SUSY $R$ parity violation and CP asymmetry in semi-leptonic $\tau$-decays

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We analyze the CP violation in the semileptonic $|\Delta S|=1$ $\tau$-decays in supersymmetric extensions of the standard model (SM) with $R$ parity violating term. We show that the CP asymmetry of $\tau$-decay is enhanced significantly and the current experimental limits obtained by CLEO collaborations can be easily accommodated. We argue that observing CP violation in semi leptonic $\tau$-decay would be a clear evidence for $R$-parity violating SUSY extension of the SM.

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I. INTRODUCTION

CP violation is one of the main open questions in high energy physics. In the standard model (SM), all CP violating observables should be explained by one complex phase $\delta_{CKM}$ in the quark mixing matrix. The effect of this phase has been first observed in kaon system and recently confirmed in $B$ decays. The quark-lepton symmetry suggests that the lepton mixing matrix should also violate CP invariance. However, the situation in the lepton sector is very different. The only evidence for flavor violation in this sector comes from the neutrino oscillations and there is no, so far, any confirmation for CP violation in leptonic decays. Hence, measuring of CP asymmetry in $\tau$ decays will open new window to study the CP violation.

Within the SM, the direct CP asymmetry rate of $\tau^\pm \rightarrow K^\pm \pi^0 \nu_\tau$ is of order $O(10^{-12})$ [1]. This nearly vanishing asymmetry implies that the observation of $\tau$-decay would be a clear signal for the presence of CP violation beyond the SM.

Supersymmetry is one of the most interesting candidates for physics beyond the SM. Supersymmetry provides a new sources of CP violation through complex couplings in the soft SUSY breaking terms. The CP asymmetry in the decay mode $\tau^\pm \rightarrow K^\pm \pi^0 \nu_\tau$, in minimal supersymmetric extension of the SM with $R$ parity conservation, has been computed in Ref. [2]. It was shown that the SUSY contributions can enhance the CP asymmetry rate in $\tau$-decay by many order of magnitude. However, the typical value is of order $O(10^{-7})$ which is still much smaller than the current experimental bound.

The aim of this paper is to show that a significant enhancement for the CP asymmetry of $\tau^\pm \rightarrow K^\pm \pi^0 \nu_\tau$ can be obtained in SUSY models with $R$ parity violating terms. It turns out that the $R$ parity violating terms (in particular, the Lepton number violating ones) induce a tree level contribution to $\tau$-decay. This contribution is proportional to the $R$ parity couplings $\lambda$ and $\lambda'$, which in general are complex. We find that this new source of CP violation enhance the asymmetry of $\tau$ decay. We impose new constraints on the couplings $\lambda$ and $\lambda'$ from the experimental limits, obtained by CLEO collaborations [3, 4, 5, 6].

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The paper is organized as follows. In section 2, some general features on \( \tau \) semi-leptonic decays are recalled. Section 3 is devoted for analyzing the CP asymmetry of \( \tau^{\pm} \to K^{\pm}\pi^{0}\nu \) in SUSY models with \( R \) parity violation. We derive the corresponding effective hamiltonian and show that terms that violate \( R \) parity may give significant contribution to the CP asymmetry in \( \tau \)-decay. Finally, we give our conclusions in section 4.

II. CP ASYMMETRY OF \( \tau \) SEMILEPTONIC DECAY IN MSSM.

In this section we analyze the CP asymmetry of \( \tau \) semileptonic decay modes within MSSM, we will focus on the decay mode \( \tau^{\pm} \to K^{\pm}\pi^{0}\nu \). The general amplitude for \( \tau^{-}(p) \to K^{-}(k)\pi^{0}(k')\nu_{\tau}(p') \) is given by

\[ M = \frac{G_F V_{u\tau}}{\sqrt{2}} \left\{ \bar{u}(p') \gamma_{\mu}(1 - \gamma_5)u(p)F_{V}(t) \left[ (k - k')_{\mu} - \frac{\Delta^2}{t} q_{\mu} \right] \right. \]

