Abstract
In the present work, the elastic scattering of the $^8$B+$^{27}$Al reaction at two different energies have been examined using the microscopic double folding potential approximation within the framework of the optical model. The real part of the optical model has been obtained by using Fermi, Gaussian and Variational Monte Carlo (VMC) density distributions in the microscopic double folding model and the imaginary part has been taken as Woods-Saxon volume type. The results of our microscopic analysis are quite compatible with the experimental cross-section data. The reaction cross sections, the volume integrals of used potentials and their $\chi^2/N$ errors have also been computed. This study is important in showing the effect of microscopically derived potentials in explaining the $^8$B+$^{27}$Al experimental data published recently.

Keywords: Double folding, proton halo nuclei, elastic scattering

1. Introduction
The radioactive ion beam (RIB) facilities have opened a way for significant improvements in the field of nuclear physics, especially those related to the halo nuclei. Structurally halo has tightly bound core with weakly bounded valence nucleons. The half-lives of these extraordinary nuclei are usually very short. Halo nuclei are evaluated in two groups; these are neutron and proton halos. These nuclei are evaluated according to their state in the stability valley. The most common proton halo is the $^8$B nuclei which has low break up threshold. The $^8$B nuclei has a weakly bound proton with a proton separation energy $S_p =0.137$ MeV and its half-life is $t_{1/2}=770$ ms. Nuclear reactions and densities involving short-lived $^8$B nuclei have been extensively investigated by both nuclear astrophysicists, experimental and theoretical nuclear physicist (Aguilera et al., 2011; Aguilera, Martinez-Quiroz, Belyaeva, Kolata, & Leyte-Gonzalez, 2008; Aguilera et al., 2009; Aguilera, Martinez-Quiroz,
In the next part, we present our theoretical model and the microscopic potentials obtained with different nuclear matter density distributions. In Section 3, we present the results of optical model calculations performed by using these two different potentials. Our conclusion is given in Section 4.

2. Theoretical Analysis

The interaction between two nuclei is generally defined as a many-body problem, and many reactions can occur as a result of this interaction. These are elastic scattering, inelastic scattering, nucleon transfer reactions and projectile fragmentations. The shape of these reactions is determined by the structure of projectile and the incoming energy, and the simplest of these reactions is the elastic scattering. Within mean-field approximation, the elastic scattering is described by using the optical model approach with either phenomenological potentials such as Woods-Saxon or Woods-Saxon derivatives or microscopic-double folding (DF) potentials. In literature, the interaction of the projectile and target nuclei is defined as optical model. The identified potential in this model has a Coulomb, centrifugal and nuclear interactions parts. In the elastic reaction, the real and imaginary parts of the nuclear potential are responsible for scattering and the lost flux, respectively (Aygun, Kocadag, & Sahin, 2015; Brandan & Satchler, 1997; Satchler, 1983). In this study, all of the computations and comments have been made according to the Microscopic Model-Double Folding method (DF). DF model is the most popular procedure for analyzing experimental angular distributions of halo nuclei on stable nuclei. In microscopic model, while imaginary potential is taken Woods-Saxon or Woods-Saxon square type potential, the real potential can be defined using double folding model.
In double folding model, the density distributions of both the projectile and target nuclei are used. The elastic scattering of $^8\text{B}$ as one-proton halo nucleus on $^{27}\text{Al}$ target was examined using the double folding model within the framework of the OM at the incident energies, $E_{\text{Lab}}=15.3$ MeV and $E_{\text{Lab}}=21.7$ MeV. The total effective potential is given in Equation 1.

$$V_{\text{total}}(r) = V_{\text{Coulomb}}(r) + V_{\text{Nuclear}}(r) + V_{\text{centrifugal}}(r)$$

(1)

In this equation the Coulomb potential term (Satchler, 1983) is owing to the interaction between the projectile and target in proton (charge) numbers over a sphere of radius $R_c$ (Equation 2).

$$V_{\text{Coulomb}}(r) = \frac{Z_p Z_T e^2}{4\pi \varepsilon_0} \begin{cases} r \geq R_c & \frac{Z_p Z_T e^2}{2 R_c} \left( \frac{r^2}{R_c^2} \right) \frac{1}{r} \\ r < R_c & \frac{Z_p Z_T e^2}{4 \pi \varepsilon_0} \end{cases}$$

(2)

Similarly, the radius of Coulomb interaction is taken as $R_c = 1.10(A_p^{1/3} + A_T^{1/3})$ fm, $Z_p$ and $Z_T$ describe the charges of the projectile and target nuclei, one by one. The centrifugal potential is given as Equation 3.

$$V_{\text{centrifugal}}(r) = \frac{\hbar^2 (l + 1)}{2 \mu r^2}$$

(3)

where $\mu$ is the reduced mass of the interaction ($^8\text{B},^2\text{7Al}$) and $l$ is angular momentum, $r$ is radius and $\hbar$ is Planck constant, respectively. The final term of the total potential is the complex nuclear potential $V_{\text{Nuclear}}(r)$ described as the double folding potential. This term with $V_{\text{NN}}$ an effective nucleon-nucleon interaction potential could be given as shown in Equation 4.

