I. INTRODUCTION

Cherenkov-drift instability was suggested by Kazbegi, Machabeli & Melikidze as a possible mechanism for generation of pulsar radio emission and later it was approved in [5]. In those works the linear theory of Cherenkov-drift instability was developed. It was shown, that in the pulsar magnetosphere due to Cherenkov-drift instability the orthogonally polarized plasma waves are excited. These waves can escape from the magnetosphere and reach observer as a pulsar radio emission. The necessary condition for the development of Cherenkov-drift instability (as for an usual Cherenkov instability) is a presence of a beam of particles in the relativistic magnetized pair plasma (consisted of relativistic electrons $e^{-}$ and positrons $e^{+}$).

Generally, Cherenkov type instabilities develop due to a resonant interaction between waves and particles of a beam. The resonance occurs, when the electric field vector $E$ and the wave vector $k$ of generated waves have got components along direction of the beam velocity $v$ ($E \cdot v \neq 0$ and $k \cdot v \neq 0$). So, in the magnetized plasma the transverse waves ($E \perp B \perp k$) propagating along the external magnetic field ($B_{0} \parallel k$) cannot be generated by an usual Cherenkov instability (because it develops on the beam particles moving along the external straight magnetic field lines: $v \parallel B_{0}$ and $E \cdot v = 0$).

Cherenkov-drift instability develops when the beam particles move along slightly curved magnetic field (SCMF) lines and, hence, drift across the plane where the curved lines lie. The drift motion of beam particles provokes generation of purely transverse as well as longitudinal-transverse waves.

Generally there are two most important effects caused by the the particle relativistic motion along the SCMF line: curvature drift and curvature radiation. The drift velocity is directed across the plane of the SCMF lines and is given by the following expression:

$$u_{d} = \frac{\gamma v_{c}^{2}}{\omega_{B} R_{B}}$$

(1)

Here $u_{d}$ denotes the drift velocity of electrons (positrons drift with the same velocity but in the opposite direction); $\omega_{B} = eB/mc$ is the cyclotron frequency of electrons; $R_{B}$ is the curvature radius of the magnetic field line; $\gamma$ is Lorentz factor of a particle; $c$ is speed of light and $v_{\parallel}$ is the component of $v$ along the magnetic field line. If $\gamma \gg 1$, the value of drift velocity $u_{d}$ could be significant.

A single particle, moving along the curved field line, radiates so called curvature radiation which can be easily described as a synchrotron radiation in an effective magnetic field (see e.g. [4]). In 1975 Blandford investigated the curvature radiation of plasma flowing along the SCMF lines. The problem was studied in the limit of infinite magnetic field $B_{0} \rightarrow \infty$ and it was shown that there is no radiation at all: the waves, radiated by each particle, are absorbed by another one. This result was confirmed later in papers [2, 3, 4, 12, 13], where spatially unlimited plasma flow was considered. Then in the paper by Asseo, Pellat and Sol the sharp boundary was assumed at the edge of the flow propagating along the curved field line and the possibility of the waves excitation was shown at this boundary. If plasma flow has zero width the instability is reduced to that of Goldreich-Keeley [13].

Development of Cherenkov type instability, taking into account the curvature drift motion, was studied and growth rate was calculated in [1, 2, 3, 4, 8, 9, 10, 11, 12, 13].
for different particular cases. These results were confirmed later after thorough investigation of the problem in [2, 3, 17]. The instability was called a Cherenkov-drift instability.

Presence of the curved magnetic field lines is the necessary condition for both the Cherenkov-drift radiation in plasma and the single particle curvature radiation in vacuum. However, the Cherenkov-drift radiation still could not be interpreted as a plasma curvature radiation, analogous to the single particle curvature radiation: as it will be shown below, the Cherenkov-drift radiation is not generated in the case of \( u_d \to 0 \). On the other hand, it is evident that the drift velocity equals to \( u_d \approx 0 \) if we assume that \( B_0 \to \infty \). However a single particle radiates even for the infinite intensity of magnetic field. Moreover, the single particle radiates the vacuum wave, while the proper waves of the medium (i.e. relativistic electron-positron plasma) are generated in the case of Cherenkov-drift instability. This particular point was not considered in the works by Blandford [2] and Melrose [3]. Polarization of these waves strongly differs from that of vacuum waves.

Brief examination of the linear theory of Cherenkov-drift instability is discussed in section 2. In section 3 the quasilinear equations for the Cherenkov-drift instability are obtained. In section 4 coefficients describing diffusion of particles in momentum space are evaluated. Alteration of plasma distribution function is studied. The results are summarized in section 5.

II. THE LINEAR THEORY

Properties of relativistic electron-positron plasma were carefully investigated in series of works [27, 3, 2, 28] and are still the subject of concern in [24, 23]. The electron-positron plasma of pulsar magnetosphere consists mainly of two components: a bulk of plasma particles with high density and low Lorentz factors (\( \gamma_p \sim 3 \div 10 \)) and a beam of the primary particles ejected from the stellar surface. Density of the beam \( n_b \sim n_{GJ} \), where \( n_{GJ} \) is so called Goldreich-Julian density. Density of secondary plasma \( n_p \sim \Sigma \cdot n_b \), where \( \Sigma \) is the Sturrock ‘multiplication factor’ (\( \Sigma \sim 10^{2} \div 10^{3} \)) [24].

The electron-positron plasma differs from the electron-ion plasma by lack of gyrotropy. Consequently, spectra of waves propagating in the \( e^- e^+ \) plasma is simpler than that in the electron-ion plasma. It consists of only four types of waves which correspond to four branches on the diagram \( \omega(k) \), where \( \omega \) is the wave frequency (see Fig.1).