\[ + \bar{u}(p')(1 + \gamma_5)u(p)m_{\tau}\Lambda F_{S}(t) \frac{\Delta^2}{t} \]

\[ + F_{T}(K\pi|\bar{s}\sigma_{\mu\nu}u|0)\bar{u}(p')\sigma^{\mu\nu}(1 + \gamma_5)u(p) \}, \]

where \( q = k + k' \) \((t = q^2)\) is the momentum transfer to the hadronic system, \( \Delta^2 \equiv m_K^2 - m_{\pi}^2 \) and \( F_{V,S,T}(t) \) are the effective form factors describing the hadronic matrix elements. In SM, \( F_{T} = 0, \Lambda = 1 \)

\[ \sum_{pol} |M|^2 \sim |F_{V}|^2 (2p.Qp'.Q - p.p'Q^2) + |A|^2|F_{S}(t)|^2 M^2 p.p' \]

\[ + 2Re \Lambda \cdot Re(F_{S}F_{V}^{*})Mm_{\tau}p'.Q - 2Im \Lambda \cdot Im(F_{S}F_{V}^{*})Mm_{\tau}p'.Q \],

where \( Q_{u} = (k - k')_{\mu} - \frac{\Delta^2}{t} q_{\mu} \).

The last two terms disappear once we integrate on the kinematical variable \( u \) unless the form factor have a \( u \) dependence. The form factors \( F_{V,S}(t) \) can receive weak phase through higher order contributions and hence it is possible to generate a CP asymmetry in total decay rates but within SM this CP asymmetry is nearly vanishing [1].

The CP asymmetry is defined as

\[ A_{CP} = \frac{\Gamma(\tau^{+} \to K^{+}\pi^{0}\nu_{\tau}) - \Gamma(\tau^{-} \to K^{-}\pi^{0}\nu_{\tau})}{\Gamma(\tau^{+} \to K^{+}\pi^{0}\nu_{\tau}) + \Gamma(\tau^{-} \to K^{-}\pi^{0}\nu_{\tau})}. \]

The effective Hamiltonian \( H_{eff} \) derived from SUSY superpotential with \( R \) parity symmetry conserved can be expressed as [2]

\[ H_{eff} = \frac{G_F}{\sqrt{2}} V_{u\tau} \sum_{i} C_{i}(\mu)Q_{i}(\mu), \]  \hspace{1cm} (1)

where \( C_{i} \) are the Wilson coefficients and \( Q_{i} \) are the relevant local operators at low energy scale \( \mu \simeq m_{\tau} \).

The operators are given by

\[ Q_{1} = (\bar{\nu}\gamma_{\mu}L\tau)(\bar{s}\gamma_{\mu}Lu), \]  \hspace{1cm} (2)

\[ Q_{2} = (\bar{\nu}\gamma_{\mu}L\tau)(\bar{s}\gamma_{\mu}Ru), \]  \hspace{1cm} (3)

\[ Q_{3} = (\bar{\nu}R\tau)(\bar{s}Lu), \]  \hspace{1cm} (4)

\[ Q_{4} = (\bar{\nu}R\tau)(\bar{s}Ru), \]  \hspace{1cm} (5)

\[ Q_{5} = (\bar{\nu}\sigma_{\mu\nu}R\tau)(\bar{s}\sigma_{\mu\nu}Ru). \]  \hspace{1cm} (6)
where \( L, R \) are defined as \( L, R = 1 \mp \gamma_5 \) and \( \sigma^{\mu\nu} = \frac{i}{2} [\gamma^\mu, \gamma^\nu] \). The SUSY contributions to the Wilson coefficients \( C_i \) can be found in Ref. [2]. For dominant \( C_3 \) and/or \( C_4 \), one finds that the decay amplitude is given by
\[
\mathcal{A}_T(\tau \to K\pi\nu) = \frac{G_F V_{us}}{\sqrt{2}} (1 + C_1) \times \left\{ f_V Q^\mu \bar{u}(p') \gamma_\mu L u(p) + \left[ m_\tau + \left( \frac{C_3 + C_4}{1 + C_1} \right) \frac{t}{m_s - m_u} \right] f_S \bar{u}(p') R u(p) \right\}.
\]

