Precise Point Positioning Algorithms based on GR models by applying CLAS

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Abstract

In this paper, we describe a method of applying CLAS (Centimeter Level Augmentation Service) correction data distributed from QZSS (Quasi-zenith Satellite System) to the algorithm based on our GR (GNSS Regression) models for double frequency PPP of GNSS (Global Navigation Satellite System). Finally, we show the positioning results by applying our algorithms for using actual measurement data.

1 Introduction

We have already presented single frequency GNSS high accuracy Precise Point Positioning (PPP) algorithms based on GNSS regression models (GR models) [1]. [2] by applying CLAS (Centimeter Level Augmentation Service) [3], [4] data [5]-[7].

In this paper, we present double frequencies (L1 and L2) PPP algorithms based on GR model with using CLAS data and show the positioning results. Throughout the numerical experiments by using the double frequencies GPS measurement data for the static position, we will show the very accurate positioning results in the Section 5, namely, 3D-RMS (root mean square) errors after starting time and after 10 minutes from the start are approximately 11 [cm] and 6 [cm], respectively.

GPS in the United States is well known as Global Navigation Satellite System (GNSS), but several national organizations are constructing similar systems. Japan launched its first quasi-zenith satellite system (QZSS; Michibiki) in 2010, and started its service in Nov. 2018 with 4 satellite systems [8].

QZSS has both complementary and augmentation services. The complementary service enhances availability by using the both US GPS and Japanese QZSS combination. The augmentation service enhances accuracy and integrity. One of the augmentation service is so-called as CLAS, which utilizes L6 signal of Quasi-Zenith Satellites (QZS) to broadcast error information (corrections) in satellite orbits and clocks derived from the navigation data (Ephemeris data), and information on ionospheric and tropospheric delays at virtual points arranged in the grid (the points with approximately 60 km intervals) , so that the receiver can use this information to perform positioning with centimeter-level accuracy.

2 Model Equations for Precise Point Positioning

2.1 Measurement Models

Similarly to [1]-[2], we describe all observed positionings by applying CLAS (Centimeter Level Augmentation Service) Algorithms based on GR models for double frequency PPP of GNSS data distributed from QZSS (Quasi-zenith Satellite System) and the pseudoranges \( \rho_{CA,u}(t) \) based on C/A and P(Y) codes, respectively, as follows:

\[
\rho_{CA,u}(t) = r_u^p(t, t - \tau_u^p) + c[\delta t_u(t) - \delta t^p(t - \tau_u^p)] + \delta t^p_u(t) + \delta T^p_u(t) + \delta \phi^p_{CA} + d_{ur,rel}^p + \Delta \phi^p_{CA,u}(t),
\]

\[
\rho_{PY,u}(t) = r_u^p(t, t - \tau_u^p) + c[\delta t_u(t) - \delta t^p(t - \tau_u^p)] + \frac{f_1^2}{f_2^2} \delta t^p_u(t) + \delta T^p_u(t) + \delta \phi^p_{PY} + d_{ur,rel}^p + \Delta \phi^p_{L2,u} + \epsilon^p_{PY,u}(t),
\]

\[
\Phi^p_{L1,u}(t) = \lambda_1 \phi^p_{L1,u}(t) = r_u^p(t, t - \tau_u^p) + c[\delta t_u(t) - \delta t^p(t - \tau_u^p)] - \delta t^p_u(t) + \delta T^p_u(t) + \delta \phi^p_{L1} + d_{ur,rel}^p + \Delta \phi^p_{L1,u} + \lambda_1 (N_{L1,u} + \Delta \phi^p_{L1,u}) + \lambda_1 \epsilon^p_{L1,u}(t),
\]

\[
\Phi^p_{L2,u}(t) = \lambda_2 \phi^p_{L2,u}(t) = r_u^p(t, t - \tau_u^p) + c[\delta t_u(t) - \delta t^p(t - \tau_u^p)] - \frac{f_1^2}{f_2^2} \delta t^p_u(t) + \delta T^p_u(t) + \delta \phi^p_{L2} + d_{ur,rel}^p + \Delta \phi^p_{L2,u} + \lambda_2 (N_{L2,u} + \Delta \phi^p_{L2,u}) + \lambda_2 \epsilon^p_{L2,u}(t).
\]

