Anti-exceptionalism and methodological pluralism in logic

Diego Tajer

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Abstract
According to methodological anti-exceptionalism, logic follows a scientific methodology. There has been some discussion about which methodology logic has. Authors such as Priest, Hjortland and Williamson have argued that logic can be characterized by an abductive methodology. We choose the logical theory that behaves better under a set of epistemic criteria (such as fit to data, simplicity, fruitfulness, or consistency). In this paper, I analyze some important discussions in the philosophy of logic (intuitionism versus classical logic, semantic paradoxes, and the meaning of conditionals), and I show that they presuppose different methodologies, involving different notions of evidence and different epistemic values. I argue that, rather than having a specific methodology such as abductivism, logic can be characterized by methodological pluralism. This position can also be seen as the application of scientific pluralism to the realm of logic.

Keywords Anti-exceptionalism · Logical methodology · Non-classical logics · Philosophy of logic · Logical pluralism

1 Introduction
Logic has often been considered the solid foundation over which every science must be built. In this view, logical principles are self-evident, and we can use them to revise the commitments of every other scientific theory: logic is part of science, but it has an exceptional place in it. Kant claimed that “General logic abstracts (...) from all content of cognition, i.e. from any relation of it to the object, and considers only the logical form in the relation of cognitions to one another, i.e., the form of thinking in general” (Kant, 1998, A55/B79). According to Wittgenstein, “It is as impossible to represent...
in language anything that ‘contradicts logic’ as it is in geometry to represent by its coordinates a figure that contradicts the laws of space, or to give the coordinates of a point that does not exist” (Wittgenstein, 1961, §3.032). In other words, both Kant and Wittgenstein argued that we cannot even think in a way that contradicts logic. Illogical thinking is just not thinking at all. More contemporarily, this foundationalist conception of logic has been associated with aprioristic methodologies (Martin & Hjortland, 2020, p. 286), where logical principles are known by rational intuition or by conceptual analysis.

Anti-exceptionalism about logic reacts against this view. According to logical anti-exceptionalism (Hjortland, 2017), logic is not exceptional with respect to science: it is continuous with science, reversible, and even revisable following a scientific method. This is not a new position. Quine defended the revisability of logic by means of empirical data, and the continuity between logic and science. In one of his most famous articles, he states as follows (Quine, 1951, p. 43):

No statement is immune to revision. Revision even of the logical law of the excluded middle has been proposed as a means of simplifying quantum mechanics; and what difference is there in principle between such a shift and the shift whereby Kepler superseded Ptolemy, or Einstein Newton, or Darwin Aristotle?

However, Quine’s position was more attached to the rejection of the synthetic/analytic distinction and to the continuity between logic and empirical science, two aspects that are not necessarily the most important ones in contemporary anti-exceptionalism. In any case, the meaning of anti-exceptionalism is still imprecise, and it has recently been used in different ways, focusing on different properties (continuity with science, scientific method, revisability, etc.).

In a recent paper, Martin and Hjortland (2020, p. 286) have described a specific version of this approach, “Methodological Anti-Exceptionalism”. According to this version, logical research works with a scientific methodology, and this makes logic non-exceptional: “Theory choice within logic is similar in important respects to that of the recognized sciences.” This is precisely the version of anti-exceptionalism that I will discuss in this paper.

The philosophy of logic has always involved questions about meaning, paradoxes and justification. For example, what is the meaning of logical constants? How are basic logical laws justified? What is the best solution to the semantic paradoxes? But part of the discussion has recently focused on methodological questions, regarding logicians’ actual practice. It is reasonable to think that these methodological observations will also illuminate these classic issues in the philosophy of logic and motivate more practice-based ideas.

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1 Martin and Hjortland (2020) identify two aprioristic methodologies: rationalism and analyticity. According to rationalism, one understands basic logical principles by rational intuition. This has been proposed as a methodology for philosophy in general (Bealer, 1998), and knowledge of logic is just an instance of it. The other methodology is analyticity (Boghossian, 2000), which is more directly connected to logic; according to it, the justification of basic logical principles is derived from the meaning of the constants (e.g., the Modus Ponens rule is justified by the meaning of “if... then...”).
The methodological approach involves two questions that are sometimes hard to distinguish:

- The descriptive question: What is the methodology of logical research?
- The normative question: What should the methodology of logical research be?

There are many factors to consider when responding to the descriptive question; indeed, every piece of logical research can be used as evidence when doing so. Hopefully, there are global and structural features of logical research that can be detected using the descriptive approach. It is somewhat harder to respond to the normative question: what evidence can we use to support that the methodology of logic should be such and such? The risk of focusing on the normative question is that we would end up defending methodological constraints that are far from real practice. On the other hand, the risk of adopting the descriptive approach is falling into conservatism.

In this paper, I will begin with a descriptive approach, using the real practice of logicians as evidence. After carrying out this descriptive approach, I will try to draw some normative conclusions about the methodology that logicians should adopt.

One version of methodological anti-exceptionalism is abductivism. The idea is that logic works, like many scientific discussions, with a broadly abductive methodology. According to abductivism about logic, we choose a logical theory whenever we know that it behaves better than the others under some epistemic criteria such as fit to data, simplicity, fruitfulness or unifying power (Priest, 2014, p. 217). Abductivism is not necessarily tied to a specific logic, as it has been proposed by both classical and non-classical logicians.

Williamson (2017), for example, argues that abductivism leads to classical logic, for classical logic is stronger than most competitors and essential for scientific research. On the other hand, Hjortland (2017) claims that abductivism paves the way for non-classical logics, for they can solve paradoxes and recapture classical logic. We will address these issues in Sect. 3.

Abductivism is not free from criticism. Martin and Hjortland (2020, pp. 286–287) have argued that abductivism is not a correct description of logical activity, for there is no conclusive reason to believe that logicians choose (or should choose) their best theory on the basis of virtues such as simplicity, strength and unifying power. This list of virtues is not necessarily suited to logical practice (ibid., p. 287): “no viable account has yet been provided of why logicians should use these virtues to dictate theory choice (that is, why such reliance upon these virtues is rational), nor even what sense we can make of these virtues within the context of logic.” They defend an alternative view, whereby logic is supposed to “predict and explain” validity sentences. The general idea is that we construct a theory (using definitions, rules and laws) to explain the validity of some intuitively valid informal arguments, and then we use the theory to

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2 So far, methodological anti-exceptionalists (such as Williamson, Hjortland or Priest) have not proposed a revision of the methodology of logic but rather an explanation of it. Therefore, it is hard to characterize their positions as “normative” or “descriptive”. They include normative and descriptive elements at the same time.

