UNITARIZATION OF STRUCTURE FUNCTIONS AT LARGE $1/x$

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Abstract

We discuss the effects of the $s$-channel unitarization on the $x$ and $Q^2$ dependence of structure functions. The unitarization is implemented at the level of photoabsorption cross sections by resorting to the light–cone wave functions of virtual photons and to the diagonalization property of the scattering matrix in a basis of Fock states of the photon with fixed transverse size. Triple pomeron effects are also explicitly taken into account. We find large unitarity corrections to the structure functions at $x < 10^{-2}$. The results are in very good agreement with the existing NMC and the preliminary HERA data.
1 Introduction.

The conventional QCD evolution predicts a rapid rise of the density of partons in the Regge limit of deep inelastic scattering, $1/x \gg 1$ and large $Q^2$ (for a review see [1]). The gluon distribution is predicted to increase as

$$xg(x, Q^2) \propto \exp \left\{ C \sqrt{\log \frac{1}{\alpha_s(Q^2)}} \log \left( \frac{1}{x} \right) \right\}.$$ (1)

and the same behaviour is expected for the sea density. This leads to violation of the $s$-channel unitarity and of the Froissart bound at large $s = Q^2/x$.

L. V. Gribov, Levin and Ryskin (GLR) proposed a modified nonlinear evolution equation to account for the recombination of partons which suppresses the increase of their densities in the region of very small $x$ [2].

In this paper we suggest to look at the unitarity problem from a different point of view, working at the level of cross sections. The starting point is that the photoabsorption cross section can be derived as an expectation value of the interaction cross sections of the multiparton Fock states of the virtual photon over the wave functions of such states [3, 4]. The $s$-channel unitarity must be imposed on the partial-wave amplitudes of interaction of these Fock states of the photon, and this unitarization of the partial-wave amplitudes can be reformulated in the parton model language as the fusion of partons. As far as the $x$ and $Q^2$ dependence of the total photoabsorption cross section is concerned, the approach [3, 4] is equivalent to the conventional Weizsäcker-Williams reinterpretation [5] of deep inelastic scattering in terms of the (tree-level) Compton scattering off the radiatively generated sea quarks, but has few technical advantages: i) an exact diagonalization of the scattering matrix in the representation of the Fock states of the photon, ii) an easy identification of the partial-wave amplitudes which are subject to the $s$-channel unitarization.

Understanding the unitarization effects becomes crucial in view of the forthcoming data from the HERA experiments, which will go down to $x \sim 10^{-4}$. In this paper we shall offer a semiquantitative analysis of the unitarity effects at large $1/x$, including the estimate of the triple pomeron (fan diagram) contributions, the driving term of which arises from the $qgq$ Fock component of the photon. We extend to small $x$, including the unitarity effects, predictions for the unitarized structure function of the proton $F_2(x, Q^2)$, based on the dynamical generation of the gluon and sea distributions starting with the three constituent quarks [6, 7, 8].

2 The color dipole cross section as a subject of unitarization

At $1/x \gg 1$ photoabsorption can be treated as the scattering on the nucleon of the $q\bar{q}$ pairs the virtual photon transforms into at large distances $\Delta z \sim 1/m_x x \gg R_N$.
upstream the target nucleon (Fig. 1). In this limit the transverse size \( \rho \) of the \( q\bar{q} \) pair and the Sudakov variable \( z \), i.e. the fraction of the photon’s light-cone momentum carried by one of the quarks of the pair \( (0 < z < 1) \), are frozen in the scattering process. The scattering matrix becomes diagonal in the \((\rho, z)\) representation, which allows one to write the transverse (T) and longitudinal (L) virtual photoabsorption cross sections as the quantum mechanical expectation value

\[
\sigma_{T,L}(Q^2) = \int_0^1 dz \int d^2\rho \left| \Psi_{T,L}(z, \rho) \right|^2 \sigma_0(\rho) .
\]  

(2)

Here \( \sigma_0(\rho) \) is the interaction cross section for the \( q\bar{q} \) color dipole of size \( \rho \), given by \[3\]

\[
\sigma_0(\rho) = \frac{16}{3} \alpha_s(\rho) \int \frac{d^2k}{(k^2 + \mu_G^2)^2} V(k) [1 - \exp(ik \cdot \rho)] .
\]  

(3)

In \( V(k) \) is the transverse momentum of the exchanged gluons (Fig.1a), \( \mu_G \approx 1/R_c \) is the effective mass of gluons introduced so that color forces do not propagate beyond the confinement radius \( R_c \). The gluon–gluon–nucleon vertex function \( V(k) = 1 - F_{ch}(3k^2) \), where \( F_{ch}(q^2) \) is the charge form factor of the proton.

