The Super-Stückelberg procedure and dS in pure supergravity

Silvia Nagy¹, Antonio Padilla¹ and Ivonne Zavala²

¹School of Physics and Astronomy, University of Nottingham, Nottingham NG7 2RD, UK
²Department of Physics, Swansea University, Singleton Park, Swansea SA2 8PP, UK

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Author for correspondence:
Silvia Nagy
e-mail: silvia.nagy@nottingham.ac.uk

Understanding de Sitter space in supergravity—and string theory—has led to an intense amount of work for more than two decades, largely motivated by the discovery of the accelerated expansion of the Universe in 1998. In this paper, we consider a non-trivial generalization of unimodular gravity to minimal $\mathcal{N}=1$ supergravity, which allows for de Sitter solutions without the need of introducing any matter. We formulate a superspace version of the Stückelberg procedure, which restores diffeomorphism and local supersymmetry invariance. This introduces the goldstino associated with spontaneous breaking of supersymmetry in a natural way. The cosmological constant and gravitino mass are related to the vacuum expectation value of the components of a Lagrange multiplier imposing a super-unimodularity condition.

1. Introduction

One of the most challenging problems in fundamental physics today is the surprising discovery about 20 years ago [1,2], that the Universe’s expansion is currently accelerating. According to the ‘standard model’ of cosmology, the $\Lambda$CDM ($\Lambda$-Cold-Dark-Matter) model, this acceleration is driven by a tiny constant energy density, $\Lambda$, whose finely tuned value is the subject of the so-called cosmological constant problem [3–8]. Regardless of how this fine tuning is resolved, be it through anthropic arguments [9–11] or some other mechanism (e.g. [12–17]), observational evidence seems to suggest that our universe is asymptotically de Sitter (dS) with a tiny vacuum energy. This has led to intense theoretical activity to construct dS vacua in string theory and supergravity (see [18] for a recent review).
In the context of supergravity, a long-standing question has been the possibility of finding dS solutions in the pure $\mathcal{N} = 1$ model (the simplest supergravity theory). The difficulty is related to the absence of scalar fields (which only appear in matter supermultiplets in $\mathcal{N} = 1$, see [19]). In string theory, the KKLT construction of de Sitter vacua proposed in 2003 [20] led to the so-called landscape (see [21] for a review). However, the consistency of these solutions has recently been called into question, so much so that stable de Sitter vacua are conjectured to be part of the string theory swampland (see [18,22,23] for recent reviews and references). Given the observational evidence in favour of a small and positive vacuum energy, the proof or disproof of this conjecture is of crucial importance. Although our immediate focus is on supergravity, one of the long-term motivations for our work is to open up new ideas for seeking de Sitter vacua in string theory.

Returning to KKLT, one of its key ingredients is the uplift from a supersymmetric AdS vacuum to a dS vacuum via the introduction of an anti-D3-brane. Recent developments have made good progress in clarifying the role of the anti-D3-brane in providing de Sitter vacua with spontaneously broken supersymmetry in terms of a 4D, $\mathcal{N} = 1$ supersymmetric action [24–26]. It corresponds to a globally supersymmetric Volkov–Akulov (VA) Goldstino theory [27,28] coupled to a supergravity background. The crucial ingredient in this approach is the use of a nilpotent constrained superfield, which is a regular chiral superfield, $X$, that satisfies the constraint, $X^2 = 0$. This constraint eliminates the scalar component of the chiral multiplet, which requires that supersymmetry be nonlinearly realized. 1 A complete local supergravity action with nonlinearly realized supersymmetry using constrained superfields was developed in [59–69]. These developments allowed for the construction of pure $\mathcal{N} = 1$ models which admit dS solutions. In particular, the strategy of [60] was to introduce a (chiral) Lagrange multiplier, $\Lambda$, in the superconformal theory. In the superconformal action, supersymmetry is linearly realized as long as the Lagrange multiplier is present. When the equation of motion for $\Lambda$ is solved, it leads to a constraint on the chiral superfield. 2 For a review of constrained superfields and applications, see [71].

In this paper, we take a very different approach, inspired by classical unimodular gravity [72–79]. Unimodular gravity is a gauge fixed version of General Relativity (GR), in which the determinant of the metric is fixed to a constant so that the resulting field equations correspond to the traceless Einstein equations. The equivalence to GR is easily demonstrated by taking the divergence of these equations and making use of the Bianchi identity. This results in the standard Einstein equations sourced by the energy momentum tensor and a cosmological constant. The only subtlety is that the cosmological constant enters as an integration constant. Although this does not help with the cosmological constant problem [79], it does suggest a new way of thinking about de Sitter solutions. In unimodular gravity, it turns out that one can identify the cosmological constant, positive or otherwise, with the vacuum expectation value (vev) of a Lagrange multiplier imposing the constraint on the determinant of the metric [75,79]. In this paper, we explore a similar idea in the context of supergravity, fixing the superdeterminant with a Lagrange multiplier, then identifying a component of its vev with a cosmological constant of arbitrary sign. As with unimodular gravity, super-unimodular gravity can be written in a way that respects an extended gauge symmetry—diffeomorphisms and local supersymmetry—with the help of a Stückelberg trick. This invariant theory admits de Sitter vacua with supersymmetry spontaneously broken by the vev of the Lagrange multiplier. As expected, the theory of fluctuations about these solutions yields a massive gravitino coupled to a goldstino on a de Sitter background (see also [60]). The goldstino is trivially identified with a Stückelberg field and, in the appropriate limit, we recover the VA action.

The rest of this paper is organized as follows: in the next section, we review unimodular gravity, the role of the cosmological constant and the Stückelberg procedure for restoring the full set of diffeomorphisms. In §3, we extend these ideas to supergravity, demonstrating explicitly

1 Constrained superfields have proved important for building cosmological and phenomenological models in the framework of supergravity and string theory, see e.g. [24,29–58].

2 See [70] for generalizations.
how one can obtain de Sitter vacua thanks to the vev of the Lagrange multiplier. We further study the theory of fluctuations about these vacua, showing how we recover the theory of a massive gravitino coupled to a goldstino. We conclude in §4 having transferred some of the calculational details to the appendices.

2. Unimodular gravity

Unimodular gravity [72–79] is obtained from a restricted variation of the Einstein–Hilbert action with matter, where the determinant of the metric is fixed to a constant value, traditionally taken to be −1. Because of this restriction, the theory is only invariant under transverse diffeomorphisms rather than the full diffeomorphism group. The resulting field equations correspond to the traceless part of Einstein equations:

$$R_{\mu\nu} - \frac{1}{4} R g_{\mu\nu} = 8\pi G_N \left( T_{\mu\nu} - \frac{1}{4} T g_{\mu\nu} \right), \quad (2.1)$$

where $G_N$ is Newton’s constant. Because the vacuum energy contributes to the energy-momentum tensor as a pure trace, it appears as if it will drop out of the field equations, thereby alleviating the cosmological constant problem. However, as explained in [78,79], this is not the case. Unimodular gravity is locally equivalent to GR. This follows trivially from the fact that the unimodularity condition, fixing the determinant of the metric, corresponds to a local gauge choice. One can see this explicitly by taking the divergence of (2.1) and then integrating the resulting equation. This reveals a hidden equation of the form $R + 8\pi G_N T = \Lambda$, where the cosmological constant, $\Lambda$, enters as an integration constant. Radiative corrections to the vacuum energy are contained in the trace of the energy-momentum tensor. As $\Lambda$ is fixed by the boundary conditions, the asymptotic curvature is as sensitive to these radiative corrections as it is in GR.

