The two-body hadronic decays of $B_c$ meson in the perturbative QCD approach: A short review

Zhen-Jun Xiao

Department of Physics and Institute of Theoretical Physics, Nanjing Normal University, Nanjing, Jiangsu 210023, People’s Republic of China

Xin Liu

School of Physics and Electronic Engineering, Jiangsu Normal University, Xuzhou, Jiangsu 221116, People’s Republic of China

(Dated: March 3, 2014)

Abstract

Along with the running of Large Hadron Collider (LHC) located at CERN in November 2009, a large number of data samples of $B_c$ meson have been collected and some hadronic $B_c$ decay modes have been measured by the LHC experiments. In view of the special and important roles of $B_c$ meson decays playing in the heavy flavor sector, we here give a short review on the status of two body hadronic decays $B_c \rightarrow M_1 M_2$ at both experimental and theoretical aspects. For the theoretical progresses, specifically, we will show lots of theoretical studies on two body hadronic $B_c$ decays involving pseudoscalar, vector, scalar, axial-vector, even tensor meson(s) in the final states by employing the perturbative QCD (pQCD) factorization approach. We will present a general analysis about the two-body hadronic decays of the heavy $B_c$ meson and also provide some expectations for the future developments.

Key Words $B_c$ meson hadronic decays; The pQCD factorization approach; Branching ratios; CP-violating asymmetries; Polarization fraction

1. INTRODUCTION

The $B_c$ meson is the lowest-lying bound state of $\bar{b}$ and $c$ quark with $J^P = 0^-$ in the standard model(SM) [1]. It is too heavy to be produced in the old $B$ factories at KEK and SLAC, but it can be produced in significant numbers in high energy hadron collisions, such as the Tevatron and LHC experiments. The heavy $B_c$ meson was first discovered by CDF collaboration at Tevatron in 1998 through the semileptonic modes $B_c \rightarrow J/\psi(\mu^+ \mu^-)l^+X(l = e, \mu)$ [2], which demonstrated the possibility for investigations on $B_c$ physics experimentally. At the Large Hadron Collider (LHC) experiments, furthermore, a large number of $B_c$ meson events could be collected. With a luminosity of about $\mathcal{L} = 10^{34} \text{cm}^{-2}\text{s}^{-1}$, around $5 \times 10^{10}$ $B_c$ events are expected to be produced each year [3]. The properties of $B_c$ meson and the dynamics involved in the $B_c$ decays would be fully exploited through the precision measurements at the LHC with its high collision energy and high luminosity. A golden era of $B_c$ physics is opened with the successful running of the LHC experiments, especially the measurements carried on by the LHCb Collaboration, where about 1% of the total $b$-related data sample are the $B_c$ events: $10^9 \sim 10^{10}$ $B_c$ decays each year.

* Electronic address: xiaozhenjun@njnu.edu.cn
† Electronic address: liuxin.physics@gmail.com
The $B_c$ meson is unique because it is flavor-asymmetric, which is very different from the symmetric heavy quarkonium states, i.e., $c\bar{c}$ and $b\bar{b}$. It is the only weakly decaying doubly heavy flavor meson since the two flavor-asymmetric quarks ($b$ and $c$) cannot annihilate into gluons or photons via strong interactions or electromagnetic interactions, which offers a novel window for studying the heavy quark dynamics that is inaccessible through the investigations on the $b\bar{b}$ and $c\bar{c}$ quarkonia. The $B_c$ meson is expected to decay through the weak interaction and has rich decay channels that could provide an ideal platform to study hadronic weak decays of heavy quark flavor \cite{4} in the SM. The decay processes of the heavy $B_c$ meson can be subdivided into three types as follows \cite{5}:

1. $\bar{b}$ weak decay modes: $\bar{b}\rightarrow (\bar{c}, \bar{u})W^+$, which will result in the final states such as $J/\psi l\bar{\nu}_l$, $J/\psi\pi^+$, etc., as shown in Fig. 1(a);

2. $c$ weak decay modes: $c\rightarrow (s, d)W^+$, which will lead to the final states such as $B_s l\bar{\nu}_l$, $B_s\pi^+$, etc., as shown in Fig. 1(b);

3. pure weak annihilation channels: $\bar{b}c\rightarrow W^+$, which will give the final states such as $B_c\rightarrow l\bar{\nu}_l$, $K^*(0)K^{(*)+}$, etc., as illustrated in Fig. 1(c).

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{fig1.png}
\caption{Typical Feynman diagrams for three types of $B_c$ decays: (a) $\bar{b}$ weak decay modes with $q = c$ or $u$, (b) $c$ weak decay modes with $q = s$ or $d$, and (c) pure weak annihilation channels, respectively.}
\end{figure}

From a theoretical point of view, the weak hadronic decays of $B_c$ meson are extremely complicated due to its heavy-heavy nature and the participation of strong interaction, which complicate the extraction of parameters in the SM, but they also provide great opportunities to study the perturbative and nonperturbative QCD, final state interactions, and heavy quarkonium properties, etc. So far, to our best knowledge, lots of hadronic $B_c$ decays have been studied extensively within various of theoretical approaches/methods in the literature, for example in Refs. \cite{6–71}.

At the quark level, the effective weak Hamiltonian $H_{\text{eff}} \propto \sum C_i(\mu)O_i(\mu)$ is theoretically well under control, where $O_i$ are local four-quark operators and $C_i(\mu)$ are the Wilson coefficients which incorporate strong-interaction effects above the scale $\mu$. However, it is a difficult task to evaluate the hadronic matrix elements of $O_i$ reliably due to the nonperturbative QCD effects involved. Since the $B_c$ meson is heavy, it is possible to describe the dynamics of hadronic decays by theories motivated by QCD. A central aspect of those theories is the factorization theorem which allows one to disentangle the short-distance QCD dynamics from the non-perturbative hadronic effects. During the past decades, theorists have made great efforts on the evaluations of hadronic matrix elements based on the QCD dynamics. So far, the QCD factorization(QCDF) approach \cite{72, 73}, the soft-collinear effective theory(SCET) \cite{74} and the perturbative QCD(pQCD) approach \cite{75–77}, have been developed to make effective evaluations of hadronic matrix elements. Furthermore, up to now, the well-defined pQCD approach \cite{77} has become one of the most popular methods in the market due to its unique features \cite{77}.

In this short review, we give an overview of the experimental measurements and the theoretical understanding of the branching ratios and CP-violating asymmetries of the two body hadronic
TABLE I. The measurements for some hadronic $B_c$ meson decays as reported by LHCb Collaboration [82, 83].

| Measured values of physical observables | Data       |
|-----------------------------------------|------------|
| $\frac{Br(B_c \to J/\psi \pi^+ \pi^-)}{Br(B_c \to J/\psi \pi^+)}$ | $2.41 \pm 0.30 \pm 0.33$ | $0.8 \text{ fb}^{-1}$ |
| $R_c/u \equiv \frac{\sigma(B_c)}{\sigma(B_c)} \frac{Br(B_c \to J/\psi \pi^+)}{Br(B_c \to J/\psi \pi^+)}$ | $0.68 \pm 0.12$ | $0.37 \text{ fb}^{-1}$ |
| $\frac{Br(B_c \to \psi(2S) \pi^+)}{Br(B_c \to \psi(2S) \pi^+)}$ | $0.250 \pm 0.068 \pm 0.014$ | $1.0 \text{ fb}^{-1}$ |
| $\frac{Br(B_c \to J/\psi D^0)}{Br(B_c \to J/\psi D^0)}$ | $2.90 \pm 0.57 \pm 0.24$ | $3 \text{ fb}^{-1}$ |
| $\frac{Br(B_c \to J/\psi D^{*+} \pi^-)}{Br(B_c \to J/\psi D^{*+} \pi^-)}$ | $2.37 \pm 0.56 \pm 0.10$ | $3 \text{ fb}^{-1}$ |
| $f_{\pi\tau} \equiv \frac{Br(B_c \to J/\psi K^+)}{Br(B_c \to J/\psi K^+)}$ | $0.069 \pm 0.019 \pm 0.005$ | $1.0 \text{ fb}^{-1}$ |
| $\frac{\sigma(B_c)}{\sigma(B_c)} \times Br(B_c \to B_c \pi^+)$ | $2.37^{+0.37}_{-0.35} \times 10^{-3}$ | $3 \text{ fb}^{-1}$ |
| $\frac{Br(B_c \to J/\psi K^+ \pi^-)}{Br(B_c \to J/\psi \pi^+)}$ | $0.53 \pm 0.10 \pm 0.05$ | $3 \text{ fb}^{-1}$ |

$B_c \to M_1 M_2$ decays (here $M_i$ denotes various mesons). We begin with a brief summary on current status about the experimental measurements of the hadronic $B_c$ decays. This is followed by an introduction to the theories for the study of hadronic $B_c$ decays, and a discussion on the choice of wave functions for doubly heavy flavor $B_c$ meson and hadrons involved in the final states. Last but not least, we present some recent investigations for the two body hadronic $B_c$ decays by employing the pQCD approach at leading order and leading power. Few of the pQCD predictions for the considered $B_c$ decays have been tested now in the experiments, but some of them will be measured soon in the LHCb experiments. Finally, we make conclusions and a short summary.

