Cooperative navigation of unmanned aerial vehicle swarm based on cooperative dilution of precision

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Abstract
Aiming at the formation problem in the cooperative navigation of unmanned aerial vehicle swarm, a cooperative position error analysis method based on cooperative dilution of precision is studied in this article. During cooperative flight, the unmanned aerial vehicle swarm can use the received position and ranging information of the adjacent unmanned aerial vehicles to calculate the position, and fuse with its own sensor position information. The final positioning accuracy depends not only on the capability of the ranging sensor but also on the position accuracy and formation of the adjacent unmanned aerial vehicles. In this article, these influence factors are combined to put forward a cooperative dilution of precision calculation method suitable for unmanned aerial vehicle swarm cooperative navigation. On this basis, a cooperative integrated navigation method based on ranging information is designed. Finally, the performance of cooperative navigation of unmanned aerial vehicles in different formations is simulated and analyzed. The simulation result shows that the cooperative dilution of precision method proposed in this article can effectively analyze the influence of formation on the positioning accuracy of unmanned aerial vehicle swarm, and the final combined positioning result is consistent with the cooperative dilution of precision analysis result.

Keywords
UAV swarm, cooperative navigation, formation geometry, cooperative dilution of precision, information screening

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Introduction
Driven by the development of control, navigation, and other related technologies, unmanned aerial vehicle (UAV) has received extensive attention in many fields. Due to the limited mission performance and damage resistance of single UAV, the research has gradually developed from single UAV to UAV swarm. With advantages of high survival, low cost, and high efficiency, UAV swarm has a broad application prospect in military and civilian fields, such as military operations, disaster relief and emergency response, precision agriculture, line patrol, surveying and mapping, and safety monitoring.¹

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Accurate navigation information is the key factor that influences whether the UAV swarm can successfully complete the task. At present, satellite navigation is the main method of UAV swarm positioning, but there are two drawbacks if only relying on satellite navigation: when UAV swarm executes tasks in areas such as urban and jungles, satellite navigation may not be available due to occlusion, and in the case of dense cluster, UAVs need higher positioning accuracy to meet the requirements of the mission, and the cost for equipping all personnel with high-precision satellite navigation equipment is very high. Therefore, the cooperative navigation method to improve the positioning accuracy of UAV swarm has become a research hotspot.

In order to solve the problem of cooperative navigation of UAV swarm, scholars have carried out targeted research. Mainstream UAV swarm cooperative navigation schemes mainly include methods based on visual assistance and wireless ranging information assistance. Vetrella et al. proposed a cooperative navigation method that integrates inertial, magnetometer, available satellite pseudorange, cooperative UAV position, and monocular camera information, effectively improving the navigation performance of UAV swarm under GPS constraints. Strader et al. used the noise estimation method of wireless ranging information and airborne navigation system to calculate the relative attitudes and azimuth of multiple UAVs.

In the process of cooperative navigation, the positioning accuracy depends not only on the capability of the ranging sensor but also on the distribution between nodes and the position accuracy of cooperative nodes. Heng and Gao analyzed the influence of the number of nodes and the accuracy of sensors on the positioning accuracy in wireless sensor network and proposed a calculation method for the lower bound of the positioning accuracy. Causa et al. studied the calculation method of cooperation formation accuracy based on visual measurement, and the experimental results showed that under the appropriate cooperative formation, the positioning accuracy of UAV swarm could reach the meter level through visual measurement assistance. Sivaneri and Gross used an unmanned ground vehicle (UGV) to aide an UAV’s navigation solution, and the UGV’s location is designed to reduce the geometric dilution of precision (GDOP) between the UGV and the UAV.

In the cooperative navigation method based on wireless ranging information, the analysis of positioning error caused by formation is mostly based on the traditional GDOP method derived from satellite navigation, without taking into account the position error of the cooperative UAV itself. Aiming at these problems, a cooperative dilution of precise (C-DOP) calculation method combining ranging error, clock error, and position error of cooperative UAVs is proposed in this article to analyze the positioning error of UAV swarm under different formations. It provides an idea for path planning and massive cooperative information screening. On this basis, a cooperative navigation algorithm of UAV based on inertial measurement unit (IMU) and cooperative ranging information is designed.

