Improvement of Background Oriented Schlieren Method Focused on Amplitude of Wavelet Transform

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Abstract
The BOS method is a visualization method of density variation in the flow field. This method is simpler and cheaper to visualize than the Schlieren method. The primary purpose of this study is to propose WA-BOS (a simpler BOS method focusing on the amplitude of the Wavelet transform). Output images by WA-BOS and WP-BOS (BOS method focused on the phase of the Wavelet transform) and the Schlieren image were compared to verify the validity of the output images. Also, output images by the captured image lost periodic information partially were verified, in order to confirm the validity of the image processing for the deviation out of the domain of phase. The targets of visualization are the supersonic jet and the shock wave ejected from the shock tube. The measurement system consists of an LED, a sine pattern background image and a high-speed camera. As a result, the supersonic jet and the shock wave ejected from the shock tube are visible in the visualized image that was obtained by WA-BOS and WP-BOS. Moreover, it is found that WA-BOS can visualize a supersonic phenomenon without containing an error.

1. Introduction
In recent years, many types of researches focusing on compressible fluid dynamics have been conducted in various fields such as aerospace mechanics [1]-[3], nuclear engineering [4], medical physics[5,6], and materials sciences[7,8], which are developing rapidly. The visualization technology is the most important technology to understand these phenomena. Representative examples are Schlieren and interferometry. Unlike these, the BOS (Background Oriented Schlieren) method, which is a unique visualization method, is proposed by Meire[9]. The visualizing principle of the BOS method is similar to the Schlieren method, which utilizes the change of the fluid density. One of the characteristics of the BOS is the ability to visualize by using straightforward experimental apparatus. The BOS system consists of a background image, a light and a camera. This is a straightforward system compared with the Schlieren method. The schlieren method requires a lot of complex components, for example, concave mirrors, pin-hole and a knife-edge. In the Schlieren method, much cost is required to expand the area of visualizing. While in the BOS, if the background image can be taken, there is no limitation on the physical size of the visualization target. This is because the BOS method depends on the area of the background image. Many experiments for visualizing have been
performed on a real scale [10]-[12]. Thus, to visualize with a simple system without depending on the size of the experimental apparatus is a major characteristic of the BOS. In the BOS, image processing is required to obtain a visualized image. This image processing is said to be a disadvantage of the BOS, as it is not required in other visualization methods like the Schlieren method.

Various image-processing methods have been proposed to increase the accuracy and efficiency of the BOS method. An output method of the visualized image by the BOS method shown in Figure 1. A positional deviation is calculated by subtracting an image having no density change from an image for measurement in which light is refracted due to the density change. The visualized image is a result obtained by reconstructing the deviation of an image. Random dot patterns were used for background images in the calculation of displacement of the background image, and PIV is used for image processing. Ota et al. [13,14] proposed CGBOS method using a color grid pattern as a background image. CGBOS method does not require a reference image and can be visualized only with captured images. Therefore, it is possible to visualize even when the position of the model is largely shifted before and after the experiment. Akatsuka [15] proposed the Wavelet-based BOS method. This technique provides a Schlieren image using the Continuous Wavelet transform for the periodic background pattern. Our studies of WA-BOS (BOS method focused on the amplitude of the Wavelet transform) have proven that it can be obtained a visualized image. However, the verification of the characteristics of the visualized image obtained by WA-BOS and the comparison of the characteristics of WA-BOS and WP-BOS (BOS method focused on the phase of the Wavelet transform) were not sufficient.

The purpose of this study is to clarify the characteristic of WA-BOS. In order to clarify the characteristics, the output images by WA-BOS and WP-BOS were compared with the Schlieren image and considered. Targets of visualizing are the supersonic jet and the shock wave that is ejected from the shock tube [16,17]. The experimental apparatus consists of the background image that has the periodic pattern, the LED and the high-speed camera.