One of them is high-frequency transverse electromagnetic wave totally located in subluminal area. Its phase velocity \( v_{ph} = \omega/k \) exceeds speed of light \( c \), hence, it is not of interest in our discussion below.

The second branch represents the dispersion of purely transverse linearly polarized electromagnetic wave. We call this dispersion curve \( t \) mode. It is totally located in subluminal area and therefore could be generated by beam particles. Its electric field vector \( \mathbf{E} \) is perpendicular to the plane of wave vector and external magnetic field \( \mathbf{k}, B_0 \).

The remaining two dispersion curves on the \( \omega(k) \) diagram describes the longitudinal-transverse waves propagating in relativistic \( e^- e^+ \) plasma. One of them is almost superluminal. This wave is purely longitudinal if it propagates strictly along the magnetic field line \( \mathbf{k} \parallel \mathbf{E} \parallel B_0 \), and, in this case called Langmuir wave associated with longitudinal oscillations of the charge density. If an angle \( \vartheta \) between \( \mathbf{k} \) and \( B_0 \) increases, the component of wave electric field \( \mathbf{E} \) starts to grow across \( k \). Langmuir wave transforms to the longitudinal-transverse wave denoted in Fig.1 as \( lt_2 \) mode. If the angle \( \vartheta \) is small enough (\( \vartheta \leq \vartheta_0 \sim \omega_p^2/\omega_B^2 \gamma_p^3 \)), where \( \omega_p = \sqrt{4\pi n_p e^2/m} \) is the Langmuir frequency), \( lt_2 \) mode is almost longitudinal and crosses \( \omega = kc \) line. In this case, \( lt_2 \) mode could be excited, if Cherenkov resonance condition \( \omega = k_\parallel \) is fulfilled. However, for the resonant particles of primary beam, growth rate of the instability is very small [27]. The wave leaves from the interaction area so quickly that no time is left for significant growth of the wave amplitude. In the case of oblique propagation, \( \vartheta > \vartheta_0 \), \( lt_2 \) mode is totally superluminal. Therefore, \( lt_2 \) mode could not be generated at all by particles of beam.

Another longitudinal-transverse wave is denoted in Fig.1 as \( lt_1 \) mode. This mode, like \( t \)-wave, is located totally in subluminal area and can easily be generated by plasma particles. Its electric field vector \( \mathbf{E}^{lt} \) is located in \( \mathbf{k}, B_0 \) plane. \( lt_1 \) mode is vacuum wave, if it propagates along the magnetic field lines \( \mathbf{k} \parallel \mathbf{B}_0 \). Its dispersion curve merges with \( t \) mode (see Fig.1) and can be arbitrarily polarized. In the case of oblique propagation, electric field of \( lt_1 \) wave has the component \( E_\parallel \) along the external magnetic field, thereby involving plasma particles in longitudinal oscillations.

Generation of \( lt_1 \)-mode, propagating in perpendicular direction to the plane of SCMF lines, is connected with the drift motion of the particles. These waves are also known as drift waves [4, 5, 28].

It should be mentioned that, while describing the waves in relativistic \( e^- e^+ \) plasma, some authors sometimes use terminology which, in our opinion, appears to be misleading. For example, since the work by Arons and Barnard [22], the superluminal longitudinal-transverse wave \( (lt_2 \) mode) was called an ordinary \( (O) \) mode, the subluminal transverse wave \( (t \) mode) – an extraordinary \( (X) \) mode and the subluminal longitudinal-transverse wave \( (lt_1 \) mode) – an Alfvén mode. However ‘ordinary’ and ‘extraordinary’ are generally related to the waves propagating across the external magnetic field in usual electron-ion plasma [29]. Moreover, \( t \) wave (so called \( X \) mode) is the purely transverse wave and its analogue does not exist in the electron-ion plasma. In electron-positron plasma the terminology ‘ordinary’ and ‘extraordinary’ are used for the waves propagating almost along the magnetic field. As for the Alfvén mode, in electron-ion plasma such a name is used for an almost linearly po-
FIG. 1: Dispersion curves of the waves in the relativistic magnetized electron-positron plasma. The solid line corresponds to \( \omega = kc \). It divides the plane \((\omega, kc)\) into superluminal and subluminal areas. The high-frequency branch of \( t \) waves (in the superluminal area) is defined as HF mode. Low-frequency and high-frequency modes of \( lt \) waves are defined as \( lt_1 \) and \( lt_2 \) modes respectively.

To obtain expression for dielectric permittivity tensor components \( \epsilon_{ij}(\omega, k) \) in the case of SCMF lines, it is handy to consider the problem in cylindrical coordinates \((x, r, \varphi)\) and direct \( x \) axis perpendicularly to the plane of the curved magnetic field lines (see Fig. 2). In the papers by Kazbegi, Machabeli and Melikidze, it was shown that \( t \) and \( lt \) waves could be generated by particles of the beam when the following resonance condition is satisfied (for the resonant values of parameters \( \omega = \omega_0; v_\varphi = v_0 \) and \( u_d = u_0 \)):

\[
\Delta \omega \equiv \omega - k_\varphi v_\varphi - k_x u_d = 0. \tag{2}
\]

Mechanism of wave generation is a modification of the well-known beam-plasma instability. However, this instability differs significantly from the usual beam-plasma instability [1].

