The Kalmag model as a candidate for IGRF-13

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Abstract

We present a new model of the Geomagnetic field spanning the last 20 years and called Kalmag. Deriving from the assimilation of CHAMP and SWARM vector field measurements, it separates the different contributions to the observable field through parameterized prior covariance matrices. To make the inverse problem numerically feasible it has been sequentialized in time though the combination of a Kalman filter and a smoothing algorithm. The model provides reliable estimates of past, present and future mean fields and associated uncertainties. The version presented here is an update of our IGRF candidates, the amount of assimilated data has been doubled and the considered time window has been extended from [2000.5, 2019.74] to [2000.5, 2020.33].

Keywords

Geomagnetic field, secular variation, assimilation, Kalman filter, machine learning

Introduction

The Earth’s magnetic field has different sources. Classically, we distinguish internal and external sources below and above the site where the field is measured. The three principal internal contributions are the core field, the lithospheric field and the magnetic fields induced in the ocean or within the crust and upper mantle. At large scales, the core field clearly dominates. Sustained by dynamo action in the liquid outer core, it is predominantly dipolar, and varies on timescales ranging from months to millennia. Magnetic field reversals, rare events on even longer times scales, are not considered here. On scales that correspond to spherical harmonics (SH) degree beyond about 16, the core field is dominated by the lithospheric field coming from the magnetized rocks of the crust. Since the Earth’s mantle evolves extremely slowly, the lithospheric field, can be considered as almost static. The external fields that are generated by electrical currents in the ionosphere and in the magnetosphere, on the other hand can vary extremely rapidly in time. Because the sources of the magnetospheric field (the ring current, the magnetopause and magnetotail currents) are distant from the Earth, only its large scale contributions can be detected by the low-orbiting magnetic satellites or by measurements on Earth’s surface. This is not the case for the ionospheric field which is generated closer to the Earth’s surface. Because the dynamical behavior
of both ionospheric and magnetospheric fields is controlled by solar radiations (and also thermospheric
winds for the former), they are closely tied to solar activity, and can vary on very short timescales. The
external fields can therefore also induce a non negligible secondary field inside the electrically conducting
parts of the mantle, crust and oceans. Other potentially important induced fields are created because
the oceans move relative to the core fields.

Disentangling the different field contributions is a difficult task since they often overlap in spatial scale
and time scale. Many field models therefore resort to a regularization in space and time and only use
selected data. Some example are the CHAOS model series by Olsen et al. (2006); Finlay et al. (2016),
the comprehensive models by Sabaka et al. (2002, 2015, 2018, 2020), the GRIMM models by Lesur et al.
(2008, 2010, 2015), the POMME models by Maus et al. (2005, 2010), or the gufm1 by (Jackson et al.
(2000)). The COV-OBS model by (Gillet et al. (2013)) and the model recently proposed by Ropp et al.
(2020) are the only models that use a Bayesian approach instead of regularization. Usually only vector
field measurements taken during night time, under geomagnetic quiet conditions, and at low to mid
magnetic latitude are considered in order to minimize the contribution of fields created by currents in the
ionosphere or in auroral regions (field-aligned currents, DP2, auroral electrojet). Four main contributions
then remain, the core, the lithospheric, the magnetospheric and the induced fields. The core and the
lithospheric fields are usually treated as one internal source described by one set of spherical harmonics
coefficients. Small scale contributions beyond degree 16 or so are supposed to be of lithospheric origin
and are static in time while the larger scales represent the varying core field. Most of the models also
treat the induced fields and the magnetetospheric field in a simplified way. The field induced by ocean
circulation and the the fields created in the magnetotail and the magnetopause are either neglected or
estimated separately. Only the magnetic field generated by the ring current and the respective induced
part then remain to be modeled. Yet, when assuming a 1d electrical conductivity profile for Earth’s
mantle, the axisymmetric magnetospheric field and the related induced field can be parameterized by the
Disturbance short-time (Dst) index proposed by Sugiuara (1963). The Dst or other similar indices are
independently estimated from observations and can serve as model input.

To achieve an optimal separation of the different contributions, proper temporal parametrization of
the different sources is mandatory. In many models (CHAOS, GRIMM, CM, COV-OBS), the time
dependency of the core field is modeled by B-splines for an a priorily fixed time step. This imposes a
relatively smooth evolution of the field, excluding the rapid variations attributed to external fields. In
addition, with algorithms based on regularized least square approaches, only the time derivatives of the core field are penalized and no constraints on the morphology of the field itself are imposed. With the COV-OBS model, Gillet et al. (2013) went a step further in characterizing a priori the spatio-temporal behavior of the core field. They assumed that its dynamical evolution was controlled by a specific second order auto regressive process which can reproduce the temporal statistical properties of the core field which have been characterized with both observatory measurements (see De Santis et al. (2003); Lesur et al. (2017)) and numerical simulations of the geodynamo (see Bouligand et al. (2016)). Although a good calibration of the temporal constraints is crucial for deriving magnetic field models from observatory and satellite data, some key information can also be extracted from the morphology and the spatial correlation structure of the different fields. Holschneider et al. (2016) have shown that the use of appropriate parameterized correlation kernels, could greatly improve the separation of the different components of the Earth’s magnetic field. Working with observatory data for a single epoch, they could detect the spatial signature of the core, lithospheric, magnetospheric and ionospheric fields. Their Bayesian approach even allowed the quantification of uncertainties.