2. HADRONIC $B_c$ DECAYS: EXPERIMENTS

Before the running of the LHC at CERN, ever since the $B_c$ meson was discovered by the CDF experiment at the Tevatron [2], only one hadronic decay mode of $B_c$ meson had been observed, $B_c \to J/\psi \pi^+$, which was utilized by CDF and D0 Collaboration [78, 79] to measure the $B_c$ mass. The mass and lifetime of $B_c$ meson as given in Particle Data Group 2012 [80] are the following:

$$m_{B_c} = (6274.5 \pm 1.8) \text{ MeV},$$
$$\tau_{B_c} = (0.452 \pm 0.033) \text{ ps}.$$  \hspace{1cm} (1)

Although the CMS and ATLAS Collaboration reported their observation of some $B_c$ decays, such as $B_c^+ \to J/\Psi \pi^+ \pi^-$ [81], most $B_c$-related measurements have been done by LHCb collaboration. In Table I, we list currently available data for the relative branching ratios of hadronic decays of $B_c$ meson and some other physical observables as reported by LHCb Collaboration in Refs. [82, 83].

In the following years, more and more hadronic decay modes of $B_c$ meson will be measured with good precision in the LHCb experiments. Meanwhile, the theoretical predictions for the hadronic $B_c$ meson decays in various approaches/methods will be greatly required in order to understand the measured results from the LHC experiments.
3. FACTORIZATION APPROACHES IN THE FRAMEWORK OF QCD

In this section, we will introduce the QCD-based factorization approaches/methods that have been adopted for studying the dynamics of hadronic $B_c \rightarrow M_1 M_2$ decays. It is worth of stressing that although charm is a “heavy” quark, the mass around 1.5 GeV makes the studies of $c \rightarrow (d, s)$ decays suffer from rather large long-distance contributions and/or final state interactions, and consequently makes the estimates of the relevant physical observables in $B_c \rightarrow B_q X$ decays with $q = (d, s)$ less trustworthy. In fact, there are no any reliable predictions for the hadronic $B_c \rightarrow B_q X$ decays based on the QCD-motivated factorization framework at present. Therefore, we will not consider the $B_c \rightarrow B_q X$ decay modes in this paper. We here will study the two body hadronic $B_c$ decays arising from the $\bar{b}$ decays or the pure annihilation processes, as shown in Figs. 1(a) and 1(c).

In the effective Hamiltonian approximation, the decay amplitude of the considered hadronic $B_c \rightarrow M_1 M_2$ decays can be written as

$$\mathcal{A}(B_c \rightarrow M_1 M_2) = \langle M_1 M_2 | H_{\text{eff}} | B_c \rangle,$$

where $H_{\text{eff}}$ is the corresponding weak effective Hamiltonian [84]

$$H_{\text{eff}} = \frac{G_F}{\sqrt{2}} \left\{ \sum_{Q=u,c} V_{Qb}^* V_{Qq} [C_1(\mu) O_1^Q(\mu) + C_2(\mu) O_2^Q(\mu)] ight. - \left. V_{tb}^* V_{tq} \sum_{i=3}^{10} C_i(\mu) O_i(\mu) \right\} + \text{H.c.},$$

with $q = (d, s)$, the Fermi constant $G_F = 1.16639 \times 10^{-5}\text{GeV}^{-2}$, CKM matrix elements $V_{ij}$, and Wilson coefficients $C_i(\mu)$ incorporating strong-interaction effects above the scale $\mu$. The local four-quark operators $O_i(i = 1, \cdots, 10)$ include the current-current(tree) operators $O_{1,2}$, the QCD penguin operators $O_{3-6}$ and the electroweak penguin operators $O_{7-10}$ [84].

The key point in the theoretical calculations for the decay amplitude is how to evaluate the hadronic matrix elements of the four-quark operators $< O_i > = \langle M_1 M_2 | O_i | B_c \rangle$ reliably. Presently, there are three popular factorization approaches: the QCDF approach[72], the SCET[74] and the pQCD approach[75, 76]. A detailed discussion for these theories goes beyond the scope of this short review, and the interested reader is referred to the original literatures. Basically, theories of hadronic $B_c$ decays are based on the “factorization theorem” under which the short-distance contributions to the decay amplitudes can be separated from the process-independent long-distance parts.

In the process of calculating the hadronic matrix elements of $B_c$ meson decays, we need to cope well with the physical scales around the so-called factorization scale, namely, $\sqrt{\Lambda_{QCD} m_b}$. Usually, scales below this factorization scale are treated as the nonperturbative physics, which is described by the transition form factors or hadron wave functions. Scales above this factorization scale are categorized as the perturbative physics, which can be evaluated as the expansion of the strong coupling constant $\alpha_s$ with various approaches/methods.

Both of the QCDF approach and the SCET are within the framework of collinear factorization. In the QCDF approach, the endpoint singularity appears at high twist calculations and the annihilation type diagrams. Those annihilation types of diagrams are later proved to be important. The SCET also leave part of the soft contribution in the form factor diagrams as nonperturbative inputs, which make it less predictive, since it requires more free parameters to be determined by experiments [85]. The predictions of the annihilation contributions in SCET are almost real with tiny
FIG. 2. Typical Feynman diagrams for $B_c \rightarrow M_1 M_2$ decays at leading order in the pQCD approach.

strong phase $[86]$, which is rather different from almost imaginary with large strong phase $[87]$ as predicted in the pQCD approach.

The basic idea of the pQCD approach is that it takes into account the transverse momentum $k_T$ of the valence quarks to kill the endpoint divergence in the calculation of the hadronic matrix elements. Therefore, we have one more scale, i.e., the quark transverse momentum than the QCDF approach and the SCET. The $b$-flavor meson transition form factors, and the spectator and annihilation contributions are then all calculable in the framework of the $k_T$ factorization, where three energy scales are involved $[75, 76, 88]$. The hard dynamics is characterized by $\sqrt{m_b \Lambda_{\text{QCD}}}$, which is to be perturbatively calculated. The harder dynamics is from $m_W$ scale to $m_B$ scale described by renormalization group equation for the four quark operators. The dynamics below $\sqrt{m_b \Lambda_{\text{QCD}}}$ is soft, which is described by the meson wave functions. The soft dynamics is not perturbative but universal for all channels. In the pQCD approach, a $B_c \rightarrow M_1 M_2$ decay amplitude is therefore factorized into the convolution of the six-quark hard kernel ($H$), the jet function ($J$) and the Sudakov factor ($S$) with the bound-state wave functions ($\Phi$) as follows,

$$A(B_c \rightarrow M_1 M_2) = \Phi_{B_c} \otimes H \otimes J \otimes S \otimes \Phi_{M_1} \otimes \Phi_{M_2}, \quad (4)$$

All nonperturbative components are organized in the form of hadron wave functions $\Phi$, which may be extracted from experimental data or other nonperturbative method, such as QCD sum rules $[89]$. Since nonperturbative dynamics has been factored out, one can evaluate all possible Feynman diagrams presented in Fig. 2 for the six-quark amplitude $H$ straightforwardly. The jet function $J$ comes from the threshold resummation, which exhibits suppression in the small $x$ (quark momentum fraction) region $[90]$. The Sudakov factor $S$ comes from the $k_T$ resummation $[91, 92]$, which exhibits suppression in the small $k_T$ region. Therefore, these resummation effects guarantee the removal of the endpoint singularities and the reliability of the pQCD approach.

4. RELEVANT HADRON WAVE FUNCTIONS

In order to calculate the analytic formulas of the decay amplitudes, we need the light cone wave functions decomposed in terms of the spin structure. In general, the light cone wave functions are decomposed into 16 independent components, $1_{\alpha\beta}$, $\gamma_{\alpha\beta}^\mu$, $\sigma_{\alpha\beta}^{\mu\nu}$, $(\gamma^\mu \gamma_5)_{\alpha\beta}$, and $\gamma_5 \delta_{\alpha\beta}$. Relative to the more heavier $b$ quark, charm can be viewed approximately as a light quark in the doubly heavy
flavor $B_c$ meson. In the leading order of $m_c/m_{B_c} \sim 0.2$ expansion, the factorization theorem is applicable to the $B_c$ system similar to the situation of $B$ meson. In analogy to the definition of the $B$ meson [75, 76], the light-cone wave function of $B_c$ meson can be defined as

$$\Phi_{B_c, \alpha\beta, ij} \equiv \langle 0 | \bar{b}_j(0) c_{\alpha i}(z) | B_c(P) \rangle$$

$$= \frac{i \delta_{ij}}{\sqrt{2 N_c}} \int dx d^2 k_T e^{-i(x P^z + k_T z)}$$

$$\times \left\{ (P + m_{B_c}) \gamma_5 \phi_{B_c}(x, k_T) \right\}_{\alpha \beta};$$

(5)

where the indices $i, j$ and $\alpha, \beta$ are the Lorentz indices and color indices, respectively, $P(m)$ is the momentum (mass) of the $B_c$ meson, $N_c$ is the color factor, and $k_T$ is the intrinsic transverse momentum of the lighter quark in $B_c$ meson. Note that, in principle, there are two Lorentz structures of the wave function to be considered in the numerical calculations, however, the contribution induced by the second Lorentz structure is numerically small and approximately negligible.

