Application of local operators for numerical reconstruction of the singular support of a vector field by its known ray transforms

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Abstract. The paper is devoted to the localization problem of a set of vector field discontinuities, and also discontinuities of its derivatives. We propose new approaches to the reconstruction of vector field singularities by its ray transforms. Alongside modification the Vainberg operator, for solution of this problem we use certain operators of vector analysis in different combinations.

1. Introduction

There are many practically important scientific and technical domains, in which objects have mathematical description by discontinuous values. Such objects frequently arise in research by means of remote sensing methods and, in particular, in tomography.

In process of tomography development as a distinct scientific branch, the reconstruction of the object with discontinuous physical properties was extracted as a special scientific problem. Theoretical description of the object with discontinuous properties in framework of integral geometry was proposed in 1962 year in monograph \cite{1}; further development of the theory, already in tomographical statements, can be find, for example, in the papers \cite{2}, \cite{3}.

At present, in tomography many approximate methods, algorithms and software tools are developed, appropriate for the reconstruction of object interior properties. Typically, approaches based on formulas of inversion, variational and algebraic methods are used. Usually they work well during reconstruction of objects with smooth properties, and at the same time they fail if the object possesses discontinuous characteristics. Thus, necessity in the development of special algorithms arises, aimed to: \(a\) localization of a set of object discontinuities; \(b\) definition of the discontinuity jump value.

Apparently, the first paper, which proposes reconstruction algorithm of a set of discontinuities, is the paper of E. Vainberg et al. \cite{4}, published in 1981. Its main idea consists of preliminary double differentiation of tomographic data, representing the two-dimensional Radon transform, with respect to the variable \(s (|s|)\) is a distance from straight line, along which the integration is done, to the coordinates origin), with following application of backprojection...
operator. Henceforth such sequence of actions, extracting a set of discontinuities, but giving only weak presentation of smooth parts of the object, is called Vainberg operator. We should note that result of application of the Vainberg operator does not give unknown function, as it is in the case of application of inversion formulas. Namely, nonlocal pseudodifferential operator, used in inversion formulas, is changed to the local differential operator of double differentiation that essentially simplify its programme implementation, but does not allow to reconstruct the smooth parts of the object.

The authors of [4] treat an application of double differentiation operator like some filtration procedure. Later (see, for example, [5]), it was proposed another justification of the Vainberg operator application. Namely, this operator reconstructs the function \((-\Delta)^{1/2}f\) (\(\Delta\) is the Laplace operator). Since \((-\Delta)^{1/2}\) is elliptic pseudodifferential operator, function \((-\Delta)^{1/2}f\) possesses the same singularities as \(f\). Development of this idea for inversion of the Radon transform \(Rf\) for non-smooth function \(f\) was proposed in [5]. First of all it is easy to check that for the Radon transform, the Laplace operator \(\Delta\) determined on functions \(f(x, y)\) goes to the operator \(\partial^2/\partial s^2\) determined on functions \((Rf)(\alpha, s)\).

In more details, if the distribution \(F(r^2)\) \((r^2 = x^2 + y^2)\) is given, then the Radon transform of \(F(-\Delta)f\) (by definition \(F(-\Delta)f\) is equal to \(F(r^2) \ast f\)) turns to a convolution by \(s:\ F(s^2) \ast (Rf)\).

Now, let a distribution \(f\) (possibly, nonsmooth one) is given. It can be represented in the form \(f = (1 - \Delta)^k p\), where \(p\) possesses necessary degree of smoothness, \(k\) is natural. Then we can define the Radon transform of function \(f\) as the convolution \(F((1+s^2)^k) \ast (Rp)\). Using inverse Radon transform, we find function \(p\). Applying differential operator \((1 - \Delta)^k\) to it, we obtain unknown function \(f\).

Further development of the reconstruction methods for a discontinuities set has been done, in the framework of local tomography, by A. Faridani et al. [6–7], A. K. Louis and P. Maass [8], A. Rieder [9], and many others. In these papers, alongside the Vainberg operator, they use tools of the microlocal analysis also. It is well known, that the Radon transform has representation as the integral Fourier operator. This allows to establish a geometric connection between the wave front of initial function and the wave front of its Radon transform by standard methods. Thus, singularity of function \(f(x, y)\) can be found straightforward from singularities of its Radon transform.

