(2+1)D surface solitons at the interface between a linear medium and a nonlocal nonlinear medium

Zhiwei Shi$^{1,2}$, Huagang Li$^3$ and Qi Guo$^{1,*}$

$^1$Key Laboratory of Photonic Information Technology of Guangdong Higher Education Institutes, South China Normal University, Guangzhou 510631, P.R.China

$^2$Faculty of Information Engineering, Guangdong University of Technology, Guangzhou 510006, P.R.China

$^3$Department of Physics, Guangdong Institute of Education, Guangzhou 510303, P.R.China and

*Corresponding author: guoq@scnu.edu.cn

We address (2+1)D surface solitons occurring at the interface between a linear medium and a nonlocal nonlinear medium whose nonlinear contribution to the refractive index has an initial value at the interface. We find that there exist stable single and dipole surface solitons which do not exhibit a power threshold. The properties of the surface solitons can be affected by the initial value and the degree of nonlocality. When a laser beam is launched away from the interface, the beam will be periodic oscillations.

© 2013 Optical Society of America

OCIS codes:
1. Introduction

Nonlocal spatial solitons have been investigated for decades both theoretically and experimentally. Spatially nonlocality is a generic property in different materials including the photorefractive media [1, 2], thermal nonlinear media [3–7], liquid crystals [8–13], and so on. Nonlocality can lead to new kinds of waves that would have been otherwise impossible in local nonlinear media. Especially, for two-dimensional media with different types of nonlocal response, they can support stationary multipoles [14–19], stable vortices [20–24], and rotating [25, 26] and spiraling [5, 27] soliton states.

Surface waves localized at the interface of two different optical materials have many novel properties, which have been studied in nonlocal media recently. The light beam trajectory can be strongly affected by the presence of interfaces, because beams propagating in nonlocal media cause refractive index changes in regions far exceeding the beam width. Under proper conditions, stationary surface waves can propagate along the interface in both local nonlinear media [28–31] and nonlocal nonlinear media [32–39].

Thermal media [34, 35, 38] and photorefractive crystals [39] have been also utilized to demonstrate (2+1)D surface solitons. In this paper, we will study (2+1)D surface solitons occurring at the interface between a linear medium and a nonlocal nonlinear medium whose nonlinear contribution to the refractive index has a initial value at the interface. We find that there exist single surface solitons at the edge and the corner of the two 2D media and dipole surface solitons at the edge of the two 2D media. These stable solitons do not exhibit a power threshold. The positions of the peak values and full width at half maximum(FWHM) of the surface solitons can be affected by the degree of nonlocality. However, the initial value can only influence the positions of the peak values of the surface solitons. In addition, when a laser beam is launched away from the interface, the beam will be periodic oscillations, even if the launch position is far away from the interface.

2. Theoretical Model

Considering a laser beam propagating along the interface between a nonlocal nonlinear medium and a linear medium, the complex amplitude $E(X, Y, Z)$ of the light field satisfies the scalar wave equation [32, 34, 39]

$$\nabla_{XY}^2 E + \frac{\partial^2 E}{\partial Z^2} + k_0^2 n^2 E = 0,$$

(1)
where $k_0 = 2\pi/\lambda$ is the wave number in vacuum, and $n = n_L + \Delta n$ for the nonlinear medium ($X \leq 0$) and $n = n_0$ for the linear medium ($X \geq 0$). $\Delta n$ represents the nonlinear contribution to the refractive index and may originate from any diffusive nonlinear effect, which can be written by [32]

$$w_m^2 \nabla_X^2 \Delta n - \Delta n + n_2 |E|^2 = 0,$$

where $w_m$ is the characteristic length of the nonlinear response and $n_2$ is the nonlinear index coefficient. For the local case, $w_m \to 0$, we have $\Delta n = n_2 |E|^2$.

