Collective coherent population trapping in a thermal field

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We analyzed the efficiency of coherent population trapping (CPT) in a superposition of the ground states of three-level atoms under the influence of the decoherence process induced by a broadband thermal field. We showed that in a single atom there is no perfect CPT when the atomic transitions are affected by the thermal field. The perfect CPT may occur when only one of the two atomic transitions is affected by the thermal field. In the case when both atomic transitions are affected by the thermal field, we demonstrated that regardless of the intensity of the thermal field the destructive effect on the CPT can be circumvented by the collective behavior of the atoms.

An analytic expression was obtained for the populations of the upper atomic levels which can be considered as a measure of the level of thermal decoherence. The results show that the collective interaction between the atoms can significantly enhance the population trapping in that the population of the upper state decreases with increased number of atoms. The physical origin of this feature was explained by the semiclassical dressed atom model of the system. We introduced the concept of multiatom collective coherent population trapping by demonstrating the existence of collective (entangled) states whose storage capacity is larger than that of the equivalent states of independent atoms.

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I. INTRODUCTION

The study of atomic coherence effects in multilevel atoms is one of the most active area in atomic spectroscopy \cite{1,2,3}. Especially, the theory of coherent population trapping (CPT) in a three-level Λ-type atom has been extensively studied and the phenomenon has been observed experimentally in a sodium vapor \cite{4,5}, photoassociation systems \cite{6}, BEC \cite{7} and solids \cite{8}. The CPT results from the formation of a coherent superposition of the ground atomic states that is decoupled from the external fields and hence referred to as a dark state. The particular interest of this phenomenon consists of the possibility of storage and coherent manipulation of the population in a coherent superposition of the ground states of the atoms \cite{9,10}. These phenomena have received greatly increased experimental attention in recent years and experimental techniques have been developed which allow a reversible transfer of quantum information from light to the dark state of the atoms \cite{11}. The coherent population trapping has also been investigated in the context of lasing without inversion \cite{12}, subcooler laser cooling \cite{13} and a search for materials that display a high index of refraction accompanied by vanishing absorption \cite{14,15,16}.

The atomic coherence effects are sensitive to decoherence. In the CPT effect, one source of decoherence is fluctuations of the laser fields used to create the coherent superposition of the atomic ground states \cite{17}. The fluctuations redistribute the population among the atomic states including the excited atomic states from which it can be spontaneously emitted resulting in optical losses. Recent investigations of decoherence processes in atomic systems have demonstrated that CPT and quantum storage in an ensemble of noninteracting atoms are limited primarily by different decoherence processes such as atomic collisions, atom loss and motion of atoms \cite{18}. The results show an interesting property that in the limit of the total number of excitations much smaller than the number of atoms, the decoherence rate of the multiatom system is of the same order of magnitude as in the single atom, i.e. is independent of the number of atoms in the sample. In an earlier study, Jyotsna and Agarwal \cite{19} showed that the CPT effect in a dense atomic medium is unaffected by local-field effects.

It is well known that the dominant contribution to the decoherence processes in the interaction of atoms with the electromagnetic field stems from the thermal fluctuations. They are present in a non-zero temperature reservoir to which the atoms are coupled. The fluctuations cause a pumping of the population stored in the dark state into the excited states of the atoms from which it can be spontaneously emitted resulting in an increase in decoherence. The magnitude of thermal fluctuations depends on temperature of the reservoir and determines the minimum level of thermal decoherence.

In this paper we propose a method to suppress the decoherences that occur due to the thermal fluctuations of the environmental electromagnetic reservoir at temperature $T$. Essentially, we examine the CPT effect in three-level Λ systems by addressing a practical question: How can
one increase the efficiency of trapping and storage of the population in the presence of thermal decoherence. In particular, we will investigate limits to the efficiency of the CPT effect in a single atom and next will explore the role of multiatom collective behavior in the reduction of the single-atom decoherence rate induced by the thermal field. The dipole-dipole interactions between the atoms will not be taken into account here assuming lower atomic densities, so that the collective behavior we consider stems entirely from the mutual coupling of all the atoms with the common radiation field [20]. Employing the analytic solution for the density operator of the system, we find that in general the single-atom coherent population trapping in the presence of thermal decoherence. Our physical interpretation of the results is based on the semiclassical dressed atom model of the collective atomic system. The collective dressed states of the system are identified, and the effect of suppression of the thermal decoherence is explained in terms of the increased capacity of these states. This is shown to arise from correlation-enhanced transition rates among the multiatom dressed states, in particular those entering the trapped state. Hence, the effects of decoherence by thermal fields may by reverted more rapidly.

