Dissipation-Assisted Quantum Information Processing with Trapped Ions

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We introduce a scheme to perform dissipation-assisted quantum information processing in ion traps considering realistic decoherence rates, for example, due to motional heating. By means of continuous sympathetic cooling, we overcome the trap heating by showing that the damped vibrational excitations can still be exploited to mediate coherent interactions, as well as collective dissipative effects. We describe how to control their relative strength experimentally, allowing for protocols of coherent or dissipative generation of entanglement. This scheme can be scaled to larger ion registers for coherent or dissipative many-body quantum simulations.

Among the platforms for quantum computation (QC) and simulations (QS) [1], trapped ions [2] stand out as excellent small-scale prototypes, which have a well-defined roadmap towards large-scale devices based on micro-fabrication [3, 4]. The success of this technology depends on the impact of various sources of decoherence, such as the anomalous heating induced by the electric noise emanating from the trap electrodes [5]. The strong ion-ion couplings, required for scalable QC/QS, demand that the ions lie closer to the electrodes of these miniaturized traps, where the heating is critical and must be carefully considered. A strategy to minimize it is to cryogenically cool the setup [6], or to clean the electrodes by laser ablation [7] or ion bombardment [8]. Although these approaches are promising, a substantial residual noise still exists. We propose to minimize it by applying sympathetic laser cooling continuously during the whole QC/QS protocol.

Sympathetic cooling requires active laser cooling of a subset of ions, and passive cooling of the remaining ions by Coulomb interaction. This technique may overcome the motional heating [9, 10], and has already been implemented between sequential gates for QC [11]. However, the larger heating rates of surface traps would require to cool also during the gates. There are different schemes along these lines: (i) In the absence of fluctuating electric gradients, interactions can be mediated by vibrational modes robust to the heating, while continuously cooling the remaining modes [9]. (ii) By using far-detuned state-dependent forces [12], the mediated interactions do not rely on the motional coherence, and can thus withstand a heating/cooling that is considerably weaker than the interactions. (iii) For ground-state cooled crystals, resolved-sideband cooling may provide a dissipative force that improves the success/fidelity of protocols that are shorter than the inverse of the heating rate [13]. Unfortunately, these requirements are not met in current surface traps: (i) Since ions lie close to the electrodes, electric gradients prevent the isolation of robust modes. (ii) Heating rates in room-temperature setups (1 phonon/ms [14]) coincide with the above protocols speed [12, 13].

In this work, we propose a dissipation-assisted protocol based on an always-on sympathetic cooling that overcomes the anomalous heating for surface traps. We identify regimes where the sympathetically-cooled vibrational modes can be used as mediators of both coherent interactions and collective dissipation. Since we only require Doppler cooling, this proposal can be applied to larger registers for QC/QS.

The model.– We consider an array of two types of ions {σ, τ} confined in a radio-frequency (rf) trap (Fig. 1(a)). Two hyperfine ground-states {↑,↓} of the σ-ions provide the playground for QC/QS, whereas the τ-ions act as an auxiliary gadget to sympathetically cool the crystal. In particular, the τ-ions are Doppler cooled by using a standing wave red-detuned from a dipole-allowed transition with decay rate Γτ [15], while the σ-ions are subjected to a spin-phonon coupling obtained from a two Raman beam in a traveling-wave configuration [16]. When the laser cooling is strong, the atomic degrees of freedom of the τ-ions can be traced out [17], and one obtains a master equation for the reduced dynamics of the σ-spins and the collective vibrations

\[
\dot{\rho} = -i[H_\sigma + H_{ph} + V_{ph}^{\sigma}, \rho] + \mathcal{G}(\rho), \quad \rho = \text{Tr}_{\tau, \text{atomic}}\{\rho\}. \tag{1}
\]

Here, we have introduced the bare spin and phonon Hamiltonians

\[
H_\sigma = \frac{1}{2} \sum_{i} \sigma_i^z \omega_i, \quad H_{ph} = \sum_{i,n} \omega_n b_i^{\dagger} b_n, \quad \omega_i^0 \quad \text{and} \quad \omega_n \quad \text{are the electronic and longitudinal normal-mode frequencies} \quad [18].
\]

Additionally, σ_i^z = |↑⟩⟨↑| – |↓⟩⟨↓|, and b_{i,n} are the operators that create-annihilate phonons. The two crucial ingredients in (1) for our dissipation-assisted protocol are:

(i) A spin-phonon coupling, provided by the Raman beams tuned to the so-called red sideband [17], leads to

\[
V_{ph}^{\sigma} = \sum_{i,n} \mathcal{F}_{in}^{\sigma} \sigma_i^z b_n e^{-i\omega_n t} + \text{H.c.,} \quad \mathcal{F}_{in}^{\sigma} = \frac{\Omega_{\sigma}}{2} \eta_i^{\sigma} a_i^{\dagger} n_i^{\sigma} \Omega_{\sigma},
\]

where σ_i^z = |↑⟩⟨↓|, and the sum is extended to all σ-ions and normal modes. Here, \(\Omega_{\sigma}\) is the Rabi frequency of the Raman.
beams, $\omega_\sigma (k_\sigma)$ is its frequency (wavevector), and $\phi = k_\sigma \cdot r^p$ is defined in terms of the ion position $r^p$. The Lamb-Dicke parameter $\eta^2 = k_\sigma \cdot e_\phi / \sqrt{2m_\sigma \omega_\sigma}$ describes the laser coupling to the $n$-th normal mode, where the $i$-th ion displacement along the direction $e_\phi$ is given by $M_{in}$, and $m_\sigma$ is the ion mass.

