Electromagnetic Wave Equation Approximation using FDTD method on Conductivity Material

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Abstract. FDTD is a method that is applied in the simulation of electromagnetic waves. This study aims to simulate the propagation of electromagnetic waves on a material with conductivity and permittivity. The approximate form of Maxwell's equations can be used to describe discrete electromagnetic waves. Signal analysis in the form of electromagnetic waves using position domains for magnetic field H and electric field E. By taking into consideration boundary conditions, stability, and boundary conditions, the proposed research employs the basic concept of differential equation method. The simulation results show that materials with high conductivity will cause the waves to decay. Under certain conditions, the relationship between the shape of the field to changes in conductivity and permittivity of the material is needed in the analysis process.

1. Introduction

Electromagnetic theory is a theory that explains the relationship between electric and magnetic fields, which cause electromagnetic waves to propagate. This theory was put forward by James Clerk Maxwell in 1865. According to Maxwell’s, shifting electric field will cause a magnetic field. Meanwhile, Faraday argued that a changing magnetic field creates an electric field. Thus, electromagnetic waves form as result of the relation between the magnetic and the electric field. In other terms, electromagnetic waves are mixed waves between electric and magnetic fields caused by accelerated moving electric charges.

Modeling of electromagnetic waveforms is difficult to visualize so an appropriate method is required to overcome this problem. One of the suitable methods to solve the problem of visualizing electromagnetic waves is the Finite Difference Time Domain (FDTD) method. The FDTD had utilized to solve problems including propagation, radiation, and electromagnetic wave propagation. Kane Yee initially proposed FDTD method in 1966 to evaluate electromagnetic fields [1]. It can analyze problems based on integral equations much easier which are very difficult to solve with the Moment Method and others. Applying this method does not require the basics of deep knowledge [2]. The FDTD method has been used as a numerical solution since 1920, and is widely used in various fields. There are many fields that had utilized the methodology, for example are fields that study electromagnetic waves, seismology, radiation and scattering and the propagation of sound waves in an acoustic space [3].
Previous research related to FDTD had mostly been done on regional hyperthermia [4], absorption of dispersive media [5], and the electromagnetic waves effect on materials of biology [6]. FDTD also analyzed how the dielectric effect in the case of electromagnetic wave propagation [7,8] and frequency dependent dispersive media [9]. Further, in the term of divergence, it was also utilized for ADI-R-FDTD 3-D modelling [10]. FDTD could also simulate the frequency shifted using radar cross-section (RCS) and CPML [11,12]. FDTD was also utilized in the material industry to estimate and calculate bands in photonic crystal [13], as well as in the telecommunications industry for acoustic and antennas analysis [14,15].

According to previous study, majority of researchers utilized FDTD method to identify material properties. However, FDTD had not been developed and calculated to explain how permittivity and conductivity variations influenced field patterns on metal plates. The simulations can be used for materials selection and physical phenomena detection associated using electromagnetic waves. Furthermore, It can be used to compute gains and return losses of antennas, especially microstrip antenna for telecommunications development. In this study, FDTD method was used to identify the influence of conductivity and permittivity for the electromagnetic wave patterns on the metal plate.

2. Method

The mathematical relation between the time-dependent electromagnetic radiation field and current as well as charge density was generally introduced by Maxwell’s. On the time-dependent equations of Maxwell’s equation [16], differential equations of propagating wave in one dimension may be explained and developed in Equations (1) and (2).

\[
\frac{\partial \vec{B}}{\partial t} = -\nabla \times \vec{E} - \vec{M} \tag{1}
\]

\[
\frac{\partial \vec{D}}{\partial t} = \nabla \times \vec{H} - \vec{J} \tag{2}
\]

Equations (1) and (2) describe the propagation of the electric field, magnetic field, and wave in the directions of the x-axis, y-axis, and z-axis, respectively. The following equation is used to solve time and space dependent differential equations.

