\(R_2\) as a single leptoquark solution to \(R_{D(\ast)}\) and \(R_{K(\ast)}\)

Oleg Popov,\(^a,b\) Michael A. Schmidt\(^c\) and Graham White\(^d\)

\(^a\)Physics and Astronomy Department, University of California, Riverside, California 92521, USA
\(^b\)Institute of Convergence Fundamental Studies, Seoul National University of Science and Technology, Seoul 139-743, Korea
\(^c\)School of Physics, The University of New South Wales, Sydney, NSW 2052, Australia
\(^d\)TRIUMF Theory Group, 4004 Wesbrook Mall, Vancouver, B.C. V6T2A3, Canada

E-mail: opopo001@ucr.edu, m.schmidt@unsw.edu.au, gwhite@triumf.ca

ABSTRACT: We show that, up to plausible uncertainties in \(\text{BR}(B_c \to \tau \mu)\), the \(R_2\) leptoquark can simultaneously explain the observation of anomalies in \(R_{K(\ast)}\) and \(R_{D(\ast)}\) without requiring large couplings. The former is achieved via a small coupling to first generation leptons which boosts the decay rate \(\Gamma(\bar{B} \to \bar{K}(\ast)e^+e^-)\). Finally we motivate a neutrino mass model that includes the \(S_3\) leptoquark which can alleviate a mild tension with the most conservative limits on \(\text{BR}(B_c \to \tau \mu)\).

KEYWORDS: Flavour physics, B physics
1 Introduction

There have recently been multiple independent anomalous measurements of semi-leptonic $B$ decays that depart from standard model (SM) predictions. Rare decays into $D^{(*)}$ mesons show a discrepancy from SM predictions in BaBar [1, 2], Belle [3–5] and LHCb [6, 7] measurements of the lepton flavour universality (LFU) ratios. New results from Belle combined with measurements from BaBar and LHCb give \[ R_D = \frac{\Gamma(\bar{B} \to D\tau\bar{\nu})}{\Gamma(\bar{B} \to De/\mu\bar{\nu})} = \begin{cases} 0.299 \pm 0.003 & \text{SM [9]} \\ 0.335 \pm 0.031 & \text{observed [8]} \end{cases} \] (1) and \[ R_{D^*} = \frac{\Gamma(\bar{B} \to D^*\tau\bar{\nu})}{\Gamma(\bar{B} \to D^*e/\mu\bar{\nu})} = \begin{cases} 0.258 \pm 0.005 & \text{SM [10]} \\ 0.298 \pm 0.015 & \text{observed [8]} \end{cases} \] (2)

When the correlation between the two observables is taken into account the significance of the anomaly is at the 3.1σ level [8]. The SM calculation is reliable as it is largely insensitive to hadronic uncertainties which cancel out in the ratios $R_{D^{(*)}}$.

LHCb has similarly found an intriguing deviation from LFU in the semileptonic $B$ meson decays to $K^{(*)}$ mesons. The LFU ratios \[ R_{K^{(*)}} = \frac{\Gamma(\bar{B} \to \bar{K}^{(*)}\mu^+\mu^-)}{\Gamma(\bar{B} \to \bar{K}^{(*)}e^+e^-)} \] (3) provide a clean probe of new physics effects because hadronic uncertainties cancel out in the ratios as long as new physics effects are small [11–13]. LHCb measured the ratios for the dilepton invariant mass range $1.1 \text{GeV}^2 < q^2 < 6 \text{GeV}^2$. A combination of run I and run II from LHCb gives \[ R_K = \begin{cases} 1.0003 \pm 0.0001 & \text{SM [14]} \\ 0.846^{+0.06}_{-0.054(\text{stat})}^{+0.016}_{-0.014(\text{sys})} & \text{observed [15]} \end{cases} \] (4) and \[ R_{K^{*}} = \begin{cases} 1.00 \pm 0.01 & \text{SM [16]} \\ 0.716^{+0.070}_{-0.057} & \text{observed [17, 18]} \end{cases} \] (5)

where we combined the LHCb measurement [18] of $R_{K^*}$ with the new Belle measurement [17] using the methods described in Ref. [19]. Experimental sensitivity to both of these anomalies is expected to improve by orders of magnitude over the next few years and make a potential confirmation of a departure from the SM imminent. The measurements are not just quantitatively different from the SM but qualitatively so as well, because the SM has no notable violation of lepton flavour universality.

The most common explanation for these anomalies is to extend the SM by leptoquarks (see Refs. [20–37] for a leptoquark solution to the $R_{K^{(*)}}$ anomalies, Refs. [34, 38–44] for the $R_{D^{(*)}}$ anomalies and Refs. [34, 45–61] for simultaneous explanations). Vector leptoquarks have issues with ultraviolet (UV) completion and their tendency is to be heavy in UV complete models. Therefore it is attractive to consider scalar leptoquark solutions to these anomalies. To date the only known candidate which simultaneously explains both sets of anomalies is the $S_1$ leptoquark [45, 56, 62],

\[ \text{The best-fit value and error bars have been extracted from the figure on slide 9.} \]
but it only satisfies $R_{K^*(\pi)}$ at $2 - \sigma$ [56]. In this work we show that the $R_2$ leptoquark can provide a simultaneous solution at $1 - \sigma$ consistent with all known constraints.