II. APPROACH

The system we consider is an ensemble of $N$ identical three-level A-type atoms each with excited state $|1\rangle$ and two nondegenerate ground states $|2\rangle$ and $|3\rangle$. The atoms are driven by two single-mode cw laser fields of Rabi frequencies $2\Omega_2$ and $2\Omega_3$ and angular frequencies $\omega_{L2}$ and $\omega_{L3}$ significantly different from each other, so that each laser is coupled only to one of the allowed transitions, as shown in Fig. 1. The transitions are associated with nonzero dipole moments $\vec{\mu}_{12}$ and $\vec{\mu}_{13}$, and the laser fields are detuned from the atomic transition frequencies, such that there is a nonzero two-photon detuning $\Delta = (\omega_{13} - \omega_{12} + \omega_{L2} - \omega_{L3})/2$. The transition $|2\rangle \rightarrow |3\rangle$ is forbidden in the electric dipole approximation ($\mu_{23} = 0$).

The master equation (1) allows to obtain equations of motion for the expectation value of an arbitrary combination of the atomic operators. The calculations can be performed without much troubles for the simple case of a single atom $(N = 1)$ and arbitrary $\Delta$. However, for $N > 1$ the calculation of the expectation value is not an easy task. In even the simplest cases of small numbers of atoms, the calculations are prohibitively difficult due to the enormity of the number of coupled equation of motion. Fortunately, for the $\Delta = 0$ case and high field strengths, $\Omega_k \gg N\gamma_k$, an approximation technique has been developed, which greatly simplifies the master equation (1) and thus able to perform analytical calculations of the expectation value of an arbitrary combination of the atomic operators. The restriction to

![FIG. 1: Energy-level diagram of a three-level A-type atom driven by two laser fields of Rabi frequencies $2\Omega_2$ and $2\Omega_3$.](image)
the $\Delta = 0$ case stems from the difficulty in obtaining a closed set of equations when the two-photon detuning is present. A full discussion of the technique is given in Refs. 22–24. In the interest of brevity only the key results will be given here. The technique is implemented by introducing dressed states of a single atom, which are obtained by a diagonalization of the single-atom interaction Hamiltonian

$$H_{0j} = \Omega_2 S_{12}^{(j)} + \Omega_3 S_{31}^{(j)}.$$  

The single-atom dressed states are of the form

$$|\Psi_j\rangle = \frac{1}{\sqrt{2}} \left( |1\rangle_j + e^{i\theta_j/2} |2\rangle_j \right),$$

$$|\Psi_0\rangle = \frac{1}{\sqrt{2}} \left( |1\rangle_j + \sqrt{2} \gamma_j \Omega_j \right),$$

$$|\Psi_3\rangle = \frac{1}{\sqrt{2}} \left( |1\rangle_j - \sqrt{2} \gamma_j \Omega_j \right),$$

where $\Omega = \sqrt{\Omega_1^2 + \Omega_3^2}$ is the generalized Rabi frequency. The idea of the approximate technique is now to replace the collective operators $S_{\alpha\beta}$ by the collective dressed-atom operators

$$R_{\alpha\beta} = \sum_{j=1}^{N} R_{\alpha\beta}^{(j)} = \sum_{j=1}^{N} |\Psi_\alpha\rangle_{jj} \langle \Psi_\beta|, \quad \alpha, \beta = 1, 2, 3,$$

and then substitute for $S_{\alpha\beta}$ into the damping terms of the master equation. Next, we make the unitary transformation of the density operator

$$\rho = \exp \left( \frac{i}{\hbar} H_0 t \right) \rho \exp \left( -\frac{i}{\hbar} H_0 t \right),$$

