$SU(3)^p$ Quiver Theories with $\mathcal{N} = 0$ for $p = 8$ and 9

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Abstract

We close a gap in previous studies of nonsupersymmetric $\mathcal{N} = 0$ quiver gauge theories from a phenomenological point of view aimed at acquiring specific proposals for models beyond the Standard Model (BSM). Because $SU(3)$ is the gauge group of QCD we fix $N = 3$ and vary only the $Z_p$ abelian orbifold. The values $1 \leq p \leq 7$ have been previously fully discussed as well as one special case, discovered by happenstance, of $p = 12$. The values $p = 8$ and $p = 9$ are discussed comprehensively in the present paper including the electroweak mixing angle, gauge coupling unification, spontaneous symmetry breakdown to the standard model, and the occurrence of three quark-lepton families. Two promising quiver node identifications are discovered for $p = 8$ and three for $p = 9$. All of these merit further study as BSM candidates.

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1 Introduction

A possible approach to generate new models beyond the Standard Model (BSM) is to use non-supersymmetric gauge theories derived from the most highly supersymmetric $N = 4$ gauge theories. Such $N = 0$ theories can be systematically constructed from $N = 4$ ones by using suitable abelian $Z_p$ orbifolding \cite{1,2}. These constructions are encoded by quiver diagrams \cite{3,4}, in which the $i^{th}$ node represents the $U(N)_i$ gauge symmetry and oriented arrows from the $i^{th}$ to the $j^{th}$ node represent fermions in the bifundamental $(N_i, \overline{N}_j)$ representation of the two gauge groups at nodes $i$ and $j$. Scalars are usually denoted by dashed lines connecting two nodes, in a related representation of the gauge group $(N, \overline{N}) + (\overline{N}, N)$, if the parameters $a_i$ of the quiver theory, that we will define below, are all nonzero. If any of the parameters $a_i$ is zero then such scalars can be in singlet or in adjoint representations of the gauge group, but it has been shown that in such a case chiral fermions are not allowed by the theory, and as such they are of no physical interest.

Explicit examples with $Z_p$ orbifolding have been considered in the past for several $p$ values. For example, in \cite{5} it has been discussed a $Z_7$ model which contains all the states of the Standard Model (SM) and in \cite{6} a $Z_{12}$ model allowing grand unification at a scale of 4 TeV. The result of this construction is a gauge theory with a gauge structure of the form $SU(3)^p$ which contains a colour gauge symmetry $SU(3)^{n_c}_C$, a weak $SU(3)^{n_W}_W$ symmetry and a $SU(3)^{n_H}_H$ of hypercolour, with $n_H + n_W + n_C = p$. This symmetry is characterised by a single coupling $g$ above the scale of grand unification (GUT) $\mu_{GUT}$, where the $p$ factors are all independent copies of $SU(3)$, with a $Z_p$ symmetry which renders the $p$ nodes of the quiver diagram identical. The issue whether such classically scale invariant theory may be conformal invariant at quantum level, with a vanishing $\beta$ function beyond one loop, has been matter of debate in the past, and conclusive arguments in this context are still missing \cite{7,8}. Recent discussions of classically conformally gauge filed theories include \cite{9,10,11,12}. The structure of the theory below $\mu_{GUT}$ ($\mu < \mu_{GUT}$) is of the form $SU(3)_c \times SU(3)_W \times SU(3)_H$, with a lumping of each of the $n_i$ ($n_i = i = C, W, H$) gauge symmetries into the product of single $SU(3)$ factors of the form $SU(3)^{n_c}_C \times SU(3)^{n_W}_W \times SU(3)^{n_H}_H$. Each of the $SU(3)$ factors, at this scale, is the surviving diagonal subgroup of the the colour, weak and hypercolour symmetries, with couplings which are renormalized and reduced by the same multiplicites $n_i$ ($g \rightarrow g_i = g/\sqrt{n_i}$).

The $SU(3)^3$ symmetry of the diagonals is indeed a trinification \cite{13}, but with gauge couplings which are different in size and that can be unified at a far smaller scale compared to the typical $10^{15} - 10^{16}$ GeV GUT scale. In ordinary trinification, the 3 couplings meet at a specific (usually very large) scale, after a large logarithmic running, which is not necessary in this case, with the result that the GUT scale can be as low as 4 TeV.