In addition to the anomalous measurements of $R_{D^{(*)}}$ and $R_{K^{(*)}}$, two other anomalies have generated interest: On the one hand, the value of the angular observable $P'_S$ [63, 64] and more generally the data of $b \to s\mu\bar{\mu}$ points to a deviation from the SM [65]. While these anomalies are intriguing they are currently less clean signals of new physics due to large hadronic uncertainties and the difficulty in estimating a signal for the $P'_S$ anomalies [64]. On the other hand, similar to the LFU ratios $R_{D^{(*)}}$ the LFU ratio $R_{J/\psi} = \Gamma(B^+_c \to J/\psi \tau\nu)/\Gamma(B^+_c \to J/\psi \mu\nu)$ points to a larger branching fraction to $\tau$ leptons compared to muons, but it is still consistent with the SM at $2 - \sigma$ due to the large error bars [66]. We therefore leave the consideration of such anomalies to future work.

The $R_2$ leptoquark has quantum numbers $(3, 2, 7/6)$ with respect to the SM gauge group $SU(3) \times SU(2) \times U(1)$ and has been proposed as a cause of the $R_{D^{(*)}}$ anomalies [10, 67, 68] with $O(1)$ couplings as well as the $R_{K^{(*)}}$ anomalies with very large couplings through a new contribution to the decay $b \to s\mu\bar{\mu}$ [69–73]. These operators are induced at the 1-loop level and thus require undesirably large couplings with at least one coupling needing to be a lot larger than 1. We reopen the case of this leptoquark and find that a more promising route to what has previously been studied is to boost to the denominator in Eq. (3), by allowing the leptoquark to couple to electrons. As the relevant operator is generated at tree level, the required couplings are quite small. The deviations in the LFU ratios $R_{D^{(*)}}$ can be explained at the same time by introducing a coupling of the $R_2$ leptoquark to $\tau$ leptons. A mild tension with the theoretically inferred constraint on $BR(B_c \to \tau\nu)$ [74] can be resolved by the introduction of the $S_3$ leptoquark, which can be motivated within a radiative neutrino mass model.

The structure of this paper is as follows. In section 2 we perform an effective field theory (EFT) analysis of the $R_2$ leptoquark. We then explain the most relevant constraints in section 3 and show that $R_2$ can explain $R_{K^{(*)}}$ and $R_{D^{(*)}}$. In section 4 we introduce a minimal model for neutrino masses based on the $R_2$ and $S_3$ leptoquarks. Finally we conclude in section 5.

2 Effective field theory analysis for the $R_2$ leptoquark

The $R_2 \sim (3, 2, 7/6)$ leptoquark is an electroweak doublet and couples to both left-handed and right-handed SM quarks and leptons. Its Yukawa couplings with SM fermions are

$$\mathcal{L}_{R_2} = -(Y_2)_{ab} \bar{u}_a R_2^\alpha \epsilon_{a\beta} P_L L_\beta^\beta - (Y_4)_{ab} \bar{e}_a R_2^\alpha P_L Q_b + h.c. \quad (6)$$

We work in the basis, where the flavour eigenstates of down-type quarks and charged leptons coincide with their mass eigenstates. In particular the component of $R_2$ with electric charge $2/3$ couples right-handed charged leptons to left-handed down-type quarks and right-handed up-type quarks to neutrinos and thus contributes to both $b \to sll$ and the $b \to c\tau$ processes.

For energies below the mass of the leptoquark, it is convenient to write an effective Lagrangian to capture the relevant contributions beyond the SM. Using the Warsaw basis [75] of the SM effective field theory (SMEFT), the relevant terms in the effective Lagrangian are

$$\mathcal{L} = C_{abcd}^{ee} (\bar{Q}_a \gamma_\mu Q_b) (\bar{e}_c \gamma^\mu e_d) + C_{abcd}^{lept1} (\bar{L}_d^\gamma e_b) \epsilon_{jk} (Q_c^\mu u_d) + C_{abcd}^{lept2} (\bar{L}_d^\gamma \sigma_{\mu\nu} e_b) \epsilon_{jk} (Q_c^k \sigma^{\mu\nu} u_d), \quad (7)$$

2
where \( \sigma_{\mu\nu} = \frac{i}{2}[\gamma_\mu, \gamma_\nu] \) with Wilson coefficients

\[
C_{bda}^{\text{leq}1} (m_{R_2}) = \frac{(Y_1)^*_{ab} (Y_4)_{cd}}{2m_{R_2}^2}, \quad C_{dla}^{\text{leq}1} (m_{R_2}) = \frac{(Y_2)^*_{ab} (Y_4)_{cd}}{2m_{R_2}^2},
\]

which are defined at the renormalization scale \( \mu = m_{R_2} \), the mass of leptoquark \( R_2 \). The vector Wilson coefficient \( C_{bsee}^{\text{eq}} \) contributes to \( b \to see \) and thus modifies the LFU ratios \( R_{K^{(*)}} \). This is illustrated in the left panel of Fig. 1. The blue-shaded region indicates the \( 1-\sigma \)-allowed region for \( R_{K^{(*)}} \). For a fixed leptoquark mass \( m_{R_2} = 1 \text{ TeV} \), the LFU ratio \( R_K \) decreases when increasing the magnitude of the Yukawa couplings \( |(Y_4)^{es} (Y_4)^{eb}| \) and thus increasing the magnitude of the Wilson coefficient \( C_{bsee}^{\text{eq}} \).