where

$$\dot{H}_0 = \hbar \Omega (R_{22} - R_{33}) = \hbar \Omega R_z$$

and on carrying out this procedure it is found that certain terms in the transformed master equation are slowly varying while the others are rapidly oscillating at frequencies $\Omega$ and $2\Omega$. The approximation then consists of dropping these rapidly oscillating terms. The master equation in the dressed state basis reduces to

$$\frac{\partial \rho}{\partial t} = -i[H_0, \rho] + \{\Gamma_0([R_{21} \rho \bar{R}_2] + [R_{32} \rho \bar{R}_3])$$

$$+ [R_{23} \rho \bar{R}_{23}] + [R_{12} \rho \bar{R}_{21}] + [R_{31} \rho \bar{R}_{31}]$$

$$+ \Gamma_2([R_{21} \rho \bar{R}_{12}] + [R_{31} \rho \bar{R}_{13}])\},$$

where

$$\Gamma_0 = \frac{1}{2} \left\{ \gamma_2 (1 + 2\bar{n}_2) \Omega_2^2 + \gamma_3 (1 + 2\bar{n}_3) \Omega_3^2 \right\},$$

$$\Gamma_1 = \frac{1}{2} \left\{ \gamma_2 (1 + \bar{n}_2) \Omega_3/\Omega_2^2 + \gamma_3 (1 + \bar{n}_3) \Omega_3/\Omega_2^2 \right\},$$

$$\Gamma_2 = \frac{1}{2} \left\{ \gamma_2 \bar{n}_2 \Omega_3/\Omega_2^2 + \gamma_3 \bar{n}_3 \Omega_2/\Omega_3^2 \right\},$$

are the transition rates between the single-atom dressed states.

Using the approximate master equation, it is straightforward to obtain a simple analytical solution for the steady-state density operator of the system. The solution can be written in the form

$$\rho_s = Z^{-1} \exp \left[-\xi R_{11} \right],$$

where

$$\xi = \ln \left( \frac{\Omega_2^2 \bar{n}_2 + \eta \Omega_2^2 \bar{n}_3}{\Omega_3^2 (1 + \bar{n}_2) + \eta \Omega_2^2 (1 + \bar{n}_3)} \right),$$

and $\eta = \gamma_3/\gamma_2$. The parameter $Z$ is the normalization constant such that $\text{Tr} \{\rho_s\} = 1$. It is easily verified that $\xi$ is always independent of the parameters used and approaches zero when $\bar{n}_2$ and/or $\bar{n}_3$ go to infinity. The solution was obtained in Refs. 22, 24, 30, and some applications are discussed there in details. In Ref. 31, the solution has been used to investigate different control schemes for collective systems of three-level atoms.

In this paper, we focus on the competition between thermal fluctuations and the collective effects that can lead to collective population trapping. The steady-state solution enables to calculate any statistical moment of the diagonal elements $R_{\alpha\alpha}$, and thus population distributions between atomic states. In particular, an $k$-th order moment of $R_{11}$ (expectation value of a product of $k$ operators $R_{11}$), is of the form

$$\langle R_{11}^k \rangle_s = (-1)^k Z^{-1} \frac{\partial^k}{\partial \gamma^k} Z, \quad k = 1, 2, \ldots,$$

and the first order statistical moments of $R_{22}$ and $R_{33}$ are

$$\langle R_{22} \rangle_s = \langle R_{33} \rangle_s = [N - \langle R_{11} \rangle_s]/2,$$

where

$$Z = \frac{N + 2 - (N + 1)e^\xi - e^{-\xi(N+1)}}{(1 - e^\xi)(1 - e^{-\xi})}.$$
a system of three-level atoms may be monitored experimentally in terms of the intensity of the fluorescence light emitted [4, 5, 6, 7, 8]. It is manifested by the disappearance of the fluorescence which, on the other side, manifests the vanishing of the population of the upper atomic states |1⟩. Therefore, we will consider first the effect of the thermal field on the so-called transparency window, i.e. the dependence of the stationary population ρ_{11}^{s} on the two-photon detuning. Next, using the stationary solution (12), we will find the analytical expression for the population at the two-photon resonance, Δ = 0, and will analyze how one could reduce the destructive effect of the thermal field on the minimum of the population at Δ = 0.