The Kalmag model we propose here combines such a technique with the sophisticated temporal correlation functions introduced by Gillet et al. (2013). Since ground based observatories and satellite missions such as Oersted, SAC-C, Champ, or Swarm, have produced or are still generating a huge amount of data, a block inversion would be numerically impossible. This is why we decided to assimilate the data sequentially using a Kalman filter approach combined with a smoothing algorithm.

The article is organised as follows: in the next section, the data selection criteria and the modeling strategy are detailed. We first present the different magnetic sources that are taken into account, we then show how they are a priori characterized, and how such prior information can be modeled through auto regressive processes. Based on these processes, the equations for the Kalman filter approach and the smoothing algorithm are then given. Finally the methodology used to derive the different candidate models for IGRF-13 is explained. In section Results and discussion, we present and discuss the outcomes of the model. However, since the spatio-temporal prior characterization of each modeled magnetic source is parameterized, we first show how these parameters are evaluated to be then incorporated in the model.

The article ends with some concluding remarks and perspectives to improve the Kalmag model.
Methods

Data

For the moment, the Kalmag model only uses the vector field measurements of the CHAMP and SWARM low orbiting satellites. We sample CHAMP data at a rate of 1 datum every 5 seconds and only use measurements where the vector field magnetometer (VFM) and the star tracker (STR) instruments were functioning nominally. Very early in the SWARM mission (September 2014), the scalar magnetometers on satellite Charlie stopped operating properly. We therefore only consider data from the Alpha and Bravo satellites, using simultaneous sampling every 10 seconds. For the construction of the IGRF-13 candidate models, which we will refer to as the Kalmag candidates, a two times lower sampling rate was used. Furthermore, we only used data up to 2019.74 for the Kalmag candidates but now extended this to 2020.33.

For latitudes between 60° north and south, only night time data are considered. Furthermore, independently of the satellites locations, the following selection criteria are also applied:

- The $z$-component of the Interplanetary magnetic field (IMF) is positive.
- The geomagnetic activity index $Kp \leq 2^0$.

All in all the dataset is composed of 2,985,442 vector field measurements for CHAMP and 4,606,159 for SWARM.

Magnetic sources

The different contributions to the observations are described in terms of magnetic sources of either internal or external origin. Except for the field produced by field-aligned currents ($b_{fac}$), each of these contributions $b_i$ is deriving from a potential $V_i$:

$$b_i(r, \theta_s, \phi_s, t) = -\nabla V_i(r, \theta_s, \phi_s, t) .$$  \hspace{1cm} (1)

For $b_{fac}$, we followed the study of Waters et al. (2001) and express it through the potential $V_{fac}$ as following:

$$b_{fac}(r, \theta_s, \phi_s, t) = -r \times \nabla V_{fac}(r, \theta_s, \phi_s, t) .$$  \hspace{1cm} (2)

Note that depending on the source, the spherical coordinate system $\{r, \theta_s, \phi_s\}$ the magnetic field is expressed in may differ. Here four types of systems are used: geographic (GEO), magnetic (MAG), solar magnetic (SM), and geocentric solar magnetospheric (GSM).

Each potential $V_i$ is then expanded in spherical harmonics (SH), which for internal and external sources
respectively read:

\[ V_i^I(r, \theta_s, \phi_s, t) = a_i \sum_{\ell \leq \ell_{\text{max}}} \sum_{m=-\tilde{m}}^{m=\tilde{m}} \left( \frac{a_i}{\ell+1} \right)^i g^I_{i,\ell,m}(t) Y_{\ell,m}(\theta_s, \phi_s), \tag{3} \]

\[ V_i^E(r, \theta_s, \phi_s, t) = a_i \sum_{\ell \leq \ell_{\text{max}}} \sum_{m=-\tilde{m}}^{m=\tilde{m}} \left( \frac{r}{a_i} \right)^i g^E_{i,\ell,m}(t) Y_{\ell,m}(\theta_s, \phi_s). \tag{4} \]

The \( Y_{\ell,m} \) are Schmidt semi-normalized spherical harmonics of degree \( \ell \) and order \( m \), \( \ell_{\text{max}} \) is the maximum of degree expansion, \( a_i \) is a reference radius, and \( g_{i,\ell,m}(t) \) (later referred as \( g_i \)) are the spherical harmonics coefficients expressed at \( a_i \). \( \tilde{m} \) is the maximum order considered for the spherical harmonics expansion.

A complete expansion, referred as standard, requires \( \tilde{m} = \ell \). However some sources, in particular external fields, are known to have a strong zonal signature (see Finlay et al. (2017)), and are therefore restricted to either zonal spherical harmonics modes with \( \tilde{m} = 0 \) or to an expansion we refer as zonal iso where \( \tilde{m} = 1 \).