Similarly the scalar and tensor Wilson coefficients \( C_{\nu\tau bc}^{\text{leq}1,3} \) contribute to \( b \to c\tau\nu \) and thus modify the LFU ratios \( R_{D^{(*)}} \). As the final state neutrino is not measured, there is a contribution from all three flavours. The coupling to \( \nu_{e,\mu} \) is accompanied by couplings to \( e \) and \( \mu \) respectively and hence there are additional constraints from lepton flavour violating processes. In order to avoid these additional constraints, we only consider couplings to \( \nu_\tau \). The dependence of \( R_D \) to the magnitude of the Yukawa couplings \( |(Y_2)^{c\nu} (Y_4)^{\tau b}| \) is illustrated in the right panel of Fig. 1. The blue-shaded region indicates the \( 1-\sigma \)-allowed region for \( R_{D^{(*)}} \). The Yukawa couplings \( |(Y_2)^{c\nu}| \) and \( |(Y_4)^{\tau b}| \) are generally of order 1 with \( \sqrt{|(Y_2)^{c\nu} (Y_4)^{\tau b}|} \sim 1 \) and thus generally larger than the Yukawa couplings required to explain \( R_{K^{(*)}} \). As there is generally operator mixing, when evolving the Wilson coefficients from the scale of the leptoquark to the scale of the \( b \)-quark, the large Wilson coefficients \( C_{\nu\tau bc}^{\text{leq}1,3} \) generally modify the result for \( R_{K^{(*)}} \) and thus the interesting parameter range for the Yukawa couplings \( (Y_4)^{es} \) and \( (Y_4)^{eb} \) differs, when attempting to explain both \( R_{K^{(*)}} \) and \( R_{D^{(*)}} \) simultaneously.

A minimal set of Yukawa couplings to accommodate a simultaneous solution to \( R_{K^{(*)}} \) and \( R_{D^{(*)}} \) is

\[
Y_2 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & (Y_2)^{c\nu} \\ 0 & 0 & 0 \end{pmatrix}, \quad Y_4 = \begin{pmatrix} 0 & (Y_4)^{es} & (Y_4)^{eb} \\ 0 & 0 & 0 \\ 0 & 0 & (Y_4)^{\tau b} \end{pmatrix},
\]

which we will focus on in the following.
Before discussing the phenomenology of the $R_2$ leptoquark we briefly make a connection to the operators in the commonly-used operator basis in $B$-physics. We limit our discussion to the operators induced after integrating out the $R_2$ leptoquark. In the weak effective theory, after integrating out the Higgs, $Z$- and $W$-bosons and the top quark, the relevant operators in the effective Lagrangians governing $b \rightarrow sll$ and $b \rightarrow c\ell\nu$ decays are

\[ \mathcal{L}_{\text{ll}} = \frac{6G_F}{\sqrt{2}} V_{tb} V_{ts}^{*} \frac{\alpha_{em}}{4\pi} \sum_{\ell} \left[ C_{ij}^{\ell} (\bar{s} \gamma_\mu P_L b) (\bar{\ell} \gamma^\mu \ell) + C_{ij}^{\ell} (\bar{\ell} \gamma_\mu P_L b) (\bar{s} \gamma^\mu \gamma_5 \ell) \right] \]

\[ \mathcal{L}_{\ell\nu} = - \frac{4G_F}{\sqrt{2}} V_{cb} \sum_{i,j} \left[ C_{ij}^{\ell\nu} (\bar{c} \gamma_\mu P_L b) (\bar{\ell} \gamma_\mu P_L \nu_j) + C_{ij}^{\ell\nu} (\bar{\ell} \gamma_\mu P_L b) (\bar{c} P_L \nu_j) + C_{ij}^{\ell\nu} (\bar{c} \gamma_\mu \gamma_5 P_L b) (\bar{\ell} \sigma_{\mu\nu} P_L \nu_j) \right] , \]

respectively, with the CKM mixing matrix elements $V_{ij}$. The Wilson coefficients in weak effective theory are related to the ones in SMEFT by

\[ C_y^c = C_{10}^e = \frac{\pi C_{\text{tree}}^{\ell e}}{2V_{tb} V_{ts}^{*} G_F \alpha_{em}} \quad C^{\ell\nu} = 4C^{\ell\nu} T - \frac{C^{\text{lequl}}_{\nu,\tau,\tau}}{2\sqrt{2} V_{cb} G_F} . \]

In our numerical analysis we use the flavio package [76] for the renormalization group evolution of the Wilson coefficients and the calculation of most processes. We vary the magnitude of the four Yukawa couplings over the range consistent with perturbativity and the explanation of the $R_D^{(\tau)}$ and $R_K^{(\tau)}$ anomalies at $1 - \sigma$ and their phases over the whole allowed range $[0, 2\pi]$ while fixing the mass $m_{R_2} = 1$ TeV.

