Abstract

Transverse expansion of centrally produced matter in Pb on Pb collisions at beam energies around 158 AGeV appears to be rather 'soft'. Two possible reasons – an extended preequilibrium stage and a first order phase transition from a quark-gluon-plasma into hadronic matter – are discussed. The softening of transverse expansion caused by preequilibrium dynamics is estimated with the aid of the transport model RQMD which does not contain a first order phase transition. It is found that the anisotropy of transverse flow in non-central reactions is very different in the preequilibrium and hydrodynamic scenarios even if the latter are based on a strong 1st order transition.
The analysis of collective flows in ultrarelativistic nucleus-nucleus collisions is one of the major tools to study the Equation of State (EoS) of strongly interacting matter, in particular the phase transition from a quark-gluon plasma (QGP) into hadronic matter [1]. In this Letter, I address the question how to distinguish mechanisms which soften the pressure and therefore the expansion in the central rapidity region of ultrarelativistic nucleus-nucleus collisions. I am going to focus here on transverse expansion, because longitudinal flows are presumably dominated by the primordial motion of the in-going projectile and target nuclei. I will discuss effects due to an extended preequilibrium stage as opposed to a first order phase transition from a thermal QGP into a hadron gas. Addressing possible causes of ‘softness’ implies that there are some hints of its presence in ultrarelativistic nucleus-nucleus collisions. Indeed, analysis of experimental data reveals that the transverse momenta of hadrons level off in heavy ion collisions between beam energies of 10 AGeV (at AGS) and 200 AGeV (at CERN) [2]. The observation means in turn that the underlying collective motion does not increase sizably. On the other side, the total energy which is dumped into the central rapidity region is approximately 70 percent larger at the higher beam energy.

A first-order phase transition is generically associated with the presence of a softest point in the EoS. The tendency of matter to expand on account of its internal pressure is reduced in the transition region [3, 4, 5]. The formation of a mixed phase seems therefore a natural candidate for an explanation if matter expands softly. The preequilibrium stage of nucleus-nucleus collisions is characterized by strong damping of transverse motion as well. How can the two sources of softness be distinguished? Here I suggest to utilize the different time orderings of when the expansion of matter becomes soft in the two cases. Preequilibrium softening is bound to happen initially. In contrast, the softness of the mixed phase evolution may be preceded by a fast evolution in the quark-gluon stage. A very hot QGP expands presumably according to a hard equation of state close to $p = c/3$ which is valid for an ideal gas of massless particles.

It is important for the idea explored here that there are two transverse flow components in nucleus-nucleus collisions with non-zero impact parameter. At sufficiently high beam energies the component in the reaction plane is expected to be stronger than the out-of-plane component. Anisotropies
as a signature for hydrodynamical motion in ultrarelativistic nucleus-nucleus reactions have been discussed some time ago by Ollitrault [6]. He suggested that a flow anisotropy is created by the anisotropy of the almond-shaped initial overlap region between projectile and target. Hydrodynamic flows are driven by pressure gradients which are mainly directed along the impact parameter in this case. One should note that this anisotropy of transverse flow has nothing to do with the so-called directed flow which has been studied very much at lower beam energies. Azimuthal average and elliptic deformation of the two flow components each exhibit different degree of sensitivity to the early pressure. It has been realized only recently that the anisotropy of the flow tensor is especially sensitive to the early pressure [7]. It is suggested here that measuring the two flow components separately will provide vital information about the timing of the softening in AA collisions at energies around and even larger than 160AGeV, e.g. at RHIC. Preequilibrium dynamics does not spoil the usefulness of such kind of analysis. Quite to the contrary, analysis of azimuthal asymmetries may be used to gain insight about the duration of preequilibrium motion.

