A Method to Measure the Unbiased Decorrelation Timescale of the AGN Variable Signal from Structure Functions

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Abstract

A simple, model-independent method to quantify the stochastic variability of active galactic nuclei (AGNs) is the structure function (SF) analysis. If the SF for the timescales shorter than the decorrelation timescale $\tau$ is a single power law and for the longer ones becomes flat (i.e., white noise), then the auto-correlation function (ACF) of the signal can have the form of the power exponential (PE). We show that the signal decorrelation timescale can be measured directly from the SF as the timescale matching the amplitude 0.795 of the flat SF part (at long timescales), and only then is the measurement independent of the ACF PE power. Typically, the timescale has been measured at an arbitrarily fixed SF amplitude, but as we prove, this approach provides biased results, because the AGN SF/power spectral density slopes, and thus the ACF shape, are not constant and depend on either the AGN luminosity and/or the black hole mass. In particular, we show that using such a method for the simulated SFs that includes a combination of empirically known dependencies between the AGN luminosity $L$ and both the SF amplitude and the PE power, and having no intrinsic $\tau/L$ dependence, produces a fake $\tau \propto L^k$ relation with $0.3 \lesssim k \lesssim 0.6$, which otherwise is expected from theoretical works ($k \equiv 0.5$). Our method provides an alternative means for analyzing AGN variability to the standard SF fitting. The caveats, for both methods, are that the light curves must be sufficiently long (with a several year rest frame) and the ensemble SF assumes AGNs to have the same underlying variability process.

Key words: accretion, accretion disks – galaxies: active – methods: data analysis – quasars: general

1. Introduction

Active galactic nuclei (AGNs) are known to be variable sources at all wavelengths (e.g., Mushotzky et al. 1993; Vanden Berk et al. 2004; Barvainis et al. 2005; McHardy et al. 2005; MacLeod et al. 2010; Ackermann et al. 2011; Kozłowski et al. 2016; Vagnetti et al. 2016), but the exact process leading to variability is still unknown, although simulations of accretion disk instabilities (Kawaguchi et al. 1998) have the closest variability pattern to observations in the optical bands (Chen & Taam 1995; Vanden Berk et al. 2004; Kozłowski 2016). What is known, however, is that a typical AGN variability is of the stochastic nature (e.g., Kelly et al. 2009; Andrae et al. 2013; Zu et al. 2013; Kozłowski 2016). This is frequently quantified by means of the power spectral density (PSD) that on the low frequencies shows a flat spectrum (the white noise; $\text{PSD} \propto \nu^0$) and on the high frequencies appears to follow the red noise ($\text{PSD} \propto \nu^{-2}$) or even steeper dependence ($\text{PSD} \propto \nu^{-\alpha}$; e.g., Mushotzky et al. 2011; Kasliwal et al. 2015; Simm et al. 2016).

A similar method of quantifying the AGN variability is the structure function (SF) analysis (e.g., Simonetti et al. 1984, 1985; Hughes et al. 1992; di Clemente et al. 1996; Collier & Peterson 2001; Emmanoulopoulos et al. 2010; MacLeod et al. 2012; Kozłowski 2016a). For a given time interval $\Delta t$ (also called the time lag), all pairs of points are identified, and then the rms of the magnitude differences is calculated. Typically SF, which measures the square root of the rms as a function of the time lag, at short time lags can be described as a single power law (SPL) with a slope of $\gamma \approx 0.5$ in optical-IR bands, corresponding to the PSD SPL slope of $\alpha = -2$ (e.g., Collier & Peterson 2001; MacLeod et al. 2012; Kozłowski 2016a; Kozłowski et al. 2016), and on the long time lags it flattens to the SPL slope of $\gamma = 0$. The time lag at which the SF changes slope is called the decorrelation timescale $\tau$ (also known as the break timescale, or the break frequency for PSD), because for short time lags the data points are correlated and for the longer ones they become uncorrelated.

