$U_L(N) \times U_R(N)$-invariant four-fermion interactions and Nambu-Goldstone mechanism at finite temperature* 

Bang-Rong Zhou† 
Department of Physics, Graduate School at Beijing University of Science and Technology of China, Academia Sinica, Beijing 100039, China‡ 
and 
CCAST (World Laboratory), Beijing 100080, China

Abstract 
In a chiral $U_L(N) \times U_R(N)$ fermion model of NJL-form, we prove that, if all the fermions are assumed to have equal masses and equal chemical potentials, then at the finite temperature $T$ below the symmetry restoration temperature $T_c$, there will be $N^2$ massive scalar composite particles and $N^2$ massless pseudoscalar composite particles (Nambu-Goldstone bosons). This shows that the Goldstone Theorem at finite temperature for spontaneous symmetry breaking $U_L(N) \times U_R(N) \rightarrow U_{L+R}(N)$ is consistent with the real-time formalism of thermal field theory in this model.

PACS numbers: 11.10.Wx, 11.30.Rd, 11.30.Qc, 14.80.Mz 
Key words: Fermion condensates, Real-time thermal field theory, $U_L(N) \times U_R(N)$ chiral symmetry breaking, Nambu-Goldstone bosons 

1 Introduction 
The Nambu-Goldstone mechanism [1-3] characterizes spontaneous breaking of a continuous symmetry. While it has been researched extensively at zero temperature field theory, it is still interesting to examine how this mechanism manifests itself at a finite temperature for a deeper understanding of symmetry breaking at high temperature, especially of the consistency between the Nambu-Goldstone mechanism and the real-time thermal field theory’s formalism [4]. This consistency is not trivial in some models. In this paper, as an example, we will take a simple model of Nambu-Jona-Lasinio (NJL) -form [2] with $U_L(N) \times U_R(N)$- chirally -invariant four-fermion interactions to explore this problem. In the fermion bubble graph approximation we will calculate the propagators of the scalar and pseudoscalar bound state modes, determine the masses of these bound states and finally confirm that, under some conditions, the Nambu-Goldstone mechanism will be consistent with the real-time formalism of thermal field theory in this model.

*This work was supported partially by National Natural Science Foundation of China and by Grant No.LWTZ-1298 of the Chinese Academy of Sciences. 
†Regular Associate of The Abdus Salam ICTP. 
‡Mailing Address.
2 Model

For a fermion system with $U_L(N) \times U_R(N)$ -invariant four-fermion interactions, its Lagrangian can be generally expressed by

$$\mathcal{L} = \sum_{j,k=1}^{N} \left[ \bar{Q}_L^j \gamma^\mu Q_{Rj} + \bar{Q}_R^j \gamma^\mu Q_{Lj} + g(\bar{Q}_L^j Q_{Rk})(\bar{Q}_R^k Q_{Lj}) + g_L(\bar{Q}_L^j Q_{Lj})(\bar{Q}_L^k \gamma_\mu Q_{Lk}) + g_R(\bar{Q}_R^j Q_{Rj})(\bar{Q}_R^k \gamma_\mu Q_{Rk}) \right],$$

(2.1)

where the fermion fields $Q_L$ and $Q_R$ are respectively assigned in the $N$-dimension representations of the symmetry group $U_L(N)$ and $U_R(N)$ and $g$, $g_L$ and $g_R$ are the real coupling constants of the corresponding four-fermion interactions. It is indicated that the independent four-fermion couplings have only the scalar and the vector coupling terms appearing in Eq. (2.1). By Fierz Rearrangement Theorem, it can be proven that the tensor coupling with $\sigma^{\mu\nu}$ does not exist and all the other couplings including the ones with $\gamma_5$, $\gamma_5\gamma_\mu$ and the vector coupling between $Q_L$ and $Q_R$ fields can be transformed into the forms shown in Eq. (2.1). By means of

$$\frac{Q_{Lj}}{Q_{Rj}} = \frac{1}{2}(1 \mp \gamma_5)Q_j$$

(2.2)

we can rewrite the scalar coupling among the chiral fields by

$$\mathcal{L}_{\text{int}}^{CS} = g \sum_{j,k=1}^{N} (\bar{Q}_L^j Q_{Rk})(\bar{Q}_R^k Q_{Lj})$$

$$\begin{aligned}
&= \frac{g}{4} \left[ \sum_{j=1}^{N} (\bar{Q}_L^j Q_{j})^2 + \sum_{j \neq k=1}^{N} (\bar{Q}_L^j Q_k)(\bar{Q}_R^k Q_j) - \sum_{j,k=1}^{N} (\bar{Q}_L^j \gamma_5 Q_k)(\bar{Q}_R^k \gamma_5 Q_j) \right].
\end{aligned}$$

