Halo growth and the NFW profile

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ABSTRACT

We perform a simple test showing that accreting halos grow inside-out and have density profiles that are statistically indistinguishable from those of halos having undergone major mergers. The test also reveals that the ability for the NFW analytical expression to fit the density profile of halos spoils towards large masses at any redshift. On view of these results, we analyze the $M-c$ relations on extreme mass and redshift domains predicted by usual toy models based on numerical data and one physical model based on the two assumptions above confirmed by the test.

Subject headings: cosmology: theory — dark matter — galaxies: halos

1. INTRODUCTION

$N$-body simulations show that dark matter halos have spherically averaged density profiles that can be fitted by the NFW law (Navarro et al. 1997)

$$\rho(r) = \rho_s \frac{4r_s^3}{r (r + r_s)^2},$$  

where $r_s$ and $\rho_s$ are the halo scale radius and characteristic density, respectively. The Einasto profile (Navarro et al. 2004; Merritt et al. 2005, 2006; Navarro et al. 2010) has also been shown to give similar or even better fits to halo density profiles. But the correlations between the NFW shape parameters, namely between the halo mass $M$ and the concentration $c$, defined as the halo radius $R$ over $r_s$, or between $r_s$ and the mass $M_s \equiv M(r_s)$ encompassed by it, at any given redshift ($z$) still play a crucial role in the modeling of structure formation.

Much effort has been done in the last decade in accurately determining these relations from $N$-body simulations (e.g. Zhao et al. 2009; Klypin et al. 2010; Munioz-Cuartas et al. 2010; Prada et al. 2011 and references therein). However, such numerical relations are not very practical because they refer to only a few redshifts and cosmologies. Moreover, they face the fundamental problem that numerical simulations have a limited dynamic range, which severely restricts the mass and redshift domains where they can be inferred. To circumvent this problem, toy models are often built that fit the numerical data and are supposed to give acceptable predictions beyond the domain covered by simulations.

On the other hand, the NFW law does not exactly fit the halo density profile, which may introduce poorly known biases in the best-fitting $r_s$ and $\rho_s$ values depending on the specific halo sample definition and fitting procedure (including the radial extent) used. Tasitsiomi et al. (2004) found indeed that very massive halos at any given $z$ have density profiles that become closer to a pure power-law with minus index equal to $\sim 1.5$-$2.0$. According to these authors, this would be the consequence that massive halos aggregate mass at a lower rate, through minor mergers contributing to smooth accretion, than less massive halos, which suffer major mergers more frequently. But this interpretation has not been proved. The idea that the density profile of halos depends on the rate at which they aggregate mass is often invoked, indeed, to explain some of the observed trends of halo density profiles (e.g. Wechsler et al. 2002;...
Zhao et al. 2003; Muñoz-Cuartas et al. 2010). However, other results of numerical simulations show the opposite result that there is no particular signature of major mergers in the halo density profiles (e.g. Huss, Jain & Steinmetz 1999; Moore et al. 1999; Wechsler et al. 2002; Nusser & Sheth 1999; Romano-Díaz et al. 2006).

Clearly, the existence of a reliable physical model of halo structure could shed light on this apparent conflict and might be used to predict accurate \( M-c \) or \( r_s-M_s \) relations at any \( M \) and \( z \) domain as well. Salvador-Solé et al. (2007; hereafter SMGH) built one simple model that relies on the assumption that halos develop from the inside out during accretion phases. The typical density profile for purely accreting halos is therefore be set by the typical accretion rate provided by the excursion set formalism, which leads to density profiles à la NFW except at very large masses where they tend to deviate from the NFW profile in the direction found by Tasitsiomi et al. (2004).

The inside-out growth of accreting halos is supported by the results of \( N \)-body simulations, which show that halos evolve by conserving the radial mapping of about 80 % of their particles (Wang et al. 2010 and references therein). The fact that 20 % of them do not preserve their radial mapping might simply be due to the effects of major mergers yielding the full rearrangement of halos. But this is hard to prove because of the difficulty to restrict the analysis to purely accreting halos. But even if this were the case, it would not be clear why the SMGH model is able to predict the right NFW profile of halos without including major mergers.

Recently, Salvador-Solé et al. (2011a) have shown from theoretical arguments that accreting halos must grow, indeed, from the inside out and that the density profile of purely accreting halos must be indistinguishable from that of halos having suffered major mergers. Hence, the typical halo density profile predicted by the SMGH model is fully justified despite the real growth of these objects both through accretion and major mergers. Unfortunately, those theoretical arguments are not straightforward, so the checking against \( N \)-body simulations of these important results is necessary.

