Power corrections of off-forward quark distributions
and harmonic operators with definite geometric twist

Bodo Geyer\(^*\), Markus Lazar\(^†\), and Dieter Robaschik\(^‡\)

\(^*\)Center for Theoretical Studies and Institute of Theoretical Physics,
Leipzig University, Augustusplatz 10, D-04109 Leipzig, Germany

\(^†\)BTU Cottbus, Fakultät 1, Postfach 101344, D-03013 Cottbus, Germany

(March 25, 2022)

We introduce a group theoretically motivated procedure of parametrizing non-forward matrix
elements of non-local QCD operators by (two-variable) distribution amplitudes of well-defined geo-
metric twist being multiplied by kinematical factors (related to the Lorentz structure of the operators
and to the target states) as well as position-dependent coefficient functions resulting from the (in-
finite) twist decomposition of the operators. These distribution amplitudes are interpreted as (sum
over) power corrections of the double distributions. — Using the technique of harmonic polynomials
for the local operators we determine the (infinite) twist decomposition of totally symmetric opera-
tors completely and for operators with non-trivial symmetry type up to twist \(\tau = 3\). This covers
the phenomenological interesting quark-antiquark operators. Using these results we determine the
power corrections to the various double distributions and the vector meson wave functions. It is
shown that the structure of the kinematical power corrections may be obtained, by harmonic exten-
sion, from the corresponding expressions for operators or distribution amplitudes, on the light-cone.

PACS number(s): 12.38 Bx, 13.85 Fb

I. INTRODUCTION

The universal, non-perturbative parton distribution amplitudes parametrizing, modulo kinematical factors, the
matrix elements of appropriate non-local quark-antiquark (as well as gluon) operators play a central role in phe-
nomenological considerations. These operators occur in the quantum field theoretic description of light-cone dom-
inated hadronic processes via the non-local light-cone expansion \([1–3]\). For deep inelastic lepton-hadron scattering
and Drell-Yan processes the parton distributions are given as forward matrix elements of bilocal light-ray operators
resulting from the time-ordered product of appropriate hadronic currents. For (deeply) virtual Compton scattering
and hadron wave functions the so-called double distributions and hadron distribution amplitudes, respectively, are
given by corresponding non-forward matrix elements. Thereby, various processes are governed by one and the same
(set of) light-ray operators.

For the sake of definiteness let us remember the amplitude of virtual Compton scattering,

\[
T_{\mu\nu}(P_1, Q; S_1) = \int d^4x \, e^{iqx} \langle P_2, S_2 | RT (J_\mu(x/2) J_\nu(-x/2) S) | P_1, S_1 \rangle,
\]

where \(Q^2 = -q^2\), \(q = q_2 - q_1\) denotes the momentum transfer and \(S\) is the (renormalized) \(S\)–matrix. Restricting to
lowest order the renormalized time-ordered product of the electromagnetic (hadronic) currents, \(J_\mu(x) = \bar{\psi}(x)\gamma_\mu \psi(x)\):
in the vicinity of the light-cone, \(x \rightarrow \tilde{x}, \tilde{x}^2 = 0\), where

\[
\tilde{x} = x - \zeta \left( (x\zeta) - \sqrt{(x\zeta)^2 - x^2} \right) \quad \text{with} \quad \zeta^2 = 1,
\]

is approximated by (a sum over) coefficient functions \(C^{(5)}_{\mu\alpha}(x^2, \kappa\tilde{x}, \mu^2)\) times the matrix elements of (renormalized)
light-ray operators (\(f\) denote the quark flavours)

\[
\langle P_2, S_2 | O^{(5)}_{\alpha}(\kappa\tilde{x}, -\kappa\tilde{x}) | P_1, S_1 \rangle = \sum_{n=0}^{\infty} \frac{(i\kappa)^n}{n!} \langle P_2, S_2 | \bar{\psi}(0)(\gamma_5)\gamma_\alpha(\tilde{x}D)^n \psi(0) | P_1, S_1 \rangle.
\]

\(^*\)E-mail: geyer@itp.uni-leipzig.de
\(^†\)E-mail: lazar@itp.uni-leipzig.de
\(^‡\)E-mail: drobasch@physik.tu-cottbus.de
Similar considerations hold for the (vector) meson wave functions occurring in processes like exclusive semi leptonic (or radiative) $B$ decays and hard electroproduction of vector mesons ($V = \rho, \omega, K^*, \phi$). The physical interest in this case arises from the fact that the following vertex function is directly observable in QCD:

$$T_{\mu\nu}(q, P; \lambda) = \int d^4x \, e^{i q \cdot x} \langle 0 | R T (J_\mu(x/2) J_\nu(-x/2) S) | V(P, \lambda) \rangle,$$

(1.4)

where $J_\mu$ can be axial or vector currents and $| V(P, \lambda) \rangle$ is the vector meson state of momentum $P$ and helicity $\lambda$. Here, the universal nonperturbative quantities, the meson distribution amplitudes (DA), sometimes also called meson wave functions, are given by means of the vacuum-to-meson matrix elements of bi-local operators which, in the limit $x \to \bar{x}$, read

$$\langle 0 | O_\Gamma(\bar{x}, -\bar{x}) | V(P, \lambda) \rangle = \sum_{n=0}^\infty \frac{\bar{\psi}(0) \Gamma(x) \psi'(0)}{n!} \langle 0 | \psi(0) V^{(\bar{x} \bar{x})} | V(P, \lambda) \rangle,$$

(1.5)

with $\Gamma = \{1, \gamma_5; \gamma_\alpha, \gamma_\alpha \gamma_5; \sigma_{\alpha\beta}, \sigma_{\alpha\beta} \gamma_5\}$.

Possibly, in the near future the experimental precision data will allow for the determination of non-leading contributions and, therefore, require for a careful analysis of the various sub-dominant effects contributing to the physical processes. When considered beyond leading order, i.e., beyond lowest twist operators in tree approximation, one is confronted not only with radiative corrections but also with power corrections resulting from higher twist as well as target mass effects. Higher twist contributions are obtained by the decomposition of the bilocal operators $O_\Gamma(x\bar{x}, -\bar{x})$ with respect to (irreducible) tensor representations of the Lorentz group having definite twist $\tau = (\text{scale})$ dimension minus Lorentz spin $\bar{\tau}$. These representations are characterized by their symmetry type (under index permutations of the traceless tensors) which is determined by corresponding Young frames having, in the case of the Lorentz group, at the most three lines, $[n] = [m_1, m_2, m_3]$.

The twist decomposition of non-local light-ray operators (‘finite on-cone decomposition’), as far as they are relevant for hadronic processes, has been completed quite recently. A comprehensive presentation of these results can be found in Refs. [3] where also earlier studies are mentioned. Recently it has been used for the definition of quark distributions functions of well-defined geometric twist together with their unique relationship to the quark distributions of dynamical twist (being introduced in Ref. [4]). However, in order to get information about the target mass contributions one is forced to consider the twist decomposition off-cone, taking into account all the trace terms leading to expressions suppressed by powers of $M^2/Q^2$.

For local operators which are characterized by symmetry type $[n]$, i.e., being determined by totally symmetric traceless tensors, there exists a well-defined group theoretical framework [3] which works on the light-cone as well as, by unique harmonic extension, also off the light-cone (‘infinite off-cone decomposition’). For any other symmetry types which, however, are relevant in the above mentioned physical applications no general theory exists. But, from our earlier papers [3] partial results can be obtained. Using them we are able to present the decomposition of the relevant off-cone operators up to twist $\tau = 3$ which already covers the cases being experimentally accessible in the future. In principle, twist-4 results also would be of interest, but there occur at least two bi-local operators of different symmetry types of which only one has been fully determined.

Let us now explicitly list down, suppressing their flavour content and, as usual, the gauge link, the non-local quark operators which will be studied in detail. We determine the twist decomposition of the following chiral-even (axial) vector operators

$$O_{\alpha}(\kappa x, -\kappa x) = \bar{\psi}(\kappa x) \gamma_\alpha \psi(-\kappa x),$$

(1.6)

$$O_{5\alpha}(\kappa x, -\kappa x) = \bar{\psi}(\kappa x) \gamma_5 \gamma_\alpha \psi(-\kappa x),$$

(1.7)

together with the corresponding (pseudo) scalar operators

$$O_{(5)}(\kappa x, -\kappa x) = x^\alpha O_{(5)\alpha}(\kappa x, -\kappa x),$$

(1.8)

and the chiral-odd scalar and skew tensor operator

$$N(\kappa x, -\kappa x) = \bar{\psi}(\kappa x) \psi(-\kappa x),$$

(1.9)

$$M_{[\alpha\beta]}(\kappa x, -\kappa x) = \bar{\psi}(\kappa x) \sigma_{\alpha\beta} \psi(-\kappa x).$$

(1.10)

together with the vector and scalar operators

$$M_{\alpha}(\kappa x, -\kappa x) = x^\beta M_{[\alpha\beta]}(\kappa x, -\kappa x),$$

(1.11)

$$M(\kappa x, -\kappa x) = x^\beta \partial^\alpha M_{[\alpha\beta]}(\kappa x, -\kappa x).$$

(1.12)
The operators \((1.6) - (1.12)\) constitute a basis not only for the (usual) parton distributions as well as meson wave functions \([4–11]\) but also for the consideration of double distribution amplitudes being relevant for the various light-cone dominated QCD processes under consideration. Here, their twist decomposition off the light-cone will be given up to twist 3 and, in the case of scalar operators, also for any twist. In fact, the ‘external’ operation of contracting with \(x^\alpha\) or \(x^\beta \partial^\alpha\) also influences the possible symmetry type of these operators and, therefore, of their twist decomposition. Notice that the external coordinates are not multiplied by \(\kappa\).

As a result these off-cone operators of definite twist are given for the local as well as the resummed nonlocal operators. The local operators given as the \(n\)-th Taylor coefficients of the non-local ones, see, Eqs. \((1.3)\) and \((1.5)\) (or, more generally, Eqs. \((2.1)\) and \((2.2)\) below), are represented by (a finite series of) Gegenbauer polynomials \(C_n(z), \nu \geq 1\). The nonlocal operators, being obtained by resummation with respect to \(n\), are represented by (a related series of) Bessel functions or, more exactly, either \(J_{\nu-\frac{1}{2}}(z)\) or \(I_{\nu-\frac{1}{2}}(z)\) depending on the values of their arguments.

The group theoretical method for the determination of target mass corrections in unpolarized deep inelastic scattering using harmonic scalar operators of definite spin and the corresponding matrix elements in terms of Gegenbauer polynomials has been used for the first time by Nachtmann \([12]\). Some years later, using the same procedure, the target mass contributions for polarized deep inelastic scattering were studied in Refs. \([13–16]\). A short review of Nachtmann’s method in deep inelastic lepton-hadron scattering was given in Ref. \([17]\). This method differs from the one being applied by Georgi and Politzer \([18]\) for the target mass corrections in unpolarized deep inelastic scattering. Here, their twist decomposition off the light-cone will be given. The off-cone operators of definite twist are given for the local as well as the resummed nonlocal operators. The latter method which is tailored to the forward case has been applied also to the study of target mass corrections of polarized structure functions \([19, 20]\).

Here, we generalize Nachtmann’s procedure to the case of non-forward matrix elements applying it to off-cone quark-antiquark operators. In this paper we restrict ourselves to a fairly complete consideration of the various distribution amplitudes and postpone a more general application, e.g., to the virtual Compton scattering (using a somewhat different approach the latter has been already considered in Ref. \([21]\)). However, we determine completely the off-cone power corrections of the meson distribution amplitudes in \(x\)-space which are much easier to handle.

The paper is organized as follows. In Sect. II we present our method by considering a typical example. The power of that approach consists in the fact that one determines the twist decomposition at first for the local and nonlocal operators and only afterwards takes the matrix elements. In Sect. III totally symmetric operators are studied thus obtaining the most general result including the whole series of infinite twist. In Sect. IV the twist decomposition of operators \((1.6) - (1.12)\) up to twist 2 and 3 is given. In Sect. V these results are applied to the double distributions and the meson distributions.

II. NON-FORWARD MATRIX ELEMENTS OF NONLOCAL OFF-CONE OPERATORS: THE METHOD

A. Parametrization of non-forward matrix elements by independent double distributions of definite twist

To begin with we consider the bilocal off-cone quark-antiquark operators \((1.6), (1.7), (1.5)\) and \((1.10)\), i.e., operators \(\mathcal{O}_\Gamma(\kappa x, -\kappa x)\) in the \(x\)-space which, generically, will be denoted by \(\mathcal{O}_\Gamma(\kappa x, -\kappa x)\). More generally, they are given as

\[
\mathcal{O}_\Gamma(y + \kappa_1 x, y + \kappa_2 x) = \bar{\psi}(y + \kappa_1 x) \Gamma U(y + \kappa_1 x, y + \kappa_2 x) \psi(y + \kappa_2 x),
\]

with \(\eta = y + (\kappa_2 + \kappa_1)(x/2)\) and \(\xi = 2\kappa x \equiv (\kappa_2 - \kappa_1)x\) being the centre and the relative coordinate, respectively,

\[
U(y + \kappa_1 x, y + \kappa_2 x) = \mathcal{P} \exp \left\{ -i g \int_{\kappa_1}^{\kappa_2} dt x^\mu A_\mu(y + tx) \right\}
\]

being the (straight) path ordered phase factor ensuring gauge invariance (which is omitted in the following).

The Fourier transforms of the ‘centred’ operators \(\mathcal{O}_\Gamma(\kappa x, -\kappa x)\) and their \(n\)-th moments \(\mathcal{O}_{\Gamma n}(x)\), i.e., their Taylor coefficients w.r.t. \(\kappa\), are related as follows:

\[
\mathcal{O}_\Gamma(\kappa x, -\kappa x) = \int d^4 q \mathcal{O}_\Gamma(q) e^{i q x} = \sum_{n=0}^{\infty} \frac{(i \kappa)^n}{n!} \mathcal{O}_{\Gamma n}(x), \tag{2.1}
\]

with

\[
\mathcal{O}_{\Gamma n}(x) = \int d^4 q \mathcal{O}_\Gamma(q) (qx)^n = (-i)^n \frac{\partial^n}{\partial \kappa^n} \mathcal{O}_\Gamma(\kappa x, -\kappa x) \bigg|_{\kappa=0}. \tag{2.2}
\]
For notational simplicity we wrote the Fourier measure without the usual factor $1/(2\pi)^4$; obviously, $q$ should not be confused with some momentum transfer. It should be remarked that the Taylor expansion of the non-local operators into local ones is only justified in a restricted region of the Hilbert space \( \mathbb{H} \). Here, we assume their existence after taking physical matrix elements.

Now, we introduce a suitable, but preliminary parametrization of the non-forward matrix elements of the bilocal operators \( \mathcal{O}_\Gamma(\kappa x, -\kappa x) \). Namely, they can be represented as (everywhere, we use summation convention w.r.t. \( a \))

\[
\langle P_2, S_2 | \mathcal{O}_\Gamma(\kappa x, -\kappa x) | P_1, S_1 \rangle = \mathcal{K}^\alpha_{\Gamma}(\mathbb{P}) \int \mathcal{D}z \mathcal{E}^{i(\kappa z)\mathbb{P}} f_a(Z, \mathbb{P}, \mathbb{P}_j, x^2; \mu^2). \tag{2.3}
\]

This parametrization which takes up and generalizes previous ones \([2,22,24]\) deserves some explanations and further comments:

- First, we introduced the notation \( \mathbb{P} = \{ P_+, P_- \} \) and \( \mathcal{Z} = \{ z_+ , z_- \} \) with \( P_\pm = P_\pm + P_1 \) and \( z_\pm = \frac{1}{2}(z_2 \pm z_1) \) thereby defining some (2-dimensional) vector space with scalar product \( \mathcal{E}Z \equiv \sum z_i = P_+ z_+ + P_- z_- \). In addition, the integration measure is defined by \( \mathcal{D}z = dz_1 dz_2 \theta(1 - z_1)\theta(1 - z_2)\theta(z_2 + 1) \).

- Next, \( \mathcal{K}^\alpha_{\Gamma}(\mathbb{P}) \) denote the linear independent spin structures being defined by the help of the (free) hadron wave functions and governed by the \( \Gamma \)-structure of the corresponding nonlocal operator \( \mathcal{O}_\Gamma \). For example, in the case of the virtual Compton scattering, there are two independent spin structures, the Dirac and the Pauli structure, \( \mathcal{K}_\Gamma^1 = \bar{u}(P_2, S_2)\gamma_\mu u(P_1, S_1) \) and \( \mathcal{K}_\Gamma^2 = \bar{u}(P_2, S_2)\sigma_{\mu\nu}P_\nu u(P_1, S_1)/M \), respectively.

- Furthermore, modulo these spin structures, the Fourier transforms \( f_a(\mathbb{P}, \mathbb{P}_j, x^2; \mu^2) \) of the matrix element \( \langle P_2, S_2 | \mathcal{O}_\Gamma(\kappa x, -\kappa x) | P_1, S_1 \rangle \) with respect to the independent variables \( \kappa(x|\mathbb{P}) \) are the associated (universal, renormalized) two-variable distribution amplitudes as introduced in \([3]\) but here extended off the light-cone. As it is obvious, also their dependence on \( \mathbb{P}, \mathbb{P}_j \), \( x^2 \) and the renormalization point \( \mu^2 \) has to be taken into account.