The expressions for the growth rates of the different waves are as follows:

\[
\Gamma_k^t = \frac{\pi}{2} \frac{\omega_b^2}{\omega_k} \frac{\gamma_b}{\gamma_p} \frac{k_x^2}{k_\perp^2} \tag{3}
\]

for transverse \((t)\) wave and

\[
\Gamma_k^{lt} = \frac{\pi}{2} \frac{\omega_b^2}{\omega_k} \frac{\gamma_b}{\gamma_p} \frac{k_x^2}{k_\perp^2} \tag{4}
\]

for longitudinal-transverse \((lt)\) wave (here \( \omega_b \) is Langmuir frequency of resonant particles of the beam). These expressions were obtained in papers [1, 3, 15]. For growth rate of the drift wave, it was obtained in paper [2]:

\[
\Gamma_d = \left( \frac{3}{2} \frac{n_b}{n_p} \right)^{1/2} \left( \frac{\gamma_p^3}{\gamma_b} \right)^{1/2} k_x u_d. \tag{5}
\]

The reason for generation of \( t \), \( lt \) and drift waves is presence of the beam in the relativistic pair plasma, although the waves cannot be excited without drift motion of particles. Indeed, expressions (3), (4) and (5) are equal to zero if \( u_d = 0 \). All these waves (see Eqs. 3-5) are purely

polarized, transverse electromagnetic wave with frequency \( \omega \ll \omega_B \), where \( \omega_B \) is the cyclotron frequency of ions).

In the case of \( k \to \infty \) and \( \omega \to \omega_B \), this wave transforms into right-hand polarized ion-cyclotron mode with frequency \( \omega \approx \omega_B \) and left-hand polarized electron-cyclotron wave with frequencies \( \omega \approx \omega_{Be} \). The latter case corresponds to the fast magnetosonic wave and is called helicon for frequencies \( \omega_B < \omega < \omega_{Be} \). Therefore, in relativistic \( e^- e^+ \) plasma, we prefer to call the dispersion curves \( t \), \( lt_2 \) and \( lt_1 \) modes respectively, hence avoiding a possible confusion with dispersion curves in electron-ion plasma.
or almost transverse waves. For \( t \) and \( t_1 \) waves the electric field vector is perpendicular to the external magnetic field (see Eqs. 3,4). As for the waves generated by the usual beam instability, both their electric field vector \( E \) and the wave vector \( k \) are directed along the external magnetic field \( B_0 \).

In the following section we study the quasilinear equations which are significantly different from those of the usual beam-plasma instability.

III. QUASI-LINEAR EQUATIONS

Let us study the quasilinear theory of Cherenkov-drift instability, using the collisionless kinetic equation in the following form:

\[
\frac{\partial f_\alpha}{\partial t} + v \frac{\partial f_\alpha}{\partial r} + \frac{\partial}{\partial p} \left[ \frac{q_\alpha}{m_\alpha c} \left( E + \frac{p \times B}{\gamma} \right) \cdot f_\alpha \right] = 0. \tag{6}
\]

Here \( f_\alpha \equiv f_\alpha(r,p,t) \) is the distribution function of particles of sort \( \alpha \), \( E(r,t) \) and \( B(r,t) \) are the electric and magnetic field vectors, respectively.

According to standard scheme, in order to obtain a system of quasilinear equations, distribution function, as well as electric and magnetic field vectors, have to be divided into the main and oscillating parts: \( f_\alpha(r,p,t) = f_{\alpha 0}(p,\mu t) + f_{\alpha 1}(r,p,t) \), \( E(r,t) = E_1(r,t) \) and \( B(r,t) = B_0(r,\mu t) + B_1(r,t) \). Then, averaging equation (1) over fast oscillations and assuming \( \langle f_{\alpha 1} \rangle = \langle B_1 \rangle = \langle E_1 \rangle = 0 \), \( \langle f_{\alpha 0} \rangle = f_{\alpha 0} \) and \( f_{\alpha 0} \gg f_{\alpha 1} \), the following equations can be obtained:

\[
\frac{\partial f_{\alpha 0}}{\partial \mu t} = -\left\langle \frac{q_\alpha}{m_\alpha c} \frac{\partial}{\partial p} \left( E_1 + \frac{p \times B_1}{\gamma} \right) \cdot f_{\alpha 1} \right\rangle \equiv QL, \tag{7a}
\]

\[
\frac{\partial f_{\alpha 1}}{\partial t} + \frac{\gamma}{\gamma} \frac{\partial f_{\alpha 1}}{\partial r} + \frac{q_\alpha}{m_\alpha c} \left( \frac{p \times B_0}{\gamma} \right) \frac{\partial f_{\alpha 1}}{\partial p} = - \frac{q_\alpha}{m_\alpha c} \left( E_1 + \frac{p \times B_1}{\gamma} \right) \frac{\partial f_{\alpha 0}}{\partial p}. \tag{7b}
\]

Here \( \langle \ldots \rangle \) denotes averaging over fast oscillations. Equation (7a) describes back reaction of the generated waves upon the non-perturbed distribution function \( f_{\alpha 0} \). In order to calculate the quasilinear term \( QL \) it is enough to substitute the solution of Eq. (7b) into Eq. (7a). The solution of Eq. (7b) in the Fourier presentation,

\[
f_{\alpha 1}(r,p,t) = \frac{1}{(2\pi)^3} \int f_{\alpha \mathbf{k}}(p) \exp(i \mathbf{k} \cdot \mathbf{r} - i \omega_k t) \, d\mathbf{k}, \tag{8}
\]

writes:
Ignoring the terms proportional to the small parameter $q$, we obtain for perturbed magnetic field $B_1$ using Maxwell equation $B_k = (c/\omega_k)(k \times E_k)$ for the Fourier transforms:

$$E_1(r, t) = \frac{1}{(2\pi)^3} \int E_k \exp(ikr - i\omega_k t) \, dk;$$

$$B_1(r, t) = \frac{1}{(2\pi)^3} \int B_k \exp(ikr - i\omega_k t) \, dk.$$ (10)

