Radial profile of dissipation rate for turbulent flow of incompressible fluid in round direct pipe based on generalized local balance model

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Abstract. The article is focused on the problem of generalized local balance turbulence model. Generalized local balance turbulence model is considered. The one-parameter turbulence model is based on semi-empirical dissipation rate transport equation with negative coefficient of diffusion. To determine a coefficient of this equation, Laplace’s method of asymptotic estimation of a double integral is used. To compute radial dissipation profile the turbulence model is applied to the pipe flow. The known wall and axial functions are used as boundary conditions. The turbulence model is applied to the pipe flow to compute radial dissipation profile, well-known wall and axial functions are used as boundary conditions. Comparisons with known experimental data are given.

1. Introduction

The model of of generalized local balance was put forward by one of the paper authors and successfully applied to turbulent Taylor – Couette flow between two coaxial rotating cylinders [1-3]. Sometimes its importance for turbulence is compared with hydrogen atom in quantum mechanics [4].

Another important example is simple turbulent flow in round direct pipe under pressure gradient. Among theoretical models we should mention such as Prandtl mixing length [5], von Karman’s theory [6], one- and –many parameters models which are supposed to use additional semi-empirical transport equations for turbulent averaged variables [7] and a very common “κ – ε” model to transport kinetic turbulence energy and an average rate of dissipation of turbulence energy. [8,9].

There are only disputable deductions of equation of the model from the initial Navier-Stokes equations [10-14]. This fact proves the exisistance of alternative models of turbulence. Sometimes new mathematical approaches are applied to describe turbulence such as group analysis of Navier - Stokes equations [15].

Direct simulation methods (dns), based on discretization of Navier - Stokes equations, are widely used with growth of supercomputer capacity [16]. The absence of a solution for 3d Navier - Stokes equation leads to the necessity to compare the dns results with the experiments.

We should mention that new experimental data on complicated spatio - temporal structure of turbulence [17] require higher standards for turbulence models.
2. Equations of the model

Extended model of local balance for fully developed turbulence of incompressible fluids put forward by first author (B.A.M.) in [1], consists of closed five differential equations for five variables: three components of large-scale velocity \( U_i \), modified pressure \( p^* \) and mean dissipation rate of turbulent energy \( \varepsilon \). We should mention that transport equation for \( \varepsilon \) is taken with negative coefficient diffusion, describing the merging of vorticity blobs.

The model belongs to one-parameter turbulence models. Closed model of generalized balance is:

\[
\partial_t U_i + U_j \partial_j U_i = -\partial_i p^* + \partial_t \left[ \frac{\varepsilon}{S^2} (\partial_i U_j + \partial_j U_i) \right] - \alpha_1 \Delta \left( \frac{\varepsilon^2}{S^3} \right) U_i, \tag{1}
\]

\[
\partial_t \varepsilon + U_j \partial_j \varepsilon = \alpha_0 \left[ \varepsilon S - \frac{1}{\kappa^2} \partial_j (\varepsilon \frac{S^2}{\kappa} \partial_j \varepsilon) \right] - \alpha_2 \Delta \left( \frac{\varepsilon^2}{S^5} \right), \tag{2}
\]

\[
\partial_i U_i = 0, \tag{3}
\]

where \( \partial_j \partial_i \) - Laplace operator, an expression for coefficient of turbulent viscosity \( \nu_t \) is deduced from approximate balance equation of turbulent energy:

\[
\nu_t = \frac{\varepsilon}{S^2}, \tag{5}
\]

diffusion coefficient \( D_\varepsilon \) of dissipation \( \varepsilon \) is defined as

\[
D_\varepsilon = -\frac{\alpha_0}{\kappa^2 S^2} \varepsilon \quad \text{from the dimensional considerations, minus sign is a consequence from a mutual compensation the first and the second terms in the right hand side of the equation (2) in a domain of validity the wall asymptotes for large-scale velocity and dissipation rate. Structure of the next terms with high order spatial derivatives, describing “hyperdiffusion” of large-scale velocity \( \mathbf{U} \) and dissipation \( \varepsilon \), follows from dimensional and regularization considerations [1], constants \( \alpha_0, \alpha_1, \alpha_2 \) are the model ones. Value of constant \( \alpha_0 \) will be determined in the next section. Values \( \alpha_1 \) and \( \alpha_2 \) can be obtained from a simulation of the non-stationary problem, connecting fluctuations of large-scale velocity \( \mathbf{U} \) and dissipation rate \( \varepsilon \) with the Kolmogorov theory [18,19].}