Above such scale, as we have already mentioned, the quiver theory is probably characterised by a quasi conformal behaviour, since the one-loop beta function vanishes, while its vanishing at two loops is not guaranteed. The appearance of double trace operators, due to the breaking of supersymmetry of the mother theory, with their non-vanishing beta-functions, has been brought up as an argument against its quantum conformal behaviour. In these theories the hierarchy is significantly ameliorated since the one loop quadratic divergences, which emerge in the Higgs sector of the SM, are absent. This is due to a precise cancellation between bosonic and fermionic contributions in the scalar 2-point function, a property which is inherited by the quiver theory from the $\mathcal{N} = 4$ mother theory.
In the models that we study below these features are all present and render them quite interesting from the phenomenological viewpoint. In the absence of any supersymmetric signal at the LHC, it is therefore tempting to reconsider such models in some generality, building on previous analysis and extending their classification, since they provide an alternative view to unification based on ordinary GUT’s. This in an energy range which can probed at the LHC or at least at the next generation of colliders.

The goal of our work is to present some additional quiver theories which are consistent with the particle content of the SM and which have not been noticed before. In the sequence of \(Z_p\) models that we consider, as we shall see, the first with chiral fermions is \(Z_4\) but the \(Z_7\) and \(Z_{12}\) examples also fall into the class we shall investigate.

2 General features of quiver theories

We consider the compactification of the type-IIB superstring on the orbifold \(AdS_5 \times S^5/\Gamma\) where \(\Gamma\) is an abelian group \(\Gamma = Z_p\) of order \(p\) with elements \(\exp(2\pi iA/p)\), \(0 \leq A \leq (p - 1)\). The resultant quiver gauge theory has \(N\) residual supersymmetries with \(N = 2, 1, 0\) depending on the details of the embedding of \(\Gamma\) in the \(SU(4)\) group which is the isotropy of the \(S^5\). This embedding is specified by the four integers \(A_m, 1 \leq m \leq 4\) with

\[
\Sigma_mA_m = 0 \pmod{p} \tag{1}
\]

which characterize the transformation of the components of the defining representation of \(SU(4)\).

We are here interested in the non-supersymmetric case \(N = 0\) which occurs if and only if all four \(A_m\) are non-vanishing. The gauge group, ignoring \(U(1)'s\), is \(U(N)^p\). The fermions are all in the bifundamental representations

\[
\Sigma_{m=1}^4 \Sigma_j^{j=p} (N_j, \bar{N}_j + A_m) \tag{2}
\]

which are manifestly non-supersymmetric because no fermions are in adjoint representations of the gauge group. Scalars appear in representations

\[
\Sigma_{i=1}^3 \Sigma_{j=1}^{j=p} (N_j, \bar{N}_j \pm a_i) \tag{3}
\]

in which the six integers \((a_i, -a_i)\) characterize the transformation of the antisymmetric second-rank tensor representation of \(SU(4)\). The \(a_i\) are given by

\[
a_1 = (A_2 + A_3), a_2 = (A_3 + A_1), a_3 = (A_1 + A_2). \tag{4}
\]

It is possible for one or more of the \(a_i\) to vanish, in which case the corresponding scalar representation in the summation in Eq.(3) is to be interpreted as an adjoint representation of one particular \(U(N)_j\). One may therefore have zero, two, four or all six of the scalar representations, in Eq.(3), in such adjoints.

Note that there is one model with all scalars in adjoints for each even value of \(p\) (see Model Nos 1,3,12). For general even \(p\) the embedding is \(A_m = (\frac{p}{2}, \frac{p}{2}, \frac{p}{2}, \frac{p}{2})\). This series is the complete list of \(N = 0\) abelian quivers with all scalars in adjoints.

To be of more phenomenological interest the model should contain chiral fermions. This requires that the embedding be complex: \(A_m \not\equiv -A_m \pmod{p}\). It has been shown that for the presence of chiral fermions all scalars must be in bifundamentals.
Table 1: List of all abelian chiral quiver models for $p \leq 7$.

