Spectroscopy of Magnetic Excitations in Magnetic Superconductors Using Vortex Motion

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In magnetic superconductors a moving vortex lattice is accompanied by an ac magnetic field which leads to the generation of spin waves. At resonance conditions the dynamics of vortices in magnetic superconductors changes drastically, resulting in strong peaks in the dc I-V characteristics at voltages at which the washboard frequency of vortex lattice matches the spin wave frequency \( \omega_s(g) \), where \( g \) are the reciprocal vortex lattice vectors. We show that if washboard frequency lies above the magnetic gap, peaks in the I-V characteristics in borocarbides and cuprate layered magnetic superconductors are strong enough to be observed over the background determined by the quasiparticles.

The coexistence of magnetism and superconductivity was observed in many crystals, such as RMnO\(_3\)Sr\(_2\), RRh\(_4\)B\(_4\), RBa\(_2\)Cu\(_3\)O\(_{7-\delta}\) and (R,A)CuO\(_{4-\delta}\) (A=Sr, Ce) with the temperatures of magnetic ordering \( T_M \) much smaller than the superconducting critical temperature \( T_c \), and also in borocarbides RT\(_2\)B\(_2\)C and ruthenocuprate RuSr\(_2\)GdCu\(_2\)O\(_8\) with \( T_M \) of the same order as \( T_c \). Here R is the rare earth element, while T=Nd, Ru, Pd, Pt. In such crystals, \( f \)-electrons of ions R give rise to localized magnetic moments, while conducting electrons exhibit the Cooper pairing. In all these crystals, except HoMo\(_6\)S\(_8\) and ErRh\(_4\)B\(_4\), magnetic moments order antiferromagnetically below \( T_M \). Such magnetic ordering coexists with superconductivity without strong interference because spin density varies on the scale much smaller than the superconducting correlation length and net magnetic moment vanishes, for review see Refs. [1][2][3]

In this Letter we consider interplay between magnetic and superconducting excitations in magnetic superconductors, particularly interaction between a moving vortex lattice and spin waves via the ac magnetic field induced by moving vortices. The energy transfer from vortices to the magnetic system leads to dissipation which is additional to that caused by quasiparticles. This results in strong current peaks in the dc I-V characteristics at voltages at which the washboard frequency of vortex lattice matches the spin wave frequency \( \omega_s(k) \) and \( k \) matches a reciprocal vortex lattice vector \( g \).

First we consider slightly anisotropic superconductors, i.e. all systems mentioned above except SmLa\(_{1-x}\)Sr\(_x\)CuO\(_{4-\delta}\) and RuSr\(_2\)GdCu\(_2\)O\(_8\) crystals, and probably also Sm\(_{2-x}\)Ce\(_x\)CuO\(_{4-\delta}\). The latter are layered superconductors with intrinsic Josephson junctions [4][5][6][7].

We assume, for simplicity, a uniaxial crystal structure with the principal axis along \( z \). The dc magnetic field is applied along the \( z \)-axis and we assume that the magnetic induction \( B(r) \), \( r = x, y \), inside the superconductor corresponds to the ideal Abrikosov square vortex lattice (such a lattice is realized in clean borocarbide crystals in field \( B \parallel c \) in some field intervals [8]). The sublattice magnetization in the case of antiferromagnetic ordering is assumed to be oriented in the \( (x,y) \) plane. The dc transport current with the density \( j \) is along the \( y \)-axis which, due to the Lorenz force, causes motion of the vortex lattice with the velocity \( \mathbf{v} \) along the \( x \)-axis.

