MHD simulations of jet acceleration from Keplerian accretion disks
The effects of disk resistivity

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\textbf{ABSTRACT}

\textbf{Context.} Accretion disks and astrophysical jets are used to model many active astrophysical objects, viz., young stars, relativistic stars, and active galactic nuclei. However, extant proposals on how these structures may transfer angular momentum and energy from disks to jets through viscous or magnetic torques do not yet provide a full understanding of the physical mechanisms involved. Thus, global stationary solutions do not permit an understanding of the stability of these structures; and global numerical simulations that include both the disk and jet physics have so far been limited to relatively short time scales and small (and possibly astrophysically unlikely) ranges of viscosity and resistivity parameters that may be crucial to define the coupling of the inflow-outflow dynamics.

\textbf{Aims.} Along these lines we present in this paper self-consistent time-dependent simulations of supersonic jets launched from magnetized accretion disks, using high resolution numerical techniques. In particular we study the effects of the disk magnetic resistivity, parametrized through an $\alpha$-prescription, in determining the properties of the inflow-outflow system. Moreover we analyze under which conditions steady state solutions of the type proposed in the self-similar models of Blandford & Payne can be reached and maintained in a self-consistent nonlinear stage.

\textbf{Methods.} We use the resistive MHD FLASH code with adaptive mesh refinement, allowing us to follow the evolution of the structure for a time scale long enough to reach steady state. A detailed analysis of the initial configuration state is given.

\textbf{Results.} We obtain the expected solutions in the axisymmetric (2.5 D) limit. Assuming a magnetic field around equipartition with the corona of the central object.

\textbf{Key words.} Accretion disks – Jets and outflows – Magnetohydrodynamics – Methods: numerical

1. Introduction

Astrophysical jets are an important component in many active astrophysical objects, from young stellar objects (YSO) to relativistic stars and active galactic nuclei. In particular, Herbig-Haro (HH) outflows are detected in stellar forming regions around T-Tauri stars, characterized mainly by optical emission line spectra (e.g., Reipurth & Bally \textsuperscript{1996}). On a much larger scale, relativistic radio jets are accelerated in the innermost cores of Active Galactic Nuclei (AGN) (e.g., Giovannini \textsuperscript{2004}); their emitting component is synchrotron relativistic electrons, with a cold proton component or, most likely, a Poynting flux electromagnetic component (De Young \textsuperscript{2006}).

Despite of being characterized by extremely different space, time and energy scales, it is commonly accepted that all these systems derive their energy from accretion onto a central object (Livio \textsuperscript{1999}); the physical origin of these supersonic outflows has been related to the dynamical evolution of magnetized accretion disks around a deep gravitational well. Even if the acceleration and collimation mechanisms of jets are still not clear, some basic models have been proposed and shown to be successful (for rather recent reviews see Pudritz et al. \textsuperscript{2006a} for YSO and Ferrari \textsuperscript{1998} \textsuperscript{2004} for AGN). The overall idea is to extract energy and angular momentum from the accreting matter and to feed it into a plasma that is accelerated in two opposite directions along the rotation axis of the disk. Lovelace \textsuperscript{1976} and Blandford \textsuperscript{1976} proposed independently that this can be done by electromagnetic forces. In particular Blandford & Payne \textsuperscript{1982} derived a steady state MHD solution for an axisymmetric magnetocentrifugally driven outflow from a Keplerian disk; for the outflow to be launched the poloidal magnetic field lines must be inclined less than 60$^\circ$ with respect to the plane of the accretion disk. On the other hand, Sauty \textsuperscript{et al.} \textsuperscript{2002}, Sauty \textsuperscript{et al.} \textsuperscript{2004} have derived meridionally similar models to study the launching mechanism from the hot corona of the central object.

The magnetocentrifugal mechanism has been the subject of a series of numerical studies (Ustyugova \textsuperscript{et al.} \textsuperscript{1999} Ouyed \textsuperscript{&} Pudritz \textsuperscript{1997} Krasnopolsky \textsuperscript{et al.} \textsuperscript{1999} Anderson \textsuperscript{et al.} \textsuperscript{2004} Fendt \textsuperscript{2006} Pudritz \textsuperscript{2006b} based on ideal MHD simulations in which the disk is treated as a boundary condition. On one
hand they show how a steady solution can be obtained in a few dynamical time scales and how the acceleration, collimation and stationarity of the outflow depend on the mass loading from the disk and on the magnetic field structure. On the other hand the back reaction of the outflow on the disk can not be taken into account.

When the structure of the magnetized accretion disk is included self-consistently in the models, a diffusive mechanism must be introduced inside the disk to balance the shearing due to differential rotation and the inward advection of field lines. Moreover also viscous torques, which can transport angular momentum radially inside the disk itself, should be, in principle, taken into account.

For instance, Königl (1989), Wardle & Königl (1993) and Li (1996) studied the structure of an ambipolar diffusion dominated disk connecting it with a Blandford & Payne solution at the disk surface. Ogilvie & Livio (2001) solved the vertical structure of a thin (optically thick) magnetized disk taking into account an effective $\alpha$ turbulent viscosity and resistivity.

In a series of papers including turbulent $\alpha$ resistivity (Ferreira & Pelletier 1995 Ferreira 1997), viscosity (Casse & Ferreira 2000a) and entropy generation (Casse & Ferreira 2000b) the authors calculated radially self-similar stationary solutions of accretion-ejection structures. Two important features emerged clearly from these latter models: first of all it was shown that, in order to balance the magnetic and gravitational compression on the disk itself, the thermal energy must be around equipartition with the magnetic energy inside the disk. With these conditions, the vertical thermal pressure gradient is the only force that can push the mass at the surface of the disk to be accelerated in the outflow. Second, the magnetic torque must change sign at the disk surface, in order to extract angular momentum from the disk and transfer it to the outflow. This condition determines a strong constraint on the resistive configuration: the magnetic diffusivity must be rather high ($\alpha \sim 1$) and anisotropic, the diffusion of the poloidal field being smaller than the diffusion of the toroidal component.

The first time-dependent numerical simulations in which the structure of the magnetized accretion disks is included (Uchida & Shibata 1985) or more recently Kato et al. (2002) showed that the interaction between a geometrically thin rotating disk and a large scale magnetic field that was initially uniform and vertical originates a transient state in which a strong toroidal field is generated that expels matter in the direction perpendicular to the disk plane (“sweeping magnetic twist mechanism”). One of the limits of these models is that the short simulated time scales and the ideal MHD approximation lead to the formation of a transient and highly unstable outflow. Moreover the small density contrast between the disk and the surrounding corona assumed in Uchida & Shibata (1985) enables the disk to lose its angular momentum on a very short time scale. These works have been recently updated in Kuwabara et al. (2005) who studied the evolution of a thick magnetized torus assuming a constant resistivity throughout the computational domain: even if the presence of such a high and uniform resistivity must be justified, the authors showed a quasi-stationary outflow launched from the inner radii of the torus.

Up to the present, the best effort to produce an accretion-ejection structure recurring to time-dependent simulations has been performed by Casse & Keppens (2002, 2004) who showed how a quasi-stationary jet can be launched from the equipartition regions of a resistive accretion disk. On the other hand, their resistive configuration, isotropic with $\alpha = 0.1$ is rather different from the one predicted by the self-similar steady models (Casse & Ferreira 2000a): despite of the use a stretched grid, the resolution of these simulations is rather low and it is likely that numerical dissipative effects are quite important.

In the present paper we present a numerical study of resistive MHD axisymmetric accretion-ejection structures performed with the high resolution code FLASH using Adaptive Mesh Refinement. The aim of the paper is to simulate the disk-jet configuration over long time scales in order to determine the effects of different configurations of an $\alpha$ resistivity, by varying its value and its degree of anisotropy, in determining the properties of the system. Moreover we want to test whether a stationary state corresponding to the Blandford & Payne self-similar solution can be reached and maintained. A critical point in the simulation is of course the choice of the initial configuration, in particular the magnetic configuration and the equilibrium of the disk. We have solved for an analytic self-similar equilibrium configuration of the disk including gravitational, centrifugal, thermal pressure and Lorentz forces; this solution defines also the initial magnetic field. We do not include physical viscosity: as it has been shown both in stationary (Casse & Ferreira 2000) and time-dependent contexts (Meliani et al. 2006), for conventional values of the disk turbulence ($\alpha \sim 0.1 - 1$, Prandtl number $\sim 1$) the viscous torque is less efficient than the magnetic in extracting angular momentum from the disk.

The paper is organized as follows. Section 2 is dedicated to illustrating the equations and the numerical code used; the initial configuration and the boundary conditions are discussed in detail. In Section 3 the numerical simulations are presented for different magnetic diffusivities and anisotropies. These parameters are linked to the mass accretion rate, required torque and liberated accretion energy. The acceleration mechanisms at the disk boundary and far above the disk are discussed, also referring to the established current circuits. The jet ejection efficiency is discussed in Section 4 in relation to the accretion rates. In Section 5 we discuss the angular momentum transport and in Section 6 the energy budget of the inflow/outflow system is evaluated. In Section 7 we test if one of our cases can be characterized as a stationary solution. Comments about the effect of Ohmic dissipation in the disk are given in Section 8. Finally, Section 9 summarizes our results and puts them in context with other analytical and numerical models.

