Updated Standard Model Prediction for $\varepsilon'/\varepsilon^*$

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Abstract

A recent lattice evaluation of $\varepsilon'/\varepsilon$, finding a 2.1 $\sigma$ deviation from the experimental value, has revived the old debate about a possible $\varepsilon'/\varepsilon$ anomaly. The unfounded claims of a too low Standard Model prediction are based on incorrect estimates that neglect the long-distance re-scattering of the final pions in $K \rightarrow 2\pi$. In view of the current situation, we have recently updated the Standard Model calculation, including all known short- and long-distance contributions. Our result, $\text{Re} \left( \frac{\varepsilon'}{\varepsilon} \right) = (15 \pm 7) \cdot 10^{-4}$ [1], is in complete agreement with the experimental measurement.

Keywords: Kaon decays, CP violation, Standard Model

1. Introduction

The CP violating ratio $\varepsilon'/\varepsilon$ constitutes a fundamental test for our understanding of flavour-changing phenomena. The present experimental world average [2–10],

$$\text{Re} \left( \frac{\varepsilon'}{\varepsilon} \right) = (16.6 \pm 2.3) \cdot 10^{-4},$$

(1)
demonstrates the existence of direct CP violation in the decay transitions $K^0 \rightarrow 2\pi$.

On the other hand, the theoretical prediction of $\varepsilon'/\varepsilon$ has been the subject of many debates. The first next-to-leading order (NLO) calculations [11–16] obtained Standard Model (SM) values one order of magnitude smaller than [1]. However, it was soon realized that the former SM predictions had missed an important ingredient: the final-state interactions (FSI) of the two emitted pions [17,18]. Once all relevant contributions were taken into account, the theoretical prediction was found to be in good agreement with the experimental value although with a large uncertainty of non-perturbative origin [19].

Lattice QCD provides a suitable tool to face non-perturbative problems. However, the lattice efforts to explain the enhancement of the $\Delta I = 1/2$ $K \rightarrow 2\pi$ amplitude remained unsuccessful for many years, while attempts to estimate $\varepsilon'/\varepsilon$ were unreliable, sometimes even obtaining negative central values. The status of lattice simulations has improved considerably with the development of more sophisticated techniques and the increasing computer power. The RBC-UKQCD collaboration has achieved a successful calculation of the $\Delta I = 3/2$ $K^+ \rightarrow \pi^+\pi^0$ amplitude [20–22], and has recently obtained the first statistically-significant signal of the $\Delta I = 1/2$ enhancement [23], in good agreement with the qualitative understanding achieved long time ago with analytical techniques [24–33].

The RBC-UKQCD group has also published a first estimate of the direct CP-violation ratio, $\text{Re} \left( \frac{\varepsilon'}{\varepsilon} \right) = (1.4 \pm 6.9) \cdot 10^{-4}$ [20,34], which exhibits a 2.1 $\sigma$ deviation from the experimental value in Eq. (1). This result has brought back the old SM approaches predicting low values of $\varepsilon'/\varepsilon$ [35–37] and has triggered many new studies of possible contributions from physics beyond the SM [38–63]. However, this discrepancy cannot be taken as evidence for new physics because the same lattice simulation fails to correctly reproduce the
and $\chi_I$ states with CP-conserving limit. Furthermore, in the isospin limit, the Hamiltonian, in the limit of isospin conservation. The $\Delta$ where the complex quantities $A_I$ incorporate the computed isospin-breaking corrections \[65-67\], which these numbers exhibit two important dynamical features that characterize the $K \to \pi\pi$ decay amplitudes:

1. Strong enhancement of the isoscalar amplitude with respect to the isoscalar one, the so-called “$\Delta I = 0 \to 1$ rule”, \[\omega \equiv \text{Re}A_2/\text{Re}A_0 \approx 1/22\]. \[4\]

2. The $S$-wave re-scattering generates a large phase-shift difference between the $I = 0$ and $I = 2$ partial waves. This implies that 50% of the $A_{1/2}/A_{3/2}$ ratio originates from the absorptive contribution: \[\text{Abs}(A_{1/2}/A_{3/2})/\text{Dis}(A_{1/2}/A_{3/2}) = 1.09\]. \[5\]

We would see later their strong implications on $\varepsilon'/\varepsilon$.

