Optimization of plane frame structure with consideration of semi-rigid connections

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Abstract. Beam-column connections play an important role in the load capacity of frame structures. In Vietnam, in the current design calculation standards for steel frame structures, to simplify calculations, the beam-column connections in the frame structures is considered as the two idealized extremes of either rigid connection or pinned connection. The actual behavior of beam-column connection is neglected. The paper presents the optimization calculation of steel frame structure taking into account the semi-rigid beam-column connections. The optimization problem is solved by Genetic algorithm method and the calculation procedure was implemented by Matlab programming software. Some numerical examples are conducted and from there the conclusions and recommendations of calculations and design of steel frames will be proposed.

1. Introduction
In the steel frame structures, the beam-column connection is the essential part and play very important role in the load carrying capacity of frame structure. The main functions of these connections are to transfer the beam and floor loads to the columns and to maintain the integrity of overall frames. In design process of steel structural frames, there are particular instructions of calculation including the behavior of semi-rigid connections, in other words, the connection flexibility in the analysis and design of structure is considered. Numerous investigations and studies of the behavior of beam-column connection with many experimental tests have been conducted and reported. These studies can be mentioned as [1, 2, 3, 4]. In the current national construction design code for steel structures in Vietnam, an approximately and simple way of analyzing internal forces and displacements for steel structured, it is often assumed that the connections between beams and columns are fully rigid or pinned connections. In fact, these connections have certain flexibility due to the deformation of the connected elements and the local deformation of the connecting elements such as bolts, connecting plates…In the case of assumption that the connections are fully rigid, the connection flexibility is neglected. Usually, the calculated displacements of the frames are smaller than displacements obtained from the real projects, the internal forces of the sections near the connections are often greater, so the cross-section and connection dimension are also greater than required. Therefore, when assuming that the beam-column connections are fully rigid in the analysis of frame structures, the maximum bending moment in the beam is usually achieved at the support position. In this case, if considering the connection flexibility, there will be a redistribution of the bending moment in the beam. In contrast, in the case of the assumption that the beam-column connections are pinned connections, the maximum bending moment in the beam will appear at the middle span of the beam and the bending moment
the support position is zero. When considering the semi-rigid connection, there will be redistribution of internal forces in beams and columns as shown in Fig.1. Hence, the consideration of the semi-rigid connections is necessary, making the structural analysis closer to the actual behavior of the real structures, giving more accurate calculation results and thereby potentially reducing the weight of frames due to the reduction of the section sizes of the elements in the frames and the selection the cross-sections more appropriately.

In the design process of engineering structures, beside the design problem of ensuring the project is capable to meet all technical requirements, the material saving for the project is always an issue of great concern in the field of construction. However, the problem of the optimization of steel structures with consideration of connection flexibility has not been mentioned much in Vietnam. Therefore, the article introduces a method of calculating the weight optimization of steel frames taking into account the connection flexibility and determines the effects of semi-rigid connections in optimal design of frame structures. The design variables are the member sections where column and beam members are distinguish. From there, the analysis and design of steel frames more appropriate and save more materials.

2. The analysis of plane frames with semi-rigid connections
The Fig.1 shows the comparison of different connection respect to distribution of bending moment in the elements

![Figure 1. Comparison of different connections with respect to bending moment distribution](image)

The problem of steel structure analysis is implemented using finite element method. When analyzing the steel frame structures the beam-column connection is usually represented as fictitious structural elements at the end of the member (usually the beam) which characterizes the behavior of the connection. These elements are assigned for determining the relationship between forces and displacements for simulation the behavior of the beam-column connection. The simplest model to analyze frames with semi-rigid connections is a linear representation of the rotational springs which in many cases will be quite adequate. The effects of connection flexibility are modeled by attaching rotational springs of moduli $S_1$ and $S_2$ normally measured in kNm.rad$^{-1}$ to the ends 1 and 2 of a member for representation the moment-rotation characteristic only, as shown in Fig. 2. [2].
The end-fixity factor $\alpha$ defines the stiffness of the beam-to-column connection in terms of the beam moment of inertia. In which $S_j$ is the end-connection spring stiffness, and $EI/L$ is the flexural stiffness of the attached member. For pinned connections, the rotational stiffness of the connection tends to zero and the value of the end-fixity factor is equal to zero as well. For rigid connections, the end-fixity factor $\alpha$ is equal to 1 ($\alpha = 1$), and in case of a more realistic design, the semi-rigid connection results in a value between 1 and zero.

