Exclusive charmless $B_s$ hadronic decays
into $\eta'$ and $\eta$

B. Tseng

Department of Physics, National Cheng-Kung University
Tainan, Taiwan 700, Republic of China

Abstract

Using the next-to-leading order QCD-corrected effective Hamiltonian, charmless exclusive non-leptonic decays of the $B_s$ meson into $\eta$ or $\eta'$ are calculated within the generalized factorization approach. Nonfactorizable contributions, which can be parametrized in terms of the effective number of colors $N_{\text{eff}}^c$ for $PP$ and $VP$ decay modes, are studied in two different schemes: (i) the one with the “homogeneous” structure in which $(N_{\text{eff}}^c)_1 \approx (N_{\text{eff}}^c)_2 \approx \cdots \approx (N_{\text{eff}}^c)_{10}$ is assumed, and (ii) the “heterogeneous” one in which the possibility of $N_{\text{eff}}^c(V + A) \neq N_{\text{eff}}^c(V - A)$ is considered, where $N_{\text{eff}}^c(V + A)$ denotes the effective value of colors for the $(V - A)(V + A)$ penguin operators and $N_{\text{eff}}^c(V - A)$ for the $(V - A)(V - A)$ ones. For processes depending on the $N_{\text{eff}}^c$-stable $a_i$ such as $\bar{B}_s \to (\pi, \rho)(\eta', \eta)$, the predicted branching ratios are not sensitive to the factorization approach we choose. While for the processes depending on the $N_{\text{eff}}^c$-sensitive $a_i$ such as $\bar{B}_s \to \omega \eta^{(')}$, there is a wide range for the branching ratios depending on the choice of the $N_{\text{eff}}^c$ involved. We have included the QCD anomaly effect in our calculations and found that it is important for $\bar{B}_s \to \eta^{(')} \eta^{(')}$. The effect of the $(c\bar{c}) \to \eta'$ mechanism is found to be tiny due to a possible CKM-suppression and the suppression in the decay constants except for the $\bar{B}_s \to \phi \eta$ decay within the “naive” factorization approach, where the internal $W$ diagram is CKM-suppressed and the penguin contributions are compensated.
1. Motivation Stimulated by the recent observations of the large inclusive and exclusive rare B decays by the CLEO Collaboration [4], there are considerable interests in the charmless B meson decays [2]. To explain the abnormally large branching ratio of the semi-inclusive process $B \to \eta' + X$, several mechanisms have been advocated [3, 4, 5, 6] and some tests of these mechanisms have been proposed [7]. It is now generally believed that the QCD anomaly [3, 4, 1] plays a vital role. The understanding of the exclusive $B \to \eta'K$, however, relies on several subtle points. First, the QCD anomaly does occur through the equation of motion [9, 13]. Third, the running strange quark mass which appears in the calculation of the matrix elements of the $(S - P)(S + P)$ penguin operator, the $SU(3)$ breaking effect in the involved $\eta'$ decay constants and the normalization of the $B \to \eta'$ matrix element involved raise the branching ratio substantially. Finally, nonfactorizable contributions, which are parametrized by the $N_c^{\text{eff}}$, gives the final answer for the largeness of exclusive $B \to \eta'K$. (We refer the reader to [14, 15] for details.)

It is very interesting to see the impacts of these subtleties mentioned above on the the exclusive charmless $B_s$ decays to an $\eta'$ or $\eta$. In addition to the essential and important QCD penguin contribution as discussed in [13, 17], it is found that the EW penguin contribution is found large and positive originally [10, 11], is now preferred to be negative and smaller than before as implied by a recent theoretical recalculation [12] and several phenomenological analyses [9, 13].

2. Theoretical Framework We begin with a brief description of the theoretical framework. The relevant effective $\Delta B = 1$ weak Hamiltonian is

$$\mathcal{H}_{\text{eff}}(\Delta B = 1) = \frac{G_F}{\sqrt{2}} \left[ V_{ub} V_{us}^* (c_1 O_{1u}^u + c_2 O_{1u}^s) + V_{cb} V_{cq}^* (c_1 O_{1c}^u + c_2 O_{1c}^s) - V_{tb} V_{tq}^* \sum_{i=3}^{10} c_i O_i \right] + \text{h.c.}, \quad (1)$$

where $q = d, s$, and

$$O_{1u}^u = (\bar{u}b)_{V-A}(\bar{q}u)_{V-A}, \quad O_{1u}^s = (\bar{c}b)_{V-A}(\bar{q}c)_{V-A},$$
$$O_{1u}^s = (\bar{q}b)_{V-A}(\bar{u}u)_{V-A}, \quad O_{1c}^s = (\bar{q}b)_{V-A}(\bar{c}c)_{V-A},$$
$$O_{3(5)} = (\bar{q}b)_{V-A} \sum_{q'} (\bar{q'}q')_{V-A(V+A)}, \quad O_{4(6)} = (\bar{q}_\alpha b_\beta)_{V-A} \sum_{q'} (\bar{q}'_\alpha q'_\beta)_{V-A(V+A)},$$
$$O_{7(9)} = \frac{3}{2} (\bar{q}b)_{V-A} \sum_{q'} e_{q'} (\bar{q}'q')_{V-A(V-A)}, \quad O_{8(10)} = \frac{3}{2} (\bar{q}_\alpha b_\beta)_{V-A} \sum_{q'} e_{q'} (\bar{q}'_\beta q'_\alpha)_{V-A(V-A)}, \quad (2)$$

with $(\bar{q}_1 q_2)_{V \pm A} \equiv \bar{q}_1 \gamma_\mu (1 \pm \gamma_5) q_2$. In Eq. (2), $O_{3-6}$ are QCD penguin operators and $O_{7-10}$ are electroweak penguin operators. $C_i(\mu)$ are the Wilson coefficients, which have been evaluated to the next-to-leading order (NLO) [13, 20]. One important feature of the NLO calculation is the renormalization-scheme and -scale dependence of the Wilson coefficients (for a...
In order to ensure the $\mu$ and renormalization scheme independence for the physical amplitude, the matrix elements, which are evaluated under the factorization hypothesis, have to be computed in the same renormalization scheme and renormalized at the same scale as $c_i(\mu)$. However, as emphasized in [14], the matrix element $\langle O \rangle_{\text{fact}}$ is scale independent under the factorization approach and hence it cannot be identified with $\langle O(\mu) \rangle$. Incorporating QCD and electroweak corrections to the four-quark operators, we can redefine $c_i(\mu)\langle O_i(\mu) \rangle = c_i^{\text{eff}}\langle O_i \rangle_{\text{tree}}$, so that $c_i^{\text{eff}}$ are renormalization scheme and scale independent. Then the factorization approximation is applied to the hadronic matrix elements of the operator $O$ at the tree level. The numerical values for $c_i^{\text{eff}}$ are shown in the last column of Table I, where $\mu = m_b(m_b)$, $\Lambda_{\overline{MS}}^{(5)} = 225$ MeV, $m_t = 170$ GeV and $k^2 = m_b^2/2$ are used [14].

