CELL-FREE MASSIVE MIMO SYSTEMS WITH MULTI-ANTENNA USERS

Trang C. Mai, Hien Quoc Ngo, and Trung Q. Duong

Institute of Electronics, Communications and Information Technology in Queen’s University Belfast, U.K.

ABSTRACT

In this paper, we investigate the impact of multiple-antenna deployment at access points (APs) and users on the performance of cell-free massive multiple-input multiple-output (MIMO). The transmission is done via time-division duplex (TDD) protocol. With this protocol, the channels are first estimated at each AP based on the received pilot signals in the training phase. Then these channel information will be used to decode the symbols before sending to all users. The simple and distributed conjugate beamforming technique is deployed. We derive a closed-form expression for the downlink spectral efficiency taking into account the imperfect channel state information (CSI), non-orthogonal pilots, and power control. This spectral efficiency can be achieved without the knowledge of instantaneous CSI at the users. In addition, the effects of the number antennas per APs and per users are analyzed in the case of using mutual orthogonal pilot sequences and data power control.

Index Terms—cell-free massive MIMO, massive MIMO, spectral efficiency.

1. INTRODUCTION

Cell-free massive MIMO is a useful and scalable version of network MIMO, where a large number of APs, which are geographically distributed, coherently serve all users in same time-frequency resource [1, 2]. Cell-free massive MIMO reaps all benefits from Massive MIMO (favorable propagation, and channel hardening when using multiple antennas at APs [3]) and network MIMO (increased macro-diversity gain), and hence, it can offer very high spectral efficiency, energy efficiency, and coverage probability. These benefits can be achieved with simple signal processing and local channel acquisition at each AP. In addition, with the cell-free topology, the excessive handover issue in small-cell systems can be resolved. Thus, cell-free massive MIMO has attracted a lot of research interest recently [4–8].

To the best of our knowledge, the aforementioned works exploit the performance of cell-free massive MIMO with single-antenna users. However, in practice, many users with moderate physical sizes (e.g. laptops, tablets, and smart vehicles) can be equipped with several antennas to increase the multiplexing gain, and hence, improve the spectral efficiency. Thus, it is important to evaluate the performance of cell-free massive MIMO with multiple antennas at the users. Understanding well the effect of multiple antennas at the users will enable us to further design the systems.

Inspired by the above discussion, in this paper, we analyze the performance of cell-free massive MIMO with multiple antennas at both APs and users. The closed-form expression of downlink spectral efficiency is derived. Moreover, effect of the number of antennas at APs and users on spectral efficiency is analyzed and exploited through the use of max-min fairness power control. We show that, comparing with the model using single antenna at users, the per-user net throughput can improves significantly with multiple antennas at both APs and users, especially when the number of users is small.

2. SYSTEM MODEL AND SPECTRAL EFFICIENCY

We consider a cell-free massive MIMO system operating in TDD mode with $M$ APs and $K$ users randomly located in a large area. Each AP has $L$ antennas, whereas each user has $N$ antennas. Let $G_{mk} \in \mathbb{C}^{L \times N}$ be the channel response matrix between the $k$-th user and the $m$-th AP. Then,

$$G_{mk} = \beta_{mk}^{1/2}H_{mk},$$

where $\beta_{mk}$ is large-scale fading between the $k$-th user and the $m$-th AP, and $H_{mk}$ is the $L \times N$ small-scale fading matrix whose elements are assumed to be independent and identically distributed (i.i.d) circularly symmetric complex Gaussian random variables (RVs) with zero mean and variance $\sigma = 1 (CN(0, 1))$. In this work, we focus on the downlink data transmission. The uplink payload data transmission is neglected. Therefore, for each coherence interval, the transmission comprises two phases: uplink channel estimation and downlink payload data transmission.