In Eq. (5), $\phi_{B_c}(x, k_T)$ is the $B_c$ meson distribution amplitude and obeys to the normalization condition: $\int_0^1 dx \, \phi_{B_c}(x, b = 0) = f_{B_c}/(2\sqrt{2N_c})$, here $b$ is the conjugate space coordinate of transverse momentum $k_T$ and $f_{B_c}$ is the decay constant of $B_c$ meson.

To our best knowledge, however, $\phi_{B_c}$ with $k_T$ is still absent now. But, the situation may become somewhat simpler, if $B_c$ meson can be approximated as a non-relativistic bound state of two sufficiently heavy quarks. In this sense we expect exclusive matrix elements, in particular, the light-cone distribution amplitude to be calculable perturbatively, since the quark masses provide an intrinsic physical infrared regulator. At the nonrelativistic scale, the leading 2-particle distribution amplitude can be approximated by delta function, fixing the light-cone momenta of the quarks according to their masses [93]. Since $B_c$ meson consists of two heavy quarks and $m_{B_c} \simeq m_b + m_c$, the distribution amplitude $\phi_{B_c}$ would thus be close to $\delta(x - m_c/m_{B_c})$ in the non-relativistic limit. We therefore adopt the non-relativistic approximation form of $\phi_{B_c}$ as [93]

$$\phi_{B_c}(x) = \frac{f_{B_c}}{2\sqrt{2N_c}} \delta(x - m_c/m_{B_c}),$$

(6)

where the value of $f_{B_c}$ is taken from the calculations in quenched lattice QCD with exact chiral symmetry [94].

For the final state wave functions, such as pseudoscalar, vector, scalar, axial-vector, even tensor mesons, we refer the readers to the papers dealing with various decay channels, for example, in Refs. [50, 51, 61]. But, we should stress here that the $k_T$ dependence of the distribution amplitudes in the final states has been neglected, since its contribution is very small as indicated in the reference [88]. The underlying reason is that the contribution from $k_T$ correlated with a soft dynamics is strongly suppressed by the Sudakov effect through resummation for the wave function, which is dominated by a collinear dynamics.

5. $B_c \to M_1 M_2$ DECAYS

In this section, we will summarize current status of the theoretical studies for the hadronic $B_c \to M_1 M_2$ decays by employing the pQCD factorization approach. In order to make numerical evaluations one need the input parameters, such as the relevant masses, decay constants and lifetimes etc. Since some parameters change from time to time, one needs to consult the original paper for specific input parameters for each predictions as given in different works.
5.1 $B_c$ decays through $b \to (c, u)$ transitions

Up to now, the decays $B_c \to D(\pi, K)$ [41, 42], $B_c \to (J/\psi, \eta_c)(\pi, K)$ [40, 52], $B_c \to D(\psi)(P, V, T)$ [60, 61] and $B_c \to (D^{(*)}, D_s^{(*)})D^{(*)}, D_s^{(*)}$ [62] have been studied in the pQCD approach, and the CP-averaged branching ratios and CP-violating asymmetries for these decay modes have been calculated. Since the charm quark in the heavy final state mesons (for example, $J/\psi, \eta_c, D^{(*)}$ and $D_s^{(*)}$) is almost at collinear state, a hard gluon is needed to transfer large momentum to the spectator charm quark. Utilizing the $k_T$ factorization instead of collinear factorization, the pQCD approach is free of endpoint singularity. Thus Feynman diagrams as illustrated in Fig.2 are all contributing and calculable.

In $B_c$ decays, there is one more intermediate energy scale, the heavy charm mass. As a result, another expansion series of $m_c/m_B$ will appear. The factorization is approved at the leading order of $m_c/m_B$ expansion [95–97]. The nonleptonic $B_c$ to $J/\psi(\eta_c)$ and a light hadron decays are similar to that of $B$ decaying into $D$ and a light meson [40].

The two body hadronic $B_c \to (J/\psi, \eta_c)(\pi, K)$ decays are tree dominated modes including factorizable emission and nonfactorizable spectator amplitudes. Furthermore, the branching ratios for the considered decays are determined by the contributions arising from the factorizable topologies. The decay amplitudes for $B_c \to (J/\psi, \eta_c)(\pi, K)$ can thus be described approximately as

$$A(B_c \to J/\psi(\eta_c) \pi) = V_{cb}^* V_{ud} \cdot f_{\pi}$$

$$\langle J/\psi(\eta_c)|V - A|B_c \rangle , \quad (7)$$

$$A(B_c \to J/\psi(\eta_c) K) = V_{cb}^* V_{us} \cdot f_{K}$$

$$\langle J/\psi(\eta_c)|V - A|B_c \rangle ; \quad (8)$$

in which the former mode is CKM favored, while the latter channel is CKM suppressed, and $f_{\pi}$ and $f_{K}$ are the decay constants of pion and kaon respectively. Then the interesting relation of branching ratios of the considered four decay modes in the limit of SU(3) flavor symmetry can be read as

$$R_{J/\psi}^{K/\pi} = R_{\eta_c}^{K/\pi} \sim \frac{|V_{us}|^2}{|V_{ud}|^2} \cdot \frac{|f_K|^2}{|f_{\pi}|^2} ; \quad (9)$$

where $R_{J/\psi}^{K/\pi}$ and $R_{\eta_c}^{K/\pi}$ are defined as

$$R_{J/\psi}^{K/\pi} = \frac{Br(B_c \to J/\psi K)}{Br(B_c \to J/\psi \pi)} ,$$

$$R_{\eta_c}^{K/\pi} = \frac{Br(B_c \to \eta_c K)}{Br(B_c \to \eta_c \pi)} ; \quad (10)$$

With the input parameters [80]: $V_{ud} = 0.97427$, $V_{us} = 0.22534$, $f_{K} = 0.16$ GeV, and $f_{\pi} = 0.13$ GeV, the expected ratios $R_{J/\psi}^{K/\pi} = R_{\eta_c}^{K/\pi} \approx 0.08$. From Ref. [52], we found that

$$Br(B_c \to J/\psi \pi)_{pQCD} = (1.35 \times 2.54) \times 10^{-3} ,$$

$$Br(B_c \to J/\psi K)_{pQCD} = (1 \times 3) \times 10^{-4} , \quad (11)$$

which lead to the ratio $R_{J/\psi}^{K/\pi} = (0.07 \sim 0.12)$ in the pQCD approach, which is consistent with the naive expectation on the above ratio. Very recently, the LHCb Collaboration has measured the
ratio of the branching ratios between $B_c \rightarrow J/\psi K$ and $B_c \rightarrow J/\psi \pi$ decays, and obtained the result [83],
\[ R_{J/\psi}^{K/\pi} = 0.069 \pm 0.020, \] (12)
which is in good agreement with the theoretical prediction in the pQCD approach.

From the pQCD prediction of $Br(B_c \rightarrow \eta_c \pi)_{\text{pQCD}} = (1.47 \sim 2.79) \times 10^{-3}$ [40], it is expected that the CP-averaged branching ratio of $B_c \rightarrow \eta_c K$ mode may be $Br(B_c \rightarrow \eta_c K)_{\text{pQCD}} = (1 \sim 3) \times 10^{-4}$, which will be tested by the forthcoming experiments. Of course, since no penguin operators are involved in these considered four channels, the direct CP asymmetries are absent here naturally.

The two body hadronic $B_c$ meson decaying into double charm hadrons [62], which are the pure tree decay modes, can be utilized particularly to extract the CKM angles because of the absence of the interference from the penguin operators. Furthermore, the decays $B_c \rightarrow D_s^+ D^0$ and $D_s^+ \bar{D}^0$ are the gold-plated modes for the extraction of CKM angle $\gamma$ through amplitude relations because their decay widths are expected to be at the same order in magnitude [8, 19, 23, 27, 30, 34].

From the numerical calculations one found that the ratio of the decay widths for $B_c \rightarrow D_s^+ D^0$ and $B_c \rightarrow D_s^+ \bar{D}^0$ is about 1.3, which indicate that the branching ratios for these two decays are really as was expected and they are indeed suitable for extracting the CKM angle $\gamma$. The theoretical predictions as given in Ref. [62] confirmed that the nonfactorizable spectator diagrams provided a remarkable contribution in the double charm decays of $B_c$ meson. The predicted branching ratios for the considered decay channels vary in the range of $10^{-8} \sim 10^{-5}$. The considered $B_c$ decays with a decay rate at the level of $10^{-6}$ or larger can be detected with a good precision at LHC experiments [44]. Meanwhile, the transverse polarization fractions of the $B_c$ meson decays with two vector $D^*$ mesons are predicted for the first time in the pQCD approach. The transverse polarization fractions are large in some channels, which mainly come from the nonfactorizable spectator diagrams.

