On the asymptotic geometry of area–preserving maps

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Preliminary Version

1 Statement and discussion of the results

Let $M$ be an open connected oriented 2–manifold endowed with an area form $\omega$. We assume that the total area of $M$ with respect to $\omega$ is infinite, i.e. $\int_M \omega = \infty$. Consider the group $\text{Ham}_c(M, \omega)$ of Hamiltonian diffeomorphisms of $M$ consisting of all time–1–maps of time–periodic compactly supported Hamiltonians $H : S^1 \times M \to \mathbb{R}$. We write $\varphi^t_H$ for the Hamiltonian flow generated by $H$.

We are interested in the asymptotic behaviour of one–parameter subgroups of $\text{Ham}_c(M, \omega)$ with respect to Hofer’s metric $d$ where

$$d(id, \varphi) = \inf \left\{ \int_0^1 \max H_t - \min H_t \, dt \mid H \text{ has compact support and } \varphi^1_H = \varphi \right\}.$$ 

Let $\mathcal{A}$ denote the Lie algebra of $\text{Ham}_c(M, \omega)$; it consists of all compactly supported time–independent Hamiltonians on $M$. Given $H \in \mathcal{A}$, we are interested in the growth of the function $r_H : [0, \infty) \to [0, \infty)$ defined by

$$r_H(t) = d(id, \varphi^t_H).$$ 

By the triangle inequality for Hofer’s metric we know that $r_H$ is subadditive, i.e.

$$r_H(t + s) \leq r_H(t) + r_H(s).$$

Therefore the limit

$$\mu(H) = \lim_{t \to \infty} \frac{r_H(t)}{t}$$

is well defined. This quantity—which is called the asymptotic non–minimality of the subgroup generated by $H$—was introduced in [BP]; see also [Po2] for further discussion.

1 We refer the reader to [HZ], [MS] and [Po2] for an introduction to Hofer’s geometry.
In particular, we see that \( r_H \) grows at most linearly in \( t \). On the other hand, if \((M, \omega)\) is the standard Euclidean plane then a theorem by Sikorav ([8], [HZ]; see also Proposition 3.2 below) states that \( r_H \) is bounded by a constant, and this constant depends only on the diameter of the support \( \text{supp}(H) \) of \( H \).

In the present note, we show that for open surfaces of infinite area the function \( r_H \) is either bounded or behaves asymptotically linear. In order to formulate our main result we need the following notion. Recall that a subset \( Z \subset M \) is called contractible in \( M \) if the inclusion \( Z \hookrightarrow M \) is homotopic to the constant map which sends the whole \( Z \) to a point in \( M \).

**Theorem 1.1 (Dichotomy Theorem).** Let \((M, \omega)\) be an open surface of infinite area and \( H : M \to \mathbb{R} \) a compactly supported Hamiltonian. Then the following dichotomy holds:

- If \( \{H \neq 0\} \) is contractible in \( M \) then the function \( r_H \) is bounded; in particular, \( \mu(H) = 0 \).
- If \( \{H \neq 0\} \) is not contractible in \( M \) then the function \( r_H \) grows asymptotically linear, i.e., \( \mu(H) > 0 \).

In fact, it is even possible to calculate the precise value of \( \mu(H) \) as the difference of two distinguished critical values of \( H \). In particular, one gets examples of one–parameter subgroups whose asymptotic non–minimality lies strictly between 0 and \( \max H - \min H \) and can be calculated precisely. As far as we know, this is the first series of examples of this type.

Assume that \( M \neq \mathbb{R}^2 \). Then \( \pi_1(M) \) is nontrivial. Let \( \mathcal{L} \) be the set of all embedded non–contractible circles in \( M \). Then we define

\[
\begin{align*}
c_+(H) &= \sup_{L \in \mathcal{L}} \min_{x \in L} H(x) \\
c_-(H) &= \inf_{L \in \mathcal{L}} \max_{x \in L} H(x)
\end{align*}
\]

Since \( H \) is compactly supported, it follows that \( c_+(H) \geq 0 \) and \( c_-(H) \leq 0 \) (see Proposition 2.4). Moreover, one can easily check (see Proposition 2.3) that \( c_+(H) \) and \( c_-(H) \) are critical values of \( H \).

