New generation of theoretical models for the thermal response of geothermal heat exchangers

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Abstract. The efficient harnessing of geothermal energy for the heating and cooling of buildings requires extensive simulations to be carried out during the detailed design phase of a new building. These simulations, performed using semi-analytical models, serve to assess the thermal response of the geothermal heat exchanger during the building’s lifetime, that can easily exceed 100 years. By using asymptotic expansion techniques, a new generation of theoretical models is being developed since 2011 that offer the flexibility and accuracy of the best performing models available nowadays, but at a fraction of their computational cost. These new models open the doors to yet untapped design and optimization possibilities.

1. Introduction
Thanks to its widespread availability and energy savings potential, geothermal energy is considered as one of the best renewable energy sources for the efficient heating and cooling of buildings [1]. A geothermal HVAC (heating, ventilation, and air conditioning) system consists in a water-to-water heat pump connected to a geothermal heat exchanger, like the one shown in Figure 1, composed of multiple vertical geothermal boreholes. Each of these boreholes is equipped with two or more pipes through which a heat carrying liquid flows and exchanges heat with the surrounding ground. To avoid the cross-contamination of aquifers, and at the same time promote the heat exchange between the pipes and the ground, boreholes are normally filled up with grout.

The correct sizing of geothermal heat exchangers plays a fundamental role in the successful harnessing of geothermal energy. If the heat exchanger is too small, lower/higher inlet temperatures for the heat carrying liquid are required for the heating/cooling of the building, negatively affecting the energy efficiency of the heat pump, and consequently of the whole HVAC system. On the contrary, if the heat exchanger is too large, the resulting initial investment costs will be too high, leading to unreasonable payback times.

The aforementioned sizing is done during the detailed design phase of a new building, using for it extensive numerical simulations to assess the thermal response of the geothermal heat exchanger during the building’s lifetime, that can easily exceed 100 years. The semi-analytical models to employ in these simulations shall therefore be as accurate as possible, to minimize the uncertainties, be as flexible as possible, to correctly reproduce the unique features of each project, and be as fast as possible, to allow different configurations and/or design choices to be explored and tested in reasonable amounts of time.
Figure 1. Sketch of a typical geothermal heat exchanger composed of multiple vertical geothermal boreholes connected to a network of distribution pipes. Each of these boreholes is equipped with two or more pipes through which a heat carrying liquid flows and exchanges heat with the surrounding ground.

Two models have dominated the research field for the past 30 years: the Superposition Borehole Model (SBM) [2, 3] and the \( g \)-function model [4]. The former one presents the desired levels of accuracy and flexibility, but its computational cost is, even nowadays, very high. As an example, the simulation of 100 years of operation of a real-world geothermal heat exchanger composed of 100 irregularly placed boreholes takes SBM several days of computing time on a standard PC. The \( g \)-function model, on the contrary, exploits a large database of precomputed heat exchanger configurations to deliver results in just seconds [5, 6]. The limited size of that database, however, whose extension with new entries is computationally expensive [7, 8, 9, 10, 11, 12, 13], implies a loss in accuracy and flexibility. Thus, none of these two models presents all the desired features, and new theoretical models need to be developed for that.

In 2011 the author started a review from scratch of the whole heat transfer problem underpinning the unsteady thermal response of geothermal heat exchangers and their surrounding ground, as the large disparity in time and length scales of the problem was susceptible to be exploited using asymptotic expansion techniques [14, 15, 16]. These mathematical methods, which extract the most relevant information from a given problem with large or small parameters, define a set of simplified problems whose sequential solution delivers the sought result.

From the work performed so far [17, 18, 19, 20, 21, 22, 23], a new semi-analytical model is being created that features the accuracy and flexibility of SBM, but with a computational cost that is more in line with the \( g \)-function model’s one. As an example, the simulation of 100 years of operation of a geothermal heat exchanger composed of 100 irregularly placed boreholes requires two minutes on a single core of an Intel Core i7-7700K processor. These simultaneous levels of accuracy, flexibility, and speed open the doors to yet untapped design and optimization possibilities. The aim of the present article is to give an overview of the on-going work and of its possibilities in the near future.

