Unitary matrix with a Penner-like potential

also yields $N_f = 2$

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Abstract

It has been known for some time that a hermitian matrix model with a Penner-like potential yields as its large-$N$ free energy the prepotential of $\mathcal{N} = 2 \ N_f = 2 \ SU(2)$ SUSY gauge theory. We give a rigorous proof that a unitary matrix model with the identical potential also yields the same prepotential, although the parameter identifications are slightly different. This result has been anticipated by Itoyama et. al.
When the Dotsenko-Fateev integral representation of correlation functions of two-dimensional CFT was derived 35 years ago [1], and also when the double-scaling limit of the one-matrix model was discovered 30 years ago [2–4], no one would have imagined that they would be related to any four-dimensional theory. However, in the early 2000’s the relationship between 4d SUSY gauge theories and matrix models was revealed [5–7], later the AGT relation between 4d SUSY gauge theories and 2d CFT was found [8], and finally it turned out that their correspondence was elegantly formulated in terms of matrix models [9–15].

Once having recognized the connection between 4d $\mathcal{N} = 2$ SUSY gauge theories and matrix models, the origin of the mysterious appearance of the Painlevé equations [2–4] in the double-scaling limit of the latter is now seen to be natural; the total space of the Seiberg-Witten curve (including the base “$u$-plane” (= an affine patch of $\mathbb{P}^1$) of the elliptic fibration) of an $\mathcal{N} = 2$ $SU(2)$ SUSY gauge theory (as well as an $E$-string) can be identified as a rational elliptic surface [16–22], and it is these particular algebraic varieties that the Painlevé equations were shown to be associated with in 2001 [23]. The latter was a geometric manifestation of the idea of constructing discrete Painlevé equations as translations of affine Weyl groups [24]. Recently, there has been an interest in the Painlevé equations in SUSY gauge theories in terms of irregular conformal blocks and double-scaled matrix models. Recent works in this direction include [25–37].

In [36], Itoyama, Oota and Yano claimed, among other things, that a unitary one-matrix model with a logarithmic potential term yields the instanton partition function [38, 39] of the $\mathcal{N} = 2 N_f = 2$ $SU(2)$ SUSY gauge theory. Prior to this, it had been explicitly confirmed by Eguchi and Maruyoshi [40] that a hermitian one-matrix model with the same potential reproduces the instanton partition function of the above same gauge theory as its large-$N$ free energy. This is quite puzzling because, even if they have the same form of the potential, they are a priori different matrix models with different Boltzmann weights defined by different integration contours of the eigenvalues. That is, in hermitian matrix models, the contour is taken to be the real axis, whereas in unitary matrix models the eigenvalue integration is performed along the unit circle around the origin. Also, one cannot expect to be able to change the contour as one does in a residue computation of a holomorphic integral.
since the integrand function is not holomorphic, and even has a logarithmic singularity on the real axis in the hermitian case.

In this paper, we examine, by an explicit calculation, whether Itoyama et.al.’s unitary matrix model with a logarithmic potential can yield the prepotential of the $N_f = 2$ theory. Our strategy is as follows: we first map Itoyama et.al.’s unitary matrix model to an equivalent hermitian matrix model giving the identical partition function by using the unitary/hermitian duality in matrix models [41, 42]. We then compute its two-cut large-$N$ free energy following the standard techniques and derive its matrix model curve. We examine whether we can find the $u$-parameter appropriately so that the $u$ derivative of the matrix model differential ($y(z)dz$ in the text) becomes proportional to the holomorphic differential. If we can find one, then the differential is identified (up to a constant of proportionality) as the Seiberg-Witten differential whose Seiberg-Witten curve is the genus-one Riemann surface associated with the holomorphic differential above. The “special geometry relation” [7, 41, 43, 44] then automatically ensures that the matrix-model free energy coincides with the prepotential of the gauge theory (up to the term linear in the Coulomb modulus $a$).

