Study of charmless $B$ decays to three-kaon final states

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We report on a study of charmless $B$ meson decays to three-kaon final states. The results are obtained with a $78.7 \text{ fb}^{-1}$ data sample collected on the $\Upsilon(4S)$ resonance by the Belle detector operating at the KEKB asymmetric energy $e^+e^-$ collider. The branching fractions for $B$ decays to three-body $K^+K^+K^-$, $K^0K^+K^-$, $K_SK_SK^+$, and $K_SK_SK^0$ final states are presented. We also make a first attempt to perform an isospin analysis of the three-kaon final states.

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INTRODUCTION

Studies of three-body $B$ decays can significantly broaden the understanding of $B$ meson decay mechanisms and provide additional possibilities for CP violation searches. Analysis of the $K\pi\pi$ and $KK\pi$ final states is presented in Refs. [1, 2]. In this paper we report results on the study of charmless $B$ meson decays to three-kaon final states. The analysis is based on a $78.7 \text{ fb}^{-1}$ data sample, which contains 85.0 million $B\bar{B}$ pairs, collected with the Belle detector operating at the KEKB asymmetric-energy $e^+e^-$ (3.5 on 8 GeV) collider [3] with a center-of-mass energy at the $\Upsilon(4S)$ resonance. All results reported here are preliminary.

THE BELLE DETECTOR

The Belle detector [4] is a large-solid-angle magnetic spectrometer that consists of a three-layer silicon vertex detector (SVD), a 50-layer central drift chamber (CDC) for charged particle tracking and specific ionization measurement ($dE/dx$), an array of aerogel threshold Čerenkov counters (ACC), time-of-flight scintillation counters (TOF), and an array of 8736 CsI(Tl) crystals for electromagnetic calorimetry (ECL) located inside a superconducting solenoid coil that provides a 1.5 T magnetic field. An iron flux return located outside the coil is instrumented to detect $K_L$ mesons and to identify muons (KLM). Electron identification is based on a combination of CDC $dE/dx$ measurements, the response of the ACC, and the position, shape and energy deposition of the associated ECL shower.

Charged hadron identification is accomplished by combining the responses of the ACC and the TOF with $dE/dx$ measurements in the CDC into a single value using the likelihood method:

$$ L(h) = \frac{L^{\text{ACC}}(h) \times L^{\text{TOF}}(h) \times L^{\text{CDC}}(h)}{L(h)} ,$$

where $h$ stands for the hadron type ($p$, $K$, $\pi$). Charged tracks are identified as protons, pions or kaons by imposing requirements on the likelihood ratios (PID):

$$ \text{PID}(p) = \frac{L(p)}{L(p) + L(K)} ; \quad \text{PID}(K) = \frac{L(K)}{L(K) + L(\pi)} ; \quad \text{PID}(\pi) = \frac{L(\pi)}{L(K) + L(\pi)} = 1 - \text{PID}(K) $$
At large momenta (>2.5 GeV/c) only the ACC and dE/dx are used since the TOF provides no significant separation of kaons and pions. We use a GEANT based Monte Carlo (MC) simulation to model the response of the detector and determine acceptance.

EVENT SELECTION

The selection criteria are similar to those used in the analysis of B decays to the $K\pi\pi$ and $KK\pi$ final states. Charged tracks are selected with a set of track quality requirements based on the average hit residual and on the distances of closest approach to the interaction point in the plane perpendicular to the beam and the plane containing the beam and the track. We also require that the transverse track momenta be greater than 0.1 GeV/c to reduce the low momentum combinatorial background. For charged kaon identification we impose a requirement on PID($K$), which has 86% efficiency and a 7% fake rate from misidentified pions. Charged tracks that are positively identified as electrons or protons are excluded. Since the muon identification efficiency and fake rate vary significantly with the momentum, we do not veto muons to avoid additional systematic errors.

Neutral kaons are reconstructed via the decay chain $K^0(\bar{K}^0) \rightarrow K_S \rightarrow \pi^+\pi^-$. The invariant mass of the two pions is required to be in the range $|M(\pi^+\pi^-) - M_{K^0}| < 12$ MeV/c$^2$. The displacement of the $\pi^+\pi^-$ vertex from the interaction point (IP) in the transverse ($r$-$\phi$) plane is required to be greater than 0.1 cm and less than 20 cm. The direction of the combined pion pair momentum in the $r$-$\phi$ plane is required to be within 0.2 rad of the direction from the IP to the displaced vertex.

