Degree stability of a minimum spanning tree of price return and volatility

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Abstract

We investigate the time series of the degree of minimum spanning trees obtained by using a correlation based clustering procedure which is starting from (i) asset return and (ii) volatility time series. The minimum spanning tree is obtained at different times by computing correlation among time series over a time window of fixed length $T$. We find that the minimum spanning tree of asset return is characterized by stock degree values, which are more stable in time than the ones obtained by analyzing a minimum spanning tree computed starting from volatility time series. Our analysis also shows that the degree of stocks has a very slow dynamics with a time-scale of several years in both cases.

PACS: 89.75Fb; 89.75Hc; 89.65Gh

Key words: Econophysics, Correlation based clustering, Volatility.

1 Introduction

The existence of correlation among price returns of different stocks traded in a financial market is a well-known fact [1–3]. Correlation based clustering procedures have been pioneered in the economic literature [4,5]. Recently, a

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new correlation based clustering procedure has been introduced in the econophysics literature. It has been shown that this correlation-based clustering procedure and some variant of it are able to filter out information which has a direct economic interpretation from the correlation coefficient matrix. In particular, the clustering procedure is able to detect clusters of stocks belonging to the same or closely related economic sectors starting from the time series of returns only \[6–10\].

In this paper we will consider the problem of the stability associated with the Minimum Spanning Tree (MST) obtained both from price return and volatility data. By investigating the stability of the value of the degree (number of links of the stock in the MST) of each stock we will show that volatility MST has less stable values of stock degree than price return MST. Moreover, by analysing the degree of elements of MSTs we will be able to show that the degree has a slow dynamics with a correlation time of several years.

The paper is organized as follows. In Sect. 2 we illustrate our results about the MST of volatility time series of a set of stocks. In Sect. 3 we comment on the stability of stock degree in the MSTs of price return and volatility time series and we discuss the time-scale associated with the slow dynamics of degree of MSTs. In Sect. 4 we briefly draw our conclusions.

2 Correlation-based clustering of volatility

We investigate the statistical properties of cross-correlation among volatility and among price return time series for the 93 most capitalized stocks traded in US equity markets during a 12 year time period. Our data cover the whole period ranging from January 1987 to April 1999 (3116 trading days). In the present study we investigate daily data. In particular, we use for our analysis the open, close, high and low price recorded for each trading day for each considered stock. The stocks were selected by considering the capitalization recorded at August 31, 1998.

Starting from the daily price data, we compute both the daily price return \( R_i(t) \) and the daily volatility \( \sigma_i(t) \) for each stock \( i = 1, \ldots, 93 \). Price returns are defined as \( R_i(t+1) = [P_i(t+1) - P_i(t)]/P_i(t) \) where \( P_i(t) \) is the close price of stock \( i \) at day \( t \). Volatility is computed by using the proxy \( \sigma_i(t) = 2 \left[ \max\{P_i(t)\} - \min\{P_i(t)\} \right]/\left[ \max\{P_i(t)\} + \min\{P_i(t)\} \right] \) where \( \max\{P_i(t)\} \) and \( \min\{P_i(t)\} \) are the highest and lowest price of the day, respectively.

The correlation based clustering procedure introduced in Ref. [6] is based on the computation of the subdominant ultrametric distance [11] associated with a metric distance that one may obtain from the correlation coefficient. The
subdominant ultrametric distance can be used to obtain a hierarchical tree and a MST. The selection of the subdominant ultrametric distance for a set of elements whose similarity measure is a metric distance is equivalent to considering the single linkage clustering procedure [12]. Further details about this clustering procedure can be found in [13].