This article is composed of three sections. The second section provides the cooperative navigation scheme based on C-DOP. The third section firstly describes the calculation method of C-DOP by combining the position error, clock error, and ranging error, and then provides an extended Kalman filter algorithm for the fusion of inertial information and ranging information. At last, a cooperative navigation method based on cooperative information screening is introduced in this section. The fourth section presents the simulation environments and the results in a series of simulation conditions. The fifth section concludes the study and introduces future research topics.

**Schematic of cooperative navigation**

As shown in Figure 1, the UAV swarm can be equipped with navigation sensors with different accuracy in the process of carrying out tasks. UAVs carrying sensors with poor accuracy or flying in areas without satellite signals can obtain the position and relative distance of UAVs with high positioning accuracy through data link. The output of inertial sensor and cooperative information can be fused through Kalman filter to obtain the final navigation result. In order to calculate the three-dimensional position, theoretically one UAV can locate itself by acquiring the position and distance information of four UAVs. Therefore, the analysis of C-DOP can screen out the cooperative information for appropriate formation, so as to reduce the calculation amount. On the other hand, the calculation of C-DOP can provide path guidance to realize optimal cluster formation design and improve cooperative positioning accuracy. The UAV with high-precision navigation equipment in the UAV swarm is called leader-UAV, while the UAV with
low-precision equipment is called wingman-UAV. The specific cooperative navigation framework is shown in Figure 2.

Cooperative navigation method of UAV swarm

Cooperative dilution of precision calculation method

In the flying process of UAV swarm, the positioning accuracy of wingman-UAV depends not only on the distribution of leader-UAVs but also on the position accuracy of leader-UAVs. Therefore, this article introduces the concept of DOP in satellite navigation and designs a C-DOP calculation method that combines ranging error and the position error of leader-UAVs.

Suppose clocks on all leader-UAVs are synchronized, and the range measurement between the $i$th leader-UAV and the wingman-UAV is shown below

$$d_i = d_i + \delta d_m + n_{range}$$  \hspace{1cm} (1)

where $d_i$ denotes the actual distance between the $i$th leader-UAV and the wingman-UAV, $\delta d_m$ is the equivalent clock error, and $n_{range}$ denotes the measurement noise and its variance is $\delta^2$. The distance between the $i$th leader-UAV and the wingman-UAV given by the inertial system satisfies the following equation

$$d_{li} = ||\mathbf{p}_w - \mathbf{p}_{Li}||_2$$

$$= \sqrt{(\hat{x}_w - \hat{x}_{Li})^2 + (\hat{y}_w - \hat{y}_{Li})^2 + (\hat{z}_w - \hat{z}_{Li})^2}$$  \hspace{1cm} (2)

where $\hat{x}_w, \hat{y}_w, \hat{z}_w$ and $\hat{x}_{Li}, \hat{y}_{Li}, \hat{z}_{Li}$ are the position output of wingman-UAV and the $i$th leader-UAV navigation system in ECEF coordinate, respectively. Considering the position error of the leader-UAV, formula (1) can be Taylor expanded and linearized as follows

$$d_{li} = ||\mathbf{p}_w - \mathbf{p}_{Li}||_2 + \frac{\partial d_{li}}{\partial x_w} \delta x_w + \frac{\partial d_{li}}{\partial y_w} \delta y_w + \frac{\partial d_{li}}{\partial z_w} \delta z_w$$

$$+ \frac{\partial d_{li}}{\partial x_{Li}} \delta x_{Li} + \frac{\partial d_{li}}{\partial y_{Li}} \delta y_{Li} + \frac{\partial d_{li}}{\partial z_{Li}} \delta z_{Li}$$

$$= d_i + \frac{\partial d_{li}}{\partial x_w} \delta x_w + \frac{\partial d_{li}}{\partial y_w} \delta y_w + \frac{\partial d_{li}}{\partial z_w} \delta z_w + \frac{\partial d_{li}}{\partial \mathbf{p}_{Li}} \delta \mathbf{p}_{Li}$$

