Acoustic impact on the laminated plates placed between barriers

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Abstract. On the basis of previously derived equations, analytical solutions are established on the forced vibrations of two-layer and three-layers rectangular plates hinged in an opening of absolutely rigid walls during the transmission of monoharmonic sound waves. It is assumed that the partition wall is situated between two absolutely rigid barriers, one of them by harmonic oscillation with a given displacements amplitude on the plate forms the incident sound wave, and the other is stationary and has a coating of deformable energy absorbing material with high damping properties. The behavior of acoustic environments in the spaces between the deformable plate and the barriers described by classical wave equation based on the ideal compressible fluid model. To describe the process of dynamic deformation of the energy absorbing coating of fixed barrier, two-dimensional equations of motion based on the use of models transversely soft layer are derived with a linear approximation of the displacement field in the thickness direction of the coating and taking into account the damping properties of the material and the hysteresis model for it. The influence of the physical and mechanical properties of the concerned mechanical system and the frequency of the incident sound wave on the parameters of its insulation properties of the plate, as well as on the parameters of the stress-strain state of the plate has been analyzed.

1. Introduction

In the second half of the last century, a scientific direction in mechanics was founded on the study of stationary and non-stationary interactions of acoustic waves with barriers in the form of solid deformable bodies and thin-walled structural elements. This direction continues still to attract the attention of researchers by its actuality, complexity and diversity of the phenomena inherent in the process of interaction of bodies with different physical fields. Related to this direction, issues about the aero-hydroelasticity of thin-walled shell structures have been covered in a number of monographs and reviews ([1–4], and et al.). However, they do not cover the issues about sound wave formation and the study on soundproofing problems as well as sound absorption by various deformable coatings, although in almost all publications devoted to the creation of various kinds of multi-layer constructions emphasizing that they have good soundproofing and sound absorption properties ([5, 6 ] et al.). Such problems of mechanics are problems acoustoelasticity, which is devoted to the rather
extensive literature in the form of scientific articles ([7-10], et al.), monographs ([11-13], et al.) and review articles ([14-18] et al.).

Subject matter of this article - the problem of forced oscillations of a rectangular plate (two-layer and three-layer), hinged to an opening in absolutely rigid partitions, under the influence of monoharmonic external pressure. Without taking into account the interaction of the plate with the surrounding acoustic environments, as a rule, researches are conducted on forced vibrations of thin-walled structural elements ignoring the fact that the real structural elements are not in vacuum, but in acoustic environments. The following shows that the correct formulation of these problems requires consideration of external damping.

2. Problem statement

Let’s consider multi-layer plate consist of \( M = N - 1 \) layers (fig.1),

![Figure 1. Multilayer plate scheme.](image)

We assume that the plate is placed in the opening of an absolutely rigid partition separating two adjacent spaces \( V_1 \) and \( V_2 \). Coordinate plane \( z = 0 \) (fig.2) is compatible with the median plane of the plate. We assume that at distances \( z = -l_1 \) and \( z = l_2 \) from the plate positioned absolutely rigid barriers of infinite extent in directions \( x \) and \( y \), coinciding with the coordinate axes of the plate \( x_1 \) and \( x_2 \), respectively. One of the barriers makes \( z \)-axis direction in the harmonic oscillation with angular frequency \( \omega \) and amplitude \( \bar{U}_1 \), and the second fixed coating has low-rigidity deformable layer with thickness \( h \). We assume that the boundary plane of coating \( z = l_2 + h \) is fixed and the point of the boundary plane \( z = l_2 \) due to the deformation of this plane undergoes the following definition of displacement

\[
U = U(x, y, \tau), \quad V = V(x, y, \tau), \quad W = W(x, y, \tau).
\]

We assume that \( V_s \) (\( s = 1, 2 \)) space filled with acoustic medium with density \( \rho_s \) and sound velocity \( c_s \). Then, due to harmonic oscillations of barriers \( z = -l_1 \), acoustic wave pressures \( p_1 \) and \( p_2 \) will be formed in spaces \( V_s \), which are reflected and emitted in the first medium and the radiated in the second medium.

