Low-temperature induction heating of flat metal objects in a traveling electromagnetic field

A N Kachanov, Y S Stepanov, N A Kachanov, V A Chernyshov and D A Korenkov

Russia, Oryol State University named after I.S. Turgenev, Russia, 302026, Komsomolskaya st., 95
E-mail: kan@ostu.ru

Annotation. The article discusses possible options for a low-temperature induction heating system (LTIHS) of flat metal products in a traveling electromagnetic field. The problem of calculating eddy currents, active and reactive powers induced in a heated flat object, as well as electromagnetic forces acting on the object moving it in a given direction, is posed and solved. A mathematical model has been developed that takes into account the dependence of the influence on the main parameters of the electromagnetic field of the following factors: geometric dimensions of the air gap between the poles of the magnetic circuit and the heated flat body; the longitudinal edge effect caused by the open circuit of the magnetic circuit of the inductor, as well as the transverse edge effect associated with the appearance of the longitudinal components of eddy currents in a heated flat object. The solution of particular problems of LTIHS in one- and two-dimensional formulation allows them to be simplified and to perform calculations for various design variants of induction heating devices with a traveling electromagnetic field, using a one-dimensional model that explicitly takes into account the features of electromagnetic processes in the systems under study.

1. Introduction
In various sectors of the economy, when carrying out a number of technological processes, the systems for low-temperature induction heating of flat products in a traveling electromagnetic field can be successfully used, shown in Figure 1 (a, b, c). The installation provides not only heating of the object 2, but also of the media inside the heated object, with their simultaneous movement in space [1,2,3,4].

2. Materials and methods
In the study of LTIHS, the problem of calculating eddy currents, active and reactive powers induced in a heated flat object and electromagnetic forces moving the object in a given direction was posed and solved. In the general case, the solution to the problem posed requires taking into account a number of specific features, such as: change in the parameters of the electromagnetic field when the heated object is removed from the inductor; longitudinal edge effect due to the open circuit of the magnetic circuit of the inductor; transverse edge effect associated with the appearance of longitudinal components of eddy currents in a heated flat object; redistribution of eddy currents due to the geometric dimensions
of the heated object and the displacement of induced currents due to the skin effect; the effect of the resulting electromagnetic force on a heated flat object, depending on the method of switching on the inductor windings (counter or concordant switching on).

Figure 1. The main types of systems "inductor - flat heated metal object"
1 - inductor; 2 - heated object; 3 - magnetic circuit.

To find a general solution to the problem posed, we will use the superposition method, which allows us to consider a complex three-dimensional problem in one- and two-dimensional formulation. Let us single out the two most characteristic methods of heating flat products in a traveling electromagnetic field - one-sided and two-sided heating. As a basic model, we will take the one-dimensional mathematical model shown in Figure 2.

Figure 2. Computational model of an inductor with a traveling electromagnetic field.

The following assumptions are made for the design model:
1. The magnetic cores of the upper and lower inductors have infinite dimensions along the X and Y axes. The parameters of the electromagnetic field do not depend on the Y and Z coordinates.
2. The magnetic permeability of the cores of the inductors \( \mu = 0 \). The cores of the inductor are laminated, which allows us to take their electrical conductivity in the transverse direction (along the Y-axis) equal to zero (\( \gamma_1 = 0 \)).

3. Primary and secondary currents are evenly distributed along the height of the air gap (\( \delta_3 \)) with densities \( J_1 \) and \( J_2 \), respectively.

4. The currents of the inductor winding, located in the slots of the magnetic circuit with a height (\( h_a \)), form a sinusoidally traveling current wave in the interpolar space, the bulk density of which is equal to [5]:

\[
J_1 = J_{lm} \cos (\omega t - \alpha x),
\]

or in complex form

\[
\dot{J}_1 = J_{lm} e^{-j \alpha x},
\]

where \( \alpha = \frac{\pi}{\tau} \) is a wave number;

\[
J_{lm} = J_\phi \frac{\sqrt{2mw_\phi} k_{nt}}{p \tau \delta_3} = J_{nm} / \delta_3,
\]

where \( J_\phi \) – effective value of the phase current of the inductor winding; \( m \) – number of phases; \( w_\phi \) – the number of turns of the phase winding of the inductor; \( k_{nt} \) – winding ratio; \( p \) – number of pole pairs; \( \tau \) – pole division; \( J_{nm} \) – the amplitude of the linear current density of the inductor.

