Decay of charged fields in de Sitter spacetime

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Abstract

We study the decay of charged scalar and spinor fields around Reissner–Nordstrom black holes in de Sitter spacetime through calculations of quasi-normal frequencies of the fields. The influence of the parameters of the black hole (charge, mass), of the decaying fields (charge, spin) and of the spacetime (cosmological constant) on the decay is analysed. The analytic formula for the calculation of quasi-normal frequencies for a large multipole number (eikonal approximation) is derived both for the spinor and scalar fields.

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(Some figures in this article are in colour only in the electronic version)

1. Introduction

It is well known that dynamical evolution of field perturbation on a black hole background can be conventionally divided into three stages [1]. The first stage is an initial wave burst coming directly from the source of perturbation and is dependent on the initial form of the original field perturbation. The characteristic feature of the second stage is the damped oscillations whose frequencies and damping times are defined by the structure of the background spacetime and are independent of the initial perturbation. This stage can be accurately described in terms of the discrete set of quasi-normal modes (QNM). And the third stage is asymptotic tail behaviour of the waves at very late time which is caused by backscattering of the gravitational field.

The original interest in the study of QNMs arises from the possibility of observing quasi-normal ringing with the use of gravitational wave detectors as follows from theoretical predictions (for a review see, e.g., [2] and references therein). Recently the interest in the study of QNMs has been reinforced in connection with their possible relation to the thermodynamic properties of black holes in loop quantum gravity [3, 4].

A new application of the quasi-normal mode spectrum has also arisen from the superstring theory [5–7]. According to the AdS/CFT correspondence, a large static black hole in
asymptotically AdS spacetime corresponds to a thermal state in CFT, and the decay of the test field in the black hole spacetime corresponds to the decay of the perturbed state in CFT, such that the quasi-normal frequencies define the thermalization time scale [8]. Therefore, many authors focused their attention on the studies of QNMs for different asymptotically AdS black holes [8–13].

There exists observational evidence that the universe is described by the general relativity equations with positive cosmological constant [14–16]. That observation attracted considerable attention of researchers to the study of QNMs in asymptotically de Sitter spacetimes [17–24, 33].

In [17, 18], the calculation of the quasi-normal frequencies for the gravitational perturbations of the Schwarzschild de Sitter (SdS) black hole was carried out. In [19], the lower overtones of QNMs for higher dimensional SdS black holes were calculated. In [20], the total spectrum of QNMs was obtained for the SdS black holes by numerical calculations and excellent coincidence with the sixth-order WKB method was shown. In [21], the low-lying quasi-normal frequencies of the SdS black hole for fields of different spin were calculated by using the sixth-order WKB and the Pöschl–Teller potential approximations. In [22], the authors derived an explicit expression for the calculation of QNMs for the case of a near extremal SdS black hole. In [23] that expression was generalized to near extremal higher dimensional SdS and Reissner–Nordström de Sitter (RNdS) black holes. In [24], an analytical method was developed to study the quasi-normal mode spectrum of SdS black holes for the scalar, electromagnetic and gravitational fields in the limit of nearly equal black hole and cosmological radii.

Perturbations of a charged massless and massive scalar field were studied by the calculation of its QNMs in the Reissner–Nordström (RN), and were investigated in [25] and [26], respectively. It was found that the neutral perturbations dominate at the stage of the ‘final ringdown’. In [27, 28], the spin 1/2 Dirac particles with positive, negative and zero charge \( e \) in the presence of RN black holes with charge \( Q \) were investigated. It was demonstrated that at late times, the neutral perturbations dominate when \( eQ > 0 \) and the charged perturbations dominate when \( eQ < 0 \).

