Decreasing critical temperature of gas BEC in spatially periodic potential and relevance to experiments treated by Mott-Hubbard model.

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It is shown that the critical temperature of gas Bose-Einstein condensation decreases in deepening periodic potential, in contrast to common regularity in a separate potential well. The physical explanation of this phenomenon is given. Characteristic scale of potential energies decaying the critical temperature is the quantum recoil energy of periodic potential.

The theory represents an alternative and direct approach to the experimental results (C. Orzel et al Science 291, 2386 (2001); M. Greiner et al, Nature 415, 39 (2002)) obtained with BEC in optical lattices and treated as the phase squeezing or Mott transition processes.

I. INTRODUCTION

One of the main characteristics of a BEC is the critical temperature of condensation [1]. As is well known, for free ideal gas (in thermodynamic limit) it can be simply determined and is presented by formula

\[ T_{co} = \frac{2\pi \hbar^2}{mk_B} \left[ \zeta \left( \frac{3}{2} \right) \right]^{-2/3} \left( \frac{N}{V} \right)^{2/3} \]  

where \( m \) is particle mass, \( k_B \) is the Boltzmann constant, \( \zeta (x) \) is the Riemann function, \( N \) is the number of gas particles in volume \( V \). This formula has clear interpretation that the condensation, as a macroscopic quantum phenomenon, starts at the temperature when the de Broglie wavelength \( \lambda_c = \sqrt{(2\pi \hbar^2/mk_BT_{co})} \) of particle’s thermal motion becomes same order quantity with the distance among the neighboring particles \( (V/N)^{1/3} \); it ensures the existence of perceptible overlapping of neighboring particle wavefunctions (wave-packets), thus making possible the manifestation of boson-inductive nature of particles.

In real situations, when one however has neither ideal nor free gas, aroused the natural question how much and in what direction changes the critical temperature compared with the one given by Eq.(I), conditioned respectively by interparticle interactions and by existence of trapping external fields (potentials). Intercparticle interactions, having crucial importance for dynamic properties of condensate, such as stability, excitations and so on, have, fortunately, feeble influence on critical temperature of gas condensation [1], [2] (note, that just this feature allows us to use the ideal gas model for critical temperature calculations in the following). The influence of the existence of trapping potential has been studied rather frequently and the result about the critical temperature can be qualitatively understood by means of Eq.(I) or reasonings connected with its physics. Actually, the existence (deepening) of trapping potential directly increases the mean density of gas relative to the free state and thereby increases the critical temperature too. Here we want to stress, however, that this reason and consequent growth of critical temperature is relevant to the single-well potential and can not be directly implemented for multi-well potentials, in particular for optical lattices [3], which attract a great deal of attention thanks to fascinating eventual applications such as matter-wave transport [4] and diffraction [5], quantum logic [6], etc. The reason, that in multi-well cases we should exercise some caution is that while in single-well case the deepening of potential leads only to drawing nearer all the atoms in the trapping potential, in multi-well potentials the reverse tendency exists too. With the deepening of potential, all the particles that have preliminarily been in barrier-type regions, fill the neighbor wells, and thereby increasing the atomic concentration in well-type regions, decrease it in barrier-type regions. In optical lattices, in addition, the mean atomic density is conserved.

The existence of two opposite tendencies for the gas density, the thickening in well-type regions and respective rarefying in intermediate barrier-type regions (see Fig.1), demands a separate considerations of the question [7], even more so it may be expected that the critical temperature of condensation in periodic in space potential must be a decreasing one depending on the depth of potential, that is it must be strictly opposite to the behavior shown in case of single-well potential. To understand the footing of this assertion one should note that the number of atoms in barrier-type region in general is less than in well-type region, and besides, the redistribution of atoms between barrier-type and well-type regions can be represented as substraction of some (definite) number of atoms from a barrier-type region and their addition into a neighbor (left-hand side or right-hand side, it doesn’t matter) well-type
region. But subtraction of a definite quantity from a smaller number is more essential for itself than the addition of the same quantity to the larger (or equal) number! That is, the decreasing of the wave-function overlapping in barrier-type regions must be more essential for the total gas condensation, than the overlapping increasing for the well-type regions, and as a consequence it would lead to mentioned decreasing of the critical temperature.