3 Two important clarifications should be made here. First, Martin and Hjortland do not believe that predicativism is the only methodology of logic (although they do not elaborate more on this point). Second, this view of prediction is obviously not diachronic: it does not work as an experiment in empirical science.
predict the validity of other arguments. We need to check whether the new predictions hold; if not, the theory (or some auxiliary hypotheses) must be revised.

Hlobil (2020) claims that abductivism cannot work in logic, for there are many different conceptions of logical consequence (semantical, proof-theoretical, normative, etc.) and they involve very different epistemic values. Those who take logic as an instrumental device for science will not share the same epistemic values as those who take logic as a maximally general standard for reasoning. For example (p. 326), capturing the practice of actual mathematicians is essential from an instrumentalist point of view, but secondary from a purely normative standpoint.\(^4\)

In this paper, I will argue that there is a healthy methodological pluralism in logic. Different philosophical problems give rise to different methodologies; and some problems are even compatible with a variety of methodologies. Labels such as “abductivism” or “predictivism” are good approximations to the issue, but they cannot explain the variety found in the methodology of logic; some discussions in logic may have an abductive nature, but this cannot be said about logic as a whole. I will analyze three different discussions and the methodology involved in them, and I will argue that some essential methodological aspects are so different in each case that it is not possible to unify everything under a common structure; moreover, some discussions in logic presuppose a common methodology, but other discussions involve internal methodological disagreements. The only way in which the methodology of logical practice can be unified is in a very trivial way, e.g., we can say that logic is “abductive” in general just because logicians use epistemic criteria for accepting or rejecting theories. But this can arguably be said about every area of research.

I agree with Martin and Hjortland (2020) that evidence for logic comes from different sources, such as informal mathematical proofs and linguistic intuitions about natural language. However, I will reject the idea that a logical theory needs to explain or predict all that evidence together; there is no need for a general “theory of validity” (Martin & Hjortland, 2020, p. 301). The predictivist view is too general: it presupposes the existence of a logical theory as a device for informally explaining valid arguments and predicting new ones, without considering what those arguments are about. Under the predictivist approach, introducing more evidence (or evidence of a different kind) makes the theory more sophisticated and accurate. However, if we observe the practice of logicians, it is mostly focused on specific problems. A theory is considered acceptable in a discussion whenever it can provide an explanation of the evidence that this discussion takes as relevant. This depends to a large extent on the problem that the theories are supposed to solve. For example, a theory which tries to reconstruct informal mathematics is not necessarily supposed to also capture the meaning of connectives in natural language. Logics might be unifying and coherent, but they cannot address all the problems with the same degree of sophistication.

The methodological pluralism that I defend will be illustrated by some classic examples from the literature. My position starts out as descriptive: I observe that logic

\(^4\) This change in epistemic values is easy to identify in the discussion about anti-exceptionalism. For example, reconstructing scientific practice is essential for Williamson (2017), and therefore the most important value is deductive strength. By contrast, Hjortland (2017) wants a general theory of validity, and he does not consider that losing deductive strength is necessarily bad.
involves a variety of methodologies. However, it also has a normative dimension, for I will argue that this pluralism is fruitful from a theoretical point of view.

My pluralistic approach is similar in some aspects to Hlobil’s. However, there is a key difference. He claims that the source of the methodological disagreement are the different conceptions of logical consequence: every conception (proof-theoretical, semantical, etc.) tries to accommodate different data. According to Hlobil (2020, p. 321): “Different conceptions of logic lead to different views about which data logics should explain.” Here I will argue that conceptions of validity are not necessarily the main source of the different methodologies in logic: two groups of logicians may share the same conception of logical consequence but diverge when it comes to methodology. For example, the theories about the conditional in natural language and the semantic theories of truth usually adopt the standard conception of validity as truth-preservation, but, as we will see, they use a completely different methodology for developing and choosing theories.

I will suggest that we should adopt a more pluralistic and “local” approach to the philosophy of logic: we should not look at logic as a homogenous activity. With a more pluralistic and local point of view, we will also obtain responses to questions about meaning or justification that are more strongly grounded on practice, and we may draw more useful directions for the future. My view is clearly connected to the main ideas of scientific pluralism (Chang, 2012; Kellert et al., 2006), a branch in the philosophy of science with a positive attitude toward the coexistence of different (and sometimes incompatible) research programmes inside one discipline.

In the next sections, I will analyze three classic discussions in philosophical logic about (i) intuitionism versus classical logic on the foundation of mathematics (Sect. 2), (ii) non-classical theories of truth (Sect. 3), and (iii) the meaning of the conditional (Sect. 4). By no means will I attempt to cover all the disputes in philosophical logic: there are arguably similar points to be made about other topics such as vagueness or modal metaphysics. However, the cases that I will discuss illuminate clearly different methodologies and, for this reason, I think that they provide strong support for the pluralistic approach that I aim to promote. In Sect. 5, I will explain my position in detail, using as evidence the examples from previous sections. In Sect. 6, I will compare my view with some versions of logical pluralism.

2 Reconstructing mathematics: classical versus intuitionistic logic

Intuitionism is one of the most discussed non-classical logics. It first emerged as a mentalistic and constructivist philosophy of mathematics (Brouwer, 1975b), but it also became a prominent philosophy of language, after the contributions by Dummett (1975). Intuitionistic philosophers and logicians share the rejection of mind-transcendent truths, which translates into the rejection of the law of excluded middle. In this section, I will analyze the methodological dimension of this discussion, using some texts by Brouwer, Heyting and Dummett. Given the complexity of the issue, I do not aim to provide a thorough analysis of their points of view, but to describe the general project of intuitionism and some of the differences with the classical approach.
Intuitionistic logicians have a very special conception of mathematics, whereby mathematical objects are interpreted as mental constructions. As Heyting claims: “every mathematical proposition must be an assertion about mental constructions” (Heyting, 1984, p. 87). They take the constructability of proofs as a fundamental value: only constructive proofs can be taken as evidence. Proofs are understood in terms of mental constructions (ibid., p. 81):

Almost every theorem can be brought in this form: Suppose that construction A has been performed, then we can also perform construction B. The proof of such a theorem consists in a construction which, joined to the construction A, yields the construction B.

