Reynolds Stress Structures in the Hybrid RANS/LES of a Planar Channel

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Abstract. The near-wall cycle of hybrid RANS/LES is studied by calculating the flow through a planar channel. Statistical results are commented on and related to instantaneous structures which are extracted from the flow field. The problematic structures in the artificial near-wall cycle, well known to be super-streaks, are identified and quantified. The calibration of such closures provides a correct mixing length argument in the logarithmic layer. However because of these overly intense streamwise streaks, it is impossible to simultaneously predict the Reynolds streamwise normal and shear stress components correctly. Further, because the location of the RANS-LES interface changes spatially and temporally, we see these structures are more free to move vertically and this further worsens statistical results.

1. Introduction
Wall-bounded flows are of great engineering importance. Unfortunately, near-wall regions are the most expensive to explicitly resolve in numerical simulations. The original concept behind combining Reynolds-Averaged Navier-Stokes (RANS) modelling with Large Eddy Simulation (LES) is to ease the near-wall grid requirements of the latter. The paradigm is often stated as, ‘perform RANS where required and LES where possible.’ This is often understood, with respect to simple wall-bounded flows, as treating the boundary layer with RANS and switching to LES in the outer layer. However, as shall be seen, this is not the case. The near-wall is in reality solved by some kind of unsteady RANS, which contains resolved fluctuations, operating in conditions much different from the original purpose. These fluctuations are well known to be large streaks that are non-physical in nature, however are necessary as boundary conditions to the LES region. Work by Jiménez et al. [1, 2] shows that, in simulations, the near-wall cycle of the small scales is reasonably localised and unaffected by the outer layer. This at least provides some hope for hybrid RANS/LES as a philosophy. Resolving large streaks whilst modelling this near-wall cycle is a possibility, although currently far from reality. This paper uses arguably the
most popular near-wall approach [3] to show that current mechanisms do not properly account for the break up of these streaks, in particular, the artificial buffer layer formed by the delay in recovering LES seriously affects statistical results. These findings are reasonably well known, however this paper provides more quantitative information regarding these structures.

Chapman [4] estimated that an LES of a flat plate boundary layer requires a total number of grid points \( N \) proportional to the Reynolds number \( \text{Re}_{L_x} \) via the relation

\[
N \sim \text{Re}_{L_x}^{9/5},
\]

which is only marginally better than Direct Numerical Simulation (DNS). The exponent was later revised by Choi and Moin [5] to be \( 13/7 \). \( L_x \) is the length of the flat plate. Strong dependence on and poor scaling with Reynolds number puts the majority of industrially relevant geometries out of reach for LES.

Work on wall-modelled LES started with the seminal work of Schumann [6] and Deardorff [7] who derived equilibrium laws to relate the wall shear stress with the velocity in the core, assuming a universal law of the wall. These laws made it possible to relax the near-wall grid requirements and good results [6] showed the feasibility of wall modelled LES. Note that Deardorff [7] used a resolution much too low in the outer layer, reducing the solution quality.

This kind of wall model is only possible when a universal law of the wall can be assumed such that the only feasible applications are simple geometries like pipes and channels. From this seminal work, varied hybrid RANS/LES methodologies have been developed that extend the wall-modelled LES philosophy. Broadly speaking, efforts can be categorised as: wall function approaches (above paragraph), two-layer models and manual/implicit controlling of the RANS-LES interface by modifying the length scale [8], subgrid viscosity [9] or turbulent kinetic energy [10]. This control of the RANS/LES switch can be achieved either by blending between turbulence and subgrid models [11, 12]; or by a unified model, where only one turbulence model is used and its contribution has to be damped to mimic a subgrid closure [8, 13, 14, 15]. Note, for completeness we mention the zonal method [16, 17] which employs two distinct regions, one LES and one RANS. The interface between the two zones is prescribed by the user and turbulent fluctuations must be generated as a boundary condition for the LES. This interface can be either perpendicular to the wall [18] or parallel [19]. The former is used when near-wall interactions are deemed crucial to the overall flow prediction and the latter when computational power cannot be spared to resolve the boundary layer.

Two-layer models, first proposed by Balaras and Benocci [20], use a coarse grid LES up to the wall, but embed a second finer grid for solving the near-wall region with the boundary layer equations, closed with an algebraic eddy viscosity. The Poisson equation is not solved near the wall and the wall normal velocity is obtained by enforcing mass conservation across the interface. The wall stress components obtained from this inner layer are then applied as a boundary condition to the LES. This additional inner layer only marginally increases the cost of the computation and obtains accurate results in planar channel flow.

