Price and capacity competition in balancing markets with energy storage

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Abstract

Energy storage can absorb variability from the rising number of wind and solar power producers. Storage is different from the conventional generators that have traditionally balanced supply and demand on fast time scales due to its hard energy capacity constraints, dynamic coupling, and low marginal costs. These differences are leading system operators to propose new mechanisms for enabling storage to participate in reserve and real-time energy markets. The persistence of market power and gaming in electricity markets suggests that these changes will expose new vulnerabilities.

We develop a new model of strategic behavior among storages in energy balancing markets. Our model is a two-stage game that generalizes a classic model of capacity followed by Bertrand-Edgeworth price competition by explicitly modeling storage dynamics and uncertainty in the pricing stage. By applying the model to balancing markets with storage, we are able to com-

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pare capacity and energy-based pricing schemes, and to analyze the dynamic effects of the market horizon and energy losses due to leakage. Our first key finding is that capacity pricing leads to higher prices and higher capacity commitments, and that energy pricing leads to lower, randomized prices and lower capacity commitments. Second, we find that a longer market horizon and higher physical efficiencies lead to lower prices by inducing the storage to compete to have their states of charge cycled more frequently.

Keywords: Energy storage, game theory, electric power systems, regulation, renewable energy

1. Introduction

High penetrations of variable renewable energy resources, such as wind and solar, call for additional regulation and load following capabilities to manage increased forecast errors [1, 2]. Energy storage and demand response are widely considered promising solutions to renewable variability [3, 4, 5]. To facilitate increased procurement and utilization of these resources, Independent System Operators (ISOs) have begun to propose new mechanisms enabling storage to participate in reserve and real-time energy markets, for example in California [6], Texas [7], and New York [8]. In this paper, we refer to reserve and real-time energy markets as balancing markets.

A variety of storage technologies are now available to power systems, such as pumped hydro, batteries, and flywheels [9]. Load aggregations that actively participate in power system operations through load shifting programs can also be approximated as virtual energy storage [10, 11]. We will henceforth refer to both physical energy storage and active load aggregations simply as storage. All types of storage share the following first-order characteristics:

- hard energy capacity constraints,
- dynamic coupling of energy states, and
- low marginal costs.

The hard energy capacity constraints arise from the physical limitations of each technology, e.g. reservoir size and flywheel material strength. These constraints couple the range of possible energy exchanges in one time period
to the amount injected or extracted in the preceding periods. Since storage does not use fuel, its costs are limited to operation, maintenance, and initial procurement, resulting in low marginal cost \([12, 13, 14]\). These features distinguish storage from conventional generators. We discuss our modeling of storage in detail in Section 2.

The increasing role and distinct characteristics of storage are changing the physical and economic nature of power systems. As electricity markets are modified to accommodate storage, new vulnerabilities to market power and gaming will surface. Indeed, the persistence of events like the California Electricity Crisis \([15]\) and the more recent market manipulation by J. P. Morgan \([16]\) indicates that system operators must always be alert for strategic behavior. The main contributions of this paper are a new model for strategic behavior among storage that captures the three aforementioned characteristics, and concrete design insights obtained from its application to balancing markets.

Our proposed model is based on capacity followed by price competition, a two-stage game in which Bertrand-Edgeworth price competition is preceded by a capacity setting stage \([17, 18]\). Bertrand-Edgeworth competition \([19, 20, 21]\) is a special case of supply function competition \([22]\), which has been widely applied to electricity markets because it captures generators’ nonlinear fuel curves \([23, 24, 25, 26]\). Bertrand-Edgeworth competition is similarly well-suited to modeling storage because it captures its hard capacity constraint and low marginal costs. In Section 3.1, we develop a novel generalization of Bertrand-Edgeworth competition, which can accommodate the dynamic coupling of storage’s energy states. The resulting generalized Bertrand-Edgeworth competition is still far more analytically tractable than general supply function competition in that it admits a complete characterization of the pricing equilibrium. The equilibrium characterization enables us to analyze the scenario where storage first bids a capacity that then parametrizes the ensuing pricing game. Our model assumes firms are paid the price they bid rather than all firms seeing the same price, which is also referred to as a discriminatory price auction. The merits of discriminatory versus uniform price auctions in power markets have been analyzed extensively, but no consensus on which format is superior has emerged \([27, 28, 29]\).

In Section 4, we apply our results on price and capacity competition to balancing markets. We first use capacity competition to compare markets in which firms are paid for capacity (as in traditional reserve markets) and for energy (as in traditional real-time energy markets). Our results show
that, similar to what has been observed in other types of markets [30, 31], firms commit more capacity and set higher prices under capacity pricing and commit less capacity and set lower, random prices under energy pricing. We then use price competition to analyze the dynamic effects of market horizon and losses due to energy leakage. We find that if the market horizon is large and leakage small enough, firms will compete to be cycled more frequently by reducing their prices, hence improving economic competitiveness. In Section 5 we state our design insights for energy balancing markets with storage and identify some future research directions.

2. Storage in balancing markets

In this section, we describe the model of storage we use and the role of storage in balancing markets.

2.1. Physical modeling

We employ the following generic storage model (see, e.g., [32]). Let $S_t^i$ be firm $i$’s stored energy at time $t$, also known as the state of charge, and let $X_t^i$ be the energy added or removed from the storage at time $t$. Between time periods, a fraction of the state of charge, $1 - \alpha_i \in [0, 1]$, is dissipated, which we refer to as leakage. The state of charge of storage $i$ then evolves according to the difference equation

$$S_{t+1}^i = \alpha_i S_t^i + X_t^i.$$ 

The state of charge must remain within the hard capacity limit $0 \leq S_t^i \leq S_i$, where $S_i$ is the maximum amount of energy that can be stored. Here we have heuristically let leakage act as a proxy for these inefficiencies because all have the effect of dissipating energy and hence reducing the state of charge.

We also neglect constraints on $X_t^i$ (analogous to a generator’s power constraint) and the derivative of $X_t^i$ (analogous to a generator’s ramp constraint) to maintain the tractability of our model. Ignoring power constraints is valid for storage technologies with high power and low energy capacities like flywheels and superconducting magnetic energy storage [9], which are most appropriate for the balancing markets we consider. Ignoring ramp constraints is justified because storage is generally able to ramp quickly. For comparison, coal, nuclear, and steam turbines can change their power output at
1–5% per minute, and gas turbines and diesel generators can ramp at 5–40% per minute [33]. In contrast, a pumped hydro storage facility in Wales, UK can move from zero to full output in less than 16 seconds [34], and newer technologies like grid-level batteries and flywheels can respond even faster [9].

In our balancing market model, an imbalance $B^t$ is realized in each time period and apportioned over a group of storage. The amount that can be allocated to a particular storage is limited by its state of charge if $B^t < 0$ and its unused capacity, $S_i - S^t_i$, if $B^t > 0$. A participating storage may be paid for both the capacity it commits to the balancing market, $S_i$, and for the imbalances it absorbs, $X^t_i$. In the latter case, storage is paid a positive amount for canceling imbalances regardless of whether $B^t$ is positive or negative.