| Model No. | $p$ | $A_m$ | $a_i$ | scalar bifunds. | scalar adjoints | chiral fermions? | Contains SM fields? |
|-----------|-----|-------|-------|----------------|----------------|------------------|-------------------|
| 4A        | 4   | (1111) | (222) | 6              | 0              | Yes              | No                |
| 5A        | 5   | (1112) | (222) | 6              | 0              | Yes              | No                |
| 5B        | 5   | (2224) | (111) | 6              | 0              | Yes              | No                |
| 6A        | 6   | (1113) | (222) | 6              | 0              | Yes              | No                |
| 6B        | 6   | (2235) | (112) | 6              | 0              | Yes              | No                |
| 6C        | 6   | (1122) | (233) | 6              | 0              | Yes              | No                |
| 7A        | 7   | (1114) | (222) | 6              | 0              | Yes              | No                |
| 7B        | 7   | (1123) | (233) | 6              | 0              | Yes              | Yes               |
| 7C        | 7   | (1222) | (333) | 6              | 0              | Yes              | No                |
| 7D        | 7   | (1355) | (113) | 6              | 0              | Yes              | Yes               |
| 7E        | 7   | (1445) | (122) | 6              | 0              | Yes              | Yes               |
| 7F        | 7   | (2444) | (111) | 6              | 0              | Yes              | No                |

The proof of this assertion follows by assuming the contrary, that there is at least one adjoint arising from, say, $a_1 = 0$. Therefore $A_3 = -A_2 \pmod{p}$. But then it follows from Eq.(1) that $A_1 = -A_4 \pmod{p}$. The fundamental representation of $SU(4)$ is thus real and fermions are non-chiral.

The converse also holds: If all $a_i \neq 0$ then there are chiral fermions. This follows since by assumption $A_1 \neq -A_2$, $A_1 \neq -A_3$, $A_1 \neq -A_4$. Therefore reality of the fundamental representation would require $A_1 \equiv -A_1$ hence, since $A_1 \neq 0$, $p$ is even and $A_1 \equiv \frac{p}{2}$; but then the other $A_m$ cannot combine to give only vector-like fermions. It follows that in an $\mathcal{N} = 0$ quiver gauge theory, chiral fermions are possible if and only if all scalars are in bifundamental representations.

For the lowest few orders of the group $\Gamma$, the members of the infinite class of $\mathcal{N} = 0$ abelian quiver gauge theories are tabulated below.

We show in Table 1 the list of quiver models for $p \leq 7$, the first is at $p = 4$. In this paper we shall discuss the cases $p = 8$ and 9. We stop at $p = 9$ because we can already satisfy all of the requisite constraints from three generations, electroweak mixing and gauge coupling unification. More mundanely this keeps the number of generators of the gauge group not above 72 which is smaller than $E_6$. In [14] it was shown that the condition necessary for the presence of chiral fermions, that all the scalars must be in bifundamentals, coincides with the condition necessary for the cancellation of one-loop quadratic divergences. This is encouraging since, if these two conditions had been contradictory, the quiver approach would be seriously compromised. The coincidence supports the idea that quiver gauge field theories are a promising and potentially fruitful future direction for BSM physics.

### 2.1 Quivers with $p > 7$

For $p \geq 7$, we shall keep only the chiral solutions because non-chiral examples are of no phenomenological interest. We continue to number the retained models sequentially. Let $n_p$ be the number of inequivalent chiral quiver theories for fixed $p$ then our search, checked by a computer
program, yields the following results: \( n_2 = n_3 = 0, n_4 = 1, n_5 = 2, n_6 = 3, n_7 = 6 \) all agreeing with the 1999 result \cite{5} and summarized above. Note that for these first 12 chiral models only the \( p = 7 \) models numbered 7B, 7D, and 7E can have their \( p \) modes labelled such that they contain the three chiral families of the SM. For \( p = 8 \), we find \( n_8 = 9 \) with the inequivalent solutions given in Table 2 (\( p=8 \)).

For \( p = 9 \), we find \( n_9 = 13 \) with the inequivalent solutions given in Table 3 (\( p=9 \)).

### 3 Model building with quiver theories

So far we have used only mathematics to arrive at potentially interesting chiral theories with gauge group \( SU(3)^p \) where \( 4 \leq p \leq 9 \). More experimental data would be very welcome to guide
us beyond the SM but for the present we have to do without.

The physics of the situation enters when we attempt to assign the \( p \) nodes to colour (C), weak (W) and hypercharge (H) preparatory to spontaneous symmetry breaking to the SM. The labels C, W, and H are for convenience with book-keeping only. More physics constraints arise from three families, the electroweak mixing, gauge coupling unification and the requirement of a scalar sector sufficient to permit spontaneous symmetry breaking to the SM.

