The State of the Universe

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Abstract

What is the quantum state of the universe? That is the central question of quantum cosmology. This essay describes the place of that quantum state in a final theory governing the regularities exhibited universally by all physical systems in the universe. It is possible that this final theory consists of two parts: (1) a dynamical theory such as superstring theory, and (2) a state of the universe such as Hawking’s no-boundary wave function. Both are necessary because prediction in quantum mechanics requires both a Hamiltonian and a state. Complete ignorance of the state leads to predictions inconsistent with observation. The simplicity observed in the early universe gives hope that there is a simple, discoverable quantum state of the universe. It may be that, like the dynamical theory, the predictions of the quantum state for late time, low energy observations can be summarized by an effective cosmological theory. That should not obscure the need for a fundamental basis for such an effective theory which provides a unified explanation of its features. It could be that there is one principle that determines both the dynamical theory and the quantum state. That would be a truly unified final theory.

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INTRODUCTION

The universe has a quantum state. What is it? That is the central question of quantum cosmology — the subject to which Stephen Hawking has contributed so many seminal ideas.

To ask this question is to assume that the universe is a quantum mechanical system. We perhaps have little direct evidence of peculiarly quantum mechanical phenomena on large and even familiar scales, but there is no evidence that the phenomena that we do see cannot be described in quantum mechanical terms and explained by quantum mechanical laws. Further, every major candidate for a fundamental dynamical law from the standard model to M-theory conforms to the quantum mechanical framework for prediction. If this framework applies to the whole thing, there must be a quantum state of the universe.

It would be even more interesting if quantum mechanics broke down on cosmological scales. But there is not a shred of evidence for that, and my guess is that, even if it does, we will only find out by pursuing the assumption that quantum mechanics is the framework for a final theory of cosmology.

My talk will not review any of the current ideas for a quantum state of the universe even the no-boundary wave function [1, 2]. The articles of Don Page and Alex Vilenkin in this volume do that. Rather, I want to concentrate on explaining why a theory of the quantum state of the universe must be part of any final theory. I am also not going to discuss the generalizations of usual quantum theory that are required for quantum cosmology [3]. For the essential points of this talk you can just imagine that the universe is a vast number of particles in a very large expanding box.

FINAL THEORIES

The final theory (to borrow Steve Weinberg’s term) predicts the regularities that are exhibited by all physical systems — without exception, without qualification, and without approximation. Much of this conference has been concerned with the search for the final theory. A possible view at present is that it consists of two parts:

- A universal dynamical law such as string theory or its successors;
- A law for the quantum state of the universe such as Hawking’s no-boundary wave function of the universe.
In the model universe in a box these two ingredients are the Hamiltonian specifying the form of the Schrödinger equation

\[ \text{itr} \frac{d|\Psi(t)\rangle}{dt} = H |\Psi(t)\rangle \]  

(1)

and the initial quantum state

\[ |\Psi(0)\rangle \]  

(2)

with which it starts off.

Both of these pieces are necessary for prediction. The Schrödinger equation by itself makes no predictions. Probabilities \( p_\alpha \) in quantum mechanics for a set of alternatives represented by projection operators \( \{ P_\alpha \} \) are given by

\[ p_\alpha = \| P_\alpha |\Psi(t)\rangle \|^2 \]  

(3)

and to compute these the quantum state is needed at least at one time. No state, no predictions.

To put the matter in a different way, if the state is arbitrary, the predictions are arbitrary. Pick any probabilities \( p_\alpha \) you like for the alternatives \( P_\alpha \). There is some state that will reproduce them. For example, you can take

\[ |\Psi(t)\rangle = \sum_\alpha p_\alpha^{\frac{1}{2}} |\Psi_\alpha\rangle \]  

(4)

where the \( |\Psi_\alpha\rangle \) are any set of eigenstates of the \( P_\alpha \)'s

\[ P_\alpha |\Psi_\beta\rangle = \delta_{\alpha\beta} |\Psi_\beta\rangle. \]  

(5)

The \( |\Psi(t)\rangle \) constructed according to (4) will reproduce the pre-assigned probabilities \( p_\alpha \) in (3).

Neither is ignorance bliss. If you assume you know nothing about the state of the universe in a box then you should make predictions with a density matrix proportional to unity

\[ \rho = \frac{I}{Tr(I)} \]  

(6)

reflecting that ignorance. But this density matrix corresponds to equilibrium at infinite temperature and its predictions are nothing like the universe we live in. In particular, there would be no evolution since \([H, \rho] = 0\). There would be no second law of thermodynamics.
since the entropy $-Tr[\rho \log \rho]$ is already at its maximum possible value. There would be no classical behavior since, although the expected value of a field averaged over a spacetime volume $R$ might be finite, its fluctuations, $\langle \phi(R)^2 \rangle$, would be infinite.

The search for a unified fundamental dynamical law has been seriously under way at least since the time of Newton with string theory or its generalizations being the most actively investigated direction today. By contrast, the search for a theory of the quantum state of the universe has only been actively under way since the time of Hawking, let us say on this occasion. Why this difference?

