Quantum Gravity near Apparent Horizon
and Two Dimensional Dilaton Gravity

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ABSTRACT

We study the Hawking radiation in two dimensional dilaton black hole by means of quantum gravity holding near the apparent horizon. First of all, we construct the canonical formalism of the dilaton gravity in two dimensions. Then the Vaidya metric corresponding to the dilaton black hole is established where it is shown that the dilaton field takes a form of the linear dilaton. Based on the canonical formalism and the Vaidya metric, we proceed to analyze quantum properties of a dynamical black hole. It is found that the mass loss rate of the Hawking radiation is independent of the black hole mass and at the same time the apparent horizon recedes to the singularity as shown in other studies of two dimensional gravity. It is interesting that one can construct quantum gravity even near the curvature singularity and draw the same conclusion with respect to the Hawking radiation as the above-mentioned picture. Unfortunately, the present formalism seems to be ignorant of the contributions from the functional measures over the gravitational field, the dilaton and the ghosts.

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1. Introduction

More than twenty years ago, Hawking [1] has shown that black holes are not completely black and emit thermal radiation with a definite temperature through quantum mechanical pair creation of particles near the horizon in a gravitational field where one member of the pair drops in a black hole while the other escapes to infinity. This result was derived in the context of “semiclassical” approach, where the effects of gravitation are still represented by a classical spacetime \((M, g^{ab})\), while matter fields are treated as quantum fields propagating in this classical spacetime. Subsequent investigations have been focused on understanding of serious problems raised by the Hawking radiation, concerning the fate of quantum information [2] and the statistical mechanical picture of black hole thermodynamics [3] e.t.c.

In recent papers, a quantum formalism has been proposed for the study of black hole quantum mechanics [4, 5]. The critical idea behind this formalism is that some essential features of quantum black holes might be intimately related to the quantum mechanical behavior of the black hole horizon, thus it might be sufficient to establish the quantum gravity holding particularly near the horizon to understand an overall picture of quantum black holes. Indeed, afterward, this formalism has been fruitfully applied to several problems associated with quantum black holes in three [6] and four [7] spacetime dimensions.

In the course of applications, we have wondered to what extent the quantum gravity near the horizon would describe quantum aspects of a black hole. To address this question, it is tempting to try to apply the formalism to well-understood model of quantum black holes, that is, the dilaton gravity in 1+1 dimensions (CGHS model) [8]. The dilaton gravity in two dimensions enjoys nice features of black-hole formation and/or evaporation shared with the spherically symmetric black hole in 3+1 dimensions. Therefore, this toy model has raised hopes that a satisfactory description of black hole evolution might be accounted for in a very simplified setting.
The article is organized as follows. In section 2, we construct the canonical formalism of the two dimensional dilaton gravity. In section 3, we derive the Vaidya metric corresponding to the dilaton black hole. The canonical formalism and the Vaidya metric are used to construct a quantum theory holding in the vicinity of the apparent horizon of the dilaton black hole in section 4. In section 5, we analyse the Hawking radiation from purely quantum mechanical viewpoint. Here it is shown that the mass loss rate is independent of black hole mass. The last section is devoted to conclusion.

2. ADM Canonical Formalism of Two Dimensional Dilaton Gravity

We begin our investigations by constructing the ADM first-order canonical formalism of two dimensional dilaton gravity.

The action that we start with has the well-known form [8]

\[
S = \frac{1}{2G} \int d^2 x \sqrt{-g} \ e^{-2\phi} \left[ R + 4(\nabla \phi)^2 + 4\lambda^2 \right] - \frac{1}{2} \int d^2 x \sqrt{-g} \ (\nabla f)^2, 
\]

(1)

with the dilaton field \( \phi \), the cosmological constant \( \lambda \), and the single massless conformal matter field \( f \). Differing from the original CGHS convention [8] where \( G = \pi \), we will take \( G = \frac{1}{2} \) in this paper. Related to this choice, we have modified the coefficient in front of the matter action from the CGHS value \( -\frac{1}{4\pi} \) to \( -\frac{1}{2} \), which is natural from the viewpoint of the spherically symmetric reduction of the four dimensional gravity [9].