In the present Letter, we perform one simple test using numerical data which confirms that: i) halos grow inside-out during accretion phases, ii) the density profile of purely accreting halos is indistinguishable from that of halos having suffered major mergers and iii) the fit by the NFW law of halo density profiles depends on \( M \) (at any given \( z \)), which causes some trends often misinterpreted as proving that the profiles of halos depend on whether they suffer major mergers or not. The implications of these results on the \( M-c \) relations predicted far away from the domain covered by numerical data is then analyzed.

2. THE TEST

If halos had density profiles strictly of the NFW form and grew inside-out by pure accretion, the values of the \( r_s \) and \( M_s \) adjusting their density profiles at different epochs would clearly not vary and the \( r_s-M_s \) relation itself would remain unchanged. Actually, halos rearrange in major mergers, so the \( r_s \) and \( M_s \) values suddenly change in those events. But, if the density profiles emerging from major mergers were indistinguishable from those of purely accreting halos, the whole \( r_s-M_s \) relation should be kept unaltered regardless of the aggregation history of halos. That is, it should be time-invariant. Moreover, as the points of the \( r_s-M_s \) relation would not depend on halo mass, the \( r_s-M_s \) curve should not privilege any particular mass. Hence, the \( r_s-M_s \) relation should also be scale-free.

Conversely, if accreting halos did not have density profiles of the NFW form or they did not grow inside-out or still the profile for purely accreting halos were distinct from that of halos suffering major mergers, the \( r_s \) and \( M_s \) values would change as they evolve and the \( r_s-M_s \) relation would vary with \( z \). Moreover, as major mergers dominate the growth of low-mass halos and smooth accretion that of very massive ones, the \( r_s-M_s \) relation would not be scale-free.

\[\text{The inside-out growth can also be tested by directly checking the constancy of the mass of halos inside any given radius. But this faces the same difficulty: the need to identify purely accreting objects.}\]

\[\text{This is not to be confused with the fact that the density profile of individual purely accreting halos depends, of course, on their past accretion rate, used in some halo age estimates.}\]
Thus, the time-invariant, scale-free behavior of the \( r_s-M_s \) relation is an unambiguous proof of the inside-out growth of accreting halos and the similarity of density profiles for purely accreting and merging halos. Hence, it is an unambiguous proof of the validity of the SMGH model (as far as the halo mass growth is reasonably well-described by the excursion set formalism).

In Figure 4, we show the \( r_s-M_s \) relations drawn from the toy models of \( M-c \) relations (see Sec. 4) proposed by Zhao et al. (2009; top-right panel), Klypin et al. (2010; bottom-left panel) and Muñoz-Cuartas et al. (2010; bottom-right panel) fitting numerical data in their simulations of the same concordance cosmology with \((\Omega_m, \Omega_{\Lambda}, h, \sigma_8) = (0.27, 0.73, 0.70, 0.82)\). (Muñoz-Cuartas et al. actually use a slightly different concordance model, but this makes no difference at the resolution of the Figure.) The curves at \( z = 0 \) are very close to straight lines in log-log as expected, but, for increasing \( z \), they progressively bend upwards, so they do not overlap.

Does this mean that the SMGH model is wrong? Not really. In the same figure (top-left panel), we plot the \( r_s-M_s \) curves predicted from the SMGH model. Again, the log-log relation at \( z = 0 \) is very approximately a straight line, but, as \( z \) increases, the relations progressively bend upwards and the curves do not overlap. As in the SMGH model all halos accrete (there is no major merger) and develop, by construction, from the inside out, this can only be due to the fact that they do not strictly have density profiles of the NFW form. Indeed, even if the density profile of a halo does not vary during accretion, its fit by the NFW law will then result in best values of \( r_s \) and \( M_s \) that slightly shift as the radial extent of the halo (setting the fitting radial range) increases.