Boundary conditions at the wall coincide with the standard two-parameters \('k-\varepsilon'\) turbulence model asymptotes:
\[ U = \frac{u_*}{\kappa} \ln \frac{C u_* x}{\nu}, \]  

where \( \kappa = 0.4 \) and \( C = 10 \) are Prandtl-von-Karman constants, and dissipation \( \varepsilon \) 

\[ \varepsilon = \frac{u_*^3}{\kappa x}, \]  

\( u_* \) - friction velocity, \( x \) - distance from the wall.

3. Determination of the model constant \( \alpha_0 \)

Let us write the transport equations for two polarization Fourier components of the small scale velocities under an influence of large-scale velocities gradients, following [20]: 

\[ (\partial_t + v k^2)u_{ij}(k,t) = a_{ij\mu}u_{\mu}(k,t) + \sum_{\rho,q,p,q,k} \Phi^{\alpha\beta\gamma}(k,p,q)u_{ij}(p,t)u_{\rho}(k,t), \]  

where Greek indexes run values 1 and 2, \( \nu \) - is molecular viscosity, 

\[ a_{ij\mu} = -e_j^\mu e_i^\eta \partial_\eta V_j. \quad \Phi^{\alpha\beta\gamma}(k,p,q) = -i k_m e_j^\eta(k) e_j^{\alpha}(p) e_m^{\beta}(q), \]  

where down indexes run values 1,2,3 and a sum over repeating indexes is assumed, \( e^1(k), e^2(k) \) - unit polarization vectors, which are perpendicular each other and the wave vector \( k \). Cartesian components of wave vector \( k \) link with standard spherical coordinates \( k, \phi, \theta \) as:

\[ k = (k \sin \theta \cos \varphi, k \sin \theta \sin \varphi, k \cos \theta), \quad k = |k|, 0 \leq \varphi < 2\pi, 0 \leq \theta \leq \pi \]  

We choose \( e^1 \) and \( e^2 \) as:

\[ e^1 = \frac{e^3 \times k}{|e^3 \times k|}, \]

\[ e^2 = \frac{k \times e^1}{|k \times e^1|}, \]

where \( e^3 = (\cos \varphi, \sin \varphi, 0) \).

Then we obtain from these formulas the next expressions:

\[ e^1 = (-\sin \varphi, \cos \varphi, 0), \quad e^2 = (\cos \varphi \cos \theta, \sin \varphi \cos \theta, -\sin \theta). \]

Coefficients in conventional spherical coordinate system \( \Phi^{\alpha\beta\gamma}(k,p,q) \) can be written in explicit form:

\[ \Phi^{111} = ik \cos(\varphi_k + \varphi_p) \sin(\varphi_k - \varphi_p), \]

\[ \Phi^{112} = ik \cos(\varphi_k + \varphi_p)[\sin \theta_k \cos \theta_p \cos(\varphi_k - \varphi_p) - \cos \theta_k \sin \theta_p], \]

\[ \Phi^{121} = -ik \sin \theta_k \sin(\varphi_q - \varphi_k) \cos \theta_p \sin(\varphi_k - \varphi_p), \]

\[ \Phi^{122} = -ik[\sin \theta_k \cos \theta_p \cos(\varphi_k - \varphi_q) - \cos \theta_k \sin \theta_p] \cos \theta_p \sin(\varphi_k - \varphi_p). \]
\Phi^{211} = -ik \sin \theta_k \cos \theta_k \sin(q - \varphi_k) \sin(p - \varphi_k)
\Phi^{212} = -ik \cos \theta_k \sin(p - \varphi_k) [\sin \theta_k \cos \theta_q \sin(\varphi_k - \varphi_q) - \cos \theta_k \sin \theta_q]
\Phi^{221} = -ik \sin \theta_k \sin(q - \varphi_k) [\cos \theta_k \cos \theta_p \sin(\varphi_k - \varphi_p) + \sin \theta_k \sin \theta_p]
\Phi^{222} = -ik [\cos \theta_k \cos \theta_p \sin(\varphi_k - \varphi_p) + \sin \theta_k \sin \theta_p] [\sin \theta_k \cos \theta_q \cos(\varphi_k - \varphi_q) - \cos \theta_k \sin \theta_q]

Anisotropy of turbulence is due to large scale gradients of large-scale velocity inserted in coefficients $a_{\mu \nu}$ in equality (8).

