We study one-loop corrections to the two-point correlation function of tensor perturbations in primordial cosmology induced by massless spectator matter fields. Using the Schwinger-Keldysh formalism in cosmological perturbation theory, we employ dimensional regularization and cutoff regularization to compute the finite quantum corrections at one-loop arising from massless isocurvature fields of various spins $< 2$ which are freely propagating on the FRW spacetime. For all cases, we find a logarithmic running of the form $C \frac{H^4}{\mu^4} \log \left( \frac{H}{\mu} \right)$ where $C$ is a negative constant, $H$ is the Hubble parameter at horizon exit and $\mu$ is the renormalization scale.
1 Introduction

The Schwinger-Keldysh formalism has been the underlying quantum field theoretic framework for computing correlation functions in cosmological perturbation theory, many crucial aspects of which were presented in the seminal paper by Weinberg in [1]. Essential cosmological observables such as the primordial spectra and bispectra are derived within the validity of this formalism which carries with it the prescription to compute loop effects for these correlation functions. In this paper, we will employ this formalism to compute quantum corrections at one-loop to the primordial tensor spectrum which arise from massless spectator/isocurvature fields freely propagating on the cosmological spacetime.

Loop corrections to the inflaton two-point function have been considered in several works after the appearance of [1]. Among the most principal results is that of Senatore and Zaldarriaga in
where they showed how the one-loop correction to the scalar spectrum is consistent with scale invariance, being of the form
\[
\langle \zeta_k^2 \rangle_{1\text{-loop}} \sim \frac{\beta}{k^3} \log \left( \frac{H}{\mu} \right) = \frac{\beta}{k^3} \log \left( \frac{k}{a(\tau_k)\mu} \right), \tag{1.1}
\]
where \( H = k/a(\tau_k) \) is the Hubble scale at horizon exit, \( \mu \) is the renormalization scale, \( a(\tau) \) is the scale factor of the FRW background, and \( \beta \) is a constant that depends on the type of matter field that couples to the inflaton - this was computed in [1], [3] and [4] for the cases of minimally and conformally coupled scalar fields, Dirac fermion and abelian gauge field. Although there has been a sizable amount of literature discussing loop corrections to cosmological correlation functions (see for example [5] for a nice review), much less attention has been devoted specifically to loop corrections to the tensor spectrum. In [6], a generalization of [3] was done to compute quantum correction at one-loop level to the tensor spectrum due to free massive and massless Dirac fermions and it led to a result similar to (1.1) with \( \beta \) being more technically involved to compute and smaller by a factor of \( H^2/M_p^2 \). There was also a similar attempt to compute the one-loop correction due to scalars in [7].

An appropriate regularization procedure for making sense of the loop integrals is required to extract the finite one-loop correction, and we found that the methods first presented in [2] were not quite accurately implemented in the papers discussing loop corrections to the scalar and tensor spectra. For example, in an earlier work in [4], we revisited the calculation for the scalar spectrum and corrected the factors of \( \beta \) in (1.1) as presented in [3]. Loop corrections also arise in the computation of potentially discoverable observables like non-gaussianity and in particular they can harbor signatures of matter fields dynamically present during inflation. Such calculations rely on the correct implementation of the Schwinger-Keldysh formalism and thus there is good motivation for understanding how to regularize divergent loop integrals correctly. In this paper, we will use both dimensional regularization and cutoff regularization. For the former, we furnish some details (crucial for concrete computations) missing (or implied) in both [1] and [2] which can be relevant for other loop computations within the general context of cosmological perturbation theory. This was first done for a related context (loop corrections to the scalar spectrum) in [4].

In this paper, we consider massless isocurvature fields minimally coupled to the FRW background, specifically the real scalar, Dirac fermions, abelian gauge field and the spin-3/2 gravitino field. We find that contrary to S-matrix elements in the standard QFT setting where loop corrections due to bosons and fermions could have opposite signs, the one-loop corrections due to fields of various spins are identically of the negative sign and take the following form in momentum space
\[
\langle h_{mn} h_{mn} \rangle_{1\text{-loop}} = \frac{C}{q^2 M_p^4} H^4 \log \left( \frac{H}{\mu} \right), \quad H = \frac{q}{a(\tau_q)}, \tag{1.2}
\]
where \( h_{mn} \) is the tensor perturbation gauge-fixed to be transverse and traceless, and \( C \) is a negative constant. Although we do not perform any resummation of higher loop effects in this work, we note that if we assume that they resum appropriately (via for example dynamical renormalization group [8]), our one-loop result naively suggests that these quantum corrections on their own lead to a small red-tilt of the tensor spectrum.

The outline of our paper is as follows: in Section 2 we provide the general scheme of the one-loop computation including the two regularization methods and some details concerning the graviton
4-point function which are universally relevant for all the matter fields considered. This is followed by Section 3 where we present some technical details specific to each type of matter field and the one-loop correction constant $C$ in (1.2) for each. In particular, we furnish some details concerning the massless gravitino which should be relevant for future analysis of gravitino loops in related contexts (such as the supersymmetric EFT of inflation presented in [9]). In Section 4, we briefly explain why all seagull vertices in the interaction Hamiltonians do not contribute to the one-loop logarithmic term in (1.2) for all the cases considered. Finally, in Section 5 we conclude with a summary of results and outlook.

2 General aspects of the one-loop computation

2.1 Schwinger-Keldysh formalism

In the Schwinger-Keldysh (or ‘in-in’) formalism\(^1\) we compute the two-point correlation function of the tensor perturbation $\langle h_{mn}(\vec{x}, \tau) h_{mn}(\vec{x}', \tau) \rangle$ evaluated at some common late time $\tau$ in the interaction picture, with the prescription

$$\langle \Omega | h_{mn}(\vec{x}, \tau) h_{mn}(\vec{x}', \tau) | \Omega \rangle = \langle 0 | \left[ T \exp \left( i \int_{-\infty}^{\tau} dt H_{\text{int}}(t) \right) \right] h_{mn}(\vec{x}, \tau) h_{mn}(\vec{x}', \tau) \left[ T \exp \left( -i \int_{-\infty}^{\tau} dt H_{\text{int}}(t) \right) \right] | 0 \rangle,$$

where the interaction Hamiltonian $H_{\text{int}}(t)$ is obtained by first carrying out a perturbation of the background metric in the Lagrangian, collecting all terms up to quadratic order and then performing the standard Legendre transform of each dynamical field. Henceforth, we take $\tau \approx 0$.

In (2.1), the vacuum state of the free theory $|0\rangle$ is obtained after projecting on the interacting vacuum state $|\Omega\rangle$ with an $i\epsilon$ prescription, with the infinities analytically continued as

$$\infty_{\pm} = \infty (1 \pm i\epsilon), \quad (2.2)$$

with $\epsilon$ being a real and positive regulator. In the interaction picture, the field operators evolve via the free Hamiltonian and thus they are expanded in terms of modes which are solutions to the Mukhanov free field equations. The vacuum expectation value is then computed in perturbation theory by expanding (2.1) to the required order. In this work, we focus on one-loop corrections to the primordial spectrum which, for the theory that we consider - Einstein gravity coupled to various matter fields - arise from terms in (2.1) up to and including the following second-order terms

$$\langle h_{mn}(\vec{x}) h_{mn}(\vec{x}') \rangle = -2 \text{Re} \left( \int_{-\infty}^{0} d\tau_2 \int_{-\infty}^{\tau_2} d\tau_1 \langle 0 | H_1 H_2 h_{mn}(\vec{x}) h_{mn}(\vec{x}') | 0 \rangle \right)$$

$$+ \int_{-\infty}^{0} d\tau_1 \int_{-\infty}^{0} d\tau_2 \langle 0 | H_1 h_{mn}(\vec{x}) h_{mn}(\vec{x}') H_2 | 0 \rangle \quad (2.3)$$

\(^1\)See for example [10] and [11] for a good review of this topic.
where \( H_{1,2} = \int d^3 x_{1,2} H_{\text{int}}(\tau_{1,2}, \vec{x}_{1,2}) \). As we shall see later in Section 4, the first-order terms of the form

\[
-2 \text{Im} \left( \int_{-\infty}^{0} d\tau \langle H_{\text{int}}(\tau) h_{mn}(\vec{x}, 0) h_{mn} \rangle \right)
\]

do not contribute to the one-loop logarithmic correction term for the matter fields that we consider. Diagrammatically, we can represent the second- and first-order terms in Figure 1a and 1b respectively.

We work in the spatially flat gauge, with the tensor perturbations defined as

\[
g_{ij} = b_{ij} + \gamma_{ij}, \quad b_{ij} = a^2 \delta_{ij}, \quad \gamma_{ij} = a^2 h_{ij},
\]

with \( h_{ij} \) being transverse and traceless. In this paper, we work in the conformal chart of the FRW geometry with vierbein \( e^\mu_a = \frac{1}{a(\tau)} \delta^\mu_a \). Relative to the primordial scalar spectrum, the tensor spectrum is smaller by a factor of the slow-roll parameter \( \epsilon \) and in this paper, we will keep to the lowest order in the slow-roll expansion. For example, the graviton and scalar mode wavefunctions are solutions of the relevant field equations defined on planar de Sitter spacetime.

In evaluating the correlation functions, we note that the terms which contribute to the one-loop quantum correction arise from terms in \( H_{\text{int}} \) which are linear in \( h_{mn} \). Thus, in (2.3), each correlation function factorizes into a product of a 4-point function of matter fields (and their derivatives) and a 4-point function of the graviton field \( h_{mn} \). There are two time integral contours in (2.3) which contain the same matter field 4-point function but slightly different graviton 4-point functions. The main technicality involved in the computation lies in the simplification of the index contractions between these two 4-point functions which will be elaborated in the subsequent sections.

### 2.2 Graviton 4-point function

The graviton field can be expanded in terms of bosonic oscillators \( \hat{a}_{q,\lambda} \), mode wavefunctions \( h_q(\tau) \) and rank-two polarization tensors \( \epsilon_{ij} \) as follows (\( q \equiv |\vec{q}| \))

\[
h_{ij} = \int d^3 q \sum_\lambda e^{i\vec{q} \cdot \vec{x}} \epsilon_{ij}(\vec{q}, \lambda) \hat{a}_{q,\lambda} h_q(\tau) + \text{c.c.}, \quad h_q(\tau) = \frac{\sqrt{16\pi G}}{(2\pi)^{3/2}} h_{3/2}^2 (1 + i q \tau) e^{-i q \tau},
\]

