A simple model for calculating magnetic nanowire domain wall fringing fields

Adam D West$^1$, Thomas J Hayward$^2$, Kevin J Weatherill$^1$, Thomas Schrefl$^3$, Dan A Allwood$^2$ and Ifan G Hughes$^1$

$^1$ Durham University, Department of Physics, South Road, Durham, DH1 3LE, UK
$^2$ University of Sheffield, Department of Materials Science and Engineering, Portobello Street, Sheffield, S1 3JD, UK
$^3$ Fachhochschule St. Pölten, Matthias Corvinus-Straße 15, 3100 St. Pölten, Austria

E-mail: adam.west@durham.ac.uk

Received 25 October 2011, in final form 30 January 2012
Published 20 February 2012
Online at stacks.iop.org/JPhysD/45/095002

Abstract

We present a new approach to calculating magnetic fringing fields from head-to-head type domain walls (DWs) in planar magnetic nanowires. In contrast to calculations based on micromagnetically simulated structures the descriptions of the fields are for the most part analytic and thus significantly less time and resource intensive. We begin with an intuitive picture of DWs, which is built upon in a phenomenological manner. The resulting models require no a priori knowledge of the magnetization structure, and facilitate calculation of fringing fields without any free parameters. Comparisons with fields calculated using micromagnetic methods show good quantitative agreement. We demonstrate that parameters key to atomic physics applications can easily be calculated with errors of around 10%. The model we present has greatest accuracy and hence utility for distances roughly greater than the width of the DW under consideration.

(Some figures may appear in colour only in the online journal)

1. Introduction

Magnetic domain walls (DWs) are boundaries between areas of differing magnetization direction, with magnetization rotating through the DW width. In patterned magnetic nanowires, DWs separate opposite magnetizations, either in a head-to-head (magnetizations point towards the wall) or tail-to-tail (magnetizations point away from the wall) configuration. The DWs form in two characteristic shapes—transverse or vortex [1, 2]. The former is more common in wires with smaller cross sections. The magnetization structures of typical DWs are shown in figure 1, as calculated via micromagnetic methods. The converging or diverging magnetization of a DW results in an associated effective magnetic monopole moment. Thus, strong magnetic fields are found in close proximity to the DWs. Directly above the walls these fields are directed out of the nanowire’s plane.

The characteristics and dynamics of nanomagnetic DWs have been the subjects of much research, e.g. [4] and are key topics in the burgeoning field of spintronics [5, 6]. The fields of atom optics and atom chips have also experienced rapid expansion and continue to strive towards improved techniques for producing arenas for atomic physics experiments and quantum information processing [7–9]. Within these research fields nanomagnetic DWs have a number of possible applications, such as an atom mirror [10] or mobile atom traps [11]. A factor common to all these areas of study is the need for an accurate knowledge of the magnetic fringing fields generated by DWs. In particular, for atomic physics applications a knowledge of the fields for distances of around 100 nm or greater is desired; at distances closer than this the attractive van der Waals interaction begins to modify significantly the atomic potential [12].

Rigorous micromagnetic methods currently allow accurate computation of magnetization configuration, and hence fields, at all distances. However they are resource and time intensive and require detailed understanding of micromagnetics. We present a more direct method of field calculation—the ‘monopole model’, which affords considerable perspicacity and utility, particularly in the regimes of interest for
applications of DWs in atomic physics experiments. In section 2 we outline the models used, extending to greater complexity in a phenomenological manner. We discuss how emulating the detailed magnetization structure of the wall is not a good way of quickly and accurately computing these fields; the models we present do not demand this and can be used without free parameters. Analysis of the models’ accuracy is described in section 3.

2. Models

Illustrations of the four different models considered are shown in figure 2. The basic ‘monopole model’ is described in section 2.1 and more detailed models are developed in subsequent sections. Derivations are provided in appendix A.

2.1. Monopole model

Although Maxwell’s equations preclude the existence of magnetic monopoles, they have been posited and observed as quasiparticles [13–15]. We will use them as a theoretical construct; at large distances, nanomagnetic DWs can be well approximated as point magnetic charges. This point acts as a source or sink of magnetic field lines—a monopole. This prescription of an effective charge is a treatment followed elsewhere, see e.g. [13, 16, 17]. The charge is then given by

\[ q_m = \pm 2\mu_0 M_s w t, \]

where \( M_s \) is the saturation magnetization, \( \mu_0 \) is the permeability, \( w \) is the wire width and \( t \) is the wire thickness. The full derivation of this expression is provided in appendix A.
a charge $q_m$ has an inverse-square dependence on distance:

$$\vec{B}(\vec{r}) = \frac{q_m}{4\pi|\vec{r}|^2} \vec{r},$$  \hspace{1cm} (2)

where $\vec{r}$ is the unit vector associated with $\vec{r}$. Note that $\vec{B}$ and $\vec{H}$ differ only by a factor of $\mu_0$.