(ii) An effective phonon damping, provided by the sympathetic Doppler cooling [17], which can be described by

$$\tilde{\mathcal{D}} (\mu) = \sum_n \{ G_n (b_n^\dagger b_n - b_n b_n^\dagger \mu) + G_n (b_n \mu b_n^\dagger - b_n^\dagger b_n \mu) \} + \text{H.c.}$$

Here, we have introduced Lorentzian-shaped cooling/heating couplings, which allow for an experimental control of the damping of the vibrational modes, and have the expression $\Gamma_n^\pm = \sum_i (\frac{1}{2} \Omega_i \eta^2 \cdot M_{in})^2 / (\frac{1}{2} \Gamma + i(\Delta_i \pm \omega_n))$, where we sum over all the $\tau$-ions. In these expressions, we have introduced the laser Rabi frequency $\Omega_n$, its detuning $\Delta_n$, and its wavevector $k_\sigma$ that determines $\eta^2 = k_\sigma \cdot e_\phi / \sqrt{2m_\sigma \omega_\sigma}$.

The master equation (1) describes an array of spins coupled to a set of damped vibrational modes. The idea now is to use the quanta of these modes, i.e. the phonons, as mediators of a coherent spin-spin interaction. However, in addition to the coherent dynamics, the phonons also provide an indirect coupling to the electromagnetic reservoir leading to some collective dissipation on the spins. Our goal is to find suitable regimes where these collective effects still allow for QC/QS.

To guide this search, note that the two-qubit gates implemented in different laboratories [19] use nearly-resonant spin-dependent forces, and rely on the motional coherence to suppress the residual spin-phonon entanglement. Since the motional coherence is absent in our case, we must work in the far off-resonant regime [12, 20], where $|\mathcal{F}_{in}^{\sigma}| \ll |\delta_n| \ll \omega_\sigma$, such that $\delta_n = \omega_\sigma - (\omega_0^\sigma - \omega_\sigma)$. In this regime, motional excitations by the spin-phonon coupling are negligible. We identify below the additional conditions to tailor the coherent/dissipative phonon-mediated processes in presence of laser cooling.

**Collective Liouvillian.** For the values considered below, the laser-cooling rates reach $W_n \approx 10^{-2} \omega_\sigma$. In this case, the cooling is very strong, and the vibrations reach the steady state very fast. Hence, we can apply the theory of Schrieffer-Wolff (SW) transformations for open systems [21] to trace out the phonons from Eq. (1), and obtain an effective Liouvillian

$$\tilde{\mathcal{L}}_{\text{eff}} (\mu_\sigma) = -i [H_{\text{eff}} (\mu_\sigma) + \mathcal{D}_{\text{eff}} (\mu_\sigma)],$$

where $\mu_\sigma = \text{Tr}_{\phi_b} \{ \mu \}$. Here, the coherent Hamiltonian is

$$H_{\text{eff}} = \sum_{i > j} \{ J_{ij}^{\text{eff}} \sigma_i^+ \sigma_j^+ + \text{H.c.} \} + \sum_{in} \frac{1}{2} B_{in}^\text{eff} \sigma_i^z,$$

which contains the phonon-mediated interactions of strength $J_{ij}^{\text{eff}}$, which describe processes where a phonon is virtually created, and then absorbed elsewhere in the chain. These interactions can be used to implement gates for QC, or to explore spin models for QS. Additionally, we also find an effective ac-Stark shift, which can be interpreted as an effective magnetic field $B_{in}^\text{eff}$, arising from the processes where the phonon is created and absorbed by the same ion. Note that the same virtual phonon exchange also introduces an indirect dissipation in (2)

$$\mathcal{D}_{\text{eff}} (\mu_\sigma) = \sum_{i,j} \Gamma_{ij}^{\text{eff}} (\sigma_i^+ \mu_\sigma \sigma_j^+ - \sigma_j^+ \sigma_i^+ \mu_\sigma + \text{H.c.}) + \sum_{i,j} \{ \Gamma_{ij}^{\text{eff}} + \Gamma_{ij}^{\text{eff}} \} (\sigma_i^- \mu_\sigma \sigma_j^+ - \sigma_j^+ \sigma_i^- \mu_\sigma + \text{H.c.}),$$

where $\Gamma_{ij}^{\text{eff}}, \Gamma_{ij}^{\text{eff}}$ are the strengths of the collective processes of spontaneous and stimulated dissipation, respectively.

