A study on the appearance of tau neutrinos from a gamma ray burst by detecting their horizontal electromagnetic showers

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Abstract

We explore the possibility of detecting horizontal electromagnetic showers of tau neutrinos from individual gamma ray bursts in large scale detectors like HiRes and Telescope Array. We study the role of the parameters of a gamma ray burst in determining the expected number of tau events from that burst. The horizontal beam of tau leptons produce visible signals in the atmosphere. It is interesting to find that there is a good chance of observing tau lepton appearances from GRBs with Telescope Array. The number of signals is strongly dependent on the Lorentz factor $\Gamma$, redshift $z$ of a GRB, energy emitted in muon neutrinos and antineutrinos $E_{\text{GRB}}$ and also on some other parameters of a GRB. It is possible to understand neutrino oscillations in astrophysical neutrinos and the mechanism or model of neutrino production inside a GRB by detection or non detection of tau lepton signals from it.
1 Introduction

The very high energy muon and electron neutrinos produced inside an astrophysical source cannot pass through the matter of Earth due to their interaction with the matter of Earth and produce signals in a detector on Earth’s surface. The very high energy upward going muon and electron neutrinos are shadowed by Earth. However, very high energy tau neutrinos can penetrate the Earth to some extent. In models of neutrino production from proton acceleration inside an astrophysical source the intrinsically produced tau neutrino flux is expected to be very small. But tau neutrinos can be generated by vacuum flavour oscillations of muon neutrinos. There is a possibility of investigating the appearance of tau neutrinos from an astrophysical source of very high energy muon neutrinos by detecting these tau neutrinos. The expected number of events from upward going high energy tau neutrinos of a gamma ray burst in a muon detector has been calculated in [1].

The suggestion of detecting tau neutrino interaction through the shower induced by tau lepton decay in the atmosphere was given in Ref.[2, 3]. If the neutrinos are passing through the surface of Earth almost horizontally the charged leptons produced by them are also emitted in the horizontal direction. The electrons produced from very high energy horizontal beam of electron neutrinos do not escape the matter of Earth. The muons produced from very high energy muon neutrinos by their interactions with the matter of Earth do not produce visible signals in the atmosphere as the electromagnetic halo that surrounds very high energy muons does not spread enough in space to produce detectable signals in arrays of detectors separated by distances of the order of a kilometer. The tau leptons produced from very high energy tau neutrinos are capable of producing clear signals if they decay above the detector [4, 6]. However, the detectability of the visible signals from high energy tau neutrinos depends on the detector geometry [4]. We need an almost horizontal beam of high energy tau neutrinos from the source and the neutrino energies as well as the distances between the point of interactions and the detector must match to satisfy the condition of observing tau decays with ground level fluorescence detectors. In Ref.[4] they have shown that 90% of the detectable signal in Auger detector comes from upward going tau neutrinos. In this case the interactions occur in the ground all around the array. The downward going tau neutrinos interact in the mountains surrounding the array and generate 10% of the detectable signal. In Ref.[7] they have discussed how the Telescope Array can explore very high energy cosmic neutrinos by detecting the earth-skimming tau leptons using a large array of bright and wide field-of-view fluorescence telescopes.

At present there are several models which have been proposed to explain the occurrence of a GRB. There are suggestions that a GRB may evolve from a merger
of a binary system of compact objects. This binary system may include two neutron stars [8], one neutron star and one black hole [9], a black hole and a Helium star or a white dwarf [10, 11]. A short duration GRB may originate in this way.

A second class of models has been discussed below. There are two models the collapsar and the supernova model which can account for the energy budget of a long duration GRB. The evolution of a GRB may happen from the death of a massive star. When a black hole is created and accretion at a rapid rate from the surrounding accretion disk feeds a strong relativistic jet in the polar regions, a GRB can evolve [12, 13]. A newborn blackhole with a large amount of energy fed from surrounding accretion disk can ultimately produce energy of the order of $10^{54}$ erg. Another suggestion [14] in this class of models is that when a massive star explodes into a supernova, it may leave behind a supra-massive neutron star of mass 2.5 to 3 $M_\odot$. This will lose rotational energy in a time scale of weeks to years and subsequently collapse into a black hole. This phenomena may also trigger a long duration GRB.

