Optimization effects of design parameter on the first frequency modal of a Bridge-type compliant mechanism flexure hinge by using the Taguchi method

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Abstract. The compliant mechanism always requests high displacement amplification ratio and high vibration frequency. Therefore, this paper optimized design parameters to obtain the maximum value of the first frequency modal shape using the Taguchi method. First, the model was designed by Solidworks. Second, effect design parameters on the first frequency modal shape were analyzed via ANSYS Workbench 18.0, and the final the maximum value of the first frequency modal was obtained by the Taguchi method. The simulation results revealed that the thickness and width of flexure hinge has a significant effect on the first frequency modal. The S/N analysis results demonstrated that the thickness of the flexure hinge has a significant affect next to the width of the flexure hinge, inline angle between two flexure hinges, input body length, and the fillet radius of the flexure hinge. The ANOVA analysis results identified that the contribution percentage of design parameters and fillet radius of the flexure hinge have a significant effect on the first frequency modal. The R-square was obtained at 99.84%, and the maximum value of the first frequency modal was obtained at 821.8267 Hz.

1. Introduction
The clearance is always present in classical joints in order to eliminate clearance, and its effects such as vibration and friction, lead to wearing of the joint. The flexure hinge was designed for many applications. The advantages of flexure hinges consist of monolithic fabrication, zero backlash and friction, reduced weight, and no lubrication. This paper researches the optimal design of the first frequency modal of a bridge-type compliant mechanism flexure hinges (BTCMFH). In recent years, researchers have designed many types of flexure hinges (FH) to replace traditional joints. For example, Qi et al. presented a displacement amplification ratio (DAR) of BTCMFH based on the kinematics theories and elastic beam theory (EBT). Xu and Li [1] calculated the DAR using the Euler-Bernoulli beam theory, and confirmed their results by FEA and experiment. In 2015, Ke-qi Qi et al. [2] analyzed the ADR of BTCMFH by using EBT and is verified by finite element method and previous experiment. Liu and Yan [3] presented a new analyzed approach using EBT to research influence of external load on the DAR and was verified by FEM in ANSYS. Ling et al. [4] used the power
preservation law and EBT to enhance a methodology approach for bridge-type, rhombus-type compliant mechanisms, and compared their results with those achieved by the FEM in ANSYS and experiments. Choi et al. [5] investigated and presented a bridge-type compliant mechanism which was fully compliant with concentration force and distribution force. Their results were verified by experiment and previous research. Ma et al. [6] noted that the DAR increased as the thickness of a flexure hinge was reduced, and this problem was explored using the FEM and a mathematic model. Ling et al. [7] presented a modular and assembled statics modeling tool for the analysis and design of a wide variety of flexure hinges used in the precision positioning stage. This method was compared with FEM and their previous investigation. The stiffness matrix method was utilized to determine the DAR and was verified by finite element method and experiment. To ensure high rigidity, large magnification, high-precision tracking, and high-accuracy positioning. Dao and Huang [8] utilized Taguchi method and grey relational analysis to optimize two degree of freedom compliant mechanism.

The target of this paper presents a design and optimizes the effects of design parameters on the first frequency modal of a BTCMFH by using the FEM in ANSYS, i.e., the Taguchi method.

2. Modeling and methodology

2.1. Modeling

The mechanism is presented in Figure 1, with dimensions of 70 mm x 25 mm x 10mm. Figure 1(a) is a 3D model, Figure 1(b) is a 2D model, and Figure 1(c) is a zoom-in medium body and two FHs to obviously see the incline angle \( \theta \) between two FHs. The mechanism has eight FHs with 4 mm length and thickness change, four medium bodies, two input bodies with input length \( L_i \) change, one fixed body, and one output body of 8 mm length and 10 mm width. The concentration force, distribution force, or displacement can be used as inputs for the mechanism. Then, the output body will be moved upward in the y-axis direction.

![Figure 1. Bridge-type compliant mechanism: (a) 3D; (b) 2D; (c) Large view medium body and two flexure hinges.](image-url)
Table 1. Material mechanical properties.

| Material   | Young's modulus (GPa) | Poisson's ratio | Yield (MPa) |
|------------|------------------------|-----------------|-------------|
| Aluminium  | 72                     | 0.33            | 503         |

2.2. Analysis finite element method

Material used for the mechanism is listed in Table 1. First, the mechanism was drawn in Solidworks and then put into the modal of ANSYS. Then, we selected material for the mechanism, i.e., aluminum AL-7075. The mechanism was meshed automatically, as presented in Figure 2(a), with the number of nodes and number of elements equal to 130524 nodes and 73352 elements, respectively. The fixed support utilized the A surface, and two input bodies input the displacement of 0 mm each on the B surface and C surface, according to three directions such as the x, y, and z axes as shown in Figure 2(b).

