HEAVY HIGGS BOSONS AT TeV e⁺e⁻ COLLIDERS

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ABSTRACT

We summarize the work done by the European working group on Higgs Particles for the Workshop “Physics with e⁺e⁻ Linear Colliders”, Annecy–Gran Sasso–Hamburg, Feb.–Sept. 1995. The main focus will be on the physics possibilities at a second phase e⁺e⁻ linear collider with a center of mass energy of ~1.5 TeV.

1. Introduction

In previous studies,¹,² it has been shown at great details that an e⁺e⁻ linear collider operating in the energy range √s = 300 to 500 GeV with a luminosity of f L = 20 fb⁻¹ is an ideal machine to search for light Higgs particles.³

In the Standard Model (SM) the whole Higgs mass range, M_H < ∼ 200 GeV, theoretically favored by the requirement that the SM be extended up the Grand Unification scale, Λ ∼ 10¹⁶ GeV, can be covered. The search can be carried out in three different channels: the Higgs–strahlung process e⁺e⁻ → ZH and the fusion mechanisms WW/ZZ → H. The cross sections are rather large [especially in the very clean Higgs–strahlung process] and the properties of the Higgs boson [in particular spin–parity quantum numbers, couplings to gauge bosons and fermions] can be thoroughly investigated, allowing for important tests of the Higgs mechanism.

In the Minimal Supersymmetric extension of the Standard Model (MSSM), in which the Higgs sector is extended to comprise three neutral h/H(CP=+), A(CP=–) and a pair of charged scalar particles H±, the lightest Higgs boson h has a mass M_h < 140 GeV and can be detected in the entire MSSM parameter space either in the Higgs–strahlung process e⁺e⁻ → hZ or in the complementary mechanism of associated production with the pseudoscalar e⁺e⁻ → hA. In addition, there is a substantial area in the MSSM parameter space where the heavier Higgs bosons can be also found; at a 500 GeV e⁺e⁻ collider, this is possible if the H, A and H± masses are less than 230 GeV. As in the case of the SM, various properties of these Higgs bosons can be investigated.

¹ Talk given by A.D. at the Workshop on Physics and Experiments with Linear Colliders, Morioka–Appi, Japan, September 8–12 1995.
Higher energies are required to sweep the entire mass range of the SM Higgs particle, $M_H \lesssim 1$ TeV. These high energies will also be needed to produce and study the heavy scalar particles of extensions of the SM, such as the MSSM, if their masses are larger than $\sim 250$ GeV. In $e^+e^-$ collisions, a second phase with a c.m. energy up to 1.5–2 TeV would be therefore mandatory.

A study of the potential of a 1.5 TeV $e^+e^-$ linear collider with an integrated luminosity of $\int L = 300$ fb$^{-1}$ per annum [to compensate for the drop in the cross sections of the interesting processes at high energies] was undertaken in Ref.[4]. In this contribution, we will summarize the main points of this study.

For the SM, we will concentrate on the mass range above $M_H \gtrsim 250$ GeV, and summarize the main production mechanisms and decay modes. The search of New Physics through the measurement of production cross sections, angular distributions and decay widths will be discussed. We will focus on the case of weakly interacting New Physics at a scale $\Lambda \gtrsim 1$ TeV [described in terms of an effective Lagrangian with dimension 6 operators]; a discussion of the strongly interacting New Physics scenario has been given in Ref.[5].

We will investigate the properties of the heavy Higgs particles of supersymmetric extensions of the SM. We will restrict ourselves to the minimal extension which is highly constrained since there are only two free parameters [at tree–level]: a Higgs mass parameter [generally $M_A$] and the ratio of the vacuum expectation values of the two doublet fields responsible for the symmetry breaking, $\tan\beta$ [which in Grand Unified Supersymmetric models with $b$–$\tau$ Yukawa coupling unification is forced to be either small, $\tan\beta \sim 1.5$, or large, $\tan\beta \sim 50$]. The various decay modes of the heavy CP–even Higgs boson $H$, the pseudoscalar $A$ and the charged Higgs particles $H^\pm$ [in particular the below–threshold three body decays and the decays into supersymmetric particles] will be discussed. The main production processes of $H, A$ and $H^\pm$ and the multiple production of the light MSSM Higgs boson [which allows the determination of Higgs trilinear couplings] will be summarized.