Fig. 2 illustrates the stationary population ρ_{11}^{s} as a function of the two-photon detuning Δ. We have obtained the population by solving numerically the master equation (1) for N = 1. It is seen that in the absence of the thermal field, n = 0, there is perfect CPT observed at Δ = 0. When the atom is in the thermal field equally affecting both transitions, the CPT effect is reduced and the thermal field washes out the transparency window as n \gg 1. Thus, the thermal field has a destructive effect on the CPT, because the thermal field is an incoherent field with random fluctuations that destroy the coherent process induced by the laser fields.

The variation with n of the minimum of the upper state population at Δ = 0 can be analyzed explicitly using Eq. (10) which for N = 1 and together with the steady-state solution (12) gives a simple analytical expression for ρ_{11}^{s} in the form

$$\rho_{11}^{s} = \frac{1}{2} (\langle R_{22} \rangle_{s} + \langle R_{33} \rangle_{s}) = \frac{\kappa}{1 + 2\kappa}. \quad (17)$$

First, we note from Eq. (14) that the population distribution between the atomic states is determined solely by the parameter ξ. Clearly, the population distribution and consequently the trapping effect depend on several parameters such as the laser intensity, spontaneous emission rates, and mean number of thermal photons. We can call the parameter ξ as a measure of efficiency of the CPT effect.

Here the efficiency of the CPT is examined in various intensity regimes of the coherent fields for equal and also unequal average numbers of thermal photons. The average numbers can be made unequal by a suitable choice of bandwidth of the thermal field. The selective excitation of the atomic transitions can be realized in practice by applying a finite bandwidth multimode thermal field whose bandwidth is much smaller than the splitting of the lower atomic levels, but large compared to the natural linewidths of the atomic transitions to satisfy the Markov approximation used in the derivation of the master equation.

In the limit of n_{2} = 0 that the thermal fluctuations affect only the |1⟩ → |3⟩ transition, the parameter ξ reduces to

$$\xi = \ln \left( \frac{\eta \Omega_{3} n_{3}}{\Omega_{3}^2 + \eta \Omega_{2}^2 (1 + n_{3})} \right). \quad (18)$$

The parameter ξ does not change substantially with the Rabi frequencies unless Ω_{3} is much larger than the Rabi frequency Ω_{2} of the other transition. In the very strong-field regime of Ω_{3} \gg \eta \Omega_{2} (1 + n_{3}), the parameter ξ approaches the limit of ξ \to -\infty. This minimum value is that one which leads to vanishing of the population of the upper atomic state, because \lim_{\xi \to -\infty} \rho_{11}^{s} = 0. This predicts that perfect coherent population trapping can be observed even in the presence of thermal decoherence on one of the two atomic transitions, which is in contrast to the result of [31]. However, it requires that the transition influenced by the decoherence is simultaneously driven by a strong laser field. It can be understood rather easily. For a large Rabi frequency Ω_{3}, the coherent processes on the |1⟩ → |3⟩ transition dominate over the incoherent thermal processes resulting in perfect transparency.

Various other intensity regimes can also be distinguished. If n_{2} \neq n_{3}, the parameter ξ can depend entirely on n_{2} or n_{3} depending on the ratio Ω_{3}/Ω_{2}. For instance, when n_{2} \Omega_{3}^2 \gg \eta n_{3} \Omega_{2}^2, we find that

$$\xi = \ln \left( \frac{n_{2}}{1 + n_{2}} \right). \quad (19)$$

This predicts that the coherent population trapping depends entirely on the thermal fluctuations at the weakly driven |1⟩ → |2⟩ transition. In the opposite limit of \eta n_{3} \Omega_{2}^2 \gg n_{2} \Omega_{3}^2, the parameter ξ now depends entirely on n_{3}. Thus, the driving fields are relatively efficient in controlling decoherence in a single atom. Again, it can be interpreted as caused by coherent processes that dominate incoherent thermal processes on the strongly driven transition. This also shows that the suppression of the thermal decoherence in a single atom is limited to the level set by the lowest thermal fluctuations affecting the atomic transitions.