$$+ \frac{\partial d_{li}}{\partial \mathbf{p}_{Li}} \delta \mathbf{p}_{Li} - d_{li} + n_{range}$$  \hspace{1cm} (3)

where $\mathbf{p}_w, \mathbf{p}_{Li}$ denote the actual position in ECEF coordinate, respectively. $\hat{x}_{Li}, \hat{y}_{Li}, \hat{z}_{Li}$ are the position error of wingman-UAV and the $i$th leader-UAV in ECEF coordinate, respectively, $\mathbf{p}_W = [x_w, y_w, z_w]^T$, $\mathbf{p}_{Li} = [x_{Li}, y_{Li}, z_{Li}]^T$.

The distance error between wingman-UAV and the $i$th leader-UAV can be obtained by differential equation (3) with the ranging measurement

$$\delta d_i = d_{li} - d_i = \frac{\partial d_{li}}{\partial x_w} \delta x_w + \frac{\partial d_{li}}{\partial y_w} \delta y_w + \frac{\partial d_{li}}{\partial z_w} \delta z_w$$

$$+ \frac{\partial d_{li}}{\partial \mathbf{p}_{Li}} \delta \mathbf{p}_{Li} - d_{li} + n_{range}$$  \hspace{1cm} (4)

The position error of leader-UAV can be modeled as white noise, and it is unrelated to the ranging measurement noise $n_{range}$. Therefore, equation (4) can be rewritten as follows
\[ \delta d_i = \frac{\partial d_{i1}}{\partial x_w} \delta x_w + \frac{\partial d_{i2}}{\partial y_w} \delta y_w + \frac{\partial d_{i3}}{\partial z_w} \delta z_w - \delta d_{in} + \nu_i \]  

(5)

where \( \nu_i \) denotes the equivalent measurement noise of distance error between wingman-UAV and the \( i \)th leader-UAV, and its variance is \( \frac{\partial d_{in}}{\partial p_{Li}} (\delta p_{Li})^2 \left( \frac{\partial d_{in}}{\partial p_{Li}} \right)^T + \delta_i^2 \).

When the wingman-UAV screens the cooperative navigation information of \( n \) leader-UAVs to assist itself, the following equation can be constructed

\[
\delta d = \begin{bmatrix}
\delta d_1 \\
\delta d_2 \\
\vdots \\
\delta d_n
\end{bmatrix} = \begin{bmatrix}
\frac{\partial d_{i1}}{\partial x_w} \\
\frac{\partial d_{i2}}{\partial y_w} \\
\frac{\partial d_{i3}}{\partial z_w} \\
\vdots \\
\frac{\partial d_{in}}{\partial x_w}
\end{bmatrix} \begin{bmatrix}
\delta x_w \\
\delta y_w \\
\delta z_w \\
\delta d_{in}
\end{bmatrix} - \begin{bmatrix}
\delta x_w \\
\delta y_w \\
\delta z_w \\
\delta d_{in}
\end{bmatrix} + v
\]

\[
+ [J]_L - \mathbf{I}_{n \times 1} \begin{bmatrix}
\delta x_w \\
\delta y_w \\
\delta z_w \\
\delta d_{in}
\end{bmatrix} + \nu
\]

\[ = H \begin{bmatrix}
\delta x_w \\
\delta y_w \\
\delta z_w \\
\delta d_{in}
\end{bmatrix} + \nu
\]

(6)

where \( \delta d \) is the vector of distance difference, \( [J]_L \) is Jacobian matrix, and \( \mathbf{I}_{n \times 1} \) defines the identity matrix of dimension \( n \times 1 \).

From equation (6), the covariance of distance difference vector in ECEF coordinate has the following relationship with the position error of wingman-UAV

\[
\text{cov}(\delta d) = E[(\delta d)(\delta d)^T] = E\left[(H \delta p)(H \delta p)^T\right] = H \cdot E\left[(\delta p)(\delta p)^T\right] \cdot H^T = H \text{cov}(\delta p)H^T
\]

(7)

where \( E[\cdot] \) represents expectation and \( \delta p_w \) satisfies the following formula: \( \delta p_w = [\delta x_w \ \delta y_w \ \delta z_w \ \delta d_{in}]^T \).

