The $D^*D\pi$ form factor is evaluated in a QCD sum rule calculation for both $D$ and $\pi$ off-shell mesons. We study the Borel sum rule for the three point function of one pseudoscalar, one axial and one vector meson currents. We find that the momentum dependence of the form factors is very different if the $D$ or the $\pi$ meson is off-shell, but they lead to the same coupling constant in the $D^*D\pi$ vertex.

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In a very recent measurement by the CLEO collaboration [1], the total width of $D^*$ meson was obtained: $\Gamma_{\text{tot}}(D^*) = 96 \pm 4 \pm 22$ keV. This measurement yields the strong $D^*D\pi$ coupling, $g_{D^*D\pi} = 17.9 \pm 0.3 \pm 1.9$, which is defined as [2]

$$
\langle D^*(p)|\pi^-(p')D^0(p-p')\rangle = g_{D^*D\pi}p'_\mu \epsilon^\mu .
$$

(1)

The $D^*D\pi$ coupling constant has been studied by several authors using different approaches of the QCD sum rules (QCDSR): two point function combined with soft pion techniques [3,4], light cone sum rules [2,5], light cone sum rules including perturbative corrections [3], sum rules in a external field [7], double momentum sum rules [8], double Borel sum rules [9]. Unfortunately, the numerical results from these calculations may differ by almost a factor two, and the upper limit of these predictions give $g_{D^*D\pi} = 13.5$ [8], which is still 25% lower than the central value of the CLEO measurement.

In ref. [9] we have estimated the $g_{D^*D\pi}(Q^2)$ form factor as a function of the off-shell pion momentum $Q^2$. Since the sum rule obtained from the used three-point function is not valid at $Q^2 = 0$, in order to determine the $D^*D\pi$ coupling we had to extrapolate the $Q^2$ behaviour of the form factor. Of course there are large uncertainties in this procedure, and, to guide us in choosing the analytical form which parametrizes our QCDSR results, we have used a
QCDSR valid only at $Q^2 = 0$, as suggested in [10] for the pion-nucleon coupling constant. We got $g_{D^*D\pi} = 5.7 \pm 0.4$, a value much smaller than the experimental result. However, in a recent calculation of the $DD\rho$ form factor [11], we have used a completely different approach to get the $DD\rho$ coupling constant: we have calculated the $DD\rho$ form factor for both $D$ and $\rho$ off-shell mesons, and the QCDSR results were parametrized by analytical forms such that the respective extrapolations at the $D$ and $\rho$ poles provided consistent values for the $g_{D^*D\rho}$ coupling constant.

In this work we use the three-point function approach to evaluate the $D^*D\pi$ form factor for an off-shell $D$ meson, and we will follow the procedure suggested in ref. [11] to parametrize the $Q^2$ behaviour of the $D^*D\pi$ form factor for both $D$ and $\pi$ off-shell mesons.

The three-point function associated with a $D^*D\pi$ vertex with an off-shell $D$ meson is given by

$$\Gamma_{\mu\nu}(p, p') = \int d^4x \, d^4y \, \langle 0| T\{j_\mu(x) j_D(y) j_D^\dagger(0)\} |0\rangle \, e^{ip'.x} \, e^{i(p-p').y} \, ,$$

(2)

where $j_D = i\bar{c}\gamma_5 u$, $j_\nu = \bar{u}\gamma_\nu \gamma_5 d$ and $j_\mu = \bar{c}\gamma_\mu d$ are the interpolating fields for $D^0$, $\pi^-$ and $D^{*-}$ respectively with $u$, $d$ and $c$ being the up, down, and charm quark fields.