Spalart et al. [21] and Speziale [8] essentially devised the same philosophy of damping the length scales in a RANS model, although their approaches were not formally described as such. These two approaches became known as Detached Eddy Simulation (DES) and Flow Simulation Methodology (FSM). DES contains an explicit switch between the RANS and LES modes,

\[
\ell_{\text{des}} = \min(y, C_{\text{des}}\Delta)
\]

where \( \ell_{\text{des}} \) replaces the dissipation length scale in the Spalart-Allmaras turbulence model [22], \( \Delta \) is the grid length scale, \( y \) is the wall distance and \( C_{\text{des}} = 0.65 \). The model becomes an ‘LES’ when \( C_{\text{des}}\Delta < y \), increasing the model dissipation and thus lowering the effective viscosity to
a level appropriate for the subgrid scales. This idea is easily generalisable to other turbulence models,

\[ \ell_{hyb} = \min(\ell_t, \ell_{sgs}) \]  

\[ (3) \]

where \( \ell_t \) is the RANS length scale and \( \ell_{sgs} \) is the subgrid length scale, which in traditional DES is defined as \( \ell_{sgs} = C_{des} \Delta \). The length \( \ell_{hyb} \) is general notation for the modified scale, denoting the hybrid length scale which replaces all occurrences of \( \ell_t \) in a given RANS model. FSM on the other hand does not have such a treatment and the switch is made continuously by damping the RANS length scale. This concept is detailed in Section 2.

Equation 3 works well when the streamwise \( \Delta_x \) and spanwise \( \Delta_z \) grid spacings are much bigger than the boundary layer thickness. Such a constraint forces the closure to act in RANS mode near the wall, through the sensitivity in \( \Delta \). Even with a \( \Delta_x^+ > 8000 \), Nikitin et al. [23] showed that turbulence is sustained in the bulk of a channel flow, lending support for the method. Despite this, hybrid RANS/LES has been shown by numerous authors to be sensitive to the streamwise grid spacing [3, 21, 23, 24, 25]. It should be noted that most of the analysis has been performed using DES; this does not reflect its shortcomings relative to other hybrid methodologies, rather the opposite. Early success of DES saw it become the industry standard in the 2000s and so the deficiencies of hybrid RANS/LES were discovered with this technique first.

When an aggressive streamwise spacing is not used, the switch between RANS-LES is pushed deep into the inner layer. This is because the cell Reynolds number is reduced and resolved eddies are allowed to form. This lowers the modelled stress contribution which is not compensated by resolved turbulence. This phenomena has become known as Model Stress Depletion (MSD) and in some cases has been shown to cause premature separation [25]. Spalart et al. [3] argue that this was slightly overstated but the sensitivity to grid spacing was acknowledged. Further, the near-wall, being a RANS solution, contains its own log-layer several wall units shy of the correct value, due to the model being artificially suppressed. Once LES takes over, the resolved stress dominates and a new log-layer is formed several wall units too big. This discrepancy, also called the log-layer mismatch, is amplified as the Reynolds number is increased [26]. Baggett [24] argues from channel flow results that both MSD and the log-layer mismatch are produced by the ‘artificial’ near-wall turbulence cycle characterised by large streamwise streaks, which is entirely correct, although he argues that this is resulting from resolved structures being entirely parametrised by the grid. This conclusion however was the result of a low resolution in the spanwise direction, and this author has numerous experiences of observing large-scale streaks on much finer meshes.

In such unified modelling, methods have surfaced that more directly control the RANS-LES interface to reduce the grid sensitivity of the switch. Piomelli et al. [27] introduced a backscatter method in the inner layer of DES, which helped in the breakdown of the large streamwise streaks. Introduced as a proof of concept only, with no physical justification to the forcing term, the modelled log-layer was removed somewhat.

In recent advances of the Partially-Averaged Navier-Stokes (PANS) method [28], wall functions have been revisited in a VLES approach [29]. The method blends integration to the wall with equilibrium wall functions [30]. The planar channel flow is very well predicted for \( Re_c = 395 \) on a fine mesh, with no evidence of a log-layer mismatch. This is unsurprising however, as the model contribution was very low and as such the modelled log-layer is absorbed fully into the buffer layer.

Spalart et al. [3] detailed the Delayed Detached Eddy Simulation (DDES) formulation which further controls the length scale, expanding on Eq. 3. Weinmann et al. [13] used a similar function that effectively acts on the model dissipation rate \( \varepsilon \). The former is the subject of this paper, in an effort to assess the near-wall behaviour of unified models.
The DDES method uses the equation,
\[ \ell_{\text{hyb}} = \ell_t - f_d \min\left(0, \ell_t - \ell_{\text{sgs}}\right), \]  
(4)

where for the standard DDES approach, \( \ell_{\text{sgs}} = C_{\text{des}} \Delta \). \( f_d \) is given as,
\[ f_d = 1 - \tanh(24r^3_d), \]  
(5)

with
\[ r_d = \frac{\nu_{\text{sgs}} + \nu}{\sqrt{\partial_x u_i \partial_x u_i \kappa^2 y^2}}. \]  
(6)

\( \kappa = 0.41 \) is the von Kármán constant and \( r_d \) controls the switch from RANS to LES, based on the effective viscosity. The length scale is then adjusted using Eq. 4.

This report is as follows. Section 2 details the hybrid RANS/LES methodology. This is formally an FSM approach, however the near-wall switching mechanism of DDES has been adopted. Consequently the methodology contain within this report is referred to as FSM. In Section 3, this modified FSM is applied to turbulent channel flow. Whilst results are reasonable, the discrepancies in the artificial buffer layer are linked to resolved structures originating in this region. Section 3.2 quantitatively looks at these structures. Finally Section 4 is a summary that briefly mentions one possible remedy.

