Parton distributions and small-$x$ QCD at the Large Hadron Electron Collider

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The proposed Large Hadron Electron Collider (LHeC) at CERN would bring Deep-Inelastic scattering into the unexplored TeV regime. The LHeC rich physics program, among other topics, includes both precision SM measurements to complement LHC physics as well as studies of QCD in the high energy limit. The present contribution reports on ongoing studies within the NNPDF framework towards the LHeC CDR. We study the impact of LHeC simulated data on PDF uncertainties, in particular the small-$x$ gluon. We also assess the LHeC potential to disentangle between various scenarios of small-$x$ QCD, including saturation models and small-$x$ resummation. Finally, we explore how deviations from DGLAP can be quantified in inclusive measurements.

Introduction

The Large Hadron Electron Collider (LHeC) \cite{1} is a proposal for a Deep-Inelastic scattering facility in the TeV range which would operate in parallel with the LHC. It would use the 7 TeV LHC proton beam colliding with a high energy electron beam, coming either from a LHC-like ring or from a linear accelerator. The kinematical coverage of such machine would extend the HERA kinematical coverage by two orders of magnitude both in $x$ and in $Q^2$.

After the experience at HERA, it is clear that the LHeC physics potential includes the capability to probe the nucleon structure and its flavour decomposition with very high precision. However, the standard approach to PDF determination \cite{2,3} suffers from several shortcomings. The most important ones are related to the fine-tuning of the PDF parametrizations and the statistical definition of the associated PDF uncertainties to the available dataset.

These shortcomings render difficult its application to extrapolation regions like the LHeC kinematics. In particular, in the standard PDF approach, PDF uncertainties are artificially reduced in extrapolation regions due to relatively simple polynomial parametrizations employed, thus making difficult a quantitative assessment of the impact of new data from unexplored regions into the PDFs. On top of that, subtle deviations from DGLAP evolution which might be present at small-$x$ are difficult to probe with simple fixed functional forms because their lack of flexibility could lead to misleading results.

A method to bypass the above problems has been proposed by the NNPDF collaboration. Within the NNPDF approach \cite{4-5,6,7,8,9} (see also \cite{10}), a combination of neural networks as universal unbiased interpolants with Monte Carlo sampling of experimental data for error propagation render the PDFs and associated uncertainties statistically faithful.

In this contribution we report on ongoing studies of PDF determination and small-$x$ QCD within the NNPDF approach. In particular, we concentrate on LHeC pseudo-data at small-$x$. We consider $F_2(x, Q^2)$ and $F_L(x, Q^2)$ simulated pseudo-data at small-$x$, in a scenario in which the LHeC machine has electron energy of $E_e = 70$ GeV and electron

* The NNPDF methodology has also been applied to other physical problems in \cite{11,12,13}.

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acceptance of $\theta_e \leq 179^\circ$, for an integrated luminosity of $\int L = 1 \text{ fb}^{-1}$. Full NC and CC cross-section simulated pseudo-data in various other machine scenarios are available, and their are under current scrutiny.

The reference baseline for the studies presented in this contribution is the recent NNPDF1.2 parton set [9], a PDF analysis of all relevant inclusive DIS data together with neutrino charm production to constrain strangeness. The kinematics of the pseudo-data, together with that of the NNPDF1.2 analysis are shown in Fig. 1. The average total uncertainty of the simulated $F_2$ pseudo-data is $\sim 2\%$, while that of $F_L$ is $\sim 8\%$.

Constraining parton distributions at small-$x$ In spite of the wealth of precision data on small-$x$ structure functions at HERA, some PDFs, most notably the gluon, have still rather large uncertainties in this region [7]. In order to quantify how these PDF uncertainties would be reduced with LHeC data, we have repeated the NNPDF1.2 analysis with the addition of the LHeC pseudo-data, with central values from the NNPDF1.0 predictions and experimental uncertainties corresponding to the simulated LHeC scenario described above. The joint data set is shown in Fig. 1.

First of all, we include only $F_2$ LHeC pseudo-data into the analysis. Although the fit is as expected perfect, the reduction of the small-$x$ uncertainties is rather moderate, as shown in Fig. 2 (left). Our results therefore indicate that only $F_2$ data, even if very accurate, is not enough to pin down the gluon at small-$x$, due to the fact that the gluon PDF only enters through scaling violations and higher order corrections.