$$V_{\text{NN}}(r) = \int dr_1 \int dr_2 \rho_p(r_1) \rho_T(r_2) V_{\text{NN}}(r_1, r_2)$$

(4)

where $\rho_p(r_1)$ and $\rho_T(r_2)$ are the nuclear matter densities of projectile and target nuclei, respectively. We have benefited from three different matter density distributions for the $^8\text{B}$ halo nucleus in both ground state and $2^+$ ground state (in VMC) to make a comparison work. The first density is 3-parameters Fermi distribution and the second density is Gaussian distribution in our analysis for the $^8\text{B}$ proton halo nucleus (Equation 5 and 6).

$$\rho(r) = \frac{\rho_0}{1 + e^{(r/a)}}, \; \text{Fermi distribution for projectile nuclei}$$

(5)

$$\rho(r) = c \exp \left( - \frac{r^2}{\alpha} \right), \; \text{Gaussian distribution for projectile nuclei}$$

(6)

where $\rho_0 = 0.1507\; (fm^{-3})$, $c = 2.000\; (fm)$ and $a = 0.486\; (fm)$.

$$\rho(r) = \frac{\rho_0}{1 + e^{(r/a)}}, \; \text{Fermi distribution for projectile nuclei}$$

(7)

where $\rho_0 = 0.159177\; (fm^{-3})$, $\alpha = 2.08207\; fm$; these coefficients can be found in normalization condition (Equation 7).

$$4\pi \int \rho(r) r^2 dr = 8 \langle A_p \rangle$$

(7)

For $^{27}\text{Al}$ nucleus, the nuclear matter density has been obtained from RIPL-3 (Capote et al., 2009). Both the projectile and target nucleus density distributions have been displayed in Figure 1 (a), Figure 1 (b) and Figure 1 (c) in logarithmic form.
The effective nucleon-nucleon potential term consists of 3 components. The first part in Equation 8 is realistic interaction parameters. We have used the most general one, the M3Y nucleon-nucleon (Michigan 3 Yukawa) realistic interaction (Brandan & Satchler, 1997; Khoa, Satchler, & Von Oertzen, 1995).

\[ V_{NN}(r) = 7999 \exp\left(\frac{-4r}{4r}\right) - 2134 \exp\left(\frac{-2.5r}{2.5r}\right) + J_{00}(E)\delta(r) \ (MeV) \]

The second term in the Equation 8 is \( J_{00}(E) \) (linear energy dependence). This term represents the nucleon exchange term (Equation 9).

\[ J_{00}(E) = -276 \left[ 1 - \frac{0.005E_{Lab}}{A_p} \right] MeV fm^3 \]

The DF potentials are obtained using the double folding computer code DFPOT (Cook, 1982). By using this double-folding approach, we have obtained the real part of the nuclear potential with these three different distributions. The shapes of the real part of the potential are shown in Figure 2. As it can be seen from Figure 2, the produced real potential with the Fermi distribution is deeper than the potential produced by using both the Gaussian distribution and VMC density distribution in the double-folding model at the same energies. In Figure 2, the calculated folding potentials of \(^{8}\text{B}+^{27}\text{Al}\) cannot be distinguished from each other especially the far periphery region 5-10 fm, the most important one when estimating cross sections. For this reason the graph was drawn in logarithmic form to see better, and also the net shapes of the potentials have been given on the side as (a) and (b)-two different graphics.

While the real part of the optical model is obtained by using the above-described double folding model, the imaginary potential is taken as in the form of Wood-Saxon shape in Equation 10 as following,
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**Figure 2.** The shapes of the real potential of the nuclear potential of $^8\text{B}$ which interacts with $^{27}\text{Al}$ target nuclei at 15.3 and 21.7 MeV.

\[ W(r) = \frac{W_0}{1 + \exp\left(\frac{r - R_W}{a_W}\right)} \]  

(10)

where $R_W$ is the normalization factor of the produced potential. All obtained parameters have been displayed as in Table 1. The FRESCO code (Thompson, 1988) has been handled to investigate the parameters of optical model via assembling with the experimental data.

| Elab (MeV) | Distribution | No Type factor | $W_0$ (MeV) | $r_w$ (fm) | $a_w$ (fm) | $\sigma_{mb}$ MeV.fm$^3$ | $\sigma_{v}$ MeV.fm$^3$ | $\chi^2/N$ |
|------------|--------------|---------------|-------------|-------------|-------------|------------------------|------------------------|-----------|
| 15.3       | Fermi        | 1.0 19.7      | 1.4 0.435   | 355         | 448.900     | 519.346                | 519.346                | 0.034396  |
|            | Gaussian     | 1.0 19.7      | 1.4 0.435   | 355         | 448.825     | 519.346                | 519.346                | 0.034698  |
|            | with VMC     | 1.0 34.7      | 1.4 0.67    | 1332        | 473.506     | 245.094                | 245.094                | 0.435061  |
| 21.7       | Fermi        | 1.0 34.7      | 1.4 0.67    | 1332        | 473.522     | 245.094                | 245.094                | 0.470026  |
|            | Gaussian     | 1.0 34.7      | 1.4 0.67    | 1332        | 473.522     | 245.094                | 245.094                | 0.470026  |
|            | with VMC     | 1.0 34.7      | 1.4 0.67    | 1332        | 473.522     | 245.094                | 245.094                | 0.470026  |