This interpretation of the actual behavior of the \( r_s-M_s \) relation is confirmed by the results plotted in Figure 2. In panel (a), we show again the \( r_s-M_s \) relations predicted at several \( z \)'s by the SMGH model but in a somewhat smaller \( r_s \) range and in panel b) we show the tracks followed for decreasing \( z \) by a few individual halos tracing such \( r_s-M_s \) relations. In the SMGH model, all halos develop inside-out. Yet, the halos still move in the \( r_s-M_s \) plane along one universal direction due, as checked, to the effect just mentioned. Remarkably, the slope of that universal shift is identical to that \((y = x^{3 \times 0.55})\) of the shift also undergone by halos in the simulations by Zhao et al. (2009) (see their Figure 22). Moreover, the shifts found by Zhao et al. end up at \( z = 0 \) along the same straight line tracing the SMGH \( r_s-M_s \) relation at the same \( z \). This unambiguously proves that such shifts have the same origin in the SMGH model as in Zhao et al. simulations. Zhao et al. interpreted them as due to a progressive change in the inner halo density profile during slow accretion phases (see also Zhao et al. 2003). However, as revealed by the SMGH model, the inner density profile for these halos does not actually change; the small shift in the \( r_s-M_s \) plane is due to the varying best-fitting \( r_s \) and \( M_s \) values obtained as the halo radial extent increases due to the fact that at massive halos are not well-fitted by the NFW profile.

When this effect is corrected for, that is, when the values of the shape parameters adjusting the density profile at any \( z \) are replaced by those adjusting the same profile extended so to encompass the whole radial range at \( z = 0 \), then the log-log
r_s—M_s relations at any z become straight lines that overlap. In other words, the test confirms that accreting halos evolve inside-out and that the density profiles of purely accreting halos are statistically indistinguishable from those of halos having suffered major mergers.

3. IMPLICATIONS FOR THE M—C RELATION

Integrating $4\pi r^2$ times the NFW profile (eq. [1]) written in terms of $M_s$ instead of $\rho_s$ and taking into account the relation $M_s(r_s) = Ar_s^{\nu}$ with $A = 9.99 \times 10^{14} M_\odot$ Mpc$^{-1}$ and $\nu = 2.481$ fitting the numerical $r_s—M_s$ relations at $z = 0$, one is led to

$$M^{3-\nu}(c, z) = \left\{ \frac{4\pi}{3} \Delta_{\text{vir}}(z) \bar{\rho}(z) \right\}^{-\nu} \times \left\{ \frac{Ac^{-\nu}}{\ln(2) - 0.5 \left[ \ln(1 + c) - \frac{c}{1 + c} \right]} \right\}^3,$$

where $\Delta_{\text{vir}}(z)$ and $\bar{\rho}(z)$ are respectively the halo overdensity and mean cosmic density at $z$. Equation [2] therefore defines the $M—c$ relation for halos strictly endowed with the NFW profile (in particular at $z = 0$), growing inside-out during accretion periods and with no signature of major mergers. Such an $M—c$ relation is from now on called “the ideal $M—c$ relation”.

In Figure 3 we compare such an ideal $M—c$ relation with the $M—c$ relations predicted by the SMGH model and the toy models by Zhao et al. (2009), Klypin et al. (2010) and Muñoz-Cuartas et al. (2010) derived from numerical data, respectively given by

$$c(M, z) = 4^8 \left( \frac{t(z)}{t_{0.04}(M)} \right)^{8.4} \times \left( \frac{M}{10^{12} h^{-1} M_\odot} \right)^{0.09} \times \left( \frac{M}{10^{12} h^{-1} M_\odot} \right) \delta(z) \left( \frac{h}{h_0} \right)^{0.25},$$

and

$$c(M, z) = 10^{b(z)} \left( \frac{M}{h^{-1} M_\odot} \right)^{a(z)},$$

where $\delta(z)$ is the critical overdensity for collapse at $z$, $a(z) \equiv 0.097 - 0.097$ and $b(z) \equiv -110.001/(z + 16.885) + 2469.72/(z + 16.885)^2$.

All the curves drawn from numerical data behave in a similar way. Contrarily to the $r_s—M_s$ relation, they show a strong dependence on $z$. At $z = 0$ they closely follow the ideal $M—c$ relation but, at larger $z$, they progressively deviate from it at large masses likely due to the effect mentioned in section 2. In fact, for large enough $M$, the ideal decreasing trend brakes and the empirical curves level off or even begin to increase again (in the case of Klypin et al. 2010; see also Prada et al. 2011). The $M—c$ relation predicted by the SMGH model shows similar trends although it remains closer to the ideal $M—c$ relation until slightly larger $M$ at each $z$. This might reflect a slight flaw of the SMGH model at very large $M$. But the possibility cannot be ruled out that the empirical $M—c$ relations are somewhat biased owing e.g. to the impact of using non-fully relaxed halos.
Fig. 3.— Comparison between the $M$–$c$ relations predicted by the SMGH model (full black lines) and proposed by Zhao et al. (2009) (red lines), Klypin et al. (2010) (blue lines) and Muñoz-Cuadras et al. (2010) (green lines) at $z = 0, 1, 2$ and 3 in the same cosmology as in Figure 1. We distinguish that part of the curves covered by simulations (thick colored lines) and their extrapolations outside it (thin colored lines). The ideal NFW $M$–$c$ relation (see text) is in dashed black lines.

