Numerical investigation for process of deformation of a multilayer shell under thermal loading

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Abstract. In this work study of deformation in multilayer element under thermal stress loading within the framework of geometrically nonlinear calculation, is carried out. The analyzed area is a multilayer truncated conical shell, which initially is under constant pressure along its outer surface, and then had been heated slowly. During the heating process, the structure loses its stability. Here the influence of kinematic fixing conditions of the shell, which is possible to be fixed as different parts of the end surfaces of the shell had been investigated moreover realized the conditions of contact interaction with rigidly fixed steel conical washer ring on which the shell is pushed before loading. Wherein, the process of attachment within the confines of the process of contact interaction is also actualized. We also investigated the possibility of replacing the multilayer shell with a single-layer shell with reduced elastic modulus.

1. Introduction
In constructing the physical model of multilayered orthotropic shell all possible approaches based on different hypotheses for each layer of the shell [1] and the uniform hypotheses for all layers of thin-walled construction [2] are consulted. In the first case, the order of resolution system depends on the number of layers. In the second case, the order of the system does not depend on the number of layers, which creates particular possibilities, to use the finite element method effectively. In calculating plates and shells with medium thickness there are widespread special elements of plates and shells which have as the nodal degrees of freedom of nodes displacement that located on the front surfaces [3, 4]. It may be noted some works [5-16] devoted to studying of such problems in which large elastoplastic deformations occurred. There are articles that calculate shell structures used in aircraft and helicopter industry [17-21].

2. Finite element for multilayer shell
Deformation of a medium is described by components of linear deformations $\varepsilon_{xx}, \varepsilon_{yy}, \varepsilon_{zz}$ and shear deformations $\gamma_{xy}, \gamma_{yz}, \gamma_{zx}$ which are expressed by displacement relationship that known as Cauchy relations. The stress state represents the Cauchy stress tensor in the form of components of normal $(\sigma_{xx}, \sigma_{yy}, \sigma_{zz})$ and tangential $(\tau_{xx}, \tau_{yy}, \tau_{zz})$ stresses. The relationship between stresses and deformations is
linear, which called the generalized Hooke’s law, used for an anisotropic material which has a complex structure and represents as a matrix component of elastic constants in $6 \times 6$ dimension. For orthotropic materials, the system of orthogonal Cartesian coordinates $\alpha, \beta, \gamma$ is used, which determines by the so-called orthotropic axes. According to these axes, deformations can be determined as $\varepsilon_{\alpha\alpha}, \varepsilon_{\beta\beta}, \varepsilon_{\gamma\gamma}, \epsilon_{\alpha\beta}, \epsilon_{\alpha\gamma}, \epsilon_{\beta\gamma}$ and stresses as $\sigma_{\alpha\alpha}, \sigma_{\beta\beta}, \sigma_{\gamma\gamma}, \tau_{\alpha\beta}, \tau_{\alpha\gamma}, \tau_{\beta\gamma}$ which are related to generalized Hooke’s law.

If we denote the given deformation vectors of the axes as $x, y, z$ and $\alpha, \beta, \gamma$ so

$$
\{\varepsilon\}^T = \{\varepsilon_{\alpha\alpha}, \varepsilon_{\beta\beta}, \varepsilon_{\gamma\gamma}, \epsilon_{\alpha\beta}, \epsilon_{\alpha\gamma}, \epsilon_{\beta\gamma}\},
$$

(1)

And given stresses vectors in the same axes will be

$$
\{\sigma\}^T = \{\sigma_{\alpha\alpha}, \sigma_{\beta\beta}, \sigma_{\gamma\gamma}, \tau_{\alpha\beta}, \tau_{\alpha\gamma}, \tau_{\beta\gamma}\},
$$

(2)

then the corresponding elasticity relations can be written in the following form

$$
\{\sigma\} = [D]\{\varepsilon\}, \quad \{\sigma\} = [D_a]\{\varepsilon\}.
$$

(3)