We use the quasistatic approach assuming that the space structure of the magnetic field is the same as in the static vortex lattice, but the field moves in the same way as vortex lattice does. Thus all quantities describing the moving vortex lattice, i.e. the magnetic field and supercurrents, have the dependence on the coordinates and time in the combination \( (r - vt) \). In the field interval \( B < H_{c2} \) the magnetic field should be found from the London equations [9][10][11]

\[
\text{curl} \mathbf{B} = \frac{4\pi}{c} \mathbf{j}_s + 4\pi \text{curl} \mathbf{M},
\]

\[
\mathbf{j}_s = \frac{e\Phi_0}{8\pi^2\lambda_\perp^2} \left( \nabla \phi - \frac{2\pi}{\Phi_0} \mathbf{A} \right), \quad \mathbf{B} = \text{curl} \mathbf{A},
\]

\[
\text{curl} \nabla \phi = \sum_n 2\pi \delta(r - r_n),
\]

where \( \mathbf{j}_s \) is the supercurrent, \( \mathbf{A} \) is the vector potential, \( \phi \) is the phase of the superconducting order parameter, \( \mathbf{M} \) is the local magnetization, \( \Phi_0 \) is the flux quantum and \( \lambda_\perp = \lambda_\parallel = \lambda_y \) is the London penetration length for currents in the \( (x,y) \) plane in the absence of magnetic moments. Further, \( r_n(t) = r_n(0) + vt \) are the coordinates of vortices and \( r_n(0) \) form a regular vortex lattice. From Eqs. [11][12] we obtain

\[
\text{curl} \text{curl} (\mathbf{B} - 4\pi \mathbf{M}) + \frac{1}{\lambda_\perp^2} \mathbf{B} = \frac{\Phi_0}{\lambda_\perp^2} \sum_n \delta(r - r_n).
\]

To relate the Fourier components of \( M_z(r,t) \equiv M \) and \( B_z(r,t) \equiv B \) we use the linear response approximation in
which supercurrents induce the “external” magnetic field,

\[ H(k, \omega) = B(k, \omega) - 4\pi M(k, \omega), \]  

acting on the magnetic moments, where \( M(k, \omega) = \chi(k, \omega)H(k, \omega) \) and \( \chi(k, \omega) \equiv \chi_{zz}(k, \omega) \) is the susceptibility of the magnetic system. This approach is valid for the magnetization harmonics \( g_n \neq 0 \) satisfying the condition

\[ |M(g, gz, v)|^2 / (\mu n M)^2 \ll 1. \]  

For an antiferromagnet with two sublattices the magnetic susceptibility is given \([11][12]\) by

\[ \chi(k, \omega) = \frac{\omega M \omega_s(k)}{\omega_s^2(k) - \omega^2 - i\omega \nu_s}. \]  

Here \( \omega_M = \mu^2 n_M/(2\hbar) \) at \( \mu B \ll k_B T_M \), the density of magnetic ions is \( n_M \) and their magnetic moment is \( \mu, \omega_s(k) \) is the magnetically active spin wave dispersion renormalized by the superconductivity \([3]\), while \( \nu_s \) is the relaxation rate of spin waves due to the interaction with phonons. Using Eqs. \([4]\) and \([5]\), we obtain for the Fourier components \((k = g \equiv 2\pi(B_0/\Phi_0)^{1/2}(n, m, 0); \omega = gz, \nu)\) of the magnetic field

\[ B(k, \omega) = \sum_{\mathbf{g}} B_0 \delta(k - g) \delta(\omega - gz) \]  

where \( B_0 \) is the average induction and \( n, m \) are integer. From Eq. \([5]\) we see that magnetic moments renormalize the London penetration length so that the effective penetration length in magnetic superconductors is given by \([2]\)

\[ \lambda_\perp(k, \omega) = \frac{\lambda_\perp}{1 + 4\pi \chi(k, \omega)^2}. \]  

Solving Eq. \([5]\) we obtain the Fourier components of the magnetic field \( B \) and the “external” field \( H \) as

\[ B(k, \omega) = B_0 \frac{\delta(k - g) \delta(\omega - gz)}{1 + \lambda_\perp^2(g, \omega)^2 g^2}, \]  

\[ H(k, \omega) = B_0 \frac{\delta(k - g) \delta(\omega - gz)}{1 + 4\pi \chi(g, \omega) + \lambda_\perp^2 g^2}. \]  

Thus the moving vortex lattice induces a spatially periodic ac “external” magnetic field \( h(r, t) = H(r, t) - B_0 \) along the \( z \)-axis characterized by momenta \( g \) and mesh frequencies \( \omega = vz_\perp \). At \( \chi = 0 \) for \( \lambda_\perp \approx 1300 \) \( \text{s}^{-1} \), typical for borocarbides, the amplitude of the main harmonic, \( n = 1, m = 0 \), is about 20 G. The moving vortex lattice induces also an electric field \( E = [v \times B]/c \) along the current direction.

