Brane Actions and String Dualities

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Abstract  

An effective action for the M9-brane is proposed. We study its relation with other branes via dualities. Among these, we find actions for branes which are not suggested by the central charges of the Type II superalgebras.

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1 Introduction

The BPS states give information about the non-perturbative structure of M- and string theory. There is a known relation between the central charge structure of the spacetime supersymmetry algebra and the occurrence of BPS states in the theory. The generic rule is that the presence of a \( p \)-form central charge in \( D \) dimensions suggests the existence of a \( p \)-brane and a \( (D-p) \)-brane \([1,2]\). The eleven-dimensional supersymmetry algebra with the maximum number of central charges is given by \((\alpha = 1, \ldots, 32; M = 0, \ldots, 10)\):

\[
\{Q_\alpha, Q_\beta\} = (\Gamma^M C)_{\alpha\beta} P_M + \frac{1}{2!} (\Gamma^{MN} C)_{\alpha\beta} Z_{MN}
+ \frac{1}{5!} (\Gamma^{MNPQR} C)_{\alpha\beta} Z_{MNPQR}.
\] (1)

The spatial component of \( P_M \) is related to an M-Wave, moreover we find an M2-brane, an M5-brane, an MKK-monopole and an M9-brane:

\[
Z_{MN} \rightarrow \text{M2 and M9}, \quad Z_{MNPQR} \rightarrow \text{M5 and MKK}.
\] (2)

The dynamics of the M2- and M5-branes have been extensively studied. However, it is only recently that the dynamics of the Kaluza-Klein monopole has been understood \([2,3]\). The nature of the M9-brane is not yet well understood.

One can perform a similar analysis of the Type IIA supersymmetry algebra with the maximal number of central charges. One then finds a gravitational wave (W-A), a fundamental string (NS-1A), \( Dp \)-branes \((p = 0, 2, 4, 6, 8)\), a solitonic five-brane (NS-5A), a Kaluza-Klein monopole (KK-5A) and a nine-brane (NS-9A). Similarly, from the Type IIB supersymmetry algebra one finds a gravitational wave (W-B), a fundamental string (NS-1B), \( Dp \)-branes \((p = 1, 3, 5, 7, 9)\), a solitonic five-brane (NS-5B), a Kaluza-Klein monopole (KK-5B) and a further nine-brane (NS-9B). We see that in particular an S-dual partner for the D9-brane is predicted \([2]\). The role of these new 9-branes in string theory is not yet well understood (see however \([4]\)).

All these branes are related to each other by T- and S- dualities, and the IIA branes can be obtained from the M-branes by dimensional reduction. In this contribution we review the dynamics of the MKK-monopole and propose an effective action for an M9-brane in similar terms. Studying their relation with other branes via dualities, we find branes which are not suggested by the supersymmetry algebras.
1.1 The M-theory KK-monopole

The Kaluza-Klein monopole is a purely gravitational solution that appears in Kaluza-Klein compactification \[5\]. In 11 dimensions, it can be schematically represented by\[2\]:

\[
\text{M KK-monopole} : \left\{ \times \times x \times x z - - - \right. \text{.} \quad (3)
\]

It has 8 isometry directions, one of which is generated by a Killing vector \( \hat{k} = \partial_z \). Thus it can be considered as an extended object. The worldvolume field content must correspond, after gauge fixing, to a 7-dimensional vector multiplet \[2\] with 3 scalars and 1 vector. However, if we consider the isometry direction \( z \) either as worldvolume or transversal, we do not obtain the right number of degrees of freedom. In order to solve this, we use a characteristic feature of the monopole. Since the monopole is localized in the transversal Taub-NUT space, the \( z \) isometry must be perpendicular to the worldvolume of the monopole action. However, we have in general \( k_M \partial_i \hat{X}^M \neq 0 \), for embedding coordinates \( \hat{X}^M \) and worldvolume coordinates \( \xi^i \). We define then a projector:

\[
\mathcal{P}^M_N = \delta^M_N + |\hat{k}|^{-2} \hat{k}_M \hat{k}^N, \quad (4)
\]

which projects any vector to the component perpendicular to \( \hat{k} \). The monopole is then described by the embeddings \( \partial_i \hat{X}^M \mathcal{P}^M_N \), which satisfy \( \hat{k}_N \partial_i X^M \mathcal{P}^M_N = 0 \).