2. The numerical model

2.1. MHD equations

We model the interaction between an accretion disk and the magnetic field which threads it within a resistive MHD framework. The system of equations that we solve numerically conveys therefore the conservation of mass:

$$\frac{\partial \rho u}{\partial t} + \nabla \cdot (\rho u u) = 0 ,$$

where $\rho$ is the mass density and $u$ is the flow speed; the momentum equation:

$$\frac{\partial \rho u}{\partial t} + \nabla \cdot \left[ \rho uu + \left( P + \frac{B \cdot B}{2} \right) I - BB \right] + \rho \nabla \Phi_g = 0 ,$$

where $P$ is the thermal pressure and $B$ is the magnetic field. This equation takes into account the action of thermal pressure gradients, Lorentz forces and gravity as determined by the potential $\Phi_g = -GM/\sqrt{r^2 + z^2}$ representative of the gravitational field of
a central object of mass $M$. The evolution of the magnetic field is determined by the induction equation (Faraday’s law):

$$\frac{\partial \mathbf{B}}{\partial t} + \nabla \times \mathbf{E} = 0,$$

where the electric field $\mathbf{E}$ is determined by the Ohm’s law $\mathbf{E} = -\mathbf{u} \times \mathbf{B} + \vec{\eta} \mathbf{J}$. Notice that the equations are written in a non-dimensional form, hence without $4\pi$ and $\mu_0$ coefficients. The electric current $\mathbf{J}$ appearing in the Ohm’s relation is determined by the Ampere’s law $\mathbf{J} = \nabla \times \mathbf{B}$. The magnetic resistivity $\vec{\eta}$ is indicated as a two-tensor to take into account anisotropic diffusive effects: in our simulations we will consider a diagonal resistivity tensor $\eta_{ij}$ whose non-zero components are $\eta_{\phi\phi} = \eta_m$ and $\eta_{zz} = \eta_m$. Finally the conservation of energy is expressed by

$$\frac{\partial e}{\partial t} + \nabla \cdot \left( e + \rho \mathbf{u} \cdot \mathbf{B} \right) \mathbf{u} = \frac{\Lambda_{\text{cool}}}{\Lambda_{\text{diss}}},$$

where the total energy density

$$e = \frac{P}{\gamma - 1} + \frac{\rho \mathbf{u} \cdot \mathbf{u}}{2} + \frac{B^2}{2} \rho + \rho \Phi_g$$

is given by the sum of thermal, kinetic, magnetic and gravitational energy. $\gamma = 5/3$ is the polytropic index of the gas. $\Lambda_{\text{cool}}$ is a cooling term defined by the parameter $0 < f < 1$:

$$f = \frac{\Lambda_{\text{cool}}}{\Lambda_{\text{diss}}}$$

given by the ratio between the specific radiated energy and the Ohmic heating term $\Lambda_{\text{diss}} = \vec{\eta} \mathbf{J} \cdot \mathbf{J}$. The parameter $f$ therefore determines the fraction of magnetic energy which is radiated away instead of being dissipated locally inside the disk increasing its entropy. Finally the system of equations is closed by the equation of state of ideal gases $P = n k T$ where $n = \rho/m_p$ ($m_p$ being the proton mass) is the number density of the gas, $T$ is its temperature and $K$ is the Boltzmann constant.

To solve the resistive MHD system of Eqs. (1)-(4) we employ a modified version of the MHD module provided with the public code FLASH\(^1\) (Fryxell et al. 2000) developed at the ASC FLASH Center at the University of Chicago, adopting its Adaptive Mesh Refinement (AMR) capabilities. The simulations have been carried out in 2.5 dimensions, that is in cylindrical geometry in the coordinates $r, z$ assuming axisymmetry around the rotation axis of the disk-jet system. The algorithm implemented belongs to the class of high resolution Godunov schemes which are the best suited to study supersonic flows: we therefore used a linear reconstruction of primitive variables with a minmod limiter on pressure and flow speed and a Van Leer limiter on density and on the magnetic field components; second order accuracy in time is obtained thanks to an Hancock predictor step on the primitive variables while, to compute the fluxes needed to update the conservative variables, we implemented an HLL-E solver, that is an approximate linearized Riemann solver, that assumes a-priori a two-wave configuration for the solution. To control the solenoidality of the magnetic field ($\nabla \cdot \mathbf{B} = 0$) the eight-wave approach (Powell et al. 1999) was used, which is known for simply advecting the monopoles, while a parabolic diffusion operator (Marder 1987) see also Dedner et al. (2002) was added to the induction equation to diffuse them. We finally ensured to use an angular momentum conserving form of the $\phi$ component of the momentum equation Eq. (2) and a poloidal current flux conserving form of the $\phi$ component of the induction equation Eq. (3).

2.2. Initial conditions

In the initial setup of our simulations we model a disk rotating with a slightly sub-Keplerian speed threaded by an initially purely poloidal magnetic field. The disk initial model is derived imposing equilibrium between the forces initially intervening inside the disk, namely, gravity, centrifugal force, thermal pressure gradients, and Lorentz force; the disk setup is therefore a solution of the following system of equations:

$$\frac{\partial P}{\partial z} = -\rho \frac{\partial \Phi_g}{\partial z} - J_\phi B_r,$$

$$\frac{\partial P}{\partial r} = -\rho \frac{\partial \Phi_g}{\partial r} + J_\phi B_z + \frac{\rho u_\phi^2}{r}$$

The system of Eqs. (6)-(7) can be easily solved in separable variables assuming radial self-similarity: with this assumption all the physical quantities $U$ are given by the product of a power law for $r$ with a function of the variable $x = z/r$

$$U = U_0 \left( \frac{r}{r_0} \right)^{\beta_\phi} \left( \frac{z}{r} \right)^{\beta_P},$$

where $z = 0$ corresponds to the midplane of the disk. $f_U$ is an even function such that $f_U(0) = 1$ or an odd function such that $f_U(0) = 0$, depending on the symmetry of the variable with respect to the midplane of the disk. For the physical quantities which have an even symmetry ($P, \rho, u_r, u_\phi, B_z$) $U_0$ therefore represents their value at $r = r_0$, $z = 0$.

Self-similarity requires that all the characteristic speeds, namely sound and Alfvén speed, and flow speeds should scale as the Keplerian velocity ($c_s(r) \sim r^{-1/2}$) on the midplane of the disk. Imposing also a polytropic relation between the disk density and pressure, that is $P = \rho_n^{\gamma} f(r)$, the power law coefficients $\beta_U$ are determined as follows:

$$\beta_\mu = \beta_u = -1/2, \quad \beta_P = -5/2, \quad \beta_B = -5/4, \quad \beta_\beta = -3/2.$$

The initial poloidal magnetic field has been set through the flux function $\Psi$ to ensure the solenoidality of the field:

$$\Psi = \frac{4}{3} B_0 \rho_0^{3/4} \left( \frac{r}{r_0} \right)^{3/4} \frac{m^{5/4}}{\left( m^2 + z^2 / r^2 \right)^{5/8}}$$

The components of the field, obtained thanks to the simple relations:

$$B_r = \frac{1}{r} \frac{\partial \Psi}{\partial r}, \quad B_z = \frac{1}{r} \frac{\partial \Psi}{\partial z}$$

obviously fulfill the self-similarity requirements. The parameter $m$ determines the height scale on which the initial magnetic field bends, where a value $m \rightarrow \infty$ gives a perfectly vertical ($B_z = 0$) field.

Given the poloidal magnetic field, the vertical equilibrium Eq. (6) can be numerically solved to give the vertical profiles of disk density ($\rho_\phi$) and pressure ($P_\phi = f_\phi^2$). On the other hand the radial equilibrium Eq. (7) can be solved to determine the disk rotation speed $u_\phi$. The solution will therefore depend on the following non-dimensional parameters:

$$\epsilon = \frac{c_s}{V_K} \bigg|_{z=0} = \sqrt{\frac{P}{\rho GM}} \bigg|_{z=0}$$

which is the ratio between the sound speed $c_s = \sqrt{P/\rho}$ and the Keplerian rotation speed $V_K = \sqrt{GM/r}$ evaluated on the disk.

\(^1\) FLASH is freely available at [http://flash.uchicago.edu]
midplane: this quantity determines the disk thermal height scale \( H \) through the relation \( H = c r \) (see Frank et al. (2002); Ferreira & Pelletier (1995)); the magnetization parameter:

\[
\mu = \frac{B^2}{2P_{\|}} \biggr|_{z=0}
\]

which gives the ratio between the magnetic and thermal pressure evaluated on the disk midplane. The disk rotation velocity at the midplane is slightly sub-Keplerian, with the deviation depending on the parameters (\( \epsilon, \mu \) and \( m \)):

\[
u_{\phi}|_{z=0} = \left[ 1 - \frac{5}{2} \epsilon^2 - 2 \epsilon^2 \mu \left( \frac{5}{4} + \frac{5}{3m^2} \right) \right] \sqrt{\frac{GM}{r}} (12)
\]

The toroidal velocity decreases towards the disk surface and it falls down to zero where the radial Lorentz force balances the gravitational pull and the thermal pressure gradient.

For the components of the magnetic diffusivity tensor \( \eta \) acting in the disk we adopt an \( \alpha \) prescription (Shakura & Sunyaev (1973) in the same vein of Ferreira (1977) and Casse & Keppens (2002, 2004). Despite of being a common way to parametrize the transport coefficients inside an accretion disk, the origin of this anomalous diffusivity is still debated. One of the most promising hypothesis states that the anomalous transport is of turbulent origin, triggered by some disk instability, being the magneto-rotational instability (MRI, Balbus & Hawley (1998)) the most accredited one. The \( \eta_{\phi \phi} = \eta_{m} \) component is parametrized as follows:

\[
\eta_m = \alpha_m V_A|_{z=0} H \exp \left( -\frac{z^2}{H^2} \right), (13)
\]

where \( \alpha_m \) is a constant parameter. \( V_A|_{z=0} = \left( B_c/\sqrt{\sqrt{\mu}} \right) |_{z=0} \) is the Alfvén speed calculated on the disk midplane and \( H = (c_s/\Omega_k)|_{z=0} \) is the thermal height scale of the disk. In the simulations both the Alfvén speed and \( H \) are allowed to evolve in time. The other components of the diffusivity tensor \( \eta_{rr} = \eta_{zz} = \eta_{m} \) are assumed to be proportional to \( \eta_m \) through an anisotropy parameter \( \chi_m \), which is the inverse of the analogous parameter introduced by Ferreira & Pelletier (1995):

\[
\chi_m = \frac{\eta_{rr}}{\eta_m} = \frac{\eta_{zz}}{\eta_m} (14)
\]

A ratio \( \chi_m = 1 \) indicates an isotropic resistive configuration. We recall that the presence of an effective resistivity inside the disk allows the magnetic field to break the “frozen-in” condition and the matter to slip through the field lines. The component \( \eta_{m} \), therefore indicated as poloidal resistivity, allows the flow to slip through the field in the poloidal plane while \( \eta'_{m} \), indicated as toroidal resistivity, controls the diffusion of the toroidal component of the field.