3. Results and Discussion

The $^8\text{B}+^{27}\text{Al}$ reaction data has been recently measured (Morcelle et al., 2017) and there has been no study by using a double-folding potential with different densities to explain this experimental data. Therefore, we have studied the elastic scattering of the $^8\text{B}$ nucleus from $^{27}\text{Al}$ target nucleus at above the Coulomb barrier within the framework of the above-described double folding model. The microscopic potential has been generated by using three different nucleon matter densities- Fermi, Gaussian and VMC density distributions for the $^8\text{B}$-the lowest energy state. The real part of the potential is shown in Figure-2 for Fermi, Gaussian and VMC density distributions. The imaginary part of the optical potential is taken as Wood-Saxon form and the best fit parameters with double-folding potential, reaction cross-sections, volume integrals and $\chi^2/N$ values for two energies are shown in Table-1. The results are presented in Figure-3 and Figure-4. As it can be seen from these figures, our microscopic potentials obtained by using Fermi, Gaussian and VMC density distributions provide very well compromise with the experimental scores and generates successfully the minimums and maximums of the elastic scattering at two different energies. The treatment of the cross-sections generated by Fermi, Gaussian and VMC density distributions are very similar at both forward and backward angles. From Table-1,
we have observed that both 15.3 MeV and 21.7 MeV incoming energies the largest cross-section is 569 mb and 1321 mb, respectively in Gaussian distribution. On the other hand, we have observed that for 15.3 MeV, with VMC obtained error results are better than the result of Fermi and Gaussian distribution, but for 21.7 MeV with Gaussian distribution obtained error results are better than the results of Fermi and VMC density distributions. Some times it may be difficult to evaluate according to the results of $\chi^2/N$, we may come across with interpretations of these results in the literature (Farag, Esmael, & Maridi, 2014).

In the double-folding model, free parameter is the normalization constant $N_R$. As shown in Table-1, we kept this value as constant to examine the behavior of the imaginary potential. As the energy of the projectile in the reaction is increased, it is expected that the flux from the elastic channel would be removed to the other channels due to the occurrence of inelastic. Therefore, as the energy increases, we expect an increase at the depth of the imaginary potential. This is what we have exactly observed in our potential parameters. As it can be seen from Table-1, while energy of $^8B$ is 15.3 MeV, the depth of imaginary potential is 19.7 MeV, as the energy increases to $E_{\text{lab}}=21.7$, the depth of imaginary potential increases to 34.7 MeV. Our finding is in very good agreement with the expectation. With this fixed real and energy dependent imaginary potentials, we have also obtained total reaction cross section and volume integrals of the real and imaginary parts of the optical potential at two energies. It should be emphasized that our results are in agreement with the findings of Morcelle et al. (Morcelle et al., 2017).

Finally, as shown in Table-1, we have computed the volume integrals of the potentials used to describe for the $^8B+^{27}Al$ elastic scattering by using following equations:

\[
J_V = \frac{4\pi}{A_P A_T} \int V(r, E)r^2 dr \quad (12)
\]

\[
J_W = \frac{4\pi}{A_P A_T} \int W(r, E)r^2 dr \quad (13)
\]

where $A_P$ is the mass number of the projectile, and $A_T$ is the mass number of the target nucleus.

**Figure 3.** Angular distributions for the $^8B+^{27}Al$ elastic scattering at $E_{\text{lab}}=15.3$ MeV.

**Figure 4.** Angular distributions for the $^8B+^{27}Al$ elastic scattering at $E_{\text{lab}}=21.7$ MeV.

### 4. Conclusion

In this study, a theoretical analysis have been conducted for the first time by using Fermi, Gaussian and VMC density distributions for the elastic scattering of the $^8B+^{27}Al$ system at $E_{\text{lab}}=15.3$ and 21.7 MeV. The real part of the complex optical potential is derived from the Fermi, Gaussian and VMC density distributions of the proton halo projectile $^8B$ and target $^{27}Al$ nuclei within double-folding potential model. The imaginary part of the potential has been taken as the form of
Woods-Saxon volume. Both at 15.3 MeV and 21.7 MeV, it has been noticed that the real potential of $^8$B+$^{27}$Al (with Gaussian distribution) goes to zero faster than the other real potentials. However, we can express that the real potential of $^8$B+$^{27}$Al (with Fermi distribution) is deeper than the other real potentials. We have investigated the effect of potentials of the elastic and reaction cross-sections as well as the change on the volume integrals of the potentials for two energies. We have shown that microscopic potentials obtained by using Fermi, Gaussian and VMC density distributions provide very good agreement with the experimental data.

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