Inclusive Higgs boson production at the LHC in the $k_T$-factorization approach

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We investigate the inclusive Higgs boson production in proton-proton collisions at the CERN LHC conditions using the $k_T$-factorization approach. Our analysis is based on the dominant off-shell gluon-gluon fusion subprocess (where the transverse momenta of initial gluons are taken into account) and covers $H\to\gamma\gamma$, $H\to ZZ^*\to 4l$ (where $l=e,\mu$) and $H\to W^+W^-\to e^\pm\mu^\mp\nu\bar{\nu}$ decay channels. The transverse momentum dependent (or unintegrated) gluon densities in a proton were derived from Ciafaloni-Catani-Fiorani-Marchesini equation, which resums large logarithmic terms proportional to $\ln s/\ln 1/x$, important at high energies. As an alternative choice, we apply the Kimber-Martin-Ryskin prescription, where the transverse momentum dependent gluon density is constructed from the known conventional parton distributions. We estimate the theoretical uncertainties of our calculations and compare our results with next-to-leading-order plus next-to-next-to-leading-logarithmic ones obtained using collinear QCD factorization. Our predictions agree well with the latest experimental data taken by the CMS and ATLAS Collaborations at $\sqrt{s}=8$ and 13 TeV.

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I. INTRODUCTION

With the startup of the Large Hadron Collider (LHC), high energy physics entered a new era. A great triumph of the Standard Model (SM) is the discovery of the Higgs boson in 2012 [1,2]. The Higgs boson $H$ was predicted more than 50 years ago as a consequence of the electroweak symmetry breaking mechanism in the SM. This mechanism introduces a single complex scalar field doublet, which gives masses to particles. Theoretical and experimental investigations of the Higgs boson production cross sections and its decay rates are an important test for possible deviations from the SM expectations [6–10].

Recently the CMS and ATLAS Collaborations have reported their measurements [11–16] of the inclusive Higgs boson total and differential cross sections at $\sqrt{s}=8$ TeV in the $H\to\gamma\gamma$, $H\to ZZ^*\to 4l$ (with $l=e,\mu$) and $H\to W^+W^-\to e^\pm\mu^\mp\nu\bar{\nu}$ decay channels. Moreover, preliminary data collected at $\sqrt{s}=13$ TeV have become available [17–20]. The measured observables, such as distributions on the transverse momentum, rapidity or scattering angle of decay particles, allow to probe fundamental properties of the Higgs boson (for example, spin and couplings to gauge bosons and fermions) and can be used to investigate the gluon dynamics in a proton since the dominant mechanism of inclusive Higgs production at the LHC is the gluon-gluon fusion¹ [6–10]. Corresponding total and differential cross sections measured at $\sqrt{s}=8$ TeV are higher than the SM estimations, obtained at next-to-next-to-leading order (NNLO) [21–26] and matched with soft-gluon resummation carried out up to next-to-next-to-leading logarithmic accuracy (NNLL) [27,28], although no significant deviations from the perturbative quantum chromodynamics (pQCD) predictions² within the experimental and theoretical uncertainties are observed [11–16]. The same conclusion was made [17–20] for preliminary data taken by the CMS and ATLAS

¹The gluon-gluon fusion and weak boson fusion (namely, $qq\to qqH$ subprocess via $t$-channel exchange of $W$ or $Z$ bosons) are also expected to be the dominant sources of semi-inclusive Higgs production at the LHC.

²The next-to-leading order perturbative electroweak corrections to the Higgs production cross section are available [29–33].
Collaborations at $\sqrt{s} = 13$ TeV. The latter were compared with the NNLOPS calculations [34,35] normalized to N$^3$LO predictions [36–38] for gluon-gluon fusion subprocess. The NNLOPS tool provides parton-level events at NNLO accuracy and is interfaced to the PYTHIA8 event generator [39] for parton showering, hadronization and multiple parton interactions.

In the present note we give a systematic comparison of the QCD predictions derived in the framework of the $k_T$-factorization approach [40,41] and latest CMS [11–13,17,18] and ATLAS [14–16,19,20] data on the inclusive Higgs production in diphoton, four-lepton, and $H \to W^+W^- \to e^\pm\mu^\mp\nu\bar{\nu}$ decay modes collected at $\sqrt{s} = 8$ and 13 TeV. The $k_T$-factorization approach is based on the Balitsky-Fadin-Kuraev-Lipatov (BFKL) [42] or Ciafaloni-Catani-Fiorani-Marchesini (CCFM) [43] gluon evolution equations, which resum large logarithmic terms proportional to $\ln s \sim \ln 1/x$, important at high energies (or, equivalently, at small proton longitudinal momentum fraction $x$ carried by gluons). The CCFM equation takes into account additional terms proportional to $\ln 1/(1-x)$ and is almost equivalent to the BFKL equation in the limit of asymptotic energies, but also similar to the conventional Dokshitzer-Gribov-Lipatov-Altarelli-Parisi (DGLAP) [44] scenario for large $x$ and high scale $\mu^2$. For inclusive Higgs production at the LHC, typical $x$ values are $x \sim m_H/\sqrt{s} \sim 0.008$–0.015 (for Higgs mass $m_H \approx 125$ GeV), so that one can reach the low $x$ domain where the BFKL-like evolution is expected to be valid. Additionally, we see certain advantages in the fact that, even with the leading-order (LO) partonic amplitudes, a large piece of higher order corrections [namely, part of next-to-leading order (NLO) + NNLO + ... terms corresponding to real gluon emissions in initial state] are included by using transverse momentum dependent (TMD) gluon densities. Special point of our interest is connected with the transverse momentum distribution of the produced Higgs boson. It is well known that traditional pQCD calculations, performed within the collinear QCD factorization, diverge at small Higgs transverse momentum $p_T \ll m_H$ with terms proportional to $\ln m_H/p_T$ appearing due to soft and collinear gluon emission. Thus, soft gluon resummation techniques [45–49] have to be used to produce reliable QCD predictions at small transverse momenta. Such resummation can be performed either in the transverse momentum space [50] or in the Fourier conjugate impact parameter space [51] (difference between these two formalisms is discussed [52]) at leading logarithmic (LL), next-to-leading logarithmic (NLL) and next-to-next-to-logarithmic (NNLL) levels. So, traditional pQCD calculations combine fixed-order perturbation theory with analytic resummation and some matching criterion. As it was shown [53], the soft gluon resummation formulas are the result of the approximate treatment of the solutions of the CCFM evolution equation. Therefore, the CCFM-evolved TMD gluon densities effectively absorb the effects of soft gluon resummation, that regularizes the infrared divergences and makes our predictions valid even at low transverse momenta. More detailed descriptions of the $k_T$-factorization formalism can be found, for example, in reviews [54].