In general, there are contributions from the nonfactorizable amplitudes. Because there is only one single form factor (or Lorentz scalar) involved in the decay amplitude of $B(D) \to PP, PV$ decays ($P$: pseudoscalar meson, $V$: vector meson), the effects of nonfactorization can be lumped into the effective parameters $a_i^{\text{eff}}$ [22]:

$$a_i^{\text{eff}} = c_i^{\text{eff}} + c_{2i} c_{2i-1} \left( \frac{1}{N_c} + \chi_{2i} \right), \quad a_i^{\text{eff}} = c_i^{\text{eff}} + c_{2i} c_{-2i} \left( \frac{1}{N_c} + \chi_{2i-1} \right),$$

where $c_{2i,2i-1}$ are the Wilson coefficients of the 4-quark operators, and nonfactorizable contributions are characterized by the parameters $\chi_{2i}$ and $\chi_{2i-1}$. We can parametrize the nonfactorizable contributions by defining an effective number of colors $N_c^{\text{eff}}$, called $1/\xi$ in [23], as $1/N_c^{\text{eff}} \equiv (1/N_c) + \chi$. Different factorization approaches used in the literature can be classified by the effective number of colors $N_c^{\text{eff}}$. The so-called “naive” factorization discards all the nonfactorizable contributions and takes $1/N_c^{\text{eff}} = 1/N_c = 1/3$, whereas the “large-$N_c$ improved” factorization [24] drops out all the subleading $1/N_c^{\text{eff}}$ terms and takes $1/N_c^{\text{eff}} = 0$. In principle, $N_c^{\text{eff}}$ can vary from channel to channel, as in the case of charm decay. However, in the energetic two-body $B$ decays, $N_c^{\text{eff}}$ is expected to be process insensitive as supported by data [25]. If $N_c^{\text{eff}}$ is process independent, then we have a generalized factorization. In this paper, we will treat the nonfactorizable contributions with two different phenomenological ways: (i) the one with “homogenous” structure, which assumes that $(N_c^{\text{eff}})_1 \approx (N_c^{\text{eff}})_2 \approx \cdots \approx (N_c^{\text{eff}})_{10}$, and (ii) the “heterogeneous” one, which considers the possibility of $N_c^{\text{eff}}(V + A) \neq N_c^{\text{eff}}(V - A)$. The consideration of the “homogenous” nonfactorizable contributions, which is commonly used in the literature, has its advantage of simplicity. However, as argued in [14], due to the different Dirac structure of the Fierz transformation, nonfactorizable effects in the matrix elements of $(V - A)(V + A)$ operators are a priori different from that of $(V - A)(V - A)$ operators, i.e. $\chi(V + A) \neq \chi(V - A)$. Since $1/N_c^{\text{eff}} = 1/N_c + \chi$, theoretically it is expected that

$$N_c^{\text{eff}}(V - A) \equiv (N_c^{\text{eff}})_1 \approx (N_c^{\text{eff}})_2 \approx (N_c^{\text{eff}})_3 \approx (N_c^{\text{eff}})_4 \approx (N_c^{\text{eff}})_9 \approx (N_c^{\text{eff}})_{10},$$

$$N_c^{\text{eff}}(V + A) \equiv (N_c^{\text{eff}})_5 \approx (N_c^{\text{eff}})_6 \approx (N_c^{\text{eff}})_7 \approx (N_c^{\text{eff}})_8.$$

To illustrate the effect of the nonfactorizable contribution, we extrapolate $N_c(V - A) \approx 2$ from $B \to D \pi(\rho)$ [23] to charmless decays. The $N_c^{\text{eff}}$-dependence of the effective parameters $a_i$’s are shown in Table I, from which we see that $a_1, a_4, a_6$ and $a_9$ are $N_c^{\text{eff}}$-stable, and the remaining ones are $N_c^{\text{eff}}$-sensitive. We would like to remark that while $a_3$ and $a_5$ are both
The QCD anomaly appears through the equation of motion mechanism involved. We first come to the QCD anomaly effect. As pointed out in \([8, 15]\), the combination of \((a_3 - a_5)\) is rather stable under the variation of the \(N_c^\text{eff}\) within the “homogeneous” picture and is still sensitive to the factorization approach taken in the “heterogeneous” scheme. This is the main difference between the “homogeneous” and “heterogeneous” approaches. While \(a_7, a_8\) can be neglected, \(a_3, a_5\) and \(a_{10}\) have some effects on the relevant processes depending on the choice of \(N_c^\text{eff}\).

Before carrying out the phenomenological analysis, we would like to discuss the dynamical mechanism involved. We first come to the QCD anomaly effect. As pointed out in \([8, 13]\), the QCD anomaly appears through the equation of motion

\[
\partial^\mu (\bar{s}\gamma_\mu \gamma_5 s) = 2m_s \bar{s}i\gamma_5 s + \frac{\alpha_s}{4\pi} G_{\mu\nu} \tilde{G}^{\mu\nu}.
\]

Neglecting the \(u\) and \(d\) quark masses in the equations of motion leads to

\[
\langle \eta | \frac{\alpha_s}{4\pi} G \tilde{G} | 0 \rangle = f_{\eta'}^u m_{\eta'}^2
\]

and hence

\[
\langle \eta' | \bar{s}\gamma_5 s | 0 \rangle = -\frac{i m_{\eta'}^2}{2m_s} \left( f_{\eta'}^s - f_{\eta'}^u \right).
\]

To determine the decay constant \(f_{\eta'}^u\), we need to know the wave functions of the physical \(\eta'\) and \(\eta\) states which are related to that of the SU(3) singlet state \(\eta_0\) and octet state \(\eta_8\) by

\[
\eta' = \eta_8 \sin \theta + \eta_0 \cos \theta, \quad \eta = \eta_8 \cos \theta - \eta_0 \sin \theta,
\]

with \(|\eta_0\rangle = \frac{1}{\sqrt{3}} |\bar{u}u + \bar{d}d + \bar{s}s\rangle, \quad |\eta_8\rangle = \frac{1}{\sqrt{6}} |\bar{u}u + \bar{d}d - 2\bar{s}s\rangle\) and \(\theta \approx -20^\circ\). When the \(\eta - \eta'\) mixing angle is \(-19.5^\circ\), the \(\eta'\) and \(\eta\) wave functions have simple expressions \([2]\): 

