A Decoder for Probabilistic Synchronous Tree Insertion Grammars

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Abstract

Synchronous tree insertion grammars (STIG) are formal models for syntax-based machine translation. We formalize a decoder for probabilistic STIG; the decoder transforms every source-language string into a target-language tree and calculates the probability of this transformation.

1 Introduction

Tree adjoining grammars (TAG) were invented in (Joshi et al. 1975) in order to better characterize the string sets of natural languages1. One of TAG’s important features is the ability to introduce two related syntactic units in a single rule, then push those two units arbitrarily far apart in subsequent derivation steps. For machine translation (MT) between two natural languages, each being generated by a TAG, the derivations of the two TAG may be synchronized (Abeille et al., 1990; Shieber and Shabes, 1990) in the spirit of syntax-directed transductions (Lewis and Stearns, 1968); this results in synchronous TAG (STAG). Recently, in (Nesson et al., 2005, 2006) probabilistic synchronous tree insertion grammars (pSTIG) were discussed as model of MT; a tree insertion grammar is a particular TAG in which the parsing problem is solvable in cubic-time (Schabes and Waters, 1994). In (DeNeefe, 2009; DeNeefe and Knight 2009) a decoder for pSTIG has been proposed which transforms source-language strings into (modifications of) derivation trees of the pSTIG. Nowadays, large-scale linguistic STAG rule bases are available.

In an independent tradition, the automata-theoretic investigation of the translation of trees led to the rich theory of tree transducers (Gécseg and Steinby, 1984, 1997). Roughly speaking, a tree transducer is a finite term rewriting system. If each rewrite rule carries a probability or, in general, a weight from some semiring, then they are weighted tree transducers (Maletti, 2006, 2006a; Fülöp and Vogler, 2009). Such weighted tree transducers have also been used for the specification of MT of natural languages (Yamada and Knight, 2001; Knight and Graehl, 2005; Graehl et al., 2008; Knight and May 2009).

Martin and Vere (1970) and Schreiber (1975) established the first connections between the two traditions; also Shieber (2004, 2006) and Maletti (2008, 2010) investigated their relationship.

The problem addressed in this paper is the decoding of source-language strings into target-language trees where the transformation is described by a pSTIG. Currently, this decoding requires two steps: first, every source string is translated into a derivation tree of the underlying pSTIG (DeNeefe, 2009; DeNeefe and Knight 2009), and second, the derivation tree is transformed into the target tree using an embedded tree transducer (Shieber, 2006). We propose a transducer model, called a bottom-up tree adjoining transducer, which performs this decoding in a single step and, simultaneously, computes the probabilities of its derivations. As a basis of our approach, we present a formal definition of pSTIG.

2 Preliminaries

For two sets Σ and A, we let $U_{Σ}(A)$ be the set of all (unranked) trees over Σ in which also elements of A may label leaves. We abbreviate $U_{Σ}(\emptyset)$ by $U_{Σ}$. We denote the set of positions, leaves, and non-leaves of $ξ ∈ U_{Σ}$ by $\text{pos}(ξ) ⊆ \mathbb{N}^*$, $\text{lv}(ξ)$, and $\text{nlv}(ξ)$, resp., where $ε$ denotes the root of $ξ$ and $ω.w$ denotes the $w$th child of position $ω$; $\text{nlv}(ξ) = \text{pos}(ξ) \setminus \text{lv}(ξ)$. For a position $w ∈ \text{pos}(ξ)$, the label of $ξ$ at $w$ (resp., subtree of $ξ$ at $w$) is denoted...
by $\xi(w)$ (resp., $\xi|_w$). If additionally $\xi \in U\Sigma(A)$, then $\xi|_w$ denotes the tree which is obtained from $\xi$ by replacing its subtree at $w$ by $\zeta$. For every $\Delta \subseteq \Sigma \cup A$, the set $\text{pos}_\Delta(\xi)$ is the set of all those positions $w \in \text{pos}(\xi)$ such that $\xi(w) \in \Delta$. Similarly, we can define $\text{lv}_\Delta(\xi)$ and $\text{nlv}_\Delta(\xi)$. The yield of $\xi$ is the sequence $\text{yield}(\xi) \in (\Sigma \cup A)^*$ of symbols that label the leaves from left to right.