5. A real flat metal object of length \( b \) (along the X-axis), width \( a \) (along the Y-axis), thickness \( d \) (along the Z-axis), with magnetic permeability \( \mu_0 \) and specific electrical conductivity \( \gamma_2 \) is replaced by an equivalent one that completely fills the equivalent working gap of the inductor (\( \delta_3' \)) on a segment of length \( b \) and having an equivalent conductivity:

\[
\gamma_2' = \gamma_2 \frac{d}{\delta_3'}.
\]

6. The active surfaces of the magnetic cores of the inductor are assumed to be smooth without taking into account the grooves.

Taking into account the accepted assumptions, the magnetic field strength in the working gap of the inductor will have only one component along the Z axis, which is equal to \( H = H_z = H_1 + H_2 \). The electric field strength and current density \( J_1 \) and \( J_2 \) have components only along the X-axis. For real induction devices with at least 6 poles, as shown in [6], the influence of the longitudinal edge effect can be neglected.

The change in the strength \( H_z \) along the Z coordinate for various design variants of inductors with a traveling magnetic field are taken into account in the design model when determining the equivalent air gap \( \delta_3' \). As an equivalence criterion, the equality of the values of the strength of the external (primary) magnetic field at the location of the heated flat object is taken:

\[
H_z = \frac{1}{d} \int_0^{\delta_3} H_z(z) \, dz,
\]

where \( H_z(z) \) - the complex value of the normal component of the primary field strength in the working gap of real LTIHS products in a traveling electromagnetic field (Figure 1 a, b, c), and \( H_z \) is the complex of the magnetic field strength in the computational model (Figure 2).
To determine $\hat{H}_z(z)$ we will use a two-dimensional computational model proposed by A.I. Voldek [5], in which the complex value of the normal component of the tension for the system under consideration is equal to:

for one-sided heating (Fig. 1, a)

$$\hat{H}_z(z) = -J_{nm} e^{j\alpha x} e^{-\alpha z}, \quad (6)$$

for double-sided heating of the plate when the windings of the upper and lower inductors are concordantly switched on (Fig. 1, b)

$$\hat{H}_z(z) = -\frac{1}{2} J_{nm} \frac{ch(z - \delta_3/2)}{sh(\alpha \delta_3/2)} e^{-j\alpha x} \quad (7)$$

for double-sided heating of the plate with the opposite connection of the windings of the upper and lower inductors (Fig. 1, c)

$$\hat{H}_z(z) = -j J_{nm} \frac{ch(\delta_3 - z)}{sh \alpha \delta_3} e^{-j\alpha x} \quad (8)$$

In the proposed calculation model, the magnetic field strength, which is a constant value along the height of the working gap of the inductor, can be calculated by the formula:

$$\hat{H}_1 = -j \frac{J_{nm}}{\alpha \delta_3 (1 + \beta^2 / \alpha^2)} e^{-j\alpha x} \quad (9)$$

where $\beta$ is a coefficient of propagation of the resulting magnetic field in the working gap of the inductor

$$\beta = \sqrt{\frac{\mu_0}{\chi}} \delta_3. \quad (10)$$

Taking into account the condition of equivalent calculation models (5) and on the basis of (6) - (7), we find the expression for the air gap ($\delta_3 = \delta + d$):

for the case of one-sided heating of the plate

$$\delta_3 = \frac{de^{\alpha x}}{(1 + \beta^2 / \alpha^2)(1 - e^{-\alpha d})}; \quad (11)$$

for double-sided heating of the plate with the consent of the upper and lower inductors

