Dynamics and control of the CO$_2$ level via a differential equation and alternative global emission strategies

Abstract

The analysis in this paper shows that the fundamental theory of the CO$_2$ level in the atmosphere, under the influence of changing CO$_2$ emissions, can be modeled as a first order linear differential equation with a forcing function, describing industrial emissions.

Observations of the CO$_2$ level at the Mauna Loa observatory and official statistics of global CO$_2$ emissions, from Edgar, the Joint Research Centre at the European Commission, are used to estimate all parameters of the forced CO$_2$ differential equation.

The estimated differential equation has a logical theoretical foundation and convincing statistical properties. It is used to reproduce the time path of the CO$_2$ data from Mauna Loa, from year 1990 to 2018, with very small errors. Furthermore, the differential equation shows that the global CO$_2$ level, without emissions, has a stable equilibrium at 280 ppm. This value has earlier been reported by IPCC as the pre-industrial CO$_2$ level.

The differential function is applied to derive four dynamic cases of the global CO$_2$ level, from the year 2020 until 2100, conditional on four different strategies concerning the development of global CO$_2$ emissions:

i. Emissions continue to increase according to the trend during 1990–2018
ii. Emissions stay for ever at the 2020 level
iii. Emissions are reduced with a linear trend to become zero year 2100
iv. Emissions are reduced with a linear trend to become zero year 2050

In case i., the CO$_2$ level year 2100 will be 688 ppm. In cases ii. and iii., the CO$_2$ levels in 2100 will be 517 ppm and 389 respectively. In case iv., the CO$_2$ level in 2050 is 408 ppm and then rapidly falls.

Introduction

The global warming and CO$_2$ dynamics issue, for very good reasons, attracts considerable global interest. The climate of our planet is of key importance to all life. The author recommends the reader to study Ramade	extsuperscript{1} in detail for a deep understanding of many of the connected issues and theories.

The first ambition is to understand the fundamental mechanisms of the dynamics of the CO$_2$ level in the atmosphere under the influence of global emissions.

We will investigate if it is possible to develop a theoretical mathematical model of the dynamics of CO$_2$. Such a model should be consistent with fundamental scientific principles. Furthermore, it should be possible to use the model to reproduce historical time series of empirical data. If such a model can be developed, it should be possible to use it also for predictions. Then, the most important application is to investigate how the global CO$_2$ level can be dynamically changed via different emissions strategies.

Statistics of the CO$_2$ level in the atmosphere and the global CO$_2$ emissions

The CO$_2$ level of the atmosphere has been recorded since 1958, at the Mauna Loa observatory. See Tans and Keeling	extsuperscript{2}. The statistical tables are well documented and freely available via the internet. In Figure 1 the annual mean values of CO$_2$ are shown. The web link connected to the reference provides access to all observations via a text file with instructions. In several cases, transformations between different physical units are necessary. O’Hara	extsuperscript{3} includes the relevant conversion factors.

In Figure 2 we find observations of global CO$_2$ emissions from fossil fuels combustion and processes. These data come from European Commission	extsuperscript{4}. The observations from 1990, 2000, 2010 and 2018 have been used in the analysis of this paper. There are two reasons for this: First, emission data were only collected with ten year intervals during the early years. Second, sufficiently long time intervals are needed if we want to be able to estimate the changes of CO$_2$ in the atmosphere with sufficiently high precision.

In the estimations of a differential equation, the following three periods will be used: 1990–2000, 2000–2010 and 2010–2018. More details about these periods are found in Table 1.

The emission forced differential equation of the global CO$_2$ level

The general theory of differential equations can be studied in Braun.	extsuperscript{5}
Let us first consider the following differential equation. We will soon discover that it has to be adjusted in order to become relevant to the CO₂ problem.

\[ x = \frac{dx}{dt} = a_0 \]  \hspace{1cm} (1)

\( x = x(t) \) is the CO₂ level in the atmosphere as a function of time. \( \frac{dx}{dt} \) is the change per time unit, or the time derivative, of \( x \). There are constant "natural" emissions, from the oceans, volcanoes and other parts of the natural environment, greater than zero. \( a_0 > 0 \). Hence, \( x = \frac{dx}{dt} \) would be strictly positive and \( x \) would increase over time, without bound, if nothing would stop that.

