COHOMOLOGICAL ASPECTS OF NON-KÄHLER MANIFOLDS

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Abstract

In studying the geometry of compact complex (especially non-Kähler) manifolds, the Bott-Chern cohomology, (as well as its dual, the Aeppli cohomology,) provides an important tool, behaving as a “bridge” between the Dolbeault cohomology and the de Rham cohomology.

In this talk, we summarize some results concerning the Bott-Chern cohomology of compact complex manifolds; finally, we look at its symplectic counterpart for compact symplectic manifolds.

More precisely, we firstly study the Bott-Chern cohomology for complex structures on the so-called nilmanifolds, namely, compact quotient of connected simply-connected nilpotent Lie groups by discrete co-compact subgroups: such manifolds turn out to be a very fruitful source of examples in non-Kähler geometry; (see: D. A., The cohomologies of the Iwasawa manifold and of its small deformations, DOI: 10.1007/s12220-011-9291-z, to appear in J. Geom. Anal.).

Once we have several examples at hand, we study Bott-Chern cohomology for general compact complex manifolds, providing an inequality à la Frölicher, which relates the dimension of the Bott-Chern cohomology to the dimension of the de Rham cohomology; furthermore, such an inequality allows to characterize the compact complex manifolds satisfying the $\partial \overline{\partial}$-Lemma (namely, the very special cohomological property that every $\partial$-closed $\overline{\partial}$-closed $\partial$-exact form is $\partial \overline{\partial}$-exact too); (see: D. A., A. Tomassini, On the $\partial \overline{\partial}$-Lemma and Bott-Chern cohomology, DOI: 10.1007/s00222-012-0406-3, to appear in Invent. Math.).

Finally, we just mention how to extend such results to the symplectic context.