The $k_T$-factorization approach has been already applied to the inclusive Higgs boson production [53,55–62]. So, the effective Lagrangian [54,63] for the Higgs coupling to gluons (valid in the large top quark mass limit, $m_t \to \infty$) was used [53,55,57–62] to calculate the amplitude of dominant gluon-gluon fusion subprocess, whereas the simplified solution of the CCFM equation in the single loop approximation (where the small-$x$ effects are neglected) was used [53]. In the framework of Monte Carlo generator CASCADE [65] the off-shell production amplitude [58] was used with the full CCFM evolution [59]. Recently, it was demonstrated [60] that the $k_T$-factorization approach supplemented with the CCFM gluon dynamics is able to describe first (preliminary) data [66] on the inclusive Higgs production in the diphoton decay mode taken by the ATLAS Collaboration at the LHC. The effect of taking into account higher-order corrections in the $k_T$-factorization approach at LO was pointed out [55,57,60,61]. The CMS [12] and ATLAS data [14] for Higgs boson production in the four-lepton decay mode were considered [61].

Our present consideration is based on the off-shell amplitude of the gluon-gluon fusion subprocess $g^*g^* \to H$ [55]. The latter was extended further to the subsequent diphoton [60] and four-lepton Higgs boson decays [61]. Below we will derive the expressions for off-shell $g^*g^* \to H \to W^+W^- \to e^\pm\mu^\mp\nu\bar{\nu}$ and $g^*g^* \to H \to ZZ^* \to 4l$ (where $l = e, \mu$) amplitudes (independently from [61]). Then, to calculate the Higgs boson production cross section we convolute these amplitudes with the TMD gluon densities in a proton, taken from the numerical solution of the CCFM equation [67]. As an alternative choice, we will use the TMD gluon densities evaluated in accordance with the KMR prescription [64]. Our main motivation is that the latest CMS [11,13] and ATLAS [14,16] data taken at $\sqrt{s} = 8$ TeV (referring to $H \to \gamma\gamma$ and $H \to W^+W^- \to e^\pm\mu^\mp\nu\bar{\nu}$ decay channels) as well as preliminary data [17–20] obtained at $\sqrt{s} = 13$ TeV have not been considered yet in the framework of $k_T$-factorization. Additionally, detailed studying of the Higgs transverse momentum

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3The preliminary ATLAS data [66] on the Higgs boson transverse momentum distribution were discussed also [62]. However, the calculations [62] are based on rather old CCFM-evolved TMD gluon density function and, in our opinion, suffer from double counting.
distributions in the different kinematical regimes of different decay channels could impose constraints on the TMD gluon density (see [55,60,61]).

The outline of our paper is as follows. In Sec. II we recall the basic formulas of $k_T$-factorization approach and briefly describe the calculation steps. In Sec. III we present our numerical results and discussion. Section IV contains our conclusions.

II. THE MODEL

Let us start from a short review of the calculation steps. We describe first the evaluation of $g'g' \to H \to ZZ^* \to 4l$ and $g'g' \to H \to W^+W^− \to e^±\mu^±\nu\bar{\nu}$ off-shell production amplitudes. The effective Lagrangian for the Higgs boson coupling to gluons in the limit of large top quark mass $m_t \to \infty$ reads [63,68]

$$\mathcal{L}_{ggH} = \frac{\alpha_s}{12\pi} (G_F \sqrt{2})^{1/2} G_{\mu\nu}^{ab} H, \quad (1)$$

where $G_F$ is the Fermi coupling constant, $G_{\mu\nu}^{ab}$ is the gluon field strength tensor, and $H$ is the Higgs scalar field. The large $m_t$ approximation is valid to an accuracy of a few percent in the mass range $m_H < 2m_t$, and, of course, is applicable at the $m_H \sim 125$ GeV [11–20]. The triangle vertex for two off-shell gluons having four-momenta $k_1$ and $k_2$ and color indices $a$ and $b$ thus takes the form [63,68]:

$$T_{ggH}^{ab} (k_1, k_2) = i\delta^{ab} \frac{\alpha_s}{3\pi} (G_F \sqrt{2})^{1/2} [(k_2^\mu k_1^\nu - (k_1 \cdot k_2) g_{\mu\nu})]. \quad (2)$$

Using (2) and taking into account the nonzero transverse momenta of initial gluons $k_{1T}^2 ≠ 0$ and $k_{2T}^2 ≠ 0$, one can easily obtain the off-shell production amplitudes squared for considered subprocesses. The latter can be written in a compact form:

$$|\mathcal{M}|^2 = \frac{8\alpha_s^2}{9\pi^2} G_F \sqrt{2} (4\pi \alpha)^3 m_Z^2 C_V \frac{(\hat{s} + p_T^2)^2}{(\hat{s} - m_H^2)^2 + m_Z^4 \Gamma_H^2} \cos^2 \phi \times \frac{2q_1^2 q_2^2 q_3^2 q_4^2 (p_1 \cdot p_4)(p_2 \cdot p_3) + (g_{(V,L)}^2 + g_{(V,R)}^2)(p_1 \cdot p_3)(p_2 \cdot p_4)}{[(q_1^2 - m_V^2)^2 + \Gamma_V^2 m_V^2][(q_2^2 - m_V^2)^2 + \Gamma_V^2 m_V^2]}, \quad (3)$$

where $g_{(V,L)} = \sin^2 \theta_W$, $g_{(V,R)} = 0$. \quad (7)

