Gravitino dark matter in hybrid gauge-gravity models

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Abstract: We study the phenomenology of generic supergravity models in which gravity mediation naturally competes with gauge mediation as the origin of supergravity-breaking. This class of hybrid models has been recently motivated in string inspired constructions and differs from usual gauge mediated supersymmetry breaking models in having messenger masses of order of the GUT scale. In these scenarios the gravitino can be the lightest supersymmetric particle in wide regions of the parameter space and therefore a potential candidate for dark matter. We investigate this possibility, imposing the WMAP bound on its relic abundance and taking into account constraints from Big Bang nucleosynthesis. We show that in these constructions viable gravitino dark matter can be obtained in specific regions of the parameter space, featuring large values of tan β and where the supersymmetry breaking mechanism is dominated by gauge mediation.

Keywords: Supersymmetry Breaking, Compactification and String Models, Cosmology of Theories beyond the SM, Supergravity Models

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1 Introduction

In the largest class of viable scenarios of supersymmetry (SUSY) breaking a hidden sector is present where the breaking of SUSY takes place, which is then communicated to the visible one by mediator fields via loop-suppressed or nonrenormalizable interactions. It is possible to parameterize the hidden sector dependence on the breaking by $\langle F_{\phi_i} \rangle$, the vacuum expectation value (vev) of the auxiliary component of the spurions fields $\phi_i$. The resulting mass scale of scalars and gauginos in the visible sector is related to the messenger scale with contributions of the order $\sum_i a_i \langle F_{\phi_i} \rangle / \langle \phi_i \rangle$, where $a_i$ are model dependent parameters which can generically be loop suppressed. Depending on whether the messenger fields which transmit the SUSY breaking into the observable sector have only gravitational or also gauge interactions, the mechanism is described as gravity mediated or gauge mediated SUSY breaking (GMSB) respectively.

In this work we concentrate on scenarios which mix both kinds of mediations, called hybrid models. Their formulation, already considered from a model building perspective in ref. [1], has been recently motivated also as effective field models of string theory constructions in ref. [2]. In these cases we can distinguish two classes of spurion fields responsible for the breaking of SUSY in the hidden sector: moduli fields, $T_i$, interacting with the Standard Model (SM) sector through gravitational interactions, and singlet chiral superfields, $X_j$, which couple directly to messenger fields in the superpotential. We can sum up these interactions and for simplicity consider for our purposes the following superpotential, depending on just one modulus $T$ and one spurion field $X$

$$ W = \lambda X M \overline{M} + W(X, T) \ . $$

(1.1)

Here $M$ and $\overline{M}$ are messenger superfields with $SU(3) \times SU(2) \times U(1)$ quantum numbers, whereas $T$ and $X$ are SM-gauge singlets. In this framework the supersymmetric mass for the messenger fields is clearly proportional to $\langle X \rangle$. We are considering a superpotential with just renormalizable (and then minimal) couplings with the spurion field but obviously
it can be further generalized, as well as the Kähler potential, for which we adopt here the standard expression.

Since the messengers lie within a representation of the SM gauge interactions, gaugino masses and scalar squared-mass parameters appear at one and two loops, respectively, being the resulting supersymmetric masses of the order \[ M^i_{\text{GMSB}} \sim \left( \frac{a_i}{16\pi^2} \right) \left( \frac{F_X}{X} \right), \]  

where \( i \) denotes the gauge index representation for the scalar or gauginos and \( a_i \) are coefficients of order unity. Concerning the gravitino, its mass is related to the fundamental SUSY breaking mass by the null cosmological constant condition \( \langle V \rangle = 0, \) 

\[ \sqrt{3} m_{3/2} M_P = \sqrt{|F_T|^2 + |F_X|^2}. \]  