Surprisingly, we will see that the unitary matrix model of Itoyama et. al. yields, through these procedures, precisely the same Seiberg-Witten curve and Seiberg-Witten differential as those obtained in the hermitian matrix model having the potential of the same form analyzed in [40]! Thus this implies that the two different - hermitian and unitary - matrix models with the same logarithmic potential computes, as their large-$N$ free energy, the instanton partition function of an identical gauge theory, the $\mathcal{N} = 2$ $N_f = 2$ $SU(2)$ theory.

This paper is organized as follows. In section 2, we revisit the Penner-like hermitian matrix model studied in [40], where we solve it by using the standard conventional technique for solving hermitian matrix models. We reproduce the results obtained in [40], such as the conditions for the positions of the end points of the cuts, the Seiberg-Witten curve and the $N_f = 2$ prepotential as its large-$N$ free energy. We then turn to the unitary matrix model with the same potential in section 3. We use the unitary/hermitian matrix model duality to convert the unitary matrix model to the equivalent hermitian matrix model. Then we solve it similarly to find that it also describes $N_f = 2$. The conclusions are summarized in Section 4.
II. HERMITIAN MATRIX MODEL WITH A PENNER-LIKE POTENTIAL RE- 
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Our convention for the matrix model partition function is

\[ Z = \frac{1}{\text{Vol}(U(N))} \int d\Phi \exp \left( -\frac{N}{\mu} \text{tr} W(\Phi) \right), \tag{1} \]

where \( \mu = g_s N \) is the t’Hooft coupling. The potential is

\[ W(z) = -\mu_3 \log z + \frac{\Lambda}{2} \left( z + \frac{1}{z} \right), \tag{2} \]

where the overall sign is flipped compared to the definition in [40]. Following the standard technique, the resolvent \( \omega(z) \) for a 2-cut solution is given by

\[ \omega(z) = \frac{\sqrt{\prod_{k=1}^{4} (z - a_k)}}{4\pi i \mu} \sum_{j=1,2} \oint_{A_j} dw \frac{W(w)}{(z - w)\sqrt{\prod_{k=1}^{4} (w - a_k)}}, \tag{3} \]

where \( A_j \ (j = 1, 2) \) are the cuts on which the eigenvalues are distributed. The definition for the resolvent is

\[ \omega(z) \equiv \frac{1}{N} \left\langle \text{tr} \frac{1}{z - \Phi} \right\rangle. \tag{4} \]

The positions of the endpoints of the cuts are constrained, as usual, by the asymptotic behavior of the resolvent

\[ \omega(z) \sim \frac{1}{z} + O \left( \frac{1}{z^2} \right). \tag{5} \]

Let

\[ \prod_{k=1}^{4} (z - a_k) \equiv z^4 + b_1 z^3 + b_2 z^2 + b_3 z + b_4, \tag{6} \]

then by Laurent-expanding (3) around \( z = \infty \), we obtain the constraints

\[ \frac{1}{2\mu} \left( -\frac{\mu_3}{\sqrt{b_4}} + \frac{\Lambda b_3}{4\sqrt{b_4}} \right) = 0, \tag{7} \]

\[ -\frac{b_1}{2\mu} \left( -\frac{\mu_3}{\sqrt{b_4}} + \frac{\Lambda b_3}{4\sqrt{b_4}} \right) + \Lambda - \frac{\Lambda}{4\mu} \cdot \frac{1}{4\sqrt{b_4}} = 0, \tag{8} \]

\[ -\frac{\mu_3}{2\mu} + \frac{1}{2\mu} \left( \frac{b_2}{2} - \frac{b_1^2}{8} \right) \left( -\frac{\mu_3}{\sqrt{b_4}} + \frac{\Lambda b_3}{4\sqrt{b_4}} \right) - \frac{\Lambda}{2\sqrt{b_4}} \cdot \frac{b_1}{4\mu} = 1, \tag{9} \]
which reproduce the relations found in \[40\]:

\[
b_3 = \frac{4\mu_3}{\Lambda}, \quad b_4 = 1, \quad b_1 = -\frac{8\mu + 4\mu_3}{\Lambda}.
\] (10)

