Coulomb gas representation of quantum Hall effect on Riemann surfaces

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Abstract

Using the correlation function of chiral vertex operators of the Coulomb gas model, we find the Laughlin wavefunctions of quantum Hall effect, with filling factor \(\nu = 1/m\), on Riemann surfaces with Poincare metric. The same is done for quasihole wavefunctions. We also discuss their plasma analogy.

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1 Introduction

Study the behaviour of electrons living on a two dimensional surface and interacting with a constant magnetic field orthogonal to the surface, is one of the most important part of physics, known as the quantum Hall effect (QHE). When the surface is plane, both integer QHE and fractional QHE have been observed experimentally, and have been described by Landau and Jastrow-type wavefunctions, respectively [1]. Also by introducing new types of many body condensates which carry fractional charge, i.e. anyons, an unified picture of integer and fractional QHE has been given. Furthermore, as it has been stressed by Laughlin, the Laughlin-Jastrow wavefunctions, both for electrons and anyons have a natural interpretation in terms of a two dimensional plasma of charges, interacting by Coulomb forces and embedded in a uniform neutralizing background [1].

A particular intriguing and interesting case occurs when the two dimensional surface is a Riemann surface of higher genus. Although it is not accessible experimentally, the problem of the physics on Riemann surfaces has deep relation with some interesting problems, like the occurrence of chaos in the surface with negative curvature [2], and recent developments in the theory of surfaces, for example, the moduli of surface and the vector bundle defined on the moduli [3]. Until now, the QHE on different non-flat surfaces have been studied. For example on the sphere [4], torus [5], and on the hyperbolic plane [6,7]. A recent and detailed investigation has been done in ref.[8] which the Landau and Laughlin levels are studied on Riemann surface with some particular metrics.

In [8], the authors have shown that the wavefunctions consist of two parts. A holomorphic part which is independent of metric and a known metric-dependent function. For Landau levels, they have shown that the holomorphic part is the Slater determinant of sections of the holomorphic line bundle, and for the Laughlin states they have an ansatz for the holomorphic part and it has been shown that this function is the determinant of the holomorphic sections of a vector bundle. This vector bundle is the tensor product of a line bundle and a flat vector bundle of rank $m$ ($\nu = 1/m$ is the filling factor).

Now, there is another approach for studying the QHE in which the conformal symmetry of QHE is used to calculate the different quantities. There are several evidences for the existence of this symmetry. For example in ref.[9], it has been shown that the Laughlin wavefunctions are related to the conformal blocks of two dimensional conformal field theories (CFT). In the same paper, the fractional statistics of quasiparticles has been related to braiding properties of the vertex operators of the Coulomb gas model, which has conformal symmetry. In ref.[10], it has been shown that the Halperin-Haldane singlet quantum Hall effect wavefunction can be split into two parts. One part is related to a state describing a one component plasma (OCP) system, and the other part behaves like a conformal block of primary fields of the $su(2)$ Wess-Zumino-Witten model. Also in ref.[11], it has been shown that the holomorphic
part of the Laughlin wavefunction on the torus, can be obtained by the correlation function of the Coulomb gas vertices. Moreover, it has been pointed out that the Coulomb gas approach and the OCP description of the Laughlin wavefunction are consistent.

In this paper we will study the QHE on arbitrary Riemann surface, in the context of CFT. Our purposes for this investigation are as follows. First to see that can this relation between QHE and Coulomb gas model be generalized to general Riemann surfaces? Second, as we will see, this approach is much easier than those which has been considered in [8]. And third, our approach can be easily generalized to the case where anyons also exist.

The plan of this paper is as follows. In section 2 we will discuss in brief the relation between OCP and CFT description of QHE. In section 3, by using the correlation function of the Coulomb gas vertices on Riemann surface (derived in [12-15]), we obtain the holomorphic part of the Laughlin wavefunction. To do this, we must appropriately determine the parameters of the corresponding Coulomb gas model. We will also discuss the different aspects of this equivalence. In section 4, we obtain an expression for quasiholes wavefunctions and determine their charges in this context.

