Charmonium Spectroscopy From Lattice NRQCD

NRQCD Collaboration presented by A.J. Lidsey

\footnote{aDepartment of Physics and Astronomy, University of Glasgow, Glasgow, G12 8QQ, Scotland, UK}

We present the first set of results for Charmonium Spectroscopy using Non-Relativistic QCD (NRQCD). For the NRQCD action the leading order spin-dependent and next to leading order spin-independent interactions have been included. Clear signals for the s and p hyperfine splittings have been observed as well as various orbital states.

1. Introduction

NRQCD is an effective field theory where a cut off of the order of the quark mass is introduced into QCD in order to remove the mass scale from the calculation. Both numerical and theoretical work has been done in developing NRQCD on the lattice where the lattice spacing provides the cut off. The NRQCD action is essentially an expansion of the usual QCD action in powers of the individual quark velocities squared $v_2 c_2$, where for the bottom and charm quarks $v_2 c_2 \approx 0.1$ and 0.3 respectively. For our action we use the lagrangian defined in Euclidean space by

$$L_{NRQCD} = -\Psi^\dagger D_t \Psi + \frac{\Psi^\dagger D^2}{2M_Q} \Psi + \delta L_{SI} + \delta L_{SD}$$

where

$$L_{SI} = -c_2 i \frac{g}{8M_Q^2} \Psi^\dagger (D.E - E.D) \Psi + \frac{c_1}{8M_Q^2} \Psi^\dagger D^4 \Psi$$

and

$$L_{SD} = c_3 \frac{g}{8M_Q^2} \Psi^\dagger \sigma (D^\wedge E - E^\wedge D) + \frac{c_4}{2M_Q^2} \Psi^\dagger \sigma.B \Psi$$

are the next to leading order spin-independent terms, and

are the leading order spin-dependent terms. Since we have included next to leading order spin-independent terms it is possible when using charm quarks to obtain an accuracy of $\approx 10\%$ for quantities such as the spin averaged S-P splitting or the 2S-1S splitting. For spin-dependent quantities like $^3S_1 - ^1S_0$ an accuracy of only $\approx 30\%$ is possible.

To define $L_{NRQCD}$ on the lattice \footnote{covariant derivatives are represented by shift operators $\Delta_i^\pm \Psi(x) = U_\mu(x) \Psi(x + i) - \Psi(x)$ and the laplacian is defined by $\Delta^2 = \sum_i \Delta_i^+ \Delta_i^-$} the usual cloverleaf expression is used to find $F_{\mu\nu}(x)$ and hence $E(x)$ and $B(x)$. The arbitrary constants appearing in eq. (2) and (3) are evaluated by matching NRQCD to full QCD at low energies. At tree level all the c’s are unity \footnote{The biggest correction coming from loop diagrams will be tadpole corrections arising due to the non-linear relation between the gluon field and the lattice gluon operator. In order to remove this we use the method suggested by \cite{7} where all the U’s are redefined by $U_\mu(x) \rightarrow \frac{U_\mu(x)}{u_0}$ with $u_0 = \langle 0\frac{1}{3} Tr U_{PLAQ} |0 \rangle \frac{1}{3}$. Since the cloverleaf expression involves the evaluation of a plaquette this renormalization will have the effect of redefining E and B via $E \rightarrow \frac{E}{u_0^2}$ B \rightarrow \frac{B}{u_0^2}$}. To evaluate the chromomagnetic and electric fields the usual cloverleaf expression is used to find $F_{\mu\nu}(x)$ and hence $E(x)$ and $B(x)$. The arbitrary constants appearing in eq. (2) and (3) are evaluated by matching NRQCD to full QCD at low energies. At tree level all the c’s are unity \footnote{The biggest correction coming from loop diagrams will be tadpole corrections arising due to the non-linear relation between the gluon field and the lattice gluon operator. In order to remove this we use the method suggested by \cite{7} where all the U’s are redefined by $U_\mu(x) \rightarrow \frac{U_\mu(x)}{u_0}$ with $u_0 = \langle 0\frac{1}{3} Tr U_{PLAQ} |0 \rangle \frac{1}{3}$. Since the cloverleaf expression involves the evaluation of a plaquette this renormalization will have the effect of redefining E and B via $E \rightarrow \frac{E}{u_0^2}$ B \rightarrow \frac{B}{u_0^2}$}.
which will strongly affect spin-dependent quantities. Not removing these tadpole contributions will result in an underestimation of the electric and magnetic fields and hence hyperfine splittings. The tadpoles are the dominant affect in radiative corrections so removing them enables us to use the tree level values for the $c$'s.

As with all lattice simulations lattice spacing errors will need to be investigated and corrected for. In NRQCD for heavy-heavy quark systems correction terms can be added as a power series in $a_s$.

In order to obtain the desired accuracy quoted above it is necessary to remove $O(a^2)$ corrections from the laplacian by redefining $\Delta^2$ as

$$\tilde{\Delta}^2 = \sum_i \Delta_i^+ \Delta_i^- - \frac{a^2}{12} \sum_i |\Delta_i^+ \Delta_i^-|^2$$

(8)

and for the temporal correction it is sufficient to redefine the kinetic operator as

$$-\frac{\tilde{\Delta}^2}{2M} \to -\frac{\tilde{\Delta}^2}{2M} - \frac{a}{4n} \frac{(\Delta^2)^2}{4M^2}$$

(9)

which still allows the evolution equation to be an initial value problem. $n$ is an even number introduced to prevent instabilities occurring in the simulation at low quark mass $a_s^2$.

