Forecasting the life of a structure relative to the operating mode

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Abstract. The question of forecasting the service life of transport machines designed by taking into consideration the load spectrum, that is close to the real one, is an important problem at the calculation. One of the ways to simulate real operating conditions at the design calculation is a method of randomization quasirandom loads. Methods of randomization are widely used in many areas of science and technique. In the article, the numeric comparison of different ways of randomization is shown at the calculation for determining the service time using two techniques: the use of a standardized function of randomization in the high-level programming language and the law of normal distribution at its different parameters. The use of the law of normal distribution makes the more exact fatigue calculation because it makes it possible to simulate the quasirandom process that corresponds to the real operation picture to a greater degree. The results presented in the work make it possible to fulfill the calculation of the service time of the metallic structure that is under cyclic asymmetric loads, at the well-known nature of the application of loading to it.

1. Introduction

The service time of any technical device or structure is determined in the element that undergoes the highest load, the destruction of which will cause the break-down of the entire structure. At the forecasting of the service time, it is necessary to take into consideration all operation loads in the calculation with maximal accuracy [1,2]. At the design of metal structures, at the unknown load distribution inside of the spectrum, most often, the calculation based on the limit condition is used, or an allowance is made that the application of load takes place within the range of certain values, and the distribution inside of these values is random. The first thing is a justified solution for increase the service time that makes it possible to secure the trouble-free operation time within the entire life cycle of the product but the second thing makes it possible to evaluate the real number of the cycles more accurately whilst the product will run until the destruction will take place.

In case the information is absent that is proven experimentally, different assumptions are brought forward in that hypotheses on the nature of operation of the designed system are created. At the calculation for determining the period of service, randomization methods can be used, but regularities, on that base they are created, are not always well-known to the end-user at the use of any programming language.
In the literary works, researches exist which are based on assumptions on the eventuality of the process [3,4], the statistically accrued information for the prevention of break-downs and forecasting of the structure service time [5] is used. However, the use of the standardized way of randomization without taking into consideration the nature of operation can result in incorrect calculation. In figure 1a, a random distribution in a certain range of values is presented that is obtained by using the standardized function of randomization in the high-level programming language Python that is a well-mixed quasirandom process that can correspond to the real image of operation. Whilst at figure 1b a well-disposed sequence of random values from figure 1a is presented. The percentage at each value remains constant, i.e. the distribution at the certain interval takes place uniformly with a certain small inaccuracy.

![Figure 1. The randomization picture obtained by using the basic function of randomization in the Python programming language: line a - in combined form; line b - in an ordered form.](image)

![Figure 2. Distribution of maximal dynamic stresses [6].](image)

In the work [6], results of the distribution of maximal dynamical equivalent stresses are presented at tensile tests of the carriage of the freight car within the year. The table of maximal dynamical equivalent stresses in the work [6] presented as a graph in figure 2 shows that the use of the uniform distribution could give incorrect forecasts at the calculation of the service time. That’s why the question of the use of other ways of randomization arises.

The most popular kinds of theoretical distributions are a normal distribution, binomial distribution, Poisson distribution, etc. Each kind of distribution has its peculiarities and is characteristic for certain cases [7]. When the area of probabilistic events is not defined, most often, the law of normal distribution is used (or the distribution based on the Gauss’s law):

\[
f(x) = \frac{1}{\sigma \sqrt{2\pi}} e^{- \frac{(x-\mu)^2}{2\sigma^2}},
\]

where \(\sigma\) – is a mean-square deviation of the distribution from the mathematical expectation; \(\mu\) – mathematical expectation (or an average value). These parameters are usually selected based on statistical observations. In 99.7% of cases, values of a random variable that has a normal distribution have a difference from the mean value that does not exceed three standard deviations [8].