Perturbations of Reissner–Nordström anti-de Sitter (RN-AdS) black holes were investigated in [29–32]. In [29] scalar QNMs of large RN-AdS black holes were computed. It was shown that scalar QNMs do not linearly scale with the black hole temperature and was found that the larger the charge of the RN-AdS black hole the sooner it returns to thermal equilibrium. In [30], along with scalar perturbations, electromagnetic and gravitational perturbations of RN-AdS spacetime were studied. It was found that different kinds of perturbations are almost exactly isospectral, electromagnetic and axial perturbations are characterized by the existence of purely damped modes. Then, in [31] the extreme limit of scalar QNMs in RN-AdS spacetime was analysed and higher overtones were studied. In [32], the Dirac QNMs of the Schwarzschild–anti-de Sitter and RN-AdS black holes were investigated. It was shown that for large black holes, the fundamental QNMs are linearly related to the Hawking temperature.

The QNMs of the massless uncharged Dirac fields for the RNAdS black hole are studied in [33] using the Pöschl–Teller potential approximation. It was found that the magnitude of the imaginary part of the quasi-normal frequencies decreases as the cosmological constant or the orbital angular momentum increases, but it increases as the charge or the overtone number increases. We note that the Dirac quasi-normal modes were first evaluated in [34] for Schwarzschild black hole spacetimes.

The decay of fields, which interact electromagnetically with the charge of a black hole in de Sitter spacetime, was not considered till now. The objective of the present paper is to study
the decay of charged fields of different spin in RNdS background and analyse how the decay is influenced by a total set of parameters: the charge $Q$ of the black hole, the cosmological constant $\Lambda$ and the charge $e$ of the decaying field.

The paper is organized as follows. In section 2 we consider the spinor field decay, in section 3 we consider the scalar field decay and in section 4 we summarize the results obtained.

2. Decay of the spinor field

In Schwarzschild coordinates, the metric for the Reissner–Nordström de Sitter black hole can be expressed as follows:

$$\text{ds}^2 = - f \, dt^2 + f^{-1} \, dr^2 + r^2 (d\theta^2 + \sin^2 \theta \, d\phi^2)$$

$$f = 1 - \frac{2M}{r} + \frac{Q^2}{r^2} - \frac{\Lambda}{3} r^2,$$

where the parameters $M, Q$ are the mass and the charge of the black hole, respectively, and $\Lambda$ is the cosmological constant. We introduce the tortoise coordinate $r_\ast = \int f^{-1} \, dr$. Then we write the metric function (2) in the form

$$f = \frac{\Lambda}{3} r^2 (r - r_-)(r - r_+)(r - r_c),$$

where $r_-$ is the cosmological horizon, $r_+$ is the inner event horizon and $r_+$ is the outer event horizon ($r_b$ can be obtained through $r_-, r_+, r_c$). One can find that the tortoise coordinate can be expressed as follows:

$$r_\ast = \frac{1}{2} \left[ \frac{1}{\kappa_-} \ln(r - r_-) + \frac{1}{\kappa_+} \ln(r - r_+) - \frac{1}{\kappa_c} \ln(r - r_c) + \frac{1}{\kappa_b} \ln(r - r_b) \right],$$

$$\kappa_\alpha = \frac{1}{2} \left. \frac{df}{dr} \right|_{r = r_\alpha}, \quad \alpha = (-, +, c, b).$$

From (3) one can see that $r_\ast \to \infty$ as $r \to r_-$ and $r_\ast \to -\infty$ as $r \to r_+$.

The wave equation of the massless Dirac field can be written as

$$\left[ \gamma^a e^a_\mu (\partial_\mu + \Gamma_\mu + eA_\mu) \right] \Psi = 0,$$

where $\gamma^a$ are the Dirac matrices, $e^a_\mu$ is the inverse of the tetrad $\Gamma_\mu = \frac{1}{8} \left[ \gamma^a, \gamma^b \right] e^a_\nu e^b_\mu$ is the spin connection. We take the tetrad $e^a_\mu$ as [35]