The covariance of position error can be obtained

\[
\text{cov}(\delta p_w) = H^{-1} \text{cov}(\delta d)(H^T)^{-1} = \left(H^T (\text{cov}(\delta d))^{-1} H\right)^{-1}
\]

(8)

The \( \text{cov}(\delta d) \) can be obtained by the following formula

\[
\text{cov}(\delta d) = \text{diag}\left(\left(\frac{\partial d_{i1}}{\partial p_{L1}} \delta p_{L1}\right)^2 \left(\frac{\partial d_{i2}}{\partial p_{L1}} \delta p_{L1}\right)^2 + \delta_i^2 \right),
\]

\[
\ldots, \left(\frac{\partial d_{in}}{\partial p_{Ln}} \delta p_{Ln}\right)^2 \left(\frac{\partial d_{in}}{\partial p_{Ln}} \delta p_{Ln}\right)^2 + \delta_i^2 \right)
\]

Convert the position error to the navigation coordinate (ENU coordinate)

\[
\delta p_w^e = \begin{bmatrix}
\delta p_e & \delta p_n & \delta p_u & \delta d_{iu}
\end{bmatrix}^T
\]

\[
= \begin{bmatrix}
C_e^w & 0_{3 \times 1} & \delta d_{iu}
\end{bmatrix}^T
\]

(9)

where \( \delta p_e, \delta p_n, \) and \( \delta p_u \) are the position errors of east, north, and up directions, respectively, and \( C_e^w \) is the direction cosine matrix from the ECEF coordinate to the navigation coordinate.

According to equation (9), equation (8) can be rewritten as follows

\[
\text{cov}(\delta p_w^e) = C_e H^{-1} \text{cov}(\delta d)(H^T)^{-1} C_e^T
\]

(10)

Isolating the ranging noise and equation (10) is changed into the following form

\[
\text{cov}(\delta p_w^e) = \left(C_e H^T \delta^2 (\text{cov}(\delta d))^{-1} C_e^T\right)^{-1}
\]

\[
= \begin{bmatrix}
G_{11} & G_{12} & G_{13} & G_{14} \\
G_{21} & G_{22} & G_{23} & G_{24} \\
G_{31} & G_{32} & G_{33} & G_{34} \\
G_{41} & G_{42} & G_{43} & G_{44}
\end{bmatrix} \delta^2
\]

(11)

For the wingman-UAV, the C-DOP can be defined as

\[
C-DOP = \sqrt{(G_{11} + G_{22} + G_{33} + G_{44})}
\]

(12)

and the C-DOP in east, north, and up directions are defined as C-E-DOP = \( G_{11} \), C-N-DOP = \( G_{22} \), and C-U-DOP = \( G_{33} \), respectively. The position component and the clock error component of the C-DOP are defined as C-P-DOP = \( G_{11}^2 + G_{22}^2 + G_{33}^2 \) and C-T-DOP = \( G_{44} \), respectively.

**Integrated algorithmic model of cooperative navigation system**

**State equation of cooperative navigation system.** The state vector of the wingman-UAV’s cooperative navigation system is modeled as

\[
\begin{align*}
\dot{x}_1 &= f(x_1, x_2, x_3, x_4) \\
\dot{x}_2 &= f(x_2, x_3, x_4) \\
\dot{x}_3 &= f(x_1, x_2, x_3) \\
\dot{x}_4 &= f(x_2, x_3, x_4)
\end{align*}
\]

where \( f \) represents the system dynamics. The initial state vector \( x_0 \) is assumed to be known. The measurement equation is given by

\[
\begin{align*}
y_1 &= h_1(x_1, x_2, x_3, x_4) \\
y_2 &= h_2(x_1, x_2, x_3, x_4) \\
y_3 &= h_3(x_1, x_2, x_3, x_4) \\
y_4 &= h_4(x_1, x_2, x_3, x_4)
\end{align*}
\]