Our interest in unimodular gravity has nothing to do with its failed assault on the cosmological constant problem. Rather, it arises from the way in which the cosmological constant enters the game—as an integration constant—which is qualitatively different to how it enters in GR. We can unpack this a little further by imposing the unimodularity condition, $\sqrt{-g} = \epsilon_0$, with a Lagrange multiplier in the Einstein–Hilbert action. The action for unimodular gravity can be written as

$$S = \frac{1}{16\pi G_N} \int d^4x \left[ \sqrt{-g} R - 2\Lambda(x) \left( \sqrt{-g} - \epsilon_0 \right) \right] + S_m[\gamma_{\mu\nu}, \Psi], \quad (2.2)$$

with $\epsilon_0$ being a constant (traditionally set to unity). Here $\Lambda(x)$ is a Lagrange multiplier field, imposing a local constant on the determinant of the metric. This version of the Lagrangian is not invariant under the full diffeomorphism group (Diff) but only under a subgroup of transformations called transverse diffeomorphism (TDiff), whose parameter satisfies $\nabla^\mu \xi_\mu = 0$. Since the Lagrange multiplier, $\Lambda(x)$, transforms like a scalar, it is obvious that Diffs are broken by the last term $\int d^4x \Lambda \epsilon_0$. We shall assume that the action for the matter fields, $S_m[\gamma_{\mu\nu}, \Psi]$ is Diff-invariant.

Variation with respect to the metric yields an equation of motion of the form

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = -\Lambda(x) g_{\mu\nu} + 8\pi G_N T_{\mu\nu}. \quad (2.3)$$

If we eliminate $\Lambda(x)$, we arrive at the traceless equations given in (2.1). Alternatively, we can take the divergence of (2.3) and combine it with the Bianchi identity, forcing the Lagrange multiplier to be a constant. Thus, we recover the equations of motion (e.o.m.) for GR with the constant vev of the Lagrange multiplier taking on the role of the cosmological constant.

(a) The Stückelberg procedure

The unimodularity condition imposed through (2.2) amounts to a local gauge choice, so we can restore the full general coordinate invariance through the Stückelberg procedure [80] of introducing an extra field, which transforms appropriately. We can do this by performing
a general coordinate transformation (gct) $x^\mu \rightarrow \hat{x}^\mu(x)$. The original part of the Lagrangian is invariant under it, and on the Diff-breaking term we get

$$2 \int d^4x \Lambda(x)\epsilon_0 \rightarrow 2 \int d^4\hat{x} \hat{\Lambda}(\hat{x})\epsilon_0 = 2 \int d^4x/J|\Lambda(x)\epsilon_0,$$  

(2.4)

where $|J| = \text{Det}(\partial \hat{x}^\mu/\partial x^\alpha)$. To perform the Stückelberg trick, we take this transformed action and promote the coordinate transformation $\hat{x}^\mu(x)$ to four new fields $s^\mu(x)$. The new action

$$S_{SI} = \frac{1}{16\pi G_N} \int d^4x \left[ \sqrt{-g}R - 2\left(\sqrt{-g} - \text{Det}\left(\frac{\partial s}{\partial \hat{x}}\right)\epsilon_0\right) \right],$$  

(2.5)

is now invariant under the full diffeomorphism group, provided that the Stückelberg fields $s^\mu(x)$ transform as scalars:

$$\hat{s}^\mu(\hat{x}(x)) = s^\mu(x).$$

(2.6)

This particular representation of unimodular gravity can be found in [76] and is also discussed in [79]. There exists a closely related version of the theory where the Stückelberg fields are repackaged in terms of a four-form field strength [75]. With Diffs fully restored, the constancy of the Lagrange multiplier arises directly from the e.o.m. associated with the Stückelberg fields.

Finally, we note that the Stückelberg trick was performed by acting with a finite transformation on the coordinates. However, in what follows, it will prove instructive to see how to restore diffeomorphism invariance order by order, by performing an infinitessimal transformation either in the passive form:

$$x^\mu \rightarrow x^\mu + \xi^\mu + \frac{1}{2} \xi^\rho \partial_\rho \xi^\mu + \cdots,$$

(2.7)

or in the active form, where we vary the scalar field $\Lambda$:

$$\delta \Lambda = -\xi^\rho \partial_\rho \Lambda - \frac{1}{2} \xi^\rho \partial_\rho \xi^\mu \partial_\mu \Lambda + \cdots.$$  

(2.8)

The perturbative form of the Stückelberg trick now involves promoting the transformation parameters to fields: $\xi^\mu \rightarrow \phi^\mu$. The Lagrangian can then be constructed order by order in the Stückelberg field $\phi^\mu$

$$\mathcal{L} = \sqrt{-g}R - 2\sqrt{-g} + 2\Lambda \epsilon_0 \left[ 1 + \partial_\mu \phi^\mu + \frac{1}{2} \phi^\rho \partial_\rho \partial_\mu \phi^\mu + \frac{1}{2} \left( \partial_\mu \phi^\mu \right) \left( \partial_\rho \phi^\rho \right) + \cdots \right],$$

(2.9)

and it will be invariant, up to the relevant order, when $\phi^\mu$ transforms as $^3$:

$$\delta \phi^\mu = -\xi^\mu - \frac{1}{2} \xi^\rho \partial_\rho \phi^\mu + \frac{1}{2} \phi^\rho \partial_\rho \xi^\mu + \cdots.$$  

(2.10)

### 3. Unimodular supergravity

We now show how to extend the procedure we discussed in §2a to the pure $\mathcal{N} = 1$ supergravity model, in the ‘old minimal’ formulation. We will see that this allows for solutions with a cosmological constant of arbitrary sign. The goldstino field will be included naturally through the Stückelberg procedure, and the cosmological constant will appear as a vev in our description.