The propagators of the intermediate Higgs and electroweak bosons are taken in the Breit-Wigner form to avoid any artificial singularities in the numerical calculations. According to the $k_T$-factorization prescription [40,41], the summation over the polarizations of initial off-shell gluons is carried out with

$$\sum e^ie^{−i} = \frac{k_T^2}{k_T^2}. \quad (8)$$

In the limit $k_T \to 0$ this expression converges to the ordinary one after averaging on the azimuthal angle. In all other respects the calculations are quite straightforward and follow the standard QCD Feynman rules. In the case of Higgs four-lepton decay $H \to ZZ^* \to 4l$, the obtained expression (3) coincides with the one [61]. The off-shell production amplitude for $g'g' \to H \to \gamma\gamma$ subprocess was calculated earlier [60].

To calculate the cross sections of the considered processes in the $k_T$-factorization approach one should convolute corresponding off-shell partonic cross sections with the TMD gluon densities in a proton. Our master formula for $H \to ZZ^* \to 4l$ and $H \to W^+W^− \to e^±\mu^±\nu\bar{\nu}$ decay channels reads:

$\sum e^ie^{−i} = \frac{k_T^2}{k_T^2}$. \quad (8)
\[
\sigma = \frac{1}{(2\pi)^8} \int \frac{\lambda^{1/2}(s, q_1^2, q_2^2)}{512 x_1 x_2 s^2 \lambda^{1/2}(\bar{s}, k_1^2, k_2^2)} f_{g}(x_1, k_{1T}^2, \mu^2) f_{g}(x_2, k_{2T}^2, \mu^2) |\hat{M}|^2 \times d\Omega_2 d\Omega_1 d\Omega d\Omega_2 \frac{d\phi_1 d\phi_2}{2\pi},
\]

where \( f_{g}(x, k_T^2, \mu^2) \) is the TMD gluon density, \( s \) is the total center-of-mass energy, \( y \) is the Higgs boson rapidity, \( \Omega \) is the decay solid angle of a vector boson in the Higgs boson rest frame, \( \Omega_1 \) and \( \Omega_2 \) are the decay solid angles of produced leptons in corresponding electroweak boson rest frame, \( \phi_1 \) and \( \phi_2 \) are the azimuthal angles of incoming off-mass shell gluons having the fractions \( x_1 \) and \( x_2 \) of the longitudinal momenta of colliding protons, \( \lambda(x, y, z) \) is the kinematical function [69]. The cross section of the inclusive Higgs production in the diphoton decay mode can be written as:

\[
\sigma = \frac{1}{2\pi} \int \frac{1}{16 x_1 x_2 s^{1/2}(\bar{s}, k_1^2, k_2^2)} f_{g}(x_1, k_{1T}^2, \mu^2) f_{g}(x_2, k_{2T}^2, \mu^2) \times |\hat{M}|^2 \times d\Omega_2 d\Omega_1 d\Omega d\Omega_2 \frac{d\phi_1 d\phi_2}{2\pi},
\]

where \( \Omega \) is the decay solid angle of produced photon in the Higgs boson rest frame. This expression is more convenient for narrow Higgs resonance than the one used earlier [60].

Concerning the TMD gluon density functions in a proton, we have tested a few sets. First of them (JH 2013 set 2) was obtained [67] from the numerical solution of the CCFM equation. The latter seems to be the most suitable tool for our consideration because it smoothly interpolates between the small-\( x \) BFKL gluon dynamics and conventional DGLAP one, as it was mentioned above. The input parameters of starting (initial) gluon distribution were fitted to describe the high-precision deep inelastic scattering data on proton structure functions \( F_2(x, Q^2) \) and \( F_2^u(x, Q^2) \) [67]. The fit is based on TMD matrix elements and involves the two-loop strong coupling constant, kinematic consistency constraint [70,71], and nonsingular terms in the CCFM gluon splitting function [72]. Below we use this TMD gluon distribution as default choice. Additionally, as an alternative choice, we apply the TMD gluon density obtained from the KMR prescription [64]. The KMR approach is a formalism to construct the TMD quark and gluon densities from well-known conventional ones. The key assumption of this approach is that the \( k_T \)-dependence of the TMD parton distributions enters at the last evolution step, so that the DGLAP evolution can be used up to this step. For the input, we used Martin-Stirling-Thorn-Watt (MSTW 2008 LO) set [74].

Other essential parameters were taken as follows: the renormalization and factorization scales \( \mu_R^2 = \xi^2 m_Z^2 \) and \( \mu_F^2 = \bar{s} + \xi Q_T^2 \), where \( Q_T^2 \) is the transverse momentum of the incoming off-shell gluon pair. To estimate the scale uncertainties of numerical calculations, we vary the unphysical parameter \( \xi \) between 1/2 and 2 about the default value \( \xi = 1 \). Following [75], we set electroweak bosons masses \( m_Z = 91.1876 \text{ GeV} \) and \( m_W = 80.403 \text{ GeV} \), their total decay widths \( \Gamma_Z = 2.4952 \text{ GeV} \) and \( \Gamma_W = 2.085 \text{ GeV} \). Additionally, we use Higgs boson mass \( m_H = 126.8 \text{ GeV} \), its full decay width \( \Gamma_H = 4.3 \text{ MeV} \), \( \sin^2 \theta_W = 0.23122 \), and adopt the LO formula for the strong coupling constant \( \alpha_s(\mu^2) \) with \( n_f = 4 \) active quark flavors at \( \Lambda_{QCD} = 200 \text{ MeV} \), so that \( \alpha_s(\mu^2) = 0.1232 \). Note that we use the running QED coupling constant \( \alpha(\mu^2) \). Finally, following [57], to take into account the nonlogarithmic loop corrections to the Higgs production cross section we apply the effective \( K \)-factor when using the KMR gluon density:

\[
K = \exp \left[ A \frac{\alpha_s(\mu^2)}{2\pi} \right],
\]

where the color factor \( A = 3 \). A particular scale choice \( \mu^2 = p_T^{13/2}/5^{2/3} \) (with \( p_T \) being the transverse momentum of produced Higgs boson) has been proposed [57] to eliminate subleading logarithmic terms. We choose this scale to evaluate the strong coupling constant in (11) only. The multidimensional integration everywhere was performed by means of a Monte Carlo technique, using the routine VEGAS [76].