The phenomenological side of the vertex function, $\Gamma_{\mu\nu}(p, p')$, is obtained by the consideration of $\pi$ and $D^*$ state contribution to the matrix element in Eq. (2):

$$\Gamma^{(\text{phen})}_{\mu\nu}(p, p') = i \frac{f_\pi f_D f_{D^*} m_D (m_D^2/m_c) g_{D^*D\pi}(q^2)}{(q^2 - m_{D^*}^2)(p^2 - m_{D^*}^2)(p'^2 - m_{D^*}^2)} \times \left( p'_\mu p'_\nu + \frac{m_{D^*}^2 + m_{\pi}^2 - q^2}{2m_{D^*}^2} p_\mu p'^\nu \right) + \text{higher resonances} \, .$$

(3)

To derive Eq. (3) we have made use of the generalization of Eq. (1) for an off-shell $D$ meson: $\langle D^{*-}(p)|\pi^-(p')D^0(q)\rangle = g_{D^*D\pi}(q^2) p'_\mu \epsilon^\mu$, where $q = p - p'$, and the decay constants $f_\pi$, $f_D$ and $f_{D^*}$ defined by the matrix elements

$$\langle 0| j_\mu |\pi(p')\rangle = i f_\pi p'_\mu \, ,$$

(4)

$$\langle 0| j_D |D\rangle = \frac{m_D^2 f_D}{m_c} \, ,$$

(5)

and

$$\langle D^*| j_\mu^\dagger |0\rangle = m_{D^*} f_{D^*} \epsilon_\mu^* \, ,$$

(6)

where $\epsilon^\mu$ is the polarization of the vector meson. The contribution of higher resonances and continuum in Eq. (3) will be taken into account as usual in the standard form of ref. [12], through the continuum thresholds $s_0$ and $u_0$, for the $D^*$ and $\pi$ mesons respectively.

The QCD side, or theoretical side, of the vertex function is evaluated by performing Wilson’s operator product expansion (OPE) of the operator in Eq. (4). Writing $\Gamma_{\mu\nu}$ in terms of the invariant amplitudes, we can write a double dispersion relation for each one of the invariant amplitudes, over the virtualities $p^2$ and $p'^2$ holding $Q^2 = -q^2$ fixed:
\[ \Gamma(p^2, p'^2, Q^2) = -\frac{1}{4\pi^2} \int_{m_Q^2}^{s_0} ds \int_0^{u_0} du \frac{\rho(s, u, Q^2)}{(s - p^2)(u - p'^2)}, \]  

where \( \rho(s, u, Q^2) \) equals the double discontinuity of the amplitude \( \Gamma(p^2, p'^2, Q^2) \) on the cuts \( m_Q^2 \leq s \leq \infty, \ 0 \leq u \leq \infty \), which can be evaluated using Cutkosky’s rules [12,13]. Finally we perform a double Borel transformation [12] in both variables \( p^2 = -p'^2 \rightarrow M^2 \) and \( p'^2 = -p^2 \rightarrow M'^2 \) and equate the two representations described above. We get one sum rule for each invariant function. In the \( p'_\mu p'_\nu \) structure the double discontinuity of the perturbative contribution reads:

\[ \rho(s, u, t) = -\frac{6im_c}{(\lambda(s, u, t))^{5/2}} \left[m_c^4(\lambda(s, u, t) + 6su) + s\left(s^2(t + u) + (t - u)^2(t + u) - 2s(t^2 - tu + u^2)\right) + m_c^2\left(-s^3 + s^2(t - 3u) - (t - u)^3 + s(t^2 - 4tu + 3u^2)\right)\right], \]  

where \( t = -Q^2 \) and \( \lambda(s, u, t) = s^2 + u^2 + t^2 - 2su - 2st - 2tu \). The integration limit condition is

\[ u \leq s + t - m_c^2 - \frac{st}{m_c^2}. \]

For consistency we use in our analysis the QCDSR expressions for the \( D^* \) and \( \pi \) decay constants up to dimension four in lowest order of \( \alpha_s \) as given in refs. [2,9,14].