We work with the conventions of [19]. The pure supergravity action in chiral superspace with coordinates $X^M = (x^\mu, \Theta^\alpha)$, is given by

$$S = \frac{6}{8\pi G_N} \int d^4x d^2\Theta \varepsilon R + h.c.$$  

(3.1)

$^3$Note that we are using $\xi$ again to denote the transformation parameter, after performing the Stückelberg trick.
The components of $R$ are given in appendix A and
\[
E = F_0 + \sqrt{2} \Theta F_1 + \Theta \Theta F_2, \quad \text{with}
\begin{align*}
F_0 &= \frac{1}{2}e, \\
F_1 &= \frac{i \sqrt{2}}{4} e \sigma^\mu \tilde{\psi}_\mu, \\
F_2 &= -\frac{1}{2} e M^* - \frac{1}{8} e \bar{\psi}_\mu (\tilde{\sigma}^\mu \sigma^v - \tilde{\sigma}^v \sigma^\mu) \bar{\psi}_v,
\end{align*}
\]
and
\[
E = \frac{1}{2} e M^* - \frac{1}{8} e \bar{\psi}_\mu (\tilde{\sigma}^\mu \sigma^v - \tilde{\sigma}^v \sigma^\mu) \bar{\psi}_v,
\]
where $e = \det e^\mu_\nu$ with $e^\mu_\nu$ the vielbein, $\psi_\mu$ the gravitino and $M$ the scalar auxiliary field in the old minimal supergravity model. Here $E$ is a chiral density superfield, characterized by the transformation law
\[
\delta E = - \partial_N \left[ (-1)^N \eta^N E \right],
\]
where
\[
(-1)^N = \begin{cases} 1, & N = \mu \\ -1, & N = \alpha \end{cases}
\]
and
\[
\eta^{\mu}(\epsilon) = \Theta^\beta y^{\mu}_{1\beta}(\epsilon) + \Theta^2 y^{\mu}_{2}(\epsilon),
\]
and
\[
\eta^\alpha(\epsilon) = \epsilon^\alpha + \Theta^\beta \Gamma^\alpha_1(\epsilon) + \Theta^2 \Gamma^\alpha_2(\epsilon),
\]
where $\epsilon$ is the parameter of local SUSY transformations. For conciseness, we introduced the following notation:
\[
\begin{align*}
y^{\mu}_{1\beta}(\epsilon) &= 2i (\sigma^\mu \epsilon)^\beta_\alpha, \\
y^{\mu}_{2}(\epsilon) &= \bar{\psi}_\mu \tilde{\alpha}^\mu e^\alpha \epsilon, \\
\Gamma^\alpha_1(\epsilon) &= i (\sigma^\mu \epsilon)^\beta_\alpha, \\
\Gamma^\alpha_2(\epsilon) &= -i \omega^\alpha_\mu (\sigma^\mu \epsilon)^\beta_\alpha + \frac{1}{3} M^* \epsilon^\alpha \\
&- \frac{1}{2} \psi^\alpha \left( \bar{\psi}_\mu \tilde{\sigma}^\mu \sigma^\alpha \bar{\epsilon} + \frac{1}{6} b^\mu_\alpha (\epsilon \sigma^\mu \epsilon)^\alpha \right),
\end{align*}
\]
where $\omega^\alpha_\mu$ is the spin connection and $b^\mu_\alpha$ is the vector auxiliary field in the old minimal model. A chiral density superfield can thus be thought of as the supersymmetric analogue of the scalar density, and $E$ in particular is the supersymmetric version of the measure $\sqrt{-g}$. We define the unimodular supergravity action to be:
\[
S = \frac{6}{8 \pi G_N} \int d^4x d^2 \Theta \left[ ER - 2 \Lambda (E - E_0) \right] + h.c.,
\]
where
\[
\Lambda = \Lambda_0 + \sqrt{2} \Theta A_1 + A_2 \Theta^2,
\]
is now a Lagrange multiplier chiral superfield, and we defined
\[
E_0 = \epsilon_0 - \frac{1}{2} m \Theta^2,
\]
with $\epsilon_0$ and $m$ real constants. Varying over $\Lambda$, we get
\[
E = E_0,
\]
which is the SUSY analogue of the unimodularity condition. In components, this condition reads:
\[
\begin{align*}
\frac{1}{2} e &= \epsilon_0, \\
\frac{i \sqrt{2}}{4} e \sigma^\mu \tilde{\psi}_\mu &= 0 \\
- \frac{1}{2} e M^* - \frac{1}{8} e \bar{\psi}_\mu (\tilde{\sigma}^\mu \sigma^v - \tilde{\sigma}^v \sigma^\mu) \bar{\psi}_v &= -\frac{1}{2} m,
\end{align*}
\]
We take the spinor component of $E_0$ to vanish for simplicity.

An alternative formulation of unimodular supergravity, in component form, was given in [81]. The model in [81] can be shown to be equivalent to the $m = 0$ limit in our approach.
The action (3.7) is invariant under a restricted set of SUSY and diffeo transformations, exactly such that they preserve the conditions in (3.11):

$$0 = \delta \mathcal{E} = -\partial_M \left[ (-1)^M \Sigma^M \mathcal{E} \right] \text{TSdiff} \quad (3.12)$$

where

$$\Sigma^M = (\xi^\mu + \eta^\mu(\epsilon), \eta^\alpha(\epsilon)),$$  

contains both the diffeo and the SUSY parameter. Note that the constraint imposed through (3.10) amounts to more than a gauge fixing; in this sense, unimodular supergravity is not a naive supersymmetrization of the unimodular gravity model in §2. As a consequence, even though the space of solutions of unimodular gravity matches that of standard Einstein gravity with a cosmological constant, this will not be the case for our model, in the sense that we will not be restricted to AdS and flat space backgrounds.

Finally, we impose the following boundary conditions on our Lagrange multiplier superfield:

$$A_0\big|_\infty = K_0, \quad A_1\big|_\infty = 0, \quad A_2\big|_\infty = K_2,$$  

with $K_0, K_2$ some constants.

(a) The Super-Stückelberg procedure

The Stückelberg trick performed in §2a can be extended to any local transformation, and here we extend it to encompass both diffeomorphisms and local supersymmetry transformations.\(^6\) We now perform a transformation $X^M \rightarrow Y^M(X)$ to the superspace coordinates\(^7\) on the non-invariant term:

$$\int d^4x d^2 \Theta \Lambda(X) \mathcal{E}_0 \rightarrow \int d^4y d^2 \Gamma \Lambda'(Y) \mathcal{E}_0 = \int d^4x d^2 \Theta |s| \Lambda(X) \mathcal{E}_0, \quad (3.15)$$

where $|s| = \text{Ber}(\partial_Y Y^M / \partial X^M)$ is the Berezinian.\(^8\) Crucially, unlike in the gravity case, $Y^M$ will not be a general superfunction of $X^M$, but will depend on the diffeomorphism and SUSY parameters in a very particular way. Its construction is given perturbatively below.

At linear order in the parameters, we have the usual transformation

$$\delta^{(1)} X^M = \Sigma^M, \quad (3.16)$$

with $\Sigma^M$ defined in (3.13) with the notation we introduced in (3.6). At second order, we have

$$\delta^{(2)} X^M = \frac{1}{2} \Sigma^R \partial_R \Sigma^M + \frac{1}{2} \delta^{(s)} \left( \Sigma^M \right), \quad (3.17)$$

where the first term is the usual one at second order, while the second one takes into account the fact that the objects (3.6) appearing in the SUSY transformations of the coordinates are not arbitrary functions, but depend on the supergravity fields $e_\mu, \psi_\mu, b_\mu, M$. The explicit form of these transformations is given in appendix B. We can thus proceed, order by order in the transformation parameters, and write:

$$X^M \rightarrow X^M + \delta^{(1)} X^M + \delta^{(2)} X^M + \cdots = Y^M(X, \Sigma). \quad (3.18)$$

\(^6\)See [82,83] for an alternative use of the Stückelberg trick in supergravity; in this formulation, the action is constructed from invariant one-forms, reminiscent of the original Volkov–Akulov construction [27].