This mentalistic conception is not part of classical mathematics (some classical mathematicians might be mentalistic but, clearly, not all are). Analogously, constructability is an epistemic value that classical logicians would hardly adopt. Other intuitionistic assumptions are even more at odds with the classical view. Heyting (ibid., p. 87), for example, claims that “when I can assert A ∨ B, then I can assert A or I can assert B.” This general view about disjunction, which is clearly coherent from an intuitionistic point of view (where the law of excluded middle is not generally accepted), is not shared by most classical researchers: they would normally claim that in some cases you can assert a disjunction even though it wouldn’t be permissible for you to assert either disjunct.5

Intuitionism is not only a constructivist philosophy of mathematics, but a constructivist metaphysics in general. According to Brouwer (1975a, p. 480), objects are “iterative complexes of sensations” that are “completely estranged from the subject”; there is no exterior world strictly speaking, but an “exterior world of the subject” (ibid., p. 481). The rejection of a mind-independent reality, although not necessarily in Brouwer’s terms, is still one of the main philosophical points of intuitionism and is frequently used as a philosophical ground for the constructability requirement. Classical mathematics, on the other hand, is not so closely linked to a specific view of reality: classical mathematicians can work and understand each other even holding radically different philosophical points of view, such as formalism or Platonism.

Dummett (1975) defended intuitionistic logic as the result of abandoning some implausible principles about meaning and communication. According to him, intuitionism is the natural consequence of taking the thesis that meaning is use seriously.6 From this perspective, the notion of truth should be replaced (or identified, depending on the interpretation) with the notion of proof. Like other concepts from natural language, whose meaning emerges from their rules of application, the meaning of connectives should emerge from the rules of elimination and introduction. This also explains why Dummett and also intuitionistic logicians usually pay more attention

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5 Furthermore, according to Grice (who defends the classical meaning of connectives), when you say “A or B”, in general you do not know at the moment whether A or B. If you knew, e.g., that A is true, then you would simply say A (Grice, 1989).

6 It is hard (if not impossible) to reconstruct Dummett’s argument in a short paragraph. In fact, a highly detailed analysis of his view would require a whole separate paper. For a nice exposition of Dummett’s intuitionistic ideas, see Placek (1999).
to the proof-theoretical side of logical systems; the meaning of connectives (or their “semantics”) must come from rules.

Dummett’s positions are far from obvious. Burgess (a classical logician) notes, as a *reductio* against Dummett, that revising mathematics goes against the idea that “meaning is use” (Burgess, 1984, p. 188):

Dummett is prepared to join Brouwer and Heyting in declaring that many of the sentences of classical mathematics are ‘incoherent’ and ‘unintelligible’. This sounds odd. For Dummett can hardly deny that the sentences of classical mathematics possess a definite usage within pure mathematics and a definite utility through applied mathematics. How can he, as a professed adherent of the slogan that meaning is use, then deny that those sentences have a meaning?

Therefore, it is fair to say that intuitionists adopt a very specific reading of the idea that meaning is use: a proof-theoretical notion of meaning, in which only constructive proofs are allowed.

Finally, intuitionistic logicians are not necessarily worried about the results of classical mathematics that they lose. Some of these strong classical results, they claim, are simply incorrect. According to Brouwer (1975b, p. 107), “logical deductions which are made independently of perception, being mathematical transformations in the mathematical system, may lead from scientifically accepted premises to an inadmissible conclusion.” As is well-known, Brouwer rejects the law of excluded middle, for he interprets that it means that every mathematical problem is solvable (Brouwer, 1975b, p. 109). According to Dummett, “classical mathematics employs forms of reasoning which are not valid in any legitimate way of construing mathematical statements” (Dummett, 1975, p. 215). On the contrary, losing mathematical results represents a huge problem for most classical logicians. Burgess, for example, claims that adopting intuitionistic mathematics would be harmful for science (p. 191): “Many physicists, mathematicians, logicians and philosophers have suggested precisely the contrary, that intuitionistic restrictions on mathematics would be detrimental to applications.”

As we can see, the epistemic values of classical and intuitionistic logicians are entirely different. For intuitionistic logicians, *constructability* is the primary epistemic value; this value is also motivated by philosophical anti-realism, and a proof-theoretical notion of meaning. By contrast, constructability is a less significant value for classical logicians, who are more worried about retaining classical mathematics as it is. A good intuitionistic theory should be able to distinguish between the most solid results in mathematics and those which depend on “doubtful” principles such as the law of excluded middle. But classical logicians in general want to capture mathematics as it is.
is normally practiced: even if they usually discuss some philosophical points about mathematical practice, they will not consider losing standard theorems an acceptable price to pay for these philosophical questions.

This has clear consequences for our view about the methodology of logic. If we take abductivism to presuppose a similar set of values across the parties, the discussion between classical and intuitionistic logicians is not abductive. There is a basic disagreement about the nature of evidence and what a theory should do. The discussion could be better described as a metaphilosophical dispute about the role of logic and the philosophical constraints that must be put upon a logical theory. We can say that intuitionistic and classical logicians use two different methodologies, with their own notions of evidence and their own epistemic values.

To summarize: in this section, I have argued that the classical and the intuitionistic approach presuppose different methodologies. In this case, the methodological differences are grounded on philosophical disagreements about reality and meaning. In the next sections, I will show that there are many other methodologies in logic, and some of them are not based on deep philosophical disagreements as in this case. We will see that other problems in logic and philosophy give rise to other methodological constraints.

3 Semantic paradoxes: the paradise of logics

The discussion about semantic paradoxes involves a large set of candidates. Tarski (1956) developed a classical theory of truth using the techniques of modern logic, and this ushered in the more contemporary axiomatic theories of truth (Halbach 2015). In parallel, there are many non-classical and substructural approaches, which reject some classical principles such as the law of excluded middle, ex falso sequitur quodlibet, or structural transitivity. In this section, we will focus on this second set of proposals.