It is of high interest to study the dependence of the decorrelation timescale on the physical parameters of AGNs, such as the black hole mass, the luminosity and/or the Eddington ratio, or its correlation with the dynamical, thermal, and/or viscous timescales in accretion disks (e.g., Siemiginowska & Czerny 1989; Collier & Peterson 2001; Czerny 2006; King 2008; Kelly et al. 2009; Edelson et al. 2014). But how does one actually measure this? Typically it has been estimated as the time lag at which the SF reaches a certain arbitrarily selected SF amplitude. We will show that this is generally an incorrect procedure (although the only available for short light curves), because the variability process changes with the changing black hole mass and/or the luminosity (Kozłowski 2016a; Simm et al. 2016), and also the SF amplitude is anti-correlated with the luminosity. As we will show, this procedure can lead to a fake relation $\tau \propto L^k$ with $0.3 \lesssim k \lesssim 0.6$, which is otherwise expected from the theory of accretion disks ($\tau \propto L^{0.5}$; e.g., Frank et al. 2002; MacLeod et al. 2010).

In this paper, we present a method that under certain conditions (discussed in Section 3) produces a correct measurement of the decorrelation timescale. If the auto-correlation function (ACF) of the stochastic process is of the power exponential (PE) form (which is a reasonable assumption, as explained in Section 2), one can measure the decorrelation timescale directly from the data via the restframe time lag at which SF reaches the amplitude 0.795 of the flat SF part at long timescales. As we will show, this is an
unbiased measure of the decorrelation timescale, because it always returns the actual decorrelation timescale (and not a biased fraction of it). One can obviously fit the SF to obtain the decorrelation timescale; however, the SF time lag bins are not independent, producing the problems described in Emmanoulopoulos et al. (2010).

In Section 2 we describe the AGN variability, while in Section 3 we discuss various problems related to the SF measurements and interpretations. The paper is summarized in Section 4.

2. Description of Variability

A light curve $y_i$ composed of $i = 1, \ldots, N$ points, measured at times $t_i$, can be represented as a sum of the signal $s_i$ and the noise $n_i$ (i.e., $y_i = s_i + n_i$; see Scargle 1981, 1982, 1989; Rybicki & Press 1992; Press et al. 1992a, 1992b). Empirically, from a light curve we know only $y_i$ and we do not know directly $s_i$. We can study the general properties of the true signal $s_i$ from the data $y_i$ using the covariance function, where we shift the copy of our light curve in time by the time difference (or the time lag) $\Delta t = t_i - t_j$, and the $j$th index is for the copied light curve

$$\text{cov}(y_i, y_j) \equiv \text{var}(y_j) - V(y_i, y_j),$$

where

$$\text{cov}(y_i, y_j) \equiv \langle (y_i - \langle y \rangle)(y_j - \langle y \rangle) \rangle,$$

$$\text{var}(y_i) \equiv \langle (y_i - \langle y \rangle)^2 \rangle,$$

$$V(y_i, y_j) \equiv \frac{1}{2} \langle (y_i - y_j)^2 \rangle.$$  

The covariance of the light curve with itself is the variance $\text{var}(y_i)$, $V(y_i, y_j)$ is the theoretical SF, and $\langle \rangle$ is the summation over all $ij$ pairs in a narrow $\Delta t$ range, divided by the number of such pairs. The theoretical SF is related to typically reported SFs via $\text{SF} = \sqrt{2}V$ (in units of magnitude that have a more natural interpretation).