If we associate with $\sigma \in \Sigma$ a rank $k \in \mathbb{N}$, then we require that in every tree $\xi \in U\Sigma(A)$ every $\sigma$-labeled position has exactly $k$ children.

3 Probabilistic STAG and STIG

First we will define probabilistic STAG, and second, as a special case, probabilistic STIG.

Let $\mathcal{N}$ and $\mathcal{T}$ be two disjoint sets of, resp., nonterminals and terminals. A substitution rule $r$ is a tuple $(\zeta_s, \zeta_t, V, W, P^r_{\text{adj}})$ where

- $\zeta_s, \zeta_t \in U\mathcal{N}(T)$ (source and target tree) and $|\text{lv}_\mathcal{N}(\zeta_s)| = |\text{lv}_\mathcal{N}(\zeta_t)|$,
- $V \subseteq \text{lv}_\mathcal{N}(\zeta_s) \times \text{lv}_\mathcal{N}(\zeta_t)$ (substitution sites), $V$ is a one-to-one relation, and $|V| = |\text{lv}_\mathcal{N}(\zeta_s)|$,
- $W \subseteq \text{nlv}_\mathcal{N}(\zeta_s) \times \text{nlv}_\mathcal{N}(\zeta_t)$ (potential adjoining sites), and
- $P^r_{\text{adj}} : W \rightarrow [0, 1]$ (adjoining probability).

An auxiliary rule $r$ is a tuple $(\zeta_s, \zeta_t, V, W, *, P^r_{\text{adj}})$ where $\zeta_s, \zeta_t, V, W$, and $P^r_{\text{adj}}$ are defined as above and

- $V$ is defined as above except that $|V| = |\text{lv}_\mathcal{N}(\zeta_s)| - 1$ and
- $* = (s, t) \in \text{lv}_\mathcal{N}(\zeta_s) \times \text{lv}_\mathcal{N}(\zeta_t)$ and neither $s$ nor $t$ occurs in any element of $V$; moreover, $\zeta_s(e) = \zeta_t(s)$ and $\zeta_t(e) = \zeta_t(t)$, and $s \neq t \neq t$; the node $s$ (and $t$) is called the foot-node of $\zeta_s$ (resp., $\zeta_t$).

An (elementary) rule is either a substitution rule or an auxiliary rule. The root-category of a rule $r$ is the tuple $(\zeta_s(\epsilon), \zeta_t(\epsilon))$, denoted by $\text{rc}(r)$.

A probabilistic synchronous tree adjoining grammar (pSTAG) is a tuple $G = (\mathcal{N}, T, (S_s, S_t), S, A, P)$ such that $\mathcal{N}$ and $T$ are two disjoint sets (resp., of nonterminals and terminals), $(S_s, S_t) \in \mathcal{N} \times \mathcal{N}$ (start nonterminal), $S$ and $A$ are finite sets of, resp., substitution rules and auxiliary rules, and $P : S \cup A \rightarrow [0, 1]$ such that for every $(A, B) \in \mathcal{N} \times \mathcal{N},$

$$\sum_{r \in S} P(r) = 1 \quad \text{and} \quad \sum_{r \in A} P(r) = 1$$

assuming that in each case the number of summands is not zero. In the following, let $G$ always denote an arbitrary pSTAG.

In Fig. 1 we show the rules of our running example pSTAG, where the capital Roman letters are the nonterminals and the small Greek letters are the terminals. The substitution site (in rule $r_1$) is indicated by $\downarrow$, and the potential adjoining sites are denoted by $a, b, c, d, e, f, g$. For instance, in formal notation the rules $r_1$ and $r_2$ are written as follows:

$$r_1 = (S_s(a, A, A(a)), S_t(B(\beta), B, \beta), \{1\}, \{a\}, P^r_{\text{adj}})$$

where $\downarrow = (2, 2)$ and $a = (3, 1)$, and

$$r_2 = (A(\gamma), B(\delta, B), \emptyset, \{b, c, e, f, g\})$$

where $b = (e, e), c = (e, 1), d = (1, 2)$.