Using CLEO limit, we can translate this bound into:
\[
-0.010 \leq \text{Im} \left( \frac{C_3 + C_4}{1 + C_1} \right) \leq 0.004,
\]
where we have used \( m_s - m_u = 100 \text{ MeV} \), and the average value \( \langle t \rangle \approx (1332.8 \text{ MeV})^2 \). However, for \( M_1 = 100 \) and \( M_2 = 200 \text{ GeV} \) and \( \mu = M_\tilde{q} = 400 \text{ GeV} \) and \( \tan \beta = 20 \), one gets
\[
\text{Im} \left( \frac{C_3 + C_4}{1 + C_1} \right) \approx 1.3 \times 10^{-5} \text{Im}(\delta_{21}^d)_{RL}
\]

It is worth noting that the mass insertions \( (\delta_{21}^d)_{RL} \) are constrained by the \( \Delta M_K \) and \( \epsilon_K \) as follows:
\[
|\,(\delta_{21}^d)_{RL} \,| \lesssim 4 \times 10^{-3}.
\]

Therefore, the resultant CP asymmetry of \( \tau \to K\pi\nu \) is smaller, by few order of magnitude, than the current experimental limit.

### III. \( \tau \) DECAY CP ASYMMETRY IN SUSY WITHOUT \( R \) PARITY

In this section we study the effect of including terms that violate lepton and baryon number on the tau decay CP asymmetry. The gauge invariance does not insure the conservations of both baryon number and lepton number and hence we can allow the SUSY superpotential to have the \( R \) parity violating terms. on the other hand side, \( R \) parity violation can be motivated by some controversial experimental observations, like events with missing energy and a hadron jet in the \( H1 \) experiment at HERA. Recall that the most general superpotential that violates the \( R \) parity symmetry can be written as [8]
\[
W_{R_p} = \frac{1}{2} \lambda_{ijk} L_i L_j \bar{E}_k + \lambda'_{ijk} L_i Q_j \bar{D}_k + \frac{1}{2} \lambda''_{ijk} \bar{U}_i \bar{D}_j \bar{D}_k + \kappa_i L_i H_2,
\]
where a summation over the generation indices \( i, j = 1, 2, 3 \) and over gauge indices is understood. The \( \lambda_{ijk} \) is anti-symmetric in \( \{i, j\} \) because of the contraction of \( SU(2) \) indices and hence \( \lambda_{ijk} \) are non-vanishing only for \( i < j \). The \( \lambda'_{ijk} \) is anti-symmetric in \( \{j, k\} \). Therefore \( j \neq k \) in \( \bar{U}_i \bar{D}_j \bar{D}_k \) and hence we can write the superpotential \( W_{R_p} \) as:
\[
W_{R_p} = \lambda_{ijk} L_i L_j \bar{E}_k + \lambda'_{ijk} L_i Q_j \bar{D}_k + \lambda''_{ijk} \bar{U}_i \bar{D}_j \bar{D}_k + \kappa_i L_i H_2.
\]

To insure that the proton is stable, we require only the conservation of baryon number and hence we forbid the term \( \lambda''_{ijk} \bar{U}_i \bar{D}_j \bar{D}_k \). Expanding \( W_{R_p} \) term into the Yukawa couplings yields
\[
\mathcal{L} = \lambda_{ijk} \left[ \bar{e}_L^i \tilde{e}_R^k e_L^j + \bar{e}_L^j \tilde{e}_R^k e_L^i + (\tilde{e}_R^k)^* (\bar{e}_L^i)^c e_L^j - (i \leftrightarrow j) \right] \\
+ \lambda'_{ijk} \left[ \bar{e}_L^i \tilde{d}_R^k d_L^j + \bar{d}_L^k \tilde{d}_R^i d_L^j + (\tilde{d}_R^k)^* (\bar{d}_L^i)^c d_L^j - \bar{e}_L^j \bar{d}_R^k u_L^i - \bar{d}_R^k \bar{d}_R^i u_L^j \right] \\
- (\tilde{d}_R^k)^* (\bar{e}_L^i)^c u_L^j + h.c.,
\]

(10)
FIG. 1: $R$ parity violation SUSY contributions to $\tau^\tau \to u\bar{u} \nu_e$ transition.

where, tilde denotes the scalar fermion superpartners. The leading diagrams for $\tau \to k\pi\nu$ are illustrated in Fig.1.

