The Aharonov–Bohm effect: A quantum or a relativistic phenomenon?

Klaus Wilhelm • Bhola N. Dwivedi

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Abstract  The Aharonov–Bohm effect is considered by most authors as a quantum effect, but a generally accepted explanation does not seem to be available. The phenomenon is studied here under the assumption that hypothetical electric dipole distributions configured by moving charges in the solenoid act on the electrons as test particles. The relative motions of the interacting charged particles introduce relativistic time dilations. The massless dipoles are postulated as part of an impact model that has recently been proposed to account for the far-reaching electrostatic forces described by Coulomb’s law. The model provides a quantitative explanation of the Aharonov–Bohm effect.

1 Introduction

The Aharonov–Bohm effect was theoretically predicted in the middle of the last century (Ehrenberg and Siday 1948; Aharonov and Bohm 1959). It was, however, implicitly derived by Glaser (1933) from Fermat’s principle and the refractive index of electron optics in 1932. Many experimental verifications have been performed since 1960 (e.g. Chambers 1960; Mollenstedt and Bayl 1962; Lenz 1962); see also Tonomura et al. (1986) and Caprez et al. (2007) for further references. Most authors consider the effect to be a purely quantum mechanical one, but there are opposing views (e.g. Boyer 2008). This has been summarized in a recent paper by Hegerfeldt and Neumann (2008), stating that a generally accepted physical understanding of the Aharonov–Bohm effect does not seem to be available. Here we will outline a solution that is based on a relativistic effect.

2 Outline of a solution

The Aharonov–Bohm effect, as illustrated and described in Fig. 1, can be understood in the context of an electric dipole model proposed by Wilhelm et al. (2014) for electrostatic forces. According to this model and its application to magnetostatic configurations (Dwivedi et al. 2013), the dipole distributions would differently be modified inside and outside of the solenoid. Of particular importance is that the outside will indeed be affected at all, in contrast to the zero magnetic field there under ideal conditions. Charged particles (electrons are generally used as test particles) moving outside of the solenoid in a plane perpendicular to its axis will react to the modified dipole distributions. On one side of the coil, a component of the velocity, $v_q$, of the charge carriers in the conductor will be in the same direction as the velocity vectors of the electrons, $v_e$, and on the other side it is oppositely directed. This is shown in Fig. 2 and will lead to different relativistic time dilations. We thus expect slightly different integrated momentum transfers in the plane of motion normal to both the axis of the coil and the trajectories of the electrons. Note that no significant acceleration or deceleration would occur along the trajectory of the test particle in line with observations. However, the transverse momentum transfers are at variance with a dispersionless interaction of the Aharonov–Bohm effect.
A very extended solenoid along the $z$ axis, with radius $R$ and $N = n a$ windings on an axial length, $a$ (i.e., a winding density of $n$), carrying a current, $I$, (realized by negative charges, $-|q|$, moving with a speed $v$) has an internal magnetic flux density of $B_i = \mu_0 n I$ and nearly no magnetic field, $B_o$, outside. Nevertheless, electrons travelling with the velocity $v_e$ parallel to the $y$ axis on either side of the coil at impact parameters $x = \pm |b|$ experience a shift of their interference pattern on a screen behind the plane of the paper (cf., Fig. 3). and a propagation of the average positions of the electron wave packets as if they would travel in free space (Caprez et al. 2007), cf., also McGregor et al. (2012) and McGregor (2013).

3 Quantitative description of the effect

In order to arrive at a quantitative description of the effect, several simplifying assumptions and approximations (considered to be reasonable) have to be made. This limits the calculations to an amount acceptable within this short communication. A number of $Z$ negative charges, $-|q|$, is assumed to be distributed along the circumference of one winding of the solenoid. The moving negative charges (i.e., electrons) constitute a current

$$I = -\frac{Z |q| v_q}{2 \pi R}.$$  

The positive charges in the conductor are at rest. With $I = 0$ ($v_q = 0$), an electron passing on the outside of the coil will, of course, experience no forces. With $I \neq 0$ and thus $v_q \neq 0$, the situation changes by relativistic time dilation in such a way that the density of the negative charges $-|q|$ is dependent on their relative velocities of with respect to the electron (Dwivedi et al. 2013).