In Figure 4, we plot the same $M$–$c$ relations extended over a much wider domain ($M$ down to $1 M_\odot$ and $z$ up to 15). We exclude the $M$–$c$ relations by Muñoz-Cuartas et al. (2010) because these authors state explicitly that their toy model is a fitting expression that cannot be extrapolated. As can be seen, the two sets of extrapolated empirical $M$–$c$ relations have a lower bound at about $\log(c) \sim 0.6$ contrarily to the ideal $M$–$c$ relations. However, in Klypin et al. (2010), the curves increase again after reaching that minimum, whereas, in Zhao et al. (2009), they level off. Notice that this discrepancy affects the part of the curves that adjusts numerical data, meaning that the disagreement between both groups is very relevant. At small $M$, the two sets of curves diverge from each other even more markedly: in Zhao et al. they show flat asymptotes, while in Klypin et al. they have increasing asymptotes similar to the ideal $M$–$c$ relations, although their dependence with $z$ is substantially less marked.

The $z$-dependence of the SMGH curves is instead quite similar (for intermediate $M$ at each $z$) to that of the ideal $M$–$c$ curves. Moreover, the $M$-dependence at any $z$ is also much closer to that of the ideal $M$–$c$ relations than for the extrapolated empirical relations. They only substantially deviate from each other at very small $M$. In brief, the SMGH $M$–$c$ relations are globally in better agreement with the ideal $M$–$c$ relations than the pseudo-empirical $M$–$c$ relations.

Certainly, the ideal relations must not necessarily trace the $M$–$c$ relations of real halos because, as shown, their density profiles are not exactly fitted by the NFW law. This translates into an apparent variation of $r_s$ and $c$ with increasing $z$, which is not present in the ideal relations. Thus, on the sole basis of the previous comparison, it is hard to conclude which $M$–$c$ relation is the most reliable...
for real simulated halos in such extreme \( M \) and \( z \) domains. As mentioned, the SMGH model might be affected by some slight flaw at the large mass end (and perhaps at the small mass end as well) at any given \( z \). But the empirical curves might be biased too, as suggested by their distinct behavior at very large masses where numerical data are available. More importantly, their extrapolation at very small \( M \) show so different behaviors at any \( z \) that they cannot be trusted. Last but not least, Salvador-Solé et al. (2011b) have recently shown that the properties of halo substructure found in high-dynamic range \( N \)-body simulations (Springel et al. 2008), very sensitive to the behavior of the \( M–c \) relation in an \( M \) and \( z \) domain as wide as the one considered here, clearly favor the SMGH model in front of the toy models by Klypin et al. and Zhao et al.

4. SUMMARY

We have performed a simple test showing unambiguously that accreting halos grow inside-out and that the density profiles of halos grown by pure accretion are indistinguishable from those of halos having suffered major mergers. The variation in the NFW shape parameters as halos accrete is due to the fact that the density profile for very massive halos is not well-fitted by the NFW law, which causes the best-fitting values of \( r_s \) and \( \rho_s \) (or \( M_s \)) to slightly vary as the radial extend of halos increases. After correction of this effect, the \( r_s–M_s \) relation is linear in log-log and overlaps for all \( z \) as expected.

Over very wide \( M \) and \( z \) domains, the ideal \( M–c \) relations for halos with exact NFW profiles and evolving inside-out by accretion deviate notably both from the extrapolated \( M–c \) relations inferred from numerical data as well as from the \( M–c \) relation predicted by the SMGH model. This difference is once again partly due to the fact that the density profile for simulated and modeled halos is strictly not of the NFW form. However, the \( M–c \) curves predicted by the SMGH model stay closer to such ideal \( M–c \) curves over a wider \( M \) and \( z \) domain than the extrapolated \( M–c \) relations drawn from toy models, which also notably deviate from each other. This suggests the better behavior of the SMGH \( M–c \) relations at such extreme \( M \) and \( z \) domains, a conclusion that meets that found on the basis of completely independent arguments by Salvador-Solé et al. (2011b).

Acknowledgments

This work was supported by the Spanish DGES AYA2009-12792-C03-01. We thank Steen H. Hansen for useful comments and discussions. One of us, JV, beneficiary of the grant BES-2007-14736, thanks the Dark Cosmology Centre for facilities during his stage at this center.

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