From the definition and properties of the covariance, we can link the data to the signal via (from Equation (1))

$$V(y_i, y_j) = \text{var}(s_i) + \text{var}(n_i) - \text{cov}(s_i, s_j) - \text{cov}(n_i, n_j) =$$

$$= \sigma_s^2 + \sigma_n^2 - \text{cov}(s_i, s_j),$$

where $\text{var}(s_i) \equiv \sigma_s^2$, $\text{var}(n_i) \equiv \sigma_n^2$ (both the signal and noise are assumed to have the Gaussian properties), and $\text{cov}(s_i, n_i) = \text{cov}(n_i, n_i) \equiv 0$ because the data are assumed here to be uncorrelated with the noise, and the noise is assumed to be uncorrelated with itself. It is also important to note that the process leading to variability must be stationary, because only then the variances and means do not change with time. The covariance function of the signal is related to the autocorrelation function as $\text{ACF}(\Delta t) \equiv \text{cov}(s_i, s_j)/\sigma_s^2$. The measured SF is then

$$\text{SF}_{\text{obs}}(\Delta t) = \sqrt{2\sigma_s^2(1 - \text{ACF}(\Delta t))} + 2\sigma_n^2.$$  

After subtracting the noise term ($2\sigma_n^2$), we have the true SF due to the variable signal only:

$$\text{SF}(\Delta t) = \sqrt{2\sigma_s^2(1 - \text{ACF}(\Delta t))} =$$

$$= \text{SF}_s \sqrt{1 - \text{ACF}(\Delta t)},$$

where $\text{SF}_s = \sqrt{2}\sigma_s$ is the SF amplitude at timescales much longer than the decorrelation timescale (Collier & Peterson 2001; Emmanoulopoulos et al. 2010; MacLeod et al. 2010; Kasliwal et al. 2015). Throughout this manuscript we will discuss the noise-subtracted SFs.

We are interested here in the ACF that has a form of the PE:

$$\text{ACF}(\Delta t) = \exp \left[ -\frac{|\Delta t|}{\tau} \right]^\beta,$$

where $0 < \beta \leq 2$ (e.g., Zu et al. 2013), because it naturally produces an SF that has one SPL slope below the decorrelation timescale and another one (flat SF) for the longer timescales, a pattern observed in AGN SFs. This can be quantified by expanding the ACF into a Taylor series, where the only non-negligible terms for $|\Delta t| \ll \tau$ are $1 - (\text{ACF}(\Delta t))$, so the SF becomes an SPL of the form $\text{SF} = \text{SF}_s/(\Delta t)^{1/\beta}$. For $|\Delta t| \gg \tau$, SF becomes simply $\text{SF} = \text{SF}_s$.

By setting the PE power to $\beta = 1$, the ACF becomes the one for the damped random walk (DRW) model (Kelly et al. 2009; Kozłowski et al. 2010; MacLeod et al. 2010, 2011, 2012; Butler & Bloom 2011; Zu et al. 2011, 2013, 2016; Ruan et al. 2012), which is the simplest of a broader class of continuous-time autoregressive moving average models (Kelly et al. 2014). DRW is currently frequently used to model individual AGN light curves, although the PE power seems to be $\beta > 1$ for bright AGNs and/or massive black holes (Simm et al. 2016; Kozłowski 2016a), causing biases in the measured DRW parameters (Kozłowski 2016b). Also, Graham et al. (2014), by
using the slepián wavelet variance method, identified a PSD break at short timescales and concluded that DRW may be too simplistic to describe AGN variability.

2.1. The Method

It is straightforward to prove that for $\Delta t = \tau$, SF is an unbiased measure (in terms of the underlying process) of the decorrelation timescale, because the exponent then does not depend on $\beta$ and all $0 < \beta < 2$ SFs cross at the same point (Figure 1). The amplitude of this point is $SF = SF_{\infty} \sqrt{1 - \exp(-1)} = 0.795 \, SF_{\infty}$. This simply means that once the measured SF reaches the flat part ($SF_{\infty}$), one can just read off the decorrelation timescale from the SF curve, and it will be correct for the case of PE ACF, regardless of the power.