We first treat one turn of the solenoid in the $x$-$y$-plane, see Fig. 2. The $y$ component of $v_q$ is $v_{q,y} = v_q \sin \phi$; integration over a half-circle with radius $R$ gives

$$M = R v_q \int_0^\pi \sin \phi \, d\phi = 2 R v_q ,$$  

and the $x$ coordinates of CG$_1$ and CG$_2$ (cf., Fig. 2) are

$$\xi^\pm = \pm \frac{R^2 v_q}{M} \int_0^\pi \sin^2 \phi \, d\phi = \pm \frac{\pi R}{4} ,$$  

respectively. We define

$$b^\tau_{\text{eff}} = b - \xi^\pm = b \mp \frac{\pi R}{4}.$$  

Fig. 2 Cross-section of the solenoid in the $(x,y)$ plane. An electron passing with a velocity $v_e$ at $x = b$ is indicated. The current in the coil is represented by charges $-|q|$ with a speed of $v_q$. Their influence on the passing electron is approximated by averaging over the angle $\phi$ to find the mean $y$ component $\langle v_q,y \rangle$ of $v_q$ and an effective value for $b$. CG$_1$ indicates the “centre of gravity” at an effective impact parameter $b^\tau_{\text{eff}}$ for the semicircle $\phi = 0$ to $\pi$ weighted with the $x$ component of $v_q$. CG$_2$ at $b^\tau_{\text{eff}}$ is the corresponding centre for $\phi = \pi$ to $2 \pi$. The positive charges in the conductor are at rest. With $I = 0$ ($v_q = 0$), an electron passing on the outside of the coil will, of course, experience no forces. With $I \neq 0$ and thus $v_q \neq 0$, the situation changes by relativistic time dilation in such a way that the density of the negative charges $-|q|$ is dependent on their relative velocities of with respect to the electron (Dwivedi et al. 2013).
as effective impact parameters. Next we calculate a mean value of the \( y \) component of \( v_q \)

\[
\langle v_{q,y} \rangle = \frac{v_q}{\pi} \int_0^\pi \sin \phi \, d\phi = \frac{2 v_q}{\pi}
\]

(5)

over \( \phi \) from 0 to \( \pi \), and \( \langle v_{q,y} \rangle = -2 v_q/\pi \) over \( \pi \) to \( 2 \pi \). The corresponding mean \( x \) components are zero. The momentum transfer between charges \( Q_1 \) and \( Q_2 \) in relative motion is according to Equation (10) of Dwivedi et al. (2013)

\[
\Delta P_b = \frac{Q_1 Q_2}{2 \varepsilon_0 b v}
\]

(6)

with \( \varepsilon_0 \) the electric constant in vacuum.

We assign portions of the negative and positive charges in the conductor to the origins \( \text{CG}_1 \) and \( \text{CG}_2 \) of the effective impact parameters, considering that the different speeds, \( v_e \pm \mp 2 v_q/\pi \), of the negative charges with respect to the electron passing at \( x = b \) must lead to an uneven partition of the positive and negative charges at these positions. Evaluating Eq. (6) separately for \( \phi \) from 0 to \( \pi \) and \( \pi \) to \( 2 \pi \) provides approximations of the charge distributions in the winding close to the \( (x, y) \) plane as seen from the electrons on either side of the solenoid. This is done in the following equation by taking the relativistic time dilations (Einstein 1905) cause by the motions into account before the determination of the momentum transfer to the electrons in the \( x \) direction. The speed \( v \) has to be taken as \( v_e \) in this average configuration:

\[
\Delta P^e_b(0) \approx \mp \frac{|e|}{2 \varepsilon_0 b^{\text{eff}} v} \frac{Z|q|}{2} (\gamma - \gamma^\mp),
\]

(7)

where the upper indices of \( \Delta P^e_b \) refer to the influence of the charges \( \Delta Q^e_b \) defined by

\[
\Delta Q^e_b = \frac{Z|q|}{2} (\gamma - \gamma^\mp)
\]

(8)

at \( \text{CG}_1 \) and \( \text{CG}_2 \), respectively, with the notations for the Lorentz factor related to the positive charges

\[
\gamma = \left(1 - \frac{v_e^2}{c^2} \right)^{-1/2} \approx 1 + \frac{1}{2} \frac{v_e^2}{c^2}
\]

(9)

and a factor of

\[
\gamma^\mp = \left[ 1 - \frac{(v_e \pm \mp |(v_{q,y})|)^2}{c^2} \right]^{-1/2}
\]

\[
\approx 1 + \frac{1}{2} \frac{(v_e \pm |(v_{q,y})|)}{c_0}
\]