Williamson (2017) claimed that the solution to paradoxes follows an abductive methodology: different solutions have different virtues, but we should choose the solution that behaves better given many epistemic virtues. According to him, the winning candidate is classical logic, mainly because classical strength is necessary for science (p. 340):

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9 Hlobil (2020) also considers the question of whether we can choose “abductively” between epistemic values, and only then choose abductively a theory. He argues that we cannot, for it leads to some form of circularity. In this case, for example, we may need to choose between constructability or strength. If we value the strictness of proofs, we may choose constructability; while if we value the reconstruction of actual mathematical practice, we may choose strength. However, according to Hlobil, going one level up is of no help here, for our choice of values is not independent from our choice of a logic (intuitionistic logicians will choose constructability and classical logicians will choose strength): as he claims, “the envisaged two-step abduction doesn’t rescue the claim that abduction can serve as a neutral arbiter in logical disputes” (p. 18).

10 Following the ideas of this paper, it would also be possible to claim that axiomatic theories of truth and model-theoretical non-classical theories of truth belong to two different projects using two different methodologies. I will go back to this topic in Sect. 5.
... any complex scientific theory, especially one that involves some mathematics, will make heavy use of negation, conjunction, disjunction, quantifiers, and identity. Thus, restricting classical logic will tend to impose widespread restrictions on its explanatory power, by blocking the derivation of its classical consequences in particular applications.

As a response, Hjortland (2017) argued that strength is not always a positive feature, and that the place of classical logic in science can be achieved even when adopting a non-classical logic (via recapture theorems); therefore, abductivism is compatible with non-classical logic.

But is this abductive picture consistent with real practice? Some authors have certainly adopted a broadly abductive methodology. According to Martin (2020), Priest’s defense of dialetheism (Priest, 2006) is a case of abductivism, for he develops not only a solution to semantic paradoxes but also a unifying approach to language and philosophical puzzles in general. However, this observation cannot be extended to all the authors in the discussion about paradoxes. As we will see, some authors do not clearly fall within the abductivist picture.

The presupposition of rivalry underlying the abductivist picture does not apply so clearly to this debate. Many authors just provide new theories that can expand our knowledge about the topic; they do not necessarily believe that their solution is the correct one, but that the solution is not worse than the others, and that it is interesting enough. For example, Beall (2009, p. 94) says this about Field’s paracomplete position in comparison with his own paraconsistent theory:

The question, in the end, is how to choose between the two given accounts—my account and that of [Field]. The short answer, of course, is that we should choose the right account. The trouble, though, is that, while I reject Field’s proposal and endorse the simple account I’ve laid out in this book, I find myself in the dubious position of enjoying precious little by way of strong objections against Field’s position. As such, I ultimately—though with genuine regret—leave the matter open for future debate.

A similar point is made in some substructural approaches. French (2016, p. 127), for example, rejects the structural rule of reflexivity, and he explains this situation in these terms:

We have judged structural reflexivity to be innocent for far too long. Moreover, much like other authors have started to argue about structural contraction and transitivity, we have always had ample reasons to be suspicious of this innocuous seeming principle, and so it is no surprise that once we dig around that such a shady character might be implicated in the paradoxes of self-reference.

As we can see, French is not saying that reflexivity is the main source of paradoxes, but that the same reasons that can lead you to reject contraction or transitivity can also

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11 Apart from the commitment to dialetheism, Priest also supports a methodological principle that he calls the “Principle of Uniform Solution”, according to which similar problems should be solved with the same solution, if possible. In a classic paper (Priest, 1994), he claims that most of the paradoxes we know (Liar, Burali-Forti, Richard, etc.) can be characterized by the same structure (the “Inclosure Schema”). It is therefore possible to use dialetheism to solve all of these paradoxes simultaneously.
lead you to reject reflexivity. He introduces another option into the market, without showing that it is strictly better than the others. In these cases, there is not a clear abductive methodology, for authors do not argue that their theories are necessarily better than the others; rather, it is enough to show that the theories are in an equal position. Novelty becomes an important virtue of a theory, and diversity becomes a virtue of the discussion as a whole.

My aim is not to argue against abductivism and to offer an alternative; I will rather defend methodological pluralism in logic. For the discussion about paradoxes is also different in many respects from the debate between classical and intuitionistic logicians. As we mentioned above, intuitionism has a strong revisionary approach: some results in classical mathematics are meant to be based on philosophical mistakes, and intuitionistic logic is supposedly the right theory to avoid them. The spirit of non-classical logicians in the discussion about paradoxes, on the contrary, is not necessarily claiming that their theory is the right one; sometimes the aim is to expand the range of acceptable positions. This debate is much more pluralistic with respect to alternative views.

Naturally, even with the tolerance shown by non-classical solutions to paradoxes, not every theory will count as equally good. The basic virtue in this debate is being able to solve paradoxes such as the Liar, which involves providing a consistent or non-trivial theory of truth. Interestingly, there are some standard techniques to prove that the theoretical virtue of non-triviality holds, which might be taken as “exemplars” in Kuhn’s (1970) sense. In his pioneering paper, Kripke (1975) proved a now standard result, the fixed-point theorem, for a non-classical theory based on Kleene’s three-valued schema. The fixed-point result appeals to some well-known facts about cardinality in set theory and shows that a theory has a transparent and non-trivial truth predicate. This result has been adapted and used by many authors in the debate about semantic paradoxes: both Field (2008) and Beall (2009), for example, provide fixed-point results.

It is also desirable in general to preserve a large portion of the deductive power of classical logic. Some proposals (Priest, 2006; Field, 2008) guarantee that, in non-paradoxical contexts, one might use classical logic. In some cases, this is proven with “recapture results”. In many paracomplete theories, for example, whenever \( A \) implies \( B \) in classical logic, \( \{ A \} \cup \Delta \) implies \( B \) in the paracomplete theory, where \( \Delta \) contains all the instances of the law of excluded middle for the premises and the conclusion. Maybe the most peculiar proposal in that respect is Ripley’s (2012) non-transitive theory, which is simply classical logic before the addition of the truth predicate. Moreover, Field, Priest and Beall provide non-classical conditionals which are closer to their classical counterparts than the pure K3 or LP material conditionals (in particular, both Identity and Modus Ponens hold for those stricter conditionals). In this sense, Williamson’s (2017) defense of “strength” as a value of a logical theory really holds in this debate: many authors want a theory as close as possible to classical logic, and they regard the weakness of their theories as undesirable. Kripke is very clear about this (Kripke, 1975, p. 700):

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12 For more on this, see Tajer (2021).
I have been amazed to hear my use of the Kleene valuation compared occasionally to the proposals of those who favor abandoning standard logic ‘for quantum mechanics,’ or positing extra truth values beyond truth and falsity, etc. (...) Conventions for handling sentences that do not express propositions are not in any philosophically significant sense ‘changes in logic.’ The term ‘three-valued logic’, occasionally used here, should not mislead. All our considerations can be formalized in a classical metalanguage.