In the presence of CP violation, the amplitudes $A_0$, $A_2$, and $A_2'$ acquire imaginary parts. To first order in CP-violating quantities,

$$
\varepsilon' = -\frac{i}{\sqrt{2}} e^{i(\varepsilon' - \chi_0)} \frac{\text{Im}A_2}{\text{Re}A_0 - \text{Re}A_2'}
$$

(6)

Thus, $\varepsilon'$ is suppressed by the ratio $\omega$ and $\varepsilon'/\varepsilon$ is approximately real since $\chi_2 - \chi_0 - \pi/2 \approx 0$. The CP-conserving amplitudes $\text{Re}A_I$ are in general fixed to their experimental values, given in Eq. (3), in order to reduce the theoretical uncertainty. A theoretical calculation is then only needed for $\text{Im}A_1$.

Eq. (6) contains a subtle numerical balance between the two isospin contributions, making the result very sensitive to the values of the CP-violating amplitudes. Naive estimates of $\text{Im}A_I$ result in a strong cancellation between the two terms, leading to unrealistically low values of $\varepsilon'/\varepsilon$, as we show in section 4.

Isospin breaking effects are very important in $\varepsilon'/\varepsilon$, due to the large ratio $1/\omega$ \[65\]. Including isospin violation, $\text{Re}(\varepsilon'/\varepsilon)$ can be written as

$$
\text{Re}\left(\frac{\varepsilon'}{\varepsilon}\right) = -\frac{\omega_+}{\sqrt{2} |\epsilon|} \frac{\text{Im}A_0^{(0)}}{\text{Re}A_0^{(0)}} \left(1 - \Omega_{\text{eff}}\right) - \frac{\text{Im}A_2^{\text{emp}}}{\text{Re}A_2^{\text{emp}}},
$$

(7)

where $\text{Im}A_2^{\text{emp}}$ contains the $I = 2$ contribution from the electromagnetic penguin operator, the superscript (0) denotes the isospin limit, and $\omega_+ \equiv \text{Re}A_2'/\text{Re}A_0$ is directly extracted from \[3\]. The parameter

$$
\Omega_{\text{eff}} = \Omega_V - \Delta_0 - \delta_{f/2} = (6.0 \pm 7.7) \cdot 10^{-2}
$$

(8)

incorporates the computed isospin-breaking corrections \[65\] \[66\].
3. Theoretical framework

The physical origin of $\varepsilon'/\varepsilon$ is at the electroweak scale where all the flavour-changing processes are described in terms of quarks and gauge bosons. Due to the presence of very different mass scales ($M_q < M_K < M_W$), the gluonic corrections to the $K \to \pi\pi$ amplitudes are amplified with large logarithms that can be summed up using the Operator Product Expansion (OPE) and the renormalization group equations (RGEs), all the way down to scales $\mu < m_c$. Finally, one gets an effective Lagrangian defined in the three-flavour theory [87],

$$\mathcal{L}_{\text{eff}}^{\Delta S=1} = \frac{G_F}{\sqrt{2}} V_{ud} V_{us}^\ast \sum_{i=1}^{10} C_i(\mu) \mathcal{Q}_i(\mu), \quad (9)$$

which is a sum of local operators weighted by short-distance coefficients $C_i(\mu)$ that depend on the heavy masses ($\mu > M$) and CKM parameters. The Wilson coefficients $C_i(\mu)$ are known at NLO [88–91]. This includes all corrections of $O(\alpha_s^t\mu^t)$ and $O(\alpha_s^{t+1}\mu^t)$, where $t \equiv \log (M_t/M_\mu)$ refers to the logarithm of any ratio of heavy mass scales $M_1, M_2 \geq \mu$. Some next-to-next-to-leading-order (NNLO) corrections are already known [92–93] and efforts towards a complete short-distance calculation at the NNLO are currently under way [94].

Below the resonance region, where perturbation theory no longer works, we can use symmetry considerations to define another effective field theory in terms of the QCD Goldstone bosons ($\pi, K, \eta$). $\chi$PT describes the pseudoscalar octet dynamics through a perturbative expansion in powers of momenta and quark masses over the chiral symmetry breaking scale $\Lambda_\chi \sim 1 \text{ GeV}$ [95–97]. At lowest order, the most general effective bosonic Lagrangian with the same $SU(3)_L \otimes SU(3)_R$ transformation properties as $\mathcal{L}_{\text{eff}}^{\Delta S=1}$ contains three terms [98]:

$$\mathcal{L}_{\text{2}}^{\Delta S=1} = G_8 \mathcal{L}_8 + G_{27} \mathcal{L}_{27} + G_{\text{gwk}} \mathcal{L}_{\text{gwk}}. \quad (10)$$

