Introducing NatPy, a simple and convenient Python module for dealing with natural units.

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In high energy physics, the standard convention for expressing physical quantities is natural units. The standard paradigm sets $c = \hbar = \epsilon_0 = 1$ and hence implicitly rescales all physical quantities that depend on unit derivatives of these quantities. We introduce NatPy, a simple Python module that defines user-friendly unit objects that can be used and converted within any predefined system of natural units. In this note, we will first introduce, then overview, the algebraic methods utilised by the NatPy module.

\section{INTRODUCTION}

In high energy physics, the common practice is to express quantities in a system where a basis of physical constants ($c$, $\hbar$, ...) are defined to be dimensionless with a value of 1, a system referred to as natural units. This practice allows for quantities that would otherwise be dimensionally incompatible in the SI, such as length and time, or mass, momentum, and energy, to be expressed in the same units and treated as dimensionally equivalent. While such a system greatly eases the complexity of physical calculations, trouble can occur when attempting to convert quantities from a system such as the SI to a system of natural units, or vice versa. One has to determine the correct combination of factors of each of the unit constants, a process that is both tedious and prone to error. To address this issue, we introduce NatPy, a Python package capable of determining the correct conversion to and from a defined system of natural units.

\begin{verbatim}
>> import natpy as nat
>> P = 1 * nat.MeV * nat.fm**(-3)
>> P.convert(nat.Pa)
<Quantity 1.60217663e+32 Pa>
\end{verbatim}

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{fig1.png}
\caption{Converting 1 MeV fm\textsuperscript{3} to pascals using NatPy, two units that are dimensionally equivalent when $c = \hbar = 1$.}
\end{figure}

NatPy leverages the astropy.units.core.Unit and astropy.units.quantity.Quantity classes from the astropy package \cite{1, 2} in order to allow users to define syntax friendly unit objects for seamless integration into any Python workflow. The power of NatPy manifests when large and/or complicated expressions are functions of many dimensional quantities. In section II we summarise the installation and usage of NatPy. In section III we outline a generic method for determining the necessary combination of factors of the unit constants for a conversion before finally discussing the implementation of this method in sections IV and V.

\section{INSTALLATION AND USAGE}

NatPy requires python $\geq$ 3.7 and can be installed via pip:

\begin{verbatim}
pip install natpy
\end{verbatim}

For instructions on usage, readers can refer to a presentation given at PyHEP 2021, which includes a Binder tag for an interactive tutorial https://github.com/AndreScaffidi/Natpy_pyhep_2021, as well as the package repository https://github.com/AndreScaffidi/NatPy and PyPI page https://pypi.org/project/NatPy/.

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\end{verbatim}
### III. CONVERSION

This method aims to determine the necessary factor required to convert a physical quantity from one set of units to another. This assumes the use of a generalised natural unit convention (e.g. $\hbar = c = ... = 1$), and includes determining the necessary combination of factors of these natural units required to obtain desired units. The outlined method provides a rigorous 2-step process to determine a conversion factor, without the trouble of remembering the correct non–natural unit versions of physical quantities. For example, converting energy to mass as $eV \rightarrow eV/c^2$.

#### A. Notation

The quantity $[q]$ with square brackets is defined as the units of the physical quantity $q$. This includes exact units maintaining metric prefixes and other normalisations. So for example

$$m_e = 512 \text{ keV} \implies [m_e] = [\text{keV}] \quad (1)$$

Second, the quantity $\{q\}$ with curly brackets is the dimensions of the quantity $q$. In the same example

$$m_e = 512 \text{ keV} \implies \{m_e\} = \text{mass}. \quad (2)$$

Importantly, dimensions removes any overall factors ($\{\text{years}\} = \{\text{seconds}\} = \{\text{time}\}$), but $\{\text{length}\} \neq \{\text{time}\}$ regardless of natural units. Finally, we have $|q|$ as the value of the quantity in units of $[q]$ (not absolute value), so

$$m_e = 512 \text{ keV} \implies |m_e| = 512. \quad (3)$$

As a result, a quantity may be decomposed as

$$q = |q|[q] \quad (4)$$

#### B. Method

Our method from here outlines a two-step process to determine the conversion factor from one set of units to another. The process begins with extracting the initial units, dropping the value of quantity and maintaining the precise units, including overall multipliers. Consider the example quantity conversion:

$$1 \text{ MeV} \quad \text{to} \quad 5.07 \times 10^3 \text{ nm}^{-1}$$

This quantity is given in units of megaelectronvolts (MeV). We wish to display this quantity in units of inverse nanometres (nm$^{-1}$). Such a conversion requires determining a relevant factor of $c$ and $\hbar$ as well as an overall scalar factor, which can be troublesome to determine. We let $|q|_i$, $[q]_i$ be the pre–conversion value and units of $q$, and $|q|_f$, $[q]_f$ the post–conversion. A conversion aims to solve the expression