3 Experimental constraints and the viable parameter space

In this section we first summarize the most significant constraints on the couplings of the $R_2$ leptoquark in Sec. 3.1, followed by a discussion of the viable parameter space in Sec. 3.2.

3.1 Constraints

We show the impact of the most relevant constraints on the parameter space satisfying a $1 - \sigma$ simultaneous solution for $R_K^{(\tau)}$ and $R_D^{(\tau)}$ in Fig. 2. The four most stringent constraints are posited by the decays $\tau \rightarrow e\gamma$, $B^+ \rightarrow K^+\tau^+\tau^-$, $Z \rightarrow \tau\tau$ and $B_c \rightarrow \tau\nu$.

$\tau^- \rightarrow e^-\gamma$

The radiative lepton-flavour-violating decay $\tau^- \rightarrow e^-\gamma$ occurs at loop level. Its branching ratio takes the form [77]

\[ \text{BR}(\tau^- \rightarrow e^-\gamma) \approx \frac{27 \alpha_{em}}{256 \pi G_F^2 m_\tau^2} \left| (Y\nu Y_\nu^T)_{\tau e} - \frac{4}{3} \sum_{q=u,c,t} (Y_2)_{q\tau} (Y_4 V)_{eq} \frac{m_q}{m_\tau} \left( 1 - \ln \frac{m_\tau^2}{m_{R_2}^2} \right) \right|^2 \]

in the limit of vanishing final state electron mass and to leading order in the quark masses in the loop. The branching ratio for the SM purely leptonic $\tau$ decay is $\text{BR}(\tau \rightarrow e\nu_\tau\bar{\nu}_e) = 0.178$. In the numerical scan, we use the exact expression and impose the current limit on the branching ratio $\text{BR}(\tau^- \rightarrow e^-\gamma) < 5.4 \times 10^{-8}$ obtained by HFLAV [78]. The HFLAV limit is less aggressive than the limit quoted in the PDG, because it combines the BaBar result [79] with the less stringent Belle result [80] while the PDG [81] relies only on the former. The combined limit is less aggressive
Figure 2: Points which simultaneously explain $R_{D(\tau)}$ and $R_{K(\tau)}$ at $1-\sigma$ with observables $\text{BR}(\tau^- \rightarrow e^-\gamma)$, $\text{BR}(B_c \rightarrow \tau\nu)$, $\text{BR}(B^- \rightarrow K^-e^+\tau^-)$ shown in the top left, top right, bottom left panels respectively. The bottom right panel shows the relative size of the largest Yukawa couplings. The orange dashed (dotted) gridlines indicate the contributions to the $Z\tau\tau$ coupling at the level of the $1-\sigma$ ($2-\sigma$) experimental error. All points explain $R_{K(\tau)}$ and $R_{D(\tau)}$ at the $1-\sigma$ level. Dark blue points satisfy all constraints. Light blue points satisfy strict limits on $\text{BR}(\tau^- \rightarrow e^-\gamma)$ but are excluded by other constraints.

as Belle saw a small excess of this process (see table 319 in Ref. [78]). Irrespective whether the Belle result is included or not, the simultaneous explanation of both $R_{K(\tau)}$ and $R_{D(\tau)}$ is viable.

In the top left panel of Fig. 2 we show the branching ratio vs $(|(Y_4)_{eb}(Y_4)_{\tau b}|)^{1/2}$. For large couplings $|(Y_4)_{eb}(Y_4)_{\tau b}|$, the branching ratio is dominated by the first term and thus increases for increasing Yukawa couplings. For small $(Y_4)_{\tau b}$, the Yukawa coupling $(Y_2)_{c\nu\tau}$ becomes large in order to explain $R_{D(\tau)}$ as shown in the bottom right plot and thus the second term in Eq. (13) dominates, which explains the increasing branching ratio for small $|(Y_4)_{eb}(Y_4)_{\tau b}|$. The Belle II experiment [82] is expected to improve the sensitivity to $\tau \rightarrow e\gamma$ by more than one order of magnitude to $3 \times 10^{-9}$ (indicated by a dotted red line) and thus probe a large part of the remaining parameter space.

$B^+ \rightarrow K^+\tau^+e^-$

Another constraint on the $\tau-e$ flavour violating processes originates from the semi-leptonic lepton flavour violating $B$ decay $B^+ \rightarrow K^+\tau^+e^-$. Its branching ratio satisfies $\text{BR}(B^+ \rightarrow K^+\tau^+e^-) < 1.5 \times 10^{-5}$ [81]. The $R_2$ leptoquark induces the vector operator

$$C_{s\nu\tau}^{qe} = -\frac{(Y_4^*_e)_{es}(Y_4)_\tau}{2m_{R_2}^2}, \quad (14)$$
which contributes to \( B^+ \to K^+\tau^+e^- \) and thus constrains the simultaneous explanation of \( R_{K^{(*)}} \) and \( R_{D^{(*)}} \). As we demonstrate in the bottom left panel of Fig. 2, it provides a moderate constraint on the parameter space that simultaneously explains \( R_{K^{(*)}} \) and \( R_{D^{(*)}} \). The region excluded by \( B^+ \to K^+\tau^+e^- \) is also excluded by \( \tau \to e\gamma \).