Fireball model of a GRB [15] belongs to the second class of models discussed above. In this work we restrict ourselves only to the fireball model and production of neutrinos from photo pions. Protons can be shock accelerated to energies more than $10^{20}$ eV by Fermi mechanism and they interact with the photons of the fireball to produce pions which will subsequently decay to produce neutrinos. Electrons are also shock accelerated in the fireball and their synchrotron radiation in a strong magnetic field can explain the observed low energy (KeV-MeV) photon spectrum from GRBs. The observations on afterglows from GRBs can be explained as a result of the collisions of expanding fireballs with the surrounding medium. Also the detections of ultra high energy cosmic rays on Earth and observations of gamma rays from GRBs suggest that the rate at which gamma rays are produced by GRBs is the same as the rate of production of ultra high energy cosmic rays in the universe. In that case proton acceleration inside a GRB can be responsible for the production of ultra high energy cosmic rays in the universe. In the future HiRes [16], AGASA (Akeno Giant Air Shower Array) [17], Auger Observatory [18], Telescope Array [7] etc. can verify the above statement by making more observations on ultra high energy cosmic rays.

For a point source like an individual gamma ray burst the nadir angle at which it will be observed from Earth is fixed at a particular instant of time. The gamma ray bursts are generally of a few seconds durations and therefore the position of a burst with respect to an observer on Earth will not change significantly during the burst. We can expect to detect horizontal electromagnetic showers on Earth from tau neutrinos of a gamma ray burst if the burst takes place near the horizon.

The detection of visible signals from horizontal showers of tau neutrinos from a GRB with a ground array of detectors is not possible unless the burst happens on
the night sky and the tau neutrinos are observed within 5° from the horizon. A tau lepton emerging at an angle more than 0.3 rad above the horizon has no chance of producing an observable shower at ground level [4].

We carry out a study on the detectability of horizontal tau neutrino showers from individual gamma ray bursts in the present work. The dependence of the number of visible tau neutrino events on the GRB parameters has been explored. The advantage of doing this study is that one can estimate the expected number of such events for a future burst from the figures of our present work.

2 The parameters of a GRB in the fireball model and the neutrino spectrum from a GRB

In proton photon interactions neutrinos can be produced inside a GRB. The low energy (KeV-MeV) photon spectrum observed from a GRB by BATSE (Burst and Transient Source Experiment) and other experimental groups is most likely due to the electron synchrotron radiations in a GRB. We define the luminosity of this photon radiation from a GRB by $L_\gamma$. Current models of GRBs like the collapsar or hypernovae model are capable of producing a total energy output of the order of $10^{53} \text{erg}$ to $10^{54} \text{erg}$. However, only a few percent of this energy may be actually available to accelerate ultra high energy protons, subsequently about 10% of the accelerated proton energies will be transferred to the muon neutrinos and antineutrinos. The total energy emitted by a GRB in muon neutrino and antineutrino emissions is denoted by $E_{\text{GRB}}$, its redshift by $z$ and its Lorentz factor by $\Gamma$. The photon spectrum break energy in MeV is $E_{\gamma, \text{MeV}}^b$ and the variability time of the source is $t_v$ seconds. $\epsilon_e$ is the fraction of internal energy transformed to electrons and $\epsilon_B$ is the fraction of internal energy carried by magnetic field in the fireball.

The maximum cutoff energy of the muon neutrino spectrum is $E_{\nu_{\text{max}}}$ at the source. By Fermi mechanism protons can be accelerated to energies more than $10^{20} \text{eV}$ inside a fireball and about 10% of that energy gets transmitted to the secondary muon neutrinos and antineutrinos. Hence we can assume $E_{\nu_{\text{max}}} = 10^{11} \text{GeV}$.

The observed photon spectrum from a GRB can be expressed as a broken power law. The data obtained by BATSE can be fitted using the parametrization given below

$$\frac{d\nu_\gamma}{d\epsilon_{\gamma, \text{ob}}} \propto \begin{cases} \epsilon_{\gamma, \text{ob}}^{-\alpha - 1} & \epsilon_{\gamma, \text{ob}} < \epsilon_{\gamma, \text{ob}}^b \\ \epsilon_{\gamma, \text{ob}}^{-\beta - 1} & \epsilon_{\gamma, \text{ob}} > \epsilon_{\gamma, \text{ob}}^b \end{cases}$$

$\epsilon_{\gamma, \text{ob}}$ is the observed photon energy and $\epsilon_{\gamma, \text{ob}}^b$ is the break energy of the photon spectrum in observer’s reference frame. In analogy with the observed photon spectrum
from a GRB the neutrino spectrum from that GRB is expressed below including the effect of energy loss via synchrotron emission by the highest energy pions [19].

