Implementation Tabu search algorithm for optimization distribution LPG

E Wati¹ and M Fauzan²

¹Student of S1 Mathematics Study Program of Department of Mathematics Education, Universitas Negeri Yogyakarta, Indonesia
²Department of Mathematics Education, Universitas Negeri Yogyakarta, Indonesia

E-mail: erau.wati2015@student.uny.ac.id, mfauzan@uny.ac.id

Abstract. PT. Kuda Mas Perkasa is a distributor company engaged in the distribution of non PSO LPG. The routes of distributing LPG at PT Kuda Mas Perkasa, is currently still intuitive so it does not provide optimal benefits. So, shortest LPG distribution route solution is needed. One of the optimal route problem-solving methods is the Tabu Search Algorithm, this method has the advantage of producing an optimal solution without getting stuck in repeating the same solution in subsequent iterations. The purpose of this study is to find the shortest route CVRP problem with the Tabu Search Algorithm. How the Tabu Search Algorithm starts with determining the initial solution using Nearest Neighbour, evaluating moves using the 2-Opt, Relocated, and Exchange methods, updating the Tabu list, then when the stop criteria are fulfilled the Tabu Search algorithm stops, if not, then returns to move evaluation. By using the Tabu Search Algorithm, the total distance travel by the truck is 146.62 km. Based on the results of the study, concluded that the completion of the distribution route using the Tabu Search Algorithm produces an optimal solution.

1. Introduction
Distribution is an activity that moves products from suppliers to customers called supply chains [1]. Almost all industrial sectors always discuss the distribution process. In industries that carry out regular frequency distribution to many locations, it takes to fuel and optimal delivery time. Liquefied Petroleum Gas (LPG) is one of the products marketed in Indonesia, so the distribution process is an important factor in determining company profits.

PT. Kuda Mas Perkasa is a company engaged in Non-PSO LPG distribution agents. The address is at Jl. Godean Km 4.5 Patran, Banyuraden, Gamping, Sleman, Yogyakarta. This study took a sample of data at PT. Kuda Mas Perkasa. The LPG products sold by the company are non-PSO LPG, including 5.5 kg Bright Gas LPG. PT. Kuda Mas Perkasa in the process of product distribution is still intuitive and does not have a fixed route, this causes the cost of distribution which includes material costs and employee salaries to be not optimal. What are the expected benefits when using the optimal route, can reduce the cost of distribution.

Based on this description, the problem that occurs in LPG distribution is a transportation problem that can be done using the Traveling Salesmen Problem (TSP) and Vehicle Routing Problem (VRP). TSP is a route search from a salesman that starts from the initial location, the cities that have been determined. Every city can only be visited once [2]. The TSP problem can be model into a weighted
graph with each destination city related to the road as weighted edges representing the length of the road between the two cities. The salesman's travel problem in a graph is the Hamilton circuit which is published in weighted graphs.

In the classic VRP, it is recommended to find the optimal route for vehicles departing from the depot. Every customer must visit now with exactly one vehicle [3]. Capacitated Vehicle Routing Problems (CVRP) is one variation of VRP problems with limited support for vehicle capacity [4]. One solution to CVRP is the Tabu Search algorithm. Tabu Search (TS) is meta-heuristic which designates local heuristic search procedures to find solution space outside local optimality [5]. Tabu Search's algorithm uses memory called taboo to avoid starting a solution. The advantage of using the taboo search algorithm is that it can produce optimal solutions without getting stuck in repeating the same solution in the next iteration.

2. Theory

2.1. Theory Graf
A graph is pair \((V, E)\) of sets, \(V\) nonempty and element of \(E\) a set of two distinct element of \(V\). The element of \(v\) are called vertices, the element of \(e\) are called edges [6]. A graph may have no edges, but a vertex must have at least one. Graphs which only have one vertex without edge are called Trivial Graphs [7].

2.2. Weighted Graph
A Weighted Graph is a graph where each edge is given a weight [8]. Weight can state the distance between two cities, the cost of travel between two cities, travel time between communication nodes to other communication nodes [7].

2.3. Hamilton Graph
A simple path in a graph \(G\) that passes through every vertex exactly once is called a Hamilton path, and a simple circuit in a graph \(G\) that passes through every vertex exactly once is called a Hamilton circuit [8]. Graf Hamilton is a graph that has a Hamilton circuit, while a graph that has a Hamilton path is called a Hamilton Semi Graph [7].

2.4. Traveling Salesman Problem
Traveling Salesmen Problem (TSP) is to find a routing of a salesman who starts from a home location, visits a prescribed set of cities and returns to the original location in such a way that the minimum total distance travelled and each city is visited exactly once. [2].