We reconstruct B mesons in the $K^+K^+K^-$, $KSK^+K^-$, $KSKS\bar{K}S$ and $KSKS\bar{K}S$ three-body final states. The inclusion of the charge conjugate mode is implied throughout this report. The candidate events are identified by their center-of-mass (c.m.) energy difference, $\Delta E = (\sum_i E_i) - E_b$, and the beam constrained mass, $M_{bc} = \sqrt{E_b^2 - (\sum_i \vec{p}_i)^2}$, where $E_b = \sqrt{s}/2$ is the beam energy in the c.m. frame, and $\vec{p}_i$ and $E_i$ are the c.m. three-momenta and energies of the candidate B meson decay products. In the first stage, we select events with $M_{bc} > 5.20$ GeV/c$^2$ and $-0.30 < \Delta E < 0.50$ GeV, which is a larger range than that used in our previous report. This allows for more detailed studies of the background. For subsequent analysis, we also define a signal region of $|M_{bc} - M_B| < 9$ MeV/c$^2$ and $|\Delta E| < 0.04$ GeV and a $\Delta E$ sideband region defined as $0.05$ GeV $< |\Delta E| < 0.15$ GeV.

To determine the signal yield, we use events with $M_{bc}$ in the signal region and fit the $\Delta E$ distribution to the sum of a signal distribution and an empirical background. The $\Delta E$ signal shape is parameterized by the sum of two Gaussian functions with the same mean. The widths and the relative fractions of the two Gaussians are determined from a MC simulation. We find that the signal MC simulation gives, in general, a narrower $\Delta E$ width than data. To correct for this, we introduce a scale factor that is determined from the comparison of the $\Delta E$ widths for $B^+ \rightarrow \bar{D}^0\pi^+$ events in MC and experimental data; this correction is about 10%. The background from $q\bar{q}$ continuum events is represented by a linear function. The $\Delta E$ shape of the $B\bar{B}$ background is determined from MC simulation, as described below.

BACKGROUND SUPPRESSION

To suppress the combinatorial background from $e^+e^- \rightarrow q\bar{q}$ continuum events, we use a set of variables that characterize the event topology. We require $|\cos\theta_{thr}| < 0.80$, where $\theta_{thr}$ is the angle between the thrust axis of the $B$ candidate and that of the rest of the event; the distribution of $|\cos\theta_{thr}|$ is peaked near 1.0 for $q\bar{q}$ and is nearly flat for $B\bar{B}$ events. We also use a Fisher discriminant, $F$, formed from nine variables of a “Virtual Calorimeter”, the angle between the candidate thrust axis and beam axis, and the angle between the $B$ candidate direction and
TABLE I: Summary of results for $B$ meson decays to three-body charmless hadronic final states. The branching fractions and 90% confidence level (CL) upper limits (UL) are quoted in units $10^{-6}$. For the modes with one neutral kaon the quoted reconstruction efficiency includes the $K^0 \rightarrow K_S \rightarrow \pi^+\pi^-$ branching fraction. For modes with more than one neutral kaon only the $K_S \rightarrow \pi^+\pi^-$ branching fraction is taken into account.