If tensor-gradient of large scale velocity has the only one distinction from zero component (the case of the simple shear) $\partial_1 V_2 = S \neq 0$, then we shall have the next expressions for these coefficients:

$$a_{11} = S \sin \varphi \cos \varphi$$
$$a_{12} = S \cos^2 \varphi \cos \theta$$
$$a_{21} = -S \sin^2 \varphi \cos \theta$$
$$a_{22} = -S \sin \varphi \cos \varphi \cos^2 \theta$$

Neglecting viscosity and nonlinear terms, we obtain system of two linear differential equations with constant coefficients, angular variables are considered in them as parameters:

$$\partial_t u_1 = a_{11} u_1 + a_{12} u_2$$  \hspace{1cm} (10)
$$\partial_t u_2 = a_{21} u_1 + a_{22} u_2$$  \hspace{1cm} (11)

General solution of these equations can be obtained by expressing variable $u_1$ from equation (12) as:

$$u_1 = \frac{1}{a_{21}} \partial_t u_2 - \frac{a_{22} u_2}{a_{21}}$$  \hspace{1cm} (13)

and inserting this expression in equation (10), we obtain linear homogeneous equation with constant coefficients for variable $u_2$. General solution of this equation is:

$$u_2 = C_1 + C_2 e^{\lambda t}$$  \hspace{1cm} (14)

Then we obtain from expression (13) a solution:

$$u_1 = -\frac{a_{22}}{a_{21}} C_1 + \frac{\lambda_2 - a_{22}}{a_{21}} C_2 e^{\lambda t}$$  \hspace{1cm} (15)

where eigenvalues $\lambda_1$ and $\lambda_2$ (let assume $\lambda_1 \neq \lambda_2$, these depend on two angles of spherical coordinate system) are given by expressions:

$$\lambda_1 = 0;$$
$$\lambda_2 = \frac{S}{4} \sin(2\varphi)[1 - \cos(2\theta)].$$
Constants \( C_1, C_2 \) can be obtained with initial conditions for two polarization components \( u_1 \) and \( u_2 \).

We can write down an expression for mean dissipation rate of kinetic energy (we denote mean dissipation as dissipation, averaging over time, much more then turnover period of large energy containing eddies \( St \gg 1 \), following [21]):

\[
\varepsilon = \nu \left\{ \frac{2\pi}{2\pi} \int_0^{2\pi} \sin \phi d\phi \int_0^{2\pi} \sin \theta d\theta \frac{u_1^* u_1 + u_2^* u_2}{2} \right\}. \tag{16}
\]

We shall seek for behavior of dissipation rate \( \varepsilon \) at \( p = St \gg 1 \). Because expressions (17) and (14) have unstable modes, corresponding to \( \lambda_2 > 0 \) at some angles \( \phi \) and \( \theta \), we can neglect neutral modes, which are proportional to \( C_1 \). Finally we can write down the next expression:

\[
\frac{u_1^* u_1 + u_2^* u_2}{2} \approx F(k, \theta, \varphi) e^{2\lambda_2^*},
\]

where \( F(k, \theta, \varphi) \) is kinetic energy at \( t = 0 \).

The problem is to estimate an integral over angular variables, that is internal one for dissipation integral (15):

\[
I(k) = \frac{2\pi}{2\pi} \int_0^{2\pi} \sin \phi d\phi \int_0^{2\pi} \sin \theta d\theta e^{2\lambda_2^*}.
\]

It can be done with Laplace’s method for double integral [21].

There are two points, corresponding to maximum of \( \lambda_2 \), \( \lambda_{2\text{max}} = \frac{S}{2} \) at each point:

\[
\theta = \pi/2; \quad \varphi = \pi/4 \quad \text{and} \quad \theta = 3\pi/2; \quad \varphi = 5\pi/4
\]

The next asymptotic expansion for the double integral over angular variables \( I(k) \):

\[
I(k) \propto \exp(St)(St)^{-3} \sum_{i=0}^{\infty} a_i (St)^{-i},
\]

\( a_i \) are known numerical coefficients \( (i=0,1,2,\ldots) \), \( a_0 \) depends on Hessian of \( \lambda_2^* (\theta, \varphi) \) function at maximum points, the expression can be differentiated for any number of times [21].