where \( h_i \) represents the measurement functions.
where $\varphi_E, \varphi_N, \varphi_U$ and $\delta v_E, \delta v_N, \delta v_U$ are platform angle error and velocity error of east, north, and up directions, respectively. $\delta L, \delta \lambda,$ and $\delta h$ are, respectively, latitude error, longitude error, and altitude error. $\varepsilon_{iz}, \varepsilon_{iz}, \varepsilon_{it}, \varepsilon_{iz}, \varepsilon_{ix}, \varepsilon_{iz}, \varepsilon_{iy}, \varepsilon_{iz}, \varepsilon_{iw}$, respectively, constant drift and first-order Markov drift of gyroscope in body frame, respectively. $e_x, e_y, e_z$ and $e_w$ are, respectively, first-order Markov drift of accelerometer. $\delta d_{nu}$ is the equivalent clock error, which is modeled as random constant.

The state equation can be constructed according to the defined state vector

$$\begin{bmatrix} \delta x \cr \delta y \cr \delta z \end{bmatrix} = \begin{bmatrix} -(R_N + h) \sin L \cos \lambda & -(R_N + h) \cos L \sin \lambda & \cos L \cos \lambda \\ -(R_N + h) \sin L \sin \lambda & (R_N + h) \cos L \cos \lambda & \cos L \sin \lambda \\ R_N (1 - f)^2 + h & \cos L & 0 \end{bmatrix} \begin{bmatrix} \delta L \\ \delta \lambda \\ \delta h \end{bmatrix}$$

(14)

According to equations (5) and (14), the measurement equation of cooperative navigation system is constructed by the ranging measurements of $n$ leader-UAVs as

$$Z_c = H_c X + v = \begin{bmatrix} 0_{n \times 6} & J_L C_H & 0_{n \times 9} & - 1_{n \times 1} \end{bmatrix} X + v$$

(15)

where $0_{n \times 6}$ defines the zero matrix of dimension $n \times 6$. A closed-loop Kalman filter can be designed by combining equations (13) and (15).

**Cooperative navigation method based on cooperative information screening**

In the process of cooperative navigation, if all the cooperative information received by the wingman-UAV enters the Kalman filter, the calculation process will be very complex. Therefore, it is necessary to screen the cooperative information before information fusion. In this section, we propose a cooperative method based on cooperative information screening, which can be divided into the following steps:

- **Step 1:** Calculating the critical C-DOP value corresponding to the required positioning accuracy in advance.
- **Step 2:** The wingman-UAV calculates the navigation information by inertial navigation system and receives cooperative information including the location of the leader-UAVs and distance.
- **Step 3:** Selecting the leader-UAV with the highest elevation as the first one, according to the azimuth angle of the first one, —four to six UAVs are selected evenly to make the difference between their azimuth angles roughly balanced, and then a new leader-UAV swarm is formed.
- **Step 4:** Calculating the C-DOP corresponding to the leader-UAV swarm. If the requirement is not met, new UAVs will be selected from the remaining leader-UAVs until the accuracy requirement is met.
- **Step 5:** The cooperative information obtained in step 4 is used for Kalman filter to update the navigation information of wingman-UAV.
- **Step 6:** Return to step 2 for inertial navigation system (INS) solution and cooperative information reception. Then calculating whether the C-DOP corresponding to the leader-UAV swarm can still meet the requirements. If yes, skip to step 5. If no, skip to step 3.

**Simulation results**

In order to verify the effectiveness of the proposed algorithm, the Monte Carlo method is firstly used to calculate the C-DOP between leader-UAVs and wingman-UAV in different positions, and then the positioning accuracy of different DOP situations is tested.
In the simulation scenario, the longitude, latitude, and altitude of a static wingman-UAV in UAV swarm are $110^{\circ}E$, $20^{\circ}N$, and 500 m, respectively. In order to simplify the analysis, it is assumed that two leader-UAVs and one satellite have been selected and fixed, and the clock error can be neglected, while the remaining leader-UAVs are distributed around the wingman-UAV in horizontal direction with a radius of 50 m and in altitude direction from 200 m to 800 m. The simulation parameters are set and shown in Table 1 and the distribution of UAV swarm is shown in Figure 3.