| Three-body mode | Efficiency (%) | Signal Yield (events) | $B$ (90\% CL UL) | Reference | Results from |
|-----------------|---------------|-----------------------|------------------|-----------|-------------|
| $K^+ K^+ K^-$   | 23.5 $\pm$ 0.50 | 565 $\pm$ 30         | 33.0 $\pm$ 1.8 $\pm$ 3.2 | This Work | 35.3 $\pm$ 3.7 $\pm$ 4.5 |
| $K^0 K^+ K^-$   | 7.20 $\pm$ 0.17  | 149 $\pm$ 15         | 29.3 $\pm$ 3.4 $\pm$ 4.1 | $\mu$ $\mu$ | $\mu$ $\mu$ |
| $K_S K_S K^+$   | 6.78 $\pm$ 0.19  | 66.5 $\pm$ 9.3       | 13.4 $\pm$ 1.9 $\pm$ 1.5 | $\mu$ $\mu$ | $\mu$ $\mu$ |
| $K_\Sigma K_S K_S$ | 3.98 $\pm$ 0.17  | 12.2$^{+4.5}$ $_{-3.8}$ | $4.3^{+1.6}_{-1.4}$ $\pm$ 0.75 | $\mu$ $\mu$ | $\mu$ $\mu$ |
| $K^+ K^- \pi^+$ | 13.8 $\pm$ 0.31  | 93.7 $\pm$ 23.2      | 9.3 $\pm$ 2.3 (<13) | $\Omega$ | < 12 |
| $K^0 K^+ \pi^-$ | 4.53 $\pm$ 0.16  | 26.8 $\pm$ 16.6      | 8.4 $\pm$ 5.2 (<15) | $\Omega$ | $\Omega$ |
| $\bar{D}^0 \pi^+, \bar{D}^0 \rightarrow K^+\pi^-$ | 28.8 $\pm$ 0.57  | 4000 $\pm$ 66        | $-$ | $-$ | $-$ |
| $D^- \pi^+, D^- \rightarrow K^0\pi^-$ | 8.24 $\pm$ 0.14  | 451 $\pm$ 25         | $-$ | $-$ | $-$ |

beam axis. We make a requirement on $\mathcal{F}$ that rejects 53\% of the remaining $q\bar{q}$ background while retaining 89\% of the signal.

We also consider backgrounds that come from other $B$ decays. We conventionally subdivide this background into two types. One type is the so called “generic” $B\bar{B}$ background that originates from the dominant $b \rightarrow c$ tree transition. The description of these decays is taken from an updated version of the CLEO group event generator $[5]$. We find that the dominant background from this source is due to $B \rightarrow Dh$ decays, where $h$ stands for a charged pion or kaon.

To suppress this background, we reject events where any two-particle invariant mass is consistent within 15 MeV ($2.5\sigma$) with $\pi^0 \rightarrow K^+ K^-$, $D^0 \rightarrow K^- \pi^+$ or $D^+ \rightarrow K^0 \pi^+$. We also reject events with a $K^+ K^-$ invariant mass that is consistent with $\chi_{c0} \rightarrow K^+ K^- (|M(K^+ K^-) - M_{\chi_{c0}}| < 50$ MeV). The other potential source of background is rare charmless $B$ decays that proceed via $b \rightarrow s(d)$ penguins or $b \rightarrow u$ tree transitions. Since these final states are not included in the main generator decay table, they are generated separately. We studied a large set of potentially dangerous two-, three-, and four-body final states. We do not find any rare charmless $B$ decay mode that produces a significant background to the three-kaon final states.

RESULTS OF THE ANALYSIS

The $\Delta E$ distributions for all three-kaon final states are shown in Fig. 1, where data (points with errors) are shown along with the background expectation (hatched histograms). To extract the three-body signal yields we fit the $\Delta E$ distributions. The results of the fits are summarized in Table II. The statistical significance of the $B^0 \rightarrow K_S K_S K_S$ signal, in terms of the number of standard deviations is $4.3\sigma$. It is calculated as $\sqrt{-2 \ln(\mathcal{L}_0 / \mathcal{L}_{\text{max}})}$, where $\mathcal{L}_{\text{max}}$ and $\mathcal{L}_0$ denote the maximum likelihood with the nominal signal yield and with the signal yield fixed at zero, respectively. The significance of the signal in all other three-kaon final states exceeds $10\sigma$. For convenience, some of the results obtained in Ref. $[2]$ are also given in Table II.