**Theorem 1.2.** The following equality holds:

\[
\mu(H) = c_+(H) - c_-(H) .
\]

We give a short outline of the proof of the two theorems. If \( \{H \neq 0\} \) is contractible then a version of the abovementioned theorem by Sikorav shows that \( r_H \) is bounded by a constant depending only on the “size” of \( \text{supp}(H) \). Thus we get the first statement of Theorem 1.1 (see Section 3 below). An elementary argument (see Proposition 2.5) shows that \( c_-(H) = c_+(H) = 0 \) if and only if \( \{H \neq 0\} \) is contractible in \( M \). Thus, the second statement of Theorem 1.1 follows from Theorem 1.2. Theorem 1.2 consists of two parts. The inequality \( \mu(H) \geq c_+(H) - c_-(H) \) follows from a Lagrangian intersection result as in [Po1] (see Section 4). The proof of the reversed inequality uses a trick, namely a decomposition of \( \varphi^t_H \) into two commuting flows:

\[
\varphi^t_H = \Phi^t \circ \Psi^t = \Psi^t \circ \Phi^t
\]
where $\Phi^t$ has contractible support. Hence, Sikorav’s theorem yields that $\Phi^t$ can be neglected in the calculation of $\mu(H)$ (see Section 3). Let us emphasize that in order to apply Sikorav’s argument we need that $M$ has infinite area.

Intuitively, the two distinguished critical values $c_\pm(H)$ correspond to the first homotopically nontrivial separatrices of $H$. More precisely, $c_+(H)$ is the infimum of energy values $E$ such that the superlevel set $H^{-1}([E, \infty))$ is contractible, and $c_-(H)$ is the supremum of $E$ with contractible sublevel set $H^{-1}((-\infty, E])$. Consequently, the asymptotic geometric behaviour of $\varphi^t_H$ depends only on the topology of the level sets of $H$.

When $M$ is the cylinder, the lower bound on the asymptotic non–minimality $\mu(H)$ in terms of the energy levels which carry non–contractible circles was known [Po2, 9.B]. Theorem 1.2 above shows that this bound is sharp!

Concerning the function $r_H(t)$, we obtain the following picture. As long as there are no non–constant periodic solutions we have

$$r_H(t) = (\max H - \min H) t,$$

see [LM, II, Cor. 1.10]. For large $t$, however,

$$r_H(t) \sim (c_+(H) - c_-(H)) t.$$

Consequently, if $\max H > c_+(H)$ or $c_-(H) > \min H$ there must be a “phase transition” in the behaviour of $r_H(t)$ from small $t$ to large $t$.

Let us conclude with a couple of open problems. First of all, it is not clear if the Dichotomy Theorem holds true for surfaces of finite area, or, even more ambitious, for higher–dimensional symplectic manifolds. We also do not know how to deal with cyclic subgroups of $\text{Ham}_c(M, \omega)$, consisting of time–1–maps of time–dependent Hamiltonians periodic in time. The reason is the lack of an integral of motion which is essential for our arguments. Finally, it would be interesting to have a dynamical interpretation of the abovementioned change in the behaviour of $r_H(t)$.

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## 2 Some properties of $c_\pm(H)$

In this section we sum up some useful elementary properties of $c_\pm(H)$ and deduce the second statement of Theorem 1.1 from Theorem 1.2. We start with some auxiliar facts and notions from topology of open surfaces.

**Proposition 2.1.** An open subset $Z \subset M$ is contractible in $M$ if and only if every embedded circle which lies in $Z$ is contractible in $M$.

**Proof.** Assume without loss of generality that $Z$ is connected. Since an open surface is a $K(\pi, 1)$–space, the inclusion $Z \hookrightarrow M$ is homotopic to a point if and only if the homomorphism $\pi_1(Z) \to \pi_1(M)$ is trivial [Sp, 8.1.11]. But $Z$ is also an open surface. Representing $Z$ as the union of an increasing chain of compact surfaces with boundary we see that there exists a countable system of embedded circles which generates $\pi_1(Z)$. Thus we get the proposition. \qed
Every contractible embedded circle $\gamma$ on $M$ bounds a unique closed disc which we denote $D(\gamma)$. For a subset $X \subset M$ set
\[
hull(X) = \text{cl}(\cup D(\gamma))
\]
where $\gamma$ runs over all contractible embedded circles which are contained in $X$.