In the present investigation, we first aim to consider the MST associated to the correlation coefficient matrix of volatility time series. It should be noted that there is an essential difference between price return and volatility probability density functions. In fact the probability density function of price return is an approximately symmetrical function whereas the volatility probability density function is significantly skewed. Bivariate variables whose marginals are very different from Gaussian functions can have linear correlation coefficients which are bounded in a subinterval of $[-1, 1]$ [14]. Since the empirical probability density function of volatility is very different from a Gaussian the use of a robust nonparametric correlation coefficient is more appropriate for quantifying volatility cross-correlation. In fact the volatility MSTs obtained starting from a Spearman rank-order correlation coefficient are more stable with respect to the dynamics of the degree of stocks than the ones obtained starting from the linear (or Pearson’s) correlation coefficient. The clustering procedure based on the Spearman rank-order correlation coefficient uses the volatility rank time series to evaluate the subdominant ultrametric distance. The time series of the rank value of volatility are obtained by substituting the volatility values with their ranks. Then one evaluates the linear correlation coefficient between each pair of the rank time series [15] and starting from this correlation coefficient matrix one obtains the associated MST. An example of the MST obtained starting from the volatility time series and by using the Spearman rank-order correlation coefficient is shown in Fig. (1). This MST is shown for illustrative purposes and it has been computed by using the widest window available in our database ($T = 3116$ trading days). A direct inspection of the MST shows the existence of well characterized clusters. Examples are the cluster of technology stocks (HON, HWP, IBM, INTC, MSFT, NSM, ORCL, SUNW, TXN and UIS) and the cluster of energy stocks (ARC, CHV, CPB, HAL, MOB, SLB, XON). As already observed in the MST obtained from the price return time series, the volatility MST of Fig. (1) shows the existence of stocks that behave as reference stocks for a group of other stocks. Examples are GE (General Electric Co), JPM (JP Morgan Chase & Co) and DD (Du Pont De Nemours Co.).

3 Stability of the price return and volatility MST

A natural question arises whether or not the structure of the MST depends on the particular time period considered. This point has been considered briefly
in [8,9] and it has also been recently addressed in [16,17]. In the present investigation we compute a MST for both volatility and price return for each trading day $t$. This is done by considering the records of the time series delimited by a sliding time window of length $T$ days ranging from day $t$ to day $t + T$. For example, by using a time window with $T = 117$ we approximately compute 3000 MSTs in our sets of data. In each MST, each stock has a number of other stocks that are linked to it. This number is usually referred to as the degree of the stock. By using the above procedure, we obtain a daily historical time series of degree for each of the considered 93 stocks both for price return and for volatility. In the following, we focus our attention on the
Fig. 2. Autocorrelation function of the degree of MSTs obtained starting from price return time series by using the correlation based clustering procedure described in the text. A different line-style indicates a different value of the sliding time window. The values of the time windows are $T = 117$ (dashed line), $T = 227$ (solid line) and $T = 357$ (dotted line) trading days. For each line, arrows indicate the length of the time window. They also indicate the point where MSTs start to be computed from non-overlapping time windows.

Each time series of the degree of each stock has about 3000 records. This number of records is not enough to detect reliably the autocorrelation function of the degree time series for each stock. Hence, we decide to investigate the properties of the degree time series obtained by joining all the 93 degree time series of each stock. This is done separately for each set of data (price return or volatility) and for each value of the time window $T$. From the time series obtained as described above we compute the autocorrelation function. The comparison of the results obtained for price return with the one obtained from volatility time series allows us to estimate the stability of MSTs of these two important financial indicators. For the sake of clarity we will first consider the two set of data separately and then we will comment on similarity and differences between them.
3.1 Price Returns

In Fig. (2) we show autocorrelation functions of 3 different time series of the degree. The analyzed MSTs are computed by investigating the linear correlation coefficient which is present among price return and by using three different time windows $T$. Specifically, we use time window of size $T = 117$, $T = 227$ and $T = 357$ trading days. In all cases the autocorrelation function shows two distinct regimes for low and high time values. The crossover between the two regimes is detected at $t = T$ (see arrows in Fig. (2)). For low time values the autocorrelation function of degree approximately decays exponentially (a straight line in the semilogarithmic plot of Fig. (2)). This behavior reflects the fact that the sliding window used to compute MSTs contains overlapping time period of records. For this reason when a day of high correlation among several pairs of stock occurs a memory of this event remains within a time interval of length $T$. This behavior is therefore simply related to the methodology used by us to compute the degree time series.

