Inventory Model with Demand Dependent on Unit Price under Fuzzy Parameters and Decision Variables

S. Ranganayaki, R. Kasthuri and P. Vasanthi

Abstract: An EOQ model with demand dependent on unit price is considered and a new approach of finding optimal demand value is done from the optimal unit cost price after defuzzification. Here the cost parameters like setup cost, holding cost and shortage cost and also the decision variables like unit price, lot size and the maximum inventory are taken under fuzzy environment. Triangular fuzzy numbers are used to fuzzify these input parameters and unknown variables. For the proposed model an optimal solution has been determined using Karush Kuhn-Tucker conditions method. Graded Mean Integration (GMI) method is used for defuzzification. Numerical solutions are obtained and sensitivity analysis is done for the chosen model.

Keywords: Crisp and Fuzzy total cost, Demand dependent on unit price, Graded Mean Integration, Triangular fuzzy numbers.

I. INTRODUCTION

The main objective of inventory control is to increase the profit and decrease the investment without affecting the manufacturing process. The growth of an organization depends on a proper inventory control.

Inventory problems help to make decisions like minimize the total cost or maximize the profit. Therefore the variables that minimize the total cost or maximize the profit are obtained by solving the inventory problem. Most of the inventory problems deal with minimizing the inventory carrying cost (3). Harris (4) first determined an inventory model with fixed demand rate. An inventory model with increasing demand under inflation was developed by Sarkar and Sana (9). In the above mentioned works the parameters are considered as crisp values. We can also find some of the Works in inventory management where the input parameters and unknown variables are taken in fuzzy environment.

Fuzzy set theory was first introduced by Zadeh in the year 1965. Later on Bellman and Zadeh (1) used the fuzzy set theory to the decision making problems. Further Tanaka et al (10) considered the objectives as fuzzy goals over the \( \alpha \)-cuts of a fuzzy constraint set. Cheng (2) applied geometric programming method to solve an EOQ model with unit price dependent on demand. Roy and Maiti (8) used both fuzzy linear and geometric programming techniques to solve inventory model with fuzzy objective function and storage area. Kazemi et al (6) developed an EOQ model with fuzzy parameters and decision variables and used the GMI method for defuzzification.

II. ASSUMPTIONS AND NOTATIONS FOR THE INVENTORY MODEL

a. Assumptions

The following assumptions have been made to solve the proposed inventory problem.

• Demand is deterministic
• Shortages are allowed and fully backlogged.
• The models a single item and warehouse space.
• At the beginning of each cycle, one placed order is manufactured and delivered in each cycle.

b. Notations

The following notations are made to develop the mentioned EOQ problem.

| Notation   | Description                  |
|------------|------------------------------|
| TC         | Total cost                   |
| Q          | Lot size or batch size       |
| I          | Maximum inventory level      |
| S          | Order cost or set up cost per order |
| D          | Demand rate units per unit of time |
| P          | Unit price per item          |
| H          | Unit holding cost per unit of item |
| M          | Penalty cost or shortage cost per unit of time |

Here S, D, H and m are input parameters and p, Q, I are decision variables.

Demand D is dependent on unit price \( p \), given by the relation

\[
D = Ap^r
\]

where \( A > 0 \) and \( 0 < \beta < 1 \) are real numbers selected to provide the best fit of the estimated price function. Therefore the annual total cost of the crisp model is the total cost = 

III. FORMULATION OF EOQ MODEL IN VARIOUS ENVIRONMENT

A. EOQ model in Crisp environment

In general, the classical inventory models are designed by assuming that the demand and unit costs are constant and independent. But conversely they are related to each other. That is the unit cost of an item is inversely proportional to the demand of that item.

