LETTER

Universal behavior of a bipartite fidelity at quantum criticality

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Abstract. We introduce the (logarithmic) bipartite fidelity of a quantum system $A \cup B$ as the (logarithm of the) overlap between its ground state wavefunction and the ground state that one would obtain if the interactions between two complementary subsystems $A$ and $B$ were switched off. We argue that it should typically satisfy an area law in dimension $d > 1$. In the case of one-dimensional quantum critical points (QCP) we find that it admits a universal scaling form, $\sim \ln \ell$, where $\ell$ is the typical size of the smaller subsystem. The prefactor is proportional to the central charge $c$ and depends on the geometry. We also argue that this quantity can be useful for locating quantum phase transitions, allows for a reliable determination of the central charge, and in general exhibits various properties that are similar to the entanglement entropy. Like the entanglement entropy, it contains subleading universal terms in the case of a 2D conformal QCP.

Keywords: conformal field theory (theory), spin chains, ladders and planes (theory), quantum phase transitions (theory), entanglement in extended quantum systems (theory)

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1. Introduction

A major challenge in the study of quantum many-body systems in condensed matter physics is the understanding and characterization of new exotic phases of matter, such as quantum critical or topological phases. For this purpose, various quantities and concepts have been introduced, some coming from quantum information theory. Amongst them, one of the most heavily studied is the entanglement entropy (EE) [1], defined through a bipartition of a total system $A \cup B$, usually in a pure state $|\psi\rangle$:

$$S = - \text{Tr} \rho_A \ln \rho_A, \quad \rho_A = \text{Tr}_B |\psi\rangle\langle\psi|.$$ (1)

The EE of a ground state is known to be universal at the one-dimensional quantum critical point (QCP) [2]–[4], and the leading term allows for an accurate determination of the central charge. In higher dimensions $d > 1$, it obeys an area law (with possible logarithmic corrections [5]): if $L$ is the typical size of the smaller subsystem, then $S$ scales as $L^{d-1}$. In this case, subleading terms [6]–[10] encode universal features of the system, characterizing quantum criticality or topological order. Despite all of these theoretical works, connecting the EE to experimentally measurable quantities remains a formidable task. One reason for that is that characterization of entanglement in a many-particle system generically requires the measurement of a prohibitively large number of observables.

A different class of quantities is the one of overlaps, or fidelities. The idea is perhaps more intuitive and can be traced back to Anderson’s orthogonality catastrophe [11]. Let $H(\lambda)$ be a Hamiltonian which depends on a physical parameter $\lambda$ that can be varied. If $|\lambda\rangle$ is its ground state, the fidelity is

$$f(\lambda, \lambda') = |\langle \lambda |\lambda' \rangle|^2.$$ (2)

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Close to a QCP the fidelity susceptibility \( \chi(\lambda) = (\partial^2 \chi/\partial \lambda \partial \lambda^\prime)|_{\lambda^\prime=\lambda} \) diverges, so this quantity can be used to detect quantum phase transitions \([12]-[14]\). The scaling behavior of the fidelity has been studied in various systems both analytically and numerically \([15,16]\). Overlaps are also interesting when considering time evolutions. Starting from an initial state \(|\Psi(0)\rangle\), one can ask what the overlap of the wavefunction with the initial state after time \(t\) is. The Loschmidt echo \(L(t) = |\langle \psi(0)|\psi(t)\rangle|^2\) \([17,18]\) has been studied in connection with NMR experiments \([19]\), and also in the context of quantum criticality \([20,21]\).

In this letter, we introduce an overlap which shares some common properties with the EE (in particular we keep the idea of cutting the system into two parts), despite being conceptually simpler. We call it the logarithmic bipartite fidelity (LBF), and claim that it provides valuable insights into quantum critical phenomena. We shall see in particular that the LBF obeys an area law in \(d>1\), can be useful for locating QCPs, and has a universal scaling form at one-dimensional QCPs that involves the central charge \(c\), much in the spirit of the EE.