In the case when the thermal field equally contributes to both atomic transitions, n_{2} = n_{3} = n, we have

$$\xi = \ln \left( \frac{n}{1 + n} \right). \quad (20)$$
The effects described in Section III can be seen in dilute atomic gases where the interatomic interactions are not important. However, a more interesting situation emerges as we have considered here atomic samples where radiative interactions between the atoms can lead to a collective (entangled) behavior of the atoms. Here, we include the multiatom effects and calculate the population $\rho_{11}$ as a function of the number of atoms and the number of thermal photons.

The upper state population $\rho_{11}$ can be evaluated using Eq. (6) which, together with the steady-state solution, gives the analytical expression for $\rho_{11}$ in terms of $\xi$ and $N$ as

$$\rho_{11} = \frac{Z^{-1}}{(1-e^{-\xi})^2} \left[ N(N + 1) - e^{-\xi N} + N e^\xi - N - 1 \right].$$

In Fig. 4, we present a three-dimensional plot which shows that in the absence of the thermal field, i.e. $\bar{n} = 0$, the stationary population $\rho_{11}$ is equal to zero independent of the number of atoms. Thus, for $\bar{n} = 0$ the collective behavior of the atoms does not affect the trapping effect. The presence of the thermal field has a destructive effect on the trapping phenomenon that the population
In other words, the thermal decoherence decreases with increasing number of atoms. Thus, the collective interactions are relatively efficient in suppression of thermal decoherence such that the atoms may remain in their ground states even in the presence of the thermal decoherence. This is a surprising result as one might expect that decoherence should increase with the increasing number of atoms.

Figure 5 shows the population $\rho_{11}$ as a function of $\bar{n}_2 = \bar{n}_3 = \bar{n}$ for different numbers of atoms. Here we see that the rate of the increase of the population decreases with the increasing number of thermal photons $\bar{n}$. However, for a suitably large $N$ the population $\rho_{11}$ may remain very small even for large $\bar{n}$. In other words, the thermal decoherence increases with increasing number of atoms. Thus, the collective population stored in the ground states is less affected by the thermal fluctuations than for the case of independent atoms. As a consequence, one has a practical scheme to reduce thermal decoherence and preserve CPT in the thermal field.

In order to obtain an insight into the physical origin of the reduction of thermal decoherence and the improvement of the population trapping, we examine the energy structure of the collective system. In general, in the absence of the driving fields, the system can be represented in terms of collective symmetric and antisymmetric states. However, in the case of $N$ identical atoms contained in a volume with linear dimensions that are small compared with the radiation wavelengths, only the symmetric states couple to external driving fields. The dipole-dipole interactions between the atoms lead to a shift of these states from the laser resonance [3]. Thus, here the Rabi frequencies should be larger than the shift caused by the dipole-dipole effects, i.e. the latter can be neglected. The antisymmetric states do not participate in the dynamics of the small sample system [21]. Therefore, we may limit the dynamics to only those involving the symmetric states. Moreover, only the lowest in energy symmetric states are of interest in the analysis of the collective population trapping. We therefore consider the lowest energy states defined as

\begin{equation}
|3\rangle = \binom{N}{0}^{-\frac{1}{2}} |3_1, 3_2, \ldots, 3_N\rangle,
\end{equation}

\begin{equation}
|2\rangle = \binom{N}{1}^{-\frac{1}{2}} \sum_{i=1}^{N} |3_1, \ldots, 2_i, \ldots, 3_N\rangle,
\end{equation}

\begin{equation}
|1\rangle = \binom{N}{1}^{-\frac{1}{2}} \sum_{i=1}^{N} |3_1, \ldots, 1_i, \ldots, 3_N\rangle,
\end{equation}

\begin{equation}
|2^2\rangle = |22\rangle = \binom{N}{2}^{-\frac{1}{2}} \sum_{i<j=1}^{N} |3_1, \ldots, 2_i, \ldots, 2_j, \ldots, 3_N\rangle,
\end{equation}