\[
E_{x}^{n+1/2}(k) = E_{x}^{n-1/2}(k) - \frac{1}{\sqrt{\varepsilon_{r}\mu_{e}}} \frac{\Delta t}{\Delta x} \frac{\Delta \varepsilon}{\Delta x} \left[ H_{y}^{n}(k+1/2) - H_{y}^{n}(k-1/2) \right] \tag{3}
\]

\[
H_{y}^{n+1/2}(k+1/2) = H_{y}^{n}(k+1/2) - \frac{1}{\sqrt{\varepsilon_{r}\mu_{e}}} \frac{\Delta t}{\Delta x} \frac{\Delta \varepsilon}{\Delta x} \left[ E_{x}^{n+1/2}(k+1) - E_{x}^{n+1/2}(k) \right] \tag{4}
\]

The relation between electric and magnetic fields as a function of time and position is shown in Equations (3) and (4).

The differential equations can be modified in different approximations for derivation of spatial and temporal dependent electric and magnetic fields to simulate electromagnetic wave propagation in the medium having both permittivity and conductivity [16]. The approximation equation can be written as in Equation (5).

\[
E_{x}^{n+1/2}(k) = \left\{ \frac{1 - \frac{\Delta t \sigma}{2\varepsilon_{0} \varepsilon_{r}}} {1 + \frac{\Delta t \sigma}{2\varepsilon_{0} \varepsilon_{r}}} \right\} E_{x}^{n-1/2}(k) - \frac{1/2}{\varepsilon_{r} \left\{ 1 + \frac{\Delta t \sigma}{2\varepsilon_{0} \varepsilon_{r}} \right\}} \left[ H_{y}^{n}(k+1/2) - H_{y}^{n}(k-1/2) \right] \tag{5}
\]

In programming language, Equation (5) was modified in Equation (6).

\[
Ex(k) = A(k) * Ex(k) + B(k) * (Hy(k - 1) - Hy(k)) \tag{6}
\]

where

\[
E = dt * Sigma / (2 * epsilon0 * epsilonr) \tag{7a}
\]

\[
A(i) = (1 - E) / (1 + E) \tag{7b}
\]

\[
B(i) = 0.5 / (epsilonr * (1 + E)) \tag{7c}
\]
The equation for the constant propagation and attenuation of electric and magnetic fields propagating on the plate is defined as in Equations (8) and (9).

$$\alpha = \frac{\omega}{c_0} \sqrt{\varepsilon_r / 2} \left[ 1 + \left( \frac{\sigma}{\omega \varepsilon_0 \varepsilon_r} \right)^2 \right]^{-1/2}$$

(8)

$$\beta = \frac{\omega}{c_0} \sqrt{\varepsilon_r / 2} \left[ 1 + \left( \frac{\sigma}{\omega \varepsilon_0 \varepsilon_r} \right)^2 + 1 \right]^{1/2}$$

(9)

Because the velocity of electromagnetic waves in a vacuum could not be faster than the light’s, it is necessary to determine time interval using FDTD stabilities. The “Courant Condition” in $n$-dimensions in Equation (10) can be used to solve the problem [17].

$$\Delta t \leq \frac{\Delta x}{\sqrt{n c_0}}$$

(10)

The theory suggested boundary condition absorption, which was necessary to avoid reflection on the electric field ($E$) and magnetic field ($H$). For the fundamental of the FDTD method, the E field calculation was obtained by considering the encircling H field value [18]. The theory was known as Theory of Absorbing Boundary Condition (ABC Theory). In the boundary conditions, it was important to consider the values in both sides [19]. However, according to the theory, it will more advantageous if so, the boundary field must spread outside due to there is no source on the outside of boundary. Both of the facts were utilized to calculate the value in the end, which was based on the value in the area. When $k = 0$, the wave will propagate to the boundary at the speed of light, just as it would in free space. While travelling, it generated one time step of the FDTD algorithm.

$$\text{displacement} = \frac{\Delta x}{2}$$

(11)

Calculation of FDTD simulation is constrained by computer resources. If wave generated from a points source was simulated as in free space, the unexpected reflections inside will be generated. It was impossible to distinguish between incident waves and reflected waves. During FDTD process, ABC theory became the appropriate solution of the complicated problem. Previous researchers developed the Perfect Match Layer (PML) to improve the flexibility and efficiency of ABC theory [19].