\[
|\eta'\rangle = \frac{1}{\sqrt{6}} |\bar{u}u + \bar{d}d + 2\bar{s}s\rangle, \quad |\eta\rangle = \frac{1}{\sqrt{3}} |\bar{u}u + \bar{d}d - \bar{s}s\rangle.
\]

\begin{table}
\centering
\begin{tabular}{|c|c|c|c|c|}
\hline
  & \(N_c^\text{eff} = 2\) & \(N_c^\text{eff} = 3\) & \(N_c^\text{eff} = 5\) & \(N_c^\text{eff} = \infty\) \\
\hline
\(a_1\) & 0.986 & 1.04 & 1.08 & 1.15 \\
\(a_2\) & 0.25 & 0.058 & -0.095 & -0.325 \\
\(a_3\) & -13.9 - 22.6i & 61 & 120 + 18i & 211 + 45.3i \\
\(a_4\) & -344 - 113i & -380 - 120i & -410 - 127i & -450 - 136i \\
\(a_5\) & -146 - 22.6i & -52.7 & 22 + 18i & 134 + 45.3i \\
\(a_6\) & -493 - 113i & -515 - 121i & -530 - 127i & -560 - 136i \\
\(a_7\) & 0.04 - 2.73i & -0.7 - 2.73i & -1.24 - 2.73i & -2.04 - 2.73i \\
\(a_8\) & 2.98 - 1.37i & 3.32 - 0.9i & 3.59 - 0.55i & 4 \\
\(a_9\) & -87.9 - 2.73i & -91.1 - 2.73i & -93.7 - 2.73i & -97.6 - 2.73i \\
\(a_{10}\) & -29.3 - 1.37i & -13 - 0.91i & -0.04 - 0.55i & 19.48 \\
\hline
\end{tabular}
\caption{Numerical values of effective coefficients \(a_i\) at \(N_c^\text{eff} = 2, 3, 5, \infty\), where \(N_c^\text{eff} = \infty\) corresponds to \(a_i^\text{eff} = c_i^\text{eff}\). The entries for \(a_3, ..., a_{10}\) have to be multiplied with \(10^{-3}\).}
\end{table}
At this specific mixing angle, \( f_{\eta'}^u = \frac{1}{2} f_{\eta'}^s \) in the SU(3) limit. Introducing the decay constants \( f_8 \) and \( f_0 \) by

\[
\langle 0 | A^{(0)}_{\mu} | \eta_0 \rangle = i f_0 p_{\mu}, \quad \langle 0 | A^{(s)}_{\mu} | \eta_s \rangle = i f_8 p_{\mu}
\]

(10)

then \( f_{\eta'}^u \) and \( f_{\eta'}^s \) are related to \( f_8 \) and \( f_0 \) by [32]

\[
f_{\eta'}^u = \frac{f_8}{\sqrt{6}} \sin \theta + \frac{f_0}{\sqrt{3}} \cos \theta, \quad f_{\eta'}^s = -2 \frac{f_8}{\sqrt{6}} \sin \theta + \frac{f_0}{\sqrt{3}} \cos \theta.
\]

(11)

Likewise, for the \( \eta \) meson

\[
f_{\eta}^u = \frac{f_8}{\sqrt{6}} \cos \theta - \frac{f_0}{\sqrt{3}} \sin \theta, \quad f_{\eta}^s = -2 \frac{f_8}{\sqrt{6}} \cos \theta - \frac{f_0}{\sqrt{3}} \sin \theta.
\]

(12)

From a recent analysis of the data of \( \eta, \eta' \to \gamma \gamma \) and \( \eta, \eta' \to \pi \gamma \gamma \) [32], \( f_{8(0)} \) and \( \theta \) have been determined to be

\[
\frac{f_8}{f_\pi} = 1.38 \pm 0.22, \quad \frac{f_0}{f_\pi} = 1.06 \pm 0.03, \quad \theta = -(22.0 \pm 3.3)^0,
\]

(13)

which lead to

\[
f_\eta^u = 99\text{MeV}, \quad f_\eta^s = -108\text{MeV}, \quad f_{\eta'}^u = 47\text{MeV}, \quad f_{\eta'}^s = 131\text{MeV}.
\]

(14)

For the \( u \) and \( d \) quarks involved, we follow [4] to use

\[
\langle \eta' | \bar{u} \gamma_5 u | 0 \rangle = \langle \eta' | d \gamma_5 d | 0 \rangle = r_{\eta'} \langle \eta' | \bar{s} \gamma_5 s | 0 \rangle,
\]

(15)

with \( r_{\eta'} \) being given by

\[
r_{\eta'} = \sqrt{2} \frac{f_0^2 - f_8^2}{f_8^2 f_0^2} \cos \theta + \frac{1}{\sqrt{2}} \sin \theta,
\]

\[
r_{\eta} = -\frac{1}{2} \frac{\sqrt{2} f_0^2 - f_8^2}{\sqrt{2} f_8^2 f_0^2} \cos \theta - \frac{1}{\sqrt{2}} \sin \theta.
\]

(16)

We next discuss the \( \bar{c}c \to \eta' \) mechanism. This new internal \( W \)-emission contribution will be important when the mixing angle involved is \( V_{cb} V_{cs}^* \), which is as large as that of the penguin amplitude and yet its effective parameter \( a_{\gamma \gamma}^{\text{eff}} \) is larger than that of penguin operators. The decay constant \( f_{\eta'}^{(\bar{c}c)} \), defined as \( \langle 0 | \bar{c} \gamma_\mu \gamma_5 c | \eta' \rangle = i f_{\eta'}^{(\bar{c}c)} q_{\mu} \), has been determined from the theoretical calculations [10, 11, 12] and from the phenomenological analysis of the data of \( J/\psi \to \eta_c \gamma \), \( J/\psi \to \eta' \gamma \) and of the \( \eta \gamma \) and \( \eta' \gamma \) transition form factors [8, 13]. In the presence of the charm content in the \( \eta_0 \), an additional mixing angle \( \theta_c \) is needed to be introduced:

\[
| \eta_0 \rangle = \frac{1}{\sqrt{3}} \cos \theta_c | u \bar{u} + d \bar{d} + s \bar{s} \rangle + \sin \theta_c | c \bar{c} \rangle,
\]

\[
| \eta_c \rangle = -\frac{1}{\sqrt{3}} \sin \theta_c | u \bar{u} + d \bar{d} + s \bar{s} \rangle + \cos \theta_c | c \bar{c} \rangle.
\]

(17)
Then \( f_{\eta}^c = \cos \theta \tan \theta_c f_{\eta c} \) and \( f_{\eta}^c = -\sin \theta \tan \theta_c f_{\eta c} \), where the decay constant \( f_{\eta c} \) can be extracted from \( \eta_c \rightarrow \gamma \gamma \), and \( \theta_c \) from \( J/\psi \rightarrow \eta_c \gamma \) and \( J/\psi \rightarrow \eta' \gamma \) \[2\]. In the present paper we shall use

\[
f_{\eta}^c = -6 \text{ MeV}, \quad f_{\eta}^c = -\tan \theta f_{\eta}^c = -2.4 \text{ MeV},
\]

for \( \theta = -22^\circ \), which are very close to the values

\[
f_{\eta}^c = -(6.3 \pm 0.6) \text{ MeV}, \quad f_{\eta}^c = -(2.4 \pm 0.2) \text{ MeV}
\]

obtained in \[13\].