The wave functions of the \( q\bar{q} \) fluctuations of the photon were derived in \[3\] and read

\[
|\Psi_T(z, \rho)|^2 = \frac{6\alpha_{em}}{(2\pi)^2} \sum_{N_f} \epsilon_f^2 \left\{ (z^2 + (1 - z)^2) \varepsilon^2 K_1(\varepsilon \rho)^2 + m_f^2 K_0(\varepsilon \rho)^2 \right\} ,
\]  

(4)

\[
|\Psi_L(z, \rho)|^2 = \frac{6\alpha_{em}}{(2\pi)^2} \sum_{N_f} \epsilon_f^2 \left( 4Q^2 \right) z^2 (1 - z)^2 K_0(\varepsilon \rho)^2 ,
\]  

(5)

where \( K_{\nu}(x) \) are modified Bessel functions, \( \varepsilon^2 = z(1 - z)Q^2 + m_f^2 \) and \( m_f \) is the quark mass. Since either \( V(k) \) and/or the factor \( [1 - \exp(ik \cdot \rho)] \) vanish as \( |k| \to 0 \), the dipole cross section \( \sigma_0(\rho) \) is free from infrared divergences. At small \( \rho \)

\[
\sigma_0(\rho) \propto \rho^2 \alpha_s(\rho) \log[1/\alpha_s(\rho)] ,
\]  

(6)

and vanishes as \( \rho \to 0 \).

Eq. \(3\) gives the driving, constant term of the QCD pomeron contribution to the dipole cross section. The \( x \) dependence of the dipole cross section \( \sigma_0(x, \rho) \) comes from the \( t \)-channel iteration of the gluon-exchange between the two gluons in the diagram of Fig.1b. These higher order contributions can be understood as the QCD evolution of the density of gluons in the nucleon, or interactions of the higher \( q\bar{q}g_1...g_n \) Fock states of the photon \[3\]. The convenient way to incorporate this \( x \)-dependence in is as follows:

The driving term \( G(x, Q^2) \) of the distribution of soft, \( x \ll 1 \), gluons with \( 0 < k^2 < Q^2 \), generated perturbatively from the nucleon, is given by \[4, 8\]

\[
G(x, Q^2) = \frac{4}{\pi x} \int_0^{Q^2} \frac{dk^2 k^2}{(k^2 + \mu_G^2)^2} \alpha_s(k^2) V(k) .
\]  

(7)
Comparing eq. (3) and eq. (7) we can rewrite the dipole cross section as

\[ \sigma_0(\rho) = \frac{4\pi^2}{3} \alpha_s(\rho) \int \frac{d\mathbf{k}^2}{k^4} \frac{dG(x, Q^2)}{d \log k^2} (1 - e^{i\mathbf{k} \cdot \rho}) \cdot (xG(x, k^2) \right) \cdot \frac{dG(x, k^2)}{d \log k^2}. \] (8)

The generalized QCD ladder diagrams of Fig.1b are summed by replacing the driving term of the gluon flux \( G(x, Q^2) \) with the full GLDAP–evolved gluon distribution \( g(\text{GLDAP}, x, Q^2) \) of our previous work \([7, 8]\), i.e.

\[ \sigma_0(x, \rho) = \frac{4\pi^2}{3} \alpha_s(\rho) \int \frac{d\mathbf{k}^2}{k^4} (1 - e^{i\mathbf{k} \cdot \rho}) \cdot dG(\text{GLDAP}, x, k^2) \cdot d \log k^2. \] (9)

This GLDAP dipole cross section (9) must then be used in calculation of the photoabsorption cross section (2) and, eventually, of the GLDAP structure function using the definition

\[ F_2(x, Q^2) = \frac{Q^2}{4\pi^2 \alpha_{em}} \sigma_\gamma^N(x, Q^2). \]

Since \( \sigma(x, \rho) \sim \rho^2 \alpha_s(\rho)g(x, Q^2) \), in view of Eq.(1) at small \( x \) it violates both the Froissart bound and the \( s \)-channel unitarity.

