CP-Violation and Baryogenesis in The Low Energy Minimal Supersymmetric Standard Model

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Abstract

In the context of the minimal supersymmetric extension of the Standard Model the effect of a realistic wall profile is studied. It has been recently showed that in the presence of light stops the electroweak scale phase transition can be strong enough for baryogenesis. In the presence of non-trivial CP-violating phases of left-handed mixing terms and Higgsino mass, the largest $n_B/s$ is created when Higgsino and gaugino mass parameters are degenerate, $\mu = M_2$. In the present paper we show that realistic wall profiles suppress the generated baryon number of the universe, so that quite a stringent bound $|\sin \phi_\mu| \gtrsim 0.2$ for $\mu$-phase $\phi_\mu$ can be inferred.
The Minimal Supersymmetric Standard Model (MSSM) has appeared to be one of the most promising candidates to explain the observed baryon asymmetry of the universe $n_B/s \sim 10^{-10}$ generated at electroweak scale. Although all requirements are included already in the Standard Model [1, 2], the phase transition [3] has appeared to be too weakly first order to preserve the generated baryon asymmetry [4]. Also it has been shown that the CP-violation needed for baryogenesis is too small in the Standard Model [5]. Therefore some new physics besides the Standard Model is necessarily needed, provided that the baryon asymmetry is generated during the electroweak phase transition.

Because MSSM is one of the most appealing extensions of the Standard Model, it has been worthwhile to study whether it is possible to generate and preserve the baryon asymmetry in it. Indeed, recent analyses show that there exists a region of the parameter space where the phase transition is strong enough [6]. It is required that $\tan \beta < 3$, the lightest stop is lighter than the top quark and the lightest Higgs must be detectable by LEP2. The bound given above may relax due to two and higher-loop effects, which seem to strengthen the phase transition [7]. Unlike the Standard Model where the source of CP-violation is solely the Cabibbo-Kobayashi-Maskawa matrix, MSSM contains an additional source due to the soft supersymmetry breaking parameters related to the stop mixing angle.

In a recent paper Carena et al. [8] has analysed the region of supersymmetric parameter space where the baryon asymmetry generated at the electroweak scale is consistent with the observed one. The generation and survival of large enough $n_B/s \approx 4 \times 10^{-11}$ seem to require that $M_Z \lesssim m_{\tilde{t}} \lesssim m_t$, the mass of the lightest Higgs boson is bounded by $m_H < 80$ GeV whereas CP-odd boson has mass $m_A \gtrsim 150$ GeV. The analysis gave dependence of $n_B/s$ on Higgsino mass parameters $|\mu|$ and its phase $\phi_\mu$ as well as gaugino mass parameters $M_1$ and $M_2$. The optimal choice of parameters showed up to be $|\mu| \simeq M_2$, $|\sin \phi_\mu| \gtrsim 0.06$, and the dependence on $M_1$ is weak, so that it can be chosen to be equal to $M_2$. In the analysis of Carena et al. it was chosen the left-handed stop mass parameter $m_Q$ to be $m_Q = 500$ GeV, effective soft supersymmetry breaking parameter $\tilde{A}_t = 0$ and right-handed stop mass parameter $m_U = -\tilde{m}_U < 0$

$$\tilde{m}_U \lesssim m_U^{\text{crit}} \equiv \left( \frac{m_{1/2}^2 v^2 g_3^2}{12} \right)^{1/4}$$

(1)

to obtain the most optimistic bounds (i.e. maximize $n_B/s$) [8]. With these parameter values the CP-violating source is generated essentially by Higgsino and gaugino currents and the right-handed stop contribution is negligible, so that without a loss of generality one can set $\sin(\phi_\mu + \phi_A) = 0$. The bound (1) is due to the colour non-breaking condition, i.e. no colour breaking minimum must be deeper than the

1 About restrictions and validity of these results, see [8].
normal electroweak breaking (and colour conserving) minimum. It is defined at zero-temperature, thus \( v_0 = 246.22 \text{ GeV} \). Also it was used \( v_w = 0.1 \) and \( L_w = 25/T \). With these conditions a large enough baryon asymmetry could have been created.

