CP violating couplings
in $Z \rightarrow 3$ jet decays revisited

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Abstract:

Possible CP violating effects in $Z \rightarrow 3$ jet decays are investigated. The analysis assumes the presence of CP violating $Zq\bar{q}G$ couplings. The contribution of these couplings to the $Z \rightarrow q\bar{q}G$ decay width is calculated for different cut algorithms, including nonzero quark masses. Various CP–odd observables are discussed and it is shown that their sensitivity can change significantly if one uses normalized or unnormalized momentum vectors for their construction. Optimal observables are proposed which allow to measure the new couplings simultaneously.
1 Introduction

The large number of $Z$ bosons collected at the LEP collider allows detailed studies of $Z$ decays. An interesting possibility is the search for CP violation beyond the one present in the Standard Model (SM). For general discussions and lists of further work we refer to [1, 2]. In this note we want to present new calculations for the $Z$ decay rate which have already been applied in a recent experimental analysis [3]. We further reinvestigate certain CP–odd observables and propose the measurement of optimal observables.

The theoretical framework is defined in ref. [1]. There, possible CP violating effects are parametrized by an effective Lagrangian, i.e. CP violating formfactors are assumed to be real and constant. This effective Lagrangian is being added to the SM and calculations are performed at tree level. Including operators of dimension 6 and demanding $SU(3)_C \times U(1)_{em}$ invariance the effective Lagrangian relevant for $Z \rightarrow 3$ jet decays contains weak and chromoelectric dipole moments and a $Zq\bar{q}G$ vertex with couplings $\hat{h}_{Vq}$ and $\hat{h}_{Aq}$.

The couplings $\hat{h}_{Vq/Aq}$ correspond to dimension 6 operators which conserve the quark chirality. A possible way to generate such couplings is through loop diagrams involving non-standard Higgs scalars [4], leading naturally to larger couplings for heavier quarks. In an analysis of $Z$ decays it is therefore legitimate (albeit not compelling) to assume that the $b$ quark couplings $\hat{h}_{Vb/Ab}$ are much larger than the corresponding couplings for light quarks. In ref. [2] such a scenario was used to explain the gap between the experimental and theoretical values of $R_b$ with the existence of large CP violating couplings for the $b$. However, new (preliminary) data [5] seem to indicate that this discrepancy is no longer that significant.

First experimental studies [6] have concentrated on the measurement of angular correlations of the jets in $Z \rightarrow 3$ jet decays. In the present framework these correlations are only sensitive to the couplings $\hat{h}_{Vq/Aq}$ (the chromoelectric dipole moment does not contribute and effects due to the weak dipole moment are suppressed by a relative factor $m_q/M_Z$). In order to obtain limits on the couplings $\hat{h}_{Vq/Aq}$ from an independent measurement, the authors of [3] used the differential two–jet rate, which requires the knowledge of the contribution of the new couplings to the decay rate. To avoid statistical uncertainties from event generators, analytic expressions for the rate are useful and shall therefore be presented in the following section.

2 Anomalous contributions to the rate

We consider the decays $Z \rightarrow 3$ jets on the $Z$ resonance. At the parton level this is equivalent to the reaction

\[ e^+e^- \rightarrow Z(p) \rightarrow q(k_-) + \bar{q}(k_+) + G(k_G), \]  

(1)
and we assume that the quark flavor (and perhaps also the jet charge) can be measured. It is convenient to describe this process in terms of production and decay matrices, $\rho$ and $R$ \textsuperscript{[1]}. Taking the $Z$ propagator in Breit–Wigner form and using the narrow width approximation, the total cross section factorizes,

$$
\sigma(e^+e^- \to Z \to q\bar{q}G) = \sigma(e^+e^- \to Z) \frac{1}{\Gamma_Z} \Gamma(Z \to q\bar{q}G),
$$

(2)

with the total $Z$ width $\Gamma_Z$ and the on-shell condition $p^2 = M_Z^2$. The partial width $\Gamma(Z \to q\bar{q}G)$ is given as integral over the trace term of the decay matrix $R$.