- The representation \([2,3]\) gives the most general expression for the non-forward matrix elements of \( \mathcal{O}_\Gamma(\kappa x, -\kappa x) \). An essential aspect of that approach consists in the support restriction, \(-1 \leq z_1 \leq 1\), of the distribution amplitudes \( f_a(\mathbb{P}, \mathbb{P}_j, x^2; \mu^2) \) since the matrix elements can be shown to be entire analytic functions with respect to \( xP_1 \) (for a detailed discussion, see, \([2,3]\)).

- Obviously, the representation \([2,3]\) may be used also for more involved matrix elements, like (scalar) meson production, by extending the final state to \( \langle P_2, S_2; k_1 \rangle \) thereby enlarging the \( \mathcal{Z}^- \)- and the \( \mathbb{P}^- \)-space and extending the set of possible spin structures \( \mathcal{K}^\alpha_{\Gamma}(\mathbb{P}) \) and related distribution amplitudes \( f_a \) (see, e.g., Ref. \([24]\)). It can be specified also to vacuum-to-hadron transition amplitudes.

Finally, let us comment on operators with external operations, i.e. when the (axial) vectors or the (skew) tensors are multiplied ‘externally’ by \( x^\beta \) and/or \( \partial^\alpha \). In these cases the ‘external’ vector \( x^\beta \) or tensor \( x^\beta \partial^\alpha \) is assumed not to be Fourier transformed and the external operations have to be applied onto both sides of Eq. \([2,3]\). Thereby the tensor structure of these external operations matches with the tensor structure of \( \mathcal{K}^\alpha_{\Gamma} \) which by itself are independent of the coordinates. Despite of this the external operations heavily influence the possible symmetry type of the local operators and their decomposition into irreducible tensor representations of the Lorentz group.

Now, let us take into account that the non-local operators \( \mathcal{O}_\Gamma(\kappa x, -\kappa x) \), formally, are given by infinite series of operators of growing (geometric) twist \( \tau \) (using summation convention also w.r.t. \( \Gamma \)),

\[
\mathcal{O}_\Gamma(\kappa x, -\kappa x) = \sum_{\tau \geq \tau_{\text{min}}} c^{(\tau)\Gamma'}(x) \mathcal{O}^{(\tau)}_{\Gamma'}(\kappa x, -\kappa x), \tag{2.4}
\]

with

\[
\mathcal{O}^{(\tau)}_{\Gamma'}(\kappa x, -\kappa x) = \mathcal{P}^{(\tau)\Gamma'}_{\Gamma} \mathcal{O}_\Gamma(\kappa x, -\kappa x), \tag{2.5}
\]

\[
(\mathcal{P}^{(\tau')} \times \mathcal{P}^{(\tau)})_{\Gamma}^{\Gamma'} = \delta^{\tau\tau'} \mathcal{P}^{(\tau)\Gamma'}_{\Gamma}, \tag{2.6}
\]

where \( \mathcal{P}^{(\tau)\Gamma'}_{\Gamma} \) are well-defined projection operators. Also here we have to add some comments in order to make the content of Eqs. \([2,3,4,6]\) more definite:

- The projection operators \( \mathcal{P}^{(\tau)\Gamma'}_{\Gamma}(x, \partial_x) \) which immediately act on the undecomposed operators \( \mathcal{O}_\Gamma(\kappa x, -\kappa x) \) depend on the coordinates and their derivatives and, eventually, contain additional integrations with respect to some auxiliary variables. Usually, the summation over \( \Gamma' \) is restricted to the same tensorial type. The specific form of the projections may be read off from the explicit twist decompositions, cf., e.g., Sec. III, where various infinite twist series have been derived, as well as Sec. IV, Eqs. \([4.17, 4.19]\).
The coefficient functions $c^{(r)\Gamma}(x)$ essentially depend on (powers of) $x^2$ but, in principle, according to the tensor structure of the operators they also could depend on $x^4$, cf., e.g., Eqs. (3.19). In addition, we remark that different operators of the same twist may occur having, of course, different coefficient functions.

Approaching the light-cone, $x \to \tilde{x}$, the series (2.4) terminates at some finite value $\tau_{\text{max}}$ since almost all of the coefficient functions vanish for $x^2 = 0$. That situation has been considered extensively, e.g., in Refs. 1–3.

Obviously, analogous decompositions exist for the local operators $O_{\Gamma}(x)$ which are uniquely related to Eqs. (2.4) – (2.6). They have been derived in parallel to the non-local operators in Secs. III and IV.

Finally, we remark that the relations (2.1) and (2.2) between the local and the non-local operators, $O_{\Gamma}(x)$ and $O_{\Gamma}(\tau x, -\kappa x)$, and the Fourier transforms, $O_{\Gamma}(q)$, also hold for the corresponding operators of definite twist $\tau$.

Introducing the decomposition (2.4) into the representation (2.3) allows for an analogous decomposition of the distribution amplitudes, $f_a(Z, P_i P_j, x^2; \mu^2)$,

$$K_{\alpha}^\Gamma(P) f_a(Z, P_i P_j, x^2; \mu^2) = \sum_{\tau \geq \tau_{\text{min}}} c^{(r)\Gamma}(x) K_{\alpha}^\Gamma(P) f_a^{(\tau)}(Z, P_i P_j, x^2; \mu^2).$$

Then, for the matrix elements of operators $O_{\Gamma}^{(\tau)}$ with definite geometric twist $\tau$ we obtain two different versions,

$$\langle P_2, S_2|O_{\Gamma}^{(\tau)}(\kappa x, -\kappa x)|P_1, S_1\rangle = K_{\alpha}^\Gamma(P) \int DZ e^{i\kappa(x P)Z} f_a^{(\tau)}(Z, P_i P_j, x^2; \mu^2)$$

$$= P^{(\tau)\Gamma}(x, \partial_x) K_{\alpha}^\Gamma(P) \int DZ e^{i\kappa(x P)Z} f_a^{(\tau)}(Z, \mu^2),$$

where the first line is only the reduction of relation (2.3) to their twist components and where, in the second line, using properties (2.6) of the projection (2.3) onto the non-local operators of definite twist we introduced the related Lorentz invariant double distributions $f_a^{(\tau)}(Z, \mu^2)$ of definite twist $\tau$ not suffering from any power corrections. Concerning that definition of double distributions let us give a heuristic justification and some of its consequences:

Reminding the Fourier representation (2.4) for the non-local operators $O_{\Gamma}^{(\tau)}(\kappa x, -\kappa x)$ it looks reasonable that the $x^2$– and $P$–dependence of the distribution amplitudes $f_a(Z, P_i P_j, x^2; \mu^2)$ should be already uniquely determined by the projection operators of (2.5) acting on the exponential on the right hand side of (2.8) and, possibly, on some external operations matching $K_{\alpha}^\Gamma(P)$. This reflects the fact that every local operator of definite twist is determined by its symmetry type expressed, within the polynomial method, through specific differential operators acting on (traceless) harmonic tensor operators containing the $x^2$–dependence, cf., Eq. (2.10) below.

As a consequence, the mass corrections of the matrix elements (2.3) are determined by Eq. (2.8). Likewise, the distribution amplitudes $f_a(Z, P_i P_j, x^2; \mu^2)$, using both forms of the matrix element (2.8), formally may be expressed through the double distributions $f_a^{(\tau)}(Z, \mu^2)$ according to:

$$f_a^{(\tau)}(Z, P_i P_j, x^2; \mu^2) = (K_{\alpha}^{-1}(P))_a^\Gamma \left( e^{-i\kappa(x P)Z} P^{(\tau)\Gamma}(x, \partial_x) e^{i\kappa(x P)Z} \right) K_{\alpha}^\Gamma(P) f_a^{(\tau)}(Z, \mu^2)$$

$$= F_{aa}^{(\tau)}((x PZ), (x^2 PZ)^2) f_a^{(\tau)}(Z, \mu^2);$$

here, $F_{aa}^{(\tau)}((x PZ), (x^2 PZ)^2)$ contains any information about the power corrections of the double distribution. In addition, we observe that, as a result of the twist projection, the $(xP)$–dependence of the matrix element is only partly determined by the double distributions $f_a^{(\tau)}(Z, \mu^2)$.

Obviously, since the decomposition of the distribution amplitudes is uniquely related to the decomposition of the corresponding (quark-antiquark) operators the power corrections of the double distributions, Eq. (2.8), are already determined by the twist decomposition of these operators. Both decompositions, besides on $(x PZ)$, also depend on the following combination of variables, $(x PZ)/(x^2 PZ)^2$ and $(x^2 PZ)^2 - x^2 (PZ)^2$, being the arguments of appropriate Gegenbauer polynomials and Bessel functions in the case of local and non-local operators, respectively; cf., Subsecs. II B and II C below.

Notice, that the double distributions of definite twist $f_a^{(\tau)}(Z, \mu^2)$ being independent on $x^2$ are already determined by the decomposition of the non-local matrix elements on the light-cone. Also their renormalization properties are determined by the light-cone operators only.
This procedure of introducing double distributions of definite geometric twist generalizes to arbitrary off-cone values of $x$ what has been introduced in [1] for the parton distribution functions of definite geometric twist, i.e., for the case of forward matrix elements restricted to the light-cone, and later on applied to the (vector) meson distribution amplitudes [10], i.e., to vacuum-to-meson matrix elements, also restricted to the light-cone.

Let us now present the arguments leading to the r.h.s. of Eq. (2.8) more explicitly. Namely, the non-forward matrix elements after performing the twist projection of relations (2.1) are given as follows:

\[
\langle P_2, S_2|\mathcal{O}_\Gamma^{(\tau)}(\kappa x, -\kappa x)|P_1, S_1\rangle = \langle P_2, S_2|\mathcal{P}_\Gamma^{(\tau)}(x, \partial)\int d^4q\mathcal{O}_\Gamma^{(\tau)}(q)e^{i\kappa x q}|P_1, S_1\rangle
\]

\[
= \int d^4q \left(\mathcal{P}_\Gamma^{(\tau)}(x, \partial) e^{i\kappa x q}\right)\langle P_2, S_2|\mathcal{O}_\Gamma^{(\tau)}(q)|P_1, S_1\rangle \tag{2.10}
\]

\[
= \int Dz \left(\mathcal{P}_\Gamma^{(\tau)}(x, \partial_2) e^{i\kappa(xz^2)}\right)K_{\Gamma}^{(\tau)}(P) f_a^{(\tau)}(z; \mu^2).
\]

Here, in the second line we simply reordered the operations of integration, taking matrix elements and twist projections appropriately. In order to get the third line we observe the symmetry in $x$ and $q$ of the local twist projections, $\mathcal{P}_\Gamma^{(\tau)}(x, \partial)(xq)^n = \mathcal{P}_\Gamma^{(\tau)}(q, \partial_q)(xq)^n$, from which one derives for the second line:

\[
\sum_{n=0}^{\infty} \frac{(i\kappa)^n}{n!} \left(\mathcal{P}_\Gamma^{(\tau)}(x, \partial) x^{\mu_1} \cdots x^{\mu_n}\right) \int d^4q \left(\mathcal{P}_\Gamma^{(\tau)}(q, \partial_q) q_{\mu_1} \cdots q_{\mu_n}\right)\langle P_2, S_2|\mathcal{O}_\Gamma^{(\tau)}(q)|P_1, S_1\rangle.
\]

Here, the integrand contains the matrix elements of the irreducible local operators of definite twist $\mathcal{O}_{\Gamma_{\mu_1, \ldots, \mu_n}}^{(\tau)}(q)$. Their reduced matrix elements $f_{a(n)}^{(\tau)}$, $n_1 + n_2 = n$, which are related to the decomposition of $(P_2 + P_1)^n$ into independent monomials may be represented by (double) moments of corresponding double distributions $f_{a(n)}^{(\tau)}(z_1, z_2)$ multiplied by the kinematical factors $K_{\Gamma}^{(\tau)}(P_1, P_2)$. Finally, since $q$ reflects the dependence of the (local) operators on $D$, after performing the integration and the resummation over $q$, $q$ within the integrand simply has to be replaced by $PZ$. That procedure will be demonstrated explicitly in Subsec. II.C.

Formally, in the various experimental situations, in any Fourier integrand having the structure of Eq. (2.11), we only have to perform the following replacements (denoted by ≃):

(A) In the case of non-forward scattering, as just explained, we obtain

\[
\langle P_2, S_2|\mathcal{O}_\Gamma(q)|P_1, S_1\rangle \equiv K_{\Gamma}^{(\tau)}(P) \int Dz \delta^{(4)}(q - PZ) f_{a}^{(\tau)}(z; \mu^2). \tag{2.11}
\]

(B) In the case of forward scattering, i.e., for $P_1 = P_2 = P$, the situation changes into

\[
\langle P, S|\mathcal{O}_\Gamma(q)|P, S\rangle \equiv K_{\Gamma}^{(\tau)}(P) \int dz \delta^{(4)}(q - 2Pz) f_{a}^{(\tau)}(z; \mu^2), \tag{2.12}
\]

with $P_+ = 2P, P_- = 0; z_+ = z$. These distributions $f_{a}^{(\tau)}(z, \mu^2)$ are obtained from the double distributions $f_{a}^{(\tau)}(z, \mu^2)$ by integrating out the independent variable $z_-$, i.e.,

\[
\tilde{f}_{a}^{(\tau)}(z; \mu^2) = \int dz_- f_{a}^{(\tau)}(z_+ = z, z_-, \mu^2). \tag{2.13}
\]

(C) In the case of vacuum-to-hadron transition amplitudes, e.g., for the meson distribution amplitudes one obtains

\[
\langle 0|\mathcal{O}_\Gamma(q)|V(P, \lambda)\rangle \equiv \tilde{K}_{\Gamma}^{(\tau)}(P) \int d\xi \delta^{(4)}(q - P\xi) \tilde{f}_{a}^{(\tau)}(\xi; \mu^2), \tag{2.14}
\]

with $P_2 = P; z_2 = \xi$. Obviously, the spin structures are different from the above cases and, by construction, only $P$ and $\xi$ occur.

From the foregoing discussion it is obvious that the mass corrections to the various physical processes are completely determined by the twist structure of the bilocal operators $\mathcal{O}_{\Gamma_{\mu_1, \ldots, \mu_n}}^{(\tau)}(\kappa x, -\kappa x) = \int d^4q \mathcal{O}_{\Gamma}^{(\tau)}(q) e^{i\kappa x q}$. Performing matrix elements according to the replacements Eqs. (2.11) – (2.14) we obtain the related expressions for the $S$-matrix elements of the physical process under consideration. Then, a further Fourier transformation with respect to the coordinate $x$
like in Eqs. (1.1) and (1.4), finally, leads to a representation of the scattering or transition amplitudes in terms of the inverse momentum transfer depending, quite generally, on \( P_1 P_2 / Q^2 \).

In the next two subsections we demonstrate our procedure for the simplest nontrivial case, first, directly for non-forward matrix elements and then, more efficiently, for the corresponding nonlocal operator itself. Thereby, we introduce the technique of expansion into (local) harmonic polynomials and their resummation to non-local harmonic operators.

