Anomalous $Wtb$ couplings from $B$-physics

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We revisit constraints on anomalous $Wtb$ couplings from $B$-physics experiments, performing a correlated analysis allowing all anomalous couplings to differ simultaneously from their Standard-Model (SM) values. We find that the SM values belong to the 95% CL allowed region obtained this way. However, the correlated analyses reveal also other allowed regions of the anomalous couplings where the latter differ sizeably from the SM values. These regions cannot be seen in a simplified analysis when only one or two anomalous couplings are allowed to differ from their SM values.

1. INTRODUCTION

The top quark is the heaviest of all known elementary particles and is expected to have large couplings with physics beyond the Standard Model (BSM). One of the possibilities to probe BSM physics is to study the anomalous structure of the $Wtb$ vertex.

Assuming that at the scale $\mu \sim M_W$ the effects of BSM interactions may be parametrized in the framework of an effective theory by a tower of higher-dimension operators constructed from the SM fields and obeying the SM symmetries $[1]$, one obtains the most general $Wtb$ vertex of the following form $[2]$

$$\mathcal{L}_{tb} = \mathcal{L}_{tb}^{SM}$$

$$+ \frac{g}{\sqrt{2}} [b\gamma^\mu (f_{VL} P_L + f_{VR} P_R) t W_\mu$$

$$- \bar{b} \sigma^{\mu\nu} \partial_\nu W_\mu (f_{TL} P_L + f_{TR} P_R) t] + h.c.,$$

$$\mathcal{L}_{tb}^{SM} = V_0 \frac{g}{\sqrt{2}} [b\gamma^\mu P_L t W_\mu + h.c.,] (1.1)$$

where $P_{L/R} = \frac{1}{2}(1 \mp \gamma_5)$, $\sigma_{\mu\nu} = \frac{i}{2}(\gamma_\mu \gamma_\nu - \gamma_\nu \gamma_\mu)$, $g$ is the $SU(2)$ gauge coupling. The Lagrangian (1.1) corresponds to the choice of the covariant derivative acting on a left quark doublet with weak hypercharge $Y$ in the form

$$D_\mu = \partial_\mu - i \frac{g}{2} \tau^a W^a_\mu = -i \frac{g}{2} Y B_\mu.$$ (1.2)

The anomalous couplings $f_{VL}$, $f_{VR}$, $f_{TL}$ and $f_{TR}$ are induced by dimension-6 operators of the effective theory. In the SM, all anomalous couplings have zero values: $^1$

$$f_{VL} = f_{VR} = f_{TL} = f_{TR} = 0.$$ (1.3)

Obtaining experimental bounds on the anomalous couplings is an important direction in the search for New Physics. Such bounds may be obtained from different sources: e.g., from the direct production of top-quarks at hadron colliders, where weak processes is the main mechanism of the $t$-production $[3]$. Another promising way is the indirect probe of the anomalous couplings from flavour-changing neutral current (FCNC) processes in $B$-physics: here, virtual top often gives the leading contribution, thus opening the possibility to constrain its anomalous couplings $[3,4,5]$. This report follows the line of the analysis of $[4]$ and studies more closely the correlations between the anomalous couplings that can be obtained from the $B$-physics data.

2. EFFECTIVE LAGRANGIAN FOR WEAK FCNC $B$-DECAYS

For the description of weak $B$-decays, an appropriate physics scale is $\mu \simeq 5$ GeV; all particles with much heavier masses are not dynamical and may be integrated out within the formalism based on the operator product expansion. For FCNC $B$-decays, this approach leads to the effective Lagrangian (for details and the full set of the basis operators see $[6,7]$):

$$\mathcal{L}_{\text{eff}}(b \to s) = \frac{4G_F}{\sqrt{2}} V_{tb} V_{ts}^{*} \sum_{i=1}^{10} C_i O_i$$

$$+ O(V_{tb} V_{ts}^{*}) + O(V_{ut} V_{us}). \quad (2.1)$$

The $\mu$-dependent Wilson coefficients $C_i(\mu)$ at the scale $\mu \sim M_W$ encode the effects of heavy particles; in the SM these are $M_W$, $Z$, and $t$-quark, but in the extensions of the SM all other heavy particles contribute thus changing the values of the Wilson coefficients. One can write

$$C_i = C_i^{SM} + \delta C_i,$$ (2.2)

where the additional terms $\delta C_i$ reflect the new physics contributions. Probing the $Wtb$ anomalous couplings is reduced to measuring the deviations of the Wilson coefficients from the SM values.

