Mathematics teacher’ perceptions and adaptations in developing online classes - ideas for teacher training

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Abstract. The wide availability of digital tools, as well as the current global pandemic has transformed the way in which teachers and students interact. This of course will have consequences on the learning of in physics and mathematics in the medium and long term. The objective of this research is to explore what are the feelings, difficulties and adaptations of the teachers to develop online classes. We reach this goal with support on the tetrahedron model for a didactical situation, which describes the interactions between student, teacher, content, and technology. Following a qualitative methodology, a survey of 48 in-service teachers was conducted to explore how teachers’ knowledge and their use of digital tools influence teacher-student-technology interactions. The results showed that 65% of the participants feel prepared for online teaching, while 35% feel. However, there is not a big difference in the interactions they promote with the students. Teachers experiment a high level of negative emotions such as frustration and anxiety, many of these emotions were recognised in teachers feeling online teaching. The findings explain how emotional and infrastructural factors are related to the interactions in online teaching. The results have implications in physics and mathematics teacher training programs, the ways we teach physical and mathematical content, insofar we highlight those aspects into account to improve the students' learning.

1. Introduction

The current global pandemic situation has forced teachers and institutions to develop online teaching, in the best of cases, when the conditions of the students allow, synchronously. This situation makes the already complex task of the teacher to develop face-to-face teaching more difficult. Just because the digital teaching environment requires the teacher to reorganize the management of the classroom, design new activities, and promote different forms of interactions, which is not a simple process [1]. This condition has revealed aspects associated with the knowledge, skills and perceptions of the teacher to develop teaching in digital environments. As well as emotional aspects associated with not being present and social inequalities that hinder access to communication technologies.

For several years now, there has been an emphasis on using digital tools to support the learning of mathematics by promoting the exploration of mathematical relationships, the formulation of conjectures, the testing process, and generalization [2]. In the case of physics interactive web-based physics lessons been used to promote the learning of concepts from this area [3]. We can also find different simulations on the web carefully designed for teaching different concepts (https://phet.colorado.edu/es/simulations/filter?sort=alpha&view=grid). However, the current need to...
carry out classes in a non-face-to-face way has shown the ways in which teachers adapt to this new reality, highlighting two forms of teaching. The asynchronous teaching where teachers send digitized material for students to read and work on mathematical activities, relying on materials on the web, such as videos, and pages with educational content. And, the synchronous teaching, where the teacher relies on some means to carry out the class virtually, and students can ask questions and receive feedback in real-time. These two environments generate different learning experiences [4]. For example, regarding to videos with conceptual content, physics students have argued that although videos are helpful in fostering understanding these resources did not orient them towards problem-solving [3]. The mediation of digital tools in teaching and learning processes naturally changes the ways in which students understand mathematical and physics objects as well as the interactions that take place between teachers, students, and the content.

Other factors such as the attitudes, perceptions, and emotions of teachers and students, also have an impact on learning [5]. In the case of physics is also well recognized the need of coordinating the constructs related to interest, motivation and values for improve teaching and learning processes [3]. With these considerations, we focus the study on the case of in-service mathematics teachers from Chile. In Chile ten months ago, we have faced the suspension of face-to-face classes due to the social protests that took place in different parts of the country, subsequently and up to the present due to the pandemic that currently affects the world. Besides, to protect the health of teachers and students, it is projected that the rest of the year 2020 classes will be held online. In this context, institutions have had to make curricular adjustments, prioritizing the teaching of some content where the use of technology acquires even more relevant role as a support tool in distance teaching and as a tool that favors new representation infrastructures.