$$e^a_\mu = \text{diag}(f^{1/2}, f^{-1/2}, r, r \sin \theta).$$

We define $\Phi = f^{-1/4} \Psi$, then using the ansatz

$$\Phi = \left( \frac{f^{(\pm)}(r)}{r} \phi^\pm_{jm}(\theta, \phi) \right) e^{-i\omega t},$$

where

$$\phi^+_{jm} = \left( \begin{array}{c} \sqrt{\frac{j+m}{2j+1}} Y_{j-l}^m \frac{1}{2}^{j+m-1/2} \\ \sqrt{\frac{j-m}{2j+1}} Y_{j+l}^m \frac{1}{2}^{j+m+1/2} \end{array} \right) \quad \text{for} \quad j = l + \frac{1}{2},$$

$$\phi^-_{jm} = \left( \begin{array}{c} \sqrt{\frac{j+1-m}{2j+1}} Y_{j-l}^{m+1} \frac{1}{2}^{j+m-1/2} \\ -\sqrt{\frac{j+1+m}{2j+1}} Y_{j+l}^{m+1} \frac{1}{2}^{j+m+1/2} \end{array} \right) \quad \text{for} \quad j = l - \frac{1}{2}.$$
we find for both signs of $F$ and $G$, 

$$
\frac{d^2 F}{dr_*^2} + (\omega^2 - V_1) F = 0 \tag{7}
$$

$$
\frac{d^2 G}{dr_*^2} + (\omega^2 - V_2) G = 0, \tag{8}
$$

where

$$
V_{1,2} = \pm \frac{dW}{dr_*} + W^2, \quad W = \frac{|k|\sqrt{J}}{r} - \frac{eQ}{r},
$$

$$
k = j + \frac{1}{2}, \quad j = l + \frac{1}{2} \quad \text{for } V_1, \tag{9}
$$

$$
k = -\left(j + \frac{1}{2}\right), \quad j = l - \frac{1}{2} \quad \text{for } V_2.
$$

The quasi-normal modes are defined as solutions of (7) and (8) satisfying the boundary conditions

$$
F(r_*) \sim A_{\pm} e^{\pm ik_* r_*}, \quad G(r_*) \sim B_{\pm} e^{\pm ik_* r_*}, \quad r_* \to \pm \infty, \tag{10}
$$

supposing $\text{Re} \, \omega > 0$ that corresponds to purely in-going waves at the event horizon and purely out-going waves at the cosmological horizon.

We note that the potentials $V_1, V_2$ give the same quasi-normal frequencies [33, 28]; therefore in what follows we focus attention on $V_1$. The effective potential $V_1$ (9) is a smooth function of $r_*$. It tends to constant values at the cosmological horizon and at the outer event horizon and has a maximum near the outer event horizon that allows us to use the WKB method to calculate quasi-normal modes. To compute QNMs from equations (7) and (9), we apply the WKB method of the sixth order [36].