In (1)-(4), \( r_u^p(t, t - \tau_u^p) \) is the geometric distance between the user’s receiver u at time t and the satellite p \((p = 1, \cdots, n_s; n_s\) is the number of visible satellites).
at time $t - \tau_p^p$, where $\tau_p^p$ is the transmission time from satellite $p$ to receiver $u$. Namely,

$$r_u^p(t, t - \tau_p^p) = \|u(t) - sp(t - \tau_p^p)\|$$

$$\equiv \left( (x_u(t) - xp(t - \tau_p^p))^2 + (y_u(t) - yp(t - \tau_p^p))^2 \right)^{1/2},$$

(5)

where receiver position $u \equiv [x_u, y_u, z_u]^T$, and satellite position $sp \equiv [xp, yp, zp]^T$. $c$ denotes the speed of light in vacuum. $\delta t_u$ is the receiver $u$ clock error, and $\delta t_p$ is the satellite $p$ clock error. $f_1(= 1575.42[\text{MHz}])$ and $f_2(= 1227.40[\text{MHz}])$ are the carrier wave frequencies of L1 and L2, respectively. $\lambda_1$ and $\lambda_2$ are corresponding wave lengths. $\delta T_u^p$ and $\delta T_p^u$ reflect the delays associated with the transmission of the signal through the ionosphere and the troposphere, respectively.

$\delta T_{CA}^p, \delta T_{PY}^p, \delta T_{L1}^p$ and $\delta T_{L2}^p$ are code biases and phase biases of the satellite $p$, respectively. $\delta t_{rel,u}^p$ is the relativity effect to the transmission signal. $\Delta p_{L1,u}^p$ and $\Delta p_{L2,u}^p$ are phase windups. $N_{L1,u}^p$ and $N_{L2,u}^p$ denote the ambiguity between the satellite $p$ and the receiver $u$. $\epsilon$ and $\delta$ are used to denote measurement errors.

### 2.2 Approximated Measurement Models

In (1)-(4), unknown parameters are the receiver 3-dimensional coordinates $(x_u, y_u, z_u)$, receiver clock error $\delta t_u(t)$, satellite clock error $\delta t_p(t - \tau_p^p)$, ionospheric refraction effect $\delta T_p(t)$, tropospheric refraction effect $\delta T_u(t)$, and integer bias $N_{L1,u}^p$. These unknown parameters are applied to GNSS linear regression (GR) model which is solved by a linear regression equation.

Then, we take the 1st order Taylor series approximation of (5) around the previous estimated value $u = \hat{u}$ and $sp = \hat{sp}$ is given as follows:

$$r_u^p \cong \hat{r}_u^p + g_u^p(u - sp - (\hat{u} - \hat{sp}))$$

$$\equiv \|\hat{u} - \hat{sp}\| + \frac{(\hat{u} - \hat{sp})^T}{\|\hat{u} - \hat{sp}\|}[u - sp - (\hat{u} - \hat{sp})]$$

$$= g_u^p(u - sp),$$

(6)

where

$$g_u^p \equiv \left[ \frac{\partial r_u^p}{\partial u} \right]_{u=\hat{u},sp=\hat{sp}} \equiv \frac{(\hat{u} - \hat{sp})^T}{\|\hat{u} - \hat{sp}\|}.$$ 

Substituting (6) into (1) - (4), we have the following approximated measurement models:

$$r_{CA,u}^p \cong g_u^p(u - sp) + c(\delta t_u - \delta t_p) + \delta T_p^u(t) + \delta T_u^p(t) + \delta T_{CA}^p + d_{rel}^p + \Delta p_{L1,u}^p + e_{CA,u}^p(t),$$

(7)

$$\Phi_{L1,u}^p \cong g_u^p(u - sp) + c(\delta t_u - \delta t_p) - \delta T_p^u(t) + \delta T_u^p(t) + \delta I_{L1,u}^p + d_{rel}^p + \Delta p_{L1,u}^p + \lambda_1(N_{L1,u}^p + \Delta \phi_{L1,u}^p) + \lambda_1\epsilon_{L1,u}(t),$$

