On model independent extraction of the Kaon Fragmentation Functions

S. Albino¹, E. Christova², a and E. Leader³

¹ Institut für Theoretische Physik, Universität Hamburg, Hamburg, Germany
² Institute for Nuclear Research and Nuclear Energy of Bulgarian Academy of Sciences, Sofia, Bulgaria
³ Imperial College, London University, London, UK

Abstract. We present a model independent approach to FFs, which allows to determine uniquely the non-singlet combination \((D_u - D_d)^{K^+ + K^-}\) of the kaon FFs, independently in the three different processes of kaon inclusive production: \(e^+e^-\) annihilation, \(pp\)-collisions and \(lN\) semi-inclusive DIS. Only C and SU(2) invariances are used and no assumptions on FFs and PDFs, the result holds in any order in QCD. The non-singlets obtained in this way, would allow to test the commonly used assumptions in global fit analysis and to test universality. This method is applied to kaon production data in \(e^+e^- \rightarrow K^+X, K = K^\pm, K^0\) and \((D_u - D_d)^{K^+ + K^-}\) is obtained without any assumptions. This result is compared to the existing parametrizations, obtained in global fit analysis.

1 Introduction

In order to extract correct information about the quark and lepton interactions from high energy experiments, the precise knowledge of two basic quantities are required – the parton distribution functions (PDFs), that describe the distribution of the nucleon momentum among its constituents – the quarks and gluons, and the fragmentation functions (FFs), that describe the distribution of the parton momentum among the hadrons, which the parton fragments into. Both the PDFs and the FFs are non-perturbative objects, to be inferred from experiment. Quantum Chromodynamics (QCD) does not provide a definite picture for their calculation, it determines only the order by order calculation in perturbation theory for their energy scale dependence, described by the \(Q^2\)-evolution DGLAP equations.

While PDFs are relatively well determined, there are lot of uncertainties about the FFs at present. The importance of the FFs has become evident only the last decade, with the start of the new generation of high-energy experiments with a final hadron \(h\) detected like semi-inclusive deep inelastic scattering (SIDIS) and hadron production in \(pp\)-collisions. The above information becomes still more viable for the LHC, where detecting different final hadrons will be our window for the New Physics expected.

a e-mail: echristo@inrne.bas.bg
At present different parametrizations about the FFs, obtained from global fit to data, exist [1234]. Three characteristic features about them hold: 1) they use different theoretical assumptions about unfavoured FFs, 2) they describe the data and 3) they don’t agree among themselves.

Recently, the problem with the kaon FFs has become most appealing when the COMPASS collaboration [5] presented two very different numbers for the strange-quark polarization \( \Delta s \) using the same data but two different parametrizations for the FFs: DSS [1] and EMC [4]:

\[
\Delta s = -0.01 \text{(DSS)}, \quad \Delta s = -0.04 \text{(EMC)}.
\]

2 The difference cross sections

2.1 Cross sections with \( K^\pm \) and \( K^0_s \)

We consider charged and neutral kaon production in the three semi-inclusive processes:

\[
e^+ + e^- \rightarrow K + X (2) \\
l + N \rightarrow l + K + X (3) \\
p + p \rightarrow K + X (4)
\]

where \( K \) stands for either charged or neutral kaons, \( K = K^+, K^-, K^0_s \). Note that measuring neutral kaons does not introduce new FFs in these cross sections as SU(2) invariance of strong interactions relates neutral and charged kaon FFs:

\[
D_u^{K^+ + K^-} = D_d^{K^+ + K^-}, \quad D_d^{K^+ + K^-} = D_u^{K^0_s + K^0_s}, \quad D_s^{K^+ + K^-} = D_s^{K^0_s + K^0_s}.
\]

We show that the measuring the difference of charged and neutral kaons:

\[
\sigma^K = \sigma^{K^+ + K^- - 2K^0_s} \equiv \sigma^{K^+} + \sigma^{K^-} - 2\sigma^{K^0_s}
\]

in any of the three processes (2 - 4), always measures the same NS combination \( (D_u - D_d)^{K^+ + K^-} \). This result is obtained without any assumptions about the PDFs and FFs, using only the SU(2) relations for the kaons and, being a consequence of a symmetry, holds in any QCD order. The results for the three processes are [6]:

1) For the \( z \)-distribution in \( e^+ e^- \rightarrow (\gamma, Z) \rightarrow K + X \) we have:

\[
\frac{d \sigma^{K^+ + K^- - 2K^0_s}}{dz} = 6 \sigma_0 (\hat{e}_q^2 - \hat{e}_d^2)(1 + \frac{\alpha_s}{2\pi} C_q \otimes D_u - D_d) \quad (7)
\]

where \( \sigma_0 = 4\pi \alpha_e^2 \nu / 3s \) and \( \hat{e}_q \) are the electroweak charges of the quark \( q \), \( C_q \) are the perturbatively calculable Wilson coefficients, \( z \) is the fraction of the momentum of the fragmenting parton transferred to the hadron \( h \); \( z = E^h / E \), where \( E^h \) and \( E \) are the CM energies of the final hadron and the initial lepton, and \( \sqrt{s} = 2E \).

