; we failed to derive a mass independent partial wave amplitude from these studies.
We have studied the Dalitz plot for $D_s^+$ decays into $\pi^-\pi^+\pi^+$. In agreement with earlier findings, we found that the largest fraction of the data stems from scalar isoscalar mesons decaying into $\pi^-\pi^+$, in particular the $f_0(980)$ meson. The $\rho(1460)$ and $f_2(1270)$ contribute significantly as well. We have made the attempt to understand the conflicting results on scalar mesons which were obtained from fits to the $D_s^+ \rightarrow \pi^-\pi^+\pi^+$ Dalitz plot. Previous fits agree that the data require scalar intensity in the mass region above the $f_0(980)$. When fitted with a Breit-Wigner amplitude, an optimal mass between 1440 and 1475 MeV/c$^2$ was found. The masses quoted were not compatible with the mass of the $f_0(1370)$ resonance, which we rather assume to have 1320 MeV/c$^2$ – nor with the $f_0(1500)$ resonance. Thus it is often argued that both, $f_0(1370)$ and $f_0(1500)$, might be required to get a good fit when nominal (PDG) masses and widths are imposed.

We reproduce these results. However, when a high mass resonance, $f_0(1710)$, is introduced, the $f_0(1500)$ resonance alone is sufficient to yield an acceptable fit. The $\chi^2$ as a function of the assumed scalar mass of the $f_0(1370)/f_0(1500)$ develops a flat floor, and the $\sigma$ mass interval extends from 1.41 to 1.53 MeV/c$^2$. Thus the observed mass is fully compatible with the hypothesis that the standard $f_0(1500)$ is produced in the reaction. An additional contribution from a $f_0(1370)$ resonance is not required. If the existence of $f_0(1370)$ is assumed, its parameters can be chosen to agree with the ‘narrow’ $f_0(1370)$ of the PDG or with the ‘wide’ red dragon of Minkowski and Ochs. The statistics we used is not sufficient to discriminate between the three alternatives ‘no $f_0(1370)$’, ‘wide $f_0(1370)$’, or ‘narrow $f_0(1370)$’. The $\sigma(500)$ leads to a marginal improvement of the fit, with a statistical evidence just above one standard deviation. In the $K$-matrix fits, the inclusion of the $f_0(1370)$ resonance was necessary in fits to a large body of different reactions. In the fits to the $D_s \rightarrow 3\pi$ data, $f_0(1370)$ does provide a notable improvement even though this data alone is not sufficient to claim its existence.

One could dream of future high-statistics high-quality data. There is however the possibility to combine existing data from different experiments. The reaction $D_s^+ \rightarrow \pi^-\pi^+\pi^+$ was studied at Fermilab by the E687 (434 events), E791 (848 events) and the FOCUS (1475 events) collaborations. The BaBar collaboration has extracted 2900 events; preliminary results were reported in a PhD thesis at Bochum [19]. Including Belle results, a factor 10 can be reached in statistics. The different background contributions would help to understand the systematic errors. We urge that enterprise should be undertaken.

Acknowledgements

We thank the E791 collaboration for providing us with their data described in [1], for simulations of their detector (for acceptance corrections) and for other related discussions. Helpful comments by B. Meadows and A. Correa dos Reis are particularly acknowledged. M.M. was supported by a grant from the Deutsche Forschungsgemeinschaft.

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