Development of static and kinematic approaches in geomechanical problems

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Abstract. The hypothesis of a linear stress field of a rock mass undisturbed by driving workings underlies the solution to many problems of rock mechanics. The problem of rock pressure control requires knowledge of the stress-strain state of the rock mass in the vicinity of workings, which causes the development of analytical and numerical methods for calculating the deformation. When the methods of numerical calculation were not sufficiently developed, the analytical capabilities of the selection of stress functions were used, which satisfied the conditions of small strain elasticity theory. The greatest success in this direction was achieved for the areas with angular points. The development of numerical methods shifted the point of influence of investigations to calculating the deformation of specific cases arising in a rock mass with workings. It is established in the work that analytical solutions for the areas with angular points are incorrect, and numerical solutions of Cauchy problems are also incorrect if an additional problem is not considered.

1. Introduction
The necessity of control of the changing stress-strain state of rock massif arises with increase in the excavated space as the stoping works advance. The linear law of stress distribution in the initial state is most common in rock mechanics. In the two-dimensional case, a rock mass with workings will be modeled using a plane with a hole. The initial stress state is taken as [1, 2]:

\[ \sigma_y = -\gamma(H - y), \quad \sigma_x = -\lambda\gamma(H - y), \]  

(1)

where \( \sigma_y \) and \( \sigma_x \) are the vertical and horizontal components of the stress tensor at the depth \( H \) from the day surface; \( \gamma \) is the specific weight of rocks; \( \lambda \) is the coefficient of lateral earth pressure. The assumptions adopted with respect to the rock mass deformations in the vicinity of workings in [1, 2] reduced the problem to the solution for a plane with mathematics section. However, no analysis have thus far been conducted for the initial stress field influence on the correctness of the boundary conditions formulation during the numerical calculations and utilization of the analytical solutions for the areas with angular points.

In the previous research works [1, 2], the rock mass weight was not taken into account. Alternatively, it is dealt with by a method proposed in [3] within the frames of the phenomenological theory, which utilizes the experimental data on rocks displacements in the roof and floor. This employs the classical solution (analytical or numerical) which is refined by involving additional information about displacements within the problem reducing to the inverse problems solution.
2. Analytical solutions

The class of analytical solutions used in rock mechanics primarily requires distinguishing solutions for areas with angular point in which the stresses turn to infinity. These solutions are well known and can be found in any textbook on the elasticity theory, particularly their specific type—a half-plane (wedge angle $\pi$), loaded with a vertically directed concentrated force. We have to admit that an attempt to combine a concentrated force (the concept does not exist in the elasticity theory) with a randomly selected stress function utterly unsuccessful

$$\phi = -\frac{P}{\pi} r \theta \sin \theta,$$

where $p$ is the constant identified with the concentrated force; $r$, $\theta$ are the polar coordinates (Figure 1). The stress components, according to (2), have the form

$$\sigma_r = -\frac{2p}{\pi} \frac{\cos \theta}{r}, \quad \sigma_{\theta} = \tau_{r\theta} = 0,$$

whereas the component of displacements in the $r$ direction will be equal to

$$u = \frac{2p}{\pi E} \cos \theta \ln r + f(\theta),$$

where $E$ is Young’s module; $f(\theta)$ is a function does not depend on $r$. Noteworthy is that there is no condition defining $f(\theta)$. The latter is proposed in [4] to be determined based on the fact that for each value of $\theta$ on the $r$ axis there is a point with no displacement occurring in the $r$ direction; however, these points are not defined in any way, i.e. the solution (2) is incorrect. As it follows from (4), with increasing distance from the origin of the coordinates, the displacements increase to infinity according to the logarithmic law. It is suggested to eliminate this incorrectness by removing the vicinity of the point $x = 0$ [4], but in this case the force $p$ disappears, which will be replaced by reactions. However, it is all but impossible to find reactions corresponding to this type of stress distribution.

![Figure 1. Scheme of the concentrated force $p$ acting on the half-plane.](image)

The solution for the half-plane, according to (3), has zero values at infinity $\sigma_r = 0$. The main stress vector which is nonzero and equal to $p$ is determined by the degree of attenuation $\sigma_r = 0$ from (3) if $r \to \infty$. The problem (2) belongs to the class of Cauchy problems according to the general theory of equations in the mathematical physics classification. The correct solution requires that the boundary conditions disappear at infinity (i.e., tend to vanish with velocity of $1/r^{1+\varepsilon}$, $\varepsilon > 0$, rather than $1/r$).