Figure 1: For the matter fields we consider in this work, all interaction Hamiltonians that give rise to the one-loop correction term are of the form diagrammatically represented by (a) whereas ‘seagull’ vertices of the form (b) do not contribute as we explain in Section 4.
with
\[
[\hat{a}_{\tilde{q}_{\lambda}}, \hat{a}^*_{\tilde{q}'_{\lambda'}}] = \delta^3(\tilde{q} - \tilde{q}')\delta_{\lambda\lambda'},
\]
and the polarization tensor satisfying (see for example [12])
\[
\sum_{\lambda} \epsilon_{ij}^* \epsilon_{kl} = \delta_{ik}\delta_{jl} + \delta_{il}\delta_{jk} - \delta_{ij}\delta_{kl} + \delta_{ij}\delta_{kl} + \delta_{kl}\delta_{ij} - \delta_{ij}\delta_{kl} - \delta_{jl}\delta_{iq} - \delta_{il}\delta_{jq} + \delta_{jl}\delta_{iq} + \delta_{il}\delta_{jq}, \tag{2.6}
\]
\[
\sum_{\lambda,\lambda'} \epsilon_{kl}(\tilde{q}, \lambda) \epsilon_{mn}^*(\tilde{q}, \lambda') \epsilon_{ij}^*(\tilde{q}', \lambda') \epsilon_{mn}(\tilde{q}', \lambda') = 2 (\delta_{ki}\delta_{lj} - \delta_{kl}\delta_{ij} + \delta_{ij}\delta_{kl} + \delta_{kl}\delta_{ij} - \delta_{jl}\delta_{iq} - \delta_{il}\delta_{jq} + \delta_{jl}\delta_{iq} + \delta_{il}\delta_{jq}) + 2 [\delta_{kij}\delta_{li} - \delta_{klj}\delta_{ji} + \delta_{lij}\delta_{jk} - \delta_{kij}\delta_{lj}] . \tag{2.7}
\]
For subsequent calculations, it is rather useful to note the following properties: (i) $\epsilon_{mn}^* (-\tilde{q}, \lambda) = \epsilon_{mn}(\tilde{q}, \lambda)$, (ii) (2.7) is invariant under the exchanges $i \leftrightarrow j$, $k \leftrightarrow l$ and $\{k \leftrightarrow i, l \leftrightarrow j\}$. Using the above mode expansion, it is straightforward to compute the graviton 4-point functions, leaving them in terms of 6D virtual momenta integrals. After summing up two Wick contractions in each correlation function, we find
\[
\langle 0 | h_{kl}(x_1) h_{ij}(x_2) h_{mn}(\tilde{x}, 0) h_{mn}(\tilde{x}', 0) | 0 \rangle = \frac{8}{(2\pi)^6} \frac{H^4}{M_p^4} \int d^3 d^3 s' \ e^{i\tilde{s}(\tilde{x} - \tilde{x}') e^{i\tilde{s}'(\tilde{x} - \tilde{x}'')}} e^{-is\tau_1 - is'\tau_2} \times \sum_{\lambda,\lambda'} \epsilon_{kl}(\tilde{s}, \lambda) \epsilon_{mn}^*(\tilde{s}, \lambda') \epsilon_{ij}(\tilde{s}', \lambda') \epsilon_{mn}(\tilde{s}', \lambda'), \tag{2.8}
\]
\[
\langle 0 | h_{kl}(x_1) h_{mn}(\tilde{x}, 0) h_{mn}(\tilde{x}', 0) h_{ij}(x_2) | 0 \rangle = \frac{8}{(2\pi)^6} \frac{H^4}{M_p^4} \int d^3 d^3 s' \ e^{i\tilde{s}(\tilde{x} - \tilde{x}') e^{i\tilde{s}'(\tilde{x} - \tilde{x}'')}} e^{-is\tau_1 + is'\tau_2} \times \sum_{\lambda,\lambda'} \epsilon_{kl}(\tilde{s}, \lambda) \epsilon_{mn}^*(\tilde{s}, \lambda') \epsilon_{ij}(\tilde{s}', \lambda') \epsilon_{mn}(\tilde{s}', \lambda'). \tag{2.9}
\]

### 2.3 A schematic outline of the one-loop computation

In the following, we present the schematic outline of our one-loop computation in particular the order of integrations in $\int d^3 x_1 d^3 x_2 \langle 0 | H_{int}(x_1) H_{int}(x_2) h_{mn}(\tilde{x}, 0) h_{mn}(\tilde{x}', 0) | 0 \rangle$ and $\int d^3 x_1 d^3 x_2 \langle 0 | H_{int}(x_1) h_{mn}(\tilde{x}, 0) h_{mn}(\tilde{x}', 0) H_{int}(x_2) | 0 \rangle$. The full one-loop result is eventually obtained after performing the time integral contours following (2.3) and performing appropriate regularizations of the momenta integrals.

(i) Each matter field 4-point function can be expressed in terms of a 6D virtual momenta integral in the form
\[
\langle G^{kl}(x_1) G^{ij}(x_2) \rangle = \int d^3 p_1 d^3 p_2 e^{i(p_1 + p_2) \cdot (\tilde{x} - \tilde{x}')} G^{klij}(p_1, p_2, \tau_1, \tau_2),
\]
where $G^{klij}$ is a function of momenta and time that depends on the specific Hamiltonian and the mode wavefunctions of the matter field. It is a sum of two Wick contractions.
Taking into account the phase factor $e^{i\vec{x}_1 \cdot \vec{\epsilon}} e^{i\vec{x}_2 \cdot \vec{\epsilon}'}$ in the graviton 4-point functions in (2.9), we first integrate over $\vec{x}_{1,2}$ to obtain the delta functions

$$(2\pi)^6 \delta^3(\vec{s} + \vec{p}_1 + \vec{p}_2) \delta^3(\vec{s}' + \vec{p}_1 + \vec{p}_2) \delta^3(\vec{s}' - \vec{p}_1 - \vec{p}_2)$$

together with a phase factor $e^{i(\vec{p}_1 + \vec{p}_2) \cdot (\vec{x} - \vec{x}')}$. 

(ii) Then we integrate over $\vec{s}, \vec{s}'$ in the graviton 4-point function using the above delta functions. The integrand is now simply a function of virtual momenta $\vec{p}_1, \vec{p}_2$.

(iii) Finally, we perform a Fourier transform $\int d^3 x e^{i\vec{q} \cdot (\vec{x} - \vec{s})} e^{i(\vec{p}_1 + \vec{p}_2) \cdot (\vec{x} - \vec{s}')} = (2\pi)^3 \delta^3(\vec{q} + \vec{p}_1 + \vec{p}_2)$, where $\vec{q}$ is the external momentum.

This leads to the eventual integral measure $\int d^3 p_1 \int d^3 p_2 \delta^3(\vec{q} + \vec{p}_1 + \vec{p}_2)$. For computational convenience, we can write this as

$$\int d^3 p_1 \int d^3 p_2 \delta^3(\vec{q} + \vec{p}_1 + \vec{p}_2) = \frac{2\pi}{q} \int_0^\infty dp_1 \int_{|p_1| - q}^{p_1 + q} dp_2 \, p_1 p_2$$

We will find that these integral diverge yet a suitable regularization parametrized by $H$ - the Hubble parameter at horizon exit and $\mu$ - the renormalization constant. This enables us to extract the finite one-loop correction which can be eventually expressed in the form

$$I = \frac{1}{(2\pi)^2} \left( \frac{H}{M_p} \right)^4 \frac{1}{q^3} F\left( \frac{H}{\mu} \right)$$

for some function $F\left( \frac{H}{\mu} \right)$.

In the following section, we will perform the above computations for various matter fields. Since the procedure is identical for all, we introduce a few other symbols to organize our presentation of results. Denoting the one-loop correction term by $I_L$, we distinguish between the two time integral contours as follows.

$$I_L = \frac{2\pi}{q} \int_0^\infty dp_1 \int_{|p_1| - q}^{p_1 + q} dp_2 \, p_1 p_2 \, G(p_1, p_2, q) \left[ -2\text{Re}(F_1(p_1, p_2, q)) + F_2(p_1, p_2, q) \right]$$

$$= \frac{1}{(2\pi)^2} \left( \frac{H}{M_p} \right)^4 \frac{1}{q^3} (I_1 + I_2)$$

where $F_1(p_1, p_2, q), F_2(p_1, p_2, q)$ are the time integrals involving the time-dependent functions that appear in the fields’ mode wavefunctions, and $G(p_1, p_2, q)$ capture all other functions and constants obtained after contracting the spacetime indices in the product

$$G^{kl\bar{i}j}(\vec{p}_1, \vec{p}_2, \tau_1, \tau_2) \left( \sum_{\lambda, \lambda'} \epsilon_k(\vec{q}, \lambda) \epsilon^*_{m'n}(\vec{q}, \lambda) \epsilon^*_{ij}(\vec{q}, \lambda') \epsilon_{m'n}(\vec{q}, \lambda') \right).$$

Some details of the computation will be presented for each matter field. The momenta integrals yield divergent results and must be regularized, after which

$$I_1 + I_2 \to F(H/\mu).$$

Specifically, both cutoff and dimensional regularization leads to $F(H/\mu)$ being some constant multiplied to $\log(H/\mu)$. 

7
2.4 Regularization methods

In the following, we explain the methods used to regularize the divergent momenta integrals in (2.12). For all matter fields we consider, we employ both dimensional regularization and cutoff regularization and in all cases, we obtain identical finite one-loop correction terms.

2.4.1 Dimensional regularization

For dimensional regularization, we begin by writing the spatial dimensionality as \( d = 3 + \delta \), and noting that the angular integration should generalize as

\[
\int d\Omega_{2+\delta} = \int_0^{\pi} d\Theta \sin^{1+\delta} \Theta \times \text{Vol}(S_{1+\delta}) = \int_0^{\pi} d\Theta \sin^{1+\delta} \Theta \times \frac{(2\pi)^{\delta/2}}{\Gamma(1 + \frac{\delta}{2})}.
\]  

The integration measure then reads

\[
\int d^{3+\delta}p_1 \int d^{3+\delta}p_2 \delta^{3+\delta}(p_1 + p_2 + q) = 2\pi q^{3+\delta} \left( \frac{\pi^{\delta/2}}{\Gamma(1 + \frac{\delta}{2})} \right) \int_0^{\infty} dp_1 p_1^{\delta} \int_{|p_1-1|}^{p_1+1} dp_2 \sin^{\delta} \Theta
\]  

where we have expressed the integrand variables in units of \( q \) and

\[
\sin^{\delta} \Theta = \left[ 1 - \left( \frac{(p_2)^2 - p_1^2 - 1}{2p_1} \right)^{\frac{\delta}{2}} \right]^{\frac{\delta}{2}}.
\]

There is also the spacetime integral measure \( \int d^4 x_{1,2} a^4(\tau_{1,2}) \) which is lifted to be \( \int d^{4+\delta} x_{1,2} a^{4+\delta}(\tau_{1,2}) \). Up to first-order in \( \delta \),

\[ a^{\delta}(\tau) = 1 - \delta \text{Log}(-H\tau) + O(\delta^2), \]

and we note that we have a product of two such measures in the correlation function that involves a product of two interaction Hamiltonians. The integrand of (2.12) involves mode wavefunctions and those running in the loop can be analytically continued to the corresponding solutions in higher dimensions. In this paper, we consider minimally coupled scalars, Dirac fermions, abelian gauge fields and the spin-\( \frac{3}{2} \) gravitino field. Below, we present their higher-dimensional modes which are crucial for dimensional regularization.

For the scalar field, the analytic continuation of the scalar mode wavefunctions

\[
\chi_k(\tau) \sim \frac{H^{1+\delta/2}(\frac{1}{2}k\tau)^{\left(\frac{3+\delta}{2}\right)}}{k^{\left(\frac{3+\delta}{2}\right)}} H^{(1)}_{\left(\frac{3+\delta}{2}\right)}(-k\tau) \]

where \( H^{(1)}_{\left(\frac{3+\delta}{2}\right)}(-k\tau) \) is the Hankel function of the first kind. For the minimally coupled scalars, the terms in \( H_{\text{int}} \) which give non-vanishing contribution to the one-loop correction are quadratic in the fields, so expanding in the parameter \( \delta \) yields

\[ 4 \times \frac{1}{2} \delta \text{log}(-H\tau). \]

The form of (2.16) is, up to some constants, identical for the graviton mode wavefunctions \( h_k(\tau) \) in (2.5). Since there is one in each of \( H_{\text{int}} \), together with the scalar mode wavefunctions, we have altogether a factor of

\[ 6 \times \frac{1}{2} \delta \text{log}(-H\tau). \]
This turns out to be identical for the other matter fields that we consider in this paper, each of them having a interaction Hamiltonian that is linear in $h_{kl}$ and quadratic in the fields (seagull vertices as depicted in Figure 1 do not contribute and are treated in Section 4).

For the fermions, in $d + 1$-dimensional de Sitter, the Dirac equation turns out to read

$$i \left( \gamma^\mu \partial_\mu - \frac{d}{2\tau} \gamma^0 \right) \Psi = 0,$$

which admits the $d-$dimensional spinor wavefunction

$$\Psi(\vec{x}, t) = (-H\tau)^{\frac{d}{2}} \int d^d k \sum_s e^{i\vec{k} \cdot \vec{x}} \left[ U_{\vec{k},s}(t) a_{\vec{k},s} + V_{-\vec{k},s}(t) \beta^\dagger_{-\vec{k},s} \right],$$

where $U_{\vec{k},s}, V_{\vec{k},s}$ are spinors with definite conformal momenta in $d + 1$-dimensional Minkowski spacetime. Expanding $d = 3 - \delta$, one obtains a correction factor of $\frac{1}{2} \delta \log(-H\tau)$, similar to what arises from the analytic continuation of scalar mode wavefunctions.

