Strongly Coupled Quark-Gluon Plasma: Equation of State near $T_c$

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We test the quark mass dependence implemented in the quasiparticle dispersion relations of our quasiparticle model for the QCD equation of state by comparing with recently available lattice QCD data near $T_c$ employing almost physical quark masses. In addition, we emphasize the capability of our model to successfully describe lattice QCD results for imaginary chemical potential and to analytically continue the latter to real chemical potential.

I. INTRODUCTION

The equation of state of strongly interacting matter is of paramount interest for understanding and describing the dynamics of relativistic heavy-ion collisions and the early universe and compact stellar objects as well. The grand canonical potential $\Omega$ depends on a set of intensive quantities (temperature $T$, various chemical potentials accumulated in $\vec{\mu}$), and parameters like quark masses, flavor number $N_f$ and color number $N_c$. For QCD as theory of hadrons and quarks and gluons these parameters are fixed, but it is instructive to study their impact on $\Omega$, given the complexity of QCD. Of particular interest is $\Omega$ near the demarcation line (phase boundary) $T_c(\vec{\mu})$ of hadrons vs. quarks and gluons. This region is probed in heavy-ion collisions [1] and is traversed in the cosmic evolution [2].

To parameterize the equation of state near $T_c$ we employ a quasiparticle model (QPM) [1, 3, 4, 5, 6, 7, 8] which is adjusted to available lattice QCD data. The striking simplicity of the model consists in the expressions for net baryon density $n_B$ and entropy density $s = \sum_{i=q,g} s_i$ reading

$$n_B(T, \mu_q) = \frac{N_c N_f}{3\pi^2} \int_0^\infty dk k^2 \left( \frac{1}{e^{(\omega_q - \mu_q)/T} + 1} - \frac{1}{e^{(\omega_q + \mu_q)/T} + 1} \right),$$

$$s_i(T, \mu_q) = \epsilon_i \frac{d_i}{\pi^2} \int_0^\infty dk k^2 \left( \ln \left[ 1 + \epsilon_i e^{-(\omega_i - \mu_i)/T} \right] + \epsilon_i \frac{(\omega_i - \mu_i)/T}{e^{(\omega_i - \mu_i)/T} + \epsilon_i} + [\mu_i \rightarrow -\mu_i] \right),$$

where $d_q = N_c N_f$, $d_g = N_c^2 - 1$, $\epsilon_q = 1$, $\epsilon_g = -1$ and $\mu_g \equiv 0$. The pressure $p = -\Omega/V$, where $V$ is the volume, is to be calculated consistently with Eqs. (1, 2), cf. [1]. We restrict here our attention onto considering one independent chemical potential $\mu_q$. $\omega_{q,g}^2 = k^2 + \Pi_{q,g}(T, \mu_q)$ are the dispersion relations for the quasiparticle excitations with self-energies $\Pi_{q,g}(T, \mu_q)$ in line with 1-loop approximations and a procedure to include nonzero quark masses according to [9]

$$\Pi_{q,g} = m^2_{q,g} + 2m_{q,g}\omega_{q,g} + 2\omega_{q,g}^2,$$

where $m_q = 0$. In the quark sector, the rest masses $m_q$ contain a "true" rest mass term $m_q^{(0)}$ and an artificial "lattice" mass term $\xi_q T$ introduced for calculational purposes on the lattice, $m_q^{(0)} = \ldots$
\[ m_q^{(0)} + \xi T^2. \] The interaction generates the self-energy contributions \( \hat{\omega}_{q,g} = G^2(\alpha_{q,g} T^2 + \beta_{q,g} \mu_q^2) \) with
\[ \alpha_q = \frac{1}{4}, \beta_q = \frac{1}{6}, \alpha_g = \frac{N_c}{24}(2N_c + N_f) \text{ and } \beta_g = \frac{N_c}{32}. \] \( G^2 \) is an effective coupling strength parametrized at \( \mu_q = 0 \) via
\[
G^2(T) = \begin{cases}
G^2_{2-\text{loop}}(\zeta(T)), & T \geq T_c, \\
G^2_{2-\text{loop}}(\zeta(T_c)) + b \left( 1 - \frac{T}{T_c} \right), & T < T_c
\end{cases}
\]
with \( \zeta(T) = \lambda(T - T_s)/T_c \), and approaches smoothly the perturbative region at large temperatures. Near \( T_c \), \( G^2 \) becomes large, and the shift parameter \( T_s \) regulates the coupling. Below \( T_c \), the coupling changes drastically its behavior.