$$\delta_3 = \frac{d}{(1 + \beta^2 / \alpha^2)sh(\alpha d/2)} \left[ ch\left(\frac{\alpha \delta_3}{2}\right) ch\left(\frac{\alpha(2\delta + d)}{2}\right) - sh\left(\frac{\alpha(2\delta + d)}{2}\right) \right]; \quad (12)$$

for double-sided heating of the plate with the opposite connection of the upper and lower inductors

$$\delta_3 = \frac{d}{2(1 + \beta^2 / \alpha^2)sh(\alpha d/2)} \left[ ch\alpha \delta_3 ch\left(\frac{\alpha(2\delta + d)}{2}\right) - sh\left(\frac{\alpha(2\delta + d)}{2}\right) \right]; \quad (13)$$

where $\delta$ is the distance from the poles of the magnetic circuit to the surface of the heated flat object in a real system.
3. Results and discussion
The possibility of solving particular problems in one- and two-dimensional formulations made it possible to simplify the task and perform calculations for various variants of induction heating devices with a traveling electromagnetic field, using a one-dimensional model that allows explicitly taking into account the features of electromagnetic processes in the systems under study.

Dividing the core of the model (Figure 2) into three areas, taking into account the accepted assumptions, we write the following system of ordinary second-order linear differential equations with constant coefficients:

\[
\begin{cases}
\frac{\partial^2 \hat{A}_m}{\partial x^2} - \gamma_1 \mu_0 v \frac{\partial \hat{A}_m}{\partial x} - j \omega \gamma_1 \mu_0 \hat{A}_m = 0 \\
\frac{\partial^2 \hat{A}_m}{\partial x^2} - \gamma_2 \mu_0 v \frac{\partial \hat{A}_m}{\partial x} - j \omega \gamma_2 \mu_0 \hat{A}_2m = -\mu_0 \hat{J}_m e^{-jx} \\
\frac{\partial^2 \hat{A}_m}{\partial x^2} - \gamma_3 \mu_0 v \frac{\partial \hat{A}_m}{\partial x} - j \omega \gamma_3 \mu_0 \hat{A}_3m = 0
\end{cases}
\]

(14)

The solution of system (14), taking into account (15) with respect to \( \hat{H}_m \), is sought in the form:

\[
\begin{align*}
\hat{H}_x &= \hat{C}_x e^{jbx}; & -\infty < x < 0; \\
\hat{H}_x &= \hat{C}_x e^{jx} + \hat{C}_2 e^{-jx} + \hat{C}_3 e^{-jbx}; & 0 < x < b; \\
\hat{H}_x &= \hat{C}_4 e^{-jbx}; & b < x < \infty \\
\lambda^2 &= \beta^2 + \sigma^2; & \sigma^2 = j \omega \mu_0 \gamma_2; & \varepsilon = \mu_0 \omega \gamma_2^2 / \pi^2; \\
\end{align*}
\]

(16)

where \( \varepsilon \) – a criterion for the quality or goodness of the device.

The method for determining the integration constants is described in sufficient detail in [5, 7], therefore, cumbersome expressions for determining the integration constants are not presented in this article.

Based on the law of the total current for the components related to the heated flat object, it is possible to write:

\[
J_z = -\frac{\partial \hat{A}}{\partial x} + \frac{\hat{H}_{2z}}{\delta_x}
\]

(18)

Taking into account (18) and (16), we obtain the formula for calculating the eddy current density in the heated plate:
It contains both a harmonic component (second term), characteristic of induction heating systems that provide heating of flat products of infinite dimensions along the X axis, and additional components due to the limited size of the conducting plate.