\[ E_{\nu}^2 \frac{dN_{\nu}}{dE_{\nu}} \propto \begin{cases} 
    (E_{\nu}/E_{b}^\nu)^\beta & E_{\nu} < E_{b}^\nu \\
    (E_{\nu}/E_{b}^\nu)^\alpha & E_{\nu} < E_{\nu} < E_{s}^\nu \\
    (E_{\nu}/E_{s}^\nu)^{\alpha}(E_{\nu}/E_{b}^\nu)^{-2} & E_{\nu} > E_{s}^\nu
\end{cases} \]  

The neutrino spectrum break energy at the source in the observer’s reference frame (which means without the correction due to the redshift of the source) is related to the photon spectrum break energy \( \epsilon_{\gamma, MeV} \) and Lorentz factor of the GRB.

\[ E_{b}^\nu = 7 \times 10^{5} \frac{\Gamma_{2.5}^2}{\epsilon_{\gamma, MeV}^b} GeV. \]  

where, \( \Gamma_{2.5} = \Gamma/10^{2.5} \). The expression for photon spectral break energy is

\[ \epsilon_{\gamma, MeV}^b \approx \epsilon_B^{1/2} \epsilon_B^{3/2} \frac{L_{\gamma,52}^{1/2}}{\Gamma_{2.5}^2 t_{v,-2}} MeV. \]  

The expression of \( E_{b}^\nu \) has been derived from the expression of proton spectrum break energy as discussed in [5]. The proton break energy is the threshold proton energy for interaction with photons of observed energy \( E_{\gamma}^b \). In the above expression \( t_{v,-2} = t_v/10^{-2} \) sec. \( E_{\nu}^s \) in eqn.(2) is the energy above which synchrotron emission by high energy pions becomes important. For production of muon neutrinos the expression of this energy is

\[ E_{\nu}^s = 10^8 \epsilon_e^{1/2} \epsilon_B^{-1/2} L_{\gamma,52}^{1/2} \Gamma_{2.5}^4 t_{v,-2} GeV \]  

For muon antineutrinos and electron neutrinos this cutoff energy is a factor of 10 lower because neutrinos from muon decay have a lifetime 100 times longer than pions. The spectrum of muon neutrino and antineutrino from a GRB can be normalised using the information of total energy emitted by that GRB in muon neutrino and antineutrino emissions (\( E_{GRB} \)). We assume half of \( E_{GRB} \) goes into the production of muon antineutrinos.

\[ E_{GRB} = \int_{E_{\nu,min}}^{E_{\nu,max}} E_{\nu} dN_{\nu}/dE_{\nu} dE_{\nu} \]  

In our case the minimum neutrino energy \( E_{\nu,min} \) is much less than the neutrino spectrum break energy \( E_{b}^\nu \). We assume that a GRB at a redshift of \( z \) is emitting neutrinos isotropically to obtain the expected number of neutrinos on Earth from that GRB.
The observed number of muon neutrinos and antineutrinos on Earth per unit area of the surface of Earth and unit energy at observed energy $E_{\nu_{obs}}$ is

$$
\frac{dM_{\nu_{obs}}}{dE_{\nu_{obs}}} = \frac{dN_{\nu}}{dE_{\nu}} \frac{1}{4\pi r^2(z)(1+z)}.
$$

(7)

$r(z)$ is the comoving radial coordinate distance of the source. In a spatially flat universe we calculate the comoving distance of a source using $\Omega_\Lambda = 0.73$, $\Omega_m = 0.27$ and $H_o = 71kmsec^{-1}Mpc^{-1}$ from [20].

If the fireball is not spherically symmetric and emits particles in a cone that cone may face towards or away from the direction of Earth. If the emissions from a GRB are not towards Earth we will not detect any signal from it. Suppose a jet-like fireball facing towards Earth has a jet opening angle $\phi_{jet}$. We denote the source size by $r$. The fireball is expanding relativistically. The time required for significant source expansion corresponds to comoving time $t_{co} \sim r/c\Gamma$. One can see Ref.[21] for discussions on fireball geometry for a jet-like fireball. The linear size of causally connected regions is $ct_{co} \sim r/\Gamma$ in the reference frame of the fireball. From the above statement it follows that the angular size of causally connected regions is $ct_{co}/r \sim \Gamma^{-1}$. As a result of relativistic beaming an observer can see only a limited portion of a GRB with angular size $\sim \Gamma^{-1}$. If jet opening angle of a fireball $\phi_{jet} > \Gamma^{-1}$ then due to relativistic beaming of emission a distant observer cannot distinguish between a sphericall fireball and a jet-like fireball. As long as Lorentz factor of the fireball $\Gamma$ remains sufficiently large such that the wind is ultrarelativistic ($\Gamma \sim 300$) the expression obtained for neutrino spectrum assuming a spherical fireball can be used also if the fireball is jet-like.