The large-$N$ expansion:

\[
Z = \exp \left( \frac{N^2}{\mu^2} F \right),
\] (11)

\[
F = F_0 + \frac{\mu^2}{N^2} F_1 + \cdots.
\] (12)

\(F_0\) is the large-$N$ free energy given by the well-known formula:

\[
\frac{1}{\mu^2} F_0 = -\frac{1}{\mu} \int_{-\infty}^{\infty} d\lambda \rho(\lambda) W(\lambda) + \int_{-\infty}^{\infty} d\lambda \int_{-\infty}^{\infty} d\lambda' \rho(\lambda) \rho(\lambda') \log |\lambda - \lambda'|,
\] (13)

where \(\rho(\lambda)\) is the eigenvalue density.

In the present case

\[
\omega(z) \equiv \frac{1}{2\mu} (W'(z) - y(z)),
\] (14)

\[
y(z) = \frac{\Lambda}{2} \sqrt{1 + \frac{b_1}{z} + \frac{b_2}{z^2} + \frac{b_3}{z^3} + \frac{1}{z^4}}.
\] (15)

Let

\[
\mu_1 \equiv -\frac{1}{4\pi i} \oint_{A_1} y(z),
\] (16)

then \(F_0\) satisfies the “special geometry relation”, meaning that

\[
\frac{\partial F_0}{\partial \mu_1} \propto \oint_B y(z),
\] (17)

where \(B\) is the other homology cycle. Therefore, if there exists some parameter variable \(u\) such that \(\frac{\partial y(z)}{\partial u} dz\) is proportional to the holomorphic differential, \(y(z) dz\) is essentially the Seiberg-Witten differential of the curve. In the present case, we can take \(b_2\) as \(u\), then (up to a term linear in \(\mu_1\)) \(F_0\) is automatically the prepotential of the special Kähler geometry.

III. UNITARY MATRIX MODEL WITH A PENNER-LIKE POTENTIAL

The partition function of a unitary matrix model is defined as usual by

\[
Z = \frac{1}{\text{Vol}(U(N))} \int dU \exp \left( -\frac{N}{\mu} \text{tr} W_U(U) \right)
\] (18)

\[
= \int \prod_{i=1}^{N} d\theta_i e^{-\frac{N}{\mu} W_U(e^{i\theta_i})} \prod_{j<k} \sin^2 \frac{\theta_j - \theta_k}{2}.
\] (19)
We take the same potential (2) as the potential for the unitary matrix model here. Then this is a Penner-like generalization of the Gross-Witten-Wadia model \[45–47\]. For convenience, we absorb the factor \(\frac{\Lambda^2}{2}\) by an overall rescaling and shift \(\theta_i\) by \(\frac{\pi}{2}\). The potential we consider is

\[
W_U(U) = i^{-1}(U - U^{-1}) + \nu \log(-U). \tag{20}
\]

If \(\nu = 0\), it reduces to the Gross-Witten-Wadia model though the potential is \(\sin\) instead of \(\cos\) due to the shift.

It has been shown that a unitary matrix model with a potential \(W_U(U)\) is equivalent \[41, 42\] to a hermitian matrix model with a potential

\[
W(\Phi) = W_\Phi(\Phi) + \mu \log(1 + \Phi^2), \tag{21}
\]

where

\[
W_\Phi(\Phi) \equiv W_U \left( \frac{i - \Phi}{i + \Phi} \right). \tag{22}
\]

In the present case, we have

\[
W(\Phi) = \frac{4\Phi}{1 + \Phi^2} + \mu_+ \log(\Phi - i) + \mu_- \log(\Phi + i), \tag{23}
\]

where \(\mu_\pm \equiv \mu \pm \nu\).

