Research of the effect of droplets deformation on the radar characteristics of rains

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Abstract. The results of calculations of the radar characteristics of rains for the averaged spectra of drops specific to the North Caucasus are presented. The results are obtained under the assumption that the droplets are deformed when falling in clouds and precipitation, taking the form of a flattened spheroid. For comparison, the radar characteristics calculated by the M\(_i\) algorithm for spherical droplets are given. For individual large droplets, which under the influence of an electric field can take extreme sizes, the reflectivity are also calculated. The analysis of the obtained results showed that when reconstructing the microstructural characteristics of hail clouds and precipitation at high intensities, it is necessary to take into account the shape of drops.

1. Introduction.
Currently, radar methods for measuring precipitation are widely used to study convective phenomena in the atmosphere and to solve applied problems related to floods and weather modification. The advantages of such methods include the ability to obtain information about precipitation parameters over large areas and over certain time intervals and the disadvantages are associated with the ambiguity of restoring precipitation parameters from radar cross section [1, 2].

To reconstruct the microstructural characteristics of rains, calculations of the scattering cross-sections are traditionally carried out under the assumption of sphericity of the droplets, which allows to use a strict solution to the problem of diffraction of electromagnetic waves on a dielectric sphere (Mie theory) [3]. However, the shape of raindrops falling freely in clouds and precipitation is markedly different from the spherical one [4]. In addition, the deformation of the spectrum of raindrops in thunderclouds is greatly influenced by the electric field initiated by natural thunderstorm activity. In such cases, the method of separation of variables for spheroid-shaped particles can be used to calculate the scattering cross-sections [5-7].

The aim of this work is to study the influence of the shape of droplets on the results of radar measurements of rain precipitation characteristics. To achieve this goal, the following tasks are set:

- to calculate the water content and intensity of liquid precipitation using the gamma function based on the drop distribution parameters known from practice;
- for the same parameters, to calculate the reflectivity under the assumption of spherical and non-spherical droplet shapes;
- to evaluate the contribution of the droplet shape to the values of the radar characteristics of precipitation.

The calculations were carried out for the electromagnetic radiation wavelength of 10 cm, which is currently most commonly used in practice.
2. Materials and methods of research

The raindrop distribution function is the main characteristic in the physics of cloud formation and precipitation development. Knowing it, it is possible to calculate the water content (g/m^3) of precipitation using the formula:

\[ W = \frac{\pi}{6} \rho \sum_{d_{\text{min}}}^{d_{\text{max}}} d^3 N(d) \Delta d \]  

(1)

where \( \rho \) – water density (g/cm^3);

\( N(d) \) – drop diameter distribution function (1/(m^3-mm));

\( d \) – drop diameter (mm).

The precipitation intensity (mm/h) can be calculated using the formula:

\[ R = \frac{\pi}{6} \sum_{d_{\text{min}}}^{d_{\text{max}}} d^3 N(d)V(d) \Delta d \]  

(2)

where \( V(d) \) – steady-state speed of drop falling (cm/s).

The reflectivity (1/cm) is calculated by the formula:

\[ \eta = \int_{d_{\text{min}}}^{d_{\text{max}}} N(d) \sigma(d, \lambda) \, dd \]  

(3)

where \( \sigma(d, \lambda) \) – cross-sections of backscattering of droplets at the electromagnetic radiation wavelength \( \lambda \). Assuming that the diffusers have a spherical shape, the well-known Mie formulas are used to calculate the cross-sections [3].

In heavy rains, when large drops are observed, their shape may be deformed. In this case, the method of separation of variables for particles of the shape of an arbitrary spheroid can be used to calculate the cross-sections of the scattering [5-7]. The scattering problem for such particles is solved on a spheroidal basis under the assumption that a plane polarized wave falls on the particle, which can be represented as a superposition of a vertically polarized (TM mode) and horizontally polarized (TE mode) wave [8, 9].

The most widely used functions are the size distribution of droplets of: Marshall-Palmer, Polyakov-Shifrin, Beta distribution. In contrast, the gamma function is three-parameter and is suitable for describing droplets in the entire size range:

\[ N(d) = \frac{N}{\Gamma(\mu + 1) \beta^{\mu+1}} d^\mu e^{-d/\beta} \]  

(4)

where \( N, \mu, \beta \) – distribution parameters.

The results of laboratory studies have shown that the shape of drops in free fall without the action of an electric field can be approximated by a flattened spheroid; and for practical use, you can apply the equation representing the ratio of the semi-axes as a function of the equivolumetric diameter of the drop [4]:

\[ b/a = 1,012 - 0,144 \cdot (d_v) - 1,03 \cdot (d_v)^2 \]  

(5)

where \( d_v \) – the diameter of the equivolumetric spherical drop (cm); \( a \) – major semi-axe; \( b \) – minor semi-axe.

In real thunderstorm and hail clouds, the growth of droplets is also influenced by the electric field initiated by natural thunderstorm activity that accompanies the formation of heavy precipitation and hail. This is confirmed by laboratory experiments. The electric field lengthens the drop along its direction, and the horizontal electric fields are more effective for deforming the drops than the vertical ones [10, 11].