To find the correct regime for QC/QS purposes, we must compare the time-scales derived from the expressions

$$J_{ij}^{\text{eff}} = \sum_n \frac{\mathcal{F}_{in}^{\sigma} (\mathcal{F}_{in}^{\sigma})^*}{\delta_n^2 + W_n^2} \tilde{\delta}_n,$$

$$\mathcal{B}_{ij}^{\text{eff}} = -\sum_n \frac{\mathcal{F}_{in}^{\sigma} (\mathcal{F}_{in}^{\sigma})^*}{\delta_n^2 + W_n^2} \tilde{\delta}_n (2n_\sigma + 1),$$

Here, the laser cooling leads to the effective cooling rates $W_n = \Re \{ \Gamma_n - \Gamma_n^\pm \}$ that damp the ion vibrations, and to a Lamb-type shift of the vibrational frequencies leading to $\delta_n = \delta_n + \Im \{ \Gamma_n^\pm - (\Gamma_n^\pm)^* \}$. Additionally, $n_\sigma = \Re (\Gamma_n^\pm)/W_n$ are the mean phonon numbers in the steady state of the laser cooling. From these expressions, it is clear that by tuning the ratio $\mathcal{R}_n = W_n (\bar{n}_\sigma + 1) / \tilde{\delta}_n$, we control if the spin interactions prevail over the dissipation $\mathcal{R}_n \ll 1$, or vice versa $\mathcal{R}_n \gg 1$.

**Coherent and dissipative generation of entanglement.** We consider the simplest scenario to test our scheme: a three-ion chain in a linear Paul trap (Fig. 1(b)). To use realistic parameters, we consider $25 \text{Mg}^{+2}, 25 \text{Mg}^{+2}, 25 \text{Mg}^{+2}$, and set the axial trap frequency to $\omega_0/2\pi = 4.1$ MHz, which is possible by optimizing the trap voltages. The dipole-allowed transition $|g\rangle = |3S_{1/2}\rangle \leftrightarrow |e\rangle = |3P_{1/2}\rangle$ of $24 \text{Mg}^{+2}$, which is characterized by $\lambda_{e\sigma} \approx 280.3$ nm and $\Gamma_e/2\pi \approx 4.14$ MHz, shall be used for continuous sympathetic cooling. By applying an external magnetic field, we can encode the spins in a couple of Zeeman sub-levels $|F, M\rangle$ of the ground-state manifold of $25 \text{Mg}^{+2}$, e.g. $|F\rangle = |2, 2\rangle$ and $|M\rangle = |3, 3\rangle$. This leads to a resonance frequency of $\omega_0^M/2\pi \approx 1.79$ GHz, and a negligible decay rate of $\Gamma_e/2\pi \approx 10^{-14}$ Hz. Finally, a pair of off-resonant lasers drive the axial red-sideband through an excited state in the $3P_{3/2}$ manifold of $25 \text{Mg}^{+2}$, such that $\eta_{1n}^2 \approx 0.16$.

The isotopic mass ratio $m_t/m_g \approx 0.96$ implies that the axial vibrational modes are almost unchanged with respect to the homogeneous chain, $\omega_\sigma/2\pi \approx \{4.1, 7.1, 10.1\}$ MHz. To attain a wide range of values for the ratio $\mathcal{R}_n$, we tune the Raman lasers closer to the highest-frequency mode, the so-called egyptian mode, such that the detunings are $\delta_n/2\pi \in \{6.2, 3.2, 0.3\}$ MHz. Note that due to the large detuning from the remaining modes, the collective effects will be mediated by the egyptian mode. To sympathetically cool it, we place the cooling isotope at the middle of the chain, such that it coincides with the node of a standing-wave laser [15]. This laser has frequency that is red-detuned from the transition, and we set the detuning to $\Delta_e = -\Gamma_e/2$. This leads to a steady-state mean phonon number $\bar{n}_3 = 0.65$ independent of the standing-wave Rabi frequency. Therefore, we can modify the Rabi frequency $0.2 \Omega_e/\Gamma_e \leq 2$ in order to control the cooling rate $W_3$, and thus the ratio $\mathcal{R}_n$, thus exploring regimes
of either dominant dissipation or interactions. We remark that the laser used for cooling $^{24}\text{Mg}^+$ will be highly detuned from the cooling transition of $^{25}\text{Mg}^+$ (i.e. $\Delta / 2\pi \approx 2.7 \text{THz}$), such that the induced decay rate for the considered regime fulfills $\Gamma_\text{eff} (\Delta^2 / 2\pi) \approx 10^{-2} \text{Hz}$. Therefore, this laser only contributes with off-resonant ac-Stark shifts that shall be considered later on. We now explore two possible applications.