As will be shown in section 3, this model is an excellent approximation at distances that are large compared with the width of the nanowire. We now extend the model to emulate more faithfully the shape of a real DW. This will significantly extend the region over which the model accurately reproduces the shape of DWs' magnetic fields.

### 2.2. 1D domain wall

The first extension to the model is to consider the wall as a 1D object, i.e. a line of charge across the width of the wire as illustrated in figure 2(b). The corresponding expressions for the field are given in equations (3)-(5). These expressions will be shown to offer a significant improvement in the accuracy of field calculations compared with the simple monopole model.

### 2.3. 2D domain wall

We now extend our model to consider the DW to be 2D as a more accurate representation of it as an extended object. In doing so we assume the magnetic charge associated with the DW is derived entirely from volume charge contributions which result from the divergence of the magnetization within the DW. We assume the volume charge to be uniformly distributed within the wall, and neglect effects of surface charges due to the normal component of the magnetization at the nanowire edges. We have observed this type of model to yield more accurate fringing fields than alternative models based on sheets of charge enclosing the DW, as explained at the end of section 2.

Modelling a DW with finite size in $z$ does not give significant improvement; the nanowire thickness is small compared with its width. From hereon we consider DWS with zero size in $z$, but finite extent in $y$, given by $s$ as illustrated in figure 2(c). The corresponding expressions for the magnetic field are shown in equations (6)-(8).

Note that equations (6) and (7) are identical under an exchange of $\{x, w\}$ with $\{y, s\}$, as expected due to symmetry.

$$B_x = \frac{\mu_0 M_I t}{2\pi s} \left[ \frac{w - 2x}{\sqrt{(w - 2x)^2 + 4(y^2 + z^2)}} + \frac{w + 2x}{2\sqrt{(w + 2x)^2 + 4(y^2 + z^2)}} \right],$$  \hspace{1cm} (3)

$$B_y = \frac{\mu_0 M_I t}{\pi(y^2 + z^2)} \left[ \frac{w - 2x}{2\sqrt{(w - 2x)^2 + 4(y^2 + z^2)}} + \frac{w + 2x}{2\sqrt{(w + 2x)^2 + 4(y^2 + z^2)}} \right] + \tan^{-1}\left\{ \frac{2(w - 2x)(s/2 - y)}{\sqrt{(w - 2x)^2 + 4(y^2 + z^2)}} \right\},$$  \hspace{1cm} (4)

$$B_z = \frac{\mu_0 M_I t}{2\pi s} \left[ \frac{s - y}{\sqrt{(s - y)^2 + 4(z^2 + y^2)}} + \frac{s + y}{\sqrt{(s + y)^2 + 4(z^2 + y^2)}} \right] + \tan^{-1}\left\{ \frac{2(w - 2x)(s/2 - y)}{\sqrt{(w - 2x)^2 + 4(y^2 + z^2)}} \right\},$$  \hspace{1cm} (5)

$$B_x = \frac{\mu_0 M_I t}{2\pi s} \left[ \frac{w - 2x}{\sqrt{(w - 2x)^2 + 4(y^2 + z^2)}} + \frac{w + 2x}{2\sqrt{(w + 2x)^2 + 4(y^2 + z^2)}} \right],$$  \hspace{1cm} (6)

$$B_y = \frac{\mu_0 M_I t}{\pi(y^2 + z^2)} \left[ \frac{w - 2x}{2\sqrt{(w - 2x)^2 + 4(y^2 + z^2)}} + \frac{w + 2x}{2\sqrt{(w + 2x)^2 + 4(y^2 + z^2)}} \right] + \tan^{-1}\left\{ \frac{2(w - 2x)(s/2 - y)}{\sqrt{(w - 2x)^2 + 4(y^2 + z^2)}} \right\},$$  \hspace{1cm} (7)

$$B_z = \frac{\mu_0 M_I t}{2\pi s} \left[ \frac{s - y}{\sqrt{(s - y)^2 + 4(z^2 + y^2)}} + \frac{s + y}{\sqrt{(s + y)^2 + 4(z^2 + y^2)}} \right] + \tan^{-1}\left\{ \frac{2(w - 2x)(s/2 - y)}{\sqrt{(w - 2x)^2 + 4(y^2 + z^2)}} \right\},$$  \hspace{1cm} (8)

