Spatial ability of student in construct volume of the solid of revolution graphic

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Abstract. Visualization ability (spatial ability) is needed in understanding integral calculus. This article discusses changes in the spatial ability of a student in constructing graphs of functions to determine the volume of the solid of revolution after obtaining treatment. The treatment is in the form of questions that encourage the subject to be able to construct the graph correctly. This research is a qualitative descriptive study with a single subject. The subject in this study was 6th semester student of mathematics education study program at one of the private universities in Yogyakarta. The selection of a subject from 44 students was done by giving a question about the volume of the solid of revolution. The subject is asked to constructing the solid of revolution graphic and determine its volume using integral. The selection of subjects by purposive random sampling based on the uniqueness of the answers to the problems given and good communication skills. The results of the study show that giving treatment can help increase a student’s spatial ability gradually. Before getting treatment, a subject is still not right in constructing volume of the solid of revolution graphic. After getting treatment, there is a gradual change in spatial ability.

1. Introduction

To be able to construct a graph requires good spatial ability because it requires the ability to imagine a space that has changes/displacement, it takes the ability to observe objects from various sides, and the ability to rotate objects properly [1–3]. A large number of students who misrepresent the rotating objects shows the lack of spatial abilities of students. Spatial ability can be increased by giving encouragement and opportunity to think not by dictating [4].

In many countries, integral calculus has been considered a subject that is not liked by students [5], whereas differential and integral are important topics for the development of science and technology [6]. Student, in general, find it difficult to understand the volume of the solid of revolution in the course of integral calculus [7–10]. Of the 44 students, 97.73% of students mis-constructed the solid of revolution, that cause mistakes in determining the volume of the solid of revolution. The problem that was found was that students were still having difficulty in constructing graphs of a function [8,9,11–13]. Constructing a graph of a function is the basic ability that students must have in determining the volume of the solid of revolution using integral techniques. If there is an error in constructing the graph, there is a high probability that there will be an error in determining the volume of the solid of revolution. Mistakes made in describing function graphs are errors in determining the points passed by the graph [14–17], not paying attention to integration boundary [8,18], errors in constructing the rotation, and errors in constructing the solid of revolution graphics [2,8,15].

In general, the problem given is to determine the volume of the solid of revolution which is formed by two functions, for example, “Determine the volume of the solid of revolution when
the area is formed by parabola-parabola \( y = x^2 \) and \( y^2 = 8x \) rotates around the x-axis”. This example is an example of a relatively simple function, but the existing problems can involve more than one function with varied forms [9]. The researcher tried to provide a problem that involved two functions with varied forms. The problem given is to determine the volume of the solid of revolution which is formed by the curve \( y = x^2 + 2 \) and the line \( y = 1 \) at the integration boundary \([0,4]\) rotated around the x-axis.

When viewed from the equation of function, the equation \( y = x^2 + 2 \) and \( y = 1 \) looks simpler than the equation \( y = x^2 \) and \( y^2 = 8x \). However, if it is depicted, the curve \( y = x^2 + 2 \) and \( y = 1 \) more varies compared to the curve \( y = x^2 \) and \( y^2 = 8x \). The picture of the two curves is shown in Figure 1.

![Figure 1. The Solid of Revolution](image)

In Figure 1a, it can be seen that the solid of revolution is formed from 2 parabolas which are depicted through the origin and only form a circular cross section on the outside of the solid of revolution. While the solid of revolution in Figure 1b is more varied where the solid of revolution is formed from a curve that is depicted through a point \( y = 2 \) and is formed from a line that is drawn through a point \( y = 1 \) so that two circular cross sections are formed on the outside of the solid of revolution.

The problem given turned out to be an obstacle for students. Most students make mistakes in constructing graphs of functions. As a result, students also make mistakes when constructing the solid of revolution and in determining the volume of the solid of revolution. The results of other studies show that not only in constructing graphs of functions, students also have difficulty in defining functions, identifying functions from statements and graphs, identifying types of functions from graphs and tables, and substituting algebraic values for given functions [12]. A lack of understanding of the concept of function can cause difficulties in studying integral calculus [19,20].

Therefore, researchers aim to identify errors so that they can minimize errors that occur in constructing graphics and the solid of revolution in the hope of minimizing errors in determining the volume of the solid of revolution. These errors need to be immediately identified so that alternative solutions can be found so that the same mistakes do not occur [21,22]. Errors in constructing graphs of functions and the solid of revolution are minimized by giving limited treatment in the form of questions that encourage subjects to be able to graph correctly in stages. There are still many who have not done research on how changes in spatial
abilities of students if given a treatment. Therefore, this study aims to determine student understanding in constructing the solid of revolution and describe changes in spatial abilities of students in constructing the solid of revolution after getting treatment.