2.5. Vehicle Routing Problem
The classic Vehicle Routing Problem (VRP) is to find the optimal route for vehicles, from a depot serving a certain number of customers. Every customer must be visited exactly one vehicle [5]. The algorithm for solving VRP can be divided into two exact and heuristic. The exact algorithm cannot consistently solve with more than two hundred customers. Therefore, meta-heuristics has been proposed to solve a larger problem. Example of a meta-heuristic method: Simulated Annealing (SA), Tabu Search (TS), Evolutionary Algorithms (EA), Ant Colony optimization (ACO) and Iterated Local Search (ILS)[5].

2.6. Capacitated Vehicle Routing Problem
Capacitated Vehicle Routing Problem (CVRP) is one variation of VRP problems with limited constraints on vehicle capacity [4]. On this issue, the total number of consumer requests in a route may not exceed the capacity of the vehicle serving the route and each consumer is only visited once by one vehicle.
The mathematical model of CVRP is represented as a graph $G = (V, E)$. Where $V = \{v_i | i = 1, 2, 3, \ldots, n\}$ can also be written with $V = \{v_1, v_2, v_3, \ldots, v_n\}$ is a set of vertices (nodes/vertex). While $E = \{e_i | i = 1, 2, 3, \ldots, n\}$ can also be written with $E = \{e_1, e_2, e_3, \ldots, e_n\}$ is a set of edges connecting the vertices. Node $v_1$ is declared a depot and the other is a consumer. Each element of the ribs declares the distance between nodes. The vehicle set $K$ is a collection of vehicles that are homogeneous with $q$ large capacity. Each nodes $i$ has a request for $d_i$. Given $v_{ij}$ is distance from node $i$ to node $j$ ($v_{ij}$ is a nonnegative number). Distance is assumed to be symmetric ($v_{ij} = v_{ji}$).

These problems can then be made into mathematical models with the aim of minimizing the total travel distance traveled.

Defined decision variable:

- $x_{ij}^k = \begin{cases} 1, & \text{if there is a trip from the node } v_i \text{ with the } k \text{ vehicle} \\ 0, & \text{if there is a trip from the node } v_i \text{ to } v_j \end{cases}$

Dan

- $y_i^k = \begin{cases} 1, & \text{if node } v_i \text{ served by } k \text{ vehicle} \\ 0, & \text{if node } v_i \text{ not served by } k \text{ vehicle} \end{cases}$

Notation:

- $G = (V, E)$
- $V$ = nodes $\{v_1, v_2, v_3, \ldots, v_n\}$, where $v_1$ is depot dan $\{v_2, v_3, \ldots, v_n\}$ is customer
- $E$ = edge set $\{e_i | i = 1, 2, 3, \ldots, n\}$
- $v_{ij}$ = Distance between nodes $i$ ke node $j$
- $d_i$ = Consumer demand $i$
- $k = \{k_1, k_2, k_3, \ldots, k_m\}$ vehicle used
- $q$ = Capacity of each vehicle $k_i, i = \{1, 2, 3, \ldots, m\}$.

The objective function of this mathematical model is to minimize the total distance traveled by the vehicle. If $Z$ is the objective function,

$$\text{Min } Z = \sum_{i=1}^{n} \sum_{j=1}^{n} \sum_{k=1}^{m} v_{ij} x_{ij}^k$$

(1)

Constraint function:

1. Each point is visited exactly once by a vehicle

$$\sum_{k=1}^{m} \sum_{j=1}^{n} x_{ij}^k = 1, \quad \forall i = \{1, 2, \ldots, n\}$$

(2)

2. Every vehicle that visits one point will leave that point

$$\sum_{i=1}^{n} x_{ij}^k - \sum_{j=2}^{n} x_{ij}^k = 0, \quad \forall k = \{1, 2, \ldots, m\}$$

(3)

3. The total demand for all points in one route does not exceed the capacity of the vehicle

$$\sum_{i=1}^{n} d_i \sum_{j=1}^{n} x_{ij}^k \leq q, \quad \forall k = \{1, 2, \ldots, m\}$$

(4)

4. Each route starts from the depot

$$\sum_{j=1}^{n} x_{ij}^k = 1, \quad \forall k = \{1, 2, \ldots, m\}$$

(5)

5. Each route ends at the depot

$$\sum_{i=1}^{n} x_{ij}^k = 1, \quad \forall k = \{1, 2, \ldots, m\}$$

(6)
6. The $x$ variable is a binary variable.