However, earlier CO₂ research has already shown that the CO₂ level has been stable during very long periods of time. Compare Ramade¹ and Solomon et al.⁶

Figure 1 CO₂ in the atmosphere, annual mean values, Mauna Loa, (ppm). Source: Tans and Keeling.²

Figure 2 Obs=Observations of global CO₂ emissions from fossil fuels combustion and processes. Source: European Commission.⁴ Approx=Linear approximation via the least squares method, by the author of this paper. Compare equation (47). Approx \( = 21.672 + 0.57366(Year - 1990) \). \( R \approx 0.984 \).
Dynamics and control of the CO₂ level via a differential equation and alternative global emission strategies

Let us assume that the oceans (and, to some degree, other parts of the natural environment) absorb a part of the CO₂ in the atmosphere. Let us also assume that the absorption is proportional to the CO₂ level in the atmosphere, $x$. This is a very reasonable assumption since the probability that a CO₂ molecule touches the surface of the sea is proportional to the CO₂ level in the atmosphere. Let the absorption be $-a_x x$. Then, we have this differential equation of global CO₂:

$$x = \frac{dx}{dt} = a_q + a_x x$$

(2)

Is there an equilibrium? Let $x = \frac{dx}{dt} = a_q + a_x x = 0$.

(3)

Yes, there is one and only one equilibrium.

$$x = 0 \quad \Rightarrow \quad x = x_{eq} = -\frac{a_q}{a_x} > 0$$

(4)

Is this equilibrium stable? Yes, if something disturbs $x$ so that $x < x_{eq}$, then $\frac{dx}{dt} > 0$, which means that $x$ increases until $x = x_{eq}$. If $x > x_{eq}$, then $x < 0$, and $x$ decreases until $x = x_{eq}$.

According to earlier research, the pre-industrial equilibrium level of CO₂ was 280 ppm (parts per million). Compare the IPCC report by Solomon et al. In this paper, we will find that the derived model confirms this finding. In other words, we will confirm that.

$$x_{eq} = -\frac{a_q}{a_x} \approx 280$$

(5)

In order to determine the parameters of a function, it is necessary to have some variation in the data. In particular, when we want to determine the values of the parameters of the differential equation of $x$, we can not do this if $x = x_{eq}$ all the time. In this respect, it is useful to observe that the industrial emissions of CO₂ during the latest decades have created earlier not available variation in $x$. Let us regard global emissions of CO₂ after the industrial revolution, $\phi(t)$, as a function of $t$. The emissions are added to the CO₂ in the atmosphere.

$$x = a_q + a_x x + \phi(t)$$

(6)

Now, since we have access to empirical data for $\left( x, \phi \right)$ in different time periods, we can estimate the parameters $\left( a_q, a_x \right)$ via the ordinary least squares method (regression analysis) in the following way:

$$y(t) = x - \phi(t) = a_q + a_x x(t)$$

(7)

Table 1 includes the transformations of the available atmospheric CO₂ raw data to a time series of $x$ that will be used in the analysis. In a similar way, in Table 2 the global emission data is developed to time series data for $\phi$.

In different statistical sources and equations, the CO₂ of the atmosphere is given in different units. Following the principles by O’Hara, the following transformation rules have been applied: 1 ppm (CO₂) can be transformed to $2.13*3.664=7.80432$ Gt CO₂. 1 g C=0.083 mole CO₂.

Table 1 includes the transformations of the available atmospheric CO₂ raw data to a time series of $x$ that will be used in the analysis. In a similar way, in Table 2 the global emission data is developed to time series data for $\phi$.

Below, a very high level of detail in the calculations has been selected. The motivation is the following: The CO₂ dynamics and global warming issue is critical to the present global political debate. It is necessary that the reader can investigate and repeat all derivations without problems.

