Modelling of inhomogeneous chamber states in rotary positive displacement vacuum pumps

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Abstract. Chamber model simulation is a common approach to simulate rotary positive displacement vacuum pumps. Therefore the pump is abstracted into working chambers and connecting clearances, whereby the clearance leakages can be identified as the major loss mechanism in such machines. The clearance mass flow rates are calculated with respect to the thermodynamic states in the adjacent chambers, which are normally considered to be homogeneous. In this work it is shown, that the chamber state is inhomogeneous for rarefied gases due to the movement of the pistons which causes a pressure gradient within the chamber. This effect increases with higher Knudsen numbers, because of the increasingly dominant friction. An efficient way to model these inhomogeneous states with a one-dimensional approach for geometrically abstracted chambers is shown. The approach is applied to chamber model simulations of a screw spindle vacuum pump (SSVP) and the results are compared to simulations with homogeneous chamber states and to measurements.

1. Introduction

Dry running vacuum pumps have become an important part of vacuum systems due to their oil-free working principle. The need for a clean vacuum, for instance in the semiconductor industry, has pushed the development of dry running positive displacement vacuum pumps, such as screw and roots pumps (see Ref. [1]). Due to contactless operation of the pistons the sealing between the working chambers is realized by small clearances. Regarding figure 1 the effective mass flow rate of a screw machine can be calculated by the delivered mass flow rate $\dot{m}_d$ of a working chamber driven by the rotation of the pistons and reduced by the clearance mass flows $\dot{m}_{\text{clearance}}$. Clearances are located between both rotary pistons like the radial clearance, the inter-lobe clearance and the blow hole and between the rotary pistons and the housing. In order to calculate the thermodynamic operation of these pumps, the knowledge of these clearance mass flow rates is essential, which represent the main loss mechanism. A common approach
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Table 1. List of most important symbols

| symbol | explanation              | unit       | symbol         | explanation               | unit   |
|--------|--------------------------|------------|----------------|---------------------------|--------|
| $A$    | cross section area       | m$^2$      | $c_m$         | most probably speed       | m/s    |
| $D$    | diameter                 | m          | $f$           | rotational speed          | Hz     |
| $G$    | reduced flow rate        | -          | $h$           | chamber/clearance height  | m      |
| $L$    | chamber Length           | m          | $m$           | mass                      | kg     |
| $\dot{m}$ | mass flow rate         | kg/s       | $p$           | pressure                  | Pa     |
| $R$    | specific gas constant    | J/(kg K)   | $s$           | rotor lead                | m      |
| $T$    | temperature              | K          | $U$           | velocity                  | m/s    |
| $V$    | volume                   | m$^3$      | $w$           | chamber width             | m      |
| $z$    | number of lobes          | -          | $\delta$      | gas rarefaction parameter | -      |
| $\eta$ | cross section ratio      | -          | $\mu$         | dynamic viscosity         | Pa s   |
| $\rho$ | density                  | kg/m$^3$   | $\psi$        | wrap angle                | -      |

to calculate the thermodynamic operation of vacuum pumps is chamber model simulation where the pump is divided in a network of chambers that transport gas from low pressure port to high pressure port and connecting clearances which cause a mass flow rate in opposite direction [2–4]. The thermodynamic state of the gas within the chambers is normally considered to be homogeneous and the clearance boundary conditions are taken from mean values within the adjacent chambers.

The calculation of clearance flows for rarefied gases including moving boundaries has previously been investigated in references [5–7]. In this paper it is shown, that these chamber states are not homogeneous for rarefied gases but a pressure gradient occurs in tangential direction of the rotor lobe.

Figure 2 shows the chamber model abstraction for a screw spindle vacuum pump (SSVP) with the sketched network of chambers and connecting clearances. In this representation pressure $p$ related to the chamber’s mean pressure $p_a$ is sketched. The inhomogeneous pressure distribution causes the clearances to not only connect two adjacent chambers, but the high pressure side and a low pressure side of adjacent chambers. Thus the boundary conditions of the clearances have to be corrected for better agreement of simulation results to the pumps’ operation performance.
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\[ U_{ax} = s \cdot f \]  

\[ U_{circ} = \pi \cdot D \cdot f \]  

Therefore the magnitude of the relative movement \( U_r \) of rotor points within the chamber’s fixed coordinate system can be obtained by

\[ U_r = \frac{\pi D \cdot f}{\cos(\varphi)} \quad \varphi(D) = \tan^{-1} \left( \frac{s}{D} \right) \quad D_l \leq D \leq D_o \]
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![Diagram](image)

Figure 3. Geometry of a 2-lobed cycloid profile and boundary velocities in the chamber fixed relative system of screw vacuum pumps.

where \( D_o \) is the vacuum pump’s outer rotor diameter and \( D_i \) the inner rotor diameter respectively.