### III. NUMERICAL RESULTS

Now we are in a position to present our numerical results and discussion. Let us consider first the Higgs boson production in the diphoton decay mode.

#### A. \( H \to \gamma\gamma \) decay mode

All cross sections were measured in a restricted part of the phase space (fiducial phase space) defined to match the

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5 There was a missing factor 1/2 in (10) of [60], which is due to identity of the final state photons. The numerical results [60] have been corrected recently, conclusions unchanged.

6 At the moment, there is a large variety of proposed TMD gluon distribution functions in a proton. Most of them is collected in the TMDLIB package [73], which is a \( C++ \) library providing a framework and an interface to the different parametrizations.

7 The special choice for \( \mu_F \) scale is connected with the CCFM evolution [67].
experimental acceptance in terms of the photon kinematics and topological event selection. We implemented the experimental setup used by the CMS and ATLAS Collaborations in our numerical program. In the CMS analysis [11] performed at $\sqrt{s} = 8$ TeV two isolated photons originating from the Higgs boson decays are required to have pseudorapidities $|\eta| < 2.5$. Additionally, photons with largest and next-to-largest transverse momentum $p_T^\gamma$ (so-called leading and subleading photons) must satisfy the conditions of $p_T^\gamma / m_{\gamma\gamma} > 1/3$ and $p_T^\gamma / m_{\gamma\gamma} > 1/4$, respectively, where $m_{\gamma\gamma}$ is the diphoton pair mass. In the ATLAS measurement [14] performed at $\sqrt{s} = 8$ TeV both of these decay photons must have pseudorapidities $|\eta| < 2.37$ with the leading (subleading) photon satisfying $p_T^\gamma / m_{\gamma\gamma} > 0.35$ (0.25), while invariant mass $m_{\gamma\gamma}$ is required to be $105 < m_{\gamma\gamma} < 160$ GeV.

The same kinematical cuts were applied in the preliminary measurements performed by the CMS [17] and ATLAS [19] Collaborations at $\sqrt{s} = 13$ TeV, with the only exception being that invariant mass $m_{\gamma\gamma}$ in the CMS analysis [17] should lie in the range $100 < m_{\gamma\gamma} < 180$ GeV. The diphoton pair transverse momentum $p_T^{\gamma\gamma}$, absolute value of the rapidity $|y^{\gamma\gamma}|$, photon helicity angle cos $\theta^*/C^3$ (in the Collins-Soper frame) and difference in azimuthal angle $\Delta \phi^{\gamma\gamma}$ between the produced photons were measured [11,14,17,19]. Both $p_T^{\gamma\gamma}$ and $y^{\gamma\gamma}$ probe the production mechanism and parton distribution functions in a proton, while cos $\theta^*$ and $\Delta \phi^{\gamma\gamma}$ are related to properties (namely, spin-CP nature) of the decaying Higgs boson.

The results of our calculations are shown in Figs. 1–3 in comparison with the LHC data. The solid histograms were obtained with the JH’2013 set 2 gluon density by fixing both the renormalization $\mu_R$ and factorization $\mu_F$ scales at the default values, while shaded regions correspond to scale uncertainties of our predictions. Following to [67], to estimate the latter we used the JH’2013 set 2+ and JH’2013 set 2-sets instead of default one. These two sets
represent a variation of the renormalization scale used in the off-shell production amplitude. The JH’2013 set 2+ set stands for a variation of $2\mu_R$, while set JH’2013 set 2- reflects $\mu_R/2$ (see also [67] for more information). One can see that the $k_T$-factorization predictions reasonably agree with the LHC data within the experimental and theoretical uncertainties for all considered kinematical observables, although some tendency to slightly underestimate the ATLAS data (see Fig. 2) and CMS data at large transverse momenta $p_T^\gamma \gamma$ is observed for both c.m. energies $\sqrt{s} = 8$ and 13 TeV. It could be due to the missing contributions from the weak boson fusion ($W^+W^- \rightarrow H$ and $ZZ \rightarrow H$) and/or associated $HZ$ or $HW^\pm$ production [62], which become important at high $p_T^\gamma \gamma$ and not taken into account in the present consideration. Our results for $y^\gamma$ and $\cos \theta^*$ distributions obtained with the JH’2013 set 2 gluon at $\sqrt{s} = 8$ TeV are consistently close to the matched NNLO + NNLL pQCD predictions obtained using the HRES routine [77] within the collinear QCD factorization (but a bit higher). Our predictions at $\sqrt{s} = 13$ TeV are similar to the NNLOPS and aMC@NLO ones.\(^8\) It can be explained by the fact that the main part of collinear QCD higher-order corrections (namely, NLO + NNLO + $N^3$LO + · · · contributions which correspond to the log $1/x$ enhanced terms in perturbative series) are effectively taken into account as a part of the CCFM gluon evolution. The corresponding variable $R$, which can be defined as a ratio between the $k_T$-factorization and LO pQCD predictions, is about of $R \sim 2.7–3.2$ in the considered kinematical region (see Fig. 4). The ratio $R$ reflects the role of log $1/x$-enhanced terms involved into these predictions. The calculations based on the alternative KMR gluon density also tend to underestimate the ATLAS data at small $p_T^\gamma \gamma$, although they describe well the CMS data and ATLAS data at high transverse momenta. Moreover, we find that these predictions (mainly for distributions in $y^\gamma$ or $\cos \theta^*$) are generally similar to the lower uncertainty bounds of matched NNLO + NNLL (and NNLOPS or aMC@NLO) predictions obtained using the CMS [11,14] and ATLAS [17,19] papers.