For the nonleptonic $B_c \rightarrow D^{(*)}_{(s)}(P, V)$ decays, their decay rates and CP-violating asymmetries, as well as the transverse polarization fractions for $B_c \rightarrow D^*_{(s)} V$ channels are calculated systematically in the pQCD approach in Ref. [60]. From the numerical calculations, one finds that the pQCD predictions for the CP-averaged branching ratios of the tree-dominant $B_c \rightarrow D^{(*)}_{(s)}(P, V)$ decays are in good agreement with that in the relativistic constituent quark model [15].

Furthermore, it is found that the nonfactorizable spectator diagrams and annihilation diagrams have remarkable effects on the physical observables in many channels, especially in the color-suppressed and annihilation-dominant decay modes. As expected, the annihilation diagrams give large contributions in the $B_c$ meson decays, because the contributions arising from annihilation diagrams are enhanced by the CKM factor $V_{cb}^* V_{cq}$. For the $b \rightarrow s$ transition process, the ratio $|V_{cb}^* V_{cs}/V_{ub}^* V_{us}| \approx 47$, which therefore results in the ratio $Br(B_c \rightarrow D^{(*)}_s K^{(*)+})/Br(B_c \rightarrow D^{(*)}_s K^{(*)0}) \approx 1$ for the considered two kinds of annihilation-dominant modes.

For $B_c \rightarrow D^*_{(s)} V$ decays, furthermore, the transverse polarization contributions are usually suppressed by the factor $r_V$ or $r_{D^{(*)}_s}$ when compared with the longitudinal part. Thus, for tree-dominant $B_c \rightarrow D^{(*)}_s \rho^+$ and pure penguin type $B_c \rightarrow D^{(*)}_s \phi$ decays, one can find the relatively small transverse polarization fractions 16.4% and 11.5%, respectively.

For other $B_c \rightarrow D^*_{(s)} V$ decays, the annihilation contributions dominate the branching ratios due to the large Wilson coefficients. Therefore, the transverse polarization contributions take a larger ratio in the branching ratios, which can reach 50% $\sim 70\%$. Because of the different weak phase and strong phase from tree diagrams, penguin diagrams, and annihilation diagrams, the possibly
large direct CP violation in some channels are predicted in the pQCD approach, for example,

\[ A_{\text{dir}}^{B_c \to D^+ \rho^0} \approx 79.8\%, \]
\[ A_{\text{dir}}^{(D^0 K^{*+})} \approx -66.2\%, \text{ etc.} \] (13)

For the hadronic \( B_c \to D^{(*)} T \) decays [61], which is slightly special compared with the above \( B_c \to D_s^{(*)}(P,V) \) decays, there are no contributions from factorizable emission diagrams because the emitted tensor meson cannot be generated from the (axial-)vector current or (pseudo-)scalar density. Thus, these \( B_c \to D^{(*)} T \) decays are forbidden in the naive factorization. One should go beyond the naive factorization to calculate the nonfactorizable spectator and annihilation diagrams. What’s more, the annihilation amplitudes are dominant in these considered \( B_c \to D^{(*)} T \) decays because they are proportional to the large CKM matrix elements \( V_{cb} \) and \( V_{cd(s)} \).

The predictions in Ref. [61] show that the CP-averaged branching ratios for hadronic \( B_c \to D^{(*)} T \) modes are in the range of \( 10^{-4} \sim 10^{-9} \). As stated in Ref. [82, 83], the LHC experiments, specifically the LHCb experiment, can produce around \( 5 \times 10^{10} B_c \) events per year. The \( B_c \) decays with a decay rate at the level of \( 10^{-6} \) can be detected with a good precision at LHC experiments. Therefore, it is of great interests that the \( B_c \) meson decays to tensor final states with branching ratios as large as \( 10^{-4} \), for example, \( B_c \to D^* K_2^*(1430) \) and \( B_c \to D_s^{*+} f_2^*(1525) \), will be easier for experiments to search than the corresponding decays with vector mesons. The modes with large branching ratios such as \( B_c \to D^0 K_2^*(1430)^+, D^+ K_2^*_0(1430) \), \( D_s^{*+} f_2^*(1525) \), \( D^* K_2^*(1430) \), and \( D_s^{*+} f_2^*(1525) \), would provide opportunities to study the properties of \( B_c \) meson and the factorization theorem in the decays with an emitted tensor meson.

Most of the direct CP asymmetries for \( B_c \to D^{(*)} T \) decays predicted in the pQCD approach are very small because the penguin contributions are too small compared with the tree annihilation contributions. The largest direct CP violation for \( B_c \to D^{(*)} T \) decays estimated with the pQCD approach is 18.2%, which belongs to the channel \( B_c \to D_s^+ a_2(1320)^0 \).

The predicted transverse polarization fractions for most annihilation-dominant \( B_c \to D^{(*)} T \) channels in the pQCD approach are larger than 50%, except for two modes \( B_c \to D_s^{*+} f_2^*(1525) \) with \( R_T \sim 45.3\% \) and \( B_c \to D_s^{*+} a_2(1320)^0 \) with \( R_T \sim 12.7\% \) [61]. Moreover, it is very interesting to note that the longitudinal polarization contributions in \( B_c \to D_s^{*+} f_2^*(1270) \) only about 1.6%.

It is worth of mentioning that the semileptonic charged decays \( B_c^+ \to D_s^{(*)}(l^+\nu, l^+l^-, \nu\bar{\nu}) \) have been studied in the pQCD approach [63]. In Ref. [63], we studied the semileptonic decays of \( B_c^+ \to D_s^{(*)}(l^+\nu, l^+l^-, \nu\bar{\nu}) \) (here \( l \) stands for \( e, \mu \) or \( \tau \)) by using the relevant form factors \( F_{0,+,T}(q^2) \), \( V(q^2) \), \( A_{0,1,2}(q^2) \) and \( T_{1,2,3}(q^2) \) for the \( B_c^+ \to (D_s^{(*)}, D_s^{(*)}) \) transitions obtained by employing the pQCD factorization approach. We calculated the decays rates for all considered semileptonic decays and found numerically that (a) the relevant transition form factors obtained in this work agree well with those from other methods; (b) the size of the pQCD predictions for the branching ratios for the decays with \( b \to s \) or \( b \to d \) transitions show clearly the effects of the CKM suppression; and (c) the pQCD predictions for the ratios of the decay rates are \( R_D \approx 0.7 \) and \( R_{D^*} \approx 0.6 \), which could be measured at LHCb soon.

\[ 5.2 \text{ Charmless hadronic } B_c \to PP, PV, VP, VV \text{ decays} \]

The charmless hadronic \( B_c \to M_1 M_2 \) decays (i.e. \( M_i \) are the charmless light mesons) can occur only via the weak annihilation diagrams in the SM. As discussed in Sec. 3, up to now, the
annihilation diagrams can be well treated only by employing the pQCD approach due to its unique features.

Although there is a different viewpoint on the evaluations of annihilation contributions proposed in the SCET, the previous predictions on the annihilation contributions in heavy flavor $B$ meson decays calculated with the pQCD approach have already been tested at various aspects, for example, branching ratios of pure annihilation $B_d \rightarrow D_s^+ K^+$, $B_d \rightarrow K^+ K^-$, and $B_s \rightarrow \pi^+ \pi^-$ decays [98–101], direct CP asymmetries of $B^0 \rightarrow \pi^+ \pi^-$, $K^+ \pi^-$ decays [75, 76, 102], and the explanation of $B \rightarrow \phi K^*$ polarization problem [103, 104], which indicate that the pQCD approach is a reliable method to deal with the annihilation diagrams.

By using the pQCD approach, the pure annihilation type of charmless hadronic $B_c \rightarrow M_1 M_2$ decays, about 200 decay modes, 

\[ B_c \rightarrow PP, \; PV, \; VV, \; AP, \; AV, \; AA, \; SP, \; SV; \]

(14) have been studied systematically in Refs. [50, 51, 53, 54, 58, 59], where the term $S$, $P$, $V$ and $A$ refers to the scalar, pseudoscalar, vector and axial-vector charmless mesons respectively. Other possible charmless $B_c \rightarrow M_1 M_2$ decays through pure annihilation topology to light mesons with $M_i M_j = SS, \; SA, \; TP, \; TV, \; TS, \; TA$ and even $TT$ are under study now by using the pQCD approach [105].

For the twenty three charmless $B_c \rightarrow PP, \; PV/VP$ decays, the decay rate can be written as

\[ \Gamma = \frac{G_F^2 m_{B_c}^3}{32\pi} |A(B_c \rightarrow M_1 M_2)|^2 \]  

(15) Using the decay amplitudes as given in Eqs.(20)-(27) and (32)-(46) in Ref. [50], it is straightforward to calculate the branching ratios with uncertainties as listed in Table II.