The relationship between deformations in the axes $\alpha, \beta, \gamma$ and $x, y, z$ can be rewritten in matrix form

$$
\{\varepsilon_{\alpha}\} = [T]\{\varepsilon\},
$$

(4)

In this case, the matrix of elastic constants $[D]$ for the given matrix $[D_a]$ is defined as follows

$$
[D] = [T]^T[D_a][T].
$$

(5)

Structurally the assumed finite element represents a curved parallelepiped, consisting of $N$ layers in thickness, each of which is an orthotropic material according to orthotropic axes. $\alpha, \beta, \gamma$. It is assumed that the plane $\alpha, \beta$ is parallel to the plane of local coordinates $\xi, \eta$ along any straight line in thickness, i.e. orthotropic axis $\gamma$ is parallel to the local coordinate $\zeta$ (Figure 1a).

![Figure 1](image_url)

**Figure 1.** a - multilayer finite element, b - triple nodes numbering

To specify the orientation of the axes $\alpha, \beta, \gamma$ there are two possibilities. If the finite element is flat and its plane coincides with the plane $x, y$ so the angle $\phi$ will determine as the angle between the $x$ axis and $\alpha$ direction. If the element is curved, then we consider that the orthogonal unit system $\hat{p}_1, \hat{p}_2, \hat{p}_3$ thereby $\hat{p}_1$ directed along the tangent coordinate line $\xi$ and $\hat{p}_3$ is perpendicular to the surface $\xi, \eta$, i.e. directed along the coordinate line $\zeta$. So it is convenient to determine this system by means of the following relations.

$$
\hat{p}_1 = \partial r / \partial \xi \frac{\partial r / \partial \zeta}{\partial r / \partial \zeta}, \quad \hat{p}_3 = \left(\partial r / \partial \zeta \times (\partial r / \partial \eta)\right)/\left((\partial r / \partial \xi)(\partial r / \partial \eta)\right), \quad \hat{p}_2 = \hat{p}_1 \times \hat{p}_3.
$$

(6)

The orientation of the axes $\alpha, \beta$ lying in plane $\hat{p}_1 - \hat{p}_2$ will be defined as an angle $\phi$ between $\hat{p}_1$ and $\alpha$ axis.
To construct the stiffness matrix, it is necessary to determine the matrix of physical relationships \([D]\) in Hooke’s law for stresses and deformations with respect to global axes \(x, y, z\) through the matrix of elastic constants \([D_e]\) defining Hooke’s law in the orthotropic axes. To do this, we must enter the "rotational deformation" matrix at angle \(\phi\) in the plane \(\alpha, \beta\) \([T_r]\).

If the finite element is flat and plane \(\alpha, \beta\) coincides with plane \(x, y\) \(\left( \phi = \alpha, x \right)\), then
\[
[D] = [T_r]^T [D_e] [T_r].
\]

If the finite element is curved, then it is necessary to enter "rotational deformation" matrix from the system \(x, y, z\) with unit vectors \(\hat{i}, \hat{j}, \hat{k}\) to system with the unit vectors \(\hat{p}_1, \hat{p}_2, \hat{p}_3\) \([T_p]\) structurally, which coincides with the matrix defining recalculated strain (4). Consequently, the matrix of elastic characteristics \([D]\) will be calculated in the following form:
\[
[D] = [T_p]^T [T_r]^T [D_e] [T_r] [T_p].
\]

The thickness of each layer \(h\) is convenient to set in the form of relative values
\[
\Delta_s = \frac{h_s}{h} , h = \sum_{s=1}^{N} h_s, \sum_{s=1}^{N} \Delta_s = 1.
\]

Then the coordinates of the interlayer surfaces of inner element will be determined as following
\[
\zeta_1 = -1 , \quad \zeta_2 = -1 + 2\Delta_1 , \quad \zeta_3 = -1 + 2\Delta_1 + 2\Delta_2, ..., \quad \zeta_s = -1 + 2\sum_{i=1}^{s-1} \Delta_i , ..., \quad \zeta_{N+1} = +1,
\]

and the coordinate of the middle \(s\) layer is defined as \(\zeta_s = (\zeta_s + \zeta_{s+1})/2\).