In a steady situation the accretion motion should be consistent with the poloidal magnetic field configuration as stated by the poloidal induction equation in its stationary form:

\[
u_r B_r - \nu_z B_z = \eta_m J_\phi \]

which expresses the balance between the advection and the diffusion of the poloidal magnetic field. Therefore we initially impose an accretion flow inside the disk solving Eq. (15) with the condition \( u_z = \frac{\nu_c}{r} u_r \). Due to the self-similarity requirements, the radial accretion speed scales initially as the Keplerian speed on the midplane of the disk:

\[
u_r|_{z=0} = -\alpha_m (2\mu)^{1/2} \left( \frac{5}{4} + \frac{5}{3m^2} \right) \frac{\sqrt{GM}}{r} \]

As the poloidal resistivity \( \eta_m \) exponentially decreases towards the disk surface, the initial accretion flow cancels out outside the disk.

It must be pointed out that in the initial conditions, since there is no toroidal magnetic field, there is no mechanism of angular momentum transport which can support the initial accretion flow; the angular momentum transport associated with the toroidal field will be triggered by torsional Alfvén waves due to the differential rotation between the midplane and the surface of the disk. Moreover, since we will assume a magnetic field around equipartition with the thermal energy on the midplane of the disk (see Section 2.3.1), the time scale on which the transport of angular momentum will become effective, given by the Alfvén crossing time of the disk thickness, is comparable to the local period of rotation of the Keplerian disk and to its epicyclic frequency.

On top of the disk we prescribe an hydrostatic spherically symmetric atmosphere for which we impose, as for the disk, a polytropic relation between pressure \( P_a \) and density \( \rho_a \): \( P_a = P_{a0}(\rho_a/\rho_{a0})^\gamma \). The density and pressure distribution will be therefore given by:

\[
\rho_a = \rho_{a0} \left( \frac{r_0}{R} \right)^\gamma, \quad P_a = P_{a0} \left( \frac{r}{r_0} \right)^\gamma \left( \frac{r_0}{R} \right)^\gamma \]

where \( \rho_{a0} \) is the value of the atmospheric density at the spherical radius \( R = r_0 \). The initial position of the disk surface is located whereas the disk and atmosphere pressures are equal. The initial position of the disk surface is therefore determined by its temperature, defined by \( \epsilon \), by the field intensity \( \mu \), its inclination, given by the parameter \( m \) and by the density contrast between the disk and the corona \( \rho_{a0}/\rho_0 \).

### 2.3. Units and normalization

Since in the formulation of the problem we did not introduce any specific physical scale, the system of Eqs. (14) and the initial conditions presented in Section 2.2 can be normalized in arbitrary units: the results will be therefore presented in non-dimensional units.

Lengths will be given in units of \( r_0 \), corresponding approximately to the inner truncation radius of the disk; speeds will be expressed in units of the Keplerian speed \( V_{K0} = \sqrt{GM/r_0} \) at \( r = r_0 \) and the densities in units of \( \rho_0 \), that is the disk initial density at \( r = r_0, z = 0 \). Assuming for \( r_0 \) the following units, appropriated for YSO or AGN systems,

\[
r_0 = 0.1 \text{ AU} \quad (\text{YSO})
\]

\[
n_0 = 10 \ R_{\text{Schw}} = 10^{-4} \left( \frac{M}{10^5 M_\odot} \right) \text{ pc} \quad (\text{AGN})
\]

where \( R_{\text{Schw}} = 2GM/c^2 \) is the Schwarzschild radius, the Keplerian speed \( V_{K0} \) is given by:

\[
V_{K0} = 94 \left( \frac{M}{10^5 M_\odot} \right)^{1/2} \left( \frac{r_0}{1 \text{ pc}} \right)^{-1/2} \text{ km s}^{-1} \quad (\text{YSO})
\]

\[
= 6.7 \times 10^4 \left( \frac{r_0}{1 \text{ pc}} \right)^{-1/2} \text{ km s}^{-1} \quad (\text{AGN})
\]
Correspondingly time will be expressed in units of \( t_0 = r_0 / V_{K0} \):

\[
t_0 = 1.7 \left( \frac{M}{M_\odot} \right)^{-1/2} \left( \frac{n_0}{0.1 \text{AU}^{-3}} \right)^{3/2} \text{ days} \quad \text{(YSO)}
\]

\[
t_0 = 0.5 \left( \frac{M}{10^9 M_\odot} \right) \left( \frac{n_0}{10^{6.5} \text{cm}^{-3}} \right)^{3/2} \text{ days} \quad \text{(AGN)}
\]  

(20)

Expressed in units of \( t_0 \) the Keplerian period of rotation at the inner radius of the disk \( r_0 \) is equal to \( 2\pi \).

Finally the normalization density \( \rho_0 \) can be chosen by determining a suitable mass accretion rate unit \( \dot{M}_0 = r_0^2 \rho_0 V_{K0} \):

\[
\dot{M}_0 = 3 \times 10^{-7} \left( \frac{\rho_0}{10^{-12} \text{ g cm}^{-3}} \right) \left( \frac{M}{M_\odot} \right)^{1/2} \left( \frac{n_0}{0.1 \text{AU}^{-3}} \right)^{3/2} M_\odot \text{yr}^{-1}
\]

\[
= 9 \left( \frac{\rho_0}{10^{-12} \text{ g cm}^{-3}} \right) \left( \frac{M}{10^9 M_\odot} \right)^{2} \left( \frac{n_0}{10^{6.5} \text{cm}^{-3}} \right)^{3/2} M_\odot \text{yr}^{-1}
\]  

(21)

where, as customary, the first expression corresponds to YSO and the second to AGN objects. The values of accretion and outflow mass rates will be presented in non-dimensional units and also assume that the system is symmetric with respect to the mid-plane of the disk.

On the outer upper boundary (\( z = 120 r_0 \)) “outflow” conditions are imposed on all the variables except for \( B_z \), whose boundary values are determined to satisfy the \( \nabla \cdot \mathbf{B} = 0 \) requirement.

We decided to prescribe the continuity of the first derivative of some variables in the radial direction since the power-law behavior of the initial conditions along \( r \) (see Eq. (8)) determines strong gradients of the physical quantities on the rightmost boundary. This boundary condition affects above all the radial component of the Lorentz force \( F_r \):

\[
F_r = -\frac{1}{2} \frac{\partial B_z^2}{\partial r} - \frac{1}{2} \frac{\partial B_r^2}{\partial r} + B_r \frac{\partial B_z}{\partial z}
\]  

(24)

and therefore the collimation of the outflow. First of all it is easy to see that a zero-gradient condition on the toroidal magnetic field component \( B_\phi \) cancels the pressure gradient of the magnetic pressure (first term in the right hand side of Eq. (24)) while the pinching force \(-B_\phi^2/r\) (second one) is still present. An outflow condition on \( B_\phi \) on the right outer boundary thus enforces the collimation due to the toroidal field. This issue has been widely discussed by Ustyugova et al. (1999) who proposed to use a “force-free” condition, where the poloidal electrical current is parallel to the magnetic field, in order to remove artificial forces originating on the boundaries. Seen from the point of view of electrical currents, a zero-gradient condition on \( B_\phi \) corresponds to have a negative collimating current component \( J_z \) flowing along the right boundary, while stationary models of accretion-ejection structures show that on the outer part of the disk the current outflows from it thus pushing the flow along the field lines without collimating it (see Fig. 13 in Ferreira 1997). The continuity of the first derivative of \( B_\phi \) on the rightmost boundary allows to develop a gradient of magnetic pressure associated with \( B_\phi \) which can counteract the toroidal pinch and generate an outflowing positive current \( J_z \).

Even if the effects are less pronounced, also the continuity of the first derivative of \( B_r \) on the rightmost boundary affects the collimation of the structure: a pressure gradient directed outwards associated with \( B_r \) (third term in the right hand side of Eq. (24)) counteracts the poloidal field tension (fourth term) thus producing slightly less collimated structures.

On the other hand, we noticed that the choice of an “outflow” condition for the toroidal field on the upper boundary did not affect the behavior of the outflow as much as the condition on the right boundary. We did not notice a huge difference between the “outflow” condition, where the poloidal current has only a \( z \) component, and the “force-free” condition proposed by Ustyugova et al. (1999), where the poloidal current is parallel to the poloidal field.

On the inner boundaries located on the edges of the rectangular region \( r \times z = [0, r_0] \times [0, 0.5 r_0] \), we adopted a similar strategy: on the \( r = r_0 \) side we extrapolated all the physical quantities imposing the continuity of the first derivative except for the poloidal components of the velocity field, for which a zero-gradient condition was used, and for the radial component of the magnetic field, which is required to fulfill the \( \nabla \cdot \mathbf{B} = 0 \) condition; on the \( z = 0.5 r_0 \) side we imposed an “outflow” condition on all the variables except for the \( z \) component of the field, determined by imposing the solenoidality of the field.

All the details on the simulation performed are given in the next Section.
2.5. The simulations

Once they are normalized with the units given in Section 2.3 the initial conditions presented in Section 2.2 depend on 6 non-dimensionnal parameters: the ratio between the sound speed and the Keplerian speed at the disk midplane \( c \); the disk magnetization parameter \( \mu \); the magnetic height scale of the initial field \( m \); the strength of magnetic diffusivity \( \alpha_m \) and its anisotropy coefficient \( \chi_m \); the ratio between the initial atmosphere and disk densities \( \rho_0/\rho_r \). All the free parameters except those describing the diffusive properties of the disk will be the same for all the simulations. We therefore assume a parameter \( \epsilon = 0.1 \), which fixes the initial thermal height scale and temperature \( T \) on the midplane of the disk, that is:

\[
T|_{z=0} = \epsilon^2 \frac{m^2 G M}{r_e} = 10^4 \left( \frac{\epsilon}{0.1} \right)^2 \left( \frac{M}{M_\odot} \right) \left( \frac{r_R}{0.1 \text{AU}} \right)^{-1} \text{K}
\]

The initial structure of the magnetic field is determined by fixing the magnetization parameters \( \mu = 0.3 \) and \( m = 0.35 \), which gives therefore a magnetic height scale 3.5 times higher than the initial thermal height scale of the disk. We recall that a value of magnetization around equipartition or slightly below it is generally required both in numerical (see Zanni et al. 2004 Casse & Keppens 2002 and analytical (Ferreira 1997, Casse & Ferreira 2000a) modeling of accretion disks launching jets; this condition is determined by the equilibrium between thermal pressure gradients, which vertically support the disk and are responsible for the initial mass loading the field lines, and Lorentz forces, which tend to pinch the disk. The magnetization parameter \( \mu \) fixes the initial value of the poloidal magnetic field on the midplane:

\[
B_l|_{z=0} = \sqrt{\mu} \frac{8 \pi P}{r} = 2.6 \left( \frac{\mu}{0.3} \right)^{1/2} \left( \frac{r}{10^{12} \text{g cm}^{-3}} \right)^{1/2} \left( \frac{M}{M_\odot} \right)^{1/2} \left( \frac{r_R}{0.1 \text{AU}} \right)^{-5/4} \text{G}
\]

Finally we assumed a density ratio between the corona and the disk \( \rho_0/\rho_r = 10^{-4} \).