By the end of 1990s, D. S. Anikonov has proposed another approach to the solution of such problem as definition of a set of function discontinuities by the ray transform, based on the theory of multidimensional singular integrals [13]. Applying the backprojection operator to the ray transform, we obtain singular integral (with sought discontinuous function as an integrand) with a weak singularity. Differentiation by spatial variable leads then to logarithmic increase when a point is moving to the discontinuity line. In particular, we can use operator \(\nabla  \cdot\). Described approach is theoretically justified in [14], [15], and numerically investigated, also with embedding of scattering in a medium model [14].

Following the logic of development of discontinuities reconstruction methods, we suggest their essential generalization. The goal of reconstruction of a discontinuities set for vector or symmetric tensor fields is stated, as well as the goal of recovering of discontinuities of their derivatives. In this case they say about singularity or about singular support.

The paper represents some results of our methods developed for singular support.
reconstruction of scalar, vector and symmetric tensor fields. Here we restrict ourselves by localization of the vector field discontinuities, given in the unit disk, and discontinuities of its first order derivatives. As initial data we use the ray transforms of vector fields, and projection scheme is parallel. Alongside with the Vainberg operator modification, for the solution of this problem we suggest certain differential operators of vector and tensor analysis and their different combinations.

Necessary mathematical tools and results are described shortly in next section. We do not present proofs of the theoretical foundations for the algorithms. In section 3 the results of numerical simulations are presented. The main goal is to compare different combinations of the operators applied for reconstruction of singular support of a vector field. This research does not include investigations how noisy data affect the reconstruction result. Partly that was done in [16], and will be published in more details elsewhere.

2. Setting of the problem and theoretical assumptions

It is well known that the operators of longitudinal and transverse ray transforms, acting on vector fields, possess nonzero kernels. By this reason we consider stated above problem in three versions. Namely, singularities of solenoidal part of required vector field are reconstructed by its known longitudinal ray transform, but singularities of the potential part are recovered by transverse ray transform. Finally, a singular support of the required vector field is reconstructed, if the compatible longitudinal and transverse ray transforms are given.

Let \( B = \{(x, y) \in \mathbb{R}^2 \mid x^2 + y^2 < 1\} \) be a unit disk centered in the origin of the Cartesian coordinate system, and \( \partial B = \{(x, y) \in \mathbb{R}^2 \mid x^2 + y^2 = 1\} \) is a unit circle. Domain \( D \subset \mathbb{R}^2 \) (possibly, multiply connected) such that \( \overline{D} \subset B \), consists from finitely many disjoint subdomains \( \{D_i\} \), \( i = 1, \ldots, N \). The union \( D_0 = \bigcup D_i \) of these subdomains is dense in \( \overline{D} \), and their boundaries are smooth of class \( C^1 \). It is easy to note that \( \partial D \subset \partial D_0 \), and boundary \( \partial D_0 \) coincide with the union of boundaries \( \bigcup D_i \) of subdomains \( D_i \), \( i = 1, \ldots, N \). Important requirement to boundaries consists in the fact that they do not contain linear segments.

Let a function \( \varphi(x) \) of class \( C^k \) is defined in \( B \), vanishes on a set \( B \setminus \overline{D} \), and its support coincides with the closure \( D \), \( \text{supp } \varphi = \overline{D} \). At points \( (x, y) \in D \) the function \( \varphi(x) \) is infinite differentiable. At points \( (x, y) \in \partial D_0 \) the function is equal to 0, and it is continuously differentiable to \( k \)-th order. Due to its smoothness in domain \( D \), the function \( \varphi \) possesses partial derivatives of any order. As for points belonging to \( \partial D_0 \), then therein all partial derivatives \( \frac{\partial^l \varphi}{\partial x^j \partial y^{l-j}}, l = 0, \ldots, k \), \( j \leq l \), up to order \( k \) inclusive are continuous, and derivatives of order \( k+1 \) have discontinuities of the 1-st kind. We say that the function \( \varphi \) is the potential of smoothness \( C^k \), or \( C^k \)-potential in \( \mathbb{R}^2 \).

It is well known that potential and solenoidal vector fields given on the plane, are determined by their potentials uniquely. Namely, potential field \( u \) has a form \( u = (u_1, u_2) = (\partial \varphi/\partial x, \partial \varphi/\partial y) \), and solenoidal field has a form \( v = (v_1, v_2) = (\partial \varphi/\partial y, -\partial \varphi/\partial x) \). To construct discontinuous vector fields we use \( C \)-potentials, and to construct fields with discontinuous derivatives we use \( C^1 \)-potentials.