2. Method
This study involved 44 6th semester students of the 2018/2019 academic year study program in one of the private universities in Yogyakarta. Students are mostly educated from vocational schools (accounting, computer, and marketing majors) so that the ability to understand mathematics, especially integral material, is low. All students are then given the problem of determining the volume of the solid of revolution which is formed by curves \( y = x^2 + 2 \) and lines \( y = 1 \) at the integration boundary \([0,4]\) to work within 20 minutes. The researcher then checks the student's answers.

In examining the answers, the researcher focused more on the solid of revolution that was constructed because the researcher wanted to see the spatial abilities of the students. Of 44 students, only 1 person is right in constructing the solid of revolution. The researcher chooses the image of the solid of revolution that is not right because it wants to know the mistakes made in constructing the function and the solid of revolution produced. Researchers choose research subject based on several things, namely the uniqueness of the answers to the problems given and good communication skills so that researchers easily dig up information. The uniqueness of the answer in question is the biggest error in constructing graphs of functions \( y = x^2 + 2 \) and lines \( y = 1 \) and errors in constructing the solid of revolution.

Based on these criteria, researchers selected one of 44 students selected as research subject. The research subject was asked to determine the volume of the solid of revolution using integrals. The volume of the solid of revolution in question is the volume of the solid of revolution formed by curves \( y = x^2 + 2 \) and lines \( y = 1 \) at the boundary of integration \([0,4]\). Student is given 10 minutes to constructing the solid of revolution and 10 minutes to calculate the volume of the solid of revolution using an integral. After that, the researcher examined the students' answers. To obtain in-depth information, researchers interviewed the answers given. The interview takes around 30-40 minutes. The purpose of this activity is to find out why student makes mistakes in constructing graphs of functions and the solid of revolution and changes in understanding of students after getting treatment from researchers to obtain the right graphic images.

3. Result and Discussion
When the researcher gave the problem of determining the volume of the solid of revolution which was formed by curves \( y = x^2 + 2 \) and line \( y = 1 \), 97.73% of students mis-constructed the solid of revolution. From these errors, there is a unique error that the line \( y = 1 \) is described as resembling a graph of function \( y = x - 1 \), the solid of revolution that is described is not at the integration boundary \([0,4]\), and the quadratic function described is not a vertical parabola (open up) but forms a horizontal parabolic (open to right).
Figure 2 Work Result of Subject

*Figure 2 shows* an image of an open satellite dish to the right that is constructed by the subject. The subject is asked to rethink the graph of the solid of revolution which is formed by functions $y = x^2 + 2$ and lines $y = 1$. There is a graphical image change function. These changes are shown in Figure 3.

After the subject reconstructed the graph of the function, then interviews are conducted to obtain information about changes in the graphic image of the function.

**Researcher** : See Figure 3a, why can the image be different from the current image (Figure 3b)?

**Subject** : Figure 3a, I draw it groggy Miss, while Figure 3b the atmosphere is relaxed so it can be more thoughtful.

Based on the interview, when constructing the function graph shown in Figure 3a, the subject has experienced learning anxiety. The subject was groggy when he had the test and concern will run out of time so the student cannot properly construct the function graph. Math-anxiety is describing as a panic when someone solve the problem. Math-anxiety is a feeling of tension that can interfere with the process of solving mathematical problems. Learning anxiety needs special attention because learning anxiety can cause learning difficulties [23]. Anxiety in learning mathematics also causes a lack of understanding of concepts. When solving problems, people who experience learning anxiety tend to have difficulty completing them [24].

The subject was asked to construct the solid of revolution referred to in the problem. There is a graphical image change function. These changes are shown in Figure 4c.
To obtain in-depth information, interviews were then conducted regarding changes in the graphic image of the function.