When the alternating magnetic field \( h(r, t) \) is not parallel to the sublattice magnetization, it excites spin waves with momenta \( g \) and frequencies \( \omega_s(g) = g \cdot v \) (this condition of resonance holds for any vortex lattice). Assuming that sublattice magnetizations are almost perpendicular to the applied magnetic field, we obtain for the power per unit volume transmitted from the vortex lattice to the magnetic system the expression \([12]\)

\[ P_M = -\left< \frac{\partial B(r, t)}{\partial t} \right> = \sum_{\mathbf{g}} 2g_v|\chi(g, gz, v)|^2 \Im\{\chi(\mathbf{g}, vz, v)\}, \]

where angular brackets denote time and space average.

Now we are in a position to find the velocity of the vortex lattice at a given transport current density \( J \). For that we equate the power per unit volume performed by the battery, \( jE \), to the sum of the power dissipated by quasiparticles, \( \eta v^2 \), and that transmitted to the magnetic system, \( P_M \). Here \( \eta \) is the viscous drag coefficient due to quasiparticles in normal vortex cores. It is given by the Bardeen-Stephen expression \( \eta = B_0 H_{\perp}^* s_n/\epsilon c^2 \), where \( s_n \) is the normal state conductivity, \( H_{\perp}^* = \Phi_0/2\pi \xi^2 \) is the orbital upper critical field and \( \xi \) is the superconducting correlation length in the direction perpendicular to the applied magnetic field. Taking into account that \( E = vB_0/c \) and \( \omega = vz_\perp = cEg_\perp/B_0 \), we find \( v, E \) and finally \( j(E) \) (i.e., \( j, \nu \)) characteristics in the intervals of \( E \), where inequality Eq. \([6]\) is fulfilled:

\[ j(E) = \frac{e^2 n_\nu}{B_0^2} + \sum_{g \neq 0} \frac{2g_v c B_0 \Im\{\chi(\mathbf{g}, cz, vz, v)\}}{1 + 4\pi \chi(\mathbf{g}, cz, vz, v) + g^2 \lambda_\perp^2 v^2}. \]

From this equation we see that the current density as a function of \( E \) has peaks corresponding to resonances between the ac magnetic field and spin waves, i.e. when \( \omega(n, m) = 2\pi v(B_0/\Phi_0)^{1/2}n \).

Let us discuss the behavior of \( j(E) \) near resonances. We introduce the frequency deviation \( \Delta \omega = \omega_s(g) - \omega \) such that \( \nu_s \ll \Delta \omega \ll \omega_s(n, m) \). Then we obtain \( \chi(\mathbf{g}, \omega) \approx \omega_M/(2\Delta \omega) \) and \( \Im\{\chi(\mathbf{g}, \omega)\} \approx \omega_M \nu_s/(2\Delta \omega)^2 \). We consider the interval of frequency deviations \( \Delta \omega \) where \( \lambda_\perp^2 g^2 \gg 4\pi \chi(\mathbf{g}, \omega) \). In this interval we estimate

\[ \frac{M(g, \omega)}{\mu n M} \approx \frac{\mu \Phi_0}{16\pi^2(n^2 + m^2)^2 \lambda_{\perp}^2 \Delta \omega}. \]

Due to the condition Eq. \([6]\) our approach is valid for \( \hbar \Delta \omega > \mu \Phi_0/(4\pi \lambda_{\perp}^2) \). The ratio of the additional current caused by spin waves over the current background is given as

\[ \frac{\Delta j(n, m)}{j} \approx \frac{\omega_M \nu_s \Phi_0 B_0}{8\pi^2 \nu_s \nu_s \Delta \omega^2 \lambda_{\perp}^2 (n^2 + m^2)^2}. \]

