Magnetic field dependent intensity variation in the hyperfine split Rb D₁ and D₂ lines

Jianping Hu and M Ummal Momeen
Department of Physics, School of Advanced Sciences, VIT University, Vellore, Tamil Nadu 632014, India

E-mail: jianpinghu@hotmail.com.

Abstract. We present the variation of intensity in Doppler-free hyperfine lines for rubidium atoms with the application of magnetic field. Different polarization configurations have been studied systematically with varying magnetic fields. There is a significant increase in the intensity variation with applied magnetic field related to different polarization configurations. These variations are explained with the theoretical calculations. The calculations are performed by adopting Nakayama’s four level model with the varying magnetic field induced transition probability.

1. Introduction
Atom-laser interaction in the presence of a magnetic field (magneto-optical effects) has been a subject of study for several decades [1]. The availability of narrow line width and tunable diode lasers made these experiments possible in many laboratories [2-3]. Conventional absorption spectroscopy techniques [4-5] results in Doppler broadened absorption spectrum. Doppler-free saturation absorption spectroscopy can be used to resolve the Zeeman lines in the case of weak or intermediate magnetic fields. Alkali metal atoms provide the simplest prototype for high-resolution spectroscopic studies of atomic energy level structure and transition probabilities under the influence of applied electric and magnetic fields. Even though the spectra of ⁸⁷Rb have already been studied extensively [6-10], there have been few systematic investigations of Zeeman spectral intensity as a function of an applied magnetic field [11]. In the present work, optical pumping effects, polarization effects and magnetic field induced transition probability have been taken into account in order to account for the complete study of the Zeeman profile.

In this paper the magnetic field dependent intensity variation of rubidium hyperfine split D₁ (5⁡²S½ → 5⁡²P₁½) and D₂ (5⁡²S½ → 5⁡²P₃½) lines have been discussed. Tremblay et al. [12] has estimated the magnetic field dependent transition probability and Nakayama [13] has developed four level model to calculate the relative intensity of hyperfine transitions. We have adopted these two models and found a unique way of calculating relative intensity in the presence of magnetic field, which is described in detail in the next section. We have also calculated the magnetic field dependent intensity variation with different polarization configurations of pump and probe beams.
2. Theoretical Formalism

The Hamiltonian which demonstrates the hyperfine structure with the magnetic field is:

\[ H = A_J (I \cdot J) + B_J \frac{3(JJ^2 + J_0^2) - J(J_0 + 1)(J + 1)}{2(J - 1)(2J - 1)} + H_B \]  

(1)

where \( A_J \) is the magnetic hyperfine structure constant and \( B_J \) is the electric quadrupole constant. \( H_B \) is the interaction Hamiltonian due to external magnetic field. The interaction Hamiltonian:

\[ H_B = -\frac{\mu_B}{\hbar} \mathbf{B} \cdot (\mathbf{L} + g_S \mathbf{S} + g_I \mathbf{I}) \]  

(2)

where \( \mu_B \) is the Bohr magneton, \( \mathbf{B} \) is the static magnetic field, \( g_S \) is the electron spin Landé factor and \( g_I \) is the nuclear spin Landé factor. \( \mathbf{L}, \mathbf{S} \) and \( \mathbf{I} \) are orbital, spin and nuclear spin quantum numbers.

The Hamiltonian can be represented in the matrix form as

\[ \langle F, m_F | H | F, m_F \rangle = H_{FS} - \mu_B g_F m_F B_z \]  

(3)

This gives the diagonal matrix elements of the Hamiltonian with the unperturbed state vectors. Here \( g_F \) is the Landé factor and \( m_F \) is the magnetic quantum number. The off-diagonal matrix elements can appear only when the selection rules satisfy this condition as \( \Delta F = \pm 1 \) and \( \Delta m_F = 0 \)

\[ \langle F - 1, m_F | H | F - 1, m_F \rangle = \frac{-\mu_B}{2} \left( g_F - g_I \right) B_z \left( \frac{(J + 1)^2 - F^2}{F} \right)^{1/2} \left( \frac{F^2 - m_F^2}{F(2F + 1)(2F - 1)} \right)^{1/2} \]  

(4)

The energy eigenvalues for different magnetic fields are obtained by diagonalizing the hyperfine structure Hamiltonian. Tremblay et al. [12] has introduced a way to calculate the transition probability with the application of magnetic field. The hyperfine total angular momentum quantum number (F) is no longer considered as a good quantum number to describe the system completely with the application of magnetic field. Hence we have to assume that the magnetic sublevels of the hyperfine levels are mixed due to the influence of magnetic field. The change of wave function for the ground state can be written as;

\[ \psi(F_g, m_g) = \sum_{F_e} \psi(F_e, m_e) \]  

and for the excited state;

\[ \psi(F_e, m_e) = \sum_{F_g} \psi(F_g, m_g) \]  

(5)

where \( \psi(F_g, m_g), \psi(F_e, m_e) \) are the state vectors for the ground and excited states. \( C_{F_g,F_e} \) and \( C_{F_e,F_g} \) are mixing coefficients depending on the field strength and magnetic quantum number \( m_F \). There are four hyperfine levels in the \( 5^2P_{3/2} \) excited state and two hyperfine levels in the \( 5^2S_{1/2} \) ground state. The linear combination of different hyperfine levels represents the new atomic states \( \psi(F_g, m_g) \), which is due to the nonlinear nature of Zeeman sublevels. The hyperfine sublevels with the same magnetic quantum numbers are likely to be mixed with the application of magnetic field. The mixing coefficients of the hyperfine states are the eigenvectors and the energies of these corresponding levels are eigenvalues. This can be obtained using the above mentioned equations.