In this paper we will afford a quantitative justification of the mentioned assertion about the decaying behavior of condensation critical temperature (and as a consequence, of the number of condensed particles) for both deepening and/or expanding periodic in space potentials. The form of periodic potential, which will be utilized to this end, will be the biparabolic form [8]. On top of that we shall present a simple and convincing physical explanation, based on regularities of zonal structure of energy spectrum, why the depth-dependence of critical temperature in standing wave trapping potential must be just the contrary to the case of single-well trapping potential.

We are coming to the problem as in ordinary statistical mechanics, that is, via the familiar relation between the number of particles (atoms), temperature and chemical potential [1]. The potential is assumed varying only in one direction, meaning that the atomic gas is free in other two directions. For the energy of interacting degree of freedom the repeated-zone scheme is used, where the quasimomentum plays the same role as the ordinary momentum plays for free degrees of freedom. A direct consequence of obtained for critical temperature regularity, the eventual diminishing of the number of condensed atoms in the deepening potential, also is elucidated: In addition we calculate the number of atoms in separate energy zones as a function of potential depth in order to enlarge the comparison with the single-potential case. Finally we work out the dependence of critical temperature on space-period of potential and find it to be decreasing too.

The paper is organized as follows: in Sec.II we shortly represent periodic in one dimension, so-called biparabolic potential and Bloch-state solutions for a particle there. In Sec.III and IV we outline the theoretical approach, illustrate the results of numerical calculations about the critical temperature of condensation, number of condensed particles and population of energy zones. Section V contains some comments and conclusions.

II. BIPARABOLIC POTENTIAL AND DISPERSION RELATION FOR A PARTICLE THERE

In order to gain an overall picture, let us start and carry the exposition in terms of interactions potential, not concretizing, in general, the mechanisms of interaction. However recalling that at the present far off-resonant standing wave laser radiation is used to trap condensed dilute atomic gases, we will also write down the expressions of potential depth and period for this special case. All the numerical simulations will be performed, having in view just this potential.

Relevant to problem Schrödinger’s stationary equation in dimensionless notations is

$$\frac{d^2\Psi(Z)}{dZ^2} + [W - U(Z)]\Psi(Z) = 0,$$

(2)

where $Z, W$ and $U(Z)$ respectively are dimensionless coordinate along the periodic potential, total energy relevant to this degree of freedom, and potential energy of the particle (atom) there. They are related with the physical coordinate $z$, energy $E$, and potential energy $V(z)$ by means of the following formulas:

$$Z = 2\pi z/l, \quad W = E/E_r, \quad U(Z) = V(z)/E_r,$$

(3)

where $l$ is the space period of potential, $E_r=(2\pi\hbar)^2/2ml^2$ is the quantum recoil energy of periodic potential, $m$ is the particle (atomic) mass.

As a model of periodic field we choose the biparabolic form [8],

$$U(Z) = \frac{1 - (-1)^m}{2}U + (-1)^m2k(Z - m\pi)^2,$$

(4)

with $(m - 1/2)\pi \leq Z \leq (m + 1/2)\pi$ for any integer $m = 0, \pm 1, \pm 2, \ldots$. Here $U$ is the depth of potential, $k = 2U/\pi^2$ is a coefficient assuring the continuity of potential, consisting of series of truncated and turned parabolas, as is shown in Fig.2 by a solid line. The potential energy is counted from the bottom of the periodic wells.

Note that this potential can also be regarded as a good approximation to a potential created for a two-level atom by a standing wave of laser radiation, far off-resonant with optical transition. This relevant standing wave potential is depicted in Fig.2 by a dashed-line, and then one has

$$2\pi/l = 2k, \quad V(Z) = 4|dE_L|^2/\hbar(\omega - \omega_0),$$

(5)
where \( k = \omega/c \) is the laser wavenumber, \( d \) is the dipole matrix element of optical transition of frequency \( \omega_0 \), \( E_L \) is the amplitude of laser electric field for both traveling waves, composing the standing wave.

