Flow of colloidal suspension and irreversibility analysis with aggregation kinematics of nanoparticles in a microchannel

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Abstract  The current exploration focuses on the ethylene glycol (EG) based nanoliquid flow in a microchannel. The effectiveness of the internal heat source and linear radiation is reflected in the present investigation. The estimation of suitable thermal conductivity model has affirmative impact on the convective heat transfer phenomenon. The examination is conceded with the nanoparticle aggregation demonstrated by the Maxwell-Bruggeman and Krieger-Dougherty models which tackle the formation of nanolayer. These models effectively describe the thermal conductivity and viscosity correspondingly. The dimensionless mathematical expressions are solved numerically by the Runge Kutta Fehlberg approach. A higher thermal field is attained for the Bruggeman model due to the formation of thermal bridge. A second law analysis is carried out to predict the sources of irreversibility associated with the thermal system. It is remarked that lesser entropy generation is obtained for the aggregation model. The entropy generation rate declines with the slip flow and the thermal heat flux. A notable enhancement in the Bejan number is attained by increasing the Biot number. It is established that the nanoparticle aggregation model exhibits a higher Bejan number in comparison with the usual flow model.

Key words  nanoparticle aggregation, nanoliquid, microchannel, slip parameter, heat flux

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Nomenclature

\begin{align*}
f, & \quad \text{axial velocity (m·s}^{-1}) ; \\
y, & \quad \text{transversal coordinate (m)} ; \\
F(Y), & \quad \text{dimensionless velocity} ; \\
Y, & \quad \text{dimensionless transverse coordinate} ; \\
T, & \quad \text{temperature of fluid (K)} ; \\
T_a, & \quad \text{ambient temperature (K)} ; \\
T_h, & \quad \text{hot fluid temperature (K)} ; \\
\theta(Y), & \quad \text{dimensionless temperature} ; \\
k^*, & \quad \text{mean absorption coefficient (m}^{-1}) ; \\
s_1, s_2, & \quad \text{slip lengths} ; \\
D, & \quad \text{fractal index} ; \\
k, & \quad \text{thermal conductivity (W·m}^{-1}·\text{K}^{-1}) ; \\
c_p, & \quad \text{specific heat at constant pressure} \quad (\text{J·kg}^{-1}·\text{K}^{-1}) ;
\end{align*}

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Re = \frac{\frac{\rho f}{\nu f} \mu_f}{\nu_f} \rho f \mu_f, \text{ Reynolds number;}

Bi_i = \frac{a_{th_i}}{k_f} i = 1, 2, \text{ Biot numbers;}

Pr = \frac{\nu_f}{\alpha_f} \nu_f \alpha_f, \text{ Prandtl number;}

\rho, \text{ modified pressure;}

P = -\frac{a_{th}}{\nu_f^2 \mu_f^2} \frac{\rho f}{\nu_f}, \text{ pressure gradient parameter;}

Nr = \frac{16\sigma^* T^4}{3k_f k_i}, \text{ radiation parameter;}

Ec = \frac{v_0^2}{a_{th}} \frac{\rho_f \alpha_f}{(T_h - T_a)}, \text{ Eckert number;}

\beta, \text{ thermal expansion coefficient (K}^{-1});

\phi_m, \text{ extreme volume fraction;}

\mu, \text{ dynamic viscosity (kg} \cdot \text{m}^{-1} \cdot \text{s}^{-1});

\rho, \text{ fluid density (kg} \cdot \text{m}^{-3});

\sigma^*, \text{ Stefan-Boltzmann constant (W} \cdot \text{m}^{-2} \cdot \text{K}^{-4});

\phi_a, \text{ effective volume fraction of aggregates;}

v_0, \text{ uniform suction/injection velocity;}

\phi, \text{ solid volume fraction of nanoparticles.}

\[ S_1, S_2, \text{ dimensionless slip parameters;}

[n], \text{ Einstein coefficient;}

h_1, h_2, \text{ convective heat transfer coefficients;}

r_a, r_p, \text{ radii of aggregates and nanoparticle, respectively.}

Subscripts

nf, \text{ nanofluid;}

s, \text{ solid particle;}

f, \text{ fluid.}

1 \text{ Introduction}

Conservation of energy in manufacturing effective thermal equipments has become a challenge to researchers and scientists over the decades. Construction of such systems has certain limitations such as maintaining a lower temperature of the surface, a reduced size of the thermal system, and certain economic enticements. Hence, the reinforcement of rate of heat transfer plays a decisive role which enables the researchers to design the thermal system with enhanced heat transfer and lower measurement of the heat exchanger.