\[
W'(x) = \frac{4(1 - x^2)}{(1 + x^2)^2} + \frac{\mu_+}{x - i} + \frac{\mu_-}{x + i}. \tag{24}
\]

\[
\omega(z) = \frac{\sqrt{\prod_{k=1}^{4}(z - a_k)}}{4\pi i \mu} \left( \oint_{x=\infty} - \oint_{x=z} - \oint_{x=i} - \oint_{x=-i} \right) dx \frac{4(1 - x^2)}{(1 + x^2)^2} + \frac{\mu_+}{x-i} + \frac{\mu_-}{x+i} \sqrt{(z-x)\prod_{k=1}^{4}(x-a_k)}. \tag{25}
\]

The 1st term = 0,

The 2nd term = \(\frac{1}{2\mu} W'(z)\),

The 3rd term = \(\frac{\sqrt{\prod_{k=1}^{4}(z - a_k)}}{2\mu(i-z)\sqrt{\prod_{k=1}^{4}(i-a_k)}} \left( \frac{2}{i - z} + \sum_{k=1}^{4} \frac{1}{i - a_k + \mu_+} \right)\),

The 4th term = \(\frac{\sqrt{\prod_{k=1}^{4}(z - a_k)}}{2\mu(-i-z)\sqrt{\prod_{k=1}^{4}(-i-a_k)}} \left( \frac{2}{-i - z} + \sum_{k=1}^{4} \frac{1}{-i - a_k + \mu_-} \right)\). \(\tag{26}\)
Again, the positions of the end points of the branch cuts are not arbitrary, but are constrained in order for the resolvent \( \omega(z) \) to have a correct asymptotic behavior at infinity.

Expanding (25) around \( z = \infty \), we find the conditions

\[
\prod_{k=1}^{4}(i - a_k) = \prod_{k=1}^{4}(-i - a_k),
\]

(27)

\[
\sum_{k=1}^{4} \frac{1}{i - a_k} + \mu_+ = -\left( \sum_{k=1}^{4} \frac{1}{-i - a_k} + \mu_- \right) = -2i.
\]

(28)

Thus, defining

\[
\omega(z) \equiv \frac{1}{2\mu}(W'(z) - y(z)),
\]

(29)

we obtain

\[
y(z) = \frac{8\sqrt{\prod_{k=1}^{4}(z - a_k)}}{A(z^2 + 1)^2},
\]

(30)

where

\[
A \equiv \sqrt{\prod_{k=1}^{4}(i - a_k)} = \sqrt{\prod_{k=1}^{4}(-i - a_k)}.
\]

(31)

Note that the explicit dependence of \( \mu_\pm \) disappears in (30), but they affect \( y(z) \) only through \( a_k \)'s by the relation (28).

To see that \( y(z)dz \) is a Seiberg-Witten differential for some \( u \), let us go back to the unitary-matrix complex coordinate by the replacements

\[
z = \frac{i(1-w)}{1+w}
\]

(32)

and

\[
a_k = \frac{i(1-T_k)}{1+T_k} \quad (k = 1, \cdots , 4).
\]

(33)

In terms of \( w \) and \( T_k \)'s, \( y(z)dz \) becomes

\[
y(z)dz = \frac{-i\sqrt{\prod_{k=1}^{4}(w - T_k)}}{w^2}dw.
\]

(34)

If we write

\[
\prod_{k=1}^{4}(w - T_k) \equiv w^4 + \sigma_1 w^3 + \sigma_2 w^2 + \sigma_3 w + \sigma_4,
\]

(35)
then by using (33) in (27) and (28) we find

\[ \sigma_4 = 1, \quad \sigma_3 = 2i\mu_+, \quad \sigma_1 = -2i\mu_- \]  

(36)