Hence \( D = Ap^r \), where \( A > 0 \) and \( 0 < \beta < 1 \) are real numbers selected to provide the best fit of the estimated price function. Therefore the annual total cost of the crisp model is the total cost = 

Revised Manuscript Received on September 15, 2019

Dr. S. Ranganayaki, Department of Mathematics, Sri Ramakrishna Engineering College, Coimbatore, India.
Dr. R. Kasthuri, Department of Mathematics, Sri Ramakrishna Engineering College, Coimbatore, India.
Dr. P. Vasanthi, Department of Mathematics, Sri Ramakrishna Engineering College, Coimbatore, India.

Retrieval Number: C401.398319/19©BEIESP
DOI: 10.35940/ijrte.C401.398319

Published By:
Blue Eyes Intelligence Engineering & Sciences Publication

784
Inventory Model with Demand Dependent on Unit Price under Fuzzy Parameters and Decision Variables

Production cost + Setup cost + Holding cost + Shortage cost
$$TC = pD + \frac{SD}{Q} + \frac{I^2H}{2Q} + \frac{(Q-I)^2m}{2Q}$$  \hspace{1cm} (1)

Decisions Variables
$$D = Ap^{-\beta}$$  \hspace{1cm} (2)

Total cost (3)
$$TC = Ap^{-\beta} + \frac{SAp^{-\beta}}{Q} + \frac{I^2H}{2Q} + \frac{(Q-I)^2m}{2Q}$$  \hspace{1cm} (3)

Where $p$, $Q$ and $I$ are the decision variables.

The crisp values of the unit cost $p$, lot size $Q$ and the maximum inventory $I$ that minimizes the total cost function are given by

$$p = \left[ \frac{mHS\beta^2}{2A(H+m)(1-\beta)^2} \right]^{1/3\beta}$$  \hspace{1cm} (4)

$$Q = \frac{S\beta}{p(1-\beta)}$$  \hspace{1cm} (5)

$$I = \frac{Qm}{H+m}$$  \hspace{1cm} (6)

IV. EOQ MODEL IN FUZZY ENVIRONMENT

An EOQ model with input parameters and decision variables under fuzzy environment is considered in this section.

Since the holding cost, setup cost and shortage cost and also the unit price, lot size and maximum inventory are fuzzy in nature, they are taken as triangular fuzzy numbers. The Triangular Fuzzy Numbers are represented by

Order cost: \hspace{1cm} $S = (S-\delta_1, S, S+\delta_2), S > \delta_1$

Holding cost: \hspace{1cm} $H = (H-\delta_3, H, H+\delta_4), H > \delta_3$

Penalty cost: \hspace{1cm} $m = (m-\delta_5, m, m+\delta_6), m > \delta_5$

Decision Variables

Maximum Inventory level: \hspace{1cm} $I = (I-\delta_7, I, I+\delta_8), I > \delta_7$

Lot size: \hspace{1cm} $Q = (Q-\delta_9, Q, Q+\delta_{10}), Q > \delta_9$

Unite Price: \hspace{1cm} $p = (p-\delta_{11}, p, p+\delta_{12}), p > \delta_{11}$

In this section, we apply GMI method to defuzzify the objective function and solved for the decision variables $p$, $Q$ and $I$ using Karush Kuhn-Tucker conditions technique. Hence the total cost under fuzzy environment is given by

$$TC = (C_1, C_2, C_3)$$  \hspace{1cm} (7)

Where

$$C_1 = A(p+\delta_{11})(p+\delta_{12})^{-\beta}$$
$$+ \frac{A(S-\delta_7)(p+\delta_{12})^{-\beta}}{Q+\delta_{10}}$$
$$+ \frac{(H-\delta_3+m-\delta_5)(I-\delta_7)^2}{2(Q+\delta_{10})}$$
$$+ \frac{(Q-\delta_9)(m-\delta_5)-(I+\delta_8)(m+\delta_6)}{2}$$  \hspace{1cm} (8)

$$C_2 = Ap^{-\beta} + \frac{Ap^{-\beta}}{Q} + \frac{(H+m)I^2}{2Q} + \frac{Qm}{2} - I$$  \hspace{1cm} (9)

$$C_3 = A(p+\delta_{12})(p-\delta_{11})^{-\beta}$$
$$\frac{A(S+\delta_2)(p-\delta_{11})^{-\beta}}{Q-\delta_9}$$
$$+ \frac{(H+\delta_4+m+\delta_5)(I+\delta_8)^2}{2(Q-\delta_9)}$$
$$\frac{(Q+\delta_{10})(m+\delta_6)}{2} - (I-\delta_7)(m-\delta_5)$$  \hspace{1cm} (10)

