Study of the Thomson effect on the performance of thermoelectric modules with application to the energy recovery

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Abstract. This paper analyzes the impact of the Thomson effect on the performance of thermoelectric modules. For this, different mathematical models are carried out that involve the relationship between temperature and the seebeck coefficient. These mathematical models are based on the equations that describe thermoelectric effects and are solved using finite element methods. Through linear and polynomial functions of the seebeck coefficient, the different behaviors that can occur in the Thomson coefficient and their effect on the power and efficiency of thermoelectric modules are analyzed. The results show that by not considering the Thomson effect, there is a variation of 31% and 32% in the power and efficiency of the thermoelectric module when the temperature conditions change, which makes it difficult to estimate the performance of the module. This problem can be solved by considering the Thomson effect since it predicts an approximately constant value of electrical power and efficiency for a wide temperature range. For the analyzed conditions, power and efficiency of 5.25 W and 13%, respectively, were observed. The proposed methodology allows an adequate estimation to determine the performance of the modules. Therefore, it could be implemented to search for materials that provide better thermoelectric characteristics.

1. Introduction
Today the energy demand is increasing as a consequence of the economic growth and consumption habits of modern society. Due to this situation, there has been an interest in looking for new ways to improve energy processes. One of the options to increase the efficiency of these processes is the recovery of residual heat, which is originated from a waste of thermal energy. By recovering and subsequently using this type of energy, it is possible to achieve significant percentages of energy savings [1].

Thermoelectric modules (TEMs) are devices with the ability to recover wasted thermal energy [2]. TEMs are composed of p-type and n-type semiconductors electrically connected in series. When a temperature difference occurs between the two TEM surfaces, the direct conversion of thermal energy into useful electrical energy is possible. The use of TEMs for the recovery of residual energy has generated significant interest due to the advantages that these devices have. These include its low complexity [3], high reliability [4], zero contaminants [5], and low maintenance [6].

The analysis of the dimensionless figure of merit allows the evaluation of the conversion efficiency of the TEMs. Despite the various advantages, TEMs generally operate at low efficiency, which has limited their massive use [1]. Due to the above, research has been carried out to look for an increase in the efficiency of TEMs, which has focused on the temperature difference of their ends [7], geometric variations [8], electrical and thermal resistances [9], the influence of heat flow [10], among others.
However, the analysis of these variables is generally carried out under ideal assumptions on the thermoelectric properties of the materials that make up the TEMs. For a more detailed study, it is necessary to consider the effect of temperature on the characteristic properties of TEMs. One of the main thermoelectric effects to consider is the Thomson effect, which arises from the strong dependence of the Seebeck coefficient on temperature [11]. Different studies investigated the Thomson effect assuming a constant Thomson coefficient. The results demonstrate a considerable impact on the efficiency of TEMs due to this effect [12]. To quantify the Thomson effect in the analysis of the performance of the TEMs, Yamashita et al. [13] studied the change in thermoelectric properties, assuming a linear behavior with temperature. Kim et al. [14] reduced general formulas are taking into account a cumulative dependence on temperature. The above in order to determine the maximum power and energy conversion efficiency of the TEMs. Other studies have focused on the Thomson effect under different operating conditions, involving the temperature difference present on the surfaces of the TEMs, various electric currents, and load resistances [15].

Previous research shows that the Thomson effect must be considered to assess the actual efficiency that TEMs can achieve. However, most of the research available in the literature describes the influence of the Thomson effect assuming simplified mathematical models, which can cause significant errors in the real estimation of the power and efficiency of thermoelectric modules. Due to the above, the present study aims to build mathematical models that allow relating the thermoelectric properties of TEMs as a function of temperature, which allows the effect of temperature on TEM performance to be directly considered. The equations used in the model are solved using the finite element method. Finally, the model is used to calculate the power and efficiency estimates of the TEM.

2. Methodology

Figure 1 shows the configuration of the thermoelectric generator (TEG) and the geometric dimensions of the TEM.

![Figure 1. Thermoelectric module diagram.](image)

The TEG is made up of the thermoelectric module and the electrical resistance (Rload). The TEM is formed by semiconductors (type p and type n), which are electrically connected in series by copper strips. These strips are subjected to a hot temperature (Th) and a cold temperature (Tc), which are generated by the flow of the heat source (Qin) and the flow of heat extracted by the cooling system (Qout). The connection made between the TEM and the electrical resistance causes the appearance of an electric current (I) and a voltage (V), which are used to calculate the power and efficiency of the TEM. To describe the electrical and thermal properties of TEM mathematically, such as heat flow (Q), electric charge, electric current density (J), dielectric medium (D), and scalar electric potential (E), the following equations are used [16,17]. The heat flow rate (Q) is determined by Equation (1).
∇ · \mathbf{Q} + \rho C \frac{\partial T}{\partial t} = \dot{Q}, \quad (1)

where \(\rho\) is the density, \(C\) is the heat capacity and \(\dot{Q}\) is the rate of heat generation; the electric charge is calculated by means of Equation (2).