The Radon transform of the potential \( \varphi \) is determined by relation

\[
\left( R\varphi \right)(\xi, s) = \int_{L(\xi, s)} \varphi(x, y) dL = \int_{-\infty}^{\infty} \varphi(s \xi + t \eta) dt.
\]

(1)

Here \( \xi \in \partial B, (\xi^1, \xi^2) = (\cos \alpha, \sin \alpha) \) is normal vector, and \( \eta \in \partial B, (\eta^1, \eta^2) = (-\sin \alpha, \cos \alpha) \) is directional one of a parallel straight lines set, by which the integration is derived. Parameter \( s, -1 < s < 1 \), characterizes a distance \( |s| \) from the straight line to the coordinates origin. The straight line \( L(\xi, s) \) is defined by its normal equation \( x \cos \alpha + y \sin \alpha - s = 0 \).
The operator of the longitudinal ray transform, acting on the vector field \( v(x, y) = (v_1(x, y), v_2(x, y)) \) given at unit disk \( B \), is determined by formula

\[
(\mathbf{P} v)(\xi, s) = \int_{L(\xi, s)} \langle v, \eta \rangle dL = \int_{-\infty}^{\infty} \langle \eta, \nu(s\xi + t\eta) \rangle dt = \int_{-\infty}^{\infty} (v_1\eta^1 + v_2\eta^2) dt. \tag{2}
\]

The operator of the transverse ray transform is determined similarly,

\[
(\mathbf{P}^\perp v)(\xi, s) = \int_{L(\xi, s)} \langle v, \xi \rangle dL = \int_{-\infty}^{\infty} \langle \xi, \nu(s\xi + t\eta) \rangle dt = \int_{-\infty}^{\infty} (v_1\xi^1 + v_2\xi^2) dt. \tag{3}
\]

There exists a simple connection between the Radon transform of \( C^k \)-potential, \( k = 0, 1, \ldots \), and the ray transforms of vector field, generated by this potential. Namely,

\[
\frac{\partial}{\partial s}(\mathbf{R} \varphi)(\xi, s) = (\mathbf{P} v)(\xi, s) = (\mathbf{P}^\perp u)(\xi, s), \tag{4}
\]

where \( v = (-\partial \varphi/\partial y, \partial \varphi/\partial x) \), \( u = (\partial \varphi/\partial x, \partial \varphi/\partial y) \) are solenoidal and potential fields, respectively, generated by the \( C^k \)-potential \( \varphi \). To prove (4) we remind that from relations \( x = s \cos \alpha - t \sin \alpha, y = s \sin \alpha + t \cos \alpha \) between \( x, y \) and \( \alpha, s, t \) it follows that \( \partial x/\partial s = \cos \alpha = \xi^1 = \eta^2 \) and \( \partial y/\partial s = \sin \alpha = \xi^2 = -\eta^1 \). Hence

\[
\frac{\partial}{\partial s}(\mathbf{R} \varphi)(\xi, s) = \int_{-\infty}^{\infty} \frac{\partial}{\partial s}\varphi(s\xi + t\eta) dt = \int_{-\infty}^{\infty} \langle \nabla \varphi, \xi \rangle dt = \int_{-\infty}^{\infty} (u_1\xi^1 + u_2\xi^2) dt = (\mathbf{P}^\perp u)(\xi, s).
\]

Similarly,

\[
\frac{\partial}{\partial s}(\mathbf{R} \varphi)(\xi, s) = \int_{-\infty}^{\infty} \left( -\frac{\partial \varphi}{\partial y}(1 - \xi^2) + \frac{\partial \varphi}{\partial x}(\xi^1) \right) dt = \int_{-\infty}^{\infty} (v_1\eta^1 + v_2\eta^2) dt = (\mathbf{P} v)(\xi, s).
\]

The operator of the \( m \)-angular moment, acting on a function \( g(\xi(\alpha), s) \) by rule

\[
\mu_{i_1, \ldots, i_m}(x, y) = \frac{1}{2\pi} \int_{0}^{2\pi} \xi^{i_1} \ldots \xi^{i_m} g(\xi(\alpha), s(x, y, \alpha)) d\alpha \tag{5}
\]

gives a symmetric \( m \)-tensor field \( \mu(x, y) \), defined everywhere in \( R^2 \). Operator of backprojection, representing a particular case of integral operator (5), to be applied to the ray transform of a vector field, gives the vector field with components

\[
\mu_1(x, y) = \frac{1}{2\pi} \int_{0}^{2\pi} \xi^1(\mathbf{T} v)(s(x, y, \alpha), \xi(\alpha)) d\alpha ,
\]
\[
\mu_2(x, y) = \frac{1}{2\pi} \int_{0}^{2\pi} \xi^2(\mathbf{T} v)(s(x, y, \alpha), \xi(\alpha)) d\alpha ,
\]