Let us adopt the ADM splitting of 1+1 dimensional spacetime given by

\[
g_{ab} = \begin{pmatrix}
-\alpha^2 + \frac{\beta^2}{\gamma} & \beta \\
\beta & \gamma
\end{pmatrix}.
\]

(2)

Then the normal unit vector \( n^a \) orthogonal to the hypersurfaces \( x^0 = \text{const} \) reads

\[
n^a = \left( \frac{1}{\alpha}, -\frac{\beta}{\alpha\gamma} \right),
\]

(3)
and the projection operator $h^{ab}$ over the $x^0 = \text{const}$ hypersurfaces becomes

$$h^{ab} = g^{ab} + n^a n^b,$$

$$= \begin{pmatrix} 0 & 0 \\ 0 & \frac{1}{\gamma} \end{pmatrix}.$$ (4)

In terms of the ADM parametrization (2), after some calculations the action (1) can be written as

$$S = \int d^2 x \, L,$$

$$= \int d^2 x \left[ 4\alpha\sqrt{-\gamma} \, e^{-2\phi} \left\{ \lambda^2 - (n^a \partial_a \phi)^2 + \frac{1}{\gamma}(\phi')^2 + Kn^a \partial_a \phi - \frac{\alpha'}{\alpha \gamma} \phi' \right\} + \frac{1}{2\alpha \sqrt{-\gamma}} \left\{ (n^a \partial_a f)^2 - \frac{1}{\gamma}(f')^2 \right\} \right],$$ (5)

where the trace of the extrinsic curvature $K = g^{ab} K_{ab}$ is

$$K = \frac{1}{\sqrt{-g}} \partial_a (\sqrt{-g} \, n^a) = \frac{\dot{\gamma}}{2\alpha \gamma} - \frac{\beta'}{\alpha \gamma} + \frac{\beta}{2\alpha \gamma^2} \gamma',$$ (6)

and $\frac{\partial}{\partial x^0} = \partial_0$ and $\frac{\partial}{\partial x^1} = \partial_1$ are also denoted by an overdot and a prime, respectively.

In deriving (5), we have used the formula [9, 10]

$$R = 2n^a \partial_a K + 2K^2 - \frac{2}{\alpha \sqrt{-\gamma}} \left( \frac{\alpha'}{\sqrt{-\gamma}} \right)'.$$ (7)

The action (5) indicates that $\alpha$ and $\beta$ are non-dynamical Lagrange multiplier fields due to the absence of the term including $x^0$-differentiation so we regard the massless matter field $f$, the dilaton field $\phi$ and the “graviton” $\gamma$ as the dynamical
fields. Then canonical conjugate momenta can be read off from the action (5):

\[ p_f = \sqrt{\gamma} n^a \partial_a f, \]
\[ p_\phi = 4\sqrt{\gamma} e^{-2\phi} (-2n^a \partial_a \phi + K), \]
\[ p_\gamma = \frac{2}{\sqrt{\gamma}} e^{-2\phi} n^a \partial_a \phi. \]  

Now it is straightforward to derive the Hamiltonian whose result is given by

\[ H = \int dx^1 (p_f \dot{f} + p_\phi \dot{\phi} + p_\gamma \dot{\gamma} - L), \]
\[ = \int dx^1 (\alpha H_0 + \beta H_1), \]  

where the constraints are explicitly of form

\[ H_0 = \frac{1}{2\sqrt{\gamma}} p_f^2 - 4\sqrt{\gamma} e^{-2\phi} \lambda^2 - \frac{4}{\sqrt{\gamma}} e^{-2\phi} (\phi')^2 - (\frac{4}{\sqrt{\gamma}} e^{-2\phi} \phi')' \]
\[ + \frac{1}{2\sqrt{\gamma}} (f')^2 + \frac{\sqrt{\gamma}}{2} e^{+2\phi} p_\phi p_\gamma + \gamma \sqrt{\gamma} e^{+2\phi} p_\gamma^2, \]
\[ H_1 = \frac{1}{\gamma} p_f f' + \frac{1}{\gamma} p_\phi \phi' - 2p_\gamma - \frac{1}{\gamma} p_\gamma \gamma'. \]  

Note that \( H_0 \) and \( H_1 \) are generators corresponding to the time translation and the spatial displacement, respectively. Also let us notice that \( \alpha \) and \( \beta \) are certainly the Lagrange multiplier fields as mentioned before. At this stage, it is easy to derive the ADM surface term via the dual Legendre transformation by following Regge and Teitelboim [11], though we now omit the detail since it is not so much important for later discussions.

3. Vaidya Metric
In this section, we will construct the Vaidya metric to the two dimensional
dilaton black hole. This Vaidya metric will be used in later sections when we wish
to discuss the Hawking radiation arising from a dynamical black hole.