Inclusion of nano-sized metallic particles into the conventional fluid enables to get the thermal efficient coolant which in turn strengthens the augmentation of convective heat transfer. Various works have been done on this concern. The previous studies have concluded that even at a low concentration of nanoparticles, the heat transfer rate is higher for the nanofluid than that for the base fluid. Colloidal suspension of ethylene glycol (EG) containing graphene nanoparticle was studied experimentally by Ijam et al.\cite{1}, and the correlation for electrical conductivity of nanoliquid was obtained. Later, Sandhya et al.\cite{2} depicted the suspension of TiO$_2$ in EG. Cabaleiro et al.\cite{3} validated the heat transfer capacity of nanofluid experimentally. A practical study on colloidal suspension of silicon dioxide was carried out by Zyla and Fal\cite{4} at different nanoparticle volume fractions. It was obtained that an increase in the nanoparticle concentration results in a better heat transfer phenomenon. Several works on heat transfer of nanofluid in distinct thermal systems can be found in Refs.\cite{5}–\cite{8}.

The value of thermal conductivity according to a typical fluid model has huger variation than the result computed experimentally. The reason for this difference is due to neglecting the effect of aggregation kinematics of nanoparticles. The fractal model which describes the interfacial structures obtained by the interaction of fluid molecules and nanoparticles\cite{9–10} illuminates the consequences of aggregation of nanoparticles on the thermal conductivity ratio. The authors demonstrated the fractal dimension for nanoparticle cluster. They remarked that thermal conductivity mainly depended on the process of nanoparticle interaction and cluster formation. The role of aggregation kinetics of nanoparticles on thermal conductivity of base fluid was reconnoitered experimentally by Prasher\cite{11}. It was found that the thermal conductivity of nanofluid relied on colloidal chemistry. Chen et al.\cite{12} reported the characteristics of EG for the inclusion of titania particles. They used the mechanism of aggregation of nanoparticles to
forecast the thermal conductivity. The impact of fractal dimension on nanoparticle aggregation was characterized by Zhou and Keller\cite{13} using ZnO. They demonstrated the influence of pH on the aggregation phenomenon. An experimental investigation on multi-walled carbon nanotubes (CNTs) was done by Halelfadl et al.\cite{14} considering the viscosity and distinct volume fraction. It was found that at a high shear rate, the viscosity of nanofluid does not fluctuate with the temperature. Features of thermal conductivity and specific heat of nanoliquid with the aid of aggregation of nanoparticles were studied by Sedighi and Mohebbi\cite{15}. They noted that during the nanoparticle aggregation, the addition of nanoparticles results in a lower specific heat, while the diffusivity and thermal conductivity enhance. The inspection on nanolubricants of ZnO was carried out by Heris et al.\cite{16}. They revealed the formation of nanoparticle aggregation in the base fluid using Krieger and Dougherty correlation. A study on thermal conductivity of nanoliquid at a lower nanoparticle volume fraction was conceded by Motevasel et al.\cite{17} with the role of nanoparticle aggregation for distinct nanofluids experimentally. Investigation of water based CNTs under the impact of nanoparticle aggregation in a cavity was shown in Ref.\cite{18}. Consequence of nanoparticle aggregation on the convective flow of nanoliquid was explored by Mackolil and Mahanthesh\cite{19} by an available contemporary model Krieger-Dougherty. They showed that the flow velocity declines with aggregation of nanoparticles.