The dipole matrix element is proportional to the square of the transfer coefficients due to the application of a magnetic field.

\[ |\langle e | D_{g_e} | g \rangle|^2 \approx a^2 |\psi(F_e, m_e); \psi(F_g, m_g); q| \]  

(6)

The transfer coefficients with the influence of magnetic field can be written as

\[ a(F_e, m_e; F_g, m_g; q) = \sum_{F'_g,F'_e} C_{F_e,F'_e} a(F'_e, m_e; F'_g, m_g; q) C_{F'_g,F_g} \]  

(7)

In equation (7), the parentheses and the curly brackets are the representation of \( 3j \) and \( 6j \) symbols.

Nakayama's four level model can be used to calculate the relative line intensities of the Doppler-free spectra [13]. Optical pumping effects are taken into consideration in this model.
The electric susceptibility can be written as

\[ \chi^{(P_a)}_{P_P P_r} \approx \sum_{u, r} \chi^{(P_a)}_{u, r} \exp \left[ -\frac{(\Delta \omega_{ur})^2}{2k^2} \right] \left( \omega - \omega_{ur} - i\Gamma \right) \]  

(8)

The indices \( u, r \) represent the pump and probe beam transitions. \( p_u, p_r \) correspond to the polarization states. \( \omega_u, \omega_r \) are the angular frequencies, \( \Delta \omega_{ur} = (\omega_u - \omega_r), \omega_{ur} = (\omega_u + \omega_r)/2 \). \( k \) is the wave number and \( \Gamma \) is the line width of the absorption signal. The exponential term is related to the Doppler factor. The most probable velocity \( v = \sqrt{\frac{2kT}{M}} \). In this approximation, saturation effects are not taken into account because we are considering only the lower laser intensity approximation. In this case the optical coherences in the atom-laser interaction can be neglected.

The relative intensity for the four-level system is

\[ I^{(P_a)}_{P_P P_r} = |\mu_u|^2 |\mu_r|^2 \left( -\delta_{u, sp} + |\mu_{sp}|^2 / \eta \right) \]  

(9)

where \( |\mu_u|^2, |\mu_r|^2 \) and \( |\mu_{sp}|^2 \) are the transition probabilities for the pump, probe and the spontaneous emission transitions. \( \eta \) is the total transition probability. The value of Kronecker’s delta \( \delta_{u, sp} \) is unity for I (two-level) and V-type (three-level) resonances. It is zero for \( \Lambda \) (three-level) and N-type (four-level) resonances. The absorption coefficient is related to the electric susceptibility, which is represented as

\[ X^{(P_a)}_{P_P P_r} \approx \text{Im} \left( \chi^{(P_a)}_{P_P P_r} \right) \]  

(10)

Nakayama used the four level model to compute the intensity variation of hyperfine lines without the magnetic field. We have extended this work by applying the magnetic field induced transition probability to calculate the intensity variation of different hyperfine lines in the presence of magnetic field.

3. Results and Discussion

3.1. Relative intensity of Doppler-free hyperfine D2 lines in the presence of magnetic field

The magnetic field dependent transition probabilities for the \( \Delta m_F=0 \) transitions in the case of \( F=2 \) to \( F'=1, 2, 3 \) for \(^{87}\text{Rb} \) (D2 line) is shown in figure 1. The dotted lines in these figures correspond to the non-dipole allowed transitions which have non zero transition probability in the presence of magnetic field. In order to explain the relative magnitude of the observed Doppler-broadened hyperfine lines in the presence of moderate magnetic field (up to 50G), Tremblay’s field induced transition probability and Nakayama’s four level model has been followed.

![Figure 1. Transition probabilities for the F = 2 to F' = 0, 1, 2, 3 transitions in π polarization of \(^{87}\text{Rb} \) D2 line as a function of the magnetic field](image-url)
The non-dipole forbidden transitions ($\Delta m_F = 0$, $\Delta F = 0$; $\Delta F = \pm 2$) will become allowed in the presence of a magnetic field. In the case of high magnetic field region, these forbidden transitions play a significant role in the absorption spectrum. In this work we focus on the weak/intermediate magnetic field region, hence it is no longer important to consider the forbidden transitions. Here we only consider the dipole allowed transitions for calculating the magnetic field dependent relative intensities.