In the following we will show the input parameters we used. One of the important parameters is the running quark mass which appears in the matrix elements of (\( S - P \))(\( S + P \)) penguin operators through the use of equations of motion. The running quark mass should be applied at the scale \( \mu \sim m_b \) because the energy release in the energetic two-body charmless decays of the \( B \) meson is of order \( m_b \). In this paper, we use \[27\]

\[
m_u(m_b) = 3.2 \text{ MeV}, \quad m_d(m_b) = 6.4 \text{ MeV}, \quad m_s(m_b) = 105 \text{ MeV},
\]

\[
m_c(m_b) = 0.95 \text{ GeV}, \quad m_b(m_b) = 4.34 \text{ GeV},
\]

in ensuing calculation, where we have applied \( m_s = 150 \text{ MeV} \) at \( \mu = 1 \text{ GeV} \).

It is convenient to parametrize the quark mixing matrix in terms of the Wolfenstein parameters: \( A, \rho, \rho' \) and \( \eta \), where \( A = 0.81 \) and \( \lambda = 0.22 \) \[28\]. A recent analysis of all available experimental constraints imposed on the Wolfenstein parameters yields \[29\]

\[
\rho = 0.156 \pm 0.090, \quad \eta = 0.328 \pm 0.054,
\]

where \( \bar{\rho} = \rho(1 - \frac{\lambda^2}{2}) \) and \( \bar{\eta} = \eta(1 - \frac{\lambda^2}{2}) \), and it implies that the negative \( \rho \) region is excluded at 93% C.L.. In this paper, we employ the representative values: \( \rho = 0.16 \) and \( \eta = 0.34 \), which satisfies the constraint \( \sqrt{\rho^2 + \eta^2} = 0.37 \).

Under the factorization approach, the decay amplitudes are expressed as the products of the decay constants and the form factors. We use the standard parametrization for decay constants and form factors \[23\]. For values of the decay constants, we use \( f_{\pi} = 132 \text{ MeV} \), \( f_K = 160 \text{ MeV} \), \( f_{\rho} = 210 \text{ MeV} \), \( f_{K^*} = 221 \text{ MeV} \), \( f_\omega = 195 \text{ MeV} \) and \( f_\phi = 237 \text{ MeV} \). Concerning the heavy-to-light mesonic form factors, we will use the results evaluated in the relativistic quark model \[23\], \[30\] by directly calculating \( B_{s(u,d)} \rightarrow P \) and \( B_{s(u,d)} \rightarrow V \) form factors at time-like momentum transfer. Denoting \( \eta_s = \bar{s}s \), the explicit values for the form factors involved are \( F_{1.0}^{B_s \eta} (0) = 0.48 \), \( F_{(0,1)}^{B_s \eta} (0) = 0.44 \), and \( A_0^{B_s K^*} (0) = 0.28 \), which are larger than BSW model’s results \[10\]. The \( q^2 \) dependence of the matrix element, parametrized under the pole dominance ansatz, are found to have a dipole behaviour for \( A_0 \), \( F_1 \), and a monopole one for \( F_0 \). In the following, we will use the exact value calculated at the relevant kinematical point in this paper. Note that these matrix elements should be used with a correct normalization \[14\], for which to a good approximation, we take \( F_{1.0}^{B_s \eta} (0) = \frac{1}{\sqrt{3}} F_{1.0}^{B_s \eta} (0) \) and \( F_{1.0}^{B_s \eta'} (0) = \frac{2}{\sqrt{6}} F_{1.0}^{B_s \eta'} (0) \).
3. Phenomenology

We are now ready to discuss the phenomenology of exclusive charmless rare $B_s$ decays. To illustrate the issue of $N_c^{\text{eff}}$ dependence (which means the different factorization approach) of theoretical predictions, we will begin with $B_s \to (\pi, \rho, \omega)\eta^{(')}$. Unlike the case of $B \to (\pi, \rho)\eta^{(')}$, $\bar{B}_s \to (\pi, \rho, \omega)\eta^{(')}$ do not receive the anomaly contribution from the $(S - P)(S + P)$ penguin operators due to the particle content of $\bar{B}_s$ and $\pi(\rho, \omega)$.

The decay amplitude for $\bar{B}_s^0 \to \pi\eta^{(')}$ reads

$$A(\bar{B}_s \to \eta^{(')}\pi) = \frac{G_F}{\sqrt{2}} \left\{ V_{u_b} V_{a_2}^* - V_{u_b} V_{a_2}^* \left[ \frac{3}{2} (-a_7 + a_9) \right] \right\} X^{(\bar{B}_s, \eta^{(')}\pi)}, \quad (22)$$

where

$$X^{(\bar{B}_s, \eta^{(')}\pi)} \equiv \langle \pi^0 | (\bar{u} u)_{V-A} | 0 \rangle \langle \eta^{(')} | (\bar{s} b)_{V-A} | \bar{B}_s \rangle = -i \frac{f_\pi}{\sqrt{2}} (m_{\bar{B}_s}^2 - m_{\eta^{(')}}^2) F_0^{B_s\eta^{(')}}(m_{\pi}^2). \quad (23)$$

Since the internal $W$-emission is CKM-suppressed and the QCD penguins are canceled out in these decay modes, $\bar{B}_s \to \pi(\rho)\eta^{(')}$ are dominated by the EW penguin diagram. The dominant EW penguin contribution proportional to $a_9$ is $N_c^{\text{eff}}$-stable, whereas the internal $W$ contribution $a_2$ is $N_c^{\text{eff}}$-sensitive. Within the “heterogeneous” nonfactorizable picture, $a_2$ is fixed and thus the predicted branching ratio is rather stable under the variation of $N_c^{\text{eff}}$ as shown in the last four columns in Table II. However, $a_2$ varies within the “homogeneous” nonfactorizable scheme and thus the predicted branching ratios do show a $N_c^{\text{eff}}$ dependence. We would like to emphasize that although $\bar{B}_s \to (\pi, \rho)\eta^{(')}$ are dominated by the EW penguin diagram, the internal $W$ diagram does make some contributions to this decay mode. Since $a_2$ changes sign from $N_c^{\text{eff}} = 2, 3$ to $N_c^{\text{eff}} = 5, \infty$, the interference pattern between the internal $W$ diagram and the EW penguin diagram will change from the destructive to the constructive one. It is thus easy to see that for $N_c^{\text{eff}} = 2$ there is a larger destructive interference between the internal $W$ diagram and the EW penguin contribution and the predicted branching ratio is the smallest one among the first four columns in Table II, whereas for $N_c^{\text{eff}} = \infty$ constructive interference takes the role and the branching ratio increases.

