Lowest eigenvalues of the Dirac operator for two-color QCD at finite density

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We investigate the eigenvalue spectrum of the staggered Dirac matrix in full QCD with two colors and finite chemical potential. Along the strong-coupling axis up to the temperature phase transition, the low-lying Dirac spectrum is well described by random matrix theory (RMT) and exhibits universal behavior. The situation is discussed in the chirally symmetric phase and no universality is seen for the microscopic spectral density.

We have continued our investigations\cite{1,2} of the small eigenvalues in the whole phase diagram. The Banks-Casher formula relates the Dirac eigenvalue density $\rho(\lambda)$ at $\lambda = 0$ to the chiral condensate, $\Sigma \equiv |\langle \bar{\psi}\psi \rangle| = \lim_{\varepsilon \to 0} \lim_{V \to \infty} \pi \rho(\varepsilon)/V$. The microscopic spectral density, $\rho_s(z) = \lim_{V \to \infty} \rho(z/V \Sigma)/V \Sigma$, should be given by the appropriate prediction of RMT. In the presence of a chemical potential $\mu > 0$, leading to complex eigenvalues, the situation is more complicated and $\rho(0)$ can be used as a lower bound for $\Sigma$.

We present results for the density of the small eigenvalues in the left plots of Fig. 1 for two-color QCD with staggered fermions on a $6^4$ lattice around the critical chemical potential $\mu_c \approx 0.3$ keeping $\beta = 1.3$ fixed\cite{3}. Since the eigenvalues move into the complex plane for $\mu > 0$, $\rho(|\lambda|)$ was constructed from the absolute value for $|\lambda|$ small. Alternatively, a band of width $\epsilon = 0.015$ parallel to the imaginary axis was considered to construct $\rho(y)$, i.e. $\rho(y) \equiv \int_{-\epsilon}^{\epsilon} dx \rho(x, y)$, where $\rho(x, y)$ is the density of the complex eigenvalues $\lambda = x + iy$. Both results are compared in the upper and lower plot.

The distribution of the lowest eigenvalue is displayed in the center plots of Fig. 1. In the confinement it was shown for $\mu = 0$ that both the microscopic spectral density $\rho_s(z)$ and the distribution $P(\lambda_{\text{min}})$ of the smallest eigenvalue agree with the RMT predictions of the chiral symplectic ensemble\cite{4}. The quality of the data from at least 2100 configurations at $\mu \neq 0$ was not sufficient for a reliable fit to a trial function for $P(\lambda_{\text{min}})$.

Nevertheless, the quasi-zero modes which are responsible for the chiral condensate $\Sigma \neq 0$ build up when we cross from the plasma into the confined phase. Both $\rho(\lambda)$ and $P(\lambda_{\text{min}})$ plotted with varying $\mu$ on identical scales can serve as an indicator for the phase transition.

In the right plots of Fig. 1 we discuss the spectrum in the quark-gluon plasma. From RMT a functional form of $\rho(\lambda) = C(\lambda - \lambda_0)^{2m+1/2}$ is expected at the onset of the eigenvalue
Figure 1. Density $\rho(\lambda)$ of small eigenvalues (left) and distribution $P(\lambda_{\text{min}})$ of the smallest eigenvalue (center) across the transition of critical chemical potential. Fit of the spectral density to $\rho(\lambda) = C(\lambda - \lambda_0)^{2m+1/2}$ at $\mu = 0.4$ (right) with the contribution of the smallest, 11th and 21st eigenvalue inserted. The upper plots refer to $\rho(\lambda)$ constructed from the absolute value of $\lambda$ while the lower plots result from averages over $\text{Re}(\lambda)$.

In summary, we studied universality concerning the low-lying spectra of the Dirac operator for two-color QCD with chemical potential. We could not obtain a relation for $\rho_s(z)$ in $0 < \mu < \mu_c$, not only because of our data but also in lack of an analytic result for non-Hermitian RMT. In the phase where chiral symmetry is restored one has to rely on ordinary RMT. Here we find universal behavior only of the macroscopic density $\rho(\lambda)$.

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