Residual uncertainty in processed line-of-sight returns from nacelle-mounted lidar due to spectral artifacts

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Abstract. An uncertainty quantification technique for nacelle-mounted lidar is developed that extends conventional error analyses to precisely account for residual uncertainty due to observed non-ideal features in processed Doppler lidar spectra. The technique is applied after quality assurance/quality control (QAQC) processing to quantify residual error, both bias and random, from solid-body interference, shot noise, and any additional uncertainty introduced to the data from the QAQC process itself. The approach follows from the one-time construction of a high-dimensional parametric database of synthetic lidar spectra and subsequent processing with an existing QAQC technique. A model of the correspondence between the spectral shape and the associated residual errors due to non-ideal features is then developed for quantities of interest (QOIs) including the geometric median and spectral standard deviation of line-of-sight velocity. The model is preliminarily implemented within a neural network framework that is then applied in post-processing to sample returns from a DTU SpinnerLidar. The initial analysis uncovers the effects of specific sources of uncertainty in the context of both individual spectra and full-field maps of the measurement domain. The technique is described in terms of application to continuous wave (CW) lidar, though it is also relevant to pulsed lidar.

Keywords: Doppler lidar, nacelle-mounted, uncertainty quantification, solid interference, shot noise, line-of-sight velocity, machine learning, neural network, wind energy

1. Introduction
Nacelle-mounted Doppler lidar instruments have made inroads in the field of wind energy with applications in site selection, power curve testing, monitoring and control, and verification/validation. Errors in lidar measurements stem both from errors in the lidar line-of-sight velocity, $v_{los}$, readings themselves and from inaccuracies in modeling approaches for reconstruction of the velocity vector [1]. This work focuses on quantification of the first, more fundamental source of lidar uncertainty that is present in all lidar measurements regardless of any flow reconstruction approach that is later applied to the data.

Beside the well-documented measurement difficulties associated with inhomogeneities within the measurement volume [2-6] (i.e. – turbulence, mean gradients, non-uniform backscatter), we find several error sources embedded in the lidar signal that can in some cases be minimized. For example, bias is introduced by interference from solid surfaces such as the ground terrain and boresight, and the severity of the interference cannot be determined a priori. Amplitude noise in the spectrum is another inherent error source and introduces a loss of precision in the velocity readings. The intensity of the noise, which
for modern lidar is due primarily to shot noise [7], depends on the range-resolved intensity of the backscatter [8]. Therefore, appropriate shot-noise error analyses should account for the unique noise content observed in each lidar return, which again cannot be determined \textit{a priori}.

The first line of attack toward reducing errors from solid interference and amplitude noise is QAQC processing. Previous work to mitigate the effect of ground interference bias was undertaken for the case of airborne pulsed lidar [9], though the approach was admittedly subject to a large degree of subjectivity in defining certain thresholds and was also unable to handle wind speeds near the interference velocity. Herges and Keyantuo [10] developed another technique that truncates the solid interference from the spectra and furthermore smooths the amplitude fluctuations, followed by estimation of QOIs. While such QAQC processing is necessary, it is not sufficient since artifacts of the unwanted features can remain in the spectra after processing as will be shown later. Both the type of QAQC algorithm [9] and subsequent mean frequency estimation [11, 12] then have bearing on the accuracy of the final QOIs.

We therefore seek an approach for estimating the end-to-end uncertainty due to solid interference and amplitude noise starting from the raw signal and finishing after the QAQC processing. The objective is to correct the bias errors and assign confidence intervals based on the random errors. Note that for the latter, lidar instruments often output only the averaged spectrum for each measurement position rather than the individual fast-sampled spectra, so some conventional techniques of estimating random uncertainty cannot be applied.

A useful template for estimating the above bias and random uncertainties can be found in the particle image velocimetry (PIV) community where so-called \textit{synthetic PIV} is used to simulate features in synthetic signals. These signals can then be run through processing algorithms to develop uncertainty estimates associated with the processing technique [13]. Importantly, synthetic PIV accounts for spatial variations in uncertainty, thus producing a full-field uncertainty map. In the lidar community, the core functionality of lidar simulators [14] offers a similar potential to that of the synthetic PIV approach.