While QCD penguin diagrams are canceled out in $\bar{B}_s \to (\pi, \rho)\eta^{(')}$, $\bar{B}_s \to \omega\eta^{(')}$ gets enhanced from the QCD penguin diagram. The decay amplitude for $\bar{B}_s \to \omega\eta^{(')}$ is

$$A(\bar{B}_s \to \omega\eta^{(')}) = \frac{G_F}{\sqrt{2}} \left\{ V_{u_b} V_{a_2}^* - V_{u_b} V_{a_2}^* \left[ 2(a_3 + a_5) + \frac{1}{2} (a_7 + a_9) \right] \right\} X^{(\bar{B}_s, \omega\eta^{(')}\omega)}, \quad (24)$$

where

$$X^{(\bar{B}_s, \omega\eta^{(')}\omega)} \equiv \langle \omega | (\bar{u} u)_{V-A} | 0 \rangle \langle \eta^{(')} | (\bar{s} b)_{V-A} | \bar{B}_s \rangle = \sqrt{2} f_\omega m_{\omega} F_1^{B_s\eta^{(')}}(m_{\omega}^2)(\varepsilon \cdot p_{B_s}). \quad (25)$$

From Table II, we see that there is a wide range of predictions for the branching ratios. This process is QCD penguin dominated, except when the “naive” factorization is used or $(N_c^{\text{eff}}(V - A), N_c^{\text{eff}}(V + A)) = (2, 5)$ where there are large cancellations between the QCD penguin contributions (i.e., $a_3 + a_5$). The largest branching ratio predicted for $B_s \to \omega\eta^{(')}$ occurs when we use the “large-$N_c$ improved” factorization, where the EW penguin and QCD penguin have constructive interference.
Next, we discuss $B_s^0 \to \eta' K^0$ decay, which has the decay amplitude

$$A(B_s^0 \to K^0 \eta') = \frac{G_F}{\sqrt{2}} \left\{ V_{ub} V_{ud}^* a_2 X_u^{(B_s,K^0 \eta')} + V_{cb} V_{cd}^* a_2 X_c^{(B_s,K^0 \eta')} \right\}$$

$$+ \left( a_3 - a_5 - a_7 + a_9 \right) X_u^{(B_s, K^0 \eta')} + \left( a_3 - a_5 + \frac{1}{2}a_7 - \frac{1}{2}a_9 \right) X_c^{(B_s, K^0 \eta')}$$

$$+ \left( a_3 - a_5 - a_7 + a_9 \right) X_c^{(B_s, K^0 \eta')}$$

$$+ \left( a_3 + a_4 - a_5 + \frac{1}{2}a_7 - \frac{1}{2}a_9 - \frac{1}{2}a_{10} \right)$$

$$+ \left( a_6 - \frac{1}{2}a_8 \right) m_s^2 (m_b - m_s) \left( \frac{f_{\eta'}^s}{f_{\eta'}^u} - 1 \right) r_{\eta'}^{(B_s, K^0 \eta')} \right\}, \quad (26)$$

where

$$X^{(B_s, K^0 \eta')} = \langle K^0 | (\bar{s}d)_{V-A} | 0 \rangle \langle \eta' | (\bar{d}b)_{V-A} | B_s^0 \rangle = -i f_K (m_{B_s}^2 - m_{\eta'}^2) F_0^{B_s, \eta'} (m_{K^0}^2),$$

$$X_{q}^{(B_s, K^0 \eta')} = \langle \eta' | (\bar{q}q)_{V-A} | 0 \rangle \langle K^0 | (\bar{s}b)_{V-A} | B_s^0 \rangle = -i f_{\eta'}^q (m_{B_s}^2 - m_{K^0}^2) F_0^{B_s, K}(m_{\eta'}^2). \quad (27)$$

Due to the QCD anomaly, there is an extra ($f_{\eta'}^s/f_{\eta'}^u - 1$) term multiplied with $a_6$ in $X_{d}^{(B_s, K^0 \eta')}$, whose presence is necessary in order to be consistent with the chiral-limit behaviour of the (S-P)/(S+P) penguin matrix elements [4]. Though penguin diagrams play the dominant role, the internal $W$ diagram and the mechanism of the $c\bar{c}$ pair into the $\eta'$ do have some nonnegligible effects when $N_{c}^{\text{eff}} = 2$ and $N_{c}^{\text{eff}} = \infty$ where $a_2$ gets the larger values. Due to the large cancellation, the EW penguin has only tiny effect and can be neglected. The monotonic decrease of the branching ratio from $N_{c}^{\text{eff}} = \infty$ to $N_{c}^{\text{eff}} = 2$ within the “homogeneous” nonfactorizable picture can be understood from the behaviour of the QCD penguin i.e. the destructive interference between $a_{(3,5)}$ and $a_{(4,6)}$: as $N_{c}^{\text{eff}}$ decreases, $a_{(3,5)}$ contributions increase and hence the branching ratios decrease. As we already mentioned before, $a_3$ and $a_5$ are $N_{c}^{\text{eff}}$-sensitive while $a_3 - a_5$ is stable under the variation of $N_{c}^{\text{eff}}$ and then the predicted branching ratio is $N_{c}^{\text{eff}}$-stable within the “homogeneous” factorization approach.

There exist some general rules for the derivation of the formula from $B \to P_a P_b$ to its corresponding $B \to V_a P_b$ and $B \to P_a V_b$. These general rules can be written as: (i) For $X^{(BP_a P_b)}$ to $X^{(BV_a P_b)}$, replace the term $m_{P_b}^2/[(m_1 + m_2)(m_3 - m_4)]$ by $-m_{P_b}^2/[(m_1 + m_2)(m_3 + m_4)]$ and the index $P_a$ by $V_a$, (ii) discard the $(S-P)(S+P)$ contribution associated with $X^{(BP_a P_b)}$ and $a_{(5,7)} \to -a_{(5,7)}$ if they contribute. Thus, the factorizable amplitude of $\bar{B}_s \to \eta' K^*$ can be readily obtained from the $\bar{B}_s \to \eta' K$ one and reads

$$A(\bar{B}_s \to \eta' K^{*0}) = \frac{G_F}{\sqrt{2}} \left\{ V_{ub} V_{ud}^* \left( a_2 X_u^{(B_s, K^{*0} \eta')} \right) + V_{cb} V_{cd}^* a_2 X_c^{(B_s, K^{*0} \eta')} \right\}$$

$$- \left( a_3 - a_5 - a_7 + a_9 \right) X_u^{(B_s, K^{*0} \eta')} + \left( a_3 + a_4 - a_5 + \frac{1}{2}a_7 - \frac{1}{2}a_9 - \frac{1}{2}a_{10} \right)$$

$$- \left( a_6 - \frac{1}{2}a_8 \right) m_s^2 (m_b - m_s) \left( \frac{f_{\eta'}^s}{f_{\eta'}^u} - 1 \right) r_{\eta'}^{(B_s, K^{*0} \eta')} \right\}, \quad (28)$$