To determine pressure values \( p_1^0 \) and \( p_2^0 \), applied to the first and \( N \)-th layers of the boundary planes of the plate, respectively, it is necessary to find solutions to the wave equation [19] \((c_1, c_2 - speed of sound in the spaces \( V_1 \) and \( V_2 \))

\[
\Phi_{s,xx} + \Phi_{s,yy} + \Phi_{s,zz} - \bar{\Phi}_s/c_s^2 = 0
\]

written with respect to the potential velocity \( \Phi_1, \Phi_2 \) of sound waves, set in space \( V_1 \) and \( V_2 \) respectively. Here and throughout \( s = 1, 2 \), the points above the functions indicate derivatives with
respect to time \( \tau \), and the subscript after the decimal point - the partial derivatives with respect to coordinates \( x, y \) and \( z \).  

![Plate fixing scheme](image)

**Figure 2. Plate fixing scheme.**

By the functions \( \Phi, \Phi_s \), pressure \( p_s \) and velocity components \( v'_x, v'_y, v'_z \) in the spaces \( V_1, V_2 \) defined by (\( \rho_s \) - density of environments in \( V_1 \) and \( V_2 \) spaces)

\[
v'_x = \Phi_{h_x}, v'_y = \Phi_{h_y}, v'_z = \Phi_{h_z}, \quad p_s = -\rho_s \Phi_s
\]

(2)

The process of dynamic deformation of the plate will be described by the equations of the theory of multilayer plates obtained in [20] with allowance for transverse compression and internal friction of the material of the plate on the model Thomson-Kelvin-Voigt. As the unknown components displacement components \( u^{(k)}_i, u^{(k)}_j, w^{(k)} \) of the points on the facial planes of the first \((k = 1)\) and the last \((k = N-1)\) layers, as well as points of interlayer planes are taken. In the case of simply supported plate in the opening of an absolutely rigid partitions, for these functions the following representations are valid

\[
u^{(k)}_i = \sum_{m=1}^{n} \overline{u}^{(k)}_{i m} \cos \lambda_m x \cdot \sin \lambda_n y, \quad u^{(k)}_j = \sum_{m=1}^{n} \overline{u}^{(k)}_{j m} \sin \lambda_m x \cdot \cos \lambda_n y,
\]

\[
w^{(k)} = \sum_{m=1}^{n} \overline{w}^{(k)}_{m} \sin \lambda_m x \cdot \sin \lambda_n y, \quad \lambda_m = m\pi/a, \quad \lambda_n = n\pi/b;
\]

(3)

Hereafter, unless otherwise stated, and the summation is to be done over \( m=1,3,... \) and \( n=1,3,... \) and plate dimensions are shown in its plan view through \( a \) and \( b \).

During the formation of the acoustic waves in space \( V_2 \) emitted by the plate under the action of pressure \( p_2^{(k)} = p_{20} \), coating layers with low stiffness on fixed barrier layer will be deformed. We assume that the main stress component will be that of in the normal direction \( \sigma_{zz} \) and the shear stresses \( \tau_{xz}, \tau_{yz} \) associated with the relevant components of the strain \( \varepsilon'_{zz}, \varepsilon'_{xz}, \varepsilon'_{yz} \) and taking into account the viscoelastic properties of the material by Thomson-Kelvin-Voigt model with elastic relations of the following kind

\[
\sigma_{zz} = E \varepsilon'_{zz}, \quad \tau_{xz} = G \varepsilon'_{xz}, \quad \tau_{yz} = G \varepsilon'_{yz}
\]

(4)

where
\[
D'_c = 1 + \frac{\delta'_c}{\pi a_0 \partial \tau}, \quad D'_d = 1 + \frac{\delta'_d}{\pi a_0 \partial \tau}
\]

(5)

Here \( E'_c, G'_c, \delta'_c, \delta'_d \) are the corresponding elastic and damping characteristics of the coating material. Considering that \( h_c << a, b \), for the displacement components in the coating we will take the approximation

\[
(u_c, v_c, w_c) = \left(1 - \frac{z - l_c}{h_c}\right) (U, V, W)
\]

(6)

allowing in necessary degree of accuracy to describe the strains \( \varepsilon'_c, \gamma'_c, \gamma'_d \). Their substitution into the Cauchy relation leads to relationships

\[
\gamma'_c = \left(1 - \frac{z - l_c}{h_c}\right) W_x - \frac{U}{h_c}, \quad \gamma'_c = \left(1 - \frac{z - l_c}{h_c}\right) W_y - \frac{V}{h_c}, \quad \varepsilon'_c = -\frac{W}{h_c}.
\]