### 2.4. Triangular domain walls

Considering the magnetic pole distributions, $\vec{V} \cdot \vec{M}$, shown in figure 1, there is a clear difference in the shape associated with the two wall types; whilst the poles, and hence volume charge, of a vortex wall are contained within an approximately rectangular section of the nanowire, the pole distribution of a transverse wall is distinctly triangular. The final extension of our model is to incorporate this characteristic shape, so as to better emulate the field from a transverse wall. This situation is illustrated in figure 2(d).

Equations (A.7) and (A.10) can be modified to incorporate a triangular shape. Unfortunately, it was not possible to derive closed expressions for $B_z$ or $|\vec{B}|$ as with previous models. Whilst numerical integration is quicker than using micromagnetically simulated structures for the same level of resolution, it is significantly slower than previous analytic expressions. Instead, the computation can be sped up considerably by utilizing the result for a rectangular wall. One can approximate the shape of the triangular wall by a series of rectangles and sum the resulting field from each. Computing the field over $10^6$ points takes around 2 s per rectangle. Negligible loss of accuracy was observed using a triangle divided into 40 rectangles.

Whilst the models presented thus far emulate the basic shape of the DWS’ volume charge distributions, they do not take into account the positive and negative regions of
edge charge present in both transverse and vortex walls (figure 1(b)). As can be seen in figure 3, at extremely short range the fields produced by the DWs mimic this complex charge structure. A number of efforts were made to emulate these more subtle features; trapezoidal walls, dominating edge charge regions and simple spatial variations in charge density/polarity were all investigated. However, none of these approaches produced overall improvements in accuracy and therefore we do not describe them in detail here. This finding corroborates the principle that volume charges, characterised by continuous magnetization variation, are seen to better describe micromagnetic systems where the size of the magnetization reversal region is much larger than the magnetic lattice size [18]. We conclude, in agreement with previous findings [19], that the overall shape of a DW’s volume charge is by far the most important feature at all but the very shortest distances. Whilst the accuracy of the models in the near field could undoubtedly be improved by incorporating a detailed magnetization structure, it is precisely this we are trying to circumvent with the development of these simple models.

In the 2D models we use the approximation $s = w$. It is possible to optimize the value of $s$ through a comparison with micromagnetic simulations, however this approach is contrary to the aim of these models and significantly detracts from their utility. We will see later that using an optimized value of $s$ confers only a very small improvement in accuracy. Unless explicitly stated otherwise, $s = w$ will be used throughout. We note for a triangular wall (as with a rectangular one) that the distribution of charge across the nanowire width (coordinate $x_N$) is independent of $s$:

$$d q_m(x_N) = 4 M_s H_{d0} / s \, dA = 4 M_s H_{d0} x_N / w \, dx_N.$$  (9)

This is equivalent to the fact that all triangles have centres of mass at barycentric coordinates $(1/3, 1/3, 1/3)$. The field maximum according to our analytic model is thus found at $(x, y) = (w/3, 0)$ when sufficiently far from the wall. Note also that integrating $d q_m(x_N)$ with respect to $x_N$ yields equation (A.5).

Real DWs exhibit a complex magnetization structure and the model we present is a simplification of this, and is indeed one of many possible representations of a DW. The chosen model assumes that volume charge is the dominant contributor to the fringing fields. We also investigated alternatives based on sheets of charge separating uniformly magnetized domains. Such models also adopt the distinctive triangular (or trapezoidal) form observed in nanomagnetic DWs, but are defined by lines of concentrated charge which

---

**Figure 3.** DW fields calculated via micromagnetic simulations. (a) and (b) are at $z = 200$ nm, whilst (c) and (d) are at $z = 12.5$ nm; $z = 0$ is located in the centre of the wire’s thickness. (a) and (c) correspond to a transverse-type DW (cross section = $200$ nm $\times$ 5 nm), whilst (b) and (d) correspond to a vortex-type wall (cross section = $200$ nm $\times$ 15 nm). At short distances the field adopts a complex shape, indicative of the magnetic charge distribution. Further away the field is significantly smoother and smaller in magnitude. Note in the far field the shape is skewed in $x$ for the transverse DW.
enclose this characteristic shape. Simple models based on such a configuration showed greater error than the one we present so are omitted from the discussion—without significant additional complexity it is unlikely that a greater fidelity is possible using such sheets of charge.