In the derivation relation of $G$ we will distinguish four types of steps:

1. substitution of a rule at a substitution site (substitution),
2. deciding to turn a potential adjoining site into an activated adjoining site (activation),
3. deciding to drop a potential adjoining site, i.e., not to adjjoin (non-adjjoining) and
4. adjoining of a rule at an activated adjoining site (adjoining).

In the sentential forms (defined below) we will maintain for every adjoining site $w$ a two-valued flag $g(w)$ indicating whether $w$ is a potential $(g(w) = p)$ or an activated site $(g(w) = a)$.

The set of sentential forms of $G$ is the set $\text{SF}(G)$ of all tuples $\kappa = (\xi_s, \xi_t, V, W, g)$ with

$$\begin{align*}
\xi_s & \text{ is the sequence of all those symbols that occur in } \xi_s, \\
\xi_t & \text{ is the sequence of all those symbols that occur in } \xi_t, \\
V & \text{ is a set of nonterminals,} \\
W & \text{ is a set of terminals,} \\
g & \text{ is a two-valued flag.}
\end{align*}$$

Figure 1: The running example pSTAG $G$. 

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\[ \text{Diagram of pSTAG G.} \]
In Fig. 2 we show a derivation of our running example pSTAG where activated adjoining sites are indicated by surrounding circles, the other adjoining sites are potential.

\[
\begin{align*}
S_1 & \quad \iff \quad S_1 \\
\text{substitution of } r_1 \text{ at } (c, c) & \quad \Downarrow \quad P(r_1) = 1 \\
\alpha & \quad A | A_a \quad \iff \quad B | B | \beta \\
\text{substitution of } r_2 \text{ at } (2, 2) & \quad \Downarrow \quad P(r_2) = 0.1 \\
\text{activation at } a = (3, 1) & \quad \Downarrow \quad P(r_3) = 0.4 \\
\text{adjoining at } r_2 \text{ at } a = (3, 1) & \quad \Downarrow \quad P(r_4) = 0.8 \\
\text{non-adjoining at } c = (3, 1) & \quad \Downarrow \quad 1 - P_{\text{adj}}(c) = 0.4 \\
\text{non-adjoining at } b = (3, 1) & \quad \Downarrow \quad 1 - P_{\text{adj}}(b) = 0.8
\end{align*}
\]

Figure 2: An example derivation with total probability 1 \times 0.1 \times 0.9 \times 0.4 \times 0.4 \times 0.8 = 0.01152.

The only initial sentential form is \( \kappa_{\text{im}} = (S_1, S_1, \{ \varepsilon, c \}, \emptyset, \emptyset) \). A sentential form \( \kappa \) is final if it has the form \( (\xi_s, \xi_t, \emptyset, \emptyset, \emptyset) \). Let \( \kappa \in SF(G) \). A derivation (of \( \kappa \)) is a sequence \( \bar{d} \) of the form \( \kappa_0, \kappa_1, \ldots, u_n \kappa_n \) with \( \kappa_0 = \kappa_{\text{im}} \) and \( n \geq 0 \), \( \kappa_{i-1} \xrightarrow{d_i} \kappa_i \) for every \( 1 \leq i \leq n \) (and \( \kappa_n = \kappa \)). We
denote $\kappa_n$ also by $\text{last}(d)$, and the set of all derivations of $\kappa$ (resp., derivations) by $D(\kappa)$ (resp., $D$). We call $d \in D$ successful if $\text{last}(d)$ is final.

The tree transformation computed by $G$ is the relation $\tau_G \subseteq U_n(T) \times U_n(T)$ with $(\xi_s, \xi_t) \in \tau_G$ iff there is a successful derivation of $(\xi_s, \xi_t, \emptyset, \emptyset, \emptyset)$.