The housing \( (h) \) does not move, so the housing’s velocity \( U_h = -U_{ax} \) is negative to the movement of the relative system. This can be divided into a part tangential and normal to the rotor lead:

\[
U_{h,\text{tangential}} = s \cdot f \cdot \sin(\varphi(D_o)) \\
U_{h,\text{normal}} = s \cdot f \cdot \cos(\varphi(D_o))
\]  

(4)

So in every cross section of a chamber there are four bounding surfaces causing a Couette flow in the same direction tangential to the rotor lobe and the housing which also causes a flow in direction normal to the rotor lobe. Because of the chamber’s encapsulation due to the other rotor and the surrounding housing this flow is dammed up. This leads to a pressure gradient which causes a Poiseuille flow in opposite direction of the Couette flow caused by the moving boundaries.

Geometrically the chamber can be abstracted as a channel with rectangular cross section with

\[
h = \frac{D_o - D_i}{2} \\
w = \frac{s}{2z} \cos(\varphi(D_{pc}))
\]  

(5)

the channel height and width is the linearized mean distance between two lobes. \( D_{pc} \) is the pitch circle diameter and mean value of inner and outer diameter. The chamber length is defined as

\[
L = (\pi - \alpha_{CL}) \cdot D_{pc} \cdot \sqrt{1 + \left(\frac{s}{\pi D_{pc}}\right)^2} \\
\alpha_{CL} = \tan^{-1}\left(\frac{\sqrt{D_o^2 - D_{pc}^2}}{D_{pc}}\right)
\]  

(6)
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with the angle of the cut lens defined in figure 3. This is the length of the helix, at the pitch circle. The end chamber geometry is defined between the intersection lines of rotor axis and cusps of the cut lens which is the intersection area of the outer diameters of both rotors (see the yellow and red lines in figure 3). The mean pressure in these intersection lines are defined as the chambers high pressure \( p_{HP} \) and low pressure \( p_{LP} \) respectively, which are used as inlet and outlet boundary values of adjacent clearances. Thus the chamber is defined with constant cross section area for constant rotor lead \( s \).

The impact of a non-constant rotor lead on the pressure distribution within the chamber is neglected in this work.

2.2. Governing equations

In order to calculate the pressure distribution within the chamber the approach of an isothermal channel flow model is used similar to the approach in [5,9–11]. Thus in steady state the clearance mass flow rate \( \dot{m}_{cl,t} \) tangential to the lobe can be expressed as the sum of the mass flow rate \( \dot{m}_C \) caused by a Couette flow and the mass flow rate \( \dot{m}_P \) caused by a Poiseuille flow:

\[
\dot{m}_{cl,t} = \dot{m}_C + \dot{m}_P
\]

The tangential clearance mass flow rate is the sum of the mass flow rates of the blow hole, the radial clearance and the intermesh clearance connected to the chamber. Furthermore when there is a difference between inflowing and outflowing housing clearance mass flow rate, this could be modelled as mass sources along the chamber length and thus \( \dot{m}_{cl,t} \) is not constant. The mass flow rate caused by a Couette flow in a rectangular channel is the sum of the mass flow rate caused by each moving wall:

\[
\dot{m}_C = \rho A \cdot \sum_{j=1}^{4} G_{C,j}(\delta, \eta) \cdot U_{w,j}
\]

\( \rho \) is the density, \( A = hw \) the cross section area, \( U_{w,i} \) is the mean velocity of the respective wall and \( G_{C,i} \) is the shape factor depending on the geometric ratio \( \eta = \frac{\tilde{w}^2}{A} \), with \( \tilde{w} \) the width of the respective moving wall within the cross section and

\[
\delta = \frac{\min(h, w) \cdot p_{ch}}{\mu \cdot c_m}
\]

is the gas rarefaction parameter with the dynamic viscosity \( \mu \) and

\[
c_m = \sqrt{2RT}
\]

the molecules’ most probable speed depending on the specific gas constant \( R \) and temperature \( T \). For \( \eta \geq 1 \) the following formula for the shape factor is used:

\[
G_C(\delta, \eta) = G_{C,mol}(\eta) + k_1 \ln(\eta) \tan^{-1}(k_2 \delta \eta + k_3 \delta) \exp(k_4 \ln^2(k_5 \eta))
\]

\[
k_1 = 86.74 \quad k_2 = 2.66 \quad k_3 = -1.07 \quad k_4 = -23.08 \cdot 10^{-3} \quad k_5 = 46.08 \cdot 10^{6}
\]

