Practical quantum error mitigation for analog quantum simulation

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Analog quantum simulation has been proposed as an efficient approach for probing classically intractable many-body physics. In contrast to digital, gate-based quantum simulation, the analog approach involves configuring the quantum hardware to directly mimic the physics of the target system. This reduces the level of control required and can offer a degree of inherent robustness, but it comes at the cost that known error-handling techniques may be inapplicable. Accumulation of error is thus a major challenge facing medium-scale and large-scale analog quantum simulation with current and near-term noisy quantum (NISQ) devices. Here we propose a hybrid error mitigation scheme that can suppress general local Markovian noise in analog quantum simulators. We employ stochastic error mitigation for physical noise and use Richardson extrapolation to compensate for model estimation error. We numerically test the error mitigation scheme for an Ising-type Hamiltonian with energy relaxation and dephasing noise, and show an improvement of simulation accuracy by two orders. We assess the resource cost of our error mitigation scheme and conclude that analog quantum simulation at scale is feasible with NISQ hardware.

As suggested by Feynman in 1982 [1], classically challenging many-body physics problems could be efficiently simulated using quantum computers. As a natural generalisation of digital classical computers, digital gate-based quantum computers offer a universal simulation capability since arbitrary gates can be composed from a small set of building blocks (generally one- and two-qubit gates) [2][4]. In contrast, analog quantum simulators directly emulate the target system by mapping it to specialised quantum hardware, achieving an isomorphism in the (relevant) physics. Sacrificing the universality of solving general problems, analog quantum simulators can instead more efficiently and robustly simulate specific systems, enabling the possibility of probing classically intractable many-body phenomena with noisy intermediate scale quantum (NISQ) hardware. In experiment, analog quantum simulation (AQS) has been applied for studying non-equilibrium dynamics [8][11], quantum scrambling [12][13], Ising and Bose-Hubbard models [14][15], dynamical quantum phase transitions [19][25], etc.

To realise a general quantum computer, it is crucial to overcome noise from the inevitable interaction with the environment [2][20][29]. Based on the universality of digital quantum simulation (DQS), many techniques ranging from error mitigation [30][33] or to full error correction [34][35] have been proposed to suppress or correct errors for either near-term quantum computing or universal quantum computing. However, owing to restricted set of allowed operations in AQS, it is more challenging to suppress or correct errors in this context. Existing methods such as dynamical decoupling are generally limited to low frequency noise [37][39] but can be very helpful in small simulations, however it remains an important open challenge to suppress errors for reliable medium- or large-scale analog quantum simulators.

In this work, we present a solution to this problem by proposing a hybrid error mitigation scheme, a combination of stochastic error mitigation and Richardson extrapolation. For any local Markovian noise, we correct it with stochastic error mitigation by randomly applying single qubit recovery operations to cancel the continuously-occurring errors. While the error mitigation procedure relies on an error model estimation and recovery operations, imperfections in error model estimation and recovery operations are suppressed by leveraging Richardson extrapolation. We numerically test the error mitigation scheme for an Ising Hamiltonian with energy relaxation and dephasing noise. We also conduct a resource estimation for near-term AQS involving up to 100 qubits and show the feasibility of our error mitigation scheme in the NISQ regime.

Framework.——We first review the basics of analog simulation and error mitigation. By mapping the target system $H_{\text{sys}}$ to a controllable quantum hardware $H_{\text{sim}}$, we focus on the ideal time evolution of $\rho_I(t)$ under $H_{\text{sim}}$:

$$\frac{d\rho_I(t)}{dt} = -i[H_{\text{sim}}(t), \rho_I(t)].$$

In practice, the implemented time evolution with Markovian noise [15] is described by

$$\frac{d\rho_N(t)}{dt} = -i[H_{\text{sim}}(t), \rho_N(t)] + \lambda \mathcal{L}_{\text{exp}}[\rho_N(t)],$$