For the gauge fields, we checked that the free $d-$dimensional Maxwell equations in conformal coordinates imply that the modes $A_k(\tau)$ satisfy

$$\frac{d^2 A_k}{d\tau^2} + k^2 A_k - (d - 3) \frac{1}{\tau} \frac{d A_k}{d\tau} = 0.$$ (2.19)

In the case of $d = 3$, we obtain plane waves which can be normalized with Bunch-Davies condition. For generic $d = 3 + \delta$, we find the solution

$$A_k \sim (-H\tau)^{\frac{1+\delta}{\frac{1}{2}}} H^{(1)}_{\frac{1}{2},\delta}(-k\tau),$$

where $H^{(1)}_{\frac{1}{2},\delta}(-k\tau)$ denotes the Hankel function of the first kind. Once again expanding $d = 3 + \delta$, one obtains the same correction factor of $\frac{1}{2} \delta \log(-H\tau)$.

For the spin-$\frac{3}{2}$ gravitino field, as we shall derive in detail later, the interaction Hamiltonian reads

$$H_{\text{int}}(\vec{x}, \tau) = a(\tau) h^{ik} \left[ \bar{\psi}_i \gamma^r \partial_r \psi_k + \frac{1}{2} \bar{\psi}_j \eta^{jm} \gamma_k \partial_i \psi_m + \bar{\psi}_j \gamma_i \partial^j \psi_k \right],$$

where it can be shown that $\psi_k = \frac{1}{\sqrt{a}} \Psi_{\text{Dirac}}$ with $\Psi_{\text{Dirac}}$ satisfying the ordinary Dirac equation in four dimensions. The analytic continuation of each gravitino field in $H_{\text{int}}$ to higher dimensions thus induces the same factor of $\frac{1}{2} \delta \log(-H\tau)$.

To summarize, for all matter fields, the analytic continuation of the mode wavefunctions as well as the scale factor in each integration of $H_{\text{int}}$ over all spacetime implies the following term that is first-order in the regularization parameter $\delta$:

$$\left( \frac{4 + 2}{2} - 2 \right) \delta \log(-H\tau) = \delta \log(-H\tau)$$

Although we shall see later that seagull vertices do not contribute to the one-loop correction, we note in passing that a similar calculation gives $\left( \frac{2+2}{2} - 1 \right) \delta \log(-H\tau) = \delta \log(-H\tau)$ which is identical to (2.22). Finally, taking into account the $q^3$ term in (2.14) (the other $\delta$-dependent terms
in (2.14) will only yield unimportant constants), we can derive the expression for the finite one-loop logarithmic correction in dimensional regularization which is the finite part of
\[
I_L \sim \frac{1}{(2\pi)^2} \left( \frac{H}{M_p} \right)^4 \frac{1}{q^3} \left( 1 + \delta \log \left( \frac{q}{\mu} \right) \right) \left( -\frac{J_1 + J_2}{\delta} + \ldots \right) \left( 1 + \delta \log \left( \frac{H}{q} \right) \right),
\]
where we have also invoked the fact that the time-integrals are dominated by the time of horizon exit, and \( J_{1,2} \) denote the residues of the momenta integral in (2.12) in each of the time integral contour after scaling all virtual momenta in units of \( q \). To pick up this residue, we find it convenient to perform a coordinate transformation as follows.

\[
P = p_1^\delta.
\]

In (2.12), we first integrate over \( p_2 \) without encountering any divergences. Then we perform the coordinate transformation in (2.24). The integral from \( p_1 = 0 \) to \( p_1 = q \) does not contribute to the logarithmic correction so we focus on the remaining domain of integration. Scaling all virtual momenta in units of \( q \), the one-loop correction is the finite part of the following expression
\[
I_L \sim -\frac{1}{(2\pi)^2} \left( \frac{H}{M_p} \right)^4 \frac{1}{q^3} \int_1^\infty dP \frac{1}{\delta} \left[ J_1(P,\delta) + J_2(P,\delta) \right] \left( 1 + \delta \log \left( \frac{H}{\mu} \right) \right)
\]
where \( J_1(P,\delta), J_2(P,\delta) \) denote the two functions arising from each time integral contour in (2.12) and \( J_{1,2} \) in (2.23) are then simply the constant terms that can be read off from the Taylor expansion of \( J_1(P,\delta) + J_2(P,\delta) \) in the small variable \( P^{-\frac{1}{2}} \). For all matter fields we consider in this paper, the one-loop correction is of this form. Finally, we note that the other \( \delta \)-dependent terms in (2.14), apart from \( q^\delta \), do not contribute to this logarithmic running apart from unimportant numerical constants. Explicitly, the factor
\[
\frac{\pi^{\delta/2}}{\Gamma(1+\frac{\delta}{2})} \approx 1 + \delta \left( \frac{\gamma}{2} + \frac{1}{2} \log(\pi) \right) + \ldots,
\]
whereas expanding the \( \sin^\delta \Theta \) term introduces a term \( \log \left( 1 - \frac{(p_2^2-p_1^2)^2}{4p_2^4} \right) \) in the integrand, which gives rise to an unimportant numerical constant. In the subsequent sections, we present some details of \( J_1(P,\delta), J_2(P,\delta) \) for each matter field and thus also the one-loop logarithmic correction.

### 2.4.2 A covariant cutoff regularization

Another regularization method that we adopt to derive our result is that of cutoff regularization. This is generally a straightforward procedure apart from perhaps some subtleties that we will now explain. In the absence of any regularization, the momenta integrals in (2.12) diverge. Suppose we wish to place a certain momentum cutoff. As first explained in [2], this momentum cutoff defined for the comoving momentum carries the same scaling ambiguity as that of the scale factor. The physical cutoff should be defined in terms of proper length scales. Denoting \( \Lambda_{phy} \) as the ‘physical’ momentum cutoff, we can write
\[
\Lambda_{phy} = \frac{L}{a(\tau_q)},
\]

where
where we have used the scale factor at horizon-crossing, noting that the integrals are dominated by dynamics at horizon-crossing where $a(\tau_q) = q/H(\tau_q)$. This implies that we can write the cutoff as

$$\frac{L}{q} = \frac{\Lambda_{\text{phy}}}{H(\tau_q)}$$

leading to the effective replacement in (3.9)

$$\log \left( \frac{q}{\mu} \right) \rightarrow \log \left( \frac{H}{\mu} \right),$$

where $\mu$ is the renormalization scale. In the subsequent section, we will present the cutoff regularized results for each matter field and we’ll see that in every case, they match consistently with the result derived in dimensional regularization.

### 3 One-loop logarithmic corrections due to massless spectator fields

#### 3.1 Dirac Fermions

We consider massless Dirac fermions coupled to the inflationary background as described by the action

$$S_{\text{fermion}} = \int d^4x \sqrt{-g} \frac{i}{2} (\bar{\varphi} \Gamma^\mu D_\mu \varphi - D_\mu \bar{\varphi} \Gamma^\mu \varphi)$$

where $\Gamma^\mu = e^a_\mu \gamma^a$ are the Dirac matrices on the FRW spacetime with vierbein $e^a_\mu = 1/\sqrt{-g} \delta^a_\mu$, $\gamma^a$ are Dirac matrices on Minkowski spacetime, and $D_\mu = \partial_\mu + \Omega_\mu$ is the covariant derivative with spin connection $\Omega$. For the FRW background, $D_0 = \partial_0$, $D_k = \partial_k + H/2 \gamma_k \gamma^0$.

With the metric perturbation switched on, one can expand the Lagrangian to first-order in metric fluctuation $h_{ij}$ and after a Legendre transform, we obtain the interaction Hamiltonian

$$H = -\frac{i}{2} \int d^3x a^3(\tau) h_{ij} \left[ \bar{\varphi} \gamma^i \partial_j \varphi - \partial_i \bar{\varphi} \gamma^j \varphi \right].$$

The spinor fields can be expanded in terms of modes labelled by spin index $\lambda$

$$\varphi = \int d^3p e^{i\vec{p} \cdot \vec{x}} \sum_\lambda \left( a_{\vec{p},\lambda} U_{\vec{p},s}(\tau) + b^\dagger_{-\vec{p},\lambda} V_{\vec{p},s}(\tau) \right),$$

with the oscillator algebras being

$$\{a_{\vec{p},\lambda}, a^\dagger_{\vec{p}',\lambda'}\} = \delta^3(\vec{p} - \vec{p}') \delta_{\lambda\lambda'}, \quad \{b_{\vec{p},\lambda}, b^\dagger_{\vec{p}',\lambda'}\} = \delta^3(\vec{p} - \vec{p}') \delta_{\lambda\lambda'},$$

and the normalization condition

$$U_{\vec{p},s} U^\dagger_{\vec{p},s} = V_{\vec{p},s} V^\dagger_{\vec{p},s} = \frac{\gamma^\mu p_\mu}{2(2\pi)^3 a^3(\tau) p}.$$  

The spinor field 4-point function can be expressed as

$$\langle \bar{\varphi}(x_1) \gamma^i \partial_k \varphi(x_1) \bar{\varphi}(x_2) \gamma^j \partial_k \varphi(x_2) \rangle = \int d^3p d^3p' e^{i(\vec{p} + \vec{p}')(\vec{x}_1 - \vec{x}_2)} p^k p'^k e^{-i(p + p')(\tau_1 - \tau_2)} \sum_{s,s'} U_{\vec{p},s} \gamma^j V_{\vec{p}',s'} \gamma^i U_{\vec{p}',s'}.$$  