3. Discussion

Measuring either the SF amplitudes at a fixed timescale (e.g., Vanden Berk et al. 2004; Schmidt et al. 2010; Morganson et al. 2014; Kozłowski et al. 2016) or the timescales at the fixed SF amplitude (e.g., Findeisen et al. 2015; Caplar et al. 2017) provide biased results, because the AGN SF slopes at short time lags (or the PSD slopes at high frequencies) are not constant and appear to depend on either the luminosity and/or the black hole mass (Kozłowski 2016a; Simm et al. 2016). However, if the data are short and/or the break in the SF is not present, this is the only justified procedure to be used.

The AGN variability amplitude is known to be anticorrelated with the luminosity (e.g., Angione & Smith 1972; Uomoto et al. 1976; Hook et al. 1994; Paltani & Courvoisier 1994; Giveon et al. 1999). In particular, Kozłowski (2016a) based on the SF analysis of the 9000 SDSS AGNs showed that the SF amplitude at long timescales (the flat part) is $SF_{\infty} \propto L^{0.3 \pm 0.05}$. This means that with the increase of brightness by one magnitude, the AGN variability amplitude decreases to about 72%. And in fact, the amplitude of the whole SF changes by this amount.

Measuring the decorrelation timescale at a fixed SF amplitude (below 0.795 $SF_{\infty}$) introduces a bias, because for fainter AGNs with higher variability, the measured decorrelation timescale will appear shorter than for the brighter ones, even for the same intrinsic decorrelation timescale (Figure 2, top-left panel). In this example, we measure the time lag at 0.3 $SF_{\infty}$ (the gray horizontal line). For the faint AGNs (that have set $SF_{\infty} = 1.0$ units) we measure 0.094 of the true decorrelation timescale, while for the brighter ones, we measure 0.151 (with set $SF_{\infty} = 0.8$ units). In other words, when SF is decreasing (along the y-axis in Figure 2) because of the increasing $L$, this can be interpreted as a fake increase of $\tau$ (with the increasing $L$) when measuring it at a constant SF level.

While there exists empirical evidence that the decorrelation timescale does not or weakly depend on the AGN luminosity but rather on the black hole mass, $\tau \propto L^{-0.05 \pm 0.17} M^{0.38 \pm 0.15}$ from Kozłowski (2016a), the theoretical predictions point to the form $\tau \propto L^{0.5}$ (Frank et al. 2002). In the top-right panel of Figure 2, we show what would happen if the decorrelation timescale had a positive correlation with the luminosity—namely, the brighter the AGN, the longer the timescale.

Simm et al. (2016) showed that the PSD slope steepens with the increasing black hole mass, and Kozłowski (2016a) showed that the SF slope ($\gamma = \beta/2$) steepens with the increasing luminosity as $\beta \propto L^{0.10 \pm 0.03}$. In the bottom panels of Figure 2, we include this effect. This causes another bias because the measured time lag at 0.3 $SF_{\infty}$ increases additionally for bright AGNs. When using the empirically measured relations $SF_{\infty} \propto L^{-0.35}$ and $\beta \propto L^{0.1}$, the measurement of the timescale at a fixed amplitude (below 0.795 $SF_{\infty}$) produces an artificial relation $\tau \propto L^{0.3}$ with $0.3 \leq \kappa \leq 0.6$ that is otherwise expected from the theoretical standpoint (namely, $\tau \propto L^{0.5}$), and the power of this artificial relation depends on what SF amplitude the $\tau$ measurement is made.

While it is not the goal of this paper to evaluate the biases of the SF amplitude at a fixed timescale, it is easy to decipher from Figure 2 what they would be. If all AGN variability was due to the same process (which is not the case) and the timescale was independent of luminosity (which appears to be the case), then the measurement of the SF amplitude would be correct and the amplitude ratio from the bright and faint AGNs would correspond to the ratio of $SF_{\infty}$ for these objects (Figure 2, top-left panel). If we added a theoretical positive correlation of the timescale with the luminosity, the SF amplitude at a fixed timescale would further decrease (Figure 2, top-right panel). Additional decreases are observed for brighter AGNs because of the steepening of the SF slope (Figure 2, bottom panels). This means that one should seek a relation of $SF_{\infty}$ with the physical AGN parameters, and not an arbitrarily selected SF amplitude below the decorrelation timescale, which will be biased.