Defuzzification of equation (7) by using Graded Mean Integration method gives

$$\theta(TC) = \frac{1}{6} [C_1 + 4C_2 + C_3]$$  \hspace{1cm} (11)

$$\theta(TC\{\tilde{p}, \tilde{Q}, \tilde{I}\}) = \frac{1}{6} \left[ \frac{A(p-\delta_7)(p+\delta_{12})^\beta + A(S-\delta_7)(p+\delta_{12})^\beta}{Q+\delta_{10}} \right.$$
$$+ \frac{A(p-\delta_7)(p+\delta_{12})^\beta}{Q-\delta_9}$$
$$+ \frac{(H+\delta_4+m-\delta_5)(I-\delta_7)^2}{2(Q+\delta_{10})}$$
$$+ \frac{(Q+\delta_{10})(m-\delta_5)}{2} - (I-\delta_7)(m-\delta_5)$$

$$\left. + \frac{1}{2} \left[ \frac{Ap^{-\beta} + \frac{Ap^{-\beta}}{2Q} + \frac{(H+m)I^2}{2Q} + \frac{Qm}{2} - I}{\beta} \right. \right.$$
$$+ \left. \frac{A(p+\delta_{12})(p+\delta_{12})^{-\beta}}{Q+\delta_{10}} \right.$$\hspace{1cm} (12)

V. COMPUTATION OF $P, Q$ AND $I$ USING KKT CONDITIONS APPROACH

Let $I_1 = I-\delta_7, I_2 = I, I_3 = I+\delta_8$

$Q_1 = Q-\delta_9, Q_2 = Q, Q_3 = Q+\delta_{10}$

$p_1 = p-\delta_{11}, p_2 = p, p_3 = p+\delta_{12}$

Substituting the above equations in (12), we get
\( \theta(\tilde{T}\tilde{C}(\tilde{p}, \tilde{Q}, \tilde{I})) = \frac{1}{6} \left[ \begin{array}{c} \frac{A_p p_1^{-\beta} + A(S - \delta_p) p_3^{-\beta}}{Q_3} \\
+ \frac{(H - \delta_3 + m - \delta_5) I_1}{2 Q_3} \\
+ \frac{Q_i (m - \delta_5) - I_3 (m + \delta_6)}{2} \\
\end{array} \right] \)

\[ + \frac{2}{3} \left[ \begin{array}{c} \frac{A_p p_1^{-\beta} + A(S - \delta_p) p_3^{-\beta}}{Q_2} \\
+ \frac{(H + m) I_2^2}{2 Q_2} \\
+ \frac{Q_i (m + \delta_6)}{2} \\
- I_3 (m - \delta_5) \\
\end{array} \right] \]

\[ + \frac{1}{6} \left[ \begin{array}{c} \frac{A_p p_1^{-\beta} + A(S + \delta_p) p_3^{-\beta}}{Q_1} \\
+ \frac{(H + \delta_3 + m + \delta_6) I_3^2}{2 Q_1} \\
+ \frac{Q_i (m + \delta_6)}{2} \\
- I_3 (m - \delta_5) \\
\end{array} \right] \]

Where

\[ 0 \leq I_1 \leq I_2 \leq I_3 \leq 0 \leq p_1 \leq p_2 \leq p_3 \text{ and} \]

\[ 0 \leq Q_i \leq Q_2 \leq Q_3 \]

