Abstract: This paper considers the design of optimal gain of set-value observer for robust fault detection (FD) from a new perspective. Set-based robust FD can be implemented by real-timely check whether the measured output is contained in the corresponding output estimation set or not. Our proposed method focuses on designing observer gains at each step to maximize the exclusion tendency of the measured output from the output estimation zonotope to achieve the goal of robust FD. The key design logic is formulated as a two-level min-max optimization problem, which is further equivalently transformed into a non-convex single-level programming based on duality theory. Finally, this non-convex single-level programming can be efficiently solved via linear programming and matrix decomposition. At the end, a circuit model is used to illustrate the effectiveness of the proposed method.

Keywords: Fault detection, set-value observer, zonotopes

1. INTRODUCTION

As modern industrial systems become more and more complex, there always exists fault occurrence during the operation of systems. Fault diagnosis techniques have attracted a great number of attentions since it plays an important role in real-time detecting, isolating and estimating (identifying) the occurred faults in sensors, actuators or the system plant itself Blanken et al. (2006). Real physical systems are always affected by system uncertainties, such as process disturbances, measurement noises, modeling error and so on. Thus, the results of fault detection (FD) should be robust against these uncertainties. Sets are naturally involved in real systems by considering system uncertainties and physical constraints. Set-based fault diagnosis can obtain robustness by using sets to bound the uncertainties and propagate their effects through system models Blanchini and Miani (2008). In the literature, there exist three commonly used set-based FD approaches, i.e., invariant-set approach Seron and Doná (2010); Seron et al. (2012), set-membership estimation Alamo et al. (2005); Combastel (2003) and set-value observer Xu et al. (2017, 2020).

Set-value observer-based methods implement robust FD by real-timely detecting whether the measured output is contained in the corresponding output estimation set or not. If the system is healthy, the real measured output should always stay in the corresponding output estimation set as long as the initial system state belongs to the initial state estimation set. In Oca et al. (2012), the design procedure of the interval set-value observer for robust fault detection is implemented via fault sensitivity analysis and residual uncertainty bounds. Pourasghar et al. (2019) formulated an on-line quadratic fractional programming to provide an optimal time-varying observer gain leading to a so-called FD-ZKF (Zonotopic Kalman Filter) that allows enhancing the fault detection properties. However, solving this quadratic fractional programming is approximated by Rayleigh quotient technique, which can not be guaranteed to obtain the global optimal solution. Further, in the authors’ work Tan et al. (2020), the optimal set-value observer gain is designed by maximizing the effect of faults with respect to that of system uncertainties on the residual set, which also leads to an on-line quadratic fractional programming problem. Different from Pourasghar et al. (2019), the obtained gain solution in Tan et al. (2020) can guarantee the global optimality via an equivalently parametric transformation.

The common point of the above works lies in that the optimal gain of the set-value observer for robust FD is
calculated by maximizing the influence of faults on the residual signal with respect to that of the system uncertainties, which finally leads to that the measured output is excluded from the corresponding output estimation set to achieve the goal of FD. Since the ultimate goal is to make the measured output out of the the corresponding output estimation set, in the context, we directly consider designing the optimal gain of set-value observer for robust FD via maximizing the exclusion tendency of the measured output from the corresponding output estimation set from a geometric perspective. The main contributions of this paper are summarized as follows:

- Solving the optimal gain of set-value observer for robust FD is formulated as a non-convex min-max optimization from a geometric perspective.
- The non-convex min-max optimization problem can be equivalently transformed into a linear programming via matrix analysis and duality theory, which lead to a global optimality of the observer gain at each time instant.

The reminder of this paper is organized as follows. Section 2 introduces the plant model, the set-value observer and the corresponding FD principle. Section 3 presents the main results of this paper regarding how to solve the observer gains. Section 4 uses a circuit model to verify the effectiveness of the proposed method. Finally, some conclusions are drawn in Section 5.

