Abstract—Wyner’s soft-handoff network is considered where transmitters simultaneously send messages of enhanced mobile broadband (eMBB) and ultra-reliable low-latency communication (URLLC) services. Due to the low-latency requirements, the URLLC messages are transmitted over fewer channel uses compared to the eMBB messages. To improve the reliability of the URLLC transmissions, we propose a coding scheme with finite blocklength codewords that exploits dirty-paper coding (DPC) to precancel the interference from eMBB transmissions. Rigorous bounds are derived for the error probabilities of eMBB and URLLC transmissions achieved by our scheme. Numerical results illustrate that they are lower than for standard time-sharing.

I. INTRODUCTION

The fifth and the forthcoming sixth generations of mobile communications have to accommodate both ultra-reliable and low-latency communication (URLLC) and enhanced mobile broadband (eMBB) services [1], [2]. URLLC services aim at guaranteeing high-reliability at a maximum end-to-end delay of 1ms and are used for delay-sensitive applications such as industrial control management as well as autonomous vehicle and remote surgery applications [2]. On the other hand, eMBB services aim to provide high data rates and are used for delay-tolerant applications such as video streaming, virtual and augmented reality applications [3], [4].

The different latency requirements of eMBB and URLLC services along with the fact that they are scheduled in the same frequency band make their coexistence challenging. Networks with such mixed-delay constraints have been studied recently. See [5]–[9] for a comprehensive review on related works. The previous studies are mostly focused on the performance of such networks in the asymptotic regime where the number of channel uses goes to infinity. Since the URLLC delay constraint limits the number of available channel uses, the problem of joint coding of messages with heterogeneous blocklengths is of an increasing interest. Notably, for the Gaussian point-to-point channel with messages of heterogeneous decoding deadlines, the work in [13], proposes a coding scheme which decodes the messages at time-instances that depend on the realizations of the random channel fading. The authors showed that significant improvements are possible over standard successive interference cancellation. In [14] achievable rates and latency of the early-decoding scheme in [13] are improved by introducing concatenated shell codes. Finally, [15] and [16] studied the uplink of the cloud radio access networks where URLLC messages are directly decoded at the base stations whereas decoding of eMBB messages can be delayed to the cloud center. In particular, [15] performs a hybrid analysis where URLLC transmissions are studied in the finite blocklength regime and eMBB transmissions in the asymptotic infinite blocklength regime.

In this paper, we consider Wyner’s soft-handoff model with \( K \) interfering transmitters and receivers pairs. Each transmitter wishes to simultaneously transmit two messages of heterogeneous blocklengths: an URLLC message and an eMBB message. The URLLC message is transmitted over a shorter blocklength compared to the eMBB message. Txs can hold a conferencing communication that depends only on the eMBB messages but not on the URLLC messages. By exploiting the DPC principle in [11], [17], we propose a coding scheme to jointly transmit the URLLC and eMBB messages. Unlike [10], [13], [15], we consider that codebooks are generated randomly according to independent uniform distributions on the power-shell. Rigorous bounds are derived for achievable error probabilities of eMBB and URLLC transmissions. To this end, Gel’fand-Pinsker analysis techniques for finite blocklengths in [12] are combined with the multiple parallel channels approach in [18]. Numerical results illustrate that our proposed scheme significantly outperforms standard time-sharing.

II. PROBLEM SETUP

Consider Wyner’s soft-handoff network with \( K \) transmitters (Txs) and \( K \) receivers (Rxs) that are aligned on two parallel lines so that each Tx \( k \) has two neighbours, Tx \( k-1 \) and Tx \( k+1 \), and each Rx \( k \) has two neighbours, Rx \( k-1 \) and Rx \( k+1 \). Define \( K := \{1, \ldots, K\} \). The signal transmitted by Tx \( k \in K \) is observed by Rx \( k \) and the neighboring Rx \( k+1 \). See Figure 1. Each Tx \( k \in K \) sends a so called eMBB type message \( M_k^{(e)} \) to its corresponding Rx \( k \), for \( M_k^{(e)} \) uniformly
distributed over $\mathcal{M}_k^{(e)} := \{1, \ldots, L_e\}$. A subset of Txs $\mathcal{K}_U \subset \mathcal{K}$ also sends additional URLLC messages $M_k^{(U)}$, for $k \in \mathcal{K}_U$, for $M_k^{(U)}$ uniformly distributed over the set $\mathcal{M}_k^{(U)} := \{1, \ldots, L_U\}$. We assume that $K$ is even and

$$\mathcal{K}_U := \{1, 3, \ldots, K-1\}, \quad \text{(1)}$$

so that URLLC transmissions are only interfered by the eMBB transmissions but not by other URLLC transmissions. (The study of sets $\mathcal{K}_U$ with interfering URLLC messages is left as a future research direction.)