We want to determine the parameters $\left( a_q, a_x \right)$ in this function:

$$y = a_q + a_x x$$

(8)

We minimize the sum of squares of the residuals:

$$\min_{a_0, a_x} Z = \sum_{i=1}^{N} (y_i - a_0 - a_x x_i)^2$$

(9)

These are the first order optimum conditions:

$$\frac{dZ}{da_0} = \sum_{i=1}^{N} 2(y_i - a_0 - a_x x_i) = 0$$

(10)

$$\frac{dZ}{da_x} = \sum_{i=1}^{N} 2(y_i - a_0 - a_x x_i) x_i = 0$$

(10)

Table 1

| $i$ (period) | $t$ (year) | $\psi_i$ (ppm) | $\Delta x_i$ (ppm) | $\Delta t$ (years) | $x_i$ (Gt CO₂) | $\bullet \frac{\Delta x_i}{\Delta t}$ (ppm per year) | $\bullet \frac{\Delta x_i}{\Delta t}$ (Gt CO₂ per year) |
|--------------|------------|----------------|--------------------|-------------------|----------------|---------------------------------|---------------------------------|
| 1            | 1990       | 354.39         |                    |                   |                |                                 |                                 |
| 2            | 2000       | 369.55         | 15.16              | 10                | 361.97         | 2824.9                          | 1.516                           |
| 3            | 2010       | 389.90         | 20.35              | 10                | 379.725        | 2963.5                          | 2.035                           |
| 4            | 2018       | 408.52         | 18.62              | 8                 | 399.21         | 3115.6                          | 2.3275                          |

Definitions in table 1: $\psi_i = CO_2$ in atmosphere, annual mean value of observations, Mauna Loa.

$x_i = CO_2$ in atmosphere, calculated mean value.

Gt denotes Giga tonnes and ppm denotes parts per million.

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Table 2: Atmospheric CO\(_2\) data transformations

| \(i\) (period) | \(f\) (year) | \(\gamma_i\) (Gt CO\(_2\)) | \(\varphi_i\) (Gt CO\(_2\) per year) | \(\phi_i\) (Gt CO\(_2\) per year) | \(\phi_i\) (ppm per year) |
|-----------------|--------------|-----------------------------|----------------------------------|--------------------------------|------------------------|
| 1990            | 22.637       |                             |                                  |                                |                        |
| 1               | 24.119       | 12.288                      | 1.5745                           |                                |                        |
| 2000            | 25.601       |                             |                                  |                                |                        |
| 2               | 29.7185      | 13.8365                     | 1.7729                           |                                |                        |
| 2010            | 33.836       |                             |                                  |                                |                        |
| 3               | 35.8615      | 17.6965                     | 2.2675                           |                                |                        |
| 2018            | 37.887       |                             |                                  |                                |                        |

Definitions in table 2: \(\gamma_i\) = Global total CO\(_2\) emission, observation
\(\varphi_i\) = Global total CO\(_2\) emission, calculated mean value
\(\phi_i = \varphi_i - x_i\)

Table 3: Regression data

| \(i\) | \(x_i\) (ppm) | \(y_i\) (Gt CO\(_2\) per year) |
|-------|---------------|-------------------------------|
| 1     | 361.97        | -12.288                       |
| 2     | 379.725       | -13.8365                      |
| 3     | 399.21        | -17.6965                      |

Definitions in table 3: \(y_i = -\phi_i = -\left(\varphi_i - x_i\right)\)

They are further developed:
\[
\begin{align*}
\frac{dZ}{da_0} &= 2 \sum_{i=1}^{N} \left( a_0 + a_i x_i - y_i \right) = 0 \\
\frac{dZ}{da_i} &= 2 \sum_{i=1}^{N} \left( a_0 x_i + a_i x_i^2 - x_i y_i \right) = 0
\end{align*}
\]