We will restrict ourselves to the $e^+e^-$ mode of the linear collider, other options such as $\gamma\gamma$ have been discussed elsewhere at this Workshop. Some aspects related to the properties of light Higgs particles in the SM and the MSSM, as well as the discussion of the Higgs sector of non-minimal extensions of the SM can be found in Ref.[4]. For more details on the topics presented in this contribution and for a complete list of references, we refer to the report Ref.[4].

2. Standard Model Higgs

2.1 Production Processes and Decay Modes

In the SM, the Higgs profile is uniquely determined once $M_H$ is fixed. For a heavy Higgs boson, $M_H \gtrsim 250$ GeV, the decay width, the branching ratios and the production cross sections are given by the strength of the Yukawa couplings to top quarks and $W/Z$ gauge bosons, the scale of which is set by the $t/W/Z$ masses.

At $e^+e^-$ linear colliders operating in the TeV energy range, the main production mechanisms for Higgs particles are
(a) bremsstrahlung process  \[ e^+e^- \rightarrow (Z) \rightarrow Z + H \]

(b) WW fusion process  \[ e^+e^- \rightarrow \bar{\nu} \nu (WW) \rightarrow \bar{\nu} \nu + H \]

(c) ZZ fusion process  \[ e^+e^- \rightarrow e^+e^- (ZZ) \rightarrow e^+e^- + H \]

Fig.1: a) Cross sections for the main production processes of Higgs bosons at \( \sqrt{s} = 1.5 \) TeV, and (b) the branching ratios and the total decay width of the Higgs particle.

The cross sections are shown in Fig.1a as a function of the Higgs mass at \( \sqrt{s} = 1.5 \) TeV. The WW fusion mechanism is by far the dominant process since the cross section grows \( \sim M_W^2 \log s \). At these energies, one can use the effective longitudinal W approximation which gives a result [dashed line] only slightly larger than the exact cross section. The cross section is peaked in the forward and backward region and the relevant energy scale at the WWH vertex is \( M_H \). Since the NC couplings are smaller than the CC couplings, the cross section for the ZZ fusion process is \( \sim 16 \cos^4 \theta_W \), i.e. one order of magnitude smaller than for WW fusion. However, the lower rate might be partly compensated by the cleaner \( He^+e^- \) final state.

The bremsstrahlung process which is dominant for moderate values of \( M_H/\sqrt{s} \), falls off like \( \sim s^{-1} \) at high energies and the cross section [which is peaked in the central region] is very small compared to the dominant WW fusion mechanism. However, as will be discussed later, this process will be very useful to study the properties of the Higgs boson since the relevant energy scale at the HHZ vertex is at \( \sqrt{s} \).

For an integrated luminosity of \( \int L = 300 \, \text{fb}^{-1} \), 60,000 (2,500) events in the WW fusion process and 1,500 (300) in the Higgs–strahlung process are to be expected for \( M_H = 250(1000) \) GeV. Note that the longitudinal polarization of the initial beam(s) will increase the effective WW luminosity by a factor 2(4).
In the high mass range, $M_H \gtrsim 250$ GeV, the Higgs bosons decay almost exclusively into $WW$ and $ZZ$ pairs, with branching ratios of $2/3$ for $WW$ and $1/3$ for $ZZ$; Fig.1b. The opening of the $t\bar{t}$ channel does not alter significantly this pattern, since for large Higgs masses, the $t\bar{t}$ decay width rises only linearly with $M_H$ while the decay widths to $W$ and $Z$ bosons grow with $M_H^3$. The latter rise of the width into longitudinal gauge bosons makes that for heavy Higgs bosons, the total width [dashed line] is very large and can be measured experimentally. For $M_H \sim 1$ TeV, the total width of the Higgs boson becomes comparable to its mass.

Once the Higgs boson has been found, it will be very important to explore its properties. The zero-spin of the Higgs particle is reflected in the angular distribution in the bremsstrahl process which asymptotically must follow the $\sin^2 \theta$ law, corresponding to the predominantly longitudinal polarization of the accompanying $Z$ boson. Correlations among the final fermions in the production process $e^+e^- \rightarrow ZH \rightarrow \bar{f}f\bar{t}t$ and in the decays processes $H \rightarrow WW/ZZ \rightarrow 4$ fermions, also allow to determine the CP properties of the Higgs particle.

Of great importance is the measurement of the couplings to gauge bosons and matter particles. The strength of the couplings to $Z$ and $W$ bosons is reflected in the magnitude of the production cross sections. The relative strength of the couplings to top quarks is accessible through the decay branching ratio which is at the level of 10% for $M_H > 350$ GeV. [For smaller Higgs masses, the $ttH$ coupling can be accessed in Higgs–strahlung off top quarks at lower energies, where the rates are more important.]