2. Bipartite fidelity

Let us consider an extended quantum system \(A \cup B\) described by the Hamiltonian

\[
H = H_A + H_B + H_{A \cup B}^{(I)}
\]

(3)

where \([H_A, H_B] = 0\) and \(H_{A \cup B}^{(I)}\) contains all the interaction between \(A\) and \(B\). We denote by \(|A\rangle\) (resp. \(|B\rangle\)) the ground state of \(H_A\) (resp. \(H_B\)), by \(|A \otimes B\rangle = |A\rangle \otimes |B\rangle\) the ground state of \(H_A + H_B\), and by \(|A \cup B\rangle\) the ground state of \(H\). We introduce the bipartite fidelity \(|\langle A \cup B|A \otimes B\rangle|^2\), the overlap between the ground state of the total Hamiltonian \(H\), and the ground state of a Hamiltonian \(H_A + H_B\) where all interactions between \(A\) and \(B\) have been switched off. A more physical way of looking at this quantity is to interpret it as a probability of measuring a given energy after a local quantum quench. Let us imagine that the system is initially disconnected (i.e. it is in the ground state of \(H_A + H_B\)), and that at time \(t = 0\) the interaction between \(A\) and \(B\) is instantaneously switched on. Then, at time \(t > 0\), the system evolves with the total Hamiltonian \(3\). If one measures the energy of the system just after the quench, the probability of finding the ground state energy is given by \(|\langle A \cup B|A \otimes B\rangle|^2\), which is the bipartite fidelity. For later convenience, we consider (minus) the logarithm of this quantity

\[
F_{A,B} = -\ln \left( |\langle A \cup B|A \otimes B\rangle|^2 \right),
\]

(4)

and call it logarithmic bipartite fidelity (LBF). The symbol \(F_{A,B}\) is chosen because it can be interpreted as a free energy in a classical system.

3. The free energy and the area law

The LBF is nothing but a linear combination of free energies of different \(d+1\)-dimensional systems. Indeed, in a Euclidean picture, the ground state \(|0\rangle\) of a Hamiltonian \(H\) can be seen as the result of an infinite (imaginary) time evolution starting from any state \(|s\rangle\), provided \(|0\rangle \neq 0\):

\[
ie^{-\tau H}|s\rangle \sim e^{-\tau E_0}|0\rangle \langle 0|s\rangle.
\]

(5)

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Figure 1. Bipartition of a 2D system (on the left), along with the three partition functions of equation (6) in $d + 1 = 3$ dimensions. Region $A$ is in blue when decoupled from $B$.

$E_0$ is the ground state energy. Making use of this, our scalar product can be expressed as a ratio of classical $d + 1$-dimensional partition functions:

$$
\langle A \cup B | A \otimes B \rangle = \lim_{\tau \to \infty} \frac{Z_{A,B}(\tau)}{\sqrt{Z_{A \cup B}(\tau)Z_{A \otimes B}(\tau)}}.
$$

(6)

$Z_{A \otimes B} = \langle s | e^{-2\tau(H_A + H_B)} | s \rangle$ is the partition function corresponding to two independent systems $A$ and $B$, $Z_{A \cup B} = \langle s | e^{-\tau(H_A + H_B + H_{A \cup B})} | s \rangle$ is the partition function of the total system, and $Z_{A,B} = \langle s | e^{-\tau(H_A + H_B)} e^{-\tau(H_A + H_B + H_{A \cup B})} | s \rangle$ corresponds to the case where $A$ and $B$ are decoupled from $-\tau$ to $0$, and coupled afterward (see the $d = 2$ example in figure 1). In terms of free energies $f = -\ln Z$, the LBF is then

$$
\mathcal{F}_{A,B} = 2f_{A,B} - f_{A \otimes B} - f_{A \cup B}.
$$

(7)

The different terms in (7) are expected to be extensive in the thermodynamic limit. There is a bulk free energy $f_{d+1}$ per unit volume, a ‘surface’ free energy $f_d$, and a ‘line’ free energy $f_{d-1}$. The bulk and surface energies are canceled out by the linear combination, and we get

$$
\mathcal{F}_{A,B} = f_{d-1}L^{d-1} + o(L^{d-1}),
$$

(8)

where $L^{d-1}$ is the ‘area’ of the boundary between $A$ and $B$ in the initial $d$-dimensional system. This is the area law for the LBF. We expect this to be true for generic systems, as is the case usually for the EE; however like for the EE [5], exceptions are possible.