We also want to investigate if the derived solution gives a unique minimum:
\[
\begin{align*}
\frac{d^2Z}{d\alpha_0^2} &= 2 \sum_1 = 2N > 0 \\
\frac{d^2Z}{d\alpha_i^2} &= 2 \sum x^2 > 0
\end{align*}
\]

\[
\Phi = \left| \begin{array}{cc}
\frac{d^2Z}{da_0 d\alpha_0} & \frac{d^2Z}{d\alpha_0 d\alpha_i} \\
\frac{d^2Z}{d\alpha_0 d\alpha_i} & \frac{d^2Z}{d\alpha_i^2}
\end{array} \right|
= \left| \begin{array}{cc}
\frac{2 \sum_1}{2 \sum x} & \frac{2 \sum x}{2 \sum x^2} \\
\frac{2 \sum x}{2 \sum x^2} & \frac{2 \sum x^2}{2 \sum x^2}
\end{array} \right|
= \frac{4 N (N - 1) N \sum x^2 - (\sum x)^2}{N \sum x^2 - (\frac{\sum x}{N})^2}
\]

The parameters can be determined from this simultaneous equation system (Table 4):

\[
\Phi = 4 N^2 \left( E \left[ x^2 \right] - E \left[ x \right]^2 \right) > 0
\]

Hence, the second order conditions of a unique minimum are satisfied. The first order conditions give a unique minimum. The first order optimum conditions imply:
\[
\begin{align*}
\left( \frac{\sum x, y_i}{\sum x^2} \right) & a_0 = \left( \frac{\sum x^2}{\sum x^2} \right) a_i = \left( \frac{\sum x_i y_i}{\sum x_i} \right)
\end{align*}
\]

The point \((a_0, a_i)\) is determined via Cramers rule:
\[
\begin{align*}
a_0 &= \frac{\sum x_i y_i}{\sum x_i} \\
&= \frac{\sum x_i y_i}{\sum x_i} = 40.951
\end{align*}
\]

\[
\begin{align*}
a_i &= \frac{\sum \sum x_i y_i}{\sum x_i^2} \\
&= \frac{\sum \sum x_i y_i}{\sum x_i^2} = -0.14609
\end{align*}
\]

If we express \(x\) in the unit Gt CO\(_2\)/year, and \(x\) in the unit ppm, we have this equation:
\[
x = 40.951 - 0.14609 x
\]

What is the equilibrium value of \(x\), via the derived function, in case there are no emissions?

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\[ \dot{x} = \frac{dx}{dt} = a_0 + a_s x_{eq} = 0 \]  
\[ x_{eq} = \frac{a_0}{a_s} \approx 280.31 \text{ ppm} \]  

Note that this value confirms the earlier empirical finding by Solomon et al.$^6$ If we express $x$ in the unit Gt CO$_2$/year, and $x$ in the unit Gt CO$_2$, we get the following differential equation (Figure 3).

\[ x = 40.951 - 0.0187191 x \]  
\[ x_{eq} = \frac{-40.951}{-0.0187191} \approx 2187.66 \text{ (Gt)} \]  

Figure 3 Determination of the global CO$_2$ differential equation via the empirical observations of CO$_2$ from Mouna Loa and the empirical observations of global CO$_2$ emissions. The estimated equilibrium value of CO$_2$ is 280 ppm, in case the global emissions of CO$_2$ are zero. This confirms the earlier findings. Compare Solomon et al.$^6$ The estimated function is: $40.951 - 0.14609 \times$ CO$_2$ (ppm). The multiple correlation coefficient R=0.977. Since the number of observations is limited, more detailed regression statistics will not be given here.

| Table 4 Parameter values |
|--------------------------|
| $N$          | 3  |
| $\sum x_i$ | 1140.905 |
| $\sum x_i^2$ | 434581.9806 |
| $\sum x_i y_i$ | -16766.57209 |
| $\sum y_i$ | -43.821 |

Determination of the differential equation of CO$_2$ in the atmosphere under the influence of changing CO$_2$ emissions

Now, the complete differential equation will be determined, giving the dynamic development of the CO$_2$ level in the atmosphere as a function of the development of the global emissions.

