Research Article

RBFNN-Based Nonuniform Trajectory Tracking Adaptive Iterative Learning Control for Uncertain Nonlinear System with Continuous Nonlinearly Input

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This paper proposes an adaptive iterative learning control (AILC) method for uncertain nonlinear system with continuous nonlinearly input to solve different target tracking problem. The method uses the radial basis function neural network (RBFNN) to approximate every uncertain term in systems. A time-varying boundary layer, a typical convergent series are introduced to deal with initial state error and unknown bounds of errors, respectively. The conclusion is that the tracking error can converge to a very small area with the number of iterations increasing. All closed-loop signals are bounded on finite-time interval $[0, T]$. Finally, the simulation result of mass-spring mechanical system shows the correctness of the theory and validity of the method.

1. Introduction

The research of the nonuniform trajectory is an interesting problem. Two new AILC methods for first-order hybrid parametric systems and high-order nonlinear hybrid parameter systems were proposed in the literature studies [1, 2], respectively. Recently, AILC was presented, the literature [3] proposed a nonuniform target tracking AILC method, and the literature [4] proposed a fault-tolerant ILC technique for mobile robot nonrepetitive target tracking with output constraints. An ILC for a flapping wing micro aerial vehicle under distributed disturbances was proposed in the literature [5]. It can be seen from the above literature studies that solving the nonuniform target tracking problem for uncertain nonlinear systems is an important problem.

Adaptive control is used to handle system control problem about uncertainties. Adaptive control schemes learn uncertainties by adaptive laws. NN and FLS are used in the method as function approximators, for example, the paper [6, 7]. The literature [8] could complete the varying control tasks by designing an adaptive fuzzy ILC for the uncertain nonlinear system. Based on RBF neural network approximation, the paper [9] proposed AILC for nonlinear pure-feedback systems to solve the nonuniform target tracking problem. The uniform AILC frame for uncertain nonlinear system was proposed in the literature [10], by Lyapunov theory, and it could prove the convergence. It should be noted that Lyapunov function-based AILC played an important role in dealing with the time-varying parameter [8, 10, 11]. However, initial state error problem is a challenging one as they need to converge to zero for keeping stability. The literature studies [8, 9, 12] considered this problem recently. It is an important problem for AILC. Because the actuator's physical is limited, the control input widely exists continuous nonlinearity such that system performance can be deteriorated. In the literature studies [13–22], control performance could change by using different technique recently. The literature [15] solved the question of adaptive stabilization for the time-delay system. The literature [16] developed an adaptive backstepping method of uncertain nonlinear systems about nonsymmetric...
dead-zone. As yet, there is no report from the literature for the AILC of nonlinear systems with continuous nonlinearly input and initial state error. This is a problem that needs to be solved urgently.

In this paper, the nonuniform trajectory tracking issue is discussed for the uncertain nonlinear systems with continuous nonlinearly input and initial state error. The contributions of the proposed control method are presented as follows:

(i) The nonuniform trajectory tracking issue is studied for uncertain nonlinear systems under continuous nonlinearly input and initial state error issues.

(ii) The AILC method is used to uncertain nonlinear systems. The RNFNN is introduced to learn unknown dynamic. A convergence order is introduced to solve the unknown bound and nonuniform target tracking problem.

Finally, simulation results of the mass-spring mechanical system are given to verify the validity of the designed controller.

This paper is organized as follows: in Section 2, the system description and related concepts are given in detail. The main results are presented in Section 3. A simulation is shown in Section 4. Section 5 is the conclusion.

2. System Description and Related Concepts

2.1. System Model. The following nonlinear systems are considered:

\[ \begin{align*}
\dot{x}_{1,k} &= x_{2,k} + f_1(x_{1,k}), \\
\dot{x}_{j,k} &= x_{j+1,k} + f_j(x_{j,k}), \\
\dot{x}_{n,k} &= N(u_k) + f_n(x_{n,k}), \\
y_{k} &= x_{1,k},
\end{align*} \]

where \( x_{j,k} = [x_{1,k}, \ldots, x_{j,k}]^T \in \mathbb{R}^j \), and \( x = x_n \) is the state that is measured. \( N(u_k) \in \mathbb{R} \) represents the actuator characteristics, and \( y_k \in \mathbb{R} \) is the system output. \( f_j(x_{j,k}), j = 1, 2, \ldots, n \), are smooth unknown nonlinear functions.

\( N(u_k) \) represents continuous nonlinearly input with \( N(0) = 0 \) which belongs to \([g_1, g_2]\), i.e.,

\[ \begin{align*}
g_1 u_k &\leq N(u_k) \leq g_2 u_k, & \text{if } u_k \geq 0, \\
g_1 u_k &\geq N(u_k) \geq g_2 u_k, & \text{if } u_k \leq 0.
\end{align*} \]

Assumption 1. \( g_1 \) and \( g_2 \) are unknown nonzero positive parameters.

Remark 1. This is a reasonable assumption, because many system constraints can satisfy this condition.