Because storage does not have a fuel source, its only costs are associated with operation, maintenance, and initial procurement. Over long time periods, these are small relative to the fuel costs incurred by generators [12, 13, 14]. For this reason and for concision, we model storage as having zero marginal costs associated with energy injections and extractions. Consequently, all prices higher than what would be necessary to recoup opportunity costs in our subsequent model are the result of gaming, which may be interpreted as undesirable additions ‘superimposed’ on top of marginal cost prices from a non-strategic model. In this regard, our approach is intended to isolate strategic behaviors, and does not account for other intended revenue streams of storage.

Storage can provide a handful of services besides balancing such as load shifting, contingency reserves, and voltage regulation [35, 36]. Here, we only consider its participation in balancing markets, and model the foregone profits from providing other services as the opportunity cost $-\gamma_i S_i$, where $\gamma_i \geq 0$ is a known parameter and $S_i$ is the capacity committed to the balancing market. Because storage cannot sell power in baseload markets, these opportunity costs may be substantially lower than those of generators participating in reserve markets [37, 38, 39].

### 2.2. Market modeling

As discussed in Section 1, balancing markets match supply with demand on faster time scales than baseload markets. For example, most ISOs operate real-time energy markets that balance supply and demand every five or fifteen minutes, and also operate frequency regulation markets that procure capacity to balance supply and demand on timescales of seconds. If
the associated baseload market is well designed, the imbalances in balancing markets should average to zero over time. Moreover, ISOs have proposed mechanisms to ensure imbalances average to zero over specific time intervals to enable participation of energy constrained resources like storage [6, 7, 8].

We consider a hypothetical balancing market in which each storage first commits a capacity, $S_i$, and then sets a price, $p_i$, which, depending on the market format, can be for energy or capacity. In each time period, the system wide energy imbalance, which we assume to be random, is allocated among the storage in merit order up to their capacities. Note that when $p_i$ is an energy price, it persists over a sequence of time periods. This represents the realistic scenario that imbalances are realized and physically responded to faster than the duration between markets, e.g., every few seconds; we describe this in detail in the next section. This setup is similar to the California Independent System Operator’s (CAISO) real-time energy market [40], as discussed in Section 4.1. Note that even if the energy imbalances exceed the combined storage capacity, there is virtually always sufficient generator reserves, which here we model simply as a more expensive storage with infinite capacity.

We remark that setting capacities prior to prices is an important difference between our setup and those used by some ISOs, wherein capacities and prices are bid simultaneously. Some theoretical results are given by [18], which finds that simultaneous bidding under deterministic demand never leads to pure strategy Nash equilibria. Further discussion on two-dimensional bidding in electricity markets is given by [41, 42]. We justify our modeling both as more tractable approximation to the case of simultaneous bidding, and as a potentially realistic model on its own. In particular, system operators often require capacity information earlier than price information, as is the case with CAISO’s residual unit commitment [40]. In a balancing market with storage, for instance, a system operator may require a capacity commitment every few hours, but allow firms to update their prices to reflect current conditions on shorter intervals.

3. Price and capacity competition

In this section, we develop tools for analyzing balancing markets with storage. First, we generalize Bertrand-Edgeworth price competition so that a single price bid determines the demand allocations for multiple dynamically coupled periods. We completely characterize the Nash equilibrium of this
Subsequently, we obtain new results for Nash equilibria when firms strategically set capacities prior to the price-setting stage when the demand is random. We primarily focus on duopolies for analytical tractability.

3.1. Price-setting stage

3.1.1. Demand allocation and payoff rules.

The firms have capacities $S_i$, $i = 1, ..., N$; these become strategic quantities in Section 3.2. Between periods, firm $i$ leaks a constant fraction of its state of charge, $1 - \alpha_i$. Each firm sets a price, $p_i \geq 0$, which persists through all time periods, $t = 0, ..., T$, where we refer to $T$ as the market horizon. In each period an imbalance $B^t \in \mathbb{R}$ is allocated amongst the firms according to the below linear program, which is parametrized by the price vector $p$.

$$\begin{align*}
\text{minimize} & \quad \sum_{i=1}^{N+1} p_i |X^t_i| \\
\text{such that} & \quad \sum_{i=1}^{N+1} X^t_i = B^t \\
& \quad 0 \leq \alpha_i S_i^t(p) + X^t_i \leq S_i, \quad i = 1, ..., N
\end{align*}$$

Here the storage are dispatched in merit order, i.e. the storage with the lowest price is used up to its capacity, then that with the second lowest, and so on until $B^t$ has been completely allocated in each time period. $X^t_i$ is the portion of $B^t$ allocated to firm $i$, and $S_i^t(p)$ its state of charge in period $t$. We display the dependence of the allocation on the price vector $p$ by denoting $X_i^t(p) = X_i^t$. Firm $i$’s state of charge evolves according to

$$S_{i+1}^t(p) = \alpha_i S_i^t(p) + X_i^t(p), \quad S_0^t(p) = S_0^t.$$  

We assume for analytical tractability that firms profit symmetrically through balancing, i.e. the payoff is the same for providing and consuming energy. Firm $i$’s total realized profit after the pricing game is then

$$p_i \sum_{t=0}^{T} |X_i^t(p)|.$$ 

Note that this payment implies that firms are paid identically for positive and negative imbalances. This is a reasonable assumption because both
positive and negative imbalances are detrimental to system operation. Also, the mileage payments some system operators have implemented in response to the Federal Energy Regulatory Commission’s Order #755 are similarly based on absolute value, albeit for ramping rather than energy or power \[43\].

Firm $N + 1$ corresponds to conventional generator reserves, which cost $p_{N+1} = R$. We assume the capacity of conventional reserves is much larger than the storage pool so that $S_{N+1} = \infty$. This implies that $p_i \leq R$ if firm $i$ is to receive any payment; for this reason, $R$ is sometimes referred to as the reservation utility. Under price ties, firms are randomly assigned priority, e.g. if $p_i = p_j$ for some $i \neq j$, firm $i$ receives first allocation with probability one half. A variety of tie-rules could be specified that would serve equivalently in our analysis, and the particular choice does not affect our results.

**Lemma 1.** $\sum_{t=0}^{T} |X_t^i(p)|$ is nonincreasing in $p_i$ and nondecreasing in $p_{-i}$.

Observe that $\sum_{t=0}^{T} |X_t^i(p)|$ only increases or decreases when the price-wise ordering of the firms changes, because otherwise the allocation (1)-(3) is unaffected.

We assume that the sequence of imbalances are random and described by the joint distribution $f(B)$. The case when $B$ is not random is simply when $f(B)$ is a discrete distribution with its entire mass on one point. Since Lemma 1 holds for any realization of $B$, it implies that $\mathbb{E} \left[ \sum_{t=0}^{T} |X_t^i(p)| \right]$ is nonincreasing in $p_i$ and nondecreasing in $p_{-i}$ as well.

Let $\mu_i$ be firm $i$’s mixed strategy. The payoff associated with $\mu_i$ is

$$
\pi_i(\mu) = \int_p \left( \prod_j \mu_j(p_j) \right) p_i \mathbb{E} \left[ \sum_{t=0}^{T} |X_t^i(p)| \right] dp.
$$

(5)

$\mu$ is a mixed strategy Nash equilibrium if for each firm $i$, $\pi_i(\mu) \geq \pi_i(\mu'_i, \mu_{-i})$ for all $\mu'_i$. We will slightly abuse notation by denoting the payoff of a pure strategy $\pi(p_i, \mu_{-i})$. Note that this definition is essentially the Bayesian equilibrium where $B$ is the only unknown parameter and all firms share the same knowledge \[44, 45\].