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\(^8\)We take these predictions from the CMS [11,14] and ATLAS [17,19] papers.
FIG. 3. The differential cross sections of inclusive Higgs boson production (in the diphoton decay mode) at $\sqrt{s} = 13$ TeV as functions of diphoton pair transverse momentum $p_T^{\gamma\gamma}$, rapidity $|y^{\gamma\gamma}|$, and photon helicity angle $\cos \theta^*\gamma$ in the Collins-Soper frame. Notation of histograms and curves is the same as in Fig. 1. The preliminary experimental data are from CMS [17] and ATLAS [19]. The NNLOPS [34,35] and aMC@NLO [70] predictions are taken from [17,19].

FIG. 4. The ratio between the $k_T$-factorization and LO pQCD predictions for inclusive Higgs boson production (in the diphoton decay mode) at $\sqrt{s} = 8$ TeV as function of photon helicity angle $\cos \theta^*\gamma$ and photon pair rapidity. Similar ratio for HRES calculations is shown for comparison. The kinematical conditions of the CMS experiment are applied. Notation of histograms and curves is the same as in Fig. 1. The HRES [69] predictions are taken from [12].
pQCD calculations. This can be explained from the fact that the KMR procedure absorbs only single gluon emission at the last step of evolution (or, in other words, initial state gluon emission closest to the produced Higgs boson), that corresponds to the taking into account of \( \ln 1/x \) enhanced NLO contributions only. One can see that the shapes of \( \phi_{\gamma\gamma} \) or \( \cos \theta \) distributions calculated using the CCFM-evolved and KMR gluon densities practically coincide, and, therefore, the difference between the JH'2013 set 2 and KMR predictions for these observables can illustrate the role of conventional high-order contributions above the NLO level. Here we demonstrate again the main advantage of the \( k_T \)-factorization approach, which gives us the possibility to estimate the size of higher-order corrections and reproduce in a straightforward manner the main features of cumbersome fixed-order pQCD calculations. In contrast, one can see that the shapes of \( p_T^{\gamma\gamma} \) distributions predicted by the JH'2013 set 2 and KMR gluon densities are very different from each other. Of course, it is not surprising since the Higgs boson transverse momentum is strongly related to the initial gluon transverse momenta [53,55–61]. The importance of this observable to distinguish between the different noncollinear evolution scenarios was pointed out [55,60,61]. Moreover, the difference in azimuthal angle \( \Delta \phi_{\gamma\gamma} \) is also very sensitive to the initial gluon transverse momenta (see Fig. 1). Such sensitivity is well-known and was demonstrated earlier for number of processes (see, for example, [54] and references therein). Thus, we confirm the previous conclusions [55,60,61] that these observables can impose constraints on the TMD gluon densities of the proton.

The estimated Higgs boson fiducial cross sections at \( \sqrt{s} = 8 \) and 13 TeV are listed in Tables I and II in comparison with the available data and conventional high-order pQCD calculations performed using the HRES [77], NNLOPS [34,35], and aMC@NLO [78] tools. One can see that the \( k_T \)-factorization predictions are close to corresponding fixed-order collinear pQCD results and agree well with the LHC data within the theoretical and experimental uncertainties. The scale dependence of the \( k_T \)-factorization predictions (especially obtained with the KMR gluon density) is significant and exceeds the uncertainties of conventional fixed-order pQCD calculations (which are about of 10–11\%).\(^9\) However, it could be easily understood because only the tree-level LO hard scattering amplitudes are involved. Moreover, it was argued [65] that amending the leading-logarithmic evolution with different kinematical constraints should lead to reasonable QCD predictions, although still formally only in leading logarithmic accuracy (see also [54]).

\(^9\)Note that scale uncertainties of the CCFM-based predictions are comparable with the ones of higher-order collinear pQCD calculations.

| Source | \( \sigma_{\text{fid}} \) (CMS) [fb] | \( \sigma_{\text{fid}} \) (ATLAS) [fb] |
|--------|-------------------------------|-------------------------------|
| \( k_T \)-fact., JH'2013 set 2 | 31.12\( ^{+7.41} \)\(-^{4.43} \) | 29.62\( ^{+4.31} \)\(-^{3.32} \) |
| \( k_T \)-fact., KMR | 22.47\( ^{+11.98} \)\(-^{8.47} \) | 21.38\( ^{+11.24} \)\(-^{8.01} \) |
| Fixed-order pQCD | 31\( ^{+3} \)\(-^{4} \) | 30.5±3.3 |
| Measurement | 32±10(stat.) | 43.2±9.4(stat.) |

Table I. The fiducial cross sections of inclusive Higgs boson production (in the diphoton decay mode) at \( \sqrt{s} = 8 \) TeV. The experimental data are from CMS [11] and ATLAS [14]. The results obtained in the collinear pQCD factorization (taken from [11,14]) are shown for comparison.

| Source | \( \sigma_{\text{fid}} \) (CMS) [fb] | \( \sigma_{\text{fid}} \) (ATLAS) [fb] |
|--------|-------------------------------|-------------------------------|
| \( k_T \)-fact., JH'2013 set 2 | 69.96\( ^{+7.11} \)\(-^{5.53} \) | 68.23\( ^{+6.69} \)\(-^{5.59} \) |
| \( k_T \)-fact., KMR | 50.78\( ^{+24.48} \)\(-^{17.99} \) | 47.91\( ^{+23.59} \)\(-^{17.30} \) |
| Fixed-order pQCD | 75±4 | 62.8\( ^{+3.4} \)\(-^{4.4} \) |
| Measurement | 84±11(stat.) | 43.2±14.9(stat.) |

Table II. The fiducial cross sections of inclusive Higgs boson production (in the diphoton decay mode) at \( \sqrt{s} = 13 \) TeV. The preliminary experimental data are from CMS [17] and ATLAS [19]. The results obtained in the collinear pQCD factorization (taken from [17,19]) are shown for comparison.