(3.5)
Simplifying further using
\[ \sum_{s,s'} U_{\vec{p},s} \gamma^j V_{\vec{p}',s'} \nabla_{\vec{p},s'} \gamma^j U_{\vec{p},s} = \sum_{s,s'} \text{Tr} \left( \gamma^j \left( \frac{1}{2(2\pi)^3 p_j} \right) \gamma^j \left( \frac{1}{2(2\pi)^3 p_j'} \right) \right) = \frac{1}{4(2\pi)^6} \text{Tr} \left( \gamma^j \gamma^j p_j' \right), \]
we can contract it with the graviton’s polarization tensors to obtain
\[ G(p_1, p_2, q) = - N (p_2^k - p_1^k)(p_1^j - p_2^j) \text{Tr}(\gamma^j p_1 \gamma^j p_2) \sum_{\lambda, \lambda'} \epsilon_{kl}(q, \lambda) \epsilon^*_{mn}(q, \lambda') \epsilon_{ij}(q, \lambda') \epsilon_{mn}(q, \lambda) \]
\[ = N (2p_1^j + q^j)(2p_1^j + q^j) \left[ \left( p_1 p_2 - \vec{p}_1 \cdot \vec{p}_2 \right) 4 \delta^{ij} + 4 (p_1^j p_2^j + p_1^j p_2^j) \right] \]
\[ \times 2 (\delta_{ki} \delta_{lj} - \delta_{kl} \delta_{ij}) \]
\[ = -N \frac{4}{q^2} \left( p_1^2 + p_2^2 - q^2 \right)^2 (p_1^2 - 4p_1^2 p_2^2 + 6p_2^2 p_1^2 - 4p_1^2 + p_2^2 - q^4) \]
where \( N = \frac{4}{(2\pi)^2 H^4 M_p^4} \) and we have invoked the useful identity \( \text{Tr}(\gamma^a \gamma^b \gamma^c \gamma^d) = 4 (\eta^{ab} \eta^{cd} - \eta^{ac} \eta^{bd} + \eta^{ad} \eta^{bc}) \)
to write
\[ \text{Tr} \left( \gamma^j \gamma^i p_j p^j \right) = pp' \text{Tr} \left( \gamma^i \gamma^j \gamma^0 \gamma^0 \right) + pp' \left( \eta^{0i} \eta^{jd} + \eta^{id} \eta^{0j} \right) \]
\[ = (pp' - \vec{p} \cdot \vec{p}') \gamma^{ij} + 4 (p^0 p^j + p^0 p^j). \]
We absorb all time-dependent functions into \( F_1(p_1, p_2, q), F_2(p_1, p_2, q) \) which represent the two time integration contours. In this case, it reads
\[ F_1(p_1, p_2, q) = \int_{-\infty}^{0} d\tau_2 \int_{-\infty}^{\tau_2} d\tau_1 e^{-i(p_1 + p_2)(\tau_1 - \tau_2)} (1 + iq \tau_1) (1 + iq \tau_2) e^{-iq(\tau_1 + \tau_2)} \]
\[ F_2(p_1, p_2, q) = \int_{-\infty}^{0} d\tau_1 \int_{-\infty}^{\tau_1} d\tau_2 e^{-i(p_1 + p_2)(\tau_1 - \tau_2)} (1 + iq \tau_1) (1 - iq \tau_2) e^{-iq(\tau_1 - \tau_2)} \]
After performing the time integrals, one can proceed to evaluating (3.12). Directly performing the momenta integration with a cutoff yields (\( \tilde{L} = L/q \))
\[ I_1 + I_2 = \frac{32}{5} \log(\tilde{L}) - \frac{32}{5} \left[ -1 + \tilde{L} \left( -5 + 2 \tilde{L}(-5 - 5 \tilde{L} + 2 \tilde{L}^3) \right) \right] \log(1 + \tilde{L}^{-1}) + \ldots - \frac{32}{5} \log(\frac{q}{\mu}) \]
where the ellipses refer to terms which are polynomial in \( \tilde{L} \). The logarithmic term yields the finite one-loop correction which is thus
\[ I_L = - \frac{H^4(\tau_c)}{(2\pi)^2 M_p^4 q^3} \times \frac{32}{5} \log \left( \frac{H}{\mu} \right). \]
We note that the first time integral contour does not contribute. This is mirrored in the alternative method of dimensional regularization. Following (2.25), we have from each time integration contour
\[ \frac{1}{5} \int_{1}^{\infty} dP J_1(P, \delta) = \frac{4}{5} \int_{1}^{\infty} dP \frac{16}{P^{-3/2}} - \frac{16}{5 P^{-3/2}} - \frac{208}{35 P^{-3/2}} + \frac{316}{105 P^{-1/2}} + \ldots \]
and
\[ \frac{1}{5} \int_{1}^{\infty} dP J_2(P, \delta) = \frac{4}{5} \int_{1}^{\infty} dP \frac{16}{5 P^{-3/2}} - \frac{316}{105 P^{-1/2}} + \frac{8}{5} - \frac{32 P^{-1/2}}{105} + \ldots \]
Combining both \( I_1 \) and \( I_2 \), we obtain the identical one-loop logarithmic correction (3.10) computed in cutoff regularization.
3.2 Minimally coupled scalars

We consider the minimally coupled scalar with action

$$S_{\text{scalar}} = \frac{1}{2} \int d^4x \sqrt{-g} g^\mu_\nu \partial_\mu \phi \partial_\nu \phi. \quad (3.13)$$

Switching on the metric perturbation and keeping to first-order, after performing a Legendre transform, we obtain the interaction Hamiltonian to read

$$H_{\text{int}} = \frac{1}{2} \int d^3x a^2(\tau) h^{ij} \partial_i \phi \partial_j \phi. \quad (3.14)$$

The scalar field can be expanded in terms of modes with Bunch-Davies initial condition as

$$\phi = \int d^3k \ e^{i\vec{k}\cdot\vec{x}} \left( \chi_k(\tau) a_k + \chi_k^*(\tau) a_k^\dagger \right) \quad (3.15)$$

with $[a_k^\dagger, a_{k'}] = \delta^3(\vec{k} - \vec{k'})$ and the modes being

$$\chi_k(\tau) = \frac{H}{\sqrt{2(2\pi)^3/2k^3/2}} e^{-ik\tau} (1 + ik\tau). \quad (3.16)$$

We compute the 4-point function of the scalar fields to be

$$\langle \partial_\kappa \phi \partial_\lambda \phi \partial_\mu \phi \partial_\nu \phi \rangle = \int d^3p_1 d^3p_2 e^{i(p_1 + p_2)(\vec{\tau}_1 - \vec{\tau}_2)} \left[ p_1^j p_2^j p_1^i p_2^i \right] \chi_{p_1}(\tau_1) \chi_{p_2}(\tau_2) \chi_{p_1}(\tau_1) \chi_{p_2}(\tau_2) \quad (3.17)$$

where we have excluded tadpole diagrams. Contracting it with the graviton’s polarization tensors yields

$$G(p_1, p_2, q) = \frac{\beta}{p_1^2 p_2^2} \left[ \left( p_1^2 - \frac{(\vec{p}_1 \cdot q)^2}{q^2} \right) \left( p_2^2 - \frac{(\vec{q} \cdot \vec{q})^2}{q^2} \right) \right]. \quad (3.18)$$

where $\beta = \frac{2}{(2\pi)^3} \left( \frac{H}{M_p} \right)^4 \frac{1}{q}$ and $\vec{p}_1 \cdot q = \frac{1}{2} (p_1^2 - p_2^2 - q^2)$. All time-dependent functions are captured in $F_1(p_1, p_2, q), F_2(p_1, p_2, q)$ which represent the two time integration contours. In this case,

$$F_1(p_1, p_2, q) = \int_{-\infty}^{0} d\tau_2 \int_{-\infty}^{\tau_2} d\tau_1 \left[ (1 + iq\tau_1)(1 + ip_1\tau_1)(1 + ip_2\tau_1)(1 + iq\tau_2)(1 - ip_1\tau_2)(1 - ip_2\tau_2) \right.$$

$$\times e^{-i(q(\tau_1 + \tau_2) + i(p_1 + p_2)(\tau_2 - \tau_1))},$$

$$F_2(p_1, p_2, q) = \int_{-\infty}^{0} d\tau_1 \int_{-\infty}^{0} d\tau_2 \left[ (1 + iq\tau_1)(1 - iq\tau_2)(1 + ip_1\tau_1)(1 + ip_2\tau_1)(1 - ip_1\tau_2)(1 - ip_2\tau_2) \right.$$

$$\times e^{i(q + p_1 + p_2)(\tau_2 - \tau_1)}. \quad (3.19)$$

After completing the time integrals, we performed the momenta integrals in (2.12) to obtain in cutoff regularization

$$I_1 + I_2 = \frac{78}{5} \log \bar{L} + 4 \left( \frac{39}{10} + \frac{8}{L} - 10\bar{L} - 12\bar{L}^3 + \frac{34\bar{L}^5}{5} \right) \log(1 + \bar{L}^{-1}) + \ldots, \quad (3.20)$$
where the ellipses are polynomials in \( \tilde{L} \). And thus, after covariantizing the cutoff, we obtain the one-loop correction to be

\[
I_t = -\frac{1}{2} \frac{H^4}{(2\pi)^2 q^3 M_p^2} \times \left( \frac{78}{5} \right) \log \left( \frac{H}{\mu} \right).
\] (3.21)

Contrary to the Dirac fermion case, this time we find that the second time integral contour does not contribute. (i.e. only \( I_1 \) in (2.12) contributes to (3.21). From dimensional regularization, we also obtain an identical finite one-loop logarithmic correction term. Following (2.25), each time integration contour gives

\[
\frac{1}{\delta} \int_1^\infty dP \lambda_1(P, \delta) = \frac{2}{\delta} \int_1^\infty dP \frac{54}{5P^{3/2}} - \frac{1093}{105P^{3/2}} + \frac{39}{5} \frac{5P^{1/2}}{3} + \ldots
\] (3.22)

and

\[
\frac{1}{\delta} \int_1^\infty dP \lambda_2(P, \delta) = \frac{2}{\delta} \int_1^\infty dP \frac{5P^{1/2}}{3} - \frac{81P^{3/2}}{28} + \ldots
\] (3.23)

Combining both \( I_1 \) and \( I_2 \), we obtain the finite term to be (3.21). The overall sign of the logarithmic correction is the same as that of fermions yet it arises from the different part of the in-in contour.

### 3.3 The abelian gauge field

We consider an abelian gauge field residing on the FRW background with action

\[
S_{\text{maxwell}} = -\int d^4x \sqrt{-g} \frac{1}{4} g^{\mu\alpha} g^{\rho\beta} F_{\alpha\beta} F_{\mu\nu}.
\] (3.24)

Keeping to first-order in metric perturbation, after performing a Legendre transform, we find the interaction Hamiltonian

\[
H_{\text{int}} = \frac{1}{2} \int d^3x \ h_{\tau\mu} F^{\mu\nu} F_{\mu\tau} \\
= -\frac{1}{2} \int d^3x \ h_{ik} \left( \partial_0 A^k \partial_0 A^i - \partial^k A^j \partial^j A^i \right) \equiv \frac{1}{2} \int d^3x \ h_{ik} G^{ki}.
\] (3.25)

The gauge field can be expanded in terms of modes as follows

\[
A_i(\vec{x}, \tau) = \int d^3q \sum_\lambda \ e^{i\vec{q}\cdot\vec{x}} \left( e_i(\vec{q}, \lambda) \ a_{\vec{q},\lambda} A_{\vec{q}}(\tau) + e_i^*(\vec{q}, \lambda) \ a_{-\vec{q},\lambda}^\dagger A_{\vec{q}}^*(\tau) \right), \quad A_0 = 0,
\] (3.26)

where \( \lambda \) labels the polarization mode with the polarization vectors satisfying

\[
\sum_\lambda e_i^*(\vec{q}, \lambda) e_j(\vec{q}, \lambda') = \delta_{ij} - \delta_{\vec{q}\vec{q}'},
\]

and the oscillators obeying the canonical relation \([a_{\vec{p},\lambda}, a_{\vec{p}',\lambda'}^\dagger] = \delta^{\vec{p}\vec{p}'} \delta_{\lambda\lambda'}\). Also, the mode wavefunctions are

\[
A_{\vec{q}}(\tau) = \frac{1}{(2\pi)^{3/2} \sqrt{2q}} e^{-iq\tau},
\] (3.27)
which solves Maxwell’s equations on a conformally flat spacetime. We have gauge-fixed the system which contains two (\(\lambda = \pm 1\)) physical propagating degrees of freedom. Substituting the mode expansion into the four-point function and omitting tadpole diagrams, we obtain after some algebra,

\[
\langle 0 \mid \mathcal{G}^{lk}(x_1)\mathcal{G}^{ji}(x_2)\langle 0 \rangle = \frac{1}{4} \int d^4 p_1 d^4 p_2 d^4 p_1' d^4 p_2' \ e^{i(\vec{p}_1 + \vec{p}_2' - \vec{x}_1 - \vec{x}_2)}
\]

\[
\left[ \delta^2(\vec{p}_2 + \vec{p}_1')\delta^2(\vec{\tilde{p}}_1 + \vec{\tilde{p}}_2') + \delta^2(\vec{\tilde{p}}_1 + \vec{\tilde{p}}_2')\delta^2(\vec{p}_2 + \vec{p}_1') \right]
\]

\[
\left( e_{p_1}^{l} e_{p_2}^{k} \hat{A}_{p_1}(\tau_1)\hat{A}_{p_2}(\tau_1) + p_{s}^{[l} e_{p_2}^{s} e_{p_2}^{k]} \hat{A}_{p_1}(\tau_1)\hat{A}_{p_2}(\tau_1) \right)
\]

\[
\times \left( e_{-p_1}^{s} e_{-p_2}^{s} \hat{A}_{p_1}'(\tau_2)\hat{A}_{p_2}'(\tau_2) + p_{m}^{[l} e_{-p_2}^{m} e_{-p_2}^{s} \hat{A}_{p_1}'(\tau_2)\hat{A}_{p_2}'(\tau_2) \right)
\]

(3.28)

where the delta functions arise from the VEV \(\langle 0 \mid a_{\vec{p}_1} a_{\vec{p}_2} a_{\vec{p}_1'} a_{\vec{p}_2'} \langle 0 \rangle\). It is convenient to express the virtual momenta in terms of unit vectors after which we are left with a remnant dimension-2 factor of \(p_{1} p_{2}\). The mode wavefunctions all yield a common phase factor \(e^{-i(\vec{p}_1 + \vec{p}_2)(\tau_1 - \tau_2)}\) and a constant factor of \(\frac{1}{4(2\pi)^{9}}\). Integrating over the primed momenta, we obtain

\[
\langle 0 \mid \mathcal{G}^{lk}(x_1)\mathcal{G}^{ji}(x_2)\langle 0 \rangle = \mathcal{N} \int d^4 p_1 d^4 p_2 e^{i(\vec{p}_1 + \vec{p}_2' - \vec{x}_1 - \vec{x}_2)} e^{-i(\vec{p}_1 + \vec{p}_2)(\tau_1 - \tau_2)} p_{1} p_{2} C^{lkji}(p_{1}, p_{2})
\]

(3.29)

where \(\mathcal{N} = \frac{1}{4(2\pi)^{9}}\) and \(C^{lkji}(p_{1}, p_{2})\) is a function of \(p_{1}, p_{2}\) that reads

\[
C^{lkji}(p_{1}, p_{2}) = P^{li}(\hat{p}_1) P^{kj}(\hat{p}_2) - P^{li}(\hat{\tilde{p}_1}) P^{kj}(\hat{\tilde{p}_2}) - P^{li}(\hat{\tilde{p}_1}) P^{kj}(\hat{\tilde{p}_2}) + P^{li}(\hat{\tilde{p}_1}) P^{kj}(\hat{\tilde{p}_2})
\]

+ \(p_{1}^{[s} P^{s]i}(\hat{\tilde{p}_1}) p_{2}^{[s} P^{s]j}(\hat{\tilde{p}_2})\).