(7)

When using the relations (4) - (7) for the case when the coating layer is subjected to the lateral load \( p_x^* \), it is possible to get the system of three differential equations of motion in standard way for the coating layer

\[
G'_c D'_c \left(U - \frac{W}{2}ight) + \frac{P}{3} h_c \dot{U} = 0, \quad G'_d D'_d \left(V - \frac{W}{2}\right) + \frac{P}{3} h_c \dot{W} = 0
\]

\[
E'_c \frac{h_c}{h} D'_c W + \frac{P}{3} h_c \dot{W} - G'_d D'_d \left[\frac{h_c}{3} \left(W_{xx} + W_{yy}\right) - \frac{1}{2} \left(U_{xx} + V_{yy}\right)\right] - p_x^* = 0
\]

(8)

wherein \( P \) - the density of the coating material.

By the time, solution of the equations of motion of the plate, and the equations (8) and (1) must satisfy the kinematic coupling conditions

\[
\dot{w}^{(1)} = v'_c\big|_{z=0}, \quad \dot{w}^{(2)} = v'_c\big|_{z=0}, \quad \dot{W} = v'_c\big|_{z=l}, \quad \dot{U}_x = v'_c\big|_{z=l}
\]

(9)

at all points of the boundary planes of the plate (the first pair of conditions (9)), and the first barrier coating.

3. Determination of aero-hydrodynamic loads acting on plate

In accordance with the assumptions made above for the functions \( U_x(\tau) \) the representation of the form [21] must be taken

\[
U_x = \tilde{U}_x e^{i\omega t}
\]

(10)

by virtue of which, taking into account the first two terms of (9) and representation (3) solutions of equations (1) should be of the form

\[
\Phi_x = e^{i\omega t} \sum \tilde{\Phi}^{mn}(z) \sin \lambda_m x \sin \lambda_n y
\]

(11)

In turn, because of the last two terms of (9) and representation (10) - (11) for seeking solutions of equations (8) should take the form

\[
U = e^{i\omega t} \sum \tilde{U}^{mn} \cos \lambda_m x \sin \lambda_n y, \quad V = e^{i\omega t} \sum \tilde{V}^{mn} \sin \lambda_m x \cos \lambda_n y
\]

\[
W = e^{i\omega t} \sum \tilde{W}^{mn} \sin \lambda_m x \sin \lambda_n y
\]

(12)

When substituting the representations (11) into the equations (1), we arrive at the equations with respect to one-dimensional functions \( \tilde{\Phi}^{mn}(z) \):

\[
\tilde{\Phi}^{mn}_{k<z} = \left(\tilde{\omega}^{mn}\right)^2 \tilde{\Phi}^{mn}_{k>z} = 0; \quad \left(\tilde{\omega}^{mn}\right)^2 = \lambda^2_m + \lambda^2_n - k^2_x; \quad k_x = \omega / c_s; \quad m,n = 1,3,\ldots
\]

(13)
Depending on the sign of \((\alpha_{s}^{m})^2\) the equation (13) will have a solution \((A_{i}^{m} \text{ and } B_{i}^{m} - \text{integration constants})\): at \((\alpha_{s}^{m})^2 > 0\),
\[
\Phi_{i}^{m} = A_{i}^{m} e^{-\alpha s_{m}} + B_{i}^{m} e^{-\alpha s_{m}}; 
\]
and in accordance with the relations (2) and (11)
\[
v_{i}^{s} = e^{i\omega t} \sum A_{s}^{m} e^{\alpha s_{m}} \sin \lambda_{m} x \sin \lambda_{n} y; 
\]
\[
p_{s}^{i} = -i \rho_{s} e^{i\omega t} \sum (A_{s}^{m} e^{\alpha s_{m}} + B_{s}^{m} e^{-\alpha s_{m}}) \sin \lambda_{m} x \sin \lambda_{n} y; 
\]
And for \((\alpha_{s}^{m})^2 < 0\) in formulas (14) - (16), the terms \(\alpha_{s}^{m}\) should be replaced by \(i\alpha_{s}^{m}\), where
\[
(\alpha_{s}^{m})^2 = k_{x}^2 - \lambda_{m}^2 - \lambda_{n}^2
\]
Due to the fact that modern computational tools (eg, Matlab) allow to perform operations with complex numbers, below all the conversions are only belong to the case \((\alpha_{s}^{m})^2 > 0\). When realizing the solutions in such computation systems, the case \((\alpha_{s}^{m})^2 < 0\) is taken into account automatically (\(\alpha_{s}^{m}\) can be either real or complex value).