In the global Cartesian coordinate system \(x, y, z\) the proposed finite element represents the curved parallelepiped which upper and lower surfaces are substantially curved, and the four side faces are linear surfaces (Figure 2) for the convenience of constructing such a finite element, the triple numbering of nodes is introduced in accordance with Figure 1b by entering covariant components of deformations:
\[
\begin{align*}
\varepsilon_{xi} &= \frac{\partial \hat{v}}{\partial \hat{x}}, \quad \varepsilon_{yi} = \frac{\partial \hat{v}}{\partial \hat{y}}, \quad \varepsilon_{zi} = \frac{\partial \hat{v}}{\partial \hat{z}}, \\
\varepsilon_{yi} &= \frac{\partial \hat{v}}{\partial \hat{y}}, \quad \varepsilon_{zi} = \frac{\partial \hat{v}}{\partial \hat{z}}.
\end{align*}
\]

The relationship between the given strain vectors
\[
\{\varepsilon\}' = \begin{bmatrix} \varepsilon_x, \varepsilon_y, \varepsilon_z, \gamma_{xy}, \gamma_{yz}, \gamma_{zx} \end{bmatrix} , \{\varepsilon\} = \begin{bmatrix} \varepsilon_{xx}, \varepsilon_{yy}, \varepsilon_{zz}, \gamma_{xy}, \gamma_{yz}, \gamma_{zx} \end{bmatrix}
\]

Similarly had written (4). By calculating deformations (12) the method of double approximation is used through displacements.

3. Settlement scheme for the problem of buckling stability of multilayer shell
On the basis of the proposed finite element, it had conducted the discretization of calculated area which is a multilayer truncated conical shell which along its outer surface part is under a constant external pressure of 1 atm and slowly heated until 60°C. In the process of increasing the temperature, the structure loses its stability, considered moment which is determined in the framework of the nonlinear deformation method, suggested by the authors earlier [22-25]. The effect of kinematic fixation conditions of the shell that has the possibility to be fixed as different parts of the end surfaces of the shell, and the conditions of contact interaction with rigidly fixed steel conical washer ring had investigated. In this case, the process of attachment, is also realized in the framework of the process of contact interaction, the finite element method is realized in [26-28]. We also investigate the possibility of replacing the multilayer shell with a single-layer shell with reduced elastic modulus.
Figure 2 shows the shell model with end rings, on which the shell is installed, figure 3 shows the scheme of the shell under loading. The layout of layers for different sections of the shell is shown in figure 4, where black color assumes the basis, blue - layers laid along the axis of the shell, turquoise - layers around circle.

![Figure 2. Model of the shell with end rings](image)

The table shows the results of calculating when structural stability was lost for single-layer and multilayer models in various boundary conditions. The initial temperature of the computational area is 20°C.

![Figure 3. Scheme of the shell under loading](image)
4. Analysis of results and conclusions
The fixing scheme 1 assumes the rigid fixing of the designed shell along the end faces, the fixing scheme 2 assumes the fixation of conical rings, with which are under contacts with the shell, the fixing scheme 3 similar to the scheme 2, only additional restrictions are set at the axial direction of the shell end faces. Figure 4 shows the shell shape at the moment of loading, prior to the moment of buckling the stability for the case of rigid fixing at the end faces of the shell (Fig. 5a) and for the case of the contact interaction of the shell with conical washer rings (Fig. 5b).

Table 1. The results of calculating the stability of shells.

| Model                      | Single layer | Multilayers |
|----------------------------|--------------|-------------|
| Buckling temperature [°C]  | 82           | 66          | 99          | 66           | 95          |
| The first principal stress $\sigma_1$ [MPa] | 113          | 91          | 111         | 159          | 105         | 158         |
| The third principal stress $\sigma_3$ [MPa] | -202         | -87         | -173        | -328         | -85         | -254        |
| Number of scheme by fixation type | 1           | 2           | 3           | 1           | 2           | 3           |
It can be noted that, in this task, for a single-layer scheme where the process of buckling the stability occurs the temperature reduced, but the maximum compressive stresses in the multilayers case are more. When between shell and washer rings made contact, the buckling temperature decrease by about 25%, wherein the stress distribution pattern in the shell at the time of loading before the loss of stability is significantly different. Final loading before the buckling is significantly different.