In order to find $f_{\alpha k}(p)$, we are using standard method of integration along non-perturbed trajectories. Following this method, in Eq. (9), $(r', p')$ are phase coordinates of the particle (along the non-perturbed trajectory) at the instant of time $t'$; they are calculated from the relativistic equations of motion of a single particle:

$$\frac{dr'}{dt'} = \gamma p', \quad \frac{dp'}{dt'} = \frac{q_a}{m_0 c} \left(\frac{p' \times B_0(r', t')}{\gamma}\right).$$ (11a, b)

Using cylindrical coordinate system, the non-perturbed magnetic field is modeled as $B_0 = B_0(0, 0, B_\parallel)$. It is supposed that non-perturbed external electric field is absent in the plasma, $E_0 = 0$. The drift velocity (see Eq. 3) is directed along $x$ axis. The particles of opposite charge $q_a$ drift in opposite directions. Hereafter the positive direction of $x$ axis is the direction of the positive charge drift.

In such a geometry, the relativistic equation of motion (11) is rewritten as the following set of equations:

$$\frac{\partial p'_\parallel}{\partial t'} - \frac{\omega_{B_\parallel} p'_\parallel}{\gamma} = 0,$$

$$\frac{\partial p'_r}{\partial t'} - p'_r \frac{d\varphi'}{dt'} + \frac{\omega_{B_\parallel} p'_r}{\gamma} = 0,$$

$$\frac{\partial p'_\varphi}{\partial t'} + p'_r \frac{d\varphi'}{dt'} = 0.$$ (12)

Ignoring the terms proportional to the small parameter $(r_L/R_B)^2$ (where $r_L \approx c/\omega_{B_\parallel}$ is the radius of Larmor circle) and assuming that $p'_\parallel^2 \gg (p'_r^2 + p'_\varphi^2)$, we can obtain the solutions of set of Eqs. (12) as was done in [17]:

$$p'_x = p_{d_\alpha} + p_\parallel \sin(\omega_{B_\parallel} \tau) + (p_x - p_{d_\alpha}) \cos(\omega_{B_\parallel} \tau),$$

$$p'_r = p_r \cos(\omega_{B_\parallel} \tau) - (p_x - p_{d_\alpha}) \sin(\omega_{B_\parallel} \tau),$$

$$p'_\varphi = p_\varphi.$$ (13)

Here $\omega_{B_\parallel} = \omega_{B_\parallel}\gamma$; the components of dimensionless momentum $(p_x, p_r, p_\varphi)$ are the values of $(p'_x, p'_r, p'_\varphi)$ at the instant of time $t' = t$, and $p_{d_\alpha} = (u_{d_\alpha}/c)\gamma$. Here we have the following integrals of motion: $\gamma, p'_\parallel - \omega_{B_\parallel} r'/c, r'p'_\varphi$. Let us notice that the particle distribution function $f_{\alpha 0}$ should only depend on the integrals of motion.

It is evident that the solutions of equation of motion (13) differ from those in the homogeneous magnetic field only by the drift term $p_{d_\alpha}$. Using cylindrical coordinates in momentum space $(p'_\parallel, p_\perp, \theta)$ (subscripts ‘\parallel’ and ‘\perp’ denote parallel and perpendicular directions to the magnetic field $B_0$, respectively), we can write

$$p_\perp \cos \theta = p_x - p_{d_\alpha},$$

$$p_\perp \sin \theta = p_r.$$ (14a, b)

Therefore Eqs. (13) are reduced to the following form:

$$p'_x = p_{d_\alpha} + p_\perp \cos(\theta - \omega_{B_\parallel} \tau),$$

$$p'_r = p_\perp \sin(\theta - \omega_{B_\parallel} \tau),$$

$$p'_\varphi = p_\parallel.$$ (14c)

Substituting Eqs. (14) into Eq. (11a) we obtain the following expressions for $\rho = r' - r$:

$$\rho_x = \frac{c p_{d_\alpha}}{\gamma} \tau - \frac{c}{\omega_{B_\parallel}} p_\perp \sin[\theta - \omega_{B_\parallel} \tau] - \sin \theta],$$

$$\rho_r = \frac{c}{\omega_{B_\parallel}} p_\perp [\cos(\theta - \omega_{B_\parallel} \tau) - \cos \theta],$$

$$\rho_\varphi = \frac{c p_\parallel}{\gamma} \tau.$$ (15)

Substituting expressions (14) and (15) for the components of $\rho$ and $p'$ in Eq. (9), we finally obtain Fourier transform of oscillating distribution function $f_{\alpha k}(p)$:

$$f_{\alpha k}(p) = -\frac{q_a}{m_0 c} \int \frac{i \exp [ib \sin(\theta - \varsigma)]}{\Delta \omega_{\alpha k}}$$

$$\times \left[ \frac{\partial f_{\alpha 0}}{\partial p_\parallel} \left( E_\parallel(k) + \frac{k_\parallel c p_{d_\alpha}}{\omega_k \gamma} E_z(k) \right) + \frac{\partial f_{\alpha 0}}{\partial p_\perp} \left( E_\parallel(k) \frac{2p_{d_\alpha} \cos \theta}{p_\parallel} + \frac{k_\parallel c p_{d_\alpha}}{\omega_k \gamma} E_z(k) \frac{2p_{d_\alpha} \cos \theta - p_\perp}{p_\parallel} \right) \right].$$ (16)
where $\Delta \omega_{\alpha k} \equiv \omega_k - k_\parallel v_\perp - k_x u_{\perp x}$. Calculating expression [14] from Eq. [13], we used the following presentation of the exponential function:

$$\exp[i k \rho - i \omega_k \tau] = \exp[i b \sin(\theta - \varsigma)] \sum_{n = -\infty}^{\infty} E_n J_n(b).$$