Let us put $E(X, Y, Z) = E_0(X, Y, Z) \exp(i\beta Z)$ and submit into the equation (1) and (2). Then, using the slowly varying envelope approximation and introducing the normalized variables $x = X/w_0, y = Y/w_0, z = Z/(\beta w_0^2), a = k_0 w_0 \sqrt{n_2 n_L E_0}$ and $\phi = k_0 w_0^2 n_L \Delta n$, we get

$$\frac{1}{2} \nabla_{\bot}^2 a + i \partial_z a + \beta_1 a + \phi a = 0 \quad \text{for} \quad x \leq 0,$$

$$\frac{1}{2} \nabla_{\bot}^2 a + i \partial_z a + \beta_2 a = 0 \quad \text{for} \quad x \geq 0,$$

and

$$d^2 \nabla_{\bot}^2 \phi - \phi + |a|^2 = 0 \quad \text{for} \quad x \leq 0,$$

where $\nabla_{\bot} = \partial_x^2 + \partial_y^2$, $\beta_1 = w_0^2 (k_0^2 n_L^2 - \beta^2)/2$, $\beta_2 = w_0^2 (k_0^2 n_0^2 - \beta^2)/2$, $\beta$ is the wave number in the medium, $w_0$ is the beam width, and $d = w_m/w_0$ stands for the degree of nonlocality of the nonlinear response. For $x \leq 0$, the equations describe a local nonlinear response as $d \to 0$ and a strongly nonlocal response as $d \to \infty$.

We search for stationary soliton solutions of Eqs. (3) and (4) numerically in the form $a(x, y, z) = u(x, y) \exp(ibz)$, where $u$ is the real function and $b$ is a real propagation constant of spatial solitons in the normalized system.

$$\frac{1}{2} \nabla_{\bot}^2 u - bu + \beta_1 u + \phi u = 0 \quad \text{for} \quad x \leq 0,$$

$$\frac{1}{2} \nabla_{\bot}^2 u - bu + \beta_2 u = 0 \quad \text{for} \quad x \geq 0,$$

and

$$d^2 \nabla_{\bot}^2 \phi - \phi + |u|^2 = 0 \quad \text{for} \quad x \leq 0.$$
3. Numerical results

A. Single surface solitons

We firstly talk about new surface-wave soliton solutions at the edge of the two 2D media. Here, we assume that the normalized nonlinear contribution to the refractive index at the interface \((y = 0)\) has a initial value \(\phi_d(\phi_d > 0)\). The boundary conditions for the fields at the interface are the continuity of the transverse field \((a(y \to 0_-) = a(y \to 0_+))\) and its derivative \((da(y \to 0_-)/dy = da(y \to 0_+)/dy)\). Because the width of the surface solitons is much smaller than the sample width, \(a\) and \(\phi\) vanishes at the other boundary.

Three different solitons are separately shown in Fig. 1(a), (b) and (c). We can easily see that the soliton is closely attached to the interface with \(\phi_d = 7\) and \(d = 20\) in Fig. 1(b), but the solitons are farther detached from the interface with \(\phi_d = 1\) and \(d = 20\) in Fig. 1(a) or \(\phi_d = 7\) and \(d = 5\) in Fig. 1(c). Obviously, \(\phi_d\) and \(d\) influence the position or shape of the solitons. To further explain this point, we see that the positions of the peak values \((y_{\text{max}})\) and FWHM of the surface wave solitons versus the boundary value \(\phi_d\) (Fig. 1(d)) at the interface or the degree of nonlocality \(d\) (Fig. 1(e)). From Fig. 1(d), we can see that, under the condition \((d = 20)\), the soliton will be attracted to the interface and more and more significant part of their optical power residing in the linear medium as \(\phi_d\) increases. That is to say, \(\phi_d\) is larger, the larger a “surface force” exerted on the beam by the interface. However, \(\phi_d\) cannot influence the beam width of solitons. From Fig. 1(e), at \(\phi_d = 7\), the changing of the refractive index of the nonlocal nonlinear medium induced by \(d\) results in the changing of FWHM and \(y_{\text{max}}\) of the solitons. Because of the changing of FWHM, \(y_{\text{max}}\) changes intricately, though a force exerted on the beam by the degree of nonlocality increases all the while.

