ROBUST CONTROL IN GREEN PRODUCTION MANAGEMENT

JIAN-XIN GUO*
Institutes of Science and Development
Chinese Academy of Sciences
Beijing 100190, China

XING-LONG QU
The Research Center of Information Technology & Social and Economic Development
Hangzhou Dianzi University
Hangzhou 310018, China

(Communicated by Gerhard-Wilhelm Weber)

Abstract. This study proposes a robust control model for a production management problem related to dynamic pricing and green investment. Contaminants produced during the production process contribute to the accumulation of pollution stochastically. We derive optimal robust controls and identify conditions under which some concerns about model misspecification are discussed. We observe that optimal price and investment control decrease in the degree of robustness. We also examine the cost of robustness and the relevant importance of contributions in the overall value function. The theoretical results are applied to a calibrated model regarding production management. Finally, we compare robust choices with those in the benchmark stochastic model. Numerical simulations show that robust decision-making can indeed adjust investment decisions based on the level of uncertainty.

1. Introduction. Production activities may give rise to several environmental problems, and regulating such an economic and environmental trade-off is still a critical issue. Firms’ green investment occurs during the product life cycle stage and interacts with other features of the product and its market performance. There are two major properties of green production management that renders theoretical analysis complex and difficult. One is the dynamic nature of production. A central problem is balancing the tradeoff between polluting emissions and abatement activities in a dynamic market. Due to potential environmental damages, management of pollutants that accompany production is indispensable. This issue is even more complex when considering the traditional market mechanism such as price stickiness, customer preference, and product innovation. Another property is the randomness or uncertainty of the pollution process. Notwithstanding the type

2020 Mathematics Subject Classification. Primary: 93E20; Secondary: 49L05.
Key words and phrases. Robust control, model misspecification, pollution accumulation, green production.
Support from National Key R&D Program of China, 2018YFC1509008; National Natural Science Foundation of China under grant No. 71801212 and 71701058.
* Corresponding author: Jian-Xin Guo, guojianxin@casisd.cn.
of uncertainties, they represent situations where decision makers lack corresponding information relating to environmental events. These uncertainties are either autonomous or may be affected by the decision to enter the system state.

In this study, we propose a stochastic dynamic robust control model to cope with the trade-off between green production and pollutant management under uncertainties. Our work mainly relates to two strands of research. The first strand originates in the research on dynamic production management. This type of work focuses on emission management issues in traditional production processes. The core issue is modeling the dynamic relationship between production and emissions that is achieved through the introduction of a certain source or accumulation process. For instance, Ouardighi et al. [7] studied the tradeoff between polluting emissions and abatement activities related to pollution accumulation in a supply chain. They view the production activities of manufacturing firms as major sources of pollution, and pollution emissions are a by-product of manufacturing processes. Moreover, they presume that abatement efforts are subject to diminishing returns and their cost is an increasing quadratic function for the manufacturer. The negative externalities of the pollution stock are valued as an increasing convex cost function of pollution. Chung et al. [4] presented a continuous time dynamic model of the spatially dispersed, oligopolistic supply chain that generated pollution resulting from the use of in-house and outsourced inputs. They assumed that pollutants follow from production activities at each stage of the spatial supply chain. The manufacturer’s choice of inputs, technology, and product design can be expected to shift pollution emissions across various stages and locations. Thus, the authors incorporated two sources of pollutants at the manufacturing stage into the dynamic process: those associated with the use of specific inputs, and those generated by operations. Pervin et al. [23] investigated the optimal retailer’s replenishment decisions for perishable items, including time-dependent demand, for demonstrating more practical circumstances within economic-order quantity frameworks. Zhang et al. [38] investigated green supply chain performance with cost learning and operational inefficiency effects. The unit production cost is jointly affected by cost learning and operational inefficiency effects, which in turn influence the dynamic cumulative effect process. The latest relevant articles on the theory and application of production management can be referred to works such as [31, 2, 24, 25].

The second strand is a broader branch of research studies that embed emission processes, contaminant characteristics, and regulation policies into traditional environmental management issues. Jorgensen et al. [17] considered two neighboring countries keen to make joint efforts to control pollution emissions. In their study, the emission rate is proportional to current output. The stock of pollution \( S_t \) evolves according to \( \dot{S}_t = -\delta S_t + E_t \), where \( E_t \) is the output/emission ratio and \( \delta > 0 \) is a constant decay rate of pollution. Masoudi et al. [21] compared the use of price-based policies or taxes and quantity-based policies or quotas for controlling emissions in a dynamic setup where the regulators face two sources of uncertainties. The related damage-cost function they preferred is a linear form. Yu et al. [37] applied a multi-period model to study the optimal hybrid policy for pollutant emissions when a permit price ceiling and emission cap are included. In their work, accumulation of emissions is equivalent to the quantity of banking. They also developed an optimal hybrid model in a multi-period framework where banking and borrowing of emission permits is allowed. Niko et al. [15] studied the economics of breakthrough energy technologies in the context of climate change and
modeled such technology breakthroughs as once-and-for-all regime shifts. Although the study focused on macro carbon emissions, the cumulative equation for carbon emissions is similar to the equation for pollutants, such as those used in [17] and [21].

In this research work, we combine the above two related strands of research and further introduce uncertainties in demand and emission processes. We use a modeling method that is similar to the traditional standard model such as [7] and [4], as we are also considering the extent of pollutant emissions due to realization of full market demand. We extend the basic model by considering the synergistic effects generated by the environmental technology investment process. Moreover, we achieve the duality of emission reduction effects and market demand increments. This insight is referred to [6], however, in a dynamic way. Thus, a dynamic pollutant process is linked through the representation of the production process and the synergistic effects within it. Moreover, we also introduce uncertainties in the robust strategy based framework. The central task for such a model is to determine a rule that is robust to model misspecification. Usually, equivalent concerns about model misspecification can be represented by either altering the decision maker’s preferences to enhance risk-sensitivity or adding a set of perturbed models and a malevolent agent [14]. For instance, Gilboa et al. [10] established the foundations of maximum and minimum (max-min) expected utility, and their theory deduces a set of models from a decision maker’s underlying preferences over risky outcomes. Hansen and Sargent [13] exploited such a model that allows an economist to take a single approximating model and manufacture from it a set of models that express a decision maker’s ambiguity. Moreover, it is easy to investigate how the penalty parameter in a robust control problem can identify a set of perturbed models that are difficult to distinguish statistically from the approximating model. Related work can also be found such as [22, 12, 35, 1].