Designing an AILC law \( u_k(t) \) on [0, T] to make the output \( y_{k}(t) \) following the target trajectory \( y_{r,k}(t) \) is the control objective of this paper; that is to say, \( \lim_{t \to \infty} \| y_{k}(t) - y_{r,k}(t) \| \leq \varrho \), where \( \varrho \) is a very small positive number. All closed-loop signals are guaranteed to be bounded. \( y_{r,k}(t) \) is the smooth desired target. \( k \) is the iteration index.

2.2. Convergent Series Sequence. The following definition and lemma are used in the controller design process.

Definition 1 (see [19]). \( \{\Delta_k\} \) is a series convergent sequence which is shown as

\[ \Delta_k = \frac{a}{k^l}, \]

where \( k = 1, 2, \ldots, a \) and \( l \) satisfies \( a > 0 \in \mathbb{R}, l \geq 2 \in \mathbb{N} \).

Lemma 1 (see [19]). For \( \{1/k^l\} \), where \( k = 1, 2, \ldots, l \geq 2 \), the following inequality holds:

\[ \lim_{k \to \infty} \sum_{j=1}^{k} \frac{1}{j^2} \leq 2. \]

2.3. Description of RBFNN Approximation (see [23])

\[ F_j(x_{j,k}) = W^T \zeta_j(x_{j,k}) + \delta_j(x_{j,k}), \]

where \( \zeta_j(x_{j,k}) = [s_1(x_{j,k}), \ldots, s_n(x_{j,k})]^T : \Omega \to \mathbb{R}^m \) are known, and \( l > 1 \) is the number of NN node. \( s_j(x_{j,k}) = r^{|x_{j,k} - \mu_j|^2/\eta_j|} \), \( \mu_j \in \Omega \), and \( \eta > 0 \) are the center and the width of \( s_j(x_{j,k}) \), respectively. The truth weight \( W = [w_{11}, \ldots, w_{l1}]^T \) is given as follows:

\[ W = \arg\min_{W \in \mathbb{R}^l} \left\{ \sup_{x_{j,k} \in \Omega} \left| F_j(x_{j,k}) - W^T \zeta_j(x_{j,k}) \right| \right\}, \]

and \( \delta(x_{j,k}) \) are the NN approximation errors. The following assumption holds.

Assumption 2. On a compact set \( \Omega \), \( \delta_j(x_{j,k}) \) are bounded and \( |\delta_j(x_{j,k})| \leq \theta_j \), where \( \theta_j (1 \leq j \leq n) \) are unknown and \( \theta_j \geq 0 \).

Remark 2. This is a general assumption for the approximation error of the neural network. The approximation error can be arbitrarily decreased by increased NN node number. So, this assumption is reasonable. The following assumptions \((4 - j), j = 1, 2, \ldots, n \) are also reasonable.

2.4. Time-Varying Boundary Layer. The following functions \( z_{j,k} \) are introduced to deal with initial state errors:

\[ \begin{align*}
&z_{j,k} = z_{j,k} - \psi_{j,k}(t) \text{sat} \left( \frac{z_{j,k}}{\phi_{j,k}(t)} \right), \\
&\phi_{j,k}(t) = \delta_{j,k} e^{-\eta t},
\end{align*} \]

where \( z_{j,k} \) and \( z_{j,k} \) are variables of \( t \), \( \delta_{j,k} \) are sequences of convergent series, and the saturation function sat is given as follows:
where \( \phi_{j,k}(t) \) are the time-varying boundary layers. The control objective can be achieved by this. See the literature [8] for specific analysis.

**Assumption 3.** The initial state errors must satisfy \( |z_{j,k}(0)| = \epsilon_{j,k} \) for some known positive parameters \( \epsilon_{j,k} \), \( j = 1, \ldots, n \), where \( z_{j,k}(0) \) are initial state errors at each iteration of state errors which are given later in this paper.

**Remark 3.** Under normal circumstances, the initial state errors at each iteration are zero, small, or fixed. Compared with the assumptions of the initial state in other articles, the assumptions in this paper are more relaxed and easy to be satisfied.

### 3. Adaptive Iterative Learning Controller Design and Convergence Analysis

In this section, we will discuss the detailed controller design process, main conclusion, and specific proof process.

#### 3.1. Adaptive Iterative Learning Controller Design

**Lemma 2.** For the controller \( u_k \) and the error \( z_{n,k} \), the following inequality holds:

\[
z_{n,k}N(u_k) \leq g z_{n,k} u_k, \quad g \in \{g_1, g_2\}.
\]

**Proof.** If both sides of equation (2) multiply \( u_k \), we obtain

\[
g_1 u_k^2 \leq u_k N(u_k) \leq g_2 u_k^2.
\]

If both sides of equation (10) multiply \( z_{n,k}^2 u_k \), we obtain

\[
g_1 u_k^2 z_{n,k}^2 \leq u_k N(u_k) z_{n,k}^2 \leq g_2 u_k^2 z_{n,k}^2,
\]

i.e.,

\[
g_1 (z_{n,k} u_k)^2 \leq z_{n,k} N(u_k) u_k \leq g_2 (z_{n,k} u_k)^2.
\]

If \( u_k \) satisfies \( z_{n,k} u_k \leq 0 \), then

\[
z_{n,k} N(u_k) \leq g_1 z_{n,k} u_k.
\]

If \( u_k \) satisfies \( z_{n,k} u_k \geq 0 \), then

\[
z_{n,k} N(u_k) \leq g_2 z_{n,k} u_k.
\]

So, the result is obtained.

