SYSTEMS & CONTROL | RESEARCH ARTICLE

Control of period doubling bifurcation in an SMIB power system using adaptive controller based on LaSalle’s invariant principle

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Abstract: This paper presents when system parameter falling into certain area, the single machine infinite bus system with excitation limits experience period doubling bifurcation. When the parameters are further changed the system shows chaotic oscillations. To control this, an adaptive controller based on LaSalle’s invariant principle is designed. The control method does not need any analytical knowledge about the system dynamics and operates without any proper information of the preferred steady state position. Numerical simulation results show the effectiveness of the controller and also it help to maintain the stable operation of the power system.

Subjects: Control Engineering; Dynamical Control Systems; Systems & Controls

Keywords: power system; bifurcation; chaos; LaSalle’s invariant principle; adaptive control

1. Introduction
Large interconnected power systems indicate non-linear dynamic phenomena. The complexity of these phenomena increases the security and stability problem, which leads to voltage collapse (Ohta & Ueda, 2002; Wang, Abed, & Hamdan, 1994). At the first place, analysts believed that the low-frequency motions brought on by negative damping were the fundamental reason for those

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PUBLIC INTEREST STATEMENT

Electric power generations and transmissions have turned out to be increasingly productive with the advancement of technology for large-scale interconnected power systems. The increasing nonlinearity and complexity of electrical power systems may show complex nonlinear dynamic phenomena for example, bifurcation and chaos. The chaotic phenomena are one type of undeterministic oscillation existing in deterministic systems. They are related to random, continuous and bounded oscillation and not dynamically stable and may face serious problems from an operating viewpoint. The presence of these phenomena not only destroys the stability operation of power system, but also can harm the entire power system in a moment. The adaptive control method helps to control the chaotic oscillation and help to maintain the stable operation of the system.
deteriorations and voltage collapse. In any case, those problems still occurred after individuals had fortified the orderly damping through application of excitation controller (Yu, 1983). By the application of bifurcation theory in power system it is found that there are different kind of bifurcations such as hopf bifurcation, saddle node bifurcation, periodic bifurcation. Subramanian, Devi, and Saravanaselvan (2011) observed various bifurcation points such as hopf bifurcation point, saddle node bifurcation and period doubling bifurcation in power system and also unearthed that chaotic motion is due to period doubling bifurcation. In the modern days it is also important to study the non-linear behavior i.e. chaos. The chaotic phenomena are one type of un-deterministic oscillation existing in deterministic systems. They are related to random, continuous and bounded oscillation and not dynamically stable and may face serious problems from an operating viewpoint. Chaotic behaviors in a simple power system have been studied and confirmed through the help of Lyapunov exponent (Chiang, Liu, Varaiya, Wu, & Lauby, 1993). The complex nonlinear phenomena in a single machine infinite bus (SMIB) system with excitation limit was observed by taking the generator electromagnetics, electro-mechanics and excitation control together by fourth order differential equation and have shown the local and global bifurcation in Ji and Venkatasubramanian (1996). Nonlinear systems and their relation to chaos, the presence of chaos in power system and their reliance on system parameter and initial conditions have been analyzed in Lal and Swarup (2011), Grillo, Massucco, Morini, Pitto, and Silvestro (2008).

It is indispensable to design a controller to suppress the occurrence of chaos and stabilize the system. The solving method of the problem to settle an unstable nonzero equilibrium point in a finite time by coordinate transformation method has been proposed (Hui, Ya-Jun, Si-Jia, Shi-Gen, & Dan, 2011). Nayfeh, Harb, and Chin (1996) have introduced a nonlinear controller to control hopf bifurcation and period doubling bifurcation in a power system model and weaken the voltage collapse. LaSalle’s invariant principle is successfully applied to design controller for different dynamical nonlinear systems such as Lorenz’s system, Van der pol oscillator and controlling pendulum (Braun, 2008). Wei, Luo, and Qin (2011) have presented an adaptive control law to control bifurcation, which is based on the LaSalle’s invariant principle in a three-bus power system. Ni, Liu, Liu, and Li (2017a) have designed a simple and robust adaptive feedback controller using an energy type Lyapunov function in power system. An adaptive backstepping sliding mode control has been proposed to control the chaotic oscillation of a power system with excitation limits (Min, Wang, Peng, Wang, & Auth, 2016).

