A criterion for estimating the probability of microporosity formation in castings

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Abstract. The study tested a new dimensionless criterion for microporosity formation, which takes a number of significant technological factors into account, including atmospheric and metallostatic pressure, and alloy gas saturation. We recommend using the criterion in systems that model casting solidification to predict microporosity formation more accurately.

Keywords: microporosity criterion, porosity estimation, Darcy law, mushy zone, melt filtration, Niyama criterion

1. Introduction
The most effective computational method of microporosity prediction during solidification is a direct simulation of pore formation based on hydrodynamic equations and physical mechanisms of pore formation. However, existing mathematical models that this simulation requires are not yet accurate enough for practical application and require further improvement.

Various forms of pore formation criteria are used in practice. The most common of these is the Niyama criterion [1, 2, 4, 5, 7]. However, all of the criteria developed do have disadvantages. One of the main problems of the existing criteria, including the Niyama criterion, is that they do not take into account a number of significant technological factors that can have a substantial effect on microporosity formation. Paper [2] lists some practical results of employing a refined Niyama criterion that takes the technological factors into account.

When developing a new criterion, we took technological factors into consideration and we did it by introducing the following into Darcy’s filtering equation: metallostatic pressure, partial pressure of gases dissolved in the liquid, and capillary pressure in a dendritic lattice. To calculate the capillary pressure, we used the geometric mean distance between secondary dendritic arms as a pore radius.

2. Model
Let us assume that dendrites form during alloy solidification. Then a Piwonka - Flemings inequality [3] can represent the condition of porosity formation at any point $x$ of a one-dimensional mushy zone $0 \leq x \leq L$ containing dendrites (Fig. 1):

$$\Delta P_E(x) < \Delta P(x),$$

(1)
where $\Delta P(x) = P_L - P(x)$ is the pressure drop in the melt at a given point $x$, $\Delta P_{cr}(x) = P_L - P_{cr}(x)$ is the critical pressure drop that creates the conditions for pore formation, $P_L$ is the melt pressure at the inlet of the mushy zone for the crystallization onset temperature (temperature $T_L$), $P_S$ is the fluid pressure for the approximate crystallization end temperature (temperature $T_S$).

$$
\Delta P(x) = P_L - P(x), \quad \Delta P_{cr}(x) = P_L - P_{cr}(x),
$$

Figure 1. One-dimensional mushy zone model.

The inequality (1) enables us to construct a pore formation criterion for an arbitrary point $x$ of the mushy zone:

$$
P_{or}(x) = \frac{\Delta P_{cr}(x)}{\Delta P(x)} < 1
$$

(2)

If the inequality (2) is true, then we say that porosity exists, if it is not true, then we say that there is no porosity.

Starting with the paper [1], works [2-7] and others reflect the assumption that the following equation determines the critical pressure $P_{cr}(x)$ at a certain point of the mushy zone:

$$
P_{cr}(x) = P_g - P_{\sigma}
$$

(3)

Here $P_g$ is the gas pressure in cavitation bubbles, and $P_{\sigma}$ is the capillary pressure, which is equal to:

$$
P_{\sigma} = \frac{2\sigma}{r_0},
$$

(4)

where $\sigma$ is the surface tension, $r_0$ is the radius of a "viable" pore nucleus.

If we take the expression (4) into consideration, the equation (3) for calculating the critical pressure takes the form:

$$
P_{cr}(x) = P_g - \frac{2\sigma}{r_0},
$$

(5)

and the critical pressure differential is given by:

$$
\Delta P_{cr}(x) = P_L - P_g + \frac{2\sigma}{r_0}
$$

(6)

The pressure in a gas bubble located in liquid metal depends on the gas diffusion rate from the liquid phase into the gas bubble. At a high diffusion rate and with a known equation of state for the gas in the bubble, the gas pressure in the bubble can be expressed in terms of its radius: $P_g = P_g(r_0)$. We should
note the fact that it is very difficult to determine in practice the value of the pore nucleus radius for a liquid metal. As a rule, the value of the pore nucleus radius remains unknown, so this fact is a constraint for employing the new criterion to accurately evaluate porosity.