In this situation, it is extremely important to know the uses that teachers give to technology [1,6] to support them in the efficient integration of these tools in their classes. Noticing that digital tools acquire singular importance in stimulating the interactions between the constitutive elements of the didactical triangle (teacher, student, and content) in synchronous and asynchronous environments. The efficient use of technology requires a technical domain of the tools, it demands aspects associated with technological, pedagogical, and content knowledge [7,8]. Given the current contingency in the teaching of mathematics and recognizing that the trend in research has been to promote the educational use of technology in overly idealized and deterministic terms [9], in this research we consider the set of facts (beliefs, infrastructure, domain and knowledge of technology) that surround the teachers and determine their perceptions of the use of technology in online teaching. With this objective, we address the research questions: what difficulties related to digital artifacts does the mathematics teacher face when conducting online classes? What are the teachers’ perceptions about online teaching and interactions in online classes? How are the adaptation processes from classroom work to online work of mathematics teachers? Although here is analyzed the case of mathematics teachers we assume, on the one hand, that epistemologically mathematics and physics have much in common. Therefore, it is possible to explain the teaching-learning processes using similar theories. On the other hand, nowadays teachers who teach these disciplines share the same challenge of online teaching. Such that, we study the case of mathematics teachers with the conviction that the results we find also describe the situation of physics teachers.

2. Research literature and conceptual framework
One element that has a strong influence on the incorporation of technology in the classroom is the teachers’ perception regarding the attitudes of students. For example, on whether technology will increase students’ interest in mathematics or the perception of the effects that technology can have on the classroom [10]. Ertmer [11] identifies two types of barriers that influence the use of technology, extrinsic and intrinsic barriers. The first one considers the structure of the classroom, the schedules, technical support, and the curriculum. The second includes the time that teacher invests in learning and preparing the class. This barrier also involves emotional aspects and beliefs about the use of technology, as well as attitudes, knowledge, and technological skills. From a constructivist point of
view, Ertmer [11] identifies that practices with a low level of technology use tend to be teacher-centered, while a high level of technology use tends to be student-centered. Unfortunately, as Ruthven and Hennessy [9] indicate and as we can see in the literature, most of the research that explores aspects related to the use of technology has been developed in countries whose educational systems have wide experience in the use of computers for mathematics teaching.

2.1. The didactical tetrahedron

From the socio-constructivist point of view, mathematical knowledge and practices are constructed as a product of the interactions between the student, the teacher, the task, and the technology [12]. Rezat and Sträßer [6] represent the relationships between the four constituents of a didactical situation (teachers, students, mathematics and artifacts) through a tetrahedron. This model incorporates different instrumental acts and highlights a different characteristic regarding the role of the artifacts on each of its faces (Figure 1). The student-artifact-mathematics face represents the student's activity mediated by the instrument for learning mathematics. The teacher-artifact-math face represents the teacher's activity mediated by the artifact to do math and plan instruction. And the teacher-artifact-student face represents the role of the teacher as a mediator and as an orchestrator of environments for learning mathematics. An artifact includes from tangible material used for teaching purposes to digital tools used for the same purpose. Therefore, in the artifact vertex, tasks, explanations, textbooks, notes, among others, are considered to allow students get involved in doing mathematics.

Three factors stand out in the model (Figure 1). The first is the complexity of the instrumental genesis [13] that individuals develop when a new artifact is introduced. This fact is related to the characteristics of the artifact, the activities, the individual’ knowledge, and with the experience [14]. The second factor is related to the schemes of use of the artifact oriented to the management and development of specific tasks [14]. Where the characteristics of the instrument may represent restrictions on its use [13]. The third factor is the use -instrumental orchestration [15] - of artifacts with a teaching purpose that the teacher develops [6]. There are also other factors affecting the interactions between the constituents of the didactical tetrahedron, these are: the teacher's beliefs regarding the use of technology for teaching and learning mathematics [16]; self-efficacy beliefs about teaching with technology [16]; and the teachers’ knowledge regarding technology, pedagogy and the content [7,8]. Student engagement (behavioral, cognitive, and emotional) depends on the learning environments [17]. Therefore, the change from face-to-face learning modality to online learning modality involves spatial and temporal adaptations on the interactions between the components of the didactical tetrahedron. In the case of teaching physics with technology some authors [3] have showed how the affective domain (motivation, interest and values) can affect the construction of knowledge. In this paper the Tetrahedron model includes the specific content of mathematics but other studies have considered a more general content [12].