3.1.2. Price equilibrium.

In this section, we characterize the price equilibrium. Discontinuous games can fail to have Nash equilibria. The next lemma guarantees that the pricing game always has a mixed-strategy price equilibrium.
Lemma 2. A mixed-strategy price equilibrium always exists.

Lemma 3. In the following two cases, the mixed strategy equilibrium reduces to a pure strategy equilibrium.

1. If for each $i$, $E\left[\sum_{t=0}^{T} |X_i^t(p)|\right] = 0$ when $i$’s price is maximal ($i = \arg\max_j p_j$), the pure strategy equilibrium is $p_i = 0$ for all $i$.

2. If $E\left[\sum_{t=0}^{T} |X_i^t(p)|\right] = E\left[\sum_{t=0}^{T} |X_i^t(p')|\right] > 0$ for all $i$ and all price vectors $p$ and $p'$, the pure strategy equilibrium is $p_i = R$ for all $i$.

In standard Bertrand-Edgeworth competition with one period and deterministic demand, the first case reduces to the firms undercutting each other to zero, and the second to the firms setting the maximum price because they are guaranteed their maximum share of the demand. When the imbalance, $B$, is random, the first case entails that any realization of $B$ be containable by any size $N - 1$ subset of the firms, and the latter that any realization exhausts each firm’s capacity in every time stage.

Let $U_i$ and $L_i$ be the upper and lower boundaries of the support of $\mu_i$. By the definition of mixed-strategy equilibrium [45], there exists a $\pi_i^*$ and a subset $P_i \subseteq [L_i, U_i], \mu_i(P_i) = 1$ for which

$$\pi_i(p_i, \mu_{-i}) \leq \pi_i^* \quad \forall p_i \in [U_i, L_i]$$

(6)

$$\pi_i(p_i, \mu_{-i}) = \pi_i^* \quad \forall p_i \in P_i$$

(7)

The payoff $\pi_i(p_i, \mu_{-i})$ is continuous at $p_i$ if $\mu_{-i}$ has no atom there ($\mu_i$ has an atom at $p_i \in [L_i, U_i]$ if $\text{Prob}(p_i) = a > 0$, or, equivalently, $\mu_i(x) = a\delta(x - p_i)$, where $\delta(x)$ is the Dirac delta function). Therefore, $\pi_i(p_i, \mu_{-i}) = \pi_i^*$ if $\mu_{-i}$ has no atom at $p_i$. The following lemma characterizes the mixed strategies.

Lemma 4. Characterization of mixed equilibria ($N$ firms)

1. For any firm’s support $[L_j, U_j]$, there are at least two firms $i$ for which $\mu_i(p) > 0$ for all $p \in [L_j, U_j]$.

2. Only one atom can exist across all firms, and if one does, it must be at $U = \max_i U_i$.

3. $U = R$.

We henceforth restrict our attention the more tractable two-firm case, and assume that $S_1 \geq S_2$. We use the notation $[X]_V^Z$ to denote the quantity
Let \( X_i \) be the sum of the magnitudes of the imbalances captured by firm \( i \) when it has first priority in the allocation of \( B_t \) in each period, and \( \overline{X}_i \) when it has second priority. Mathematically, let \( p' \) denote a price vector in which \( p_i < p_{-i}, \) and \( p'' \) vice versa. Then

\[
X_i = E \left[ \sum_{t=0}^{T} \left| X^t_i (p') \right| \right]
\]

and

\[
\overline{X}_i = E \left[ \sum_{t=0}^{T} \left| X^t_i (p'') \right| \right]
\]

Clearly, \( X_i \geq \overline{X}_i, X_1 \geq X_2, \) and \( \overline{X}_1 \geq \overline{X}_2. \)

Note that \( X_i \) and \( \overline{X}_i \) can be directly computed from each \( S_i, \alpha_i \) and the distribution of \( B. \) For example, when there is only one time period and \( S_i^0 = 0, \) we have

\[
X_i = \int_0^{S_i} B f(B) dB + S_i (1 - F(S_i)) \quad (8)
\]

\[
\overline{X}_i = \int_0^{S_i} B f(B + S_{-i}) dB + S_i (1 - F(S_1 + S_2)) \quad (9)
\]

When \( B \) is also not random,

\[
X_i = [B]_0^{S_i} \quad (10)
\]

\[
\overline{X}_i = [B - [B]_0^{S_{-i}}]_0^{S_i} \quad (11)
\]

**Theorem 1.** Assume \( S_1 \geq S_2. \) The equilibrium payoffs are then

\[
\pi^*_1 = R \overline{X}_1
\]

\[
\pi^*_2 = \frac{R \overline{X}_1 X_2}{X_1}
\]

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The mixed strategies are
\[
\mu_1(x) = \frac{RX_1X_2}{X_1(X_2 - X_1)x^2} + \frac{X_2X_1 - X_2X_1}{X_1(X_2 - X_1)} \delta(x - R), \quad x \in [L, R]
\]
\[
\mu_2(x) = \frac{RX_1}{(X_1 - X_1)x^2}, \quad x \in [L, R]
\]
where \(\delta(x)\) is the Dirac delta function and \(L = R\frac{X_1}{X_1}\).

Observe that the equilibrium in Theorem 1 is the only possible equilibrium because our derivation constructively identifies the properties an equilibrium must have. It is straightforward to show that the pure strategies in Lemma 3 are special cases of Theorem 1 by considering two limiting cases. In the first, \(X_i = 0, \pi^*_i = \pi^*_2 = 0\), and each \(\mu_i(x)\) has an atom of mass one at \(x = 0\). In the latter, \(\bar{X}_i = X_i, \pi^*_i = RX_1, \pi^*_2 = RX_2\), and each mixed strategy has an atom of mass one at \(x = R\).

### 3.2. Capacity-setting stage

We now turn to the strategic determination of storage capacities. For analytical tractability, we restrict our attention to a single time period, and consider both when \(B\) is deterministic and random. Since there is no dynamic coupling over a single time period, we can assume \(B \geq 0\) and \(S_i^0 = 0\) for both firms without loss of generality. In these cases, the price equilibria are given by (8)-(11) and Theorem 1. For tractability, we only seek pure strategy capacity equilibria, as is standard in almost all supply function-based approaches, cf. [23, 24, 25, 26].

Prior to price setting, the firms choose capacities \(S_1, S_2 \geq 0\) to maximize expected profits. Each firm incurs an opportunity cost as discussed in Section 2, \(-\gamma_iS_i\), which must satisfy \(0 < \gamma_i \leq R\) for the game to be nontrivial. The price equilibrium payoff of the larger firm is denoted by \(\pi_i(S_i, S_{-i})\) and of the smaller firm by \(\pi(S_i, S_{-i})\), where \(S_i \geq S_{-i}\). The expected net profit of firm \(i\) is the payoff from the pricing stage minus the opportunity cost:

\[
\tilde{\pi}_i(S) = -\gamma_iS_i + \begin{cases} 
\pi_i(S_i, S_{-i}) & S_i \geq S_{-i} \\
\pi(S_{-i}, S_i) & S_i < S_{-i}
\end{cases}
\]

**Lemma 5.** When \(B\) is deterministic, the capacity equilibria occupy the continuous range of values described by

\[
S_1 + S_2 = B, \quad \frac{R - \gamma_i}{2R - \gamma_i}B \leq S_i \leq S_{-i}, \quad i = 1, 2.
\]
The $n$-firm generalization of Lemma 5 and its proof are given in [18]. Observe that the sum of the firms capacity exactly equals the total demand, $B$, which, according to the first part of Lemma 3, means that the firms will both set their price at $R$.