2 QHE on the plane

To obtain an insight about the relation between QHE and CFT, let us recall the Laughlin wavefunction. It has been shown by Laughlin [1], that the wavefunction of the QHE is the many particle wavefunction which looks like:

$$\psi(z_1, \bar{z}_1, \cdots, z_N, \bar{z}_N) = N \prod_{i=1}^{N} e^{-\int A_{\bar{z}_i} d\bar{z}_i} F(z_1, \cdots, z_N).$$

$$z_j = x_j + i y_j,$$ is the position of $j$-th particle and $A_{\bar{z}} = \frac{1}{2}(A_x + i A_y)$ is the gauge potential. The main purpose of theoretical investigations of QHE is to determine the holomorphic function $F(z_1, \cdots, z_N)$ which must obey the Fermi statistics. Laughlin chose this function to be the eigenfunction of angular momentum, and showed that $\prod_{i<j} (z_i - z_j)^m$ is appropriate function for $\nu = 1/m$ filling factor. The final result, in $A_x = -\frac{B}{2} y$ and $A_y = \frac{B}{2} x$ gauge, is

$$\psi(z_1, \bar{z}_1, \cdots, z_N, \bar{z}_N) = \prod_{i<j} (z_i - z_j)^m \prod_{i=1}^{N} e^{-\frac{1}{4}|z_i|^2},$$

By introducing the classical potential energy $U$ through $|\psi|^2 = e^{-\beta U}$, where $\beta^{-1}$ is an arbitrary effective temperature, Laughlin showed that this system is equivalent to a two dimensional plasma of particles with electric charge $m$, interacting by Coulomb forces and embedded in a uniform neutralizing background.

Now the interesting point is that, this OCP description of QHE can also be achieved by considering the Coulomb gas model. This model is a free massless scalar field which modified
with a background charge at infinity. The two point function of these fields satisfies in \[16\]
\[\partial_z\partial_{\bar{z}} < \Phi(z, \bar{z})\Phi(w, \bar{w}) > = \pi\delta^2(z, w), \tag{3}\]
and \(\Phi(z, \bar{z})\) splits into holomorphic and antiholomorphic parts. Note that this equation is nothing but the Laplace equation for the Coulomb interaction. If \(\varphi(z)\) is the holomorphic part of \(\Phi(z, \bar{z})\), then the expectation value of the product of vertex operators \(e^{iq\varphi(z_i)}:\)
is
\[F(z_1, \cdots, z_N) = < \cdots e^{iq\varphi(z_i)} e^{iq\varphi(z_j)} \cdots > = e^{-q_i\varphi_i\sum_{i<j}<\varphi(z_i)\varphi(z_j)>}. \tag{4}\]
Now as on the plane \(<\varphi(z)\varphi(z) > = -\ln(z_i - z_j)\), which is the Coulomb potential on two dimension, the holomorphic part of eq.(2) is recovered by choosing \(q_i = \sqrt{m}\) for all \(i\)'s (as the particles are identical) and \(\beta = 1/m\). In this manner the Coulomb gas and QHE relate to each other on the plane, that is each vertex corresponds to an electron and its conformal charge \(\sqrt{m}\) relates to the electric charge of particles of plasma. In summary as the Coulomb gas model is effectively a theory of Coulomb interaction and QHE has a plasma analogy which again is based on Coulomb interaction, therefore the result of these two theories coincide. In the next section we will use this correspondence and also the braiding properties of vertex operators, to find the Laughlin wavefunctions on Riemann surfaces.