The appropriate tool to address preequilibrium phenomena in nucleus-nucleus collisions is transport theory. Here I am employing the relativistic quantum molecular dynamics (RQMD) approach [8]. In this Letter, I am going to study one particular reaction, Pb(159AGeV) on Pb collisions. Such reactions are currently explored by various experimental groups at CERN. It should be mentioned that the ‘inertial confinement’ effect from spectators discussed in Ref. [7] is very weak for the collisions at CERN energy, at least in the model. Sizable pressure builds up only after the spectators have disappeared from the central region. Furthermore, it can be expected that heavy-ion reactions at this beam energy create energy densities in the central region which may be sufficient for QGP formation. This Letter should be viewed as complementary to the analysis presented in Ref. [9]. Corrections to the ideal hydrodynamic evolution in the later dilute stages (post-equilibrium) were discussed in the earlier work. The motivation for the present study has been to get a handle on the role of the preequilibrium stage for transverse expansion dynamics. Subtracting the nonequilibrium effects would enable one to extract the ‘thermal’ properties of the quark-hadron transition from experimental data.
The outline of this Letter is as follows. Using RQMD the softening of transverse expansion caused by preequilibrium effects is estimated. By comparing final hadron spectra to recent preliminary NA49 data it is checked that the expansion from RQMD for Pb on Pb collisions at CERN energy is compatible with experimental observations. The RQMD calculation provides an example of an expansion dynamics without a first order phase transition. I will demonstrate that elliptic flow distinguishes the RQMD-type preequilibrium scenario from hydrodynamical evolution based on a strong first order phase transition.

A detailed overview of the RQMD model can be found elsewhere [8]. Here I summarize only how the ingredients affect the transverse expansion. RQMD is based on string and resonance excitations in the primary collisions of nucleons from target and projectile. Overlapping color strings fuse into ropes, flux-tubes with sources of larger than fundamental color charges. Color strings and ropes model the prehadronic stage in 1+1 dimensions. By construction, they do not exert any transverse pressure on their environment. Indeed, Trottier and Woloshyn have shown that according to their lattice gauge simulations color ropes do not expand in transverse dimensions, contrary to naive bag model expectations [10]. Transverse expansion in the central region starts only after hadronization, because the initially generated transverse momenta from string and rope fragmentation are oriented randomly. The hadronic expansion stage in RQMD starts off far from kinetic equilibrium. One reason is that the nuclear thickness sets a minimum time for nonequilibrium due to the finite crossing time of projectile and target. The crossing time of the two Pb nuclei in an observer frame with CMS rapidity is 1.4 fm/c at 160 AGeV and therefore even larger than the hadronization time from string and rope fragmentation. Hadrons in the same space-time area may have very different rapidity initially – just because they are produced in elementary collisions with different locations along the beam axis. The spreading of longitudinal velocities $\delta \beta$ from the dispersion of collision

1 Sometimes the crossing time is unjustifiably ignored in the literature. E.g. in Ref. [11] it is assumed that all entropy is produced already after 1 fm/c – before the two Pb nuclei have even completely passed through each other. The large densities which result from this choice of initial conditions form the base of the claim in Ref. [11] that a QGP is a more ‘natural’ thermal state than a resonance gas at CERN energies.
points can be easily estimated in the Bjorken scenario \[12\] (with formation time \(\tau_0\) taken to be 1 fm/c) which bears some resemblance to string-type approaches. \(\delta \beta\) may take values up to

\[
\delta \beta = \left( 1 + \left( \frac{\tau_0 \gamma}{2 R_A} \right)^2 \right)^{-1/2}.
\]

The corresponding difference in rapidities \(\delta y = 1/2 \ln((1 + \delta \beta)/(1 - \delta \beta))\) amounts to approximately 1.1 units for Pb(159AGeV)+Pb collisions. This dispersion comes on top of rapidity fluctuations from hadronization which have the same order of magnitude. The total dispersion of local hadron rapidities is clearly much larger than in thermal equilibrium in which the width of rapidity distributions is restricted not to exceed 0.7 units. Initially, the local momentum distributions in RQMD are therefore elongated along the beam axis. This diminishes expansion in transverse direction in comparison to the kinetic equilibrium case. One can define an effective pressure \(p\) employing the spatial diagonal components of the energy-momentum tensor (see Ref. \[9\]). The softening of the transverse expansion from pre-equilibrium anisotropies shows up as a reduction of the transverse pressure.

In the hadronic stage of RQMD, the fragmentation products from rope, string and resonance decays interact with each other and the original nucleons, mostly via binary collisions. These interactions drive the system towards equilibrium \[9\]. The equilibrium state in RQMD is an ideal gas of hadrons and resonances, up to small corrections from strings and neglecting contributions from mean-field type potentials between baryons (which have not been employed for the present study). The relevant quantity for hydrodynamic expansion is the pressure as a function of energy density. At relevant temperatures around 150-180 MeV the ratio \(p/e\) stays approximately around 1:6 (for \(\mu_B=0\)) if all experimentally well confirmed hadronic states with masses below 2 GeV/c\(^2\) are included \[13\]. This is rather close to the spectrum of states included in RQMD \[8\]. The inclusion of resonances already strongly softens the EoS. A pion gas at same energy density would provide twice as much pressure as the resonance gas.