Table 1. Magnetic sources considered in the model. The second column corresponds to the coordinate system each field is expressed in. GEO stands for geographic, SM for solar magnetic, MAG for magnetic and GSM for geocentric solar magnetotetospheric. \( \ell_{\text{max}} \) is the maximum degree of the SH expansion, for the three following types of decomposition: standard with \( m = [-\ell, \ell] \), zonal with \( m = 0 \) and zonal iso where \( m = \{0, 1, -1\} \).

| Source                  | Coordinate | \( \ell_{\text{max}} \) | SH decomposition |
|-------------------------|------------|-------------------------|-----------------|
| Core \( g_c \)          | GEO        | 20                      | Standard        |
| Lithospheric \( g_l \)  | GEO        | 76                      | Standard        |
| Remote magnetospheric \( g_{rm} \) | GSM        | 1                       | Zonal           |
| Close magnetospheric \( g_m \) | SM         | 15                      | Zonal           |
| Fluctuating magnetospheric \( g_{fm} \) | SM         | 15                      | Zonal iso       |
| Residual ionospheric/ induced \( g_{ii} \) | MAG        | 50                      | Zonal iso       |
| Field-aligned currents \( g_{fac} \) | SM         | 15                      | Zonal iso       |

The Kalmag model is composed of 7 sources. 3 of them are of internal origin, the core field \( (g_c) \), the lithospheric field \( (g_l) \) and the induced / residual ionospheric field \( (g_{ii}) \). \( g_c \) and \( g_l \) are expressed in the geographic coordinate system and expanded in SH with the standard decomposition. \( g_{ii} \) is expressed in...
the solar magnetic coordinate system and its SH decomposition is restricted to $\tilde{m} = 1$ (zonal iso). 3 sources are used to characterize the magnetospheric field. A remote one ($g_{rm}$) in GSM which is purely dipolar and zonal, and 2 close sources ($g_m$ and $g_{fm}$) expressed in the SM coordinate system. $g_m$ is purely zonal and it is accompanied by $g_{fm}$ a fluctuating source expanded with the zonal iso SH decomposition. Finally, the source associated with field-aligned currents is expressed in the SM coordinate system and restricted to the zonal iso SH expansion. The nature of the 7 sources composing the Kalmag model, the coordinate system they are expressed in, and their spherical harmonics truncation level are listed in table 1.

Prior characterization of spatial and temporal correlations

To obtain an optimal separation of the various contributions to geomagnetic observations, proper prior characterization of the different magnetic sources is mandatory. Following the studies of Hulot and Le Mouël (1994); Gillet et al. (2013); Holschneider et al. (2016), full space-time covariance matrices are used to characterize each magnetic source $g_i$. Assuming that $E[g_i] = 0$ the latter read:

$$E \begin{pmatrix} g_i(t) \\ g_i(t + \Delta t) \end{pmatrix} \begin{pmatrix} g_i(t)^T \\ g_i(t + \Delta t)^T \end{pmatrix} = \begin{pmatrix} \Sigma_{g_i} & c_i(\Delta t)\Sigma_{g_i}^\infty \\ \Sigma_{g_i}^\infty c_i(\Delta t)^T & \Sigma_{g_i}^\infty \end{pmatrix},$$

(5)

where the matrix $\Sigma_{g_i}^\infty$ corresponds to the stationary spatial covariance, and $c(\Delta t)$ is a temporal correlation matrix depending on the time lag $\Delta t$. $\Sigma_{g_i}^\infty$ is assumed to derive from energy spectra $E_i^\infty(\ell, a_i)$, expressed at given radii $a_i$, such as:

$$\Sigma_{g_i}^\infty(\ell, m, \ell', m', r = a_i) = E[g_i(\ell, m)g_i(\ell', m')] = E_i^\infty(\ell, a_i) \frac{N_m}{F(\ell)} \delta(\ell - \ell')\delta(m - m')$$

(6)

where $N_m$ is the number of modeled spherical harmonics coefficients per degree $\ell$, and $F$ is the pre-factor of the energy spectra given by $F(\ell) = \ell + 1$ and $F(\ell) = \ell$ for internal and external sources respectively.

Two types of spectra are used for the model, flat ones, with $E_i^\infty(\ell) = A_i^2$ where $A_i$ is a magnitude, and spectra of the form $E_i^\infty(\ell) = A_i^2(2\ell + 1)F(\ell)$, referred as C-based spectra, making equation 6 equivalent to the correlation kernels proposed by Holschneider et al. (2016). Only 2 sources are characterized by a flat spectrum, the core field and the induced / residual ionospheric field. This choice was driven by the evaluation we performed in section Parameter estimation where such a parametrization enabled us to better explain the data.

Note that the covariance matrices of equation 6 are diagonal. More complex covariance structures could
be used, accounting for the correlations between different magnetic modes as they appear in dynamo simulations for instance (see Sanchez et al. (2019)). However, the large amount of data available for this study are sufficient to properly constrain the model and permits such general prior assumptions.

Temporal constraints are prescribed by the type of correlation functions introduced by Gillet et al. (2013, 2015) in the context of geomagnetic modeling. They are deriving from the auto-regressive processes further discussed below, and read:

\[ c_i(\Delta t) = \exp \left[ -|\Delta t| / \tau_i(\ell) \right] \]  

(7)

for first order processes and:

\[ c_i(\Delta t) = \left( 1 + \left( |\Delta t| / \tau_i(\ell) \right) \right) \exp \left[ - (|\Delta t| / \tau_i(\ell)) \right] \]  

(8)

for second order processes. \( \tau_i(\ell) \) are scale dependent characteristic timescales. For the core field, Christensen and Tilgner (2004); Lhuillier et al. (2011) have shown that its characteristic timescales \( \tau_c(\ell) \) could be approximated by a power law such as \( \tau_c(\ell) = \tau_{SV} \ell^{-1} \) with \( \tau_{SV} \) the secular variation timescale. In this study we decided to use such a power law description of \( \tau_i \) for each magnetic source except for the lithospheric field, leading to:

\[ \tau_i(\ell) = M_i \ell^{-\alpha_i} \]  

(9)

with amplitudes \( M_i \) and exponent \( \alpha_i \). In our parametrization of the problem we therefore use four main parameters to characterize each source: 1) the amplitude \( A_i \), 2) the (virtual) source radius \( a_i \), 3) the time scale amplitude \( M_i \) and 4) the time scale slope \( \alpha_i \). In addition, because of the specific behavior of the dipole components, for the core field (see Christensen and Tilgner (2004); Lhuillier et al. (2011)) but also for the magnetospheric sources (see Sugiuara (1963); Finlay et al. (2016)), the spatial and temporal properties of each source’s dipole (except for the lithospheric field) is treated separately from the remaining SH coefficients. These parameters are directly estimated with a subsample of the dataset following the procedure described in section Parameter estimation.