Now assume that $B$ is random and that $f(B)$ is positive and decreasing.

**Lemma 6.** $\pi(S_i, S_{-i}) - \gamma_i S_i$ and $\pi(S_{-i}, S_i) - \gamma_i S_i$ are respectively strictly concave and strictly quasiconcave in $S_i$.

Because each segment of $\tilde{\pi}_i(S)$ is quasiconcave and differentiable in $S_i$ when $S_i \neq S_{-i}$, pure strategy capacity equilibria are liable to occur where their derivatives are zero, enabling us to obtain constructive characterizations. Note, however, that $\tilde{\pi}_i(S)$ may not be quasiconcave, and in some cases a pure strategy capacity equilibrium may not exist. Let the cumulative distribution of $B$ be given by $F(B) = \int_0^B f(x)dx$, and define

$$\Lambda_1^i = F^{-1}(1 - \gamma_i/R) \quad (12)$$

$$\Lambda_2^i = S_i \text{ such that } \frac{d\pi(S_{-i}, S_i)}{dS_i} \bigg|_{S_{-i} = \Lambda_1^{i-1} - S_i = \gamma_i} = 0 \quad (13)$$

Setting the derivative of $\pi(S) - \gamma_i S_i$ with respect to $S_i$ equal to zero, we have that at equilibrium $S_i + S_{-i} = \Lambda_1^i$ if $S_i \geq S_{-i}$. Similarly if $S_i < S_{-i}$, $S_i = \Lambda_2^i$ at an equilibrium. Define

$$\hat{S}_i = (\Lambda_1^i - \Lambda_2^i, \Lambda_2^i).$$

$\hat{S}_i$ is a Nash equilibrium if it satisfies the standard definition:

$$\hat{S}_j^i = \arg\max_{S_j} \pi_j(S_j, \hat{S}_j^{i-1}), \quad i, j = 1, 2.$$ 

Note that we have not proven that $\Lambda_2^i$ is uniquely defined, i.e. that there is a single solution to (13); however, this is the case in all of our numerical examples in Section 4.1.

**Lemma 7.** The $\hat{S}_i$ are the only possible pure-strategy equilibria.

We remark that since $\tilde{\pi}_i$ is continuous, a mixed-strategy equilibrium must exist even if both of the $\hat{S}_i$ fail to be an equilibrium [46]. In our subsequent analysis in Section 4.1, we limit our attention to the above pure strategy equilibria, which do exist in our examples. While non-ideal, it is common in practice to only consider pure strategy equilibria for tractability reasons, for example in supply function competition and its application to power markets [22, 24].
4. Analysis of electricity markets

In this section, price and capacity competition are applied to energy imbalance markets with storage. We first use capacity competition to compare markets with capacity and energy-based payments. Then, we restrict our attention to energy-based markets and examine the effect of market horizon and leakage. As discussed in Section 2, prices and profits beyond that necessary to recoup opportunity costs in our model are the exclusive result of strategic behavior. Hence, in this section we will equate economic competitiveness with lower firm prices and profits.

Our application of price and capacity competition most closely parallels supply function modeling of generator competition (see the introduction) in that here firms submit what is essentially a two-parameter bid curve consisting of a single price and a capacity limit. As discussed in Section 2, the price and capacity competition framework is highly appropriate for storage because it precisely captures its hard capacity limits, dynamic coupling, and low operating costs while admitting more nuanced analyses than would be obtainable from generally supply function competition.

4.1. Pricing capacity versus energy

Under capacity pricing, firms are paid for committed capacity in forward markets (FM). Under energy pricing, firms are paid for absorbing energy imbalances in real-time markets (RTM). The key mathematical difference between each format is that the system operator’s capacity requirement is deterministic and energy imbalances are random. Tables 1 and 2 summarize the sequence of events under each type of pricing. In both cases, firms first set their maximum capacities, \( S_i \), in the FM. Under capacity pricing, firms then set prices for capacity (also known as ‘capacity premiums’) in the FM, and the system operator purchases a portion of each firm’s capacity at these prices in the FM. Under energy pricing, firms set prices for energy (also known as ‘imbalance fees’) in the FM, and firms are paid for absorbing energy imbalances in the RTM. While some energy markets allow prices and capacities to be set in the RTM, our assumption is consistent with markets like CAISO’s real-time (5-minute) energy market, in which capacities and prices are set on the hour, and generators are dispatched price-wise every 5-minutes.

We now use our results on price and capacity competition to compare capacity and energy prices. We first discuss the qualitative properties of
the two formats. Ascribing Lemma 5 to capacity markets, firms will declare capacities that sum exactly to the total capacity requirement. Consequently, the price equilibrium reduces to the second scenario in Lemma 3, which says that each firm’s pure strategy is to bid the maximum price, the least economically competitive outcome. Moreover, there is a potentially wide range of capacity equilibria, some of which may be inequitable to some firms [18].

Energy-based markets are described by Lemma 7 on capacity equilibria when the demand in the pricing stage is random. Here, under certain conditions, at most two pure-strategy capacity equilibria can exist, but in some scenarios there may only be mixed equilibria. This implies that firm capacity commitments are more predictable under energy pricing, and is reminiscent of a central result of [22] showing that uncertainty reduces the number of pure-strategy supply function equilibria. It is easy to see from Theorem 1 that the price equilibrium is mixed and result in expected firm profits below pricing at the maximum.

Next, we numerically compare the capacity equilibria under capacity and energy pricing. We assume that the real-time energy imbalance obeys the half-normal distribution \( f(B) = \sqrt{2/\pi}e^{-B^2/2}, B \geq 0 \), because normal distributions have long been used to model mismatches in supply and demand in power systems [47]. The system operator’s capacity requirement is just the mean of \( B, \sqrt{2/\pi} \); we comment more on this choice later. In both market formats, the reservation utility, \( R \), is based on the cost of generator reserves. Specifically, suppose that \( x \) units of generator capacity costs \( r(x) = ax^2 + bx \).
The reservation price for capacity is then $R_1 = r(x)/x$, and for expected reserve energy $R_2 = r(x)/E[\min\{B, x\}]$. If we assume that all capacity will be procured from storage, i.e. $x = 0$, we have

$$R_2 = \lim_{x \to 0} \frac{r(x)}{E[\min\{B, x\}]}$$

$$= \lim_{x \to 0} \frac{2ax + b}{1 - F(x)} \text{ by l’Hôpital’s rule}$$

$$= \lim_{x \to 0} \frac{r(x)}{x}$$

$$= R_1$$

Hence, we may assume that both formats have the same reservation utility. For simplicity, we set $R_1 = R_2 = 1$.