This is the differential equation in general form:

\[ \dot{x} = a_0 + a_s x + \varphi(t) \]  

We will consider the special case of emissions that grow with a linear trend, since that is supported by the available empirical data. (Note that the forcing function could be generalized to almost any form, if considered relevant.)

\[ \varphi(t) = m_0 + m_1 t \]  

The differential equation becomes:

\[ \dot{x} = a_s x = a_0 + m_0 + m_1 t \]  

Solution of the homogenous equation:

\[ x_i - a_s x_{eq} = 0 \]  
\[ x_i = A e^{s t} \]  
\[ s = a_s \]  
\[ (s - a_s) x_i = 0 \]  
\[ (s - a_s) x_i = 0 \]  

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(x₀ ≠ 0) ⇒ s = aᵢ

\( x(t) = Ae^{s \cdot t} \) 

(36)

Determination of the particular solution:

\[ x_p = k_0 + k_1 \cdot t \]

(37)

\[ x_p - a_1 \cdot x_p = a_0 + m_0 + m_1 \cdot t \]

(38)

\[ k_1 - a_1 \cdot (k_0 + k_1 \cdot t) = a_0 + m_0 + m_1 \cdot t \]

(39)

\[ \begin{cases} 
  k_1 - a_1 \cdot k_0 = a_0 + m_0 \\
  -a_1 \cdot k_1 = m_1 
\end{cases} \]

(40)

\[ (-a_1 \cdot k_1 = m_1) \Rightarrow k_1 = -\frac{m_1}{a_1} \]

(41)

\[ (k_1 - a_1 \cdot k_0 = a_0 + m_0) \land k_1 = -\frac{m_1}{a_1} \Rightarrow \left( -\frac{m_1}{a_1} - a_1 \cdot k_0 = a_0 + m_0 \right) \]

(42)

\[ k_0 = \left( -\frac{a_0 + m_0 + m_1}{a_1} \right) \]

(43)

\[ m_0 = \left( \frac{\sum \phi_j \cdot \sum t_j}{N \cdot \sum t_j^2} \right) \approx 38403.096 \]

(45)

\[ m_1 = \left( \frac{N \cdot \sum \phi_j \cdot \sum t_j}{N \cdot \sum t_j^2} \right) \approx 1016.526 \]

(46)

\[ \text{(The multiple correlation coefficient: } R = 0.984 \text{ )} \]

The point \((m_0, m_1)\) is determined via Cramer's rule:

\[ \varphi(t) = -0.57366 -0.0187191 \approx 30.646 \]

(48)

\[ \varphi(t) = 21.672 + 0.57366 \cdot t \]

(47)

\[ x(t) = 1057.52 \]

(54)

\[ x(t) = 1057.52e^{-0.0187191 \cdot t} + 1708.27 + 30.646 \cdot t \] (Gt)

(55)

\[ x(t) = 135.50e^{-0.0187191 \cdot t} + 218.89 + 3.927 \cdot t \] (ppm)

(56)

In Figure 4 we find that the estimated function can reproduce the CO₂ observations from Mauna Loa extremely well. Most years during the period 1990 to 2018, the deviations are less than 1 ppm.

Predictions into the future

Now, the estimated differential equation will be used to predict the future development of the CO₂ level, conditional on the following four alternative global emission strategies:

- Cont: During the period 2020 to 2100, the emissions continue to increase according to the trend estimated during the period 1990 to 2018.

- Lev 2020: The emissions 2020 are estimated from the trend 1990 to 2018. Then, the emissions stay at that level until 2100.

- Stop 2100: The emissions 2020 are estimated from the trend 1990 to 2018. Then, the emissions are reduced with a constant amount each year, such that the emissions are zero in 2100.

- Stop 2050: The emissions 2020 are estimated from the trend 1990 to 2018. Then, the emissions are reduced with a constant amount each year, such that the emissions are zero in 2050.