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References
[1] Solov'ev S S 1989 Izv. VUZ. Aviationsnaya Tekhnika 4 71–75
[2] Rickards R B 1988 The finite element method in the theory of shells and plates (Riga, Znanie) 284
[3] Berezhnoi D V, Sagdatullin M K and Golovanov A I 2011 Vestnik Saratov State Technical University 3 (57) 9-19
[4] Burman Ya Z and Solov'ev S S 1990 Issledovania po teorii plastin i obolochek 22 98–107
[5] Gerasimov O, Shigapova F, Konoplev Y and Sachenkov O 2016 IOP Conference Series: Materials Science and Engineering 158 012036
[6] Gerasimov O, Shigapova F, Konoplev Y and Sachenkov O 2016 IOP Conference Series: Materials Science and Engineering 158 012037
[7] Kharin N V, Vorobyev O V, Berezhnoi D V and Sachenkov O A 2018 PNRPU Mechanics Bulletin 3 95-102
[8] Sachenkov O A, Hasanov R F, Andreev P S and Konoplev Y G 2016 Russian Journal of Biomechanics 20 (3) 220-232
[9] Badriev I B, Makarov M V and Paimushin V N 2015 Russian Mathematics 59 (10) 57–60
[10] Badriev I B, Garipova G Z, Makarov M V, Paymushin V N and Khabibullin R F 2015 Lobachevskii Journal of Mathematics 36 (4) 474-481
[11] Badriev I B and Banderov V V 2014 Lobachevskii Journal of Mathematics 35 (4) 371–383
[12] Badriev I B, Makarov M V and Paymushin V N 2017 Russian Mathematics 61 (1) 69–75
[13] Sultanov L U and Fakhрутдинов Л.Р 2013 Magazine of Civil Engineering 44 (9) 69–74
[14] Sultanov L U and Davydov R L 2013 Magazine of Civil Engineering 44 (9) 64–68
[15] Sultanov L U 2016 Lobachevskii Journal of Mathematics 37 (6) 787–793
[16] Golovanov A I and Sultanov L U 2005 Prikladnaya Mekhanika 41 (6) 36–43
[17] Kasumov E V and Gajnutdinov V G 2013 Vestnik KGTU im. A.N.Tupoleva 4 7–12
[18] Kasumov E V 2013 Izv. VUZ. Aviationsnaya Tekhnika 3 11–14
[19] Kasumov E V 2014 Izv. VUZ. Aviationsnaya Tekhnika 1 24–29
[20] Kasumov E V 2015 TsAGI Science Journal 46 (2) 63–75
[21] Kasumov E V 2016 TsAGI Science Journal 47 (4) 62–74
[22] Berezhnoi D V, Balafendieva I S, Sachenkov A A and Sekaeva L R 2016 IOP Conference Series: Materials Science and Engineering 158 012018
[23] Shamim M R and Berezhnoi D V 2016 IOP Conference Series: Materials Science and Engineering 158 012083
[24] Berezhnoi D V, Balafendieva I S, Sachenkov A A and Sekaeva L R 2017 IOP Conference Series: Materials Science and Engineering 204 012005
[25] Berezhnoi D V, Shamim R and Balafendieva I S 2017 MATEC Web of Conferences 129 03023
[26] Berezhnoi D V and Shamim R 2017 Procedia Engineering 206 1056–1062
[27] Berezhnoi D V and Paymushin V N 2005 Naukoyemkiye tekhnologii 6 (8-9) 59–64
[28] Berezhnoi D V, Kuznetsova I S and Sachenkov A A 2010 Uchenye Zapiski Kazanskogo Universiteta. Seriya Fiziko-Matematicheskie Nauki 152 (1) 116–125