The next step consists of the addition of the complete $F_2$ and $F_L$ pseudo-data. In this case, the joint fit with $F_2$ and $F_L$ pseudo-data leads to a sizable decrease of the small-$x$
gluon uncertainties, which can be understood from the greater sensitivity of $F_L$ to the gluon PDF. To quantify more these results, we have generated LHeC pseudo-data in three different scenarios: one where the gluon is the central NNPDF1.0 gluon and two more were the gluon sits near the associated $\pm 1$-\(\sigma\) envelope. We have repeated the joint $F_2 + F_L$ analysis in these three cases: results are shown in Fig. 2 (right). As expected, after the fit the three extreme scenarios for the small-\(x\) gluon can be precisely disentangled. Therefore, it is clear that the LHeC has the potential to pin down with great precision the behaviour of the low-\(x\) gluon, but only after accurate measurements of $F_L$ are performed.

**Probing small-\(x\) QCD** The LHeC would also provide us with an improvement of our understanding of the small-\(x\) dynamics of QCD. Indeed, even after years of intensive study at HERA, no convincing evidences for departures from standard DGLAP evolution has been found. For example, geometric scaling of HERA data, which was thought to provide a clear signal for saturation, was recently shown to be consistent with linear QCD evolution as well [14]. The situation could be different at the LHeC, with its extended kinematical coverage at small-\(x\) (see Fig. 1).

In order to test whether or not a DGLAP analysis can reproduce theoretical predictions which deviate from pure DGLAP in inclusive measurements, LHeC pseudo-data has been generated not within the DGLAP framework, as in the previous sections, but rather from two different models: the AAMS09 model [15], which is based on BK evolution with running coupling, and the FS04 model [16], based on the dipole model.

We have repeated the PDF analysis of the previous section but with these new pseudo-data. Although clearly the procedure is not consistent (for example, PDF error reduction would be meaningless in this case), it provides an illustration of a potential analysis technique which ultimately should be applied to experimental data. For both the AAMS09 and the FS04 models the conclusions of the study are the same: the DGLAP analysis reproduces perfectly the $F_2(x, Q^2)$ pseudo-data, which implies that although the underlying physical theories are different, from a practical point of view the small-\(x\) extrapolations of AAMS09 and FS04 for $F_2$ are rather similar to DGLAP-based extrapolations.
The situation however is different for $F_L(x, Q^2)$: provided the level arm in $Q^2$ is large enough, the DGLAP analysis fails to reproduce simultaneously $F_L$ in all the $Q^2$ bins, and thus the overall $\chi^2$ is very large, a clear signal of the departure from fixed order DGLAP of the simulated pseudo-data. This effect is illustrated in Fig. 3 where the results of the DGLAP analysis are compared with the LHeC pseudo-data generated from the AAMS09 model.

There exists however other scenarios for QCD at small-$x$ than saturation/dipole models. In particular, linear QCD evolution with resummation of BFKL small-$x$ logarithms is the natural extension of standard DGLAP evolution. Recently, the full set of small-$x$ resummed splitting functions and coefficient functions became available [17]. The results of Ref. [17] have been used to compute resummed K-factors [18], defined as the ratio of structure functions NLO small-$x$ resummed over fixed order NLO, as a function of $(x, Q^2)$. These K-factors can be used for realistic, though qualitative, phenomenological studies of the impact of small-$x$ resummation.

We have used these resummed K-factors to estimate the feasibility of the LHeC to disentangle between scenarios for small-$x$ linear QCD: NLO, NNLO and NLOres. In Fig. 4 we show the LHeC pseudo data for $F_2(x, Q^2)$ at small-$x$ compared with the NLO NNPDF1.0 prediction (including the associated PDF uncertainties) and the corresponding NNLO and NLOres computations, obtained from the NLO one with these K-factors. Fig. 4 seems to indicate that a PDF analysis capable of implementing both the the NNLO and NLOres computations of physical observables has the potential to disentangle between these two scenarios of small-$x$ QCD, given the foreseen experimental accuracy at the LHeC.