The model may be considered as a resummed expression for the thermodynamic potential, as a formal power expansion in \( G^2 \) generates an infinite series of terms, including also a term proportional to the plasmon term, for instance. It goes beyond a perturbative expansion scheme as result of the effective coupling \( G^2 \), which also may repair possible shortcomings of the 1-loop inspired parameterizations of the dispersion relations.

In this form, the model does not contain critical point (cf. [10]) or color-flavor locking effects (cf. [11]).

Despite of its simplicity, the equation of state for real and imaginary chemical potential as well as various susceptibilities are described very well [12, 13] in the QPM. Here, we describe one new aspect of our model, namely a naive chiral extrapolation of the equation of state. We emphasize also the capability to extrapolate lattice QCD results from imaginary \( \mu_q \) to real \( \mu_q \). We focus on the region \( T \sim T_c \), where interaction effects are strong. It is the region of the strongly coupled quark-gluon plasma [14, 15, 16] presently explored experimentally at RHIC and in near future at LHC and later on at FAIR.

II. CHIRAL EXTRAPOLATION

The QPM parametrization of the lattice QCD results [17] for \( N_f = 2 + 1 \) with fairly large quark masses, i.e. \( \xi_{u,d} = 0.4 \) and \( \xi_s = 1 \), was already presented in [1] for the scaled pressure \( p/T^4 \) at \( n_B = 0 \). The QPM parameters read \( \lambda = 7.8, T_s/T_c = 0.8, \) \( b = 347 \) for \( m_{u,d,s}^{(0)} = 0 \).

Now we try to accomplish a chiral extrapolation by means of Eq. [18]. Neglecting naively a conceivable dependence of \( \lambda, T_s/T_c \) and \( b \) on \( m_q \), the extrapolated results are exhibited in the left panel of Figure 1 (dashed curve) for \( \xi_{u,d} = 0.015 \) and \( \xi_s = 0.15 \), corresponding to the set-up in [18]. These lattice QCD results [18] (squares) are astonishingly well reproduced, however, by the price of changing the pressure integration constant, \( B(T_c) \), which needs to be readjusted in order to generate the small pressure below \( T_c \). We note that putting \( \xi_{u,d,s} = 0 \) in the strict chiral limit does not change noticeably the dashed curve on the scale exhibited in Figure 1.

Another important test [19] of the suitability of our hydrodynamic EoS is the interaction measure \( (e - 3p) \), where \( e \) denotes the energy density. As shown in the right panel of Figure 1 the QPM for almost physical quark masses (i.e., \( \xi_{u,d} = 0.015 \) and \( \xi_s = 0.15 \)) faithfully reproduces corresponding lattice QCD data [18] of \( (e - 3p)/T^4 \). The peak, which is related to the softest point in the QCD equation of state, is located at \( T/T_c = 1.08 \). For larger temperatures, the interaction measure approaches logarithmically zero according to the temperature dependence in the effective coupling \( G^2 \), though, is close to the conformal limit \( e = 3p \) already for \( T/T_c \geq 10 \).
III. IMAGINARY CHEMICAL POTENTIAL

In this section, QCD thermodynamics for \( N_f = 4 \) at nonzero imaginary chemical potential \( \mu_i = i \mu_i \) is considered. The net quark number density \( n_q \) in the QPM \[ n_q(T, i\mu_i) = i \frac{d}{d\mu} \int_0^\infty dk k^2 \left( \frac{e^{\omega_i/T} \sin(\mu_i/T)}{e^{2\omega_i/T} + 2 e^{\omega_i/T} \cos(\mu_i/T)} + 1 \right), \tag{5} \]
which is purely imaginary and an odd function in \( \mu_i \). In Figure 2 (left panel), QPM results for \(-i\mu_i/T^3\), employing the parametrization from \[13\], are compared with the lattice QCD results \[20, 21\]. In particular, the pronounced bending for \( T = 1.1 T_c \) close to \( \mu_c/T = \pi/3 \), which signals the onset of the first-order Roberge-Weiss transition \[23\], is accurately represented by a sensible test of the QPM at nonzero \( n_B \). In the QPM, the bending is driven by the increasing \( \mu_i \) dependence in the quasiparticle dispersion relation \( \omega_q \) close to \( \mu_c \). For temperatures \( T \geq 1.5 T_c \) we observe an independence of \( n_q/T^3 \) considered as a function of \( \mu_i/T \) on the explicit value of \( T \). This independence follows from Eq. (5) as long as \( \omega_q \) is approximately independent of \( \mu_i \).