3 Coulomb gas approach of QHE on Riemann surface with Poincare metric

Now we study the QHE on a two dimensional compact and orientable Riemann surface \(\Sigma\). On this surface, the charged particles interact with constant orthogonal magnetic field produced by monopoles. We choose, as in [8], the Poincare metric \(g_{z\bar{z}} = y^{-2}\). The simply connected covering space of \(\Sigma\) is the upper half plane \(H\) and \(\Sigma = H/\Gamma\), where \(\Gamma\) is a discrete subgroup of the isometry group of \(H\). \(\Gamma\) is generated by Fuchsian transformations around canonical homology basis. For a covariantly constant magnetic field \(B\), and in the symmetric gauge \(A_z = A_{\bar{z}} = B/2\), the one particle Hamiltonian is [8]
\[H = -g^{z\bar{z}}D\bar{D} + B/4, \tag{5}\]
where \(D = \partial - \frac{B}{2}\partial\ln g_{z\bar{z}}\) and \(\bar{D} = \bar{\partial} + \frac{B}{2}\bar{\partial}\ln g_{z\bar{z}}\) and we take the electron mass \(m = 2\) for simplicity. The ground state wavefunction satisfies in
\[\bar{D}\psi = 0, \tag{6}\]
with solution \(\psi(z, \bar{z}) = g_{z\bar{z}}^{-\frac{B}{4}}F(z) = y^B F(z)\), where \(F(z)\) is a holomorphic function. The behaviour of \(F(z)\) under Fuchsian transformation have been discussed in [8]. But here, we
want to solve this problem in the context of CFT, so we need to find the behaviour of the wavefunction under a larger transformation, that is the general conformal transformation which the Fuchsian transformation is a subclass of it. To do so, we note that for two dimensional surface with Poincare metric, the conformal transformation, which leaves the metric invariant up to a scale change

\[ g_{\tilde{z}\tilde{\bar{z}}}d\tilde{z}d\tilde{\bar{z}} = \Omega g_{z\bar{z}}dzd\bar{z}, \]  

reduces to analytic coordinate transformations

\[ \tilde{z} = f(z) \quad ; \quad \tilde{\bar{z}} = \bar{f}(\bar{z}), \]  

where \( f(\bar{f}) \) is a holomorphic (antiholomorphic) function. Under conformal transformation \( D \) and \( \bar{D} \) change as

\[ \tilde{D} = \frac{dz}{d\tilde{z}}U^{-1}DU \quad ; \quad \bar{D} = \frac{d\bar{z}}{d\tilde{\bar{z}}}U'^{-1}DU', \]  

where

\[ U(z, \bar{z}) = \Omega^{-B/2} \left( \frac{dz}{d\tilde{z}} \right)^{-B/2} \left( \frac{d\bar{z}}{d\tilde{\bar{z}}} \right)^{B/2}; \quad U'(z, \bar{z}) = \Omega^{B/2} \left( \frac{dz}{d\tilde{z}} \right)^{-B/2} \left( \frac{d\bar{z}}{d\tilde{\bar{z}}} \right)^{B/2}. \]  

The Hamiltonian (5) in new coordinate is

\[ H = -g_{\tilde{z}\tilde{\bar{z}}}\tilde{D}\tilde{\bar{D}} + B/4, \]  

and the transformed ground state wavefunction \( \tilde{\psi} \) satisfies in \( \tilde{D}\tilde{\bar{D}}\tilde{\psi} = 0 \). Using (6) we find \( \psi = U'\tilde{\psi} \) and then using (10) we obtain:

\[ \tilde{\psi}\Omega^{B/2}dz^{B/2}d\tilde{\bar{z}}^{-B/2} = \psi dz^{B/2}d\bar{z}^{-B/2}. \]  

Now considering the decomposition \( \psi(z, \bar{z}) = y^B F(z) \), and putting it in (12), we find that \( F(z) \) must be a primary field of weight \( B \), i.e. a \( B \)-form under general conformal transformation.

As was mentioned in introduction, the authors of [8] have found the holomorphic part of Landau and Laughlin wavefunction by lengthy calculations. Here we want to calculate these functions by using the plasma analogy of QHE, that is using again the Green functions of Coulomb gas, but now on Riemann surface. So let us first bring a quick review about the Coulomb gas model on Riemann surface. This model is defined by a bosonic scalar field coupled to a background charge \( Q \) and described by the following action [15]

\[ S = \frac{1}{2\pi} \int d^2 z (\partial \Phi(z, \bar{z})\bar{\partial} \Phi(z, \bar{z}) + \frac{1}{4} Q \sqrt{g} R\Phi(z, \bar{z})), \]  

where \( R \) is the scalar curvature of the surface and \( g = \det g_{\mu\nu} \). In the following we shall only consider the holomorphic part of the correlation functions and hence we require \( \varphi (}
the holomorphic part of $\Phi$) to compactify on a unit circle $R/2\pi Z$ [15]. $R$ is real line and $Z$ denotes integer numbers. The correlation function of vertex fields $<\prod_{j=1}^{N} : e^{i\alpha_j \varphi(z_j)} : >$ is calculated in different context [12-15]. In [12], it is obtained by successive application of Wick theorem and by considering the effect of zero modes. In [13], it is shown that this correlation function can be derived by splitting $\varphi(z)$ to its zero and nonzero mode components

$$\varphi(z) = 2\pi \sum_{i=1}^{g} p_i \int \omega_i(\nu) d\nu + \hat{\varphi}(z),$$