In the paper [8] the baryon asymmetry was inferred by first calculating the CP-violating sources and then solving the relevant Boltzmann equations. It was shown that the baryon to entropy ratio reads

\[
\frac{n_B}{s} = -g(k) \frac{\mathcal{A} \bar{D} \Gamma_{ws}}{v_w^2 s},
\]

where \( g(k) \) is a numerical coefficient depending on the degrees of freedom, \( \bar{D} \) the effective diffusion rate, \( \Gamma_{ws} = 6\kappa \alpha_w^4 T (\kappa = 1) \) [9] the weak sphaleron rate, \( v_w \) the wall velocity. The entropy density \( s \) is given by

\[
s = \frac{2\pi^2 g_{s,s} T^3}{45},
\]

where \( g_{s,s} \) is the effective number of relativistic degrees of freedom. The coefficient \( \mathcal{A} \) was shown to be a certain integral over the source \( \tilde{\gamma}(u) = v_w f(k) \partial_u J^0(u) \):

\[
\mathcal{A} = \frac{1}{D \lambda_+} \int_0^\infty du \tilde{\gamma}(u) e^{-\lambda_+ u},
\]

where \( \lambda_+ = (v_w + \sqrt{v_w^2 + 4\bar{D} \Gamma}/(2\bar{D}) \) and the wall was defined so that it begins at \( u = 0 \). Here \( u \) is the co-moving coordinate \( u = z + v_w t \) supposing that the wall moves in the direction of \( z \)-axis. \( f(k) \) is again a coefficient depending on the number of degrees of freedom present in thermal path and related to the definition of the effective source [8, 10].) Thus the coefficient \( \mathcal{A} \) is dependent on the actual wall shape via

\[
\mathcal{A} \propto I \equiv \int_0^\infty du \frac{\partial}{\partial u} (H(u)^2 \partial \beta(u) \partial u) e^{-\lambda_+ u},
\]

where \( H = \sqrt{H_1^2 + H_2^2} \), \( \tan \beta = H_1/H_2 \) and \( H_i \)'s are the real parts of the neutral components of the Higgs doublets. In [8] the wall shape was, however, taken \textit{ad hoc}. It was assumed to have a simple sinusoidal form so that the field \( H(u) \) can be given by

\[
H_s(u) = \frac{v}{2} \left[ 1 - \cos \left( \frac{u \pi}{L_w} \right) \right] [\theta(u) - \theta(u - L_w)] + v \theta(u - L_w)
\]

and the field angle \( \beta(u) \) by

\[
\beta_s(u) = \frac{\Delta \beta}{2} \left[ 1 - \cos \left( \frac{u \pi}{L_w} \right) \right] [\theta(u) - \theta(u - L_w)] + \Delta \beta \theta(u - L_w),
\]

where \( \Delta \beta \) is given by \( \Delta \beta = \beta(T_0) - \arctan(m_1(T_0)/m_2(T_0)) \), calculated at the temperature where curvature of the one-loop effective potential vanishes at the origin.
Inserting these Anz"atse to Eq. (3), we obtain a corresponding contribution to the CP-violation

\[ I_s = \int_0^\infty du \frac{\partial}{\partial u} (H_s(u)^2 \frac{\partial \beta_s(u)}{\partial u}) e^{-\lambda u}. \] (8)

Using this approximation, the results of [8] was inferred.