Communication takes place in two phases.

**Tx-cooperation phase:**
The encoding starts with a first Tx-cooperation phase in which Txs not in $\mathcal{K}_U$ share their eMBB message with their left-neighbouring Txs in $\mathcal{K}_U$. (For example over high-rate optical fibers if the Txs are BSs.) The URLLC messages, which are subject to stringent delay constraints, are only generated after the Tx-cooperation phase, at the beginning of the subsequent channel transmission phase.

**Channel transmission phase:**
URLLC messages are transmitted over $n_U$ channel uses and eMBB messages over $n_e > n_U$ channel uses. The blocklengths $n_U$ and $n_e$ are assumed to be fixed constants. Notice that while the transmission delay of URLLC messages is determined by the $n_U$ channel uses, transmission delay of eMBB consists of both the delay of the Tx-cooperation phase as well as the delay induced by the $n_e$ channel uses.

For each $k \in \mathcal{K}$, Tx $k$ computes its time-$t$ channel input $X_{k,t}$ with $t \in \{1, \ldots, n_e\}$ as

$$X_{k,t} = \begin{cases} f_k^{(b)}(M_k^{(U)}, M_k^{(e)}, M_{k-1}^{(e)}), & k \in \mathcal{K}_U \text{ and } t \leq n_U, \\ f_k^{(e)}(M_k^{(e)}), & k \notin \mathcal{K}_U \text{ or } n_U < t \leq n_e, \end{cases}$$

for some encoding functions $f_k^{(b)}$ and $f_k^{(e)}$ on appropriate domains satisfying the average block-power constraint

$$\frac{1}{n_e} \sum_{t=1}^{n_e} X_{k,t}^2 \leq P, \quad \forall k \in \mathcal{K}, \quad \text{almost surely.} \quad \text{(2)}$$

The input-output relation of the network is described as

$$Y_{k,t} = h_{k,k} X_{k,t} + h_{k-1,k} X_{k-1,t} + Z_{k,t}, \quad \text{(3)}$$

where $\{Z_{k,t}\}$ are independent and identically distributed (i.i.d.) standard Gaussian for all $k$ and $t$ and independent of all messages; $h_{k,t} > 0$ is the fixed channel coefficient between Tx $k$ and Rx $\ell$; and we define $X_{0,t} = 0$ for all $t$.

After $n_U$ channel uses, each Rx $k \in \mathcal{K}_U$ decodes the URLLC message $M_k^{(U)}$ based on its own channel outputs $Y_{k,t}^{(U)} := \{Y_{k,1}, \ldots, Y_{k,n_U}\}$. It produces:

$$\hat{M}_k^{(U)} = g_k^{(n_U)}(Y_{k,t}^{(U)}), \quad \text{(4)}$$

for some decoding function $g_k^{(n_U)}$ on appropriate domains. The average error probability for each message $M_k^{(U)}$ is given by

$$\epsilon_{U,k} := P\{\hat{M}_k^{(U)} \neq M_k^{(U)}\}, \quad \text{for } k \in \mathcal{K}_U. \quad \text{(5)}$$

After $n_e$ channel uses, each Rx $k$ decodes its desired eMBB messages as

$$\hat{M}_k^{(e)} = b_k^{(n_e)}(Y_{k,t}^{(e)}), \quad \text{(6)}$$

where $b_k^{(n_e)}$ is a decoding function on appropriate domains. The average error probability for message $M_k^{(e)}$ is given by

$$\epsilon_{e,k} := P\{\hat{M}_k^{(e)} \neq M_k^{(e)}\}, \quad \text{for } k \in \mathcal{K}. \quad \text{(7)}$$

We will be interested in the average URLLC and eMBB error probabilities

$$\epsilon_U := \frac{1}{|\mathcal{K}_U|} \sum_{k \in \mathcal{K}_U} \epsilon_{U,k}, \quad \text{(8)}$$

$$\epsilon_e := \frac{1}{|\mathcal{K}|} \sum_{k \in \mathcal{K}} \epsilon_{e,k}. \quad \text{(9)}$$

**III. CODING SCHEME**

Txs in $\mathcal{K}_U$ use DPC to precancel the interference of eMBB transmissions from their neighbouring transmissions and from their own eMBB transmissions on their URLLC transmissions. (Recall that during the Tx-cooperation rounds Txs in $\mathcal{K}_U$ learn the eMBB messages of their neighbouring Txs.)