Departures from DGLAP It is clear from the previous discussion that there is some contradiction between the two goals of our study: either we determine the PDFs or we find evidence for saturation or resummation. However, both these goals require the same first step: we have to determine the kinematic region where saturation/resummation effects, or more general, departures from fixed-order DGLAP evolution, start to play a role, if any.

The idea is therefore to single out a safe region, where the standard PDFs extraction via fixed order DGLAP is reliable, and a small-$x$ region where deviations from pure fixed order DGLAP could provide evidence for saturation or resummation. The determination of these kinematic regions is a highly non trivial task: both BFKL and non-linear effects are known to be rather moderate in the HERA region, and thus are difficult to observe in inclusive observables. In particular, they could be absorbed in the initial condition for flexible enough parametrizations of the PDFs. This might already be the case at HERA for $F_2$, and if so even more at the LHeC.

A possible approach to this problem is the following. First we repeat the global PDF analysis removing subsets of data where small-$x$ effects could play some role. Then we determine whether NLO DGLAP is able to reproduce the excluded data or not. A tension between the actual data and the DGLAP prediction should mark the onset of some saturation/resummation effect. As a cross check, we can assess the NLO DGLAP fit quality in the fitted data region: the fit quality should improve if there is some tension between DGLAP and the actual data in the excluded region. Note that deviations from DGLAP are known to be rather moderate, hence our approach is meaningful only on statistical grounds. It is therefore mandatory to perform a PDF analysis with no parametrisation bias and with faithful uncertainty estimation [17]. Related studies of the stability of global analysis within the standard PDF approach have been reported in [19, 20].
Figure 3: The results of the combined DGLAP analysis of the NNPDF1.2 data set and the LHeC pseudo-data for $F_L(x, Q^2)$ in various $Q^2$ bins generated with the AAMS09 model.

Figure 4: A comparison of various approximations to linear low-$x$ QCD for $F_2$ at the LHeC: the NNPDF1.0 prediction which includes PDF uncertainties (green lines) and the NNPDF1.0 result corrected with the NNLO (black, dot-dashed) and NLOres (violet, short-dashed) K-factors. The expected experimental precision at the LHeC is also shown for illustration.
This approach has been applied to search for DGLAP deviations in the small-\(x\) HERA data. Taking as a reference the NNPDF1.2 analysis [9], we excluded data points with a saturation-inspired cut \(Q^2 \geq Q^2_S(x) \equiv A x^{-0.3}\), with \(A\) ranging from 0.2 to 1.5. In order to quantify deviations from DGLAP we computed the distance \(d(x, Q^2)\) between the DGLAP extrapolation \(F^\text{fit}\) for an observable \(F\) and the actual data \(F^\text{data}\), defined as

\[
d(x, Q^2) = \sqrt{\frac{F^\text{fit}(x, Q^2) - F^\text{data}(x, Q^2)}{\sigma^2_{\text{fit}} + \sigma^2_{\text{data}}} \times \text{sign}\left(F^\text{data} - F^\text{fit}\right)}.
\]

A typical result for the cut \(Q^2 \geq 1.5 x^{-0.3}\) is shown in Fig. 5. Note that while in the global fit distances seem uncorrelated, in the fit with the kinematical cut \(Q^2 \geq 1.5 x^{-0.3}\) there seems to be a hint of a correlation, that is, the NLO DGLAP prediction tends to be smaller than actual data. A systematic study is in progress using these methods in order to determine the statistical significance, if any, of departures from DGLAP in inclusive small-\(x\) data. The ultimate validation of the method will be its application to LHeC pseudo-data, where there the underlying physics can be varied within various scenarios.

**Outlook** This contribution summarizes some of the studies performed within the NNPDF framework in order to assess the physics potential of the LHeC as a probe of the nucleon structure and of small-\(x\) QCD dynamics. From these preliminary studies, one solid conclusion is that the importance of accurate measurements of \(F_L(x, Q^2)\) should be emphasized. Ongoing work towards the LHeC Conceptual Design Report includes the generalization of the PDF analysis to the complete LHeC data set for various scenarios and the impact of the reduction in PDF uncertainties on LHC phenomenology.

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