Chiral doublers of heavy-light baryons

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Abstract

We discuss the consequences of the chiral doubling scenario for baryons built of heavy and light quarks. In particular, we use the soliton description for baryons, demonstrating why each heavy-light baryon should be accompanied by the opposite parity partner. Our argumentation holds both for ordinary baryons and for exotic heavy pentaquarks which are required by the symmetries of QCD to appear in parity doublets, separated by the mass shift of the chiral origin. Interpreting the recently observed by BaBaR, CLEO and Belle charmed mesons with assignment \((0^+, 1^+)\) as the chiral partners of known \(D\) and \(D^*\) mesons, allows us to estimate the parameters of the mesonic effective lagrangian, and in consequence, estimate the masses of ground states and excited states of both parities. In particular, we interpret the state recently reported by the H1 experiment at HERA as a chiral partner \(\tilde{\Theta}^0_c(3099)\) of yet undiscovered ground state pentaquark \(\Theta^0_c(2700)\).

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Recently, experimental physics of hadrons with open charm has provided several spectacular discoveries:

- First, BaBar [1] has announced new, narrow meson $D_{sJ}^*(2317)^+$, decaying into $D_s^+$ and $\pi^0$. This observation was then confirmed by CLEO [2], which also noticed another narrow state, $D_{sJ}(2463)^+$, decaying into $D_s^*$ and $\pi^0$. Both states were confirmed by Belle [3], and finally, the CLEO observation was also confirmed by BaBar [4].

- Second, Belle has not only measured the narrow excited states $D_1, D_2$ with foreseen quantum numbers $(1^+, 2^+)$, but provided also first evidence for two new, broad states $D_0^*$ $(2308 \pm 17 \pm 15 \pm 28)$ and $D_1^*$ $(2427 \pm 26 \pm 20 \pm 17)$ [5]. Both of them are approximately 400 MeV above the usual $D_0, D^*$ states and seem to have opposite to them parity.

- Third, Selex has provided preliminary data for doubly charmed baryons [6]. On top of known since 2002 $cc\bar{d}$ state (3520), four other cascade $j = 1/2$ states are visible, in particular the pair of opposite parity $cc\bar{u}$ states separated by the mass gap of the order 337MeV.

- Fourth, H1 experiment at DESY has announced [7] a signature for charmed pentaquark $\bar{c}udud$ at mass 3099 MeV, i.e. approximately 400 MeV higher than the expected estimates known in the literature [8, 9, 10, 11].

The above states and in particular the decay patterns of all this particles are a challenge for standard estimations based on quark potential models and triggered a flurry of activity among the theorists.

An appealing possibility is that the presence of the above states is the consequence of so-called chiral doublers scenario, theoretically anticipated [12, 13] already in 1992 and 1993. In brief, the scenario, based on simultaneous constraints of the spontaneous breakdown of chiral symmetry for light quarks and of heavy quark spin symmetry (Isgur-Wise symmetry) [14] for heavy quarks, leads to parity duplication of all heavy-light hadrons, with mass shift fixed by the pattern of the spontaneous breakdown of the chiral symmetry. Very recently, in light of Babar, CLEO and Belle discoveries, chiral doublers scenario was reminded [15], pointing that the $D_s(2317)$ is a $0^+$ chiral partner of the usual $(0^-)$ $D_s$ state and the $D_{sJ}^*(2463)$ is a $1^+$ partner of the known vector $D_s^*$. Similar arguments were suggested for new $(0^+, 1^+)$ states observed by Belle [5] and similar kind of doubling is also expected for excited non-strange heavy-light mesons.

In this note, we discuss the extension of the chiral doublers scenario for all baryons, including the exotic states (pentaquark). To avoid any new parameters, we simply view
baryons as solitons of the effective mesonic lagrangian including both chiral copies of heavy-light mesons, a point addressed already in [12]. We are working in large $N_c$ limit, which justifies the soliton picture, and large heavy quark mass limit, where we exploit the Isgur-Wise symmetry. This approach could be viewed as a starting point for including $1/m_h$ corrections from the finite mass of the heavy quark, explicit breaking of chiral symmetry, etc.

The description of baryons as solitons of the mesonic lagrangians has a long history. Original Skyrme [17] idea was elaborated by Witten [18], and Adkins, Nappi and Witten [19] for $SU(2)_{flavor}$ with enormous success and hundreds of followers. The extension to $SU(3)$ was more tricky. Simple embedding of Skyrme ansatz and proper inclusion of the Wess-Zumino-Witten-Novikov term led to the appearance of octet, decuplet and antidecuplet baryons [20]. On one side, this approach, including the explicit mass of the strange quark seemed to be less successful phenomenologically then the $SU(2)$ version [21]. On the other side, it is one of very few dynamical models, which predicted the presence of strange pentaquark [22], a state widely discussed nowadays and observed by several experiments [23].