Notations: The symbol $I_n \in \mathbb{R}^{n \times n}$ and $O$ denote the identity matrix and the null matrix, respectively. $1$ and $0$ are vectors whose elements are composed of all $1$ and $0$, respectively. The Minkowski sum of two sets $X$ and $Y$ is given as $X \oplus Y = \{x + y | x \in X, y \in Y\}$. A zonotope $Z$ is defined as $Z = g \oplus \mathbb{B}^n$, where $g$ and $H$ are its center and generator matrix, respectively, $\mathbb{B}^n$ is an interval vector composed of $t$ unitary intervals $[-1, 1]$. Denote $Z = g \oplus \mathbb{B}^n$ as $Z = \{g | x \in \mathbb{B}^n\}$. Given two zonotopes $Z_1 = (g_1, H_1)$ and $Z_2 = (g_2, H_2)$, $Z_1 \oplus Z_2 = (g_1 + g_2, [H_1, H_2])$. Given a zonotope $Z = \{g | x \in \mathbb{B}^n\}$ and a compatible matrix $K$, $KZ = \{Kg | g \in KH\}$.

2. PROBLEM FORMULATION

2.1 Plant Model

Let us consider the following discrete linear time-invariant system:

$$x_{k+1} = Ax_k + Bu_k + Ew_k, \quad (1a)$$
$$y_k = Cx_k + Fv_k, \quad (1b)$$

where $x_k \in \mathbb{R}^n$, $u_k \in \mathbb{R}^m$ and $y_k \in \mathbb{R}^p$ denote the state, input and output of system at time instant $k$. $w_k \in \mathbb{R}^r$ and $v_k \in \mathbb{R}^q$ represent the process disturbance and measurement error, respectively. $A \in \mathbb{R}^{n \times n}$, $B \in \mathbb{R}^{n \times m}$, $E \in \mathbb{R}^{n \times r}$, $C \in \mathbb{R}^{p \times n}$ and $F \in \mathbb{R}^{q \times r}$ are constant matrices with appropriate dimensions. It is assumed that the disturbance $w_k$ and the measurement error $v_k$ are bounded by given zonotopes:

$$w_k \in W = \{w_c | H_w \}, \quad v_k \in V = \{v_c | H_v \},$$

where $w_c \in \mathbb{R}^r$, $v_c \in \mathbb{R}^q$, $H_w \in \mathbb{R}^{q \times r}$ and $H_v \in \mathbb{R}^{q \times q}$ are given constant vectors and matrices.

2.2 Set-value Observer

In order to implement robust FD, we construct the following set-value observer:

$$X_{k+1} = (A - L_k C)X_k \oplus \{B u_k \} \oplus \{L_k y_k \} \oplus \mathbb{E} \oplus (-L_k F) V \quad (2a)$$
$$Y_k = C X_k \oplus F V, \quad (2b)$$

where $X_k$ and $Y_k$ are the state-estimation set and output-estimation set of the system (1), respectively. $L_k \in \mathbb{R}^{n \times n}$ is the gain matrix of the set-value observer (2). Furthermore, the set-wise dynamics (2) can be rewritten as the following center-generator matrix form:

$$H_{k+1} = [(A - L_k C)H_k E H_w - L_k F H_v], \quad (3a)$$
$$x_{k+1}^g (A - L_k C)x_k^g + Bu_k + L_k y_k + E w_c - L_k F v_c, \quad (3b)$$
$$M_k = [C H_k F H_v], \quad (3c)$$
$$y_k^g = C x_k^g + F v_c, \quad (3d)$$

where $x_k^g$ and $y_k^g$, and $H_k$ and $M_k$ are the centers and generator matrices of $X_k$ and $Y_k$, respectively. It is assumed that the initial state $x_0$ is contained in the initial state-estimation set $X_0$ of the set-value observer (2), i.e., $x_0 \in X_0$.