The objective of this article is to introduce a technique to calculate \textit{a posteriori} estimates of the uncertainty remaining after QAQC processing by objectively accounting for the non-ideal spectral features including solid interference and amplitude noise. In the remainder of the article, the methodology for developing a synthetic lidar signal and associated error model is first outlined in Section 2, followed by an overview of the demonstration experiment in Section 3, example analysis of the uncertainty of measured returns in Section 4, and concluding remarks in Section 5.

2. Methodology

The approach developed here leverages the richness of information encoded in each individual lidar return via the Doppler power spectral density (PSD). Specifically, the analysis is accomplished by quantifying critical features of individual lidar spectra to infer the

![Figure 1](image-url). Example spectrum of a measured (raw) lidar return illustrating the contamination of the region of interest (ROI) by solid interference and amplitude noise. The raw geometric median contains bias error due to the solid interference and random error due to the amplitude noise. The error band represents the range of true ROI medians which are possible given the measured median.
The source of truth during this process is synthetic spectra with known statistics.

The features of note in the spectra are the region of interest (ROI) whose width stems primarily from the distribution of flow-field velocities, region of interference from solid-body returns that are often from the surrounding terrain, and amplitude noise. An example spectrum is shown in figure 1.

To understand the relationships between these features and their impacts on the QOIs, this section describes the creation of an idealized PSD for synthetic processing, parameterization of the processed spectra, and subsequent training of an error model to apply to measured returns. Note that the PSDs described below are functions of \( v_{\text{los}} \) rather than the Doppler frequency shift, \( f \), and the two are related

\[
v_{\text{los}} = \lambda f / 2 ,
\]

where \( \lambda \) is the wavelength of the laser.

### 2.1. Generation of Synthetic Spectra

The synthetic PSD of the ROI, \( s_{\text{ROI}} \), is generated as a function of \( v_{\text{los}} \) from a scaled epsilon-skew-normal distribution \([15]\) as in equation (1)

\[
s_{\text{ROI}} = \frac{m_{0,\text{ROI}}}{m_{2,\text{ROI}}^{1/2} \sqrt{2\pi}} e^{-\frac{1}{2} \left( \frac{v_{\text{los}} - m_{3,\text{ROI}} / \sqrt{m_{2,\text{ROI}}} \right)^{2}} ,
\]

where \( m_{0,\text{ROI}} \) is a magnitude parameter, \( m_{3,\text{ROI}} \) is a location parameter, \( m_{2,\text{ROI}} \) is a width parameter, and \( m_{3,\text{ROI}} \) is a skew parameter whose absolute value is less than one, and the \( \mp \) takes the sign opposite of the numerator of the exponent. Note that the four \( m \) parameters are similar in nature but not identical to the conventional zeroth, first, second-central, and third-central moments which are commonly referred to as area, centroid, spectral standard deviation, and spectral skewness. For the differences, see \([15]\).

The PSD of the solid interference, \( s_{\text{solid}} \), is generated as a function of \( v_{\text{los}} \) based on an inverse function according to equation (2)

\[
s_{\text{solid}} = \frac{p_{\text{solid}}}{1 + (v_{\text{los}} - v_{\text{solid}})/w_{\text{solid}}} ,
\]

where \( p_{\text{solid}} \) is the prominence of the solid interference, \( v_{\text{solid}} \) is the velocity at \( p_{\text{solid}} \) (which, in the case of CW lidar, may be the minimum \( v_{\text{los}} \) that can be sensed by the lidar), and \( w_{\text{solid}} \) is the full-width half-maximum of the interference spectrum. We have initially found when examining measured PSDs that \( w_{\text{solid}} \) does not show as much variability as \( p_{\text{solid}} \) between returns, so it could suffice as a simplification to vary only \( p_{\text{solid}} \) when generating the synthetic spectra.

Modeling of the PSD must also include amplitude noise that adheres to the probability density function of measured noise content. The statistics of the noise within each PSD bin follow a scaled chi-square distribution \([16, 17]\). By the central limit theorem, the chi-square distribution asymptotes to a Gaussian distribution for sufficiently large sample sizes such as for the hundreds of individual spectra which are averaged in typical lidar measurements. In such cases, randomized instances of the time-averaged noise spectrum, \( s_{\text{noise}} \), can be generated given a variance within each spectral bin. The variance is here taken to be uniform over the spectrum.