The mixed success of the first order perturbation theory in $m_s$ in the Skyrme model [21] led in two directions. One consisting in enriching the splitting hamiltonian by terms subleading in $N_c$ [24] gave very good description of hyperon mass spectra and produced successful prediction of the strange pentaquark mass. The other one was based on the simple observation that the mass of the strange quark is of the same order as the inverse moment of inertia of the soliton. This fact tempted Callan and Klebanov [25] to consider an alternative scheme for $SU(3)$. They looked at the binding of the kaon in the field of the $SU(2)$ soliton, and then collectively quantized the bound state as a whole. Although this approach was phenomenologically successful, its extension to strange pentaquarks revealed some fundamental difficulties [26].

The bound state approach was expected to work even better for baryons including heavy quark, i.e. the charm one for example [27]. In [28] it was pointed out that such an approach does not respect the Isgur-Wise symmetry – for infinitely heavy quarks the soliton should bound the degenerate in the IW limit pair of a pseudoscalar and a vector, i.e. $D$ and $D^*$. Charmed hyperons emerge therefore as bound states of $D$ and $D^*$ in the presence of the $SU(2)$ Skyrme background. The related approach [29] used the Born-Oppenheimer approximation. First the pseudoscalar-vector heavy meson pair was bound in the background of
the static soliton, generating the $O(N_c^0)$ binding. Vibrational modes were the "fast degrees" of the freedom. The adiabatical rotation of the bound system by quantization of collective coordinates of the $SU(2)$ skyrmions corresponds then to "slow degrees" of freedom. It is well known, that in this case the rotation is not the free one. Fast degrees of freedom in Born-Oppenheimer approximation generate the effective "gauge" potential, of a Berry phase [30] type. In the case of degenerate pesudoscalar and vector mesons (IW limit) the phases coming from $D$ meson and $D^*$ meson are equal, but opposite. Their cancellation corresponds to the realization of the Isgur-Wise symmetry at the baryonic level, therefore degeneration of spin 1/2 and 3/2 multiplets.

In the following, we choose this philosophy, but contrary to other approaches known in literature [8, 11, 28], we consider the full heavy-light effective lagrangian with both chiral copies [12, 13]. A related approach was considered in [31], however the effects of chiral shift were not included. To avoid unnecessary repetitions we rewrite this lagrangian using the conventions applied in solitonic calculations in the $D$ meson sector [8]. The full lagrangian reads now

$$L = L_H + L_G + L_{GH}$$

(1)

where

$$L_H = -i\text{Tr}(H\gamma^\mu D_\mu H) + g_H\text{Tr}\gamma^\mu\gamma_5 A_\mu H$$

$$+ m_H(\Sigma) \text{Tr}H\dot{H}H$$

(2)

is the usual [16] lagrangian (modulo the last $O(m_h^0)$ mass term, depending on constituent mass of the light quark [12, 13]) for the standard $(0^-, 1^-)$ multiplet

$$H = \frac{1 + \gamma^\mu}{2}(\gamma_5 D - \gamma^\mu D_\mu^*)$$

(3)

and

$$L_G = -i\text{Tr}(\bar{G}\gamma^\mu D_\mu G) + g_G\text{Tr}G\gamma^\mu\gamma_5 A_\mu \bar{G}$$

$$+ m_G(\Sigma) \text{Tr}\bar{G}G$$

(4)

is the chiral doubler lagrangian for $(0^+, 1^+)$ chiral partner

$$G = \frac{1 + \gamma^\mu}{2}(\bar{D} - \gamma^\mu\gamma_5 \bar{D}_\mu^*)$$

(5)
Chiral partners communicate with each other via light axial currents

\[ L_{HG} = g_{GH} \text{Tr}(\gamma_5 \bar{G}H \gamma^\mu A_\mu) + \text{(h.c.)} \]  

with no vector mixing because of the parity. The axial \( A_\mu \) reads

\[ A_\mu = \frac{i}{2} (\xi^\dagger \partial_\mu \xi - \xi \partial_\mu \xi^\dagger) \]

where \( \xi^2 = U = \exp(i\pi \cdot \bar{\tau}) \) and \( v_\mu \) is the four-velocity of the heavy quark. In our case, we take the pion field as the Skyrme hedgehog ansatz \( \pi_i = F(r)n_i \).