Sequentialization

For the following developments, the parameters characterizing the prior covariance structures (\( A_i \), \( a_i \), \( M_i \) and \( \alpha_i \)) are assumed to be known. Instead of performing a full Bayesian block inversion with the covariance matrices given by equation 5 as a prior information, we proceed in a recursive way through the Kalman filter approach proposed by Kalman (1960). To do so, dynamical equations are required
to forecast the statistical properties of the different modeled sources. As previously mentioned, the covariance structures we wish to a priori impose are deriving from autoregressive processes. In their continuous form the latter are given for first and second orders by respectively:

$$\begin{align*}
\partial_t g_{i,\ell,m}(t) + \frac{1}{\tau_i(\ell)} g_{i,\ell,m}(t) &= \sigma_{i1}(\ell) \dot{\omega}_{i1}(t) \\
\partial_t^2 g_{i,\ell,m}(t) + \frac{2}{\tau_i(\ell)} \partial_t g_{i,\ell,m}(t) + \frac{1}{\tau_i^2(\ell)} g_{i,\ell,m}(t) &= \sigma_{i2}(\ell) \dot{\omega}_{i2}(t)
\end{align*}$$

(10)

(11)

where $\dot{\omega}_{i1}(t)$ and $\dot{\omega}_{i2}(t)$ are Gaussian white noises scaled by the factors $\sigma_{i1}(\ell)$ and $\sigma_{i2}(\ell)$ respectively.

These equations have explicit solutions which satisfy:

$$z_i(t + \Delta t) = F_i(\Delta t) z_i(t) + \xi_i(t, \Delta t)$$

(12)

where the temporal Gaussian white noise $\xi_i$ is spatially (in terms SH coefficients) characterized by the distribution $\mathcal{N}(0, \Sigma^\infty_{z_i} - F_i(\Sigma^\infty_{z_i} F_i^T))$, and where $\Delta t$ can either be positive or negative.

For magnetic sources characterized by first order auto regressive processes, $z_i = g_i$ and $F_i$ is given by:

$$F_i(\ell, \Delta t) = \exp\left[-|\Delta t|/\tau_i(\ell)\right].$$

(13)

The core field evolution is prescribed by a second order auto regressive process, so the field itself and the secular variation are dynamically tied together. In this case, $z_i = (g_i, \partial_t g_i)^T$ and:

$$F_i(\ell, \Delta t) = \begin{pmatrix}
1 + |\Delta t|/\tau_i(\ell) & \Delta t \\
-\Delta t/\tau_i^2(\ell) & 1 - |\Delta t|/\tau_i(\ell)
\end{pmatrix} \exp\left[-|\Delta t|/\tau_i(\ell)\right].$$

(14)

Note that the covariance matrix of the core field when the later is assumed to be in a stationary state is given by:

$$\Sigma^\infty_{g,\partial_t g} = \begin{pmatrix}
\Sigma^\infty_{g_c} & 0 \\
0 & \Sigma^\infty_{g_c}/\tau_i^2(\ell)
\end{pmatrix},$$

(15)

as shown by Hulot and Le Mouël (1994).

**Sequential assimilation**

The Kalmag model consists in a vector $z$ containing the spherical harmonics coefficients of every magnetic source (including the SH expansion of the secular variation). With the decomposition detailed in section Magnetic sources, $z$ contains 6624 SH coefficient entries. Its evaluation is performed with a Kalman filter algorithm which proceeds sequentially in two steps. In the first step, the forecast, the evolution of the mean model $E[z]$ together with its associated covariance matrix $\Sigma_z$ are predicted until observations
become available. In the second step, namely the analysis, the model is corrected to better reflect the data through a Bayesian inversion.

To predict the simultaneous evolution of the different magnetic sources with the auto-regressive processes presented in the previous section, a matrix $\mathbf{F}$ containing all the matrices $F_i$, and a matrix $\tilde{\Sigma} = \Sigma^\infty - \mathbf{F} \Sigma^\infty \mathbf{F}^T$ characterizing the white noise of the complete evolution model, are constructed. The evolution of the mean model and its covariance from time step $k - 1$ to step $k$ is then given by the forecast:

$$E[z_{k|k-1}] = \mathbf{F}_{k-1} E[z_{k-1}]$$

$$\Sigma_{z_{k|k-1}} = \mathbf{F}_{k-1} \Sigma_{z_{k-1}} \mathbf{F}_{k-1}^T + \tilde{\Sigma}.$$  \hspace{1cm} (16)