3. Comparison of models

To assess the accuracy of the models presented comparison was made with results from calculations based on micromagnetic simulations. We use a proprietary micromagnetic code that solves the Landau–Lifshitz–Gilbert equation of motion and quasistatic Maxwell equations within a finite element/boundary element framework [20]. We simulate 8.4 μm long nanowire structures, discretised within the DW region into tetrahedral meshes with a 5 nm characteristic size. Physically appropriate DW structures are introduced by imposing simple bi-domain states and then allowing relaxation to equilibrium. The magnetic field profiles above the DWs are calculated analytically from the equivalent dipole charges on the nanowires’ surfaces [11, 21]. Magnetic fields created by effective magnetic charges at the nanowire ends are subtracted from the data using the simple point monopole approximation, which, as we will show later, is extremely accurate in the far field. Figure 3 shows the increase in complexity of the field shape at small heights above the DW. Because of this complex shape the models presented here will always experience inaccuracy when calculating fields at very short distances.

We consider the fields calculated by all four models for a transverse-type DW in a wire with w = 200 nm and t = 5 nm. A saturation magnetization, \( M_s \), of 8.6 \( \times 10^5 \) A m\(^{-1} \) was used throughout [22]. The models were examined over a 1 \( \mu \)m \( \times 1 \) \( \mu \)m \( \times 1 \) \( \mu \)m cube divided evenly into 10\(^6\) points, with the DW centred at the bottom of the cube. This is representative of distances within which atomic physics applications of DWs aim to work; outside this region the field is less than 1 G.

The following analysis examines only the field magnitude for the sake of brevity. Very good accuracy was also observed for the field direction as is discussed briefly towards the end of the section. A summary of all the figures of merit that will be presented is provided in appendix B. Initial comparison was made by considering the maximum field at a given height, shown in figure 4. This is an important quantity in atomic physics applications of DWs, and shows well how the models scale with distance.

From figure 4 it can be seen that at heights greater than 50 nm there is very good agreement (<10% error) with micromagnetic simulations from all but the simplest ‘monopole’ model. There is a stark improvement moving to the 1D case—for \( z \lesssim 300 \) nm there is an order of magnitude improvement in the error. Above this height all the models converge with the micromagnetic simulations, having an error of less than 5%. Extension beyond the 1D model does not afford significantly more accuracy. At distances of <100 nm we see a region where the 1D model offers an improvement in the accuracy of the maximum field compared to the 2D model (the triangular model is better than both the 1D and 2D models), yet we already know that our models suffer in this near-field region. The 2D model could be improved in this region by a different choice of \( s \), but this would be to the detriment of the overall accuracy, which is a more important figure of merit and one we consider now.

A more thorough analysis of the models examines the field over the entire region of space, so the shape of the field is tested. Figure 5 shows the root mean square (RMS) error, \( E_{\text{RMS}} \), over all points at a given height for the different models considered. For the analytic field \( |\vec{B}| \) and the field from micromagnetics, \( |\vec{B}_M| \), over a region \( \{\vec{r}_i\} \) we have

\[
E_{\text{RMS}} = \sqrt{\frac{100^2}{N} \sum_{i=1}^{N} \frac{\left|\vec{B}_M(\vec{r}_i) - |\vec{B}_M(\vec{r}_i)|\right|^2}{|\vec{B}_M(\vec{r}_i)|^2}}.
\]  

(10)

We see that there is a big improvement in adopting a triangular shape; more accurately imitating the shape of the DW produces a more accurate field shape. By mimicking the distribution of the volume charge in the DW the shape of the field is more
accurate—weighting the fringing fields according to this shape is advantageous and lends support to our choice of basing our model on the DW’s volume charge. Typical values of the RMS and mean percentage errors over the 1 $\mu$m$^3$ region are also provided in appendix B.