(a) Coherent generation of entanglement: Our goal is to use the coherent phonon-mediated interaction in Eq. (2) to generate entanglement between the $^{25}\text{Mg}^+$ ions. By setting $\Omega_\text{eff} = 0.15 \Gamma_\text{eff}$, we obtain a cooling rate of $W_3 / \omega_3 \approx 4.3 \cdot 10^{-3}$, such that $\beta_3 \approx 7 \cdot 10^{-3}$, and the Hamiltonian part of the Liouvilian (2) thus dominates. Initializing the spin state in $|\psi_0(0)\rangle = |\uparrow\downarrow\rangle$, and setting $\Omega_\text{eff} \epsilon_3 \approx 10 W_3$, such that the distance between the ions is an integer multiple of the effective Raman wavelength, we obtain the Bell state $|\psi_\text{Bell}\rangle = 1/\sqrt{2} (|\uparrow\downarrow\rangle - i|\downarrow\uparrow\rangle)$ for $t_0 \approx 4$ ms (Fig. 2(a)). In the numerical simulations, we have considered a realistic heating rate for macroscopic rf-traps of $\Gamma_\text{heating} \approx 0.1$ phonon/ms, by substituting $\Gamma_\text{eff} \rightarrow \Gamma_\text{eff} + \Gamma_\text{heating}$ in the dissipator of (1). From this figure, we observe that, even if the process is slower than the usual gates [19], it prevails over the phonon-mediated decoherence leading to errors as low as $\epsilon_\text{t} \approx 10^{-2}$. Note that such errors are not sufficient for fault-tolerance QC, which require $\epsilon_\text{t} \approx 10^{-2} \cdot 10^{-4}$. On the one hand, we can achieve lower error rates by working with larger detunings. On the other hand, this leads to slower gates, which require an additional scheme to decouple from other sources of decoherence that shall be introduced below. Finally, for the anomalous heating rates in micro-fabricated surface traps $\Gamma_\text{heating} \approx 1$ phonon/ms, the same parameters lead to errors $\epsilon_\text{t} \approx 2 \cdot 10^{-2}$ for $t_0 \approx 5$ ms, which illustrates the robustness of our scheme with respect to motional heating. Let us also advance that our protocol might be scaled directly to many ions for QS [17], which do not require such small error rates.

(b) Dissipative generation of entanglement: A different possibility would be to exploit the collective dissipation in Eq. (2) to generate entanglement in the steady state. The idea is to set $\Omega_\text{eff} = 2 \Gamma_\text{eff}$, such that the dissipative part of the Liouvilian (2) becomes dominating $\beta_3 \approx 2.3$, and we can exploit a superradiant/subradiant phenomenon [22]. By controlling the ion distance with respect to the Raman wavelength such that $k_\text{eff} \cdot (r^\sigma_1 - r^\sigma_2) = p \pi$, where $p \in \mathbf{Z}$, the decay rates fulfill $\Gamma_\text{eff} = \Gamma_\text{eff}^{33} = \Gamma_\text{eff}^{11} = \Gamma_\text{eff}^{13} = \Gamma_\text{eff}^{31} = (-1)^p \Gamma_\text{eff}^{ij}$. In this limit, the dissipator in (2) can be written as:

$$\mathcal{D}(\mu_\text{eff}) = L_+ \mu_\text{eff} L_+^\dagger + L_- \mu_\text{eff} L_-^\dagger - L_- L_+ \mu_\text{eff} - L_+ L_- \mu_\text{eff} + \text{H.c.},$$

where we have introduced the collective jump operators $L_+ = \sqrt{\Gamma_\text{eff}^{33}} (\sigma^+_{13} + (-1)^p \sigma^-_{13})$, and $L_- = \sqrt{\Gamma_\text{eff}^{33}} (\sigma^+_{13} + (-1)^p \sigma^-_{13})$. One can check that the symmetric/antisymmetric Bell states $|\phi_+\rangle = 1/\sqrt{2} (|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle)$ are dark states of these jump operators for $p$ odd, or $p$ even, respectively. These are the so-called sub-radiant decay channels [23], which allows us to get a mixed stationary state that is partially entangled. In particular, starting from $|\psi_0(0)\rangle = |\uparrow\downarrow\rangle$ for $p$ even, and $\Omega_\text{eff} \epsilon_3 \approx W_3$, we obtain a decoherence-free entangled steady-state $|\phi_+\rangle$ for $t \gg t_\text{fs} \approx 50 \mu$s with fidelities around $30\%$ (Fig. 2(b))[24]. Note that this phononic subradiance is not affected by limitations in the ratio of the ion-ion distance with respect to the wavelength of the emitted light. The collective nature of the vibrations that mediate the subradiance allows to surpass the limitations of the pioneering trapped-ion experiments [26]. We also note that the ultimate limit of $50\%$ cannot be achieved due to the thermal contribution to Eq. (2). However, schemes originally formulated for cavities [27] can be adapted for our trapped-ion setting to reach unit fidelities.