As mentioned in the introduction, we shall use the notation

\[
SU(3)^p \equiv SU(3)_{C}^{n_{C}} \times SU(3)_{W}^{n_{W}} \times SU(3)_{H}^{n_{H}}
\]

for the general gauge structure of a quiver theory. The general understanding will be that the \( n_{C}, n_{W}, n_{H} \) sectors will undergo spontaneously symmetry breaking to the corresponding diagonal subgroups

\[
SU(3)^p \rightarrow SU(3)_{C} \times SU(3)_{W} \times SU(3)_{H}
\]

in which, by virtue of the choices of the diagonal subgroups, the gauge couplings of the C, W, and H sectors are related to the original common quiver gauge coupling by

\[
g_{C} = \left( \frac{g}{\sqrt{n_{C}}} \right), \quad g_{W} = \left( \frac{g}{\sqrt{n_{W}}} \right), \quad g_{H} = \left( \frac{g}{\sqrt{n_{H}}} \right).
\]

If we define \( \alpha_{i} \equiv g_{i}^{2}/(4\pi) \) then we have from Eq. (7)

\[
\left( \frac{\alpha_{C}}{\alpha_{W}} \right) = \left( \frac{n_{W}}{n_{C}} \right),
\]

which will play a role in gauge coupling unification. Notice that the other two independent ratios involving \( \alpha_{1} \) are not necessary given the fact that the normalization of the U(1) generator is arbitrary. This will only occur if the \( U(1) \) is embedded in a non abelian gauge symmetry, which is not the case here, since \( U(1)_{Y} \) emerges both from \( SU(3)_{W} \) and \( SU(3)_{H} \) after the lumping of the original symmetry to diagonal.

The electroweak mixing angle \( \Theta_{W} \) depends on \( g_{W} \) and on \( g_{Y} \) where \( Y \) is the weak hypercharge according to

\[
\sin^{2} \Theta_{W} = \left( \frac{g_{Y}^{2}}{g_{W}^{2} + g_{Y}^{2}} \right).
\]

From the PDG tables [15], the values of \( \alpha_{C}(M_{Z}^{2}) \), \( \alpha_{W}(M_{Z}^{2}) \) and \( \alpha_{Y}(M_{Z}^{2}) \) at \( \mu = M_{Z}^{2} = (91.19 \text{ GeV})^{2} \) are

\[
\alpha_{C}(M_{Z}^{2}) = 0.1193 \\
\alpha_{W}(M_{Z}^{2}) = 0.03379 \\
\alpha_{Y}(M_{Z}^{2}) = 0.010166.
\]

We shall use the RG equations

\[
\alpha_{i}^{-1}(M) = \alpha_{i}^{-1}(\mu) - \left( \frac{b_{i}}{2\pi} \right) \ln \left( \frac{M}{\mu} \right)
\]

\[(11)\]
for $I = C, W, Y$ where the RG $\beta$-functions are, at one-loop order \cite{16}

\[
\begin{align*}
  b_C &= -11 + \frac{4}{3} N_{fam} \\
  b_W &= -\frac{22}{3} + \frac{4}{3} N_{fam} + \frac{1}{6} \\
  b_Y &= +\frac{4}{3} N_{fam} + \frac{1}{10}
\end{align*}
\]  

(12)

with $N_{fam} = \frac{5}{2}$ for $M \leq M_t = 173.2$ GeV and $N_{fam} = 3$ for $M > M_t$. Using these relations, we can determine that

\[
R(\mu) = \left( \frac{\alpha_C(\mu)}{\alpha_W(\mu)} \right)
\]

(13)

has the value $R(\mu) = 3, 2$ for the $\mu$ values $\mu \simeq 800$ GeV, $\mu \simeq 200$ TeV, respectively, and that

\[
\sin^2 \Theta_W(\mu) = \left( \frac{g_Y^2(\mu)}{g_Y^2(\mu) + g_W^2(\mu)} \right)
\]

(14)

has the value $\sin^2 \Theta(\mu) = \frac{1}{4}$ for $\mu \simeq 3.8$ TeV. In general, for a large value of $p$, one could explore various possibilities for $R(\mu)$, linked to the ratio \cite{5}, which would fix appropriately the unification scale $\mu_{GUT}$.