As a simple illustration, let us recall how to build the Vaidya metric to the
Schwarzschild black hole in four dimensions. Neglecting the irrelevant angular
parts ($\theta, \varphi$), the Schwarzschild geometry has the famous form

$$ds^2 = -(1 - \frac{2M}{r})dt^2 + \frac{1}{1 - \frac{2M}{r}}dr^2.$$  \hspace{1cm} (12)

Introducing the advanced time coordinate $v = t + r^*$ with the tortoise coordinate
$dr^* = \frac{dr}{g_{00}}$, in the $(v, r)$ coordinates the Schwarzschild metric can be transformed
to

$$ds^2 = -(1 - \frac{2M}{r})dv^2 + 2dvdr.$$  \hspace{1cm} (13)

A generalization of a constant mass $M$ to the mass function $M(v)$ gives rise to the
Vaidya metric corresponding to the Schwarzschild black hole. The reason why we
prefer the Vaidya metric to the Schwarzschild one is that the former satisfies the
classical field equations as it is when there is a flow of matters with a form of the
energy-momentum tensor $T(v)$, while in the latter the mass function must have
a complicated dependence on the coordinates to satisfy them, which makes the
following analysis ugly. In other words, the Vaidya form of a black hole provides
us a convenient playground to discuss the properties of a dynamical black hole.

In order to have a close relationship with CGHS work [8], let us start by their
black hole solution in the lightcone coordinates

$$ds^2 = -e^{2\rho}dx^+dx^-,$$

$$= -\frac{1}{M} \frac{1}{\lambda^2 x^+ x^-} dx^+ dx^-.$$  \hspace{1cm} (14)

Transforming the coordinates from the $(x^+, x^-)$ to the $(v, r)$ which are related to
each other

\[ x^+ = \frac{\sqrt{M}}{\lambda \sqrt{\lambda}} e^{\lambda v}, \]
\[ x^- = -\frac{1}{\sqrt{\lambda M}} (e^{2\lambda r} - \frac{M}{\lambda}) e^{-\lambda v}, \] (15)

the two dimensional line element (14) reduces to

\[ ds^2 = -(1 - \frac{M}{\lambda} e^{-2\lambda r}) dv^2 + 2dvdr. \] (16)

Then the Vaidya metric corresponding to the dilaton black hole can be obtained by promoting a constant mass to the mass function \( M(v) \) depending on only the \( v \) coordinate. Here it is worthwhile to notice that in the newly introduced coordinates \((v, r)\), the dilaton field which was given by \( \phi = \rho \) in the lightcone coordinates \((x^+, x^-)\), takes a remarkably simple form, that is, a linear dilaton form

\[ \phi = -\lambda r. \] (17)

This would lead to a great advantage in analyzing the constraints as well as the field equations in the below.

Indeed, it is verified that the solutions (16) and (17) are an extremum of the action (1) near the apparent horizon

\[ r_{AH} = -\frac{1}{2\lambda} \log \frac{\lambda}{M}, \] (18)

whose definition arises from the condition \( g_{vv} = 0 \) which is also consistent with the usual definition \((\nabla \phi)^2 = 0\) in the two dimensional dilaton gravity. The classical
field equations are easily obtained from the action (1):

\[
2e^{-2\phi} \left[ \nabla_a \nabla_b \phi + g_{ab} \left\{ (\nabla \phi)^2 - \nabla^2 \phi - \lambda^2 \right\} \right] = \frac{1}{2} \left[ \nabla_a f \nabla_b f - \frac{1}{2} g_{ab} (\nabla f)^2 \right],
\]

(19)

\[
R + 4\lambda^2 + 4\nabla^2 \phi - 4(\nabla \phi)^2 = 0,
\]

(20)

\[
\nabla^2 f = 0.
\]

(21)

Since we are interested in the physics only in the vicinity of the apparent horizon, it is sufficient to verify that the Vaidya metric (16) and the dilaton field (17) are consistent with the field equations (19)-(21) near the apparent horizon. After some manipulation, the field equations require to satisfy

\[
\partial_r f = \partial_v \partial_r f \approx 0,
\]

(22)

\[
\partial_v M \approx \frac{1}{2} (\partial_v f)^2,
\]

(23)

where we shall use \(\approx\) to express the equalities holding approximately near the apparent horizons from now on. From (22) and (23), we find the general solution

\[
f(v) \approx \pm \int dv \sqrt{2\partial_v M}.
\]

