Kurano, Kazuhiko

Equations of negative curves of blow-ups of Ehrhart rings of rational convex polygons.
(English) Zbl 1484.14029
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Summary: Finite generation of the symbolic Rees ring of a space monomial prime ideal of a 3-dimensional weighted polynomial ring is a very interesting problem. Negative curves play important roles in finite generation of these rings. We are interested in the structure of the negative curve. We shall prove that negative curves are rational in many cases. We also see that the Cox ring of the blow-up of a toric variety at the point $(1, 1, \ldots, 1)$ coincides with the extended symbolic Rees ring of an ideal of a polynomial ring. For example, Roberts’ second counterexample to Cowsik’s question (and Hilbert’s 14th problem) coincides with the Cox ring of some normal projective variety (Remark 2.7).

MSC:
14E08 Rationality questions in algebraic geometry
13A30 Associated graded rings of ideals (Rees ring, form ring), analytic spread and related topics
14E30 Minimal model program (Mori theory, extremal rays)
14M25 Toric varieties, Newton polyhedra, Okounkov bodies
13E05 Commutative Noetherian rings and modules
13E15 Commutative rings and modules of finite generation or presentation; number of generators

Keywords:
Cox ring; symbolic Rees ring; finite generation; Ehrhart ring; toric variety

Full Text: DOI arXiv

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