Bloch-solutions of Eq. (2) with potential (4) results in a following dispersion relation:

\[
\cos[2\pi P] = 1 + 2G_{11}(W)G_{22}(W) = 1 - 2G_{12}(W)G_{21}(W),
\]

where \( P \) is the particle’s quasimomentum in units \( 2\pi \hbar/l \) (\( = 2\hbar k \) in laser standing wave case), and the normalized energy \( W \) is presented in the second equation of (3). Present in (6) functions \( G_{ij}(W) \) \((i, j = 1, 2)\) are expressed via the linearly independent solutions \( u_1(Z) \) and \( u_2(Z) \) in the well-type region, \( \tilde{u}_1(Z) \) and \( \tilde{u}_2(Z) \) in the neighbor barrier-type region, and their first derivatives at the connective point \( Z = \pi/2 \) as

\[
G_{ij}(W) = \left[ u_i(Z) \frac{d\tilde{u}_j(Z)}{dZ} - \tilde{u}_i(Z) \frac{d\tilde{u}_j(Z)}{dZ} \right]_{Z=\pi/2}, \quad i, j = 1, 2,
\]

\[
u_1(Z) = \exp \left[ -\frac{\sqrt{Z^2}}{2} \right] \Phi(\alpha, 1/2; \sqrt{\kappa}Z^2),
\]

\[
u_2(Z) = Z \exp \left[ -\frac{\sqrt{Z^2}}{2} \right] \Phi(\alpha + 1/2, 3/2; \sqrt{\kappa}Z^2),
\]

\[
\alpha = \frac{1}{4} \left[ 1 - \frac{W}{\sqrt{\kappa}} \right]
\]

and \( \tilde{u}_{1,2}(Z) \) are obtained from \( u_{1,2}(Z) \) respectively by means of substitutions \( Z^2 \rightarrow -i(Z - \pi)^2, \quad \alpha \rightarrow \beta = \frac{1}{4} \left[ 1 - \frac{W}{\sqrt{\kappa}} \right] \). Here \( \Phi(\ldots, \ldots; \ldots) \) is the confluent hypergeometric function. Let us note that the functions \( \tilde{u}_{1,2}(Z) \) also are real functions of coordinate \( Z \) and physical parameters, although they contain imaginary argument \( -i(Z - \pi)^2 \) and complex parameter \( \beta \) [8].

III. CRITICAL TEMPERATURE AND NUMBER OF CONDENSED ATOMS AS A FUNCTION OF DEPTH OF PERIODIC IN SPACE POTENTIAL

Our analysis of ideal gas critical temperature and number of condensed particles (atoms) will be worked out by the familiar relation between the mean number of particles, temperature and chemical potential [1], turning from the summation over energy spectrum to integration over the momenta. For the energies corresponding to along the potential’s periodicity degree of freedom, it is convenient use the repeated-zone scheme of energies where each value of energy specifies a single value of quasimomentum (see Fig. 3). Quasimomentum naturally plays the role of ordinary momentum. In this representation the mentioned relation can be written in form

\[
2V \sum_{i=1}^{\infty} dP \int_{-\infty}^{\infty} \frac{dP_{\perp}}{k_B T} \exp \left[ \frac{W + W - \mu}{k_B T} \right] - 1 = N.
\]

\( V \) and \( N \) are the gas volume and the mean number of particles. The momentum \( dP_{\perp} \) (in the free-motion \( X0Y \) plane) is normalized, as the quasimomentum \( P \), in \( 2\pi \hbar/l \) units (\( = 2\hbar k \) in laser standing wave case), and the energetic quantities \( W_{\perp}, \mu \) and \( k_B T \) are normalized in units \( E_r = (2\hbar k)^2/2m \) in laser standing wave case) as the energy \( W \).

Note that in normalized form \( W_{\perp} = P_{\perp}^2. \) Integration over the \( P \)-momenta is divided into integration over consequent zones \( i = 1, 2, \ldots \). And finally, integration over the negative values is changed into positive-values too, due to which the coefficient 2 has appeared in left-hand side of (11). It is to be noted here that the state density is zero at \( P = 0 \) (as for a free gas) and the twofold repeating of the point \( P = 0 \) in (11) does not matter.

Performing the integration over \( P_{\perp} \) (free degrees of freedom), instead of (11) we arrive at
\[-2^{1/2} \left( \frac{\pi}{d} \right)^3 V k_B T \sum_{i=1}^{\infty} \int_{i-1/2}^{i+1/2} dP \ln[1 - \exp[\frac{\mu - W(P)}{k_B T}]] = N. \]  

(12)

Because of the zero state-density at \( P = 0, \beta \rightarrow 0 \) (ground state), the critical temperature \( T_c \) and the number of condensed particles \( N \) are afforded to be determined in full analogy with the free ideal gas, that is to consider (12) as a determination of \( \mu \) for a given \( N \), if \( T \geq T_c \), but for lower temperatures \( T \leq T_c \) to fix the value of \( \mu(T \leq T_c) = \mu(T = T_c) \), and consider (12) as a relation for determination of the number of noncondensed particles \( N_{nc} \). For the last case, when the gas is cooled lower than the temperature of condensation, the number of condensed atoms is [9]

\[ N_c = N - N_{nc}. \]  

(13)

Equations (12), (6) and (13), the second of which determines a one-valued implicit function \( W(P) \) in repeated-zone picture, are the basis of the reminder of this paper.