Thus, $\mathcal{L}_{\text{2}}^{\Delta S=1}$ determines the $K^0 \to \pi\pi$ amplitudes at $O(p^2)$ in terms of the LECs $G_8, G_{27}$ and $G_{\text{gwk}}$. A first-principle computation of these three LECs requires to perform a matching between the short-distance and effective Lagrangians in Eqs. (9) and (10). This can be easily done in the limit of an infinite number of quark colours, where the four-quark operators factorize into currents with well-known chiral realizations. Since the large-$N_C$ limit is only applied in the matching between the two effective field theories, the only missing contributions are $1/N_C$ corrections that are not enhanced by any large logarithms. Figure 1 shows schematically the chain of effective theories entering the analysis of the kaon decay dynamics.

| Energy | Fields | Effective Theory |
|--------|--------|------------------|
| $M_W$  | $W, Z, \gamma, G_8$ | Standard Model |
| $t, b, c, s, d, u$ | | |
| $\mathcal{O}$ | $C_{\text{pert}}$, $C_{\text{gwk}}$, $C_{\text{gwk}}$ | OPE |
| $\mathcal{L}_{\text{QCD}}$, $\mathcal{L}_{\text{ad}}$ | $N_C \to \infty$ | $\chi$PT |

![Figure 1](image)

4. Strong cancellation in simplified analysis

The CP-odd amplitudes in Eq. (7) are dominated by the penguin operators $Q_6$ and $Q_8$, due to their chiral enhancement. Taking only into account these two operators and ignoring all others in the estimation of $\text{Im} A_{\ell}$, one finds [99, 100]:

$$\text{Im} A_{1,2}|_{Q_6} = \tilde{x}_6 \left( F_K - F_\pi \right) B_6^{1/2}, \quad (11)$$

$$\text{Im} A_{1,2}|_{Q_8} = - \tilde{x}_8 \left( 2 F_\pi B_8^{3/2} \right), \quad (12)$$

where $\tilde{x}_i \equiv \frac{G_F^2}{\sqrt{2}} \Lambda^2 \varepsilon \eta(\mu) \left[ \frac{M^2}{m_\pi^2} \right]$, $F_K$ is the kaon decay constant and the factors $B_6^{1/2}$ and $B_8^{3/2}$ parametrize the deviations of the true hadronic matrix elements from their large-$N_C$ approximations. Notice, in the definition of $\tilde{x}_i$, that the short-distance scale of $\gamma_6(\mu)$ and $\gamma_8(\mu)$ is cancelled by the scale dependence of the quark masses since, at $N_C \to \infty$, the only elements of the anomalous dimension matrix $\gamma_{ij}$ that survive are $\gamma_{66}$ and $\gamma_{88}$ [101]. This nice scale cancellation illustrates how the product of two colour-singlet quark currents factorizes and, in addition, that the product $m_\ell \delta_{\ell q}$ is renormalization-scale invariant.

Introducing this rough prediction of the CP-odd amplitudes in Eq. (7), one obtains

$$\text{Re} \left[ \frac{\varepsilon'}{\varepsilon} \right] \approx 2.2 \times 10^{-3} \left[ B_6^{1/2} (1 - \Omega_{\text{eff}}) - 0.48 B_8^{3/2} \right]. \quad (13)$$

With $B_6^{1/2} = B_8^{3/2} = 1$ (large-$N_C$ values) and $\Omega_{\text{eff}} = 0.06$, the naive estimation gives $\text{Re} (\varepsilon'/\varepsilon) \approx 1.0 \times 10^{-3}$ as the order of magnitude for the SM prediction. An interesting observation is the delicate cancellation among the different terms in (13) which makes the final number very sensitive to the chosen inputs [102–106]. With the
values adopted in Ref. [37], $B_6^{1/2} = 0.57$, $B_8^{3/2} = 0.76$ and $\Omega_{\text{eff}} = 0.15$, one obtains $\text{Re}(\epsilon'/\epsilon) \approx 2.6 \cdot 10^{-4}$, which is an order of magnitude smaller and in clear conflict with the experimental value in Eq. (1).