We present the results graphically for the fixed overtone number $n = 0$ and $k = 1$ in figures 1–5 for the charge of the spinor field $e = 0, \pm 0.1$. The dependence of QN frequencies (figure 1) and their damping rates (figure 2) on the charge of the black hole $Q$ for different values of the cosmological constant $\Lambda$ is shown. In figures 1 and 2, the dependences are shown for three values of the charge of the spinor field $e = 0, \pm 0.1$, which in the figures looks like a triplet of curves beginning at the same point at $Q = 0$. The curves of different densities on the graphs correspond to different $e$. In figure 1, the upper curve in the triplets corresponds to $e = -0.1$, the lower curve in the triplets corresponds to $e = 0.1$ and the middle curve corresponds to $e = 0$. In figure 2, the lower curve in the triplets corresponds to $e = -0.1$, and
the upper curve in the triplets corresponds to $e = 0.1$ and the middle curve corresponds to $e = 0$. The QN frequencies are growing monotonically with $Q$ for $e = 0$, $-0.1$, although for $e = 0.1$ the behaviour of the curves is qualitatively different. The curves for $e = 0.1$ have a minimum at some point $Q_0$. The value of $Q_0$ shifts to smaller values as $\Lambda$ grows. Besides, figure 1 demonstrates that the QN frequencies become lower as $\Lambda$ increases. The position of a curve in the triplet says that the QN frequencies are lower for the field charge $e > 0$ and higher for $e < 0$ ($Q > 0$). The curves displaying QNM damping rate are similar for all $\Lambda$ and $e$. They have a maximum at $Q_{\text{max}} \sim 0.9$. Until reaching $Q_{\text{max}}$, the curves grow
slowly and after reaching $Q_{\text{max}}$ they fall sharply. The greater $\Lambda$ is the greater the rise of the curves is until reaching $Q_{\text{max}}$. The value of $Q_{\text{max}}$ shifts to greater values as $\Lambda$ increases. The position of a curve in the triplet says that the decay of the field is slower for the field charge $e < 0$ and faster for $e > 0$ ($Q > 0$). The decay of the spinor fields becomes slower as $\Lambda$ grows. Since the charges $e, Q$ appear in $V_1$ only as $e^2, Q^2$ or $eQ$ one can also conclude that for $Q < 0$ the conclusions are inverted. These observations generalize results [28] obtained for asymptotically flat spacetimes. The peculiarities of the dependence of $\text{Im} \omega$, $\text{Re} \omega$ on $Q$ are more obvious when depicted on diagrams of the dependence of $\text{Im} \omega$ on $\text{Re} \omega$ and parametrized by the black hole charge $Q$. These curious pictures are presented in figure 3.

The dependence of $\text{Im} \omega$ on $\text{Re} \omega$ parametrized by the cosmological constant $\Lambda$, which changes along the curves, is depicted in figures 4 and 5 for the fixed black hole charge $Q$. The curves are approximately straight lines. This observation generalizes conclusions [33] obtained for the uncharged spinor field. Both the QN frequencies and their damping rates decrease with $\Lambda$, although the manner of the decrease for $\text{Re} \omega$ and $\text{Im} \omega$ is different.

Making use of the first-order WKB method, we also found the asymptotic formula of large $k$ for the calculation of quasi-normal modes $\omega$,

$$\omega = C_0 k - i(n + 1/2)C_i^{\text{sp}} + C_r^{\text{sp}} + O(1/k),$$

(11)

where

$$C_0 = \left( \frac{Q^2}{r_0^2} - \frac{2M}{r_0^3} + \frac{1}{r_0^2} - \frac{\Lambda}{3} \right)^{1/2},$$

$$C_i^{\text{sp}} = \frac{1}{\sqrt{3}r_0} \left[ -42Q^4 + 3Q^2 r_0 (42M - 15r_0 + 2\Lambda r_0^3) \
+ r_0^2 (90M^2 - 60Mr_0 + 9r_0^2 + 6M\Lambda r_0^3 - \Lambda r_0^5) \right]^{1/2},$$

(12)

$$C_r^{\text{sp}} = -\frac{1}{2r_0} \left[ 4r_0^{\text{sp}} (r_0 - 3M) \
+ \frac{2}{3r_0^2} C_0^{1/2} \left( Q^2 + eQr_0^2 + r_1^{\text{sp}} (3M - 3r_0 + 2r_0^3 \Lambda) \right) \right],$$

$$r_0 = \frac{3}{2} M \left( 1 + \sqrt{1 - \frac{8Q^2}{9M^2}} \right).$$
\[ r_{1}^{sp} = -\frac{1}{2} r_{0}^{2} C_{0}^{1/2} \left( -30 Q^4 + 90 M Q^2 r_{0} - 63 M^2 r_{0}^2 - 33 Q^2 r_{0}^2 - 18 e Q^3 r_{0}^2 + 42 M r_{0}^3 ight. \\
\left. + 30 e M Q r_{0}^3 - 6 r_{0}^4 + 12 e Q r_{0}^4 + 6 Q^2 r_{0}^4 - 6 M r_{0}^5 \Lambda + r_{0}^6 \Lambda + 2 e Q r_{0}^6 \right) \\
\times \left( -42 Q^4 + 126 M Q^2 r_{0} - 90 M^2 r_{0}^2 - 45 Q^2 r_{0}^2 + 60 M r_{0}^3 - 9 r_{0}^4 + 6 Q^2 r_{0}^4 - 6 M r_{0}^5 \Lambda + r_{0}^6 \Lambda \right)^{-1}. \]