The specific process about designing controller is given as follows.

**Step 1.** Let \( z_{1,k} = x_{1,k} - y_{r,k} \), \( z_{2,k} = x_{2,k} - \alpha_{1,k} \), \( F_1(x_{1,k}) = f_1(x_{1,k}) \), \( \alpha_{1,k} \) is the virtual controller. Introduce error function \( z_{1,k} \) and \( z_{2,k} \) by Section 2.4 to deal with initial state error as

\[
z_{1,k} = z_{1,k} - \phi_{1,k}(t) \text{sat}(z_{1,k}^{\frac{1}{\phi_{1,k}(t)}}),
\]

\[
\phi_{1,k}(t) = e_{1,k} e^{-\eta_1 t};
\]

\[
z_{2,k} = z_{2,k} - \phi_{2,k}(t) \text{sat}(z_{2,k}^{\frac{1}{\phi_{2,k}(t)}}),
\]

\[
\phi_{2,k}(t) = e_{2,k} e^{-\eta_1 t}.
\]

Recall that

\[
\dot{x}_{1,k} = z_{2,k} + F_1(x_{1,k}).
\]

The derivative of \( z_{1,k} \) is as follows:

\[
\dot{z}_{1,k} = z_{2,k} + \alpha_{1,k} + F_1(x_{1,k}) - \dot{y}_{r,k} - \text{sgn}(z_{1,k}) \phi_{1,k}.
\]

According to Section 2.3, by RBFNN, \( F_1(x_{1,k}) \) is approximated and approximate error \( \delta_1(x_{1,k}) \) is as follows:

\[
F_1(x_{1,k}) = W_1^T \zeta(x_{1,k}) + \delta_1(x_{1,k}),
\]

where \( W_1 \) is optimal weight vector.

Denote \( N_1 = \omega_1^2 M_1 \), which is needed later, \( \Delta_k = a/k \), \( a > 0 \), and \( l \leq 2 \). The virtual controller is taken as

\[
\alpha_{1,k} = -\bar{W}_1^{\top} \zeta(x_{1,k}) - \frac{1}{\Delta_k} z_{1,k} + \bar{N}_1 \bar{z}_{1,k} + \eta_1 \bar{z}_{1,k}.
\]

When (18) and (19) are substituted into (17), we have

\[
\dot{z}_{1,k} = z_{2,k} - \bar{N}_1 \bar{z}_{1,k} - \frac{1}{\Delta_k} z_{1,k} + \bar{W}_1^{\top} \zeta(x_{1,k}) - \bar{W}_1^{\top} \zeta_1(x_{1,k})
\]

\[
- \eta_1 \bar{z}_{1,k} - \text{sgn}(z_{1,k}) \phi_{1,k}(t)
\]

\[
= z_{2,k} - \bar{N}_1 \bar{z}_{1,k} - \bar{W}_1^{\top} \zeta(x_{1,k})
\]

\[
+ \delta_1(x_{1,k}) + \phi_{2,k}(t) \text{sat}(z_{2,k}^{\frac{1}{\phi_{2,k}(t)}})
\]

\[
- \eta_1 \bar{z}_{1,k} - \text{sgn}(z_{1,k}) \phi_{1,k}(t),
\]

where estimated \( W_1 \) and \( N_1 \) are \( \bar{W}_1 \) and \( \bar{N}_1 \), respectively. \( \bar{W}_1 = W_1 - W_1 \) and \( \bar{N}_1 = N_1 - N_1 \) are the errors of estimated parameters. The last two terms of (20) can be changed to
where
\[
\dot{z}_{j,k} = z_{j,k} - \frac{1}{\Delta_k} z_{j,k} - \eta_1 z_{j,k}
\]
(23)

Let \( \omega_1 = \delta_1(\mathbf{x}_{1,k}) + \phi_{2,k}(t) \text{sat}(z_{2,k}(t)/\phi_{2,k}(t)) \), then (22) becomes
\[
\dot{z}_{1,k} = z_{2,k} - \frac{1}{\Delta_k} z_{2,k} - \eta_1 z_{1,k} + \frac{1}{\Delta_k} \omega_1 + \frac{1}{\Delta_k} \omega_2
\]
(24)

**Assumption 4.** 1: the bounded term \( \omega_1 \) satisfies \( |\omega_1| \leq \omega_{M1} \), where \( \omega_{M1} \) is a positive parameter.