In order to investigate the effects of magnetic resistivity we performed a series of simulations varying the value of the \( \alpha_m \) parameter and the anisotropy factor \( \chi_m \). The summary of all the simulations performed is given in Table 1

| Simulation | \( \alpha_m \) | \( \chi_m \) | \( f \) | \( M/M_\odot \) | \( J/J_0 \) | \( \dot{E}/\dot{E}_0 \) | Resolution |
|------------|---------------|-------------|-----|-------------|-------|-----------------|-------------|
| 1          | 0.1           | 1           | 1   | \( 7.3 \times 10^{-1} \) | \( 1.4 \times 10^{-1} \) | \( 3.3 \times 10^{-1} \) | 512 \times 1536 |
| 2          | 0.1           | 1           | 0   | \( 7.3 \times 10^{-1} \) | \( 1.4 \times 10^{-1} \) | \( 3.3 \times 10^{-1} \) | 128 \times 384 |
| 3          | 0.1           | 1           | 0   | \( 7.3 \times 10^{-1} \) | \( 1.4 \times 10^{-1} \) | \( 3.3 \times 10^{-1} \) | 512 \times 1536 |
| 4          | 0.1           | 0           | 1   | \( 7.3 \times 10^{-1} \) | \( 1.4 \times 10^{-1} \) | \( 3.3 \times 10^{-1} \) | 512 \times 1536 |
| 5          | 0.1           | 1           | 1   | \( 7.3 \times 10^{-1} \) | \( 1.4 \times 10^{-1} \) | \( 3.3 \times 10^{-1} \) | 512 \times 1536 |
| 6          | 0.1           | 0           | 1   | \( 7.3 \times 10^{-1} \) | \( 1.4 \times 10^{-1} \) | \( 3.3 \times 10^{-1} \) | 512 \times 1536 |

3. Production of jets from magnetized accretion disks

Table 1. Initial parameters of the simulations performed: poloidal resistivity parameter \( \alpha_m \), anisotropy of magnetic resistivity \( \chi_m \), cooling parameter \( f \), initial accretion rate \( M/M_\odot \), torque \( J/J_0 \) needed to support the accretion rate between \( r_l = r_0 \) and \( r_e = 10r_0 \), accretion energy \( \dot{E}/\dot{E}_0 \) liberated between the same radii, equivalent resolution of the adaptive grid.

All the simulations were carried on up to a time \( t = 400 \) which corresponds to \( \sim 63 \) periods of rotation of the disk at its inner radius. On the other hand the final time \( t = 400 \) corresponds to only 0.25 rotations at the outer boundary \( r = 40r_0 \). It is very unlikely that the outer part of the disk has reached an equilibrium stage at the end of the simulation. Therefore the study of the
accretion-ejection system will be restricted mainly to the inner part of the disk \( r < 10r_0 \), see Section 4) and to the outflow coming from it.

In all the cases studied we observed a robust outflow to emerge from the underlying accretion disk: the solutions show a hollow jet, where the central hole corresponds to the “sink” region \( r < r_0 \). The outflows are not completely collimated at the end of the runs, that is some matter is outflowing from the computational box from the outer cylinder at \( r = 40r_0 \). Nevertheless in all the solutions found the part of the outflow coming from the inner part of the disk crosses the Alfvenic and fast-magnetosonic critical surfaces inside the domain (see Fig. 1 and 2). Since no disturbance produced in the super-fast region of the outflow can propagate upstream towards the accretion disk, this condition obviously ensures that the outer boundary conditions do not affect the launching region of this part of the outflow. On the other hand perturbations can still propagate in a direction transverse to the magnetic field lines: if the fast-Mach cones at the boundaries intersects the computational box, the boundary conditions can still affect the radial structure and therefore the collimation of the outflow (see Ustyugova et al., 1999), as already discussed in Section 2.4.

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**Fig. 1.** Time evolution of density maps in logarithmic scale of the simulation characterized by \( (\alpha_m = 1, \chi_m = 3, f = 1) \). Time is given in units of \( t_0 \) (see text). In these units the Keplerian period at the inner radius of the disk \( r = r_0 \) is equal to \( 2\pi \). Superimposed are sample magnetic field lines: the distance between the field lines is proportional to the intensity of the field. In the last panel \( (t = 400) \) are also plotted the critical Alfven (dashed line) and fast-magnetosonic (dotted line) surfaces.

**Fig. 2.** Same as Fig. 1 but for the case characterized by \( (\alpha_m = 0.1, \chi_m = 1, f = 1) \).
To overcome the problem of the influence of boundary conditions on the launching phase, Krasnopolsky et al. (2003) and Anderson et al. (2004) restricted the launching of the wind to a narrow region of the inner accretion disk and chose an initial magnetic configuration as to contain the entire fast-magnetosonic critical surface inside the computational domain. The closed shape of the critical surfaces observed in the cited papers characterizes those solutions more as “X-winds” (Shu et al. 1994), that is wide-angle outflows from the corotation radius of a stellar magnetosphere, while the almost conical shape of the critical surfaces of our solutions is typical of extended disk-winds. Moreover, if on one hand it cannot be taken for granted that an underlying accretion disk can support the magnetic structure and provide the mass load imposed as a boundary condition by the authors, on the other hand it has been shown (Ferreira et al. 2006) that “X-winds” have kinematical properties which are inconsistent with observations of T-Tauri microjets.

The structure of the magnetic field allows to distinguish two classes of solutions: the cases characterized by \( \alpha_m = 1 \) show at the end of the computation an “ordered” magnetic configuration (Fig. 1), while the poloidal field lines are strongly warped and distorted if \( \alpha_m = 0.1 \) (Fig. 2). An exception is represented by the case performed with a lower resolution: despite of having a small resistivity parameter \( \alpha_m = 0.1 \), it does not present the characteristic field inversions (Fig. 3). This anomalous behavior can be reasonably ascribed to the higher numerical dissipation determined by the lower resolution.

Other differences can be noticed in Fig. [1][2] in the more diffusive case the jet is asymptotically less dense than in the less diffusive one; both outflows become super-Alfvénic (dashed line) and super-fast-magnetosonic (dotted line) but in the \( \alpha_m = 0.1 \) simulation both characteristic surfaces lie closer to the disk.

Despite of these morphological differences between the more and less dissipative cases, the mechanism which drives the outflows from the disk is qualitatively the same: in the following Section we will make a few general considerations about the forces which determine both the accretion and the ejection flow.

3.1. Acceleration mechanism

Since the accretion-ejection mechanism is mainly magnetically driven it is worth writing explicitly the Lorentz forces which are acting on the disk-jet system. Following Ferreira (1997) we decompose the Lorentz force \( F = J \times B \) in the directions parallel and perpendicular to the poloidal field:

\[
F_\parallel = \frac{B_p}{r} \nabla_\parallel (r B_\phi) \\
F_\perp = -\frac{B_\phi}{r} \nabla_\perp (r B_\phi) \\
F_\phi = \frac{B_\phi}{r} \nabla_\phi (r B_\phi) + J_\phi B_p
\] (29)

where \( B_\parallel \) is the poloidal magnetic field while \( \nabla_\parallel \) and \( \nabla_\perp \) indicate the derivatives parallel and perpendicular to the poloidal field respectively. These expressions show clearly that the poloidal component of the Lorentz force associated with the toroidal field is perpendicular to the isosurfaces \( r B_\phi = \text{const.} \) which are the surfaces along which the poloidal electric current flows. Moreover they show that the component parallel to the poloidal field \( F_\parallel \) and the toroidal one \( F_\phi \) are linked by the simple relation (see also Casse & Keppens 2002, 2004):

\[
F_\parallel = -\frac{B_\phi}{B_p} F_\phi
\] (30)

(notice that the toroidal field \( B_\phi \) assumes negative values in our simulations). This clearly shows that a poloidal current-field configuration that accelerates the outflow along the field lines (\( F_\parallel > 0 \)) is accelerating the plasma also in the toroidal direction (\( F_\phi > 0 \)) thus providing an additional centrifugal force. This represents the core of the so called magneto-centrifugal mechanism: the magnetic energy stored in the toroidal field at the base of the outflow both accelerates the plasma along the field lines and increases its angular momentum thus providing a centrifugal acceleration. As stated by the above relation the relative importance of the two mechanisms, poloidal and centrifugal acceleration, is given by the ratio \( |B_\phi|/B_p \); when the toroidal field is stronger than the poloidal one, the gradient of \( B_\phi \) along the field lines is the main accelerating mechanism while if \( |B_\phi|/B_p < 1 \) the plasma tends to corotate with the mainly poloidal magnetic field and the centrifugal force is dominant.

Inside the accretion disk the conditions are reversed: as in a unipolar inductor, a radial electric current develops inside the conducting plasma, thus providing a toroidal force \( -J \times B \) which acts to slow down the rotation. According to Eq. (30), since the Lorentz force brakes the matter in the toroidal direction, a poloidal pinching also occurs. Therefore, since the disk is vertically pinched by the gravity as well as by the magnetic pressure, only the thermal pressure gradient can ensure a quasi-static vertical equilibrium gently lifting the accreting matter towards the surface of the disk where it can be accelerated to form the outflow. As pointed out in Ferreira (1997), the transition from accretion to ejection can be therefore achieved if the sign of the magnetic torque \( F_\phi \) changes sign at the disk surface in order to provide a magnetic acceleration.