(17)

Here $E_n \equiv \exp[i n(\varsigma - \theta) - i \tau(\Delta \omega_{\alpha k} - n \tilde{\omega}_{B_x})]$; $J_n(b)$ is the Bessel function of integer order [31];

$$b = \frac{k_\perp c}{\omega_{B_x}} p_\perp$$

(18)

and $\varsigma$ is defined as follows:

$$k_x = k_\perp \cos \varsigma, \quad k_\tau = k_\perp \sin \varsigma.$$  

(19)

In derivation of Eq. [14], we have assumed that $b \ll 1$ and kept only the terms with $n = 0$ in Eq. [13]. This approximation leaves only the terms describing contribution of Cherenkov-drift resonance. Let us mention that $(\partial f_{\alpha 0}/\partial \theta) = 0$ since the distribution function possesses an axial symmetry. Therefore, corresponding terms do not contribute into Eq. [19].

IV. QUASI-LINEAR DIFFUSION

The next step is to study alteration of slowly varying part of distribution function $f_{\alpha 0}(p, \mu t)$ due to development of Cherenkov-drift instability. Generally, alteration is described by diffusion coefficients involved in the quasilinear term. The coefficients show the rate of particle diffusion in momentum space along, as well as across, the magnetic field. Substituting Eq. [16] into Eq. [7a] and using Maxwell equation for Fourier transforms, we obtain $QL$ term in the following form:

$$QL \equiv \frac{q_\alpha}{m_\alpha c} \left( \frac{\partial}{\partial \rho} \left( E_\parallel + \frac{p \times B_1}{\gamma} \right) f_{\alpha 1} \right)$$

$$= \frac{\partial}{\partial p_\parallel} \left( D_{\parallel \parallel} \frac{\partial f_{\alpha 0}}{\partial p_\parallel} \right) + \frac{\partial}{\partial p_\parallel} \left( p_\perp D_{\parallel \perp} \frac{\partial f_{\alpha 0}}{\partial p_\perp} \right)$$

$$+ \frac{1}{p_\perp} \frac{\partial}{\partial p_\perp} \left( p_\perp^2 D_{\perp \perp} \frac{\partial f_{\alpha 0}}{\partial p_\perp} \right) + \frac{1}{p_\perp} \frac{\partial}{\partial p_\perp} \left( p_\perp D_{\perp \parallel} \frac{\partial f_{\alpha 0}}{\partial p_\perp} \right).$$

(20)

Here $D_{\parallel \parallel}, D_{\parallel \perp}, D_{\perp \parallel}$ and $D_{\perp \perp}$ are the diffusion coefficients. Below we calculate particular expressions for diffusion coefficients corresponding to $t$ and $lt$ waves in the case of Cherenkov-drift instability. Let us notice that Eq. (21) has rather general meaning and can as well be used for other type of instabilities. However, diffusion coefficients will differ for different instabilities.

It is convenient to consider quasilinear development of $t$ and $lt$ waves separately. Using simple geometrical assumptions, the following relations between components of electric field and wave vector can be written as

$$E^{t}_\parallel = 0, \quad E^{t}_\perp k_x = -E^{t}_\parallel k_r$$

(21)

for purely electromagnetic ($E \perp k$) $t$ waves and

$$E^{lt}_\parallel k_r = E^{lt}_\perp k_x, \quad E^{lt}_\parallel k_x = -E^{lt}_\parallel k_r$$

(22)

for $lt$ waves, which are almost electromagnetic if we assume $k_x \gg k_r$. Using Eqs. (21) and (22) we can write the diffusion coefficients for $t$ waves and $lt$ waves as

$$D^{t}_{\parallel \parallel} = \int_{-\infty}^{\infty} \left( \frac{k_\parallel c}{\omega_k} \right)^2 \frac{p_\parallel^3}{\gamma^2} I_t(k) \, dk$$

(23a)

$$D^{t}_{\parallel \perp} = \int_{-\infty}^{\infty} \frac{p_\perp}{\gamma} \left( \frac{k_\parallel c}{\omega_k} \right) I_t(k) \, dk$$

(23b)

$$D^{t}_{\perp \parallel} = D^{t}_{\parallel \perp} = 0$$

(23c)

for transverse $t$ waves and

$$D^{lt}_{\parallel \parallel} = \int_{-\infty}^{\infty} \left( \frac{k_\parallel c}{\omega_k} + \frac{k_\parallel c \rho_{da}}{\omega_k} \right)^2 I_{lt}(k) \, dk$$

(24a)

$$D^{lt}_{\parallel \perp} = \int_{-\infty}^{\infty} \frac{p_\perp}{\gamma} \left( \frac{k_\parallel c}{\omega_k} + \frac{k_\parallel c \rho_{da}}{\omega_k} \right)$$

$$\times \left( 1 - \frac{k_\parallel c}{\omega_k} \right) I_{lt}(k) \, dk$$

(24b)

$$D^{lt}_{\perp \parallel} = D^{lt}_{\perp \perp} = 0.$$  

(24c)

for longitudinal-transverse $lt$ waves. Here we use the following definition:

$$I_{lt}(k) = i \frac{1}{\Delta \omega_{\alpha k}} \left( \frac{q_\alpha}{m_\alpha c} \right)^2 \frac{E^{lt}_\times (-k) E^{lt}_\parallel (k)}{V (2\pi)^3}.$$