The relativistic beaming has two effects [22]. If the beaming factor is $b$ then the energy emitted by a GRB is actually smaller by a factor of $b$ than the energy emitted assuming an isotropic emission of energy. The second effect is that the actual GRB rate is larger than the observed GRB rate by a factor of $b$.

We ultimately want to know the tau neutrino spectrum from a GRB due to $\nu_\mu \rightarrow \nu_\tau$ oscillations. The probablity of vacuum flavour oscillation from $\nu_\mu$ to $\nu_\tau$ is

$$
Prob(\nu_\mu \rightarrow \nu_\tau) = \sin^22\theta\sin^2\left(\frac{\delta m^2d}{4E_\nu}\right).
$$

(8)

In the above expression $d$ is the distance the neutrinos travel from the source to the detector. Super-Kamiokande [23] results on GeV energy atmospheric neutrinos give us the best fitted values for mass difference ($\delta m^2$) and mixing parameter ($\sin^22\theta$) for $\nu_\mu \rightarrow \nu_\tau$ oscillations.

$\delta m^2 \sim 10^{-3}eV^2, \sin^22\theta \sim 1.$
If the source is at a distance of a megaparsec to thousands of megaparsecs away from us the above expression of oscillation probability averages to about half for all relevant neutrino energies to be considered for detection.

Number of $\nu_\tau$s produced from a GRB due to $\nu_\mu \rightarrow \nu_\tau$ oscillation = Number of $\nu_\mu$s from the GRB $\times \text{Prob}(\nu_\mu \rightarrow \nu_\tau)$.

In this way we obtain the expected number of tau neutrinos from a GRB produced due to vacuum oscillations of muon neutrinos. Originally there were almost no tau neutrinos in the GRB compared to the number of muon and electron neutrinos. If we can detect tau neutrinos from a GRB by large scale experiments it will be a tau appearance experiment. These are very high energy tau neutrinos therefore they will preserve the directionality of the source.

3 The tau neutrino signals visible near the horizon of Earth

We follow the procedure discussed in Ref. [6] to calculate the number of tau neutrino signals in a ground array of detectors. The probability for a tau neutrino with energy $E_\nu$ which is coming from a direction of nadir angle $\theta$ to survive after travelling a distance $X$ is

$$P_{\text{surv},\nu_\tau} = \exp \left[-\int_0^X \frac{dX'}{L_{\nu_\tau\nu}(E_\nu, \theta, X')}\right]. \tag{9}$$

In the above expression

$$L_{\nu_\tau\nu}(E_\nu, \theta, X) = [\sigma_{\nu_\tau\nu}(E_\nu) \rho(r(\theta, X))]^{-1}.$$ 

is the charge current interaction length. $\sigma_{\nu_\tau\nu}(E_\nu)$ is the neutrino nucleon charge current interaction cross section. Here the interactions are

$$\nu_\tau(\bar{\nu}_\tau) + N(\text{nucleon}) \rightarrow \tau^- (\tau^+) + \text{anything}.$$ 

The tau neutrino interacts with a nucleon in the matter of Earth. The density of Earth is $\rho(r)$ at a distance of $r$ from the center of Earth, and $N_A = 6.022 \times 10^{23}$. The distance from the center of Earth is given by

$$r(\theta, X) = R_e^2 + X^2 - 2R_e X \cos \theta.$$ 

The radius of Earth is $R_e, R_e \approx 6371 \text{km}$. We are only considering tau neutrinos with energy above $10^8 \text{GeV}$. For $E_\nu \gtrsim 10^8 \text{GeV}$ the charge current cross sections for $\nu_\tau$ and $\bar{\nu}_\tau$ are virtually identical.
The probability for a tau lepton production from a tau neutrino in a distance between $X$ and $X + dX$ is $dX/L_{CC}^\nu(E_{\nu^\tau}, \theta, X)$. We are interested only on the tau leptons which travel nearly horizontally so that they can decay in the atmosphere and produce visible signals in ground array detectors, therefore in our case the generation of tau leptons from tau neutrinos occur near Earth’s surface where Earth’s density is $\rho_s = 2.65 gm/cm^3$. The probability of conversion for tau neutrinos to tau leptons can be expressed as

$$P_{\text{conv}} = \frac{dX}{L_{CC}^\nu(E_{\nu^\tau})}. \quad (10)$$

The expression for charge current interaction length gets simplified in this case because the tau neutrino beam is passing the surface of Earth almost horizontally.