Generically, GMSB models lead to small values for the gravitino mass if the only source of SUSY breaking comes from the spurion field. Indeed, if the dominant term of eq. (1.3) is \( |F_X| \), using eq. (1.2) we see that in order to have a MSSM mass spectrum of order of the TeV, the gravitino mass reads

\[ m_{3/2} \sim \frac{16\pi^2 \langle X \rangle M^i_{\text{GMSB}}}{M_P} \sim 10^{-13} \langle X \rangle, \]  

with \( M_P = (8\pi G_N)^{1/2} \sim 2.4 \times 10^{18} \) GeV and \( \langle X \rangle \sim M_{\text{Mess}} \). We see that in the usual GMSB case, where the messenger masses are of order of 100 TeV, the resulting gravitino mass is of the order of 1 eV. Heavier gravitinos are allowed if one assumes other dominant sources for the SUSY breaking which generates the gravitino mass (for example \( |F_T| \) in models with secluded sectors). This adds new degrees of freedom and alters the direct proportionality of eq. (1.4) between the gravitino mass and \( \langle X \rangle \).

Recently another possibility was proposed in ref. [2], namely that the dynamics of the model forces the spurion \( X \) to be stabilized at a near-GUT scale. In fact in this type of models, the presence of a Fayet-Iliopoulos term, \( \xi \), generated at a string scale (typically \( \sim 10^{-1} \sim 10^{-3} M_P \)), requires that \( X \) takes a vev which cancels the \( \xi \) contribution to the potential, through the D-term flatness condition. In other words, the dynamics of the theory pushes \( \langle X \rangle \) towards large values of the order of \( 10^{16} \) GeV. However, in order to obtain a MSSM spectrum within the TeV range, a fundamental scale \( \Lambda = F_X/X \sim 10^5 \) GeV is needed. This in turn requires higher values for \( F_X \), and consequently heavier gravitinos, \( m_{3/2} \sim 10^3 \) GeV, also implying the interference of gravity mediation with the gauge mediation mechanism. This class of constructions was called hybrid models, which were introduced in ref. [1]. The phenomenological consequences are numerous and “interpolate” between gravity-mediated models (such as the usual Constrained MSSM) and GMSB scenarios.

In this sense, a very appealing feature of supersymmetric theories is that they can provide candidates to solve the problem of the dark matter in the Universe in terms of the lightest supersymmetric particle (LSP). A discrete symmetry, \( R \)-parity, is often imposed in
order to forbid lepton and baryon violating processes which could lead, e.g., to rapid proton decay. This implies that SUSY particles are only produced or destroyed in pairs, thus rendering the LSP stable. Among the most interesting possibilities for supersymmetric dark matter are the lightest neutralino [4, 5], which enters the category of weakly-interacting massive particle, and the gravitino [6], which only has gravitational couplings and is therefore extremely weakly-interacting.

The viability of gravitino dark matter has been widely studied within the context of supergravity models in which the gravitino mass enters as a free parameter [7]. These supergravity scenarios can be thought of as appearing as the low-energy limit of some more fundamental string models. However, in most of the stringy inspired scenarios studied so far the gravitino is not the LSP.\(^1\) As we will show, this is not the case in hybrid models, in which both the neutralino and gravitino can be the LSP in different areas of the parameter space. The regions with neutralino dark matter were already studied in ref. [2] but the possibility of gravitino dark matter has not been addressed yet. In this work we investigate the viability of the gravitino as a dark matter candidate in this class of hybrid models, calculating its relic abundance and imposing Big Bang nucleosynthesis (BBN) constraints.

The paper is organized as follows. In section 2 we summarize the parametrization used in our phenomenological analysis and we discuss the theoretical motivations and the peculiarities of the hybrid models with respect to the usual gravity and gauge mediated scenarios. In section 3 we explore the conditions under which the gravitino can be the LSP and a good dark matter candidate. Finally, in section 4 we expose our conclusions.