Note that expressions (23) and (24) are obtained after averaging Eq. (7a) over the angle $\theta$. This procedure nullifies the diffusion coefficients $D^{t}_{\parallel \perp}$ and $D^{lt}_{\perp \parallel}$; we also ignored $D^{lt}_{\parallel \parallel} = 0$ taking into account that the terms

$$\frac{\partial}{\partial p_\parallel} \left( D_{\parallel \parallel} \frac{\partial f_{\alpha 0}}{\partial p_\parallel} \right) \quad \text{and} \quad \frac{1}{p_\perp} \frac{\partial}{\partial p_\perp} \left( p_\perp D_{\perp \parallel} \frac{\partial f_{\alpha 0}}{\partial p_\parallel} \right)$$

are larger then the term

$$\frac{\partial}{\partial p_\parallel} \left( p_\perp D_{\parallel \perp} \frac{\partial f_{\alpha 0}}{\partial p_\perp} \right)$$

by a factor $(p_\parallel/p_{da})^2$. In Eqs. (23) and (24) $E_x(-k)$ identifies the complex conjugate to the $x$ component of electric field vector $E_k$, and $V = \int_{-\infty}^{\infty} dr$.

The multiplier

$$i \frac{1}{\Delta \omega_{\alpha k}} \to \frac{\Gamma_k}{(\Delta \omega)^2 + \Gamma_k^2}$$

$$= \left\{ \begin{array}{ll}
\pi \delta(\Delta \omega), & \text{if } (\Delta \omega)^2 \gg \Gamma_k^2; \\
1/\Gamma_k, & \text{if } (\Delta \omega)^2 \ll \Gamma_k^2.
\end{array} \right.$$  

(25)
For estimation of diffusion coefficients \([23, 24]\) and \([24]\) depends on the type of instability: the upper case corresponds to the kinetic approximation and the lower case to hydrodynamic one. Consequently they should be used for instabilities in kinetic and hydrodynamic approximations respectively.

It is worth to note that the drift velocity \([1]\) depends on the Lorentz factors of the particles, \(\gamma\). Hence, thermal spread in Lorentz factors of resonant particles \((\gamma_T = |\gamma - \gamma_0|)\) results in scatter of drift velocities, increasing the resonant width of instability, \(\Delta \omega = -\frac{k_1 v_1 - k_2 u_d}{\gamma_T}\). It allows to consider the kinetic approximation of Cherenkov-drift instability. However, the resonant width of usual Cherenkov instability is smaller than the corresponding width of Cherenkov-drift instability. Therefore, in the presence of narrow relativistic beam (with low value of \(\gamma_T\)) the kinetic approximation for usual Cherenkov instability is not valid. The growth rate of the instability, \(\Gamma_k\), is small for hydrodynamic approximation as well. Therefore usual Cherenkov instability, as opposed to Cherenkov-drift instability, cannot develop in relativistic magnetized pair plasma \([23, 27]\).

Particle diffusion in momentum space appears both in parallel and perpendicular directions with respect to the magnetic field \(B_0\). Diffusion process causes alteration of particle distribution function until the quasilinear relaxation of instability is saturated \((\partial f_0 / \partial \mu t = 0)\), where \(f_0\) is distribution function of resonant particles. In order to investigate the stage of saturation of quasilinear relaxation, it is worth to rewrite Eq. \((23)\) in the following form:

\[
\frac{\partial f_0}{\partial \mu t} = \frac{\partial}{\partial \rho_{||}} \left( D_{||} \frac{\partial f_0}{\partial \rho_{||}} \right) - \frac{1}{p_{\perp}} \frac{\partial}{\partial p_{\perp}} p_{\perp} \left( D_\perp \frac{\partial f_0}{\partial p_{\perp}} \right) . \tag{26}
\]

For estimation of diffusion coefficients \(D_{\perp}\) and \(D_{||}\), we suppose that \(k_x/k_{||} \approx p_d/\gamma\) and \(k_x c/\omega_k \approx \omega_0\) and use the following approximations in Eqs. \((22, 23)\):

\[
\frac{p_{d\perp}}{\gamma} \sim \left( \frac{k_x c}{\omega_k} \right)^2 \frac{p_{d\perp}}{\gamma^2} ,
\]

\[
\frac{p_{d\perp}}{\gamma} \sim \left( \frac{k_x c}{\omega_k} \right)^2 \left( \frac{1}{\omega_k} \right) ,
\]

\[
4 \frac{p_{d\perp}}{\gamma} \sim \left( \frac{k_x}{k_{||}} + \frac{k_{||} c p_{d\perp}}{\omega_k \gamma} \right)^2 ,
\]

\[
4 \frac{p_{d\perp}}{\gamma} \sim \frac{p_{d\perp}}{\gamma} \left( \frac{k_x}{k_{||}} + \frac{k_{||} c p_{d\perp}}{\omega_k \gamma} \right) \times \left( \frac{1}{\omega_k} + \frac{k_{||} c k_x^2}{\omega_k k_{||}^2} \right) . \tag{27}
\]