**Proposition 2.2.** Let $N \subset M$ be a $2$-dimensional compact submanifold with boundary. Assume that $\partial N$ is contractible in $M$. Then $\text{hull}(N)$ is the union of a finite number of pairwise disjoint closed discs whose boundaries are connected components of $\partial N$. In particular, $N$ is contractible in $M$.

**Proof.** It suffices to show that there exist pairwise disjoint closed embedded discs $D_1, \ldots, D_k \subset M$ such that $\partial D_j$ is a component of $\partial N$ and $N \subset \cup_j D_j$. Since $N$ is compact it has only finitely many connected components which we denote by $N_i$; let $\gamma_{ij}$ denote the boundary components of $N_i$. Now fix some $i$. Since $M$ is open and $\partial N$ contractible, at least one of the discs $D(\gamma_{ij})$ intersects the interior of $N_i$. Denote this disc by $D_i$.

We claim that $D_i$ contains $N_i$. Indeed, pick any point $x \in \text{int}(D_i \cap N_i)$, and assume on the contrary that there exists a point $y \in \text{int}(N_i) \setminus D_i$. Since $N_i$ is connected, there exists a path $\alpha$ in the interior of $N_i$ which joins $x$ and $y$. But, since $x$ lies inside $D_i$ and $y$ outside, $\alpha$ must intersect $\partial D_i \subset \partial N$ which is impossible. This contradiction proves the claim.

Notice that for $i \neq j$ either $D_i$ and $D_j$ are disjoint, or one contains the other. So choose from the set $\{D_1, \ldots, D_p\}$ those discs which are maximal with respect to inclusion. This family of discs clearly satisfies all the requirements above. $\square$

As an immediate corollary of the proposition let us mention that the hull of a compact subset is compact. Indeed, each compact subset is contained in some compact submanifold with boundary.

Let us return now to the quantities $c_{\pm}$. As before we assume that $M \neq \mathbb{R}^2$. The next proposition is quite standard in the calculus of variations and could be formulated, for instance, in the setting of the Minimax Lemma in [HZ, Sect. 3.2]. For the convenience of the reader we give a slightly more direct proof here.

**Proposition 2.3.** $c_+(H)$ are critical values of $H$.

**Proof.** Assume on the contrary that $c_+(H)$ is a regular value of $H$. Then there exists a segment $[E_1, E_2]$ which consists of regular values of $H$ and such that $E_1 < c_+(H) < E_2$. By definition of $c_+(H)$, the set $\{H \geq E_1\}$ contains a non-contractible circle. Since the gradient flow of $H$ takes $\{H \geq E_1\}$ into $\{H \geq E_2\}$ we conclude that $\{H \geq E_2\}$ contains a non-contractible circle, too. Hence $c_+(H) \geq E_2$, in contradiction to the choice of $E_2$. The proof for $c_-(H)$ is analogous. $\square$

**Proposition 2.4.** $c_+(H) \geq 0$ and $c_-(H) \leq 0$.

**Proof.** It suffices to show that $M \setminus \text{supp}(H)$ contains a non-contractible curve from $\mathcal{L}$. Write $M = \bigcup N_i$, where $N_i$ are compact surfaces with boundary such that $\text{supp}(H) \subset \text{int}(N_i)$ and $N_i \subset \text{int}(N_{i+1})$ for all $i \geq 1$.

\textsuperscript{2}We write cl($Z$) and int($Z$) for the closure and the interior of a subset $Z$, respectively.
If some boundary component of some $N_i$ is non-contractible in $M$ we are done. Assume therefore that all of them are contractible. Then Proposition 2.2 implies that all $N_i$ are contractible in $M$. We conclude that $\pi_1(M) = 0$, i.e. $M = \mathbb{R}^2$, in contradiction to our standing assumption.

**Proposition 2.5.** $c_-(H) = c_+(H) = 0$ if and only if $\{H \neq 0\}$ is contractible in $M$.

**Proof.** We are going to apply Proposition 2.1 with $Z = Z_H = \{H \neq 0\}$.

If $Z_H$ is not contractible in $M$ then it contains a curve from $\mathcal{L}$. This curve lies either in $\{H > 0\}$, which implies $c_+(H) > 0$, or in $\{H < 0\}$, in which case $c_-(H) < 0$.

