Chandrasekhar-Clogston limit and critical polarization in a Fermi-Bose superfluid mixture

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We study mixtures of a population-imbalanced strongly-interacting Fermi gas and of a Bose-Einstein condensed gas at zero temperature. In the homogeneous case, we find that the Chandrasekhar-Clogston critical polarization for the onset of instability of Fermi superfluidity is enhanced due to the interaction with the bosons. Predictions for the critical polarization are given in the trapped case, with a special focus to the situation of equal Fermi-Bose and Bose-Bose coupling constants, where the density of fermions becomes flat in the center of the trap. This regime can be realized experimentally using Feshbach resonances and is well suited to investigate the emergence of exotic configurations, such as the occurrence of spin domains or the FFLO phase.

I. INTRODUCTION

The property of fermions interacting with a Bose fluid has been a long-standing subject of research in condensed matter physics, dating back to the study of $^3$He-$^4$He mixtures [1]. With the recent development of research activity in ultracold gases, it is now possible to experimentally create mixtures of degenerate bosonic and fermionic atomic gases [2–11]. Very recently, the first experimental realization of a superfluid Bose-Fermi mixture was reported [12], the Fermi gas being at the unitarity limit.

There are several theoretical works on mixtures of superfluid Bose gases interacting with spin-1/2 Fermi gases [13–17], but the behavior of coexisting superfluid Fermi and Bose gases in the case of strong Fermi-Fermi interaction has not yet been considered in the literature. Furthermore, since spin-imbalanced fermions are predicted to give rise to exotic phases such as the Fulde-Ferrell-Larkin-Ovchinnikov (FFLO) phase [18–20], it is of great interest to investigate how their behavior is modified by the interaction with bosons.

In this paper, we show that in a homogeneous configuration the Chandrasekhar-Clogston critical polarization for the breakdown of superfluidity is larger than in the absence of the bosonic component [21]. We then consider the case of a harmonically trapped configuration: when the Bose-Bose and Bose-Fermi interactions are equal, the fermionic density in the region of coexistence with bosons becomes flat, because the interaction with bosons exactly compensates the external trapping potential [22]. We investigate the phase diagram of the trapped gas when the fermion imbalance is varied, and show that, for a finite range of polarization, the fermionic density in the Bose-Fermi coexistence region can become inhomogeneous.

II. HOMOGENEOUS SYSTEM

The balanced unitary Fermi gas is known to be fully superfluid at zero temperature. As one increases the polarization, it has been observed that the system phase separates into a balanced superfluid phase and an imbalanced normal phase [23]. The two phases have different densities, and the equilibrium conditions between the two phases fix the ratio $x$ between the density of the minority species over the density of the majority species in the normal phase, which determine the Chandrasekhar-Clogston limit. At zero temperature, this critical ratio turns out to be, at unitarity, $x \approx 0.4$ [21, 24, 25]. As we show, this value is modified by the interaction with bosons.

We assume that the Fermi gas is phase separated into a superfluid phase with density $n_s$ for both species and a normal phase with density $n_{\uparrow}$ and $n_{\downarrow}$ for the spin-up (majority) and spin-down (minority) fermions, respectively. The density of the coexisting bosons in the Fermi superfluid phase is $n_{bn}$ and that in the normal phase is $n_{bn}$. Later we discuss the stability conditions for such configurations. We assume that both the bosonic and fermionic species can be described within the local density approximation and both the Bose-Bose and the Bose-Fermi interactions are weak enough to be treated within the mean field approximation. Then the energy density in the superfluid phase ($E_s$) and in the normal phase ($E_n$) takes the form

$$E_s = \frac{gh_b}{2} n_{bn}^2 + 2g_{bf}n_{bn}n_s + e_s[n_s],$$

$$E_n = \frac{gh_b}{2} n_{bn}^2 + g_{bf}n_{bn}(n_{\uparrow} + n_{\downarrow}) + e_n[n_{\uparrow}, n_{\downarrow}],$$