The Higgs boson self-coupling can be measured in the processes

$$e^+e^- \rightarrow Z + HH \ , \text{ and } WW/ZZ \rightarrow HH$$

(1)

While the cross section is rather small in the $e^+e^- \rightarrow Z + HH$ process [less than 1 fb already at $\sqrt{s} = 500$ GeV for $M_H \sim 60$ GeV], it is sizeable in the $WW$ fusion mechanism: at a 1.5 TeV collider, it is the order of 1 fb for $M_H \lesssim 200$ GeV [see next section]. With $\mathcal{L} = 300$ fb$^{-1}$, a few hundred events can be expected and a measurement of the trilinear coupling is possible, allowing to reconstruct the scalar potential; a decisive test of the Higgs mechanism.

2.2 New Physics Effects

The large sample of Higgs bosons that can be produced in the 1.5 TeV $e^+e^-$ mode at high luminosity allows for tests of the Standard Model predictions at the percent level. Thus one can search for indirect effects of New Physics on the Higgs sector which are associated to a scale $\Lambda$ beyond the reach of the collider for direct particle production. They can be parameterized in terms of effective non-renormalizable interactions among the Higgs boson and the other SM particles, and are suppressed by powers of $\Lambda^{-2}$. The largest effects are induced by dimension-six operators when interference with the Standard Model amplitudes is possible. Otherwise the New Physics contributions are suppressed at least by $\Lambda^{-4}$.

In terms of the operator basis introduced by Buchmüller and Wyler, the operator $(\partial(\phi^\dagger \phi))^2$ amounts to a common renormalization of all Higgs couplings. A non-zero
coefficient would result in a deviation of the total Higgs production rate from the Standard Model prediction. Similarly, a deviation in the ratio of $ZZ$ fusion over $WW$ fusion, which in the SM is approximately equal to 0.1 over the whole Higgs mass range, would indicate a contribution of the operator $(D\phi)^\dagger\phi\phi(D\phi)$. This operator breaks the custodial $SU_2$ symmetry of the Higgs sector; hence a measurement of $\sigma(ZZ \to H)/\sigma(WW \to H)$ is complementary to the measurement of the $\rho$ parameter at lower energies. Obviously, the branching ratios $BR(H \to WW)$ and $BR(H \to ZZ)$ are affected by the same corrections.

Although the Higgs-strahlung process $e^+e^- \to ZH$ plays only a minor role at high energies, it is important for the search for new interactions, since the effective c.m. energy in the Higgs production is equal to the total collider energy. In contrast, in the fusion processes the relevant scale is set by the Higgs mass. The total cross section for Higgs-strahlung is affected by contact terms $eeZH$ which are induced by operators that also modify the $eeZ$ couplings $g_V$ and $g_A$. The effect of the contact terms grows with increasing c.m. energy, so that the limits on the effective scale $\Lambda$ obtained by LEP can be improved by more than one order of magnitude at a 1.5 TeV linear collider. On the other hand, in the fusion processes contact terms will probably be unobservable.

An analysis of the angular distribution of Higgs bosons will give information on anomalous couplings to transversal $W$ and $Z$ gauge bosons. However, according to the equivalence principle Higgs bosons are produced mostly by fusion of longitudinal gauge bosons, and the interference with production by transversal gauge bosons due to new interactions is suppressed. Similarly, anomalous couplings to photons are difficult to observe since the process $e^+e^- \to H\gamma$ and the decay $H \to \gamma\gamma$ occur in the Standard Model only at the loop level. Anomalous couplings of the Higgs boson to top quarks can be accessed through the analysis of the branching ratio which is $\mathcal{O}(10\%)$ when the decay is kinematically allowed.

With increasing Higgs mass ($\gtrsim 400$ GeV) the lower production rate diminishes the resolving power on New Physics effects. In particular, the cross section for double Higgs production falls below the limit of observability, so that the trilinear Higgs coupling is no longer directly measurable. On the other hand, radiative corrections to the production and decay processes are determined by the size of the Higgs self-coupling and become more important for a heavier Higgs boson, so that New Physics contributions may become accessible through their loop effects. In order to be sensitive to these corrections it is crucial to measure the shape of the Higgs resonance in the $WW$ and $ZZ$ channels beyond the peak, where due to the rapidly decreasing effective luminosity of $W$ and $Z$ bosons in the initial state the event rate will be low.

