(Anti-)self-dual homogeneous gluon field and axial anomaly in QCD

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Abstract

The transition form factor $F_{\gamma\pi}(Q^2)$, decay width $\Gamma(\pi^0 \rightarrow \gamma\gamma)$ and charge form factor of pion $F_{\pi^0}(Q^2)$ are calculated within the model of induced nonlocal quark currents based on the assumption that the nonperturbative QCD vacuum can be characterized by a homogeneous (anti-)self-dual gluon field. It is shown that the interaction of the quark spin with the vacuum gluon field, being responsible for the chiral symmetry breaking and the spectrum of light mesons, can also play the decisive role in forming the form factor $F_{\gamma\pi}(Q^2)$ and decay width $\Gamma(\pi^0 \rightarrow \gamma\gamma)$. An asymptotic behavior of quark loops in the presence of the background gluon field for large $Q^2$ is discussed.

1 Electromagnetic interactions within the model of induced nonlocal quark currents

The model of induced nonlocal quark currents [1, 2] is based on the assumption that the (anti-)self-dual homogeneous field [3]

$$
\hat{B}_\mu(x) = \hat{n} B_{\mu\nu} x^\nu, \quad \hat{n} = \lambda_3 \cos \xi + \lambda_8 \sin \xi,
$$

$$
\hat{B}_{\mu\nu} = \pm B_{\mu\nu}, \quad B_{\mu\rho} B_{\rho\mu} = -B^2 \delta_{\mu\nu}
$$

(1)

can be considered as a dominating gluon configuration in the QCD vacuum.

Interaction of quarks with electromagnetic field $A_\mu$ can be introduced within the model by means of the minimal substitution (see also [4]) both in the free and interaction parts of quark Lagrangian.

By means of the bosonization procedure [1, 2] the functional integral $Z[A]$ in the presence of electromagnetic field $A_\mu$ can be represented in terms of composite meson fields $\Phi_N$ [5]

$$
Z[A] = N \int \prod_N D\Phi_N \exp \left\{ \frac{1}{2} \int \int d^4 x d^4 y \Phi_N(x) \left[ \left( \square - M_N^2 \right) \delta(x-y) - h_N^2 \Pi_{xy}^N \right] \Phi_N(y) + I_{\text{int}}[\Phi \mid A] \right\},
$$

(2)

$$
I_{\text{int}} = -\int d^4 x h_N \Pi^N(x) \left[ \Gamma_N(x \mid A) - \Gamma_N(x \mid 0) \right]
$$
\[ -\frac{1}{2} \int d^4x_1 \int d^4x_2 h_N h_{N'} \Phi_{N'}(x_1) \left[ \Gamma_{N,N'}(x_1, x_2 | A) - \delta_{N,N'} \Pi_N(x_1 - x_2) \right] \Phi_{N'}(x_2) \]

\[ - \sum_{m=3}^{m} \frac{1}{m} \int d^4x_1... \int d^4x_m \prod_{k=1}^{m} h_N \Phi_{N_k}(x_k) \Gamma_{N_1...N_m}(x_1, ..., x_m | A), \]

\[ \Gamma_{N_1...N_m} = \int d\sigma B \text{Tr} V_{N_1}(x_1 | A) S(x_1, x_2 | A)... V_{N_m}(x_m | A) S(x_m, x_1 | A), \] (3)

where the vertex functionals \( \Gamma_{N_1...N_m} \) include electromagnetic field \( A \) through the covariant derivatives

\[ V^b = m \Gamma^J \left\{ F_{\mu\nu} \left( \frac{g^{\mu\nu}}{\Lambda^2} \right) T^J \left( \frac{1}{i} D^\cdot (x) \right) \right\}, \] (4)

\[ D_{\mu}^\cdot = \xi_f \left[ \nabla^\cdot + ie_f A^\mu(x) \right] - \xi_f' \left[ \nabla^\cdot - ie_f A^\mu(x) \right]. \]

The action in (2) is invariant under \( U(1) \) transformations.