With this construction we have gauged the translations along \( \hat{k} \), since the projected pullback is in fact equivalent to a covariant derivative:

\[
D_i \hat{X}^M = \partial_i \hat{X}^M + \hat{A}_i \hat{k}^M, \quad (5)
\]

with gauge field \( \hat{A}_i = |\hat{k}|^{-2} \partial_i \hat{X}^M \hat{k}_M \). Therefore, after gauge fixing, the embedding scalar \( \hat{X}^z \) disappears. Consequently we achieve the right number of degrees of freedom. The kinetic term of the action has the following form \[3\]:

\[
\hat{S} = -T_{\text{M KK}} \int d^7 \xi |\hat{k}|^2 \sqrt{|\det (\hat{\Pi} + l_p^2 |\hat{k}|^{-1} \hat{K}^{(2)})|}, \quad (6)
\]

where the field-strength \( \hat{K}^{(2)} \) of the Born-Infeld-like 1-form \( \hat{\omega}^{(1)} \) is defined by

\[
\hat{K}^{(2)} = d\hat{\omega}^{(1)} + l_p^{-2} \partial \hat{X}^M \partial \hat{X}^N (i \hat{k} \hat{C}^{(3)})_{MN}, \quad (7)
\]

\(^2\)We use a notation where \( \times (-) \) denotes a worldvolume (transversal) direction and \( z \) corresponds to the isometry direction.

\(^3\)We use hats for 11-dimensional fields and no hats for 10-dimensional ones.
and \( \hat{\Pi} \) is the pullback of the spacetime metric:

\[
\hat{\Pi} = D\hat{X}^M D\hat{X}^N \hat{g}_{MN} = \partial\hat{X}^M \partial\hat{X}^N \left( \hat{g}_{MN} + |\hat{k}|^{-2} \hat{k}_M \hat{k}_N \right). \tag{8}
\]

In general, for all KK-monopoles, there is a coupling to the Killing vector through the scalar

\[
|\hat{k}|^2 = \left( \frac{R_k}{l_p} \right)^2, \tag{9}
\]

where \( R_k \) is the radius of the compact isometry and \( |\hat{k}|^2 = -\hat{k}^M \hat{k}^N \hat{g}_{MN} \). Accordingly, the tension of the KK-monopole has the following form:

\[
\mathcal{T}_{\text{MKK}} = \frac{R_k^2}{(2\pi)^6 l_p^9}. \tag{10}
\]

The complete action for the KK-monopole in different backgrounds has been given in [6, 7, 8]. The 1-form field \( \hat{\omega}^{(1)} \) describes the flux of an M2-brane wrapped around the isometry direction. This corresponds to the intersection of an M2-brane with the MKK-monopole over a 0-brane, such that one of the worldvolume directions of the M2-brane coincides with the isometry direction \( z \) of the Taub-NUT space [9]. A reduction of this intersection along the \( z \)-direction gives the configuration \( (0| F1, D6) \). The reduction of \( \hat{\omega}^{(1)} \) leads in this case to the BI field of the D6-brane, describing the tension of the fundamental string.

1.2 The M9-brane

The M9-brane is believed to be the strong coupling limit of the D8-brane [10]. However, the D8-brane is only consistent in the context of a Type I’ construction [11], where there are 2 orientifold 8-planes at the ends of one interval and 32 D8-branes located along the interval. In general, the background between two D8-branes is given by massive IIA supergravity. On the other hand, it is known [12] that only in the case where 16 D8-branes are sitting on top of each orientifold, there exists a strong coupling limit where the theory becomes eleven-dimensional. In this situation, the background in the bulk is standard IIA supergravity. This is related to the fact that massive IIA supergravity can only be given an eleven dimensional interpretation in terms of compactified M-theory [13].

\[\text{We use conventions for which } G_{11} = 16\pi^7 l_p^8.\]
It is then natural to consider an M9-brane as part of a composite of sixteen M9-branes and one M-theory orientifold plane, given that an M9-brane cannot be singled out in eleven uncompactified dimensions. As discussed in [4] this system would describe the end of the world branes of Hořava and Witten [14].