where $g_{bb} \equiv 4\pi\hbar^2 a_{bb}/m_b$, assumed to be positive, and $g_{bf} \equiv 2\pi\hbar^2 a_{bf}/m_f$ are, respectively, the Bose-Bose and spin-independent Bose-Fermi interaction coupling constants. The Bose-Bose and Bose-Fermi scattering lengths are $a_{bb}$ and $a_{bf}$, respectively, and $m_s \equiv m_b + m_f$ where $m_b$ and $m_f$ are the boson and fermion masses, respectively. The Fermi energy density in the superfluid phase is given by the universal form

$$e_s[n_s] \equiv \frac{6}{5} \frac{\hbar^2}{2m_f} (6\pi^2 n_s)^{2/3} n_s,$$
where $\xi = 0.370$ is the Bertsch parameter. For the normal phase we use the expansion in the parameter $x \equiv n_b/n_f$ introduced in [21]:

$$e_n[n_f,n_b] \equiv \frac{3}{5} \varepsilon_{F_f} n_f \left( 1 - \frac{5}{3} A_x + \frac{m_f}{m_b} x^{5/3} + F x^2 \right) \equiv \frac{3}{5} \varepsilon_{F_f} n_f \varepsilon(x),$$

(3)

where $\varepsilon_{F_f} \equiv (\hbar^2 / 2m_f)(6\pi^2 n_f)^{2/3}$ is the non-interacting Fermi energy of the majority species and for the parameters in $\varepsilon(x)$ we use $A = 0.615$, $m^*/m_f = 1.20$, and $F = (5/9)A^2$, determined by diagramatic methods and Monte-Carlo calculations [28–30]. Using different sets of parameters would not change our results significantly. The equilibrium between the two phases is determined by matching the pressure and the chemical potentials for both bosons and fermions at the interface, which leads to the following conditions for $x$ and $y \equiv n_s/n_f$:

$$\xi y^{2/3} - 2G y - \frac{1}{2} \varepsilon(x) - \frac{3}{10} \varepsilon'(x)(1-x) + G(1+x) = 0,$$

$$2G y^2 - \frac{4}{5} \xi y^{5/3} - G \frac{(1+x)^2}{2} + \frac{2}{5} \varepsilon(x) = 0,$$

(4)

where $\varepsilon'(x) \equiv d\varepsilon(x)/dx$ and $G \equiv n_f g_{ff}^2 / (\varepsilon_{F_f} g_{bb})$ is a dimensionless parameter independent of the bosonic density. As a consequence also the critical ratios $x$ and $y$ are independent of the boson density, provided there are background bosons with nonzero densities in both phases. The parameter $G$ has an important physical meaning, corresponding to the ratio between the change in the energy of fermions caused by the induced interaction $-g_{ff}^2 / g_{bb}$ in the static limit and the non-interacting Fermi energy. The existence of two real solutions for $x$ and $y$ for $G$ is ensured for $0 \leq G \leq G_{\text{max}} \approx 0.089$ and in Fig. 1 we plot the resulting values of $x$ and $y$ as a function of $G$. When $G = 0$, the critical ratio $x \approx 0.40$ coincides with the value obtained in the absence of Bose-Fermi interaction ($g_{ff} = 0$). As $G$ becomes larger, the value of $x$ decreases reaching the minimum value of $x \approx 0.30$, which means that the superfluid phase of fermions is stabilized by the interaction with bosons. The ratio $y$, on the other hand, increases with $G$ reaching the maximum value of $y \approx 2.68$, which implies that the density jump at the interface of the two phases becomes larger; the maximum value of the jump, corresponding to $G = G_{\text{max}}$, is $2n_s/(n_f+n_b) \approx 4.1$ to be compared with the value $\approx 1.5$ when $G = 0$.