This scenario is confirmed by our simulations: in Fig. 4 we plot, at the end of our simulations, the total force, given by the sum of gravity, Lorentz force and thermal pressure gradient, acting along the \( z \) direction on the disk height scale; these curves are obtained by averaging inside the region \( r_0 < r < 10r_0 \) for the cases (\( \alpha_m = 1, \chi_m = 3, f = 1, \text{solid line} \), (\( \alpha_m = 1, \chi_m = 1, \text{dashed line} \).
Important differences can be noticed between the three cases shown: decreasing the value of the poloidal or the toroidal magnetic diffusivity the location of the points where the total force and the magnetic torque change their sign are located at lower height scales. A possible explanation of this behavior can be found in Ferreira (1997): to change the sign of the magnetic torque it is in fact necessary that the radial current \( J \) responsible for the braking of the disk decreases vertically on a disk scale height; it has been shown that in a stationary situation the radial current falls off on a height scale proportional to \( \sqrt{\alpha_m \chi_m} \) (see Eq. (B1) in Ferreira 1997). Consistently with our simulations, the change of sign of the torque therefore occurs at higher height scales increasing the value of \( \alpha_m \) or the anisotropy factor \( \chi_m \). A second noticeable difference is that while in the two higher diffusivity cases (\( \alpha_m = 1 \)) the torque changes sign when the disk is still pinched by the poloidal magnetic pressure (squares), in the \( \alpha_m = 0.1 \) simulation it changes above this point: in this case the \( z \)-component of the Lorentz force provides an additional source of mass loading before the magnetocentrifugal effect becomes effective.

Another important difference between these three simulations is shown in Fig. 5 where we plotted a radial average of the ratio between the toroidal and poloidal fields \( |B_t|/B_p \) along the outflows of the three cases: according to Eq. (30), these curves show that the centrifugal effects are stronger when the magnetic diffusivity is anisotropic (solid line), while for a low diffusivity (dashed line) the toroidal field pressure gradient is the dominant accelerating force. Moreover the increase of this ratio with \( z \) shows that while near the disk surface the centrifugal force is effectively contributing to the acceleration of the outflow, it becomes negligible far from it.

### 3.2. Current circuits

A suitable way to understand how the accretion-ejection mechanism works, is to analyze the circulation of the poloidal current in the disk-outflow system: in Fig. 5 we show the poloidal current circuits (solid thin lines) superimposed to density maps in logarithmic scale at \( t = 400 \). Sample field lines (solid thick lines) and the poloidal speed vectors are also plotted; the left panel refers to the (\( \alpha_m = 0.1, \chi_m = 1, f = 1 \)) case while the right one to the (\( \alpha_m = 1, \chi_m = 3, f = 1 \)) simulation. The current circuits are given by the \( rB_\theta = \text{const.} \) isosurfaces: the poloidal currents circulate counter-clockwise and the Lorentz forces are directed outwards perpendicularly to the closed circuits. As discussed before, a strong radial positive current flows along the disk midplane, thus providing a braking and vertically pinching force; at the disk surface the current changes direction until a force component accelerating the plasma along the poloidal field appears.

Some differences can be noticed between the two panels: due to a strong advection determined by the accretion flow, the poloidal field lines are much more inclined inside the disk in the less diffusive case (left panel) thus providing a vertically directed magnetic tension which mass loads the outflow as it was shown in Fig. 4 (see the position of the square symbol along the dashed line); Moreover, it is possible to see that the different shape of the current circuits, together with the higher inclination of the poloidal field lines, develops in the \( \alpha_m = 0.1 \) a strong vertical Lorentz force which strongly bends the field lines, as it was already shown in Fig. 2.

This effect is even more evident if we look at a three-dimensional rendering of the two simulations discussed here: in
Fig. 7 we show a 3D rendering of density maps in logarithmic scale for the two cases (α_m = 0.1, χ_m = 1, f = 1, left panel) and (α_m = 1, χ_m = 3, f = 1, right panel) at a time t = 130. It is possible to see that in the more diffusive case the field lines (solid blue lines) are gently wrapped around the magnetic surfaces. On the other hand, in the α_m = 0.1 simulation the footpoints of the field lines are advected towards the central object, not balanced by a strong enough diffusion; a strong differential rotation along the field line therefore appears, the footpoint of the line rotating much faster than its opposite end; a strong toroidal field develops at the footpoint which gives the strong force directed vertically shown in Fig. 6 that completely distorts the field lines.

The mainly toroidal feature visible in the left panel of Fig. 7 propagates vertically along the outflow axis similarly to a “magnetic tower” (Li et al. 2001, Lynden-Bell 2003). Moreover this mechanism is not transient, repeating itself on every field line when its footpoint is advected towards the faster rotating central part of the disk. Since the α_m = 0.1 simulation at a low resolution shows a magnetic configuration similar to the right panel of Fig. 7 it is possible to speculate that the higher numerical diffusion can effectively balance the advection of the footpoints so that no strong differential rotation along the field lines is present.

Thanks to the current circuits shown in Fig. 6 we can also better understand how the toroidal field can act to collimate the outflow against the centrifugal force of the rotating jet. It is in fact usually assumed that the self-generated toroidal field can automatically ensure the collimation of the outflow through the so called “hoop stress” due to the magnetic tension of the field and equal to −B_z^2/r. What emerges clearly from both the panels of Fig. 6 is that in the outer part of the outflow, where the currents flow out from the disk with a positive J_z component, the Lorentz force associated with the toroidal field pushes the plasma outwards, thus uncollimating the jet; in fact, in this situation the outward directed pressure gradient of the toroidal field is stronger than the collimating tension. It must be pointed out that the shape of the electric current circuits opposing the collimation of the outer layers of the outflow strongly depends on the outer boundary conditions on B_φ, as discussed already in Section 2.4. On the other hand this behavior is not just a numerical artifact: the current circuits of our simulations have the typical butterfly-like shape of analytical models of disk-winds (see Fig. 13 in Ferreira 1997). The outflow starts to be collimated where the circuit closes back with a negative J_z component so that the “hoop stress” exceeds the pressure gradient. As the poloidal velocity vectors in Fig. 6 show, in our solutions only the inner part of the outflow is almost cylindrically collimated inside the computational domain.

4. Accretion rates and ejection efficiency

After having analyzed the acceleration and collimation mechanisms we now take into account the accretion and outflow rates which characterize our solutions. In Fig. 8 we plot for all the four simulations in which we suppressed the Ohmic heating, the accretion rates M_a at t = 400, defined as:

\[ M_a = -2\pi r \int_{-1.6H}^{1.6H} \rho u_z \, dz \]

as a function of the cylindrical radius r. H(r) is the thermal height scale of the disk defined as H = (c_s/Ω_K)_infall; the above integral is calculated up to 1.6H, where the magnetic diffusivity defined by Eq. 13 becomes negligible and the plasma, now in an ideal MHD regime should flow out from the disk almost parallel to the poloidal magnetic field. It is possible to see that for all the curves shown, the accretion rate has a sudden decrease
Fig. 7. Three dimensional rendering of density at $t = 130$ for the cases ($\alpha_m = 0.1, \chi_m = 1, f = 1$, left panel) and ($\alpha_m = 1, \chi_m = 3, f = 1$, right panel). We also plotted four sample magnetic field lines wrapped around the same magnetic surface.

for $r \gtrsim 10$: this is due to the fact that the outflow is extracting efficiently mass and angular momentum only for $r < 10$, which is therefore identified as the “launching region”. As already pointed out in Section 3 at the end of our simulations the outer part of the disk has still not reached a dynamically relevant time scale and therefore had no possibility to fully develop an outflow. Moreover the flow emerging from the inner radii of the accretion disk becomes super-fast-magnetosonic inside the computational domain and therefore the “launching region” will not be affected by the outer boundary conditions. We can therefore define a control volume delimiting this part of the disk: this volume is delimited by an inner cylinder of radius $r_i = 1$ and height $-1.6H(r_i) < z < 1.6H(r_i)$ and by an outer cylinder of radius $r_e = 10$ and height $-1.6H(r_e) < z < 1.6H(r_e)$. These surfaces will be indicated as $S_i$ and $S_e$ respectively. Above the disk the control volume is closed at the surface determined by the height scale $1.6H(r)$ between $r_i = 1$ and $r_e = 10$. This surface will be indicated as $S_s$.

The four cases shown in Fig. 8 are characterized by a different outer accretion rate (calculated through $S_e$, i.e. at $r_e = 10$): when the poloidal resistivity parameter $\alpha_m$ or the anisotropy coefficient $\chi_m$ are increased the outer accretion rate, indicated as $\dot{M}_{ae}$, becomes smaller. Moreover it is evident that in the cases characterized by $\alpha_m = 0.1$ (dotted and dash-dotted line) the accretion rate is at least one order of magnitude higher than the initial one (see Table 1) which was determined to balance the advection and the diffusion of poloidal field lines inside the disk: it is therefore clear that in these cases the advection of the field should dominate thus explaining the peculiar dynamical effects of the $\alpha_m = 0.1$ simulations shown in the previous Section. Despite of having such a high accretion rate, we recall that the low resistivity case at low resolution (dash-dotted line) does not show these features, thus supporting the idea that the strong advection is balanced by a high numerical diffusion.

Besides of being characterized by a different $M_{ae}$, the solutions show also a different slope of the accretion rate inside the “launching region”. This slope is clearly linked to the amount of
mass that is extracted from the disk and goes into the outflow: defining a simplified power-law radial behavior for the accretion rate (Ferreira & Pelletier 1995)

\[ M_a(r) = M_{ae} \left( \frac{r}{r_e} \right)^\xi \]

we can find, imposing mass conservation inside the accretion disk, a simple relation between the ejection efficiency and the ejection parameter $\xi$:

\[ \frac{2M_j}{M_{ae}} = 1 - \left( \frac{r_e}{r} \right)^\xi \] (31)

Here the ejection efficiency $2M_j/M_{ae}$ is defined as the ratio of the mass outflow from the disk surface $S_s$ and the outer accretion rate calculated at $r_e = 10$; the factor 2 takes into account the two bipolar jets. It is easy to see that the higher the ejection efficiency the higher the ejection parameter $\xi$ and therefore the steeper is the slope of the accretion rate.

In Fig. 9 we plot the ejection efficiency $2M_j/M_{ae}$ as a function of time for the four simulations taken into account in this Section: it is possible to see that after an initial transient, this efficiency reaches an almost constant value, suggesting the attainment of a stationary final state. On the other hand the solutions reach different efficiency values, increasing from 20% for the $(\alpha_m = 1, \chi_m = 3)$ case to 55% for the $(\alpha_m = 0.1, \chi_m = 1)$ simulation. This result is consistent with the plot shown in Fig. 8 where we showed that the forces acting vertically inside the disk change sign at lower height scales in less diffusive cases: since the disks become denser towards the midplane this determines also a higher outflow rate. As usual the low resistivity case at low resolution shows an anomalous behavior (dash-dotted line) having an ejection efficiency typical of the $\alpha_m = 1$ cases.