At iteration $k$, whenever measurements are available, the model is updated with the formulations:

$$K_k = \Sigma_{z_{k|k-1}} \mathbf{H}_k^T \left( \mathbf{H}_k \Sigma_{z_{k|k-1}} \mathbf{H}_k^T \right)^{-1}$$

$$E[z_{k|d_k}] = E[z_{k|k-1}] + K_k \left( d_k - \mathbf{H}_k E[z_{k|k-1}] \right)$$

$$\Sigma_{z_{k|d_k}} = \left( I - K_k \mathbf{H}_k \right) \Sigma_{z_{k|k-1}}$$  \hspace{1cm} (18)

where $K_k$ is the Kalman gain matrix and $\mathbf{H}_k$ is the operator projecting the model to the data $d_k$ at iteration $k$. Note that the time step of the algorithm has been set to $\Delta t = 30$ minutes. Within this time window most of the magnetic sources are assumed to be static. However, the spatio-temporal correlations of the FAC source as well as the non dipolar part of the close magnetospheric field are modeled within this time window for reasons detailed in section Parameter estimation.

**Smoothing**

With the Kalman filter algorithm, one gets access to the distribution $p(z_k|d_k)$, where $d_k$ corresponds to all the measurements up to iteration $k$. To obtain $p(z_k|d)$ the posterior distribution of the model at iteration $k$ given the entire dataset $d$, one can apply a smoothing algorithm. In this study we chose the formulation of Rauch et al. (1965). Starting at the last iteration of the Kalman filter algorithm, the smoothing algorithm performs iteratively backward in time accordingly to the following steps:

$$G_{k-1} = \Sigma_{z_{k-1}|d_k} \mathbf{F}_k^T \Sigma_{z_{k|k-1}}^{-1}$$

$$E[z_{k-1|d}] = E[z_{k-1|d_k}] + G_k \left( E[z_k|d] - E[z_{k|k-1}] \right)$$

$$\Sigma_{z_{k-1|d}} = \Sigma_{z_{k-1|d_k}} + G_{k-1} \left( \Sigma_{z_{k}|d} - \Sigma_{z_{k|k-1}} \right) G_{k-1}^T.$$  \hspace{1cm} (21)
The combination Kalman filter-smoothing algorithm was also chosen by Ropp et al. (2020) for their IGRF-13 main field candidate. However, their approach differs from ours in many aspects. In particular, their core field evolution is prescribed by an Euler scheme and they estimate the secular variation through the fluctuation of the field within 3 month time windows. Here the secular variation is tied to the core field evolution through the AR2 process. Its evaluation is therefore achieved by through dynamical link and correlation with the core field.

**Candidate models**

The models that we proposed as candidates for the IGRF-13 in 2020.0 are the Kalman filter solutions after the last analysis step in 2019.74 (September the 27th). This solution was forwarded in time until 2020.0 using the forecast of equations 16 and 17 with propagators $F$ and noise covariance $\Sigma$ for a time step of $\Delta t = 0.26\, \text{yr}$. The secular variation candidate is the mean secular variation estimation in 2020.0. The associated uncertainties were obtained by taking the square root of the diagonal elements of the covariance matrix $\Sigma_{\partial g_c}(t = 2020)$, providing the standard deviation corresponding to each SH coefficients of $\partial g_c$.

Our internal field candidate model in 2020.0 contains the sum of the mean core field and the mean lithospheric field at this epoch ($E[g_c] + E[g_l]$). The uncertainties estimates were derived from the covariance matrix $\Sigma_{g_c+g_l} = \Sigma_{g_c} + \Sigma_{g_l} + \Sigma_{g_c g_l} + \Sigma_{g_l}^T$ in 2020.0, where $\Sigma_{g_c g_l}$ is the cross covariance between the core field and the lithospheric field. The square root of each diagonal elements of $\Sigma_{g_c+g_l}$ provides the standard deviation associated with $E[g_c] + E[g_l]$.

Finally, our candidate for the DGRF 2015.0 model was constructed as our 2020.0 internal field model, except that the core and the lithospheric fields were taken from the smoothing solution.

The Kalmag model presented below uses additional data from September 27 2019 until April 2020.

### Results and discussion

#### Parameter estimation

In this section the parameters characterizing the different magnetic sources, the spectra amplitudes $A_i$ and radius $a_i$, the characteristic timescales magnitudes $M_i$ and slopes $\alpha_i$ are evaluated. The spectral resolution as well as the spherical harmonics expansion chosen to model the different fields are a priori imposed (see table 1). The spherical harmonics coefficients of the different sources at the different models time are calculated with our Kalman filter scheme with a subsample of the data between 2001.0 and 2018.0.

In order to avoid measurements taken by CHAMP or SWARM during strongly magnetically disturbed
epochs, and such as no permanent bias due to the static part of the magnetic field generated by the ring current remain in the data, measurements deviating by 60 nT in intensity from the CHAOS-6 internal field model and a yearly estimation of a degree 1 external field expressed in the SM coordinate system are removed from the set. After this operation, a sample of \( N_{\text{est}} = 247,453 \) vector field measurements regularly spaced in time is kept and used to estimate the parameters for the different sources.

This estimation procedure is initialized with a first guess for each parameter. For internal (or external) sources, radii lower (or larger) than the Earth’s radius are chosen. For the external sources we assume a characteristic time scale of one day and set the slopes associated with \( \tau_i(\ell) \) to \( \alpha_i = 0 \). The same is used for the induced and the ionospheric field. The lithospheric field, on the other hand, is assumed to be static. As mentioned above, several authors report that the core field time scales are inversely proportional to the spherical harmonics degree (except for the axial dipole), implying \( \alpha_c = 1 \). We start our estimation with an initial guess of \( \alpha_c = 0 \) and \( M_c = 30 \) years and check whether we nevertheless recover the results suggested by the other authors.