As can be seen in figure 5, the accuracy of all the models is much worse at small distances, due to the dramatic change in the shape of the fields. An examination of e.g. figure 3 suggests highly localised regions of magnetic charge for both wall types. However, as previously mentioned, improved accuracy is not gained by using a simple representation of these charge distributions. Whilst very good accuracy is achieved for all wire geometries at heights $>100$ nm, some spintronics applications require knowledge of the field very close to the wire, e.g. read/write heads can be located at flying heights of $\sim 10$ nm [20]. This is within the regime where these analytic models break down.

These initial figures of merit are strong indicators of the utility of the model we present, allowing non-expert users to calculate fields at all but the closest distances with a very good level of accuracy. Whilst mean errors indicate well the overall accuracy of models, the best overall representation of the accuracy of the models is achieved by examining the distribution of the error. This is shown for the 200 nm $\times$ 5 nm wire, using the triangle model, in figure 6. At all heights the large majority of points have an error $<20\%$. The error directly above the triangle barycentre (white line) is quite high within the points at a given height; this is the point closest to the wall for a given height.

In an effort to present a figure of merit independent of sample size and distance from the wall centre, we now perform analysis over regions of specific field strength. This removes biases due to changes in the volume of space examined, and will also be independent of nanowire size, whilst still assessing the fidelity of the models to the true field shape.

The regions analysed are shells centred on field isosurfaces. Whilst for some applications the field at a given distance is the focus, there are a number of applications, e.g. atom trapping [11], where the working regime is defined by the field strength itself. The following analysis illustrates the error across these different regimes of field. We compute the error over a region where, according to micromagnetic simulations, $0.9B_0 \leq |B| \leq 1.1B_0$, for some $B_0$. The results of this analysis are shown in figure 7. A familiar trend is observed; the error increases for larger fields (shorter distances), and the error decreases for models which more faithfully replicate the shape of real DWs. For very large fields the differences between models become less clear. This is expected as the shape is not accurately reproduced by any of the models in the near field. Again there is convergence at low fields as the approximation of the DW as a point object becomes increasingly valid. The biggest benefit of using a more complex model is seen in the range $\sim 10$–100 G. It is precisely this region where the shape of the field begins to reflect the asymmetry of the DW shape, but does not show the complex behaviour observed at very short distances.

The analysis presented has used the assumption $s = w$. In appendix B we provide the minimum possible RMS percentage error, $E_{\text{RMS}}$, corresponding to an optimized value of $s$. Comparing this with the RMS error using $s = w$, $E_{\text{RMS}}$, it is clear that there is only a very small loss of accuracy when using this approximation.

Discussion up to this point has been based upon the field size. Similar analysis was also performed for the field direction. This has inherent difficulties as $B_x$ and $B_y$ have zero points at all $z$. None of the models reproduces well the complex field shape at short distances. The error is very similar for all models for a vortex wall. For example, for a 200 nm $\times$ 15 nm wire we observe a mean error of less than $10^\circ$ ($5^\circ$) at heights above 73 nm (195 nm). For a transverse wall there is a significant increase in accuracy when moving to the triangular model. This is because this model reproduces the ‘skewed’ shape of the magnetic field present due to the characteristic

**Figure 6.** The distribution of the percentage error in the field at all heights as calculated by the triangular model for a 200 nm $\times$ 5 nm nanowire. At each height the data are grouped into 100 evenly distributed bins. The white line shows the error directly above the wall barycentre.

**Figure 7.** RMS accuracy of the field within regions centred around field isosurfaces above a transverse DW (200 nm $\times$ 5 nm wire). For a given $B_0$, the error is averaged over points with field within 10% of $B_0$. 

| $B_0$ (G) | RMS Error (%) |
|----------|---------------|
| 10^2      | 10            |
| 10^3      | 10            |
| 10^4      | 10            |
| 10^5      | 10            |
| 10^6      | 10            |

In appendix B we provide the minimum possible RMS accuracy of the field within regions centred around field isosurfaces above a transverse DW (200 nm $\times$ 5 nm wire). For a given $B_0$, the error is averaged over points with field within 10% of $B_0$.
triangular shape of the transverse DW. For example, for the 200 nm × 5 nm we observe a mean error of less than 10^−5 (9°) at heights above 25 nm (125 nm) when using the triangular model. For the rectangular model the corresponding error is 14° and 9°. The remaining inaccuracy is due to the fact that real DWs produce small regions where there is a negative effective charge. For a transverse DW this region is found at the point of the triangle. Because of this the z component of the field has a large associated error, particularly for z < 100 nm; higher than this there is excellent agreement, as with the field magnitude.