**A. Encoding at Txs in $\mathcal{K} \setminus \mathcal{K}_U$**

Each Tx $k \in \mathcal{K} \setminus \mathcal{K}_U$ transmits only the eMBB message $M_k^{(e)}$ over the entire block of $n_e$ channel uses. Over the first $n_U$ channel uses, it transmits a codeword $X_{k,t}^{(e1)}(M_k^{(e)})$ that is uniformly distributed on the centered $n_U$-dimensional sphere of radius $\sqrt{n_U/\beta_e}P$, for some $\beta_e \in [0, 1]$, independently of all other codewords. Tx $k$ also describes its message $M_k^{(e)}$, and thus its input signal $X_{k,t}^{(e1)}$, to the neighbouring Tx to its right during the only Tx-cooperation round.

To encode $M_k^{(e)}$ over the following $(n_e - n_U)$ channel uses, Tx $k$ employs a second codeword $X_{k,t}^{(e2)}(M_k^{(e)})$ that is uniformly distributed on the centered $(n_e - n_U)$-dimensional sphere of radius $\sqrt{(n_e - n_U)(1 - \beta_e)}P$, independently of all other codewords.
B. Encoding at Txs in $K_U$

Each Tx $k$ in $K_U$ has both eMBB and URLLC messages to transmit. To transmit its URLLC message $M_k^{(U)}$, Tx $k$ employs DPC encoding to precancel the interference of the eMBB transmission of the Tx to its left and its own eMBB transmission. Tx $k$ transmits its URLLC message over only $n_U$ channel uses whereas it sends its eMBB message over the entire block of $n_a$ channel uses. To transmit both messages while satisfying (2), we divide the total transmit power $P$ into three parts $\beta_0 P$, $\beta_{e1} P$, $\beta_{e2} P$, where power $\beta_0 P$ is used for eMBB transmission, power $\beta_{e1} P$ for eMBB transmission during the first $n_U$ channel uses, and power $\beta_{e2} P$ for eMBB transmission during the last $n_e - n_U$ channel uses. The coefficients $\beta_0$, $\beta_{e1}$, $\beta_{e2} \in [0,1]$ are chosen such that

$$\beta_0 + \beta_{e1} + \beta_{e2} = 1. \quad (10)$$

Transmitting $M_k^{(e)}$ and $M_k^{(U)}$: Over the first $n_U$ channel uses, Tx $k$ sends its eMBB message $M_k^{(e)}$ jointly with its URLLC message $M_k^{(U)}$. To this end, it encodes $M_k^{(e)}$ using a codeword $X_k^{(e)}(M_k^{(e)})$ that is uniformly distributed on the centered $n_U$-dimensional sphere of radius $\sqrt{n_U \beta_{e1} P}$. To encode $M_k^{(U)}$, for each realization $m$ of message $M_k^{(U)}$, $[2 n_U R_U]$ codewords $V_k(m,i)$, $i = 1, \ldots, [2 n_U R_U]$, are drawn uniformly from a centered $n_U$-dimensional sphere of radius $\sqrt{n_U R_U}$ independently of each other and of all other codewords, where

$$r_k := \beta_0 + \alpha_1^2 \beta_{e1} + \alpha_2^2 \beta_{e2}. \quad (11)$$

Tx $k$ then chooses a codeword $V_k(M_k^{(U)}, i)$ such that the sequence

$$X_k^{(U)} := V_k(M_k^{(U)}, i) - \alpha_1 k \ X_k^{(e)}(\beta_{e1}) - \alpha_2 k \ X_k^{(e)}(\beta_{e2}) \quad (12)$$

lies in the set

$$D_k := \begin{cases} x_k^{(U)} : n_U \beta_{e1} P - \delta_k \leq \| x_k^{(U)} \|^2 \leq n_U \beta_{e1} P \end{cases} \quad (13)$$

for a given $\delta_k > 0$. If multiple such codewords exist, one of them is chosen at random, and if no appropriate codeword exists, an error is declared.