\(^7\)We will often omit the spinor indices for simplicity.

\(^8\)The Berezinian, Ber, is the generalization of the determinant to supermatrices—see (3.22) for its definition.
The power of the Stückelberg procedure is that it automatically constructs a density superfield (as given in (B 1)), but the Stückelberg fields which transform nonlinearly are embedded in the components of a standard chiral superfield such a way that it will be transformed when we perform the Stückelberg procedure.

\[
\xi^\mu \rightarrow \phi^\mu \\
\epsilon \rightarrow \zeta
\]
\[
\mathcal{S}^M = (\xi^\mu + \chi^\mu (\epsilon), \eta^\alpha (\epsilon)) \rightarrow \phi^M = (\phi^\mu + \eta^\mu (\zeta), \eta^\alpha (\zeta))
\]
\[
\gamma^M(X, \mathcal{S}) \rightarrow \Phi^M(X, \varphi).
\]

At this point, we can construct the Lagrangian:
\[
S = \frac{6}{8\pi G_N} \int d^4 x d^2 \theta \left[ E R - 2 \Delta \left( E - \text{Ber} \left( \frac{\partial\Phi}{\partial X} \right) \epsilon_0 (\Phi) \right) \right] + h.c.
\]

(3.19)

In components, the Berezinian superfield is given by
\[
S = S_0 + \sqrt{2} \Theta S_1 + S_2 \Theta^2.
\]

(3.23)

where
\[
S_0 = 1 + \frac{1}{2} \partial_\mu \phi^\mu - \text{Tr}(\Gamma_1 (\xi)) + \frac{1}{2} \partial_\mu \left[ \phi^\mu \partial_\nu \phi^\nu - \phi^\mu \text{Tr}(\Gamma_1 (\xi)) \right]
\]
\[
- \frac{1}{2} \text{Tr}(\Gamma_1 (\xi)) \partial_\mu \phi^\mu + \frac{1}{2} \phi^\rho \partial_\rho \left[ b_\mu \partial_\nu \phi^\nu \right] + \frac{1}{2} M^\nu \partial_\nu \xi
\]
\[
+ \frac{1}{2} \bar{M}^\nu \partial_\nu \bar{\xi} + \frac{1}{8} b_\mu \xi \partial_\mu \bar{\xi},
\]
\[
\sqrt{2} S_1 = \partial_\mu y_1^\mu (\xi) + 2 \Gamma_2 (\xi) + \frac{1}{2} \partial_\mu \left[ \phi^\mu \partial_\nu y_1^\nu (\xi) + y_1^\nu (\xi) \partial_\nu \phi^\mu \right]
\]
\[
+ 2 \phi^\mu \partial_\nu \left[ \frac{1}{2} \xi^\nu \partial_\mu \phi^\nu \right] + \Gamma_2 (\xi) \partial_\mu \phi^\mu
\]
\[
- \frac{1}{2} \partial_\mu \left[ \phi^\rho \partial_\rho \left[ 2 i \sigma^\mu \right] \xi - \partial_\rho \phi^\mu y_1^\nu (\xi) \right]
\]
\[
- \phi^\rho \partial_\rho \left[ - i \epsilon \sigma^\mu \partial_\mu \right] \xi - \phi^\rho \partial_\rho M^\nu \frac{1}{2} \xi - \frac{1}{8} \phi^\rho \partial_\rho \left[ b_\mu \epsilon \sigma^\mu \right] \bar{\xi},
\]
\[
\sqrt{2} S_2 = \partial_\mu y_2^\mu (\xi) + \frac{1}{2} \partial_\mu \left[ \phi^\mu \partial_\nu y_2^\nu (\xi) + y_2^\nu (\xi) \partial_\nu \phi^\mu \right]
\]
\[
- \frac{1}{2} y_1^\nu (\xi) \partial_\nu y_2^\nu (\xi) - \text{Tr}(\Gamma_1 (\xi) D_\nu y_2^\nu (\xi))
\]
\[
- \phi^\rho \partial_\rho \left[ \psi_i \bar{\sigma}^\mu \sigma^\nu \right] \bar{\xi} + \partial_\rho \phi^\mu y_2^\rho (\xi) - \frac{4}{3} M^\nu \xi \omega^\mu \bar{\xi}
\]
\[
- i b_\nu \bar{\xi} \bar{\sigma}^\mu \sigma^\nu \bar{\xi} + \frac{4}{3} b^\nu \bar{\xi}.
\]

The crucial point here is that $S_0$, $S_1$ and $S_2$ transform like the components of a standard chiral density superfield (as given in (B 1)), but the Stückelberg fields $\xi$ and $\phi^\mu$ transform nonlinearly. The power of the Stückelberg procedure is that it automatically constructs $S = \text{Ber}(\partial \Phi^M / \partial X^N)$ in such a way that fields which transform nonlinearly are embedded in the components of a standard chiral superfield.

\[\text{Note that, unlike in the gravity case, the symmetry breaking term } \xi_0 = \epsilon_0 - \frac{1}{2} m \theta^2 \text{ depends on the superspace coordinates, so it will be transformed when we perform the Stückelberg procedure.}\]
Finally, the SUSY transformations of the Stückelberg fields $\phi^\mu$ and $\zeta$ are:

$$\delta \phi^\mu = \frac{1}{2} \left[ \xi^\mu (\epsilon) - \epsilon y_1^\mu (\zeta) \right]$$

and

$$\delta \zeta = -\epsilon + \frac{1}{2} \phi^\rho \partial_\rho \epsilon.$$  \hspace{1cm} (3.25)

Above we performed the Stückelberg trick in the passive form. Just as in the gravity case, the invariant action can alternatively be obtained through an active transformation on the components of the Lagrange multiplier superfield, $\Lambda$. In components, the symmetry-breaking terms following from (3.7) are:

$$2\Lambda_2 \epsilon_0 - m\Lambda_0 + h.c.$$  \hspace{1cm} (3.26)

We want to perform the Super-Stückelberg trick up to second order, so we need to know the second-order transformation of the component of a chiral superfield in curved space. We can work this out from the general rule [84]:

$$\Lambda'(X') = \Lambda(X),$$  \hspace{1cm} (3.27)

and making use of (3.17) we get (working up to second order in the fermions):

$$\delta \Lambda_0 = -\xi^\mu \partial_\mu \lambda_0 - \sqrt{2} \epsilon \lambda_1 + \frac{1}{2} \xi^\rho \partial_\rho \big( \xi^\mu \partial_\mu \lambda_0 \big) + \frac{1}{2} \epsilon y_1^\mu (\epsilon) \partial_\mu \lambda_0$$

$$+ \frac{\sqrt{2}}{2} \xi^\mu \partial_\mu \big( \epsilon \lambda_1 \big) + \frac{\sqrt{2}}{2} \xi^\mu \epsilon \partial_\mu \lambda_1 + A_2 \epsilon^2,$n