Now, let us identify the corresponding partial waves using the conventional impact parameter representation for the scattering amplitude

\[ f_0(\mathbf{q}) = 2i \int d^2\mathbf{b} \Gamma_0(\mathbf{b}) \exp(-i\mathbf{b} \cdot \mathbf{q}). \] (10)

For the predominantly imaginary scattering amplitude \( f_0(\mathbf{q}) = i\sigma_0(x, \rho) \exp(-B_0q^2/2) \) the profile function (partial-wave amplitude) \( \Gamma_0(\mathbf{b}) \) of the GLDAP dipole cross section (9) equals

\[ \Gamma_0(\mathbf{b}) = \frac{\sigma_0(x, \rho)}{4\pi B_0} \exp \left( -\frac{\mathbf{b}^2}{2B_0} \right) \] (11)

and violates the \( s \)-channel unitarity constraint \( \Gamma(\mathbf{b}) \leq 1 \) when \( \sigma_0(x, \rho) \) becomes large. (By geometrical considerations, the diffraction slope \( B_0 \) can be related to the conventional hadronic slope for \( \pi p \) scattering \( B_{\pi p} \), through \( B_0 = B_{\pi p} / 2 + \rho^2 / 4 \).)

We encounter, in fact, the as yet unsolved problem of unitarization of the rising strong interaction cross section. Two different schemes are commonly adopted for construction of the unitarized partial-wave amplitude \( \Gamma(\mathbf{b}) \) : i) the eikonal unitarization

\[ \Gamma(\mathbf{b}) = 1 - \exp[-\Gamma_0(\mathbf{b})], \] (12)

and the \( K \) matrix unitarization

\[ \Gamma(\mathbf{b}) = \frac{\Gamma_0(\mathbf{b})}{1 + \Gamma_0(\mathbf{b})}. \] (13)

In both cases as the bare profile function \( \Gamma_0(\mathbf{b}) \) rises, the unitarized one \( \Gamma(\mathbf{b}) \) tends to the black disk limit \( \Gamma(\mathbf{b}) \to 1 \).
The unitarized total cross sections \( \sigma(x, \rho) = 2 \int d^2 b \Gamma(b) \) reads

\[
\text{eikonal: } \sigma(x, \rho) = 4\pi B [\log(\eta(x, \rho)) + E_1(\eta) + \gamma] ;
\]

\[
\text{K matrix: } \sigma(x, \rho) = 4\pi B \log(1 + \eta(x, \rho)),
\]

where \( \gamma \) is the Euler–Mascheroni constant, \( E_1 \) is the integral exponential function. The quantity \( \eta(x, \rho) = \sigma_0(x, \rho)/4\pi B_0 \) shown in Fig.2, controls the effect of the unitarization:

\[
\sigma(x, \rho) \approx \sigma_0(x, \rho) \text{ at } \eta(x, \rho) \ll 1, \text{ and at } \eta(x, \rho) \gg 1 \text{ the unitarization suppresses the cross section, } \sigma(x, \rho) \ll \sigma_0(x, \rho).
\]

This plot shows that the unitarization effects are important only at large \( \rho \). The difference between the two cross sections (14) and (15) can be taken as a measure of the theoretical uncertainty on the unitarization procedure. For convenience we shall adopt hereafter the \( K \)-matrix procedure, the eikonal unitarization gives similar, if somewhat weaker unitarization effects.

3 Unitarity and the triple pomeron contribution

To the leading order in the \( s \)-channel unitarization \( \Gamma(b) = \Gamma_0(b) - \chi \Gamma_0(b)^2/2 \), where \( \chi = 1(2) \) for the eikonal (K-matrix) unitarization, and the unitarized photoabsorption cross section reads

\[
\sigma_{\gamma N}(x, Q^2) = 2 \int d^2 b \Gamma_0(b) = \langle \sigma_0 \rangle - \chi \langle \int d^2 b \Gamma_0(b)^2 \rangle,
\]

where we have denoted by \( \langle \cdot \rangle \) the average over the photon wave function. The first term in eq. (16) gives the GLDAP cross section and the conventional GLDAP structure function, the second term is the unitarity (shadowing) correction, which in principle is a nonlinear functional of the GLDAP cross section. The unitarity correction can be related to the cross section of the forward diffraction dissociation of the virtual photons \( \gamma^* + p \to X + p \), where \( X = q\bar{q} \), as follows [4]:

\[
\sigma_D(\gamma^* \to q\bar{q}) = \langle \int d^2 b \Gamma_0(b) \rangle = \int dt dM^2 \frac{d\sigma_D(\gamma^* \to q\bar{q})}{dM^2 dt} \bigg|_{t=0},
\]

\[
\left. \frac{d\sigma_D(\gamma^* \to q\bar{q})}{dt} \right|_{t=0} = \int dM^2 \frac{d\sigma_D(\gamma^* \to q\bar{q})}{dM^2 dt} \bigg|_{t=0} = \frac{1}{16\pi} \langle \sigma_0^2 \rangle.
\]