If we consider one compact direction, we can single out one of the M9-branes from the composite, trading the Wilson lines of the heterotic theory for positions of the M9-branes [15, 4]. The background between the M9-branes is the massive 11-dimensional supergravity of [13]. Accordingly, the dynamics of a single M9-brane will have associated a Killing isometry describing this compact direction, which means that it will be naturally described by a gauged sigma-model[4]. According to this discussion, an M9-brane can be represented schematically by:

\[
\text{M9 – brane : } \{ \times \times \times \times \times \times \times \times \times - ,
\]

where \(z\) indicates the direction of the Killing isometry. We propose the following gauged sigma-model to describe the dynamics of a single M9-brane:

\[
\hat{S} = -T_{M9} \int d^9 \xi |\hat{k}|^3 \sqrt{|\text{det} \left( \hat{\Pi} + |\hat{k}|^{-1} \hat{k}^{(2)} \right)|},
\]

(11)

with \(\hat{\Pi}\) and \(\hat{k}^{(2)}\) defined as for the MKK-monopole. Notice that, strictly speaking, this M9-brane is actually an 8-brane with one extra isometry. The M9-brane worldvolume corresponds to a 9-dimensional vector multiplet with one vector and one scalar, where one transversal scalar has been eliminated by the gauging. The action is such that dimensionally reducing along the Killing vector gives the D8-brane action. The effective tension of the M9-brane can be written as [17]:

\[
T_{M9} = \frac{R_k^3}{(2\pi)^8 l_p^2}.
\]

(12)

Similarly to the MKK-monopole case, the 1-form field \(\hat{\omega}^{(1)}\) appearing in the M9-brane action describes the flux of an M2-brane which has one direction wrapped around the isometry direction \(z\). This corresponds to the intersection of an M2-brane with an M9-brane, which gives upon reduction along the isometry direction the intersection of a fundamental string with a D8-brane.

\(^5\)This was also suggested in [8, 18], based on other arguments.
2 Gauged sigma-models and Type II dualities

Since the MKK-monopole and the M9-brane have similar actions, we can describe the reduction to Type IIA branes in a uniform fashion for both branes. The reduction of the gauged sigma-models (6) and (11) along the Killing isometry gives a D$p$-brane. A D6-brane from the MKK and a D8-brane from the M9. Any other type of reduction gives again a gauged sigma-model in Type IIA. In general, the Killing vector $\hat{k}$ becomes, after reduction along an eleventh-coordinate $Y$, the Killing vector associated to the isometry of the reduced brane: $\hat{k}^y = 0, \hat{k}^\mu = k^\mu$. We consider first the double dimensional reduction of the MKK/M9 action. We find:

$$S = - \int d^{p+1}\xi \ e^{-\frac{a}{\pi} \phi} |k|^{\frac{p-1}{2}} \left( 1 + e^{2\phi} |k|^{-2} (i_k C^{(1)})^2 \right)^{\frac{p-3}{4}} \times \frac{2\pi \alpha'}{|k|^{\frac{1}{2}} e^{2\phi} (i_k C^{(1)})^2 |\mathcal{K}_{ij}^{(1)}|}{\sqrt{1 + e^{2\phi} |k|^{-2} (i_k C^{(1)})^2} \mathcal{K}_{ij}^{(2)}}.$$  