Meson-quark coupling constants are defined by the relations:

\[ h_N^2 = 1/\left( \bar{\Pi} N(p^2) |_{p^2 = -M_N^2} \right). \] (5)

To get various meson-photon amplitudes one has to decompose the vertex functionals \( \Gamma_{N_1...N_m} \) into a series over the electromagnetic field \( A \).

Fourier transform of the translation invariant part \( H_f \) of the quark propagator in the homogeneous (anti-)self-dual gluon field reads [2, 3]

\[ \tilde{H}_f(p) = \frac{1}{2\nu \Lambda^2} \int_0^1 ds e^{-\frac{s^2}{2\Lambda^2}} \left( \frac{1 - s}{1 + s} \right)^{m_f^2} [p_\alpha \gamma_\alpha \pm is \gamma_5 \gamma_\alpha f_{\alpha\beta} p_\beta \]

\[ + m_f \left( P_\pm + P_\mp + 1 - s^2 - \frac{i}{2} \gamma_\alpha f_{\alpha\beta} \gamma_\beta \frac{s}{1 - s^2} \right). \] (6)

Contribution of zero modes to the propagator is seen in Eq. (3) as a singularity of the integrand at \( s = 1 \) which is integrable unless \( m_f = 0 \). Zero modes are due to interaction of a quark spin with the vacuum field.

### 2 Decay \( \pi^0 \rightarrow \gamma\gamma \) and \( \gamma^*\pi^0 \rightarrow \gamma \) transition form factor

The contribution of the triangle diagram to the form factor \( F_{\gamma\pi}(Q^2) \) can be represented as an integral over proper times \( t \) and \( s_1, s_2, s_3 \), corresponding to the vertex and propagators, respectively [3]:

\[ F_{\gamma\pi}(Q^2) = \frac{1}{\Lambda^2 2\pi^2} \text{Tr}_v \int_0^1 dt d\sigma_1 d\sigma_2 d\sigma_3 \left[ \left( \frac{1 - s_1}{1 + s_1} \right) \left( \frac{1 - s_2}{1 + s_2} \right) \left( \frac{1 - s_3}{1 + s_3} \right) \right]^{\frac{m^2}{2\nu}} \times \]

\[ \sum_{i=1,2,3} \frac{m_u}{1 - s_i} \Phi_i(M_i^2, Q^2; s, t) \exp \left[ M_i^2 \varphi(s, t) - Q^2 \varphi(s, t) \right]. \] (7)
The singularities \((1-s_i)^{-1}\) of the integrand in Eq. (7) at \(s_i \to 1\) appear from the zero mode contribution to the quark propagator (see the second line in Eq. (6)). These singularities lead to the \(1/m_u\)-dependence of the integral in Eq. (7) in the limit \(m_u \ll \Lambda\), while the effective coupling constant \(h_\pi\) in Eq. (5) behaves as \(h_\pi \sim m_u\), and \(F_{\gamma\pi}(Q^2)\) does not vanish as \(m_u \to 0\) but approaches a constant value. The same is valid for decay width \(\Gamma(\pi^0 \to \gamma\gamma)\). This asymptotic analysis illustrates an appearance of axial anomaly as a result of interaction of a quark spin with the vacuum field.

Numerical results for the form factor and decay width are represented in Fig. a and Table 1. The model parameters are fitted from the meson masses: \(m_u = m_d = 198.3\) MeV, \(\Lambda = 319.5\) MeV that gives \(M_\pi = 140\) MeV, \(h_\pi = 6.51\). The radius for \(\gamma^*\pi^0 \to \gamma\) transition is equal to \(0.57\) fm, that have to be compared with \(r_{\gamma\pi}^{\exp} = 0.65 \pm 0.03\) fm.