**Z decays**

The \( R_2 \) leptoquark also contributes to several \( Z \)-boson decay processes. In particular, its contribution to \( Z \to \tau\tau \) is significant due to the large couplings to \( \tau \) leptons. Approximate expressions for the left-handed and right-handed couplings of the \( Z \)-boson to \( \tau \) leptons

\[
\text{Re}(\delta g^2_L) \simeq \frac{|(Y_1)_{\tau b}|^2}{16\pi^2} \left\{ -\frac{3}{2} x_t \left[ 1 + \ln x_t \right] + x_Z \left[ \frac{23}{12} + \frac{128}{9} + 8 \ln x_t - \frac{1}{3} \ln x_Z \right] \sin^2 \theta_W \right\}
\]

\[
\text{Re}(\delta g^2_R) \simeq \frac{|(Y_2)_{\tau \nu}|^2}{16\pi^2} x_Z \left[ \frac{1}{12} - \frac{1}{2} \ln x_Z + \left( \frac{1}{18} + \frac{2}{3} \ln x_Z \right) \sin^2 \theta_W \right]
\]

in terms of the Weinberg angle \( \theta_W \) and the ratios \( x_t = (m_t/m_{R_2})^2 \) and \( x_Z = (m_Z/m_{R_2})^2 \) are obtained by expanding the expressions given in Ref. [83] to leading order in \( x_t, x_Z \) and the quark mixing angles by taking \( V_{tb} \simeq 1 \).

The LEP experiments measured the \( Z \)-boson couplings precisely [84] with \( 1 - \sigma \) uncertainties of \( |\text{Re}(\delta g^2_L)| < 5.8 \times 10^{-4} \) for couplings to left-handed \( \tau \) leptons and \( |\text{Re}(\delta g^2_R)| \leq 6.2 \times 10^{-4} \) for right-handed \( \tau \) leptons. This translates to a constraint on the magnitude of the Yukawa couplings \( |(Y_1)_{\tau b}| \) and \( |(Y_2)_{\tau \nu}| \) of \( |(Y_1)_{\tau b}| \leq 1.0(1.4) \) and \( |(Y_2)_{\tau \nu}| \leq 2.6(3.7) \) using \( 1 - \sigma \) (2 - \( \sigma \)) experimental uncertainties respectively. This is indicated in the bottom right panel of Fig. 2 as orange dashed (dotted) lines. The dark blue points in the numerical scan do not lead to any correction larger than the \( 1 - \sigma \) experimental uncertainties. In reality, a full global fit to all electroweak observables is needed to impose a reliable constraint, and it is probable that significantly larger deviations to effective \( Z \) couplings can be accommodated. We leave such a work to the future and comment here that even our pessimistic approach does not rule out our model.

**\( B_c \to \tau\nu \)**

The \( R_2 \) leptoquark contributes to \( B_c \to \tau\nu \) via the same couplings relevant to \( R_{D^{(*)}} \), since the same scalar operator contributes to both \( R_{D^{(*)}} \) and \( B_c \to \tau\nu \). In the top right panel of Fig. 2 we show the prediction for \( \text{BR}(B_c \to \tau\nu) \). We find branching ratios between 15% and 23% for the region of parameter space which explains both \( R_{D^{(*)}} \) and \( R_{K^{(*)}} \) at \( 1 - \sigma \). Thus limits on this process pose a direct constraint on the explanation of \( R_{D^{(*)}} \).

Several groups inferred limits on \( \text{BR}(B_c \to \tau\nu) < [0.1, 0.6] \) [51, 74, 85–87] via different theoretical arguments. In particular, Ref. [74] found that the branching ratio can be at most 10% which is in tension with the viable parameter space of the \( R_2 \) leptoquark explanation of \( R_{D^{(*)}} \). However, there is some controversy over this constraint: To derive this bound they needed to first extract the probability that a bottom quark hadronizes with a charmed quark using methods which were recently critiqued [86, 87]. Furthermore, in deriving the aggressive bound [74], they combined data from \( Z \) decays in LEP with \( p - p \) collisions at CMS and LHCb. However there are \( B_c \) production processes in the latter that have no counterpart to \( Z \) decays. The authors of Ref. [86, 87] also critique the less aggressive bound of 30% however, we note that any controversy over this bound
will have no bearing on our analysis as it is always satisfied. Even the most stringent constraint of 10\% may be avoided by extending the model with a $S_3$ leptoquark, as we discuss in Sec. 4.