We choose one of the candidate leader-UAV to combine with the selected leader-UAVs and satellite, and then calculate the corresponding C-DOP. The C-DOP corresponding to each candidate leader-UAV position was calculated through 8000 Monte Carlo simulations. The relationship between C-DOP and selected candidate leader-UAV positions is shown in Figure 4.

As can be seen from Figure 4, the C-DOP calculated by selecting candidate leader-UAVs in different positions is also different. Therefore, in the process of cooperative navigation, it is necessary to select the appropriate location of UAV to assist in order to achieve better positioning performance.

### Filter results

According to the C-DOP given in Figure 4, the cooperative information under different C-DOP is combined with the inertial information of the wingman-UAV itself. The simulation parameters of the IMU are listed in Table 2.

According to Figure 4, three conditions with different C-DOP are selected to investigate the positioning accuracy of the wingman-UAV. The corresponding relationship between C-DOP and the position of the candidate leader-UAV is shown in Table 3.

The position errors of the wingman-UAV and the corresponding C-DOP are shown in Figures 5 to 7, respectively. In order to compare the performance of the C-DOP and the traditional DOP in position error analysis, the position errors of the wingman-UAV, the C-DOP, and the DOP in three directions are calculated. The calculation method of DOP can be seen in the work of Sivaneri and Gross, and the DOP in east, north, and up directions are represented by E-DOP, N-DOP, and U-DOP, respectively. Statistical results of the position error of the wingman-UAV are shown in Table 4. From the comparison results, it can be seen that the positioning accuracy of the wingman-UAV is
positively correlated with the C-DOP, and the position errors in longitude, latitude, and altitude are positively correlated with C-E-DOP, C-N-DOP, and C-U-DOP, respectively.

According to Table 4, in the case of better formation (C-DOP is small), the traditional DOP calculation method does not accurately describe the size of the error in longitude and latitude, and the C-DOP algorithm proposed in this article can more accurately describe the position accuracy of the wingman-UAV in all directions than the traditional DOP calculation method. In order to further verify the effectiveness of the method, three scenarios are designed for comparison.

**Scenario 1:** Choosing another better formation (C-DOP = 3.32) with unchanged parameters such as the position and error of the leader-UAV 1, leader-UAV 2, and wingman-UAV.

| C-DOP | Position |
|-------|----------|
| 7.74  | Longitude 109.999613484192°C, Latitude 19.9997345438209°C, Altitude 475 m |
| 5.06  | Longitude 110.000454356036°C, Latitude 20.0001395518624°C, Altitude 780 m |
| 3.28  | Longitude 109.99982362753°C, Latitude 20.0004295191517°C, Altitude 445 m |

RMSE: root mean square error; C-DOP: cooperative dilution of precision; UAV: unmanned aerial vehicle.

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**Table 3. C-DOP and position of candidate leader-UAV.**

| C-DOP | Position |
|-------|----------|
| 7.74  | Longitude 109.999613484192°C, Latitude 19.9997345438209°C, Altitude 475 m |
| 5.06  | Longitude 110.000454356036°C, Latitude 20.0001395518624°C, Altitude 780 m |
| 3.28  | Longitude 109.99982362753°C, Latitude 20.0004295191517°C, Altitude 445 m |

C-DOP: cooperative dilution of precision; UAV: unmanned aerial vehicle.

**Figure 5.** Position error of wingman-UAV (C-DOP = 7.74). C-DOP: cooperative dilution of precision; UAV: unmanned aerial vehicle.

**Figure 6.** Position error of wingman-UAV (C-DOP = 5.06). C-DOP: cooperative dilution of precision; UAV: unmanned aerial vehicle.

**Figure 7.** Position error of wingman-UAV (C-DOP = 3.28). C-DOP: cooperative dilution of precision; UAV: unmanned aerial vehicle.