2. Design criteria for geothermal heat exchangers
As pointed out before, the correct sizing of geothermal heat exchangers represents a crucial step in the successful harnessing of geothermal energy. The three main criteria steering that sizing
process are the prescribed minimum energy efficiency for the geothermal HVAC system, the maximum acceptable payback time for the whole installation, and the maximum and minimum admissible operating temperatures for the heat carrying liquid.

2.1. Prescribed minimum energy efficiency
The continuous exchange of heat with the ground alters the temperatures in the vicinity of the boreholes. Due to the limited heat conduction capability of the ground, these thermal perturbations take centuries to dissipate, which forces the temperature of the heat carrying liquid to be continuously raised/lowered in order to keep injecting/extracting the same amounts of heat to/from the ground. This drift in the inlet temperature of the geothermal heat exchanger, which worsens over time, has negative consequences for the energy efficiency of the heat pump, and therefore also for the energy efficiency of the whole HVAC system.

In order to ensure a prescribed minimum energy efficiency is fulfilled at all times, geothermal HVAC designers look at the performance of the system at the end of its lifespan, of typically 100 years, as it represents the worst case scenario. This is accomplished by numerically time-marching a theoretical model for the thermal response of the geothermal heat exchanger, from its initial commissioning till the end of its expected lifespan. Depending on the complexity, and therefore accuracy, of the employed model, the required computing times vary, being of a few seconds for the $g$-function model and of a few days for the SBM.

2.2. Maximum acceptable payback time
The just described drift in the inlet temperature diminishes with the number of boreholes and their depth. But increasing the size of the geothermal heat exchanger implies higher initial investment costs, which brings up the second design criteria, the payback time. It represents the time it takes to compensate the higher initial investment costs with the extra savings brought in by the higher energy efficiency. In Spain, for instance, a property developer considers a geothermal HVAC system to be economically viable, if its payback time is below 10 years.

Thus, to assess the payback time of a geothermal HVAC system it is sufficient to look at its first 10 to 15 years of operation. The thermal response of the geothermal heat exchanger during that timespan is automatically obtained from the numerical time-marching of the theoretical model performed for the enforcement of the previous design criteria. The 85 to 90 years in between, however, are not relevant for the design process, but must nevertheless be computed in order to get from the initial commissioning of the HVAC system till its expected lifespan.

2.3. Maximum and minimum admissible fluid temperatures
The most challenging heating and coolings needs of a building are caused by the hottest and coldest moments of the year. Fortunately, these only occur during short periods of time, of hours or days, so that their energy relevance in the overall heating and cooling of the building is small. Hence, to design a geothermal HVAC system to perform optimally during these peak loads is not advisable, as it leads to excessively large installations with unacceptable payback times. Instead, a geothermal HVAC system is sized to perform optimally during the rest of the year, accepting its subpar performance during these peak loads.

To extract/inject the extra amounts of heat demanded by the peak heating/cooling loads, lower/higher than usual temperatures are required at the inlet of the geothermal heat exchanger. Certain temperature limitations need to be obeyed, though, in order to ensure the integrity and safety of the installation and the environment. So, temperatures below the freezing point of the heat carrying liquid must be avoided. Without the use of anti-freezing additives, this means $0^\circ$C. On the other hand, temperatures above the maximum operating temperature of the pipe materials, for example $40^\circ$C for PE, are also forbidden. Even though higher inlet
temperatures are possible with other pipe materials like PEX, environmental concerns regarding the mechanical and chemical impact onto the surrounding ground start to play a role.

2.4. Optimization process

To simultaneously satisfy the prescribed minimum energy efficiency, the maximum acceptable payback time, and the maximum and minimum admissible fluid temperatures represents a challenging optimization problem that is solved by trying out different configurations and design choices for the geothermal heat exchanger. One such design choice, for instance, is the incorporation of a secondary HVAC system, so that the extra amounts of heat demanded by the peak loads are supplied by other means than geothermal energy.

Each try of a possible configuration and/or design choice implies the numerical time-marching of the already mentioned theoretical model, which highlights the strong impact its computational cost has on the design process of geothermal HVAC systems. Hence, the availability of an accurate, flexible, and fast theoretical model for the thermal response of geothermal heat exchangers would have a profound impact on the successful harnessing of geothermal energy.