Let us now present the scheme of calculations, which will be used here. To determine \( T_c \) we first insert the values of interest of \( N \), \( V \) and \( l \) into (12) and consider it as a relation, which determines the chemical potential \( \mu \) as a function of temperature \( T \) for a given dispersion relation \( W(P) \). Since the seeking functional dependence \( \mu(T) \) can be obtained only by means of numerical integration int(12) and \( \mu \) sits in integrand, we preliminarily cast the \( T(\mu) \) dependence and hereupon turn to \( \mu(T) \) function numerically, using the one-valued behavior dependences. In its turn to obtain the \( T(\mu) \) dependence we give values to \( \mu \) and performing the numerical integration in (12) find the corresponding values of \( T \).

The stage of integration demands of course to fix the depth of the potential too, to have the \( W(P) \)--dependence. The latter we get by reversing the dispersion relation (6) numerically, where the repeated-zone picture is very convenient.

Construction of the \( T(\mu) \) dependence in practice starts from some value of \( \mu \), greater than the numerical value of the particle energy \( W_{min} \) for the chosen potential, and gradually move to smaller values of \( \mu \). The possible minimal (having solution of (12)) value of \( \mu \) is \( \mu = W_{min} \), and corresponding to it value of temperature, as solution of (12), just specifies the critical temperature \( T_c \). Some graphs of the \( \mu(T) \) function for five values of potential depth \( U \), constructed in mentioned above way, are depicted in Fig.4. In accordance with the described procedure of calculations, the value of critical temperature for each graph-conditions is determined by the corresponding left-hand side border point of graph.

Now let us concentrate attention on the fact that the graphs from right to left correspond to lifting values of potential depth, which immediately shows the anticipated in Introduction behavior: the critical temperature of ideal gas condensation decreases as the potential depth increases.

It should be noted specially, that the determination of the border points of graphs (and corresponding values of \( T_c \)) demands a great caution of numerical integration procedure, since the integrand at \( \mu < W_{min} \) (near-border values) rapidly increases near the lower edge of the first energy zone, that is near \( P = 0 \), where \( W(P = 0) = W_{min}, \mu - W(P = 0) \rightarrow 0, 1 - \exp[\mu / k_B T] \ll 1 \), and respectively \( - \ln[1 - \exp[\mu / k_B T]] \gg 1 \). Fortunately, the theoretical value of the border (maximal) value of \( \mu \) is determined by exact condition \( \mu_{max} = W_{min} \) which can be fulfilled as precisely as it is possible within the range of computer capability. The minimal accuracy for the (dimensionless) difference has been chosen \( 10^{-9} \). Besides, the procedure of integration in (12) was replaced by corresponding summation, dividing the first zone quasimomentum range into \( 10^3 \) equal parts, and the first of them, which is about the \( P = 0 \), already into \( 10^4 \) equal parts. As a result, the time consumption of our PC for each point on the presented graphs was about 30-40 minutes.

In Fig.5 we explicitly present the dependance of gas critical temperature on the depth of periodic potential. It has a monotonically decreasing nature everywhere, rather minor in small-depth range and rapid in intermediate-depth range of the potential. In the range of greater depths the rate of decreasing, naturally, slows down.

The number of condensed atoms, as it directly follows from the mentioned behavior of critical temperature, decreases with the growth of potential depth too. In addition, since the critical temperature monotonically tends to zero as a function of \( U \), for arbitrary chosen low temperature \( T \) there is always some potential depth, for which the critical temperature becomes equal to \( T \), and starting from which no more condensation takes place. This behavior of the number of condensed atoms is shown in Fig.6. The point where the graph touches the depth axis, just corresponds to the peculiar value of potential depth, for which the critical temperature becomes equal to the chosen temperature of the gas (0.1\( T_0 \) in this case). For dipper potentials \( (U > 13) \) the critical temperature is lower then the gas temperature and because of it no condensation takes place.