(9)

$$\Phi_{L2,u}^p \cong g_u^p(u - sp) + c(\delta t_u - \delta t_p) - \frac{f_2^2}{f_1^2} \delta T_p^u(t) + \delta T_u^p(t) + \delta I_{L2,u}^p + d_{rel}^p + \Delta p_{L2,u}^p + \lambda_2(N_{L2,u}^p + \Delta \phi_{L2,u}^p) + \lambda_2\epsilon_{L2,u}(t),$$

(10)

### 3 CLAS and CLASLIB

#### 3.1 Centimeter-Level Augmentation Service

CLAS is a service that estimates satellite orbit and clock errors and local atmospheric delay in State-Space Representation (SSR) [9] using data from multiple Continuously Operating Reference Stations (CORS) in GNSS Earth Observation Network System (GEONET) in Japan, and broadcasts these correction data from QZSS to the whole of Japan using L6 signals. The target satellite systems include GPS, Galileo, and GLONASS (Schedule) as well as QZSS. This method can be called PPP in the sense that it enables positioning without directly acquiring observation data of nearby CORS, but it is called PPP-RTK because observation data of CORS in Japan are used for generating correction data.

#### 3.2 CLAS Test Library and SSR2OSR

CLAS Test Library (CLASLIB) is an open source software developed based on RTKLIB and GSI-LIB as a reference data generation tool for those considering implementation algorithms for CLAS-enabled products. It consists mainly of 2 tools: SSR2OSR, which decodes L6 signals, i.e., converts the correction amount from the SSR to the observation space representation (OSR) at a positioning point, and RNX2RTKP, which further performs positioning calculation by PPP-RTK and outputs the positioning result. The L6 signals used in CLASLIB and these tools can be obtained from the QZSS website [10], [11]. In this paper, however, we use only SSR2OSR and perform positioning by our own PPP algorithms base on GR-models in Section 2, which had been developed in [1], [2].

#### 3.3 Apply to GR measurement models

Applying the correction values obtained by the SSR2OSR to the GR models (7) - (10), we have the linear GR models as follows:
\[ \rho_{CA,u}^p \cong g^p_u(u - s_{brd}^p) + c(\delta t_u - \delta t_{brd}^p) + \delta s_{CL}^p - \delta t_{CL}^p + \delta I_{u,CL}^p + \delta T_{u,CL}^p + \Delta p_{L1,u}^p + \epsilon_{CA,u}^p, \]

\[ \rho_{PY,u}^p \cong g^p_u(u - s_{brd}^p) + c(\delta t_u - \delta t_{brd}^p) + \delta s_{CL}^p - \delta t_{CL}^p + \frac{f_2^p}{f_1^p} \delta I_{u,CL}^p + \delta T_{u,CL}^p + \delta \phi_{PY,CL}^p + \Delta p_{L2,u}^p + \epsilon_{PY,u}^p, \]  

(11)

\[ \Phi_{L1,u}^p \cong g^p_u(u - s_{brd}^p) + c(\delta t_u - \delta t_{brd}^p) + \lambda_1 N_{L1,u}^p + \delta s_{CL}^p - \delta t_{CL}^p - \delta I_{u,CL}^p + \delta T_{u,CL}^p + \delta \phi_{L1,CL}^p + \Delta p_{L1,u}^p + \delta \phi_{L1,u}^p, \]

(12)

\[ \Phi_{L2,u}^p \cong g^p_u(u - s_{brd}^p) + c(\delta t_u - \delta t_{brd}^p) + \lambda_2 N_{L2,u}^p + \delta s_{CL}^p - \delta t_{CL}^p - \delta I_{u,CL}^p + \delta T_{u,CL}^p + \delta \phi_{L2,CL}^p + \Delta p_{L2,u}^p + \delta \phi_{L2,u}^p, \]  

(13)

where the position of the satellite \( p: s_{brd}^p \) and the clock error of the satellite \( p: \delta t_{brd}^p \) are obtained from the navigation data (\( : \text{Ephemeres} \)).

In the case of post-processing, multiple effective ephemerides exist at the same time, and the accuracy cannot be obtained unless the same ephemeris is used in the CLAS correction data generating process (can be confirmed by IODE, and will necessarily be the same ephemeris for real-time processing).