(14)

where $p_i$ and $\hat{\varphi}(z)$ are independent free fields and $p_i$ are zero mode oscillators. The contraction rule for $\hat{\varphi}(z)$ is $<\hat{\varphi}(z)\hat{\varphi}(w)> = -\ln E(z,w)$ [13,15], which is the Green function of two charges located at $z$ and $w$, interacting via a Coulomb potential in two dimension. In [14], the correlation function is obtained by using the $b-c$ system, which is described by the first order action $S = \int d^2z b\partial \bar{c}$. $b$ and $c$ are conformal fields with weights $\lambda$ and $\lambda -1$, respectively.

By calculating the correlation function of the vertex fields (vertex insertions), the authors of [14] have shown that they are the same as the corresponding one in the Coulomb gas model. The result which is obtained in all above papers is

$$< \prod_{j=1}^{N} V_{q_j}(z_j) > = < \prod_{j=1}^{N} : e^{i\alpha_j \varphi(z_j)} : > \prod_{k=1}^{N} \sigma^{Q_{q_k}}(z_k) \prod_{i<j}^{N} E^{q_i q_j}(z_i, z_j) \theta \left( \begin{array}{c} \delta \\ \epsilon \end{array} \right) (cv|d\Omega).$$

(15)

$\sigma(z)$ is a holomorphic $(g/2,0)$-form, without zero or pole, where $g$ is the genus of the surface and $\sigma(g = 1) = 1$. $E(z_i, z_j)$ is a holomorphic $(-1/2,0)$-form, which is antisymmetric under interchanging of its coordinates and is zero for $z_i = \gamma(z_j); \gamma \in \Gamma, \Gamma \subset PSL(2R)$. $v = \sum_i q_i z_i - Q\Delta$, in which the Riemann class $\Delta$ is a $(g - 1)$ degree divisor. Theta characteristics $(\delta, \epsilon)$ and $c$ and $d$ must be consistent with the boundary condition of $V_{q_i}(z_i)$. By boundary condition we mean the behaviour of $< V_{q_i}(z_i) >$ under winding the point $z_i$ around the homology cycles of our Riemann surface. It can also be shown that the correlation function (15) vanishes, unless the total charges $q_i$ cancel the background charge $Q$ [12]

$$\sum_i q_i = -\frac{Q}{8\pi} \int d^2z \sqrt{g} R(z) = Q(g - 1).$$

(16)

Now if we want the correlation function (15) describes a fermionic wavefunction, $q_i q_j$ must be an odd integer (as $E(z_i, z_j)$ is antisymmetric). Also if we demand that all fermions are identical, we must choose all $q_i$’s equal to $\sqrt{m}$, where $m$ is an odd integer. In this way $< \prod_{i=1}^{N} V_{q_i}(z_i) >$ becomes a Jastrow type wavefunction.

Another necessary condition for $< \prod_{i=1}^{N} V_{q_i}(z_i) >$ to be a Laughlin wavefunction, is that its behaviour under the action of conformal group transformation must be consistent with the conformal weight of electrons wavefunctions, which as was mentioned after eq.(12) is
equal to $B$. Now as the conformal weight of $e^{iq\phi}$ is $\frac{1}{2}q(q + Q) = \frac{1}{2}\sqrt{m}(\sqrt{m} + Q)$, we obtain

$$B = \frac{(m + \sqrt{m}Q)}{2}. \quad (17)$$

Following the above discussion, the appropriate wavefunction for a Laughlin state, which satisfies in eqs.(12) and (16) for each of its coordinates, is

$$\psi(z_1, \cdots, z_N) = \prod_{i=1}^{N} y_i^B \prod_{i<j} V_{q_j}(z_j) \rangle = \prod_{i=1}^{N} y_i^B \prod_{i<j} \sigma^{2B-m}(z_i) \prod_{i<j} E^m(z_i, z_j) \theta \left[ \frac{\delta}{\epsilon} \right] (m \sum_{i=1}^{N} z_i - (2B - m)\Delta|m\Omega). \quad (18)$$