2 The model

The contribution from the two different mediation mechanisms we are considering can be parametrized in a general way by the gravitino mass, \(m_{3/2}\), and two dimensionless parameters, \(\alpha\) and \(\delta\), which measure the relative sizes of standard (F-term induced) and non-standard (D-term induced [9]) gauge mediation contributions in units of \(m_{3/2}\) respectively. The latter, in particular, have to be taken into account when extra abelian gauge groups enter in the computation (see, e.g., ref. [2] for details). The soft supersymmetry-breaking terms can then be written as

\[
M_a = M_a^{\text{Grav}} + M_a^{\text{GMSB}} = m_{3/2} \left( \tilde{\epsilon} + g_a^2 S_Q \alpha \right),
\]

\[
m_i^2 = (m_i^{\text{Grav}})^2 + (m_i^{\text{GMSB}})^2 = m_{3/2}^2 \left( 1 + C_i S_Q \left( \delta + \frac{\alpha^2}{N} \right) \right),
\]

(2.1)

where \(N\) is the effective number of messenger fields contributing to gauge mediation, \(S_Q\) is the Dynkin index of the messenger representation \((1/2\) for the fundamental representation of \(SU(N)\), \(g_a\) are the gauge couplings and \(C_i = \sum_a g_a^2 C_a^{i}\), \(C_a^{i}\) being the Casimir of the MSSM scalar fields representations (in our normalization the Casimir of the fundamental representation of \(SU(N)\) is \((N^2 - 1)/(2N)\), that of \(U_Y(1)\) is simply \(Y^2\)). In our

\(^1\) Notice in this sense that a class of 6D chiral gauged supergravity was studied in [8] which presents regions with gravitino LSP.
phenomenological analysis we consider a flavor universal case, where the gravity-mediated contributions are dominated by the term
\[(m_{ij}^{\text{Grav}})^2 \simeq m_{3/2}^2 \delta_{ij}, \tag{2.2}\]
keeping in mind that in principle there could be some flavor-mixing effects from the gravity side. However, this assumption is justified in generic supergravity constructions. The extra parameter $\tilde{\epsilon}$ includes the effects of gravity mediation for gauginos. In this case the gravitational contributions are present only if the gauge kinetic function depends on the modulus field $T$. Moreover, since these contributions are proportional to the ratio $F_T/T$, in the cases under consideration in our analysis this universal coefficient is naturally of order $\tilde{\epsilon} \sim \mathcal{O}(10^{-1})$. It is therefore suppressed with respect to the above mentioned universal contribution from gravity mediation to the scalar masses (which is of order 1). Different values should be taken into account in the cases where extra (for instance secluded) sectors are included in the model.

Unlike the classical GMSB at low energy, gauge mediation in hybrid models occurs around the GUT scale, where the gauge contributions to the gaugino masses $M_a$ (proportional to their gauge couplings $g_a$) are approximately gauge universal. Thus, the gauge non-universality only affects scalars masses. Concerning the trilinear couplings $A_i = t, b, \tau$, there is no 1-loop messenger contribution to the SUSY-breaking trilinear terms. However, $A_i$ terms are generated in the leading-log approximation by the RG evolution and are proportional to gaugino masses. For simplicity, in our analysis we will assume that the trilinear terms are universal at the GUT scale and given by a unique parameter $A$. In order to study the effects of variations in the trilinear term, we will consider the two examples with $A = 0$ and $A = -3m_{3/2}$. The reader can find in the appendix of ref. [2] the explicit expressions of the mass terms for each generation of squarks and sleptons.

Such a hybrid model has several peculiarities.

- The value of the GUT scale for the messengers sector appears in a natural way. In fact, the dynamics of supersymmetry breaking itself justifies very heavy masses for the messengers (of the order of the Fayet-Illiopoulos term for instance) and the rest of the spectrum, at least qualitatively, turns out to be strongly constrained by this first peculiarity. Moreover, as it was pointed out for example in ref. [2], it seems difficult to avoid hybrid scenarios in any stringy inspired supergravity scenario with extra $U(1)$.

- The regions with viable neutralino dark matter and allowed by WMAP constraints have quite distinctive phenomenological consequences which in principle could be observable at LHC [2]. For example, the measurable non-universality in the scalar soft breaking terms makes it possible to distinguish this scenario from the Constrained MSSM and the fact of generating trilinear couplings makes it possible to distinguish it from pure GMSB.