In a nutshell, a theory needs to be able to solve paradigmatic semantic paradoxes, and to prove that this solution works (usually with fixed-point theorems). As far as possible, the deductive power of classical mathematics must also be preserved. It is hard to determine that a non-classical solution to paradoxes is strictly better than the others; there are some basic requirements to be eligible as a candidate, but no unified criterion about when a theory is better or worse.

For the purpose of this paper, it is enough to notice that the epistemic values in this discussion are completely different from the notions that appear in the debate between intuitionistic and classical logicians. Intuitionistic logicians intend to revise classical mathematics, and to give a different philosophical foundation to logic in general; for them, a good theory should capture those strong desiderata. On the contrary, most non-classical solutions to paradoxes are conservative with respect to classical mathematics and as regards the philosophical foundations of logic: their main aim is to provide a nice truth predicate, without sacrificing much of the classical background. Weakening classical mathematics is here considered a problem, not a virtue. One clear attitude that intuitionistic logicians and non-classical truth-theorists share is the rejection of classical logic as a general theory of validity; however, the problems they seek to solve are completely different, and so are their epistemic values. Moreover, as we have mentioned, intuitionistic logicians usually believe that they use the correct logic, while most non-classical logicians in the discussion about paradoxes have a more open and pluralistic stance toward alternative solutions.

In the next section, we will look at the discussion about the logic of conditionals. As we will see, even though the conception of consequence is similar to the one in the paradox-solving debate, the kind of evidence and epistemic values that are in play are different. Again, this will work as evidence for methodological pluralism in logic.

4 Conditionals

In this section, we will focus our attention on the discussion about conditionals. There is an enormous amount of literature about this topic, so I do not intend to cover all of it, or even most of it. I will simply mention some relevant contributions and analyze the methodology that philosophers apply in those cases, in order to show that it is different from the methodologies that we mentioned earlier. The discussion about the meaning of the conditional changed drastically after Stalnaker (1981) and Lewis (1973). As we know, the material conditional is very problematic as an analysis of the everyday conditional: among other problems, it makes every conditional with a false antecedent
true, so it cannot capture our intuitions about sentences with uncertain antecedents, or our intuitions about counterfactuals (Edgington 2020).

According to Stalnaker, a conditional statement “if A then B” says that, in the closest world in which A is true, B is also true.\textsuperscript{13} The theory has a detailed axiomatic and semantic structure based on modal logic: worlds are connected by a similarity relation, and for every world \(w\) and every consistent sentence A, there is a closest world \(w\) where the sentence A is true. This property is usually called Uniqueness. In response, Lewis developed a similar theory, but with different constraints: in particular, Uniqueness is rejected (there might be different closest worlds where the antecedent is true). The theories have different predictions with respect to counterfactuals. For example, Stalnaker’s approach satisfies Conditional Excluded Middle: “If A then B, or if A then \(\neg B\)”. This is a direct consequence of the Uniqueness property. As we may expect, this principle fails in Lewis’ theory. Lewis appeals to a linguistic intuition: according to him, neither “if Bizet and Verdi were compatriots, they would be Italian” nor “if Bizet and Verdi were compatriots, they would not be Italian” are intuitively true (Lewis 1973, p. 421).

From a very different perspective, McGee (1985) claimed that Modus Ponens is invalid, for there is an intuitive case where the premises of a Modus Ponens inference are acceptable and the conclusion is not acceptable. The well-known argument describes a situation where Reagan (Republican), Carter (Democratic) and Anderson (Republican) are candidates for the elections. The polls show that Reagan leads, followed by Carter, while Anderson is in a distant third. In this context, we would accept that “If a Republican wins the election, then if it’s not Reagan who wins, it will be Anderson.” We could also accept that “A Republican will win the election.” However, we cannot accept that “If it’s not Reagan who wins, it will be Anderson” (since, naturally, if Reagan is not the winner, it will be Carter). McGee did not provide an alternative theory about the conditional, but some brief suggestions.

The discussion about this issue continued for decades. Kolodny and MacFarlane (2010), for example, provided an informational theory about the conditional which explains why Modus Ponens fails in McGee’s example and in other interesting cases such as the Miners Paradox. In this paradox, you know that there is going to be a flood in either shaft A or shaft B (formally, A \(\lor\) B), and you know that there are 10 miners in one shaft. You can choose to block one shaft: if you prevent the flood there, you save the 10 miners. However, if you block a shaft and the flood affects the other one, all 10 miners will die. And if you don’t block any shaft, only one miner will die. In this context, it is reasonable to assert that “If the flood is in shaft A, you should block shaft A” (A \(\rightarrow\) O(Block A)), and “if the flood is in shaft B, you should block shaft B” (B \(\rightarrow\) O(Block B)). You also know that A \(\lor\) B. Following just classical logic and deontic logic, you ought to block one shaft. But, intuitively, since you are in a state of ignorance, you ought not to block any shaft, by a simple utility calculus. The solution suggested by MacFarlane and Kolodny is to accept the premises but reject Modus Ponens in these contexts, and to adopt an informational interpretation of the conditional.

\textsuperscript{13} Formally speaking, the idea of world similarity is described using a sphere-system. See Priest (2008) for details about this.
There are yet many other theories about conditionals which I do not cover here, such as Adams’ (1975) probabilistic approach, or Jackson’s (1979) pragmatic reconstruction. However, I consider that the cases I have analyzed provide a reasonable sociological description of the discussion.

It is important to look at what counts as evidence in this discussion. In general, authors use as evidence some intuitions about conditional sentences from natural language. Stalnaker uses “if this match were struck, it would light” (Stalnaker, Stalnaker 1981, p. 38), while Lewis argues against Uniqueness using the intuition about Verdi and Bizet. McGee also provided a very colloquial example from natural language, and the same applies to the Miners Paradox. These authors did not provide as basic evidence complex philosophical arguments or self-referential paradoxes, but pre-theoretical examples that everyone can understand.