Fig. 2(a) shows that the energy flow \(U = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} |a|^2 dx dy\) of the single 2D surface solitons monotonically increases with \(b\) where \(dU/db > 0\). This shows that the solitons are stable [33, 34]. Here, to further elucidate the stability of the surface solitons, we do the direct numerical simulations of Eqs. (3) and (4) with input conditions \(a|_{Z=0} = u(1 + \rho)\), where \(\rho(x, y)\) is a broadband random perturbation. The fact which is shown in Fig. 2(b) confirms the result of Fig. 2(a). Then we proceed to address the dynamics behavior of the propagation of surface solitons. For convenience, the (1+1)D circumstance is considered. Fig. 2(c) depicts that a narrow beam is launched \(y = -1.99 \mu m\) away from the interface. The beam maintains a localized shape. However, it is oscillation in a fully periodic fashion in the
virtue of the cooperation of the forces exerted by the boundary and the nonlocal nonlinearity, even if the launch position is far away from the position of the surface soliton [34].

Next, we consider that new soliton solutions at the corner of the two 2D media. Here, we assume that the normalized nonlinear contribution to the refractive index at \(x = 0\) and \(y = 0\) have an initial value \(\phi = \phi_d (\phi_d > 0)\). The boundary conditions for the fields at the interface meet the continuity conditions.

Fig. 3(a), (b) and (c) separately show three different solitons. The soliton is closely attached to the interface with \(\phi_d = 10\) and \(d = 20\) (see Fig. 3(b)), but the solitons are farther detached from the interface with \(\phi_d = 5\) and \(d = 20\) (Fig. 3(a)) or \(\phi_d = 10\) and \(d = 10\) (Fig. 3(c)). To further illustrate the influence of \(\phi_d\) and \(d\) on the solitons, we display that \(y_{\text{max}}\) and FWHM versus \(\phi_d\) in Fig. 3(d) or \(d\) in Fig. 3(e). The energy flow \(U\) monotonically increasing with \(b\) shown in Fig. 4(a) and the direct numerical simulations of Eqs. (3) and (4) with noise \(\sigma^2_{\text{noise}} = 0.05\) shown in Fig. 4(b) explain that the solitons are stable. The results can be similarly illustrated as the solitons at the edge of the interface.

B. Two dimensions dipole surface solitons

In addition to single surface solitons, we also find a 2D stationary dipole surface solitons. The surface solitons are found numerically by a standard relaxation method which converges to a stationary solution after some iterations provided that a suitable guess for initial field distribution. The boundary conditions are the same as the single surface solitons at the edge of the interface.

Fig. 5(b) depicts that the amplitude for a dipole soliton which is closely attached to the interface with \(\phi_d = 10\) and \(d = 20\). However, the solitons are farther detached from the interface with \(\phi_d = 1\) and \(d = 20\) in Fig. 5(a) or \(\phi_d = 10\) and \(d = 10\) in Fig. 5(c). Because the poles of solitons are almost symmetric in the \(x\) direction, the changing of \(y_{\text{max}}\) and FWHM of one pole of the solitons can explain the changing of the position and shape of the dipoles. Fig. 5(d) and (e) shows \(y_{\text{max}}\) and FWHM as functions of the boundary value \(\phi_d\) and the degree of nonlocality \(d\), respectively.

