Analysis of student’s error in resolving the Pythagoras problems

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Abstract. This study aims to describe the error made by students in solving Pythagoras questions, to analyse the factors that cause the students to make those errors, and to identify whether students are able to associate trigonometric concepts with Pythagoras. The subject of this study was the eleventh grade students of SMP Negeri 1 Yogyakarta Academic Year 2018/2019. There were 31 samples of students taken as the object of the research. The data collection used was a test. The classification of students’ errors type was done based on Newman process skill errors, inaccuracy, errors in understanding the questions’ intention and errors in the use of notation or misconception. The data analysis technique of the research was conducted through data reduction, data presentation and conclusion. The results showed that the errors percentage in the process was 38%. The less accurate percentage was 25%. The percentage of errors in understanding the intent of the problem was 10%. The percentage of errors in the use of notation was 15%. The percentage in misconception was 12%. The type of errors made by the students were: (1) students not know the meaning of symbols. (2) students not know the purpose of the question. (3) students not know the formula used in problem solving. (4) students cannot solve problems in the algorithm. (5) students cannot solve the problem according to the questions and some students are not able to associate the concept of pythagoras with trigonometry.

1. Introduction

Education has become an important component in the current era of globalization. Education experiences rapid development, along with the development of civilization, knowledge and technology [1]. Thus, education experiences a dynamic process of change. Various kinds of changes in the learning process and competencies achieved are gradually introduced in school [2]. In education, assessment is an important component for identifying educational attainment [3]. The forms of assessment exams can vary, depending on what components will be assessed. Thus, an evaluation is needed to determine the extent to which learning objectives are achieved at each level of the school [4]. One of the subjects that becomes the object of assessment is mathematics.

Mathematics is a learning material that has logical and structured characteristics. With mathematics, one can solve a problem in life. According to [5] that mathematics is a science that is
processed from the activities of measurement, calculation and consists of measurable and systematic learning with abstraction processes on a problem to get a solution using logical and systematic thinking. In today's modern era, many challenges faced by students are increasingly complex. Thus, not only logical and systematic abilities are needed, but also creative, critical and innovative abilities. Mathematics is a component of tools for students to achieve creative, logical, critical, innovative and systematic thinking skills [6]. Thus, mathematics is an important science to be developed because it can develop students’ ability of logical, creative, critical, analytical, critical and systematic skills which are needed in every age development, including the era of globalization.

But the fact shows that in the learning process, mathematics becomes a subject that is difficult to learn. As stated in the research conducted by [7] that in solving the questions given, there are many students who have difficulty in understanding mathematical questions. Hence, they often make errors related to the problems that have been presented. When students experience errors in learning mathematics, it becomes an indicator that students experience difficulties in their learning. According to [8] that learning difficulties is a condition experienced by children who are significantly disturbed both in general tasks and special assignments which are thought to be caused by not functioning nervous system or neurological, psychological processes or other causes that lead to low learning achievement in children who have learning difficulties. Thus, learning difficulties must be immediately overcome by the teacher, so that students are able to follow mathematics learning well and be able to solve mathematical problems in daily life.

To find out the learning difficulties that can lead to students’ errors in solving mathematical problems, teacher is expected to know about the characteristics of students' difficulties in learning mathematics. Hence, it can be seen from the opinions expressed by Wood [9] that there are students who experience difficulties in differentiating numbers, building space and symbols, (2) students who have difficulties in remembering mathematical formulas or propositions, (3) students who write numbers that cannot be read or who write in small sizes, (4) students who not understand mathematical symbols , (5) students who have abstract thinking which is still weak, (6) students who are still weak in the ability of metacognition (weak ability to identify and utilize algorithms in mathematic problem solving).

After the teacher knows the learning difficulties of the students, then the teacher can identify the types of students’ errors that are often done in order to minimize the similar errors in the future. According to [10] there are errors often made by students that consist of errors in using everyday language, the ability in spatial fields, the ability to master a material prerequisite, errors in mastering a theory that has been studied and errors in applying the relevant rules. The use of elements in pythagoras is one of the subject learned by students which often leads to some errors in solving mathematical problems.