**B. Non-forward matrix elements of bilocal operators having definite twist: A simple nontrivial example**

Let us now demonstrate our procedure by the simplest nontrivial example, the non-forward matrix element of the twist-2 part of the (pseudo)scalar operator, Eq. (1.8), which is relevant for the leading terms of virtual Compton scattering \([25]\). This operator containing the ‘external’ truncation by \( \mu \) is given by \([26,5]\)

\[
O^{tw2}(\kappa x, -\kappa x) = \bar{\psi}(\kappa x)(x\gamma)(-\kappa x) + \sum_{k=1}^{\infty} \int_0^1 dt \left( \frac{1-t}{t} \right)^{k-1} \left( \frac{x^2}{4} \right)^{k/2} \bar{\psi}(\kappa tx)(x\gamma)(-\kappa tx). \tag{2.15}
\]

Its matrix elements, taking into account the Dirac structure only, read

\[
\langle P_2 | O^{tw2}(\kappa x, -\kappa x) | P_1 \rangle = \bar{u}(P_2)(x\gamma)u(P_1) \int D\mathcal{Z} e^{i\kappa x\mathcal{Z}} F_D^{(2)}(\mathcal{Z})
\]

\[
+ \sum_{k=1}^{\infty} \int_0^1 dt \left( \frac{1-t}{t} \right)^{k-1} \left( \frac{x^2}{4} \right)^{k/2} \bar{u}(P_2)(x\gamma)u(P_1) \int D\mathcal{Z} e^{i\kappa x\mathcal{Z}} F_D^{(2)}(\mathcal{Z}), \tag{2.16}
\]

where \( F_D^{(2)}(\mathcal{Z}) \) denotes the twist-2 Dirac type double distribution. Obviously, on the light-cone only the first term survives. Here, we take the complete expansion into account. Performing the differentiations,

\[
\Box^k(x\gamma)e^{i\kappa tx\mathcal{Z}} = \left[ 2^{2k}(x\gamma)(i\kappa x\mathcal{Z})^{2k} + 2k(2k-1)(i\kappa x\mathcal{Z})^{2(2k-1)} \right] e^{i\kappa tx\mathcal{Z}},
\]

and using

\[
\int_0^1 dt \left( \frac{1-t}{t} \right)^{k-1}t^{\ell+1} e^{i\kappa x\mathcal{Z}} = \sum_{n=0}^{\infty} \frac{(i\kappa x\mathcal{Z})^n}{n!} \frac{(n+\ell+1)!}{(n+k+\ell+1)!},
\]

this can be rewritten as follows:

\[
\langle P_2 | O^{tw2}(\kappa x, -\kappa x) | P_1 \rangle = \int D\mathcal{Z} F_D^{(2)}(\mathcal{Z}) \left\{ \bar{u}(P_2)(x\gamma)u(P_1) \sum_{k=0}^{\infty} \sum_{n=0}^{\infty} \frac{(i\kappa x\mathcal{Z})^n}{n!} \frac{(n+k+1)!}{(n+2k+1)!k!} \left( \frac{(\kappa x)^2(\mathcal{Z})^{2}}{4} \right)^k \right. 
\]

\[
- \frac{1}{2} i\kappa x^2 \bar{u}(P_2)(\gamma x\mathcal{Z})u(P_1) \sum_{k=0}^{\infty} \sum_{n=0}^{\infty} \frac{(i\kappa x\mathcal{Z})^n}{n!} \frac{(n+k)!}{(n+2k)!} \frac{(\kappa x)^2(\mathcal{Z})^{2}}{4} \frac{(\kappa x)^2(\mathcal{Z})^{2}}{4} \right\}.
\]

Now, shifting \( k \to k + 1 \) in the second \( k \)-summation, and observing the series representation of the modified Bessel functions of the first kind (cf., Ref. \([27]\), Eq. I.5.2.13.27),

\[
\sum_{n=0}^{\infty} \frac{\Gamma(n+nu+1)}{\Gamma(n+nu+1)n!} z^n = \sqrt{\pi} e^{z/2} z^{-v} I_v(z/2) \tag{2.17}
\]

for \( \nu = k + 3/2 \) and \( z = i\kappa(x\mathcal{Z}) \) we arrive at

\[
\langle P_2 | O^{tw2}(\kappa x, -\kappa x) | P_1 \rangle = \int D\mathcal{Z} F_D^{(2)}(\mathcal{Z}) \left\{ \bar{u}(P_2)(x\gamma)u(P_1)(2 + x\partial) - \frac{1}{2} i\kappa x^2 \bar{u}(P_2)(\gamma x\mathcal{Z})u(P_1) \right\} (3 + x\partial)
\]

\[
\times \sum_{k=0}^{\infty} \frac{(\kappa x)^2(\mathcal{Z})^{2}/4}{k!} \sum_{n=0}^{\infty} \frac{(n+k+1)!}{(n+2k+3)!} \frac{(i\kappa x\mathcal{Z})^n}{n!} \]

\[
\times \int D\mathcal{Z} F_D^{(2)}(\mathcal{Z}) \left\{ x_\mu(2 + x\partial) - \frac{1}{2} i\kappa x\mathcal{Z} x^2 \right\} (3 + x\partial)
\]

\[
\times \sum_{k=0}^{\infty} \frac{(\kappa x)^2(\mathcal{Z})^{2}/4}{k!} \sqrt{\pi} e^{i\kappa x\mathcal{Z}/2} (i\kappa x\mathcal{Z})^{-k-3/2} I_{k+3/2}(i\kappa x\mathcal{Z}/2).
\]
Now, using $I_p(e^{\pi i/2}z) = e^{\pi i/2} J_p(z) \text{ with the (usual) Bessel functions } J_p(z)$ as well as (cf. Ref. [27], Eq. II.5.7.6.1)

$$\sum_{k=0}^{\infty} \frac{t^k}{k!} J_{k+p}(z) = z^{\nu/2} (z - 2t)^{-\nu/2} J_{\nu}(\sqrt{z^2 - 2zt}) \quad \text{for} \quad |2t| < z, \quad (2.18)$$

and substituting $t = \kappa x q^2/(4xq)$ and $z = \kappa x q/2$ we finally arrive at

$$\langle P_2 | O^{tw2}(\kappa x, -\kappa x) | P_1 \rangle = \sqrt{\pi} \bar{u}(P_2) \gamma^\mu u(P_1) \int DZ F^{(2)}_D(z) \left\{ x_\mu (2 + x\partial) - \frac{1}{2} i \kappa \gamma_\mu Z x^2 \right\} (3 + x\partial)$$

$$\times \left( \kappa \sqrt{(xP^2 - x^2 P^2)^2} \right)^{-3/2} J_{3/2} \left( \frac{\sqrt{2}}{\sqrt{(xP^2 - x^2 P^2)^2}} \right) e^{i\kappa x P \zeta /2}. \quad (2.19)$$

This holds in the case $(xP^2 - x^2 P^2)^2 \geq 0$, otherwise we have to change into $I_{\nu}(z)$. That result exactly corresponds to Eq. (2.18) for the example under consideration. Observe that another half of the exponential is concealed within the Bessel function!

In the case of forward scattering because of $\bar{u}(P) \gamma^\mu u(P) = 2P^\mu$ we obtain

$$\langle P | O^{tw2}(\kappa x, -\kappa x) | P \rangle = 2 \sqrt{\pi} \int dz \bar{F}^{(2)}_D(z) \left\{ (xP) (2 + x\partial) - i \kappa x P^2 x^2 \right\} (3 + x\partial)$$

$$\times \left( 2\kappa z \sqrt{(xP^2 - x^2 P^2)^2} \right)^{-3/2} J_{3/2} \left( \kappa \sqrt{(xP^2 - x^2 P^2)^2} \right) e^{i\kappa x z}. \quad (2.20)$$

If this expression is restricted to the light-cone, $x^2 = 0$, the well-known parton distribution is obtained. Namely, using the Poisson integral for the Bessel functions (cf., Ref. [28], Eq. II.7.12.7),

$$\Gamma(\nu + \frac{1}{2}) J_{\nu}(z) = \frac{1}{\sqrt{\pi}} \left( \frac{z}{2} \right)^\nu \int_{-1}^{1} dt (1 - t^2)^{\nu - 1/2} e^{itz} \quad \text{for} \quad \text{Re} \nu > -\frac{1}{2}, \quad (2.21)$$

we obtain ($\xi = \kappa x(\bar{x} P)$)

$$\langle P | O^{tw2}(\kappa \bar{x}, -\kappa \bar{x}) | P \rangle = 2(\bar{x} P) \sqrt{\pi} \int dz \bar{F}^{(2)}_D(z) (2 + \xi \partial_\xi)(3 + \xi \partial_\xi)(2\xi)^{-3/2} J_{3/2}(\xi) e^{i\xi}$$

$$= 2(\bar{x} P) \int dz \bar{F}^{(2)}_D(z) \int_0^1 dt t(1 - t)(3 + t\partial_t)(2 + t\partial_t) e^{2it\xi}$$

$$= 2(\bar{x} P) \int dz \bar{F}^{(2)}_D(z) \int_0^1 dt t(1 + t\partial_t) e^{2it\xi} = 2(\bar{x} P) \int dz F^{(2)}_D(z) e^{2i\kappa z(\bar{x} P)}; \quad (2.22)$$

in the second line we introduced the Poisson integral after shifting the integration variable $t \to (t + 1)/2$ and followed by changing $\xi \partial_\xi \to t\partial_t$, then we partially integrated two times retaining finally only the surface term at $t = 1$. As a result we arrived at the twist-2 parton distribution as introduced in [4], $\bar{F}^{(2)}_D(z) \equiv F^{(2)}(z)$, which coincides with $f_1(z)$ in the notation adopted by Jaffe and Ji [7].

The power corrections of the parton distribution $F^{(2)}(z)$ are obtained according to the relation (2.9). Namely, using the matrix element (2.20) and, again, taking into account the representation (2.21), one gets ($Y = \sqrt{1 - x^2 P^2/(xP^2)}$)

$$F^{(2)}(z, xP, x^2 P^2; \mu^2) = F^{(2)}(z) e^{-2i\kappa z(xP)z} \frac{1}{8} \left\{ (2 + x\partial) - i \kappa x \frac{x^2 P^2}{(xP^2)} \right\} (3 + x\partial) \int_{-1}^{1} dt (1 - t^2)^e^{i\kappa z(xP)z}$$

$$\left\{ (1 + Y)^2 e^{i\kappa z(xP)z(1 - Y)} - (1 - Y)^2 e^{i\kappa z(xP)z(1 - Y)} \right\} \quad (2.23)$$

Analogous expressions result for the pseudo scalar case by observing $\bar{u}(P) \gamma^\mu \gamma_5 u(P) = 2 S^\mu$.

The situation in the non-forward case is somewhat more complicated and, therefore, will be omitted here. There, not only the Dirac but also the Pauli structure has to be taken into account whose general form coincides with the expression (2.19) with $K_{\mu}$ being replaced by $K_{\mu}^\nu$, but, can be proven by using the hadrons equation of motion, in accordance with Eq. (2.18) both the Dirac and Pauli structures mix with each other.

Finally, let us point to the fact that expressions similar to Eq. (2.20) have been obtained in Ref. [29] for inclusive particle production in $e^+ e^-$-annihilation. This work is based on a procedure [29] which, in order to obtain higher twists, avoids the consideration of group representations and, instead, makes use of the quark (and gluon) equations of motion. As already mentioned [4] in the case of vector (and tensor) operators by this approach the tracelessness of the (local) operators and, consequently, also the definiteness of their geometric twist may be missing.
C. Nonlocal off-cone operators of definite twist: The technique of harmonic polynomials

Obviously, the $x-$dependence of the expressions (2.13) – (2.22) is completely determined by the twist structure of the operator from which the matrix element has to be built. Therefore, let us derive these results directly from the related twist-2 operator. But, instead starting from the nonlocal expression (2.13) we use its local version being given by harmonic polynomials of order $n$ in $x$. This polynomial technique uses the vector $x \in \mathbb{R}^4$ as a device for writing tensors with special symmetries in analytic form \cite{59,61}. It has the advantage to be directly related to the irreducible tensor representations of the Lorentz group. Its group theoretical background as far as it is related to totally symmetric tensors has been given in Ref. \cite{8}.

The local scalar operator according to (2.2) is given by

$$O_{n+1}(x) = \bar{\psi}(0)(x\gamma)(\mp \hat{D})^n\psi(0) = \int d^4q \left( \bar{\psi}\gamma^\mu \psi \right)(q) x_\mu (qx)^n,$$

(2.24)

which shows that, in this connection, $q$ formally replaces the covariant derivative $\hat{D}$ sandwiched between the quark operators. This local operator has a (finite) twist decomposition whose complete series will be given later on, cf., Eq. (3.11). Its twist-2 part is simply given by the harmonic polynomials \cite{12,8},

$$O^{tw}_{n+1}(x) = \sum_{k=0}^{[n+1]/2} \frac{(-1)^k(n+1-k)!}{4^k k!(n+1)!} x^{2k} D^k O_{n+1}(x) \equiv P^{(2)}_{n+1}(x^2, \partial^2) O_{n+1}(x),$$

(2.25)

being characterized by traceless, totally symmetric tensors of rank $n+1$ whose indices are completely contracted by $x^{\mu_1}\cdots x^{\mu_{n+1}}$. They obey the condition of harmonicity: $\Box O^{tw}_{n+1}(x) = 0$.

Performing the derivatives,

$$x^{2k} D^k (\gamma x)(qx)^n = \frac{n!}{(n-2k)!} (\gamma x)(q^2 x^2)^k (qx)^{n-2k} \theta_{n-2k} + \frac{2kn!}{(n+1-2k)!} x^2 (\gamma q)(q^2 x^2)^{k-1} (qx)^{n+1-2k} \theta_{n+1-2k},$$

(2.26)

and using the series expansion of the Gegenbauer polynomials (see, e.g., Ref. \cite{7}, Appendix II.11),

$$C_n^\nu(z) = \frac{1}{(\nu-1)!} \sum_{k=0}^{[\nu]/2} \frac{(-1)^k (n-k+\nu-1)!}{k!(n-2k)!} (2z)^{n-2k},$$

(2.27)

we obtain

$$O_{n+1}^{tw}(x) = \frac{1}{n+1} \int d^4q \left( \bar{\psi}\gamma^\mu \psi \right)(q) \left\{ x_\mu \left( \frac{1}{2} \sqrt{q^2 x^2} \right)^n C_n^\nu \left( \frac{q x}{\sqrt{q^2 x^2}} \right) - \frac{1}{2} q_\mu x^2 \left( \frac{1}{2} \sqrt{q^2 x^2} \right)^{n-1} C^\nu_{n-1} \left( \frac{q x}{\sqrt{q^2 x^2}} \right) \right\}. \quad (2.28)$$

Again, some remarks are in order:

- The factor $1/(n+1)$ in front of the integral results from the equality $x_\mu (qx)^n = (1/n+1) \partial_\mu (qx)^{n+1}$; it corresponds to the normalization of the decomposition of the local operators into irreducible representations, i.e., to the normalization of the associated Young operators. In the nonlocal formulation it leads to the $t-$integration in Eq. (2.16), cf., also, Ref. \cite{8}.

- The (discrete) $\theta-$functions guarantee that both the $k-$summations do not contain undefined factorials thus correctly leading to the corresponding Gegenbauer polynomials. In the following this is circumvented by the convention that Gegenbauer polynomials of negative order are equal to zero.

- Obviously, Eq. (2.28) is the analytic continuation from Euclidean spacetime — where the Gegenbauer polynomials obey the well-known orthonormality relations within the region $-1 \leq z \leq 1$ — to the Minkowski spacetime where the arguments of the square roots and of the Gegenbauer polynomials, depending on whether $q^2$ and/or $x^2$ are space-like or time-like, may be imaginary and, furthermore, could take values outside that region. This, however, has no influence on the validity of the above result since these polynomials being entire analytic functions are well defined on the whole complex plane.

- Notice that the second term in Eq. (2.28) is due to the exterior $x-$dependence of the scalar operator. Being proportional to $x^2$ it vanishes on the light-cone. Furthermore, on the light-cone only the highest power $z^n$ of the Gegenbauer polynomial $C_n^\nu(z)$ survives and, as it should be because of (2.25), leads exactly to the expression (2.24) with $x \to \hat{x}$!
Finally, according to the above mentioned interpretation of $q$ as some kind of ‘operator symbol’ within the Fourier integral, Eq. (2.28) operationally has to be understood as if the polynomial inside the curly brackets had been written in terms of $\tilde{D}$ instead of $q$ and inserted into the operator $\bar{\psi}(0)\gamma^{\mu}\psi(0)$.

Now, let us remind the generating function of the Gegenbauer polynomials (see, e.g., Ref. 27, Eq. II.5.13.1.3),

$$\sum_{n=0}^{\infty} \frac{a^n}{(2\nu)_n} C_n^{(2\nu)}(z) = \Gamma\left(\nu + \frac{1}{2}\right) \left(\frac{a}{2}\sqrt{1-z^2}\right)^{1/2-\nu} J_{\nu-1/2}\left(a\sqrt{1-z^2}\right)e^{zn},$$

(2.29)

where $(2\nu)_n = (2\nu + 1) \cdots (2\nu + n - 1) = \Gamma(n+2\nu)/\Gamma(2\nu)$ is the Pochhammer symbol. Choosing $z = (qx)/\sqrt{q^2x^2}$ and $a = i\sqrt{q^2x^2}/2$, the local scalar operators of twist–2, Eq. (2.28), can be summed up according to Eq. (2.1) to the bilocal scalar operator of twist–2:

$$O^{tw2}(\kappa x, -\kappa x) = \sum_{n=0}^{\infty} \frac{(ik)^n}{n!} O^{tw2}_{n+1}(x) = \sqrt{\pi} \int d^4 q \left(\tilde{\psi}\gamma^{\mu}\psi\right)(q) \left\{ x^{\mu} (2 + x\partial) - \frac{i}{2} q^{\mu} x^2 \right\} (3 + x\partial) \times \left(\kappa \sqrt{(qx)^2 - q^2x^2}\right)^{-3/2} J_{3/2}\left(\frac{k}{2}\sqrt{(qx)^2 - q^2x^2}\right) e^{iqx/2}.$$  

(2.30)

Here, the homogeneous derivations $(c + x\partial)$ are required to compensate for some extra factors $(c + n)$ which are necessary in order to be able to introduce the Pochhammer symbols in the denominator. Obviously, for the second term in Eq. (2.28), after shifting $n \rightarrow n + 1$ in the series over $n$, only one additional factor is required. — Also here $q$ has to be considered as a symbol replacing the covariant derivatives sandwiched between the quark operators.