- The FCNC problem, inherent to gravity mediated supergravity constructions, is alleviated by the gauge mediated contributions. In particular, for large values of $\alpha$
the assumption made in eq. (2.2) concerning the flavor dependence of the gravity contribution is not so relevant since the gauge mediated contribution dominates. Interestingly, as we will see in the next section, it is precisely in this region of the parameter space that the gravitino is a viable dark matter candidate.

- The gravitino is naturally heavy (TeV scale) without the need of extra supersymmetry breaking sectors. Indeed, from eq. (1.4) we clearly see that heavy messengers (i.e., large values of \(\langle X \rangle \sim M_{GU T} \)) can easily be consistent with large values for \(m_{3/2} \).

- Concerning the \(\mu/B\mu\) problem characteristic of pure GMSB constructions, hybrid models can help finding a solution through the Giudice-Masiero mechanism [10]. One of the main problems arising in gauge mediation constructions, where the Higgs fields directly couple to the spurion, is the fact that it is difficult to satisfy the MSSM-induced relation

\[
\sin 2\beta = \frac{B\mu}{m_{H_1}^2 + m_{H_2}^2 + 2\mu^2} \sim \frac{\mu \Lambda}{m_{H_1}^2 + m_{H_2}^2 + 2\mu^2},
\]

(2.3)

(where \(m_{H_i}\) are the Higgs soft masses) due to the fact that \(\Lambda \gg \mu\). However, hybrid models can address this issue, giving an extra gravitational contribution to the \(\mu\)-term. Indeed, one can show that if a Kähler term of the form

\[
K = \int d^4\theta \, Z(T, \bar{T}, X, \bar{X}) H_1 H_2,
\]

(2.4)
is introduced, where \(Z(T, \bar{T}, X, \bar{X})\) is a modular function ensuring the modular invariance of the term \(Z(T, \bar{T}, X, \bar{X}) H_1 H_2\), one can generate a \(\mu\) and \(B\mu\)-term after SUSY breaking of the order

\[
\mu \sim m_{3/2}(Z), \quad B\mu \sim 2 \frac{m_{3/2}^2}{\langle Z \rangle}.
\]

Then the values of \(m_{H_1}\) and \(m_{H_2}\) (including their contributions from GMSB) can be easily arranged to fulfill \(\sin 2\beta < 1\). Notice that in this case a direct coupling between the Higgs and the spurion fields \(\lambda' X H_1 H_2\), would require an unreasonably small coupling \(\lambda' \sim 10^{-16}\) to obtain a TeV scale \(\mu\)-term. However, such a direct coupling can be easily avoided imposing, for example, suitable charges for the Higgs fields under an extra abelian gauge group. Even in the absence of such an interaction, the supergravity sector provides a \(\mu\)-term and \(B\mu\)-term of the right order of magnitude since the gravitino mass is already approximately 100 GeV to 1TeV.

Notice however that large values of \(\alpha\) imply the dominance of gauge mediation contributions, which implies that gauginos and scalars are at the same mass scale. In particular, we will show in the next section that for a gravitino of order 100 GeV and \(\alpha \sim 50 - 100\), the whole soft spectrum is very heavy, and the resulting values for \(\mu\) are of order of the TeV. This implies a tension, from a theoretical point of view, with the predicted value of \(\mu\) from the Giudice-Masiero mechanism. A possible way-out could be provided by the presence of the non-standard gauge mediation contributions parametrized by \(\delta\). In fact, \(\delta\) acts just for the scalar mass contributions and
could then help in approaching the so-called “focus-point” region where a small \( \mu \) is expected. In any case, it is obvious that this kind of solution needs a \( \delta \) at least comparable with \( \frac{\alpha}{\pi} \), which seems unnatural in the class of UV model considered here as examples.

- The contributions from anomaly mediation [11] can be neglected in this kind of models. In fact, since these are proportional to \( \frac{\alpha^2}{16\pi^2} \frac{m_{3/2}}{2} \), they are naturally loop-suppressed with respect to the universal terms generated by the gravity mediation, for \( \tilde{\epsilon} \sim \mathcal{O}(10^{-1}) \). Therefore we will not consider effects like mirage mediation [12], or deflected mirage mediation [13], which appear when the anomaly and gravity contributions are of the same order because of the suppression of the gravity mediation due to the moduli couplings. Among other things, from a phenomenological point of view this means that in hybrid models, the gravitino mass scale is typically in the range of 100 GeV to 1 TeV, whereas in mirage mediation a 100 TeV gravitino is required to obtain a TeV-scale SUSY spectrum [12, 13].