**Table 4. Statistics of C-DOP and DOP.**

| C-DOP | DOP | Position error (RMSE) |
|-------|-----|-----------------------|
| 7.74  | 6.4790 E-DOP 1.8066 Longitude 6.6565 m |
|       | C-N-DOP 71.4746 N-DOP 18.3431 Latitude 51.5557 m |
|       | C-U-DOP 14.5006 U-DOP 2.9302 Altitude 10.1609 m |
| 5.06  | 5.2787 E-DOP 1.4859 Longitude 2.5042 m |
|       | C-N-DOP 17.5207 N-DOP 9.6529 Latitude 7.1811 m |
|       | C-U-DOP 3.2910 U-DOP 0.6908 Altitude 1.7732 m |
| 3.28  | 4.7194 E-DOP 1.2312 Longitude 2.3925 m |
|       | C-N-DOP 4.6200 N-DOP 1.6293 Latitude 2.1485 m |
|       | C-U-DOP 1.4431 U-DOP 0.5641 Altitude 1.1234 m |

RMSE: root mean square error; C-DOP: cooperative dilution of precision.
Scenario 2: The position of the selected candidate leader-UAV is consistent with that of C-DOP at 3.28, and the standard deviation of the position error white noise of the leader-UAV in three directions is changed to 0.5 m, 0.5 m, and 0.5 m, respectively.

Scenario 3: The position of the selected candidate leader-UAV is consistent with that of C-DOP at 3.28, and the standard deviation of the position error white noise of the leader-UAV in three directions is changed to 20 m, 25 m, and 20 m, respectively.

Combining the above three scenarios, the simulation results are shown in Table 5.

As can be seen from Table 5, when the position error of the leader-UAV is small, that is, the impact of the position error is small, the results of the C-DOP method proposed in this article are approximately the same as those of the traditional DOP method. However, the C-DOP can better describe the level of positioning accuracy achieved by wingman-UAV when the position errors of leader-UAVs are large.

In order to verify the effectiveness of the proposed method in reducing the amount of computation, the following scenario is designed.

In this scene, a wingman-UAV can receive the cooperative information of 20 leader-UAVs around and filter the information through the method proposed in the “Cooperative navigation method based on cooperative information screening” section. The sensor parameters of wingman-UAV and leader-UAVs are the same as the previous simulation parameters. We choose 4 as the critical value of C-P-DOP. The spatial distribution of the UAV group is shown in Figure 8.

Two hundred Monte Carlo simulation trials are carried out for the above scene, and the running time of codes (the simulation time of each trial is 8000 s) obtained by the “tic” and “toc” functions of MATLAB (2016b) is used as the performance metric. The simulation results in the two cases of no information screening and information screening by the method proposed in this article are shown in Table 6.

The position error in Table 6 is calculated by

\[ \text{Err} = \sqrt{\text{Err}_E^2 + \text{Err}_N^2 + \text{Err}_U^2} \]  

where \( \text{Err}_E \), \( \text{Err}_N \), and \( \text{Err}_U \) are estimation errors in three directions of east, north, and up, respectively.
In the 200 times Monte Carlo simulation trials, the relationship between the number of leader-UAVs after information screening and the times corresponding to each number is shown in Figure 9.
It can be seen from Figure 9 and Table 6 that after information screening, the number of leader-UAVs participating in information fusion has decreased, while the positioning accuracy level of the wingman-UAV has not reduced significantly. Therefore, the cooperative navigation method proposed in this article can effectively reduce the computational complexity while ensuring the positioning accuracy.

Conclusion
Aiming at the problem that cooperative positioning accuracy of UAV swarm is affected by formation geometries in cooperative navigation process, a cooperative navigation method based on C-DOP is proposed. The calculation of C-DOP depends not only on formation geometries but also on ranging measurement error and position error of leader-UAVs. Therefore, compared with the traditional DOP method, C-DOP can more accurately describe the positioning performance of the wingman-UAV. On the basis of precision factor analysis, Kalman filter is used to fuse cooperative information and inertial information. The simulation results show that the proposed method can effectively analyze the positioning accuracy of UAV swarm with different formation geometries and realize the localization of wingman-UAV.
Future work will focus on cooperative path planning for UAV swarm, so as to provide better cooperative positioning service for wingman-UAV in real time.

Author contributions
Conceptualization, ZX and MC; methodology, MC and ZX; validation, MC; writing—original draft, MC and ZX; writing—review and editing, JL, RW, and JX.

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