For Higgs masses near the upper limit ($\gtrsim 700$ GeV) the Standard Model predictions become increasingly unreliable, since higher-order corrections approach the lowest-order contribution in size. In this mass range an effective Lagrangian with a linear Higgs representation is no longer appropriate. The usual approach is to use a nonlinear representation of the symmetry-breaking sector instead, which leaves freedom to incorporate arbitrary Higgs couplings, or to dispense of a scalar resonance altogether.\textsuperscript{5}
3. Minimal Supersymmetric Standard Model

3.1 Decay Modes

The decay pattern of the SUSY Higgs bosons is determined to a large extent by their couplings to fermions and gauge bosons; these couplings will in general depend strongly on the angles $\alpha$ and $\beta$. The pseudoscalar and charged Higgs boson couplings to down (up) type fermions are (inversely) proportional to $\tan \beta$; $A$ has no tree level couplings to gauge bosons. For the CP–even Higgs bosons, the couplings to down (up) type fermions are enhanced (suppressed) compared to the SM Higgs couplings [$\tan \beta > 1$]; the couplings to gauge bosons are suppressed by $\sin \beta / \cos (\beta - \alpha)$ factors.

In the following, we will summarize the various decay modes of the heavy Higgs bosons, first in the case where the decay channels into supersymmetric particles are shut, and then when these decays are allowed. We will discuss the two scenarios of small and large $\tan \beta$ values, favored by SUSY–GUT models with $b-\tau$ Yukawa coupling unification. The QCD corrections to the hadronic modes have been recently summarized [see [4] for details] and will not be discussed here.

Non–SUSY Two– and Three–Body Decay modes

For large values of $\tan \beta$ the pattern is simple, a result of the strong enhancement of the Higgs couplings to down–type fermions. The neutral Higgs bosons will decay into $b\bar{b}$ and $\tau^+\tau^-$ pairs, the charged Higgs bosons into $\tau \nu_\tau$ pairs below and $tb$ pairs above the top–bottom threshold. For $H$, only when $M_H$ approaches its minimal value is this simple rule modified; in this case it will mainly decay into $hh$ and $AA$ final states.

For small values of $\tan \beta \sim 1$ the decay pattern of the heavy neutral Higgs bosons is much more complicated. The $b$ decays are in general not dominant any more; instead, cascade decays to pairs of light Higgs bosons and mixed pairs of Higgs and gauge bosons are important. Moreover, decays to gauge boson pairs play a major role. However, for very large masses, the neutral Higgs bosons decay almost exclusively to top quark pairs. The decay pattern of the charged Higgs bosons for small $\tan \beta$ is similar to that at large $\tan \beta$ except in the intermediate mass range where cascade decays to light Higgs and $W$ bosons are dominant.

Besides these two–body decays, below–threshold three–body Higgs decay modes can play an important role. It is well–known that SM Higgs decays into real and virtual $Z$ pairs are quite substantial: the suppression by the off–shell propagator and the additional $Zff$ coupling is at least partly compensated by the large Higgs coupling to the $Z$ bosons. For the same reason, three–body decays of MSSM Higgs particles mediated by gauge bosons, heavy Higgs bosons and top quarks, are of physical interest. Important three-body decays for the heavy CP–even, the CP–odd and the charged Higgs bosons, analyzed recently are

\[
H \rightarrow VV^* \rightarrow Vff^* \quad AZ^* \rightarrow Af^{*} \quad H^\pm W^\mp* \rightarrow H^\pm f^{*} f^* \quad \bar{t}t^* \rightarrow \bar{t}bW^+ \quad (2)
\]

\[
A \rightarrow hZ^* \rightarrow hf \quad \bar{t}t^* \rightarrow \bar{t}bW^+ \quad (3)
\]

\[
H^\pm \rightarrow hW^* \rightarrow hf \quad AW^* \rightarrow Af^{*} \quad \bar{b}t^* \rightarrow \bar{b}bW \quad (4)
\]
Fig. 2: Branching ratios for the heavy CP–even, the CP–odd and the charged Higgs bosons, including the three–body decays, for $\tan\beta = 1.5$ and no stop mixing.

The branching ratios for $h$, $A$ and $H^\pm$ decays are shown in Fig. 2 for $\tan\beta = 1.5$, in the case where the mixing in the stop sector is neglected.
For the heavy Higgs boson $H$, the decay $H \to hh$ is the dominant channel, superseded by $t\bar{t}$ decays above the threshold [for the latter, the inclusion of the three–body modes provides a smooth transition from below to above threshold]. This rule is only broken for Higgs masses of about 140 GeV where an accidentally small value of the $\lambda_{Hhh}$ coupling allows the $b\bar{b}$ and $WW^*$ decay modes to become dominant. Important channels in general, below the $t\bar{t}$ threshold, are decays to pairs of gauge bosons and $b\bar{b}$ decays. In a restricted range of $M_H$, below–threshold $AZ^*$ and $H^\pm W^{\pm*}$ also play a non–negligible role.