where $\rho_N(t)$ is the noisy state, and $\mathcal{L}_{\text{exp}}$ is the time-independent noise super-operator with an error strength $\lambda$. Suppose we are interested in measuring the state at time $T$ with an observable $O$. Then the task of error mitigation is to efficiently recover the noiseless measurement outcome $\langle O \rangle_I = \text{Tr}[O \rho_I(t)]$ via the noisy process in Eq. 2. We also require that the error mitigation process must be independent of the observable $O$.
In general, it would be difficult to efficiently mitigate arbitrary noise with any noise strength. In this work, we assume that the noise operators only act weakly, locally, and time-independently on small subsystems. Note that nevertheless, local noise operators at instant time $t$ can easily propagate to become global noise after integrating time. While different analog simulators may have different sets of controllable operations, we assume that individual single qubit controls are allowed. This assumption is valid in various practical analog simulators with superconducting qubits [40–42], ion trap systems [19, 43], and Rydberg atoms [44]. In this paper, we mainly focus on qubit systems, whereas the discussion can be naturally generalised to multi-level systems.

**Stochastic error mitigation for AQS.**—We first introduce stochastic error mitigation for suppressing physical errors. As an intermediate step, we first introduce the 'continuous' error mitigation scheme in Fig. 1(a), which is a generalisation of the probabilistic error mitigation method [30, 31] from DQS to AQS. Consider a finite small time step $\delta t$, the discretised time evolution of Eqs. (1) and (2) can be represented as

$$\rho_\alpha(t + \delta t) = \mathcal{E}_\alpha(t)\rho_\alpha(t).$$

Here $\alpha = I, N$ and $\mathcal{E}_\alpha(t)$ denotes the ideal (I) or noisy (N) channels that evolve the state from $t$ to $t + \delta t$. In theory, we can find a recovery operation $\mathcal{E}_N$ that approximately maps the noisy evolution back to the noiseless one, i.e.,

$$\mathcal{E}_I(t) = \mathcal{E}_Q\mathcal{E}_N(t) + \mathcal{O}(\delta t^2).$$

However, the operation $\mathcal{E}_Q$ is in general not completely positive, hence cannot be physically realised by a quantum channel. Nevertheless, with the assumption of local noise operators, following [30, 31] we can efficiently decompose $\mathcal{E}_Q$ as a linear sum of a polynomial number of physical operators $\{B_j\}$ that are tensor products of qubit operators,

$$\mathcal{E}_Q = c \sum_j \alpha_j p_j B_j,$$

with coefficients $c \approx 1$ and $\alpha_j = \pm 1$, and a normalised probability distribution $p_j$. We refer to Supplemental Materials for details of the decomposition and its optimisation via linear programming. Under this decomposition, the whole ideal evolution from 0 to $T$ can be mathematically decomposed as

$$\prod_{k=0}^{n-1} \mathcal{E}_I(k\delta t) \approx C \sum_j \alpha_j p_j \prod_{k=0}^{n-1} B_{jk} \mathcal{E}_N(k\delta t) + \mathcal{O}(T\delta t),$$

where $n = T/\delta t$, $C = c^n$, $\alpha_j = \prod_{k=0}^{n-1} \alpha_{jk}$, $p_j = \prod_{k=0}^{n-1} p_{jk}$, and $j = (j_1, \ldots, j_{n-1})$. Denote the ideally evolved state as $\rho_I(T) = \prod_{k=0}^{n-1} \mathcal{E}_I(k\delta t)\rho(0)$ and the noisily evolved and corrected state as $\rho_{Q,j}(T) = \prod_{k=0}^{n-1} B_{jk} \mathcal{E}_N(k\delta t)\rho(0)$, then we can approximate the ideal state $\rho_I(T)$ as a linear sum of noisy states $\rho_{Q,j}(T)$,

$$\rho_I(T) = C \sum_j \alpha_j p_j \rho_{Q,j}(T) + \mathcal{O}(T\delta t).$$

When measuring an observable $O$ of the ideal state, the ideal measurement outcome $\langle O \rangle_I = \text{Tr}[\rho_I(T)O]$ is also approximated as a linear sum of the noisy measurement outcomes $\langle O \rangle_{Q,j} = \text{Tr}[\rho_{Q,j}(T)O]$ as

$$\langle O \rangle_I = C \sum_j \alpha_j p_j \langle O \rangle_{Q,j} + \mathcal{O}(T\delta t).$$

In practice, we can randomly prepare $\rho_{Q,j}(T)$ with probability $p_j$, measure the observable $O$, and multiply the outcome with the coefficient $C\alpha_j$. Then the average measurement outcome of the noisy and corrected states is exactly the noiseless measurement outcome. Suppose we aim to measure the average outcome to an additive error $\varepsilon$ with failure probability $\delta$, we would need $T \propto C^2 \log(\delta^{-1})/\varepsilon^2$ samples according to the Hoeffding inequality. Since the number of samples needed given access to $\rho_I(T)$ is $T_0 \propto \log(\delta^{-1})/\varepsilon^2$, the error mitigation scheme introduces a sampling overhead $C^2$. We will presently discuss the magnitude of the overhead with NISQ hardware, and we describe the analytical upper and lower bounds in the Supplemental Materials.