The constraints are

\[ I_1 - I_2 \leq 0 \]

\[ I_2 - I_3 \leq 0 \]

\[ -I_1 < 0 \]

\[ p_i - p_{i+1} \leq 0 \]

\[ p_1 - p_2 \leq 0 \]

\[ Q_i - Q_{i+1} \leq 0 \]

\[ Q_2 - Q_3 \leq 0 \]

\[ Q_1 < 0 \]

By KKT conditions approach,

\[ \frac{1}{6} \left[ \begin{array}{c} \frac{(H - \delta_3 + m - \delta_5) I_1}{Q_i} \\
- (m - \delta_5) \\
\end{array} \right] - \lambda_1 + \lambda_3 \leq 0 \] \hspace{1cm} (14.1)

\[ \frac{2}{3} \left[ \begin{array}{c} \frac{(H + m) I_2^2}{Q_2} - m \\
+ \frac{(m + \delta_6)}{2} \\
\end{array} \right] + \lambda_1 - \lambda_2 \leq 0 \] \hspace{1cm} (14.2)

\[ \frac{1}{6} \left[ \begin{array}{c} \frac{(H + \delta_3 + m + \delta_6) I_3^2}{Q_i} \\
+ \frac{A(S + \delta_p) p_3^{-\beta}}{Q_1} \\
\end{array} \right] + \lambda_2 \leq 0 \] \hspace{1cm} (14.3)

\[ \frac{1}{6} \left[ \begin{array}{c} \frac{2}{Q_3} \\
\frac{(H + \delta_3 + m + \delta_6) I_3^2}{2 Q_3} \\
\end{array} \right] - \lambda_4 + \lambda_6 \leq 0 \] \hspace{1cm} (14.4)

\[ \frac{2}{3} \left[ \begin{array}{c} \frac{-A S p_2^{-\beta} - (H + m) I_2^2}{Q_2} \\
+ \frac{m}{2} \\
\end{array} \right] + \lambda_4 - \lambda_3 \leq 0 \] \hspace{1cm} (14.5)

\[ \frac{1}{6} \left[ \begin{array}{c} \frac{A(S - \delta_p) (p_3)^{-\beta}}{Q_3^2} \\
+ \frac{(H - \delta_3 + m - \delta_5) I_3^2}{2 Q_3} \\
\end{array} \right] + \lambda_4 \leq 0 \] \hspace{1cm} (14.6)

\[ \frac{2}{3} \left[ \begin{array}{c} \frac{-A S p_2^{-\beta} - (H + m) I_2^2}{Q_2} \\
+ \frac{m}{2} \\
\end{array} \right] + \lambda_5 \leq 0 \] \hspace{1cm} (14.7)

\[ \frac{1}{6} \left[ \begin{array}{c} \frac{-A \beta(S + \delta_p) p_3^{-\beta}}{Q_1} \\
+ \frac{(1 - \beta) p_2^{-\beta}}{Q_2} \\
\end{array} \right] - \lambda_4 + \lambda_5 \leq 0 \] \hspace{1cm} (14.8)

Also

\[ I_1 \frac{\partial g}{\partial I_1} = 0 \] \hspace{1cm} (14.10)

\[ Q_1 \frac{\partial g}{\partial Q_1} = 0 \] \hspace{1cm} (14.11)

\[ p_1 \frac{\partial g}{\partial p_1} = 0 \] \hspace{1cm} (14.12)

\[ I_2 \frac{\partial g}{\partial I_2} = 0 \] \hspace{1cm} (14.13)

\[ Q_2 \frac{\partial g}{\partial Q_2} = 0 \] \hspace{1cm} (14.14)

\[ p_2 \frac{\partial g}{\partial p_2} = 0 \] \hspace{1cm} (14.15)

\[ I_3 \frac{\partial g}{\partial I_3} = 0 \] \hspace{1cm} (14.16)

\[ Q_3 \frac{\partial g}{\partial Q_3} = 0 \] \hspace{1cm} (14.17)