2.3 Robust Fault Detection

As for the system (1) and the corresponding set-value observer (2), if the initial condition $x_0 \in X_0$ holds, it always follows $y_k \in Y_k \forall k > 0$ as long as the system (1) operates in the healthy situation. In other words, if we detect that $y_k \notin Y_k$ at some time instant, it means that the system (1) becomes faulty. Indeed, we can construct the following residual set

$$R_k = \{ -y_k \} \oplus Y_k. \quad (4)$$

Then, we can real-timely check whether the inclusion relationship

$$0 \in R_k \quad (5)$$

holds or not. If (5) violates, i.e., $0 \notin R_k$, it means that the system (1) is faulty. Otherwise, we consider that the system (1) still operates in the healthy situation. Therefore, in the following, we will consider designing the optimal observer gain $L_k$ such that the origin $0$ does not belong to the residual set $R_{k+1}$ as much as possible.

3. MAIN RESULTS

3.1 Solving Optimal Gain

Based on the previous analysis, our key goal is to design the observer gain $L_k$ such that the constraint condition $0 \notin R_{k+1}$ holds as much as possible at each time instant. We first introduce the following lemma to equivalently transform the constraint $0 \notin R_{k+1}$ into a linear programming problem.

Lemma 3.1. The FD problem $0 \notin R_{k+1}$ holds if and only if $\hat{\phi} < 0$, where $\hat{\phi}$ is computed by the following linear programming:

$$\hat{\phi} \triangleq \max_{\xi \in \mathbb{R}^p, \phi \in \mathbb{R}} \phi \quad (6a)$$
$$s.t. \quad A_\xi \xi + v_L = 0, \quad (6b)$$
$$\phi + \|\xi\|_\infty \leq 1,$$
\[ A_L = [-CL_kCH_k + CAH_k CEH_w - CL_kFH_v FH_v] \]

\[ v_L = CL_k(-C\tau_k + y_k - Fv') - y_{k+1} + CAx_k + CBu_k + CEw + Fv'. \]

**Proof 1.** By combing (3) and (4), the residual set \( R_{k+1} \) can be computed as

\[ R_{k+1} = \{ v_L, A_L \} \]

where \( A_L \in \mathbb{R}^{n \times n} \) and \( v_L \in \mathbb{R}^n \) are denoted as in (7). Firstly prove \( 0 \notin R_{k+1} \Rightarrow \hat{\phi} < 0 \). If \( 0 \notin R_{k+1} \), then \( \hat{\phi} \in R^n \) such that \( \|\xi\|_{\infty} \leq 1 \) and \( A_L \xi + v_L = 0 \). In this case, the optimization problem (6) has no feasible solution with \( \phi \geq 0 \). Thus it follows \( \hat{\phi} < 0 \). Secondly prove \( \hat{\phi} < 0 \Rightarrow 0 \notin R_{k+1} \). If \( \hat{\phi} < 0 \), then there cannot exist a feasible solution of the optimization problem (6) with \( \phi \geq 0 \), and hence \( \hat{\phi} \in R^n \) such that \( \|\xi\|_{\infty} \leq 1 \) and \( A_L \xi + v_L = 0 \). That is \( 0 \notin R_{k+1} \). \( \Box \)

Based on Lemma 3.1, we need to design the observer gain \( L_k \) such that \( \hat{\phi} < 0 \) (i.e., \( 0 \notin R_{k+1} \)) holds as much as possible. That means, we could design the observer gain \( L_k \) to minimize \( \phi \) such that the exclusion tendency of the origin 0 from the residual set \( R_{k+1} \) could be maximized. Thus, a min-max optimization problem can be formulated as

\[ \min \hat{\phi} = \min_{L_k} \max_{\phi} \phi \quad \text{s.t.} \quad (6a) \to (6b). \]

**Theorem 3.1.** As for the min-max two-level optimization problem (8), it can be equivalently transformed into the following single-level minimization problem:

\[ \min_{L_k, \lambda_1, \lambda_2, \mu} -\lambda_1^T \mathbf{1} + \lambda_2^T \mathbf{1} + \mu^T v_L \quad \text{s.t.} \quad (9a) \to (9b), \]

\[ \mu^T C E H_w - \lambda_1^T \mathbf{1} + \lambda_2^T \mathbf{1} = 0, \]

\[ \mu^T C L_k C H_k + \mu^T C A H_k - \lambda_1^T \mathbf{1} + \lambda_2^T \mathbf{1} = 0, \]

\[ \mu^T C L_k F H_v - \lambda_1^T \mathbf{1} + \lambda_2^T \mathbf{1} = 0, \]

\[ \lambda_1^T = \left[ \lambda_1^T \lambda_2^T \lambda_1^T \lambda_2^T \right], \]

\[ \lambda_2^T = \left[ \lambda_1^T \lambda_2^T \lambda_1^T \lambda_2^T \right], \]

\[ \lambda_1 \leq 0, \lambda_2 \leq 0. \]