In dealing with uncertain problems, robust theory is a good strategy. Robust control methods are especially useful for dynamic systems that are highly unstable and where catastrophic events are difficult to revert. The use of robust investment strategies can make precautionary investments in the face of these catastrophic events. This idea has already been adopted in many applications such as [32, 11, 33, 34]. However, empirical studies typically assume a unique and explicitly specified dynamic statistical model. Concerns about model misspecification recognize that an unknown member of a set of alternative models may govern the data. The central issue is how to specify those alternative models. In many production management problems, the description of uncertain information is often distributed. Thus, applying the conventional robust control theory leaves those models only vaguely specified, which may enable shock feedback on state variables arbitrarily. Recently, a practical attraction of the robust control theory, proposed by Hansen and Sargent [13], is the way it enables an economist to take a single approximating model and manufacture from it a set of models that express a decision maker’s ambiguity. Thus, it allows the approximating model to capture the serial correlation of exogenous variables and the dynamics of how they impinge on endogenous state variables by introducing this modeling method into our production management problems. Therefore, the decision-maker model has the same special status as the approximation model in the robust control theory. Further, through this modeling technique, we can exploit the feature of robust control to construct a multiple agent
model in which a common approximating model embodies the role of an equilibrium model in a rational expectations model.

Based on the above discussion, we apply such a framework of model misspecification to study the influence of uncertainties in green production management issues. We find that the model has an analytical solution when the cumulative emission process is represented by a class of linear loss functions. In fact, according to the necessary condition from Hansen’s work, the equivalent feedback solution to the reduced the Hamilton–Jacobi–Bellman (HJB) function can be expressed through an extremely complex parameter form. Although the proofing process is tedious with many calculations and inequality scaling techniques, the analytical form enables us to derive further properties—such as sensitivity and its key parameters—of the closed solution. In addition, some attempts have also been made to analyze the robust perturbation boundary. The convergence, progressiveness, and boundary features of the robust solution are further analyzed. The degree of precautionary decision making due to robustness can be further characterized. We also conduct related numerical calculations to confirm the analytical results we obtain, and illustrate the circumstances that induce robustness. Specifically, we verified the sensitivity analysis of the synergistic parameters and find that synergy stimulates price increase and abatement, while the effect decreases as robustness increases.

In sum, this work could be useful in clarifying the uncertainty modeling and analysis of traditional production operations management, and the robust analysis method for the solution can be extended to similar problems. The rest of this paper is organized as follows. In Section 2, we present our basic model to describe the problem. In Section 3, we provide a solution process and characterize the robust properties. In Section 4, we present the numerical method to specify our results. In Section 5, we summarize our conclusions.

2. The model. We consider a firm selling a certain type of product in a consumer market. Assume that the firm’s production quantity for each time $t$ is price dependent, denoted by $d_t = \alpha - \beta p_t$ in which $p_t$ denotes the market price of a product sold by the firm at time $t$, and $\alpha, \beta$ are two related coefficients. Note that the production activities of manufacturing firms are major sources of pollution, and pollution emissions $P_t = P_t(d_t)$ are the by-products of the manufacturing processes at time $t$. During this process, the firm aims to implement abatement investment $e_t$ at time $t$. The change in pollution level is governed by the dynamical equation as follows:

$$\dot{S}_t = -\delta S_t + P_t - e_t, S(0) = S_0 > 0,$$

where $\delta$ reflects the environment’s self-cleaning capacity satisfying $0 < \delta < 1^1$.

We assume that the process of green investment has a synergistic effect, that is, in addition to reducing pollutant emissions, it increases market demand as $d_t + se_t$. Here, $s$ is the synergetic yield coefficient that satisfies $s < 1^2$. Accumulation of

---

1 This specification was introduced in Keeler et al.[18] and is extensively used in the existing literature, notably to determine an optimal economic growth path that is compatible with environmental protection. Recent work can be found in Ouaddighi et al.[6] and Masoudi et al.[21].

2 One such example is from Yenipazarli [36], where investment in water-saving technology to improve the product’s water efficiency in the water-use stage (e.g., Caroma dual flush with its half-flush and full-flush technology reduces water usage by up to 67 percent compared to traditional toilets 9) may appeal to consumers’ self-interest, leading to enhanced demand.
pollutants will incur $\tau$ cost per unit for environmental loss\(^3\), and we also assume that the abatement effort will incur $cc^2_t$ cost\(^4\), with $c$ being the unit parameter\(^5\).

Thus, the firm’s total profit $\Pi_t$ at time $t$ can be expressed as:

$$\Pi_t = p_t(\alpha - \beta p_t + se_t) - \tau S_t - cc^2_t.$$ \tag{2}

Usually, the parameters satisfy that $\alpha \gg \beta, s, \tau, c$.

Risk is introduced in the standard model by allowing the pollutant to accumulate according to a diffusion process. If we assume that the $P_t = d_t$, that is, one unit production will emit one unit pollutant, then considering Eq.1, the stochastic accumulation in the risk version can be expressed as:

$$dS_t = (-\delta S_t + \alpha - \beta p_t + (s - 1)e_t)dt + \sigma dW_t,$$ \tag{3}

where $\{W(t) : t \geq 0\}$ is a Brownian motion on an underlying probability space $(\Omega, F, G)$.

Thus, with uncertainty and the control policy, the firm’s objective is to maximize the expected profit:

$$\text{Max } J = \mathbb{E}_G \int_0^\infty e^{-rt}[p_t(\alpha - \beta p_t + se_t) - \tau S_t - cc^2_t]dt$$

s.t.

$$dS_t = (-\delta S_t + \alpha - \beta p_t + (s - 1)e_t)dt + \sigma dW_t,$$

$$S(0) = S_0,$$ \tag{4}

where $r$ is a discount rate. Here, $\mathbb{E}_G(\cdot)$ takes expectation on $F$ with the probability measure $G$.

Following Hansen [14], optimization Problem.4 is regarded as a benchmark model. The above benchmark model can also be found in a direct stochastic process modeling such as [20, 5, 16, 9]. Model misspecification can be reflected by a family of stochastic perturbations to the Brownian motion so that the probabilistic structure implied by stochastic differential Eq.3 is distorted, and the probability measure $G$ is replaced by another probability measure $\tilde{G}$.

The key idea is that stochastic processes under $\tilde{G}$ will be difficult to distinguish from those under $G$ using a finite amount of data. The perturbed model is constructed by replacing $W(t)$ in Eq.3 with

$$W_t = \tilde{W}_t + \int_0^t h_s, \left\{ \tilde{W}_t : t \geq 0 \right\},$$ \tag{5}

\(^3\)The firm internalizes cost by introducing an environmental tax or damage payment. There are many relevant examples such as food production, apparel production, smelting plants, and other enterprises that are often accompanied by discharge of industrial wastewater during processing.

\(^4\)For the environmental loss function, it is more often used as a state variable. Therefore, the linear relationship can characterize the main features of the system, thereby ensuring the solvability of the model. Some of the literature is based on quadratic functions [7], but not every model has good analytical features in this case.