2. Hybrid RANS/LES Methodology

The unified framework adopted in this work is the latest incarnation of the FSM blending philosophy [8, 13]. The grid-filtered momentum and continuity equations for an incompressible fluid,
\[ \partial_t \tilde{u}_i + \tilde{u}_j \partial_x \tilde{u}_i = -\partial_x \tilde{p} + \nu \partial_x^2 \tilde{u}_i - \partial_x \tau_{\text{hyb}}^{ij}, \]  
(7)
\[ \partial_x \tilde{u}_i = 0, \]  
(8)

are solved. \( \tilde{\cdot} \) is the grid-filter and \( \tau_{\text{hyb}}^{ij} \) is the model term that must pick up the subgrid scales caused by this cut-off. In the coarse grid limit, \( \tau_{\text{hyb}}^{ij} \rightarrow \tau_{\text{rans}}^{ij} \), no turbulence may be explicitly resolved and the grid-filter satisfies the definition of a Reynolds operator. As the grid scale \( \Delta \) becomes sufficiently small, say \( \Delta \sim \ell_k \) (\( \ell_k \) being the Kolmogorov length scale), then \( \tau_{\text{hyb}}^{ij} \rightarrow 0 \) and Eq. 8 reduces to the Navier-Stokes. Thus it is the philosophy of unified hybrid RANS/LES to operate as a DNS, RANS or anywhere in between; depending on the local values of \( \Delta \) and \( \ell_k \).

To respect the coarse grid RANS limit, \( \tau_{\text{hyb}}^{ij} \) is a modified RANS turbulence model. Away from this limit, the RANS model acts as in subgrid mode and the formulation can be thought of as a non-classical LES. Consider the classical RANS stress-strain relationship,
\[ \tau_{\text{rans}}^{ij} = \frac{2}{3} k \delta_{ij} - 2 \nu_t S_{ij} + a_{ij}^R, \]  
(9)

where \( k \) is the turbulent kinetic energy, \( \nu_t \) the eddy viscosity and \( S_{ij} \) the mean strain rate. By setting the extra anisotropy \( a_{ij}^R \) to 0, one recovers the linear Boussinesq approximation. In general \( a_{ij}^R \) is a non-linear polynomial in the velocity gradient tensor. For more detail see the work of Pope [31] or Speziale et al. [32].

Equation 9 can be coerced into a hybrid formulation by,
\[ \tau_{\text{hyb}}^{ij} = \frac{2}{3} k \delta_{ij} - 2 \nu_{\text{sgs}} S_{ij} + F a_{ij}^R, \]  
(10)
where the subgrid viscosity is given as,

$$
\nu_{sgs} = F \nu_t \sim F \cdot (u_t \ell_t) = u_t \cdot (F \ell_t) = u_t \ell_{hyb},
$$

where $\ell_t$ and $u_t$ are characteristic velocity and length scales of the largest unresolved motions. Note, $k$ is now the turbulent kinetic energy of the unresolved scales. $F$ is a damping function that is responsible for reducing $\ell_t$ to be characteristic of the smallest resolved eddies. Scales below $\ell_{hyb}$ are therefore not resolved and $\nu_{sgs}$ is appropriately defined as a subgrid viscosity. Note, $u_t$ is not damped by $F$ because the unsteady flow field is already a characteristic velocity scale of the resolved motions.

By similar arguments, consider the dissipation rate $\varepsilon$ in first a RANS framework,

$$
\varepsilon_{rans} \sim \frac{u_t^3}{\ell_t}.
$$

Then, for the hybrid formulation,

$$
\varepsilon \sim \frac{u_t^3}{F\ell_t}.
$$

Therefore to formulate a unified hybrid model from RANS, one may simply define

$$
\ell_{hyb} = F\ell_t
$$

in place of all instances of $\ell_t$ throughout a RANS closure. Note that this is a key difference between the DES and FSM philosophies, the former only damps $\ell_t$ in the dissipation term.

The damping function $F$ is the crucial element for success. Apart from being bound between 0 and 1, the DNS and RANS limits, there is very little one can write down. We assume that $F$ is a function of the turbulent scales $\ell_k$ and $\ell_t$ and also $\Delta$. This is intuitive; through this functional dependency, $F$ can assess the local and instantaneous turbulence and decide if the grid resolution is sufficient. The grid-filter is mesh dependent and a formal definition does not exist, therefore no derivation exists of a fully analytical $F = F(\Delta, \ell_t, \ell_k, \ldots)$. One must always begin a derivation from assumptions, which do not hold for every situation. For example, by considering Eq. 3 one can define the (D)DES damping function away from solid boundaries as $F = 0.65\Delta/\ell_t$. Clearly if $\Delta \approx \ell_t$, one desires the model in RANS mode, however the constant prevents this — $\Delta \approx \ell_t \implies F \approx 0.65 < 1$. Therefore on coarse meshes DES acts as a poorly calibrated RANS model.

