The development of the problem-based learning module to facilitate students’ mathematical reasoning

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Abstract. The instructional materials which are distributed in schools are not developed to facilitate the students’ abilities, instead of only consisting of concepts with suitable formulas and the questions without solutions. Based on the problem, the aim of the research is to develop an instructional material in the form of the module which can facilitate the students’ mathematical reasoning. The research used the development model design of Dick and Carey consisting of nine steps. The module was developed with problem-based learning model in the circle lesson and was evaluated in the validity test. The validity test was examined by four validators dealing with content, construct, language and technique. The module was evaluated in two stages consisting of small and big group tests by students of Senior High School 10 Pekanbaru. After having learned with the developed module, students were tested to solve the problems to know their mathematical reasoning. The results were that the developed problem-based learning module is valid (94.21%) and practical (88.85%), and the students’ reasoning test is high (88.97%). So, it is concluded that the developed module can facilitate the students’ mathematical reasoning.

1. Introduction

Mathematics is formed as the human intuition results relating to objects, processes and reasoning [1,2]. Taking this into account, the mathematical reasoning is one of the important abilities for students to master in solving the problems [3,4]. Therefore, mathematics and reasoning cannot be separated in the student learning in the school.

Reasoning is the process of thinking that associates the given facts to produce new conclusions [5,6]. Some indicators to identify students’ mathematical reasoning in this research are identifying the assumptions or variables with presenting in the mathematics model, selecting the characteristic pattern or mathematical indication to generalize the problem, manipulating the problem mathematically, and checking the argument validity [7,8]. Each of these four indicators must be mastered by students in order to help them solving the problems either given in the school or the daily life.

However, the mathematical reasoning is not really mastered by students. From the interview with two mathematics teachers in Senior High School 10 Pekanbaru, we got that many students could not manipulate the mathematical problems which variables were rearranged; explain the solving procedures; and draw the conclusions. These facts indicated that students actually had understood the concepts of problems, but they did not master the mathematical reasoning.

The mathematical reasoning can be mastered with problem-based learning which leads students to solve the open-ended problem with the open answer and solution [9]. Problem-based learning is also the
learning model where students solve the authentic problem to construct their own knowledge, develop their inquiry and high order thinking skills, improve their independence and confidence [10,11]. Problem-based learning has five steps, namely, orient the problem, organize the instruction, guide the investigation, develop and present the work task, and analyse and evaluate the problem solving [12,13].

The mathematical reasoning can be embedded in the instructional material which is suitable for the demanded curriculum and the student development [14]. The instructional material can be the module which is developed with the problem-based learning model. The module is a self-contained book written to make students learn individually without the significance of the teacher guiding [15]. This developed module can facilitate the students’ mathematical reasoning.

Based on the problem background, thus we conduct the study of the development of the problem-based learning module to facilitate the students’ mathematical reasoning. The purpose of this study are as follows (1) to develop and produce problem-based learning module to facilitate the students' mathematical reasoning that fulfils the valid criteria, (2) to develop and produce problem-based learning module to facilitate the students' mathematical reasoning that fulfils the practical criteria, and (3) to know the students' mathematical reasoning of Senior High School 10 Pekanbaru after having learned with the problem-based learning module.

2. Method
This study used the methodology of the research and development to develop the mathematics module. The research used the development model design of Dick and Carey having ten steps: identify instructional goals, conduct instructional analysis, analyse learners and context, write performance goals, develop assessment instrument, develop instructional strategy, develop and select instructional materials, develop and conduct formative evaluation, revise instruction, and develop and conduct summative evaluation [16]. For the last step, we did not conduct it because of the limitation research. The formative evaluation was divided into three categories: validity test, small group test and big group test. The validity test was conducted by four validators arranging from two instructional content experts and two educational technology experts. The small group test was conducted by six students and the big group test was conducted by one class, in this research, the grade 11 consisting of 34 students. Both last two tests were come from students’ Senior High School 10 Pekanbaru and they were also the practicality test.

The data collection instruments were the validity and practicality questionnaires with rating scale, the interview and the test. The data analysis techniques were divided into three categories consisting of validity test, practicality test, and mathematical reasoning test. Both validity and practicality tests analysed the validity and practicality of the module, and they were evaluated by ideal percentage of mean score; organized by validity and practicality categories; and interpreted by descriptive technique to know the degree of validity and practicality of problem-based learning module. The mathematical reasoning test consisting of four problems about the circle lesson was the test for knowing the students’ mathematical reasoning, and it was evaluated by ideal percentage of mean score; organized by general criteria of ability; and interpreted by descriptive technique to know the degree of mastery of students’ mathematical reasoning after having learned with the module [17].

3. Result and discussion
This section presents results of nine-step model design of Dick and Carey. We identified the instructional goals, i.e. the mastery of circle lesson with the realistic problem and knowing the students’ mathematical reasoning after learning the module. The instructional analysis of this learning was obtained from the syllabus and the study plan about circle. After analysing the learners and context, we got that students of Senior High School 10 Pekanbaru are able to identify the variables and to select the right formulas, but they are not able to solve the different problem in the same concept, it means the students’ reasoning are not facilitated well. From the first three steps, we wrote the performance goals from the four indicators of the mathematical reasoning which students must do. The performance goals are used to assess the students’ mathematical reasoning.
To evaluate the product, the developed material, we developed the instruments consisting of instructional content questionnaire and educational technology questionnaire for validity questionnaire, and students’ response questionnaire for practicality questionnaire. We developed the instructional strategy for the instructional materials with five steps of problem-based learning model. Then, we developed the instructional materials, in the form of learning module. Figure 1 shows two pages of the module including cover and beginning of learning activities with the problem.