\(^5\)In some inventory related problems, due to the market demand uncertainty, we must consider the demand surplus or shortage. Thus, the cost of excess or loss due to inventory status needs to be depicted [30, 26, 27, 28]. However, demand uncertainty is ignored here, implying that supply is always satisfied. This assumption is also reasonable, for example, in some literature [7, 19]. The crux of the matter is what our model intends to portray. As explained above, our model aims to characterize the uncertainty caused by emissions, and thus it can only distract our attention by introducing demand uncertainty at same time. Also, the situation will be more complex if we do this, thereby rendering the model unsolvable since another state process must be introduced. Solving these two types of uncertainty is a good topic and remains to be discussed in future research studies.
where \( \{ \hat{W}_t : t \geq 0 \} \) is a Brownian motion and \( \{ h_t : t \geq 0 \} \) is a measurable drift distortion. Thus, changes in the distribution of \( W(t) \) will be parametrized as drift distortions to a fixed Brownian motion \( \{ \hat{W}_t : t \geq 0 \} \). The distortions will be zero under the measure \( G \), in which case \( W_t \) and \( \hat{W}_t \) coincide. Thus, pollution dynamics under model misspecification are given by:

\[
dS_t = (-\delta S_t + \alpha - \beta p_t + (s-1)e_t + \sigma h_t)dt + \sigma d\hat{W}_t.
\]

(6)

As shown in [14], the discrepancy between the distributions \( G \) and \( \hat{G} \) is measured as the relative entropy:

\[
R(\hat{G}) = \int_{0}^{\infty} e^{-rt} \frac{1}{2} \mathbb{E}_{\hat{G}}[h(t)^2] \, dt.
\]

(7)

To allow for the notion that even when the model is misspecified the benchmark model remains a better approximation, the misspecification error is constrained so that we only consider the distorted probability measures \( \hat{G} \) such that:

\[
R(\hat{G}) = \int_{0}^{\infty} e^{-rt} \frac{1}{2} \mathbb{E}_{\hat{G}}[h(t)^2] \, dt \leq \eta < \infty.
\]

(8)

Under model misspecification, Eq.3 is replaced by Eq.6. Two robust control problems can be associated with the problem of maximizing Eq.2 subject to Eq.6, a constraint robust control problem. The robust control problem is defined as:

\[
\begin{align*}
\max_{p_t, e_t} \min_{h_t} J &= \mathbb{E}_{\hat{G}} \int_{0}^{\infty} e^{-rt} \frac{1}{2} \mathbb{E}_{\hat{G}}[h(t)^2] \, dt \\
\text{s.t.} & \quad dS_t = (-\delta S_t + \alpha - \beta p_t + (s-1)e_t + \sigma h_t)dt + \sigma d\hat{W}_t.
\end{align*}
\]

(9)

The robustness parameter takes nonnegative values, \( \theta \geq 0 \), and will be zero if the constraint Eq.8 is inactive or infinity if the constraint Eq.8 is violated. A value of \( \theta = \infty \) indicates that the manager is confident about the benchmark model and is not concerned about a possible model misspecification model, with no preference for robustness.

3. Problem solution. Hansen et al. [14] show that a decision problem for promoting robustness to model misspecification is a zero-sum, two-player game in which a maximizing player (“the decision maker”) chooses the best response to a malevolent player (“nature”) who can alter the stochastic process within the prescribed limits. They show that a Problem.9 can be transformed into a stochastic infinite horizon two-player game where the Bellman-Isaacs conditions imply that the value function is given by the following equation:

\[
rV(S_t; \theta) = \max_{p_t, e_t} \min_{h_t} \left( p_t(\alpha - \beta p_t + se_t) - \tau S_t - \alpha^2 + \theta \frac{h_t^2}{2} + V_S(-\delta S_t + \alpha - \beta p_t + (s-1)e_t + \sigma h_t) + \frac{1}{2} \sigma^2 V_{SS} \right).
\]

(10)

Here, we denote the right side to be optimized as:

\[
J = p_t(\alpha - \beta p_t + se_t) - \tau S_t - \alpha^2 + \theta \frac{h_t^2}{2} + V_S(-\delta S_t + \alpha - \beta p_t + (s-1)e_t + \sigma h_t) + \frac{1}{2} \sigma^2 V_{SS}.
\]

(11)
where $V_S = \frac{dV}{\delta S} = x$ and $V_{SS} = \frac{d^2V}{\delta S^2} = y$. Thus, the first order conditions that determine the optimal feedback rules for $p_t$, $e_t$, and $h_t$ are:

\[
\begin{align*}
\frac{\partial J}{\partial h_t} &= 0 \iff \theta h_t + V_S \sigma = 0 \iff h_t = -\frac{\sigma x}{\theta}, \\
\frac{\partial J}{\partial p_t} &= 0 \iff (\alpha - \beta p_t + se_t) - \beta p_t - \beta x = 0 \iff p_t = \frac{-2 \beta c x + s^2 x + 2 \alpha c - sx}{4 \beta c - s^2}, \\
\frac{\partial J}{\partial e_t} &= 0 \iff p_t s - 2ce_t + V_S(s - 1) = 0 \iff e_t = \frac{\beta sx + \alpha s - 2x \beta}{4 \beta c - s^2}.
\end{align*}
\]

We conjecture that value function $V$, satisfying Eq.\ref{eq:10}, admits the following simple quadratic form:

\[V = l + mS + nS^2.\]  \hfill (13)

Thus, $V_S = m + 2nS$ and $V_{SS} = 2n$.

Substitute $x = m + 2nS$, Eq.\ref{eq:12}, and Eq.\ref{eq:13} into Eq.\ref{eq:10} we obtain

\[
\text{Arg } J = AS^2 + BS + C,
\]

where

\[
\begin{align*}
A &= \frac{16 ns^2}{4 \beta c - s^2} - \frac{8 \beta cn^2}{\theta (4 \beta c - s^2)} - \frac{8 \beta cn(\beta c - s^2)}{4 \beta c - s^2} + \frac{4 \beta c}{4 \beta c - s^2}, \\
B &= \frac{\delta nms^2}{4 \beta c - s^2} - \frac{\delta mms}{4 \beta c - s^2} - \frac{\delta mmsn}{4 \beta c - s^2} + \frac{\beta m^2}{4 \beta c - s^2} + \frac{\alpha \beta cm}{4 \beta c - s^2} - \frac{\alpha \beta cmn}{4 \beta c - s^2} + \frac{\alpha^2 c}{4 \beta c - s^2}, \\
C &= \frac{4 \beta c}{4 \beta c - s^2} + \frac{4 \beta c}{4 \beta c - s^2} + \frac{2 \alpha \beta cm}{4 \beta c - s^2} + \frac{2 \alpha \beta cm}{4 \beta c - s^2} + \frac{8 \beta c - 2s^2}{4 \beta c - s^2}.
\end{align*}
\]

