Randall-Sundrum Black Holes at Colliders

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In this paper we calculate the evolution of black holes at colliders in the Randall-Sundrum scenario, taking the effect of accretion from the quark-gluon plasma into account. We calculate the evolution using both the canonical and microcanonical ensemble approaches and compare the results to each other and to the well-known corresponding lifetimes for ADD. We find that for an initial mass of 10 TeV the canonical lifetime is of the order of $10^{-25}$ s, which is of the same order of magnitude as its ADD counterpart. In the microcanonical approach, like in ADD, the microcanonical deviations from thermality slow down the evaporation of the black hole, but, unlike in ADD, the black hole does not completely evaporate; it tends to a stable final state of 0.16 GeV. As far as we are aware this is the first prediction of a stable black hole remnant using semiclassical gravity.

I. INTRODUCTION

With the imminent arrival of data from the LHC, much recent interest in black hole events has been focussed on detailed calculations of their signatures [1, 2, 3, 4, 5, 6]. In this paper we return to a more basic question - that of the lifetimes of the black holes. An important question is whether instead of harmlessly evaporating away the black holes might grow. An argument used to dispatch these worries is that cosmic rays bombard white dwarfs in much higher energy collisions than those at the LHC, and so the white dwarfs’ observed lifetimes can be used to put a very low upper bound on the probability of a catastrophic collision [7, 8]. Nevertheless, it is still interesting to calculate the black holes’ lifetimes, as the requirement that they must not grow too large may constrain the models of extra-dimensional physics which give rise to them.

Mini black holes have been quite extensively studied in the ADD scenario [1, 2, 3, 4, 5, 6, 9, 10, 11, 12]. Their lifetimes have been calculated and convincingly shown to be small even when accretion from the quark-gluon plasma is taken into account [13].

Mini Randall-Sundrum (RS) black holes in colliders on the other hand have been much less studied. Most of the literature that exists has considered the warping to be negligible and so the analysis is more or less the same as that for ADD black holes [4, 14].

In [13], Casadio and Harms considered a different kind of black hole solution [16], where the warping gives rise to a tidal term on the four-dimensional brane. They calculated the evolution without accretion of this black hole and found an exponential decay with typical lifetime $> 10^9$ s. This is a surprisingly long time and led us to wonder whether when accretion is taken into account the black hole might be stable or even grow.

Therefore, in this paper we calculate the lifetime of this black hole, including the effect of accretion from the quark-gluon plasma. We give two alternative calculations. The first is the standard, canonical, calculation of the evaporation rate, where the black hole is taken to be in thermal equilibrium with its radiation. The second is the more accurate, microcanonical, calculation, where the effect on the black hole mass of losing radiation is taken into account.

The paper is structured as follows. In section II we set up notation. In section III we derive the accretion rate in RS, which is the same for both the canonical and microcanonical calculations. Then in section IV we calculate the canonical lifetime. In section V we look at the microcanonical black hole evolution, and in section VI we conclude.

II. RS BLACK HOLES

The black hole solution we consider was discovered by Dadhich et al [16]. Its induced metric on the brane is given by

$$ds^2 = -A(r)dt^2 + A(r)^{-1}dr^2 + r^2d\Omega^2,$$

where $A(r) = 1 + \frac{\alpha}{r} + \frac{\beta}{r^2}$, $\alpha = -\frac{2M_{pl}}{m_p}$ and $\beta = -\frac{m_{ew}^2}{m_5}$. $q = \frac{M}{m_{ew}} \left(\frac{m_{ew}}{m_5}\right)^\alpha$ is the tidal term, where $\alpha$ must be $\geq 0$ for black holes of mass $\sim m_{ew}$ to exist. The horizon is at

$$r_H = \frac{M}{m_p} \left(1 + \sqrt{1 + \frac{qm_{ew}^4}{M^2m_5^2}}\right).$$

Some attempts have been made to extend [11] off the brane [17], but as yet no consistent solution has been found. Hence in this paper we consider only on-brane evaporation, and hope that, as in ADD [12], the on-brane radiation is dominant.