8
with

\[
X^{(B_s \eta', K*)} = \langle K^*| (\bar{s}u)_{V-A}|0\rangle \langle \eta'| (\bar{u}b)_{V-A}|\bar{B}_s\rangle = 2f_{K*}m_{K*}F_{1}^{B_s \eta'} (m_{K*}^2) (\varepsilon \cdot p_{B_s}),
\]

\[
X^{(B_s \eta, \eta')}_q = \langle \eta'| (\bar{q}q)_{V-A}|0\rangle \langle K^*| (\bar{s}b)_{V-A}|\bar{B}_s\rangle = 2f_{\eta'}^q m_{K*} A_0^{B_s K*} (m_{\eta'}^2) (\varepsilon \cdot p_{B_s}).
\]

(29)

It is interesting to see that since there is no \(a_6\) term in \(X^{(B_s \eta', \eta')}\), the penguin contribution is reduced substantially and so does the branching ratio. With a reduced penguin contribution, the involved internal \(W\) diagram and the mechanism of the \(c\bar{c}\) pair into the \(\eta'\) become more important than those of the \(\bar{B}_s \to K\eta'\). The larger branching ratios in columns denoted by \(I_a\) and \(II_b\) are due to the constructive interference between the internal \(W\) diagram and the penguin contribution.

We are now coming to the most complicated process \(\bar{B}_s \to \eta\eta'\), which has the decay amplitude

\[
A(\bar{B}_s \to \eta\eta') = \frac{G_F}{\sqrt{2}} \left\{ V_{ub}V_{us}^* \left[ a_2X_{u}^{(B_s \eta, \eta')} + a_2X_{u}^{(B_s \eta', \eta)} \right] + V_{cb}V_{cs}^* \left[ a_2X_{c}^{(B_s \eta, \eta')} + a_2X_{c}^{(B_s \eta', \eta)} \right] \right. 
\]

\[
- V_{ub}V_{ts}^* \left[ \left( a_3 + a_4 - a_5 + \frac{1}{2}a_7 - \frac{1}{2}a_9 - \frac{1}{2}a_{10} \right) X_{s}^{(B_s \eta, \eta')} + \frac{(a_6 - \frac{1}{2}a_8)}{m_s (m_b - m_s)} \left( 1 - \frac{f_{\eta}^u}{f_{\eta}^s} \right) X_{s}^{(B_s \eta, \eta')} \right] 
\]

\[
+ \left( 2a_3 - 2a_5 + \frac{1}{2}a_7 + \frac{1}{2}a_9 \right) X_{u}^{(B_s \eta, \eta')} + (a_3 - a_5 - a_7 + a_9) X_{c}^{(B_s \eta, \eta')} 
\]

\[
+ \left( a_3 + a_4 - a_5 + \frac{1}{2}a_7 - \frac{1}{2}a_9 - \frac{1}{2}a_{10} \right) X_{s}^{(B_s \eta', \eta)} + \frac{(a_6 - \frac{1}{2}a_8)}{m_s (m_b - m_s)} \left( 1 - \frac{f_{\eta}^c}{f_{\eta}^s} \right) X_{s}^{(B_s \eta', \eta)} 
\]

\[
+ \left( 2a_3 - 2a_5 + \frac{1}{2}a_7 + \frac{1}{2}a_9 \right) X_{u}^{(B_s \eta', \eta')} + (a_3 - a_5 - a_7 + a_9) X_{c}^{(B_s \eta', \eta')} \right\},
\]

(30)

with

\[
X_{q}^{(B_s \eta, \eta')} = \langle \eta'| (\bar{q}q)_{V-A}|0\rangle \langle \eta| (\bar{s}b)_{V-A}|_{\bar{B}_s}\rangle = -i f_{\eta'}^q (m_{B_s}^2 - m_{\eta}^2) F_0^{B_s \eta} (m_{\eta'}^2),
\]

\[
X_{q}^{(B_s \eta', \eta)} = \langle \eta| (\bar{q}q)_{V-A}|0\rangle \langle \eta'| (\bar{s}b)_{V-A}|_{\bar{B}_s}\rangle = -i f_{\eta}^q (m_{B_s}^2 - m_{\eta}'^2) F_0^{B_s \eta'} (m_{\eta}^2).
\]

The destructive interference between \(X_{q=\{u,c\}}^{(B_s \eta, \eta')}\) and \(X_{q=\{u,c\}}^{(B_s \eta', \eta)}\) makes the internal \(W\)-emission, \(c\bar{c} \to \eta(\prime)\) and the corresponding penguin contributions smaller. The EW penguin is smaller than the QCD penguin by an order of magnitude at the amplitude level and hence can be neglected. The dominant QCD penguin contributions are governed by \(X_{s}^{(B_s \eta, \eta')}\) and \(X_{s}^{(B_s \eta', \eta)}\), which have the constructive interference. The \((a_3 - a_5)\) term is positive, contrary to the
negative $a_4$ and $a_6$ terms, and becomes smaller when $N_c^{\text{eff}}$ increases within the “homogeneous” nonfactorizable picture. Thus a monotonic increase of the branching ratio when $N_c^{\text{eff}}$ increases comes mainly from this reduced destructive interference, within the “homogeneous” nonfactorizable structure. Similar arguments are also applied to the “heterogeneous” structure.