Given a set of parameters, we perform the Kalman filter assimilation described above with the data subset. Before each analysis step we can calculate how well the model predict the data with the relation:

\[
F_{k}^{\text{pred}} = -\log \left| H_k \Sigma_k|_{k-1} H_k^T \right| - \left( d_k - H_k E[z_k|_{k-1}] \right)^T \left( H_k \Sigma_k|_{k-1} H_k^T \right)^{-1} \left( d_k - H_k E[z_k|_{k-1}] \right). \tag{24}
\]

Summing \( F_{k}^{\text{pred}} \) over all \( k \) iterations provides the measure for the model compatibility with the data. We randomly explore the multi-dimensional parameter space, seeking to maximise \( \sum_k F_{k}^{\text{pred}} \). The final values from this parameter search are given in table 2. Remember that for each source with the exception of the lithospheric field, we distinguish between the dipole spatial and time scales and the spatial and time scales of the other harmonics.

Figure 1 shows the static energy spectra projected at the Earth’s surface that define the spatial covariance structure of equation 6 for the optimal parameters. A comparison with the CHAOS-6.9 core field model of Finlay et al. (2016) (black circles) and the LCS-1 lithospheric field model of Olsen et al. (2017) demonstrates the close agreement. The other internal source taken into account in our model is the residual ionospheric/induced field \( (g_{ii}) \). It is dipole dominated and exhibits an almost flat spectrum at the Earth’s surface as illustrated by the blue line in figure 1. Without the restriction to magnetically quiet data, this source would be much more energetic. This is also the case for the magnetic field generated by field-aligned currents \( (g_{fac}) \), which reaches a similar amplitude as \( g_{ii} \) (dashed line in figure 1). The
Table 2. Magnetic sources parameters as described in section magnetic sources. The prior spatial covariance matrices are deriving from energy spectra expressed at some radii $a_i$ which are are either flat with $E_i^\infty(\ell) = A_i^2$ or of the C-based type with the form $E_i^\infty(\ell) = A_i^2(2\ell + 1)F(\ell)$ where $F(\ell) = \ell$ for respectively internal and external sources. The characteristic timescales of equations 5, 7 and 8 are parameterized by $\tau_i(\ell) = M_i\ell^{-\alpha_i}$.

| Field                        | Spectrum | radius $a$(km) | $A$ (nT) | $M$                     | $\alpha$ |
|------------------------------|----------|----------------|----------|-------------------------|----------|
| Core                         | Flat     | 3456           |          |                         |          |
|                              |          |                |          | $\tau_c(1): 935$ yrs    |          |
|                              |          |                |          | $M(\ell \geq 2) = 514$ yrs | 1.06     |
| Lithospheric                 | C-Based  | 6287           | 0.16     | $\tau_m(1): 1.54$ days  | 0        |
|                              |          |                |          | $M(\ell \geq 2) = 18$ min | 0        |
| Close magnetospheric        | C-Based  | 12524          |          |                         |          |
|                              |          |                |          | $\tau_{fm}(1): 0.36$ day | 1.15     |
|                              |          |                |          | $\tau_{fm}(2): 0.55$ days | 0        |
| Remote magnetospheric       | C-Based  | 235570         | 7.3      | $\tau_i(1): 0.71$ day   | 0.93     |
|                              |          |                |          | $M(\ell \geq 2) = 1.76$ day |          |
| Fluctuating magnetospheric  | C-Based  | 13028          |          |                         |          |
|                              |          |                |          | $\tau_{fac}(1): 0$      |          |
|                              |          |                |          | $M(\ell \geq 2) = 1$ min | 0        |
| Residual ionospheric/ induced| Flat     | 6324           |          |                         |          |
|                              |          |                |          | $\tau_i(1): 0.71$ day   | 0.93     |
| Field-aligned currents      | C-Based  | 7917           |          |                         |          |
|                              |          |                |          | $\tau_{fac}(1): 0$      |          |
|                              |          |                |          | $M(\ell \geq 2) = 1$ min | 0        |
last sources are the external magnetospheric fields, which we model with a close \((g_m)\), a remote \((g_{rm})\) and a fluctuation components \((g_{fm})\). Together they exhibit a strong dipole and their energy spectra are rapidly decaying.