\[ p_3 \frac{\partial g}{\partial p_3} = 0 \] \hspace{1cm} (14.18)
Inventory Model with Demand Dependent on Unit Price under Fuzzy Parameters and Decision Variables

\[ I_1 - I_2 \leq 0, I_2 - I_3 \leq 0 \]  
\[ -I_1 \leq 0 \]  
\[ Q_2 - Q_2 \leq 0, Q_2 - Q_1 \leq 0 \]  
\[ -Q_1 \leq 0 \]  
\[ p_1 - p_2 \leq 0, p_2 - p_3 \leq 0 \]  
\[ -p_1 \leq 0 \]  
\[ \lambda_1(I_1 - I_2) = 0, \lambda_2(I_2 - I_3) = 0 \]  
\[ -\lambda_3I_1 = 0 \]  
\[ \lambda_4(Q_1 - Q_2) = 0, \lambda_5(Q_2 - Q_1) = 0 \]  
\[ -\lambda_6Q_1 = 0 \]  
\[ \lambda_7(p_1 - p_2) = 0, \lambda_8(p_2 - p_1) = 0 \]  
\[ -\lambda_9p_1 = 0 \]  

Solving the equations (14.1) to (14.30), the optimum values of p, Q and I are as follows:

\[ p = \frac{\beta^2(H - \delta_3 + 4H + H + \delta_4)}{(S + \delta_2 + 4S + S - \delta_1)} \]

\[ Q = \frac{\beta(S + \delta_2 + 4S + S - \delta_1)}{6p(1 - \beta)} \]

\[ I = \frac{(m - \delta_2 + 4m + m + \delta_5)}{6p(1 - \beta)(H - \delta_1 + m - \delta_5 + 4(H + m) + H + \delta_4 + m + \delta_6)} \]

VII. NUMERICAL EXAMPLE

The developed EOQ model under various environments is illustrated by assuming the following data:

\[ \beta = (30,150,195), \tilde{H} = (0.06,0.25,0.36), \bar{m} = (1,5,7.6) \]

The demand D is dependent on unit price \( p \) is given by

\[ D = \alpha p^\theta \]

Where \( A = 100 \). The decision variables \( p, Q \) and \( I \) are calculated for different values of \( \beta \) and hence the corresponding demand and the total cost are also calculated (both crisp and fuzzy values).

VIII. SENSITIVITY ANALYSIS

Table 1:

| \( \beta \) | \( \bar{p} \) | \( \tilde{D} \) | \( \bar{I} \) | \( \tilde{Q} \) | TC (crisp) | TC (Fuzzy) |
|---|---|---|---|---|---|---|
| 0.86 | 4.71 | 26.38 | 170.85 | 179.33 | 167.52 | 164.66 |
| 0.87 | 5.56 | 22.48 | 157.67 | 165.50 | 164.95 | 162.31 |
| 0.88 | 6.44 | 18.90 | 144.67 | 151.86 | 162.18 | 159.73 |
| 0.89 | 8.07 | 15.59 | 131.33 | 137.85 | 159.14 | 156.92 |
| 0.90 | 9.97 | 12.62 | 118.25 | 124.12 | 155.80 | 153.82 |

The table shows that for different values of \( \beta \), as the unit price value increases, values of lot size and maximum inventory decreases and hence the total cost value also decreases both in crisp and fuzzy environment.

IX. CONCLUSION

Our investigation deals with attaining both analytical solution and fuzzy optimal solution and a comparative study which meets the needs of the real world situation. It is concluded from this investigation that the impreciseness in inventory management can be overcome by using fuzzy input parameters and decision variables. Here the demand dependent on unit price is considered. The total annual cost for both crisp and fuzzy values are calculated which shows that the fuzzy values are better than the crisp one. Hence the conclusion drawn from this result is that the optimal solution is obtained under fuzzy environment compared to the crisp case.