Other constraints

Apart from the discussed constraints we also studied possible constraints from several other processes and we briefly summarize the results. The limits obtained from lepton-flavour-violating semi-leptonic $\tau$ decays, $\tau \rightarrow eP$ with a pseudo-scalar meson $P = K, \pi$, are always substantially weaker than the limit from $\tau \rightarrow e\gamma$ and thus we do not report them here. Furthermore, we considered leptonic meson decays, in particular $B_s \rightarrow ee$ and $D_s \rightarrow e\nu$, using flavio and as expected neither of them provides a relevant constraint. As the couplings required for an explanation of $R_{K^{(*)}}$ are small, the contribution to $B_s \rightarrow ee$ is suppressed. The dominant contribution to $D_s \rightarrow e\nu$ is controlled by $(Y_2)_{c\nu}(Y_4)_{es}$. While $(Y_4)_{es}$ is small, $(Y_2)_{c\nu}$ is constrained by its contribution to $\tau \rightarrow e\gamma$. Moreover, the contribution to the LFU ratios $R_{\mu/e} \equiv \Gamma(B \rightarrow D\mu\mu)/\Gamma(B \rightarrow Dep)$ and $R_{e/\mu} \equiv \Gamma(B \rightarrow D^*ee)/\Gamma(B \rightarrow D^*\mu\mu)$ are generally small, because the couplings $(Y_4)_{es}$ and $(Y_4)_{eb}$ which are responsible for explaining $R_{K^{(*)}}$ are small. Finally, let us turn our attention to $B_s - \bar{B}_s$ mixing. Matching the full theory with the $R_{2}^{(*)}$ leptoquark to SMEFT induces an effective four-quark interaction in SMEFT

$$
\mathcal{L} = -\frac{(Y_4^\dagger Y_4)_{ij}}{128\pi^2 m_R^2}(\bar{Q}_i \gamma_\mu Q_j)(\bar{Q}_i \gamma_\mu Q_j).
$$

In particular, this four-quark interaction induces a new contribution to $B_s - \bar{B}_s$ mixing which can be parameterized by

$$
\mathcal{L} = C^{bsbs}_{VLL} \langle s\gamma_\mu P_L b \rangle \langle s\gamma_\mu P_L b \rangle C^{bsbs}_{VLL,R_2} = -\frac{(Y_4^\dagger Y_4)_{sb}^2}{128\pi^2 m_R^2}.
$$

in weak effective theory. It interferes with the SM contribution (see e.g. [88])

$$
C^{bsbs}_{VLL,SM} = \frac{G_F^2 m_W^2}{4\pi^2} (V_{tb}^* V_{ts})^2 S_0(m_t^2/m_W^2)
$$

where $S_0$ is the Inami-Lim function [89]

$$
S_0(x) = \frac{x^3 - 11x^2 + 4x}{4(x - 1)^2} - \frac{3x^3}{2(x - 1)^3} \ln x.
$$

The contribution of $R_2$ to $C^{bsbs}_{VLL}$ can be expressed in terms of the Wilson coefficient $C_{VLL}$. A simple order of magnitude estimate shows that $C^{bsbs}_{VLL,R_2}$ is several orders of magnitude smaller as the SM contribution for the interesting $R_2$ leptoquark mass range

$$
\left| \frac{C^{bsbs}_{VLL,R_2}}{C^{bsbs}_{VLL,SM}} \right| \simeq \left( \frac{\alpha_{em}}{2\pi} \frac{m_{R_2}}{m_W} C_{VLL} \right)^2.
$$

We independently checked the contribution to $B_s - \bar{B}_s$ mixing using flavio with the same result.

3.2 Viable parameter space

We show the viable parameter space in Fig. 3 and the bottom right panel of Fig. 2. For a fixed leptoquark mass of $m_{R_2} = 1 \text{ TeV}$, the bottom right panel of Fig. 2 shows that an aggressive
constraint from $Z$ decays restricts two of the Yukawa couplings to the range $|(Y_4)_{rb}| \in [0.44, 1.0]$ and $|(Y_2)_{c\nu_s}| \in [1.0, 2.6]$ respectively. All quoted ranges are approximate and are only intended to give an indication. The product is also constrained by the need to explain $R_{\Delta(\gamma)}$ at the $1 - \sigma$ level to the range $|(Y_2)_{c\nu_s} (Y_4)_{rb}| \in [0.88, 1.3]$. The product is almost purely imaginary with $\arg((Y_2)_{c\nu_s} (Y_4)_{rb}) \in \pm [0.45, 0.54] \pi$, irrespective of the experimental constraints, which confirms previous findings [10, 67, 68].

The other two Yukawa couplings are generally smaller with $|(Y_4)_{eb}| \in [0.11, 0.37]$, $|(Y_4)_{es}| \in [0.015, 0.055]$. Their product constrained to the narrow range $|(Y_4)_{eb} (Y_4)_{es}| \in [0.0047, 0.0070]$ after imposing all experimental constraints. This is shown in the bottom panel of Fig. 3. For the points which satisfy all experimental constraints, the real part of the product is generally negative, the argument is weakly constrained to the range $\arg((Y_4)_{eb} (Y_4)_{es}) \in \pm [0.47, 1.0]$. This implies that the Wilson coefficient $C_{s\nu e e}^{\text{le}}$ is generally positive. The absolute value of the product is by contrast, confined to a narrow range $\sqrt{|(Y_4)_{eb} (Y_4)_{es}|} \in [4.7, 7.0] \times 10^{-3}$. The hierarchy $|(Y_4)_{es}| \ll |(Y_4)_{eb}|$ can be understood as follows. The constraint from $Z$-boson decays constrains the coupling $(Y_4)_{rb} \lesssim 1.0$ and thus the coupling $(Y_2)_{c\nu_s}$ has to be larger than 1 in order to explain $R_{\Delta(\gamma)}$. This in turn leads to a stronger constraint on $|(Y_2)_{c\nu_s}|$ from $\tau \to e\gamma$, because the suppression from the ratio $m_\tau/m_\tau$ is compensated by the large logarithm, $\ln(m_\tau^2/m_{\tau}^2)$. 