Over the first $n_U$ channel uses, Tx $k$ transmits

$$X_k^{(U)} + X_k^{(e)}(\beta) . \quad (14)$$

Over the last $(n_e - n_U)$ channel uses, Tx $k$ simply encodes $M_k^{(e)}$ using a codeword $X_k^{(e)}(M_k^{(e)})$ that is uniformly distributed on the centered $(n_e - n_U)$-dimensional sphere of radius $\sqrt{(n_e - n_U) \beta_{e2} P}.

C. Decoding at Rxs in $K \setminus K_U$

Each Rx $k$ in $K \setminus K_U$ only has an eMBB message to decode. Rx $k \in K \setminus K_U$ decomposes its channel outputs into two output blocks consisting of the first $n_U$ and the last $(n_e - n_U)$ channel uses, respectively. These blocks are of the form:

$$Y_{k,1} = h_{k,k} k X_k^{(e)}(\beta_{e1}) + h_{k,k} X_k^{(e)}(\beta_{e2}) + Z_{k,1}, \quad (15a)$$

$$Y_{k,2} = h_{k,k} X_k^{(e)}(\beta_{e2}) + h_{k,k} X_k^{(e)}(\beta_{e2}) + Z_{k,2}, \quad (15b)$$

where $Z_{k,1}$ and $Z_{k,2}$ are independent i.i.d. standard Gaussian noise sequences. For $Y_{k,1} = y_{k,1}$ and $Y_{k,2} = y_{k,2}$, Rx $k$ estimates $M_k^{(e)}$ as an index $m$ for which the corresponding codewords $x_k^{(e)}(\beta_{e1})(m)$ and $x_k^{(e)}(\beta_{e2})(m)$ maximize the information density

$$i_1(x_k^{(e)}(\beta_{e1}), x_k^{(e)}(\beta_{e2}); y_{k,1}, y_{k,2}) := \ln \frac{\int_{Y_{k,1}} f_{Y_{k,1}| X_k^{(e)}(\beta_{e1})}(y_{k,1}| x_k^{(e)}(\beta_{e1})) f_{Y_{k,2}| X_k^{(e)}(\beta_{e2})}(y_{k,2}| x_k^{(e)}(\beta_{e2}))}{\int_{Y_{k,1}} f_{Y_{k,1}| X_k^{(e)}(\beta_{e1})}(y_{k,1}| x_k^{(e)}(\beta_{e1})) f_{Y_{k,2}| X_k^{(e)}(\beta_{e2})}(y_{k,2}| x_k^{(e)}(\beta_{e2}))}, \quad (16)$$

among all codeword pairs $x_k^{(e)}(\beta_{e1}) = x_k^{(e)}(\beta_{e1})(m')$ and $x_k^{(e)}(\beta_{e2}) = x_k^{(e)}(\beta_{e2})(m')$.

D. Decoding at Rxs in $K_U$

Similarly to the previous subsection, also Rxs in $K_U$ decompose their channel outputs into two output blocks consisting of the first $n_U$ and the last $(n_e - n_U)$ channel uses, respectively. For a Rx $k \in K_U$, these blocks are of the form:

$$Y_{k,1} = h_{k,k} X_k^{(e)}(\beta_{e1}) + h_{k,k} X_k^{(e)}(\beta_{e2}) + Z_{k,1}, \quad (17a)$$

$$Y_{k,2} = h_{k,k} X_k^{(e)}(\beta_{e2}) + h_{k,k} X_k^{(e)}(\beta_{e2}) + Z_{k,2}, \quad (17b)$$

where $Z_{k,1}$ and $Z_{k,2}$ are independent i.i.d. standard Gaussian noise sequences.

1) Decoding $M_k^{(U)}$: Rx $k$ decodes $M_k^{(U)}$ based on the outputs of the first channel inputs $Y_{k,1}$ defined in (17a). Rx $k$ estimates $M_k^{(U)}$ as an index $m$ for which the corresponding codeword $v_k(m, i)$ maximizes the information density

$$i(v_k; y_{k,1}) := \ln \frac{\int_{Y_{k,1}} f_{Y_{k,1}| X_k^{(e)}(\beta_{e1})}(y_{k,1}| v_k) f_{Y_{k,2}| X_k^{(e)}(\beta_{e2})}(y_{k,2}| v_k)}{\int_{Y_{k,1}} f_{Y_{k,1}| X_k^{(e)}(\beta_{e1})}(y_{k,1}| v_k) f_{Y_{k,2}| X_k^{(e)}(\beta_{e2})}(y_{k,2}| v_k)} \quad (18)$$

among all codewords $v_k = v_k(m', j)$.