Inserting the expression in dissipation integral (16), we obtain an asymptotic expansion for dissipation rate under shear in linear approximation:

\[
\varepsilon \propto \exp(St)[a_0(St)^{-1} + a_1(St)^{-2} + a_2(St)^{-3} + \ldots] \tag{17}
\]

We can rewrite expansion (16) as:

\[
\varepsilon(p) = \varepsilon_0 \exp(p)[a_0 p^{-1} + a_1 p^{-2} + \ldots] \tag{18}
\]

where \( \varepsilon_0 \) is dissipation rate at initial moment of time, we assume that \( p = St \gg 1 \).

Taking derivative of \( \varepsilon \) with respect to \( p \), we obtain:
\[
\frac{\partial \varepsilon}{\partial p} = \varepsilon_0 \exp(p)[a_0 p^{-1} + a_1 p^{-2} + \ldots] + \varepsilon_0 \exp(p)[-a_0 p^{-2} - 2a_1 p^{-3} - \ldots]
\] (19)

Comparing right hand sides of (18) and (19), we see that leading terms in them coincide, and we can write down the following

\[
\frac{\partial \varepsilon}{\partial p} = \varepsilon(1 + O\left(\frac{1}{p}\right))
\] (20)

Since we consider that invariant \( S \) does not change with time, we can rewrite expression (19) as:

\[
\frac{\partial \varepsilon}{\partial t} = \varepsilon S(1 + O\left(\frac{1}{St}\right))
\] (21)

Therefore, we conclude that constant of extended local balance model is \( \alpha_0 = 1.0 \). Positive value of this constant justify the introduction of negative coefficient diffusion for dissipation rate, otherwise, formulas (6) and (7) will contradict to the model near channel walls. It seems, that invariant \( S \) is adequate in reproducing an influence tensor-gradient of large scale velocity, if one its component prevails over eight others. In general, we should use the main invariant of turbulence, determining by most unstable modes [22]. (The case of rotating turbulence under prevailing rotation against shear requires a special consideration.) We should note that corresponding term exists in exact dissipation rate transport equation. Derivation of maximum simplified form of this variable can be found in [23]. The resulting equation is:

\[
\frac{\partial \varepsilon}{\partial t} + U_i \frac{\partial \varepsilon}{\partial x_i} = T_1 + T_2 + T_3 + T_4,
\]

where

\[
T_1 = -2\nu \left< \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) \frac{\partial u_k}{\partial x_k} \right> \frac{\partial U_i}{\partial x_k} - \text{production of dissipation},
\]

\[
T_2 = -\frac{2\nu}{\rho} \left< \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) \frac{\partial^2 p}{\partial x_j \partial x_j} \right> - \text{tending to isotropy by pressure},
\]

\[
T_3 = -2\nu \left< \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) \frac{\partial^3 (u_i u_k)}{\partial x_j \partial x_j \partial x_k} \right> - \text{nonlinear term},
\]

\[
T_4 = -\nu^2 \left< \left( \frac{\partial^3 (u_i u_k)}{\partial x_j \partial x_j \partial x_k} \right) \frac{\partial u_i}{\partial x_j} \frac{\partial u_j}{\partial x_k} \right> - \text{dissipation of dissipation rate},
\]

\( p \) - small-scale pressure, \( \rho \) - constant density of fluid, \( \nu \) - molecule viscosity.

