On the maximum running time in graph bootstrap percolation

Michal Przykucki

Abstract

Graph bootstrap percolation is a simple cellular automaton introduced by Bollobás in 1968. Given a graph $H$ and a set $G \in E(K_n)$ we initially “infect” all edges in $G$ and then, in consecutive steps, we infect every $e \in K_n$ that completes a new infected copy of $H$ in $K_n$. We say that $G$ percolates if eventually every edge in $K_n$ is infected. The extremal question about the size of the smallest percolating sets when $H = K_r$ was answered independently by Alon, Kalai and Frankl. In this paper we investigate further the extremal properties of $K_r$-bootstrap percolation and we analyse the maximum time the process can run before it stabilizes. It is an easy observation that for $r = 3$ this maximum is $\lceil \log_2(n - 1) \rceil$. However, a new phenomenon occurs for $r = 4$ when, as we show, the maximum time of the process is $n - 3$. For $r \geq 5$ the behaviour of the dynamics is even more complex which we demonstrate by showing that the $K_r$-bootstrap process can run for at least $n^{2-\varepsilon_r}$ time steps for some $\varepsilon_r$ that tends to 0 as $r \to \infty$.

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