\begin{equation}
|12\rangle = \frac{1}{\sqrt{2}} \binom{N}{2}^{-\frac{1}{2}} \sum_{i \neq j=1}^{N} |3_1, \ldots, 1_i, \ldots, 2_j, \ldots, 3_N\rangle,
\end{equation}

\begin{equation}
|2^3\rangle = |222\rangle = \binom{N}{3}^{-\frac{1}{2}} \sum_{i<j<k=1}^{N} |3_1, \ldots, 2_i, \ldots, 2_j, \ldots, 2_k, \ldots, 3_N\rangle,
\end{equation}

etc.,

where the binomial coefficients are the normalization constants. The states $|22\rangle$ are superpositions of single-atom
product states \( |m\rangle_i \otimes |n\rangle_j \otimes \ldots \otimes |k\rangle_l \) that are symmetric under the exchange of any pair of atoms. For example, the state \( |2\rangle \) is a linear superposition of the product states in which atom \( i \) is in the state \( |2\rangle_i \) and the remaining \( N - 1 \) atoms are in their states \( |3\rangle_j \).

If we now allow the atoms to interact with the laser fields, each state \( |2^k\rangle \) couples to the first excited states \( |12^{k-1}\rangle \), \( |2^k\rangle \) with the Rabi frequencies \( \Omega_2 \) and \( \Omega_3 \), respectively. Figure 6 shows the collective symmetric states and possible couplings of the two laser fields. As we have already mentioned, we limit the presentation to the lowest energy levels which will be mixed together by the interaction leading to a ground collective dressed state, which is of the main interest here. The lowest energy state is the product state \( |3\rangle = |3\rangle_3 |3\rangle_2 \ldots |3\rangle_N \).

Each succeeding state \( |2^k\rangle \) is of energy higher by successive increments of \( \delta = \omega_{13} - \omega_{12} \). Similarly, each succeeding state \( |2^k\rangle \) is of energy higher by successive increments of \( \delta \). It is interesting to note that the rotating-wave approximation, which we are assuming to be valid, ignores coupling of states which differ in excitation by two and higher. In other words, the laser fields couple only the neighboring ground states through the first excited states. It forms a two-dimensional chain of \( \Lambda \) configurations. With the state ordering \( |3\rangle, |1\rangle, |2\rangle, |12\rangle, |22\rangle, \ldots, |2^N\rangle \), corresponding to the path of successive excitations of the states \( |2^k\rangle \) by the laser fields, the interaction Hamiltonian \( H_0 \) can be expressed as an infinite tridiagonal matrix

\[
H_0/h = \begin{pmatrix}
-N\Delta & \Omega_3\sqrt{N} & 0 & 0 & 0 & \cdots \\
\Omega_3\sqrt{N} & -(N-1)\Delta & \Omega_2\sqrt{\Delta} & 0 & 0 & \cdots \\
0 & \Omega_2\sqrt{\Delta} & -(N-1)\Delta & \Omega_3\sqrt{N-1} & 0 & \cdots \\
0 & 0 & \Omega_3\sqrt{N-1} & -(N-1)\Delta & \Omega_2\sqrt{\Delta} & \cdots \\
0 & 0 & 0 & \Omega_2\sqrt{\Delta} & -(N-1)\Delta & \cdots \\
\vdots & \vdots & \vdots & \vdots & \vdots & \ddots 
\end{pmatrix}
\]

(23)

Substituting Eq. (25) into Eq. (24) yields the eigenvalue equation

\[
(\lambda_n + N\Delta)\{[\lambda_n + (N-1)\Delta](\cdots) + \Omega_2^2|\lambda_n + (N-1)\Delta](\cdots) + \Omega_3^2N[(\lambda_n + N\Delta)(\cdots) + \Omega_3^2(N-1)(\cdots)] = 0
\]

(25)

where the \((\cdots)\) refers to terms of which the explicit form is not needed apart from that those are polynomial functions of \(\lambda_n\). It is easily to show that in the case of \(\Delta = 0\), the eigenvalue equation reduces to

\[
\lambda_n \{[\lambda_n (\cdots) + \Omega_2^2\lambda_n (\cdots) + \Omega_3^3N (\cdots)] = 0
\]

(26)

from which we see that \(\lambda_n = 0\) is one eigenvalue of \(H_0\).