1. Here, we use $m_p$, $l_p$ to denote the 4d Planck mass and Planck length respectively, $m_5$ to denote the 5d Planck mass and $m_{ew}$ the electroweak symmetry breaking scale.
III. ACCRETION

We use the same accretion rate as [13]:
\[ \dot{M}_{\text{accr}} = F \pi r_e^2 \epsilon(t), \]
except that our \( r_e \) is different, as [13] consider a (4+d)d Schwarzschild black hole. For us, the greybody factor, \( \Gamma \), is composed of two parts: spin-dependent and geometry-dependent. The spin-dependent part by the geometric optics approximation is given by [14]:
\[ \Gamma_s = \frac{1}{4} \left( \frac{6M \frac{l_p}{m_p} + \sqrt{\left(6M \frac{l_p}{m_p}\right)^2 + 32M \frac{m_p^2 + \alpha r_s^3}{m_{\text{ew}}}}}{M} \right)^{-1}, \]
where
\[ r_c = \frac{1}{4} \left( 6M \frac{l_p}{m_p} + \sqrt{\left(6M \frac{l_p}{m_p}\right)^2 + 32M \frac{m_p^2 + \alpha r_s^3}{m_{\text{ew}}}} \right). \]
Like [13] we take \( \epsilon(t) = 517 GeV/fm^3 \), and the maximum possible value of \( F, F = 1 \), to obtain an upper limit on the lifetimes of the black holes.

IV. CANONICAL EVAPORATION

In 4d the canonical evaporation rate is
\[ \dot{M}_{\text{evap}} = - \sum_s \Gamma_s g^{s}_{\infty} A_3 T^4, \]
where \( T \) is the temperature of the black hole, \( A_3 \) is its event horizon area on the brane, \( \Gamma_s \) is a sum over the spins of the Standard Model particles into which the greybody factor, \( \Gamma_s \), is the effective number of degrees of freedom of each spin and \( \Gamma_s \) is the corresponding greybody factor.

Like [13] we take \( \epsilon(t) = 517 GeV/fm^3 \), and the maximum possible value of \( F, F = 1 \), to obtain an upper limit on the lifetimes of the black holes.

V. MICROcanonical EVAPORATION

The microcanonical luminosity is given by [15]:
\[ \mathcal{L}_{\text{micro}}(M) = A_3 \sum_s \Gamma_s \int n(\omega) \omega^3 d\omega, \]
where
\[ n(\omega) = C \sum_{i=1}^{\frac{\omega}{\alpha}} \exp(S_E(M - i\omega) - S_E(M)), \]
where \( C \) is assumed constant, and \( S_E(x) \) is the Euclidean action of a black hole of mass \( x \).

Hence to find \( \mathcal{L}_{\text{micro}}(M) \), we firstly need to calculate the Euclidean action of [14]. This turns out to be
\[ S_E = \frac{\frac{l_p}{m_p}}{\pi \alpha} \left( \alpha + \frac{2 \beta}{r_H} \right)^{-1} \frac{r_H^2}{m_{\text{ew}}}. \]
For \( M < 10 TeV \), it can be approximated by
\[ S_E = \frac{\pi l_p^2 m_p}{m_{\text{ew}} r_H} M^\frac{3}{2} = \left( \frac{M}{m_{\text{ew}}} \right)^\frac{3}{2} = m^\frac{3}{2}. \]
This disagrees with the action [15] for the RS black hole. Their action is given by [16] for example in [19]. The standard RN Euclidean action includes a contribution from the electromagnetic action,
\[ S_{\text{em}} = \frac{1}{16\pi} \int d^5 x \sqrt{|g|} \frac{1}{2} A_\mu \nabla_{\nu} F^{\mu\nu} + \frac{1}{8\pi} \int d^4 x A_\mu n_\mu F^{\mu\nu}. \]
In the case of Dadhich et al’s black hole, this contribution should not be included as there is only an apparent electric charge in the metric which is not a source for the electromagnetic field.
Substituting (5) into (4), we now obtain
\[ \mathcal{L}_{\text{micro}} = K_0 K_{\text{small}} m^2 e^{-m^2} \int_0^m e^{x^2} (m - x)^3 \, dx, \]  
(6)
In this equation \( K_0 \) and \( K_{\text{small}} \) have between them absorbed all the multiplicative constants. \( K_{\text{small}} = \frac{\sqrt{\pi} m_p^2}{l_p^2} m_{\text{eff}}^5 \), and so can be evaluated, but \( K_0 \) contains a constant which depends on the quantum description of the black hole and so cannot be straightforwardly calculated. This poses something of a problem as we would like to numerically solve the differential equation
\[ \frac{dM}{dt} = -\mathcal{L}_{\text{micro}}(M) + \dot{M}_{\text{accr}} \]
for \( M(t) \). However, in the large \( M \) limit the microcanonical and canonical descriptions coincide, and so we can find \( K \) by matching the microcanonical luminosity to the canonical luminosity in this limit.