$$|q|_f/[q]_f = |q|_i/[q]_i. \quad (5)$$

The method aims to find a conversion factor $x$ such that

$$|q|_f = x \cdot |q|_i, \quad (6)$$

by solving

$$[q]_f = x^{-1} \cdot [q]_i. \quad (7)$$

The factor $x$ has two contributions; one from the overall conversion factor between “same dimensional” quantities, denoted $f$ (e.g. metres to centimetres, seconds to years), and a factor due to the “natural dimensions” denoted $d$ (e.g. metres to seconds, mass to energy). The first step in this method is to determine the “natural dimensionality” of our conversion, i.e. the combination of factors of basis physical constants ($\hbar$, $c$, etc.) necessary to achieve our desired conversion. We determine the necessary product of natural units that has the same dimension of $\{q\}_f/\{q\}_i$, so

$$d = h^a c^b \ldots, \quad \text{such that} \quad \{d\} = \{q\}_f/\{q\}_i. \quad (8)$$
Hence we must first obtain $d$ from $\{q\}_f / \{q\}_i$, which in our example is

$$\{d\} = \frac{\{\text{nm}^{-1}\}}{\{\text{MeV}\}} = \frac{1}{\{\text{length}\}\{\text{energy}\}} = \frac{1}{\{\text{energy}\}\{\text{time}\}} = \{h\}^{-1}\{c\}^{-1}$$

From here we carefully consider $d$, as this can be expressed in a variety of units. We have from the convention of natural units, that $d = |d| = 1$ for any $d$ composed of basis physical constants. Due to this, equation (5) can be written

$$|q|_f |q|_f = |d| |q|_i |q|_i,$$

(9)

giving our conversion equations

$$|q|_f = f |d| |q|_i,$$

(10)

and

$$|q|_f = f^{-1} |d| |q|_i,$$

(11)

where $|d|$ corresponds to the value of $d$ in units of $[d]$. The second step from here is to determine the multiplicative factor $f$ to match the remaining units. We have by construction of $d$ that $|q|_f$ and $|d| |q|_i$ have the same dimensions, so any difference in units is simply a scalar multiplicative factor. So $f$ is determined as in equation (11). In our original example, if $d = (hc)^{-1} = 5.07 \text{ eV}^{-1} \text{ nm}^{-1}$,

$$f = |q|_f^{-1} |d| |q|_i$$

$$= ([\text{nm}])((\text{eV}) \cdot [\text{nm}])^{-1}([\text{MeV}])$$

$$= \frac{[\text{nm}]}{[\text{MeV}]} \frac{[\text{MeV}]}{[\text{eV}]} = 10^{-3} \times 10^6 = 10^3$$

Finally, we have that $x = \overline{d} \cdot f$, which is our final conversion factor. In our example,

$$x = 5.07 \times 10^3.$$

So as a result,

$$|q|_f = 5.07 \times 10^3 |q|_i,$$

where $|q|_i = \text{MeV}$, and $|q|_f = \text{nm}^{-1}$,

$$\implies 1 \text{ MeV} = 5.07 \times 10^3 \text{ nm}^{-1}$$

To summarise the method;

1. Find the dimensions of the quotient of the final and initial units, $\{d\} = \{q\}_f / \{q\}_i$,

2. Determine the combination of factors that has the same dimensions as that quotient, $d = \hbar n c e^\ldots$,

3. Find the resulting scaling factor from cancelling any remaining units, $f = |q|_f^{-1} |d| |q|_i$,

4. Obtain the final conversion factor, $x = |d| \cdot f$.

IV. ALGEBRAIC IMPLEMENTATION

The method expressed above is still somewhat tedious. While it does give a systematic framework from which to find conversion factors, step 2 in which $d$ is determined from the dimensionality is not automatic, requiring somewhat arbitrary algebraic manipulations until the correct factor is found. Instead, we propose a direct computation to determine the factor $d$ using simple linear algebra. It is this implementation on which NatPy is developed, applying this computation to dimensional quantities to obtain conversion factors.
A. Notation

In this algebraic framework, we shall define two sets of vectors: The powers of dimensionality (PoD) of a quantity \( q \), denoted \( \tilde{q} \), and the powers of natural units (PoNU) of a dimensional conversion factor \( d \), denoted \( \vec{d} \). The PoD of a quantity \( q \) is the vector of multiplicities of the dimensions of \( q \) in terms of a set of base units. The seven SI base units are used in NatPy.

\[
\{q\} = \{\text{length}\}^{n_{\text{length}}} \{\text{time}\}^{n_{\text{time}}} \{\text{mass}\}^{n_{\text{mass}}} \ldots \quad \Rightarrow \quad \tilde{q} := \begin{pmatrix} n_{\text{length}} \\ n_{\text{time}} \\ n_{\text{mass}} \\ \vdots \end{pmatrix} .
\] (12)

For example, a force \( F \) has a PoD given by:

\[
\{F\} = \{\text{newton}\} = \{\text{mass}\}\{\text{length}\}\{\text{time}\}^{2} \Rightarrow \tilde{F} = \begin{pmatrix} F_{\text{length}} \\ F_{\text{time}} \\ F_{\text{mass}} \end{pmatrix} = \begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix} .
\]