Following the freedom of choosing the characteristics of the theta function of eq.(15) (as discussed in [14]), we can choose $c = \sqrt{m}$ and $d = m$ in our case. This choice of $c$ and $d$ is consistent with the wavefunction on the torus [5,11], and also ensures that the phase of $\psi$, when the points $z_i$'s wind around the homology cycles, does not depend on $z_i$. This independence comes from the invariance of eq.(6) under this windings [8]. Now what are the characteristics $\delta$ and $\epsilon$? As was discussed in [8], by comparing the behaviour of wavefunction under Fuchian transformations $z \rightarrow \gamma z$, with behaviour of theta functions under similar transformations, one leads to $\delta = \delta_0 + l/m, (l_i = 1, \cdots, m$ and $i = 1, \cdots, g)$ and $\epsilon = \epsilon_0$, where $\delta_0$ and $\epsilon_0$ are $g$-component constant vectors with components in the interval [0,1]. These values of $\delta$ and $\epsilon$ give the correct degeneracy number of the Laughlin wavefunctions, that is $mg - g + 1$. The explicit values of $\delta_0$ and $\epsilon_0$ depend on our explicit choice of the phase which appears from wavefunction under $z \rightarrow \gamma z$. For example in [5,24], these values have been fixed by choosing $u(\gamma_j, z_j) = e^{i\phi_j} (j = 1, \cdots, 2g)$, where $u(\gamma, z)$ is defined through $\psi(\gamma z) = u(\gamma, z)\psi(z)$, and $\gamma_j$ is a transformation identifying sides in the fundamental polygon which represents our Riemann surface in covering space, and $\phi_j$ is the flux through the $j$-th cycle. In this way our final result (18) becomes exactly the same as one has been obtained in [8].

Now let us investigate the plasma description of the the wavefunction (18). We write this wavefunction as $\psi = \psi_1 \psi_2$ where

$$\psi_1 = \prod_{i=1}^{N} y_i^B \prod_{i<j} E^m(z_i, z_j), \quad (19)$$

and

$$\psi_2 = \prod_{i=1}^{N} \sigma^{2B-m}(z_i) \theta \left[ \frac{\delta}{\epsilon} \right] (m \sum_{i=1}^{N} z_i - (2B - m)\Delta|m\Omega). \quad (20)$$

$\psi_1$ only depends on interaction part of the wavefunction, i.e. Coulomb interaction, and $\psi_2$ is related to spin structure of the electrons wavefunction on this surface. Therefore the
interaction potential $U$, which can be defined in

$$|\psi_1|^2 = e^{-\beta U},$$

becomes

$$U = -\frac{1}{\beta} \left( \sum_{i=1}^{N} \ln y_i^2 + \sum_{i<j} \ln |E(z_i - z_j)|^{2m}. \right)$$

Now as in the fundamental domain of the Riemann surface we have

$$\partial_z \partial_{\bar{z}} \ln |E(z, w)|^{2m} = \pi m \delta^2(z - w),$$

by choosing $\frac{1}{\beta} = m$, we see that $U$ is a Coulomb potential of particles with charge $m$, interacting with themselves and with a uniform background charge $\rho_0 = B/2\pi$ [7]. Charge neutrality of the plasma requires that the plasma particles spread out in the surface with density $\rho_m = \rho_0/m$, which corresponds to filling factor $\nu = 1/m$.

To determine the precise value of $m$, we use eqs.(16) and (17) to obtain:

$$m(N + g - 1) = 2B(g - 1).$$

This equation gives the value of $m$ in terms of magnetic field $B$, the genus $g$, and the electrons number $N$. It is also interesting to see the geometrical meaning of eq.(24). By Riemann vanishing theorem, the number of zeros of the theta function of eq. (18) is $mg$ and as $\prod_{i<j}^N E(z_i, z_j)^m$ (as a function of $z_i$), has $m(N - 1)$ zeros, so the number of zeros of the wavefunction (18), with respect to each of its coordinates, is $mg + m(N - 1)$, which from eq.(24) is equal to the magnetic flux $\phi = 2B(g - 1)$. This shows that the degree of line (for $m = 1$) or vector (for $m > 1$) bundle is equal to the first Chern number of the gauge field, as we have expected.