The phase space for the decay $Z \to q\bar{q}G$ can be written as

$$
d\phi = \frac{M_Z}{2^{3/2}\pi^3} dy_+ dy_- \Theta\left(y_+ y_- (1 - y_+ - y_-) - r^2 (y_+ + y_-)^2\right) \Theta(\text{cuts}).
$$

(3)

The (positive) integration variables $y_{\pm}$ are defined in terms of the $\bar{q}/q$ energies $E_{\pm}$ in the $Z$ rest system as $y_{\pm} = 1 - 2E_{\pm}/M_Z$; the gluon energy is $E_G = M_Z (y_+ + y_-)/2$. We further defined $r = m_q/M_Z$. The second $\Theta$-function implements the presence of cuts with cut parameter $y$,

$$
\Theta(\text{cuts}) = \Theta(y_+ - y) \Theta(y_G - y) \Theta(y_+ G - y),
$$

(4)

with

$$
y_{ij} = \begin{cases} 2M_Z^{-2} E_i E_j (1 - \cos \vartheta_{ij}) & \text{for JADE cuts,} \\ 2M_Z^{-2} \min\{E_i^2, E_j^2\} (1 - \cos \vartheta_{ij}) & \text{for DURHAM cuts.} \end{cases}
$$

(5)

Here $\vartheta_{ij}$ is the angle between the momentum directions $\hat{k}_i$ and $\hat{k}_j$ ($i, j = +, -, G$) measured in the $Z$ rest system. Note that the JADE algorithm agrees with the so-called ‘$E$-cuts’ $y_{ij} = (k_i + k_j)^2/M_Z^2$ only for massless fermions. These prescriptions can of course be applied to the decay $Z \to f\bar{f}\gamma$ as well.

In Fig. 1a) we show the shape of the phase space for massless fermions (the dashed triangle) and for $b$ quarks (i.e. $r = 1/20$), both without cuts. Fig. 1b) shows the effect of JADE and DURHAM cuts on the phase space for $b$ quarks. In this case the border lines are nontrivial functions given implicitly by the zeroes of $\Theta(\text{cuts})$ (although they look like straight lines).

The SM result for the rate $\Gamma(Z \to q\bar{q}G)$ has been calculated to order $\alpha_s^2$ in \textsuperscript{[7]}. Since the new couplings $\hat{h}_{Vq/Aq}$ are CP violating, their contribution to the width adds incoherently \textsuperscript{[2]},

$$
\Gamma(Z \to q\bar{q}G) = \Gamma_{SM}^{q\bar{q}G} + \Delta\Gamma_{q\bar{q}G}.
$$

(6)

Note that in the differential two–jet rate

$$
D_2(y) = \frac{1}{\Gamma(Z \to q\bar{q}G)} \frac{d\Gamma(Z \to q\bar{q}G)}{dy}
$$

(7)
the anomalous part is not additive, since also the normalization is affected. The
additional contribution to the decay width can be written as

$$\Delta \Gamma_{qqG} = \frac{\alpha_s}{\pi} \Gamma_{\nu\bar{\nu}} \left( f_+(y, r^2) [\hat{h}_{V_{q}}^2 + \hat{h}_{A_{q}}^2] + f_-(y, r^2) [\hat{h}_{V_{q}}^2 - \hat{h}_{A_{q}}^2] \right)$$  \hspace{1cm} (8)

with \( \Gamma_{\nu\bar{\nu}} = \alpha M_Z/(24 \sin^2 \theta_W \cos^2 \theta_W) \). The coefficient functions \( f_{\pm}(y, r^2) \) are obtained from integrating the relevant parts of the trace term \( a \) of the decay matrix \( R \) (cf. eq. (A.4)) over the phase space volumes shown in Fig. 1b). Note that we consider only the contributions from the couplings \( \hat{h}_{V_{q}/A_{q}} \), i.e. we assume the dipole moments to vanish. Their contributions to the rate can be computed in a similar way with the full trace term \( a \).

