Calculation of the prismatic thin-walled structure on the concentrated effect

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Abstract: When creating original structures, one of the main goals is to create rational schemes of engineering structures, improving the methods of their calculation, allowing them to present their actual work adequately. Therefore, the authors have considered the classical problem of the stress-strain state of a thin-walled prismatic structure, which is a dihedral right angle, composed of two plates (half-planes). The problem is solved by the traditional method. An inversely symmetric load from the concentrated forces $F$ with a period $l$ in the plane of one of the faces is applied along the edge of the structure. The general solution is presented in the form of a single trigonometric series and enables more accurately to determine the stresses near the point of applied force. A comparison of the numerical results obtained in the case when the bending of the faces from its plane is taken into account and in the case when the bending is not taken into account is given. It is shown that the difference between the results is insignificant.

Keywords: stress-strain state, inversely symmetric load, bending moment, boundary conditions, modulus of elasticity, prismatic thin-walled structure.

Introduction. At all times improving the quality and ensuring high efficiency of building structures used in fundamental building and engineering in the development of new building structures is of great importance. To achieve these goals, there is a need to create rational schemes of engineering structures, to improve the methods of their calculation, which will allow us to present the actual work of the structures more fully. One of the areas in which further development and refinement of calculation methods are important is the calculation of thin-walled structures consisting of plates and shells of different shapes and configurations. Due to its lightness and aesthetics of architectural forms, such structures are widely used in civil engineering work. Many scientists reviewed thin-walled prismatic structures in their research articles [1-3], but in fact only the foundations of new sections of structural mechanics related to the calculation of thin-walled structures are put in their research results. When calculating thin-walled structures, issues related to the determination of the stress-strain state (SSS) in the areas of application of local and concentrated loads are important. The definition of SSS in plates and shells from concentrated loads was represented by V. Z. Vlasov [1], B. M. Browde [5], V. V. V. Vlasov [6], B. G. Lampsi [7], P. F. Papkovich [8], etc.

It should be noted that the calculation of thin-walled structures on concentrated effects is developed using various mathematical methods. The finite element method [9 Pro finite element method] is widely used.

Recent scientific studies have mainly considered beam-type structures [1]. And at the moment there is a need to use thin-walled structures in order to save expenses in construction.

We consider a thin-walled structure, which is a dihedral right angle composed of two plates (half-planes). An inverse symmetric load of concentrated forces $F$ with period $l$ in the plane of one of the faces is applied along the edge (Fig. 1).
Figure 1. Thin-walled structure in the form of a dihedral right angle.

We consider that the $x$ coordinate is counted along the edge passing in the middle between the points of application of two adjacent forces, of the faces from the cross section of the structure, the $y_1$ – coordinate – in the plane of the vertical face, the $y_2$–coordinate in the plane of the horizontal face. From the condition of the inverse symmetry of the problem at the ends of the plot (Fig.1 it is shaded) for $x=0$ and $x=l$ we have the following boundary conditions:

$$
\sigma_x^{(1)} \bigg|_{x=0} = \sigma_x^{(2)} \bigg|_{x=0} = 0, \\
V_x^{(1)} \bigg|_{x=0} = V_x^{(2)} \bigg|_{x=0} = 0, (1)
$$

These conditions are also valid for an isolated section of the structure, if at its edges ($x=0$ $ux=l$) the faces of the angle are continuously supported by diagrams absolutely rigid in their plane and absolutely flexible in the plane.

The general solution of the inverse symmetric problem under consideration in single trigonometric series decaying with distance from the edge, in dimensionless coordinates

$$
\xi = \frac{x}{l}, \eta_1 = \frac{y_1}{l}, \eta_2 = \frac{y_2}{l}
$$

(2)

for each of the faces is defined by the formula 3:

$$
\sigma_x^{(i)} = \sum_{m=1,3,5}^{\infty} \left[A_{1m}^{(i)} + (m\eta_i - 2)A_{2m}^{(i)}\right] e^{-m\eta_i \sin m\xi}
$$

$$
\sigma_y^{(i)} = \sum_{m=1,3,5}^{\infty} \left[A_{1m}^{(i)} + m\eta_i A_{2m}^{(i)}\right] e^{-m\eta_i \sin m\xi}
$$

$$
\tau_{xy}^{(i)} = \sum_{m=1,3,5}^{\infty} \left[A_{1m}^{(i)} - (1 - m\eta_i)A_{2m}^{(i)}\right] e^{-m\eta_i \cos m\xi}
$$