3 Gravitino dark matter in hybrid models

As we emphasized in the introduction, in the hybrid models that we are studying both the neutralino and gravitino can be the LSP. Indeed, eq. (2.1) shows that, depending on the value of \( \alpha \), the LSP can be either a neutralino (for small values of \( \alpha \)) or a gravitino (when \( \alpha \) increases). Notice that the scalar soft breaking terms are dominated by the flavor dependent gravity mediation in the former case and flavor-blind gauge mediation in the latter. We investigate here the possibility that the gravitino LSP is a viable dark matter.

In scenarios with gravitino dark matter the late decay of the NLSP into the LSP produces electromagnetic and hadronic showers. If the decay takes place after Big Bang nucleosynthesis (BBN), the products of these showers may alter the primordial abundances of light elements [14]. Also, the late injection of electromagnetic energy may distort the frequency dependence of the cosmic microwave background spectrum from its observed blackbody shape [15–17].

It has been shown that hadronic BBN constraints rule out the possibility of neutralino NLSP for gravitino masses above \( m_{3/2} \gtrsim 100 \text{ MeV} \) [18–23]. However, if the NLSP is the lightest stau, \( \tilde{\tau}_1 \), we should also take into account the effect of bound-states effects on the primordial \( ^6\text{Li} \) abundance. Indeed, it has been shown [24] that bound-state formation of \( \tilde{\tau}_1 \) with \( ^4\text{He} \) can lead to an overproduction of \( ^6\text{Li} \) via the catalyzed BBN (CBBN) reaction \( ^4\text{He} X^- + D \rightarrow ^6\text{Li} + X^- \) [25–30], which has a serious impact on the regions with viable gravitino dark matter [21, 29, 31–33]. In fact, the observationally inferred upper limit on the primordial \( ^6\text{Li} \) abundance [14] implies a stringent upper bound on the stau NLSP lifetime

\[
\tau_{\tilde{\tau}_1} \lesssim 5 \times 10^3 \text{ s}.
\]

A similar bound can be extracted using the same arguments to avoid overproduction of \( ^9\text{Be} \) (see, e.g., [24, 29]). In the case of a stau NLSP decays, the stau decays primarily to the

\[ ^2 \text{ This bound can be relaxed} [34] \text{ for } m_{\tilde{\tau}_1} \lesssim 200 \text{ GeV and a very large } \mu \text{-term if the stau mass eigenstates present a substantial left-right mixing, due to a reduction in the density of primordial staus.} \]
gravitino and a τ lepton at tree level, via gravitational interactions with a lifetime \[ \tau_{\tilde{\tau}} \simeq \tau_1 \approx \Gamma_{\tilde{\tau} \rightarrow \tilde{G}\tau} = 6.1 \times 10^6 \left( \frac{m_{\tilde{G}}}{100 \text{ GeV}} \right)^2 \left( \frac{100 \text{ GeV}}{m_{\tilde{\tau}}} \right)^5 \left( 1 - \frac{m_{\tilde{G}}^2}{m_{\tilde{\tau}}^2} \right)^{-4} \text{s}. \] (3.2)

The relic abundance of gravitinos receives contributions from two different sources. First, there is a non-thermal production (NTP) \[ \Omega_{\tilde{G}}^{\text{NTP}} h^2 = \frac{m_{3/2}}{m_{\text{NLSP}}} \Omega_{\text{NLSP}}^{\text{TP}} h^2. \] (3.3)

Second, gravitinos are also thermally produced (TP) through scatterings in the plasma, the resulting relic abundance being proportional to the reheating temperature, \( T_R \), of the Universe after inflation \[ \Omega_{\tilde{G}}^{\text{TP}} h^2 \simeq 0.32 \left( \frac{100 \text{ GeV}}{m_{3/2}} \right) \left( \frac{m_{\tilde{g}}}{1 \text{ TeV}} \right)^2 \left( \frac{T_R}{10^7 \text{ GeV}} \right). \] (3.4)