The couplings \( \hat{h}_{V_{q}/A_{q}} \) correspond to a pointlike interaction which is not present in the SM Lagrangian. Since there are no internal propagators, infrared singularities are absent and the anomalous contribution \( \Delta \Gamma_{qqG} \) is therefore finite even if no cuts are applied. In this case one finds

$$f_+(0, r^2) = \frac{2}{5} \left( 1 - \frac{41}{2} r^2 - 29 r^4 + 5 r^6 + 30 r^8 \right) \sqrt{1 - 4 r^2}$$

$$f_-(0, r^2) = \frac{r^2}{3} \left( 3 + 26 r^2 - 62 r^4 + 60 r^6 \right) \sqrt{1 - 4 r^2}$$

$$f_+(y, r^2) = \frac{2}{5} \left( 1 - \frac{41}{2} r^2 - 29 r^4 + 5 r^6 + 30 r^8 \right) \sqrt{1 - 4 r^2} + 48 r^4 \left( 1 - r^2 + r^6 \right) \ln \left( \frac{1 + \sqrt{1 - 4 r^2}}{2 r} \right),$$

$$f_-(y, r^2) = \frac{r^2}{3} \left( 3 + 26 r^2 - 62 r^4 + 60 r^6 \right) \sqrt{1 - 4 r^2} + 16 r^4 \left( 1 - 3 r^2 - 6 r^4 + 5 r^6 \right) \ln \left( \frac{1 + \sqrt{1 - 4 r^2}}{2 r} \right).$$

In the presence of cuts the coefficient functions \( f_{\pm} \) can be computed analytically only for massless fermions. For nonzero fermion masses one can at least give a series expansion

$$f_{\pm}(y, r^2) = f_{\pm}^{(0)}(y) + r^2 f_{\pm}^{(1)}(y) + O(r^4, r^4 \ln r^2),$$

where the neglected terms for the heaviest fermion in question – the b quark – are of the order \( 4 \cdot 10^{-5} \). In (10), \( f_{\pm}^{(0)}(y) \) represents the exact result for massless fermions. For JADE cuts, the coefficient functions are

$$f_{\pm}^{(0)}(y) = \frac{14}{27} (1 - 3y)^2 - \frac{1}{9} (1 - 3y)^4 - \frac{1}{135} (1 - 3y)^5,$$

$$f_{\pm}^{(1)}(y) = (1 - 3y) \left( -9 - \frac{107}{3} y + 13 y^2 - 3 y^3 \right) + 8y(5 - y^2) \ln \left( \frac{1 - y}{2y} \right),$$

$$f_{-}^{(0)}(y) = 0,$$

$$f_{-}^{(1)}(y) = \frac{8}{9} (1 - 3y)^2 + \frac{1}{9} (1 - 3y)^4.$$
For DURHAM cuts we obtain

\[
\begin{align*}
\frac{d^2 \bar{\theta}}{d\phi^2} &= \frac{1}{60} \left( 24 - 350y - 720y^2 + 2570y^3 + 525y^4 - 2997y^5 \right) \\
&+ \frac{y}{60} \left( 400 - 754y - 321y^2 + 999y^3 \right) \sqrt{y(8 + y)} \\
&+ 8y^2 \left( -3 + 5y + 4y^2 - 6y^3 \right) \ln \left( \frac{3y + \sqrt{y(8 + y)}}{8y} \right), \\
\frac{d^2 \bar{\theta}}{d\phi^2} &= \frac{1}{6} \left( -54 - 170y + 1104y^2 + 29y^3 + 351y^4 \right) \\
&+ \frac{1}{6} \left( 120 - 380y - 5y^2 - 117y^3 \right) \sqrt{y(8 + y)} \\
&+ 8y \left( -13 + 20y - 3y^2 + 7y^3 \right) \ln \left( \frac{3y + \sqrt{y(8 + y)}}{8y} \right), \\
\frac{d^2 \bar{\theta}}{d\phi^2} &= 0, \\
\frac{d^2 \bar{\theta}}{d\phi^2} &= \frac{1}{6} \left( 6 + 28y + 132y^2 - 322y^3 + 351y^4 \right) \\
&+ \frac{y}{6} \left( -68 + 122y - 117y^2 \right) \sqrt{y(8 + y)} \\
&+ 8y \left( -2 + 5y - 10y^2 + 7y^3 \right) \ln \left( \frac{3y + \sqrt{y(8 + y)}}{8y} \right).
\end{align*}
\]  

As a nontrivial check on these results we have \( f_{\pm}^{(0,1)}(1/3) = 0 \) since for the cut parameter \( y = 1/3 \) the phase space has shrunk to a point.