4 Model Building for $p = 8$

There are $n_8 = 9$ possibilities for the $A_m$ and $a_i$ listed in Table 2 (p=8) which we may label (8A) through (8I) and analyse them in turn. We will be labelling the nodes in a quiver clockwise as nodes on a heptagon, according to their $C, W$ or $H$ nature and represent them in a sequence, with the edges represented by hyphens. For $p = 8$

(8A) $A_m = (1115), a_i = (222)$. With one color (C) node and two weak (W) node, the node assignments allowed, when we require that there are three families and sufficient scalars to break $SU(3)_W \times SU(3)_W$ to its diagonal subgroup, are

C - W - H - W - H - H - H - H - W,

but in neither identification can the five $SU(2)_H$ be broken to a single $SU(2)_H$ subgroup because there are insufficient scalars. In order to break a product of $SU(N)$s to their diagonal subgroup, it is necessary to have bifundamental scalars linking all the $SU(N)$s together without dividing into subclusters. Thus, the (8A) quiver does not allow a 3-family SM to arise by its SSB. Note that an oriented line C - W transforms as $(3, 3^*, 1)$ under $SU(3)_C \times SU(3)_W \times SU(3)_H$ and the automatic anomaly cancellation dictates that it comes only as the combination $(3, 3^*, 1)+(1, 3, 3^*)+(3^*, 1, 3)$ which is one family. This is how to see quickly the number of families by the number of chiral C - W links.

(8B) $A_m = (1124), a_i = (233)$. This gives the unique possible node assignment
C - W - H - H - W - H - H - H
and, for this case, there are sufficient scalars for SSB to the SM.

(8C). $A_m = (1133), a_i = (334)$. For the W modes, a consistent node assignment would be

C - W - H - W - H - H - H - H
but the $SU(3)_H$ groups cannot be broken to the diagonal subgroup using the scalar bifundamentals which correspond to $a_i = (334)$.

(8D). $A_m = (1223), a_i = (334)$.
In this case, 3-family assignments such as

C - W - W - H - H - H - H - H or C - H - W - W - H - H - H - H
do not permit SSB of the $SU(3)_W \times SU(3)_W$ to its diagonal subgroup.

(8E). $A_m = (1366), a_i = (114)$.
As in (8D), the two possible 3-family arrangements

C - W - H - H - H - H - W - H and C - H - H - W - H - H - W - H
do not have the right scalars to break to $SU(3)_W$.

(8F). $A_m = (1456), a_i = (123)$.
To obtain three families, the $A_m$ dictate at least 3 weak W nodes whereupon possibilities are

C - W - H - H - W - W - H - H, C - W - H - H - H - W - W - H,
C - W - H - H - W - H - W - H and C - H - H - H - W - W - W - H.
For all four of these, there are sufficient scalar bifundamentals to break the symmetry.

(8G). $A_m = (1555), a_i = (222)$.
If we try node assignments such as

C - H - H - W - H - W - H - H
it is easy to see that there is no hope appropriately to break the $SU(3)_H$’s.

(8H). $A_m = (2222), a_i = (444)$.
Four families are possible, and appropriate $SU(3)_W$ breaking, by assigning nodes as

C - H - W - H - H - H - W - H
but then the $SU(3)_H$ breaking is impossible for the reasons explained under (8A).

(8I). $A_m = (2455), a_i = (112)$.
For this case, we may try either
C - H - W - H - H - W - H - H or C - H - H - W - W - H - H - H.

In both assignments, however, the breaking of the H's fails.

5 Model Building for $p = 9$

There are $n_9 = 13$ possibilities for the $A_m$ and $a_i$ listed in Table 2 (p=9) which we may label (9A) through (9M) and analyse them in turn.

(9A) $A_m = (1115)$, $a_i = (222)$. With one color (C) node and two weak (W) node, the node assignments allowed, when we require that there are three families and sufficient scalars to break $SU(3)_W \times SU(3)_W$ to its diagonal subgroup, are

C - W - H - H - H - H - H - H - H or C - W - H - H - H - H - H - H - W

but in neither case are there sufficient scalars to allow appropriate SSB of the $SU(3)_H^6$.

(9B). $A_m = (1125)$. $a_i = (244)$. Three-family node identifications suggested by $A_m$ and $a_i$ are

C - W - W - H - H - H - H - H and C - W - H - H - W - H - H - H - H

but the $SU(3)_W^2$ fails to break to its diagonal subgroup.

(9C) $A_m = (1134)$. $a_i = (244)$.