Value function coefficients are derived from the following system of equations:

\[
\begin{align*}
\beta n &= \frac{4n((\beta c)^2 + ((-s + 1)n - 2c\beta)\beta + 1/2s^2\beta)}{\theta (4\beta c - s^2) + s(\delta ms - 2\alpha n + s)}, \\
\beta m &= \frac{4\beta^2 cmn + ((-s + 4n - 4c\beta)\beta + 4\beta c\alpha n - \tau)\beta}{\theta (4\beta c - s^2) + s(\delta ms - 2\alpha n + s)}, \\
\beta l &= \frac{2\beta^2 c^2 + ((-2s + 4c\alpha n + 4c\beta \alpha n)\beta)}{\theta (4\beta c - s^2) + s(\delta ms - 2\alpha n + s)}.
\end{align*}
\]

We can obtain the coefficients as follows:

\[
\begin{align*}
l &= L_2 \tau^2 + L_1 \tau + L_0 + l_0 \\
L_2 &= \frac{2 \theta (4 \beta c - s^2) r \delta^2 (2 \beta^2 c \theta - 4 \beta c \alpha^2 + 2 \beta^2 s \theta + 2 \beta \theta \delta)}{2 \beta^2 c \theta - 4 \beta c \alpha^2 + 2 \beta^2 s \theta + 2 \beta \theta \delta} \\
L_1 &= -2 (2 \beta^2 c \theta - 4 \beta c \alpha^2 + 2 \beta^2 s \theta + 2 \beta \theta \delta) (r + \delta) \alpha (2 \beta c - s) \theta \\
L_0 &= (r + \delta) (2 \beta c - s)^2 \theta^2 \\
l_0 &= 16 \delta^2 (2 \beta c - s)^2 (\beta c - 1/4 s^2) (\beta c - 1/4 s^2) \theta (r + \delta) (2 \beta c - s)^2 + 1/4 \alpha^2 \theta - 1/2 \alpha^2 \delta^2 \theta \alpha^2 \delta^2 \theta.
\end{align*}
\]
\[ m = \frac{(2c^2\theta^2 - 4\beta c^2\sigma^2 + s^2\sigma^2 - 2\beta s\theta + 2\beta\theta )\tau - 4\alpha \beta c\delta \theta - 2\alpha \beta c\theta + 2\alpha \delta s\theta + \alpha rs\theta}{\delta (2c^2\theta^2 - 4\beta c^2\sigma^2 + s^2\sigma^2 - 2\beta s\theta + 2\beta\theta)} \quad (18) \]

\[ n = \frac{\theta (8\beta c\delta + 4\beta c\tau - 2s^2\delta - rs^2)}{4\beta^2c - 8\beta c\sigma^2 + 2s^2\sigma^2 - 4\beta s\theta + 4\beta\theta}. \quad (19) \]

Since \( \beta c - s + 1 > 0 \), thus when \( \theta < \infty \), \( n \geq 0 \) if and only if \( (4c\beta - s^2)\theta - (4\beta c - s^2)^2 \sigma^2 \geq 0 \). Since \( s \ll 1 \), \( 4\beta c - s^2 \geq 0 \) is a nonsingular case. Thus, \( \theta > \frac{2\beta(\beta c - s + 1)}{2\beta(\beta c - s + 1)} \) or infinity (non-robust solution). Based on the solution Eq.17, Eq.18, Eq.19, we can analyze the analytical properties of these parameters, which will help us to understand the robustness of the value function.

**Proposition 1.** Under the nonsingular case, if \( \text{Max}\{2\beta^2c\tau, \beta\} < \delta\alpha \), then there exists \( \theta > \theta \)

(a) \( n, l > 0 \) and \( m < 0 \). Moreover \( \frac{\partial m}{\partial \theta} < 0 \), \( \frac{\partial m}{\partial \theta} > 0 \).

(b) \( u_t \) is the control variable, which stands for \( p_t, e_t \) and \( h_t \). Thus, \( u_t - \frac{\partial u_t}{\partial S_t}S_t > 0 \) and \( \frac{\partial u_t}{\partial S_t} < 0 \).

**Proof.** For Part(a), reshaping the formulation \( n \) as:

\[ n = \frac{8\beta c\delta + 4\beta c\tau - 2s^2\delta - rs^2}{2\beta^2c + \frac{s^2 - 4\beta c}{\theta}c\sigma^2 - 2\beta s + 2\beta}. \quad (20) \]

Thus, in the nonsingular case,

\[ 8\beta c\delta + 4\beta c\tau - 2s^2\delta - rs^2 = (4\beta c - s^2)(2\delta + r) > 0, \]

and

\[ 2\beta^2c + \frac{s^2 - 4\beta c}{\theta}c\sigma^2 - 2\beta s + 2\beta = 2\beta(\beta c + 1 - s) - \frac{4\beta c - s^2}{\theta}\sigma^2 > 0 \]

if and only if \( \theta > \frac{(4\beta c - s^2)^2\sigma^2}{2\beta(\beta c - s + 1)} = \theta_1 \). Hence, \( n > 0 \), and in this case \( \frac{\partial n}{\partial \theta} < 0 \).

Similarly,

\[ m = \frac{(2\beta^2c\theta - 4\beta c\sigma^2 + s^2\sigma^2 - 2\beta s\theta + 2\beta\theta )\tau - \alpha\theta (r + 2\delta)(2c\beta - s)}{\alpha (r + 2\delta)(2c\beta - s)} \]

\[ = \frac{\tau}{\delta} - \frac{(2\beta^2c + \frac{s^2 - 4\beta c}{\theta}c\sigma^2 - 2\beta s + 2\beta)}{\delta}. \quad (21) \]

It is not easy to check that \( m < 0 \) if \( 2\beta^2c\tau < \delta\alpha \) for all \( \theta > \theta_1 \). In fact, if \( \beta c > 1 \) then

\[ < 2\beta(\beta c - s + 1)\tau < 4\beta^2c\tau < 2\delta\alpha < \alpha (r + 2\delta)(2c\beta - s). \]

Otherwise, the inequality still holds when \( \beta c \leq 1 \). In this case, it is easy to verify that \( \frac{\partial m}{\partial \theta} > 0 \).
Since
\[ L_2 \tau^2 + L_1 \tau + L_0 = \left( (2c(r+\delta)\alpha + s - 1) \beta - 2\beta^2c - \alpha(s(r+\delta)) \theta + (4c\beta - s^2) \sigma^2 \right)^2 \geq 0, \]
thus, the sign of \( l \) depends on that of \( l_0 \). Noticing that,
\[ l_0 = (4c\beta - s^2)(r + 2\delta) \sigma^2 \theta + 2\alpha \theta - 2c\alpha^2 \sigma^2 > 0 \]
\( \Leftrightarrow \theta > \frac{2c\alpha^2 \sigma^2}{(4c\beta - s^2)(r + 2\delta) \sigma^2 + \alpha^2} = \theta_2. \)
Therefore, in a nonsingular situation, \( \theta > \theta_2 \), where \( \theta = \text{Max}\{\theta_1, \theta_2\} \), the parameters \( l, n \) are all positive.