To determine branching fractions, we normalize our results to the observed $B^+ \rightarrow \bar{D}^0 \pi^+$, $\bar{D}^0 \rightarrow K^+ \pi^-$ and $B^0 \rightarrow D^- \pi^+$, $D^- \rightarrow K^0 \pi^-$ signals. This reduces the systematic errors associated with the particle identification efficiency, charged track reconstruction efficiency, and the event shape variables requirements. We calculate the branching fraction for $B$ meson decay to a particular final state $f$ via the relation
FIG. 1: $\Delta E$ distributions for $B \to KKK$ three-body final states. Points with error bars are data; the open histogram is the fit result; the hatched histogram is the background. The straight line shows the $q\bar{q}$ continuum background contribution.

$$B(B \to f) = \frac{N_f}{N_{D\pi}} \frac{\varepsilon_{D\pi}}{\varepsilon_f} \times B(B \to D\pi)B(D \to K\pi),$$

where $N_f$ and $N_{D\pi}$ are the numbers of reconstructed signal events for the final state $f$ and that for the $D\pi$ reference process, and $\varepsilon_f$ and $\varepsilon_{D\pi}$ are the corresponding reconstruction efficiencies determined from MC. We use the recently updated results on $B^+ \to \bar{D}^0\pi^+$ and $B^0 \to D^-\pi^+$ branching fractions from the CLEO Collaboration [8]. The $\Delta E$ distributions for the reference processes $B^+ \to \bar{D}^0\pi^+$, $\bar{D}^0 \to K^+\pi^-$ and $B^0 \to D^-\pi^+$, $D^- \to K^0\pi^-$ are shown in Fig. [3]. The results of the fits are summarized in Table I.

The results of the three-kaon branching fraction measurements are presented in Table I. To determine the reconstruction efficiencies for $K^+K^+K^-$ and $K^0K^+K^-$ final states, we use a simple model [1] that takes into account the non-uniform distribution of signal events over the Dalitz plot. The three-body signal in this model is parameterized by a $\phi K$ intermediate state and a $f_X K$ state, where $f_X$ is a hypothetical wide scalar state. For the $K_SK_SK^+$ and $K_SK_SK$ final states, the reconstruction efficiencies are determined from MC simulated events that are generated with a uniform (phase space) distribution over the Dalitz plot. The dominant sources of systematic error are listed
FIG. 2: $\Delta E$ distributions for $B^+ \to \bar{D}^0 \pi^+$, $\bar{D}^0 \to K^+ \pi^-$ (left) and $B^0 \to D^- \pi^+$, $D^- \to K^0 \pi^-$ (right) events. The open histogram is the fit result and the hatched histogram is the background. The straight line shows the $q\bar{q}$ continuum background contribution.

TABLE II: List of systematic errors (in percent) for the $B \to KKK$ branching fractions.

| Source | $K^+K^-K^+$ | $K_SK^+K^+$ | $K_SK^+K^+$ | $K_SK_SK$ |
|--------|-------------|-------------|-------------|------------|
| $B \to D\pi$ and $D \to K\pi$ branching fractions | 7.7 | 11.0 | 7.7 | 11.0 |
| Efficiency non-uniformity over the Dalitz plot | 2.2 | 3.6 | - | - |
| Signal parameterization | 2.1 | 5.6 | 4.8 | 7.6 |
| Particle identification | 4.0 | 4.0 | 2.0 | 2.0 |
| $K_S$ reconstruction | - | - | 5.0 | 10.0 |
| MC statistics | 2.1 | 2.4 | 2.8 | 4.3 |
| Total | 9.4 | 13.7 | 10.9 | 17.4 |

in Table II. We estimate the systematic uncertainty due to variations of reconstruction efficiency over the Dalitz plot by varying the relative fractions of quasi-two-body states. The uncertainty in the reconstruction efficiencies due to the limited MC statistics is also included in systematic error. The uncertainty due to the particle identification is estimated using pure samples of kaons and pions from $D^0 \to K^-\pi^+$ decays, where the $D^0$ flavor is tagged using $D^{*+} \to D^0\pi^+$ decays. The systematic error due to uncertainty in the $K_S$ reconstruction efficiency is estimated from the study of $D^{*+} \to D^0\pi^+$, $D^0 \to K_S\pi^+\pi^-$ decays. We estimate the uncertainty due to the signal shape parameterization by varying the parameters of the fitting function within their errors.

The BaBar Collaboration has recently presented results on charmless three-body $B$ decays [9]. The reported value, $B(B^+ \to K^+K^+K^-) = (34.7 \pm 2.0 \pm 1.8) \times 10^{-6}$, is in agreement with the result presented here and our previously published measurement [1].