To alleviate this, Weatheritt and Sandberg [15] used an evolutionary algorithm to create a functional form not from assumptions, but from physical flow data. This bases the damping function on scenarios that actually transpired, not on ones assumed to occur. This damping function is used throughout this work and is given as,

$$
F = \min \left[ \log \left( 1 + c_1 \frac{\Delta}{\ell_t} \right), 1.0 \right] \min \left[ c_2 \frac{\Delta}{\ell_t}, 1.0 \right],
$$

with the constants $c_1 = 0.75$ and $c_2 = 2.1$. In order to make use of the DDES near-wall switch, Eq. 4 is used as the definition of $\ell_{hyb}$, in favour of Eq. 14, where we define $\ell_{sgs} = F\ell_t$. This implies that as $y \to \infty$, $\ell_{hyb} = F\ell_t$ and as $y \to 0$, $\ell_{hyb}$ is found by Eq. 4 which includes the delayed switch to LES feature discussed in Section 1.

The rest of this section details the specific RANS model used in this work. The authors choose the pressure-strain correlation of Speziale, Sarkar and Gatski [32], as the starting point for the turbulence model, because it is derived for homogeneous turbulence. In other words an assumption of slowly changing mean shear seems reasonable for the subgrid scales. This model
is then inserted into an explicit algebraic equation for the Reynolds stress. The model is defined by Eq. 10. The extra anisotropy is found from,

\[ a_{ij}^* = \beta_2 \tau^2 \left( \widetilde{S}_{ikj} \widetilde{S}_{kji} - \frac{1}{3} \tilde{S}_{mn} \tilde{S}_{nm} \delta_{ij} \right) + \beta_4 \tau^2 \left( \widetilde{S}_{ik} \tilde{\Omega}_{kj} - \tilde{\Omega}_{ik} \tilde{S}_{kj} \right) \]  

(16)

where \( \Omega_{ij} \) is the rotation rate tensor and \( \tau = (1/\beta^* \omega) \) is the turbulent time scale. The coefficients \( \beta \) and the specific dissipation rate \( \omega \) will be defined below. The subgrid viscosity required for Eq. 10 is defined as,

\[ \nu_{sgs} = -\frac{1}{2} F \beta_1 k \tau. \]  

(17)

The dissipation rate \( \varepsilon \) is evaluated from

\[ \varepsilon = \frac{k^{3/2}}{F \ell_t}, \]  

(18)

where the characteristic length scale is \( \ell_t = k^{1/2}/\beta^* \omega \).

The coefficients \( \beta \) are respectively,

\[ \beta^* = 0.09, \quad \beta_1 = -A_1 N/Q, \quad \beta_2 = 2A_1 A_2/Q, \quad \beta_4 = -A_1/Q. \]  

(19)

Where the constants \( A \) are,

\[ A_1 = 1.22, \quad A_2 = 0.47, \quad A_3 = 0.88, \quad A_4 = 2.37. \]  

(20)

\( Q \) is found from,

\[ Q = N^2 - 2 \tau^2 \tilde{\Omega}_{mn} \tilde{\Omega}_{nm} - \frac{2}{3} \tau^2 A_2^2 \tilde{S}_{mn} \tilde{S}_{nm}, \]  

(21)

where \( N \) is,

\[ N = \begin{cases} 4A_3 + (P_1 + \sqrt{P_2})^{1/3} + (P_1 - \sqrt{P_2})^{1/3}, & P_2 \geq 0, \\ \frac{4A_3}{3} + 2(P_1^2 - P_2)^{1/6} \cos \left[ \frac{1}{3} \arccos \left( \frac{1}{P_1^2 - P_2}^{-1/2} \right) \right], & P_2 < 0, \end{cases} \]  

(22)

and the \( P \),

\[ P_1 = \left[ \frac{A_3^2}{27} + \left( \frac{A_1 A_4}{6} - \frac{2}{9} A_2^2 \right) \tau^2 \tilde{S}_{mn} \tilde{S}_{nm} - \frac{2}{3} \tau^2 \tilde{\Omega}_{mn} \tilde{\Omega}_{nm} \right] A_3, \]  

(23)

\[ P_2 = P_1^2 - \left[ \frac{A_3^2}{9} + \left( \frac{A_1 A_4}{6} + \frac{2}{9} A_2^2 \right) \tau^2 \tilde{S}_{mn} \tilde{S}_{nm} + \frac{2}{3} \tau^2 \tilde{\Omega}_{mn} \tilde{\Omega}_{nm} \right]^3. \]  

(24)

The relations for \( N, P \) and \( Q \) are the result of assuming the anisotropy tensor \( a_{ij} = \tau_{ij}^{hyb} - 2/3k \delta_{ij} \) is a function of \( S_{ij}, \Omega_{ij} \) and \( P_k/\varepsilon \). \( P_k = -\tau_{ij}^{hyb} \partial_x \tilde{u}_i \) is the production of turbulent kinetic energy. By further assuming two dimensional mean flow, the tensor basis required to span \( a_{ij} \) reduces to just three components. Finally, because \( a_{ij} = 0 \), there are only two non-zero scalar invariants. \( N \) is then an algebraic approximation of the ratio \( P_k/\varepsilon \). For a full and guided derivation see the detailed work of Wallin and Johansen [33].