In the case of the pseudoscalar $A$, the dominant modes are the $A \to b\bar{b}$ and $A \to t\bar{t}$ decays below the $hZ$ and $t\bar{t}$ thresholds respectively; in the intermediate mass region, $M_A = 200$ to 300 GeV, the decay $A \to hZ^*$ [which reaches $\sim 1\%$ already at $M_A = 130$ GeV] dominates. The gluonic decays are significant around the $t\bar{t}$ threshold.

For the charged Higgs boson, the inclusion of the three–body decay modes will reduce the branching ratio for the $\tau\nu$ channel quite significantly. Indeed, this decay does not overwhelm all the other modes since the three–body decay channels $H^+ \to hW^*$ as well as $H^+ \to AW^*$ in the low mass range and $H^+ \to bt^*$ in the intermediate mass range have appreciable branching ratios.

SUSY Decay modes

In the previous discussion, we have assumed that decay channels into neutralinos, charginos and sfermions are shut. However, these channels could play a significant role, since some of these particles [at least the lightest neutralinos, charginos, stop squarks and the sleptons] can have masses in the $\mathcal{O}(100 \text{ GeV})$ range or less.

To discuss these decays in the general case is an almost impossible task because of the many parameters that one has to deal with. We have therefore performed the analysis in the MSSM constrained by minimal Supergravity, in which the SUSY sector is described in terms of five universal parameters at the GUT scale: the common scalar mass $m_0$, the common gaugino mass $M_{1/2}$, the trilinear coupling $A$, the bilinear coupling $B$ and the higgsino mass $\mu$. These parameters evolve according to the RGEs, forming the supersymmetric particle spectrum at low energy.

The requirement of radiative electroweak symmetry breaking further constrains the SUSY spectrum, since the minimization of the one–loop Higgs potential specifies the parameter $\mu$ [to within a sign] and also $B$. The unification of the $b$ and $\tau$ Yukawa couplings gives another constraint: in the $\lambda_t$ fixed–point region, the value of $tg\beta$ is fixed by the top quark mass through: $m_t \simeq (200 \text{ GeV}) \sin \beta$, leading to $tg\beta \simeq 1.75$. There also exists a high–$tg\beta$ [$\lambda_3$ and $\lambda_\tau$ fixed–point] region for which $tg\beta \sim 50$–60 [though disfavoured by $b \to s\gamma$ and Dark Matter constraints]. If one also notes that moderate values of the trilinear coupling $A$ [e.g. $|A| \lesssim 500 \text{ GeV}$ at the GUT scale] have little effect on the resulting spectrum, then the whole SUSY spectrum will be a function of only a few parameters: $tg\beta$ which we take to be 1.75 and 50, the sign of $\mu$ [we will use the convention of Haber and Kane], $m_0$ which in practice we replace with $M_A$ taking the two illustrative values $M_A = 300$ and 600 GeV [note that for these large values, $M_H \sim M_{H^\pm} \sim M_A$], and finally the common gaugino mass $M_{1/2}$ that we will freely vary.
Fig. 3: Decay widths for the SUSY decay modes of the heavy CP–even, the CP–odd and the charged Higgs bosons, for $\tan \beta = 1.75$. The total and the non–SUSY widths are also shown.
The decay widths of the heavy CP-even, the CP–odd and the charged Higgs bosons, \( H, A \) and \( H^\pm \), into pairs of neutralinos and charginos [dashed lines], squarks [long–dashed lines] and sleptons [dot–dashed lines], as well as the total [solid lines] and non–SUSY [dotted–lines] decay widths, are shown in Fig.3 for \( \tan \beta = 1.75, \mu > 0 \) and two values of \( M_A = 300 \) [left curves] and 600 GeV [right curves].

For \( M_A = 300 \) GeV, i.e. below the \( t\bar{t} \) threshold, the widths of the decays of \( H \) into inos, sleptons and squarks are much larger than the non–SUSY decays. In particular, squark [in fact stop and sbottom only] decays are almost two–orders of magnitude larger when kinematically allowed. The situation changes dramatically for larger \( A \) masses when the \( t\bar{t} \) channel opens up: only the decays into stop pairs, when allowed, are competitive with the dominant \( H \to t\bar{t} \) channel. Nevertheless, the decays into inos are still substantial having branching ratios at the level of 20%; the decay widths into sleptons never exceed the few percent level.