To examine possible quasi-two-body intermediate states in the observed $B \to KKK$ signals, we analyze the two-kaon invariant mass spectra. The Dalitz plot for $B^+ \to K^+K^+K^-$ candidate events in the $M_{bc}\Delta E$ signal region is shown in Fig. 3. Since there are two same-charge kaons in this case, we distinguish the $K^+K^-$ combinations with smaller, $M(K^+K^-)_{\text{min}}$, and larger, $M(K^+K^-)_{\text{max}}$, invariant masses. We avoid double entries by forming the Dalitz plot as $M^2(K^+K^-)_{\text{max}}$ versus $M^2(K^+K^-)_{\text{min}}$.

The $K^+K^-$ invariant mass spectra for events from the $B$ signal region are shown as open histograms in Figs. 3(a)-
FIG. 3: Dalitz plot for $B^+ \to K^+ K^- K^-$ candidates in the $B$ signal region.

FIG. 4: Two-particle invariant mass spectra for $B^+ \to K^+ K^+ K^-$ candidates from the $B$ signal region (open histograms) and for background events in the $\Delta E$ sidebands (hatched histograms). (a) $M(K^+ K^-)_{\text{min}}$ invariant mass spectrum. The inset in (a) shows the $\phi(1020)$ mass region in 2 MeV/$c^2$ bins. (b) $M(K^+ K^-)_{\text{max}}$ spectrum with $M(K^+ K^-)_{\text{min}} < 1.1$ GeV/$c^2$ and (c) $M(K^+ K^-)_{\text{max}}$ with 1.1 GeV/$c^2 < M(K^+ K^-)_{\text{min}} < 2.0$ GeV/$c^2$.

The hatched histograms show the corresponding spectra for background events in the $\Delta E$ sidebands, normalized to the estimated number of background events from the $\Delta E$ fit. The $M(K^+ K^-)_{\text{min}}$ spectrum, shown in Fig. 4(a), is characterized by a narrow peak at 1.02 GeV/$c^2$ corresponding to the $\phi(1020)$ meson and a broad structure around 1.5 GeV/$c^2$ that is consistent with a scalar state. In contrast to the $B^+ \to K^+ \pi^+ \pi^-$ three-body decay, we also
observe a strong "non-resonant" enhancement in the $K^+K^+K^-$ final state that extends over the full $M(K^+K^-)_{\text{min}}$ mass range in Fig. 4(a). To plot the $M(K^+K^-)_{\text{max}}$ mass spectrum we subdivide the $M(K^+K^-)_{\text{min}}$ mass region into two ranges: $M(K^+K^-)_{\text{min}} < 1.1 \text{ GeV}/c^2$ and $1.1 \text{ GeV}/c^2 < M(K^+K^-)_{\text{min}} < 2.0 \text{ GeV}/c^2$. The $M(K^+K^-)_{\text{max}}$ mass spectra for these two regions are shown separately in Fig. 4(b) and Fig. 4(c), respectively. The prominent two-peak
structure apparent in Fig. 4(b) is due to the 100% $\phi$ meson polarization in the $B^+ \to \phi K^+$ decay, which results from angular momentum conservation. The Dalitz plot for $B^0 \to K^+K^-K^+$ candidate events in the $M_{bc}-\Delta E$ signal region is shown in Fig. 5. The $KK$ invariant mass spectra for these events are shown as open histograms in Fig. 6, and the hatched histograms show the corresponding spectra for background events in the $\Delta E$ sidebands. The structure observed in the $K^+K^-$ mass spectrum is very similar to that observed in $M(K^+K^-)_{\text{min}}$ spectrum for the $K^+K^-K^+$ final state (see Fig. 4): a prominent peak that corresponds to the $\phi$ meson and a broad enhancement in the higher $K^+K^-$ mass region.

The numbers of reconstructed $B^+ \to K_S K_SK^+$ and $B^0 \to K_SK_S K_S$ signal events are significantly smaller because of the additional suppression due to the $K^0 \to K_S \pi^+\pi^-$ branching fraction. A Dalitz plot for the $B^+ \to K_S K_SK^+$ candidate events in the $M_{bc}-\Delta E$ signal region is shown in Fig. 7 along with two-kaon invariant mass spectra. It is apparent in Fig. 7 that the $K_S K_S$ invariant mass spectrum is quite similar to the $K^+K^-$ mass spectrum observed in the $K_SK^+K^-$ final state. Except for the absence of the $\phi$ meson, which cannot decay to $K_SK_S$, we observe a similar broad structure in the higher $K_SK_S$ mass region. The low mass region of the $K_SK^+$ mass spectrum shown in Fig. 7(b) agrees with background and exhibits no prominent structures.