Before proceeding to other results obtained, let us turn to other language of presentation, explaining why the number of condensed particles decreases with the increasing depths of the periodic potential, which is the equivalent of critical temperature decreasing. This representation is based on the behavior of energy spectrum for increasing depths of potential and gives, perhaps, more detailed and transparent explanation, than the previous one, based on behavior of particle density. It easily explains the studying regularity also in single-potential case. Indeed, in a
single-potential the particle has discrete set of energy levels \( E_n \) \((n = 0, 1, \ldots)\) of bounded states (only they are relevant to the problem under hand). The probability of population for each excited level, relative to population of the ground energy level, is determined by the Boltzmann factor \( \exp\left[-E_n/k_BT\right] \) in thermally equilibrium state. The energy \( E_n \) of each excited energy level can be also interpreted as the distance from ground energy level, choosing without loss of generality the energy of the latter zero. The deepening of a single potential, as is well known increases distances between energy levels and therefore, between each of them and ground energy level. As a result, the population of all excited energy levels decreases. In addition taking into account that the total probability of level occupation (population) remains the same unity, we immediately arrive to the result, that the occupation probability of the ground energy level must increase due to this deepening of the potential. But the condensed gas is just the particles in ground energy level and therefore, the deepening of a single-potential results in increasing of condensed particles, or which is equivalent, in increasing of condensation critical temperature.

In periodic-potential case the situation is, nevertheless, different. The spectrum of a particle has a zonal structure. And while the deepening of potential removes from each other the energy bands, it simultaneously narrows all these bands, in particular the first one, which plays the dominant role in phenomenon of condensation. In order to easily understand the resultant seeking behavior in this situation, it is convenient to present the first energy band as a collection of discrete energy levels \((\text{in case of finite}, \text{even arbitrary large}, \text{number of repeated potential wells this approach is absolutely correct})\). To the phenomenon of condensation is relevant, by definition, only the ground state. Repeating the above discussion about the behavior of level occupation probabilities for deepening potentials we see, that the occupation probabilities increase in all excited levels in the first energy band and decrease in all upper energy bands. But since the condensation appears at low temperatures, when the low-energy levels are mainly populated, the increase of population in excited first-bound levels plays more important role, than the decrease of population in upper energy levels. Therefore the increase of potential depth, at least for low temperatures, must result in increment of population in the total excited states, including all the excited levels of the first energy band and all excited energy bands. This implies that one has decrement of population in ground energy level, relevant to the state of condensation. So, regularity of the ground state population for a series of deepening periodic potentials is opposite to the case of a single potential: the number of condensed particles is decreasing and the critical temperature of condensation is respectively decreasing. It is not needless to note, that the presented discussion, based on the behavior of energy spectrum and relevant Boltzmann function can help us understand qualitatively the behavior of number of condensed atoms, or equivalently, the critical temperature in other potential forms too, particularly in double well potential.

In order to give somewhat auxiliary elucidation to the problem under hand, we have calculated the populations of each energy band as a function of depth of periodic potential for some, over-critical, temperature, when not only the first energy band is effectively populated. The results are presented in Fig.7(point lines). As is seen, the population of the first energy band increases monotonically as the depth of periodic potential increases, while the populations of the all excited energy bands, vice versa, decrease. It also gives an opportunity to see, that these regularities for energy band populations in periodic-wells potential precisely coincide with the regularities of the level-populations for single-well potential. This implies, in particular, that the growing behavior of the critical temperature of condensation would have been obtained in periodic potential case, if one had taken as a ground level the first energy zone totally, which is however incorrect by definition, even for very deep potentials.

In envisioned circumstances some interest may also present the total number of particles, trapped by the periodic potential. That are the particles, energies of which are less than the depth of potential. Because of a zonal structure of energy spectrum, in general we should expect a stepwise dependence of this number on the potential depth. The results are illustrated in Fig.8. At the origin \((\text{point } a_1)\) with \( U = 0 \), of course there are no trapped particles. The piece \(a_1b_1\) corresponds to situation when the rising potential captures more and more of the first, narrowing at that, energy band, naturally increasing the number of trapped particles. Since the energy bands are distanced from each other by energy gaps, the pieces corresponding to the second, third, etc. bands are \(a_2b_2, a_3b_3, etc\) (the upper range is not presented), distanted from each other by intermediate pieces \(b_1a_2, b_2a_3, etc\). All the regularities of the graph, such as the rate of increasing, saturation, etc., are in agreement with the presented picture, viewing the distribution of quantum states within the energy bands and their thermal population as a function of potential depth.