Moreover it is clear the direct correlation between the ejection rate and the ratio $|B_d|/B_p$ plotted in Fig. 5 This means that the main mechanism accelerating the outflow depends strongly on the outflow rate: the less mass loaded jets are efficiently centrifugally accelerated, while in the more loaded jets the higher inertia strongly bends the magnetic field in the azimuthal direction right above the disk surface; as stated by Eq. (30), the outflow is therefore mainly accelerated by the pressure gradients of the toroidal field (see also Anderson et al. 2004).

Consistently the solutions characterized by a higher ejection efficiency show a steeper slope of the accretion rate as shown by Eq. (31); as an example, we plotted in Fig. 8 the power-laws calculated solving Eq. (31) for $\xi$ for the cases $(\alpha_m = 1, \chi_m = 3)$ and $(\alpha_m = 0.1, \chi_m = 1)$ (long-dashed lines). The ejection parameters $\xi$ calculated for these two cases are $\xi = 0.09$ and $\xi = 0.17$ respectively.

5. Angular momentum transport

It is commonly accepted that disk-winds are a viable mechanism to extract angular momentum from accretion disks without recurring to viscous torques to allow accretion. In this Section we show how the bipolar jets can extract angular momentum from the underlying disk and through which channel.

The angular momentum transport is regulated by the angular momentum flux through the surface delimiting the control volume defined in Section 4. We can therefore define an accretion torque $J_{acc} = J_{acc,kin} + J_{acc,mag}$ defined by the sum of the two terms:

\[ J_{acc,kin} = \int_S r \rho u_d \cdot dS - \int_S r \rho u_d \cdot dS \] (32)

and

\[ J_{acc,mag} = \int_S r |B_s| \cdot dS - \int_S r |B_s| \cdot dS \] (33)

For the accretion torque we use a positive sign if it increases the angular momentum contained inside the control volume and a negative sign if it extracts it. The term $J_{acc,kin}$ defines the flux of angular momentum inside the control volume due to the accretion motion, while $J_{acc,mag}$ is the magnetic torque which can transport angular momentum along the radial direction inside the disk. This magnetic torque represents a small contribution to $J_{acc}$, in all of our simulations the ratio $J_{acc,mag}/J_{acc,kin}$ is always negligible, around -10%. A small fraction of the disk angular momentum is extracted by the magnetic torque also radially, thus helping accretion. On the other hand, the main contribution to $J_{acc,kin}$ is given by the integral on the outer radius, where both the accretion rate and the specific angular momentum of the disk are higher. Assuming a Keplerian rotation, a reasonable approximation for $J_{acc}$ is therefore given by (see also Eq. (28)):

\[ J_{acc} \sim M_{ae} \sqrt{GM_r}l \] (34)

In the same way we can define the torque exerted by the outflow on the disk $J_{jet} = J_{jet,kin} + J_{jet,mag}$ as the sum of the flux of angular momentum $J_{jet,kin}$ and of the magnetic torque $J_{jet,mag}$ at the disk surface:

\[ J_{jet,kin} = \int_S r \rho u_d \cdot dS \] (35)

and

\[ J_{jet,mag} = \int_S r |B_s| \cdot dS \] (36)

For the jet torque we use a positive sign if extracts angular momentum from the disk.

In a steady situation the accretion and the jet torque should be equal: $2J_{jet} = J_{acc}$. Using the approximation made in Eq. (34), the relation

\[ M_{ae} \sqrt{GM_r}l \sim 2J_{jet} \] (37)
shows clearly that the accretion rate at the outer radius of the launching region $r_e$ is mainly controlled by the torque exerted by the outflow on the accretion disk. In the central panel of Fig. 10 we plot the temporal evolution of the total torque $2\dot{J}_{jet}$ exerted by the bipolar jets of the four simulations performed without Joule heating: it is clear the correlation between the outer accretion rate shown in Fig. 8 and the jet torque plotted here, as stated by Eq. (37). The high accretion rate of the $\alpha_m = 0.1$, cases, which determines the peculiar behavior of these solutions, is determined by the high torque exerted by the jet, at least ten times higher than the torque needed to maintain the initial rate (see Table 1).

The curves plotted in the left panel of Fig. 10 show a slow decrease in time, thus suggesting that our solutions did not reach a final steady state. On the other hand, the ratio $2\dot{J}_{jet}/\dot{J}_{acc}$ plotted in the central panel of Fig. 10 stays approximately constant in time and equal to one, after an initial transient: this shows that the accretion rate of the disk reacts quickly to the slowly varying jet torque suggesting that our simulations are evolving through a series of quasi-stationary states.

Even if the outflows of the different simulations are exerting almost the same torque (left panel, Fig. 10), being a factor two smaller just in the case characterized by a high and anisotropic magnetic diffusivity (solid line), the ratio between the magnetic $J_{jet, mag}$ and the mechanical torque $J_{jet, kin}$ shows noticeable differences (right panel in Fig. 10). The ratio goes from a value around unity for the less dissipative case ($\alpha_m = 0.1, \chi_m = 1$) up to a value $\gtrsim 4$ for the ($\alpha_m = 1, \chi_m = 3$) simulation. Once again we point out that the low resistivity simulation at low resolution (dot-dashed line) behaves like a higher resistivity case, showing $J_{jet, mag}/J_{jet, kin} \sim 4$. This quantity has an easy interpretation: the angular momentum extracted from the disk is in fact stored initially in the toroidal magnetic field and it is then transferred starting from the disk surface to the outflowing plasma which is therefore centrifugally accelerated. The ratio $J_{jet, mag}/J_{jet, kin}$ is therefore a measure of the efficiency of the magneto-centrifugal mechanism: the higher is this ratio, the more specific angular momentum is available at the disk surface, centrifugally accelerating the plasma to higher poloidal terminal speeds (see Section 6).

### 6. Energy budget of the disk-jet system

In this Section we finally study the energetics of the disk-jet system. In order to study the energy balance of our simulations we consider the energy fluxes through the surfaces of the control volume defined in Section 4. We then define an accretion power $\dot{E}_{acc} = \dot{E}_{acc, mec} + \dot{E}_{acc, mag} + \dot{E}_{acc, thm}$ given by the sum of the terms:

$$\dot{E}_{acc, mec} = \int_{S_i} \left( \frac{u_x^2 + \Phi_g}{2} \right) \rho u \cdot dS - \int_{S_i} \left( \frac{u_x^2 + \Phi_g}{2} \right) \rho u \cdot dS$$

$$\dot{E}_{acc, mag} = \int_{S_i} E \times B \cdot dS - \int_{S_i} E \times B \cdot dS$$

$$\dot{E}_{acc, thm} = \int_{S_i} \gamma - 1 \rho u \cdot dS - \int_{S_i} \gamma - 1 \rho u \cdot dS$$

where $\dot{E}_{acc, mec}$, $\dot{E}_{acc, mag}$ and $\dot{E}_{acc, thm}$ represent the mechanical (kinetic + gravitational), Poynting and enthalpy flux through the cylindrical surfaces at the external and internal radius of the launching region. For the accretion power we will use a positive sign if it increases the energy contained inside the control volume.

In the left panel of Fig. 11 we plot the temporal evolution of the total accretion power while in Table 2 we show the different contributions to $\dot{E}_{acc}$ at $t = 400$ for the cases taken into account in this Section. We can see that the dominant term is always the mechanical power liberated in the accretion: the Poynting flux increases slightly the total energy contained in the control volume, while the enthalpy flux always acts to decrease the accretion power, advecting the thermal energy at the inner radius. It is important to notice that the simulation with a low magnetic diffusivity performed at a lower resolution (second line) shows a higher enthalpy flux due to the higher numerical dissipation. Neglecting the enthalpy and the magnetic contribution and assuming a Keplerian rotation, the energy liberated by accretion can be safely approximated by (see also Eq. (28)):

$$\dot{E}_{acc} \sim \frac{GM}{2r_1} - \frac{M_i GM}{2r_e}$$

### Table 2. Values of the different contributions to the total accretion power $\dot{E}_{acc}$ calculated at $t = 400$. The simulations are characterized by their diffusivity parameters $\alpha_m$ and $\chi_m$.

| Simulation | $\dot{E}_{acc, mec}/\dot{E}_{acc}$ | $\dot{E}_{acc, mag}/\dot{E}_{acc}$ | $\dot{E}_{acc, thm}/\dot{E}_{acc}$ |
|------------|----------------------------------|----------------------------------|----------------------------------|
| ($\alpha_m, \chi_m$) |                   |                                  |                                  |
| (0.1, 1)   | 0.97                             | 0.14                             | -0.11                            |
| (0.1, 1) low r. | 1.29                             | 0.09                             | -0.38                            |
| (1, 1)     | 0.97                             | 0.10                             | -0.07                            |
| (1, 3)     | 0.99                             | 0.05                             | -0.04                            |
Fig. 11. Temporal evolution of the accretion power $\dot{E}_{\text{acc}}$ (left panel); the ratio between the jet and accretion power $2\dot{E}_{\text{jet}}/\dot{E}_{\text{acc}}$ and the ratio between the radiated and accretion power $2\dot{E}_{\text{jet}}/\dot{E}_{\text{acc}}$ (central panel); the ratio between the jet magnetic and kinetic power $\dot{E}_{\text{jet, mag}}/\dot{E}_{\text{jet, kin}}$ (right panel). The plots refer to the cases ($\alpha_m = 1, \chi_m = 3$ solid line), ($\alpha_m = 1, \chi_m = 1$ dotted line), ($\alpha_m = 0.1, \chi_m = 1$ dashed line) and ($\alpha_m = 0.1, \chi_m = 1$, low resolution dot-dashed line).