**DISCUSSION & CONCLUSION**

Charmless $B$ meson decays have attracted a considerable amount of attention in recent years, primarily because they provide a possible way to extract weak phases. An important check of the Standard Model would be provided by measurements of the same CP-violating parameter in different weak interaction processes. A good example is the comparison of the measurement of the coefficient of the CP violating $\sin(\Delta m_d t)$ term in the time dependent analysis of neutral $B$ meson decays. In $B^0 \to (c\bar{c})K^0$ decays (where $(c\bar{c})$ denotes a charmonium state) this coefficient is $\sin(2\phi_1)$. Precise measurements of $\sin(2\phi_1)$ (also known as $\sin(2\beta)$) have recently been reported by the Belle and
BaBar experiments [10]. The best known candidates for $b \to s$ penguin dominated processes where this quantity can be measured independently are $B^0 \to \phi K^0$ and $B^0 \to \eta' K^0$ decays. However, these modes have small branching fractions of order $10^{-6} - 10^{-5}$ (including secondary branching fractions). Thus, very large numbers of $B$ mesons are required to perform these measurements. This is especially true for the $\phi K^0$ final state. The large signal observed in the three-body $B^0 \to K_S K^+ K^-$ decay mode, where the $\phi K_S$ two-body intermediate state gives a relatively small contribution, would significantly increase statistics if these events could be used. There are two possible complications: (1) Whilst the $\phi K_S$ state has fixed CP, the CP-parity of the three-body $K_S K^+ K^-$ final state is not fixed. If the fractions of CP-even and CP-odd components are comparable, the $K_S K^+ K^-$ state will not be useful for a CP violation measurement; (2) Possible $b \to u$ tree contributions may introduce an additional weak phase in the $B^0 \to K_S K^+ K^-$ amplitude and cloud the interpretation of any observed CP violation. The $b \to u$ contribution in $B^0 \to \phi K_S$ is expected to be negligible (since $\phi$ is a pure $s\bar{s}$ state), but this is not necessarily the case for the three-body $K_S K^+ K^-$ final state. Here we discuss the possibility of the use of the three-body $B^0 \to K_S K^+ K^-$ decay mode for CP measurements.

The decays of $B$ mesons to three-body $K\phi K$ final states can be described by $b \to u$ tree-level spectator and $b \to s(d)g$ one-loop penguin diagrams. Although $b \to u$ $W$-exchange and annihilation diagrams can also contribute to these final states, they are expected to be much smaller and we neglect them in the following discussion.

$B$ meson decays to final states with odd numbers of kaons ($s$-quarks) are expected to proceed dominantly via the $b \to sg$ penguin transition since, for these cases, the $b \to u$ tree contribution has an additional CKM suppression. In contrast, decays with two final state kaons proceed via the $b \to u$ tree transition with no $b \to sg$ penguin contribution. This allows us to estimate the $b \to u$ tree contribution to final states with three kaons via the analysis of $KK\pi$ final states. It is also important to note that the $b \to sg$ penguin transition is an isospin conserving process, while the $b \to u$ tree and $b \to dq$ penguin transitions are isospin violating.

This is illustrated for $B^+ \to K^+ K^+ K^-$ decay in Fig. 3. The diagram that corresponds to the main $b \to s$ penguin contribution is shown in Fig. 3(a). The $b \to u$ tree contribution, Fig. 3(b), has an additional Cabibbo suppression due to the $W^+ \to s\bar{u}$ vertex. The corresponding diagram without Cabibbo suppression ($W^+ \to d\bar{u}$) is shown in Fig. 3(d) and expected to be the dominant contributor to the $K^+ K^-\pi^+$ final state. A quantitative estimate of the size of the $b \to u$ tree amplitude is provided by the ratio