Let us emphasize that, although we have considered a particular example, the scheme is also applicable to other ion species. For the regime of dominant dissipation, any ion will work equally well. Conversely, for dominant coherent interactions, the required strong sympathetic-cooling strengths and detunings are likely to be optimized for crystals with two light isotopes. For the regime of dominant dissipation, any ion will work equally well. Conversely, for dominant coherent interactions, any ion will work equally well. Conversely, for dominant coherent interactions, any ion will work equally well. Conversely, for dominant coherent interactions, any ion will work equally well.
dephasing term \( H_\text{d} = \sum_i \frac{1}{2} (\sum_a B_i^a(t) + F_i(t)) \sigma_i^a \). Here, \( B_i^a \) in Eq. (2) introduces noise via fluctuations over the phonon steady state, and \( F_i(t) \) is a random process that models the noise of external magnetic fields, or uncompensated ac-Stark shifts. This random noise, which typically has a short correlation time \( \tau_\text{c} \), leads to a dephasing rate \( \Gamma_\text{d}/2\pi \sim 0.1-1\text{kHz} \) [17].

For any practical implementation of the proposed protocol, this dephasing should be carefully considered. The standard approach for prolonging the coherence of a system consists of a sequence of refocusing pulses, a technique known as pulsed dynamical decoupling [29]. Another approach, known as continuous dynamical decoupling, produces a similar effect by continuous drivings [30, 31]. As recently demonstrated experimentally [32], these techniques allow to implement robust 2-qubit gates exploiting a single sideband [31]. As a byproduct, we show that this tool allows for slower gates, and thus smaller errors in principle. Moreover, it also provides a new gadget to tailor the collective Liouvillian (2).

We apply a continuous driving resonant with the spins, such that the bare spin Hamiltonian reads
\[
H_\text{d} = 2 \sum_i \Omega_i \sigma_i^\mu \sigma_i^\nu + \text{h.c.}
\]
with \( \Omega_i = \Omega_0 \). In this regime, a modified SW transformation leads to
\[
\mathcal{L}_\text{eff}(\mu_\sigma) = -i[H_\text{eff} + H_\alpha, \mu_\sigma] + \mathcal{D}_\text{eff}(\mu_\sigma) + \mathcal{D}_\alpha(\mu_\sigma).
\] (3)

In the limit of a strong driving [17], the Hamiltonian above corresponds to an interacting Ising model
\[
H_{\text{eff}} = \sum_{i \neq j} \tilde{B}_{ij}^{\text{off}} \sigma_i^\sigma_j + \sum_i \frac{1}{2} \tilde{\Omega}_i \sigma_i^z,
\]
and we obtain a collective phonon-mediated dephasing
\[
\mathcal{D}_\text{eff}(\mu_\sigma) = \sum_{i \neq j} \Gamma_{ij}^{\text{off}} (2 \sigma_i^\mu \sigma_j^\mu - \sigma_i^\nu \sigma_j^\nu). \]

Assuming that the surface-trap array is designed such that \( k_L \cdot r_i^p = 2\pi p \) with \( p \in \mathbb{Z} \), we emphasize that the interaction strengths and dissipation rates,
\[
f_{ij}^{\text{off}} = \sum_n \frac{|\tilde{F}_{ij}^n|^2}{\tilde{\delta}_n^2 + \tilde{W}_n^2} \tilde{\delta}_n \frac{1}{2}, \quad f_{ij}^{\text{on}} = \sum_n \frac{|\tilde{F}_{ij}^n|^2}{\tilde{\delta}_n^2 + \tilde{W}_n^2} W_n \tilde{\delta}_n \left( \tilde{n}_n + \frac{1}{2} \right),
\]
lead to a very similar control parameter \( \tilde{\delta}_n = W_n \tilde{n}_n \left( \tilde{n}_n + \frac{1}{2} \right)/|\tilde{\delta}_n| \).

Accordingly, under the same assumptions as we considered above, we can interpolate between regimes where the coherent Ising interactions dominate over the collective dephasing, or vice versa. In addition to the new range of possibilities offered by this collective Liouvillian, note that the dephasing noise terms contribute to the new Liouvillian (3) with
\[
\tilde{H}_\alpha = \sum_i \frac{1}{2} \tilde{\Omega}_i \sigma_i^z, \quad \tilde{\mathcal{D}}_\alpha(\mu_\sigma) = \sum_{i \neq j, \zeta} \frac{1}{2} \Gamma_{ij}^{\zeta} (\sigma_i^\mu \mu_\sigma \sigma_j^\mu - \mu_\sigma),
\]
where we have assumed that the noise is local, and introduced \( \tilde{\Omega}_i = (B_i^{\text{off}} \tilde{\delta}_i + \Gamma_i)/2\tilde{\Omega}_i \tau_\text{c} \), and \( \Gamma_i = \Gamma_i/(\tilde{\Omega}_i \tau_\text{c})^2 \) in the limit of a strong driving \( \tilde{\Omega}_i \tau_\text{c} \gg 1 \) [17]. According to the above constraints, these noisy terms are suppressed by a sufficiently-strong driving. To be more specific, for noise correlation times on the order of \( \tau_\text{c} = 10^{-2}/\tilde{\Omega}_i \), and detunings \( \tilde{\delta}_i/2\pi \sim 0.1-1\text{MHz} \), it suffices to apply drivings with \( \tilde{\Omega}_i/2\pi \approx 10\text{MHz} \) to reduce the noise by more than two orders of magnitude. Therefore, we can decouple from the noise efficiently, while preserving the collective part of the Liouvillian for QC/QS. In contrast to Fig. 2(a), the new Liouvillian (3) allows for the coherent generation of all four Bell states.