The goal of the work is a research of the influence of parameters of the law of normal distribution on the quasirandom application of load. As an illustrated example, a calculation of the service-time will be analyzed for a tank wagon in the dangerous cross-section. The unit under consideration is a place of attachment of the boiler bottom to the main beam. Initial data for calculation were obtained in the work [9]. The calculation will be fulfilled based on the linear hypothesis [10] and based on a model of cyclic degradation of the material [9,11,12] at different parameters of the law of normal distribution that corresponds to the mode of operation of the tank wagon.
2. Methodology of fatigue calculation

Fatigue calculations based on two techniques are presented: the rule of the linear summation of damages (or the Palmgren-Miner hypothesis) [10] and the model of cyclic degradation of the material [12]. The structure damage $\omega$ is calculated as a relation of the current number of cycles to the limit number

$$\omega = \sum_{i=1}^{k} \frac{n_i}{N_i},$$

where $n_i$ is the number of cycles at stress $i$, and $N_i$ is the service time at stress $i$. In addition, as a criterion it is the classical understanding of the rule of linear summation, the equation $\omega=1$ serves. This hypothesis is insensitive to the reorganization of cycles that makes it universal and easy to use but has its inaccuracy at multiple changes of the cyclic load and the presence of rare peak glideslopes.

An alternative approach proposed in works [9,11,12] is sensitive to the reorganization of cycles; this makes it possible to obtain a clarified fatigue forecast in case of the change of the nature of structure operation or the impact of the external environment on it. This method of the fatigue calculation based on the model of the cyclic degradation involving a manner of randomization for the asymmetric cycle of application of load was described in detail in works [9,11]. In this chapter, we will briefly repeat the highlights of the used method. For this calculation, it is necessary to compose a software where three calculations are fulfilled in parallel:

- calculation based on a linear hypothesis (2) of the damage $\omega$;
- calculation based on a model of cyclic degradation of the breaking strength change $S_B$, and
- calculation based on a model of cyclic degradation of the damage $\omega_C$.

As a criterion of the occurrence of the limit condition at the linear hypotheses an equation $\omega=1$ is regarded; for the model of cyclic degradation, the change of the descending branch of the breaking strength $S_{B0}(\sigma_M, n)$ will not be equal to the current stress $\sigma_M$ (figure 3) for the time being. The calculation of the kinetic curve is carried out through the expression in the Corten-Dolan form [9,11,12]:

$$S_B(\sigma_M, n) = S_{B0} - k_\sigma n^m,$$

where $k_\sigma$ is a kinetic coefficient that is calculated based on the condition of fatigue and statistical destruction $S_B(\sigma_M, N) = \sigma_M$ [12]:

$$k_\sigma = \frac{S_{B0} - \sigma_M}{N^m},$$

where $N$ is a service time of the product for one stress level which is calculated based on the Weller’s curve. The function is non-linear (3) is non-linear; this excludes the reorganization of cycles that is allowed due to the linear hypothesis (2). The transition from one level of the stress to another level is performed based on the equation of equivalent conditions of the material $S_B(\sigma_{M2}, n_2) = S_B(\sigma_{M1}, n_1)$ (figure 4). When setting equal equations (3) at different values $\sigma_{M1}$ and $\sigma_{M2}$, the formula for calculation of the equivalent number of cycles can be obtained

$$n_e = n_2 \left( \frac{S_{B0} - \sigma_{M2}}{S_{B0} - \sigma_{M1}} \right)^{\frac{1}{m}}.$$

As far as the technology of the fatigue calculation was based on tests of samples of pulsating cycles, the asymmetric cycle of the application of load shall be brought to the same form [13]:

$$\sigma_{ij} = \sigma_i \frac{\sigma_{-1K}}{\sigma_{RKi}}.$$
where $\sigma_{ri}$ – is the stress of the pulsating cycle; $\sigma_i$ – is the current stress of the cycle; $\sigma_{1K}$ – the fatigue endurance limit at the symmetric limit; $\sigma_{RKi}$ – the fatigue endurance limit at the random loading cycle which is calculated based on a well-known formula:

$$
\sigma_{RK} = \frac{2\sigma_{1K}}{1 - R + \psi_k (1 + R)},
$$

where $R$ is an asymmetry factor of the cycle.