By setting \( Q = 0 \) one can make sure that formula (11) reproduces the corresponding formula [21] obtained for the SdS black hole. We note that \( \text{Im} \omega \) does not depend on the charge of the spinor field \( e \) in the limit of large \( k \). Therefore, the damping rate is equal for the arbitrary spinor field charge for large enough multipole moment. The dependence of \( \text{Re} \omega \) on \( k \) is linear for large \( k \). The slope of the line on the plane \((\text{Re} \omega, k)\) is defined by the coefficient \( C_{0} \).

These conclusions are confirmed by the results of [28] where the particular case \( \Lambda = 0 \) was considered.

3. Decay of the scalar field

The wave equation of the complex scalar field has the form [37]

\[ \phi_{ab} - i e A_{a} g^{ab} (2 \phi_{b} - i e A_{b} \phi) - i e A_{a} g^{ab} = 0. \]

Here we choose the electromagnetic potential as \( A_{t} = -Q/r \). Then, decomposing the wavefunction \( \phi \) into spherical harmonics \( \phi = \sum_{l,m} u_{l}^{m}(t, r) Y^{lm}(\theta, \phi)/r \), the wave equation for each multipole moment takes the form

\[ \partial_{r}^{2} u + 2 i e \frac{Q}{r} \partial_{r} u - \partial_{r}^{2} u + V u = 0 \]

\[ V = f \left( \frac{l(l+1)}{r} + \frac{2 M}{r^3} \right) - \frac{2 \Lambda}{r} \frac{Q^2}{r^2}, \]

where \( r_{s} \) is the tortoise coordinate and \( u(t, r) = e^{-i \omega t} u(r) \). Therefore we arrive at the equation for \( u(r) \),

\[ \partial_{r}^{2} u + [\omega^{2} - V_{\text{eff}}] u = 0, \quad V_{\text{eff}} = V + \frac{2 e Q \omega}{r}. \]

Supposing \( \text{Re} \omega > 0 \) solutions of (17) have to satisfy the boundary conditions

\[ u(r_{s}) \sim C_{\pm} e^{\pm i \omega r_{s}}, \quad r_{s} \rightarrow \pm \infty. \]

To calculate quasi-normal modes, we apply the WKB method of the third order [38] in the scalar field case. The motivation for the use of the WKB method is the same as that in section 2. The use of the calculation method of higher order [36] is overly cumbersome as in the case of scalar field \( V_{\text{eff}} \) includes \( \omega \) so that it is necessary to solve simultaneously the equation for the search of an extremum and the equations of the WKB method itself.

We present the results graphically for the fixed overtone number \( n = 0 \) and multipole number \( l = 1 \) in figures 6–8. The dependence of QN frequencies (figure 6) and their damping rates (figure 7) on the charge of the black hole \( Q \) for different values of the cosmological constant \( \Lambda \) is shown. In figures 6 and 7, the dependences are shown for two values of the charge of the scalar field \( e = 0.1, 0.05 \), which in the figures looks like a couple of curves beginning at the same point at \( Q = 0 \). The upper curve in the couples corresponds to \( e = 0.1 \), and the lower curve in the couples corresponds to \( e = 0.05 \).