The corresponding effective Hamiltonian, $H_{\text{eff}}$, derived from SUSY $R$ parity violating terms can be expressed as

$$H_{\text{eff}} = \frac{G_F}{\sqrt{2}} V_{us} \sum_i C_i(\mu) Q_i(\mu),$$

where $C_i$ are the Wilson coefficients and $Q_i$ are the relevant local operators at low energy scale $\mu \simeq m_\tau$. These operators are given by

$$Q_1 = (\bar{s}_\mu L\tau)(\bar{\nu}_\mu Lu),$$

$$Q_2 = (\bar{\nu}_R \tau)(\bar{s} Lu),$$

$$Q_3 = (\bar{\nu}_L \tau)(\bar{s} Lu).$$

The Wilson coefficients $C_i$, at the electroweak scale, can be expressed as $C_i = C_i^{SM} + C_i^{SUSY}$. For $i = 2, 3$ $C_i^{SM}$ vanish identically. In this respect, the Wilson coefficients $C_i$ are given by

$$C_1 = 1$$

$$C_2 = \frac{\sqrt{2}}{G_F V_{us}} \left( \frac{1}{m^2} \right) (\lambda_{1333} \lambda_{312}^2 + \lambda_{123} \lambda_{212}),$$

$$C_3 = \frac{\sqrt{2}}{G_F V_{us}} \left( \frac{1}{m^4} \right) (\lambda_{13M} \lambda_{112}^2)(\delta_{LR})_{ik}.$$  

The couplings $\lambda_{ijk}$ and $\lambda'_{ijk}$ are generally complex numbers. At a value for $\tilde{m} = 100$ GeV, the upper bounds on these couplings are given as follows:

$$|\lambda_{131}| = |\lambda_{132}| = 0.06,$$

$$|\lambda_{133}| = 0.004, \quad |\lambda_{123}| = 0.05,$$

$$|\lambda'_{212}| = 0.09, \quad |\lambda'_{312}| = 0.16,$$

$$|\lambda'_{112}| = 0.02, \quad |\lambda'_{312}| = 0.16.$$  

As can be seen, $C_3$ can be dropped comparing to $C_1$ and $C_2$. The decay amplitude for the decay $\tau^\tau(p) \to K^-(k)\pi^0(k')\nu(p')$ including SM and SUSY contributions becomes:

$$\mathcal{M} = \frac{G_F}{\sqrt{2}} V_{us} \left\{ \langle K\pi|\bar{s}\gamma_\mu u|0\rangle \bar{\nu}\gamma_\mu L\tau + C_2 \langle K\pi|\bar{s}u|0\rangle \bar{\nu}R\tau + C_3 \langle K\pi|\bar{s}u|0\rangle \bar{\nu}L\tau \right\}$$

(19)
Using
\[ \langle K\pi |\bar{s}\gamma_\mu u|0\rangle = f_V(t)Q_\mu + f_S(t)(k+k')_\mu , \tag{20} \]

and
\[ \langle K\pi |\bar{s}u|0\rangle = \frac{t}{m_s-m_u}f_S(t) . \tag{21} \]

The amplitude can be written as:
\[ A_T(\tau\to K\pi\nu) = \frac{G_FV_{us}}{\sqrt{2}} \left\{ f_V(t)Q_\mu \bar{\nu}(p')\gamma_\mu L\tau(p) + \left[ m_\tau + C_2 \frac{t}{m_s-m_u} \right] f_S(t)\bar{\nu}(p')R\tau(p) \right\} . \tag{22} \]