The above scheme requires one to 'continuously' interchange the noisy evolution and the recovery operation with a sufficiently small time step $\delta t$, which could be challenging in practice. Notice that $\mathcal{E}_I(t) \approx \mathcal{E}_N(t)$ and hence that the recovery operation at each time is almost...
an identity operation,
\[ \mathcal{E}_Q = c \left( p_0 I + \sum_{j \geq 1} \alpha_j p_j \delta t B_j \right), \]  
with \( B_0 \) being the identity channel \( I \) and \( p_0 \approx 1 \). We can thus further apply the Monte Carlo method to stochastically realise the continuous recovery operations. In particular, we initialise \( p = 1 \) and randomly generate \( q \in [0, 1] \) at time \( t = 0 \). We then evolve the state according to the noisy evolution \( \mathcal{E}_N \) until time \( t_{jp} \) by solving \( p(t_{jp}) = q \) with \( p(t) = \exp(-\Gamma(t)) \) and \( \Gamma(t) = t \sum_{j \geq 1} p_j \). At time \( t_{jp} \), we generate another uniformly distributed random number \( q' \in [0, 1] \), apply the recovery operation \( B_j \) if \( q' \in [s_{j-1}, s_j] \), and update the coefficient as \( \alpha = \alpha_j \alpha \). Here \( s_j(t) = \left( \sum_{i=1}^j p_i \right) / \left( \sum_{i=1}^{N_{\text{op}}} p_i \right) \), \( N_{\text{op}} \) is the number of basis operations, and the sum omits the identity channel. Then, we randomly initialise \( q \), and repeat this procedure until the measurement time \( t = T \). On average, this stochastic error mitigation scheme is equivalent to the above 'continuous' scheme. Furthermore, since we can determine the jump time and the recovery operations before the experiment, the error mitigation operations can be pre-engineered as parts of the evolution operations. We summarise the scheme as follows.

**Algorithm 1** Stochastic error mitigation for AQS.

Input: initial state \( \rho(0) \), number of samples \( N_s \), noisy evolution \( \mathcal{E}_N \), basis operations \( B_j \); Output: \( \hat{O} \).

1. Get \( C, \{\alpha_j\}, \{p_j\} \) of Eq. [5], set \( s_j = \frac{\sum_{i=1}^j p_i}{\sum_{i=1}^{N_{\text{op}}} p_i} \).
2. for \( m = 1 \) to \( N_s \) do
3. Randomly generate \( q_0 \in [0, 1] \), set \( t = 0, n = 0, \alpha = 1 \).
4. while \( t \leq T \) do
5. Get \( t_{jp} \) by solving \( -\Gamma(t_{jp}) = q_n \).
6. Randomly generate \( q'_n \in [0, 1] \).
7. Set \( s_n = j \) if \( q'_n \in [s_{j-1}, s_j] \) and update \( \alpha = \alpha_j \alpha_n \).
8. Update \( t = t + t_{jp} \) and \( n = n + 1 \).
9. end while
10. Set \( \rho_Q = \rho(0) \) and \( \hat{O} = 0 \).
11. for \( k = 0 : n - 1 \) do
12. Evolve \( \rho_Q \) under \( \mathcal{E}_N \) for time \( t_{jk} \) and apply \( B_{jk} \).
13. end for
14. Evolve \( \rho_Q \) under \( \mathcal{E}_N \) for time \( T - \sum_{k=0}^{n-1} t_{jk} \).
15. Measure \( O \) of \( \rho_Q \) to get \( O_m \).
16. Update \( \hat{O} = \hat{O} + C a O_m / N_s \).
17. end for

We remark that the stochastic error mitigation scheme can also suppress time-dependent Markovian noise and coherent error in the implementation of the Hamiltonian.