![Figure 3. Intersection of the kinetic curve $S_B(\sigma_M, n)$ and the current stress $\sigma_M$.](image)

![Figure 4. Example of calculation at one change of stress level.](image)

The procedure of calculation based on the model of cyclic degradation of the material is based on the presupposition that the descending branch of the breaking strength $S_B(\sigma_M, n)$ has to cross the current stress level; that’s why it is necessary to introduce a criterion of the fatigue loading at the calculation of damage based on the model of the cyclic degradation of the material [15]:

$$
\omega_{KR} = 1 - \frac{\sigma_M}{S_{B0}}.
$$

The calculation of the damage based on the model of cyclic degradation of the material by itself is executed based on the formula [15]

$$
\omega_c = \frac{S_{B0} - \sigma_M}{S_{B0}} \left( \frac{n}{N(\sigma_M)} \right)^\alpha,
$$

(4)

where $N(\sigma_M)$ – is the service time of the product at the constant stress $\sigma_M$, recorded as a function calculated based on the Weller's curve

$$
\sigma_{RK}^{\alpha} N_0 = \sigma_{M}^{\alpha} N_M,
$$

where $\alpha$ – is an indicator of the fatigue curve.

3. Methodology of the randomization process

When execution of fatigue calculations based on techniques described earlier an assumption is made that the operation process is a random process that depends on plenty of factors; that’s why it is impossible to forecast how exactly the designed product will be operated.

During the work, a standardized function of randomization in the Python programming language is used that gives the uniform distribution percentages of different stress levels (figure 1a) in the mixed form (figure 1b), as well as the law of normal distribution (1).

With the use of the law of normal distribution, a well-known formula (1) was used at different parameters of $\sigma$ and $\mu$. The peculiarity of this way of randomization is that the spread in values $\pm 3\sigma$ lies
within the range of stresses $[\sigma_{\text{min}};\sigma_{\text{max}}]$ of the structure that makes it possible to set another spectra of randomization range by changing the parameters $\sigma$ and $\mu$ of the formula (1). In figure 5, the normal distribution at parameters $\sigma = 1$ and $\mu = -2$ is shown, which corresponds to the smooth operation mode, which is non-intensive, within its physical meaning. This distribution (figure 5) lies mostly close to the kind of statistically obtained information taken from the work [6] (figure 2).

4. Example of the fatigue calculation with peak glideslopes

Let’s perform the fatigue calculation based on the cyclic degradation model [11,12] and the linear hypothesis [10] in the dangerous cross-section of the considered unit of the tank wagon according to the technique described above. For example, structure parameters from the work [9] for the similar case were taken: maximal stress $\sigma_{\text{max}}=90$ MPa, randomization range $[60;90]$ MPa, average stress in the cycle $\sigma_M=\text{const}=60$ MPa, fatigue endurance limit for the symmetrical cycle $\sigma_{1K}=30$ MPa, breaking strength of the material $S_B=470$ MPa, fatigue curve index $\alpha = 3.5$. The kinetic curve index $m$ shows the rate of descent of the breaking strength curve $S_B(\sigma_M,n)$; it is calculated in an experimental way for the specified mark of the material [12]. In the considered example, it has been taken as a parameter $[0.5; 1; 2; 6]$. The maximal accumulated damage based on the formula (4) is equal to $\omega_{\text{CR}}=0.81$.

The results of the calculation are presented in table 1. The calculations are presented at different values of the kinetic curve (3) $m \in [0.5; 1; 2; 6]$ and on the linear hypothesis (L.G.). Under figure I, a calculation at the quasirandom application loading is presented using the standardized function of randomization in Python programming language (uniform distribution), under figures II–IV for the normal distribution law: II $- \sigma = 1$ and $\mu = 0$, III $- \sigma = 1$ and $\mu = -2$, IV $- \sigma = 1$ and $\mu = 2$. The calculation time takes from 1 to 14 minutes, depending on the loading mode.

| $m$   | 0.5 | 1  | 2  | 6  | L.G. ($\omega=1$) |
|-------|-----|----|----|----|-------------------|
| I     | 9.239 | 9.388 | 9.462 | 9.512 | 9.538             |
| II    | 13.412 | 13.776 | 13.961 | 14.087 | 14.149            |
| III   | 92.125 | 96.713 | 99.087 | 100.699 | 101.515           |
| IV    | 3.859 | 3.901 | 3.922 | 3.937 | 3.944             |

Table 1. Results of calculation at different parameters of the law of normal distribution and the standardized function of randomization (millions of cycles).