This action corresponds to the KK-5A monopole for $p = 5$. For $p = 7$ we call it a KK-7A brane. The field-strengths are defined as follows:

$$\mathcal{K}^{(1)} = d\omega^{(0)} - \frac{1}{2\pi \alpha'} (i_k B),$$

$$\mathcal{K}^{(2)} = d\omega^{(1)} + \frac{1}{2\pi \alpha'} (i_k C^{(3)}) - \mathcal{K}^{(1)} \wedge DX^\mu C^{(1)}_{\mu},$$  

where $B$ is the NS-NS 2-form and $C^{(p)}$ denotes a R-R $p$-form. The covariant derivatives are defined as usual. The effective tension for these branes is given by:

$$T_p = - \frac{R^{rac{p-1}{2}}}{(2\pi)^{p-1} g_s^{\frac{p+1}{2}}} \ , \quad p = 5, 7.$$  

On the other hand, the reduction of the MKK/M9 along one transversal direction gives rise to a worldvolume scalar $c^{(0)} = (2\pi \alpha')^{-1} Y$. The action for this brane is given by

$$S = - \int d^{p+1}\xi \ e^{-\frac{a}{\pi} \phi} |k|^\frac{p}{2} \left( 1 + |k|^{-2} e^{2\phi} (i_k C^{(1)}) \right)^{\frac{p-3}{4}} \times \frac{2\pi \alpha' |k|^{-1} e^{\phi} (i_k C^{(1)})^2 |\mathcal{G}_{ij}^{(1)}|}{\sqrt{1 + |k|^{-2} e^{2\phi} (i_k C^{(1)})^2} \mathcal{H}_{ij}^{(2)}}.$$  

$^6$We use the notation of [3].

6
For $p = 8$ it corresponds to the NS-9A brane (as shown explicitly in [4]). We call it here a KK-8A brane to make clear that it contains a gauged isometry. For $p = 6$, it gives a new brane which is not predicted by the Type IIA supersymmetry algebra. We call it a KK-6A brane. The field-strengths are now defined as follows:

$$G^{(1)} = \frac{1}{2\pi\alpha'} DX^\mu C^{(1)}_{\mu},$$

$$H^{(2)} = \frac{1}{2\pi\alpha'} (i_k C^{(3)}) - (i_k B) \wedge dc^{(0)},$$

(17)

The fields $\omega^{(1)}$ and $\nu^{(1)}$ have the interpretation of the flux of a D2-brane ending on the corresponding gauged sigma-model. The effective tension for the KK-6A/KK-8A branes is given by:

$$T_p = \frac{\hat{R}^{\frac{3-p}{2}}}{(2\pi)^p g_s^{\frac{p}{2}} l_s^{3-p}}, \quad p = 6, 8.$$  

(18)

At the level of the solutions of the Type II supergravities it is known that T-duality relates the KK-monopole with the NS-5 brane, where the duality is performed in the isometry direction of the monopole and a transversal direction of the 5-brane. This also works at the level of the worldvolume effective actions. Then one can construct the effective action of the NS-5B brane by applying T-duality to the KK-A monopole action. The result is exactly the S-dual of the D5-brane action [3]. Similarly, applying T-duality to the action of the NS-5A brane, one finds the action for the Type IIB KK-monopole. The solutions corresponding to the KK-6A and KK-7A branes and their relations with other branes via dualities have been considered in [18].

Here we are going to show that the KK-$pA$ branes described by (14) and (17) are in general T-dual to the solitonic $p$-branes which are S-dual partners of the Type IIB $Dp$-branes ($p$ odd and $p > 3$) (see Figure 1), whose actions are given by [8]:

$$\int d^{p+1}\xi \ e^{-(\frac{p-1}{3})r(1 + e^{2r}C^{(0)2})\frac{p-3}{3}} \sqrt{|\det \left( g + \frac{(2\pi\alpha')e^r}{\sqrt{1 + e^{2r}C^{(0)2}} F} \right)|}.$$  

(19)