More relevant information is obtained from the degree autocorrelation at time $t$ equal or longer than the time window size $T$. For $t \approx T$ the autocorrelation function assumes a non negligible value approximately equals to 0.31, 0.39 and 0.42 for a time window of $T = 117$, $T = 227$ and $T = 357$ trading days respectively. These results indicate that the information carried by the degree of the stocks in the MSTs is robust in spite of the presence of some noise dressing. The increase of the value of the autocorrelation function at $t \approx T$, which is detected by increasing $T$ indicates that the noise dressing decreases when $T$ increases. For time $t$ longer than $T$ the degree autocorrelation function approximately decays exponentially with a very long time-scale $\tau_R$. For instance, in the case of $T = 117$ an exponential best fit of the autocorrelation function is obtained with the time-scale $\tau_R = 714$ trading days. This time-scale approximately corresponds to 2.8 calendar years. It should be noticed that the 3 autocorrelation functions obtained for different values of $T$ are approximately parallel to each other and follow an exponential function with the same time-scale. Before we move to consider the analogous results obtained for volatility time series we wish to point out that the results presented in Fig. (2) are essentially independent on the methodology used to compute the correlation coefficient matrix. In fact we obtain the same results when we use the Spearman rank-order correlation coefficient.

3.2 Volatility

In Fig. (3) we show the results of the same analysis performed on volatility time series. In the case of volatility the MSTs are obtained starting from
Fig. 3. Autocorrelation function of the degree of MSTs obtained starting from
volatility time series by using a correlation based clustering based on the Spear-
man rank-order correlation coefficient and the procedure described in the text. A
different line-style indicates a different value of the sliding time window. The values
are the same as in Fig. (1). For each line, arrows indicate the size of the time window.
They also indicate the point where MSTs start to be computed from non-overlapping
time windows.

the Spearman rank-order correlation coefficient. In fact if we compute MSTs
and degree time series by using a linear correlation coefficient the results are
much less reliable and the degree autocorrelation function for \( t > T \) seems
to be more affected by noise. MSTs obtained starting from the Spearman
rank-order correlation coefficient are more statistically robust and the degree
autocorrelation function shows the same general behavior as in the case of
price return time series. However some important differences are detected.
The first one concerns the amount of correlation observed at \( t \approx T \). The
values of the autocorrelation function are approximately equals to 0.19, 0.21
and 0.22 for a time window of \( T = 117, T = 227 \) and \( T = 357 \) trading days
respectively. These results indicate that the information carried by the degree
of the stocks in the MSTs of volatility is less stable over time than the one
detected in the MSTs of price returns. Moreover, the increase of the value of
the degree autocorrelation time series with the time window \( T \) is much less
pronounced for volatility than for price return. Another difference concerns the
slow decay of the correlation observed for \( t > T \). For large values of \( t \) the decay
of the degree autocorrelation function is again approximately exponential but
the time-scale $\tau_\sigma$ obtained by best fitting the autocorrelation function with an exponential function is $\tau_\sigma = 1510$ trading days in this time interval. This value is approximately double than the time-scale $\tau_R$ detected in the analysis of price return.

4 Conclusions

The parallel investigation of MSTs obtained from (i) price return and (ii) volatility time series of a set of stocks allows us to conclude that the stability of the degree of MSTs is lower for volatility time series than for price return time series. For price return time series, the stability of stock degree dynamics increases when the time window $T$ used to compute MSTs is increased. A similar but much weaker trend is also observed in MSTs obtained starting from volatility time series.

The dynamics of the degree of stocks in the MSTs is of statistical nature with a time memory which is approximately close to 700 trading days for price return and 1500 trading days for volatility time series. The time-scale of the degree of MSTs of price return is much less than the maximal time-scale of our investigation $T_{max} \approx 3000$ and therefore it should not be significantly affected by it. On the other hand the time-scale found for the degree of MSTs obtained starting from volatility time series is just $\approx T_{max}/2$, which implies that the detection of this specific time-scale could be an artifact of the procedure we used to compute the degree autocorrelation function. However, it should be noted that, the detected value of $\tau_\sigma = 1510$ trading days is certainly a lower bound of the true time-scale of the degree autocorrelation function of volatility MST.

In summary, relevant economic information is stored in the degree of MSTs obtained from price return and volatility time series. The dynamics of stock degree is statistically more stable for price return than for volatility MSTs and it has a slow dynamics characterized by a time-scale of the order of 3 calendar years for price return MSTs and longer than 6 calendar years for volatility MSTs.

5 Acknowledgments

The authors thank INFM and MIUR for financial support. This article is part of the MIUR-FIRB project on “Cellular self-organization nets and chaotic non-linear dynamics to model and control complex systems”. G.B. acknowledges financial support from FET open project COSIN IST-2001-33555.
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