Obtaining a meaningful SF from a single AGN light curve that typically is short and not well sampled is problematic, if not impossible. Emmanoulopoulos et al. (2010) have already studied and discussed various problems regarding this topic. In particular, they investigate the impact of data sampling and gaps, as well as data length, on the SF measurements. The most interesting finding is that for light curves with no intrinsic
decorrelation timescales (featureless PSD), breaks will appear in the SFs of almost all light curves, and they provide a rough guide for what timescales should appear as a function of the experiment length and the PSD slope (their Figure 5). While for all considered types of samplings (dense, sparse, with/without gaps) the short time lag SF part appears to be nearly independent of the sampling, the SFs differ in shape after the spurious break.

To explore some of these problems, we simulate three sets of 50 AGN light curves spanning 5000 days (13.7 years), with the same process having $\beta = 1.0$, $S_{\text{IC}} = 0.25$ mag, and for the decorrelation timescales of $\tau = 0.5$, 1, and 3 years, sampled every 10 days, thus having 500 data points (using the prescription from Kozowski et al. 2010)). For every light curve, we calculate its SF (Figure 3, thin gray lines). The SF for the input process is shown as the thick black line in Figure 3. It is obvious that each individual SF differs from the input SF, because of the data sampling and due to different light curve realizations of the same process. We calculate the ensemble SF for the 50 light curves, shown as the dotted black line in Figure 3. It closely resembles the input SF, and we show that the measurement of $\tau$ at 0.795 $S_{\text{IC}}$ is adequate (as indicated by the uncertainties). Note, however, we assumed here a simplification by using the exact same process for all 50 light curves (with identical process parameters but different light curve realizations). It is not clear if this assumption holds for the variability processes for a collection of true AGNs with similar physical parameters, although this is what is commonly assumed.

While this question still awaits to be answered, MacLeod et al. (2008) show that ensemble SFs from two-epoch data provide quantitatively similar results to those based on light curves with many epochs.

Another potential problem is mentioned by Emmanoulopoulos et al. (2010), who argued that fitting a model to the SFs is an intrinsically incorrect procedure because the time lag bins are not independent, the SF uncertainties appear too small, and the bootstrap method yields statistically meaningless SF error bars (these problems were also identified and discussed in Kozowski (2016a)). We provide here a method of determination of the decorrelation timescale that is not based on SF fitting, so once the flat part of the SF can be identified, $\tau$ can be just “read off” from the SF at the 0.795 $S_{\text{IC}}$ level. In practice, however, reaching the $S_{\text{IC}}$ level may be problematic, because one needs to collect many light curves that are several years long in rest frame, so for distant AGNs, this plausibly means decades. In addition, as noted previously, the assumption that an ensemble of light curves for many AGNs can be treated as representative for the group has not been verified. It is plausible that AGNs with similar or identical physical parameters (the BH mass and luminosity) will have variability that is due to different processes, so ensemble variability studies may not be valid.

Kelly et al. (2011) proposed a sophisticated method of analyzing individual AGN light curves with a mixture of DRW processes, and pointed out that such a mixture can result in a range of PSD slopes. It is likely, however, that most near-future individual light curves will be either short or not well sampled to enable secure determination of the model parameters for large AGN samples, so ensemble SFs will be a necessity (but see the caveats from the previous paragraph).