Now, let us discuss the matrix elements of the twist-2 operators (2.28) and (2.30) starting with the forward case $P_1 = P_2 = P$. The $q$–integral representation of the local twist-2 operator, observing the symmetry in $q$ and $x$ of the twist-2 projection of $(qx)^{n+1}$,

$$\mathcal{P}^{(2)}_{n+1}(x^2, \partial^2) (qx)^{n+1} = \left(\frac{1}{2} \sqrt{q^2x^2}\right)^{n+1} C_{n+1}^{(2)} \left(\frac{qx}{\sqrt{q^2x^2}}\right),$$

may be rewritten as follows

$$O^{tw2}_{n+1}(x) = \frac{1}{n+1} \int d^4 q \left(\bar{\psi}\gamma^{\mu}\psi\right)(q) \frac{\partial}{\partial q^\mu} \left\{ \left(\frac{1}{2} \sqrt{q^2x^2}\right)^n C_n^{(2)} \left(\frac{qx}{\sqrt{q^2x^2}}\right) \right\}$$

$$= \left\{ \mathcal{P}^{(2)}_{n+1}(x^2, \partial^2) (x^{\mu_1} \cdots x^{\mu_{n+1}}) \right\} \frac{1}{n+1} \int d^4 q \left(\bar{\psi}\gamma^{\mu_1}\psi\right)(q) \frac{\partial}{\partial q^{\mu_1}} (q^{\mu_2} \cdots q^{\mu_{n+1}})$$

$$= \left\{ \mathcal{P}^{(2)}_{n+1}(x^2, \partial^2) (x^{\mu_1} \cdots x^{\mu_{n+1}}) \right\} \frac{1}{n+1} \int d^4 q \left(\bar{\psi}\gamma^{\mu_1}\psi\right)(q) \frac{\partial}{\partial q^{\mu_1}} \left\{ \mathcal{P}^{(2)}_{n+1}(q^2, \partial^2) (q^{\mu_2} \cdots q^{\mu_{n+1}}) \right\}.$$  

(2.31)

Obviously, the integrand is an irreducible tensor operator of order $n + 1$. Its forward matrix elements are given by the reduced matrix element $f^{(2)}_n$ times the irreducible tensor written in terms of the momentum $P_+ = 2P$. As the result we finally obtain

$$\langle P, S|O^{tw2}(\kappa x, -\kappa x)|P, S\rangle = \langle P(\gamma^{\mu}u(P))\partial\partial(2P^\nu)\left\{ \left(\sqrt{P^2x^2}\right)^{n+1} C_{n+1}^{(2)} \left(\frac{Px}{\sqrt{P^2x^2}}\right) \right\}$$

$$= f^{(2)}_n (2Px) \left\{ \left(\sqrt{P^2x^2}\right)^n C_n^{(2)} \left(\frac{Px}{\sqrt{P^2x^2}}\right) - \frac{P^2x^2}{(xP)} \left(\sqrt{P^2x^2}\right)^{n-1} C^{(2)}_{n-1} \left(\frac{Px}{\sqrt{P^2x^2}}\right) \right\}.$$  

(2.32)

Now, as usual, the reduced matrix elements $f^{(2)}_n$ are considered as the moments of a distribution $f^{(2)}(z)$,

$$f^{(2)}_n = \int_{-1}^{1} dz z^n f^{(2)}(z).$$

(2.33)

Taking this into account we obtain, after summing up over $n$, the following expression of the forward matrix elements of the non-local twist-2 operators

$$\langle P, S|O^{tw2}(\kappa x, -\kappa x)|P, S\rangle = \bar{u}(P)(\gamma^{\mu}u(P)) \int_{-1}^{1} dz \left\{ (2 + x\partial) - \frac{i}{2} \frac{P^2x^2}{(xP)} \right\} (3 + x\partial)$$

$$\times \sqrt{\pi} \left(2\kappa z \sqrt{(Px)^2 - P^2x^2}\right)^{-3/2} J_{3/2}\left(\kappa z \sqrt{(Px)^2 - P^2x^2}\right) e^{i\kappa zPx},$$

(2.33)
which coincides with the expression (2.20).

More generally, when non-forward matrix elements are taken the argumentation is almost the same. But now, in the case of the local operators, there occur \( n + 1 \) different reduced matrix elements, \( f_{n,m}^{(2)} \), related to the monomials \( P_1^n P_2^{n-m} \), \( 0 \leq m \leq n \) which are obtained from \((P_2 + P_1)^n\), and the corresponding non-forward matrix element reads:

\[
\langle P_2, S_2|O_{n+1}^{tw}(x)|P_1, S_1 \rangle = \sum_{m=0}^{n} \binom{n}{m} f_{n,m}^{(2)} \left( \bar{u}(P_2) \gamma_\mu u(P_1) \right) \mathcal{P}_{n+1}^{(2)}(x^2, \partial^2) \left\{ x^\mu (xP_1)^m (xP_2)^{n-m} \right\}.
\] (2.34)

Now let us rewrite the reduced matrix elements \( f_{n,m}^{(2)} \) as the double moments of some double distribution \( f^{(2)}(z_1, z_2) \),

\[
f_{n,m}^{(2)} = \int_{-1}^{1} \int_{-1}^{1} dz_1 \int_{-1}^{1} dz_2 \ z_1^n z_2^{-m} f^{(2)}(z_1, z_2).
\] (2.35)

After resumming w.r.t. \( n \) and rewriting \( f^{(2)}(z_1, z_2) = E_L^{(2)}(z_+, z_-) \) we finally arrive at the expression (2.19) for the non-forward matrix element of the nonlocal operator \( O^{tw(3+2\tau)}(x, -x) \).

Obviously, this second approach is more appropriate since, as an intermediate step, also knowledge about the local operators of definite twist is obtained which is related to the corresponding moments of the double distributions. From this approach it becomes also obvious that the mass corrections of different physical processes are related to each other if they can be traced back to the same operator content.

In the following we use this polynomial technique to determine the local as well as nonlocal quark operators of definite twist. Thereby, we generalize Nachtmann’s approach not only to non-forward matrix elements of (non)local operators. In addition we consider also the case of more general tensor operators having nontrivial symmetry types.

### III. TOTALLY SYMMETRIC NONLOCAL HARMONIC OPERATORS OF ANY GEOMETRIC TWIST

The determination of the complete twist decomposition of non-local operators into the infinite tower of harmonic operators of definite twist is possible only for the case when their \( n \)-th moments are related to irreducible tensor representations of the Lorentz group having symmetry type \([n]\), i.e., being completely symmetric and traceless. The corresponding group theoretical background underlying this polynomial technique has been formulated by Bargmann and Todorov \([8]\). There, also the projection property (2.6) is proven by explicit construction. In order to demonstrate the efficiency of the polynomial technique and because of their physical relevance we apply it to the complete twist decomposition of the scalar operators \( N(x, -x) \), \( M(x, -x) \) and \( O(x, -x) \), Eqs. (1.9), (1.12) and (1.8), having minimal twist \( \tau = 3 \) and \( \tau = 2 \), respectively, as well as to the related vector and tensor operators. At the same time the general aspects of the twist projection, Eqs. (2.4) – (2.6), together with the related comments, will be exemplified.

#### A. Scalar harmonic operators without external operations

To begin with the simplest case let us study the local operators \( N_n(x) \) of degree \( n \) which are generating polynomials of symmetric tensors of degree \( n \). The general formula for the decomposition of the local scalar operator \( N_n(x) \) into its harmonic polynomials of definite twist \( \tau = 3 + 2j \), \( j = 0, 1, \ldots \), reads \([8]\)

\[
N_n(x) = \sum_{j=0}^{n/2} \frac{(-1)^j (n - 2j)!}{4^j j! (n + 1 - j)!} x^{2j} N_{n-2j}^{tw(3+2j)}(x),
\] (3.1)

where the harmonic operators of definite twist are defined by (see also Ref. [32], Eq. 9.3.2(3))

\[
N_{n-2j}^{tw(3+2j)}(x) = \sum_{k=0}^{[n-2j]/2} \frac{(-1)^k (n - 2j - 2k)!}{4^k k! (n - 2j)!} x^{2k} \Box^k \left( \Box^j N_n(x) \right),
\] (3.2)

they satisfy

\[
\Box N_{n-2j}^{tw(3+2j)}(x) = 0.
\]

Therefore, the polynomials of Eq. (3.2) span the space of homogeneous harmonic polynomials of degree \( n - 2j \).
After substituting $N_n(x) = \int dq N(q)(xq)^n$ into Eq. (3.2), and observing

$$\Box^{k+j}(xq)^n = \frac{n!}{(n-2k-2j)!} (q^2)^{k+j}(xq)^{n-2k-2j},$$

we can rewrite these harmonic operators in terms of Gegenbauer polynomials as follows:

$$N_{n-2j}^{tw(3+2j)}(x) = \frac{n!}{(n-2j)!} \int dq N(q) q^{2j} \left( \frac{1}{2} \sqrt{q^2 - x^2} \right)^{n-2j} C_{n-2j}^1 \left( \frac{xq}{\sqrt{q^2 - x^2}} \right). \quad (3.3)$$

Here, the factorial in front of the integral are normalization coefficients and the $q$—integration introduces a superposition of the Fourier transforms $N(q)$ times $q^{2j}$ with coefficients being (two-sided) harmonic polynomials,

$$h_{n-2j}^1(q|x) = \left( \frac{1}{2} \sqrt{q^2 - x^2} \right)^{n-2j} C_{n-2j}^1 (qx/\sqrt{q^2 - x^2}) \quad \text{with} \quad \Box_x h_{n-2j}^1(q|x) = 0 = \Box_q h_{n-2j}^1(q|x). \quad (3.4)$$

The resummation of the moments $N_n(x)$ according to $N(x,x) = \sum_{n=0}^{\infty} (i^n/n!) N_n(x)$ is obtained by using the well-known integral representation of Euler’s beta function,

$$B(n,m) = \frac{\Gamma(n)\Gamma(m)}{\Gamma(n+m)} = \int_0^1 dt t^{n-1}(1-t)^{m-1}. \quad (3.5)$$

The (infinite) twist decomposition of the nonlocal scalar operator $N(x,-x)$ into nonlocal harmonic operators $N_{n-2j}^{tw(3+2j)}(x,-x)$ of twist $\tau = 3 + 2j$, $j = 0, 1, 2, \ldots$, reads (remind that the summation over $n$ for $N_{n-2j}^{tw(3+2j)}$ starts at $n = 2j$),

$$N(x,-x) = N^{tw}_{3}(x,-x) + \sum_{j=1}^{\infty} \frac{(-1)^j}{4j(j+1)!} \int_0^1 dt t(1-t)^{j-1} N_{n-2j}^{tw(3+2j)}(tx,-tx), \quad (3.6)$$

with

$$N_{n-2j}^{tw(3+2j)}(x,-x) = \sqrt{\pi} \int dq N(q) (-q^2)^j (1 + q\partial_q) \left( \sqrt{(qx)^2 - q^2 x^2} \right)^{1/2} J_{j/2} \left( \frac{1}{2} \sqrt{(qx)^2 - q^2 x^2} \right) e^{iq x/2}. \quad (3.7)$$

Let us remark that, for notational simplicity, in Eq. (3.6) we wrote the nonlocal harmonic operators for $j \geq 1$ without including the integration over $t$ which is due to the normalization of the local harmonic operators, cf. Eqs. (3.1) and (3.3). The normalization of the nonlocal harmonic operators may be read off from the terms in front of the $t$—integral in Eq. (3.4). Let us point also to the remarkable fact that the property of harmonicity is independent of $j$; this obtains immediately from the fact that the following expressions are (two-sided) harmonic functions,

$$\mathcal{H}_1(q|x) = \sqrt{\pi} \left( \sqrt{(qx)^2 - q^2 x^2} \right)^{1/2} J_{j/2} \left( \frac{1}{2} \sqrt{(qx)^2 - q^2 x^2} \right) e^{iq x/2} \quad \text{with} \quad \Box_x \mathcal{H}_1(q|x) = 0 = \Box_q \mathcal{H}_1(q|x). \quad (3.8)$$

Furthermore, we observe that the factor $(1 + q\partial_q)$ in Eq. (3.7) may be changed into $(1 + x\partial / \partial x)$ after which it can be taken outside the $q$—integration and, in Eq. (3.4), could be changed into $(1 + t\partial / \partial t)$; if desired, this could be used for a partial integration.

The resulting expression (3.7) can be rewritten in a form which, in the case $j = 0$, has been already introduced by Eq. (3.3). Namely, using

$$z^{-n-1/2} J_{n+1/2}(z) = (-1)^n \sqrt{\frac{2}{\pi}} \left( \frac{1}{z \pi} \right)^n \left( \frac{\sin z}{z} \right), \quad (3.9)$$

we obtain

$$N_{n-2j}^{tw(3+2j)}(x,-x) = \int dq N(q) (-q^2)^j \frac{2}{\sqrt{(qx)^2 - q^2 x^2}} q \partial_q \left( \sin \left( \frac{1}{2} \sqrt{(qx)^2 - q^2 x^2} \right) e^{iq x/2} \right)
$$

$$= \int dq N(q) (-q^2)^j \frac{1}{2 \sqrt{(qx)^2 - q^2 x^2}} \left( (qx + \sqrt{(qx)^2 - q^2 x^2}) e^{i/2}(qx + \sqrt{(qx)^2 - q^2 x^2}) \right)
$$

$$- \left( (qx - \sqrt{(qx)^2 - q^2 x^2}) e^{i/2}(qx - \sqrt{(qx)^2 - q^2 x^2}) \right). \quad (3.10)$$

Analogous results may be obtained also in the more complicated cases which will be considered below.
B. Scalar harmonic operators with external operations

Now, let us consider the case of $O(x, -x) = x^n O_n(x, -x)$, where an additional power of $x$ occurs through contraction of the vector operator with $x^n$. Therefore, the formulae (3.1) and (3.2) are to be written down for the scalar operator $O_{n+1}(x) = \int dq x_o O^n(q)(xq)^n$, i.e., by replacing $n$ by $n + 1$ within these expressions for the decomposition into harmonic operators of any twist $\tau = 2 + 2j$. Then the decomposition of the scalar local operators $O_{n+1}(x)$ reads:

$$O_{n+1}(x) = \sum_{j=0}^{n} \frac{(-1)^j (n + 2 - 2j)!}{4^j j!(n + 2 - j)!} x^{2j} O_{n+1-2j}^{tw(2+2j)}(x),$$  \hspace{1cm} (3.11)

where the local harmonic operators of twist $\tau = 2 + 2j$ are given by

$$O_{n+1-2j}^{tw(2+2j)}(x) = \sum_{k=0}^{n-2j} \frac{(-1)^k (n + 1 - 2j - k)!}{4^k k!(n + 1 - 2j)!} x^{2k} \Box_k \left(\Box O_{n+1}(x)\right).$$  \hspace{1cm} (3.12)

After performing the differentiations $\Box^{k+j} O_{n+1}(x)$ we can rewrite these harmonic operators in terms of Gegenbauer polynomials as follows:

$$O_{n+1-2j}^{tw(2+2j)}(x) = \frac{n!}{(n + 1 - 2j)!} \int dq O^n(q) q^{2j} \left\{ x_{\mu} \left( \frac{1}{2} \sqrt{q^2 x^2} \right)^{-n-2j} C_{n-2j}^2 \left( \frac{q x}{\sqrt{q^2 x^2}} \right) \right. - \frac{1}{2} q \mu x^2 \left( \frac{1}{2} \sqrt{q^2 x^2} \right)^{-n-2j} C_{n-2j}^2 \left( \frac{q x}{\sqrt{q^2 x^2}} \right) + 2j q \mu x^2 \left( \frac{1}{2} \sqrt{q^2 x^2} \right)^{n+1-2j} C_{n+1-2j}^1 \left( \frac{q x}{\sqrt{q^2 x^2}} \right) \right\},$$  \hspace{1cm} (3.13)

where the additional terms proportional to $q \mu$ occur because of the appearance of derivatives with respect to $(x \gamma)$. The change in the order of the Gegenbauer polynomials and, consequently, in the order of the Bessel functions and the accompanying factors $(q x)^2 - q^2 x^2$ occurring below, is also due to that fact. Of course, for $j = 0$ we re-obtain the expression (2.28).