The analysis presented here has been for one particular wire geometry. To examine the flexibility of our approach to model nanowires with other dimensions we also considered five other wire geometries, detailed in table 1. Note that the three geometries with larger wire thicknesses correspond to wires which host vortex-type DWs. For these wires the triangular model is no longer appropriate since, as can be seen from figure 1, the magnetic pole distribution for a vortex-type DW is approximately rectangular in shape.

A general trend is observed that there is a loss of accuracy with an increase in nanowire size, shown explicitly in appendix B. This is due to larger wires having larger and more prominent ‘near field’ regions. The distributions of error for all the wires considered were found to be very similar to those shown explicitly earlier.

To demonstrate the utility of the models we consider an example application. Work elsewhere [23] has used micromagnetic techniques to compute time orbiting potentials (TOPs) [24] based on DW fringing fields. Atoms placed in these potentials are harmonically confined with an associated trap frequency, ω, which is directly determined by the shape and size of the fringing fields. We consider the case of a trap formed at a height of 1 μm above a DW and TOP fields in the range 2–10 G. We calculate the trap frequencies using our analytic model and observe good agreement when compared with calculations based on micromagnetics. The average and maximum errors between the analytic and micromagnetic models are provided in table 2. We see that the analytic model works well for wire widths up to 400 nm, with errors of less than 15%. As the distance from the wall becomes comparable to its width, and the region in which we compute enters the ‘near field’ region of the DW, the accuracy of the model reduces, as observed in the case of a vortex type 800 nm × 10 nm wall. However, there is a large range of parameters and distances over which the model exhibits utility.

### 4. Conclusions

A series of analytic models of fields from DWs in nanomagnetic structures have been derived and developed to incorporate the characteristic shape of transverse-type walls. The results obtained from these models were compared with micromagnetic simulations. Improved accuracy was observed for models incorporating higher dimensionality in DW structure, reflecting the fact that DWs are not simple point-like objects. We have also shown that no a priori knowledge is required that might limit the usefulness of these models; assuming the wall width (along the wire) to be equal to the wire width results in negligible loss of accuracy.

Data from analytic models show better agreement at larger distances from the nanomagnetic structure—regions where the field is smaller. This is intuitively expected as further from the DW the approximation of it being a point object is increasingly accurate. At very short range more detailed structures are observed in micromagnetic simulations. The models presented in this paper do not reproduce this.

Examples have been presented where an RMS error over an extended region of interest of less than 5% is achieved. For fields up to ∼100 G an error of less than 10% can be achieved. The maximum field for a given height is reproduced accurately by all but the simplest of models. Similar accuracy is also obtained when considering the direction of the field.

The models presented give an efficient and intuitive way of calculating the fields from DWs in nanowires. The loss of accuracy compared with detailed micromagnetic simulations is small. The methods provided are quicker and much more accessible than existing techniques. We anticipate that the models presented will be useful to those studying nanomagnetic fringing fields as an efficient method of calculating such fields away from the ‘near field’ region. There may also be use for this type of model in studying magnetic thin films which exhibit domains with high shape anisotropy, recasting interesting phenomena such as topological phase transitions in terms of magnetic charges.

### Acknowledgments

The authors gratefully acknowledge financial support from EPSRC under grants EP/F025459/1 and EP/F024886/1.
Appendix A. Fringing field expressions

Here we provide the derivations of the expressions for the magnetic field for all the presented models. The symbols used throughout are defined as follows: $M_s =$ saturation magnetization, $\mu_0 =$ permeability of free space, $w =$ wire/DW width, $t =$ wire/DW thickness, $s =$ DW length and $\{x, y, z\} =$ spatial coordinates.

A.1. Simple monopole

In the presence of a magnetic medium we have from Maxwell’s equations:

$$\mu_0 \nabla \cdot \left( \vec{H} + \vec{M} \right) = 0,$$

(A.1)

where $\vec{H}$ is the magnetic field and $\vec{M}$ the magnetization. Considering the wall as an extended object (figure 2(a)) with a volumetric magnetic charge density $\rho_m$ we have in analogy to Gauss’ law for electrostatics

$$\nabla \cdot \vec{H} = \rho_m/\mu_0,$$

(A.2)