![Figure 1. Schematic image of the BOS method](image)

2. Principle

2.1 Principle of the background-oriented Schlieren method

Figure 2, shows the principle of the BOS method and measurement system. The measurement system consists of a background image, light and a camera. The red line and the blue line in the figure are light paths from the background image. The red line is the light path in the case of no density gradient. The blue line is the refracted light path by the density gradient. In the BOS method, $\Delta h$ is obtained by using image processing applications a captured image that is taken by a camera. In this paper, the
image processing method is the main focus. The equation of change in refractive index by density change is given as

\[ \int_0^\Delta r \frac{\partial n}{\partial z} \, dx = n_0 \frac{\Delta h}{L_B - \Delta r / 2} \]  

(1)

where \( \Delta h \) is the positional deviation amount, \( \partial n / \partial z \) is the refractive index gradient, \( n_0 \) is the refractive index of air, \( L_B, L_C \) and \( f \) are the distance from the background image to the density change area, from the measurement target to the camera and distance of focal length. \( \Delta r \) and \( \varepsilon \) are the width and refraction angle of the density change area. Also, the equation of the relationship between refractive index and density is given as

\[ n = G \rho + 1 \]  

(2)

where \( n, \rho \) and \( G \) are the refractive index, the density of air and the Gladstone-Dale constant respectively. The density information can be determined quantitatively from these relational equations.

**Figure 2.** Optical set up of the BOS method

### 2.2 The WA-BOS method and the WP-BOS method

In this study, the Wavelet transform is applied to the BOS method. The Wavelet transform is spatial frequency analysis. The most major method of frequency analysis is FFT. The Fourier transform maps the target signal to the basis of sine or cosine. In this analysis, the periodic information is obtained clearly, but time information is lost. That is because of using the signal on the assumption that it repeats infinitely. On the other hand, the Wavelet transform uses a Mother wavelet that is temporally localized. Thus, the Wavelet transform can obtain the frequency band that has information of a specific time. Many Mother wavelet function is proposed. Gabor wavelet that is a good balance between the resolution of time and frequency is used in this study. The amplitude and phase can be obtained by applying the Wavelet transform for the target signal. The Wavelet transform is applied to the BOS method uses this information to visualize the flow field. In the BOS method, a positional deviation is needed to visualize quantitatively. That deviation can be calculated by a phase. The
visualized image from the phase of the Wavelet transform is called the WP-BOS. In the WP-BOS, an image equivalent to the Schlieren image can be obtained. Also, the visualized image from the amplitude of the Wavelet transform is called the WA-BOS. Thus, In WA-BOS, an image equivalent to the Shadow Graph image can be obtained.

3. The experimental setup and image processing method

Figure 3 shows the experimental apparatus which consists of the shock tube and the BOS system. The jet flowing towards the test section from the shock tube is measured by the BOS system. The measurement system of the BOS method consists of the background image, light and high-speed camera. The background image has the periodic pattern that the period is 2 mm. The LED light is SELMIC SLS-75-08W and the high-speed camera is Vision Research Phantom v7.3.

Figure 4 shows the shock tube which is an experimental device that is suitable to study for a characteristic of the supersonic jet and the shock wave. The shock tube has a long tube of constant cross-section, which is divided into the high-pressure chamber and low-pressure chamber by diaphragm. In Figure 4, the x-axis is parallel to the jet flow, the shock tube diameter \( D = 10 \text{mm} \), the high-pressure chamber length \( L_h = 100 \text{mm} \), and the low-pressure chamber length \( L_l = 10 \text{mm} \). \( P_h \) and \( P_l \) are the pressure of the high-pressure chamber and the low-pressure chamber. By breaking the diaphragm, the high-pressure air was discharged. Then, the supersonic jet and the shock wave are formed by discharged air. Both supersonic phenomena are visualized by the BOS system.

Figure 5 shows the image processing procedure to visualize the supersonic phenomena. Time \( t = 0 \mu s \) is one frame before immediately after the jet is ejected. Figure 5(a) is the reference image at \( t = 0 \mu s \). Figure 5(a') is the measurement image at \( t = 195 \mu s \). Figures 5 (b) and (b') show the luminance on \( y/D = 0 \) in Figures 5 (a) and (a'). Figures 5 (c) and (c') show the amplitude that is obtained by using the Wavelet transform for the luminance information of (b) and (b'). The y-axis shows the amplitude of the Wavelet transform. Figure 5 (d) shows the value obtained by subtracting (c) and (c'). The y-axis shows the amplitude difference. This image processing has performed all rows of images and is reconstructed to obtain the output image. Figure 5 (e) is the output image that has finished a process of image processing.
4. Results and Discussions

Figure 6 shows a captured image of the supersonic jet ejected from the shock tube and the output image. The thickness of the diaphragm is 100 μm and $P_a/P_b$ is 50.2. Figures 6(a), (b) and (c) are the reference images at $t = 0$ μs, the measurement image at $t = 145$ μs and output image by WA-BOS respectively. In Figure 6(b), there is a large distortion of the background image at $x/D = 0$ to $1.5$ and $y/D = -0.6$ to $0.6$. The distortion amount of the background image confirms to the density gradient that is formed by the supersonic jet from the shock tube. In Figure 6(c), there is the head of the shock wave at $x/D = 6.8$, $y/D = 0$ and the head of the supersonic jet on $x/D = 5.0$, $y/D = 0$. While applying the image processing of this study to the captured images, the shock wave and the supersonic jet become visible.