In the present paper a more realistic prescription of wall shape is used. Working at the critical temperature \( T_c \) and using the one-loop resummed effective potential [11] we numerically find the path of smallest gradient \( \gamma_g \) from \( (H_1, H_2) = (0, 0) \) to \( (v_1, v_2) \equiv (v_1(T_c), v_2(T_c)) \), which well approximates the true solution. Moreover, using path \( \gamma_g \), an upper bound for \( I \) is necessarily obtained. The true solution lies necessarily between \( \gamma_g \) and straight line from \( (0, 0) \) to \( (v_1, v_2) \) (which leads to \( n_B = 0 \)) as can be concluded by studying the Lagrangean[4]. Thus \( |I| \) along such a path is smaller than along \( \gamma_g \).

The effective potential for MSSM at finite temperature can be expressed in three parts [11]

\[ V_{\text{eff}}(\phi, T) = V_0(\phi) + V_1(\phi) + V_{1,T}(\phi) + \Delta V_T(\phi), \] (9)

where \( V_0 \) is the tree level zero-temperature potential, \( V_1 \) the renormalized 1-loop zero-temperature potential, \( V_{1,T} \) the 1-loop finite temperature potential and \( \Delta V_T \) the daisy-resummed part. They are given by

\[ V_0(H) = m_1^2 H_1^2 + m_2^2 H_2^2 + 2 m_{12}^2 H_1 H_2 + \frac{g^2 + g'^2}{8} (H_1^2 - H_2^2)^2, \] (10)

\[ V_1(H) = \sum_{t, \tilde{t}, \tilde{t}' \equiv W, Z} \frac{n_i}{64 \pi^2} m_i^4(H) \left( \ln \frac{m_i^2(H)}{m_Z^2} - \frac{3}{2} \right), \] (11)

\[ V_{1,T}(H) = \frac{T^4}{2 \pi^2} \sum_{t, \tilde{t}, \tilde{t}' \equiv W, Z} n_i J_i \left[ \frac{m_i^2(H)}{T^2} \right] \] (12)

and

\[ \Delta V_T(H) = - \frac{T}{12 \pi} \sum_i n_i [m_i^3(H) - m_i^3(H)], \] (13)

where \( m_i(H) \) and \( m_i(H,T) \) are the zero temperature and temperature corrected field dependent masses, respectively, \( n_i \) are the degrees of freedom of each particle (including -- sign for fermions) and

\[ J_i(x^2) = \int dy y^2 \ln(1 \pm e^{-\sqrt{y^2 + x^2}}). \] (14)

Note, that we have neglected the b-quark, \( \tilde{b} \)-squark as well as other generation contributions as small ones. The heavy supersymmetric particles do not contribute neither.

\(^2\)This is true providing that the field is not strongly oscillating within the wall, but is a smooth configuration.
The mass parameters $m_1, m_2, m_{12}$, are related to $\beta, m_A, m_H$ and other parameters of the theory, as given in [11]. Using these formulas, we can solve $\gamma_g$ and define the corresponding CP-violation integral

$$I_\gamma = \int_0^\infty du \frac{\partial}{\partial u} (H(u)^2 \frac{\partial \beta(u)}{\partial u}) e^{-\lambda_u}$$

(15)

at the critical temperature along the path $\gamma_g$. It also shows up that the form of profiles is, with good accuracy, the form of a kink. Indeed, if we reparametrise the field $(H_1, H_2)$ to a component pointing towards $(v_1, v_2)$, $H_\parallel$ and to a component orthogonal to that, $H_\perp$, it appears that the ratio of maximum value of $H_\perp$ to $v = \sqrt{v_1^2 + v_2^2}$ is in any case smaller than 0.004. Thus the bending of the path, i.e. the deviation from a straight line is small. (Note, that here $v$ is the value of vacuum at $T_c$, thus not equal to $v_0 = 247$ GeV.)