Table 3. Values of the different contributions to the total jet power $\dot{E}_{\text{jet}}$ calculated at $t = 400$. The simulations are characterized by their diffusivity parameters $\alpha_{\text{m}}$ and $\chi_{\text{m}}$.

| Simulation ($\alpha_{\text{m}}, \chi_{\text{m}}$) | $\dot{E}_{\text{jet, kin}}/\dot{E}_{\text{jet}}$ | $\dot{E}_{\text{jet, grv}}/\dot{E}_{\text{jet}}$ | $\dot{E}_{\text{jet, mag}}/\dot{E}_{\text{jet}}$ | $\dot{E}_{\text{jet, thm}}/\dot{E}_{\text{jet}}$ |
|-----------------------------------------------|---------------------------------|---------------------------------|---------------------------------|---------------------------------|
| (0.1, 1)                                      | 0.42                            | -0.75                           | 1.28                            | 0.03                            |
| (0.1, 1) low r.                               | 0.13                            | -0.24                           | 1.05                            | 0.06                            |
| (1, 1)                                        | 0.25                            | -0.49                           | 1.22                            | 0.02                            |
| (1, 3)                                        | 0.13                            | -0.26                           | 1.12                            | 0.01                            |

This approximation shows clearly that the liberated power is mainly determined by the inner accretion rate: the values of $\dot{E}_{\text{acc}}$ of our simulations at $t = 400$ (left panel of Fig. 11) are clearly correlated with the inner accretion rates visible in Fig. 8.

To characterize the energy extracted by the outflow we define a jet power $\dot{E}_{\text{jet}} = \dot{E}_{\text{jet, kin}} + \dot{E}_{\text{jet, grv}} + \dot{E}_{\text{jet, mag}} + \dot{E}_{\text{jet, thm}}$ given by the sum of the fluxes of kinetic, gravitational, magnetic and thermal energy at the disk surface $S_c$:

$$\dot{E}_{\text{jet, kin}} = \int_{S_c} \frac{u^2}{2} \rho \mathbf{u} \cdot d\mathbf{S}$$  \hspace{1cm} (42)$$

$$\dot{E}_{\text{jet, grv}} = \int_{S_c} \Phi_g \rho \mathbf{u} \cdot d\mathbf{S}$$  \hspace{1cm} (43)$$

$$\dot{E}_{\text{jet, mag}} = \int_{S_c} \mathbf{E} \times \mathbf{B} \cdot d\mathbf{S}$$  \hspace{1cm} (44)$$

$$\dot{E}_{\text{jet, thm}} = \int_{S_c} \frac{\gamma}{\gamma - 1} P \mathbf{u} \cdot d\mathbf{S}$$  \hspace{1cm} (45)$$

For the jet power we will use a positive sign if it extracts energy from the control volume.

Since in this Section we are taking into account cases characterized by $f = 1$, we can also define the radiated power $\dot{E}_{\text{cool}}$ as

$$\dot{E}_{\text{cool}} = \int_{V_c} \Lambda_{\text{cool}} dV = \int_{V_c} \tilde{\mathcal{H}} J \cdot dV$$  \hspace{1cm} (46)$$

where $V_c$ is the control volume. In a steady situation the energy fluxes should balance to give $2\dot{E}_{\text{jet}} = \dot{E}_{\text{acc}} - \dot{E}_{\text{cool}}$. In the central panel of Fig. 11, we plot the time evolution of the ratios $2\dot{E}_{\text{jet}}/\dot{E}_{\text{acc}}$ and $\dot{E}_{\text{cool}}/\dot{E}_{\text{acc}}$. We can see that most of the accretion power is liberated in the jet, increasing from $\sim 90\%$ for the more dissipative case ($\alpha_m = 1, \chi_m = 3$) up to $\sim 98\%$ for the less dissipative one ($\alpha_m = 0.1, \chi_m = 1$). Correspondingly the radiated power decreases from $\sim 10\%$ down to $\sim 2\%$ respectively.

Anyway the two contributions sum up to approximately one, suggesting a quasi-stationary situation.

As it was noticed for the jet torques, also the jet powers do not show a huge difference between the simulations. On the other hand the different contributions to the total jet power depend more clearly on the simulation parameters. In Table 3 we see that the greater contribution to the jet power at the surface of the disk comes from the Poynting flux, since most of the energy is stored in the toroidal magnetic field. The enthalpy flux is always negligible: our jets are cold, undergoing a continuous adiabatic expansion starting from the disk surface. The kinetic and the gravitational fluxes are strongly correlated with the outflow rates of the different cases: the gravitational contribution is negative since at the disk surface the outflow is still inside the potential well of the central object.

In the right panel of Fig. 11, we plot the temporal evolution of the ratio between the jet Poynting and kinetic flux $\dot{E}_{\text{jet, mag}}/\dot{E}_{\text{jet, kin}}$. A higher $\dot{E}_{\text{jet, mag}}/\dot{E}_{\text{jet, kin}}$ ratio indicates that more magnetic energy per particle is available at the disk surface to accelerate the outflow. The Poynting flux available at the disk surface is then converted into kinetic energy along the outflow and at the upper end of the computational box the kinetic flux is dominant, as confirmed by the fact that the flow is super-fast-magnetosonic. The asymptotic speed of the outflow is therefore higher for cases characterized by a higher $\dot{E}_{\text{jet, mag}}/\dot{E}_{\text{jet, kin}}$, as confirmed by Fig. 12 where we plot an average value of the poloidal speed normalized to the toroidal (Keplerian) speed at the base of each streamline. The poloidal velocity measured at the upper end of the computational domain is a few times the Keplerian speed at the base of the streamlines, consistently with the fact that different classes of astrophysical jets show velocities of the same order of the escape velocity from the central object (Livio 1999).
7. A quasi-steady solution

In this Section we will try to characterize one of the simulations performed as a steady solution. We already pointed out that none of our solutions are perfectly steady: the low diffusivity case shows a continuously evolving magnetic structure, while all the simulations show a slowly evolving jet torque (left panel in Fig. 10) and accretion power (left panel in Fig. 11). On the other hand the constant values assumed by the accretion efficiency (Fig. 9), by the ratio between the jet and the accretion torque (central panel in Fig. 10) and by the ratio between the jet and the accretion power (central panel in Fig. 11), indicate that our solutions, at least those with $\alpha_m = 1$, are slowly evolving through a series of quasi-stationary states. In this Section we will analyze the simulation characterized by $\alpha_m = 1, \chi_m = 3$ and $f = 1$: besides of showing some features of a stationary solution, its parameters $\alpha_m, \chi_m, \mu$ and $\epsilon$ are typical of the cold self-similar steady solutions found by Casse & Ferreira (2000a), allowing a direct comparison between steady and time-dependent solutions.

We recall a few relations which are valid for a steady axisymmetric solution of the ideal MHD equations. The poloidal speed $u_p$ is related to the poloidal magnetic field $B_p$ by

$$u_p = \frac{K(\Psi)}{\rho} B_p$$

(47)

where $K(\Psi)$ is a constant along every field line marked by the flux function $\Psi$ and it represents the ratio between the mass flux and the magnetic flux. This relation clearly states that in a steady situation the poloidal speed and magnetic field are parallel. The toroidal speed $u_\theta$ is related to the rotation rate of the magnetic surface $\Omega(\Psi)$ by

$$u_\theta = \Omega(\Psi) r + \frac{K(\Psi)}{\rho} B_\theta$$

(48)

The total specific angular momentum $l(\Psi)$, which is constant along each field line, is given by

$$l(\Psi) = ru_\theta - \frac{r B_\theta}{K(\Psi)} = \Omega^2 \lambda^2$$

(49)

where $r_A$ is the Alfven radius. Each field line is therefore characterized by the non-dimensional constants $k$, or mass loading parameter, and $\lambda$, which gives a measure of the magnetic lever arm $r_A/r_0$:

$$k = \frac{K(\Psi) r_0 \Omega}{B_0}$$

(50)

$$\lambda = \frac{l(\Psi)}{\Omega^2 r_0} = \left( \frac{r_A}{r_0} \right)^2$$

(51)

where $r_0$ is the cylindrical radius of the footpoint of the field line and $B_0$ is the value of the magnetic field at $r_0$.

We therefore tried to characterize our simulation computing these steady invariants along three sample field lines whose footpoints are located at $r_0 = 2$ (solid line in the next Figures), 4 (dashed line) and 8 (dot-dashed line).

**Fig. 12.** Average poloidal speed $u_{pol}$ along the jets calculated at $t = 400$ normalized to the value of the toroidal (Keplerian) speed $u_{\phi 0}$ at the base of each streamline. These curves have been obtained averaging the ratio $u_{pol}/u_{\phi 0}$ on all the streamlines outflowing from the upper boundary of the computational box. The line style used refers to the same cases as in Fig. 11.

**Fig. 13.** Angle between poloidal speed and poloidal magnetic field for the case characterized by $\alpha_m = 1, \chi_m = 3$ and with the Ohmic heating suppressed. This quantity is calculated at $t = 400$ along three sample field lines whose footpoints are located at $r_0 = 2$ (solid line), 4 (dashed line) and 8 (dot-dashed line).

**Fig. 14.** Rotation rate of the magnetic field calculated along the fieldlines whose footpoints are located at $r_0 = 2$ (solid line), 4 (dashed line) and 8 (dot-dashed line).
Fig. 15. Mass loading parameter calculated along the fieldlines whose footpoints are located at $r_0 = 2$ (solid line), 4 (dashed line) and 8 (dot-dashed line).

Fig. 16. Magnetic lever arm parameter calculated along the fieldlines whose footpoints are located at $r_0 = 2$ (solid line), 4 (dashed line) and 8 (dot-dashed line).

Fig. 17. $k$-$\lambda$ relation (squares). The values are calculated at the Alfvén point along different fieldlines anchored in the launching region of the disk ($1 < r < 10$). The value of $k$ increases going from the outer to the inner fieldlines. For comparison it is also plotted the relation Eq. (52) found by Weber & Davis (1967) for a radial wind geometry (dot-dashed line).

We plotted an averaged value over the disk surface of these two ratios in Fig. 10 and 11. The magnetic lever arm is therefore a measure of the efficiency of the magneto-centrifugal mechanism: assuming that all the Poynting flux available at the disk surface is completely converted into poloidal kinetic energy, the following relation for the poloidal asymptotic speed of the outflow $u_{p,\infty}$ holds:

$$u_{p,\infty} = \Omega r_0 \sqrt{2\lambda - 1 - 3}$$

Therefore the poloidal speed, expressed in units of the rotation speed at the footpoint, asymptotically assumes higher values on the outer field lines.

Finally, it is interesting to compare our solution with the cold self-similar steady solutions found by Casse & Ferreira (2000a) characterized by the same parameters $\alpha_m, \chi_m, \epsilon$ and $\mu$. The simulation presented in this section shows greater values of the mass...
loading $k$ and of the ejection parameter $\xi \sim 0.1$ (calculated in Section 4) and lower magnetic lever arms. The steady solution, characterized approximately by $k \sim 2 \times 10^{-2}$, $\xi \sim 10^{-2}$ and $\lambda \sim 35$, has therefore a much lower outflow rate and higher terminal speed.