Figure 3: Relevant parameter range of the Yukawa couplings. Top left panel shows the phase vs. the magnitude for $(Y_4)_{eb} (Y_4^*)_{es}$ and the top right panel the phase vs. the magnitude of $(Y_2)_{c\nu_s} (Y_4)_{rb}$. In the bottom panel we plot the absolute values of the Yukawa couplings entering $R_{K(\pi)}$ against each other. All points explain $R_{K(\pi)}$ and $R_{\Delta(\gamma)}$ at the $1 - \sigma$ level. Dark blue points satisfy all experimental constraints. Light blue points satisfy strict limits on $\tau^{-} \to e^{-}\gamma$ but are excluded by other constraints.
4 Alleviating the (possible) tension with BR($B_c \rightarrow \tau \nu$) via a neutrino mass model

It is well known that the $S_3 \sim (3, 3, 1/3)$ leptoquark with the Yukawa interaction

$$\mathcal{L}_{S3} = - (Y_S)_{ab} \bar{Q}_a \gamma^\mu P_L L_{bk} S_{3;jl} \epsilon^{ij} \epsilon^{kl} + \text{h.c.} \quad (22)$$

and mass $m_S$ contributes to the Wilson coefficients

$$C_{d_{bca}}^{ql1} = \frac{3(Y_S)_{ab}(Y^*_S)_{cd}}{8m_S^2} \quad C_{d_{bca}}^{ql3} = \frac{(Y_S)_{ab}(Y^*_S)_{cd}}{8m_S^2} \quad (23)$$

of the vector operators $[90–92]$

$$\mathcal{L} = C_{abcd}^{ql1}(\bar{L}_a \gamma_\mu L_b)(\bar{Q}_c \gamma^\mu Q_d) + C_{abcd}^{ql3}(\bar{L}_a \gamma_\mu \tau^I L_b)(\bar{Q}_c \gamma^\mu \tau^I Q_d) \quad (24)$$

which can help alleviate the possible tension with BR($B_c \rightarrow \tau \nu$) at the cost of a contribution to the decay $B \rightarrow K\nu\nu$. Such a model involving two leptoquarks can be motivated by a neutrino mass model. In this section we sketch out how this is possible leaving a detailed analysis to future work.

4.1 Neutrino masses

Just extending the SM with $R_2$ and $S_3$ leptoquarks is not sufficient to generate non-zero neutrino masses. To keep our model minimal we will extend our two leptoquark extension of the SM by a single particle which is a SU(2)$_L$ quadruplet with quantum numbers $\Sigma \sim (1, 4, \frac{3}{2})$. Then neutrino masses are generated at the 1-loop level as shown in the left panel of Fig. 4. There is also a 2-loop contribution which is shown on the right.

![Figure 4: Neutrino mass from leptoquarks in the loop. The superscripts $q$ of $R_2^q$ and $S_3^q$ denote the electromagnetic charge of the different components of $R_2$ and $S_3$, respectively.](image)

If the 2-loop diagram can be neglected, neutrino masses are approximately given by their 1-loop contribution

$$\langle m_{\nu}\rangle_{ij} \simeq \frac{1}{16\pi^2} \frac{\mu\langle\Sigma^0\rangle}{m_{R_2}^2 - m_S^2} \left( Y_2 \right)_{ki} \left\{ m_k \left[ F \left( \frac{m_{R_2}^2}{m_k^2} \right) - F \left( \frac{m_S^2}{m_k^2} \right) \right] V_{kl}^* \right\} (Y_S)_{ij} + (i \leftrightarrow j) \quad (25)$$

in the limit of small mixing between $R_2$ and $S_3$ leptoquarks, which is generated by the trilinear potential term $\mu \Sigma^{*ijk} R_{2i} S_{3jk}$. The Yukawa coupling matrices are defined in the basis where

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*Further details we relegate to the appendix.*
charged lepton and down-type quark mass matrices are diagonal. Thus the loop diagram with up-type quarks in the loop is proportional to the CKM mixing matrix element $V_{kl}^\ast$. Roman indices $i,j,k,l$ indicate flavour, $m_k$ the up-type quark mass, $m_{R_2}, m_S$ are $R_2$ and $S_3$ leptoquark masses respectively, and $\langle \Sigma^0 \rangle$ the VEV of the neutral component of $\Sigma$. The loop function $F(x)$ is defined as

$$F(x) = \frac{x \ln x}{1-x}$$

The more general expression for a general mixing angle between the $R_2$ and $S_3$ leptoquarks is given in appendix A. The 2-loop contribution features a similar flavour structure.