We represent the amplitude values \(U_{s}\) in the form of Fourier series
\[
\tilde{U}_{s} = \tilde{U}_{s} \sum f_{s} \sin \lambda_{m} x \sin \lambda_{n} y, \quad f_{s} = \frac{16}{\pi^2 m n}; 
\]
and subject the expression (15), the representation (18) and (12) (9). As a result, after a series of transformations, we obtain dependencies
\[
U_{s} = U_{s} \sum f_{s} \sin \lambda_{m} x \sin \lambda_{n} y, \quad f_{s} = \frac{16}{\pi^2 m n} 
\]
where
\[
a_{1}^{m} = b_{1}^{m} = i \frac{\omega f_{s} e^{\alpha s_{m}}}{\Delta_{1}^{m}}, \quad a_{2}^{m} = b_{2}^{m} = i \frac{\omega e^{-\alpha s_{m}}}{\Delta_{2}^{m}} 
\]
\[
\Delta_{s}^{m} = \alpha_{s}^{m} \left(1 - e^{-2\alpha s_{m}}\right) 
\]
After introducing the expression (19) in (16), we arrive at the dependencies
\[
p_{s}^{i} = p_{s}^{i} \frac{\omega f_{s} e^{\alpha s_{m}}}{\Delta_{1}^{m}}, \quad \tilde{R}_{s}^{m} = \frac{\omega f_{s} e^{\alpha s_{m}}}{\Delta_{1}^{m}} \left(1 + e^{-2\alpha s_{m}}\right) 
\]
where for \((\alpha_{s}^{m})^2 > 0\)
\[
R_{s}^{m} = 2 \rho_{s} e^{\alpha s_{m}} e^{-\alpha s_{m}} \frac{\omega f_{s} e^{2\alpha s_{m}}}{\Delta_{1}^{m}}, \quad \tilde{R}_{s}^{m} = \frac{\omega f_{s} e^{2\alpha s_{m}}}{\Delta_{1}^{m}} \left(1 + e^{-2\alpha s_{m}}\right) 
\]
Substituting the functions (12) of the first two equations of system (8), the following algebraic dependencies are obtained
\[
\vec{U}_{mn} = A_{mn} \lambda_m \vec{W}_{mn}, \quad \vec{V}_{mn} = A_{mn} \lambda_m \vec{W}_{mn}; \quad A_m = \frac{G_m \left(1 + i \frac{\delta_m^2}{\pi}\right)}{2 \left(\frac{G_m}{h_m} \left(1 + i \frac{\delta_m^2}{\pi}\right) + \frac{\rho_m h_m}{3} \omega^2\right)}
\]

(24)

the use of which by virtue of (22), the third equation of the system leads to dependencies

\[
\vec{W}_{mn} = \frac{\vec{P}_{mn}^{(N)}}{C_{mn}} \vec{\xi}_{n}^{(N)} - \frac{R_{2mn}^{0}}{C_{mn}} \vec{\xi}_{n}^{(N)}
\]

(25)

where

\[
C_{mn} = \frac{E_m}{h_m} \left(1 + i \frac{\delta_m^2}{\pi}\right) - \frac{\rho_m h_m}{3} \omega^2 + G_m \left(1 + i \frac{\delta_m^2}{\pi}\right) \left(\lambda_m^2 + \lambda_m^2\right) \left(\frac{h_m}{h_m} - \frac{A_m}{2}\right) - P_{2mn}^{0}
\]

(26)

Thus, using equation (26) in accordance with (21) and (22) we obtain the dependencies

\[
\vec{P}_{1mn}^{0} = R_{1mn}^{0} \vec{U}_{*} + \vec{R}_{1mn}^{0} \vec{W}_{mn}^{(1)}
\]