In Fig. 2 we show the coefficient functions \( f_{\pm}(y, r^2) \) of (8) for \( b \) quarks, i.e. for \( r = 1/20 \), both for the JADE and the DURHAM algorithm. For massless quarks, \( f_{+}(0,0) = 2/5 \) whereas \( f_{-}(y, 0) \) vanishes identically. For nonzero quark masses the first expansion coefficients in (10) are larger than the leading terms by a factor of \( \sim 20 \). As a consequence, the mass corrections for \( b \) quarks amount to 5\% (and not – as expected – to \( r^2 \approx 0.25\% \)).

For practical purposes one can neglect the contribution proportional to \( \hat{h}^2_{Vq} - \hat{h}^2_{Ag} \) in (8). Assume first that \( f_{-} \) vanishes. An experimental limit on the size of \( \Delta \Gamma \) translates then into a bound on \( (\hat{h}^2_{Vq} + \hat{h}^2_{Ag})^{1/2} \) which means that allowed couplings lie within a circle with a certain radius \( R \) in the \( \hat{h}_{Vq} \cdot \hat{h}_{Ag} \)-plane. If \( f_{-} \) is turned on, this circle is deformed to an ellipse with half axes \( R/\sqrt{1 \pm \varepsilon} \) with \( \varepsilon = f_{-}/f_{+} \). A numerical study shows however that \( 0.004 \leq \varepsilon \leq 0.006 \) for all values of the cut parameter \( y \), both for JADE and DURHAM cuts.
3 CP–odd observables

The best way to measure CP violating couplings is of course with CP–odd observables. Due to fragmentation effects, the parton spins cannot be reconstructed, and therefore CP–odd observables have to be built from parton momentum directions (the parton energies are determined once the angles are known [3]). This is in contrast to $t\bar{t}$ production (see e.g. [8]) where the top spin can be traced through the decay. Observables that use only momentum directions, i.e. pure angular correlations, were proposed and investigated in [1]:

$$V_i^{(1)} = \left( \frac{\hat{k}_+ \times \hat{k}_-}{|\hat{k}_+ \times \hat{k}_-|} \right)_i,$$

$$T_{ij}^{(1)} = (\hat{k}_+ - \hat{k}_-)_i \left( \frac{\hat{k}_+ \times \hat{k}_-}{|\hat{k}_+ \times \hat{k}_-|} \right)_j + (i \leftrightarrow j),$$

where $i, j$ denote Cartesian indices in the $Z$ rest system. In addition we consider the following set of CP–odd observables:

$$V_i^{(2)} = (\hat{k}_+ \times \hat{k}_-)_i,$$

$$V_i^{(3)} = |\hat{k}_+ \times \hat{k}_-| (\hat{k}_+ \times \hat{k}_-)_i,$$

$$T_{ij}^{(2)} = (\hat{k}_+ - \hat{k}_-)_i (\hat{k}_+ \times \hat{k}_-)_j + (i \leftrightarrow j),$$

$$T_{ij}^{(3)} = |\hat{k}_+ \times \hat{k}_-| (\hat{k}_+ - \hat{k}_-)_i (\hat{k}_+ \times \hat{k}_-)_j + (i \leftrightarrow j).$$

These differ from (13), (14) only by factors of $|\hat{k}_+ \times \hat{k}_-| = \sin \vartheta_{\pm\pm}$. In the effective Lagrangian approach, the anomalous couplings $\hat{h}_V q/Aq$ correspond to pointlike interactions, which typically lead to event topologies which are not collinear. An additional weight factor $\sin \vartheta_{\pm\pm}$ suppresses collinear events and should therefore increase the sensitivity to $\hat{h}_V q/Aq$.

The observables listed above have definite transformation behaviour (as vectors and tensors), thus their expectation values are proportional to the $Z$ vector and tensor polarization $s$ and $s_{ij}$. For unpolarized beams and taking the $e^+$ beam as 3-direction we have $s = (0, 0, \gamma_{\alpha})$ and $s_{ij} = \frac{1}{6} \text{diag}(-1, -1, 2)$ (here $\gamma_{\alpha} = 2g_{Ve} g_{Ae}/(g_{Ve}^2 + g_{Ae}^2)$ with the weak vector and axial vector $Ze^+e^-$ couplings $g_{Ve}$ and $g_{Ae}$). Thus what one really should measure are the components $V_3^{(n)}$ and $T_{33}^{(n)}$. Also note that the tensor observables $T_{ij}^{(n)}$ do not change sign upon charge misidentification $\hat{k}_+ \leftrightarrow \hat{k}_-$, whereas for the vector observables the correct assignment of the jet charge is required, which makes them less useful experimentally.