The total relic density is the sum of both contributions \( \Omega_{\tilde{G}} h^2 = \Omega_{\tilde{G}}^{\text{NTP}} h^2 + \Omega_{\tilde{G}}^{\text{TP}} h^2 \), to which we will apply the constraint extracted from the WMAP data \[ \Omega_{\text{CDM}} h^2 \leq 0.025. \]

Thus, in order to check the viability of gravitino dark matter in hybrid models, we have studied the parameter space, imposing the upper bound of eq. (3.1) to the stau lifetime and the WMAP constraint to the relic abundance. We show that these stringent conditions can be realized in a very particular region of the parameter space.

It should be stressed that since in these hybrid scenarios the messenger scale is much larger than in the standard GMSB models, the cosmological effect of mediators is completely different. Concerning the influence of heavy messengers on the reheating temperature, as underlined by the authors of [31], if the post-inflationary reheating temperature is larger than the mass of the lightest messenger, \( M_{M_1} \), the "messenger number" which ensures the stability of the lightest messenger should be violated, otherwise the messenger population would overclose the universe if \( M_{M_1} \gtrsim 30 \text{ TeV} \). The consequences of the decays of messengers after their freeze out temperature, would be the dilution of the dark matter gravitino component by the late time increasing of the entropy. However, in hybrid models, GUT-scale messenger masses are well above \( T_R \) and its population is naturally suppressed by the Boltzmann factor in the primordial thermal bath. Thus they should not have any late time effects on the dark matter population.

### 3.1 Results

We have performed a scan in the parameter space of a general hybrid model. For concreteness, we have fixed the number of mediators to \( N = 6 \), which leaves only two input parameters in the equations describing the soft masses in eq. (2.1), namely the gravitino mass and the parameter \( \alpha \). Moreover, we also fixed the effects coming from non-standard contributions, taking a typical value for \( \delta = -1.8 \). As stressed in section 2, a more detailed
analysis taking into account a scan over $\delta$ could be useful in order to investigate the little hierarchy problems in this kind of model, but this is beyond the scope of the paper.

We have calculated the low energy spectrum solving the renormalization group equations with the code SPheno [41], taking into account the LEP constraints on the masses of supersymmetric particles. We also included the current experimental bounds on low energy observables, such as on the branching ratios of rare decays $b \rightarrow s\gamma$ and $B_S \rightarrow \mu^+\mu^-$. In particular, we imposed $2.85 \times 10^{-4} \leq \text{BR}(b \rightarrow s\gamma) \leq 4.25 \times 10^{-4}$, which is obtained from the experimental world average reported by the Heavy Flavor Averaging Group [42], and the theoretical calculation in the Standard Model [43], with errors combined in quadrature. We have also taken into account the upper constraint on the $(B_S^0 \rightarrow \mu^+\mu^-)$ branching ratio obtained by CDF, $\text{BR}(B_S^0 \rightarrow \mu^+\mu^-) < 5.8 \times 10^{-8}$ at 95% c.l. [44]. Given the current discrepancy between the experimental measurements of the muon anomalous magnetic moment, $a_\mu \equiv (g - 2)_\mu$, from $e^+e^-$ or tau data, we have not imposed any constraint on the resulting supersymmetric correction, $a^\text{SUSY}_\mu$. We nevertheless comment on the regions which are favoured by the current $e^+e^-$ result [45] and the present evaluations of the Standard Model contributions [46–48], which lead to $a^\text{SUSY}_\mu = (27.6 \pm 8) \times 10^{-10}$.

An important part of the $(\alpha, m_{3/2})$ plane of hybrid models was already studied in ref. [2], where regions with viable neutralino dark matter were obtained with $1 \lesssim \alpha \lesssim 8$ and moderate values of the gravitino mass. Here we are interested in exploring a complementary region of the parameter space, with larger values of $\alpha$, where the gravitino is the LSP and therefore a potential dark matter candidate.