**Proof 2.** Considering the inner optimization problem (6) of the min-max optimization problem (8), it is equivalent to

\[ \hat{\phi} = \max_{\xi \in \mathbb{R}^n, \phi \in \mathbb{R}} \phi \quad \text{s.t.} \quad (6a), \phi - 1 - \xi_i \leq 0, \phi - 1 + \xi_i \leq 0, 1 \leq i \leq n, \]

where \( \xi_i \) is the i-th element of \( \xi \). By constructing Lagrange multipliers \( \lambda_1 \leq 0, \lambda_2 \leq 0 \) and \( \mu \), Lagrange function for the optimization problem (10) can be obtained as

\[ L(\xi, \phi, \lambda_1, \lambda_2, \mu) = \phi + \sum_{i=1}^{n} \lambda_{1,i}(\phi - 1 - \xi_i) + \mu^T (A_L \xi + v_L). \]

where \( \lambda_{1,i} \) and \( \lambda_{2,i} \) are the i-th elements of the Lagrange multipliers \( \lambda_1 \) and \( \lambda_2 \), respectively. Then, the corresponding dual function can be computed as

\[ D(\lambda_1, \lambda_2, \mu) = \sup_{\xi, \phi} L(\xi, \phi, \lambda_1, \lambda_2, \mu) \]

\[ = \left\{ \begin{array}{ll} \phi_d + 1 + \lambda_1^T \mathbf{1} + \lambda_2^T \mathbf{1} = 0, & \phi_d < 0, \\
\infty, & \text{otherwise}, \end{array} \right. \]

where \( \phi_d = -\lambda_1^T \mathbf{1} + \lambda_2^T \mathbf{1} + \mu^T v_L \). Then the dual problem of (10) can be obtained as

\[ \min_{\lambda_1, \lambda_2, \mu} \phi_d \quad \text{s.t.} \quad (9a), (9b), \]

\[ -\lambda_1^T + \lambda_2^T + \mu^T A_L = 0. \]

It must point out that since the optimization problem (10) is a linear programming, the dual problem (13) and the original problem (10) should share the same optimal value, i.e., \( \phi = \hat{\phi} \). Therefore, the min-max optimization problem (8) can be further transformed into the following single-level programming:

\[ \min_{L_k, \lambda_1, \lambda_2, \mu} \phi_d \quad \text{s.t.} \quad (9a), (9b), (13b). \]

Considering \( A_L = [-CL_kCH_k + CAH_k CEH_w - CL_kFH_v FH_v] \), the equality constraint (13b) can be split into a series of constraints (9b)-9g). The proof is completed. \( \Box \)

Although the min-max two-level optimization problem (8) can be equivalently transformed into the single-level programming (9) based on Theorem 3.1, the single-level programming (9) is still not a convex optimization problem and can not be solved directly owing to the involvement of the quadratic term \( \mu^T C L_k \). In order to deal with this problem, we first revisit the following lemma in White and Speyer (1987).

**Lemma 3.2.** (White and Speyer, 1987) If \( D, S \) and \( Q \) are matrices of dimension \( n \times m, m \times n \) and \( n \times r \), respectively, where \( n \geq m \geq r \) and \( \text{rank}(S) = r \), then the general solution of \( DS = Q \) is given by \( D = Q S^* + R(1 - SS^*) \), where \( R \) is an arbitrary \( n \times m \) matrix and represents the freedom left in \( D \) after satisfying \( DS = Q \), and \( S^* \equiv (SS^*)^{-1}S^T \) is the Moore-Penrose pseudo left inverse.