3. Results and Discussion

Figure 1 describes the visual appearance of the FDTD program that had been made to illustrate electromagnetic waves. The procedure for making the proposed electromagnetic wave FDTD program includes determining the input parameters, the number of N-Steps, calculating the attenuation and propagation constants either manually or based on simulations. In addition, it is also necessary to determine the variation of the thickness of the conductor plate and the type of pulse used.

The input parameters present the position domain length, time domain, wave pulse position, permittivity, conductivity, and plate thickness. Pulse variations can use gaussian or sinusoidal wave pulse types. Program-bound variables include the domain length of position 620, frequency of 700 MHz, the initial position of the wave pulse at position 1. While the independent variables that we use include plate thickness, permittivity (epsilon), and conductivity (sigma).
Figure 1. FDTD program display

Figure 2 shows the symptoms of electromagnetic waves when passing through a conductor plate. The relative permittivity of the plate is 4 and the conductivity value is 0.04 and the plate thickness is 100. The sinusoidal electromagnetic wave pulses \( E_x \) (red) and \( H_y \) (yellow) decay with \( B \) values ranging from 0.12383450 Equation (7c) and \( A \) from 0.98135203 in Equation (7b). Based on the simulation results, the \( E_x \)-value decays first and then \( H_y \) is followed according to Equation (5).

Figure 2. Electromagnetic wave simulation results on the plate
The calculation results are taken at N-step 620 with the attenuation value in Equation (8) showing a value ranging from 3.7344977 Np/m while the simulation results show 4.1632605 Np/m, this difference is due to the position factor of the location of the test point taking the $E_x$ value in the initial position and end. While the value of the propagation constant according to Equation (9) shows 29.54364880 rad/m.

![Figure 3. Simulation using Gaussian wave pulse](image1)

![Figure 4. Simulation using Gaussian wave pulse](image2)
Figure 3 illustrates how gaussian pulses form in electromagnetic waves. The simulation results show that the reflection results of the $E_x$ and $H_y$ field components experience different wave phases, while the transmission results after passing through the plate show the same phase and experience attenuation so that the waveform decays. In addition, the values of $B$ and $A$ on N-step 400 showed constant values. The decay of electric and magnetic fields is caused by a decrease in the intensity or energy of the wave due to the absorption of energy by the permittivity of the plate material. Previous studies have not shown this physical phenomenon [19].

Figure 4, which shows the plate change to 50, shows that for the permittivity value of 4 and the conductivity of 0.04 the wave pulse still penetrates the conductor plate. While attenuation and wave propagation are not affected by plate thickness. These results indicate that what affects the attenuation and wave propagation values is the permittivity and conductivity of the material [8].

Figure 5. Simulation using Gaussian wave pulse with different permittivity

Figure 6. The Simulation uses Gaussian wave pulses with different conductivity
Figure 5, shows the change in permittivity of the material to 12 at a thickness of 50. The simulation results show that changes in the permittivity value cause the wave superposition in the $E_x$ field to decrease while the $H_y$ field component increases. This is different from Figure 6, which shows an increase in the conductivity value to 0.08 causing the $E_x$ and $H_y$ components to rapidly decay so that no wave pulses are transmitted.

4. Conclusion
The conclusion that can be drawn based on the simulation results is that the permittivity and conductivity properties affect the wave propagation on the plate, both reflected and transmitted. Changes in the thickness of the plate have no effect on attenuation and wave propagation. Meanwhile, changes in the permittivity value cause the field components $E_x$ and $H_y$ to have different wave superposition results in the destructive $E_x$ and constructive $E_y$ components. Meanwhile, the change in the conductivity value causes the $E_x$ and $H_y$ field components to decay faster before passing through the plate.

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