3. Thermal response of geothermal heat exchangers

As shown in Figure 1, a geothermal heat exchanger is composed of multiple vertical geothermal boreholes separated a distance \( r_{ij} \) of the order of meters. Typical geothermal boreholes present depths \( H \) in the range of a hundred meters and radii \( r_b \) in the range of tens of centimeters, leading to aspect ratios \( \Lambda = H/r_b \) in the range of thousands. Three characteristic times can be defined with the just introduced length scales and the thermal diffusivity \( \alpha \) of the ground, namely the characteristic longitudinal diffusion time \( t_H \sim H^2/\alpha \), the characteristic transversal diffusion time \( t_b \sim r_b^2/\alpha \), and the characteristic inter-borehole diffusion time \( t_d \sim r_{ij}^2/\alpha \). Taking into account the typical values for the different variables involved, these characteristic times are of the order of centuries, hours, and years, respectively [17, 18, 19, 21].

Two additional characteristic times can be identified in the problem. The first one is the residence time \( t_r \sim H/V \) of the heat carrying liquid in the pipes, where \( V \) is the bulk velocity of the flow. Since that flow in the pipes is kept turbulent, in order to promote the energy exchange between the heat carrying liquid and the pipe walls, \( t_r \) is of the order of tens of minutes, leading to the following sequence of characteristic times:

\[
t_r \ll t_b \ll t_d \ll t_H.
\]

The use of asymptotic expansion techniques for the analysis of the thermal response of geothermal heat exchangers is motivated by precisely this sequence of characteristic times, and especially by their large disparity in time scales.

The fifth and last characteristic time is related to the heat injection rate \( Q(t) \) imposed onto the geothermal heat exchanger. The value of \( Q(t) \) varies depending on the heating and cooling needs of the building, which change on an hourly, daily, weekly, monthly, and yearly basis. Therefore, it presents a wide spectrum of characteristic heat injection times \( t_q \) that goes from minutes up to decades. How these \( t_q \) compare to the previous sequence of characteristic times defines the different regimes of operation of the geothermal heat exchanger. As shown next, these can be linked to the three design criteria presented before.

3.1. Time-periodic thermal response

To enforce the prescribed minimum energy efficiency, the thermal response of the geothermal heat exchanger at the end of its lifetime, for instance 100 years, is required. In this case, the heat injection rate presents two different ranges of characteristic heat injection times \( t_q \). First, a range that goes from minutes up to one year, related to the variations experienced by the
heating and cooling needs of the building during that 100\textsuperscript{th} year. Second, a single value of order 100 years, related to the buildup of the thermal perturbation in the ground caused by the 100 years of continuous operation of the geothermal HVAC system.

Since the established strategy of time-marching a theoretical model through the whole lifetime of the system is computationally expensive, a different approach was pursued by the author [19, 21]. By assuming the thermal response of the geothermal heat exchanger to be time-periodic, it is possible to directly analyze the long-term thermal response of the system without requiring the expensive time-marching of a theoretical model. The time-periodic assumption, in fact, corresponds to analyzing the geothermal HVAC system after an infinite number of years, which in turn represents a conservative approach to the design problem.

The main advantage of the time-periodic assumption is that the whole problem can be expanded in Fourier series, which is very convenient for the development of theoretical models for the thermal response of geothermal heat exchangers. The Fourier series expansion of a time-periodic variable, for instance the heat injection rate $Q(t)$, is given by

$$Q(t) = \sum_{n=-\infty}^{\infty} \hat{Q}_n e^{i\omega_n t},$$

where $\hat{Q}_n$ is the $n$\textsuperscript{th} harmonic, $i = \sqrt{-1}$, $\omega_n = 2\pi n/T$ is the angular frequency of the $n$\textsuperscript{th} harmonic, making $T$ the annual period, and $t$ is time.