![Figure 1: Each storage’s equilibrium capacity and payoff for $\gamma_2 = 0.5$. The two equilibria are identical when $\gamma_1 = \gamma_2 = 0.5$.](image)

Figure 1 shows the pure strategy equilibria under energy pricing. Two equilibria can coexist, but only when the firms’ opportunity costs are similar, i.e. $\gamma_1 \approx \gamma_2$. Otherwise, the single equilibrium is $\hat{S}_i$ where $\gamma_i < \gamma_{-i}$, which corresponds to the firm with the lower opportunity cost committing a higher capacity. The associated equilibrium profits correspond to prices well below
the reservation utility, \( R = 1 \); for instance, when \( \gamma_1 \approx \gamma_2 \), the profits are approximately one quarter the committed capacity, corresponding to average prices of approximately \( 1/4 \).

Figure 2 shows the total profits and capacity commitment over the same range as Figure 1 along with the deterministic capacity pricing case. Here, profits under capacity pricing are substantially higher, even in the low opportunity cost regime where energy pricing leads to greater capacity commitment. We remark that the system operator would typically procure far more than the mean, \( \sqrt{2/\pi} \), so we regard the capacity commitment and profit under capacity pricing to be a conservative approximation.

The salient differences between the formats are summarized below:

- Capacity pricing leads to larger capacity commitments and consistent (pure strategy), high prices.
- Energy pricing leads to smaller capacity commitments and inconsistent (mixed strategy), lower prices.

Except for the inconsistency of prices under energy pricing, these are well-known characteristics in other markets (e.g. conventional energy and reserves), and reflect the tradeoff between robust (capacity-based) approaches
that guarantee supply adequacy and average (energy-based) cost approaches that achieve better economic competitiveness (see, e.g., [30] or [31]). To our knowledge, this is the first such analysis capturing this tradeoff in balancing markets with storage.

4.2. Dynamic effects

In this section, restrict our attention to energy-based pricing and examine the dynamic effects of the market horizon and efficiency. Note that although our formulation can accommodate serial correlations in the sequence of energy imbalances, we do not consider their effects for concision and because we regard market horizon and efficiency to be more influential parameters.

4.2.1. Market horizon

We now restrict our attention to energy-based pricing and examine the effect of the number of periods on equilibria. Note that this relationship can only be observed through energy pricing because the number of periods does not (directly) affect firm profits under capacity commitments. We look at the expected equilibrium prices, which can be calculated analytically from Theorem 1 as

$$
\rho_1^* = \frac{R}{\bar{X}_1 (\bar{X}_2 - \bar{X}_1)} \left( \frac{\bar{X}_1 \bar{X}_2}{\bar{X}_2} \left( 1 + \ln \left( \frac{\bar{X}_1}{\bar{X}_2} \right) \right) - \bar{X}_2 \bar{X}_1 \right)
$$

$$
\rho_2^* = \frac{R \bar{X}_1}{\bar{X}_1 - \bar{X}_1} \ln \left( \frac{\bar{X}_1}{\bar{X}_1} \right)
$$

Figure 3 shows $\rho_1^*$ and $\rho_2^*$ as a function of the market horizon, $T$. The firm parameters are $S_1 = 1.5$ and $S_2 = 1$ and $\alpha_1 = \alpha_2 = 1$. Each period’s energy imbalance is drawn from an independent, zero mean Gaussian distribution with variance $\sigma$, where $\sigma = 1/4$ in the top plot and $\sigma = 4$ in the bottom. Each plot shows the result for three different initial states of charge, which we assume satisfy $S_1^0/S_1 = S_2^0/S_2$. This assumption is consistent with mechanisms used by system operators to ‘reset’ storage to ensure that imbalances average to zero over time [6, 7, 8]. The integrals necessary to compute the equilibria were evaluated numerically using $10^5$ Monte Carlo points.

As is to be expected, regardless of initial condition the expected prices converge to the same long-horizon value for each storage, which increases with $\sigma$. However, the initial condition can lead to qualitatively different transient behaviors. When the firms begin empty ($S_0^0 = 0$), each $\rho_i^*$ monotonically
increases with $T$. This is because the firms cannot capture profits when the early imbalances are negative, and thus have a relatively high probability of not exhausting their capacity, leading to undercutting. In numerical experiments, we observed that this behavior only occurs when the initial state of charge is under a tenth the total capacity (or within a tenth of full), and thus does not account for the majority of cases.

When the initial state of charge is between ten and ninety percent of the capacity, we observe three different equilibrium regimes as $T$ is increased:

1. *(Small $T$)* In this range, each firm rarely hits its energy capacity limit before $T$. Consequently, either firm can physically accommodate most of the demand, causing them to engage in undercutting. This equilibrium is closer to the first case in Lemma 3.

2. *(Medium $T$)* As $T$ is increased, firms are likely to have their capacities exhausted exactly once regardless of their price. Consequently, firms set higher prices closer to the reservation utility. This equilibrium is closer to the second case in Lemma 3.
3. *(Large T)* Further increasing $T$ causes the firms to enter a regime where the lower priced firm cycles faster, i.e. hits its capacity limits more than the other, and hence profits more. As a result, the firms resume undercutting. There is a ‘knee’ in the curve, beyond which increasing $T$ further contributes little to economic competitiveness.

In the top plot, the first regime appears within the first couple periods, the second between three and seven periods, and the third past the seventh period. In the bottom plot, only the latter two regimes are present, with the transition occurring around the third period. The knee at which a larger market horizon does not significantly reduce firm profits occurs around $T = 20$ in the top plot and $T = 7$ in the bottom. Generally, the location of the knee can be expected to increase as a function of storage size and decrease as a function of energy imbalance variance.

Market horizon is an important design parameter for balancing markets because too large a horizon will make adapting to changes difficult and too short a horizon could add price volatility. For the majority of initial conditions, the expected prices decrease with $T$. Therefore, the market horizon should be chosen to be large enough that the expected prices have nearly converged to the long-horizon value. From Figure 3 we see that this convergence occurs relatively early at the knee in each plot. Hence, our analysis enables the identification of market horizons that are small but still minimize gaming.

4.2.2. Leakage

We lastly consider the effect of losses due to the leakage parameter, $\alpha$, which was defined in Section 2. Like the market horizon, leakage can only be analyzed in a dynamic setting because it manifests between periods. As discussed in Section 2 we use leakage as an analytically tractable proxy for general inefficiencies like injection and extraction losses, though we note there are important differences between the two mechanisms; we leave detailed analysis of the effects of inefficiency to future work.

We again study the expected equilibrium price, $\rho^*_i$, for $T = 100$, which represents the long-horizon value. The firms have capacities $S_1 = 1.5$ and $S_2 = 1$ with $S^0_i = S_i/2$. Figure 4 shows $\rho^*_i$ as a function of $\alpha$ for $\sigma = \{1/10, 1, 10\}$, now with thicker lines corresponding to larger variances.

At the left side of the plot, the state of charge drops to zero at the end of each time period, effectively resulting in a single period model. Consequently,
at low variances, the firms undercut intensely, and at high variances where capacity is expected to be exhausted, the firms set prices near the reservation utility. We remark that inefficient storage is less useful to power systems due to high energy waste, which is not penalized in our model. As the physical efficiency improves, the prices increase in the lower variance cases, and then decrease just before reaching zero leakage. This transition from increasing to decreasing prices occurs because the profit gain from cycling more frequently by being the lower priced firm begins to outweigh the profits lost to leakage. On the other hand, in the high variance case, the price decreases monotonically as the profit gains from cycling more frequently overtake those of charging high prices in each individual period.