(2.3)

Assuming that the scalar couplings $(g/4) \sum_{j=1}^{N} (\bar{Q}_L^j Q_{j})^2$ among the same $Q_j$ fields could lead to formation of the condensates $\langle \bar{Q}_L^j Q_{j} \rangle$ ($j = 1, \ldots, N$) and generation of the fermion masses $m_j$ ($j = 1, \ldots, N$). At finite temperature $T$, the condensate $\langle \bar{Q}_L^j Q_{j} \rangle$ must be replaced by the corresponding thermal expectation value $\langle \bar{Q}_L^j Q_{j} \rangle_T$, thus we will obtain the gap equation for the dynamical fermion mass at $T \neq 0$

$$m_j(T, \mu_j) = -\frac{g}{2} \langle \bar{Q}_L^j Q_{j} \rangle_T.$$

(2.4)

A natural supposition is that

$$\mu_1 = \mu_2 = \ldots = \mu_N = \mu \quad \text{and} \quad m_1 = m_2 = \ldots = m_N = m \equiv m(T, \mu),$$

(2.5)

i.e. the chemical potentials and the dynamical masses of all the fermions are equal, then the gap equation will take the form

$$gI = 1$$

(2.6)

with

$$I = \frac{1}{2m} \int \frac{d^4l}{(2\pi)^4} tr[i S^{11}(l, m)]$$

$$= 2 \int \frac{d^4l}{(2\pi)^4} \left[ \frac{i}{l^2 - m^2 + i\varepsilon} - 2\pi \delta(l^2 - m^2) \sin^2 \theta(l^0, \mu) \right],$$

(2.7)
where we have used the fermion matrix propagator in the real-time thermal field theory [4]

\[
\begin{pmatrix} iS^{11}(l, m) & iS^{12}(l, m) \\ iS^{21}(l, m) & iS^{22}(l, m) \end{pmatrix} = \begin{pmatrix} i/(\beta - m + i\epsilon) & 0 \\ 0 & -i/(\beta - m - i\epsilon) \end{pmatrix} - 2\pi(\beta + m)\delta(l^2 - m^2) \begin{pmatrix} \sin^2\theta(l^0, \mu) & \frac{1}{2}e^{\beta\mu/2}\sin2\theta(l^0, \mu) \\ \frac{1}{2}e^{-\beta\mu/2}\sin2\theta(l^0, \mu) & \sin^2\theta(l^0, \mu) \end{pmatrix}
\]

with \( \beta = 1/T \) and

\[
\sin^2\theta(l^0, \mu) = \frac{\theta(l^0)}{e^{\beta(l^0-\mu)} + 1} + \frac{\theta(-l^0)}{e^{\beta(-l^0+\mu)} + 1}. \tag{2.9}\]

It is pointed out that the gap equation (2.6) could be satisfied only at the temperature \( T < T_c \), where \( T_c \) is the critical temperature for chiral symmetry restoration in a model of NJL-form [5]. In the following discussions of the propagators of bound states we will confine ourselves to the temperature below \( T_c \), i.e. assume Eq.(2.6) is satisfied.

### 3 Scalar bound state modes

The propagators for scalar bound states relate to the scalar four-point functions of fermions. To calculate them in the real-time formalism of thermal field theory, we must take the doubled scalar four-fermion interaction Lagrangian [4]

\[
L^S_{\text{int}} = \frac{g^4}{4} \sum_{a=1,2} \sum_{j,k=1}^N (-1)^{a+1}(\bar{Q}^j Q_k)^{(a)}(\bar{Q}^k Q_j)^{(a)}, \tag{3.1}\]

where \( a = 1 \) means physical fields and \( a = 2 \) ghost fields. The physical and the ghost fields can interact only through propagators. Consider scalar bound state \((\bar{Q}^k Q_j)\). The scalar four-point function from \( a \)-type vertex to \( b \)-type vertex will be denoted by

\[
\Gamma^{(\bar{Q}^j Q_k)^{(a)}(\bar{Q}^k Q_j)^{(a)}}_S(p) = \Gamma^{ba}_S(p),
\]