![Figure 1. Tetrahedron model for a didactical situation [6].](image-url)
3. Method
The global pandemic situation highlighted different challenges and difficulties in mathematics teacher's practices. In this context, an exploratory and descriptive research was carried out to reconstruct teachers’ perceptions through characterizing the knowledge and uses of technology declared by them when teaching without face-to-face interaction. For this aim, an online questionnaire was constructed to know what the difficulties and adaptations of teachers are when conducting online classes. Three aspects associated with the tetrahedron model were considered: (a) knowledge of digital tools, (b) use of digital tools to promote cognitive development, and (c) teacher-student-technology interactions. Furthermore, from a predominantly qualitative perspective, the questionnaire collects information on the types of software used and how teachers have responded to the transitions between face-to-face classes and online classes. The questionnaire was directed to teachers who had their classes to be developed in online mode, 48 mathematics teachers (men and women) voluntarily answered. They teach students aged 13–18 years in central Chile.

Data analysis was carried out in two cyclic stages. First, we identify teachers feeling prepared to teach with the support of technology. Then an exploratory and frequential analysis was carried out to know the perceptions about the use of software in online teaching, the level of mastery and interactions. Subsequently, a discursive analysis was carried out to analyze the answers to the following questions: Explain how have you had to adapt to online teaching? In addition to the technical artifacts, what other aspects do you consider have a negative impact on online teaching? The Figure 2 shows the protocol for the analysis and the instruments used in each stage.

![Figure 2. Protocol for the analysis.](image)

4. Data analysis and results
To get a general impression of the scenario in which the participants were working at that time, they were initially asked: Do you feel prepared to teach with the support of technology? 65% of the teachers declared feeling prepared; this group will be called G1. 35% declared that they do not feel prepared, which we will call the G2 group. This allows us to focus the analysis on both groups of teachers to establish characteristics about the mathematics teachers’ practice and their use of technologies according to their self-efficacy perceptions. Considering that emotions play an important role in teachers’ perceptions another factor that motivated us to compare both groups was the high difference observed in the emotions expressed by the participants. When asked how about doing online classes? 82% of the participants in G2 negative emotions, such as frustration and anxiety, while only 29% of G1 indicated these same emotions.

Regarding the knowledge of technological tools as mediators of learning, participants were asked about their mastery of software traditionally used in mathematics education, such as Excel, Geogebra, Derive, Cabri, Mathematica, Maple, Matlab, between others. Although the answers showed that teachers have a generalized knowledge in software such as Excel (96.8% and 94.1% for G1 and G2 respectively), there is evidence of higher mastery in G1 for each software questioned, especially, Excel, Geogebra, Derive and Cabri (the last three with 93.5%). While 76.5% of G2 has mastery in the...
same software, highlighting a weakness in the group G2 that could affect participants’ self-efficacy perception.

The questions associated with the use of digital tools to promote cognitive development [2] measure the importance that the teacher assigns to cognitive development processes approached with digital tools. The measurement was developed using Likert scale, where 1 means less importance and 5 greater importance. To better visualize the distributive behavior of preferences, we decided to categorize in three levels: minor importance, medium importance, and major importance. No significant differences are recognized when averaging the results of each group in the questions: (1) representing mathematical concepts, (2) constructing mathematical concepts, and (3) searching for information. However, it was observed that G1 gives greater importance than G2 to (4) exploration of mathematical concepts (3.9 and 3.6, respectively). And G2 give more importance to (5) visualization of mathematical concepts and their relationships and (6) explaining mathematical concepts than G1 (4.5 and 4) (see Table 1).

Related to teacher-student-technology interactions, variables commonly found in face-to-face mathematics class [9,15] were measured, but in this case, mediated by the screen and projected to students online. The variables were (1) technical demonstration of software use, (2) tool use and relationships with mathematical content, (3) explaining different mathematical representations and their relationship with pencil and paper work, (4) discuss with students about what is shown in the screen, (5) discuss the students’ reasoning, and (6) students showing their work and reasoning to others. The results showed a high level of importance assigned to each variable, with average varying in each response between 3.9 and 4.5. This suggests that the teachers’ feelings regarding whether or not they are prepared to teach non-face-to-face classes are not strongly related to their use of resources and to the forms of interaction with students. However, we can see that the majority of teachers in G1 give more importance to technical demonstration and discussion than teachers in G2. Table 1 summarizes the results of this part of the analysis. Qualitative analysis allows us to know adaptation processes and difficulties (extrinsic and intrinsic barriers) experimented by the teacher.