Take the following nonnegative function:
\[
V_{1,k} = \frac{1}{2} z_{1,k}^2 + \frac{1}{2} \tilde{W}_{1,k} \mathbf{1}_{-1} \tilde{W}_{1,k} + \frac{1}{2} \tilde{N}_{1,k}^2
\]
(25)

where \( \Gamma_{11} \) and \( \Gamma_{21} \) are the symmetric positive definite matrices. The derivative of \( V_{1,k} \) along (23) is as follows:
\[
\dot{V}_{1,k} = z_{1,k} \dot{z}_{1,k} - \eta_1 z_{1,k}^2
\]
(26)

\[
\dot{V}_{1,k} = \frac{1}{\Delta_k} z_{1,k}^2 + \omega_1 z_{1,k} + \Gamma_{21} \tilde{N}_{1,k} \dot{\mathbf{x}}_{1,k}
\]
(27)

\[
\dot{V}_{1,k} = \tilde{W}_{1,k} \mathbf{1}_{-1} \tilde{W}_{1,k} + \frac{1}{\Delta_k} \omega_1 z_{1,k} - \eta_1 z_{1,k}^2
\]
(28)

In previous equation, \( mn \leq (1/r)m^2 + (1/4)n^2r \) is used, where \( r = \Delta_k \).

Step 2, j: (2 \leq j \leq n - 1). Denote \( F_j(\mathbf{x}_{j,k}) = f_j(\mathbf{x}_{j,k}) - \sum_{i=1}^{j-1} \alpha_{j-1,k} \partial z_i(\mathbf{x}_{j,k}) \dot{\mathbf{x}}_{i,k} \), which is given later. \( z_{j+1,k} = \mathbf{x}_{j+1,k} - \mathbf{\lambda}_{j,k} \), similar to Step 1, and \( z_{j,k} \) by Section 2.4 are introduced as follows:
\[
z_{j,k} = z_{j,k} - \phi_{j,k}(t) \text{sat}(\frac{z_{j,k}}{\phi_{j,k}(t)})
\]
(29)

\[
\phi_{j,k}(t) = \varepsilon_{j,k} e^{-\eta_{j,k}}
\]
(30)

\[
z_{j+1,k} = z_{j+1,k} - \phi_{j+1,k}(t) \text{sat}(\frac{z_{j+1,k}}{\phi_{j+1,k}(t)})
\]
(31)

The derivative of \( z_{j,k} \) is as follows:
\[
\dot{z}_{j,k} = z_{j,k} + \alpha_{j,k} + F_j(\mathbf{x}_{j,k})
\]
(32)

\[
\dot{z}_{j,k} = \varepsilon_{j,k} e^{-\eta_{j,k}} + P_{j-1,k} - \text{sgn}(z_{j,k}(t)) \phi_{j,k}
\]
(33)

According to Section 2.3, \( F_j(\mathbf{x}_{j,k}) \) can become by the RBFNN
\[
F_j(\mathbf{x}_{j,k}) = W_j^T \mathbf{1}_{-1} \mathbf{x}_{j,k} + \delta_j(\mathbf{x}_{j,k})
\]
(34)

where \( \delta_j(\mathbf{x}_{j,k}) \) are the approximation errors and \( W_j \) are optimal weight vectors.

The virtual controller is chosen as follows:
\[
\alpha_{j,k} = -z_{j-1,k} - \tilde{W}_{j-1,k} \mathbf{1}_{-1} \mathbf{x}_{j,k} - \tilde{N}_{j,k} \frac{1}{\Delta_k} z_{j,k}
\]
(35)

\[
\alpha_{j,k} = -z_{j-1,k} - \tilde{W}_{j-1,k} \mathbf{1}_{-1} \mathbf{x}_{j,k} - \tilde{N}_{j,k} \frac{1}{\Delta_k} z_{j,k} + P_{j-1,k} - \text{sgn}(z_{j,k}(t)) \phi_{j,k}
\]
(36)

When (29) and (30) are substituted into (28), we have
\[
\dot{z}_{j,k} = z_{j,k} - z_{j-1,k} - \tilde{N}_{j,k} \frac{1}{\Delta_k} z_{j,k} - \eta_j z_{j,k} - \text{sgn}(z_{j,k}(t)) \phi_{j,k}
\]
(37)

\[
\dot{z}_{j,k} = z_{j,k} - z_{j-1,k} - \tilde{N}_{j,k} \frac{1}{\Delta_k} z_{j,k} - \tilde{W}_{j-1,k} \mathbf{1}_{-1} \mathbf{x}_{j,k} + \delta_j(\mathbf{x}_{j,k})
\]
(38)

\[
\dot{z}_{j,k} = z_{j,k} - z_{j-1,k} - \tilde{N}_{j,k} \frac{1}{\Delta_k} z_{j,k} - \tilde{W}_{j-1,k} \mathbf{1}_{-1} \mathbf{x}_{j,k} + \delta_j(\mathbf{x}_{j,k})
\]
(39)

\[
\dot{z}_{j,k} = z_{j,k} + \alpha_{j,k} + F_j(\mathbf{x}_{j,k}) \text{sat}(\frac{z_{j+1,k}}{\phi_{j+1,k}(t)})
\]
(40)