The QN frequencies grow monotonically with \( Q \) and become lower as \( \Lambda \) increases. The QN frequencies are higher for the fields with greater charge \( e \). The curves displaying the
QNMs damping rates have a pronounced maximum which is located at $Q_{\text{max}} \approx 0.8$ and the location of the maximum shifts from $Q \approx 0.7$ to $Q \approx 0.9$ as $\Lambda$ increases. Until reaching $Q_{\text{max}}$, the curves grow slowly and after reaching $Q_{\text{max}}$ they fall sharply. Figure 7 shows that the field decays faster if it has greater charge. And similar to the spinor case the decay of the scalar fields becomes slower as $\Lambda$ grows. Both the QN frequencies and the damping rates decrease with $\Lambda$, although the manner of the decrease is different.

Figure 8 displays QNMs for different fixed $Q$. The curves in figure 8 are parametrized by the cosmological constant which changes along the curves. We note that $\text{Im } \omega$ is not related linearly to $\text{Re } \omega$, in contrast to the spinor field case (figures 4 and 5).

Making use of the first-order WKB method, it is also possible to find the asymptotic formula of large $l$ for the calculation of quasi-normal modes $\omega$ for the scalar field,

$$\omega = C_0(l + 1/2) - i(n + 1/2)C_i^\text{sc} + C_r^\text{sc} + O(1/l),$$

(19)
where
\[C_{sc} = \frac{1}{3} r_0^{-3} \left[ -126 Q^4 r_0^2 + Q^2 r_0^2 \left( 378 M r_0 + r_0^2 \left( -135 + 18 \Lambda r_0^2 \right) \right) \right] + r_0^3 \left[ -270 M^2 r_0 - 6 M r_0^2 \left( -30 + 3 \Lambda r_0^2 \right) + r_0^3 \left( -27 + 3 \Lambda r_0^2 \right) \right]^{1/2}. \]

\[C_{sc}^r = \frac{1}{r_0^3} \left( -1 - \frac{2 Q^2}{r_0^2} + \frac{3 M}{r_0} \right) C_0^{-1}, \]

\[r_1^{sc} = -r_0 \left( -6 Q^4 + 21 M Q^2 r_0 - 18 M^2 r_0^2 - 9 Q^2 r_0^2 + 15 M r_0^3 - 3 r_0^4 + 2 Q^2 \Lambda r_0^4 \right) - 3 M \Lambda r_0^2 + \Lambda r_0^6) (42 Q^3 - 12 M Q^2 r_0 + 90 M^2 r_0^2 + 45 Q^2 r_0^2 \right) - 60 M r_0^3 + 9 r_0^4 - 6 Q^2 \Lambda r_0^3 + 6 M \Lambda r_0^5 - \Lambda r_0^6). \]

In expressions (19)–(21) \( C_0, r_0 \) are the same as those in formulae (12), (13) of section 2. By setting \( Q = 0 \) one can make sure that formula (11) reproduces the corresponding formula [21] obtained for the SdS black hole for the scalar field. We note that the asymptotic formula for the RN black hole in asymptotically flat spacetime for the charged scalar field was not found till now. The formula for such a particular case can be obtained by setting \( \Lambda = 0 \) in (19)–(21).

4. Conclusions

We considered the decay of the charged spinor and scalar field near the Reissner–Nordström black hole in de Sitter spacetime. We calculated the quasi-normal frequencies of the fields and their damping rates for the lower overtone number. The analysis of the results allows us to formulate the following conclusions. (a) In the spinor case for a fixed cosmological constant \( \Lambda \), the quasi-normal frequencies of the charged field are lower and the damping rates are higher than those of the neutral field if \( e Q > 0 \) and vice versa if \( e Q < 0 \); (b) In the scalar case for a fixed cosmological constant \( \Lambda \), the fields possessing greater charge have higher quasi-normal frequencies and decay faster; (c) The quasi-normal frequencies of both the spinor and scalar fields are lower in the spacetimes with greater cosmological constant \( \Lambda \). The decay of both the spinor and scalar fields is slower in the spacetimes with greater cosmological constant \( \Lambda \); (d) In the case of the charged spinor field, the dependence of \( \text{Im} \omega \) on \( \text{Re} \omega \) is approximately linear on the diagram parametrized by \( \Lambda \); (e) In the case of the charged scalar field the dependence of \( \text{Im} \omega \) on \( \text{Re} \omega \) is not linear on the diagram parametrized by \( \Lambda \); (f) The dependence of \( \text{Im} \omega \) and \( \text{Re} \omega \) of the charged spinor field on the black hole charge \( Q \) is different for \( e > 0 \) and \( e < 0 \) for all values of \( \Lambda \).