The charge within the DW takes the form of a bulk region of volume charge and concentrated regions of edge charge (cf figure 1). If we consider a volume that encloses the entire DW the total magnetic charge within this volume, $q_m$, is then given by

$$q_m = -\mu_0 \int \nabla \cdot \vec{M} \, dV.$$

(A.3)

By assuming that the DW is a discontinuous magnetization reversal we can reformulate the total charge solely in terms of the magnetization flux through a surface $S$ enclosing $V$, thus yielding a simple expression for $q_m$. Using the divergence theorem:

$$q_m = -\mu_0 \int \vec{M} \cdot \hat{n} \, dS,$$

(A.4)

where $\hat{n}$ is the unit normal of the surface element $dS$ with $S$ enclosing $V$. Evaluation of equation (A.4) yields the charge associated with a head-to-head wall,

$$q_m = 2\mu_0 M_s w t.$$

(A.5)

A tail-to-tail wall has a charge of $-q_m$. Equation (A.5) has a form similar to other expressions for effective magnetic charge, cf e.g. [16]. The magnetic field at a position $\vec{r}$ is then given, in direct analogy to Coulomb’s law, by

$$\vec{B}(\vec{r}) = \frac{\mu_0 M_s w t}{2\pi |\vec{r}|} \hat{r}.$$  

(A.6)

A similar application of the divergence theorem can also be used to show that the surface charge density due to the normal component of magnetization at a surface is given by $\sigma_m = -\mu_0 \vec{M} \cdot \hat{n}$. The total charge distribution of a magnetic object is thus generally due to both discontinuous changes in $\vec{M}$ at the object’s edges ($\sigma_m$), and continuous changes within the volume of the object ($\rho_m$). The relative contributions of volume and surface charges are governed by the system’s characteristic size, and hence, the relative sizes of the exchange and anisotropy energies [18]. For planar magnetic nanowires volume charge is the dominant contributor.

A.2. 1D model

We now extend the DW from a point charge to a line of charge. Infinitesimal elements of this charge, $dq_m$, contained in a length $dx_N$, are given by

$$dq_m = 2\mu_0 M_s dx_N.$$  

(A.7)

The contributions from across the entire wire width are summed in the following integral to give the components of magnetic field, $B_i$:

$$B_i = \frac{\mu_0 M_s t}{2\pi} \int_{-w/2}^{w/2} r_i \frac{1}{|\vec{r}|^3} \, dx_N,$$

(A.8)

where we define $\vec{r} = (r_x, r_y, r_z) = (x - x_N, y - y_N, z)$ the vector from an infinitesimal charge element located at $(x_N, y_N, 0)$ to the point under consideration $(y_N = 0$ in this case). This then yields the expressions given in equations (3)–(5).4

A.3. 2D rectangle

The final extension to the model which we derive analytically is to consider the DW as a 2D object in the $xy$ plane, as in figure 2(c). In doing so we assume that the contributions of magnetic charge come solely from volume charge. Alternative representations using sheets of surface charge to represent the DW were investigated and were seen to be less accurate. The wall size can now vary arbitrarily along the wire length, to have width $s$. We choose to fix the value of $s$ to be $w$, the wire width. We rescale $dq_m$ to reflect the change in DW size. By analogy to equation (A.7) we have

$$dq_m = 2\mu_0 M_s t/w \, dx_N dy_N,$$

(A.9)

which is the infinitesimal element of charge associated with an area element $dA = dx_N dy_N$. Using this expression the total field is then given by integrating over all $dA$:

$$B_i = \int_{-w/2}^{w/2} \int_{-w/2}^{w/2} \frac{\mu_0 M_s t r_i}{2\pi w |\vec{r}|^3} \, dx_N dy_N.$$  

(A.10)

Evaluation of these integrals was performed symbolically [25], see also e.g. [26].

Appendix B. Figures of merit

The following table shows all the figures of merit for the six different wire sizes according to the various models.