Our definition of the probability of a derivation is based on the following observation. Let $d \in D(\kappa)$ for some $\kappa = (\xi_s, \xi_t, V, W, g)$. Then, for every $w \in W$, the rule which created $w$ and the corresponding local position in that rule can be retrieved from $d$. Let us denote this rule by $r(d, \kappa, w)$ and the local position by $l(d, \kappa, w)$.

Now let $d$ be the derivation $\kappa_0u_1\kappa_1 \ldots u_n\kappa_n$. Then the probability of $d$ is defined by

$$P(d) = \prod_{1 \leq i \leq n} P_d(\kappa_{i-1} \Rightarrow \kappa_i)$$

where

1. (substitution) $P_d(\kappa_{i-1} \xrightarrow{u, t} \kappa_i) = P(r)$
2. (activation)
   $$P_d(\kappa_{i-1} \xrightarrow{w, r} \kappa_i) = P'_{\text{adj}}(w')$$
   where $r' = r(d, \kappa_{i-1}, w)$ and $w' = l(d, \kappa_{i-1}, w)$
3. (non-adjoining)
   $$P_d(\kappa_{i-1} \xrightarrow{w, t} \kappa_i) = 1 - P'_{\text{adj}}(w')$$
   where $r'$ and $w'$ are defined as in the activation case
4. (adjoining)
   $$P_d(\kappa_{i-1} \xrightarrow{w, r} \kappa_i) = P(r)$$

In order to describe the generative model of $G$, we impose a deterministic strategy $\text{sel}$ on the derivation relation in order to obtain, for every sentential form, a probability distribution among the follow-up sentential forms. A deterministic derivation strategy is a mapping $\text{sel} : \text{SF}(G) \rightarrow (N^* \times N^*) \cup \{\perp\}$ such that for every $\kappa = (\xi_s, \xi_t, V, W, g) \in \text{SF}(G)$, we have that $\text{sel}(\kappa) \in V \cup W$ if $V \cup W \neq \emptyset$, and $\text{sel}(\kappa) = \perp$ otherwise.

In other words, $\text{sel}$ chooses the next site to operate on. Then we define $\Rightarrow_{\text{sel}}$ in the same way as $\Rightarrow$ but in each of the cases we require that $w = \text{sel}(\kappa_1)$.

Moreover, for every derivation $d \in D$, we denote by $\text{next}(d)$ the set of all derivations of the form $du w_k$ where $\text{last}(d) \Rightarrow_{\text{sel}} \kappa$.

The generative model of $G$ comprises all the generative stories of $G$. A generative story is a tree $t \in U_D$; the root of $t$ is labeled by $\kappa_{\text{init}}$. Let $w \in \text{pos}(t)$ and $t(w) = d$. Then either $w$ is a leaf, because we have stopped the generative story at $w$, or $w$ has $|\text{next}(d)|$ children, each one represents exactly one possible decision about how to extend $d$ by a single derivation step (where their order does not matter). Then, for every generative story $t$, we have that

$$\sum_{w \in \text{lv}(t)} P(t(w)) = 1$$

We note that $(D, \text{next}, \mu)$ can be considered as a discrete Markov chain (cf., e.g., (Baier et al., 2009)) where the initial probability distribution $\mu : D \rightarrow [0, 1]$ maps $d = \kappa_{\text{init}}$ to 1, and all the other derivations to 0.