From a physical perspective, losses are always undesirable, and modern batteries can achieve roundtrip efficiencies of over 70% [48] and up to 95% for technologies like superconducting magnetic energy storage [9]. Therefore, focusing on the right side of the plot, we conclude that lower leakage reduces gaming. More precisely, lower leakage strengthens the state of charge’s dynamic coupling. This increases the value of being cycled more frequently relative to setting a high price in each individual period, thus inducing the
firms to lower their prices. This conforms with the results of [32], which also finds through an inventory control-based model that increased leakage reduces competition. Note that this is a similar outcome to that achieved by increasing the market horizon.

5. Conclusion and future work

Energy storage and aggregations of shiftable loads could soon compete alongside traditional generators in energy balancing markets. As in traditional generation markets, storage may have opportunities to exert market power, justifying analysis of strategic behavior in these new markets. Price and capacity competition is a game theoretic model of strategic behavior that precisely captures three essential characteristics of storage: hard capacity limits, dynamically coupled energy states, and low marginal costs. Using our new results on price and capacity competition, we obtain the following insights about designing energy balancing markets with storage.

- Energy pricing leads to lower, randomized prices and lower capacity commitments. Capacity pricing leads to higher, consistent prices and higher capacity commitments. These two formats represent opposing extremes of possible market designs, which may have both energy and capacity payments. Choosing how much to pay for energy versus capacity is therefore a tradeoff between economically efficient pricing with low capacity commitment and robust capacity commitment with high prices. Despite the fact that, as discussed in Section 2.1, high prices in our model are the result of strategic behavior, capacity pricing may nevertheless be useful when the objective is to encourage investment in storage and its participation in balancing markets. Likewise, energy pricing may be well-suited to maintaining the competitiveness of markets with established participants.

- Longer market horizons and lower leakage (and, by proxy, higher physical efficiency) both reduce gaming by inducing firms to compete to be cycled more frequently by lowering their prices (under energy pricing). The benefits of a longer horizon are easily obtainable because equilibrium prices converge rapidly with horizon length.

We now discuss some relevant future research directions. First, there are many additional questions that could be addressed with the framework.
in this paper, for instance the effect of temporal correlations in the energy imbalances, and sequences of non-zero mean energy imbalances. Since we have assumed a discriminatory payment mechanism, it is of interest and would likely be a similar exercise to analyze a uniform payment mechanism. Barrier to entry is another important market feature; however, this would likely be more difficult to analyze because it involves the non-trivial extension of our framework to the n-firm case. Models in which price and capacity are bid simultaneously or in which hybrid payments are made for both price and capacity may be a better match for real markets, but may be harder to analyze because each firm’s strategy space will be two-dimensional. Finally, a dynamic stochastic game framework like that of [49] could enable the analysis of sequences of markets with price and capacity bidding.

6. Appendix

Proof. (Lemma 1). Suppose that \( p_j \) is the smallest price larger than \( p_i \), and consider an increase in \( p_i \) to \( p_i + \epsilon, \epsilon < 0 \). If \( p_i + \epsilon < p_j \), then \( \sum_{t=0}^{T} |X_t^i(p)| \) does not change. Assume that \( p_i + \epsilon > p_j \). We proceed inductively.

Without loss of generality, assume \( \beta^0 \geq 0 \). By construction, \( |S_i^1(p) - S_i^1(p_{-i}, p_i + \epsilon)| = |X_i^0(p)| - |X_i^0(p_{-i}, p_i + \epsilon)| \geq 0 \), which establishes the base case. Now assume that \( \sum_{k=0}^{k} |X_i^k(p)| - \sum_{t=0}^{T} |X_i^t(p_{-i}, p_i + \epsilon)| \geq |S_i^{k+1}(p) - S_i^{k+1}(p_{-i}, p_i + \epsilon)| \). Without loss of generality, assume that \( B^{k+1} > 0 \) and thus \( X_i^{k+1}(p) \geq 0 \) and \( X_i^{k+1}(p_{-i}, p_i + \epsilon) \geq 0 \). First, consider the case that \( X_i^{k+1}(p) - X_i^{k+1}(p_{-i}, p_i + \epsilon) \leq 0 \). This implies \( S_i^{k+1}(p) - S_i^{k+1}(p_{-i}, p_i + \epsilon) \geq 0 \) because \( S_i^{k+1}(p) = S_i \).

By assumption, we then have

\[
\sum_{t=0}^{k+1} |X_i^t(p)| - \sum_{t=0}^{k+1} |X_i^t(p_{-i}, p_i + \epsilon)| \geq \alpha_i |S_i^{k+1}(p) - S_i^{k+1}(p_{-i}, p_i + \epsilon)|
\]

\[
+ |X_i^{k+1}(p)| - |X_i^{k+1}(p_{-i}, p_i + \epsilon)|
\]

\[
= \alpha_i S_i^{k+1}(p) - \alpha_i S_i^{k+1}(p_{-i}, p_i + \epsilon)
\]

\[
+ X_i^{k+1}(p) - X_i^{k+1}(p_{-i}, p_i + \epsilon)
\]

\[
= |S_i^{k+2}(p) - S_i^{k+2}(p_{-i}, p_i + \epsilon)|,
\]

where the last line is due to (4) and the fact that \( |X_i^{k+1}(p_{-i}, p_i + \epsilon)| - |X_i^{k+1}(p)| \leq \alpha_i |S_i^{k+1}(p) - S_i^{k+1}(p_{-i}, p_i + \epsilon)| \) due to the capacity limit.
Now assume that that $X_i^{k+1}(p) - X_i^{k+1}(p_{-i}, p_i + \epsilon) \geq 0$. Then

$$\sum_{t=0}^{k+1} |X_i^t(p)| - \sum_{t=0}^{k+1} |X_i^t(p_{-i}, p_i + \epsilon)| \geq \alpha_i |S_i^{k+1}(p) - S_i^{k+1}(p_{-i}, p_i + \epsilon)|$$

$$+ |X_i^{k+1}(p)| - |X_i^{k+1}(p_{-i}, p_i + \epsilon)|$$

$$= \alpha_i |S_i^{k+1}(p) - S_i^{k+1}(p_{-i}, p_i + \epsilon)|$$

$$+ |X_i^{k+1}(p) - X_i^{k+1}(p_{-i}, p_i + \epsilon)|$$

$$\geq |\alpha_i S_i^{k+1}(p) + X_i^{k+1}(p)|$$

$$- \alpha_i S_i^{k+1}(p_{-i}, p_i + \epsilon) - X_i^{k+1}(p_{-i}, p_i + \epsilon)$$

$$= |S_i^{k+2}(p) - S_i^{k+2}(p_{-i}, p_i + \epsilon)|,$$

where the third line is due to the triangle inequality. Since $|S_i^{k+2}(p) - S_i^{k+2}(p_{-i}, p_i + \epsilon)| \geq 0$, we have by induction that $\sum_{t=0}^{k} |X_i^t(p)| - \sum_{t=0}^{k} |X_i^t(p_{-i}, p_i + \epsilon)| \geq 0$ for all $k$, which establishes the desired result that $\sum_{t=0}^{T} |X_i^t(p)|$ is nonincreasing in $p_i$.