Fig. 6(a) shows that the energy flow monotonically increases with the propagation constant for dipole surface solitons. With increasing energy flow, surface dipoles become more localized, i.e., the distance between poles along the \(x\) axis and their widths decrease. To further elucidate the stability of the surface dipoles, we do the direct numerical simulations of Eqs. (3) and (4) with noise \(\sigma^2_{\text{noise}} = 0.05\). Particularly, the complex surface solitons are
stable in the entire existence domain. Fig. 6(b) depicts a typical evolution dynamic. The considerable input perturbations cannot almost cause oscillations of amplitudes of the two poles forming the dipole, but the dipoles remain their internal structures over huge distances. Contrarily, in a bulk diffusive medium, it is known that weak input perturbations can cause slow but progressively increasing oscillations of the bright spots forming a dipole, resulting specially in their slow decay into fundamental solitons [35]. So, we think that the presence of a interface possessing a initial value for $\phi$ leads to stabilization of dipole solitons. This is further illustrated by the results of Fig. 6(c). when the beam is launched $y = -1.91 \mu m$ away from the interface, the stationary soliton cannot be formed, but it is periodically oscillating. This are very good description of the boundaries of the role of solitons.

4. Conclusion

In conclusion, we have addressed (2+1)D surface solitons occurring at the interface between a linear medium and a nonlocal nonlinear medium whose nonlinear contribution to the refractive index has a initial value at the interface. We find that there exist stable single and dipole surface solitons which do not exhibit a power threshold. The degree of nonlocality have influence on the positions of the peak value and FWHM of the surface solitons, but the initial value can only influence the positions of the peak values of the surface solitons. In addition, when a laser beam is launched away from the interface, the beam will be periodic oscillations, even if the launch position is far away from the interface.

Acknowledgments

This research was supported by the National Natural Science Foundation of China (Grants No. 11074080 and 10904041), the Specialized Research Fund for the Doctoral Program of Higher Education (Grant No. 20094407110008), and the Natural Science Foundation of Guangdong Province of China (Grant No. 1015106310100017).

References

1. M. Segev, B. Crosignani, A. Yariv, and B. Fischer, “Spatial solitons in photorefractive media,” Phys. Rev. Lett. 68, 923-926 (1992).
2. W. Krolikowsky, B. Luther-Davies, and C. Denz, “Photorefractive solitons,” Quantum Electron. 39, 3-12(2003).
3. C. Rotschild, O. Cohen, O. Manela, M. Segev, and T. Carmon, “Solitons in nonlinear media with an infinite range of nonlocality: first observation of coherent elliptic solitons and of vortex-ring solitons,” Phys. Rev. Lett. 95, 213904 (2005).

4. A. Dreischuh, D. N. Neshev, D. E. Petersen, O. Bang, and W. Krolikowski, “Observation of attraction between dark solitons,” Phys. Rev. Lett. 96, 043901 (2006).

5. C. Rotschild, B. Alfassi, O. Cohen, and M. Segev, “Long-range interactions between optical solitons,” Nat. Phys. 2, 769 (2006).

6. F. Ye, Bambi. Hu, Y. V. Kartashov, and L. Torner, “Nonlinearity-mediated soliton ejection from trapping potentials in nonlocal media,” Phys. Rev. A 82, 023822 (2010).

7. D. Buccoliero, A. S. Desyatnikov, W. Krolikowski, and Y. S. Kivshar, “Spiraling multivortex solitons in nonlocal nonlinear media,” Opt. Lett. 33, 198-201 (2008).

8. C. Conti, M. Peccianti, and G. Assanto, “Observation of optical spatial solitons in a highly nonlocal medium,” Phys. Rev. Lett. 92, 113902 (2004).

9. M. Peccianti, K. Brzadkiewicz, and G. Assanto, “Nonlocal spatial soliton interactions in nematic liquid crystals,” Opt. Lett. 27, 1460-1462 (2002).

10. M. Peccianti, C. Conti, G. Assanto, A. De Luca, and C. Umeton, “Routing of anisotropic spatial solitons and modulational instability in nematic liquid crystals,” Nature 432, 733C737 (2004).

11. A. Alberucci, M. Peccianti, G. Assanto, A. Dyadyusha, and M. Kaczmarek, “Two-color vector solitons in nonlocal media,” Phys. Rev. Lett. 97, 153903 (2006).