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Table 5 Regression data

| j | Year | t | \( \varphi(t) \) (Gt CO₂ per year) |
|---|------|---|--------------------------------|
| 1 | 1990 | 0 | 22.637 |
| 2 | 2000 | 10 | 25.601 |
| 3 | 2010 | 20 | 33.836 |
| 4 | 2018 | 28 | 37.887 |

Definitions in table 5: \( t = \text{Year} - 1990 \)

Table 6 Parameter values

| \( N \) | 4 |
| \( \sum t_j \) | 58 |
| \( \sum t_j^2 \) | 1284 |
| \( \sum t_j \cdot \phi_j \) | 1993.566 |
| \( \sum \phi_j \) | 119.961 |

The parameters can be determined from this simultaneous equation system:

\[ \left[ \begin{array}{cc} N \cdot \sum t_j & \sum \phi_j \\ \sum t_j \cdot \sum t_j^2 & m_0 \cdot m_1 \end{array} \right] = \left[ \begin{array}{c} \sum \phi_j \\ \sum t_j \cdot \phi_j \end{array} \right] \]

(44)
In Figure 5 we see the graphs of the four emission scenarios and in Table 7, we find more details about the four scenarios. The general principles derived and described in the earlier sections of this paper have been used to derive the equations of the CO$_2$ level that are consistent with the four different emission scenarios. The parameters are presented in Table 8 for the unit Gt, and in Table 9 for the unit ppm.

\[ x(t) = 135.50e^{-0.0187t} + 218.89 + 3.927t \text{ (ppm)} \]

**Figure 4** Mauna Loa= CO$_2$ observations from 1990 to 2018. Model= CO$_2$ prediction model. The empirical CO$_2$ observations from Mauna Loa, compare Figure 1 and the prediction according to the derived differential equation model are almost identical. The graph was derived with the following equation:

**Figure 5** Four different alternative scenarios for the future development of global CO$_2$ emissions, during the time interval 2020 to 2100. The emission level 2020 is estimated via the linear approximation based on data from the time interval 1990 to 2018. The scenarios are used to predict the future development of CO$_2$ in the atmosphere. Compare figure 6. Cont=The emissions continue to develop according to the trend during 1990 to 2018. Lev 2020=The emissions stay, for ever, at the level of 2020. Stop 2100=The emissions are reduced with the same amount each year, during the time interval 2020 until 2100. Then, the total emission is zero. Stop 2050=The emissions are reduced with the same amount each year, during the time interval 2020 until 2050. (Observation: The negative emissions after 2050 are technically possible but not necessarily optimal and relevant.)

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Table 7 Parameter values for predictions

| Alternative | Year when t=0 | x(0)$_{ppm}$ | a0 | ax | m0 | m1 |
|-------------|---------------|--------------|----|----|----|----|
| Cont        | 1990          | 354.39       | 40.951 | -0.01872 | 21.672 | 0.57366 |
| Lev 2020    | 2020          | 413.96911    | 40.951 | -0.01872 | 38.8818 | 0 |
| Stop 2100   | 2020          | 413.96911    | 40.951 | -0.01872 | 38.8818 | -0.48602 |
| Stop 2050   | 2020          | 413.96911    | 40.951 | -0.01872 | 38.8818 | -1.29606 |

Table 8 Parameter values for predictions

| Alternative | k0 (Gt) | k1 (Gt) | A (Gt) |
|-------------|---------|---------|--------|
| Cont        | 1708.27101 | 30.64570412 | 1057.501954 |
| Lev 2020    | 4264.777687 | 0 | -1034.030282 |
| Stop 2100   | 5651.809577 | -25.96398865 | -2421.062173 |
| Stop 2050   | 7963.529394 | -69.23730308 | -4732.781989 |

