Cluster formation and dissolution in a generalized relativistic density functional approach for dense matter

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Abstract. The formation and dissolution of clusters in dense matter is described in a generalized relativistic mean-field model with density-dependent nucleon-meson couplings. Nucleons, light clusters and two-body correlations in the continuum are treated as effective degrees of freedom with medium dependent properties. The resulting generalized relativistic density functional successfully describes the transition from clusterized matter at low densities to uniform neutron-proton matter at high densities with the correct low-density limits. The formation of heavy clusters is modeled by employing inhomogeneous calculations in the Thomas-Fermi approximation within spherical Wigner-Seitz cells.

1. Introduction
Correlations in nuclear matter modify the thermodynamical properties of the equation of state (EoS) and affect the chemical composition. At low densities, two-, three-, ..., many-body correlations due to the short-range nuclear interaction lead to the formation of bound states that appear as new particle species. At high densities around and above nuclear saturation density, homogeneous and isotropic matter is expected that is composed of neutron and proton quasiparticles and mean-field effects dominate. In between at low temperatures, there is a transition from inhomogenous to homogeneous matter passing through various “pasta” phases where the interplay of surface effects and the long-range Coulomb interaction strongly affects the spatial distribution of the nucleons. For the application in astrophysical models, e.g. simulations of core-collapse supernovae or neutron stars, an EoS of dense matter is required that takes correlations into account and that consistently interpolates between the low-density and high-density limits. The effects strongly depend on the temperature and the neutron-proton asymmetry of the system.

2. Theoretical approaches
There are a number of models for the dense matter EoS with different points of view in the approach. In a chemical picture, the system is described as a mixture of different inert nuclei and nucleons in chemical equilibrium. A typical example are nuclear statistical equilibrium
(NSE) approaches, e.g. [1]. Interactions between the particles can be included in a virial EoS that encodes two-body correlations in the second virial coefficient. In the quantum mechanical formulation, the two-body density of states enters. It depends on experimentally known quantities such as the binding energies of the nuclei and the scattering phase shifts [2]. However, the properties of the constituents are independent of the medium and the Mott effect, i.e. the dissolution of the nuclei at high densities, is not described. It has to be modeled by simple concepts like the excluded volume mechanism [1]. In consequence, these models are limited to low densities.

In a physical picture, nucleons can be regarded as the basic constituents and the formation of bound states and resonances in the continuum is explicitly considered. Here, the main problems are the treatment of three-, ..., many-body correlations and the choice of an appropriate interaction. Considering the different paths to describe dense nuclear matter, it is tempting to formulate a model that combines the various approaches.

A step beyond the standard virial EoS is the generalized Beth-Uhlenbeck approach that is able to describe the Mott effect. It is a quantum statistical method employing a finite-temperature Green’s function formalism and an expansion of the spectral function beyond the quasiparticle approximation [3]. The main features are medium-dependent shifts of the nucleon self-energies, mainly due to the Pauli principle, and the use of generalized scattering phase shifts from the in-medium T-matrix.

Around nuclear saturation density, phenomenological approaches with effective nuclear interactions depending on parameters that are fitted to properties of finite nuclei are very successful. Typical examples are nonrelativistic Hartree-Fock or relativistic mean-field (RMF) models that give information on the nucleon self-energies. Another class of models are ab initio approaches, e.g. (Dirac) Brueckner-Hartree-Fock calculations, that use a given realistic nucleon-nucleon interaction. Unfortunately, these models are limited to the description of homogenous and isotropic nuclear matter.

3. Generalized relativistic density functional

The approach of the present contribution is an extension of an RMF model with density-dependent meson-nucleon couplings with parameters that are fitted to properties of finite nuclei [4]. In addition to nucleons, light clusters (deuterons, tritons, helions, α particles) are introduced as additional degrees of freedom [5]. Continuum contributions of two-body correlations are represented by effective, temperature-dependent resonances. The binding and resonance energies are shifted depending on density, temperature and asymmetry. Theses shifts are taken in parametrized form from generalized Beth-Uhlenbeck calculations. A density functional is constructed that depends on temperature, neutron and proton chemical potentials, the meson fields and their spatial derivatives. The field equations can be derived and contain additional rearrangement contributions due to the medium dependence of the self-energy shifts, reflecting the interaction of the clusters inside the medium. All thermodynamical quantities can be derived consistently. In addition to the bound states of light clusters in Ref. [5] also two-body scattering correlations are included. The formation of heavy clusters is described by applying the Thomas-Fermi (TF) approximation in spherical Wigner-Seitz cells.

4. Results

The generalized RMF (gRMF) model has to approach the correct limits at low densities. For temperature $T = 0$ MeV, only a single heavy cluster appears, possibly surrounded by a gas of nucleons for large isospin asymmetries beyond the drip lines. Since the original RMF parametrisation DD2 [5] was based on a Hartree approximation for the description of nuclei and the gRMF model for the EoS uses the TF approximation, different results are obtained. Experimental binding energies can be reproduced by a rescaling of the $\sigma$ meson mass and of the
coupling in a modified parametrization DD2 mod such that the properties of uniform nuclear matter are not affected. As depicted in figure 1, an excellent agreement with NSE calculations at low density is obtained.

At finite temperatures, only nucleons and light clusters survive at low densities and the standard virial EoS results are reproduced. This is achieved by exploiting new relations that connect the temperature dependent resonance energies in the continuum contributions and the scattering lengths in the various nucleon-nucleon scattering channels. In addition, a relativistic correction is found that becomes more important than the virial two-body correlation correction at low densities. In figure 2 the various effects that modify the low-density behavior of the internal energy per nucleon in neutron matter are compared. It can be seen that the generalized relativistic density functional (gRDF) nicely reproduces the relativistically corrected virial EoS.

The nucleon-nucleon correlations in the continuum, that are explicitly included in the present model, also affect the particle fractions of the nucleons and light clusters, in particular for the deuteron as depicted in figure 3. At low-densities, two-body correlations, that are not considered in standard EoS for astrophysical applications, are most important. With increasing densities, the dissolution of the clusters, i.e. the Mott effect, is observed in the gRDF approach. Thus both low- and high-density limits are correctly reproduced in this model.

At low densities and not too high temperatures, nuclear matter becomes inhomogeneous. Heavy nuclei are formed that are surrounded by a low-density background of nucleons and light clusters. The charge of the hadronic system is compensated by an almost constant distribution of electrons, hence total charge neutrality is achieved. In the present approach, the density distribution of nucleons, clusters and electrons inside a spherical Wigner-Seitz cell is calculated using a Thomas-Fermi approximation beyond the local density approximation since gradient terms in the functional are considered. The formation of clusters reduces the size of the heavy nucleus. A specific feature of the model is the enhanced probability of finding clusters at the surface of the heavy nucleus.

Another consequence of the cluster formation at low densities and temperatures is the increase...
of the symmetry energy as compared to pure mean-field models. See Ref. [6] for the experimental
evidence of this effect and a comparison with models that consider cluster formation.

5. Conclusions

The relativistic mean-field model with density-dependent nucleon-meson couplings was extended
to a relativistic density functional that includes light clusters as additional degrees of freedom
and considers two-body correlations in the continuum. The model describes the formation
and dissolution of nuclei on a more microscopic level than NSE models with excluded volume
mechanism. Using a suitable parametrization of the effective interaction, the correct limits
at low and high densities are obtained. In the future, the new approach will be used in the
construction of an EoS table for astrophysical applications.

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