In the case of JH'2013 gluon density, the uncertainty band of our predictions is rather asymmetric (see Figs. 1–3 and Tables I and II), that is, in contrast with the KMR gluon and conventional high-order pQCD calculations. The source of these asymmetrical uncertainties is connected with using the different TMD gluon densities (JH'2013 set 2- and JH'2013 set 2+) when varying the renormalization scale. In fact, the scale variation as described above but with default JH'2013 set 2 gluon results in more symmetric uncertainty band: \( \sigma = 31.12\( ^{+6.63} \)\(-^{4.99} \) fb (compare with \( \sigma = 31.12\( ^{+7.41} \)\(-^{4.43} \) fb from Table I) and \( \sigma = 29.62\( ^{+6.31} \)\(-^{4.75} \) fb (compare with \( \sigma = 29.62\( ^{+4.31} \)\(-^{3.32} \) fb) at \( \sqrt{s} = 8 \) TeV in the CMS and ATLAS fiducial regions, respectively. The method to estimate the theoretical uncertainties in the CCFM-based approach is described in detail [67]. This method is somewhat different from the one usually applied in determinations of conventional parton density functions. So, fit procedure to generate JH'2013 set 2- and JH'2013 set 2+ gluons leads to the observed almost asymmetric uncertainty band, at least in the kinematical region probed (see also [79]).

**B. \( H \to ZZ^* \to 4l \) and \( H \to W^+W^- \to e^\pm \mu^\mp \bar{\nu}\nu \) decay channels**

Now we turn to the \( H \to ZZ^* \to 4l \) and \( H \to W^+W^- \to e^\pm \mu^\mp \bar{\nu}\nu \) decay channels. The data for the first of them come
The invariant mass \( m \) events with a four-lepton invariant mass \( m_{ll} > 118 \) GeV and the remaining ones having \( m_{ll} > 120 \) GeV are kept and each lepton (electron or muon) must satisfy transverse momentum cut \( p_T > 6 \) GeV and be in the pseudorapidity range \( |\eta| < 2.47 \). The highest-\( p_T \) lepton in the quadruplet must have \( p_T > 20 \) GeV and the second (third) lepton in \( p_T \) order must satisfy \( p_T > 15(10) \) GeV. These leptons are required to be separated from each other by \( \Delta R = \sqrt{(\Delta \eta)^2 + (\Delta \phi)^2} > 0.1(0.2) \) when having the same (different) lepton flavors. The invariant mass \( m_{12} \) of the lepton pair closest to the Z boson mass (leading pair) is required to be \( 50 < m_{12} < 106 \) GeV. The subleading pair is chosen as the remaining lepton pair with invariant mass \( m_{34} \) closest to the Z boson mass and satisfying the requirement \( 12 < m_{34} < 115 \) GeV. The CMS measurement [12] performed at the same energy \( \sqrt{s} = 8 \) TeV requires at least four leptons in the event with at least one lepton having \( p_T > 20 \) GeV, another lepton having \( p_T > 10 \) GeV and the remaining ones having \( p_T > 7 \) and 5 GeV, respectively. All leptons must have the pseudorapidity \( |\eta| < 2.4 \), the leading pair invariant mass \( m_{12} \) must be \( 40 < m_{12} < 120 \) GeV, and subleading one should be \( 12 < m_{34} < 120 \) GeV. Finally, the four-lepton invariant mass \( m_{ll} \) must satisfy \( 105 < m_{ll} < 140 \) GeV cut. Such cuts allow one to identify the decay leptons as originating from different Z bosons (real and virtual) and the interference effects in case of the production of identical leptons thus can be neglected.10 Similar to the diphoton decay, the measurements are performed in several observables related to the Higgs boson production and decay, namely the Higgs transverse momentum \( p_T^H \) and rapidity \( |y^H| \), invariant mass of the subleading lepton pair \( m_{34} \), and cosine of the leading lepton pair decay angle \( \cos \theta^l \) in the four-lepton rest frame with respect to the beam axis. While the distributions in the \( p_T^H \) and \( |y^H| \) observables are sensitive to the production mechanism and gluon densities in a proton, the distributions in the decay variables \( m_{34} \) and \( |\cos \theta^l| \) are sensitive to the Lagrangian structure of Higgs interaction (spin/CP quantum numbers and higher-dimensional operators). In the ATLAS analysis [16] performed at \( \sqrt{s} = 8 \) TeV for the \( H \rightarrow W^+W^- \rightarrow e^\pm \mu^\mp \nu \bar{\nu} \) decay channel, events are selected from those with exactly one electron and one muon with opposite charge, a dilepton invariant mass \( 10 < m_{ll} < 55 \) GeV, azimuthal angle difference \( \Delta \phi^{ll} < 1.8 \) and missing transverse momentum (which is produced by the two neutrinos from the W boson decays) \( p_T^{miss} > 20 \) GeV. The leading lepton is required to have \( p_T > 22 \) GeV, the other one is required to have \( p_T > 15 \) GeV and both of them should be in the range \( |\eta| < 2.47 \). The CMS analysis [13] requires \( p_T > 20(10) \) GeV for the leading (subleading) leptons with \( |\eta| < 2.5 \), lepton pair invariant mass \( m_{ll} > 12 \) GeV, their transverse momentum \( p_T^{ll} > 30 \) GeV, and invariant mass of the leptonic system in the transverse plane \( m_{ll}^{trans} > 50 \) GeV. The differential cross sections were measured as functions of Higgs boson transverse momentum \( p_T^H \) and absolute value of the dilepton rapidity \( |y^{ll}| \). The latter is highly correlated to the Higgs boson rapidity \( y^H \) which can not be reconstructed experimentally in the \( H \rightarrow W^+W^- \rightarrow e^\pm \mu^\mp \nu \bar{\nu} \) final state. Of course, all the experimental cuts listed above are taken into account in the numerical evaluations. The preliminary data reported by the CMS [18] and ATLAS [20] Collaborations at \( \sqrt{s} = 13 \) TeV were obtained using similar analysis strategy.

