The Nucleon-Nucleon Problem in Quark Models

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Abstract. In the first part we summarize the status of the nucleon-nucleon (NN) problem in the context of Hamiltonian based constituent quark models and present results for the $\ell = 0$ phase shifts obtained from the Goldstone-boson exchange model by applying the resonating group method. The second part deals with the construction of local shallow and deep equivalent potentials based on a Supersymmetric Quantum Mechanics approach.

1 Introduction

The strong interaction between two nucleons is the basic ingredient of nuclear physics. Since about 25 years there have been many efforts to derive the nuclear forces from the underlying theory of strong interactions, the Quantum Chromodynamics (QCD). As QCD cannot be solved exactly in the low energy regime, QCD-inspired models have been used both in baryon spectroscopy and the NN problem. We first briefly describe the situation in the framework of Hamiltonian based quark models. Next, we present recent progress obtained from using the Goldstone-boson exchange model.

After some pioneering work in the late ’70 a major breakthrough in the microscopical derivation of the NN interaction has been achieved at the beginning of the 80’s with the work of Oka and Yazaki who used the resonating group method to derive phase shifts from the one-gluon exchange model, the work of Harvey on the symmetries of the most important six-quark states and the work of Golli, Rosina and collaborators on local effective nucleon-nucleon interactions.

Presently there are several review papers available on the subject. They show that the following steps are important: 1) the choice of the quark model, 2) the choice of the six-quark basis states, 3) the method to calculate the phase shifts. A challenge is to describe both the nucleon, as a three-quark system, and the NN interaction, as a six-quark problem, using the same quark model.

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2 Quark models

The quark models used so far in the NN problem are mostly nonrelativistic. Studies based on the one-gluon exchange (OGE) model explained the short-range repulsion as due to the color-magnetic interaction combined with quark interchange between $q^3$ clusters. But in order to reproduce the scattering data some extra medium- and long-range repulsion was necessary. This was added at the baryon level. That is why more consistent models, called hybrid, have subsequently been introduced where all interactions are introduced at the quark level. In such models the short-range repulsion is still attributed to the OGE interaction and the medium- and long-range attraction is due to scalar and pseudoscalar meson exchange. To our knowledge, these models have problems in fitting the nucleon resonances and the NN interaction with the same parameters.

Here we present results derived from the Goldstone-boson exchange (GBE) model which has been successful in describing the baryon properties. The pseudoscalar meson exchange interaction between quarks, which is spin and flavour dependent, contains both a long and a short range part, appropriate for the NN problem. In this work we use the parametrization of ref. The technique proposed by Kamimura is employed the resonating group method to calculate the $\ell = 0$ phase shifts by using the technique proposed by Kamimura.

3 Phase shifts in the GBE model

![Figure 1](image.png)

**Figure 1.** The $^1S_0$ NN scattering phase shift. The coupled channel calculations are from Ref. The crosses are the Nijmegen fit to the data.

The phase shifts obtained in Ref. from the GBE model also imply that the NN potential is strongly repulsive at short range. This means that
the repulsion can equally be explained as due to a flavour-spin interaction combined with quark interchange. The repulsion obtained from the GBE model is somewhat stronger than that produced by the OGE model. However, to reproduce the experimental $^1S_0$ phase shift a certain amount of middle-range attraction was necessary. This has been provided by the addition of a scalar meson exchange interaction of the form \[1\]

$$V_\sigma = -\frac{g_\sigma^2}{4\pi} \left( \frac{e^{-\mu_\sigma r}}{r} - \frac{e^{-A_\sigma r}}{r} \right)$$

where we chose $\frac{g_\sigma^2}{4\pi} = \frac{g^2}{4\pi} = 1.24$, $\mu_\sigma = 278$ MeV, $A_\sigma = 337$ MeV. Interestingly, the fit to the data favours $\mu_\sigma = 2m_\pi$, consistent with the findings of Ref. \[12\]. Fig. 1 shows that the addition of \[1\] leads to a good agreement with the data over a large energy interval, the best result being obtained with three coupled channels NN+$\Delta\Delta$+CC. The quality of the baryon spectrum remains unchanged after the addition of \[1\].

\[Figure 2.\] The $^3S_1$ NN scattering phase shift: with $\sigma$-meson exchange only (full line), with $\sigma$-meson exchange + full tensor interaction with \[2\] with $G_f = 33$ (dotted line), with $\sigma$-meson exchange + the first term of the tensor interaction \[2\] only, still with $G_f = 33$ (dot-dashed line). The crosses are the Nijmegen fit \[13\] to the data.