$$x_{i,k}^k \in \{0,1\}, \quad i = \{1,2,\ldots,n\}, \forall k = \{1,2,\ldots,m\}$$

(7)

2.7. Tabu Search

Tabu Search (TS) is a meta-heuristic that guides a local heuristic search procedure to explore the solution space beyond local optimality. Widespread successes in practical applications of optimization include finding better solutions to problems in scheduling, sequencing, resource allocation, investment planning, telecommunications and many other areas. The Tabu Search algorithm has five steps used to complete VRP [5]. Fred Glover was the first person to introduce the Tabu Search Algorithm in 1986. The word taboo comes from the Tongan language, which is a Polynesian language used by the native inhabitants of the island of Tonga to reveal something that should not be touched because it is something sacred [9].

2.7.1. Representation Solution. The representation of the solution used by the Tabu Search algorithm to solve VRP is a feasible solution written as a sequence of nodes, where each node is only seen once in sequence. The node represents depots and consumers.

2.7.2. Initial Solution. Initial Solution is the first step taken in the Tabu Search Algorithm process. The initial solution can be formed using a random method or heuristic method that will be corrected in the next iteration.

2.7.3. Neighborhood Solution. Neighborhood solutions are alternative solutions obtained by moving nodes (move). Moving nodes (move) can be done using the heuristic method, namely Exchange, 2-Opt, and Relocated [11]. The Relocated method is a node in a route moved in the order of visits. The 2-Opt method moves two paths on one route, then reconnects the path with different pairs of nodes. The Exchange method is to exchange the order of two node visits in one route.

2.7.4. Tabu List. In order to avoid repetition of the steps taken, a tabu test is performed by using the existing Tabu list. The tabu list contains the move attributes that have been found before. The size of the Tabu list will increase with increasing size of the problem.

2.7.5. Aspiration criteria. Aspiration criteria are method for cancel tabu status [12]. The basic rules used in the aspiration criteria in the Tabu Search Algorithm are the quality of the best neighbourhood solutions and the resulting solutions are not the same as the existing solutions.

2.7.6. Termination criteria. Termination criteria are condition in which the calculation process of the Tabu Search Algorithm stops. There are three types of dismissal criteria commonly used in Tabu Search as follows: All previously specified iterations are fulfilled, after several iterations without any improvement in the objective function value and when the objective function reaches a predetermined value [13].

3. Methods

The data used in this study are consumer demand data, the distance between consumers, the distance of depots to consumers, the capacity of vehicles PT. Kuda Mas Perkasa is one of the 5.5 Kg bright gas distributors for the Yogyakarta Special Region. Data obtained by the number of consumers as many as 15, as many as one depot, the maximum capacity is 150 units, data distribution in January 2019, as well as the distance between consumers. Then the calculation process can be carried out with the following steps:
3.1. Initial Solution
The initial solution for LPG Gas distribution was formed using the Nearest Neighbour heuristic method. In this method, the visit starts from the depot to the location that has the closest distance from the depot. Then do the next visit to the location closest to the location of the first consumer, and so on until all consumers are visited and return to the depot.

3.2. Neighborhood Solution
After getting the initial solution, the next step is to determine the environmental solution. Then the best environmental solution is used to update tabu status. Environmental solutions are obtained by three heuristic methods, namely Relocation, exchange, 2-Opt. The selection of these three methods is randomly selected. Furthermore, after all the iterations have been fulfilled, the resulting solution is limited to vehicle capacity. The best solution is chosen based on the closest distance.

3.3. Termination Criteria
A new solution with tabu status in the Neighbourhood solution is better than the previous solution, so the new solution replaces the previous solution. Then after the ledge is repeated as many as the specified iterations are fulfilled. Termination Criteria Tabu Search algorithm is a condition where all predefined tabus are fulfilled.

4. Result and Discussion
CVRP problems in the distribution of 5.5 kg LPG at PT. Kuda Mas Perkasa can be defined as a graph \( G = (V,E) \). A graph \( G \) consists of a set of nodes \( V\)\( = \{v_i | i = 1,2,3,...,16\} \) stating the set of customers and depot, while the set of vertices \( E = \{e_{ij} | i,j = 1,2,3,...,16\} \). Every customer has a request for \( d_i \), so the route length is limited by capacity.

Assumptions that can be formed in the problem of distribution of 5.5 kg LPG at PT. Kuda Mas Perkasa are as follows:
- Each customer location is only visited once and is assumed to be a point.
- The fleet used in shipping is a number of two fleets with each carrying capacity of 150 5.5 kg LPG units.
- The distance between two points is obtained from the shortest distance shown by Google My Maps.
- Data in the form of the number of customer requests in one month.
- The road being passed is not a one-way lane, than in this studied, \( v_{ij} = v_{ji} \).