\n
∇ \cdot \left( \frac{\partial \mathbf{D}}{\partial t} + \mathbf{J} \right), \quad (2)

where \(D\) and \(J\) are the vectors of electric flux density and electric current density.

Heat flow rate \((Q)\) and electric current density \((J)\) can be expressed, as shown in Equation (3) and Equation (4).

\begin{align*}
Q &= \Pi \cdot J - k \cdot \nabla T, \\
J &= \sigma \cdot (E - S \cdot \nabla T),
\end{align*}

(3)

(4)

where \(k\) is the thermal conductivity, and \(\Pi\) is the Peltier coefficient; the dielectric medium \((D)\) and the scalar electric potential \((E)\) are defined by Equation (5) and Equation (6).

\begin{align*}
D &= \varepsilon \cdot E, \quad (5) \\
E &= -\nabla \phi, \quad (6)
\end{align*}

where \(\phi\) is the scalar electric potential; by unifying the previously described equations, it is possible to form Equation (7) and Equation (8), which describe the thermal and electrical phenomena of TEM [18].

\begin{align*}
\rho C \frac{\partial T}{\partial t} + \nabla \cdot (\Pi \cdot J) - \nabla \cdot (k \cdot \nabla T) &= \dot{Q}, \\
\nabla \left( \varepsilon \cdot \nabla \frac{\partial \phi}{\partial t} \right) + \nabla \cdot (\sigma \cdot S \nabla T) + \nabla (\sigma \cdot \nabla \phi) &= 0.
\end{align*}

(7)

(8)

Finally, the calculation of the electrical power and the efficiency of the TEM was carried out using Equation (9) and Equation (10).

\begin{align*}
P_{\text{elec}} &= I^2 \cdot R_{\text{load}}, \\
\eta &= \frac{P_{\text{elec}}}{Q_{\text{in}}}.
\end{align*}

(9)

(10)

To solve this series of equations, the finite element method was used, which allows simulating the electrical and thermal effects. In addition, it allows considering the effects of seebeck, Joule, Peltier, and Thomson. The thermal effects of radiation and convection are ignored in the study, as they are depreciable in most conditions.

3. Results

This section shows the linear and polynomial fit of the seebeck coefficient and its effect on the Thomson coefficient as a function of different temperature levels. Additionally, the estimate of electrical power and TEM efficiency with and without the Thomson effect is shown.
3.1. Lineal variation of the seebeck coefficient

As a first approximation, it was assumed that the Seebeck coefficient ($S_{pn}$) varies linearly with temperature, as shown in Equation (11) [1].

$$S_{pn} = b_0 + b_1 T. \quad (11)$$

By changing the parameters $b_0$ and $b_1$ it is possible to obtain different values of the Seebeck coefficient. Figure 2 shows the relationship of this coefficient with different temperature conditions.

The results in Figure 2 show that the seebeck coefficient increases with increasing parameter $b_1$. In the case of $b_1 = 0$, an average Seebeck coefficient is obtained for the analyzed temperature range. This behavior is directly related to the mathematical model used to calculate $S_{pn}$. Therefore, a higher value of $b_1$ implies an increase of equal magnitude in the slope of the linear functions shown in Figure 2. Based on Equation (11), the Thomson coefficient ($\tau_{pn}$) is calculated, as shown in Equation (12) [19].

$$\tau_{pn} = T \cdot \frac{dS_{pn}}{dT} = T \cdot b_1. \quad (12)$$

The relationship between the Thomson coefficient and the temperature is shown in Figure 3. The results show a considerable increase in the Thomson coefficient with increasing temperature levels. For an increase in the $b_1$ value from 0.25 to 0.28, it was observed that the Thomson coefficient increases 52% faster for each change in temperature. This drastic increase in $\tau_{pn}$ implies a considerable decrease in TEM efficiency. For the value of $b_1 = 0$, it is obtained that $\tau_{pn}$ does not present any change in the temperature range, which is a consequence of the absence of variation in the value of the Seebeck coefficient.

![Figure 2. Linear dependence of the seebeck coefficient with temperature.](image1)

![Figure 3. Linear dependence of the Thomson coefficient with temperature.](image2)

3.2. Polynomial variation of the seebeck coefficient

Although linear dependence allows one to observe the effect of temperature on TEM performance, it is an unrealistic assumption of the process. Because of this, an adjustment of the Seebeck coefficient is performed using a third-degree polynomial function, as shown in Equation (13) [1].

$$S_{pn} = a_0 - a_1 T + a_2 T^2 - a_3 T^3, \quad (13)$$
where \(a_0, a_1, a_2\) and \(a_3\) are positive parameters. Among these four factors, it is observed that \(a_2\) is the most influential in the value of the Seebeck coefficient. Through changes in \(a_2\) different values of the Seebeck coefficient are obtained in relation to various temperature values. The results obtained are shown in Figure 4.