where \( \mathbf{T} \) is one of the operators (2) or (3). An application of the 1-angular moment operator to the ray transform of a vector field leads to the multidimensional singular integrals [13]. These integrals theory is a base for the result obtained in [15]. Let a ray transform (2) or (3) of a vector field generated by \( C^k \)-potential \( \varphi \) be given. We act on this transform by the backprojection operator (6), and then by operator of partial derivation, \( \partial \varphi/\partial x \) or \( \partial \varphi/\partial y \). If a point \( P(x, y) \in D \) tends to the boundary point \( P_0(x_0, y_0) \in \partial D_0 \), then partial derivative behaves like \( \ln|P - P_0| \). The additional requirement here is that the partial derivative has to be transversal to the line of discontinuity.

The above formulated result is generalized by natural way to the case of \( C^k \)-potentials. Then, discontinuities arise in derivatives of (\( k + 1 \)) order only. Together with the partial derivatives we can use also other differential operators, for which the condition of transversality is not necessary.
3. Numerical simulations

We suggest tools for reconstruction of singular support of vector field. As for the authors' best knowledge they were not used in tomography before. Alongside differentiation with respect to $s$ of ray transforms of a vector field, we use, after application of the backprojection operator, operators $\nabla$, $|\nabla(\cdot)|$, directional derivative $\langle \nabla(\cdot), \nu \rangle$, and also the operators div and rot. Listed operators are applied not only by themselves, but also in different combinations and sequences. This approach gives several ways for reconstruction of the vector field discontinuities or discontinuities of its derivatives.

Let us assume the following agreement. As values of all potentials and corresponding vector fields vanish outside domain $D$, which is a disk of radius $R < 1$, so we reduce the formulas only for values of variables, lying inside this disk. Similarly, for the Radon and ray transforms we present the formulas only for values $s$ such that $-R \leq s \leq R$. For values $s$ outside this interval these transforms are equal to 0, and we shall not mention this fact again below.

Below results of numerical simulation are described. It should be mentioned that in the first test $\partial D_0 = \partial D \cup \{(0, 0)\}$. In the second test $\partial D_0 = \partial D$.

In the first numerical simulation we recover a set of discontinuous points of a solenoidal vector field by its known longitudinal ray transform. The vector field is constituted by means of potential

$$\varphi(x, y) = h - \frac{h}{R} \sqrt{x^2 + y^2}, \quad x^2 + y^2 < R^2,$$

(7)

$h > 0$, $R > 0$ are parameters) with discontinuous 1-st derivatives at the origin and on the boundary. The Radon transform of this potential is

$$(R\varphi)(\xi, s) = h\sqrt{R^2 - s^2} - \frac{hs^2}{2R} \ln \frac{R + \sqrt{R^2 - s^2}}{R - \sqrt{R^2 - s^2}}.$$  

(8)

Figure 1. C-potential (7) (a) of vector field (9) (d); 1-st component $-\partial \varphi/\partial y$ of the field (b); 2-nd component $\partial \varphi/\partial x$ of the field (c).

The solenoidal vector field generated by potential (7) is

$$v = (v_1, v_2) = \left( \frac{h}{R} \frac{y}{\sqrt{x^2 + y^2}}, -\frac{h}{R} \frac{x}{\sqrt{x^2 + y^2}} \right).$$

(9)

The longitudinal ray transform of this field is calculated straightforward, and it coincides with partial derivative by $s$ of the Radon transform (8) of potential (7),

$$(Pv)(\xi, s) = -\frac{h}{R} s \ln \frac{R + \sqrt{R^2 - s^2}}{R - \sqrt{R^2 - s^2}}.$$  

(10)

We reconstruct a set of discontinuities points of the vector field by four different ways. In two versions on the first step components $\mu_1, \mu_2$ by formulas (6) are calculated. Henceforth (version 1,
Fig. 2a), we calculate $Q_1(x, y) = \nabla \mu_1(x, y)$, $Q_2(x, y) = \nabla \mu_2(x, y)$ and $Q(x, y) = \sqrt{|Q_1|^2 + |Q_2|^2}$.

In the second version (Fig. 2b) $|\text{div} \mu|$ is calculated. In the 3-th and the 4-th versions on the first step the first derivatives $\partial / \partial s$ of the ray transform (10) of the field (9) are calculated. Henceforth, we apply the backprojection operator to the Radon transform of the function (Fig. 2c). In the last version (Fig. 2d), we use the operator of 2-angular moment. We obtain a symmetric 2-tensor field with components $\mu_{11}$, $\mu_{12} = \mu_{21}$, $\mu_{22}$ as a result. The resulted field is averaged by the formula $(\mu_{11}^2 + 2\mu_{12}^2 + \mu_{22}^2)^{1/2}$.