A probabilistic synchronous tree insertion grammar (pSTIG) $G$ is a pSTIG except that for every rule $r = (\xi_s, \xi_t, V, W, \star_{\text{adj}})$ or $r = (\xi_s, \xi_t, V, W, \star, \star_{\text{adj}})$ we have that

- if $r \in A$, then $|\text{lv}(\xi_s)| \geq 2$ and $|\text{lv}(\xi_t)| \geq 2$.
- for $* = (\star, \star)$ we have that $\star$ is either the rightmost leaf of $\xi_s$ or its leftmost one; then we call $r$, resp., $L$-auxiliary in the source and $R$-auxiliary in the source; similarly, we restrict $\star$; the source-spine of $r$ (target-spine of $r$) is the set of prefixes of $\star$ (resp., of $\star$).
- $W \subseteq \text{nlv}_N(\xi_s) \times \{L, R\} \times \text{nlv}_N(\xi_t) \times \{L, R\}$ where the new components are the direction-type of the potential adjoining site, and
- for every $(w_s, \delta_s, w_t, \delta_t) \in W$, if $w_s$ lies on the source-spine of $r$ and $r$ is $L$-auxiliary ($R$-auxiliary) in the source, then $\delta_s = L$ (resp., $\delta_s = R$), and corresponding restrictions hold for the target component.

According to the four possibilities for the foot-node * we call $r$ $LL_*$, $LR_*$, $RL_*$, or $RR_*$-auxiliary. The restriction for the probability distribution $P$ of $G$ is modified such that for every $\{A, B\} \in N \times N$ and $x, y \in \{L, R\}$:

$$\sum_{r \in A, \text{ty}(r) = (A, B), \text{r is xy-} \text{auxiliary}} P(r) = 1$$

In the derivation relation of the pSTIG $G$ we will have to make sure that the direction-type of the chosen adjoining site $w$ matches with the type of auxiliary of the auxiliary rule. Again we assume that the data structure $\text{SF}(G)$ is enriched such that for every potential adjoining site $w$ of $\kappa \in \text{SF}(G)$ we know its direction-type $\text{dir}(w)$.

We define the derivation relation of the pSTIG $G$ to be the binary relation $\Rightarrow I \subseteq \text{SF}(G) \times \text{SF}(G)$ such that we have $\kappa_1 \Rightarrow I \kappa_2$ iff (i) $\kappa_1 \Rightarrow \kappa_2$ and
4 Bottom-up tree adjoining transducer

Here we introduce the concept of a bottom-up tree adjoining transducer (BUTAT), which will be used to formalize a decoder for a pSTIG.

A BUTAT is a finite state machine which translates strings into trees. The left-hand side of each rule has a string over terminal symbols and a sub-variable combination. A variable is either a substitution variable or an adjoining variable.

In fact, BUTAT can be viewed as the string-to-tree transducer of a pSTIG, where variables have direction and subscript, which transform strings directly into trees after productions.

**The derivation shown in Fig. 2 is a pSTIG along the pSTIG defined in [18].**

5 Decoder for pSTIG

Now we construct the decoder dec(C) for a pSTIG C that transforms source strings directly into parse trees and simultaneously computes the probability of the corresponding derivation of C. This decoder is formalized as a BUTAT.

A bottom-up tree adjoining transducer (BUTAT) T (C) is a triple $M = (Q,F \Delta,Q_0,R)$, where $Q$ is a finite set of rules of the form
\[
\xi = \phi(\zeta_1, \ldots, \zeta_n|\zeta_{n+1}, \ldots, \zeta_r)
\]
\[
\textit{IR}(M) = \{\xi \mid \xi \in \textit{RHS}(\zeta)|\zeta \in \Delta(M)\}
\]
\[
\textit{RHS}(\zeta) = \{\phi(\xi_1, \ldots, \xi_n | \zeta_{n+1}, \ldots, \zeta_r)|\zeta \in \Delta(M)\}
\]

\[
\textit{RHS}(\zeta) = \{\phi(\xi_1, \ldots, \xi_n | \zeta_{n+1}, \ldots, \zeta_r)|\zeta \in \Delta(M)\}
\]

\[
\Delta(M) = \{\zeta_1, \ldots, \zeta_r \mid \zeta_1, \ldots, \zeta_r \in \Phi, \zeta_1, \ldots, \zeta_r \in \Phi, \zeta_1, \ldots, \zeta_r \in \Phi\}
\]