Moreover, the entropy generation analysis has a vibrant part in engineering heat transfer. Scrutinization of entropy relies on irreversibility associated with the thermal system. Every heat transfer equipment and thermal system, such as the heat exchanger and heat sink, are accompanying with the irreversibility. It is essential to determine the source of irreversibility and to minimize it to some extent. Makinde and Eegunjobi\cite{20} scrutinized the generation of entropy in a vertical microchannel due to the nanofluid flow. They obtained that the Grashof number strengthens the entropy rate. Mahdavi et al.\cite{21} reported that the Darcy number enhances the rate of entropy generation due to the fluid flow in a pipe. Later, this work was extended by using the nanofluid flow in a microchannel which has been perceived in Refs.\cite{22} and \cite{23} with the effect of thermal radiation, slip, and convective boundary. They demonstrated that minimization of entropy can be achieved via a higher solid volume fraction of nanoparticle and suction-injection Reynolds number. Torabi et al.\cite{24} demonstrated the nanofluid flow in porous media considering Al$_2$O$_3$/water. They noted that the Bejan number increased with the nanoparticle concentration. Liakopoulos et al.\cite{25} and Miao et al.\cite{26} explored the nanoscale liquid transport phenomenon in a channel. They found that in the pressure-driven flow, when the solubility is low, the dominant dissipation is the viscous dissipation. Their results helped to control the flow patterns.

Modeling of the thermal characteristics which describes the physical situation has always been a challenge to the researchers and inventors. By employing the distinct flow model, Ibanez et al.\cite{27} studied the entropy generation in a microchannel driven by the nanofluid. The consequences of the effective parameter on nanofluid were found\cite{28-29}. The role of viscous dissipation and thermal heat flux was studied for distinct nanoparticles\cite{30}. They remarked that increasing the volume fraction of nanoparticles has an advanced effect on the thermal efficiency of the system. The magnetohydrodynamic (MHD) stream of nanoliquid over L-shaped ribs was investigated by Toghraie et al.\cite{31}. The contemporary models\cite{32-33} which described the efficient thermal conductivity ratio were given to analyze the nanofluid flow in a microchannel.

Inclusion of nanoparticles to enhance the efficiency of micro systems such as micro pumps, microchannels, and micro heat exchangers has significant impact on the heat transfer phenomenon. In the present work, we envision to explore the consequences of considering the Maxwell-Bruggeman and Krieger-Dougherty models which include aggregation kinematics of nanoparticles. Physical effects of nanofluid flow in a microchannel are investigated for both cases, i.e., (i) without nanoparticle aggregation and (ii) with nanoparticle aggregation (the Fractal model).

The EG based titania nanofluid is considered in the current study. The effects of the volume
fraction of nanoparticle, the thermal radiation parameter, the heat source, the Eckert number, and the Biot number on the flow and thermal features are validated and depicted through graphical figures. The solutions are obtained by means of numerical computation.

The main objectives of the present work are as follows.

(i) Interpret the flow and thermal field characteristics in the presence and absence of nanoparticle aggregation kinematics.

(ii) Scrutinize the Maxwell-Bruggeman and Krieger-Dougherty models for TiO$_2$/EG nanoliquid.

(iii) Examine the source of irreversibility associated with the heat transfer phenomenon.

(iv) Find out the physical factors which contribute towards the entropy generation.

(v) Find out the possible way of minimizing entropy generation via controlling the flow parameters.

2 Mathematical modeling of the problem

A laminar steady flow of an incompressible nanoliquid in a microchannel is considered in the present investigation. The horizontal microchannel consists of two parallel plates parted by a distance $a$, as shown in Fig. 1. The following assumptions are made to obtain the mathematical model.

(i) The microchannel is composed of two horizontal plates of an infinite length.

(ii) The flow of nanoliquid is induced due to the pressure gradient.

(iii) The phenomenon of suction/injection occurs at the upper and lower horizontal plates of the microchannel, respectively.

(iv) Thermal and hydrodynamical fully developed circumstances (see Fig. 1) are presumed.

(v) Due to the flow of hot liquid of temperature $T_h$, the lower plate of the microchannel gets heated.

(vi) The upper plate exchanges heat with the ambient surrounding of temperature $T_a$.

(vii) Since the roughness of the surface is distinct for each plate, the slip regime is considered.

(viii) The combined effects of the viscous dissipation, the heat source, and the linear thermal heat flux are considered to obtain the energy balance equation.