(24)

Consequently, it has been checked that the solutions (16) and (17) are at least classically consistent with the field equations near the apparent horizon as long as (24) is satisfied. Incidentally, (24) represents the physical fact that the increase of the black hole mass, \(\partial_v M > 0\), is classically allowed, but the loss of it, \(\partial_v M < 0\), i.e., the Hawking radiation, is classically forbidden and can occur only through the quantum tunneling effects owing to \(f(v)\) being the real scalar field.
4. Quantum Gravity near Apparent Horizon

We now consider the dynamical black hole (16) and the linear dilaton (17). Our main concern in this section is to construct a quantum theory of two dimensional dilaton gravity holding near the apparent horizon.

Let us begin by introducing the coordinates

\[ x^a = (x^0, x^1) = (v - r, r). \]

(25)

Next we set up the gauge conditions such that the gauge symmetries associated with the two dimensional reparametrization invariances are completely fixed

\[ g_{ab} = \begin{pmatrix} -\alpha^2 + \frac{\beta^2}{\gamma} & \beta \\ \beta & \gamma \end{pmatrix}, \]

(26)

\[ = \begin{pmatrix} -\left(1 - \frac{M}{X}e^{-2\lambda r}\right) & \frac{M}{X}e^{-2\lambda r} \\ \frac{M}{X}e^{-2\lambda r} & 1 + \frac{M}{X}e^{-2\lambda r} \end{pmatrix}, \]

where the black hole mass \( M \) is the function of the two dimensional coordinates \( x^a \). Notice that we have chosen these gauge conditions to correspond to the Vaidya metric built in the previous section. Of course, at this stage, we cannot restrict the mass function to be the function depending on only the \( v \) coordinate from an argument of symmetries. Near the apparent horizons (18), (26) yields

\[ \alpha \approx \frac{1}{\sqrt{2}}, \beta \approx 1, \gamma = \frac{1}{\alpha^2} \approx 2. \]

(27)

Note that the dynamical degrees of freedom representing “graviton” \( \gamma \) are effectively fixed in (27). At this point, let us make physically plausible assumptions [4, 5] near the apparent horizon

\[ f \approx f(v), M \approx M(v), \phi \approx -\lambda r. \]

(28)

As shown in the section 3, these assumptions are consistent with the field equations, but their quantum-mechanical meaning is not clear at present. Given the
assumptions (28), near the apparent horizon the canonical conjugate momenta (8)
become
\[ p_f \approx \partial_v f, \]
\[ p_\phi \approx -2M - \frac{1}{\lambda} \partial_v M, \]
\[ p_\gamma \approx M. \]  
(29)

Then after a little lengthy calculation, one arrives at a remarkable relation that
the Hamiltonian constraint \( H_0 = 0 \) becomes proportional to the supermomentum
constraint \( H_1 = 0 \)
\[ \sqrt{2} H_0 \approx 2H_1, \]
\[ \approx p_f^2 + 2\lambda p_\phi + 4\lambda M. \]  
(30)

This relation can be understood from an observation that the time translation
generated by the Hamiltonian constraint is frozen on the apparent horizon due to
gravitational time dilation in the present coordinate system [6].

We are now ready to carry out the canonical quantization of the model. Following Dirac’s quantization procedure of the first-class constraints [12], the residual
symmetry (30) is imposed on the state
\[ (-\frac{\partial^2}{\partial f^2} - 2i\lambda \frac{\partial}{\partial \phi} + 4\lambda M)\Psi = 0, \]  
(31)

which is nothing but the Wheeler-DeWitt equation. A special solution can be
found to be
\[ \Psi = (Be^{\sqrt{A}f(v)} + Ce^{-\sqrt{A}f(v)})e^{i\frac{\lambda - 4\lambda M}{\lambda} \phi}, \]  
(32)

where \( A, B, \) and \( C \) are integration constants. Without losing generality, we shall
choose the boundary condition \( B = 0. \)

5. Hawking Radiation
We now turn our attention to an application of the quantum gravity near the apparent horizon for understanding the Hawking radiation in the two dimensional dilaton gravity. A similar analysis was carried out in four [4, 5] and three [6] dimensional black holes. The main motivation behind the present work is to clarify to what extent the quantum gravity holding near the apparent horizon reflects the physical properties of quantum black holes since we have a better grasp of the quantum mechanical features of a black hole in the two dimensional dilaton gravity compared to in the four and three dimensional gravities.