In (11) - (14), the following quantities are provided as the ORS correction values distributed by CLAS (Be output by SSR2OSR from CLAS).

\[ \delta s_{CL}^p \quad : \text{LOS error of } \delta s_{brd}^p \]

\[ \delta t_{CL}^p \quad : \text{range dimension error of } \delta t_{brd}^p \]

\[ \delta I_{u,CL}^p(t) \quad : \text{Ionsospheric delay} \]

\[ \delta T_{u,CL}^p(t) \quad : \text{Tropospheric delay} \]

\[ \delta \phi_{CA,CL}^p \quad : \text{C/A code bias} \]

\[ \delta \phi_{PY,CL}^p \quad : \text{P(Y) code bias} \]

\[ \delta \phi_{L1,CL}^p \quad : \text{L1 carrier phase bias} \]

\[ \delta \phi_{L2,CL}^p \quad : \text{L2 carrier phase bias}. \]

The estimated value of \( \Delta p_{L1,u}^p, \Delta p_{L2,u}^p, \Delta \phi_{L1,u}^p, \) and \( \Delta \phi_{L2,u}^p \) are not the correction values distributed by CLAS, but SSR2OSR outputs the value calculated accurately by the built-in model. Therefore, these values may be used, or the values calculated by the built-in model of the own positioning program may be used.

These estimated values are subtracted from both sides in (11) - (14), then these terms are eliminated. Therefore, the unknown values are the three-dimensional coordinate \( (x_u, y_u, z_u) \), receiver clock error \( \delta t_u(t) \), and integer bias \( N_{L1,u}^p, N_{L2,u}^p \).

Namely, define new observables as follows.

\[ \tilde{\rho}_{CA,u}^p = \rho_{CA,u}^p + g_u^p s_{brd}^p + c \delta t_{brd}^p - \delta s_{CL}^p + \delta t_{CL}^p - \delta I_{u,CL}^p + \delta T_{u,CL}^p - \delta \phi_{CA,CL}^p - \Delta p_{L1,u}^p - \epsilon_{CA,u}^p, \]  

(15)

\[ \tilde{\rho}_{PY,u}^p = \rho_{PY,u}^p + g_u^p s_{brd}^p + c \delta t_{brd}^p - \delta s_{CL}^p + \delta t_{CL}^p + \frac{f_2^p}{f_1^p} \delta I_{u,CL}^p + \delta T_{u,CL}^p - \delta \phi_{PY,CL}^p - \Delta p_{L2,u}^p + \epsilon_{PY,u}^p, \]  

(16)

\[ \tilde{\Phi}_{L1,u}^p = \Phi_{L1,u}^p + g_u^p s_{brd}^p + c \delta t_{brd}^p - \delta s_{CL}^p + \delta t_{CL}^p + \delta I_{u,CL}^p + \delta T_{u,CL}^p - \delta \phi_{L1,CL}^p - \delta \phi_{L1,u}^p - \lambda_1 \Delta \phi_{L1,u}^p, \]  

(17)

\[ \tilde{\Phi}_{L2,u}^p = \Phi_{L2,u}^p + g_u^p s_{brd}^p + c \delta t_{brd}^p - \delta s_{CL}^p + \delta t_{CL}^p + \frac{f_2^p}{f_1^p} \delta I_{u,CL}^p + \delta T_{u,CL}^p - \delta \phi_{L2,CL}^p - \delta \phi_{L2,u}^p - \lambda_2 \Delta \phi_{L2,u}^p, \]  

(18)

Then, the measurement equations in (11) - (14) are expressed by

\[ \tilde{\rho}_{CA,u}^p \cong g_u^p u + c \delta t_u + \epsilon_{CA,u}^p, \]  

(19)

\[ \tilde{\rho}_{PY,u}^p \cong g_u^p u + c \delta t_u + \epsilon_{PY,u}^p, \]  

(20)

\[ \tilde{\Phi}_{L1,u}^p \cong g_u^p u + c \delta t_u + \lambda_1 N_{L1,u}^p + \lambda_1 \epsilon_{L1,u}^p, \]  

(21)

\[ \tilde{\Phi}_{L2,u}^p \cong g_u^p u + c \delta t_u + \lambda_2 N_{L2,u}^p + \lambda_2 \epsilon_{L2,u}^p. \]  

(22)

Namely, we have the following vectors and matrices measurement equations:

\[ y_t = H_t x_t + v_t, \]