Approximately 300 Pb(160AGeV) on Pb collisions with an impact parameter \(b=6\) fm have been calculated with the RQMD model (version 2.3) for the present study. The hadronic energy-momentum tensor has been evaluated in the collision center. Fig. \[1\] displays the time evolution of the event-averaged
local transverse pressure and energy density. Initially, the energy density is very large, close to 3 GeV/fm$^3$. However, a rather large fraction of this energy resides in the ‘hidden’ collective motion along the beam axis. As a consequence, the transverse pressure is considerably softened for a time interval of about 4 fm/c. Usually, hydrodynamic simulations of ultrarelativistic collisions assume that there is no relevant transverse expansion before local equilibrium has been achieved. This approximation can be justified only if the equilibration time would be small compared to both the total interaction time and to the transverse size of the collision region divided by $c$. Taking the prehadronic and hadronic evolution together the preequilibrium stage lasts for approximately 5 fm/c. Such large values make the preequilibrium evolution utterly relevant for the transverse expansion dynamics. Note that $e$ and $p$ evolve rather similarly in the other areas of the collision region although the maximum densities are somewhat lower than in the center.

After equilibration, the ratio $p/e$ approaches values around 1:6 which are expected for the hadronic system in kinetic and chemical equilibrium. It provides indirect evidence that chemical equilibrium must be close at this later stage. Finally, the $p/e$ ratio drops again signaling the break-down of near-equilibrium dynamics. The pressure goes down faster than the energy density, because the dilute gas in the central region is characterized by more massive constituents. One reason is loss of chemical equilibrium. Finally, the system in the center becomes a dilute gas dominated by nucleons and not—contrary to naive expectation—by pions. Increasingly with time, interactions are ceasing and being replaced by free streaming. The lighter mass particles leave the collision center faster than the other ones.

Transverse momentum spectra of different hadron species can be employed to check whether the average transverse (radial) flow generated by RQMD has the appropriate strength. Fortunately, the often discussed ambiguity in the hydrodynamic model concerning trade-off between temperature and flow [14] which is related to the choice of freeze-out criteria does not exist for RQMD. Transport models like RQMD which have included two-body scattering in accordance with free-space data become accurate in the dilute gas limit. Fig. 2 contains a comparison of RQMD predictions for hadron spectra at central rapidity which were taken from Ref. [9] with recent preliminary NA49 data [15]. The shapes of the calculated spectra which directly
reflect the flow effects (and also the particle ratios) agree well with the data.

In order to assess the importance of preequilibrium dynamics on the transverse expansion the ‘effective EoS’ \( p(e) \) from RQMD is compared to an equilibrium EoS with a 1st order phase transition (PT) in Fig. 1. The equilibrium EoS with critical temperature at 160 MeV was employed for the hydrodynamic studies in Ref. [4]. The comparison reveals that preequilibrium in RQMD softens its effective EoS as much as the latent heat does for the equilibrium EoS. Thus it seems conceivable that the RQMD nonequilibrium dynamics without and a hydrodynamic evolution with a strong PT included generate comparable average transverse flows if initial energy densities around 2-4 GeV/fm\(^3\) are chosen. In fact, results of some hydrodynamic studies incorporating an EoS with phase transition have been published which fit measured slope parameters for collisions at CERN energy reasonably well [11], [16]-[18].

How can one distinguish the equilibrium scenario with strong 1st order transition from the RQMD-type preequilibrium softening of transverse expansion? The different time ordering of hard and soft expansion stage in the two cases which is visible from Fig. 1 is quite suggestive for a solution. One may look for observables other than radial flow which are also sensitive to the pressure, however, with different relative weight of early and late stage. Here the anisotropies of the transverse flow in non-central collisions are a promising candidate, because they are arguably more sensitive to early pressure. They evolve only as far as the system retains some memory of the initial anisotropy, because the anisotropic overlap zone of projectile and target nucleus is responsible for the asymmetries. Furthermore, the developing stronger in-plane flow acts against its cause, the asymmetry of the collision zone. In contrast, the late stage is weighted more heavily in the evolution of the average flow. The weight is essentially proportional to the system size (due to the \(pdV\) term in the thermodynamic approximation). The different sensitivity of elliptic deformation and average flow to the earlier pressure explains some amusing results of hydrodynamic calculations in Ref. [4]. Ollitrault has found that additional expansion in the longitudinal direction strengthens the azimuthal asymmetry as compared to a 2-dimensional transverse expansion. He discusses this in terms of hydrodynamics approaching eventually a scaling solution faster in two than in three dimensions. On the
other side, the radial flow is getting weaker if another dimension for expansion is open. Earlier freeze-out due to transport into longitudinal direction causes these opposite trends.