The derivation relation defined by $M$ is the binary relation $\Delta(M)$ in which the nullary rule must be $\Delta(w)$-auxiliary. Since $\Delta$ is a subset of $\Delta(w)$, the concepts of derivation, success, and failure for a pSTIG, where $\zeta$ and $\zeta'$ are $\Delta(w)$-auxiliary and $\Delta$-auxiliary, are defined in the same manner.
have to transform the source tree \( \zeta_s \) of a rule \( r \) into a left-hand side \( \rho \) of a \( \text{dec}(G) \)-rule. This is done similarly to (DeNeefe and Knight, 2009) by traversing \( \zeta_s \) via recursive descent using a mapping \( \varphi \) (see an example after Theorem 1); this creates appropriate state-variable combinations for all substitution sites and potential adjoining sites of \( r \). In particular, the source component of the direction-type of a potential adjoining site determines the position of the corresponding combination in \( \rho \). If there are several potential adjoining sites with the same source component, then we create a \( \rho \) for every permutation of these sites. The right-hand side of a \( \text{dec}(G) \)-rule is obtained by traversing the target tree \( \zeta_t \) via recursive descent using a mapping \( \psi_p \) and, whenever a nonterminal with a potential adjoining site \( w \) is met, a new position labeled by \( f_w \) is inserted.\(^5\) If there is more than one potential adjoining site, then the set of all those sites is ordered as in the left-hand side \( \rho \) from top to bottom.

Apart from these main rules we will employ rules which implement the decision of whether or not to turn a potential adjoining site \( w \) into an activated adjoining site. Rules for the first purpose just pass the already computed output tree through from left to right, whereas rules for the second purpose create for an empty left-hand side the output tree \( * \).

We will use the state behavior of \( \text{dec}(G) \) in order to check that (i) the nonterminals of a substitution or potential adjoining site match the root-category of the used rule, (ii) the direction-type of an adjoining site matches the auxiliarity of the chosen auxiliary rule, and (iii) the decisions of whether or not to adjoin for each rule \( r \) of \( G \) are kept separate.

Whereas each pair \( (\zeta_s, \zeta_t) \) in the translation of \( G \) is computed in a top-down way, starting at the initial sentential form and substituting and adjoining to the present sentential form, \( \text{dec}(G) \) builds \( \zeta_t \) in a bottom-up way. This change of direction is legitimate, because adjoining is associative (Vijay-Shanker and Weir, 1994), i.e., it leads to the same result whether we first adjoin \( r_3 \) to \( r_1 \), and then align \( r_3 \) to the resulting tree, or first adjoin \( r_3 \) to \( r_2 \), and then adjoin the resulting tree to \( r_1 \).

In Fig. 3 we show some rules of the decoder of our running example pSTIG and in Fig. 4 the derivation of this decoder which corresponds to the derivation in Fig. 2.

**Theorem 1.** Let \( G \) be a pSTIG over \( N \) and \( T \). Then there is a BUTAT \( \text{dec}(G) \) such that for every \( (\xi_s, \xi_t) \in U_N(T) \times U_N(T) \) and \( p \in [0, 1] \) the following two statements are equivalent:

1. there is a successful derivation of \( (\xi_s, \xi_t, \emptyset, \emptyset, \emptyset) \) by \( G \) with probability \( p \).

2. there is a derivation from \( \text{yield}(\zeta_s) \) to \( [S_s, S_t](\xi_t) \) by \( \text{dec}(G) \) with probability \( p \).

**Proof.** Let \( G = (N, T, [S_s, S_t], S, A, P) \) be a pSTIG. We will construct the BUTAT \( \text{dec}(G) = (Q, T, N \cup T, \{[S_s, S_t]\}, R) \) as follows (where the mappings \( \varphi \) and \( \psi_p \) will be defined below):