Finally, to close the equation set, transport equations for \( k \) and \( \omega \) are required. These are written down in the formulation of the Menter Shear Stress Transport (SST) [34] model, modified by Hellsten [35]

\[ \partial_t k + \tilde{u}_j \partial_x k = P_k - \varepsilon + \partial_x \left[ (\nu + \sigma_k \nu_l) \partial_x k \right] \]  

(25)

\[ \partial_t \omega + \tilde{u}_j \partial_x \omega = \gamma \frac{\omega}{k} P_k - \zeta \omega^2 + \partial_x \left[ (\nu + \sigma_\omega \nu_l) \partial_x \omega \right] + \sigma_D C D_{k\omega}^+. \]  

(26)
Left to be defined is,

\[ CD_{k\omega} = \partial_{x_j} k \partial_{x_j} \omega, \]
\[ \psi = \psi_2 + F_1(\psi_1 - \psi_2), \]
\[ \Gamma = \min \left[ \max \left[ \frac{k^2}{3\omega y}, \frac{500\nu}{y^2\omega} \right], \varphi \right], \]
\[ \varphi = \frac{20k}{\max \left[ y^2\omega, 200k \right]} \].

(27)

\( \psi \) is a coefficient in Eqs. 25-26 that has two definitions, defined by a subscript 1 or 2. Each \( \psi \) is blended between two values \( \psi_1 \) and \( \psi_2 \), such that \( \psi_1 \) is active near the wall and \( \psi_2 \) far from it. These coefficients, along with \( \sigma_k \), are given as,

\[ \sigma_{\omega 1} = 0.53, \quad \sigma_{\omega 2} = 1.00, \quad \sigma_{d 1} = 1.0, \quad \sigma_{d 2} = 0.4, \quad \sigma_k = 1.1, \]
\[ \gamma_1 = 0.518, \quad \gamma_2 = 0.44, \quad \zeta_1 = 0.0747, \quad \zeta_2 = 0.0828. \]

(28)

This hybrid closure is implemented into OpenFOAM [36], this limits numerical schemes to 2\(^{nd}\) order. Further for the convection of \( \tilde{u}_i \), a blending of 95\% central differencing and 5\% upwinding is performed, sacrificing this second order accuracy.\(^1\) Being an unstructured code, \( \Delta \) is defined as the cube root volume of each cell,

\[ \Delta = (\Delta_x \Delta_y \Delta_z)^{1/3}. \]

(29)

3. Planar Channel

The turbulent channel simulation used in this work is described as follows. The flow is driven at a constant mass rate between two parallel zero slip flat plates of infinite extent. In practice this is achieved by periodic boundary conditions in the streamwise and spanwise directions and a body force added to the momentum equation. The streamwise, wall normal and spanwise directions are respectively parallel to the \( x, y \) and \( z \) coordinates. The Reynolds number based on friction velocity \( u_\tau \) and channel half height \( h \) is \( \text{Re}_\tau = 590 \). The grid is deliberately prescribed such that the RANS-LES switch is ‘sensitive’ to the streamwise spacing — see Section 1. To achieve this \( \Delta_x = h/20 \) and \( \Delta_z = \Delta_x/2 \). The streamwise spacing is set to match the original diagnosis by Spalart \textit{et al.} [3]. Normally \( \Delta_x \geq \Delta_z \), which assumes that the smallest structures are streaks. Note, the grid here is set such that near-wall super-streaks of domain length are not governed by an overly coarse spanwise spacing. Because the hybrid RANS/LES formulation does not utilise wall functions, \( y^+ \) is set to 0.7. These grid requirements are satisfied by a computational domain of \( 2\pi \times 2h \times \pi \) spanned by \( 108 \times 108 \times 108 \) cells.

The reference DNS data for statistical comparison is that of Moser, Kim and Mansour [37], whilst the reference structures are from the DNS data of Lozano-Durán and Jiménez [38]. This latter data set was analysed with the method outlined by Lozano-Durán \textit{et al.} [39] and is followed within this report. Both reference sets [37, 38] are denoted purely by DNS without ambiguity.

3.1. Statistical Results

Presented in this section are the time-, span- and stream-wise averaged quantities of typical interest in a planar channel flow. Table 1 is a list of global results for the hybrid case. Because of the low Reynolds number, the buffer layer is defined by the interval bounded by the modelled and resolved peaks \((y^+(\tau_{2gs}^{\text{max}}), y^+(\tau_{2res}^{\text{max}}))\). From Fig. 1(a), it is evident the buffer layer is

\(^1\) This is not required for the channel simulation here, but merely an artefact of the calibration considerations of more complex geometries.
Table 1. Location of the peak modelled, resolved and total stresses, the average RANS-LES interface and the size of the buffer layer. The RANS-LES interface is defined by the smallest value of $y^+$ such that $F < 1$.

|       | $y^+(\tau^{\text{max}}_{12, \text{sgs}})$ | $y^+(\tau^{\text{max}}_{12, \text{res}})$ | $y^+(\tau^{\text{max}}_{12, \text{tot}})$ | $F^{\text{switch}}_+$ | $\delta^{\text{buff}}_+$ |
|-------|------------------------------------------|------------------------------------------|------------------------------------------|----------------------|----------------------|
| FSM   | 15.69                                    | 84.27                                    | 53.89                                    | 17.67                | 68.58                |

incorrect with a modelled log-layer partially developing from the near-wall RANS model. It is this region that is known as the artificial buffer layer. The resolved log-layer intercept is correctly predicted, as is the shear stress, and consequently the skin friction is under predicted due to this buffer layer discrepancy.