Figure 2. Reconstruction of the field (9) discontinuities.

The line of the vector field discontinuities is observed perfectly in all pictures. A singular point $(0, 0)$ is recovered with usage of the first two methods. Perhaps, we can explain it by the fact that in the first two methods the averaging was done before the differentiation.

In the second numerical simulation the goal is to reconstruct a set of discontinuities points of the first derivative of vector field. To this end, we choose $C^1$-potential with discontinuous 2 derivatives on the boundary,

$$\varphi(x, y) = xy \left(R^2 - x^2 - y^2\right)^2, \quad x^2 + y^2 < R^2, \quad R < 1,$$

which generates potential vector field

$$u = (u_1, u_2) = (R^2 - x^2 - y^2) \left(y(R^2 - 5x^2 - y^2)\right), x(R^2 - x^2 - 5y^2)).$$

(12)

The transverse ray transform of potential field (12) is

$$(P^\perp u)(\xi, s) = \frac{16}{15}s \left(3R^2 - 8s^2\right) \left(R^2 - s^2\right)^{3/2} \xi_1 \xi_2.$$ 

(13)

Figure 3. Potential vector field (d) generated by $C^1$-potential (11) (a); 1-st component $\partial \varphi / \partial x$ of the field (b); 2-nd component $\partial \varphi / \partial y$ of the field (c).
Fig. 4 depicts images, which demonstrate the set of discontinuities of the 1-st derivatives of the potential vector field (12) reconstructed by different methods. In Fig. 4(a,b,c) to the known transverse ray transform we apply the operator of 1-angular moment, which coincides with the backprojection operator by relation to the ray transform of vector field. Consequently, we obtain the vector field \((\mu_1, \mu_2)\). Henceforth, we act on the field twice by the operator of gradient and calculate modulus of obtained 3-tensor field (Fig. 4a); by the operator of gradient, then by divergence and calculate modulus of obtained vector field (Fig. 4b). Finally, we act on the field by the operator of divergence, then by gradient and calculate modulus of the obtained vector field (Fig. 4c).

![Figure 4](image)

Figure 4. Reconstruction of the first derivatives discontinuities of the field (12).

Fig. 4(d,e,f) show results, obtained by means of another order of actions. Namely, first of all we apply the operator of differentiation with respect to \(s\) to the transverse ray transform. Consequently, we obtain transverse ray transform from symmetric potential 2-tensor field, generated by the potential (11). Obtained function \(g(\xi(\alpha), s)\), as well as the Radon transform of this potential, are even. Therefore, we can apply to them the operators of angular moments of even orders (the operators of angular moments of odd order give identical 0). We apply the operator of 0-angular moment to the result of differentiation with respect to \(s\) (like to the Radon transform of the scalar field), then calculate gradient and its modulus (Fig. 4d). Fig. 4(e,f) correspond to the application of the operator of 2-angular moment, which coincides with the backprojection operator applied to the ray transform of 2-tensor field. Henceforth, we apply the operator of divergence to this result and then calculate modulus of obtained vector field (Fig. 4e): apply the operator of gradient and then calculate modulus of obtained 3-tensor field (Fig. 4f).

In Fig. 4(g,h) the results, obtained on the first step, by the application of the operator of double differentiation with respect to \(s\) to transverse ray transform (13), are depicted. Consequently, we obtain transverse ray transform of potential symmetric 3-tensor field generated by potential (11). Obtained function \(g(\xi(\alpha), s)\), as well as the transverse ray transform of the vector field (12), are odd. Therefore, we can apply the operators of angular moments of odd orders (in this case the operators of angular moments of even order give identical 0). We
apply to the result the operator of 1-angular moment (like to the ray transform of the vector field), then modulus of the obtained vector field is calculated (Fig. 4g). Fig. 5h corresponds to the application of the operator of 3-angular moment, which coincides with the backprojection operator applied to the ray transform of the potential symmetric 3-tensor field. Henceforth, we calculate modulus of the obtained 3-tensor field.

Fig. 4 shows that the most pronounced resolution of discontinuities is done by the operator of divergence modulus. This conclusion, due to our numerical simulations, is valid for singularities of the vector field itself as well as for singularities of its first derivatives. We suppose, that the same procedures will also work appropriately for resolution of vector field singularities of higher order.

Future research consists of a rigorous analysis of the applied local operators as well as the investigation of the stability of our methods. This includes numerical tests with noise contaminated data.

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