(23)

where,

\[ y_t = \begin{bmatrix} \tilde{\rho}_{CA,u}^p \\ \tilde{\rho}_{PY,u}^p \\ \tilde{\Phi}_{L1,u}^p \\ \tilde{\Phi}_{L2,u}^p \end{bmatrix}, \]

(24)

\[ \hat{\rho}_{CA,u}^p = [\hat{\rho}_{CA,u}^1, \cdots, \hat{\rho}_{CA,u}^n]^T, \]

\[ \hat{\rho}_{PY,u}^p = [\hat{\rho}_{PY,u}^1, \cdots, \hat{\rho}_{PY,u}^n]^T, \]

\[ \hat{\Phi}_{L1,u}^p = [\hat{\Phi}_{L1,u}^1, \cdots, \hat{\Phi}_{L1,u}^n]^T, \]

\[ \hat{\Phi}_{L2,u}^p = [\hat{\Phi}_{L2,u}^1, \cdots, \hat{\Phi}_{L2,u}^n]^T. \]
the observation data used are described in Table 1. We have already proposed and examined many cases
of movements of the user in (12) - (15), similarly to
Singer’s models ([16]). Also several state models for the
unknown states \( x \) are depended on the occasions of the user positions.

Therefore we should examine to the proper dynami-
cal models for the states of \( x \) and then we will applying the recursive estimation of
measurement noise vector \( v \), we have following state equation:
\[
x_{t+1} = F_t x_t + w_t, \quad (28)
\]
where
\[
F_t \equiv \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & \alpha \\ 0 & 0 & 0 & I_{2n_s} \end{bmatrix} : \text{state transition matrix}
\]
\( w_t \) : process noise.

For accurate estimation by Kalman filtering, it is nec-
essary to properly set covariance matrixes of the ob-
servation noises, the process noises, and initial values of states.

Table 2 shows the existence and distribution of vari-
cations. The first five items correspond to the
correction amounts distributed from CLAS, and the fol-
lowing three items use the model of SSR2OSR. The last
earth solid tide was calculated and corrected by the
positioning program, because it was considered to be
a non-negligible value although it was not specified in
the above equation. On the other hand, the correction
of ocean tidal load deformation and polar motion tide
were omitted.

Figs. 1 - 4 show the positioning errors in the ENU
coordinate system in time series. Table 3 shows the
RMS value of the error for one hour in each case.

The positioning errors were calculated by (estimated
values – true values) using the precise analysis result
(F3 solution) provided by GSJ(Geospatial Information
Authority of Japan) as “daily coordinate value” [18] as
the reference (true value).

Compared with our previous results in [5] - [7], the accuracy is generally improved by using double frequency
signals.

6 Conclusions

We discussed double-frequencies GNSS PPP meth-
ods by applying CLAS correction data. We also added
some CLAS correction functions shown in Table 2 to
our GNSS PPP programs, and evaluated the position-
ing errors from GEONET points. From Table 3, we
can say that the positioning accuracy of 3D-RMS after
starting time and after 10 minutes from the start are
approximately 11 [cm] and 6 [cm], respectively.

In the future, we will examine whether the accuracy
and speed of our estimators for GNSS PPP.

4 Kalman Filtering for PPP positioning by applying CLAS

Now we have get the measurement equations in (23)
and then we will applying the recursive estimation of
the unknown states \( x_t \) by applying the Kalman filter.
Therefore we should examine to the proper dynamical
models for the states of \( x_t \), especially of \( u_t \), which are
depending on the occasions of the user positions.
We have already proposed and examined many cases
of movements of the user in ([12]) - ([15]), similarly to
Singer’s models ([16]). Also several state models for the
clock errors \( \delta t_u \) are considered in ([17]).

5 Experimental Results

The positioning performances of the method de-
scribed above are carried out; by using the received
data from GEONET (GNSS Earth Observation Net-
work System) which are provided by the Geospatial
Information Authority of Japan (GSJ). The properties of
the observation data used are described in Table 1.