\[
\dot{z}_{j,k} = z_{j,k} - \text{sgn}(z_{j,k}(t)) \phi_{j,k}
\]
(41)
where $\hat{W}_{j,k}$ and $\hat{N}_{j,k}$ are the estimated parameters of $W_j$ and $N_j$, respectively. $\hat{W}_{j,k} = \tilde{W}_{j,k} - W_j$ and $\hat{N}_{j,k} = \tilde{N}_{j,k} - N_j$ are estimated parameter errors. The last two terms of (31) can be changed as

$$-\eta_j z_{j,k} - \text{sgn}(z_{j,k}) \dot{\phi}_{j,k}(t)$$

$$= -\eta_j z_{j,k} - \eta_j \dot{\phi}_{j,k}(t) \frac{z_{j,k}}{\phi_{j,k}(t)} - \text{sgn}(z_{j,k}) \dot{\phi}_{j,k}(t)$$

$$= -\eta_j z_{j,k} \cdot \dot{\phi}_{j,k}.$$  

(32)

Let $\omega_j = \delta_j(\pi_{j,k}) + \phi_{(j+1),k}(t) \text{sat}(z_{(j+1),k} / \phi_{(j+1),k} (t))$, then (31) becomes

$$\dot{z}_{j,k} = z_{(j+1),k} - \eta_j z_{j,k} - \eta_j \dot{z}_{j,k} - \hat{N}_{j,k} \frac{1}{\Delta_k} z_{j,k}$$

$$+ \hat{W}_{j,k}^T \pi_{j,k} + \omega_j.$$  

(33)

Assumption 5. $\gamma_j$: $|\omega_j| \leq \omega_{M_j},$ here $\omega_{M_j}$ is unknown.

The following positive definite function is chosen:

$$V_{j,k} = V_{j-1,k} + \frac{1}{2} z_{j,k}^2 + \frac{1}{2} \hat{W}_{j,k}^T \hat{\Gamma}_{j-1} \hat{W}_{j,k} + \frac{1}{2} z_{j,k}^2.$$  

(34)

The derivative of $V_{j,k}$ is as follows by substituting (33):

$$V_{j,k} \leq z_{j,k}^2 + \sum_{i=1}^{j} \eta_i z_{i,k}^2$$

$$- \sum_{i=1}^{j} \hat{W}_{i,k}^T \hat{\Gamma}_{i-1} \pi_{i,k} z_{i,k} - \hat{W}_{i,k}^T.$$  

(35)

Step 3. Define $z_{n,k} = x_{n,k} - \alpha_{n-1,k}, z_{n,k}$ by Section 2.4 is introduced as follows:

$$z_{n,k} = \frac{x_{n,k} - \phi_{n,k}(t)}{\phi_{n,k}(t)}.$$  

(36)

The derivative of $z_{n,k}$ is as follows:

$$\dot{z}_{n,k} = N(u_k) + \dot{f}(x_{n,k}) - \hat{\alpha}_{n-1,k}$$

$$- \text{sgn}(z_{n,k}) \dot{\phi}_{n,k}(t).$$  

(37)

where $\hat{\alpha}_{n-1,k} = \sum_{i=1}^{n-1} \hat{\alpha}_{n-1,k} / \partial x_{i,k} \pi_{i,k} x_{i,k} + f_i(\pi_{i,k}) + \partial \alpha_{n-1,k} / \partial \hat{W}_{i,k} \hat{W}_{i,k} + \partial \alpha_{n-1,k} / \partial \hat{N}_{i,k} \hat{N}_{i,k} + \alpha_{n-1,k} / \partial t$.

Denote $F_n(\pi_{n,k}) = f_n(x_{n,k}) - \sum_{i=1}^{n} \hat{\alpha}_{n-1,k} / \partial x_{i,k} \pi_{i,k} x_{i,k} + f_i(\pi_{i,k}), p_{n-1,k} = \partial \alpha_{n-1,k} / \partial \hat{W}_n - \hat{N}_{n-1,k} + \partial \alpha_{n-1,k} / \partial \hat{N}_{n-1,k} \hat{N}_{n-1,k} + \alpha_{n-1,k} / \partial t$, and then (37) can be rewritten as follows:

$$\dot{z}_{n,k} = N(u_k) + F_n(\pi_{n,k}) - P_{n-1,k} - \text{sgn}(z_{n,k}) \dot{\phi}_{n,k}(t).$$  

(38)

$F_n(\pi_{n,k})$ can be rewritten by RBFNN approximation as follows:

$$F_n(\pi_{n,k}) = W_n^T \pi_n(\pi_{n,k}) + \delta_n(\pi_{n,k}),$$  

(39)

where $\delta_n(\pi_{n,k})$ is the approximation error and $W_n$ is an optimal weight vector.