Resuming with respect to $n$ ($\geq 2j + 1$ or $2j$, respectively) and, again, using the representation (3.5) of Euler’s beta function we obtain the following (infinite) twist decomposition

$$O(x, -x) = O^{tw(2+2j)}(x, -x) + \sum_{j=0}^{\infty} \frac{(-1)^j x^{2j}}{4^j j!(j - 1)!} \int_0^1 dt (1 - t)^{j-1} O^{tw(2+2j)}(tx, -tx),$$  \hspace{1cm} (3.14)

with

$$O^{tw(2+2j)}(x, -x) = \sqrt{\pi} \int dq O^n(q) (-q)^j \left\{ x_{\mu} (2 + q \partial_q) - \frac{1}{2} i q \mu x^2 \right\} \times (3 + q \partial_q) \left( \sqrt{(q x)^2 - q^2 x^2} \right)^{-3/2} J_{3/2} \left( \frac{1}{2} \sqrt{(q x)^2 - q^2 x^2} \right) - 2j i q \mu \left( 1 + q \partial_q \right) \left( \sqrt{(q x)^2 - q^2 x^2} \right)^{-1/2} J_{1/2} \left( \frac{1}{2} \sqrt{(q x)^2 - q^2 x^2} \right) e^{iq x/2}. \hspace{1cm} (3.15)$$

Again, by convention, the resummed nonlocal harmonic operators $O^{tw(2+2j)}(tx, -tx)$ of twist $\tau = 2 + 2j$, $j = 0, 1, 2, \cdots$, are obtained by changing any $x \to tx$. Also here analogous comments are in order as for the simpler case of the operator $N(x, -x)$. Harmonicity of the nonlocal operators (3.15), $\Box O^{tw(2+2j)}(x, -x) = 0$, holds by construction and, again, it is independent of $j$; namely, it holds separately for the last, $j$-dependent term due to the equality (3.8) and, therefore, must be fulfilled also for the combination of the first and second term. Obviously, for $j = 0$ we recover the expression (2.30).

Secondly, let us consider the scalar operator $M(x, -x) = x^\nu \partial^\mu M_{\mu\nu}(x, -x)$ resulting from the skew tensor operator $M_{\mu\nu}(x, -x)$. This operator is governed by totally symmetric tensors since the skew tensors $M_{\mu\nu}[n]$ having symmetry type $[n+1, 1]$ are composed with another skew tensor $x^{[\nu} \partial_{\mu]}$ having symmetry type $[1, 1]$ and the Clebsch-Gordan series of their direct product consists only of one symmetry type, $[n]$.

The local operators are decomposed according to formulae (3.1) and (3.2). However, because of the special ‘external’ structure the resulting expressions simplify considerably. Namely, after performing the differentiations

$$\Box^{k+j} \partial^\mu \sigma_{\mu\nu} x^{(q x)^n} = \left( n! / (n - 2j)! \right) (q^\mu \sigma_{\mu\nu} x^{q x}) q^{2j} (q x)^{n-1-2k-2j},$$

13
we arrive at the following harmonic operators in terms of Gegenbauer polynomials:

\[ M_{n-2j}^{\text{tw}(3+2j)}(x) = \frac{n!}{(n-1-2j)!} \int dq M_{[\mu \nu]}(q) q^\mu x^\nu q^{2j} \left( \frac{1}{2} \sqrt{q^2 x^2} \right)^{n-2j} C_{n-1-2j}^2 \left( \frac{q x}{\sqrt{q^2 x^2}} \right). \]  

(3.16)

Now, resumming with respect to \( n \) (\( \geq 2j+1 \)) we finally get the infinite twist decomposition:

\[ M(x, -x) = M_{\text{tw}3}(x, -x) + \sum_{j=1}^{\infty} (1)^j x^{2j} \int_0^1 dt (1-t)^{j-1} M_{\text{tw}(3+2j)}(tx, -tx), \]

(3.17)

with

\[ M_{\text{tw}(3+2j)}(x, -x) = \sqrt{\pi} \int dq M_{[\mu \nu]}(q) (q^2)^j i q^n x^n \times (2 + q \partial_q) (3 + q \partial_q) \left( \sqrt{(q x)^2 - q^2 x^2} \right)^{-3/2} J_{3/2} \left( \frac{1}{2} \frac{q x}{\sqrt{(q x)^2 - q^2 x^2}} \right) e^{i q x / 2}. \]  

(3.18)

Again, harmonicity of the nonlocal operators \( M_{\text{tw}(3+2j)}(x, -x) \) of twist \( \tau = 3+2j \), \( j = 0, 1, 2, \cdots \), is fulfilled without taking care of \( j \).

C. Harmonic vector and tensor operators related to totally symmetric local operators

The prescribed procedure may be used also for the twist decomposition of any totally symmetric (tensor) operator with an arbitrary number of free tensor indices which, by construction, are obtained from the corresponding scalar operators simply by applying appropriate derivatives with respect to \( x \).

The simplest of these operators, whose tower of infinite twist part starts with \( \tau = 2 \) and which contains the leading contributions to virtual Compton scattering is the operator \( O^S_\alpha(x, -x) \). Its symmetry type \( S \) is characterized by the Young frames \( [n+1], 1 \leq n \leq \infty \) (cf. also Ref. 3 where we denoted that symmetry type by (i)). Its moments, \( O_{\alpha n}^S(x) \), are obtained from the local scalar operators (3.11) of degree \( n+1 \) by applying \((1/(n+1))\partial_\alpha\). For the moment leaving aside the factor \((n+1)^{-1}\) we get

\[ \partial_\alpha O_{n+1}^S(x) = \sum_{j=0}^{[n+1]} \frac{(-1)^j (n - 2j)!}{4j!(n + 1 - j)!} 2x_\alpha x^{2(j-1)} O_{n-1-2j}^{\text{tw}(2+2j)}(x) + \sum_{j=0}^{[n+1]} \frac{(-1)^j (n + 2 - 2j)!}{4j!(n + 2 - j)!} x^{2j} \left( \partial_\alpha O_{n+1-2j}^{\text{tw}(2+2j)}(x) \right). \]

Using the representation (3.12) of the beta function both sums may be rewritten as

\[ \frac{1}{2} x_\alpha \sum_{j=0}^{[n+1]} \frac{(-1)^j (n - 2j)!}{4j!(n + 1 - j)!} x^{2j} O_{n-1-2j}^{\text{tw}(4+2j)}(x) = -\frac{1}{2} x_\alpha \sum_{j=0}^{[n+1]} \frac{(-1)^j x^{2j}}{4j!(j+1)!} \int_0^1 dt (1-t)^j O_{n-1-2j}^{\text{tw}(4+2j)}(tx), \]

\[ \sum_{j=0}^{[n+1]} \frac{(-1)^j (n + 2 - 2j)!}{4j!(n + 2 - j)!} x^{2j} \partial_\alpha O_{n-1-2j}^{\text{tw}(4+2j)}(x) = \partial_\alpha O_{n+1}^{\text{tw}(2+2j)}(x) + \sum_{j=1}^{[n+1]} \frac{(-1)^j x^{2j}}{4j!(j-1)!} \int_0^1 dt (1-t)^{j-1} \left( \partial_\alpha O_{n+1-2j}^{\text{tw}(2+2j)}(tx) \right), \]

respectively, where \( O_{n-1-2j}^{\text{tw}(4+2j)}(x) \) is given by Eq. (3.13) with \( j \to j + 1 \). From this it becomes obvious that, beginning with twist-4, one obtains two different contributions of the same twist, namely a vector and a scalar part.

Now, putting together these terms and resumming over \( n \), thereby representing the normalizing coefficient \(1/(n+1)\) as \( \int_0^1 d\lambda \lambda^n \), we obtain

\[ O^S_\alpha(x, -x) = \partial_\alpha \int_0^1 d\lambda \frac{1}{\lambda} O(\lambda x, -\lambda x) \]

\[ = \int_0^1 d\lambda \left\{ (\partial_\alpha O^{(2)})(\lambda x, -\lambda x) + \sum_{j=1}^{\infty} \frac{(-1)^j x^{2j} \lambda^{2j}}{4j!(j-1)!} \int_0^1 dt (1-t)^{j-1} \left( \partial_\alpha O^{(2+2j)}(\lambda tx, -\lambda tx) \right) \right. 

\left. - \frac{1}{2} x_\alpha \sum_{j=0}^{\infty} \frac{(-1)^j x^{2j} \lambda^{2j+1}}{4j!(j+1)!} \int_0^1 dt (1-t)^j O^{(4+2j)}(\lambda tx, -\lambda tx) \right\}, \]

(3.19)
where \( O^{\text{tw}(4+2j)}(x,-x) \equiv O^{\text{tw}(2+2(j+1))}(x,-x) \) and \( O^{\text{tw}(2+2j)}(x,-x) \) are given by Eq. (3.13). Both the integrals over \( \lambda \) and \( \ell \) are due to normalizations and, therefore, it depends on the personal taste if they are to be included into the definition of the harmonic operators of twist \( \tau \) or if they are considered as part of the twist decomposition. It is interesting to note that in Eq. (3.19) only the twist-2 vector and the twist-4 scalar operator survives on the light-cone. All other higher twist operators for \( j \geq 1 \) are cancelled due to the factor \( x^2 \).

In the same manner we are able to define the twist decomposition of (completely symmetric) tensor operators which result from the expression

\[
O^{\text{S}}_{(\alpha_1 \alpha_2 \cdots \alpha_n)}(x) = \frac{\partial_{\alpha_1}}{(n+1)} \cdots \frac{\partial_{\alpha_r}}{(n+r)} \sum_{j=0}^{n+r} \frac{(-1)^j(n+1+r-2j)!}{4^j j!(n+1+r-j)!} x^{2j} O^{\text{tw}(2+2j)}_{n+r-2j}(x).
\]

The general procedure is obvious from the consideration of the vector operator. Completely analogous to that case with only one derivative we would obtain a finite number of different operators of a given twist \( \tau \).

However, in the case of operators having more complicated symmetry type, e.g., being characterized by the symmetry type \([n+1,1]\), which are necessary for the consideration, e.g., of skew symmetric tensor operators like \( M_{[\alpha\beta]}(x,-x) \), a theory which generalizes the formalism of Ref. [8] is not available up to now. There exist only results for the lowest values of \( \tau \) which, however, are sufficient for the consideration of the physically relevant cases. They will be considered in the next chapter.

**IV. HARMONIC QUARK-ANTIQUARK OPERATORS OF GEOMETRIC TWIST 2 AND 3**

In this section we determine the harmonic quark-antiquark operators up to twist \( \tau = 2 \) and \( 3 \) which are relevant for the generalization of the parton distributions of definite (geometric) twist \([8]\) to the double distributions appearing in the parametrization in the non-forward matrix elements of the corresponding non-local operators off the light-cone. Unfortunately, in the case of non-trivial Young frames where, up to now, no general group theoretical study being comparable with that of Ref. [8]. However, harmonic twist operators which are sufficient for the physically relevant off-cone operators have been obtained in Refs. [5,6]. They allow for the consideration of the vector and skew tensor operators up to twist 3. According to the procedure of Sect. II.C we start with their moments, i.e., the local off-cone harmonic operators of definite twist. These (pseudo) scalar, (axial) vector and skew tensor operators have been already determined (implicitly) in Ref. [8]. Appendix B. They are obtained by applying onto the harmonic tensor operators – which ensure tracelessness – the appropriate differential operators which take care of the required symmetry types.

**A. Local harmonic operators: Series representation**

To begin with we introduce the local harmonic operators of geometric twist-2 and twist-3 for both the chiral-even (axial) vector operators,

\[
O_{\alpha n}(x) = \bar{\psi}(0) \gamma_{\alpha} (x D)^n \psi(0), \quad (4.1)
\]

\[
O_{5\alpha n}(x) = \bar{\psi}(0) \gamma_{\alpha} \gamma_5 (x D)^n \psi(0), \quad (4.2)
\]

and the chiral-odd scalar and skew tensor operators,

\[
N_{\alpha}(x) = \bar{\psi}(0) (x D)^n \psi(0), \quad (4.3)
\]

\[
M_{[\alpha\beta]}(x) = \bar{\psi}(0) \sigma_{\alpha\beta} (x D)^n \psi(0), \quad (4.4)
\]

together with their related scalar and vector operators

\[
O_{n+1}(x) = x^\alpha O_{\alpha n}(x), \quad M_{\mu n+1}(x) = x^\nu M_{\mu\nu n}|_{n}(x), \quad M_n(x) = \partial^\mu M_{\mu n+1}(x). \quad (4.5)
\]

First we consider the vector operators \( O_{\alpha n}(x) \) which allow for two different symmetry types \([n+1,1]\). The irreducible vector operators \( O^{\nu n}_{\alpha n}(x) \) of Lorentz type \([(n+1)\to n+1]\) have symmetry types \([n+1\to n+1]\) and can be taken over from the previous section. On the other hand, the twist-3 vector operators transform according to \([(n+1)\to n+1]\) $\oplus$ \([(n+1)\to n+1]\).
and have symmetry types \([n,1]\). They may be summed up to the lowest term of another infinite tower of non-local operators of twist \(\tau = 3,4,\ldots\). The conditions of tracelessness for harmonic vector operators are as follows:

\[
\Box O^{\text{tw} \tau}_{\alpha\beta}(x) = 0, \quad \partial^\alpha O^{\text{tw} \tau}_{\alpha\beta}(x) = 0 \quad \text{for} \quad \tau = 2,3. \tag{4.6}
\]

For the sake of completeness and in order to introduce a new constructive element let us reconsider the twist-2 vector operator. It is obtained from the scalar twist-2 operator, cf. Eq. (3.11):

\[
O^{\text{tw}2}_{n+1}(x) = \sum_{k=0}^{\lfloor \frac{n+2}{2} \rfloor} \frac{(-1)^k(n+1-k)!}{4k^k!(n+1)!} x^{2k} \Box O_{n+1}(x).
\]

by means of a partial differentiation, \(\partial_\alpha\), together with the normalization factor \(1/(n+1)\) as follows:

\[
O^{\text{tw}2}_{\alpha\beta}(x) = \frac{1}{n+1} \partial_\alpha O^{\text{tw}2}_{n+1}(x)
\]

\[
= \frac{1}{n+1} \sum_{k=0}^{\lfloor \frac{n+2}{2} \rfloor} \frac{(-1)^k(n-k)!}{4k^k!(n+1)!} x^{2k} D_\alpha(k) \Box O_{n+1}(x). \tag{4.7}
\]

Here, we introduced the operation

\[
D_\alpha(k) = (k + 1 + x\partial)\partial_\alpha - \frac{1}{2} x\Box
\]

which is a generalization of the inner derivative (for the latter, cf. Ref. [1] and, more detailed, Refs. [8,9]) off the light-cone and its extension to arbitrary values of \(k \geq 0\). (A similar derivative in momentum space already has been introduced in Ref. [3].) As a side remark we mention that with the help of this generalized derivative also symmetric tensor operators of rank 2 and higher can easily be constructed.

The twist-3 vector operator which satisfies the conditions (4.6) is, with the convention \(A_{\mu\nu} \equiv \frac{1}{2} (A_{\mu\nu} - A_{\nu\mu})\),

\[
O^{\text{tw}3}_{\alpha\beta}(x) = \frac{2}{n+1} x^\beta \left\{ \delta^\mu_\alpha \partial_\beta - \frac{1}{n+1} x^{[\alpha} \partial_\beta \partial^\mu \right\} \sum_{k=0}^{\lfloor \frac{n+2}{2} \rfloor} \frac{(-1)^k(n-k)!}{4k^k!(n+1)!} x^{2k} \Box O_{\mu\nu}(x),
\]

\[
= \frac{2}{n+1} \sum_{k=0}^{\lfloor \frac{n+2}{2} \rfloor} \frac{(-1)^k(n-k)!}{4k^k!(n+1)!} x^{2k} x^\beta \left\{ \delta^\mu_\alpha D_\beta(k) - \frac{1}{n+1} x^{[\alpha} \partial_\beta D^\mu(k) \right\} \Box O_{\mu\nu}(x). \tag{4.9}
\]

The twist-4 vector operators having symmetry type \([n+1]\) can be read off from the general result of Sec. III.C. However, there occur also twist-4 vector operators having symmetry type \([n,1]\) which have been determined on the light-cone only \([11]\). The scalar twist-3 operator is of Lorentz type \((\frac{n}{2},\frac{n}{2})\) and also has been considered in Sec. III; it reads

\[
N^{\text{tw}3}_n(x) = \sum_{k=0}^{\lfloor \frac{n+1}{2} \rfloor} \frac{(-1)^k(n-k)!}{4k^k!(n+1)!} x^{2k} \Box N_n(x). \tag{4.10}
\]