A. Microcanonical/Canonical Matching

The large \( M \) limit of the canonical luminosity is easy to calculate as we made no assumptions about the size of \( M \) in deriving (3). So we can take the large \( M \) limit of (3) to obtain
\[ \mathcal{L}_{\text{can}}(M) = \frac{139}{296.720 \pi} \left( \frac{m_p}{l_p} \right)^2 M^{-2}. \]
(7)
The large \( M \) limit of the microcanonical luminosity on the other hand is not so easy, as our derivation is only valid for small mass. For large \( x \),
\[ S_m(x) = 4\pi \left( \frac{l_p}{m_p} \right)^2 m_p x^2 = \left( \frac{x}{m_{\text{eff}}(2)} \right)^2. \]
Therefore, for large \( M \), (4) becomes
\[ \mathcal{L}_{\text{micro}}(M) = K_0 K_{\text{large}} m^2 e^{-m^2} \int_0^m e^{S(x)} (m - x)^3 \, dx, \]
where \( m = \frac{M}{m_{\text{eff}}} \), \( K_0 \) is as in (4), and \( K_{\text{large}} = 4\pi \left( \frac{m}{m_{\text{eff}}} \right)^2 m_{\text{eff}}^{(2)} \). This integral is dominated by wherever \( S(x) \) is largest. \( S \) increases with \( x \) and so we can use the large \( x \) approximation of \( S(x) \sim x^2 \) throughout the integral.
Approximating \( \int_0^m e^{x^2} \, dx \) by an asymptotic series we obtain
\[ \mathcal{L}_{\text{micro}}(M) = \frac{3}{8} K_0 K_{\text{large}} \left( \frac{1}{m^2} + O \left( \frac{1}{m^4} \right) \right). \]
Matching this to equation (7), we finally find:
\[ K_0 = \frac{139}{9 \cdot 120} \pi^2. \]
(8)
Now the integral in (6) can be evaluated.

B. Evolution

(6) cannot be evaluated analytically and so we use Mathematica to solve it numerically. The resulting evolution is shown in Fig. 2.

The graph demonstrates very clearly the change in behaviour from the canonical decay shape to the microcanonical decay shape which prolongs the life of the black hole. The canonical behaviour dominates until \( t \approx 0.22 GeV^{-1} = 1.4 \times 10^{-25} \text{s} \) and the shape of the graph fits with the lifetime of \( 1.6 \times 10^{-25} \text{s} \) that we obtained for the canonical evolution in section IV. After \( t \approx 0.22 GeV^{-1} \), the behaviour looks like an exponential decay asymptoting to zero, as it is in Casadio and Harms’ calculation.

However, the decay is in fact slower than exponential. For very small mass one can calculate an approximate evolution analytically, and find that \( M \sim t^{-\frac{3}{4}} \). It is not clear how to make a meaningful comparison to Casadio and Harms’ result. This is just a different kind of decay. A further difference from Casadio and Harms’ result is that our black hole does not completely decay. It evaporates down to a tiny stable final state of mass 0.16 GeV.
As far as we are aware, this is the first prediction of a stable final state using semiclassical gravity. Previous discussion of stable remnants in the literature have either postulated that instead of semiclassical gravity being valid below the Planck scale and so the black hole evaporating completely, the semiclassical description of black holes breaks down at the Planck scale and a Planck mass remnant remains [10, 21], or they have worked with modifications of semiclassical gravity which have then predicted Planck mass size remnants [10, 21]. Since the mass of our stable remnant is less than the Planck mass, our prediction will only hold if the semiclassical description of black holes is valid below the Planck scale.

It is not clear how different the experimental signature of such a small remnant compared to a complete decay would be. It would be interesting to look at this, but we leave it to future research.
VI. CONCLUSIONS

We have calculated the evolution of black holes at colliders in the RS scenario taking accretion into account, which, as far as we are aware, has not been done before. We calculated the evolution using both the standard canonical method and the more accurate microcanonical method. In the canonical case we found that a black hole of initial mass 10 TeV decayed with a lifetime of $\sim 10^{-25}$ s. This is of the same order of magnitude as the lifetimes of similar sized black holes in ADD. In the microcanonical case we found that as in ADD the microcanonical deviations from thermality act to prolong the life of the black hole. We corrected a mistake in a previous calculation of the lifetime without accretion by Casadio and Harms and found that when accretion is taken into account the microcanonical evolution leaves a stable remnant of 0.16 GeV. As far as we are aware this is the first prediction of a black hole final state using semiclassical gravity. It would be interesting to study the phenomenological consequences of such a remnant.

In this calculation, we only considered radiation on the brane, as the black hole metric has not yet been extended off the brane, so our lifetimes are upper bounds on the actual lifetimes. It may be that once off-brane radiation is taken into account the black hole is found to decay completely in the microcanonical calculation. Nonetheless, we feel that these calculations are interesting progress towards understanding the RS black hole evolution more fully.

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