2) The considered difference cross-sections [3] for SIDIS on proton and deuteron targets are given by:

\[
\frac{d \sigma^{K^+ + K^- - 2K^0_s}}{d \hat{z}} = \frac{1}{3} [(\hat{u} + \hat{d}) (1 + \frac{\alpha_s}{2\pi} C_q \otimes D_u - D_d) \quad \sigma^{K^+ + K^- - 2K^0_s} \quad (8)
\]

\[
\frac{d \sigma^{K^+ + K^- - 2K^0_s}}{d \hat{z}} = \frac{1}{3} [(\hat{u} + \hat{d}) (1 + \frac{\alpha_s}{2\pi} C_q \otimes D_u - D_d) \quad \sigma^{K^+ + K^- - 2K^0_s} \quad (9)
\]
Here \( \bar{q} \equiv u + \bar{u} \) and \( C_{ab} \) are the perturbatively calculable Wilson coefficients.

3) For \( pp \to K + X, K = K^\pm, K^0_s \) we obtain:

\[
E^K \frac{d\sigma_{pp}^{K^+ + K^- - 2K^0_s}}{d^3pK} = \frac{1}{\pi} \sum_{a,b} \int dx_a \int dx_b \int \frac{dz}{z} f_a(x_a) f_b(x_b) \times \left( d\tilde{\sigma}_{ab}^{uX} + d\tilde{\sigma}_{ab}^{qX} - d\tilde{\sigma}_{ab}^{dX} - d\tilde{\sigma}_{ab}^{dX} \right) D_{u-d}^{K^+ + K^-}. \tag{10}
\]

Here the sum over \( a, b \) is over all partons that contribute, \( f_{a,b} \) are the PDFs, and \( d\tilde{\sigma}_{ab}^{qX} (s,t,u) \) are the partonic cross sections for the inclusive processes \( a + b \to c + X \) that contribute. They are functions of the corresponding Mandelstam variables:

\[
s = (p_a + p_b)^2, \quad t = (p_a - p_c)^2, \quad u = (p_b - p_c)^2
\]

\[
p_a = x_a P_A, \quad p_b = x_b P_B, \quad p_c = P^K / z. \tag{11}
\]

(In NLO \( s, t \) and \( u \) are independent variables.) In addition, that the expression for the difference cross section \( \sigma_{pp}^{K^+} \) is considerably simpler than the corresponding cross sections for single kaon production – in NLO it is only 8 partonic cross sections that contribute to \( \sigma_{pp}^{K^+} \) versus 21 to \( \sigma_{pp}^{K^+} \).

Eqs. 7–10 are four independent measurements that determine uniquely the NS combination of the kaon FF \( D_{u-d}^{K^+ + K^-} \). As \( D_{u-d}^{K^+ + K^-} \) is a NS, no new FFs will enter through its \( Q^2 \) evolution. As these expressions use only SU(2) invariance, the verification of any of them might serve as a test for SU(2) invariance of the kaon FFs, used in all analysis.

In addition, eqs. 7–9 allow to compare the NS obtained in \( e^+ e^- \) at rather high \( Q^2 \approx m_e^2 \), with those from SIDIS at quite low \( Q^2 \). This would provide a challenging test of the \( Q^2 \)-evolution and universality of the FFs.

As these expressions are model independent, it would be interesting to compare the resulting NS to the NS obtained with the existing parametrizations extracted from \( e^+ e^- \) data, obtained with various assumptions. This we shall do in Section 3.

### 2.2 Cross sections with \( K^\pm \)

Let’s consider charged kaon production in:

\[
l + N \to l + K + X \tag{12}
\]

\[
p + p \to K + X \tag{13}
\]

\( K = K^+, K^- \). We show that, using only \( C \)-invariance, the difference cross sections \( \sigma_{K^+ + K^-} = \sigma_{K^+} - \sigma_{K^-} \) in \( 12 \) and \( 13 \) determine uniquely the NSs \( D_{u-d}^{K^+ + K^-} \) and \( D_{u-d}^{K^+ - K^-} \) without any assumptions. The expressions in NLO are [6]:

\[
\sigma_{K^+ - K^-} \simeq [4uV \otimes D_{u}^{K^+ - K^-} + dV \otimes D_{d}^{K^+ - K^-}] \otimes (1 + \alpha_s C_{qq}), \tag{14}
\]

\[
\sigma_{d}^{K^+ - K^-} \simeq [(uV + dV) \otimes (4D_{u} + D_{d})^{K^+ - K^-}] \otimes (1 + \alpha_s C_{qq}) \tag{15}
\]

\[
\sigma_{pp}^{K^+ - K^-} \simeq [L_u \otimes uV \otimes D_u + L_d \otimes dV \otimes D_d]^{K^+ - K^-}. \tag{16}
\]