(3)
\[ E_u^l = -\frac{l(1 + \nu)}{\pi} \sum_{m=1,3,5} \infty \frac{1}{m} \left[ A_{1m}^{(l)} + \left( m\eta_l - \frac{2}{1 + \nu} \right) A_{2m}^{(l)} \right] e^{-m\eta_1 \cos \mu \xi} \]

\[ E_v^l = -\frac{l(1 + \nu)}{\pi} \sum_{m=1,3,5} \infty \frac{1}{m} \left[ A_{1m}^{(l)} + \left( m\eta_l + \frac{1 - \nu}{1 + \nu} \right) A_{2m}^{(l)} \right] e^{-m\eta_1 \sin \mu \xi} \]

Here \( \nu \)–Poisson’s ratio, \( E \) – modulus of elasticity of the first kind, \( A_{1m}^{1}, A_{2m}^{1}, A_{1m}^{2} \) \( (m=1,3,5,...,\infty) \) are derivatives of integration constants. The solution with index \( l \) refers to the vertical face; the solution with index 2 refers to the horizontal face.

We set ourselves the task to take into account also the bending of the faces of the structure as plates. Taking into account the above, we will look for deflection functions for vertical and horizontal plates in the form of single trigonometric series,

\[ W^{(l)} = \sum_{m=1,3,5, \infty} \left( B_{1m}^{(l)} + \eta_l B_{2m}^{(l)} \right) e^{-m\eta_1 \sin \mu \xi} \quad (4) \]

thus \( B_{1m}^{(1)}, B_{2m}^{(1)}, B_{1m}^{(2)}, B_{2m}^{(2)} \)–arbitrary integration constants. Solutions with index \( l \) are applied to the vertical face, solution with index 2 refers to the horizontal face.

The deflection function (4) satisfies a homogeneous biharmonic plate bending equation. The corresponding formulas for the rotation angles \( \frac{d w^{(l)}}{d y_i}, \frac{d w^{(l)}}{d x_i} \), bending moments\( M_x^{(l)}, M_y^{(l)} \) and lateral forces\( Q_y^{(l)} \) in each of the plates have the form:

\[ \frac{d w^{(l)}}{d x} = \frac{\pi^2}{l} \sum_{m=1,3,5} \infty m \left( B_{1m}^{(l)} + \eta_l B_{2m}^{(l)} \right) e^{-m\eta_1 \cos \mu \xi} \]

\[ \frac{d w^{(l)}}{d y} = \frac{\pi^2}{l} \sum_{m=1,3,5} \infty \left[ -mB_{1m}^{(l)} + (1 - m\eta_l) B_{2m}^{(l)} \right] e^{-m\eta_1 \sin \mu \xi} \]

\[ M_x^{(l)} = D_l \frac{\pi^2 (1 - \nu)}{l^2} \sum_{m=1,3,5} \infty m \left[ mB_{1m}^{(l)} + \left( \frac{2\nu}{1 - \nu} + m\eta_l \right) B_{2m}^{(l)} \right] e^{-m\eta_1 \sin \mu \xi} \]

\[ M_y^{(l)} = D_l \frac{\pi^2 (1 - \nu)}{l^2} \sum_{m=1,3,5} \infty \left[ mB_{1m}^{(l)} + \left( \frac{2\nu}{1 - \nu} + m\eta_l \right) B_{2m}^{(l)} \right] e^{-m\eta_1 \sin \mu \xi} \]

\[ Q_y^{(l)} = D_l \frac{\pi^2 (1 - \nu)}{l^2} \sum_{m=1,3,5} \infty \left[ mB_{1m}^{(l)} + \left( \frac{(1 + \nu)}{1 - \nu} + m\eta_l \right) B_{2m}^{(l)} \right] e^{-m\eta_1 \sin \mu \xi} \]

Here \( Q_y^{(l)} \)–generalized transverse forces in the Kirchhoff sense; \( D_l \) – cylindrical plate stiffness calculated by the formula:

\[ D_l = \frac{E_i \delta_i^3}{12(1 - \nu^2)} \quad (6) \]

Arbitrary constants \( A_{1m}^{1}, A_{2m}^{1}, B_{1m}^{1}, B_{2m}^{1} \) \( (i = 1,2) \), included into expressions (3), (4), (5), we determine from the boundary conditions on the contact line of faces \( at y_1 = y_2 = 0 \):

\[ v_{y_1}^{(1)} = 0, v_{y_2}^{(2)} = 0, v_{y_1}^{(2)} = 0, v_{y_2}^{(1)} = 0, u_{y_1}^{(1)} = u_{y_2}^{(2)} = 0, u_{y_1}^{(2)} = u_{y_2}^{(1)} = 0 \]
\[ \delta_1 \tau_{xy}^{(1)} |_{y_1=0} = \delta_2 \tau_{xy}^{(2)} |_{y_2=0} \]  

\[ \frac{d\omega^{(1)}}{dy_1 |_{y_1=0}} = \frac{d\omega^{(2)}}{dy_2 |_{y_2=0}} \]

\[ M_{y1}^{(1)} = -M_{y2}^{(2)}, Q_{y1}^{(1)} = -\delta_2 \sigma_{y1}^{(2)} |_{y_2=0} \]

\[ -F_m + Q_{y0}^{(2)} = \delta_1 \sigma_{y1}^{(1)} \]

where \( \delta_1, \delta_2 \) - are the thickness of the vertical and horizontal faces, respectively.