The key difference in the chiral copy is the opposite sign of the constituent mass contribution in (4) \( m_G(\Sigma) \approx -\Sigma \), with respect to the similar term for the \( H \) multiplet, \( m_H(\Sigma) \approx \Sigma \), where \( \Sigma \) denotes one loop heavy meson self-energy [12, 13, 15]. The sign flip follows from the \( \gamma_5 \) difference in the definition of the fields \( H \) and \( G \). In other words: it is sensitive to the parity content of the heavy-light field since \( H\phi = -H \) and \( G\phi = +G \). The result is a split between the heavy-light mesons of opposite chirality.

Standard approach [8, 11, 28] ignores (“integrates”) the heavier chiral copy \( G \), and the heavy hyperon spectrum comes only from \( L_H \) part of the lagrangian leading to [8]

\[ M = M_{sol} + m_D - \frac{3}{2}g_H F'(0) + a/I_1 \]

where \( M_{sol} \) is the \( O(N_c) \) classical mass of the Skyrmion, \( m_D = (3M_{D^*} + M_D)/4 \) is the averaged mass of heavy-light mesons, \( g_H \) is the axial coupling constant responsible for the \( D^* \) decays into a \( D \) and a pion, and the inverse of moment of inertia of the Skyrmion \( 1/I_1 \) provides the splitting between the various isospin states. We follow here the conventions of [8]. Since for isosinglet \( a = 3/8 \) and for isotriplet \( a = 11/8 \), one immediately recovers the remarkable formula [28]

\[ M(\Sigma_h) - M(\Lambda_h) = \frac{1}{I_1} = \frac{2}{3} (M(\Delta) - M(N)) \]

where the r.h.s. comes from the \( SU(2) \) Skyrme model.

The pentaquark spectrum comes [8] from replacing the meson by antimeson in the field of soliton with baryon number one, and for the isosinglet pentaquark mass formula reads:

\[ M_5 = M_{sol} + m_D - \frac{1}{2}g_H F'(0) + 3/(8I_1) \]

so pentaquark in this model is three times less bound that the heavy hyperon. Since numerically [8] \( M_{sol} = 866 \text{ MeV}, \) averaged \( m_D = 1973 \text{ MeV}, \) binding strength \( g_H F'(0) = 419 \text{ MeV}, \)
\( I^{-1} = 195 \, \text{MeV} \), the estimate for the \( \Theta_c = \bar{c}udud \) pentaquark mass is 2702 MeV, in agreement with recent estimates of the correlated quark model [9] and SU(3) soliton calculations [11].

The spectrum of pentaquarks [8] is strongly degenerate in mass. However, there is no mixing between these states because they differ in parity of the state, the parity of the light degrees of freedom and/or isospin. The mixing is suppressed by the powers of the heavy quark mass and by the number of colors.

Let us consider now the full lagrangian (1). First we observe, that due to the properties of the heavy spin symmetry, one can trade \( \gamma^\mu A_\mu \) into \( v^\mu A_\mu \) in (6). This implies, that in the rest frame static Skyrmion background decouples the \( G \) and \( H \) lagrangians. Similar observation holds for the version of binding in the scenario of ref. [28], where the coupling vanishes due to the \( r\delta(r) \) term from the wave function of the infinitely heavy charmed meson. This decoupling allows immediately to write down the mass formula for opposite chirality partner of the isoscalar baryon and for opposite chirality partner of the isoscalar pentaquark (denoted by tilde)

\[
\tilde{M} = M_\text{sol} + m_{\tilde{D}} - 3/2g_GF'(0) + 3/(8I_1)
\]

\[
\tilde{M}_5 = M_\text{sol} + m_{\tilde{D}} - 1/2g_GF'(0) + 3/(8I_1)
\]  

(11)

It is of primary importance that, despite the additional \( \gamma_5 \) in the definition of the \( G \) field (4), both hamiltonians have the same functional form of lowest eigenvalue: \( M_5 \) for \( H \) and \( \tilde{M}_5 \) for \( G \). Hence both chiral partners emerge as \( H \) and \( G \) bound states in the SU(2) solitonic background. The mass difference comes in the first approximation solely from the difference of the coupling constants \( g_G - g_H \) and meson mass difference \( m_{\tilde{D}} - m_D \) where \( m_{\tilde{D}} = (3M_{\tilde{D}} + M_D)/4 \) is the averaged over heavy-spin mass of the \( (1^+, 0^+) \) mesons. Constant \( g_G \) is the axial coupling constant in the opposite parity channel, responsible for pionic decays of the \( 1^+ \) axial states into \( 0^+ \) scalars. Using recent Belle data [3], i.e. \( 0^+ \) candidate \( D_0^* \) \( (2308 \pm 17 \pm 15 \pm 28) \) and \( 1^+ \) candidate \( D_1^* \) \( (2427 \pm 26 \pm 20 \pm 17) \), we get \( M_{\tilde{D}} = 2397 \, \text{MeV} \), unfortunately with still large errors.