$$\delta \Lambda_1 = -\xi^\mu \partial_\mu \lambda_1 - \sqrt{2} \lambda_1 (\epsilon) \partial_\mu \lambda_0 - \sqrt{2} \lambda_2 \epsilon + \frac{\sqrt{2}}{2} \xi^\rho \partial_\rho \big( y_1^\mu (\epsilon) \partial_\mu \lambda_0 \big) + \frac{\sqrt{2}}{2} y_1^\mu (\epsilon) \partial_\rho \big( \xi^\mu \partial_\mu \lambda_0 \big)$$

$$+ \frac{1}{2} \xi^\rho \partial_\rho \big( \xi^\mu \partial_\mu \lambda_1 \big) + \frac{\sqrt{2}}{2} \xi^\rho \partial_\rho \big( \epsilon \lambda_2 \big) + \frac{\sqrt{2}}{2} \epsilon \xi^\mu \partial_\mu \lambda_2$$

$$+ \frac{i\sqrt{2}}{4} \xi^\rho \partial_\rho [\sigma^\mu] \epsilon - \frac{\sqrt{2}}{4} \partial_\rho \xi^\mu y_1^\rho (\epsilon),$$

$$\delta \Lambda_2 = -\xi^\mu \partial_\mu \lambda_2 + \frac{\sqrt{2}}{2} y_1^\mu (\epsilon) \partial_\mu \lambda_1 - y_2^\mu (\epsilon) \partial_\mu \lambda_0 - \text{Tr}(\Gamma_1(\epsilon)) \lambda_2 - \sqrt{2} \Gamma_2(\epsilon) \lambda_1$$

$$+ \frac{1}{2} \xi^\rho \partial_\rho \bigg[ \xi^\mu \partial_\mu \lambda_2 - \frac{\sqrt{2}}{2} y_1^\mu (\epsilon) \partial_\mu \lambda_1 + y_2^\mu (\epsilon) \partial_\mu \lambda_0 \bigg]$$

$$+ \text{Tr}(\Gamma_1(\epsilon)) \lambda_2 + \sqrt{2} \Gamma_2(\epsilon) \xi^\mu \partial_\mu \lambda_1$$

$$+ y_1^\mu (\epsilon) \partial_\mu \lambda_0 + 2 \epsilon \lambda_2 \bigg] + \frac{1}{2} y_\rho^\mu (\epsilon) \partial_\rho \bigg[ \phi^\mu \partial_\mu \lambda_0 \bigg]$$

$$+ \frac{1}{2} \text{Tr}(\Gamma_1(\epsilon)) \bigg[ \xi^\mu \partial_\mu \lambda_2 + \frac{\sqrt{2}}{2} \Gamma_2(\epsilon) \xi^\mu \partial_\mu \lambda_1 \bigg]$$

$$+ \frac{\sqrt{2}}{4} \bigg[ -\xi^\rho \partial_\rho \bigg[ 2i\sigma^\mu \epsilon \bigg] + \partial_\rho \xi^\mu y_\rho^\mu (\epsilon) \bigg] \partial_\mu \lambda_1 \lambda_a$$

$$- \frac{1}{2} \left[ -\frac{1}{2} y_1^\mu (\epsilon) D_\mu y_1^\mu (\epsilon) - \xi^\rho \partial_\rho \bigg[ \bar{\psi}_\mu \bar{\sigma}^\mu \sigma^\nu \bigg] \bar{\epsilon} + \partial_\rho \xi^\mu y_\rho^\mu (\epsilon) - \frac{i}{2} M^a \epsilon \sigma^\mu \bar{\epsilon} \right.$$

$$- ib\bar{\epsilon} \bar{\sigma}^\mu \sigma^\nu \bar{\epsilon} + \frac{i}{2} b \bar{\epsilon} \bar{\psi}_\mu \epsilon \bigg] \lambda_0 - \frac{1}{2} \bigg[ -\partial_\mu \epsilon y_\mu^\mu (\epsilon) - \xi^\rho \partial_\rho \bigg[ i\psi_\mu \sigma^\mu \bigg] \bar{\epsilon} \bigg.$$
Finally, the full action, up to second order in the Stückelberg fields, obtained either through a passive (3.20) or an active (3.29) transformation (setting $\epsilon_0 = \frac{1}{2}$) is

$$S = \frac{1}{16\pi G_N} \left[ \sqrt{-g} \left[ R - \frac{2}{3} M^* M + \frac{2}{3} b^{\mu} b_{\mu} + \epsilon^{\mu \nu \rho \sigma} \left( \bar{\psi}_\mu \tilde{D}_\nu \bar{\psi}_\sigma - \bar{\psi}_\mu \sigma_{\nu} \tilde{D}_\rho \bar{\psi}_\sigma \right) \right] ight. $$

$$ + \frac{1}{2} \sqrt{-g} \left[ - 2 A_2 + \sqrt{2} i \Lambda_1 \sigma^\mu \bar{\psi}_\mu + 2 A_0 (\bar{\psi}_\mu \tilde{\sigma}^{\mu \nu} \psi_\nu + M^*) + h.c. \right] $$

$$ + \left[ \left\{ \begin{array}{l}
A_0 \left( \bar{\psi}_\mu \tilde{D}_\nu \psi_\sigma - m - m \partial_\nu \phi^{\mu \nu} \right) - \Lambda_1 \left( \frac{\sqrt{2}}{2} \partial_\nu \psi_\sigma + \sqrt{2} G_2 (\psi) - \sqrt{2} m \zeta \right) \\
A_2 \left[ 1 + \partial_\mu \phi^{\mu \nu} - \mathrm{Tr} (\Gamma_1 (\zeta)) \right] + A_2 \left[ \frac{1}{2} \partial_\mu (\phi^{\nu \rho} \partial_\nu \partial_\rho - \phi^{\nu \rho} \Gamma_1 (\zeta)) \right] \\
- \frac{1}{2} \partial_\mu \phi^{\nu \rho} \partial_\rho (\Gamma_1 (\zeta)) + \frac{1}{2} \partial_\mu (\psi^\nu \partial_\nu + 2 \partial_\mu \Gamma_2 (\zeta)) \right) + \right\}
\right] $$

Note that, upon application of the Stückelberg procedure, the boundary conditions (3.14) are modified (to linear order in the Stückelberg field) as

$$A_0 - \phi^{\mu \nu} \partial_\mu A_0 - \sqrt{2} \zeta A_1 \Bigg|_\infty = K_0, \quad A_1 - \phi^{\mu \nu} \partial_\mu A_1 - \sqrt{2} \zeta A_2 + \frac{1}{2} \psi^\nu \partial_\nu + \frac{1}{2} \partial_\nu \phi^{\mu \nu} \Bigg|_\infty = 0 $$

and

$$A_2 - \phi^{\mu \nu} \partial_\mu A_2 + 1 \frac{1}{2} \psi^\nu \partial_\nu A_1 - \sqrt{2} \zeta A_1 \Bigg|_\infty = K_2. $$

One can recover the boundary conditions up to second order in the Stückelberg field by making use of (3.28).