From now on we assume that only the $b$ quarks have nonvanishing $\hat{h}_V q/Aq$ couplings. The expectation values of the observables should then be evaluated in an event sample containing $Z \rightarrow b\bar{b}G$ decays whereas $Z$ decays into light quarks can be used to test the CP blindness of the detector.
For the number $N_{\text{cut}}$ of $Z \to b\bar{b}G$ events within cuts needed to see a 1 s.d. effect we have (writing generically $\mathcal{O}$ for one of the observables)

$$N_{\text{cut}} = \frac{\langle \mathcal{O}^2 \rangle}{\langle \mathcal{O} \rangle^2}.$$  \hspace{1cm} (19)

Following ref. [4] we scale this number

$$N = \frac{\Gamma_Z}{\Gamma(Z \to b\bar{b}G)} N_{\text{cut}}$$ \hspace{1cm} (20)

in order to give an estimate of the total number $N$ of $Z$ bosons (note however that this procedure disregards detector resolutions, $b$ tagging efficiencies etc.).

For the analysis of CP–odd observables we neglect terms quadratic in CP violating couplings. The variances $\langle \mathcal{O}^2 \rangle$ are then determined from the SM amplitude, whereas the expectation values are linear functions of the anomalous couplings. In this approximation the sensitivity of the observable $\mathcal{O}$ defines a band in the $\sqrt{N} \hat{h}_{Vb} - \sqrt{N} \hat{h}_{Ab}$ plane. In the case of the tensor observables $T_{ij}^{(n)}$, the slope of this band is given by the ratio $g_{Vb}/g_{Ab}$ of weak $Zb\bar{b}$ couplings, since the expectation values $\langle T_{ij}^{(n)} \rangle$ are proportional to the combination $\hat{h}_b = \hat{h}_{Vb}g_{Ab} - \hat{h}_{Ab}g_{Vb}$ [4] (the latter remains true also if nonzero dipole moments are allowed, cf. eq. (A.2)).

In Fig. 3 we show the result for the observables $V_{3}^{(1)}$, $T_{33}^{(1)}$ (long dashed curves). The best observables are $V_{3}^{(2)}$ and $T_{33}^{(2)}$ (solid lines) whereas the sensitivity of $V_{3}^{(3)}$ and $T_{33}^{(3)}$ is slightly reduced. The gain in sensitivity when going from $V_{3}^{(1)}$ ($T_{33}^{(1)}$) to $V_{3}^{(2)}$ ($T_{33}^{(2)}$) can be easily understood: The unit vector $\hat{n} = (\hat{k}_+ \times \hat{k}_-)/|\hat{k}_+ \times \hat{k}_-|$ used in the construction of the observables $V_{3}^{(1)}$ and $T_{33}^{(1)}$ gives only the direction normal to the decay plane, whereas the unnormalized vector $\hat{k}_+ \times \hat{k}_-$ has an additional information content, namely the angle between $\hat{k}_+$ and $\hat{k}_-$. As a byproduct we have also recalculated the results of ref. [2] for nonzero $b$ quark mass (in [2] $m_b = 0$ was used throughout) and found that the numbers $N$ given there increase typically by 3% – 8%.

## 4 Optimal observables

In this section we construct a set of optimal observables to measure the couplings $\hat{h}_{Vb/Ab}$ separately, following the procedure developed in [1], [2]. Central assumptions are that one can neglect higher orders in the anomalous couplings and that the usual Gaussian error analysis applies. In linear approximation one writes the differential cross section for the reaction (1) as

$$d\sigma(e^+ e^- \to Z \to b\bar{b}G) = d\phi' \left( S_0 + S_{V} \hat{h}_{Vb} + S_{A} \hat{h}_{Ab} + O(\hat{h}_{Vb/Ab}^2) \right), \hspace{1cm} (21)$$
where $S_0$ represents the SM contribution. The phase space measure $d\phi'$ includes in addition to $d\phi$ of eq. (3) two integrations over angles relative to the $e^+$ beam direction $\hat{p}_+$. According to [10], the optimal observables are then given by the ratios