In this paper we propose to estimate the value $r_0$ according to the distance between secondary dendritic arms $\lambda_2$, which represents a space in which a micropore can be possibly located in terms of geometry. This space can be characterized by the size $r_{\text{max}} = 0.5 f_1 \lambda_2$, where $f_1$ is the proportion of the liquid phase. In fact $r_{\text{max}}$ is the radius of the sphere inscribed in the liquid portion of the interdendritic space between secondary dendritic arms (Fig. 2).

**Figure 2.** A geometric model of placing micropores in a dendritic lattice.

So, if there is a "viable" critical pore nucleus in the mushy zone, the condition $r_0 \leq r_{\text{max}}$ is true. Taking into account the ratio for the micropore radius, the critical pressure differential is equal to:

$$\Delta P_{\text{cr}}(x) = P_L - P_g + \frac{4 \sigma}{f_1(x) \lambda_2}$$

To describe the filtration flow of the melt in a one-dimensional mushy zone, we use the following form of Darcy’s equation:

$$u f_1(x) = - \frac{K}{\mu} \frac{dP}{dx},$$

where $u$ is the filtering rate, $\mu$ is the dynamic viscosity, $K = \frac{\lambda_2^2}{180 (1 - f_1)^3}$ is the Carman - Kozeny filtration coefficient [8,9], $\lambda_2$ is the distance between secondary dendritic arms (fig. 2).

From the mass conservation equation for a stationary one-dimensional mushy zone the following is derived:

$$u(x) = - \beta V = \text{const},$$

where $V$ is the crystallization front advancement velocity (fig. 1); $\beta = (\rho_s - \rho_l)/\rho_l$ is the volumetric shrinkage coefficient, $\rho_s$ and $\rho_l$ are the densities of the solid and liquid phases respectively.

If we insert (9) into (8), we get:

$$\beta V f_1(x) = \frac{K}{\mu} \frac{dP}{dx}$$

If the crystallization front velocity is constant, separating the variables and integrating the expression (10) over $x$, we obtain the required pressure differential at an arbitrary point $x$ of the one-dimensional mushy zone $0 \leq x \leq L$ (fig. 1):

$$\Delta P(x) = \mu \beta V \int_x^L \frac{f_1(x)}{K(f_1)} dx$$

(11)
Substituting a variable \( f_i \) for the variable \( x \) as well as assuming that the following coefficients \( \mu, \beta, \lambda \) are constant within the mushy zone, and that a value \( G = (T_L - T_s) / L \) is the average temperature gradient, taking into account the filtration coefficient \( K(f_i) \), we obtain the expression for the pressure differential in the mushy zone:

\[
\Delta P(x) = \mu \beta \Delta T \frac{V}{\lambda^2 G} I(f_i(x)).
\]

(12)

where \( I(f_i(x)) = 180 \int_{f_i(x)}^{1} \left( \frac{1 - f_i}{f_i} \right)^2 df_i \).

Then, taking (12) and (7) into consideration, the criterion of porosity formation (2) at any point in the stationary mushy zone takes the form:

\[
Por(x) = \frac{\Delta P_{cr}(x)}{\Delta P(x)} = \frac{P_L - P_g + \frac{4\sigma}{f_i(x)\lambda^2}}{\mu \beta \Delta T \frac{V}{\lambda^2 G} I(f_i(x))} < 1
\]

(13)

If the inequality (13) is true for at least one point of the stationary mushy zone, porosity will appear.

3. Conclusion

1. We obtained a new criterion for microporosity formation in castings. The criterion has a dimensionless form and takes into account not only thermal conditions of the solidification process, but also significant technological parameters, such as metallostatic pressure and alloy gas saturation.

2. In formulating the conditions of microporosity formation, instead of the critical pore nucleus radius we suggest using its estimation determined by the distance between secondary dendritic arms and the proportion of the liquid phase. This eliminates technically complex experimental determination of the critical micropore nucleus radius in metal alloys.

3. The resulting criterion does not contain difficult to measure physical parameters and does not require "experimental" tuning, which is of practical use in systems simulating casting processes when evaluating the conditions of microporosity formation in the castings.

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