Here \( B_D \) stands for the diffraction slope of the diffraction dissociation. The driving term of diffraction dissociation is excitation of the \( q\bar{q} \) component of the photon, which has the \( \propto 1/(Q^2 + M^2)^2 \) mass spectrum and gives the \( x \)-independent shadowing term. Diffraction excitation of the higher \( q\bar{q}g_1...g_n \) Fock states of the photon gives rise to the triple pomeron component of the mass spectrum at \( M^2 \gg Q^2 \):

\[
\frac{d\sigma_D(\gamma^* \to q\bar{q}g...)}{dt dM^2} \bigg|_{t=0} = G_{3\text{FP}} \frac{1}{M^2},
\]
and to the logarithmically, $\propto \log(s) \propto \log(1/x)$ , rising shadowing component.

We shall present a brief discussion of the effects of the $q\bar{q}g$ Fock state, which is the driving term of the triple-pomeron mass spectrum. We apply to the $q\bar{q}g$ system the same wave function formalism already used for the $q\bar{q}$ component:

$$
\int dM^2 \frac{d\sigma_D(\gamma^* \rightarrow q\bar{q}g)}{dt dM^2} \bigg|_{t=0} = \frac{1}{16\pi} \int_z^1 dz_g \int d\rho \int d^2 \rho |\Phi(r, R, \rho, z, z_g)|^2 \sigma_0(r, R, \rho)^2.
$$

(20)

Here we have introduced the three-particle wave function $\Phi$ and the three-particle interaction cross section $\sigma_0(r, R, \rho)$, where $r$, $R$ and $\rho$ are the $q\bar{q}$, $qg$ and $q\bar{q}$ separations, respectively, and $z_g$ is the fraction of the photon’s light–cone momentum carried by the gluon. It is possible to show \[9\] that the leading contribution to the triple pomeron component comes from the ordering $1/Q^2 \ll \rho^2 \ll r^2 \sim R^2$. In this case the $q\bar{q}$-pair of the $q\bar{q}g$ Fock state can be treated as the pointlike color-octet charge, so that the three-particle cross section $\sigma_0(r, R, \rho)$ reduces to

$$
\sigma_0(r, R, \rho) \rightarrow \frac{9}{4} \sigma_0(r),
$$

(21)

where $9/4$ is the familiar ratio of the octet and the triplet strong couplings. The three-particle wave function takes on the factorized form

$$
|\Phi|^2 = \frac{2}{3\pi^2 z_g} \frac{1}{\alpha_s(\rho)} |\Psi(z, \rho)|^2 \frac{\rho^2}{r_4} F(\mu G r),
$$

(22)

which holds when the gluon of the $q\bar{q}g$ Fock state participates in the interaction, so that only the diagram of Fig. 1c (and not the one of Fig. 1d) should be taken into account and the corresponding three-particle cross section is reduced to half of (21), we refer to \[9\] for a derivation and more details. Here $\Psi(z, \rho)$ is the wave function of the $q\bar{q}$ state and $F(x) = (x^2 K'_1(x))^2$ satisfies $F(0) = 1$ and $F(x) \sim \exp(-2x)$ at large $x$.

Since at $M^2 \gg Q^2$ one has $dM^2/M^2 = dz_g/z_g$, the $1/z_g$ dependence of the wave function (22), which has its origin in the spin 1 of gluons, leads to the mass spectrum of the form (19) and to the $\propto \log(1/x)$ dependence of the shadowing term. Using the wave function (22) and the cross section (21) it can be shown that the triple-pomeron component of the diffraction dissociation cross section (20) satisfies an approximate factorization property:

$$
\frac{d\sigma_D(\gamma^* \rightarrow X)}{dt dM^2} \bigg|_{t=0} \approx \sigma_{\gamma^* N} A_{3IP} \frac{1}{M^2}
$$