(3.30)

In obtaining \(\mathcal{G}(p_{1}, p_{2}, q)\) in (2.12), we find the following identities useful:

\[
P^{li}(\hat{p}_1) P^{li}(\hat{p}_2) = 1 + (\hat{p}_1 \cdot \hat{q})^2,
\]

\[
P^{sli}(\hat{\tilde{p}_1}) P^{sli}(\hat{\tilde{p}_2}) = 1 - (\hat{p}_1 \cdot \hat{q})^2,
\]

\[
P^{jsi}(\hat{\tilde{p}_2}) P^{sii}(\hat{\tilde{p}_1}) = P^{ji}(\hat{\tilde{p}_2}) + \hat{p}_1 \left( \hat{p}_2 \cdot \hat{p}_1 - \hat{p}_1 \right).
\]

(3.31)

We find that contracting the spacetime indices with those of the graviton polarization tensors yields (below, we introduce the symbols \(S = \hat{p}_1 \cdot \hat{p}_2, R_{1} = \hat{p}_1 \cdot \hat{q}, R_{2} = \hat{p}_2 \cdot \hat{q}\) to simplify our notation)

\[
\mathcal{G}(p_{1}, p_{2}, q) = \mathcal{N} \left[ (1 + S^2)(1 + R_{1}^2)(1 + R_{2}^2) + (1 - R_{1}^2)(1 - R_{2}^2)(1 + S^2) \right]
\]

\[
+ 2S \left[ R_{1}^2 - 1 + (1 + S)(1 - R_{1}^2)^2 + (1 - R_{2}^2)[R_{1} R_{2} - S] \right]
\]

\[
-(1 - R_{1}^2)(1 + R_{1}^2 - (1 + R_{2})(1 - S^2) + 2(R_{2} - S R_{1})(S R_{2} - R_{1})
\]

\[
-(2S(1 - S^2) + 2(1 - S^2)(1 - R_{1}^2)(1 + R_{2}^2) - 4(1 + S)(R_{1}^2 - 1)S(1 + R_{2}^2)
\]

\[
-4(1 - R_{2}^2)(R_{2} - S R_{1})S R_{1} - 2S(1 + R_{1}^2)(1 + R_{2}^2) + 4(R_{1}^2 - 1)(1 + S)(1 + R_{2}^2)
\]

\[
-4R_{1}^2(R_{1}^2 - 1) - 2(1 - R_{1}^2) - 2(1 - R_{2}^2)(1 + S)(1 - R_{1}^2)
\]

(3.32)
where \( N = \frac{2p_1p_2}{(2\pi)^4 q^4 M_p^9} \). We assemble all time-dependent parts of various modes wavefunctions into the two time integrals which read

\[
F_1(p_1, p_2, q) = \int_{-\infty}^{0} d\tau_2 \int_{-\infty}^{\tau_2} d\tau_1 e^{-iq(\tau_1+\tau_2)} e^{-i(p_1+p_2)(\tau_1-\tau_2)} (1 + iq\tau_1)(1 + iq\tau_2) \quad (3.33)
\]

\[
F_2(p_1, p_2, q) = \int_{-\infty}^{0} d\tau_1 \int_{-\infty}^{0} d\tau_2 e^{-iq(\tau_1-\tau_2)} e^{-i(p_1+p_2)(\tau_1-\tau_2)} (1 + iq\tau_1)(1 - iq\tau_2) \quad (3.34)
\]

Upon performing the momenta integrals in (2.12), we find the logarithmic divergence via cutoff regularization to be

\[
I_L = -\frac{1}{(2\pi)^2} \frac{H^4}{M_p^4 q^4} \frac{2657}{315} \log \left( \frac{H}{\mu} \right). \quad (3.35)
\]

We find identical results from dimensional regularization. Following (2.25), for \( I_1 \) in (2.12), we have

\[
\frac{1}{\delta} \int_{1}^{\infty} dP J_1(P, \delta) = 2 \frac{1}{\delta} \int_{1}^{\infty} dP \left( \frac{70}{3P^{-\frac{1}{2}}} - \frac{14}{3P^{-\frac{3}{2}}} - \frac{233}{21P^{-\frac{5}{2}}} + \frac{493}{70P^{-\frac{7}{2}}} + \frac{199}{126} - \frac{779}{315}P^{-\frac{9}{2}} + \ldots \right),
\]

whereas for \( I_2 \), we have

\[
\frac{1}{\delta} \int_{1}^{\infty} dP J_2(P, \delta) = 2 \frac{1}{\delta} \int_{1}^{\infty} dP \left( \frac{14}{3P^{-\frac{1}{2}}} - \frac{373}{70P^{-\frac{3}{2}}} + \frac{277}{105} + \frac{383P^{-\frac{5}{2}}}{315} + \ldots \right). \quad (3.36)
\]

Combining both \( I_1 \) and \( I_2 \), we obtain the finite term to be \(-\frac{2657}{315\delta}\) after integrating and taking into account \( q^0 \) term, which leads to (3.35).

### 3.4 Spin-3/2 field: the massless gravitino

In the following, we compute the one-loop correction to the primordial tensor spectrum due to a free, massless spin-3/2 field as described by the Rarita-Schwinger Lagrangian defined on the FRW background. Compared to the previous cases, the computational process is somewhat more intricate essentially due to an enhanced gauge symmetry that arises in the massless case which makes extracting the physical degrees of freedom of the spin-3/2 field and the derivation of the interaction Hamiltonian somewhat more complicated.

In SUGRA theories, the gravitino field is typically described by a Majorana spinor field with a vector index \( \psi^\mu \) that belongs to the \( \left[ (\frac{1}{2}, 0) \oplus (0, \frac{1}{2}) \right] \times (\frac{1}{2}, \frac{1}{2}) \) representation of the Lorentz group.\(^2\) One can extract the \( (1, \frac{1}{2}) \oplus (\frac{1}{2}, 1) \) components by imposing a suitable gauge. In the massive case, the gravitino field possesses both a spin-3/2 component and a spin-1/2 longitudinal mode. In the massless case, the spin-1/2 goldstino field can be projected away by imposing another constraint which we take to be \( \psi^0 = 0 \).

What is crucial for our one-loop computation is an explicit expression for the spin-3/2 helicity sum which we develop in this section and by which we demonstrate how each propagating degree of freedom can be neatly written as the product of a spin-1 and spin-1/2 field components. After deriving the interaction Hamiltonian and the helicity sum formula for the gravitino field, we then compute the one-loop correction just as in the preceding sections.

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\(^2\)See for example [13] for a nice review on this point.
3.4.1 Gravitino propagating on the FRW background and a helicity sum formula for the massless spin-$\frac{3}{2}$ field.

We first recall that in Minkowski spacetime, the massive Rarita-Schwinger equation reads
\[ \frac{i}{2} \left( \gamma^\alpha \gamma^\mu \gamma_\beta - \gamma_\beta \gamma^\mu \gamma^\alpha \right) \partial_\beta \psi_\mu + \frac{m}{2} [\gamma^\alpha, \gamma^\beta] \psi_\beta = 0, \]
where $\psi^\alpha$ is a set of four Majorana spinors. Contracting the LHS of (3.38) with $\gamma_\alpha$ and $\partial_\alpha$, we obtain the equations
\[ \gamma^\alpha \psi_\alpha = \partial^\alpha \psi_\alpha = 0 \]
which imply that we have two dynamical spinor fields and they correspond to spin-$\frac{3}{2}$ and spin-$\frac{1}{2}$ fields with each spinor satisfying the Dirac equation. In this paper, for simplicity, we focus on the massless case in which SUSY is unbroken. Now in the massless case, the equation of motion is invariant under
\[ \psi^\alpha \rightarrow \psi^\alpha + \partial^\alpha \epsilon \]
for some spacetime-dependent spinor $\epsilon$. This gauge symmetry can be partially fixed by choosing $\gamma^\alpha \psi_\alpha = 0$ (which unlike the massive case does not necessarily follow from the equation of motion). The residual gauge symmetry is parametrized by some $\epsilon$ satisfying $\gamma^\alpha \partial_\alpha \epsilon = 0$. We can thus impose one more constraint to fully fix the gauge ending up with only spin-$\frac{3}{2}$ fields capturing the physical degrees of freedom. In our paper, we adopt
\[ \psi^0 = 0, \quad \gamma^\alpha \psi_\alpha = 0 \]
to be our gauge conditions for the massless gravitino field.

On the cosmological FRW background, much of the features discussed above remain the same. The partial derivative is replaced by a covariant one as follows
\[ D_\mu = \partial_\mu + \frac{1}{8} \omega_{\mu ab} \left[ \gamma^a, \gamma^b \right], \quad \omega_{\mu ab} = \frac{1}{2} \left( -C_{\mu ab} + C_{aba} + C_{bma} \right), \quad C^{\alpha}_{\mu \nu} = \partial_\mu \epsilon^\alpha_\nu - \partial_\nu \epsilon^\alpha_\mu - \frac{1}{2 M_p^2} \bar{\psi}_\mu \gamma^\alpha \psi_\nu, \]
where $\frac{1}{2 M_p^2} \bar{\psi}_\mu \gamma^\alpha \psi_\nu$ is the torsion term in the connection. The torsion could only modify the primordial tensor spectrum at three-loop level and for our purpose here we will not consider it further.