Pythagoras can be used to solve various mathematical problems related to building flat and building space, so that pythagorass is one of the main foundations in mathematic problem solving [11]. There are various problems in daily life that can be related to pythagoras. Many life problems are solved by pythagoras concepts and algorithms. One of the mathematical concepts related to the pythagoras concept is geometry. The relationship between pythagoras and other geometric concepts is that, the stronger the students’ understanding in learning pythagoras, the stronger the students' understanding in learning geometry [12]. Looking at the importance of pythagoras learned in the mathematics learning process, it is important to make an evaluation by analyzing students’ errors in pythagoras problem solving.

There are various types of errors made by students in mathematics learning material, such as the results of research conducted by [13] which shows that students in government-assisted schools do a much higher level of errors than students in other school categories. Besides, students experience errors in understanding questions because of foreign languages that were used. In addition, male students experience greater levels of mathematical operations in algebra. When doing mathematical operations, students need to master the understanding of symbols, graphics and problem solving. Various reasons were put forward by students when they made errors in understanding symbols,
graphics and problem solving. As research conducted by [14] that the main reason put forward when making a error is the lack of understanding, forgetting about the procedure, neglecting in copying the question information, carelessness and guesswork. One of the procedures to determine errors in completing mathematics is the NEA procedure (Newman Error Analysis). Based on research [15] the NEA procedure provides a way for teachers to evaluate the students’ development more comprehensively. The results of their research show that the errors often made by the students lie in the modeling phase (transformation) and the ability of the process with a 20% and 15% overall percentage distribution.

So, researchers are interested in doing research with this study to analyze the errors made by students in solving problems related to pythagoras. In addition, this study also looked at whether students were able to related trigonometric concepts in pythagoras. Thus, it is important to conduct an analysis related to the errors that are made by the students in other materials, especially in pythagoras material and identify whether students are able to associate trigonometric concepts in pythagoras. The purpose of this study is to find out the students' errors in solving questions related to pythagoras and identify whether students are able to associate trigonometric concepts in pythagoras. The analysis of the data used in the study is based on NEA (Newman Analysis Error) which is a series of work with a simple examination, including (1) decoding, (2) comprehension, (3) transformation, (4) process skill and (5) encoding. With this examination method based on Newman, the error categories in solving mathematical problems from a description test can be identified. The data collection technique in this study was written tests containing questions about pythagoras. The subjects of this study were 31 students of class IX Yogyakarta State Middle School Academic Year 2018/2019. The benefit of this research is to provide an overview for teachers and educators as well as students in order to be able to avoid the same errors in conducting procedure for solving trigonometry problems.

2. Trigonometry in the Pythagoras Concept

The Pythagoras Theorem was pioneered by a Greek mathematician named Pythagoras. The definition of the Pythagoras Theorem is a relationship of the three sides of a right triangle in Euclidean geometry. The trigonometry concepts in pythagoras can be seen in Figure 1.

It can be seen that figure 1 is a right triangle used in the Pythagoras theorem. One of the corners is called as angle a. A right triangle has three sides hypotenuse, front and side. The side is the side next to the corner a. The front side is the side in front of the corner (on the image above, located in front of the angle a). Angled side is the side that is located in front of the 90 degree angle. In right triangles, the trigonometric comparison applies as follows:

\[
\sin \alpha = \frac{\text{front}}{\text{hypotenuse}} \\
\cos \alpha = \frac{\text{side}}{\text{hypotenuse}} \\
\tan \alpha = \frac{\text{front}}{\text{side}}
\]

Figure 1. Pythagoras Theorem
That is, the value of \( \sin \propto \) is equal to the length of the front side of the angle \( \propto \) divided by the length of the hypotenuse. Likewise for the values of \( \cos \) and \( \tan \). This rule applies only to right triangles.

3. Method
This study is a qualitative descriptive research. The purpose of the study is describing the errors made by students in solving pythagoras problems. The data used in this study is essays, in the form of answers sheet containing explanations made by the students. The explanations are obtained from the written examinations using pythagoras materials.

