Application of Background Oriented Schlieren for quantitative measurement of transonic flows

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Abstract. Flow visualization techniques have progressed from methods based on qualitative description of flow-field into techniques providing both qualitative and quantitative results. In this paper we present implementation of Background Oriented Schlieren (BOS) technique for measurements of density values at transonic speeds. The density values obtained by BOS were then compared with results from CFD analysis of the same flow characteristics. In both qualitative and quantitative terms, the BOS and CFD results agreed, which proves the applicability of BOS for transonic flows.

1. Introduction

Flow visualization techniques have been more frequently applied to wind tunnel tests, replacing the usual setup of pressure taps and other intrusive instrumentation. These techniques are an improvement in terms of whole flow-field visualization and the possibility of processing complex types of flow.

One of these methods, the Background Oriented Schlieren (BOS) technique [3,7] is based on imaging of background distortion caused by changes of the refraction index $n$ of medium in the region between camera and background. The changes of refraction index are proportional to the density change as stated in equation 1 and therefore determination of apparent movement of background pattern can be used for quantitative determination of the density gradients in the investigated flow.

$$\frac{n - 1}{\rho} = G(\lambda)$$

The constant $G(\lambda)$, in equation 1, is called the Gladstone-Dale constant. It depends on characteristics of the gaseous media and also on the frequency of light used.

The technique is also based on image displacements obtained by comparing wind-off and wind-on pictures\(^1\). The refraction index $n$ is related to the obtained image displacements [1], $(\Delta x$ and $\Delta y)$, by the partial differential equation:

$$\frac{\partial^2 n}{\partial x^2} + \frac{\partial^2 n}{\partial y^2} = \frac{n_0}{MZ_\varnothing h} \left( \frac{\partial \Delta x}{\partial x} + \frac{\partial \Delta y}{\partial y} \right)$$

\(^1\) Wind-off and Wind-on pictures stand for pictures taken of the test section of a wind tunnel with flow at rest and during wind tunnel operation respectively.
The constants $n_0$, $M$, $Z_d$ and $h$ in equation 2 represent refractive index of undisturbed flow, optical magnification, distance between background and schlieren object and object thickness respectively. Once this equation is integrated, equation 1 can be used to obtain density.

This visualization technique is therefore not only capable of extracting qualitative information about the flow-field but also quantitatively extract density values. This makes BOS an attractive method for analyzing complex flow types.

This shows that visualization techniques have progressed from methods based on qualitative description of fluid flow into techniques providing both qualitative and quantitative results.

Transonic flows are a good example of such complex phenomenon where significant gradients occur. These occur due to the mixed subsonic and supersonic regions which are both present in the same flow-field. There is very little analytical theory available on the behavior of transonic flow, which makes the Background Oriented Schlieren technique suitable for this type of application.

In this paper we present the implementation of Background Oriented Schlieren technique for measurements of the density gradients at transonic speeds. We perform our analysis on a conical body submitted to transonic flow of Mach 0.9 inside a wind tunnel test section. In the past decade, the BOS technique has been applied for measurements of compressible flows in wind tunnels, supersonic jets [4] and full-scale measurements, such as visualization of shock waves surrounding jet fighter [10] and path of rotor blade vortices in helicopter flight [4,10]. Although, the possibility of using density measurements with BOS technique at supersonic speeds [4,9,10] has been confirmed, the feasibility of BOS for density field reconstruction at transonic flow regime has not been explored. The aim of the presented research was to perform BOS flow visualization and develop a procedure for determination of density field at transonic speeds. The results obtained by this procedure are then validated by Computational Fluid Dynamics (CFD) simulations that numerically determine the density field around the conical body. In the paper, development of BOS measurement setup as well as data reduction procedures are presented. A dedicated procedure was developed for retrieving the density field from the experimental data. The presented methodology can be used for investigations of compressible flows at transonic speeds as well as can be extended for supersonic flows.

2. Experimental Setup
The flow over a cone placed in the test section of a transonic wind tunnel was investigated at Mach number 0.9 by the BOS technique.

The N-3 wind tunnel is a closed circuit blow down type wind tunnel with partial recirculation of the flow. The cross-section of the test chamber is a square of side $H = 0.6$ m. The Mach number can be set with accuracy 0.01.

The Background Oriented Schlieren is based on digital analysis of image displacements. The background pattern is illuminated by source of light providing constant level intensity over the whole test time and imaged through a fluid containing spatial density gradients (figure 1). Since coherent beam of light is not required, the BOS setup is much simpler than classical Toepler schlieren or shadowgraph schlieren apparatus.

The images were captured with Nikon d800 digital SLR camera with 36 MP CMOS sensor and Nikon Nikkor AF-S 70-200 mm f2.8 lenses. The focal length was set to 200 mm. The background was created by spraying black paint on white paper. A series of images was created by varying the pressure in the airbrush. The series varied from very small (1-2 mm in diameter) and dense spots to big (5-8 mm) and sparse dots. The optimal density, providing images of dots size from 1 to 3 pixels on wind-off images, was chosen from the series of background. In order to provide stable illumination of the background, a HardSoft IL-106X pulsed LED illuminator [5] with a green LED was utilized. During the test, the illuminator was in continuous work mode and the images were acquired by remote shutter release.
3. BOS Algorithm
The Background Oriented Schlieren technique can be summarized in the following steps:

(i) Acquisition of reference image (wind-off) of background without flow and images during the wind tunnel run (wind-on).
(ii) Determination of the apparent image displacements between the wind-off and wind-on images by digital image processing.
(iii) Integrate Poisson equation (equation 2) in order to obtain the density field.