In the Fourier series expansion of $Q(t)$, the summand for $n = 0$ represents the mean annual heat injection rate, which is a continuous and uninterrupted injection/extraction of heat to/from the ground. Hence, after 100 years of operation, it is the primary responsible for the buildup of the thermal perturbation in the ground. In the time-periodic assumption used here, the geothermal heat exchanger responds in a steady-state manner to that mean annual heat injection rate, and the full sequence of characteristic times becomes

$$t_r \ll t_b \ll t_d \ll t_H \ll t_q.$$

This sequence of characteristic times has already been exploited using asymptotic expansion techniques to obtain the steady-state thermal response of a single borehole [18] and of multiple thermally-interacting boreholes [20]. In the first case fully analytical expressions are obtained, while semi-analytical results are derived in the second case.

The remaining terms in (1) represent subannual harmonic heat injection rates related to the inter-annual variations of the heating and cooling needs of the building. Expressed in terms of the sequence of characteristic times of the problem, and postponing to Subsection 3.3 the case in which $t_q$ is of the order of minutes or hours,

$$t_r \ll t_b \ll t_q \sim t_d \ll t_H.$$

By using asymptotic expansion techniques, this large disparity in time scales has successfully been exploited to obtain the time-harmonic thermal response of geothermal heat exchangers. In this case, fully analytical expressions have been obtained for both configurations, a single borehole [21] and multiple thermally-interacting boreholes [19].

3.2. Unsteady thermal response

To analyze the payback time of the installation, the first 10 to 15 years of operation of the geothermal HVAC system are considered. Expressed in terms of the characteristic heat injection time $t_q$, a continuous range of time scales, spanning from minutes up to 10 or 15 years, is expected
in this case. Postponing again to Subsection 3.3 the case in which \( t_q \) is of the order of minutes or hours, the resulting full sequence of characteristic times of the problem is

\[ t_r \ll t_b \ll t_q \sim t_d \ll t_H. \]

Again, this large disparity in time scales allows the problem to be tackled using asymptotic expansion techniques. However, the resulting analysis of the unsteady thermal response of geothermal heat exchangers has not been completed yet. Therefore, what comes next is a brief outline of the on-going work.

In order to handle the time dependency of the problem, which represents the main new burden to overcome, the Laplace transform is used to convert time derivatives and integrals into algebraic operators. The resulting system of integro-differential equations presents strong similarities with the one describing the time-harmonic thermal response of geothermal heat exchangers [19, 21]. This is no coincidence, as the sequence of characteristic times is the same. Hence, the thermal behavior of the geothermal heat exchanger is governed by the same physical phenomena, the same simplifying assumptions, and the same resulting equations. Once the solution in the Laplace plane is known, the temporal evolution of the geothermal heat exchanger is recovered. For it, a fast numerical Laplace inversion formula has recently been developed by the author [22].

The theoretical analysis of the unsteady thermal response of a single geothermal borehole has almost been completed now, and a publication describing the developed theory is being prepared [24]. On the contrary, the extension to multiple thermally-interacting boreholes is still at an early stage. Nevertheless, some preliminary results will be shown in Section 4.

3.3. Transient thermal response

The last case to consider is when the characteristic heat injection time \( t_q \) is of the order of minutes or hours. It represents the thermal response of a geothermal heat exchanger to the peak heating and cooling loads, triggered by the coldest and hottest moments of the year, respectively. Assuming the residence time \( t_r \) is comparable to the characteristic transversal diffusion time \( t_b \), which is the most challenging case to analyze, the full sequence of characteristic times becomes

\[ t_r \sim t_b \sim t_q \ll t_d \ll t_H. \]

The analysis of this regime of operation differs from the ones discussed so far, in which the characteristic heat injection time \( t_q \) is large compared to the characteristic transversal diffusion time \( t_b \). In them, the borehole and the ground located at distances from the borehole comparable to the borehole radius \( r_b \) respond in a quasi-steady manner to the applied heat injection rate. This allows important simplifications to be applied to the governing equations, which ultimately contribute to the successful theoretical analysis and modeling of those regimes [17, 18, 19, 21].