2.1. Uplink Channel Estimation

In this phase, all users will send pilot signals to the APs. Then each AP will estimate its channels to all users using the received pilot signals. Let $\tau$ be the length of training duration
per coherence interval, and \( \Phi_k \in \mathbb{C}^{\tau \times N} \) be a pilot matrix of the \( k \)-th user. Then, the received signal at the \( m \)-th AP is

\[
Y_m = \sum_{k=1}^{K} \sqrt{\tau \rho_p} \Phi_{mk} \Phi_k^H + W_m,
\]

where \( \rho_p \) is the normalized signal-to-noise ratio (SNR) of each pilot symbol and \( W_m \) is the \( L \times \tau \) matrix of additive noise at the \( m \)-th AP. We assume that the elements of \( W_m \) are i.i.d. \( \mathcal{C}\mathcal{N}(0, 1) \) RVs. A projection of \( Y_m \) onto \( \Phi_k \) is

\[
Y_{mk} = Y_m \Phi_k
\]

where \( \Phi_{ik} \triangleq \Phi_k^H \Phi_k \), and \( W_{mk} \triangleq W_m \Phi_k \). By stacking all columns of \( Y_{mk} \) on top of each other, we have

\[
\text{vec}(Y_{mk}) = \sqrt{\tau \rho_p} \sum_{i=1}^{K} (\Phi_i^T \otimes I_L) \text{vec}(G_{mi}) + \text{vec}(W_{mk})
\]

\[
= \sqrt{\tau \rho_p} \sum_{i=1}^{K} \Phi_{ik} \text{vec}(G_{mi}) + \text{vec}(W_{mk}),
\]

where \( \text{vec}(\cdot) \) is the vectorization operation, and \( \Phi_{ik} \triangleq \Phi_k^H \otimes I_L \). Assuming that orthogonal pilot sequences are assigned for antennas of each user, but these pilot sequences can be reused in other users, then MMSE estimation of \( \text{vec}(G_{mk}) \) given \( \text{vec}(Y_{mk}) \) is

\[
\text{vec}(\hat{G}_{mk}) = \sqrt{\tau \rho_p} \beta_{mk} \text{vec}(Y_{mk}) \left( \tau \rho_p \sum_{i=1}^{K} \Phi_{ik} \beta_{mi} I_L \Phi_k^H + I_{LN} \right)^{-1}.
\]

**Lemma 1** The estimate of the channel matrix \( G_{mk} \) is given by

\[
\hat{G}_{mk} = Y_{mk} A_{mk},
\]

where

\[
A_{mk} \triangleq \sqrt{\tau \rho_p} \beta_{mk} \left( \tau \rho_p \sum_{i=1}^{K} \beta_{mi} \Phi_{ik}^H \Phi_k + I_N \right)^{-1}.
\]

**Proof:** See Appendix A.1.

### 2.2. Downlink Data Transmission

In this phase, each AP uses the channel estimates in the uplink channel estimation phase together with the conventional conjugate beamforming technique to precode the desired symbols. Then the precoded signal will be sent to all users. The \( L \times 1 \) transmitted signal from the \( m \)-th AP is

\[
x_m = \sqrt{\rho_d} \sum_{k=1}^{K} \eta_{mk}^{1/2} \hat{G}_{mk} q_k,
\]

where \( q_k \in \mathbb{C}^L \times 1 \) is the vector of symbols intended for the \( k \)-th user which satisfies \( \mathbb{E}[q_k q_k^H] = I_N \), \( \rho_d \) is the normalized SNR of each data symbol, and \( \eta_{mk} \) is power control coefficient between the \( m \)-th AP and the \( k \)-th user. The power control coefficients \( \eta_{mk} \) are chosen to satisfy the power constraint at each AP, i.e., \( \mathbb{E}[\|x_m\|^2] \leq \rho_d \) which is equivalent to

\[
\sqrt{\tau \rho_p} \sum_{k=1}^{K} \eta_{mk} \beta_{mk} \text{tr}(A_{mk}) \leq 1.
\]

The received signal at the \( k \)-th user is

\[
r_k = \sum_{m=1}^{M} G_{mk}^H x_m + n_k
\]

\[
= \sqrt{\rho_d} \sum_{m=1}^{M} \sum_{k'=1}^{K} \eta_{mk'}^{1/2} G_{mk'}^H \hat{G}_{mk} q_k' + n_k,
\]

where \( n_k \) is the noise vector. The elements of \( n_k \) are assumed to be i.i.d. \( \mathcal{C}\mathcal{N}(0, 1) \).

### 2.3. Spectral Efficiency

In this section, we derive an achievable spectral efficiency for each user. To obtain this spectral efficiency we use a simple bounding technique in massive MIMO [9]. This technique is widely used because it yields a simple, insightful, and tight spectral efficiency expression. Furthermore, this spectral efficiency can be achieved without any requirements of instantaneous CSI at the users.