Suppose now that $Z_H$ is contractible in $M$. Then $Z_H$ cannot contain a curve from $\mathcal{L}$. This means that every curve from $\mathcal{L}$ intersects the set $\{H = 0\}$, so $c_+(H) \leq 0$ and $c_-(H) \geq 0$. But, as we have seen in Proposition 2.4, $c_+(H)$ is non-negative and $c_-(H)$ is non-positive. Therefore $c_-(H) = c_+(H) = 0$.

As a consequence we see that the second statement of Theorem 1.1 follows from Theorem 1.2.

### 3 Decomposing Hamiltonian flows

In this section we prove the following result.

**Theorem 3.1.** The following inequality holds:

$$\mu(H) \leq c_+(H) - c_-(H).$$

Moreover, if $c_- = c_+ = 0$ then $r_H$ is bounded.

Together with Propositions 2.4 and 2.5 this implies the first statement of Theorem 1.1.

An essential ingredient of our approach is the following version of Sikorav’s theorem [Si]; see also [HZ, Sect. 5.6].

**Proposition 3.2.** Let $X \subset M$ be a finite union of pairwise disjoint closed discs, and let $F \in \mathcal{A}$ be a Hamiltonian function on $M$ whose support is contained in the interior of $X$. Then

$$d(id, \varphi_t^F) \leq 16 \text{area}(X)$$

for every $t$.

**Proof.** When $M = (\mathbb{R}^2, dp \wedge dq)$ this is proved in [Si, HZ]. The case of a general open surface of infinite area can be reduced to this one as follows. Assume without loss of generality that $X \subset (M, \omega)$ consists of just one disc of area $A$. Let $D \subset (\mathbb{R}^2, dp \wedge dq)$ be the closed standard disc of area $A$. Since $M$ has infinite area, it is an easy consequence of the Dacorogna–Moser theorem ([DM], see also [HZ, Sect. 1.6]) that there exists a symplectic embedding

$$i : (\mathbb{R}^2, dp \wedge dq) \hookrightarrow (M, \omega)$$
such that \( i(D) = X \). Clearly, \( i \) induces the natural homomorphism

\[
i_* : \text{Ham}_c(\mathbb{R}^2, dp \wedge dq) \to \text{Ham}_c(M, \omega) .
\]

It is important to notice that \( i_* \) does not increase the corresponding Hofer distances. Our flow \( \varphi^t_F \) lies in the image of \( i_* \), i.e., \( \varphi^t_F = i_*(f_t) \) where \( f_t \) is a one–parameter subgroup of \( \text{Ham}_c(\mathbb{R}^2, dp \wedge dq) \) whose Hamiltonian is supported in \( \text{int}(D) \). Thus, the desired inequality follows from Sikorav’s original theorem since \( d(id, \varphi^t_F) \leq d(id, f_t) \leq 16A \).

**Proof of Theorem 3.1.** Let us decompose the flow \( \varphi^t_H \) into two commuting flows as follows. Fix any \( \epsilon > 0 \), and choose a smooth function \( \rho : \mathbb{R} \to \mathbb{R} \) satisfying the following properties:

1. \( \rho(s) = s \) if \( c_-(H) - \epsilon \leq s \leq c_+(H) + \epsilon \)

2. \( \rho(s) = c_+(H) + 2\epsilon \) if \( s \geq c_+(H) + 3\epsilon \)

3. \( \rho(s) = c_-(H) - 2\epsilon \) if \( s \leq c_-(H) - 3\epsilon \)

4. \( 0 < \rho'(s) < 1 \) if \( c_-(H) - 3\epsilon < s < c_-(H) - \epsilon \) or \( c_+(H) + \epsilon < s < c_+(H) + 3\epsilon \)

Define the new Hamiltonians \( K = \rho \circ H \) and \( H_0 = H - K \), and denote their flows by \( \Psi^t \) and \( \Phi^t \), respectively. Then

\[
\varphi^t_H = \Phi^t \circ \Psi^t = \Psi^t \circ \Phi^t .
\]