The phase boundary \( T_c(\mu_B) \) for \( N_f = 4 \) is shown in the right panel of Figure 2 for imaginary as well as for real baryo-chemical potential \( \mu_B = 3\mu_q \), which is accessible by analytic continuation. Within the QPM, an estimate for \( T_c(\mu_B) \) follows as self-consistent solution of a partial differential equation based on Maxwell’s relation emerging at \( T_c(\mu_B = 0) \) (cf. \[13\]). \( T_c(\mu_B) \) for imaginary \( \mu_B \) and the first Roberge-Weiss transition line cross each other at \( T^*/T = 1.112 \) and \( (\mu_B^*)^2/T^2 = -12.214 \) being close to the lattice QCD results \( T^*/T = 1.095 \) and \( (\mu_B^*)^2/T^2 = -11.834 \) \[20, 21\].

The features of the phase diagram in the imaginary chemical potential sector can be described as follows. Dashed curves represent the analytic sections of the first two Roberge-Weiss transition lines \( \mu_B^2/T_c^2 = -4 T^2 \pi^2 (2 k + 1)^2/T_c^2 \) (here \( k = 1, 2 \)) turning into first-order transitions (solid section) while the dotted curve shows the first \( Z_3 \) center symmetry line \( \mu_B^3/T_c^3 = -4 T^2 \pi^2/T_c^2 \). The Roberge-Weiss periodicity \[23\] implies that sectors between \( \mu_i/T = 2 \pi k/3 \) and \( 2 \pi(k + 1)/3 \) (sector II for \( k = 1 \)) are repeated copies of sector I between \( \mu_i/T = 0 \) and \( 2 \pi/3 \). The subsector between \( \mu_i/T = \pi/3 \) and \( 2 \pi/3 \) (sector \( \Gamma' \)) is a reflected copy of the subsector between \( \mu_i/T = 0 \) and \( \pi/3 \) (sector \( \Gamma' \)) mirrored at the first Roberge-Weiss transition line.
FIG. 2: Left: Comparison of QPM \cite{13} (solid curves) with continuum estimated lattice QCD results \cite{20,21} (symbols) for the imaginary part of the scaled net quark number density $n_q/T^3$ for $N_f = 4$ as a function of $\mu_i/T$ for temperatures $T = 1.1, 1.5, 2.5, 3.5 T_c$ (diamonds, circles, squares and triangles, respectively). The discontinuity in $n_q/T^3$ takes place at the Robere-Weiss critical chemical potential $\mu_c/T = \pi/3$. For $T \geq 1.5 T_c$, $n_q/T^3$ as a function of $\mu_i/T$ exhibits an interesting scaling behavior with $T$. Right: QCD phase diagram for $N_f = 4$ at imaginary and real $\mu_B$. Diamonds represent a polynomial fit to the lattice QCD data in \cite{22} in the sector of imaginary chemical potential and its analytic continuation, while the solid curve is the QPM result for the parametrization of the pseudo-critical line from \cite{13}. See text for details of the phase structure in the imaginary sector.

IV. SUMMARY

In summary we show that the quasiparticle model with the chosen dispersion relations for quarks and gluons accounts fairly well for the quark mass dependence in the QCD equation of state near $T_c$ as delivered by selected lattice QCD results. We emphasize further that lattice QCD calculations at imaginary chemical potential, avoiding the sign problem of the fermion determinant, give valuable information which can be continued to real chemical potential within our model. The feasibility of exploring larger net baryon densities is relevant for future heavy-ion experiments at FAIR.

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