In the first-order analysis, the flexibility stiffness matrix of a planar member $i$ with semi-rigid connection having stiffness modulus $S_j$ ($j=1,2$) at the ends can be represented by the stiffness matrix $K_i$ for the member with rigid connections modified by a correction matrix $[7]$: \[ K_i = K_{si} + K_{ci} \] (2)

In which $K_i$ – the stiffness matrix of member $i$ with semi-rigid connections,
$K_{si}$ – the stiffness matrix of member $i$ with rigid connections
$K_{ci}$ – the correction stiffness matrix
The matrix $K_{si}$ and $K_{ci}$ can be expressed as following:

\[
K_{si} = \begin{bmatrix}
\frac{EA}{l} & 0 & 0 & -\frac{EA}{l} & 0 & 0 \\
0 & \frac{12EI}{l^3} & 6EI & 0 & -\frac{12EI}{l^3} & 6EI \\
0 & \frac{4EI}{l} & 0 & -6EI & 2EI & 0 \\
Sym.
\end{bmatrix}
\] (3)
In which, $E$ - Young’s modulus, and $l$, $A$ and $I$ are respectively the length, cross-section area and moment of inertia of the member. The parameters $\alpha_1$ and $\alpha_2$ in (1) are fixity factors at the two ends 1 and 2 of the member, and are related to the corresponding rotational spring stiffness $S_1$, and $S_2$ as following:

$$\alpha_1 = \frac{1}{1 + \frac{3EI}{S_1 \cdot l}}; \quad \alpha_2 = \frac{1}{1 + \frac{3EI}{S_2 \cdot l}} \quad (5)$$

In the linear elastic analysis, the connection and members are assumed to have linear force-displacement relationship and the effect of deformation on the equilibrium of the frame is disregarded. In its usual form, the analysis requires the solution of the set of linear equations:

$$K \cdot u = P \quad (6)$$

Where $K$ – Stiffness matrix of element in global coordinate;

$U$ – Nodal displacement vector in global coordinate;

$P$ – Nodal load vector in global coordinate.

From equations (6) the displacements are obtained, and then the internal forces in elements are calculated.

3. The optimization problem of plane frames with semi-rigid connections

3.1. Establishment of objective function

The optimization problem is illustrated shortly: Find the minimize volume of structure $V$ with semi-rigid connections, in the analysis considering only flexural behavior of structure subjected to applied load. The problem is discrete optimization problem in terms of the member sections $A_i$, $A_j$ and in terms of the rotational stiffness of the connections, $S_k$.

The objective function of the optimization problem can be expressed as following:

$$Z(A_i, A_j) = W_{fr} = \sum \rho_i \cdot \left( A_i \cdot l_i + A_j \cdot l_j \right) \rightarrow \min$$

$$i=1, 2, \ldots, n; \quad j=1, 2, \ldots, m \quad (7)$$

In which: $n$ is the number of column, $m$ is the number of beam elements, $k$ is number of the connections in the frame.

The variables of the optimization problem are $A_i$ selected from a discrete set of the predetermined $A_i \in B = \{ B^1, B^2, \ldots, B^N \}$ - Cross-sectional areas of column elements;
A_j \in C = \{C^1, C^2, ..., C^M\} - Cross-sectional areas of beam elements;

S_k is rotational stiffness of the ends of connection. These parameters are changing in the given range of values:

S_k \in S = \{S^1, S^2, ..., S^K\}

Where k – Number of the connections in the frames

N – Numbers of cross-sectional catalogue values for columns

M – Numbers of cross-sectional catalogue values for beam elements

K – Number of rotational stiffness values

3.2. The constraints conditions

3.2.1. The displacement constraints

\[ y_p \leq [y] \] (8)
Where \([y]\) is allowable displacement of elements of frames subjected to different loads;
Displacement \(y\) is determined by finite element method

\[ y_p = K^{-1}P \] (9)

3.2.2. The bending and axial tension (compression) constraints

\[ \frac{N}{A_n} + \frac{M_x}{C_x \cdot W_{nx,min}} + \frac{M_y}{C_y \cdot W_{ny,min}} \leq f_y \cdot \gamma_c \] (10)

In which \(N, M_x, M_y\) – Correspondingly axial forces, bending moments of extreme internal force combination;

\(C_x, C_y\) – Coefficients determined by Appendix C in [4];

\(W_{n,min}, W_{ny,min}\) - Minimal values of section modulus for the axes x and y;

\(f_y\) – Yield strength of material;

\(\gamma_c\) – Safety factor.