The powers of natural units (PoNU) of the dimensional conversion factor \( d \) is defined similarly, but by the powers of the basis physical constants rather than base units. So

\[
d = \hbar^{n_{\hbar}} c^{n_{c}} \ldots \Rightarrow \vec{d} := \begin{pmatrix} n_{\hbar} \\ n_{c} \end{pmatrix} .
\] (13)

B. Implementation

Given the previously defined notation, the goal of the outlined method is simple: find \( \vec{d} \) given \( \tilde{d} \). Since \( d \) is defined entirely by \( \vec{d} \), the PoNU is now the target quantity to calculate. The dimensionality factor \( d \) is found such that \( \{d\} = \{q\} f/\{q\} i \). Due to the property that \( a^{b} a^{c} = a^{b+c} \), it follows that \( d = \tilde{q} f - \tilde{q} i \), giving a simple method to determine \( d \). Furthermore, applying this same property to the definition of \( d = \hbar^{n_{\hbar}} c^{n_{c}} \ldots \), we have that

\[
\begin{pmatrix} d_{\text{length}} \\ d_{\text{time}} \\ d_{\text{mass}} \\ \vdots \end{pmatrix} = n_{\hbar} \begin{pmatrix} \hbar_{\text{length}} \\ \hbar_{\text{time}} \\ \hbar_{\text{mass}} \end{pmatrix} + n_{c} \begin{pmatrix} c_{\text{length}} \\ c_{\text{time}} \\ c_{\text{mass}} \end{pmatrix} + \ldots
\]

\[
\vec{d} = \begin{pmatrix} \hbar_{\text{length}} & c_{\text{length}} & \ldots \\ \hbar_{\text{time}} & c_{\text{time}} & \ldots \\ \hbar_{\text{mass}} & c_{\text{mass}} & \ldots \end{pmatrix} = A\vec{d}
\]

\[
\Rightarrow \quad \vec{d} = A^{+} \vec{d}
\]

We shall define \( A \) the change of dimensionality (CoD) matrix. The final result depends on calculating the left pseudo–inverse of \( A \), denoted \( A^{+} \), obtainable as the Moore–Penrose inverse of \( A \). Notice the CoD matrix is entirely independent of the initial or final units, being a function only of the chosen system of natural units. This results in \( A^{+} \) needing only to be calculated once, and is usable for all natural dimensional conversions in a given system of natural units.

The goal of determining \( \vec{d} \) from \( \tilde{d} \) has become one of simple algebra:

1. The CoD matrix \( A \) is constructed from the PoD vectors of each of the basis physical constants, \( (\hbar, c, \ldots) \),
2. The Moore–Penrose pseudo–inverse \( A^{+} \) of \( A \) is calculated,
3. Apply \( A^{+} \) to the PoD vector of the dimensionality factor \( \tilde{d} \) to obtain the the PoNU vector of this conversion, \( \vec{d} \),
4. Calculate \( d \) from \( \vec{d} \).
V. TECHNICAL IMPLEMENTATION

NatPy implements the above conversion process by leveraging the astropy Python module. It makes use of the astropy.units.core.Unit and astropy.units.quantity.Quantity classes to define dimensional objects. It then draws on the astropy.constants submodule to define the list of constants on which to form the natural unit basis. From this list the change of dimensionality (CoD) matrix is constructed, and pseudo–inverted by the numpy.linalg submodule [3]. The natpy.convert method is constructed to calculate the power of dimensionality (PoD) vector between initial and target units, so as to find the power of natural units (PoNU) vector of the dimensional conversion factor $d$. The correct factors of natural units are multiplied to the initial quantity, and astropy is used for the final scalar conversion to the target units.

By utilising the astropy module, numpy is fully incorporated. Quantity objects may store numpy.ndarray objects as the quantity, providing full access to numpy’s ufunc functionality. In storing an array of quantities with the same units, such an array may be converted between compatible units by only calculating a conversion factor once, applying the same factor across the array.

VI. SUMMARY

NatPy provides a computational framework for calculations involving dimensional quantities in a way that properly considers equivalences due to the conventions of a given natural units scheme. Quantities may be converted from the conventional units of high energy physics, such as masses in GeV or times in fm, to the standard SI units kg and s and vice versa, when considering a system of natural units. NatPy automates this process, reducing the likelihood of simple algebraic errors. The framework provided can be incorporated into existing analysis, either by using NatPy to find relevant conversion factors, or by storing quantities in Quantity objects and using NatPy to convert such an object between compatible units. NatPy is fully incorporated with numpy to allow for powerful functionality.

VII. ACKNOWLEDGEMENTS

TLH acknowledges support from an Australian Government Research Training Program Scholarship. AS acknowledges support from the research grant “The Dark Universe: A Synergic Multimessenger Approach” No. 2017X7X85K funded by MIUR and the project “Theoretical Astroparticle Physics (TAsP)” funded by the INFN. We thank all funding agencies.

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