4. 1D conformal QCPs

In general, $\mathcal{F}_{A,B}$ should be finite away from a QCP, because the correlation length $\xi$ is small and the two ground states are very close to each other, except on a thin region of typical size $\xi$. At a QCP however, this is no longer true and $\mathcal{F}_{A,B}$ can become large. As the calculation of the overlap boils down to a free energy, the scaling behavior should be controlled by the conformal symmetry only. This is indeed the case, and this result constitutes the central point of our work. The geometries considered are shown in table 1, along with the corresponding formulas that we derived for the bipartite fidelity. For example, geometry (a) consists in a finite chain of length $\ell$ connected to another finite chain of length $L - \ell$. When $\ell \ll L$, the scaling of the fidelity takes the following simple form:

$$
\mathcal{F}_{A,B} \sim \frac{c}{8} \ln \ell.
$$

(9)
Figure 2. The three geometries giving the terms $f_{A,B}$, $f_{A \otimes B}$ and $f_{A \cup B}$ for the case (a) in table 1. The blue intervals $[-\Lambda, +\Lambda]$ are used for the regularization.

Table 1. Geometries considered, along with the leading term for the two LBFs ($F_a$ and $F_b$), as a function of $L$ and $x = \ell/L$.

| Geometry (a) | Geometry (b) |
|--------------|--------------|
| $L - \ell$  | $L - \ell$  |
| $\ell$      | $\ell$      |

$F_a \sim \frac{c}{8} [\ln L + g_a(x) + g_a(1 - x)]$

$F_b \sim \frac{c}{4} [\ln L + g_b(x) + g_b(1 - x)]$

$g_a(x) = \frac{3 - 3x + 2x^2}{3(1 - x)} \ln x$

$g_b(x) = \frac{3 - 6x + 4x^2}{6(1 - x)} \ln x$

This result is similar to the one for the EE [4]: $S \sim (c/6) \ln \ell$. Another simple result is for the symmetric case $\ell = L/2$, where

$$F_{A,B} \sim \frac{c}{8} \ln L,$$

(10)

whereas the EE behaves as $S \sim (c/6) \ln L$ [4]. There is no general relation between the LBF and the EE though. Our analytical results (table 1) do not match the ones for the EE, $S \sim (c/6) \ln [(L/\pi) \sin(\pi \ell/L)]$, which can be traced back to the fact that the Cardy–Calabrese derivation [4] of the EE involves a local twist operator, whereas the LBF cannot be expressed as a correlator of a local field. The two quantities have similar qualitative behavior however.

5. The conformal field theory derivation

The results in table 1 assume that the boundary conditions at the boundaries of $A \cup B$ (if any), $A$ and $B$ are conformal boundary conditions [22]. Moreover, for simplicity, we assume that these boundary conditions are the same everywhere. For different boundary conditions, the scaling dimensions of the different boundary condition changing operators [22] would modify our results [23]. With those two assumptions at hand, the two special cases of geometry (a) given in (9)–(10) are straightforward applications of the celebrated Cardy–Peschel formula [24]: in the three geometries shown in figure 2 we have one corner with angle $2\pi$ and several corners with angle 0 at infinity. The contributions at infinity cancel, and we are left with the contribution of the corner with angle $2\pi$, which gives a total factor of $2 \times (c/16) \ln \ell$ (or $2 \times (c/16) \ln L$).

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In the general case (a) in table 1, the calculation goes as follows. $f_{A,B}$, $f_{A\otimes B}$ and $f_{A\cup B}$ in (7) are the free energies in the geometries shown in figure 2. Let $w = x + iy$ be the complex coordinate such that the lower (resp. upper) boundary corresponds to $y = \Re mw = 0$ (resp. $y = L$). Let us consider the mapping $w \mapsto w + i\delta \ell$ if $\Im mw \in (0, L)$, and $w \mapsto w$ otherwise. Such a mapping keeps $L$ fixed but changes $\ell$ into $\ell + \delta \ell$. The variation of the free energy can be expressed in terms of the $T_{yy}$ component of the stress tensor as [22]

$$
\delta f_{A,B} = \lim_{\Lambda \to \infty} \frac{\delta \ell}{2\pi} \int_{-\Lambda}^{+\Lambda} \left[ \langle T_{yy}(y=0) \rangle - \langle T_{yy}(y=L) \rangle \right] dx,
$$

and there are similar expressions for $f_{A\otimes B}$ and $f_{A\cup B}$. Each of these expressions diverges when $\Lambda \to \infty$, but the combination (7) is finite. To evaluate $\delta f_{A,B}$ one needs the stress tensor in the pants-like geometry (figure 2 left). We use a conformal mapping $z \mapsto w(z)$ from the upper half-plane $z \in \mathbb{C}$, $\Im mz > 0$ to the pants-like geometry

$$
w(z) = \frac{\ell}{\pi} \ln(1 + z) + \frac{L - \ell}{\pi} \ln \left( z \frac{\ell}{L - \ell} - 1 \right).
$$