Then we rewrite Eq. \((23)\) in the form of quasilinear integral of Cherenkov-drift instability:

\[
\frac{\partial f_0}{\partial \mu t} = \frac{\partial}{\partial \rho_{||}} \left[ \frac{A_{lt}}{3 \gamma_T} \omega_0^2 \gamma_T \frac{u_d}{c} \right] \frac{\partial f_0}{\partial \rho_{||}} + \frac{1}{p_{\perp}} \frac{\partial}{\partial p_{\perp}} p_{\perp} \left[ \frac{A_{lt}}{3 \gamma_T} \omega_0^2 \gamma_T \frac{u_d}{c} \right] \frac{\partial f_0}{\partial p_{\perp}} \tag{28}
\]

Here \(\omega_0\) is the resonant frequency of \(t\) and \(lt\) waves excited on Cherenkov-drift instability (see e.g. \([13]\)):

\[
\omega_0 \approx \frac{\omega_p c}{\gamma_T} \left( \frac{\gamma_0}{\gamma_p} \right)^{1/2} . \tag{29}
\]

\(A_{lt}\) is a numerical coefficient \((A_t = 2\) and \(A_{lt} = 8)\); \(\tilde{W}^{lt}\) denotes the ratio of wave energy \(W^{lt}\) and kinetic energy of plasma particles \((W_p = mc^2 n_p \gamma_p)\) for \(t\) and \(lt\) waves respectively:

\[
\tilde{W}^{lt} = \frac{W^{lt}}{W_p} = \int_{-\infty}^{\infty} \frac{W_k^{lt}}{W_p} d\mathbf{k} = \int_{-\infty}^{\infty} \tilde{W}^{lt}_{kk} d\mathbf{k} = \frac{1}{mc^2 n_p \gamma_p V(2\pi)^d} \int_{-\infty}^{\infty} E^{lt}(-\mathbf{k}) \omega^{lt}_{kk}(\mathbf{k}) d\mathbf{k} ; \tag{30}
\]

the multiplier \((i/\Delta \omega_{\omega_k})\) is replaced by \(1/\Gamma_k\) \([23]\), where \(\Gamma_k\) is determined from Eqs. \((3)\) and \((1)\) assuming \((k_x/k_{||}) \approx (k_x/k_{\perp}) \approx 1\).

Stationary state \((\partial f_0 / \partial \mu t = 0)\) is reached in the following two cases:

\(a)\) \(p_d \ll p_{\perp}\). In this case the first term in Eq. \((28)\), containing alteration of distribution function over parallel momenta \((\partial f_0 / \partial \rho_{||})\), is significant. Process of quasilinear relaxation will be saturated when plateau is formed on parallel distribution function of resonant particles \((\partial f_0 / \partial \rho_{||} = 0)\);

\(b)\) \(p_d \gg p_{\perp}\). In this case the second term of Eq. \((28)\), containing alteration of distribution function over perpendicular momenta \((\partial f_0 / \partial p_{\perp})\), appears significant. During the process of quasilinear relaxation, energy transfers from parallel to perpendicular motion of the particles, hence increasing \(p_{\perp}\). Relaxation will be saturated when \(p_\perp \sim p_d\). In this case the right hand side terms of Eq. \((28)\) cancel each other. Indeed, they are of the same order but with opposite signs (for the beam particle distribution function: \(\partial f_0 / \partial p_{\perp} < 0\)).

V. CONCLUSION

In this paper we discuss the development of Cherenkov-drift instability in relativistic magnetized pair plasma. We are taking into account particle drift motion across the plane of SCMF lines, which is significant for the particles of relativistic beam penetrating the bulk pair plasma. We studied quasilinear stage of the instability – quasilinear interaction of excited waves over plasma particles and
corresponding redistribution of particle momenta. As a result, diffusion of particle momenta takes place along, as well as across, the magnetic field lines.

The linear stage of Cherenkov-drift instability develops similarly to the usual Cherenkov instability. The mechanism of wave excitation is based on the wellknown Cherenkov wave-particle interaction. Same as in the case of Cherenkov instability, presence of high energy particle beam (with positive slope on the shape of distribution function) is necessary condition for developing of Cherenkov-drift instability. However, in the case of Cherenkov-drift instability, the generation of oscillations with \( E \perp B_0 \) is possible only due to particle drift motion across \( B_0 \) (contrary to the case of usual Cherenkov instability generating only longitudinal oscillations with \( E \parallel k \parallel B_0 \)). Cherenkov-drift instability generates both \( t \) (with \( E_\perp \)) and \( \ell t \) (with \( E_\perp \) and \( E_\parallel \)) waves; electromagnetic oscillations with \( E_\parallel \) are generated by particle longitudinal motion with velocities \( v_\parallel \), as in the case of usual Cherenkov instability.

Back reaction of excited waves over resonant particles should suppress the reason of wave excitation: in the case of Cherenkov-drift instability, the process causes, at the same time, formation of plateau on the distribution function of parallel momenta (similar to the quasilinear case of usual Cherenkov instability) and energy transfer from parallel motion of particles to their motion across the magnetic field.

The later process, inhibiting anisotropy in momentum space, is similar to the quasilinear relaxation of cyclotron instability. The reason for development of cyclotron instability – anisotropy in momentum space (\( p_\perp \ll p_\parallel \)) – is suppressed by particle diffusion over perpendicular momenta. Perpendicular diffusion is described by nonzero diffusion coefficients \( D_\perp \) and \( D_\parallel \). As a result, the energy of parallel motion of beam particles is transferring into the perpendicular energy until \( p_\perp \sim p_\parallel \). As for Cherenkov-drift instability, relaxation is saturated since \( p_\perp \sim p_\ell \) and the rate of alteration of distribution function over perpendicular momenta is described by nonzero perpendicular diffusion coefficients \( D_\perp \) (23b) and (24b).