(27)

\[
\vec{P}_{2mn}^{0} = R_{2mn}^{0} \vec{W}_{mn} + \vec{R}_{2mn}^{0} \vec{\xi}_{n}^{(N)} = \left(\frac{C_{mn}^{(1)}}{C_{mn}} \vec{P}_{mn}^{(1)} + \vec{R}_{2mn}^{(N)} \vec{\xi}_{n}^{(N)}
\]

(28)

Closing the equations of motion of the plate, using (27), (28) where in accordance with adopted problem statement \(\vec{P}_{1mn}^{(1)} = \vec{P}_{1mn}^{0}, \quad \vec{P}_{2mn}^{(N)} = \vec{P}_{2mn}^{0}\), we obtain the governing system of \(3N+2\) linear equations with respect to the unknowns \(U^{(k)} = \{u^{(k)}_{1mn}, u^{(k)}_{2mn}, w^{(k)}_{mn}, P^{(k)}_{1mn}, P^{(k)}_{2mn}\}\). For the obtained values of \(\vec{P}_{1mn}^{(1)}, \vec{P}_{2mn}^{(N)}\), it is possible to determine the amplitude values of the pressures \(\vec{p}^{0}_{1}\) and \(\vec{p}^{0}_{2}\) acting on the boundary plane of the plate

\[
\vec{p}^{0}_{1} = \sum p^{0}_{1 \ell} \sin \lambda_{m \ell} x \sin \lambda_{n \ell} y
\]

(29)

In this case, at the point \(x = a/2, \quad y = b/2\) of plate, the soundproofing parameter measured in decibels will be equal to

\[
R_{p}^{0} = -20 \log \left|\sum p^{0}_{2 \ell} / \sum p^{0}_{1 \ell}\right|
\]

(30)

4. Results of computations and their analysis.

In [22], on the basis of experiments conducted in the acoustic laboratory of reverberation type, sound insulation parameters were determined for a rectangular plate made of steel and having a thickness of 20 mm, 480mm length, 560mm width. Tests were performed with both energy absorbing plate coating (rubber plate mark 2H-1-S-MBS with \(t_c = 2 \text{mm}\) thickness), and without it. For a qualitative comparison of the theoretical results with experimental data four computational cases are considered: the first corresponds to a single-layer steel plate with properties

\[
E_p = 200 \text{GPa}, \quad \nu_p = 0.3, \quad \rho_p = 7800 \text{kg/m}^3, \quad \delta_p = 0.02 \text{;}\n\]

the second and third - a two-layer plate having on one side a coating of rubber-like material \(t_c = 2 \text{mm}\) thickness and acts as a first or second layer of the plate within the established models, respectively, with the following properties of the coating

\[
E_c = 5 \text{MPa}, \quad \nu_c = 0.36, \quad \rho_c = 500 \text{kg/m}^3, \quad \delta_c = 1.2 \text{;}\n\]

fourth - three-layer plate with the outer layers are made of rubber and middle layer is from steel. It is believed that the plate is surrounded on both sides by air having the properties

\[
\rho_1 = \rho_2 = \rho = 1.225 \text{kg/m}^3, \quad c_1 = c_2 = c = 340 \text{m/s}\]
The calculations were performed for given size $l_1 = l_2 = 2.3$ m, the value of the parameter $U_\ast = 0.01$ mm and the thickness of layer rigid partitions $h_k = 0.3$ m (fig. 2) having elastic properties, density and internal damping parameter

$E_p = 16$ GPa, $\nu_p = 0.17$, $\rho_p = 500$ kg/m$^3$, $\delta_p = 0.02$

Each layer of the plate was divided into 20 layers. As shown by numerical experiments adopted by the mesh size in the direction of plate thickness allows to obtain numerical solutions, almost approaching to solutions on the basis of three-dimensional equations of elasticity.

In Figure 3 graphs of dependencies of the soundproofing parameter $R_p$ on the frequency $f = \omega/(2\pi)$ are illustrated for the four cases described above. Here, the calculation results for the first case shown by the solid line, the second - a bar, for the third - dashed for the fourth - the dotted dash. Almost complete coincidence of the lines for the examined four cases give evidence of that the presence energy absorbing rubber layer on the steel plate has no influence on the parameter $R_p$ of the considered mechanical system.