![Figure 2](image)

(a) The meshed mechanism, (b) Fixed support and input displacement for the mechanism.

3. Optimization

Advantages of the Taguchi method include fewer required experiments or simulations by utilizing orthogonal arrays, and its ability to minimize influences of parameters that cannot be controlled. Moreover, it is a simple method and easy to use. In this method, a loss function is used to determine the error between simulation or experiment and actual values. This function is the signal-to-noise (S/N) ratio [8, 9]. It has three categories, but only the “the larger the better” categories were used in this paper to optimize the first frequency modal shape, as follows.

“The larger the better” approach,

\[
S / N = -10 \log \left( \frac{1}{n} \sum_{i=1}^{n} \frac{1}{y_i^2} \right) 
\]

where, the variance of y is the observed data of each characteristic, n is the number of experiments, respectively. In this paper, the Minitab 18 program was used to create the Taguchi method, S/N analysis, and analysis of variance (ANOVA). The obtained results are stated in the Results and Discussion section of this paper.

4. Results and discussions

4.1. Influence of thickness

The first frequency modal shape is presented in Figure 3, which identifies this frequency increase from 190.46 to 429.34 Hz when thickness of flexure hinge (TOFH) increases from 0.4 to 0.6 mm with input displacement equal to zero according to the x-, y-, and z-axes directions, with an input body length of 5 mm, incline angle between two flexure hinges (FH) of 0.7 degrees, width of FH of 2 mm, and fillet radius of FH of 0 mm. The simulation results are higher references [1, 2] and smaller than in reference [6].
Figure 3. FEM result for the 1st frequency modal shape with different thickness of flexure hinge: (a) Thickness of 0.4 mm; (b) Thickness of 0.6 mm; (c) Thickness of 0.8 mm.

Figure 4. FEM result for the 1st frequency modal shape with different width of flexure hinge: (a) Width of 2 mm; (b) Width of 4 mm; (c) Width of 6 mm.
4.2. Influence of width

The first frequency modal shape is presented in Figure 4, which identifies this frequency increase from 302.34 to 515.61 Hz when width of FH increases from 2 to 6 mm, with input displacement equaling to zero according to the x, y, and z axes directions, and with an input body length of 5 mm, incline angle between the two flexure hinges (FH) of 0.7 degrees, TOFH of 0.6 mm, and fillet radius of FH of 0.2 mm. The simulation results are higher References [1, 2, 6].

4.3. Optimization of the first frequency modal shape

In Section 4.1 and 4.2 revealed that the first frequency modal shape is significantly affected by TOFH and width of FH. In order to maximum the first frequency modal, the design parameters and their level need to optimize as listed in Table 2. Table 3 presented orthogonal arrays, simulation results and S/N ratio which the orthogonal arrays were designed by Minitab 18.0, the simulation results were obtained by ANSYS 18.0 and S/N ratios results were obtained by Minitab 18.0 or by using Equation (1).

The S/N plot as shown in Figure 5(a) and response table for S/N ratios for the first frequency modal as listed in Table 4 revealed that the maximum value of the first frequency modal shape was obtained at input body length of 10 mm (level 2), TOFH of 0.8 mm (level 3), incline angle of 1.3 degree (level 3), fillet radius of FH of 0.4 mm (level 3), and width of FH (level 3). The mean values for the first frequency modal as drawn in Figure 5(b) and response table for means as listed in Table 5. The mean values in Table 5 were used to determine the optimal value for the first frequency modal. Table 4 and Table 5 identified the design parameters has effected on the first frequency modal such as the first is TOFH has significantly affected, the second is the width of FH, the third is incline angle between two FH, the fourth is input body length and the final is fillet radius of FH.

![Figure 5](image_url)

Figure 5 (a) S/N plot and (b) Mean plot for the first frequency modal.
Table 2. Design parameters and their levels.

| Factor       | Unit | Levels 1 | Levels 2 | Levels 3 |
|--------------|------|----------|----------|----------|
| Input body length | mm   | 5        | 10       | 15       |
| TOFH         | mm   | 0.4      | 0.6      | 0.8      |
| Incline angle | degree | 0.7     | 1.0      | 1.3      |
| Fillet radius | mm   | 0.0      | 0.2      | 0.4      |
| WOFH         | mm   | 2        | 4        | 6        |

The ANOVA results were presented in Table 6. It identified that contribution percent of regression equation and design parameters a, b, c, d, e is 99.84%, 83.01%, 14.75%, 0.8%, 0.03%, 0.07%, respectively and error percent is 0.16%. F-value is the larger is the better but is greater than 2 and P-value must be smaller than 0.05, but F-value and P-value of d are 0.49 and 0.489 respectively. It demonstrated d parameter is not significant.