However, these simplified estimates neglect completely the strong FSI present in $K^0 \rightarrow \pi\pi$ decays, and miss the very large absorptive contribution giving rise to the measured phase-shift difference. The two relevant decay amplitudes get large logarithmic corrections from pion loops [17,19] that can be rigorously calculated using the usual $\chi$PT methods. They turn out to be positive for $A_0|_{Q_0}$, while negative for $A_1|_{Q_0}$. Consequently, the numerical cancellation in Eq. (12) disappears and one gets a sizeable enhancement of the SM prediction for $\epsilon'/\epsilon$, in good agreement with its experimental value.

5. $K \rightarrow \pi\pi$ amplitudes in $\chi$PT

With the Lagrangian [10], the kaon decay amplitudes are easily obtained at $O(p^4)$ through a simple perturbative calculation in $\chi$PT. One only needs to consider tree-level Feynman diagrams with one insertion of $L_{\chi}^{\Delta S=1}$. Assuming isospin conservation, the $\mathcal{A}_M$ amplitudes defined in Eq. (2) are given by [19,66]

$$\mathcal{A}_{1/2} = -\sqrt{2} G_F \left( M_K^2 - M_\pi^2 \right) - 2 G_F^2 \epsilon^2 \epsilon_{\text{ewk}},$$

$$\mathcal{A}_{3/2} = -\frac{10}{9} G_F^2 \left( \frac{M_K^2 - M_\pi^2}{M_\pi^2} \right) + \frac{3}{2} G_F^3 \epsilon^2 \epsilon_{\text{ewk}},$$

$$\mathcal{A}_{5/2} = 0.$$  \hspace{1cm} (14)

From the measured amplitudes in Eq. (3), one immediately obtains the tree-level determinations $g_8 = 5.0$ and $g_{27} = 0.25$ for the octet and 27-plet chiral couplings, respectively, with the normalization $G_{8,27} = -\frac{M_K^2}{\sqrt{2}} V_{ud}^* V_{us}^* g_{8,27}$. The large numerical difference between these two LECs just reflects the smallness of the measured ratio $\omega \approx 5 \sqrt{2} g_{27}/(9 g_8)$.

At LO in $\chi$PT, the phase shifts are predicted to be zero, because they are generated through loop diagrams with $\pi\pi$ absorptive cuts. Figure 2 displays the only one-loop topology contributing to the absorptive amplitudes. The large value of the measured phase-shift difference in Eq. (3) indicates a very large absorptive contribution. Analyticity relates the absorptive and dispersive parts of the one-loop diagram, which implies that the dispersive correction is also very large. A proper calculation of chiral loop corrections is then compulsory in order to obtain a reliable prediction for $\epsilon'/\epsilon$.

The incorrect estimates, claiming small SM values of $\epsilon'/\epsilon$ [35,37], are flawed because they totally ignore the presence of absorptive cuts. They are based on (model-dependent) real $K \rightarrow \pi\pi$ amplitudes that fail to comply with the experimental constraint in Eq. (5).

At the NLO in $\chi$PT, the $\mathcal{A}_M$ amplitudes can be written in the form

$$\mathcal{A}_M = -G_F \left( M_K^2 - M_\pi^2 \right) \mathcal{A}_M^{(8)} - \epsilon^2 F_\pi^2 \epsilon_{\text{ewk}} \mathcal{A}_M^{(27)},$$

$$-G_{27} F_\pi (M_K^2 - M_\pi^2) \mathcal{A}_M^{(27)},$$

where $\mathcal{A}_M^{(8)}$ and $\mathcal{A}_M^{(27)}$ represent the octet and 27-plet components, and $\mathcal{A}_M^{(27)}$ contains the electroweak penguin contributions. Moreover, these quantities can be further decomposed as

$$\mathcal{A}_M^{(X)} = a_{M,1/2}^{(X)} \left[ 1 + \Delta L \mathcal{A}_M^{(X)} + \Delta C \mathcal{A}_M^{(X)} \right].$$  \hspace{1cm} (16)

with $a_{M,1/2}^{(X)}$ the tree-level contributions, $\Delta L \mathcal{A}_M^{(X)}$ the one-loop chiral corrections and $\Delta C \mathcal{A}_M^{(X)}$ the NLO local corrections at $O(p^6)$. The numerical values of the different $a_{M,1/2}^{(X)}$ and $\Delta C \mathcal{A}_M^{(X)}$ components are displayed in tables 2 and 2 respectively.