4. Stationary fully-developed flow in direct round pipe

Fully-developed flow in a round pipe is considered in scope of the extended local balance model. Differential operators of high order with respect to spatial derivatives in equations (1)-(3) will be omitted. Cylindrical system of coordinates is used.
We shall assume, that there are only one component – velocity, directed along axis of the pipe \( U_z = U(r) \), and dissipation rate \( \varepsilon = \varepsilon(r) \), depending on distance \( r \) from the pipe axis. For stationary flow \( \partial_z < u U_z > = 0 \). The corresponding equation of the model will be:

\[
- \frac{1}{\rho} \partial_z p + \frac{1}{r} \partial_r (r \frac{\varepsilon}{S^2} \partial_r U) = 0
\]  

(22)

\[
\varepsilon S - \frac{1}{\kappa^2 r} \partial_r (r \frac{\varepsilon}{S^2} \partial_r \varepsilon) = 0
\]  

(23)

It is well known [24], that for stationary fully-developed turbulent flow we can obtain linear profile for shear stresses \( \tau \) from Reynolds equation (except viscous sublayer near the wall):

\[
\tau = u_*^2 r / r_0
\]  

(24)

where \( u_* \) - friction velocity, \( r_0 \) - pipe radius.

On the other hand (excluding small domain along pipe axis), equation of local kinetic energy balance is valid:

\[
\varepsilon S = \xi
\]  

(25)

Inserting expression (24) in formula (25), we obtain dependence between \( S \) and dissipation \( \varepsilon \) (in the same way, as it was done for turbulent Couette -Taylor [1]):

\[
S = \frac{r_0 \varepsilon}{u_*^2 r}
\]  

(26)

Inserting this expression in the equation (23), we obtain an equation for dissipation rate of turbulent energy , which in dimensionless variables \( \varepsilon_i = \frac{\varepsilon r_0}{u_*^3} \) and \( \eta = \frac{r}{r_0} \) has the next form:

\[
\varepsilon_i^2 - \frac{1}{\kappa^2} \partial_\eta (\eta^3 \partial_\eta \ln \varepsilon_i) = 0
\]  

(27)

As local balance of turbulent energy is broken near the pipe axis (we believe that local balance of turbulent energy is valid there in 3-dimensional flow simulation ), we can use empiric dissipation and velocity profiles.

We shall assume validity of Darcy’s law [25] near the pipe axis:

\[
\frac{U_{max} - U(r)}{u_*} = 5.08 \left( \frac{r}{r_0} \right)^{3/2}
\]  

(28)

We should note that sometimes another mean velocity profile is used near the pipe axis:

\[
V = V_{\max} \left( 1 - \frac{r}{r_0} \right)^{1/n}
\]

where \( n = 7 \) (see , for example, [26]), \( V_{\max} \) - velocity at the center line.
Differentiating the expression (28), we obtain:

$$\frac{\partial U}{\partial r} = - \frac{7.62 u_s}{r_0} \left( \frac{r}{r_0} \right)^{1/2}$$

(29)

It can be shown that using cylindrical coordinates, we can write

$$S = \left| \frac{\partial V_z}{\partial r} \right|, \quad u_z \cdot \frac{r}{r_0} \cdot 7.62 \frac{u_s}{r_0} \left( \frac{r}{r_0} \right)^{1/2} = \varepsilon,$$

(30)

that gives asymptote for dissipation near the pipe axis and permits a shift from the axis, where the local balance does not hold.

Thus, asymptote for dissipation (convex function near pipe axis as in experiments [24]) is:

$$\varepsilon = \frac{7.62 u_s^3}{r_0} \left( \frac{r}{r_0} \right)^{3/2}$$

(31)

We have in dimensionless form the next expression:

$$\varepsilon_1 = 7.62 \eta^{3/2}$$

(32)

Von Karman constant $\kappa = 0.4$ can be excluded from the equation (17) by introducing a new variable:

$$\varepsilon_2 = \kappa \varepsilon_1$$

(33)

The wall function for the new variable $\varepsilon_2$ will be an expression:

$$\varepsilon_2 = \frac{1}{1-\eta}$$

(34)

Coaxial function instead of the expression (21) will be a function:

$$\varepsilon_2 = 3.048 \eta^{3/2}$$

Further simplification of the equation (15) consists in changes of dependent and independent variables:

$$g = \ln \varepsilon_2$$

(35)

or $\varepsilon_2 = \exp(g)$

and $q = \frac{1}{2\eta^2}$

(36)

or $\eta = \frac{1}{\sqrt{2q}}$

Since $\eta \in [0,1]$, therefore $q \in [0.5, +\infty)$. 
Unifying formulas (22-24),
an asymptote near wall \((q \to 0.5)\) will be a function:
\[
g = -\ln(1 - \frac{1}{\sqrt{2q}}).
\] (37)

We shall have the near pipe axis \((q \to +\infty)\):
\[
g = \ln(3.048/2^{3/4}) - \frac{3}{4} \ln q
\]
or
\[
g = 0.595 - 0.75 \ln q
\] (38)

Differential equation will be:
\[
g'' = (2q)^{-3/2} \exp(2g)
\] (39)

In can be easily shown, that the equation (27) transforms to generalized Emden–Fowler equation, but general analytic solution for power function with index \((-3/2)\) is unknown [27].