10Incorrect identification is possible but happens only in approximately 5% of events [15].
The results of our calculations are shown in Figs. 5–9 in comparison with the data. The estimated total cross sections are listed in Tables III–V. Similar to $H \to \gamma\gamma$ decay, the $k_T$-factorization predictions for $H \to ZZ \to 4l$ and $H \to W^+W^- \to e^\pm\mu^\mp\nu\bar{\nu}$ decay modes agree well with the LHC data taken at $\sqrt{s} = 8$ TeV for all considered kinematical observables within the theoretical and experimental uncertainties. The best description of the data is achieved with the CCFM-evolved JH’2013 set 2 gluon density. Moreover, the overall agreement between these predictions and the preliminary ATLAS data [20] taken at $\sqrt{s} = 13$ TeV looks to be even a bit better than the one given by the NNLO pQCD calculations (see Fig. 9), that could be essentially due to the small-$x$ region probed. The KMR approach results in lower cross sections compared to the JH’2013 set 2 calculations since only single gluon emission in the initial state is taken into account here. Good agreement is also observed in the normalized differential cross sections $1/\sigma d\sigma/dp_T^H$ and $1/\sigma d\sigma/d|y^H|$ (see Fig. 8).

Studying of the normalized differential cross sections leads to a more stringent comparison between data and theory due to reduced experimental (mainly systematic) uncertainties. As it was expected, the distributions on the Higgs boson transverse momentum are highly sensitive to the TMD gluon densities applied in the numerical calculations and therefore can be used to discriminate between the latter. In contrast, the predicted shapes of rapidity and $\cos \theta^*$ distributions are almost insensitive to the TMD gluon density in a proton. The KMR predictions for these distributions are rather similar to the lower uncertainty bounds of the NNLOPS calculations, whereas the JH’2013 set 2 ones slightly overshoot them. This fact demonstrates again the role of $\ln 1/x$-enhanced NNLO + $N^3$LO + $\cdots$ terms taken into account in the CCFM gluon evolution.

Finally, we would like to note that a similar study (but using the $H \to ZZ \to 4l$ decay channel only) was done very recently [61]. Unlike our choice, older version of CCFM-evolved gluon density in a proton (namely, set

FIG. 6. The differential cross sections of inclusive Higgs boson production (in the $H \to ZZ \to 4l$ decay mode) at $\sqrt{s} = 8$ TeV as functions of Higgs transverse momentum $p_T^H$, rapidity $|y^H|$, leading lepton pair decay angle $\cos \theta^*$ (in the Collins-Soper frame), and invariant mass $m_{34}$ of the subleading lepton pair. Notation of histograms and curves is the same as in Fig. 1. The experimental data are from ATLAS [15]. The HRES [69] and MINLO HJ [71] predictions are taken from [15].

The results of our calculations are shown in Figs. 5–9 in comparison with the data. The estimated total cross sections are listed in Tables III–V. Similar to $H \to \gamma\gamma$ decay, the $k_T$-factorization predictions for $H \to ZZ \to 4l$ and $H \to W^+W^- \to e^\pm\mu^\mp\nu\bar{\nu}$ decay modes agree well with the LHC data taken at $\sqrt{s} = 8$ TeV for all considered kinematical observables within the theoretical and experimental uncertainties. The best description of the data is achieved with the CCFM-evolved JH’2013 set 2 gluon density. Moreover, the overall agreement between these predictions and the preliminary ATLAS data [20] taken at $\sqrt{s} = 13$ TeV looks to be even a bit better than the one given by the NNLO pQCD calculations (see Fig. 9), that could be essentially due to the small-$x$ region probed. The KMR approach results in lower cross sections compared to the JH’2013 set 2 calculations since only single gluon emission in the initial state is taken into account here. Good agreement is also observed in the normalized differential cross sections $1/\sigma d\sigma/dp_T^H$ and $1/\sigma d\sigma/d|y^H|$ (see Fig. 8).
FIG. 7. The differential cross sections of inclusive Higgs production (in the $H \to W^+W^- \to e^\pm\mu^\mp\nu\bar{\nu}$ decay mode) at $\sqrt{s} = 8$ TeV as functions of Higgs transverse momentum and lepton pair rapidity. Notation of histograms and curves is the same as in Fig. 1. The experimental data are from CMS [13] and ATLAS [16]. The HRES [69] and NNLOPS [34,35] predictions are taken from [13,16].

FIG. 8. The normalized differential cross sections of inclusive Higgs production (in the $H \to W^+W^- \to e^\pm\mu^\mp\nu\bar{\nu}$ decay mode) at $\sqrt{s} = 8$ TeV as functions of Higgs transverse momentum and lepton pair rapidity. Notation of histograms and curves is the same as in Fig. 1. The experimental data are from ATLAS [16]. The NNLOPS [34,35] predictions are taken from [16].
A0) [80] was applied in these calculations. We reproduce the results [61] when using the A0 gluon. We note also that we do not try to give better predictions than the fixed-order pQCD calculations. One of our goals is to extend the applicability of the $k_T$-factorization approach by including the $H \rightarrow W^+W^- \rightarrow e^\pm \mu^\mp \nu \bar{\nu}$ decay mode, not investigated yet in the $k_T$-factorization formalism, into the consideration.