12. A. A. Minzoni, N. F. Smyth, and Z. Xu, “Stability of an optical vortex in a circular nematic cell,” Phys. Rev. A 81, 033816 (2010)

13. A. Alberucci, A. Piccardi, M. Peccianti, M. Kaczmarek, and G. Assanto, “Propagation of spatial optical solitons in a dielectric with adjustable nonlinearity,” Phys. Rev. A 82, 023806 (2010)

14. C. Rotschild, M. Segev, Z. Xu, Y. V. Kartashov, L. Torner, and O. Cohen, “Two-dimensional multipole solitons in nonlocal nonlinear media,” Opt. Lett. 31, 3312-3314(2006).

15. S. Skupin, O. Bang, D. Edmundson, and W. Krolikowski, “Stability of two-dimensional spatial solitons in nonlocal nonlinear media ,” Phys. Rev. E 73, 066603(2006).

16. A. I. Yakimenko, V. M. Lashkin, and O. O. Prikhodko, “Dynamics of two-dimensional coherent structures in nonlocal nonlinear media,” Phys. Rev. E 73, 066605(2006).
17. Y. V. Kartashov, L. Torner, V. A. Vysloukh, and D. Mihalache, “Multipole vector solitons in nonlocal nonlinear media,” Opt. Lett. 31, 1483-1485 (2006).
18. D. Buccoliero, A. S. Desyatnikov, W. Krolikowski, and Y. S. Kivshar, “Laguerre and Hermite soliton clusters in nonlocal nonlinear media,” Phys. Rev. Lett. 98, 053901 (2007).
19. V. M. Lashkin, “Two-dimensional nonlocal vortices, multipole solitons, and rotating multisoliotons in dipolar Bose-Einstein condensates,” Phys. Rev. A 75, 043607 (2007).
20. V. I. Kruglov, Y. A. Logvin, and V. M. Volkov, “The theory of spiral laser beams in nonlinear media,” J. Mod. Opt. 39, 2277-2291 (1992).
21. D. Briedis, D. Petersen, D. Edmundson, W. Krolikowski, and O. Bang, “Ring vortex solitons in nonlocal nonlinear media,” Opt. Express 13, 435-443 (2005).
22. A. I. Yakimenko, Y. A. Zaliznyak, and Y. Kivshar, “Stable vortex solitons in nonlocal self-focusing nonlinear media,” Phys. Rev. E 71, 065603 (R) (2005).
23. C. Rotschild, O. Cohen, O. Manela, M. Segev, and T. Carmon, “Two-dimensional multipole solitons in nonlocal nonlinear media,” Phys. Rev. Lett. 95, 213904 (2005).
24. Y. V. Kartashov, V. A. Vysloukh, and L. Torner, “Stability of vortex solitons in thermal nonlinear media with cylindrical symmetry,” Opt. Express 23, 9378-9384 (2007).
25. S. Lopez-Aguayo, A. S. Desyatnikov, Y. S. Kivshar, S. Skupin, W. Krolikowski, and O. Bang, “Stable rotating dipole solitons in nonlocal optical media,” Opt. Lett. 31, 1100-1102 (2006).
26. S. Lopez-Aguayo, A. S. Desyatnikov, and Y. S. Kivshar, “Azimuthons in nonlocal nonlinear media,” Opt. Express 14, 7903-7908 (2006).
27. A. Fratalocchi, A. Piccardi, M. Peccianti, and G. Assanto, “Nonlinearly controlled angular momentum of soliton clusters,” Opt. Lett. 32, 1447-1449 (2007).
28. W. J. Tomlinson, “Surface wave at a nonlinear interface,” Opt. Lett. 5, 323-325 (1980).
29. N. N. Akhmediev, V. I. Korneev, and Y. V. Kuzmenko, “Excitation of nonlinear surface waves by Gaussian light beams,” Sov. Phys. JETP 61, 62-67 (1985).
30. A. D. Boardman, A. A. Maradudin, G. I. Stegeman, T. Twardowski and E. M. Wright, “Exact theory of nonlinear p-polarized optical waves,” Phys. Rev. A 35, 1159-1164 (1987).
31. D. Mihalache, G. Stegeman, C. T. Seaton, R. zanoni, A. D. Boardman and T. Twardowski, “Exact dispersion relations for transverse magnetic polarized guided waves at
a nonlinear interface,” Opt. Lett. **12**, 187-189 (1987).