We now turn to Part (b). According to the Eq.12 and Eq.13, we obtain:
\[
\begin{aligned}
    h_t &= -\left( \frac{2\alpha}{\theta} \right) (2nS + m), \\
n_t &= \frac{2n(s^2 - s - 2\beta c)S + (s^2 - s - 2\beta c)m + 2\alpha c}{4\beta c - s^2}, \\
e_t &= \frac{2n(\beta s - 2\beta)S + (\beta s - 2\beta)m + \alpha s}{4\beta c - s^2}.
\end{aligned}
\]
Considering the related results Part(a), we can arrive at the conclusion. \( \square \)

**Remark 1.** The entire conclusion provides us a description of the perturbation of the parameters that are understood. Although they are not comprehensive (it does not contain \( l \)), the properties of \( m \) and \( n \) are clear. In addition, the conclusion of (b) helps us to better understand the analytical nature of the feedback solution, indicating that the control process is a decreasing function of the state and this trend will gradually weaken.

Consider the solution to the problem when \( \theta \to \infty \), which is the case where the regulator is not concerned about model misspecification and trusts the benchmark model. In this case, we denote the optimal feedback control \( u^\infty \), which is different from the robust case \( u^\theta \). According to the denotation in [3], the expected minimum cost of the regulator when the benchmark model is trusted will be:
\[
C^\infty = \mathbb{E}_\mathcal{G} \int_0^\infty e^{-rt}(\alpha - \beta p^\infty_t + se^\infty_t) - \tau S^\infty_t - c(e^\infty_t)^2)\,dt. \tag{22}
\]
Here, parameters \( p^\infty_t, e^\infty_t \), and \( S^\infty_t \) are the corresponding price, abatement effort, and pollution when \( \theta \to \infty \).

Now, consider the solution of the robust control problem for a \( \theta \to \infty \) such that \( \theta \in \Lambda \). The expected minimum cost of the regulator under robust control will be:
\[
C^\theta = \mathbb{E}_\mathcal{G} \int_0^\infty e^{-rt}(\alpha - \beta p^\theta_t + se^\theta_t) - \tau S^\theta_t - c(e^\theta_t)^2 + \theta \frac{h^2_t}{2})\,dt. \tag{23}
\]
Parameters \( p^\theta_t, e^\theta_t \), and \( S^\theta_t \) are the corresponding price, abatement effort, and pollution given \( \theta \). Thus, the cost of robustness can be defined as \( C_R(\theta) = C^\theta - C^\infty \), and the effort of robustness can be defined as \( u_R(\theta) = u^\theta - u^\infty \).

The corresponding value function is of the form \( V^\theta = l^\theta + m^\theta S_0 + n^\theta S_0^2 \), and \( V^\infty = l^\infty + m^\infty S_0 + n^\infty S_0^2 \). Here, \( V^\theta, l^\theta, m^\theta, \) and \( n^\theta \) are the corresponding parameters given \( \theta \). \( V^\infty, l^\infty, m^\infty, \) and \( n^\infty \) are the corresponding parameters when \( \theta \to \infty \).

This gives us the total cost of the minimum possible deviation from the desired goal and it is made up from the contributions of three terms. The value functions
depend on all these three contributions. This can be clearly seen since \( C_R \) and \( u_R \) are in fact a function of the parameters \( l, m, n \). Thus, it is feasible for us to analyze the relevant importance of each of these contributions in the overall value function, or even if one term dominates over the others. We characterize in the following result.

**Proposition 2.** Define \( \Delta_\xi = \xi^\theta - \xi^\infty, \xi = l, m, n \).

(a) \( \Delta_n > 0, \frac{\partial \Delta_n}{\partial \theta} < 0; \Delta_m < 0, \frac{\partial \Delta_m}{\partial \theta} > 0; \) and there exists \( \hat{\theta}, \Delta_t > 0, \frac{\partial \Delta_t}{\partial \theta} < 0 \) if \( \theta > \hat{\theta}, \) and otherwise.

(b) \( |\Delta_m| = \frac{\partial |\Delta_m|}{\partial \Delta_n} > 1. \)

(c) \( \lim_{\theta \to \infty} C_R(\theta) = 0, \lim_{\theta \to \infty} u_R(\theta) = 0. \) If \( \theta = \max\{\hat{\theta}, \theta_0\}, |C_R(\theta)| \leq |l^\theta| + |m^\theta| S_0 + |n^\theta| S_0^2. \)

**Proof.** According to Eq.17, Eq.18, Eq.19, when \( \theta \to \infty \), the parameters in the non-robust solution are:

\[
l^\infty = \frac{L_2^\infty \tau^2 + L_1^\infty \tau + L_0^\infty + l_0^\infty}{4\beta \delta (4\beta c - s^2) r\delta^2 (\beta c - s + 1)^{1}}, \tag{24}\]

where

\[
\begin{align*}
L_2^\infty &= (2\beta^2 c - 2\beta s + 2\beta)^2, \\
L_1^\infty &= -2(2\beta^2 c - 2\beta s + 2\beta)(r + \delta)\alpha(2\beta c - s), \\
L_0^\infty &= (r + \delta)^2\alpha^2(2\beta c - s)^2, \\
l_0^\infty &= \delta^2(4\beta c - s^2)((4\beta c - s^2)(r + 2\delta)\sigma^2 + \alpha^2).
\end{align*}
\]

\[
m^\infty = \frac{\tau}{\delta} - \frac{\alpha(r + 2\delta)(2\beta c - s)}{(2\beta^2 c - 2\beta s + 2\beta)\delta}, \tag{25}\]

\[
n^\infty = \frac{(8\beta \delta + 4\beta c r - 2s^2\delta - rs^2)}{4\beta \delta (\beta c - s + 1)}. \tag{26}\]

Define \( l = l^\theta, m = m^\theta \), and \( n = n^\theta \) in the Eq.17, Eq.18, Eq.19. Denote

\[
\Delta_n = n^\theta - n^\infty = \frac{(4\beta c - s^2)^2(2\delta + r)\sigma^2}{(\beta c + 1 - s)(2\beta(\beta c + 1 - s) - \frac{4\beta c - s^2}{\theta}\sigma^2)} > 0.
\]

It is easy to check that \( \frac{\partial \Delta_n}{\partial \theta} < 0. \)

\[
\Delta_m = m^\theta - m^\infty = -\frac{\alpha(4\beta c - s^2)(2\beta c - s)(r + 2\delta)\sigma^2}{2\delta \beta(\beta c - s + 1)(2\beta(\beta c + 1 - s) - \frac{4\beta c - s^2}{\theta}\sigma^2)} < 0.
\]

It is easy to check that \( \frac{\partial \Delta_m}{\partial \theta} > 0. \)