It should be noted that intuitions about ordinary sentences of natural language do not have this amount of weight in the discussions I mentioned in the previous sections. Intuitionistic logicians would arguably not care about McGee’s example, because Modus Ponens is presumably essential for constructive mathematical reasoning. A similar point could be made regarding discussions about paradoxes: those involved in these discussions worry about the conditional, but not necessarily about the conditional of natural language. Priest (2006) wants to recover the Modus Ponens rule, which is invalid in the basic LP logic, and he provides an additional system with stronger conditionals where the rule is valid. But, normally, truth-theorists want the theory to satisfy Modus Ponens in order to be able to make this kind of inference:

Every theorem of arithmetic is true. “S” is a theorem of arithmetic. Therefore, S.

The ability to perform these infinite truth ascriptions (such as ‘every theorem of arithmetic is true’) is normally regarded as a fundamental aim of a theory of truth. In general, the main problem with the conditional in discussions about paradoxes is Curry’s paradox, i.e., “if this sentence is true, then falsum” (Shapiro & Beall, 2018). Moreover, Beall, Field and Priest do not criticize Stalnaker’s or Lewis’ theories about the conditional, presumably because they take them to be solving a different problem. Field (2008, p. 373) does mention Stalnaker’s conditional, but he admits that Stalnaker’s considerations have not been introduced in his own theory (p. 374):

To do this one would presumably have to generalize the Stalnaker connective a bit to contexts where excluded middle isn’t assumed, just as one has to generalize the classical connective a bit. This would be a worthy project, which I have not undertaken.

Indeed, in Field’s approach, the conditional is classical for the grounded language. Moreover, Priest (2006, p. 71) mentions Stalnaker’s proposal, but only as an inspiration for an intensional non-contraposible conditional, which he later discusses mostly in

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14 In Tarski’s famous paper (Tarski 1944), the two main functions of a truth predicate are (a) infinite conjunctions and (b) blind truth ascriptions (e.g., “all of what you said last night was true”). In both cases, the truth predicate is not redundant (as the truth operator arguably is in “it is true that it rains”), for there is no direct way of saying the same without it. In a recent paper, Picollo and Schindler (2017) have argued that, in order to perform infinite conjunctions, it is enough to have one direction of the T-schema.
relation to the truth predicate and semantic paradoxes. Beall (2009) devotes an entire chapter to the conditional operator, but Lewis and Stalnaker are not even mentioned.

Similarly, the discussions about the meaning of the conditional in natural language are in general not concerned with typical semantic paradoxes such as Curry’s paradox. This paradox probably holds in some systems of conditional logic supplemented with a transparent truth predicate, and I assume that most authors do not care so much about it (after all, it can be solved by restricting self-reference). Interestingly enough, the current entries about “Indicative Conditionals” and “The Logic of Conditionals” in the *Stanford Encyclopedia of Philosophy* (Arlo-Costa, 2007; Edgington, 2020) do not mention Curry’s paradox, while the entry about “Curry’s Paradox” (Shapiro & Beall, 2018) does not mention Stalnaker or Lewis.

Unlike what happens in discussions about self-referential paradoxes, the basic evidence in discussions about the conditional in natural language (as exemplified by Stalnaker, Lewis and McGee) is mostly linguistic, making extensive use of pre-theoretical linguistic intuitions. Now we can look at the epistemic values in discussions about the conditional. The best theory about the conditional is the one that can capture all those linguistic intuitions, or most of them, with a good explanation of why conditional sentences are true or false (or assertable, acceptable, etc.). This would be considered too mundane in other philosophical discussions. Intuitionistic philosophers, for example, normally care about an idealized conditional interpreted in terms of proofs. And, as we mentioned above, in discussions about semantic paradoxes, the development of a suitable conditional is important, but the aim is not to capture natural language but rather to have a stable and strong connective which may be sufficiently strong to perform some basic tasks.

Some of the examples from the paper (which are far from exhaustive) are summarized in Table 1. This analysis shows again how various philosophical problems and approaches can involve completely different kinds of evidence and epistemic values. The central evidence in one discussion may be secondary evidence in a different discussion. The main epistemic value in one discussion may be overlooked in a differ-

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**Table 1** Some methodologies in logic

| Problem                | Approach      | Evidence               | Epistemic Values                      |
|------------------------|---------------|------------------------|---------------------------------------|
| Foundations of arithmetic | Intuitionistic | Constructive proofs    | Constructability                      |
|                        | Classical     | Proofs in classical logic | Continuity with mathematics, usefulness for science |
| Semantic paradoxes     | Model-theoretical | Fixed-point proofs      | Consistency, recapture, novelty, non-triviality |
| Conditionals           | Semantics     | Linguistic intuitions  | Adequacy to (linguistic) data          |

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15 As an exception to this general tendency, Nolan (2016) claimed that a semantics for natural language conditionals can also be a solution to Curry’s paradox. According to his view, Curry’s paradox can be solved if we adopt a counterfactual semantics with impossible worlds. For example, “If C is true, then 2+2=5” will be false. This is problematic in classical logic, for it implies that C is true. But in counterfactual semantics this does not imply that C is true; it just implies that, in the closest world in which C is true, 2+2 is not 5.
ent discussion. And one problem may involve two completely different approaches. Therefore, it is not possible to describe the diversity of logical practice under a general structure. In other words, there is a clear methodological pluralism in logic. In the next section, we will discuss the consequences of this pluralistic approach.

5 Methodological pluralism in logic

I have argued in this paper that there is methodological pluralism in logic. Different methodologies involve (at least) different kinds of evidence and different epistemic values. The adoption of a specific methodology depends mostly on the philosophical problem that logicians want to solve: a theory of conditionals for truth-theories will not look at the same evidence as a theory of conditionals for natural language. Moreover, in some cases the same problem can give rise to different methodologies; for example, as we saw in Sect. 2, the problem of the foundation of mathematics involves many different approaches, each with its own epistemic criteria.