For $B_c \rightarrow VV$ decays, the decay rate can be written explicitly as,

\[ \Gamma = \frac{G_F^2 P_c}{16\pi m_{B_c}^2} \sum_{\sigma=L,T} \mathcal{M}^{(\sigma)\dagger} \mathcal{M}^{(\sigma)} \]  

(16) where $P_c \equiv |P_{z,2}| = |P_{3z}|$ is the momentum of either of the outgoing vector mesons. Based on the helicity amplitudes as defined in Eq. (48) of Ref. [50], we can define the transverse amplitudes,

\[ A_L = -\xi m_{B_c}^2 \mathcal{M}_L, \quad A_{\|} = \xi \sqrt{2} m_{B_c}^2 \mathcal{M}_N, \]

\[ A_{\perp} = \xi m_{B_c}^2 \sqrt{2(r^2 - 1)} \mathcal{M}_T \]  

(17) for the longitudinal, parallel, and perpendicular polarizations, respectively, with the normalization factor $\xi = \sqrt{G_F^2 P_c / (16\pi m_{B_c}^2 \Gamma)}$ and the ratio $r = P_2 \cdot P_3 / (m_{M_1} \cdot m_{M_2})$. These amplitudes satisfy the relation,

\[ |A_L|^2 + |A_{\|}|^2 + |A_{\perp}|^2 = 1 \]  

(18) following the summation in Eq. (16).

Since the transverse-helicity contributions manifest themselves in polarization observables, we here define two kinds of polarization observables, i.e., polarization fractions $(f_L, f_{\|}, f_{\perp})$ and relative phases $(\phi_{\|}, \phi_{\perp})$ as

\[ f_L(\|, \perp) = \frac{|A_L(\|, \perp)|^2}{|A_L|^2 + |A_{\|}|^2 + |A_{\perp}|^2}, \]

(19) \[ \phi_{\|}(\perp) \equiv \arg \frac{A_{\|}(\perp)}{A_L}; \]

(20)
TABLE II. The pQCD predictions of branching ratios for eight $B_c \rightarrow PP$ modes and fifteen $B_c \rightarrow (PV, VP)$ decays. The dominant errors come from charm quark mass $m_c = 1.5 \pm 0.15$ GeV, combined Gegenbauer moments $a_i$, and chiral enhancement factors $m_0^6 = 1.4 \pm 0.3$ GeV and $m_0^K = 1.6 \pm 0.1$ GeV, respectively.

| Decay Modes ($\Delta S = 0$) | $BR'(s)(10^{-8})$ | Decay Modes ($\Delta S = 1$) | $BR'(s)(10^{-8})$ |
|-----------------------------|--------------------|-----------------------------|--------------------|
| $B_c \rightarrow \pi^+\pi^0$ | 0                  | $B_c \rightarrow \pi^+K^0$  | $4.0^{+1.0}_{-0.6}(m_c)^{+2.3}_{-1.6}(a_i)^{+0.5}_{-0.3}(m_0)$ |
| $B_c \rightarrow \pi^+\eta$ | $22.8^{+6.9}_{-4.6}(m_c)^{+7.2}_{-4.5}(a_i)^{+3.4}_{-2.2}(m_0)$ | $B_c \rightarrow K^+\eta$ | $0.6^{+0.0}_{-0.0}(m_c)^{+0.6}_{-0.5}(a_i)^{+0.2}_{-0.1}(m_0)$ |
| $B_c \rightarrow \pi^+\eta'$ | $15.3^{+4.6}_{-3.3}(m_c)^{+4.8}_{-3.0}(a_i)^{+2.2}_{-1.8}(m_0)$ | $B_c \rightarrow K^+\eta'$ | $5.7^{+0.9}_{-0.9}(m_c)^{+1.0}_{-1.0}(a_i)^{+0.0}_{-0.0}(m_0)$ |
| $B_c \rightarrow K^0\bar{K}^0$ | $24.0^{+2.4}_{-2.0}(m_c)^{+7.3}_{-6.0}(a_i)^{+6.8}_{-5.8}(m_0)$ | $B_c \rightarrow K^+\pi^0$ | $2.0^{+0.5}_{-0.3}(m_c)^{+1.2}_{-0.8}(a_i)^{+0.3}_{-0.3}(m_0)$ |

It should be noted that the final results of relative phases will plus one value, i.e., $\pi$, due to an additional minus sign in the definition of $A_2$.

In Table III, we present the pQCD predictions for CP-averaged branching ratios, the longitudinal polarization fractions ($f_L$) and relative phases of the considered nine $B_c \rightarrow VV$ decays. The dominant theoretical errors comes from the uncertainties of the charm quark mass $m_c = 1.5 \pm 0.15$ GeV, and the Gegenbauer moments $a_i$ of related meson distribution amplitudes, respectively. The total error is the combination of individual errors in quadrature.

TABLE III. The pQCD predictions of branching ratios (BRs), $f_L$, and the relative phases $\phi_\parallel$ and $\phi_\perp$ for $B_c \rightarrow VV$ decays.

| Decay Modes | BRs($10^{-7}$) | $f_L(\%)$ | $\phi_\parallel$ (rad) | $\phi_\perp$ (rad) |
|-------------|----------------|-----------|------------------------|------------------------|
| $B_c \rightarrow \rho^+\rho^0$ | 0 | 10.6^{+3.8}_{-3.0} | 92.9^{+2.0}_{-0.1} | 3.86^{+0.40}_{-0.32} | 4.43^{+0.30}_{-0.25} |
| $B_c \rightarrow \rho^+\omega$ | 10.0^{+8.1}_{-4.8} | 92.0^{+3.6}_{-1.5} | 4.11^{+0.28}_{-0.28} | 4.20^{+0.33}_{-0.22} |
| $B_c \rightarrow K^{*0}K^{*+}$ | 0.6^{+0.2}_{-0.1} | 94.9^{+2.2}_{-1.5} | 4.11^{+0.28}_{-0.28} | 4.20^{+0.33}_{-0.22} |
| $B_c \rightarrow K^{*+}\rho^0$ | 0.3^{+0.1}_{-0.0} | 94.9^{+1.4}_{-1.5} | 4.11^{+0.34}_{-0.28} | 4.20^{+0.33}_{-0.22} |
| $B_c \rightarrow K^{*+}\omega$ | 0.3^{+0.0}_{-0.0} | 94.8^{+1.2}_{-1.3} | 4.15^{+0.28}_{-0.35} | 4.23^{+0.33}_{-0.26} |
| $B_c \rightarrow \phi K^+$ | 0.5^{+0.1}_{-0.0} | 86.4^{+4.9}_{-9.1} | 3.80^{+0.51}_{-0.39} | 3.89^{+0.48}_{-0.28} |

From the numerical results in Table II, one can see that...
1 Analogous to $B \to K\eta^{(*)}$ decays, the branching ratios of $B_c \to K\eta^{(*)}$ modes also show an approximate relation:

$$Br(B_c \to K^+\eta') \sim 10 \times Br(B_c \to K^+\eta).$$

(21)

This large difference can be understood by the destructive and constructive interference between the $\eta_q$ and $\eta_s$ contributions to the $B_c \to K^+\eta$ and $B_c \to K^+\eta'$ decay.

2 The pQCD predictions for the branching ratios vary in the range of $10^{-6} \sim 10^{-8}$ [50, 53], basically agree with the predictions obtained by using the exact SU(3) flavor symmetry. The $B_c \to \bar{K}^0 K^+$ and other decays with a decay rate at $10^{-6}$ or larger could be measured at the LHC experiment.

3 For $B_c \to (\pi^+, \rho^+)(\eta, \eta')$ decays, the final state mesons $\eta$ and $\eta'$ contain the same component $\bar{u}u + \bar{d}d$, and the differences among their branching ratios mainly come from the mixing coefficients, i.e., $\cos \phi$ and $\sin \phi$.

4 Among the thirty $B_c$ decays considered in this subsection, $B_c \to \rho^+ \omega$, $\bar{K}^0 K^{*+}$ and $B_c \to \bar{K}^0 K^{*+}$ have the largest branching ratios and are in the order of $10^{-6}$. This means that the annihilation contributions to $B_c$ meson decays may be rather important. We suggest the LHCb experiment to search for such decay modes.

5 For $B_c \to VV$ decays, the contributions coming from the longitudinal polarization play the dominant role and the longitudinal polarization fractions $f_L \approx 95\%$, except for $B_c \to \phi K^{*+}$ ($f_L \sim 86\%$). Unfortunately, it is very hard to measure these decays, due to the smallness of their decay rates: in the range of $10^{-8} - 10^{-7}$.