With the general rules (i) and (ii) mentioned before, the decay amplitude for $B_s \to \phi \eta'$ can be easily obtained from $B_s \to \eta \eta'$:

$$A(B_s \to \phi \eta') = \frac{G_F}{\sqrt{2}} \left\{ V_{ub} V_{us}^* a_2 X_u^{(B_s \phi \eta')} + V_{cb} V_{cs}^* a_2 X_c^{(B_s \phi \eta')} \right. \right.
- \left. \left. V_{tb} V_{ts}^* \left[ \left( a_3 + a_4 - a_5 + \frac{1}{2} a_7 - \frac{1}{2} a_9 - \frac{1}{2} a_{10} \right) - \left( a_6 - \frac{1}{2} a_8 \right) \frac{m_\eta^2}{m_s (m_b + m_s)} \left( 1 - \frac{f_\eta^u}{f_\eta^s} \right) \right] X_s^{(B_s \phi \eta')} \right.
+ \left. \left( 2 a_3 - 2 a_5 - \frac{1}{2} a_7 + \frac{1}{2} a_9 \right) X_u^{(B_s \phi \eta')} + \left( a_3 - a_5 - a_7 + a_9 \right) X_c^{(B_s \phi \eta')} \right. \right.
+ \left. \left. \left( a_3 + a_4 + a_5 - \frac{1}{2} a_7 - \frac{1}{2} a_9 - \frac{1}{2} a_{10} \right) X_s^{(B_s \phi \eta', \phi)} \right\}, \quad (31)$$

with

$$X_q^{(B_s \phi \eta', \phi)} \equiv \langle \eta' | (\bar{q} q)_{\nu-A} | 0 \rangle \langle \phi | (\bar{s} b)_{\nu-A} | B_s \rangle = 2 f^{s q}_{\eta'} m_\phi A_0^{B_s \phi} (m_\eta^2) (\varepsilon \cdot p_{B_s}),$$

$$X_q^{(B_s \eta', \phi)} \equiv \langle \phi | (\bar{q} q)_{\nu-A} | 0 \rangle \langle \eta' | (\bar{s} b)_{\nu-A} | B_s \rangle = 2 f_{\phi} m_\phi F_1^{B_s \eta'} (m_\phi^2) (\varepsilon \cdot p_{B_s}).$$

While the internal $W$ diagram is subject to the CKM-suppression, the $c\bar{c} \to \eta'$ mechanism suffers from the suppression in the decay constant and thus $B_s \to \phi \eta'$ is dominated by the penguin contribution. Due to the cancellation among the different $a_i (i = 3, 4, 5, 6)$’s, effect of the QCD penguin, though still dominant, are reduced substantially. Within the “homogeneous” nonfactorizable picture, we find a monotonic decrease of the branching ratios when $N_c^{\text{eff}}$ increases, which comes from a monotonic decrease of the QCD penguin contributions as $N_c^{\text{eff}}$ increases. Since the QCD penguin contributions are reduced, the EW penguin contributions become important. It is found that the interference pattern between the QCD and EW penguin is destructive except for the “large-$N_c$ improved” factorization approach, where a constructive interference exists and a very dramatically suppressed QCD penguin contribution appears. The strength of the destructive interference depends on $N_c^{\text{eff}}$, and its effect is to reduce the QCD penguin contribution without changing the trend of reduced penguin when $N_c^{\text{eff}}$ increases. A dramatic destructive interference among the penguin contributions occurs for $B_s \to \phi \eta'$ when $N_c^{\text{eff}} (V - A) = N_c^{\text{eff}} (V + A) = \infty$ and $(N_c^{\text{eff}} (V - A), N_c^{\text{eff}} (V + A)) = (2, \infty)$, thus the internal $W$ diagram and the $c\bar{c} \to \eta'$ mechanism contribution becomes important relative to the other cases and the branching ratio is the smallest in these situations.

4. Summary and Discussions  To summarize, we have studied charmless exclusive non-leptonic $B_s$ meson decay into an $\eta$ or $\eta'$ within the generalized factorization approach. Non-factorizable contributions are parametrized in terms of the effective number of colors $N_c^{\text{eff}}$
Table II. Average branching ratios (in units of $10^{-6}$) for charmless $B_s$ decays to $\eta'$ and $\eta$. Predictions are for $k^2 = m_b^2/2$, $\eta = 0.34$, $\rho = 0.16$. I denotes the “homogeneous” nonfactorizable contributions i.e. $N_c^{\text{eff}}(V - A) = N_c^{\text{eff}}(V + A)$ and (a,b,c,d) represent the cases for $N_c^{\text{eff}}=(\infty,5,3,2)$. II denotes the “heterogeneous” nonfactorizable contributions, i.e. $N_c^{\text{eff}}(V - A) \neq N_c^{\text{eff}}(V + A)$ and (a',b',c') represent the cases for $N_c^{\text{eff}}(V + A)=(3,5,\infty)$, where we have fixed $N_c^{\text{eff}}(V - A)=2$ (see the text).

| Decay         | $I_a$ | $I_b$ | $I_c$ | $I_d$ | II_{0'} | II_{0''} | II_{0'''} |
|---------------|-------|-------|-------|-------|---------|----------|-----------|
| $B_s \to \pi \eta'$ | 0.25  | 0.17  | 0.13  | 0.11  | 0.11    | 0.11     | 0.10      |
| $B_s \to \pi \eta$   | 0.16  | 0.11  | 0.08  | 0.07  | 0.07    | 0.068    | 0.067     |
| $B_s \to \rho \eta'$ | 0.70  | 0.47  | 0.36  | 0.30  | 0.30    | 0.30     | 0.31      |
| $B_s \to \rho \eta$   | 0.45  | 0.30  | 0.24  | 0.19  | 0.19    | 0.19     | 0.20      |
| $B_s \to \omega \eta'$ | 6.9   | 0.9   | 0.012 | 2.14  | 0.48    | 0.03     | 0.83      |
| $B_s \to \omega \eta$   | 4.45  | 0.63  | 0.008 | 1.39  | 0.31    | 0.02     | 0.54      |
| $B_s \to \eta'K^0$   | 1.25  | 1.07  | 1.01  | 1.00  | 1.27    | 1.51     | 1.90      |
| $B_s \to \eta K^0$   | 1.35  | 0.81  | 0.68  | 0.76  | 0.75    | 0.74     | 0.72      |
| $B_s \to \eta' K^*0$ | 0.49  | 0.35  | 0.32  | 0.26  | 0.49    | 0.60     | 0.80      |
| $B_s \to \eta K^*0$  | 0.45  | 0.05  | 0.02  | 0.24  | 0.24    | 0.24     | 0.25      |
| $B_s \to \eta' \eta$ | 47.4  | 41.8  | 38.3  | 34.4  | 39.5    | 44.1     | 51.5      |
| $B_s \to \eta' \eta'$ | 26.6  | 24.9  | 23.8  | 22.4  | 33.8    | 43.9     | 62.2      |
| $B_s \to \eta \eta$   | 20.3  | 17.1  | 15.1  | 12.8  | 11.6    | 10.7     | 9.1       |
| $B_s \to \phi \eta'$ | 0.44  | 0.59  | 2.29  | 6.20  | 4.41    | 3.11     | 1.66      |
| $B_s \to \phi \eta$   | 0.04  | 0.02  | 2.29  | 4.92  | 2.28    | 0.92     | 0.10      |

and predictions using different factorization approaches are shown with the $N_c^{\text{eff}}$ dependence. It is found that for processes depending on the $N_c^{\text{eff}}$-stable $a_i$’s such as $B_s \to (\pi, \rho)\eta^0$, the branching ratios are not sensitive to the factorization approach we used. While for the processes depending on the $N_c^{\text{eff}}$-sensitive $a_i$’s such as the $\bar{B}_s \to \omega \eta^0$, the predicted branching ratios have a wide range depending on the choice of the factorization approach. The effect of the QCD anomaly, which is not discussed in the earlier literature, is found to be important for the $\bar{B}_s \to \eta^0(\rho^0)\eta^0$. We also found that the mechanism $(c\bar{c}) \to \eta'$, in general, has smaller effects due to a possible CKM-suppression and the suppression in the decay constants except for the $\bar{B}_s \to \phi \eta$ under the “large-$N_c$ improved” factorization approach, where the internal $W$ diagram is CKM-suppressed and the penguin contributions are compensated.