In any case, it is not easy to determine a specific fact which gives rise to a new methodology. This often depends on the topic or on the general philosophical assumptions. But not always. For example, axiomatic theories of truth and substructural theories of truth could be taken as two different approaches to the same phenomenon (i.e., truth), but the main disagreement is about the possibility of restricting the T-schema (usually, axiomatic theories restrict the T-schema while non-classical theories assume Transparency); it is a methodological disagreement about how to handle the truth predicate, but not necessarily a deep disagreement regarding truth as such. Indeed, on both sides of the debate many authors take themselves as deflationists (Beall & Armour-Garb 2003; Halbach & Horsten, 2003; 2005).

If the analogy with science is correct, the emergence of new methodologies in logical research is largely unpredictable. After all, logic is predominantly an abstract discipline and there are infinitely many possible logical systems, and a huge variety of proof methods. In addition, philosophical logic is also affected by new trends and problems of philosophy. For example, the logic of metaphysical grounding became a popular logical problem once philosophers started to take grounding more seriously.

Methodological pluralism has interesting consequences for the philosophy of logic. Fortunately, we are paying increasingly more attention to actual logical practice. However, it is easy to fall into the temptation of cherry-picking some examples and characterizing logical practice by one or two decisive features. I would claim that the general aim of research in the methodology of logic should be more modest. There is no “methodology of logic” that applies to every debate. Logical research involves a set of different methodologies, and it is important to understand how those methodologies work. This pluralism can also provide more solid ground for a normative approach, as we may ask which methodology is better for solving this or that philo-

16 Also Cook (2010) defends a similar view. He claims that logics work as models of natural language. In that sense, different models may emphasize one aspect or another (pp. 500–501): “Different logics, viewed as models of various linguistic [phenomena], are correct relative to different theoretical goals, or relative to different ways of simplifying, idealizing, or precisifying the phenomena in question.” Unlike Cook, we do not use the notion of correctness (an essential notion in the debate about logical pluralism) and we do not
sophical problem. Understanding the structure of a problem and the theories that have been proposed for solving it can certainly give some hints about the methodology that we should adopt. There is no need to choose a unique methodology for solving every logical problem at the same time.

A natural question, from a more normative perspective, is whether this situation is desirable. It might be better to unify the procedures and the epistemic values. However, this depends on our theoretical goal. Should natural language experts be worried about Curry’s paradox? Should truth theorists be worried about McGee’s counterexample to Modus Ponens? Should intuitionistic logicians try to solve the Miners Paradox?

A significant problem for this sort of methodological unification is that some discussions presuppose a specific machinery which cannot deal with all problems at the same time. For example, the epistemic theories about the conditional that philosophers such as Stalnaker and Lewis developed are not obviously appropriate for representing infinitary iterations of the truth predicate. Analogously, the usual non-classical theories of truth are not obviously appropriate for representing the informational and epistemic complexities of conditionals in natural language. If we accept that different theories are devoted to solving different problems, methodological pluralism makes much more sense. Instead of a competition between very different theories, we can think of the field as consisting of coexisting research programmes (Lakatos, 1978). There is no reason to think that a general unified theory that can deal with every problem is possible; trying to construct such a theory is probably worthwhile, but the result can be even more problematic than the individual theories that we had before.

This does not mean that unification should not be taken seriously. It can still be seen as an ideal feature of a logical theory. Arguably, a theory which can deal with many phenomena successfully will be preferred over a theory which can only deal with one. What I suggest here is that logicians normally work in a more specialized way, and we should not regard the lack of unification as a reason for rejecting a logical theory. This is consistent with taking unification as an epistemic value and as a general goal. Moreover, the project of a unified logical theory can be interesting in itself: how can a theory deal with all the relevant data at the same time? This requires paying attention to many research styles, methods and communities at the same time, and trying to put together many different desiderata.

My pluralistic approach toward logic is very similar to the pluralistic stance in the philosophy of science, developed by Kellert et al. (2006). There are at least three important similarities. First, we claimed that, in logic, the epistemic values depend on the theoretical project that researchers adopt. According to Kellert et al., “a pluralist solution (...) would claim that which virtues should hold what degree of regulative status in any given research project is a function of features specific to the problem and of the particular aims of the research” (p. ix). Secondly, we stated that the coexistence of different research programmes in logic with different methodologies should not be regarded as a problem but as a virtue. Scientific pluralism also accepts that “the multiplicity of approaches that presently characterizes many areas of scientific investigation does not necessarily constitute a deficiency” (p. x). Finally, we held that presuppose that logics are models of natural language. Moreover, we are not so concerned about how logics can achieve different goals, but about how different goals presuppose different scientific methodologies.

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the project of finding a “genuine logic” which can explain everything at the same time is not necessarily promising and does not represent our actual practice. According to what Kellert et al. call the “pluralist stance”, “a complete account is not possible in the same representational idiom and is not forthcoming from any single investigative approach” (p. xiv).

Another pluralistic perspective in the philosophy of science has been provided by Chang (2012). He claims that maintaining different approaches in the same area of study is good for science. He introduces the notion of a “system of practice”, i.e., a “coherent and interacting set of epistemic activities performed with a view to achieve certain aims” (p. 260). Systems of practice sometimes have the form of Kuhnian paradigms or coexisting Lakatosian research programmes. According to Chang, a scientific field usually benefits from having different systems of practice at the same time. The benefits of toleration arise from the fact that every system of practice can make its own contribution: “all the benefits of Kuhnian normal science can be had within each paradigm, while we retain multiple paradigms” (p. 264). This becomes natural once we admit the multiplicity of theoretical aims or values (p. 274): “Once we grant that there are multiple human needs that science is called upon to satisfy, it is easy to recognize that we will most likely not be able to come up with the perfect scientific system that satisfies all the needs”. Chang calls this picture “pluri-axial”: there are multiple legitimate values/aims that drive scientific work. Finally, he also sees the pluralistic approach as being opposed to abductivism (p. 288):

The very idea of inference to the best explanation will start to seem quite futile. Even setting aside the difficulty of needing to know about all plausible alternatives before we can say which one is the best, we have to ask why we should be so concerned about making that monistic inference down to one best alternative.

The acceptance of some kind of scientific pluralism does not mean, however, that every problem also has many solutions; the different discussions can be internally more or less pluralistic, depending on the methodology and the evidence adopted by the researchers.