**Reduction of model estimation error.** The above error mitigation scheme assumes a prior knowledge of the noise model. In reality the actual noise \( \mathcal{L}_{\text{exp}} \) and the estimated noise \( \mathcal{L}_{\text{est}} \) may differ due to imprecise knowledge of the noise model. Here we show how to mitigate model estimation error by using the extrapolation method. The effective evolution after applying the error mitigation method with \( \mathcal{L}_{\text{est}} \) is
\[ \frac{d}{dt} \rho_{\alpha}^{(Q)}(t) = -i[H(t), \rho_{\alpha}^{(Q)}(t)] + \lambda \Delta \mathcal{L}[\rho_{\alpha}^{(Q)}(t)], \]
where \( \rho_{\alpha}^{(Q)}(t) \) is the effective density matrix and \( \Delta \mathcal{L} = \mathcal{L}_{\text{exp}} - \mathcal{L}_{\text{est}} \). By re-scaling \( H(t) \rightarrow \frac{1}{\lambda} H(t) \), the evolution for a rescaled time \( rt \) is equivalent to
\[ \frac{d}{dt} \rho_{\alpha}^{(Q)}(t) = -i[H(t), \rho_{\alpha}^{(Q)}(t)] + rt \Delta \mathcal{L}[\rho_{\alpha}^{(Q)}(t)]. \]

The above dynamics can be experimentally implemented by re-running the error-mitigated experiment with the re-scaled Hamiltonian for a re-scaled time \( rt \). As we can freely tune the value of \( r \geq 1 \), we can choose several different values of \( r \) and suppress the model estimation error via Richardson extrapolation. Similar ideas have been applied for suppressing physical errors in DQS [30, 32]. In particular, we can use more than two values of \( r \) denoted as \( \{r_j\} \) and introduce constants \( \beta_j = \prod_{l \neq j} r_l (r_l - r_j)^{-1} \).

Denote the error mitigated measurement outcome for \( \rho_{\alpha}^{(Q)} \) as \( \langle O \rangle_{\alpha} \), then we have
\[ \langle O \rangle_I \approx \sum_{j=0}^{n} \beta_j \langle O \rangle_{\alpha j}, \]
with an error in the order of \( O(\lambda^{n+1}) \). The Richardson extrapolation procedure can also correct imperfections of the recovery operations \( B_i \) in stochastic error mitigation. This is because the error for \( B_i \) also leads to the deviation of \( \mathcal{L}_{\text{est}} \), which can be regarded as a special case of model estimation error.

**Numerical simulation.** Now we show a numerical test of our error mitigation scheme for the scenario where a two qubit Ising-type Hamiltonian \( H = J_z \sigma_z^{(1)} \sigma_z^{(2)} - h \sum_{j=1}^2 \sigma_z^{(j)} \) is implemented on a noisy superconducting quantum simulator [15, 45]. In superconducting quantum hardware, the dominant noise types are typically energy relaxation and dephasing processes, described by \( \mathcal{L}_{\beta} \rho = \sum_{j} \lambda_{\beta} L^{(j)} \rho L^{(j)}(\rho) = \frac{1}{2} \{ L^{(j)}(\rho) L^{(j)}(\rho) \} \) with \( \sigma_z^{(j)} = \sigma_z^{(j)} - i a_{\sigma_z^{(j)}} \) and \( L_z^{(j)} = \sigma_z^{(j)} \), respectively. Such a noise model is also relevant for other quantum simulators such as trapped ions [9, 13] [25, 47], NMR [13, 14], ultracold atoms [20, 24], optical lattices apparatus [22], etc. The noise process can be characterised by measuring the energy relaxation time \( T_1 \) and the dephasing time \( T_2 \) without full process tomography [27, 47, 49]. We also consider physical errors for the single qubit recovery operations as single-qubit inhomogeneous Pauli error, \( \mathcal{E}_{\text{inh}} = (1 - p_x - p_y - p_z) I + p_x X + p_y Y + p_z Z \) with \( I, X, Y, Z \) being the Pauli channel and \( p_x \) being the error probability.
In our simulation, we set \( h = J = 2\pi \times 3 \text{ MHz}, p_\chi = p_\eta = 0.5\% \) and \( p_\zeta = 1\%, \) which can be easily achieved with current superconducting simulators \[^{50, 51}\]. We also considered the effect of the model estimation error by setting the real noise strength to be 20% greater than the estimated one, i.e., \( \lambda_{\text{exp}} = 1.2\lambda_{\text{est}}, \) and used linear extrapolation with \( r_\eta = 1 \) and \( r_\zeta = 2. \) In our simulation, we set the initial state to \( (\ket{0} + \ket{1})/\sqrt{2} \otimes \), evolve it to time \( T = 7\pi/J, \) and measure the expectation value of the Pauli \( X \) operator \( \sigma_\chi^{(1)} \) on qubit 1. The total number of samples of the measurement is fixed to be \( 10^8. \) In practice we can determine the number of samples depending on the variance, and efficiently obtain the error-mitigated expectation value within given sampling errors; c.f. Supplemental Materials.