REFERENCES

1. Bellman, R.E., and Zadeh, L.A., ‘Decision Making in a Fuzzy Environment’, Management Science, (1970), 17(4), B141-B164.
2. Cheng, T.C.E., ‘An Economic Quantity Model with demand dependent unit cost’, European Journal of Operations Research, (1989), 40, 252-256.
3. Donalson, W.A., ‘Inventory replenishment policy for linear trending demand, an analytic solution’, Operations Research Quarterly (1977), 28, 663-670.
4. Harris, F.W. (1915), ‘Operations and Cost Factory’, Management Series.
5. USA : A.W.Shaw Co.
6. Kasthuri, R and Seshaiah, C.V., ’Multi item fuzzy inventory model involving three constraints: A Karush-Kuhn-Tucker conditions approach’, American Journal of Operations Research, (2011), vol. 1, pp. 155-159.
7. Kazemi, N., Eshani, E., and Javer, M.Y, ‘An Inventory model with backorders with fuzzy parameters and decision variables’, International Journal of Approximate Reasoning, (2010), 51, 964-972.
8. Journal of Contemporary Mathematical Sciences, (2012), 7(18), 865-871.
9. Multi Objective Fuzzy Inventory model with limited storage space and investment through Karush Kuhn Tucker conditions’, International Journal of Contemporary Mathematical Sciences, (2012), 7(18), 865-871.
10. Roy, T.K., and Maiti, M., ‘Multi Objective inventory models of deteriorating items with some constraints in a fuzzy environment’, Computers and Operations Research, (1998), 25(12), 1085-1095.
11. Sarkar, B., and Sana, S., ‘A Finite Replenishment with increasing demand under Inflation’, International of Mathematics in Operations Research, (2010), 2(3), 347-385.
12. Tanaka, H., Okuda, T., and Asai, K., ‘On Fuzzy Mathematical Programming’, Journal of Cybernet, (1974), 3(4), 37-46.
13. Vasanthi, P & Seshiaah, CV, ‘Solution of Multi-Objective fuzzy inventory model with limited storage space and investment through Karush Kuhn Tucker conditions’, International Journal of Mathematical Archive, (2013), vol. 4, no. 7, pp. 216-220.
14. Zadeh, L.A. ‘Fuzzy Sets’, Information and Control, (1965), 8, 338-356.

AUTHORS PROFILE

Dr. S. Ranganayaki completed her Ph.D in Science and Humanities discipline under Anna University, Chennai in October 2014 and has published 7 papers in reputed journals. She received her M.Phil degree from Alagappa University, Karaikudi in 2005 and Master’s degree from PSG College of Arts and Science, Coimbatore in 2002. She completed her Bachelor’s degree in Mathematics at PSGR Krishnammal College for women, Coimbatore in 2000. She has participated in Seminars, Workshops and Conferences.

Dr. R. Kasthuri completed her Ph.D in Science and Humanities discipline under Anna University, Chennai in January 2015. She received her M.Phil degree from PSG College of Arts and Science in April 2005. She completed her Master’s degree from Nirmala College, Coimbatore in 2003, and Undergraduate degree from PSGR Krishnammal College for women, Coimbatore in 2001. She did her research work in the area of Inventory model and has 7 published papers in well reputed journals. She has participated in Seminars, Workshops, Conferences and has also presented a paper in an International Conference.

Dr. P. Vasanthi completed her Ph.D in Science and Humanities discipline under Anna University, Chennai in July 2014. She received her M.Phil degree from Madurai Kamaraj University in April 2003 and Master’s degree in April 2001 from PSG College of Arts and Science, Coimbatore. She completed her Bachelor’s degree in May 1999 at PSGR Krishnammal College for women, Coimbatore. She did her research work in the area of Inventory model and has published 7 papers papers in well reputed journals. She has participated in Seminars, Workshops, Conferences and has also presented a paper in an International Conference.

The table shows that for different values of $\beta$, as the unit price value increases, values of lot size and maximum inventory decreases and hence the total cost value also decreases both in crisp and fuzzy environment.