According to the above flow assumptions, the corresponding flow field, the thermal field, and the corresponding boundary conditions are \cite{28}

\begin{equation}
\rho_{nf} \frac{df}{dy} = -\frac{dp}{dx} + \mu_{nf} \frac{d^2f}{dy^2},
\end{equation}

\begin{equation}
(p_c)_{nf} \frac{dT}{dy} = \frac{d}{dy} \left( k_{nf} \frac{dT}{dy} - q_f \right) + \mu_{nf} \left( \frac{df}{dy} \right)^2 + Q_T^* (T - T_a),
\end{equation}

Fig. 1 Pictographic representation of the physical phenomenon (color online)
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\[ f - s_1 \frac{df}{dy} = 0 \quad \text{at} \quad y = 0, \quad (3) \]

\[ f + s_2 \frac{df}{dy} = 0 \quad \text{at} \quad y = a, \quad (4) \]

\[ k_{nf} \frac{dT}{dy} - h_1(T - T_h) \quad \text{at} \quad y = 0, \quad (5) \]

\[ k_{nf} \frac{dT}{dy} + h_2(T - T_a) \quad \text{at} \quad y = a. \quad (6) \]

Here, \( f \) is the axial velocity, \( c_p \) is the specific heat at constant pressure, \( \rho \) is the density, \( \mu \) is the dynamic viscosity, \( \sigma \) is the electrical conductivity, \( p \) is the modified pressure, \( T \) is the temperature of the nanoliquid, \( T_a \) is the ambient temperature, \( T_h \) is the hot fluid temperature, \( Q_T^* \) is the heat source coefficient, \( s_1 \) and \( s_2 \) are the slip lengths, and \( h_1 \) and \( h_2 \) are the convective heat transfer coefficients.

The radiative heat flux can be given as \(^{[28]}\) (the Rosseland diffusion approximation)

\[ q_r = -\frac{16\sigma^*}{3k^*} T_a^2 \frac{dT}{dy}. \quad (7) \]

The subsequent nondimensional entities are used to make the governing momentum and temperature equation as a dimensionless one as follows:

\[ F = \frac{\rho_a f}{\mu_t}, \quad Y = \frac{y}{a}, \quad S_1 = \frac{s_1}{a}, \quad S_2 = \frac{s_2}{a}, \quad \theta = \frac{T - T_a}{T_h - T_a}. \quad (8) \]

Equations (1)–(6) become

\[ \frac{\rho_{nf} Re}{\rho_t} \frac{dF}{dY} = P + \frac{\mu_{nf}}{\mu_t} \frac{d^2F}{dY^2}, \quad (9) \]

\[ \frac{(\rho c_p)_{nf}}{(\rho c_p)_t} Re Pr \frac{d\theta}{dY} = \left( \frac{k_{nf}}{k_t} + Nr \right) \frac{d^2\theta}{dY^2} + Ec Pr \left( \frac{\mu_{nf}}{\mu_t} \left( \frac{dF}{dY} \right)^2 \right) + H\theta, \quad (10) \]

\[ F - S_1 \frac{dF}{dY} = 0 \quad \text{at} \quad Y = 0, \quad (11) \]

\[ F + S_2 \frac{dF}{dY} = 0 \quad \text{at} \quad Y = 1, \quad (12) \]

\[ \frac{k_{nf}}{k_t} \frac{d\theta}{dY} - Bi(\theta - 1) = 0 \quad \text{at} \quad Y = 0, \quad (13) \]

\[ \frac{k_{nf}}{k_t} \frac{d\theta}{dY} + Bi\theta = 0 \quad \text{at} \quad Y = 1. \quad (14) \]

### 2.1 The conventional model without aggregation \(^{[17,19]}\) (Case 1)

\[ \frac{\rho_{nf}}{\rho_t} = (1 - \phi) + \phi \frac{\rho_s}{\rho_t}, \quad (15) \]

\[ \frac{(\rho c_p)_{nf}}{(\rho c_p)_t} = (1 - \phi) + \phi \frac{(\rho c_p)_s}{(\rho c_p)_t}, \quad (16) \]

\[ \frac{\mu_{nf}}{\mu_t} = \left( 1 - \phi \right)^{2.5}, \quad (17) \]

\[ \frac{k_{nf}}{k_t} = \frac{k_s + 2k_t + 2\phi(k_s - k_t)}{k_s + 2k_t - \phi(k_s - k_t)}. \quad (18) \]
2.2 The conventional model with aggregation\textsuperscript{[17,19]} (fractal model, Case 2)