To begin with, let us define the expectation value $<O>$ of an operator $O$ by

$$<O> = \frac{1}{\int df |\Psi|^2} \int df \, \Psi^\dagger O \Psi.$$  \hspace{1cm} (33)

Under this definition, it is straightforward to evaluate the expectation value of the change rate in black hole mass

$$<\partial_v M> = -\frac{A}{2},$$  \hspace{1cm} (34)

where the constraint (30) (or $p_\phi$ in (29)) and the physical state (32) were used. Moreover, in a similar manner one can calculate that in the radius in the apparent horizon

$$<\partial_v r_{AH}> = -\frac{A}{4\lambda <M>}.$$  \hspace{1cm} (35)

To represent the Hawking radiation, we have to select the integration constant $A$ to be a positive constant, for example, $k_1^2$, then (32), (34) and (35) reduce to

$$\Psi = C \, e^{-|k_1| f(v) + i\frac{A - 4\lambda M}{2\lambda} \phi}.$$  \hspace{1cm} (36)

$$<\partial_v M> = -\frac{k_1^2}{2},$$  \hspace{1cm} (37)
To see explicitly that this is in fact the Hawking radiation carried by the matter field $f$, it is useful to argue the expectation value of the energy-momentum tensor of the matter field, which is defined as

$$< T^f_{ab} > = \frac{1}{\sqrt{-g}} \delta S_f \delta g_{ab}$$

$$= -\frac{1}{2} < \nabla_a f \nabla_b f - \frac{1}{2} g_{ab} (\nabla f)^2 > ,$$

where $S_f$ is the matter part in the action (1). Then $T_{vv}$ in which we are interested is calculated to be

$$< T^f_{vv} > = \frac{k_1^2}{2} ,$$

which is precisely equal to the opposite sign of (37), thus means that the matter flux is equivalent to the Hawking radiation as expected. Alternatively, if we choose $A$ to be a negative constant, e.g., $-k_2^2$, we gain a physical situation where external neutral matters flow into a black hole across the horizon.

Several comments about (36)-(38) are now in order. Firstly, (36) shows that the physical state has an exponentially damping-like form in the classically forbidden region implying the quantum tunneling process. This behavior seems to match our interpretation of the present situation as the Hawking radiation. Secondly, from (37), the Hawking radiation rate is independent of the black hole mass in contrast with the case of the four dimensional Schwarzschild black hole where it is shown that $< \partial_v M > \propto -\frac{1}{<M^2>}$ [4, 5] inferred by Hawking in his semiclassical approach [1]. This surprising result, however, has been already found in studies of the two dimensional dilaton and the other two dimensional gravities [13]. This is because in two dimensions there exists a beautiful relation between the trace anomaly and the Hawking radiation [14]. Namely, although for $N$ species of massless scalar field the
trace of the energy-momentum tensor vanishes classically, quantum-mechanically
there is the trace anomaly

\[ < g^{ab} T_{ab}^f > = \frac{N}{24} R. \]  (41)

(Up to now we take account of the case \( N = 1 \).) As a result, in the coordinates
where the metric is asymptotically constant on the null infinities \( I^\pm_R \), the Hawking
radiation rate is asymptotically independent of the black hole mass and approaches
the constant value \( \frac{N \lambda^2}{48} \) \[8\]. One might ask what would happen if one replaces a
single scalar matter \( f \) with \( N \) species of scalar \( f_i (i = 1, 2, ..., N) \) in our formalism
as considered in the CGHS original work. In this case, the physical state (32) is
replaced by

\[ \Psi = \prod_{i=1}^{N} (B_i e^{\sqrt{A_i} f_i(v)} + C_i e^{-\sqrt{A_i} f_i(v)}) e^{i \frac{A_i \lambda M}{2 \lambda} \phi}, \]  (42)

with

\[ \sum_{i=1}^{N} A_i = A, \]  (43)

and the result with respect to the Hawking radiation (34) remains unchanged. If
we consider completely identical \( N \) scalar matters such that \( A = N \bar{A}, (A_1 = A_2 = \ldots = A_N \equiv \bar{A}) \), the Hawking radiation rate becomes to scale with \( N \)

\[ < \partial_v M > = -\frac{N \bar{A}}{2}, \]  (44)

which is equal to the result derived by CGHS up to a numerical constant.