2) Decoding $M_k^{(e)}$: Rx $k$ decodes $M_k^{(e)}$ based on the channel outputs of the first and second channels $Y_{k,1}$ and $Y_{k,2}$ by looking for the index $m$ for which the corresponding codewords $x_k^{(e)}(\beta_{e1})(m)$ and $x_k^{(e)}(\beta_{e2})(m)$ maximize the information density

$$i_2(x_k^{(e)}(\beta_{e1}), x_k^{(e)}(\beta_{e2}); y_{k,1}, y_{k,2}) := \ln \frac{\int_{Y_{k,1}} f_{Y_{k,1}| X_k^{(e)}(\beta_{e1})}(y_{k,1}| x_k^{(e)}(\beta_{e1})) f_{Y_{k,2}| X_k^{(e)}(\beta_{e2})}(y_{k,2}| x_k^{(e)}(\beta_{e2}))}{\int_{Y_{k,1}} f_{Y_{k,1}| X_k^{(e)}(\beta_{e1})}(y_{k,1}| x_k^{(e)}(\beta_{e1})) f_{Y_{k,2}| X_k^{(e)}(\beta_{e2})}(y_{k,2}| x_k^{(e)}(\beta_{e2}))}, \quad (19)$$

among all codeword pairs $x_k^{(e)}(\beta_{e1})(m')$ and $x_k^{(e)}(\beta_{e2})(m')$.

IV. MAIN RESULT

Fix $\beta_e, \beta_{e1}, \beta_{e2}, \beta_0 \in [0,1]$ such that (10) is satisfied. Define

$$\sigma_1^2 := h_{k,k}^2 (r_k + (1 - \alpha_1 k)^2 \beta_{e1}) + (h_{k,k} - h_{k,k})^2 \beta_{e1} P + 1, \quad (24a)$$

$$\sigma_2^2 := h_{k,k}^2 (r_k + (1 - \alpha_1 k)^2 \beta_{e1}) + (h_{k,k} - h_{k,k})^2 \beta_{e1} P + 1, \quad (24b)$$

$$\sigma_3^2 := h_{k,k} (1 - \beta_{e1}) + h_{k,k}^2 \beta_{e1} P + 1, \quad (24c)$$

$$\sigma_4^2 := (h_{k,k}^2 \beta_{e2} + h_{k,k}^2 (1 - \beta_{e1})) + 1, \quad (24d)$$

$$c_1 := (h_{k,k} \sqrt{\beta_e} + h_{k,k}^2 (\sqrt{\beta_0} + \sqrt{\beta_{e1}})) \quad (24e)$$
By employing the scheme proposed in Section III, we have the following theorem on the upper bounds on the average URLLC and eMBB error probabilities $\epsilon_u$ and $\epsilon_e$.

**Theorem 1:** For fixed message set sizes $L_u$ and $L_e$, the average error probabilities $\epsilon_u$ and $\epsilon_e$ are bounded by

$$
\epsilon_u \leq \frac{2}{K} \sum_{k \in K_u} \left(1 - F(u_{k,2} - u_{k,1}) + F(-u_{k,2} - u_{k,1})
+ (1 - \max\{\mathcal{L}_{k,1}, \mathcal{L}_{k,2}\})^{2n_u R_e} \right) + L_u [2^{n_u R_e}] e^{-n_u},
$$

\(25\)

where

$$
\epsilon_e \leq \frac{1}{K} \sum_{k \in K_e} \left(\gamma_{k,1} \epsilon_{k,1} + \gamma_{k,2} \epsilon_{k,2} + n_u l_3 + (n_e - n_u) l_4\right)
+ \frac{1}{K} \sum_{k \in K_e} \left(\gamma_{k,2} \epsilon_{k,1} + \gamma_{k,2} \epsilon_{k,2} + n_d d_3 + (n_e - n_u) d_4\right)
+ L_e \left(e^{\gamma_{k,1} - 1} + e^{-\gamma_{k,2}}\right),
$$