Now, let us give the decomposition of the skew tensor operator \(M_{[\alpha\beta]\mu\nu}(x)\) into its twist-2 and twist-3 parts. This determination is simplified by the observation that these operators are related to the vector operator \(M_{\mu\nu+1}(x) = x^\nu M_{[\mu\nu](x)}\) and the scalar twist-3 operator \(M_n(x) = \partial^\mu M_{\mu\nu+1}(x)\), respectively. The twist-2 tensor operator \(M^{\text{tw}2}_{[\alpha\beta]\mu\nu}(x)\) transforms according to \((\frac{n+2}{2},\frac{n}{2}) \oplus (\frac{n}{2},\frac{n+2}{2})\), whereas the twist-3 operator which is obtained from the trace terms of \(M^{\text{tw}2}_{[\alpha\beta]\mu\nu}(x)\) is of Lorentz type \((\frac{n}{2},\frac{n}{2})\). They have symmetry types \([n+1,1]\) and \([n]\), respectively, and are given by

\[
M^{\text{tw}2}_{[\alpha\beta]\mu\nu}(x) = \frac{2}{n+2} \left\{ \delta^\mu_\alpha \partial_\beta - \frac{1}{n+2} x^{[\alpha} \partial_\beta \partial^\mu \right\} \sum_{k=0}^{\lfloor \frac{n+2}{2} \rfloor} \frac{(-1)^k(n+1-k)!}{4k^k!(n+1)!} x^{2k} \Box M_{\mu\nu+1}(x),
\]

\[
= \frac{2}{n+2} \sum_{k=0}^{\lfloor \frac{n}{2} \rfloor} \frac{(-1)^k(n-k)!}{4k^k!(n+1)!} x^{2k} \left\{ \delta^\mu_\alpha D_\beta(k) - \frac{1}{n+2} x^{[\alpha} \partial_\beta D^\mu(k) \right\} \Box M_{\mu\nu+1}(x). \tag{4.11}
\]
and

\[ M_{[\alpha\beta]\mu\nu}(x) = \frac{2}{(n+2)} x_{[\alpha} \hat{M}_{\beta\mu\nu]n-1}(x) \],

\[ M_{[\alpha\beta]\mu\nu}(x) = \frac{2}{(n+2)} x_{[\alpha} \hat{M}_{\beta\mu\nu]n-1}(x) \],

with

\[
\hat{M}_{\beta\mu\nu}(x) = \frac{1}{n} \frac{\partial}{\partial x} \sum_{k=0}^{n} \frac{(-1)^k(n-k)!}{4^k k! n!} x^{2k} \nabla^k M_n(x)
\]

\[
= \frac{1}{n} \sum_{k=0}^{n} \frac{(-1)^k(n-k)!}{4^k k! n!} x^{2k} \nabla^k \nabla^k M_n(x)
\]

Besides the twist-3 operator (4.12) resulting from the trace terms of the twist-2 operator (4.11) there exists also a genuine twist-3 operator having Lorentz type (\(\frac{3}{2}, \frac{3}{2}\)) and being governed by the symmetry type \([n, 1, 1]\). It is given by the following expression (with the convention \(A_{\alpha\beta\gamma} \equiv \frac{1}{3!}(A_{[\alpha[\beta][\gamma]} + A_{[\beta[\gamma][[\alpha]} + A_{[\gamma[[\alpha][\beta]}])\):

\[
\hat{M}_{\alpha\beta\mu\nu}(x) = \frac{3}{n+2} \sum_{k=0}^{n} \frac{(-1)^k(n-k)!}{4^k k! n!} x^{2k} \nabla^k M_{[\alpha\beta]\mu\nu](n)}(x)
\]

\[
= \frac{3}{n+2} \sum_{k=0}^{n} \frac{(-1)^k(n-k)!}{4^k k! n!} x^{2k} \nabla^k \{ \delta^\mu_{[\alpha} \delta^\nu_{\beta]} \partial_{\gamma}] - \frac{2}{n} \delta^\nu_{[\alpha} x_{\beta] \partial_{\gamma]} \partial^\gamma(k) - \frac{2}{n} \delta^\nu_{[\alpha} x_{\beta] \partial_{\gamma]} \partial^\gamma(k) \} \nabla^k M_{[\alpha\beta]\mu\nu](n)}(x).
\]

By construction, Eqs. (4.11), (4.13) and (4.14) are (traceless) harmonic polynomials obeying

\[
\nabla^k M_{[\alpha\beta]\mu\nu](n)}(x) = 0, \quad \partial^\alpha M_{[\alpha\beta]\mu\nu](n)}(x) = 0 = \partial^\beta M_{[\alpha\beta]\mu\nu](n)}(x), \quad \tau = 2, 3,
\]

and

\[
\nabla^k \hat{M}_{[\alpha\beta]\mu\nu](n)}(x) = 0, \quad \partial^\alpha \hat{M}_{[\alpha\beta]\mu\nu](n)}(x) = 0.
\]

Let us point to the structural similarities of the operators (4.7) and (4.13) on the one hand and (4.9) and (4.11) on the other hand. Furthermore, we observe that when organizing the various expressions such that everywhere the partial derivatives appear to the right of the \(k\)th power of \(x^2\) the differential operations ensuring the symmetry type are transformed according to \(\partial_\alpha \rightarrow D_\alpha(k)\) whereas the generalized (first order) inner derivative \(x_{[\alpha} \partial_{\beta]}\) remains unchanged because it commutes with \(x^2\). This might be used as a hint how to extend harmonically the operators when they are given on the light-cone in terms of inner derivatives \(d_{\alpha} \equiv D_\beta(0)\) and \(\tilde{x}_{[\alpha} \partial_{\beta]}\). Namely, a short look on our results, Eqs. (4.7), (4.9), (4.10), (4.11), (4.12) and (4.14), shows that the expressions on-cone are obtained by restricting the sums to those terms with \(k = 0\) and replacing \(x \rightarrow \tilde{x}, \partial_\alpha \rightarrow \partial / \partial \tilde{x}^\alpha\). Let us mention that this is nothing else but another form of the harmonic extension which is well-known in group theory (80,51).

In the case of symmetric traceless tensors the harmonic extension is uniquely defined and proved in Ref. (80). In all the other cases it can be used heuristically according to the above observation. This harmonic extension gives an one-to-one relation between homogeneous polynomials on the light-cone and the corresponding harmonic polynomials off-cone. It is important to note that the unique harmonic extension must not destroy the type of the Lorentz representation.

According to Eqs. (4.7), (4.9), (4.10), (4.11), (4.12) and (4.14) we may introduce the orthogonal projection operators onto the subspaces with definite spin and, therefore, with definite geometric twist \(\tau = 2, 3\) as follows:

\[ O_{(5)\alpha\beta}(\tau) = P_{(5)\alpha\beta}(\tau) O_{(5)\alpha\beta}(x), \]

\[ N_{(\tau)\alpha}(x) = P_{(\tau)\alpha}(x), \]

\[ M_{[\alpha\beta]\mu\nu}(\tau) = P_{[\alpha\beta]\mu\nu}(x). \]

The corresponding twist projectors onto the light-cone are discussed in more detail in Ref. (80).
B. Resummed harmonic operators with twist $\tau = 2, 3$

In this subsection we rewrite the harmonic operators of definite twist $\tau = 2, 3$ which have been given in Subsect. A in terms of Gegenbauer polynomials analogous to the presentation in Sect. II for the scalar harmonic operators. In a second step we resum the infinite series (for $n$) of Gegenbauer polynomials to the related Bessel functions thereby obtaining nonlocal harmonic operators of (the same) definite twist.

For notational simplicity we introduce the following abbreviations for the homogeneous polynomials and their related functions:

$$h_n^\nu(q|x) = \left(\frac{1}{2}\sqrt{q^2 x^2}\right)^n C_n^\nu \left(\frac{q x}{\sqrt{q^2 x^2}}\right), \quad (4.20)$$

$$H_\nu(q|x) = \sqrt{\pi} \left(\sqrt{(q x)^2 - q^2 x^2}\right)^{1/2-\nu} J_{\nu-1/2} \left(\frac{1}{2}\sqrt{(q x)^2 - q^2 x^2}\right) e^{iq x/2}. \quad (4.21)$$

Using the following functional relations of the Gegenbauer and Bessel functions [27],

$$mC_n^\nu(z) = 2\nu (zC_{n-1}^{\nu+1}(z) - C_{n-2}^{\nu+1}(z)),$$

$$\frac{d}{dz} (z^{-\lambda} J_\lambda(z)) = -z^{-\lambda} J_{\lambda+1}(z),$$

$$z \frac{d}{dz} J_\lambda(z) = -\lambda J_\lambda(z) + z J_{\lambda-1}(z),$$

respectively, one obtains the following results for the partial derivations of these functions:

$$\partial_\alpha h_n^\nu(q|x) = \nu \left( q_\alpha h_{n-1}^{\nu+1}(q|x) - \frac{1}{2} q^2 x_\alpha h_{n-2}^{\nu+1}(q|x) \right),$$

$$\partial_\alpha H_\nu(q|x) = \left( i q_\alpha (2\nu + 1 + x \partial) - \frac{1}{2} i (iq)^2 x_\alpha \right) H_{\nu+1}(q|x). \quad (4.23)$$

The local twist-2 vector operator is obtained through derivation with respect to $x^\alpha$ of the local twist-2 scalar operator as follows, cf., Eq. (1.7),

$$O^{tw2}_a(x) = \frac{1}{(n+1)^2} \partial_a \int d^4q \left( \bar{\psi} \gamma_\mu \psi \right)(q) \left\{ x^\mu h_n^2(q|x) - \frac{1}{2} q^a x^2 h_n^2(q|x) \right\}$$

$$= \frac{1}{(n+1)^2} \int d^4q \left( \bar{\psi} \gamma_\mu \psi \right)(q) \left\{ \delta^\mu_a h_n^2(q|x) - q^a x_\alpha h_n^2(q|x) \right\}$$

$$+ 2 x^\mu q_\alpha h_{n-1}^3(q|x) - (x^\mu x_\alpha q^2 + q^a q_\alpha x^2) h_{n-2}^3(q|x) + \frac{1}{2} q^a x^2 q^2 h_{n-3}^3(q|x), \quad (4.24)$$

and the corresponding resummed nonlocal twist-2 vector operator reads

$$O^{tw2}_a(x, -x) = \partial_a \int_0^1 dt \int d^4q \left( \bar{\psi} \gamma_\mu \psi \right)(q) \left\{ x^\mu (2 + q \partial_q) - \frac{1}{2} it q^2 x^2 \right\} (3 + q \partial_q) H_2(q|tx)$$

$$= \int d^4q \left( \bar{\psi} \gamma_\mu \psi \right)(q) (2 + q \partial_q) \int_0^1 dt \left\{ (3 + q \partial_q) \delta^\mu_a - it q^a x_\alpha \right\} H_2(q|tx)$$

$$+ \left\{ (3 + q \partial_q) (4 + q \partial_q) it q^a x_\alpha - \frac{1}{2} (it)^2 (q^2 x^2 + x^2 q^2), \right\} + \frac{1}{4} (it)^3 q^a q^2 x^2 h_3^3(q|tx). \quad (4.25)$$

Here, the equality $1/(n+1) = \int_0^1 dt t^n$ has been used; analogous equalities will be used in the following.

The resummed local twist-3 vector operator reads, cf., Eq. (1.9):

$$O^{tw3}_a(x) = \int d^4q \left( \bar{\psi} \gamma_\mu \psi \right)(q) x_\beta \left\{ \delta^\mu_{[a} \delta^\mu_{\beta]} - \frac{1}{n+1} x_{[a} \partial_{\beta]} \hat{\partial}^\mu \right\} \int d^4q \left( \bar{\psi} \gamma_\mu \psi \right)(q) h_n^a(q|x)$$

$$= \int d^4q \left( \bar{\psi} \gamma_\mu \psi \right)(q) x_\beta \left\{ 2(n+1) \delta^\mu_{[a} \delta^\mu_{\beta]} h_{n-1}^2(q|x) + (n+2) x_{[a} \delta^\mu_{\beta]} q^2 h_{n-2}^2(q|x) \right.$$

$$- 4 x_{[a} q_{\beta]} q^2 h_{n-2}^2(q|x) + 2 x_{[a} q_{\beta]} x_\mu q^2 h_{n-3}^3(q|x) \right\}. \quad (4.26)$$

---

18
and the bilocal twist-3 vector operator is given by

\[ O_{t}^{\text{tw3}}(x, -x) = 2 \int d^4q \left( \bar{\psi} \gamma_\mu \psi(q) \right) \int_0^1 dt x^\beta \left\{ \delta^\mu_{[\alpha} \partial_{\beta]} (1 + q \partial_\beta) - x_{[\alpha} \partial_{\beta]} \partial^\mu \right\} H_1(q |tx) \]

\[ = \int d^4q \left( \bar{\psi} \gamma_\mu \psi(q) \right) \int_0^1 dt (2 + q \partial_\beta) x^\beta \left\{ 2i t \delta^\mu_{[\alpha} q_{\beta]} (2 + q \partial_\beta) - (it)^2 \delta^\mu_{[\alpha} x_{\beta]} q^2 \right\} H_2(q |tx) \]

\[ - i t x_{[\alpha} q_{\beta]} \left[ 2i t q^\mu (5 + q \partial_\beta) - (it)^2 x^\mu q^2 \right] H_3(q |tx) \] (4.27)

Let us remark that in the expression (1.27) by partial integration the term containing \((1 + t \partial_\beta)\) contributes only through its surface terms for \(t = 1\). Remind also that according to the special structure of \(H_\nu(q |tx) \equiv H_\nu(tq |x)\) in Eqs. (4.25) and (4.27), the homogeneous derivation \(q \partial_q\) may be replaced by either \(x \partial_x\) or \(t \partial_t\). Analogous observations could be made below; however, there the \(x\)–derivatives in the local as well as the nonlocal case will be not taken explicitly.

The local twist-3 chiral-odd skew tensor operator will be given only for completeness, cf., Eq. (3.3):

\[ N_{t}^{\text{tw3}}(x) = \int d^4q \left( \bar{\psi} \psi(q) \right) H_1^1(q |x), \] (4.28)

and for the corresponding bilocal one, one gets

\[ N^{\text{tw3}}(x, -x) = \int d^4q \left( \bar{\psi} \psi(q) \right) (1 + q \partial_q) \ H_1(q |x). \] (4.29)

The local resummed twist-2 chiral-odd skew tensor operator is given by, cf., Eq. (4.11),

\[ M_{t}^{\text{tw2}}(x) = \frac{2}{n + 2} \left\{ \delta^\mu_{[\alpha} \partial_{\beta]} - \frac{1}{n + 2} x_{[\alpha} \partial_{\beta]} \partial^\mu \right\} \int d^4q \left( \bar{\psi} \gamma_\mu \psi(q) \right) \left\{ x^\nu h^2_n(q |x) - \frac{1}{2} q^\nu x^2 h^2_{n-1}(q |x) \right\}, \] (4.30)

and the related bilocal twist-2 skew tensor operator reads

\[ M_{t}^{\text{tw2}}(x, -x) = 2 \int d^4q \left( \bar{\psi} \sigma_{\mu \nu} \psi(q) \right) \int_0^1 dt (1 + q \partial_\beta) \left\{ (2 + q \partial_\beta) \delta^\mu_{[\alpha} \partial_{\beta]} - x_{[\alpha} \partial_{\beta]} \partial^\mu \right\} \left\{ x^\nu (3 + q \partial_\beta) - \frac{1}{2} q^\nu x^2 \right\} H_2(q |tx). \] (4.31)

For the related, resummed local twist-3 skew tensor operator resulting from the trace terms we obtain

\[ M_{t}^{\text{tw3}}(x) = \frac{2}{2n(n + 2)} x_{[\alpha} \partial_{\beta]} \int d^4q \left( \bar{\psi} \sigma_{\mu \nu} \psi(q) \right) q^\mu x^\nu h^2_{n-1}(q |x), \] (4.32)

leading to the following twist-3 bilocal skew tensor

\[ M_{t}^{\text{tw3}}(x, -x) = 2x_{[\alpha} \partial_{\beta]} \int d^4q \left( \bar{\psi} \sigma_{\mu \nu} \psi(q) \right) i q^\mu x^\nu (2 + q \partial_\beta) \int_0^1 dt H_2(q |tx). \] (4.33)

In addition, the independent twist-3 skew tensor having symmetry type \([n, 1, 1]\) has the following local form:

\[ \tilde{M}_{t}^{\text{tw3}}(x) = \frac{3}{n + 2} x^\gamma \left\{ \delta^\mu_{[\alpha} \delta^\nu_{\beta]} \partial_\gamma - \frac{2}{n} \delta^\mu_{[\alpha} x_\beta \partial_\gamma \partial^\nu \right\} \int d^4q \left( \bar{\psi} \sigma_{\mu \nu} \psi(q) \right) h^1_n(q |x) \] (4.34)

leading to the following expression for the twist-3 bilocal skew tensor,

\[ \tilde{M}_{t}^{\text{tw3}}(x, -x) = 3x^\gamma \left\{ \delta^\mu_{[\alpha} \delta^\nu_{\beta]} x^\partial - 2 \delta^\mu_{[\alpha} x_\beta \partial_\gamma \partial^\nu \right\} \int d^4q \left( \bar{\psi} \sigma_{\mu \nu} \psi(q) \right) (1 + q \partial_\gamma) \int_0^1 dt \frac{1 - t^2}{2t} H_1(q |tx). \] (4.35)
V. POWER CORRECTIONS OF NON-FORWARD MATRIX ELEMENTS

A. Double distributions from quark–antiquark operators with definite twist \( \tau \)

According to the different approaches exemplified in Subsect. II.B and II.C in order to obtain the non-forward matrix elements for the quark–antiquark operators with definite twist \( \tau \) we have only to take matrix elements of the expressions given in the last Subsect. IV.B and to use formula (2.11) of Subsect. II.A. Let us emphasize that the spin-dependence of the hadronic states \( |P_i, S_i\rangle \), \( i = 1, 2 \), is completely contained in the independent kinematical structures \( K_\alpha^\tau(P) \) being relevant for the processes under consideration, e.g., the Dirac and the Pauli structure \( \bar{u}(P_2, S_2)\gamma_\mu u(P_1, S_1) \) and \( \bar{u}(P_2, S_2)\gamma_\mu P_\mu u(P_1, S_1)/M \), respectively, in case of the virtual Compton scattering (see, e.g., [25]).