$$L_{CC}^\nu(E_{\nu^\tau}) = \frac{1}{\sigma_{CC}(E_{\nu^\tau})\rho_s N_A}. \quad (11)$$

The tau leptons take 80% of the tau neutrino energy [24], we have used this result in our calculation. A charged tau lepton loses energy as it moves through the Earth. The energy loss rate can be parametrised as

$$dE_{\tau}/dX = -(\alpha_{\tau} + \beta_{\tau}E_{\tau})\rho[r(\theta, X)] \quad (11)$$

The above equation parametrises lepton energy loss through bremsstrahlung, pair production, and photonuclear interactions under the assumption of uniform energy loss. Here $\beta_{\tau} \approx 0.8 \times 10^{-6} cm^2/gm$ [25] and the effect of $\alpha_{\tau}$ at these neutrino energies is negligible. The probability of survival for a charged lepton which is losing energy at the rate described above is

$$dP_{\text{surv,\tau}}/dX = -P_{\text{surv,\tau}}/(ct_{\tau}E_{\tau}/m_{\tau}). \quad (12)$$

In the above expression $t_{\tau}$ and $m_{\tau}$ are the lifetime and mass of tau lepton, $c$ is the speed of light. Finally solving eqn.(11) and (12) one gets the expression of survival probability of a tau lepton assuming a constant density $\rho_s$ of the surface of Earth.

$$P_{\text{surv,\tau}} = \exp \left[ -\frac{m_{\tau}}{ct_{\tau}\beta_{\tau}\rho_s} \left( \frac{1}{0.8E_{\nu^\tau}} - \frac{1}{E_{\tau}} \right) \right] \quad (13)$$

In the above expression of survival probability of a tau lepton we have used the values of lifetime of a tau lepton $t_{\tau} = 2.96 \times 10^{-13} sec$ and mass of a tau lepton $m_{\tau} = 1.777 GeV$. A tau lepton, produced from a tau neutrino of energy $E_{\nu^\tau}$ at a distance of $X'$ after the tau neutrino enters the Earth, loses energy and then exits the surface of Earth with an energy $E_{\tau}$. The condition of consistency of the exit energy of a tau lepton with its original energy and location gives

$$P_{\text{cond}} = \delta(E_{\tau} - 0.8E_{\nu^\tau}e^{-\beta_{\tau}\rho_s(2R_e\cos\theta-X')}). \quad (14)$$
In writing the above expression the Earth’s surface has been assumed to have a constant density $\rho_s$. $\theta$ is the nadir angle at which the neutrino is coming to Earth. The combined effect of all the probabilities described in equation (9), (10), (13) and (14) can be written as

$$K(E_{\nu_\tau}, \theta; E_\tau) = \int_0^{2R_e \cos \theta} P_{\text{surv},\nu_{\tau}} P_{\text{conv}} P_{\text{surv},\tau} P_{\text{cond}} dX.$$  \hspace{1cm} (15)

The range of a tau lepton in Earth is far less than the typical interaction length of a neutrino, therefore the kernel described above is dominated by the contribution from $X \approx 2R_e \cos \theta$. In equation (9) $X$ is replaced by $2R_e \cos \theta$. The expression in equation (14) can be simplified after doing the integration in $X$ using $\int dX \delta[h(X)] = |dh/dX|_{h=0}^{-1}$.

$$K(E_{\nu_\tau}, \theta; E_\tau) \approx \frac{1}{L_{CC}(E_{\nu_\tau})} e^{-\int_0^{2R_e \cos \theta} dX'(/[L_{CC}(E_{\nu_\tau}, \theta, X')] \times \exp \left[ \frac{m_\tau}{c t_\tau \beta_\tau \rho_s} \left( \frac{1}{0.8 E_{\nu_\tau}} - \frac{1}{E_\tau} \right) \right]} \frac{1}{E_\tau \beta_\tau \rho_s}.$$  \hspace{1cm} (16)