From (10) and by using [10, Theorem 2], we can obtain an achievable downlink spectral efficiency for the \( k \)-th user as

\[
R_k = (1 - \tau / \tau_c) \log_2 |I_N + \bar{H}_k^H \Xi_k \bar{H}_k|,
\]

where \( \tau_c \) is the coherence interval (in samples),

\[
\bar{H}_k \triangleq \mathbb{E} \left\{ \sqrt{\rho_d} \sum_{m=1}^{M} \eta_{mk}^{1/2} G_{mk}^H \hat{G}_{mk} \right\},
\]

and

\[
\Xi_k \triangleq \left( \mathbb{E} \left\{ \rho_d \sum_{m=1}^{M} \sum_{n=1}^{K} \eta_{mk}^{1/2} G_{mk}^H G_{mk} G_{nk} G_{nk}^H \right\} - \bar{H}_k \bar{H}_k^H + I_N \right)^{-1}.
\]

We next provide a closed-form expression for the spectral efficiency given by (11).
Theorem 1 With conjugate beamforming, the achievable downlink spectral efficiency for the k-th user can be represented in closed-form as

$$R_k = (1 - \tau/\tau_c) \log_2 |I_N + H_k^H (S + I_N)^{-1} H_k|,$$  \hspace{1cm} (14)

where

$$H_k = L\sqrt{\tau p_d \rho_d} \sum_{m=1}^{M} \eta_{mk}^{1/2} \beta_{mk} A_{mk},$$ \hspace{1cm} (15)

and

$$S = L^2 \tau p_d \rho_d \sum_{m=1}^{M} \sum_{m' \neq m} \sum_{k' \neq k}^{K} \eta_{mk'}^{1/2} \eta_{mk}^{1/2} \beta_{mk} \beta_{mk'} \Phi_{kk'}^H \times$$

$$\times A_{mk'} A_{mk}^H \Phi_{kk'}^H - L^2 \tau p_d \rho_d \sum_{m=1}^{M} \eta_{mk}^{1/2} \beta_{mk} \Phi_{kk'}^H (S + I_N)^{-1} H_k,$$  \hspace{1cm} (16)

Proof: See Appendix A.3. 
Remark 1: In the special case that all APs and users have a single antenna, i.e., $L = N = 1$, the spectral efficiency (14) is identical to the one in [1].

3. NUMERICAL RESULTS AND DISCUSSION

In this section, we provide the numerical results to evaluate the performance of cell-free massive MIMO with multi-antenna users. We consider the net throughput which is defined as $T_k = B \times R_k$, where $B$ is the spectral bandwidth.

The APs and the users are uniformly distributed at random within a square of size $1 \times 1$ km$^2$. In all examples we assume that $\tau = K \times N$ and pilot sequences used during the training phase are mutually orthogonal. With this assumption, the resulting spectral efficiency has a similar form as the one in [1]. Therefore, max-min fairness data power control proposed in [1] can be applied.

Figures show per-user net throughputs as the function of the number of antennas per user and the number of antennas per AP with $M = 50$. 95%-likely per-user net throughput is used to evaluate the performance of the system. Some important insights obtained from these results are as follows:

- As expected, the 95%-likely per-user net throughput increases proportionally with the number of antennas per AP. This comes from the fact that when $L$ increases, the channel is more favorable, and hence, the inter-user interference reduces. At the same time, the array gain increases.

- The per-user net throughput first increases when the number of antennas per user increases as the increasing of independent channels (or degrees of freedom) per user. However, this throughput will reach to the maximum value and then it will decrease when the number of antennas per user increases. The reason is that although the number of independent channels per user increases, the channel estimation overhead (the training duration relative to the coherence interval) also increases. This channel estimation overhead may be dominated when $N$ is large.

- Using multiple antennas at the users greatly enhances the per-user net throughput, especially with small number of users in the system. With $M = 50$, $K = 20$, by using 3 antennas per user, we can double the 95%-likely net throughput, compared to single-antenna user systems.
4. CONCLUSION

In this paper, we derived a general closed-form expression of the downlink spectral efficiency for a cell-free massive MIMO system with finite numbers of APs, users and arbitrary numbers of antennas at the APs and the users. This expression took into account the simple conjugate beamforming scheme, imperfect channel estimation, non-orthogonal pilot sequences, and power control coefficients. It will enable us for further design of cell-free massive MIMO. In addition, the analysis shows that with multiple antennas at the users, per-user net throughput of the system can greatly enhance, especially in the case of a small number of users in the system.