The output images by WA-BOS and WP-BOS and the Schlieren image are compared to verify the validity of the output images. Figures 7 (a), (b) and (c) are the output images by WA-BOS and WP-BOS for $P_a/P_b = 50.2$ and the Schlieren image for $P_a/P_b = 40.8$, respectively. In Figures 7 (a) and (b), there is the head of the shock wave at $x/D = 6.8$, $y/D = 0$ and the head of the supersonic jet at $x/D = 5.0$, $y/D = 0$. In Figures 7(c), there is the head of the shock wave and the head of the supersonic jet at the same points of (a) and (b). Therefore, the output images of WA-BOS and WP-BOS are valid. The shock wave in Figure 7 (a) is visualized more clearly than that in (b). This could be because of the differences in image processing between WA-BOS and WP-BOS. The phase is the position of a point in time on a periodic cycle. In the case of visualizing by using phase, that deals with the displacement of the background image as the vector that has luminance and information of position. While the amplitude is maximum displacement in a periodic signal and non-negative scalar. The vector deals
with different values as a gradient. Yet, the scalar cannot do it. Therefore, it is obvious that the output image from the phase has less noise than the output image from the amplitude.

The wavelet transform deals with the argument of a complex function as a phase. The domain of the argument is limited ±π; it becomes discontinuous bordering on ±π. To calculate a phase difference requires phase unwrapping, in case of the deviation of background image exceeds ±π, it is not calculated correctly. Output images by the captured image lost periodic information partially are shown in Figure 8, in order to confirm the validity for the image processing for the deviation out of the domain of phase. The thickness of the diaphragm is 50μm and $P_b/P_s$ is 23.5. Figures 8 (a), (b) and (c) are the measurement image at $t = 145μs$, the output image by WP-BOS and WA-BOS. In Figure 8 (a), the background image is lost on the range of $x/D = 0~3$ and $y/D = 0.2~0.5$. In Figure 8 (b), the pattern is confirmed in the same range; in (c) it is filled in black. The zebra pattern in Figure 8(b) shows the density gradient that does not exist the actual phenomenon. This is because the deviation exceeds the domain of phase. While the amplitude needs to calculate only the luminance at that pixel and does not depend on the periodic information, therefore, the advantage of WA-BOS is error resistance in image processing and able to visualize the captured image that does not depend on the background image.

Figure 6. Visualization of captured images for $P_b/P_s = 50.2$. (a) Reference image $t = 0μs$, (b) Measurement image $t = 145μs$, and (c) WA-BOS image.
5. Conclusions

The purpose of this study is to clarify the characteristic of WA-BOS (BOS method focused on the amplitude of the Wavelet transform). The visualized image was obtained by applying the Wavelet transform as an image processing. Targets of visualizing are the supersonic jet and the shock wave ejected from the shock tube. Output images by WA-BOS and WP-BOS (BOS method focused on the phase of the Wavelet transform) and the Schlieren image were compared in order to verify the validity of the output images. Also, the output images by the captured image lost periodic information partially were verified, in order to confirm the validity for the image processing for the deviation out of the domain of phase. The results are as follows.

1. WA-BOS and WP-BOS can obtain the visualized image of the supersonic jet and the shock wave ejected from the shock tube.
2. The head position of the supersonic jet and the shock wave in the output image by WA-BOS and WP-BOS correspond to the Schlieren image. Thus, visualizing by WA-BOS and WP-BOS is valid.
3. The output image by WP-BOS has less noise than the output image by WA-BOS, which is visualized more clearly.
4. In WP-BOS, an error occurs in the case of the periodic information of the background image which is lost.
5. In WA-BOS, no error was detected in the amplitude calculation. Therefore, WA-BOS can obtain a visualized image faithful to the actual phenomenon.
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