To suppress the chaotic oscillation in power system Ni, Liu, Liu, and Pang (2014) designed a sliding mode control. They have projected a variable speed synergetic control to eliminate the chattering phenomenon arises in sliding mode control. To solve the problem of chaos control and voltage stabilization in three bus power system model, CSC-STATCOM controller is designed in Ni, Liu, Liu, Hu, and Shen (2016). To design the controller they have taken the help of fixed-time stable dynamic surface higher order sliding mode control. Ni, Liu, Liu, Hu, and Li (2017b) have designed Static var compensator controller incorporated with energy storage device controller based on fast fixed-time nonsingular terminal sliding mode control to stifle the chaos in power system.

Motivated by the above study, the present work proposes an independent controller to stabilize unstable state variable of an SMIB model, which shows a period doubling bifurcation routes to chaos. The control technique is based on a closed loop controller with the help of an adaptive steady state estimator. An adaptive robust control based on Lyapunov stability theory without the help of steady state estimator is considered to control the chaotic oscillation in power system with excitation limit by Du-Qu and Xiao-Shu (2007). The limitation in their designed controller is to pre-define the equilibrium point, which is overcome in the proposed control technique. If multiple equilibrium points are available, the proposed control technique automatically stable the state variables of the system to one of the equilibrium point. Here the proposed controller does not need any proper information of steady state position. In light of the LaSalle’s invariant theory (LaSalle, 1960), the proposed controller stabilizes the unstable state variables.
The remainder of the paper is organized as follows. In Section 2, system model is described. Section 3 describes the occurrence of period doubling bifurcation. In Section 4 LaSalle’s invariant principle based adaptive control technique is given and its implementation in an SMIB system is analyzed. Section 5 presents simulation results and conclusions are given in Section 6.

2. System model

In this section an SMIB power system model with excitation system as shown in Figure 1 is considered (Ji & Venkatasubramanian, 1996). In the Figure 1 $V_{net}$ represents voltage at infinite bus whose value is 1 p.u. and its phase angle is 0°.

The generator is modeled as a third order model. There are two mechanical state variables i.e. the angular frequency deviation $\omega$ and the rotor angle $\delta$ and an electrical state variable i.e. the transient emf $E$. There is also an additional state variable $E_{fdr}$ due to the automatic voltage regulator (AVR). The synchronous generator has a proportional voltage regulator with a gain $K_A$ and a delay $T_A$ together with a static wind-up voltage limiter.

The voltage controlled method is shown in Figure 2 where $E_{fdr}$ signifies the ideal field voltage of the controller, $E_{fd}$ denotes the real field voltage, $E_{fdo}$ is the reference field voltage, $V_{ref}$ is the voltage set point of the AVR, $E_{fdmin}$ and $E_{fdmax}$ are the minimum and maximum threshold voltage of the limiter respectively.

The dynamic equations describing the system are:

$$\delta = 2\pi f_0 \cdot \omega$$
$$\dot{\omega} = \frac{-D\omega + P_t - \frac{E'}{x_d' + x} \sin \delta}{M}$$
$$E' = \frac{-x + x_d' E' + \frac{x_d - x_d'}{x + x_d} \cos \delta + E_{fdr}}{T_{do}}$$
$$E_{fdr} = \begin{cases} 
E_{fdmax} & \text{if } E_{fdr} > E_{fdmax} \\
E_{fd} & \text{if } E_{fdmin} \leq E_{fdr} \leq E_{fdmax} \\
E_{fdmin} & \text{if } E_{fdr} < E_{fdmin} 
\end{cases}$$

With

$$V = \frac{1}{x + x_d'} \left( \sqrt{(x_d' + xE' \cos \delta)^2 + (xE' \sin \delta)^2} \right)$$

Figure 1. Power system under test.
Here, $x_d'$, $x_d''$ and $x$ symbolizes the synchronous reactance, transient reactance and the line series reactance on system base. $M$ is the moment of inertia and $T_{do}'$ is the $d$-axis transient time constant.

The values of the system parameters are chosen to be

$M = 10$ s, $x_d = 1.0$ p.u., $x_d' = 0.4$ p.u., $x = 0.5$ p.u., $T_A = 1$ s, $T_{do}' = 10$ s,

$E_{fd0} = 2$ p.u., $V_{ref} = 1.05$ p.u., $f_o = 60$ Hz, $E_{fmax} = 5$ p.u., $E_{fmin} = 0$ p.u.