In the case of the pseudoscalar \( A \), because of CP–invariance and the fact that sfermion mixing is small except in the stop sector, only the decays into inos and \( A \to \tilde{t}_1 \tilde{t}_2 \) decays are allowed. For these channels, the situation is quite similar to the case of \( H \): below the \( t\bar{t} \) threshold the decay width into ino pairs is much larger than the non–SUSY decay widths [here \( \tilde{t}_2 \) is too heavy for the \( A \to \tilde{t}_1 \tilde{t}_2 \) decay to be allowed], but above \( 2m_t \) only the \( A \to \tilde{t}_1 \tilde{t}_2 \) channel competes with the \( t\bar{t} \) decays.

For the charged Higgs boson \( H^\pm \), only the decay channel \( H^+ \to \tilde{t}_1 b_1 \) [when kinematically allowed] competes with the dominant decay mode \( H^+ \to t\bar{b} \), yet the \( \tilde{\chi}^+ \tilde{\chi}^0 \) decays have a branching ratio of a few ten percent; the decays into sleptons are at most of the order of a few percent.

In the case where \( \mu < 0 \), the situation is quite similar as above. For large \( \tan \beta \) values, \( \tan \beta \sim 50 \), all gauginos and sfermions are very heavy and therefore kinematically inaccessible, except for the lightest neutralino and the \( \tau \) slepton. Moreover, the \( b\bar{b}/\tau\tau \) and \( t\bar{b}/\tau\nu \) [for the neutral and charged Higgs bosons respectively] are enhanced so strongly, that they leave no chance for the SUSY decay modes to be significant. Therefore, for large \( \tan \beta \), the simple pattern of \( b\bar{b}/\tau\tau \) and \( t\bar{b} \) decays for heavy neutral and charged Higgs bosons still holds true even when the SUSY decays are allowed.

**Total Decay Widths**

The total widths of the SUSY Higgs particles are in general considerably smaller than the width of the SM Higgs, due to the absence or the suppression of the decays to longitudinal \( W/Z \) bosons which grow as \( G_F M_H^3 \). The dominant decay modes are built-up by top quarks so that the widths rise only linearly with the Higgs masses \( \sim G_F m_t^2 M_H \). Including the SUSY modes will not change the situation considerably since the widths still grow only linearly with the Higgs mass. However, for large \( \tan \beta \) values, the decay widths of all the five Higgs bosons are determined by \( b \)–quark final states and they scale like \( \tan^2 \beta \) [except for \( h \) and \( H \) near their maximal and minimal mass values, respectively]; the \( H, A \) and \( H^+ \) widths therefore become experimentally significant, for \( \tan \beta \) values of order \( \gtrsim 30 \) and for large Higgs masses.
3.2 Production Processes

Main Production mechanisms

At $e^+e^-$ linear colliders operating in the TeV energy range, the main production mechanism for the heavy neutral Higgs particles is the associated $H$ and $A$ production

$$e^+e^- \rightarrow Z^* \rightarrow AH$$

This cross section is proportional to $\sin^2(\beta - \alpha)$ which is close to one near the decoupling limit. If $H$ and $A$ are very heavy, the cross sections for the Higgs–strahlung process $e^+e^- \rightarrow HZ$, the fusion processes $e^+e^- \rightarrow W^*W^*(Z^*Z^*) \rightarrow H\nu_e\bar{\nu}_e(e^+e^-)$ and the pair production process $e^+e^- \rightarrow hA$ are very small since they are suppressed by a factor $\cos^2(\alpha - \beta)$ which is then close to zero. These cross sections are significant only for $H$ masses below 400 GeV and small values of $\tan\beta \sim 1.5$.

Fig.4: Cross sections for $e^+e^- \rightarrow HA$ and $e^+e^- \rightarrow H^+H^-$ as a function of the c.m. energy.