Data reports on requests for Bright Gas product shipments to companies in January 2019. This report data is included with consumer demand. The following are data reports on product delivery requests obtained from the company:

| Table 1. Demand by customers PT Kuda Mas Perkasa (unit) |
|------------------------|--------|--------|--------|
| Customer | Demand | Customer | Demand |
| 1        | 0      | 9       | 30     |
| 2        | 6      | 10      | 7      |
| 3        | 50     | 11      | 38     |
| 4        | 35     | 12      | 7      |
| 5        | 5      | 13      | 22     |
| 6        | 15     | 14      | 15     |
| 7        | 23     | 15      | 10     |
| 8        | 39     | 16      | 5      |
The application of the Tabu Search Algorithm in the case of distributing Bright Gas products uses distance data as follows:

Table 2. The distance between node (km)

|   | 1  | 2  | 3  | 4  | 5  | 6  | 7  | 8  | 9  | 10 | 11 | 12 | 13 | 14 | 15 | 16 |
|---|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|
| 1 | 0  | 33.3 | 6 | 5.21 | 15.3 | 11.2 | 24.1 | 2 | 2.72 | 11.5 | 9.82 | 0.69 | 3.55 | 4.07 | 7.77 | 2.17 |
| 2 | 0  | 32 | 36.4 | 44.6 | 39.8 | 51.9 | 31.9 | 35.8 | 41.2 | 30.4 | 33 | 36.6 | 37.4 | 40.4 | 32.8 |
| 3 | 0  | 11.1 | 21 | 8.21 | 30 | 4.42 | 8 | 17.3 | 14.8 | 6.49 | 6.76 | 7.77 | 12.9 | 3.84 |
| 4 | 0  | 10.1 | 14.4 | 19.1 | 7.12 | 3.35 | 10.9 | 11.2 | 19.1 | 21 | 11.9 | 8.74 | 9.52 | 13.6 | 9.74 |
| 5 | 0  | 22.1 | 9.04 | 17.2 | 13 | 3.82 | 14.2 | 15.1 | 14.6 | 13.5 | 8.66 | 17.3 |
| 6 | 0  | 30.6 | 10.9 | 11.2 | 19.1 | 21 | 11.9 | 8.74 | 9.52 | 13.6 | 9.74 |
| 7 | 0  | 26.3 | 22 | 12.8 | 21.7 | 24.1 | 23.5 | 22.4 | 17.5 | 26.3 |
| 8 | 0  | 4.61 | 13.5 | 10.6 | 2.21 | 4.75 | 5.6 | 9.71 | 1.14 |
| 9 | 0  | 9.3 | 10.3 | 2.91 | 2.49 | 2.12 | 5.1 | 4.36 |
| 10 | 0 | 11.1 | 11.3 | 11.2 | 10 | 5.62 | 13.5 |
| 11 | 0 | 9.13 | 12.7 | 12.4 | 12.3 | 11.5 |
| 12 | 0 | 4.14 | 4.55 | 7.87 | 2.65 |
| 13 | 0 | 1.21 | 6.12 | 3.95 |
| 14 | 0 | 4.92 | 4.93 |
| 15 | 0 | 9.39 |
| 16 | 0 |

4.1. Application Of Manual Tabu Search Algorithms For LPG Distribution

In the case of LPG distribution at PT Kuda Mas Perkasa, based on company data, the demand is 15 nodes and one depot. Then set 4 tabu list sizes and many Neighbourhood solutions are 16 solutions. Next is the process of manually calculating the Tabu Search Algorithm with the following steps.

4.1.1. Forming of initial solution. The first step is to determine the initial solution in the 0 iterations. The initial solution is obtained using the Nearest Neighbor method on the TSP route. This initial solution will be a temporary route that will be fixed in the next iteration.

Table 3. Initial Solution Nearest Neighbour

| Route | VRP Solution | Capacity | Distance |
|-------|--------------|----------|----------|
| 1     | 1-12-8-16-3-13-14-1 | 138 | (7+39+5+50+22+15) | 19.92 km |
|       |               |         | (0.69+2.21+1.14+3.84+6.76+1.21+4.07) |
| 2     | 1-9-4-15-10-5-7-11-1 | 148 | (30+35+10+7+5+23+38) | 60.2 km |
|       |               |         | (2.72+3.35+4.13+5.62+3.82+9.04+21.7+9.82) |
| 3     | 1-6-2-1      | 21      | (15+6)   | 84.3 km |
|       |               |         | (11.2+39.8+33.3) |
|       | Total        | 307 unit|          | 164.42 km |