This action is obtained by directly applying S-duality on the $Dp$-brane action. The curvature $\tilde{F}$ is given by:

$$\tilde{F} = dc^{(1)} + \frac{1}{2\pi\alpha'} C^{(2)},$$  

(20)
where the 1-form \( c^{(1)} \) is a Born-Infeld-like field representing the flux of a D-string ending on the brane \([19]\). The effective tension for these branes is given by:

\[
T_p = \frac{1}{(2\pi)^p g_s \frac{1}{\ell_s} p^{p+1}}, \quad p = 5, 7, 9.
\]

The action \([19]\) represents a NS-5B brane for \( p = 5 \), as derived in \([8]\). For \( p = 7 \), it corresponds to a NS-7B. However, this is not predicted by the Type IIB supersymmetry algebra and indeed there seems to be only one seven-brane in Type IIB \([20, 21]\). It is not yet clear what the role of this NS-7B brane is. Finally, for \( p = 9 \) it gives the action of the NS-9B brane \([4]\).

T-duality on the gauged sigma-models \([14]\) and \([17]\) is performed along their Killing direction. For the Type IIB branes \([19]\), the T-duality is performed along a transversal direction when it connects them to the KK-5A and KK-7A branes. However, for the case of the KK-6A and KK-8A branes, the T-duality is performed along a worldvolume direction. For instance, the KK-5A/KK-7A are T-dual to the NS-5B/NS-7B branes, respectively. The relation between the worldvolume fields is given by:

\[
\omega^{(0)\nu} = \frac{1}{2\pi \alpha'} Z, \quad \omega^{(1)\nu} = -c^{(1)},
\]

where \( Z \) represents a transversal embedding coordinate of the NS-5B/NS-7B brane. On the other hand, the KK-6A/KK-8A are T-dual to the NS-7B/NS-9B branes, respectively. In this case the T-duality rules are given by

\[
v_i^{(1)\nu} = -c_i^{(1)}, \quad c^{(0)\nu} = -c^{(1)} \sigma,
\]

where \( \sigma \) is the wrapped worldvolume direction of the NS-7B/NS-9B brane.

The Type II supersymmetry algebras also predict the presence of two 9-branes in Type IIB. These 9-branes and the D8- and KK-8A branes are related by a chain of S- and T- dualities (See Figure \([1]\)). The D8-brane is known to be the T-dual of the D9-brane. On the other hand, by the IIB supersymmetry algebra there is an S-dual partner of the D9-brane, the NS-9B brane \([3]\), which is given by \([19]\) for \( p = 9 \). This 9-brane is T-dual to the KK-8A brane. Accordingly, the Killing isometry has the interpretation of an eleventh circular coordinate, from the point of view of the D8-branes; or a tenth circular coordinate, from the point of view of the KK-8A brane \([4]\).

The D8- and D9- branes play a role in the orientifold constructions of the Type IIA and IIB theories. The duality relations above suggest that the NS-
9B and KK-8A branes should play a similar role in the duality related theories (this is extensively studied in [4]).

3 Conclusions

We have presented a dynamical description for a single M9-brane in terms of a gauged sigma-model, similar to that describing the MKK-monopole. The descendants of these two branes have been considered in a unified fashion, with the result that two of them are not predicted by the Type IIA supersymmetry algebra: the KK-6A and KK-7A branes. The KK-\(p\) branes for \(p = 6, 7, 8\), have an exotic dependence in the string coupling, proportional to \(1/g_s^3\) and \(1/g_s^4\). Thus they do not have an obvious interpretation in weakly coupled string theory, where the most singular behaviour is expected to be \(1/g_s^2\). Nevertheless, U-duality studied at the algebraic level requires these extra states in order to fill up multiplets of BPS states in representations of the U-duality symmetry group of M-theory on a d-torus [17, 21]. These KK-\(p\)A branes are typically T-dual to the NS-\(p\)B branes defined by (19).

The description of the M9-brane is not yet complete, since we are forced to have a compactified 11-dimensional background. However, one expects that there is a formulation for which, putting 16 M9-branes together, one can get rid of the gauged isometry and go over to the ends of the world description of [14].
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