In the frequency interval \( \hbar \Delta \omega > \mu \Phi_0/(4\pi \lambda_{\perp}^2) \) we obtain the inequality

\[ \frac{\Delta j(n, m)}{j} < \frac{16\pi^2 h M n M \nu_s B_0}{\omega \Phi_0} \frac{n^2}{(n^2 + m^2)^2}. \]

In magnetic insulators \( \nu_s \) is typically of order \( 10^6 \) \( \text{s}^{-1} \). One can anticipate the same value in magnetic superconducting
crystals, as conducting electrons are gapped. For HoNi$_2$B$_2$C, taking $H_{J2} \approx 10$ T, $n_M = 10^{22}$ cm$^{-3}$, $\sigma_n = 10^5$ (ohm cm)$^{-1}$ at $\omega = 10^{10}$ s$^{-1}$ we derive $\Delta j(n,m)/j < 0.8n^2/(n^2 + n^2)$. Thus, the peak is $n = 1, m = 0$ is observable even in the frequency interval where our linear response approach is valid. Here the magnetic system deviates only slightly from equilibrium as energy is transformed further to phonon bath.

Closer to the resonance the linear response approach breaks down. Here the dominant contribution to dissipation comes from generation of spin waves by vortices which leads to a strong deviation of the magnetic system from equilibrium. For quantitative description of the j-E characteristics at different magnetic fields and currents may be changed independently by varying $B_0$ and $j$, but an important question is what are limitations on the variations of the magnetic field and the current density. Momentum $k \sim 2\pi(B_0/\Phi_0)^{1/2}$ is of order $10^6$ cm$^{-1}$ in fields $B_0 \leq 1$ T and increases as one approaches $H_{J2}$, but then harmonic amplitudes $h(g,\omega)$ drop. Limitations on frequencies are due to limitations on the current density, which should be lower than the depairing current density, and also should not lead to excessive heating. From Eq. 13, to reach frequency $\omega$ one needs current density $j(\omega) \geq \sigma_n\omega H_{J2}/cg_s$ and the electric field $E(\omega) = \omega B_0/cg_s$. For $B_0 = 1$ T we obtain $j(\omega) \approx 10^7 n^{-1}(\hbar\omega/1K)$ A/cm$^2$ if the lowest harmonics are used, while for higher harmonics higher frequencies may be reached. The depairing current density for borocarbides is of order $10^7$ A/cm$^2$ and thus spin waves with energies $\hbar\omega \lesssim 1$ K may be probed without strong suppression of superconducting properties by the transport current. For the dissipation power per unit volume, $P_{dis} = jE \geq \sigma_n\omega^2\phi_0 H_{J2}^2/4\pi^2 c^2$, we estimate $P_{dis} \sim 10^8 n^{-2}(\hbar\omega/1K)^2$ W/cm$^3$. To diminish heating the pulse technique may be used, as in I-V measurements by Kunchur 14.

As only the low energy part of the spin wave spectrum may be probed by I-V measurements, the important question is what is the minimum energy of spin waves. For a Neel antiferromagnetic state there is always magnetic gap in the spectrum due to the magnetic anisotropy slightly renormalized by the interlayer coupling in this crystal has been confirmed by observation of the double Josephson plasma resonance stemming from two layers in a unit cell 3. The specific heat measurements 6 show that magnetic ordering is absent down to a temperature of 0.3 K and a magnetic gap, if any, lies below 0.3 K. They reveal also a broad peak near the temperature 1 K and the height of this peak indicates the presence of competing interactions that might be described by the two-dimensional $J_1$-$J_2$ Heisenberg model with $J_2/J_1 > 0.4$ 2,15. Such a model has very complex dynamics and contains a variety of transitions down to zero temperature, making it an ideal testing ground for the theory of quantum phase transitions. The most interesting part of the phase diagram is in the region $0.4 \lesssim J_2/J_1 \lesssim 0.55$, where a gapped phase without magnetic ordering is likely to be taking place. However, its characterization has been one of the most intriguing puzzles of the physics of strongly correlated systems 16.