$$ F = \frac{|A_{b\to uu}^{KKK}|^2}{|A_{b\to uu}^{KKK}|^2} \approx \frac{\mathcal{B}(B^+ \to K^+ K^- \pi^+)}{\mathcal{B}(B^+ \to K^+ K^+ K^-)} \times \left( \frac{f_K}{f_\pi} \right)^2 \times \tan^2 \theta_C, $$

where $A_{b\to uu}^{KKK}$ is the total amplitude for the $B^+ \to K^+ K^+ K^-$ decay and $A_{b\to uu}^{KKK}$ is its $b \to u$ tree contribution. The $(f_K/f_\pi)^2$ factor, where $f_\pi = 131$ MeV and $f_K = 160$ MeV are pion and kaon decay constants, respectively, takes into account the corrections for SU(3) breaking effects in the factorization approximation, and $\theta_C$ is the Cabibbo angle ($\sin \theta_C = 0.2205 \pm 0.0018$) [13]. Using the results for $B^+ \to K^+ K^-\pi^+$ and $B^+ \to K^+ K^+ K^-$ branching fractions from Table 3, we obtain $F = 0.022 \pm 0.005$. Similarly, for $B^0$ decays to $K_S K^+ K^-$ and $K_S K^+\pi^-$ final states, respectively, we find $F = 0.023 \pm 0.013$ ($< 0.037$), where the second number is obtained using the upper limit for the $B^0 \to K_S K^+\pi^-$ branching fraction. The small value of $F$ indicates that we can neglect the $b \to u$ tree contribution to the $B \to KKK$ rates and perform an isospin analysis of the three-kaon final states. From an isospin decomposition of $B$ mesons wave functions we obtain the following relations between three-kaon branching fractions,

$$ \mathcal{B}(B^0 \to K^0 K^+ K^-) = \mathcal{B}(B^+ \to K^+ K^- K^-) \times \frac{\tau_{B^0}}{\tau_{B^+}}; \quad (1) $$

$$ \mathcal{B}(B^0 \to K^0 K^+ K^-) = \mathcal{B}(B^+ \to K^+ K^0 K^-) \times \frac{\tau_{B^0}}{\tau_{B^+}}, \quad (2) $$

where the factor $\tau_{B^0}/\tau_{B^+}$ takes into account the difference in total widths of charged and neutral $B$ mesons. We use the first relation, Eq. 1, as a check of our assumption that the isospin violating contribution is small, and calculate...
\[ R = \frac{B(B^0 \to K^0 K^+ K^-)}{B(B^+ \to K^+ K^+ K^-)} \times \frac{\tau_{B^+}}{\tau_{B^0}} = \frac{N_{K_S K^+ K^-}}{N_{K^+ K^+ K^-}} \times \frac{\varepsilon_{K^0 K^+ K^-}}{\varepsilon_{K^+ K^+ K^-}} \times \frac{\tau_{B^+}}{\tau_{B^0}} = 0.95 \pm 0.11 \pm 0.06, \]

where we use signal yields \((N)\) and reconstruction efficiencies \((\varepsilon)\) from Table 1 instead of branching fractions to reduce the systematic error, and \(\tau_{B^+}/\tau_{B^0} = 1.091 \pm 0.023 \pm 0.014\). The calculated value agrees with unity within its statistical error.

Let us now consider the second isospin relation, Eq. 2, in more detail. The \(B^+ \to K^+ K^0 \bar{K}^0\) decay results in three different observable states: \(K^+ K_S K_S, K^+ K_L K_L\) and \(K^+ K_S K_L\). The relative fractions of these states depend on the relative fractions of states with even and odd orbital momenta in the \(K^0 \bar{K}^0\) system. Bose statistics requires that the \(K^0 \bar{K}^0\) wave function be symmetric (and, therefore, CP even), independently of the relative orbital momentum, \(l\), of the neutral kaons. As a result, a \(K^0 \bar{K}^0\) system with even orbital momenta can only decay to \(K_S K_S\) or \(K_L K_L\) final states (with equal fractions), while a \(K^0 \bar{K}^0\) system with odd orbital momenta can only decay to the \(K_S K_L\) final state. Thus, the \(K^+ K^0 \bar{K}^0\) wave function can be written in the following form

\[ |K^+ K^0 \bar{K}^0\rangle = \alpha |K^+ K_S K_S\rangle + |K^+ K_L K_L\rangle + \beta |K^+ K_S K_L\rangle, \]

(3)

where \(\alpha\) and \(\beta\) are unknown coefficients constrained by \(\alpha^2 + \beta^2 = 1\).