(b) dS solutions

Let us now look at the possible solutions to the e.o.m. derived from (3.30) for the bosonic fields, $g^{\mu \nu}, M, b_\mu, \phi^{\mu \nu}, A_0$ and $A_2$, which are:\footnote{We omit the fermion terms and e.o.m. because we are seeking a background solution with vanishing fermions.}

$$G_{\mu \nu} + g_{\mu \nu} \left[ \frac{1}{2} MM^* + \frac{3}{2} b^{\mu} b_\mu + \mathrm{Re} (A_2 - A_0 M^*) \right] $$

$$ + b_\mu b_\nu = 0, $$

$$- \frac{1}{2} M + A_0 = 0, $$

$$b_\mu = 0, $$

$$\partial_\mu A_2 + m \partial_\mu A_0 - \frac{1}{2} \partial_\nu \phi^{\nu \mu} \partial_\mu A_2 - \frac{1}{2} \partial_\mu (\phi^{\nu \mu} \partial_\nu A_2) $$

$$+ \frac{m}{2} \partial_\nu \phi^{\nu \mu} A_0 - \frac{m}{2} \partial_\mu (\phi^{\nu \mu} A_0) = 0, $$

$$M^* - m - m \partial_\nu \phi^{\mu \nu} + \frac{m}{2} \partial_\mu (\phi^{\mu \nu} \partial_\nu A_2) = 0 $$

and

$$- \frac{1}{2} A_2 + \partial_\mu A_2 + \frac{1}{2} \partial_\nu \phi^{\mu \nu} + \frac{1}{2} \partial_\mu (\phi^{\mu \nu} \partial_\nu A_2) = 0.
These admit the solution
\[
g_{\mu \nu} = \bar{g}_{\mu \nu}, \quad \text{with} \quad \sqrt{-\bar{g}} = 1,
\]
\[
M = m,
\]
\[
\Lambda_0 = \frac{2}{3} m
\]
and
\[
\Lambda_2 = \Lambda_2, \quad \text{with} \quad \text{Im}(\Lambda_2) = 0,
\]
with all other fields vanishing on the background. The cosmological constant is
\[
c.c. = \Lambda_2 - \frac{1}{3} m^2.
\]

Thus, our model allows for a cosmological constant of either sign, similar to the results in the constrained superfields literature [59–68]. However, we stress that in our approach, the cosmological constant appears as the combination of the vevs of the Lagrange multiplier superfield components \( \Lambda_0 \) and \( \Lambda_2 \).

\( \text{(c) Perturbative treatment and gravitino mass} \)

We now perform a perturbative expansion around the background solution (3.33), in order to study the mass of the gravitino:

\[
g_{\mu \nu} = \bar{g}_{\mu \nu} + h_{\mu \nu},
\]
\[
\sqrt{-\bar{g}} = 1 + \frac{1}{2} h + \frac{1}{4} \left( \frac{1}{2} h^2 - h_{\mu \nu} h^{\mu \nu} \right),
\]
\[
\psi^m_{\mu} = 0 + \psi^m_{\mu},
\]
\[
b_{\mu} = 0 + b_{\mu},
\]
\[
M = M + M,
\]
\[
\Lambda_0 = \frac{2}{3} m + \lambda_0,
\]
\[
\Lambda_1 = 0 + \lambda_1,
\]
\[
\Lambda_2 = \Lambda_2 + \lambda_2, \quad \text{Im}(\Lambda_2) = 0,
\]
\[
\phi^\mu = 0 + \phi^\mu
\]
and
\[
\zeta = 0 + \zeta.
\]

Working to second order in the small perturbations, the action becomes:

\[
S = \frac{1}{16\pi G_N} \int d^4x \left[ 2 \Lambda_2 - \frac{3}{2} m^2 + \frac{1}{4} \bar{\nabla}_\mu h_{\rho \lambda} \bar{\nabla}^{\mu \rho \lambda} - \frac{1}{2} \bar{\nabla}_\mu h_{\rho \lambda} \bar{\nabla}^{\rho \lambda} h^{\mu \nu} + \frac{1}{2} \bar{\nabla}_\mu h^{\nu \mu} \bar{\nabla}_\nu h \right.
\]
\[
- \frac{1}{3} \bar{\nabla}_\mu h \bar{\nabla}^{\mu} h - \frac{3}{2} M^* M + \frac{3}{2} b_{\mu} b_{\nu} + \frac{1}{2} \left( \Lambda_2 - \frac{1}{3} m^2 \right) \left( h^{\mu \nu} h_{\mu \nu} - \frac{1}{2} h^2 \right)
\]
\[
+ \varepsilon^{\mu \nu \rho} \left( \bar{\psi}_\mu \bar{\sigma}_\nu \mathcal{D}_{\rho} \psi_{\sigma} \right) - \psi_{\mu} \sigma_{\nu} (\mathcal{D}_{\rho} \psi_{\sigma}) - \frac{2}{3} m^2 + \frac{2}{3} m \bar{\psi}_\mu \bar{\sigma}^{\mu \nu} \psi_\nu + \frac{2}{3} m \psi_{\mu} \sigma^{\mu \nu} \psi_\nu + \left[ \Lambda_2 - \Lambda_2 \text{Tr}(f_1(\xi)) \right]
\]
\[
- \lambda_2 \left( \frac{\sqrt{2}}{2} \bar{\psi}_\mu \psi_\mu(\xi) + \sqrt{2} \left( f_{12}(\xi) + \frac{1}{2} m \xi \right) - \sqrt{2} m \xi \right)
\]
\[
+ \sqrt{2} \lambda_1 \sigma^{\mu} \bar{\psi}_\mu + \frac{2}{3} m A_{\xi}^2 + \frac{1}{3} m A_{\zeta}^2 - m A_{\zeta}^2 + \lambda_2 \left( \partial_\mu \mu - \frac{1}{2} h \right)
\]
\[
+ \lambda_0 \left( M^* - m \left( \partial_\mu \mu - \frac{1}{2} h \right) \right) + h.c.] \right],
\]

(3.36)
where $D_\mu$ is the covariant derivative on the background. The linearized local supersymmetry transformations are

$$
\delta \psi_\mu = -2D_\mu \varepsilon + i \frac{1}{3} m (\varepsilon \sigma_\mu \varepsilon),
\delta (2) \zeta = -\varepsilon,
\delta \lambda = -\sqrt{2} \varepsilon A_2
\delta h_{\mu \nu} = \delta M = \delta b_\mu = \delta t^\mu = \delta \lambda_0 = \delta \lambda_2 = 0,
$$

and

where $D_\mu \varepsilon$ is defined in (A 2) and the boundary conditions on the Lagrange multiplier superfield reduce to:

$$
\lambda_0 \bigg|_\infty = 0, \quad (\lambda_1 - \sqrt{2} A_2 \zeta) \bigg|_\infty = 0, \quad \lambda_2 = 0 \bigg|_\infty.
$$