A. APPENDIX

A.1. Proof of Lemma 1

From (5), we have

\[
\text{vec}(G_{mk}) = \sqrt{\tau_p \rho} \beta_{mk} I_N \otimes I_L \times \\
\times (\tau_p \rho \sum_{i=1}^{K} \beta_{mi} \Phi_{ik}^T \Phi_{ik}^* + I_N)^{-1} \otimes I_L \text{vec}(Y_{mk}) \\
= \sqrt{\tau_p \rho} \beta_{mk} (\tau_p \rho \sum_{i=1}^{K} \beta_{mi} \Phi_{ik}^T \Phi_{ik}^* + I_N)^{-1} \otimes I_L \text{vec}(Y_{mk}).
\]

(17)

Finally, (6) is obtained by applying the following identity

\[
\text{vec}(ABC) = C^T \otimes \text{Avec}(B)
\]

on (17).

A.2. Lemma 2

This Lemma will be used to proof Theorem 1.

Lemma 2 Let \( B = Y^H X \), where \( X, Y \) are \( M \times N \) random matrix which its elements are assumed to be i.i.d. \( \mathcal{CN}(0, 1) \) and \( C \) is \( N \times N \) matrix. Then

\[
\mathbb{E} \{ B^H CB \} = M \text{tr}(C) I_N.
\]

(18)

Proof:

\[
\mathbb{E} \{ B^H CB \} = \mathbb{E} \left\{ \begin{bmatrix} b_1^H \\ b_2^H \\ \vdots \\ b_N^H \end{bmatrix} C \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_N \end{bmatrix} \right\}
\]

\[
= \mathbb{E} \left\{ \begin{bmatrix} \text{tr}(C b_1 b_1^H) \\ \text{tr}(C b_2 b_2^H) \\ \vdots \\ \text{tr}(C b_N b_N^H) \end{bmatrix} \right\}
\]

\[
= \begin{bmatrix} \mathbb{E} \{ \text{tr}(C b_1 b_1^H) \} \\ \mathbb{E} \{ \text{tr}(C b_2 b_2^H) \} \\ \vdots \\ \mathbb{E} \{ \text{tr}(C b_N b_N^H) \} \end{bmatrix}
\]

\[
= \begin{bmatrix} \text{tr}(C \mathbb{E} \{ b_1 b_1^H \}) \\ \text{tr}(C \mathbb{E} \{ b_2 b_2^H \}) \\ \vdots \\ \text{tr}(C \mathbb{E} \{ b_N b_N^H \}) \end{bmatrix}
\]

(19)

Then, calculate \( \mathbb{E} \{ b_k b_k^H \} \) where \( b_k^H = X_k^H Y \)

\[
\mathbb{E} \{ b_k b_k^H \} = \mathbb{E} \{ Y^H X_k X_k^H Y \}
\]

\[
= \mathbb{E} \left\{ \begin{bmatrix} y_1^H \\ y_2^H \\ \vdots \\ y_N^H \end{bmatrix} x_k X_k^H \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_N \end{bmatrix} \right\}
\]

\[
= \begin{bmatrix} \mathbb{E} \{ |y_1 x_k|^2 \} \\ \mathbb{E} \{ |y_2 x_k|^2 \} \\ \vdots \\ \mathbb{E} \{ |y_N x_k|^2 \} \end{bmatrix}
\]

\[
= M I_N.
\]

(20)

The substitution of (20) into (19) yields (18).

A.3. Proof of Theorem 1

A.3.1. Compute \( H_k \)

From (12), we have

\[
\hat{H}_k = \mathbb{E} \left\{ \sqrt{\tau_p d} \sum_{m=1}^{M} \eta_{mk}^{1/2} G_{mk}^H (Y_{mk} A_{mk}) \right\}
\]

\[
= \sqrt{\tau_p d} \sum_{m=1}^{M} \eta_{mk}^{1/2} \mathbb{E} \left\{ G_{mk}^H G_{mk} \right\} A_{mk}
\]

\[
= L \sqrt{\tau_p d} \sum_{m=1}^{M} \eta_{mk}^{1/2} \beta_{mk} A_{mk}.
\]

(21)

A.3.2. Compute \( S \)

Due to the page constraint, the detail of the proof is omitted here. However, the main idea is that (16) is computed by applying Lemma 2.
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