Conclusions.— We have proposed a scheme based on sympathetic cooling to overcome the anomalous heating in surface traps, while allowing for QC/QS. The sympathetic cooling becomes a tool to tailor the collective effects of the Liouvillian. By controlling a single parameter, namely the laser-cooling power, we have shown how the Liouvillian interpolates between regimes of dominating coherent interactions/collective dissipation, both of which allow for generation of entanglement. Moreover, we have this control can also be exploited for coherent/dissipative many-body QS.

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[1] T. D. Ladd, et al., Nature **464**, 45 (2010); J. I. Cirac and P. Zoller, Nat. Phys. **8**, 264 (2012).
[2] R. Blatt and D. Wineland, Nature **453**, 1008 (2008); R. Blatt and C. F. Roos, Nat. Phys. **8**, 277 (2012).
[3] See D. J. Wineland and D. Leibfried, Laser Phys. Lett. **8**, 175 (2011) and references therein.
[4] See Ch. Schneider, D. Porras, and T. Schaez, Rep. Prog. Phys. **75**, 024401 (2012), and references therein.
[5] Q. A. Turchette, et al., Phys. Rev. A **61**, 063418 (2000).
[6] L. Deslauriers, et al., Phys. Rev. Lett. **97**, 103007 (2006); J. Labaziewicz, et al., Phys. Rev. Lett. **100**, 013001 (2008).
[7] D. T. C. Allcock, et al., New J. Phys. **13**, 123023 (2011).
[8] D. A. Hite, et al., Phys. Rev. Lett. **109**, 103001 (2012).
[9] D. Kielpinski, et al., Phys. Rev. A **61**, 032310 (2000); G. Morigi and H. Walther, Eur. Phys. J. D **13**, 261 (2001).
[10] H. Rohde, et al., J. Opt. B **3**, S3 (2001); M. D. Barrett, et al., Phys. Rev. A **68**, 042302 (2003); J. P. Home, et al., Phys. Rev. A **79**, 050305(R) (2009).
[11] J. P. Home, et al., Science **325**, 1227 (2009).
[12] A. Sørensen and K. Mølmer, Phys. Rev. Lett. **82**, 1971 (1999).
[13] A. Beige, Phys. Rev. A **67**, 020301(R) (2003); ibid. **69**, 012303 (2004).
[14] K. R. Brown, et al., Nature **471**, 196 (2011); M. Harlander, et al., Nature **471**, 200 (2011).
[15] Standing-wave laser cooling is considered for generality. However, for the protocol of dissipative QIP, the standard travelling-wave cooling may suffice (see a more detailed discussion below).
[16] See D. J. Wineland, et al., J. Res. Natl. Inst. Tech. **103**, 259.
(1998), and references therein.
[17] See the Supplemental Material.
[18] The scheme works for any vibrational branch of the crystal (e.g. axial and radial in a chain) provided that the Coulomb couplings and sympathetic cooling rates are of comparable magnitude.
[19] See K.-A. Brickman Soderberg and C. Monroe, Rep. Prog. Phys. 73, 036401(2010), and references therein.
[20] D. Porras and J. I. Cirac, Phys. Rev. Lett. 92, 207901 (2004).
[21] E. M. Kessler, Phys. Rev. A 86, 012126 (2012).
[22] R. H. Dicke, Phys. Rev. 93, 99 (1954); R. H. Lemberg, Phys. Rev. A 2, 889 (1970); Z. Ficek and R. Tanas, Phys. Rep. 372, 369 (2002).
[23] M. Gross and S. Haroche, Phys. Rep. 93, 301 (1982).
[24] We have also computed the an entanglement measure for the steady state, namely the logarithmic negativity [25] $LN(\mu_{ss}) \approx 0.12$, to rule out that this corresponds to a product state.
[25] M. B. Plenio, Phys. Rev. Lett. 95, 090503 (2005).
[26] R. G. DeVoe and R. G. Brewer, Phys. Rev. Lett. 76, 2049 (1996).
[27] M. J. Kastoryano, F. Reiter, and A. S. Sørensen, Phys. Rev. Lett. 106, 090502 (2011).
[28] A. Walther, U. Poschinger, K. Singer, and F. Schmidt-Kahler, Appl. Phys. B 107, 1061 (2012).
[29] E. L. Hahn, Phys. Rev. 80, 580 (1950); L. Viola and S. Lloyd, Phys. Rev. A 58, 2733 (1998); M. J. Biercuk, et al., Nature 458, 996 (2009); D. J. Szwer, S. C. Webster, A. M. Steane, and D. M. Lucas, J. Phys. B 44, 02550 (2011).
[30] N. Timoney, et al., Nature 476, 185 (2011); A. Noguchi, S. Haze, K. Toyoda, and S. Urabe, Phys. Rev. Lett. 108, 060503 (2012).
[31] A. Bermudez, P. O. Schmidt, M. B. Plenio, and A. Retzker, Phys. Rev. A 85, 040302(R) (2012).
[32] T. R. Tan, et al., arXiv:1301.3786 (2013).
1. SUPPLEMENTAL MATERIAL: DISSIPATION-ASSISTED QUANTUM INFORMATION PROCESSING WITH TRAPPED IONS