4 An alternative route to deriving these expressions is to use the magnetic scalar potential and integrating in an entirely analogous manner; the magnetic field is then simply the negative of the gradient of the scalar potential. Whilst entirely equivalent, within this paper we integrate expressions for the magnetic field directly throughout for the sake of clarity.
Table B1. A summary of the figures of merit. $E_{\text{RMS}}$ is the RMS error over all points, $E'_{\text{RMS}}$ is the RMS error with optimal $s$, $E_m$ is the mean percentage error, $E_{\text{MaxB}}$ is the RMS error in the maximum field for a given height. The labels A-F refer to the wire geometries detailed in Table 1.

| Model            | Quantity | A  | B  | C  | D  | E  | F  | A  | B  | C  | D  | E  | F  | A  | B  | C  | D  | E  | F  |
|------------------|----------|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|
| Monopole         | $E_{\text{RMS}}$ (%) | 11 | 20 | 67 | 6  | 21 | 129| 11 | 18 | 35 | 5  | 17 | 59 | 10 | 16 | 30 | 4  | 10 | 26 |
|                  | $E'_{\text{RMS}}$ (%) |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    | 10 | 16 |
|                  | $E_m$ (%) | 7  | 10 | 21 | 2  | 6  | 24 | 7  | 10 | 19 | 2  | 6  | 22 | 7  | 10 | 17 | 1  | 3  | 8  |
|                  | $E_{\text{MaxB}}$ (%)| 110| 169| 355| 116| 330| 720| 15 | 10 | 14 | 27 | 52 | 85 | 3  | 4  | 10 | 7  | 10 | 15 |

References

[1] McMichael R D and Donahue M J 1997 *IEEE Trans. Magn.* 33 4167–9
[2] Nakatani Y, Thiailee A and Miltat J 2005 *J. Magn. Magn. Mater.* 290–291 750–3
[3] http://math.nist.gov/oommf/
[4] Atkinson D, Allwood D A, Xiong G, Cooke M D, Faulkner C C and Cowburn R P 2003 *Nature Mater.* 2 85–7
[5] Allwood D A, Xiong G, Faulkner C C, Atkinson D, Petit D and Cowburn R P 2005 *Science* 309 1688–92
[6] Parkin S S P, Hayashi M and Thomas L 2008 *Science* 320 190–4
[7] Hinds E A and Hughes I G 1999 *J. Phys. D: Appl. Phys.* 32 R119–46
[8] Fortágh J and Zimmermann C 2007 *Rev. Mod. Phys.* 79 235–89
[9] Adams C S, Sigel M and Mlynek J 1994 *Phys. Rep.* 240 143–210
[10] Hayward T J et al 2010 *J. Appl. Phys.* 108 043906
[11] Allwood D A, Schrefl T, Hrkac G, Hughes I G and Adams C S 2006 *Appl. Phys. Lett.* 89 014102
[12] Mohapatra A K and Unnikrishnan C S 2006 *Europhys. Lett.* 73 839–45
[13] Castelnovo C, Moessner R and Sondhi S L 2008 *Nature* 451 42–45
[14] Bramwell S T, Gihlbl S R, Calder S, Aldus R, Prabhakaran D and Fennell T 2009 *Nature* 461 956–9
[15] Mengotti E, Heyderman L J, Rodriguez A F, Nolting F, Hugli R V and Braun H-B 2011 *Nature Phys.* 7 68–74
[16] Ladak S, Read D, Tyliszczak T, Branford W R and Cohen L F 2011 *New J. Phys.* 13 023023
[17] Hayward T J, Bryan M T, Fry P W, Fundi P M, Gibbs M R J, Im M-Y, Fischer P and Allwood D A 2010 *Appl. Phys. Lett.* 96 052502
[18] Barbara B 1993 *J. Magn. Magn. Mater.* 129 79–86
[19] Hayward T J, Bryan M T, Fry P W, Fundi P M, Gibbs M R J, Allwood D A, Im M-Y and Fischer P 2010 *Phys. Rev.* B 81 020410
[20] Schrefl T, Schabes M E, Susse D and Stehno M 2004 *IEEE Trans. Magn.* 40 2341–234
[21] Rech M 2002 *Diplomarbeit Thesis* Rheinisch Friedrich-Wilhelms Universität
[22] Chikazumi S 1997 *Physics of Ferromagnetism* (New York: Oxford University Press) p 603
[23] Hayward T J, West A D, Weatherill K J, Schrefl T, Hughes I G and Allwood D A 2011 *J. Appl. Phys.* 110 123918
[24] Petrich W, Anderson M H, Ensher J R and Cornell E A 1995 *Phys. Rev. Lett.* 74 3352–5
[25] Wolfram Research, Inc., 2008 *Mathematica*, Version 7.0, Champaign, IL
[26] Gradshteyn I S and Ryzhik I M 1980 *Table of Integrals, Series, and Products*, corrected and enlarged edition ed A Jeffrey (San Diego, CA: Academic) p 81