(23)

where $A_{3IP}$ only weakly depends on $x$, $Q^2$ and on the flavour of quarks in the $q\bar{q}$ state. This approximate factorization allows one to write down the unitarized structure function in the form

$$
F_2(x, Q^2) = F_2(GLDAP, x, Q^2) \left( 1 - \Delta Sh(q\bar{q}) - \chi \frac{A_{3IP}}{B_{3IP}} \log(1 + \frac{x_0}{x}) \right).
$$

(24)
Here we singled out the shadowing correction from excitation of the $q\bar{q}$ Fock state, which has a strong flavour dependence, $\Delta S h(q\bar{q}) \propto 1/m_f^2$, and vanishes $\propto 1/Q^2$ for the longitudinally polarized photons \cite{4,15}. In Eq. (24) $B_{3\text{IP}}$ stands for the diffraction slope of the diffraction dissociation in the triple-pomeron region, and $x_0 \approx 0.1$ corresponds to the usual definition of the diffraction excitation as interactions in which the target nucleon receives little recoil and emerges in the final state separated from the virtual photon’s debris by the rapidity gap $\Delta \eta = \log(1/x_0) \gtrsim 2$. Diffraction excitation of the $q\bar{q}$ Fock state dominates the mass spectrum at $M^2 \sim Q^2$ and is the counterpart of excitation of resonances in the hadronic scattering, which has the diffraction slope $B_D$ close to that of the elastic scattering, so that here we take $B_D = 10 \left(GeV/c\right)^{-2} \approx B_{\pi N}$, whereas in the triple-pomeron region $B_D = B_{3\text{IP}} \approx B_{\pi N}/2$ (for the review see \cite{13}). More complete treatment of the shadowing, in which one describes the diffraction dissociation in terms of the structure function of the pomeron, does not change much the above simple estimate of the triple-pomeron effect.

4 Discussion of the results

In our approach to DIS at small-$x$ the fundamental quantity is the dipole cross section $\sigma_0(x, \rho)$. We assume that at large $\rho \sim R_N$ it is practically flat as a function of $x$, which gives a smooth connection with the slow rise of the hadronic cross sections \cite{2}. This corresponds also to our treatment of the glue in nucleons at $Q^2 \leq Q_0^2$ as a radiative effect, starting from three valence quarks at the spectroscopic scale, as explained in detail in our previous papers \cite{4,6}. Beyond $Q_0^2$ we calculate $g(x, Q^2)$ using the conventional GLDAP evolution. At small $Q^2$ we use the freezing coupling constant $\alpha_s(Q^2 \leq Q_f^2) = \alpha_s(Q_f^2)$, where $Q_f^2$ is a scale of the same order in magnitude as $Q_0^2$. Also, at $Q^2 \leq Q_0^2$ we neglect the splitting of gluons and the momentum flow from glue to sea. The absolute normalization of the sea structure function is determined by the absolute normalization of the dipole cross section $\sigma_0(\rho)$, which is fixed by the confinement radius $R_c = 1/\mu_G$ and the frozen strong coupling $\alpha_s(Q_f^2)$. These two parameters $\mu_G$ and $Q_f$ are strongly correlated when we ask for simultaneous description of the $\pi N$ total cross section. This procedure corresponds to the minimal infrared regularization and has been already applied with success to various structure function calculations \cite{4,6,3,12}. The choice of parameters used in the present calculations is:

$Q_0^2 = 1.0$ GeV$^2$/c$^2$, $Q_f^2 = 0.5$ GeV$^2$/c$^2$, $\mu_G = 300$ MeV/c$^2$.

With the above set of parameters, in the region of small $Q^2$ we find $A_{3\text{IP}} \approx 0.15$ GeV$^{-2}$, to be compared with the experimental result for the diffraction dissociation of real photons $A_{3\text{IP}} = 0.16$ GeV$^{-2}$ \cite{13}. As stated above, our calculations show that $A_{3\text{IP}}$ changes little, $\lesssim 10\%$, with $Q^2$ and on the flavour of quarks.