Thus, incorrectness of solutions (2) – (4) is characterized by: the infinity of stresses and uncertainty of displacements at the point $x = 0$; an increase to infinity of displacements $v$ as $x$ tends to zero ($x = 0$); the uncertainty of a small radius when cutting out the vicinity of the point $x = 0$; and their contradiction to the class of Cauchy problems.

3. The Cauchy problem for cracks

A classical solution for a plane weakened by finite straight line section simulating the crack was discussed in [5], which has shown incorrectness of the solution. In the absence of analytical solution to this problem this requires application of numerical calculations. Let’s consider that the crack-path lies
in the rock’s gravity field (1). The calculation area contains an infinitely remote point and therefore the problem falls into the Cauchy problem class. Compressive forces on the edges of the section
\[ \sigma_y = \sigma_0 = \text{const}, \]
where \( \sigma_0 \) is determined from (1). We solve the problem for a weightless plane by formulating boundary conditions for the crack written as
\[ \sigma_y = \sigma_1, \quad \tau = 0, \quad (5) \]
when we know \( \sigma_1 \) opening the section (mine opening). The problem can be considered by passing on to two half-planes, as shown in Figure 2, thereby reducing complexity of the calculation. If the difference in rocks which characterize the half-planes for \( y \geq 0 \) and \( y \leq 0 \) is taken into account we will have to return to the boundary conditions formulation for a piecewise homogeneous plane shown in Figure 3. Since in the general case the problems for the crack fall into the category of Cauchy problems we need to proceed to the additional problem solution (5). The considered static calculation allows taking account the rock mass weight.

Based on the kinematic theory [3], an additional solution (5) is used for calculation of the stress-strain state of the rock massif, with establishing the zone of the crack influence within it. The initial parameters characterizing the rock massif containing a crack are refined using the inverse problems formulation, proceeding from [3]. Given that the section opens at low values of \( \sigma_2 \), this enables realization of the rock mass weight. The displacement caused by the rock mass weight is significantly greater than by the mine opening from \( \sigma_2 \). As the value of \( \sigma_2 \) grows, there comes a point of time when the mine opening from \( \sigma_2 \) exceeds the displacement caused by its mass weight. The value that causes the crack to open can be defined. In this case, the real displacements of the section edges for which the calculation is performed will be critical.

4. Numerical methods of solving the Cauchy problem
The rock pressure control during the stoping works requires the knowledge of the stress-strain state of the rock massif near the workings, which has led to a necessity of the development of analytical and numerical methods for solving problems in geomechanics. Their development was accompanied by
the appearance of a class of incorrect problems that involved infinite stresses and displacements and that belong to the category of the elasticity theory problems. Initially, incorrect problems arose when considering analytical solutions. With introduction of the numerical calculations (e.g. the finite element method) the number of incorrect problems has considerably increased [1, 2, 6, 7]. In any numerical calculation the workings configuration is not essential and therefore the analytical solution generally fails to be obtained, which, however, does not mean that the proof of existence, uniqueness, and the solution’s continuous of dependence on the original data can be ignored.

Cauchy problems include the problems, whose calculation domain contains an infinitely remote point. Many problems of rock mechanics are reduced to the consideration of a plane having a hole or a space with a cavity, i.e. they automatically fall into the class of Cauchy problems in mathematical modeling of stress-strain state near “weak” zones. According to the general theory of equations in mathematical physics, the boundary conditions should vanish at infinity, to exclude possible incorrectness. The boundary conditions formulation assumes two types of problems in numerical calculations: (1) problems that assume zero values of normal and shear stresses to be set for the excavated contour after driving workings (such solution would be incorrect, since the real existence of zero boundary conditions at the contour needs to be proven); (2) problems when nonzero boundary conditions are applied on the contour in addition to the conditions at infinity (all known solutions of this type are incorrect).

Obtaining a correct solution to the Cauchy problem requires analyzing an additional problem, as shown in [5]. Such consideration is also necessary in the problem of taking into account the rock mass weight, i.e. the kinematic aspect, which uses an additional solution, thereby essentially increasing possibilities of the mathematical modeling in geomechanics. Numerical calculation of the additional problem and taking into account of the rock mass weight allow to refine the a priori data on the mechanical characteristics and stress field of the intact rock mass.

5. Conclusions

The incorrectness of the analytical solutions to the problems for areas with angular points including mathematical sections modeling cracks and theories based on them (failures, fractures, etc.) is established. The Cauchy problem for a crack in a linear field of the initial stresses is analyzed, and the algorithm for its correct formulation is proposed. Numerical solutions to geomechanics problems which are not reduced to solving an additional problem have proven to be incorrect.

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