The calculation showed that the use of the normal distribution law at standardized coefficients $\sigma = 1$ and $\mu = 0$, in comparison with the linear distribution, leads to the increase of the service time by half as much again. The shift of the mathematical expectation towards the highest stresses that is equal to the more intensive operation mode within its mathematical meaning influences the reduction of the service time because the percentage of stresses of the higher-order has increased. However, the shift of the mathematical expectation to the opposite side leads to the increase of the service time what correlates well with data obtained in the experimental way [6] presented in figure 2.
5. Conclusion
The use of the randomization process in the designing of machines is justified when the statistically accumulated information on the nature of the operation is present. The use of the linear distribution gives the incorrect forecast of the service time if real operation loads don’t correspond to the uniform distribution in the percentage ratio for the entire spectrum of stresses. The idea of using the law of the normal distribution makes it possible, to select such distribution parameters \( \sigma \) and \( \mu \), that can to the greatest degree correspond to the real image of the stress distribution.

The fatigue calculation of the tank wagon has shown that the use of the law of the normal distribution at the randomization of stresses in the process of operation leads to the increase of the forecasted service time of the structure, in comparison with the uniform distribution, and to obtaining the more correct result of calculation because the randomized spectrum of stresses corresponds to the real image of operation more precisely.

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References
[1] Abdullah L, Singh S S K, Abdullah S, Azman A H and Ariffin A K 2021 J. Eng Fail Anal 123 105314
[2] Liu R, Zhang P, Zhang Z J, Wang B and Zhang Z F 2021 J. Materials Science & Technology 70 250-67
[3] Ermakov S M 2017 J. Vestnik of Saint Petersburg University Mathematics Mechanics Astronomy 4 570-6
[4] Mitrichev I I, Zhensa A V and Kol'cova E M 2016 Proc. Int. Conf.-School on Chemical Technology (Volgograd State Technical University) 103-5
[5] Litvinenko R S, Bagaev A V and Yamshchikov A S 2016 J. Scientific almanac 9-1(23) 427-31
[6] Yakushev A V 2007 Forecasting the fatigue life of cast parts of a freight car bogie (Dissertation for the degree of candidate of technical sciences) (Yekaterinburg: Ural State Transport University)
[7] Tret'yak L N and Vorob'ev 2015 Fundamentals of theory and practice of experimental data processing (Orenburg: Orenburg State University) 215
[8] Wang B, Shi W and Miao Z 2015 J. PLoS ONE 10(3) e0118537
[9] Emelyanov I, Mironov V and Ogorelkov D 2020 Proc. International Scientific Siberian Transport Forum: TransSiberia 2019 (Novosibirsk) 340-8
[10] Callins J A 1984 Failure of Materials in Mechanical Design: Analysis Prediction Prevention (Moscow : World) 624
[11] Mironov V I, Ogorelkov D A and Yakovlev V V 2018 Influence of structural damping on the durability of a crane metal construction J. AIP Conference Proceedings 2053 40060
[12] Emelianov I G and Mironov V I 2012 Durability of Shell Structures (Yekaterinburg: UrO RAN Publishing) 217
[13] Sokolov S A 2011 Structural Mechanics and Metal Structures of Machines (Saint Petersburg: Politehnika) 450
[14] Gusev A S 1989 Resistance to fatigue and survivability of structures under random loads (Moscow: Mechanical Engineering) 248
[15] Ogorelkov D, Mironov V and Lukashuk O 2018 J. MATEC Web of Conferences 224 02091