The Effect of the Third Dimension on Rough Surfaces Formed by Sedimenting Particles in Quasi-Two-Dimensions

K. V. McCoubrey
Department of Physics and Engineering, Xavier University of Louisiana, New Orleans, LA 70125

M. L. Kumar
Department of Physics, Bogazici University, 80815 Bebek Istanbul

The roughness exponent of surfaces obtained by dispersing silica spheres into a quasi-two-dimensional cell is examined. The cell consists of two glass plates separated by a gap, which is comparable in size to the diameter of the beads. Previous work has shown that the quasi-one-dimensional surfaces formed have two distinct roughness exponents in two well-defined length scales, which have a crossover length about 1 cm. We have studied the effect of changing the gap between the plates to a limit of about twice the diameter of the beads.

PACS numbers: PACS numbers: 05.40.+j, 47.15.Cf, 47.53.+n, 81.15.Lm

I. INTRODUCTION

The formation of rough surfaces is a problem of theoretical and practical importance in many areas, ranging from the study of fluid entanglement to nonequilibrium statistical physics to various industrial processes such as the growth of films by deposition [1,2]. Rough surfaces formed by the sedimentation of particles through a viscous fluid are particularly interesting, since interactions between the sedimenting particles, and between the particles and the walls of the container, can be expected to have an effect on the wall interface. Surfaces formed by sedimentation are close to the original problem of sedimentation of particles sedimenting along straight vertical lines first studied by Edwards and Wilkison [3]. However, since the hydrodynamic particle/particle and particle/wall forces are in principle long-range, the rough surfaces formed by particles sedimenting in a viscous fluid are a different growth situation from the simpler vertical deposition. The situation is further complicated by the presence of back-phenomena caused by the motion of significant numbers of particles [4,5].

In this work we are primarily interested in studying experimentally the effect of the particle/wall interactions on the roughness of the wall interface. The motion of a sphere parallel to a single wall is of interest as the limiting case of motion of a small sphere in a cylindrical container when the sphere approaches the cylinder wall. This problem and the more general one of the motion of a sphere parallel to two external walls were treated in the 1920s by Faxen [6]. Unfortunately, it is very difficult to comment on the exact nature of the interactions between the particles in the presence of the walls. A nontypical sedimentation theory has succeeded only in analyzing the effective settling velocity of particles in a dilute regime in the presence of the walls [1,2,12,13,14,15] and some features of any-body interactions between the particles [14,15] when there are no walls [16]. Recent theoretical work [3,17,18] and experimental [19,20,21] work hold out some hope of determining particle interactions through a wide range of volume fractions and Peclet numbers in sedimentation problems. However, so far there has been no experimental or theoretical work to distinguish between the effects of the interaction between the particles versus the interactions between the particles and the wall, and to correlate these interactions with the surface growth problem.

In our previous work [22,23] on the quasi-one-dimensional surfaces formed by particles sedimenting through a viscous fluid in a quasi-two-dimensional cell, we have found that the surfaces formed by sedimenting particles are rough on all scale lengths between the particle size and the cell size. However, different roughness exponents were found in two well-defined length-scale regimes, with a well-defined crossover length scale. These roughness exponents and the crossover length scale have been found to be independent of the cell aspect ratio or the viscosity of the fluid through which the particles settle. The exponent found at long length scales has been shown to depend on the rate at which particles are deposited into the cell (hence to the strength of the interaction between the particles [24]). This leads to the conclusion that the scaling exponent seen at long length scales depends on the nature of the hydrodynamic interactions between the particles, while the exponent seen at smaller length scales, which remained relatively unchanged by changes in the deposition rate, may be due to more universal considerations.

In the present work, we are continuing to study the rough surfaces formed by sedimenting particles, but we are focusing specifically on the effect of the particle/wall interactions on the wall interface while keeping the average volume fraction of the particles at the low value of 0.01 to restrict the particle/particle interactions to a minimum level allowed by the experimental setup. All of our work to date has taken place in quasi-two-dimensional sedimentation cells, which consist of two glass plates separated by a gap set by Teflon spacers, through which glass beads are allowed to settle to form a quasi-one-dimensional rough surface at the bottom of
the cell (Fig. 1). There are two types of cells, open and closed, which will be discussed in more detail below. In each type of cell, the gap between the plates, although variable, has been of the order of the diameter of the sedimenting particles.