As a last point, we know that the Coulomb gas model, by suitable choosing of $Q$, can be considered as minimal models. The minimal models characterized by two positive coprime integers $p$ and $q$ with central charge $c(Q) = 1 - 6(p - q)^2/pq$. Now as the central charge of Coulomb gas is $c(Q) = 1 - 3Q^2$, eq. (17) shows that if $B$ and $m$ satisfy in

$$\left( \frac{2B - m}{\sqrt{m}} \right)^2 = \frac{2(p - q)^2}{pq},$$

our QHE is a $(p, q)$ minimal model. For example for $p = q + 1$ unitary minimal models, any odd integer $m$ which satisfies in

$$m = \frac{q(q + 1)}{2} \left( \frac{r}{s} \right)^2,$$

where $r$ and $s$ are integers, has a $(q + 1, q)$ corresponding minimal model description. At $Q = 0$, $c$ is equal to one and $B$ is $m/2$. A detailed discussion about this case can be found in [18].
4 Quasiholes on Riemann surface

In this section we want to study the aspect of quasihole states in the context of conformal field theory. Laughlin argued that the lowing excited state of QHE are produced by creation of quasiparticles (quasiholes) in the system. These are fractional statistics particles, i.e., by interchanging two of them, the wavefunction takes the \( e^{i\theta} \) phase. This phase for quasiholes is \( \pi/m \) where \( \nu = 1/m \) is the filling factor [1,17]. If we want to express the fractional statistics particles in terms of vertex fields, we must choose the appropriate charges for these vertices. Using (15), it is seen that by interchanging two vertices we obtain

\[
<V_{q_i}(z_i)V_{q_j}(z_j)> = e^{i\pi q_i q_j} <V_{q_i}(z_j)V_{q_j}(z_i)>,
\]

so we must choose \( q_i = 1/\sqrt{m} \) to relate the vertex fields to the quasiholes. Now consider a system containing \( N \) electrons (represented by vertices with charge \( \sqrt{m} \)), and \( N_q \) quasiholes (represented by vertices with charges \( 1/\sqrt{m} \)), then eq. (16) leads to

\[
N\sqrt{m} + \frac{N_q}{\sqrt{m}} = Q(g - 1).
\]

Using eqs. (16) and (17) (which also holds in this case), we determine the filling factor

\[
m(N + g - 1) + N_q = 2B(g - 1) = \phi.
\]

This relation is consistent with the result pointed out in [17], and can be used to obtain the electric charge of quasiholes with the method that was introduced in [4]. If the system of \( N \) electrons, in Laughlin state, is excited by removal of an electron, at fixed magnetic field, the final state has the following flux

\[
\phi(N; m) = \phi(N - 1; m) + m,
\]

where \( \phi(N; m) = m(N + g - 1) \). Comparing eqs. (29) and (30), showing that the new system (30) is composed of \( N - 1 \) electrons and \( m \) quasiholes. Hence the quasiholes carry the charge \( e^* = e/m \) (\( e > 0 \)). To reproduce this result in another way, we note that the charge of particles can also be determined by using the OPE of the current and corresponding fields [9]. On Riemann surface, the above OPE is [19]

\[
J(z)e^{i\varphi(w)/\sqrt{m}} = \frac{1/m}{z-w}e^{i\varphi(w)/\sqrt{m}} + \ldots.
\]

Therefore following the steps of [9], the charge of quasihole corresponding to vertex \( e^{i\varphi(w)/\sqrt{m}} \) is \( e^* = e/m \).