Dynamical laws govern regularities in time and its an empirical fact that the basic dynamical laws are local in space on scales above the Planck length. The laws that govern regularities in time across the whole universe are therefore discoverable and testable in laboratories on earth. By contrast many of the regularities predicted with near certainty by the quantum state occur mostly in space on large cosmological scales. Only recently has there been enough data to confront theory with observation. That difference in the nature of the predicted regularities, or their difference in scales, should not obscure the fact that the state is just as much a part of the final theory as is its Hamiltonian.

Given these differences, what grounds do we have to hope that we can discover the quantum state of the universe? There are two: The first is the simplicity of the early universe revealed by observation — more homogeneous, more isotropic, more nearly in thermal equilibrium than the universe is today. It’s therefore possible that the universe has a simple, discoverable initial quantum state and that all of the complex universe of galaxies, stars, planets, and life today arose from quantum accidents that have happened since and the action of gravitational attraction. The second reason is the idea that the quantum state and the dynamical theory may be naturally connected as in Hawking’s no-boundary theory.

**EFFECTIVE THEORIES**

We are used to the idea of effective dynamical theories that accurately describe limited ranges of phenomena. The Navier-Stokes equations, non-relativistic quantum mechanics, general relativity, quantum electrodynamics, and the standard model of particle physics are all familiar examples. To construct an effective theory we typically assume a coarse-grained description (restricting attention to energies below the Planck scale for instance) and assume
some simple property that the state might predict there (classical spacetime, for example).

Cosmology too has its effective theories and its standard model. This is summarized neatly by Martin Rees in his cosmologists’ credo[4]. I reproduce it here with unauthorized additions.

• spacetime is classical,

• our universe is expanding,

• from a hot big bang,

• in which light elements were synthesized,

• there was a period of inflation,

• which led to a flat universe today,

• structure was seeded by gaussian irregularities,

• which are relics of quantum fluctuations,

• the dominant matter is cold and dark,

• but a cosmological constant (or quintessence) is dynamically dominant.

Possibly all current observations in cosmology, at least the large scale ones, can be compressed into this list of ten assumptions and a few cosmological parameters. That is not unlike the situation in particle physics where most observations can be compressed into the Lagrangian of the standard model and its eighteen or so parameters.

However, the success of such effective theories which operate in limited ranges of phenomena should not obscure the need to find fundamental ones which apply to all phenomena without qualification and without approximation. It would be inconsistent, I believe, to pursue a fundamental dynamical theory in the face of a successful standard model, and not pursue a fundamental theory of the state of the universe because of the success of its standard model. That not least because the fundamental theory could provide a unified explanation of its assumptions.

It must be said, however, that when the natural domains of fundamental theories are as far from controllable experiments as string theory and the quantum initial condition the
possibility of definitive tests seems to recede. It could be that the predictions of string theory are limited to general relativity, gauge theories, supersymmetry, and the parameters of the standard particle model. In a similar way the predictions of the state of the universe could be limited to classical spacetime, the initial conditions for inflation, and the quantum fluctuations that satisfy large scale structure. Perhaps that is prediction enough.

DIRECTIONS

The instructions of the organizers were to discuss “future directions in theoretical physics and cosmology”. Continuing the search for a final theory incorporating dynamics and the initial quantum state is certainly one direction. But I would like to mention three questions that might lead to different approaches to the main one.

What’s Environmental?

Which features of the observed universe follow entirely from the dynamical theory \((H)\) and which follow entirely from the initial condition \(\langle \Psi(0) \rangle\), and which are the result of quantum accidents that occurred over the course of the universe’s history with probabilities specified by the combination of \(H\) and \(\langle \Psi(0) \rangle\). Those that depend significantly on \(\langle \Psi(0) \rangle\) are called “environmental”. Some version of this question was number one on the list of top ten questions for the next millennium prepared by string theorists at the Strings 2000 conference.

Take the coupling constants in effective dynamical theories for instance. The viscosities and equation-of-state in the Navier-Stokes equation are certainly environmental. They vary with system, place, and time. But at a given energy do the coupling constants of the standard model of the elementary particle interactions vary with place and time or with the possible history of the universe? If so then the initial quantum state is central to determining their probabilities.

Why Quantum Mechanics?

The founders of quantum mechanics thought that the inherent indeterminancy of quantum theory “reflected the unavoidable interference in measurement dictated by the magni-
tude of the quantum of the action [3]. But why then do we live in a quantum mechanical universe which by definition is never measured from the outside?

The most striking general feature of quantum mechanics is its exact linearity — the principle of superposition. But why should there be a principle of superposition in quantum cosmology which has only a single quantum state?

Why a Division into Dynamics and Initial Condition?

The schema for a final theory which I have been describing posits a separate theory of dynamics and quantum state. Could they be connected? They already are in Hawking’s no-boundary wave function

$$\Psi \left[ h_{ij}, \chi \right] = \int \delta g \delta \phi \, e^{-I[g, \phi]} \quad (7)$$

where the action for metric $g_{\alpha \beta}(x)$ coupled to matter $\phi(x)$ determines both the state and quantum dynamics. Is there a principle that determines both? Is there a connection between superstring theory and its successors and a unique quantum state?

A unified quantum theory of state and dynamics would be truly a final theory. That is surely a direction for theoretical physics and cosmology.

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