3.2.3. The buckling constraints

\[ \frac{N}{\phi_e \cdot A} \leq f_y \cdot \gamma_c \] (11)

In which \(\phi_e\) - Overall buckling factor.

3.3. The optimization procedure

For the discrete optimization problem, it can be solved by many different methods. Among those methods, the Genetic Algorithm method solves the research problem mentioned in this article relatively thoroughly. Therefore, the author uses the Genetic Algorithm method to solve the above mention problem.

Since GA method is used to solve unconstrained optimization problems, we must transform the constrained problem to an unconstrained problem by using penalty functions.

In this study, we use the following penalty function:

\[ F(A_i, A_j) = Z(1+C) \] (12)
In which, F – fitness function, C – constraint violation functions, determined as sum of the constraints conditions (8), (10) and (11).

The GA procedure used in this study for optimization frames with semi-rigid connections can be illustrated as the following steps.

Step 1: Input parameters of the problem. The parameters are population size, string length for individual design variable, crossover, mutation rate.

Step 2: Generate the initial population

Step 3: Decode the binary design variables and generate input file for Finite element analysis

Step 4: Implementing finite element analysis using Sap2000, check the given constraints, calculate the value of the penalty function, in contrast the calculation process is continued.

Step 5: Check the convergence of the problem. The optimization process is terminated if it is satisfied

Step 6: Calculate the penalized fitness function for every individual of the population and generate the next generation through reproduction, crossover and mutation.

The above procedure of calculation may be expressed in the block scheme as shown in Fig. 3.

**Figure 3.** Block-scheme of calculation optimization procedure of frame with semi-rigid connections.

4. **Numerical example**

Using the calculation procedure that is mentioned in paragraph 3.3, the authors implemented the optimization problem of steel frames with accounting of the semi-rigid connections [8]

Requirement: Calculating the optimal size of the element in the frame subjected to loads shown in Fig. 4. Given elastic modulus E=2*10^6 kPa, l=5m, h=4m, f_y=355 MPa, ρ=7,85.10^3 kg/m^3. The rotational stiffness of the end connections are changeable in a range of S_j={10^4, 5.10^4, 10^5, 5.10^5, 10^6, 5.10^6, 10^7, 5.10^7}

The calculation procedure for this problem is implemented using Matlab programming software
Table 1. Optimal results of elements with rigid connections

| Element | Cross-section | Weight | Fixity factor |
|---------|---------------|--------|---------------|
| Beam    | C150x70       | 215.8  | 1             |
| Column  | I150x75       |        |               |

Table 2. Optimal results of elements with semi-rigid connections

| Element | Cross-section | Weight | Fixity factor |
|---------|---------------|--------|---------------|
| Beam    | C150x70       | 207.8  | 0.57          |
| Column  | I125x75       |        |               |

From the calculation results, it can be seen that for each case of connections the optimal sizes of elements always will be found to minimize the weight of the frames under the given loads. The subroutine established by the authors using Matlab programming software helps to solve the research problem relatively thoroughly without any mathematical problems.

The optimization problem is solved with two cases for showing the effect of fixity factor to the value of weight of frame. From the results it can be concluded that it is necessary to account the flexibility of the beam-column connection in design and optimization process of steel frames.

5. Conclusions
The optimization problem of steel frames with semi-rigid connections using genetic algorithm established by Matlab programming software helps the research issues effectively.

The above mentioned optimization procedure provides an effective way to account for the weight of frames. From calculation result, accounting for the actual semi-rigid behavior of connections gives less weight than the results obtained from the traditional way as the connection are idealized as being fully rigid. This is because semi-rigid connections allow the redistribution of internal forces in all element of the system, that influences in more economical in material to bear the given applied loads.

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