In this experiment, we observe only the \(K^+ K_S K_S\) component of the \(K^+ K^0 \bar{K}^0\) final state. Measuring the \(B^+ \to K^+ K^+ K^-\) and \(B^+ \to K^+ K_S K_S\) branching fractions and using the isospin relation, Eq. 2, with the wave function decomposition, Eq. 3, we can determine the parameter \(\alpha^2\),

\[ \alpha^2 = 2 \frac{B(B^+ \to K^+ K_S K_S)}{B(B^0 \to K^0 K^+ K^-)} \times \frac{\tau_{B^0}}{\tau_{B^+}} = 2 \frac{N_{K^+ K_S K_S}}{N_{K^0 K^+ K^-}} \times \frac{\varepsilon_{K^0 K^+ K^-}}{\varepsilon_{K^+ K^+ K^-}} \times \frac{\tau_{B^0}}{\tau_{B^+}}. \]
Here, the parameter $\alpha^2$ characterizes the fraction of states with even orbital momenta in the $K^0\bar{K}^0$ system in the three-body $K^+K^0\bar{K}^0$ final state. At the same time, due to isospin symmetry, $\alpha^2$ also gives the fraction of states with even orbital momenta in the $K^+K^-$ system of the three-body $K^0K^+K^-$ final state. Since the total angular momentum of the $K^0K^+K^-$ system is zero, the orbital momentum of the $K^+K^-$ pair relative to the remaining neutral kaon, $l'$, is equal to $l$. Thus, the CP-parity of the $K_SK^+K^-$ three-body system is $(-1)^l$, and $\alpha^2$ also gives the fraction of CP-even component of the three-body $K_SK^+K^-$ final state. Using the information from Table I, we obtain: $\alpha^2 = 0.86 \pm 0.15 \pm 0.05$. Note that the $K_SK^+K^-$ three-body final state includes the $\phi K_S$ state which is CP-odd. We remove $B^0 \to \phi K_S$ events by requiring $|M(K^+K^-) - M_\phi| > 15$ MeV; the number of remaining $K_SK^+K^-$ events is 123 $\pm$ 14. The value of the $\alpha^2$ for remaining events is: $\alpha^2_{\text{non } \phi} = 1.04 \pm 0.19 \pm 0.06$. Assuming isospin symmetry, we can also use the $B^+ \to K^+K^+K^-$ final state instead of $B^0 \to K^0K^+K^-$ to determine $\alpha^2$. In this case Eq. [4] becomes

$$\alpha^2 = 2 \frac{B(B^+ \to K^+K_SK_S)}{B(B^+ \to K^+K^+K^-)} = 2 \frac{N_{K^+K_SK_S}}{N_{K^+K^+K^-}} \times \frac{\varepsilon_{K^+K^+K^-}}{\varepsilon_{K^+K^+K^-}},$$

(5)

which gives $\alpha^2 = 0.82 \pm 0.12 \pm 0.06$ and $\alpha^2_{\text{non } \phi} = 0.97 \pm 0.15 \pm 0.07$. The two ways of computing $\alpha^2$ and $\alpha^2_{\text{non } \phi}$ are in good agreement. This is evidence for the dominance of the CP-even component in the three-body non-$\phi$ $K_SK^+K^-$ final state.

In conclusion, we have measured branching fractions for charmless $B$ mesons decays to the three-kaon $K^+K^+K^-$, $K_SK^+K^-$, $K_SK^+K^+$, and $K_SK_SK_S$ final states. The isospin analysis of the three-kaon final states presented here implies several conclusions that should be carefully examined further. The three-body $K_SK^+K^-$ final state may be a good candidate for a CP violation measurement in $b \to s$ penguin transitions. In this case a significant increase (by a factor of about four) in the statistics as compared to the $B^0 \to \phi(K^+K^-)K_S$ final state, which is a well known mode for the measurement of CP violation in $b \to s$ penguin dominated decays, is possible.

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