In the half-plane, one has $\langle T(z) \rangle = 0$, so using the transformation law for the stress tensor [22] we get $\langle T(w) \rangle = -c/12(dw/dz)^{-2}\{w, z\}$, where $\{w, z\} = (w''/w') - (3/2)(w''/w')^2$ is the Schwarzian derivative of $w$ with respect to $z$. Since $\langle T_{yy} \rangle = \langle T(w) \rangle + \langle T(\bar{w}) \rangle$, we find

$$
\frac{12\pi \delta f_{A,B}}{c} = \int_{x_1}^{x_2} dz \{w, z\} \left( \frac{dw}{dz} \right)^{-1} - \int_{x_3}^{x_4} dz \{w, z\} \left( \frac{dw}{dz} \right)^{-1}
$$

where $w(x_1) = \Lambda$, $w(x_2) = -\Lambda$, $w(x_3) = \Lambda + iL$, $w(x_4) = -\Lambda + iL$. In a strip of width $\ell$ the stress tensor is [22] $\langle T(w) \rangle = -(\pi^2c/24\ell^2)$, so $\delta f_{A\otimes B} = 2\Lambda(\pi c \delta \ell/24)(1/(L - \ell)^2 - 1/\ell^2)$ and $\delta f_{A\cup B} = 0$. Finally, introducing the aspect ratio $x = \ell/L$ and taking the $\Lambda \to \infty$ limit in the sum (7), we get the following equation for $\mathcal{F} = \mathcal{F}_{A,B}$:

$$
\frac{\delta \mathcal{F}}{\delta x} = c \left[ \frac{x^2(2 - x)}{2(1 - x)^2} \ln x + \frac{x^2 - 1}{2x} \ln(1 - x) + \frac{1 + x}{4 - 4x} \right] - (x \to 1 - x).
$$

This can be integrated to give the formula (a) in table 1. For the periodic case (b), we need a conformal transformation which maps the upper half-plane onto a cylinder with two slits:

$$
w(z) = \frac{L}{2\pi} \ln \left( 1 + \frac{\ell}{L}(z^2 - 1) \right) - \frac{\ell}{L} \ln z^2,
$$

and the result follows from a similar calculation.

6. Numerical checks

We study the $XY$ chain in transverse field, with boundary conditions $\sigma^x_{L+1} = \sigma^y_{L+1} = 0$:

$$
H = -\sum_{i=1}^{L} \left( \frac{1+r}{2} \sigma_i^x \sigma_{i+1}^x + \frac{1-r}{2} \sigma_i^y \sigma_{i+1}^y + h \sigma_i^z \right).
$$

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Figure 3. Left panel: $XX$ chain numerical results for the LBF $F = F_{A,B}$ in geometry (a). Right panel: geometry (a) with $\ell = L/2$ for the ICTF. The rescaled LBF $F(L(h-1)) - (c/8) \ln L$ can be seen to collapse onto a single universal curve in the vicinity of the critical point. Inset: $F(L, h)$ as a function of $h$.

Two cases are of special interest: $r = 0$ and $h = 0$ gives the critical $XX$ chain, in the universality class of the free boson ($c = 1$), whereas $r = 1$ gives the Ising chain in transverse field (ICTF), critical at $h = 1$ with $c = 1/2$. Using a Jordan–Wigner transformation, $H_A$, $H_B$ and $H_{A \cup B}$ can be recast as free fermion Hamiltonians, and diagonalized by a Bogoliubov transformation. Keeping track of the changes of basis, the overlap can be expressed as a fermionic correlator, and reduced to a $L \times L$ determinant after some algebra.

Results for the $XX$ chain are shown in figure 3 for geometry (a), and agree very well with the CFT prediction. We also checked our formula for geometry (b).

Results for the overlap as a function of $h$ in the ICTF are also shown in figure 3. The quantum phase transition at $h = 1$ can be clearly seen, even with relatively small system sizes. In the vicinity of the critical point, the correlation length is known to diverge as $\xi \sim |h - 1|^{-1}$, and the rescaled overlaps can be made to collapse onto a universal curve. Like for the EE [25], we also have the exact relation $F^{(XX)}(L, \ell) = 2F^{(Ising)}(L/2, \ell/2, h = 1)$ on the lattice.