The external concentrated normal load is presented in the form of a trigonometric series:

\[ F_m = \frac{2F}{l} \sum_{m=1,3,5} \infty (-1)^{\frac{m-1}{2}} \sin m\xi \]  

Satisfying boundary conditions (7) using dependencies (3), (4), (5), and (8), taking into account the orthogonality of trigonometric functions, we obtain a system of algebraic equations with respect to unknown coefficients \( A_{1m}^{(i)}, A_{2m}^{(i)}, B_{1m}^{(i)}, B_{2m}^{(i)} \) \( (i = 1,2) \), for each number \( m = 1,3,5 \ldots \infty \):

\[ \frac{(1 + \nu)_l}{Em \pi} \left( A_{1m}^{(1)} + \frac{1 - \nu}{1 + \nu} A_{2m}^{(1)} \right) = -B_{1m}^{(2)}, \]

\[ \frac{(1 + \nu)_l}{Em \pi} \left( A_{1m}^{(2)} + \frac{1 - \nu}{1 + \nu} A_{2m}^{(2)} \right) = B_{1m}^{(1)}, \]

\[ \frac{(1 + \nu)_l}{Em \pi} \left( A_{1m}^{(1)} - \frac{2}{1 + \nu} A_{2m}^{(1)} \right) = \frac{(1 + \nu)_l}{Em \pi} \left( A_{1m}^{(2)} - \frac{2}{1 + \nu} \right) \]

\[ \delta_1 A_{1m}^{(1)} - (A_{2m}^{(1)}) = -\delta_2 \left( A_{1m}^{(2)} - A_{2m}^{(2)} \right), \]

\[ -mB_{1m}^{(1)} + B_{2m}^{(1)} = -mB_{1m}^{(2)} + B_{2m}^{(2)} \]

\[ D_1 \left( (1 - \nu)m^2 B_{1m}^{(1)} - 2m B_{2m}^{(1)} \right) = D_2 \left( (1 - \nu)m^2 B_{1m}^{(2)} - 2m B_{2m}^{(2)} \right) \]

\[ D_1 \pi^3 l^3 \left( 1 - \nu \right) m^2 B_{1m}^{(1)} + (1 + \nu) m B_{2m}^{(1)} = \delta_2 A_{1m}^{(2)} \]

\[ -2F = \frac{(-1)^{\frac{m-1}{2}}}{l} \frac{D_1 \pi^3 l^3}{m^2} \left( 1 - \nu \right) m B_{1m}^{(2)} + (1 + \nu) B_{2m}^{(2)} = -\delta_1 A_{1m}^{(1)} \]

In the case of the same thickness on both faces \( \delta_1 = \delta_2 = \delta \), \( D_1 = D_2 = D \) the system of equations (9) an example of more simple view:

\[ \frac{\alpha}{m} \left( (1 + \nu) A_{1m}^{(1)} + (1 - \nu) A_{2m}^{(1)} \right) = -B_{1m}^{(2)} \]

\[ \frac{\alpha}{m} \left( (1 + \nu) A_{1m}^{(2)} + (1 - \nu) A_{2m}^{(2)} \right) = -B_{1m}^{(1)} \]

\[ A_{1m}^{(1)} - \frac{2}{1 + \nu} A_{2m}^{(2)} = A_{1m}^{(2)} - \frac{2}{1 + \nu} A_{2m}^{(2)} \]
\[
\left( A_{1m}^{(1)} - A_{2m}^{(1)} \right) = - \left( A_{1m}^{(2)} - A_{2m}^{(2)} \right) (10)
\]
\[
-mB_{1m}^{(1)} + B_{2m}^{(1)} = -mB_{1m}^{(2)} + B_{2m}^{(2)}
\]
\[
(1 - \nu) mB_{1m}^{(1)} - 2B_{2m}^{(1)} = (1 - \nu) mB_{1m}^{(2)} - 2B_{2m}^{(2)}
\]
\[
\beta m^2 \left[ (1 - \nu) mB_{1m}^{(1)} + (1 + \nu) B_{2m}^{(1)} \right] = A_{1m}^{(2)}
\]
\[
-\frac{2F}{\delta l} (1) m^2 \left[ (1 - \nu) mB_{1m}^{(2)} + (1 + \nu) B_{2m}^{(2)} \right] = -A_{1m}^{(1)}
\]

where, \( \alpha = l/E\pi \), \( \beta = D\pi^3/l^3 \delta \).