Finally, (38) shows that as a black hole emits the Hawking radiation and loses
the mass the apparent horizon recedes toward the singularity. This is a very plaus-
ible picture from physical consideration. In the limit \( < M > \rightarrow 0 \), this equation
indicates that $< \partial_v r_{AH} > \rightarrow -\infty$, suggesting that the apparent horizon disappears and the curvature singularity might be visible to external observers (violation of weak cosmic censorship). However, in order to get a definite answer to this problem, it seems that we need a more improved and sophisticated model as discussed in the next section.

Before closing this section, it is valuable to inquire what would happen near the origin $r = 0$. To keep the role of $x^0$ coordinate as the time, we here assume the inequality $M < \lambda$. Surprisingly enough, despite $r = 0$ being the curvature singularity [15], it turns out that one can also establish quantum gravity in this vicinity in a perfectly similar way to the case of the apparent horizon. For example, analogous equations to (27), (29), (30), (32) and (34) can be deduced as follows:

$$\alpha \approx \frac{1}{\sqrt{1 + \frac{M}{\lambda}}}, \quad \beta \approx \frac{M}{\lambda}, \quad \gamma = \frac{1}{\alpha^2} \approx 1 + \frac{M}{\lambda}, \quad (45)$$

$$p_f \approx \partial_v f, \quad p_\phi \approx -\frac{2}{\gamma \lambda} (\partial_v M + 2M^2), \quad p_\gamma \approx \frac{2M}{\gamma}, \quad (46)$$

$$\sqrt{\gamma} H_0 \approx \gamma H_1, \approx p_f^2 + \lambda \gamma p_\phi + 4M^2, \quad (47)$$

$$\Psi = (Be^{\sqrt{A}f(v)} + Ce^{-\sqrt{A}f(v)})e^{\frac{i\Delta - M^2}{\lambda \gamma} \phi}, \quad (48)$$

$$< \partial_v M > = -\frac{A}{2}. \quad (49)$$

These results, in particular, (49), are obviously consistent with results obtained by means of quantum gravity holding near the apparent horizon. One interesting problem in future would be to find the physical state holding in the region between the curvature singularity and the apparent horizon by connecting the two states (32) and (49) consistently.
6. Conclusion

So far we have investigated the Hawking radiation in terms of quantum gravity holding near the apparent horizon. In particular, it was found that the Hawking radiation rate is independent of the black hole mass, and it scales with the number of massless scalar fields when there are $N$ identical matters as shown in the other analysis of the two dimensional dilaton gravity.

Since the main motivation in the present study is to compare the results obtained in the quantum gravity holding near the apparent horizon with those of fully quantized two dimensional dilaton gravity, we should make comments on their relationship more closely. First of all, we notice that the value of the coefficient of the Hawking flux cannot be determined in the present formalism treating only the region in the vicinity of the horizon since it is fixed by imposing the boundary conditions at the null past infinities $I^-_R$ and $I^-_L$ [8, 13]. Nevertheless, it is remarkable that the present formalism describes qualitative features of the Hawking radiation without paying much attention to the interior and the exterior regions of a black hole. Actually, the quantum gravity near the horizon has provided a nice description of the Hawking radiation in three dimensional de Sitter black hole [16] where there is no asymptotically flat regions so that the usual technique cannot be applied to this case [6].

Next as mentioned before, it was known that in two dimensions the Hawking radiation stems from the trace anomaly [14]. Since the present formalism gives the qualitatively same picture as the semiclassical approach, it is certain that our formalism respects the contribution from the trace anomaly properly. However, it is conjectured that the functional measures over the gravitational field, the dilaton and the ghosts give rise to the nontrivial contributions which are different from the form of the trace anomaly, and eventually yield the backreaction of the Hawking radiation on the geometry although we have not yet reached the fully consistent model even in the two dimensional dilaton gravity [13]. In this respect, unfortu-
nately it seems that our formalism largely ignores this issue. But this problem is shared in the more general ADM or the Wheeler-DeWitt formalisms where we have no idea how to evaluate the functional measures. It is likely that one should set up the Wheeler-DeWitt equation after estimating the contributions from the functional measures carefully though we do not know how to accomplish this procedure except in two dimensions. Although we have a lot of things to overcome in the future, we believe that an improvement of the formalism at hand would give us a useful analytical method for understanding various properties of quantum black holes and might to a certain extent realize the Membrane Paradigm of a black hole [17-19].

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