Besides a spin-spin, the pseudoscalar meson exchange gives rise to a tensor term as well

$$V_\gamma^T(r_{ij}) = G_f \frac{g_\gamma^2}{4\pi} \frac{1}{12m_im_j} \times \left\{ \frac{\mu_\gamma^2(1 + \frac{3}{\mu_\gamma^2r^2})}{r} - \frac{A_\gamma^2(1 + \frac{3}{A_\gamma^2r^2})}{r} \right\}$$

(2)
where $\gamma = \pi, \eta$ and $\eta'$. It was found [13] that the tensor term used in baryon spectroscopy with $G_f = 1$ had to have $G_f = 33$ in order to fit the experimental $^3S_1$ phase shift. This is shown in Fig. 2. The tensor term (2) has a usual Yukawa type part, depending on the pseudoscalar meson masses $\mu_\gamma$ and a regularized part, containing cut-off parameters $\Lambda_\gamma$. If the latter term is removed, the attraction increases. Then a smaller factor, $G_f = 12$, is required to reproduce the data. Further details of these studies can be found in Ref. [13]. It would certainly be useful to search for alternative parametrizations of the GBE model which could better fit the NN phase shifts.

4 SUSY approach to local phase shift equivalent potentials.

A microscopic derivation of the NN interaction, as above, leads to a nonlocal potential. This is a consequence of the complex structure of the interacting nucleons. As in the $\alpha - \alpha$ scattering [15], the wave function of this nonlocal potential presents a node in the $l = 0$ partial wave [10]. It means that if one would try to describe the interaction between two nucleons by an equivalent local potential, this potential would have an extra $l = 0$ state. This is an unphysical state known as Pauli forbidden state. The mathematical relation between such deep potentials and phenomenological shallow potentials (no bound state), as e.g. the Reid soft core potential [16], can interestingly and successfully be described through a Supersymmetric (SUSY) Quantum Mechanics approach. The procedure has been originated by Sukumar [17] and applied as a two-step SUSY transformation to the NN scattering by Sparenberg and Baye, see e.g. ref. [18].

Figure 3. The shallow potential $V_6$ (solid line) compared to Reid68 [16] (dotted line), Baye & Sparenberg [18] (dashed line) and Reid93 [14] (dots) potentials.

Here we present an alternatively new procedure [19] based on phase equivalent chains of Darboux (or SUSY) transformations at fixed $\ell$. These chains contain $N$ successive transformations instead of two, as in ref. [18]. For poten-
Figure 4. Asymptotic behaviour of the absolute value of $V_6$ (solid line), Reid68 [16] (dotted line), Baye & Sparenberg [18] (dashed line) in natural logarithmic scale.

...tials related by a chain of transformations we derive an analytic expression for the Jost function and hence for the phase shift. They both contain N parameters, related to the N unphysical eigenvalues of a starting Hamiltonian, used to construct the chain. It is convenient to choose this Hamiltonian as the free particle Hamiltonian. These parameters are also poles of the S-matrix and can be fixed by a fit to the experimental phase shift. They can be distributed between poles and zeros of a Jost function and each distribution corresponds to a different potential. We applied this method to derive a shallow and a family of deep phase equivalent potentials for the $l = 0$ neutron-proton scattering. The $^1S_0$ phase shift has optimally been fitted with $N = 6$ S-matrix poles. The shallow potential, denoted by $V_6$, can be seen in Fig. 3. It is very close to the Reid soft core potential, denoted by Reid68, and also close to its updated version, called Reid93 [14]. The major improvement over the results of Ref. [18] can be seen in Fig. 2. The unwanted oscillations in the potential tail [18] have disappeared so that the behaviour of $V_6$ is consistent with Yukawa’s theory. A deep phase equivalent potential, accommodating a (Pauli forbidden) bound state, was found by adding two more poles. This is the supersymmetric partner of $V_6$. Varying the available parameters we brought this potential close to that of Ref. [20], inspired from microscopic calculations as the sum of a Gaussian plus a Yukawa potential tail.

Thus the use of chains of Darboux (SUSY) transformations at fixed $\ell$ provides a powerful method for getting shallow and deep phase equivalent potentials for $l = 0$ partial waves. Studies of $l \neq 0$ phase equivalent potentials based on $\ell$-changing Darboux transformations are underway.

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