Relative magnitude of Doppler-free hyperfine lines have been measured using saturation spectroscopy. All possible polarization configurations of pump and probe beams such as ($\pi$, $\sigma$); ($\sigma_+$, $\sigma$); ($\sigma_+$, $\sigma_+$); and ($\pi$, $\pi$) have been considered. In the four level model, number of separate resonances for ($\pi$, $\sigma_+$) polarization configuration is 214 for $^{87}$Rb atoms. Likewise ($\sigma_+$, $\sigma$), ($\pi$, $\pi$) and ($\sigma_+$, $\sigma_+$) have 107, 106 and 95 resonances. We have calculated the magnetic field dependent transition probabilities for each of these resonances and added them together for particular pump-probe polarization scheme.

![Graph](image1)

![Graph](image2)

![Graph](image3)

![Graph](image4)

**Figure 2.** Relative intensities as a function of magnetic field for the transition $F=2$ to $F'=3$ in $^{87}$Rb for different pump-probe polarization schemes.

The variation of relative intensities for the closed transition ($F=2$ to $F'=3$) for $^{87}$Rb is shown in figure 2. In the case of ($\pi$, $\sigma$) and ($\sigma_+$, $\sigma_+$) pump- probe polarization configurations, the relative intensity increases and reaches a maximum around 10 G and then decreases. This has been predicted in the theoretical calculation as well. The mixing of different hyperfine F levels with the same $m_F$ value and the influence of optical pumping is responsible for this intensity variation. This has been observed only for the closed transition of $^{85}$Rb. The other hyperfine transitions of the D$_2$ line ($F=2$ to $F'=1$ and $F=2$ to $F'=2$) do not have such characteristics with the magnetic field. Hence it is an interesting feature which can be taken into consideration while designing rubidium atomic clock and vapor cell magnetometer.
3.2. Relative intensity of Doppler-free hyperfine D1 lines in the presence of magnetic field

The magnetic field induced transition probability for the rubidium D1 line is calculated and is shown in figure 3. The transition probability varies linearly with respect to magnetic field. There are two different F levels and the number of Zeeman sublevels in 5\(^2\)P\(_{1/2}\) state is 8 for \(^{87}\)Rb. The hyperfine energy levels are widely spaced (~812 MHz for \(^{87}\)Rb) in comparison with 5\(^2\)P\(_{3/2}\) state. This makes the study of 5\(^2\)P\(_{1/2}\) state less of an issue with respect to magnetic field.

![Figure 3. Transition probabilities for the F = 1, 2 to F' = 1, 2 transitions in \(\sigma_+\) polarization of \(^{87}\)Rb D1 line as a function of the magnetic field](image)

In the case of D1 line \(^{87}\)Rb spectrum, the field induced transition probability varies linearly with respect to magnetic field. All the four hyperfine transitions for the D1 line are open transitions and there is an equal distribution of population among these appropriate hyperfine ground levels. According to four level model the intensity varies linearly with respect to magnetic field, which is depicted in figure 4.

4. Summary and conclusions

The relative intensity is computed using theoretical models of field induced transition probability and the four level model. The study of magnetic field dependent intensity variation shows the role of non-
linearity of Zeeman levels, optical pumping and polarization effects. The relative magnitude of intensity is maximum at a field of 10 G in the $^{87}$Rb $D_2$ line transition $F=2$ to $F'=3$ for the polarization configurations ($\sigma_+\pi$) and ($\pi_+\sigma$). This is purely a non-linear effect which needs to be taken into account while designing magnetic devices with rubidium atoms.

References
[1] Arimondo E, Inguscio M and Violino P 1977 Rev. Mod. Phys. 49 31
[2] MacAdam K B, Steinbach A and Wieman C 1992 Am. J. Phys. 60 1098
[3] Wieman C E and Hollberg L 1991 Rev. Sci. Instrum. 62 1
[4] Lee H S, Park S E, Park J D and Cho H 1995 J. Opt. Soc. Am. B 11 558
[5] Hänisch T W, Lyons D R, Schawlow A L, Siegel A, Wang Z-Y and Yan G-Y 1981 Opt. Commun. 37 87
[6] Nakayama S 1984 Jpn. J. Appl. Phys 23 879
[7] Scotto S, Ciampini D, Rizzo C and Arimondo E 2015 Phys. Rev. A 92 063810
[8] Sargsyan A, Tonoyan A, Mirzoyan R, Sarkisyan D, Wojciechowski A M, Stabrawa A and Gawlik W 2014 Opt. Lett. 39 2270
[9] Jeong T, Won J Y and Noh H- R 2013 Opt. Commun. 292 106
[10] Sargsyan A, Tonoyan A, Hakhumyan G, Leroy C, Pashayan- Leroy Y and Sarkisyan D 2015 Opt. Commun. 334 208
[11] Ummal Momeen M, Rangarajan M and Deshmukh P C 2007 J. Phys. B: At. Mol. Opt. Phys. 40 3163
[12] Tremblay P, Michaud A, Levesque M, Thériault S, Breton M, Beaubien J and Cyr N 1990 Phys. Rev. A 42 2766
[13] Nakayama S 1997 Phys. Scripta T70 64