5.3 Charmless hadronic $B_c \to SP, SV, AP, AV, AA$ decays

For charmless $B_c \to SP$ and $SV$ decays, the pQCD predictions for the branching ratios are in the range of $10^{-5}$ to $10^{-8}$ [54]. Many decays with a decay rate at $10^{-6}$ or larger could be measured at the LHC experiment. Similar to $B \to K^*\eta^{(*)}$ decays, the branching ratios of $B_c \to \kappa\eta^{(*)}$ channels also exhibit the interesting relation:

$$Br(B_c \to \kappa^+\eta) \sim 5 \times Br(B_c \to \kappa^+\eta').$$

(22)

This difference can be understood by the destructive and constructive interference between the $\eta_d$ and $\eta_s$ contributions to the $B_c \to \kappa^+\eta'$ and $B_c \to \kappa^+\eta$ decay, respectively.

For $B_c \to K_0^*(1430)\eta$ and $B_c \to K_0^*(1430)\eta'$ decays, the pQCD predictions for their branching ratios [54] are similar in size, since the factorizable contributions of $\eta_s$ term play the dominant role. This feature will be tested by the near future experiments. If $a_0$ and $\kappa$ are the $q\bar{q}$ bound states, the pQCD predictions for $Br(B_c \to a_0(\pi, \rho))$ and $Br(B_c \to \kappa K^{(*)})$ will be in the range of $10^{-6} \sim 10^{-5}$, which are within the reach of the LHCb experiments and expected to be measured.

For the $a_0(1450)$ and $K_0^*(1430)$ channels, the branching ratios for $B_c \to a_0(1450)(\pi, \rho)$ and $B_c \to K_0^*(1430)K^{(*)}$ modes in the pQCD approach are found to be of order $(5 \sim 47) \times 10^{-6}$ and $(0.7 \sim 36) \times 10^{-6}$ respectively [54]. A measurement of them at the predicted level will favor the structure of $q\bar{q}$ for the $a_0(1450)$ and $K_0^*(1430)$ and identify which scenario is preferred.
For the pure annihilation $B_c \rightarrow AP, AV, AA$ decays, such as

\[
B_c \rightarrow K^0(K_1(1270)^+, K_1(1400)^+), \quad a_1(1260)^+ \omega, \\
b_1(1235)\rho, \quad (K_1(1270), K_1(1400))K^*, \\
\rho^+ f_1(1285), \quad a_1(1260)b_1(1235), \\
(\bar{K}_1(1270)^0, \bar{K}_1(1400)^0)(\bar{K}_1(1270)^+, K_1(1400)^+), \\
\text{etc.}
\] (23)

the pQCD predictions for their branching ratios are in the range of $10^{-5}$ to $10^{-9}$ [51, 58, 59]. The decay modes with a sizable decay rate at $10^{-6}$ or larger could be measured at the LHC experiments.

Since the QCD behavior of the $^1P_1$ meson is rather different from that of the $^3P_1$ meson, the branching ratios in the pQCD approach of pure annihilation $B_c \rightarrow A(^1P_1) (P, V, A(^1P_1))$ are basically larger than that of $B_c \rightarrow A(^3P_1) (P, V, A(^3P_1))$ with a factor around $10 \sim 100$ [51, 58, 59], for example,

\[
10 \times Br(B_c \rightarrow a_1(1260)(\pi, \rho)) \\
\approx Br(B_c \rightarrow b_1(1235)(\pi, \rho)), \\
100 \times Br(B_c \rightarrow a_1(1260)^+ f_1(1285)) \\
\approx Br(B_c \rightarrow b_1(1235)^+ h_1(1170)).
\] (24)

These relations can be tested by the LHC experiments.

The pQCD predictions [51, 59] about the branching ratios of some $B_c$ decays, such as

\[
B_c \rightarrow K_1(1270)^+ \eta^{(l)}, K_1(1400)^+ \eta^{(l)}, \\
K_1(1270)K, K_1(1400)K, \bar{K}_1(1270)^0 K_1(1270)^+, \\
\bar{K}_1(1400)^0 K_1(1270)^+, \bar{K}_1(1270)^0 K_1(1400)^+, \\
\bar{K}_1(1400)^0 K_1(1400)^+,
\]

are rather sensitive to the value of the mixing angle $\theta_K$, which will be tested by the running LHC experiments. One can determine $\theta_K$ through the measurement of these decays if enough $B_c$ events become available at the LHC experiments. The pQCD predictions for several decays involving the mixtures of $^3P_1$ and/or $^1P_1$ mesons are rather sensitive to the values of the mixing angles, which can provide the important information on both of sign and size of the mixing angles if they are detected in the future experiments. For $B_c \rightarrow VV, AV(VA), AA$ decays [58, 59] the longitudinal contributions play a dominant role in most of those considered modes, which will be tested by the ongoing LHC experiments in the near future.

Once the above predictions on the physical quantities in the pQCD approach can be confirmed at the predicted level by the precision experimental measurements in the future, which can also provide indirect evidence for the important but controversial issues (See Refs. [86, 87] for detail) on the evaluation of annihilation contributions at leading power, one can ask whether it is almost real with a tiny strong phase in SCET or almost imaginary with a large strong phase in the pQCD approach.

For all the considered charmless hadronic $B_c$ decays, the branching ratios of $\Delta S = 0$ processes are basically larger than those of $\Delta S = 1$ ones. Such differences are mainly induced by the CKM factors involved: $|V_{ud}| \sim 1$ for the former decays while $|V_{us}| \approx \lambda \sim 0.22$ for the latter ones. Because only tree operators are involved, the $CP$-violating asymmetries for these considered $B_c$ decays are absent naturally.
It should be stressed that the pQCD predictions still have large theoretical uncertainties, mainly induced by the errors of the Gegenbauer moments in the hadron distribution amplitudes and the errors of decay constants $f_{B_c}$ and $f_{M_i}$. By reducing these uncertainties, one can improve the precision of the theoretical predictions effectively. Moreover, only the short-distance contributions in the aforementioned hadronic $B_c$ decays are considered and perturbatively calculated by employing the pQCD approach.

The possible long-distance contributions to $B_c$ hadronic decays, such as the rescattering effects, have been neglected in our calculations since such contributions should be small based on the general expectations: the perturbative contribution most possibly dominate the heavy $B_c$ meson decay. One of the effective methods to decrease the theoretical error is to define the ratios between branching ratios for suitable decay modes, such as those connected through various $SU(3)$ flavor symmetries.

It is believed that all hadronic $B_c$ meson decays will provide important platform for studying the mechanism of annihilation contributions, understanding the helicity structure of the considered channels with vector and/or axial-vector meson(s) and the content of the involved light scalar and axial-vector mesons.

6. SUMMARY AND EXPECTATIONS

In the following years, the LHC experiments will collect more and more $B_c$ production and decay events. The analysis of the huge number of $B_c$ events do require precision theoretical predictions. On the other hand, the properties of $B_c$ meson and many other different kinds of light or heavy mesons, such as the relevant decay constants, the internal structure and the mixing angles, etc., will be measured in the LHCb, CMS and ATLAS experiments.

In this short review, we firstly summarize the recent progress of hadronic $B_c$ decays at both experimental and theoretical aspects. As aforementioned in Sec. 2, some hadronic $B_c$ decay channels have been detected by the LHCb experiment in the past four years. The $B_c$ decays with a decay rate at the level of $10^{-6}$ can be detected with a good precision at LHC experiments.

We then provide an outline about theoretical studies of hadronic $B_c \to M_1M_2$ decays with $M_i = (S,P,V,A,T)$ in the framework of the pQCD approach, at the leading order and leading power. Up to now, about four hundred such decay modes have been studied in the pQCD approach, some most important results are discussed here explicitly. For more details of specific decays, one can see the original paper cited here.

For those considered $B_c$ decays, besides the emission diagrams, the nonfactorizable spectator diagrams and the annihilation diagrams can also be evaluated in the pQCD approach. Furthermore, phenomenologically, it is found that the dominant contributions to the branching ratios in many decay channels arise from the nonfactorizable spectator and/or annihilation amplitudes. Such decay channels can be classified into two sets: (a) The $B_c$ decays with an emitted scalar or tensor meson, which cannot be produced from the vacuum by the (axial-)vector current or (pseudo-)scalar density in the SM; and (b) the charmless hadronic $B_c$ meson decays, which can only occur through the pure annihilation topology in the SM.

For the hadronic $B_c \to M_1M_2$ decays considered in this short review, the CP-averaged branching ratios vary in the range of $10^{-3} \sim 10^{-9}$. Those decays with a decay rate at or larger than $10^{-6}$ can be measured in the near future LHC experiments.

For the considered annihilation dominant modes or pure annihilation channels, the confirmation at the pQCD predictions through the precision experimental measurements will provide important information to the controversial issues on how effectively and accurately to evaluate the
annihilation diagrams at leading power, which will provide more important evidence on the sizable annihilation contributions in heavy $B$ meson physics and further shed light on the underlying mechanism of the annihilated $B$ meson decays.

ACKNOWLEDGMENTS

This work is supported by the National Natural Science Foundation of China under Grants No. 11205072, 10975074 and 11235005.