**IV. CRITICAL TEMPERATURE AS A FUNCTION OF PERIOD OF PERIODIC POTENTIAL**

Another side of interest presents the dependence of critical temperature of condensation on the period of periodic in space potential. In case of laser-induced standing wave potentials the variation of the period can be obtained varying the angle between two waves, composing the standing wave. Denoting by \( \theta \) the half of supplementary angle between these two wave vectors (see Fig.9), we get \( k_{SW} = 2k\cos \theta \) and respectively \( l = \pi/k \cos \theta \). \( \theta = 0 \) corresponds
to counterpropagating case, where the period is minimal. We have found convenient to present the $l$-dependence of critical temperature as a $\theta$-dependence. The scheme of calculations is the same with the one described in previous section. Numerical calculations had been worked out for two $((a)U = 1$ and $(b)U = 6$) depths of potential and the results are plotted in Fig.10. The critical temperature, as is seen decreases as the period of potential increases, monotonically approaching zero.

We have not been able to understand this result on the base of picture about the overlapping of particles' material waves (see Introduction), but it is easy to do, using the picture of energy spectrum and thermal population of energy levels there. Really, the spatial expansion of periodic potential results in narrowing of all energy bands and simultaneously in widening of the energy gaps. But it is the same regularity as it was for the deepening potential, and therefore the resultant functional dependance also is the same: the critical temperature decreases as the spatial period increases.

Of course, an analogous regularity exists also for the number of condensed atoms, but we do not see the necessity of presenting this concrete form.

### V. CONCLUDING REMARKS

So, the critical temperature of condensation and respectivelly the number of condensed particles in a trapping periodic potential must be less than it is in free state.

An important question is, of course, how much "dangerous" are these results for experimental situations, typical for condensates trapped in optical standing wave potentials. The critical value of thermal energy of free gas condensation in typical range of densities $10^{11} - 10^{12}$ cm$^{-3}$, as is well known is far subrecoi (is on the order of one percent or less). This implies that such a gas may be trapped, in principle, by means of an optical standing wave potential with the depth of the same order of magnitude. Looking at the Figs.5 and 6 we see that this subrecoi range of potentials lies close to the origin of horizontal axis, where the lowering of graphs (critical temperature or/and number of condensed particles) is unessential. Moreover, the characteristic parameters of condensation in practice stay unchanged up to depths $U/E_r \approx 1$, that is in so called shallow potentials (note that our scale of recoil energy $E_r = (2\hbar k)^2/2m$ is four times greater than the usually using scale $\hbar^2 k^2/2m$). However, in deeper periodic potentials with $U/E_r \approx 5 \div 12$(in usual scale $\approx 20 \div 50$), which are exploited for strong localization of super-cooled condensed gases within separate wells, the critical temperature of condensation falls about one order and the gas can get out from the condensation state.

The comparison with the real experimental situation involves another important parameter too: it is the time, especially in cases when the gas is filled into the standing wave potential already being in condensed state (obtained in magneto-optical traps). The matter is that the above presented theory pertains to thermally equilibrium gas in the given external potential field. The gas, finding itself in the periodic field, preliminarily is not in that state, but it would tend to that state in the course of time. The dynamics of approaching to the thermally equilibrium state is, in general, complicated and requires a separate and accurate consideration, but on qualitative level it may be expected, that two processes would determine the transfer of gas into statistical-mechanical equilibrium state in the case under consideration. The first one is the process of interparticle collisions, leading to thermal equilibrium, and the second one is the tunneling of particle through the potential barrier between neighboring potential wells, providing the establishment of zonal structure for particle energy spectrum. Denoting their characteristic times by $\tau_{col}$ and $\tau_{tun}$ respectively, it will be expected the ultimate thermally equilibrium distribution (with lower critical temperature and less number of condensed particles) should be attained in times $t \geq \tau_{col}$, $\tau_{tun}$.