The results shown in Figure 4 indicate that the seebeck coefficient begins to decrease for a temperature value greater than 550 K. The decrease in \(S_{pn}\) is greater for high-temperature levels, which is a consequence of the polynomial function described in Equation (13). For this condition, the Thomson coefficient is defined, as shown in Equation (14) [19].

\[
\tau_{pn} = T \cdot \frac{dS_{pn}}{dt} = -T \cdot (a_1 - 2a_2T + 3a_3T^2)
\]  

Thomson coefficient values for different temperature conditions are shown in Figure 5. It is observed that the \(\tau_{pn}\) values grow in an approximately linear way for low-temperature levels. However, above 650 K, the Thomson coefficient values show a reduction. This is a consequence of the increase in the influence of the quadratic and cubic term of Equation (14).

![Figure 4](image1.png)  
**Figure 4.** Polynomial dependency of the seebeck coefficient.

![Figure 5](image2.png)  
**Figure 5.** Polynomial dependence of the Thomson coefficient with temperature.

To consider the influence of the Thomson effect, the results of TEM power and efficiency are compared in three different cases, which are defined in Table 1.

| Case | Condition | Equation |
|------|-----------|----------|
| Case 1 | Without Thomson effect [20] | \(S_{pn} = S_{pn}(T_h) + S_{pn}(T_c)\) |
| Case 2 | Without Thomson effect [21] | \(S_{pn} = S_{pn}(T_h) + S_{pn}(T_c)\) |
| Case 3 | With Thomson effect [1] | \(S_{pn} = a_0 - a_1T + a_2T^2 - a_3T^3\) |

The first two cases shown in Table 1 indicate a situation without the Thomson effect. Therefore, a constant value is assumed for the seebeck coefficient considering the temperatures \(T_h\) and \(T_c\) located at the extremes of the TEM (see Figure 1). In the third case, the Thomson effect is considered using Equation (13). The results of electrical power and efficiency TEM for the three cases are shown in Figure 6. Figure 6 (a) show the TEM electrical power and Figure 6(b) show the efficiency with \(T_c=300\) K and
Th= 950 K. Additionally the Figure 6, shows that the electrical power and energy conversion efficiency of TEM is reduced when the Thomson effect is considered, as it is in a range of the parameter $a_2$ between 2.6 - 2.75. However, for higher values of $a_2$, the estimation of power and efficiency of the TEM is higher when considering the Thomson effect. This indicates that ignoring the Thomson effect can lead to a higher or lower estimate of the actual performance of the TEM. This behavior is attributed to the cumulative effect of heat flow at the hot end of TEM [22].

In general, considering the Thomson effect allows an average estimate of the TEM performance to be made. It was observed that in case 3, the power and efficiency are approximately 5.25 W and 13% for the range analyzed. For the cases without Thomson effect (case 1 and case 2) a variation of 31% and 32% is observed in the electric power and efficiency of the TEM.

![Figure 6](image)

**Figure 6.** (a) TEM electrical power and (b) efficiency with $T_c=300$ K and $T_h=950$ K.

### 4. Conclusions

In the present paper, a mathematical model was performed to analyze the Thomson effect on the general performance of thermoelectric modules. Using finite element methods, the equations that describe the thermal and electrical phenomena characteristic of TEM were solved. Two types of relationships between temperature and the seebeck coefficient were analyzed in the study, which was linear and polynomial. The results obtained allowed concluding that the Thomson effect causes a negative impact on the performance of the TEM when it operates at a low-temperature difference, and the seebeck coefficient is high. However, for large temperature differences and small seebeck coefficients, the prediction of TEM performance is higher when considering the Thomson effect.

On average, by not considering the Thomson effect, an overestimation of the electrical power and the efficiency of the TEM is observed, which can reach values close to 6 W and 14.5%. However, depending on the temperature level, ignoring the Thomson effect causes an underestimation of the TEM performance, reaching a maximum value of power and efficiency of 4.5 W and 12%. The results show that the previous problem can be solved by considering the Thomson effect since it predicts an approximately constant value of electrical power and efficiency for a wide temperature range. Therefore, this last estimate of TEM performance is more appropriate, since TEMs are normally exposed to temperature variations in their operating conditions. The average values obtained for power and efficiency when considering the Thomson effect are 5.25 W and 13%. In general, the inclusion of the Thomson effect allows a more precise definition of the performance that can be obtained in thermoelectric modules. Therefore, the methodology developed in the study provides a guide for identifying the appropriate characteristics of TEMs in order to achieve high levels of efficiency.
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