The shear stress, with the modelled and resolved component dissection, is seen correctly predicted in Fig. 1(c). This is unsurprising, the methodology was originally calibrated for this very quantity. The normal stress components are also included, $u_i^{\text{rms}}$ denotes the root mean square of the velocity fluctuations about the mean. The added turning points at $y^+ \approx 80$ in the $v^{\text{rms}}$ and $w^{\text{rms}}$ hint at destruction of the model component and the delayed growth of resolved normal stress. This delay is because initially, at the RANS-LES interface $y^+ = F^{\text{switch}}_+$, resolved motion is produced in the streamwise direction. Consider the transport equations for the fluctuations about the effective filtered velocity $u'_i = u_i - \bar{u}_i$. In the limit $F \to 1$, the production terms reduce to,

$$
P_{u'} = -v'\partial_y \bar{u}
$$

$$
P_{v'} = -v'\partial_y v'
$$

$$
P_{w'} = -v'\partial_y w'.
$$

(30)

Note that in the RANS limit, $\langle \cdot \rangle$ satisfies the properties of a Reynolds operator and $\bar{u}_i \to u_i$. $\bar{u}_i$ is the velocity of the RANS part of the flow field. As $F \to 0.5$, see Fig. 1(d), the effective viscosity is not low enough for proper structures to form, instead $\bar{u} \to \bar{u}$ guides more energy into $u'$. It is not until $F$ is a function of $\Delta$ and not the consequence of the DDES wall switching mechanism (Eq. 4), that the anisotropy levels recover somewhat. Clearly, near the wall at least, an isotropic damping of the length scale $\ell_t$ is insufficient.

The anisotropy is shown best through the invariant map, displayed in Fig. 1(b). The bounding lines, known as the Lumley triangle [40], enclose all realisable states of turbulence. $II$ and $III$ are the second and third invariants of $a_{ij}/2k$ given as $a_{ij}a_{ji}/8k$ and $\det(a_{ij}/2k)$ respectively. For more information, see for example the paper by Simonsen and Krogstad [41]. The right-hand edge corresponds to a state where one component of $a_{ii}$ (no summation) dominates (cigar shaped stress tensor) and the left-hand edge is where one component is negligible (pancake shaped stress tensor). The upper boundary is all other possible states where only two components exist and $(0, 0)$ is the point of perfect isotropy. Walking the DNS profile, from the origin, we see that we begin with an isotropic state (on average) in the bulk of the channel, moving to the small kink at $y^+ = 100$ and then as we move closer to the wall we finally see two-component turbulence. For the FSM, we see the largest discrepancy in the artificial buffer layer. Further, the profile is translated to the right of the DNS because of a global over prediction of $u^{\text{rms}}$ and isotropy is never fully recovered in the bulk. Finally, the two-component state of turbulence is never achieved; the underlying RANS model, see Section 2, was derived assuming homogeneous turbulence, a condition that is strongly violated near the wall.
3.2. The Instantaneous Field
To better understand the shape of the statistics profiles, the instantaneous fields are now inspected. Figure 2 are profiles of $\tilde{u}$ and $\tilde{v}$ in the $x$-$z$ planes. Note initially that $y^+ = 10$ is deep within the ‘RANS’ zone ($F_{\text{switch}}^+ = 17.67$) and the fluctuations one observes provide the boundary condition for the LES region.

The streamwise fluctuations are heavily correlated. The familiar super-streaks can be seen in the artificial buffer layer, which we define to be the region between the peaks of modelled...
y^+ = 15.69 and resolved y^+ = 84.27 stress — see Table 1. Further, the streaks at y^+ = 10 are visually correlated with those at y^+ = 50; implying the presence of structures as tall as this region.

Within the artificial buffer layer, one can see an anticorrelation between u' and v' — red u' values indicates the lift up of these streaks and blue is the downwash of more homogeneous shaped structures. In order to analyse these apparent structures further, momentum transport structures are extracted from the flow and compared with DNS simulations [42], following the method of Lozano-Durán et al. [39]. The DNS Reynolds number is Reτ = 950. Despite being significantly higher than the hybrid case, it was shown in the original analysis [39] that channel flows at Reτ = 950 and Reτ = 2000 yielded similar results. As such, the FSM is contemplated alongside the Reτ = 950 DNS with some confidence of drawing meaningful comparisons.

A structure S is defined as a connected region of space that satisfies,

\[ |u'v'(x, y, z)| \geq H \cdot u_{\text{rms}}^r(y)v_{\text{rms}}^r(y). \]  

Connectivity is defined by considering the six adjacent points in the mesh including the periodic boundaries. Percolation analysis (initially developed for isotropic turbulence [43] and later adapted to channels [39]) showed that a hyperbolic hole size H = 1.75 yielded the threshold value of the ratio of the volume of the largest structure and the total volume of all structures. For the FSM, percolation analysis revealed H ≈ 1.25 ∼ 1.75 yielded similar behaviour and the upper bound is taken in this report to match the reference data.