While solving for the equilibrium points of the system substitute $E_{id} = E_{fdr}$. The equilibrium point of the system can be obtained by solving the following equation:

$0 = 2\pi f_o \omega$

$0 = -D\omega + P_t - \frac{E'}{X_d' + X} \sin \delta$

$0 = -\frac{x + x_d}{x + x_d'} E' + \frac{x_d - x_d'}{x + x_d} \cos \delta + E_{id}$

$0 = -K_A (V - V_{ref}) - (E_{fdr} - E_{fd0})$

where

$V = \frac{1}{X + X_d} \left( \sqrt{(x_d' + xE' \cos \delta)^2 + (xE' \sin \delta)^2} \right)$

The above equation can be written as

$\omega = 0$

$E_{id} = \frac{5}{3} E' - \frac{2}{3} \cos \delta$

$0.9P_t = E' \sin \delta$

$E' \cos \delta = \frac{73.3025 - 25(E')^2}{40}$

(3)

After further simplification, the equation will be

$(E')^4 - 8.4249(E')^2 + 8.5980 + 2.0737P_t^2 = 0$

(4)
It is effortlessly observed that this quadratic condition in $E'$ has positive genuine arrangements at whatever point $P_t$ exists in the area $-2.08 < P_t < 2.08$. So by putting $P_t = 1.3$ in (4) the two unstable equilibrium points are calculated as

$$X_1^* = (1.0416, 0, 1.3554, 1.9224)$$  

$$X_2^* = (2.6682, 0, 2.5665, 4.8707)$$

However, $X_1^*$ is the permissible operating equilibrium point for the system.

3. Period doubling bifurcation analysis

In this section period doubling bifurcation route to chaos in SMIB power system model is studied. A period doubling bifurcation is a series of doublings and further doublings of the repeating period. The period doubling route to chaos, when there is change in system parameters causes the loss of stability.

The term chaos has the following properties (Tan, Varghese, Varaiya, & Wu, 1995):

1. Have sensitive to initial conditions.
2. Contains infinitely many nonperiodic orbits.
3. There is an orbit which is dense in nature.
4. Have a positive Lyapunov exponent.

Here, there are two bifurcation parameters i.e. damping parameter ($D$) and mechanical input power ($P_t$) are considered. So to study the bifurcation in details first keeping $P_t$ constant, parameter $D$ is varied and in next varying $P_t$ and keeping $D$ constant is considered.

3.1. By varying the parameter $D$

In this sub-section it is seen that when the damping parameter $D$ is decreased, the stable periodic orbits experience a sequence of period double bifurcations, which are eventually leads to the birth of chaos. All the results in this section are demonstrated through Matlab.

The presence of period doubling bifurcations in the SMIB model is justified by the series of limit cycles shown in the Figure 3(a)–(e) for the variation of damping parameter $D$. From Figure 3(a) it is seen that at $D = 20$ there is a stable limit cycle. When the parameter value decreased to $D = 5$, the stable periodic orbit has doubled its period indicating a period doubling bifurcation in Figure 3(b). The further period doubling bifurcation occurs at $D = 4$ in Figure 3(c). From Figure 3(d) it is viewed as $D$ continues to decrease, the period of periodic orbit gets further doubled and “double twist” of the limit cycle evolves in a denser manner at $D = 3$, which eventually leads to the birth of chaos at $D = 2.4$ in Figure 3(e).

3.2. By varying the parameter $P_t$

In this sub-section it is seen that when the damping parameter $D$ is decreased, the stable periodic orbits experience a sequence of period double bifurcations, which are eventually leads to the birth of chaos. All the results in this section are demonstrated through Matlab.