Observe that \( \text{supp}(H_0) \) is contained in the set

\[
Z(\epsilon) = H^{-1}((-\infty, c_-(H) - \epsilon] \cup [c_+(H) + \epsilon, \infty)) .
\]

Pick any regular value \( \kappa \in (0, \epsilon) \) of \( H \). Then \( Z(\kappa) \) is a compact 2–dimensional submanifold with contractible boundary. Denote by \( X \) the hull of \( Z(\kappa) \). Proposition 2.2 implies that \( X \) is a finite union of pairwise disjoint closed discs. Moreover, \( Z(\kappa) \) is a subset of \( \text{supp}(H) \), so \( X \) is contained in the hull of \( \text{supp}(H) \). Recall that this hull is compact. Combining this with Proposition 3.2 above, we conclude that there is a constant \( C > 0 \), depending only on \( \text{supp}(H) \) but not on \( \epsilon \), such that

\[
d(id, \Phi^t) \leq C
\]

for every \( t \).

On the other hand, \( \Psi^t \) is generated by \( K \) with \( \max K - \min K = c_+(H) - c_-(H) + 4\epsilon \), hence

\[
d(id, \Psi^t) \leq t (c_+(H) - c_-(H) + 4\epsilon) .
\]

Now, the relation (1) implies that

\[
d(id, \varphi^t_H) \leq d(id, \Phi^t) + d(id, \Psi^t) .
\]

Therefore

\[
d(id, \varphi^t_H) \leq C + t (c_+(H) - c_-(H) + 4\epsilon)
\]

for every \( \epsilon > 0 \), and the inequality in Theorem 3.1 follows. Moreover, if \( c_-(H) = c_+(H) = 0 \) we get that \( r_H(t) \leq C \) for all \( t \geq 0 \), and Theorem 3.1 is proven. \( \square \)
4 A lower bound on $\mu$ via Lagrangian intersections

Recall from the introduction that $\mu(H) = \lim_{t \to \infty} d(\text{id}, \varphi^1_H)/t$. In the present section we prove the following result.

**Theorem 4.1.** We have the following inequality:

$$\mu(H) \geq c_+ (H) - c_- (H).$$

Together with Theorem 3.1 this completes the proof of Theorem 1.2, and thus that of Theorem 1.1.

We will make use of the Lagrangian suspension construction and Lagrangian intersection theory, similar to what is done in [Po1]. It is convenient to split Hofer’s original definition for $d$ into two parts separating the maximum and minimum. Define

$$d_+ (\text{id}, \varphi) = \inf_F \left\{ \int_0^1 \max_x F_t \, dt \mid \varphi^1_F = \varphi \right\}$$

$$d_- (\text{id}, \varphi) = \inf_F \left\{ \int_0^1 \min_x F_t \, dt \mid \varphi^1_F = \varphi \right\}$$

where $F : S^1 \times M \to \mathbb{R}$ runs over all compactly supported Hamiltonians generating $\varphi$. Then

$$d(\text{id}, \varphi) \geq d_+ (\text{id}, \varphi) + d_- (\text{id}, \varphi).$$

Recall that we consider $\varphi = \varphi^1_H$ where $H$ is autonomous.

**Lemma 4.2.**

$$d_+ (\text{id}, \varphi) = \inf_F \left\{ \max_{t,x} F_t \mid \varphi^1_F = \varphi \right\}$$

$$= \inf_G \left\{ \max_{t,x} (H - G) \mid \varphi^1_G = \text{id} \right\}$$

$$d_- (\text{id}, \varphi) = \inf_F \left\{ \min_{t,x} F_t \mid \varphi^1_F = \varphi \right\}$$

$$= \inf_G \left\{ \min_{t,x} (H - G) \mid \varphi^1_G = \text{id} \right\}$$

**Proof.** The first equalities are proved in [Po1, §7], and the second ones in [Po1, Lemma 3.A].

**Lemma 4.3.** Suppose that $G : S^1 \times M \to \mathbb{R}$ is a compactly supported Hamiltonian which generates the identity: $\varphi^1_G = \text{id}$. Let $L \subset M$ be an embedded non-contractible circle. Then there exist $x_0 \in L$ and $t_0 \in S^1$ such that $G(t_0, x_0) = 0$.