Our starting point is the Rarita-Schwinger action
\[ S_{rs} = i \int d^4x \sqrt{-g} \frac{1}{3!} \bar{\psi}_\mu \Gamma^{\mu \rho \nu} D_\rho \psi_\nu, \]
where $\Gamma^{\mu \rho \nu} = \epsilon^\rho_\mu \gamma^\alpha$ and $\Gamma^{\mu \nu \rho} = \Gamma^{[\mu \nu} \Gamma^{\rho]}$ is the completely anti-symmetrized product of the Dirac matrices. Again, in the absence of torsion, we note that $D_0 = \partial_0$, $D_k = \partial_k + \frac{H}{2} \gamma_k \gamma^0$. The equations of motion can be simply expressed as
\[ \gamma^{\mu \rho} \left( \partial_\rho + \Omega_\rho \right) \psi_\rho = 0. \]
We find that upon imposing the gauge conditions $\psi^0 = 0, \gamma^\alpha \psi_\alpha = 0$ and working in conformal time, the $\mu = 0$ equation is identically satisfied whereas taking $\mu$ to be a spatial index yields
\[ \gamma^0 \partial_\tau \psi_k^{(T)} + \gamma^j \partial_j \psi_k^{(T)} + \frac{H}{2} \gamma^0 \psi_k^{(T)} = 0, \]
\[ \text{See for example [14] and [15] for a nice exposition of the Rarita-Schwinger equation in relation to cosmology.} \]
where we have used the superscript on $\psi^{(T)}$ to denote the fact that the spinor has been gauge-fixed. In particular (3.45) implies that if we define

$$\psi_k^{(T)} = \frac{1}{\sqrt{a}} \Psi_k,$$  \tag{3.46}

then $\gamma^\mu \partial_\mu \Psi_k = 0$ so we can understand $\Psi_k$ to be satisfying the ordinary Dirac equation on flat spacetime.\footnote{We note that our result is largely the same as an expression presented in [16] which carried out a different derivation and of which final result differs from ours possibly due to spinor normalization among other reasons.}

We note that the gauge constraints can be expressed through a projection. In momentum space, we find that we can decompose the spinor into the gauge-fixed piece and remaining degrees of freedom as follows.

$$\psi_i = \psi_i^{(T)} + \left( \frac{1}{2} \gamma_i - \frac{1}{2} \hat{k}_i (\hat{k} \cdot \vec{\gamma}) \right) \gamma^j \psi_j - \left( \frac{3}{2} \hat{k}_i + \frac{1}{2} \gamma_i (\hat{k} \cdot \vec{\gamma}) \right) \hat{k}^j \psi_j,$$

or $\psi^{(T)i} = \left[ \delta^i_j - \left( \frac{1}{2} \gamma_i - \frac{1}{2} \hat{k}_i (\hat{k} \cdot \vec{\gamma}) \right) \gamma^j + \left( \frac{3}{2} \hat{k}_i + \frac{1}{2} \gamma_i (\hat{k} \cdot \vec{\gamma}) \right) \hat{k}^j \right] \psi^j$. \tag{3.47}

One can check that (3.47) implies the gauge conditions $\gamma^\alpha \psi^{(T)}_\alpha = k^\alpha \psi^{(T)}_\alpha = 0$. Henceforth, we will be working with the gauge-fixed spinor and hence will drop the superscript on the spinor to ease notations.

In momentum space, the fourier components of the projected spinors satisfy the following sum rule which is essential for our computation of the loop corrections later.

$$P^{ij}(\vec{k}) = \sum_{\lambda = \pm \frac{3}{2}} \psi^i(\vec{k},\lambda) \bar{\psi}^j(\vec{k},\lambda) = \frac{N_k}{2} \left( \gamma^i - \hat{k}^i \left( \hat{k} \cdot \vec{\gamma} \right) \right) \gamma^\mu k_\mu \left( \gamma^j - \hat{k}^j \left( \hat{k} \cdot \vec{\gamma} \right) \right),$$ \tag{3.48}

where $\lambda$ labels the spin-$\frac{3}{2}$ helicity states, and $N_k = \frac{1}{2k(2\pi)^{3/2}}$ is a normalization factor identical to the case of massless Dirac fermions on a conformally flat spacetime. The projector satisfies the gauge contraints

$$\gamma_i P^{ij} = \hat{k}_j P^{ij} = 0.$$

In the ordinary Dirac fermion theory, the spinor field has no vectorial index and the analogue of (3.48) is simply the propagator. We will need (3.48) in computing the correlation function of the spinor fields in our calculation of the one-loop correction.

In the following, we derive (3.48) and in the process demonstrate how each of the two spin-$\frac{3}{2}$ components of the projected gravitino spinor can be understood as arising from taking a suitable product of the massless spin-1 field polarization vector and a chiral spinor.\footnote{Now the spacetime index on $\psi$ is raised/lowered using the metric tensor, so this implies that (3.45) can also be written as $\gamma^0 \partial_\mu \psi^{(T)k} + \gamma^i \partial_i \psi^{(T)k} + \frac{2H}{\tau} \gamma^0 \psi^{(T)k} = 0$, which implies the identification $\psi^{(T)k} = \frac{1}{\sigma^{3/2}} \Psi^k$.}

We will work in momentum space and for the purpose of deriving (3.48), we need a specific representation of the Dirac matrices. For convenience, we pick the Weyl representation in which

$$\gamma^\mu = \begin{pmatrix} 0 & \sigma^\mu \\ \overline{\sigma}^\mu & 0 \end{pmatrix}, \quad \sigma^\mu = (1, \vec{\sigma}), \quad \overline{\sigma}^\mu = (1, -\vec{\sigma}).$$ \tag{3.49}
A 4-dimensional Dirac spinor $U(\vec{k})$ can be decomposed in terms of a pair of massless left- and right-handed spinors by writing

$$U(\vec{k}) = \begin{pmatrix} u_L(\vec{k}) \\ u_R(\vec{k}) \end{pmatrix} \equiv U_L(\vec{k}) \oplus U_R(\vec{k}),$$

where we normalize the two-component spinors as follows

$$u_R(\vec{k})u_R^\dagger(\vec{k}) = N_k \kappa_{\mu} \sigma^\mu, \quad u_L(\vec{k})u_L^\dagger(\vec{k}) = N_k \kappa_{\mu} \sigma^\mu.$$ \hfill (3.51)

On the other hand, it is known that the massless spin-1 polarization vectors can be expressed in terms of these chiral spinors as follows (see for example \[17\]).

$$e_i^j(\vec{k}) = -\frac{1}{\sqrt{2}} \frac{U_R(-\vec{k})\gamma^i U_R(\vec{k})}{U_R(-\vec{k})U_L(\vec{k})}, \quad e_i^j(\vec{k}) = -\frac{1}{\sqrt{2}} \frac{U_L(-\vec{k})\gamma^i U_L(\vec{k})}{U_L(-\vec{k})U_R(\vec{k})}.$$ \hfill (3.52)

Substituting (3.51) into (3.52) yields the following useful relations for the spin-1 polarization vectors.

$$e_{i-1}^j(\vec{k})e_{-j}^{i+1}(\vec{k}) = \frac{U_R^\dagger(-\vec{k})\gamma^i U_R^\dagger(\vec{k})U_R^\dagger(\vec{k})\gamma^j U_R(-\vec{k})}{2U_R^\dagger(-\vec{k})U_R^\dagger(\vec{k})U_R^\dagger(\vec{k})U_R(-\vec{k})} = \frac{1}{8} \text{Tr} \left( \sigma^i u_R(\vec{k})u_R^\dagger(\vec{k})\sigma^j u_R(-\vec{k})u_R^\dagger(-\vec{k}) \right) = \frac{1}{2} \left( \delta^{ij} - \hat{k}^i \hat{k}^j + i \epsilon^{ijm} \hat{k}_m \right) \equiv \frac{1}{2} P^{ij}_\perp(\vec{k}),$$ \hfill (3.53)

and similarly,

$$e_i^j(\vec{k})e_{-j}^{i+1}(\vec{k}) = \frac{1}{2} \left( \delta^{ij} - \hat{k}^i \hat{k}^j - i \epsilon^{ijm} \hat{k}_m \right) \equiv \frac{1}{2} P^{ij}_\parallel(\vec{k}).$$ \hfill (3.54)

We note that (3.53) and (3.54) imply that $\sum_{\lambda = \{-1, 1\}} e^j_\lambda e^{i}_\lambda = \delta^{ij} - \hat{k}^i \hat{k}^j = P_{ij} = \frac{1}{2} \left( P^{ij}_\perp(\vec{k}) + P^{ij}_\parallel(\vec{k}) \right).$

In the Weyl representation, we find that (3.48) can be written as

$$\mathcal{P}^{ij} = \begin{pmatrix} 0 & u_R u_L^\dagger e_{i-1}^j e_{+1}^i \\ u_R u_L^\dagger e_{i+1}^j e_{-1}^i & 0 \end{pmatrix} = e_{i-1}^j(\vec{k})U_R^\dagger(\vec{k})U_L + e_i^j(\vec{k})e_{-j}^{i+1}(\vec{k})U_L^\dagger U_R.$$ \hfill (3.55)

Comparing (3.55) with (3.48), we see that the massless spin-$\frac{3}{2}$ field has components that can be simply decomposed in terms of massless spin-1 and spin-$\frac{1}{2}$ degrees of freedom, i.e. we can identify

$$\psi^i \left( \vec{k}, \frac{3}{2} \right) = e_i^1(\vec{k})U_L(\vec{k}), \quad \psi^i \left( \vec{k}, -\frac{3}{2} \right) = e_{i-1}^1(\vec{k})U_R(\vec{k}),$$ \hfill (3.56)

where $U_{L,R}(\vec{k})$ are the left- and right-handed massless Dirac spinors of spin $\frac{3}{2}$ respectively. This completes our derivation of the helicity sum formula (3.48) which we need for calculating our one-loop correction term.

### 3.4.2 The interaction Hamiltonian from linear fluctuations

In this Section, we derive the interaction Hamiltonian for the massless gravitino. Our final result is the following expression:

$$H_{\text{int}} = i \int d^3x a(\tau) \ h^{ik} \left[ \bar{\psi}_j \gamma^* \partial_{\tau} \psi_k + \frac{1}{2} \left( \bar{\psi}_j \eta^{im} \gamma_k \partial_{\tau} \psi_m + \bar{\psi}_j \gamma_0 \partial^j \psi_k \right) \right],$$ \hfill (3.57)
where in our notational conventions, the indices (apart from those on the spinors) in (3.57) are raised/lowered via Minkowski metric. In the following, we present the outline for the derivation of (3.57).

Up to first order, the spin connection remains unchanged and we only need to consider the linear fluctuations of the vielbeins. From

\[ \Gamma^{\mu\nu\rho} \mathcal{D}_\nu \psi_\rho = e_\alpha^b e_\beta^c \gamma^{abc} (\partial_\nu + \Omega_\nu) \varphi_\rho, \quad e_\alpha^\mu = \frac{1}{2} \left[ \delta_\alpha^\mu - \frac{1}{2} h_\alpha^\mu + \ldots \right] \]  

we obtain the interaction Lagrangian to be

\[ \mathcal{L}_{\text{int}} = i \int d^3 x a(\tau) \bar{\psi}_i \mathcal{P}_{abc}^\mu \gamma^{abc} \mathcal{D}_\nu \psi_\rho, \]  

where

\[ \mathcal{P}_{abc}^\mu = -\frac{1}{2} \left( \delta_\mu^\nu \delta_\rho^\sigma h_\sigma^\nu + \delta_\rho^\nu \delta_\mu^\sigma h_\sigma^\nu + \delta_\nu^\sigma \delta_\mu^\rho h_\rho^\sigma \right). \]

Imposing \( \psi^0 = 0 \), (3.59) then becomes

\[ \mathcal{L}_{\text{int}} = i \int d^3 x a(\tau) \bar{\psi}_i \left( \mathcal{P}_{abc}^{ijk} \gamma^{abc} \partial_\nu \psi_k + \mathcal{P}_{abc}^{ijk} \gamma^{abc} (\partial_j + \Omega_j) \psi_k \right). \]  

In the following, we furnish some details as to how we obtain (3.57) from (3.61):

(i) We find that the term involving the spin connection \( \Omega \) in (3.61) vanishes. This term is

\[ \int \bar{\psi}_i \gamma \gamma^{abc} \partial_\nu \psi_k \]

For each term in (3.62), we find

\[ \bar{\psi}_i \gamma^{abc} \gamma^{0} \psi_k = -12 h_b^{ik} \bar{\psi}_i \gamma^{0} \psi_k, \]

\[ \bar{\psi}_i \gamma^{ijk} h_b^{i} \gamma^{0} \psi_k = 6 h_b^{ik} \gamma^{0} \psi_k, \]

\[ \bar{\psi}_i \gamma^{ijk} h_b^{i} \gamma^{0} \psi_k = 6 h_b^{ik} \gamma^{0} \psi_k, \]

and thus \( \bar{\psi}_i \mathcal{P}_{abc}^{ijk} \gamma^{abc} \Omega_j \psi_k = 0 \).