The combined synthetic PSD, \( s \), is constructed in equation (3),

\[
s = s_{\text{ROI}} + s_{\text{solid}} + s_{\text{noise}} .
\]

Collectively, there are at least six parameters implicit to equation (3) including four from equation (1), one from equation (2), and one from the noise contribution. A full-factorial sweep across a range of values that have been observed in measurements for each parameter results in a database of lidar spectra. For each combination of parameters, hundreds of spectra with different instances of noise are generated to ensure statistical convergence of the random uncertainty estimates to be made.
The $s$ curves are then each processed with the QAQC code developed by Herges and Keyantuo [10], which includes a filter for removal of solid interference and bilateral smoothing to reduce amplitude noise. The outputs of this process are two QOIs: geometric median line-of-sight velocity, $v_m$, (chosen over other mean frequency estimators due to the results of Held [18]) and spectral standard deviation of line-of-sight velocity, $v_σ$. For both QOIs, we now have a database of the correspondence between the shape of the input spectra and the output QOI, as well as any deviation in the output QOI from its original input value.

With the ultimate goal of training a model to predict the residual error in the QOIs, predictors of the model must first be calculated that will depend on the shape of the spectra. Whereas the spectral shape parameters were specified during the generation of synthetic spectra above, the spectral shape returned from the QAQC code is not known precisely and must be parameterized as described below.

### 2.2. Parameterization of Processed Spectra

Moments of the spectra and noise are employed as a robust means of parameterizing the PSDs instead of relying on a high-dimensional curve-fitting procedure over the large number of unknowns in equation (3). In truth, the number of unknowns in equation (3) is not the governing factor for how many unknowns are required to fully describe the processed spectra because the QAQC processing will likely alter the spectra from its original, idealized shape. Increasing the dimensionality of the parameterization to account for such alterations is tempting, but including too many dimensions in the parameterization could exaggerate the effect of amplitude noise. As will be discussed below, the number of parameters required to faithfully describe the shape of the ROI, solid interference, and amplitude noise, respectively, was found to be four, one, and one, as also in the initial parameterization. The parameters related to the spectra in this section are denoted with capital letters to avoid any confusion with those of the original synthetic spectra above.

The conventional zeroth, first, second-central, and third-central moments are given in discrete form in table 1 and can be calculated for both $S_{ROI}$ and $S_{solid}$. To describe the shape of a given ROI, the values $M_{0_{ROI}}, M_{1_{ROI}}, M_{2_{ROI}},$ and $M_{3_{ROI}}$ are computed where the prime symbol indicates that $S_{ROI}$ has already been QAQC-processed.

| Moment Type          | Form                                  |
|----------------------|---------------------------------------|
| 0th moment (area)    | $M_{0_{(c)}} = \sum (S_{(c)} - \overline{S_{noise}}) \Delta v_{los}$ |
| 1st moment (centroid)| $M_{1_{(c)}} = \sum (S_{(c)} - \overline{S_{noise}}) v_{los} \Delta v_{los} / M_{0}$ |
| 2nd-central mo.      | $M_{2_{(c)}} = \sum (S_{(c)} - \overline{S_{noise}})^2 v_{los} \Delta v_{los} / M_{0}$ |
| 3rd-central mo.      | $M_{3_{(c)}} = \sum (S_{(c)} - \overline{S_{noise}})^3 v_{los} / M_{0} / M_{2}^{1.5}$ |

To describe the shape of a given solid-interference spectrum, the extent of the interference as determined from the location of the first trough in the raw spectra (i.e. – the first intersection of the green and red lines in figure 1) is input to a curve-fitting algorithm for $P_{solid}$ of equation (2). For consistency with the approach of parameterizing the ROI above, the value used for the final parameterization of the solid interference region is $M_{0_{solid}}$ where $S_{solid}$ is the curve-fit spectrum.

To describe the magnitude of the amplitude noise, the parameterization approach is to estimate the variance, $s_{noise}^2$, within each bin of the noise spectrum, $S_{noise}$, from the fluctuations in the PSD using the last 100 spectral bins similar to [14]. These last 100 bins are in the tail of the spectrum sufficiently
away from the ROI (beyond the right edge of figure 1). Since the calculation must be made for each measured PSD, we are able to gather enough samples to estimate $\sigma^2_{\text{noise}}$ within each bin by assuming there is no correlation between spectral bins so that each of the final 100 bins represents an independent observation of spectrally uniform noise.