| X | $a_{M,1/2}^{(X)}$ | $\Delta L \mathcal{A}_M^{(X)}$ | $\Delta C \mathcal{A}_M^{(X)}$ |
|---|----------------|-----------------|----------------|
| 8 | $\sqrt{2}$ | $0.27 \pm 0.47 i$ | $0.01 \pm 0.05$ | $0.02 \pm 0.05$ |
| g | $\frac{g}{\sqrt{2}}$ | $0.27 \pm 0.47 i$ | $-0.19 \pm 0.01$ | $-0.19 \pm 0.01$ |
| 27 | $\frac{27}{2}$ | $1.03 \pm 0.47 i$ | $0.01 \pm 0.06$ | $0.01 \pm 0.63$ |

Table 1: Numerical predictions for the $a_{M,1/2}$ components. The local NLO correction to the CP-even $(|\Delta L \mathcal{A}_M^{(X)}|)$ and CP-odd $(|\Delta C \mathcal{A}_M^{(X)}|)$ amplitudes is only different in the octet case.

| X | $a_{M,3/2}^{(X)}$ | $\Delta L \mathcal{A}_M^{(X)}$ | $\Delta C \mathcal{A}_M^{(X)}$ |
|---|----------------|-----------------|----------------|
| g | $\frac{g}{2}$ | $-0.50 \pm 0.21 i$ | $-0.19 \pm 0.19$ |
| 27 | $\frac{27}{9}$ | $-0.04 \pm 0.21 i$ | $0.01 \pm 0.05$ |

Table 2: Numerical predictions for the $a_{M,3/2}$ components.
negative for $\Delta I = 3/2$. Furthermore, they do not depend on the chiral renormalization scale $v_k$. Besides, table 1 shows a huge dispersive one-loop correction to the $\mathcal{A}^{(g)}_{1/2}$ amplitude. However, since $\text{Im}(g_{2\gamma}) = 0$, the 27-plet components do not contribute to the CP-odd amplitudes and, therefore, do not introduce any uncertainty in the final numerical value of $\text{Im}A_0$.

The relevant NLO loop corrections for $\epsilon'/\epsilon$ are $\Delta_L\mathcal{A}^{(g)}_{1/2}$ and $\Delta_L\mathcal{A}^{(g)}_{3/2}$. The first one generates a significant enhancement of $\text{Im}A_0$, $|1 + \Delta_L\mathcal{A}^{(g)}_{1/2}| \approx 1.35$, while the second one produces a suppression in $\text{Im}A_2^{\text{emp}}$, $|1 + \Delta_L\mathcal{A}^{(g)}_{3/2}| \approx 0.54$. Consequently, the numerical cancellation between the $I = 0$ and $I = 2$ terms in Eq. (13) is completely destroyed by the chiral loop corrections.

Tables 1 and 2 show also the numerical predictions for the NLO local corrections $\Delta_L\mathcal{A}^{(X)}_M$, which have been estimated in the large-$N_C$ limit. The dependence with $v_k$ (absent at large $N_C$) is our main source of uncertainty. In order to estimate the errors, we have varied $v_k$ between 0.6 and 1 GeV in the corresponding loop contributions $\Delta_L\mathcal{A}^{(M)}_M$. In addition, we have taken the uncertainty associated to the short-distance scale varying $\mu$ between $M_\rho$ and $m_\pi$, but the impact on the $\Delta_L\mathcal{A}^{(M)}_M$ corrections is negligible compared with the $v_k$ uncertainty. The most significant local corrections for $\epsilon'/\epsilon$ are $[\Delta_L\mathcal{A}^{(g)}_{1/2}]^{-}$ and $\Delta_L\mathcal{A}^{(g)}_{3/2}$, nevertheless, they are much smaller than the loop contributions.

6. The SM prediction for $\epsilon'/\epsilon$

Taking into account all computed corrections in Eq. (7), our SM prediction for $\epsilon'/\epsilon$ is

$$\text{Re} \left( \frac{\epsilon'}{\epsilon} \right) = \left( 15 \pm 2 \mu \pm 2_{\text{emp}} \pm 6_{1/N_C} \right) \times 10^{-4}$$

$$= \left( 15 \pm 7 \right) \times 10^{-4}.$$  \hspace{1cm} (17)

The first uncertainty has been estimated by varying the short-distance renormalization scale $\mu$ between $M_\rho$ and $m_\pi$. The second error shows the sensitivity to the strange quark mass, within its allowed range, while the third one displays the uncertainty from the isospin-breaking parameter $\Omega_{\text{eff}}$. The last error is our dominant source of uncertainty and reflects our ignorance about $1/N_C$ suppressed contributions that we have missed in the matching process.