2. Simulation results

In the simulation we use a lattice of size $12^3 \times 24$ at $\beta = 5.7$ with quenched gauge configurations supplied by the UKQCD collaboration, fixed to Coulomb gauge. The operators are smeared using a coulombic wavefunction as a smearing function. Due to the relatively small size of charmonium it is possible to use more than one starting site on the lattice and since our evolution equation is a simple difference one, different starting times can be used too. For our particular lattice we use 8 different spatial origins and 2 different starting times.

For NRQCD the dispersion relation has the form

$$E_p = M_1 + \frac{p^2}{2M_2} + ....$$

(10)

where, because we have removed the $M_Q\Psi^\dagger \Psi$ term from the original Lagrangian, $M_1 \neq M_2$. $M_1$ is a redundant zero of energy and it is $M_2$ which is the important dynamical mass determining spin averaged splittings. The bare quark mass $M_Q$ appearing in the Lagrangian is tuned in the simulation so that $M_2$ appearing in the dispersion relation for the $^1S_0$ state is equal to the experimental value of the $\eta_c$. This is done by Fourier transforming the correlation function into momentum space from which $E_{p_{\text{min}}}$ can be found, where $p_{\text{min}}$ is the lowest allowed non-zero momentum on the lattice. Using eq. \ref{dispersion_rel} $M_2$ is evaluated from $E_{p_{\text{min}}} - E_0$.

In our simulation we have used 100 configurations and a bare quark mass in lattice units of 0.8. Extracting the lattice spacing from the spin averaged S-P splitting, a value known to be insensitive to the bare quark mass $M_Q$, a value of $a^{-1} = 1.23 (7)$ GeV is obtained. Using this we find $M_2 = 3.00$ Gev for the $^1S_0$ to be compared to the experimental value of 2.98 GeV.

Shown in figure \ref{fig:splitting} are values for various spin-independent splittings where mostly single exponential fits have been used. The $^1D_2 - ^1S_0$ splitting has to be compared to the experimental $^3D_1 - ^1S_0$ result since the $^1D_2$ has not been observed experimentally. The $^3D_1$ has the same quantum numbers as the $^3S_1$ and will appear as a third excited state state in that channel making it difficult to extract a value for its mass. The simulation result is slightly higher as one would expect and certainly in the right area considering the $^1D_2$ is near the threshold for decay into D mesons. For the 2S-1S splitting the 2S was obtained by performing a three exponential fit to two

| Splitting     | Lattice result (MeV) | Exp (MeV) |
|---------------|----------------------|-----------|
| $^3S_1 - ^1S_0$ | 96 (4)               | 117       |
| $^3P_2 - ^3P_1$ | 60 (30)              | 46        |
| $^3P_1 - ^3P_0$ | 32 (9)               | 95        |

Table 1 Results for s and p hyperfine splittings for the 1s and 1p states.
correlation functions. The result matches the experiment although the error bar is large. Table 1 shows values for spin-dependent splittings. For the S hyperfine splitting the value is well within the expected systematic accuracy even though it is smaller than the experimental value. Since this splitting is caused by a local-local interaction the reduction might be caused by quenching. This effect has been estimated to cause a reduction of \( \approx 20\% \). The fact that the S hyperfine is within the expected accuracy strongly suggests that the improved tadpole coupling constant in the \( \sigma B \) interaction is the correct value to use. For the p hyperfines the statistical errors have increased but still a signal can be seen. The agreement with experiment suggests that the \( \sigma D^E \) term can correctly account for the p hyperfine splitting, again using a tadpole improved coupling.

3. Conclusion

Using NRQCD it is possible to obtain a spectrum for Charmonium in good agreement with experiment.

In the future more configurations will be used to reduce statistical errors. This will provide a stringent test of our perturbatively improved action and so of QCD itself. Using perturbation theory with the numerical results will then enable a value for the charm quark mass to be determined.

4. Acknowledgements

A.J. Lidsey and C.T.H Davies would like to thank the UK SERC for supporting this work and the UKQCD collaboration for supplying the configurations. They were generated on a Meiko i860 computing surface supported by SERC grant GR/G32779, Meiko Limited, and the University of Edinburgh. The computer simulations presented here were performed on the ynp8 at the Ohio Supercomputer centre and at the Atlas centre RAL, UK.

REFERENCES

1. B.A. Thacker and G.P. Lepage, Phys. Rev. D 43, 196 (1991).
2. G.P. Lepage, L. Magnea, C. Nakhleh, U. Magnea, and K. Hornbostel, Phys. Rev. D 46, 4052 (1992).
3. C.T.H Davies and B.A. Thacker, Nucl. Phys. B 405, 593 (1993).
4. C.J. Morningstar, Phys. Rev. D 48, 2265 (1993); and these proceedings.
5. NRQCD collaboration presented by G.P. Lepage and J. Sloan. See these proceedings.
6. C.T.H. Davies, Review of Heavy-Heavy Spectroscopy. See these proceedings.
7. G.P. Lepage and P.B. Mackenzie, Phys. Rev D48, 2250 (1993).