5 Conclusion

We demonstrate that the $R_2 \sim (3,2,7/6)$ leptoquark is a new single particle candidate for explaining the anomalous lepton-flavour-universality ratios $R_K(\ast)$ and $R_D(\ast)$. There is possibly a mild tension with the theoretically-derived limit on the branching ratio $\text{BR}(B_c \rightarrow \tau \nu)$. Since we require the branching ratio of $B_c \rightarrow \tau \nu$ to be within a relatively narrow range, the viability of the $R_2$ leptoquark as a single particle solution to these anomalies is a directly falsifiable scenario. Another promising probe of the viable parameter space of our model is BR($\tau \rightarrow e\gamma$) where the projected sensitivity for Belle II is expected to improve by an order of magnitude [82].

The tension with the disputed aggressive limit on BR($B_c \rightarrow \tau \nu$) can be alleviated through the introduction of a $S_3$ leptoquark which can be motivated by a neutrino mass model as discussed in Sec. A. This suggests that even if future analysis indeed rules out the $R_2$ leptoquark as a single leptoquark solution to anomalous $B$ decays, it still can play a substantial role in an extended model.

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A Leptoquark mixing and neutrino masses

The relevant terms in the scalar potential $V = V_0 + V_1$ are

$$V_0 = \sum_{H,R_2,S_3, \Sigma \in \mathbb{R}} \left( (-1)^{q_x} m_x^2 |x|^2 + \frac{\lambda_x}{2} |x|^4 \right) + \sum_{H,R_2,S_3, \Sigma \in \{<0\}} \lambda_{xy} |x|^2 |y|^2$$

$$V_1 = \mu \Sigma^{*ijk} R_{2i} S_{3jk} + \lambda_3 \Sigma H \Sigma^{*ijk} H_i H_j H_k + \lambda_2 H S_{2i} S_{3ijk} H m_{ijl} \epsilon^{lm} + \text{h.c.}$$

where $(-1)^{q_x}$ is $-1$ for $H$ and $\Sigma$ and $+1$ for the other scalar fields. Thus the general form for neutrino masses at 1-loop order is given by

$$(m_\nu)_{mn} = \frac{1}{16\pi^2} \left( U_s^\dagger \right)_{R_2 s_i} (Y_2)_{km} \left[ m_k F \left( \frac{m_S^2}{m_k^2} \right) V_{kl}^\ast \right]^{kl} (Y_S)_{ln} (U_s)_{s_i S_3} + (m \leftrightarrow n).$$
The mixing between $R_2^{2/3}$ and $S_3^{2/3}$ is generated by $\langle \Sigma^0 \rangle$ and is obtained by diagonalizing the charge $2/3$ leptoquark mass matrix which is given in the $(R_2^{2/3}, S_3^{2/3})$ basis

$$M_3^2 = \begin{pmatrix}
\mu_R^2 + \lambda_{HR} \frac{v_H^2}{2} + \lambda_{RS} \frac{v_S^2}{2} & \mu_{H^2}^2 \\
\mu_{H^2}^2 & \mu_S^2 + \lambda_{HS} \frac{v_H^2}{2} + \lambda_{SS} \frac{v_S^2}{2}
\end{pmatrix} = U_3^T \text{Diag} \left( m_{S_1}^2, m_{S_2}^2 \right) U_3 $$

with $v_H = \langle H^0 \rangle / \sqrt{2}$ and $v_S = \langle \Sigma^0 \rangle / \sqrt{2}$, the masses $m_{S_i}$ and the $2 \times 2$ unitary mixing matrix $U_3$ which defines the mass eigenstates $S_i$ in terms of the flavour eigenstates

$$\begin{pmatrix} S_1 \\ S_2 \end{pmatrix} = U_3 \begin{pmatrix} R_2^{2/3} \\ S_3^{2/3} \end{pmatrix} \quad U_3 = \begin{pmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{pmatrix}.$$ (31)

A straightforward calculation results in the following expressions for the rotation angle and the masses

$$\tan (2\theta) = \frac{\sqrt{2} \mu_{V^2}}{\mu_R^2 - \mu_S^2 + \frac{v_H^2}{2} (\lambda_{HR} - \lambda_{HS}) + \frac{v_S^2}{2} (\lambda_{RS} - \lambda_{SS})}$$ (32)

$$m_{S_1,2}^2 = \frac{\mu_R^2 + \mu_S^2 + \frac{v_H^2}{2} (\lambda_{HR} + \lambda_{HS}) + \frac{v_S^2}{2} (\lambda_{RS} + \lambda_{SS})}{2}$$ (33)

$$\pm \frac{1}{2} \sqrt{\left[ \left( \mu_R^2 - \mu_S^2 + \frac{v_H^2}{2} (\lambda_{HR} - \lambda_{HS}) + \frac{v_S^2}{2} (\lambda_{RS} - \lambda_{SS}) \right)^2 + 4 \mu_{H^2}^2 \right]}.$$

For small $\mu$ and thus small mixing, the square of the masses $m_{R_2}$ and $m_S$ in the main part of the text can be identified with the diagonal elements of the scalar mass matrix $M_3^2$, $m_{R_2}^2 = \mu_R^2 + \lambda_{HR} v_H^2 / 2 + \lambda_{RS} v_S^2 / 2$ and $m_S^2 = \mu_S^2 + \lambda_{HS} v_H^2 / 2 + \lambda_{SS} v_S^2 / 2$.

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