with the analytical molecular solution with diffuse wall scattering

\[
G_{C,mol}(\eta) = \frac{1}{4\pi} \left[ \left( \eta - \eta^{-1} \right) \ln \left( \eta^2 + 1 \right) - 2\eta \ln(\eta) + 4 \tan^{-1}(\eta) \right]
\]
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The factors in equation 12 are results of a non linear regression with the Levenberg Marquardt algorithm which hold a maximum error smaller than 1% and an average error of 0.27% to analytical solutions of the hydrodynamic regime ($\delta \rightarrow \infty$) and the free molecular regime ($\delta = 0$) and solutions of the DSMC method in a large variety of $0.01 \leq \delta \leq 100$ and $1 \leq \eta \leq 100$ in between. [12]

For $\eta < 1$ the relation

$$G_C(\delta, \eta) = \frac{1}{2} - G_C(\delta, \eta^{-1}) \quad \forall \quad \eta < 1 \quad (14)$$

is used. The mass flow rate of the Poiseuille flow reads as follows

$$\dot{m}_P = -\frac{\min(h, w) \cdot A}{c_m} \cdot G_P(\delta, \eta) \cdot \frac{\partial p}{\partial x} \quad (15)$$

and is related to the pressure gradient in flow direction (in this case the tangential lobe direction, see the local coordinate system in figure 3). The dimensionless flow rates $G_P$ used in this model are described in a previous work [5] and the values are related from references [11,13,14]. Note, that due to symmetry $G_P(\delta, \eta) = G_P(\delta, \eta^{-1})$ holds.

Inserting equations 8 and 15 in equation 7 leads to a differential equation in tangential lobe direction which is solved by an explicit Euler method

$$p_{i+1} = p_i + \frac{2\Delta x}{G_P(\delta_i, \eta) \min(h, w)c_m} \left( p_i \sum_{i=1}^{4} G_C, \eta \cdot U_w, - \frac{\dot{m}_{cl,t}RT}{hw} \right) \quad (16)$$

where the local rarefaction parameter $\delta_i(p_i)$ is used. To solve this equation the tangential clearance mass flow rate $\dot{m}_{cl,t}$ needs to be known, which can principally be used from the last time step within chamber model simulation. In this work it is assumed to be small compared to the couette flow within the chamber and is therefore neglected.

3. Chamber model simulation

3.1. Geometric and thermodynamic model

As sketched in figure 2 for the chamber model the vacuum pump is divided into chambers that transport mass from low pressure port to high pressure port and connecting clearances. Therefore the geometric properties of the chambers and clearances as a function of the rotation angle are needed. The geometric model used in this paper is adapted from references [15] and [16]. As a new feature the clearances are not simply connected with two chambers, but with a certain chamber position. Therefore the blow hole connects the high pressure side of two chambers, while the radial and inter-lobe clearance connect the high pressure side of the chamber at higher rotation angle with the low pressure side of the chamber with lower rotation angle considering the rotation angle to start at the low pressure port. The boundary conditions of the housing clearance are taken from mass averaged mean values of the certain chamber. The mass flow rates for given geometric and thermodynamic boundary conditions are obtained by tabulated data of measurements and simulations of the specific clearance shapes investigated and described in references [3] and [17].
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Due to the chamber’s geometric abstraction for the 1D model, there is a discrepancy between the 1D model’s chamber volume $V_{1D}$ and the total chamber volume $V_t$, where the volume within the cut lense is neglected. Thus for chambers, that are not connected to the high or low pressure port, the total chamber volume is divided as follows

$$V_t = V_s + V_{1D} + V_e$$

where $V_s = V_e$ are the chamber’s start and end volume respectively. This classification is performed as the total mass $m_t$ within the chamber is divided on these three volumes with the linearization, that the pressure within the end volumes is constant with

$$p(V_s) = p_{LP} \quad \text{and} \quad p(V_e) = p_{HP}$$

being the high and low pressure boundaries calculated by the 1D model. Thus the pressure distribution calculated with equation 16 has to match the constraint

$$m_t = \frac{1}{RT} \left( V_s p_{LP} + A \int_0^L p \, dx + V_e p_{HP} \right)$$

where $p_{LP}$ is found by a shooting method according to reference [18]. The integral can be solved numerically

$$m_{1D} = \frac{A}{RT} \int_0^L p \, dx \approx \frac{A}{RT} \sum_{i=1}^N p_i \Delta x_i$$

where $N$ is the number of increments used to solve the differential equation 16. For a chamber which is open to low pressure or high pressure port the effective chamber length is reduced, because the pressure at inlet or outlet area is fixed. The new model is implemented in the chamber simulation software $KaSim$ developed at the Chair of Fluidics of TU Dortmund University described in reference [19].