In figure 3, we plot the prediction for $\epsilon'/\epsilon$ as function of the $\gamma\gamma'$ coupling $L_9$, which clearly shows a strong dependence on this parameter. The experimental 1 $\sigma$ range is indicated by the horizontal band, while the dashed vertical lines display the current lattice determination of $L_9(M_\rho)$. The measured value of $\epsilon'/\epsilon$ is nicely reproduced with the preferred lattice inputs.

7. Final remarks

Our SM prediction for $\epsilon'/\epsilon$ is in perfect agreement with the measured experimental value. We have shown the important role of FSI in $K^0 \to \pi\pi$. When $\pi\pi$ rescattering corrections are taken into account, the numerical cancellation between the $Q_8$ and $Q_9$ terms in Eq. (13) is completely destroyed because of the positive enhancement of the $Q_9$ amplitude and the negative suppression of the $Q_8$ contribution. Once these important corrections are included, the contributions from other four-quark operators to $\text{Im}A_0^{(0)}$ and $\text{Im}A_2^{\text{emp}}$ become numerically less relevant, since the cancellation is no longer operative.

The claims [35–37] of a flavour anomaly in $\epsilon'/\epsilon$ originate in naive approximations that overlook the important role of pion chiral loops. These incorrect estimates are using simplified ansatzs for the $K \to \pi\pi$ amplitudes, without any absorptive contributions, in complete disagreement with the strong experimental evidence of a very large phase shift difference.

The recent lattice results look quite encouraging, since it is the first time that a clear signal of the $\Delta I = 1/2$ enhancement seems to emerge from lattice data [23]. RBC-UKQCD has obtained a first numerical estimate of $\epsilon'/\epsilon$ with a quite small central value, $2.1 \sigma$ lower than the experimental measurement. However, the same lattice simulation finds a $(\pi\pi)_{\text{emp}}$ phase shift 2.9 $\sigma$ away from its physical value, which indicates that these results are still in a very premature stage. Substantial
improvements, with much larger statistics and a better control of the final pion dynamics, are expected soon.

Our prediction of $\epsilon'/\epsilon$ agrees well with the measured value, providing a qualitative confirmation of the SM mechanism of CP violation. Although the theoretical error is still large, improvements can be achieved in the next years via a combination of analytical calculations, numerical simulations and data analyses. For instance,

- A computation of the Wilson coefficients at NNLO is currently been performed [94].
- The isospin-breaking effects are vital for a correct $\epsilon'/\epsilon$ prediction. A complete re-analysis with updated inputs is currently under way [107].
- The $O(\epsilon^2 p^0)$ coupling $g_\pi g_{\text{vac}}$ can be expressed as a dispersive integral over the hadronic vector and axial-vector spectral functions. The $\tau$ decay data can then be used to perform a direct determination of this LEC. A new phenomenological analysis is close to being finalized [108].
- The dominant $\chi$PT corrections originate from large chiral logarithms. A reliable estimate of higher-order contributions should be feasible either through explicit two-loop calculations or with dispersive techniques [17,19,109,110].
- A matching calculation of the weak LECs at NLO in $1/N_C$ remains a very challenging task. A fresh view to previous attempts [24–33,102–104,111,112], with a modern perspective, could suggest new ways to face this unsolved problem.
- Lattice QCD simulations are expected to provide new improved data on $K^0 \rightarrow \pi\pi$ transitions in the next years [64,113]. Combined with appropriate $\chi$PT techniques, a better control of systematic uncertainties could be achieved.