Knowing $J_2$ according to (19), it is possible to determine the active specific power released per unit volume of the heated plate:

$$ P_{ jos } = J_2^2 / \gamma_2 $$

The force acting on the plate can be calculated, taking into account $\dot{H}_1$ according to (9), and the formula known from the theory for linear induction motors [7]:

$$ F = \frac{1}{2} \mu_0 a \delta \int_0^b \text{Re}[H_1 J_2] dx. $$

For flat bodies with infinite dimensions along the X coordinate, it is necessary to know the basic force per section of the heated tape with a length equal to double pole division:

$$ F_{bas} = \frac{H_0}{\pi} r^2 a \delta J_{1a}^2 \frac{e}{1 + e^2}, $$

which, taking into account the additional coefficient $D$, which represents the ratio of the real length of the heated flat object $b$ to $2\tau$, makes it possible to find the total value of the force acting on the object:

$$ F = F_{bas} \text{Re} \left[ (1 - j\varepsilon) \left( D - \frac{\sqrt{\varepsilon}}{\pi (1 + j\varepsilon)} \cdot \frac{\text{ch} \left( 2\pi D \sqrt{\varepsilon} \right) - \cos(2\pi D)}{\text{sh} \left( 2\pi D \sqrt{\varepsilon} \right)} \right) \right]. $$

Let's denote the factor in square brackets $- \chi$. In the LIM theory, a similar coefficient takes into account the phenomenon of the transverse edge effect in the medium being moved along the Y-axis. In the general case, this coefficient is equal to:

$$ \chi = \text{Re} \left[ (1 - j\varepsilon) \left( D - \frac{\sqrt{\varepsilon}}{\pi (1 + j\varepsilon)} \cdot \frac{\text{ch} \left( 2\pi D \sqrt{\varepsilon} \right) - \cos(2\pi D)}{\text{sh} \left( 2\pi D \sqrt{\varepsilon} \right)} \right) \right]. $$

As follows from the analysis of the curves shown in Figure 3, the coefficient $\chi$ depends on the quality factor $\varepsilon$ and the ratio $a/\tau$. Thus, with an increase of $\varepsilon$, the value of $\chi$ decreases, which is explained by the displacement of eddy currents from the center to the edges of the heated plate. For real induction devices, the width of the heated plates $a \geq \tau$. Therefore, an increase of $\chi$ to a certain maximum value in the range of changes for $a/\tau$ ratio from 0 to 1.0 can be neglected. In [7, 9, 10], the question of taking
into account the transverse edge effect was studied quite fully. The following regularities are revealed that are of interest for the practice of induction heating:

• with an increase in slip, the effect of displacing the parameters of the electromagnetic field is more pronounced. With an increase in the speed of movement of the heated medium \((s \to 0)\), the process of displacement of eddy currents from the center to the peripheral zones sharply weakens;

• weakening of the transverse edge effect is also observed in the case when the width of the plate is greater than the width of the poles of the magnetic circuit along the Y-axis. In this case, it is advisable to increase the size \(a\) only up to 40% of the value of the pole division \((\tau)\). A further increase in the size of the "protrusion" a greater than the indicated value has no practical effect on the redistribution of the parameters of the electromagnetic field and induced eddy currents.

![Figure 3](image.png)

**Figure 3.** Dependence of the coefficient \(\chi\) on \(a/\tau\) and the quality criterion of the heating device.

When calculating induction devices that create a traveling electromagnetic field, it is of practical interest to solve the problem of calculating the active and reactive power released in a heated flat object. From the theory of electrical machines it is known that the total power transmitted from the primary circuit to the heated object is equal to:

\[
\dot{S}_{12} = -E_{12}I_1, \tag{25}
\]

where \(E_{12} = jX_{12}I_2\) is an effective value of the EMF induced by the secondary current in the primary circuit; \(X_{12}\) is an inductive reactance of mutual induction taking into account \(E_{12}\).