Here \( L_{qq} \) are known functions of the unpolarized PDFs and partonic cross sections:

\[
L_u(x, t, u) = \tilde{u}(x) d\tilde{\Sigma}(s, t, u) + [\tilde{d}(x) + \tilde{s}(x)] d\tilde{\sigma}_{qq}^{qX}(s, t, u) + g(x) d\tilde{\sigma}_{gg}^{qX}(s, t, u)
\]

\[
L_d(x, t, u) = \tilde{d}(x) d\tilde{\Sigma}(s, t, u) + [\tilde{u}(x) + \tilde{s}(x)] d\tilde{\sigma}_{qq}^{qX}(s, t, u) + g(x) d\tilde{\sigma}_{gg}^{qX}(s, t, u)
\]

\[
L_s(x, t, u) = \tilde{s}(x) d\tilde{\Sigma}(s, t, u) + [\tilde{u}(x) + \tilde{d}(x)] d\tilde{\sigma}_{qq}^{qX}(s, t, u) + g(x) d\tilde{\sigma}_{gg}^{qX}(s, t, u)
\]
where \(d\hat{\Sigma}\) is the combination:

\[
d\hat{\Sigma} \equiv \left[ d\hat{\sigma} X_q(s,t,u) + \frac{1}{2} d\hat{\sigma}^{(q\bar{q})}X(s,t,u) \right], \quad \hat{q} \equiv q + \bar{q}.
\] (17)

If we assume \(D_d^{K^+ - K^-} = 0\), used in all global analysis, from (14 - (16)) we obtain:

\[
\sigma_{pp}^{K^+ - K^-} = \frac{4}{9} \left[ (u_V + d_V) \otimes (1 + \alpha_s C_{qq}) \otimes D_u^{K^+ - K^-} \right] + \frac{4}{9} \left[ (u_V + d_V) \otimes (1 + \alpha_s C_{qq}) \otimes D_u^{K^+ - K^-} \right] + \frac{4}{9} \left[ (u_V + d_V) \otimes (1 + \alpha_s C_{qq}) \otimes D_u^{K^+ - K^-} \right].
\] (19)

Eqs. (18) - (20) present 3 independent measurements for the NS \(D_d^{K^+ - K^-}\), obtained with the only assumption \(D_d^{K^+ - K^-} = 0\) and all \(\sigma^{K^+ - K^-}\) are fitted with just one FF. As \(D_d^{K^+ - K^-} = 0\) has been the only assumption, each of these equations would be a test of just this assumption.

3 \(D_{u-d}^{K^+ + K^-}\) from \(e^+e^-\) data

On basis of eq. (7), analyzing the available data on \(K^\pm\) and \(K^0\)-production in \(e^+e^-\)-annihilation, we extracted for the first time \(D_{u-d}^{K^+ + K^-}\) without any correlations to other FFs. There are several arguments for choosing \(e^+e^-\) data: 1) these are the most accurate and precise data, 2) the NS component doesn’t contain unresummed soft gluons divergences, which allows to use lower values of \(z\) \((z > 0.001)\) than in global fit analysis \((z > 0.1, \text{DSS} [1]; z > 0.05, \text{HKNS} [2] \text{and AKK08} [3]), 3) for the NS component a next-next to leading order (NNLO) fit is possible as both the splitting and coefficient functions are known to NNLO, while in global fit analysis only NLO calculations are available at present.

The data used in our analysis are in the energy range \(\sqrt{s} = 12 - 189\) GeV, but not all data appear equally important. Eq. (7) implies that the sensitivity of \(\sigma^{K^\pm}\) to \(D_{u-d}^{K^+ + K^-}\) is determined by the \(s\)-dependence of \((\hat{c}_u^2 - \hat{c}_d^2)(s)\). This difference is the biggest away from the Z-pole, i.e. most important for our studies are the data with small \(s\), \(\sqrt{s} \leq 60\) GeV and big \(s\), \(\sqrt{s} \geq 110\) GeV. Unfortunately, the data around the Z-pole, where the most abundant and precise data exist, almost does not contribute.