2.3. Model Training

The six parameters described in the previous section are the predictors of the uncertainty model. As conventional curve fitting becomes computationally difficult for higher-dimensional problems, we employ a simple supervised machine learning regression via neural networks. The network architecture is two layers of perceptrons with 12 nodes each. Each node features a sigmoid symmetric transfer function, and model learning is based on mean-square error evaluation and backpropagation using Levenberg-Marquardt.

Once the one-time training of the model is complete, processing of measured lidar data proceeds as depicted in figure 2. The model includes both bias and random components of the residual error for the two QOIs, which results in a set of four outputs: residual bias in $v_m$ and $v_\sigma$ are given as $\hat{\epsilon}_m$ and $\hat{\epsilon}_\sigma$, respectively, and residual random uncertainty in $v_m$ and $v_\sigma$ are given as $\hat{\epsilon}_m$ and $\hat{\epsilon}_\sigma$, respectively, and reported at 95% confidence. Further models are generated, as shown, for the five unknown parameters of equations (1) and (2), which then allows visualization of corrected spectra.

3. Experimental Setup

3.1. Facility

A test case for the uncertainty analysis procedure is derived from data at the Scaled Wind Farm Technology (SWiFT) facility in Lubbock, Texas, USA as illustrated in figure 3. The site features level
terrain and minimal surface roughness, and characterization of the atmospheric conditions is given in [19] with recent benchmarking activities given in [20]. Each of the three V27 wind turbine rotors on the site are 27 m in diameter and stand 32 m off the ground. The case considered below was taken in the morning of July 5, 2017 during a mean hub-height wind velocity of 6 m/s in an unstable atmospheric boundary layer (ABL).

3.2. Lidar
A single scan from the DTU SpinnerLidar [21] mounted on WTGa1 will be considered below. The scan was focused 105 m from WTGa1 along the axis of the turbine rotor. A rosette pattern is completed in 2 s and consists of 984 measurement locations, some of which are eliminated from the measurement domain when the focus distance falls below the surface of the ground. Within each 2 s scan, the lidar samples at 100 MHz, and power spectra are calculated from sequences of 512 samples to yield 256 FFT bins so that the returned power spectrum for each measurement location is the average of 400 consecutive spectra.

4. Results
This section describes the preliminary results of the uncertainty quantification technique on a simplified parametric space.

4.1. Development of Uncertainty Model
The training cases for the uncertainty model can be generated from parametric sweeps through a gridded six-dimensional space as noted previously. Initial experimentation revealed that coupling between $\sigma^2_{\text{noise}}$ and some other parameters was negligible. Therefore, to reduce the dimensionality of the neural networks, $\sigma^2_{\text{noise}}$ was taken as uncoupled from $M_{\text{solid}}$ and furthermore as having no effect on the bias component of error. Likewise, $M_{\text{solid}}$ was taken as having no effect on the random component of error. The result is that the neural networks for $\bar{\varepsilon}_m$ and $\bar{\varepsilon}_\sigma$ are functions of $M_{\text{ROI}}, M_{\text{ROI}}', M_{\text{ROI}}, M_{\text{ROI}}', M_{\text{ROI}}, M_{\text{ROI}}', M_{\text{ROI}}^3$, and $M_{\text{ROI}}^3$ while those for $\hat{\varepsilon}_m$ and $\hat{\varepsilon}_\sigma$ are functions of $M_{\text{ROI}}, M_{\text{ROI}}', M_{\text{ROI}}, M_{\text{ROI}}', M_{\text{ROI}}, M_{\text{ROI}}', M_{\text{ROI}}^3$, and $\sigma^2_{\text{noise}}$. This simplification allowed the original six-dimensional space to be reduced to two five-dimensional subspaces, which lessens the difficulty of the regression problem. After filtering some cases near the extreme bounds of the parametric space, the two subspaces were sized 800 and 2600 for the bias and random error components, respectively.

A randomized set of some 150-250 test cases within the limits of the parametric space was also generated to evaluate the trained model. The root-mean-square (rms) error of the predicted versus actual values for the four metrics are given in table 2.

| Metric | rms error (m/s) |
|--------|-----------------|
| $\bar{\varepsilon}_m$ | 0.0129 |
| $\bar{\varepsilon}_\sigma$ | 0.0245 |
| $\hat{\varepsilon}_m$ | 0.00846 |
| $\hat{\varepsilon}_\sigma$ | 0.00762 |

4.2. Demonstration of Uncertainty Quantification
This section presents the first implementation of the uncertainty quantification technique as applied to an actual lidar scan. First, several example spectra will be examined followed by analysis of full-field maps of the residual uncertainty.