$\sum_{t=0}^{T} |X_i^t(p)|$ can be shown to be nondecreasing in $p_{-i}$ by reducing some $p_j$, $j \neq i$, by $\epsilon > 0$ and repeating the above argument. \(\square\)

**Proof.** (Lemma 2). We now show that a mixed strategy equilibrium exists in the pricing game. Existence for a single period when $B$ is not random is proved in Prop. 4.3 [50] using Theorem 5 of [51]. We may straightforwardly adapt their approach to the present scenario. Let $\hat{\pi}(p) = \sum_{i=1}^{N} \pi_i(p)$. For each firm $i$, let $G_i \in \mathbb{N}$. For each $G$ with $0 \leq G \leq G_i$ and $j \neq i$, $1 \leq j \leq N$, let $g_{ij}^G$ be a one-to-one continuous function. Let $\hat{P}(i)$ be defined as

$$\hat{P}(i) = \{p | \exists j \neq i, \exists G, 0 \leq G \leq G_i \text{ s.t. } p_j = g_{ij}^G(p_i)\}.$$

Theorem 5 in [51] is as follows:

**Theorem 2.** Suppose that $\pi_i(p, p_{-i})$ is bounded, continuous in $p$ except on a subset $P^*$ of $\hat{P}(i)$, and weakly lower semicontinuous in $p_i$ for all $i$, and that $\hat{\pi}(p)$ is upper semicontinuous in $p$. Then a mixed-strategy equilibrium exists.

Using the argument in [50], [18], it can be shown that $\pi_i(p)$ is bounded, continuous in $p$ except at a subset $P^*$, and weakly lower semicontinuous when $B$ is deterministic. All of these properties are preserved by taking the expectation over $B$ and thus hold for $\pi_i(p)$ when $B$ is random. Clearly, $\hat{\pi}(p)$ is continuous as well since it is the sum of continuous functions. This establishes the existence of a mixed-strategy equilibrium. \(\square\)

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Proof. (Lemma 3). We proceed case-wise as in the lemma.

1. Let $i = \arg\max_j p_j$ and suppose $p_i > 0$. If $i$ is not a strict maximum, profits can be made through undercutting, so we assume that it is a strict maximum. Then $\mathbb{E} \left[ \sum_{t=0}^T |X_t^i(p)| \right] = 0$, and thus a profitable deviation exists to $p_i - \epsilon$ for some $\epsilon > 0$. Since $p_i$ is maximal and a deviation exists if it is positive, the pure strategy equilibrium must be $p_j = 0$ for all $j$.

2. Since the allocation to each firm is independent of the price vector in this case, the pure strategy equilibrium is for each firm to set the maximum price, $p_i = R$.

\[\qed\]

Proof. (Lemma 4).

1. First suppose that $\mu_i(p) = 0$ for all $i$ and all $p_i \in [p, \bar{p}]$, $L_j < p < \bar{p} < U_j$ for at least one firm $j$. Then from $p$ to $p \in (p, \bar{p})$ is a profitable deviation for any firm with probability mass below $p$. Now suppose that only one firm $i$ has $\mu_i(p) > 0$ for $p_i \in [p, \bar{p}]$. Then the distribution

$$\mu'_i(p) = \begin{cases} 
\mu_i(p) & p_i < \underline{p} \\
0 & \underline{p} \leq p_i < \bar{p} \\
\mu_i \left( [\underline{p}, \bar{p}] \right) & p_i = \bar{p} \\
\mu_i(p) & p_i > \bar{p} 
\end{cases}$$

is a profitable deviation for $i$; essentially, all of firm $i$’s probability mass in $[p, \bar{p}]$ has been shifted to $\bar{p}$. Thus at least two firms $i$ must have $\mu_i(p) > 0$ for all $j$ with $p_i \in [L_j, U_j]$.

2. First we show that no two firms may have an atom at the same location. Suppose multiple firms have an atom at $p'$. Then with positive probability all such firms set $p_i = p'$. For a given firm $i$, there exists an $\epsilon > 0$ small enough that $\pi_i(p' - \epsilon, \mu_{-i}) > \pi_i(p', \mu_{-i})$, a profitable deviation for firm $i$.

We now show that no firm can have an atom in $[L, U)$. Suppose firm $i$ has an atom at $p$. If $p \notin [L_j, U_j)$ for all $j \neq i$, then $p + \epsilon, \epsilon > 0$ is a profitable deviation for firm $i$. Now assume firm $i$ has an atom of mass $\alpha \in (0, 1]$ at $p' \in (L_j, U_j)$ for some $j \neq i$, and let $\hat{\mu}$ denote $\mu$ with the
atom subtracted off. Consider the difference between firm \( j \)'s profits at \( p' - \epsilon \) and \( p' + \epsilon \):

\[
\pi_j (p' - \epsilon, \mu_{-j}) - \pi_j (p' + \epsilon, \mu_{-j}) =
\]

\[
(p' - \epsilon) \left[ \int_{p-j} \left( \prod_{k \neq j} \hat{\mu}_k (p_k) \right) \mathbb{E} \left[ \sum_{t=0}^{T} |X^t_j (p' - \epsilon, p_{-j})| \right] dp \right]
\]

\[
+ a \int_{p-i,j} \left( \prod_{k \neq i,j} \hat{\mu}_k (p_k) \right) \mathbb{E} \left[ \sum_{t=0}^{T} |X^t_i (p' - \epsilon, p_{-i,j})| \right] dp \right]
\]

\[
- (p' + \epsilon) \left[ \int_{p-j} \left( \prod_{k \neq j} \hat{\mu}_k (p_k) \right) \mathbb{E} \left[ \sum_{t=0}^{T} |X^t_j (p' + \epsilon, p_{-j})| \right] dp \right]
\]

\[
+ a \int_{p-i,j} \left( \prod_{k \neq i,j} \hat{\mu}_k (p_k) \right) \mathbb{E} \left[ \sum_{t=0}^{T} |X^t_i (p' + \epsilon, p_{-i,j})| \right] dp \right]
\]

Letting \( \epsilon \) tend to zero, the first and third terms cancel, while the sum of the remaining terms remains positive because \( \sum_{t=0}^{T} |X^t_j (p)| \) (by Lemma 1) and hence \( \mathbb{E} \left[ \sum_{t=0}^{T} |X^t_j (p)| \right] \) are nonincreasing in \( p_j \). Since firm \( j \) has no have an atom at \( p' - \epsilon \) or \( p' + \epsilon \) for some \( \epsilon \), its strategy is continuous at \( p' + \epsilon \) and \( \pi_j (p' + \epsilon, \mu_{-j}) = \pi^*_j \). But \( p' - \epsilon \) is a profitable deviation from \( p' + \epsilon \) for firm \( j \), establishing that an atom can only exist at \( U \).

3. Suppose \( U < R \). First consider the case that a firm, \( i \), has an atom at \( U \). Then \( \pi_i (R, \mu_{-i}) > \pi_i (U, \mu_{-i}) \), and a profitable deviation exists. Now suppose no firm has an atom at \( U \). Then similarly for any firm \( i \) with upper support \( U \), \( \pi_i (R, \mu_{-i}) > \pi_i (U, \mu_{-i}) \), establishing the existence of a profitable deviation.