It is worth to note that, in the Cherenkov-drift instability, the plateau on the parallel distribution function forms not only due to parallel diffusion of particles (which transfers energy from high velocity resonant particles to those with low velocities as for usual Cherenkov instability), but also because of perpendicular diffusion which transfers energy from parallel to perpendicular motion of particles. This scenario works if the other factors which can balance quasilinear diffusion, are not taken into account. Such factors could be, on the one hand, the radiation reaction force (acting on synchrotron emitting particle, spiraling in strong magnetic field) and, on the other hand, the force arising due to particle motion in weekly inhomogeneous field. We plan to include these factors into consideration in the future works.

Acknowledgments

GMa thanks D. Melrose for stimulating discussions. DSh and GMa acknowledge hospitality of Institute of Astronomy (Zielona Góra University, Poland). GMa and DSh were supported by KBN grants 2 P03D 008 19 and 5 P03D 010 21. The work was partially done at Abdus Salam International Center for Theoretical Physics (Trieste, Italy).

[1] A. Z. Kazbegi, G. Z. Machabeli and G. I. Melikidze, in Joint Varenna-Abastumani International School & Workshop on Plasma Astrophysics, ESA SP-285, edited by T. D. Guyenne, (European Space Agency, Paris, 1989), p. 277.
[2] A. Z. Kazbegi, G. Z. Machabeli and G. I. Melikidze, Aust. J. Phys. 44, 573 (1991).
[3] A. Z. Kazbegi, G. Z. Machabeli and G. I. Melikidze, in Magnetospheric Structure and Emission Mechanisms of Radio Pulsars, proceedings of IAU Colloquium no. 128, Lagow, Poland, 1990, edited by T.H. Hankins, J.M. Rankin and J.A. Gil (Pedagogical University Press Zielona Góra, Poland, 1992), p. 373.
[4] A. Z. Kazbegi, G. Z. Machabeli, G. I. Melikidze and C. Shukre, A&A 309, 515 (1996).
[5] M. Lyutikov, G. Machablei and R. Blandford, ApJ 512, 804 (1999).
[6] V. V. Zhelezniakov, Radiation in Astrophysical Plasmas, (Kluwer, Dordrecht London 1996).
[7] R. D. Blandford, MNRAS 170, 551 (1975).
[8] V. V. Zhelezniakov and V. E. Shaposhnikov, Aust. J. Phys. 32, 49 (1979).
[9] D. B. Melrose, in Proceedings of Symposium on Radio Physics of the Sun, College park, Md, 1979, (D. Reidel Publishing Co, Dordrecht, 1980), p. 149.
[10] Q. H. Luo and D. B. Melrose, MNRAS 258, 616 (1992).
[11] Y. V. Chugunov and V.E Shaposhnikov, Astrophysics 28, 98 (1988).
[12] E. Asseo, R. Pellat and H. Sol, ApJ 266, 201 (1983).
[13] P. Goldreich and D.A. Keeley, ApJ 170, 463 (1971).
[14] A. Z. Kazbegi, G. Z. Machabeli and G. I. Melikidze, in International Summer School and Workshop on Plasma Astrophysics, volume ESA SP-251, edited by T.D. Guyenne (European Space Agency, Paris, France, 1986) pp. 405-410.
[15] A. Z. Kazbegi, G. Z. Machabeli, G. I. Melikidze and T. V. Smirnova, Astrofizika 34, 433 (1991).
[16] Ph. Pajot-El Abed, G. Melikidze and M. Tagger, A&A 292, L9, (1992).
[17] Ph. Pajot-El Abed, G. Melikidze and M. Tagger, A&A 291, 687, (1994).
[18] A. Z. Kazbegi, G. Z. Machabeli and G. I. Melikidze, 1992, in Magnetospheric Structure and Emission Mechanisms of Radio Pulsars, proceedings of IAU Colloquium no. 128, Lagow, Poland, 1990, edited by T.H. Hankins, J.M. Rankin and J.A. Gil (Pedagogical University Press
Zielona Góra, Poland, 1992), p. 232.

[19] M. Lyutikov, R. Blandford and G. Machabeli, MNRAS 305, 338 (1999).

[20] P. E. Hardee and W. K. Rose, ApJ 219, 274 (1978).

[21] A. S. Volokitin, V. V. Krasnoseľskikh and G. Z. Machabeli, Sov. J. Plasma Phys. 11, 310 (1985).

[22] J. Arons and J. J. Barnard, ApJ 302, 120 (1986).

[23] J. G. Lominadze, G. Z. Machabeli, G. I. Melikidze, A. D. Pataraia, Sov. J. Plasma Phys. 12, 712 (1986).

[24] M. Gedalin, D. B. Melrose and E. Gruman, Phys. Rev. E 57, 3399 (1998).

[25] D. B. Melrose and M. E. Gedalin, ApJ 521, 351 (1999).

[26] P. A. Sturrock, ApJ 164, 529 (1971).

[27] V. D. Egorenkov, J. G. Lominadze and P. G. Mamradze, Astrofizika 19, 753 (1983).

[28] O. Chedia, D. Lominadze, G. Machabeli, G. Mchedlishvili and D. Shapakidze, ApJ 479, 313 (1997).

[29] N. A. Krall and A. W. Trivelpiece, Principles of Plasma Physics (McGraw-Hill, New York, 1973).

[30] A. Z. Kazbegi, G. Z. Machabeli and G. I. Melikidze, MN- RAS 253, 377, (1991).

[31] M. Abramowitz and I. Stegun, Handbook of Mathematical Functions (Dover Publications, Inc., New York, 1965), p. 360.