In these experiments, we use the following state equa-
tion, namely the user position is static, therefore we use
the model: \( u_{t+1} = u_t \). Also we apply the model of
the receiver’s clock error is assumed as the first or-
der markov model such as
\[
\begin{align*}
G^s_u &= \begin{bmatrix}
\mathbf{g}_1^s \\
\mathbf{g}_2^s \\
\mathbf{g}_3^s \\
\mathbf{g}_4^s \\
\end{bmatrix} : n_s \times 3 \text{ gradient matrix}, \\
\end{align*}
\]
State vector:
\[
x_t = \begin{bmatrix}
u \\
\delta t_u \\
N_{L1} \\
N_{L2} \\
\end{bmatrix}, \quad (26)
\]
Measurement noise vector
\[
v_t \equiv \begin{bmatrix}
\epsilon_{CA,u}^p \\
\epsilon_{PP,u}^p \\
\lambda_1^p_{L1,u} \\
\lambda_2^p_{L2,u} \\
\end{bmatrix}.
\]
Here, it is assumed that the observed quantities of the
n_s satellite have been obtained, and \( 1_{n_s} \equiv [1, 1, \cdots, 1]^T \)
is the \( n_s \times 1 \) vector, \( I_{n_s} \) is the unit matrix of \( n_s \times n_s \),
and \( O \) is the zero matrix of \( n_s \times n_s \).

6 Conclusions

We discussed double-frequencies GNSS PPP meth-
ods by applying CLAS correction data. We also added
some CLAS correction functions shown in Table 2 to
our GNSS PPP programs, and evaluated the position-
ing errors from GEONET points. From Table 3, we
can say that the positioning accuracy of 3D-RMS after
starting time and after 10 minutes from the start are
approximately 11 [cm] and 6 [cm], respectively.

In the future, we will examine whether the accuracy
and speed of our estimators for GNSS PPP.
Table 1: Conditions for Observed Data

| Date        | February 5, 2019 |
|-------------|------------------|
| Data1(GPST) | (06:00'20 ~ 06:59'59) |
| Data2(GPST) | (07:00'20 ~ 07:59'59) |
| Data3(GPST) | (18:00'20 ~ 18:59'59) |
| Data4(GPST) | (19:00'20 ~ 19:59'59) |
| Location    | Tsukuba1, Japan (GEONET) |
| Antenna(Ant)| TPSCR.G5 GSI |
| Receiver    | TRIMBLE NETR9 |
| Epoch interval | 1 [s] |
| Satellite System | GPS |
| Measurement Data | C/A code, L1 carrier-phase |
|                 | P(Y) code, L2 carrier-phase |
| Elevation angle mask | 15 [deg.] |

Table 2: Correction Items and Correction Methods

| correction item       | correction method                                                                 |
|-----------------------|-----------------------------------------------------------------------------------|
| Orbit correction      | use CLAS correction $\delta s_{CL}$                                               |
| Clock correction      | use CLAS correction $\delta t_{CL}$                                               |
| Ionosphere delay      | use CLAS correction $\delta I_{CL}$                                               |
| Troposphere delay     | use CLAS correction $\delta t_{CL}$                                               |
| satellite H/W bias    | use CLAS correction $\delta b_{CL}$                                               |
| Phase Wind-up         | use CLASLIB model $\Delta \phi_{1,1,u}$                                           |
| General relativistic delay | use CLASLIB model $\Delta \phi_{1,2,u}$                                           |
| antenna PCO/PCV       | use CLASLIB model $\Delta P_{CL}$                                                |
| solid Earth tide      | use positioning software model (IERS)                                             |

Fig. 1: Positioning Errors at Data1

Fig. 2: Positioning Errors at Data2

Fig. 3: Positioning Errors at Data3

Fig. 4: Positioning Errors at Data4
Table 3: RMS-ERRORS

|        | RMS [m] | RMS [m] after 10 minutes |
|--------|---------|--------------------------|
| Data1  | E 0.049 | 0.114                    |
|        | N 0.035 | 0.025                    |
|        | U 0.097 | 0.013                    |
| Data2  | E 0.050 | 0.112                    |
|        | N 0.055 | 0.017                    |
|        | U 0.084 | 0.023                    |
| Data3  | E 0.058 | 0.111                    |
|        | N 0.039 | 0.045                    |
|        | U 0.086 | 0.019                    |
| Data4  | E 0.023 | 0.087                    |
|        | N 0.049 | 0.033                    |
|        | U 0.068 | 0.043                    |

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