The nonexistence of real solutions when $G > G_{\text{max}}$ is related to the occurrence of dynamical instability in the fermionic superfluid phase caused by the interaction with bosons. The dynamical stability of the superfluid phase requires that the following inequality be obeyed [31]:

$$\frac{\delta^2 e_s[n_s]}{\delta n_s^2} - 4 \frac{g_{ff}^2}{g_{bb}} > 0,$$

(5)

which is equivalent to imposing $\xi/3y^{1/3} > G$. We have checked that the condition for having real solutions for $x$ and $y$ coincides with the one ensuring dynamical stability. If $G$ becomes larger than $G_{\text{max}}$, the superfluid Fermi gas and the Bose gas are expected to phase separate.

### III. TRAPPED SYSTEM

Let us now consider the case of a trapped quantum mixture. In the absence of bosons, it is known that as one introduces a small imbalance between the two species, the central part of the trap remains superfluid, and the outer shell is turned into a normal state [21, 23]. When the imbalance is large enough, the whole Fermi gas is in the normal state.

In the presence of bosons, the situation can change significantly. The energy of a highly polarized Fermi gas interacting with a BEC gas is given, within the local density approximation (LDA), by

$$E = \int_{r < R_b} d^3r \left[ \frac{g_{bb}}{2} n_b^2(r) + [V_b(r) - \mu_b] n_b(r) ight.$$ 

$$+ g_{ff} n_b(r) [n_t(r) + n_s(r)] + e_n[n_t(r), n_s(r)]$$ 

$$+ [V_f(r) - \mu_t] n_t(r) + [V_f(r) - \mu_s] n_s(r)]$$ 

$$+ \int_{R_b < r} d^3r \left\{ e_n[n_t(r), n_s(r)] + [V_f(r) - \mu_t] n_t(r) ight.$$ 

$$+ [V_f(r) - \mu_s] n_s(r) \right\},$$

(6)

where $R_b$ is the radius at which the boson density vanishes and $V_b(r)$ and $V_f(r)$ are the harmonic traps for bosons and fermions, respectively [33]. The densities of boson, spin-up fermion, and spin-down fermion are $n_b(r)$, $n_t(r)$, and $n_s(r)$, respectively, and the corresponding chemical potentials are labelled, respectively, with $\mu_b$, $\mu_t$, and $\mu_s$.

Taking the variation of the energy with respect to $n_b(r)$, $n_t(r)$, and $n_s(r)$ in the Bose-Fermi coexistence
When the imbalance is small, most of the fermions are in the superfluid phase, and one can write down a similar equation for the region in the core. As in the highly polarized case, one can choose an external potential of harmonic form, the bosonic density, for the fermions feel an anti-trapping potential in the core and their density will increase when one moves away from the center.

For intermediate values of the population imbalance, coexistence of the superfluid and the normal phase takes place in the core region, giving rise to inhomogeneity and new interesting physics. Inhomogeneity in the core can be reached either by starting with a balanced superfluid-gas and gradually decrease the number of minority fermions till the normal part enters the core, or by starting with a completely polarized gas and gradually increase the number of minority fermions till a superfluid phase region in the core is favorable. In Fig. 3, the critical polarization as a function of \( \rho \) for different values of \( N \) and \( N_f \) is shown in units of 1/\( l_{ho} \). The length is in units of \( l_{ho} \), and the density of particles is in units of 1/\( l_{ho} \).

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We now discuss the possible scenarios characterizing the inhomogeneous phase for intermediate values of population imbalance (see Fig. 3). The simplest possibility, hereafter called the superfluid-normal (S-N) scenario, is that the core is phase separated into a central superfluid and an outer normal phase. The equilibrium condition between the superfluid phase and the normal phase turns out to be determined by the same conditions holding for the homogeneous mixture. Another possibility, hereafter called the normal-superfluid-normal (N-S-N) scenario, is that the core is phase separated into a central normal phase and an outer superfluid phase, while the tail is normal. The two scenarios have very similar energies, and can be easily distinguished in experiments by measuring the doubly-integrated column densities.

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The emergence of the FFLO phase. Indeed the local chemical potential for fermions is constant over the flat region, therefore phases which can exist only within a narrow range in the chemical potential could be observed in the core.

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