[1] Abulencia A et al. (CDF Collaboration) (2006) Evidence for the Exclusive Decay $B_c \rightarrow J/\psi \pi^\pm$ and Measurement of the Mass of the $B_c$ Meson. Phys Rev Lett96: 082002
[2] Abe F et al. (CDF Collaboration) (1998) Observation of the $B_c$ meson in $p\bar{p}$ collisions at $\sqrt{S} = 1.8$ TeV. Phys Rev Lett81: 2432
[3] Gouz I P, Kiselev V V, Likhoded A K, Romanovsky V I, Yushchenko O P (2004) Prospects for the $B_c$ studies at LHCb. Phys. Atom. Nucl., 67: 1559-1570
[4] Brambilla N et al. (Quarkonium Working Group) (2011) Heavy quarkonium: progress, puzzles, and opportunities. Eur Phys J C71: 1534
[5] Ji W (2010) Prospects of Observing the Decay $B_c \rightarrow J/\psi \pi$ and the Alignment Performance in the ATLAS Experiment. Ph.D Thesis, CERN-THESIS-2010-127, 2010
[6] Hussain F, Scadron M D (1984) Two-body nonleptonic weak decays of charm and bottom mesons. Phys Rev D30: 1492
[7] Du D, Wang Z (1989) Predictions of the standard model for $B_c^\pm$ weak decays. Phys Rev D39: 1342
[8] Masetti M (1992) CP violation in $B_c$ decays. Phys Lett B286: 160-164
[9] Xu Q P, Kamal A N (1992) Nonleptonic charmed-baryon decays: $B_c \rightarrow B((3/2)^+, \text{decuplet}) + P(0^-)$ or $V(1^-)$. Phys Rev D46: 3836
[10] Chang C H, Chen Y Q (1994) Decays of the $B_c$ meson. Phys Rev D49: 3399
[11] Gershtein S S, Kiselev V V, Likhoded A K, Tkabladze A V (1995) Physics of $B_c$ mesons. Phys.Usp. 38:1-37 [Usp.Fiz.Nauk. 165: 3-40]
[12] Sanchis-Lozano M A (1995) Weak decays of doubly heavy hadrons. Nucl Phys B440: 251-275
[13] Kiselev V V (1996) Hard-soft factorization in $B_c^+ \rightarrow \psi \pi^+$ decay. Phys Lett B372: 326-330
[14] Du D, Lu G, Yang Y (1996) Perturbative QCD estimation of the $B_c$ exclusive nonleptonic decays. Phys Lett B387: 187-190
[15] Liu J F, Chao K T (1997) $B_c$ meson weak decays and CP violation. Phys Rev D56: 4133-4145
[16] Du D S, Wei Z T (1998) Space-like Penguin effects in $B_c$ decays. Eur Phys J C5: 705-709
[17] Dai Y, Du D (1999) CP violation in two-body hadronic decays of $B_c$ mesons. Eur Phys J C9: 557-564
[18] El-Hady A A, Muñoz J H, Vary J P (2000) Semileptonic and nonleptonic $B_c$ decays. Phys Rev D62: 014019
[19] Fleischer R, Wyler D (2000) Exploring CP violation with $B_c$ decays. Phys Rev D62: 057503
[20] Guo L B, Du D S (2001) Perturbative Quantum Chromodynamics effects in $B_c \rightarrow PP$ decays. Chin. Phys. Lett. 18: 498-500
[21] Pakhomova O N, Saleev V A (2000) Spin effects in two particle hadronic decays of $B_c$ mesons. Phys. At. Nucl. 63: 1999-2003 [Yad. Fiz. 63: 2091-2095]
16

[22] Saleev V A (2001) Production of $P$-wave charmonium states in two-particle decays of $B_c$ mesons. Phys. At. Nucl. 64: 2027-2031 [Yad. Fiz. 64: 2113-2117]

[23] Giri A K, Mohanta R, Khanna M P (2002) Determining the CKM angle $\gamma$ with $B_c$ decays. Phys Rev D65: 034016

[24] Verma R C, Sharma A (2002) Quark diagram analysis of weak hadronic decays of the $B_c^+$ meson. Phys Rev D65: 114007

[25] Kiselev V V, Pakhomova O N, Saleev V A (2002) Two-particle decays of the $B_c$ meson into charmonium states. J Phys G28: 595

[26] Castro G L, Mayorga H B, Muñoz J H (2002) Non-leptonic decays of the $B_c^+$ into tensor mesons. J Phys G28: 2241

[27] Ivanov M A, Körner J G, Pakhomova O N (2003) The nonleptonic decays $B_c^+ \to D_s^+ D^0$ and $B_c^+ \to D_s^+ D^0$ in a relativistic quark model. Phys Lett B555: 189-196

[28] Ebert D, Faustov R N, Galkin V O (2003) Weak decays of the $B_c$ meson to charmonium and $D$ mesons in the relativistic quark model. Phys Rev D65: 094020

[29] Ebert D, Faustov R N, Galkin V O (2003) Weak decays of the $B_c$ meson to $B_s$ and $B$ mesons in the relativistic quark model. Eur Phys J C32: 29-43

[30] Kiselev V V (2004) Gold-plated mode of CP-violation in decays of $B_c$ meson from QCD sum rules. J Phys G30: 1445

[31] Fajfer S, Kamenik J F, Singer P (2004) New-physics scenarios in $\Delta S = 2$ decays of the $B_c$ meson. Phys Rev D70: 074022

[32] Ivanov M A, Körner J G, Santorelli P (2006) Exclusive semileptonic and nonleptonic decays of the $B_c$ meson. Phys Rev D73: 054024

[33] Hernández E, Nieves J, Verde-Velasco J M (2006) Study of exclusive semileptonic and nonleptonic decays of $B_c^-$ in a nonrelativistic quark model. Phys Rev D74: 074008

[34] Giri A K, Mawlong B, Mohanta R (2007) Determination of the angle $\gamma$ from nonleptonic $B_c \to D_s D^0$ decays. Phys Rev D75: 097304

[35] Wang W, Shen Y L, Lü C D (2007) Study of $B_c^- \to X(3872)\pi^-(K^-)$ decays in the covariant light-front approach. Eur Phys J C51: 841-847

[36] Dhir R, Sharma N, Verma R C (2008) Flavor dependence of $B_c$ meson form factors and $B_c \to PP$ decays. J Phys G35: 085002

[37] Sun J, Yang Y, Du W, Ma H (2008) Study of $B_c \to B^{(*)} P, BV$ decays with QCD factorization. Phys Rev D77: 114004

[38] Sun J, Xue G, Yang Y, Lu G, Du D (2008) Study of $B_c^- \to J/\psi\pi^-\eta_c\pi^- \eta_c\pi^- \eta_c\pi^-$ decays with QCD factorization. Phys Rev D77: 074013

[39] Liu X, Li X Q (2008) Effects of hadronic loops on the direct CP violation of $B_c$. Phys Rev D77: 096010

[40] Sun J, Du D, Yang Y (2009) Study of $B_c \to J/\psi\pi, \eta_c\pi$ decays with perturbative QCD approach. Eur Phys J C60: 107-117

[41] Cheng J F, Du D S, Lü C D (2006) Study of $B_c \to D\pi$ in the perturbative QCD approach. Eur Phys J C45: 711-720

[42] Zhang J, Yu X Q (2009) Branching ratio and CP violation of $B_c \to DK$ decays in the perturbative QCD approach. Eur Phys J C63: 435-442

[43] Choi H M, Ji C R (2009) Non-leptonic two-body decays of the $B_c$ meson in light-front quark model and QCD factorization approach. Phys Rev D80: 114003