In the single atom case the dressed state \(|\Psi_j\rangle\) corresponding to the zero eigenvalue was of very special significance as corresponding to a trapping (dark) state completely decoupled from the fields. Let us investigate this possibility in the multiatom case. If we represent the eigenvector \(|D_n\rangle_N\) by the column vec-
where the coefficients $c_n$ for even $n$ are all zero, whereas for odd $n$ the coefficients are given by the recurrence relation

$$c_{2k+1} = \left( -\frac{\Omega_3}{\Omega_2} \right)^k \frac{N!}{k!(N-k)!} c_k, \quad k = 1, 2, 3, \ldots$$

and $c_1$ is found from the normalization condition.

The dressed state corresponding to the eigenvalue $\lambda_n = 0$ can thus be written as

$$|D_n\rangle_N = \left( \begin{array}{c} c_1 \\ c_2 \\ \vdots \\ c_i \\ \vdots \\ c_n \end{array} \right), \quad (27)$$

we find by substituting into Eq. (24) that for $\lambda_n = 0$ the coefficients $c_n$ for even $n$ are all zero, whereas for odd $n$ the coefficients are given by the recurrence relation

$$c_{2k+1} = \left( -\frac{\Omega_3}{\Omega_2} \right)^k \frac{N!}{k!(N-k)!} c_k, \quad k = 1, 2, 3, \ldots$$

and $c_1$ is found from the normalization condition. The dressed state corresponding to the eigenvalue $\lambda_n = 0$ can thus be written as

$$|D_n\rangle_N = |D_0\rangle_N$$

$$= \left( \cos \theta \right)^N \sum_{k=0}^{N} \left( \begin{array}{c} N \\ k \end{array} \right) \frac{1}{\sqrt{N}} \left( -\tan \theta \right)^k |2^k\rangle, \quad (29)$$

where $|2^0\rangle = |3\rangle$, and

$$\tan \theta = \frac{\Omega_3}{\Omega_2}. \quad (30)$$

The collective dressed state (29) is a linear combination of the state $|3\rangle$ and $N$ of the states $|2^k\rangle$. The important feature of the state is that it does not contain the excited states of the atoms and thus does not radiate. The dressed state is a stationary state of the Hamiltonian $H_0$ describing the atoms driven by two coherent fields. Therefore, if nothing else is allowed to interact with this system, the state (29) will never change in time.

We can write the multi-atom dressed state (29) in the basis of the single-atom dressed states (6). Surprisingly, we find that the state is of the form

$$|D\rangle_N = |\Psi_1\rangle_1 \otimes |\Psi_1\rangle_2 \otimes \cdots \otimes |\Psi_1\rangle_N, \quad (31)$$

which is a product of the single-atom trapping states $|\Psi_1\rangle_j$. Obviously, the state (31) is not entangled, which shows that trapping of the population in all of the atomic ground states is equally effective in destroying collective (entangled) properties of the system. Thus, the improvement of the CPT in the collective multiatom system, seen in Figs.4 and 5 does not arise from collective excitations of the dark state $|D\rangle_N$.

In fact, the master equation (10) leads to a set of $N(N+1)/(N+2)$ coupled equations of motion for the

$$\dot{\rho}_{DD} = -4N\Gamma_2\rho_{DD} + 2N\Gamma_1(\rho_{D2} + \rho_{D3}), \quad (32)$$

where $\rho_{DD}$ is the population of the state $|D\rangle_N$ and $\rho_{D2}$ and $\rho_{D3}$ are populations of the following superposition states

$$|D_2\rangle_N = \frac{1}{\sqrt{N}} \sum_{j=1}^{N} |\Psi_1^{(1)}\rangle \otimes |\Psi_1^{(2)}\rangle \otimes \cdots \otimes |\Psi_1^{(N)}\rangle, \quad (33)$$

and

$$|D_3\rangle_N = \frac{1}{\sqrt{N}} \sum_{j=1}^{N} |\Psi_1^{(1)}\rangle \otimes |\Psi_2^{(2)}\rangle \otimes \cdots \otimes |\Psi_3^{(N)}\rangle, \quad (33)$$