Throughout the previous sub-section the parameter $P_t$ was kept constant at $P_t = 1.3$ and the damping parameter $D$ was varied. In this sub-section $P_t$ is varied and parameter $D$ is kept constant. From Figure 4(a)–(e) it is seen that there exist sequence of period doubling bifurcation when parameter $P_t$ is gradually increased. From Figure 4(a) it is seen that at $P_t = 1.02$, a stable limit cycle exits. It can be seen from the Figure 4(b) that when parameter $P_t$ ranges from $P_t = 1.02$ to $P_t = 1.15$, double twist of the limit cycle occurred, indicating the period of the limiting cycle is doubled and period-doubling bifurcation occurs. From Figure 4(c) it is viewed as when $P_t$ continue to increase from $P_t = 1.15$ to $P_t = 1.3$ the period of periodic orbit gets further doubled and “double twist” of the limit cycle evolves in a denser manner at $P_t = 1.31$ in Figure 4(d), and at $P_t = 1.321$ it become more denser, which ultimately hints the birth of chaos in Figure 4(e).
Figure 3. Period doubling bifurcation study by varying parameter $D$. 

(a) 

(i) Phase Portrait at $D = 20$

![Phase Portrait at $D = 20$](image)

(ii) Time domain waveform at $D = 20$

![Time domain waveform at $D = 20$](image)

(b) 

(i) Phase Portrait at $D = 5$

![Phase Portrait at $D = 5$](image)

(ii) Time domain waveform at $D = 5$

![Time domain waveform at $D = 5$](image)
Figure 3. (Continued).
Figure 5 shows the system experienced unstable oscillation at $P_t = 1.3$ and $D = 2$. The unpredictable and periodic oscillatory conduct is troublesome for power system operation and control. It might offer ascent to serious harm to power system. The attractor has a wide frequency range, which may incorporate unsafe consonant transient in synchronous machine. Accordingly, it is basic to plan a possible controller to stifle chaotic motion.

4. Adaptive control method based on LaSalle’s invariant principle

4.1. Basic concept of LaSalle’s invariant principle

Consider a nonlinear system

$$\dot{x} = f(x)$$  \hspace{1cm} (5)

where $x \in \mathbb{R}^n$ is the state variable, while $f: D \rightarrow \mathbb{R}^n$ is a time invariant vector field defined on $D \subseteq \mathbb{R}^n$. Assume that the system has no less than one fixed point $x^* \in D$ and $f$ is sufficiently smooth (i.e. Lipschitz on $D$), which ensures the presence and uniqueness of the solution $x(t)$ with any initial condition $x_0 \in D$.

The vector function $f(x)$ satisfies Lipschitz condition

$$|f(x) - f(y)| \leq k|x - y|$$

where $k$ is the Lipschitz constant and $y$ is the control target of state variable $x$.

Chaotic systems fulfill the above condition because chaotic system has bounded attractor (Zhao, Ma, Liu, & Zhong, 2011). Assume that the considered physical system is complex and we are unable to rebuild a reasonable analytical model (5) to describe its behavior. Also assume that there exist an equilibrium state $x^*$ and its position is not known for control design.
Figure 4. Period doubling bifurcation study by varying parameter $P_t$.

(a) Phase Portrait at $P_t = 1.02$

(b) Phase Portrait at $P_t = 1.15$

(ii) Time domain waveform at $P_t = 1.02$

(iii) Time domain waveform at $P_t = 1.15$
Figure 4. (Continued).

(i) Phase Portrait at $P_t = 1.3$

Angular frequency deviation (in p.u.)

(ii) Time domain waveform at $P_t = 1.3$

(i) Phase Portrait at $P_t = 1.31$

Angular frequency deviation (in p.u.)

(ii) Time domain waveform at $P_t = 1.31$
**Theorem 1**  
LaSalle (1960), Khalil (2002), LaSalle's Invariant Principle  
Let $\Omega \subseteq D$ be a compact (i.e. closed and bounded) set that is positively invariant with respect to the dynamics (5). Let $V(x)$ be a continuously differentiable function on $D$ such that $\dot{V}(x) \leq 0$ in $\Omega$. Let $E$ be the set of all points in $\Omega$ where $\dot{V}(x) = 0$ and let $M$ be the largest invariant set contained $E$. Then every solution starting in $\Omega$ converges to $M$ as $t \to \infty$.

For a detail proof, refer (LaSalle, 1960).
Corollary 1  if \( V > 0 \) and \( M = \{0\} \), then the origin is asymptotically stable.