In order to study the gravitino mass, we will need the e.o.m. for the fermion fields in our theory. The gravitino e.o.m. is

$$
e^{\mu \nu \rho \sigma} \bar{\sigma}_\nu D_\rho \psi_\sigma + \frac{2}{3} m \bar{\sigma}^{\mu \nu} \bar{\psi}_\nu - i \sqrt{2} \bar{\sigma}^{\mu \nu} \lambda_1 - i \frac{A_2}{2} \bar{\sigma}^{\mu} \zeta = j_\mu,
$$

where the source $j_\mu$ accounts for the contributions from higher-order terms in the perturbative expansions. Note that the local symmetry of the gravitino (3.37) implies the source conservation condition

$$
2D_\mu j_\mu + i \frac{2}{3} m \bar{\sigma}^{\mu} j_\mu = 0.
$$

The equations for $\lambda_1$ and $\zeta$, respectively, give:

$$
i \bar{\sigma}^{\mu} \psi_\mu = 2i \bar{D} \zeta - \frac{4}{3} m \bar{\zeta}
\quad \text{and}
\quad i \bar{\sigma}^{\mu} \psi_\mu = \frac{\sqrt{2}}{2} \left( i \bar{D} \lambda_1 - \frac{2}{3} m \lambda_1 \right)
$$

Taking the trace and the divergence of the gravitino equation, in conjunction with (3.41) and (3.40) we obtain

$$
i \bar{\sigma}^{\mu} \psi_\mu = i \bar{D} \left( \zeta + \frac{\sqrt{2}}{2 A_2} \lambda_1 \right) - \frac{2}{3} m \left( \zeta + \frac{\sqrt{2}}{2 A_2} \lambda_1 \right)
\quad \text{and}
\quad -i m \bar{\sigma}^{\mu} \psi_\mu + \bar{\sigma}^{\mu \nu} D_\mu \psi_\nu = \frac{A_2}{2} \left( \zeta + \frac{\sqrt{2}}{2 A_2} \lambda_1 \right) + \frac{1}{3} \bar{\sigma}^{\mu} j_\mu.
$$

Thus the effective goldstino at linear level is identified as the combination

$$
\mathcal{G} \equiv \frac{1}{2} \left( \zeta + \frac{\sqrt{2}}{2 A_2} \lambda_1 \right).
$$

We note that the orthogonal mode to $\mathcal{G}$

$$
\tau \equiv \frac{1}{2} \left( \zeta - \frac{\sqrt{2}}{2 A_2} \lambda_1 \right),
$$

satisfies the equation

$$
i \bar{D} \tau - \frac{2}{3} m \bar{\tau} = 0,
$$

and is thus eliminated via the boundary conditions (3.38). This is analogous to the elimination of the extra mode for the Weyl-Rarita Schwinger field in $[85]$. Finally, this allows us to recover the standard equations for a massive gravitino coupled to a goldstino. The e.o.m. in our model are then equivalent to those arising from the perturbative action

$$
\mathcal{L} = \epsilon^{\mu \nu \rho \sigma} \bar{\psi}_\mu \sigma_\nu D_\rho \psi_\sigma + \frac{2}{3} m \bar{\psi}_\mu \bar{\sigma}^{\mu \nu} \psi_\nu
+ 2i A_2 \mathcal{G} \bar{\sigma}^{\mu} \psi_\mu - 2i A_2 \mathcal{G} \bar{\mathcal{G}} + \frac{4}{3} m A_2 \mathcal{G}^2 + \text{h.c.}
$$

In the flat limit, and taking $m \to 0$ we recover the kinetic term of the Volkov–Akulov action for the goldstino, as expected (since we are working here in the linearized approximation). We further note that supersymmetry is broken by the vev of $A_2$. In the limit where it goes to zero, supersymmetry is restored, as evidenced by the fact that the goldstino drops out of (3.46). Of course, in this supersymmetric limit the backgrounds will be AdS, as revealed by equation (3.34).
4. Conclusion

In this paper, we developed a superspace version of unimodular gravity, where the chiral density superfield is constrained. This is similar to the way in which the determinant of the metric is constrained in standard unimodular gravity, although it differs in that the constraint does not quite correspond to a local gauge fixing. In any event, the constraint is most elegantly imposed using a superspace Lagrange multiplier in the form of a chiral superfield. By restoring general coordinate invariance and local supersymmetry order by order using the Stückelberg trick, we have shown how the vev of the Lagrange multiplier can contribute a positive vacuum energy and allow for de Sitter vacua. More precisely, it is its top component that contributes to the vacuum energy, whilst at the same time spontaneously breaking supersymmetry. Fluctuations about these vacua yield a massive gravitino coupled to a goldstino. The latter can be identified with a Stückelberg field introduced in the usual way to restore local supersymmetry.

At first glance, our work differs from previous formulations of pure de Sitter supergravity which make use of a nilpotency constraint on a chiral superfield [60]. There the nilpotency constraint is imposed directly using a Lagrange multiplier, motivated by the fact that the goldstino is expected to form part of a nilpotent superfield [86]. The same ought to be true of the goldstino that emerges in our formulation, although it is not immediately obvious how this is actually realized. We think a better understanding of the connection between the two formulations could enhance our understanding of both, perhaps leading to a unified description.

Our work can be extended in several other directions. The first of these is to develop a supersymmetric version of unimodular gravity in the Henneaux Teitelboim formulation [75]. This makes use of four-form field strengths rather than Jacobians of Stückelberg fields, although the two representations are closely related. A supersymmetric version of [75] could open up a natural path towards string theory realization of our work, since four-forms are ubiquitous in flux compactifications [87]. A second motivation is to extend our ideas to a supersymmetric version of the sequestering proposal [12–17,88–91] for tackling the cosmological constant problem, again with a view towards a stringy embedding.

Data accessibility. This article has no additional data.

Authors’ contributions. S.N. led the project in terms of calculations, contributing expertise in the area of supergravity. She wrote up §3 and the appendices of the paper. A.P. came up with the idea for the project and performed the early exploratory calculations, contributing expertise on unimodular gravity. He wrote up §2 of the paper. I.Z. performed the checks on the calculations, and contributed expertise in the area of nonlinearly realized supersymmetry. She also wrote the introduction for the paper. Additionally, the authors met regularly to discuss strategy and contributed equally to these discussions and to the developing strategy.

Competing interests. We declare we have no competing interests.

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Appendix A. Conventions and useful quantities

We are using the conventions of [19]. We work with the mostly plus signature metric $\eta_{\mu\nu} = \text{diag}(-1, 1, 1, 1)$ and

$$\varepsilon_{21} = \varepsilon^{12} = 1, \quad \varepsilon_{12} = \varepsilon^{21} = -1, \quad \varepsilon_{11} = \varepsilon_{22} = 0.$$  \hspace{1cm} (A 1)

The Pauli matrices are

$$\sigma^0 = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}, \quad \sigma^1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma^2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \sigma^3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$  \hspace{1cm} (A 2)

and

$$\tilde{\sigma}^0 = \sigma^0, \quad \tilde{\sigma}^1 = -\sigma^{1,2,3}.$$  \hspace{1cm} (A 3)
The chiral supergravity superfield in component form is
\[
R = -\frac{1}{6} \left[ M + \Theta \left[ \sigma^\mu \tilde{\sigma}^\nu \psi_{\mu \nu} - i \sigma^\mu \tilde{\psi}_\mu M + i \nu b^\mu \right] + \Theta^2 \left[ - \frac{1}{2} R + i \tilde{\psi}^\mu \tilde{\sigma}^\nu \psi_{\mu \nu} + \frac{\tau}{2} MM^* + \frac{1}{3} b^\mu b_\mu \right. \\
- i \nu^\mu D_\mu b^\alpha + \frac{1}{2} \tilde{\psi}^\mu M - \frac{1}{2} \nu \sigma^\mu \tilde{\psi}_\mu b^\nu + \left. \frac{1}{8} e^{\mu \nu \rho \sigma} \left[ \tilde{\psi}_\mu \tilde{\sigma}_\nu \psi_{\rho \sigma} + \psi_{\mu \nu} \sigma^\rho \psi_{\rho \sigma} \right] \right],
\]
(A 4)

with
\[
\psi_{\mu \nu} = 2 \tilde{D}_{[\mu} \psi_{\nu]}, \quad \tilde{D}_\mu \psi_\nu = \partial_\mu \psi_\nu + \psi_\nu \omega_\mu.
\]
(A 5)