which differ in energy from the state $|D\rangle_N$ by $+\Omega$ and $-\Omega$, respectively. The states $|D_2\rangle_N$ and $|D_3\rangle_N$ are linear superpositions of the product states in which $N-1$ atoms are in state $|\Psi_1\rangle_j$ and one atom is in the state $|\Psi_2\rangle_j$ and $|\Psi_3\rangle_j$, respectively. It is interesting to note from Eq. (32) that both spontaneous emission and the thermal field couple the state $|D\rangle_N$ to only those states which differ in the excitation by one. Moreover, the transition rates between these states are $N$ times larger than that for single atoms. This shows that the system is superradiant despite the fact that the state $|D\rangle_N$ is the product state of the single-atom trapping states. In addition, the collective decay rate of the radiators entering the state $|D\rangle_N$ is larger than that describing the atoms escaping from it. Thus, the collective properties of the system are preserved due to the presence of the superposition states involving the single-atom states $|\Psi_2\rangle_j$ and $|\Psi_3\rangle_j$. In fact, the master equation (10) leads to a set of $(N + 1)(N + 2)/2$ coupled equations of motion for the
populations of the collective dressed states. Fortunately, however, an explanation of the enhancement of the CPT effect, seen in Figs. 4 and 5, does not require a complete solution for the populations of the dressed states. It is enough to consider only the population of the state $|D2\rangle_N$ or $|D3\rangle_N$. Thus, using Eqs. (33) and (34), we can show that for $N = 1$ the stationary population of the state $|D2\rangle_N$ is simply equal to $\langle R_{22} \rangle$, and for $N > 1$ is given by the following expectation value

$$\rho_{D2} = \frac{1}{N!} (R_{12} R_{21} + R_{22} - R_{11}) R_{11} (R_{11} - 1) \times (R_{11} - 2) \cdots (R_{11} - N + 2),$$

which can be evaluated using the steady-state solution (32).

We also calculate the population of the state $|D2\rangle_N$ in the case of independent atoms and find

$$\rho_{D2}^{in} = \frac{\Gamma_2 (\Gamma_1)^{N-1}}{(\Gamma_1 + 2\Gamma_2)^N}. \quad (35)$$

To see the difference between the populations (34) and (35), we study the ratio $\rho_{D2}^{in}/\rho_{D2}$. Figure 7 shows the ratio for different numbers of atoms. For $N = 1$, the ratio is equal to one, but for $N > 1$ the ratio is smaller than one and decreases with $N$. This shows that the population of the collective states is larger than the population of the equivalent states of independent atoms. In other words, we may say that the capacity of the collective states is larger than the capacity of the equivalent states of independent atoms.

The above analysis give us a simple physical interpretation of the collective trapping effect. We may conclude that the improvement of trapping effect by multiatom system is due simply to the increased storage capacity of the collective (entangled) states compared with the storage capacity of the equivalent states of independent atoms.

V. SUMMARY

We have investigated the coherent population trapping effect in a collective system of three level atoms driven by two coherent laser fields and simultaneously coupled to the reservoir of a non-zero temperature. The thermal reservoir causes thermal decoherence which affects the trapping effect. We have shown that in a single atom there is no perfect CPT when both atomic transitions are affected by thermal decoherence. The perfect CPT may occur when only one of the two atomic transitions is affected by thermal decoherence. Extending the analysis to multi-atom systems, we have shown that the destructive effect of the thermal decoherence on the CPT can be circumvented by the collective behavior of the atoms. Unlike the case of noninteracting atoms in which decoherence processes are independent of the number of atoms, we have found that the collective behavior of the atoms can substantially improve the trapping effect destroyed by the thermal decoherence. In the collective atomic system the trapping effect increases with increasing number of atoms. If number of atoms is large enough, an almost complete CPT is observed even at high temperatures of the reservoir. This feature is explained in terms of the semiclassical dressed atom model. We have shown that the improvement of the CPT trapping in the multiatom system arises from the presence of collective (entangled) states whose capacity of storage of the atomic population is larger than the corresponding states of independent atoms.

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