Thus we finally obtain

\[
y(z)dz = \sqrt{\hat{\omega}^4 + 2\mu_-\hat{\omega}^3 - \sigma_2\hat{\omega}^2 + 2\mu_+\hat{\omega} + 1} \, d\hat{\omega} 
\equiv \hat{y}(\hat{\omega})d\hat{\omega},
\]

(37)

where \( \hat{\omega} \equiv iw \). This is precisely the same expression as \( y(z) \) (15) of the hermitian matrix model we saw in the previous section, up to an overall constant factor. Again, we can take \(-\sigma_2\) as \( u \), then

\[
\frac{\partial \hat{y}(\hat{\omega})d\hat{\omega}}{\partial u} = \frac{d\hat{\omega}}{2\sqrt{\hat{\omega}^4 + 2\mu_-\hat{\omega}^3 + u\hat{\omega}^2 + 2\mu_+\hat{\omega} + 1}} 
\equiv \frac{d\hat{\omega}}{2v(\hat{\omega})},
\]

(38)

which is a holomorphic differential of the \( N_f = 2 \) curve

\[
v^2 = \hat{\omega}^4 + 2\mu_-\hat{\omega}^3 + u\hat{\omega}^2 + 2\mu_+\hat{\omega} + 1.
\]

(39)

It has been shown [41] that, even in the presence of the log potential terms, the periods of the differential (37) satisfy

\[
\frac{\partial F_0}{\partial \mu_j} = \frac{1}{2} \lim_{\epsilon \to 0} \left[ \int_{\gamma(j \text{th cut})} \hat{y}(z) - W(i - i\epsilon) - W(-i + i\epsilon) \right],
\]

(40)

\[
\oint_{A_j} dz y(z) \equiv -4\pi i\mu_j \quad (j = 1, 2),
\]

(41)

where \( \sim \) denotes the point on the second sheet specified by the value of \( z \). The relation (40) holds in the case where the two periods \( \mu_1 \) and \( \mu_2 \) are treated as independent variables. If, instead, \( \mu_1 \) and \( \mu(= \mu_1 + \mu_2) \) are treated as independent, then

\[
\frac{\partial F_0}{\partial \mu_1}\bigg|_\mu = \frac{\partial F_0}{\partial \mu_1}\bigg|_{\mu_2} - \frac{\partial F_0}{\partial \mu_2}\bigg|_{\mu_1} 
\equiv -\frac{1}{2} \oint_B dz y(z),
\]

(42)

which shows that the unitary-matrix free energy \( F_0 \) is the prepotential of the \( N_f = 2 \) (up to a term linear in \( \mu_1 \)).
IV. CONCLUSIONS

We have shown that a unitary matrix model with a Penner-like potential identical to the one used in Eguchi-Maruyoshi’s hermitian matrix model also yields the prepotential of $\mathcal{N} = 2 \, N_f = 2 \, SU(2)$ SUSY gauge theory as its large-$N$ free energy, the fact anticipated by Itoyama et. al.

Although the two matrix models describe the same gauge theory, the parameter identifications are slightly different. For instance, in the hermitian model, the coefficient of the log term in the potential corresponds to one of the mass of the flavor, while in the unitary model, the coefficient of log represents the difference of the masses of the two flavors. (This fact was already noted in \[36\].) After all, we can say that the two matrix models are different models with different Boltzmann weights, but they can describe the same gauge theory if the parameter identifications are changed in a suitable way.

Itoyama et. al.’s proposal \[36\] that the Penner-like unitary matrix model also yields $N_f = 2$ was concluded by considering an irregular limit of the AGT relation, in which they used a few assumptions. (See Appendix F of \[37\].) Our result implies the validity of the assumptions.

Finally, we would like to emphasize that what we have shown in this paper implies more than the fact the unitary and hermitian matrix models belong to the same universality class \[48, 49\] in the critical limit. Our proof holds true even away from the criticality, though, of course, being in the same class will be a necessary condition for their equivalence.

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