7. Time evolution after a local quench

Let us consider again the system in table 1 (geometry a). It is prepared at time $t = 0$ in the state $|A \otimes B\rangle$. Then for $t > 0$ the two parts $A$ and $B$ interact, and the system $A \cup B$ evolves with the total Hamiltonian (3). It is well known that the EE for such a system grows as $[26] S \sim (c/3) \ln t$ for $a \ll v_F t \ll L$ (a is the lattice spacing and $v_F$ the Fermi velocity). This logarithmic growth has given rise to speculations about a possible relation between the EE and the statistics of fluctuations of the current between the two parts in certain fermionic systems [27], which would open the route to an experimental measure of the EE. Here we stress the fact that the time-dependent LBF grows logarithmically as well. Actually, the bipartite fidelity in that case is nothing but a Loschmidt echo

$$L(t) = \langle A \otimes B| e^{\mathcal{H}t} |A \otimes B\rangle^2$$

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so the LBF is \( F_{A,B}(t) = -\ln \mathcal{L}(t) \). Its universal behavior can be derived in CFT as follows. In imaginary time the scalar product \( \langle A \otimes B | e^{-\gamma H} | A \otimes B \rangle \) is the partition function of a 2D statistical system in a strip with two slits separated by a distance \( v_F \tau \). In the limit \( v_F \tau \ll \ell, L \) the two slits almost touch each other. Again, the Cardy–Peschel formula shows that the contribution to the free energy of each of these corners scales as \( (c/16) \ln v_F \tau \). The LBF behaves then as \( F_{A,B}(\tau) \sim (c/4) \ln |\tau| \). Going back to real time \( \tau \rightarrow \epsilon - it \), we find \( F_{A,B}(t) \sim c/8 \ln \left(1 + \frac{t^2}{\epsilon^2}\right) \sim c/4 \ln t \). (17)

8. 2D conformal QCPs

As discussed before, in 2D the bipartite fidelity should scale linearly with the system size \( F = f_1 L + o(L) \), where \( f_1 \) depends on the microscopic details of the theory. Universal quantities, if present, have to be looked for in subleading corrections. We consider the simple example of critical quantum dimer wavefunctions, whose amplitudes are given by the Boltzmann weights of a 2D classical dimer model. In the continuum limit this wavefunction is related to a free boson CFT with compactification radius \( R \) [28]. For the geometry of a cylinder of height \( L_y \gg L_x \) cut into two parts (see [10]), the LBF can be expressed using classical partition functions for the dimers [10], \( F = \ln Z_{DD}(L_x, L_y) - 2 \ln Z_{DD}(L_x, L_y/2) \), \( D \) stands for Dirichlet and encodes the conformal invariant boundary condition at both end of the cylinders in the continuum limit. The first subleading term is a constant related to the (Dirichlet) Affleck–Ludwig boundary entropy [29] \( s_D \) computed in [30]:

\[
F \sim f_1 L - 2s_D = f_1 L + \ln R. \tag{18}
\]

It would certainly be interesting to study this idea in more complicated models.

As is the case for the EE [6], we also speculate that subleading terms in the LBF might be used to identify topological order. For certain trial wavefunctions, such as the Rokhsar–Kivelson triangular lattice quantum dimer and Levin–Wen string net wavefunctions, the LBF is nothing but the \( n \rightarrow \infty \) Rényi entropy. Then the arguments in [6] would yield the same subleading constant in the LBF and in the EE. Another way of looking at this would be to compare the eigenstate of the reduced density matrix associated with its largest eigenvalue to the actual ground state of the physical Hamiltonian \( H_A \). For a generic Hamiltonian there is no reason why there should be any relation between those two states. However, for special Hamiltonians associated with trial wavefunctions, we speculate that they might be closely related to each other. We leave this important open question for future studies.

9. Conclusion

We have introduced the LBF of an extended quantum system \( A \cup B \), and studied some of its properties. We have shown in particular that it generically obeys an area law, and exhibits universal behavior at 1D and 2D QCPs, like the EE. We note that its simple definition
makes it easier to grasp intuitively than the EE, and convenient to study using standard analytical and numerical methods. This could be particularly useful in dimensions $d > 1$, where quantum Monte Carlo algorithms allow us to compute efficiently ground state overlaps, as opposed to the von Neummann entropy. Therefore, we believe that the LBF can be a useful and general tool in the study of quantum many-body systems.

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