4.2.1. Example Spectra. The rosette scan pattern of the lidar is pictured in figure 4(a). Out of the 984 scan locations in figure 4(a), three are displayed in figures 4(b) - (d) as examples that contain

![Figure 3. Illustration of the SWiFT facility in Lubbock, Texas, USA. The nacelle of WTGa1 was outfitted with a rear-mounted DTU SpinnerLidar which scanned the turbine wake at different downstream positions. Image from [20].](image-url)
several recurring features. For each of the examples, the raw and QAQC-processed spectra are compared, and the reconstructions of the ROI and solid interference are shown for visual reference.

Figure 4(b) represents a spectrum with relatively simple analysis. $\bar{\varepsilon}_m$ is negligibly small, and $\sigma_\varepsilon$ indicates that the processed spectrum underestimates $v_\sigma$ by 0.024 m/s (or 0.072 m/s for $3\sigma_\varepsilon$, as plotted), or 8.3%. This latter effect may be a result of the tails of the ROI, which are low enough in magnitude to potentially be misinterpreted as part of the noise signature by the QAQC processing. The random error components are relatively small at $\pm 0.023$ m/s ($\pm 0.42\%$ of $v_m$) and $\pm 0.034$ m/s ($\pm 12\%$ of $v_\sigma$) for $\bar{\varepsilon}_m$ and $\sigma_\varepsilon$, respectively, at 95% confidence. Note that the magnitudes of these errors are on the same order as the uncertainties in the error model from table 2, so the preliminary implementation of the model training techniques presented in this paper stands to improve in future efforts.

Figure 4(c) represents a spectrum with a low prominence. The lower prominence is associated with a higher relative magnitude of $\sigma_{\text{noise}}^2$, which has two apparent effects. Following the reasoning described above, one effect is to exaggerate the underprediction of $v_\sigma$ by the QAQC processing because the tails

![Figure 4](image)

**Figure 4.** (a) Rosette scan pattern of the 984 measurement locations overlaid on an interpolated field of line-of-sight velocity, $v_{\text{los}}$. The normalized spectra, $S/S_{\text{max}}$, for the three locations annotated by number are shown in (b) - (d) which plot the residual bias and random error components of the geometric median and spectral width, the latter of which is taken as three times the spectral standard deviation. The error bands represent the range of true QOIs which are possible given the measured QOIs, and the confidence interval is 95%. The values indicated for $3\bar{\varepsilon}_\sigma$ are the mean of the errors on either side of the ROI which differ when $\bar{\varepsilon}_m$ is non-zero. The reported $M_0$ and $\sigma_{\text{noise}}^2$ values have been normalized on the maximum of $S$ in accordance with the ordinate.
of the ROI become lower relative to the amplitude noise. The $v_\sigma$ is thus underpredicted by 0.064 m/s, a 170\% larger bias than for the spectrum in figure 4(b) with higher prominence. Another effect is to increase the values of $\bar{\varepsilon}_m$ and $\bar{\varepsilon}_\sigma$ which are $\pm 0.11$ m/s (\pm 2.2\% of $v_m$) and $\pm 0.11$ m/s (\pm 3.8\% of $v_\sigma$), respectively.

Figure 4(d) represents a spectrum where a small amount of residual distortion from a partial solid return remains in the spectra after QAQC processing. This effect is visible as the thin stretch of heightened level around $v_{los} = 3$ m/s. In contrast to the spectra in figures 4(b) and (c), $\bar{\varepsilon}_m$ here is significant at -0.079 m/s, or 1.8\% of $v_m$. The underprediction of $v_m$ appears to be common when the QAQC processing is not able to completely segregate and remove the solid interference from the ROI. This residual interreference is also responsible for the reversed direction of the $\bar{\varepsilon}_\sigma$ bias from the previous cases.