- \( Q = [N \times N] \cup [N \times \{L, R\} \times N \times \{L, R\}] \cup \{(r, w) \mid r \in A, w \text{ is an adjoining site of } r\} \).
- \( R \) is the smallest set \( R' \) of rules such that for every \( r \in S \cup A \) of the form \( (\zeta_s, \zeta_t, V, W, P_{\text{adj}}^r) \) or \( (\zeta_s, \zeta_t, V, W, *, P_{\text{adj}}^r) \):
  - for every \( \rho \in \varphi(\varepsilon) \), if \( r \in S \), then the main rule
    \[ \rho \xrightarrow{P_r(r)} \text{[x]}(\varepsilon, \zeta_t(\varepsilon))(\psi_p(\varepsilon)) \]
  is in \( R' \), and if \( r \in A \) and \( r \) is \( \delta_s \delta_t \)-auxiliary, then the main rule
    \[ \rho \xrightarrow{P_r(r)} \text{[x]}(\varepsilon, \zeta_s(\varepsilon), \zeta_t(\varepsilon))(\psi_p(\varepsilon)) \]
  is in \( R' \), and
  - for every \( w = (w_s, \delta_s, w_t, \delta_t) \in W \) the rules
    \[ q_w(f_w) \xrightarrow{P_{\text{adj}}^r(w)} (r, w)(f_w(*) \}
    with \( q_w = [\zeta(w_s), \delta_s, \zeta_t(w_t), \delta_t] \) for activation at \( w \), and the rule
    \[ \varepsilon \xrightarrow{1-P_{\text{adj}}^r(w)} (r, w)(* \}
    for non-adjointing at \( w \) are in \( R' \).

We define the mapping
\[ \varphi : \text{pos}(\zeta_s) \rightarrow \mathcal{P}(\mathcal{T}) \] with \( \mathcal{Q}(X \cup F) = \{q(z) \mid q \in \mathcal{Q}, z \in X \cup F\} \) inductively on its argument as follows. Let \( w \in \text{pos}(\zeta_s) \) and let \( w \) have \( n \) children.

(a) Let \( \zeta_s(w) \in T \). Then \( \varphi(w) = \{\zeta_s(w)\} \).

\(^5\)We will allow variables to have structured indices that are not elements of \( N \). However, by applying a bijective renaming, we can always obtain rules of the form \((\dag)\).
(b) (substitution site) Let $\zeta_s(w) \in N$ and let $w' \in \text{pos}(\zeta_t)$ such that $(w, w') \in V$. Then
\[
\varphi(w) = \{[\zeta_s(w), \zeta_t(w')]|x(w, w')\}.
\]
(c) (adjoining site) Let $\zeta_s(w) \in N$ and let there be an adjoining site in $W$ with $w$ as first component. Then, we define $\varphi(w)$ to be the smallest set such that for every permutation $(u_1, \ldots, u_t)$ (resp., $(v_1, \ldots, v_m)$) of all the L-adjoining (resp., R-adjoining) sites in $W$ with $w$ as first component, the set
\[
J \circ \varphi(w).1 \circ \ldots \circ \varphi(w.n) \circ K
\]
is a subset of $\varphi(w)$, where $J = \{u'_1 \ldots u'_l\}$ and $K = \{v'_m \ldots v'_1\}$ and
\[
u'_i = [r, u_i](f_{u_i}) \text{ and } v'_j = [r, v_j](f_{v_j})
\]
for $1 \leq i \leq l$ and $1 \leq j \leq m$.

(d) Let $\zeta_s(w) \in N$, $w \neq \varepsilon$, and let $w$ be neither the first component of a substitution site in $V$ nor the first component of an adjoining site in $W$. Then
\[
\varphi(w) = \varphi(w.1) \circ \ldots \circ \varphi(w.n).
\]

(e) Let $w = \varepsilon$. Then we define $\varphi(w) = \{\varepsilon\}$.

For every $\rho \in \varphi(\varepsilon)$, we define the mapping
\[
\psi_\rho : \text{pos}(\zeta_t) \to U_{\text{pos}}(T \cup \{\varepsilon\})
\]
inductively on its argument as follows. Let $w \in \text{pos}(\zeta_t)$ and let $w$ have $n$ children.

(a) Let $\zeta_t(w) \in T$. Then $\psi_\rho(w) = \zeta_t(w)$.