This position is not necessarily new: similar versions of this idea have been proposed, in particular regarding intuitionistic and classical logic. In his *Logical Syntax of Language*, Carnap (2000, p. 51) recommends the “simultaneous investigation (analogous to that of Euclidean and non-Euclidean geometries) of language-forms of different kinds –for instance, a definite and an indefinite language, or a language admitting and one not admitting the Law of Excluded Middle.” According to Bell and Hellman (2006, p. 66), “as both purposes—truth-preservation simpliciter and constructive interpretability—are worthy and important, we should certainly have peaceful coexistence and even cooperation.” Shapiro (2014) also holds a similar view, which we will discuss in the next section.

Before ending this section, I would like to consider a last point. One may claim that some of the discussions analyzed here do not belong to logic strictly speaking. For example, discussions about the conditional are sometimes taken as part of linguistics or semantics, not as part of logic. Arguably, there is a sociological divide (though not a sharp one) between linguistics and logic, and understanding the demarcation is in itself an interesting philosophical problem. However, these divisions are often
tricky and can change with time; e.g., are the conditional logics based on Kratzerian semantics logic or linguistics? Moreover, the diversity of problems and methods in contemporary philosophical logic cannot be underestimated. Just to give an example, the most recent issue of the *Journal of Philosophical Logic* (December 2021) includes papers about epistemic logic, Gödel’s theorem, logics for causation, mathematical ontology, connexive logics and hyper-intensionality. It is hard to find something in common between all these proposals (except for the rather trivial observation that they use consequence relations). This large logical diversity requires a specific philosophy, and this is what I am trying to provide in this paper; arguing that most papers in some logic journals are not logic would miss the point. The diversity of methodologies in logic does not depend on one case it is by now a structural feature of logic as a discipline.

### 6 Discussion of similar proposals

Similar pluralistic views about logic have been proposed by Beall and Restall (2006) and Shapiro (2014). In this section I will compare their views with the one defended in this paper.

According to Beall and Restall’s influential version of logical pluralism, different logics are correct at the same time. They defend pluralism among intuitionistic, relevant and classical logics. The three logics share a minimal core, which is the notion of validity as truth preservation. But they preserve truth over a different range of “cases”: possibly inconsistent and incomplete situations (relevant logic), possibly incomplete constructions (intuitionism), and consistent and complete models (classical logic).

The theory defended in this paper is different from Beall and Restall’s approach in many ways. First, I do not propose here that legitimate logics are always concerned about truth-preservation over cases. As Shapiro (2014, p. 33) observes, not all intuitionistic logicians, for example, are concerned about semantic issues such as truth-preservation. Some of them just adopt a proof-theoretical approach. Arguably, some methodologies in logic will only focus on the proof-theoretical part, ignoring the model-theoretical aspect, and there is nothing intrinsically problematic about that. Second, Beall and Restall emphasize the generality of correct logics: they should be applicable to every context. In contrast, I claim here that the generality of logic is not to be taken at face value, for logics are usually designed to speak about certain phenomena and solve specific problems. Finally, from a methodological point of view, I am not trying to give a unified theory of logic which accommodates a plurality of logics. Rather, I suggest a different kind of pluralism, according to which logics are used to address different phenomena, and I argue that unification is not an essential feature of logical pluralism.

Now I will discuss Shapiro’s view. In his book *Varieties of Logic*, Shapiro proposes an “eclectic” pluralism for logic. According to him, the ordinary notion of “validity” is a cluster concept (Shapiro 2014, p. 25), and different logics emphasize different aspects of it. This theory is more representational than mine, as I do not claim that different logics necessarily capture the intuitive notion of “validity”. This depends on
the problem that the logics are supposed to solve, and some theories can be truly far from the common usage of the notion.

Shapiro’s idea is that logic is relative to a structure (p. 96). He pays special attention to the coexistence between intuitionistic and classical mathematics. Shapiro claims that classical and intuitionistic mathematics are both respectable theories that can coexist peacefully. The criterion for existence is inspired by Hilbert: every consistent theory can be mathematically studied. But Shapiro goes one step further, for he takes consistency to be also relative to a logic. In particular, some theories such as Smooth Infinitesimal Analysis are inconsistent with classical logic (i.e., one cannot add the Law of Excluded Middle). However, smooth infinitesimal analysis is still a respectable mathematical theory.

In contrast to Shapiro, I don’t maintain that logics are only correct relative to a structure. Rather, logics can be used for different theoretical aims, and they will be evaluated according to different criteria. I share Shapiro’s spirit regarding intuitionistic and classical logic; but the other cases that I mention here do not clearly fit into his narrative. For example, theories about the conditional in natural language or solutions to the Liar paradox do not necessarily refer to a “structure” in any substantial way. Claiming, e.g., that paracomplete solutions to the Liar are correct relative to paracomplete structures is true but rather trivial. Unlike intuitionistic arithmetic, some non-classical structures are not supposed to be intrinsically interesting or illuminating, and they just serve as support for specific theories which solve specific problems.

7 Conclusion

In this paper, I have explored some central topics in logic: the rivalry between intuitionistic and classical logic, the problem of self-referential paradoxes, and the analysis of conditional sentences in natural language. I showed that different problems usually motivate different methodologies, with their own notions of evidence and epistemic values; and even the same problem can often be addressed using very different approaches.

On the one hand, this implies that logic as a whole does not work with an abductive methodology, insofar as abductivism presupposes certain degree of agreement about what a theory should do. On the other hand, and more importantly, this implies that there is no unique methodology in logic; there are many coexisting methodologies. Hopefully, this pluralistic approach can help us better understand logical practice, give us a more precise account of what logical evidence is, and motivate interesting new directions for some classic discussions in the philosophy of logic.

What is the moral for anti-exceptionalism about logic? Following the ideas held by Chang (2012) and other scientific pluralists, the main point is that philosophers of logic should not be afraid to adopt more pluralistic views regarding the methodology of logic. Here we are only discussing one specific aspect of anti-exceptionalism, which is methodological anti-exceptionalism, i.e., the idea that logic works as a scientific theory. If my arguments are right, then logic does work as a scientific theory, insofar as scientific theories are conceived from a pluralistic approach, where many different
methodologies coexist. This does not make logic different from science, but brings it closer to the diverse and pluralistic view of science that scientific pluralism offers.

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