This expression should be compared with the decay amplitude given in Eq. (2) of Ref. [6]:
\[ A(\tau^-\to K\pi\nu_\tau) \sim \bar{u}(p')\gamma_\mu L\nu(p)f_VQ_\mu + \Lambda\bar{u}(p')R\nu(p)f_SM , \tag{23} \]

where \( M = 1 \text{ GeV} \) is a normalization mass scale. Hence one finds the relation:
\[ \Lambda M = m_\tau + C_2 \frac{t}{m_s-m_u} . \tag{24} \]

The first term in the last equation is the usual contribution of the SM, which is real, and the second term arises from the SUSY contributions.

Using the bound obtained by the CLEO collaboration: \(-0.172 < \text{Im}(\Lambda) < 0.067 \) at 90% C.L. [8], we can translate this bound into:
\[ -0.010 < \text{Im}C_2 < 0.004 , \tag{25} \]

where we have used again as before, \( m_s-m_u = 100 \text{ MeV} \), and the average value \( \langle t \rangle \approx (1332.8 \text{ MeV})^2 \). After substitution, we can write
\[ -0.010 < \text{Im} \left[ \frac{\sqrt{2}}{G_FV_{us}} \left( \frac{1}{m^2} \right) (\lambda_{133}\lambda'_{312} + \lambda_{123}\lambda'_{212}) \right] < 0.004 . \tag{26} \]

For \( G_F = 1.166 \times 10^{-5} \text{ GeV}, \) \( \bar{m} = 100 \text{ GeV}, \) and \( V_{us} = 0.22, \) one obtains the following bound
\[ -1.8 \times 10^{-4} < \text{Im}(\lambda_{133}\lambda'_{312} + \lambda_{123}\lambda'_{212}) < 7.2 \times 10^{-5} . \tag{27} \]

From the upper bounds on the Yukawa couplings: \( \lambda_{ijk} \) and \( \lambda'_{ijk}, \) reported in Eq. [15], one finds that \( |\lambda_{133}\lambda'_{312}| \lesssim 10^{-4}, \) while the coupling \( \lambda_{123} \) is unconstrained. Thus, the above equation leads to the following bound on \( \text{Im}(\lambda_{123}\lambda'_{212}): \)
\[ -1.8 \times 10^{-4} < \text{Im}(\lambda_{123}\lambda'_{212}) < 7.2 \times 10^{-5} . \tag{28} \]

Thus, with order \( 10^{-2} \) complex Yukawa couplings \( \lambda_{123} \) and/or \( \lambda'_{212}, \) the CP asymmetry experimental limits obtained by CLEO collaborations can be easily accommodated. This result is an intrinsic feature for \( R \) parity SUSY contribution to the CP asymmetry of \( \tau \to k\pi\nu. \) It is important to stress that this is the only model, to our knowledge, that enhance CP asymmetry of \( \tau \) decay significantly and account for the CLEO limits. Therefore, a confirmation for CLEO measurements would be a clear evidence of \( R \) parity violating SUSY extension of the SM.
IV. CONCLUSION

We have studied the supersymmetric contributions to the CP asymmetry of $\tau \to k\pi\nu$ decay. We emphasized that CP asymmetry in this decay is nearly vanishing within the SM. Therefore, any non-vanishing CP asymmetry in this decay channel will be a clear evidence for physics beyond the SM. We have shown how physics beyond standard model as supersymmetric extensions of the SM could induce CP violating asymmetry in the double differential distribution as CLEO collaboration did. In case of Supersymmetry with conserved R parity, it has been found that the CP asymmetry is enhanced by several orders of magnitude than the SM expectations. However, the resulting asymmetry is still well below the current experimental limits obtained by CLEO collaborations\cite{2}. Within R-parity violation SUSY models, we found that the CP asymmetry of $\tau$-decay is enhanced significantly and the current experimental limits obtained by CLEO collaborations can be easily accommodated.

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