Figure 4 The Change of All Graph Function

| Researcher | Subject |
|------------|---------|
| Try to explain Figure 4c. | For images $y = x^2 + 2$, the image is like Figure 4b Miss, but now it is adjusted at the same interval, from 0 to 4. |
| Next, how can you describe the rotating object? | If this (pointing to Figure 4c) the results of the reflection. Let it be easy, Miss. Later the results of the rotation will be obtained from the results of the mirroring. When rotated, it will form an ellipse at the end here, Miss. |
| Ellipse? Are you sure it’s an ellipse? Are there 2? | Eh circle Miss. From the end here (pointing to the image), until the x-axis, there is 1, from the x-axis to the end here (pointing to the image) there is also 1, so everything is 2. |
| What is an object that when rotated forms 2 circles? | Only one Miss. |
| Is the picture $y = 1$ like that? Which y axis? Where is $y = 1$? | This y-axis Miss (pointing to the y-axis), if $y = 1$ here, Miss. |
| Then, how is the line $y = 1$ drawing?? | (depicting back), like this Miss (pointing to Figure 4d) |
| Now what is the image of the white matter? | (Redrawing), This is the image of the rotating object (pointing to Figure 4d) |

Based on the interviews, subject was still confused in constructing graphs, especially graphs of functions which consisted of only one variable such as $y = 1$, cause cannot be determined cut points in x-axis and y-axis. Subject also still confused in construct cross sections of the solid of revolution from the front and rear sides because there is a line (x-axis) that lies between two curves and still did not understand that the cross section was a circle not an ellipse. When viewed from the picture, it looks like an ellipse not a circle.

When constructing the graph $y = x^2 + 2$, subject sketch the graph at first (Figure 4b). It done by subject to make easier to imagine/visualize how the graph $y = x^2 + 2$ can be constructed, then subject construct the graph adjusted to the integration boundary $[0,4]$ and construct the rotating object by illustrating the result of rotation. At first, subject was constructing the graph of functions $y = 1$ resembling graphs of functions $y = x - 1$. Subject think that the graph of functions $y = 1$ located on the x-axis and passed the point 1 on the x-axis.

In interviews, researchers gave limited treatment. The limited treatment given is in the form of questions that encourage subject to construct the correct graph. The results showed, after getting limited treatment there was a change in understanding in constructing the graph. This change in understanding is meant as a change in spatial ability. Changes in the spatial ability of subject in constructing the graph are shown in Figure 4.

In Figure 4b, the research subjects no longer construct the quadratic function of $y = x^2 + 2$ forming a horizontal parabola but it has formed a vertical parabola. In Figure 4c, it can be seen that the subject correctly depicted the graph $y = x^2 + 2$ forming an open parabolic upward adjusted to the integration boundary $[0,4]$ as requested in the problem and illustrating the results of the rotation. In addition, the changes that appear after getting limited
treatment are in constructing the function graph \( y = 1 \). The subject no longer construct graphs of functions \( y = 1 \) resembling graphs of functions \( y = x - 1 \). But at this stage, the research subject is still not precise in constructing graphs of functions \( y = 1 \) where graphs of functions \( y = 1 \) are described as \( x = 1 \). \textit{Figure 4d} shows, the research subject was correct in constructing both graphics and the solid of revolution but were still not precise in constructing circles at the end and base. The research subject described a circle in the center of the solid of revolution.

4. Conclusion
Subject still has difficulty in solving integral calculus problems, namely regarding the volume of the solid of revolution, especially in constructing the solid of revolution. In constructing the graph, the subject did not determine in advance the points passed by a graph of function so that the graph of the wrong function is obtained. In addition, the error is not paying attention to the interval specified in the problem. Subject is also less able to visualize images where subject see the outer cross section of the solid of revolution in the form of an ellipse, not a circle. With limited treatment, there is a change in the spatial ability of the subject slowly, then it can properly construct the graph of functions and the solid of revolution.