The parameter values used in all calculations are \( m_c = 1.5 \text{ GeV}, \ m_\pi = 140 \text{ MeV}, \ m_D = 1.87 \text{ GeV}, \ m_{D^*} = 2.01 \text{ GeV}, \ f_D = 160 \text{ MeV}, \ \langle \bar{q}q \rangle = -(0.23)^3 \text{ GeV}^3, \ \langle g^2 G^2 \rangle = 0.5 \text{ GeV}^4, \ s_0 = 6.3 \text{ GeV}^2 \) and \( u_0 = 2.0 \text{ GeV}^2 \).

In the calculation of the \( D^* D \pi \) form factor with the off-shell pion \([4]\) we have included, besides the perturbative contribution, the gluon condensate contribution. We have found that the gluon condensate is small, as compared with the perturbative contribution and decreases with the Borel mass. The most important feature of the gluon condensate is the fact that it improves the stability of the result as a function of the Borel mass. Since its contribution at \( M^2 = 7 \text{ GeV}^2 \), is less than 5% of the perturbative contribution, in this work we will neglect the gluon condensate. In order to be sure that the absence of the gluon condensate will not affect our results, we will extract the value of the form factor at a higher value of the Borel mass, where we expect the gluon condensate contribution to be negligible.

In refs. [15][16] it was found that relating the Borel parameters in the two- \( (M_M^2) \) and three-point functions \( (M^2) \) as

\[ 2M_M^2 = M^2, \]

is a crucial ingredient for the incorporation of the HQET symmetries, and leads to a considerable reduction of the sensitivity to input parameters, such as continuum thresholds \( s_0 \) and \( u_0 \), and to radiative corrections. Therefore, in this work we will use Eq. (10) to relate the Borel masses.

Fixing \( M^2 = 7 \text{ GeV}^2 \) (at a fixed ratio \( M^2/M^2 = m^2/(m_{D^*}^2 - m_c^2) \) which corresponds to \( M^2 = 2.5 \text{ GeV}^2 \)) we show, in Fig. 1, the momentum dependence of the form factor (circles for an off-shell \( D \) meson) in the interval \( -0.5 \leq Q^2 \leq 5 \text{ GeV}^2 \), where we expect the

\[ 4\pi^2 \int_{m_Q^2}^{s_0} ds \int_0^{u_0} du \frac{\rho(s, u, Q^2)}{(s - p^2)(u - p'^2)}, \]  

\[ 4\pi^2 \int_{m_Q^2}^{s_0} ds \int_0^{u_0} du \frac{\rho(s, u, Q^2)}{(s - p^2)(u - p'^2)}, \]
sum rules to be valid (since in this case the cut in the $t$ channel starts at $t \sim m_c^2$ and thus the Euclidian region stretches up to that threshold). From this figure we can see that the $Q^2$ dependence of the form factor represented by the circles can be well reproduced by the monopole parametrization (solid line)

$$g_{D^*D\pi}^{(D)}(Q^2) = \frac{126.1}{Q^2 + 11.95}.$$  \hfill (11)

In Fig. 1 we also show, through the squares, the momentum dependence of the $g_{D^*D\pi}^{(\pi)}(Q^2)$ form factor for an off-shell pion, obtained in ref. [9], in the interval $2 \leq Q^2 \leq 5 \text{ GeV}$. In ref. [9] the $Q^2$ dependence of the form factor, represented by the squares, was parametrized by a gaussian form (dashed line)

$$g_{D^*D\pi}^{(\pi)}(Q^2) = 5.7 e^{-Q^4/9.17}.$$  \hfill (12)

However, as can be seen by the dot-dashed line, the $Q^2$ dependence of the QCDSR results for $g_{D^*D\pi}^{(\pi)}(Q^2)$ can also be well reproduced by the exponential parametrization

$$g_{D^*D\pi}^{(\pi)}(Q^2) = 15.5 e^{-Q^2/1.48}.$$  \hfill (13)

Off course, the two parametrizations in Eqs. (12) and (13) lead to very different values for the $D^*D\pi$ coupling constant, defined as the value of the form factor at the pole of the off-shell meson ($Q^2 = -m^2_\pi \sim 0$ in the case of the off-shell pion):

$$g_{D^*D\pi} = \begin{cases} 5.7 & \text{with the gaussian parametrization} \\ 15.5 & \text{with the exponential parametrization} \end{cases}$$ \hfill (14)