The fluid models such as the Brinkman model, the Einstein model, and the Maxwell model depend on the nanoparticle volume fraction only. The normalized shear viscosity of the nanofluid does not depend on the temperature. Such characteristics are elucidated by the aggregation model. In addition to this, elevation in the thermal conductivity of nanoliquids is higher in case of experimental results than that obtained by employing common fluid models. The reason for this variation is neglecting the impact of aggregation of nanoparticles (see Refs. [9], [10], [17], and [19]). Hence, it is essential to explore the ratio of thermal conductivity by considering the aggregation kinematics of nanoparticles. The fractal model is the appropriate model to describe the thermal conductivity with the aid of nanoparticle aggregation. This model remarks that the thermal conductivity mainly relies on the interaction of nanoparticles and clustering phenomenon. We consider the Krieger-Dougherty model to describe the viscosity of the nanofluid with aggregation and Maxwell-Bruggeman to obtain the effective thermal conductivity.

\[ \mu_{nf} = \left( 1 - \frac{\phi_a}{\phi_m} \right)^{[n] \phi_m}. \]  \hspace{1cm} (19)

Here, \( \phi_m \) is the extreme volume fraction, \([n]\) is the Einstein coefficient, and \( \phi_a \) is the effective volume fraction of aggregates, where

\[ \phi_a = \phi \left( \frac{r_a}{r_p} \right)^{3-D}, \]  \hspace{1cm} (20)

in which \( D \) is the fractal index.

By utilizing the above entities, we obtain

\[ \frac{\mu_{nf}}{\mu_f} = \left( 1 - \frac{\phi}{\phi_m} \left( \frac{r_a}{r_p} \right)^{3-D} \right)^{[n] \phi_m}. \]  \hspace{1cm} (21)

The widely accepted values of \( D = 1.8, \frac{r_a}{r_p} = 3.34, \) and \( \phi_m = 0.605 \) represent high rate flows. For the monodisperse system, \([n] = 2.5 \) (see Ref. [19]).

![Pictorial representation of nanocluster, nanolayer, and the base fluid medium](image)

**Fig. 2** Pictorial representation of nanocluster, nanolayer, and the base fluid medium (color online)

The fluid molecules close to the nanoparticle surface form a layered structure which behaves like a solid and has a magnitude of nanometer. The formation of nanolayer between particles and aggregates of fluids augments the thermal conductivity. The structural model of nanofluid
is shown in Fig. 2. It contains a solid nanoparticle cluster, bulk fluid, and a nanolayer which covers the clustering of nanoparticles. Thus, the thermal conductivity according to the fractal model constitutes the effect of cluster radius, nanolayer thickness, and thermal resistance of the interfacial layer between the particle surface and the fluid aggregates. 

Nanoparticle aggregation is incorporated by Bruggeman to deduce the thermal conductivity, and is given by

\[
\frac{k_{nf}}{k_f} = \frac{(k_a + 2k_f) - 2\phi_a(k_f - k_a)}{(k_a + 2k_f) + \phi_a(k_f - k_a)}
\]  

(22)

where \(k_a\) is the thermal aggregation conductivity, and its value is obtained by the Bruggeman model as

\[
\frac{k_a}{k_f} = \frac{1}{4} \left( (3\phi_m - 1) \frac{k_a}{k_f} + (3(1 - \phi_m) - 1) \right) + \left( \left( (3\phi_m - 1) \frac{k_a}{k_f} + (3(1 - \phi_m) - 1) \right)^2 + 8 \left( \frac{k_a}{k_f} \right)^2 \right)^{\frac{1}{2}},
\]

(23)

\[
\phi_m = \left( \frac{r_a}{r_p} \right)^{D=3}.
\]

(24)

2.3 Entropy analysis

The generation of entropy comprises the existing irreversibility in the physical phenomenon. In this investigation, the major contribution towards irreversibility is the viscous dissipation and thermal radiation. The expression for entropy generation is

\[
S' = \frac{1}{T_a} \left( k_{nf} + \frac{16\sigma^* T_a^3}{3k^*} \right) \left( \frac{dT}{dy} \right)^2 + \frac{\mu_{nf}}{T_n} \left( \frac{dF}{dy} \right)^2.
\]

(25)

The nondimensional entropy generation equation is

\[
E_G = \left( \frac{k_{nf}}{k_f} + Nr \right) \left( \frac{d\theta}{dy} \right)^2 + EcPrw \left( \frac{\mu_{nf}}{\mu_f} \left( \frac{dF}{dy} \right)^2 \right),
\]

(26)

where \(w = \frac{T_a}{T_h - T_a}\) is the characteristic ratio of temperature. \(I_{HT} = \left( \frac{k_{nf}}{k_f} + Nr \right) \left( \frac{d\theta}{dy} \right)^2\) is the heat transfer irreversibility, and \(I_D = EcPrw \left( \frac{\mu_{nf}}{\mu_f} \left( \frac{dF}{dy} \right)^2 \right)\) is the dissipative irreversibility.