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6. References
[1] Yilmaz B 2017 On the development and measurement of spatial ability \textit{Int. Electron. J. Elem. Educ.} \textbf{1} 83–96
[2] Education O M of 2014 Paying Attention to Spatial Reasoning, K–12 Support Document for Paying Attention to Mathematics Education (Ontario: ServiceOntario) pp 1–28
[3] Boonen A J H, Van Wesel F, Jolles J and Van der Schoot M 2014 The role of visual representation type, spatial ability, and reading comprehension in word problem solving: An item-level analysis in elementary school children \textit{Int. J. Educ. Res.} \textbf{68} 15–26
[4] Harmony J and Theis R 2012 Pengaruh Kemampuan Spasial Terhadap Hasil Belajar Matematika Siswa Kelas VII SMP Negeri 9 Kota Jambi \textit{Edumatika} \textbf{02} 11–9
[5] Li V L, Julaihi N H and Eng T H 2017 Misconceptions and Errors in Learning Integral Calculus \textit{Asian J. Univ. Educ.} \textbf{13} 17–39
[6] Tall D 2011 Looking for the Bigger Picture \textit{Learn. Math.} \textbf{31} 17–8
[7] Adiraaksiwi A G, Warmi A and Imani A I 2018 Penerapan Pendekatan Kontekstual Terhadap Penguasaan Konsep Dasar Materi Volume Benda Putar \textit{J. Penelit. dan Pembelajaran Mat.} \textbf{11} 1–10
[8] Khoiriyah S 2016 Kemampuan Komunikasi Matematis Mahasiswa dalam Pemecahan Masalah Kalkulus II \textit{J. e-DuMath} \textbf{2} 202–9
[9] Romadiastri Y 2013 Penerapan Pembelajaran Kontekstual pada Kalkulus 2 Bahasan Volum Benda Putar \textit{J. Phenom.} \textbf{1} 131–43
[10] Salmina M 2017 Analisis Kekeliruan dalam Menyelesaikan Soal Kalkulus pada Mahasiswa Pendidikan Matematika \textit{Numer. J.} \textbf{4} 62–70
[11] Rimo I H E 2018 Analisis Kesulitan Mahasiswa Pendidikan Fisika FKIP-UNDANA Dalam Memahami Materi Volume Benda Putar \textit{J. Ilm. Soulmath J. Edukasi Pendidik. Mat.} \textbf{6} 91
[12] Bardini C, Pierce R, Vincent J and King D 2014 Undergraduate Mathematics Students’ Understanding of the Concept of Function \textit{Indones. Math. Soc. J. Math. Educ.} \textbf{5} 85–107
[13] Subanji S and Supratman A M 2015 The Pseudo-Covariational Reasoning Thought Processes in Constructing Graph Function of Reversible Event Dynamics Based on
Assimilation and Accomodation Frameworks *Korean Soc. Math. Educ.* **19** 61–79

[14] Ningsih Y L 2016 Kemampuan Pemahaman Konsep Matematika Mahasiswa Melalui Penerapan Lembar Aktivitas Mahasiswa (LAM) Berbasis Teori APOS Pada Materi Turunan *Edumatica* **06** 1–8

[15] Cai J and Wang T 2006 U.S. and Chinese teachers’ conceptions and constructions of representations: A case of teaching ratio concept *Int. J. Sci. Math. Educ.* **4** 145–86

[16] van Garderen D and Montague M 2003 Visual-Spatial Representation, Mathematical Problem Solving, and Students of Varying Abilities *Learn. Disabil. Res. Pract.* **18** 246–54

[17] Hegarty M and Kozhevnikov M 1999 Types of visual-spatial representations and mathematical problem solving. *J. Educ. Psychol.* **91** 684–9

[18] Norton S 2006 Pedagogies for the engagement of girls in the learning of proportional reasoning through technology practice *Math. Educ. Res. J.* **18** 69–99

[19] Dane A, Çetin Ö F, Bas F and Özturan Sağırlı M 2016 A Conceptual and Procedural Research on the Hierarchical Structure of Mathematics Emerging in the Minds of University Students: An Example of Limit-Continuity-Integral-Derivative *Int. J. High. Educ.* **5** 82–91

[20] Irfan M, Setiana D S, Ningsih E F, Kusumaningtyas W and Widodo S A 2019 Traditional ceremony ki ageng wonolelo as mathematics learning media *J. Phys. Conf. Ser.* **1175** 1–6

[21] Zenal Mutakin T 2013 Jurnal Formatif 3(1): 49-60 Mutakin – Analisis Kesulitan Belajar Kalkulus … ANALISIS KESULITAN BELAJAR KALKULUS 1 MAHASISWA TEKNIK INFORMATIKA *Form. J. Ilm. Pendidik. MIPA* **3** 49–60

[22] Irfan M, Nusantara T, Subanji S and Sisworo 2018 Why Did the Students Make Mistakes in Solving Direct and Inverse Proportion Problem? *Int. J. Insights Math. Teach.* **01** 25–34

[23] Irfan M 2018 Proses Berpikir Siswa yang Mengalami Math-Anxiety dalam Menyelesaikan Masalah Sistem Persamaan Dua Variabel *Kalamatika J. Pendidik. Mat.* **3** 27–38

[24] Irfan M 2017 Analisis Kesalahan Siswa dalam Pemecahan Masalah Berdasarkan Kecemasan Belajar Matematika *Kreano, J. Mat. Kreat.* **8** 143–9