A possible physical explanation is given by the shape of the current circuits shown in Section 3.2. Ferreira (1997) has shown that high ejection efficiencies ($\xi > 0.5$) are obtained if the poloidal current enters the disk at its surface and that no steady solution crossing all the critical points is possible. Its solutions are always characterized by a lower ejection parameter $\xi < 0.5$ and by a current outflowing from the disk surface. In our solutions the current returns and enters the disk in small region $1 < r < 2$ near the central sink region: due to the internal boundary itself the rotation of the outflow goes suddenly to zero at the inner edge of the jet, creating a current sheet which is forced to flow back inside the disk. This is therefore a numerical effect and a proper treatment of the region of interaction between the disk and the central object is required to understand how the currents close in the inner region of the system. Anyway it is possible to speculate that the returning current increases the ejection efficiency as proposed by Ferreira (1997): this could be also confirmed by the fact that in our solution the mass loading parameter $k$ increases towards the center, where the current flows back into the disk.

Another possible explanation is determined by the numerical dissipation which can increase the thermal energy and the pressure in the launching region, increasing the outflow rate as shown in Section 3.1 (see also Casse & Ferreira 2000b). If on one hand we showed that a low resolution yields the effects of a high disk diffusivity, on the other hand we cannot be confident that the standard resolution used in the simulations is sufficient to accurately resolve the vertical structure of the accretion disk: it is well known that it is difficult for the type of algorithms as the one used in this paper to handle low densities, as in the launching region, where the disk expands a lot. This is also suggested by the behavior of the invariants $k$ and $\lambda$, e.g. on the inner field line considered: they show a slow transition to their more or less constant values, indicating that the transition between the resistive and the ideal MHD regimes happens on a scale larger than the one determined by the vertical profile of the diffusivity (Eq. (13)).

### 8. Effects of Ohmic heating

We finally make a few considerations about the two simulations performed in which all the dissipated magnetic energy is released locally inside the disk ($f = 0$) and we will make a comparison with the corresponding cases in which the dissipated energy is radiated away ($f = 1$). We recall that these two simulations are characterized by $(\alpha_m = 0.1, \chi_m = 1)$ and $(\alpha_m = 1, \chi_m = 3)$.

The dissipative Ohmic term $\Lambda = \vec{\eta} \cdot \vec{J}$ acts now to increase the thermal energy and the entropy of the plasma inside the disk. This has two important consequences: the energy dissipation at the midplane changes the disk thickness and modifies its structure while an increase of the pressure gradient at the disk surface allows more matter to be loaded in the outflow.

In Fig. 18 we plot the time evolution of the ejection efficiency of the two “hot” solutions and of the two corresponding “cold” simulations: both the “hot” simulations are characterized by a higher efficiency; moreover, the case with $\alpha_m = 1$ (solid line) shows at the end of the simulation a steep increase. This is due both to a decrease of the outer accretion rate and to an increase of the outflow rate: the energy dissipation steepens the radial gradient of the thermal pressure inside the disk, thus slowing down the outer accretion; at the inner radius the gradient is so steep that the disk does not accrete anymore and all the matter is pushed by the pressure gradient in a highly unsteady outflow.

The disk thermal energy of the “cold” cases is almost constant in time. The “hot” case with $\alpha_m = 0.1$ shows a similar behavior, with a disk thermal energy just slightly higher than the corresponding “cold” simulation. It is likely that the higher outflow rate can balance the small increase of the disk thermal energy which is equivalent to the 2% of the accretion power that was “radiated” in the corresponding “cold” simulation (see Section 6). On the other hand the disk thermal energy of the “hot” counterpart of the $\alpha_m = 1$ simulation continuously increases, not balanced by the energy extracted by the outflow, leading to the unsteady behavior shown in Fig. 18. It is important to notice that the behavior of this simulation depends a lot on the expression that we used for the magnetic diffusivity (Eq. (13)) which is proportional to the disk thermal height scale: the Ohmic heating determines in fact an increase of the disk thickness that in turn implies a higher magnetic diffusivity.

### 9. Summary and conclusions

Our calculations have followed the long-term evolution of an axisymmetric quasi-Keplerian magnetized disk up to the establishment of an inflow/outflow configuration. The accretion flow is driven by extraction of angular momentum of the disk by the jet; this is shown by reaching balance between the accretion and jet torques. The magnetic torque of the jet is most efficient close to the surface of the disk extracting angular momentum from the accretion flow; it stores angular momentum in the toroidal magnetic field that then accelerates the outflowing plasma. Therefore the simulations have succeeded in demonstrating that the magnetocentrifugal mechanism originally proposed by Blandford & Payne can launch jets, provided certain physical conditions on the magnetic resistivity and initial field configuration are satisfied.
In particular we have shown that an isotropic ($\chi_m = 1$) resistive configuration with $\alpha_m = 0.1$, due to the stronger advection of the field compared to its diffusion, produces highly unsteady magnetic structures, like it was displayed in Fig. 4 and in the left panel of Fig. 7. This is in agreement with the stationary models of Casse & Ferreira (2000a) according to which it is not possible to obtain a steady outflow with such a low $\alpha_m$ parameter except for highly anisotropic configurations ($\chi_m > 10^3$). This would be in agreement with the results shown in Section 5 a much stronger toroidal resistivity should reduce the torque exerted by the jet on the disk and thereby its accretion rate (Eq. (57)). On the other hand we have also shown how this unsteady behavior depends strongly on the resolution assumed and therefore on the numerical dissipation: the same case $\alpha_m = 0.1$ performed with a resolution four times lower presents many of the characteristics of a solution with $\alpha_m = 1$. It can be presumed that the quasi-stationary behavior found by Casse & Keppens (2002) 2004 with an isotropic $\alpha_m = 0.1$ parameter can be affected by the resolution used.

On the other hand, also the cases characterized by $\alpha_m = 1$ are not perfectly stationary, despite of showing an ordered magnetic configuration favorable for a steady launching: in all the simulations performed some integrated quantities, like the jet torque (Fig. 10) or the accretion energy (Fig. 11), are still slowly evolving. Even the case characterized by an anisotropic ($\chi_m = 3$) diffusivity, needed by the stationary models, does not show a perfectly stationary behavior; moreover it can be characterized by adimensional quantities like $\xi$, $k$ or $\lambda$ which differ a lot from the analogous found in the cold solutions of Casse & Ferreira (2000a). In Section 7 we proposed some physical and numerical reasons to explain this behavior.

Nevertheless the constant value assumed in time by some quantities, like the ejection efficiency (see Fig. 9) or the ratio between the accretion power and the jet energy flux (central panel of Fig. 11) suggests that our solutions are slowly evolving through a series of quasi-stationary states. Some clear trend emerges: increasing the poloidal and/or the toroidal resistivity the ejection efficiency decreases, going from 55% in the ($\alpha_m = 0.1$, $\chi_m = 1$) case to 20% for the ($\alpha_m = 1$, $\chi_m = 3$) simulation. From the energetic point of view, our simulations show that more than 90% of the energy liberated in the accretion inflow is released in the jet and that less dissipative cases produce slightly more powerful jets. Correspondingly, the radiated power that must take care of the Joule heating dissipation is less than 10%. In Section 8 we have also shown that in the more dissipative case with $\alpha_m = 1$, if this energy is released inside the disk instead of being radiated, the accretion flow is disrupted and a highly unsteady outflow is formed.

We have also demonstrated how the efficiency of the magneto-centrifugal mechanism, as measured by the ratio of the Poynting flux to the kinetic flux at the disk surface, is affected by the resistive configuration. A higher value of the poloidal and/or toroidal resistivity determines a greater magnetic lever arm, linked to the energy flux ratio as stated by Eq. (53). Our simulations show also that the $\lambda$ parameter, as measured by Eq. (53), is related with a good approximation to the ejection parameter $\xi$ (in the cases where this parameter has been measured):

$$\lambda \sim 1 + \frac{1}{2\xi}$$  

as stated by the self-similar models by Ferreira (1997).

We finally try to test if observations of outflows from classical T-Tauri stars can give some constraint on the parameters of our simulations. The size of the launching region measured in our simulations (from 0.1 to 1 AU) is consistent with the estimates derived from observations (Bacciotti et al. 2000). On the other hand, the one-sided ejection efficiency inferred from the observations gives values, even with high uncertainty, in the range 0.01-1 (Cabrit 2002): this result seems to favor our solution characterized by ($\alpha_m = 1$, $\chi_m = 3$). Even if, in agreement with observations, all our solutions are characterized by a speed at the upper boundary of the computational domain which is a few times the escape speed from the potential well of the central object (Fig. 17), a quantitative comparison with the poloidal speed (Bacciotti et al. 2000) and rotation signatures (Bacciotti et al. 2002) Coffey et al. 2004 Woitas et al. 2005 of the observed outflows is much more difficult: the size of our computational domain, which reaches a physical scale along $z$ equal to 12 AU, is around three times smaller than the distance from the source investigated by current HST observations (> 30 AU). Anyway, as it has been pointed out by Ferreira et al. 2006, a solution characterized by a magnetic lever arm $\lambda \sim 10$ successfully reproduces the values of both the poloidal and toroidal speeds at the currently observed spatial scale: the highest lever arm value found in our simulations ($\lambda \sim 9$) is observed in the outer part of the launching region of the case ($\alpha_m = 1$, $\chi_m = 3$) (Fig. 17).

Two specific conclusions and difficulties of our calculations must be mentioned. First, the magneto-centrifugal process appears to operate only with relatively strong magnetic fields in the disk; a strong field tends to inhibit the development of turbulence that could be the origin of magnetic resistivity. Instabilities (shear, Kelvin-Helmholtz) other than magneto-rotational might produce the required turbulence. On the other hand, the resistivity inside a protostellar disk can be extremely high between 0.1 and 10 AU, where the grains become the dominant charge carriers (Wardle 1997). The coupling between the accreting plasma and the magnetic field is therefore strong, perhaps too much, reduced. The second point is that outflow rates measured in our simulations turn out to be rather high, which applies well to jets in star formation regions, where extended bipolar structures are observed; for AGN nuclei one must go to the relativistic limit (which we have not done here), and also discuss how the (possibly subsonic) wind from the disk outer part interacts with the surrounding galaxy.

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