Adaptive control in power system:

Add a controller \( u \) to system (5), the controlled system can be expressed as

\[
\dot{x} = f(x) + u_i, \quad i \in \{1, 2, \ldots, n\}
\]

The basic approach to get the adaptive control \( u \) is to make it proportional to the distance between the equilibrium point and the system's present state (\( x \sim x^* \)). But implementation of this method is not realized without clear knowledge of the steady-state position \( x^* \). This limitation can be overcome by estimating the equilibrium point \( x^* \) obtained by a first order estimator

\[
y_i = \alpha_i (x_i - y_i)
\]

where \( y_i \in \mathbb{R}^n \) and \( \alpha_i \in \mathbb{R}_{+}^n \).

The adaptive controller \( u \) is proposed as follows:

\[
u_i = -\beta_i (x_i - y_i)
\]

\[
\dot{\beta}_i = \gamma_i (x_i - y_i)^2
\]

where \( \gamma \in \mathbb{R}_+ \), with zero initial gain \( \beta(0) = 0 \), each component in \( \beta(t) \in \mathbb{R}_+^n \) is a nondecreasing positive function which tends monotonically to its maximum value when the desired state is reached.

Theorem 1  System (5) can be asymptotically stable at equilibrium point with adaptive controller \( u \).

Proof:  Let the Lyapunov function be

\[
V = \frac{1}{2} \sum_{i=1}^{n} (x_i - y_i)^2 + \frac{1}{2} \sum_{i=1}^{n} \frac{1}{\beta_i} (G - \beta_i)^2
\]

where \( G \) is a positive constant parameter.

The time derivative of \( V \) is given by

\[
\dot{V} = \sum_{i=1}^{n} (x_i - y_i) f(x_i) - \sum_{i=1}^{n} (G + \alpha_i) (x_i - y_i)^2
\]

Since \( f(x_i) \) is locally Lipschitz, it is bounded on its domain \( D \) which implies, there exists a positive constant parameter \( g \), such that \( \forall i, f(x_i) \leq g|x_i - y_i| \).

We can choose \( G > g \) in (11) to obtain the inequality

\[
\dot{V} \leq W = -(G - g) \sum_{i=1}^{n} (x_i - y_i)^2 \leq 0
\]

By LaSalle’s invariant principle (LaSalle, 1960) (12) secures that for \( t \to \infty \) the largest invariant set \( M = \{(x,y,\beta)x = y = x^*, \beta = \beta^*\} \) of (6) to (9) only contains the equilibrium state \( x^* \) of (5). So the unstable fixed point of the plant becomes asymptotically stable under the controller.

4.2. Application of adaptive controller in SMIB Power system

In conventional power system design, power system stabilizer (PSS) signal is used as input and it is directly added to the voltage reference \( V_{ref} \) (Du-Qu & Xiao-Shu, 2007). With the above knowledge
described in Section 4.1, the adaptive controller in SMIB power system is applied and the adaptive control input $u$ has been added to $V_{\text{ref}}$ for stabilization of the power system to its equilibrium state. The control input $u$ has been defined in (14) based on the proposed method. The procedure for finding the equilibrium point (whether single or multiple) using first order estimator has been described in details in (Braun, 2008) for nonlinear dynamical system. The first order estimator is defined in (14).

Now the controlled system is expressed as

$$\begin{align*}
\dot{\delta} &= 2\pi f_0 \cdot \omega \\
\dot{\omega} &= \left(-D\omega + P_t - \frac{E'}{x_d + x} \sin \delta\right)/M \\
E' &= \left(-\frac{x + x_d}{x + x_d} E' + \frac{x_d - x'}{x + x_d} \cos \delta + E_{\text{fd}}\right)/T_{do} \\
E_{\text{fd}} &= (-K_A (V - V_{\text{ref}} - u) - (E_{\text{fd}} - E_{\text{fdo}}))/T_A
\end{align*}$$

(13)

where control input

$$u = -\beta(E_{\text{fd}} - y)$$

$$\dot{\beta} = \gamma(E_{\text{fd}} - y)^2$$

$$\dot{y} = \alpha(E_{\text{fd}} - y)$$

(14)

As per LaSalle’s invariant rule, all together to secure asymptotic conduct of the controlled solution at the equilibrium point, the uncontrolled solution of (1) should be bounded. It is demonstrated in past work that power system can without much of a stretch fulfill Lipschitz condition, so the uncontrolled solution of (1) is limited (Du-Qu & Xiao-Shu, 2007). Once $E_{\text{fd}}$ is asymptotically stable, the output signal of the excitation limiter $E_{\text{a}}$ becomes stable, then oscillations in power system are controlled. The effectiveness of the controller is shown by numerical simulation.