We show the performance of stochastic error mitigation, Richardson extrapolation, and their combination in Fig. 2. In particular, we compare the time evolution of the expectation value of \( \sigma_\chi^{(1)} \) with and without error mitigation in Fig. 2 (a) with noise strength \( \lambda_1 = 0.1 \text{ MHz} \) and \( \lambda_2 = 0.05 \text{ MHz} \). The decay of the amplitude of the expectation value is significantly suppressed under error mitigation. Fig. 2 (b) shows the simulation error versus different noise strengths \( \lambda. \) We find that the stochastic error mitigation method can eliminate a large portion of errors, and with a combination of the extrapolation method, errors can be suppressed by two orders of magnitude even with a quite large model estimation error. Thus our error mitigation scheme can be robust to the drift of noise \[^{52, 53}\]. In addition, our method can be applied to correlated noise; we leave the discussion of this point for the Supplemental Materials.

**Resource cost for NISQ hardware.**—Here, we estimate the resource cost for stochastic error mitigation with NISQ hardware. Given precise noise models, the stochastic error mitigation method in principle enables exact recovery of the ideal evolution. However, to achieve the same accuracy of the measurement on the ideal evolution, we need \( C^2 \) times more samples or experiment runs with the error mitigated noisy evolution. The overhead \( C^2 \) is likely to prove prohibitively large with a significant amount of noise on a medium-scale quantum computer. Nevertheless, we show that the overhead can be reasonably small (less than 100) when the total error (defined below) is less than 1. In particular, we consider a noisy superconducting simulator with up to \( N = 100 \) qubits, which suffers from single qubit relaxation and dephasing noise with noise strength \( \lambda_1 \) and \( \lambda_2 = 0.5\lambda_1, \) respectively. While the noise strength is defined as the noise rate at instant time, we define the total noise strength \( \Lambda = NT(\lambda_1 + \lambda_2) \) to be the noise of the whole \( N \)-qubit system within time \( T. \) Fig. 3 illustrates the dependence of the overhead \( C^2 \) on the total noise strength \( \Lambda. \) The cost is 55.3 when \( \Lambda = 1. \) We also show the dependence of the cost on the number of qubits in the inset of Fig. 3 (a). For \( \Lambda = 1, \) we consider a special case with \( T = 1 \text{ µs}, \) \( \lambda_1 + \lambda_2 = 0.01 \text{ MHz}, \) and \( N = 100 (50) \) qubits, and show the number of measurements needed to achieve a given simulation accuracy in Fig. 3 (b). Note that the overhead \( C^2 \) is independent of the Hamiltonian \( H_{\text{sim}}, \) so the results apply for general AQS with NISQ hardware.

**Discussion.**—To summarise, we propose a hybrid error mitigation scheme for analog quantum simulation. We numerically test it with a two qubit system under energy relaxing and dephasing noise. We also show its feasibility with general noisy-intermediate-scaled-quantum hardware. Although our error mitigation scheme is designed for analog quantum simulation, it can be naturally applied for digital gate-based quantum computing. In
particular, our scheme can be applied to mitigate errors for multi-qubit gates (which generally have large errors) and thus extend the computation capability for noisy gate-based quantum processors. Furthermore, resolving the drifting of noise is challenging for existing error mitigation methods. As our hybrid scheme can mitigate model estimation error, it can also be robust to the drift of noise. Although our discussion focused on local time-independent noise, it can be potentially generalised to general non-local time-dependent noise. We leave the detailed discussion to a future work.

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