The different sources cover a large variety of timescales, ranging from minutes to centuries. For the core field, the characteristic time associated with its non dipolar part reads \(\tau_c(\ell) = 514\ell^{-1.06}\). This power law is close to the slower estimate of *Lhuillier et al.* (2011) given by \(\tau(\ell) = 470\ell^{-1}\), but suggests about 10\% longer time scales. For the core dipole, the estimation algorithm yields \(\tau_c(1) = 935\) years. Since we only consider data over a 17 year period, this estimate is likely not very precise but nevertheless illustrates that the dipole evolves much slower than the other harmonics. The residual ionospheric and induced fields vary very rapidly in comparison, with time scales between one hour and one day, independent of the length scale. Sometimes assumed to be static (see *Olsen et al.* (2014); *Finlay et al.* (2016)), the remote magnetospheric field has a time scale of \(\tau_{rm} \sim 10.3\) years in our study, a value close to the solar cycle. The part of the external fields typically associated with the ring current are the degree \(\ell = 1\) contribution of the close and fluctuating magnetetospheric fields. Whereas the purely zonal part \(g_m\) exhibits a characteristic timescale of \(\tau_m(1) = 1.5\) days, the fluctuating part \(g_{fm}\), assumed to have SH order \(m = \{0, 1, -1\}\) here, varies faster with \(\tau_{fm}(1) \sim 8\) hours. For the small scales magnetospheric field, the time scales of the zonal contributions are shorter than those of the degree one contributions. Note that for \(g_m\), \(\tau_m(\ell \geq 2) = 18\) minutes, a characteristic time lower than the 30 minutes time step of the Kalman filter algorithm. In such a case, where the estimation of \(\tau\) was leading to lower values than the algorithm time step, the slope of the parameterized timescale was set to zero, and the source was only characterized by a spatio-temporal covariance structure of the form given by equation 5. Its evolution from one time step to the other was also treated as a temporal white noise. Finally, the fastest varying source is the field-aligned currents with \(\tau_{fac}(\ell) = 1\) minute. Together with the non dipolar part of \(g_m\), \(g_{fac}\), with its zonal structure and short memory of its past, strongly resembles the observed disturbance along satellite tracks discussed in *Finlay et al.* (2017), and generally affecting the construction of small scale lithopsheric field models (see *Thébault et al.* (2017)).

**Model results**

The optimal model parameters described in the previous section are fixed in the sequential Kalman filter assimilation. The model seeks to describe the data with the spherical harmonics source coefficients \(g_i(t)\), which we call the Kalmag geomagnetic field model. Because the parameters were derived from data at
low geomagnetic activity, we also have to restrict the final model data. This is done on the fly by testing
how much a forecast differs from the data. Whenever the difference lies outside the 95.4% confidence
interval predicted by the slow varying sources (the ones exhibiting some characteristic time larger or
equal than a day at some degree $\ell$) the associated data points are dismissed. All in all, 28.6% of the
originally selected data (see section Data) were dismissed.

We recall that the forecast time step is set to 30 minutes. The entire model (mean and covariance of
$z$) is stored every 0.25 year, although outputs could be saved down to every time steps. Figure 2 shows
the Kalman energy spectra at the Earth’s surface for the core and the lithospheric field for the epoch
2015.0. For degree $\ell \leq 15$, the standard deviations (SD) of both fields are comparable and exceed the
mean value of the lithospheric field. Moreover, the SD of the combined field is smaller than the SD of
the individual fields. This illustrates that we cannot separate core and lithospheric contribution at these
large scales. It also indicates that the prior level of variance of the lithospheric field, as estimated in the
previous section, is therefore simply the extrapolation of the small scale stationary spectrum towards the
larges sales.

Figure 3 compares energy spectra for three types of solutions for the main field (left) and the secular
variation (right) in 2015.0. The Kalman filter solution (thin gray lines and symbols), the solution after the
smoothing algorithm (thick black lines and symbols), and a third solution for a 5 year forecast from 2010.0
(thin black lines and symbols). Continuous lines show the mean, dashed lines the standard deviation,
triangles the differences to the DGRF-13 final field model, and circles the difference to the CHAOS-6.9
SV model.

Not surprisingly, the forecast yields the largest uncertainties. The smallest uncertainties are achieved
in the smoothed solution, since the smoothing process allows to take information from the future into
account. For the field itself, the fact that the differences to the DGRF-13 final model are similar to the
model uncertainties, indicates that these uncertainties are reliably estimated. For the secular variation,
the predicted uncertainty levels seems to be slightly overestimated, at least for the Kalman filter and the
smoothing solutions. The maximum resolution achieved for the SV is $\ell = 16$ for the smoothing solution,
beyond this value the SD becomes larger that the mean signal.

On figure 4 are displayed various estimations of the radial (left), azimuthal (middle) and longitudinal
(right) secular variation at the level of several ground based observatories over the period 2000.0–2025.0.
Blue dots correspond to SV estimations deriving from ground based observatory measurements. They are
obtained by taking annual differences of the measured magnetic field averaged over 0.1 years. The black
lines are evaluations of the SV through the CHAOS-6.9 model. The blue and yellow lines are respectively
the IGRF-13 secular variation and the Kalmag candidate SV. The red area is the Kalmag mean secular
variation plus and minus 2 standard deviation ($\sigma$). Between 2000.6 and 2020.33 the outcomes of the
smoothing solution are shown whereas outside this time window the secular variation is estimated with
the forecast step the Kalman filter. Finally, the red dashed lines are the mean SV $\pm 2\sigma$ coming from 5
year forecast simulations.

The first observation one can make is that whenever the secular variation deriving from observatory
data exhibits a smooth evolution, the latter is well reproduced by the Kalmag model. We can also
notice that at least until 2019.0, the CHAOS-6.9 SV is always lying within 95.4% confidence interval
($E[\partial_t B] \pm 2\sigma$) predicted by our model. Because the Kalmag model is only deriving from the CHAMP
and SWARM measurements, data are missing between 2010.7 and 2013.8. This translates into a global
increase of uncertainty predictions as it can clearly be witnessed for the longitudinal component of the
SV in Mawson or Tuntungan. However, with the combination of the Kalman filter with the smoothing
algorithm, the data gap does not lead to any particular issue to connect the two satellite eras since such
an approach enables us to account for any time space correlations. As already shown through the energy
spectra of figure 3, the forecast algorithm is quite accurate to predict the future states of the secular
variation. The three hindcast simulations covering the periods 2005–2010, 2010–2015 and 2015–2020
are confirming it. Nevertheless, in particular locations where the SV exhibits rapid variations as in
M’Bour, the simple auto regressive dynamics propagating the core field, fails to not only reproduce but
also bound the real evolution of the SV. This calls for using more complex forecast models, able for
example to account for the nonlinear interactions between the core field and a time dependent outer core
flow as in Barrois et al. (2017); Bärenzung et al. (2018); Sanchez et al. (2019). For the incoming 5 years,
both the IGRF-13 or our candidate SV models (which are everywhere quite close to one another) are lying
well between the $\pm 2\sigma$ predicted error bars of the updated Kalmag model. In M’Bour however, recent
observations tend to show a rapid increase of the azimuthal component of the SV. If it is not followed by
a decrease, core field predictions at this location using the IGRF model may rapidly deviate from reality.