Within the model of induced nonlocal currents the \(\gamma^*\pi^0 \to \gamma\) transition form factor and two photon decay constant, a smallness of pion mass, splitting of the masses of vector and pseudoscalar light mesons and weak decay constants of pions and kaons are explained by the same reason – an interaction of a quark spin with the vacuum homogeneous gluon field.

A behaviour of the triangle diagram for the transition form factor in the limit \(Q^2 \gg \Lambda^2\) can be easily estimated, using the Laplace method. As a result, we arrive at relation

\[
Q^2 F_{\gamma\pi}(Q^2/\Lambda^2) = C\Lambda + O(\Lambda^2/Q^2) \approx 0.2 \text{ Gev} + O(\Lambda^2/Q^2),
\]

that is consistent with the analysis within factorization hypothesis and QCD sum rule approaches. This result has to be compared with the Brodsky-Lepage limit \(\lim_{Q^2} Q^2 F_{\gamma\pi}(Q^2) \to 2F_\pi = 0.186\) Gev.

### 3 The pion charge form factor

According to the effective action (3), the one-loop amplitude for the processes \(\pi^+\gamma^* \to \pi^\pm\) is described by the triangle and bubble diagrams. The analytical expressions for the contributions of these diagrams to the pion charge form factor can be found in [5].

For the parameter values given in [2] the charge form factor is plotted in Fig. b by the solid line. The electromagnetic radius takes the value \(r_\pi = 0.524\) fm that is in agreement with experimental data \(r_\pi^{\exp} = 0.656\) fm. Numerically the contribution of the bubble diagrams is negligible at small and intermediate \(Q^2\), but is the leading contribution for the asymptotically large \(Q^2\).

In our case, a naive estimation of the asymptotic behaviour of the triangle diagram based on the ultraviolet behaviour of the quark propagator (6) and vertex (4) gives \((Q^2)^{-2}\).
The asymptotic formula for the triangle diagram in the limit \( Q^2 \gg \Lambda^2 \) for the parameter values from [2] reads

\[
F_\pi^\Delta(Q^2/\Lambda^2) = \frac{2.96}{(Q^2/\Lambda^2)^{1.1435}}.
\]  

This is clearly due to the specific interplay of translation and color gauge invariance in the quark loops in the presence of vacuum homogeneous gluon field. Asymptotic formula (9) fits well the solid curve in Fig. b for \( Q^2 > 5 \text{Gev}^2 \).

The contribution of bubble graph to asymptotic behaviour takes the form

\[
F_\pi^\circ(Q^2/\Lambda^2) \approx 0.3\Lambda^2/Q^2 + O\left((\Lambda^2/Q^2)^2\right).
\]

The asymptotic behaviour of the pion form factor is determined by the one-gluon exchange between quarks inside a pion. This is consistent with the mechanism of hard rescattering and quark counting rules [10]. However, in the experimentally observed region the triangle diagram dominates in the form factor.

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Table 1: The two-photon decay constant $g_{\pi\gamma\gamma}$ (Gev$^{-1}$) and decay width $\Gamma(\pi^0 \rightarrow \gamma\gamma)$ (ev); $g_{\pi\gamma\gamma}^*, \Gamma^*$ are the values calculated without taking into account the spin-field interaction

| $g_{\pi\gamma\gamma}$ | $g_{\pi\gamma\gamma}^*$ | $g_{\pi\gamma\gamma}^{exp}$ | $\Gamma$ | $\Gamma^*$ | $\Gamma^{exp}$ |
|-----------------------|------------------------|----------------------------|---------|-----------|-------------|
| 0.235                 | 0.108                  | 0.276                      | 6.3     | 1.34      | 8.74        |

Transition pion form factor (a). Solid curve represents a result of the model of induced nonlocal quark currents. Long dashed line - calculation without taking into account the spin-field interaction. Experimental fit is given by short dashed line, and Brodsky-Lepage limit ($\approx 1.186$ Gev) is shown by solid straight line.

The pion charge form factor (b) calculated in the present model (solid line) compared with experimental fit (dashed line)