Dissipative ion crystals.— We consider a collection of atomic ions confined in radio-frequency traps [1]. In particular, we will explore dissipative-assisted protocols for an array of $N_l = N_{a} + N_{b}$ trapped ions in equilibrium positions $\{\mathbf{r}_{a}, \mathbf{r}_{b}\}$. The geometry of the array will depend on the particular trap under consideration. For micro-fabricated surface traps, we will consider arbitrary geometries (Fig. 1(a)), whereas for the more standard rf-traps, we restrict to one-dimensional chains (Fig. 1(b)). A fraction of the ions, $N_l$, is laser cooled via a dipole-allowed transition $|g\rangle \leftrightarrow |e\rangle$ with frequency $\omega_0$ and decay rate $\Gamma_0$, providing the sympathetic cooling of the remaining ions, $N_{a}$. Two hyperfine ground-states of the $\sigma$-ions $\{\uparrow, \downarrow\}$ form the spins/qubits for QS/QC purposes, such that their decay rate is negligible, and their energy spacing is $\Delta_0 \approx \hbar (n - 1)$. To minimize the action of the cooling laser on the spins, one may use beams tightly focused on the $\tau$-species, or exploit two different ion species/isotopes.

As customary in the theory of open quantum-optical systems [2], one obtains the master equation

$$\dot{\rho} = -i[H, \rho] + \mathcal{D}(\rho),$$

after tracing out the electromagnetic bath. Here, we have introduced the Hamiltonian $H$ and dissipative $\mathcal{D}(\rho)$ parts. Let us start by describing the Hamiltonian

$$H = H_\tau + H_\sigma + H_{ph} + V_{\tau}^{ph} + V_{\sigma}^{ph}.$$  

where $H_\tau$ is the twophonon Rabi frequency, $\omega_0(k_\sigma) = \text{Raman frequency}$ (wavevector), and $\phi = k_\sigma \cdot \mathbf{r}$. The Lamb-Dicke parameter can be defined as $\eta_\sigma = \omega_0 \phi / \sqrt{2m_\sigma \omega_0}$, describes the laser coupling to the $n$-th mode with displacements $\mathcal{M}_n$ along $\mathbf{e}_n$, where $m_\sigma$ is the ion mass. This spin-phonon coupling arises from the dipole laser-ion interaction after expanding $\eta_\sigma^{\sigma} \ll 1$, such that $\omega_0 \approx \omega_0 - \omega_\sigma$. We set $|\Omega_\sigma| \ll \omega_\sigma$ to neglect the contribution of other terms (i.e. carrier and blue sideband) from the laser-ion interaction.

We laser cool the $\tau$-species of the node of a standing wave [3], which shall be red-detuned with respect to the atomic transition, and can be described by

$$V_{\tau}^{ph} = \sum_{in} \mathcal{F}_n^{\tau} \frac{\tau^{\dagger}_n}{\tau^{\dagger}_n} + \text{H.c.,} \quad \mathcal{F}_n^{\tau} = -\frac{\Omega_\tau^{\tau}}{\eta_\tau^{\tau}} \mathcal{M}_n.$$
whose major contribution may be caused by instabilities in the laser intensity of the cooling lasers. These three terms can be modeled by

$$H_n = \sum_i \frac{1}{2} \left( \sum_n B_{n}^{\text{eff}} + F_i(t) \right) \sigma_i^z, \quad (13)$$

where $B_{n}^{\text{eff}}$ in Eq. (2) yields the thermal noise, and $F_i(t)$ is a random process for the magnetic/laser-intensity dephasing. We assume a local Gaussian noise with a short correlation time $\tau_c$, which determines the stochastic average of two-time correlators $\langle F_j(s)F_i(t) \rangle_{st} = \frac{\tau_c}{\tau} e^{-s/\tau} \delta_{ij}$, and in turn the dephasing rate of the spin dynamics $\Gamma_d = \frac{1}{\tau} \int_0^\tau ds \langle F_j(s)F_j(s) \rangle_{st} = 1/2T_2$. Let us note that typical decoherence times in trapped-ion experiments are $T_2 \approx 1-10$ ms ($\Gamma_d \approx 0.05-0.5$ kHz), and that the regimes considered above yield $B_{n}^{\text{eff}}/\bar{n}_n \approx 0.3-3$ kHz. Comparing these values to the time-scales of the dissipation-assisted protocols ($\sim 0.1-1$ ms), it emphasizes that we need a scheme to actively decouple from this noise.