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References

[1] H. Gisbert and A. Pich, “Direct CP violation in $K^0 \rightarrow \pi\pi$: Standard Model Status”, Rept. Prog. Phys. 81 (2018) no.7, 076201 [arXiv:1712.06147 [hep-ph]].
[2] J. R. Batley et al. [NA48 Collaboration], “A Precision measurement of direct CP violation in the decay of neutral kaons into two pions”, Phys. Lett. B 544 (2002) 97 [hep-ex/0208009].
[3] A. Lai et al. [NA48 Collaboration], “A Precise measurement of the direct CP violation parameter R($\epsilon'/\epsilon$)”, Eur. Phys. J. C 22 (2001) 231 [hep-ex/0101019].
[4] V. Fanti et al. [NA48 Collaboration], “A New measurement of direct CP violation in two pion decays of the neutral kaon”, Phys. Lett. B 465 (1999) 335 [hep-ex/9909022].
[5] G. D. Barr et al. [NA31 Collaboration], “A New measurement of direct CP violation in the neutral kaon system”, Phys. Lett. B 317 (1993) 233.
[6] H. Burkhardt et al. [NA31 Collaboration], “First Evidence for Direct CP Violation”, Phys. Lett. B 206 (1988) 169.
[7] E. Abouzaid et al. [KTeV Collaboration], “Precise Measurements of Direct CP Violation, CPT Symmetry, and Other Parameters in the Neutral Kaon System”, Phys. Rev. D 83 (2011) 092001 [arXiv:1011.0127 [hep-ex]].
[8] A. Alavi-Harati et al. [KTeV Collaboration], “Measurements of direct CP violation, CPT symmetry, and other parameters in the neutral kaon system”, Phys. Rev. D 67 (2003) 012005 [Erratum-ibid. D 70 (2004) 079904] [hep-ex/0208007].
[9] A. Alavi-Harati et al. [KTeV Collaboration], “Observation of direct CP violation in $K_{\ell 4}$ $\rightarrow$ $\pi\pi$ decays”, Phys. Rev. Lett. 83 (1999) 22 [hep-ex/9905060].
[10] L. K. Gibbons et al. [E731 Collaboration], “Measurement of the CP violation parameter $R(\epsilon'/\epsilon)$”, Phys. Rev. Lett. 70 (1993) 1203.
[11] A. J. Buras, M. Jamin and M. E. Lautenbacher, “The Anatomy of $\epsilon'/\epsilon$ beyond leading logarithms with improved hadronic matrix elements”, Nucl. Phys. B 408 (1993) 209 [hep-ph/9303284].
[12] A. J. Buras, M. Jamin and M. E. Lautenbacher, “A 1996 analysis of the CP violating ratio $\epsilon'/\epsilon$”, Phys. Lett. B 389 (1996) 749 [hep-ph/9608365].
[13] S. Bosch et al., “Standard model confronting new results for $\epsilon'/\epsilon$”, Nucl. Phys. B 565 (2000) 3 [hep-ph/9904408].
[14] A. J. Buras et al., “$\epsilon'/\epsilon$ and rare $K$ and $B$ decays in the MSSM”, Nucl. Phys. B 592 (2001) 55 [hep-ph/0007313].
[15] M. Ciuchini et al., “An Upgraded analysis of $\epsilon'/\epsilon$ at the next-to-leading order”, Z. Phys. C 68 (1995) 239 [hep-ph/9502125].
[16] M. Ciuchini et al., “$\epsilon'/\epsilon$ at the Next-to-leading order in QCD and QED”, Phys. Lett. B 301 (1993) 263 [hep-ph/9212203].
[17] E. Pallante and A. Pich, “Strong enhancement of $\epsilon'/\epsilon$ through final state interactions”, Phys. Rev. Lett. 84 (2000) 2568 [hep-ph/9911233].
[18] E. Pallante and A. Pich, “Final state interactions in kaon decays”, Nucl. Phys. B 592 (2001) 294 [hep-ph/0007208].
[19] E. Pallante, A. Pich and I. Scimemi, “The Standard model prediction for $\epsilon'/\epsilon$”, Nucl. Phys. B 617 (2001) 441 [hep-ph/0105011].
[20] T. Blum et al., “$K \rightarrow \pi\Delta\pi = 3/2$ decay amplitude in the continuum limit”, Phys. Rev. D 91 (2015) no.7, 074502 [arXiv:1502.00263 [hep-lat]].
[21] T. Blum et al. [RBC and UKQCD Collaborations], “The $K \rightarrow \pi\Delta\pi$
A. J. Buras, "New physics patterns in $\epsilon'/\epsilon$ and $\epsilon_K$ with implications for rare kaon decays and $\Delta M_{K^0}$", JHEP 1604 (2016) 071 [arXiv:1601.00005 [hep-ph]].

T. Kitahara, U. Nierste and P. Templer, "Supersymmetric Explanation of CP Violation in $K \to \pi$ Decays", Phys. Rev. Lett. 117 (2016) no.9, 091802 [arXiv:1604.07400 [hep-ph]].

T. Kitahara, U. Nierste and P. Templer, "Singularity-free next-to-leading order $\Delta S = 1$ renormalization group evolution and $\epsilon'_K/\epsilon_K$ in the Standard Model and beyond", JHEP 1612 (2016) 078 [arXiv:1607.09727 [hep-ph]].