Reformulating \( l^\theta \) as the following form:

\[
l^\theta = \frac{L_2^\theta \tau^2 + L_1^\theta \tau + L_0^\theta + l_0^\theta}{2(4\beta c - s^2) r\delta^2 \left(2\beta^2 c + \frac{-4\beta c r + s^2\sigma^2}{\theta} - 2\beta s + 2\beta\right)\delta}, \tag{27}\]
where

\[
\begin{align*}
L_2^\theta &= \left(2\beta^2c + \frac{\alpha^2r^2 - 4\beta \sigma^2}{\theta} - 2 \beta s + 2 \beta \right)^2, \\
L_1^\theta &= -2 \left(2\beta^2c + \frac{\alpha^2r^2 - 4\beta \sigma^2}{\theta} - 2 \beta s + 2 \beta \right)(r + \delta) \alpha (2 \beta c - s), \\
L_0^\theta &= (r + \delta)^2 \alpha^2 (2 \beta c - s)^2, \\
l_0^\theta &= 16 \delta^2 \left(\beta c - 1/4 s^2\right) \left(\left((\beta c - 1/4 s^2) (r + 2 \delta) \sigma^2 + 1/4 \alpha^2\right) - \frac{\alpha^2 r^2}{2\theta}\right).
\end{align*}
\]

\[\Delta_l = l_0^\theta - l^\infty = \frac{D \theta + 2 \beta \left(\beta c - s + 1\right) \alpha^1 \left(\beta c - \frac{s^2}{4}\right) \tau^2}{2 \delta^2 \theta^2 \beta \left(\beta c - s + 1\right) r \left(2 \beta^2c + \frac{-4 \beta \sigma^2 + s^2 \alpha^2}{\theta} - 2 \beta s + 2 \beta\right)},\]

(28)

where

\[D = -\sigma^2 \beta^2 \left(\beta c - s + 1\right)^2 \tau^2 + 4 \alpha^2 \left(\beta c - \frac{s^2}{4}\right)^2 \delta^2 + \alpha^4 \left(\beta c - \frac{s^2}{4}\right)^2 (r + 2 \delta)\].

(29)

Obviously, the sign of \(\Delta_l\) depends on the sign of \(D\). When \(D > 0\), there exists a \(\hat{\theta}\), if \(\theta > \hat{\theta}, \Delta_l > 0\) and \(\frac{\partial \Delta_l}{\partial \theta} < 0\); if \(\theta < \hat{\theta}, \Delta_l < 0\) and \(\frac{\partial \Delta_l}{\partial \theta} > 0\).

We now refer to Part(b). The first inequality can be derived based on the fact that:

\[\left|\frac{\Delta_m}{\Delta_n}\right| = \frac{(2\beta c - s) \alpha}{2 \delta \beta \left(4 \beta c - s^2\right)} > 1,\]

considering the relationship specified in Eq.2. It is also easy to check that

\[\frac{\partial |\Delta_m|}{\partial |\Delta_n|} = \frac{\partial |\Delta_m|}{\partial \theta} \cdot \frac{\partial |\Delta_n|}{\partial \theta} = \frac{(2\beta c - s) \alpha}{2 \delta \beta \left(4 \beta c - s^2\right)} \frac{\partial |\Delta_m|}{\partial |\Delta_n|} > 1.\]

Part(c) can be established instantly considering the results in Part(a) and Prop.1.

We find that \(\Delta_c\) measures the degree of deviation of the robust solution. Result (a) indicates the direction of convergence. It can be observed that parameters \(m\) and \(n\) are uniformly convergent, while for parameter \(l\) it is conditional convergence. (b) is an interesting conclusion, which shows that the convergence speed and acceleration of the two parameters with respect to the robustness parameters are the same, and \(m\) is larger than \(n\). (c) shows that when \(theta\) tends to infinity, the robust solution converges to the general solution, and provides a previous term of deviation in this process. In fact, these conclusions and the results from Prop.1 will be further confirmed in the next numerical experiments.

4. Numerical examples.

4.1. Basic results. To provide insights into the impact of robustness, a numerical solution is presented in Table.1 to summarize the values of all the model parameters. Specifically, the parameters here are the values used in the numerical experiment. Through numerical simulation, we verify the effectiveness and properties of our model. Most of these parameters are similar to the values in the related literature as our citation [38, 17, 5, 7, 29], and they are all within the acceptable range of this model. Thus, it is worth noting that the analytical solution of the model is a linear feedback form, and the disturbances from these parameters will not have a nonlinear effect on the results. Therefore, we choose more representative data as the
input value of the model parameters, which are sufficient to ensure that the relative sizes between these parameters are reasonable. The model parameters $l, m, n$, and control processes $h_t, p_t, e_t$ are presented in Table.2, Table.4.1, respectively. All the results are implemented through Matlab using a laptop with a 1.99 GHz processor and 8 MB of RAM. The corresponding results can be calculated from Eq.17, Eq.18, Eq.19 and Eq.24, Eq.25, Eq.26. $|\Delta_m|$ and $|\Delta_n|$ can also be calculated from the expression in Prop.2.

On one hand, it is easy to check the parameters satisfying the nonsingular case. The absolute values of $l, m, n$ are decreasing; however, $m$ is always a negative number. This result is in agreement with Prop.1. On the other hand, $|\frac{\partial u_t}{\partial S_t}|$ and $u_t - |\frac{\partial u_t}{\partial S_t}|S_t$ are both decreasing as $\theta$ is increasing. The corresponding results are illustrated in Fig.1 and Fig.2. These figures show the same results as were discussed. The robustness costs $u_t$ for low values of $\theta$ but using relatively more for high levels of $\theta$.

As seen from the results in Prop.2, the absolute values of the cost of the robustness are decreasing. Moreover, sensitivity to the parameter $\theta$ of $|\Delta_m|$ is larger than that of $|\Delta_n|$. This result continues to be valid for $\frac{\partial |\Delta_m|}{\partial \theta}$ and $\frac{\partial |\Delta_n|}{\partial \theta}$ according to the result, which is not presented here.

### Table 1. Model parameters used in the simulation

| Parameter | Description | Value |
|-----------|-------------|-------|
| $T$       | Time Duration | 20    |
| $S_0$     | Pollution stock in the initial year | 100   |
| $r$       | Compound Rate | 5%    |
| $\alpha$  | Potential market size | 100$^1$ |
| $\beta$   | Coefficient in the demand function associated with the sales price | $1^1$ |
| $s$       | Co-benefit of the abatement effort | 0.1$^2$ |
| $\tau$    | Coefficient of environmental damage caused by accumulation of pollution | 0.5$^3$ |
| $c$       | Cost coefficient associated with firm’s pollution abatement effort | $1^4$ |
| $\delta$  | Pollution decay rate | 0.1$^3$ |
| $\sigma$  | Volatility parameter in $S_t$ | $10^5$ |
| $\theta$  | Robust parameter | depends |

$^1$ Parameters in the demand function mainly refer to the relevant literature [38, 5, 7] and are corrected. The special relative relationship must be kept reasonable.