A partial solution would be the use of states that are insensitive to linear Zeeman shifts, such as $|\uparrow\rangle = |2,1\rangle$ and $|\downarrow\rangle = |3,1\rangle$ at a field of $B_0 = 213$ G for $^{25}$Mg$^{+}$. However, since we still need to mitigate the other sources of dephasing, we will exploit a different mechanism. We introduce a continuous driving of the spins, such that the bare spin Hamiltonian

$$H_d = \frac{1}{2} \sum_i \omega_i \sigma_i^z + (\Omega_d \sigma_i^+ \cos(\omega_d t) + \text{H.c.}), \quad (14)$$

may be provided by a microwave source. Here, the driving parameters are $\omega_d = \omega_i^0$, and $|\Omega_d| \ll \omega_d$. Since the resonance frequencies are $\omega_d/2\pi \approx 1$ GHz, the driving can still be strong enough to fulfill strong-driving conditions

$$\{W_n, \tilde{\sigma}_i\} \ll |\Omega_d|, \quad \Gamma_d, \tau_c^{-1} \ll |\Omega_d|. \quad (15)$$

The first of these conditions allows us to use a modified Schriefer-Wolff transformation, which leads us to the new phonon-mediated terms $\hat{H}_n$ and $\hat{G}_n$ of the Liouvillian in Eq. (3) of the main text, after neglecting of-resonant contributions for such a strong driving. The second condition allows us to obtain the effect of the residual noise $\hat{H}_n$ and $\hat{G}_n$ in Eq. (3), by using a Born-Markov approximation

$$\hat{\rho} = -\int_0^\infty ds \langle [\hat{H}_n(t), [\hat{H}_n(t-s), \hat{\rho}(t)]]_H \rangle_{st}, \quad (16)$$

where we perform a stochastic average, and work in the interaction picture with respect to the resonant driving $\hat{H}_n(t) = U H_n U^\dagger$, where $U = \exp[i\int_0^t \sum_i \frac{1}{2} \Omega_i \sigma_i^+].$

Many-body physics.– The collective Liouvillians in Eqs. (2) and (3) define our toolbox for dissipation-assisted QS. In the main text, we have considered quantum information processing by means of an isolated vibrational mode that mediates the collective dynamics. In the many-ion scenario, this would lead to fully-connected spin networks that become very interesting in the presence of magnetic frustration [6]. However, to achieve strongly-correlated models, it is better to work with a full vibrational branch of a small frequency width. The small ratios $\mathcal{R}_n \approx 10^{-3}$ obtained for trapping frequencies $\omega_n/2\pi \approx 10$ MHz, indicate that the coherent dynamics can be dominating also in this case, while minimizing the heating. This would allow for the QS of exotic models with 3-body interactions [7] or topological order [8] in surface ion traps.

These many-body QS focus on the Hamiltonian while minimizing the influence of the environment. However, the dissipative dynamics may also lead to interesting many-body phenomena [9]. The possibility to control the relative strength of the coherent/dissipative parts in (2) and (3) is very appealing in this respect. In particular, we note that for intermediate driving strengths $\Omega_d$, competing dissipative terms in (3) may lead to purely-dissipative quantum phase transitions.

[1] See D. J. Wineland, et al., J. Res. Natl. Inst. St. Tech. 103, 259 (1998), and references therein.
[2] H. P. Breuer and F. Petruccione, The theory of open quantum systems, (Oxford University Press, Oxford, 2003); A. Rivas and S. F. Huelga, Open Quantum Systems: An Introduction (Springer, Heidelberg, 2012).
[3] J. I. Cirac, R. Blatt, P. Zoller, and W. D. Phillips, Phys. Rev. A 46, 2668 (1992).
[4] J. Dalibard and C. Cohen-Tannoudji, J. Phys. B: At. Mol. Phys. 18, 166 (1985); Y. Castin, H. Wallis, and J. Dalibard, J. Opt. Soc. Am. B. 6, 2046 (1989).
[5] G. Lindblad, Commun. Math. Phys. 48, 119 (1976).
[6] G.-D. Lin, C. Monroe, L.-M. Duan, Phys. Rev. Lett. 106, 230402 (2011).
[7] A. Bermudez, D. Porras, and M. A. Martin-Delgado, Phys. Rev. A 79, 060303(R) (2009).
[8] R. Schmied, J. H. Wesenberg, and D. Leibfried, New J. Phys. 13, 115011 (2011).
[9] M.B. Plenio and S.F. Huelga, Phys. Rev. Lett. 88, 197901 (2002); S.F. Huelga and M.B. Plenio, Phys. Rev. Lett. 98, 170601 (2007); M. Müller, S.Diehl, G. Pupillo, P. Zoller, Adv. At. Mol. Opt. Phys. 61, 1 (2012).