Thus, this boundary problem will be numerically solved with given the wall and the axis asymtoes (37), (38).

The condition (37) was set at \(q = 2.0\) and the condition (45) was set at \(q = 12.5\), corresponding dimensionless distance from the pipe axis \(r/r_0 = 0.5\) and \(r/r_0 = 0.2\), namely
\[
g(2.0) = 0.693
\] (40)
\[
g(12.5) = -1.30
\] (41)

An ordinary equation of the second order (46) was split into a system of two differential equation of the first order:
\[
g' = p
\] (42)
\[
p' = (2q)^{-3/2} \exp(2g)
\] (43)

This system was solved by a shooting method: by varying a derivative \(p(2.0)\)
at left end of closed interval \([2,12.5]\) and an integrating system (42), (43) by stiff equation solver \texttt{ode23tb} in Mathlab. It was established that fulfilling boundary condition (41) at the right end of the closed interval was achieved for \(p(2.0) = -0.550\).

This solution is depicted on figure 1.

Obtained theoretical dependence, depicted on figure 1, was used for comparison with the experiments [24] and with the experiments [28], given in [16]. Since the data on dissipation profile in the pipe were not given in this paper [28], these data were estimated by way of differentiating mean velocity with respect to the distance from the pipe axis. Since we know the profile of turbulent stresses (linear dependence on distance from pipe axis) and can assume validity of local balance approach: creation of turbulent energy is equal to its dissipation, it is possible to determine dissipation profile in radial direction. Results of direct numerical simulation [16] were not used for comparison with our theory, since these results corresponded to not so high Reynolds numbers.
Figure 1. Dependence of $g$ (dimensionless dissipation rate $\varepsilon_0/u^2 = \kappa^{-1} \exp(g)$, $\kappa = 0.4$— von Karman constant) on independent variable $q$ (dimensionless distance from pipe axis $r/r_0 = 1/\sqrt{2q}$).

Table 1. Dependence of dimensionless dissipation rate on dimensionless radial distance.

| $r/r_0$ | 0.2 | 0.3 | 0.4 | 0.5 |
|---------|-----|-----|-----|-----|
| $\varepsilon_0/u^2$ theory | 0.69 | 2.17 | 3.37 | 5.03 |
| experiment [24], a | 3.7 | 4.9 | 4.9 | 8.0 |
| experiment [24], b | 2.5 | 3.7 | 4.3 | 5.5 |
| experiment [28] | 0.73 | 1.59 | 2.75 | 3.81 |

Dimensionless dissipation rate was obtained in experiments [24]: a- from Kolmogorov hypothesis, b- from micro-scale.

5. Conclusions

Obtained results of numerical simulation of fully developed turbulent flow in a round pipe, based on one-parameter turbulence model, given in table 1, demonstrate that the theory gives values of dimensionless dissipation rate between the data [24] and [28], excluding its value at $r/r_0 = 0.2$, where theoretical prediction coincides with the data [28]. Our model has a simple physical sense, since it is applied to equilibrium turbulent flows, where dissipation is equal to the creation of turbulent energy. The model, used here, depends on two von Karman-Prandtl constants and the constant of Darcy’s law.

New considerations, supporting negativity of diffusion coefficient for dissipation rate transport equation are given. Indeed, fully-developed turbulence can be considered as a spatio-temporal chaos. It is well-known, that negative coefficient diffusion permits to reproduce that phenomenon [29]. We are planning to conduct a numeric simulation of three-dimensional turbulent flow with system of equation (1-3) without given axial function, based on Darcy’s law (16). It can be assumed that local balance approach will be valid. It is unlikely that all nine components of large scale velocity nullify in time-dependent flow. But it is quite possible to predict broken axial symmetry for turbulent flow in a pipe.

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