On the contrary, when \( t_r \sim t_b \sim t_q \), the borehole and its surrounding ground respond in an unsteady way, with the thermal inertia of the heat carrying liquid, grout, and ground playing a central role. To tackle this regime of operation, the author’s research group is developing an enhanced version of the classical multipole method used for the computation of thermal resistances in geothermal boreholes [25, 26]. It is already clear, from the work performed so far, that the theory under development will be successful, but no preliminary results can be shown at the time of writing the present article.

4. Analysis of a geothermal heat exchanger composed of 100 boreholes

The capabilities of the theoretical model under development are demonstrated in the present section by analyzing the geothermal heat exchanger shown in Figure 1. The 100 identical boreholes, of depth \( H = 120 \) m and diameter \( 2r_b = 152 \) mm, are placed in a \( 10 \times 10 \) array with a
4 m spacing between adjacent boreholes. The considered geographical location presents a mean annual temperature of 14°C and a geothermal heat flux of 0.06 W/m². Due to the light-sand character of the considered ground, with a thermal conductivity of 1.4 W/(mK) and a thermal diffusivity of \( \alpha = 8.68 \cdot 10^{-7} \text{m}^2/\text{s} \), the boreholes are filled with grout up to half a meter from the ground surface, leading to a so-called buried depth of 0.5 m. The employed grout has a thermal conductivity of 1.5 W/(mK) and a thermal diffusivity of \( 4.63 \cdot 10^{-7} \text{m}^2/\text{s} \).

Each borehole is equipped with two straight pipes connected at their bottom, forming a so-called U-shaped probe. Both pipes have an outer diameter of 40 mm and a wall thickness of 3.7 mm, and are made of PE with a thermal conductivity of 0.42 W/(mK). The placement of the pipes inside each borehole is as shown in Figure 1, with a gap of 5 mm between them and between each pipe and the borehole wall. The upper ends of each U-shaped probe are connected in parallel to a network of distribution pipes, through which a mass flow rate of 0.150 kg/s is supplied to each borehole. The employed heat carrying liquid is pure water with a density of 999 kg/m³, a specific heat capacity of 4184 J/(kgK), a thermal conductivity of 0.577 W/(mK), and a dynamic viscosity of \( 1.138 \cdot 10^{-3} \text{kg/(m s)} \).

Using Gnielinski’s correlations for the computation of convective heat transfer coefficients in pipes [27], the formulae in [17, 18, 21] for the computation of pipe’s inner thermal resistances, and the multipole method for the determination of borehole thermal resistances [25, 26], the following values for the so-called borehole’s thermal resistance \( R_b \), borehole’s inner thermal resistance \( R_a \), and borehole’s thermal skewness parameter \( S \) are obtained:

\[
R_b = 0.145 \text{ (m K)/W}, \quad R_a = 0.353 \text{ (m K)/W}, \quad S = 0.
\]

The water-to-water heat pump feeding the network of distribution pipes imposes onto the geothermal heat exchanger the heat injection rates shown in Table 1, where negative values mean the heat is extracted from the ground. For the sake of simplicity, constant values for the heat injection rate are assumed for each month, and the proposed pattern is repeated for each year (365.25 days) during the whole lifetime of the building.

|       | Jan | Feb | Mar | Apr | May | Jun | Jul | Aug | Sep | Oct | Nov | Dec |
|-------|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| \( Q \) [kW] | -200 | -175 | -100 | -25 | 75  | 175 | 250 | 200 | 125 | 25  | -75 | -175 |

4.1. Time-periodic thermal response

Figure 2 shows the time-periodic thermal response of the considered geothermal HVAC system throughout the 12 months of the year. The imposed heat injection rate \( Q \) is represented as vertical bars and read from the left vertical axis, whereas the resulting inlet temperature \( T_{in} \) is represented as a solid line and read from the right vertical axis.

The considered geographical location, which resembles Madrid, has a relatively high mean annual temperature of 14°C, that combined with the higher cooling than heating needs of the building leads to relatively high inlet temperatures. Even in wintertime, where the building requires up to 200 kW of heating power, the lowest inlet temperature of 10°C is well above the freezing temperature of the heat carrying liquid. On the contrary, during the summer, the 250 kW of cooling needs raise the inlet temperature to almost 34°C, close to the maximum admissible temperature of the pipe materials.
Figure 2. Time-periodic thermal response of the considered geothermal heat exchanger composed of 100 identical boreholes. The imposed heat injection rate $Q$ is plotted as vertical bars and read from the left vertical axis, whereas the resulting inlet temperature $T_{in}$ of the geothermal heat exchanger is plotted as a solid line and read from the right vertical axis.