3.2. Comparison to measurements

In order to work out the impact of the new investigated model for inhomogeneous chamber states, simulations for a test machine are performed and compared to measurements. This machine is an isochoric two-lobed cycloid profiled SSVP. The dimensionless data is given in table 2, where $L_R$ is the rotor length, $h_h$, $h_{ilc}$ and $h_{rc}$ are clearance heights of the housing clearance, the inter-lobe clearance and the radial clearance respectively and $\psi_t$ is the machines total wrap angle. As the machine is cooled it is assumed to be fully isothermal at room temperature $T = 293 K$. The machine neither has a radial inlet area nor a pressure relief valve.

| $D_o/D_i$ | $D_o/s$ | $D_o/L_R$ | $D_o/h_h$ | $D_o/h_{ilc}$ | $D_o/h_{rc}$ | $\psi_t$ | $z$ |
|------------|---------|------------|------------|---------------|---------------|----------|-----|
| 1.571      | 5.641   | 0.667      | 880        | 733.33        | 733.33        | 3474°    | 2  |
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Figure 4 shows the ratio of the suction speed $\dot{V}$ to the theoretical suction speed $\dot{V}_{th}$

$$\dot{V} = \frac{\dot{m}_{\text{effective}} \cdot R \cdot T}{p_{suction}} \quad \dot{V}_{th} = 2 \cdot V_{ch,max} \cdot f \cdot z \quad (21)$$

as a function of the suction pressure $p_{suction}$. $\dot{m}_{\text{effective}}$ is the effective mass flow rate of the machine. The discharge pressure is always $10^5$ Pa. Comparing the results of $KaSim$ without the new model (red line) to the measurements, a good agreement is obtained for large values of the suction pressure. But with the decrease of the suction pressure those two curves diverge. The measurements show that the machine cannot evacuate the low pressure volume lower than about 6 Pa at the given rotational speed, while the simulated suction speed is still at a high value. Regarding the new model (blue line) the curve is not affected in the high pressure regime, but collapses at low suction pressures in qualitatively good agreement with the measurements. But the new model in the current implementation slightly seems to overrate the effect, as the results indicate a worse suction speed than the measurements.

![Figure 4](image_url)

Figure 4. Comparison of the suction speed curves of measurements and chamber model simulation with homogeneous and inhomogeneous pressure distribution within the chambers for a rotational speed $f = 83.33$ Hz at constant temperature $T = 293$ K.

Figure 5 shows the pressure distribution within the machine as a function of the wrap angle $\psi$ for a suction pressure $p_{suction} = 10$ Pa and $p_{suction} = 100$ Pa. The straight lines show the pressure distribution of the old model with homogeneous chamber states while the dashed lines show the mass averaged mean pressure distribution of the new model. The dotted lines show the chamber’s low and high pressure distribution respectively. As expected all lines converge to the same line during the transport phase when the pressure is high enough. Big differences are obtained at the low pressure regime especially during the suction phase. As the model with homogeneous chamber states delivers a mean chamber pressure which is not significantly lower than the
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![Graph showing pressure distribution](image)

**Figure 5.** Comparison of the pressure distribution within the vacuum pump for homogeneous and inhomogeneous pressure distribution within the chambers.

suction pressure, the new model simulates a pressure drop until the chamber is fully encapsulated. Thus in this regime an immense suction throttling is depicted with this approach. Nevertheless the maximum pressure ratios within the chamber seem very high. Especially in the suction phase an improvement is expected when the impact of the clearance mass flows on the pressure distribution is considered. In the current implementation the clearances just affect the chamber’s mass and therefore the mean pressure but the last term in equation 16 is neglected.

4. Conclusion

In this work a one-dimensional model in order to calculate the pressure distribution within the chambers of an SSVP has been investigated. This model has been implemented into the chamber model simulation software *KaSim*. A comparison with measurements shows a great improvement of the simulation results due to the new model as especially the suction phase at low suction pressure is highly affected by inhomogeneous chamber states. In order to have a further improvement of the chamber model simulation, the impact of clearance mass flow rates on the chamber’s pressure distribution should be included. Furthermore a better accuracy is expected if the mass flow rates of the clearances are calculated by the model presented in reference [6]. In order to validate the one-dimensional approach of the new model in detail three-dimensional CFD simulations including Maxwell velocity slip boundary conditions as described in reference [20] should be performed.
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