Let us now give the final expressions for matrix elements of those nonlocal operators which are relevant for the power corrections of the various processes. The corresponding expressions for the local operators are easily obtained by taking moments.

For the non-forward matrix element of the nonlocal twist-2 and twist-3 vector operator, Eqs. (4.25) and (4.27), obeying the replacement (2.11), one gets

\[
\langle P_2, S_2 | O_{a}^{\text{tw2}}(\kappa x, -\kappa x) | P_1, S_1 \rangle = \mathcal{K}_a^\tau(P) \int \left( F_a^{(2)}(Z) \partial_\alpha \right) \int_0^1 dt \left\{ x^\mu (2 + x \partial) - \frac{1}{2} i \kappa t (P^\mu Z) x^2 \right\} (3 + x \partial) \mathcal{H}_2(PZ|\kappa tx)
\]

\[
= \mathcal{K}_a^\tau(P) \int \left( F_a^{(2)}(Z) \right) \int_0^1 dt (2 + x \partial) \left[ (3 + x \partial) \delta_\alpha^\mu - i \kappa t (P^\mu Z) x_\alpha \right] \mathcal{H}_2(PZ|\kappa tx)
\]

\[\quad + \left( 3 + x \partial \right) \left( 4 + x \partial \right) i \kappa t (P_\alpha Z)(2 + 2x \partial) \left( (P^\mu Z)(P_\alpha Z) x^2 \right)
\]

\[\quad + \frac{1}{4} (i\kappa t)^2 (P^\mu Z)^2 x_\alpha x^2 \right] \mathcal{H}_3(PZ|\kappa tx) \right\},
\]

(5.1)

and

\[
\langle P_2, S_2 | O_{a}^{\text{tw3}}(\kappa x, -\kappa x) | P_1, S_1 \rangle = 2\mathcal{K}_a^\tau(P) \int \left( F_a^{(3)}(Z) \right) \int_0^1 dt x^\beta \left\{ \delta_\alpha^\mu \partial_\beta \right\} \left( 1 + x \partial \right) x_\alpha \partial_\beta \partial^\mu \right] \mathcal{H}_1(PZ|\kappa tx)
\]

\[= \mathcal{K}_a^\tau(P) \int \left( F_a^{(3)}(Z) \right) (2 + x \partial) \int_0^1 dt x^\beta \left[ 2 i \kappa t \delta_\alpha^\mu \partial_\beta \left( P^\mu \partial_\beta \right) - (i\kappa t)^2 \delta_\alpha^\mu x_\beta (PZ)^2 \right] \mathcal{H}_2(PZ|\kappa tx)
\]

\[\quad - i \kappa t x_\alpha (P_\beta Z) \left( 2 i \kappa t (P^\mu Z)(5 + x \partial) - (i\kappa t)^2 x_\mu (PZ)^2 \right) \mathcal{H}_3(PZ|\kappa tx) \right\}.
\]

(5.2)

In the case of the axial vector operators \( O_{a,\alpha}^\tau(\kappa x, -\kappa x) \) the kinematical structures \( K_{5\alpha}(P) \) contain an additional \( \gamma_5 \) and the double distributions should be denoted by \( G_{\alpha}^{(\tau)}(Z) \) but otherwise the structure will be exactly the same.

The non-forward matrix elements of the nonlocal twist-3 chiral-odd scalar operator, Eq. (1.29), reads:

\[
\langle P_2, S_2 | N_{a}^{\text{tw3}}(\kappa x, -\kappa x) | P_1, S_1 \rangle = \mathcal{K}_a^{\tau}(P) \int \left( E_a^{(3)}(Z) \left( 1 + x \partial \right) \mathcal{H}_1(PZ|\kappa x). \right)
\]

(5.3)

The non-forward matrix elements nonlocal twist-2 chiral-odd skew tensor operator and of the related twist-3 operator, Eqs. (4.31) and (4.33), are given by

\[
\langle P_2, S_2 | M_{a[\alpha,\beta]}^{\text{tw2}}(\kappa x, -\kappa x) | P_1, S_1 \rangle = 2\mathcal{K}_{[\alpha,\beta]}^\tau(P) \int \left( H_a^{(2)}(Z) \right) \int_0^1 dt \left( 1 + x \partial \right) \left[ (2 + x \partial) \delta_\alpha^\mu \partial_\beta - x_\alpha \partial_\beta \partial^\mu \right]
\]

\[\times \left( x^\nu (3 + x \partial) - \frac{1}{2} i \kappa t (P^\nu Z)x^2 \right) \mathcal{H}_2(PZ|\kappa tx),
\]

(5.4)

and

\[
\langle P_2, S_2 | M_{a[\alpha,\beta]}^{\text{tw3}}(\kappa x, -\kappa x) | P_1, S_1 \rangle = 2\mathcal{K}_{[\alpha,\beta]}^\tau(P) \int \left( H_a^{(3)}(Z) \right) x_\alpha \partial_\beta \ln (P^\mu Z)x^\mu (2 + x \partial) \int_0^1 dt \mathcal{H}_2(PZ|\kappa tx).
\]

(5.5)
Finally, the non-forward matrix element of the independent nonlocal twist-3 skew tensor, Eq. (3.15), reads:

\[
\langle P_2, S_2 | \tilde{M}_{[\alpha \beta]}^{tw}(\kappa x, -\kappa x) | P_1, S_1 \rangle = 3 \tilde{K}_{[\mu \nu]}^{a}(P) \int D\zeta H_{a}^{(3)}(\zeta) x^\gamma \{ \delta_{[\alpha}^{\mu} \delta_{\beta]}^{\nu} \partial_{\gamma} x \theta - 2 \delta_{[\alpha}^{\mu} x_{\beta]}^{\nu} \partial_{\gamma} \theta \} \times (1 + x \theta) \int_{0}^{1} dt \frac{1 - t^{2}}{2t} H_{1}(P\zeta | ktx). \tag{5.6}
\]

The matrix elements of the scalar and vector operators are obtained by multiplying the above expressions by \(x^a\) (of course, for the operator \(M_{[\alpha \beta]}^{twb}(\kappa x, -\kappa x)\) no vector operator exists because of its symmetry type). For completeness let us also give the power corrections of the double distributions of higher twist in the scalar cases. Using the expressions (3.15), (3.7) and (3.18) we obtain

\[
\langle P_2, S_2 | O_{a}^{tw(2+2j)}(\kappa x, -\kappa x) | P_1, S_1 \rangle = K_{a}^{\mu}(P) \int D\zeta F_{a}^{(2+2j)}(\zeta) (i\zeta P)^{2j}
\times \left\{ x^{\mu} (2 + x \theta) - \frac{1}{2} i \kappa (\zeta P z)^{2j} (3 + x \theta) H_{2}(P\zeta | kx) - 2j (i\zeta P z)^{2j} (1 + x \theta) H_{1}(P\zeta | kx) \right\}, \tag{5.7}
\]

\[
\langle P_2, S_2 | N_{a}^{tw(3+2j)}(\kappa x, -\kappa x) | P_1, S_1 \rangle = K_{a}^{\mu}(P) \int D\zeta E_{a}^{(3+2j)}(\zeta) (i\zeta P)^{2j} (1 + x \theta) H_{1}(P\zeta | kx) \tag{5.8}
\]

and

\[
\langle P_2, S_2 | M_{[\alpha \beta]}^{tw(3+2j)}(\kappa x, -\kappa x) | P_1, S_1 \rangle = K_{[\mu \nu]}^{a}(P) \int D\zeta H_{a}^{(3+2j)}(\zeta) i\zeta P z x^{\nu} (i\zeta P z)^{2j} (2 + x \theta) (3 + x \theta) H_{2}(P\zeta | kx), \tag{5.9}
\]

respectively. Let us remind that, eventually, the \(t\)-integration which appeared, e.g., in Eq. (3.6) should be included into the definition of the operators of definite twist and, therefore, it could appear also here.

Now, a couple of remarks are in order. First, as indicated, different tensorial structures of the operators lead to different kinematical structures. In the forward case they simplify or eventually disappear, e.g., the Dirac structures are to be replaced by \(P_{\mu}\) and \(S_{\mu}\) for the vector and axial vector case, respectively, whereas the Pauli structures vanish. Second, going on-cone the double distributions remain the same. Therefore, their evolution is determined by the anomalous dimensions resulting from the renormalization group equation of the corresponding light-ray operators. Third, taking the forward limit on the light-cone the double distributions are projected onto the quark distribution functions determined in Ref. [3]. In this connection we emphasize that these distribution functions are uniquely determined by the geometric twist which differs from the dynamic twist being used for phenomenological considerations.

Furthermore, we have to mention that – contrary to the case when the non-forward matrix elements are restricted to the light-cone where the decomposition of the non-local quark-antiquark operators into operators of definite twist terminates at finite values \(\tau_{max}\) – the off-cone decomposition results in an infinite series of any twist. Therefore, the decomposition of the double distributions related to the undecomposed operators, \(f_{\alpha}(Z, P_{1}P_{j}, x^{2}, \mu^{2})\), results also in an infinite series. This becomes obvious especially in the scalar cases considered in Sec. III.

The double distributions occur in the scattering amplitudes of various physical processes, like virtual Compton scattering in the generalized Bjorken region. There, after multiplication with appropriate coefficient functions \(C_{\gamma}(x)\) a Fourier transformation with respect to \(x\) has to performed relating \(x\) to the (inverse) momentum transfer \(1/Q\) of the process. Thereby, the variables \((xP\zeta)\) and \(x^{2}(P\zeta)^{2}\) are changed into \((QP\zeta)/Q^{2}\) and \((P\zeta)^{2}/Q^{2}\). The Fourier transformation will be simplified when the Poisson integral is used for the Bessel functions. This will be considered in more detail in another paper.

Another application of our general procedure is the consideration of the meson distribution functions which does not require such an additional Fourier transformation.

### B. Mass corrections of vector meson distributions

Based on bilocal light-cone operators of definite twist-2, 3 and 4 the distribution amplitudes (DAs) of the \(\rho\)-meson have been already studied in Refs. [10, 11]. The relationship between the DAs of geometric and dynamical twist has also been discussed (see also, Ref. [34]). These considerations can be extended to the other vector mesons by trivial substitutions. It is an advantage of this approach that operators of well-defined twist are used and, therefore, it is not necessary to use operator relations, like dynamical Wandzura-Wilczek relations, in order to isolate contributions of
These distribution amplitudes are dimensionless functions of \( \xi \) inverting the moment integral, \( u \) (quark) and (1 meson DAs, \( \Phi(x) \), of twist-2 and twist-3 off-cone operators in order to get the power corrections to the results obtained earlier. Vector meson mass corrections of order \( x^2 \) were already discussed by Ball and Braun [35] and resummed meson mass corrections in the scalar case was given by Ball [33] (see also Ref. [24]).

The structure of the mass corrections of \( \rho \)-meson DAs of geometric twist are, from the group theoretical point of view, similar to the target mass corrections in deep inelastic scattering, which can be resummed using Nachtmann’s method [21]. Also here, the basic tool, in order to obtain the mass corrections, are the harmonic operators of definite geometric twist which have been determined in Sec. IV. Obviously, they are the harmonic extensions of the corresponding light-cone operators which already are used in Refs. [10,11] for the classification of the corresponding meson light cone DAs with respect to geometric twist.

Let us now introduce the distribution functions for the harmonic operators of geometric twist sandwiched between the vacuum and the meson state, \( \langle 0 | O^{(\tau)}(x, -x) | \rho(P, \lambda) \rangle \). Thereby, we adopt the definitions of Chernyak and Zhitnitsky [30] in the terminology of Ref. [37]. As usual, these matrix elements are related to the momentum \( P_\alpha \) and polarization vector \( e^{(\lambda)} \) of the meson with helicity \( \lambda \), \( P^2 = m^2, e^{(\lambda)} \cdot e^{(\lambda)} = -1, P \cdot e^{(\lambda)} = 0 \), \( m \) denotes the meson mass.

First, we consider the chiral-even vector operator. Using the twist projections we introduce the moments of the meson DAs, \( \Phi_n^{(\tau)}(x) \), of twist \( \tau \) according to

\[
\langle 0 | O^{(\tau)}(x) | \rho(P, \lambda) \rangle = f_\rho m \mathcal{P}^{(\tau)}_n \left( e^{(\lambda)}_\beta h^{\beta}_{n}(P|x) + 2P_\alpha (e^{(\lambda)}|x) h_{n-1}^{\alpha}(P|x) - m^2 x_\alpha (e^{(\lambda)}|x) h_{n-2}^{\alpha}(P|x) \right),
\]

(5.10)

where \( f_\rho \) is the vector meson decay constant. The corresponding meson distribution amplitudes \( \Phi_n^{(\tau)}(\xi) \) are given by inverting the moment integral,

\[
\Phi_n^{(\tau)}(\xi) = \int_{-1}^{1} d\xi \xi^n \Phi^{(\tau)}(\xi), \quad \xi = u - (1 - u) = 2u - 1.
\]

(5.11)

These distribution amplitudes are dimensionless functions of \( \xi \) and describe the probability amplitudes to find the \( \rho \)-meson in a state with minimal number of constituents (quark and antiquark) which carry the momentum fractions \( u \) (quark) and \( 1 - u \) (antiquark).

Taking into account expression (4.24), we obtain the local twist-2 matrix element as follows:

\[
\langle 0 | O^{(w2)}_{\alpha n}(x) | \rho(P, \lambda) \rangle = \frac{1}{(n + 1)^2} f_\rho m \Phi^{(2)}_n \left\{ e^{(\lambda)}_\alpha h^{2}_{n}(P|x) + 2P_\alpha (e^{(\lambda)}|x) h^{3}_{n-1}(P|x) - m^2 x_\alpha (e^{(\lambda)}|x) h_{n-2}^{\alpha}(P|x) \right\},
\]

(5.12)

which is analogous to Wandzura’s expression [4] for the target mass corrections in deep inelastic scattering in the \( x \)-space. After resummation we get the bilocal matrix element of geometric twist-2:

\[
\langle 0 | O^{(w2)}_{\alpha n}(x, -x) | \rho(P, \lambda) \rangle = f_\rho m \int_0^1 dt \int_{-1}^{1} d\xi \Phi^{(2)}(\xi) (3 + \xi \partial_\xi)(2 + \xi \partial_\xi)
\]

\[
\times \left\{ e^{(\lambda)}_\alpha H_2(P|x) + ((4 + \xi \partial_\xi)(i\xi t)P_\alpha - \frac{1}{2}(i\xi t)^2 m^2 x_\alpha) (e^{(\lambda)}|x) H_3(P|x) \right\}.
\]

(5.13)

An analogous calculation using expression (4.24) gives the local twist-3 matrix element

\[
\langle 0 | O^{(w3)}_{\alpha n}(x) | \rho(P, \lambda) \rangle = \frac{1}{(n + 1)^2} f_\rho m \Phi^{(3)}_n \left\{ 2(n + 1) e^{(\lambda)}_\alpha P_\beta h^{2}_{n-1}(P|x) + (n + 2) m^2 x_\alpha (e^{(\lambda)}|x) h_{n-2}^{\alpha}(P|x) + 2 m^2 x_\alpha P_\beta (e^{(\lambda)}|x) h_{n-3}^{\alpha}(P|x) \right\};
\]

(5.14)

the resummed bilocal matrix element of twist-3 reads

\[
\langle 0 | O^{(w3)}_{\alpha n}(x, -x) | \rho(P, \lambda) \rangle = f_\rho m \int_0^1 d\xi \Phi^{(3)}(\xi) (2 + \xi \partial_\xi)x^\beta \left\{ 2(i\xi t) e^{(\lambda)}_\alpha P_\beta H_2(P|x)
\]

\[
+ m^2 \int_0^1 dt \left\{ (i\xi t)^2 x_\alpha (e^{(\lambda)}|x) h_{n-2}^{\alpha}(P|x) + (i\xi t)^3 x_\alpha P_\beta (e^{(\lambda)}|x) H_3(P|x) \right\}.
\]

(5.15)
It is well-known that the twist-3 operator $O_{\alpha}^{tw3}(x, -x)$ is related to other twist-3 operators containing total derivatives and operators of Shuryak-Vainshtein type by means of QCD equations of motion \cite{26,33}. Therefore, the matrix elements \((5.14)\) and \((5.15)\) include the contribution of the twist-3 operator containing total derivatives which is as large as those from twist-2 (see also \cite{34}). In addition the next higher twist contributions of the chiral-even vector operator are of twist-4.