\( \square \)

**Proof.** (Theorem 1). Let \( \Upsilon_i \) be the cumulative distribution of firm \( i \)'s mixed-strategy. Since, by Lemma 4 neither firm has an atom in \([L, R] \), we have
that

$$\pi_1^* = x \left( \int_L^x \mu_2(p_2) \mathbb{E} \left[ \sum_{t=0}^T |X_t^1(x, p_2)| \right] dp_2 + \int_x^R \mu_2(p_2) \mathbb{E} \left[ \sum_{t=0}^T |X_t^1(x, p_2)| \right] dp_2 \right)$$

$$= x \left( \overline{X}_1 \int_L^x \mu_2(p_2) dp_2 + \overline{X}_1 \int_x^R \mu_2(p_2) dp_2 \right)$$

$$= x \left( \overline{X}_1 \gamma_2(x) + \overline{X}_1 (1 - \gamma_2(x)) \right).$$

Solving for $\gamma_2(x)$ over $[L, R)$, we have

$$\gamma_2(x) = \frac{\overline{X}_1}{\overline{X}_1 - \overline{X}_1} - \frac{\pi_1^*}{(\overline{X}_1 - \overline{X}_1)} x.$$

By the same argument, we similarly have

$$\gamma_1(x) = \frac{\overline{X}_2}{\overline{X}_2 - \overline{X}_2} - \frac{\pi_2^*}{(\overline{X}_2 - \overline{X}_2)} x,$$

also over $[L, R)$.

From Lemma 4, only one firm can have an atom at $R$. If $S_1 = S_2$, they are interchangeable. Assume now that $S_1 > S_2$ and that $\overline{X}_1 \geq \overline{X}_2$ and $\overline{X}_1 \geq \overline{X}_2$. Suppose for the sake of contradiction that firm two, which has smaller capacity, has an atom at $R$. Then, since firm one cannot have an atom at $R$,

$$\pi_2^* = R \overline{X}_2$$

$$\pi_1^* = \frac{R \overline{X}_2 \overline{X}_1}{\overline{X}_2}$$

Subbing $\pi(C, R, S)$ into the above expression for $\gamma_2(R)$, we have

$$\gamma_2(R) = \frac{\overline{X}_1}{\overline{X}_1 - \overline{X}_1} - \frac{\overline{X}_2 \overline{X}_1}{\overline{X}_2 (\overline{X}_1 - \overline{X}_1)}$$

$$> \frac{\overline{X}_1}{\overline{X}_1 - \overline{X}_1} - \frac{\overline{X}_2 \overline{X}_1}{\overline{X}_1 (\overline{X}_1 - \overline{X}_1)} \quad \text{(because } \overline{X}_1 > \overline{X}_2)$$

$$= \frac{\overline{X}_1 - \overline{X}_2}{\overline{X}_1 - \overline{X}_1}$$

$$> 1 \quad \text{(because } \overline{X}_1 > \overline{X}_2).$$
This contradicts the assumption that $\mu_2$ has an atom at $R$. Therefore, the equilibrium payoffs are

$$\pi^*_1 = \frac{RX_1}{X_1}$$

$$\pi^*_2 = \frac{RX_1X_2}{X_1}$$

Now setting $\Upsilon_2(L) = 0$, we have that $L = \frac{RX_1}{X_1}$. The atom at $R$ in the large firm’s strategy can be shown to be

$$\frac{X_2X_1 - X_2X_1}{X_1 (X_2 - X_2)}$$

The mixed strategies, can be obtained by substituting the above into $\Upsilon_i(x)$ and differentiating.

Proof. (Lemma 6). The second derivative of $\pi(S_i, S_{-i}) - \gamma_i S_i$ with respect to $S_i$ is $-Rf(S_i + S_{-i})$, and hence it is strictly concave because $f$ is positive. Since $\pi(S_{-i}, S_i)$ is also differentiable, it suffices for quasiconcavity to show that its first derivative with respect to $S_i$ is initially positive and crosses zero exactly once \[52\]. Let $S_i = \int_0^S Bf(B)dB + S_i(1 - F(S_i))$. The first derivative is given by

$$\frac{d\pi(S_{-i}, S_i)}{dS_i} = \frac{1}{S_{-i}} [(1 - F(S_i))\pi(S_{-i}, S_i) + R(F(S_i) - F(S_i + S_{-i}))]$$

where $S_i = \int_0^\infty \min(S_i, D)f(D)dD$. Dividing through by $S_i$, define

$$M(S_i) = \frac{(1 - F(S_i))\pi(S_{-i}, S_i)}{S_i}$$

$$N(S_i) = R(F(S_i) - F(S_i + S_{-i}))$$

$\pi(S_{-i}, S_i)$ is equal to zero at $S_i = 0$ and $S_i = \infty$ but is not constantly zero. Therefore, by the mean value theorem, \[14\] is zero at least once in between, which implies that $M(S_i) + N(S_i) = 0$ at any such point.

Observe that $M(S_i)$ is always positive and $N(S_i)$ always negative. One may straightforwardly differentiate to see that the magnitude of $(1 - F(S_i))$ shrinks faster than that of $(F(S_i) - F(S_i + S_{-i}))$ as $S_i$ increases; since the fraction $\frac{\pi(S_{-i}, S_i)}{S_i}$ also approaches zero with $S_i$, we may conclude that the
magnitude of $M(S_i)$ shrinks faster than that of $N(S_i)$. Since the magnitudes are identical at any point $S'_i$ where $M(S'_i) + N(S'_i) = 0$, then $M(S''_i) + N(S''_i) < 0$ for any $S''_i > S'_i$. Therefore (14) is zero at only one finite value, henceforth denoted $S'_i$, establishing the quasiconcavity of $\pi(S_{-i}, S_i)$ with respect to $S_i$.

Because the magnitude of $M(S_i)$ shrinks faster than that of $N(S_i)$, $\frac{d\pi(S_{-i}, S_i)}{dS_i} - \gamma_i$ is also strictly decreasing in $S_i$ before crossing zero and then remains negative, implying quasiconcavity of $\pi(S_{-i}, S_i) - \gamma_iS_{-i}$ with respect to $S_i$.

\textbf{Proof.} (Theorem 7). We prove that $\hat{S}^i$ are the only possible pure-strategy equilibria by showing that an equilibrium can only exist where $\bar{\pi}_i$ has zero slope in $S_i$. This is equivalent to showing that no ‘ridges’ exist in $\bar{\pi}_i$. A sufficient condition is

$$\frac{d\pi(S_{-i}, S_i)}{dS_i} \bigg|_{S_i = S_{-i}} \leq \frac{d\bar{\pi}(S_i, S_{-i})}{dS_i} \bigg|_{S_i = S_{-i}}. \quad (15)$$

We have that

$$\frac{d\bar{\pi}(S_{-i}, S_i)}{dS_i} \bigg|_{S_{-i} = S_i} = (1 - F(S_i)) \frac{\pi(S_i, S_i)}{S_i} + R(F(S_i) - F(2S_i))$$
$$\leq R(1 - F(S_i)) + R(F(S_i) - F(2S_i))$$
$$= R(1 - F(2S_i))$$
$$= \frac{d\bar{\pi}(S_i, S_{-i})}{dS_i} \bigg|_{S_{-i} = S_i},$$

establishing the claim.

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