The Bejan number \(Be\) gives the share of heat transmission and fluid friction in net generation of entropy and is represented as follows:

\[
Be = \frac{I_{HT}}{I_{HT} + I_D}.
\]

(27)

3 Numerical method

The nonlinear differential equations are solved by employing the Runge Kutta Fehlberg method united with the shooting technique. During the mathematical simulation, the step size is set to be 0.001 in order to acquire the result. The criterion of convergence is \(10^{-6}\). The symbolic software MAPLE is used to obtain the numerical solution. The subsequent procedure is visualized through Fig. 3.

4 Results and analysis

The physical effects of the distinct flow parameters such as the Biot number \(Bi\), the Eckert number \(Ec\), the nanoparticle volume fraction \(\phi\), the thermal radiation parameter \(Nr\), the heat generation parameter \(H\), and the slip flow parameter \(S\) on the dimensionless flow field \(F(Y)\), the temperature field \(\theta(Y)\), the generation of entropy \(E_G\), and the Bejan number \(Be\) are exhibited
through graphical illustration. Here, we consider the colloidal suspension of TiO$_2$ nanoparticles in the EG fluid. Table 1 shows the thermophysical characteristics of nanoparticle and base fluid. The flow constraints are maintained as $Bi_1 = Bi_2 = Bi = 1$, $Ec = 0.5$, $Re = 1$, $Pr = 6.8$, $P = 1$, $Nr = 0.5$, $H = 0.5$, $\phi = 0.04$, and $S_1 = S_2 = S = 0.05$ during calculation of solution. The verification of the employed numerical results with the analytical solution for the limiting case is displayed in Table 2. The obtained results are in good agreement.

**Table 1** Thermophysical characteristics of the conventional fluid and nanoparticle$^{[19]}$

| Physical property       | EG     | TiO$_2$ |
|-------------------------|--------|---------|
| $\rho$/kg·m$^{-3}$      | 1 114  | 4 250  |
| $c_p$/J·kg$^{-1}$K$^{-1}$ | 2 415  | 686.2  |
| $k$/W·m$^{-1}$K$^{-1}$  | 0.252  | 8.953 8 |

**Table 2** Validations of numerical results with the analytical solution for the limiting case, when $Re = 1.0$ and $\phi = P = 0$ along with the boundary conditions $F(0) = 0$ and $F(1) = 1$

| Y          | $F(Y)$      | Y          | $F(Y)$      |
|------------|-------------|------------|-------------|
| 0.0        | 0.000 000 000 | 0.0        | 0.000 000 000 |
| 0.2        | 0.128 851 248 | 0.2        | 0.128 851 252 |
| 0.4        | 0.286 230 517 | 0.4        | 0.286 230 530 |
| 0.6        | 0.478 453 992 | 0.6        | 0.478 454 902 |
| 0.8        | 0.713 236 273 | 0.8        | 0.713 236 277 |
| 1.0        | 1.000 000 000 | 1.0        | 1.000 000 000 |

Figures 4 and 5 define the impact of the pressure gradient parameter $P$ and the slip flow parameter $S$ on the velocity of the nanofluid with and without nanoparticle aggregation. It is visualised from the figures that the flow velocity increases with $P$ and $S$. The slip regime enhances the fluid velocity. This is because when the slip parameter increases, the friction force (an opposing force to the nanofluid motion) at the walls of the microchannel diminishes, which favours the flow of nanofluid in a microchannel. Hence, $F(Y)$ increases with $S$. 