However, $B$-meson observables, in addition to the Wilson coefficients, involve the amplitudes of the effective operators; the latter involve complicated hadron effects.

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$^1$ For convenience, we provide a comparison with the anomalous couplings used in other papers: $f_{VL} = V_L \frac{2}{2} = v_L - 1 \frac{2}{2}$, $f_{VR} = V_R \frac{4}{2} = v_R \frac{2}{2}$, $f_{TL} = G_L \frac{4}{2} = g_L \frac{2}{2}$, $f_{TR} = G_R \frac{4}{2} = g_R \frac{2}{2}$, and $f_{TR} = G_R \frac{4}{2} = g_R \frac{2}{2}$. 

So, in practice, only those processes involving $B$-mesons, where the hadron uncertainties are kept under good theoretical control, may be used for probing the anomalous $Wtb$ couplings.

The operators $O_{1-6}$ in (2.1) are four-quark operators, and $O_8$ is the gluon penguin operator; their contributions to the amplitudes of FCNC $B$-decays are very difficult to calculate with good precision; therefore, a plausible strategy for the purpose of searching New Physics is to avoid observables influenced by these operators.

The operators relevant for our analysis are

\[ O_7 = \frac{c_{m_b}}{16\pi^2} (s_L\sigma_{\mu\nu}b_R)F^{\mu\nu}, \]

\[ O_9 = \frac{c^2}{16\pi^2} (s_L\gamma_{\mu}b_L)(\bar{f}_\mu l), \]

\[ O_{10} = \frac{c^2}{16\pi^2} (s_L\gamma_{\mu}b_L)(\bar{f}_\mu\gamma_5 l). \]

The additions to the corresponding Wilson coefficients due to the anomalous couplings were obtained in [3, 4]: they are linear functions of the following anomalous couplings:

\[ \delta C_7(f_{VL}, f_{VR}, f_{TL}, f_{TR}), \]

\[ \delta C_9(f_{VL}, f_{TR}), \]

\[ \delta C_{10}(f_{VL}, f_{TR}). \]

For the explicit formulas, we refer to Appendix B of Ref. [4]. We also use the amplitude of $B$-$\bar{B}$ oscillations; noteworthy, it is a linear function of only two anomalous couplings, $f_{VL}$ and $f_{TR}$ [6].

For our further analysis, it is important to pay attention to the following facts:

(i) the coefficients $\delta C_9$ and $\delta C_{10}$, as well as the amplitude of the $B$-$\bar{B}$ oscillations do not get contributions from $f_{VR}$ and $f_{TL}$.

(ii) the coefficient $\delta C_7$ involves only one linear combination of $f_{VR}$ and $f_{TL}$.

So, in practice, precision measurements of $\delta C_1$ ($i = 7, 9, 10$) from $B$-physics allows one to get access to $f_{VL}$, $f_{TR}$ and one specific linear combination of $f_{VL}$ and $f_{TR}$, determined by $\delta C_7$.

3. BOUNDS ON ANOMALOUS Wtb COUPLINGS

When studying constraints on the anomalous couplings, both from direct top-quark production and from $B$-physics, one often considers different scenarios, depending on how many couplings are allowed to vary from their SM values. For instance, one-dimensional scenarios, when only one coupling differs from its SM value, are well known [3, 6]. Using such an approach, however, one does not access those regions where different anomalous couplings may have strongly correlated values far away from their SM values. We therefore follow here a different strategy: we allow all couplings to differ from their SM values and obtain the corresponding bounds from the data.

As already noticed above, only those $B$-decay channels, where theoretical QCD uncertainties are kept under control, may be used for the extraction of the anomalous couplings. In $B$-physics, the following three channels, that we use in our analysis, give the most stringent constraints on the anomalous $Wtb$ couplings:

- $B_{d,s} - \bar{B}_{d,s}$ oscillations (we use data from [10] and theoretical results from [9]);
- $Br(\bar{B} \to X_s\gamma)|_{E_\gamma > 1.6 \text{ GeV}}$ (data from [11] and theoretical estimates from [12]);
- $Br(\bar{B} \to X_s\mu^+\mu^-)|_{\mu^+\mu^- \text{ pair}}$ (data from [11] and theoretical inputs from [4]).