then in the bubble graph approximation, they submit to the following equations

\[
\Gamma^{ba}_S(p) = i\frac{g}{2}(-1)^{a+1}\delta^{ba} + \sum_{c=1,2} \Gamma^{bc}_S(p)L^{ca}(p)i\frac{g}{2}(-1)^{a+1}, \quad a, b = 1, 2, \tag{3.2}\]

which are extension of the similar equations at zero temperature [6] to finite temperature, where \( p \) is the four-momentum of the bound state \((\bar{Q}^k Q_j)\), \( L^{ca}(p) \) expresses the contribution of the \( Q_j - Q^k \) fermion loop with an \( a \)-type and a \( c \)-type scalar interaction vertex,

\[
L^{ca}(p) \equiv L^{ca}_{\bar{Q}^k Q_j}(p) = -\int \frac{d^4l}{(2\pi)^4} tr[iS^{ca}(l, m)iS^{ac}(l + p, m)]. \tag{3.3}\]

Eq.(3.2) has the solutions

\[
\begin{align*}
\Gamma^{b1}_S(p) &= \frac{1}{\Delta(p)} \left\{ i\frac{g}{2} \left[ 1 + i\frac{g^2}{2}L^{22}(p) \right] \delta^{b1} + \frac{g^2}{4}L^{21}(p) \delta^{b2} \right\}, \\
\Gamma^{b2}_S(p) &= \frac{1}{\Delta(p)} \left\{ \frac{g^2}{4}L^{12}(p) \delta^{b1} - i\frac{g}{2} \left[ 1 - i\frac{g}{2}L^{11}(p) \right] \delta^{b2} \right\} \tag{3.4} \end{align*}
\]
with
\[
\Delta(p) = \left[1 - i\frac{g}{2}L^{11}(p)\right]\left[1 + i\frac{g}{2}L^{22}(p)\right] - \frac{g^2}{4}L^{12}(p)L^{21}(p). \tag{3.5}
\]

The propagators for physical scalar bound states \((\bar{Q}^k Q_j)(j,k = 1, \ldots, N)\) are
\[
\Gamma_s^{11}(p) = i\frac{g}{2}\left\{1 - i\frac{g}{2}L^{11}(p)\right\} - \frac{g^2}{4}L^{12}(p)L^{21}(p) + \left[1 + i\frac{g}{2}L^{22}(p)\right]. \tag{3.6}
\]

The problem is reduced to the calculation of the fermionic loop \(L^{ca}(p)\). From Eqs.(3.3) and (2.8), by direct but rather lengthy derivation we obtain
\[
L^{11}(p) = -2iI + (4m^2 - p^2 - i\varepsilon)\{i[K(p) + H(p)] + S(p)\} = [L^{22}(p)]^*,
\]
where \(K(p), H(p), S(p)\) and \(R(p)\) are all real functions and expressed by
\[
K(p) = \frac{1}{8\pi^2} \int_0^1 dx \ln \frac{\Lambda^2 + M^2(p)}{M^2(p)} - \frac{\Lambda^2}{\Lambda^2 + M^2(p)}, \quad M^2(p) = m^2 - p^2 x(1 - x) \tag{3.8}
\]
with the four-fermion Euclidean momentum cut-off \(\Lambda\),
\[
H(p) = 4\pi \int \frac{d^4l}{(2\pi)^4} \left\{\frac{(l + p)^2 - m^2}{[(l + p)^2 - m^2]^2 + \varepsilon^2} + (p \to -p)\right\} \delta(l^2 - m^2)\sin^2\theta(l^0, \mu), \tag{3.9}
\]
\[
S(p) = 4\pi^2 \int \frac{d^4l}{(2\pi)^4}\delta(l^2 - m^2)\delta[(l + p)^2 - m^2]
\[
\left[\sin^2\theta(l^0 + p^0, \mu)\cos^2\theta(l^0, \mu) + \sin^2\theta(l^0, \mu)\cos^2\theta(l^0 + p^0, \mu)\right] \tag{3.10}
\]
and
\[
R(p) = 2\pi^2 \int \frac{d^4l}{(2\pi)^4}\delta(l^2 - m^2)\delta[(l + p)^2 - m^2][\sin2\theta(l^0, \mu)\sin2\theta(l^0 + p^0, \mu)). \tag{3.11}
\]