Variation of deflection at the center of face surface of first layer of plate $\tilde{w}^{(1)}$ at two different bands of frequency $f$ are illustrated in fig.4-5. It is obvious that there exist such frequencies of sound wave, at which significant bursts in the deflection values $\tilde{w}^{(1)}$ are observed (fig.4) due to the coincidence of the frequency of natural oscillations of the concerned mechanical system with the oscillation frequency of the incident harmonic waves. Analyzing the results, it can be seen that the addition of the energy absorbing rubber layer onto the plate leads to a substantial reduction of the deflection values at resonance frequencies $f^R$ (fig.5). This conclusion is consistent with the results presented in [23, 24].

Along with this, there is also a slight change in the frequencies $f^R$ themselves. We note that in fig.5 plot of dependence $\tilde{w}^{(1)} = \tilde{w}^{(1)}(f)$ for the third computation case is not listed as it fully coincided with the curve calculated for the second case.

There were also computational experiments to determine the law of variation for parameters of the stress-strain state of the plate through the thickness. For illustrative purposes, in Table 1, for the center point of the plate, the variation of the deflection values $\tilde{w}$, resulted normal stresses $\sigma_{xx} = \sigma_{11}$, $\sigma_{zz} = \sigma_{33}$ and shear stresses $\tau_{xz} = \sigma_{13}$ through the plate thickness are shown. The calculation results are
presented for the second, third and fourth cases calculated at fixed frequency $f$. Results for the first computational case are not presented, as they do not differ from the results obtained on the basis of the classical Kirchhoff-Love model.

Analyzing the graphs, it can be seen that the complicated law of variation exists for stress-strain state through the thickness of the plate, which indicates the necessity for describing the deformation mechanics of plates within the present class of problems on the basis of the refined equations [20]. It should be noted that under certain combinations of mechanical and geometrical parameters of the studied mechanical system, there are significant shifts of emerging stresses on the interfaces of the layers of materials with different mechanical properties. They, in particular, due to the complex nature of interaction of different waves propagating in the plate, including those reflected from interfaces between layers in thickness direction.

It is supposed that the results are of fundamental importance. They suggest that at high frequencies of the acoustic interaction, the damage of laminated plates of this class is likely to occur at the layer interfaces. It should be noted that at low frequencies (see e.g. $f = 50 \text{ Hz}$), the main stresses in the plate are normal stresses and shear stresses. At high frequencies (see e.g. $f = 1000 \text{ Hz}$) the maximum magnitude of the stress $\sigma_z$, corresponding to the deformation of the transverse compression, are comparable or even higher than the magnitudes of stress $\sigma_x$.

**Table 1.** The law of variation for parameters of the stress-strain state of the plate through the thickness

| $f$ | $\tilde{w}$ | $\tilde{w}$ |
|-----|-------------|-------------|
| 50 Hz | $\cdot 10^{-15}$ m | $\cdot 10^{-7}$ m |
| 1000 Hz | $\cdot 10^{-15}$ m | $\cdot 10^{-7}$ m |

**Figure 4.** Relationship $\tilde{w}^{(1)} (f)$

**Figure 5.** Relationship $\tilde{w}^{(1)} (f)$
third computational case

\[ \delta_x \cdot 10^{-5} \text{Pa} \]

\[ \delta_y \cdot 10^{-5} \text{Pa} \]

\[ \delta_z \cdot 10^{-5} \text{Pa} \]

\[ \sigma_{xz} \cdot 10^{-5} \text{Pa} \]

\[ \tau_{yz} \cdot 10^{-5} \text{Pa} \]

\[ \tau_{xz} \cdot 10^{-5} \text{Pa} \]

\[ \hat{w} \cdot 10^{-5} \text{m} \]

\[ \hat{w} \cdot 10^{-7} \text{m} \]

\[ \delta_x \cdot 10^{-5} \text{Pa} \]

\[ \delta_y \cdot 10^{-5} \text{Pa} \]

\[ \delta_z \cdot 10^{-5} \text{Pa} \]
fourth computational case
We note that stresses resulted in rubber layers $\sigma_x$, $\sigma_y$, $\tau_{xy}$, at low frequencies are much less than the similar stresses resulted in the steel plate. In this regard, in the presented figures on the scale of the graph axes they look like zero. At high frequencies, stress shifts formed at the boundaries between the layers differing significantly by elastic characteristics are much more similar to stresses emerging in other areas.

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