Once structures have been extracted from uncorrelated snapshots, properties that define each S can be calculated. The structure velocity is defined as,

\[ \langle u \rangle_i = \frac{\int_S u'dV}{\int_S dV}. \]  

Quadrant analysis [44, 45] can be performed by considering the signs of \langle u \rangle and \langle v \rangle. The most common types of structures are ejections (Q2's, \langle u \rangle < 0, \langle v \rangle > 0) and sweeps (Q4's, \langle u \rangle > 0, \langle v \rangle < 0), smaller interactions of outward (Q1, \langle u \rangle > 0, \langle v \rangle > 0) and inward (Q3, \langle u \rangle < 0, \langle v \rangle < 0) natures complete the quadrant. Also of particular interest in this work are the dimensions of the bounding box and the minimum and maximum wall distance of each structure. These are denoted respectively as: (\ell_x, \ell_y, \ell_z), y_{\text{min}} and y_{\text{max}}. Special care is taken to ensure these quantities are properly defined across periodic boundaries and that the normal velocity points away from the nearest wall.

A total of 10^6 structures were extracted from 208 DNS snapshots, whilst 6000 structures were extracted from 71 FSM snapshots. Each structure can be reduced to a data point, such that it is described by its list of features (\langle u \rangle, \langle v \rangle, \ell_x, \ell_y, \ell_z, y_{\text{min}}, y_{\text{max}}, \ldots). This results in a high dimensionality data set.

The velocity quadrant of all structures is displayed in Fig. 3(a). The filled contours are the DNS and the red the FSM. From this point, DNS structures will be denoted DNS-S and similarly the FSM as FSM-S. The inward and outward interactions are almost absent from the hybrid flow field. This is because they are much smaller in nature than Q2's and Q4's and are removed by the effective filter. The hybrid case has produced sweeps and ejections with much too great streamwise velocity. Generally the wall normal velocity is in much better agreement, although some sweeps are too intense in this direction. These erroneous structures are those closest to the wall. Knowing that the outer boundary of each histogram is made up of the near-wall structures, it is deduced that the velocity in the streamwise streaks of the hybrid RANS/LES is approximately 1.5-1.75 times too great. This is not immediately apparent in Fig 1(c) because the RANS-LES interface fluctuates vertically and RANS regions with no structures persist (making the resolved streaks rarer). Once deep in the artificial buffer layer, this no longer occurs and one
can see an overprediction in $u'_{rms}$. Currently this phenomena is mesh/methodology/Reynolds number dependent, however further simulations should make their discrepancy parametrisable.

Figure 3(b) is a plot of the structure averaged shear stress against its averaged wall normal velocity for FSM-$S$ that live in the log-layer only. This is defined as $y < 0.2h$. To extract data for these log-layer plots, structures satisfying $y_{max} > 0.2h$ are removed. We see the FSM structures closely follow the included lines. These lines are $|u'v'| = \pm 2.5v^+$. (33)

This is based on the mixing length argument, that $\ell \propto y$ and the fluctuation (displacement of
Figure 3. Histograms of structures, all contour levels are with respect to the maximum density on a logarithmic scale. Colours are consistent across all plots and plot type. Contours/lines: filled/black, DNS-S; red, FSM-S; blue, DNS-S$^+$; green, FSM-S$^+$. (a) $\langle v \rangle$ against $\langle u \rangle$. Contour levels: 30%, 50%, 70%, 90%. (b) $|\langle uv \rangle^+|$ against $\langle v \rangle^+$ for structures in the logarithmic layer only. Contour levels 10% increments. Lines: Eq. 33. (c) $\ell_z$ against $\ell_x$. Contour levels: 40%, 60%, 80%. Lines: least squares regression fits $\ell_z = m\ell_x + c$. (d) $y_{\max}$ against $y_{\min}$. Contour levels: filled, 30%, 50%, 70%, 90%; blue and green, 60%, 90%. Diagonal Lines: least squares regression for all structures above Eq. 36 such that $y_{\min} > 0.1h$; vertical line, $y_{\min} = F_{\text{switch}}$. 
velocity mean) is proportional to this eddy size $\ell$,

$$\overline{uv^+} \propto (\overline{v\partial_y u})^+ \propto \overline{v^+}. \quad (34)$$

The last simplification is based on the log-law equation,

$$\partial_y \overline{u} = \frac{u_r}{k y}. \quad (35)$$

Therefore resolved structures in hybrid RANS/LES, once developed, do reproduce the correct mixing. The model has been calibrated such that this is reproduced, however because of the well documented increase in streamwise fluctuations, occurring from the RANS-LES interface, it becomes an impossibility to tune the model for both $u_{rms}$ and $w_{rms}$ in its current state and we see an over prediction in the normal stress (Fig. 1(c)).