In this Letter, we, following the standard approach, have neglected the $W$-exchange and the space-like penguin contributions. Another major source of uncertainties comes from the form factors we used, which are larger than the BSW model’s calculations. Although the Wolfenstein parameter $\rho$ ranges from the negative region to the positive one, we have “fixed” it to some representative values. The interference pattern between the internal $W$ diagram and the penguin contributions will change when we take a different sign of $\rho$. We will study these form factor- and CKM- dependence involved and all the $B_s \to PP, VP, VV$ decay modes in a separate publication.

ACKNOWLEDGMENT: We are very grateful to Prof. H.Y. Cheng for helpful discussions. This work is supported in part by the National Science Council of the Republic of China.
under Grant NSC87-2112-M006-018.

References

[1] For a recent review, see K. Lingel, T. Skwarnicki, and J. G. Smith, hep-ex/9804015.

[2] For earlier studies, see L.L. Chau, H.Y. Cheng, W.K. Sze, H. Yao, and B. Tseng, Phys. Rev. D43, 2176 (1991); ibid D45, 3143 (1992); G. Kramer, W.F. Palmer, and H. Simma, Z. Phys. C66, 429 (1995); Nucl. Phys. B428, 77 (1994); J.-M. Gérard and W.S. Hou, Phys. Rev. Lett. 62, 855 (1989); Phys. Rev. D43, 2909 (1991).

[3] D. Atwood and A. Soni, Phys. Lett. B405, 150 (1997); Phys. Rev. Lett. 79, 5206 (1997).

[4] W.S. Hou and B. Tseng, Phys. Rev. Lett. 80, 434 (1998).

[5] H. Fritzsch, Phys. Lett. B415, 83 (1997).

[6] A.S. Dighe, M. Gronau, and J.L. Rosner, Phys. Rev. Lett. 79, 4333 (1997); I. Halperin and A. Zhitnitsky, Phys. Rev. Lett. 80, 438 (1998); F. Yuan and K.T. Chao, Phys. Rev. D56, 2495 (1998); M.R. Ahmady, E. Kou, and A. Sugamoto, hep-ph/9710509; D.S. Du, C.S. Kim, and Y.D. Yang, Phys. Lett. B426, 133 (1998) and hep-ph/9805451.

[7] D.S. Du and M.Z Yang, Phys. Rev. D57, R5332 (1998).

[8] A.L Kagan and A.A. Petrov, hep-ph/9707354.

[9] A. Ali, J. Chay, C. Greub, and P. Ko, Phys. Lett. B424, 161 (1998); A. Ali and C. Greub, Phys. Rev. D57, 2996 (1998).

[10] I. Halperin and A. Zhitnitsky, Phys. Rev. D56, 7247 (1997).

[11] E.V. Shuryak and A.R. Zhitnitsky, Phys. Rev. D57, 2001 (1998).

[12] F. Araki, M. Musakhanov, and H. Toki, hep-ph/9803356.

[13] T. Feldmann and P. Kroll, hep-ph/9711231, to appear in Eur. Phys. J. (1998); T. Feldmann, P. Kroll, and B. Stech, hep-ph/9802409.

[14] H.Y. Cheng and B. Tseng, hep-ph/9803457; H.Y. Cheng, talk presented at the First APCTP Workshop on Pacific Particle Physics Phenomenology, Seoul, Oct. 31-Nov. 2, 1997 [hep-ph/9712244].

[15] A. Ali, G. Kramer and C.-D. Lü, hep-ph/9804363.

[16] D.S. Du and Z.Z. Xing, Phys. Rev. D48, 3400 (1993).

[17] A. Deandrea, N. Di Bartolomeo, R. Gatto, F. Feruglio, and G. Nardulli, Phys. Lett. B320, 170 (1994).
[18] R. Fleischer, Phys. Lett. B332, 419 (1994); N.G. Deshpande and X.G. He, Phys. Lett. B336, 471 (1994); N.G. Deshpande, X.G. He, and Trampetic, Phys. Lett. B345, 847 (1994); D.S. Du and M.Z. Yang, Phys. Lett. B358, 123 (1995).

[19] A.J. Buras, M. Jamin, M.E. Lautenbacher, and P.H. Weisz, Nucl. Phys. B370, 69 (1992); A.J. Buras, M. Jamin, and M.E. Lautenbacher, Nucl. Phys. B408, 209 (1993).

[20] M. Ciuchini, E. Franco, G. Martinelli, L. Reina, and L. Silvestrini, Z. Phys. C68, 255 (1995).

[21] G. Buchalla, A.J. Buras, and M.E. Lautenbacher, Rev. Mod. Phys. 68, 1125 (1996).

[22] H.Y. Cheng, Int. J. Mod. Phys. A4, 495 (1989); Phys. Lett. B335, 428 (1994); Phys. Lett. B395, 345 (1997).

[23] M. Wirbel, B. Stech, and M. Bauer, Z. Phys. C29, 637 (1985).

[24] A.J. Buras, J.-M. Gérard, and R. Rückl, Nucl. Phys. B268, 16 (1986).

[25] M. Neubert and B. Stech, hep-ph/9705292.

[26] H.Y. Cheng and B. Tseng, Phys. Rev. D51, 6295 (1995).

[27] H. Fusaoku and Y. Koide, Phys. Rev. D57, 3986 (1998).

[28] Particle Data Group, Phys. Rev. D54, 1 (1996).

[29] F. Parodi, P. Roudeau, and A. Stocchi, hep-ph/9802289.

[30] H.Y. Cheng, C.Y. Cheung, and C.W. Hwang, Phys. Rev. D55, 1559 (1997). We are very grateful to Dr. C.W. Hwang for providing the light-front form factors relevant to the present paper.

[31] P. Ball, J.-M. Frère, and M. Tytgat, Phys. Lett. B365, 367 (1996); R. Akhoury and J.-M. Frère, Phys. Lett. B220, 258 (1989).

[32] E.P. Venugopal and B.R. Holstein, Phys. Rev. D57, 4397 (1998).