(ii) For the remaining terms involving derivatives of \( \psi \), we find

\[ \bar{\psi}_i \gamma^{abc} \partial_\nu \psi_k = 6 \bar{\psi}_i \gamma^{0} \partial_\nu \psi_k, \]

\[ \bar{\psi}_i \gamma^{ijk} h_b^{i} \partial_\nu \psi_k = 6 h_b^{ik} \gamma^{0} \partial_\nu \psi_k, \]

\[ \bar{\psi}_i \gamma^{ijk} h_b^{i} \partial_\nu \psi_k = 6 h_b^{ik} \gamma^{0} \partial_\nu \psi_k. \]

Assembling all the terms together and after a Legendre transformation, we then obtain (3.57).  

\[ 6 \]  

We note in passing that we can make the reality condition of the classical Hamiltonian real by writing \( H_{\text{int}}(\vec{x}, \tau) = \frac{1}{2} a(\tau) h^{ik} \left[ \bar{\psi}_i \gamma^0 \partial_\nu \psi_k - \partial_\nu \bar{\psi}_i \gamma^0 \partial_\nu \psi_k + \frac{1}{2} \left( \bar{\psi}_i \gamma^m \gamma_k \partial_\nu \psi_m - \partial_\nu \bar{\psi}_i \gamma^m \gamma_k \psi_m + \bar{\psi}_j \gamma_i \partial_\nu \psi_k - \partial_\nu \bar{\psi}_j \gamma_i \psi_k \right) \right] \). One can check that this yields the same results as adopting (3.57).
3.4.3 The gravitino correction at one-loop

In the following, we compute the one-loop correction to the tensor spectrum due to the free massless gravitino propagating on the FRW background. We can write its mode expansion as

\[ \psi_i(\vec{x}, \tau) = \int d^3p \sum_{\lambda} e^{i\vec{p} \cdot \vec{x}} \psi_i(\vec{p}, \lambda) e^{-ip\tau} a_{\vec{p}, \lambda} + e^{-i\vec{p} \cdot \vec{x}} \psi^c_i(\vec{p}, \lambda) e^{ip\tau} a^\dagger_{\vec{p}, \lambda} \]  

(3.65)

of which form ensures that \( \psi^c_i(\vec{x}, \tau) = \psi_i(\vec{x}, \tau) \), with the superscript ‘c’ denoting charge conjugation and the fermionic oscillators obey the anti-commutation relation

\[ \{a_{\vec{p}, \lambda}, a^\dagger_{\vec{k}, \lambda'}\} = \delta^3(\vec{p} - \vec{k}) \delta_{\lambda\lambda'} \].

From (3.57), the interaction Hamiltonian density is of the form

\[ \psi_m(x) D_{mnklr}^{ijkl}(x) \gamma^r \psi_n(x) \],

where we define

\[ D_{mnklr}^{ijkl}(x) \equiv \delta^m_{jl} \delta^{kn} \gamma^r \delta_{\lambda\lambda'} - \frac{1}{2} (\delta^m_{ln} \gamma_k \partial_l + \delta^n_{lk} \gamma_l \partial_n) \].

In the absence of any differential/matrix-valued operator, the 4-point function involving \( \psi, \bar{\psi} \) reads

\[ \langle \bar{\psi}_l \gamma^a \psi_k \bar{\psi}_i \gamma^b \psi_j \rangle = \sum_{\lambda_1, \lambda_2} \int d^3p_1 d^3p_2 e^{-i(p_1 + p_2)(\tau_1 - \tau_2)} e^{i(p_1 + p_2) \cdot (\vec{x}_1 - \vec{x}_2)} \times \bar{\psi}^c_i(\vec{p_1}, \lambda_1) \psi_k(\vec{p_2}, \lambda_2) \left[ \bar{\psi}_l(\vec{p_2}, \lambda_2) \psi_i(\vec{p_1}, \lambda_1) - \bar{\psi}_l(\vec{p_1}, \lambda_1) \psi_i^c(\vec{p_2}, \lambda_2) \right] \],

(3.66)

where we note that the first and second term in the bracket corresponds to the contraction between the second and third spinor field and that between the second and last spinor field respectively, the negative sign due to the anti-commuting nature of the oscillators.

To proceed, we commit to various specific representations of the Dirac algebra at various points in the following computation. Earlier on, we had worked in the Weyl representation when demonstrating how the form of \( P^{ij} \) can be derived from decomposing a spin-\( \frac{3}{2} \) field in terms of lower spin ones. For the purpose of computing the VEV, we now pick the Majorana representation\(^7\) in which all the Dirac matrices are purely imaginary and

\[ \varphi^c = \varphi^* \].

(3.67)

Let us proceed to simplify the gravitino correlation function. The interaction Hamiltonian contains terms of the following form

\[ \langle \bar{\psi}_l \gamma^a \psi_k \bar{\psi}_i \gamma^b \psi_j \rangle \].

Using the anticommuting nature of the oscillators, the first term \( \bar{\psi}^c_i(\vec{p_1}, \lambda_1) \gamma^a \psi_k(\vec{p_2}, \lambda_2) \psi_i(\vec{p_2}, \lambda_2) \gamma^b \psi_j^c(\vec{p_1}, \lambda_1) \) can be easily seen to be equivalent to

\[ \text{Tr} \left[ P^{ij}(\vec{p_1}) \gamma^a P^{kl}(\vec{p_2}) \gamma^b \right] \].

\(^7\)In this representation, we have \( \gamma^0 \) being Hermitian and \( \gamma^k \) being anti-Hermitian. For example, \( \gamma^0 = \sigma^2 \otimes \sigma^1, \gamma^2 = \sigma^2 \otimes i\sigma^2, \gamma^1 = i\sigma^1 \otimes 1, \gamma^3 = i\sigma^3 \times 1 \). But we won’t actually need any particular choice in the rest of our workings.
The second term $\bar{\psi}_i(\vec{p}_1, \lambda_1) \gamma^a \psi_b(\vec{p}_2, \lambda_2) \bar{\psi}_j(\vec{p}_1, \lambda_1) \gamma^b \psi^c_j(\vec{p}_2, \lambda_2)$ is not so immediately obvious in terms of how it can be expressed in terms of the projector $P_{ij}$. Making visible the matrix component indices (with capitalized Roman indices), we find that we can massage it to be in the form

$$
(\gamma^0)_{MN} P_{MS}^i(\vec{p}_1) (\gamma^a)_{NJ} P_{Qj}^k(\vec{p}_2) (\gamma^b)_{SK} (\gamma^0)_{QK} \\
= \text{Tr} \left( \gamma^a P^{kj}(\vec{p}_2) \gamma^0 \left( \gamma^b \right)^T P^{(i)(T)}(\vec{p}_1) \gamma^0 \right) \\
= -\text{Tr} \left( \gamma^a P^{kj}(\vec{p}_2) \gamma^b P^{ij}(\vec{p}_1) \right), \quad (3.68)
$$

where we have invoked the useful relation

$$
P^{ij(T)}(\vec{p}) = -P^{ji(-\vec{p})}, \quad \gamma^0 P^{ij}(\vec{p}) \gamma^0 = P^{ij}(-\vec{p}). \quad (3.69)
$$

So this implies that the second term is related to the first one by switching $i \leftrightarrow j$. But when differential operators are involved, we need to switch the momenta sign for the last two spinors. Taking into account the full Hamiltonian, after some algebra, we find the gravitino correlation function to be

$$
\int \int d^3p_1 d^3p_2 e^{-i(p_1+p_2)(\vec{r}_1-\vec{r}_2)} (p_1^s + p_2^s) \\
\times \text{Tr} \left[ \left( \gamma^a P^{ji}(\vec{p}_1) \gamma^b P^{kj}(\vec{p}_2) \right) p_1^a(p_2^s + p_2^s) + \left( \gamma^a P^{jb}(\vec{p}_1) \gamma^b P^{ai}(\vec{p}_2) \right) p_2^b(p_1^s + p_2^s) \eta^{ab} \right. \\
+ \left( \gamma^a P^{jr}(\vec{p}_1) \gamma^b P^{il}(\vec{p}_2) \right) p_2^b(p_1^i + p_2^i) + \frac{1}{2} \left( \gamma^a P^{ib}(\vec{p}_1) \gamma^b P^{ac}(\vec{p}_2) \right) p_2^b(p_1^i + p_2^i) \eta^{ab} \eta^{cd} \right. \\
+ \frac{1}{2} \left( \gamma^a P^{dr}(\vec{p}_1) \gamma^b P^{lc}(\vec{p}_2) \right) p_2^b(p_1^i + p_2^i) \eta^{cd} + \frac{1}{4} \left( \gamma^a P^{jr}(\vec{p}_1) \gamma^b P^{ls}(\vec{p}_2) \right) p_2^b p_1^a \right. \\
+ \frac{1}{4} \left( \gamma^a P^{sr}(\vec{p}_1) \gamma^b P^{lj}(\vec{p}_2) \right) p_2^b p_1^a, \quad (3.70)
$$

where we have in the process of simplification used (3.69) repetitively and the fact that we are integrating over all momenta space of $p_1, p_2$ which can be interchanged as dummy variables. We also used the fact that switching the sign of the momenta doesn’t matter since it is multiplied to the graviton 4-point function which eventually depends on $\vec{q} = -\vec{p}_1 - \vec{p}_2$. At this point, we judiciously turn to the Weyl representation for an easier computation of the trace. \footnote{Otherwise, without further simplification/insight, the algebra involves manipulating the spacetime indices contained in the trace of 8 gamma matrices which involve 105 terms involving products of kronecker delta tensor.} In terms of Pauli matrices and the spin-1 projectors, each matrix trace reads

$$
\text{Tr} \left[ \gamma^a P^{bs}(\vec{p}_1) \gamma^b P^{ij}(\vec{p}_2) \right] = \text{Tr} \left( \sigma^a \gamma^b \gamma^c \gamma^d \right) P^{bs}_l(\vec{p}_1) P^{ij}_l(\vec{p}_2) + \text{Tr} \left( \sigma^a \gamma^b \gamma^c \gamma^d \right) P^{bs}_l(\vec{p}_1) P^{ij}_l(\vec{p}_2) \quad (3.71)
$$

with

$$
\text{Tr} \left[ \sigma^a \gamma^b \gamma^c \gamma^d \right] = 2 \delta^{ac} (p_1 p_2 - \vec{p}_1 \cdot \vec{p}_2) + 2 p_1^a p_2^d + 2 p_1^d p_2^a + 2i p_1 p_2 \epsilon^{acr}(\vec{p}_1 l - \vec{p}_2 l). \quad (3.72)
$$

Defining

$$
M^{ar} = \delta^{ar} (1 - \vec{p}_1 \cdot \vec{p}_2) + \vec{p}_1 \gamma^a \vec{p}_2 \gamma^a + \vec{p}_2 \gamma^a \vec{p}_1 \gamma^a + i \epsilon^{acr}(\vec{p}_1 l - \vec{p}_2 l),
$$
after some algebra, we find that the product of the gravitino and graviton correlation functions reads

\[
\begin{align*}
\int \int d^3p_1 d^3p_2 e^{-i(p_1+p_2)(\tau_1-\tau_2)} &+ (\hat{p}_1+\hat{p}_2)(\vec{x}_1-\vec{x}_2) (p_1^2 + p_2^2) \\
\times \left[ 2M^{sr} P^i_{\uparrow}(\vec{p}_2) P^j_{\uparrow}(\vec{q}_1) q^s q^j + 2M^{sk} P^i_{\uparrow}(\vec{q}_1) P^j_{\uparrow}(\vec{p}_2) q^s q^j \\
+ M^{ik} P^j_{\uparrow}(\vec{p}_1) P^s_{\uparrow}(\vec{p}_2) q^i q^s + +M^{ik} P^j_{\uparrow}(\vec{q}_1) P^s_{\uparrow}(\vec{p}_2) q^i q^s \\
+ 2M^l P^i_{\uparrow}(\vec{p}_1) P^j_{\uparrow}(\vec{p}_2) s^{ab} q^k q^j + M^{ij} P^s_{\uparrow}(\vec{p}_1) P^s_{\uparrow}(\vec{p}_2) s^{ab} q^k q^j \\
+ M^{ik} P^j_{\uparrow}(\vec{p}_1) P^s_{\uparrow}(\vec{p}_2) e^{a\ell} q^i q^j \right] \times \langle h^{kl} h^{ij} h_{mn} h_{mn} \rangle \tag{3.73}
\end{align*}
\]