Our analysis would be easy if we had data on \(K^\pm\) and \(K^0\) production at the same \(z\) and \(s\). However, as this is not the case, we proceed in three steps (7): 1) we combine the measurements into 7 energy intervals: \(\sqrt{s} = 12-14.8; 21.5-22; 29-35; 42.6-44; 58; 91.2; 183-189\) GeV, 2) in each interval we fix \(K^0\) data purely phenomenologically and 3) we parametrize \(D_{u-d}^{K^+ + K^-}\) at a factorization scale \(\mu_0 = \sqrt{2}\) GeV:

\[
D_{u-d}^{K^+ + K^-}(z, \mu_0^2) = n z^\alpha(1-z)^b + n' z'^\alpha(1-z)'b'
\] (21)

and fit simultaneously the parameters in the parametrizations for \(\sigma^{K^0}\) and \(D_{u-d}^{K^+ + K^-}\) in a combined analysis of the data on \(K^\pm\) and \(K^0\). In our analysis we keep the hadron mass \(m_h\) as a fitting parameter.

Small \(z\)-values imply small hadron energies, which may become of the order or smaller than the mass of the hadron. That’s why including the small-\(z\) and/or small-\(s\)
data the effects of the hadron mass, i.e. \( m_h \neq 0 \), become significant and have to be taken into account. In our analysis we must distinguish kinematically between the experimentally measured fractions of the final hadron energy \( E_h \) and the modulus of its momentum \( |p_h| \) to \( \sqrt{s} \):

\[
x_E = \frac{E_h}{2\sqrt{s}} = x \left( 1 + \frac{m_h^2}{x^2 s} \right), \quad x_p = \frac{|p_h|}{2\sqrt{s}} = x \left( 1 - \frac{m_h^2}{x^2 s} \right).
\]

These quantities are equal and equal to the light cone scaling variable \( x \), \( x_E = x_p = x \), only at \( m_h = 0 \), \( x = z \) in LO in QCD.

The NS \( D_{u-d}^{K^+ + K^-} \) is presented on Fig. 1 left. On the same figure \( D_y^{K^+ + K^-} \), obtained from the global fits of DSS, AKK08 and HKNS, are presented as well. The differences between our result for \( D_{u-d}^{K^+ + K^-} \) and those of global fit analysis might be due mainly to 1) the different assumptions – in DSS and HKNS \( D_y^{K^+ + K^-} = D_d^{K^+} = D_d^{K^+} = D_d^{K^+} \) is assumed; in AKK08 \( D_y^{K^+} = D_y^{K^+} \) is used; we make no assumptions; 2) the small \( z \)-data, for the first time included. In order to get insight on the effect of the low-\( z \)-data, on Fig 1 right we present \( D_{u-d}^{K^+ + K^-} \) with different cuts on the lower values of \( z \). The result shows that data at \( z \geq 0.01 \) is not enough to impose constraints on the NS.

At the end we point to a striking feature of our result – the negative value for \( D_{u-d}^{K^+ + K^-} \) at about \( z \lesssim 0.3 \). In order to check this result and find out its origin we performed our analysis with different parametrizations for \( D_{u-d}^{K^+ + K^-} \) and this does not seem a genuine feature of the NS – choosing \( D_{u-d}^{K^+ + K^-} = n z^{\alpha} (1 - z)^{b} \) in stead of \( 21 \) yields \( D_{u-d}^{K^+ + K^-} > 0 \), \( D_{u-d}^{K^+ + K^-} = 0 \) appears also compatible with the data. The quality of the fit only slightly worsens: from \( \chi^2_{dof} = \chi^2 / N = 2.2 \) for \( D_{u-d}^{K^+ + K^-} < 0 \), to \( \chi^2_{dof} \approx 2.4 \) for positive and zero values of the NS, \( N = 730 \) is the number of points in the fit. However, in all cases the quality of the fit is worse than the one obtained in the purely phenomenological fit to \( K_y^0 \) - \( \chi^2_{dof} = 1.1 \).

Also the value of the fitted kaon mass, obtained in the purely phenomenological fit to \( K_y^0 \) - \( m_K = 320 \) MeV, differs significantly from the one obtained in the combined \( K_y^0 \) and \( K^\pm \) analysis based on the QCD formula \( 4 \) - \( m_K = 124 \) MeV. (The true kaon mass is \( m_K = 494 \) MeV.) A possible reason for this big difference might be that other low-\( x \) and small-\( x \) effects, such as higher twists and mass effects of resonances from which kaons may be produced, that have not been accounted for in our analysis, can also be absorbed in the fitted value for \( m_K \). The performed analysis are very sensitive to hadron mass effects and these effects strongly affect our fits. More accurate data will be needed to determine how important these other effects are.

Our analysis shows that the non-singlet \( D_{u-d}^{K} \) can be determined uniquely in a combined fit to \( e^+ e^- \) data, but more precise data is needed to fix it better.

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Fig. 1. up: $D_{u-d}^{K^+ + K^-}$ in this paper at NLO with resumed logarithms [label AC], and from DSS, HKNS and AKK08 sets; down: the NS in this paper with different cuts on the data

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