\(26\)

for any $\gamma_u$, $\gamma_{e,1}$ and $\gamma_{e,2}$, and where

$$
u_{k,1} := \sqrt{n_u P} \left(c_2 + \frac{h_{k,k\sqrt{\beta_{\epsilon}}}^2}{\sigma_1^2} \right),
$$

\(27\)

$$
u_{k,2} := \sqrt{\sigma_1^2 \left(\frac{1}{\sigma_1^2 - 1} \left(n_u \ln(\sigma_1^2) - 2\gamma_1 + \frac{n_u P r_{k,h_{k,k}}}{\sigma_2^2}\right)\right)},
$$

\(28\)

$$
l_{k,1} := 2 \sqrt{n_u P} \left(c_1 + h_{k,k\sqrt{\beta_{\epsilon}}} - c_1^2\right),
$$

\(29\)

$$
l_{k,2} := 2 \sqrt{(n_e - n_u) P} \left(c_3 + h_{k,k\sqrt{1 - \beta_{\epsilon}}} - c_3^2\right)
+ 2 \sqrt{n_u P} \left(c_2 + h_{k,k\sqrt{\beta_{\epsilon}}} - c_2^2\right),
$$

\(30\)

and

$$
\gamma_{k,1} := \frac{d_1}{\sigma_1^2 - 1},
$$

\(32\)

$$
\gamma_{k,2} := \frac{d_2}{\sigma_1^2 - 1},
$$

\(33\)

where $c_{1,2}$ are defined in (20) to (23). $J_0$ is defined in (46), $J_{e,1}$ and $J_{e,2}$ are defined in equations (79) and (96) of [21], and $F(\cdot)$ represents the cumulative distribution function (CDF) of a chi distribution of degree $n_u$.

**Proof:** See Appendix A for the proof of the bound in (25) and [21] for the proof of the bound in (26).

In Figure 2, we numerically compare the bounds in Theorem 1 with the time-sharing scheme where only Txs in $K_u$ send URLLC messages over $n_u$ channel uses whereas all the Txs, including Txs in $K_u$, send eMBB messages but over only the remaining $n_e - n_u$ channel uses. In this plot, the value of $n_u$ varies from $90$ to $10$ with step size $10$, while the value of $n_e$ is fixed at $100$. In our simulations, the values of the parameters $\beta_{\epsilon}$, $\beta_0$, $\beta_{e,1}$, $\beta_{e,2}$, $\alpha_{k,1}$, and $\alpha_{k,2}$ are optimized to minimize $\epsilon_e$ for a given $\epsilon_u$. As can be seen from this figure, our scheme outperforms the time-sharing scheme.

**V. CONCLUSIONS**

We considered Wyner's soft-handoff model where transmitters simultaneously send eMBB and URLLC messages of heterogeneous blocklengths. We proposed a coding scheme to jointly transmit URLLC and eMBB messages in such a network. We derived rigorous upper bounds on the error probability of eMBB and URLLC transmissions. Our numerical analysis
showed that the proposed scheme significantly improves over the standard time-sharing.

An interesting future line of work is to study this network under the assumption that \( n_e \) is much larger than \( n_U \). This assumption allows the eMBB transmissions to benefit from their delay-tolerance feature. Another interesting scenario is to let all the Txs send URLLC messages which requires dealing with the interference from the URLLC messages on the URLLC transmissions as well. Finally, for more complex networks, employing rate-splitting multiple access based schemes [22] which allows suitable users to split messages depending on the service requirements is also of interest.

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**APPENDIX A**

**PROOF OF THEOREM 1**

In the following, we sketch the proof of the achievability bound (25). See [21] for full details and for the proof of the bound in (26).