Fig. 3 displays how the azimuthal asymmetry of transverse flows develops with time according to the RQMD calculation. The azimuthal asymmetry of transverse hadron momenta can be quantified by defining the dimensionless variable $\alpha$ via

$$\alpha = \frac{\langle p_x^2 \rangle - \langle p_y^2 \rangle}{\langle p_x^2 \rangle + \langle p_y^2 \rangle}.$$  \hspace{1cm} (1)

$p_x$ ($p_y$) denotes the transverse momentum component of hadrons parallel (orthogonal) to the impact parameter vector. The time evolution of $\alpha$ in Pb(159AGeV) on Pb collisions with $b=6$ fm generated from RQMD has been calculated and is plotted in Fig. 3. A central rapidity cut ($y_{CMS} \pm 0.7$) has been applied. $\alpha(t)$ stays close to zero in the preequilibrium stage which reflects the small transverse pressure during this stage. Only after the system approaches kinetic equilibrium the flow develops a stronger asymmetry. The final $\alpha$ value is reached after approximately 10 fm/c. Fig. 3 exhibits also the final values of $\alpha$ for the same system but calculated with boost-invariant hydrodynamics [6]. Since thermal pressure which drives the expansion in these hydrodynamic calculations starts already at very early times (1 fm/c), the anisotropies become much stronger than in RQMD. The asymmetry parameter has been calculated by Ollitrault for different equations of state, of a pure $\pi$ gas and with 1st order phase transition into a QGP (and critical temperature $T_c$ either 150 or 200 MeV). Comparing the hydrodynamic and non-equilibrium RQMD values for $\alpha$ one finds that the $\pi$ gas value is larger by a factor 3.84 and the QGP values by factors 1.74 (150 MeV) or 2.84 (200 MeV). It should be noted that the same hydrodynamic calculation with EoS as in the equilibrium limit of RQMD would give an $\alpha$ value between $\pi$ gas and QGP value. We find indeed that the anisotropies are more sensitive to an early softening from preequilibrium than to a later phase transition.

In this paper I have discussed how to distinguish whether transverse expansion in $AA$ collisions at 160AGeV is softened by extended preequilibrium motion or by a 1st order first transition from a thermal QGP into hadronic matter. The strength of elliptic flow looks like a promising candidate to shed light on this question. Fortunately, important progress has also been
achieved recently on the experimental side. Several experimental collaborations have found signals of azimuthal asymmetries in the central collision region. NA49 has reported preliminary data on elliptic transverse energy flow patterns for non-central Pb(158AGeV) on Pb reactions [19]. Energy flows in neighboring pseudo-rapidity windows are clearly correlated. Similar correlations in photon emission (mostly $\pi^0$ decays) have been observed by WA93 [20]. Recent E877 data for Au on Au collisions show that already at the lower beam energy of 11.5 AGeV the main flow direction is indeed parallel to the impact parameter [21]. First comparisons to models indicate that the experimental signals may be strong enough to distinguish between different scenarios [19]. Hopefully, in the near future the combined experimental and theoretical analysis of elliptic flow will considerably narrow down the range of ‘allowed’ expansion scenarios and implicitly of the EoS in the quark-hadron transition region.

The author thanks M. Hung for providing the input data to the equilibrium EoS which has been displayed in Fig. 1. This work has been supported by DOE grant No. DE-FG02-88ER40388.

References

[1] L.van Hove: Z. f. Phys. C21 (1983) 93; K. Kajantie, M. Kataja, L. McLerran, and P.V. Ruuskanen: Phys. Rev. D34 (1986) 2746; S. Chakrabarty, J. Alam, D.K. Srivastava, B. Sinha: Phys. Rev. D46 (1992) 3802.

[2] N. Xu (for the NA44 collaboration): Proc. of Quark Matter 96, Nucl. Phys. A (1996) in print.

[3] E. Shuryak and O.V. Zhirov: Phys. Lett. B89 (1979) 253.

[4] C.M. Hung and E.V. Shuryak: Phys. Rev. Lett. 75 (1995) 4003.

[5] D. Rischke and M. Gyulassy: Nucl. Phys. A 597 (1996) 701.