(10)

for the negative charges. \( c_0 \) is the speed of light in vacuum. An evaluation with \( v_e \gg v_q \) yields

\[
\Delta Q^{\mp}_b = \pm \frac{Z|q| v_e v_q}{\pi c_0}.
\]

(11)

If \( dN = 1 \) represents one turn of the solenoid shown in Fig. 1 the total effective charge, \( Q_{\text{eff}} \), can be approximated by an integration along the length \( a \) in the \( z \)-direction

\[
Q^{\mp}_b \approx \int_{-\infty}^{\infty} \Delta Q^{\mp}_b \frac{b}{\rho} \cos \psi \, d\psi \, dN = \Delta Q^{\mp}_b \int_{-\infty}^{\infty} \cos^2 \psi \, d\psi = nb \Delta Q^{\mp}_b,
\]

(12)

where we have used

\[
a = b \tan \psi = \frac{N}{n}
\]

(13)

and, after differentiation,

\[
dN = \frac{nb \Delta Q^{\mp}_b}{\cos^2 \psi} \, d\psi.
\]

(14)

Eqs. (7) and (8) together with Eqs. (11) and (12) yield after a short calculation

\[
P^{\mp}_b \approx \mp \frac{|e|}{2 \pi b^{\text{eff}}} \int_{-\infty}^{\infty} \frac{Z|q| v_q}{2} \, d\psi.
\]

(15)

With \( \pi R^2 = S \), the area of the solenoid cross-section; the magnetic flux density, \( B_i = \mu_0 n I \); as well as Eqs. (1) and (3), the total momentum transfer is:

\[
P^\mp_b = P^+_b + P^-_b \approx -\frac{e S B_i}{2 b[1 - (\pi R)^2/(4 b^2)]},
\]

(16)

A corresponding calculation for an electron passing the solenoid at \( x = -b \) gives the same result. The axial components cancel each other for reasons of symmetry, see Fig. 1 and Eq. (12). It might be in order to mention that the slightly asymmetric configuration during the approach leads to a minute differential momentum transfer according to Eq. (16) of Dwivedi et al. (2013).

It can, however, be neglected against the momentum of an electron of typically 30 keV. This can be confirmed by considering the relative variation of the de Broglie wavelength during the flyby due to the change of the refractive index (cf., Glaser 1933; Ehrenberg and Siday 1934). It is of the order of \( 10^{-28} \).

Electrons with a momentum

\[
\rho_e = \frac{h}{\lambda_e},
\]

(17)
Fig. 3  Geometry of the electron diffraction near a solenoid. Electrons with momentum $p_e$ pass at impact parameters $\pm b$ with respect to the origin of the $(x, y)$ coordinate system. The momentum changes are $P_b$, each. The deflection angle $\delta$ on either side leads to a shift of the interference pattern on the screen $F$—an appropriate focusing device is assumed, for instance, an electro-optic bi-prisma (Möllenstedt and Dücker 1956).

where $\lambda_e$ is the de Broglie wavelength, passing in Fig. 3 at $x = \pm b$ will thus be diffracted according to

$$\tan \delta = -\frac{P_b^*}{p_e}.$$  

(18)

The fringes of the interference pattern will be shifted by one order on the screen $F$, if

$$2b \sin \delta_1 = \lambda_e,$$  

(19)

or—with $\sin \delta_1 \approx \tan \delta_1$ for the small angle $\delta_1$ under consideration and Eqs. (16) to (19)—if

$$S [B_1]_1 \approx \frac{\hbar}{e} \left[ 1 - (\pi R)^2/(4 b)^2 \right].$$  

(20)

For $b/R \gg 1$ it is

$$S [B_1]_1 \approx \frac{\hbar}{e},$$  

(21)

where $[B_1]_1$ denotes the magnetic field required for a shift of one order.

4 Conclusion

The electric dipole model [Wilhelm et al. 2014] and its application to magnetostatic configurations [Dwivedi et al. 2013] provide an explanation of the Aharonov–Bohm effect based on relativistic time dilations with reasonable accuracy considering that—despite the many approximations made—the result in Eq. (20) is for $b > 2 R$ within a factor of $\approx 1.2$ of the expected value given in the literature (e.g. Aharonov and Bohm 1959; Caprez et al. 2007).
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