We are going to fold this kernel with the tau neutrino spectrum on Earth and the aperture of the detector to calculate the number of tau lepton events or signals in the detector.

$$M_\tau = \int dE_{\nu_\tau} dE_\tau \frac{dM_{\nu_\tau}}{dE_{\nu_\tau}} K(E_{\nu_\tau}, \theta; E_\tau)(A\Omega)_{\text{eff}}$$  \hspace{1cm} (17)

We integrate over tau neutrino energies between $10^8 \text{GeV}$ to $10^{11} \text{GeV}$ to calculated the number of visible tau events in the atmosphere. The signal from an electromagnetic shower must compete with the background noise from the night sky. It is possible to calculate the energy required for an electromagnetic shower to be detectable by considering the signal to noise ratio in individual photomultiplier tubes. The effective apertures for earth-skimming tau leptons for HiRes and Telescope Array are given in [6]. These apertures are calculated for diffused sources of tau leptons. These tau leptons are coming from all directions. The apertures are expressed in unit of $km^2sr$.

However in our work we are considering individual GRBs as our source of neutrinos, therefore the effective apertures given in [6] have to be modified to use them for a point source. If the point source is in the field of view of the detector it will be observed otherwise it will not be observed. From the homepages of HiRes and Telescope Array we can find the angular coverages of these detectors. The azimuthal angular coverage for HiRes is 240° and for Telescope Array it is 360°, which means tau leptons coming from directions covering 240° and 360° will be detectable by HiRes and Telescope Array respectively. But of course only those tau leptons which travel
within 0.3 rad from the surface of the Earth will produce visible signals in the atmosphere. So tau signals which are coming within \(240^\circ \times (0.3/\pi)180^\circ\) to HiRes will be detectable by HiRes.

If we divide the effective apertures given in [6] by the solid angle covered by these detectors for detecting high energy \(\tau\) leptons passing close to the surface of the Earth, we can find the effective apertures for tau leptons produced from neutrinos emitted by “point sources”.

### 4 Results and Discussions

We have calculated the expected number of tau neutrino events from individual GRBs for detectors like HiRes and Telescope Array. These tau neutrinos are coming to the detectors almost horizontally and producing visible signals in the atmosphere. In the present work we have assumed that the burst is occurring at a nadir angle of \(\theta = 0.017\) and we have considered the neutrinos which have energies between \(10^8 GeV\) to \(10^{11} GeV\) to calculate the total number of expected tau events from the GRB.

Each ground array of detectors has a duty cycle. Each GRB is usually of a few seconds duration. If the occurrence of the bursts near the horizon coincides with the requirement of clear moonless nights for fluorescence detection then only those bursts will be detectable by the detectors.

In the first figure Figure. 1 we have just plotted the expected number of tau neutrinos from a GRB in a generic detector of 1 km\(^2\) area. One should note that this figure does not show results for HiRes or Telescope Array. We have varied the threshold energy of the detector to show that the threshold energy has a role in determining the shape of the curve, if we plot the number of tau neutrinos against the Lorentz factor \(\Gamma\) of the GRB. While calculating the expected number of tau neutrinos from a GRB we have assumed \(\alpha = 0.01, \beta = 1, z = 0.5, E_{GRB} = 10^{51} erg\). The GRB is assumed to occur at nadir angle 0.017rad. The tau neutrino spectrum is described by three functional forms in three regions of energies in eq.2 of our text. If the threshold energy of the detector is above the neutrino break energy \(E_b\) then the first part of the spectrum does not contribute to the total number of tau neutrinos. The break energies of the neutrino spectrum are proportional to \(\Gamma_2^{4.5}\), so a slight change in \(\Gamma\) can produce significant changes in the break energies as well as the number of tau neutrinos. This is the reason why there is an extreme jump in the number of tau signals for \(E_{th} = 10^6 GeV\) when it is displayed as a function of \(\Gamma\).