As explained in the previous section, BBN constraints strongly disfavour the regions with neutralino NLSP [19–23] and consequently we have excluded these from the parameter space and studied only the regions with stau NLSP. In these areas, the lifetime of the stau is calculated and condition (3.1) is used as an extra constraint. Finally, the relic abundance of gravitinos is evaluated with the code micrOMEGAs [49] and bounded using the WMAP result [40].

As a first example, we fixed $A = 0$ and performed a scan in the $(\alpha, m_{3/2})$ plane for various choices of $\tan \beta$. The results are displayed in figure 1. As already shown in ref. [2], the gravitino in hybrid models becomes the LSP for $\alpha > \sim 10$, due to the increase in both the gaugino and scalar mass parameters at the GUT scale (see eq. (2.1)). In the figure the light grey area indicates the points in which the gravitino is not the LSP. The region with gravitino LSP corresponds to the area on the right of the almost vertical line$^3$ at $\alpha \sim 10$. Within that region, the grided area corresponds to points with neutralino NLSP and, as argued before, is excluded. In the remaining regions of the parameter space the stau is the NLSP. The ruled area corresponds to the points in which the condition in the stau lifetime given by eq. (3.2) is not fulfilled and we therefore also consider it excluded by BBN constraints. Finally, in the remaining white area (towards large values of $\alpha$) the stau decays rapidly enough and BBN constraints are satisfied.

In the dark grey area at least one experimental constraint is violated (LEP lower bound on the Higgs or SUSY masses, and branching ratios of rare decays $\text{BR}(b \rightarrow s\gamma)$ and

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$^3$ To the left of this line the stau (neutralino) is the LSP for light (heavy) gravitino masses.
Figure 1. Gravitino mass as a function of $\alpha$ for $A = 0$ and $\tan \beta = 35$, 40, 45, and 50 from left to right and top to bottom. The coefficient for non-standard GMSB contributions is always taken $\delta = -1.8$. The line and colour code is explained in the text.

$\text{BR}(B_S \rightarrow \mu^+ \mu^-)$. Notice that these are generically confined to the area with neutralino LSP, since the spectrum is lighter. Finally, to the right of the dot-dashed line, the supersymmetric contribution to the muon anomalous magnetic moment would be too small to account for the observed $e^+e^-$ data. Notice however that it would not necessarily be in contradiction with tau data. In figure 2 we represent the same example but for a different soft trilinear term, $A = -3m_{3/2}$.

A first thing to notice is that large values for $\tan \beta$ are needed in order to obtain regions of the parameter space in which the stau is the NLSP instead of the neutralino. Indeed, for large $\tan \beta$ the bottom Yukawa increases, thereby inducing a larger negative contribution to the running of the stau mass parameters. As we see in both cases, $A = 0$ and $A = -3m_{3/2}$, a value of $\tan \beta \gtrsim 40$ is enough. Also the areas with stau NLSP are wider for $A = -3m_{3/2}$ since the negative contribution to the running of the stau mass
parameters and the larger off-diagonal elements in the mass matrix (which also increase with $\tan \beta$) imply lighter staus.

As discussed previously, the most stringent constraint stems from $^6$Li and $^9$Be overproduction. In order for the stau lifetime to be short enough, the stau has to be sufficiently heavy (while still being lighter than the lightest neutralino). For a given gravitino mass $m_{3/2}$, this implies a lower bound on $\alpha$. This can be qualitatively understood from eq. (3.2) and eq. (2.1) as follows. An increase in $\alpha$ for fixed $m_{3/2}$ implies an increase in the stau mass and consequently a decrease in its lifetime. For small values of $\beta$ and fixed $N$ we can approximate $\tau_{\tilde{\tau}_1} \propto \alpha^{-\frac{5}{2}} m_{3/2}^{-3}$, which is in agreement with the slope of the dashed line in figure 1.