4.1.2. Neighborhood Solution. Step 2nd is to determine the next iteration and look for Neighborhood solutions. Neighborhood solutions are obtained by moving nodes using three methods, Exchange, Relocated or 2-Opt which are randomly selected so that the best Neighborhood solutions are obtained. The number of Neighborhood solutions, in this case, is 16 solutions.
Table 4. A Neighbourhood Solution VRP in first iteration

| Route | VRP Solution | Capacity | Distance |
|-------|--------------|----------|----------|
| 1     | 1-6-8-16-3-13-14-1 | 133 | 28,57 |
|       | (7+39+5+15+22+15+30) | (0,69 + 2,21 + 1,14 + 9,74 + 8,74 + 1,21 + 2,12) |
| 2     | 1-9-4-15-10-5-7-11-1 | 118 | 59,34 |
|       | (118+35+10+7+5+23+38) | (2,72 +5,21 + 4,13 + 5,62 + 3,82 + 9,04 + 21,7 + 9,82) |
| 3     | 1-12-2-1 | 56 | 71,3 |
|       | (50+6) | (6+32+33,3) |
| Total | 307 unit | 159,21 |

4.1.3. Tabu list. After obtaining the best Neighborhood Solution in the first iteration, the next step is to move the node to the best solution into the Tabu list. This is done so that the first optimal iteration solution is not used in subsequent iterations.

Table 5. Tabu List

| 1 | 2-Opt | 3 | 4 |
|---|-------|---|---|
| Exchange | Relocated | Relocated | |
| 6 | 3 | 2 | 2 |
| 3 | 16 | 12 | 1 |

4.1.4. Aspiration criteria. If the previous step is to determine the tabu list for the first iteration, then the next step is to determine the Neighborhood VRP solution for the next iterations until the dismissal criteria are met, that is, if all the iterations specified have been fulfilled. In this case, the calculation process for the Tabu Search algorithm stops at the 4th iteration.

Table 6. Final Solution

| Route | VRP Solution | Capacity | Distance |
|-------|--------------|----------|----------|
| 1     | 1-12-2-11-7-5-10-15-4-1 | 131 | 113,61 |
|       | (7+6+38+23+5+7+10+35) | (0,69+33+30,4+21,7+9,04+3,82+5,62+4,13+5,21) |
| 2     | 1-9-14-13-6-3-16-1 | 137 | 29,01 |
|       | (30+15+22+15+50+5) | (2,72+2,12+1,21+8,74+8,21+3,84+2,17) |
| 3     | 1-8-1 | 39 | 4 |
|       |       | (2+2) |
| Total | 307 unit | 146,62 |

The best Neighborhood Solution in the 4th iteration is the best solution to the problem of distributing 5.5 kg Bright Gas LPG at PT. Kuda Mas Perkasa. Based on Table 6. shows that there are three distribution routes:

- On the first route the distribution starts from the depot to node 12, node 2 and so on until node 4 with a demand of 131 units.
- Then the second vehicle distributes the product to each node on route two. On route two distribution starts from node 9, node 14 and so on until node 16 with a demand of 137 units, then returns to the depot to fill the product load.
- Distribution on route three to the last node, node 8 with a request of 39 units.

The total distance of 5.5 kg Bright Gas LPG distribution at PT. Kuda Mas Perkasa using the Tabu Search Algorithm is 146.62 km.
4.2. Application of Tabu Search Algorithm by Using MATLAB
The GUI program is made with input data in the form of coordinates \((x, y)\), capacity, maximum iteration request, number of neighborhood solutions, length of tabu list and distance matrix between nodes. Then with the same completion process using the tabu search algorithm, so that the output results are in the form of routes, total route distances, number of route requests and solution representations.

The output is the optimal solution or the shortest route using the Tabu Search Algorithm. In addition, the number of requests for each distribution route is shown. The GUI Output results in Figure 1. also display map images and graph representations of the optimal solution. Tabu Search algorithm produces 3 routes with each request per route is 131 units, 137 units, and 39 units. The shortest total distance obtained is 146.62 km.

![Figure 1. Output GUI Tabu Search Algorithm](image_url)

5. Conclusion
Based on the results of the analysis of the shortest route search Tabu Search algorithm produces 3 routes with each request per route is 131 units, 137 units, and 39 units. The shortest total distance obtained is 146.62 km. The uniqueness of this method is the existence of a flexible taboo list, user-specified iteration, a certain amount of time or a number of sequential iterations without increasing the value of the best objective function.

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