| Table 1. Analysis summary. | G1 (%) | G2 (%) |
|---------------------------|-------|-------|
| Negative emotions         | 29.0  | 35.0  |
| Mastery in software       | 96.8  | 94.0  |
| Promotion of cognitive development |       |       |
| Exploring                 | 64.5  | 52.9  |
| Visualizing               | 71.0  | 76.5  |
| Explaining                | 71.0  | 76.5  |
| Uses of technology and interactions |       |       |
| Technical demonstration   | 90.3  | 82.4  |
| Explaining mathematical representations | 74.2  | 82.4  |
| Discussion with students about what is shown in the screen | 80.6  | 58.8  |

4.1. Adaptations

Subsequently, the teachers were asked to explain their adaptation processes to online teaching. Answers from teachers in G1 highlight the need for in-depth knowledge about the use of technological tools. This represents an economic and time cost for teachers “To improve the quality of my courses I must look for information about technology, I also have to explore different applications and web pages to do classes and formative assessments”. This situation also affects the teachers’ lesson plans, due to remote work reduce the teaching time. Although a process of familiarization with the teaching platforms can be observed in a short time: “[...] after the first classes, one begins to become familiar with the different platforms and the students also [...]”.

The teachers’ attitudes toward change play a fundamental role. In teachers who affirm feeling prepared for teaching with technology, a positive attitude the study and learning of technological resources was evidenced. They positively explained their adaptation process: “I have had to innovate
in new web class platforms that are easily accessible for the students and friendly in their use, because most of the low-income students do not know this type of platforms”. Teachers also recognize the importance of the classes being motivating for the students: "learning to use software that allows making explanatory capsules, attractive to my students and with simple language for them."

4.2. Difficulties
Among the many difficulties that teachers experience from remote work, solutions with some degree of conformity are evident, and others that still require attention, after fourth months of online work. Collaborative groups have emerged to share knowledge to facilitate adaptation to online work: "We have a network of teachers who have been collaborating to overcome difficulties (Teacher in G2)." However, these types of networks do not always reach all teachers, demanding systematic actions that enable collaborations: "there is a lack of support networks, by cities, communes, by areas [there is a lack of] teachers specializing in geometry (teacher in G2)."

Other difficulties of structural nature are related to economic efforts, both on the part of students and teachers. A teacher explains it “because of the little connectivity that my students have, I have had to make explanatory guides, videos on YouTube and provide feedback creating different strategies to solve exercises”, this shows the teachers' effort to adapt the classes to the conditions of the students. These limitations are also typical of low socioeconomic sectors (where COVID has exacerbated and made the technology gap even more big) “In my case, I work in a rural school. About 20% of students have access to the Internet. The main means of communication at first was WhatsApp. The school decided not to send material [learning material to students]. So, I have not been able to work in this modality”. Also, the lack of appropriate technological resources on the part of the professor was evident, which showed another way of adaptation: "[I am] making capsules because doing face-to-face feedback would imply having a tablet."

In summary, the responses of the teachers in groups G1 (they feel prepared) and G2 (they feel prepared) show a strong relationship between the self-efficacy perception about the use of technology, and attitudes and concerns they express. Most of the teachers from group G2 express feeling frustration and anxiety regarding online teaching (higher than G1). Teachers in G1 show a strong commitment to finding new ways and tools to motivate students. Most teachers are concerned about the lack of technological infrastructure students face. They are also more concerned about the emotional distance caused by online teaching (demotivation of students, lack of contact, the impossibility of feedback, etc.). These teachers are more motivated than teachers in G2, towards the search for information (technological, didactic, pedagogical) to improve their teaching. Regarding the materials used for teaching, the teachers of both groups have used videos, digitized documents, and tutorials from the web to explain the mathematical content to the students.