We predict the GLDAP and unitarized structure functions with absolute normalization. The predictions for the unitarized structure function $F_2(x, Q^2)$ at different $x$ and $Q^2$ are shown in Fig. 3 and Fig. 4, compared to the existing NMC data \cite{15} at
$x = 0.025$ and to the preliminary HERA data \[17, 16\] at $x = 1.5 \cdot 10^{-3}, 4 \cdot 10^{-4}$. The unitarity correction is numerically large and makes the rise of the structure function with $1/x$ much less steep. Considering the accuracy of the NMC data, the shadowing correction is rather large already at $x \sim 10^{-2}$, where the effect of shadowing of the $q\bar{q}$ Fock state of the photon and the triple-pomeron contribution are of comparable magnitude. At smaller $x$ the triple-pomeron component starts taking over, but the $q\bar{q}$ component also rises slowly as it is proportional to $\langle \sigma(x, \rho)^2 \rangle$. We find very good agreement between our predictions for the unitarized structure function and the experiment. Notice, that the triple pomeron component tends to dominate as $x$ decreases.

In Fig. 5 we compare our prediction for $F_2$ in the very small $x$ region with some parametrizations \[18, 19\], based on fits to the previously published experimental data on structure functions. At very small $x \sim 10^{-4}$ our structure functions follow an approximate law $F_2(x, Q^2) \propto (1/x)^{\Delta(Q^2)}$, where the exponent $\Delta(Q^2) \approx 0.21$ at $Q^2 = 4 \text{GeV}^2/c^2$ and $\Delta(Q^2) \approx 0.311$ at $Q^2 = 15 \text{GeV}^2/c^2$.

In Fig. 6 we present the effects of the unitarization on the charm component of the structure function. We take $m_c = 1.7 \text{GeV}/c^2$. Here the shadowing of the $q\bar{q}$ Fock state is negligible, and the unitarity correction is completely dominated by the triple-pomeron component. The situation with the ratio $R = \sigma_L/\sigma_T$ is similar: the $\Delta Sh(q\bar{q})$ component is small, and the unitarization of the longitudinal structure function is dominated by the triple pomeron component. We find that $R$ is quite structureless and depends very weakly on $Q^2$ at very small $x$. At $x \lesssim 10^{-2}$ we predict $R \approx 0.20 - 0.24$.

After this work was completed, there appeared an estimate of the unitarity effects by Askew et al. \[20\], who use a very different formalism. In \[20\] the parameter which controls the shadowing is the radius $R_g$ of the region of the proton in which the gluons are concentrated. With $R_g = 2 \text{GeV}^{-1}$, which is significantly smaller than the proton's size, Askew et al. find unitarity corrections close to ours at $x \lesssim 10^{-3}$, and have no shadowing at $x \sim 10^{-2}$.

## 5 Conclusions

We have presented a simple approach to the unitarization of rising structure functions at small $x$. The shadowing term in the unitarized structure function is dominated by the triple-pomeron term, which is approximately flavour- and $Q^2$-independent. We find large shadowing corrections already at relatively large $x \sim 10^{-2}$. We find that the dynamical generation of the glue and sea from the valence quarks, which predicts the small-$x$ structure functions with absolute normalization, is in good agreement with the preliminary data from HERA \[17, 16\].
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Figure captions:

Fig.1 - a) Driving term of the QCD pomeron,  b) Generalized ladder diagrams for the QCD pomeron,  c) Interaction of the $q\bar{q}g$ Fock state of the photon which gives the rising contribution to the total cross section,  d) Interaction with the color-octet $q\bar{q}$ component of the $q\bar{q}g$ state which does not contribute to the cross section growth.

Fig.2 - The unitarization parameter $\eta(x, \rho)$.

Fig.3 - Predictions for the GLDAP (dashed curve) and unitarized (solid curve) structure functions at small $x$ compared to the NMC [15] data at $x = 0.025$ and the preliminary H1 [17] and ZEUS [16] data at small $x$. The dot-dashed curve shows the effect of shadowing of the $q\bar{q}$ Fock state of the photon.

Fig.4 - Our prediction for the $x$ dependence of the unitarized structure function at $Q^2 = 15 (GeV/c)^2$ compared to the NMC [14], H1 [17] and ZEUS [16] data.

Fig.5 - Comparison of our prediction for the small-$x$ behaviour of $F_2(x, Q^2)$ (solid curve) with some of the recent MRS [18] and DGJLP [19] parametrizations based on fits to the published data.

Fig.6 - Our prediction for the GLDAP (dashed curve) and the unitarized (solid curve) charm distribution at $Q^2 = 15 (GeV/c)^2$. The effect of shadowing of the $c\bar{c}$ Fock state of the photon is shown by the dot-dashed curve.