Now we consider the chiral-even axial vector operator $O_{5\alpha n}(x)$ with the same twist projectors as for the chiral-even vector operator $O_{\alpha n}(x)$. We define the corresponding moments of the meson DAs $\Xi_n^{(7)}$ by

$$
\langle 0|O_{5\alpha n}^{tw3}(x)|\rho(P, \lambda)\rangle = \frac{1}{2} \left( f_\rho - f_\rho^T \frac{m_u + m_d}{m} \right) m \mathcal{P}_n^{(7)} \gamma_\lambda \epsilon_\alpha \gamma_\rho \epsilon_\beta P_{\mu} x_{\nu} (P x)^n \Xi_n^{(7)},
$$

(5.16)

where $f_\rho^T$ denotes the tensor decay constant. First, we observe that the twist-2 contribution vanishes. The nontrivial local vacuum-to-meson matrix elements of this axial vector operator are of twist-3:

$$
\langle 0|O_{5\alpha n}^{tw3}(x)|\rho(P, \lambda)\rangle = \frac{1}{2} \left( f_\rho - f_\rho^T \frac{m_u + m_d}{m} \right) m \epsilon_\alpha \beta \epsilon_\beta \gamma_\rho \epsilon_\gamma P_{\mu} x_{\nu} \Xi_n^{(3)} h_n^1(P|x),
$$

(5.17)

and the bilocal matrix element of twist-3 reads

$$
\langle 0|O_{5\alpha n}^{tw3}(x, -x)|\rho(P, \lambda)\rangle = \frac{1}{2} \left( f_\rho - f_\rho^T \frac{m_u + m_d}{m} \right) m \epsilon_\alpha \beta \epsilon_\beta \gamma_\rho \epsilon_\gamma P_{\mu} x_{\nu} \int_{-1}^{1} d\xi \Xi_n^{(3)}(\xi) (1 + \xi \partial_\xi) \mathcal{H}_1(P_\xi|x).
$$

(5.18)

By the way the twist-4 matrix element also vanishes and the next higher twist contribution would be of twist-5. Also here, the matrix elements \((5.17)\) and \((5.18)\) include the contribution of the twist-3 operator containing total derivatives.

The matrix element of the chiral-odd scalar operator is defined as

$$
\langle 0|N_{\alpha n}^{tw3}(x)|\rho(P, \lambda)\rangle = -i \left( f_\rho^T - f_\rho \frac{m_u + m_d}{m} \right) m^2 \mathcal{P}_n^{(3)} \left( (\epsilon^{(\lambda)} x)(P x)^n \right) \mathcal{Y}_n^{(3)}(x),
$$

(5.19)

where $\mathcal{Y}_n^{(3)}$ is the moment of a spin-independent twist-3 distribution function. Using expression \((4.28)\) the local matrix element is given as

$$
\langle 0|N_{\alpha n}^{tw3}(x)|\rho(P, \lambda)\rangle = -i \left( f_\rho^T - f_\rho \frac{m_u + m_d}{m} \right) (\epsilon^{(\lambda)} x) m^2 \mathcal{Y}_n^{(3)} h_n^1(P|x),
$$

(5.20)

and for the bilocal matrix element of twist-3 we obtain

$$
\langle 0|N_{\alpha n}^{tw3}(x, -x)|\rho(P, \lambda)\rangle = -i \left( f_\rho^T - f_\rho \frac{m_u + m_d}{m} \right) (\epsilon^{(\lambda)} x) m^2 \int_{-1}^{1} d\xi \mathcal{Y}_n^{(3)}(\xi) (1 + \xi \partial_\xi) \mathcal{H}_1(P_\xi|x).
$$

(5.21)

The next higher twist contributions of the scalar chiral-odd operator are of order twist-5.

Now, we consider the matrix elements of the chiral-odd skew tensor operators. The corresponding moments of the wave function are introduced by

$$
\langle 0|M_{(\alpha\beta)\alpha n}^{tw3}(x)|\rho(P, \lambda)\rangle = i f_\rho^T \mathcal{P}_{(\alpha\beta)\alpha n} \left( \epsilon^{(\lambda)} P_{\mu} - \epsilon^{(\lambda)} P_{\mu} \right) (P x)^n \psi^{(r)} n.
$$

(5.22)

The local matrix element of the skew tensor operator of twist-2, performing the differentiations in the expression \((4.30)\), is given by

$$
\langle 0|M_{(\alpha\beta)\alpha n}^{tw2}(x)|\rho(P, \lambda)\rangle = i f_\rho^T \mathcal{P}_{(\alpha\beta)\alpha n} \left\{ \frac{e^{(\lambda)} P_{\mu} h_n^2(P|x)}{n + 1} + \frac{n + 3}{(n + 2)^2} m^2 x_{(\alpha} e^{(\lambda) \beta)} h_{n-1}^2(P|x) 
+ \frac{m^2}{(n + 2)^2 (n + 1)} \left\{ 4 x_{(\alpha} e^{(\lambda) \beta)} h_{n-1}^3(P|x) - \left( 2 x_{(\alpha} e^{(\lambda) \beta)} + 4 x_{(\alpha} P_{\beta}) (P x) + 4 x_{(\alpha} P_{\beta}) (P x) \right) h_{n-2}^3(P|x) 
+ 12 x_{(\alpha} P_{\beta}) (e^{(\lambda)} x) h_{n-2}^4(P|x) - 6 x_{(\alpha} P_{\beta}) (e^{(\lambda)} x)(P x) h_{n-3}^4(P|x) \right\},
$$

(5.23)
and the local matrix element of the skew tensor operator of twist-3 is obtained from (4.32) as follows:

\[ \langle 0 | M^{\text{tw}3}_{[\alpha\beta]}(x,-x) | \rho(P, \lambda) \rangle = i f^T_\rho \int_{-1}^{1} \text{d} \xi \, \bar{\Psi}^{(2)}(\xi) (3 + \xi \partial_\xi) \left\{ 2 (2 + \xi \partial_\xi) e^{(\lambda)}_{[\alpha} P_\beta] H_2(P \xi | x) + m^2 \int_{0}^{1} \text{d} t \left[ (1 + \xi \partial_\xi) (i \xi t) x_{[\alpha \epsilon^{(\lambda)}_\beta]} H_2(P \xi | xt) + \left( 2 (4 + \xi \partial_\xi) (i \xi t) x_{[\alpha \epsilon^{(\lambda)}_\beta]} (P x) + 2 x_{[\alpha \beta]} (e^{(\lambda)} x) \right) H_3(P \xi | xt) + \left( 2 (5 + \xi \partial_\xi) (4 + \xi \partial_\xi) (i \xi t)^2 x_{[\alpha \beta]} (e^{(\lambda)} x) - (4 + \xi \partial_\xi) (i \xi t)^2 x_{[\alpha \beta]} (e^{(\lambda)} x) (P x) \right) H_4(P \xi | xt) \right]\} \right\}. \tag{5.24} \]

The local matrix element of the skew tensor operator of twist-3 is obtained from (4.33) as follows:

\[ \langle 0 | M^{\text{tw}3}_{[\alpha\beta]}(x) | \rho(P, \lambda) \rangle = -\frac{2}{n+2}_n i f^T_\rho \Psi^{(3)}_n m^2 \left\{ x_{[\alpha \epsilon^{(\lambda)}_\beta]} h^2_{n-1}(P | x) + 2 x_{[\alpha \beta]} (e^{(\lambda)} x) h^3_{n-2}(P | x) \right\}, \tag{5.25} \]

and the corresponding bilocal matrix element reads

\[ \langle 0 | M^{\text{tw}3}_{[\alpha\beta]}(x,-x) | \rho(P, \lambda) \rangle = -2 i f^T_\rho m^2 \int_{0}^{1} \text{d} t \int_{-1}^{1} \text{d} \xi \, \bar{\Psi}^{(3)}(\xi) (1 + \xi \partial_\xi) \left\{ (i \xi t) x_{[\alpha \epsilon^{(\lambda)}_\beta]} H_2(P \xi | xt) + (3 + \xi \partial_\xi) (i \xi t)^2 x_{[\alpha \beta]} (e^{(\lambda)} x) H_3(P \xi | xt) \right\}. \tag{5.26} \]

The next higher twist contributions of the skew tensor operator would be of twist-4. Again, the matrix elements (5.20), (5.21), (5.23) and (5.26) include the contribution of the twist-3 operator containing total derivatives. Let us note, that the contribution of the additional twist-3 operator, Eq. (4.34), vanishes.

Finally, we remark that after projection onto the light-cone the matrix elements, which have been introduced in Ref. [10], are recovered. This can be easily checked by using the expansion of the Gegenbauer polynomials or the theoretical origin and is equivalent to the decomposition of the local operators into irreducible tensor representations of the Lorentz group. In the case of operators whose moments are related to totally symmetric tensors, using the procedure of harmonic extension \(^3\), we were able to determine the decomposition completely. For operators with non-trivial symmetry type, due to the lack of a general theoretical framework, only the terms up to twist \( \tau = 3 \) have been determined. However, this covers already almost all the phenomenological interesting cases of quark-antiquark operators.

Using these results we determined the off-cone power corrections to various double distributions and the vector meson wave functions being the inputs of the corresponding scattering amplitudes and hadronic form factors, respectively. These power corrections, in the case of local operators, are expressed by a finite sum in terms of Gegenbauer polynomials.

VI. CONCLUSIONS AND COMMENTS

In this paper, we introduced a general procedure of parametrizing non-forward matrix elements of non-local QCD operators \( O_{\tau}(x \epsilon, -x \epsilon) \) by multi-variable distribution amplitudes \( f^T_{\alpha}(x \epsilon) \) of well-defined geometric twist \( \tau \), namely, single variable hadron distributions, double distributions, triple distributions etc., times position-dependent coefficient functions \( e^{(\tau)}_\alpha \) as well as kinematical factors \( K_\tau \) related to the Lorentz structure of the QCD operators and the hadron states sandwiching the operators \( O_{\tau}^{(\tau)} \). The procedure relies on the unique twist decomposition of non-local operators off the light-cone leading to an infinite series of operators with growing twist. This decomposition is completely of group theoretical origin and is equivalent to the decomposition of the local operators into irreducible tensor representations of the Lorentz group. In the case of operators whose moments are related to totally symmetric tensors, using the procedure of harmonic extension \(^3\), we were able to determine the decomposition completely. For operators with non-trivial symmetry type, due to the lack of a general theoretical framework, only the terms up to twist \( \tau = 3 \) have been determined. However, this covers already almost all the phenomenological interesting cases of quark-antiquark operators.

Using these results we determined the off-cone power corrections to various double distributions and the vector meson wave functions being the inputs of the corresponding scattering amplitudes and hadronic form factors, respectively. These power corrections, in the case of local operators, are expressed by a finite sum in terms of Gegenbauer polynomials.
polynomials being multiplied with the moments of the distribution amplitudes and, in the case of non-local operators, by a finite sum in terms of Bessel functions now multiplied with the distribution amplitudes directly. As a remarkable fact, which may be read off from Sec. V, we observe that (the moments of) the distribution amplitudes are independent of whether the operators are taken off-cone or on-cone. Furthermore, the off-cone expressions are obtained from the on-cone ones by harmonic extension.

This very encouraging behaviour strongly relies on the definition of the distribution amplitudes through matrix elements of operators with definite geometric twist – contrary to their possible definition with respect to ‘dynamical’ twist. This, of course, would extend a procedure to the non-forward case which has been introduced for the forward matrix elements (on the light-cone) in Ref. [9]. As a consequence it is not necessary to use any further dynamical input, like equations of motion, and to take into account operators containing total derivatives in order to get expressions of well-defined geometric twist: their contribution is already contained in the expressions of given (geometric) twist.

Concerning the computation of the scattering amplitudes resp. form factors of physical relevance some Fourier transformation has to be carried out whose result mainly depends on the (singular) coefficient functions as well as on the various Bessel functions. However, modulo specific tensorial structures, one simply has formally to substitute \( x \rightarrow \frac{i q}{Q^2} \) leading to the replacements of \( (xPZ)^2 \) → \( (qPZ)^2/Q^2 \) and \( x^2/(xPZ)^2 \) → \( ((PZ)^2/Q^2)^2((qPZ)^2/Q^2)^2 \), especially in the expressions \( h_\nu(\nu x) \) and \( H_\nu(\nu x) \). The currently most interesting example is the (deeply) virtual Compton scattering whose consideration has been postponed to another paper.

There is a further point deserving attention. Since the distribution amplitudes of definite twist are given already on-cone their renormalization group induced \( Q^2 \)–evolution, which is determined by the (non-local) anomalous dimension of the non-local light-ray operators of definite twist, should be clearly separated from the kinematical power-like target momentum resp. mass corrections.

**ACKNOWLEDGMENTS**

The authors are grateful to J. Blümlein, J. Eilers and C. Weiss for various discussions. In addition, M.L. is grateful to P. Ball for stimulating discussions. He also acknowledges the Graduate College “Quantum field theory” at Center for Theoretical Sciences of Leipzig University for financial support.

---

[1] S.A. Anikin and O.I. Zavialov, Ann. Phys. (N.Y.) 116 (1978) 135.
[2] O.I. Zavialov, Renormalized Feynman Diagrams, Nauka, Moscow 1979 (in Russian); (extended) English edition: Renormalized Quantum Field Theory, Kluwer, Dortrecht 1990.
[3] D. Müller, D. Robaschik, B. Geyer, F.-M. Dittes, and J. Horejsi Fortschr. Phys. 42 (1994) 101.
[4] D.J. Gross and S.B. Treiman, Phys. Rev. D 4 (1971) 1059.
[5] B. Geyer, M. Lazar, and D. Robaschik, Nucl. Phys. B 559 (1999) 339.
[6] B. Geyer and M. Lazar, Nucl. Phys. B 581 (2000) 341.
[7] R.L. Jaffe and X. Ji, Nucl. Phys. B 375 (1991) 527.
[8] V. Bargmann and I.T. Todorov, J. Math. Phys. 18 (1977) 1141.
[9] B. Geyer and M. Lazar, Phys. Rev. D 63 (2001) 094003.
[10] M. Lazar, Phys. Lett. B 497 (2001) 62; Phys. Lett. B 506 (2001) 385 (Erratum).
[11] M. Lazar, JHEP 0105 (2001) 029.
[12] O. Nachtmann, Nucl. Phys. B 63 (1973) 273.
[13] V. Baluni and E. Eichten, Phys. Rev. Lett. 37 (1976) 1181; Phys. Rev. D 14 (1976) 3045.
[14] S. Wandzura, Nucl. Phys. B 122 (1977) 412.
[15] S. Matsuda and T. Uematsu, Nucl. Phys. B 168 (1980) 181.
[16] H. Kawamura and T. Uematsu, Phys. Lett. B 343 (1995) 346.
[17] B. Geyer, D. Robaschik, and E. Wieczorek, Fortschr. Phys. 27 (1979) 75.
[18] H. Georgi and H.D. Politzer, Phys. Rev. D 14 (1976) 1829.
[19] A. Piccione and G. Ridolfi, Nucl. Phys. B 513 (1998) 301.
[20] J. Blümlein and A. Tkabladze, Nucl. Phys. B 553 (1999) 427.
[21] A.V. Belitsky and D. Müller, Phys. Lett. B 507 (2001) 173.
[22] X. Ji, Phys. Rev. Lett., 78 (1997) 610, Phys. Rev. D. 55 (1997) 7114, J. Phys. G 24 (1998) 1181.
[23] A. Radyushkin, Phys. Lett. B 385 (1996) 333, Phys. Rev. D. 56 (1997) 5524.
[24] J. Blümlein, J. Eilers, B. Geyer, and D. Robaschik, to appear.
[25] J. Blümlein, B. Geyer, and D. Robaschik, Nucl. Phys. B 560 (1999) 283.
[26] I.I. Balitsky and V.M. Braun, Nucl. Phys. B 311 (1988/89) 541.
[27] P. Prudnikov, Yu.A. Brychov, and O.I. Marichev, Integrals and Series, Vol. 2, Special Functions, Gordon & Breach, New York 1988.
[28] H. Bateman and A. Erdelyi, Higher Transcendental Functions, Vol. 2, New York 1953.
[29] I.I. Balitsky and V.M. Braun, Nucl. Phys. B 361 (1991) 93.
[30] V.K. Dobrev, G. Mack, V.B. Petkova, S.G. Petrova, and I.T. Todorov, Harmonic Analysis of the n-Dimensional Lorentz Group and its Applications to Conformal Quantum Field Theory, Lecture Notes in Physics 63, Springer, Berlin 1977.
[31] V.K. Dobrev and A.Ch. Ganchev, Conformal operators from spinor fields: Antisymmetric tensor case, Dubna Report E2-82-881 (1982).
[32] N.Ya. Vilenkin, Special functions and the theory of group representations, Moscow 1965 (in Russian), Ch. IX; translated in: Translations of Mathematical Monographs Vol. 22, AMS, Providence 1968,
N.Ya. Vilenkin and A.U. Klimyk, Representations of Lie groups and Special Functions, Vol. 2, Kluwer, Dordrecht 1993.
[33] P. Ball, JHEP 9901 (1999) 010.
[34] P. Ball and M. Lazar, Phys. Lett. B 515 (2001) 131.
[35] P. Ball and V.M. Braun, Nucl. Phys. B 543 (1999) 201.
[36] V.L. Chernyak and A.R.Zhitnitsky, Phys. Rept. 112 (1984) 173.
[37] P. Ball, V.M. Braun, Y. Koike, and K. Tanaka, Nucl. Phys. B 311 (1988/89) 541.