In the above calculation, we have used the formula
\[
\frac{1}{X + i\varepsilon} = \frac{X}{X^2 + \varepsilon^2} - i\pi\delta(X) \tag{3.12}
\]
and the result
\[
4\pi \int \frac{id^4l}{(2\pi)^4} \left\{\frac{[(l - p)^2 - m^2]^2}{[(l - p)^2 - m^2]^2 + \varepsilon^2} - (p \to -p)\right\} \delta(l^2 - m^2)\sin^2\theta(l^0, \mu)\]
\[
= 4\pi \int \frac{id^4l}{(2\pi)^4} \left\{[(l - p)^2 - m^2]^2 - (p \to -p)\right\} \pi^2\delta[(l + p)^2 - m^2]
\[
\delta(l^2 - m^2)\delta[(l - p)^2 - m^2]\sin^2\theta(l^0, \mu) = 0 \tag{3.13}
\]
owing to the fact that the arguments of the three \(\delta\)-functions in the integrand can not be equal to zeros simultaneously. We notice that the pinch singularities will appear in \(S(p)\)
and \( R(p) \) when \( p \to 0 \). Substituting Eq. (3.7) into Eq. (3.6) and taking the gap equation (2.6) into account, we obtain the propagator for physical scalar bound state \((\bar{Q}^kQ_j)\)

\[
\Gamma_S(p) \equiv \Gamma_S^{(\bar{Q}^jQ_k)^{(1)}}(\bar{Q}^kQ_j)^{(1)} = -i/(p^2 - 4m^2 + i\varepsilon) \left[ K(p) + H(p) - iS(p) - \frac{R^2(p)}{K(p) + H(p) + iS(p)} \right].
\]

(3.14)

It seems that \( p^2 = 4m^2 \) is the simple pole of \( \Gamma_S(p) \). To verify this, we must examine the behavior of \( K(p), H(p), S(p) \) and \( R(p) \) at \( p^2 = 4m^2 \). It is seen from Eq. (3.8) that \( K(p)\big|_{p^2=4m^2} \) is a finite constant when \( \Lambda \) is fixed. By means of Eq. (2.9), we may rewrite \( H(p) \) in Eq. (3.9) by

\[
H(p) = \frac{1}{16\pi^2} \frac{1}{|\bar{p}|} \int_0^\infty \frac{d \tilde{l}}{\omega_l} \ln \left( \frac{(p^2 - 2\omega_l p^0 + 2|\tilde{l}||\bar{p}|^2 + \varepsilon^2)}{(p^2 - 2\omega_l p^0 - 2|\tilde{l}||\bar{p}|^2 + \varepsilon^2)} + (p^0 \to -p^0) \right)
\]

\[
\left\{ 1/[e^{\beta(\omega_l-\mu)} + 1] + 1/[e^{\beta(\omega_l+\mu)} + 1] \right\}, \quad \omega_l = \sqrt{l^2 + m^2},
\]

(3.15)

where the zero points of the arguments of all the logarithmic functions must be removed from the integral because these functions come from the integration of the principal parts of the integrand. It is indicated that \( H(p) \) in Eq. (3.15) contains no singularity when \( |\tilde{l}| \to 0 \). In fact, if we set \( p^2 = \lambda^2 \), then when \( |\tilde{l}| \to 0 \), \( p^0 = \lambda \) and it can be proven that

\[
\lim_{|\tilde{l}| \to 0} \frac{1}{|\tilde{l}|} \ln \left( \frac{(p^2 + 2\omega_l p^0 + 2|\tilde{l}||\bar{p}|^2 + \varepsilon^2)}{(p^2 + 2\omega_l p^0 - 2|\tilde{l}||\bar{p}|^2 + \varepsilon^2)} = \frac{8|\tilde{l}|(\lambda^2 + 2\omega_l\lambda)}{(\lambda^2 + 2\omega_l\lambda)^2 + \varepsilon^2}
\]

(3.16)

which are finite even if when \( \lambda = 0 \). It is easy to find that when \( p^2 = 4m^2 \) the arguments of the logarithmic functions in Eq. (3.15) have no zero except \( p^2 - 2\omega_l p^0 + 2|\tilde{l}||\bar{p}| = 0 \) if \( |\tilde{l}| = |\bar{p}|/2 \). However, now that this point has been removed from the integral, it will not lead any singularity of \( H(p) \).