For charged Higgs particles, the production process [the cross section of which does not depend on any parameter other than $M_{H^\pm}$] proceeds through virtual photon and $Z$–boson exchange

$$e^+e^- \rightarrow \gamma^*, Z^* \rightarrow H^+H^-$$

The cross sections for the $(AH)$ and $(H^+H^-)$ processes are shown in Fig.4 as a function of the c.m. energy for $M_A [\simeq M_H \simeq M_{H^\pm}] = 300$ and 600 GeV. [In the case of $(AH)$ two values of $\tan\beta$ are shown; but as can be seen, the cross section depend only slightly on the value of $\tan\beta$, since we are close to the decoupling limit]. Since the processes are mediated by $s$–channel exchanges, the cross sections scale like $1/s$ sufficiently above the thresholds. Due to the presence of the additional photon
channel, the cross section for \((H^+H^-)\) is twice as large as for \((AH)\). At \(\sqrt{s} \sim 1.5\) TeV, the cross sections are of the order of a few fb: for a luminosity \(L = 300\) fb\(^{-1}\)/year, 500 to 1000 events could be expected. For masses above 350 GeV and in the absence of SUSY decays, the final states consist of \(b\bar{b}b\bar{b}\) [for large \(tg\beta\)] and \(tt\bar{t}t\) [for small \(tg\beta\)] in the case of \((HA)\) production and \(b\bar{b}tt\) in the case of \((H^+H^-)\) production. Efficient \(b\)-tagging through \(\mu\)-vertexing is therefore required to separate the signals from the backgrounds. Rich final states maybe be investigated if the decay channels into SUSY particle are open.

**Multiple Higgs Production**

One of the main motivations to proceed to higher c.m. energies is the availability of enough phase space to produce Higgs particles in association with heavy states. For instance, at sufficiently high energies, one can have multiple Higgs production, the cross sections of which allow the trilinear couplings among the Higgs particles to be determined. By measuring these trilinear couplings the scalar potential can be reconstructed, allowing a fundamental test of the Higgs mechanism. The multiple production of the lightest SUSY Higgs boson has been analyzed recently [see [4] for details]; the results are summarized below.

The most copious source of multiple light Higgs boson final states is the cascade decay \(H \rightarrow hh\) where the heavy CP–even neutral Higgs boson is produced either by Higgs–strahlung and associated pair production, or in the \(WW(ZZ)\) fusion mechanisms. The cross sections are sizeable for small \(tg\beta\) values and \(H\) masses below \(\sim 400\) GeV; and in this range the \(H \rightarrow hh\) branching ratio is neither too small nor too close to unity [Fig.2] if the SUSY decay channels are closed. Apart from small mass intervals, the other important decay modes are \(WW^*/ZZ^*\) decays; the \(HVV\) couplings can be measured through the production cross sections of the fusion and Higgs–strahlung processes, the branching ratio \(BR(H \rightarrow hh)\) can be exploited to measure the coupling \(\lambda_{Hhh}\).

Besides the previous process, multiple light Higgs bosons \(h\) can [in principle] be generated in the MSSM by three mechanisms: (i) double Higgs–strahlung in the continuum, with a final state \(Z\) boson \(e^+e^- \rightarrow hhZ\); (ii) associated production with the pseudoscalar \(A\) in the continuum, \(e^+e^- \rightarrow hhA\); and (iii) non–resonant \(WW\) fusion in the continuum, \(e^+e^- \rightarrow \nu_\ell\nu_\ell hh\). These processes are disfavoured by an additional power of the electroweak coupling compared to the resonance processes; nevertheless, they must be analyzed in order to measure the values of the \(hhh\) and \(hAA\) couplings. The situation for double Higgs–strahlung and \(WW\) fusion is analogous to the SM, while the process \(Ahh\) is novel.

The cross section \(\sigma(e^+e^- \rightarrow hhZ)\) is shown for \(\sqrt{s} = 500\) GeV at \(tg\beta = 1.5\) as a function of \(M_h\) in Fig.5a. For small masses, the cross section is built–up almost exclusively by \(H \rightarrow hh\) decays [dashed curve], except close to the point where \(\lambda_{Hhh} \sim 0\). For intermediate masses, the resonance contribution is reduced and, in particular above 90 GeV where the decoupling limit will be approached, the continuum \(hh\) production becomes dominant, finally falling down to the SM cross section [dotted line]. After subtracting \(hh\) decays, the continuum cross section is of the same order,
\( \sim 0.5 \text{ fb} \) as in the SM. Very high luminosity is therefore needed to measure the trilinear \( hhh \) coupling. At higher energies, since the cross section scales like \( 1/s \), the rates are smaller. Prospects are similar for large \( tg\beta \) values. The cascade decay \( H \rightarrow hh \) is restricted to a range \( M_h \lesssim 70 \text{ GeV} \), with a cross section of \( \sim 20 \text{ fb} \) at \( \sqrt{s} = 500 \text{ GeV} \) and \( \sim 3 \text{ fb} \) at 1.5 TeV. The continuum cross sections are of the order of 0.1 fb at both energies, so that very high luminosities will be needed.