**A. Bounding \( \epsilon_{U,k} \)**

We start by bounding the decoding error probability of a URLLC message at a given Rx \( k \) in \( K_U \). Define the decoding error event \( \mathcal{E}^{(U)}_{k,U} := \{ M^{(U)}_k \neq M^{(U)}_i \} \) and \( \mathcal{E}_{k,\ell} \) be the encoding error event that no appropriate codeword \( V_k(M^{(U)}_i, i) \) can be found so that \( X^{(U)}_k(M^{(U)}_i) \in D_k \). Then we have

\[
\epsilon_{U,k} \leq \mathbb{P}[\mathcal{E}_{k,\ell}] + \mathbb{P}[\mathcal{E}^{(U)}_{k,U} | \mathcal{E}_{k,\ell}].
\]  

(33)

1) **Analyzing \( \mathbb{P}[\mathcal{E}^{(U)}_{k,U} | \mathcal{E}_{k,\ell}] \):** To calculate this probability, from (13) we notice that \( V_k - \alpha_{k,1}X^{(e,1)}_k - \alpha_{k,2}X^{(e,1)}_{k-1} \in D_k \) if and only if

\[
n_U (r_k - \beta_0)P - \delta_k \leq ||V_k - \alpha_{k,1}X^{(e,1)}_k - \alpha_{k,2}X^{(e,1)}_{k-1}||^2 \leq n_U \beta_0 P.
\]

Recall that \( ||V_k||^2 = n_U r_k P \) almost surely. Thus event \( \mathcal{E}_{k,\ell} \) holds except when the following condition is satisfied.

\[
n_U (r_k - \beta_0)P + ||\alpha_{k,1}X^{(e,1)}_k + \alpha_{k,2}X^{(e,1)}_{k-1}||^2 \\
\leq 2\alpha_{k,1} ||V_k, X^{(e,1)}_k + \alpha_{k,2}X^{(e,1)}_{k-1}|| + \delta_k
\]

(34)

Equation (34) is equivalent to

\[
C_k \leq \langle V_k, X^{(e,1)}_k \rangle \leq C_k + \frac{\delta_k}{2\alpha_{k,1}}.
\]  

(35)

where

\[
C_k := \frac{n_U (r_k - \beta_0) P}{2\alpha_{k,1}} - \frac{\alpha_{k,2}}{\alpha_{k,1}} \langle V_k, X^{(e,1)}_{k-1} \rangle \\
+ \frac{||\alpha_{k,1}X^{(e,1)}_k + \alpha_{k,2}X^{(e,1)}_{k-1}||^2}{2\alpha_{k,1}}.
\]  

(36)

Since \( X^{(e,1)}_k \) is drawn uniformly from the sphere, the distribution of \( \langle V_k, X^{(e,1)}_k \rangle \) depends on \( V_k \) only through its magnitude, this is seen by noting that the inner product of two vectors is unchanged when an orthogonal transformation is applied to both arguments, and the distribution of \( X^{(e,1)}_k \) is unchanged under any orthogonal transformation. Thus, assuming \( V_k = (||V_k||, 0, \ldots, 0) \) and following the same arguments as in [12, Appendix E], we prove that

\[
\mathbb{P}[\mathcal{E}^{(U)}_{k,U} | \mathcal{E}_{k,\ell}] \leq |L_{U,2}^{(k,2)}| = 2^{m_0 R_x}.
\]

Since the \([2^{m_0 R_x}]\) codewords are generated independently,

\[
\mathbb{P}[\mathcal{E}_{k,\ell}] \leq (1 - \max(L_1, L_2)) \cdot 2^{m_0 R_x}.
\]

(37)

(38)

See [21] for the detailed proof.

2) **Analyzing \( \mathbb{P}[\mathcal{E}_{k,U} | \mathcal{E}_{k,\ell}] \):** To evaluate this error event, we use the threshold bound for maximum-metric decoding.

\[
\mathbb{P}[\mathcal{E}_{k,U} | \mathcal{E}_{k,\ell}] \leq \mathbb{P}[i(V_k; Y_{k,1}) \leq \gamma_0] \\
+ L_{U}^{(2^{m_0 R_x}, 1)} \cdot \mathbb{P}[i(V_k; Y_{k,1}) > \gamma_0]
\]

(39)

for any \( \gamma_0 \), where \( V_k \sim f_{V_k} \) and is independent of \( (V_k, Y_{k,1}) \).