$^2$ This parameter mainly refers to [5], whose magnitude corresponds to $\alpha, \beta$.

$^3$ This parameter mainly refers to [17] and is adjusted.

$^4$ This parameter mainly refers to [7] and is adjusted.

$^5$ This parameter mainly refers to [29] and is adjusted.

### 4.2. Sensitivity analysis.
In this section, we investigate the sensitivity of some key parameters for the control variables. It would be significant to understand how the resulting price $p_t$ and the abatement effort $e_t$ capture customers’ preferences toward green products. Thus, the main test parameters are the robustness parameter
Table 2. Model parameters with different levels of robustness

|     | l      | m     | n     | Δm   | Δn   |
|-----|--------|-------|-------|------|------|
| θ=200 | 106304.82 | -258.15 | 0.27  | -138.15 | 0.11 |
| θ=500  | 81176.12  | -153.22 | 0.16  | -33.22  | 0.03 |
| θ=800  | 77749.76  | -138.88 | 0.15  | -18.88  | 0.02 |
| θ=∞     | 73244.95  | -120.0 | 0.13  | 0.00    | 0.00 |

Table 3. Control processes with different levels of robustness

|     | h_t    | p_t    | e_t   |
|-----|--------|--------|-------|
| θ=200 | -0.03S + 12.90 | -0.28S + 185.34 | -0.25S + 125.43 |
| θ=500 | -0.006S + 3.06  | -0.16S + 130.38  | -0.15S + 75.46  |
| θ=800 | -0.003S + 1.73  | -0.15S + 122.87  | -0.14S + 68.63  |
| θ=∞    | 0.00    | -0.13S + 112.98 | -0.12S + 59.64  |

Figure 1. $p^*(S)$ with different levels of robustness.

θ and synergetic effect s. The remaining parameters used are the same as those in Table.1. The relationship between $p_t$ and $S_t$ or $e_t$ and $S_t$ are given by Eq.12, that is,

\[
\begin{align*}
    p_t &= \frac{2n(s^2 - s - 2\beta c)S + (s^2 - s - 2\beta c)m + 2\alpha c}{4\beta c - s^2}, \\
    e_t &= \frac{2n(\beta s - 2\beta)S + (\beta s - 2\beta)m + \alpha s}{4\beta c - s^2}, \\
    d_t &= \alpha - \beta p_t + se_t.
\end{align*}
\]

We denote them as separate parts: $p_t = k_pS + b_p$, $e_t = k_eS + b_e$ and $d_t = k_dS + b_d$; $b_p$, $b_e$, and $b_d$ contain no $S$ items. Thus, we test the sensitivity of the slope terms
We find that the slope term and the intercept term are more sensitive to $s$ when the robust parameters are smaller. For the same value of $\theta$, $k_p$ and $k_e$ are decreasing with $s$; $b_p$ and $b_e$ are increasing with $s$. Combining these phenomena, we can conclude that at the same level of robustness, the response change of the decision is stronger. That is, under the same emission accumulation state $S_t$, the large synergy coefficient $s$ can stimulate the price and abatement interest. As $\theta$ increases, this stimulation weakens. These conclusions are exactly opposite to demand. In fact, from Fig.5, although the robust model pays more cost, it brings about a considerable increase in demand. Similarly, even at the same level of robustness, the increase in synergistic yield improves demand significantly.

4.3. Managerial insights and practical implications. First, from the perspective of long- and short-term investment, our analysis shows that it is still advisable to control the cost of technology and degree of uncertainty based on the level of pollution, and implement preventive measures through early investment in emission reduction and price control. In this case, the current investment and price levels and future mitigation measures cannot replace each other. On the other hand, when the investment cost of emission reduction is high, the price mechanism can be used as a supplement to the current mitigation measures, and the increase in uncertainty will lead to the prevention of these two policy measures. The synergetic reduction of emissions has a greater impact on changes in robust decision making. Under a high level of synergy, emission reduction measures and price mechanism in terms of optimal cost are relatively sensitive to pollution levels. Therefore, in economic decision making at sensible levels, a synergistic process of emission reduction is required, which can be improved by implementing advanced technology at an early stage. This can also be reflected in the selection of robust decision parameters.
In fact, in case of a large risk appetite, policy makers are more inclined to adopt non-equal emission reduction targets and price strategies. Only when prices and emission reduction control strategies are likely to be very expensive and unrealistic, can emissions increase indefinitely. Overall, decision makers must fully consider the long- and short-term effects of risk and the rate of risk attenuation to make a comprehensive decision.

5. Conclusion. This study develops a robust control model, in which a decision maker considers intertemporal profit maximization while adopting an optimal selling price and investment related to green production. The pollutants produced in the production process have cumulative effects, and the environmental problems they generate further affect the company’s total profit. Due to the uncertainty of the cumulative process of pollutants, decision makers must make robust decisions based on future uncertainties. By establishing the HJB equation, the entire model
can obtain the analytical solution of the relevant decision about robust preference. We find that robust decision making tends to decrease with increasing preference parameters. Besides, robust decision making brings a corresponding cost and increases as the preference improves. We further analyze the key factors affecting these costs. The theory shows that some important parameters have greater flexibility for robust preferences, while others are relatively small. Under a weak assumption, we also provide the cost of a bound in robust decision making. Finally, the numerical results are compared with the benchmark case to verify the relevant theoretical results.

The study limitations include selection of estimation items and error analysis. The associated detection error probability can be calculated for a given sample using the likelihood ratios when the approximating model generates the data and when the worst case model is true. Thus, further research must be dedicated toward analyzing detection error probabilities using real data samples. Additionally, we believe that stochastic models in many environmental and resource-related areas must leverage this methodology and focus on the available analytical models involving dynamic damage-control investment and nonlinear pollution dynamics.

It is worth noting that despite the above limitations, the proposed model is still a good choice for actual decision-making process. The model framework here can provide investment strategy enlightenment for the meso and micro decision makers charging green operation management. Especially for micro enterprises, a robust investment strategy will most likely hedge against the risk generated from the future uncertainties. For environmental departments at the meso level, this model can also be used in regional decision-making and industry planning.

REFERENCES

[1] S. Athanassoglou and A. Xepapadeas, Pollution control with uncertain stock dynamics: When, and how, to be precautious, *Journal of Environmental Economics and Management*, 63 (2012), 304–320.

[2] H. Barman, M. Pervin, S. K. Roy and G. W. Weber, Back-ordered inventory model with inflation in a cloudy-fuzzy environment, *Journal of Industrial and Management Optimization*,
[3] W. A. Brock, A. Xepapadeas and A. N. Yannacopoulos, Robust control and hot spots in spatiotemporal economic systems, *Dyn. Games Appl.*, 4 (2014), 257–289.