**Fig. 3** Flow diagram of the numerical procedure
The physical impacts of the nanoparticle volume fraction $\phi$, the Eckert number $Ec$, the heat generation parameter $H$, the thermal radiation parameter $Nr$, and the Biot number $Bi$ on the thermal profile $\theta(Y)$ are depicted through Figs. 6–10 for both cases. The consequences of $\phi$ on $\theta(Y)$ are revealed in Fig. 6. By increasing the volume fraction $\phi$ from 1% to 5%, the temperature increases due to the enhanced thermal conductivity. Because of the development of cluster by nanoparticles, the fluid near the vicinity of the nanoparticles develops a nanolayer which acts as a thermal bridge between the particle and aggregates of fluids. The thickness of nanolayer plays a vital role in enhancing the thermal conductivity. This nanolayer has a substantial role in heat transportation from solid to adjacent fluid. The dimensionless thermal field for various $Ec$ (as 0.5, 1.0, and 1.5) is represented in Fig. 7. We observe an increase in the thermal field $\theta(Y)$ for a larger Eckert number. This is due to the augmentation of viscous dissipation for uplifting values of $Ec$. Figure 8 displays the physical impact of the heat generation on the thermal profile. It is remarked that for a larger $H$, the temperature increases due to the internal heating. It is noted from Fig. 9 that the temperature field declines with the variation of the radiation parameter $Nr$ as 0.5, 1.0, and 1.5. This is because by increasing $Nr$, heat transfer from the lower microchannel plate to the ambience increases through the radiative heat flux at the upper plate. As a result, the temperature of the nanofluid declines in a microchannel. Hence, the thermal profile decays. Figure 10 displays the impact of the Biot number on $\theta(Y)$. The thermal field shows a decrease in the trend with a higher $Bi$. Physically, the augmentation of $Bi$ results in an increase in the convective heat transport from ethylene based titania nanoliquid to the surrounding.
The consequences of flow constraints $S$, $Ec$, $Nr$, and $Bi$ on $E_G$ and $Be$ are revealed in Figs. 11–16. Scrutiny of entropy represents the irreversibility associated with the system.

Figure 11 shows the deviation in $E_G$ for distinct $S$ as 0.01, 0.03, 0.05. It is noted that the dimensionless entropy generation declines for the slip parameter. Physically, as the slip parameter increases, the fluid friction at the plates of the microchannel decreases which causes a reduction in the fluid friction irreversibility. The strengthening of entropy for distinct $Ec$ is shown in Fig. 12. This is because, when $Ec$ upsurges, the temperature of the nanofluid intensifies due to viscous heating of nanoliquid which in turn uplifts the heat transfer irreversibility. Figure 13 depicts the impact of the radiation parameter on $E_G$. It is visualized that as the radiation parameter increases, the entropy generation decays. This is due to a lower heat transmission irreversibility. It is illustrated that a lower entropy generation is attained for the nanoparticle aggregation model.

Figures 14–16 reveal the consequences of the flow parameters on $Be$. The impact of viscous heating $Ec$ on the Bejan number is presented in Fig. 14. It is visible that the Bejan number declines with the Eckert number. Intensification of the Bejan number with $Nr$ and $Bi$ is depicted through Figs. 15 and 16, respectively, in a microchannel. It is noted that the Bejan number upsurges for uplifting values of the radiation parameter and the Biot number. From these graphs, it is established that the Bejan number is larger for the aggregation model.
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5 Concluding remarks

The flow of EG based titania nanofluid through the horizontal microchannel is scrutinized. The normalized shear viscosity of the nano mixtures with the corresponding conventional fluid is independent of the temperature. We consider the aggregation kinematics model to analyze the nanoliquid flow in a microchannel. The significance of distinct parameters on the velocity profile, the thermal profile, the entropy generation, and the Bejan number is deliberated.

The key upshots of the existing research exertion are listed below.
(i) The slip flow regime contributes a higher flow velocity of the nanoliquid in a microchannel.
(ii) The thermal field of nano mixtures increases with the enhanced concentration of nanoparticles.
(iii) The phenomenon of radiation and convection results in a lower temperature profile due to the increased heat transfer.
(iv) It is emphasized that a higher temperature profile is observed for the aggregation model compared with usual models.
(v) The minimum entropy generation can be achieved through the increased slip length.
(vi) Due to the viscous heating, the entropy generation in a microchannel increases for both models.
(vii) A reduction in the entropy generation is achieved for the nanoparticle aggregation model.
(viii) The Bejan number is intensified for the radiation parameter and the Biot number.
(ix) It is emphasized that the nanoparticle aggregation model demonstrates a higher Bejan number.

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