3. Research results
In this research, the radar characteristics of rains, where temperatures of about 10 °C predominate are considered. In such areas, large drops of up to 8 mm can exist. The gamma function was used as the distribution function, the parameters of which were obtained from the averaged spectra specific to the North Caucasus (table 1) [12]. For these spectra, the water content and intensity of precipitation and reflectivity by Mie were calculated (table 1).

As can be seen from table 1 an increase in the intensity and water content of precipitation is accompanied by an increase in the concentration and an increase in the average size of drops. Therefore, there is a dependence between the concentration of drops, the characteristic size of drops, the intensity and water content in clouds and precipitation. Table 1 shows that with increasing water content and precipitation intensity, radar cross section also increases.

Table 1. Radar characteristics of precipitation.

| $N$ | $\mu$ | $\beta$ | $W_\gamma$, g/m$^3$ | $R$, mm/h | $Z$, dBZ (by Mie) |
|-----|-------|---------|---------------------|-----------|------------------|
| 121 |  3    | 0.14    | 0.17                | 3.71      | 35.30            |
| 146 |  4    | 0.13    | 0.28                | 6.53      | 38.16            |
| 178 |  4    | 0.15    | 0.53                | 20.23     | 42.67            |
| 215 |  4    | 0.16    | 0.77                | 19.96     | 45.13            |
| 250 |  3    | 0.21    | 1.16                | 31.64     | 48.70            |
| 265 |  3    | 0.20    | 1.07                | 28.40     | 48.12            |
| 375 |  4    | 0.22    | 3.50                | 102.31    | 55.41            |
| 395 |  1    | 0.39    | 2.30                | 67.20     | 54.95            |
| 520 |  2    | 0.31    | 3.85                | 112.52    | 56.36            |

Next, radar reflectivity were calculated taking into account the drops form factor in the absence of an external electric field. Figure 1 shows the dependence of precipitation intensity on radar cross section, calculated for spherical drops (Mie) and spheroidal drops at different polarizations of the probed electromagnetic wave (TM – vertical, TE – horizontal). Thus, the curve obtained by the Mie formulas for spherical droplets is consistent with the $R(Z)$ relation widely used in practice (figure 1). The curves obtained for spheroidal droplets with horizontal and vertical polarization of the electromagnetic wave differ from it. Moreover, with increasing reflectivity, the difference increases. This trend is due to the fact that there are large drops at high intensities in precipitation.

For the used droplet spectra, the relative error in calculating the reflectivity, taking into account the deformation, reaches up to 5 dBZ at high intensities (in the absence of an external electric field).

![Figure 1. Dependence of precipitation intensity on radar cross section for different droplet shapes: Mie – spherical droplet, spheroidal drop with vertical (TM) and horizontal (TE) polarizations.](image-url)
Under laboratory conditions [10, 11], large droplets with diameters of 7 and 7.3 mm were considered under the influence of an electric field with intensity equal to $H = 500 \text{ kV} \cdot \text{m}^{-1}$. It turned out that under the influence of a horizontal electric field, the droplets are more elongated along the main axis, reaching extreme sizes, than in the vertical direction of the electric field and in the absence of an electric field (table 2). Based on the results of these experiments, the reflectivity for individual droplets were calculated due to the lack of data for the entire spectrum of droplets. The results of the calculations are shown in figure 2.

**Table 2.** Extreme deformation of droplets in a horizontal electric field.

| Equivolumetric diameter, mm | Axes of the spheroid | Form factor, $a/b$ |
|----------------------------|----------------------|-------------------|
| 1                          | 2                    | 3                 | 4                  |
| 7                          | 29                   | 3,44              | 8,4302             |
| 7.3                        | 42                   | 3,01              | 13,9535            |

**Figure 2.** Contribution of individual droplets to reflectivity at different precipitation intensities for different shapes: Mie – spherical drop, spheroid drop at vertical (TM) and horizontal (TE) polarizations.

In figure 2, a set of lines and dots in green color corresponds to the precipitation intensity of 110 mm/h, red color corresponds to the precipitation intensity of 30 mm/h, blue color corresponds to 13 mm/h. The solid lines correspond to the reflectivity of spherical droplets by Mie. For spheroidal droplets, dotted lines are plotted for horizontal polarization of the electromagnetic wave, and dashed lines are plotted for vertical polarization. The reflectivity of individual droplets (7 and 7.3 mm) at extreme elongation under the influence of a horizontal electric field are marked with separate points.

The analysis of figure 2 showed that the values of the reflectivity of spheroidal droplets growing in a horizontal electric field differ from similar values for equivolumetric spheres in the absence of an electric field. The reflectivity of spheroidal droplets are almost an order of magnitude different from those of spherical droplets in the absence of an electric field. This will introduce a significant error in the $R(Z)$ relation compared to the deformation of the droplets in the absence of an electric field.

4. **Conclusion**

In fine-dispersed systems (clouds without precipitation, fine rain, fog, etc.), the drop form factor does not significantly affect the radar reflectivity. But in thunderclouds, large droplets in free fall in the...
atmosphere can deform, taking extreme sizes, especially under the influence of a horizontal electric field. Such droplets reflect back the electromagnetic wave, which differs significantly from similar characteristics for equivolumetric spheres. Therefore, when restoring the microstructural characteristics of hail clouds and precipitation, including the R(Z) relation, it is necessary to introduce a correction for the shape of drops.

The results obtained can be useful in the development of methods for radar research of clouds and precipitation.

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