The results shown in Figure 2 are obtained using the formulae and methods described in Subsection 3.1 and explained in [17, 18, 19, 20, 21, 23]. Without exploiting any of the spatial symmetries of the geothermal heat exchanger, the required computing time on a single core of an Intel Core i7-7700K processor is 27 s. Although the current implementation of the theoretical model still has some room for improvement, the actual state of the simulation tool is already competitive in terms of accuracy, flexibility, and speed.

4.2. Unsteady thermal response

Figure 3 shows the unsteady thermal response of the considered geothermal HVAC system during its first 10 years of operation. Again, the imposed heat injection rate $Q$ is represented as vertical bars and read from the left vertical axis, whereas the resulting inlet temperature $T_{in}$ is represented as a solid line and read from the right vertical axis.

Since the cooling needs of the building exceed its heating needs, a net injection of heat into the ground occurs after each year of operation. This imbalance progressively heats up the ground surrounding the boreholes, forcing the inlet temperature of the geothermal heat exchanger to be raised over time in order to keep exchanging the same amounts of heat with the ground. This phenomenon can perfectly be observed in Figure 3, where after the initial 10 years of operation the inlet temperature has to be raised by almost 4°C. Most of the temperature raise takes place during the first years of operation. In fact, for the shown example, the inlet temperature only raises 1°C more during the rest of the geothermal HVAC system’s lifetime.

The results shown in Figure 3, which are obtained using the approach and methods described in Subsection 3.2, are very preliminary. As such, the desired levels of accuracy, flexibility, and speed are not met yet. Nevertheless, the required computing time of 98 s on a single core of an
Figure 3. Unsteady thermal response of the considered geothermal heat exchanger during its first 10 years of operation. The imposed heat injection rate $Q$ is plotted as vertical bars and read from the left vertical axis, whereas the resulting inlet temperature $T_{in}$ of the geothermal heat exchanger is plotted as a solid line and read from the right vertical axis.

Intel Core i7-7700K processor is already competitive in terms of speed. As before, no spatial symmetries of the geothermal heat exchanger have been exploited, so that the quoted computing time corresponds to a geothermal heat exchanger composed of 100 irregularly placed boreholes.

5. Conclusions
The successful harnessing of geothermal energy for the efficient heating and cooling of buildings requires the accurate forecasting of the thermal response of geothermal heat exchangers for the whole lifetime of the buildings. This makes the theoretical models employed for that forecasting play a crucial role in the design process of geothermal HVAC systems.

To achieve the desired levels of accuracy, flexibility, and speed, the author initiated in 2011 a review from scratch of the whole heat transfer problem underpinning the thermal response of geothermal heat exchangers and their surrounding ground. By using asymptotic expansion techniques, which exploit the large disparity of time and length scales present in the problem, a new generation of theoretical models is being developed at the moment.

In the present article, an overview of the on-going work has been given by first introducing and discussing the typical criteria used for the design of geothermal HVAC systems. Then, the thermal response of geothermal heat exchangers has been discussed in terms of their characteristic times. By realizing the existence of a well defined sequence of characteristic times, with a large disparity in their values, different regimes of operation of geothermal heat exchangers have been identified, described, and matched with the typical design criteria for geothermal HVAC systems.

For each of the identified regimes of operation, a tailored theoretical modeling approach has been proposed, for which fully analytical, or semi-analytical, expressions for the thermal response
of geothermal heat exchangers can be derived by using asymptotic expansion techniques. Finally, a teaser of the capabilities of the theoretical models under development has been given by analyzing the thermal response of a typical geothermal heat exchanger composed of 100 identical boreholes.

Once completed, and thanks to the targeted levels of accuracy, flexibility, and speed, the theoretical models under development will unleash new design and optimization possibilities that will ultimately lead to an even more successful harnessing of geothermal energy for the efficient heating and cooling of buildings.

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