5. Numerical simulation

In this section, simulation results of the system with controller and without controller are presented. For simulation work the system parameters are set to be $P_t = 1.3$ and $D = 2$, with the unstable equilibrium point is $X_{eq} = (1.0416, 0, 1.3554, 1.9224)$. Controller input $u$ is put into effect at $t = 40$ s. The controller parameter are set as, $\gamma = 30$, $\alpha = 30$. The initial condition is taken as $\left(\delta, \omega, E, E_{\text{fd}}, y, \beta\right) = (1.0416, 0, 1.3554, 1.9224, 0, 0)$ as found. From Figure 6, it is illustrated that when the controller is put into effect at $t = 40$ s the amplitude of the oscillatory response of the state variable is reduced and each state variable converges to its steady-state operating point. Figure 7 displays a clear view of the time domain waveform of state variables of the system under the proposed adaptive controller. This proved the proposed controller can successfully suppress the chaotic oscillation in a power system.

The robustness of the controller with respect to parameters variation of the system and external disturbance is analyzed by doing further simulation. Firstly, the robustness of the designed controller with respect to parameter variation is studied by changing the mechanical input parameter and damping parameter values from $P_t = 1.3$ to $P_t = 1.321$ and $D = 2$ to $D = 2.5$ respectively and keep all of the other parameter values (including controller parameters) unchanged. From the Figure 8, it is observed that, each state variable is stabilized to its steady-state operating point, which verifies the robustness of the designed controller against parameters variation.
Furthermore, to verify the robustness of the present controller with respect to external disturbance, a disturbance of \( n = 0.5 \) is exercised to the first equation of system (13). For simulation the mechanical input parameter and damping parameter are set as \( P_t = 1.3 \) and \( D = 2 \) respectively and keep all of other system parameters and controller parameters values unchanged. It is clearly seen from Figure 9 that the amplitude of oscillatory response of the state variable is reduced and each state variable is stabilized to its equilibrium point, which shows the effectiveness of the proposed controller.

Figure 6. Time response of state variables without controller and with proposed adaptive controller.

Figure 7. Time response of state variables in controlled state under proposed adaptive controller.
From Table 1 it is clearly seen that although the proposed adaptive controller and Ni et al. (2017a) proposed adaptive feedback controller require nearly same time for stabilization of each state variables. However, the proposed adaptive controller does not need any information about the system’s stable state and it works without utilizing any detail mathematical analysis of the system’s dynamics.

Figure 8. Time response of state variables under adaptive controller at $P_i = 1.321$ and $D = 2.5$. 

Figure 9. Time response of state variables under adaptive controller in presence of external disturbance.
without utilizing any detail mathematical analysis of the system’s dynamics. Targets, first it does not need any information about the system’s stable state and second it works the parameters variation in system and external disturbance. The controller possesses two advantages. Simulation results show that the proposed controller is very effective and robust with respect to both ensure stable operation an adaptive controller based on LaSalle’s invariant principle is designed.

6. Conclusion
In this paper, period doubling bifurcation diagrams that leads to chaos in the SMIB power system are observed by varying the damping parameter D and mechanical input power P, to the system. To ensure stable operation an adaptive controller based on LaSalle’s invariant principle is designed. Simulation results show that the proposed controller is very effective and robust with respect to both the parameters variation in system and external disturbance. The controller possesses two advantages, first it does not need any information about the system’s stable state and second it works without utilizing any detail mathematical analysis of the system’s dynamics.

Table 1. Comparison of settling time (sec) of different controller

| State variables          | Proposed adaptive controller | Adaptive feedback controller designed by Ni et al. (2017a) |
|--------------------------|-------------------------------|---------------------------------------------------------|
| Rotor angle              | 28.58                         | 33.25                                                   |
| Angular frequency deviation | 30.12                         | 30.01                                                   |
| Transient emf            | 26.71                         | 36.05                                                   |
| Ideal field voltage      | 25.44                         | 0.20                                                    |

Funding
The authors received no direct funding for this research.

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Citation information
Cite this article as: Control of period doubling bifurcation in an SMIB power system using adaptive controller based on LaSalle’s invariant principle, Bimalesh Chandra Rout, Deepak Kumar Lal & A.K. Barisal, Cogent Engineering (2017), 4: 1362804.
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