$$O_{V/A} = \frac{S_{V/A}}{S_0},$$

and any nonzero expectation value $\langle O_{V/A} \rangle \neq 0$ signals CP violation. From the explicit formulae one easily derives

$$S_0 = a^{(0)} + \gamma_{el} \left[ (\hat{p}_+ \hat{k}_+) b_1^{(0)} + (\hat{p}_+ \hat{k}_-) b_2^{(0)} \right] + \frac{1}{2} \left[ (\hat{p}_+ \hat{k}_+) - \frac{1}{3} \right] c_1^{(0)} + \frac{1}{2} \left[ (\hat{p}_+ \hat{k}_-) - \frac{1}{3} \right] c_2^{(0)} + \left[ (\hat{p}_+ \hat{k}_+) (\hat{p}_+ \hat{k}_-) - \frac{1}{3} (\hat{k}_+ \hat{k}_-) \right] c_3^{(0)},$$

$$S_{V/A} = \pm \beta^2 C_F N_c \frac{2}{M_Z} \hat{p}_+ (k_+ \times k_-) \left\{ g_{Ab/Vb} \frac{2}{M_Z} \left[ \frac{\hat{p}_+ \hat{k}_+}{y_-} - \frac{\hat{p}_+ \hat{k}_-}{y_+} \right] + g_{Vb/Ab} \gamma_{el} \left[ -\frac{y_+}{y_-} \frac{y_-}{y_+} + (1 \pm 4r^2) \left( \frac{1}{y_+} + \frac{1}{y_-} \right) \right] \right\}.$$

The term $S_0$ contains only SM quantities; the relevant contributions $a^{(0)}, \ldots, c_3^{(0)}$ to the coefficients of the decay matrix $R$ can be found in [1]. The global factors $\beta$, $C_F$, and $N_c$ are defined in the Appendix.

The general formula given in [10] for the sensitivity of these observables simplifies for the case of CP violating couplings. Since there are no contributions to the rate linear in $\hat{h}_{Vb/Ab}$, we have immediately $\int d\phi' S_V = \int d\phi' S_A = 0$. Moreover, only the trace term $a$ contributes to the rate, i.e. $\int d\phi' S_0 = \int d\phi' a$. The sensitivity is then determined by the matrix

$$M_{ij} = (\int d\phi' a)^{-1} \int d\phi' \frac{S_i S_j}{S_0},$$

which defines – after scaling according to eq. (20) – an ellipse in the $\sqrt{N} \hat{h}_{Vb} - \sqrt{N} \hat{h}_{Ab}$ plane. In Fig. 4 we show the results of our numerical evaluation. The strong correlation in direction of the line $\hat{h}_b = 0$ is not surprising, since the other combination of couplings, $\hat{h}_{Vb} g_{Vb} - \hat{h}_{Ab} g_{Ab}$, is suppressed by a factor of $\gamma_{el} \simeq 0.16$. In the same plot we compare the results for the best vector observable $V_3^{(2)}$ (long dashed) and the best tensor observable $T_{33}^{(2)}$ (short dashed). The optimal observables have a higher sensitivity, although the difference is only small. Note however that the construction of the optimal observables requires a measurement of the jet charge (cf. eq. (23)), similar to the vector observables.
5 Conclusions

In this paper we have presented various calculations which can be applied in a search for CP violation in $Z \rightarrow 3$ jet decays. The effect of CP violating $Zq\bar{q}G$ couplings on the partial width $\Gamma(Z \rightarrow q\bar{q}G)$ was computed for different cut algorithms and for nonzero quark masses. An important result of our investigation of CP–odd observables was that the sensitivity of the conventional vector and tensor observables changes significantly depending on whether normalized or unnormalized momentum vectors are used for their construction. The analysis of optimal observables showed that the gain in sensitivity is only small. Nevertheless their experimental exploration might be useful in order to perform a complete analysis of the data.

Acknowledgments

I would like to thank S. Dhamotharan, J. v. Krogh, M. Steiert, and M. Wunsch for their continuous interest in this work and in particular H. Stenzel for discussions which lead to part of this project. Special thanks are due to O. Nachtmann for valuable discussions and a critical reading of the manuscript.