At the end, we present an expression for the holomorphic part of the wavefunction containing \( N \) electrons and one quasihole

\[
\psi(z, z_1, \ldots, z_N) = <V_q(z) \prod_{i=1}^{N} V_{q_i}(z_i)> = \sigma^{2B-m}(z) \prod_{i=1}^{N} \sigma^{2B-m}(z_i)
\]
\[
\times \prod_{i=1}^{N} E(z_i, z) \prod_{i<j}^{N} E^m(z_i, z_j) \theta \left[ \frac{\delta}{\epsilon} \right] (m \prod_{i=1}^{N} z_i + z - (2B - m) \Delta |m\Omega),
\]

which is obtained by eq. (15). Factorize this wavefunction as \( \psi = \psi_1 \psi_2 \), where

\[
\psi_1 = \prod_{i=1}^{N} E(z_i, z_j) \prod_{i<j}^{N} E^m(z_i, z_j),
\]

and \( \psi_2 \) other terms, and again by considering \( |\psi_1|^2 = \exp(-U/m) \), one can see that \( U \) is the Coulomb potential of a system of particles of charge \( m \), interacting with themselves and with a particle of charge one located at \( z \) (which again proves that \( e^* = e/m \)).

At the end we would like to add a point. One of the important point in the physics of QHE, is to understand the incompressibility feature of the Laughlin wavefunctions, which may be related to quantum group symmetry of the Laughlin states [7,20-22]. On the other hand, there is a deep connection between the conformal and quantum group symmetries [23]. Our procedure in expressing the Laughlin states in the context of CFT, may shed some light on these connections on Riemann surfaces. We will discuss these elsewhere.

5 Conclusion

Using the analogy of the Coulomb gas and plasma description of QHE, the conformal symmetry of Laughlin states, and the results that have been found for the Coulomb gas model on Riemann surface, we obtain the Laughlin wavefunction on an arbitrary compact and orientable Riemann surface. We have also determined the filling factor and degeneracy of these wavefunctions. In the case of Poincare metric, we find the plasma description of QHE on these surfaces and also state the relation between FQHE and minimal models. Finally, for the cases where the quasiholes are also present, we find the wavefunctions.

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References

[1] R. Prange, S. Girvin The Quantum Hall Effect. 2nd edn ( Springer New York, Springer, 1990).

[2] M. C. Gutzwiller, Chaos in Classical and Quantum Mechanics ( Springer–Verlag, 1990).
[3] J. Fay, American Mathematical Society, Memories, No. 464 (Providence, Rhode Island, 1992).

[4] F. D. M. Haldane; Phys. Rev. Lett. 51 (1983) 605.

[5] F. D. M. Haldane, E. H. Rezayi; Phys. Rev. Lett. B31 (1985) 2529.

[6] A. Comtet; Ann. Phys. 173 (1987) 185.

[7] M. Alimohammadi and H. Mohseni Sadjadi; J. phys. A A29 (1996) 5551.

[8] R. Iengo and D. Li; Nucl. Phys. B413 (1994) 735.

[9] G. Moore and N. Read; Nucl. Phys. B360 (1991) 362.

[10] A. Balatsky and E. Fradkin; Phys. Rev. B43 (1990) 10622.

[11] G. Cristofano, G Maiella, R. Musto, and F. Nicodemi; Phys. Lett. B262 (1991) 88.

[12] E. Verlinde and H. Verlinde; Phys. Lett. B192 (1987) 95, Nucl. Phys B288 (1987) 357.

[13] T. Eguchi and H. Ooguri; Phys. Lett. B187 (1987) 127.

[14] L. Bonora, M. Matone, F. Toppan, and K. Wu; Nucl. Phys. B334 (1990) 717.

[15] O. Lechtenfeld; Phys. Lett. B232 (1989) 193.

[16] P. Christ and M. Henkel; Lecture Notes in Physics m16 (1993),
    P. Ginsparg in: Les Houches (1988), Fields, Strings and Critical Phenomena.

[17] D. Li; Modern. Phys. Lett. B7 (1993) 1103.

[18] R. Dijkgraf, E. Verlinde, and H. Verlinde; Commun. Math. Phys 115 (1988) 649.

[19] T. Eguchi and H. Ooguri; Nucl. Phys. B289 (1987) 308.

[20] H. T. Sato; Mod. Phys. Lett. A9 (1991) 451.

[21] N. Aizawa, S. Sachse, and H. T. Sato; Mod. Phys. Lett. A10 (1995) 853.

[22] M. Alimohammadi and A. Shafei Deh Abad; J. Phys. A A29 (1996) 559.

[23] C. Gomez and G. Sierra; Lecture Notes in Physics 375 (1990).

[24] J. E. Avron, M. Klein, and A. Pnueli; Phys. Rev. Lett. 69 (1992) 128.