The data sources used in this study were 31 students of class IX of SMP Negeri Yogyakarta in the odd semester of Academic Year 2018/2019 that were given written tests related to pythagoras material. The informants in this study were several teachers interviewed and produced data to be discussed in this study. The data collection technique used in this study was essay questions containing pythagoras material. This essay questions are prepared by the teacher based on the applicable curriculum. The data analysis was done based on the Flow Model of Miles and Huberman [16] which consists of (1) data reduction, (2) data presentation and (3) conclusion / verification.

The data analysis used in this study was based on NEA (Newman Analysis Error), a series of work with a simple examination, which includes (1) decoding, (2) comprehension, (3) transformation, (4) process skill and (5) encoding. With this method based on Newman inspection, the researchers can identify mathematical problems from a description test. The descriptions are as follows:

(1) Errors because students not know the meaning of the symbol or term in the problem (Type R or Reading Errors).
(2) Errors because students not understand the meaning of the problem. Students fail to write back what is known and what is asked (Type C or Comprehension Errors).
(3) Errors because students forget the formula that will be used or what strategy / procedure that will be done (Type T or Transformation Errors).
(4) Errors because students cannot solve algorithm problems in sequence, detail and correct (type P or Process Skills Errors).
(5) Errors because students cannot answer the questions because they are based on questions (Type E or Encoding Errors).
(6) Errors because students not succeed in translating properly, especially questions written in English (Type L or Language Errors).
(7) Errors occur due to inadvertence (Careless Errors).

4. Results and Discussion
Based on the results of student error analysis conducted at school, percentage of each type of error was obtained. The results showed that the errors percentage in the process was 38%. The less accurate percentage was 25%. The percentage of errors in understanding the intent of the problem was 10%. The percentage of errors in the use of notation was 15%. The percentage in misconception was 12%. Thus, a more in-depth analysis is needed regarding the types of errors made by students. The problems related to pythagoras are represented by sailing vessels in Figure 2.

![Figure 2. Question of Pythagoras](image)

From the results of students' work, it can be seen that there are 5 types of student errors in answering pythagoras questions. For the first type of error that is errors because students not know the
meaning of the symbol or term in the problem (Type R or Reading Errors) represented in Figure 3 which can be seen that results of the analysis of student work, it was revealed that the cause of the error was because students not know the meaning of the symbol. This is revealed when students use the "@" symbol to find the angle value. In Figure 3, students not understand the trigonometry concept. Students are still wrong in putting the angle on the problem. The same type of error is also done by students in Figure 4 because students not know the meaning of the symbol. In this problem it can be seen that students divide numbers by the angle size. Students should transform the angle value into numbers. So, the students' answer sheet can be seen at Figure 3 and Figure 4.

For the second type of error that is errors because students not understand the meaning of the problem, students fail to write back what is known and what is asked (Type C or Comprehension Errors) represented in Figure 5 and Figure 6. From the results of the analysis in Figure 5, it is known that students it was revealed that the cause of errors in solving the problem because students not understand the intent and meaning stated in the problem, thus affecting the difficulties or misunderstanding of students in identifying what is known or asked. As stated in the student's answer, that students fill out the components known as "wind power vessels" which are not referred to in the problem. As for what is meant is the size of the screen angle is 45° and the screen height is 150 m. The results of the analysis in Figure 6 are known students fill in the "known" component by "pulling the ship", where it does not match what is expected of the question, which should be filled with "angles". So, the students' answer sheet can be seen at Figure 5 and Figure 6.

For the third type of error that is errors because students forget the formula that will be used or what strategy/procedure will be done (Type T or Transformation Errors) represented in Figure 7. From the results of the analysis in Figure 7 it is known that one of the causes of errors is that students forgot the formula to be used in solving the problem.
not know what formulas are used in solving problems that result in errors in mathematical problem solving. Like the picture beside, where a student directly enters the number, without entering the formula what should be entered, so that it impacts on the error in solving the problem.

For the fourth type of errors, that is errors because students cannot solve algorithm problems in sequence, detail and correct (type P or Process Skills Errors) represented in Figure 8. From the results of the analysis in Figure 8 is known that students not yet understand how to put numbers in accordance with the circumstances. Students division algorithms by equating between angles with numbers. This is not yet in accordance with the procedure, where the value of angles cannot be equated with numbers and the angles must be replaced first in numbers. So, the students’ answer sheet can be seen at Figure 7 and Figure 8.