[44] Descotes-Genon S, He J, Kou E, Robbe P (2009) Nonleptonic charmless $B_c$ decays and their search at LHCb. Phys Rev D80: 114031
[45] Rakitin A, Koshkarev S (2010) Hadronic $B_c$ decays as a test of $B_c$ cross section. Phys Rev D81: 014005
[46] Likhoded A K, Luchinsky A V (2010) Light hadron production in $B_c \to J/\psi + X$ decays. Phys Rev D81: 014015
[47] Sharma N, Dhir R, Verma R C (2010) Branching ratios of $B_c$ meson decaying to pseudoscalar and axial-vector mesons. J Phys G37: 075013
[48] Sharma N (2010) Branching ratios of $B_c$ meson decays into tensor meson in the final state. Phys Rev D81: 014027
[49] Sharma N, Verma R C (2010) Predictions of $B_c$ meson decay emitting pseudoscalar and heavy scalar mesons using ISGW II model. Phys Rev D82: 094014
[50] Liu X, Xiao Z J, Lü C D (2010) Pure annihilation type $B_c \to M_2 M_3$ decays in the perturbative QCD approach. Phys Rev D81: 014022
[51] Liu X, Xiao Z J (2010) Branching ratios of $B_c \to A P$ decays in the perturbative QCD approach. Phys Rev D81: 074017
[52] Yu X Q, Zhou X L (2010) Study of $B_c \rightarrow J/\psi K$ decays in the perturbative QCD approach. Phys Rev D81: 037501
[53] Yang Y, Sun J, Wang N (2010) Study of $B_c \to KK$ decay with perturbative QCD approach. Phys Rev D81: 074012
[54] Liu X, Xiao Z J (2009) Light scalar mesons and charmless hadronic $B_c \rightarrow SP, SV$ decays in the perturbative QCD approach. Phys Rev D82: 054029
[55] Ebert D, Faustov R N, Galkin V O (2010) Semileptonic and nonleptonic decays of $B_c$ mesons to orbitally excited heavy mesons in the relativistic quark model. Phys Rev D82: 034019
[56] Fu H, Jiang Y, Kim C S, Wang G L (2011) Probing non-leptonic two-body decays of $B_c$ meson. J High Energy Phys06: 015
[57] Wang Z, Wang G L, Chang C H (2012) The $B_c$ decays to $P$-wave charmonium by improved Bethe-Salpeter approach. J Phys G39: 015009
[58] Liu X, Xiao Z J (2011) Studies on charmless hadronic $B_c \rightarrow AV(VA)$ decays in the perturbative QCD approach. J Phys G38: 035009
[59] Xiao Z J, Liu X (2011) Study of the pure annihilation $B_c \to A_2 A_3$ decays. Phys Rev D84: 074033
[60] Zhou R, Zou Z T, Lü C D (2012) Two-body $B_c \rightarrow D(s)P, D(s)V$ decays in the perturbative QCD approach. Phys Rev D86: 074008
[61] Zou Z T, Yu X, Lü C D (2013) $B_c \rightarrow D(s)T$ decays in perturbative QCD approach. Phys Rev D87: 074027
[62] Zhou R, Zou Z T, Lü C D (2012) Double charm decays of $B_c$ meson in the perturbative QCD approach. Phys Rev D86: 074019
[63] Wang W F, Yu X, Lü C D, Xiao Z J (2014) The semileptonic decays $B_c^+ \rightarrow D(s)(l^+ \nu, l^+ l^-, \nu \bar{\nu})$ in the perturbative QCD approach. arXiv:1401.0391 [hep-ph].
[64] Wang Z G (2012) The $B_c$-decays $B_c^+ \to J/\psi \pi^+ \pi^- \pi^+$, $\eta_c \pi^+ \pi^- \pi^+$. Phys Rev D86: 054010
[65] Luchinsky A V (2012) Production of charged $\pi$-mesons in exclusive $B_c \to V(P) + n\pi$ decays. Phys Rev D86: 074024
[66] Qiao C F, Sun P, Yang D, Zhu R L (2012) $B_c$ Exclusive decays to charmonia and light mesons in QCD factorization at next-to-leading order accuracy. arXiv:1209.5859[hep-ph]
[67] Naimuddin Sk, Kar S, Priyadarshini M, Barik N, Dash P C (2012) Nonleptonic two-body $B_c$-meson decays. Phys Rev D86: 094028
[68] Dhir R, Kim C S (2013) Branching ratios of $B_c$ meson decaying to vector and axial-vector mesons. Phys Rev D87: 034004
[69] Dhir R, Kim C S (2013) First estimates of nonleptonic $B_c \to AT$ weak decays. Phys Rev D88: 034024
[70] Esposito A, Papinutto M, Pilloni A, Polosa A D, Tantalo N (2013) Doubly Charmed Tetraquarks in $B_c$ and $\Xi_{bc}$ Decays. Phys Rev D88: 054029
[71] Kar S, Dash P C, Priyadarsini M, Naimuddin Sk, Barik N (2013) Nonleptonic $B_c \to VV$ decays. Phys Rev D88: 094014
[72] Beneke M, Buchalla G, Neubert M, Sachrajda C T (1999) QCD factorization for $B \to \pi\pi$ decays: Strong phases and CP violation in the heavy quark limit. Phys Rev Lett83: 1914-1917
[73] Du D, Gong H, Sun J, Yang D, Zhu G (2002) Phenomenological analysis of charmless decays $B \to PV$ with QCD factorization. Phys Rev D65: 094025
[74] Bauer C W, Pirjol D, Rothstein I Z, Stewart I W (2004) $B \to M_1M_2$: Factorization, charming penguins, strong phases, and polarization. Phys Rev D70: 054015
[75] Keum Y Y, Li H N, Sanda A I (2001) Fat penguins and imaginary penguins in perturbative QCD. Phys Lett B504: 6-14
[76] Lü C D, Ukai K, Yang M Z (2001) Branching ratio and CP violation of $B \to \pi\pi$ decays in perturbative QCD approach. Phys Rev D63: 074009
[77] Li H N (2003) QCD aspects of exclusive $B$ meson decays. Prog Part Nucl Phys51: 85-171 and reference therein
[78] Abazov V M et al. (CDF Collaboration) (2008) Observation of the decay $B_c^+ \to J/\psi\pi^+$ and measurement of the $B_c^+$ mass. Phys Rev Lett100: 182002
[79] Abazov V M et al. (D0 Collaboration) (2008) Observation of the $B_c$ meson in the exclusive decay $B_c \to J/\psi\pi$. Phys Rev Lett101: 012001
[80] Beringer J et al. (Particle Data Group) (2012) Review of particle physics. Phys Rev D86: 010001
[81] ATLAS Collaboration, (2012) Observation of the $B_c$ meson in the decay $B_c \to J/\Psi(\mu^+\mu^-)\pi^+$. J High Energy Phys1309: 075
[82] Bachmann S, (LHCb Collaboration) (2013) The LHCb Experiment: Recent results and prospects, talk given at Theory meeting on particle physics phenomenology, Sep.30 - Oct.3, 2013, KEK, Japan.
[83] Arnesen C M, Ligeti Z, Rothstein I Z, Stewart I W (2008) Power corrections in charmless nonleptonic $B$-decays: Annihilation is factorizable and real. Phys Rev D77: 054006
[84] Chay J, Li H N, Mishima S (2008) Possible complex annihilation and $B \to K\pi$ direct CP asymmetry. Phys Rev D78: 034037
[85] Li H N, Yu H L (1995) PQCD analysis of exclusive charmless $B$ meson decay spectra. Phys Lett B353: 301-305
[86] Shifman M A, Vainshtein A I, Zakharov V I (1979) QCD and resonance physics: theoretical foundations. Nucl Phys B147: 385-447
[87] Li H N (2002) Threshold resummation for exclusive $B$ meson decays. Phys Rev D66: 094010
[88] Li H N, Sterman G (1992) The perturbative pion form factor with Sudakov suppression. Nucl Phys B381: 129-140
[89] Huang T, Shen Q X (1991) The applicability of perturbative QCD to the pion form factor and the pionic wavefunction. Z Phys C50: 139-144
[93] Bell G, Feldmann Th (2008) Modelling light-cone distribution amplitudes from non-relativistic bound states. J High Energy Phys04: 061
[94] Chiu T W et al. (TWQCD Collaboration) (2007) Beauty mesons in lattice QCD with exact chiral symmetry. Phys Lett B651: 171-176
[95] Lü C D (2002) Calculation of pure annihilation type decay $B^+ \rightarrow D^+_s \phi$. Eur Phys J C24: 121-126
[96] Keum Y Y, Kurimoto T, Li H N, Lü C D, Sanda A I (2004) Nonfactorizable contributions to $B \rightarrow D^{(*)} M$ decays. Phys Rev D69: 094018
[97] Li R H, Lü C D, Zou H (2008) The $B(B_s) \rightarrow D_{(s)} P, D_{(s)} V, D^{*}_{(s)} P$ and $D^{*}_{(s)} V$ decays in the perturbative QCD approach. Phys Rev D78: 014018
[98] Lü C D, Ukai K (2003) Branching ratios of $B \rightarrow D_s K$ decays in perturbative QCD approach. Eur Phys J C28: 305-312
[99] Li Y, Lü C D, Xiao Z J, Yu X Q (2004) Branching ratio and CP asymmetry of $B_s \rightarrow \pi^+ \pi^-$ decays in the perturbative QCD approach. Phys Rev D70: 034009
[100] Ali A, Kramer G, Li Y, Lü C D, Shen Y L, Wang W, Wang Y W (2007) Charmless nonleptonic $B_s$ decays to $P P, P V$, and $V V$ final states in the perturbative QCD approach. Phys Rev D76: 074018
[101] Xiao Z J, Wang W F, Fan Y Y (2012) Revisiting the pure annihilation decays $B_s \rightarrow \pi^+ \pi^-$ and $B^0 \rightarrow K^+ K^-$: The data and the perturbative QCD predictions. Phys Rev D85: 094003
[102] Hong B H, Lü C D (2006) Direct CP violation in hadronic $B$ decays. Sci. China G 49: 357-366
[103] Li H N, Mishima S (2005) Polarizations in $B \rightarrow V V$ decays. Phys Rev D71, 054025 (2005)
[104] Li H N (2005) Resolution to the $B \rightarrow \phi K^*$ polarization puzzle. Phys Lett B622: 63
[105] Liu X, Xiao Z J, Zou Z T (2014) in preparation