Let’s note that from the viewpoint of our results the distortion of gas BEC-state in a deepening periodic potential and its consequent revival in a shallowing one, observed experimentally in [10,?] as a loss and revival of phase, may be accounted simply as a respective transfer to noncondensed state and consequent return into the condensed one, as is predicted by the above presented theory for a rather slowly deepening and shallowing periodic potential. All the more, that the totally distorting potential depths $U/E_r^2 = 22$ (≈ 5.5 in our $E_r$-scale) in [10] and 44 (≈ 11 in our $E_r$-scale) in [11], lie in the region of our curve of Fig5, where the condensed state must be depressed in practice. Plus, that in three-dimensional standing wave case [10] the analogous to Fig5 curve must decay more rapidly than our, calculated for one-dimensional standing wave.

The detailed elucidation of results of these experiments in view of presented here theory will be accomplished and presented elsewhere.
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FIG. 1. Scheme of gas spacial distribution for (a) a free state, (b) intermediate depth and (c) great depth of periodic potential. The mean concentration of gas stays unchanged with the potential depth evolution.

FIG. 2. Biparabolic form of potential (solid line), used in our calculations. For comparison is also presented the respective sinusoid (dashed line) of laser standing wave potential.

FIG. 3. Repeated -zone scheme of single particle energy spectrum, which has been used in our calculations. The picture is for the case $U = 1$ (in recoil energy units).

FIG. 4. The temperature dependence of chemical potential $\mu(N/V, T)$ for different depths of potential (a) $U = 0$, (b) 0.5, (c) 2.5, (d) 7, and (e) 12) and for $N/V = 10^{11}$ cm$^{-3}$. Note that the values of $\mu$ (vertical axis) are presented relative to minimal energy $E_{\text{min}}$ of particle in periodic potential. Both axis are normalized by recoil energy $E_r$.

FIG. 5. The behavior of critical temperature $T_c$ of ideal gas condensation as a function of depth $U$ in external periodic potential. The vertical axis $T_c$ is normalized by critical temperature $T_{c0}$ of free gas (with $N/V = 10^{11}$ cm$^{-3}$), horizontal axis $U$ by recoil energy $E_r$, as it is done along all the paper. The discovery of this behavior is the main result of the presented study.

FIG. 6. The periodic potential depth dependence of the number $N_c$ of condensed particles (atoms). The gas temperature is $T = 0.1T_{c0}$, that is ten times below the free gas critical temperature $T_{c0}$ of condensation (in case under consideration $k_B T_{c0} = 0.0021637774 E_r$, or equivalently $T_{c0} = 1.03819 \times 10^{-6} K$ if one is concerned with D2 optical transition in sodium atom with $\lambda = 5896 \AA$). The vertical axis is normalized by the total number $N$ of particles in the gas.

FIG. 7. The periodic potential depth dependence of the numbers $N_{\text{zone}}$ of particles, populating first, second, and third zones respectively. The latter are given in units of total number $N$. The gas temperature is chosen $k_B T = 1.5$ and is far over the critical value of condensation. The points represent the numbers, irrelevant to the fact are the particles trapped by the periodic potential or not, whereas the dashed line represents the number of trapped particles (that is particles with $E < U$). In deep potential regions, when the zone is entirely within the potential, both quantities coincide. As is seen (points) the population of the first zone monotonically increases as potential becomes deeper, while the population of the others decreases, in full analogy with the regularity of energy-level populations in single -well potential case.

FIG. 8. The periodic potential depth dependence of the number $N_{tr}$ of particles, trapped by the potential. The deepening of the potential includes more and more number of zones, thereby increasing the number $N_{tr}$. The stepwise behavior of the dependence is conditioned by the zonal structure of energy spectrum (for details see the text).

FIG. 9. The geometry of two laser waves, creating the standing-wave periodic potential. $\theta = 0$ corresponds to the counter-propagating case. Variation of $\theta$ changes the spatial period, but keeps the depth constant.

FIG. 10. The behavior of critical temperature $T_c$ of ideal gas condensation as a function of angle $\theta$ (see previous Figure). The spatial expansion of periodic potential, as the deepening decreases the value of critical temperature of condensation.
\frac{\pi}{2} 0 2 \frac{\pi}{2} \pi 2 \pi 5 \frac{\pi}{2}

U(Z)/U

well type

barrier type
\[
\frac{N_c}{N} = T = 0.1T_c 0
\]
$N_{\text{zone}} \quad N \quad k_B T = 1.5$

- first zone
- second zone
- third zone
$N_{tr}/N \quad k_B T = 1.5$

$a_1 \quad b_1$

$a_2 \quad b_2$

$a_3 \quad b_3$
Laser standing wave