If the magnetic field is applied perpendicular to the layers (along the c-axis), it induces pancake vortices which do not form a regular lattice in magnetic fields above 20 G as they order along the c-axis only due to weak Josephson and magnetic interactions 17. This makes excitation of spin waves ineffective by moving vortex lattice induced by a perpendicular magnetic field. When a magnetic field is applied parallel to the layers (in the ab-plane, along the y-axis), the situation is drastically different, because now Josephson vortices 16,18,20,21,22 are induced. In high fields they form a lattice which is quite regular in the x-direction (parallel to the layers). Josephson vortices do not have normal cores and so only thermally induced quasiparticles (or those near the nodes in the case of d-wave pairing) cause dissipation. A weak interlayer tunneling transport current, which leads to vortex motion in the x-direction, cannot destroy superconductivity and produces much less heating than in the case of isotropic or weakly anisotropic superconductors.

The distribution of the magnetic field $B(r)$ inside intrinsic Josephson junctions is described by coupled finite-difference differential equations for the phase difference $\varphi_n$ and for the magnetic field $B_n$ inside the junction n between layers n and $n + 1$ 19,20,22. Accounting for the magnetization $M_n$ of ions inside intrinsic Josephson junction $n$ we obtain equations for the dimensionless variables $\varphi_n$, $b_n = B_n/2\pi\lambda_{ab}\lambda_c/\Phi_0$, $m_n = M_n/2\pi\lambda_{ab}\lambda_c/\Phi_0$ and $h_n = b_n - 4\pi m_n$:

$$\frac{\partial^2 \varphi_n}{\partial \tau^2} + \nu_2 \frac{\partial \varphi_n}{\partial \tau} + \sin \varphi_n - \frac{\partial h_n}{\partial u} = 0,$$

$$\nabla^2 b_n - \frac{b_n}{\tau^2} + \frac{\partial \varphi_n}{\partial u} + \nu_2 \frac{\partial (\partial \varphi_n/\partial u - b_n/\tau^2)}{\partial u} = 0,$$

where $u = x/\lambda_J$, $\tau = t\omega_p$, $\lambda_J = \gamma_s$, $s$ is the interlayer distance, $\gamma = \lambda_c/\lambda_{ab}$ is the anisotropy ratio, $\lambda_c$ and $\lambda_{ab}$ are the London penetration lengths for currents along the c-axis and in the ab-plane, respectively, $\omega_p = c/\lambda_{\sqrt{\tau}}$ is the
Josephson frequency, \( \epsilon_J \), is the dielectric function along the c-axis, \( \nu_c = 4\pi\epsilon_c/(\omega_p\epsilon_c) \), \( \nu_{ab} = 4\pi\epsilon_{ab}/(\gamma^2\epsilon\omega_p) \), \( \sigma_c \) and \( \sigma_{ab} \) are quasiparticle conductivities along the c-axis and in the ab-plane, respectively. Using linear response approximation, \( m_n = \chi b_n/(1 + 4\pi\chi) \), where \( \chi = \chi_{xy} \), we see that \( h_n = b_n/(1 + 4\pi\chi) \) satisfies the same equations as \( b_n \), but with the renormalized parameter \( \ell^2 = (1 + 4\pi\chi)\ell'^2 \).

For SmLa_{1-x}Sr_xCuO_4-\( \delta \) we estimate \( \ell^2 \gg 1 \) because \( \omega_M \approx 10^5 \text{s}^{-1} \), \( \ell^2 \approx 2 \times 10^{-4} \) at \( \mu_B = 0.8\mu_B \), \( n_M = 5 \times 10^{21} \text{cm}^{-3} \) and \( \lambda_{ab} \approx 2000 \text{ Å} \).

In the following we consider large enough fields \( B > B_J \equiv \Phi_0/(2\pi\lambda_J) \). Then the Josephson vortices fill all intrinsic magnetic fields, higher spin wave energies may be probed by use of moving Josephson vortices. This is sufficient to study almost complete spin wave spectrum in SmLa_{1-x}Sr_xCuO_4-\( \delta \) with exotic magnetic ordering.

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