Fig. 5: Cross sections for double Higgs production as a function of \( M_h \) at \( tg\beta = 1.5 \): a) \( e^+e^- \rightarrow hhZ \) at \( \sqrt{s} = 500 \text{ GeV} \), and (b) \( WW \rightarrow hh \) at \( \sqrt{s} = 1.5 \text{ TeV} \).

The analysis has been repeated for the continuum process \( e^+e^- \rightarrow Ahh \). However, it turned out that the cross section is built–up almost exclusively by the resonant process \( e^+e^- \rightarrow AH \rightarrow Ahh \), with a very small continuum contribution, so that the coupling \( \lambda_{hAA} \) cannot be measured in this process.

The total cross section \( \sigma(e^+e^- \rightarrow \nu\bar{\nu}hh) \) [obtained in the effective longitudinal \( W \) approximation] is shown in Fig. 5b as a function of \( M_h \) for \( tg\beta = 1.5 \) at \( \sqrt{s} = 1.5 \text{ TeV} \). It is significantly larger than for double Higgs–strahlung in the continuum. Again, for very light Higgs masses, most of the events are \( H \rightarrow hh \) decays [dashed line]. The continuum \( hh \) production is of the same size as pair production of SM Higgs bosons [dotted line] which as anticipated, is being approached near the upper limit of the \( h \) mass in the decoupling limit. The size of the continuum \( hh \) fusion cross section renders this channel more promising than double Higgs–strahlung for the measurement of the trilinear \( hhh \) coupling. For large \( tg\beta \) values, strong destructive interference effects reduce the cross section in the continuum to very small values, of order \( 10^{-2} \text{ fb} \), before the SM cross section is reached again in the decoupling limit. As before, the \( hh \) final state is almost exclusively built–up by the \( H \rightarrow hh \) decays.
4. Summary

$e^+e^-$ linear colliders with center of mass energies larger than $\sim 500$ GeV will be needed to search for Higgs particles in the mass range above $M_H \sim 250$ GeV. We have summarized the rather rich potential of an $e^+e^-$ linear collider operating at a center of mass energy of $\sqrt{s} \sim 1.5$ TeV.

The search for the SM Higgs particle can be extended to the entire mass range below $O(1$ TeV). This search can be carried out in several complementary channels. The clean environment of the collider allows to investigate thoroughly the properties of the Higgs boson: the mass and decay widths can be precisely measured and the measurement of its couplings to gauge bosons and heavy fermions as well as its self–coupling can be performed. These measurements will allow a stringent test of the Higgs mechanism, and provide a window for new phenomena beyond the SM.

In Supersymmetric extensions of the SM, one would have access to the heavy Higgs particles. In the MSSM, the heavy CP even, the pseudoscalar and the charged Higgs bosons with masses up to the beam energy can be found if the luminosity is high–enough, $\int L \gtrsim 100$ fb$^{-1}$ at $\sqrt{s} = 1.5$ TeV. The measurement of production cross sections [including the cross sections for multiple Higgs production], branching ratios [especially in the case where decay modes into supersymmetric particles are present] allows fundamental tests of this extension of the SM.

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References

1. For summaries, see H.E. Haber, Proc. “Physics and Experiments with Linear Colliders”, Saariselkä 1991; F. Zwirner, ibid; S. Komamiya, ibid; J.F. Gunion, Proc. “Physics and Experiments with Linear Colliders”, Waikolaoa, 1993; P. Janot, ibid; Y. Okada, these proceedings; R. van Kooten, ibid.
2. Proceedings of the Workshop “$e^+e^-$ Collisions at 500 GeV: the Physics Potential”, DESY Report 92–123, P. Zerwas ed.
3. For reviews on Higgs Physics, see J. Gunion, H. Haber, G. Kane and S. Dawson, The Higgs Hunter’s Guide, Addison–Wesley, Reading 1990; P.M. Zerwas, Proceedings of the International Conference on High–Energy Physics, Marseille, 1993; A. Djouadi, Int. J. Mod. Phys. A10 (1995) 1.
4. A. Djouadi, H.E. Haber, P. Igo–Kemenes, P. Janot and P.M. Zerwas [conv.] et al., Proceedings of the European Workshop “Physics with $e^+e^-$ Linear Colliders”, Annecy–Gran Sasso–Hamburg, 1995.
5. Talk given by T. Barklow, these proceedings.
6. Talk given by D. Miller, these proceedings.