In Figure.2 we have plotted the expected number visible tau signals against Lorentz factor of a GRB for HiRes detector. The burst is assumed to be occur-
ing at a nadir angle $\theta = 0.017 rad$. We assume the burst is occurring at a distance of redshift $z = 0.5$, total energy emitted by the GRB in muon neutrinos and muon antineutrinos is $E_{\text{GRB}} = 10^{51} \text{erg}$. The luminosity of the burst is $L_\gamma = 10^{51} \text{erg/sec}$, the spectral indices of the photon spectrum are $\alpha = 0.01$ and $\beta = 1$. The values of the equipartition parameters are assumed to be $\epsilon_e = 0.45$ and $\epsilon_B = 0.1$. The break energies of the neutrino spectrum $E_{\nu}^b$ and $E_{\bar{\nu}}^b$ are proportional to $\Gamma_{2.5}^4$. In the figure the changes in the shape of the visible tau spectrum are due to the changes in the break energies of the spectrum of neutrinos as we vary $\Gamma$. Also the break energies vary linearly with the variability time $t_v$. As we increase $\Gamma$ the number of signals initially increase and after reaching a peak value it falls down. The variation in $t_v$ gives a linear displacement to the curve.

The redshift of a GRB has also an important role in determining the number of signals from that GRB. Figure.3 shows how the number of visible signals from a GRB falls down drastically as the distance of the GRB from us increases. We have assumed $\alpha = -0.3$ and considered three cases $\beta = 1, 1.5$ and $2$. The Lorentz factor $\Gamma$ of the GRB is assumed to be 300. The total energy emitted in muon neutrinos and antineutrinos is $E_{\text{GRB}} = 10^{52} \text{erg}$ and $L_\gamma = 10^{52} \text{erg/sec}$ as in Figure.1. We have calculated the number of tau events for HiRes detector assuming the burst is occurring at nadir angle $\theta = 0.017$.

The next figure shows the role of $\alpha$ in determining the number of visible tau signals in HiRes. Comparing Figure.3 and Figure.4 we find that the variation in number of tau signals is more due to the variation in the value of $\alpha$ than due to the variation in the value of $\beta$.

In Figure.5 we observe the dependence of the number of tau signals on the equipartition parameters $\epsilon_e$ and $\epsilon_B$. The equipartition parameters enter in the expressions of break energies of the neutrino spectrum. The variation in the number of visible tau signals is more due to the variation in the value of $\epsilon_e$ than due to the variation in the value of $\epsilon_B$ because the neutrino spectrum break energy $E_{\nu}^b$ is proportional to $\epsilon_e^{-3/2}$ and $\epsilon_B^{-1/2}$.

In Figure.6 and Figure.7 we have calculated number of visible tau signals expected to be detected by Telescope Array. Here we have assumed 11 and 2 detectors for Telescope Array and HiRes respectively as in [6]. In Figure.7 as we vary $L_\gamma$, the break energies of the neutrino spectrum change and it is reflected in the linear displacement of the curve.

The number of tau lepton signals increases linearly with the increase in $E_{\text{GRB}}$.

From our study we find that there is a good chance of detecting tau appearances
from GRBs with Telescope Array detector. If the GRB has a Lorentz factor about 300, it emits energy in neutrinos of the order of $10^{52}\text{erg}$ we expect to detect tau signals from it with Telescope Array even if it occurs at a redshift of 2.5. HiRes can also observe tau appearances if the burst is at a redshift of less than 0.5 and emit energies in neutrinos of the order of $10^{52}\text{erg}$ with its Lorentz factor $\Gamma = 300$. But the important point is that the geometrical configuration to produce a visible tau signal must be met. The neutrinos should come to the detector from the source almost in a horizontal direction and the leptons would decay in the atmosphere to produce visible signals.

We can study neutrino oscillations in astrophysical neutrinos, production of ultra high energy cosmic rays by GRBs, verify the model of high energy neutrino production inside a GRB if we can detect visible tau signals from a GRB by HiRes or Telescope Array or other similar detectors.

5 Conclusions

Our calculation shows that HiRes is capable of detecting visible tau signals at nadir angle $0.017\text{rad}$ from a GRB of Lorentz factor 300 at a redshift of 0.5, which is emitting energy in neutrinos of the order of $10^{52}\text{erg}$. If the burst is occurring at redshift $z = 2.5$, emitting energy $10^{52}\text{erg}$ in neutrino emissions and its $\Gamma$ is 300, it should be able to produce observable tau lepton signal in Telescope Array detector. The number of tau signals depends on Lorentz factor of the GRB $\Gamma$, redshift of the GRB $z$, energy emitted in neutrinos $E_{\text{GRB}}$, variability time $t_v$, photon luminosity $L_{\gamma}$, equipartition parameters $\epsilon_e$, $\epsilon_B$ and also on the photon spectral indices. There is an opportunity of understanding neutrino oscillations in astrophysical neutrinos, production of ultra high energy cosmic rays by GRBs and the model of high energy neutrino production inside a GRB by detecting visible tau signals from it.