As discussed in the introduction, the parametrization of Eq. (12), adopted in ref. [9], was oriented by the QCDSR valid only at $Q^2 = 0$, as suggested in [10] for the pion-nucleon coupling constant. It consists in neglecting the pion mass in the denominator of the phenomenological side and working at $Q^2 = 0$, making a single Borel transformation to both external momenta $P^2 = P'^2 \rightarrow M^2$. The problem of doing a single Borel transformation in a three-point function is the fact that the single pole contribution, associated with the pole-continuum transitions, is not suppressed [3,2,17]. In ref. [17] it was explicitly shown that the pole-continuum transition has a different behavior as a function of the Borel mass, as compared with the double pole contribution and continuum contribution: it grows with $M^2$ as compared with the double pole contribution. Therefore, the single pole contribution can be taken into account through the introduction of a parameter $A$, in the phenomenological side of the sum rule [17]. The value of the coupling constant is obtained by the extrapolation of the line $g_{D^*D\pi} + AM^2$ to $M^2 = 0$ [17]. Off course this procedure also involves large uncertainties if $A$ is not much smaller than 1, which was the case. Also, if $A \sim 1$, this may be an indication that the sum rule is dominated by the pole-continuum transitions and, therefore, is not a good sum rule to extract informations about the low-energy states.

From the parametrization in Eq. (14) we can also extract the $D^*D\pi$ coupling constant, which now is defined as the value of the form factor at the $D$ pole ($Q^2 = -m^2_D$). We get

$$g_{D^*D\pi} = 14.9,$$ \hfill (15)
in an excellent agreement with the exponential parametrization of $g_{D^*D\pi}^{(\pi)}(Q^2)$.

There is another important information that we can extract from the parametrization of the QCDSR results which is the value of the cut-off. Defining the coupling constant as the value of the form factor at $Q^2 = -m^2_M$, where $m_M$ is the mass of the off-shell meson, the monopole and the exponential parametrizations of the form factor can be written as (neglecting $m^2_\pi$):

$$g_{D^*D\pi}^{(D)}(Q^2) = g_{D^*D\pi} \frac{\Lambda^2_D - m^2_D}{Q^2 + \Lambda^2_D},$$  \hspace{1cm} (16)

$$g_{D^*D\pi}^{(\pi)}(Q^2) = g_{D^*D\pi} e^{-\frac{Q^2}{\Lambda^2_\pi}},$$  \hspace{1cm} (17)

and from Eqs. (11) and (13) we get

$$\Lambda_D = 3.5 \text{ GeV},$$  \hspace{1cm} (18)

$$\Lambda_\pi = 1.2 \text{ GeV}.$$  \hspace{1cm} (19)

Therefore, the form factor is harder if the off-shell meson is heavy, implying that the size of the vertex depends on the exchanged meson, in agreement with our findings in refs. [11,18]. This means that a heavy meson will see the vertex as pointlike, whereas a light meson will see its extension. The value obtained for the cut-offs are also in a very good agreement with the values of the cut-offs in the $DD\rho$ vertex [11].

The same calculation can be done for the $B^*B\pi$ form factor and one has only to change the $D^*$, $D$ and quark $c$ masses by $B^*$, $B$ and quark $b$ masses, that we take as: $m_{B^*} = 5.33 \text{ GeV}$, $m_B = 5.28 \text{ GeV}$ and $m_b = 4.7 \text{ GeV}$. In Fig. 2 we show, through the circles and through the squares, the QCDSR results for the $B^*B\pi$ form factor with the $B$ and $\pi$ off-shell mesons respectively. Using Eqs. (16) and (17) to fit ours QCDSR results we get the couplings and cut-offs shown in Table I.

|           | $g_{B^*B\pi}^{(M)}(Q^2 = -m^2_M)$ | $\Lambda_M (\text{GeV})$ |
|-----------|-----------------------------------|--------------------------|
| $B$ off-shell | 42.3                              | 6.8                      |
| $\pi$ off-shell | 45.1                              | 1.3                      |

**TABLE I:** Values of the coupling constants and cut-offs which reproduce the QCDSR results for $g_{B^*B\pi}^{(M)}(Q^2)$.