4.2.2. Full-Field Error Maps. Figure 5 presents the full-field maps of residual bias error for a single lidar scan. The area of rejected returns below $z/D \approx 0$ is due to the probe volume intersecting the ground. Immediately above the region of rejected solid returns, the area of partial solid returns is found with $\bar{\varepsilon}_m$ biases as negative as -0.62 m/s, or 13\% of the local $v_m$, in figure 5(a) and with positive values for $\bar{\varepsilon}_\sigma$ in figure 5(b). Over the remainder of the scan field, biases are typically negligible except for those returns contaminated by the boresight or nearby towers and for those near the wake region. For the former, $\bar{\varepsilon}_m$ as negative as -0.35 m/s, or 7.1\% of the local $v_m$ are observed. For the latter, $\bar{\varepsilon}_\sigma$ as negative as -0.18 m/s, or 6.6\% of the local $v_\sigma$, are observed. Some unexpected scatter remains throughout the field, which suggests that the training of the error model could be refined.

![Figure 5](image)

**Figure 5.** Full-field map of residual bias error in (a) geometric median line-of-sight velocity, $\bar{\varepsilon}_m$, and (b) spectral standard deviation of line-of-sight velocity, $\bar{\varepsilon}_\sigma$. Symbols: ● (clean return), ■ (partial solid return), × (rejected return)

Figure 6 presents the full-field maps of residual random uncertainty at 95\% confidence for the lidar scan. Note that the foreground color-scale is logarithmically scaled to highlight small differences across the scan field. As with the bias errors above, the random uncertainties are influenced by partial solid returns. The more significant influence, however, is proximity to the wake region. The driving factor behind this effect may be the flattening of the spectra in this region due to the presence of a broader range of flow scales. As discussed in figure 4(c), spectral signatures with lower prominence make
estimation of QOIs more susceptible to noise. Within the wake region, the metrics $\dot{\varepsilon}_m$ and $\dot{\varepsilon}_\sigma$ reach maxima of $\pm 0.11$ m/s ($\pm 2.5\%$ of $\bar{v}_m$) and $\pm 0.12$ m/s ($\pm 13\%$ of $\bar{v}_\sigma$), respectively, so the peak residual uncertainties of the random component for this case are smaller than those of the bias component.

5. Conclusions and Future Work
The uncertainty quantification technique described above extends conventional lidar error analyses to allow estimation of measurement errors, both bias and random, due to non-ideal features remaining in the spectra after QAQC processing. The technique was described in the context of wind turbine nacelle-mounted, CW lidar. The approach featured the construction of a six-dimensional parametric database of synthetic lidar spectra. Each case was processed with an existing QAQC technique, and a model of the correspondence between the spectral shape and the associated measurement errors in terms of the geometric median and spectral standard deviation of line-of-sight velocity was trained via a backpropagating neural network. Using sample measured lidar returns, the preliminary error model was used to explore specific sources of uncertainty in the context of individual spectra and to demonstrate the full-field prediction of uncertainties due to non-ideal spectral features. Notably, the proposed technique accounts for residual uncertainty due to solid interference and amplitude noise that remains after the QAQC processing. Given sufficient training data, the technique also implicitly handles other sources of uncertainty, such as truncation of the spectra resulting from solid interference filtering and any small biases introduced by the QAQC code, that are inherent in the estimation of QOIs. For the examined dataset, residual bias errors due to solid interference were found to produce error magnitudes up to 0.62 m/s, or 13% of the local $v_m$, for $\dot{\varepsilon}_m$, though these were located away from the wake region. For measurements in the wake region itself, the more significant bias source was $\dot{\varepsilon}_\sigma$, the magnitude of which was as large as 0.18 m/s, or 6.6% of the local $v_\sigma$. In comparison, residual random uncertainties within the wake region due to amplitude noise were only $\pm 0.11$ m/s ($\pm 2.5\%$ of $\bar{v}_m$) and $\pm 0.12$ m/s ($\pm 13\%$ of $\bar{v}_\sigma$) at 95% confidence for $\dot{\varepsilon}_m$ and $\dot{\varepsilon}_\sigma$, respectively. Future work should include expansion of the parametric database to include a wider range of realistic spectral return shapes.

Figure 6. Full-field map of residual random uncertainty in (a) geometric median line-of-sight velocity, $\dot{\varepsilon}_m$, and (b) spectral standard deviation of line-of-sight velocity, $\dot{\varepsilon}_\sigma$. Random uncertainty bound given at 95% confidence. Symbols: ● (clean return), □ (partial solid return), × (rejected return).
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