FIG. 9. The differential cross sections of inclusive Higgs production (in the $H \rightarrow ZZ^* \rightarrow 4l$ decay mode) at $\sqrt{s} = 13$ TeV as functions of Higgs boson transverse momentum $p_T$, rapidity $|y_H|$, leading lepton pair decay angle $\cos \theta^*$ (in the Collins-Soper frame), and invariant mass $m_{34}$ of the subleading lepton pair. Notation of histograms and curves is the same as in Fig. 1. The preliminary experimental data are from CMS [18] and ATLAS [20]. The HRES [69] and NNLOPS [34,35] predictions are taken from [18,20].
TABLE III. The fiducial cross sections of inclusive Higgs production (in the $H \rightarrow ZZ^* \rightarrow 4l$ decay channel) at $\sqrt{s} = 8$ TeV. The experimental data are from CMS [12] and ATLAS [15]. The results obtained in the collinear pQCD factorization (taken from [12,15]) are shown for comparison.

| Source                  | $\sigma_{\text{fid}}$ (CMS) [fb] | $\sigma_{\text{fid}}$ (ATLAS) [fb] |
|-------------------------|---------------------------------|-----------------------------------|
| $k_T$-fact., JH'2013 set 2 | $1.61^{+0.22}_{-0.01}$          | $1.58^{+0.23}_{-0.01}$            |
| $k_T$-fact., KMR         | $1.22^{+0.59}_{-0.42}$          | $1.20^{+0.58}_{-0.43}$            |
| Fixed-order pQCD         | $1.15^{+0.12}_{-0.13}$          | $1.30 \pm 0.13$                   |
| Measurement              | $1.11^{+0.41}_{-0.35}$ (stat.)$^{+0.08}_{-0.02}$ (mod.) | $2.11^{+0.53}_{-0.45}$ (stat.)$^{+0.08}_{-0.02}$ (syst.) |

TABLE IV. The fiducial cross sections of inclusive Higgs production (in the $H \rightarrow W^+W^- \rightarrow e^+\mu^+\nu\bar{\nu}$ decay channel) at $\sqrt{s} = 8$ TeV. The experimental data are from CMS [13] and ATLAS [16]. The results obtained in the collinear pQCD factorization (taken from [13,16]) are shown for comparison.

| Source                  | $\sigma_{\text{fid}}$ (CMS) [fb] | $\sigma_{\text{fid}}$ (ATLAS) [fb] |
|-------------------------|---------------------------------|-----------------------------------|
| $k_T$-fact., JH'2013 set 2 | $54.47^{+8.20}_{-8.48}$         | $34.02^{+5.88}_{-5.38}$           |
| $k_T$-fact., KMR         | $40.80^{+2.15}_{-2.21}$         | $27.38^{+1.37}_{-1.37}$           |
| Fixed-order pQCD         | $48 \pm 8$                      | $25.1 \pm 2.6$                    |
| Measurement              | $39 \pm 8$ (stat.)$^{+9}_{-9}$ (syst.) | $36.0 \pm 7.2$ (stat.)$^{+6.4}_{-6.4}$ (syst.)$^{+1.0}_{-1.0}$ (lumi.) |

TABLE V. The fiducial cross sections of inclusive Higgs production (in the $H \rightarrow ZZ^* \rightarrow 4l$ decay channel) at $\sqrt{s} = 13$ TeV. The preliminary experimental data are from CMS [18] and ATLAS [20]. The results obtained in the collinear pQCD factorization (taken from [18,20]) are shown for comparison.

| Source                  | $\sigma_{\text{fid}}$ (CMS) [fb] | $\sigma_{\text{fid}}$ (ATLAS) [fb] |
|-------------------------|---------------------------------|-----------------------------------|
| $k_T$-fact., JH'2013 set 2 | $3.61^{+0.33}_{-0.01}$          | $3.84^{+0.38}_{-0.02}$            |
| $k_T$-fact., KMR         | $2.71^{+0.17}_{-0.09}$          | $2.83^{+0.18}_{-0.09}$            |
| Fixed-order pQCD         | $2.76 \pm 0.14$                 | $2.91 \pm 0.13$                   |
| Measurement              | $2.92^{+0.28}_{-0.24}$ (stat.)$^{+0.25}_{-0.20}$ (syst.) | $3.62^{+0.53}_{-0.45}$ (stat.)$^{+0.25}_{-0.20}$ (syst.) |

IV. CONCLUSIONS

We investigated the inclusive Higgs boson production in $pp$ collisions at the LHC using the $H \rightarrow \gamma\gamma$, $H \rightarrow ZZ^* \rightarrow 4l$ and $H \rightarrow W^+W^- \rightarrow e^+\mu^+\nu\bar{\nu}$ decay channels in the framework of the $k_T$-factorization approach. Our consideration was based on the dominant off-shell gluon-gluon fusion subprocess where the transverse momenta of initial gluons are taken into account. The essential part of our analysis was using of the TMD gluon density derived from the CCFM evolution equation. The latter seems to be the most suitable tool for our consideration because it smoothly interpolates between the small-$x$ BFKL gluon dynamics and conventional DGLAP one, which is valid at large $x$. Using the CCFM-evolved gluon density, we have achieved a reasonably good description of the latest data taken by the CMS and ATLAS Collaborations at $\sqrt{s} = 8$ TeV and recent preliminary data taken at $\sqrt{s} = 13$ TeV. The theoretical uncertainties of our calculations were estimated and comparison with the high-order pQCD predictions (up to NNLO + NNLL level) obtained within the collinear factorization was done. We have illustrated the effect of taking into account $\ln 1/x$-enhanced higher-order terms in our calculations and demonstrated the strong sensitivity of predicted Higgs transverse momentum distributions to the TMD gluon densities used. Such observables could impose constraints on the latter.

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