II. EXPERIMENTAL METHOD

Two types of cells were used in the previous work, denoted as "closed" and "open" cells. Closed cells were constructed of 1/4 in. oate glass, held 1 mm apart by sealed side frames of precision machined Plexiglas. Around 10,000 0.6 mm -diam et er monodisperse silica spheres were placed in the cell, which was then filled with a viscous uid (such as glycerin) and closed. Each cell could be rotated about a horizontal axis perpendicular to the gap direction. When the cell was rotated, the particles which had been at rest on the bottom fell through the viscous uid, slowly building up a new surface at the bottom of the cell. In the closed cells we only had a fixed gap size between the cell walls. The open cell was constructed of in. oate glass, separated by strips of Te on of known thickness. It had dimensions comparable to the closed cell, but was open at the top, so that beads could be dispensed through a funnel which steadily dropped beads as it traveled back and forth across the top of the cell (Fig. 1). In this way, the deposition rate of the beads into the cell could be controlled precisely by varying the speed and the size of the funnel. The experiments were designed to test the effect of changing the distance between the walls on the roughness of the interface while keeping the interparticle distance constant. The experiments took place in the open cell and we investigated the effect of variability in the gap by setting the gap at different values, and measuring the effect of the walls on the roughness of the interface formed after sedimentation. We investigated gaps ranging from 0.8 mm to 2.0 mm. The ratio of the gap thickness to the bead diameter was defined as a dimensionless parameter R, and our experiments spanned a gap/bead diameter ratio of R = 1.33 to R = 3.33. The previous experiments took place at either a gap of 1 mm (R = 1.66) (closed cell) or 0.8 mm (R = 1.33) (open cell), so this range of gap values went far beyond our earlier measurements. In all cases, the deposition rate of beads into the cell was controlled at about 4 beads/sec so the average distance between the particles was about 20R. The surface was photographed during and at the end of the deposition process and the photographs were digitized by a Nikon LS-2000 scanner. Individual particles were typically resolvable and thus the position of the particles on the interface could be traced accurately. There is a limit to the extent over which the gap can be widened without changing the method of analysis, since at one point it will no longer be possible to analyze the rough surface as a one-dimensional interface. We believe that we are already past that limit at R = 2, but to give an estimate of the extent of the wall separation to the interested reader we have included the data for R > 2.

III. DISCUSSION

As in the previous work, we have analyzed these rough surfaces using the scaling ansatz proposed by Family and Vicsek [23]. In this ansatz, the rms thickness of the interface is defined to be:

\[ \delta_x = \left( \frac{1}{N} \sum_{i=1}^{N} \delta_x \right)^{1/2} \]

where

\[ \delta_x(x,t) = h(x,t) - h(t) \]

FIG. 2: The sedimentation cell consists of two glass plates with a small gap between them, of the order of the diameter of the glass beads. The cell is filled with a viscous uid such as oil and a funnel sweeps across the top of the cell, delivering a mixture of oil and beads to the cell. The beads settle to the bottom of the cell and build a rough surface.
and

\[ h(t) = \frac{1}{N} \sum_{i=1}^{N} h(x_i; t) \]  

(3)

The scaling ansatz predicts that:

\[ W(L; t) = L^{\alpha_t} f(t/L^\alpha_s) \]  

(4)

where the exponents \( \alpha_t \) and \( \alpha_s \) are the static and dynamic scaling exponents. The function \( f(t/L^\alpha_s) \) is expected to have an asymptotic form such that

\[ W(L; t) \sim t \]  

for \( t \ll L^\alpha_s \)  

(5)

and

\[ W(L; t) \sim L \]  

for \( t \gg L^\alpha_s \)  

(6)

Fig. 3 shows an example of \( W(L,t) \) at a typical gap/bead ratio. To minimize the wall effects at the horizontal edges, we have used only the middle 70% of each interface for our analysis.