The only node identifications to try are

C - W - H - W - H - H - H - H - H or C - W - H - H - W - H - H - H - H,

but in the first the spontaneous symmetry breaking of $SU(3)_W^2$ fails, while in the second the $SU(3)_H^6$ fails to break properly.

(9D). $A_m = (1224)$. $a_i = (334)$.

Here, for three families each of which involves a chiral C - W link we may try

C - W - W - H - H - H - H - H - H or C - H - W - H - W - H - H - H - H

but in both cases the breaking of $SU(3)_W^2$ is impossible.

(9E). $A_m = (1233)$. $a_i = (345)$.

The three family structure dictates either C - W - H - W - H - H - H - H - H or

C - H - W - W - H - H - H - H

but, for both node identification choices, the $SU(3)_W^2$ symmetry breaking fails.

(9F). $A_m = (1377)$. $a_i = (114)$. 

8
One choice C - W - H - H - H - H - W - H fails because of $SU(3)^2_W$ but

C - H - H - W - H - H - W - H

succeeds in that there are sufficient scalars to allow diagonal SSB of both $SU(3)^2_W$ and $SU(3)^6_H$ and thence breaking to the SM.

(9G). $A_m = (1467)$. $a_i = (124)$.

Because the four components of $A_m$ are all different, three families requires three W nodes and, with one C node, there are four node identification choices. One, which fails because of $SU(3)^3_W$ breaking, is

C - W - H - H - W - H - H - W - H.

The other three node identifications all work. They are

C - W - H - H - W - H - W - H - H,  C - W - H - H - H - W - W - H - H and
C - H - H - H - W - H - W - W - H.

(9H). $A_m = (1557)$. $a_i = (133)$. Both of the three family assignments fail in the $SU(3)^2_W$ breaking; they are

C - W - H - H - W - H - W - H - H and C - H - H - H - W - H - W - H.

(9I). $A_m = (1566)$. $a_i = (223)$.

As in (9H), the $SU(3)^2_W$ fails for both

C - W - H - H - H - W - W - H and C - H - H - H - W - W - H - H

(9J). $A_m = (2223)$. $a_i = (444)$.

Here it is the spontaneous symmetry breaking of $SU(3)^6_H$ which is problematic for

C - H - W - H - H - H - W - H.

(9K). $A_m = (2466)$. $a_i = (113)$.

$SU(3)^2_W$ symmetry breaking is impossible for both

C - H - H - H - W - H - H - W - H - H and C - H - H - W - H - W - H - H.

(9L). $A_m = (2556)$. $a_i = (122)$. 

9
Here there is one unique node choice with enough scalars to break all the required symmetries down to the SM gauge group. It is

\[ C - H - H - H - W - W - H - H \]

(9M). \( A_m = (3558). \ a_i = (111). \)

The diagonal subgroup of \( SU(3)_H^6 \) is inaccessible in

\[ C - H - H - W - H - W - H - H \]

or

\[ C - H - H - H - H - W - H - W \]

which are the only possible 3-family choices of nodes.

6 Discussion

For \( p = 8 \), only the cases (8B) and (8F) allow the spontaneous symmetry breaking to the three-family standard model, and these both have unification possible between the gauge couplings, provided that the energy scale \( \mu \) is chosen correctly.

In (8B), the C and W embeddings require the matching condition

\[
\frac{\alpha_W(\mu)}{\alpha_C(\mu)} = \left( \frac{1}{2} \right). \tag{15}
\]

In (8F), on the other hand, the SM embedding requires the different condition

\[
\frac{\alpha_W(\mu)}{\alpha_C(\mu)} = \left( \frac{1}{3} \right). \tag{16}
\]

Using an RGE running of the couplings \( \alpha_i(\mu) \) up from the Z mass gives the energy scales \( \mu = M_{GUT} \simeq 200 \text{ TeV} \) and \( \mu = M_{GUT} \simeq 800 \text{ GeV} \) corresponding to Eqs. (15) and (16) respectively.

For \( p = 9 \), we have identified (9F), (9G) and (9L) as the only consistent node identifications. The first and third require the unification implied by Eq.(15) while the second needs Eq.(16) for unification. These \( p = 8 \) and \( p = 9 \) quivers merit further study, including whether there is the possibility of a conformal window for at least a part of the extensive energy range between \( M_{GUT} \) and \( M_{Planck} \).