[4] S. H. Chung, R. D. Weaver and T. L. Friesz, Strategic response to pollution taxes in supply chain networks: Dynamic, spatial, and organizational dimensions, *European J. Oper. Res.*, 231 (2013), 314–327.

[5] C. Dong, B. Shen, P.-S. Chow, L. Yang and C. T. Ng, Sustainability investment under cap-and-trade regulation, *Ann. Oper. Res.*, 240 (2016), 509–531.

[6] F. El Ouardighi, H. Benchekroun and D. Grass, Controlling pollution and environmental absorption capacity, *Ann. Oper. Res.*, 283 (2014), 111–133.

[7] F. El Ouardighi, J. E. Sim and B. Kim, Pollution accumulation and abatement policy in a supply chain, *European J. Oper. Res.*, 248 (2016), 982–996.

[8] M. Funke and M. Paetz, Environmental policy under model uncertainty: A robust optimal control approach, *Climatic Change*, 107 (2011), 225–239.

[9] I. Gilboa and D. Schmeidler, Maxmin expected utility with non-unique prior, *J. Math. Econom.*, 18 (1989), 141–151.

[10] H. Golpira and E. B. Tirkolaee, Stable maintenance tasks scheduling: A bi-objective robust optimization model, *Computers & Industrial Engineering*, 137 (2019), Sourced from Microsoft Academic - https://academic.microsoft.com/paper/2964639183.

[11] L. P. Hansen and T. J. Sargent, Robustness and ambiguity in continuous time, *J. Econom. Theory*, 146 (2011), 1195–1223.

[12] L. P. Hansen and T. J. Sargent, Robust control of forward-looking models, *Journal of Monetary Economics*, 50 (2003), 581–604.

[13] L. P. Hansen, T. J. Sargent, G. Turmhambetova and N. Williams, Robust control and model misspecification, *J. Econom. Theory*, 128 (2006), 45–90.

[14] N. Jaakkola and F. van der Ploeg, Non-cooperative and cooperative climate policies with anticipated breakthrough technology, *Journal of Environmental Economics and Management*, 97 (2019), 42–66.

[15] K. Jiang, R. Merrill, D. You, P. Pan and Z. Li, Optimal control for transboundary pollution under ecological compensation: A stochastic differential game approach, *Journal of Cleaner Production*, 241 (2019), 118391.

[16] S. Jørgensen and G. Zaccour, Incentive equilibrium strategies and welfare allocation in a dynamic game of pollution control, *Automatica J. IFAC*, 37 (2001), 29–36.

[17] E. Keeler, M. Spence and R. Zeckhauser, *The Optimal Control of Pollution*, Taylor & Francis Oxford, 1971.

[18] K. Kogan, F. El Ouardighi and A Herbon, Production with learning and forgetting in a competitive environment, *International Journal of Production Economics*, 189 (2017), 52–62.

[19] N. Masoudi, M. Santugini and G. Zaccour, A dynamic game of emissions pollution with uncertainty and learning, *Environmental and Resource Economics*, 64 (2016), 349–372.

[20] N. Masoudi and G. Zaccour, Emissions control policies under uncertainty and rational learning in a linear-state dynamic model, *Automatica J. IFAC*, 50 (2014), 719–726.

[21] J. Miao and A. Rivera, Robust contracts in continuous time, *Econometrica*, 84 (2016), 1405–1440.

[22] M. Pervin, S. K. Roy and G.-W. Weber, Analysis of inventory control model with shortage under time-dependent demand and time-varying holding cost including stochastic deterioration, *Ann. Oper. Res.*, 260 (2018), 437–460.

[23] M. Pervin, S. K. Roy and G. W. Weber, An integrated vendor-buyer model with quadratic demand under inspection policy and preservation technology, *Hacettepe Journal of Mathematics and Statistics*, 49 (2020), 1168-1189.

[24] M. Pervin, S. K. Roy and G. W. Weber, An integrated inventory model with variable holding cost under two levels of trade-credit policy, *Numer. Algebra Control Optim.*, 8 (2018), 169–191.

[25] M. Pervin, S. K. Roy and G. W. Weber, Deteriorating inventory with preservation technology under price-and stock-sensitive demand, *J. Ind. Manag. Optim.*, 16 (2020), 1585–1612.

[26] M. Pervin, S. K. Roy and G. W. Weber, Multi-item deteriorating two-echelon inventory model with price-and stock-dependent demand: A trade-credit policy, *J. Ind. Manag. Optim.*, 15 (2019), 1345–1373.
[27] M. Pervin, S. K. Roy and G. W. Weber, A Two-echelon inventory model with stock-dependent demand and variable holding cost for deteriorating items, Numer. Algebra Control Optim., 7 (2017), 21–50.
[28] C. Roseta-Palma and A. Xepapadeas, Robust control in water management, Journal of Risk and Uncertainty, 29 (2004), 21–34.
[29] S. K. Roy, M. Pervin and G. W. Weber, Imperfection with inspection policy and variable demand under trade-credit: A deteriorating inventory model, Numer. Algebra Control Optim., 10 (2020), 45–74.
[30] S. K. Roy, M. Pervin and G. W. Weber, A two-warehouse probabilistic model with price discount on backorders under two levels of trade-credit policy, J. Ind. Manag. Optim., 16 (2020), 553–578.
[31] A. K. Sangaiah, E. B. Tirkolaee, A. Goli and S. Dehnavi-Arani, Robust optimization and mixed-integer linear programming model for LNG supply chain planning problem, Soft Computing, 24 (2020), 7885–7905.
[32] E. B. Tirkolaee, S. Hadjan, G.-W. Weber and I. Mahdavi, A robust green traffic-based routing problem for perishable products distribution, Computational Intelligence, 36 (2020), 80–101.
[33] E. B. Tirkolaee, I. Mahdavi, M. M. S. Esfahani and G.-W. Weber, A robust green location-allocation-inventory problem to design an urban waste management system under uncertainty, Waste Management, 102 (2020), 340–350.
[34] G. Vardas and A. Xepapadeas, Model uncertainty, ambiguity and the precautionary principle: implications for biodiversity management, Environmental and Resource Economics, 45 (2010), 379–404.
[35] A. Yenipazarli, To collaborate or not to collaborate: Prompting upstream eco-efficient innovation in a supply chain, European J. Oper. Res., 260 (2017), 571–587.
[36] J. Yu and M. L. Mallory, An optimal hybrid emission control system in a multiple compliance period model, Resource and Energy Economics, 39 (2015), 16–28.
[37] Q. Zhang, W. Tang and J. Zhang, Green supply chain performance with cost learning and operational inefficiency effects, Journal of Cleaner Production, 112 (2016), 3267–3284.

Received February 2020; 1st revision May 2020; final revision October 2020.

E-mail address: guojianxin@casisd.cn
E-mail address: quxinglong@amss.ac.cn