Regarding the resulting relic abundance, we display two examples with different values for the reheating temperature. Black dots represent the results with $T_R = 10^6$ GeV and empty circles correspond to $T_R = 10^8$ GeV. In fact, for $T_R = 10^6$ GeV the thermal contribution is very small and these points can be understood as coming from purely non-thermal production. On the one hand, from eq. (3.4) one can qualitatively infer that for a fixed

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure2}
\caption{The same as in figure 1 but for $A = -3 m_{3/2}$.}
\end{figure}
number of messengers the thermal production can be approximated as \( \Omega_{\text{TP}} h^2 \propto m_{3/2}^2 \alpha^2 \). This is consistent with the slope of the region with \( T_R = 10^8 \) GeV where the relic abundance of gravitino is mainly thermal. On the other hand, using the same qualitative arguments, the non-thermal contribution to the gravitino relic abundance should behave as \( \Omega_{\text{NTP}} h^2 \propto m_3^2 \). As already pointed out [18, 19, 33, 35, 36] within the context of the CMSSM, in order to fulfill the BBN constraints the mass of the light stau needs to be \( m_{\tilde{\tau}} \gtrsim 2 \) TeV. The regions with viable gravitino dark matter that lie between the two limiting values of \( T_R \), correspond in our case to \( m_{\tilde{\tau}} \propto m_{3/2} \alpha \gtrsim 1.6 \) TeV. An example of the characteristic spectrum that would be expected in these models is shown in figure 3 for \( A = 0 \) and \( \tan \beta = 45 \) with \( \alpha = 80 \) and \( m_{3/2} = 200 \) GeV. As expected, the spectrum is very heavy, only the lightest stau and the lightest neutralino have masses below 2 TeV and the whole squark sector above 6 TeV.

To sum up, regions with viable gravitino dark matter can be found in this general class of hybrid models. They correspond to areas with a large value of \( \alpha \), of order 45 (typically this implies \( \langle X \rangle \lesssim 10^{16} \)), with gravitinos in the mass range of several hundred GeV and \( T_R \lesssim 10^8 \) GeV. The rest of the supersymmetric spectrum is rather heavy, with staus in the mass-range of 2 TeV and with \( \tan \beta \gtrsim 40 \).

Therefore the allowed region in the parameter space corresponds to the opposite of the range considered in [2], where a very small value of the Fayet-Iliopoulos term is required and GMSB begins to be dominant. In particular, the construction discussed there seems to slightly disfavor these points with gravitino dark matter, even if still possible in the framework of such models.
Another interesting point would be to see some collider signatures of such class of models. Indeed, the stau lifetime in eq. (3.2) depends on the messengers mass through $F_X$, contrary to secluded sector breaking. Thus, any information on the stau lifetime would give information on the messenger mass scale. For instance, 100 TeV messengers imply a short stau lifetime, whereas $10^{16}$ GeV messengers mass lead to a stau lifetime of seconds. Variations of orders of magnitude in the messenger masses directly induce differences of orders of magnitude in the stau lifetime [50].

4 Conclusions

In this work we have studied the phenomenology of a generic class of string motivated scenarios in which gravity mediation naturally competes with gauge mediation as the origin of supergravity-breaking. An interesting feature of these constructions is that the messenger masses are of order of the GUT scale, contrary to standard GMSB models. In these scenarios the neutralino is typically the lightest supersymmetric particle when gauge and gravity contributions are of the same order. However, when gauge-mediation becomes dominant, the gravitino easily becomes the LSP and therefore a potential dark matter candidate. We have shown that even without secluded breaking sector, heavy messengers induce indirectly a GeV/TeV gravitino mass if the contribution to the cosmological constant comes from the spurion field.

We have then investigated the viability of the gravitino as dark matter, calculating its relic abundance and imposing the WMAP result. Furthermore, we have taken into account existing bounds from Big Bang nucleosynthesis. The latter play a leading role in constraining the parameter space. Regions with viable gravitino dark matter can be obtained when the SUSY breaking mechanism is mostly dominated by gauge mediation ($\alpha > \sim 45$) and with $\tan \beta > \sim 40$. The resulting spectrum is relatively heavy, with squark masses larger or of order of 6 TeV and slepton masses above 2 TeV.

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