Appendix

The decay matrix $R$ for the process $Z \rightarrow q\bar{q}G$ is defined as

$$R_{ij} = \sum \langle Z(e_i)|T|q\bar{q}G\rangle \langle q\bar{q}G|T|Z(e_j)\rangle,$$

where the summation extends over the $q$ and $\bar{q}$ spins and the gluon polarisation. We work in the $Z$ rest system, where $Z(e_i)$ denotes a $Z$ with polarization vector along the $i^{th}$ Cartesian coordinate. The decomposition of $R$ with the help of the $q$ and $\bar{q}$ momentum unit vectors $\hat{k}_-$ and $\hat{k}_+$ is given in [1]. Note however that we did not normalize $R$ with $\Gamma_{q\bar{q}G}$ and therefore we have $\frac{1}{2} \int d\phi \text{Tr}(R) = \Gamma_{q\bar{q}G}$ (instead of $\frac{1}{3} \int d\phi \text{Tr}(R) = 1$ in [1]).

The coefficients of that decomposition are calculated at tree level using in addition to the SM Lagrangian the effective CP violating Lagrangian of [1] which contains weak and chromoelectric dipole moments $\hat{d}_q$ and $\hat{d}_q'$ and the $Zq\bar{q}G$ couplings $\hat{h}_{Vq}$ and $\hat{h}_{Aq}$. Contributions from the SM and terms linear in CP violating
couplings have already been given in \[1\]. In order to make this article self-contained and to illustrate the different notations, we include the latter here as well:

\[
a^{(1)}(y_+, y_-) = b_1^{(1)}(y_+, y_-) = b_2^{(1)}(y_+, y_-) = 0 ,
\]

\[
b_3^{(1)}(y_+, y_-) = \frac{w(y_+, y_-)}{y_+ y_-} \left[ (\hat{h}_{Vq}g_{Vq} - \hat{h}_{Aq}g_{Aq})(y_+(1 - y_+) + y_-(1 - y_-)) + 4(\hat{r} \hat{d}_{q}g_{Aq} + r^2(\hat{h}_{Vq}g_{Vq} + \hat{h}_{Aq}g_{Aq}))(y_+ + y_-) \right] ,
\]

\[
c_1^{(1)}(y_+, y_-) = c_2^{(1)}(y_+, y_-) = c_3^{(1)}(y_+, y_-) = 0 ,
\]

\[
c_4^{(1)}(y_+, y_-) = -c_5^{(1)}(y_-, y_+) = \frac{\hat{h}_{q}w(y_+, y_-)}{y_-} \sqrt{(1 - y_+)^2 - 4r^2} .
\]

where

\[
\hat{h}_{q} = \hat{h}_{Vq}g_{Aq} - \hat{h}_{Aq}g_{Vq} ,
\]

\[
w(y_+, y_-) = \sqrt{y_+ y_-(1 - y_+ - y_-) - r^2(y_+ + y_-)^2} = 2\frac{\|k_+ \times k_-\|}{M_Z^2} . \tag{A.3}
\]

Here $g_{Vq} = T_3 - 2Q_q \sin^2 \theta_W$ and $g_{Aq} = T_3$ are the SM $Zq\bar{q}$ couplings; for the definition of $y_\pm$ and $r$ see eq. (3). Out of the contributions quadratic in CP violating couplings we list only the trace term $a^{(2)}(y_+, y_-)$ and the vanishing coefficients:

\[
a^{(2)}(y_+, y_-) = \frac{8}{3} \left\{ \hat{d}_{q} \left[ \frac{(y_+ + y_-)^2}{2y_+ y_-} - 2(y_+ + y_-) + (1 - 4r^2) \frac{1 - y_+ - y_-}{y_+ y_-} \right] - r^2(1 - 4r^2) \frac{(y_+ + y_-)^2}{(y_+ y_-)^2} + \hat{d}_{q} \hat{d}_{q} g_{Vq} \left[ 3 - 2y_+ - 2y_- - 2r^2(y_+ + y_-)^2 \right] \right. \\
+ \hat{d}_{q} \hat{h}_{Aq} \left[ - \frac{r}{2} (y_+ + y_-)^3 \right] + 2r(1 + y_+ + y_-) - 2r^3 \frac{(y_+ + y_-)^2}{y_+ y_-} \left. \right] + \hat{d}_{q}^2 g_{Aq} \left[ \frac{1}{2} y_+ y_- + r^2(-3 + y_+ + y_-) + r^4 \frac{(y_+ + y_-)^2}{y_+ y_-} \right] \\
+ \hat{d}_{q}^2 (g_{Vq}^2 + g_{Aq}^2) \frac{1}{2} (1 - y_+ - y_-) + \hat{d}_{q} \hat{h}_{Aq} g_{Vq} r(y_+ + y_-) \\
+ \hat{d}_{q} \hat{h}_{Vq} g_{Aq} \left[ \frac{r}{2} (y_+ + y_-) - \frac{r^3 (y_+ + y_-)^3}{2y_+ y_-} \right] + \hat{h}_{Vq}^2 - \hat{h}_{Aq}^2 \frac{r^2}{4} (y_+ + y_-)^2 \\
+ (\hat{h}_{Vq}^2 + \hat{h}_{Aq}^2) \frac{1}{8} (4y_+ y_- + (y_+ + y_-)^2 (1 - y_+ - y_-)) \right\} ,
\]

\[
b_3^{(2)}(y_+, y_-) = c_4^{(2)}(y_+, y_-) = c_5^{(2)}(y_+, y_-) = 0 .
\]
The nonvanishing coefficients $b_1^{(2)}$, $b_2^{(2)}$, $c_1^{(2)}$, $c_2^{(2)}$, and $c_3^{(2)}$ which do not contribute to the rate but are relevant for other (CP–even) observables have been calculated as well and are available on request.

All coefficients in (A.2), (A.4) have to be multiplied with the global factor $\beta^2 C_F N_c$, with the coupling $\beta = e g_s / (\sin \theta_W \cos \theta_W)$ and the color factor $C_F \cdot N_c = \frac{4}{3} \cdot 3$. The coefficients can of course be applied to the process $Z \rightarrow f \bar{f} \gamma$ as well. In this case one has to replace the chromoelectric dipole moment $\hat{d}'_q$ with the electric dipole moment $-\hat{d}_f$ and the $Zq\bar{q}G$ couplings $\hat{h}_{Vq/Aq}$ with the $Zf\bar{f}G$ couplings $-\hat{f}_{Vf/Af}$ (the minus signs stem from the normalization in [1]). Moreover, the global factor is obtained from replacing in $\beta$ the strong coupling constant $g_s$ with $eQ_f$ and changing the color factor $C_F \cdot N_c$ to 1·1 (1·3) for leptons (quarks).

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Figure Captions

Figure 1: Allowed phase space regions (Dalitz plots) for the decays $Z \rightarrow f \bar{f}V$ ($V = \text{photon or gluon}$) in the $y^+-y^-$ plane. a) Without cuts, for massless fermions (dashed) and for $b$ quarks (solid). b) With cut $y = 0.1$, for $b$ quarks with JADE (solid) and DURHAM algorithm (dashed).

Figure 2: The coefficients $f_+(y, r^2)$ and $f_-(y, r^2)$ in the decomposition (8) shown as function of the cut parameter $y$ for JADE (solid) and DURHAM cuts (dashed). The parameter $r = m_q/M_Z$ is put to $1/20$ ($b$ quark).

Figure 3: a) The half width of the sensitivity band in the $\sqrt{N} \hat{h}_{Vb} - \sqrt{N} \hat{h}_{Ab}$ plane as function of the cut parameter $y$. Shown are the results for the vector observables $V_3^{(1)}$ (long dashed), $V_3^{(2)}$ (solid) and $V_3^{(3)}$ (short dashed). b) The same for the tensor observables $T_{33}^{(1)}$ (long dashed), $T_{33}^{(2)}$ (solid) and $T_{33}^{(3)}$ (short dashed).

Figure 4: Sensitivity contours of different observables in the $\sqrt{N} \hat{h}_{Vb} - \sqrt{N} \hat{h}_{Ab}$ plane. The results for the best vector observable $V_3^{(2)}$ (long dashed) and the best tensor observable $T_{33}^{(2)}$ (short dashed) are compared to the ellipse from the optimal observables $O_V$, $O_A$ (solid).