Thus, we find the values of the constants:

\[
A_{1m}^{(1)} = \frac{F}{\delta l} (-1)^{m-1} \Delta_m \left[ B + m^2 \alpha \beta (5 - \nu^2)^2 \right]
\]
\[
A_{2m}^{(1)} = \frac{F}{\delta l} (-1)^{m-1} \Delta_m \left[ 2(5 + \nu) + m^2 \alpha \beta (1 + \nu)(\nu^3 - \nu^2 - 9\nu + 17) \right]
\]
\[
A_{1m}^{(2)} = \frac{F}{\delta l} (-1)^{m-1} \Delta_m m^2 \alpha \beta [3 - \nu^2] - 32
\]
\[
A_{2m}^{(2)} = \frac{F}{\delta l} (-1)^{m-1} \Delta_m \left[ 2(1 - \nu) + m^2 \alpha \beta (1 + \nu)(\nu^3 - \nu^2 - 9\nu + 1) \right]
\]
\[
B_{1m}^{(1)} = \frac{F}{\delta l} (-1)^{m-1} \Delta_m \frac{2\alpha}{m} \left[ (1 - \nu)^2 - m^2 \alpha \beta (3 - \nu) (1 + \nu)^3 \right]
\]
\[
B_{2m}^{(1)} = \frac{F}{\delta l} (-1)^{m-1} \Delta_m \frac{2\alpha}{m} \left[ (\nu^3 - 3\nu^2 - \nu - 5) - 2m^2 \alpha \beta (1 - \nu) (1 + \nu)(3 - \nu) \right]
\]
\[
B_{1m}^{(2)} = \frac{F}{\delta l} (-1)^{m-1} \Delta_m \frac{2\alpha}{m} \left[ (\nu^2 - 2\nu - 7) - m^2 \alpha \beta (7 - 2\nu - \nu^2)(3 - \nu)(1 + \nu) \right],
\]
\[
B_{2m}^{(2)} = \frac{F}{\delta l} (-1)^{m-1} \Delta_m \alpha \left[ (\nu^3 - 3\nu^2 - \nu + 11) + 2m^2 \alpha \beta (1 - \nu)(5 + \nu)(3 - \nu) \right]
\]

Here, \( \Delta_m = \frac{l}{[4 + m^2 \alpha \beta (5 - \nu^2)^2 + 4m^2 \alpha^2 \beta^2 (1 - \nu^2)(9 - \nu^2)]} \).

Substituting these coefficient values into expressions (3), (4), (5), we obtain the required dependences that determine the stress-strain state of the faces of the structure on their median surface.

Using the known dependencies, we calculate the stresses in the plates by bending from its plane:

\[
\sigma_x^i = \pm \frac{12M_x^{(i)}}{\delta l^3} z_i
\]
\[
\sigma_y^i = \pm \frac{12M_y^{(i)}}{\delta l^3} z_i
\]

The maximum values of these stresses are determined by the formulas:

\[
\sigma_x^i = \pm \frac{6M_x^{(i)}}{\delta l^2} z_i
\]

\[
\tau_{xy}^i = \pm \frac{12H^{(i)}}{\delta l^3} z_i
\]
\[
\sigma_y = \pm \frac{6M_y^i}{\delta_i^2} (13)
\]

\[
\tau_{xy} = \pm \frac{6H^{(i)}}{\delta_i^2}
\]

Adding up the stress dependences (3) and (13), we obtain the maximum stress values in the faces of a thin-walled prismatic structure, taking into account the plane stress state and bending.

If the bending of faces as plates is not taken into account, the cylindrical stiffness for each of the plates equal to zero should be taken into account in formulas (11). In this case, the faces of the structure are in a plane stress state. The relevant solution of the problem is given in [3].

In order to implement the obtained solutions on the PC, a calculation program was compiled, with the help of which the results given in figures 2, 3 were obtained.

Figure 2. Plots $\sigma_x$ in various cross sections of a vertical face.

Figure 3. Plots $\sigma_y$ in various cross sections of the vertical face.
The solid line represents the stress diagrams $\sigma_x^{(1)}, \sigma_y^{(1)}$, arising in the vertical face along the median surface in different sections, as shown in Fig. 2 and 3. The dashed line shows the values of the same stresses in the case when the cylindrical stiffness $D=0$.

It can be seen from the figures that there is a slight difference between the stresses calculated taking into account the bending of the faces from the plane and without it (within 1%). Therefore, in such problems, it is more expedient to use a simpler solution obtained without taking into account the bending ($D=0$) [3].

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