4. Summary

In this paper, from basic properties of the covariance of the variable signal in the data, we derived a method of measuring the decorrelation timescale for AGN light curves that always provides the actual and process-independent value. It is valid for SFs that at short time lags show a single power-law behavior and on the long ones appear to be flat; hence the ACF of the process can be of the PE type. The decorrelation timescale should be measured at 0.795 of the SF amplitude at the long timescales (after the photometric noise is removed). We also showed that when using the empirically established relations $S_{\text{IC}} \propto L^{-0.35}$ and $\beta \propto L^{0.1}$, the measurement of the timescale at a fixed SF amplitude (below 0.795 $S_{\text{IC}}$) produces an artificial non-existing relation, $\tau \propto L^\kappa$ with $0.3 \lesssim \kappa \lesssim 0.6$ (e.g., $\kappa = 0.4$ found by Caplar et al. (2017)), which is
otherwise expected from the theory of accretion disks (i.e., $\kappa \equiv 0.5$).

While individual SFs for typical AGN light curves that are short and sparsely sampled are rarely meaningful (Emmanoulopoulos et al. 2010), we showed that ensemble SFs from many AGNs would yield reliable decorrelation timescales for a whole class (having assumed identical variability parameters for individual objects). This is of particular importance because deep, large, optical sky surveys aiming at variability ratios such as Shen et al. 2011; Kozłowski 2017b distributed over a quarter of the sky, enabling unprecedented studies of the connection between the AGN variability and the underlying AGN physics. The forthcoming decades are guaranteed to bring many new and exciting developments in this field of research.