The ratio $I_\gamma / I_s$ gives immediately the supression of $n_B/s$ with respect to the results of Carena et al. [8]. Hence the value needed for soft supersymmetry mixing phase $\sin \phi_\mu$ is increased by factor $I_\gamma / I_s$. In the Figure 1. we have presented the value of the integral $I_s$ as a function of $\mu$ for several values of $m_A$. From the figure it can be read out that increasing $\mu$ decreases the (absolute) value of $I_\gamma$ so that for $\mu > 250$ GeV inequality $|I_\gamma| < 1$ holds. Also it can be found that increasing $m_A$ with factor $r$ decreases $I_\gamma$ roughly by factor $1/r$ within the parameter range studied. This behaviour is likely be more general than just restricted to analysis of [8], because the amount of CP-violation is in general proportional to the change of $\beta$ on the path from the origin to the non-trivial vacuum.

In Figure 2. a comparison to the result of Carena et al. [8] is made. We have plotted the ratio $I_\gamma / I_s$ also as a function of $\mu$ with several values of $m_A$. There appears a clear tendency of additional suppression: for very small $\mu$, $I_\gamma / I_s \simeq 0.4$, whereas for $\mu \simeq 250$ GeV, $I_\gamma / I_s \simeq 0.1$. For optimal values of $\mu$, $150$ GeV $\leq \mu \leq 250$ GeV the supression factor is at least 0.3. Also it appers that the dependence of this supression factor on CP-odd boson mass $m_A$ is relatively weak. Taking in to the account that according to the analysis of [8] it is required that $|\sin \phi_\mu| \gtrsim 0.06$, this is now converted to a more stringent bound $|\sin \phi_\mu| \gtrsim 0.2$ which remarkably weakens the possibility of baryogenesis in MSSM. However, for $\mu \simeq 250$ GeV the bound rises to $|\sin \phi_\mu| \gtrsim 0.6$ which possibly is already too large.

In the present paper we have calculated the amount of CP-violation in the bubble wall of the minimal supersymmetric extension of the Standard Model at the electroweak scale phase transition. It has shown up that the previous estimates tend to be too optimistic and an extra supression of (at least) 0.3 is found. This tends to make the electroweak baryogenesis in MSSM more difficult and less likely. However, more analysis on the model is needed, in particular to clarify how higher corrections (more scalar insertions) to the CP-violating source behave. If the higher corrections to
the source expand in the powers of \((H_1\partial H_2 - H_2\partial H_1)\) no help from them is expected. If they, however, order by some other expansion parameter, their contribution to the source may be remarkably large. Unfortunately this may also lead to the situation where non-perturbative effects are important. Also the effect of higher order corrected effective potential remains to be studied. The two-loop corrections tend to strenghten the phase transition and thus relax the bounds \([6]\). The two-loop contributions may, however, affect also directly the value of the integral \(I\) needed in the calculation of CP-violating source. On the other hand, also the sphaleron rate \(\Gamma_{ws}\) is under discussion \([12]\) and changes on that may change significantly the conclusions made about baryogenesis.

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Figure captions

Figure 1. Values of the integral $I_\gamma = \int_0^\infty (H^2 \beta')' e^{-\lambda u} \mu$ (where comma stands for $u$-derivative) as a function of the soft supersymmetry parameter $\mu = M_1 = M_2$ with $v_w = 0.1$, $m_Q = 500 \text{ GeV}$, $\tilde{m}_U = \tilde{m}_U^{\text{crit}}$, $\tan \beta = 2$ and $\tilde{A}_t = 0$.

Figure 2. Values of the ratio $I_\gamma/I_s$ as a function of $\mu = M_1 = M_2$ with $v_w = 0.1$, $m_Q = 500 \text{ GeV}$, $\tilde{m}_U = \tilde{m}_U^{\text{crit}}$, $\tan \beta = 2$ and $\tilde{A}_t = 0$. For $I_s$ wall width $L_w = 25/T$ is used.
$I_\gamma$ vs $\mu/GeV$

- $m_\lambda = 150$ GeV
- $m_\lambda = 175$ GeV
- $m_\lambda = 200$ GeV
- $m_\lambda = 225$ GeV

Fig. 1
Fig. 2