The regression equation was obtained by Minitab 18.0 as presented in Equation (2). The residual plot for the first frequency modal shape was illustrated in Figure 6. It demonstrated the graph of the regression equation was plotted is close to the straight line. The error is equal to from -2 to 2 and was confirmed by Equation (4) at 95% confidence interval.

\[ f = (0.691a + 18.72b + 2.331c + 0.82d + 1.261e - 0.348a^2)^2 \] (2)

Table 3. Orthogonal arrays, simulation results and S/N ratio for the first frequency modal shape.

| Trial No. | a  | b  | c  | d  | e  | 1st frequency mode (Hz) | S/N 1st frequency mode |
|-----------|----|----|----|----|----|--------------------------|-------------------------|
| 1         | 5  | 0.4| 0.7| 0  | 2  | 190.77                   | 45.6102                 |
| 2         | 5  | 0.4| 1  | 0.2| 4  | 313.86                   | 49.9347                 |
| 3         | 5  | 0.4| 1.3| 0.4| 6  | 448.49                   | 50.7559                 |
| 4         | 5  | 0.6| 0.7| 0.2| 6  | 515.47                   | 54.2441                 |
| 5         | 5  | 0.6| 1  | 0.4| 2  | 344.98                   | 50.7765                 |
| 6         | 5  | 0.6| 1.3| 0  | 4  | 508.31                   | 54.1226                 |
| 7         | 5  | 0.8| 0.7| 0.2| 4  | 627                      | 55.9454                 |
| 8         | 5  | 0.8| 1  | 0  | 6  | 771.26                   | 57.7440                 |
| 9         | 5  | 0.8| 1.3| 0.2| 2  | 497.54                   | 53.9366                 |
| 10        | 10 | 0.4| 0.7| 0  | 2  | 269.98                   | 48.6266                 |
| 11        | 10 | 0.4| 1  | 0.2| 4  | 323.95                   | 50.2096                 |
| 12        | 10 | 0.4| 1.3| 0.4| 6  | 472.09                   | 53.4805                 |
| 13        | 10 | 0.6| 0.7| 0.2| 6  | 523.45                   | 54.3775                 |
| 14        | 10 | 0.6| 1  | 0.4| 2  | 350.64                   | 50.8972                 |
| 15        | 10 | 0.6| 1.3| 0  | 4  | 525.26                   | 54.4075                 |
| 16        | 10 | 0.8| 0.7| 0.4| 4  | 632.87                   | 56.0263                 |
| 17        | 10 | 0.8| 1  | 0  | 6  | 786.59                   | 57.9150                 |
| 18        | 10 | 0.8| 1.3| 0.2| 2  | 702.71                   | 56.9355                 |
| 19        | 15 | 0.4| 0.7| 0  | 2  | 194.35                   | 45.7717                 |
| 20        | 15 | 0.4| 1  | 0.2| 4  | 329.16                   | 50.3481                 |
| 21        | 15 | 0.4| 1.3| 0.4| 6  | 483.75                   | 53.6924                 |
| 22        | 15 | 0.6| 0.7| 0.2| 6  | 528.57                   | 54.4621                 |
| 23        | 15 | 0.6| 1  | 0.4| 2  | 353.61                   | 50.9705                 |
| 24        | 15 | 0.6| 1.3| 0  | 4  | 384.34                   | 51.6943                 |
| 25        | 15 | 0.8| 0.7| 0.4| 4  | 636.84                   | 56.0806                 |
| 26        | 15 | 0.8| 1  | 0  | 6  | 796.48                   | 58.0235                 |
| 27        | 15 | 0.8| 1.3| 0.2| 2  | 512.3                    | 54.1905                 |
Table 4. Response table for signal to noise ratios.

| Level | a   | b   | c   | d   | e   |
|-------|-----|-----|-----|-----|-----|
| 1     | 52.81 | 50.08 | 52.35 | 52.66 | 50.85 |
| 2     | 53.65 | 52.88 | 52.98 | 53.18 | 53.20 |
| 3     | 52.80 | 56.31 | 53.94 | 53.43 | 55.22 |
| Delta | 0.85 | 6.23 | 1.59 | 0.77 | 4.36 |
| Rank  | 4   | 1   | 3   | 5   | 2   |