**Proof of Theorem 4.1.** Since $\varphi^1_H = \varphi^1_{H'}$, it suffices to show that

$$d(\text{id}, \varphi^1_H) \geq c_+ (H) - c_- (H).$$

Fix an arbitrary $\epsilon > 0$. Choose $L$ to be a non-contractible circle on $M$ such that $H|_L \geq c_+ (H) - \epsilon$. Lemmata 1.2 and 1.3 imply that $d_+ (\text{id}, \varphi^1_H) \geq c_+(H) - \epsilon$. Analogously, $d_- (\text{id}, \varphi^1_H) \geq - c_- (H) - \epsilon$. Thus $d(\text{id}, \varphi^1_H) \geq c_+ (H) - c_- (H) - 2\epsilon$, for every $\epsilon > 0$. Thus we get the desired inequality.
Proof of Lemma 4.3. The proof goes along the lines of [Po1], and we only give a sketch here. The argument is divided into three steps.

1) Choose a compact connected submanifold with boundary \( N \subset M \) whose interior contains both \( L \) and \( \cup_{t \in \mathbb{R}} \text{supp}(G(t, \cdot)) \). Let us perform the following surgery on \((M, \omega)\). We remove the complement to \( N \) and attach to each boundary component of \( N \) a cylindrical end of infinite area. Note that the loop of Hamiltonian diffeomorphisms \( \varphi^G_t \) extends to this new surface. Therefore we can assume from the very beginning that \((M, \omega)\) has a finite number of ends and each end has infinite area. Such a surface \((M, \omega)\) is geometrically bounded (or tame) in the sense of Gromov’s theory of pseudo–holomorphic curves (see [AL]). This will enable us to apply Floer theory in Step 3 below.

2) We claim that \( \pi_1(\text{Ham}_c(M, \omega)) = 0 \).

This fact is well known to experts, however, as far as we know, no reference is available. Here is a sketch of the argument. Denote by \( \text{Diff}_{c, \text{0}}(M) \), respectively \( \text{Symp}_{c, \text{0}}(M, \omega) \), the identity component of the group of compactly supported diffeomorphisms, respectively symplectomorphisms, of \( M \). Consider the sequence

\[
\pi_1(\text{Ham}_c(M, \omega)) \to \pi_1(\text{Diff}_{c, \text{0}}(M)) \to \pi_1(\text{Symp}_{c, \text{0}}(M, \omega)).
\]

The left arrow is a monomorphism (see [MS, Cor. 10.18(iii)] adjusted to the non-compact case along the lines mentioned in the book). The right arrow is an isomorphism; this follows from Moser’s deformation argument with parameters (cf. [MS, Sect. 3.2]). But it is shown in [ES] that \( \pi_1(\text{Diff}_{c, \text{0}}(M)) = 0 \). This completes the proof sketch of the claim.

3) Finally, recall the so–called Lagrangian suspension construction for Lagrangian submanifolds \( L \) in a symplectic manifold \((M, \omega)\). Given \( G : S^1 \times M \to \mathbb{R} \) with \( \varphi^G_1 = \text{id} \), we consider the embedding

\[
L \times S^1 \to M \times T^*S^1
\]

\[
(x, t) \mapsto (\varphi^G_t(x), t, -G(t, \varphi^G_t(x)))
\]

If we equip \( M \times T^*S^1 \) with the split symplectic form \( \omega \oplus dr \wedge dt \) then the above map is a Lagrangian embedding. In our case \( L \) is a circle, and the image of the embedding is a Lagrangian torus which we denote by \( T(G) \).

In view of Step 2 we know that the loop \( \varphi^G_t, 0 \leq t \leq 1 \), is homotopic to the constant loop at the identity. Hence the Lagrangian torus \( T(G) \) is exact Lagrangian isotopic to \( T(0) = \{(x, t, 0) \mid x \in L, t \in S^1\} \). Moreover, since \( L \) is non–contractible, \( \pi_2(M \times T^*S^1, T(0)) = 0 \). Then Floer theory [Fl] guarantees the existence of an intersection point in \( T(G) \cap T(0) \), i.e., there are \( x_0 \in L \) and \( t_0 \in S^1 \) such that

\[
G(t_0, x_0) = 0.
\]

This completes the proof of the lemma and finishes the proof of Theorem 4.1. 

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