We start by calculating \( \mathbb{P}[i(V_k; Y_{k,1}) > \gamma_0] \):

\[
= \int_{\tilde{V}_k} \mathbb{P}[i(\tilde{V}_k; Y_{k,1}) > \gamma_0] \\
\times \exp(-i(\tilde{V}_k; Y_{k,1})) f_{V_k}(Y_{k,1}| \tilde{V}_k) d\tilde{V}_k
\]

(40)

Now we calculate \( \mathbb{P}[i(V_k; Y_{k,1}) \leq \gamma_0] \). Note that \( Y_{k,1} \) and \( V_k; Y_{k,1} \) do not follow a Gaussian distribution. We however define equivalent Gaussian distributions \( Q^{(U)}(y_{k,1}) = N(y_{k,1}; 0, I_{k,1} \sigma^2_{v}) \) and \( W^{(U)}(y_{k,1}| v_k) = N(y_{k,1}; h_{k,k} V_k, I_{k,1} \sigma^2_{v}) \) where \( \sigma^2_{v} \) is defined in (24). We also introduce

\[
\tilde{i}(v_k; y_{k,1}) := \ln \frac{W^{(U)}(y_{k,1}| v_k)}{Q^{(U)}(y_{k,1})}.
\]

(44)

**Lemma 1:** It holds that

\[
\frac{\tilde{i}(v_k; y_{k,1})}{\tilde{i}(v_k; y_{k,1})} \geq J_U
\]  

(45)
where
\[ J_U := (n_U - 2) \ln(2a_1a_2) \]
\[-2nuP(a_1^2\beta_e,1 + a_2^2\beta_e) - \frac{e^{\gamma_U}a_1^2\beta_eP}{\sqrt{2\pi a_1^2\beta_e,1P}} - \kappa, \]  
(46)
where \( a_1 = h_{k,k}(1 - \alpha_{k,1}), a_2 = h_{k,k-1} - h_{k,k}\alpha_{k,2}, \kappa = \ln(\frac{1}{2}) + c_1 + \ln(\sqrt{\frac{V}{n}}) - 2\ln(h_{k,k}) \) with \( c_1 \leq 2 \).

**Proof:** See [21].

As a result, we have:
\[ P[i(V_k; Y_{k,1}) \leq \mu_U] \]
\[ \leq P[i(V_k; Y_{k,1}) \leq \gamma_U] \]
\[ = P \left[ \ln \left( \frac{1}{(\sqrt{2\pi})^{n_U}} \exp \left( -\frac{||Y_{k,1} - h_{k,k}V_k||^2}{2\sigma_{V_k}^2} \right) \right) \leq \frac{\gamma_U}{J_U} \right] \]
\[ \leq P \left[ \frac{h_{k,k}^2n_U\beta_eP + ||Z_{k,1}||^2 + 2h_{k,k}n_U\beta_e||Z_{k,1}||}{\beta_e,1}\right] \]
\[ + 2h_{k,k}n_U\beta_e||Z_{k,1}|| + 2h_{k,k-1}n_U\beta_e||Z_{k,1}|| \]
\[ + 2h_{k,k}n_U\beta_e||Z_{k,1}|| + 2h_{k,k-1}n_U\beta_e||Z_{k,1}|| \]
\[ \geq \gamma_U - n_U P(h_{k,k}^2\beta_e,1 + h_{k,k-1,1}\beta_e) \]
\[ = P \left[ ||Z_{k,1}||^2 + b_1||Z_{k,1}|| \geq \gamma_U \right] \]
\[ = 1 - F \left( \sqrt{\frac{b_1^2}{2}} - \frac{b_1}{2} \right) + F \left( \frac{\gamma_U + \frac{b_1^2}{4} - \frac{b_1}{2}}{2} \right) \]
(51)
where
\[ \gamma_U := \frac{\sigma_{Z,1}^2}{\sigma_{V_k}^2 - 1} \left( n_U \ln(\sigma_{Z,1}^2) - \frac{2\gamma_U}{J_U} - h_{k,k}r_kn_U\beta_e \right) \]
(53a)
\[ \gamma_U := \gamma_U - n_U P c_2^2 - \frac{2h_{k,k}^2n_U\beta_e||Z_{k,1}||}{\sigma_{Z,1}^2} \]
(53b)
\[ b_1 := 2\sqrt{n_U}P \left( c_2 + \frac{h_{k,k}r_k\sigma_{Z,1}^2}{\sigma_{Z,1}^2 - 1} \right) \]
(53c)
where \( c_2 \) is defined in (24f). Note that \( ||Z_{k,1}|| \) follows a chi distribution of degree \( n_U \) and \( F(\cdot) \) is its corresponding CDF.

By defining \( u_{k,1} := \frac{b_1}{2} \) and \( u_{k,2} := \sqrt{\gamma_U + \frac{b_1^2}{4}} \) and combining this bound with the bound in (43), we proved upper bound (25).

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