Table 5. Response table for means.

| Level | a   | b   | c   | d   | e   |
|-------|-----|-----|-----|-----|-----|
| 1     | 468.6 | 336.3 | 457.7 | 491.9 | 379.7 |
| 2     | 509.7 | 448.3 | 485.6 | 471.9 | 475.7 |
| 3     | 468.8 | 662.6 | 503.9 | 483.4 | 591.8 |
| Delta | 41.1 | 326.4 | 46.2 | 20.0 | 212.1 |
| Rank  | 4   | 1   | 3   | 5   | 2   |

Table 6. Analysis of variance for transformed response.

| Source    | DF | Seq SS  | Contribution | Adj SS  | Adj MS  | F-Value | P-Value |
|-----------|----|---------|--------------|---------|---------|---------|---------|
| Regression| 6  | 13003.7 | 99.84%       | 13003.7 | 2167.29 | 2180.61 | 0.000   |
| a         | 1  | 10811.2 | 83.01%       | 10.2    | 10.20   | 10.26   | 0.004   |
| b         | 1  | 1920.7  | 14.75%       | 294.4   | 294.41  | 296.22  | 0.000   |
| c         | 1  | 104.6   | 0.80%        | 10.7    | 10.69   | 10.75   | 0.004   |
| d         | 1  | 4.2     | 0.03%        | 0.5     | 0.49    | 0.49    | 0.489   |
| e         | 1  | 153.5   | 1.18%        | 122.2   | 122.23  | 122.98  | 0.000   |
| a*a       | 1  | 9.6     | 0.07%        | 9.6     | 9.62    | 9.68    | 0.005   |
| Error     | 21 | 20.9    | 0.16%        | 20.9    | 0.99    |         |         |
| Total     | 27 | 13024.6 | 100.00%      |         |         |         |         |

R-sq =99.84%, R-sq(adj) =99.79, R-sq(pred) =99.73%

4.4. Predicted optimization

In this section, the forecasted value is calculated and verified by the optimization models was simulation in ANSYS, as based the optimal combination parameters. First, the mean value of the first frequency modal were listed in Table 5, Second, \( \mu_f \) was obtained by using Equation (3) and Table 5.

The total mean value is equal to 482.3933 and \( \mu_f \) is obtained, as follows:

\[
\mu_f = f_m + \sum_{i=1}^{q} (f_i - f_m) = a_2 + b_3 + c_3 + d_3 + e_3 - 4f_m
\]

\[
= 509.7 + 662.6 + 503.9 + 483.4 + 591.8 - 4 \times 482.3933 = 821.8267 (Hz)
\]
The predicted result is larger than PSO optimization method as presented in reference [1]. The 95% confidence interval to verify (CI) was obtained using Equation (4), at $\alpha=0.05$, $f_e=21$, $F_{0.05}(1,21)=4.3248$ [9], $V_e=0.99$, $R=6$, $R_e=1$, $n=27$

\[
CI_{CE} = \pm F_{\alpha}(1, f_e) V_e \left( \frac{1}{n_{\text{eff}}} + \frac{1}{R_e} \right)
\]

\[
= \pm 4.3248 \times 0.99 \times \left( \frac{1}{27/7} + 1 \right) = \pm 2.322 , \quad 819.5047 < f_{\text{confirmation}} < 824.1486
\]

5. Conclusions
The effects of design parameters on the first frequency modal were analyzed using ANSYS. The optimal value of the first frequency modal was determined by the Taguchi method. The S/N ratios analysis stated that the design parameters have a significant effect on the first frequency modal. The first is TOFH, the second is WOFH, the third is an incline angle of two FHs, the fourth is input body length, and the final is the fillet radius of FH. The ANOVA results demonstrated that the fillet radius is not significant and presented a contribution percentage of design parameters. The obtained optimal value was 821.8267 Hz. With the optimal combination parameter $a_2b_3c_3d_3e_3$, the simulation value of the first frequency modal was obtained 873.99 Hz, and deviation between simulation value and the optimal value was 5.97%. The first frequency modal also verified at 95% confidence interval of $819.5047 < f_{\text{confirmation}} < 824.1486$.

Acknowledgments
The authors appreciate the partly funding supported by MOST 107-2622-E-992-012-CC3 from Ministry of Sciences and Technology, and support from National Kaohsiung University of Science and Technology in Taiwan.

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