M. Endo et al., "Chargino contributions in light of recent $\epsilon'/\epsilon$", Phys. Lett. B 762 (2016) 493 [arXiv:1608.01444 [hep-ph]].

M. Endo et al., "Revisiting Kaon Physics in General Z Scenario", Phys. Lett. B 771 (2017) 37 [arXiv:1612.08839 [hep-ph]].

V. Cirigliano et al., "An $\epsilon'/\epsilon$ improvement from right-handed currents", Phys. Lett. B 767 (2017) 1 [arXiv:1612.03914 [hep-ph]].

S. Aoi et al., "Right-handed charged currents in the era of the Large Hadron Collider", JHEP 1705 (2017) 086 [arXiv:1703.04751 [hep-ph]].

C. Bobeth et al., "Patterns of Flavour Violation in Models with Vector-Like Quarks", JHEP 1704 (2017) 079 [arXiv:1609.04783 [hep-ph]].

C. Bobeth et al., "Yukawa enhancement of Z-mediated new physics in $\Delta S = 2$ and $\Delta B = 2$ processes", JHEP 1707 (2017) 124 [arXiv:1703.04753 [hep-ph]].

A. Crivellin et al., "$K \to \pi\phi$ in the MSSM in light of the $\epsilon'_K/\epsilon_K$ anomaly", Phys. Rev. D 96 (2017) no.1, 015023 [arXiv:1703.05786 [hep-ph]].

V. Chobanova et al., "Probing SUSY effects in $K^0_L/\mu^-\mu^+$", JHEP 1805 (2018) 024 [arXiv:1711.11030 [hep-ph]].

C. Bobeth and A. J. Buras, "Leptquarks meet $\epsilon'/\epsilon$ and rare Kaon processes", JHEP 1802 (2018) 101 [arXiv:1712.01295 [hep-ph]].

M. Endo et al., "Gluino-mediated electroweak penguin with flavor-violating trilinear couplings", JHEP 1804 (2018) 019 [arXiv:1712.04959 [hep-ph]].

C. H. Chen and T. Nomura, "$\epsilon_K$ and $\epsilon'/\epsilon$ in a disquark model", Phys. Lett. B 1808.0409 [hep-ph].

J. Aebischer et al., "Master formula for $\epsilon'/\epsilon$ beyond the Standard Model", JHEP 1807.02520 [hep-ph].

J. Aebischer, A. J. Buras and J. M. Gérard, "BSM Hadronic Matrix Elements for $\epsilon'/\epsilon$ and $K \to \pi$ Decays in the Dual QCD Approach", JHEP 1807.01709 [hep-ph].

N. Haba, T. Umeda and T. Yamada, "Direct CP Violation in Cabibbo-Favored Charged Meson Decays and $\epsilon'/\epsilon$ in $S(U(2)) \times S(U(2))_R \times U(1)_{P-L}$ Model", JHEP 1806.03424 [hep-ph].

S. Matsuizaki, K. Nishiwaki and K. Yamamoto, "Simultaneous interpretation of $K$ and $B$ anomalies in terms of chiral-flavorful vectors", arXiv:1806.02332 [hep-ph].

C. H. Chen and T. Nomura, "$\epsilon'/\epsilon$ from charged-Higgs-induced gluonic dipole operators", arXiv:1805.07522 [hep-ph].

C. H. Chen and T. Nomura, "$\epsilon'/\epsilon$ from $S(U(2)_C \times S(U(2)_R \times U(1))_{P-L}$ model with Charge Symmetry", JHEP 2018 (2018) 052 [arXiv:1802.09993 [hep-ph]].

C. Kelley, "Progress in lattice in the kaon system", talk at CKM 2018 (Heidelberg, September 17th).

V. Cirigliano, A. Pich, G. Ecker and H. Neufeld, "$\epsilon'_K$ anomalous and $\epsilon_K$ with $\text{Neutron EDM in } S(U(2)_C \times S(U(2)_R \times U(1)_{P-L}$ model with Charge Symmetry", JHEP 2018 (2018) 052 [arXiv:1802.09993 [hep-ph]].

V. Cirigliano, G. Ecker, H. Neufeld and A. Pich, "$\epsilon'/\epsilon$ and $\epsilon_K$ in $\pi$ Decays", ECO. Phys. J. C 33 (2004) 369 [hep-ph].