The images acquired for both wind-off and wind-on conditions are displayed in figure 2.

![Figure 1. Experimental setup.](image)

![Figure 2. Images acquired of the wind tunnel test section: (a) Wind-off. (b) Wind-on](image)

There is no apparent difference between wind-off and wind-on images as one can easily notice in figure 2. However, when both images are represented by matrices of pixel values, the differences become clear. Based on that, the PIV Cross-Correlation Analysis [6] computes the difference of both images pixel by pixel. The maximum difference corresponds to the most probable displacement, and generates a corresponding vector in each interrogation area of the image.

Adaptive PIV algorithm [8] was applied to both images, where the areas with no displacement (body area) was removed. The resultant displacement field is shown in figure 3. One can easily notice that the greater displacements are concentrated in the regions where expansion and shock waves appear (greater density gradients).
Once the displacement field is obtained, we have all the information required to integrate equation 2. In order to restrict our analysis to the flow area of interest, we crop the domain as demonstrated in figure 4.

Equation 2 is then integrated over the cropped domain using the five point finite difference method with successive over relaxation [2]. Dirichilet and Neumann boundary conditions are applied to the edges parallel and perpendicular to the flow respectively [1] as shown in figure 5. The Dirichilet boundaries were set to be equal the undisturbed flow refractive index ($n = n_0$) and the Neumann boundaries were set to zero. Once the refractive index field was obtained, density was calculated using the Gladstone-Dale equation (1).

A Computational Fluid Dynamics (CFD) simulation was also performed using the commercial code Ansys Fluent in order to check density values feasibility. Fluent is a Pressure based solver, where the governing equations are solved sequentially (segregated from one another).

The simulation domain consisted in a 2-D wedge that represents the cross section of the conical shaped body under analysis. The simulation domain and the mesh are represented in figure 6 and 7. A total of 26400 elements were used. The boundary conditions were set as follows:
• Farfield of Mach 0.9 on the right, top and left edges.
• Physical walls on the diagonal edge and the adjacent to it.
• Symmetry line ahead of the conical body.

![Simulation domain](image1.png)

**Figure 6.** Simulation domain.

![Simulation Mesh](image2.png)

**Figure 7.** Simulation Mesh.

### 4. Results and Discussion

In this section, the outputs obtained from the BOS algorithm are discussed and compared to CFD results in both qualitative and quantitative terms. The density contours obtained by CFD are displayed in figure 8. In this figure it is possible to notice the presence of characteristic elements of transonic flows such as shock and expansion waves.

![Density field - CFD](image3.png)

**Figure 8.** Density field - CFD

However, a comparative analysis is only possible if the two flow domains of BOS and CFD are the same. In order to do so, the CFD is cropped identically to the BOS displacement field as shown in figures 9 and 10.

Once the domains are equal, they can be compared as shown in figure 11. Although the CFD contours are smoother, the similarity between CFD and BOS is clear in terms of location of flow gradients. The location of density decrease is similar in both cases as show in blue in figure 11. Re-compression of the flow is also evident in both plots on the left hand side of the blue part. Therefore, both plots demonstrate the qualitative similarity between BOS and CFD.

In order to compare both techniques in quantitative terms, lines were traced on both domains at different locations in the $y$ direction. Density was acquired across those lines for both domains, and the values were plotted in figure 12. The density lines show good similarity between BOS
and CFD results, as well as almost the same behavior in terms of increase and decrease of density.

Table 1 compares the maximum, minimum and average values extracted from both domains, as well as the difference between them. The average and maximum values are within a small error range when compared to the minimum values. This happens because of wall effects that considerably affect CFD results. Comparing the numerical values shows that the BOS outputs match the CFD results within an acceptable error range. This technique is therefore capable of not only displaying qualitative characteristics of the flow, but also quantitatively calculating density values. BOS is indeed a proof that flow visualization techniques are powerful tools that can measure flow properties.

**Figure 9.** CFD domain cropped.  
**Figure 10.** BOS region of interest.

**Figure 11.** Density field comparison: (a) Density field - BOS. (b) Density field - CFD
Figure 12. Density lines at different $y$ locations: (a) $y = 0.2$. (b) $y = 0.4$. (c) $y = 0.6$. (d) $y = 0.8$.

Table 1. Density values comparison.

|                | Density (BOS) | Density (CFD) | Difference |
|----------------|---------------|---------------|------------|
| Maximum Value  | 1.89          | 1.76          | 7.60 %     |
| Minimum Value  | 1.26          | 1.06          | 18.4 %     |
| Average Value  | 1.66          | 1.60          | 3.73 %     |

5. Conclusions
Flow visualization techniques have greatly progressed from methods of qualitative description to methods capable of measuring flow properties.

The results demonstrated by figures 11 and 12 show that the BOS technique was able to match CFD results within an acceptable error range in both qualitative and quantitative terms. Background Oriented Schlieren together with the PIV Correlation technique are capable of not only providing qualitative information about the flow, but also compute the density flow-field within a small error range. This technique also proved to be very convenient for the analysis of transonic flows, and can be further applied to more complex types of flow. The simplicity of the BOS setup makes it attractive to use for comparison of the CFD results with Wind Tunnel experimental results for verification of the numerical simulations results.
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