Then, (38) can be rewritten by substituting (39):

$$\dot{z}_{n,k} = N(u_k) + W_n^T \pi_n(\pi_{n,k}) + \delta_n(\pi_{n,k})$$

$$- P_{n-1,k} - \text{sgn}(z_{n,k}) \dot{\phi}_{n,k}(t).$$  

(40)

Lyapunov function is chosen as follows:

$$V_{n,k} = V_{n-1,k} + \frac{1}{2} z_{n,k}^2 + \frac{1}{2} \hat{W}_{n,k}^T \hat{\Gamma}_{n-1} \hat{W}_{n,k}$$

$$+ \frac{1}{2} z_{n,k}^2 + \frac{1}{2} z_{n,k}^2.$$  

(41)

where estimated $W_{n,n}, N_n$ and $\varphi = 1 / g$ are $\hat{W}_{n,k}, \hat{N}_{n,k}$ and $\hat{\varphi}_k(t)$, respectively. $\hat{W}_{n,k} = \hat{W}_{n,k} - W_n, \hat{N}_{n,k} = N_{n,k} - N_n$, and $\hat{\varphi}_k(t) = x_k(t) - \varphi$ are parameter estimation errors. And $\Gamma_k$ is learning gain to be designed.

Using the derivative of $V_{n,k}$ along (40), then we have

$$V_{n,k} \leq z_{(n-1),k} z_{n,k} - \sum_{i=1}^{n-1} \eta_i z_{i,k}^2$$

$$- \sum_{i=1}^{n-1} \hat{W}_{i,k}^T \hat{\Gamma}_{i-1} \pi_{i,k} z_{i,k} - \hat{W}_{i,k}^T$$

$$- \sum_{i=1}^{n-1} \hat{N}_{i,k} \hat{\Gamma}_{i-1} \hat{\pi}_{i,k} z_{i,k}^2 - \hat{N}_{i,k}$$

$$+ (n - 1) \frac{1}{4} \hat{\Delta}_k + z_{n,k} \dot{z}_{n,k} + \hat{W}_{n,k}^T \hat{\Gamma}_{n-1} \hat{W}_{n,k}$$

$$+ \hat{\Gamma}_{n-1} \hat{N}_{n,k} \hat{N}_{n,k} + \hat{\Gamma}_{n} \hat{\varphi}_k(\hat{\varphi}_k)$$

$$\leq z_{(n-1),k} z_{n,k} - \sum_{i=1}^{n-1} \eta_i z_{i,k}^2$$

$$- \sum_{i=1}^{n-1} \hat{W}_{i,k}^T \hat{\Gamma}_{i-1} \pi_{i,k} z_{i,k} - \hat{W}_{i,k}^T$$

$$- \sum_{i=1}^{n-1} \hat{N}_{i,k} \hat{\Gamma}_{i-1} \hat{\pi}_{i,k} z_{i,k}^2 - \hat{N}_{i,k}$$

$$+ (n - 1) \frac{1}{4} \hat{\Delta}_k + (u_k + (g - 1) u_k)$$

$$+ W_{n-1,k}^T \hat{\pi}_{n-1,k} \hat{\pi}_{n-1,k} - P_{n-1,k}$$

$$- \text{sgn}(z_{n,k}) \dot{\phi}_{n,k}(t) z_{n,k} + \hat{W}_{n,k}^T \hat{\Gamma}_{n-1} \hat{W}_{n,k}$$

$$+ \hat{\Gamma}_{n-1} \hat{N}_{n,k} \hat{N}_{n,k} + \frac{1}{2} \hat{\varphi}_k(\hat{\varphi}_k).$$
Choose 
\[ u_{1,k} = -z_{(n-1)\phi,k} - \tilde{W}_{n,k}^T \zeta_n(\vec{x}_{n,k}) - \tilde{N}_{n,k}(1/\Delta_k) \]
where 
\[ u_{2,k} = -\text{sat}(z_{n,k}/\phi_n(t)) (\hat{\varrho}_k(t) + 1)|u_{1,k}|, \]
and then (46) becomes

\[ \dot{V}_{n,k} \leq -\sum_{i=1}^{n} \eta_i z_{\phi,k}^2 \]

Then, (46) becomes

\[ \dot{V}_{n,k} \leq z_{(n-1)\phi,k} \bar{z}_{n,k} - \sum_{i=1}^{n} \eta_i z_{\phi,k}^2 \]

When \( \omega_n = \delta_n(\vec{x}_{n,k}) \), the following equation holds:

\[ -\eta_n z_{n,k} - \text{sgn}(z_{n,k}) \psi_k(t) \]

and then we have

\[ \dot{V}_{n,k} \leq -\sum_{i=1}^{n} \eta_i z_{\phi,k}^2 \]

Assumption 6. \( n \): bounded \( \omega_n \) satisfies \( |\omega_n| \leq \omega_{\Delta n} \), here \( \omega_{\Delta n} \) is an unknown parameter.

Choose the AILC laws as follows:

\[ \tilde{W}_{j,k} = \Gamma_1 \zeta_j(\vec{x}_{j,k}) \bar{z}_{\phi,k}, \quad j = 1, \ldots, m, \]

and then we obtain

\[ \dot{V}_{n,k} \leq -\sum_{i=1}^{n} \eta_i z_{\phi,k}^2 \]

and then (46) becomes

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and then we obtain

\[ \dot{V}_{n,k} \leq -\sum_{i=1}^{n} \eta_i z_{\phi,k}^2 \]

3.2 Convergence Analysis. According to the above design process, we can get the following conclusion expressed by Theorem 1. The following assumption is needed.
Theorem 1. Under Assumptions 1–7, by controller (43) and where $\varsigma$ is a small positive parameter.

\[
\lim_{k \to \infty} z_{\phi,k}(t) = 0, \quad l = 1, 2, \ldots, n,
\]

i.e., \( \lim_{k \to \infty} z_{\phi,k}(t) = 0, \) \( \| z_{\phi,k}(t) \| < \phi_{i,\infty}(t) + \varsigma \) as \( k \to \infty, \)

where \( \varsigma \) is a small positive parameter.