The last result analyzed in this study, is a comparison of the different candidates for the IGRF-13 main
field in 2020.0, and the field as it can be evaluated with measurements taken after 2020.0. As shown
with figure 3 and discussed previously, the accuracy of the model deriving from the smoothing algorithm,
which takes into account knowledge beyond the epoch of evaluation, is higher than the Kalman filter solution where the model derivation only accounts for previously assimilated data. We could also observe that the longer the forecasts the lower the accuracy of the model. For the construction of the IGRF-13 model, measurements were only available up to maximum 2019.75, so the difference between the updated Kalmag model (which derives from data assimilated up to 2020.33) and the various candidates can be considered as the errors of the candidates predictions. These errors are displayed in figure 5 through their energy spectra evaluated at the Earth’s surface. Whereas the error spectrum of the Kalmag candidate is drawn with a thick black line, the ones associated with the other candidates are shown with thin gray lines. Because the Kalmag model may exhibit a permanent bias, the difference between the model and the candidate may represent an erroneous evaluation of the error. Therefore the candidates errors were also computed using another model taking recent data into account (up to March 2020), the CHAOS-7.2 model of Finlay et al. (2020) which is also, in its first version, the parent model of the DTU candidate for IGRF-13. The spectra of these error evaluations are shown with dashed lines on figure 5. When compared to the Kalmag model, the Kalmag candidate appears to be the most accurate prediction of the main field in 2020.0 with an error level lower than every other candidate at any SH degree $\ell$. When the comparison is performed with the CHAOS-7.2 model, the Kalmag candidate globally remains the most precise estimation of the 2020.0 field up to $\ell = 8$. However, at smaller scales the DTU candidate is closer to its parent model CHAOS-7.2, but the level of approximated error of the Kalmag candidate remains extremely low.

**Conclusion**

We presented in this study a new approach to derive a Geomagnetic field model from direct measurements of the Earth’s magnetic field. Performing sequentially in time, the Kalmag model, which is the combination of a Kalman filter and a smoothing algorithm, enables us to consider complex prior covariance structure to characterize both spatially and temporally the different magnetic sources composing the observable field. The evaluation of the parameters controlling the statistical properties of each modeled source reveals the large variety of spatial and timescales populating the Earth’s magnetic field, and reinforces the idea of treating the assimilation of geomagnetic data sequentially in time. By allowing the presence of a large scale lithospheric field independent from the core field, we could show that with the prior characterization we chose, the two sources could not be separated. Furthermore, although the sum
of the two fields can be very accurately estimated, the level of uncertainty associated with each individual
source is directly linked to the prior variance of the lithospheric field. This implies a maximum resolution
for the core field of spherical harmonics degree $\ell \sim 15$. Its time derivative however can be accurately
estimated up to $\ell = 16$. Globally, the model provides reliable uncertainty quantification for whether
past, present or future field estimates. It also permits, through the spatio temporal correlations a priori
imposed, to consistently connect the CHAMP and the SWARM satellite eras.

For short term forecasts, as the derivation of the IGRF model requires it, we could observe that our
approach can be more accurate than other existing methods. This is certainly due to the fact that the
secular variation is estimated through its dynamical correlation with the core field and is not a fit to
the past evolution. There is nevertheless still some room for improvement. Considering more physically
based dynamical equations to constrain the evolution of the various fields, such as dynamo simulations for
the core field, would certainly improve the separation of the different sources, and provide more accurate
predictions of future states. The temporal window covered by the model could also be extended by taking
data from previous satellite missions but also ground based observatories or magnetic surveys.

List of abbreviations

- SH: Spherical harmonics.
- SV: Secular variation.
- SD: Standard deviation.

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Availability of data and materials

Champ data can be downloaded at https://isdc.gfz-potsdam.de/champ-isdc/access-to-the-champ-data/
Swarm data can be downloaded at ftp://swarm-diss.eo.esa.int/Level1b/Entire_mission_data/MAGx_LR/
The $K_p$ index can be downloaded at ftp://ftp.gfz-potsdam.de/pub/home/obs/kp-ap/
The IMF indices can be downloaded at https://spdf.gsfc.nasa.gov/pub/data/omni/low_res_omni/
The model presented here is available upon request.
Competing interests

The authors declare that they have no competing interests.

Author’s contributions

Baerenzung Julien produced the Kalmag model. Holschneider Matthias contributed to the theoretical developments. Lesur Vincent provided his expertise on satellite data, and the algorithms to calculate the different coordinate transforms required for the model. Wicht Johannes and Sanchez Sabrina participated to the elaboration of the model requirements, and to the redaction of the manuscript.

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