For the calculated mathematical model, the total electromagnetic power is equal to:
\[
\hat{S}_{em} = -\frac{E_{12}}{2} \int_{-\hat{y}/2}^{\hat{y}/2} d\hat{z} \cdot \int_{-\hat{x}/2}^{\hat{x}/2} d\hat{y} \cdot \int_{\hat{p}}^{\hat{p} + \hat{y} / \sqrt{2}} \int_{\hat{p}}^{\hat{p} + \hat{x} / \sqrt{2}} \frac{E_{12}}{\sqrt{2}} d\hat{x}.
\] (26)

It should also be borne in mind that the induction of the secondary traveling magnetic field moves relative to the stationary windings of the inductor at a speed \( \vartheta = 2\tau f_1 \) and induces an EMF in them, which is equal to:

\[
\hat{E}_{12} = \mu_0 \hat{h}_z 2\tau f_1.
\] (27)

Using relations (2), (8) and (27), according to (26), we obtain the formula for calculating the electromagnetic power:

\[
\hat{S}_{em} = \hat{S}_{em0} \chi,
\] (28)

\[
\hat{S}_{em0} = \frac{\mu_0 p r \delta^e_j A J_{1m}^2 2\tau f_1}{\alpha},
\] (29)

where \( \hat{S}_{em0} \) is the total power in the absence of the transverse edge effect and demagnetizing action of the secondary eddy current field.

Using the relation \( \alpha = \pi / \tau, (3) \) and (17) for \( \hat{S}_{em0} \), according to (29):

\[
\hat{S}_{em0} = 2p r^2 \sigma f_1 \gamma^2 a \delta^e_1 \left\{ \frac{I_m \mu_{w_{d}} s}{p r \delta^e}\right\}^2 2\tau f_1.
\] (30)

Considering the above, the active (\( P_2 \)) and reactive (\( P_{2q} \)) powers transmitted from the primary electrical circuit to the secondary are respectively equal:

\[
P_2 = \hat{S}_{em0} \chi;
\]

\[
P_{2q} = \hat{S}_{em0} \chi_p,
\] (31)

where \( \chi_p \) is a coefficient that allows you to separate the share of reactive power from the total electromagnetic power transmitted to the secondary circuit:

\[
\chi_p = \text{Im} \left[ (1 - j\varepsilon) \left\{ D - \frac{\sqrt{\hat{E}}}{\pi (1 + j\varepsilon)} \cdot c h \left( \frac{2\pi D \sqrt{\hat{E}}}{\hat{E}} \right) - \cos (2\pi D) \right\} \right].
\] (32)

Figure 4 shows the relation \( \chi_p = f (a/\tau, \varepsilon) \).

It follows from the graph that the absolute value of \( \chi_p \) as a function of \( a/\tau \) increases monotonically for induction devices with a quality factor \( \varepsilon = 0.1 \div 1.0 \). For values \( \varepsilon > 1.0 \), the coefficient \( \chi_p \) decreases monotonically.
Figure 4. Dependence of the coefficient $\chi_p$ on $a/\tau$ and the quality criterion $\varepsilon$ of the heating device.

For an idealized induction heating device, in which the active and reactive resistances of the inductor winding are equal to each other, and the transformation ratio of the EMF is equal to one, you can also write formulas for determining technical and economic indicators, which are a function of the electromagnetic quality factor:

- Device power factor

$$\cos \phi = \frac{\varepsilon}{\sqrt{1 + \varepsilon^2}};$$  \hspace{1cm} (33)

- Device efficiency

$$\eta = 1 - S.$$  \hspace{1cm} (34)

4. Conclusions

1. The developed mathematical model of the "inductor - heated flat metal object" system and the obtained graphical dependencies allow us to make a preliminary assessment of the efficiency of the technical indicators of induction heating devices for heating flat metal products in a traveling electromagnetic field.

2. From the analysis of the obtained mathematical expressions and the graphical dependencies given in the article, the following remark can be made. One of the main criteria for evaluating a designed or operated induction device for low-temperature induction heating of flat metal products in
a traveling electromagnetic field is the electromagnetic quality factor, which characterizes the quality of the secondary electrical and magnetic circuits of the heating device, i.e. characterizes its ability to convert energy from one type to another. So, for example, the greater the value of the product of quality factor ($\varepsilon$) and slip ($s$), the more energy consumed is converted into mechanical energy, thereby reducing the heating efficiency.

5. References

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