Proof. By Section 2.4, \( \| z_{\phi,k}(0) \|^2 \leq 0 \leq \| z_{\phi,k}(T) \|^2, \)

where \( z_{\phi,k} = [z_{1,\phi,k}, z_{2,\phi,k}, \ldots, z_{n,\phi,k}]^T. \)

By (41), we get

\[
V_{n,k}(z_{\phi,k}(0), \dot{W}_k(T), \tilde{N}_k(T), \tilde{\varrho}_k(T)) \leq V_{n,k}(0) + \int_0^T V_{n,k}(t) dt,
\]

where \( \dot{V}_k = [W_{1,k}, W_{2,k}, \ldots, N_{n,k}]^T \) and \( \tilde{N}_k = [\tilde{N}_{1,k}, \tilde{N}_{2,k}, \ldots, \tilde{N}_{n,k}]^T. \) (53) is substituted into (53), and we get

\[
V_{n,k}(z_{\phi,k}(0), \dot{W}_k(T), \tilde{N}_k(T), \tilde{\varrho}_k(T)) \leq V_{n,1}(0) - \sum_{j=1}^m \int_0^T \eta_j^2(z_{\phi,j}(t))^2 + n \left( \frac{1}{4} \right)^\tau \sum_{j=1}^m (\Delta_j).
\]

(54)

Denote \( V_{0,k} = V_{n,1}(0) + n(1/4)^\tau (\sum_{j=1}^m \Delta_j), \) and then (54) can be rewritten as

\[
\sum_{j=1}^m \int_0^T \eta_j^2(z_{\phi,j}(t))^2 dt \leq V_{0,k} - V_{n,k}(z_{\phi,k}(0), \dot{W}_k(T), \tilde{N}_k(T), \tilde{\varrho}_k(T)).
\]

By (41), \( \lim_{k \to \infty} V_{0,k} \leq V_{n,1}(0) + 2an(1/4)^\tau, \) then \( V_{0,k} \) is bounded, and by Assumption 7 \( V_{n,k}(0) \) is bounded. \( V_{n,k}(z_{\phi,k}(0), \dot{W}_k(T), \tilde{N}_k(T), \tilde{\varrho}_k(T)) \geq 0, \) so

\[
\lim_{k \to \infty} \sum_{j=1}^m \int_0^T \eta_j^2(z_{\phi,j}(t))^2 dt = 0.
\]

By (41), for any \( k, V_{n,k}(t) = V_{n,1}(0) + \int_0^t V_{n,k}(\tau) d\tau, \)

\[
V_{n,k}(t) \leq V_{n,1}(0) - \sum_{j=1}^m \int_0^t \eta_j^2(z_{\phi,j}(\tau))^2 d\tau + n(\frac{1}{4})^\tau \Delta_k.
\]

By (56), \( \sum_{j=1}^m \int_0^t \eta_j^2(z_{\phi,j}(\tau))^2 d\tau \) is bounded. By Definition 1, \( \Delta_k \) is bounded, and \( t \in [0, T], \) \( n(1/4)^\tau \Delta_k \) is also bounded. From above all, for any \( k, V_{n,k}(t) \) is bounded, then we have \( x_{j,k}, \dot{W}_k(t) \) and \( \tilde{N}_k(t) \) are bounded. By (43), \( u_k \)

is bounded. By (33), \( \dot{z}_{j,k} \) is bounded, \( z_{j,k} \) is continuous uniformly, according to Barbalat lemma, \( z_{j,k} \) \( \to \) 0, when \( k \to \infty, \) i.e., \( k \to \infty, z_{j,k} \to 0, \) then \( \| z_{j,k} \| < \varsigma, \) \( \| \dot{z}_{j,k} \| = \| z_{j,k} + \text{sgn}(z_{j,k}) \dot{z}_{j,k}(t) \| \leq \| z_{j,k} \| + \| \dot{z}_{j,k}(t) \| < \| \dot{z}_{j,k}(t) \| + \varsigma, \)

so \( k \to \infty, \) \( \| z_{j,k} \| < \phi_{i,\infty}(t) + \varsigma, \) and the proof is finished. \( \square \)

4. Simulation

In this section, a mass-spring mechanical system is considered to show the effectivity of the proposed controller. \( m \) is a mass, and assume that resistive force caused by friction is zero. The external force \( u_k \) drives the mass, which is a control variable. \( y_k \) is the displacement from a reference position, and the motion equation of the system with continuous nonlinearly input is as follows:

\[
\overline{m} \ddot{y}_k + F_{ms}(y_k) = N(u_k),
\]

(58)

where \( t \in [0, \pi] \) and \( F_{ms}(\cdot) \) is the restoring force of the spring, \( k \) denotes the iteration index.

