"Gradient marker" -- a universal wave pattern in inhomogeneous continuum

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Outline:

- Wave patterns -- a beauty and a science
- Riding over a slow gradient with no retro-reflection
- Wave equation with inhomogeneous parameters & a gradient marker hump
- The lineage of a gradient marker
- Quantum well: GM and resonant scattering
- Feasible applications: from medical tomography to sub-marine detection

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Wave patterns -- not just a toy...

They show up in *inhomogeneous media* or *confining structures*

in quantum mechanics, optics and electrodynamics,
acoustics, hydrodynamics, and chemistry.

**Examples:**

- wave packets in atoms,
- Gladny patterns in acoustics,
- EM resonator and waveguide modes,
- Anderson localization in disordered systems,
- soliton formation due to nonlinearity,
  including atomic solitons in bosonic gas,
- Belousov-Zhabotinskii waves in chemical reactions,
- dark-soliton grids,
- "scars" in "quantum billiard",
- "quantum carpets" in QM potentials,
- nano-stratification of local field in finite lattices, etc.

A pre-requisite for *interference* and pattern formation: *multi-modes* or a broad-bend spectrum.
FIG. 1: QM carpet development in time after $\delta$ kicks of varying intensity in a soft potential, $U \sim x^4$

FIG. 2: QM carpet development in time after a double $\pm \delta$ kick in a soft potential, $U \sim x^4$
Local-field patterns in 2D lattices
(nonuniform local field, uniform incident field)

2D triangular lattice:

- simultaneous 3rd order resonances in x and y directions
- vortices formation

The simplest “magic” shape in 2D

nano-sample geometry: six-point star, \( N = 13 \)

local-field distribution

zero field
Wave equation in an inhomogeneous medium

\[ \frac{d^2 E}{dx^2} + n^2(\xi)E = 0; \quad \xi = k_0x; \quad k_0 = \frac{\omega}{c} = \frac{2\pi}{\lambda_0} \]

Typical wave configurations in various fields:

Conversion from optics/electrodynamics to quantum mechanics:

\[ E \rightarrow \psi; \quad H \rightarrow -i \frac{d\psi}{d\xi} = \frac{p_Q}{\hbar k_0} \quad (p_Q -- QM \text{ momentum}) \]

\[ n \rightarrow \sqrt{2m[E_0 - U(x)]} = \frac{p_C}{\hbar k_0} \quad (E_0 -- \text{energy}; \quad p_C -- \text{classical momentum}) \]
Gradient marker -- a "patterned" traveling wave in an adiabatically slow-inhomogeneous continuum

gradient parameter \( \mu \sim \frac{1}{k_0 L \ln_{\text{min}}} \ll 1 \); \( L \) -- inhomogeneity scale
vanishing reflectivity: \( R = e^{-A/\mu} \ll 1 \)

A solution of interest -- strictly \textit{traveling} wave;
slightly perturbed WKB approximation:

\[
E = \frac{1 + \Delta(\xi)}{\sqrt{n(\xi)}} e^{-i \int n d\xi}
\]
an analytical solution for \( \mu \ll 1 \): \( \Delta = \gamma + i \beta \)

\[
\beta' = -\left(\frac{n'/n^{3/2}}{4\sqrt{n}}\right); \quad \gamma = -\left(\frac{\beta^2 + \beta'/n}{2}\right)
\]
with full intensity: \( I \equiv n|E|^2 = (1 + \gamma)^2 + \beta^2 \)

Gradient marker:

\[
\delta I(\xi) = I(\xi) - 1 = \frac{1}{4n^{3/2}} \left(\frac{n'}{n^{3/2}}\right); \quad \left( X' \equiv \frac{dX}{d\xi} \right)
\]

A model for numerics: \( n(x) = n_1 + \frac{n_2 - n_1}{2} \left[ 1 + \tanh \left( \frac{2x}{L} \right) \right] \)

gradient parameter: \( \mu = \frac{1}{k_0 L} \left( \frac{1}{n_1} + \frac{1}{n_2} \right); \quad \mu_{cr} = \frac{1}{2\pi} \)
FIG. 1: (a) Refractive index, \( n \) (and potential \( U \)) soft-step spatial profiles, \( n_1 = 1.5 \) and \( n_2 = 3 \), \( \mu_{cr} \approx 0.24 \); (b-d) G-marker intensity, \( \delta I \), vs distance \( x/2L \) for various parameters \( \mu \); curves: \( \delta I_N \) - numerical, and \( \delta I_A \) - analytical.
A physics behind a G-marker: how it is formed?

In lieu of explanation: a visual evolution/morphing of a breaker wave into a G-marker:

A G-marker is a "remnant", or "far relative" of a breaker wave
Gradient markers in a quantum well

A model for symmetrical quantum well with controllable slopes:

\[
n(x) = n_1 + \left( \frac{n_2 - n_1}{2} \right) \frac{\tanh[(2x + D)/L] - \tanh[(2x - D)/L]}{\tanh(D/L)}
\]
Full-transmission resonances in continuum

\[ \frac{\mu}{\mu_{cr}} > 1 \quad (= 2); \quad \frac{E_0}{U_0} = \frac{4}{3} \]

\[ \mu = \frac{n_{\text{min}} - 1 + n_{\text{max}} - 1}{Lk_0}; \quad \frac{\mu_{cr}}{2\pi} \sim \frac{1}{2\pi} \]

There is a wave reflection between G-markers, BUT no reflection from the entire potential well.

This is similar to full transparency resonances in a Fabry – Perot resonator with semi-transparent mirrors, the same as in a continuum over a finite width box of the size D.

\[ E_N = \frac{(N^2\hbar^2\pi)^2}{2mD^2} \]
Potential applications/uses

- the tomography of quantum landscape in disordered solid-state at above-critical temperature
- observation of quantum "traces" in continuum, i.e. beyond "quantum carpets" in potential wells,
- detection and control of slight changes of optical fiber parameters
- a bulk tomography of opaque fluids (e.g. oil or muddy water) by using nonpenetrating surface EM or acoustic waves,
- or of solid-state bodies (e.g. in "introvision" of computer chips, or lacunas in blobs of metallic alloys or glass),
- the diagnostics of cold under-dense plasma,
- medical surface wave ultrasound tomography,
- detection of the movement of near-shelf profiles of the bottom of oceans and rivers by space- or air-borne photography of the patterns of wind-driven gravitation waves, as well as
- contour-detection and tracing of submerged large moving man-made objects or whales in the ocean.

Future research: 2D and 3D expansion of the theory for potential plasma or astrophysics applications
(1) Tomography by surface waves -- in medicine and geophysics

(2) "I see you!" -- contour-detecting submerged objects in the ocean from a satellite
Conclusions

• Found a universal wave pattern in an inhomogeneous media: gradient marker located near max/min of a gradient of refractive index or of potential function in a continuum.

• Found a critical condition for a G-marker to be resolved on the background of residual reflection.

• In a trapping potential, also found resonant states in the continuum above photo-ionization and formulated a condition for them to exist.