At all values of \( R \) (gap/bead ratio) studied, we still see two well-defined roughness exponents. These roughness exponents have a crossover length scale at about 1 cm, which is typical from the previous work. Our earlier work corresponded to a gap width of 1.0 mm, and the present work gives the same results at this gap width as expected. As the value of \( R \) is increased (Fig. 4), we do not see any significant change in the value of either exponent. While there is a slight increase in the scaling exponents around \( R = 2 \), this slight increase is less significant than the increase in the roughness exponents when we increase the mean particle separation by increasing the feeding rate of the particles in our previous experiments, where the large length scale roughness rose from 0.2 at an average deposition rate of one bead/sec to 0.5 at an average deposition rate of 35 beads/sec.

We have investigated the effect of the interaction between the walls of the container and the sedimenting particles on the roughness exponent of the surface formed by this quasi-two-dimensional sedimentation. The roughness exponent is found to be robust to the changes in the separation between the walls of the container. The reasons for the slight increase in the roughness exponent at \( R = 2 \) is under further investigation using computational models to simulate the effects of the wall separation, as well as direct experimental investigations of correlations between particles in the fluid.

Acknowledgments

The authors would like to acknowledge the advice and loan of equipment graciously provided by Dr. James Maher at the University of Pittsburgh.
[1] T. Vicsek, Fractal Growth Phenomena (World Scientific, Singapore, 1992).
[2] A.-L. Barabasi and H. E. Stanley, Fractal concepts in Surface Growth (Cambridge University Press, Cambridge, 1995).
[3] S. F. Edwards and D. R. Wilkinson, Proc. R. Soc. Lond. A 381, 17 (1982).
[4] R. H. Davis and A. Acivos, Ann. Rev. Fluid Mech. 17, 91 (1985).
[5] F. M. Auzerais, R. Jackson, and W. B. Russell, J. Fluid Mech. 195, 437 (1988).
[6] H. Faxen, Arkiv. Mat. Astron. Fys. 17, 23 (1923).
[7] H. Smoluchowski, Bull. Acad. Sci. Cracow 1a, 27 (1911).
[8] P. M. Azur and W. Van Saarloos, Physica A 115, 21 (1982).
[9] J. M. Burgers, Proc. Koningl. Akad. Weten schap 44, 1045 (1941).
[10] H. Brenner, Phys. Fluids 1, 338 (1958).
[11] G. J. Lynch, J. Fluid Mech. 5, 193 (1959).
[12] H. Hasimoto, J. Fluid Mech. 5, 317 (1959).
[13] H. Faxen, Arkiv. Mat. Astron. Fys. 19A, 22 (1925).
[14] P. M. Azur and W. Van Saarloos, Physica A 115, 21 (1982).
[15] W. Van Saarloos and P. M. Azur, Physica A 120, 77 (1983).
[16] J. H. Happe and H. Breuner, Low Reynolds Number Hydromechanics (Prentice-Hall, Englewood Cliffs, N.J., 1965).
[17] J. F. Brady and L. J. Durbin, Phys. Fluids 31, 717 (1988).
[18] D. L. Koch and E. S. G. Shaikh, J. Fluid Mech. 224, 275 (1991).
[19] H. H. Nikolai and E. E. Guazzelli, Phys. Fluids 7, 3 (1995).
[20] J.-Z. Xue, D. J. Pine, S. T. Milner, X. L. Wu, and P. M. Chaikin, Phys. Rev. A 46, 6550 (1992).
[21] H. E. Segre, P. N. and P. M. Chaikin, Phys. Rev. Lett. 79, 2574 (1997).
[22] M. K. V. Kumaz, M. L. and J. V. Maher, Fractals 1, 583 (1993).
[23] M. L. Kumaz and J. V. Maher, Phys. Rev. E 53, 978 (1997).
[24] K. M. L. M. Coud, K. V. and J. V. Maher, Phys. Rev. E 56, 5768 (1997).
[25] F. Famuly and T. Vicsek, J. Phys. A: Math. Gen. 18, 75 (1985).