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References

Ackermann, M., Ajello, M., Allafort, A., et al. 2011, ApJ, 743, 171
Andrae, R., Kim, D.-W., & Bailier-Jones, C. A. L. 2013, A&A, 554, A137
Angione, R. J., & Smith, H. J. 1972, in IAU Symp. 44, External Galaxies and Quasi-Stellar Objects, ed. D. S. Evans, D. Wills, & B. J. Wills (Dordrecht: Reidel), 171
Barvains, R., Lehár, J., Birkinshaw, M., Falcke, H., & Blundell, K. M. 2005, ApJ, 618, 108
Butler, N. R., & Bloom, J. S. 2011, AJ, 141, 93
Caplar, N., Lilly, S. J., & Trakhtenbrot, B. 2017, ApJ, 834, 111
Chen, X., & Taam, R. E. 1995, ApJ, 441, 354
Collier, S., & Peterson, B. M. 2001, ApJ, 555, 775
Czerny, B. 2006, in ASP Conf. Ser. 360, AGN Variability from X-Rays to Radio Waves, ed. C. Martin Gaskell et al. (San Francisco, CA: ASP), 265
di Clemente, A., Giallonardo, E., Natali, G., Trevese, D., & Vagnetti, F. 1996, ApJ, 463, 466
Edelson, R., Vaughan, S., Malkan, M., et al. 2014, ApJ, 795, 2
Emmanoulopoulos, D., McHardy, I. M., & Uttley, P. 2010, MNRAS, 404, 931
Findeisen, K., Casy, A. M., & Hillenbrand, L. 2015, ApJ, 798, 89
Frank, J., King, A., & Raine, D. J. 2002, Accretion Power in Astrophysics (Cambridge: Cambridge University Press)
Giveon, U., Maoz, D., Kaspi, S., Netzer, H., & Smith, P. S. 1999, MNRAS, 306, 637
Graham, M. J., Djorgovski, S. G., Drake, A. J., et al. 2014, MNRAS, 439, 703
Hook, I. M., McMahon, R. G., Boyle, B. J., & Irwin, M. J. 1994, MNRAS, 268, 305
Hughes, P. A., Aller, H. D., & Aller, M. F. 1992, ApJ, 396, 469
Kasiwal, V. P., Vogel, M. S., & Richards, G. T. 2015, MNRAS, 451, 4328
Kawaguchi, T., Mineshige, S., Umemura, M., & Turner, E. L. 1998, ApJ, 504, 671
Kelly, B. C., Bechtold, J., & Siemiginowska, A. 2009, ApJ, 698, 895
Kelly, B. C., Becker, A. C., Sobolewska, M., Siemiginowska, A., & Uttley, P. 2014, ApJ, 788, 33
Kelly, B. C., Sobolewska, M., & Siemiginowska, A. 2011, ApJ, 730, 52
King, A. 2008, NewAR, 52, 253
Kozłowski, S. 2016a, ApJ, 826, 118
Kozłowski, S. 2016b, MNRAS, 459, 2787
Kozłowski, S. 2017a, A&A, 597, A128
Kozłowski, S. 2017b, ApJ, 828, 9
Kozłowski, S., Kochanek, C. S., Ashby, M. L. N., et al. 2016, ApJ, 817, 119
Kozłowski, S., Kochanek, C. S., Udalski, A., et al. 2010, ApJ, 708, 927
MacLeod, C., Izvek, Z., de Vries, W., Sesar, B., & Becker, A. 2008, in AIP Conf. Ser. 1082, Classification and Discovery in Large Astronomical Surveys (Cambridge: Cambridge Univ. Press), 282
MacLeod, C. L., Brooks, K., Izvek, Z., et al. 2011, ApJ, 728, 26
MacLeod, C. L., Izvek, Z., Kochanek, C. S., et al. 2010, ApJ, 721, 1014
MacLeod, C. L., Izvek, Z., Sesar, B., et al. 2012, ApJ, 753, 106
McHardy, I. M., Gunn, K. F., Uttley, P., & Goad, M. R. 2005, MNRAS, 359, 1469
Morganson, E., Burgett, W. S., Chambers, K. C., et al. 2014, ApJ, 784, 92
Mushotzky, R. F., Done, C., & Pounds, K. A. 1993, ARA&A, 31, 717
Mushotzky, R. F., Edelson, R., Baumgartner, W., & Gandhi, P. 2011, ApJ, 743, L12
Paltani, S., & Courvoisier, T. J.-L. 1994, A&A, 291, 74
Páris, I., Petitjean, P., Ross, N. P., et al. 2017, A&A, 597, A79
Press, W. H., Rybicki, G. B., & Hewitt, J. N. 1992a, ApJ, 385, 404
Press, W. H., Rybicki, G. B., & Hewitt, J. N. 1992b, ApJ, 385, 416
Ruan, J. J., Anderson, S. F., MacLeod, C. L., et al. 2012, ApJ, 760, 51
Rybicki, G. B., & Press, W. H. 1992, ApJ, 398, 169
Scargle, J. D. 1981, ApJS, 45, 1
Scargle, J. D. 1982, ApJ, 263, 835
Scargle, J. D. 1989, ApJ, 343, 874
Schmit, K. B., Marshall, P. J., Rix, H.-W., et al. 2010, ApJ, 714, 1194
Schneider, D. P., Richards, G. T., Hall, P. B., et al. 2010, A&A, 139, 2360
Shen, Y., Richards, G. T., Strauss, M. A., et al. 2011, ApJS, 194, 45
Siemiginowska, A., & Czerny, B. 1989, MNRAS, 239, 289
Simm, T., Salvato, M., Saglia, R., et al. 2016, A&A, 585, A129
Simonetti, J. H., Cordes, J. M., & Ho, S. S. 1985, ApJ, 296, 46
Simionett, J. H., Cordes, J. M., & Spangler, S. R. 1984, ApJ, 284, 126
Uomoto, A. K., Wills, B. J., & Wills, D. 1976, AJ, 81, 905
Vagetti, F., Fadda, R., Antonucci, M., Paolillo, M., & Serafinelli, R. 2016, A&A, 593, A55
Vanden Berk, D. E., Wilhite, B. C., Kron, R. G., et al. 2004, ApJ, 601, 692
Zu, Y., Kochanek, C. S., Kozłowski, S., & Peterson, B. M. 2016, ApJ, 819, 122
Zu, Y., Kochanek, C. S., Kozłowski, S., & Udalski, A. 2013, ApJ, 765, 106
Zu, Y., Kochanek, C. S., & Peterson, B. M. 2011, ApJ, 735, 80