When \( p^2 = 4m^2 \), the general form of \( S(p) \) and \( R(p) \) may be expressed by

\[
A(p)|_{p^2=4m^2} = \int d^4l \delta(l^2 - m^2) \delta((l + p)^2 - m^2) f(l^0, p^0, \mu)|_{p^2=4m^2}
\]

\[
= \int \frac{d^3l}{4\omega_l} \left[ \delta(\omega_l p^0 - |\tilde{l}||\bar{p}|^2 + 2m^2) f(\omega_l, p^0, \mu) + \delta(-\omega_l p^0 - |\tilde{l}||\bar{p}|^2 + 2m^2) f(-\omega_l, p^0, \mu) \right] p^0 = \sqrt{\bar{p}^2 + 4m^2}.
\]

(3.17)

Since \( |\cos\theta| \leq 1 \), the argument of the first \( \delta \)-function in Eq. (3.17) could not be zero for any value of \( |\tilde{l}| \) and the second \( \delta \)-function could have zero argument only if \( |\tilde{l}| = |\bar{p}|/2 \) \((\cos\theta = -1)\), thus we obtain

\[
A(p)|_{p^2=4m^2} = \pi \int_0^\infty \frac{d\tilde{l}}{4\omega_l} f(-\omega_l, p^0, \mu) \delta_{|\tilde{l}|, |\bar{p}|/2} = 0.
\]

(3.18)
This means that
\[ S(p)|_{p^2=4m^2} = R(p)|_{p^2=4m^2} = 0. \]  
(3.19)
As a result, the propagator of the scalar bound state \( \langle \bar{Q}^k Q_j \rangle \)
\[ \Gamma_S(p) \to -i/(p^2 - 4m^2 + i\varepsilon)[K(p) + H(p)], \quad \text{when } p^2 \to 4m^2 \]  
(3.20)
and \( p^2 = 4m^2 \) is its simple pole indeed. In this way, we obtain \( N^2 \) scalar bound states \( \langle \bar{Q}^k Q_j \rangle \) \((j, k = 1, ..., N)\) with the mass \( 2m \).

It may be verified that \( \Gamma_S(p) \) expressed by Eq.(3.14) contains no pinch singularity. We notice that when \( p \to 0 \) \( K(p) \) is still a finite constant, and \( H(p) = 0 \) by Eq.(3.9) and \( S(p) - R(p) = 0 \) from Eqs.(3.10) and (3.11). These results indicate that the terms containing pinch singularity in the denominator of \( \Gamma_S(p) \) will become
\[ -iS(p) - \frac{R^2(p)}{K(p) + iS(p)} \to \frac{-iS(p)K(p) + S^2(p) - R^2(p)}{S^2(p)} \to 0, \]  
(3.21)
i.e. the pinch singularities in the propagator \( \Gamma_S(p) \) will be cancelled by each other and do not appear in the final expression.

4 Pseudoscalar bound state modes

A parallell discussion to scalar bound states can be applied to the case with pseudoscalar bound states. The relevant four-fermion interactions are now expressed by the Lagrangian
\[ \mathcal{L}_{\text{int}}^P = \frac{g}{4} \sum_{a=1,2} \sum_{j,k=1}^{N} (-1)^{a+1} (\bar{Q}^j i\gamma_5 Q_k)^{(a)} (\bar{Q}^k i\gamma_5 Q_j)^{(a)}. \]  
(4.1)

For pseudoscalar bound state \( \langle \bar{Q}^k i\gamma_5 Q_j \rangle \), the corresponding pseudoscalar four-point function from \( a \)-type vertex to \( b \)-type vertex can be denoted by
\[ \Gamma_P^{(Q^i i\gamma_5 Q_k)^{(b)} (\bar{Q}^k i\gamma_5 Q_j)^{(a)}}(p) \equiv \Gamma_p^{b,a}(p) \]
and submit to the algebraic equations
\[ \Gamma_p^{b,a}(p) = i g/2 (-1)^{a+1} \delta^{ba} + \sum_{c=1,2} \Gamma_p^{c}(p) N^{ca}(p) i g/2 (-1)^{a+1}, \quad a, b = 1, 2 \]  
(4.2)
where \( N^{ca}(p) \) expresses the contribution of the \( Q_j - \bar{Q}^k \) fermion loop with an \( a \)-type and a \( c \)-type pseudoscalar interaction vertex, i.e.
\[ N^{ca}(p) \equiv N^{ca}_{\bar{Q}^k Q_j}(p) = - \int \frac{d^4l}{(2\pi)^4} tr[i\gamma_5 iS^{ca}(l,m)i\gamma_5 iS^{ac}(l+p,m)]. \]  
(4.3)
It is easy to see that Eq.(4.2) has the same form as Eq.(3.2) after the substitutions \( \Gamma_p^{b,a}(p) \to \Gamma_S^{b,a}(p) \) and \( N^{ca}(p) \to L^{ca}(p) \). Hence we can directly put down the propagators for physical pseudoscalar bound states \( \langle \bar{Q}^k i\gamma_5 Q_j \rangle \) \((j, k = 1, ..., N)\)
\[ \Gamma_p(p) \equiv \Gamma_p^{11}(p) = i g/2 / \left\{ \left[ 1 - i g/2 N^{11}(p) \right] - \frac{g^2}{4} N^{12}(p) N^{21}(p) / \left[ 1 + i g/2 N^{22}(p) \right] \right\}. \]  
(4.4)
The results of calculations of $N^{ca}(p)$ are