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Figure 1: The expected number of tau neutrinos from a GRB in a detector of $Km^2$ area has been plotted against the Lorentz factor of that GRB. We have varied the threshold energy $E_{th}$ of the detector to show how the shape of the spectrum changes due to the change in the threshold energy of the detector. The GRB is observed at a nadir angle of $0.017 rad$ and at a redshift of $z = 0.5$. We assume $\epsilon_e = 0.45$ and $\epsilon_B = 0.1$. 

$E_{GRB} = 10^{51} erg$, $L_\gamma = 10^{51} erg/sec$ 

$z = 0.5$, $t_v = 1. sec$ 

$\alpha = 0.01$, $\beta = 1.$, $\epsilon_e = 0.45$, $\epsilon_B = 0.1$
Figure 2: The expected number of visible tau lepton events from a GRB has been plotted against its Lorentz factor $\Gamma$ for HiRes detector. The burst is assumed to occur at $\theta = 0.017 \text{rad}$. The variability time of the GRB has been varied. We assume $z = 0.5$, $E_{\text{GRB}} = 10^{51} \text{erg}$, $L_{\gamma} = 10^{51} \text{erg/sec}$, $\epsilon_e = 0.45$, $\epsilon_B = 0.1$, $\alpha = 0.01$ and $\beta = 1$.
Figure 3: This figure shows how the expected number of visible tau lepton events falls off as we go to higher redshifts. The burst is assumed to be observed at a nadir angle of $\theta = 0.017\,\text{rad}$ with HiRes detector. Lorentz factor of the GRB is assumed to be $\Gamma = 300$, and total energy emitted by the GRB is $E_{\text{GRB}} = 10^{52}\,\text{erg}$. We consider three values of $\beta$ in our plot $\beta = 1, 1.5, 2$. We also assume $\alpha = -0.3$, $\epsilon_e = 0.45$, $\epsilon_B = 0.1$, $t_v = 1\,\text{sec}$. The luminosity of the burst is assumed to be $L_\gamma = 10^{52}\,\text{erg/sec}$. 

$E_{\text{GRB}} = 10^{52}\,\text{erg}$, $L_\gamma = 10^{52}\,\text{erg/sec}$, $\Gamma = 300$

$\alpha = -0.3$, $t_v = 1\,\text{sec}$, $\epsilon_e = 0.45$, $\epsilon_B = 0.1$
Figure 4: This figure is same as figure 3. The only difference is that in this case we assume $\beta = 1$. and we consider three cases $\alpha = -0.7, -0.3$ and 0.01.
Figure 5: The expected number of visible tau signals has been plotted against Lorentz factor $\Gamma$ of the GRB. The GRB is assumed to be observed at nadir angle $\theta = 0.017\text{rad}$ with HiRes. We assume redshift of the GRB is $z = 0.5$, the total energy emitted initially in muon neutrinos is $E_{GRB} = 10^{51}\text{erg}$ and $\alpha = -0.3$, $\beta = 1.5$. The variability time is assumed to be $t_v = 1.\text{sec}$. 

$$E_{GRB} = 10^{51}\text{erg}, L_\gamma = 10^{51}\text{erg/sec}$$
Figure 6: The expected number of visible tau lepton events in the atmosphere has been plotted for Telescope Array against redshift of the GRB. The burst is assumed to be observed near the horizon at nadir angle $\theta = 0.017\,rad$. We assume $\alpha = -0.3$, $\Gamma = 300$, $E_{GRB} = 10^{52}\,erg$, $t_v = 1\,sec$, $\epsilon_e = 0.45$, $\epsilon_B = 0.1$ and $L_\gamma = 10^{52}\,erg/sec$.
Figure 7: This figure shows the dependence of expected number of visible tau signals on the Lorentz factor of the GRB $\Gamma$. The burst is assumed to be observed at nadir angle $\theta = 0.017\,\text{rad}$ as earlier with Telescope Array. We consider two cases $L_\gamma = 10^{50}\,\text{erg/sec}$ and $L_\gamma = 10^{51}\,\text{erg/sec}$. The other parameters are $z = 0.5$, $E_{\text{GRB}} = 10^{51}\,\text{erg}$, $\alpha = -0.3$, $\beta = 1.5$, $t_v = 1.\,\text{sec}$, $\epsilon_e = 0.45$ and $\epsilon_B = 0.1$. 