32. P. Varatharajah, A. Aceves, J. V. Moloney, D. R. Heatley, and E. M. Wright, “Stationary nonlinear surface waves and their stability in diffusive Kerr media,” Opt. Lett. **13**, 690-692 (1988).

33. Y. V. Kartashov, L. Torner, and V. A. Vysloukh, “Lattice-supported surface solitons in nonlocal nonlinear media,” Opt. Lett. **31**, 2595-2597 (2006).

34. B. Alfassi, C. Rotschild, O. Manela, M. Segev, and D. N. Christodoulides, “Nonlocal surface-wave solitons,” Phys. Rev. Lett. **98**, 213901 (2007).

35. F. Ye, Y. V. Kartashov, and L. Torner, “Nonlocal surface dipoles and vortices,” Phys. Rev. A **77**, 033829 (2008).

36. Y. V. Kartashov, F. Ye, V. A. Vysloukh, and L. Torner, “Surface waves in decofusing thermal media,” Opt. Lett. **32**, 2260-2262 (2007).

37. Y. V. Kartashov, V. A. Vysloukh, and L. Torner, “Multipole surface solitons in thermal media,” Opt. Lett. **34**, 283-285 (2009).

38. Y. V. Kartashov, V. A. Vysloukh, and L. Torner, “Ring surface waves in thermal nonlinear media,” Opt. Express **15**, 16216-16221 (2007).

39. H. Z. Kang, T. H. Zhang, B. H. Wang, C. B. Lou, B. G. Zhu, H. H. Ma, S. M. Liu, J. G. Tian, and J. J. Xu, “(2+1)D surface solitons in virtue of the cooperation of nonlocal and local nonlinearities,” Opt. Lett. **34**, 3298-3300 (2009).
List of Figure Captions

Fig. 1. Sketch of 2D single surface solitons at the edge of the interface with (a) $\phi_d = 1$, $d = 20$, (b) $\phi_d = 7$, $d = 20$ and (c) $\phi_d = 7$, $d = 5$. The positions of the peak values $y_{\text{max}}$ and FWHM versus (d) the boundary value $\phi_d$ and (e) the nonlocal degree $d$. White dashed line indicates interface position. All quantities are plotted in arbitrary dimensionless units.

Fig. 2. (a) Energy flow $U$ versus the propagation constant $b$ with $d = 20$ and $\phi_d = 7$. (b) Stable propagation of surface solitons in Fig.1(b) with noise $\sigma_{\text{noise}}^2 = 0.05$ for a distance of 15 diffraction lengths. (c) Trajectories of the incident beam with the beam center coordinates $y = -1.99\mu m$. White dashed line indicates interface position. All quantities are plotted in arbitrary dimensionless units.

Fig. 3. Sketch of 2D single surface solitons at the corner of the interface with (a) $\phi_d = 5$, $d = 20$, (b) $\phi_d = 10$, $d = 20$ and (c) $\phi_d = 10$, $d = 10$. The positions of the peak values $y_{\text{max}}$ and FWHM versus (d) the boundary value $\phi_d$ and (e) the nonlocal degree $d$. White dashed line indicates interface position. All quantities are plotted in arbitrary dimensionless units.