**Appendix B. Supergravity transformations**

The SUSY transformations of the supergravity fields (up to linear order in \( \epsilon \)) are
\[
\delta \epsilon^A_\mu = i \left( \psi_\mu \sigma^A \tilde{\epsilon} - \epsilon \sigma^A \tilde{\psi}_\mu \right),
\]
\[
\delta \psi_\mu = -2 D_\mu \epsilon^m + \frac{\tau}{4} M (\epsilon \sigma_\mu \tilde{\epsilon})^m + i b_\mu \epsilon^m + \frac{1}{3} b^m \left( \epsilon \sigma_\mu \tilde{\sigma}_\mu \right)^m,
\]
\[
\delta M = -\epsilon \left( \sigma^\mu \tilde{\sigma}_\nu \psi_{\alpha \beta} + i b^\mu \psi_\alpha - i \sigma^\mu \tilde{\psi}_\alpha M \right),
\]
\[
\delta b_{\alpha \beta} = \epsilon F(\psi; M, b)
\]
\[
= \epsilon^B \left[ \frac{3}{4} \tilde{\psi}_\alpha \tilde{\psi}_\beta \tilde{\gamma} \tilde{\gamma} + \frac{1}{4} \epsilon \delta_\alpha \tilde{\psi}_\beta \tilde{\gamma} \tilde{\gamma} - \frac{1}{2} M^\ast \psi_{\alpha \beta} \right]
\]
\[
+ \frac{1}{4} \left( \tilde{\psi}_\alpha \tilde{\psi}_\beta \tilde{b}_{\tilde{a} \tilde{a}} + \tilde{\psi}_\beta \tilde{b}_{\alpha \tilde{a}} - \tilde{\psi}_\alpha \tilde{b}_{\beta \tilde{a}} \right)
\]
\[
- \epsilon \left[ \frac{3}{4} \tilde{\psi}_\beta \tilde{\gamma} \tilde{\gamma} \tilde{\gamma} + \frac{1}{4} \epsilon \delta_\beta \tilde{\psi}_\alpha \tilde{\gamma} \tilde{\gamma} + \frac{1}{2} M \psi_{\alpha \beta} \right]
\]
\[
+ \frac{1}{4} \left( \psi_{\alpha \beta} \tilde{b}_{\alpha \tilde{a}} + \psi_{\beta \tilde{a}} \tilde{b}_{\alpha \tilde{a}} - \psi_{\alpha \beta} \tilde{b}_{\tilde{a} \tilde{a}} \right),
\]
(B1)

where space–time indices have been converted to spinor indices through contraction with the Pauli matrices (A 2) and:
\[
D_\mu \epsilon^m = \partial_\mu \epsilon^m + \epsilon^B \omega^m_{\mu b}.
\]
(B 2)

Working to second order in the fermions, the supergravity transformations of the quantities defined in (3.6) are then:
\[
\delta^{(s)}[y^1_1(\zeta)] = -\phi^\rho \partial_\rho \left[ 2i \sigma^{\mu \nu} \right] \tilde{\zeta} + \partial_\rho \phi^\mu \tilde{y}^1_1(\zeta),
\]
\[
\delta^{(s)}[y^1_2(\zeta)] = -\frac{1}{2} y^1(\zeta) D_\rho y^1_2(\zeta) + \Gamma_2 y^1_2(\zeta)
\]
\[
- \phi^\rho \partial_\rho \left[ \tilde{\psi}_{\nu} \tilde{\sigma}_\nu \sigma^\mu \tilde{\zeta} \right] + \partial_\rho \phi^\mu \tilde{y}^1_2(\zeta) - \frac{4}{3} M^\ast \zeta \sigma^\mu \tilde{\zeta},
\]
\[
- \frac{1}{3} b_\mu \tilde{\sigma}_\nu \tilde{\sigma}_\nu \tilde{\sigma}_\nu \tilde{\zeta},
\]
\[
\delta^{(s)}[\Gamma_1(\zeta)] = -\partial_\mu \tilde{y}^1_1(\zeta) + 2 \xi \Gamma_2(\zeta) - \phi^\rho \partial_\rho \left[ i \psi_{\mu \nu} \sigma^\mu \right] \tilde{\zeta}
\]
\[
- \frac{3}{2} M^\ast \xi \tilde{\zeta}^2 - \frac{1}{3} b_\mu \zeta \sigma^\mu \tilde{\zeta},
\]
\[
\delta^{(s)}[\Gamma_2(\zeta)] = -\phi^\rho \partial_\rho \left[ i \omega_{\mu \nu} \sigma^\mu \right] \tilde{\zeta} - \phi^\rho \partial_\rho M^\ast \frac{1}{3} \zeta
\]
\[
- \frac{1}{6} \phi^\rho \partial_\rho \left[ b_\mu \epsilon \sigma^\mu \right] \tilde{\zeta}.
\]
(B 3)

A chiral density superfield \( \Delta \) is defined by its transformation law
\[
\delta \Delta = -\partial_N \left[ (-1)^N \eta^N \right],
\]
(B 4)
with $\eta^\mu$, $\eta^\nu$ defined in (3.5). In components,
\[
\Delta = a + \sqrt{2}\Theta \rho + \Theta^2 f,
\]
(B5)
and then the transformation rules of the components will be
\[
\begin{align*}
\delta a &= -\sqrt{2}\epsilon \eta + ia\psi_{\mu}\sigma^\mu \bar{\epsilon}, \\
\delta \rho &= -\sqrt{2}\epsilon \psi - i\sqrt{2}D_\mu (\sigma^\mu \bar{\epsilon} a) + i\psi_{\mu}\sigma^\mu \epsilon \rho \\
&\quad + i(\sigma^\mu \bar{\epsilon}) \psi_{\mu}\rho - \frac{\sqrt{2}}{2}\epsilon M^a a \\
&\quad - \frac{\sqrt{2}}{6}\epsilon a \sigma^\mu \bar{\epsilon} b_\mu + \frac{\sqrt{2}}{2}\psi_{\nu}\bar{\psi}_{\mu}\bar{\sigma}^\nu \sigma^\mu \bar{\epsilon} a,
\end{align*}
\]
(B6)
and
\[
\delta f = \partial_\mu \left[ -a\bar{\psi}_{\nu}\bar{\sigma}^\nu \sigma^\mu \bar{\epsilon} + i\sqrt{2}\rho \sigma^\mu \bar{\epsilon} \right].
\]

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