7 Conclusions

The objective of this work has been to present some additional examples of quivers which are compatible with the spectrum of the Standard Model. At the same time they involve scalars which can have VEVs to break the products of the SU(3)’s to the diagonal subgroups, and as such they merit further analysis. The surviving models are (8B), (8F), (9F), (9G) and (9L). They have a type of grand unification which is quite different than the way it was envisioned long ago [17,18], where a single group contained the Standard Model group and that there was
a desert between the weak scale and the GUT scale. The predictions of that approach were connected to proton decay and neutrino masses. In this approach, by contrast, there is no need for assumption of a desert extending over 10 or more orders of magnitude in energy. In these models the unification takes place at 800 GeV or 200 TeV which are scales within the foreseeable realm of accelerators in existence or of the next generation. They predict a wealth of new particles, including gauge bosons and further quarks and leptons. We eagerly await more data from the LHC to identify which BSM is chosen by Nature.

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References

[1] L.J. Dixon, J.A. Harvey, C. Vafa and E. Witten. Strings on Orbifolds. Nucl. Phys. B261, 678 (1985).
[2] L.J. Dixon, J.A. Harvey, C. Vafa and E. Witten. Strings on Orbifolds. 2. Nucl. Phys. B274, 285 (1986).
[3] M.R. Douglas and G.W. Moore, D-Branes, Quivers and ALE Instantons. arXiv:hep-th/9603167.
[4] S. Kachru and E. Silverstein, 4-D Conformal Theories and Strings on Orbifolds. Phys. Rev. Lett. 80, 4855 (1998). arXiv:hep-th/9802183.
[5] P.H. Frampton, Conformality from Field-String Duality on Abelian Orbifolds. Phys. Rev. D60, 121901 (1999). arXiv:hep-th/9907051.
[6] P.H. Frampton, Strong-Electroweak Unification at About 4 TeV.; Mod. Phys. Lett. A18, 1377 (2003). arXiv:hep-ph/020844.
[7] A. Dymarsky, I. R. Klebanov and R. Roiban, Perturbative Search for Fixed Lines in Large N Gauge Theories. JHEP 0508, 011 (2005). arXiv:hep-th/0505099.
[8] A. Dymarsky, I. R. Klebanov and R. Roiban, Perturbative Gauge Theory and Closed String Tachyons. JHEP 0511, 038 (2005) arXiv:hep-th/0509132.
[9] N. Haba, H. Ishida, N. Okada and Y. Yamaguchi, Bosonic Seesaw Mechanism in a Classically Conformal Extension of the Standard Model. Phys. Lett. B754, 349 (2016). arXiv:1506.06828[hep-ph].
[10] N. Haba, H. Ishida, N. Okada and Y. Yamaguchi, Electroweak Symmetry Breaking through Bosonic Seesaw Mechanism in a Classically Conformal Extension of the Standard Model. arXiv:1509.01923[hep-ph].
[11] N. Haba, H. Ishida, R. Takahashi and Y. Yamaguchi, Gauge Coupling Unification in a Classically Scale Invariant Model. JHEP 1602:058 (2016). arXiv:1511.02107[hep-ph].
[12] N. Haba, H. Ishida, R. Takahashi and Y. Yamaguchi, *A New Dynamics of Electroweak Symmetry Breaking with Classically Scale Invariance*. Phys. Lett. **B755**, 439 (2016). [arXiv:1512.05061[hep-ph]].

[13] S.L. Glashow, *Trinification of All Elementary Particle Forces* in Fifth Workshop on Grand Unification, Editors: P.H. Frampton, H. Fried and K.Kang. World Scientific Publishing (1984).

[14] X. Calmet, P.H. Frampton and R.M. Rohm, *Chiral Fermions and Quadratic Divergences*. Phys. Rev. **D72**, 055003 (2005). [arXiv:hep-th/0412176].

[15] C. Patrignani *et al.* (Particle Data Group). Chin. Phys. **C40**, 100001 (2016).

[16] M.B. Einhorn and D.R.T. Jones, *The Weak Mixing Angle and Unification Mass in Supersymmetric SU(5)*. Nucl. Phys. **B196**, 475 (1982).

[17] H. Georgi and S.L. Glashow, *Unity of All Elementary Particle Forces*. Phys. Rev. Lett. **32**, 438 (1974).

[18] H. Georgi, H.R. Quinn and S. Weinberg, *Hierarchy of Interactions in Unified Gauge Theories*. Phys. Rev. Lett. **33**, 451 (1974).