THE FEFFERMAN–STEIN TYPE INEQUALITIES FOR
THE MULTILINEAR STRONG MAXIMAL FUNCTIONS

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Abstract. Let \( \vec{\omega} = (\omega_1, \ldots, \omega_m) \) be a multiple weight and \( \{\Psi_j\}_{j=1}^m \) be a sequence of Young functions. Let \( \mathcal{M}_{\vec{\omega}}^{\vec{\Psi}} \) be the multilinear strong maximal function with Orlicz norms which is defined by

\[
\mathcal{M}_{\vec{\omega}}^{\vec{\Psi}}(\vec{f})(x) = \sup_{R \ni x} \prod_{j=1}^m \|f_j\|_{\Psi_j,R},
\]

where the supremum is taken over all rectangles with sides parallel to the coordinate axes. If \( \Psi_j(t) = t \), then \( \mathcal{M}_{\vec{\omega}}^{\vec{\Psi}} \) coincides with the multilinear strong maximal function \( \mathcal{M}_{\vec{\omega}}^\vec{\psi} \) defined and studied by Grafakos et al. In this paper, we first investigated the Fefferman-Stein type inequality for \( \mathcal{M}_{\vec{\omega}}^{\vec{\Psi}} \) when \( \vec{\omega} \) satisfies the \( A_{\infty}^\mathbb{R} \) condition. Then, for arbitrary \( \vec{\omega} \geq 0 \) (each \( \omega_j \geq 0 \)), the Fefferman-Stein type inequality for the multilinear strong maximal function \( \mathcal{M}_{\vec{\omega}}^\vec{\psi} \) associated with rectangles will be given.

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