According to the above, the results in this research shows how the incorporation of online teaching challenges teachers to innovate in their teaching methods. The adaptation process is not easy, it does not depend only on good intentions but is subject to extrinsic factors, such as the connectivity and availability of computers with different capacities (for both teachers and students). This factor has a direct impact on the interactions between student-teacher-content mediated by technology. Other extrinsic factor is given by the curricular modifications that online teaching requires. Besides, economizing time is required to achieve progress in the knowledge of students in less time [19,20].

Other aspect involved in the student-teacher-content interaction is the access that students have to different media for searching information, communication, and support in solving problems. From here the teacher must to support students in the critical and reflective use of digital tools. The teacher also needs to know how technology can be used in a complementary way since it is not the same to learn through technology than to learn with technology. In learning through technology, the student is a passive receiver of information, where the content is shown, for example, through videos or digitized material. Whereas in learning with technology the students use technology as a tool to explore, analyze, interpret the world and organize their knowledge; instead of simply consuming media [19,20]. Therefore, the tasks for the students must represent a challenge for them.
5. Conclusions
Although, in this case, the research has been carried out with mathematics teachers, we consider that the results can be extrapolated to teachers of other sciences since the theme of online teaching is a global problem. On the one hand, physics teachers as mathematics teachers require support to incorporate technology into teaching. On the other hand, the learning of mathematics and physics are strongly related, mathematical objects present in physics share meaning with the problems that arise in this area. Therefore, students must understand the meaning of mathematical objects to be successful in learning physics and other sciences. In addition, it has been reported that physics teachers do not use technology in their classes, so it is to be expected they have the same difficulties as mathematics teachers. We conclude by calling attention to the appropriate use of technologies in order to encourage students in doing, avoiding placing them as passive receivers of information.

References
[1] Hoyles C, Lagrange J B 2010 Mathematics Education and Technology: Rethinking the Terrain (Berlin: Springer)
[2] Santos-Trigo M 2019 Mathematical Problem Solving ed Liljedahl P, Santos-Trigo M (Cham: Springer)
[3] Ramma Y, Bholoa A, Watts M, Sylvain P 2018 Education Inquiry 9 210
[4] Borba M, Linares S 2012 ZDM Math. Educ 44 697
[5] Akkaya R 2016 Eurasia J Math Sci Tech Educ 12 861
[6] Rezat S, Sträßer R 2012 ZDM Math. Educ 44 641
[7] Koehler M, Mishra P 2009 Contemporary Issues in Tech and Teacher Educ 9 60
[8] Muhtadi D, Wahyudin, Kartasasmita B G, Prahmana R C I 2017 Journal of Physics: Conference Series 943 012020:1
[9] Ruthven K, Hennessy S 2002 Educ Stud in Math 49 47
[10] Pierce R, Ball L 2009 Educ Stud in Math 71 299
[11] Ertmer P, Ottenbreit-Leftwich A 2010 J. of Research Tech Educ 42 255
[12] Olive J, Makar K, Hoyos V, Kor L, Kosheleva O, Sträßer R 2010 Mathematics Education and Technology-Rethinking the Terrain ed C Hoyles C, Lagrange J (New York: Springer)
[13] Rabardel P 2002 People and Technology: A Cognitive Approach to Contemporary Instruments Hal (Paris: Universite Paris 8)
[14] Trouche L 2004 Int. J. Comp Math. Learning 9 281
[15] Drijvers P, Doorman M, Boon P, Reed H, Gravemeijer K 2010 Educ Stud in Math 75 213
[16] Thurm D, Barzel B 2020 ZDM Mathematics Education https://doi.org/10.1007/s11858-020-01158-6
[17] Louwrens N, Hartnett M 2015 J. of Open, Flexible, and Distance Learning 19 27
[18] Santander P 2011 Cinta de Moebio 41 207
[19] Niederhauser D S 2013 The Nature of Tech. ed Clough M P, Olson M K, Niederhauser D S (Rotterdam: SensePublishers)
[20] Catalán L 2012 Revista Científica Electrónica de Educación y Comunicación en la Sociedad del Conocimiento 12 284