We define \( x_{1,k} = y_k, x_{2,k} = \dot{y}_k, \) and \( \overline{m} = 1 \) which transform (58) into the state-space form

\[
\dot{x}_{1,k} = x_{2,k},
\]

\[
\dot{x}_{2,k} = N(u_k) - F_{ms}(x_{1,k}),
\]

(59)

The restoring force of the spring can be modeled as

\[
F_{ms}(x_{1,k}) = kx_{2,k} \left( \sum_{i=0}^n a_i x_{1,k}^i \right).
\]

(60)

In the system, we have \( k = 1, \) \( a_0 = 0, \)

\( a_1 = a_2 = a_3 = a_4 = a_5 = 1, \) and \( q = 4. \)

Continuous nonlinearly input \( N(u) \) is shown as \( N(u) = (0.5 + 0.1 \sin(u))u. \) System object is the that of (59) can follow the reference trajectory \( y_{rk} \) on \( [0, \pi] \) when \( k \to \infty. \)

In the different target case, \( y_{rk} = g_k \sin(2t) \) as \( k \) is even, and \( y_{rk} = g_k \cos(t) \) as \( k \) is odd, where \( g_k \) is \( \text{rand}(0, 1). \)

By Theorem 2, the AILC is chosen as

\[
\alpha_{1,k} = -\overline{W}_{1,h} \tilde{\zeta}_k(x_{1,k}) - \tilde{N}_{1,k} \Delta_k z_{1,k} + \dot{y}_r - \eta_1 z_{1,k},
\]

\[
u_{1,k} = u_{1,k} + u_{2,k} - \overline{W}_{2,k} \tilde{z}_k(x_{2,k}) - \tilde{N}_{2,k} \Delta_k z_{2,k} + P_{1,k} - \eta_2 z_{2,k},
\]

\[
u_{2,k} = -\text{sat} \left( z_{n,k} \phi_{n,k}(t) \right) \left( \tilde{\varrho}_k(t) + 1 \right) |u_{1,k}|
\]

(61)

where \( P_{1,k} = (\partial \alpha_{1,k}/\partial \tilde{W}_{1,k}) \overline{W}_{1,k} + (\partial \alpha_{1,k}/\partial \tilde{N}_{1,k}) \tilde{N}_{1,k} + (\alpha_{1,k}/\partial \dot{y}_r) \dot{y}_r \) and \( \alpha_{1,k}/\partial y_r \dot{y}_r \) and the parameter estimation laws are represent by (48)–(50), where \( \eta_1 = 10, \eta_2 = 5, \Delta_k = a/k^2, \)

\( a = 100/3, \quad \Gamma_{11} = \text{diag} \left\{ 1, 0.1, \ldots, 0.1 \right\}, \quad \Gamma_{21} = 1, \)
Figure 1: Curve graph of $\|z_{1,k}\|$ along $k$.

Figure 2: Curve graph of $\|u_k\|$ along $k$.

Figure 3: Curve graph of $\|\hat{W}_{1,k}\|$, $\|\hat{W}_{2,k}\|$ along $k$. 

Variables \( x_{1,k}, x_{2,k} \) are used as the inputs of the RBFNNS; the neural network consists of 31 neurons, the center of basis function uniform coverage of \([-1, 1]\), and the width of basis function is set to 1.

Here, choose the parameters and initial value as follows:
\[
\begin{align*}
    z_{1,k}(0) &= 1/k^2, \\
    z_{2,k}(0) &= 1/k^2, \\
    \hat{W}_{1,1}(0) &= [0.01, 0, \ldots, 0]^T, \\
    \hat{W}_{2,1}(0) &= [0.01, 0, \ldots, 0]^T, \\
    \hat{N}_{1,1}(0) &= -0.1, \\
    \hat{N}_{2,1}(0) &= -0.1, \\
    \hat{\varrho}_1(0) &= 0.1.
\end{align*}
\]
Taking \( k = 40 \), Figures 1–5 are the simulation figures.

Figure 1 shows the convergence of error. Moreover, on the interval \([0, \pi]\), Figures 2–5 give that \( \|\hat{u}_k\|, \|\hat{W}_{1,k}\|, \|\hat{W}_{2,k}\|, \|\hat{N}_{1,k}\|, \|\hat{N}_{2,k}\|, \|\hat{\varrho}_k\| \) are bounded. Figures 1–5 confirm the validity of control scheme which is developed in this article from the simulation results.

5. Conclusions

The different target tracking problem for unknown nonlinear systems with continuous nonlinearity input is solved in this paper. We introduce the RBFNN to deal with the uncertain dynamics. The problems of approximation error and initial state error can be efficiently solved by suitable means. This paper can keep all signals being bounded on \([0, T]\), and errors can converge with the number of iterations increasing. Finally, the effectiveness of the proposed control method is verified via a simulation example.

Data Availability

No data were used to support this study.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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