In the case of strange \( D_s \), the impressive evidence for such states comes from BaBar [1], Cleo [2] and Belle [3] data.

Let us combine now the above formulae. Fist, we notice, that the mass splitting between the usual baryons of opposite parity leads to

\[
\Delta_B = \Delta_M + 3/2F'(0)g_H\delta g
\]  

(12)
where $\Delta_M = M_\tilde{D} - M_D$ is the mass shift between the opposite parity heavy-light mesons and $\delta_g = 1 - g_G/g_H$ measures the difference between the axial couplings for both copies. Similar reasoning leads to the formula for the chiral splitting between the opposite parity pentaquarks:

$$\Delta_P = \Delta_M + 1/2 F'(0)g_H\delta_g .$$  \hspace{1cm} \text{(13)}

Combining both formulae we get

$$\Delta_P = \frac{\Delta_B + 2\Delta_M}{3} .$$  \hspace{1cm} \text{(14)}

Let us turn now towards the data. Comparing the mass shift between the lowest $\Lambda_c$ states of opposite parities, $\Lambda_c(1/2^+, 2285)$ and $\Lambda_c(1/2^-, 2593)$ we arrive at $\Delta_B = 310 \text{ MeV}$. Similarly, $\Xi_c(1/2^+, 2470)$ and $\Xi_c(1/2^-, 2790)$ give $\Delta_B = 320 \text{ MeV}$. Comparing the shift of the opposite parity heavy charmed mesons from very recent Belle [5] data we arrive at $\Delta_M = 425 \text{ MeV}$ unfortunately with still large errors. These two numbers allow us to estimate $\Delta_P = 350 \text{ MeV} \pm 60 \text{ MeV}$, i.e. we get the mass of the chiral partner of the pentaquark as high as $3052 \pm 60 \text{ MeV}$. We note that the argument proposed here is based on the leading approximation in large $N_c$ and large $m_h$ limit, and is intended to demonstrate the order of magnitude for chiral splitting for heavy pentaquarks.

One is therefore tempted to interpret the recent H1 state [7] as a chiral partner $\tilde{\Theta}_c$ of the yet undiscovered isosinglet pentaquark $\Theta_c$ of opposite parity and $M_5 \approx 2700 \text{ MeV}$. Similar reasoning applies to other isospin channels, stranged charmed pentaquarks and to extensions for $b$ quarks. Despite Babar and Cleo data yield with the impressive accuracy the chiral mesonic shift to be equal to $350 \text{ MeV}$, no charmed strange baryon data for both parities do exist by now, so one cannot make similar estimation for strange charmed pentaquarks.

Unfortunately present accuracy of the soliton models does not allow to estimate splittings between pentaquarks of various parities and spins [8]. Assuming that spin $3/2$ pentaquarks will be shifted in mass by $1/m_h$ corrections, there are still two $1/2^-$ and one $1/2^+$ degenerate isosinglet states of the $(D, D^*)$ mesons bound in the soliton background, and similarly three $1/2$ states of opposite parities in the $(\tilde{D}, \tilde{D}^*)$ sector.

Understanding the narrow width of the new state reported by H1 remains a challenge. Our scenario offers, however, a qualitative explanation. Let us first observe that the natural channel for the decay of this state into a nucleon and chiral partners of the standard $D(D^*)$
mesons is kinematically blocked. De-excitation of $\tilde{\Theta}_c$ into $\Theta_c$ and a pion is isospin forbidden and to $\eta^0$ kinematically blocked. Three body decay into $\Theta_c 2\pi$ has very small phase space. Therefore the only way the decay process may proceed, is a chiral fluctuation of a bound $\tilde{D}$ into $D$ by virtual interaction with a pion from the nucleon cloud. That requires, however, spacial rearrangement, since the $D$ meson must be in a partial wave of opposite parity with respect to the partial wave of $\tilde{D}$. Hence the overlap of the $\tilde{D}$-soliton bound state wave function with the one of the $D$-soliton is expected to be small.

In this note, we pointed out that the surprisingly heavy mass of the new charmed pentaquark state may be naturally interpreted in the chiral doubler scenario, forcing each heavy-light hadron to have the opposite parity partner. This pattern seems to be confirmed by now for strange charmed mesons, and is very likely for the recently observed charmed non-strange mesons. In the baryonic sector the universal shift of approximately $310 \pm 10$ MeV seems to separate heavy-light-light and heavy-heavy-light conventional states with opposite parity. If heavy pentaquarks exist, similar pattern of chiral doubling forces them to appear in opposite parity pairs.

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