$$N^{11}(p) = -2iI - i(p^2 + i\varepsilon)[K(p) + H(p)] - iS(p)] = [N^{22}(p)]^*,$$

$$N^{12}(p) = N^{21}(p) = -p^2 R(p).$$

Thus we obtain

$$\Gamma_P(p) = -i/(p^2 + i\varepsilon) \left[ K(p) + H(p) - iS(p) - \frac{R^2(p)}{K(p) + H(p) + iS(p)} \right].$$

We observe that when $p^2 \to 0$, $K(p)$ is finite, $H(p)$ in Eq.(3.9) is equal to zero and both $S(p)$ and $R(p)$ are also equal to zeroes because the arguments of $\delta(l^2 - m^2)$ and $\delta[(l + p)^2 - m^2]$ in Eqs.(3.10) and (3.11) can not be zeroes simultaneously. Consequently, we have

$$\Gamma_P(p) \xrightarrow{p^2 \to 0} -i/(p^2 + i\varepsilon)K(p)$$

which is of the same form as the propagator for pseudoscalar bound state at $T = 0$. Therefore, $p^2 = 0$ is the simple pole of $\Gamma_P(p)$ and we will have $N^2$ massless pseudoscalar bound states $(\bar{Q}^k i\gamma_5 Q_j)$ $(j,k = 1, ..., N)$. By comparing Eq.(4.6) with Eq.(3.14) we see that $\Gamma_P(p)$ and $\Gamma_S(p)$ have the identical form except the position of the pole. Hence the same cancellation mechanism of the pinch singularities as in $\Gamma_S(p)$ certainly exists in $\Gamma_P(p)$ as well and we need not worry about the problem of pinch singularity here.

5 Conclusion

The above discussions show that under the assumption (2.5) i.e. all the fermions have equal masses and equal chemical potentials, at the finite temperature $T < T_c$, the critical temperature below which the gap equation (2.6) is satisfied, we may obtain $N^2$ scalar bound states $(\bar{Q}^k Q_j)$ $(j,k = 1, ..., N)$ with the mass $2m$ and $N^2$ massless pseudoscalar bound states $(\bar{Q}^k i\gamma_5 Q_j)$ $(j,k = 1, ..., N)$. These results characterize spontaneous symmetry breaking of the chiral group $U_L(N) \times U_R(N)$ down to the vector-like group $U_{L+R}(N)$. The $N^2$ massive scalar composite particles will correspond to the generators of the unbroken group $U_{L+R}(N)$. The $N^2$ massless pseudoscalar composite particles will correspond to the generators of the broken axial-vector group $U_{L-R}(N)$ and can be identified with the Nambu-Goldstone bosons. This shows the Goldstone Theorem at finite temperature. Here the theorem is proven in the chiral $U_L(N) \times U_R(N)$ model of NJL-form by means of the real-time formalism of thermal field theory without any inconsistency. However, we emphasize that the assumption (2.5) is decisive for validity of such consistency between the Goldstone Theorem at finite temperature and the real-time thermal field theory. For the model discussed in this paper, the assumption (2.5), especially that the fermions have the same masses, can be natural and plausible. As for the models in which the assumption (2.5) could not satisfied, we will research them elsewhere.

References

[1] Y. Nambu, Phys. Rev. Lett. 4 (1960) 380; J. Goldstone, Nuovo Cimento 19 (1961) 154.

[2] Y. Nambu and G. Jona-Lasinio, Phys. Rev. 122 (1961) 345; 124 (1961) 246.
[3] J. Goldstone, A. Salam and S. Weiberg, Phys. Rev. 127 (1962) 965; S. Bludman and A. Klein, Phys. Rev. 131 (1962) 2363.

[4] N. P. Landsman and van Ch. G. Weert, Phys. Rep. 145 (1987) 141 and references therein.

[5] B. R. Zhou, Phys. Rev. D 57 (1998) 3171; Commun. Theor. Phys. to be published.

[6] B. R. Zhou, Phys. Rev. D 47 (1993) 5038.