Fig. 4. (a) Energy flow $U$ versus the propagation constant $b$ with $d = 20$ and $\phi_d = 10$. (b) Stable propagation of surface solitons in Fig.3(b) with noise $\sigma_{\text{noise}}^2 = 0.05$ for a distance of 15 diffraction lengths. White dashed line indicates interface position. All quantities are plotted in arbitrary dimensionless units.

Fig. 5. Sketch of 2D dipole surface solitons with (a) $\phi_d = 1$, $d = 20$, (b) $\phi_d = 10$, $d = 20$ and (c) $\phi_d = 10$, $d = 10$. The positions of the peak values $y_{\text{max}}$ and FWHM versus (d) the boundary value $\phi_d$ and (e) the nonlocal degree $d$. White dashed line indicates interface position. All quantities are plotted in arbitrary dimensionless units.

Fig. 6. (a) Energy flow $U$ versus the propagation constant $b$ with $d = 20$ and $\phi_d = 10$. (b) Stable propagation of surface solitons in Fig.5(b) with noise $\sigma_{\text{noise}}^2 = 0.05$ for a distance of 15 diffraction lengths. (c) Trajectories of the incident beam with the beam center coordinates $y = -1.91\mu m$. White dashed line indicates interface position. All quantities are plotted in arbitrary dimensionless units.
Fig. 1. Sketch of 2D single surface solitons at the edge of the interface with (a) $\phi_d = 1$, $d = 20$, (b) $\phi_d = 7$, $d = 20$ and (c) $\phi_d = 7$, $d = 5$. The positions of the peak values $y_{\text{max}}$ and FWHM versus (d) the boundary value $\phi_d$ and (e) the nonlocal degree $d$. White dashed line indicates interface position. All quantities are plotted in arbitrary dimensionless units.
Fig. 2. (a) Energy flow $U$ versus the propagation constant $b$ with $d = 20$ and $\phi_d = 7$. (b) Stable propagation of surface solitons in Fig.1(b) with noise $\sigma_{\text{noise}}^2 = 0.05$ for a distance of 15 diffraction lengths. (c) Trajectories of the incident beam with the beam center coordinates $y = -1.99\mu m$. White dashed line indicates interface position. All quantities are plotted in arbitrary dimensionless units.
Fig. 3. Sketch of 2D single surface solitons at the corner of the interface with (a) $\phi_d = 5$, $d = 20$, (b) $\phi_d = 10$, $d = 20$ and (c) $\phi_d = 10$, $d = 10$. The positions of the peak values $y_{\text{max}}$ and FWHM versus (d) the boundary value $\phi_d$ and (e) the nonlocal degree $d$. White dashed line indicates interface position. All quantities are plotted in arbitrary dimensionless units.
Fig. 4. (a) Energy flow $U$ versus the propagation constant $b$ with $d = 20$ and $\phi_d = 10$. (b) Stable propagation of surface solitons in Fig.3(b) with noise $\sigma^2_{\text{noise}} = 0.05$ for a distance of 15 diffraction lengths. White dashed line indicates interface position. All quantities are plotted in arbitrary dimensionless units.
Fig. 5. Sketch of 2D dipole surface solitons with (a) $\phi_d = 1$, $d = 20$, (b) $\phi_d = 10$, $d = 20$ and (c) $\phi_d = 10$, $d = 10$. The positions of the peak values $y_{max}$ and FWHM versus (d) the boundary value $\phi_d$ and (e) the nonlocal degree $d$. White dashed line indicates interface position. All quantities are plotted in arbitrary dimensionless units.
Fig. 6. (a) Energy flow $U$ versus the propagation constant $b$ with $d = 20$ and $\phi_d = 10$. (b) Stable propagation of surface solitons in Fig.5(b) with noise $\sigma_{\text{noise}}^2 = 0.05$ for a distance of 15 diffraction lengths. (c) Trajectories of the incident beam with the beam center coordinates $y = -1.91\mu m$. White dashed line indicates interface position. All quantities are plotted in arbitrary dimensionless units.