**Theorem 3.2.** As for the single-level optimization problem (9), the optimal gain \( L_k^* \) can be computed as

\[ L_k^* = a^T b^T + (1 - a^T a^T) R^T, \]

\[ a = C^T \mu^*, \]

\[ b = U S^{-1} Q V^T (\lambda_2^* - \lambda_1^* + H_k^T A^T \mu^*), \]

where \( R \) is an arbitrary matrix with appropriate dimensions, and the pair \((U, \Sigma, V)\) constructs Singular Value Decomposition (SVD) of \( CH_k \), i.e., \( CH_k = U \Sigma O V^T \), and \( \mu^*, \lambda_1^* \) and \( \lambda_2^* \) are the optimal solutions of the following linear programming problem:

\[ \min_{\lambda_1, \lambda_2, \mu} -\lambda_1^T \mathbf{1} + \lambda_2^T \mathbf{1} - (\lambda_1^T - \lambda_2^T d_1 + \mu^T d_2), \quad \text{s.t.} \quad (9a), (9c), (9e) - (9h), \]

\[ (\lambda_1^T - \lambda_2^T - \mu^T C A H_k) V Q_2 = O, \]

\[ (\lambda_1^T - \lambda_2^T - \mu^T C A H_k) V Q_1 S^{-1} U^T F H_v - \lambda_1^T + \lambda_2^T = 0, \]
with
\[ Q_1 = [I \ O], Q_2 = [O \ I], \]
\[ d_1 = VQ_1\Sigma^{-1}UT(Cx_k - y_k + Fv^c), \]
\[ d_2 = CAHd_1 - y_{k+1} + CAzx_k + CBu_k + CEwc + FVc. \]

**Proof 3.** As for the constraint (9b), let us consider SVD of \( CH_k \) as \( CH_k = U(\Sigma \ O)V^T \), where both \( U \) and \( V \) are orthogonal matrices, and \( \Sigma \) is a diagonal matrix with positive diagonal elements. Then the constraint (9b) is rewritten as
\[ \begin{bmatrix} -\mu^T CL_k (\Sigma \ O) V^T \end{bmatrix} + \mu^T CAH_k - \lambda_1^{tr} + \lambda_2^{tr} = 0. \] (17)

By letting \( Q_1 = [I \ O], Q_2 = [O \ I] \), the equality constraint (17) can be equivalently transformed into the following two constraints:
\[ (\lambda_1^{tr} - \lambda_2^{tr} - \mu^T CAH_k)VQ_1 = 0, \] (18a)
\[ (\lambda_1^{tr} - \lambda_2^{tr} - \mu^T CAH_k)VQ_2 = 0. \] (18b)

We can further solve \( \mu^T CL_k \) from (18a) as
\[ \mu^T CL_k = (\lambda_2^{tr} - \lambda_1^{tr}) + \mu^T CAH_k)VQ_1 \Sigma^{-1}U^T. \] (19)

By substituting (19) into the objective \(-\lambda_1^{tr}1 + \lambda_2^{tr}1 + \mu^T v_L \) of the optimization problem (9), we can obtain the equivalent objective in the optimization problem (16). Similarly, by substituting (19) into the constraint (9d), the equivalent constraint (16c) can be obtained. Then, by removing the quadratic equality constraint (19), we can obtain the linear programming (16) based on the single-level optimization problem (9). The unique difference between the optimization problem (9) and the linear programming (16) lies in that there is an extra quadratic equality constraint (19) for the optimization problem (9). Furthermore, it can be found that the linear programming (16) no longer contains the decision variable \( L_k \). Therefore, we can first compute the optimal solutions \( \lambda_1^{tr}, \lambda_2^{tr} \) and \( \mu^* \) for the optimization problem (16) and then substitute them into the equality constraint (19) to solve \( L_k^* \). Based on Lemma 3.2, \( L_k^* \) can be directly computed as in (15). In this case, obviously, \( \lambda_1^{tr}, \lambda_2^{tr}, \mu^* \) and \( L_k^* \) are also the optimal solutions of the single-level optimization problem (9). The proof is completed. \( \square \)