The black and red lines in Fig. 3(c) are linear regressions of FSM-$S$ and DNS-$S$ respectively. An approximate relationship between the spanwise and streamwise length, $\ell_x = m \ell_z$, can be formulated. For the DNS-$S$ this gradient is $m \approx 2.6$, whilst for the FSM-$S$ the value is considerably larger at $m \approx 4.5$. This indicates that FSM-$S$ constitutes structures whose ratio $\ell_x/\ell_z$ is approximately 1.75 times greater than equivalent structures in DNS-$S$. Although not shown in this preliminary report, the strong, small near-wall ejections in DNS-$S$ satisfy $\ell_x/\ell_z \approx 5.3$ but the corresponding structures in FSM-$S$ satisfy $\ell_x/\ell_z \approx 7.8$ — showing that the streakiness is a feature at the smallest scales also. This comparison is tentatively made however; the bounding boxes of FSM-$S$ of this type are approximately 10 times their DNS-$S$ counterparts.

In order to probe the DNS data further, a slightly modified version of an evolutionary clustering algorithm [46] is employed to divide the structures into groups. Evolutionary Algorithms (EAs) are non-deterministic optimisation tools that iterate towards a solution via survival of the fittest. The specifics and the optimality of solutions are not dwelt on here, rather a few comments about the structure groupings will be discussed below. Typically, DNS structures can be categorised as wall attached or wall detached. For example the filled contours in Fig. 3(c) show two arms; the thin, left arm are wall attached structures that have grown tall and the much larger, right arm are structures that are not as tall but exist away from $y = 0$. Instead, the EA finds the distinction,

$$\frac{4}{5} \ell_x + \ell_z = h, \quad (36)$$

which categorises the structure based implicitly on its volume. Figure 3(c) is the joint probability density function showing the distribution of the DNS structures with respect to $\ell_z$ and $\ell_x$. The blue contours are those that lie above Eq. 36. It is the latter grouping that is the primary focus of the rest of this report, used to contrast with equivalent FSM structures. The DNS structures of this sort shall now be denoted by DNS-$S_+$. The grey dashed line in this figure is a regression line of the detached DNS-$S_+$, included as there are too few to show up in the histogram.

Equivalently, the red contours of Fig. 3(d) are the FSM-$S_+$ and the red line is a linear regression of those detached. The term detached has a slightly different meaning for the FSM. Because of the RANS zone, structures cannot properly form near the wall that satisfy Eq. 31 and instead structures really take shape above the RANS-LES switch. The grey dashed line...
of the same figure highlights $y_{\text{min}} = F_{\text{switch}}$. In this sense, attached means ‘interface’ attached. From Table 1 we can read that the artificial buffer layer begins at $y^+ = 15.69$ ($y/h = 0.027$) and ends at $y^+ = 84.27$ ($y/h = 0.14$). Figure 3(d) shows that between these bounds, interface attached structures grow much taller than their DNS counterparts, with many extending beyond the half height of the channel. Interestingly, their number is slightly fewer — 26% of DNS-$S_+$ satisfy $y_{\text{max}} > h$ but only 22% of FSM-$S_+$. Crucially however, the differences lie in when they detach and also that the wall is a solid boundary whilst the interface is not. The tall FSM-$S_+$ structures exist over a wider band of possible $y_{\text{min}}$’s and become detached from the interface much more sporadically than the DNS-$S_+$. At a height of $\ell_y/h \approx 0.6$, the DNS-$S_+$ begin to detach and follow the regression line outlined in Fig. 3(d). Remarkably the FSM-$S_+$ follow a similar line, the gradients differ by only 0.01 $h$. However, due to the freedom allowed by the interface over the wall, the standard deviation from this line in the artificial buffer layer is 0.27 $h$ for FSM-$S_+$ and only 0.21 $h$ for the DNS-$S_+$. This increase of 33% is indicative of the sporadic detachment, which pollutes the bulk flow with the elongated structures that contribute to the regression lines in Fig. 3(c).

4. Conclusions and Future Avenues

In short we are now in a better position to understand the discrepancies of Fig. 1. The correct shear stress comes at the price of an incorrectly predicted streamwise stress. The streamwise streaks that cause the issues are these very tall, thin structures that exist in the artificial buffer layer. These are malformed due to the delay in dropping the length scale to an LES value and are allowed more freedom to move around because they are only interface attached, not wall attached. This has the knock-on effect of never recovering isotropy in the bulk and a smeared $u_{\text{rms}}$ peak.

These structures (FSM-$S_+$) characterise the incorrect buffer layer, which spans from $y^+(\tau_{12}^{\text{max}})$ to $y^+(\tau_{12}^{\text{max}})$. In this region, the incorrect velocity profile is caused by the set in of the underlying RANS model or in other words the delay in turbulence generation. To improve the hybrid RANS/LES approach, the break-up of the interface attached FSM-$S_+$ must be tackled. Piomelli et al. [26] have added white noise forcing to the interface with some success but without physical justification. This work is the very early beginnings of motivating a physical argument to such a backscatter method. Namely a forcing method that breakups up the interface attached FSM-$S_+$ and reduces the variation in their height when they detach. Such a methodology could be incorporated as a body forcing term on the right hand side of Eq. 8 or alternatively each Reynolds stress component $\tau_{ij}^{\text{hyb}}$ could be damped individually to account for the anisotropy of the resolved structures at the RANS-LES interface.

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