The amplitudes of these processes involve different linear combinations of the anomalous couplings $f_{VL}$, $f_{TR}$ but only one linear combination of the couplings $f_{VR}$ and $f_{TL}$:

\[ f \equiv a_{VR}f_{VR} + a_{TR}f_{TR}, \]

with $a_{VR} \simeq 98.6$ and $a_{TR} \simeq -50.1$ at the scale 5 GeV [4]. In what follows we obtain experimental bounds on $f$, $f_{VR}$, and $f_{TR}$ from the $B$-physics data.

With the theoretical expressions for the observables of interest as functions of the anomalous couplings at hand, we proceed as follows: for a combination of the independent observables $A_j$ with the calculated theoretical dependence on the set of the anomalous couplings $F$ in the form $A_j(F)$, the experimental averages $\bar{A}_j$, and the experimental uncertainties $\Delta A_j$, we obtain the combined probability distribution of $F$ as follows

\[ \rho(F) \propto \prod_j \exp \left[ -\left( \frac{A_j(F) - \bar{A}_j}{\Delta A_j} \right)^2 \right]. \]

Integrating these distributions over one of the couplings, we obtain 2D distributions of two other couplings $\rho(f_1, f_2)$ shown in Fig. 1. Figure 2 exhibits the regions of the two couplings corresponding to the probability of 95%. Obviously, in all cases the SM values belong to the 95% CL area; still, it might be noticed that the SM value is not the most probable value in the 95% CL region: there is an area with strongly correlated values of the anomalous couplings different from their SM values, which has a higher probability. Such interesting correlated solutions cannot be seen within 1D or 2D scenarios: a full correlated 3D analysis is necessary to observe them.

Finally, Fig. 3 gives a one-dimensional distribution of the individual coupling, obtained by integrating the 3D distribution over other couplings. We emphasize that these 1D distributions have a complicated shape and are much broader than the 1D distributions obtained by setting all other couplings to their SM values.
4. DISCUSSION AND CONCLUSIONS

We presented the analysis of the anomalous $Wtb$ couplings based on $B$-physics data without a priori making any assumptions about these couplings and allowing all of them simultaneously to differ from their SM values. Our conclusions may be summarized as follows:

1. Taking into account that any analysis of the $B$-physics data involves the theoretical calculation of complicated nonperturbative QCD effects (e.g., related to $B$-meson in the initial state, light mesons in the final state, charming-loops and charmonia resonances) only those modes where such effects may be controlled with good accuracy provide the possibility to probe the anomalous $Wtb$ couplings. Presently, such effects are limited to FCNC ra-

Fig. 1: 2D distributions obtained by integrating the 3D distribution (a): $f_{VL}$-$f_{TR}$; (b): $f_{VL}$-$f$; (c): $f_{TR}$-$f$.

Fig. 2: 95% CL allowed 2D regions corresponding to the 2D distributions in Fig. 1: (a): $f_{VL}$-$f_{TR}$; (b): $f_{VL}$-$f$; (c): $f_{TR}$-$f$. 
The momentum transfers of the $l^+l^-$ pair, and the oscillation of neutral $B$-mesons. In other interesting processes, where the gluon penguin operator $O_8$ or four-quark operators provide sizeable contributions, the nonperturbative QCD effects are very difficult to calculate with good accuracy; therefore such processes are not suitable for the analysis of the anomalous couplings from the data. Consequently, $B$-decays can provide bounds on three quantities: the anomalous couplings $f_{VL}$, $f_{TR}$ and one linear combination $f$ of two other couplings, $f_{VR}$ and $f_{TL}$, that appears in the Wilson coefficient $\delta C_7$.

2. Allowing all three couplings $f_{VL}$, $f_{TR}$, and $f$ to be different from their SM values reveals interesting regions of these couplings where they all take values different from the SM, see Fig. 2. Such solutions are missed within those scenarios that assume that only one or two couplings are different from their SM values \[12\]. Also, the latter scenarios lead to strongly underestimated uncertainties of the anomalous couplings compared to the general analysis when all couplings are allowed to take nonzero values.

3. In all considered 2D and 1D distributions, the SM values of the couplings belong to the region allowed at the 95% CL. Notice, however, that the SM values of the anomalous couplings are not the most probable set of the anomalous couplings. Thus, combining bounds on the anomalous couplings from the $B$-physics data with the direct bounds form top-quark production may be a promising route to new physics.

Fig. 3: The 1D distributions obtained by integrating the 3D distributions (red solid lines). (a): $f$; (b): $f_{VL}$; (c): $f_{TR}$. For comparison, the blue dashed lines show the distributions obtained by setting all other anomalous couplings to zero.

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