4. APPLICATION TO A CIRCUIT MODEL

In order to illustrate the effectiveness of the proposed method, we use an electric circuit taken from Tan et al. (2019) as a case study to show the results of robust FD and the whole electric circuit chart is shown in Fig. 1, where \( R_i (i = 1, \ldots, 8) \) and \( L_i (i = 1, 2) \) represent the resistors and inductors, respectively. \( R_1 = 10\Omega, R_2 = 8\Omega, R_k = 109\Omega, L_1 = 0.3H, L_2 = 0.65H, c_1(t) \) and \( c_2(t) \) are the voltage sources used as inputs. The system states are the loop currents \( i_1(t) \) and \( i_2(t) \), and the measured outputs are the voltages of resistors \( R_1 \) and \( R_6 \), i.e., \( U_{R_1} \) and \( U_{R_6} \). The sampling period is set as \( T_s = 0.01s \). The related parametric matrices are given as
\[ A = \begin{bmatrix} 0.5187 & 0.0467 \\ 0.0215 & 0.4646 \end{bmatrix}, B = \begin{bmatrix} 0.0033 & 0.0040 \\ 0.0062 & 0.0031 \end{bmatrix}, C = \begin{bmatrix} 10 & 0 \\ 0 & 26 \end{bmatrix}, \]
\[ E = \begin{bmatrix} 0.6324 & 0.2785 \\ 0.0975 & 0.5469 \end{bmatrix}, F = \begin{bmatrix} 0.9575 & 0.1576 \\ 0.9649 & 0.9760 \end{bmatrix}. \]

The process disturbance set \( W \) and the measurement noise set \( V \) are set as \( W = \{0, 0.02I_2\} \) and \( V = \{0, 0.02I_2\} \), respectively. The system inputs are given as \( u_k = [25\sin(\pi k/25) \ 2k]^T \). The initial state set is \( X_0 = \{0, 0.5I_2\} \). We set the following fault scenario: from \( k = 1 \) to \( k = 30 \), the system (1) operates in the healthy situation, and then the system (1) becomes faulty after \( k = 30 \). Here we consider the following three cases to compute the observer gain \( L_k \) of the set-value observer (2) for robust FD:

- Given a fixed observer gain \( L_k = \begin{bmatrix} -0.0181 & 0.0018 \\ 0.0022 & -0.0091 \end{bmatrix} \).
- Computing the optimal gain \( L_k^* \) using the method in Tan et al. (2020).
- Computing the optimal gain \( L_k^* \) using the proposed method in the context.

The FD results of the above three cases are shown in Figs. 2, 3 and 4, respectively. Regarding the fixed observer gain, it can be found from Fig. 2 that the system fault

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**Fig. 1.** An electric circuit chart.

**Fig. 2.** Robust FD by using a fixed observer gain.

**Fig. 3.** Robust FD based on the method in Tan et al. (2020).
is not detected until the time instant $k = 38$, i.e., $y_{k} \notin Y_{38}$. Compared to the fixed observer gain, the method computing the optimal gain $L_{k}^{*}$ in Tan et al. (2020) has a higher sensitivity of FD and the fault is detected at time instant $k = 34$, i.e., $y_{34} \notin Y_{34}$, which is shown in Fig. 3. By using the proposed method in this context, it can be found from Fig. 4 that the fault is immediately detected at time instant $k = 31$ after the fault occurrence, i.e., $y_{31} \notin Y_{31}$, and there is no time delay for FD, which indicates a better performance of robust FD.

5. CONCLUSIONS

This paper proposes a novel method designing the optimal gain of set-value observer for robust fault detection (FD). The optimal observer gain is designed such that the exclusion tendency of the origin from the residual set could be maximized, which formulates a two-level min-max optimization problem. Although this non-convex optimization problem can not be solved directly, it is proved that this problem can be equivalently transformed into a linear programming based on matrix decomposition and duality theory. In the future, we will consider extending the results to linear parameter varying systems and fault-tolerant control field.

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