[6] J.Y. Ollitrault: Phys. Rev. D46 (1992) 229; Phys. Rev. D48 (1993) 1132.
[7] H. Sorge: preprint SUNY-NTG 96-40, nucl-th/9610026, subm. to Phys. Rev. Lett. (1996).

[8] H. Sorge: Phys. Rev. C52 (1995) 3291.

[9] H. Sorge: Phys. Lett. B 373 (1996) 16.

[10] H.D. Trottier and R.M. Woloshyn: Phys. Rev. D48 (1993) 2290.

[11] J. Cleymans, K. Redlich, and D.K. Srivastava: preprint nucl-th/9611047.

[12] J.D. Bjorken: Phys. Rev. D27 (1983) 140.

[13] H. Bebie, P. Gerber, J.L. Goity, and H. Leutwyler: Nucl. Phys. B 378 (1992) 95.

[14] E. Schnedermann and U. Heinz: Phys. Rev. Lett. 69 (1992) 2908.

[15] P.G. Jones (for the NA49 collaboration): Proc. of Quark Matter 96, Nucl. Phys. A (1996) in print.

[16] A. Dumitru, U. Katscher, J. Maruhn, H. Stöcker, W. Greiner, and D. Rischke: Phys. Rev. C51 (1995) 2166.

[17] B. Schlei, U. Ornik, M. Plümer, D. Strottman, and R. Weiner: Phys. Lett. B376 (1996) 212.

[18] J. Sollfrank, P. Huovinen, M. Kataja, P.V. Ruuskanen, M. Prakash, and R. Venugopalan: preprint nucl-th/9607023.

[19] T. Wienold (for the NA49 collaboration): Proc. of Quark Matter 96, Nucl. Phys. A (1996) in print.

[20] V.P. Viyogi (for the WA93 collaboration): Proc. of Quark Matter 95, Nucl. Phys. A 590 (1995) 503c.

[21] T. Hemmick (for the E877 collaboration): Proc. of Quark Matter 96, Nucl. Phys. A (1996) in print.
Figure Captions:

Figure 1:
Time evolution of local transverse pressure $p$ and energy density $\epsilon$ in the collision center of the system Pb(159AGeV) on Pb with impact parameter 6 fm. Symmetry under reflections enforces that the spatial components of the (energy and baryon) flow velocities are zero at this point. $p$ is determined as the average of the spatial transverse components in the diagonal of the hadronic stress tensor. These quantities have been extracted from approximately 300 RQMD events. Square symbols represent calculated $p/\epsilon$ values versus $\epsilon$ starting 1 fm/c after initial touching of the two Pb nuclei in time intervals of 1 fm/c each (in the CMS of the two nuclei). The straight line is only to guide the eye. The dashed line represents the equilibrium equation of state which was used for hydrodynamic studies in Ref. [4]. This EoS contains a 1st order phase transition with $T_c=160$ MeV.

Figure 2:
RQMD prediction for the transverse mass spectrum $1/2\pi m_t \, d^2N/dm_t dy$ at central rapidity in the reaction Pb(160AGeV) on Pb as a function of $\Delta m_t=m_t-m_q$: results have been taken from Ref. [9] and are compared with preliminary NA49 data [15]. The histograms represent the RQMD spectra for protons (straight line), neutral (anti-)kaons (dashed-dotted line) and charged pions (dashed line), the symbols the data for protons (full circles), $2*K_{short}$ (stars) and $2*\pi^+$ (squares). In addition, the data have been multiplied with 1.2 which probably reflects the tighter centrality cut ($b<1$ fm) used in the calculation.

Figure 3:
Time evolution of transverse momentum anisotropy parameter $\alpha$ calculated with RQMD for the same system as in Fig. 1. The variable $\alpha$ is defined in eq. (3). Only hadrons with rapidities in the window $y_{CMS} \pm 0.7$ are included. The squares represent the RQMD values, the arrows the final $\alpha$ values from hydrodynamic calculations for the same system and three different EoS which have been published in the first of Refs. [6].
Figure 1:
Figure 2:
Figure 3:

\[
\text{asymmetry } \alpha
\]

\begin{align*}
Pb(159\text{AGeV}) + Pb, b = 6 \text{ fm} \\
\text{thermal} & \quad \pi \text{ gas} \\
\left\{ \begin{array}{l}
\text{QGP } (T_c = 200\text{MeV}) \\
\text{QGP } (T_c = 150\text{MeV})
\end{array} \right.
\end{align*}