(b) (substitution site) Let $\zeta_t(w) \in N$ and let $w' \in \text{pos}(\zeta_t)$ such that $(w', w) \in V$. Then
\[
\psi_\rho(w) = x(w, w').
\]

(c) (adjoining site) Let $\zeta_t(w) \in N$ and let there be an adjoining site in $W$ with $w$ as third component. Then let $\{u_1, \ldots, u_t\} \subseteq W$ be the set of all potential adjoining sites with $w$ as third component, and we define
\[
\psi_\rho(w) = f_{u_1}(\ldots f_{u_t}(\zeta \ldots)
\]
where $\zeta = \zeta_t(w)(\psi_\rho(w.1), \ldots, \psi_\rho(w.n))$ and the $u_i$'s occur in $\psi_\rho(w)$ (from the root towards the leaves) in exactly the same order as they occur in $\rho$ (from left to right).

(d) Let $\zeta_t(w) \in N$, $w \neq \varepsilon$, and let $w$ be neither the second component of a substitution site in $V$ nor the third component of an adjoining site in $W$. Then
\[
\psi_\rho(w) = \zeta_t(w)(\psi_\rho(w.1), \ldots, \psi_\rho(w.n)).
\]

Using the usual concatenation $\circ$ of formal languages,

As illustration of the construction in Theorem 1 let us apply the mappings $\varphi$ and $\psi_\rho$ to rule 2 of Fig. 1, i.e., to $r_2 = (\zeta_s, \zeta_t, \emptyset, \{b, c\}, \{\emptyset\}_{\text{adj}})$ with $\zeta_s = A(\gamma, \alpha), \zeta_t = B(\delta, \beta)$, $b = (\varepsilon, R, \varepsilon, L), c = (\varepsilon, R, 1, L)$, and $* = (1, 2)$.

Let us calculate $\varphi(\varepsilon)$ on $\zeta_s$. Due to (e),
\[
\varphi(\varepsilon) = J \circ \varphi(1) \circ \varphi(2) \circ K.
\]
Since there are no L-adjoining at $\varepsilon$, we have that $J = \{\varepsilon\}$. Since there are the R-adjoinings $b$ and $c$ at $\varepsilon$, we have the two permutations $(b, c)$ and $(c, b)$.
\[
\begin{align*}
(v_1, v_2) &= (b, c) : K = \{[r_2, c](f_c)[r_2, b](f_b)\} \quad \text{and} \quad (v_1, v_2) = (c, b) : K = \{[r_2, b](f_b)[r_2, c](f_c)\}
\end{align*}
\]
Due to (e) and (a), we have that $\varphi(1) = \{\varepsilon\}$ and $\varphi(2) = \{\gamma\}$, resp. Thus, $\varphi(\varepsilon)$ is the set:
\[
\{\gamma [r_2, c](f_c) [r_2, b](f_b), \gamma [r_2, b](f_b) [r_2, c](f_c)\}.
\]
Figure 3: Some rules of the running example decoder.

Now let $\rho = \gamma [r_2, b] (f_b) [r_2, c] (f_c)$. Let us calculate $\psi_\rho(\varepsilon)$ on $\zeta_r$.

$$\begin{align*}
\psi_\rho(\varepsilon) &\stackrel{(c)}{=} f_b(B(\psi_\rho(1), \psi_\rho(2))) \\
&\stackrel{(c)}{=} f_b(B(f_c(B(\psi_\rho(1.1)), \psi_\rho(2)))) \\
&\stackrel{(a)}{=} f_b(B(f_c(B(\delta)), \psi_\rho(2))) \\
&\stackrel{(c)}{=} f_b(B(f_c(B(\delta)), *))
\end{align*}$$

Hence we obtain the rule

$$\gamma [r_2, b] (f_b) [r_2, c] (f_c) \rightarrow [A, R, B, L](f_b(B(f_c(B(\delta)), *)))$$

which is also shown in Fig. 3.

Figure 4: Derivation of the decoder corresponding to the derivation in Fig. 2.
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