In Fig. 2 we also show, for completeness, the gaussian fit obtained in ref. [9], which leads to a much smaller value to the coupling constant. It is interesting to notice that the value of the cut-off for a off-shell pion is of the same order in both, $B^*B\pi$ and $D^*D\pi$, vertices. However, in the case of an off-shell $B$ meson, the cut-off is much bigger, as expected from the discussion above.
From $g_{B^*B\pi}$ we can extract the effective scale-independent coupling constant $g$, which controls the interaction of the pion with infinitely heavy fields in effective lagrangian approaches \cite{13,20}, defined as $g = \frac{f_\pi}{2m_B} g_{B^*B\pi}$. During the last years, a large number of theoretical papers has been devoted to the calculation of $g$. However, the variation of the value obtained for $g$, even within a single class of models, turns out to be quite large. For instance, using different quark models one obtains $1/3 \leq g \leq 1$ \cite{20} while QCDSR calculations points in the direction of small $g$, with a typical value in the range $g \simeq 0.13 - 0.35$ \cite{2–5,7,8}.

Using the values for $g_{B^*B\pi}$ given in Table I we get, at order $\alpha_s = 0$:

$$g = 0.59 - 0.63,$$ (20)

therefore, our number is much bigger than the other QCDSR calculations, and is in a better agreement with quark models.

In conclusion, we have extracted the $D^*D\pi$ coupling constant using two different QCDSR for the $D^*D\pi$ form factor for the $D$ and the $\pi$ off-shell mesons. We have obtained for the coupling constant:

$$g_{D^*D\pi} = 14.0 \pm 1.5,$$ (21)

where the errors reflect variations in the continuum thresholds, different parametrizations of the form factors and the use of different relations between the Borel masses in the two- and three-point functions. There are still sources of errors in the values of the condensates and in the choice of the Borel mass to extract the form factor, which were not considered here. Therefore, the errors quoted are probably underestimated. As for the form factors, we obtain a harder (softer) form factor when the off-shell particle is heavier (lighter).

In Table II we present a compilation of the estimates of the coupling constants $g_{D^*D\pi}$ and $g_{B^*B\pi}$ from distinct QCDSR calculations.

| approach | $g_{D^*D\pi}$ | $g_{B^*B\pi}$ |
|----------|---------------|---------------|
| this work | $14.0 \pm 1.5$ | $42.5 \pm 2.6$ |
| two-point function + soft pion techniques (2PFSP) | 9 ± 2 | 20 ± 4 |
| 2PFSP + perturbative corrections | 7 ± 2 | 15 ± 4 |
| light cone sum rules (LCSR) | 11 ± 2 | 28 ± 6 |
| LCSR + perturbative corrections | 10.5 ± 3 | 22 ± 9 |
| double momentum sum rule | 6.3 ± 1.9 | 14 ± 4 |

**TABLE II:** Summary of QCDSR estimates for $g_{D^*D\pi}$ and $g_{B^*B\pi}$.

From this Table we see that our result is in a fair agreement with the LCSR calculation in refs. \cite{2,3}, but is still smaller than the experimental value \cite{1}: $g_{D^*D\pi} = 17.9 \pm 0.3 \pm 1.9$.

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FIG. 1. Momentum dependence of the $D^*D\pi$ form factor. The solid, dashed and dot-dashed lines give the parametrization of the QCDSR results through Eq. (11) for the circles, and Eqs. (12) and (13) for the squares.

FIG. 2. Momentum dependence of the $B^*B\pi$ form factor. The solid, dashed and dot-dashed lines give the parametrization of the QCDSR results through Eq. (11) for the circles, and Eqs. (12) and (13) for the squares.