Results and discussion

The developed model will now be used to investigate the dynamic effects of four different alternative scenarios for the future development of global CO$_2$ emissions, during the time interval 2020 to 2100. In Figure 5 we find the four emission scenarios. The predictions of the future CO$_2$ level, conditional on the different emission strategies, are found in Figure 6. The predictions function, (57) is used. Then, $t$ is defined according to the information in Table 7 and the parameter values $A, k_0, k_1$ from Table 9 are used.

$$x(t) = Ae^{-0.0187191t} + k_0 + k_1 t \quad (ppm)$$ (57)

Figure 6 Four different alternative scenarios for the future development of CO$_2$ level in the atmosphere, during the time interval 2020 to 2100. The scenarios are conditional on the global emission scenarios found in figure 5. The emission level 2020 is estimated via the linear approximation based on data from the time interval 1990 to 2018. Cont=The emissions continue to develop according to the trend during 1990 to 2018. Lev 2020=The emissions stay, for ever, at the level of 2020. Stop 2100=The emissions are reduced with the same amount each year, during the time interval 2020 until 2100. Then, the total emission is zero. Stop 2050=The emissions are reduced with the same amount each year, during the time interval 2020 until 2050. After 2050, the net emission is strictly negative and follows the same trend as before 2050. (Observation: The negative emissions after 2050 contribute to the dramatic fall of the CO$_2$ level after 2050 in this scenario. If the emissions would be zero after 2050, the CO$_2$ level would converge to the pre-industrial level of 280 ppm. Alternative scenarios may easily be constructed.)

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Table 9 Parameter values for predictions

| Alternative | k0 (ppm) | k1 (ppm) | A (ppm) |
|-------------|----------|----------|---------|
| Cont        | 218,88778738 | 3,926761604 | 135,5021262 |
| Lev 2020    | 546,4637133 | 0        | -132,4946033 |
| Stop 2100   | 724,1898816 | -3,326873918 | -310,2207716 |
| Stop 2050   | 1020,400162 | -8,871663781 | -606,4310522 |

Conclusion

Now, it is possible to understand the fundamental mechanisms of the dynamics of the CO$_2$ level of the atmosphere, under the influence of global emissions.

A theoretical mathematical model of the dynamics of CO$_2$ has been developed. This model is consistent with fundamental scientific principles. Furthermore, we can use the model to reproduce historical time series of empirical data. We can even use the model to calculate the pre-industrial level of CO$_2$ and discover that the calculated equilibrium value is consistent with earlier research findings. The model can also be used for Predictions. We have investigated how the global CO$_2$ level can be dynamically changed via different emissions strategies. Detailed predictions of possible future developments have been produced and described.

The CO$_2$ and global warming topic is central to the present global political agenda. It is necessary to create a fundamental understanding of the principles and methods that can be used to handle the problems and to stabilize our global climate. The model developed in this paper can hopefully make it possible for a large part of the human population to really understand how the CO$_2$ dynamics and emissions are connected. Without this fundamental understanding, it is difficult to convince critical persons that large investments in emission reductions may be necessary in order to stabilize the global climate.

The model developed in this paper should be possible to understand, investigate and to reproduce, in every detail by every person that has a PhD or masters degree in engineering, mathematics, mathematical statistics or mathematical economics. Earlier models presented on similar topics are not presented with all the details. Completeness and transparancy are necessary for complete understanding and acceptance.

According to the Occams razor, a scientific model should not be more complicated than necessary. In this paper, a differential equation is developed that is only based on very fundamental principles from physical science and mathematics. Two highly reliable sources of empirical data have been used to estimate the parameters. In the analysis, we have seen that a first order differential equation with emission forcing has been able to explain the development of the dynamics of the CO$_2$ level in the atmosphere, with very high precision. Furthermore, the function shows that the CO$_2$ equilibrium level, before the industrial revolution, should be 280 ppm, which confirms earlier empirical research. According to the opinion of the author, it is hardly possible to develop a more simple scientific model that explains the CO$_2$ dynamics in a better way.

Finally, the author hopes that the new model will be used to optimize and control global emission reductions, in order to give our planet the optimal climate.

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Conflicts of interest

The author declares that there was no conflict of interest.

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