The integrand terms outside the square bracket can be shown to vanish since they involve momenta 4-vectors with the free indices and they are multiplied to the graviton correlation function which contains polarization tensors satisfying

\[ \sum_{\lambda, \lambda'} \epsilon_{k\ell}(q, \lambda') \epsilon_{*mn}(q, \lambda) \epsilon_{*ij}(q, \lambda') \epsilon_{mn}(q, \lambda') = 2 \left( P^{k\ell}(q) P^{ij}(q) + P^{kij}(q) P^{\ell}(q) - P^{kij}(q) P^{\ell}(q) \right). \tag{3.74} \]

After using (3.74) in the midst of some heavy algebra, we find that the contraction between the gravitino and graviton correlation function yields the following function \( \mathcal{G}(p_1, p_2, q) \) in the master formula \[ \textbf{[2.12]} \):

\[
\begin{align*}
\mathcal{G}(p_1, p_2, q) &= N_G \left[ 1 - \hat{p}_1 \cdot \hat{p}_2 + 2\hat{q} \cdot \hat{p}_1 \hat{q} \cdot \hat{p}_2 \left[ (1 + (\hat{q} \cdot \hat{p}_1)^2)(1 + (\hat{q} \cdot \hat{p}_2)^2) - 4\hat{q} \cdot \hat{p}_2 \hat{q} \cdot \hat{p}_1 \right] \\
&+ 2 \left( (\hat{p}_1 \cdot \hat{p}_2)^2 - 1 \right) - \hat{q} \cdot \hat{p}_1 \hat{q} \cdot \hat{p}_2 - (\hat{q} \cdot \hat{p}_1)^2 \left[ \hat{p}_1 \cdot \hat{p}_2 - 2\hat{q} \cdot \hat{p}_1 \hat{q} \cdot \hat{p}_2 \right] \\
&- 2(1 - \hat{p}_1 \cdot \hat{p}_2) + 2(\hat{q} \cdot \hat{p}_1 \hat{q} \cdot \hat{p}_2) \left[ (1 + (\hat{q} \cdot \hat{p}_1)^2)(1 + (\hat{q} \cdot \hat{p}_2)^2) - 2\hat{q} \cdot \hat{p}_2 (\hat{q} \cdot \hat{p}_1 - \hat{q} \cdot \hat{p}_2) \right] \\
&+ (1 - \hat{q} \cdot \hat{p}_1 \hat{q} \cdot \hat{p}_2) \left[ (\hat{p}_1 \cdot \hat{p}_2 - \hat{q} \cdot \hat{p}_1 \hat{q} \cdot \hat{p}_2) (\hat{q} \cdot \hat{p}_1 \hat{q} \cdot \hat{p}_2 - 1) + (1 - (\hat{q} \cdot \hat{p}_1)^2)(1 + (\hat{q} \cdot \hat{p}_2)^2) \right] \right],
\end{align*}
\] \[ \tag{3.75} \]

where \( N_G = -\frac{1}{(2\pi)^2 M_p^4} \frac{H^4}{M_p^4} \) takes into account all the normalization constants. For the time integrals, from the gravitino correlation function, we have the factor \( e^{-i(p_1+p_2)(\tau_1-\tau_2)} \) just like the case of the gauge field and fermions. Hence the time integrals \( F_1(p_1, p_2, q) \) and \( F_2(p_1, p_2, q) \) are the same as in those cases:

\[
\begin{align*}
F_1(p_1, p_2, q) &= \int_{-\infty}^{0} d\tau_2 \int_{-\infty}^{\tau_2} d\tau_1 e^{-i(p_1+p_2)(\tau_1-\tau_2)} (1 + iq\tau_1)(1 + iq\tau_2) e^{-iq(\tau_1+\tau_2)}, \\
F_2(p_1, p_2, q) &= \int_{-\infty}^{0} d\tau_1 \int_{-\infty}^{0} d\tau_2 e^{-i(p_1+p_2)(\tau_1-\tau_2)} (1 + iq\tau_1)(1 - iq\tau_2) e^{-iq(\tau_1-\tau_2)}. \tag{3.76}
\end{align*}
\]

Performing the momenta integral yields the logarithmic one-loop correction

\[
I_{\text{gravitino}} = -\frac{1}{(2\pi)^2 M_p^4} \frac{5107}{315} \frac{1}{\mu} \log \left( \frac{H}{\mu} \right), \tag{3.77}
\]
a result which we also obtain from dimensional regularization. Following (2.25), we find for \( I_1 \) in (2.12)
\[
\frac{1}{\delta} \int_1^\infty dP P_1(P, \delta) = \frac{1}{\delta} \int_1^\infty dP \left( \frac{218}{21P^{-\frac{1}{2}}} + \frac{17}{70P^{-\frac{1}{2}}} + \frac{5653}{630} + \ldots \right)
\]
whereas for \( I_2 \), we have
\[
\frac{1}{\delta} \int_1^\infty dP P_2(P, \delta) = -\frac{1}{\delta} \int_1^\infty dP \left( \frac{218}{105P^{-\frac{1}{2}}} + \frac{13}{15} + \ldots \right).
\]
Combining both \( I_1 \) and \( I_2 \), we obtain the finite term to be \(-\frac{5107}{630}\) after integrating which leads to (3.77).

4 On Seagull diagrams

Seagull vertices (see Figure 1b) can potentially contribute to the one-loop logarithmic correction term since from (2.1), the first-order term is at one-loop order if \( H_{int} \) is quadratic in \( h_{ij} \),
\[
I_L = -2\mathrm{Im} \left( \int_{-\infty}^0 d\tau \langle H_{int}(\tau) h_{mn}(\vec{x}, 0) h_{mn}(\vec{x}', 0) \rangle \right).
\]
On a closer inspection, we find that this cannot contribute to the one-loop logarithmic correction. In the following, we elaborate on this point.

In (4.1), the correlation function factorizes into a graviton 4-point function and a 2-point function of the matter fields. The two-point function can be expressed as a momenta-integral, the integrand having two spacetime indices (to be contracted with two other in the graviton’s 4-point function) and of which scaling dimension reveals if it potentially harbors the logarithmic running. By assembling all terms quadratic in the graviton fields, one obtains the interaction Hamiltonian. After integrating over the dummy spatial coordinate and the virtual momenta of the gravitons, one finds that for all cases, one ends up with the following schematic form for \( I_L \):
\[
I_L = \int_{-\infty}^0 d\tau \int d^3p \ \Phi^{ij}(\vec{p}, \tau) H^{ij}(\vec{q}, \tau),
\]
where \( \Phi^{ij}(\vec{p}, \tau), H^{ij}(\vec{q}, \tau) \) are functions arising from the matter and graviton correlation functions respectively. The interaction Hamiltonian in (4.1) can be obtained by expanding
\[
\sqrt{-g} = \sqrt{-b} \left( 1 - \frac{1}{4} \gamma_{\mu} \gamma^{\mu} + \ldots \right), \ g^{ij} = b^{ij} - \gamma^{ij} + \gamma_{k}^{i} \gamma^{kj} + \ldots,
\]
and for the fermions, we also have relevant terms arising from the spin connection \( \Omega_{\mu} = e_{a}^{\nu} \nabla_{\mu} e_{\nu b} = \frac{1}{2} (h_{b}^{a} \partial_{\mu} h_{aa} - h_{a}^{a} \partial_{\mu} h_{ab} + \ldots). \) For the fermions and gauge field, \( \Phi^{ij} \) is time-independent and from the scaling dimension we can deduce that there is no logarithmic correction term from (4.2).

For the scalar field, the time integral induces some momentum-dependence since \( \Phi^{ij} \) is time-dependent, with the relevant seagull vertices arising from
\[
H_{int} \sim \int d^3y \left( h_{ik} h_{kj} \partial_{i} \phi \partial_{j} \phi - \frac{1}{4} h_{ik} h_{kl} \delta^{ij} \partial_{i} \phi \partial_{j} \phi \right).
\]
For the first term in the bracket in (4.3), after substituting the modes and taking the VEV, we find

\[
\frac{8 H^4}{(2\pi)^3 M_p^4 q^4} \text{Re} \left( i \int_{-\infty}^{0} d\tau \int d^3 p \frac{p^i p^j}{p^2} (1 + p^2 \tau^2) e^{-2i q \tau} (1 + i q \tau)^2 (4 \delta^{ij}(\hat{q})) \right).
\]

After performing the time integral we obtain

\[
\int d^3 p \frac{p^i p^j (5q^2 - 7p^2)}{p^2} F^{ij}(\vec{q}),
\]

where \( F^{ij}(\vec{q}) \) is some function of \( \vec{q} \) only. Taking into account the second term which differs from the first only by the index structure, we find

\[
I_L = \int d^3 p \left[ \frac{p^i p^j (5q^2 - 7p^2)}{p^2} F^{ij}(\vec{q}) - \frac{1}{4} \frac{p^m p^n (5q^2 - 7p^2)}{p^2} F^{mn}(\vec{q}) \right],
\]

from which we deduce that there is no logarithmic correction term.

5 Discussion

We have implemented dimensional regularization and cutoff regularization in the Schwinger-Keldysh formalism following the broad framework first presented in [2] to compute one-loop corrections to the two-point correlation function of tensor perturbations in primordial cosmology induced by massless isocurvature fields of spins \(< 2\). For all cases, we found a logarithmic running of the form

\[
\langle h_{mn} h_{mn} \rangle_{1\text{-loop}} = \frac{C}{q^3 M_p^4} \log \left( \frac{H}{\mu} \right), \quad H = \frac{q}{a(\tau_q)},
\]

and determined the constant \( C \) for each isocurvature field. It is notable that this constant is negative for all cases considered in this paper and it would be interesting to explore if this is universally true for all physically relevant fields and if there is a deeper reason. This is not the case for one-loop corrections to the primordial scalar spectrum as presented recently in [4]. In [9], it was pointed out that the results of [18] appear to suggest that after resummation of higher loop corrections, the scalar spectrum should be corrected to be of the form \( \langle \zeta \zeta \rangle \sim \frac{1}{q^{n_s + c}} \) where \( n_s \) is the scalar tilt that depends on the slow-roll parameters, and \( c \) is a constant of order \( \frac{H^2}{M_p^2} \). Although we do not perform any resummation of higher loop effects in this work, we note that if we assume that they can be summed up appropriately, our one-loop result would naively suggest that these quantum corrections on their own lead to a small red-tilt of the tensor spectrum. For example, in a different yet related context of loop corrections to propagators of isocurvature fields, it was argued in [19] that via the method of dynamical renormalization group [8], one can view the one-loop late-time divergence as the first term of an exponential series. If a variant of this approach can be applied to our one-loop result, this suggests that the quantum corrections to the tensor spectrum induce a red tilt of the form \( \frac{1}{q^c} \) where \( c \) is a constant of order \( \frac{H^4}{M_p^4} \), similar to the suggestion in [9] for the loop-corrected scalar spectrum. It would be interesting to explore this speculation further.

Moving beyond the context of our work, we hope that our explanation of the regularization procedures will be relevant and useful for computing loop corrections in the broader picture of
cosmological perturbation theory. Recent explorations of the cosmological collider program ([20], [21]) have indicated that the mass and spin of particles present during inflation can leave their imprints on various cosmological correlation functions and loop effects play a certain role. In [9], it was argued that gravitino and goldstino loops to the inflaton bispectra bear the imprints of these particles. Some of our results in Section 3.4 such as the spin- helicity sum could potentially be useful towards extracting these signatures in a deeper analysis of [9].

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