We derived the analytic formula for the calculation of \( \omega \) for large values of the multipole number both for the spinor and scalar fields. The formula for the spinor field shows that for asymptotically large multipole number, the damping rate does not depend on the charge \( e \) of the spinor field.

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References

[1] Frolov V P and Novikov I D 1998 Black Hole Physics: Basic Concepts and New Developments (Dordrecht: Kluwer)
[2] Kokkotas K and Schmidt B 1999 *Liv. Rev. Rel.* 2 2
[3] Nollert H P 1999 *Class. Quantum Grav.* 16 R159
[4] Hod S 1998 *Phys. Rev. Lett.* 81 4293
[5] Dreyer O 2003 *Phys. Rev. Lett.* 90 081301
[6] Cardoso V and Lemos J P S 2001 *Phys. Rev.* D 63 124015
[7] Birmingham D, Sachs I and Solodukhin S N 2002 *Phys. Rev. Lett.* 88 151301
[8] Konoplya R A 2002 *Phys. Rev.* D 66 044009 (Preprint hep-th/0205142)
[9] Cardoso V, Konoplya R and Lemos J P S 2003 *Phys. Rev.* D 68 044024 (Preprint gr-qc/0305037)
[10] Horowitz G T and Hubeny V E 2000 *Phys. Rev.* D 62 024027
[11] Konoplya R A 2002 *Phys. Rev.* D 66 044009 (Preprint hep-th/0205142)
[12] Konoplya R A 2002 *Phys. Rev.* D 66 044009 (Preprint hep-th/0205142)
[13] Perlmutter S et al 1997 *Astrophys. J.* 483 565
[14] Maassen van den Brink A 2003 *Phys. Rev.* D 68 064007
[15] Konoplya R A 2003 *Phys. Rev.* D 68 047501
[16] Maassen van den Brink A 2003 *Phys. Rev.* D 68 064007
[17] Konoplya R A 2003 *Phys. Rev.* D 68 047501
[18] Konoplya R A 2002 *Phys. Rev.* D 66 044007 (Preprint gr-qc/0207028)
[19] Konoplya R A 2002 *Phys. Lett.* B 550 117 (Preprint gr-qc/0210105)
[20] Zhang H and Zhou W 2004 *Class. Quantum Grav.* 21 917 (Preprint gr-qc/0312029)
[21] Zhidenko A 2004 *Class. Quantum Grav.* 21 273 (Preprint gr-qc/0307012)
[22] Wang B, Lin C-Y and Abdalla E 2000 *Phys. Lett.* B 481 79
[23] Berti E and Kokkotas Kostas D 2003 *Phys. Rev.* D 68 064020
[24] Jing J and Pan Q 2005 *Phys. Rev.* D 71 124011
[25] Jing J 2004 *Phys. Rev.* D 69 084009 (Preprint gr-qc/0312079)
[26] Cho H T 2003 *Phys. Rev.* D 68 024003 (Preprint gr-qc/0303078).
[27] Brill D R and Wheeler J A 1957 *Rev. Mod. Phys.* 29 465
[28] Konoplya R A 2003 *Phys. Rev.* D 68 024018 (Preprint gr-qc/0303052)
[29] Hawking S W and Ellis G F R 1973 *The Large Scale Structure of Space Time* (Cambridge: Cambridge University Press)
[30] Iyer S and Will C M 1987 *Phys. Rev.* D 35 3621