For the fifth type of error that is errors because students cannot answer questions because they are based on questions (Type E or Encoding Errors) represented in Figure 9. From the results of the analysis in Figure 9 is known that students failed to answer questions according to the problem. Students operate angles by adding angles, then the conclusions taken are different from what is intended, where at the end of the conclusion, the angle is changed to the length of the rope. Students are also error in understanding angles in trigonometry concepts, where in the student's final answer, the angle is changed to the length of the rope. In Figure 10, students also experience the same error as Figure 9 because students misunderstand what is meant in the problem. In addition, students also experience a misconception of trigonometric formulas to look for the sloping side of an object. Thus, students fail to associate the concept between of pythagoras and trigonometry. So, the students’ answer sheet can be seen at Figure 9 and Figure 10.

As for the sixth type of error that is error that occurs because students not succeed in translating properly, especially questions written in English (Type L or Language Errors) is no students
experience this type of error. This is because using Indonesian language is relatively understandable for students. The seventh type of error that is errors occur due to inadvertence (Careless Errors) no students experience errors in this type.

From the results of the analysis of the types of errors in solving the pythagoras problem, it can be seen that some students experience the first to fifth error in solving the pythagoras problem. While, no students experience errors in this type. From the results of the analysis of the types of errors in solving the pythagoras problem, it can be seen that some students experience the first to fifth error in solving the pythagoras problem.

From the results of the analysis of the types of errors in solving the pythagoras problem, it can be seen that some students experience the first to fifth error in solving the pythagoras problem. Whereas, no students make errors in the sixth and seventh types in solving pythagoras questions.

The results of the analysis of student errors in solving the pythagoras problem, in line with the results of research conducted by [17] that errors often made by students are errors in using everyday symbols, abilities in spatial fields, ability to master a prerequisite material, errors in mastering a theories that have been studied and errors in applying relevant rules.

Students' errors in solving pythagoras questions are expected to be known by the teacher. This is done so that the teacher can evaluate the students who have errors in solving the Pythagoras problem, especially for the first to fifth type errors. The teacher can also do therapy for students to prevent the same type of error. As for according to [18] to avoid error in solving math problems, the steps are: (1) before starting learning, students are accustomed to learning in advance about the prerequisite material and are trained to understand the usefulness of the symbols that form the basis of the solution to the problem. This action is taken to avoid type R errors. (2) Students are trained to understand the problem by practicing writing about what is known and what is asked well and correctly. This action is done to avoid type C errors. (3) Students are trained to write formulas or develop resolution strategies so that problem solving is clear and directed. This action is carried out to avoid type T errors. (4) Students are trained to solve problems based on procedures that have been correctly chosen, meticulously, detailed and complete. This action is an attempt to avoid all causes of type P errors. (5) Students are trained to review the questions and their work in solving questions that are appropriate to the problem being asked. This action is done to avoid the type E error.

By making the efforts described above, it is expected that learning therapy can solve the problem correctly, so that students' error in solving pythagoras problems can be minimized and can also minimize the number of students who make error.

5. Conclusion

Based on the results of the analysis of student errors in solving the pythagoras problem, it can be seen that: (1) Students make type R errors because students do not know the meaning of the symbol. Thus, before learning begins, students learn the prerequisite material and practice recognizing symbols. (2) students make type C error because students do not know the meaning and meaning of the question. Thus, students should be trained to understand the intent and meaning of a problem. (3) Students make T-type error because students do not know what formulas are used in solving problems. Thus, students should be trained to write formulas and develop resolution strategies so that problem solving is clear and directed. (4) Students make type P errors because students cannot solve problems in the algorithm. Thus, students should be trained to solve problems based on correct, careful, detailed and complete procedures. (5) Students make type E error because students do not solve problems according to questions. Thus, students should be trained to review questions and work results that have been completed and there are some students who have not succeeded in associating trigonometric concepts with pythagoras.
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