![Module Cover and Begin of Learning Activities](image)

**Figure 1.** The Appearance of the developed module.

The last two steps of developing and conducting formative evaluation, and revising learning were done one after another. From data of the one-to-one test or validity test in Table 1, both assessments of instructional content and educational technology indicated that the module was very valid and had the close score of the ideal percentage. It means that the content of the circle lesson in the module fulfils the didactical and constructional criteria, and the problem-based learning steps; and the module design fulfils the technical criteria. Validators also gave a few suggestions in both assessments of instructional content and educational technology shown in Error! Reference source not found. and they were already revised.

| No. | Type of validities          | Ideal percentage | Categories    |
|-----|----------------------------|------------------|---------------|
| 1.  | Instructional content      | 93.75%           | Very Valid    |
| 2.  | Educational technology     | 94.67%           | Very Valid    |
|     | Overall ideal percentage   | 94.21%           | Very Valid    |

Table 1. Module validity data.
Table 2. Validators’ suggestions of instructional content and educational technology.

| No. | Validators                              | Suggestions                                                                 |
|-----|----------------------------------------|-----------------------------------------------------------------------------|
| 1.  | Instructional content expert 1         | Adjust the provided task with the desired competencies.                     |
| 2.  | Instructional content expert 2         | Show solutions of problems within short version, in the feedback section.    |
| 3.  | Educational technology expert 1        | Move the figure legends downward.                                           |
| 4.  | Educational technology expert 2        | Increase the font size of the module writing to easily read and understand. |

Table 3. Module practicality data.

| No. | Type of practicalities | Ideal percentage | Categories       |
|-----|------------------------|------------------|------------------|
| 1.  | Small group test       | 85.17%           | Very Practical   |
| 2.  | Big group test         | 92.53%           | Very Practical   |
|     | Overall ideal percentage | 88.85%          | Very Practical   |

From data of the small and big group tests or the practicality test in Table 3, both tests indicated that the module was very practical. It means that the appearance and presentation of the module are very good and understandable to students; and the module gives students easiness and motivation in learning the circle lesson. Students from small group test gave a few suggestions which were already revised (see Table 4), while students from another test were very satisfied with the developed module with the highest ideal percentage 92.53%.

Table 4. Students’ suggestions from small group test.

| No. | Suggestions                           |
|-----|---------------------------------------|
| 1.  | Brighten the colour of the module cover. |
| 2.  | Add more animations or figures in order to be more interested. |

Having finished developing the module with the development model design of Dick and Carey, we conducted the test to know the degree of mastery of students’ mathematical reasoning after they had learned with the developed module. The test consists of four problems about the circle lesson, those are the circle equation in the problem 1 and 2, the position of points and lines to the circle in the problem 3, and the tangent line at the circle in the problem 4. The test data in Table 5 shows that students have highly understood the mathematical reasoning and mastered it on solving the problem. The indicator of selecting the characteristic pattern or mathematical indication to generalize the problem is the highest ideal percentage 92.28% which means that students can generalize the given problem of the circle lesson.

Table 5. Mathematical reasoning test data.

| No. | Indicators of the mathematical reasoning                                      | Ideal percentage | Degrees of mastery |
|-----|-------------------------------------------------------------------------------|------------------|--------------------|
| 1.  | Identifying the assumptions with presenting in the mathematics model.         | 89.46%           | High               |
| 2.  | Selecting the characteristic pattern or mathematical indication to generalize the problem. | 92.28%           | High               |
| 3.  | Manipulating the problem mathematically.                                     | 89.71%           | High               |
| 4.  | Checking the argument validity.                                              | 83.82%           | High               |
|     | Overall ideal percentage                                                      | 88.97%           | High               |
To know the students’ mathematical reasoning, we describe one of the student test answers on the problem 3 in Figure 2. The solution and the solving procedure are correct and are divided into four sections as the four indicators in Table 5. In the first section, the student could make the analyses about the problem into what the given variables (points and an equation) are and what the desired aim of the problem (Does the position of points A become a circle? If true, what are the radius and the centre point). In the second section, the student could find the relation between the given variables (using the concept of the distance of two points) and formulate it to solve the problem. Then, the student could calculate the formulated equation without mistakes until find the desired equation in the third section. Eventually, in the final section about making and proofing the conclusion, the student could apply the equation into the Cartesian diagram so the answer meets the problem criteria. All of sections indicated that the student has mastered the mathematical reasoning with the problem-based learning module as the other researches also support these results [18,19].

**Figure 2.** One of the student test answers in the problem 3.

4. **Conclusion**
Based on the conducted research, we can conclude that (1) problem-based learning module is very valid on the one-to-one-test or the validity test with ideal percentage is 94.21%. This indicates that the developed module can be used as instructional materials. (2) Problem-based learning module is very practical on both the practicality tests with ideal percentage of 88.85%. This shows that the developed
A module can attract students and is easily used in the learning process. (3) After having learned with problem-based learning module, the students highly understood and mastered the mathematical reasoning with the overall ideal percentage 88.97% on the circle lesson. Especially, this occurs on the indicator of selecting the characteristic pattern or mathematical indication to generalize the problem.

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